A modified economic order quantity of vendor – buyer non-cooperative supply chain model for imperfect quantity and inspection error using the nash equilibrium concept

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Abstract. In this paper, the Economic Order Quantity (EOQ) of two-level supply-chain systems has been analyzed using the Nash Equilibrium concept. In this paper, a two-echelon supply chain consisting of a single vendor and a single buyer that both under the non-cooperative strategy. A mathematical analysis method, which according to appropriate literature, data and recent research result, is used to determine the mathematical formula of expected individual total cost both vendor and buyer, expected total cost of inventory system and also EOQ of decision variables. The non-cooperative strategy is used by the vendor and buyer to gain their optimal revenue. Thereby, game theoretical solution which is Nash equilibrium concept is used to determine the economic order quantity of this system. In this study, several assumptions are considered like the existence of the imperfect quality product in the manufacturing process and inspection error (Type I and Type II) in buyer’s place. To the best of authors’ knowledge, this is the first paper that deals with a game theoretical solution, which is especially Nash equilibrium, in probabilistic supply chain under the imperfect condition and inspection error.

1. Introduction
Supply chain analysis is one of the interesting research topics in operation research in which industrial engineering problem as a primary reference. There are many result about supply chain with specific assumption [1-12]. The objective of supply chain analysis is to find the mathematical model, often called by inventory model, that represents the relationship between a firm in supply chain system and to find the optimal condition for all of the decision variables. In recent decades, the game theory approach introduced as an alternative way to find the optimal solution of a supply chain system. Because it can be analyzed the possibility of various strategy from each party in the supply chain system. There are also many results about the application of game theory in supply chain management analysis, for example [13-16]. One of the conditions in the supply chain system is a condition when there is no coordination and collaborative in term of optimal strategy between each player in the supply chain system. It’s different with an integrative scheme that each player prefers to take coordination to gain mutual long-term profit. There is also no dominant player. First, each player observes the other player strategy and take his own optimal strategy. This condition leads to the Nash equilibrium concept. There is some reference to the application of the Nash equilibrium concept to determine the economic order quantity of inventory model, like in Elyasi et al.[10], Alaei et al. [11] and Esmaeili et al. [12]. But according to best of our knowledge, analysis this EOQ using Nash equilibrium is still limited to the deterministic model, so in this paper we propose the analysis of the
application of Nash equilibrium in probabilistic two level inventory model. Based on background and research progress above, in this paper, we will extend and investigate the continuous review of the probabilistic integrated vendor-buyer system for imperfect quality that has been analyzed by H-J. Lin, (2013) [5]. We extend those work under the condition that inspection process policy at the buyer’s side is not 100 % perfect (Type I and Type II inspection error) and also under the condition that there is no coordination strategy between vendor and buyer.

2. Method

General Assumptions In this paper, we use the vendor-buyer inventory model as a basic model to formulate the expected total cost of continuous review supply chain system. The system is consist of a single vendor and a single buyer with a single product. It is assumed that defective items only occur in the production process and it exists in random number of defective items in retailer arriving order lot. On the vendor’s side, the defective items exist in an arriving lot of size \( Q \) with defective percentage \( y \) and probability function \( f(y) \). Vendor’s production rate for non-defective items is greater than buyer’s demand rate or \( (1 - y)P > D \). Shortage conditions are allowed in the relationship between the vendor and the buyer. Shortages are partially back-ordered with a fraction of the demand \( \beta, \beta \in [0,1] \) during the stock out period that will be backordered. The inspection process at the buyer is imperfect. The probability of classifying a non-defective item as defective is \( e_1 \). Meanwhile, the probability of classifying a defective item as non-defective is \( e_2 \). The buyer will return all item classified as defective and will be given a full price refund as a warranty from the vendor at the end of 100% screening process. The vendor incurs a cost of per unit defective item. The vendor will be sell the returned items from the buyer at the reduced price (discount) to the secondary market. There is \( c_r \), which is cost of rejecting the non-defective item. It is due to error type II in inspection process. Furthermore, \( c_r \) is the difference between the regular and the discounted price. The customers who buys the defective item will detect the quality problem and return them to the buyer, and the buyer must be replace with a good item. Both the vendor and the buyer incur a post-sale failure cost for the items returned from the market. The buyer will return all defective items to the vendor upon receipt of the next lot. In this research, we proposed the expected total cost of the supply chain system under the condition when there is no cooperative strategy and responses between the vendor and the buyer.

