Robust Restless Bandits: Tackling Interval Uncertainty with Deep Reinforcement Learning

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Abstract

We introduce Robust Restless Bandits, a challenging generalization of restless multi-arm bandits (RMAB). RMABs have been widely studied for intervention planning with limited resources. However, most works make the unrealistic assumption that the transition dynamics are known perfectly, restricting the applicability of existing methods to real-world scenarios. To make RMABs more useful in settings with uncertain dynamics: (i) We introduce the Robust RMAB problem and develop solutions for a minimax regret objective when transitions are given by interval uncertainties; (ii) We develop a double oracle algorithm for solving Robust RMABs and demonstrate its effectiveness on three experimental domains; (iii) To enable our double oracle approach, we introduce RMABPPO, a novel deep reinforcement learning algorithm for solving RMABs. RMABPPO hinges on learning an auxiliary \( \lambda \)-network that allows each arm’s learning to decouple, greatly reducing sample complexity required for training; (iv) Under minimax regret, the adversary in the double oracle approach is notoriously difficult to implement due to non-stationarity. To address this, we formulate the adversary oracle as a multi-agent reinforcement learning problem and solve it with a multi-agent extension of RMABPPO, which may be of independent interest as the first known algorithm for this setting. Code is available at https://github.com/killian-34/RobustRMAB.

1 Introduction

The restless multi-armed bandit (RMAB) problem models sequential decision making tasks under a limited resource constraint. An RMAB instance consists of a budget \( B \) and a set of \( N \) arms, where an arm corresponds to an action-dependent Markov state-transition process. At every timestep, a planner selects a subset of \( N \) arms and decides what actions to take on each selected arm such that the total cost of actions per timestep is less than a budget \( B \). The arms transition to new states and generate rewards, depending on the action taken on them. The goal is to plan a policy that maximizes the total expected reward.

This problem naturally captures a wide range of real-world problems. For example, in a healthcare intervention problem [26], a health worker (planner) is in charge of the well-being of a set of patients (arms) who have enrolled in a health program, e.g., by ensuring adherence to daily tuberculosis medication. The healthcare worker provides timely interventions to the patients via phone call or in-person visit to ensure that the patients remain in the adhering state. However, phone calls and
in-person visits require dedicated time from the health worker, thereby imposing a budget $B$ on the
total number of patients she can call or visit in-person each day. The problem is challenging because of
three key reasons: (1) the state of each patient representing the extent of adherence evolves over
time; (2) the intervention decisions at each timestep influence the state transitions of all patients and,
consequently, affect future rewards; and (3) different actions incur different costs, and the total cost
of allocation must be at most $B$ at every timestep. The combinatorial nature of the problem makes
it computationally expensive to solve optimally [31]. Prior works have considered specific variants
of the RMAB problem that are applicable to real-world scenarios, including healthcare intervention
planning [23, 26], anti-poaching patrol planning [34], sensor monitoring tasks [11, 17], machine
replacement [35], and many more.

A common assumption across existing literature on RMABs is that the state transition dynamics of all
the arms are known beforehand. Unfortunately, in real-world scenarios, even in the presence of prior
data, there is still often significant uncertainty in transition dynamics. A natural way to encode this
uncertainty is interval uncertainty—rather than assuming that the probability of transitioning from
one state to another is fixed to one value, we assume that it can take any value from a given closed
continuous interval. This additional uncertainty makes the RMAB problem even more challenging
since any solution must be robust to variations in transition probabilities within the given interval.

In the presence of interval uncertainty, we formulate a generalization of RMAB that we call Robust
Restless Bandits. Our goal is to achieve minimax regret—minimize the worst-case regret against
the optimal policy over all possible values of transition probabilities within the given uncertainty
intervals. Our contributions are as follows:

1. We introduce the Robust RMAB problem for interval uncertainty settings and optimize for
minimax regret. We are the first to provide a minimax-regret solution for the restless bandits
problem, while also generalizing to multi-action RMABs.
2. We develop a double oracle algorithm for solving Robust RMABs and demonstrate its effectiveness
on experimental domains, including two inspired by real-world problems.
3. To enable our double oracle approach, we propose RMABPPO, a deep reinforcement learning
(RL) algorithm for solving multi-action RMABs. RMABPPO hinges on learning an auxiliary
“$\lambda$-network” allowing us to decouple each arm’s learning, greatly reducing sample complexity.
4. Under minimax regret, the adversary in the double oracle approach is notoriously difficult to
implement due to non-stationarity. To address this, we formulate the adversary oracle as a multi-
agent reinforcement learning problem and solve it with a multi-agent extension of RMABPPO.

2 Related Work

Restless bandits Whittle [40] introduced RMABs, following which, there is a vast literature on
binary-action RMABs. In the classic setting, a planner selects $k$ out of $N$ state-transitioning arms,
resulting in a two-action decision problem with the goal of maximizing reward. Although the
problem is PSPACE-Hard [31], heuristics based on Lagrangian relaxation of the optimization have
been proposed [40] which turns out to be optimal under the indexability criterion. However, the
Whittle index solution does not extend to budgeted multi-action settings, where there are more than
two actions, each with an associated cost. Glazebrook et al. [12] and Hodge and Glazebrook [16]
have extended index-based solutions to multi-action RMABs for instances with special monotonic
structure. Along similar lines, Killian et al. [20] proposed a method that leverages the convexity of an
approximate Lagrangian version of the multi-action RMAB problem. Another relevant category
of research is taking decisions on weakly coupled Markov decision processes (WCMDP). Hawkins
[15] proposed a Lagrangian relaxation of the general WCMDP problem and proposed an LP for
minimizing the Lagrange bound. Adelman and Mersereau [3] and Gocgun and Ghate [13] provided
less scalable solutions that provide better approximations to WCMDPs. All these papers have
assumed perfect knowledge of the state transition dynamics to compute policies. A few recent works
have studied RMABs in online settings with unknown transition probabilities but make the limiting
assumptions that the planner takes only one non-combinatorial action [8] or that states periodically
reset [18]. None of the above papers consider robust planning under environment uncertainty, which
we address.

Despite the recent successes of reinforcement learning (RL) for solving large-scale games [28, 37],
RL has so far seen little application to RMABs, except for a few recent works that learn Whittle
indices for indexable binary-action RMABs using (i) deep RL [29] and (ii) Q-learning when states are observable [5, 7] or when arms are homogeneous [4]. In contrast, our deep RL approach provides a more general solution to binary and multi-action RMAB domains that performs well regardless of indexability. One recent work does also address learning in the multi-action setting [19], but their approach is based on tabular Q-learning which can be sample inefficient and scale poorly, unlike function-approximation methods like ours.

**Robust planning** The RL literature has a large body of work on robust planning, mainly focused on maximizing the minimum (maximin) reward through robust adversarial RL [33] or multi-agent RL settings [22, 24]. The minimax regret criterion [6] has been offered as a compelling alternative to maximin reward, which often leads to overly conservative policies that buffer against worst-possible realizations. A challenge to computing minimax regret–optimal strategies is that such games often involve very large and sometimes continuous strategy spaces, rendering it impossible to explore the entire strategy space. The double oracle approach [27] has been applied as an effective means by which to optimally explore only a small subset of these large spaces while still guaranteeing optimal performance [30, 10]. Double oracle has been extended to optimize multi-agent RL problems with multiple selfish agents [22]. More recently, Xu et al. [41] offered an algorithm for minimax regret planning for a non-RMAB problem using RL to instantiate the two oracles. However, their approach cannot handle budget-constrained discrete actions, which we have in the RMAB setting. Additionally, we formulate the nature oracle as a multi-agent RL problem, allowing us to handle the joint discrete and continuous action spaces that arise in the minimax regret nature oracle.

