Nuclear matter properties at finite temperatures from effective interactions

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(Dated: April 23, 2019)

We study if commonly used nucleon-nucleon effective interactions, obtained from fitting the properties of cold nuclear matter and of finite nuclei, can properly describe the hot dense nuclear matter produced in intermediate-energy heavy-ion collisions. Using two representative effective interactions, i.e., an improved isospin- and momentum-dependent interaction with its isovector part calibrated by the results from the ab initio non-perturbative self-consistent Green’s function (SCGF) approach with chiral forces, and a Skyrme-type interaction fitted to the equation of state of cold nuclear matter from chiral effective many-body perturbation theory and the binding energy of finite nuclei, we evaluate in the mean-field approximation the equation of state and the single-nucleon potential for nuclear matter at finite temperatures and compare them to those from the SCGF approach. We find that the former reproduces reasonably well the SCGF results due to its weaker momentum dependence of the mean-field potential. Our study thus indicates that effective interactions with the correct momentum dependence of the mean-field potential can properly describe the properties of hot dense nuclear matter and are thus suitable for use in transport models to study heavy-ion collisions at intermediate energies.

I. INTRODUCTION

One of the main motivations for pursuing experiments on heavy-ion collisions at intermediate energies is to study the equation of state (EOS) of nuclear matter. Its knowledge is in fact essential for understanding the properties of systems ranging from finite nuclei [1, 2] to neutron stars [3, 4] as well as the gravitational-wave signal from neutron star mergers [5–7]. Because of the complexities of heavy-ion collision dynamics, transport models have been indispensable tools to extract the information on the nuclear EOS, particularly at high densities that exist during the early stage of the collisions, from various observables measured in experiments [8–11]. In transport models, which are based on either the Boltzmann-Uehling-Uhlenbeck equation [12] or the quantum molecular dynamics [13], the time evolution of nucleon phase-space distribution functions in a heavy-ion collision is determined by both the mean-field potential acting on nucleons and their scatterings. The nucleon mean-field potential is usually obtained from nucleon-nucleon (NN) effective interactions that are constructed from fitting the properties of cold nuclear matter and of finite nuclei. Thus, the mean-field potential does not include explicitly the temperature effect on the NN effective interactions in the nuclear medium, which is needed to describe the hot nuclear matter produced in heavy-ion collisions. As a result, the mean-field potential extracted from comparing results of transport models with the experimental data is not exactly that of cold nuclear matter. Using this mean-field potential as well as the resulting EOS to describe the properties of neutron stars can therefore lead to possible misleading conclusions.

The above problem can be alleviated if the mean-field potential obtained from NN effective interactions and used in transport models can also fit the finite temperature single-nucleon potential obtained from microscopic calculations, such as that based on the self-consistent Green’s function (SCGF) approach [14] or the many-body perturbation theory [15–16] employing chiral nuclear forces [17–18]. In the present study, we choose NN effective interactions that correspond to two energy-density functionals based on Hartree-Fock calculations. One is obtained from an improved isospin- and momentum-dependent interaction (ImMDI) model [19–20], which is constructed from fitting cold nuclear matter properties at saturation density and the empirical nuclear optical potential. The other is the Skyrme-Hartree-Fock (SHF) model [2, 21, 22] using the Skym* force, which is constructed from fitting the properties of cold nuclear matter from chiral effective many-body perturbation theory (χEMBPT) and the binding energies of finite nuclei [23]. The properties of cold neutron matter from the SCGF approach were further used to constrain the isovector part of the ImMDI model, and the new parametrization of this effective interaction is dubbed as ImMDI-GF. Using these two effective interactions, which have been used in transport models to study heavy-ion

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collisions at intermediate energies [24, 25], we then evaluate in the non-relativistic mean-field approximation the properties of symmetric nuclear matter (SNM) and pure neutron matter (PNM) at finite temperatures. These results are then compared with those from the SCGF approach [14, 26] and the χEMBPT approach [13, 16] within their theoretical uncertainties, which are mainly due to the variation of the high-momentum cutoff in nuclear interactions and the three-body forces included in these microscopic studies.

