O.V. Selyugin · O.V. Teryaev

Gravitational Potential with Extra-dimensions and Spin Effects In Hadronic Reactions

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Abstract The impact of the KK-modes in d-brane models of gravity with large compactification radii and TeV-scale quantum gravity on the hadronic potential at small impact parameters is examined. The effects of the gravitational hadronic formfactors obtained from the hadronic generalized parton distributions (GPDs) on the behavior of the gravitational potential and the possible spin correlation effects are also analysed.

Keywords high energy · hadron · gravitation · n-dimensional · spin correlation

1 Introduction

Standard Regge representation shows that the energy dependence of the scattering amplitude depends on the spin of the exchanged t-channel particles. So the exchange of a graviton of spin 2 leads to a growth of the scattering
amplitude with energy proportional to $s$ [1]. As the usual gravitational interaction is very small, this growth can be seen at the Plank scale. However, a modern development of the fundamental theory, introduced long ago [2, 3], is connected with the fruitful idea [4] that spacetime has a dimension higher than $D = 4$ manifested already at TeV scale [5]. Now there are many different paths in the development of these ideas [6]. A number of studies of higher-dimensional (Kaluza-Klein) field theories were carried out [7,8,9,10]. In a modern context, the Kaluza-Klein (KK) theories arise naturally from (super)string theories in the limit where the relevant energies $E$ are much smaller than the string mass scale $M_s \sim (\alpha')^{-1/2}$, $\alpha'$ being the slope parameter. Since field theories of gravity behave badly in the ultraviolet limit, Kaluza-Klein formulations should in general be regarded as effective actions, with an implicit or explicit ultraviolet cutoff $\Lambda$ [11]. As a first approximation, we may suppose that all Standard Model fields are confined to a four-dimensional brane world-volume. In the Arkani-Hamed, Dimopoulos and Dvali approach (ADD) [12] a large number $d$ of extra dimensions is responsible for a lower Planck scale, down to a TeV and only the graviton propagates in the $4 + d$ dimensions. This propagation manifests itself in the standard 4 dimensions as a tower of massive KK-modes. The effective coupling is obtained after summing over all the KK modes and, due to the high multiplicity of the KK modes, the effective interaction has strength $1/M_s$ [13,14]. Setting $M_{4+d}^2 = (2\pi)^d M_4^{d+2}$, as in Ref. [12] (motivated by toroidal compactification, in which the volume of the compactified space is $V_d = (2\pi r_d)^d$) and applying Gauss’s law at $r << r_d$ and $r >> r_d$, one finds that $M_{Pl}^2 = r_d^d M_{4+d}^{2+d}$, so that

$$r_d = \left(\frac{M_{Pl}}{M_{4+d}}\right)^{2/d} / M_{4+d}.$$

In the higher-dimensional models with a warped extra dimension [10], the first KK mode of the graviton can have a mass of the order of 1 TeV and the coupling with matter on the visible brane is of the order $1 \text{TeV}^{-1}$. There are also "intermediate" models with a small warp which consider the brane as almost flat. Such models remove some cosmological bounds on the number
of additional dimensions. All these models provide some experimental possibilities to check (or discover) the impact of the extra dimensions on our 4-dimensional world. Now in many papers new effects are examined which in principle can be seen at future colliders.

In this work, we show that these effects can also be discovered in experiments on elastic polarized hadron scattering. We explore in this case the sensitivity of interference spin effects to small corrections to the scattering amplitude, linear rather than quadratic (as in the case of cross sections) functions of a small parameter. We show that there will be some additional growth of the analyzing power \( A_N \) which is connected with an additional term in the real part of the scattering amplitude, due to the graviton part of the hadron interaction. We will focus on 2 extra dimensions, as different models give for that case coinciding results and as the TeV-scale mass does not contradict existing accelerator and cosmological bounds.

