Abstract
Consumers learn their valuation for most goods and services sooner than the price or the availability. In such markets, the optimal price of each firm falls in the search cost of the consumers, despite the exit of lower-value consumers when search becomes costlier. The reason is that a greater search cost causes inframarginal consumers to exit instead of switching firms. The marginal consumers respond less and may become more numerous. The more elastic demand raises prices. At a high enough search cost, no consumer switches. Each firm is a monopolist but sets a lower price than under competition over the switchers because demand changes shape when some consumers exit. Total surplus, demand and profits fall in the search cost. Consumer surplus and total surplus are higher when consumers do not know their valuations. The results are robust to various changes of the assumptions, for example some consumers having zero search cost or firms running out of stock.

Keywords Search cost · Congestion · Imperfect information · Price competition

1 Introduction
Car dealerships, gas stations and department stores co-locate to reduce search costs, as discussed below. OTC financial markets invest in the speed at which buyers get seller quotes (faster search and comparison) but do not establish a public price registry (an exchange). Easier comparison and switching between firms seemingly increases competition and reduces profits—an intuition confirmed by most industrial organisation models. By contrast, this paper shows how differentiated firms can in fact increase profits by reducing search costs and making the surplus offered more comparable across firms. Providing greater product transparency without full price transparency is
unambiguously anticompetitive. The message to regulators is to provide price comparisons or to mandate that industry associations do so, especially if they already display product comparisons.

The economic causes can be elucidated in a simple model of a duopoly competing on price but extend more broadly, including to any number of firms. Firms simultaneously set prices. Each consumer visits a firm, observes its price and the valuations offered by all firms.\(^1\) The valuations are private to the consumer.

One interpretation is that the consumer has a vague need, visits a random seller and learns the relative worth of the products when examining one of these. The salesperson or the firm’s website explains the difference between its product and the rival’s (blender vs juicer, gravel bike vs hybrid, chainsaw vs disc-saw, geothermal heat pump vs air-source, drugs or medical devices for the consumer’s symptoms). The rival’s product may suit the buyer better. Other examples in the Applications section below include how the time of a service suits the buyer’s schedule or an investment complements her or his portfolio.

A second interpretation is that the consumer knows the valuations perfectly from the start. The motives to first visit a seller expected to offer a non-maximal value are (a) this seller happens to be on the way, (b) sells other goods the buyer also wants or (c) has a lower probability of a stock-out.

The third interpretation is that the consumer first visits the seller expected to offer the best value, but a negative preference shock (unpleasant people, smells or spills at the first seller) sometimes causes the consumer to go to the rival. The shocks may be combined with the other interpretations. For example, the seller that is on the way changes when the consumer’s destination changes (the buyer remembers an errand or gets a call requiring her to go somewhere).

Once the consumer visits a firm, she or he may buy immediately, exit, or learn the price of the competing firm. After learning, the consumer may buy from either firm or exit. Firms cannot distinguish customers who buy immediately from those who first learn and then buy.\(^2\)

The consumers who learn can be held up because their willingness to pay the search cost implies that their valuation for the firm they arrive at is above its equilibrium price. This hold-up motive increases prices.\(^3\) A greater search cost weakens the hold-up motive because fewer consumers search. When a smaller fraction of a firm’s demand is composed of customers switching from the competitor, the hold-up motive and the price are lower.

Another economic force in the same direction is that a higher search cost causes some consumers to exit who previously would have switched firms. More exit leaves fewer inframarginal consumers (those whose valuations strictly exceed the price) to the firms on average, so some firm’s inframarginal demand falls. If the firms are

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\(^1\) The first price observation being free is the standard assumption to avoid a market breakdown resulting from the Diamond paradox.

\(^2\) Websites try to track the browsing history of the buyers to segment them into switchers and captive customers, but the buyers may take countermeasures (using a VPN, the Tor browser, or searching on different devices). Also, the segmentation may be illegal or create negative publicity, making it not worthwhile.

\(^3\) The companion paper Heinsalu (2021) solves the observable prices case and proves that prices are lower in that case. Similar comparative statics occur: prices fall in the transport or switching cost.

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symmetric enough, then each of them receives fewer inframarginal buyers. The mass of marginal consumers (those whose valuations equal the price) may increase or decrease in the search cost, but this change is smaller. As the ratio of the inframarginal to the marginal consumers falls, so does the optimal price. The prices of the firms are strategic complements (Lemma 1), so one firm’s price cut motivates others to follow suit.

When the search cost is low, the consumers who purchase from a firm have a high valuation for that firm. When the search cost is high, the consumers who have a low valuation for their initial firm but above its price still buy from that firm because they are unwilling to search. This lowers the values of the consumers at the firm, suggesting a lower price.

The equilibrium is proved to be pure and unique under weak conditions, so the comparative statics are unambiguous.

The same comparative statics occur when the buyer knows her initial value for both goods and first visits the firm offering greater value. The valuations may be determined by the distance from home, brand or the preferred time for a service. With positive probability, the first firm is closed, out of stock or too congested. In that case, the buyer visits the next firm. This model is solved in Sect. 4.

The demand and the total surplus decrease in the search cost because some consumers exit and costlier search makes the final allocation of the buyers to the firms less efficient. Given that the prices and the total surplus fall in the search cost, the profits decrease.

Applications  A gasoline buyer’s valuation depends on the distances to the stations and on preference shocks. If, as during the truck driver shortage in the UK in 2021, a station has a line or is out of stock (Sect. 4), then the buyer drives to the next.

The prices decrease in the search cost in the German gasoline market (Martin 2020). Nishida and Remer (2018b) find that for isolated gas stations in California, Florida, New Jersey and Texas, reducing the search cost by 20% leads to a price increase in 32% of markets. The average change is +5.2 cents per gallon. Nishida and Remer (2018a) shows that prices are slightly lower at isolated gas stations. These are costlier to search.

Car sellers reveal the technical specifications (the valuations) but hide the real price using discounts and extra fees. They also reduce the search costs of the consumers by co-locating, often in a row next to a major road. In Murry and Zhou (2020) Table 11, the effect of closing a co-located car dealer is to reduce its competitors’ prices by 0.1%.

In a cluster of sellers like a shopping mall, similar shops locate close to each other (groceries on the ground floor, jewellery, cosmetics and clothes on higher floors). In the absence of search costs, similar shops should prefer to locate far from each other. A buyer is unlikely to know which shop offers the best value before visiting one. The current paper explains within-cluster agglomeration: prices and profits decrease in the search cost whenever it is positive. Vitorino (2012) Table 8 shows that the profits of midscale department stores (e.g., Mervyn’s, JC Penney) increase in the number of midscale stores in the mall.

4 For across-cluster agglomeration applied to malls, see (Parakhonyak and Titova 2018).
Industry associations often provide a member directory\(^5\) that reduces search costs and lists each firm’s characteristics but not the price. This suggests that firms in the industry profit from cheaper search and revealing the valuations.

Airlines seemingly allow price comparisons on travel websites (Kayak, Bing Travel, Google Flights), but bait and switch: after clicking, cheap flights are unavailable or higher-priced due to credit card fees or airport taxes. A flyer knows how well each flight fits her schedule but visits the airline websites to learn the real prices.

Over-the-counter financial assets require asking for quotes. This imposes a small delay cost. The buyer privately knows how the asset correlates with her portfolio, which determines her valuation.

The next section introduces the framework and derives the main result. Extensions and generalisations (distributions of search costs and valuations, observable switching, unknown valuations, advertising) are discussed in Sect. 4. The literature and conclusions are discussed in Sect. 5. The online appendix generalises to \(n \geq 2\) asymmetric firms and proves additional results, e.g., prices increase in \(n\), so competition is price-increasing.

### 2 Horizontally differentiated duopoly

Two symmetric\(^6\) firms called \(X\) and \(Y\) simultaneously set prices. A generic firm is denoted by \(i\) and its price by \(P_i\). There is a mass 1 of consumers. Each has valuations \(v = (v_X, v_Y)\) for the firms, where \(v_i \in [0, 1]\). The valuations \(v_X, v_Y\) are assumed independent.\(^7\) The firms only have the common prior belief that \(v_i\) is distributed according to \(f\), which is positive with interval support. The corresponding cdf is denoted \(F\).

Half of the consumers initially observe \(P_X\) and half \(P_Y\).\(^8\) Call the firm whose price a consumer initially observes the initial firm of the consumer. Together with the initial firm’s price, the consumer privately observes her valuations \(v\).\(^9\) Each consumer decides whether to buy from her initial firm, learn the price of the other firm at cost \(s > 0\)\(^10\) or exit. The consumer knows her valuation when deciding whether to learn (Sect. 4 relaxes this assumption). After learning, the consumer decides whether to buy from firm \(X\), firm \(Y\) or exit. Free recall is assumed for simplicity. The online appendix proves the results are unchanged when returning to the previous firm is costly.

