Transverse Single Spin Asymmetries and Charmonium Production

Rohini M. Godbole\textsuperscript{a}, Anuradha Misra\textsuperscript{b,}\textsuperscript{*}, Asmita Mukherjee\textsuperscript{c}, Vaibhav S. Rawoot\textsuperscript{b}

\textsuperscript{a}Centre for High Energy Physics, Indian Institute of Science, Bangalore, India-560012
\textsuperscript{b}Department of Physics, University of Mumbai, Santa Cruz(E), Mumbai, India-400098
\textsuperscript{c}Department of Physics, Indian Institute of Technology, Bombay, Mumbai, India-400076

Abstract

We estimate transverse spin single spin asymmetry (TSSA) in the process $e + p^{↑} \rightarrow J/\psi + X$ using color evaporation model of charmonium production. We take into account transverse momentum dependent (TMD) evolution of Sivers function and parton distribution function and show that there is a reduction in the asymmetry as compared to our earlier estimates wherein the $Q^2$ - evolution was implemented only through DGLAP evolution of unpolarized gluon densities.

Keywords: Single Spin Asymmetry, Charmonium

1. Introduction

Single spin asymmetries (SSA’s) arise in the scattering of transversely polarized nucleons off an unpolarized nucleon (or virtual photon) target, when the final state hadrons have asymmetric distribution in the transverse plane perpendicular to the beam direction. SSA for inclusive process $A^{↑} + B \rightarrow C + X$ depends on the polarization vector of the scattering hadron $A$ and is defined by

$$A_N = \frac{d\sigma^{↑} - d\sigma^{↓}}{d\sigma^{↑} + d\sigma^{↓}}$$  \hspace{1cm} (1)

Non-zero SSAs have been observed over the years in pion production at Fermilab\textsuperscript{[1]} and at RHIC\textsuperscript{[2]} in $pp^{↑}$ collisions as well as in Semi-inclusive deep inelastic scattering (SIDIS) experiments at HERMES\textsuperscript{[3]} and COMPASS\textsuperscript{[4]}. These results have generated a lot of interest amongst theoreticians to investigate the mechanism involved and to understand the underlying physics.

The initial attempts to provide theoretical predictions of asymmetry, based on collinear factorization of pQCD, led to estimates which were too small as compared to the experimental results\textsuperscript{[5]}. In collinear factorization formalism, the parton distribution functions (PDF’s) and fragmentation functions (FF’s) are integrated over intrinsic transverse momentum of the partons and hence depend only on longitudinal momentum fraction $x$. The observation that SSAs calculated within collinear formalism were almost vanishing suggested that these asymmetries may be due to parton’s transverse motion and spin orbit correlation. A generalization of factorization theorem, in the form of transverse momentum dependent (TMD) factorization which includes the transverse momentum dependence of PDF’s and FF’s, was proposed as a possible approach to account for the asymmetries\textsuperscript{[6]}.

One of the TMD PDF’s of interest is Sivers function, which gives the probability of finding an unpolarized quark inside a transversely polarized proton. The Sivers function, $\Delta N f_{u/p↑}(x, k_{⊥a})$, defined by

$$\Delta N f_{u/p↑}(x, k_{⊥a}) \equiv \hat{f}_{u/p↑}(x, k_{⊥a}) - \hat{f}_{u/p↑}(x, -k_{⊥a}) = \hat{f}_{u/p↑}(x, k_{⊥a}) - \hat{f}_{u/p↑}(x, -k_{⊥a})$$  \hspace{1cm} (2)
is related to the number density of partons inside a proton with transverse polarization $S$, three momentum $p$ and intrinsic transverse momentum $k_\perp$ of partons, and its spin dependence is given by

$$\Delta^N f_{u/p\uparrow}(x, k_\perp) = \Delta^N f_{u/p\downarrow}(x, k_\perp) \ S \cdot (\hat{p} \times \hat{k}_\perp) \quad (3)$$

Parametrizations of quark Sivers distributions have been obtained from fits of SSA in SIDIS experiments\cite{7}. However, not much information is available on gluon Sivers function. Processes that have been studied with the aim of getting information about this TMD are back to back correlations in azimuthal angles of of jet produced in $pp$ scattering\cite{8} and D meson production in $pp$ scattering\cite{9}. Heavy quark and quarkonium systems have also been proposed as non-zero in ep collisions only in color-octet model. Thus, SSA in charmonium production can be used to throw some light on the issue of production mechanism. In this work, we present estimates of SSA in the process $e^+ p^\uparrow \rightarrow J/\psi + X$ and compare the results obtained using TMD evolution of PDF’s with our earlier results which were obtained using DGLAP evolution only.