Robustness objectives have been considered for bandit applications in the two-action stochastic setting, where each pull of an arm draws a reward sampled from an unknown Bernoulli distribution. Wei and Srivastava [39] address minimax regret of time-varying reward shifts using heuristics to trade off remembering vs. forgetting, and Garivier et al. [9] consider a maximin objective to guide exploration in Monte Carlo Tree Search. However, RMABs are significantly harder than the stochastic settings since, in RMABs, the rewards are dependent on the current state which in turn depends on the actions taken on the arms.

### 3 Problem Statement

We consider the multi-action RMAB setting with \( N \) arms. Each arm \( n \in [N] \) follows a Markov decision process (MDP) \((S_n, A_n, C_n, T_n, R_n, \beta)\), where \( S_n \) is a set of finite, discrete states, \( A_n \) is a set of finite, discrete actions, \( C_n : A_n \rightarrow \mathbb{R} \) corresponds to action costs, \( T_n : S_n \times A_n \times S_n \rightarrow [0, 1] \) gives the probability of transitioning from one state to another given an action, \( R_n : S_n \times A_n \times S_n \rightarrow \mathbb{R} \) is a reward function, and \( \beta \in [0, 1) \) is the discount parameter. The agent must select actions for each arm, each round, such that the sum cost of actions does not exceed a per-round budget \( B \). The aim of multi-action RMABs is to maximize total reward over a fixed number of \( H \) rounds, subject to this budget constraint, generalizing the well-studied binary-action RMAB.

In this work, we extend multi-action RMABs to the robust setting in which the exact transition probabilities are unknown. Instead, the transitions of each arm are determined by a set of parameters \( \mathcal{P}_n \). For a given arm \( n \in [N] \) and parameter \( p_n \in \mathcal{P}_n \), let \( \langle \omega_{n, p_n} \rangle := [\omega_{n, p_n}, \overline{\omega}_{n, p_n}] \) represent the range, i.e., uncertainty over transition probabilities. We ask the following question: how to find a solution to the multi-action RMAB problem that is robust to such uncertainties?

We consider an objective that incorporates our uncertainty over \( \mathcal{P}_n \). Let \( \omega \) be a given realization of the transition probabilities such that \( \omega_{n, p_n} \in \langle \omega_{n, p_n} \rangle \) for all \( n \in [N] \) and \( p_n \in \mathcal{P}_n \). Let \( G(\pi, \omega) \) be the planner’s expected reward under policy \( \pi \) and a realization \( \omega \) of the uncertainty. Regret is defined:

\[
L(\pi, \omega) = G(\pi^*_\omega, \omega) - G(\pi, \omega),
\]

where \( \pi^*_\omega \) is the optimal policy under \( \omega \). In our robust planning formulation, our objective is to compute a policy \( \pi \) that minimizes the maximum regret \( L \) possible for any realization of \( \omega \), leading to the following minimax objective:

\[
\min_{\pi} \max_{\omega} L(\pi, \omega).
\]

This problem is computationally expensive to solve since simply computing a policy \( \pi \) that maximizes the expected reward \( G(\pi, \omega) \) is PSPACE-Hard [31] even for the special case when the transition rules
are known i.e., \( \omega \) is given. This challenge is likely a key reason why the robust formulation has not yet been addressed in the literature. To overcome the complexity of the minimax optimization, we take a double oracle approach \([27]\), which requires key innovations to work in the RMAB setting.

4 Preliminaries

The double oracle approach achieves the minimax regret objective in Eq.\([2]\) by casting the optimization problem as a zero-sum game between two players, the agent and nature \([41]\). The agent selects a policy that minimizes regret for some realization of the transition probabilities. Nature then adversarially selects the values of \( P_n \) that maximize regret for a given policy of the planner. This framework is desirable since it converges to an \( \varepsilon \)-optimal solution \([2, 41]\), assuming there are oracles that return the best response for both players. The key technical contributions of this paper arise from the design of the agent and nature oracles for best response computations.

For the agent, minimizing regret with respect to a fixed nature strategy is equivalent to maximizing reward w.r.t. that strategy, so the agent objective is the same as solving a multi-action RMAB to find the best policy. A policy \( \pi \) maps states to decision matrices \( A \in \{0, 1\}^{N \times |A|} \) where the total number of active actions is constrained by a budget for each round. Let \( s = (s_1, \ldots, s_N) \) represent the initial state of each arm. Then, for a given parameter \( \omega \), the optimal policy \( \pi_\omega^* \) maximizes the expected discounted sum of rewards of all arms as given by the constrained Bellman equation:

\[
J(s) = \max_A \left\{ \sum_{n=1}^N R_n(s_n) + \beta \mathbb{E}_\omega[J(s') \mid s, A] \right\}
\]

\[
\text{s.t. } \sum_{j=1}^{|A|} a_{nj} = 1 \quad \forall n \in [N] \quad \sum_{n=1}^N \sum_{j=1}^{|A|} a_{nj} c_{nj} \leq B
\]

where \( a_{nj} \in A_n \) and \( c_{nj} \) are the corresponding action costs in \( C_n \). We then relax the problem by taking the Lagrangian relaxation of the budget constraint \([15]\), giving:

\[
J(s, \lambda^*) = \min_\lambda \left( \frac{\lambda B}{1 - \beta} + \sum_{n=1}^N \max_{a_{nj} \in A_n} Q_n(s_n, a_{nj}, \lambda) \right)
\]

where \( Q_n(s_n, a_{nj}, \lambda) = R_n(s_n) - \lambda c_{nj} + \beta \mathbb{E}_\omega \left[ Q_n(s', a_{nj}, \lambda) \mid \pi^L_\omega(\lambda) \right] \).

Here, \( Q \) is the value function and \( \pi^L_\omega(\lambda) \) is the optimal policy for a given \( \lambda \). See Adelman and Mersereau \([3]\) for a detailed derivation. Note that for a given value of \( \lambda \), Eq.\([3]\) could be solved using \( N \) individual value iterations. However, setting \( \lambda := \lambda^* \) is critical to finding good policies for a multi-action RMAB and is known to be asymptotically optimal in the binary-action case \([38]\), i.e., \( \pi^L_\omega(\lambda^*) \rightarrow \pi_\omega^* \). Given this relationship, in the remainder of the paper, we focus of computing \( \pi^L_\omega(\lambda^*) \) and denote it as \( \pi_\omega^* \) for convenience.

5 Solving Robust Restless Bandits

We now build up our approach for finding robust RMAB policies. The underlying pure strategy space for the agent is the set of all feasible policies \( \pi : \mathcal{S}_1 \times \cdots \times \mathcal{S}_N \mapsto A_1 \times \cdots \times A_N \). The pure strategy space for nature is a closed set of parameters \( \omega \) within the given uncertainty intervals.

