The Evolution of Cellar Automaton based on Dilemmma Games with Selfish Strategy

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Abstract

We have proposed two new evolutionary rules on spatio-iterated games that is not mimic evolution of strategies, and mainly discussed the Prisoner’s Dilemma game [10] by the two evolutionary rules [11]. In this paper we focus the first rule, that is, the selfish evolutionary rule for various dilemma games. In contrast to the Prisoner’s Dilemma, there are generally rich phase structures in the dilemma games. First we analytically clear the structure to present phase diagrams in the various dilemma games. Furthermore we simulate the time evolution of the spatio-games in the some representatives of the parameters according to the phase diagrams. Including some mutations, detail investigations are made by a computer simulation for five kinds of initial configurations. As results we find some dualities and game invariant properties. They show a sort of bifurcation as a mutation parameter are varied. In the path from one period to two one some common features are observed in most of games and some chaotic behaviors appear in the middle of the transition. Lastly we estimate the total hamiltonian, which is defined by the sum of the total payoff of all agents in the system, and show that the chaotic period is best from the perspective of the payoff. We also made some primitive discussions on them.

key words: Chicken Game, Prisoner’s Dilemma, Stag Hunt Game, Cellar Automaton, Spatio-Evolutionary Game, Phase Structure

1 Introduction

Traditionally differential equations have been applied to understand various phenomena in the nature and the artificial, especially physical phenomena, since they were used to construct the classical dynamics by I. Newton. It, however, seems that some phenomena are rather relevant to be studied as they follow discrete dynamics. They may include biological, social, ecological phenomena and so on, which have a close relationship to "information". The game theory among many aproaches give one of interesting view of them. In this paper, we discuss various games with Dilemma, some kinds of the Chicken games, Prisoner’s Dilemma (PD), the Stag Hunt game which have been pointed out to be dilemma games by Poundstone [2],[4].

They are symmetric games played by two agents and the payoff function acquired by taking each action is discribed by a bimatrix form. A set of the Chicken games, which include the so-called the Chicken game, the Hero game, the Leader game, are dilemma games because they have two Nash equilibria. In this paper we use the term "the Chicken game" in a narrow sense. In the meanwhile, PD faces dilemma in the sense that pareto optimal ≠ Nash equilibrium [1](the deadlock game does not face any dilemma [2]).

Especially many researches have been made to resolve the dilemma, pareto optimal ≠ Nash equilibrium, and how can we obtain cooperation rationally in PD. Based on PD, iterated games have been studied and Axelrod [3] has shown that cooperation can emerge as a norm in a society comprised of individuals with selfish motives. Moreover spatial version of PD has been discussed so that a defection is to be only evolutionary stable strategy (ESS) [7] if each agent interacts with any other agents. This aspect drastically changes if a spatial structure of the population is considered. If the interaction between agents is locally restricted to
their neighbors\,[5]\,\,[6], a stable coexistence between cooperators and defectors become possible under certain conditions\,[5]\,\,[6]. In the case, one agent plays a game with their neighbors and in next step the agent take the same strategy as that of the agent that acquired highest payoff among the neighbors, which reflects Darwin’s theory, sometimes including ”mutation”\,[8]. Then it is assumed that all agents play at same time and follow the same way. Recently the evolution of the spatio-structured PD has been systematically explored in details in Ref.\,[9].

We have discussed two other evolutionary rules within the framework of the spatio-structured games where agents interact locally on the neighborhood cells\,[10]\,\,[11]\,\,[12]. One way to realize it is to change his(her) action to new one when the agent would get larger payoff if he(she) had chosen the opposite action. This is called the selfish rule in this paper. Second one is to change the action like totalitarian, that is, if the total payoff yielded in the whole system increases when an agent changes the strategy. We call this evolutionary rule the totalitarian rule. This evolutionary role is expected to lead to full cooperative action but it do not so\,[10]\,\,[11].

In this paper we only discuss the former case in various type dilemma games and argue general properties lurking in the time evolution of them. Then we introduce the rule to a kind of mutation based on the Gibbs distribution. As a result of the simulations, we find some general properties when one periodic motion of the population with cooperation undergoes a transition to two periodic one, regardless of changing payoff parameters. In the bifurcation, some chaotic behaviours generally appears. Some considerations will be given for it.

