Distribution of electromagnetic fields in heterogeneous composite structures when exposed to direct current.

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Abstract. In this paper, we study the electromagnetic fields in composite structures that occur when exposed to an external source of direct current, located on the surface of the composite. The article proposes an algorithm for solving the direct problem, based on the application of the finite element method, and a method for solving the inverse problem, which is described as an optimization problem. To search for the extremum of the target functional, an empirically developed method is used. The task is considered in a quasistatic approximation. The developed software was tested on a three-layer composite. The developed algorithms made it possible to solve a given problem with high accuracy.

Introduction
The solution of inverse problems and the study of electromagnetic fields are described in [1-14]. The purpose of the research in this paper is to determine the geometric and physical parameters of heterogeneous composite structures by comparing the measurement data of electromagnetic fields with the corresponding results of their numerical simulation. In the case of the application of electromagnetic fields, the problem is reduced to the determination of the coefficients of Maxwell's equations from some indirect data and refers to the classical inverse problems studied in numerous works, for example [8], [10], [13].

Now, the solution of inverse problems of restoring the parameters of multilayer structures is based on optimization principles. The essence of these methods is to minimize the objective functional of the difference between the measured values and the values obtained in the simulation in the presence of linear or non-linear constraints on the theoretical values of the parameters of the composite structure. Consequently, the search for a solution to the inverse problem is reduced to the multiple solution of the direct problem, which requires a lot of time. Also the complexity is added by a large number of variables and a multi-character character.

In this paper, we consider the problem of the distribution of the electric field to solve direct and inverse problems in composites characterized by the location of sources and measuring devices on the surface of the composite. Electrostatic or electromagnetic fields are investigated in various frequency ranges. The simulated fields are described by the Maxwell system of differential equations.

Solved problems for heterogeneous composite structures are considered in the 3-dimensional approximation. The purpose of research in this area is to determine the geometric characteristics and conductive properties of composites in their study.

The proposed computational algorithms are based on the approximation of multidimensional mixed boundary value problems by the finite element method (FEM) of high accuracy, on fast iterative processes of solving the generated systems of linear algebraic equations (SLAE) with sparse matrices.

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of large order, and also on effective optimization methods. To solve the obtained SLAU, we used the biconjugate gradient method.

Mathematical formulation of a direct problem.
At points A and B, two electrodes are located on the surface of the composite. A current is supplied to these electrodes with opposite values \( I_k \) and \(-I_k\), respectively. It is required to find the distribution of the electrostatic potential in a given region.

Electromagnetic fields described by the system of Maxwell's equations are investigated in various applications in papers [6], [7], [13], [14]. We study the fields constant in time. In the specified fields there are no volumetric sources, therefore, the electrostatic potential in the computational domain \( \Omega \) is determined by the equation

\[
L\phi \equiv \nabla \sigma \nabla \phi(x) = 0, \quad x \in \Omega
\]  

(1)

here the differential operator \( L \) is described in cylindrical coordinates for axisymmetric problems and in Cartesian coordinates for general three-dimensional statements. In the case of anisotropic media with different conductivities \( \sigma_x, \sigma_y, \sigma_z \) in the directions of the Cartesian axes \( x, y, z \), we have

\[
L\phi = \frac{\partial}{\partial x} \sigma_x \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial y} \sigma_y \frac{\partial \phi}{\partial y} + \frac{\partial}{\partial z} \sigma_z \frac{\partial \phi}{\partial z} = 0
\]  

(2)

At different parts of the outer boundary of the computational domain, for physical reasons, boundary conditions of the first or second kind are set:

\[
\phi|_{\Gamma} = \phi|_{\Gamma}, \quad \frac{\partial \phi}{\partial n}|_{\Gamma_i} = 0, \quad \sigma \frac{\partial \phi}{\partial n}|_{\Gamma_i} = I_k, \quad \sigma \frac{\partial \phi}{\partial n}|_{\Gamma_i} = -I_k.
\]  

(3)

We use the finite element method to solve this problem. We also use the Bubnov-Galerkin method. Then equation (2) can be rewritten as follows.

\[
\int_{\Omega} [N]^T \left( \frac{\partial}{\partial x} \sigma_x \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial y} \sigma_y \frac{\partial \phi}{\partial y} + \frac{\partial}{\partial z} \sigma_z \frac{\partial \phi}{\partial z} \right) d\Omega = 0
\]  

(4)

To begin with, we transform an equation containing only the first derivatives with respect to \( x, y \) and \( z \). We do this for the \( x \) term using the first Green formula.

\[
\frac{\partial}{\partial x} \left( \sigma_x [N]^T \frac{\partial \phi}{\partial x} \right) = [N]^T \frac{\partial}{\partial x} \sigma_x \frac{\partial \phi}{\partial x} + \frac{\partial [N]^T}{\partial x} \sigma_x \frac{\partial \phi}{\partial x}
\]  

(5)

Using the expression, we can write

\[
[N]^T \frac{\partial}{\partial x} \sigma_x \frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x} \left( \left[ N \right]^T \sigma_x \frac{\partial \phi}{\partial x} \right) - \frac{\partial}{\partial x} \left[ N \right]^T \sigma_x \frac{\partial \phi}{\partial x}
\]  

(6)

Then the first term is transformed to the form

\[
\int_{\Omega} [N]^T \frac{\partial}{\partial x} \sigma_x \frac{\partial \phi}{\partial x} d\Omega = \int_{\Omega} \frac{\partial}{\partial x} \left( \left[ N \right]^T \sigma_x \frac{\partial \phi}{\partial x} \right) d\Omega - \int_{\Omega} \frac{\partial [N]^T}{\partial x} \sigma_x \frac{\partial \phi}{\partial x} d\Omega
\]  