The remaining part of the paper is organized as follows. Section II gives the details on the theoretical framework for the ImMDI model and the SHF model as well as the SCGF approach. In Sec. III we compare and discuss the results for the occupation probabilities, the EOSs, and the mean-field potentials for SNM and PNM obtained from these different approaches. Finally, a summary is given in Sec. IV.

II. THEORETICAL FRAMEWORK

A. Effective interactions in Hartree-Fock calculations

The effective interaction between two nucleons at coordinates \( \vec{r}_1 \) and \( \vec{r}_2 \) in the ImMDI model includes a zero-range density-dependent term and a Yukawa-type finite-range term [27], i.e.,

\[
v_{\text{ImMDI}}(\vec{r}_1, \vec{r}_2) = \frac{1}{6}t_3(1 + x_3 P_r) \rho \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right) \delta(\vec{r}_1 - \vec{r}_2) + (W + BP_\sigma - HP_\tau - MP_\alpha P_r) \frac{e^{-\rho|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|},
\]

where \( \rho \) is the nucleon number density, \( P_r \) and \( P_\tau \) are the spin and the isospin exchange operators, respectively, and \( t_3, x_3, \alpha, W, B, H, M, \) and \( \mu \) are parameters.

The standard Skyrme interaction [21] without the spin-orbit coupling has the form of

\[
v_{\text{SHF}}(\vec{r}_1, \vec{r}_2) = t_0(1 + x_0 P_\sigma) \delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2}t_1(1 + x_1 P_\sigma) |\vec{k}|^2 \delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2) |\vec{k}|^2 \] 
\[+ t_2(1 + x_2 P_\sigma) \vec{k} \cdot \delta(\vec{r}_1 - \vec{r}_2) \vec{k} + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^3 \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right) \delta(\vec{r}_1 - \vec{r}_2), \]

where \( \vec{k} = \frac{1}{2}(\nabla_1 - \nabla_2) \) is the relative momentum operator acting on the right-hand side, \( \vec{k}^\dagger \) is the complex conjugate of \( \vec{k} \) acting on the left-hand side, and \( t_0, t_1, x_1, t_2, x_2, t_3, x_3, \) and \( \alpha \) are parameters.

In the Hartree-Fock approach, the total potential energy of nuclear matter is calculated according to

\[
E_p = \frac{1}{2} \sum_{ij} |i(j) > < i|v(1 - P_r P_\sigma P_\tau)|i(j) >, \tag{3}
\]

where \( P_r \) is the space exchange operator, \( |i(j) > \) is the quantum state of \( i(j) \)th nucleon, and \( v \) is the NN effective interaction.

The potential energy density from the ImMDI model is then given by [20]

\[
V_{\text{ImMDI}} = \frac{A_u \rho_n \rho_p}{\rho_0} + \frac{A_t}{2 \rho_0}(\rho_n^2 + \rho_p^2) + \frac{B}{\sigma + 1} \frac{\rho^\sigma + 1}{\rho_0^2} - \frac{1}{2} x \delta^2 \sum_{q,q'} C_{q,q'}^2 \rho_q \rho_{q'} \frac{1}{(\vec{p} - \vec{p}')^2} + \frac{1}{16 \pi \rho_0^2} \int d^3p d^3p' \int \int \frac{f_q(\vec{r}, \vec{p}) f_{q'}(\vec{r}, \vec{p}')}{(\vec{p} - \vec{p}')^2} \dfrac{\rho_q \rho_{q'}}{\Lambda^2},
\]

where \( \rho_n \) and \( \rho_p \) are the neutron and proton number densities, respectively, \( \rho_0 = 0.16 \text{ fm}^{-3} \) is a constant density, \( \delta = (\rho_n - \rho_p)/\rho \) is the isospin asymmetry of the nuclear matter with \( \rho = \rho_n + \rho_p \), and \( f_q(\vec{r}, \vec{p}) \) is the nucleon phase-space distribution function from the Wigner transformation of its density matrix with \( q = 1 \) for neutrons and \( -1 \) for protons. For the detailed derivation of above expression, we refer the reader to Ref. [27], where the relation between values of the parameter sets \((t_3, x_3, \alpha, W, B, H, M, \alpha)\) and \((A_u, A_t, B, C_{q-q}, C_{q-q}, \Lambda, \sigma, x)\) can be found. In Ref. [20], an optimized parameter set \((A_0, B, C_U, C_V, \Lambda, \sigma, x, y)\) was introduced by using following relations

