How to observe the localization law $\sigma(\omega) \propto -i\omega$ for conductivity?

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The Berezinskii localization law $\sigma(\omega) \propto -i\omega$ for frequency-dependent conductivity was never questioned from the theoretical side, but never observed experimentally. In fact, this result is valid for closed systems, while most of actual systems are open. We discuss several possibilities for observation of this law and experimental difficulties arising at this way.

It is well-known [1, 2] that the electron states in disordered systems can be extended or localized. In the latter case, when a system is an Anderson dielectric, its frequency-dependent conductivity is believed to obey the Mott law, $\text{Re} \sigma(\omega) \propto \omega^2 \ln^0 \omega$. In fact, in the low-frequency limit conductivity is dominated by its imaginary part, and dependence $\sigma(\omega) \propto -i\omega$ is expected.

The localization law $\sigma(\omega) \propto -i\omega$ was predicted by Berezinskii in 1973 [3] for one-dimensional disordered systems. According to self-consistent theory by Vollhardt and Wölfle [4], the same result is valid in the localization phase for systems of arbitrary dimension $d$. In the recent paper [5] of the present author the same behavior of conductivity was established for systems of finite size $L$ at the arbitrary extent of disorder. The latter is a consequence of the fact that a finite system is topologically zero-dimensional, and its effective dimensionality is less than lower critical one ($d_{c1} = 2$ [6]).

The Berezinskii law was never questioned in the theoretical community; however, it was never observed experimentally. This paradoxical situation was clarified in Ref.5: Berezinskii’s result is valid in closed systems, while most of actual systems are open. In open systems, replacement $-i\omega \to -i\omega + \gamma$ occurs (where $\gamma$ is inelastic damping) and dependence $\sigma(\omega) \propto -i\omega$ transforms into the usual metallic behavior.

A possibility of realization of closed systems became clear after observation of the persistent current in disordered systems (in the Aharonov–Bohm geometry) [7, 8, 9], in accordance with its prediction in [10]. In fact, the persistent current is a consequence of the Berezinskii law, establishing the dissipativeless character of conductance. Its observation is possible, when a size $L$ of the disordered ring is small in comparison with the inelastic length $L_{in}$, depending on temperature $T$. The typical scales in the indicated experiments were $L \sim 1\mu m, T \sim \text{100mK}$. If one accepts that $L_{in} \propto T^{-2}$ (as for e-e interaction), then a system is closed for $L \lesssim 10\text{nm}$ in the helium region ($T \sim 1\text{K}$).

Let discuss several experimental situations, where observation of the Berezinskii law is possible.

1. The first variant is the island film of a disordered metal lying on the dielectric substrate (Fig.1). We suppose for clearance that all islands are of the same size $L$, which increases monotonically in the course of the film deposition [6]. Then for $L \lesssim L_{in}$ the Berezinskii law is valid (Fig.1.a), while in the opposite case $L \gtrsim L_{in}$ the usual metallic conductivity takes place (Fig.1.b). A transition from one regime to another can be provided by the change of $L$ or the temperature.

At first glance, the described experiment is simple. However, there is a bottleneck in it. It is clear from relation $\epsilon \sim i\sigma/\omega$, that behavior $\sigma \propto -i\omega$ corresponds to the frequency-independent permittivity $\epsilon$, so a disordered system is an ordinary dielectric. The properties of the film in the Berezinskii law regime are the same as those of the dielectric substrate, hence the former gives a negligible contribution to conductivity in comparison with the latter. The width of the film is by 6–7 orders less than the width of the substrate, but the corresponding smallness can be partially compensated by a large value of the film permittivity $\epsilon_1$ in comparison with

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1 In fact, there is a distribution of islands in size, which shifts to the large $L$ region in the course of deposition.
Figure 1: In the case of the island metallic film, the Berezinskii law is observable when the island size $L$ is small in comparison with the inelastic length $L_{\text{in}}$ (a), while in the opposite case the metallic behavior is valid (b).

its substrate value $\epsilon_0$. By the order of magnitude, $\epsilon_1 \sim \xi^2/a_0^2$ (where $\xi$ is the localization length for wave functions, and $a_0$ is the atomic space), and saturates by a value $L^2/a_0^2$ for large $\xi$. If the metallic film is weakly disordered, then for $L \sim 10\text{nm}$ its permittivity $\epsilon_1$ can exceed $\epsilon_0$ by 3–4 orders.

The experimental procedure looks as follows. The experiment is carried out in situ and begins with a measurement of the frequency and temperature dependencies of the substrate conductivity, with saving results in a file. Then a small amount of metallic atoms is deposited, and again conductivity is measured and saved; again deposition is made and so on. Proceeding by small steps, one should reach a regime, when the film contribution is clearly seen in the substrate background. Then the actual measurements can be made.

