Improved neural network adaptive control for compound helicopter with uncertain cross-coupling in multimodal maneuver

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Abstract The main goal of this study is to create a robust control system that could guide or replace pilots in the tracking of the commanded velocity and attitude in multimodal maneuver, while complex dynamics and uncertain aerodynamic cross-coupling among control surfaces of compound helicopter are considered. To this end, a Pi-Sigma neural network (PSNN) adaptive controller is proposed based upon the certainty-equivalence (CE) principle, where a novel Lyapunov-based weight self-tuning algorithm augmented with e-modification is designed to realize efficient uncertainty approximation and guarantee robustness of convergence process. Compared with traditional neural networks in control field, stronger generalization ability of PSNN must be balanced against weaker stability, which leads to inevitable parameters perturbation. Therefore, an incremental nonlinear dynamic inversion (INDI) framework is established to decouple original overactuated system and reject parameters perturbation in PSNN. Meanwhile, by incorporating Lagrange multiplier method into allocation, an original incremental allocation method is designed to get globally ideal control input according to time-varying working capability of each surface. In terms of Lyapunov theorem, it is demonstrated that the closed-loop augmented system driven by the proposed control scheme is semi-global uniformly ultimately bounded. Finally, by comparing with existing methods, the simulation validates the effectiveness of proposed control scheme.

Keywords Compound helicopter · Pi-Sigma neural network · Adaptive control · Incremental dynamic inversion

1 Introduction

Compound helicopter is a kind of rotorcraft designed to break through maximum speed limit of traditional helicopters (approximately 160–170 knots) and retain capability of vertical take-off/landing. Different from conventional helicopters, the auxiliary propulsion system and wings can help the main rotor from dynamic stall on the relating side in high-speed mode. During the past decade, compound helicopters have grown dramatically in both military and civilian fields. However, due to the unique characteristics of high-speed rotor dynamic, these benefits must be balanced against many disadvantages including time-varying dynamics, mechanical difficulty, unknown aerodynamic interfer-
ence of additional components, control difficulty and so on [1,2].

In particular, complex aerodynamic cross-coupling among redundancy control surfaces may severely degrade the performance and the stability of compound helicopters. Thus, it is critical to characterize the effect carefully and to alleviate them for reliable autonomous command tracking in multimodal maneuver [3,4]. Besides, with the increase in complexity of the flight environment, multisource disturbances such as the external disturbances, model uncertainties and unmodeled dynamics also seriously limit the tracking precision of compound helicopters [5,6]. Due to factors mentioned above, most missions of compound helicopters can be hardly achieved by traditional controller and design of multimodal command tracking control strategy becomes a core technique of compound helicopters [7].

Recently, much literature revealed the complex aerodynamic characteristics of compound helicopter and made it possible to build an accurate compound helicopter nominal model which is helpful for control system design [8–11]. However, there are still few related research results to deal with the challenges in advanced controller design. These related studies mainly consider high-speed mode flight [12] or focus on analysis and optimization of redundant control surfaces [13]. The control laws they used are mainly traditional model-based approaches including linear control laws [13–15], sliding mode control [16] and traditional non-linear dynamic inversion [17]. However, these methods ignored nonaffine property of compound helicopter and can hardly overcome uncertain cross-coupling perturbation throughout flight envelope.

In recent years, there have been various excellent approaches for reference to solve similar problems in command tracking control of complex multi-mode aircraft, such as tilt rotorcraft [18–21], near-space vehicles [22], hypersonic vehicles [23–25] and so on. Within these advanced control methods, neural network (NN) adaptive control is an effective way to handle complex disturbance and model uncertainty in multimodal vehicles [26–29] However, some factors limit the extension of traditional NN adaptive control. On the one hand, although universal approximation theorem of simple NN has been proved in [30,31], existing NN in control field including single hidden layer perceptron (SHL) and radial basis function network (RBFNN) are too simple to efficiently handle uncertain nonlinear function online in complex systems [32]. Furthermore, the introduction of excessive neurons in these NN leads to severe co-adaptation and overfitting. On the other hand, some powerful NN such as cerebellar model articulation controller (CMAC) and ridge polynomial neural network (RPNN) usually have heavy structures, which bring too many hyperparameters to design and make these NN much unreliable in engineering [33]. Different from above NN, PSNN is a kind of high-order neural network and has received considerable attention recently [34,35] due to its ability of realizing faster nonlinear approximation by introducing both sum and multiplication neurons [36,37]. Meanwhile, simple structure and few hyperparameters are needed in PSNN to improve convergence efficiency compared with other complex neural networks [38]. Nowadays PSNN has been applied in various fields [39] and is more suitable for solving control problem in compound helicopter.

In spite of excellent performance of PSNN, research of PSNN adaptive control is still in its infancy. Within these related studies, PSNN adaptive fuzzy controller proposed in [40,41] is most popular to deal with tracking problem. An offline training process is needed in these fuzzy controllers to approach the optimal membership function. However, such offline training cannot adapt to complex environment and unknown interference unavoidable in aerospace engineering. Other literature mainly focuses on PSNN backstepping approach for a relatively simple system like hydraulic control system [42] to improve control accuracy. Although these methods have been proofed stability, the simple gradient descent adaptive law used in these studies cannot guarantee the robustness of convergence in more complex systems. The improved PSNN adaptive control strategy for the compound helicopter should consider the following problems: (1) Stability should be ensured strictly in adaptive algorithm since multiplication neurons seriously damage the robustness of PSNN; (2) the adaptive PSNN should be augmented with a robust controller architecture to keep robustness to inevitable parameters perturbation in the converge process; (3) practical control input to redundant control surfaces of compound helicopters should be allocated carefully to obtain a efficient tracking performance.

This paper proposes a novel adaptive control scheme for compound helicopters in multimodal maneuver, where aerodynamic cross-coupling among each con-
control surface is considered as partly uncertain disturbances. More specifically, we firstly formulate nonlinear dynamics of a typical compound helicopter and reveal the effect of cross-coupling on control strategy by trim analysis throughout the flight envelope. Then, an INDI framework composed of an inner attitude control loop and an outer velocity control loop is proposed to decouple the original overactuated system and keep robustness to the inevitable weight perturbation of PSNN. Considering uncertain coupling disturbances, a PSNN adaptive controller augmented with a novel self-tuning adaptive algorithm based on CE principle and Lyapunov theory is proposed to approximate the undesirable uncertainties. To ensure efficiency and robustness, the weight updating law combined with e-modification can improve convergence performance and correct the potential parameter drift in absence of PE. In particular, an original incremental allocation method is determined based on Lagrange multiplier optimization to develop globally optimal input for each control surface based on its time-varying working capability in multimodal maneuver according to a designed allocation matrix satisfying various tasks. It is shown that the tracking errors and the weight parameters are SGUUB in terms of Lyapunov theorem. Finally, this study presents a complete Lyapunov stability proof and verifies the efficacy of the proposed approach by numerical simulations.

As compared with previous work, the main contributions of this work can be summarized as follows:

(1) Instead of traditional simple NN adaptive control, PSNN is utilized to deal with uncertain coupling in the compound helicopter. To keep robustness to uncertain cross-coupling and model error, this study proposes a PSNN Lyapunov-based adaptive algorithm based upon CE principle. Robust adaptive term and e-modification are introduced to correct the potential parameters drift and guarantee robustness in the convergence process especially in absence of PE. This design can better trade off stability and nonlinear mapping capability of PSNN compared to existing adaptive law.

(2) Stronger generalization ability of PSNN must be balanced with weaker stability. To address this, an INDI-PSNN controller architecture is designed to decouple the original overactuated system and minimize the effects caused by uncertainties. This controller architecture can significantly enhance robustness to the weight parameters perturbation without the requirement of precise mathematical model of the compound helicopter.

(3) An incremental Lagrange optimal allocation method is designed to solve the control allocation problem considering different working capability of each control surface throughout flight envelope. Based on traditional optimal control allocation methods, this approach optimizes global control efficiency by regulating incremental control input. Through selecting the appropriate allocation matrix, the ideal control of various task requirements can be achieved.

The remainder of this paper is organized as follows. In Sect. 2, we briefly describe nonlinear dynamic model of compound helicopter and analyze cross-coupling between control surfaces. In Sect. 3, we explain the details of the improved PSNN adaptive control strategy in INDI framework and the proof of stability. In Sect. 4, we present comparative numerical simulation results with respect to different control strategies. In Sect. 5, we conclude the whole paper.

2 Model statement and preliminaries

Notation: $\mathbb{R}^n$ is the $n$-dimensional Euclidean space. $(\cdot)^T$ represents matrix transpose. $\|\cdot\|$ stands for the 2-norm of a vector or Frobenius norm of a matrix. $\sup(\cdot)$ denotes the supremum of $(\cdot)$. $(\cdot)^{-1}$ stands for the inverse matrix for a non-singular matrix or one of the pseudo-inverse matrices for a singular matrix. Supposing $x \in \mathbb{R}^n$ and $f(x) \in \mathbb{R}^m$, $\nabla_x f(x_0) \in \mathbb{R}^{m \times n}$ represents the gradient of $f(x)$ to $x$ at the point $x_0$.

In this section, a typical compound helicopter nonlinear dynamic system is first briefly presented with undesirable cross-coupling considered. Thereafter, a trim analysis adjusting to full envelope is given in order to further design the control strategy.
For convenience, choosing the vectored thrust ducted propeller (VTDP) as an auxiliary propulsion device, the main structure of a typical compound helicopter is shown in Fig. 1. The body-fixed coordinate system used throughout this paper is also shown in Fig. 1. The components of the velocity vector in $x, y, z$ axes of the coordinate system are respectively defined as $u, v, w$. Similarly, the components of angular velocity are respectively defined as $p, q, r$ in body-fixed coordinate system. In engineering, these dynamic states are easy to measure accurately.