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\(^5\) Grocers: [http://www.agbr.com/store-locator/](http://www.agbr.com/store-locator/), notaries: [https://www.thenotariessociety.org.uk/notary-search](https://www.thenotariessociety.org.uk/notary-search), restoration contractors: [https://www.iicrc.org/page/IICRCGlobalLocator](https://www.iicrc.org/page/IICRCGlobalLocator).

\(^6\) The case of firms with asymmetric costs, initial demands and valuation distributions is solved in an earlier version of this paper, available on [https://sanderheinsalu.com/](https://sanderheinsalu.com/). The direction of comparative statics is the same.

\(^7\) Section 4 considers correlated valuations and other extensions. The online appendix solves for \(n\) asymmetric firms.

\(^8\) Heinsalu (2021) proves similar results under publicly observed prices. The extension to price advertising in Sect. 4 is related.

\(^9\) The results are unchanged if a consumer privately observes \(v\) before visiting the first firm.

\(^10\) Zero search cost is qualitatively different (Bertrand competition), discussed in Sect. 4. High enough search cost makes the firms monopolists, as discussed at the end of Sect. 3.
A sufficient condition for the main theorem and the lemmas is

**Assumption 1** $v_i f'(v_i) \in [-f(v_i), 0]$ for all $v_i \in (0, 1)$.

This is simpler but stronger than the assumptions below, e.g. the sufficient condition for prices to decrease in the search cost:

**Assumption 2** $f' \leq 0$ or $f(P_i - s + w) + P_i \frac{\partial f(P_i - s + w)}{\partial P_i} \leq f(P_i + s + w)$ for all $P_i \in (0, 1), w \in [0, 1 - P_i + s)$

The pointwise Assumption 2 on $f$ is simpler but stronger than the integral condition used in the proof of the main theorem.

A sufficient condition for the strategic complementarity of prices is

**Assumption 3** $P_i \frac{\partial f(P_i + w)}{\partial P_i} \geq -f(P_i + w)$ for all $P_i \in (0, 1), w \in [0, 1 - P_i)$.

Assumption 1 implies Assumption 3: rewrite 1 as $P_i \frac{\partial f(P_i + w)}{\partial P_i} + w \frac{\partial f(P_i + w)}{\partial P_i} \geq -f(P_i + w)$ and $\frac{\partial f(P_i + w)}{\partial P_i} \leq 0 \leq w$. Assumption 3 in turn implies the sufficient condition for the uniqueness of equilibrium (defined below):

**Assumption 4** $P_i \frac{\partial f(P_i)}{\partial P_i} \geq -2 f(P_i)$ for all $P_i \in (0, 1)$.

Uniform valuations satisfy Assumption 1, as does a truncated exponential distribution, a beta distribution $f(v_i) = B(a, \beta) v_i^{a-1}(1-v_i)^{\beta-1}$ with $a \in [0, 1], \beta = 1$ (i.e., a power distribution), a truncated gamma distribution $f(v_i) = k \left( \frac{v_i}{\beta} \right)^{c-1} \exp\left( -\frac{v_i}{\beta} \right)$ with $c \in \left[ \frac{1}{\beta}, 1 \right]$ and any nonincreasing $f(v_i)$ with slope magnitude bounded by $\frac{1}{v_i}$.

The payoff of a consumer from not buying is normalised to zero. A consumer with valuation $v$ who buys from firm $i$ at price $P_i$ without searching obtains payoff $v_i - P_i$ but after search obtains $v_i - P_i - s$ from buying and $-s$ from exiting. Firm $i$ that sets price $P_i$ resulting in ex post demand $D_i$ gets ex post profit $\pi_i := P_i D_i$. The marginal cost is assumed constant and normalised to zero. W.l.o.g. restrict $P_i \in [0, 1]$ because a price that is negative or above the maximal valuation of consumers is never a unique best response. Mixed prices on or off the equilibrium path are ruled out in the online appendix.

Subgame perfect equilibrium consists of the firms’ pricing strategies and the decisions of the continuum of consumers such that (i) each firm maximises its profit given the decisions it expects from the consumers and the rival firm. (ii) Each consumer maximises her expected payoff by choosing to buy, learn or exit based on the prices she sees and expects. Consumers who learn choose which firm, if any, to buy from to maximise their expected payoff. (iii) The expectations of the firms and consumers are correct.11

Denote the decision of a consumer with valuations $v$ who observes price $P_i$ and expects $P_j^*$ by $\delta_1(v, P_i) \in \{b, l, 0\}$ (buy, learn, exit) and after learning by $\delta_2(v, P_X, P_Y) \in \{X, Y, 0\}$.

11 As is standard, expectations are correct on the equilibrium path but do not anticipate a deviation, so may be incorrect off-path. A deviation by a firm does not change consumers’ expectations about the other firm. Consumers put zero probability on a coalitional deviation even conditional on observing one firm deviating.
**Definition 1** Equilibrium consists of $P^* \in [0, 1]^2$, $\delta_1^*: [0, 1]^3 \rightarrow \{b, l, 0\}$ and $\delta_2^*: [0, 1]^4 \rightarrow \{X, Y, 0\}$ such that for $i \in \{X, Y\}$ and $v \in [0, 1]^2$,

\[(i)\] $P_i^* \in \arg \max_{P_i} \left[ \frac{1}{2} \int \int \{v: \delta_1^*(v, P_i) = b \lor \delta_1^*(v, P_i) = l \land \delta_2^*(v, P_i, P_j^*) = i\} dF(v_i) dF(v_j) \right.$

$+ \frac{1}{2} \int \int \{v: \delta_1^*(v, P_i) = l \land \delta_2^*(v, P_i, P_j^*) = i\} dF(v_i) dF(v_j) \left. \right]$, \n
\[(ii)\] $\delta_1^*(v, P_i) = b \Rightarrow v_i - P_i \geq \max \left\{0, v_j - P_j^* - s\right\}$,

$\delta_1^*(v, P_i) = l \Rightarrow v_j - P_j^* - s \geq \max \{0, v_i - P_i\}$,

$\delta_1^*(v, P_i) = 0 \Rightarrow \max \{v_j - P_j^* - s, v_i - P_i\} \leq 0$,

$\delta_2^*(v, P_i, P_j) = k \Rightarrow v_k - P_k = \max_m \{v_m - P_m\} \geq 0$,

$\delta_2^*(v, P_i, P_j) = 0 \Rightarrow \max_m \{v_m - P_m\} \leq 0$,

\[(iii)\] $P_i = P_i^*$.

Asymmetric equilibria are allowed. With symmetric firms, Lemma 1 provides sufficient conditions for all equilibria to be symmetric.

The next section first finds the optimal decisions of consumers, which determine the demands for the firms. Then the profit-maximising prices are calculated, followed by the main comparative static of prices decreasing in the search cost.

### 3 Demand, profit and comparative statics

To find the optimal prices, start with the decisions of the consumers. These determine the demands for the firms, which are then used to find the best-response prices.

Consumer $v$ who observes firm $j$’s price $P_j$ and expects firm $i$ to choose $P_i^*$ learns $P_i$ if the payoff $\max \left\{0, v_i - P_i^*, v_j - P_j\right\} - s$ from learning is greater than the payoff $\max \{0, v_j - P_j\}$ from either choosing to exit (payoff zero) or buying immediately at price $P_j$. The payoff from learning includes the options to exit, buy from firm $i$ or buy from firm $j$ after learning, minus the search cost $s$.

The demand for a firm consists of consumers initially at that firm who either buy immediately or learn and then buy from that firm and consumers initially at the rival firm who learn and switch. Figure 1 depicts demands for each firm from customers initially at each firm (left panel: buyers initially at firm $X$, right panel: $Y$). The marginal customers for firm $Y$ are marked as the thick blue line in the right panel and the marginal buyers for $X$ as the thick orange line in the left panel. Consumers initially at $X$ are not marginal for $Y$ and vice versa.
Fig. 1 Demands at the pure prices $P_X = P_X^* = 0.6$ and $P_Y = P_Y^* = 0.45$ and search cost $s = 0.1$. Left panel: consumers initially at firm $X$, right panel: $Y$

Customers initially at firm $i$ buy immediately from $i$ if $v_i - P_i \geq \max\{0, v_j - P_j^* - s\}$. They learn and then buy from firm $i$ when both $v_j - P_j^* - s \geq \max\{0, v_i - P_i\}$ and $v_i - P_i \geq \max\{0, v_j - P_j\}$.

Consumers starting at $j$ buy from $i$ if both their expected payoff $v_i - P_i^* - s$ from learning and the observed payoff $v_i - P_i$ from $i$ after learning are larger than the payoff $\max\{0, v_j - P_j\}$ from exiting or buying from $j$. Combining these conditions results in $v_i \geq \max\{P_i, P_i^* + s\} + \max\{0, v_j - P_j\}$.

