Can a Black Hole Collapse to a Space-time Singularity?

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**Abstract** A critique of the singularity theorems of Penrose, Hawking, and Geroch is given. It is pointed out that a gravitationally collapsing black hole acts as an ultrahigh energy particle accelerator that can accelerate particles to energies inconceivable in any terrestrial particle accelerator, and that when the energy $E$ of the particles comprising matter in a black hole is $\sim 10^2\text{GeV}$ or more, or equivalently, the temperature $T$ is $\sim 10^{15}\text{K}$ or more, the entire matter in the black hole is converted into quark-gluon plasma permeated by leptons. As quarks and leptons are fermions, it is emphasized that the collapse of a black-hole to a space-time singularity is inhibited by Pauli’s exclusion principle. It is also suggested that ultimately a black hole may end up either as a stable quark star, or as a pulsating quark star which may be a source of gravitational radiation, or it may simply explode with a mini bang of a sort.

**Keywords** black hole · gravitational collapse · space-time singularity · quark star

1 Introduction

When all the thermonuclear sources of energy of a star are exhausted, the core of the star begins to contract gravitationally because, practically, there is no radiation pressure to arrest the contraction, the pressure of matter being inadequate for this purpose. If the mass of the core is less than the Chandrasekhar limit ($\sim 1.44M_\odot$), the contraction stops when the density of matter in the core, $\rho > 2 \times 10^6\text{g cm}^{-3}$; at this stage the pressure of the relativistically degenerate electron gas in the core is enough to withstand the force of gravitation. When this happens, the core becomes a stable white dwarf. However, when the mass of the core is greater than the Chandrasekhar limit, the pressure of the relativistically degenerate electron gas is no longer sufficient to arrest the gravitational contraction, the core continues to contract and becomes...
denser and denser; and when the density reaches the value $\rho \sim 10^7 \text{ g cm}^{-3}$, the process of neutronization sets in; electrons and protons in the core begin to combine into neutrons through the reaction

$$p + e^- \rightarrow n + \nu_e$$

The electron neutrinos $\nu_e$ so produced escape from the core of the star. The gravitational contraction continues and eventually, when the density of the core reaches the value $\rho \sim 10^{14} \text{ g cm}^{-3}$, the core consists almost entirely of neutrons. If the mass of the core is less than the Oppenheimer-Volkoff limit ($\sim 3M_\odot$), then at this stage the contraction stops; the pressure of the degenerate neutron gas is enough to withstand the gravitational force. When this happens, the core becomes a stable neutron star. Of course, enough electrons and protons must remain in the neutron star so that Pauli’s exclusion principle prevents neutron beta decay

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Where $\bar{\nu}_e$ is the electron antineutrino (Weinberg 1972a). This requirement sets a lower limit $\sim 0.2M_\odot$ on the mass of a stable neutron star.

If, however, after the end of the thermonuclear evolution, the mass of the core of a star is greater than the Chandrasekhar and Oppenheimer-Volkoff limit, the star may eject enough matter so that the mass of the core drops below the Chandrasekhar and Oppenheimer-Volkoff limit as a result of which it may settle as a stable white dwarf or a stable neutron star. If not, the core will gravitationally collapse and end up as a black hole.

As is well known, the event horizon of a black hole of mass $M$ is a spherical surface located at a distance $r = r_g = 2GM/c^2$ from the centre, where $G$ is Newton’s gravitational constant and $c$ the speed of light in vacuum; $r_g$ is called gravitational radius or Schwarzschild radius. An external observer cannot observe anything that is happening inside the event horizon, nothing, not even light or any other electromagnetic signal can escape outside the event horizon from inside. However, anything that enters the event horizon from outside is swallowed by the black hole; it can never escape outside the event horizon again.

Attempts have been made, using the general theory of relativity (GTR), to understand what happens inside a black hole. In so doing, various simplifying assumptions have been made. In the simplest treatment (Oppenheimer and Snyder 1939; Weinberg 1972b) a black hole is considered to be a ball of dust with negligible pressure, uniform density $\rho = \rho(t)$, and at rest at $t = 0$. These assumptions lead to the unique solution of the Einstein field equations, and in the comoving co-ordinates the metric inside the black hole is given by

$$ds^2 = dt^2 - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$ (1)

in units in which speed of light in vacuum, $c=1$, and where $k$ is a constant. The requirement of energy conservation implies that $\rho(t)R^3(t)$ remains constant. On normalizing the radial co-ordinate $r$ so that

$$R(0) = 1$$ (2)
one gets

$$\rho(t) = \rho(0)R^{-3}(t)$$

(3)

The fluid is assumed to be at rest at $t = 0$, so

$$\dot{R}(0) = 0$$

(4)

Consequently, the field equations give

$$k = \frac{8\pi G}{3}\rho(0)$$

(5)

Finally, the solution of the field equations is given by the parametric equations of a cycloid:

$$t = \frac{\psi + \sin \psi}{2\sqrt{k}}$$

$$R = \frac{1}{2}(1 + \cos \psi)$$

(6)

From equation (6) it is obvious that when $\psi = \pi$, i.e., when

$$t = t_s = \frac{\pi}{2\sqrt{k}} = \frac{\pi}{2} \left( \frac{3}{8\pi G\rho(0)} \right)^{1/2}$$

(7)

a space-time singularity occurs; the scale factor $R(t)$ vanishes. In other words, a black hole of uniform density having the initial values $\rho(0)$, and zero pressure collapses from rest to a point in 3 - subspace, i.e., to a 3 - subspace of infinite curvature and zero proper volume, in a finite time $t_s$; the collapsed state being a state of infinite proper energy density. The same result is obtained in the Newtonian collapse of a ball of dust under the same set of assumptions (Narlikar 1978).