Different from conventional helicopters, the compound helicopter is under the action of various forces involving the main rotor force $F_R$ (mainly acting on low-speed mode), the VTDP force $F_T$, the fuselage force $F_f$, the wing force $F_w$ (mainly acting on high-speed mode) and gravity $G$. Based on the momentum theorem and the moment of momentum theorem, the compound helicopter dynamic model can be obtained as

$$m\begin{bmatrix} \dot{u} + qw - vr \\ \dot{v} + ur - pw \\ \dot{w} + pv - uq \end{bmatrix} = F_R + F_T + F_f + F_w + G$$

Different from linear assumption in other literature [14, 15], the flapping motion of main rotor across flight envelope is described by a second-order differential equation as

$$\ddot{\beta}_i = \frac{M_b}{I_b} \sin \beta_i \left(-e\Omega^2\right) - \cos \beta_i \sin \beta_i \Omega^2 + \left(M_{FAB}\right)_i - k_\beta \dot{\beta}_i$$

(2)

where the subscript $i$ represents the parameters on the $i$th blade, $\beta$ denotes blade flapping angle, $M_b$ is the aerodynamic torque acting on blade, $I_b$ is blade inertia, $e$ is flapping hinge offset, $\Omega$ is main rotor speed, $M_{FAB}$ is the sum of the aerodynamic torque acting on the flapping hinge, and $k_\beta$ is flapping coefficient. Then, according to blade-element theory, the aerodynamic force and torque expressions of main rotor are calculated as

$$\begin{align*}
(F_{Pb})_i &= \sum_{k=1}^{NS} \left[ \frac{1}{2} \rho \Omega^2 R^3 U (C_d U_T |\cos \gamma| + C_d U_P) \right]_k \\
(F_{Tb})_i &= \sum_{k=1}^{NS} \left[ \frac{1}{2} \rho \Omega^2 R^3 U \left( C_d U_T - C_d U_P |\cos \gamma| \right) \right]_k \\
(F_{Rb})_i &= \sum_{k=1}^{NS} \left[ \frac{1}{2} \rho \Omega^2 R^3 U \left( C_d U_T |\cos \gamma| \right) \right]_k \\
(F_X)_i &= -(F_{Tb})_i + 2M_b \Omega \dot{\beta}_i \sin \beta_i \\
(F_Y)_i &= (F_{Rb} \cos \beta - F_{Pb} \cos \beta)_i + M_b [\cos \beta_i (\dot{\beta}_i^2 + \Omega^2)] + \sin \beta_i \beta_i \\
(F_Z)_i &= (F_{Pb} \sin \beta - F_{Rb} \sin \beta)_i + M_b [\dot{\beta}_i \cos \beta_i - \beta_i \dot{\beta}_i \cos \beta_i - \beta_i^2 \sin \beta_i]
\end{align*}$$

(3)

where $F_{Pb}$, $F_{Tb}$ and $F_{Rb}$ are the blade aerodynamic forces in vertical, tangential and spanwise directions, respectively; $F_X$, $F_Y$ and $F_Z$ are the load components of blade in rotation axis; $NS$ is the number of elements on blade; $\rho$ is air density; $R$ is blade elements rotation radius; $U_T$, $U_P$ and $U_R$ are the tangential, vertical and spanwise velocity components at an element of the blade; $U = (U_T^2 + U_P^2 + U_R^2)^{1/2}$ is total velocity of blade; $\gamma$ is blade profile drift angle; $C_d$ and $C_d$ are the lift coefficient and drag coefficient got from wind tunnel test data; and $ds$ is the area of each blade element.

$$\begin{align*}
J_z \ddot{p} + qr (J_x - J_z) \\
J_y \ddot{q} + pr (J_x - J_z) \\
J_x \ddot{r} + pq (J_y - J_z)
\end{align*} = M_R + M_T + M_f + M_w$$

(1)

where $(J_x, J_y, J_z)$ are the inertia moment in the body-fixed frame, $M_R$, $M_T$, $M_f$ and $M_w$ are the corresponding torques. In this study, to analyze the control characteristics of compound helicopter, the nonlinear dynamics as well as coupling effects of the main rotor, the wings and the VTDP will be briefly introduced. The complete dynamic model is introduced in [15].
To sum up, the force and torque generated by the main rotor throughout flight envelope can be written as

\[
F_R = \begin{bmatrix}
- \sum_{i=1}^{N} (F_z)_i \\
N \sum_{i=1}^{N} (F_Y \cos \varphi - F_X \sin \varphi)_i \\
- \sum_{i=1}^{N} (F_X \cos \varphi + F_Y \sin \varphi)_i \\
\sum_{i=1}^{N} e(F_Z \cos \varphi)_i \\
\sum_{i=1}^{N} e(F_Z \sin \varphi)_i \\
- \sum_{i=1}^{N} (eF_X - M_{LAB} \cos \beta)_i
\end{bmatrix}
\]

(4)

where \( M_{LAB} \) is the sum of the aerodynamic moment acting on the shimmy hinge, \( \varphi \) is rotor azimuth, and \( N \) is the number of blades.

2.2 Wings nonlinear dynamic

Supposing that the right-wing center is \((x_{wr}, y_{wr}, z_{wr})\) in the body-fixed frame, the velocity of right wing can be described as

\[
V_{wr} = \begin{bmatrix}
u + qz_{wr} - ry_{wr} + v_l \zeta_{XMWR} \\
v - pz_{wr} - rx_{wr} \\
w + py_{wr} - qx_{wr} + v_l \zeta_{XMWR}
\end{bmatrix}
\]

(5)

where \( v_l \) is rotor inducted velocity, \( \zeta_{XMWR} \) and \( \zeta_{XMWB} \) are the cross-coupling factors of rotor downwash shown in Figs. 2 and 3,

where \( \beta_{1k} \) is rotor flapping skew angle.

The aerodynamic force and torque of the wind frame can be expressed as

\[
\begin{bmatrix}
F_{x_{wr}}^w \\
F_{y_{wr}}^w \\
F_{z_{wr}}^w \\
M_{x_{wr}}^w \\
M_{y_{wr}}^w \\
M_{z_{wr}}^w
\end{bmatrix} = L_B^W (V_{wr}) \begin{bmatrix}
-C_{D_{w_{wr}}} (\alpha_{u_{wr}}, \delta_r) q_{u_{wr}} S \\
-C_{L_{w_{wr}}} (\alpha_{u_{wr}}, \delta_r) q_{u_{wr}} S \b \\
C_{M_{w_{wr}}} (\alpha_{u_{wr}}, \delta_r) q_{u_{wr}} S b
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

(6)

where \( L_B^W (V_{wr}) \) is the coordinate transformation matrix of wind frame in body-fixed frame; \( C_{D_{w_{wr}}}, C_{L_{w_{wr}}}, C_{M_{w_{wr}}} \) are the wings coefficient functions of drag, lift and yawing torque; \( S \) is stabilizing surface area; \( b \) is wing span; \( \alpha_{u_{wr}} \) is the attack angle of right wing; and \( q_{u_{wr}} \) is the dynamic pressure of right wing.

In the same way, the aerodynamic force and torque of left-wing \( F_{x_{wl}}^l, F_{y_{wl}}^l, F_{z_{wl}}^l, M_{x_{wl}}^l, M_{y_{wl}}^l, M_{z_{wl}}^l \) can be obtained, and the dynamic of wings can be described as

\[
F_w = \begin{bmatrix}
F_{x_{wr}}^w \\
F_{y_{wr}}^w \\
F_{z_{wr}}^w \\
F_{x_{wl}}^l \\
F_{y_{wl}}^l \\
F_{z_{wl}}^l
\end{bmatrix} + \begin{bmatrix}
F_{x_{wr}}^w \\
F_{y_{wr}}^w \\
F_{z_{wr}}^w \\
F_{x_{wl}}^l \\
F_{y_{wl}}^l \\
F_{z_{wl}}^l
\end{bmatrix}
\]

\[
M_w = \begin{bmatrix}
M_{x_{wr}}^w \\
M_{y_{wr}}^w \\
M_{z_{wr}}^w \\
M_{x_{wl}}^l \\
M_{y_{wl}}^l \\
M_{z_{wl}}^l
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
-F_{x_{wr}}^w z_{u_{wr}} + F_{x_{wl}}^l y_{u_{wl}} \\
-F_{u_{wl}}^l x_{u_{wl}} + F_{w_{wl}}^l z_{u_{wl}} \\
-F_{x_{wl}}^l z_{u_{wl}} + F_{x_{wl}}^l y_{u_{wl}} \\
-F_{u_{wl}}^l x_{u_{wl}} + F_{w_{wl}}^l z_{u_{wl}} \\
F_{w_{wl}}^l z_{u_{wl}}
\end{bmatrix}
\]

(7)
2.3 VTDP nonlinear dynamic

The VTDP dynamic in the body-fixed frame can be obtained by assuming that the VTDP can make small angle variation of two degree of freedom in the thrust direction. Let the pressure center of VTDP be \((x_t, y_t, z_t)\). The approaching velocity in the duct frame is

\[
V_T = \begin{bmatrix}
k_p u + q z_t - r y_t + v_i \xi_{3MRT} \\
k_p v + p z_t - r x_t \\
k_p w + p y_t - q x_t + v_i \xi_{2MRT}
\end{bmatrix}
= \begin{bmatrix}
V_{xt} \\
V_{yt} \\
V_{zt}
\end{bmatrix}
\]

where \(k_p\) is the pressure-lose coefficient of the ducted fan, \(\xi_{3MRT}\) and \(\xi_{2MRT}\) are the cross-coupling factors of rotor to VTDP shown in Figs. 4 and 5. Transform the components of the VTDP thrust in duct frame to the body-fixed frame, and dynamic of VTDP can be described as

\[
F_T = L_B^T (\theta_1, \theta_2, V_T)
\]

\[
M_T = \begin{bmatrix}
0 & -z_t & y_t \\
z_t & 0 & -x_t \\
y_t & z_t & 0
\end{bmatrix}
\]

where \(Q_{xy}\) and \(Q_{xz}\) are airflow slip coefficients, the \(L_B^T\) is the coordinate transformation matrix controlled by the VTDP deflection \(\theta_1, \theta_2\) and \(V_T\).

