The $^{14}N(p, \gamma)^{15}O$ reaction, solar neutrinos and the age of the globular clusters

S. Degl’Innocenti$^{1,2}$, G. Fiorentini$^{3,4}$, B. Ricci$^{3,4}$ and F.L. Villante$^{3,4}$

$^1$Dipartimento di Fisica dell’Università di Pisa, via Buonarroti 2, I-56126 Pisa, Italy
$^2$Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, via Buonarroti 2, I-56126 Pisa, Italy.
$^3$Istituto Nazionale di Fisica Nucleare, Sezione di Ferrara, via Paradiso 12, I-44100 Ferrara, Italy,
$^4$Dipartimento di Fisica dell’Università di Ferrara, via Paradiso 12, I-44100 Ferrara, Italy

Abstract

We discuss implications of a new measurement of $^{14}N(p, \gamma)^{15}O$ concerning solar neutrinos, solar models and globular cluster dating. Predictions for the gallium and chlorine experiments are reduced by 2 and 0.1 SNU respectively. Predictions for helioseismic observables are unchanged within uncertainties. The age of globular clusters as deduced from the Turn-Off luminosity is increased by about 0.7 Gyr.

A. Introduction

The last few years have presented us with a remarkable progress in understanding hydrogen burning in stars: the solution of the solar neutrino puzzle provided by SNO [1] and KamLAND [2] experiments has allowed a precise determination of the $^8B$ neutrino flux, in agreement with the theoretical prediction of Standard Solar Model (SSM) calculations. The predicted signals for gallium and chlorine detectors are also in good agreement with SSMs, once the survival probability of electron neutrinos is calculated according to the Large Mixing Angle (LMA) solution, see e.g. [3] $^1$. In this way, the nuclear energy source of the sun, proposed by Eddington [5], Bethe [6] and von Weizsäcker [7] long ago, has been checked with observation: the production rate of nuclear energy in the sun, as deduced from neutrino observations, agrees with the observed photon luminosity to within about 20% [3,8].

On the laboratory side, recent experiments have provided refined measurements of nuclear cross sections relevant to hydrogen burning: the $^3He(^3He, 2p)^4He$ astrophysical $S$-factor has been measured well in the solar Gamow peak with 6% accuracy [9] and that of

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$^1$Actually, there is a slight ($\simeq 2\sigma$) tension between prediction and measurement for the chlorine experiment [3,4].
$^7\text{Be}(p, \gamma)^8\text{B}$ is presently known with an accuracy of about 5% as a result of several recent measurements, see [8] and refs. therein.

Although rather precise concerning the $pp$ chain, neutrino observations and laboratory experiments shed little light on the role of the CNO bi-cycle in the sun. This cycle, producing a tiny ($\approx 1.5\%$ according to SSM) contribution to the solar energy output, is most important in more advanced burning phases, where it sustains a large fraction of the stellar luminosity by hydrogen burning in shells as soon as hydrogen is exhausted at the stellar center.

The age of the globular clusters, the oldest systems in the Galaxy, is determined by locating the turning point on the Hertzsprung-Russell diagram, i.e. the point which signals hydrogen exhaustion at the stellar center. The efficiency of the CNO cycle is thus expected to be relevant for globular cluster dating.

No experiment aimed at a direct determination of CNO neutrinos from the sun has been performed so far. Gallium and chlorine experiments are sensitive to neutrinos from the CNO cycle, however these provide an undistinguishable contribution to the total signal, dominated by neutrinos from the $pp$ chain. A combined analysis of all solar and reactor neutrino experiments could only provide an upper bound for the CNO contribution to solar luminosity, $L_{(CNO)}/L_0 < 7.3\%$ at 3$\sigma$ [10].

As well known, the key reaction for deciding the efficiency of the CNO cycle (see Fig. 1) is the radiative proton capture $^{14}\text{N}(p, \gamma)^{15}\text{O}$. It has been measured by several groups in the last fifty years, but only the measurements of Schröder et al. [11] extend over a wide energy range, above $E_{cm} = 200$ keV. For the extrapolation to lower energies, the effect of a subthreshold resonance at $E_r = -504$ keV is important. The zero energy astrophysical $S$-factor recommended by the NACRE compilation [12]

$$S_{SSM} = (3.2 \pm 0.8) \text{keVb}$$

is mainly based on data from [11]. This value, which we shall refer to as the SSM value, agrees with the estimate in the compilation by Adelberger et al. [13], $S = (3.5^{+0.4}_{-1.0})\text{keVb}$.