Although the black hole collapses completely to a point at a finite co-ordinate time $t = t_s$, any electromagnetic signal coming to an observer on the earth from the surface of the collapsing star before it crosses its event horizon will be delayed by its gravitational field, so an observer on the earth will not see the star suddenly vanish. Actually, the collapse to the Schwarzschild radius $r_g$ appears to an outside observer to take an infinite time, and the collapse to $R = 0$ is not at all observable from outside the event horizon.

The internal dynamics of a non-idealized, real black hole is very complex. Even in the case of a spherically symmetric collapsing black hole with non-zero pressure the details of the interior dynamics are not well understood, though major advances in the understanding of the interior dynamics are now being made by means of numerical computations and analytic analyses. But in these computations and analyses no new features have emerged beyond those that occur in the simple uniform-density, free-fall collapse considered above (Misner, Thorne, and Wheeler 1973). However, using topological methods, Penrose (1965, 1969), Hawking (1996a, 1966b, 1967a, 1967b), Hawking and Penrose (1970), and Geroch (1966, 1967, 1968) have proved a number
of singularity theorems purporting that if an object contracts to dimensions smaller than $r_g$, and if other reasonable conditions - namely, validity of the GTR, positivity of energy, ubiquity of matter and causality - are satisfied, its collapse to a singularity is inevitable.

2 A critique of the singularity theorems

As mentioned above, the singularity theorems are based, inter alia, on the assumption that the GTR is universally valid. But the question is: Has the validity of the GTR been established experimentally in the case of strong fields? Actually, the GTR has been experimentally verified only in the limiting case of weak fields, it has not been experimentally validated in the case of strong fields. Moreover, it has been demonstrated that when curvatures exceed the critical value $C_g = 1/L_g^4$, where $L_g = (\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33}$ cm corresponding to the critical density $\rho_g = 5 \times 10^{33}$ g cm$^{-3}$, the GTR is no longer valid; quantum effects must enter the picture (Zeldovich and Novikov 1971). Therefore, it is clear that the GTR breaks down before a gravitationally collapsing object collapses to a singularity. Consequently, the conclusion based on the GTR that in comoving co-ordinates any gravitationally collapsing object in general, and a black hole in particular, collapses to a point in 3-space need not be held sacrosanct, as a matter of fact it may not be correct at all.

Furthermore, while arriving at the singularity theorems attention has mostly been focused on the space-time geometry and geometrodynamics; matter has been tacitly treated as a classical entity. However, as will be shown later, this is not justified; quantum mechanical behavior of matter at high energies and high densities must be taken into account. Even if we regard matter as a classical entity of a sort, it can be easily seen that the collapse of a black hole to a space-time singularity is inhibited by Pauli’s exclusion principle. As mentioned earlier, a collapsing black hole consists, almost entirely, of neutrons apart from traces of protons and electrons; and neutrons as well as protons and electrons are fermions; they obey Pauli’s exclusion principle. If a black hole collapses to a point in 3-space, all the neutrons in the black hole would be squeezed into just two quantum states available at that point, one for spin up and the other for spin down neutron. This would violate Pauli’s exclusion principle, according to which not more than one fermion of a given species can occupy any quantum state. So would be the case with the protons and the electrons in the black hole. Consequently, a black hole cannot collapse to a space-time singularity in contravention to Pauli’s exclusion principle.

Besides, another valid question is: What happens to a black hole after $t > t_s$, i.e., after it has collapsed to a point in 3-space to a state of infinite proper energy density, if at all such a collapse occurs? Will it remain frozen forever at that point? If yes, then uncertainties in the position co-ordinates of each of the particles - namely, neutrons, protons, and electrons - comprising the black hole would be zero. Consequently, according to Heisenberg’s uncertainty principle, uncertainties in the momentum co-ordinates of each of the particles would be infinite. However, it is physically inconceivable how particles of infinite momentum and energy would remain frozen forever at a point. From this consideration also collapse of a black hole to a singularity appears to be quite unlikely.

Earlier, it was suggested by the author that the very strong ‘hard-core’ repulsive interaction between nucleons, which has a range $l_c \sim 0.4 \times 10^{-13}$ cm, might set a limit
on the gravitational collapse of a black hole and avert its collapse to a singularity (Thakur 1983). The existence of this hard-core interaction was pointed out by Jastro (1951) after the analysis of the data from high energy nucleon-nucleon scattering experiments. It has been shown that this very strong short range repulsive interaction arises due to the exchange of isoscalar vector mesons $\omega$ and $\phi$ between two nucleons (Scotti and Wong 1965). Phenomenologically, that part of the nucleon-nucleon potential which corresponds to the repulsive hard core interaction may be taken as

$$V_c(r) = \infty \quad \text{for} \quad r < r_c$$

where $r$ is the distance between the two interacting nucleons. Taking this into account, the author concluded that no spherical object of mass $M$ could collapse to a sphere of radius smaller than $R_{\text{min}} = 1.68 \times 10^{-6} M^{1/3} \text{cm}$, or of the density greater than $\rho_{\text{max}} = 5.0 \times 10^{16} \text{g cm}^{-3}$. It was also pointed out that an object of mass smaller than $M_c \sim 1.21 \times 10^{33} \text{gm}$ could not cross the event horizon and become a black hole; the only course left to an object of mass smaller than $M_c$ was to reach equilibrium as either a white dwarf or a neutron star. However, one may not regard these conclusions as reliable because they are based on the hard core repulsive interaction (8) between nucleons which has been arrived at phenomenologically by high energy nuclear physicists while accounting for the high energy nucleon-nucleon scattering data; but it must be noted that, as mentioned above, the existence of the hard core interaction has been demonstrated theoretically also by Scotti and Wong in 1965. Moreover, it is interesting to note that the upper limit $M_c \sim 1.21 \times 10^{33} \text{g} = 0.69 M_\odot$ on the masses of objects that cannot gravitationally collapse to form black holes is of the same order of magnitude as the Chandrasekhar and the Oppenheimer-Volkoff limits.