The demand that firm $i$ expects from price $P_i$ is

$$D_i(P_i, P_j^*, P_i^*) = \frac{1}{2} \int_0^{v_i} \int_{P_i + \max\{0, v_j - \max\{P_j^* + s, P_j^*\}\}}^1 f(v_i) f(v_j) dv_i dv_j$$

$$+ \frac{1}{2} \int_0^{v_i} \int_{\max\{P_i, P_i^* + s\} + \max\{0, v_j - P_j^*\}}^1 f(v_i) f(v_j) dv_i dv_j. \quad (1)$$

The two integrals in the demand aggregate the consumers initially at each firm over the region of valuations that result in these consumers eventually buying from $i$, given the prices.

Having derived the demand, the next lemma establishes the strategic complementarity of prices on and off the equilibrium path. Several alternative sufficient conditions are available: either the demand is log concave or the ccdf $1 - F$ is concave or the density of the valuations does not decrease too fast. Any weakly increasing density satisfies the latter, for example, uniform, triangular $f(v_i) = 2v_i$, trapezoidal $f(v_i) = \min\{kv_i, k - \sqrt{k^2 - 2k}\}$ for $k > 2$. The truncated exponential and the truncated normal distribution also satisfy the conditions, as does the truncated Pareto with a negative location parameter.

Lemma 1 (a) If one of the following holds:

(i) Assumption 3 or
(ii) demand is log concave or
(iii) $1 - F$ concave or
(iv) \( f \) is log concave and \( f'(1) \geq -f(1) \),

then the prices are strategic complements.

(b) If the prices are strategic complements and the firms are symmetric, then any equilibrium is symmetric.

The proof in the appendix shows that the cross-partial derivatives of the profits w.r.t. the prices are positive. Subsequent proofs omitted from the text are also in the appendix. Concave \( 1 - F \) implies Assumption 3, as does \( f \) log concave with \( f'(1) \geq -f(1) \).

The conditions in Lemma 1 are sufficient but not necessary for the prices to be strategic complements. Strategic complementarity makes the best response of each firm to the price of any other an increasing function (the prices of the firms move together). Strategic complements also imply that the equilibria with the lowest and the highest prices are stable and all stable equilibria have the same comparative statics (Milgrom and Roberts 1990 Theorems 6,8). Symmetry implies an odd number of equilibria and that stable and unstable equilibria alternate (Hefti 2017).

A firm’s profit increases in a rival’s price, so the firms impose positive externalities on each other by raising prices. This implies that for the firms, the equilibria are Pareto ordered by price (Milgrom and Roberts 1990 Theorem 7). The highest-price equilibrium is the natural focus of coordination if multiple equilibria exist. Given the stability and Pareto dominance of the highest-price equilibrium, uniqueness is not needed for comparative statics. To strengthen the results, the next lemma shows that the equilibrium is unique if the consumer valuation pdf does not jump up (implied by continuity) and does not decrease too fast (e.g., uniform, truncated exponential, normal, truncated Pareto with a negative location parameter). The inequality in Lemma 2 is implied by log concave demand or a concave ccdf or the sufficient condition \( P_i \frac{\partial f(P_i + w)}{\partial P_i} \geq -f(P_i + w) \) for strategic complementarity in Lemma 1.

**Lemma 2** If Assumption 4 holds, then the equilibrium is unique.

Assumption 4 is implied by log concave \( f \) with \( f'(1) \geq -2f(1) \). The conditions in Lemma 2 are sufficient but not necessary for uniqueness (see the proof). Further, uniqueness is not necessary for the main result because strategic complementarity makes the direction of comparative statics in all stable equilibria the same.

The main theorem establishes that if the consumer valuation pdf is decreasing or does not vary too fast, then each firm’s price decreases in the search cost of the consumers. Uniform valuations satisfy the condition, as does a truncated exponential distribution and any nonincreasing \( f(v_i) \) that does not decreases faster than \(-\ln v_i\). The proof uses the Implicit Function Theorem to calculate the derivatives of the equilibrium prices w.r.t. the search cost because the prices have no closed form in general.

**Theorem 3** If Assumption 2 holds, then \( \frac{dP^*_i}{ds} \leq 0 \) for both firms in any stable equilibrium. If further \( s < 1 - P^*_i \), then \( \frac{dP^*_i}{ds} < 0 \).

The economic interpretation of \( f(P_i - s + w) + P_i \frac{\partial f(P_i - s + w)}{\partial P_i} \leq f(P_i + s + w) \) is that as \( s \) rises, the marginal profit of a firm changes for three reasons. The integral of \( f(P_i - s + w) \) measures the reduced switching away by the inframarginal consumers initially at the firm. The decrease in incoming switchers is the integral of
- \( f(P_i + s + w) \). The integral of \( \frac{\partial f(P_i - s + w)}{\partial P_i} \) is the effect on the marginal profit of changing the marginal demand. If the overall effect is that the marginal profit function decreases, then the price falls because the SOC implies that the marginal profit crosses zero from above at an optimum.

One economic force in Theorem 3 is that the fraction of incoming switchers among a firm’s customers falls in the search cost. The outgoing switchers also decrease, but they cannot be held up; they only influence the price via changing the marginal and the inframarginal demand, discussed below. The incoming switchers can be held up because they are willing to pay the price plus the search cost. They will all still buy if the firm’s chosen price exceeds the expected price slightly. This hold-up motive increases a firm’s optimal price. Greater search cost decreases the hold-up motive and thus the price. When the search cost becomes so large that no consumers switch, then each firm’s price falls to its monopoly level.\(^\text{12}\) This price best responds to the demand that remains at the large search cost when the low-valuation customers have exited. This demand is smaller than at a low search cost and contains relatively more high-valuation customers.

Figure 2 shows the effect of a greater search cost on the demands, fixing the prices. In the left panel (consumers initially at firm \( X \)), the light blue rectangle below \( P_Y + s \) is the consumers who stop buying from \( Y \) and exit when \( s \) increases. The bluish band of height \( s \) below the diagonal part of the thick orange line consists of the consumers who start buying from \( X \) instead of \( Y \). In the right panel, the light orange rectangle to the left of \( P_X + s \) shows the consumers who stop buying from \( X \) and exit, while the orange band above the diagonal part of the thick blue line depicts those who stay with \( Y \) instead of switching to \( X \). Compared to before the search cost increase, each firm loses some incoming switchers who could be held up and gains some demand from consumers initially at itself. The latter respond to any price increase, however small. With a uniform valuation distribution and symmetric initial demands, firm \( X \) gains marginal consumers and firm \( Y \) loses a small measure, but both firms lose a substantial mass of inframarginal customers.

\(^{12}\) In a monopoly, all consumers observe the price and the valuation and those with \( v_i \geq P_i \) buy. Demand is twice that of a duopolist at \( s \geq 1 \). Multiplying demand by a positive constant does not change the optimal price.
If the search cost is large enough, then no consumer searches. The demand for firm X is \( \frac{1}{2} [1 - F_X (P_X)] \), which in Fig. 2 is the rectangle to the right of \( P_X \) only in the left panel. The demand for firm Y is the rectangle above \( P_Y \) only in the right panel. Each firm is a monopolist over the consumers initially at that firm. The corresponding monopoly price is lower than when the search cost is low enough that some consumers switch because the demand at high \( s \) for a firm includes relatively more customers with a low valuation for that firm than the demand at low \( s \). For example, in Fig. 2, firm X at low search cost loses the customers above the diagonal orange line segment but at high search cost sells to them. The expected \( v_X \) above the diagonal segment is lower than the expected \( v_X \) below. If the prices fall in the search cost and each firm becomes a monopoly at high search cost, then the prices decrease to the monopoly level.

If a single owner controlled both firms and the search cost was high enough, then prices would be the same as with the two monopolists in the previous paragraph because the demands of the firms are independent at high \( s \). With a single owner and a low search cost, the prices would be higher than in duopoly because some of the consumers leaving firm X due to its high price would buy from Y. The prices thus decrease in the search cost under a single owner also.

This concludes the discussion of how the search cost affects prices. The following subsection examines the change of the profits, the welfare and the consumer surplus in the search cost, as well as how the prices respond to the initial allocation of consumers.

### 3.1 Other comparative statics

Exit increases in the search cost, as proved next, so the demand and the total surplus (the buyer value minus the seller cost integrated over all trading buyers and sellers) in the market fall. Conditional on the consumers who trade, the price is a transfer that does not affect the total surplus, only the profits and the consumer surplus. Costlier search also makes the final allocation of the buyers to the firms less efficient (ideally, consumers above the diagonal in Fig. 2 would buy from firm Y and below the diagonal from X). Therefore, the welfare and the profits decrease in the search cost.

The total demand decreases because each firm gets less than the full benefit of keeping consumers in the market, so does not reduce price enough to fully offset the higher search cost. Oligopoly market power is intermediate between monopoly and perfect competition, so the oligopolists internalise part of the efficiency loss from a higher search cost but leave part of it to the consumers. The resulting smaller benefit from trade makes some consumers exit. The reasoning above is formalised in the following result.

