Study of the resummation of chiral logarithms in the exponentiated expression for the pion form factor

Francisco Guerrero

Departament de Física Teòrica, Universitat de València and IFIC, CSIC - Universitat de València, C/ del Dr. Moliner 50, E-46100 Burjassot (València), Spain

Abstract

From the properties of analyticity and unitarity it has been recently obtained an exponentiated expression for the pion form factor. In this work I show the validity of this expression comparing its order $p^6$ term with the one exactly calculated in ChPT.

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1. Introduction

ChPT allows us to compute hadronic matrix elements at low energy, where QCD is not perturbative. The corresponding results are obtained organized as a power expansion in momenta and masses in a perturbative series, but the predictions are valid only near the threshold. Different ways of summing the series have been developed in order to extend the range of validity to higher energies.

In particular, this occurs in the case of the vectorial pion form factor. At the moment there at two calculations to the order $p^6$ (i.e. until the 2-loop contribution): one numerical [1] and one analytical [2]. With this results is possible to explain the experimental data from threshold until some 400 MeV in energy, where the effects of the $\rho$ resonance are already visible. To raise this maximum energy is necessary to include the $\rho$ resonance explicitly and to resum the series.

Being based in the properties of analyticity and unitarity some resummands exist in the literature. For instance, the Gounaris-Sakurai parametrization [3] starts from an extended effective range formula and imposing certain conditions on the phase shift obtains a propagator-like expression for the form factor that resums all the orders. Another example is that of the inverse amplitude method. It applies a dispersion relation to $F^{-1}$, calculated with ChPT, instead of applying it to $F$, the form factor. It works better because the imaginary part of $F^{-1}$ in the elastic case is a better approximation to data than just with $F$ [4, 5, 6].

In this letter I will deal with the exponentiated parametrization obtained in [7], where using Watson theorem [8], Vector Meson Dominance (VMD) and the Omnès equation [9] we were able to resum the contribution of the final state interaction of the pions into an exponential multiplied by the $\rho$ propagator.

To obtain that parametrization we started with the pion form factor calculated at order $p^4$ in ChPT plus the contribution from the $\rho$ exchange, obtained with an effective chiral theory including the resonances of the lightest vectorial octet. After using Vector Meson Dominance and avoiding to include twice the order $p^4$ contribution from the local terms in both ChPT calculation (i.e. the $L_9$ constant) and the $\rho$ propagator we got the expression (restricted to two flavours)

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2} A(m_\pi^2/s, m_\pi^2/M_\rho^2) \quad (1)$$

where

$$A(m_\pi^2/s, m_\pi^2/M_\rho^2) = \ln(m_\pi^2/M_\rho^2) + \frac{8m_\pi^2}{s} - \frac{5}{3} + \sigma^3 \ln\left(\frac{\sigma + 1}{\sigma - 1}\right) \quad (2)$$

with

$$\sigma = \sqrt{1 - 4m_\pi^2/s} \quad (3)$$

The first term sums the local terms to all orders whereas the second term gives the contribution of the final state interaction of the pions to order $p^4$. The next step
was to sum that pion interaction to all orders too. Using the Watson theorem \[8\] we obtained the imaginary part of the form factor and using the Omnès equation \[9\] we were able to obtain an exponentiated expression for the pion form factor. The needed phase shift used in the Omnès solution is the one coming from the $\pi\pi$ scattering at tree level in ChPT.

Matching the Omnès solution with the equation (2) we got the following expression

$$F(s) = \frac{M^2_\rho}{M^2_\rho - s} \exp \left\{ \frac{-s}{96\pi^2 f^2} A(m^2_\pi/s, m^2_\pi/M^2_\rho) \right\}$$

(4)

Finally we included the $\rho$ width in the parametrization. Calculating with the effective chiral theory with resonances mentioned above and making a Dyson summation we saw that the result obtained was equivalent to shift the imaginary part from the exponent in equation (4) to the denominator in the propagator. Making the shift we got our final result for the parametrization,

$$F(s) = \frac{M^2_\rho}{M^2_\rho - s - iM_\rho \Gamma_\rho(s)} \exp \left\{ \frac{-s}{96\pi^2 f^2} \text{Re} A(m^2_\pi/s, m^2_\pi/M^2_\rho) \right\}$$

(5)

where

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi f^2_\pi} \sigma^3 \theta(s - 4m^2_\pi)$$

(6)

This parametrization fits the experimental data perfectly up to 1 GeV for both the modulus squared and the phase shift of the pion form factor. The final expression has two different parts: the $\rho$ propagator, which sums the local terms to all orders in the low energy chiral expansion, and the exponential which sums the final state interaction between the two pions.

