The growth index of matter perturbations and modified gravity

Spyros Basilakos\textsuperscript{1} and Athina Pouri\textsuperscript{1,2}

\textsuperscript{1}Academy of Athens, Research Center for Astronomy & Applied Mathematics, Soranou Efessiou 4, 11-527 Athens, Greece
\textsuperscript{2}Faculty of Physics, Department of Astrophysics, Astronomy and Mechanics, University of Athens, Panepistimiopolis, 157 83 Athens, Greece

Accepted 2012 April 24. Received 2012 April 24; in original form 2012 March 27

ABSTRACT

We place tight constraints on the growth index $\gamma$ by using the recent growth history results of 2 degree Field Galaxy Redshift Survey (2dFGRS), Sloan Digital Sky Survey luminous red galaxy (SDSS-LRG), VIMOS-VLT Deep Survey (VVDS) and WiggleZ data sets. In particular, we investigate several parametrizations of the growth index $\gamma(z)$ by comparing their cosmological evolution using observational growth rate data at different redshifts. Utilizing a standard likelihood analysis we find that the use of the combined growth data provided by the 2dFGRS, SDSS-LRG, VVDS and WiggleZ galaxy surveys puts the most stringent constraints on the value of the growth index. As an example, assuming a constant growth index we obtain that $\gamma = 0.602 \pm 0.055$ for the concordance $\Lambda$ cold dark matter (LCDM) expansion model. Concerning the Dvali–Gabadadze–Porrati (DGP) gravity model, we find $\gamma = 0.503 \pm 0.06$, which is lower and almost $3\sigma$ away from the theoretically predicted value of $\gamma_{\text{DGP}} \simeq 11/16$. Finally, based on a time varying growth index we also confirm that the combined growth data disfavour the DGP gravity.

Key words: cosmological parameters.

1 INTRODUCTION

Recent studies in observational cosmology, using all the available high-quality cosmological data [Type Ia supernovae (SNeIa), cosmic microwave background (CMB), baryonic acoustic oscillations (BAOs), etc.], converge to an emerging ‘standard model’. This cosmological model is spatially flat with a cosmic dark sector usually formed by cold dark matter (CDM) and some sort of dark energy (DE), associated with large negative pressure, in order to explain the observed accelerating expansion of the Universe (cf. Lima & Alcaniz 2000; Tegmark et al. 2004; Davis et al. 2007; Spergel et al. 2007; Kowalski et al. 2008; Hicken et al. 2009; Hinshaw et al. 2009; Jesus & Cunha 2009; Komatsu et al. 2009, 2011; Basilakos & Plionis 2010, and references therein). Despite the mounting observational evidence on the existence of the DE component in the universe, its nature and fundamental origin remains an intriguing enigma challenging the very foundations of theoretical physics. Indeed, during the last decade there has been an intense theoretical debate among cosmologists regarding the nature of the exotic ’DE’. The absence of a fundamental physical theory, concerning the mechanism inducing the cosmic acceleration, has opened a window to a plethora of alternative cosmological scenarios. Most are based either on the existence of new fields in nature (DE) or in some modification of Einstein’s general relativity, with the present accelerating stage appearing as a sort of geometric effect (for reviews, see Copeland, Sami & Tsujikawa 2006; Caldwell & Kamionkowski 2009; Amendola & Tsujikawa 2010, and references therein).

In order to test the validity of general relativity on cosmological scales, it has been proposed that measuring the so-called growth index, $\gamma$, could provide an efficient way to discriminate between scalar field DE models which admit to general relativity and modified gravity models (cf. Ferreira & Skordis 2010, and references therein). Linder & Cahn (2007) have shown that there is only a weak dependence of $\gamma$ on the equation of state (EoS) parameter $w(z)$, implying that one can separate the background expansion history, $H(z)$, constrained by a large body of cosmological data (SNIa, BAO, CMB, etc.), from the fluctuation growth history, given by $\gamma$. In this framework, it was theoretically found that for those DE models which adhere to general relativity the growth index $\gamma$ is well approximated by $\gamma \simeq 3(w-1)/(6w-5)$ (see Silveira & Waga 1994; Wang & Steinhardt 1998; Linder 2004; Linder & Cahn 2007; Nesseris & Perivolaropoulos 2008; Lee & Kim-Wang 2010), which reduces to $\gamma_{\Lambda} \simeq 6/11$ for the traditional CDM cosmology $w(z) = -1$. On the other hand, in the case of the braneworld model of Dvali, Gabadadze & Porrati (2000; hereafter DGP) the growth index becomes $\gamma_{\text{DGP}} \simeq 11/16$ (see also Linder 2004; Linder & Cahn 2007; Gong 2008; Wei 2008; Fu, Wu & Hu 2009), while for the $f(R)$ gravity models we have $\gamma \simeq 0.41-0.21z$ for $\Omega_{\text{m0}} = 0.27$ (Gannouji, Moraes & Polarski 2009; Tsujikawa et al. 2009; Motohashi, Starobinsky & Yokoyama 2010). Indirect methods to determine $\gamma$ have also been proposed (mostly using a constant $\gamma$), based either on the observed growth rate of clustering (Di Porto & Amendola 2008; Gong 2008; Guzzo et al. 2008; Nesseris & Perivolaropoulos 2008; Dosset et al. 2010; Hudson & Turnbull...
2012; Samushia, Percival & Raccanelli 2012) providing a wide range of $\gamma$ values, $\gamma = (0.58-0.67) \pm 0.11 \pm 0.17$, or on the massive galaxy clusters of Vikhlinin et al. (2009) and Rapetti et al. (2010), with the latter study providing $\gamma = 0.42 \pm 0.16$. or even on the weak gravitational lensing (Daniel et al. 2010). Gaztanaga et al. (2012) performed a cross-correlation analysis between probes of weak gravitational lensing and redshift space distortions, and found no evidence for deviations from general relativity. With the next generation of surveys, based on Euclid and BigBOSS, we will be able to put strong constraints on $\gamma$ (see e.g. Bellomo, Garcia-Bellido & Sapone 2011; Linder 2011; Di Porto, Amendola & Branchini 2012, and references therein) and thus to test the validity of general relativity on extragalactic scales.

