Analysis of a point on line segments in geometry analytical concepts

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Abstract. Points and lines are the most basic elements of geometry. If there are two points, one line can be made through these two points. In the Euclidean field every point \( P \) has a coordinate in the form of a real number \((x_1, y_1)\) and is called a coordinate point. Vector is a trending line segment, for example vector \( v = (x_v, y_v) \) can be determined as a vector with a starting point \( O = (0,0) \) and ends at point \( P = (x_1, y_1) \). A line in analytic concepts can be expressed as a combination of vectors and equal points. The purpose of the study is to examine the position of a point located on the line segment. This point is between or extension of a line of two different points and is represented in the form of a parameter equation.

1. Introduction
Mathematics is one of the disciplines that provides an important role in solving problems in everyday life and mathematics is also one of the sciences whose existence is always used in almost all other disciplines [1]. One branch of mathematics is algebra and geometry. Geometry presented in algebraic form is called geometric analytic. Analytical geometry is the basic course of geometry that studies the two-dimensional object. This course aims to develop the ability of students to understand the geometry equation in 2D plane in the form of vector equations, canonic, and parameters, the position of the line against other lines, and the position of the line against the cone slices [2]. The geometry studied in elementary, middle and upper schools is a geometry based on the Euclid postulate. Euclid's own geometry was introduced by Euclid of Alexandria in 300 BC [3].

Euclidean's geometry provides a view of points, lines, and angles that form the basis of other geometric elements such as geometry in a flat plane and space [4]. A field in Euclidean geometry or can be called Euclid field is denoted by \( E^2 \). A coordinate system at \( E^2 \), the set all point in line is correspondent 1-1 with set of the real number [5]. Defines that every point \( P \) has a coordinate in the form of a pair of real numbers \((x_1, y_1)\) and is called a coordinate point. In addition, a point \( P = (x_1, y_1) \) can also state the position of a vector \( v = (x, y) \) with a starting \( O \); 0.0 and end at point \( P = (x_1, y_1) \). However, in point geometry and vectors have different meanings [6].
The set of vectors form a vector space denoted by $R^2$. Vector space is an algebraic structure in which two algebraic operations apply, namely scalar multiplication and multiplication operations. A vector is generally also expressed as a line segment that starts from one point and ends at another [7].

Two vectors $v$ and $w$ is said to be the same if it has the same length and direction, ($v = w$). Suppose there are two vectors $v = (x_1, y_1)$ and vectors $w = (x_2, y_2)$, then the sum of vectors $v + w = (x_1 + x_2, y_1 + y_2)$.

It assumed that a vector can be shifted without changing its length and direction. Suppose vector $v = \overrightarrow{OA}$ and $w = \overrightarrow{OB}$, then $v + w = \overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$, by shifting the $\overrightarrow{OB}$ vector into an vector $\overrightarrow{AC}$, see Figure 2 then the vector sum becomes, $v + w = \overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$. Thus from this property an vector can be determined $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$.

A vector $v = (x_1, y_1)$ and scalar $k \in R$, the scalar multiplication is $kv = (kv_1, kv_2)$. 
Figure 4. Scalar multiplication

Especially if $k = -1$, then $kv = -v$, which is a vector that has the same length as $v$, but has the opposite direction. Based on the scalar addition and multiplication, can be defined as $vw = v + (-1)w$

Then $vw = (x_1 - x_2, y_1 - y_2)$.

If there are two points, suppose $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ point at $E^2$, to determine the distance between point $P$ and $Q$ used the following equation $d(P, Q) = |QP|$. If both points are on the horizontal line then $d(P, Q) = |x_2 - x_1|$

Figure 5. The two points are on the horizontal line

and if both points are on the vertical line, then $d(P, Q) = |y_2 - y_1|$.

Figure 6. The two points are in the vertical line.