The agent oracle’s goal is to find a policy \( \pi \), or pure strategy, to minimize regret (Eq.\([1]\)) given a mixed strategy \( \bar{\omega} \), where a mixed strategy is a probability distribution over a set of pure strategies. That is, the agent minimizes \( L(\pi, \bar{\omega}) \) w.r.t. \( \pi \), while \( \bar{\omega} \) is constant. Recall that \( L(\pi, \bar{\omega}) = G(\pi_\omega^*, \bar{\omega}) - G(\pi, \bar{\omega}) \). Since the first term \( G(\pi_\omega^*, \bar{\omega}) \) is constant, minimizing \( L(\pi, \bar{\omega}) \) is equivalent to maximizing the second term \( G(\pi, \bar{\omega}) \), which is maximal at \( \pi = \pi_\omega^* \). In other words, the agent oracle must compute an optimal reward-maximizing policy w.r.t. \( \bar{\omega} \). The objective of maximizing reward is standard in reinforcement learning, enabling us to build off existing RL techniques as we show in Section 5.1.

On the other hand, the nature oracle’s goal is to find a parameter setting \( \omega \), or pure strategy, that maximizes the agent’s regret given a mixed strategy \( \pi \), i.e., maximize \( L(\pi, \omega) \) with respect to \( \omega \), while \( \pi \) is fixed. This objective is far more challenging because both \( G(\pi_\omega^*, \omega) \) and \( G(\pi, \omega) \) are
functions of $\omega$. Moreover, computing $G(\hat{\pi}_n^*, \omega)$ requires obtaining an optimal policy $\hat{\pi}_n^*$ as $\omega$ changes in the optimization. Unfortunately, $\omega$ comes from an infinite continuous strategy space, making this problem difficult. However, as one of our main contributions, we propose a novel method for implementing the regret-maximizing nature oracle by casting it as a multi-agent reinforcement learning problem, simultaneously solving for policies $\pi_n^*$ while computing worst-case parameters $\omega$ to maximize $L(\pi, \omega)$. Solving a multi-agent reinforcement learning problem first requires an RL algorithm to optimize the underlying policy; hence we first introduce a novel RL approach, RMABPPO, to solve RMABs (Sec. 5.1) as a part of our agent oracle and then use the algorithm as the backbone of our nature oracle (Sec. 5.2).

5.1 Agent Oracle: Deep RL for RMABs via RMABPPO

Existing deep RL approaches can be applied to the objective in Eq. 3 but they fail to scale past trivially sized RMAB problems since the action and state spaces grow exponentially in $N$. For example, for a binary-action RMAB with $N = 50$ and $B = 20$, the action space would be of size ${50 \choose 20} \approx 10^{12}$, which is not feasible to learn, even with a neural network. To overcome this, we develop a novel deep RL algorithm that instead solves the decoupled problem (Eq. 4). The key benefit of decoupling is to render policies and $Q$ values of each arm independent, allowing us to learn $N$ independent networks with linearly sized state and action spaces, relieving the combinatorial burden of the learning problem — the above example would now only have $N \times 2 = 100$ actions. However, this approach introduces a new technical challenge in solving the dual objective which maximizes over policies but minimizes over $\lambda$, as discussed in Sec. 4.

We derive a dual gradient update procedure that iteratively optimizes each objective as follows: (1) holding $\lambda$ constant, learn $N$ independent policies via a policy gradient procedure, augmenting the state space to include $\lambda$ as input, as in Eq. 4; (2) use sampled trajectories from those learned policies as an estimate to update $\lambda$ towards its minimizing value via a novel gradient update rule. Another challenge is that $\lambda^*$ of Eq. 4 depends on the current state of each arm — therefore, a key element of our approach is to learn this function concurrently with our iterative optimization, using a neural network we call the $\lambda$-network that is parameterized by $\Lambda$. To train the $\lambda$-network, we use the following gradient update rule.

**Proposition 1.** A gradient rule for updating the $\lambda$-network, parameterized by $\Lambda$, such that for a state $s$, the $\lambda$-network predicts the value $\lambda$ that minimizes Eq. 4 is as follows:

$$\lambda_t = \lambda_{t-1} - \alpha \left( \frac{B}{1 - \beta} + \sum_{n=1}^{N} D_n(s_n, \lambda_{t-1}(s)) \right)$$

(6)

where $\alpha$ is the learning rate and $D_n(s_n, \lambda)$ is the negative of the expected $\beta$-discounted sum of action costs for arm $n$ starting at state $s_n$ under the optimal policy for arm $n$ for a given value of $\lambda$.

Although $D_n$ cannot be computed exactly as we do not know the optimal policy, it can be estimated from samples of multiple rollouts of the policy during training. As long as arm policies are trained for adequate time on the given value of $\lambda$, the gradient estimate will be accurate, i.e., $D_n(s_n, \lambda_{t-1}(s)) \approx -\sum_{k=1}^{K} \beta^k c_{nk}$ where $K$ is the number of samples collected in an epoch and $c_{nk}$ is the action cost of arm $n$ in round $k$. Moreover, this procedure will converge to the optimal parameters $\Lambda$ if the arm policies are optimal.

**Proposition 2.** Given arm policies corresponding to optimal $Q$-functions, the gradient update rule of Prop. 1 will lead $\lambda$ to converge to the optimal as the number of training epochs and $K \to \infty$.

Note that to collect samples that reflect the proper gradient, the RMAB budget must not be imposed at training time — rather the policy networks and $\lambda$-network must learn to play the Lagrange policy of Eq. 4 which spends the correct budget in expectation. At training time, actions are sampled randomly according to the actor network distributions. At test time, actions are taken deterministically by greedily selecting the highest probability actions until the budget is spent. In theory, the policy networks could be trained via any deep RL procedure, as long as the above characteristics for training the $\lambda$-network are ensured. In practice, we train with proximal policy optimization (PPO) [36], which has been demonstrated to have state-of-the-art performance while being relatively simple to implement, and will allow flexibility to handle both discrete and continuous actions—the latter will be important for the nature oracle. We also navigate the important trade-off between exploring new
We use RMABPPO to instantiate the agent oracle and MA-RMABPPO for the nature oracle. The \( \epsilon \)-double oracle problem is defined as follows: given two agents \( A \) and \( B \) with \( A \)’s action set \( \mathcal{A}_A \) and \( B \)’s action set \( \mathcal{A}_B \), the goal is to learn policies \( \tilde{\pi}_A \) and \( \tilde{\pi}_B \) that maximize the expected reward for each agent while\( B \) selects actions following \( \tilde{\pi}_B \) and \( A \) observes \( B \)’s response \( \tilde{\omega}_B \). We now have all the pieces we need to present our robust algorithm RMAB Double Oracle (RMABDO), with pseudocode presented in Algorithm 2, adapted from the MIRROR framework [41].

This objective is challenging because it requires computing and optimizing over the returns of the \( \tilde{\pi}_A \) and \( \tilde{\pi}_B \) strategies. We solve for a mixed Nash equilibrium in the regret game between the agent and nature to learn a mixed strategy \( \tilde{\pi}_A \) and \( \tilde{\omega}_B \). The shared environment transition function is \( T: \mathcal{S} \times \mathcal{A}_A \times \mathcal{A}_B \rightarrow \mathcal{S} \). The action space \( \mathcal{A}_A \) will be the same as the action space of multi-action RMABs. At a given state \( s \), the action space \( \mathcal{A}_B \) will allow agent \( B \) to select \( \omega \) which, in general, depends on \( s \). That is, at each step, agent \( B \) will select environment parameters \( \omega \), and thus state/action transition probabilities that will determine the outcome of agent \( A \)’s actions. We adopt the centralized critic idea from multi-agent PPO [43] to our RMAB setting to create MA-RMABPPO. Since the policy space of agent \( A \) is discrete while that of agent \( B \) is continuous, a notable benefit to PPO is that it offers a convenient way to train both policies.

