The link between the electroweak gauge boson masses and the Fermi constant via the muon lifetime measurement is instrumental for constraining and eventually pinning down new physics. We consider the simplest extension of the Standard Model with an additional real scalar $SU(2)_L \otimes U(1)_Y$ singlet and compute the electroweak precision parameter $\Delta r$, along with the corresponding theoretical prediction for the W–boson mass. When confronted with the experimental W–boson mass measurement, our predictions impose limits on the singlet model parameter space. We identify regions where these correspond to the most stringent experimental constraints that are currently available.

1 Introduction

The relation between the Electroweak (EW) gauge boson masses, the Fermi constant $|G_F|$ and the fine structure constant $[\alpha_{\text{em}}]$ is anchored experimentally via the muon lifetime measurement and constitutes a prominent tool for testing the quantum structure of the Standard Model (SM) and its manifold conceivable extensions. This relation is conventionally expressed in the literature by means of the $\Delta r$ parameter [1–6] and plays a major role in placing bounds on, and eventually unveiling new physics coupled to the standard electroweak Lagrangian.

Aside from being interesting on its own, the quantum effects traded by $\Delta r$ are part of the electroweak radiative corrections to production and decay processes in the SM and beyond. In particular, the knowledge of $\Delta r$ is a required footstep towards a full one–loop electroweak characterization of the Higgs boson decay modes in the singlet extension of the SM [7].

The calculation of electroweak precision observables (EWPO) and its role in constraining manifold extensions of the SM has been object of dedicated attention in the literature [1,2,5,6,8–19]. Theoretical predictions for $\Delta r$ and for the W–boson mass $[m_W^{\text{th}}]$ were first derived in the context
of the SM \cite{20,21} and later on extended to new physics models such as the Two-Higgs-Doublet Model (2HDM) \cite{22,30} and the Minimal Supersymmetric Standard Model (MSSM) \cite{17,31,39}. These predictions have proven to be relevant not only to impose parameter space constraints, but also to identify new physics structures capable to in part reconcile the well-known tension between the SM prediction and the experimental value, $|m^{\text{SM}}_W - m^{\text{exp}}_W| \approx 20$ MeV. For instance, in Ref. \cite{30} it was shown that the extended Higgs sector of the general Two–Higgs–Doublet Model (2HDM) could yield $m^{2\text{HDM}}_W \gtrsim m^{\text{SM}}_W$, thus potentially alleviating the present discrepancy.

Our main endeavour in this note is to provide a one–loop evaluation of the electroweak parameter $\Delta r$ and the W–boson mass in the presence of one extra real scalar $SU(2)_L \otimes U(1)_Y$ singlet. This model, which incorporates an additional neutral, CP-even spinless state, corresponds to the simplest renormalizable extension of the SM, and can also be viewed as an effective description of the low–energy Higgs sector of a more fundamental UV completion. Pioneered by Refs. \cite{40,42}, this class of models has undergone dedicated scrutiny for the past two decades, revealing rich phenomenological implications, especially in the context of collider physics, see e.g. \cite{43,55,56,67}.

Our starting point is the current most precise theoretical prediction for the SM W–boson mass $m^{\text{SM}}_W$, which is known exactly at two–loop accuracy, including up to leading three–loop contributions \cite{36,68,74}. We combine these pure SM effects with the genuine singlet model one–loop contributions and analyse their dependences on the relevant model parameters. Next we correlate our results with the experimental measurement of the W–boson mass and derive constraints on the singlet model parameter space. Finally, we compare them to complementary constraints from direct collider searches, as well as to the more conventional tests based on global fits to electroweak precision observables.

### 2 $\Delta r$ and $m_W$ as Electroweak precision measurements

In the so–called “$G_F$ scheme”, electroweak precision calculations use the experimentally measured $Z$–boson mass $m_Z$, the fine–structure constant at zero momentum $[\alpha_{\text{em}}(0)]$, and the Fermi constant $[G_F]$ as input values. The latter is linked to the muon lifetime via \cite{2,3,5,6}

$$
\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} F \left( \frac{m_\mu^2}{m_e^2} \right) \left( 1 + \frac{3 m_\mu^2}{5 m_W^2} \right) (1 + \Delta_{\text{QED}}),
$$

where $F(x) = 1 - 8x - 12x^2 \ln x + 8x^3 - x^4$. Following the standard conventions in the literature, the above defining relation for $G_F$ includes the finite QED contributions $\Delta_{\text{QED}}$ obtained within the Fermi model – which are known to two–loop accuracy \cite{75,79}. Matching the muon lifetime in the Fermi model to the equivalent calculation within the full–fledged SM yields the relation:

$$
m^2_W \left( 1 - \frac{m^2_W}{m^2_Z} \right) = \frac{\pi \alpha_{\text{em}}}{\sqrt{2} G_F} (1 + \Delta r) \quad \text{with} \quad \Delta r \equiv \frac{\hat{\Sigma}_W(0)}{m^2_W} + \Delta r^{[\text{vert,box}]},
$$

which is the conventional definition of $\Delta r$, with $m_{W,Z}$ being the renormalized gauge boson masses in the on–shell scheme. Accordingly, we introduce the on–shell definition of the electroweak mixing angle \cite{20} $\sin^2 \theta_W = 1 - m^2_W/m^2_Z$, along with the shorthand notations $s^2_W \equiv \sin^2 \theta_W$, $c^2_W \equiv 1 - s^2_W$. In turn, $\hat{\Sigma}_W(k^2)$ denotes the on–shell renormalized W-boson self–energy. The latter accounts for the oblique part of the electroweak radiative corrections to the muon decay. The non–universal (i.e. process–dependent) corrections rely on the vertex and box contributions and are subsumed into $\Delta r^{[\text{vert,box}]}$. The explicit expression for $\Delta r$ after renormalization in the on–shell scheme may
be written as a combination of loop diagrams and counterterms as follows:

\[
\Delta r = \Pi_\gamma(0) - \frac{c_W^2}{s_W^2} \left( \frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} \right) + \frac{\Sigma_W(0) - \delta m_W^2}{m_W^2} + 2 \frac{c_W}{s_W} \Sigma_Z(0) + \Delta r_{\text{vert.box}},
\]

(3)

where \( \Pi_\gamma(0) \) stands for the photon vacuum polarization, while \( \delta m_{W,Z}^2 \) denote the gauge boson mass counterterms. Additional degrees of freedom and/or modified interactions will enter the loop diagrams describing the muon decay, making \( \Delta r \) (and so \( m_W \)) model–dependent quantities. At present, the calculation of \( \Delta r \) in the SM is complete up to two loops \[36,68–73,80–88\] and includes also the leading three \[74, 89–93\] and four–loop pieces \[94, 95\]. The dominant contribution stems from QED fermion loop corrections and is absorbed into the renormalization group running of the fine structure constant.