The scope of the present study is along the same lines, i.e. to place constraints on the growth index using a single cosmologically relevant experiment, i.e. that of the recently derived growth data of the 2dFGRS, Sloan Digital Sky Survey luminous red galaxy (SDSS-LRG), VIMOS-VLT Deep Survey (VVDS) and WiggleZ galaxy surveys. Note that for the background we use two reference expansion models, namely flat $\Lambda$CDM and DGP. The interesting aspect of the latter scenario is that the corresponding functional forms of the Hubble parameters are affected only by one free parameter, that of the dimensionless matter density at the present time $\Omega_{m0}$. The structure of the article is as follows. In Section 2, we briefly discuss the background cosmological equations. The theoretical elements of the growth index are presented in Section 3 in which we extend the original Polarski & Gannouji (2008) method for a large family of $\gamma(z)$ parametrizations. In Section 4 we briefly discuss the growth data. In Section 5, we perform a likelihood analysis in order to constrain the growth index model free parameters. Finally, the main conclusions are summarized in Section 6.

2 THE BACKGROUND EVOLUTION

For homogeneous and isotropic flat cosmologies, driven by non-relativistic matter and an exotic fluid (DE models) with EoS, $p_{DE} = w(a)p_{DE}$, the first Friedmann equation can be written as

$$H^2(a) = \Omega_{m0} a^{-3} + \Omega_{DE,0} a^{-3} f_w(1+w(a)),$$

(1)

where $E(a)$ is the normalized Hubble flow, $a(z) = 1/(1+z)$ is the scale factor of the universe, $w(a)$ is the EoS parameter, $\Omega_{m0}$ is the dimensionless matter density at the present time and $\Omega_{DE,0} = 1 - \Omega_{m0}$ denotes the DE density parameter. Using the Friedmann equations, it is straightforward to write the EoS parameter in terms of $E(a) = H(a)/H_0$ (Saini et al. 2000; Huterer & Turner 2001):

$$w(a) = -1 - \frac{\Omega_{m0} a^{-3}}{E^2(a)},$$

(2)

where

$$\Omega_{m0} = \frac{\Omega_{m0} a^{-3}}{E^2(a)}.$$  

(3)

Differentiating the latter and taking into account equation (2), we obtain

$$\frac{d\Omega_{m0}}{da} = -3 w(a)\Omega_{m0}(1 - \Omega_{m0}).$$

(4)

Since the exact nature of the DE is unknown, the above DE EoS parameter includes our ignorance regarding the physical mechanism powering the late-time cosmic acceleration. It is also worth noting that the concordance $\Lambda$CDM cosmology is described by a DE model with $w(a) = -1$. Interestingly, the above method can be generalized to the context of modified gravity. Indeed, instead of using the exact Hubble flow through a modification of the Friedmann equation one may consider an equivalent Hubble flow somewhat mimicking equation (1). The ingredient here is that the accelerating expansion can be attributed to a kind of ‘geometrical’ DE contribution. Now, due to the fact that the matter density (baryonic+dark) cannot accelerate the cosmic expansion, it is fair to utilize the following parametrization (Linder & Jenkins 2003; Linder 2004):

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_{m0} a^{-3} + \Delta H^2.$$  

(5)

It becomes clear that any modification to the Friedmann equation of general relativity is included in the last term of the above expression. Now using equations (2) and (5) one can derive the effective (‘geometrical’) DE EoS parameter:

$$w(a) = -1 - \frac{1}{3} \frac{d \ln \Delta H^2}{d \ln a}.$$  

(6)

In the context of a flat DGP cosmological model the ‘accelerated’ expansion of the universe can be explained by a modification of the gravitational interaction in which gravity itself becomes weak at very large distances (close to the Hubble scale) due to the fact that our four-dimensional brane survives into an extra dimensional manifold (see Deffayet, Dvali & Cabadadze 2002, and references therein). An interesting feature of this pattern is that the corresponding functional form of the normalized Hubble function as given by equation (5) contains only one free parameter, $\Omega_{m0}$. The quantity $\Delta H^2$ is given by

$$\Delta H^2 = 2\Omega_{m0} + 2\sqrt{\Omega_{m0}} \sqrt{\Omega_{m0}} a^{-3} + \Omega_{m0},$$

(7)

where $\Omega_{m0} = (1 - \Omega_{m0})^2/4$. From equation (6), it is readily checked that the geometrical (effective) DE EoS parameter reduces to

$$w(a) = -1 - \frac{1}{1 + \Omega_{m0}(a)}.$$  

(8)