The final step is to define the rewards for each agent \( A \) and \( B \) to match their respective objectives. Since agent \( A \)’s objective is to maximize \( \pi_{\omega}^\ast \), it simply adopts the reward defined by the RMAB, i.e., \( R_{A}^{(A)} = \sum_{n=1}^{N} R_n \). However, agent \( B \)’s objective is to learn the regret-maximizing parameters \( \omega \). This objective is challenging because it requires computing and optimizing over the returns of the fixed input policy \( \tilde{\pi} \) with respect to all possible \( \omega \) values, which in general is non-convex. In practice, to estimate the returns of \( \tilde{\pi} \), we execute a series of single-step roll-outs for computational efficiency. That is, given \( s \) and \( a \) at a given round, we sample the next state profile \( s' \) and define the reward of agent \( B \) (i.e., the regret of input policy \( \tilde{\pi} \)) as \( R_{B}^{(B)} = \sum_{n=1}^{N} R_n(s_n, a_n, s'_n) - \frac{1}{Y} \sum_{y=1}^{Y} r_{y}^{\tilde{\pi}} \), where \( r_{y}^{\tilde{\pi}} \) is the reward obtained from each of \( Y \) 1-step Monte Carlo simulations of the mixed strategy \( \tilde{\pi} \).

Since agent \( A \) has the same policy network architecture as RMABPPO, i.e., \( N \) discrete policy networks and one \( \lambda \)-network. The agent \( B \)’s network is a single continuous-action policy network. Since agent \( A \) and \( B \) have separate objectives, they each have their own centralized critic networks which take the actions of the other as inputs to their state space. For agent \( A \), this is implemented as one critic network per arm, where each network is augmented with agent \( B \)’s actions. Finally, to ensure good gradient estimates for the \( \lambda \)-network in MA-RMABPPO, we keep agent \( B \)’s network — and thus the environment — constant between \( \lambda \) updates, updating \( B \)’s network at the same frequency as the \( \lambda \)-network updates. Pseudocode for MA-RMABPPO and further details of its implementation are given in the appendix.

5.3 The Minimax Regret–Robust RMAB Double Oracle

We now have all the pieces we need to present our robust algorithm RMAB Double Oracle (RMABDO), with pseudocode presented in Algorithm 2 adapted from the MIRROR framework [41]. We use RMABPPO to instantiate the agent oracle and MA-RMABPPO for the nature oracle. The double oracle approach proceeds as follows. The agent maintains strategy set \( \Pi \), initialized empty, and nature maintains strategy set \( \Omega \), initialized with an arbitrary parameter setting. In each iteration, we solve for a mixed Nash equilibrium in the regret game between the agent and nature to learn a mixed strategy \( (\tilde{\pi}, \tilde{\omega}) \) for each player. We then call the agent and nature oracles to each compute a best response \( \pi \) and \( \omega \) to their opponent’s strategy, which get added to their respective strategy sets \( \Pi \) and \( \Omega \). We repeat this process until the improvement in value for each player is within the tolerance \( \varepsilon \), which is guaranteed to converge within finite steps to within \( \epsilon \) value of the minimax regret–optimal policy (Theorem 2 of [41]), or until a set number of iterations. Additionally, we show that a policy that maximizes reward assuming a fixed parameter set can incur arbitrarily large regret when the parameters are changed (proof in appendix). Formally,
We compare our algorithm against five baselines. Namely, we compare against three variations of the Algorithm 2: RMABPPO:

1. Randomly initialize a policy network $\pi_{\theta_n}$ for each arm $n \in [N]
2. Randomly initialize $\lambda$-network LAMBDA
3. Initialize an empty buffer
4. for epoch = 1, 2, ..., n_epochs do
5. Sample $\lambda = \text{LAMBDA}(s)$
6. for subepoch = 1, ..., n_subepochs do
7. for timestep $t = 1, ..., n_{\text{simulation steps}}$ do
8. Sample action $a_n = \pi_{\theta_n}(s_n, \lambda)$ for all $n \in [N]
9. Add trajectories $(s, a, r, s', \lambda)$ to buffer
10. Update policy network $\pi_{\theta_n}$ of all arms via PPO, using trajectories in buffer
11. Update LAMBDA using discounted sum of trajectories of final subepoch
12. return $\pi_{\theta_1}, ..., \pi_{\theta_N}$ and LAMBDA

Algorithm 2: RMABDO

1. Input: Environment simulator and parameter uncertainty interval $\langle \omega_n, p_n \rangle$ for all $n \in [N]
2. Parameters: Convergence threshold $\varepsilon$
3. Output: Best agent mixed strategy $\tilde{\pi}$

We compare our algorithm against five baselines. Namely, we compare against three variations of the approach from Hawkins [15], which computes a reward-maximizing Lagrange policy for each step of a multi-action RMAB problem for fixed transition probabilities. The three variations are, pessimistic (HP), mean (HM), and optimistic (HO), which assume the transition probabilities have been set to lower bound, mean, and upper bound of the intervals, respectively. We also compare against RLvMId, which learns a policy via RMABPPO assuming mean parameter values, and Rand, which acts randomly each round. All results are averaged over 50 random seeds and were executed on a cluster running CentOS with Intel(R) Xeon(R) CPU E5-2683 v4 @ 2.1 GHz with 8GB of RAM using Python 3.7.10. Our RMABPPO implementation builds on OpenAI Spinning Up [1] and RMABDO builds on the MIRROR implementation [41], computing Nash equilibria using Nashpy 0.0.21 [21]. Code is available on github [4]. Hyperparameter settings are included in the appendix. We evaluate all the approaches on three experimental domains, which are as follows.

**Synthetic domain:** We create this setup to show that the reward-maximizing Lagrange policies (HP, HM, and HO) may incur large regret. There are three arm types $\{U, V, W\}$, each with two actions, $C = \{0, 1\}$, two states $S = \{0, 1\}$, $R(s) = s$, and the following transition probabilities, with rows and

$\begin{align*}
&\text{Transition to lower bound, mean, and upper bound of the intervals, respectively.}
&\text{We also compare against RMABDO assuming mean parameter values, and Rand,}
&\text{which acts randomly each round. All results are averaged over 50 random seeds}
&\text{and were executed on a cluster running CentOS with Intel(R) Xeon(R) CPU}
&\text{E5-2683 v4 @ 2.1 GHz with 8GB of RAM using Python 3.7.10. Our RMABPPO}
&\text{implementation builds on OpenAI Spinning Up [1] and RMABDO builds on the}
&\text{MIRROR implementation [41], computing Nash equilibria using Nashpy 0.0.21 [21].}
&\text{Code is available on github [4]. Hyperparameter settings are included in the appendix.}
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$\begin{align*}
&\text{for action } a_n \text{ and state } s_n \text{ of all arms via PPO, using trajectories in buffer}
&\text{Update LAMBDA using discounted sum of trajectories of final subepoch}
&\text{return } \pi_{\theta_1}, ..., \pi_{\theta_N} \text{ and LAMBDA}

Proposition 3. In the RMAB problem with interval uncertainty, the max regret of a reward-maximizing policy can be arbitrarily large compared to a minimax regret-optimal policy.