Even if we disregard the role of the hard core, short range repulsive interaction in arresting the collapse of a black hole to a space-time singularity in comoving co-ordinates, it must be noted that unlike leptons which appear to be point-like particles - the experimental upper bound on their radii being $10^{-16} \text{cm}$ (Barber et al. 1979) - nucleons have finite dimensions. It has been experimentally demonstrated that the radius $r_0$ of the proton is about $10^{-13} \text{cm}$(Hofstadter & McAllister 1955). Therefore, it is natural to assume that the radius $r_0$ of the neutron is also about $10^{-13} \text{cm}$. This means the minimum volume $v_{\text{min}}$ occupied by a neutron is $\frac{4\pi}{3} r_0^3$. Ignoring the “mass defect” arising from the release of energy during the gravitational contraction (before crossing the event horizon), the number of neutrons $N$ in a collapsing black hole of mass $M$ is, obviously, $\frac{M}{m_n}$ where $m_n$ is the mass of the neutron. Assuming that neutrons are impregnable particles, the minimum volume that the black hole can occupy is $V_{\text{min}} = N v_{\text{min}} = v_{\text{min}} \frac{M}{m_n}$, for neutrons cannot be more closely packed than this in a black hole. However, $V_{\text{min}} = \frac{4\pi R_{\text{min}}^3}{3}$ where $R_{\text{min}}$ is the radius of the minimum volume to which the black hole can collapse. Consequently, $R_{\text{min}} = r_0 \left( \frac{M}{m_n} \right)^{1/3}$. On substituting $10^{-13} \text{cm}$ for $r_0$ and $1.67 \times 10^{-24} \text{g}$ for $m_n$ one finds that $R_{\text{min}} \approx 8.40 \times 10^{-6} M^{1/3}$. This means a collapsing black hole cannot collapse to a density greater than $\rho_{\text{max}} = \frac{M}{V_{\text{min}}} = \frac{N m_n}{V_{\text{min}}^2} = 3.99 \times 10^{14} \text{g cm}^{-3}$. The critical mass $M_c$ of the object for which the gravitational radius $R_g = R_{\text{min}}$ is obtained from the equation

$$\frac{2GM_c}{c^2} = r_0 \left( \frac{M_c}{m_n} \right)^{1/3}$$
This gives

\[ M_c = 1.35 \times 10^{34} \text{ g} = 8.68 M_\odot \]  

(10)

Obviously, for \( M > M_c \), \( R_g > R_{\text{min}} \), and for \( M < M_c \), \( R_g < R_{\text{min}} \).

Consequently, objects of mass \( M < M_c \) cannot cross the event horizon and become a black hole whereas those of mass \( M > M_c \) can. Objects of mass \( M < M_c \) will, depending on their mass, reach equilibrium as either white dwarfs or neutron stars. Of course, these conclusions are based on the assumption that neutrons are impregnable particles and have radius \( r_0 = 10^{-13} \text{ cm} \) each. Also implicit is the assumption that neutrons are fundamental particles; they are not composite particles made up of other smaller constituents. But this assumption is not correct; neutrons as well as protons and other hadrons are not fundamental particles; they are made up of smaller constituents called quarks as will be explained in section 4. In section 5 it will be shown how, at ultrahigh energy and ultrahigh density, the entire matter in a collapsing black hole is eventually converted into quark-gluon plasma permeated by leptons.

3 Gravitationally collapsing black hole as a particle accelerator

We consider a gravitationally collapsing black hole. On neglecting mutual interactions the energy \( E \) of any one of the particles comprising the black hole is given by \( E^2 = p^2 + m^2 > p^2 \), in units in which the speed of light in vacuum \( c = 1 \), where \( p \) is the magnitude of the 3-momentum of the particle and \( m \) its rest mass. But \( p = \frac{h}{\lambda} \), where \( \lambda \) is the de Broglie wavelength of the particle and \( h \) Planck's constant of action. Since all lengths in the collapsing black hole scale down in proportion to the scale factor \( R(t) \) in equation (1), it is obvious that \( \lambda \propto R(t) \). Therefore it follows that \( p \propto R^{-1}(t) \), and hence \( p = a R^{-1}(t) \), where \( a \) is the constant of proportionality. From this it follows that \( E > a/R \). Consequently, \( E \) as well as \( p \) increases continually as \( R \) decreases. It is also obvious that \( E \) and \( p \), the magnitude of the 3-momentum, \( \rightarrow \infty \) as \( R \rightarrow 0 \). Thus, in effect, we have an ultra-high energy particle accelerator, so far inconceivable in any terrestrial laboratory, in the form of a collapsing black hole, which can, in the absence of any physical process inhibiting the collapse, accelerate particles to an arbitrarily high energy and momentum without any limit.

What has been concluded above can also be demonstrated alternatively, without resorting to GTR, as follows. As an object collapses under its selfgravitation, the interparticle distance \( s \) between any pair of particles in the object decreases. Obviously, the de Broglie's wavelength \( \lambda \) of any particle in the object is less than or equal to \( s \), a simple consequence of Heisenberg's uncertainty principle. Therefore, \( s \geq h/p \), where \( h \) is Planck's constant and \( p \) the magnitude of 3-momentum of the particle. Consequently, \( p \geq h/s \) and hence \( E \geq h/s \). Since during the collapse of the object \( s \) decreases, the energy \( E \) as well as the momentum \( p \) of each of the particles in the object increases. Moreover, from \( E \geq h/s \) and \( p \geq h/s \) it follows that \( E \) and \( p \rightarrow \infty \) as \( s \rightarrow 0 \). Thus, any gravitationally collapsing object in general, and a black hole in particular, acts as an ultrahigh energy particle accelerator.