\[
A_t(x, y) = A_0 + y + x \frac{2B}{\sigma + 1}, \tag{5}
\]
\[
A_u(x, y) = A_0 - y - x \frac{2B}{\sigma + 1}, \tag{6}
\]
\[
C_{q-q}(y) = C_{10} - 2(y - 2z) \frac{p_{f0}^2}{\Lambda^2 \ln[(4p_{f0}^2 + \Lambda^2)/\Lambda^2]}, \tag{7}
\]
\[
C_{q-q}(y) = C_{10} + 2(y - 2z) \frac{p_{f0}^2}{\Lambda^2 \ln[(4p_{f0}^2 + \Lambda^2)/\Lambda^2]}, \tag{8}
\]

where \( p_{f0} = h(3\pi^2 \rho_0/2)^{1/3} \) is the nucleon Fermi momentum in SNM at \( \rho_0 \). The number of independent parameters in the new set is the same as before. The parameters \( x, y, \) and \( z \) then characterize the slope parameter of the symmetry energy, the momentum dependence of the symmetry potential, and the symmetry energy at \( \rho_0 \), respectively.

The potential energy density in uniform nuclear matter for the SHF model is given by

\[
V_{\text{SHF}} = t_0[(2 + x_0)\rho^2 - (2x_0 + 1)(\rho_n^2 + \rho_p^2)]/4
\]
\[
+ [t_1(2 + x_1) + t_2(2 + x_2)]\tau_{\rho}/8
\]
\[
+ [t_2(2x_2 + 1) - t_1(2x_1 + 1)](\tau_{\rho} \rho_n + \tau_{\rho} \rho_p)/8
\]
\[
+ t_3\rho^3[(2 + x_3)\rho^2 - (2x_3 + 1)(\rho_n^2 + \rho_p^2)]/24. \tag{9}
\]
where \( \tau_q = \int p^2 f_q(\vec{r}, \vec{p}) d^3p/(2\pi)^3 \) is the kinetic density.

Through the variational principle, the mean-field potential for a nucleon with momentum \( \vec{p} \) and isospin \( q \) in the asymmetric nuclear matter of isospin asymmetry \( \delta \) and nucleon number density \( \rho \) from the ImMDI model can be expressed as [20]

\[
U_{\text{ImMDI}} = A_\sigma \frac{\rho - \rho_0}{\rho_0} + A_\tau \frac{\rho_\tau}{\rho_0} + B \frac{\rho - \rho_0}{\rho_0} (1 - x \delta^2) - 4q_x \frac{B}{\rho + 1} \frac{\rho^\sigma - 1}{\rho_0} \delta \rho - \tau
\]

\[
+ 2C_{\gamma q} \int d^3p' \frac{f_q(\vec{r}, \vec{p})}{1 + (\vec{p} - \vec{p'})^2/\Lambda^2}
\]

\[
+ 2C_{\gamma q} \frac{\rho_0}{\rho_0} \int d^3p' \frac{\tilde{f}_q(\vec{r}, \vec{p})}{1 + (\vec{p} - \vec{p'})^2/\Lambda^2}.
\]

Similarly, the mean-field potential in the standard SHF model can be expressed as

\[
U_{\text{SHF}} = \frac{\rho^2}{2m^*_q} - \frac{\rho^2}{2m} + t_0 (1 + \frac{x_0}{2}) \rho - t_0 (\frac{1}{2} + x_0) \rho_q
\]

\[
+ \frac{1}{4} \left[ t_1 (1 + \frac{x_1}{2}) + t_2 (1 + \frac{x_2}{2}) \right] \rho
\]

\[
- \frac{1}{4} \left[ t_1 (1 + x_1) - t_2 (1 + x_2) \right] \rho_q
\]

\[
+ \frac{1}{12} t_3 \rho^2 \left[ (2 + \alpha)(1 + \frac{x_3}{2}) \rho - (1 + 2x_3) \rho_q \right]
\]

\[
- \frac{\alpha (1 + x_3) \rho^2 + \rho^2}{\rho}.
\]