2 The graviton contribution with KK-modes

Assuming that the higher-dimensional theory at short distances is a string theory, one expects that the fundamental string scale \( M_s \) and the Planck mass \( M_{4+d} \) are not too different (a perturbative expectation is that \( M_s \sim g_s M_{4+d} \)). As of now, only known framework that allows a self-consistent description of quantum gravity is string theory [15]. Thus, a compactification radius \( r_d << M_{Pl}^{-1} \) corresponds to a short-distance Planck scale and string mass \( M_s \) which are \( << M_{Pl} \). So, we apply constrain on the quantum gravity scale \( M_D \) to the string scale \( M_s \). There are different estimations of the mass \( M_s \) from the particle reactions and the cosmic data. The particle data lead to the low bound of the mass \( M_s > 0.5 \) GeV, see for example [16][17]. The cosmic data gave significantly larger bound \( M_s > 100 \) TeV [18]. However, these estimations heavily depend on the model approaches. The large bounds were obtained at the tree level. If loop corrections are taken into account, the bound significantly decreases [19]. So, as in other works, we take the upper bound of the integrals proportional to \( M_D = M_{4+d} \) of an order of 1 TeV. Following [13][14], the amplitude taking into account the KK-modes can be
The profile function of proton-proton scattering at $\sqrt{s} = 4$ TeV (hard line - (5) and dashed line - (5) with correction term equal zero). b) The graviton potential at small $r$ for $d = 2$ (hard line - our calculations, dashed line $V(r) \sim 1/r^3$).

written as

$$A_{grav.} \sim \int_0^{M_D^d} \frac{d^{d-1}q_T}{q^2 + q_T^2}$$

$$= \frac{M_D^d}{d} q^2 F_1[1, d/2, 1 + d/2, -M_D^2 m^2/q^2]) \quad (1)$$

The hypergeometric function $2 F_1$ has a smooth behavior but the upper limit of the integral appears as a multiplicative coefficient $M_D^d$ which leads to a divergence of the Born amplitude if $M_D^d \to \infty$ and $2 \leq d$.

$$\tilde{A}_{Born grav.} = \frac{\pi s^2}{M_D^{d+2}} \ln \left(1 + \frac{M_D^2}{q^2}\right) \quad (2)$$

In this work, we restrict our consideration only to the case $d = 2$. If our particles live on the 3-dimensional brane, we can obtain the amplitude in an impact parameter representation, where the Born amplitude corresponds to the eikonal of the scattering amplitude [20]

$$\chi(s, b) = \frac{1}{2\pi} \int_0^\infty b J_0(qb) A_{\text{Born}}(q^2) \, db. \quad (3)$$

The potential of the two body interaction corresponding to this eikonal can be obtained immediately:

$$V(r) = -\frac{2}{\pi r} \frac{d}{dr} \int_r^\infty \frac{b \chi(b)}{\sqrt{b^2 - r^2}} \, db. \quad (4)$$
An exact calculation of (3) for \(d = 2\) gives (see fig.1a)

\[
\chi(b) = \frac{s}{2M_D^2} (1 - b M_D K_1(b M_D))/b^2. \tag{5}
\]

Some detailed calculations can be found in our work [21]. It is interesting to note that when one takes the \(KK\) modes (see fig. 1b) into account, one obtains the gravitational potential

\[
V(r) \sim \frac{1}{r^3} (1 - e^{-M_D r} - M_D r e^{-M_D r}) \tag{6}
\]

### 3 Nucleon-graviton interaction

First of all, we need to examine the gravitational interactions of point particles. There are several points of view on this. Some authors suppose that the interaction will be the same for different particles, and independent of their structure. At the same time, the equivalence principle [22] requires that the nucleon-graviton interactions are described by the matrix elements of energy momentum tensors, related to moments of Generalized Parton Distributions (GPDs) [23][24].

In [25], the electromagnetic and gravitational form factors were calculated for the electron. It was shown that both form factors have practically the same form. Of course, the proton has a complicated structure and there is no evidence for the equality of the electromagnetic and hadronic form factors. In some papers they have a different dependence on the momentum transferred to the hadron form factors. Based on the standard form of the parton distribution functions with a the specific \(t\) dependence, we obtained a good description of the electromagnetic form factors of the proton and neutron [26][27]. On this basis we obtained the gravitational form factors \(A(t)\) and \(B(t)\) which correspond to GPDs \(H(t, x)\) and \(E(t, x)\). It can be shown that in the first approximation the form factor \(A(t)\) can be taken in the standard dipole form

\[
G(t) = 1/(1 - t/A^2)^2, \tag{7}
\]
Fig. 2 a) [left] Gravi-potential without (hard line) and with (dashed line) taking into account the hadron form factor $A(t)$. b) [right] Behaviour of the eikonal over $b$ corresponds to the Born amplitude with $d = 2$ ($1/b^2$ - hard line; with gravitational hadron form factors - dashed line).