2.4 Trim analysis

In summary, the compound helicopter dynamic model is obtained as

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix} = \begin{bmatrix}
vr - qw - mg \sin \theta \\
pw - ur - mg \cos \theta \sin \phi \\
pv + qw + mg \cos \phi \cos \theta + F_f + F_R (\varphi_7, A_{1s}, B_{1s}) + F_T (T, \theta_1, \theta_2, v_i) + F_w (\delta_r, \delta_l, v_i)
\end{bmatrix}
+ \begin{bmatrix}
q_r \\
q_p \\
q_r - q_p
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
(J_y - J_z) qr \\
(J_z - J_x) pr \\
(J_x - J_y) pq
\end{bmatrix}
\]

\[
M_f + M_R (\varphi_7, A_{1s}, B_{1s}) + M_T (T, \theta_1, \theta_2, v_i) + M_w (\delta_r, \delta_l, v_i)
\]

where the control surfaces include collective \(\varphi_7\), lateral cyclic \(A_{1s}\), longitudinal cyclic \(B_{1s}\), deflection angle of VTDP relative to longitudinal symmetry plane \(\theta_1\), deflection angle of VTDP projection on longitudinal symmetry plane relative to transverse symmetry plane \(\theta_2\), left-wing deflection angle \(\delta_l\), right-wing deflection angle \(\delta_r\) and VTDP thrust \(T\). The forces and torques are not only the nonlinear functions of control surfaces, but also the functions of flight states and the rotor inducted velocity \(v_i\), which is shown above.

The efficiencies of each control surface vary in multimodal maneuver. When flight speed increases and dynamic pressure \(q\) increases, the lift and torques generated by the wing grow in quadratic function. Mean-
while, the aerodynamic characteristics of the rotor gradually show strong nonlinearity, and blade dynamic stall occur due to unsteady flapping motion $\beta$. To be specific, a detailed trim analysis is given in this study.

The trim optimization of flight state from hovering to 360 km/h is conducted using sequential quadratic programming (SQP), and optimal variety of control surfaces deflection in different flight mode is obtained. Under constraint of ensuring the balance of external force and torque, the objective function of trim optimization is designed as $J_{\text{trim}} = u^T H u$, where $H$ is optimization matrix. The trim results are given below.

Figures 6 and 7 show that the lift and thrust are mainly provided by the main rotor in low-speed mode. The loading on the rotor reduces and the wings auxiliary lift as dynamic pressure increases. In high-speed mode, the rotor takes only 40% of lift, while the wing supplements 60%. Meanwhile, the thrust is completely supplemented by VTDP, which embodies the compounding of the lift and thrust. It is worthwhile noting that these variations are nonlinear.

Figures 8 and 9 show the trim values of control surfaces in different flight mode. Figure 8 shows that optimal collective, lateral cyclic and longitudinal cyclic approach zero in high-speed mode to reduce the negative effect of blade stall. Figure 9 shows that the wings control surfaces keep large deflection angle in low-speed flight to reduce wings area and influence of rotor downwash. With the increase in dynamic pressure, they gradually undertake the task of attitude control and approach their normal workspace. Furthermore, in low-speed mode, VTDP deflection maintains large angle to undertake the task of helicopter tail rotor with low
thrust. As the airspeed increases, VTDP power also increases and the deflection decreases to provide sufficient thrust.

From the above analysis, it is known that the control efficiency of each control surface has time-varying nonlinear characteristics in multimodal maneuver, and complex cross-couplings between each part also bring lots of challenges to controller design.

3 Main results

For convenience, the compound helicopter nonaffine nonlinear dynamic system (10), (11) can be represented as two subsystems

\[
\dot{x}_1 = f_1(x_1, x_2) + g_1(x_1, x_2, u_1) \\
\dot{x}_2 = f_2(x_2) + g_2(x_2, u_2) \tag{12}
\]

where \(x_1 = [u, v, w]^T\) are relatively slow translational state variables, \(x_2 = [p, q, r]^T\) are relatively fast rotational state variables, \(u_1 = [\phi_{cmd}, \theta_{cmd}, \varphi, T]^T\) are control inputs in translational dynamics and \(u_2 = [A_{1x}, B_{1x}, \delta_l, \delta_r, \theta_1, \theta_2]^T\) are control inputs in rotational dynamics. Owing to partly uncertain coupling and variations of aerodynamic models, it is difficult to obtain system parameters by measurement and theoretical calculation. A nominal model is used to approximate the real compound helicopter model

\[
\dot{x}_1 = \hat{f}_1(x_1, x_2) + \hat{g}_1(x_1, x_2, u_1) + \hat{A}_1(x_1, x_2, u_1) \\
\dot{x}_2 = \hat{f}_2(x_2) + \hat{g}_2(x_2, u_2) + \hat{A}_2(x_2, u_2) \tag{13}
\]

where the superscript " denotes nominal model and the function \(\hat{A}_i\) is uncertain model defined as

\[
\hat{A}_i(x_i, u_i) \triangleq f_i(x_i) + g_i(x_i, u_i) - \hat{f}_i(x_i) - \hat{g}_i(x_i, u_i) \tag{14}
\]

**Assumption 1** The considered nonlinear function \(f_i(\cdot)\) satisfies global Lipschitz condition, that is

\[
\| f_i(x_1(k_1), x_2(k_1)) - f_i(x_1(k_2), x_2(k_2)) \| \\
\leq L_1|x_1(k_1) - x_1(k_2)| + L_2|x_2(k_1) - x_2(k_2) |
\]

Similar conclusions apply to \(\hat{f}_i(\cdot), g_i(\cdot), \hat{g}_i(\cdot)\).

**Assumption 2** The partial derivative of \(\hat{g}_i(\cdot)\) with respect to control input exits and the sign is unchanged. That is \(\frac{\partial \hat{g}_i(\cdot)}{\partial u_i} \geq l\) (or \(\frac{\partial \hat{g}_i(\cdot)}{\partial u_i} \leq -l\)), where \(u_i\) denotes control input, \(l > 0\) is a small constant. In following discussion, we consider the case of \(\frac{\partial \hat{g}_i(\cdot)}{\partial u_i} \geq l\)

In this paper, a nonlinear controller is designed to realize the command tracking control of the compound helicopter in multimodal maneuver. The objective is to find control inputs \(u_1(t)\) and \(u_2(t)\), such that \(x_1(t)\) accurately tracks the desired command \(x_{1cmd}(t) = [u_{cmd}(t), v_{cmd}(t), w_{cmd}(t)]^T\) and \(\psi_{cmd}\) in multimodal maneuver.

**Assumption 3** Let the external commanded input and its first and second derivatives be bounded, such that

\[
\| (u_{cmd}, \dot{u}_{cmd}, \ddot{u}_{cmd})^T \| \leq \bar{U}_C
\]

Similar assumptions are made for all channels in \(x_{1cmd}\) and \(\psi_{cmd}\).

According to singular perturbations theory, the controlled dynamic states of compound helicopter are separated into fast rotational dynamic states and slow translational dynamic states based on time-scale. Correspondingly, the two sets of state variables are controlled by two different control loops, namely inner attitude control loop and outer velocity control loop. Thereafter, in both two control loops, improved PSNN adaptive controllers with INDI architecture are proposed to realize robust tracking control in multimodal maneuver. Finally, in terms of Lyapunov theorem, it is demonstrated that the close-loop system is SGUUB. To summarize the proposed controller, a flow diagram is shown in Fig. 10.
3.1 INDI control framework

3.1.1 Outer control loop design

The objective of the outer-loop velocity controller is to follow the velocity command \(x_{cmd}\) and generate desired commands \(\theta_{cmd}\) and \(\phi_{cmd}\) for the attitude controller. In addition, to follow the velocity command, it generates the collective input \(\varphi_f\) for the rotor and the desired thrust \(T\) for the thrust ducted propeller.