### 2. Transverse Single Spin Asymmetry in $e^+ p^\uparrow \rightarrow J/\psi + X$

The first estimate of SSA in photoproduction (i.e. low virtuality electroproduction) of $J/\psi$ in the scattering of electrons off transversely polarized protons were provided by us in Ref.\cite{15} using Color Evaporation Model. In the process under consideration, at LO, there is contribution only from a single partonic subprocess and therefore, it can be used as a clean probe of gluon Sivers function.

Color Evolution Model (CEM) was introduced in 1977 by Fritsch and was revived in 1996 by Halzen\cite{13}. This model gives a good description of photoproduction data after inclusion of higher order QCD corrections\cite{17} and also of the hadroproduction CDF data\cite{18} after inclusion of $k_T$ smearing. In CEM, the cross-section for a quarkonium state $H$ is some fraction $F_H$ of the cross-section for producing $Q\bar{Q}$ pair with invariant mass below the $MM$ threshold, where $M$ is the lowest mass meson containing the heavy quark $Q$:

$$\sigma_{CEM}[h_{\uparrow}\bar{h}_B \rightarrow H + X] = F_H \int_{4m^2_{CB}}^{4m^2_{h_{\uparrow}}} d\delta \times \int dx_1 dx_2 f_1(x_1, \mu) f_2(x_2, \mu) \bar{\sigma}_{\gamma^*}(\delta)(\delta - x_1 x_2 \delta) \quad (4)$$

We have used a generalization of CEM expression for electroproduction of $J/\psi$ by taking into account the transverse momentum dependence of the gluon distribution function and the William Weizsacker (WW) function which gives the photon distribution of the electron in equivalent photon approximation\cite{19}. The cross section for the process $e^+ p^\uparrow \rightarrow J/\psi + X$ is then given by

$$\sigma_{e^+p^\uparrow \rightarrow J/\psi + X} = \int_{4m^2_{CB}}^{4m^2_{h_{\uparrow}}} dM^2_{\gamma e} \bar{\sigma}_{\gamma^*}(\delta)(\delta - x_1 x_2 \delta) \quad (5)$$

where $f_{\gamma/e}(x, k_\perp)$ is the distribution function of the photon in the electron. We assume a gaussian form for the $k_\perp$ dependence of pdf’s\cite{7},

$$f(x, k_\perp) = \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2/\langle k_\perp^2 \rangle} \quad (6)$$

where $\langle k_\perp^2 \rangle = 0.25GeV^2$. $f_{\gamma/e}(x, k_\perp)$ is also assumed to have a similar $k_\perp$ dependence and is given by

$$f_{\gamma/e}(x, k_\perp) = f_{\gamma/e}(x_0) \frac{1}{\pi \langle k_{\perp_0}^2 \rangle} e^{-k_\perp^2/\langle k_{\perp_0}^2 \rangle} \quad (7)$$

where $f_{\gamma/e}(x_0)$ is the William Weizsacker function given by\cite{20}:

$$f_{\gamma/e}(y, E) = \frac{\alpha}{\pi} \left[ 1 + \frac{1}{y} + \frac{1}{2y} \ln \frac{2y - 2}{2y - 2y} \right] \quad (8)$$

$y$ being the energy fraction of the electron carried by the photon.
Using Eq. [3] the expression for the numerator of the asymmetry reduces to [16]
\[
\frac{d^3q}{dyd^2q_T} = \frac{d^3q}{dyd^2q_T} = \frac{1}{2} \int_{4\pi}^0 dM^2 \int [dx_\gamma dx_\gamma] \\
\times \int [d^2k_{\perp}d^2k_{\perp}] |\Delta f_{g/p}(x_\gamma, k_{\perp})| \\
\times f_{g/\gamma}(x_\gamma, k_{\perp}) \sigma \left( p_\perp + p_\gamma - q \right) \frac{\gamma}{\sin^2(\gamma/2)} (M^2) \\
\text{where } q = p_x + p_t \text{ and } \sigma_0^{\gamma-\gamma}(M^2) \text{ is the partonic cross section}[21].
\]