6 Experimental Evaluation

We compare our algorithm against five baselines. Namely, we compare against three variations of the approach from Hawkins [15], which computes a reward-maximizing Lagrange policy for each step of a multi-action RMAB problem for fixed transition probabilities. The three variations are, pessimistic (HP), mean (HM), and optimistic (HO), which assume the transition probabilities have been set to lower bound, mean, and upper bound of the intervals, respectively. We also compare against RLvMId, which learns a policy via RMABPPO assuming mean parameter values, and Rand, which acts randomly each round. All results are averaged over 50 random seeds and were executed on a cluster running CentOS with Intel(R) Xeon(R) CPU E5-2683 v4 @ 2.1 GHz with 8GB of RAM using Python 3.7.10. Our RMABPPO implementation builds on OpenAI Spinning Up [1] and RMABDO builds on the MIRROR implementation [41], computing Nash equilibria using Nashpy 0.0.21 [21]. Code is available on github [4]. Hyperparameter settings are included in the appendix. We evaluate all the approaches on three experimental domains, which are as follows.

**Synthetic domain:** We create this setup to show that the reward-maximizing Lagrange policies (HP, HM, and HO) may incur large regret. There are three arm types $\{U, V, W\}$, each with two actions, $C = \{0, 1\}$, two states $S = \{0, 1\}$, $R(s) = s$, and the following transition probabilities, with rows and

$\begin{align*}
&\text{for action } a_n \text{ and state } s_n \text{ of all arms via PPO, using trajectories in buffer}
&\text{Update LAMBDA using discounted sum of trajectories of final subepoch}
&\text{return } \pi_{\theta_1}, ..., \pi_{\theta_N} \text{ and LAMBDA}

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6 Experimental Evaluation

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**Synthetic domain:** We create this setup to show that the reward-maximizing Lagrange policies (HP, HM, and HO) may incur large regret. There are three arm types $\{U, V, W\}$, each with two actions, $C = \{0, 1\}$, two states $S = \{0, 1\}$, $R(s) = s$, and the following transition probabilities, with rows and
The regret of an agent’s strategy against a nature pure strategy \( \omega \) is computed. To evaluate the algorithms for robustness, we compute the regret of an agent’s strategy against a nature pure strategy \( \omega \).

**ARMMAN**: The maternal healthcare intervention problem \([5]\) is modeled as a binary-action RMAB that selects a subset of beneficiaries each week to intervene on with tailored maternal health messaging to encourage engagement. The behavior of each enrolled woman is modeled by an MDP with three states: Self-motivated, Persuadable, and Lost Cause. We use the summary statistics mentioned in their paper and assume uncertainty intervals of 0.5 centered around the transition parameters, resulting in 6 uncertain parameters per arm (details in appendix). Similar to the setup in \([5]\), we assume 1:1:3 split of arms with high, medium, and low probability of increasing their engagement upon intervention. In our experiments, we scale the value of \( N \) in multiples of 5 to keep the same split of arm categories of 1:1:3.

**SIS Epidemic Model**: A discrete-state model in which arms represent distinct geographic regions and each member of an arm’s population of size \( N_p \) is tracked to measure whether they are already infected by a disease or are susceptible. For a particular region, the fraction of susceptible members represents the state of the region. The model is defined by parameters \( \lambda_c \), the average number of contacts per timestep, and \( r_{\text{infect}} \), the probability of infection given contact with an infectious person. Details on computing discrete state transition probabilities are derived from Yaesoubi and Cohen \([42]\) and given in the appendix. We augment the model to include three actions \( \{a_0, a_1, a_2\} \) with costs \( c = \{0, 1, 2\} \). Action \( a_0 \) represents no action, \( a_1 \) divides the contacts per day \( \lambda_c \) by \( a_{1}^{\text{eff}} \), e.g., by communicating messaging about physical distancing, and \( a_2 \) divides the probability of infection given contact \( r_{\text{infect}} \) by \( a_{2}^{\text{eff}} \), e.g., by distributing face masks. We impose the following uncertainty intervals: \( \lambda_c \in [1, 10] , r_{\text{infect}} \in [0.5, 0.99] , a_{1}^{\text{eff}} \in [1, 10] , \) and \( a_{2}^{\text{eff}} \in [1, 10] .\)

### 6.1 Robust Double Oracle

To evaluate the algorithms for robustness, we compute the regret of an agent’s strategy against a nature pure strategy \( \omega \) as the difference between the average reward obtained by the agent’s strategy...
on $\omega$ and the average reward of the optimal strategy against $\omega$. The average reward is computed as the discounted sum of rewards over all arms for a horizon of length 10, averaged over 25 simulations. In each setting, double oracle runs for 6 iterations, using 100 rollout steps and 100 training epochs for each oracle. After completion, each baseline strategy is evaluated, querying the nature oracle for the best response against that strategy. We report the max regret against all nature’s pure strategies.

Figure 1 shows that our double oracle method has the lowest regret-per-arm and beats all the baselines across all experimental settings. The top row shows the results on the synthetic domain, demonstrating that our approach can reduce regret by about 50% against existing benchmarks, across various values of $N$ and $B$. This is expected because the domain is designed to ensure large regret for HP, HM, and HO baselines. Here, a benefit of 50% in the regret corresponds to, in the worst case, keeping $1/3$ of the arms in the good state for an extra round compared to the baseline. Similarly, for the ARMAMAN domain, our algorithm performs consistently better than the baselines, achieving regret that is around 50% lower than the best baselines. The third domain (SIS) is interesting since the state space is large (number of states was restricted to 2 and 3 in the previous domains). Similar to the earlier results, in the SIS model with populations size of 50, our algorithm outperforms in terms of regret. We discuss the runtime of Figure 1 in Section 6.2. Additional results included in the appendix.

