Universal pattern in \((e,e'p)\) at large missing momenta: quasi-deuteron or diffractive final state interactions?

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Abstract

The intrinsic single particle momentum distributions in nuclei are supposed to show a universal behavior at large momenta, dominated by short-range correlated pairs, or quasi-deuterons. We discuss whether the quasi-deuteron universality survives the final state interaction effects, which are present in the missing momentum spectra measured in \(A(e,e'p)\) experiments at GeV energies. We demonstrate that in the observed missing momentum spectra an approximate universality is present, but originating from the universal pattern of diffractive final state interactions of the struck proton independent of the target nucleus.
In the past decades, a great theoretical effort has been devoted to the study of the nuclear single particle momentum distribution \( N(k) \). Particular importance is attributed to the large momentum tail of \( N(k) \), which is expected to be dominated by two nucleon short-range correlations, or “quasi-deuteron” configurations \([1]\) (QD onwards). The quasi-deuteron idea was first discussed quantitatively in \([2]\) (see also \([3]\)). This name originates in the predicted analogous behavior of short-range interactions in all nuclei, from deuteron to nuclear matter. This should be reflected in a similar shape for the large-\( k \) tail \((k > 1.5 \div 2 \text{ fm}^{-1})\) of \( N(k) \) in deuteron or in any other nucleus (hereafter the universality of \( N(k) \) refers to the \( k \)-dependence, apart from a \( k \)-independent normalization factor). An example can be seen, e.g., in Fig.5 of \([4]\) where the ratio between \(^4\text{He}\) and \( D \) distributions varies roughly between 2.5 and 4.5 in the range 1 fm\(^{-1} < k < 4 \text{ fm}^{-1}\).

However, the pressing issue is how this important intrinsic feature of the nuclear structure can be tested in the missing momentum distribution \( W(\vec{p}_m) \) measured experimentally in \((e, e'p)\) reactions at large missing momentum \( \vec{p}_m \). In two previous papers \([5, 6]\) we have shown that final state interactions (FSI) between the struck nucleon and the spectator nucleons take over at large \( \vec{p}_m \), making the observed \( W(\vec{p}_m) \) substantially different from the ground state distribution \( N(k) \). Furthermore, the sensitivity to the details of the nuclear structure is lost to a large extent. In this communication we wish to focus on an approximate universality of the observed \( W(\vec{p}_m) \) for \(^4\text{He}\) and \( D \) targets, which is driven by the target independence of FSI between the struck and spectator nucleons.

We confine ourselves to large \( Q^2 \) and high kinetic energy of the struck proton \( T_{\text{kin}} \approx Q^2/2m_p \). The very nature of nucleon-nucleon interaction changes from the purely elastic potential scattering at low energies to a strongly absorptive, diffractive small angle scattering at \( T_{\text{kin}} \gtrsim 0.5-1 \text{ GeV} \). In this high energy regime, the Compton wavelength of the struck proton is much smaller than the size of a nucleon and/or average nucleon-nucleon separation in nuclei. The Glauber model \([7]\) becomes a natural framework for quantitative description of FSI.

The single particle momentum distribution \( N(k) \) coincides with the longitudinal reduced response in \((e,e'p)\) scattering in the Plane Wave Impulse Approximation (i.e. ne-
glecting FSI) integrated over all the missing energies. This can be written as:

\[ N(p_m) = \sum_n \left| \langle \Psi_n | e^{i\vec{p}_m \cdot \vec{r}} | \Psi_A \rangle \right|^2 \]  \hspace{1cm} (1)

where \( \Psi_A \) is the ground-state wave function of the target nucleus (mass number \( A \)), the sum goes over all the possible states \( \Psi_n \) for the recoiling system of \( A-1 \) particles, \( \vec{r} \) is the position of the struck proton, \( \vec{p}_m \equiv \vec{q} - \vec{p} \) is the missing momentum (in PWIA, the missing momentum \( p_m \) coincides with the initial nucleon momentum \( k \)), where \( \vec{q} \) is the momentum transfer from the electron to the target and \( \vec{p} \) is the momentum of the detected proton. The angle between \( \vec{p}_m \) and \( \vec{q} \) is denoted by \( \theta \).