In this work I will check the usefulness of this parametrization. Since we started from the order $p^4$ form factor to obtain the exponential I will compare the $p^6$ contribution predicted by the exponential with the exact calculation in ChPT \[2\].

2. Order $p^6$ term in ChPT

In the exact and analytical result \[3\] of the vectorial pion form factor in ChPT with $SU(2)_L \times SU(2)_R$ the expression given by the authors is defined with the help of some functions.

Here I modify slightly the presentation in order to show clearer the comparison with the exponentiated parametrization. I will write only the $p^6$ term, that I will denote by $F^{(6)}_{ChPT}(s)$, as an expansion in powers of the logarithm

$$L(s) = \ln \frac{1 + \sigma}{1 - \sigma}$$

(7)
for values of $s$ above the $2\pi$ threshold.

I will have an expression of the following form

$$F_{\text{ChPT}}^{(6)}(s) = a_0 + a_1 L(s) + a_2 (L(s))^2 + a_3 (L(s))^3$$

(8)

It has to be remembered that the 1-loop calculation only contributes to the first two terms $a_0$ and $a_1$, therefore $a_2$ and $a_3$ are strictly coming from order $p^6$.

Taking the result from [2] the functions $a_i$ are, at 2-loop order, the following ones

$$a_0 = \left[ \frac{sm_\pi^2}{6(16\pi^2f^2)^2} T_1 + \frac{s^2}{(16\pi^2f^2)^2} T_2 + \left( \frac{m_\pi^2}{16\pi^2f^2} \right)^2 \left\{ \left( \ell_2 - \ell_1 + \frac{\ell_6}{2} \right) \frac{x^2}{27} (1 + 3\sigma^2) - \frac{x^2}{30} \frac{3191}{6480} x^2 + \frac{223}{216} x - \frac{16}{9} - \frac{\pi^2 x}{540} (37x + 15) + \frac{1}{54} (7x^2 - 151x + 99) - \frac{\pi^2}{72x} (x^3 - 30x^2 + 78x - 128) + 8\pi^2 (x^2 - \frac{13}{3} x - 2) \left( \frac{1}{192} - \frac{1}{32\pi^2} - \frac{1}{48x\sigma^2} \right) \right\} \right] + i \left[ \left( \frac{m_\pi^2}{16\pi^2f^2} \right)^2 \left\{ \left( \ell_2 - \ell_1 + \frac{\ell_6}{2} \right) \frac{\pi x^2 \sigma^3}{18} + \frac{\pi \sigma}{108} (7x^2 - 151x + 99) + 8\pi^2 (x^2 - \frac{13}{3} x - 2) \frac{1}{16\pi x\sigma} \right\} \right]$$

(9)

$$a_1 = \left( \frac{m_\pi^2}{16\pi^2f^2} \right)^2 \left\{ \left[ - \left( \ell_2 - \ell_1 + \frac{\ell_6}{2} \right) \frac{3 \ell_3}{2} \frac{x^2 \sigma^3}{18} - \frac{\sigma}{108} (7x^2 - 151x + 99) + 8\pi^2 \left( x^2 - \frac{13}{3} x - 2 \right) \right] \frac{2\pi^2 - 3x\sigma^2}{48\pi^2x^2\sigma^3} \right\} + i \left[ \left( \frac{-\pi}{36x\sigma^2} \right) (x^3 - 16x^2 + 120x - 476 + 512/x) \right]$$

(10)

$$a_2 = \left( \frac{m_\pi^2}{16\pi^2f^2} \right)^2 \left\{ \left[ \frac{1}{72x\sigma^2} (x^3 - 16x^2 + 120x - 476 + 512/x) \right] + i \left[ \frac{\pi}{2x^2\sigma^3} \right] \left( x^2 - \frac{13}{3} x - 2 \right) \right\}$$

(11)

$$a_3 = - \left( \frac{m_\pi^2}{16\pi^2f^2} \right)^2 \left[ \frac{1}{6x^2\sigma^3} \right]$$

(12)