**Proposition 4** If Assumption 3 holds, then the total surplus and each firm’s profit and demand decrease in the search cost.

The proof is in the appendix. The result holds in any equilibrium if there are multiple (for multiplicity, the conditions in Lemma 2 must fail).

The effect on the consumer surplus could have either sign in general because both the prices and the allocative efficiency decrease. With a uniform valuation distribution, the consumer surplus strictly decreases in the search cost.
Suppose all consumers are initially at one firm (the incumbent) instead of a half-half split. Then the unique equilibrium prices are such that the incumbent remains a monopolist. The consumers expect a high enough price from the other firm (the entrant) that learning is not optimal. Any price cut by the entrant is not observed by the consumers, so they cannot start learning in response to it. Suppose the consumers expected the entrant to set a low enough price to make learning worthwhile at some valuations. Then the entrant would hold up all arriving switchers by choosing a price greater than they expected. This holds for any expected price and any positive search cost, which contradicts the assumption that some consumers learn the entrant’s price.

Modifications of the baseline model are considered next. The results remain robust to a distribution of search costs, unattached consumers or many firms and are continuous in the correlation of valuations.

4 Extensions and generalisations

Search costs At a zero search cost, the prices are discretely lower than at an arbitrarily small positive search cost, as in the Diamond paradox. Learning both prices becomes weakly dominant for consumers, discretely increasing the mass of buyers who learn and are on the margin of switching. Discretely more marginal consumers and approximately the same mass of inframarginals reduces the optimal price.

Suppose a fraction $\alpha$ of consumers are ‘shoppers’ whose search cost is zero and $1 - \alpha$ are captive or loyal with $s > 0$. Then the demand is the weighed average of the benchmark demand (1) of captive customers and the demand of shoppers who are unaffected by a change in the search cost. The profits, the FOCs and other formulas are similarly weighted averages. The comparative statics decrease in magnitude linearly in $\alpha$ but retain their sign.

Because the firms are horizontally differentiated, a Bertrand outcome is avoided even when all consumers are shoppers. The mass of marginal consumers increases in $\alpha$ for each firm because the shoppers switching to the firm respond to its price, not just the expected price. The inframarginal consumers also increase in $\alpha$ because the shoppers are more likely to switch firms instead of exiting. Demand rises in $\alpha$, given the prices. The prices and the profits may rise or fall because the inframarginal and the marginal consumers change in the same direction. If the opposite of one sufficient condition for strategic complementarity holds, then the prices increase in the fraction of shoppers, as the next result shows.

Proposition 5 If $f_i'(P_i + w) + P_i f_i'(P_i + w) \leq 0$ for all $P_i \in [0, 1]$, $w \in [0, 1 - P_i]$, then $\frac{dP_i^*}{d\alpha} \geq 0$.

The proof shows that each firm’s marginal profit increases in the fraction of shoppers, and thus the optimal price increases.

Uniform valuation distributions lead to the intuitive result that the prices and the profits decrease in the fraction of shoppers, unlike in Proposition 5.

A distribution $G(s)$ of search costs independent of the valuations yields the same results as a known $s$. To see this, interpret the demand (1) as conditional on $s$ and integrate it w.r.t. $G$ to obtain the expected demand of firm $i$. Similarly, the firm’s
FOC (3) and the cross-partial derivative (4) in the proofs are integrated w.r.t. $G$. If the distribution of search costs is translated upward by adding $\Delta s > 0$ to each $s$, then the comparative statics (using the integral of (6) w.r.t. $G$) are the same as in Theorem 3: the prices decrease in $\Delta s$. It is not surprising that if a result holds pointwise for every $s$ in the support of the distribution $G$, then it holds for shifts of $G$. All the formulas are continuous in $s$, so by the Mean Value Theorem, for each result using $G$, there exists a known search cost delivering the same result.

Suppose a fraction $v$ of customers are initially at neither firm and have to pay the search cost no matter which price they first learn about. Then the hold-up motive is strengthened for both firms, and thus prices are higher at any $s > 0$ at which some consumers search. At large enough $s$, no consumers search, so each firm charges its monopoly price, which is unaffected by multiplying demand by $1 - v$. The price decrease in the search cost becomes steeper as $v$ increases because the prices fall from a higher level at small $s > 0$ to the same monopoly level. The following proposition formalises this intuition.

**Proposition 6** For both firms, $P_i$ and $\left| \frac{dP_i}{ds} \right|$ increase in $v$.

The proof uses the Implicit Function Theorem to show that the derivative of the equilibrium prices w.r.t. the fraction $v$ of unattached consumers is positive.

If no consumers know any price before search ($v = 1$) but know their valuations and have to pay a cost for their first price observation, then the market breaks down (Stiglitz 1979). No matter what price the arriving consumers expect, each firm strictly prefers to charge more, for the same reason as the entrant discussed in Sect. 3.1.

The market does not break down if the consumers learn their valuations only at the first firm and the first price observation costs $s_1 \in [0, \mathbb{E} \max\{0, v_i - P_i^*, v_j - P_j^* - s\})$, where $P_i^* = P_j^*$ are the duopoly equilibrium prices satisfying (3). All consumers optimally pay $s_1$ to visit a firm, given their expectation $P_i^*$. Because the consumers do not know $v$ before visiting, they pick a firm uniformly at random. Thus, for the firms, the valuation distribution of the arriving consumers is the ex ante $F$, same as in the baseline model, so the optimal price is $P_i^*$, which justifies the expectation.

Firm $i$ is deterred from raising its price towards monopoly by the consumers who learn $v_i \approx v_j - s$ and are on the margin of leaving to the rival firm $j$. If the consumers learn their valuations upon arrival, then arrival does not signal a valuation strictly above the price, which would provide an opportunity for hold-up.

**Observable switching** If the firms can distinguish their initial customers from the incoming switchers, then they charge the switchers a prohibitively high price. Equivalently, each firm would operate in two markets—in one as an incumbent serving its initial customers and in one as an entrant selling to the switchers. As explained at the end of Sect. 3, hold-up leads to market breakdown for the entrant because no matter what price the switchers expect, the firm at which they arrive strictly prefers to charge more. Both the price for the initial customers and the price for the switchers are thus independent of the search cost.

**Stock-outs** Each consumer type $v$ first visits the firm offering the greatest expected net valuation $\max_i \{v_i - P_i^*\}$ if this is positive, otherwise exits. With independent
probability \( p_0 \), each firm runs out of product, is closed, congested or the consumer gets some other negative shock to her valuation that reduces it below the marginal cost of the firm. The consumer then either exits the market or continues to the firm with the next highest \( v_j - P_j^* \). With probability \( 1 - p_0 \), the consumer buys at the initial firm.

In a duopoly with stock-outs, the demand that firm \( i \) expects is

\[
(1 - p_0) \int_0^1 \left[ 1 - F \left( \max \left\{ P_i^*, P_i^* + v_j - P_j^*, P_i + v_j - P_j^* - s \right\} \right) \right] dF(v_j) + (1 - p_0) p_0 \int_{P_j^*}^1 \max \left\{ 0, F \left( P_i^* + v_j - P_j^* \right) - F \left( \max \left\{ P_i^* + s, P_i \right\} \right) \right\} dF(v_j).
\]

The first line is the consumers who visit \( i \) initially and find the price low enough to buy instead of exiting or visiting \( j \). The second line are the consumers initially choosing \( j \) (valuation for \( i \) below \( P_i^* + v_j - P_j^* \)) who face a stock-out, visit \( i \) (valuation above \( P_i^* + s \)) and buy when \( i \) is not out of stock. Depicting demand by modifying Fig. 2, the marginal consumers are only the vertical orange and the horizontal blue line segments. No buyer returns to the initial firm because the shock is assumed large enough to put her valuation below the firm’s marginal cost.

The next result proves that the prices decrease in the search cost for any valuation distribution when consumers sort themselves by valuation and face stock-outs.

**Proposition 7** In the stock-out model, \( \frac{dP_i^*}{ds} \leq 0 \), strictly if \( s \in (0, 1 - P_i^*) \).

The proof takes the cross-partial derivative of the profit under stock-outs w.r.t. the price and the search cost and shows that this is negative. Thus, even if the prices are observable, the main comparative static holds—the prices decrease in the search cost.

Observable prices are solved in a general oligopoly framework in Heinsalu (2021) and are related to the price advertising section below.

**Joint distribution of valuations** Correlated valuations of consumers may change the results, depending on the joint distribution of the valuations. The only modification in the proofs is replacing \( F(v_i) \) in all formulas by \( F(v_i|v_j) \). The modified sufficient conditions may be harder or easier to satisfy than the original assumptions, depending on the joint distribution of the valuations. If \( v_X \) and \( v_Y \) are perfectly positively correlated, then the model with \( s = 0 \) is Bertrand competition but with \( s > 0 \) is the original Diamond (1971) paradox, where the prices stay constant in \( s \). Perfect negative correlation of \( v_X \) and \( v_Y \) reduces the environment with \( s = 0 \) to the Hotelling model.

