Black hole with a scalar field as a particle accelerator

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We consider stationary axially symmetric black holes with the background scalar field and test particles that can interact with this field directly. Then, particle collision near a black hole can lead to the unbounded energy $E_{c.m.}$ in the centre of mass frame (contrary to some recent claims in literature). This happens always if one of particles is neutral whereas another one has nonzero scalar charge. Kinematically, two cases occur here. (i) A neutral particle approaches the horizon with the speed of light while the velocity of the charged one remains separated from it (this is direct analogue of the situation with collision of geodesic particles.). (ii) Both particles approach the horizon with the speed almost equal to that of light but with different rates. As a result, in both cases the relative velocity also approaches the speed of light, so that $E_{c.m.}$ becomes unbounded. We consider also a case when the metric coefficient $g_{\phi\phi} \to 0$ near a black hole. Then, overlap between the geometric factor and the presence of the scalar field opens additional scenarios in which unbounded energy $E_{c.m.}$ is possible as well. We give a full list of possible scenarios of high-energy collisions for the situations considered.

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I. INTRODUCTION

Several years ago, Bañados, Silk and West found that under certain conditions, collision of particles near the Kerr black hole can give rise to the unbounded energy $E_{c.m.}$ in the centre of mass frame [1]. This is called the BSW effect after the names of its authors. It turned out that for phenomenon of this kind, rotation is an essential ingredient, the effect being universal [2]. Meanwhile, there are other versions and analogues of this effect caused by the influence of the electric [3] or magnetic [4] field. One more dynamic factor that potentially may lead to unbounded $E_{c.m.}$ is the scalar field. In [5], [6] the possibility of unbounded $E_{c.m.}$ was examined for black holes with the scalar (dilaton) field. However, the most part of corresponding results can be considered as particular cases or slight modification of the general scheme [2], [3] since the scalar field acts as a source for metric only, whereas particles themselves do not interact with this background field.

Another interesting situation arises when collisions with unbounded $E_{c.m.}$ occur near naked singularities [7], [8]. It was shown later [9] that the main reason giving rise to ultrahigh $E_{c.m.}$ in these papers, is related not to the scalar field (or any other kind of a source) by itself but, rather, arises due to specific features of the geometry near the horizon.

A possible dynamic role of the scalar field in the acceleration of particles by black hole to ultrahigh $E_{c.m.}$ due to their interaction with the background scalar field was examined, for the first time, in [10], [11]. The situations considered in both these papers are quite different and require separate attention. In [10], collisions near static black holes were studied. It was argued that the effect under discussion is absent in this case. We show in the present paper that this is not so. In [10], only collisions between two geodesic particles and two ones with the scalar charge (charged, for brevity) were considered. The conclusion about finiteness of $E_{c.m.}$ in both cases made in [10] is correct. However, the most interesting case that does lead to the effect of ultrahigh energies was overlooked. It consists of collision between a neutral particle and a charged particle, as it will be seen below.

In [11], similar scenarios were considered for a rotating metric with the same conclusion. Meanwhile, in the end of [11], a brief remark was made that in the absence of interaction, unbounded $E_{c.m.}$ are possible if both colliding particles are uncharged. This interesting properties has no analogue in [10]. It was shown in [9] in the general setting that the collision of two geodesic particles near the horizon indeed leads to unbounded $E_{c.m.}$, provided
the horizon is highly anisotropic (the metric coefficient $g_{\phi\phi} \to 0$ where $\phi$ is the azimuthal angle). The particular example considered in \cite{11} (the counterpart of the Kerr metric in the Brans-Dicke theory \cite{12}, \cite{13}, \cite{14}) belongs just to this class of metrics and this explains the result for the collision of two free particles. Thus two different factors overlap in the example in Ref. \cite{11} - anisotropy of horizons and interaction between a scalar particle and the background scalar field. As far as the role of the second factor is concerned, the main effect was overlooked for rotating metrics as well as for static ones since collision between the charged and neural particles (not considered in \cite{11}) gives rise to unbounded $E_{\text{c.m.}}$.

The aim of the present work is to develop a general scheme describing high energy collisions near scalar black holes. It applies to generic axially symmetric rotating black holes. This includes also, as a particular case, static black holes. In a model-independent approach, we show that collision between a neutral and the charged particles near scalar black holes leads, for an irregular background field, to unbounded $E_{\text{c.m.}}$. If, additionally, $g_{\phi\phi} \to 0$, the overlap between geometric and dynamic factors (due to the scalar field) allows also unbounded $E_{\text{c.m.}}$ for collision between two charged particles as well. We give a full list of scenarios of high-energy collisions.

Another aim of the present work is to elucidate the general physical mechanism and trace how interaction of a scalar particle with the background scalar field leads to unbounded $E_{\text{c.m.}}$. We do not consider applications to realistic astrophysics and discuss particular examples for the illustrative purposes only.

The paper is organized as follows. In Sec. II, we list the general formulas for the metric of a rotating axially symmetric black hole and equations of motion for a particle interacting with a background scalar field in this background. In Sec. III, we list general formulas for the energy in the centre of mass frame of two colliding particles. In Sec. IV, we study the near-horizon behavior of relevant quantities that enter the expression for $E_{\text{c.m.}}$. In Sec. V, we consider different scenarios of collisions and show in a general form that (i) collision between two charged particles give bounded $E_{\text{c.m.}}$, (ii) collisions between a neutral and a charged particles lead to unbounded $E_{\text{c.m.}}$. In Sec. VI, we reveal the underlying kinematic picture that explains the appearance of unbounded $E_{\text{c.m.}}$. In Sec. VII, we discuss the behavior of the scalar field and a proper time for near-horizon trajectories and relate them to the kinematic censorship that forbids infinite energies. In Sec. VIII, the combined effect of dynamic and geometric factors is considered due to irregular scalar field and $g_{\phi\phi} \to 0$. In Sec. IX, we
illustrate the obtained results using the Bocharova, Bronnikov Melnikov and Bekenstein (BBMB) black hole [15], [16], [17] as exact solution of field equations. We show that particle collision in this background can indeed lead to unbounded $E_{c.m.}$. In Sec. X, we exploit the exact solutions in the Brans-Dicke theory as example and describe near-horizon behavior of relevant quantities used in next section. In Sec. XI, we consider particle collisions in this background and demonstrate explicitly that collision between a neutral and the charged particles leads to unbounded $E_{c.m.}$ In Sec. XII, we give summary of the results.

Throughout the paper, we put fundamental constants $G = c = 1$.

II. ROTATING BLACK HOLES

Let us consider the metric

$$ds^2 = -N^2 dt^2 + g_{\phi}(d\phi - \Omega dt)^2 + \frac{dr^2}{A} + g_{\theta\theta} d\theta^2,$$

(1)

where all coefficients do not depend on $t$ and $\phi$. We suppose that it describes a black hole, so $N = 0$ on the horizon. We assume the linear interaction between a particle and the background scalar field $\psi$ described by the simplest action

$$S = -\int (m + s\psi) d\tau,$$

(2)

where $s$ is a coupling constant (the scalar charge of a particle), $m$ is the mass, $\tau$ is the proper time. In what follows, we assume $s > 0$, so that the combination $m + s\psi > 0$ as well.

