Modeling Complex Social Systems: A New Network Point of View in Labour Markets

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\textbf{ABSTRACT} Complex Systems is a new field of science studying how parts of a system give rise to the collective behaviours of the system, and how the system interacts with its environment. Graph theory is a fundamental tool in the study of social systems and economic issues. The input-output tables are precisely one of the main examples of it. We use the interpretation of labour market through networks to get a better understanding on its overall functioning. One benefit of the network perspective is that a large body of mathematics exists to help analyse many forms of networks models. If an economic system has a suitable model, then it becomes possible to utilize relevant mathematical tools, such as general systems theory, graph theory and discrete chaos theory, to better understand the way the labour market works. In this article, we apply concepts including structural functions, coverage and invariant sets to a social system’s modelling.

\textbf{INDEX TERMS} General systems theory, labour markets, social system modelling, coverage, invariability.

I. INTRODUCTION

Deterioration of the employment rate is one of the most serious problems in developed countries. The economic crisis in Spain with its associated difficulties raised unemployment of the active population to 26\% in the first quarter of 2013 [1]. The Spanish labour market has traditionally been perceived as a very rigid one for two main reasons: unemployment rates are always very high and the time an individual spends unemployed is very long [2]. This paper presents a new approach to explore this problem. Understanding the labour market is crucial to manage the modern economy. The operation of the labour market is a complicated process which is not easily addressed by governments and has become a big challenge for modern societies.

For these reasons the study of labour markets must be promoted by administrations and governments who must provide the necessary resources since any progress in understanding the labour market mechanisms will improve the correctness of the economic decisions and so the impact on the society’s welfare.

Research with varying approaches has outlined the underlying phenomena to explain market behaviour in order to correctly match supply and demand [3]–[8]. A rigorous analysis of all these models is beyond the scope of this paper. Initially we highlight aspects of the considerable research activity in this field.

The simplest labour market model has some basic features. For example, let’s consider an unemployed person searching for a job. A job offer consists of an offer of employment at a stated wage rate. Classical theories of economic analysis have performed computer simulations to reveal the essential features of this problem [9], [10]. However, the labour market is quite different from other spot markets analysed in economic theory because there are aspects involved that can influence their performance [11], [12] so these models are not suitable [13]. Specifying labour market variables is complicated with incomplete information and uncertainty [14]. For this reason many approaches use probabilistic and/or econometric models of labour markets based on statistical mechanics to explain the empirical evidence [15]–[19].
Other labour market studies examine job searching [13], [20], [21] and job offering [22]. These studies identify the dynamic discrete choice variables to describe their behaviour and try to explain them with discrete variables. In other approaches new elements have been added to the analysis such as migrations [23], [24] or social networking considered as a new technology [25] to report the effects of larger connectivity between social agents and markets. Finally, the latest research uses new models based on human behaviour theories to explain motivations for decision making in labour markets [26]–[28].

Most of these studies try to build a model of the labour market which allows study of the effects of applied policies as well as the consequences of rules affecting different aspects of it. However, very little work has been done to encompass the whole labour market of a region or country in a conceptual framework with the capability to cope with the set of relationships in order to predict the behaviours and to design suitable policies for improving effectiveness.

Many problems in theoretical economics are mathematically formalized as dynamic systems. Recently, an open approach to studying the dynamic behaviour of these models has appeared. In this sense, dynamic systems are often linked to the notion of an attractor. An attractor consists of a subset of the system capable of attracting the rest of the variables that are part of its basin of attraction. Therefore, the attractor would reflect where the variables of our system are tending to. Attractors can appear in dynamic systems belonging to multiple fields, such as the industrial sector [29], product design [30], or even in electronic components such as the memristor [31], [32].

In this paper, we present a mathematical model of the labour market based on a similarity with ecosystems, such as the relationships of predator-prey and competition [33] that can be equated with their analogues supply-demand and competition [34]. The model has exploited the knowledge and the progress made by the research group in modelling of complex systems [35]–[38].