Taking \( m_Z \) and \( G_F \) as experimental inputs, and using Eq. (2), the evaluation of \( \Delta r \) within the SM or beyond can be translated into a theoretical prediction for the W–boson mass \( m_{W}^{\text{th}} \). For this we need to (iteratively) solve the equation

\[
m_W^2 = \frac{1}{2} m_Z^2 \left[ 1 + \sqrt{1 - \frac{4 \pi \alpha_{\text{em}}}{\sqrt{2} G_F m_Z^2} \left[ 1 + \Delta r(m_W^2) \right]} \right].
\]

(4)

To first–order accuracy, Eq. (4) implies that a shift \( \delta(\Delta r) \) promotes to the W–boson mass through

\[
\Delta m_W \simeq -\frac{1}{2} m_W \frac{s_W^2}{c_W^2 - s_W^2} \delta(\Delta r).
\]

(5)

For \( \Delta r = 0 \) one retrieves the tree-level value \( m_{W}^{\text{tree}} \approx 80.94 \) GeV. But the full theoretical result is smaller in the SM since quantum effects yield \( \Delta r > 0 \) of order few percent. Once we identify the \( \sim 126 \) GeV resonance with the SM Higgs boson, all experimental input values in Eq. (4) are fixed and thereby the theoretical prediction for the W–boson mass is fully determined. Setting the SM Higgs boson mass to the \text{HiggsSignals} \[96–98\] best–fit value \( m_H = 125.7 \) GeV \[96\], one gets \( \Delta r \simeq 0.038 > 0 \), wherefrom \( m_{W}^{\text{SM}} = 80.360 \) GeV. The estimated theoretical uncertainty reads \( \Delta m_{W}^{\text{th}} \simeq 4 \) MeV \[72\] and stems mainly from the top mass measurement \[99\]. This prediction needs to be confronted with the experimental W–boson mass measurement, whose present world–average combines the available results from LEP \[100\], CDF \[101\] and D0 \[102\] and renders

\[
m_{W}^{\exp} = 80.385 \pm 0.015 \text{ GeV}.
\]

(6)

This represents an accuracy at the \( \simeq 0.02\% \) level. The corresponding discrepancy with the SM theoretical prediction \( |m_{W}^{\exp} - m_{W}^{\text{SM}}| \simeq 20 \) MeV falls within the 1\( \sigma \)–level ballpark; however, it is as large as roughly 5 times the estimated theoretical error. On the other hand, these differences should be accessible by the upcoming W–boson mass measurements at the LHC, which are expected to pull the current uncertainty down to \( \Delta m_{W}^{\exp} \simeq 10 \) MeV \[103,104\]. Furthermore, a high–luminosity linear collider running in a low–energy mode at the \( W^+ W^- \) threshold should be able to reduce it even further, namely at the level of \( \Delta m_{W}^{\exp} \simeq 5 \) MeV or even below \[105\]. This strongly justifies, if not simply demands, precision calculations of \( \Delta r \) and \( m_W \) to probe, constrain, or even unveil, new physics structures linked to the electroweak sector of the SM.

As a byproduct, the task of computing \( \Delta r \) involves the evaluation of the so–called \( \delta \rho \) parameter \[106,109\]. The latter is defined upon the static contribution to the gauge boson self–energies,
\[
\frac{\Sigma_Z(0)}{m_Z^2} - \frac{\Sigma_W(0)}{m_W^2} \equiv \delta \rho,
\]
and measures the ratio of the neutral–to–charged weak current strength. Quantum effects yielding \(\delta \rho \neq 0\) may be traced back to the mass splitting between the partners of a given weak isospin doublet, and so to the degree of departure from the global custodial \(SU(2)\) invariance of the SM Lagrangian. The \(\delta \rho\) parameter is finite for each doublet of SM matter fermions and is dominated by the top quark loops

\[
\delta \rho_{[t]}^{[\text{SM}]} = \frac{3 G_F m_t^2}{8 \sqrt{2} \pi^2}.
\]

In terms of \(\delta \rho\), the general expression for \(\Delta r\) can be recast as \([2,5,6]\):

\[
\Delta r = \Delta \alpha - \frac{c_W^2}{s_W^2} \delta \rho + \Delta r_{\text{rem}} = \Delta \alpha + \Delta r^{[\delta \rho]} + \Delta r_{\text{rem}},
\]

where \(\Delta r^{[\delta \rho]} \equiv -\frac{c_W^2}{s_W^2} \delta \rho\) denotes the individual contribution from the static part of the self–energies. The \(\Delta \alpha\) piece accounts for the (leading) QED light–fermion corrections, while the so–called “remainder” term \([\Delta r_{\text{rem}}]\) condenses the remaining (though not negligible) effects. In fact, in the SM we have \(\Delta \alpha \simeq 0.06\) and \(\Delta r_{\text{rem}} \simeq 0.01\), while \(\Delta r^{[\delta \rho]} \simeq -0.03\) \([3,5,6]\).

At variance with this significant contribution, the counterpart Higgs boson–mediated effects are comparably milder in the SM and feature a trademark logarithmic dependence on the Higgs mass \(107\) \(^a\).

\[
\delta \rho^{[H]} \simeq -\frac{3 \sqrt{2} G_F m_W^2}{16 \pi^2} c_W^2 s_W^2 \left\{ \ln \frac{m_H^2}{m_W^2} - \frac{5}{6} \right\} + \ldots.
\]

Remarkably, this telltale screening behavior does not hold in general for extended Higgs sectors – viz. in the general 2HDM \([30]\).

### 3 \(\Delta r\) and \(m_W\) in the singlet model

#### 3.1 Model parametrization at leading–order

Our starting point is the most general form of the gauge invariant, renormalizable potential involving one real \(SU(2)_L \otimes U(1)_Y\) singlet \(S\) and one doublet \(\Phi\), the latter carrying the quantum numbers of the SM Higgs weak isospin doublet (see e.g. \([41,42,110]\)):

\[
L_s = (D^\mu \Phi)^\dagger D_\mu \Phi + \partial^\mu S \partial_\mu S - V(\Phi, S),
\]

with the potential

\[
V(\Phi, S) = -\mu_1^2 \Phi^\dagger \Phi - \mu_2^2 S^2 + \left( \Phi^\dagger \Phi \cdot S^2 \right) \left( \frac{\lambda_1}{\lambda_1 + \frac{\lambda_3}{2}} \frac{\lambda_2}{\lambda_2 + \frac{\lambda_3}{2}} \right) \left( \Phi^\dagger \Phi \right) S^2
\]

\[
= -\mu_1^2 \Phi^\dagger \Phi - \mu_2^2 S^2 + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 S^4 + \lambda_3 \Phi^\dagger \Phi S^2.
\]