Equations of motion in this case read [18]

$$(m + s\psi)u^\alpha_{\beta\gamma}u^\beta = -s[\psi^\alpha + u^\alpha(\psi_{\beta\gamma}u^\beta)].$$

(3)

Here, $u^\mu = \frac{dx^\mu}{d\tau}$ is the four-velocity. Due to the independence of the metric on $t$, there is the integral of motion that may differ from the standard definition of energy due to interaction of a particle with a scalar field:

$$E = -(m + s\psi)u_0.$$

(4)

In a similar way, the angular momentum $L$ is also conserved,

$$L = (m + s\psi)u_{\phi}.$$

(5)
For simplicity, we restrict ourselves by the motion in the equatorial plane \( \theta = \frac{\pi}{2} \). Within this plane, one can always redefine the radial coordinate to achieve \( A = N^2 \). Then, the equations of motion read (dot denotes derivative with respect to \( \tau \))

\[
\begin{align*}
m \dot{t} &= \frac{\tilde{X}}{N^2}, \\
m \dot{\phi} &= \frac{\tilde{L}}{g_\phi} + \frac{\Omega \tilde{X}}{N^2}.
\end{align*}
\]

Here,

\[
\begin{align*}
\tilde{X} &= \frac{Xm}{m + s\psi}, \\
X &= E - \Omega L, \\
\tilde{L} &= \frac{Lm}{m + s\psi},
\end{align*}
\]

\[
m \dot{r} = \sigma \tilde{Z},
\]

where \( \sigma = \pm 1 \),

\[
\tilde{Z} = \sqrt{\tilde{X}^2 - \left( m^2 + \frac{\tilde{L}^2}{g_\phi} \right) N^2}.
\]

These equations can be formally obtained from equations of motion for geodesic particles if one replaces \( E, L \) and \( X \) with their tilted counterparts.

### III. PARTICLE COLLISIONS: GENERAL FORMULAS

Now, let two particles 1 and 2 collide. One can define their energy in the centre of mass in the standard way:

\[
E_{c.m.}^2 = -(m_1u_1u_2 + m_2u_2u_2)(m_1u_1^\mu + m_2u_2^\mu) = m_1^2 + m_2^2 + 2m_1m_2\gamma,
\]

where

\[
\gamma = -u_1u_2^\mu
\]

is the Lorentz factor of relative motion. In what follows, we will consider collision of two particle moving towards a black hole, so \( \sigma_1 = \sigma_2 = -1 \). Then, using the equations of motion listed above, one obtains

\[
m_1m_2\gamma = \frac{\tilde{X}_1\tilde{X}_2 - \tilde{Z}_1\tilde{Z}_2}{N^2} - \frac{\tilde{L}_1\tilde{L}_2}{g_\phi}.
\]

If, say, particle 1 is neutral, \( \tilde{L}_1 = L_1, \tilde{E}_1 = E_1, \tilde{X}_1 = X_1 \).
IV. NEAR-HORIZON DYNAMICS OF CHARGED PARTICLES

We are interested in the near-horizon region since it is this region where a small denominator in the first term in (15) may potentially lead to unbounded $\gamma$. Dynamics of neutral (geodesic) particles in the context of high energy collisions was studied in [2]. If a charged particle participates in collision, the properties of $\gamma$ depend not only on the metric but also on the behavior of the scalar field. If this field remains bounded near the horizon, there is no qualitative difference between the presence or absence of the scalar field, and we return to the known situation to which the general analysis of [2] applies with minor modifications. Therefore, we assume that near the horizon, the scalar field diverges. For $N \to 0$, let the scalar field behave like

$$\psi \approx c N^{-\beta}$$

with

$$\beta > 0,$$

(17)

c is some constant related to the scalar charge of the background configuration. We assume that $c > 0$, so $\psi > 0$.

The fact that we deal with an infinite scalar field does not necessarily mean that something pathological arises here. As is explained by J. D. Bekenstein in [18] (with the reference to private communication to B. De Witt), "it is not associated with an infinite potential barrier for test scalar charges; it does not cause the termination of any trajectories of these test particles at finite proper time; and it is not connected with unbounded tidal accelerations between neighboring trajectories". More precisely, all this is also valid now for $\beta = 1$. If $\beta < 1$, the acceleration diverges when the horizon is approached (see Sec. VII below). Nonetheless, we include this case into consideration as well since in the context of particle acceleration to unbounded energies all factors that cause this effect (including the singular features in the metric or particle dynamics) are of interest (see, e.g. [7], [8], [20]). It is also worth noting that divergent scalar field arises for some other exact solutions that have a physical meaning of the counterparts of the Kerr and Schwarzschild black holes in Brans-Dicke (see Sec. X and Sec. XI below). In general, in scalar-tensor theories of gravity there is a freedom of redefining the scalar field $\tilde{\psi} = \psi(\bar{\psi})$ and using different conformal frames according to $g_{\mu\nu} = \tilde{g}_{\mu\nu} e^{2\rho}$, so what was finite in one conformal frame can in general be divergent in another one and vice versa (in particular, on the black hole horizon). To fix the
class of frame to which our consideration applies, we assume that in the frame where the asymptotic behavior (16) is valid, the interaction between a particle and the scalar field is described by the action (2).

Now, near the horizon $\tilde{L} \to 0$ according to (10). We also assume that 
\[
\lim_{N \to 0} g_\phi \neq 0. \tag{18}
\]
(The case when (18) is violated is considered in Sec. VIII and X, XI below, see also [9].)

There are two possible cases to be discussed separately (subscript "H" denotes quantities calculated on the horizon). Below, we assume that a particle is charged.

1) $X_H \neq 0$.

If $\beta > 1$, the negative term dominates (12), so that the condition $Z^2 > 0$ is violated. Therefore, instead, we assume that
\[
0 < \beta \leq 1. \tag{19}
\]

Let $\beta < 1$. Then, near the horizon we have from (8), (16)
\[
\tilde{X} \approx \frac{X_H m}{cs} N^\beta, \tag{20}
\]
so $mN \ll \tilde{X}$. Taking also into account that $\frac{L^2}{g_\phi} \ll m^2$, we obtain
\[
\tilde{Z} \approx \tilde{X} - \frac{m^2 N^2}{2\tilde{X}} \approx \frac{mcs}{2X_H} N^{2-\beta}, \tag{21}
\]
\[
\tilde{X} - \tilde{Z} = O(N^{2-\beta}).
\]

If $\beta = 1$,
\[
\tilde{X} \approx \frac{X_H m}{cs} N, \tag{22}
\]
\[
\tilde{Z} \approx Nm \sqrt{\frac{X_H^2}{c^2 s^2} - 1}. \tag{23}
\]

2) $X_H = 0$

We can use the near-horizon expansion for $X$. For extremal black holes [21], it reads
\[
X = B_1 LN + O(N^2), \tag{24}
\]
where $B_1$ is a constant and we took into account that $X_H = 0$. By substitution into (12), we formally obtain
\[
Z^2 \approx m^2 \left[ \frac{L^2}{s^2 c^2} N^{2+2\beta} (B_1^2 - \frac{1}{g_{\phi H}}) - N^2 \right]. \tag{25}
\]
The positivity of $Z^2$ is inconsistent with (17) since the positive term in (25) is much smaller than the negative one. This means that a particle cannot reach the horizon, so we reject this case.