In this model due to its gravity nature, the effective Newton’s parameter $G_{eff}$ is not any more the usual constant $G_N$, but it takes the following form (Lue, Scoccimarro & Starkman 2004):

$$G_{eff}(a) = G_N Q(a),$$

(9)

$$Q(a) = 2 + 4 \Omega_{m0}(a)/3 + 3 \Omega_{m0}(a).$$

3 THE LINEAR GROWTH RATE

For the purpose of the present study, we first discuss the basic equation which governs the evolution of the matter perturbations within the framework of any DE model (scalar or geometrical). An important ingredient in this analysis is the fact that at the sub-Hubble scales, the DE component is expected to be smooth and thus one can use perturbations only on the matter component of the cosmic fluid (Dave, Caldwell & Steinhardt 2002). In particular, following the notations of Lue et al. (2004), Linder (2005), Stabenau & Jain (2006), Uzan (2007), Linder & Cahn (2007), Tsujikawa, Uddin & Tavakol (2008) and Dent, Dutta & Perivolaropoulos (2009), we can derive the well-known scale-independent equation of the linear matter overdensity $\delta_m \equiv \rho_m/\rho_m$:

$$\delta_m + 2H\delta_m = 4\pi G_{eff} \rho_m \delta_m,$$

(10)

a solution of which is $\delta_m(t) \propto D(t)$, with $D(t)$ denoting the linear growing mode (usually scaled to unity at the present time). Note
that \( \rho_m \) is the matter density. Of course, for the scalar field DE models \([G_{\text{eff}} = G_{\Lambda}, Q(a) = 1]\), the above equation reduces to the usual time evolution equation for the mass density contrast (Peebles 1993), while in the case of modified gravity models (see Lue et al. 2004; Linder & Cahn 2007; Tsujikawa et al. 2008; Gannouji et al. 2009) we have \( G_{\text{eff}} \neq G_{\Omega} \) or \( Q(a) \neq 1 \). Transforming equation (10) to \( t \) to \( a \left( \frac{d a}{d t} = H \frac{d}{d a} \right) \), we simply derive the evolution equation of the growth factor \( D(a) \):

\[
\frac{d^2 D}{D \, da^2} + \left( 3 \, a \, \frac{d}{d a} E(z) \right) \frac{d D}{d a} = \frac{3}{2} \Omega_m(a) Q(a) .
\]

We would like to remind the reader here that solving equation (11) for the concordance \( \Lambda \) cosmology, \( ^1 \) we derive the well-known perturbation growth factor (see Peebles 1993):

\[
D(z) = \frac{5 \Omega_m E(z)}{2} \left[ \int_0^z \left( 1 + a \right) \frac{d a}{E(u)} \right] .
\]

We would like to stress that for the \( \Lambda CDM \) cosmological model we use the above equation normalized to unity at the present time.

### 3.1 The evolution of the growth index

As we have mentioned in Section 1, for any type of DE, an efficient parametrization of the matter perturbations is based on the growth rate of clustering originally introduced by Peebles (1993). This is

\[
f(a) = \frac{d \ln D}{d \ln a} \equiv \Omega_m'(a) ,
\]

which implies

\[
D(a) = \exp \left[ \int_1^a \frac{\Omega_m'(x)}{x} \, dx \right] ,
\]

where \( \gamma \) is the so-called growth index (see Silveira & Waga 1994; Wang & Steinhardt 1998; Linder 2004; Lue et al. 2004; Linder & Cahn 2007; Nesseris & Perivolaropoulos 2008).

Combining equations (13), (11) and (2), we find after some simple algebra

\[
a \frac{df}{da} + f^2 + X(a) f = \frac{3}{2} \Omega_m(a) Q(a) ,
\]

where

\[
X(a) = 1 - \frac{3}{2} \gamma(a) \left[ 1 - \Omega_m(a) \right] .
\]

If we change variables in equation (15) from \( a \) to redshift \( \frac{d a}{d z} = -(1 + z)^2 \frac{d z}{d a} \) and utilizing equations (4) and (13), then we can derive the evolution equation of the growth index \( \gamma = \gamma(z) \) (see also Polarski & Gannouji 2008):

\[
-(1 + z) \gamma' \ln(\Omega_m) + \Omega_m' + 3 w(1 - \Omega_m) \left( \gamma - \frac{1}{2} \right) + \frac{1}{2} = \frac{3}{2} Q \Omega_m^{1-\gamma} .
\]

Evaluating equation (17) at \( z = 0 \), we have

\[
-\gamma'(0) \ln(\Omega_{\text{eff}}) + \Omega_m^{\gamma(0)} + 3 w_0(1 - \Omega_{\text{eff}}) \left[ \gamma(0) - \frac{1}{2} \right] + \frac{1}{2} = \frac{3}{2} Q_0 \Omega_m^{1-\gamma(0)} ,
\]

where \( Q_0 = Q(z = 0) \) and \( w_0 = w(z = 0) \).