### 6.2 Agent Oracle

We evaluate the performance of RMABPPO, our novel RL approach to find a reward-maximizing policy for multi-action RMABs, against an upper bound, namely, the solution by Hawkins, given exact transition probabilities. This will create an upper bound because, given access to the exact transition dynamics, Hawkins will compute the exact Lagrange policy, whereas RMABPPO must learn to approximate the Lagrange policy from samples. Each setting instantiates the environment with a random sample of valid parameter settings for each seed. Figure 2 shows that the reward accumulated by our agent oracle is comparable to the Hawkins algorithm. In the synthetic domain (top row), the policy learns to act on the 33% of arms who belong to category $W$. The mean reward of RMABPPO almost matches that of Hawkins algorithm as $N$ scales with a commensurate budget (Fig. 2 left). As we fix $N$ and vary the budget (Fig. 2 right), the optimal policy accumulates more reward, and RMABPPO is almost equal to the optimal. We observe similar results on the ARMAMAN domain (middle row), where it is optimal to act on 20% of arms (that forms category $A$; details in appendix). Additionally, on the third large-scale domain (SIS, bottom left), we show the strong performance of RMABPPO holds in a multi-action setting even as we increase the number of states from 50 to 500 to test the scalability. Moreover, computationally, RMABPPO beats Hawkins: in the bottom-right of Fig. 2, a single rollout (10 time steps) of the Hawkins policy takes around 100 seconds when there are 500 states (population size), and steeply increases with more states. This demonstrates that it would be prohibitive to run Hawkins within the loop of the double oracle, since the agent policy needs to be evaluated thousands of times to compute the regret matrices—for merely 25 simulations, computation would take around 42 minutes to evaluate a single cell in the regret matrix over all pure strategy combinations, where the matrix has size $|\Pi| \times |\Omega|$.
**Limitations** We believe these advancements have the potential to improve resource allocation in low-resource settings, but acknowledge they are not without tradeoffs. For example, the baseline methods we compare against, while less robust, can provide interpretable ‘index’ policies that capture the value for acting on an arm, whereas our solution’s output can be difficult for a domain expert to interpret. Further, such optimization tools have the risk of amplifying underlying biases in the data and translating that to unfair resource allocation. However, by addressing the robust version of the problem, we directly address this concern by providing a flexible tool for mitigating biases, by allowing users to tune their uncertainties and thus, providing natural ways to develop good policies even when data availability is skewed.

7 Conclusion

In this paper, we address a blocking limitation that inhibits restless bandits to be used for many real-world planning problems: we often lack perfect knowledge of the environment dynamics. To plan effective and risk-averse policies, it is therefore essential to take a robust approach to account for this uncertainty. Our approach enables us to learn minimax regret–optimal policies by providing RMABPPO, a novel RL algorithm to decouple the budget-constrained problem by training an auxiliary \( \lambda \)-network, and MA-RMABPPO, a new approach to instantiating the nature oracle of an adversarial double oracle game setup by using multi-agent RL to handle nonstationary in nature’s challenging optimization problem. Notably, RMABPPO can also extend to solving RMAB problems with continuous states and/or actions, a setting which has not previously been addressed in the literature. We hope these contributions bring us closer to deploying restless bandits in the real world.

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A Proofs

A.1 Proof of Proposition 1

Proposition 1. A gradient rule for updating the $\lambda$-network, parameterized by $\Lambda$, such that for a state $s$, the $\lambda$-network predicts the value $\lambda$ that minimizes Eq. 4 is as follows:

$$\Lambda_t = \Lambda_{t-1} - \alpha \left( \frac{B}{1-\beta} + \sum_{n=1}^{N} D_n(s_n, \lambda_{t-1}(s)) \right) \quad (6)$$

where $\alpha$ is the learning rate and $D_n(s_n, \lambda)$ is the negative of the expected $\beta$-discounted sum of action costs for arm $n$ starting at state $s_n$ under the optimal policy for arm $n$ for a given value of $\lambda$.

Proof. This follows from taking the gradient of Eq. 4 with respect to $\lambda$. To compute the gradient of $Q_n$, we simply rollout $Q_n$ over time then take the derivative, yielding $\frac{dQ_n}{d\lambda} = \sum_{t=0}^{T} \beta^t c_n(t)$, i.e., the negative of the expected discounted sum of action costs under the optimal policy.

A.2 Proof of Proposition 2

Proposition 2. Given arm policies corresponding to optimal $Q$-functions, the gradient update rule of Prop. 1 will lead $\Lambda$ to converge to the optimal as the number of training epochs and $K \to \infty$.

Proof. This follows from the convexity of Eq. 4 with respect to $\lambda$. The convexity of Eq. 4 can be seen from the definition of $Q_n$, i.e., the max over piece-wise linear functions of $\lambda$ is convex.

A.3 Proof of Proposition 3

Proposition 3. In the RMAB problem with interval uncertainty, the max regret of a reward-maximizing policy can be arbitrarily large compared to a minimax regret–optimal policy.

Proof. Consider a binary-action RMAB problem with two arms A and B. Let the reward from each arm be $R$ when the arm is in a good state and 0 in a bad state. Our problem is to plan the best action with a budget of 1 and horizon of 1. Supposing the initial state is bad for each arm, the transition probabilities for the transition matrix for each arm $n$ is

$$\begin{pmatrix} 1 & 0 \\ 1-p_n & p_n \end{pmatrix}$$

where the uncertain variable $p_n$ is constrained to be within $p_A, p_B \in [0, 1]$. Each value in the matrix corresponds to the probability of an arm at state bad transitioning to bad (column 1) or good (column 2) if we take the passive (row 1) or active action (row 2).

To compute a reward-maximizing policy that does not consider robustness to uncertainty, we must optimize for one instantiation of the uncertainty set, which requires making one of three assumptions.

- **Case 1:** If we assume $p_A = p_B$, then an optimal policy is to act with probability $a_A$ on arm A and $a_B$ on arm B as long as $a_A + a_B = 1$. W.l.o.g., suppose $a_A \geq a_B$; then nature would set $p_A = 0$ and $p_B = 1$, imposing regret at least $R/2$.
- **Case 2:** If $p_A > p_B$, then the optimal policy would be to always act on arm A with probability $a_A = 1$ and never act on B ($a_B = 0$). Nature would then set $p_A = 0$ and $p_B = 1$ to impose regret $R$.
We provide the pseudocode for MA-RMABPPO in Alg. 3, which is used to implement the nature oracle. A slight modification to this problem renders Case 1 non-optimal. Let the reward be $R$. When we are in a good state and $R < 1$, the optimal policy only under Case 1. Following Case 2 or 3, the difference between our regret and the minimax regret is $R / 2$, which grows arbitrarily higher as $R \to \infty$.

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**MA-RMABPPO: Nature Oracle Algorithm**

We provide the pseudocode for MA-RMABPPO in Alg. 3, which is used to implement the nature oracle.

**Experimental Domain Details**

**ARMMAN**

The MDPs in the ARMMAN domain have three ordered states representing the level of engagement of the beneficiaries in the previous week. Rewards are better for lower states, i.e., $R(0) = 1, R(1) = 0.5, R(2) = 0$. At each step, the beneficiary may only change by one level, e.g., low-to-medium or high-to-medium but not low-to-high. They also assume that beneficiaries follow one of three typical patterns, A, B, and C, resulting in three MDPs with different transition probabilities. There are two patterns of effects present that differentiate the beneficiary types. (1) For each of the above types, the planner can only make a difference when the patient is in state 1. Type A responds very positively to interventions, but regresses to low reward states in absence. Type B has

