Atom localization using Laguerre–Gaussian beams

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Abstract

Use of the Laguerre–Gaussian (LG) beams in an atom-light interaction makes the linewidth of the optical spectrum narrow. We exploit this fact for providing the ability to accomplish simultaneous ultra-high precision and spatial resolution atom localization in a double-$\Lambda$ atomic system; under multi-photon resonance condition, the resolution of the localization is remarkably improved so that the atom can be localized in a region smaller than $\lambda/100 \times \lambda/100$. Most prominently, the probability of finding the atom at a particular position is always 100%, when a photon with certain frequencies is absorbed or amplified. Such features are mainly attributed to radial dependence associated with the LG beams in a spatially dependent atom-light interaction.

Keywords: 2D atom localization, Laguerre–Gaussian beams, probe absorption

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent years, many efforts have been devoted to optical techniques for precision position measurement of atoms, mainly due to the fact that these methods provide better spatial resolution. An illustrative example is the Heisenberg microscope [1] of which the resolving power is limited by optical wavelength and it allows position measurement of moving atoms as they pass through the optical fields. Further, optical methods for atom localization have been developed via an internal structure of the atomic levels. Several schemes have been proposed to achieve sub-wavelength localization of atoms [2–6] which has attracted wide scientific attention owing to its importance in the area of nanolithography at the Heisenberg limit [7] along with its fundamental importance in laser cooling [8], laser trapping of atoms [9, 10], atom optics [11], Bose–Einstein condensation [12], measurements of the center-of-mass wave function of the moving atom [13], and fluorescence microscopy [14]. Some earlier proposals for the localization include the measurement of either phase shift [3–5] or entanglement between an atom’s position and its internal state [6] after passage of the atom through standing-wave field. Besides, sub-wavelength atom localization has been shown to be possible through atomic coherence, for example, detection of spontaneously emitted photons [15], population [16], absorption [17, 18] and gain [19]. The main advantage of these schemes over the earlier ones is that the atom can be immediately localized during its motion, in the domain of the standing-wave field. The chief drawback of majority of these methods, however, is the fact that they yield different positions of the atom in a unit wavelength region ($\lambda$) and for a single measurement and, in favorable circumstances, the probability of finding the atom at certain positions may be $1/4$.

Two-dimensional (2D) atom localization, having broader and better prospects of application as compared with its one-dimensional (1D) counterpart, is recently studied in several papers [20–28]. For instance, population, absorption and spontaneous emission from a driven atomic system are used to achieve a 2D localization by interacting with two-orthogonal standing-wave fields. Generally, it is possible to take advantage of closed-loop atomic system for enhancing the precision of atom localization. In 2012, Ding et al proposed a scheme for high-precision atom localization with a cyclic configuration via controlled spontaneous emission [26]. In another related study, the use of phase-dependent absorption for approaching 2D atom localization in a four-level double-$\Lambda$ system was suggested [27]. In spite of the pronounced
success, there still exists a continuing need for increased precision along with higher spatial resolution of atom localization than conventional techniques can provide.

On the other hand, Laguerre–Gaussian (LG) light beams [29] having a doughnut-shaped intensity distribution and zero intensity at beam center, have given birth to various excellent applications, such as evidence of electric-field phase singularities [30], creation of a waveguide in an atomic vapor [31], rotating trapped microscopic particles [32], and readily formation of trap split [33]. As mentioned before, there have been numerous works on atom localization in which standing-wave fields were adopted in order to have a position-dependent atom-field interaction; however, based on our knowledge, none of them have considered LG beams. In this paper, we present a novel scheme of atom localization using the LG beams rather than the standing-wave fields, which has allowed us to always achieve 100% probability of the finding the atom at a particular position. Another prominent parameter, as far as atom localization is concerned, is spatial resolution in the probability distribution. It is already found that profile of driving fields plays a vital role in the narrowing of the linewidth of the optical spectrum [34, 35]. To proceed, we will show how using of the LG beams can reduce the width of the atom localization peak (dip) so that the probability distribution of the atom can be confined in a region smaller than $\lambda/100 \times \lambda/100$, for the parameters explored here. It is pointed out that the effects arise mainly due to radial dependence associated with LG beams. The LG-induced ultra-high precision and spatial resolution localization peak (dip) may simplify a possible implementation of sub-wavelength atom localization.