It is also obvious that \( \rho \), the density of matter in the black hole, increases as it collapses. In fact, \( \rho \propto R^{-3} \), and hence \( \rho \rightarrow \infty \) as \( R \rightarrow 0 \).
4 Quarks: The building blocks of matter

In order to understand eventually what happens to matter in a collapsing black hole one has to take into account the microscopic behavior of matter at high energies and high densities; one has to consider the role played by the electromagnetic, weak, and strong interactions - apart from the gravitational interaction - between the particles comprising the matter. For a brief account of this the reader is referred to Thakur (1995), for greater detail to Huang (1992), or at a more elementary level to Hughes (1991).

As has been mentioned in Section 2, unlike leptons, hadrons are not point-like particles, but are of finite size; they have structures which have been revealed in experiments that probe hadronic structures by means of electromagnetic and weak interactions. The discovery of a very large number of apparently elementary (fundamental) hadrons led to the search for a pattern amongst them with a view to understanding their nature. This resulted in attempts to group together hadrons having the same baryon number, spin, and parity but different strangeness \( S \) (or equivalently hypercharge \( Y = B + S \), where \( B \) is the baryon number) into I-spin (isospin) multiplets. In a plot of \( Y \) against \( I_3 \) (z-component of isospin \( I \)), members of I-spin multiplets are represented by points. The existence of several such hadron (baryon and meson) multiplets is a manifestation of underlying internal symmetries.

In 1961 Gell-Mann, and independently Neemann, pointed out that each of these multiplets can be looked upon as the realization of an irreducible representation of an internal symmetry group \( SU(3) \) (Gell-Mann and Neemann 1964). This fact together with the fact that hadrons have finite size and inner structure led Gell-Mann, and independently Zweig, in 1964 to hypothesize that hadrons are not elementary particles, rather they are composed of more elementary constituents called quarks \( (q) \) by Gell-Mann (Zweig called them aces). Baryons are composed of three quarks \( (qqq) \) and antibaryons of three antiquarks \( (\bar{q}\bar{q}\bar{q}) \) while mesons are composed of a quark and an antiquark each. In the beginning, to account for the multiplets of baryons and mesons, quarks of only three flavours, namely, u(up), d (down), and s(strange) were postulated, and they together formed the basic triplet \( \begin{pmatrix} u \\ d \\ s \end{pmatrix} \) of the internal symmetry group \( SU(3) \). All these three quarks u, d, and s have spin 1/2 and baryon number 1/3. The u quark has charge \( 2/3e \) whereas the d and s quarks have charge \(-1/3e\) where \( e \) is the charge of the proton. The strangeness quantum number of the u and d quarks is zero whereas that of the s quark is -1. The antiquarks \( (\bar{u}, \bar{d}, \bar{s}) \) have charges \(-2/3e, 1/3e, 1/3e\) and strangeness quantum numbers 0, 0, 1 respectively. They all have spin 1/2 and baryon number -1/3. Both u and d quarks have the same mass, namely, one third that of the nucleon, i.e., \( \simeq 310 MeV/c^2 \) whereas the mass of the s quark is \( \simeq 500 MeV/c^2 \). The proton is composed of two up and one down quarks (p: uud) and the neutron of one up and two down quarks (n: udd).

Motivated by certain theoretical considerations Glashow, Iliopoulos and Maiani (1970) proposed that, in addition to u, d, s quarks, there should be another quark flavour which they named charm (c). Gaillard and Lee (1974) estimated its mass to be \( \simeq 1.5 GeV/c^2 \). In 1974 two teams, one led by S.C.C. Ting at SLAC (Aubert et al. 1974) and another led by B. Richter at Brookhaven (Augustin et al. 1974) independently discovered the \( J/\Psi \), a particle remarkable in that its mass \( (3.1 GeV/c^2) \) is more than three times that of the proton. Since then, four more particles of the same family, namely, \( \psi(3684), \psi(3950), \psi(4150), \psi(4400) \) have been found. It is now established
that these particles are bound states of charmonium \((\bar{c}c)\), \(J/\psi\) being the ground state. On adopting non-relativistic independent quark model with a linear potential between \(c\) and \(\bar{c}\), and taking the mass of \(c\) to be approximately half the mass of \(J/\psi\), i.e., \(1.5\text{GeV}/c^2\), one can account for the \(J/\psi\) family of particles. The \(c\) has spin \(1/2\), charge \(2/3\ e\), baryon number \(1/3\), strangeness \(-1\), and a new quantum number charm \((c)\) equal to \(1\). The \(u, d, s\) quarks have \(c = 0\). It may be pointed out here that charmed mesons and baryons, i.e., the bound states like \((c\bar{d})\), and \((cdu)\) have also been found. Thus the existence of the \(c\) quark has been established experimentally beyond any shade of doubt.