\(^a\)One should bear in mind that the Higgs boson contribution in the SM \([\delta \rho_{\text{SM}}^{[H]}]\) is neither UV finite nor gauge invariant on its own, but only in combination with the remaining bosonic contributions.
In this minimal version we assume an additional $\mathbb{Z}_2$ symmetry which forbids additional terms in the potential. The neutral components of these fields can be expanded around their respective Vacuum Expectation Values (VEVs) as follows:

$$
\Phi = \begin{pmatrix} G^\pm \\ v_d + l^0 + iG^0 \end{pmatrix} \quad \text{and} \quad S = \frac{v_s + s^0}{\sqrt{2}}.
$$

The minimum of the above potential is achieved under the conditions

$$
\mu_1^2 = \lambda_1 v_d^2 + \frac{\lambda_3 v_s^2}{2} ; \quad \mu_2^2 = \lambda_2 v_s^2 + \frac{\lambda_3 v_d^2}{2},
$$

while the quadratic terms in the fields generate the mass–squared matrix

$$
\mathcal{M}_{ls}^2 = \begin{pmatrix} 2\lambda_1 v_d^2 & \lambda_3 v_d v_s \\ \lambda_3 v_d v_s & 2\lambda_2 v_s^2 \end{pmatrix}.
$$

Requiring this matrix to be positively–defined leads to the stability conditions

$$
\lambda_1, \lambda_2 > 0; \quad 4\lambda_1 \lambda_2 - \lambda_3^2 > 0.
$$

The above mass matrix in the gauge basis $\mathcal{M}_{ls}^2$ can be transformed into the (tree–level) mass basis through the rotation $R(\alpha) \mathcal{M}_{ls}^2 R^{-1}(\alpha) = \mathcal{M}_{HH}^2 = \text{diag}(m_{h^0}^2, m_{H^0}^2)$, with

$$
R(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \quad \text{and} \quad \tan(2\alpha) = \frac{\lambda_3 v_d v_s}{\lambda_1 v_d^2 - \lambda_2 v_s^2}.
$$

Its eigenvalues then read

$$
m_{h^0,H^0}^2 = \lambda_1 v_d^2 + \lambda_2 v_s^2 \pm |\lambda_1 v_d^2 - \lambda_2 v_s^2| \sqrt{1 + \tan^2(2\alpha)} \quad \text{with the convention} \quad m_{H^0}^2 > m_{h^0}^2, \quad (18)
$$

and correspond to a light [$h^0$] and a heavy [$H^0$] $\mathcal{C}\mathcal{P}$-even mass–eigenstate. From Eq. (17), we see that both are admixtures of the doublet [$l^0$] and the singlet [$s^0$] neutral components

$$
h^0 = l^0 \cos \alpha - s^0 \sin \alpha \quad \text{and} \quad H^0 = l^0 \sin \alpha + s^0 \cos \alpha.
$$

The Higgs sector in this model is determined by five independent parameters, which can be chosen as

$$
m_{h^0}, m_{H^0}, \sin \alpha, v_d, \tan \beta \equiv \frac{v_d}{v_s},
$$

where the doublet VEV is fixed in terms of the Fermi constant through $v_d^2 = G_F^{-1}/\sqrt{2}$. Furthermore, we fix one of the Higgs masses to the LHC value of 125.7 GeV; therefore, three parameters of the model are presently not determined by any experimental measurement.

As only the doublet component can couple to the fermions (via ordinary Yukawa interactions) and the gauge bosons (via the gauge covariant derivative), all of the Higgs couplings to SM particles are rescaled universally, yielding

$$
g_{xxh} = g_{xxh}^\text{SM}(1 + \Delta_{xh}) \quad \text{with} \quad 1 + \Delta_{xh} = \begin{cases} \cos \alpha & h = h^0 \\ \sin \alpha & h = H^0 \end{cases}.
$$

\footnote{Cf. e.g. \cite{110} for a more detailed discussion.}
3.2 Calculation details

Let us now focus on the calculation of $\Delta r$ and $m_W$ in the singlet extension of the SM. The pure SM contributions $[\Delta r_{\text{SM}}]$ and the genuine singlet model effects $[\delta(\Delta r_{\text{sing}})]$ can be split into two UV-finite, gauge-invariant subsets and treated separately:

$$\Delta r_{\text{sing}} = \Delta r_{\text{SM}} + \delta(\Delta r_{\text{sing}}).$$

We here include the state-of-the-art $\Delta r_{\text{SM}}$ evaluation, extracted from Eq. (2) and the numerical parametrization given in Ref. [72], which renders the central values

$$m_W^{\text{SM}} = 80.360 \text{ GeV} \quad \text{and} \quad \Delta r_{\text{SM}} = 37.939 \times 10^{-3}.$$  \hspace{1cm} (22)

We set the top-quark mass $[m_t = 173.07 \text{ GeV}]$ and the Z-boson mass $[m_Z = 91.1875 \text{ GeV}]$ at their current best average values [111]. The SM Higgs mass is fixed to the HiggsSignals best-fit value of 125.7 GeV. This result for $\Delta r_{\text{SM}}$ includes the full set of available contributions, combining the full-fledged two-loop bosonic [73, 88] and fermionic [36, 72, 87] effects, alongside the leading three-loop corrections at $\mathcal{O}(G_F^3 m_t^6)$ and $\mathcal{O}(G_F^2 \alpha_s m_t^4)$ [74].

The genuine singlet model contributions $[\delta(\Delta r_{\text{sing}})]$ originate from the Higgs-boson mediated loops building up the weak gauge boson self-energies, which are shown in Fig. 1. This model-dependent part relies on the Higgs masses $[m_{h^0}, m_{H^0}]$ and the mixing angle $[\sin \alpha]$, and we compute it analytically to one-loop order. As the Higgs self-interactions do not feature at one-loop, the results are insensitive to $\tan \beta$.

At this point, care must be taken not to double-count the pure SM Higgs-mediated contributions. To that aim we define $[\delta(\Delta r_{\text{sing}})]$ in Eq. (21) upon subtraction of the SM contribution:

$$\delta(\Delta r_{\text{sing}}) \equiv \Delta r_{\text{sing}}^\text{[H]} - \Delta r_{\text{SM}}^\text{[H]} \quad \text{where} \quad \Delta r_{\text{SM}}^\text{[H]} = \Delta r_{\text{sing}}^\text{[H]} |_{\sin \alpha = 0},$$

while the superscript $[\text{H}]$ selects the Higgs-mediated contributions in each case. In this expression we explicitly identify the SM-like Higgs boson with the lighter of the two mass-eigenstates $[h^0]$, while the second eigenstate $[H^0]$ is assumed to describe a (so far unobserved) heavier Higgs companion. Analogous expressions can be derived for the complementary case $[m_{h^0} = 125.7 \text{ GeV} > m_{h^0}]$, wherein the SM limit corresponds to $\cos \alpha = 0$. The phenomenology of both possibilities is analysed separately in section 3.3.