Where $x = s/m_\pi^2$, and $\ell_i$ are the scale-independent coupling constants of $SU(2)_L \times SU(2)_R$ ChPT at order $p^4$ and $f_1$ and $f_2$ at order $p^6$. As it can be seen $a_3$ is real (its imaginary part appears at order $p^8$), whereas $a_0$, $a_1$ and $a_2$ are complex.
Another important characteristic is that these functions are divergent at threshold (i.e. \( \sigma = 0 \)). Obviously \( F_{\text{ChPT}}^{(6)}(4m^2_\pi) \) is not, since the divergencies from the functions, combined with the powers on \( \sigma \) appearing in the expansion of \( L(s) \) for small \( \sigma \), cancel each other. This divergences are originated in the loops exchanged in the u and t-channels. This implies that they are not expected to appear in the exponenciated resummation, since the latter sums only the final state interaction of the pions in the s-channel.

3. Order \( p^6 \) term in the exponential parametrization

Expanding in powers of momenta I get the order \( p^6 \) term given by the equation (5). Now, if I do the same as in the case of the ChPT calculation and I expand in powers of the logarithm \( L(s) \) in the form

\[
F^{(6)}_{\text{exp}}(s) = b_0 + b_1 L(s) + b_2 (L(s))^2 + b_3 (L(s))^3
\]

(13)

the functions, denoted this time by \( b_i \), are

\[
b_0 = \left[ \frac{s^2}{M^2_\rho} - \frac{\Gamma^2}{M^2_\rho} + \frac{1}{2} \frac{1}{(96\pi^2 f^2)^2} \left( s \ln \left( \frac{m^2_\pi}{M^2_\rho} \right) + 8m^2_\pi - \frac{5}{3} s \right) \right]^2
\]

\[
- \frac{s}{96\pi^2 f^2 M^2_\rho} \left( s \ln \left( \frac{m^2_\pi}{M^2_\rho} \right) + 8m^2_\pi - \frac{5}{3} s \right)
\]

\[
i \left[ \frac{2s^2 \sigma^3}{96\pi f^2 M^2_\rho} - \frac{\pi s^3}{(96\pi^2 f^2)^2} \left( s \ln \left( \frac{m^2_\pi}{M^2_\rho} \right) + 8m^2_\pi - \frac{5}{3} s \right) \right]
\]

(14)

\[
b_1 = \frac{s^2 \sigma^3}{(96\pi^2 f^2)^2} \left\{ \ln \left( \frac{m^2_\pi}{M^2_\rho} \right) + 8m^2_\pi - \frac{5}{3} s - \frac{96\pi^2 f^2}{M^2_\rho} \right\} - i\pi \sigma^3
\]

(15)

\[
b_2 = \frac{1}{2} \frac{s^2 \sigma^6}{(96\pi^2 f^2)^2}
\]

(16)

\[
b_3 = 0
\]

(17)

As it was expected, the values of the \( b_i \) functions are not divergent at threshold. Therefore the direct comparison between \( a_i \) and \( b_i \) has no sense for \( \sigma = 0 \). I will compare \( F^{(6)}_{\text{exp}}(4m^2_\pi) \) with \( F^{(6)}_{\text{ChPT}}(4m^2_\pi) \) because now both quantities are finite.

4. Comparison

Now that I have introduced the necessary formulae I can proceed to the comparison of both results.

In the way the functions \( a_i \) and \( b_i \) are written, that is, with the complete analytic expression, no similarity can be seen at first sight between them.
To understand better the physics behind these formulae is convenient to go to the chiral limit \( (m_\pi = 0) \) where the similarities become more evident. In this limit the \( a_i \) functions are finite at threshold so we can compare them directly with the \( b_i \) functions.

I begin with the highest power of the logarithm, \( a_3 \) and \( b_3 \). Here the chiral limit is easy

\[
\hat{a}_3 = \hat{b}_3 = 0
\]

The hat means that the quantity is in the chiral limit.

The coefficients for the second power of the logarithm are also equal in this limit.

\[
\hat{a}_2 = \hat{b}_2 = \frac{1}{72} \left( \frac{s}{16\pi^2 f^2} \right)^2 \]

(19)

This is an important result because it means that in the chiral limit the dominant logarithms to order \( p^6 \) are correctly resummed. This fact does not occur in resummations based on the \([0,1]\) Padé approximants like for instance those from the inverse amplitude method \([4, 5, 6]\) or the Gounaris-Sakurai parametrization \([3]\).