The discovery of the \(c\) quark stimulated the search for more new quarks. An additional motivation for such a search was provided by the fact that there are three generations of lepton weak doublets: \((\nu_e, \bar{e})\), \((\nu_\mu, \bar{\mu})\), and \((\nu_\tau, \bar{\tau})\) where \(\nu_e, \nu_\mu,\) and \(\nu_\tau\) are electron \((e)\), muon \((\mu)\), and tau lepton \((\tau)\) neutrinos respectively. Hence, by analogy, one expects that there should be three generations of quark weak doublets also: \((u, \bar{u})\), \((d, \bar{d})\), and \((s, \bar{s})\). It may be mentioned here that weak interaction does not distinguish between the upper and the lower members of each of these doublets. In analogy with the isospin \(1/2\) of the strong doublet \((u, d)\), the weak doublets are regarded as possessing weak isospin \(I_W = 1/2\), the third component \((I_W)\) of this weak isospin being \(+1/2\) for the upper components of these doublets and \(-1/2\) for the lower components. These statements apply to the left-handed quarks and leptons, i.e., those with negative helicity (i.e., with the spin antiparallel to the momentum) only. The right-handed leptons and quarks, i.e., those with positive helicity (i.e., with the spin parallel to the momentum), are weak singlets having weak isospin zero.

The discovery, at Fermi Laboratory, of a new family of vector mesons, the upsilon family, starting at a mass of \(9.46\text{GeV}/c^2\) gave an evidence for a new quark flavour called bottom or beauty \((b)\) (Herb et al. 1997; Innes et al. 1977). These vector mesons are in fact, bound states of bottomonium \((\bar{b}b)\). These states have since been studied in detail at the Cornell electron accelerator in an electron-positron storage ring of energy ideally matched to this mass range. Four such states with masses \(9.46, 10.02, 10.35,\) and \(10.58\ \text{GeV}/c^2\) have been found, the state with mass \(9.46\text{GeV}/c^2\) being the ground state (Andrews et al. 1980). This implies that the mass of the \(b\) quark is \(\simeq 4.73\text{GeV}/c^2\). The \(b\) quark has spin \(1/2\) and charge \(-1/3\ e\). Furthermore, the \(b\) flavoured mesons have been found with exactly the expected properties (Beherend et al. 1983).

After the discovery of the \(b\) quark, the confidence in the existence of the sixth quark flavour called top or truth \((t)\) increased and it became almost certain that, like leptons, the quarks also occur in three generations of weak isospin doublets, namely, \((u, \bar{u})\), \((d, \bar{d})\), and \((s, \bar{s})\). In view of this, intensive search was made for the \(t\) quark. But the discovery of the \(t\) quark eluded for eighteen years. However, eventually in 1995, two groups, the CDF (Collider Detector at Fermi lab) Collaboration (Abe et al. 1995) and the \(D\) Collaboration (Abachi et al. 1995) succeeded in detecting toponium \(\bar{t}t\) in very high energy \(p\bar{p}\) collisions at Fermi Laboratory’s 1.8TeV Tevatron collider. The toponium \(\bar{t}t\) is the bound state of \(t\) and \(\bar{t}\). The mass of \(t\) has been estimated to be \(176.0 \pm 2.0\text{GeV}/c^2\), and thus it is the most massive elementary particle known so far. The \(t\) quark has spin \(1/2\) and charge \(2/3\ e\).

Moreover, in order to account for the apparent breaking of the spin-statistics theorem in certain members of the \(J^P = \frac{1}{2}^+\) decuplet (spin \(3/2\),parity even), e.g., \(\Delta^{++}\) (uuu), and \(\Omega^- (sss)\), Greenberg (1964) postulated that quark of each flavour comes in three colours, namely, red, green, and blue, and that real particles are always colour
**singlets.** This implies that real particles must contain quarks of all the three colours or colour-anticolour combinations such that they are overall **white** or **colourless.** **White** or **colourless** means all the three primary colours are equally mixed or there should be a combination of a quark of a given colour and an antiquark of the corresponding anticolour. This means each baryon contains quarks of all the three colours (but not necessarily of the same flavour) whereas a meson contains a quark of a given colour and an antiquark having the corresponding anticolour so that each combination is overall white. Leptons have no colour. Of course, in this context the word ‘colour’ has nothing to do with the actual visual colour, it is just a quantum number specifying a new internal degree of freedom of a quark.

The concept of colour plays a fundamental role in accounting for the interaction between quarks. The remarkable success of quantum electrodynamics (QED) in explaining the interaction between electric charges to an extremely high degree of precision motivated physicists to explore a similar theory for strong interaction. The result is quantum chromodynamics (QCD), a non-Abelian gauge theory (Yang-Mills theory), which closely parallels QED. Drawing analogy from electrodynamics, Nambu (1966) postulated that the three quark colours are the charges (the Yang-Mills charges) responsible for the force between quarks just as electric charges are responsible for the electromagnetic force between charged particles. The analogue of the rule that like charges repel and unlike charges attract each other is the rule that like colours repel, and colour and anticolour attract each other. Apart from this, there is another rule in QCD which states that different colours attract if the quantum state is antisymmetric, and repel if it is symmetric under exchange of quarks. An important consequence of this is that if we take three possible pairs, red-green, green-blue, and blue-red, then a third quark is attracted only if its colour is different and if the quantum state of the resulting combination is antisymmetric under the exchange of a pair of quarks thus resulting in red-green-blue baryons. Another consequence of this rule is that a fourth quark is repelled by one quark of the same colour and attracted by two of different colours in a baryon but only in antisymmetric combinations. This introduces a factor of 1/2 in the attractive component and as such the overall force is zero, i.e., the fourth quark is neither attracted nor repelled by a combination of red-green-blue quarks. In spite of the fact that hadrons are overall colourless, they feel a residual strong force due to their coloured constituents.