With this in mind, the purely singlet model contributions to the gauge boson self-energies give
The loop integrals in the above equations are expressed in terms of the standard Passarino–Veltman coefficients in the conventions of [113]. The overlined notation \( \overline{\Sigma} \) indicates that the overlap with the SM Higgs–mediated contribution has been removed according to Eq. (23). Analogous expressions where \( m_{h^0} \leftrightarrow m_{H^0} \) and \( \cos \alpha \leftrightarrow \sin \alpha \) are valid if we identify the heavy scalar eigenstate \( H^0 \) with the SM–like Higgs boson.

The presence of the additional singlet has a twofold impact: i) first, via the novel one–loop diagrams mediated by the exchange of the additional Higgs boson, as displayed in Fig. 1; ii) second, via the reduced coupling strength of the SM–like Higgs to the weak gauge bosons, rescaled by the mixing angle (cf. Eq. 20).

At this stage, we in fact do not yet have to specify a complete renormalization scheme for the model. It suffices to consider the weak gauge boson field and mass renormalization entering Eq. (2). The relevant counterterms therewith are fixed in the on–shell scheme [2,20,114,115], i.e. by requiring the real part of the transverse renormalized self–energies to vanish at the respective gauge boson pole masses, while setting the propagator residues to unity:

\[
\text{Re} \hat{\Sigma}^W_T (m_W^2) = 0, \quad \text{Re} \hat{\Sigma}^Z_T (m_Z^2) = 0, \\
\text{Re} \frac{\partial \hat{\Sigma}^W_T (p^2)}{\partial p^2} \bigg|_{p^2=m_W^2} = 0, \quad \text{Re} \frac{\partial \hat{\Sigma}^Z_T (p^2)}{\partial p^2} \bigg|_{p^2=m_Z^2} = 0.
\]

The complete singlet model prediction in Eq. (21) is exact to one–loop order and, as alluded to above, it includes in addition all known higher order SM effects up to leading three–loop precision. Finally, let us also remark that the additional singlet–mediated contributions to the vertex and box diagrams contained in \( \Delta r^{\text{vert,box}} \) (cf. Eq. (3)) are suppressed by the light fermion Yukawa couplings and therefore negligible.

In turn, the static contributions traded by the \( \delta \rho \) parameter, as defined in Eq. (7), can be obtained by taking the limit \( p^2 \to 0 \) on Eqs. (24)-(25) and are given by

\footnote{On–shell mass renormalization in theories with mixing between the gauge eigenstates, as in the Higgs sector of the singlet model, must be addressed with care. In these cases, quantum effects generate off–diagonal terms in the loop–corrected propagators, which can be absorbed into the renormalization of the mixing angle. However, it can be shown that, regardless of the specific renormalization scheme chosen for the mixing angle, the on–shell renormalized masses coincide with the physical (pole) masses to one–loop accuracy. A detailed discussion on this issue as well as on the complete renormalization scheme as such for the singlet model will be presented in [7].}
\[
\Delta(\delta\rho_{\text{sing}}) = \delta \rho_{\text{sing}}^{[H]} - \delta \rho_{\text{SM}}^{[H]}
\]

\[
\frac{G_F \sin^2 \alpha}{2\sqrt{2}\pi^2} \left\{ m_Z^2 \left[ \log \left( \frac{m_{h^0}^2}{m_{H^0}^2} \right) + \frac{m_{h^0}^2}{m_{h^0}^2 - m_Z^2} \log \left( \frac{m_{h^0}^2}{m_Z^2} \right) - \frac{m_{H^0}^2}{m_{H^0}^2 - m_Z^2} \log \left( \frac{m_{H^0}^2}{m_Z^2} \right) \right] + \frac{m_{H^0}^2}{4(m_{h^0}^2 - m_Z^2)} \log \left( \frac{m_{h^0}^2}{m_Z^2} \right) - \frac{m_{H^0}^2}{4(m_{h^0}^2 - m_Z^2)} \log \left( \frac{m_{H^0}^2}{m_Z^2} \right) \right\}
- m_W^2 \left[ \log \left( \frac{m_{h^0}^2}{m_{H^0}^2} \right) + \frac{m_{h^0}^2}{m_{h^0}^2 - m_W^2} \log \left( \frac{m_{h^0}^2}{m_W^2} \right) - \frac{m_{W}^2}{m_{W}^2 - m_W^2} \log \left( \frac{m_{W}^2}{m_W^2} \right) \right] + \frac{m_{H^0}^2}{4(m_{H^0}^2 - m_W^2)} \log \left( \frac{m_{H^0}^2}{m_W^2} \right) - \frac{m_{H^0}^2}{4(m_{H^0}^2 - m_W^2)} \log \left( \frac{m_{H^0}^2}{m_W^2} \right) \right\}.
\]

(26)

The logarithmic dependence on both the light and the heavy Higgs masses follows the same screening–like pattern of the SM, as shown in Eq. (10). The model–specific new physics imprints are again to be found in i) the additional Higgs contribution; and ii) the universally rescaled Higgs couplings to the gauge bosons. The size of \(\Delta(\delta\rho_{\text{sing}})\) is controlled by the overall factor \(\sim \sin^2 \alpha\), while its sign, which is fixed by the respective Higgs and gauge boson mass ratios, is negative in all cases. Equation (26) therefore predicts a systematic, negative yield from the new physics effects \(\Delta(\delta\rho_{\text{sing}}) < 0\), which implies \(\delta\rho_{\text{sing}} < \delta\rho_{\text{SM}}\). Finally, and owing to the fact that \(\delta \rho\) is linked to \(\Delta r\) via Eq. (9), we may foresee \(\Delta r_{\text{sing}} = \Delta r_{\text{SM}} + \delta(\Delta r_{\text{sing}}) > \Delta r_{\text{SM}}\) and hence \(m_{W}^{\text{sing}} < m_{W}^{\text{SM}}\). Keeping in mind the current \(|m_{W}^{\text{exp}} - m_{W}^{\text{SM}}| \simeq 20\text{ MeV}\), (1σ level) tension, this result anticipates tight constraints on the singlet model parameter space – at the level of, if not stronger than, those stemming from the global fits based on the oblique parameters \([S,T,U]\) [10–12] (cf. discussion in section 3.4). Conversely, when considering \(m_{H^0} \sim 126\text{ GeV}\) and a light Higgs companion \([h^0]\), similar arguments predict a systematic upward shift \(\Delta(\delta\rho_{\text{sing}}) > 0\) with a global \(\cos^2 \alpha\) rescaling. In this case, the singlet model has the potential to bring the theoretical value \(m_{W}^{\text{sing}}\) closer to the experimental measurement \(m_{W}^{\text{exp}}\). In the next subsection we quantitatively justify all these statements.