For nonextremal black holes [21], we would have

\[
X = C_1 LN^2 + O(N^3),
\]

where $C_1$ is a constant,

\[
Z^2 \approx m^2 \left( \frac{C_1^2 L^2}{s_2 c^2} N^{4+2\beta} - N^2 \right) - \frac{N^2 \bar{L}^2}{g_{\phi H}},
\]

so it also would become negative and we arrive at the same conclusion as for extremal black holes.

V. WHEN IS $E_{c.m.}$ UNBOUNDED?

Now, we apply the formalism under consideration looking for a possibility of getting unbounded $E_{c.m.}$ in the horizon limit $N \to 0$. Let particles have the coupling constants $s_1$ and $s_2$, respectively.

A. $s_{1,2} \neq 0$

It follows from the previous results that it is sufficient to consider the case when $X_H \neq 0$ for both particles. We consider two situations separately depending on the allowed value of $\beta$.

1) $\beta < 1$.

Using (20), (21) we obtain from (15) that

\[
\gamma \approx \frac{1}{2} \left[ \frac{s_2 (X_H)_1}{s_1 (X_H)_2} + \frac{s_1 (X_H)_2}{s_2 (X_H)_1} \right]
\]

is finite.

2) $\beta = 1$.

Then, (22), (23), (15) give us that

\[
\gamma \approx \frac{X_1 X_2 - \sqrt{X_1^2 - c^2 s_2^2} \sqrt{X_2^2 - c^2 s_1^2}}{c^2 s_1 s_2}
\]

is finite again.
B. $s_1 = 0, s_2 = s \neq 0$

In the same manner, we obtain that

1) $\beta < 1$

$$\gamma \approx \frac{(X_1)_H^{cs}}{2(X_2)_H} m_1^{-\beta} N^{-\beta}. \quad (30)$$

2) $\beta = 1$

$$\gamma \approx \frac{(X_1)_H}{m_1 N^{cs}} [(X_2)_H - \sqrt{(X_2)_H^2 - c^2 s^2}]. \quad (31)$$

Thus both for $\beta < 1$ and $\beta = 1$ the Lorentz factor $\gamma$ diverges.

Thus, collisions between two charged particles or one charged and one neutral particles lead to qualitatively different results. Is it possible to pass from one case to another? From the formal viewpoint, this implies the comparison of double limits $\lim_{N \to 0} \lim_{s_1 \to 0} \gamma(N, s_1, s_2)$ and $\lim_{s_1 \to 0} \lim_{N \to 0} \gamma(N, s_1, s_2)$. (For a moment, we showed explicitly the dependence of the gamma-factor on relevant quantities.) We see from (28), (29) and (30), (31) that, indeed, in both cases we obtain divergent $E_{c.m.}$ To some extent, this resembles the situation with double limits for the standard BSW effect - see eqs. (11), (15) in [2].

It is seen from (11), (19), (21) and (23) that

$$u^r = \dot{\tau} \to 0$$

(32)

for any charged particle when $N \to 0$. This generalizes the observation made in [11] for the Kerr-like solution in the Brans-Dicke theory. Formally, there is also the combination $s_1 = 0 = s_2$ but in this case we return to the standard BSW context [1], [2].

C. Collisions with participation of critical neutral particle

Let us call a particle usual if $X_H \neq 0$ and critical if

$$X_H = 0. \quad (33)$$

Up to now, we discussed collisions in which both particles are usual, so $(X_1)_H \neq 0, (X_2)_H \neq 0$. If a charged particle is critical, it cannot approach the horizon at all, see eq. (25) and subsequent discussion. However, this is not excluded for a neutral particle. It is worth reminding that it is collision between a critical and a charged particles gives rise to the
standard BSW effect \cite{1, 2}. Therefore, it makes sense to examine also collision between a charged usual one 1 and a neutral critical particle 2 \( (X_2)_H = 0 \). Then, for particle 2 eq. (24) is valid near the horizon. By substitution into eq. (12), where \( \tilde{X} = X \) and \( \tilde{L} = L \), one has near the horizon
\[
Z_2 \approx N \sqrt{(B_1^2 - \frac{1}{g_\phi})L_2^2 - m_2^2}.
\] (34)

It is seen from that the case under discussion is reasonable for \( (g_\phi)_H \neq 0 \) only. For \( \beta < 1 \), using previous formulas (20) and (21) for particle 1, one obtains after simple transformations that
\[
m_1 m_2 \gamma \approx (X_1)_H \left( B_1 L_2 - \sqrt{(B_1^2 - \frac{1}{g_\phi})L_2^2 - m_2^2} \right) N^{\beta - 1}.
\] (35)

Taking into account condition (19) we see that the factor \( \gamma \) diverges although somewhat slower than for the standard BSW effect due to an additional factor \( N^\beta \).

If \( \beta = 1 \), eq. (22), (23) should be used for particle 1. Then, it is seen that \( \gamma \) remains finite.

For nonextremal black holes, \( X_1 = O(N^2) \) \cite{26} and according to (12), the condition \( Z^2 > 0 \) is violated. Therefore, such a neutral particle cannot reach the horizon.

**VI. KINEMATIC UNDERLYING REASON FOR UNBOUNDED E_{c.m.}**

We consider a collision between a neutral (geodesic) particle 1 and the charged one 2. For a geodesic particle moving in the background \cite{1}, there is the relation \cite{22}
\[
X = \frac{m N}{\sqrt{1 - V^2}}.
\] (36)

Here, \( V \) is the velocity measured by the local zero angular momentum observer \cite{23}. Then, in the vicinity of the horizon, we have for neutral particle 1:
\[
V_1^2 \approx 1 - \frac{m N^2}{(X_1)_H},
\] (37)
so \( V_1 \to 1 \) in the limit \( N \to 0 \).

The tetrad components of velocity
\[
V_1^{(1)} = \sqrt{1 - \frac{N^2}{X_1^2} \left( m_1^2 + \frac{L_1^2}{g_\phi} \right)},
\] (38)
\[ V_1^{(3)} = \frac{L_1 N}{\sqrt{g_\phi X_1}}, \]  

(39)

where \( V^{(1)} \) is the component in the radial direction and \( V^{(3)} \) is that in the azimuthal direction (see [22] for details). Near the horizon, \( V_1^{(3)} = O(N) \to 0 \),

\[ V_1^{(1)} = 1 - O(N^2). \]  

(40)

A particle hits the horizon perpendicularly with the speed approaching that of light.