An interesting case occurs with the consumers uniformly distributed on two crossing streets (on a + shape) with firm \( Y \) at the north and \( X \) at the east end of the +. In this case, both firms charge \( P_i = \frac{1}{2} \) for all \( s \geq 0 \), so each firm’s competitive and monopoly price are equal.

No outside option for the consumers (a covered market) leads to similar results as the baseline model—simply replace \( \max \{ 0, x \} \) by \( x \) in the formulas. Then the extensive margin (more consumers entering as the search cost falls) that drives the results in Moraga-González et al. (2017) and Section 6.2 of Choi et al. (2018) is absent and not needed for the prices to fall in the search cost.
Fig. 3 Demands at the pure prices $P_X = P_X^* = 0.6$ and $P_Y = P_Y^* = 0.45$ and the search cost $s = 0.1$. Left panel: the consumers initially at firm $X$, right panel: $Y$. Thick vertical line, including dashed diagonal: the marginal consumers for $X$; thick horizontal line, including dashed diagonal: the marginal consumers for $Y$.

The valuations may be correlated with the initial visit probability. If the correlation is weak enough, then the results still hold. For uniform valuation distributions, it is sufficient that the probability $p_{init}$ of initially visiting the higher-valuation firm is less than $\frac{2}{3}$, as may be seen from (6) by replacing $\frac{1}{2}$ in the first line with $p_{init}$ and in the second line with $1 - p_{init}$. If the valuations are learned only at the first firm, as the baseline model assumes, then these are uncorrelated with the visit probability. Consumers cannot condition their visit on what they do not know.

**Unknown valuation** If the consumers initially at firm $i$ do not know their valuation $v_j$ for the rival firm but can learn $v_j$ and $P_j$ together, then the market segmentation is depicted in Fig. 3.

Then valuation distributions close to uniform result in intuitive comparative statics—the prices increase in the search cost. However, for symmetric firms, a sufficiently fast decrease in the valuation pdf implies that the prices decrease in the search cost over some range of $s$. A numerical example has $f_i(v_i) = \begin{cases} \frac{3}{2} & \text{if } v_i \in [0, \frac{1}{2}], \\ \frac{1}{2} & \text{if } v_i \in (\frac{1}{2}, 1], \end{cases}$ for both firms. The equilibrium prices at $s = 0$ are approximately 0.31. The monopoly price as $s$ becomes large is 0.25 and is thus lower than the duopoly price. As $s$ increases from 0.13 to 0.19, the prices decrease linearly from 0.491 to 0.384.

At large $s$, the environments with known and unknown valuation for the other firm are identical because no consumers switch. At $s = 0$, these models are also identical because it is weakly dominant for all consumers to search. If the consumers know both valuations, then the prices jump up when the search cost becomes positive, but if the valuation for the other firm is unknown, then the prices are continuous at costless search because the mass of marginal consumers changes continuously. Thus, for low positive search costs, the consumers obtain a greater utility with less information.

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13 If the initial price observation also costs $s$ and firms are restricted to symmetric strategies, but $n \geq 2$ firms are allowed, then the model is Wolinsky (1986), which also assumes uniform valuation distributions for its main results.

14 If $f_i = 1$, then the monopoly prices at large $s$ are $P_i = \frac{1}{2}$ and the competitive prices at $s = 0$ are $P_i \approx 0.414$. 

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Firms correspondingly make lower profits, so would prefer to inform the consumers about their valuation for the rival.

*Price advertising* Advertising changes the market outcome if the ad commits the firm to the advertised price. Without commitment, the firms may use hidden fees to increase the actual price above the advertised level. If the price in the ad is cheap talk, then the consumers do not respond to it because each firm wants to attract as many consumers as possible by claiming to charge the lowest price. The claim is then uninformative.

By revealed preference, if a firm can commit to a price and the competitor does not know of this option, then the firm’s profit cannot decrease. If the competitor knows, then its anticipatory change of price may decrease the advertiser’s profit. The next proposition proves that the profit of a firm strictly increases from deviating to advertising. However, if the competitor also advertises, then both prices and profits are lower than without advertising. The greater demand at lower prices does not compensate for the reduced price-cost margin.

**Proposition 8** *In any equilibrium of the baseline model, giving a firm a cheap enough option to advertise its price causes it to deviate to advertising and a lower price. If both firms can advertise, then both do, and prices and profits are lower than without advertising.*

The proof is in the appendix.

Proposition 8 implies that all firms advertise in equilibrium and the prices are revealed, same as if the search cost was zero. The resulting profit of each firm is lower than when no firm advertises. Industry associations thus prefer to prevent members from advertising their prices and to only reveal valuations to consumers. Observable prices in a general oligopoly framework are studied in Heinsalu (2021).

The following section concludes with a discussion of the literature, the predictions and the policy implications from the main model.

## 5 Discussion

**Literature**

In most of the literature from Diamond (1971) on, the prices increase in the search cost. Exceptions assume either multiperiod or multiproduct markets or add a countervailing force (higher switching cost, seller obfuscation, buyer entry). The current work fills a gap in the literature by using a simpler one-shot, one-product framework with different driving forces (reduced hold-up, inframarginal consumer exit) and proving a stronger result—the prices strictly decrease in the search cost for any positive search cost at which some consumers still switch firms.

**Multiple markets** In a multiperiod market with a switching cost after the first purchase, higher switching costs may reduce the prices below cost initially (Klemperer 1987) because the firms compete to lock in customers for later monopolization. The second-period prices weakly increase in the switching cost.
In Choi et al. (2018), the prices fall in the search cost under ordered search when firms advertise their prices. The firms compete to be the first to be visited because the search cost locks in the consumer. Instead of lock-in, the current paper studies the exit of the inframarginal buyers and the moral hazard of holding up the consumers who have already learned about both firms. The online appendix covers asymmetric firms, unlike Choi et al. (2018).

In Dubé et al. (2009), Cabral (2009), an intermediate switching cost leads to lower prices than a zero switching cost. The incentive to cut price to ‘invest’ in customer acquisition outweighs the incentive to ‘harvest’ with a high price. However, for large enough switching costs, the prices rise. In Cabral (2016), if trades have high frequency or the market is close to a symmetric duopoly, then switching costs increase competition, otherwise decrease. The one-shot model of the present paper focusses on the exit of the inframarginal customers and on hold-up, not consumer acquisition over time. The prices globally decrease in the search cost.

In a multiproduct market, search makes the products complements (Zhou 2014): a price cut on one increases the demand for both, more so at a greater search cost. Thus, the prices may fall in the search cost. In the present work with a single product, the prices decrease in the search cost due to the exit of the inframarginal buyers and the hold-up of the arriving ones instead of search deterrence. In Rhodes (2015), a multiproduct monopolist advertises one price to signal the others. The prices increase in the search cost.

Haan et al. (2018) use directed search for both observably and unobservably horizontally differentiated products. The prices increase in the search cost if unobserved before search. Under observable prices and other assumptions, the prices decrease in the search cost because the buyers who still search at a high cost are more likely indifferent between the firms, which strengthens price competition. The inability of the buyers to exit is key to increasing the number indifferent. The present work assumes unobserved prices before search, allows exit and relies on a different mechanism.

**Countervailing forces** Lal and Sarvary (1999) model a firm which may add a web shop to a physical store. The web shop reduces the search cost but increases the switching cost (easy to re-order). This may raise prices and reduce search. In the current paper, the search cost is the only force and always reduces prices.

Consumer entry may overturn the usual effect of the search cost (Moraga-González et al. 2017). Lower search costs make the existing consumers search more (the usual intensive margin) and attract new consumers with higher search costs (the extensive margin). If the extensive margin dominates, then greater variance of the search costs reduces the price. In the present work, the extensive margin is not needed. The intensive margin reverses if the consumers learn their valuations at the first firm.

**Other related work** In Mauring (2021), the firms have a positive probability of knowing a consumer’s search cost, which may be positive or zero. The valuations are common knowledge. The consumers learn prices. A firm’s profit may increase and the consumer surplus fall in the number of firms.

Wolinsky (1986) assumes the consumers do not know their valuation for any firm before visiting it, which Sect. 4 proves causes the opposite comparative statics (the prices rise in the search cost) for most valuation distributions.
Board and Pycia (2014) Equation (1) shows the prices are independent of the discount factor (equivalent to a search cost) when buyers cannot exit. The prices converge to competitive when the discount factor rises to 1, unlike the discontinuity at zero search cost in the current work where the buyers can exit.

Anderson and Renault (2000) study the negative externality that the consumers informed about their valuations impose on the uninformed by making demand more inelastic. The prices increase in the fraction of informed consumers. If all are uninformed, then the prices increase in the search cost under general assumptions.