In the subleading term (i.e. the one linear in the logarithm) we have the first differences. The values for the functions are

\[
\hat{a}_1 = \left( \frac{s}{16\pi^2 f^2} \right)^2 \left[ - \frac{1}{18} \left( \frac{L_2}{L_1} + \frac{L_6}{2} \right) + \frac{7}{108} \right] - \frac{i\pi}{36}
\]

\[
\hat{b}_1 = \left( \frac{s}{16\pi^2 f^2} \right)^2 \left[ \frac{1}{36} \left\{ \ln \left( \frac{m_\pi^2}{M_\rho^2} \right) - \frac{5}{3} - \frac{96\pi^2 f^2}{M_\rho^2} \right\} - \frac{i\pi}{36} \right]
\]

(20)

In order to establish a good comparison I have to manipulate the real part of \( a_1 \), in particular the \( L_i \) constants. In \([2]\) we can find how to rewrite them in terms of the usual \( L_i \) from Gasser and Leutwyler \([10]\), and also in the latter we find how to pass to the \( SU(3)_L \times SU(3)_R \) constants denoted by \( L'_i \). The equivalence is

\[
\overline{L}_2 - \overline{L}_1 + \frac{\overline{L}_6}{2} = 96\pi^2 \left( 2\overline{L}'_2 - 4\overline{L}'_1 - 4\overline{L}'_3 + 2\overline{L}'_5 \right) - \frac{1}{2} \ln \left( \frac{m_\pi^2}{\mu^2} \right)
\]

(21)

Applying now Vector Meson Dominance in accordance with \([12]\) at the scale \( \mu^2 = M_\rho^2 \) (remember that to derive the exponentiated parametrization we used VMD) I obtain

\[
\overline{L}_2 - \overline{L}_1 + \frac{\overline{L}_6}{2} = \frac{120\pi^2 f^2}{M_\rho^2} - \frac{1}{2} \ln \left( \frac{m_\pi^2}{M_\rho^2} \right)
\]

(22)

In this way the real part of \( \hat{a}_1 \) now is, in the VMD approximation,
\[ \text{Re } \hat{a}_1 = \left( \frac{s}{16\pi^2 f^2} \right)^2 \frac{1}{36} \left\{ \ln \left( \frac{m^2_\pi}{M^2_\rho} \right) - \frac{7}{3} - \frac{240\pi^2 f^2}{M^2_\rho} \right\} \]  
(23)

The correct logarithm is reproduced, however there is a difference with \( \hat{b}_1 \)

\[ \hat{a}_1 - \hat{b}_1 = \left( \frac{s}{16\pi^2 f^2} \right)^2 \frac{1}{36} \left\{ \frac{-2}{3} - \frac{144\pi^2 f^2}{M^2_\rho} \right\} \]  
(24)

This difference comes from the contribution due to the exchange of one \( \rho \) in the t-channel in the interaction between the final pions. It has to be remembered that the final state interaction, resummed in the exponential, comes here from the tree level phase shift \( \delta^1_1(s) \) calculated from \( \pi \pi \) scattering. It would be necessary to include the 1-loop term in \( \delta^1_1(s) \) to obtain that contribution.

Finally I compare the polynomial terms. The chiral limits for \( a_0 \) and \( b_0 \) are

\[ \hat{a}_0 = \left( \frac{s}{16\pi^2 f^2} \right)^2 \left\{ \left[ \mathcal{F}_2 + \frac{4}{27} \left( \mathcal{F}_4 - \mathcal{F}_3 + \frac{7\pi}{2} \right) \right] - \frac{\mathcal{F}_4}{30} + \frac{2411}{6480} - \frac{11\pi^2}{270} \right\} \]

\[ i \left\{ \left( \mathcal{F}_2 - \mathcal{F}_3 + \frac{7\pi}{18} \right) \right\} \]

\[ \hat{b}_0 = s^2 \left[ \frac{4}{M^2_\rho} - \frac{1}{(96\pi^2 f^2)^2} + \frac{1}{2(96\pi^2 f^2)^2} \left( \ln \left( \frac{m^2_\pi}{M^2_\rho} \right) - \frac{5}{3} \right)^2 \right] \]

\[ -\frac{1}{96\pi^2 f^2 M^2_\rho} \left( \ln \left( \frac{m^2_\pi}{M^2_\rho} \right) - \frac{5}{3} \right) \]

\[ + is^2 \left[ \frac{1}{48\pi f^2 M^2_\rho} - \frac{\pi}{(96\pi^2 f^2)^2} \left( \ln \left( \frac{m^2_\pi}{M^2_\rho} \right) - \frac{5}{3} \right) \right] \]  
(25)