Networks play an important role in a wide range of economic phenomena [39]–[43]. The diffusion of information across a network only requires a single contact between nodes, making network connectivity the crucial determinant of whether or not these “simple contagions” will spread [44]. Despite this fact, standard economic theory rarely considers economic networks explicitly in its analysis. A wide range of empirical studies of labour markets have shown that a significant fraction of all jobs are found through social networks. The role of informal social networks in labour markets has been emphasized initially by Granovetter [45]. He found that over 50% of jobs were found through personal contacts. In a recent paper, Calvó-Armengol and Jackson [46] introduced a network model of job information transmission. Network sampling is only useful if a researcher can produce accurate global network estimates. Recently, Smith [47] explored the practicality of making network inferences and examined networks with a skewed degree distribution surveying the limit that the number of social ties a respondent can list.

Also, network theory analysis methods are widely used in social media. Lloret-Climent et al. [48] apply network theory and several concepts of chaos theory (such as orbits, coverage, invariance and circular flow) to the labour market of a region of Spain. Thus, they obtain a systemic explanation of the mechanism of this type of market, finding a dependence between the decisions of supply and demand of labour. Lv et al. [49] have studied existing colative networks in a sample of 67 scientific articles, according to 3 hierarchical levels: researcher-level, institution-level and country-level. These networks are quantified using the parameters of centralization, average degree and clustering coefficient. Finally, they conclude that although co-authorship networks are scarce among many countries, the USA and China publish most of the articles and present a great deal of collaboration. Wang [50] makes use of social network analysis (SNA), the concepts of observability and controllability, and the 0-1 programming optimization model, in order to establish monitoring in food supply networks, with the aim of controlling harmful substances in these networks and thus ensuring food safety. Also interesting is the study by Vimalajeewa et al. [51], in which they use SNA techniques to characterize behavior in groups of cows. In particular, thanks to node centrality measures, network entropy, animal social interaction and nearest-neighbour frequency matrix, they show that social affiliation of sick cows is lower than that of healthy cows.

On the other hand, Ghahramani et al. [52], analyse the patterns underlying the network of mobile phone interactions between people in different areas of Macau, concluding that these people tend to communicate in regions that are close to each other. Finally, thanks to the development of social networks, the Internet of Things (IoT), cloud-based computing and 3D printing, Xiong et al. [53] propose the implementation of a new mode of manufacturing: social manufacturing (SM). This new form of personalized manufacturing would allow the consumer to materialize an idea into a final product, starting with the design of the product and ending with its manufacture and final use.

We interpret the labour market through networks. One benefit of the network perspective is that a large body of mathematics exists to help analyse many forms of network models. If an economic system has got a suitable model, then it becomes possible to use relevant mathematical tools, such as graph theory, to better understand the way the labour market works. This interpretation allows us to use the concepts of coverage and invariance alongside other related concepts. The latter will allow us to present the two most important relations in a labour market –supply-demand and competition– in a different way.

In the present work, several mathematical concepts and formalisms are proposed in order to model a general labour market. These concepts are constructed from three
determination and causal interactions may be of two classes: actions between the elements. Real determination is causal. People (employed or unemployed) and by determining inter-relationships. We define a labour market as a pair of companies and the labour market simply without reference to its proper-ty and discrete chaos theory. Next, the terms of coverage and invariance applied to the field of the labour market are presented, extracting from these some interesting properties. Finally, we make use of the concepts seen above to study a particular case of the labour market: the Spanish tourism sector.

With regard to the innovative contributions of the following study, we can highlight the following: (1) the description and modelling of the relations of a labour system is carried out using structural functions, i.e. functions that give us back the direct relations of one or more variables related to the labour system under analysis; and (2) new concepts of coverage and invariance applied to the field of the labour market are used, which allow us to discover certain patterns and trends between sets of variables in the system, as would be the case of sets of variables influenced only by other sets (coverage), or sets of variables whose relations do not escape from them (invariability).

However, the study carried out would also have a number of limitations, namely:

- The proposed modelling of complex social systems (such as labour markets) involving a large number of variables can be complicated, requiring in these cases the use of causal analysis software capable of working with large amounts of data.
- Structural functions, coverage and invariance are of a qualitative nature, unlike many concepts related to network theory (such as centralization, network density, average degree, etc.), which focus on providing quantitative results on systems of this nature.
- The study carried out in this article constitutes a first approach to the modeling of complex social systems through the concepts and properties suggested. Therefore, it is essential to introduce new terms, such as orbits, attractors and basins of attraction, which will allow us to model such systems in a more comprehensive way.