We note that our approach for atom localization has the following key advantages: (1) from an experimental point of view, it is more convenient to deal with LG beams than with two-orthogonal standing-wave fields. (2) The probability of finding the atom at a particular position is always 100%, independent of the system parameters. We must reiterate the importance of the fact that maximal probability of finding an atom at certain positions can be always obtained, while in the above studies involving atom localization with standing-wave fields, the probability of finding the atom at the certain position may be 100%, under favorable circumstances, only for limited parameters and limited applicability. For instance, Kou’s group found that the probability of detecting the atom can be 100% merely for specific probe detunings [27]. (3) The main feature of our suggested scheme, besides appearing just an ultra-narrow localization peak (dip) in the absorption spectra, is that spatial resolution is remarkably improved compared with similar works [22–28]. Here, it is worth comparing our scheme with a previous one on 2D atom localization in a five-level M-type atomic system, which leads to a 2D sub-half-wavelength atom localization [28]; in addition to the obvious difference of the coupling the atom to laser fields, there are also differences in the condition of the localization. They showed that precision and spatial resolution of atom localization can be improved in the presence of quantum interference induced by spontaneous emission, i.e. spontaneously generated coherence (SGC) and by adjusting Rabi frequency of microwave field and the detuning of spontaneously emitted photon. Noting that rigorous condition of the non-orthogonal dipole moments and also near degenerated states are barely met in real atomic systems. Another merit of our approach is that it is based on the probe-absorption measurement which is much easier to carry out or much less time consuming, compared to the measurement of spontaneous emission. These advantages may provide a possibility to observe a 2D sub-wavelength atom localization in the experiment.

2. The model

As shown in figure 1, we consider a closed-loop four-level double-Λ atomic system interacting with four coherent fields, whose level structure consists of two metastable lower states $|1\rangle$ and $|2\rangle$ plus two excited states $|3\rangle$ and $|4\rangle$. All of the allowed electric dipole transitions, i.e., $|1\rangle \rightarrow |3\rangle$, $|2\rangle \rightarrow |3\rangle$, $|1\rangle \rightarrow |4\rangle$ and $|2\rangle \rightarrow |4\rangle$ are driven by laser fields; the ground level $|1\rangle$ is coupled to excited levels $|3\rangle$ and $|4\rangle$ by control and probe fields $E_{31}$ and $E_{41}$ with carrier frequencies $\omega_{31}$ and $\omega_{41}$, respectively. Two coupling fields $E_{23}(x, y)$ and $E_{24}(x, y)$ with carrier frequencies $\omega_{23}$ and $\omega_{24}$ couple excited states to another lower state $|2\rangle$. The spontaneous decay rates on the dipole-allowed transitions are denoted by $\gamma_{13}$, $\gamma_{23}$, $\gamma_{14}$, and $\gamma_{24}$.

The electromagnetic driving fields applied to transition $|i\rangle$ and $|j\rangle$ ($i \in \{3, 4\}$ and $j \in \{1, 2\}$) can be written as

$$\hat{E}_{ij} = E_{ij} e^{-i(\omega_{ij} - \tilde{\omega}_{ij} + \phi_{ij})} + \text{c.c.},$$

where $E_{ij}$, $\tilde{\omega}_{ij}$, and $\phi_{ij}$ are amplitude, unit polarization vector, wave vector and absolute phase, respectively. Also, c.c. denotes the complex conjugate of the first term in equation (1). The Hamiltonian in the dipole and rotating-wave approximation is given by [36]

$$H = \sum_{k=1}^{4} E_k |k\rangle \langle k| - \sum_{i,j} \left( \hbar g_{ij} e^{-i\Delta} |i\rangle \langle j| + \text{h.c.} \right).$$

Here, h.c. corresponds to the Hermitian conjugate of the terms explicitly written in the Hamiltonian and corresponding Rabi

![Figure 1. The figure illustrates the considered energy scheme of the four-level double-Λ atomic system in which wavy lines show the spontaneous decays from the excited states.](image-url)
frequencies are denoted by \( g_{ij} = E_i (\hat{\epsilon}_{ji}, \hat{\mu}_{ij}) / \hbar \) with \( \hat{\mu}_{ij} \) being the atomic dipole moment of the corresponding transition. The energy of the involved state is denoted by \( E_k \) \((k \in \{1, \ldots, 4\})\) and \( \omega_{ij} \) is the argument of exponential functions function appeared in equation (1).