It was soon realized that if the three colours are to serve as the Yang-Mills charges, each quark flavour must transform as a triplet of $SU_c(3)$ that causes transitions between quarks of the same flavour but of different colours (the $SU(3)$ mentioned earlier causes transitions between quarks of different flavours and hence may more appropriately be denoted by $SU_f(3)$). However, the $SU_c(3)$ Yang-Mills theory requires the introduction of eight new spin 1 gauge bosons called *gluons.* Moreover, it is reasonable to stipulate that the gluons couple to **left-handed** and **right-handed** quarks in the same manner since the strong interactions do not violate the law of conservation of parity. Just as the force between electric charges arise due to the exchange of a photon, a massless vector (spin 1) boson, the force between coloured quarks arises due to the exchange of a gluon. Gluons are also massless vector (spin 1) bosons. A quark may change its colour by emitting a gluon. For example, a **red** quark $q_R$ may change to a **blue** quark $q_B$ by emitting a gluon which may be thought to have taken away the **red** ($R$) **colour** from the quark and given it the **blue** ($B$) colour, or, equivalently, the gluon may be thought to have taken away the **red** ($R$) and the **antiblue** ($\overline{B}$) colours from the quark. Consequently, the *gluon* $G_{RB}$ emitted in the process $q_R \rightarrow q_B$ may be regarded as
the composite having the colour $R \overline{B}$ so that the emitted gluon $G_{RB} = q_R \overline{q}_B$. In general, when a quark $q_i$ of colour $i$ changes to a quark $q_j$ of colour $j$ by emitting a gluon $G_{ij}$, then $G_{ij}$ is the composite state of $q_i$ and $\overline{q}_j$, i.e., $G_{ij} = q_i \overline{q}_j$. Since there are three colours and three anticolours, there are $3 \times 3 = 9$ possible combinations (gluons) of the form $G_{ij} = q_i \overline{q}_j$. However, one of the nine combinations is a special combination corresponding to the white colour, namely, $G_W = q_R \overline{q}_R = q_G \overline{q}_G = q_B \overline{q}_B$. But there is no interaction between a coloured object and a white (colourless) object. Consequently, gluon $G_W$ may be thought not to exist. This leads to the conclusion that only $9 - 1 = 8$ kinds of gluons exist. This is a heuristic explanation of the fact that $SU_c(3)$ Yang-Mills gauge theory requires the existence of eight gauge bosons, i.e., the gluons. Moreover, as the gluons themselves carry colour, gluons may also emit gluons. Another important consequence of gluons possessing colour is that several gluons may come together and form gluonium or glue balls. Glueballs have integral spin and no colour and as such they belong to the meson family.

Though the actual existence of quarks has been indirectly confirmed by experiments that probe hardronic structure by means of electromagnetic and weak interactions, and by the production of various quarkonia ($\overline{q}q$) in high energy collisions made possible by various particle accelerators, no free quark has been detected in experiments at these accelerators so far. This fact has been attributed to the infrared slavery of quarks, i.e., to the nature of the interaction between quarks responsible for their confinement inside hadrons. Perhaps enormous amount of energy, much more than what is available in the existing terrestrial accelerators, is required to liberate the quarks from confinement. This means the force of attraction between quarks increases with increase in their separation. This is reminiscent of the force between two bodies connected by an elastic string.

On the contrary, the results of deep inelastic scattering experiments reveal an altogether different feature of the interaction between quarks. If one examines quarks at very short distances ($< 10^{-13}$ cm) by observing the scattering of a nonhadronic probe, e.g., an electron or a neutrino, one finds that quarks move almost freely inside baryons and mesons as though they are not bound at all. This phenomenon is called the asymptotic freedom of quarks. In fact Gross and Wilczek (1973 a,b) and Politzer (1973) have shown that the running coupling constant of interaction between two quarks vanishes in the limit of infinite momentum (or equivalently in the limit of zero separation).

5 Eventually what happens to matter in a collapsing black hole?

As mentioned in Section 3 the energy $E$ of the particles comprising the matter in a collapsing black hole continually increases and so does the density $\rho$ of the matter whereas the separation $s$ between any pair of particles decreases. During the continual collapse of the black hole a stage will be reached when $E$ and $\rho$ will be so large and $s$ so small that the quarks confined in the hadrons will be liberated from the infrared slavery and will enjoy asymptotic freedom, i.e., the quark deconfinement will occur. In fact, it has been shown that when the energy $E$ of the particle $\sim 10^2$ GeV ($s \sim 10^{-16}$ cm) corresponding to a temperature $T \sim 10^{15}K$ all interactions are of the Yang-Mills type with $SU_c(3) \times SU_I_W(2) \times U_Y_W(1)$ gauge symmetry, where $c$ stands for colour, $I_W$ for weak isospin, and $Y_W$ for weak hypercharge, and at this stage quark deconfinement occurs as a result of which matter now consists of its fundamental constituents: spin 1/2 leptons, namely, the electrons, the muons, the tau leptons, and their neutrinos, which
interact only through the electroweak interaction (i.e., the unified electromagnetic and weak interactions); and the spin 1/2 quarks, u, d, s, c, b, t, which interact electroweakly as well as through the colour force generated by gluons (Ramond, 1983). In other words, when $E \geq 10^2$ GeV ($s \leq 10^{-16}$ cm) corresponding to $T \geq 10^{15}$K, the entire matter in the collapsing black hole will be in the form of quark-gluon plasma permeated by leptons as suggested by the author earlier (Thakur 1993).