3.3 Numerical analysis

In the following we present an upshot of our numerical analysis. Figures 2 and 3 illustrate the behavior of \(\Delta r \equiv \Delta r_{\text{sing}}\) and \(\Delta m_W \equiv m_{W}^{\text{sing}} - m_{W}^{\text{exp}}\) under variations of the relevant singlet model parameters. In Figure 2 we portray the evolution of both quantities with the mixing angle, for illustrative Higgs companion masses. In the upper panels the SM–like Higgs particle is identified with the lightest singlet model mass–eigenstate \([h^0]\). We fix its mass to \(m_{h^0} = 125.7\text{ GeV}\) and sweep over a heavy Higgs mass range \(m_{H^0} = 200 - 1000\text{ GeV}\). The complementary case \([m_{H^0} = 125.7\text{ GeV} > m_{h^0}]\) is examined in the lower panels, with a variable mass for the second (light) Higgs spanning \(m_{h^0} = 5 - 125\text{ GeV}\). The results shown for \(\Delta r\) are referred to both the SM prediction \([\Delta r_{\text{SM}}]\) and the experimental value \([\Delta r_{\text{exp}}]\). The latter follows from Eq. (2) with the experimental inputs [111]

\[
\begin{align*}
m_{W}^{\text{exp}} &= 80.385 \pm 0.015\text{ GeV} \\
\alpha_{\text{em}}(0) &= 1/137.035999074(44) \\
m_Z &= 91.1876 \pm 0.0021\text{ GeV} \\
G_F &= 1.663787(6) \times 10^{-5}\text{ GeV}^{-2},
\end{align*}
\]

wherefrom we get
Figure 2: Upper panels: full one–loop evaluation of $\Delta r \equiv \Delta r_{\text{sing}}$ (left) and $\Delta m_W \equiv m_{\text{sing}} - m_{\text{exp}}$ (right) for different heavy Higgs masses $[m_{H^0}]$ with fixed $[m_{h^0} = 125.7 \text{ GeV}]$, as a function of the mixing angle $[\sin \alpha]$. Lower panels: likewise, for different light Higgs masses $[m_{h^0}]$ and fixed $[m_{H^0} = 125.7 \text{ GeV}]$. The corresponding SM predictions (the experimental values) are displayed in dashed (dotted) lines. The shaded bands illustrate the 1$\sigma$ and 2$\sigma$ C.L. exclusion regions. Compatibility with the LHC signal strength measurements requires $|\sin \alpha| \lesssim 0.42$ (upper panels) and $|\sin \alpha| \gtrsim 0.91$ (lower panels) (c.f. section 3.4).

\[
\Delta r_{\text{exp}} = \frac{\sqrt{2} G_F}{\pi \alpha_{\text{em}}} m_W^2 \left( 1 - \frac{m_W^2}{m_Z^2} \right) - 1 = (36.320 \pm 0.976) \times 10^{-3}. \quad (28)
\]

The 1$\sigma$ and 2$\sigma$ C.L. regions in $\Delta r_{\text{exp}}$ are derived from the $m_{W}^{\text{exp}}$ uncertainty bands using standard error propagation.

Figure 3 provides a complementary view of the $\Delta r$ and $\Delta m_W$ dependence on the additional Higgs boson mass assuming mild ($\sin \alpha = 0.2$), moderate ($\sin \alpha = 0.5$) and strong ($\sin \alpha = 0.7$) mixing.

These plots nicely illustrate the parameter dependences anticipated earlier e.g. in Eqs. (24)-(25). On the one hand, the quadratic $\sin^2 \alpha$ ($\cos^2 \alpha$) dependence reflects the global rescaling of the light (heavy) SM–like Higgs coupling to the weak gauge bosons. Accordingly, the values of $\Delta r$ and $m_{W}^{\text{sing}}$ converge to the SM predictions in the limit $\sin \alpha = 0$ ($\sin \alpha = \pm 1$) in which the new physics effects decouple. The growing departure from the SM as we raise (lower) the mass of the heavy (lighter) Higgs companion follows the logarithmic behavior singled out in Eq. (26), and can be traced back to the increasing breaking of the (approximate) custodial invariance.

In the case where $m_{h^0} = 125.7 \text{ GeV}$ and $m_{H^0} > 130 \text{ GeV}$ (cf. upper panels of Figs. 2 and 3).
Figure 3: Full one–loop evaluation of $\Delta r \equiv \Delta r^{\text{sing}}$ (left) and $\Delta m_W \equiv m^{\text{sing}}_W - m^{\text{exp}}_W$ (right) for different mixing angle values, as a function of the heavy Higgs mass $m_{h^0}$ (upper panels) and the light Higgs mass $m_{h^0}$ (lower panels). The corresponding SM predictions (the experimental values) are displayed in dashed (dotted) lines. The shaded bands illustrate the $1\sigma$ and $2\sigma$ C.L. exclusion regions.

we pin down positive (negative) deviations of $\Delta r^{\text{sing}}$ ($m^{\text{sing}}_W$) with respect to the corresponding SM predictions. These increase systematically for larger mixing angles and heavier Higgs companions. The stark dependence on $\sin\alpha$ and $m_{h^0}$, combined with the fact that $m^{\text{sing}}_W - m^{\text{SM}}_W < 0$ and that $m^{\text{SM}}_W$ already lies 20 GeV below the experimental measurement, explains why the results obtained in this case can easily lie outside of the $2\sigma$ C.L. exclusion region. In the complementary scenario (cf. lower panels), in which we set $m_{h^0} = 125.7$ GeV and vary the light Higgs mass $m_{h^0} \leq 125$ GeV, we find analogous trends – but with interchanged dependences. Here the additional one–loop effects from the light Higgs companion help to release the $m^{\text{sing}}_W - m^{\text{exp}}_W$ tension. On the other hand, the onset of $2\sigma$–level constraints appears for $m_{h^0} \lesssim 30$ GeV. These results spotlight a significant mass range in which the singlet model contributions could in principle achieve $m^{\text{sing}}_W \simeq m^{\text{exp}}_W$. The viability of these scenarios is nevertheless hindered in practice, due to the direct collider mass bounds and the LHC signal strength measurements. The impact of these additional constraints, which at this point we have not yet included, will be addressed in section 3.4.