Following the introduction, Section II outlines the basic concepts of the mathematical model and defines the structural functions associated with competition and supply-demand. Section III introduces the concepts of coverage and invariance in labour markets. Sections IV and V show an application of the theory described in part of the labour market, the Spanish wholesale tourism sector. Finally, Section VI summarizes and presents concluding remarks.

II. BASIC CONCEPTS

Our basis for studying economic systems was the General Systems Theory of Bertalanffy [54], who defined a system as “a set of elements standing in interrelation among themselves and with the environment.” Here, we present the labour market simply without reference to its properties. We define a labour market as a pair of companies and people (employed or unemployed) and by determining interactions between the elements. Real determination is causal determination and causal interactions may be of two classes: transactions, with material and monetary changes; and relations, which are indirect consequences of transactions such as competition and supply-demand relations.

Generally, network theory is represented by a square matrix of interactions between agents. In this paper, network theory is replaced by structural functionalism and the concepts of coverage and invariability. But there is a distinction between structural functionalism, coverage, invariability, and network theory. Network theory is quantitative, whereas structural functionalism, coverage, and invariability are qualitative. In economic systems, a mathematical model means a formula, a matrix or an equation or a system of equations that provide an accurate description of a phenomenon or behaviour. They also characterize and/or simulate and predict in space and/or time the dynamics of economic systems.

Economics is a social science in which the phenomena are produced by the interaction of thousands of personal and business decisions in many social areas. It is appropriate to think that many of these phenomena are related, although these cause-effect relationships are not evident. In this sense, the application of network theory to analyze these phenomena may help to explain the operation of social activities or economic sectors. Labour markets are a good example for this. In this connection, it is crucial to establish correct analogies between theory and elements of the labour market to find explanations that help to understand how socio-economic reality works. Therefore, in the development of this research we have established analogies between network theory and the labour market that fit observable social behaviour.

The structural function is a mathematical function that works in a qualitative way. A mathematical function usually assigns numerical values to numerical values whereas the structural function assigns to each set of agents another set of agents.

In our model, the state equations are represented by the structural functions associated with competition and supply-demand relationships. The structural function associated with competition assigns to the state variable of each competitor the set of all of its competitors. Thus, our first analogy of the model with the labour market is to consider any employed or unemployed people associated with the set of all the people that compete with them according to competition and supply-demand relationships.

Here, we present the labour market by adapting the concept of system-linkage [35], [55]. We only give a simple view of them and avoid analysing their features.

Definition 1: Labour market $S = (M, R)$ is the pair formed by object set $M$, determined by all people offering their skills to employers in exchange for wages, salaries and other forms of compensation. Participants in the labour market include any person $x_i$ who is seeking to work for compensation and any person or company $y_j$ that is looking for people to perform labour and a set of binary relations, so that $R \subseteq P(M \times M) = P(M^2)$ (P means “parts of a set”).
Thus we assume that $R$ is a set of relations between elements of $M$. $R$ is formed of interactions that determine the elements. For example, if $r_c$ is the competition relation, $(x_i, x_j) \in r_c$ will signify that the persons $x_i$ and $x_j$ compete among themselves for a job or a better position in the company; $(y_i, y_j) \in r_c$ will signify that the companies $y_i$ and $y_j$ compete among themselves for more market share or, if $r_sd$ is the supply-demand relation, $(y_i, x_j) \in r_sd$ means that $x_j$ works with any company $y_i$; and $(y_i, y_j) \in r_sd$ means that company $y_i$ has outsourced company $y_j$ performing some activity.

We know that the labour markets are full of relationships that are not limited to pairwise interactions, but as seen in [55], an equivalence between an n-sized relationship system and 2-sized relationship system has been proved so limiting the labour markets to only binary relationships makes sense [56]. This assumption may not be evident in the socioeconomic context in which, for example, there are collective bargaining processes both to hire and to fire workers.