LG beam (LG\(_l^p\)) defines a solution of the paraxial wave function in a cylindrical coordinate which its indices \( l \) and \( p \) are the number of times the phase completes \( 2 \pi \) on a closed loop around the axis of propagation and the number of radial node for radius \( r > 0 \), respectively [34, 37]. The field amplitude for the LG mode can be defined in terms of mode amplitude \( \hat{\epsilon}_{lkp} \) and phase factor \( \Theta_{lkp} \) as [37, 38]

\[
E_{LG} = \hat{\epsilon}_{lkp} e^{i\Theta_{lkp}} - \text{c.c.,}
\]

by ignoring the \( z \) dependence in the region \( z \ll \text{Rayleigh length} \) \( (\hat{\epsilon}_{R}) \). \( E_{LG} \) reduces to

\[
E_{LG} = E_{0LG} e^{-ih2} \left( \frac{x^2 + y^2}{w} \right)^l \exp \left( -\frac{x^2 + y^2}{w^2} \right) - \text{c.c.,}
\]

where \( e^{-ih2} \) with \( \varphi = \arctan(y/x) \) denotes the phase factor and its coefficient represents the reduced mode amplitude of the electric field. In this paper, the azimuthal mode and the radial mode indexes associated with LG beams are taken as \( l = 1 \) and \( p = 0 \), respectively. So, the Rabi frequency for LG beams (LG\(_1^0\)) is written as

\[
g_{l2}(x, y) = g'_{l2} e^{-ih2} \left( \frac{x^2 + y^2}{w} \right)^l \exp \left( -\frac{x^2 + y^2}{w^2} \right),
\]

where \( g'_{l2} \) is the Rabi frequency constant. Also, \( w \) represents the beam waist for a beam width of \( \text{w}_0 \) \((w \approx \text{w}_0 \) for \( z \ll \text{z}_R \) [37]. Note that we have assumed \( p = 0 = 1 \) and \( l = 1 \), allowing only for so-called doughnut modes of order \( l = 1 \) but the treatment can accommodate any field belonging to the well-known family of LG beams.

In our suggested scheme for 2D atom localization, two fields \( E_{31} \) and \( E_{41} \) propagate in the \( y \) direction and the coupling fields \( E_{32} \) and \( E_{42} \) have LG profile. As an atom passes through LG beams with high enough velocity and interaction with fields does not affect its motion along the \( z \) direction, atomic motion is treated as a classical case. However, the velocity of the atom along the \( x(y) \) direction is very small and its center-of-mass position is nearly constant during the interaction time. Thus, we can neglect the kinetic energy in the Hamiltonian by applying the Raman-Nath approximation. In a suitable reference frame, the Hamiltonian can be written as [39]

\[
V = h (\Delta_{32} - \Delta_{31})\hat{p}_{22} - h \Delta_{31}\hat{p}_{33} + h (\Delta_{32} - \Delta_{31} - \Delta_{42})\hat{p}_{34} - h (\Delta_{31}\hat{p}_{31} + \Delta_{32}\hat{p}_{32} + \Delta_{42}\hat{p}_{42} + \Delta_{41}\hat{p}_{41} e^{i\varphi} + \text{h.c.}).
\]

(6)

The parameter \( \rho_{num} = |m\rangle \langle n| \) and \( \rho_{num} \) \((m, n \in \{1, \ldots, 4\})\) denote the corresponding operator in the new reference frame. The parameters \( \Delta_{ij} = \omega_{ij} - \omega_{ij} \) and \( \omega_{ij} = (E_i - E_j) / \hbar \) denote the detuning of the laser field from the corresponding transition and transition frequency, respectively. Note that the residual time dependence in the system appears only together with the Rabi frequency \( g_{41} \) and the parameter \( \Phi \) is given by

\[
\Phi = \Delta t - \vec{K} \vec{r} + \phi_0.
\]

\[
\Delta = (\Delta_{32} + \Delta_{41}) - (\Delta_{31} + \Delta_{42}),
\]

\[
\vec{K} = (\vec{K}_{32} + \vec{K}_{41}) - (\vec{K}_{31} + \vec{K}_{42}),
\]