\( ^1 \) For the usual \( \Lambda CDM \) cosmological model, we have \( w(a) = -1, \Omega_m(a) = 1 - \Omega_m(a) \) and \( Q(a) = 1 \).

\[ \text{© 2012 The Authors, MNRAS 423, 3761–3767} \]

\[ \text{Monthly Notices of the Royal Astronomical Society © 2012 RAS} \]

It is interesting to mention here that in the last few years there have been many theoretical speculations regarding the functional form of the growth index, and indeed various candidates have been proposed in the literature. In this work, we decide to phenomenologically treat the functional form of the growth index \( \gamma(z) \) as follows:

\[
\gamma(z) = \gamma_0 + \gamma_1 \gamma(z) .
\]

In other words, the above equation can be viewed as a first-order Taylor expansion around some cosmological quantity such as \( a(z), z \) and \( \Omega_m(z) \). Interestingly, for those \( \gamma(z) \) functions which satisfy \( \gamma(0) = 0 \) or \( \gamma(0) = \gamma(0) \), one can write the parameter \( \gamma_1 \) in terms of the \( \gamma_0 \). In this case \( [\gamma'(0) = \gamma_1], \) using equation (18) we obtain

\[
\gamma_1 = \frac{3 \Omega_m^{\gamma(0)} + 3 w_0(1 - \Omega_{\text{eff}}) - \frac{3}{2} Q_0 \Omega_m^{1-\gamma(0)} + \frac{1}{2}}{\gamma(0) \ln \Omega_{\text{eff}}} .
\]

In brief, we present various forms of \( \gamma(z), \Psi, \zeta \):

(i) Constant growth index (hereafter \( \Gamma_0 \) model): here we set \( \gamma_1 = 0 \).

(ii) Expansion around \( z = 0 \) (hereafter \( \Gamma_1 \) model; see Polarski & Gannouji 2008): in this case we have \( \gamma(z) = \gamma_0 \). However, the latter parametrization is valid at relatively low redshifts \( 0 \leq z \leq 0.5 \). In the statistical analysis presented below, we use a constant growth index, namely \( \gamma_0 + 0.5 \gamma_1 \), for \( z > 0.5 \).

(iii) Interpolated parametrization (hereafter \( \Gamma_2 \) model): owing to the fact that the \( \Gamma_1 \) model is valid at low redshifts, we propose to use a new formula \( \gamma(z) = \gamma_0 \left[ 1 + \gamma_1 (z + 1) \right] \) which connects smoothly low- and high-redshift ranges. The latter \( \gamma(z) \) formula can be seen as a combination of \( \Gamma_1 \) and \( \Gamma_2 \) model with that of Dosset et al. (2010). Obviously, at large redshifts \( z \gg 1 \) we have \( \gamma(z) \gamma_0 + \gamma_1 \gamma_0 \).

(iv) Expansion around \( a = 1 \) (hereafter \( \Gamma_3 \) model; Ishak & Dosset 2009; Wu, Yu & Fu 2009; Bellos et al. 2011; Di Porto et al. 2012): here we use \( \gamma(z) = \gamma_0 \left[ 1 - \gamma_1 (1 + z) \right] \), which implies that for \( z \gg 1 \) we get \( \gamma(z) \gamma_0 + \gamma_1 \gamma_0 \).

(v) Expansion around \( \Omega_m = 1 \) (hereafter \( \Gamma_4 \) model; Wang & Steinhardt 1998): now we parametrize \( \gamma(z) \) as follows: \( \gamma(z) = 1 - \Omega_m(z) \). For the DE models with a constant EoS parameter \( w(z) \equiv w_0, \) one can write \( \gamma(z) \gamma_0 \gamma_1 \gamma_0 \) only in terms of \( w_0 \):

\[
\gamma_1 = \frac{3(1 - w_0)}{6 |w_0|} \gamma_0 = \frac{3}{125} \left( \frac{1 - w_0}{1 - 6w_0/5} \right) .
\]

At large redshifts \( \Omega_m(z) \approx 1 \), we get \( \gamma(z) \approx \gamma_0 \). Note that the DGP cosmological model predicts \( \gamma(z) \gamma_0 \gamma_0 \approx 11/16, 7/256 \) (Linder 2004; Linder et al. 2007; Gong 2008).

From the above presentation it becomes evident that for the \( \Gamma_{1-3} \) parametrizations, we have \( \gamma_0 = 0 \) and \( \gamma_1 = 1 \). Therefore, for the case of the \( \Lambda CDM \) cosmology with (\( \Omega_m, \gamma_0 = 0.273, 6/11 \)) equation (20) provides \( \gamma_1 \approx -0.0478 \), while for the case of the \( \Lambda \) model we obtain \( \gamma_1 \approx 0.11 \) (see equation 21). In addition, based on the DGP gravity with (\( \Omega_m, \gamma_0 = 0.273, 11/16 \)), the \( \Gamma_{1-3} \) models give \( \gamma_1 \approx 0.05 \).

### 4 THE GROWTH DATA

The growth data that we will use in this work are based on 2dF, SDSS and WiggleZ galaxy surveys, for which their combination parameter of the growth rate of structure, \( f(z) \), and the redshift-dependent rms fluctuations of the linear density field, \( \sigma_8(z) \), is available as a function of redshift, \( f(z) \sigma_8(z) \). The \( f \sigma_8 \) is an estimator almost a model-independent way of expressing the observed growth history of the universe (Song & Percival 2009). In particular, we will use the following.
(i) The 2dF (Percival et al. 2004), SDSS-LRG (Tegmark et al. 2006) and VVDS (Guzzo et al. 2008) based growth results as collected by Song & Percival (2009). This sample contains three entries.

(ii) The SDSS [Data Release 7 (DR7)] results (two entries) of Samushia et al. (2012) based on spectroscopic data of \( \sim 106,000 \) LRGs in the redshift bin \( 0.16 < z < 0.44 \).

(iii) The WiggleZ results of Blake et al. (2011) based on spectroscopic data of \( \sim 152,000 \) galaxies in the redshift bin \( 0.1 < z < 0.9 \). This data set contains four entries.

In Table 1 we list the precise numerical values of the data points with the corresponding error bars.