In order to obtain the incremental form of the compound helicopter velocity dynamic model, (12) are rewritten by applying the Taylor series expansion to the \(\hat{g}_i(x_1, x_2, u_1)\) at the beginning of each sampling interval (denoted by superscript 0)

\[
\dot{x}_1 = \hat{f}_1 (x_1, x_2) + \hat{g}_1(x_1^0, x_2^0, u_1^0) + \nabla_{(x_1, x_2, u_1)} \hat{g}_1(x_1^0, x_2^0, u_1^0) \cdot (\Delta x_1, \Delta x_2, \Delta u_1) + R'(\Delta x_1, \Delta x_2, \Delta u_1) + \bar{\Delta}_1 (x_1, x_2, u_1) \tag{15}
\]

in which the increments of the variables with respect to their current values are denoted by the notation \(\Delta\), \(R'(\Delta x_1, \Delta x_2, \Delta u_1)\) is the higher-order remainder of Taylor expansion. By introducing acceleration feedback from tracking differentiator, (15) is then written as

\[
\dot{x}_1 = \hat{f}_1 \Delta x_1^0 - \hat{f}_1 (x_1, x_2) - \bar{\Delta}_1 (x_1^0, x_2^0, u_1^0) + \hat{f}_1 (x_1, x_2) + \nabla_{(x_1, x_2, u_1)} \hat{g}_1(x_1^0, x_2^0, u_1^0) \cdot (\Delta x_1, \Delta x_2, \Delta u_1) + R'(\Delta x_1, \Delta x_2, \Delta u_1) + \bar{\Delta}_1 (x_1, x_2, u_1) \tag{16}
\]

Assumption 4 Without loss of generality, the changes of system states including \(x_1\) and \(x_2\) are approximately negligible in a short time increment \(T_s\). Such that

\[
\lim_{T_s \to 0} x_i = x_i^0.
\]

Based on this assumption, (16) becomes

\[
\dot{x}_1 = x_1^0 + \nabla_{u_1} \hat{g}_1 (x_1, x_2, u_1) \cdot \Delta u_1 + R_1(\Delta u_1) + \bar{\Delta}_1(\Delta u_1) \tag{17}
\]

where \(\Delta u_1\) is defined as incremental model error depending on \(\Delta u_1\) and \(R_1\) is defined as the incremental higher-order perturbing term mainly depending on higher-order term of \(\Delta u_1\). Thus, the INDI control law of velocity loop is designed by using the NDI based on (17) in every sampling interval given by

\[
\Delta u_1 = \left[ \nabla_{u_1} \hat{g}_1 (x_1, x_2, u_1^0) \right]^{-1} (\dot{x}_1 - v_{L1}) \tag{18}
\]

where \(\dot{x}_1\) is obtained from the reference model designed according to the requirements of performance index, \(u_1^0 = u_1 (t - T_s)\) is the \(u_1\) before a time increment \(T_s\), \(\left[ \nabla_{u_1} \hat{g}_1 (x_1, x_2, u_1^0) \right]^{-1}\) denotes one of the pseudo-inverse matrices designed below, the linear pseudo-control signal \(v_{L1}\) is designed as follows.

\[
v_{L1} = K_{P1} \left[ \begin{array}{c} u_c - u \\ u_c - v \\ w_c - w \end{array} \right] + K_{I1} \int_0^t \left[ \begin{array}{c} u_c - u \\ u_c - v \\ w_c - w \end{array} \right] d\tau \tag{19}
\]

where \(K_{P1}\) and \(K_{I1}\) are linear control coefficients.

3.1.2 Inner control loop design

To follow the Euler angle command \(\phi_{cmd}(t), \theta_{cmd}(t)\) and \(\psi_{cmd}(t)\) obtained from the velocity controller and the yaw angle command, the angular velocity command is achieved by Euler conversation \(\dot{x}_{cmd} = (\Omega_r [\phi_{cmd}(t), \theta_{cmd}(t), \psi_{cmd}(t)]^T)^T\), where \(\Omega_r\) is corresponding coordinate rotation matrix.

Similar to the velocity control, Taylor series expansion is applied again to obtain the incremental form of the compound helicopter velocity dynamic model

\[
\dot{x}_2 = \hat{f}_2 (x_2) + \nabla_{u_2} \hat{g}_2 (x_2^0, u_2^0) \cdot (\Delta x_2, \Delta u_2) + \hat{g}_2 (x_2^0, u_2^0) + R'_2(\Delta x_2, \Delta u_2) + \bar{\Delta}_2(x_2, u_2) \tag{20}
\]

where \(R'_2(\Delta x_2, \Delta u_1)\) is the higher-order remainder of the Taylor expansion. Through the introduction of angular acceleration feedback, (20) is then written as

\[
\dot{x}_2 = \dot{x}_2^0 - \hat{f}_2 (x_2^0) - \bar{\Delta}_2(x_2^0, u_2^0) + \hat{f}_2 (x_2) + R'_2(\Delta x_2, \Delta u_2) + \bar{\Delta}_2(x_2, u_2) + \nabla_{(x_2, u_2)} \hat{g}_2(x_2^0, u_2^0) \cdot (\Delta x_2, \Delta u_2) \tag{21}
\]
Based on Assumption 4, (21) becomes

\[
\dot{x}_2 = \dot{x}_2^0 + \nabla u_2 \tilde{g}_2 (x_2, u_2) \cdot \Delta u_2 + R_2 (\Delta u_2) + \Delta_2 (\Delta u_2) \tag{22}
\]

where \( \Delta_2 \) is defined as the incremental model error depending on \( \Delta u_2 \) and \( R_2 \) is defined as the incremental higher-order perturbing term mainly depending on high-order term of \( \Delta u_2 \). INDI control law of attitude loop is designed by using the NDI in every sampling interval

\[
\Delta u_2 = \left[ \nabla u_2 \tilde{g}_2 \left( x_2, u_2^0 \right) \right]^{-1} (\dot{x}_2 - \dot{x}_2^0 + v_{l2} - \dot{x}_2) \tag{23}
\]

where \( u_2^0 = u_2 (t - T_s) \) is the \( u_2 \) before a time increment \( T_s \), \( \dot{x}_2 \) is obtained from the reference model designed according to the requirements of performance index, the pseudo-control signal \( v_{l2} \) is designed as

\[
v_{l2} = K_{p2} \begin{bmatrix} p_c - p \\ q_c - q \\ r_c - r \end{bmatrix} + K_{l2} \int_0^t \begin{bmatrix} p_c - p \\ q_c - q \\ r_c - r \end{bmatrix} d\tau \tag{24}
\]

\subsection*{3.2 Improved PSNN adaptive control design}

Substituting the proposed control law (18) and (23) to compound helicopter dynamics, the following closed-loop augmented compound helicopter dynamic system is derived as

\[
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} v_{l1} \\ v_{l2} \end{bmatrix} + \begin{bmatrix} R_1 (\Delta u_1) + \Delta_1 (\Delta u_1) \\ R_2 (\Delta u_2) + \Delta_2 (\Delta u_2) \end{bmatrix} \tag{25}
\]

Throughout flight envelope, even if the time interval is small, the dynamic inverse is not accurate enough when the amount of \( \Delta u \) is large due to the strong non-linearity of the control efficiency. In pure mathematical sense, that is reflected by the remarkable influence of the higher-order perturbing term \( R_i (\Delta u_i) \) as well as model uncertainty \( \Delta_i (\Delta u_i) \). Therefore, an improved PSNN adaptive compensation is designed to overcome these undesirable influences.

According to CE principle, an adaptive term can be designed in pseudo-control signal to overcome the uncertainty. Thus, 

\[
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} v_{l1} \\ v_{l2} \end{bmatrix} + \begin{bmatrix} v_{ad1} \\ v_{ad2} \end{bmatrix} + \begin{bmatrix} R_1 (\Delta u_1) + \Delta_1 (\Delta u_1) \\ R_2 (\Delta u_2) + \Delta_2 (\Delta u_2) \end{bmatrix} \tag{26}
\]

where \( v_{ad1} \) and \( v_{ad2} \) are outputs of the adaptive controller.

\textbf{Remark 1} In fact, the strength of the INDI control architecture lies in the cancellation of a nonlinear and possibly multidimensional uncertainty which may include coupling of multiple states and control effects. In this paper, the state vector \( x \) and the corresponding control input \( u \) and \( v_{ad} \) will be used as a detailed example for design and analysis, and the same conclusions can be extended to all subsystems including \( x_1 \) and \( x_2 \). The decoupled close-loop dynamics can be obtained from (26)

\[
\dot{x} = \dot{x}_c + v_l - v_{ad} + R (\Delta u) + \Delta (\Delta u)
\]

\[
= \dot{x}_c + K_p \ddot{x} + K_1 \int_0^t \ddot{x} d\tau - v_{ad} + \delta (\Delta u) \tag{27}
\]

in which \( \ddot{x} \) is defined as \( \ddot{x} \equiv x_c - x \) and the total uncertainty is defined as

\[
\delta (\Delta u) \equiv R (\Delta u) + \Delta (\Delta u) = \dot{\tilde{g}} (x, u) - \dot{\tilde{g}} (x, u^0)
\]

\[
- \frac{\partial \dot{\tilde{g}}}{\partial u} (u^0) \Delta u + \tilde{\Delta} (x, u) - \Delta (x, u^0)
\]

\[
= \dot{g} (x, u) - \dot{g} (x, u^0) - \frac{\partial \dot{g}}{\partial u} (u^0) \Delta u + g (x, u) - g (x, u^0)
\]

\[
- \dot{q} (x, u^0) - g (x, u^0)
\]

\[
= g (x, u) - g (x, u^0) - \frac{\partial \dot{g}}{\partial u} (u^0) \Delta u \tag{28}
\]

Combining (27) and (28), the close-loop system with respect to state tracking error can be obtained as

\[
\dot{\hat{e}} = A e + b (v_{ad} - \delta) \tag{29}
\]

with 

\[
e \equiv \begin{bmatrix} \int_0^t \ddot{x} d\tau \\hat{x} \end{bmatrix} A \equiv \begin{bmatrix} 0 & 1 \\ -K_1 & -K_P \end{bmatrix} b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]
Assumption 5 A in (29) is Hurwitz. By selecting appropriate control parameters \( K_P \) and \( K_1 \), the system matrix \( A \) can be made Hurwitz. Therefore, this assumption is usually satisfied.