In \( \hat{a}_0 \) I have already taken \( \mathcal{F}_1 = 0 \) as it is suggested by all the authors. This choice is justified because the contribution to the electromagnetic radius of the pion from \( \mathcal{F}_1 \) is negligible, since this radius is saturated by the \( \rho \) resonance (i.e. the \( L_0 \) constant).

To establish numerical comparisons I will take the following experimental values for the \( \mathcal{F}_i \) constants \[ 3 \]

\[ \mathcal{F}_1 = -1.7 \pm 1.0 \]
\[ \mathcal{F}_2 = 6.1 \pm 0.5 \]
\[ \mathcal{F}_3 = 2.9 \pm 2.4 \]
\[ \mathcal{F}_4 = 4.3 \pm 0.9 \]
\[ \mathcal{F}_6 = 16.5 \pm 1.1 \]  
(27)
With these values I obtain

\[ \hat{a}_0 = \left[ (1.16 \pm 0.16 + 0.53 \overline{f}_2) + i(1.59 \pm 0.4) \right] s^2 \]
\[ \hat{b}_0 = (3.84 + 1.52 i) s^2 \] \hspace{1cm} (28)

As we can see the imaginary parts coincide. The real parts are equal for \( \overline{f}_2 = 5.1 \pm 0.3 \). This value agrees with previous estimations. Some of these estimations are \( \overline{f}_2 \sim 4.8 \), \( \overline{f}_2 = 6.9 \), \( \overline{f}_2 = 6.6 \), \( \overline{f}_2 = 3.7 \).

After all this collection of formulae it can be concluded that the exponentiated parametrization is, in the chiral limit, a good extrapolation for ChPT at higher energies. The prediction made for the order \( p^6 \) term is basically equal to the ChPT result (the order \( p^4 \) term is exact by construction). It is different only in the real part of \( a_1 \) (that is, since the functions are complex, in one of six), where it was expected due to the arguments given above.

Once the underlying physics has been seen, analyzing the chiral limit, we can study numerically the complete expressions (without any limit) for \( a_i \) and \( b_i \).

We have to remember that \( a_i \) is divergent at \( \sigma = 0 \), so in that point the comparison between ChPT and the exponential has to be done for the total sum at order \( p^6 \) and not term by term.

The values are

\[ F_{\text{ChPT}}^{(6)}(s = 4m_\pi^2) = 0.0227 \pm 0.0009 \]
\[ F_{\exp}^{(6)}(s = 4m_\pi^2) = 0.0216 \] \hspace{1cm} (29)

They are completely equivalent. For \( F_{\text{ChPT}}^{(6)} \) I have taken the constant \( \overline{f}_2 \) equal to the value obtained above.

When we increase the energy \( \sqrt{s} \) the difference between \( F_{\text{ChPT}}^{(6)} \) and \( F_{\exp}^{(6)} \) also increases. For \( \sqrt{s} \sim 0.7 \) GeV the real part in the exponentiated expression is only a 15% larger than that of ChPT, reaching 33% around 1 GeV. In the imaginary part the comparison is even better keeping the difference around the 3% for \( \sqrt{s} = 1 \) GeV.

However at so high energies the expansion in ChPT is not valid anymore. We have just to remember that around 0.7 GeV the order \( p^4 \) correction has the same value that the tree level one (order \( p^2 \)), and the same happens around 0.8 GeV between the orders \( p^4 \) and \( p^6 \).

The numerical study allows us to conclude that the most important piece at order \( p^6 \) is the polynomial, followed by the term linear in the logarithm, the quadratic and so on.

One delicate point of the exponentiated parametrization is the shift of the imaginary part from the exponent to the propagator \( [7] \). If we do not do the shift and keep the imaginary part in the exponent the terms with logarithms are not
modified at all (i.e. the functions $b_1, b_2$ and $b_3$ remain the same). However the $b_0$ function changes substantially in its imaginary part. In the chiral limit its value passes from $\text{Im} \hat{b}_0 = 1.52 s^2$ to $\text{Im} \hat{b}_0 = 0.88 s^2$, a difference of the 50%. Thus, higher-order corrections are more efficiently summed doing the shift, as expected from the Dyson summation of the $\rho$ self-energy.