After the Introduction, we discuss our evolutionary rule and give the phase structure of games discussed here based on an analytical study in the section 2. In the section 3 simulation results for a set of Chicken games are obtained based on the results in the section 2. There the time serie of the population of each action is studied in details and some analytical arguments are also given. The section 4 is devoted to the analysis of other dilemma games. Concluding remarks are given in the final section 5.

2 Phase Structure of Dilemma Games

2.1 Spatial structured game as cellar automaton

Two type agents, cooperators C and defectors D are considered on cells with the size \( N = n \times n \). The distinction is made by means of the suffixes C and D. The total number of agents are given by

\[
N = N_C + N_D,
\]

where \( N_C \) and \( N_D \) are the population number of C-agents and D-agents, respectively. The spatial distribution of agents are considered as a two dimensional cellar automaton (CA) consisting of \( N \) cells, where each cell is identified by the index \( i \in N \) refering its spatial position and the state C or D sits on the cell. The state space of all possible configurations is of order \( 2^N \). We assume that an agent \( i \) simultaneously plays with 4 neighbors of \( i \) with a same strategy C or D, and so the game essentially reduces to 2-person game. All agents in the system asynchronously play a game every round. Then the game can be described by a payoff bimatrix such as Table 1.
2.2 Formulation of dilemma games

We give some general and analytic discussions on symmetric games played by two agents and give the phase diagrams of them. The symmetric games with two choices played by two agents are generally defined by a payoff table which is often represented a bimatrix such as Table 1, where two actions C and D show Cooperation and Defection, respectively. $i$ and $j$ distinguishes two agents.

|       | $C_i$ | $D_i$ |
|-------|-------|-------|
| $C_j$ | $(R, R)$ | $(S, T)$ |
| $D_j$ | $(T, S)$ | $(P, P)$ |

Table 1: General form of a payoff table of $2 \times 2$ symmetric game. The left and right variables in the parenthesis show the payoffs of the agent $i$ and $j$, respectively.

Each game is classified according to the magnitude of $R$, $S$, $T$, and $P$. The list of games explored in this paper is the following.

1. Chicken (exploiter) game for $T > R > S > P$.
2. Hero game for $S > T > R > P$.
3. Leader game (which essentially includes the battle of the sexes) for $T > S > R > P$.
4. PD for $T > R > P > S$.
5. Stag Hunt game for $R > T > P > S$.

(1), (2), and (3) are included within a set of the Chicken games in a broad sense, and they and (5) have two Nash equilibria.

We consider a sort of 2 dimensional cellar automaton. One agent exits at a cell on torus (i.e. we consider a periodic boundary condition) with $n \times n = N$ cells. A agent play a game with the four agents stood in the Neumann neighborhood with a fixed action C or D at each step. The payoff acquired in each step is estimated by using Table. 1. Under this situation, if an agent tried a different choice, the agent might get more payoff from the Neumann neighborhood. If so, the agent changes to the opposite action at the next step in the selfish rule.

Let’s estimate the increment of a payoff, $\Delta P(C \rightarrow D)$, when an agent change the action from C to D. It generally depends on the population of C agents of his(her) Neumann neighborhood. They are given as follows;

\[
\begin{align*}
\Delta P(C \rightarrow D)[0, 4] &= 4(P - S), \\
\Delta P(C \rightarrow D)[1, 3] &= (T - R) + 3(P - S), \\
\Delta P(C \rightarrow D)[2, 2] &= T - R + P - S, \\
\Delta P(C \rightarrow D)[3, 1] &= 3(T - R) + (P - S), \\
\Delta P(C \rightarrow D)[4, 0] &= 4(T - R),
\end{align*}
\]

where the numbers of the inside of the square brackets the represent the populations of C and D in the Neumann neighborhood, respectively. That is, $\Delta P(C \rightarrow D)[k, \ell]$ means the increment of the payoff obtained by an agent when the target agent is surrounded by $k$ cooperators and $\ell$ defectors. When an agent change the action from D to C, the signs in the Eqs. (2)-(6) reverse. While the order of the magnitude of $R$, $S$, $T$, and $P$ is only essential in usual game theories, the values of $R$, $S$, $T$, and $P$ themselves play an important role in
an iterated game. By changing the four values, the signs in above equations may go into reverse. To vary all of them independently is really too complex to analyze them. Instead of varying all of them, we change only largest parameter and lowest parameter in each game. For example of the Chicken game with $T > R > S > P$, we change only $T$ and $P$, and fix $R$ and $S$ to some correct values, because $T$ can take from $R$ to $\infty$ and $P$ from $S$ to $-\infty$. $R$ and $S$ are restricted by $T$ and $P$, which they can move so freely. In this paper we basically follow values given by Okada [1] for restricted parameters.