B. Green’s function approach using chiral forces

The SCGF method is a nonperturbative many-body approach based on the calculation of the dressed nucleon propagator, i.e., its Green’s function \( G \) [24]. The single-particle propagator provides access to microscopic properties of the many-body system, such as the nucleon spectral function or momentum distribution, and also to bulk thermodynamical quantities, such as internal energy, entropy, pressure, etc. Within this approach, the dressed propagator \( G \) is obtained via the iterative solution of the Dyson’s equation

\[
G(\mathbf{p}, \omega) = G_0(\mathbf{p}, \omega) + G_0(\mathbf{p}, \omega) \Sigma^*(\mathbf{p}, \omega) G(\mathbf{p}, \omega),
\]

where a nonperturbative self-energy \( \Sigma^*(\mathbf{p}, \omega) \) is employed, with \( \mathbf{p} \) and \( \omega \) being the single-particle momentum and energy. The self-energy is obtained within the so-called ladder approximation, where an infinite resummation of particle-particle and hole-hole intermediate states is considered. Hence the method is nonperturbative and self-consistent, providing a fully correlated description of the many-body system beyond the mean-field level [30]. In recent years the SCGF approach has been extended to consistently include two- and three-body forces [31]. Within this improved approach, the energy per nucleon can be obtained via an extended energy sum rule that reads [31]:

\[
E/A = \frac{\nu}{\rho} \int \frac{dp}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{p^2}{2m} \Omega(\mathbf{p}, \omega) f(\omega) - \frac{1}{2} \langle \hat{W} \rangle;
\]

here \( \nu = 2 \) for PNM and 4 for SNM is the nucleon degeneracy; \( \rho \) is the total nucleon number density; \( \Omega(\mathbf{p}, \omega) \) is the spectral function and is related to the imaginary part of \( G(\mathbf{p}, \omega) \); \( f(\omega) \) is the Fermi-Dirac distribution; and \( \langle \hat{W} \rangle \) is the expectation value of the three-body operator. The spectral function \( \Omega(\mathbf{p}, \omega) \), which enters the calculation of the energy sum rule, is directly connected with the single-particle propagator \( G \), being proportional to its imaginary part [29]. From the spectral function one has direct access to the nucleon momentum distribution

\[
n(\mathbf{p}) = \int \frac{d\omega}{2\pi} \Omega(\mathbf{p}, \omega) f(\omega).
\]

One can then evaluate the kinetic energy contribution \( E_k \) to the energy per nucleon according to

\[
E_k = \nu A \int \frac{dp}{(2\pi)^3} \frac{p^2}{2m} n(\mathbf{p}),
\]

as well as the potential energy contribution \( E_p \) via subtraction of Eq. (17) from Eq. (15). For the nucleon energy spectrum, it is obtained by solving consistently the equation

\[
\varepsilon(\mathbf{p}) = \frac{p^2}{2m} + \text{Re} \Sigma^*(\mathbf{p}, \varepsilon(\mathbf{p})].
\]
The second term on the right-hand side only selects the on-shell part of the real self-energy, and it is what corresponds to a mean-field potential. For further details on the calculation of the finite-temperature properties of infinite matter within the SCGF method, we refer the reader to Ref. [30].

The extension of the SCGF method to include three-body forces paved the way to the possibility of using consistently nuclear interactions derived from the chiral effective field theory. These interactions, being derived from a low-energy effective theory of QCD, have a cutoff in momentum usually around ~ 500 MeV/c. The high-energy physics, which is integrated out, is then encoded in low-energy constants, which need to be fitted to nuclear matter properties [17,18]. Studies of the properties of infinite matter at both zero and finite temperatures have been presented within the SCGF method considering several different chiral interactions [14,26]. In this work we make use of three different chiral interactions. These have been chosen because they predict reasonably well the empirical saturation properties of symmetric nuclear matter [32]. We are then able to provide an error band on our theoretical results based on the nuclear interaction. Two of these interactions, i.e., 2.0/2.0(EM) and 2.0/2.5(EM), have been obtained by fitting the two-body part to nucleon-nucleon phase shifts and deuteron properties, while the three-body part has been constructed to reproduce the binding energy of the triton and the radius of alpha particle. The two-body part has been further softened with the similarity renormalization group technique, in order to improve the convergency of many-body calculations, as detailed in Ref. [33]. The third interaction is called NNLOsat with the whole two- and three-body parts fitted consistently, and it can reproduce reasonably well the properties of light nuclei as well as those of medium-mass nuclei, such as the radii of carbon and oxygen isotopes [34].

