Anomalous photon noise levels predicted for CMB measurements made by the Planck satellite mission

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Abstract

A fundamental assumption inherent in the standard ΛCDM Hot Big Bang (HBB) model is that photons lose energy as they are redshifted due to the expansion of the universe. We show that for the Quasi-Static Universe (QSU) model, in which photon energy is an invariant in the cosmological reference frame, the photon number density in the universe today is a factor of approximately 1600 less than in the standard model. We examine some of the consequences for a number of processes that occur during the thermal history of the early universe, including primordial nucleosynthesis, the formation of neutral hydrogen (recombination), and the evolution of the Cosmic Microwave Background (CMB) radiation. We show that the QSU model predicts that the measured CMB photon noise level will be a factor of ∼ 40 higher than the level that would be observed assuming the standard HBB model. The CMB data that will be collected by the recently launched Planck satellite mission provides an ideal opportunity to test the validity of this prediction.

Subject headings: cosmic microwave background, photon noise

1. Introduction

1.1. The Hot Big Bang Model

The standard Hot Big Bang (HBB) model has formed the cornerstone of cosmology for over 40 years, since the discovery of the 2.7K Cosmic Microwave Background (CMB) by Penzias and Wilson in 1965. Over time, the model has been extended and enhanced to provide a better fit with the mass of data obtained from numerous ground and satellite based astronomical observations. The main ingredients of the current so-called ‘Concordance Model’ include a hot Big Bang plus inflation, with the addition of dark energy and cold dark matter to the mix. This is often referred to as the Lambda + Cold Dark Matter (ΛCDM) model. One of the key features of the HBB model is the assumption that the universe started off in a state of near infinite density and temperature, and subsequently cooled as it expanded. This leads to the concept of an early radiation dominated phase. As the universe cooled it became possible for nucleons to condense out of the primordial

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quark-gluon plasma. The standard model assumes that near equal amounts of matter and anti-matter were formed at this stage. Subsequently, these matter and anti-matter particles annihilated each other, releasing energetic photons. It is these same photons that constitute the CMB that we now observe. The small net imbalance between matter and anti-matter resulted in the matter dominated universe that we now inhabit. Following this matter generation epoch, the universe continued to expand and cool until it became energetically favourable for nucleons to combine to form heavier elements in a process of primordial nucleosynthesis.

Further expansion reduced the temperature to a point at which previously free electrons and protons were able to combine into neutral hydrogen. As the temperature and matter density dropped further, a point was reached at which the mean free path of CMB photons became comparable to the size of the universe. At this stage, matter and radiation became decoupled and fell out of thermal equilibrium and the CMB photons were free to stream - the so-called Last Scattering Surface (LSS). The CMB power spectrum that we observe today is therefore a snapshot of the CMB at the LSS.

An important principle inherent in the standard model is that matter density falls in proportion to $T^3$, whereas the energy density of photons in the CMB radiation falls as $T^4$ because of the additional energy loss as the photons are redshifted.