(7)

Further, using the Ostragradsky-Gauss formula we get
Transforming in a similar way all the terms in (5) we get the following expression:

\[
\int_{\Omega} \frac{\partial}{\partial x}\left([N]^T \sigma_x \frac{\partial \varphi}{\partial x}\right) d\Omega = \int_{S} [N]^T \sigma_x \frac{\partial \varphi}{\partial x} l_x dS
\]

(8)

in which the integral over the surface is expressed in terms of the derivative along the normal \( \frac{\partial \varphi}{\partial n} \)

\[
\int_{S} \left( \sigma_x \frac{\partial \varphi}{\partial x} l_x + \sigma_y \frac{\partial \varphi}{\partial y} l_y + \sigma_z \frac{\partial \varphi}{\partial z} l_z \right) dS - \int_{\Omega} \frac{\partial [N]^T}{\partial x} \sigma_x \frac{\partial \varphi}{\partial x} + \frac{\partial [N]^T}{\partial y} \sigma_y \frac{\partial \varphi}{\partial y} + \frac{\partial [N]^T}{\partial z} \sigma_z \frac{\partial \varphi}{\partial z} d\Omega = 0
\]

(9)

The first integral in the resulting expression contributes to the stiffness matrix, the second integral contributes to the column vector of the free terms. An unknown function \( \varphi \) is defined by the relation.

\[
\varphi = [N][\Phi]
\]

(10)

To solve the problem, we divide the original domain into tetrahedral finite elements.

**Calculations of the direct problem**

Expression (10) is a system of linear algebraic equations. To solve the resulting SLAE, we used several methods, namely the Seidel method, the Cholesky method, and the biconjugate gradient method. As a result of the comparison, it was found that the results obtained using the method of biconjugate gradient method have a higher accuracy.

To test the developed algorithm, we chose a three-layer composite. The resistivity is respectively equal \( \sigma_1 = 10 \text{ Ohm}\cdot\text{m}, \sigma_2 = 0.1 \text{ Ohm}\cdot\text{m}, \sigma_3 = 1000 \text{ Ohm}\cdot\text{m} \). The thickness of the composite was 0.2 meters.

The composite used for testing is shown in Fig. 1.
When solving the problem, the potential value was obtained which corresponds to the physical meaning. As an illustration of the solution, Fig. 2 shows the lines of the equal potential level. In practice, only the potential values on the surface of the composite can be measured. Clots on the surface of the composite correspond to the location of the electrodes.

Figure 1. Heterogeneous composite
Inverse task

The inverse problem can be formulated as an optimization problem with the objective functional

\[ F(\sigma_1, \ldots, \sigma_n) = \min \left\{ \sum_{i=1}^{n} (\varphi_i - \varphi_{i+1}(\sigma_1, \ldots, \sigma_n))^2 \right\} \]

Optimization methods for solving inverse problems are applied in various fields [1], [2], [3]. When solving the inverse problem, we used an algorithm developed experimentally. This solution method is described in [13]. The algorithm is as follows: for a start, suppose that the computational domain contains one layer with useful resistance \( \sigma \), we obtain a solution for a given computational domain. Then the computational domain is randomly divided into two parts with resistivity \( \sigma_1 \), \( \sigma_2 \), and solve the inverse problem for the obtained computational domain. Then, in the same way, we continue to break the computational domain into new regions with specific resistivity \( \sigma_1 \), \( \sigma_2 \), \( \ldots \), \( \sigma_n \). One of the possible schemes for the sequential separation of the computational domain is presented in Fig. 3.

![Figure 2. Lines of equal medium potential](image)

![Figure 3. Successive division of the computational domain](image)
The optimization problem was solved by the Hook-Jeeves method, as well as the brute force method. For the condition of stopping the optimization, we take the difference between the values of resistivity in the adjacent steps. To complete it is necessary that the deviation was less \( \delta = 0.001 \). The criterion for stopping the search for the solution of the inverse problem is the difference between the theoretical and experimental potential values at the nodal points. Provided that the difference is less \( \epsilon = 0.0001 \), then the search for composite parameters is terminated. In the case when the minimum difference between the theoretical and numerical values does not change significantly over 10 steps, the search for a solution to the inverse problem stops and returns to the beginning. This algorithm is based on the analysis of existing optimization methods for solving inverse task [1], [8], [13]. The program implementation was carried out using existing methods [4], [5].

### Calculations of the inverse problem

For the experimental data in solving the inverse problem, we take the values obtained in solving the direct problem for the composite selected above. As an illustration of the results of solving the inverse problem, we present the composite model obtained by solving the inverse problem. In fig. 4 depicts a composite with physical and geometric, obtained by solving the inverse problem. The error of the solution in all parameters does not exceed 5 percent; the most significant deviation in the analysis of the results obtained was found in the conductivity of the third layer.

![Figure 4. The composite model obtained by solving the inverse problem](image)

### Conclusions

The paper proposed a method for studying electromagnetic fields in heterogeneous composite structures, similar to geological exploration methods. To solve the direct and inverse problems, algorithms were
developed that allow one to calculate the potential values and the physical parameters of the composite with a different number of layers and in a wide range of thicknesses. As a result of applying the new algorithm for solving the inverse problem, it was possible to significantly reduce the time spent on finding a solution, but even in this case, the task remains very resource-intensive.

It should be noted that the results of this work can also find application in the study of composites with a nonlinear structure. It is assumed that the described algorithms can help in determining the characteristics of laying fabric composites, as well as, when finalizing, the developed methods can be used to study cracks in composites, taking the crack as a separate layer.

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