This paper studies unordered consumer search, unlike Armstrong et al. (2009) in which one firm is sampled first by all buyers and sets the lowest price. The consumers do not know their valuations. Savelle (2019) proves the equivalence of ordered search to discrete choice.

Schultz (2005) studies a repeated Hotelling duopoly. The more consumers observe the prices, the harder collusion is to sustain, so the prices increase in the search cost.

5.1 Conclusion

Because the prices and the profits decrease in the search cost and increase in the information that the consumers have about their valuations, industry associations naturally want to help customers compare the association members. An online directory achieves this, which justifies the cost of creating and maintaining the member database. Notably, such searchable directories do not provide price comparisons, even though these would be easy to add. A simple explanation for the lack of price information is that reducing the search cost to zero by price transparency would discontinuously decrease the prices and the profits compared to a small positive cost.

At low positive search costs, the prices and the profits are discretely higher when consumers know their valuation for each firm before the learning decision than when they learn each valuation together with the price. This is an additional motive for industry groups to inform consumers in detail about what each member provides.

For a large enough search cost, each firm is a monopolist over its initial customers, which would be a reason for high prices, especially when the exit of the low-valuation buyers increases the average willingness to pay among the remaining ones. However, the exit of many consumers (who are inefficiently allocated to a firm which they value little) reduces the total surplus enough to outweigh the larger share of surplus that a monopolist can obtain using its market power. Therefore, the firms prefer a more efficient allocation even if it means more competition.

As Adam Smith already noted, industry associations tend to collude to increase the profits of their members at the expense of the consumers. A regulator maximising the consumer surplus prefers a zero search cost. The second-best for some valuation distributions, or if the consumers with higher valuations receive greater welfare weight, is maximal search cost, which results in the lowest prices. Prohibiting information release by an association is difficult, so the regulator should instead provide price comparisons directly. Examples already implemented are government-run health insurance exchanges, websites listing pension funds ordered by their total fee loading and public gasoline price comparison databases. Of course, the industry can counter by obfuscat-
ing prices with hidden add-on costs and private discounts. Antitrust legislation may make sharing price data among firms illegal to stop tacit collusion and prevent cartels from detecting deviations, but the tradeoff is higher prices if the consumers have a search cost.

A regulator maximising total surplus unambiguously prefers a lower search cost. At small positive search costs, both kinds of regulators prefer that the consumers do not know their valuation for the rival firm. However, providing price information to the consumers dominates removing their valuation information even if the latter was possible.

A Proofs and results omitted from the main text

Proof of Lemma 1 Using (1), firm $i$’s expected profit has the derivative (FOC)

$$\frac{\partial \pi_i(P_i, P^*)}{\partial P_i} = \frac{1}{2} \int_0^1 \left[ 2 - F \left( P_i + \max \left\{ 0, v_j - P_j^* - s \right\} \right) - F \left( \max \left\{ P_i, P_j^* + s \right\} + \max \left\{ 0, v_j - P_j^* \right\} \right) - P_i f \left( P_i + \max \left\{ 0, v_j - P_j^* \right\} \right) \right] dF(v_j).$$

(3)

Near any equilibrium $P_i \approx P_i^*$, clearly $1_{\{P_i > P_i^* + s\}} = 0$. By Milgrom and Roberts (1990) Theorem 4, the game is supermodular if $\frac{\partial^2 \pi_i}{\partial P_i \partial P_j} \geq 0 \ \forall i, j$, in which case the prices are strategic complements. Log concave demand is sufficient for supermodularity. The cross-partial derivative is

$$\frac{\partial^2 \pi_i(P_i, P^*)}{\partial P_i \partial P_j^*} = \frac{1}{2} \int_{P_j^* + s}^1 f \left( P_i + \max \left\{ 0, v_j - P_j^* - s \right\} \right) dF(v_j) + \frac{1}{2} \int_{P_j^*}^1 f \left( \max \left\{ P_i, P_i^* + s \right\} + \max \left\{ 0, v_j - P_j^* \right\} \right) dF(v_j) + \frac{1}{2} \int_{P_j^* + s}^1 P_i f' \left( P_i + \max \left\{ 0, v_j - P_j^* - s \right\} \right) dF(v_j).$$

(4)

A sufficient condition for strategic complementarity is $P_i f'(P_i + w) \geq -f'(P_i + w)$ for all $P_i \in (0, 1), w \in [0, 1 - P_i]$. Dividing by $P_i f(P_i + w)$ yields $\frac{f'(P_i + w)}{f(P_i + w)} + \frac{1}{P_i} \geq 0$. Differentiating, $\frac{f''(P_i + w) f'(P_i + w) - (f'(P_i + w))^2}{f'(P_i + w)^2} - \frac{1}{P_i} \geq 0$, negative if $(\ln f)' \leq 0$, in which case $\frac{f'(P_i + w)}{f(P_i + w)} + \frac{1}{P_i}$ is decreasing. Thus, if $f'(1) \geq -f(1)$, then the sufficient condition holds.

Any asymmetric equilibrium in a symmetric game remains an equilibrium after any permutation of player labels, so if an asymmetric equilibrium exists, then there exist at least two. Suppose $P_X^* > P_Y^*$ w.l.o.g. and the firms are symmetric. Strategic
complements then imply that the best response to \(P_Y^*\) is less than the best response to \(P_X^*\), contradicting \(P_X^* > P_Y^*\).

**Proof of Lemma 2** Profit after imposing the equilibrium condition \(P_i = P_i^*\) on both firms is denoted \(\pi_i^*\). Sufficient for uniqueness is that the best responses do not jump up and the slopes of best responses are below 1 at any equilibrium, i.e.,

\[
\left| \frac{\partial^2 \pi_i^*}{\partial P_i \partial P_j} \right| < 1 \text{ for each } i \neq j.
\]

Equivalently, \(\frac{\partial^2 \pi_i^*}{\partial P_i \partial P_j} + \frac{\partial^2 \pi_i^*}{\partial P_i^2} < 0\).

If the slope of a function with no upward jumps is < 1 at any fixed point, then there is a unique fixed point.

Use \(1 \{P_i > P_i^* + s\} = 0\) in (3) and (4) to obtain

\[
\frac{\partial^2 \pi_i^*}{\partial P_i \partial P_j} = \frac{1}{2} \int f(P_i + v_j - P_j - s) + P_i f'(P_i + v_j - P_j - s)\,dF(v_j) + \frac{1}{2} \int f(P_i + v_j - P_j + s)\,dF(v_j) - \frac{1}{2} \int_{\{0\}}^{1} \left[ f(P_i + \max \{0, v_j - P_j - s\}) + P_i f'(P_i + \max \{0, v_j - P_j - s\})\right] dF(v_j),
\]

which equals

\[
-\frac{1}{2} \int_{0}^{1} P_i f'(P_i)\,dF(v_j) - \frac{1}{2} \int_{0}^{1} \left[ f(P_i + \max \{0, v_j - P_j - s\}) + P_i f'(P_i + \max \{0, v_j - P_j - s\})\right] dF(v_j).
\]

This further simplifies to

\[
-\frac{1}{2} \left[ 2 f(P_i) + P_i \frac{\partial f(P_i)}{\partial P_i} \right] F(P_j + s) + \frac{1}{2} \int_{0}^{1} f(P_i + v_j - P_j + s) - f(P_i + v_j - P_j - s)\,dF(v_j).
\]

Sufficient for uniqueness are \(P_i \frac{\partial f(P_i)}{\partial P_i} \geq -2 f(P_i)\) and \(f'(x) \leq 0 \forall x\).

Use \(P_i = P_j\) as in any equilibrium by Lemma 1 and substitute the FOC (3) into (5): \(-\frac{1}{2} \left[ 2 f(P_i) + P_i \frac{\partial f(P_i)}{\partial P_i} \right] F(P_i + s) + \frac{1}{2} \int_{0}^{1} \left[ 1 - F(v_j + s) - [1 - F(v_j - s)]\right] dF(v_j).\)

Sufficient for this to be negative and thus for uniqueness is that \(f\) has no upward jumps and \(P_i \frac{\partial f(P_i)}{\partial P_i} \geq -2 f(P_i),\) which is implied by the condition in Lemma 1. The proof of sufficiency of \(f''(1) \geq -2 f(1)\) and \((\ln f)'' \leq 0\) follows Lemma 1.