According to the above explanations, the equations of motion for a charged particle can be obtained from the geodesic ones by replacement of relevant quantities with their tilted counterparts. As a result, we obtain:

\[ \tilde{X} = \frac{Xm}{m + s \psi} = \frac{mN}{\sqrt{1 - V^2}}, \]  

(41)

For particle 2,

\[ V_2^{(1)} = \sqrt{1 - \frac{N^2}{X_2^2} (m_2^2 + \frac{\tilde{L}_2^2}{g_\phi})}. \]  

(42)

\[ V_2^{(3)} = \frac{\tilde{L}_2 N}{\sqrt{g_\phi X_2}} = \frac{L_2 N}{\sqrt{g_\phi X_2}}. \]  

(43)

Two subcases for the charged particle should be considered separately.

\[ \text{A. } \beta = 1 \]

The situation is completely similar to that in the standard case [2], [22]. Then, it follows from [22] and [12], [13] that both components \( V_2^{(3)} \) and \( V_2^{(1)} \) are separated from zero. As a result, the charged particle hits the horizon under some nonzero angle relative to the normal direction, the absolute value \( V_2 < 1 \). Thus we have collision between a rapid neutral particle and the slow charged one, so explanation is completely similar to that for geodesics particles [22]. In doing so, the neutral particle in the scenario under discussion is a counterpart of a usual one in the BSW effect, the charged particle corresponds to the critical geodesic particle.

As a result, \( \gamma \) diverges. According to [31], \( \gamma = O(N^{-1}) \).
B. $\beta < 1$

Now it is seen from (42), (43) that in the vicinity of the horizon $V_2^{(1)} \approx 1, V_2^{(3)} = O(N^{1-\beta}) \to 0$, so $V_2^{(3)} \ll V_2^{(1)}$. Thus the charged particle, similarly to a neutral one, hits the horizon perpendicularly. As far as the absolute value of $V_2$ is concerned, we obtain from (20) and (41) that

$$V_2^2 \approx 1 - \left[ \frac{sc}{(X_2)_H} \right]^2 N^{2(1-\beta)}.$$  

(44)

Then, in the limit $N \to 0$ the velocity $V_2 \to 1$. Again, we see similarity with the case of a neutral particle. However, there is also difference in that the charged particle approaches the horizon more slowly than a neutral one because of the additional factor $N^{-2\beta}$ in (44).

The behavior of the Lorentz gamma factor can be explained in terms of the relative velocity $w$, 

$$\gamma = \frac{1}{\sqrt{1 - w^2}}.$$  

(45)

According to general formula (see. e.g., problem 1.3. in [24]), the relative velocity of particles $w$ obeys the relation

$$w^2 = 1 - \frac{(1 - V_1^2)(1 - V_2^2)}{(1 - \vec{V}_1 \cdot \vec{V}_2)^2},$$  

(46)

where vectors and the scalar product are defined in the tangent space in terms of tetrad components.

It follows from (38), (39), (42), (43) that

$$1 - \vec{V}_1 \cdot \vec{V}_2 = O(N^{2-2\beta}).$$  

(47)

Then, using (37), (41), (46), one finds that

$$w^2 = 1 - O(N^{2\beta}).$$  

(48)

Thus the relative velocity of two particles approaches the speed of light and this is the reason why $E_{c.m.}$ grows unbounded.

For the case $\beta < 1$ under discussion, both particles approach the horizon almost with the speed of light but these two velocities do it with essentially different rates. Therefore, in this case the kinematic mechanism is different from that for the standard BSW effect [22].
VII. PROPER TIME, ACCELERATION AND KINEMATIC CENSORSHIP

The appearance of the unbounded energy leads to some subtleties. It is clear, on physical ground, that in any process the relevant energy cannot be infinite literally (the kinematic censorship), although it can be as large as one likes. In the collision of two geodesic particles, one of such particles is fine-tuned and this leads to an infinite proper time required to reach the horizon \[25\], \[26\], \[27\], \[2\]. Any actual collision occurs outside the horizon, so \(\tau\) and \(E_{c.m.}\) remain finite (although can be as large as one likes). What happens in the present case?

According to (11), the proper time required for travelling between a given point and the point of collision \(r_c < r\) is equal to

\[
\tau = \int_{r_c}^{r} \frac{mdr}{Z}.
\]

Let us consider the limit when \(r_c \to r_+\), where \(r_+\) is the horizon radius. For a neutral particle it is finite since \(Z_H \neq 0\). For the charged particle, \(\tilde{Z} = O(N^\beta)\) according to (21) or (23). Let, for definiteness, the horizon be extremal, so

\[
N = O(r - r_+).
\]

If \(\beta = 1\), it is seen from (49) that \(\tau\) is infinite similarly to the case when a particle is geodesic. However, for \(\beta < 1\) it is finite. How to reconcile infinite \(E_{c.m.}\) with finite \(\tau\)?

The crucial point is the behavior of the acceleration \(a_\mu\) of the charged particle. For the action (2), the acceleration is given by (3), hence

\[
a^2 \equiv a_\mu a^\mu = \frac{s^2}{(m + s\psi)^2} h^{\mu\nu} \psi^\mu \psi^\nu, \quad (51)
\]

\[
h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu. \quad (52)
\]

For the extremal black hole (1), evaluation of \(a^2\) with (51) taken into account shows that \(a\) is finite, provided \(\beta = 1\) in (16). However, if \(\beta < 1\),

\[
a^2 \sim N^{2(\beta - 1)} \quad (53)
\]

diverges. For nonextremal black holes, we have, by definition,

\[
N^2 \sim r - r_+ \quad (54)
\]
instead of (50). Then, divergences become even stronger, \( a^2 \sim N^{2(\beta - 2)} \). Thus a particle experiences the action of infinite force that should be considered as a singularity. Moreover, as the gradient of \( a \) also diverges, any physical object of a finite size could not withstand such infinite forces caused by the scalar field that would tear it. Therefore, any actual collision should occur not exactly on the horizon but outside it, so the kinematic censorship is preserved.

In the case \( \beta = 1 \), the acceleration is finite, the proper time is infinite, so the total configuration (geometry plus the scalar field) exhibits no singular properties in spite of divergent scalar field (cf. [18]). This is direct generalization of what happens in the case of the BBMB black hole. If \( \beta < 1 \), singular properties of the scalar field reveal themselves in dynamics of a charged particle but the geometry itself can be quite regular, neutral particles feel no singularity.

**VIII. COMBINED FACTORS NEAR HORIZON: DIVERGENT SCALAR FIELD AND VANISHING \( g_\phi \)**

As far as the role of the scalar field is concerned, we are faced with two situations which are in sense complimentary to each other. Either (i) \( (g_\phi)_H \neq 0 \) and \( \psi_H = \infty \) or (ii) \( (g_\phi)_H = 0 \) and \( \psi_H < \infty \). Variant (i) is analyzed above. Variant (ii) is the particular case of what was considered in [9], where the material source (the scalar field or something else was unimportant, provided \( (g_\phi)_H = 0 \). Meanwhile, in [7], [11] one deals with the combination of both properties \( (g_\phi)_H = 0 \) and \( \psi_H = \infty \). Therefore, for completeness, we will consider such a case as well. To this end, we repeat briefly the analysis of eq. (15) carried out in Sec. V. However, now apart from (16) we must take into account also the condition \( (g_\phi)_H = 0 \). It is convenient to introduce the parameter

\[
 b = \lim_{N \to 0} \frac{N}{\sqrt{g_\phi}} \tag{55}
\]

similarly to what has been done in [9].