The closeness of the approximation is captured by the total uncertainty, which we may express in terms of the pseudo-control signal as

\[
\delta (\Delta u) = \hat{x} - K_P \hat{x} - K_1 \int_{t_0}^{t} \hat{x} dt + v_{ad} \tag{30}
\]

The total uncertainty \( \delta \) depends on \( v_{ad} \), whereas \( v_{ad} \) will be designed to cancel \( \delta \). This poses a fixed-point problem with existence and uniqueness of its solution \( v_{ad} \) guaranteed with the following assumption

Assumption 6 The mapping \( v_{ad} \rightarrow \delta \) is a contraction over the entire input domain of interest satisfied by

\[
\left| \frac{\partial \delta (\Delta u)}{\partial v_{ad}} \right| = \left| \frac{\partial}{\partial \Delta u} \left[ g(x,u) - g(x,u^0) - \frac{\partial \hat{g}}{\partial u}(u^0) \Delta u \right] \right| \cdot \left| \frac{\partial \Delta u}{\partial (\hat{x} + v_L - v_{ad})} \right| \cdot \left| \frac{\partial (\hat{x} + v_L - v_{ad})}{\partial v_{ad}} \right| \cdot \left| \frac{\partial}{\partial \Delta u} \left[ g(x,u) - g(x,u^0) - \frac{\partial \hat{g}}{\partial u}(u^0) \Delta u \right] \right| \left| \frac{\partial}{\partial \Delta u} \left[ g(x,u) + \frac{\partial \hat{g}}{\partial u}(u^0) \Delta u \right] \right| < 1 \tag{31}
\]

which can be rewritten as follows

\[
\left| \frac{\partial g}{\partial \Delta u} \right| < 1 \tag{32}
\]

Pi-Sigma neural networks is a kind of feedforward network with a single hidden layer and product units at the output layer. The weight from the hidden layer to the output layer is fixed to 1 while the weight from the input layer to the hidden layer is adjustable. The input–output map of a PSNN can be represented as

\[
h_i = \sum_{j=1}^{N} w_{ij} x_j
\]

\[
y = \sigma \left( \prod_{i=1}^{K} h_i \right) \tag{33}
\]

where \( w_{ij} \) are the adjustable weights, \( x_j \) are the scalar inputs, \( K \) is the number of summing units, \( N \) is the number of input nodes, \( y \) is the PSNN scalar output, \( h_i \) is the output of each summing unit, and \( \sigma \) is a suitable nonlinear transfer function. For convenience, we define the weight vector and matrix as

\[
W_i = \left[ \theta_{ui} \ w_{i1} \ w_{i2} \ldots \ w_{IN} \right]^T \tag{34}
\]

\[
W = \begin{bmatrix} W_1 & W_2 & \cdots & W_K \end{bmatrix}
\]

\[
= \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_K \\ w_{11} & w_{12} & \cdots & w_{K1} \\ \vdots & \vdots & \ddots & \vdots \\ w_{1N} & w_{2N} & \cdots & w_{KN} \end{bmatrix} \tag{35}
\]

It is also convenient to define a vector

\[
\tilde{x} = \begin{bmatrix} b_x \ x_1 \ x_2 \ \cdots \ x_N \end{bmatrix}^T \tag{36}
\]

where \( b_x \geq 0 \) allows for thresholds \( \theta_{ui} \) to be included in the weight vector. With the previous definitions, the input–output map of a PSNN can be written in matrix form as

\[
y = \sigma \left( \prod_{i=1}^{K} W_i^T \tilde{x} \right) \tag{37}
\]

We consider the uncertainty \( \delta \) is a function mainly depending on \( \Delta u \), and

\[
\Delta u = \left[ \frac{\partial \hat{g}}{\partial u}(u^0) \right]^{-1} \left( \hat{x}_c + v_L - v_{ad} - \hat{x} \right)
\]

\[
= \left[ \frac{\partial \hat{g}}{\partial u}(u^0) \right]^{-1} \left( \hat{x} + K_P \hat{x} + K_1 \int_{t_0}^{t} \hat{x} dt - v_{ad} \right) \tag{38}
\]
The functional dependence of $\delta$ is
$$\delta \sim \delta(\hat{x}, \dot{x}, x, \int_{0}^{t} \ddot{x} \, dt, v_{ad}, v_{l})$$

Therefore, PSNN input is chosen as
$$\hat{x} = \left[ b_{x}, \hat{x}, \dot{x}, x, \int_{0}^{t} \ddot{x} \, dt, v_{L}, v_{ad}, \| \hat{W} \| \right]^T$$

(39)
where $\hat{W}$ is the estimate of ideal weight matrix and $b_{x}$ is the constant bias.

Remark 2 Due to nonaffinity of the compound helicopter with respect to control, the total uncertainty $\delta$ is a general function of the pseudo-control, including the output of the PSNN. Since the output of the PSNN is providing compensation for the interference, a fixed-point problem occurs. Therefore, to ensure that Assumption 6 holds, $v_{L}$ and $v_{ad}$ are chosen as inputs to the PSNN through a squashing function.

The adaptive law for the weight matrix $W$ is derived based on the Lyapunov theory. Firstly, the adaptive term constructed with the PSNN output is defined as follows

$$v_{ad} = \hat{v}_{ad} + v_{r}$$

$$\hat{v}_{ad} = \sigma \left( \prod_{i=1}^{K} \hat{W}_{i}^T \frac{\dot{x}}{\| \dot{x} \|} \right)$$

(40)
where $v_{r}$ is a term that robustifies against the effects of higher order terms in the Taylor series approximation in what follows. For convenience, we define the function

$$h \left( \hat{W} \right) = \sum_{i=1}^{K} \prod_{j \neq i}^{K} \| \hat{W}_{j} \|$$

where $h \left( \hat{W} \right)$ is strictly positive due to each weight vector is designed to contain a constant bias, and $v_{r}$ is designed as

$$v_{r} = -K_{r1} \xi \left( \| \hat{W} \| + \hat{W} \right) h \left( \hat{W} \right)$$

$$-K_{r2} \left( \| \hat{W} \| + \hat{W} \right) h \left( \hat{W} \right) \xi$$

(41)
where $\xi (\cdot)$, $\zeta$ and $\hat{W}$ are known terms defined later, and $K_{r1}$, $K_{r2}$ are robust gains. $W_{0} = \left[ W_{10} W_{20} \cdots W_{K0} \right]$ is defined as a pretrained weight matrix with

$$\| W^{*} - W_{0} \| \leq \hat{W}_{0}$$. The update law of the weight matrix $\hat{W}$ is designed as

$$\dot{\hat{W}} = -\Gamma_{w} \left[ \hat{x} \cdot \nabla_{x} \frac{\sigma}{\| \hat{W} \|} \left( \prod_{i}^{K} \hat{W}_{i}^T \right) \right] \xi$$

$$+ \lambda_{h} \left( \hat{W} \right) | \xi | \left( \hat{W} - W_{0} \right)$$

(42)
where $\Gamma_{w}$ is the learning rate. The damping term $\lambda_{h} \left( \hat{W} \right) | \xi | \left( \hat{W} - W_{0} \right)$ defined as e-modification of PSNN adaptive control is mainly used to guarantee the robustness while approximation error of the network exists.

Remark 3 Persistent excitation (PE) condition is very hard to provide for all but the simplest NNs. The e-modification corrects the potential parameter drift that may occur in the absence of PE.

The structure of the proposed close-loop system is shown in Fig. 11.

To obtain the optimal pseudoinverse in (18) and (23), we next design the actual control input. In particular, consider the following optimization problem in close-loop system

$$\min \ u_{i} \Gamma_{i}^{-1} u_{i}$$

s.t. $\nabla_{u_{i}} \hat{g}_{i} \left( u_{i} \right) \Delta u_{i} = (\dot{x}_{i} + v_{Li} - v_{adi})$

(43)
where subscript $i = 1$ denotes variables in velocity control loop, $i = 2$ denotes variables in attitude control loop, $\Gamma_{i}$ is the allocation matrix. Based on Assumption 2, $\nabla_{u_{i}} \hat{g}_{i} \left( u_{i} \right)$ is a non-square matrix with full rank. Using Lagrange multiplier method, we define a Lagrange function

$$H \left( \Delta u_{i}, \mu_{i} \right) \triangleq \left( u_{i}^{0} + \Delta u_{i} \right) \Gamma_{i}^{-1} \left( u_{i}^{0} + \Delta u_{i} \right)$$

$$+ \mu_{i} \left[ \nabla_{u_{i}} \hat{g}_{i} \left( u_{i}^{0} \right) \Delta u_{i} - \dot{x}_{i} - v_{Li} + v_{adi} \right]$$

(44)
where $\mu_{i}$ is Lagrange parameter. By solving $\partial H / \partial \Delta u_{i} = 0$, $\Delta u_{i}^{*}$ can be obtained as

$$\Delta u_{i}^{*} = -\frac{1}{2} \Gamma_{i} \nabla_{u_{i}}^{T} \hat{g}_{i} \left( u_{i}^{0} \right) \mu_{i} - u_{i}^{0}$$

(45)
Substituting (45) into constraint, the complete optimal control law can be given as

\[ u_i^{k+1} = \Gamma_i \nabla_{u_i} \hat{g}_i \left( u_i^k \right) \left\{ \nabla_{u_i} \hat{g}_i \left( u_i^k \right) \Gamma_i \nabla_{u_i} \hat{g}_i \left( u_i^k \right) - v_{adi} + \nabla_{u_i} \hat{g}_i \left( u_i^k \right) u_i^k \right\}^{-1} \]

where \( k \) denotes controlling time sequence.