2. The Quasi-Static Universe model

The Quasi-Static Universe (QSU) is used here as a shorthand to refer to the paradigm described in [Booth (2002)]. The main feature of this model is that the Planck scale is decoupled from the atomic scale conventionally used as the as the basis for our measurement system. The changes in the dynamical behaviour of the universe that result from this modification can not only account for most of the problems associated with the Big Bang model, but are also able provide a very simple explanation for the apparent acceleration of the expansion rate of the universe that has been observed in various high redshift supernovae studies in recent years. One of the principal consequences of the QSU paradigm is that the atomic scale, as defined by the de Broglie wavelength of sub-atomic particles, is not an appropriate reference frame for measuring gravitational phenomena or the behaviour of photons. The QSU model is founded on the postulate that the correct reference frame for these phenomena is in fact a cosmological frame based on the size and energy content of the universe as a whole. In such a frame, photons do not undergo any change in frequency, since as far as they are concerned, the universe is static. Hence, it is not meaningful to talk in terms of photons losing energy in this reference frame. In transforming from the cosmological reference frame to our conventional atomic frame, photons will be perceived to exhibit a redshift as the scale factor of the universe with respect to the atomic frame increases with time. The crucial difference between the QSU model and the conventional formalism is that the relationship $E = h\nu$ no longer holds true for photons emitted at times $t < t_0$, where $t_0$ signifies the present time. The energy of such ‘old’ photons will remain constant, at the same value they possessed when they were first
emitted. However their power, i.e. the energy transferred per unit time, will be reduced in proportion to their redshift, such that $W = W_0 \nu / \nu_0 = W_0(z_0 + 1)$, where $W_0$, $\nu_0$ and $z_0$ are respectively the photon power, photon frequency and cosmological redshift at the time of emission. Clearly, such a modification to one the most fundamental equations in physics will have a very significant impact on any phenomena that involve redshifted photons, and in particular, the CMBR.

To understand the implications of the QSU model for the CMBR, we need to review the way in which the standard Planck black-body distribution law is applied. A summary of the standard derivation of this law is provided in Appendix A. Present day observations of the CMB give a value for the energy density of $U \simeq 4 \times 10^{-14} J m^{-3}$, which from (A4) corresponds to a temperature of $T = 2.7^\circ K$. Conventionally, the next step is to take this temperature and use equation (A6) to calculate the photon number density, giving a result of $N \simeq 4 \times 10^8 m^{-3}$. However, these formulae are only valid for black-body radiation that is in equilibrium with its surroundings. It would be perfectly correct to use these equations if we wished to deduce the photon number density for a black-body with this temperature today. In erroneously applying them in the context of the relic CMBR generated by the Big Bang, we are perpetuating the assumption inherent in going from (A3) to (A5) - that photon energy is always equal to $h\nu$.

In the QSU model, red-shifted photons do not lose energy, so it is not possible to determine the energy of an observed photon merely by measuring its frequency. It is also necessary to know the thermal history of the photon, i.e. its frequency when it was originally emitted. It is this that determines the energy of the photon. In order to obtain the correct result for the photon number density today that corresponds to an observed energy density, we need to correct for the fact that the photon energy $h\nu$ applied in going from (A3) to (A5) should reflect the energy at the time of emission, or more precisely, its energy at the time it was in black-body equilibrium with its surroundings.

We therefore need to apply a factor of $T_{\text{obs}}/T_{\text{equi}}$ to the expression for photon number density in (A5), where $T_{\text{obs}}$ is the observed absolute temperature of the CMBR today, and $T_{\text{equi}}$ is the temperature at the time of black-body equilibrium. Clearly we do not know this figure with any precision. However we can take an educated guess that is at least consistent with other observational data and with plausible models for primordial nucleosynthesis. One such model is the neutron decay variant of the Cold Big Bang (CBB), which predicts a maximum reaction temperature of $\approx 10^{10} K$, with an equilibrium temperature of the order of $\approx 10^9 K$. Applying the correction factor to equation (A6) gives a calculated photon number density of $\approx 0.3 m^{-3}$ - a factor of $\sim 10^9$ lower than for the standard Hot Big Bang (HBB) model. This is very close to the measured baryon number density of the universe, giving a photon-barion ratio of $\eta_\gamma \equiv n_\gamma/n_B \simeq 1$. It should be noted that this simplistic calculation is based on the assumption that there are no intervening process that might change the photon number density. As we shall see in the next section, this assumption is unlikely to be valid.
3. Implications for cosmological processes

Having established that the initial value of $\eta_\gamma$ in the QSU model is many orders of magnitude less than the conventionally accepted value, the natural question to ask is: how does this affect other cosmological processes? We now review the consequences of a lower photon-barion ratio for three of the key stages in the thermal history of the early universe:

1. Primordial nucleosynthesis
2. Recombination
3. CMB cooling

3.1. Primordial nucleosynthesis

Arguably, one of the processes most sensitive to changes in $\eta_\gamma$ is that of primordial nucleosynthesis. Applying the conventional value of $\eta_\gamma \simeq 2 \times 10^9$ to the standard model for HBB nucleosynthesis results in predicted element abundances that are in good accord with observational data. Relatively small changes in $\eta_\gamma$ will result in large changes to the predicted abundances, and would therefore appear not to be compatible with observations. However, it has been pointed out by Aguirre (2001) that, provided that appropriate changes are made to a range of initial parameters, including $\eta_\gamma$, it is possible to construct alternative models for primordial nucleosynthesis that will produce predicted element abundances that are still in accord with observational data. One such model is the Cold Big Bang (CBB), which takes a value of $\eta_\gamma \sim 1$ as one of its initial conditions. It is perhaps worth mentioning in passing that the neutron decay variant of the CBB model predicts a photon energy of $0.78\, MeV$, giving a baryon to photon energy ratio of $E_\gamma/E_B \simeq 1200$. Under the QSU scenario, this ratio should persist from the nucleosynthesis epoch to the present day, and indeed, the observed value of $E_\gamma/E_B$ is very close to this value.

3.2. Hydrogen ionization

Another astrophysical measurement that is linked to $\eta_\gamma$ is the hydrogen ionization fraction. As the universe expands and cools, protons combine with electrons to form neutral hydrogen when the energy of CMB photons falls below the hydrogen ionization energy threshold. The equilibrium ionization fraction $\chi_e$ as a function of temperature is given by the Saha equation

$$\frac{1 - \chi_e}{\chi_e} = \frac{4\sqrt{2}\xi(3)}{\pi\eta_\gamma} \left( \frac{T}{m_e} \right)^{\frac{3}{2}} e^{E_B/T}$$

from which it can be seen that the ionization fraction is also dependent on $\eta_\gamma$. The ionization
fraction can be expressed as a function of redshift using the relation $T = 2.73(1 + z)K$. This is plotted in Figure 1 for a standard HBB cosmology with $\eta_\gamma = 2 \times 10^9$, and a CBB cosmology with $\eta_\gamma = 1$. This illustrates that a reduction in the photon to baryon ratio will cause recombination to take place at a much higher redshift, with the surface of last scattering for the CMB occurring at $z \simeq 2600$, as compared to $z \simeq 1100$ for the standard model.

![Figure 1: Ionization fraction as a function of redshift](image)

In practice, the situation is somewhat more complicated than that depicted by the Saha equation because the system will not in fact be in equilibrium. This is due to the fact that the process of electron-proton recombination will itself release an energetic photon that will go on to re-ionize another hydrogen atom. A more detailed analysis (see Peacock (1999) for example) reveals that recombination proceeds via a two-photon decay process, with the rate of re-ionization as a function of redshift being given by

$$\frac{d\ln \chi}{d\ln z} = 60\chi z \sqrt{\Omega_B h^2}$$

where $\Omega_B$ is the barionic density parameter, $\Omega$ is the total density parameter, and $h$ is the Hubble parameter.

The fact that the process of recombination results in the creation of new photons is of particular relevance for the QSU model. In the standard HBB model, the fact that the photon-barion ratio, at $\eta_\gamma = 2 \times 10^9$, is already very high before the beginning of the recombination era means that the
additional photons generated by the recombination process will be of little consequence. Conversely, if one assumes an initially low value for $\eta_\gamma = 1$, such as might occur in the CBB scenario, then the process of recombination will result in a large increase in the photon number density. A value for $\eta_\gamma$ after the recombination era can be estimated by taking the initial photon energy for the CBB scenario, $0.78\, MeV$, and dividing it by the ionization energy of the hydrogen atom, $13.6\, eV$. This gives a photon-barion ratio of $\eta_\gamma \approx 5 \times 10^4$. The ionization fraction as a function of redshift for this scenario is also plotted in Figure 1.