Meanwhile, if the two points are not on the horizontal or vertical lines, then determine the distance between the two points using the phytagoras formula is $d(P, Q) = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$
The concept of distance in geometry is very basic, therefore the important properties of distance are set forth in the following theorem:

Let \( P, Q \) and \( R \) points be at \( E^2 \). Then

i. \( d(P, Q) \geq 0 \).

ii. \( d(P, Q) = 0 \) if only if \( P = Q \).

iii. \( d(P, Q) = d(Q, P) \).

iv. \( d(P, Q) + d(Q, R) \geq d(P, R) \) (triangle inequality).

Proof

i. Because the value of distance is the absolute value, the value of distance is not negative.

ii. a. \( d(P, Q) = |QP| \)

\[ 0 = |QP| \]

\[ P = Q \]

b. \( 0 = |QP| \)

\[ P = Q \]

\[ 0 = d(P, Q) \]

iii. \( d(P, Q) = |QP| = |-(QP)| = |PQ| = d(Q, P) \)

iv. \( |QP| + |RQ| \geq |(QP) + (RQ)| = |RP| = d(P, R) \)

Suppose \( P \) and \( Q \) are two different points. The set that contains points \( P \) and \( Q \) and all points between them are called line segments and denoted by \( PQ \). \( P \) and \( Q \) are called end points and other points are called interior points.

Let \( M \) be the interior point on the line segment \( PQ \). \( M \) is called the midpoint of \( PQ \) if \( d(P, M) = d(M, Q) = \frac{1}{2} d(P, Q) \) and each line segment has exactly one midpoint.

![Figure 7. The two points are not on the horizontal or vertical lines.](image)

The of points and vectors that match can form a line. The set of all vectors is \( v \) written as follows, \( [v] = \{tv \mid t \in R\} \). If \( P \) is a point and \( v \) vector is not zero, then \( l: \{X \mid X - P \in [v]\} \), called the line through point \( P \) with the direction vector \([v]\). When a line is determined by \( l: P + tv \), means \( l \) is the line through point \( P \) and \( v \) is the direction vector of line \( l \).
If \( l \) is a line and \( P \) is a point, then there are many expressions used to express it namely

i. \( P \in l \)

ii. \( l \) contains \( P \)

iii. \( P \) located at \( l \)

iv. \( l \) through \( P \)

v. \( P \) is at \( l \)

vi. \( P \) along with \( l \)

vii. \( l \) along with \( P \)

If point \( P \) and \( Q \) are different, then there is only one line through \( P \) and \( Q \), denoted by \( l = \overrightarrow{PQ} \).

Proof:
Take vector \( v \), then the line \( P + [v] \) through \( Q \) if only if \( Q - P \in [v] \), meaning \( [Q - P] = [v] \). So that there is only a line \( P + [Q - P] \) in other words, line \( l \) which passes through point \( P \) and \( Q \) singular.

2. Methods
The method used in this study is a literature study method or literature study that is a research method whose data collection technique is to search for information through books, magazines, newspapers, and other literature that aims to form a particular theory or concept. The concepts obtained include the concept of vectors, dots, and lines as a theoretical basis in supporting this research. The collected data is then reviewed and processed so that a generalization is obtained.

3. Results and discussion
Position of point \( R \) which is at line \( l = \overrightarrow{PQ} \). Suppose there are three different points and the line is \( P, Q \) and \( R \) Based on the previous definition, a vector can be expressed as a directed line segment. Whereas vectors that originate at point \( O = (0,0) \) and end at a certain point, can be expressed as the position of a point. Therefore, in this section, the definition will be used to determine the position of point \( R \) where
points $P$ and $Q$ have been determined, in this case it will be reviewed in three cases, where point $R$ will be expressed in the form $R (t) = (1 - t)P + tQ$ with $t$ a parameter.