with $A^2 = 1.8 \text{ GeV}^2$. In this case, our Born gravitational amplitude for $d = 2$ will be

$$\tilde{A}_{\text{Born grav.}} = \frac{\pi s}{M_B^2} \ln \left(1 + \frac{M_B^2}{q^2}\right) G^2(t)$$

(8)

The corresponding gravitational potential is

$$\tilde{V}_{\text{Born grav.}}(r) \sim \frac{1}{r^3} \left(1 - (1 + \frac{A r}{20} (20 + A r (9 + A r))) e^{-A r}\right).$$

(9)

This leads to a change of the gravitational potential at small distances. For $d = 0$, the gravitational potential between two protons with taking into account the gravitational form factor of the hadrons leads to the very different potential at distances of the order of the size of the hadron. This behavior has no divergence at $r = 0$.

For $d = 2$, the gravitational potential corresponding to the Born amplitude is shown on Fig 2a. In this case, the form factors modify the amplitude at large distances, up to 20 times the size of the proton. The decrease in the eikonal at small impact parameter is also large. This reflects the different behavior of the eikonal at small impact parameters. The eikonal for point particles has the form $1/b^2$ for $d = 2$. The corresponding eikonal, including the hadron form factor but dropping the 1 in the logarithm is

$$\chi_{\text{grav.}}(b) \sim \int_0^\infty q J_0(qb) \ln \left(\frac{M_B^2}{q^2}\right) G^2(t)$$
\[
\mathcal{X}_{\text{grav}}(b) = \frac{s}{M_D^4} \left[ \frac{\ln(M_D^2)A b^3}{1.3855 \times 48} K_3(A b)(1 + \frac{1}{2.2 + 10b^4}) + \frac{0.5}{1 + b^{1.5}} \right].
\]

Using this representation for the eikonal we can numerically calculate the full graviton amplitude in the tree approximation and calculate the corresponding spin-correlation parameter \(A_N\).

4 The spin correlation parameter \(A_N\)

The differential cross section and the analyzing power \(A_N\) are defined as follows:

\[
\frac{d\sigma}{dt} = \frac{2\pi}{s^2} (|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2),
\]

\[
A_N \frac{d\sigma}{dt} = -\frac{4\pi}{s^2} \text{Im}[\phi_1 + \phi_2 + \phi_3 - \phi_4 \phi_5^*],
\]

in terms of the usual helicity amplitudes \(\phi_i\). These amplitudes can be written as

\[
\phi_i(s, t) = \phi_i^h(s, t) + \phi_i^{em}(t) \exp[i\alpha_{em}\varphi_{\text{cn}}(s, t)],
\]

where \(\alpha_{em} = 1/137\) is the fine structure constant, \(\phi_i^h(s, t)\) describes the strong interaction of hadrons, \(\phi_i^{em}(t)\) their electromagnetic interaction, and \(\varphi_{\text{cn}}(s, t)\) is the electromagnetic-strong interference phase factor. This Coulomb-nuclear phase was calculated in the entire diffraction domain taking into account the form factors of the nucleons [28]. In this work, we define the hadronic and electromagnetic spin-non-flip amplitudes as

\[
F_{\text{hf}}^h(s, t) = \frac{1}{2s} \left[ \phi_1^h(s, t) + \phi_3^h(s, t) \right]; \quad F_{\text{hf}}^{em}(s, t) = \frac{1}{2s} \left[ \phi_1^{em}(s, t) + \phi_3^{em}(s, t) \right],
\]

and the spin-flip amplitudes as

\[
F_{\text{sf}}^h(s, t) = \frac{1}{2s} \phi_5^h(s, t); \quad F_{\text{sf}}^{em}(s, t) = \frac{1}{2s} \phi_5^{em}(s, t).
\]
Equation (13) was applied at high energy and at small angles. To take into account the unitarization effects, we used the standard eikonal representation for the spin-non-flip parts of the scattering amplitude [29]. The phase $\chi(s,b)$ is connected to the interaction quasi-potential which can have real and imaginary parts

$$\chi(s,b) = F_{\text{Born}}(s,b) \approx \frac{1}{k} \int \hat{V} \left( \sqrt{b^2 + z^2} \right) dz.$$  (17)