The Planck satellite will have sufficient sensitivity to measure the angular power spectrum to a level of accuracy that will permit a more accurate picture of the recombination era to be determined (ESA (2005)).

### 3.3. CMB measurements

Measurements of the CMB spectrum can provide a wealth of information about the early history of the universe (ESA (2005)). The WMAP satellite mission has already helped to refine the value of the main parameters in the concordance model, including $\Omega$, $\Omega_\Lambda$, and the Hubble constant. The recently launched Planck mission aims to refine these measurements using the greater sensitivity of its bolometric detectors to map the CMB at higher angular resolutions. The satellite will also include polarization sensitive detectors, enabling it to perform measurements of CMB polarization, which can be used, amongst other things, to probe for gravitational waves that might have been generated during an era of cosmic inflation. The sensitivity of the detectors used on these satellite missions is determined largely by the noise levels present in the detector chain. This includes thermal noise for the detector chain itself, $1/f$ noise, Johnson noise, and CMB photon counting noise. The performance of the bolometric detectors on the Planck satellite has improved to the extent that sensitivity is now limited primarily by the photon noise of the CMB (see appendix B).

The performance goals for the Planck HFI are summarized in Table 1. The sensitivity figures represent the quadrature sum of the combined sensitivities of all the bolometric detectors in that frequency band. Figure 2 shows the sensitivities per pixel per HFI detector, together with the theoretical CMB Background Limited Infrared Photons (BLIP) according to the standard HBB model. From this it can be seen that the noise performance of the most sensitive detectors is at a level that is comparable to the CMB BLIP.

From Appendix B we see that $NEP \approx h\nu \sqrt{N}$, where $N$ is the photon count. However, this is based on the assumption inherent in the standard Big Bang model: that CMB photons lose energy as they are red-shifted by the expansion of the universe. In the QSU, no such expansion takes place in the cosmological reference frame inhabited by these photons, and hence they can not be said to lose energy. We therefore need to apply a redshift modification factor to the apparent photon energy to give the correct value in the present epoch, such that $E = h\nu(z_0 + 1)$, where $z_0$ is the
redshift at the time that the CMB photons were in blackbody equilibrium with their surroundings, i.e. the era of recombination. Now if we apply this corrected photon energy to the equation for the CMB energy density (Equation A3), we see that the predicted photon number density (Equation A6) will be reduced proportionately, such that
\[ N = N_0 / (z_0 + 1). \]
So the CMB photon noise level, using the QSU model, will be given by
\[ NEP \approx h \nu \sqrt{N(z_0 + 1)}. \]

Applying this now to Equation 12 gives us:
\[ \frac{\Delta T}{T} = \frac{(e^x - 1)^2}{xe^x} \sqrt{\frac{(z_0 + 1)(n^2 + n)}{\Delta \nu \tau}} \]  \hspace{1cm} (2)

This is plotted as the upper curve of Figure 2, assuming a redshift at LSS of \( z \approx 1800. \) From this, it can be seen that the predicted CMB photon noise level for a single detector now lies well above the sensitivities of the Planck HFI bolometers. In principle, therefore, it should be possible to measure this photon noise in order to act as a test of the QSU model.

4. Measurement strategy

The scanning strategy employed on the Planck mission (see ESA (2005)) is ideally suited to the generation of a data set that can be readily analyzed to determine the level of photon noise in the CMB. The use of multiple detectors in each HFI frequency band further extends the options for analysis. The Planck satellite spins at one revolution per minute, which with a beam size of 5 arc mins (FBHW), gives an integration period per pixel per scan of approximately \( 14 \times 10^{-3} \) s. The

\[ ^1 \text{It is worth noting that the same result can be obtained by assuming that the CMB is emitted from a body at about } 4800K, \text{ but the integration time in the reference frame of the CMB photons is reduced by a factor of } 1 / (z_0 + 1). \]
Fig. 2.— CMB photon noise