**Proof of Theorem 3** The prices are interior, i.e. \(P_i \in (0, 1)\) and satisfy the first order conditions (3). The support of valuations is \([0, 1],\) i.e., \(v_i \notin [0, 1] \Rightarrow f(v_i) = f''(v_i) = 0\). Impose the equilibrium condition \(P_i = P_i^*\) in the FOC (3) and take the derivative:
\[
\frac{\partial FOC_i^*}{\partial s} = \frac{1}{2} \int_{P_j + s}^{P_j + s + \delta} \left[ f(P_i + s + \delta) - f(P_i + s) - f(P_i + v_j - P_j - s) - f(P_i + v_j - P_j - s - \delta) \right] dF(v_j)
\]

The second integral in (6) starts at 0, the first at \(P_j + s\), so if \(f' \leq 0\), then \(\frac{\partial FOC_i^*}{\partial s} < 0\). Sufficient for \(\frac{\partial FOC_i^*}{\partial s} < 0\) is \(f_i \approx 1\) (nearly uniform) or \(f(P_i - s + w) + P_i \frac{\partial f(P_i - s + w)}{\partial P_i} \leq f(P_i + s + w)\) for all \(P_i - s + w \in (0, 1), w \geq 0\). For specific distributions and two firms, (6) can be calculated explicitly: sufficient for \(\frac{\partial FOC_i^*}{\partial s} < 0\) is that \(f\) is uniform, or truncated exponential and \((1 - P_i) \exp(-P_i - 1 + P_j + s) \leq \exp(-P_i - 1 + P_j - s)\).

By the Implicit Function Theorem, \[
\begin{bmatrix}
\frac{dP^*_X}{dP^*_Y} \\
\frac{dP^*_Y}{dP^*_X}
\end{bmatrix}
= -\begin{bmatrix}
\frac{\partial FOC_i^*}{\partial P_i} \\
\frac{\partial FOC_i^*}{\partial P_j}
\end{bmatrix}
^{-1}
\begin{bmatrix}
\frac{\partial FOC_i^*}{\partial s} \\
\frac{\partial FOC_i^*}{\partial s}
\end{bmatrix}.
\]
The matrix is negative semidefinite iff the equilibrium is stable. Therefore, a sufficient condition for \(\frac{dP^*_X}{dP^*_Y} < 0\) \(\forall i\) in any stable equilibrium is \(\frac{\partial FOC_i^*}{\partial s} < 0\) \(\forall i\).

If no consumers learn, then \(s\) does not affect prices, so \(\frac{\partial FOC_i^*}{\partial s} = 0 \leq 0\). The condition \(1 - P_i^* - s > -P_j^*\) ensures that some consumers at \(j\) learn about \(i\), given the equilibrium prices. In a symmetric equilibrium \(P_i^* = P_j^*\), so some consumers at any firm learn if \(s < 1\).

**Proof of Prop. 4** Let \(\rho_k := \frac{dP^*_X}{dP^*_Y}\). Write the equilibrium condition \(FOC_i^*\) (3) of firm \(i\) in terms of a change \(\delta\) in the search cost from some baseline (e.g., initial equilibrium before a change in parameters) and a change \(\rho_k\delta\) in \(P_k\) as

\[
FOC_i^* = \frac{1}{2} \int_{0}^{P_j + \rho_j\delta + s + \delta} \left[ 1 - F(P_i + \rho_i\delta) \right] dF(v_j)
+ \frac{1}{2} \int_{P_j + \rho_j\delta + s + \delta}^{1} \left[ 1 - F(P_i + \rho_i\delta + v_j - P_j - \rho_j\delta - s - \delta) \right] dF(v_j)
+ \frac{1}{2} \int_{0}^{P_j + \rho_j\delta} \left[ 1 - F(P_i + \rho_i\delta + s + \delta) \right] dF(v_j)
+ \frac{1}{2} \int_{P_j + \rho_j\delta}^{1} \left[ 1 - F(P_i + \rho_i\delta + s + \delta + v_j - P_j - \rho_j\delta) \right] dF(v_j)
+ \frac{1}{2} \int_{0}^{P_j + \rho_j\delta + s + \delta} (P_i + \rho_i\delta) f(P_i + \rho_i\delta) dF(v_j)
- \frac{1}{2} \int_{P_j + \rho_j\delta + s + \delta}^{1} (P_i + \rho_i\delta) f(P_i + \rho_i\delta + v_j - P_j - \rho_j\delta - s - \delta) dF(v_j).
\]
The derivative of FOC* w.r.t. $\delta$ is (because the derivatives w.r.t. the bounds of the integrals cancel)

$$-ho_i \frac{1}{2} \int_0^{P_i + \rho_j \delta + s + \delta} f (P_i + \rho_i \delta) dF(v_j)$$

$$- (\rho_i - \rho_j - 1) \frac{1}{2} \int_0^{P_i + \rho_j \delta + s + \delta} f (P_i + \rho_i \delta + v_j - P_j - \rho_j \delta - s - \delta) dF(v_j)$$

$$- (\rho_i + 1) \frac{1}{2} \int_0^{P_i + \rho_j \delta} f (P_i + \rho_i \delta + s + \delta) dF(v_j)$$

$$- (\rho_i - \rho_j + 1) \frac{1}{2} \int_0^{P_i + \rho_j \delta} f (P_i + \rho_i \delta + s + \delta + v_j - P_j - \rho_j \delta) dF(v_j)$$

$$- \rho_i \frac{1}{2} \int_0^{P_i + \rho_j \delta + s + \delta} f (P_i + \rho_i \delta) + (P_i + \rho_i \delta) \frac{\partial f (P_i + \rho_i \delta)}{\partial P_i} dF(v_j)$$

$$- \frac{1}{2} \int_0^{P_i + \rho_j \delta + s + \delta} [\rho_i f (P_i + \rho_i \delta + v_j - P_j - \rho_j \delta - s - \delta)$$

$$+ (\rho_i - \rho_j - 1)(P_i + \rho \delta) \frac{\partial f (P_i + \rho_i \delta + v_j - P_j - \rho_j \delta - s - \delta)}{\partial P_i} dF(v_j).$$

(8)

Equilibrium implies $\frac{dFOC_i^*}{d\delta} = 0$ for both firms because $FOC_i^* = 0$ at any $s$. The first four lines of (7) are $D_i$ and the first four lines of (8) are $\frac{dD_i}{d\delta}$. If $f (P_i) + P_i \frac{\partial f (P_i)}{\partial P_i} \geq 0$ for all $P_i$ and $|\rho_i - \rho_j| < 1$ (e.g., in a symmetric equilibrium), then the last three lines of (8) are positive because $\rho_k = \frac{dP_k}{d\delta} < 0$ by Theorem 3. Then $\frac{dD_i}{d\delta} < 0$ for both firms and the mass of exiting consumers increases in the search cost.

Lemma 1 proved that log concave $f$ and $f'(1) \geq -f(1)$ are sufficient for $f (P_i) + P_i \frac{\partial f (P_i)}{\partial P_i} \geq 0$.

The total surplus $TS$ is

$$TS := \mu_i \int_{v_i - P_i \geq \max \{0, v_j - \max \{P_j^* + s, P_j^*\}\}} (v_i - c_i) dF_i(v_i) dF_j(v_j)$$

$$+ \mu_j \int_{v_j - \max \{P_i, P_j^* + s\} > \max \{0, v_j - P_j^*\}} (v_i - c_i) dF_i(v_i) dF_j(v_j)$$

$$+ \mu_j \int_{v_j - P_j \geq \max \{0, v_i - \max \{P_i^* + s, P_i^*\}\}} (v_j - c_j) dF_j(v_j) dF_i(v_i)$$

$$+ \mu_i \int_{v_i - \max \{P_j, P_j^* + s\} > \max \{0, v_i - P_i^*\}} (v_j - c_j) dF_j(v_j) dF_i(v_i)$$

(9)

If $\frac{dD_i}{d\delta} < 0$ for both firms, then $\frac{dTS}{d\delta} < 0$ because the integration region in $TS$ is the same as in $D_i + D_j$ and on it, the integrated functions $v_i - c_i, v_j - c_j$ are nonnegative.
and do not depend on \( \delta \). Profit is \( P_i D_i \). Because \( \frac{dP_i}{ds} < 0 \) and \( \frac{dD_i}{ds} = \frac{dDi}{d8} < 0 \), profit falls in \( s \).

Consumer surplus \( CS \) just replaces \( c_i \) with \( P_i \) in (9). Because prices fall in the search cost, \( \frac{dCS}{d8} \gtrless 0 \).