Incidentally, it may be mentioned that efforts are being made to create quark-gluon plasma in terrestrial laboratories. A report released by CERN, the European Organization for Nuclear Research, at Geneva, on February 10, 2000, said that by smashing together lead ions at CERN’s accelerator at temperatures 100,000 times as hot as the Sun’s centre, i.e., at $T \sim 1.5 \times 10^{12}$K, and energy densities never before reached in laboratory experiments, a team of 350 scientists from institutes in 20 countries succeeded in isolating tiny components called quarks from more complex particles such as protons and neutrons. “A series of experiments using CERN’s lead beam have presented compelling evidence for the existence of a new state of matter 20 times denser than nuclear matter, in which quarks instead of being bound up into more complex particles such as protons and neutrons, are liberated to roam freely ” the report said. However, the evidence of the creation of quark gluon plasma at CERN is indirect, involving detection of particles produced when the quark-gluon plasma changes back to hadrons. The production of these particles can be explained alternatively without having to have quark-gluon plasma. Therefore, Ulrich Heinz at CERN is of the opinion that the evidence of the creation of quark-gluon plasma at CERN is not enough and conclusive. In view of this, CERN will start a new experiment, ALICE, soon (around 2007-2008) at its Large Hadron Collider (LHC) in order to definitively and conclusively create QGP.

In the meantime the focus of research on quark-gluon plasma has shifted to the Relativistic Heavy Ion Collider (RHIC), the world’s newest and largest particle accelerator for nuclear research, at Brookhaven National Laboratory in Upton, New York. RHIC’s goal is to create and study quark-gluon plasma. RHIC’s aim is to create quark-gluon plasma by head-on collisions of two beams of gold ions at energies 10 times those of CERN’s programme, which ought to produce a quark-gluon plasma with higher temperature and longer lifetime thereby allowing much clearer and direct observation. RHIC’s quark-gluon plasma is expected to be well above the transition temperature for transition between the ordinary hadronic matter phase and the quark-gluon plasma phase. This will enable scientists to perform numerous advanced experiments in order to study the properties of the plasma. The programme at RHIC began in the summer of 2000 and after two years Thomas Kirk, Brookhaven’s Associate Laboratory Director for High Energy Nuclear Physics, remarked, “It is too early to say that we have discovered the quark-gluon plasma, but not too early to mark the tantalizing hints of its existence.” Other definitive evidence of quark-gluon plasma will come from experimental comparisons of the behavior in hot, dense nuclear matter with that in cold nuclear matter. In order to accomplish this, the next round of experimental measurements at RHIC will involve collisions between heavy ions and light ions, namely, between gold nuclei and deuterons.

Later, on June 18, 2003 a special scientific colloquium was held at Brookhaven National Laboratory (BNL) to discuss the latest findings at RHIC. At the colloquium, it was announced that in the detector system known as STAR (Solenoidal Tracker AT RHIC) head-on collision between two beams of gold nuclei of energies of 130 GeV per nuclei resulted in the phenomenon called “jet quenching“. STAR as well as three other
experiments at RHIC viz., PHENIX, BRAHMS, and PHOBOS, detected suppression of “leading particles”, highly energetic individual particles that emerge from nuclear fireballs, in gold-gold collisions. Jet quenching and leading particle suppression are signs of QGP formation. The findings of the STAR experiment were presented at the BNL colloquium by Berkeley Laboratory’s NSD (Nuclear Science Division) physicist Peter Jacobs.

6 Collapse of a black hole to a space-time singularity is inhibited by Pauli’s exclusion principle

As quarks and leptons in the quark-gluon plasma permeated by leptons into which the entire matter in a collapsing black hole is eventually converted are fermions, the collapse of a black hole to a space-time singularity in a finite time in a comoving coordinate system, as stipulated by the singularity theorems of Penrose, Hawking and Geroch, is inhibited by Pauli’s exclusion principle. For, if a black hole collapses to a point in 3-space, all the quarks of a given flavour and colour would be squeezed into just two quantum states available at that point, one for spin up and the other for spin down quark of that flavour and colour. This would violate Pauli’s exclusion principle according to which not more than one fermion of a given species can occupy any quantum state. So would be the case with quarks of each distinct combination of colour and flavour as well as with leptons of each species, namely, \( e, \mu, \tau, \nu_e, \nu_\mu, \text{ and } \nu_\tau \). Consequently, a black hole cannot collapse to a space-time singularity in contravention to Pauli’s exclusion principle. Then the question arises: If a black hole does not collapse to a space-time singularity, what is its ultimate fate? In section 7 three possibilities have been suggested.

7 Ultimately how does a black hole end up?

The pressure \( P \) inside a black hole is given by

\[
P = P_r + \sum_{i,j} P_{ij} + \sum_k P_k + \sum_{i,j} \mathcal{T}_{ij} + \sum_k \mathcal{T}_k
\]

where \( P_r \) is the radiation pressure, \( P_{ij} \) the pressure of the relativistically degenerate quarks of the \( i^{th} \) flavour and \( j^{th} \) colour, \( P_k \) the pressure of the relativistically degenerate leptons of the \( k^{th} \) species, \( \mathcal{T}_{ij} \) the pressure of relativistically degenerate antiquarks of the \( i^{th} \) flavour and \( j^{th} \) colour, \( \mathcal{T}_k \) that of the relativistically degenerate antileptons of the \( k^{th} \) species. In equation (11) the summations over \( i \) and \( j \) extend over all the six flavours and the three colours of quarks, and that over \( k \) extend over all the six species of leptons. However, calculation of these pressures is prohibitively difficult for several reasons. For example, the standard methods of statistical mechanics for calculation of pressure and equation of state are applicable when the system is in thermodynamics equilibrium and when its volume is very large, so large that for practical purpose we may treat it as infinite. Obviously, in a gravitationally collapsing black hole, the photon, quark and lepton gases cannot be in thermodynamic equilibrium nor can their volume be treated as infinite. Moreover, at ultrahigh energies and densities, because of the \( SU_{1w}(2) \) gauge symmetry, transitions between the upper and lower components...
of quark and lepton doublets occur very frequently. In addition to this, because of the \(SU_f(3)\) and \(SU_c(3)\) gauge symmetries transitions between quarks of different flavours and colours also occur. Furthermore, pair production and pair annihilation of quarks and leptons create additional complications. Apart from these, various other nuclear reactions may as well occur. Consequently, it is practically impossible to determine the number density and hence the contribution to the overall pressure \(P\) inside the black hole by any species of elementary particle in a collapsing black hole when \(E \geq 10^2\) Gev \((s \leq 10^{-16}\) cm), or equivalently, \(T \geq 10^{15}\) K. However, it may not be unreasonable to assume that, during the gravitational collapse, the pressure \(P\) inside a black hole increases monotonically with the increase in the density of matter \(\rho\). Actually, it might be given by the polytrope, \(P = k\rho^{\frac{n+1}{n}}\), where \(K\) is a constant and \(n\) is polytropic index. Consequently, \(P \to \infty\) as \(\rho \to \infty\), i.e., \(P \to \infty\) as the scale factor \(R(t) \to 0\) (or equivalently \(s \to 0\)). In view of this, there are three possible ways in which a black hole may end up.