### 3.3 Stability analysis of proposed controller

Consider a PSNN approximation of uncertainty

\[ \sigma \left( \prod_{i=1}^{K} W_i^* T \bar{x} \right) = \delta - \varepsilon \]

where \( W_i^* \in \mathbb{R}^n \) are column vectors of ideal weight matrix \( W^* = \left[ W_1^* \cdots W_K^* \right] \) is defined as the value of adaptable weight \( W \) over \( \Omega_W \) that minimizes \( |\epsilon| \) for all \( \bar{x} \in \Omega_x \) such that

\[ W^* = \Delta \arg \min_{W \in \Omega_W} \left\{ \sup_{\bar{x} \in \Omega_x} \left| \sigma \left( \prod_{i=1}^{K} W_i^* T \bar{x} \right) - \delta \right| \right\} \]

and upper bound \( \tilde{\varepsilon} \) is

\[ \tilde{\varepsilon} = \Delta \sup_{\bar{x} \in \Omega_x} \left| \sigma \left( \prod_{i=1}^{K} W_i^* T \bar{x} \right) - \delta \right| \]

The Frobenius norm of ideal PSNN weight matrix is bounded by a known positive value

\[ \| W^* \| \leq \tilde{W} \]

and

\[ \| \hat{W} \| = \| W^* + \tilde{W} \| \leq \tilde{W} + \| \hat{W} \| \]

\[ \| \hat{W}_i \| = \| W_i^* + \hat{W}_i \| \leq \tilde{W} + \| \hat{W}_i \| \]

where \( \tilde{W} \) and \( \hat{W}_i \) are weight estimation errors.

Using a Taylor series expansion about the current estimate of the output, the approximation error can be
expressed as

\[
\sigma \left( \prod_{i=1}^{K} W_i^T \tilde{x} \right) = \sigma \left( \prod_{i=1}^{K} \tilde{W}_i^T \tilde{x} \right) - \nabla_{\tilde{W}} \sigma \left( \prod_{i=1}^{K} \tilde{W}_i^T \tilde{x} \right) \cdot \tilde{W}^T \tilde{x} + \alpha \left( \tilde{W}^T \tilde{x} \right)^2
\]

(51)

where

\[
\nabla_{\tilde{W}} \sigma \left( \prod_{i=1}^{K} \tilde{W}_i^T \tilde{x} \right) = \sigma' \left( \prod_{i=1}^{K} \tilde{W}_i^T \tilde{x} \right) \cdot \left[ \prod_{j \neq 1}^{K} \tilde{W}_j^T \tilde{x} \prod_{j \neq 2}^{K} \tilde{W}_j^T \tilde{x} \cdots \prod_{j \neq K}^{K} \tilde{W}_j^T \tilde{x} \right]
\]

(52)

which can be bounded by

\[
\left\| \nabla_{\tilde{W}} \sigma \left( \prod_{i=1}^{K} \tilde{W}_i^T \tilde{x} \right) \right\| \leq \sigma' \left( \prod_{i=1}^{K} \tilde{W}_i^T \tilde{x} \right) \cdot K \left\| \tilde{x} \right\|^{K-1} h \left( \tilde{W} \right)
\]

(53)

Thus, the higher-order residual term is given by

\[
o(\tilde{W}^T \tilde{x})^2 = -\nabla_{\tilde{W}} \sigma \left( \prod_{i=1}^{K} \tilde{W}_i^T \tilde{x} \right) \cdot \tilde{W}^T \tilde{x} + \sigma \left( \prod_{i=1}^{K} W_i^T \tilde{x} \right) - \sigma \left( \prod_{i=1}^{K} \tilde{W}_i^T \tilde{x} \right)
\]

(54)

Using (50), the NN output is bounded by

\[
\left\| \tilde{u}_{ad} \right\| < \sigma \left[ c' \prod_{i=1}^{K} \left( \tilde{W} + \left\| \tilde{W}_i \right\| \right) \right]
\]

(55)

where \(c' > 0\). The design as in (39) and Assumption 3 imply that the NN input can be maximally bounded in terms of the tracking performance by

\[
\left\| \tilde{x} \right\| \leq c'_0 + c'_1 \left| \xi \right| + c'_2 \left\| \tilde{W} \right\|
\]

(56)

The scalar \(\xi\) is filtered error term

\[
\xi = e^T P b
\]

(57)

where matrix \(P\) is the solution to the Lyapunov equation \(A^T P + PA = -Q\) and \(P = P^T > 0\). The construction of \(\xi\) in (56) can be seen as an error filter. From Cauchy–Schwarz inequality and the compatibility of the Frobenius norm with the vector 2-norm, it is clear that

\[
\left| \xi \right| \leq \left\| e \right\| \left\| P \right\|
\]

(58)

Using design of PSNN input (39), we have

\[
\left\| \tilde{x} \right\|^K \leq \left( c'_0 + c'_1 \left| \xi \right| + c'_2 \left\| \tilde{W} \right\| \right)^K \\
= c'_3 + c'_4 \left| \xi \right| + c'_5 \left\| \tilde{W} \right\| + \xi \left( \left| \xi \right| , \left\| \tilde{W} \right\| \right)
\]

(59)

where \(c'_i > 0, \xi (a, b)\) is defined as a polynomial function of \(a\) and \(b\).

From (53), the high-order term is bounded by

\[
\left| o(\tilde{W}^T \tilde{x})^2 \right| < c'_6 + c'_7 \left\| \tilde{W} \right\| h \left( \tilde{W} \right)
\]

(60)

where \(c'_i > 0\). Let \(w_p\) be defined as the NN approximation error plus the higher-order effects of back propagation through the PSNN, that is

\[
w_p = -o(\tilde{W}^T \tilde{x})^2 + \epsilon
\]

(61)

Consider the properties of the PSNN structure and (60), an upper bound on \(w_p\) is

\[
\left| w_p \right| \leq \epsilon + c'_6 + c'_7 \left\| \tilde{W} \right\| h \left( \tilde{W} \right)
\]

(62)

Combine this with (56), then

\[
\left| w_p \right| < \epsilon + c'_6 + c'_7 \left\| \tilde{W} \right\| h \left( \tilde{W} \right) \\
\leq \epsilon + c'_6 + c'_7 \left( c'_0 + c'_1 \left| \xi \right| \right) + c'_2 \left\| \tilde{W} \right\| K \cdot \left\| \tilde{W} \right\| h \left( \tilde{W} \right) \\
= \epsilon + c'_6 + c'_7 \left( c'_3 + c'_4 \left| \xi \right| + c'_5 \right) \left\| \tilde{W} \right\| + \xi \left( \left| \xi \right| , \left\| \tilde{W} \right\| \right)
\]
\[ \begin{align*}
&\cdot \|\hat{W}\| h(\hat{W}) \\
&= c_0 + c_1 \|\hat{W}\| h(\hat{W}) + c_2 \|\xi\| \|\hat{W}\| h(\hat{W}) \\
&+ c_3 \|\hat{W}\|^2 h(\hat{W}) \\
&+ c_4 \xi (\|\xi\|, \|\hat{W}\|) h(\hat{W})
\end{align*} \]

(63)

where \( c_i > 0 \) are known.

**Theorem 1** Consider the compound helicopter nonlinear dynamics given by (12) together with the control law given by (18) and (23), and the adaptive law given by (42) satisfying assumptions 1–6 as well as the following condition

\[ \begin{align*}
\Gamma_W > 0 \\
K_{r_1} \geq c_4 \\
K_{r_2} \geq c_2 \\
c_3 + \frac{1}{2} c_0 \geq \lambda \geq c_3
\end{align*} \]

(64)

Such that for initial conditions \( x(0) \), \( \hat{W}(0) \) belonging to compact set \( \Omega_0 \), it can be guaranteed that all the signals in the proposed closed-loop system are semi-globally uniformly ultimately bounded (SGUUB) and the tracking error \( e(t) \) and the weight error \( \hat{W}(t) \) converge to the zero point.

**Proof of theorem 1** The tracking error dynamics are given by

\[ \begin{align*}
\dot{e} &= A e + b(v_{ad} - \delta) = A e \\
&+ b \left\{ v_r + \sigma \left( \prod_{i=1}^{K} \hat{W}^T_i \bar{x} \right) - \sigma \left( \prod_{i=1}^{K} W_i^T \bar{x} \right) - e \right\}
\end{align*} \]

(65)

Consider the candidate Lyapunov function

\[ V(e(t), \hat{W}(t)) = \frac{1}{2} e^T P e + \frac{1}{2 \Gamma_W} tr(\hat{W}^T \hat{W}) \]

(66)

Combining with the Lyapunov function \( A^T P + PA = -Q \) for \( Q = Q^T > 0 \) and substituting the update law, the derivative of the Lyapunov function with respect to time of this expression may be expressed as

\[ \dot{V}(e(t), \hat{W}(t)) = \frac{1}{2} e^T P [A e + b(v_{ad} - \delta)] + \frac{1}{2} [A e + b(v_{ad} - \delta)]^T P e + tr \left\{ \hat{W}^T \Gamma_W^{-1} \hat{\dot{W}} \right\} \]

\[ = e^T \left( A^T P + PA \right) e + (v_{ad} - \delta) b^T P e \]

\[ + tr \left\{ \hat{W}^T \Gamma_W^{-1} \hat{\dot{W}} \right\} \]

\[ = \left\{ \sigma \left( \prod_{i=1}^{K} \hat{W}^T_i \bar{x} \right) + v_r - \sigma \left( \prod_{i=1}^{K} W_i^T \bar{x} \right) + e \right\} \xi \]

\[ - e^T Q e + tr \left\{ \hat{W}^T \Gamma_W^{-1} \hat{\dot{W}} \right\} \]

\[ = \left\{ v_r - \sigma (\hat{W}^T \bar{x})^2 + e \right\} \xi - e^T Q e \]

\[ + tr \left\{ \hat{W}^T \left[ \bar{x} \cdot \nabla_{\bar{x} \times} \sigma \left( \prod_{i=1}^{K} \hat{W}^T_i \bar{x} \right) \xi + \Gamma_W^{-1} \hat{\dot{W}} \right] \right\} \]

\[ = -e^T Q e + \left\{ v_r + v_p (\bar{x}, \hat{W}) \right\} \xi \]

\[ - \lambda \|\xi\| h(\hat{W}) tr \left\{ \hat{W}^T (\hat{W} - W_0) \right\} \]