2.3 Phase structure of dilemma games

We investigate the phase structure of each game based on the knowledge given above.

(1) Chicken Game

We adopt $R = 5$ and $S = -4$ (according to Okada [1] and variables are $T$ and $P$. Then

\begin{align*}
\Delta P(C \rightarrow D)[0, 4] &= 4(P - S) < 0 \quad \text{and} \quad \Delta P(C \rightarrow D)[4, 0] = 4(T - R) > 0, \quad (7)
\end{align*}

where the inequality are due to the condition of $T > R > S > P$ of the Chicken Game. In order to explore other equations, we observe the points where the values of the equations are zero;

\begin{align*}
\Delta P(C \rightarrow D)[1, 3] &= (T - R) + 3(P - S) = 0, \quad (8) \\
\Delta P(C \rightarrow D)[2, 2] &= T - R + P - S = 0, \quad (9) \\
\Delta P(C \rightarrow D)[3, 1] &= 3(T - R) + (P - S) = 0. \quad (10)
\end{align*}

The different evolutionary behaviour is expected to appear in each region divided by the boundaries given by Eqs. (8)-(10) on $T - P$ space. They are summarized in Fig.1.

![Fig1. Phase diagram of the Chicken game with the selfish evolutionary rule.](image1)

![Fig2. Phase diagram of the Hero game with the selfish evolutionary rule.](image2)

The part with oblique lines in each Fig. is only available for the corresponding game. The figures in parentheses on real lines in Figs. correspond to the numbers of equations.

(2) Hero game

We adopt $T = 6$ and $R = 5$ and variables are $P$ and $S$. Then equation (7) also holds. For other equations, we
obtain in a similar way to the Chicken game:

\[
\Delta P(C \rightarrow D)[1, 3] = a + 3(P - S) = 0, \tag{11}
\]

\[
\Delta P(C \rightarrow D)[2, 2] = a + (P - S) = 0, \tag{12}
\]

\[
\Delta P(C \rightarrow D)[3, 1] = 3a + (P - S) = 0, \tag{13}
\]

where we generally introduce \( a \equiv (T - R) \) and in this case \( a = 1 \). Fig.2 shows the phase structure of Hero game.

(3) Leader game

Eq. (7) also holds, which is a common property in a set of the Chicken games. For other equations with \( R = -4 \) and \( S = 5 \), we obtain,

\[
\Delta P(C \rightarrow D)[1, 3] = T + 3P - 11 = 0, \tag{14}
\]

\[
\Delta P(C \rightarrow D)[2, 2] = T + P - 1 = 0, \tag{15}
\]

\[
\Delta P(C \rightarrow D)[3, 1] = 3T + P + 7 = 0, \tag{16}
\]

Fig.3 shows the phase structure of the Leader game.

(4) PD game

In PD, the payoffs have to satisfy the following condition: \( T > R > P > S \), and so Eqs. (2)-(6) are trivially positive. So any significant structure can be found. We here make a brief comment on PD. For iteration of PD, an additional condition is usually imposed:

\[
2R > S + T. \tag{17}
\]

The second condition, however, is considered to have not any essential meaning for the structured spatio-game. That the breakdown of this condition (17) really induces some interesting phenomena in spatio PD with totalitarian rule has been pointed out \[10, 11\].

![Fig3. Phase structure of the Leader game with selfish evolutionary rule.](image1)

![Fig4. Phase structure of the Stag Hunt game with selfish evolutionary rule](image2)
Eqs. (2) and (6) are positive and negative in this game, respectively. For other equations, we choose $P = -3$ and $T = 5$ and obtain

$$
\Delta P(C \rightarrow D)[1,3] = 12 - 3R - S = 0,
$$

(18)

$$
\Delta P(C \rightarrow D)[2,2] = 2(2 - R - S) = 0,
$$

(19)

$$
\Delta P(C \rightarrow D)[3,1] = -4 - R - 3S = 0.
$$

(20)

We give the phase structure of this game in Fig.4.