### 5 FITTING MODELS TO THE DATA

In order to quantify the free parameters of the growth index, we perform a standard \( \chi^2 \) minimization procedure between \( N = 9 \) growth data measurements, \( A_{\text{obs}} = f_{\text{obs}}(z)\sigma_{\text{obs}}(z) \), with the growth values predicted by the models at the corresponding redshifts, \( A(p, z) = f(p, z)\sigma(p, z) \) with \( \sigma(p, z) = \sigma_{\text{obs}}D(p, z) \). The vector \( p \) contains the free parameters of the model and parameters depending on the model. In particular, the essential free parameters that enter in the theoretical expectation are \( p \equiv (\gamma_0, \gamma_1, \Omega_0) \). The \( \chi^2 \) function\(^2\) is defined as

\[
\chi^2(z_i|p) = \sum_{i=1}^{N} \left[ \frac{A_{\text{obs}}(z_i) - A(p, z_i)}{\sigma_i} \right]^2,
\]

where \( \sigma_i \) is the observed growth rate uncertainty. To this end, we will use the, relevant to our case, corrected Akaike information criterion for small sample size (AIC\(_c\); Akaide 1974; Sugiuira 1978), defined, for the case of Gaussian errors, as

\[
\text{AIC}_c = \chi^2_{\text{min}} + 2k + \frac{2k(k - 1)}{N - k - 1},
\]

where \( k \) is the number of free parameters, and thus when \( k = 1 \), \( \text{AIC}_c = \chi^2_{\text{min}} + 2 \). A smaller value of AIC\(_c\) indicates a better model data fit. However, small differences in AIC\(_c\) are not necessarily significant and therefore in order to assess the effectiveness of the different models in reproducing the data, one has to investigate the model pair difference \( \Delta \text{AIC}_c = \text{AIC}_{c, \gamma} - \text{AIC}_{c, \chi} \). The higher the value of \( \Delta \text{AIC}_c \), the higher the evidence against the model with higher value of AIC\(_c\), with a difference \( | \Delta \text{AIC}_c | > 2 \) indicating positive such evidence and \( | \Delta \text{AIC}_c | > 6 \) indicating strong such evidence, while a value \( \leq 2 \) indicates consistency among the two comparison models. A numerical summary of the statistical analysis for various \( \gamma(z) \) parametrizations is shown in Table 2. In general, we find that our results are in agreement, within 1\( \sigma \) uncertainties, with previous studies (Di Porto & Amendola 2008; Gong 2008; Nesseris & Perivolaropoulos 2008; Fus et al. 2009; Dosset et al. 2010; Basilakos 2012).

#### 5.1 Constant growth index

First of all, we consider the \( \Gamma_0 \) parametrization \( (\gamma = \gamma_0, \gamma_1 = 0; \text{see Section 3.1}) \), which implies that the corresponding statistical vector becomes \( p \equiv (\gamma_0, 0, \Omega_0) \). We will restrict our present analysis to the choice \( (\Omega_0, \sigma_{\text{obs}}) = (0.273, 0.811) \) provided by WMAP7 (Komatsu et al. 2011).\(^3\) Note that we sample \( \gamma \in [0.1, 1.3] \) in steps of 0.001.

In the left-hand panel of Fig. 1 we show the variation of \( \Delta \chi^2 = \chi^2(\gamma) - \chi^2_{\text{min}}(\gamma) \) around the best-fitting \( \gamma \) value for the concordance \( \Lambda \) cosmology. We find that the likelihood function of the growth data peaks at \( \gamma = 0.602 \pm 0.055 \) with \( \chi^2_{\text{min}} \approx 7.1 \) for 7 d.o.f.\(^4\) Alternatively, considering the \( \Lambda \text{CDM} \) theoretical value of \( \gamma \equiv 6/11 \) and minimizing with respect to \( \Omega_0 \), we find \( \Omega_0 = 0.243 \pm 0.034 \) (see also Nesseris & Perivolaropoulos 2008) with \( \chi^2_{\text{min}}/\text{d.o.f.} \approx 7.37/7 \). Our growth index results are in agreement within 1\( \sigma \) errors to those of Samushia et al. (2012) who found \( \gamma = 0.584 \pm 0.112 \). However, our best-fitting value is somewhat greater and almost \( 1\sigma \) \( (\Delta \chi^2 \approx 1) \) away from the theoretically predicted value of \( \gamma \approx 6/11 \) (see the cross in the left-hand panel of Fig. 1). It is interesting to mention here that such a small discrepancy between the theoretical \( \Lambda \text{CDM} \) and observationally fitted value of \( \gamma \) has also been found by other authors. For example, Di Porto et al. (2008) obtained \( \gamma = 0.60^{+0.09}_{-0.03} \) Gong (2008) measured \( \gamma = 0.64^{+0.17}_{-0.17} \), while Nesseris & Perivolaropoulos (2008) found \( \gamma = 0.67^{+0.17}_{-0.17} \). Recently, Basilakos (2012) and Hudson & Turnbull (2012) using a similar analysis found \( \gamma = 0.613^{+0.083}_{-0.083} \) and \( \gamma = 0.619 \pm 0.054 \) respectively.

\(^2\) Likelihoods are normalized to their maximum values. In the present analysis we always report 1\( \sigma \) uncertainties on the fitted parameters. Note also that the total number of data points used here is \( N = 9 \), while the associated degrees of freedom is d.o.f. = \( N - k - 1 \), where \( k \) is the model-dependent number of fitted parameters. The uncertainty of the fitted parameters will be estimated, in the case of more than one such parameter, by marginalizing one with respect to the others.