Our discussion is complemented by specific numerical predictions for $\Delta r$ and $m^{\text{sing}}_W - m^{\text{exp}}_W$, which we list in Table 1 for representative parameter choices. For small mixing $|\sin\alpha| \lesssim 0.2$ and $d$ Let us recall that both instances $m^{\text{th}}_W - m^{\text{exp}}_W \leq 0$ are possible in the 2HDM for a large variety of Higgs mass spectra. However, unlike the singlet model case, these situations are not attached to a specific mass hierarchy $^{30}$. $e$ A fully comprehensive analysis of the model combining all currently available constraints deserves a dedicated study and will be presented elsewhere $^{116}$.
Table 1: Parameter space survey of the electroweak parameters $\Delta r_{\text{sing}}$, $\Delta m_W \equiv m_{W}^{\text{sing}} - m_{W}^{\text{exp}}$ and $\Delta(\delta\rho_{\text{sing}})$ in the singlet model for representative Higgs masses and mixing angle choices.

| $m_{H^0}$ [GeV] | $\Delta r_{\text{sing}} \times 10^4$ | $m_{W}^{\text{sing}} - m_{W}^{\text{exp}}$ [MeV] | $\Delta(\delta\rho_{\text{sing}}) \times 10^4$ |
|-----------------|----------------------------------|----------------------------------|----------------------------------|
| $h^0$ SM–like   | $m_{h^0} = 125.7$ GeV            | $m_{h^0} = 125.7$ GeV            |
| $\sin\alpha = 0.2$ | $38.067$ | $38.153$ | $38.277$ | $-27$ | $-29$ | $-31$ | $-0.241$ | $-0.428$ | $-0.711$ |
| $\sin\alpha = 0.5$ | $38.744$ | $39.281$ | $40.056$ | $-38$ | $-46$ | $-58$ | $-1.508$ | $-2.674$ | $-4.450$ |
| $\sin\alpha = 0.7$ | $39.515$ | $40.565$ | $42.077$ | $-50$ | $-66$ | $-90$ | $-2.956$ | $-5.244$ | $-8.730$ |
| $H^0$ SM–like   | $m_{H^0} = 125.7$ GeV            | $m_{H^0} = 125.7$ GeV            |
| $\sin\alpha = 0.2$ | $34.305$ | $35.824$ | $36.921$ | $31$ | $8$ | $9$ | $3.798$ | $2.707$ | $1.466$ |
| $\sin\alpha = 0.5$ | $35.103$ | $36.288$ | $37.144$ | $19$ | $1$ | $-13$ | $2.968$ | $2.115$ | $1.146$ |
| $\sin\alpha = 0.7$ | $36.012$ | $36.816$ | $37.398$ | $5$ | $-8$ | $-17$ | $2.019$ | $1.439$ | $0.779$ |

heavy Higgs masses of few hundred GeV, $\Delta r_{\text{sing}}$ departs from $\Delta r_{\text{SM}}$ at the $O(0.1)\%$ level. These deviations may increase up to $O(10)\%$ for mixing angles above $|\sin\alpha| \gtrsim 0.5$ and $O(1)\ TeV$ scalar companions. Not surprisingly, these are the parameter space configurations that maximize the non–standard singlet model imprints, viz. the rescaled Higgs boson interactions and the non–decoupling mass dependence of the Higgs–mediated loops. As we have seen in Figures 2–3 and according to Eq. (5), these shifts pull the resulting prediction $|m_{W}^{\text{sing}}|$ down to $\sim 1 - 70$ MeV below the SM result. Staying within $1\sigma$ C.L. we find $|m_{W}^{\text{sing}} - m_{W}^{\text{SM}}| \sim 10$ MeV ($m_{W}^{\text{sing}} < m_{W}^{\text{SM}}$) for relatively tempered mixing ($|\sin\alpha| \lesssim 0.2$) and heavy Higgs masses up to $1$ TeV. Larger mixings of typically $|\sin\alpha| \gtrsim 0.4$ push $m_{W}^{\text{sing}}$ into the $2\sigma$–level exclusion region. These results once more illustrate that, for a second heavy Higgs resonance, the singlet model effects tend to sharpen the $m_{h^0} - m_{H^0}$ tension even further, and more so as we increasingly depart from the SM–like limit. Alternatively, for $m_{h^0} < m_{H^0} = 125.7$ GeV we find that relative deviations of $\sim 5\%$ in $\Delta r_{\text{sing}}$ (with $\Delta r_{\text{sing}} < \Delta r_{\text{SM}}$) are attainable for $50 - 100$ GeV light Higgs companion masses and mixing angles above $|\sin\alpha| \sim 0.5$.

Alongside with the calculation of $\Delta r$ and $m_{W}^{\text{sing}}$ we compute the new physics one–loop contributions to the $\delta\rho$ parameter (cf. Eq. (7)). The behavior of $\Delta(\delta\rho_{\text{sing}})$ as a function of the relevant singlet model parameters $|\sin\alpha|$ and $[m_{h^0, H^0}]$ is illustrated in Figure 4. Again, we separately examine the two complementary situations in which either the lighter (top–row panels) or the heavier (bottom–row panels) singlet model mass–eigenstate describes the SM–like Higgs boson. As expected, $|\Delta(\delta\rho_{\text{sing}})|$ enwraps as we progressively separate from the SM limit. The strong dependence in the additional Higgs mass displays the increasing deviation from the custodial symmetry limit, which is enhanced by the mass splitting between the Higgs mass–eigenstates. Conversely, we recover $\Delta(\delta\rho_{\text{sing}}) \rightarrow 0$ in the $m_{h^0} \rightarrow m_{H^0}$ limit. The relative size of the static one–loop effects encapsulated in $\Delta(\delta\rho_{\text{sing}})$ is quantified in the lower subpannels of Fig. 4 through the ratio

$$\Delta r_{\text{rel}}^{[\delta\rho]} = \Delta r_{\text{sing}}^{[\delta\rho]} / \Delta r_{\text{sing}} = -\frac{2}{s_W^2} \Delta(\delta\rho_{\text{sing}}) / \Delta r_{\text{sing}},$$

(29)

which we construct from the different pieces singled out in Eq. (9), retaining the singlet model contributions only.