### 2.4 Selfish evolution

For an iterated game, we assume the selfish evolutionary rule in this paper. The selfish evolution is to change present action to opposite one at next step only in the situation that the (target) agent would get larger payoff if he(she) took the opposite action, independent of other agents. Suppose that all the agents follow this rule and each agent updates its rule in regular order on cells, asynchronously.

In the simulation experiment of the next section, we introduce mutant agents with some mutation probability $\mu$ with respect to the Boltzmann-like distribution among regular agents that obey the selfish rule;

$$
\mu \sim \exp[-|\Delta P|q],
$$

(21)

where $q$ corresponds to a temperature in thermodynamics. Agents do not obey the selfish rule with the probability $\mu$. In a simulation we take $\mu$ from 0 to $\infty$.

### 3 Simulation of Chicken like games

In this section we mainly discuss the Chicken game as a representative in details. The investigations can be made in a similar way for the other games. For $T$ and $P$, we choose some representatives from regions in Fig.1 and on the borderlines between two or three regions. Really we explore the following nine points

$$(P, T) = (-12, 5), (-11, 6), (-10, 7), (-9, 8), (-8, 9), (-7, 10), (-6, 11), (-5, 12), (-4, 13).$$

These are noted in Fig. 1.

As initial states we explore the following five cases;

(a) All states are C.
(b) All states but one cell in the center are C.
(c) All states are D.
(d) All states but one cell in the center are D.
(e) Random state with $N_C : N_D = 1 : 1$.

These will uncover outcome of initial state dependence. Agents play asynchronously in order on the lattice (a random order will also discussed later) with the size $N = 12 \times 12$. Though the lattice size $n = 12$ seems to be too small, we have ascertained that essential results are unchanged by magnifying the lattice size except for some cases pointed specially. While configurations immediately converge in almost every case, some interesting cases show rather complex behaviors such as chaotic-like behaviors.
3.1 Chaos-like behaviour and bifurcation

First of all, we study the cases (a)-(d) as initial states. Any phase transition does not occur contrasted with PD game in totalitarian rule[10]. At $q = 0$, in the all cases except for the peculiar points, $(P, T) = (-12, 5)$ and $(P, T) = (-4, 13)$ which exist on the border of the Chicken game with others, the population of cooerators converges to $N_C(t) = N/2$ after large step $t$. Because of one period, we call the state $P_1$. This makes a checked pattern consist of C and D. On the one hand, at $q \rightarrow \infty$, all cases come to 2 period state $P_2$. How about intermediate $q$? We note that in all cases common behaviors are observed in the intermediate value of $q$. Some chaotic behaviour and intermittent chaos appear. These are showed in Figs.5 and 6.

At a large $q$, the state converge to $P_1$, and as $q$ becomes smaller, $C(t)$ begin fluctuating and comes to a chaotic state. As $q$ is made further smaller, the amplitude of $C(t)$ inreses and chaotic behaviour (C-state) become more and more conspicuous (Fig. 5). Of course since we now consider a finite system, it can not be actually chaos. As $q$ is made further and further smaller, an intermittent like chaos ($CI$ − $state$) appears (Fig. 6). So the parts with small amplitude begin clustering but the parts with a large amplitude also appears at small $t$.

After that, the $BI$ − $state$, which has initially $P2$-like state and releases some chaotic lumps as $t$ grows larger, appears (see Fig. 7). At next stage of smaller $q$, a linerly damped oscillation arises but it does not completely reduces to one value but fluctuates around $N_C = N/2$ protractedly($D$ − $state$, Fig. 8). In further smaller $q$, the small fluctuation grows larger ($DP2$ − $state$, Fig. 9) and it spreads over $t$. In the end it converges.

Figure 5: Typical chaos-like state in $C(t)$ with $n = 14$, $q = 0.25$ and $(P, T) = (-11, 6)$.

Figure 6: Typical $CI$ state in $C(t)$ with $n = 14$, $q = 0.025$ and $(P, T) = (-11, 6)$. 
Figure 7: Typical DI state in $C(t)$ with $n = 14$, $q = 0.005$ and $(P,T) = (-10,7)$.