5. Phase shift $\delta_1^I(s)$

There is also another test that we can do with the exponentiated parametrization and it has to do with the phase shift.

The resummation presented in [7] introduces a prediction for the phase shift which, as we saw then, fits the data perfectly.

Here I present this graphically reconstructing the pion form factor from the obtained phase shift

$$\delta_1^I(s) = \arctan \left( \frac{M_\rho \Gamma_\rho(s)}{M_\rho^2 - s} \right)$$

and using dispersion relations

$$F(s) = \exp \left\{ \sum_{k=1}^{n-1} \frac{[\ln F(0)]^{(k)} s^k}{k!} \right\} \exp \left\{ \frac{s^n}{\pi} \int \frac{dz}{z^n} \frac{\delta_1^I(z)}{z - s} \right\}$$

with

$$[\ln F(0)]^{(k)} = \left. \frac{d^{(k)} \ln (-iF(s)/2)}{ds^k} \right|_{s=0}$$

Its experimental values are

$$[\ln F(0)]^{(1)} = \frac{1}{6} \left\langle r_V^2 \right\rangle = 1.98 \text{ GeV}^{-2}$$

$$[\ln F(0)]^{(2)} = 2c_V - \left( \frac{1}{6} \left\langle r_V^2 \right\rangle \right)^2 = 4.13 \text{ GeV}^{-4}$$

The result shown in the figure is the numerical solution for the dispersion integral, eq. (31), with one, two and three subtractions (respectively the lower, the upper and the middle curves).

The fit obtained for the data is pretty good, and improves with the number of subtractions. This is due to the fact that increasing the number of subtractions the contribution from lower energies (the better understood region) becomes more and more important.

This plot shows that the prediction given for the phase shift by the exponential parametrization is correct.
6. Conclusion

In [7] we obtained an exponentiated expression for the pion form factor based on its properties of analyticity and unitarity.

We started there from the tree level phase shift $\delta_1$ and, applying a dispersion relation with subtractions, we obtained the Omnès solution in form of an exponential. Its expansion in powers of momentum gives the form factor to order $p^4$ exactly in ChPT, as it had to be, because the complex phase of the form factor is the same as the one of the $\pi\pi$ scattering amplitude, i.e. $\delta_1(s)$. If the latter is to order $p^2$ then its dispersion integral has to give the form factor to order $p^4$.

However, the exponentiated parametrization contains all the orders in powers of momenta. In particular it contains the order $p^6$ term. We can take advantage of this to compare this predicted term with the one coming from the already existing calculation of the pion form factor at two loops in ChPT. In this work I have shown that comparison to study the validity of the exponentiated parametrization.

In the chiral limit I have proved qualitatively that the resummation is correct. It reproduces the correct factor for the dominant logarithm and a value compatible with those in the literature for $f_2$.

Numerically we observe that both contributions (ChPT and exponential) at order $p^6$ differ only in a few per cent in the real parts and less than one per cent in the imaginary ones, for the region of energies where the expansion of ChPT has sense ($\sqrt{s} < 500$ MeV). This indicates that the parametrization contains the most relevant contributions at higher orders. The ones not included (loops and resonances in t and u-channels) are not quantitatively so important.

The difference observed in the functions $a_1$ and $b_1$ between ChPT and the exponential can be corrected introducing $\delta_1(s)$ to order $p^4$. In that case the correspondence between $F_{\text{ChPT}}^{(6)}$ and $F_{\text{exp}}^{(6)}$ would be exact.

The study done in this work suggests that the resummation obtained to that order would be equally useful, and it would give a prediction for the order $p^8$ where no exact calculation in ChPT has been done. Remember that to calculate the pion form factor in ChPT to order $p^8$ is not possible for the moment because of its complexity and the unknowledgement of the many constants appearing in the order $p^6$ and $p^8$ terms in the lagrangian of ChPT.

We can finally conclude from this study that the exponentiated parametrization gives a resummation correctly defined that includes the most relevant pieces at higher orders.

Future calculations should work out the resummation with the order $p^4$ phase shift. It would be also worthwhile to try to apply this parametrization to other channels.

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