**Definition 2:** In the labour market $S = (M, R)$, if the pair of person and company satisfies $(y_i, x_j) \in r_sd$ when $r_sd$ is the supply-demand relation, the company $y_i$ directly influences the person $x_j$, that is to say, $x_j$ is an employee of company $y_i$. Similarly, if $(x_i, x_j) \in r_c$ or $(y_i, y_j) \in r_c$ when $r_c$ is a competitive relationship, then $x_j$ directly influences $x_i$ or $y_i$ respectively. That is to say, $x_i$ and $x_j$ compete with each other for a job or a better position in the company or, companies $y_i$ and $y_j$ compete in the same sector.

**Definition 3:** We will say that the person $x_j$ influences the person $x_i$ indirectly by means of a competitive relationship when the persons $x_1, x_2, \ldots, x_n \in M$ satisfy: $(x_i, x_1) \in r_c, (x_1, x_2) \in r_c, \ldots, (x_{n-1}, x_n) \in r_c, (x_n, x_i) \in r_c$.

Similarly, we will say that the company $y_i$ influences the company $y_j$ indirectly by means of a competitive relationship when the companies $y_1, y_2, \ldots, y_n \in M$ satisfy: $(y_i, y_1) \in r_c, (y_1, y_2) \in r_c, \ldots, (y_{n-1}, y_n) \in r_c, (y_n, y_i) \in r_c$.

That is to say both people and companies compete in the same economic sector. In the same way, we will say that the company $y_j$ influences the company $y_i$ indirectly by means of the supply-demand relationship when the companies $y_1, y_2, \ldots, y_n \in M$ satisfy: $(y_i, y_1) \in r_sd, (y_1, y_2) \in r_sd, \ldots, (y_{n-1}, y_n) \in r_sd, (y_n, y_i) \in r_sd$.

That means company $y_i$ has outsourced company $y_j$ performing some activity; company $y_j$ has outsourced company $y_i$ and so on until $y_n$ outsources company $y_1$.

**Definition 4:** In a labour market $S = (M, R)$, let $f_{sd} : M \rightarrow P(M)$ be the input-output structural function associated with the supply-demand relationship. We defined it as follows: $\forall y_i \in M, f_{sd}(y_i) = M_{y_i} \in P(M)$, where $M_{y_i}$ is a set defined by: $M_{y_i} = \{x_j \in M / (y_i, x_j) \in r_sd\}$.

Any company $y_i$ is associated to the set of all companies that compete with one another for their market share.

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Any company $y_i$ is associated with the set of all its employees and with the set of all companies that perform some work for it.

### III. COVERAGE AND INVARIANCE IN LABOUR MARKETS

The concepts of coverage and invariance are necessary to explain phenomena observed more adequately, predict effects of disturbances, and provide a more reliable basis for possible economic management.

**Definition 6:** Let the labour market be $S = (M, R)$ and the set of persons $A, B \in P(M)$, with an input-output structural function $f_c$. We will say that $A$ covers $B$ if $f_c(A) = B$.

This concept must be understood as any set of employees in a company covering the set of all employees that compete and any set of unemployed persons in an economic sector covering the set of all unemployed persons in this sector since they are all competing for the same job.

If $A, B \in P(M)$ are companies, then $f_c(A) = B$ means that the first set of companies can be understood to compete with the second one.

In the same way, let the labour market be $S = (M, R)$, the set of companies be $A \in P(M)$, the set of companies and/or persons be $B \in P(M)$ and the input-output structural function be $f_{sd}$. We will say that $A$ covers $B$ if $f_{sd}(A) = B$.

This property means that the companies belonging to $A$ have outsourced services and have subcontracted workers to company $B$.

**Definition 7:** Let the labour market be $S = (M, R)$, and the set of companies or persons be $A \in P(M)$, with the input-output structural function $f_c$. If $f_c(A) \subseteq A$, then $A$ is invariant.

The concept of invariance can be interpreted as a type of endogamy between the elements of a single set which are only going to relate to each other maintaining the set’s structure and status in respect of any type of relation. The characteristic example of an invariant set is the set of competitors in the labour market.