We have for the spin-non-flip

$$F_{nf}^b(s,t) = i \int_0^\infty b J_0(bq) \left[ 1 - e^{\chi(s,b)} \right] db.$$  (18)

According to the standard opinion, the hadron spin-flip amplitude is connected with quark exchange between the scattering hadrons, and at large energy and small angles it can be neglected. Some models, which take into account non-perturbation effects, lead to a non-vanishing hadron spin-flip amplitude [30,31,32,33]. Another complicated question is related to the difference between the phases of the spin-non-flip amplitude and of the spin-flip one. To estimate the possible graviton effect let us take the hadron-spin-flip amplitude to be proportional to the gravitational form factor $B(t)$ which is related to the GPD $E(t,x)$ [34]:

$$F_{sf}^b(s,t) = iK(s)B(t),$$  (19)

where $K(s) = 5.10^{-4}(ln(s) - i\pi/2)$.

Elastic nucleon scattering can occur in the region of $t$ after the second diffraction maximum but still at small angles ($t/s << 1$). The analysis of the differential cross sections at ISR energies has revealed scaling properties [35]: the differential cross section can be described in a simple form proportional to the electromagnetic form factor in the standard dipole form with $\Lambda = 0.71$ GeV which leads to the correct asymptotic behavior of the scattering amplitude $\sim 1/t^4$. In this case, the eikonal will be represented as $\Lambda^5b^3K_3(b,\Lambda)$. The corresponding interaction constant is chosen to obtain the measured differential cross sections at $\sqrt{s} = 52.8$ GeV and $-t = 10$ GeV$^2$. Taking into account an additional contribution from the graviton amplitude, we obtain
for the spin-non-flip amplitude

\[ \chi^{nf}(s, t) = \chi_h^{nf}(s, t) + \chi_{\text{grav}}(s, t). \]  

(20)

The additional contribution of the graviton changes the real part of the spin-non-flip amplitude. Hence, a difference appears between the spin-flip and the spin-non-flip phases. So that the analyzing power differs from zero (see Fig. 3). Of course, there is the question of the size of the mass cut \( M_D \). The astronomical data lead to the maximum bound \( M_D \geq 1500 \text{ TeV} \). However, the particle physics data lead to the possible size \( M_D \sim 1 \text{ TeV} \) (see the corresponding discussion [36,37,9]). So we will examine the possible polarization effects for \( M_D \) in the region of a few TeVs. Of course, larger \( M_D \) require interactions at larger energy.

Figure 4 displays \( A_N \) calculated for two values of \( M_D \), 1.5 TeV and \( M_D = 2 \text{ TeV} \). Despite the fact that the measurement of the elastic cross section at such values of the momentum transfer is not a simple task, we can see that the size of the effect may be sufficient for a discovery. The dashed line in this figure shows the analyzing power without gravitation interaction. A slight difference of this curve from zero is due to the small contribution of the electromagnetic amplitude. The main characteristic of the analyzing power
is its $t$ dependence. Such strong $t$ dependence will be present in all cases. The form of $A_{N}^{gr}$ is dictated by the form of the graviton amplitude, and in the cases when the number of additional dimensions exceeds two ($d > 2$), such a behavior will be more pronounced.

5 Conclusion

Our calculations of the impact of two additional dimensions on the gravitational potential at small distances show that it differs from a simple power at $r \leq 3 \text{ GeV}^{-1}$ and it changes the profile function of proton-proton scattering. Including the effects of Kaluza-Klein modes of graviton scattering amplitudes, with two extra dimensions and taking into account the gravitational form factor, which we calculated from the GPDs of the nucleons, it was shown that the impact-parameter dependence of the gravitational eikonal strongly deviates from the standard $1/b^2$ dependence. This is the main result of our paper. We think that this effect has to be taken into account in the calculation of the production of Black Holes at super-high energy accelerators.

We have shown that the gravitational interaction additional dimensions and the possible small spin-flip amplitude, proportional to the gravitational form factor $B(t)$, lead to large spin correlation effects at small angles and
\[ -t \sim 10 - 30 \text{ GeV}^2 \]. However, the inclusion of the gravitational form factors \( A(b) \) decreases this effect and drastically changes its form.

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