\[
\phi_0 = (\phi_{32} + \phi_{41}) - (\phi_{31} + \phi_{42}),
\]

where \( \Delta, \vec{K} \), and \( \phi_0 \) are the multiphoton resonance detuning, wave vector mismatch, and initial phase difference, respectively. Using equations (7a)-(7d), the transformation of the operator \( \rho_{41} \), which will be of particular interest later on, to the chosen interaction picture can be written as \( \rho_{41} = e^{i(\alpha_{41} - \gamma_0)} \rho_{41} \). We further define \( \rho_{41} \) as the coherence \( \rho_{41} \) in a reference frame oscillating in phase with the probe field \( E_{41} \) as \( \rho_{41} = e^{i(\alpha_{41} - \gamma_0)} \rho_{41} \).

From the Hamiltonian equation (6), and including the spontaneous decay in Born–Markov approximation, the density matrix equations for the considered four-level system are [39]

\[
\frac{d}{dt} \rho_{11} = ig_{31}^* \rho_{31} - ig_{31}^* \rho_{13} + ig_{41}^* \rho_{41} e^{i\varphi} - ig_{41} \rho_{14} e^{i\varphi} + 2(\gamma_{31}\rho_{13} + 2\gamma_{32}\rho_{12}),
\]

\[
\frac{d}{dt} \rho_{12} = ig_{32} \rho_{32} - ig_{33} \rho_{22} + ig_{42} \rho_{42} - ig_{42} \rho_{44} + 2(\gamma_{32}\rho_{33} + 2\gamma_{34}\rho_{34}),
\]

\[
\frac{d}{dt} \rho_{13} = -i(\Delta_{32} - \Delta_{31})\rho_{13} + ig_{31}^* \rho_{33} - ig_{31}^* \rho_{12} + ig_{41} \rho_{43} e^{i\varphi} - \Gamma_{13}\rho_{13},
\]

\[
\frac{d}{dt} \rho_{14} = i(\Delta_{32} - \Delta_{31} - \Delta_{42})\rho_{14} + ig_{31}^* \rho_{34} e^{i\varphi} - ig_{41} \rho_{44} \rho_{41} - \Gamma_{41}\rho_{14},
\]

\[
\frac{d}{dt} \rho_{22} = -i(\Delta_{32}\rho_{22} + ig_{32}^* \rho_{32} - ig_{32} \rho_{22} + ig_{42} \rho_{42} + \gamma_{32}\rho_{32} - \Gamma_{32}\rho_{22},
\]

\[
\frac{d}{dt} \rho_{23} = -i(\Delta_{32}\rho_{23} + ig_{32}^* \rho_{33} - ig_{32} \rho_{23} + ig_{42} \rho_{43} + \gamma_{32}\rho_{33} - \Gamma_{32}\rho_{23},
\]

\[
\frac{d}{dt} \rho_{24} = -i(\Delta_{32}\rho_{24} + ig_{32}^* \rho_{34} - ig_{32} \rho_{24} + ig_{42} \rho_{44} + \gamma_{32}\rho_{34} - \Gamma_{32}\rho_{24},
\]

\[
\frac{d}{dt} \rho_{34} = -i(\Delta_{42} - \Delta_{32})\rho_{34} + ig_{31} \rho_{14} + ig_{41} \rho_{34} - ig_{41} \rho_{34} e^{i\varphi} - \Gamma_{34}\rho_{34}.
\]

(7m)

The remaining equations follow from the constraints \( \rho_{num} = \rho_{num}^* \) and \( \sum_n \rho_{num} = 1 \). We have also defined \( \gamma_j = \gamma_{ij} + \gamma_{ji} \) and \( \Gamma_{ji} = (\gamma_{ji} + \gamma_{ij}) / 2 \) as the damping rate of
coherences on transition $|j\rangle - |i\rangle$. Note that we have simplified the notation $g_{2j}(x, y)$ to $g_{2}$. It is clear that in general the system does not have a constant steady-state solution, due to the explicit time dependence of equations (7e)–(7m), however, in the case of multiphoton resonance condition, i.e. $\Delta = 0, \Phi = 0$, the equations of motion have constant coefficients and thus a stationary solution in the long-time limit can be found.