The satellite is re-pointed by an angle of 2.5 arc mins once every hour (which provides two complete sky scans over a one year period), so that each pixel will be scanned 60 times between re-pointings, giving a total integration time of about 0.8s. The data analysis is somewhat complicated by the fact that the 2.5 arc min re-pointing angle is designed to give a beam overlap between successive re-pointings such that each sky pixel is sampled approximately 2.4 times. Various alternative analysis strategies present themselves. For example, the mean $\Delta T/T$ value for each detector chain (or pair of orthogonal detectors in the case of polarization sensitive bolometers) could be determined for each of the 4320 pixels in a 60 min sky scan. The RMS deviation of each detector output from the mean value of all the detectors would then provide a measure of the photon noise level.

Since the precise orientation of the satellite in space is not critical for the purposes of analyzing CMB photon noise, it should in principle be possible to obtain the necessary data for this analysis from the test measurements that will be carried out during the 3-4 months that it will take the Plank satellite to reach its final orbital position at the L2 point of the Earth-Sun system.
5. Conclusions

The Planck CMB mission will provide a wealth of detailed measurement data that will help to pin down the value of various cosmological parameters to hitherto unachievable levels of precision. Intriguingly, the sensitivity of its detectors provides us with an ideal opportunity to determine whether some of the fundamental assumptions underlying the standard Big Bang model are in fact correct, or whether an alternative paradigm might provide a more accurate description of our universe.
A. Black Body Radiation

Planck’s black-body distribution law can be derived in two steps. First, by considering the number of radiation modes that can be supported in a black-body cavity of unit volume, an expression for the mode density can be obtained

$$dN(\nu) = \frac{8\pi\nu^2}{c^3}d\nu$$  \hspace{1cm} (A1)

The energy density as a function of frequency can then be calculated by multiplying the mode density by the average energy per mode. In classical terms this would simply be $kT$, where $k$ is the Boltzmann factor and $T$ is the absolute temperature. However, the quantum nature of photons results in the probability distribution being skewed, so that the correct mean energy per mode must be calculated from the Boltzmann distribution, giving

$$\bar{E} = \frac{h\nu}{e^{h\nu/kT} - 1}$$  \hspace{1cm} (A2)

Combining (A1) and (A2) we obtain the final form of Planck’s law for the energy density of black-body radiation as a function of frequency

$$dU(\nu) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{d\nu}{e^{h\nu/kT} - 1}$$  \hspace{1cm} (A3)

The total energy density per unit volume is then simply obtained by integrating (A3) to give

$$U = \frac{8\pi^5 k^4 T^4}{15c^3 h^3}$$  \hspace{1cm} (A4)

The final expression for the photon number density as a function of frequency is merely (A3) divided by the photon energy, $h\nu$

$$dN(\nu) = \frac{8\pi\nu^2}{c^3} \cdot \frac{d\nu}{e^{h\nu/kT} - 1}$$  \hspace{1cm} (A5)

The photon number density per unit volume is therefore given by

$$N = \int_0^\infty \frac{8\pi\nu^2}{c^3} \cdot \frac{d\nu}{e^{h\nu/kT} - 1}$$  \hspace{1cm} (A6)

$$= \frac{16\pi k^3 T^3 \zeta(3)}{c^3 h^3}$$  \hspace{1cm} (A7)

The energy intensity as a function of frequency and temperature is given by
\[ I(\nu, T) = \frac{U(\nu, T)c}{4\pi} \]

Applying this to (A3) yields

\[ I(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\hbar\nu/kT} - 1} \quad (A8) \]

B. CMB Photon Noise

B.1. Noise Equivalent Power

Noise Equivalent Power (NEP) is defined as the incident signal power required to obtain a signal-to-noise ratio of unity in a 1 Hz bandwidth. This derivation follows the methodology used in [Benford (1998)](http://example.com).