2. The second example is the nanocomposite system [12, 13], which is a dielectric sample with the metallic granules embedded in it (Fig. 2). The volume fraction $p$ of a metal can be rather large and its effect should be easily observable, being of the order of unity. However, a "hidden rock" is present here. Let exploit the formula from the Landau and Lifshitz book [15]

$$\bar{\epsilon} - \epsilon_0 = p \frac{3(\epsilon_1 - \epsilon_0)}{2\epsilon_0 + \epsilon_1}, \quad (1)$$

which is valid for a small concentration of spherical granules: it gives the average permittivity $\bar{\epsilon}$ (for the system of Fig. 2) in terms of its values for a dielectric ($\epsilon_0$) and a metal ($\epsilon_1$). Since $\epsilon_1 \gg \epsilon_0$, then

$$\frac{\bar{\epsilon} - \epsilon_0}{\epsilon_0} \approx 3p - 9p\frac{\epsilon_0}{\epsilon_1}, \quad (2)$$

and the main contribution $3p$ is an uninteresting constant, while the useful effect, depending on $\epsilon_1$, is determining by two small parameters $p$ and $\epsilon_0/\epsilon_1$. As a result, the problem of a reference arises, i.e. a necessity to have the identical sample without metallic granules. Fortunately, such a problem is absent for a specific technology [12, 13, 14], when nanocomposites are produced on the base of a porous glass, whose pores are filled by metallic granules (of suitable size about 7nm); so the same sample can be measured in absence and in presence of granules. It is useful to note that for the system of Fig. 2 (in contrast to that of Fig. 1) the strongly disordered metal is desirable, in order to increase the ratio $\epsilon_0/\epsilon_1$.

3. Derivation of Eq.1 is based on a solution of the well-known problem on a dielectric ball in the...
external electric field. The analogous problem is solvable for an ellipsoid with arbitrary ratios of its semi-axes $a$, $b$, $c$, and generalization of (1) is possible for granules of ellipsoidal form:

$$\frac{\bar{\epsilon} - \epsilon_0}{\epsilon_0} = p \frac{\epsilon_1 - \epsilon_0}{A\epsilon_0 + B\epsilon_1},$$

(3)

where $A = 1 - B$, and

$$B = \frac{abc}{2} \int_0^\infty \frac{dx}{(x + a^2)^{3/2}(x + b^2)^{1/2}(x + c^2)^{1/2}},$$

(4)

if the electric field $\mathbf{E}$ is directed along the axis $a$.

In reality, the metallic granules are not strictly spherical. For modelling of such situation, one can suggest that granules are ellipsoids with fluctuating ratios of semi-axes. Then for $\epsilon_1 \gg \epsilon_0$ one has

$$\frac{\bar{\epsilon} - \epsilon_0}{\epsilon_0} \approx p \langle B^{-1} \rangle - p \langle B^{-2} \rangle \frac{\epsilon_0}{\epsilon_1},$$

(5)

($\langle \ldots \rangle$ is averaging over fluctuations), so the structure of Eq.2 is preserved but the coefficients are changed.

Parameter $B$ decreases when $a$ becomes greater than $b$ and $c$. In the limit of a strongly oblong ellipsoid ($a \gg b \sim c$) one has $B \rightarrow 0$ and Eq.3 takes a form

$$\frac{\bar{\epsilon} - \epsilon_0}{\epsilon_0} = p \frac{\epsilon_1 - \epsilon_0}{\epsilon_0},$$

(6)

i.e. optimal conditions for observation correspond to the needle-shaped granules (Fig.3). In this case one can provide a sufficient smallness of $p$ (which is necessary for validity of Eq.3 and a transparent interpretation of the experiment) and its compensation by a large parameter $\epsilon_1/\epsilon_0$. As a result, the effect is of the order of unity, or even more. Such systems can be fabricated on the base of chrysotile asbestos, which is a stack of the parallel nanotubes with a typical pore diameter 5 nm; since the length of the granules should be essentially greater than one is induced to work in the millikelvin range of temperatures.

4. In relation with the latter, we can indicate one exotic possibility. If a vessel with superfluid helium is rotated, then a set of the parallel vortices arises. If metallic atoms are injected in helium, they are localized at the vortex cores and form nanowires. Regulating the length of the latter, one can create the desired system (Fig.4). A concentration of the metallic phase is strongly restricted in this case but at sufficiently low temperatures one can deal with large $L$ scales and, as a consequence, with enormous values of permeability $\epsilon_1$.

Analogously, parameter $B$ tends to zero in the case of pancake-shaped granules ($a \sim b \gg c$), if their plane is oriented along the electric field; this case is also described by formula (6). In particular, it is valid in the situation of Fig.1, where the volume concentration $p$ is inevitably small.

In conclusion, the Berezinskii law was not observed previously, since it refers to closed systems, while most of actual systems are open. We have suggested several possibilities for its observation, and shown that experimental difficulties are present in all situations. The latter is rather natural, since in the opposite case this law would be discovered experimentally long ago.

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3 This length can reach 1 mm.

4 For realistic conditions, one can have about $10^4$ vortices per $1cm^2$, while a diameter of nanowires varies from $a_0$ till several nanometers.
Figure 4: An exotic realization of the system represented in Fig.3. If a vessel with superfluid helium is rotated, then a set of the parallel vortices arises, and the injected metallic atoms are localized on the vortex cores.

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