If $A$ is the set of employees in a company or unemployed persons in an economic sector, then $f_c(A) = A$ and therefore, the set of employees in a company or the unemployed persons in an economic sector is invariant for the structural function associated with the competitive relationship. Likewise, in some economy $f_c(O) = O$ is verified and thus the sets of companies can also be invariants for the structural function associated with the competitive relationship.

Now, we start proving a series of immediate results stemming from the previous definition.

**Proposition 1:** Let the labour market be $S = (M, R)$, and let the set of companies be $O \in P(M)$, with the input-output structural function associated with the competitive relationship $f_c$. Therefore, if $O$ is invariant, then $f_c(O)$ is also invariant.
Proof: Let the company be \( y_i \in f_i(\mathcal{O}) \); therefore, another company \( \exists y_j \in f_i(\mathcal{O}) \) must exist so that \( f_i(y_j) = y_i \). Since \( \mathcal{O} \) is invariant, \( y_j \in \mathcal{O} \) and then \( y_i \in f_i(\mathcal{O}) \). This proves that \( f_i(\mathcal{O}) \) is invariant.

In addition to Proposition 1, the following propositions are satisfied.

Proposition 2: Let the labour market be \( S = (M, R) \), and let the set of persons be \( P \in \mathcal{P}(M) \) with the input-output structural function associated with the competitive relationship \( f_i \). Therefore, if \( P \) is invariant, \( f_i(P) \) is also invariant.

From Proposition 1, we obtain the following corollary:

Corollary 1: Let the labour market be \( S = (M, R) \), and let the set of companies be \( \mathcal{O} \in \mathcal{P}(M) \), where \( \mathcal{O} \) is invariant and the input-output structural function is \( f_i \). Then, any iteration of \( f_i \) over \( \mathcal{O} \) will also be invariant.

Proof: It is trivial by applying Proposition 1 repeatedly (according to the iteration sought).

Similar results can be obtained by replacing the set of companies \( \mathcal{O} \) with the set of persons \( P \).

Corollary 2: Let the labour market be \( S = (M, R) \), and let the set of persons be \( P \in \mathcal{P}(M) \), where \( P \) is invariant and the input-output structural function is \( f_i \). Then, any iteration of \( f_i \) over \( P \) will also be invariant.

The above propositions lead us to interpret the reality of labour markets in a particular way. If the group of workers (or companies) that operate in a particular economic sector are invariant, we can deduce that competition is limited by the set of actors in this economic sector. That is, individuals of an economic sector are not in labour competition with individuals from other sectors. This may indeed occur in sectors requiring highly skilled labour, where workers are well paid (e.g., surgeons) and unemployment is low. However, it does not exist in other sectors where competition is more open (for example, supermarket stockers), because it is a type of work that many more people can do.

Identifying what kind of sectors have invariant sets of firms and/or workers in such relationships can help define employment policies towards the training of workers and unemployed to encourage job opportunities with higher wages.

Regarding firms, the existence of invariant sets of companies competing for the same workers is a sign that there are entry barriers to the sector that prevent hiring or working to the entities outside this sector. For example, in the sector of hospitals there is competition for hiring surgeons.

In this way, the advantage of using the concepts of coverage and invariability to deal with these issues instead of analysing the economic relations from a classical point of view is that they enable us to use a mathematical function, and in this manner, it makes sense to work with (mathematical) unions and intersections to create varied compositions of structural functions over different sets. In this manner, we are creating a Mathematical Formulation of Economic Systems which uses the concepts of coverage and invariability to obtain conclusions regarding the behaviour of these sets. The supply-demand relation will act in an equivalent fashion to the concept of coverage, whereas the competition relation will be equivalent to the concept of invariant set.

Here we present some results about the behaviour of invariant sets.

Proposition 3: Let the labour market be \( S = (M, R) \), and let the sets of companies and workers be \( \mathcal{O}, P \in \mathcal{P}(M) \), with both invariant, and the input-output structural function \( f_i \). Then, \( \mathcal{O} \cup P \) is invariant.