We will assume that \( (X_H)_1 \neq 0 \), \( (X_H)_2 \neq 0 \). If both particles are neutral, we return to the situation already analyzed in Ref. [9]. It remains to study the following combinations.
A. Both particles are charged

1. \( b \neq 0 \)

Let in (16) \( \beta = 1 \). It follows from (15) that in the horizon limit \( N \to 0 \) (22) is valid and
\[
\hat{Z} \approx \frac{N m}{c s} \sqrt{X_1^2 - b^2 L^2},
\]
where it is implied that \( b |L| \leq X_H \) for each particle. One can see that the Lorentz factor \( \gamma \) remains finite.

Let \( 0 < \beta < 1 \). Then, (20) is still valid. One also obtains that for each particle
\[
\hat{Z} \approx N^\beta m \frac{m}{c s} \sqrt{X_1^2 - b^2 L^2}. \tag{57}
\]
As a result,
\[
\gamma \approx \frac{(X_1 X_2 - \sqrt{X_1^2 - b^2 L_1^2} \sqrt{X_2^2 - b^2 L_2^2})_H - L_1 L_2 b^2}{N^2 - 2 \beta c^2 s_1 s_2} \tag{58}
\]
diverges.

2. \( b = 0 \)

Let \( g_\phi \to 0 \) in the horizon limit in such a way that
\[
\frac{N^2}{g_\phi} \approx b_1^2 N^{2\alpha}, \quad 0 < \alpha \leq 1 \tag{59}
\]
near the horizon. Now, one can examine different possibilities.

If \( 1 - \alpha \leq \beta \leq 1 \), \( \gamma \) turns out to be finite. If
\[
\beta < 1 - \alpha, \tag{60}
\]
\[
\gamma \approx \frac{b_1^2 [(X_2)_H L_1 - (X_1)_H L_2]^2}{2 (X_1)_H (X_2)_H c^2 s_1 s_2} N^{2(\alpha + \beta - 1)} \tag{61}
\]
diverges. We see that if \((g_\phi)_H = 0\) a new possibility opens for getting unbounded \( \gamma \) that was absent for \((g_\phi)_H \neq 0\) when collisions of two charged particles do not give the effect under discussion.

The case of finite nonzero \((g_\phi)_H\) falls into this scheme if we put \( \alpha = 1 \) in (59). Then, the necessary condition (60) for the unbounded \( \gamma \) reduces formally to \( \beta < 0 \). Obviously, this cannot be realized since it is inconsistent with (17). Thus we confirm that collision of two charged particles near the horizon with \((g_\phi)_H \neq 0\) cannot produce divergent \( \gamma \) and hence \( E_{c.m.} \).
B. Particle 1 is neutral, particle 2 is charged

C. \( b \neq 0 \)

Repeating calculations step by step, we arrive at the result

\[
\gamma \approx \frac{1}{scm_1} \left[ X_1 (X_2 - \sqrt{X_2^2 - b^2 L_2^2}) - bL_1 L_2 \right] N^{\beta-2}. \tag{62}
\]

We see that \( \gamma \) diverges according to (19).

1. \( b = 0 \)

We assume that the asymptotic form (59) is valid. If condition (60) is fulfilled, one can infer that for particle 2

\[
\tilde{Z} \approx \frac{m_2 N^\beta}{cs} [(X_2)_H - \frac{b_1^2}{2(X_2)_H^2} N^{2\alpha} L_2^2]. \tag{63}
\]

For particle 1 we have from (12) with \( \tilde{X}_1 = X_1 \) and \( \tilde{L}_1 = L_1 \) and (59)

\[
Z \approx X_1 - N^{2\alpha} b_1^2 L_1^2 \tag{64}
\]

Then, we obtain the following result from (15):

\[
\gamma \approx \frac{b_1^2 [(X_1)_H L_2 - (X_2)_H L_1]^2}{2 (X_1)_H^2 (X_2)_H^2 m_1 cs} N^{\beta+2(\alpha-1)}. \tag{65}
\]

We see that \( \gamma \) diverges.

If \( \alpha + \beta > 1 \), the contribution of the term containing \( g_\phi \) to \( \tilde{Z} \) is negligible and we return to (30) for \( \beta < 1 \) or (31) for \( \beta = 1 \). In this sense, there is no difference between the effect of unbounded \( \gamma \) for nonzero or vanishing \( (g_\phi)_H \).

In the marginal case \( \beta = 1 - \alpha \) we obtain that \( \gamma = O(N^{-\beta}) \) that agrees with (65) and (30), (31).

Thus collision between a charged particle and a neutral one does give the unbounded \( \gamma \) both for nonzero and zero \( (g_\phi)_H \).

D. Physical origin of unbounded Lorentz gamma-factor

Both the special feature \( (g_\phi)_H = 0 \) of the geometry and the action of the scalar field can be considered as potential sources of particles’ acceleration. When they act together,
one can ask: is it possible to disentangle both factors and single out the main reason of the effect? The answer depends strongly on a type of scenario. If both particles are charged, characteristics of the geometry $b, b_1, \alpha$ enter the expressions (58), (61) for $\gamma$ along with those of the scalar field $c, s_1, s_2$. Therefore, it is impossible to disentangle the roles of both factors that produce the combined effect.

Let us discuss now another scenario when a neutral particle collides with a charged one. We see from (62) that $b \neq 0$ enters the expression for $\gamma$ as well as $c$ and $s$, so both factors are entangled as well as in the previous case. If $b = 0$, the situation is different. If (60) is valid, entanglement takes place also. However, if it is violated, the final answer coincides with previous formulas (30), (31), so the effect arises due to the scalar field only and has nothing to do with the peculiarity of the geometry in question, according to which $(g_\phi)_{H} = 0$.

**IX. EXAMPLE WITH EXACT SOLUTIONS: BOCHAROVA, BRONNIKOV MELNIKOV AND BEKENSTEIN (BBMB) BLACK HOLE**

In this section, we illustrate general formalism using the example of metrics that are exact solutions of field equations. As we saw, rotation did not play an essential role in the effect under discussion. Therefore, to simplify matter, we restrict ourselves with static metrics, so we put $\Omega = 0$ in (1). We choose here the solution describing the BBMB black hole [15], [16], [17]. Its metric can be written in the form

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2,$$

(66)

where $d\omega^2$ is the metric on unit 2-sphere,

$$f = (1 - \frac{M}{r})^2.$$  

(67)

The scalar field

$$\psi = \frac{q}{r\sqrt{f}},$$

(68)

the quantity $q$ has the meaning of the scalar charge. In what follows, we assume $q > 0$, so $\psi > 0$ as well. For the solution under discussion $q = \sqrt{\frac{2}{4\pi}} M$.