\[
\sigma_0^{\gamma-\gamma}(M^2) = \frac{1}{2} \frac{4\pi\alpha_s}{M^2} \left( 1 + \frac{1}{1 - \gamma^2} \right) \ln \left( \frac{1 + \sqrt{1 - \gamma}}{1 - \sqrt{1 - \gamma}} \right)
\]

\[
\gamma = 4m_T^2/M^2 \text{ and } q_T \text{ and } k_{\perp} \text{ are the transverse momenta of the gluon and } J/\psi \text{ respectively with azimuthal angles } \phi_\eta \text{ and } \phi_\phi.
\]

\[
A_N^{\sin(\phi_\eta - \phi_\phi)} = \frac{\int d\phi_\eta [d\sigma^1 - d\sigma^1] \sin(\phi_\eta - \phi_\phi)}{\int d\phi_\eta [d\sigma^1 + d\sigma^1]}
\]

which finally leads to

\[
A_N = \frac{\int d\phi_\eta [d\sigma^1] |dM^2| \int [d^2k_{\perp}] |\Delta f_{g/p}(x_\gamma, k_{\perp})| f_{g/\gamma}(x_\gamma, q_T - k_{\perp}) \sigma_0 \sin(\phi_\eta - \phi_\phi)}{2 \int d\phi_\eta [d\sigma^1] |dM^2| \int [d^2k_{\perp}] |\Delta f_{g/p}(x_\gamma, k_{\perp})| f_{g/\gamma}(x_\gamma, q_T - k_{\perp}) \sigma_0}
\]

where

\[
d\sigma = \frac{d^3\sigma}{dyd^2q_T} , \quad x_{\gamma, \gamma} = \frac{M}{\sqrt{s}} e^{\gamma y}
\]

3. Models for Sivers function

In our analysis, we have used the following parameterization for the gluon Sivers function [7]

\[
\Delta^N f_{g/p}(x, k_{\perp}) = 2N_c(x) h(k_{\perp}) f_{g/\gamma}(x) \\
\times \frac{e^{-k_{\perp}^2/(\Delta_{k_{\perp}}^2)}}{\pi(k_{\perp}^2)} \cos \phi_{k_{\perp}}
\]

where \(N_c(x)\) is an x dependent normalization. We have used two different models for the functional forms of \(h(k_{\perp})\): In Model(1)[23]

\[
h(k_{\perp}) = \sqrt{2e} k_{\perp} M_1 e^{-k_{\perp}^2/M_1^2}
\]

whereas in Model(2)[9]

\[
h(k_{\perp}) = \frac{2k_{\perp} M_0}{k_{\perp}^2 + M_0^2}
\]

where \(M_0 = \sqrt{\langle k_{\perp}^2 \rangle} \) and \(M_1 \) are best fit parameters. Here, we will present the results for Model I only. The results for Model II and a comparison of the two models can be found in Ref.[16]. For \(N_c(x)\) also, we have used two kinds of parametrizations [8]

(a) \(N_c(x) = (N_c(x) + N_c(x)) / 2\)

(b) \(N_c(x) = N_d(x)\)

where \(N_u(x)\) and \(N_d(x)\) are the normalizations for u and d quarks given by [8]

\[
N_f(x) = N_f(x) = (1 - x)^x \frac{(a_f + b_f)^{(a_f + b_f)}}{a_f a_f b_f b_f}
\]

Here, \(a_f, b_f\) and \(N_f\) are best fit parameters fitted from new HERMES and COMPASS data[24] fitted at \(\langle Q^2 \rangle = 2.4 GeV^2\) as given below:

\[
N_u = 0.4, a_u = 0.35, b_u = 2.6
\]

\[
N_d = -0.97, a_d = 0.44, b_d = 0.90
\]

\[
M_1 = 0.19.
\]

We have estimated SSA using both Model I and II and parameterizations (a) and (b). The detailed results can be found in Ref. [16].