Proof: Let the company or person be \( x \in \mathcal{O} \cup P \); so, \( x \in \mathcal{O} \) or \( x \in P \). If \( x \in \mathcal{O} \), because \( \mathcal{O} \) is invariant, then \( f_i(x) \subseteq \mathcal{O} \); then, \( f_i(x) \subseteq \mathcal{O} \cup P \). In the case of \( x \in P \), we will use the same method and thereby complete the proof.

Proposition 4: Let the labour market be \( S = (M, R) \), and let the sets of companies, employees and unemployed persons of an economic sector be respectively \( \mathcal{O}, E, D \in \mathcal{P}(M) \) and invariant, with the input-output structural function \( f_i \). Then, \( \mathcal{O} \cup E \cup D \) is invariant.

Proof: Let the company, employee or unemployed person be \( x \in \mathcal{O} \cup E \cup D \); so, \( x \in \mathcal{O} \), \( x \in E \) or \( x \in D \). If \( x \in \mathcal{O} \), since \( \mathcal{O} \) is invariant, then \( f_i(x) \subseteq \mathcal{O} \); thus, \( f_i(x) \subseteq \mathcal{O} \cup E \cup D \). In the case that \( x \in E \) or \( x \in D \), we will employ the same method and thereby complete the proof.

The joint consideration of firms, workers and unemployed persons of economic sectors with invariant sets remains invariant. In this case, an analogy with the labour market can be made with self-employment in professions in which the worker is self-employed or he can hire others workers to do the same job (e.g., lawyers with law firms). The existence of invariant sets in these sectors indicates that there is a combination of barriers to entry (need licenses to work) and a specialized type of work. The identification of these sectors by public authorities can be helpful to reduce or eliminate the need to practice and increase competition to allow hiring more workers.

The same results are obtained with the intersection of invariant sets, as we can see below.

Proposition 5: Let the labour market be \( S = (M, R) \), and let the sets of companies and persons be \( \mathcal{O}, P \in \mathcal{P}(M) \), both \( \mathcal{O} \) and \( P \) are invariant; the input-output structural function is \( f_i \). Therefore, \( \mathcal{O} \) and \( P \) are not disjoint, and \( \mathcal{O} \cap P \) is invariant.

Proof: Let the company or person be \( x \in \mathcal{O} \cap P \). Since \( x \in \mathcal{O} \) and \( P \) is invariant, then \( f_i(x) \subseteq \mathcal{O} \). We use the same method with \( x \in P \) and we obtain \( f_i(x) \subseteq \mathcal{P} \). Therefore, \( f_i(x) \subseteq \mathcal{O} \cap P \) and thus we have \( f_i(\mathcal{O} \cap P) \subseteq \mathcal{O} \cap P \), so that \( \mathcal{O} \cap P \) is invariant.

Proposition 6: Let the labour market be \( S = (M, R) \), and let the sets of companies, employees and unemployed in an economic sector be invariant \( \mathcal{O}, E, D \in \mathcal{P}(M) \), with the input-output structural function \( f_i \). If \( \mathcal{O} \cap E \cap D \neq \emptyset \), then \( \mathcal{O} \cap E \cap D \) is invariant.

In this case the same interpretation can be applied as in the previous propositions, wherein the non-empty intersection between the sets of workers and job seekers may be due to unemployed workers who are actually working in the black economy or employees looking for a better job in the same sector.
Proof: Let the company, employee and the unemployed be \( x \in O \cap E \cap D \). Since \( x \in O \) and \( O \) is invariant, then \( f_c(x) \subseteq O \). □

We use the same method for \( x \in E \) and \( x \in D \), and we obtain \( f_c(x) \subseteq E \) and \( f_c(x) \subseteq D \). Therefore, \( f_c(x) \subseteq O \cap E \cap D \) and thus we have \( f_c(O \cap E \cap D) \subseteq O \cap E \cap D \), and so \( O \cap E \cap D \) is invariant.

Now we will present a series of results proving the relationship between the properties that have been described here and that appear in diverse forms. The following statement confirms how the concepts of coverage and invariability can act jointly.

**Proposition 7:** Let the labour market be \( S = (M, R) \), and let \( O \) be the set of companies and \( P \) be the set of persons. Let \( O, P \in P(M) \); let \( P \) be invariant, with the input-output structural function \( f_{sd} \). So, if \( O \) covers \( P \), \( O \cup P \) will be invariant.