Although the solution under discussion is known to be unstable [28], here we exploit it for the illustration of general formulas describing particle collisions near a black hole. These formulas are insensitive to whether or not a black hole is stable.
For simplicity, we will consider here pure radial motion within the plane $\theta = \frac{\pi}{2}$, so the angular momentum is equal to zero. Then, the equations give us

$$\dot{t} = \frac{\tilde{E}}{m f},$$  \hspace{1cm} (69)$$

$$\tilde{E} = \frac{Em}{m + s\psi}.$$  \hspace{1cm} (70)

Using the normalization condition $u_\mu u^\mu = -1$, one obtains eq. (11) with

$$\tilde{z} = \sqrt{\tilde{E}^2 - fm^2},$$  \hspace{1cm} (71)

It follows from (11) that

$$f + (u^r)^2 = \frac{\tilde{E}^2}{m^2}.$$  \hspace{1cm} (72)

If $s = 0$, there is no coupling between a particle and the background scalar field. Then, $\tilde{E} = E, \tilde{z} = Z = \sqrt{E^2 - fm^2}$, and eqs. (69), (11), (71) turn into the geodesic equations.

We assume that particle 1 is neutral and particle 2 is charged. Using the equations of motion, one finds from (15) that for small $f$

$$\gamma \approx \frac{Y}{\sqrt{f}}, \hspace{0.5cm} Y = \frac{E_1}{m_1} \sqrt{\frac{4\pi}{3}} - \sqrt{\frac{4\pi E_2^2}{3s^2} - 1}. \hspace{1cm} (73)$$

It is implied that

$$E_2 \geq s \sqrt{\frac{3}{4\pi}}, \hspace{1cm} (74)$$

otherwise the particle cannot approach the horizon. Thus $E_{c.m.} \sim f^{-1/2} \to \infty$.

It is worth noting that one can use eq. (38) of [10] where collision between particles with different scalar charges $f_1$ and $f_2$ (our $s_1$ and $s_2$) was considered. Then, one can take the limit $f_1 \to 0$ while $f_2$ is kept fixed to confirm that $E_{c.m.}$ diverges in agreement with our general results.

In [10], another example of exact spherically symmetric solution, the so-called MTZ black hole [19] was mentioned. For this solution, the scalar field is regular on the horizon. Therefore, the described mechanism does not work and $E_{c.m.}$ remains bounded not only for collision between two charged or between two neutral particles [10] but also for collision between a neutral and the charged one.
X. BRANS-DICKE ANALOGUE OF SCHWARZSCHILD SOLUTION

Here, we take advantage of the exact solution found within the Brans-Dicke theory \[12\], \[13\], \[14\]. In general, it describes a rotating black hole with a scalar field that represents the analogue of the Kerr solution in the Brans-Dicke theory (the BDK solution). In the limit, when rotation is absent, it turns to the counterpart of the Schwarzschild solution (the so-called BDS solution). In a given context, this metric was discussed in \[11\] with the conclusion that collision of charged particles gives bounded $E_{c.m.}$. We would like to stress again that collision between the charged and a neutral particles does indeed produce unbounded $E_{c.m.}$.

Meanwhile, one should bear in mind the following subtlety. Now, as will be seen below, the coefficient $g_\phi \to 0$ on the horizon. By itself, this leads to the possibility of high energy collisions, even without interaction with the background field ($s = 0$) \[9\]. However, now we are interested just in the role of this interaction, so one of colliding particles is assumed to be charged. Thus in the case under discussion, there is overlap between two completely different factors - dynamic interaction with the background field and the properties of the horizon geometry. Because of this overlap, the situation is a particular case of what was considered in Sec. VIII.

By contrary to the standard BSW effect \[1\], rotation is not essential here, as is explained above and in \[9\]. Therefore, we restrict ourselves by the nonrotating version of the BDK solution given by the BDS one.

The metric can be written in the form (see eq. (13) of \[14\])

$$ds^2 = \Delta r^{-\frac{2}{2\omega + 3}} \sin \theta \left[-dt^2 \left(1 - \frac{2M}{r}\right) + r^2 \sin^2 \theta d\phi^2\right] + \Delta r^{-\frac{2}{2\omega + 3}} \sin \frac{4}{2\omega + 3} \theta \left(\frac{dr^2}{1 - \frac{2M}{r}} + d\theta^2\right),$$

(75)

$\Delta = r(r - 2M)$, the horizon is located at $r = r_+ = 2M$.

$$\psi = \Delta r^{-\frac{2}{2\omega + 3}} \sin \frac{4}{2\omega + 3} \theta.$$  

(76)

We restrict ourselves by motion within the plane $\theta = \frac{\pi}{2}$. Then, equations of motion \(6\) - \(12\) read

$$mu^t = \tilde{E} r^2 \Delta r^{-\frac{2}{2\omega + 3}},$$

(77)

$$mu^\phi = \frac{\tilde{L}}{r^2} \Delta r^{-\frac{2}{2\omega + 3}},$$

(78)

where $\tilde{E}$ and $\tilde{L}$ are given by eqs. \(70\), \(10\). Eq. \(12\) takes the form

$$\tilde{Z} = \sqrt{\tilde{E}^2 - N^2 (m^2 + \frac{\tilde{L}^2}{r^2} \Delta r^{-\frac{2}{2\omega + 3}})},$$

(79)
where
\[ N^2 = \frac{\Delta^{2\omega+1}}{r^2}, \]  
(80)

the scalar field is equal now to
\[ \psi = \Delta^{\frac{2}{2\omega+3}} r^{\frac{4}{2\omega+1}} N^{-\beta}, \quad \beta = -\frac{4}{2\omega + 1}, \]  
(81)

Now,
\[ \tilde{E} = \frac{Em}{m + s\Delta^{\frac{2}{2\omega+1}}}, \quad \tilde{L} = \frac{Lm}{m + s\Delta^{\frac{2}{2\omega+1}}}, \]  
(82)
\[ g_\phi = r^2 \Delta^{-\frac{2}{2\omega+1}}. \]  
(83)

Comparing this to (16), (19) and demanding that \( N \to 0 \) when \( \Delta \to 0 \), we see that
\[ \omega \leq -\frac{5}{2}. \]  
(84)

Although the interval (84) seems to be unrealistic from the astrophysical point of view, we use it as a simple example to illustrate general features of particle acceleration near scalar black holes.

It follows from (80), (83) that eq. (59) holds true with
\[ \alpha = \frac{2\omega + 3}{2\omega + 1} > 0. \]  
(85)

### A. Near-horizon behavior

Near the horizon, \( \Delta \to 0 \). Then, for the charged particle
\[ \tilde{E} \approx \frac{s}{m} \Delta^{-\frac{2}{2\omega+1}}, \quad \tilde{L} \approx \frac{s}{m} \Delta^{-\frac{2}{2\omega+1}}. \]  
(86)

The term \( \frac{\hat{L}^2}{r^2} \Delta^{-\frac{2}{2\omega+1}} \) in (79) has the order \( \Delta^{-\frac{2}{2\omega+1}} \) and represents a small correction to \( m^2 \) in (79), so it can be neglected.