### III. Results and Discussions

In the following, we compare some properties of infinite nuclear matter obtained from the two effective interactions ImMDI-GF and Skχm* using the Hartree-Fock approach to those from the microscopic SCGF calculations, whose uncertainties mainly come from those in the three-body forces and the high-momentum cutoffs. As stated in the introduction, the ImMDI model is fitted to the empirical properties of SNM, which are approximately reproduced by the SCGF approach using the chiral forces. As an improvement of the ImMDI model, we adjust the parameters of its isovector part, i.e., $x$, $y$, and $z$, to reproduce the results from the SCGF approach for the properties of PNM at zero temperature, and this new parameter set is dubbed as ImMDI-GF. For the SHF energy density functional, the Skχm* interaction used in the present study is constructed from fitting the EOS and nucleon effective masses of cold nuclear matter from χEMBPT and the binding energies of finite nuclei [23]. Details on the values of the parameters in ImMDI-GF and Skχm* interactions as well as some of their predicted physical quantities are listed in Table I.

#### A. Nucleon occupation probability

We first show in Fig. 1 the nucleon occupation probability at saturation density, i.e., \(1/\{\exp[(p^2/2m + U_q - \mu_q)/T] + 1\}\), in both symmetric and pure neutron matter at temperatures of 10 MeV, 30 MeV, and 50 MeV from ImMDI-GF and Skχm* by solid and dashed lines, respectively. These results are compared with those from the SCGF calculations as given in Eq. (16) and shown by shaded bands, which represent the uncertainties in this approach using the three chiral forces 2.0/2.0(EM), 2.0/2.5(EM), and NNLOsat. While the occupation probabilities at zero temperature are simply $\Theta(p_f - p)$ in the mean-field models, the sharp discontinuity at the Fermi momentum is smoothed in the SCGF calculations by correlations in the nuclear many-body system [35].

For SNM, the occupation probabilities obtained from ImMDI-GF and Skχm* differ from those from the SCGF

### Table I: Values of parameters and some physical quantities for ImMDI-GF and Skχm* with $\rho_{sat}$ the saturation density, $E_0(\rho_{sat})$ the energy per nucleon at saturation density, $K_0$ the incompressibility, $U_0^\infty$ the mean-field potential for SNM at saturation density and infinitely large nucleon momentum, $m^*_i$ the isoscalar and the isovector effective mass, $E_{sym}(\rho_{sat})$ and $L$ the value and the slope parameter of the symmetry energy at saturation density, and $G_S$ and $G_V$ the isoscalar and the isovector density gradient coefficient.

| Parameter | ImMDI-GF | Skχm* |
|-----------|----------|--------|
| $A_0$ (MeV) | -66.963 | -2260.7 |
| $B$ (MeV) | 141.963 | 0.327488 |
| $C_{\alpha 0}$ (MeV) | -99.70 | 433.189 |
| $C_{\alpha 0}$ (MeV) | -60.49 | -1.088968 |
| $\sigma$ | 1.2652 | 274.553 |
| $\Lambda (p_{\rho 0})$ | 2.424 | -1.822404 |
| $x$ | 0.5 | -12984.4 |
| $y (MeV)$ | -6.0 | 0.442900 |
| $z (MeV)$ | -2.5 | 0.198029 |
| $\rho_{sat}$ (fm$^{-3}$) | 0.16 | 0.1651 |
| $E_0(\rho_{sat})$ (MeV) | -16 | -16.07 |
| $K_0$ (MeV) | 230 | 230.4 |
| $U_0^\infty$ (MeV) | 75 | N/A |
| $m^*_i$ (m) | 0.70 | 0.750 |
| $E_{sym}(\rho_{sat})$ (MeV) | 30 | 30.94 |
| $L$ (MeV) | 40 | 45.6 |
| $m^*_i$ (m) | 0.59 | 0.694 |
| $G_S$ (MeVfm$^3$) | N/A | 141.5 |
| $G_V$ (MeVfm$^3$) | N/A | -70.5 |
FIG. 1: (Color online) Nucleon occupation probability at \(\rho_0 = 0.16 \text{ fm}^{-3}\) as a function of nucleon momentum in symmetric nuclear matter (left) and pure neutron matter (right) at various temperatures from the ImMDI-GF, Sk\(\chi_m^*\), and SCGF calculations.