(67)

Use the known bound \( \|W^* - W_0\| \leq \bar{W}_0 \), Cauchy–Schwarz inequality and note that

\[ - tr \left\{ \hat{W}^T (\hat{W} - W_0) \right\} \]

\[ = - tr \left\{ \hat{W}^T (\hat{W} + W^* - W_0) \right\} \]

\[ = - \|W\|^2 - tr \left\{ \hat{W}^T (W^* - W_0) \right\} \]

\[ = - \|\hat{W}\|^2 - \sum_{i=1}^{K} \hat{W}_i^T (W_i^* - W_{10}) \]

\[ \leq - \|\hat{W}\|^2 + \|\hat{W}\| \bar{W}_0 \]

(68)

For convenience of analysis, choose \( Q = I \). Observe that

\[ e^T P e \leq \rho (P) \|e\|^2 \Rightarrow -e^T e = -\|e\|^2 \leq - \frac{e^T P e}{\rho (P)} \]

(69)
where \( \rho(P) \) signifies the spectral radius of positive-definite matrix. Therefore,

\[
\dot{V}(\hat{e}(t), \hat{\theta}(t)) \\
\leq \left\{ v_r + w_P(\hat{x}, \hat{\theta}) \right\} \xi - \lambda h(\hat{\theta}) |\xi| tr \left\{ \hat{W}^T \hat{W} \right\} \\
- \frac{\dot{e}^T Pe}{\rho(P)} \\
- \frac{\lambda h(\hat{\theta}) |\xi| tr \left\{ \hat{W}^T (\hat{W} + W^* - W_0) \right\}}{\rho(P)} \\
\leq v_r \xi - \lambda \|\hat{W}\|^2 h(\hat{\theta}) |\xi| - \frac{\dot{e}^T Pe}{\rho(P)} \\
+ \lambda \|\hat{\theta}\| h(\hat{\theta}) |\xi| \\
+ \sup \left\{ w_P(\hat{x}, \hat{\theta}) \right\} \xi \\
= - \frac{\dot{e}^T Pe}{\rho(P)} - \lambda |\xi| \|\hat{W}\|^2 h(\hat{\theta}) \\
+ \lambda |\xi| \|\hat{\theta}\| h(\hat{\theta}) \\
- K_{r1} \xi \left( |\xi|, \|\hat{W}\| + \hat{\theta}_0 \right) h(\hat{\theta}) \xi \\
- K_{r2} \left( \|\hat{W}\| + \hat{\theta}_0 \right) \\
h(\hat{\theta}) \xi^2 + \left\{ c_0 + c_1 \|\hat{\theta}\| h(\hat{\theta}) \right\} \\
+ c_2 |\xi| \|\hat{W}\| h(\hat{\theta}) \\
+ c_3 \|\hat{W}\|^2 h(\hat{\theta}) + c_4 \xi \left( |\xi|, \|\hat{W}\| \right) h(\hat{\theta}) \xi \\
\leq - \frac{\dot{e}^T Pe}{\rho(P)} + c_0 |\xi| + (c_1 + \lambda \hat{\theta}_0) |\xi| \|\hat{W}\| h(\hat{\theta}) \xi \\
+ (c_2 - K_{r2}) \|\hat{W}\| h(\hat{\theta}) \xi^2 \\
+ (c_3 - \lambda) \|\hat{W}\|^2 h(\hat{\theta}) |\xi| \\
+ (c_4 - K_{r1}) \xi \left( |\xi|, \|\hat{W}\| + \hat{\theta}_0 \right) h(\hat{\theta}) \xi \tag{70}
\]

With the following condition

\[
\begin{cases}
\Gamma_W > 0 \\
K_{r1} \geq c_4 \\
K_{r2} \geq c_2
\end{cases}
\tag{71}
\]

such that

\[
\dot{V}(\hat{e}(t), \hat{\theta}(t)) \leq - \frac{\dot{e}^T Pe}{\rho(P)} + \left\{ c_0 + (c_1 + \lambda \hat{\theta}_0) \|\hat{W}\| \right\} h(\hat{\theta}) |\xi| \\
+ (c_3 - \lambda) \|\hat{W}\|^2 h(\hat{\theta}) |\xi| \tag{72}
\]

with \( \lambda > c_3 \) this can be written as a quadratic form

\[
\dot{V}(\hat{e}(t), \hat{\theta}(t)) \leq - \frac{\dot{e}^T Pe}{\rho(P)} + \left\{ (c_3 - \lambda) \left[ \|\hat{\theta}\| - \frac{(c_1 + \lambda \hat{\theta}_0)}{2(c_3 - \lambda)} \right]^2 \right\} \\
- \frac{(c_1 + \lambda \hat{\theta}_0)^2}{4(c_3 - \lambda)^2} + c_0 \right\} h(\hat{\theta}) |\xi| \tag{73}
\]

when \( \lambda \) satisfies the condition that

\[
c_3 + \frac{c_1}{2\sqrt{c_0}} \geq \lambda > c_3
\]

thus

\[
\frac{(c_1 + \lambda \hat{\theta}_0)^2}{4(c_3 - \lambda)^2} > \frac{c_1}{4(c_3 - \lambda)^2} \geq c_0
\]

which implies that \( \dot{V} \) will be negative with each term in the formula is strictly negative, thus the error \( e \) and the weight estimation error \( \hat{W} \) will converge to the zero point. All the signals in the closed-loop system are proved SGUUB.

\( \square \)

Remark 4 Note that \( \lambda, \Gamma_W, K_{r1} \) and \( K_{r2} \) are the design parameters and can determine a trade-off between control performance and robustness. The external command signals were assumed to be bounded, as in
assumption 2. This shows that commands of larger magnitude imply smaller values for $\Gamma_{\text{psnn}}$. This may be interpreted to mean that to limit the closed-loop bandwidth, smaller PSNN learning rates may be required when allowing for more aggressive command tracking.

4 Numerical simulation

In this section, numerical examples are provided to demonstrate the effectiveness and improved performance of the proposed approach. The proposed approach is designed to solve a class of compound helicopter automatic control problem. In this study, X-49 known as a classical compound helicopter is taken as an example in this simulation. The main rotor and the fuselage’s nonlinear mathematical models are built using data from General Helicopter (GENHEL) Flight Dynamics Simulation [43]. The main parameters are shown in the table below (Table 1).

An approximate nominal nonlinear model $\hat{g}_1(x, u)$ is used to get incremental dynamic inversion. The nominal model is established by combining known parts of each component model and ignoring random aerodynamic interference, control surface coupling as well as model uncertainty varying with flight state, which are considerable and may cause fatal instability during actual flight.

The initial state is set to hovering equilibrium state, of which all states set to zero. The forward flight command $u_{\text{cmd}}$ is given by as a multimodal maneuver signal, which contains two aggressive maneuver processes and two cruise processes.

$$u_{\text{cmd}}(t) = \begin{cases} 
36.7t & t < 30s \\
110 & 30s < t < 50s \\
610 - 10t & 50s < t < 55s \\
60 & 55s < t
\end{cases}$$

Furthermore, realistic strong aerodynamic cross-coupling among various control surfaces is designed through the experimental data to make it more difficult for the controller to keep the compound helicopter stable. The velocity reference model is designed as an inertial link

$$x_{1c}(s) = \frac{1}{0.3s + 1}x_{1\text{cmd}}(s)$$

where all parameters in reference are designed according to performance index. The velocity optimization matrix is designed as

$$\Gamma_1 = \text{diag}(1, 1, 2, 5000)$$

To ensure Assumption 4 and reduce system burden, the short time increment $T_s = 20$ ms. Designed as (46), the velocity control input recursion is

$$u_1^{k+1} = \Gamma_1 \nabla_{u_1}^T \hat{g}_1(x, u_1^k)$$

$$= \left[ \nabla_{u_1} \hat{g}_1(x, u_1^k) \Gamma_1 \cdot \nabla_{u_1}^T \hat{g}_1(x, u_1^k) \right]^{-1}$$

$$\left[ \dot{x}_1 + v_{l1} - v_{u_1} + \nabla_{u_1} \hat{g}_1(x, u_1^k) \right] u_1^k$$

(74)

Similarly, the angular velocity reference model is designed as a second-order system.

$$x_{2c}(s) = \frac{17.64}{s^2 + 8.4s + 17.64}x_{2\text{cmd}}(s)$$

The optimization matrix is

$$\Gamma_2 = \text{diag}(0.33, 1, 1.25, 1.25, 2, 0.66)$$

Attitude control input recursion is

$$u_2^{k+1} = \Gamma_2 \nabla_{u_2}^T \hat{g}_2(x, u_2^k)$$

$$= \left[ \nabla_{u_2} \hat{g}_2(x, u_2^k) \Gamma_2 \cdot \nabla_{u_2}^T \hat{g}_2(x, u_2^k) \right]^{-1}$$

$$\left[ \dot{x}_2 + v_{l2} - v_{u_2} + \nabla_{u_2} \hat{g}_2(x, u_2^k) \right] u_2^k$$