If \( \omega < -\frac{5}{2} \), we have
\[ \tilde{Z} \approx \tilde{E} - \frac{m^2}{2E} N^2 \approx \frac{Em}{s} \Delta^{-\frac{2}{2\omega+1}} - \frac{ms}{2Er_+^2} \Delta, \]  
(87)

where the second term is a small correction to the first one. We see that the term with \( L \) does not affect the near-horizon expression for \( Z \) in the leading and sub-leading terms giving only unessential higher-order corrections.
If \( \omega = -\frac{5}{2} (\beta = 1) \),
\[
\tilde{Z} \approx m \Delta \sqrt{\frac{E^2}{s^2} - \frac{1}{r_+^2}}. \tag{88}
\]

By contrast, for a neutral particle the near-horizon expansion depends on the angular momentum more strongly. If \( L = 0 \),
\[
Z \approx E - \frac{m^2}{2 Er_+^2} \Delta^{\frac{2\omega+1}{2\omega+3}}, \tag{89}
\]
For a neutral particle with \( L \neq 0 \),
\[
Z \approx E - \frac{1}{2 r_+^4} E \Delta. \tag{90}
\]
It turns out that the curvature invariants remain finite in the interval of the Brans-Dicke parameter \[14\]
\[-\frac{5}{2} \leq \omega < -\frac{3}{2}. \tag{91}\]

According to (83), in the horizon limit \( \Delta \to 0 \) the metric coefficient \( g_{\phi} \to 0 \) as well if \( 2\omega + 3 < 0 \) that is compatible with (91). However, the coefficient \( g_{\theta\theta} \to 0 \), so that the horizon area \( A = 4\pi r_+^2 \) remains finite.

As we are interested in the possibility of acceleration of particles by the scalar field, eq. (84) should be obeyed. In combination with (91), it gives that either \( \omega = -\frac{5}{2} \) (the horizon is regular) or \( \omega < -\frac{5}{2} \) (then instead of a regular horizon we have a singularity).

XI. PARTICLE COLLISIONS IN THE BDS METRIC

Now, we consider different possible situations.

A. Both particles 1 and 2 are charged

One can check that collisions between two charged particles gives the bounded Lorentz factor \( \gamma \) that agrees with [11]. It also agrees with our previous general treatment since now
\[
\alpha + \beta - 1 = \frac{-2}{2\omega + 1} > 0 \tag{92}
\]
according to (81) and (85). Therefore, the necessary condition (60) of unbounded \( \gamma \) cannot be satisfied.
B. Particle 1 is neutral, particle 2 is charged

This is the most interesting case in our context not considered in \([11]\).

If \(\omega = -\frac{5}{2}\), (15) gives us

\[
\gamma \approx \frac{r^2}{\Delta m_1} E_1 \left( \frac{E_2}{s} - \sqrt{\frac{E^2}{s^2} - \frac{1}{r^2}} \right),
\]

so \(\gamma\) and \(E_{c.m.}\) diverge. Thus, in full analogy with the case of the BBMB black hole (see above), the collision between a neutral and the charged particles leads to unbounded \(E_{c.m.}\).

Let \(\omega < -\frac{5}{2}\) (singular horizon). Then, using (82), (87) for a charged particle, we obtain that

\[
\gamma \approx \frac{E_1 s}{2E_2 m_1} \Delta^2 \omega + 3^2
\]

diverges. It is worth noting that both for \(\omega < -\frac{5}{2}\) and \(\omega = \frac{5}{2}\) the values \(L_1\) and \(L_2\) do not enter the asymptotic forms (93) and (94).

It follows from (92) that, according to explanations given after eq. (65), the results are insensitive to the fact that \((g_\phi)_H = 0\). Indeed, using expressions (81), (85) it is easy to check that the results (93), (94) agree with general formulas (31), (30) which were derived without account for \((g_\phi)_H = 0\). In this sense, in the scenario under consideration the unbounded \(\gamma\) is achieved due to the scalar field (that acts to particles 1 and 2 differently), whereas the effects of curvature singularity are of the secondary importance. In particular, this applies to the results of \([7], [11]\).

C. Both particles 1 and 2 are neutral

If at least one of angular momenta does not vanish, \(\gamma = O(\Delta^{2\omega+3})\) diverges with the same rate as in (94). However, if \(L_1 = 0 = L_2\), it turns out that \(\gamma\) is finite. This is in agreement with a general case when in metric (11) the coefficient \(g_\phi \to 0\) on the horizon \([9]\).

In this context, it is the case \(L_1 = L_2 = 0\) for which the properties under discussion become especially pronounced: the effect of unbounded \(E_{c.m.}\) is absent both for collisions between two charged and two neutral particles but reveals itself in collision between one charged and one neutral particles.
In the present paper we gave a comprehensive analysis of the situations when the effect of unbounded $E_{c.m.}$ in particle collisions can arise due to interaction between the background scalar field and test scalar particles in the vicinity of black holes. The summary of the results is given in Table 1 below, where we list only the scenarios capable to produce indefinitely large $\gamma$ and $E_{c.m.}$. We also indicate which factor (the scalar field or/and geometry) is relevant for a corresponding scenario. By relevance of the geometric factor we imply that $(g_\phi)_H = 0$ that corresponds to $0 < \alpha < 1$ in the table according to (59). If $\alpha = 1$, $(g_\phi)_H \neq 0$. The index $\beta$ is responsible for the action of the scalar field. Thus the presence or absence of $0 < \beta \leq 1$ and $\alpha$ enables us to see which factor (or both) is relevant in the effect of unbounded $\gamma$.

For completeness, we also put there in line 6 the results of our previous work [9] and in line 7 the results inherent to the standard BSW effect [1]. They apply not only to the case of rotating black holes [2] but also to static electrically charged black holes (with minor modifications of definitions of critical and usual particles - see [3] for details). We see that inclusion of the mechanism under discussion due to the scalar field gives rise to new scenarios of unbounded $\gamma$ (lines 1 - 5) that were absent in the standard case 7. Lines 1, 2 and 5 correspond to the effect due to the scalar field in itself, line 6 represents the effect due to the geometry, cases 3 and 4 give the combined effect of the scalar field and the geometry when both factors cannot be disentangled from each other. Main scenarios of collisions in the MDK metric considered in [11] correspond to both usual charged particles with $\alpha + \beta > 1$, so that they do not lead to unbounded $\gamma$ and, therefore, do not fall into this table. Meanwhile, the example typical of line 6 was mentioned in passing in [11] after

| Colliding particles                                      | $g_\phi \sim N^{2(1-\alpha)}$, $0 \leq \alpha \leq 1$ | Relevant factor(s) of two                  |
|----------------------------------------------------------|---------------------------------------------------------|--------------------------------------------|
| 1 usual neutral and usual charged                        | $(g_\phi)_H \neq 0$, $\gamma \sim N^{-\beta}$         | scalar field                               |
| 2 usual neutral and usual charged                        | $\alpha + \beta > 1$, $\gamma \sim N^{-\beta}$        | scalar field                               |
| 3 usual neutral and usual charged                        | $\beta + \alpha \leq 1$, $\gamma \sim N^{\beta+2(\alpha-1)}$ | scalar field and geometry                  |
| 4 usual charged and usual charged                        | $\alpha + \beta < 1$, $\gamma \sim N^{2(\alpha+\beta-1)}$ | scalar field and geometry                  |
| 5 critical neutral and usual charged                     | $\beta < 1$, $(g_\phi)_H \neq 0$, $\gamma \sim N^{-1}$ | scalar field                               |
| 6 usual neutral and usual neutral                        | $\gamma \sim \frac{1}{g_\phi} \sim N^{2(\alpha-1)}$  | geometry                                  |
| 7 critical neutral and usual neutral                     | $(g_\phi)_H \neq 0$, $\gamma \sim N^{-1}$             | absent (standard BSW effect)               |

Table 1. Types of collisions that lead to unbounded $E_{c.m.}$.
their eq. (24).