Notations. In this paper, we use some following parameter with some specific notation

\( Q \): The size of the shipments from the vendor to the buyer.
\( n \): The number of lots in which the product is delivered from the vendor to the buyer.
\( L \): Length of lead time for the buyer.
\( r \): Reorder point for the buyer.
\( k \): Safety factor for the buyer.
\( D \): Expected demand per unit timeonthebuyer.
\( P \): The production rate at the vendor.
\( F \): The freight (transportation) cost per shipment.
\( S_b \): Buyer’s ordering cost per order.
\( S_v \): Vendor’s setup cost per production.
\( h_{b1} \): Buyer’s holding cost for a non-defective item per unit per cycle.
\( h_{b2} \): Buyer’s holding cost for a defective item per unit per cycle.
\( c_w \): Vendor’s unit warranty cost per defective item.
\( c_t \): Inspection cost at the buyer.
\( c_r \): The cost of rejecting a non-defective item.
\( c_{pb} \): The pose—sale defective item cost for the buyer.
\( c_{pv} \): The pose—sale defective item cost for the vendor.
\( I_1 \): The number of the item that are classified as defective in each delivery of \( Q \) units.

\( I_2 \): The number of items that are returned from the market in each delivery of \( Q \) units.

\( y \): Defective rate in order lot.

\( \beta \): Fraction of demand during stock out period that will be ordered, \( \beta \in [0,1] \)

\( x \): Buyer’s unit screening cost.

\( e_1 \): The probability of a Type I inspection error (classifying a non-defective item as defective).

\( e_2 \): The probability of a Type II inspection error (classifying a defective item as non-defective).

\( \pi \): Buyer’s shortage cost per unit item.

\( \pi_0 \): Buyer’s marginal’s profit (cost of fractoflostofdemand) per unit.

3. Result and Discussion

In this paper, we will extend and investigate the continuous review of the probabilistic integrated vendor-buyer system for imperfect quality that has been analyzed by H-J. Lin, (2013) [5]. We extend those work under the condition that inspection process policy at the buyer’s side is not 100% perfect (Type I and Type II inspection error) and also under the condition that there is no coordination strategy between vendor and buyer. In this research, we use Nash equilibrium concept to determine the economic order quantity.

3.1. Buyer’s expected the average total cost per unit time

While inspecting the items, errors may occur because of human activity. It’s reached an error type I and type II. The inspectormaymisclassify non-defective item as defective item with probability \( e_1 \), or misclassify defective items as non-defective with probability \( e_2 \). Then, the vendor will incurs a cost \( c_{vw} \) from each defective item and incurs a cost of each non-defective item classified as defective. The buyer incurs a post-sale cost \( c_{pb} \) per unit item returned from the market while the vendor incurs \( c_{pu} \) per unit item. The defective item which are found by inspector in buyer’s inspection process and each defective item that returned from the market to the vendor as a single batch at the time of delivery of the next lot. By definition, we calculate \( I_1 \) and \( I_2 \) as follows: \( I_1 = Q(1 - y)e_1 + Qye_2 \) and \( I_2 = Qye_2 \). The buyer has three kinds of holding costs: defective item holding cost, non-defective item holding cost, and returned items from the market (including defective item that are returned from the market in end of inspection process because ofan error type II) holding cost. There are defective items in an arriving order of size \( Q \) with probability \( y \) and \( \theta \) (\( \theta \) is percentage rate from \( Q(1 - y) \) items). Furthermore, the number of non-defective item in each shipment, including error by human activity in a inspection process, is \( I_1 - I_2 \). The buyer’s average inventory of defective items per cycle with inspection period \( Q/x \), can be formulated by \( \frac{Q^2(1-y)e_1}{x} + \frac{Q^2ye_2}{x} \).

Withholding cost \( h_{b_1} \left( \frac{Q^2(1-y)e_1}{x} + \frac{Q^2ye_2}{x} \right) \). Meanwhile, the average inventory level of non-defective item for buyer per cycle can be formulated by \( \frac{Q(1-y)e_1}{2} + k \sigma \sqrt{L} + (1 - \beta)E[(X - r)^+] \).