For blackbody radiation, the photon occupation number per mode is given by:

\[ n = \frac{1}{e^{\hbar\nu/kT} - 1} \]

and the variance in occupation number is:

\[ \langle (\Delta n)^2 \rangle = n(1 + n) \]

For a practical detector implementation, the mean square fluctuation of photons detected per mode is given by:

\[ \langle (\Delta n)^2 \rangle = n(1 + \eta(\nu)n) \]

where \( \eta(\nu) \) is the overall detector efficiency.

The spectral density of the detected power is obtained by multiplying this expression by the number of photon modes, \( N \), and the energy per photon, \( h\nu \).

\[ NEP^2 = \frac{2N\eta(\nu)(h\nu)^2}{(e^x - 1)} \left( 1 + \frac{\eta(\nu)}{e^x - 1} \right) \]

where we have made the substitution \( x = \hbar\nu/kT \).

To obtain the total NEP we must integrate this expression over the detector frequency band:
\[ \text{NEP}^2 = \frac{1}{\tau} \int \frac{2N\eta(\nu)(h\nu)^2d\nu}{(e^x - 1)} \left(1 + \frac{\eta(\nu)}{(e^x - 1)}\right) \]

where \( \tau \) is the integration time of the detector measurement.

The number of photon modes is given by \( N = \frac{4\Omega}{A^2} \), where \( A \) is the area of the telescope and \( \Omega \) is the solid angle of the detector beam. In the diffraction limited case, \( \lambda \simeq \sqrt{A\Omega} \) so \( N \simeq 1 \). For a narrowband filter, where \( \Delta\nu \ll \nu \), we can approximate integrals so that this expression becomes:

\[ \text{NEP}^2 = \frac{\eta(h\nu)^2\Delta\nu}{\tau(e^x - 1)} \left(1 + \frac{\eta}{(e^x - 1)}\right) \]

giving us a final formula for the CMB photon noise power:

\[ \text{NEP} = h\nu \sqrt{\frac{\Delta\nu(n^2 + n)}{\tau}} \quad \text{(B1)} \]

where \( n = \eta/(e^{h\nu/kT} - 1) \).

Applying the appropriate values for the Planck satellite: \( T_{CMB} = 2.726 \text{K} \), frequency = 100 GHz, bandwidth = 30\%, detector efficiency \( \eta = 60\% \), gives a \( \text{NEP} \approx 1 \times 10^{-17} \text{W/}\sqrt{\text{Hz}} \). NEP and CMB photon flux as a function of frequency are plotted in Figure 3. For frequencies higher than the Rayleigh-Jeans limit, given by \( \nu = kT_{CMB}/h \), it can be seen that the approximation \( \text{NEP} \approx h\nu \sqrt{\frac{n^2 + n}{\tau}} \) is valid.

**B.2. Thermodynamic units**

In the field of CMB astronomy, it is conventional to use thermodynamic units in which variations in CMB brightness are expressed as temperature variations with respect to the CMB background level. To derive an expression for photon noise in thermodynamic units we start by differentiating the equation for the mean energy per mode (A2):

\[ \bar{E} = \frac{h\nu}{e^{h\nu/kT} - 1} \]

Making the substitution \( x = h\nu/kT \), and differentiating with respect to \( T \) gives:

\[ \frac{d\bar{E}}{dT} = \frac{d}{dx} \frac{h\nu}{e^x - 1} \cdot \frac{dx}{dT} \]


Applying this factor to the equation for NEP (31) we obtain an expression for CMB photon noise in thermodynamic units:

\[
\frac{\Delta T}{T} = \frac{(x^e - 1)^2}{xe^x} \sqrt{\frac{n^2 + n}{\Delta \nu \tau}}
\]  

(B2)

where \( \Delta \nu \) is the bandwidth of the observation, \( x = h\nu/kT \) is the dimensionless frequency, \( \tau \) is the integration time, \( n = \eta/(e^x - 1) \) is the absorbed photon occupation number and \( \eta \) is the absorption efficiency.
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