Thus the scalar field does act as a particle accelerator in that it leads to new scenarios of high-energy collisions that were impossible without it. Case 5 is somewhat special in that the scalar field accelerates the particles but does it weaker than in the standard BSW effect. Some suppression of collision energy takes place in case 5 but it does not cancel the effect itself, provided \( \alpha + \beta < 1 \). The latter condition is not satisfied for the BDK solution and this is the reason of significant suppression of collision energy because of which the effect of unbounded energy \( E_{c.m.} \) disappears \[11\]. In other words, the accelerator due to the scalar field exists but it is not universal.

Cases 1 and 5 are the most interesting ones in that they are universal, provided \( (g_\phi)_H \neq 0 \), so geometry of the horizon is quite standard. Then, collision between a neutral and charged particles always gives the effect of unbounded \( \gamma \).

The results from Table 1 related to collisions between usual particles, are valid for nonextremal and extremal black holes. This is not so for cases 5 and 7 since the approximate form of \( X \) for the critical particle in both cases are different \[21\] because of which a critical particle cannot reach the nonextremal horizon at all \[2\].

The research carried out in the present paper clearly revealed that a scalar field can, in principle, be a particle accelerator. However, in contrast to, say, the electromagnetic field \[3\], it requires stronger conditions such as divergence of the scalar field on the horizon.

An interesting issue for further research is whether and how the effect under discussion reveals itself in cosmology where the scalar field can play an essential role.

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[1] M. Bañados, J. Silk and S.M. West, Kerr black Holes as particle accelerators to arbitrarily high Energy, Phys. Rev. Lett. 103 (2009) 111102 [arXiv:0909.0169].

[2] O.B. Zaslavskii, Acceleration of particles as universal property of rotating black holes, Phys. Rev. D 82 (2010) 083004 [arXiv:1007.3678].
[3] O. Zaslavskii, Acceleration of particles by nonrotating charged black holes, Pis’ma ZhETF 92, 635 (2010) (JETP Letters 92, 571 (2010)), [arXiv:1007.4598].

[4] V. P. Frolov, Weakly magnetized black holes as particle accelerators, Phys. Rev. D 85, 024020 (2012) [arXiv:1110.6274].

[5] S.W. Wei, Y. X. Liu, H. T. Li, and F.W. Chen, Particle Collisions on stringy black hole background, J. High Energy Phys. 12, 066 (2010), [arXiv:1007.4333].

[6] P. J. Mao, R. Li, L.-Yu Jia, Ji-R. Ren, Acceleration of particles in Einstein-Maxwell-Dilaton black hole [arXiv:1008.2660].

[7] M. Patil and P. Joshi, Acceleration of particles by Janis-Newman-Winicour singularities, Phys. Rev. D 85, 104014 (2012), [arXiv:1112.2525].

[8] S. Fernando, String black hole: can it be a particle accelerator ?, Gen. Relativ. Gravit. 46, 1634 (2014), [arXiv:1311.1455].

[9] O. B. Zaslavskii, High energy particle collisions and geometry of horizon, Int. J. Mod. Phys. D 25, 1650095 (2016), [arXiv:1602.03336].

[10] J. Sultana and B. Bose, Scalar fields and particle accelerators, Phys. Rev. D 91, 124046 (2015).

[11] J. Sultana and B. Bose, Particle collisions near a Kerr-like black hole in Brans-Dicke theory, Phys. Rev. D 92, 104022 (2015).

[12] R. N. Tiwari and B. K. Nayak, Class of the Brans-Dicke Maxwell fields, Phys. Rev. D 14, 2502 (1976).

[13] T. Singh and T. Singh, Ann. Phys. Hungr. 56, 55 (1984).

[14] H. Kim, New black hole solutions in Brans-Dicke theory of gravity, Phys. Rev. D 60, 024001 (1999) [arXiv:gr-qc/9811012].

[15] N. Bocharova, K. Bronnikov, and V. Melnikov, On an exact solution of the EinsteIn and massless scalar field equations, Vestn.Mosk. Univ., Ser. 3: Fiz., Astron. 6, 706 (1970) (English transl.: Moscow Univ. Phys. Bull. 25, 6, 80 (1970)).

[16] K. A. Bronnikov, Scalar-tensor theory and scalar charge, Acta Phys. Polon. B 4, 251 (1973).

[17] J. D. Bekenstein, Exact solutions of Einstein-conformal scalar equations, Ann. Phys. (N.Y.) 82, 535 (1974).

[18] J. D. Bekenstein, Black holes with a scalar charge, Ann. Phys. (N.Y.) 91, 75 (1975).

[19] C. Martinez, R. Troncoso, and J. Zanelli, de Sitter black hole with a conformally coupled scalar field in four dimensions, Phys. Rev. D 67, 024008 (2003), [arXiv:hep-th/0205319].
[20] M. Patil and P. Joshi, Naked singularities as particle accelerators, Phys. Rev. D 82, 104049 (2010), arXiv:1011.5550.

[21] I. V. Tanatarov and O. B. Zaslavskii, Dirty rotating black holes: regularity conditions on stationary horizons, Phys. Rev. D 86, 044019 (2012) arXiv:1206.2580.

[22] O. B. Zaslavskii, Acceleration of particles by black holes: kinematic explanation, Phys. Rev. D 84, 024007 (2011) arXiv:1104.4802.

[23] J. M. Bardeen, W. H. Press, and S. A. Teukolsky, Rotating black holes: locally nonrotating frames, energy extraction and scalar synchrotron radiation, Astrophys. J. 178, 347 (1972).

[24] A. P. Lightman, W. H. Press, R. H. Price, and S. A. Teukolsky, Problem book in Relativity and Gravitation (Princeton University Press, Princeton, New Jersey, 1975).

[25] A. A. Grib and Yu.V. Pavlov, On particle collisions in the gravitational field of the Kerr black hole, Astropart. Phys. 34, 581 (2011) arXiv:1001.0756.

[26] T. Jacobson, T.P. Sotiriou, Spinning black holes as particle accelerators, Phys. Rev. Lett. 104, 021101 (2010), arXiv:0911.3363.

[27] E. Berti, V. Cardoso, L. Gualtieri, F. Pretorius, U. Sperhake, Comment on ”Kerr Black Holes as Particle Accelerators to Arbitrarily High Energy, Phys. Rev.Lett. 103, 239001 (2009), arXiv:0911.2243.

[28] K. A. Bronnikov and Yu. N. Kireyev, Phys. Lett. A 67, 95 (1978).