**Proof of Proposition 5** Denote the equilibrium price at \( \alpha = 0 \) by \( P_i^0 \). Define \( P_i^{0w} := P_i^0 + \max\left\{ 0, v_j - P_j^0 \right\} \), \( P_i^{0w-s} := P_i^0 + \max\left\{ 0, v_j - P_j^0 - s \right\} \) and \( P_i^{0w+s} := P_i^0 + s + \max\left\{ 0, v_j - P_j^0 \right\} \). At \( P_i^0 \), (3) equals zero, so with \( \alpha \) fraction of shoppers,

\[
\frac{\partial \pi_i(P_i^0, P_j^*, P_j^*)}{\partial P_i^0} = \frac{\alpha}{2} \int_0^1 \left[ 1 - F\left( P_i^{0w} \right) - P_i^0 f\left( P_i^{0w} \right) \right] dF(v_j). \tag{10}
\]

Because \( 1 - F\left( P_i^{0w+s} \right) > 0 \), (3) implies \( \int_0^1 \left[ 1 - F\left( P_i^{0w-s} \right) - P_i^0 f\left( P_i^{0w-s} \right) \right] dF(v_j) > 0 \). If \( f_i(P_i + w) + P_i f_i'(P_i + w) \leq 0 \) for all \( P_i, w \), then \( \int_0^1 \left[ 1 - F\left( P_i^{0w} \right) - P_i^0 f\left( P_i^{0w} \right) \right] dF(v_j) \geq \int_0^1 \left[ 1 - F\left( P_i^{0w-s} \right) - P_i^0 f\left( P_i^{0w-s} \right) \right] dF(v_j) > 0 \) because \( P_i^{0w-s} < P_i^{0w} \). Thus, the FOC (10) is positive at \( P_i^0 \), so the equilibrium price strictly exceeds \( P_i^0 \). \( \square \)

Despite prices rising in \( \alpha \) when \( f_i(P_i + w) + P_i f_i'(P_i + w) \leq 0 \) and at given prices, demand rising, profits may change either way because demand decreases in response to the higher price.

**Proof of Prop. 6** Compared to (1), the mass of consumers initially at each firm is multiplied by \( 1 - \nu \). The mass \( \nu \) of unattached customers with \( v_i - P_i^* > v_j - P_j^* \) learn firm \( i \)'s price if \( v_i - P_i^* \geq s \) and then buy from \( i \) if both \( v_i \geq P_i \) and \( v_i - P_i \geq v_j - \max\left\{ P_j^* + s, P_j^* \right\} \). Demand is

\[
\frac{1 - \nu}{2} \int_0^1 \int_0^1 f(v_i) f(v_j) dv_i dv_j 
+ \frac{1 - \nu}{2} \int_0^1 \int_{\max\left\{ P_i, P_i^* + s \right\}}^{\max\left\{ 0, v_j - P_j^* \right\}} f(v_i) f(v_j) dv_i dv_j 
+ \nu \int_0^1 \int_{\max\left\{ P_i^* + v_j - P_j^*, P_j^* + s, P_i, P_i + v_j - \max\left\{ P_j^* + s, P_j^* \right\} \right\}} f(v_i) f(v_j) dv_i dv_j.
\]

The FOC after imposing the equilibrium condition is

\[
FOC_i^* = \frac{1 - \nu}{2} \int_0^1 \left[ 1 - F\left( P_i + \max\left\{ 0, v_j - P_j - s \right\} \right) \right] dF(v_j) 
+ \frac{1 - \nu}{2} \int_0^1 \left[ 1 - F\left( P_i + s + \max\left\{ 0, v_j - P_j \right\} \right) \right] 
\]
Greater search cost...

\[- P_i f \left( P_i + \max \left\{ 0, v_j - P_j - s \right\} \right) dF(v_j) + v \int_0^1 \left[ 1 - F \left( P_i + \max \left\{ s, v_j - P_j \right\} \right) \right] dF(v_j) = 0.\]

Compared to (3) in which \( v = 0 \), the derivative \( \frac{\partial FOC^s}{\partial s} < 0 \) is larger in magnitude and the matrix \( \left[ \frac{\partial FOC^s}{\partial P_i} \right] \) the same. The Implicit Function Theorem then yields \( \frac{dP_i}{ds} < 0 \) larger in magnitude.

**Proof of Prop. 7** The FOC times \( 1 - p_0 \) is

\[
\begin{align*}
&\int_0^1 \left[ 1 - F \left( \max \left\{ P_i^*, P_i^* + v_j - P_j^* - P_j, P_i, P_i + v_j - P_j^* - s \right\} \right) \right] dF(v_j) \\
&+ p_0 \int_{P_j^*}^1 \max \left\{ 0, F \left( P_i^* + v_j - P_j^* \right) - F \left( \max \left\{ P_i^* + s, P_i \right\} \right) \right\} dF(v_j) \\
&- P_i \int_0^1 \left[ \max \left\{ P_i, P_i + v_j - P_j^* - s \right\} \right] dF(v_j) \\
&- p_0 P_i \int_{P_j^*}^1 \left[ v_j - P_j^* \right] dF(v_j) \left[ v_j - P_j^* \right] dF(v_j) = 0.
\end{align*}
\]

The last line of (11) is zero if \( P_i < P_i^* + s \). The penultimate line is zero if \( v_j > P_j^* \) and \( P_i < P_i^* + s \), so rewrite it as \( -(1 - p_0) P_i \int_{P_j^*}^{P_j^*} \frac{dF}{dF(v_j)} \left( P_i^* \right) dF(v_j) \), which is the only negative term in the FOC and thus averts a corner solution. It shows the reason the market does not break down due to hold-up: the initial consumers with low values for the other firm exit in response to a price rise.

The search cost changes (11) by

\[
\frac{\partial^2 \pi^s}{\partial P_i \partial s} = -(1 - p_0) \int_0^1 \left[ v_j \geq P_j^* + s \wedge P_i - s \geq P_i^* \right] f \left( P_i + v_j - P_j^* - s \right) dF(v_j) \\
- (1 - p_0) p_0 \int_{P_j^* + s}^{P_j^*} f \left( P_i^* + s \right) dF(v_j) \\
- (1 - p_0) P_i \int_{P_j^* + s}^{P_j^* + s} \left[ P_i - s \geq P_i^* \right] f_i^j \left( P_i + v_j - P_j^* - s \right) dF(v_j),
\]

which is negative because the first and last lines are zero near equilibrium \( P_i \approx P_i^* \). Thus, prices always fall in the search cost, strictly if \( P_j^* + s < 1 \).

**Proof of Prop. 8** For firm \( i \) who advertises when firm \( j \) does not, demand is

\[
D_i^o(P_i, P_j^*) = \frac{1}{2} \int_0^1 \int_{P_i + \max \left\{ 0, v_j - P_j^* - s \right\}}^1 dF_i(v_i) dF_j(v_j).
\]
\[ + \frac{1}{2} \int_0^1 \int_{P_i + s + \max\{0, v_j - P_j^*\}} F_i(v_i) dF_j(v_j). \] (12)

Demand \( D_i \) from not advertising only replaces the last \( P_i \) with \( P_i^* \). Denote the profit of firm \( i \) from advertising and committing to price \( P_i \) by \( \pi_i^a(P_i, P_j^*) \). The FOC for a unilaterally advertising firm \( i \) who expects \( j \) to set \( P_j^* \) is

\[
\frac{\partial \pi_i^a(P_i, P_j^*)}{\partial P_i} = \int_0^1 \left[ \frac{1}{2} F \left( P_i + \max\{0, v_j - P_j^* - s\} \right) + \frac{1}{2} F \left( P_i + s + \max\{0, v_j - P_j^*\} \right) - P_i \frac{1}{2} f \left( P_i + \max\{0, v_j - P_j^* - s\} \right) - P_i \frac{1}{2} f \left( P_i + s + \max\{0, v_j - P_j^*\} \right) \right] dF(v_j). \tag{13}
\]

The only change in the FOC compared to unadvertised \( P_i \) is that the last line of (13) is negative instead of zero. Thus, the marginal profit at any \( P_i \), including \( P_i^* \), is lower for a firm deviating to advertising, so the best response to any \( P_j^* \) is lower. If a firm deviates to advertising, then it sets a lower price than without advertising.

Prices are strategic complements, so both firms cut price if one advertises. If the second firm also advertises, then again by (13), prices are even lower.

Advertising \( P_i^* \) (the no-advertising equilibrium price) yields the same profit as not advertising it because consumer decisions and demand (12) are the same when \( P_i^* \) is observed as when it is expected in equilibrium. By (13) with \( i \) and \( j \) switched, the rival who expects \( P_i^* \) to be advertised responds the same way as when it expects \( P_i^* \) to be chosen without advertising. If \( \pi_i^a(P_i^*, P_j^*) = \pi_i(P_i^*, P_j^*, P_i^*) \) and \( \frac{\partial \pi_i^a(P_i, P_j)}{\partial P_i} < \frac{\partial \pi_i(P_i, P_j, P_i^*)}{\partial P_i} \) for any \( P_i, P_j \), then \( \pi_i^a(P_i, P_j^*) > \pi_i(P_i, P_j^*, P_i^*) \) for any \( P_i < P_i^* \). So profit is higher from deviating to advertising a lower price than from equilibrium without ads. Of course, in the new equilibrium the competitor anticipates advertising, so both prices fall.

If both firms advertise, then profits are lower than if both do not because (i) both prices are lower, (ii) a firm’s profit always increases in a rival’s price (first order effect), (iii) a firm cutting its own price from its equilibrium level decreases its profit (second order effect), and (iv) the marginal profit increases in a rival’s price by strategic complementarity.

\[ \square \]

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