Figure 8: Typical $D-$state in $C(t)$ with $n = 14$, $q = 0.005$ and $(P,T) = (-11,6)$.

to the complete $P2$ (Fig. 10). The all-C and the all-D repeat by turns. These are the fundamental pattern in
the bifurcation of $P1 \rightarrow P2$. They are summarized as follows;

$$P1 \rightarrow C \rightarrow CI \rightarrow DI \rightarrow D \rightarrow P2.$$ 

In the peculiar points pointed out previously, the states $D1$ only appear. These may give a deep
understanding of a bifurcation phenomenon, as the lattice size grows larger to be infinite degree of freedom.

3.2 p duality

The reason why there is not any phase-transition like phenomenon exists mainly in the fact that the signs of
Eqs. (2) and (6) are determined in the Chicken game. This shows that an agent surrounded by all cooperaters
or all defectors does not change the action by no means in any cases for various initial configurations of the
Chicken game. So a checked pattern consist of C and D is stable. Though the trendency to C or D depends on
the parameter $P$ and $T$, the state necessarily trends to the checked pattern at $q \rightarrow \infty$. This is the reason why
the state becomes $P1$ in $q \rightarrow \infty$ independent of various parameters and the initial states. They only reflects
on the speed of the transition from $P1$ to $P2$. In all cases, however, that the stste transits through geometrical
pattern to $P2$ can be understood analytically.
As \( q \) becomes smaller, \( P1 \) is disturbed and then at \( q = 0 \) the all C and the all D states repeat by turns in the cases of initial states (a)-(d). In the intermediate \( q \), the trend toward converging on a checked pattern competes with the one toward escaping from it stochastically and chaotic behaviors emerge.

As we can also observed from simulation results, there is the following symmetry with respect to \( p \) which is the initial population ratio of C:

\[
p \leftrightarrow 1 - p, \\
C \leftrightarrow D.
\]

We call this symmetry of the considering system \( p \) duality. This show that \( p = 0.5 \) is a singular point, that is, a fixed point.

From above considerations, the properties in the subsection 3.1 result from many facors involved in complicate way. Next we investigate the effect of the factors in the subsequent subsections. First of all, the reflection of C and D arise only when C or D stands alone and is surrounded completely by the opposite action. What happens in the configuration that C or D is adjacency to each other? An extereme point where this happens is the fixed point \( p = 0.5 \). This will be discussed in the next subsection 3.3.

The order of a target agent is also essential for the above results. The case that the order of a target agent is taken on the cells at random is explored in 3.4. The situation that the size \( n \) is even is needed for the stability of a checked pattern on Torus. When \( n \) is taken as an odd number, a frustration prevents from making a complete checked pattern. How about this case? We are going to study it in 3.5.
3.3 Random initial configuration

We discuss here the initial condition (e). the essential difference from (a)∼(d) arise when D change into P2, where a new pattern A appears (Fig. 11). So the transition pattern when q becomes smaller is generally

\[ P_1 \to C \to CI \to DI \to A \to P_2, \]

while for two singular points on the borderline it is

\[ P_1 \to C \to CI \to CID \to D \to A \to AP_2 \to P_2. \]

Though the state A is similar to the state D from appearances alone, it begins with the behavior like the inverse of D and A−like behavior is restored after a while (Fig. 11). When q grows larger, the revival inverse D like behavior spreads over the time series and it becomes to P2 in the end. AP2 is a middle state between A and p2 (Fig. 12). Except for this point, there is not great difference between the behaviors with the random initial configuration and the others in subsection 3.2. It is considered that the effect of mutual interference of an isolated C or D is not so drastic in general. The power of absorption into the checked pattern is so strong that there are not various roots in the transition from \( P_1 \) to \( P_2 \).

3.4 Random selection and P-T duality

The results in the section 3.2 depends strongly on which agent is choosen first, that is, the order of a target agent. Here we study the case of a random ordering. Then the dependence of q in the transition is very simple.
For all initial configuration, (a) $\sim$ (e), the transition is

$$P1 \rightarrow C.$$ 

In the Chicken game on the borderline, however, only $C$ appears.