Interestingly, the analysis of $\delta\rho$ provides a handle for estimating the size of higher–order corrections. The leading singlet model two–loop effects $[\Delta(\delta\rho_{\text{sing}})]$ arise from the exchange of virtual top
quarks and Higgs bosons. This type of mixed $O(G_F^2 m_t^4)$ Yukawa corrections was first computed within the SM in the small Higgs boson mass limit in Ref. [117] and later on extended to arbitrary masses [118,119]. The analytical expressions therewith can be readily exported to our case. Taking into account the rescaled top–quark interactions with the light (heavy) Higgs mass–eigenstate by an overall factor $\sim \cos^2 \alpha (\sim \sin^2 \alpha)$; and removing as usual the overlap with the SM contribution (which we identify here with $h^0$ in the $\sin \alpha = 0$ limit) we find

$$
\Delta (\delta \rho_{\text{sing}}^{[2]}) = 3 G_F^2 m_t^4 \sin^2 \alpha \left( f(m_t^2/m_{h^0}^2) - f(m_t^2/m_{h^0}^2) \right) \sim \frac{3 G_F^2 m_t^4 \sin^2 \alpha}{128 \pi^4} \left[ 27 \log \left( \frac{m_t}{m_{h^0}} \right) + \frac{4 \pi m_{h^0}}{m_t} \right],
$$

where in the latter step we have introduced the asymptotic expansions of $f(r)$ [118,119]. The above estimate $\Delta (\delta \rho_{\text{sing}}^{[2]})$ stagnates around $O(10^{-4})$ for fiducial parameter choices with $|\sin \alpha| \lesssim 0.5$. 

Figure 4: Singlet model contribution to the $\delta \rho$ parameter at one loop $[\Delta (\delta \rho_{\text{sing}})]$ for representative mixing angles and Higgs companion masses. In the bottom subpannels we quantify the relative size of these contributions to the overall $\Delta r$ prediction, following Eq. (29).
$m_H$ [GeV] | $| \sin \alpha |_{\text{max}}$
\hline
1000 & 0.19 \\
900 & 0.20 \\
800 & 0.20 \\
700 & 0.21 \\
600 & 0.22 \\
500 & 0.24 \\
400 & 0.26 \\
300 & 0.31 \\
200 & 0.43 \\
150 & 0.70 \\
130 & 1.00 \\
\hline

Table 2: Upper limits on the mixing angle compatible with $m_{\text{exp}}^{W}$ at the 2$\sigma$–level, for $m_{h^0} = 125.7$ GeV and representative heavy Higgs masses. Consistency with the LHC signal strength measurement implies $| \sin \alpha | \leq 0.42$, cf. Fig. 6.

When promoted to the W–boson mass prediction through Eqs. (5) and (9) we find

$$\left[ \Delta (m_{[2]}^W) \right]_{\text{sing}} \sim -\frac{1}{2} m_W \frac{s_W^2}{c_W^2 - s_W^2} \delta (\Delta r_{\text{sing}}^{\delta \rho}) \lesssim O(1) \text{MeV},$$

which we can interpreted as an estimate on the theoretical uncertainty on $m_{W}^{\text{sing}}$ due to the quantum effects beyond the one–loop order.

3.4 Comparison to complementary model constraints

In this section, we first confront the model constraints imposed by the $[m_{W}^{\text{sing}} - m_{W}^{\text{exp}}]$ comparison to those following from global fits to electroweak precision data, commonly expressed in terms of the oblique parameters $[S, T, U]$. In the standard conventions [111], and retaining the one–loop singlet model contributions only, the latter are given by

$$\alpha_{\text{em}} T = \frac{\Sigma_W(m_W^2) - \Sigma_W(0)}{m_W^2} - \frac{\Sigma_Z(0)}{m_Z^2};$$

$$\alpha_{\text{em}} U = \frac{\Sigma_W(m_W^2) - \Sigma_W(0)}{m_W^2} - \frac{c_W^2 \Sigma_Z(m_Z^2) - \Sigma_Z(0)}{m_Z^2}.$$ (32)

Notice that genuine singlet model contributions to the photon and the mixed photon–Z vacuum polarization are absent at one loop. The overlined notation $\Sigma$ is once more tracking down the consistent subtraction of the overlap with the SM Higgs–mediated contributions, as specified by Eq. (23). The $T$ parameter can obviously be related to the $\delta \rho$ parameter in Eq. (7), yielding $\alpha_{\text{em}} T = -\delta \rho$. Likewise, we may rewrite $\Delta r_{\text{sing}}$ as

$$\Delta r_{\text{sing}} = \frac{\alpha_{\text{em}}}{c_W^2} \left( -\frac{1}{2} S + c_W^2 T + \frac{c_W^2 - s_W^2}{4 s_W^2} U \right).$$ (33)

In Fig. 5 we survey $[S, T, U]$ as a function of the relevant singlet model parameters. The $1\sigma$ and $2\sigma$ level contours have been taken from Ref. [120], in which the LHC Higgs mass measurement is already included as an input quantity, and reports a best–fit point
Figure 5: Singlet model contributions to the oblique parameters $S$ (left), $T$ (center) and $U$ (right) as a function of the mixing angle for representative heavy (upper row) and light (lower row) Higgs companion masses. The shaded areas indicate the $1\sigma$ and $2\sigma$ exclusion regions [120]. The dashed–dotted line represents the fiducial SM reference value $S, T, U = 0$.

\[
S = 0.03 \pm 0.10; \quad T = 0.05 \pm 0.12; \quad U = 0.03 \pm 0.10. \quad (34)
\]

Allowing for independent variations of up to $2\sigma$ in each of these parameters, we find no constraints in the entire mass range $m_{H^0} = 30 - 1000$ GeV. The onset of the $2\sigma$–level exclusion is found for $m_{H^0} \gtrsim 1130$ GeV, triggered by the limits on the $T$ parameter. These constraints are, however, manifestly milder than those resulting from $m_W^\text{exp}$. This is after all not surprising, since $[S, T, U]$ are estimated from a global simultaneous fit to several electroweak precision observables. The resulting C.L. limits are thus smeared with respect to electroweak precision tests based on the individual observables.

Next we compare the above constraints to the upper limits on the mixing angle stemming from i) direct collider searches; and ii) the averaged LHC Higgs signal strength measurements $[\bar{\mu}^\text{exp}]$. For the former, we use HIGGSBOUNDS [121] to extract upper limits $|\sin\alpha|_{\text{max}}$ as a function of $m_{H^0}$. In the mass range $m_{H^0} = 200 - 1000$ GeV, the primary collider limits follow from the CMS four-lepton mode search [122]; for lower masses additional channels are equally important [123, 124]. Concerning the Higgs signal strength, we use the most recent values reported in [125, 126].

\[
\mu_{\text{ATLAS}} = 1.30 \pm 0.18, \quad \mu_{\text{CMS}} = 0.80 \pm 0.14 \quad \text{wherefrom} \quad \bar{\mu}^\text{exp} = 1.05 \pm 0.11. \quad (35)
\]

A word of caution should be given here. Note that these best–fit estimates and C.L. limits are not tailored to any particular model. This means for instance that, although the singlet model can only yield a suppressed Higgs signal strength $\mu^\text{sing} \leq 1$, such restriction is not enforced beforehand.
Figure 6: Upper limits on the mixing angle $|\sin \alpha|_{\text{max}}$ from i) the $m_W^{\text{exp}}$ measurement; ii) direct collider searches; and iii) compatibility with the $m_{h^0} = 125.7$ GeV LHC Higgs signal strength measurements.

when deriving the results in Eq. (35). Dedicated model–specific analyses should therefore include such model–dependent fit priors, which would eventually modify the resulting bounds.

