DÉNES MOLNÁR
Physics Department, Columbia University
538 West 120th Street, New York, NY 10027, U.S.A.
E-mail: molnard@phys.columbia.edu

Differential elliptic flow and particle spectra are calculated from covariant Boltzmann transport theory taking into account the finite transport opacity of the gluon plasma produced in Au+Au at \( E_{\text{cm}} \sim 130 \text{ A GeV} \) at RHIC. The solutions are shown to depend mainly on the transport opacity, \( \chi = \int dz\sigma_{t} \rho_{g} \). The elliptic flow saturation pattern reported by STAR indicates that \( \chi_{b=0} \sim 25 \), i.e., the gluon plasma is \( \sim 80 \) times more opaque than the pQCD estimate based on HIJING. Such large opacities are also consistent with the measured charged hadron spectra.

1 Introduction

At present we have a very limited understanding of the properties of the partonic environment created in heavy-ion collisions at RHIC. Predictions for the density of the gluons produced vary by a factor of five depending on the model considered. Elliptic flow, \( v_{2}(p_{\perp}) = \langle \cos(2\phi) \rangle_{p_{\perp}} \), the differential second moment of the azimuthal momentum distribution, and the high-\( p_{\perp} \) suppression of the particle spectra have been the subject of increasing interest because they provide important constraints on the density and effective energy loss of partons.

The origin of the remarkable saturation of elliptic flow \( v_{2}(p_{\perp}) \to 0.2 \) above \( p_{\perp} \sim 2 \text{ GeV} \) reported by STAR at Quark Matter 2001 is an open question for theory. Calculations based on inelastic parton energy loss do predict saturation or decreasing \( v_{2} \) at high \( p_{\perp} \). These calculations are valid for high \( p_{\perp} \), where collective transverse flow from lower-\( p_{\perp} \) partons can be neglected and Eikonal dynamics is applicable. However, a constant spatial anisotropy was assumed throughout the evolution, while in reality, it decreases and probably even changes sign. This is likely to reduce the generated elliptic flow much below the preliminary data.

Ideal hydrodynamics, the simplest theoretical framework to study elliptic flow, agrees remarkably well with the measured elliptic flow data up to transverse momenta \( \sim 1.5 \text{ GeV}/c \). However, it fails to saturate at high \( p_{\perp} > 2 \text{ GeV} \) as does the preliminary data.

A theoretical problem with ideal hydrodynamics is that it assumes local equilibrium throughout the whole evolution. This idealization is marginal for conditions encountered in heavy ion collisions. Covariant Boltzmann trans-
port theory provides a convenient framework for nonequilibrium dynamics that depends on the local mean free path \( \lambda(x) \equiv 1/\sigma_n(x) \).

Parton cascade simulations\(^2,^3\) show on the other hand, that the initial parton density based on HIJING\(^8\) is too low to produce the observed elliptic flow unless the pQCD cross sections are artificially enhanced by a factor \( \sim 2 - 3 \). However, gluon saturation models\(^9\) predict up to five times higher initial densities, and these may be dense enough to generate the observed collective flow even with pQCD elastic cross sections.

In this study, we explore the dependence of elliptic flow and the high-\( p_\perp \) suppression of the particle spectra on the initial density and the elastic \( gg \) cross section. Though parton cascades lack at present covariant inelastic energy loss, elastic energy loss alone may account for the observed high-\( p_\perp \) azimuthal flow pattern as long as the number of elastic collisions is large enough\(^3\).

### 2 Covariant parton transport theory

We consider here, as in\(^6,^7,^10,^11\), the simplest nonlinear form of Lorentz-covariant Boltzmann transport theory in which the on-shell phase space density, evolves with an elastic \( 2 \rightarrow 2 \) rate. We solve the transport equation via the MPC algorithm\(^7\), which maintains Lorentz covariance using the parton subdivision technique\(^10,^12\). See Ref. \(^3\) and references therein for details.

For a given nuclear geometry and formation time, the solutions of the nonlinear transport equation has been shown\(^3\) to depend mainly on the transport opacity \( \chi \equiv \int dz \sigma_t \rho = N \langle \sin^2 \theta_{cm} \rangle \) and the impact parameter \( b \). Here \( \sigma_t \) is the elastic transport cross section, \( N \) is the average number of collisions per parton during the whole evolution, while \( \theta_{cm} \) is the collision deflection angle in the c.m. frame. To good accuracy the transport opacity factorizes as \( \chi = C(b) \sigma_t(T_0) dN_g/d\eta \).

We label our results by the transport opacity \( \chi \) and impact parameter \( b \). Furthermore, we quote our transport opacities relative to that for the pQCD minijet gluons predicted by HIJING \( [dN/d\eta = 210, \sigma_{gg \rightarrow gg} = 3 \text{ mb} \Rightarrow \sigma_t \approx 1 \text{ mb}, \chi_{b=0} \approx 0.3] \).

### 3 Numerical results

The initial condition was a longitudinally boost invariant thermal Bjorken tube at proper time \( \tau_0 = 0.1 \text{ fm}/c \) with uniform pseudo-rapidity distribution between \( |\eta| < 5 \) and transverse density distribution proportional to the binary collision distribution for the two gold nuclei. Based on HIJING, the pQCD
jet cross section was normalized to yield \( dN_{g}/dy = 210 \) in central collisions and the initial temperature was chosen to be \( T_0 = 700 \) MeV.

Two different hadronization schemes were applied. One is based on local parton-hadron duality, where each gluon is assumed to convert to a pion. The other hadronization prescription is independent fragmentation, where we considered only the \( g \to \pi^\pm \) channel. See Ref. \( \text{[3]} \) for details.

Fig. 1 shows the impact-parameter-averaged elliptic flow as a function of \( p_\perp \) for Au+Au at \( \sqrt{s} = 130 \) GeV for different transport opacities with hadronization via local parton-hadron duality (left) and independent fragmentation (right).

Fig. 2 shows that such large transport opacities are also consistent with the pQCD prediction from HIJING. Surprisingly, the results show no sensitivity to the applied hadronization prescription.
the preliminary charged hadron spectra measured by STAR. Hadronization via parton-hadron duality yields too little suppression at high $p_T$ because it only incorporates quenching due to elastic energy loss. However, with the additional quenching due to independent fragmentation, the parton cascade results approach the preliminary STAR data.

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