Proof: Let the company or person be \( x \in O \cup P \); therefore, \( x \in O \) or \( x \in P \). If \( x \in O \), since \( O \) covers \( P \), \( f_{sd}(x) \subseteq P \subseteq O \cup P \). Furthermore, if \( x \in P \), since it is invariant, then \( f_{sd}(x) \subseteq P \subseteq O \cup P \). □

In this way we have proven that \( O \cup P \) is invariant.

**Corollary 3:** Under the same conditions as above, \( O \cup P \) covers \( P \) (that is, the set of companies and persons covers the set of persons).

Proof: It is trivial, by applying properties. □

This last proposition allows us to make interesting interpretations of the functioning of labour markets. Using the same previous example, the set of medical companies of hospitals covers the set of surgeons. If the latter is invariant, then so is the combined set. Indeed, no other companies that do not provide medical services contract surgeons. So an invariance is produced. Identifying these relationships can be very useful to the authorities, for example, by detecting abusive situations in hiring these skilled workers who can establish the invariant set of professionals.

**IV. APPLICATION**

The purpose of this example is to illustrate an application of the theory described in part of the labour market, the wholesale Spanish Tourism sector. The example is not intended to be exhaustive, but rather to show the possibilities of the model to explain the labour market. In recent years this sector has been transformed into highly competitive domains. In this example, the labour market will be segmented into five levels, positioning the firms in a circular flow of income (Fig. 1).

In the outer part of the diagram showing contacts with the majority of customers are tour operators (level 5), these companies compete with each other in attracting tourists and maintaining supply-demand relationships with particular geographical areas with large numbers of tourists (for example, the Greek Mediterranean islands, north Africa, the Spanish “costa blanca”, the Atlantic islands, etc.) (level 4). Each geographical area competes with each other and at the same time offers its catalogue of services to tour operators. The geographical areas are specified by a number of providers of tourist services such as hotels (level 3) competing with each other and offering their services to the whole of the area as part of its bid. Hotels require firm-oriented service providers, e.g., telecommunication companies (level 2) compete with each other offering their range of services to hotels.

This trend is in the majority in those specialized areas where the practice of outsourcing is becoming more frequent. Specialized companies need professionals that compete with each other (level 1). Agents within each concentric circle are competitors with each other offering their products and/or services to the adjacent outer circle and demanding products and/or services from the adjacent inner circle.

The labour market system is composed of tour operators, tourist areas, hotels, specialized firms and specialized workers (see Table 1). These entities are in both supply-demand relationships and relationships based on competition. The direct relationships between entities are paired relationships; for example, hotels demand specialized firms. Generalizing this relationship between pairs, we obtain the indirect relationship; for example, hotels are in indirect demand with regard to specialized workers.

**V. DISCUSSION**

These concepts form the basis for interpreting the structural functions associated with supply-demand and competition relationships: the competition function associates each entity with the group comprising all of the entities that compete with each other. Hence, the competition function associated with one hotel will be composed of all the hotels at the same level and so in the same concentric circle. The supply-demand function associates each entity with the group comprising all those demanding employment from the interior
adjacent circle. So, the supply-demand function of the tourist areas would be the group consisting of all hotels.

The concepts of coverage and invariance are defined as follows: when as a result of applying one of these functions in a group of entities another group of entities is obtained, we will say that the first group covers the second. The coverage between groups is entirely connected to the supply-demand relationship since when we act on a group at one level we obtain the group at the preceding level. Hence, when we act on tour operators, we obtain tourist areas, as a result of which level 5 tour operators cover level 4 tourist areas. When we act on tourist areas, we obtain hotels, as a result of which tourist areas cover hotels, and when we act on hotels, we obtain specialized firms; in other words, hotels cover specialized firms and so on.

When we apply these functions to a group of entities and obtain a portion of the same group, the group is said to be invariant. Invariant groups are connected to competitive relationships since when we act on a group of entities at the same level we obtain entities at the same level. Hence, the group composed of tour operators (5), the group composed of tourist areas (4), and the group composed of hotels (3) would be examples of invariant groups with regard to competitive relationships.