B. Kinetic and potential energy contributions to the nuclear matter EOS

The kinetic energy contribution, which is uniquely determined by the nucleon occupation probability as indicated in Eq. (17), and the potential energy contribution to the EOS obtained from different approaches, are compared in Fig. 2. Since ImMDI-GF and Sk\(\chi_m^*\) are constructed from fitting similar nuclear EOSs at zero temperature, and they also have same nucleon occupation probabilities at zero temperature, the kinetic energy and the potential energy contribution to the EOS of cold nuclear matter from these two effective interactions are almost identical. The kinetic energy contributions to the EOS from these two effective interactions start to deviate as temperature increases, especially for PNM, with Sk\(\chi_m^*\) always giving smaller values. The fact that the kinetic energy contributions in SNM and PNM from effective interactions based on Hartree-Fock calculations are always below those from SCGF is consistent with the behavior of nucleon occupation probabilities shown in Fig. 1. where the SCGF always gives a larger population of high-momentum states and thus a larger kinetic energy, as a result of correlation effects. Deviations between the results on kinetic energy contributions in both SNM and PNM from these two effective interactions increase with both increasing density and temperature as a result of the different momentum dependence in their mean-field potentials.

For the potential energy contribution, which depends on both the density and the nucleon occupation probability, results between the two effective interactions are in good agreement at all temperatures for SNM, while they start to deviate for PNM as temperature increases. This is consistent with the results on the nucleon occupation probability shown in Fig. 1. The potential energy contribution from the SCGF approach in both SNM and PNM is, however, always lower compared to that from the two effective interactions due to its larger nucleon occupation probability at high momenta as shown in Fig. 1.
The density dependence of the total energy per nucleon for SNM and PNM from ImMDI-GF and Skχm* are similar for cold and low-temperature SNM and PNM. Except for small deviations at very low densities, the EOSs from these two effective interactions are within the SCGF uncertainty band (see panels (a) and (b) in Figs. 3 and 4). However, results start to deviate at higher temperatures, with the EOS of SNM from ImMDI-GF remaining within the uncertainty band of SCGF but that from Skχm* standing slightly lower. For the EOS of PNM, ImMDI-GF gives slightly larger values at very high temperatures, while that from Skχm* is within the uncertainty band of SCGF. These results can be partially understood from the relative contributions of the kinetic energy and the potential energy to the EOS, as shown in Fig. 2. Also shown in panels (a) and (b) for both SNM and PNM are results from χEMBPT calculations using n3lo414 and n3lo450 chiral forces, which are taken from Figs. 1 and 2 of Ref. [13]. The uncertainty band in the latter approach for SNM is smaller than that given by SCGF due to a smaller range of variation in the high-momentum cutoff and similar three-body forces for the two potentials employed. For PNM at low temperatures, results from χEMBPT using n3lo414 and n3lo450 forces give almost the same EOS, due to reduced regulator dependence in the three-body forces [36]. The EOSs of both SNM and PNM from the χEMBPT are seen to be well reproduced by ImMDI-GF and Skχm* at both $T = 0$ and 10 MeV.

D. Nucleon mean-field potentials in symmetric and pure neutron matter

In transport simulations of intermediate-energy heavy-ion collisions, the direct input is the mean-field potential instead of the EOS. The temperature dependence of the mean-field potential is thus important in determining the evolution of the hot nuclear matter produced in these collisions. We compare in this subsection the momentum dependence of the mean-field potential at $\rho_0$ obtained from ImMDI-GF and Skχm* with that from the SCGF approach in Figs. 3 and 4 for SNM and PNM, respectively. The mean-field potentials from the SCGF approach in all these different cases are seen to always approach zero at nucleon momenta $\sim 1000\text{MeV/c}$. This is due to the high momentum cutoff in the regulator functions used in constructing these chiral forces [37]. With its proper isoscalar and isovector effective masses as well as the mean-field potential $U_0^{\text{m}}$ at saturation density and infinite large nucleon momentum, the ImMDI model can well reproduce the momentum dependence of the optical potential extracted by Hama et al. [38, 39] from the proton-nucleus scattering data as shown in Fig. 1 of Ref. [20].