\(^3\) For the DGP model, Gong (2008) found \( \Omega_{\text{m0}} = 0.278 \).

\(^4\) Using equation (14) in the likelihood analysis for the usual \( \Lambda \) cosmology, we obtain \( \gamma = 0.595 \pm 0.071 \) with \( \chi^2_{\text{min}}/\text{d.o.f.} \approx 7.59/7 \). Note that for the DGP model, we only use equation (14).
Table 2. Statistical results for the combined growth data (see Table 1). The first column indicates the expansion model, while the second column corresponds to $\gamma(z)$ parametrizations appearing in Section 3.1. The third and fourth columns show the $\gamma_0$ and $\gamma_1$ best values. The remaining columns present the goodness-of-fit statistics (reduced $\chi^2$ and $\text{AIC}_C$).

| Expansion model | Parametrization model | $\gamma_0$ | $\gamma_1$ | $\chi^2_{\text{min}}$/d.o.f. | $\text{AIC}_C$ |
|-----------------|-----------------------|------------|------------|-----------------------------|----------------|
| $\Lambda$CDM    | $\Gamma_0$            | 0.602 ± 0.055 | 0          | 7.10/7                     | 9.10           |
|                 | $\Gamma_1$            | 0.400 ± 0.006 | 0.603 ± 0.241 | 5.74/6                    | 10.41         |
|                 | $\Gamma_2$            | 0.311 ± 0.005 | 1.221 ± 0.343 | 5.26/6                    | 9.94           |
|                 | $\Gamma_3$            | 0.345 ± 0.005 | 1.006 ± 0.314 | 5.06/6                    | 9.74           |
| DGP             | $\Gamma_0$            | 0.503 ± 0.060 | 0          | 5.32/7                     | 7.32           |
|                 | $\Gamma_1$            | 0.441 ± 0.004 | 0.164 ± 0.221 | 5.10/6                    | 9.73           |
|                 | $\Gamma_2$            | 0.401 ± 0.004 | 0.384 ± 0.320 | 5.00/6                    | 9.66           |
|                 | $\Gamma_3$            | 0.412 ± 0.003 | 0.321 ± 0.290 | 4.94/6                    | 9.60           |

Concerning the DGP model (see the right-hand panel of Fig. 1), the best-fitting parameter is $\gamma = 0.503 ± 0.06$ with $\chi^2_{\text{min}}$/d.o.f. ≃ 5.32/7. If we fix the value of $\gamma (= 11/16)$ to that predicted by the DGP model, we find a rather large value of the dimensionless matter density at the present time, $\Omega_{\text{m}0} = 0.380 ± 0.042$ with $\chi^2_{\text{min}}$/d.o.f. ≃ 5.38/7.

The value of $\text{AIC}_{C,\text{DGP}}$ (≃7.32) is smaller than the corresponding $\Lambda$CDM value which indicates that the DGP model ($\gamma_{\text{DGP}} = 0.503$) appears now to fit slightly better than the usual $\Lambda$ cosmology the growth data. However, the small $|\Delta \text{AIC}_{C}|$ value (i.e. $< 1.8$) indicates that the two comparison models represent the growth data at a statistically equivalent level. On the other hand, form the right-hand panel of Fig. 1, it becomes clear that the best-fitting $\gamma$ value is much lower and almost $3\sigma$ ($\Delta \chi^2_{\gamma} \simeq 9$) away from $\gamma_{\text{DGP}} \simeq 11/16$ (see the cross in the right-hand panel of Fig. 1) implying that the growth data disfavour the DGP gravity. We would like to stress here that the above observational DGP constraints are in excellent agreement with previous studies. Indeed, Wei (2008) found $\gamma = 0.438^{+0.126}_{-0.116}$, Also Gong (2008) and Doss et al. (2010) obtained $\gamma = 0.55^{+0.12}_{-0.11}$ and $0.483^{+0.113}_{-0.088}$, respectively. In Fig. 2, we plot the measured $A_{\text{obs}}(z)$ with the estimated growth rate function, $A(z) = f(z)\sigma_8(z)$ (see $\Lambda$CDM – solid line and DGP – dashed line).

The goal from the above discussion is to give the reader the opportunity to appreciate the relative strength and precision of the different methods used in order to constrain the growth index. It becomes evident that with the combined high-precision $f\sigma_8$ growth rate data of Song & Percival (2009), Samushia et al. (2012) and Blake et al. (2011), we have managed to place quite stringent constraints on $\gamma$.