In the calculation of the experimentally measured \((e,e'p)\) coincidence distribution one must include FSI distortions,

\[ W(\vec{p}_m) = \sum_n \left| \langle \Psi_n | e^{i\vec{p}_m \cdot \vec{r}} S(1,...A) | \Psi_A \rangle \right|^2 , \]  \hspace{1cm} (2)

where the operator \( S(1,...A) \) describes interactions between the struck proton and the remaining \((A-1)\) spectator nucleons. In the diffractive regime, the Glauber model gives

\[ S(1,...A) = \prod_{n=2}^{A} \left\{ 1 - \Gamma(\vec{b} - \vec{b}_n) \theta(z_n - z) \right\} . \]  \hspace{1cm} (3)

Here we decompose \( \vec{r}_i = (\vec{b}_i, z_i) \), taking the \( z \) axis along \( \vec{q} \). The profile function of the \( pN \) interaction, \( \Gamma(\vec{b}) \), is usually parameterized as

\[ \Gamma(\vec{b}) = \frac{\sigma_{\text{tot}}(1 - i\rho)}{4\pi b_o^2} \exp \left( -\frac{\vec{b}^2}{2b_o^2} \right). \]  \hspace{1cm} (4)

Here \( \rho \) is the \( Re/Im \) ratio for the forward elastic \( pN \) scattering amplitude, \( b_o^2 \) is the diffraction slope. In the GeV energy range, \( \sigma_{\text{tot}} \approx 40 \text{ mb}, \rho \approx 0.3 \div 0.4, b_o \approx 0.5 \text{ fm} \) \cite{8, 9}.

For the deuteron target, the FSI distortion factor takes on the particularly simple form

\[ S(\vec{r}) = 1 - \Gamma(\vec{b}) \theta(-z) . \]  \hspace{1cm} (5)

Here \( \vec{r} \) is the proton-neutron separation. Eq. (5) illustrates basic features of FSI at high energy and momentum transfer as reflected in the Glauber formalism: (i) The \( \theta(-z) \) tells that FSI is possible only provided that the spectator nucleon was in the forward
hemisphere with respect to the struck proton. (ii) The form of $\Gamma(\vec{b})$ tells that the struck proton wave is distorted only at small $|\vec{b}| \lesssim b_o$. (iii) Because $\rho$ is small, $\Gamma(\vec{b})$ is dominated by the imaginary part of the $p$-$n$ elastic scattering amplitude and the distortion factor $S(\vec{r})$ predominantly gives an absorption of the struck proton wave at small $\vec{b}$. At lower energies, the term ”inelastic event” indicates that the target nucleus breaks up, whereas at the considered high four-momentum transfer $(e, e'p)$ experiments, ”inelastic” means that the proton breaks up when undergoing FSI with the residual nucleus. The fact that $\rho$ is small and $\sigma_{el} < \sigma_{in}$ means that those inelastic events are dominating at GeV energies. In contrast to FSI at low energies, which is S-wave dominated and therefore isotropic, the above basic properties of FSI at high energies demonstrate that the transverse and longitudinal directions, and the forward and backward hemisphere, have different roles, with the phenomenological consequences of a marked angular anisotropy, and forward-backward asymmetry.

In the Glauber model the FSI factor has no free parameters, it is fully specified in terms of the free nucleon scattering amplitude. At the energies which are relevant for this work, the Glauber model description of hadron-nucleus scattering is well tested [10, 11].

The above expression has been used for calculating the longitudinal response in $(e,e'p)$ on deuteron [6], with realistic Bonn [12] and Paris [13] wave functions. For the $^4He$ target, one must use a wave function with realistic Jastrow-type correlations:

$$\Psi = \prod_{i<j} (1 - F_{ij}) \Psi_o. \quad (6)$$

Here $\Psi_o = \exp\{-r_1^2 + r_2^2 + r_3^2 + r_4^2/2R_o^2\}$ is a harmonic oscillator mean field wave function, and $1 - F_{ij} = 1 - C_o \exp(-r_{ij}^2/2r_c^2)$ is a correlation operator expressing hard ($C_o=1$) or soft ($C_o < 1$) core repulsion when two nucleon centers are within a distance $r_c$ (the practical calculations are done in Jacobi coordinates). This function contains the dominating features of the $^4He$ ground state, with two exceptions, namely 3-body and d-wave correlations (see e.g. [11]). However, we show that the sensitivity towards those corrections tends to be lost when FSI are included. In the Jastrow-type correlation $F_{ij}$, we take a standard value of $r_c = 0.5$ fm. Then, the choice $R_o = 1.29$ fm correctly reproduces the experimental
charge radius (taking into account the finite nucleon size [14]). The qualitative agreement of our PWIA distribution with the results of the Monte Carlo calculation of [4] and the parametrization given by [15] is satisfactory. An extensive discussion of the results of the full calculation of the distribution (2) with the wave function (3) is presented elsewhere [16]. Here we only wish to compare certain common features of the deuteron and 4He (e,e’p) distributions at large $p_m$.