3. Results

In what follows, we assume the multiphoton resonance condition to be fulfilled. Then, the coefficients of the density matrix equations, equations (7e)–(7m), do not have an explicit time dependence, and the system has a stationary steady-state in the long-time limit. Because of position-dependent Rabi frequencies $g_{12}$ and $g_{2}$, the Hamiltonian and the dynamics of the system are position dependent and in principle, the 2D position information of the atom can be extracted via the susceptibility of the probe field which can be written as $\chi = N_{\eta_{\phi}}\rho_{43}/(\epsilon_{0}E_{\text{4}})$ with $N$ and $\eta_{\phi}$ being as, respectively, atom number density in the medium and the probe transition dipole moment [36]. Real and imaginary parts of the susceptibility correspond to dispersion and absorption, respectively. Notice that, throughout the discussion, for simplicity we have written the susceptibility as

$$\Im \chi = (N\eta_{\phi}/\epsilon_{0})F(x, y)$$

via definition a filter function $F(x, y)$ which determines the position probability distribution of the atom [25, 27].

As mentioned before, in the previously proposed schemes based on the phase-dependent absorption [27] and the SGC effect [28], the number of the localization peaks in a unit wavelength domain of the classical standing-wave fields can be reduced to 1 and the detection probability at a certain position can be improved to 100%. Keeping in mind their innovative ideas, the main question here is: is there any way to readily make high precision position measurement of the moving atom? In the following, we show how we can have just an ultra-narrow localization peak (dip) as well as the improved spatial resolution without encountering difficulty in realizing the SGC or controlling detunings. As the probe absorption spectrum depends on $x, y$ from the coupling fields, it is possible to extract 2D position information of the atom when it is passing through fields via measuring the probe absorption or the gain. Our results are represented in scaled quantities to obtain the best possible comparison with other localization schemes; positions are divided by $\lambda$ with a typical value of the wavelength $\lambda = 253$ nm.

In figure 2, we analyze the filter function $F(x, y)$ which shows the atomic position probability distribution. This function is obtained by numerically solving the density matrix equations, equations (7e)–(7m), in the long-time limit. The common parameters are chosen as $\gamma_{13} = \gamma_{14} = \gamma_{23} = \gamma_{24} = \gamma$, $g_{31} = 0.001\gamma$, $g_{41} = 10^{-4}\gamma$, and $g_{23} = g_{24} = \gamma$ [40]. We also assume that carrier frequencies of fields satisfy the multi-photon resonance condition, i.e. $\Delta_{31} = \Delta_{32} = \Delta_{42} = \Delta = 0$ [41]. Further, all calculations are performed by assuming that the beam width of each LG beam is given by $w = 1\mu m$ [42]. Due to the closed-loop configuration, the absorption and gain properties of the system depend on the relative phase of the fields: $\Phi = \phi_{32} + \phi_{41} - (\phi_{31} + \phi_{24})$. According to the definition of the filter function, $F(x, y) > 0$ means that the probe field is absorbed by the atom, while on the condition $F(x, y) < 0$ corresponds to the amplification. In order to describe clearly the localization properties, three-dimensional plots of the filter function are presented in figures 2(d)–(f) corresponding to the figures 2(a)–(c), respectively.

First of all, we consider the case of $\Phi = 0$ and plot the filter function versus the normalized position $(x/\lambda, y/\lambda)$. As can be seen in figures 2(a) and (d), an ultra-narrow localization peak with a full width at half-maximum (FWHM) of less than 0.004 $\lambda$ (about 1 nm) is found and the probability of detecting the atom, determined by the gain of the probe field, is 100%. On the condition of $\Phi = \pi/2$, the filter function has a narrow dip (see figures 2(b) and (e)). For the case of $\Phi = \pi$, it takes the forms similar to that for the previous case: exhibiting a narrow dip, but with a significantly larger depth (figures 2(c) and (f)). That is to say, when a probe photon is absorbed, we can be sure that the atom is passing through the fields from the position determined by the circle in the figure and the probability of finding the atom at such position also is 100% which is greatly improved comparing with previous schemes [21–25, 27]. It is also obvious that the atom is located in a region smaller than $\lambda/100 \times \lambda/100$, for the parameters explored in this work, which compares favorably to previous works [22–28]. It is imperative to point out that the ultra-high precision and spatial resolution atom localization is independent of choosing the detunings in such a way that similar behavior is found for the case of nonzero detunings, but peak with smaller height (dip with smaller depth) for the case of $\Delta > \gamma$.