In the simulation under (e), we find another duality. At small $q$ in the Chicken game, the ratio of the population of $C$ agents to total population in $P1$ is given by Table 2. This show that there is the following approximate duality;

$$Region \ I(II) \iff Region \ IV(III), \quad C \iff D \quad (23)$$

Table 2. The ratio of the population of agents with the strategy $C$ in each region.

| Region | I  | II | III | IV  |
|--------|----|----|-----|-----|
| Ratio of $C$ | 0.65 | 0.52 | 0.45 | 0.33 |

Since the regions in the Chicken game are specified by $P$ and $T$, we call this duality $P-T$ duality. This holds independently of the initial configurations and even on the borderline. This duality can be approximately understood by a simple analytic way but it is not so nice to describe it due to the asynchronous time evolution. The analyses by the mean field method or a replicator equation [14] may give better understanding for the duality but they are beyond this paper.

### 3.5 Odd lattice

We discuss odd lattice, on which the checked pattern is not well defined and so a frustration arises. The simulation shows that the complete checked pattern can not merely exist but the statistical properties are invariant. As the size of the lattice grows larger, eventually these differences will disappear. Mainly we explore the case with initial condition (a). As a result, the states converge into $P1$ at $q \rightarrow \infty$ and the $P-T$ duality can be also observed in the population of $C$ manifestly. A complete periodic pattern, however, curiously appears on the borderline between two areas. An example is given in Fig. 13. Then we also observe the $P-T$ duality in the periodic behavior itself.

![Figure 13: $C(t)$-state in the Chicken game with $n = 11$, and $(P,T) = (-8,9)$.](image-url)
4 Other Dilemma Games

4.1 Hero game and Leader game

The Hero game and the Leader game also face a dilemma because they have two Nash equilibria. All simulation results, however, are same as those of the Chicken game. Studying the details, the fact that these games also satisfy the Eqs. (2) and (6) seems to be crucial. We see that we need only explore the Chicken game in a narrow sense but not study its relative games. Strangely we observe that rough behaviors even in some non dilemmma games are same as those of the Chichen game.

4.2 Stag Hunt game

This game face a dilemma in the sense that it has two Nash equilibria, but the one of which is a pareto optimal. It is not necessary that the relative games of the Stag Hunt game are explored in the similar reason to the Chichen game.

We simulate on 20 points based on the Fig. 4 for five kinds of initial state; (a)-(e). The points are taken from the regions I ~ VI, their border lines, I-II, II,III, III-IV, IV-V, V-VI, VI-I, and the border line between this game and its outside N, I-N, II-N, III-N, IV-N.

We find a period two behavior for small $q$ where all C and all D appear alternately for (a) and (c) cases (in the case (b) ((d)), one D (C) and all C (D) alternate repeat), and a convergence into one state at large $q$ in $C(t)$. This properties is almost common to all initial configurations without the (e). In the intermediate $q$, some chaotic behaviors appears like the Chicken game. The transition is the same as the $P1 \rightarrow C \rightarrow CI \rightarrow DI \rightarrow D \rightarrow P2$.

There, however, is one exception. It is the peculiar point, at which three border lines intersect each other, in Fig. 4. The configuration oscillates between all C and all D all over $q$-values in this point.

There is one difference from the chicken in the convergent state, too. Though the convergent state in the Chicken game is a checked pattern, we find two convergent states in this game which depend on the regions shown in the Fig. 4. One is a checked pattern same as in the Chicken game. Another is the all C (D) for the initial state (a) and (b) ((c) and (d)). The former arises in areas II and III, on their boundaries (I-II, II-III, III-IV) and on the border line between the this game and its outside N. The latter arises in all other regions. The signs of Eqs. (2) and (6) in this game are opposite to those of Chiken game. An agent tends to imitate the actions of the neighborhoods, that is, if the neighborhoods of the agent are all C(D), the agent takes C(D) strategy in the next step. Then a checked pattern, in which every C and D flip every inning, seems to be attractive. This occurs completely in regions II and III, where an agent abide by the decision of the majority (see Eqs. (3) and (5)). It is actually this time that the checked pattern arises.

It is considered that the difference between the Chiken game and the Stag Hunt game is due to the asynchronous time evolution. Simulating under random ordering of a target agent in time evolution, the difference disappear. The both game show the same behaviors as the section 3.3.