3.1. Point $R$ between points $P$ and $Q$

![Figure 11. Point $R$ between point $P$ and $Q$.](image)

Line $l = PQ$, it can be stated that the vector is $\overrightarrow{PR}$ in the direction of the vector $\overrightarrow{RQ}$, it can be written $\overrightarrow{PR} = r(\overrightarrow{RQ})$, with $0 < r < 1$, using the definition of vector and addition properties and scalar multiplication, the vector $\overrightarrow{PR}$ and $\overrightarrow{RQ}$ can be written as, $\overrightarrow{OR} - \overrightarrow{OP} = r(\overrightarrow{OQ} - \overrightarrow{OR})\overrightarrow{OR} = \frac{1}{r^2}\overrightarrow{OP} + \frac{r}{(1 + r)} \overrightarrow{OQ}$.

Let $t = \frac{r}{1 + r}$ and $1 - t = \frac{1}{1 + r}$ so that the vector $\overrightarrow{OR}$ can be expressed as $R (t) = (1 - t)P + tQ$, with $0 < t < 1$, which means $R$ between $P$ and $Q$. Thus, if $R$ is between $P$ and $Q$, then $d (P, R) + d (R, Q) = d (P, Q)$

3.2. Point $R$ is located in the extension of the line $PQ$.

![Figure 12. Point $R$ is located in line extension $PQ$.](image)

There is a line $l = PQ$ based on Figure 12. It can be stated that the $\overrightarrow{PQ}$ vector is in the direction of the vector $\overrightarrow{QR}$, and can be written $\overrightarrow{PQ} = r(\overrightarrow{QR})$, with $r > 1$. Using the vector definition addition and scalar multiplication, the vector equation can be written as, $\overrightarrow{OQ} - \overrightarrow{OP} = r(\overrightarrow{QR} - \overrightarrow{OR})\overrightarrow{QR} = \frac{-1}{r}\overrightarrow{OP} + \left(\frac{1}{r} + 1\right) \overrightarrow{OQ}$.

Determine $t = \frac{1}{r} + 1$ and $1 - t = -\frac{1}{r}$ thus the vector $\overrightarrow{OR}$ can expressed as point $R (t) = (1 - t)P + tQ$, with $t > 1$, which means that $R$ is in the extension of the line $PQ$ and in the direction of the vector $\overrightarrow{PQ}$. 
3.3. **Point R is in the extension of the line segment PQ, with point P in between points R and Q.**

Seeing the position of three points \(P, Q, R\), it can be stated that the vector is the \(PQ\) opposite of the vector \(PR\), so \(\overrightarrow{PQ} = r \overrightarrow{PR}\), with \(r < 0\). Thus the vector equation can be expressed as \(\overrightarrow{OQ} - \overrightarrow{OP} = r(\overrightarrow{OR} - \overrightarrow{OP})\overrightarrow{OR} = (1 - \frac{1}{r}) \overrightarrow{OP} + \frac{1}{r} \overrightarrow{OQ}\).

![Figure 13](image)

**Figure 13.** \(R\) is in the extension of the line \(PQ\), where \(P\) lies between \(R\) and \(Q\).

Let determined \(t = \frac{1}{r}\) equation of the vector \(\overrightarrow{OR}\) can be changed in the form of point \(R(t) = (1 - t)P + tQ\), with \(t < 0\).

4. **Conclusion**

Based on the results and discussion that has been described. Point position can be viewed using the concept of vectors and represented in the form of parameters divided into three cases. Suppose there are three different points namely point \(P, Q\) and \(R\) which are in line. Obtained the following equation is:

1. The point \(R\) between points \(P\) and \(Q\).
   
   \[R(t) = (1 - t)P + tQ, \text{ with } 0 < t < 1.\]

2. Point \(R\) is located in the extension of the line segment \(PQ\).
   
   \[R(t) = (1 - t)P + tQ, \text{ with } t > 1.\]

3. Point \(R\) is in the extension of the line segment \(PQ\), with point \(P\) in between points \(R\) and \(Q\).
   
   \[R(t) = (1 - t)P + tQ, \text{ with } t < 0.\]

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