5.2 The $\Gamma_{1-4}$ parametrizations

After we have presented the simplest version of the growth index, it seems appropriate to discuss the observational constraints on the time varying growth index, $\gamma(z)$. Following the considerations exposed in Section 3.1, hereafter we will set $p = (\gamma_0, \gamma_1, 0.273)$ in equation (22). In Fig. 3 ($\Lambda$CDM model) and Fig. 4 (DGP model), we present the results of our statistical analysis for the $\Gamma_1$ (upper-left panel), $\Gamma_2$ (upper-right panel), $\Gamma_3$ (bottom-left panel) and $\Gamma_4$ (bottom-right panel) parametrizations in the $(\gamma_0, \gamma_1)$ plane in which the corresponding contours are plotted for $1\sigma, 2\sigma$ and $3\sigma$ confidence levels. Note that we sample $\gamma_0 \in [0.1, 1.3]$ and $\gamma_1 \in [−2.2, 2.2]$ in steps of 0.001. The theoretical $(\gamma_0, \gamma_1)$ values (see Section 3.1) in the $\Lambda$CDM and DGP expansion models are indicated by the crosses. Overall, we find that the predicted $\Lambda$CDM $(\gamma_0, \gamma_1)$ solutions of the $\Gamma_{1-4}$ parametrizations remain close to the $1\sigma$ borders ($\Delta \chi^2_{\gamma} \simeq 2.30$; see the crosses in Fig. 3). Regarding the DGP model (see Fig. 4), we would like to stress that the predicted $(\gamma_0, \gamma_1)$ values approach the $3\sigma$ borders ($\Delta \chi^2_{\gamma} \simeq 11.83$; see the crosses in Fig. 4) of the $\gamma_0-\gamma_1$ contours. Obviously, this is a clear indication that the current growth data cannot accommodate the DGP gravity model.
Such an improvement is to be expected because the WiggleZ and the SDSS DR7 surveys measure \( f(z)\sigma_8(z) \) to within 8–17 per cent (Blake et al. 2011; Samushia et al. 2012) in every redshift bin, in contrast to the old growth rate data (Song & Percival 2009) in which the corresponding accuracy lies in the interval of 12–37 per cent.

(b) Now we concentrate on the \( \Gamma_2 \) and \( \Gamma_3 \) parametrizations: we find that within 1σ errors we can put some constraints on the free parameters. In particular, the best-fitting values are the following:

(i) \( \Lambda CDM \) – for \( \Gamma_2 \) we have \( \gamma_0 = 0.311_{-0.085}^{+0.085} \), \( \gamma_1 = 1.221 \pm 0.343 \) \( (\chi^2_{\text{min}}/\text{d.o.f.} \simeq 5.26/6) \), while for \( \Gamma_3 \) we get \( \gamma_0 = 0.345_{-0.085}^{+0.085} \), \( \gamma_1 = 1.006 \pm 0.314 \) \( (\chi^2_{\text{min}}/\text{d.o.f.} \simeq 5.06/6) \); and (ii) DGP – in the case of \( \Gamma_2 \) model, we obtain \( \gamma_0 = 0.401_{-0.090}^{+0.094} \), \( \gamma_1 = 0.384 \pm 0.320 \)

Finally, as we have already mentioned in Table 2, one may see a more compact presentation of our statistical results. For both cosmological (\( \Lambda CDM \) and DGP) models, the information theory pair model characterization parameter, \( \Delta \text{AIC}_C \), indicates that all the \( \gamma(z) \) functional forms explored in this study are statistically equivalent in representing the growth rate data, since \( |\Delta \text{AIC}_C| < 2 \) for any parametrization pair. In Fig. 5 we present the evolution of the growth index for various parametrizations. In the case of the concordance \( \Lambda \) cosmology (upper panel of Fig. 5), the relative growth index difference of the various fitted \( \gamma(z) \) models indicates that the \( \Gamma_{1–3} \) models have a very similar redshift dependence for \( z \lesssim 0.5 \), while the \( \Gamma_3 \) parametrization shows very large such deviations for \( z > 0.5 \). Based on the DGP gravity model (bottom panel of Fig. 5), we observe that the \( \Gamma_{1–2} \) parametrizations provide a similar evolution of the growth index. The \( \Gamma_3 \) parametrization shows large deviations at large redshifts \( z \geq 1.5 \). However, the large \( \gamma(z) \) errors appearing in Fig. 5 are due to the large uncertainty of the \( \gamma_1 \) fitted parameter, implying that more and accurate data are essential in order to distinguish among the different \( \gamma(z) \) functional forms.

## 6 Conclusions

It is well known that the so-called growth index \( \gamma \) plays a key role in cosmological studies because it can be used as a useful tool in order to test Einstein’s general relativity on cosmological scales. We have
utilized the recent growth rate data provided by the 2dFGRS, SDSS-LRG, VVDS and WiggleZ galaxy surveys, in order to constrain the growth index. Performing a likelihood analysis for various $\gamma(z)$ parametrizations, we argue that the use of the above combined growth data places the most stringent constraints on the value of the growth index. Overall, considering a $\Lambda$CDM expansion model, we find that the observed growth index is in agreement, within 1$\sigma$ errors, with the theoretically predicted value of $\gamma_A \simeq 6/11$. In contrast, for the DGP expansion model we find that the measured growth index is almost 3$\sigma$ away from the corresponding theoretical value $\gamma_{DGP} \simeq 11/16$, which implies that the present growth data cannot accommodate the DGP gravity model. Finally, considering a time varying growth index parametrization, namely $\gamma(z) = \gamma_0 + \gamma_1\alpha(z)$ [where $\alpha(z) = z, ze^{-z}, 1 - \alpha(z)$ and $1 - \Omega_m(z)$], we find that although the $\gamma_0$ parameter is tightly constrained, the $\gamma_1$ parameter remains weakly constrained. Hopefully, with the next generation of surveys, based on Euclid and BigBOSS, we will be able to put strong constraints on $\gamma_1$ and thus to check departures from $\gamma = \text{constant}$.