4. TMD Evolution of PDF’s and Sivers Function

Early phenomenological fits of Sivers function were performed using experimental data at fixed scales and estimates of asymmetry were also performed either neglecting QCD evolution of TMD PDF’s or by applying DGLAP evolution only to the collinear part of TMD parametrization. In our earlier estimates of asymmetry
in Ref.\[16\] also, we have assumed the $Q^2$-dependence of PDF’s and the Sivers function to be of the form,

$$f_{g/p}(x, k^2_T; Q) = f_{g/p}(x; Q) \frac{1}{\pi (k^2_T)} e^{-k^2_T/(2k^2_0)}$$  \hspace{1cm} (19)$$

and

$$\Delta^N f_{g/p}(x, k^2_T; Q) = 2N_g(x) f_{g/p}(x; Q) \times \sqrt{2} e \frac{k}{M_1} \frac{1}{\pi (k^2_T)} e^{-k^2_T/(2k^2_0)}$$  \hspace{1cm} (20)$$

where $(k^2_0) = 0.25 \text{ GeV}^2$. Note that the $Q^2$ dependence of PDF comes from collinear PDF $f_{g/p}(x; Q)$ only which have been evolved using DGLAP evolution. More recently, energy evolution of TMD’s has been studied by various authors and a TMD evolution formalism has been developed and implemented \[25, 26\].

TMD evolution is more complicated as compared to collinear counterpart because unlike collinear distributions TMDs have rapidity divergences in addition to collinear singularities. Thus TMD evolution describes how the form of distribution changes and also how the width changes in momentum space. A strategy to extract Sivers function from SIDIS data taking into account the TMD $Q^2$ evolution has been proposed \[27\]. We have estimated SSA in electroproduction of $J/\psi$ taking into account this strategy. In this formalism, the $Q^2$ dependence of PDF’s is given by

$$f_{g/p}(x, k^2_T; Q) = f_{g/p}(x, Q_0) R(Q, Q_0) \frac{e^{-k^2_T/(2w^2)}}{\pi w^2} e^{\gamma_{K}(2k^2_0)},$$  \hspace{1cm} (21)$$

where, $f_{g/p}(x, Q_0)$ is the usual integrated PDF evaluated at the initial scale $Q_0$ and $w^2 = k^2_0 + 2g_2 \ln \frac{Q}{Q_0}$. \hspace{1cm} (22)$$

$R(Q, Q_0)$ is the limiting value of a function $R(Q, Q_0, b_T)$ that drives the $Q^2$-evolution of TMD’s in coordinate space and is driven by

$$R(Q, Q_0, b_T) \equiv \exp[\ln \frac{Q}{Q_0} \int_{0}^{\mu} \frac{d\mu'}{\mu} \gamma_{K}(\mu') + \int_{0}^{\mu} \frac{d\mu}{\mu} \gamma_{F}(\mu, \frac{Q^2}{\mu^2})].$$  \hspace{1cm} (23)$$

where $b_T$ is the parton impact parameter,

$$b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}}, \quad \mu_T = C_1 \frac{b_T}{b_*(b_T)}$$  \hspace{1cm} (24)$$

with $C_1 = 2e^{-\gamma_E}$ where $\gamma_E = 0.577$, $b_* \rightarrow b_{max}$. $\gamma_F$ and $\gamma_K$ are anomalous dimensions which are given at $O(\alpha_s)$ by

$$\gamma_F(\mu, \frac{Q^2}{\mu^2}) = \alpha_s(\mu) \frac{C_F}{\pi} \left( \frac{3}{2} - \ln \frac{Q^2}{\mu^2} \right)$$  \hspace{1cm} (25)$$

$$\gamma_K(\mu) = \alpha_s(\mu) \frac{2 C_F}{\pi}.$$  \hspace{1cm} (26)$$

In the limit $b_T \rightarrow \infty$, $R(Q, Q_0, b_T) \rightarrow R(Q, Q_0)$. \hspace{1cm} (27)$$

5. Numerical Estimates

We have estimated SSA in electroproduction of $J/\psi$ for JLab, HERMES, COMPASS and eRHIC energies. Our earlier calculation of asymmetry\[16\] had taken into account energy evolution of PDF’s and Sivers function using DGLAP evolution. The details can be found in Ref.\[16\].