Withholding cost \( h_{b_2} \frac{Q(1-y)(1-e_1)}{D} \left( \frac{Q[e_1-e_2]}{2} \right) + k \sigma \sqrt{L} + (1 - \beta)E[(X - r)^+] \). Then, the average inventory of item that are returned from the market to the vendor per cycle per unit time is given by \( \frac{Q^2(1-y)(1-e_1)e_2}{2D} \). Withholding cost \( h_{b_3} \frac{Q^2(1-y)(1-e_1)e_2}{2D} \). In this research, we use lead free time which is also used by Lin [5]. Therefore, buyer’s total cost per cycle per unit time is the sum of the cost of due to the order (ordering cost), transportation cost, screening cost (and also error in inspection) cost, total holding cost, expected shortage cost, and lead time crashing cost.

\[
TC_b(Q, k, L) := S_b + F + c_{ib}Q + c_{pb}ye_2Q + h_{b_1} \left( \frac{Q^2(1-y)e_1}{x} + \frac{Q^2ye_2}{x} \right) + h_{b_2} \frac{Q(1-y)(1-e_1)}{D} \left( \frac{Q[e_1-e_2]}{2} \right) + k \sigma \sqrt{L} + (1 - \beta)E[(X - r)^+] + h_{b_3} \frac{Q^2(1-y)(1-e_1)e_2}{2D} + [\pi + \pi_0(1 - \beta)]E[(X - r)^+] + C(L)
\]
The expected length of the cycle time under the lot of size \( Q \) is \( E[T] = \frac{Q(1-\gamma)(1-e_1)}{D} \). Finally, using renewal reward theorem, the expected average total cost per unit time for the buyer is

\[
ETC_b(\, Q, k, L) = \mathbb{E}[T] \cdot E[T] = \frac{ETC_b(\, Q, k, L)}{Q(1-\gamma)(1-e_1)}
\]

\[
\equiv ETC_b(\, Q, k, L) = \frac{D}{Q(1-\gamma)(1-e_1)} \left( S_b + F + \bar{E}E(X - r)^* + C(L) \right)
+ \frac{D}{(1-\gamma)(1-e_1)} (c_{ib} + c_{ib} \gamma e_2) + \frac{h_v \frac{D}{x}}{(1-\gamma)(1-e_1)} (Q(1-\gamma)e_1 + \gamma e_2)
+ h_b \frac{Q(\gamma e_1 - e_2)}{2} + k\sigma L + (1-\beta)E[(X - r)^*]
\]

(2)

where \( \bar{E} = \pi + \pi_0 (1 - \beta) \)

3.2. Vendor’s expected average total cost per unit time
The production rate of vendor’s non-defective items must be greater than the buyer’s demand rate. The vendor’s inventory level will increase gradually. But, if the total required amount \( nQ \) is fulfilled, then the vendor stop producing items immediately. The vendor’s inventory per production cycle can be obtained by subtracting the accumulated buyer inventory level from the accumulated vendor inventory level as follows

\[
\frac{nQ}{2} \left\{ (n-1)T + \frac{Q}{P} \right\} + \left\{ (n-1)T + \frac{Q}{P} - \frac{nQ}{P} \right\} - [1 + 2 + \cdots + (n-1)]QT
\]

\[
= \left\{ nQ \left( \frac{Q}{P} + (n-1)T \right) - \frac{nQ(nQ/P)}{2} \right\} - T[Q + 2Q + \cdots + (n-1)Q]
\]

\[
= \frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)}{2D}
\]

(3)

and then, the holding cost for the vendor is

\[
h_v \left\{ \frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)}{2D} \right\}
\]

(4)

The vendor will incur a cost \( c_{wu} \) from each defective item and incurs a cost of \( c_r \) for each non-defective items classified as defective. After adding the setup and warranty cost, Type I and Type II error costs, the vendor’s total cost per cycle per unit time \( TC_v(\, Q, n) \) is

\[
TC_v(\, Q, n) = S_v + c_w \gamma + c_{wu} \gamma e_2 + c_r \gamma e_1 + h_v \left\{ \frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)}{2D} \right\}
\]

(5)

Therefore, the expected average total cost per unit time for the vendor can be obtained as

\[
ETC_v(\, Q, n)
= \frac{D \left\{ S_v + c_w \gamma + c_{wu} \gamma e_2 + c_r \gamma e_1 + h_v \left\{ \frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)}{2D} \right\} \right\}}{nQ(1-\gamma)(1-e_1)}
\]

(6)

3.3. Economic Order Quantity using Nash Equilibrium Scheme
According to the assumption, we analyzed the relationship between manufacturer and retailer in a supply chain system when they not willing to take a cooperative strategy. Related to the game optimal
solution, we considered the noncooperative static game and follow the Nash equilibrium scheme. For doing so, the first-order partial derivative of best responses of the player with respect to $Q$, $k$, and $n$, is set equal to zero. Then, by solving this equation set, we have a solution of Nash equilibrium for each decision variables as follows.