In the case that begin with initial state (e), three patterns appears at large $q$. The first pattern is that $C(t)$ fluctuates around the $N/2$ with period two. This occurs in the interval $[I,III]$ where the symbol $[.,]$ means that the interval includes both ends. The second is to converge into all D state which occurs in the interval $(IV,V)$ where the symbol $(.,)$ means that the interval does not include both ends. The third one is to converge into all C state which occurs in the region IV, the border between V and IV and the border between IV and
I. On the border line between this game and its outside, the first pattern appears basically. The behavior of \( C(t) \) in the intermediate \( q \) is essentially the same as those in the Chicken game. At small \( q \) \((q \to 0)\), \( C(t) \) fluctuates around the \( N/2 \).

4.3 Prisoner’s dilemma game

In PD, as stated before, Eqs. (1)-(5) are trivially positive and there is not any significant phase structure. The simulation results, however, are same as those of the Chicken game wholly. Only difference from the Chicken game is that \( P1 \) is all D because the action D is a dominant action in PD.

In the following subsection we investigate total hamiltonian in PD in behalf of all game discussed in this paper.

4.4 Complexity and total hamiltonian

We study the total hamiltonian in PD in which the essential properties do not depend on the explicit value of the parameters included in the payoff matrix. The total hamiltonian is defined as the sum of the payoffs acquired by all agents at each step. This shows that how the behaviors of selfish agents have an influence on the whole system.

The time series of the total hamiltonian and that of the population of C (or) is roughly alike (see Fig. 14). A little difference arise only in the chaotic phase. There is an evident differences in Fig. 15. The population of C is increasing with (damped) oscillation during the initial (damped) oscillation in \( C(t) \). The reformation caused by the oscillation increase the total payoff in the system.

![Figure 14: Total hamiltonian (left) and \( C(t) \) (right) in the PD game with \((R, P, T, S) = (5, -3, 50, -4) \) and \( q = 0.5 \).](image)

We estimate the time average of the total hamiltonian at each \( q \) in PD. This is particularly significant when the configuration of the system is not convergent. Looking at Fig. 16, we find that the time average of the total hamiltonian reaches a maximum in the chaotic phase. It is very suggestive that such complex disturbance induces the maximum of efficiency on an average. It may give the evolutionary understanding of the relation between a complexity and the \( \lambda \) parameter discussed by Langton [13]. The real maximum of the total hamiltonian is nearly equal to 7000, which is close to the convergent value in Fig. 15. Then the damping oscillation in \( C(t) \) makes the total hamiltonian in the system reach a potential maximum. As the oscillation becomes regularly periodic, that is to say small \( q \), the average value of the total hamiltonian decreases, again. In a complete two period where all C and all D arise repeatedly, the total hamiltonian goes up and down.
between two values. The average is reduced to about 700 in PD. We should notice that same results apply for the others games.

Figure 16: Total hamiltonian v.s. log $q$ in the PD game.

5 Concluding Remarks

We study various types of spatial dilemma games under the evolution with the selfish rule on cells in this paper. Thus we find a kind of game universality, that is, the properties of time evolution are game invariant in wide range. The behaviors show some common characteristics even in non-dilemma games.

We must note that the explicit values in a payoff matrix themselves should play an important role, differently from the usual game theory. It leads to the varied phase diagrams (Figs.1-4), which satisfy the corresponding conditions made among $R$, $S$, $T$ and $P$ to the games. This is an important and theoretical defference between spatio-iterated games and non iterated games. The simulation results made under the selfish rule, however, mostly are the same as the Chicken game in a narrow sense.

A common behavior of them are that a sort of bifurcation is observed universally. Though we see one periodic begavior in $C(t)$ when a fluctuation parameter $q$ is large, as the parameters become smaller, $C(t)$ is chaotic and finally reaches a two period state at $q = 0$. If a continuous limit of the infinite degrees of freedom is taken, the present analysis may lead to a deep understanding of a period double bifurcation or it also uncovers the difference between continous and discrete systems.

We pointed out that in the chaotic phase the total hamiltonian reaches a maximum. This result should
be compared with the one made under totalitarian strategy. The totalitarian may not necessarily still more contribute to society than egoist. Investigating this is so interesting and will be a next our work.

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