The graphical interpretation of propositions 1 and 2 can be represented in Fig. 2.

From the previous results, we obtain the following consequences (see corollary 1 & 2): if the set of employment offers is invariant, any iteration of the structural function associated with the competition relation over the conjunction of offers of employment will also be invariant. Similar results are obtained by replacing the set of employment offers with the set of employment demands or the set of competitors (see Fig. 3).

The properties projected in the above paragraph can be developed for the case of the tourism sector. Among these, we mention that the group offering employment: \{specialized firms\} and to the jobseekers: \{specialized workers\}, and given that both groups are invariant we can deduce that the joint group of \{specialized firms, specialized workers\} will also be invariant for competitive relationships (proposition 3). This can be generalized for different connections between a range of groups (proposition 4).

The intersection also functions between different types of groups; hence, we could consider the groups of jobseekers: \{tourist areas, hotels, specialized firms, specialized workers\}, which is an invariant group for competitive relationships, and as a group offering employment: \{tour operators, tourist areas, hotels, specialized firms\}, which is an invariant group for competitive relationships, the intersection of these groups \{tourist areas, hotels, specialized firms\} is invariant for competitive relations (proposition 5 & 6) (see Fig. 4).

The most important properties taken from the tourism sector are established when two relationships are involved or when the concepts of coverage and invariance are jointly involved. Hence, we can consider the whole group of employment offers: \{tour operators, tourist areas, hotels,
specialized firms) and the whole group of employment requests (tourist areas, hotels, specialized firms, specialized workers). The first employment offers group covers the employment requests group; therefore, the joint group containing all of the entities is invariant (proposition 7).

VI. CONCLUSION

The present work provides the first attempt to address the interactions occurring in the labour market using the concepts of coverage and invariability. The structural functions associated with labour market relationships, along with the concepts of coverage and invariance, allow us to make a different and original presentation of labour markets.

In fact, the concept of invariant set is not unusual in economics since it is directly linked to the concept of vicious and virtuous circles [57].

The shortest circle \( A \rightarrow B \rightarrow A \) implies that the set \( \{A, B\} \) is invariant. In other words, when there is a circle within a set of variables, this set will also be invariant. These properties illustrate the importance of the concepts of coverage and invariance because they allow us to study the typical relationships occurring in a labour market in a much more precise manner. Furthermore, these concepts can be extrapolated to the analysis of other economic relationships.

One benefit of the network perspective is that a large body of mathematics exists to help analyse networks models. If an economic system is suitably modelled, then it becomes possible to use relevant mathematical tools, such as graph theory, to better understand the way the system works. The role of network theory is replaced by structural functions in this article and we also use the concepts of coverage and invariance from graph theory. There is an important distinction between structural functions, coverage, invariance on the one hand, and network theory. Network theory is quantitative, whereas structural functions, coverage, and invariance are qualitative.

The advantage of using the concepts of coverage and invariability to deal with these issues, instead of analysing the labour market relationships from a classical point of view, is that they enable us to use a mathematical function and, in this manner, we work with unions and intersections or create varied compositions of the structural function over different sets, as is demonstrated by the results presented in this article. We are creating a Mathematical Formulation of the labour market which uses the concepts of coverage and invariability to obtain conclusions regarding the behaviour of these sets. The majority of the analytical techniques are based on a mathematical modelling process which does not always faithfully reflect the real model. On the other hand, the type of analysis involving coverage and invariability is accurately based on the real behaviour of the labour market. Relations with examples and analogies to the labour market have been described, which, without wishing to be exhaustive, illustrate certain behaviours of the real world that can be explained with the functions and proven properties. A more rigorous analysis may identify other situations in labour sectors of the economy that can be explained with the proposed theoretical model.

Economic networks may occur with indirect effects. These indirect effects appear in the compositions of the structural functions over the concepts of coverage and invariability. The successive iterations tell us the direction of the evolving economic system. This can be used in the analysis of other questions such as stability within an economic system, because if one important part of the system shows invariance, this really means that a part maintains its status, increasing system resistance so helping the system to adapt to change.

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