To estimate the mixing angle range for which $\bar{\mu}_{\text{sing}}$ is compatible with the LHC observations [$\bar{\mu}_{\text{exp}}^{\text{sing}}$], we identify the light (heavy) singlet model mass–eigenstate with the SM Higgs boson and assume a global rescaling $\bar{\mu}_{\text{sing}}/\bar{\mu}_{\text{SM}} \simeq \cos^2 \alpha (\sin^2 \alpha)$. In this simple estimate, we do not entertain the possibility that two eigenstates with almost degenerate masses $m_{h^0} \simeq m_{h^0}$ could contribute to the LHC Higgs signal. Allowing up to 2$\sigma$–level deviations, we obtain upper (lower) mixing angle limits of $|\sin \alpha| \leq 0.42$ ($|\sin \alpha| \geq 0.91$). These imply that the parameter space for the case of $m_{h^0} = 125.7$ GeV $> m_{h^0}$ is severely restricted. Consequently, most regions in Figures 2–3 for which quantum corrections would shrink the $[m_W^{\text{th}} - m_W^{\text{exp}}]$ discrepancy below the 1$\sigma$–level are in practice precluded by the LHC signal strength measurements.

On the other hand, larger regions of parameter space are still allowed when $m_{h^0} = 125.7$ GeV $< m_{h^0}$. For this case, compared vistas of the different model constraints are displayed in Figure 6. We sweep the heavy Higgs masses in the range 130 – 1000 GeV and overlay the upper bounds on the mixing angle $|\sin \alpha|_{\text{max}}$ from each constraint individually. While direct search bounds and signal strenght measurements dominate in the low–mass region, both are superseded by the W–boson mass measurement $[m_W^{\text{exp}}]$ for $m_{h^0} \gtrsim 300$ GeV. This can once again be attributed to the Higgs–mediated corrections encoded within $\Delta r$, which increase with $m_{h^0}$ and are ultimately linked to the custodial symmetry breaking. In turn, the limits on the oblique $[S, T, U]$ parameters do not feature for the mass range under scrutiny, as discussed earlier. We conclude that these different sources of constraints are highly complementary to each other in the different heavy Higgs mass regions, and in all cases rule out substantial deviations from the SM–like limit, viz. mixing angles of $|\sin \alpha| \gtrsim 0.2 – 0.4$.

4 Summary

We have reported on the computation of the electroweak precision parameter $\Delta r$, along with the theoretical prediction of the W–boson mass, in the presence of one additional real scalar $SU(2)_L \otimes U(1)_Y$ singlet. The $\Delta r$ parameter trades the relation between the electroweak gauge

\[\text{We thank A. Straessner for clarifying comments regarding this point.}\]
boson masses, the Fermi constant and the muon lifetime. Its precise theoretical knowledge plays a salient role in the quest for physics beyond the SM. The reason is twofold: first, because $\Delta r$ and $m_W$ constitute a probe of electroweak quantum effects and are therefore sensitive to, and able to constrain, extended Higgs sectors; and second, due to the current 1σ discrepancy $|m^{SM}_W - m^{exp}_W| \sim 20$ MeV which, if eventually growing with the more accurate upcoming $W$–boson mass measurements, it could become a smoking gun for new physics.

In this work we have combined the state–of–the–art SM prediction (available up to leading three–loop accuracy) with the one–loop evaluation of the genuine singlet model effects. The two possible realizations of the singlet–extended SM Higgs sector, viz. featuring a heavy or a light Higgs companion, have been separately examined. Finally, we have confronted the constraints on the parameter space stemming from $[m^{exp}_W]$ to the limits imposed by i) direct collider searches; ii) Higgs signal strength measurements; and iii) the bounds on $[S,T,U]$ based on global fits to electroweak precision data.

Our conclusions may be outlined as follows:

- The singlet model contributions to $\Delta r$ and $m_W$ are characterized by: i) a global rescaling factor which depends on the mixing between the two scalar mass–eigenstates and reflects the universal suppression of all Higgs boson couplings in this model; ii) the additional exchange of the second Higgs boson, which exhibits a logarithmic screening–like non-decoupling dependence with the Higgs mass.

- The singlet–induced new physics effects may typically yield up to $O(10)\%$ deviations in the $\Delta r$ parameter with respect to the SM prediction. Due to the characteristic dependence on the Higgs masses, these departures are bound to be positive if the lightest mass eigenstate is identified with the SM Higgs boson. Such a shift $\Delta r^{sing} > \Delta r^{SM}$ implies $m^{sing}_W - m^{SM}_W \sim 1–70$ MeV with $m^{sing}_W < m^{SM}_W$, which raises the tension with the current $[m^{exp}_W]$ measurement. These trends are reverted if we exchange the roles of the two mass–eigenstates and consider instead a light Higgs companion with $m^{h_0}_W < m^{H_0}_W = 125.7$ GeV. In that case we retrieve $\Delta r^{sing} < \Delta r^{SM}$ and hence $m^{sing}_W > m^{SM}_W$, which makes in principle possible to satisfy $m^{sing}_W \simeq m^{exp}_W$. The viability of these scenarios is nonetheless limited in practice, as they are hardly compatible with the Higgs signal strength measurements.

- Tight upper bounds on the mixing angle parameter $|\sin \alpha|_{\text{max}}$ can be derived when confronting $[m^{sing}_W]$ to $[m^{exp}_W]$. These are particularly stringent for $m^{h_0}_W = 125.7$ GeV and $m^{H_0}_W \gtrsim 300$ GeV, and reflect the enhanced breaking of the (approximate) custodial symmetry of the SM. In this mass range they dominate over the additional model constraints from direct collider searches and Higgs signal strength measurements, as well as from global electroweak fits traded by the oblique parameters $[S,T,U]$.

With the calculation of the $\Delta r$ parameter, we have taken one step towards a complete characterization of the one–loop electroweak effects in the singlet extension of the SM. The knowledge of $\Delta r$ is a key element in the evaluation of the electroweak quantum corrections to the Higgs boson decays. Work in this direction is underway [7].

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