In Figs.1-5, we have presented a comparison of SSA’s calculated using DGLAP evolution and TMD evolution of TMD PDF’s at various energies for Model I with parametrization (a). For TMD evolved Sivers function, we have used the parameter set fitted at $Q_0 = 1 \text{ GeV}$ given in Ref.\[27\],

$$N_u = 0.75, N_d = -1.00, \quad b = 4.0, a_u = 0.82, a_d = 1.36, \quad M_1^2 = 0.34 \text{ GeV}^2, g_2 = 0.68.$$  \hspace{1cm} (28)$$

It is found that the asymmetry is substantially reduced in all cases when TMD evolution of PDF’s and Sivers function is taken into account. Here, we have used parametrization (a) for our estimates. A more detailed analysis with parametrization (b) and a comparison of the two parametrizations as well as of various parameter sets can be found in Ref\[28\].

6. Summary

Transverse SSA in electroproduction of $J/\psi$ has been calculated using color evaporation model of charmonium production. A TMD factorization formalism has been used first with DGLAP evolved PDF’s and then with TMD evolved PDF’s and Sivers function. Sizable asymmetry is predicted at energies of JLab, HERMES, COMPASS and eRHIC experiments in both cases. However, it is found that there is a substantial reduction in asymmetry when TMD evolution is taken into account. Substantial magnitude of asymmetry indicate that it may be worthwhile to look at SSA’s in charmonium production both from the point of view of comparing different models of charmonium production.
as well as comparing the different models of gluon Sivers function. It is also clear that TMD evolution effects are substantial and one must take them into account for accurate predictions.

ACKNOWLEDGEMENTS

I would like to thank the organizers of LC2012 Delhi for their kind hospitality. I would also like to thank DAE BRNS, India for financial support during this project under the grant No. 2010/37P/47/BRNS.

References

[1] D. L. Adams et al. [FNAL-E704 Collaboration], Phys. Lett. B 264, 462 (1991); A. Bravar et al. [Fermilab E704 Collaboration], Phys. Rev. Lett. 77, 2626 (1996).

[2] K. Krueger, C. Allgower, T. Kasprzyk, H. Spinka, D. Underwood, A. Yokosawa, G. Bunce and H. Huang et al., Phys. Lett. B 459, 412 (1999); C. E. Allgower, K. W. Krueger, T. E. Kasprzyk, H. M. Spinka, D. G. Underwood, A. Yokosawa, G. Bunce and H. Huang et al., Phys. Rev. D 65, 092008 (2002).

[3] A. Airapetian et al. [HERMES Collaboration], Phys. Rev. Lett. 84, 4047 (2000) [hep-ex/0003075]; Phys. Rev. D 64, 097101 (2001) [hep-ex/0104005].

[4] V. Y. Alexakhin et al. [COMPASS Collaboration], Phys. Rev. Lett. 94, 202002 (2005) [hep-ex/0503002].

[5] U. D’Alesio and F. Murgia, Prog. Part. Nucl. Phys. 61, 394 (2008) [arXiv:0712.4328 [hep-ph]].

[6] D. W. Sivers, Phys. Rev. D 41, 83 (1990); Phys. Rev. D 43, 261 (1991).

[7] M. Anselmino, M. Boglione, U. D’Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin and C. Turk, Eur. Phys. J. A 39, 89 (2009), [arXiv:0805.2677 [hep-ph]].

[8] D. Boer and W. Vogelsang, Phys. Rev. D 69, 094025 (2004) [hep-ph/0312320].

[9] M. Anselmino, M. Boglione, U. D’Alesio, E. Leader and F. Murgia, Phys. Rev. D 70, 074028 (2004); [hep-ph/0407100].

[10] F. Yuan, Phys. Rev. D 78, 014024 (2008) [arXiv:0804.2457 [hep-ph]].

[11] S. J. Brodsky, D. s. Hwang and I. Schmidt, Phys.Lett. B530,99(2002);Nucl.Phys.B642,344(2002).

[12] E. L. Berger, D. Jones, Phys. Rev D 23 , 1521 (1981); R. Baur, R. Rückl, Phys. Lett. B 102, 364 (1981), Nucl. Phys. B 201, 1 (1982).

[13] F. Halzen, Phys. Lett. B 69, 105 (1997); F. Halzen, S. Matsuda, Phys. Rev. D 17, 1344 (1978), H. Fritsch, Phys. Lett. B 67, 217 (1977), O. J. P. Ebioli, E. M. Gregores, F. Halzen, Phys. Rev. D 67, 054002 (2003).