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**Algorithm 3: MA-RMABPPO**

1. **Input**: Agent mixed strategy $\tilde{\pi}$
2. **Parameters**: $n_{epochs}, n_{subepochs}, n_{simsteps}, n_{sims}$
3. Initialize agent A: arm policies $\pi_n^{(A)} \forall n \in [N]$, $\lambda$-network LAMBDA, arm critic networks $\phi_n \forall n \in [N]$
4. Initialize agent B: nature parameter policy $\pi^{(B)}$, critic network $\phi^{(B)}$
5. Initialize empty buffer
6. Sample $s$ at random
7. for epoch = 1, ..., $n_{epochs}$ do
8. for subepoch = 1, ..., $n_{subepochs}$ do
9. Sample agent A action $a_n^{(A)} \sim \pi_n^{(A)}(s_n, \lambda)$ for each $n \in [N]$
10. Sample agent B action $\omega^{(B)} \sim \pi^{(B)}$ ($\pi^{(B)}$ may take $s$ as input)
11. $r^{(A)}, s' = \text{SIMULATE}(s, \omega^{(A)}, \omega^{(B)})$
12. $\tilde{r} = \text{SIMULATE}(s, \tilde{\pi}(s), \omega^{(B)}, n_{sims})$ (mean of 1-step rollouts of $\tilde{\pi}$)
13. $r^{(B)} = r^{(B)} - \tilde{r}$ (agent regret)
14. Store $(s, a^{(A)}), \omega^{(B)}, r^{(A)}, r^{(B)}, s')$ in buffer
15. for $t = 1, \ldots, n_{simsteps}$ do
16. Update $\pi_n^{(A)}, \phi_n^{(A)}$ for all $n$ using trajectories in buffer. $\pi_n^{(A)}$ gets $\omega^{(B)}$ as part of state
17. Update LAMBDA via discounted trajectories in buffer
18. Update $\pi^{(B)}, \phi^{(B)}$ from trajectories in buffer. $\phi^{(B)}$ gets $a^{(A)}$ as part of state
19. return $\pi^{(B)}$

**Case 3**: If $p_A < p_B$, the case is symmetric to Case 2 and result in regret $R$. Clearly, max regret is minimized when our action is such that $a_A + a_B = 1$; in this setting, we learn this optimal policy only under Case 1. Following Case 2 or 3, the difference between our regret and the minimax regret is $R / 2$, which grows arbitrarily higher as $R \to \infty$.

B **MA-RMABPPO: Nature Oracle Algorithm**

We provide the pseudocode for MA-RMABPPO in Alg. 3, which is used to implement the nature oracle.

C **Experimental Domain Details**

C.1 **ARMMAN**

The MDPs in the ARMMAN domain have three ordered states representing the level of engagement of the beneficiaries in the previous week. Rewards are better for lower states, i.e., $R(0) = 1, R(1) = 0.5, R(2) = 0$. At each step, the beneficiary may only change by one level, e.g., low-to-medium or high-to-medium but not low-to-high. They also assume that beneficiaries follow one of three typical patterns, A, B, and C, resulting in three MDPs with different transition probabilities. There are two patterns of effects present that differentiate the beneficiary types. (1) For each of the above types, the planner can only make a difference when the patient is in state 1. Type A responds very positively to interventions, but regresses to low reward states in absence. Type B has
We converted these patient types to robust versions where the transition probabilities are uncertain as follows:

\[
T_{s=0}^i = \begin{bmatrix}
p_{000}^i & 1 - p_{000}^i & 0.0 \\
p_{010}^i & 1 - p_{010}^i & 0.0 \\
\end{bmatrix}, \quad T_{s=1}^i = \begin{bmatrix}
0.0 & 1 - p_{102}^i & p_{102}^i \\
p_{110}^i & 1 - p_{110}^i & 0.0 \\
\end{bmatrix}, \quad T_{s=2}^i = \begin{bmatrix}
0.0 & 1 - p_{202}^i & p_{202}^i \\
0.0 & 1 - p_{212}^i & p_{212}^i \\
\end{bmatrix},
\]

where \( i \) indexes the type (i.e., A, B or C). We then set each \( p_{sas'}^i \) to be in a range of width 0.5 centered on the entries from each of the A, B, C beneficiary types for \( s \in \{1, 2\} \). To add additional heterogeneity to the experiments, for \( s = 0 \), we set the range to 1.0 so that any beneficiary type can be made to have some non-negligible chance of staying in the good state, rather than only type B beneficiaries. The full set of parameter ranges are given in the Table 1 below.

| parameter | lower | upper | lower | upper | lower | upper |
|-----------|-------|-------|-------|-------|-------|-------|
| Type A    |       |       |       |       |       |       |
| \( p_{000}^i \) | 0.00  | 1.00  | 0.00  | 1.00  | 0.00  | 1.00  |
| \( p_{010}^i \) | 0.00  | 1.00  | 0.00  | 1.00  | 0.00  | 1.00  |
| \( p_{102}^i \) | 0.50  | 1.00  | 0.35  | 0.85  | 0.35  | 0.85  |
| \( p_{110}^i \) | 0.50  | 1.00  | 0.15  | 0.65  | 0.00  | 0.50  |
| \( p_{202}^i \) | 0.35  | 0.85  | 0.35  | 0.85  | 0.35  | 0.85  |
| \( p_{212}^i \) | 0.35  | 0.85  | 0.35  | 0.85  | 0.35  | 0.85  |

Table 1: Upper and lower parameter ranges for the robust ARMMAN environment.

In all experiments, 20% of arms were sampled from type A, 20% from type B and 60% for type C. To add additional heterogeneity, for each of the 50 random seeds we uniformly sample a sub-range contained within the ranges given in Table 1. In the agent oracle experiments, for each of the 50 random seeds, since these require fully instantiated transition matrices, we uniformly sample each parameter value for each arm according to its type such that the values are contained in the ranges given in Table 1.

C.2 SIS Epidemic Model

In this domain, each arm follows its own compartmental SIS epidemic model. Each arm’s SIS model tracks whether each of \( N_p \) members of a population is in a susceptible (S) or infectious (I) state. This can be tracked with \( N \) states, since it can be computed how many people are in state I if only the number of people in state S and the population size \( N_p \) is known.

To define a discrete SIS model, we instantiate the model given in Yaesoubi and Cohen [42] section 4.1 with a \( \Delta t \) of 1. We also augment the model to include action effects and rewards. Specifically, \( R(N_S) = N_S/N_p \), where \( N_S \) is the number of susceptible (non-infected) people. Further, there are three actions \( \{a_0, a_1, a_2\} \) with costs \( c = \{0, 1, 2\} \). Action \( a_0 \) represents no action, \( a_1 \) divides the contacts per day \( \lambda_c \) (\( \lambda \) in Yaesoubi and Cohen [42]) by \( a_1^{eff} \), and \( a_2 \) divides the infectiousness \( r_{infect} \) (\( r(t) \) in Yaesoubi and Cohen [42]) by \( a_2^{eff} \). That is, taking action \( a_1 \) will reduce the average number of contacts per day in a given arm, and taking action \( a_2 \) will reduce the probability of infection given contact in a given arm, thus reducing the expected number of people that will become infected in the next round. However, to make this a robust problem, the relative effect sizes of each action for each arm will not be known to the planner, nor will the \( \lambda_c \) or \( r_{infect} \). We impose the following uncertainty intervals for all arms: \( \lambda_c \in [1, 10] \), \( r_{infect} \in [0.5, 0.99] \), \( a_1^{eff} \in [1, 10] \), and \( a_2^{eff} \in [1, 10] \).

In the robust double oracle experiments, to add additional heterogeneity, for each of the 50 random seeds we uniformly sample a sub-range contained within the ranges given above for each arm. In
the agent oracle experiments, for each of the 50 random seeds, since these require fully instantiated transition matrices, we uniformly sample each parameter value for each arm such that the values are contained in the ranges given above.