A new insight into FSI effects is needed in the regime of diffractive $N$-$N$ scattering. At lower energies, FSI act like a correlated response of the full residual nucleus to the passage of the ejectile. The characteristic parameter of FSI is the nuclear radius. In the GeV energy range, the wavelength of the struck proton is short and the crucial parameter is the radius of diffractive $pN$ scattering, which is much smaller than the nuclear radii, $b_0 \ll R_A$, and approximately equal to the correlation radius, $b_0 \approx r_c$. For this reason one can expect a large FSI contribution at large $p_m \sim 1/b_0 \approx 1/r_c$. For instance, for transverse $p_m$, the FSI factor (4) gives rise to a large Fourier transform in (2) and to a large $p_m\perp$ tail of the missing momentum distribution $W(\vec{p}_m)$:

$$ W(\vec{p}_m) \propto \left| \int d^2\vec{b} \Gamma(\vec{b}) \exp(i\vec{p}_\perp \vec{b}) \right|^2 = 4\pi \frac{d\sigma_{el}}{dp_\perp^2} = \frac{1}{4} \sigma_{tot}^2 (1 + \rho^2) \exp(-b_0^2 p_\perp^2). \tag{7} $$

This contribution to $W(\vec{p}_m)$ can be attributed to elastic rescattering of the struck proton on spectator nucleons. To a crude approximation, the $p_{m\perp}$ dependence in (7) does not depend on the target nucleus, as $b_0^2 \ll R_A^2$. In the absence of FSI, both in deuteron and in larger nuclei the large $p_m$ tail of the single particle distribution would have been dominated by short-distance nucleon-nucleon interactions in the nuclear ground state. However our results [5, 6] show that this is not the case when FSI are included.

In Fig. 1a we show the PWIA and the full transverse momentum distribution $W(\vec{p}_m)$ including FSI for the deuteron wave functions calculated from the Bonn and Paris models. The well known smaller D-wave content of the Bonn model makes the corresponding PWIA much smaller than in the Paris model at large $p_m$. This difference gives an estimate of the uncertainties in the predictions of modern theories of the $NN$ interaction at large $p_m \gtrsim 1.5 fm^{-1}$. However, when FSI are included the differences between the
two distributions disappear: FSI, which distort the struck proton’s wave function only at small distances $|\vec{b}| \lesssim b_o$, are hardly sensitive to the deuteron D-wave (where the centrifugal barrier keeps nucleons apart). In Fig. 1, we show how the distribution $W(\vec{p}_m)$ for the $^4He(e,e'p)$ reaction is changed for different types of correlations, going from $C_o = 0$ (pure mean field) to $C_o = 1$ (hard core). For transverse kinematics, the FSI-dominated distributions are not very sensitive to such drastic changes in the ground state wave function. From Fig. 1 we learn that in the leading order the FSI are not too sensitive to the large-$k$ components of the nuclear ground state $\Psi_A$. In transverse kinematics, the FSI redistribute strength from small and intermediate momenta to the large $p_m$ tail of $W(\vec{p}_m)$, and therefore the transverse missing momentum distribution including FSI effects is mainly sensitive to the bulk of the nuclear ground state (in deuteron the S-wave, in complex nuclei the mean-field orbitals).

In Fig. 2, we come to the main point: in transverse and parallel kinematics (both forward and backward) we show the distributions (including FSI) for $^4He$ and for the deuteron. The latter is multiplied by 3 to take into account that in $^4He$ we have 3 FSI scatterers. The similarity between the FSI-inclusive distributions for $^4He$ and deuteron at $p_m$ larger than 1.5 fm$^{-1}$ is impressing, especially in transverse kinematics. It is even more striking if one compares the transverse and longitudinal distributions corresponding to one and the same nucleus (either deuteron or $^4He$). They are equal in the fully isotropic PWIA prediction. On the contrary, at large $p_m$ the full distributions $W(\vec{p}_m)$ differ by one order of magnitude, attesting the FSI dominance. So, at last, universality appears, but it is driven by FSI rather than by an intrinsic feature of the nuclear ground state wave function.