[14] M. B. Gay Ducati and C. Brenner Mariotto, Phys. Lett. B 464, 266 (1999) [hep-ph/9906407].

[15] G. T. Bodwin, E. Braaten, G. P. Leapage, Phys. Rev. D 43, 1914 (1992).

[16] R. M. Godbole, A. Misra, A. Mukherjee and V. S. Rwoot, Phys. Rev. D 85, 049013 (2012) [arXiv:1201.1066 [hep-ph]].

[17] O. J. P. Ebioli, E. M. Gregores and F. Halzen, [hep-ph/0211161].

[18] G. T. Bodwin, E. Braaten and J. Lee, Phys. Rev. D 72, 014004 (2005) [hep-ph/0504014].

[19] C. F. Weissacker, Z. Phys. 88 (1934), E. J. Williams, Phys. Rev. 45 729(1934).

[20] B. A. Kniehl, Phys. Lett. B 254, 267 (1991).

[21] M. Gluck and E. Reya, Phys. Lett. B 79, 453 (1978).

[22] W. Vogelsang, and F. Yuan, Phys. Rev. D 72, 054028 (2005) [arXiv:hep-ph/0501296]; J. C. Collins et al., Phys. Rev. D 73, 094023 (2006).

[23] M. Anselmino, M. Boglione, U. D’Alesio, S. Melis, F. Murgia and A. Prokudin, Phys. Rev. D 79, 054010 (2009) [arXiv:0901.3075 [hep-ph]].

[24] M. Anselmino, M. Boglione, U. D’Alesio, S. Melis, F. Murgia and A. Prokudin, [hep-ph/1107.4446].

[25] J. C. Collins, Foundations of Perturbative QCD, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, No. 32, Cambridge University Press, Cambridge, 2011.

[26] S. M. Aybat and T. C. Rogers, Phys. Rev. D 83, 114042 (2011) [arXiv:1101.5057 [hep-ph]]; S. M. Aybat, J. C. Collins, J. W. Qiu and T. C. Rogers, [arXiv:1110.6428 [hep-ph]]; S. M. Aybat, A. Prokudin and T. C. Rogers, [arXiv:1112.4423 [hep-ph]].

[27] M. Anselmino, M. Boglione and S. Melis, Phys. Rev. D 86, 014028 (2012) [arXiv:1204.1239 [hep-ph]].

[28] R. M. Godbole, A. Misra, A. Mukherjee and V. S. Rwoot, Phys. Rev. D 88, 014029 (2013) [arXiv:1304.2584 [hep-ph]].

Figure 1: The Sivers asymmetry $A_T^{up(\uparrow \downarrow)}$ for $e^+ p \rightarrow e^+ J/\psi X$ at JLab energy ($\sqrt{s} = 4.7$ GeV) as a function of $y$ (top panel) and $q_T$ (bottom panel) for parametrization (a).
Figure 2: The Sivers asymmetry $A_{2N}^{\sin(\phi_q - \phi_S)}$ for $e + p^1 \rightarrow e + J/\psi + X$ at HEMRES energy ($\sqrt{s} = 7.2$ GeV) as a function of $y$ (top panel) and $q_T$ (bottom panel) for parametrization (a).

Figure 3: The Sivers asymmetry $A_{2N}^{\sin(\phi_q - \phi_S)}$ for $e + p^1 \rightarrow e + J/\psi + X$ at COMPASS energy ($\sqrt{s} = 17.33$ GeV) as a function of $y$ (top panel) and $q_T$ (bottom panel) for parametrization (a).

Figure 4: The Sivers asymmetry $A_{2N}^{\sin(\phi_q - \phi_S)}$ for $e + p^1 \rightarrow e + J/\psi + X$ at eRHIC-1 energy ($\sqrt{s} = 31.6$ GeV) as a function of $y$ (top panel) and $q_T$ (bottom panel) for parametrization (a).

Figure 5: The Sivers asymmetry $A_{2N}^{\sin(\phi_q - \phi_S)}$ for $e + p^1 \rightarrow e + J/\psi + X$ at eRHIC-2 energy ($\sqrt{s} = 158.1$ GeV) as a function of $y$ (top panel) and $q_T$ (bottom panel) for parametrization (b).