1. During the gravitational collapse of a black hole, at a certain stage, the pressure \(P\) may be enough to withstand the gravitational force and the object may become gravitationally stable. Since at this stage the object consists entirely of quark-gluon plasma permeated by leptons, it means it would end up as a stable quark star. Indeed, such a possibility seems to exist. Recently, two teams - one led by David Helfand of Columbia University, NewYork (Slane, Helfand, and Murray 2002) and another led by Jeremy Drake of Harvard-Smithsonian Centre for Astrophysics, Cambridge, Mass. USA (Drake et al. 2002) studied independently two objects, 3C58 in Cassiopeia, and RXJ1856.5-3754 in Corona Australis respectively by combining data from the NASA’s Chandra X-ray Observatory and the Hubble Space Telescope, that seemed, at first, to be neutron stars, but, on closer look, each of these objects showed evidence of being an even smaller and denser object, possibly a quark star.

2. Since the collapse of a black hole is inhibited by Pauli’s exclusion principle, it can collapse only up to a certain minimum radius, say, \(r_{\text{min}}\). After this, because of the tremendous amount of kinetic energy, it would bounce back and expand, but only up to the event horizon, i.e., up to the gravitational (Schwarzschild) radius \(r_g\) since, according to the GTR, it cannot cross the event horizon. Thereafter it would collapse again up to the radius \(r_{\text{min}}\) and then bounce back up to the radius \(r_g\). This process of collapse up to the radius \(r_{\text{min}}\) and bounce up to the radius \(r_g\) would occur repeatedly. In other words, the black hole would continually pulsate radially between the radii \(r_{\text{min}}\) and \(r_g\) and thus become a pulsating quark star. However, this pulsation would cause periodic variations in the gravitational field outside the event horizon and thus produce gravitational waves which would propagate radially outwards in all directions from just outside the event horizon. In this way the pulsating quark star would act as a source of gravitational waves. The pulsation may take a very long time to damp out since the energy of the quark star (black hole) cannot escape outside the event horizon except via the gravitational radiation produced outside the event horizon. However, gluons in the quark-gluon plasma may also act as a damping agent. In the absence of damping, which is quite unlikely, the black hole would end up as a perpetually pulsating quark star.

3. The third possibility is that eventually a black hole may explode; a mini bang of a sort may occur, and it may, after the explosion, expand beyond the event horizon though it has been emphasized by Zeldovich and Novikov (1971) that after a collapsing sphere’s radius decreases to \(r < r_g\) in a finite proper time, its expansion into the external space
from which the contraction originated is impossible, even if the passage of matter through infinite density is assumed.

Notwithstanding Zeldovich and Novikov’s contention based on the very concept of event horizon, a gravitationally collapsing black hole may also explode by the very same mechanism by which the big bang occurred, if indeed it did occur. This can be seen as follows. At the present epoch the volume of the universe is \( \sim 1.5 \times 10^{85} \text{ cm}^3 \) and the density of the galactic material throughout the universe is \( \sim 2 \times 10^{-31} \text{ g cm}^{-3} \) (Allen 1973). Hence, a conservative estimate of the mass of the universe is \( \sim 1.5 \times 10^{85} \times 2 \times 10^{-31} \text{ g} = 3 \times 10^{54} \text{ g} \). However, according to the big bang model, before the big bang, the entire matter in the universe was contained in an "ylem" which occupied very very small volume. The gravitational radius of the ylem of mass \( 3 \times 10^{54} \text{ g} \) was \( 4.45 \times 10^{21} \text{ km} \) (it must have been larger if the actual mass of the universe were taken into account which is greater than \( 3 \times 10^{54} \text{ g} \)). Obviously, the radius of the ylem was many orders of magnitude smaller than its gravitational radius, and yet the ylem exploded with a big bang, and in due course of time crossed the event horizon and expanded beyond it up to the present Hubble distance \( c/H_0 \sim 1.5 \times 10^{23} \text{ km} \) where \( c \) is the speed of light in vacuum and \( H_0 \) the Hubble constant at the present epoch. Consequently, if the ylem could explode in spite of Zeldovich and Novikov’s contention, a gravitationally collapsing black hole can also explode, and in due course of time expand beyond the event horizon. The origin of the big bang, i.e., the mechanism by which the ylem exploded, is not definitively known. However, the author has, earlier proposed a viable mechanism (Thakur 1992) based on supersymmetry/supergravity. But supersymmetry/supergravity have not yet been validated experimentally.

8 Conclusion

From the foregoing three inferences may be drawn. One, eventually the entire matter in a collapsing black hole is converted into quark-gluon plasma permeated by leptons. Two, the collapse of a black hole to a space-time singularity is inhibited by Pauli’s exclusion principle. Three, ultimately a black hole may end up in one of the three possible ways suggested in section 7.

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