$$-rac{D}{Q^2(1-\gamma)(1-e_1)} \left( S_b + F + \pi \sigma \sqrt{L} \left( \sqrt{1+k^2-k} \right) + C(L) \right) + \frac{h_{b_3} D(1-\gamma)e_1 + ye_2}{x(1-\gamma)(1-e_1)}$$

$$\left( h_{b_2} \xi_1 - e_2 \right) + \frac{h_{b_2} ye_2}{2} = 0 \quad (7)$$

$$\frac{1}{2}\sigma \sqrt{L} \left( \frac{k}{\sqrt{1+k^2}} - 1 \right) \left[ \frac{\bar{D} Q(1-\gamma)(1-e_1)}{Q(1-\gamma)(1-e_1)} + h_{b_2}(1-\beta) \right] + h_{b_2} \sigma \sqrt{L}$$

$$= 0 \quad (8)$$

$$\frac{D \bar{\pi} \sigma \sqrt{L} \left( \sqrt{1+k^2-k} \right)}{4Q \sqrt{L}(1-\gamma)(1-e_1)} + \frac{h_{b_2}(1-\beta) \left( \sqrt{1+k^2-k} \right) \sigma}{4\sqrt{L}} = 0 \quad (9)$$

$$\frac{DS_v}{-nQ^2(1-\gamma)(1-e_1)} + \frac{D \left( 1 - \frac{e_1}{2} \right) h_v}{(1-\gamma)(1-e_1)P} + \frac{Ph_v(n-1)}{2(1-e_1)} = 0 \quad (10)$$

$$\frac{D [S_v + c_u Qy + c_m Qye_2 + c_r Q(1-\gamma)e_1]}{n^2 Q(1-\gamma)(1-e_1)} + \frac{Dh_vQ}{2P(1-\gamma)(1-e_1)} + \frac{Qh_v}{2(1-e_1)} = 0 \quad (11)$$

By solving equation (7), (8), (9), (10) and (11) in all decision variable ($Q$, $k$, $n$, $L$), we can get a solution which can be concluded as Nash equilibrium of supply chain system. Furthermore, those equilibrium is also an economic order quantity of the respective supply chain system.

4. Conclusion

In this paper we consider with the probabilistic vendor-buyer inventory model for imperfect quality with lead-free demand, it’s mean that lead time demand is uncertainty and information about lead time distribution are limited only in first and two moments. The inspection process on the buyer’s side is not 100% perfect yet, so there are two types of error that occur in the inspection process. We also consider the partial back ordering process with a certain rate. We investigate the optimal order quantity under the condition that there is no coordination between two parties in the system. For doing so, we use the Nash equilibrium concept to get the optimal decision of each party. In this paper, we propose something new about the application of Nash equilibrium to determine the economic order quantity (EOQ) of inventory model under probabilistic assumptions and also kinds of error of production and inspection process in supply chain system.

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References

[1] Goyal S K 1977 *Int.J.Prod.Res.* **15** 107
[2] Banerjee A 1986 *Decision Sci.* **17** 292
[3] Huang C K 2002 *Prod.Plan.Control* **13** 355
[4] Hsu J T and Hsu L F 2012 *Int.J.Eng.Comp.* **3** 703
[5] Lin H J 2012 *Yugosl. J. Oper. Res.* **23** 87
[6] Salameh M K and Jaber M Y 2000 *Int. J. Prod. Econ.* **64** 59
[7] Abad P L and Jaggi C K 2003 *Int. J. of Prod. Econ.* **83** 115
[8] Abad P L 1994 *Eur. J. of Oper.R.* **78** 334
[9] Bylka S 2003 *Int. J.of Prod Econ* **82** 533
[10] Elyasi M, Khoshalhan F, and Khanmirzae M 2014 *Int. J. Eng. Comp.* **5** 211
[11] AlaeiS, Hajji A, Alaei R and Behvaresh M 2015 *Acta Polytech. Hung.* **12** 221
[12] Esmaeili M, Aryanezhad M B, and Zeephongsekul P 2009 *Eur.J.Oper.Res.* **1** 95
[13] Setiawan R and Triyanto 2016 *Far East J. Math.Sci.* **99** 109
[14] Setiawan R 2016 *Far East J.Math.Sci.* **100** 1695
[15] Setiawan R 2018 *J.Phys.: Conf.Ser.* **983**
[16] Setiawan R 2018 *J.Phys.: Conf.Ser.* **1022**