It is seen that in SNM at low temperatures the mean-field potentials from both ImMDI-GF and Skχm* are consistent with results from the SCGF approach up to $p = 500\text{MeV/c}$, while in PNM these results start to deviate already below $p = 500\text{MeV/c}$ [42]. With its isovector effective mass adjusted to be about $m_v^* = 0.59m$, the ImMDI-GF interaction is seen to give mean-field poten-
tials in PNM that are consistent with those from the SCGF approach all the way to higher momenta. This is different for the mean-field potentials from Sk\(\chi_m^*\), which are seen to increase quadratically with nucleon momentum and become more repulsive around \(p = 500\) MeV/c in SNM. This is especially so for PNM where the deviations appear already at lower momenta. Both ImMDI-GF and Sk\(\chi_m^*\) reproduce very well the mean-field potential at low momenta from the \(\chi\)EMBPT using the n3lo450 force at \(T = 0\) MeV for SNM [40]. The temperature effect on the mean-field potentials from ImMDI-GF and Sk\(\chi_m^*\) is seen to be stronger than that from the SCGF approach, especially at lower nucleon momenta. It is remarkable that the momentum dependence of the mean-field potentials from the SCGF approach for both SNM and PNM are reproduced reasonably well by ImMDI-GF at various temperatures.

IV. SUMMARY

To study if the commonly used nucleon-nucleon effective interactions, which are usually constructed from fitting the properties of cold nuclear matter and of finite nuclei, can properly describe nuclear matter at finite temperatures, we have used the improved isospin- and momentum-dependent interaction ImMDI-GF and the recently constructed Skyrme interaction Sk\(\chi_m^*\) to evaluate the nucleon occupation probabilities, the equations of state, and the mean-field potentials in symmetric nuclear matter and pure neutron matter at finite temperatures using the Hartree-Fock approach. These results have been compared with those from the microscopic self-consistent Green’s function method and the chiral effective many-body perturbation theory using chiral nuclear forces. We find that differences start to appear more strongly at higher temperatures between results for nuclear matter properties from the two effective interactions and also between these two and those from the microscopic theories. The deviations seen in the nucleon occupation probabilities have been understood from the different momentum dependence in the single-nucleon potential obtained within these approaches, while the latter is strongly suppressed at high momenta in the microscopic calculations based on chiral forces compared to those from the effective interactions, especially for Sk\(\chi_m^*\) that has a quadratic momentum dependence. These differences in the nucleon momentum distributions lead to deviations in the kinetic energy contribution and also partially in the potential energy contribution to the nuclear equation of state. The energies per nucleon for symmetric nuclear matter and pure neutron matter from these two effective interactions are roughly consistent with those from the self-consistent Green’s function approach, although the equation of state for symmetric nuclear matter from Sk\(\chi_m^*\) remains softer at higher temperatures. Using the ImMDI-GF model in the Hartree-Fock calculation reproduces remarkably well the mean-field potential from the microscopic approaches at various temperatures for both symmetric nuclear matter and pure neutron matter. Our study thus shows that effective interactions with the correct momentum dependence in the mean-field potential, such as the one from the ImMDI-GF model, can properly describe the properties of hot dense nuclear matter and is thus suitable for use in transport models to extract the equation of state of cold nuclear matter, which is needed for describing the properties of neutron stars, from intermediate-energy heavy-ion collisions [41, 42].
Acknowledgments

We thank Jeremy Holt for providing results from the many-body perturbation theory using nuclear chiral forces. J.X. acknowledges support from the Major State Basic Research Development Program (973 Program) of China under Contract No. 2015CB856904 and the National Natural Science Foundation of China under Grant No. 11421505. C.M.K. acknowledges support from the US Department of Energy under Contract No. de-sc0015266 and the Welch Foundation under Grant No. A-1358.

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[43] It must be noted that the mean-field potential from the SCGF approach is not calculated at $T = 0$ MeV but at $T = 4$ MeV to avoid pairing instability, since thermal effects are very small at such low temperatures.