D Hyperparameter Settings and Implementation Details

**Neural networks:** All neural networks in experiments are implemented using PyTorch 1.3.1 with 2 fully connected layers each with 16 units and tanh activation functions, and a final layer of appropriate size for the relevant output dimension with an identity activation function. The output of discrete actor networks (i.e., the policy network from the agent oracle, and the policy network of agent A in the nature oracle) pass through a categorical distribution from which actions are randomly sampled at training time, without a budget imposed. It is critical not to impose the budget at training time, so that the budget spent by the optimal policy under a given $\lambda$ will result in a meaningful gradient for updating the $\lambda$-network. The output of continuous actor networks (i.e., agent B in the nature oracle which selects environment parameter settings) instead are passed as the means of Gaussian distributions – with the log standard deviations learned as individual parameters separate from the network – from which continuous actions are sampled at training time. At test time, actions are sampled from both types of networks deterministically. For categorical distributions, we greedily select the highest probability actions. For Gaussian distributions, we act according to the means. All discount factors were set to 0.9. The remaining hyperparameters that were constant for all experiments for the agent and nature oracles are indicated in Table 2. For Robust Double Oracle experiments, all agent and nature oracles were run for 100 training epochs. For Agent Oracle experiments, RMABPPO was run for 100 training epochs for the synthetic and ARMMAN domains and 200 epochs for the SIS domain.

**$\lambda$-network:** Critical to training the $\lambda$-network is cyclical control of the temperature parameter that weights the entropy term in the actor loss functions. Recall that the $\lambda$-network is only updated every $n_{\text{subepochs}}$. In general, after each update to the $\lambda$-network, we want to encourage exploration so that actor networks explore the new part of the state space defined by updated predictions of $\lambda$. However, after $n_{\text{subepochs}}$ rounds, we will use the cost of the sampled actor policies as a gradient for updating the $\lambda$-network, and that gradient will only be accurate if the actor policy has converged to the optimal policies for the given $\lambda$ predictions. Therefore, we also want to have little or no exploration in the round before we update the $\lambda$-network. In general, we would also like the entropy of the policy network to reduce over time so that the actor networks and $\lambda$-networks eventually both converge.

To accomplish both of these tasks, the weight (temperature) of the entropy regularization term in the loss function of the actor network will decay/reset according to two processes. The first process will linearly decay the temperature from some positive, but time-decaying starting value (see next process) $\tau_t$ immediately after each $\lambda$-network update, down to 0 after $n_{\text{subepochs}}$. The second process will linearly decay the temperature from a maximum $\tau_0$ (start entropy coeff in Table 2) down to $\tau_{\min}$ (end entropy coeff in Table 2) by the end of training.

We found that it also helps to train the actor network with no entropy and with the $\lambda$-network frozen for some number of rounds before training is stopped (lambda freeze epochs in Table 2).

**Double Oracle:** In all experiments in the main text, we initialize the agent strategy list with HO, HM, and HP, and the nature strategy list with pessimistic, mean, and optimistic nature strategies, then run RMABDO for 5 iterations. This produces a set of 8 agent strategies, 8 nature strategies, a table where each entry represents the regret of each agent pure strategy (row) against each nature pure strategy (column), and an optimal mixed strategy over each set that represents a Nash equilibrium of the minimax regret game given in the table. The regret table is computed by first computing the returns of each agent/nature pure strategy combination, then subtracting the max value of each column from all entries in that column (i.e., the best agent strategy for a given nature strategy gets 0 regret). The regret of RMABDO is reported as the expected utility corresponding to the Nash equilibrium of the regret game given by the table, once that regret table is normalized to account for the returns of baselines (see next paragraph).

After this main loop completes, we then compute the regret of the baselines by evaluating each baseline policy against each pure strategy in the nature strategy list. Then, we also run the nature oracle against each baseline policy to find a nature strategy that should maximize the regret of that
baseline. The regret for each baseline is reported as the max regret against this new nature strategy, as well as all pure nature strategies from the main RMABDO loop.

**Hawkins Baselines:** The Hawkins policies are implemented with gurobipy 9.1.2, a Python wrapper for Gurobi (9.0.3) following the LP given in Hawkins [15] equation 2.5 to compute $\lambda$ and $Q(s, a, \lambda)$ for each arm and the integer program in equation 2.12 to select actions.

**RLvMid Baseline:** We found that RLvMid found effective policies for the nature strategy it was trained against (as evidenced in Figure 2), but that that learned policy could be brittle against other nature strategies. This is likely because different nature strategies produce different distributions of states, meaning RLvMid would fit policies well to states seen when planning against the mean nature strategy, but underfit its policies for states seen more often in different distributions. However, the lone RLvMid baseline policy can somewhat correct for this effect by training an ensemble of policies against slight perturbations of the mean nature strategy that adjust the parameter values output by nature by a small $\epsilon$. In all experiments we train 3 RLvMid policies against 3 random perturbations of the mean nature strategy, then report the regret of RLvMid as the minimum of the max regrets returned by any of the 3.

| parameter               | value   |
|-------------------------|---------|
| **agent**               |         |
| clip ratio              | 2.0e+00 |
| lambda freeze epochs    | 2.0e+01 |
| start entropy coeff     | 5.0e-01 |
| end entropy coeff       | 0.0e+00 |
| actor learning rate     | 2.0e-03 |
| critic learning rate    | 2.0e-03 |
| lambda learning rate    | 2.0e-03 |
| trains per epoch        | 2.0e+01 |
| n_subepochs             | 4.0e+00 |
| **nature**              |         |
| clip ratio              | 2.0e+00 |
| lambda freeze epochs    | 2.0e+01 |
| start entropy coeff     | 5.0e-01 |
| end entropy coeff       | 0.0e+00 |
| actorA learning rate    | 1.0e-03 |
| criticA learning rate   | 1.0e-03 |
| actorB learning rate    | 5.0e-03 |
| criticB learning rate   | 5.0e-03 |
| lambda learning rate    | 2.0e-03 |
| trains per epoch        | 2.0e+01 |
| n_subepochs             | 4.0e+00 |
| n_sims                  | 2.5e+01 |

Table 2: Hyperparameter settings for agent and nature oracles for all experiments.

**E Additional Experimental Results**

Fig. 3 shows regret results from additional robust experiments which scale up the number of arms for Synthetic (top left), ARMMAN (top right) and SIS (bottom left; $N_p = 50$), as well as and the size of the state space of SIS (bottom right; $N = 5, B = 4$). RMABDO continues to outperform all baselines by a large margin. When the size of the state space is scaled up for SIS, it becomes infeasible to run the Hawkins baselines due to its long query time which grows quadratically with the size of the state space. Because of this, we exclude the Hawkins baselines from the main double oracle loop in these experiments. Further, Hawkins cannot even be evaluated as a baseline as the state space increases since to compute its maximum regret, we must get one best response from the nature oracle against the baseline, which requires querying the Hawkins baseline policies tens of thousands of times, which is prohibitive when the query time takes even $\sim 1s$ to run.
Figure 3: Additional experiments showing maximum policy regret in robust setting for Synthetic (top left), ARMMAN (top right) and SIS (bottom) domains, respectively. Synthetic is scaled by 3 and ARMMAN by 5 to maintain the distributions of arm types specified in Section 6. RMABDO beats all baselines by a large margin across various parameter settings. When the state space is scaled up (bottom right) Hawkins baselines become infeasible to run due to its long query time (see Section 6.1 for a discussion), even for a small number of arms ($N = 5, B = 4$).