We then proceed to further analyze the atom localization, by investigating analytical solution for the filter function. By solving equations of motion in the steady-state situation, which can be obtained from those equations for vanishing time derivatives, we can derive analytical solution for the probe susceptibility and, consequently, the filter function. In the particular case of $\Delta = 0$ and $g_{31}(x, y) = g_{42}(x, y) = g(x, y)$, the expression for the $\rho_{41}$ is given as

$$m[\rho_{41}(x, y)] = \frac{-\gamma}{2D^2}(Dg_{31}|g^2(x, y)||\Im[ie^{i\phi}] - g_{41}g_{31}^2|g^2(x, y)||g_{31}^2 + 2|g^2(x, y)|)$$

where $D = g_{31}^2|g^2(x, y)| + g_{41}^2|g^2(x, y)||\Im[ie^{2\phi}]$, (9)

It is worth mentioning that analytical solutions can, in principle, be found for more general conditions, but we concentrate on the case of $\Delta = 0$, in which more optimal results, i.e. peaks with larger height (dips with larger depth) can be found. These terms have a simple interpretation in terms of the physical processes [39]. The first one, proportional to $g_{31}|g^2(x, y)| = g_{31}g_{31}^2(x, y)g_{42}(x, y)$, corresponds to the
closed interaction loop and also a scattering of the driving field modes to the probe field mode. The second component proportional to $g_{41}$ represents a direct scattering of the probe field off of the probe transition. The last term, proportional to $g_{41}$, can be interpreted as a counter-rotating propagation.

This expression also allows us to analyze the atom localization: the height of the peak (the depth of the dip) is remarkably affected by the relative phase of the applied fields, since terms of the form $e^{i\Phi}$, can be considered as a counter-rotating propagation.

For the case of $\Phi = \pi/2$ also is approximately reverse to that of $\Phi = 0$, as two last terms are very small. It may be mentioned that here we present results for incident LG beams with equal azimuthal mode indices ($l$), as optimal results, i.e. high precision and high resolution atom localization, can be achieved for such beams. Neither the depth nor the FWHM of the dips created by LG beams with different magnetic quantum numbers and $m_F$ stands for the corresponding state. The closed-loop configuration can be established with a similar scheme of the experimental setup in [44].

Finally, we discuss a real atomic system that can be used for possible experimental realization. The cold atom $^{85}$Rb ($D_1$ line) can be a possible candidate; the control (probe) field drives the transition $|F = 3, m_F = +1\rangle \rightarrow |F' = 2, m_F = +2\rangle$, while two coupling fields drive $|F = 3, m_F = +3\rangle \rightarrow |F' = 2, m_F = +2\rangle$ and $|F = 3, m_F = +3\rangle \rightarrow |F' = 3, m_F = +2\rangle$ with Rabi frequencies $\Omega_{42}$ and $g_{42}$, respectively [43]. Here, $F$ and $F'$ are the total atomic angular momentum quantum numbers and $m_F$ stands for the magnetic quantum number of the corresponding state. The closed-loop configuration can be established with a similar scheme of the experimental setup in [44].

Last but not the least, the source of the ultra-high precision and spatial resolution atom localization originates in the presence of the spatially-dependent Rabi frequency associated with LG beams. To understand the origin of such outstanding properties, we recomputed the filter function without the phase factor ($e^{-i\varphi}$), the same results are obtained. On the other hand, this point is clearly evident from equation (9); the filter function for our suggested scheme becomes independent of the phase factor of the coupling fields, since terms of the form $|g^2|$ appear in the equation.
Thus, by attending to these points, it can be claimed that the radial dependence, not the phase factor, associated with LG beams brings about observed ultra-high precision and spatial resolution atom localization.

4. Conclusions

In conclusion, this paper has proposed and analyzed a novel atom localization scheme permitting simultaneous ultra-high precision and spatial resolution which takes advantage of the sharp spectra induced by the LG beams. Under multi-photon resonance condition and for the parameters explored in this work, the atom can be localized in a region smaller than $\lambda/100 \times \lambda/100$ and probability of the finding the atom at a particular position is always 100%. Taking into account the fact that the spatial resolution as well as the precision of the atom localization are significantly improved over those provided by conventional localization techniques, such features are mainly dependent on radial dependence associated with LG beams.

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