(75)
where $u^0_i$ is initial control input, the structure of pseudo-control $v_{Li}$ and $v_{adi}$ is given before, all linear control parameters in two control loops are uniformly designed as $K_P = 5$, $K_I = 0.2$. To ensure the accuracy of feedback differential signals, the second-order tracking differentiator is used to remove noise as

$$y(s) = \frac{s}{2.4s^2 + 3.8s + 1} v(s)$$

During the simulation, forth order PSNN is selected in both outer control loop and inner control loop. The control of neural network is designed in (39). Used as controls, classical INDI controller (INDIC) without adaptive compensation and traditional single layer perceptron controller (SHLC) in [33] are built in the simulation. Compared with these classical controllers, the improvement of proposed methods can be effectively verified. To make the comparison fair and persuasive, the basic controller structure and constant parameters of control groups are consistent with (74) and (75) (except adaptive controller output $v_{ad}$), where $v_{ad} = 0$ in INDIC and $v_{ad}$ in SHLC is presented as

$$v_{ad} = v_{SHL} = \hat{W}^T \sigma \left( V^T \dot{x} \right) - K_r \left( \| \dot{Z} \| + \hat{Z}_0 \right) \xi$$

$$\dot{\hat{W}} = -\Gamma_w \left[ \zeta \left( \sigma \left( V^T \dot{x} \right) - \sigma_r \left( V^T r V^T \dot{x} \right) \right) 
+ \lambda |\xi| (W - W_0) \right]$$

$$\dot{\hat{V}} = -\Gamma_v \left( r \dot{\xi} \hat{W}^T \sigma_r \left( V^T \dot{x} \right) + \lambda |\xi| (V - V_0) \right)$$

$$Z = \begin{bmatrix} \hat{W} & 0 \\ 0 & \hat{V} \end{bmatrix}$$

The meaning of all variables in (76) is explained in [33]. All constant parameters of neural network in proposed controller and control groups are given in Table 2.

The simulation process is carried out in MATLAB. During the simulation, the proposed method has worked and weight of PSNN has converged. The final steady weight of PSNN in attitude control loop is

$$\hat{W}_{final} = \begin{bmatrix} 0.060 & 0.105 & 0.112 & 0.045 & 0.101 & 0.109 & -0.111 & 0.162 \\ 0.238 & -0.117 & -0.164 & 0.135 & -0.219 & 0.071 & 0.036 & -0.084 \\ 0.031 & 0.189 & 0.203 & -0.117 & 0.213 & -0.081 & 0.143 & 0.017 \\ -0.172 & 0.021 & 0.167 & 0.147 & -0.069 & -0.041 & 0.102 & 0.129 \end{bmatrix}$$

The results are shown in figures below. Figure 12a shows that proposed controller successfully tracked desired forward velocity command and made it possible to improve tracking performance by designing reference model according to performance index of time domain or frequency domain. Nevertheless, INDI controller and SHL adaptive controller only followed the command barely (Figs. 12 and 13).

Figures 12b and 14 show convergence performance related to attitude and velocity. It can be observed that attitude, angular velocity and velocity errors driven by proposed control scheme asymptotically converge to origin even though flight state rapid change, dynamics inaccuracy, as well as aerodynamic coupling interference. However, in contrast, the other two methods just guarantee bounded results within simulation time and cannot eliminate violent shock during the maneuver process.
Table 2  Initial parameters setting in all adaptive controllers

| Controllers | NN parameters | Adaptive parameters |
|-------------|---------------|---------------------|
| SHLC        | Number of hidden layer $\Gamma_w = 100$, $\Gamma_r = 70$, nodes: 10, $K_r = 1.6$, $\lambda = 0.8$ | $u_0 = 0.01$, $v_0 = 0.01$, $b_t = 0.1$, $b_w = 0.1$ |
| SLRBFC      | Number of hidden layer $\Gamma_w = 100$, $\Gamma_p = 1$, nodes: 8, $\Gamma_q = 1$, $\tau = 0.1$ | $b = 5$, $w_0 = 0.01$, $p_0 = 0.01$, $q_0 = 0.01$, $b = 0.1$, $o = [-10, -5, -2.5, -1.25, 1.25, 2.5, 5, 10]$ |
| Proposed    | Number of hidden layer $\Gamma_w = 100$, $\lambda = 0.8$ nodes: 4, $K_{r1} = 1.4$, $K_{r2} = 1.28$, $K_{NN} = 2$, $b_t = 0.1$, $b_w = 0.1$, $w_0 = 0.01$, $\theta_{ui} = 0.02$ |

Fig. 13 Time responses of the Euler angle state variables of compound helicopter

Fig. 14 Time responses of the angular velocity state variables of compound helicopter

Fig. 15 Time histories for control surfaces including $\psi_7$, $A_{1x}$, $B_{1x}$, $\delta_l$ and $\delta_r$

Figures 15 and 16 show that proposed controller ensures that multiple redundant control surfaces can complete the maneuver task in an efficient way during the whole maneuver process of the compound helicopter, and guarantee the smoothness of mode conversion. The helicopter control surfaces mainly work in low-speed flight and keep minimum mode in high-
speed flight. On the contrary, fixed wing control surfaces play important roles in high-speed flight. Figures 17 and 18 show that the proposed controller relied on the output of linear controller at first, while the adaptive controller realized the cancellation of perturbation term and convergence of error in a short time (about 10 sec), and had a vital catalytic role in the process of maintaining stability. It can also be shown that tracking errors tend to be flat due to PSNN. Along with the flight states varying, the PSNN adaptive controller output changed fast enough to maintain robustness to disturbances, which is superior to the traditional neural networks.

Figures 19 and 20 show that the weight matrix norms converge in 10 second with great robustness. Although in the process of network convergence, the weight pro-
Fig. 20 Time histories for Frobenius norm of weight matrices to attitude control

Fig. 21 Time responses of the velocity state variables of compound helicopter (Compared with SLRBFC)

Fig. 22 Time responses of the Euler angle of compound helicopter (Compared with SLRBFC)

Produced inevitable oscillation phenomenon, the robust adaptive term designed in (41) and the damping item designed in (42) still helped keep PSNN stable. During first maneuver process, the convergence of weights was completed after a series of small shocks. Moreover, during the second maneuver process, although the process is more violent, the norm of weight matrix still maintained a steady process, which implies that the uncertain nonlinear perturbation has been successfully canceled by PSNN and local minimum has been successfully avoided. In engineering, storing the PSNN parameters can further improve the stability and robustness of the close-loop system.

At present, the popular advanced adaptive network algorithms have strong autonomous learning and generalization ability. Self-learning radial basis function neural network controller (SLRBFC) in [44] is one of the advanced approaches for tracking control. Taking it as an example, an additional simulation example is designed to verify the superior performance of proposed method compared with mainstream advanced NN controller.
Similar to other control groups, the basic controller architecture and constant parameters of SLRBFC is consistent with (74) and (75) (except adaptive controller output $v_{ad}$), where $v_{ad}$ is presented as

$$v_{ad} = v_{RBF} = \hat{W}^T \Pi (\bar{x})$$

$$\Pi (r) = \exp \left( -\frac{\|q \bar{x} - o\|^2}{2b^2} \right)$$

(77)

Self-learning algorithm is designed as

$$\Delta p = \Gamma_p \left( \frac{1}{p} - \frac{1}{2b^2} \left( 1 + \frac{\Pi}{\tau} \right) \|q \bar{x} - o\|^2 + \frac{1}{q_p} \|q' \| (q \bar{x} - o) \right)$$

$$\Delta q = \Gamma_q \left( \frac{\Pi \bar{x}}{|q' \| (q \bar{x} - o)|} - \frac{\Pi \bar{x}}{\tau b^2} q \bar{x} - o \right)$$

$$\dot{W} = -\Gamma_w e^T P b \Pi$$

(78)

The meaning of each parameter in (77) and (78) is illustrated in [44] and the main constant parameters in neural network are shown in Table 2. The multimodal velocity and attitude commands are given as

$$u_{cmd} = 70 + 40 \sin(0.05t)$$

$$\psi_{cmd} = \frac{\pi}{2} \sin(0.05t)$$

The tracking results of different controllers are depicted in Figs. 21 and 22. Compared with SLRBFC, the convergence time of proposed has been reduced by over 30% and the tracking accuracy has been greatly improved. By comparing the tracking performance, it can be concluded that SHLC and SLRBFC are limited by the topology of traditional neural network and cannot meet the convergence requirements of multimodal control. It is worth mentioning that although more hidden layer nodes and more adjustable weights are involved in SHLC and SLRBFC, the proposed method can perform better with its unique structure and better nonlinear approximation ability.

5 Conclusion

By incorporating adaptive control method into PSNN controller design, the novel method with global stability, fast convergence ability and strong robustness have been proposed for solving command tracking problem of compound helicopters with aerodynamic cross-coupling in multimodal maneuver. The weights of PSNN are adjusted online according to a novel adaptive algorithm based on CE principle such that the effects of the uncertain coupling disturbance can be mitigated under the condition of close-loop augmented system SGUUB. INDI control framework including a couple of control loop has been built to decouple the overactuated system and keep robustness to instability of PSNN. Moreover, on the basis of an incremental Lagrange multiplier optimization, the globally optimal input for each control surface is allocated based on its time-varying working capability. It has been proven in terms of Lyapunov theorem that the proposed control scheme achieves the compound helicopter multimodal tracking objective with an asymptotically stable performance. By comparing with existing methods, numerical simulation finally has verified the efficacy of the proposed approach.

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Declarations

Conflicts of interest The authors declare that they have no conflict of interest.

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