**REFERENCES**

Akaïke H., 1974, IEEE Trans. Automatic Control, 19, 716

Amendola L., Tsujikawa S., 2010, Dark Energy Theory and Observations. Cambridge Univ. Press, Cambridge

Basilakos S., 2012, preprint (arXiv:1202.1637)

Basilakos S., Plionis M., 2010, ApJ, 714, L185

Belloso A. B., Garcia-Bellido J., Sapone D., 2011, J. Cosmol. Astropart. Phys., 1110, 010

Blake C. et al., 2011, MNRAS, 415, 2876

Caldwell R. R., Kamionkowski M., 2009, Annu. Rev. Nuclear Particle Sci., 59, 397

Copeland E. J., Sami M., Tsujikawa S., 2006, Int. J. Modern Phys. D, 15, 1753

Daniel S. F., Linder E. V., Smith T. L., Caldwell R. R., Cooray A., Leauthaud A., Lombriser L., 2010, Phys. Rev. D, 81, 123508

Dave R., Caldwell R. R., Steinhardt P. J., 2002, Phys. Rev. D, 66, 023516

Davis T. M. et al., 2007, ApJ, 666, 716

Deffayet C., Dvali G., Cabadadze G., 2002, Phys. Rev. D, 65, 044023

Dent J. B., Dutta S., Perivolaropoulos L., 2009, Phys. Rev. D, 80, 023514

Di Porto C., Amendola L., 2008, Phys. Rev. D, 77, 083508

Di Porto C., Amendola L., Branchini E., 2012, MNRAS, 419, 985

Dosset J., Ishak M., Moldenhaver J., Gong Y., Wang A., 2010, J. Cosmol. Astropart. Phys., 1004, 022

Dvali G., Gabadadze G., Porrati M., 2000, Phys. Lett. B, 485, 208

Ferreira P. D., Skordis C., 2010, Phys. Rev. D, 81, 104020

Fu X.-Y., Wu P.-X., Hu W., 2009, Phys. Lett. B, 677, 12

Gannouji R., Moraes B., Polarski D., 2009, J. Cosmol. Astropart. Phys., 100462, 034

Gaztanaga E., Eriksen M., Crocce M., Castander F. J., Fosalba P., Marti P., Miquel R., Cabre A., 2012, MNRAS, in press (arXiv:1109.4852v3)

Gong Y., 2008, Phys. Rev. D, 78, 123010

Guzzo L., 2008, Nat, 451, 541

Hicken M., Wood-Vasey M. W., Blondin S., Challis P., Jha S., Kelly P. L., Rest A., Kirshner R. P., 2009, ApJ, 700, 1097

Hinshaw G. et al., 2009, ApJ, 180, 225

Hudson M. J., Turnbull S. J., 2012, ApJ, in press (arXiv:1203.4814)

Huterer D., Turner M. S., 2001, Phys. Rev. D, 64, 123527

Ishak M., Dosset J., 2009, Phys. Rev. D, 80, 043004

Jesus F. J., Cunha J. V., 2009, ApJ, 690, L85

Komatsu E. et al., 2009, ApJ, 180, 330

Komatsu E. et al., 2011, ApJS, 192, 18

Kowalski M. et al., 2008, ApJ, 686, 749

Lee S., Kin-Wang N., 2010, Phys. Lett. B, 688, 1

Lima J. A. S., Alcaniz J. S., 2000, MNRAS, 317, 893

Linder E. V., 2004, Phys. Rev. D, 70, 023511

Linder E. V., 2005, Phys. Rev. D, 72, 043529

Linder E. V., 2011, Philos. Trans. R. Soc. A, 369, 4985

Linder E. V., Cahn R. N., 2007, Astropart. Phys., 28, 481

Linder E. V., Jenkins A., 2003, MNRAS, 346, 573

Lue A., Scoccimarro R., Starkman G. D., 2004, Phys. Rev. D, 69, 124015

Motohashi H., Starobinsky A. A., Yokoyama J., 2010, Progress Theor. Phys., 123, 887

Nesseris S., Perivolaropoulos L., 2008, Phys. Rev. D, 77, 023504

Peebles P. J. E., 1993, Principles of Physical Cosmology. Princeton Univ. Press, Princeton, NJ

Percival W. J. et al., 2004, MNRAS, 353, 1201

Polarski D., Gannouji R., 2008, Phys. Lett. B, 660, 349

Rapetti D., Allen S. W., Mantz A., Ebeling H., 2010, MNRAS, 406, 179

Saini T. D., Raychaudhury S., Sahni V., Starobinsky A. A., 2000, Phys. Rev. Lett., 85, 1162

Samushia L., Percival W. J., Raccanelli A., 2012, MNRAS, 420, 2102

Silveira V., Waga L., 1994, Phys. Rev. D, 64, 4890

Song Y.-S., Percival W. J., 2009, J. Cosmol. Astropart. Phys., 10, 4

Spergel D. N. et al., 2007, ApJ, 170, 377

Stabenau F. H., Jain B., 2006, Phys. Rev. D, 74, 084007

Sugiyama N., 1978, Commun. Statistics A, Theor. Methods, 7, 13

Tegmark M. et al., 2004, ApJ, 606, 702

Tegmark M. et al., 2006, Phys. Rev. D, 74, 123507

Tsujikawa S., Gannouji R., Moraes B., Polarski D., 2009, Phys. Rev. D, 80, 084044

Uzan J. F., Cunha J. V., 2009, ApJ, 700, 1097

Vikhlinin A. et al., 2009, ApJ, 692, 1060

Wang L., Steinhardt J. P., 1998, ApJ, 508, 483

Wei H., 2008, Phys. Lett. B, 664, 1

Wu P., Yu H., Fu X., 2009, J. Cosmol. Astropart. Phys., 0906, 019

This paper has been typeset from a TEX file prepared by the author.