In transverse kinematics, the PWIA curve is overwhelmed by the FSI effect by orders of magnitude. There is no hope to find direct quasi-deuteron effects in experiments performed in transverse kinematics. Even more confusion can arise from experiments in which events which are taken at different angles $\theta$ of the missing momentum $\vec{p}_m$ are put together as a function of $|\vec{p}_m|$ and are not presented in separate distributions. In transverse kinematics the FSI dominance is so marked that it seems difficult to think that
a more refined $^4He$ ground state can modify this situation, especially as the transverse missing momentum distribution for the $D(e, e'p)$ reaction calculated with realistic wave functions is also dominated by FSI and as the FSI effects in $^4He$ should be stronger than in the rather dilute deuteron.

In longitudinal kinematics, the situation seems more interesting. There, our results suggest that PWIA and FSI effects are in competition, and a quantitative understanding of their interplay can become decisive. At these kinematics, at large $p_{m,z}$, FSI effects are mainly due to the $\theta(-z)$ factor in (5). The related discontinuity introduces high-momentum longitudinal components. This is again a universal property of the FSI, the presence of which does not depend on the specific target nucleus, although there is a slight sensitivity to the form of the correlation function.

Apart from this large $p_{m,z}$ tail, the interference between the PWIA amplitude and the $\propto \rho$ component of the FSI amplitude, leads to a forward-backward (F/B onwards) asymmetry, shown in Figs. 3a,b for $^4He$ and $D$. The mere presence of a large F/B asymmetry suggests strong FSI effects, and again its $p_m$ dependence for deuteron and $^4He$ with soft core/hard core correlations included is qualitatively similar. The realistic deuteron wave functions do already include effects of the short range $NN$ interaction. However, as the F/B asymmetry is a PWIA-FSI interference effect, it also contains some information on the nuclear ground state, as was shown in detail in [17] for the special case of polarized deuteron, and therefore and due to the different values of $\rho$ for $D$ and $^4He$ the similarity is less pronounced than in transverse kinematics.

Concluding, we have shown that FSI create strong similarity patterns in the energy-integrated momentum distributions for $(e,e'p)$ on $^4He$ and deuteron at large missing momenta. This will allow for detailed studies of universal features of nuclear FSI in the diffractive regime, but even make testing short range structures in the nuclear ground state much more difficult. In particular, in transverse kinematics this task seems hopeless. It has to be stressed that FSI effects cannot be described by a simple overall renormalization factor as they depend strongly on the specific kinematics. A further analysis of the situation in longitudinal kinematics requires more sophisticated models for the $^4He$, due
to the complex interplay of FSI and PWIA effects there.

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Figure Captions

Figure 1: a) The PWIA missing momentum distribution \( N(p_m) \) in the \( D(e,e'p) \) reaction calculated with the Bonn wave function \cite{12} (dashed line) and with the Paris wave function \cite{13} (dotted line) and the full missing momentum distribution \( W(\vec{p}_m) \) including FSI in the \( D(e,e'p) \) reaction at \( \theta = 90^\circ \) calculated with the Bonn wave function (solid line) and the Paris wave function (dash-dotted line). b) The full missing momentum distribution \( W(\vec{p}_m) \) including FSI in the \( ^4He(e,e'p) \) reaction for hard core correlations \( C_o = 1 \) (solid line), soft core correlations \( C_o = 0.5 \) (dashed line), and pure mean field, \( C_o = 0 \) (dotted line).

Figure 2: The full missing momentum distribution \( W(\vec{p}_m) \) for the \( ^4He(e,e'p) \) reaction (solid line) and for the \( D(e,e'p) \) reaction (dashed line) multiplied with a factor of 3 to account for the higher number of FSI scatterers in \( ^4He \). For comparison, the PWIA missing momentum distribution \( N(p_m) \) for \( ^4He \) is also shown (dotted line). On top, we show transverse kinematics, \( \theta = 90^\circ \), and in the middle and the lower panel we show parallel and antiparallel kinematics, \( \theta = 0^\circ \) and \( \theta = 180^\circ \).

Figure 3: The forward-backward asymmetry \( A_{FB} = \frac{W(\theta=0^\circ;p_m)-W(\theta=180^\circ;p_m)}{W(\theta=0^\circ;p_m)+W(\theta=180^\circ;p_m)} \) is shown for the reaction \( D(e,e'p) \) in a) and for the reaction \( ^4He(e,e'p) \) in b). For \(^4He\), different correlations were used: hard core, \( C_o = 1 \), (solid line), soft core, \( C_o = 0.5 \), (dashed line), and pure mean field, \( C_o = 0 \) (dotted line).