Angulo and Descouvemont [14] reconsidered the data of [11] within the framework of an R-matrix model. The $S$-factor which is extracted from their analysis is essentially halved with respect to that of NACRE. In conclusion, whereas all the $S$-factors for the $pp$ chain have been determined with accuracy of 10% or better, until recently the key cross section for the CNO cycle was known with an uncertainty of a factor two.

A new experiment, performed at the underground Gran Sasso laboratory by the LUNA collaboration, has just presented its first results [15]. The new value of the astrophysical $S$-factor

$$S_{LUNA} = (1.7 \pm 0.2)\text{keVb}$$

is in good agreement with that found in ref. [14].

The present letter addresses the following questions:

i) what is the impact on solar neutrinos, produced by change of a factor two in the astrophysical $S$-factor of $^{14}\text{N}(p, \gamma)^{15}\text{O}$?

ii) which are the consequences for the estimated globular cluster ages?
B. The sun and $^{14}N(p,\gamma)^{15}O$

As a first approximation, one expects that the produced fluxes of CNO neutrinos are proportional to the astrophysical $S$-factor, since the production rates depend linearly on $S$ whereas the density of reacting nuclei and temperature are weakly dependent on it, being essentially fixed by the solar mass, composition and age and by the main mechanism - the $pp$ chain - for energy production.

A decrease by a factor two in $S$ will thus be accompanied by a corresponding decrease of the CNO produced fluxes. Since the total produced neutrino flux is fixed by the observed solar luminosisty, this has to be accompanied by an increase of neutrinos from the $pp$ chain.

This qualitative picture is confirmed by the solar models which we have built with FRANEC [16] for different values of $S$, see Figs. 2 and 3. We find a linear dependence of the CNO produced flux which is essentially compensated by the variation of $\Phi(pp)$, the Beryllium flux being practically unchanged. The Boron flux slightly decreases with decreasing $S$.

The sensitivity of the produced fluxes to changes of the physical and chemical inputs of SSM is usually parametrized in terms of power laws [8,17,18]:

$$\Phi(i) = \Phi_{SSM}(i)(S/S_{SSM})^{a_i}$$ (3)

These laws are valid for small changes of the input with respect to the SSM adopted value. In our case it is preferable to resort to a linear parametrization:

$$\Phi(i) = \Phi_{SSM}(i)[1 + a_i(S - S_{SSM})/S_{SSM}]$$ (4)

The values of $a_i$, as obtained by least square fitting, are presented in Table I.

The produced signal rates $R_{pr}$ in radiochemical experiments are defined as:

$$R_{pr} = \Sigma_i \sigma(i)\Phi(i)$$ (5)

where $\sigma(i)$ is the capture cross section of the $i$-th neutrinos in the detector. Their dependence on $S$ is shown in Tables II and III and Fig. 4. One sees that the linear $S$-dependence of the fluxes translates into a similar behaviour of the signals. For gallium detectors, the reduction of $S$ corresponds to the fact that CNO neutrinos are replaced with a similar number of lower energy $pp$ neutrinos. Since the cross section decreases when energy decreases, the resulting signal is smaller. We remind that $pp$ neutrinos are below threshold for chlorine detectors, sensitive to B, Be and CNO neutrinos. For this reason, a decrease of CNO neutrinos results in a signal decrease.

These effects are enhanced when considering the effective signal rates, which include neutrino oscillations,

2Here and in the following we refer to “produced fluxes” and “produced signals” to indicate predictions in the absence of oscillation.
\[ R = \sum_i \sigma(i) \Phi(i) P_{ee}(i) \]  

(6)

where \( P_{ee}(i) \) is the average survival probability of the \( i \)-th neutrinos. The LMA solution predicts \( P_{ee} \) decreasing with energy, so that the percentage contribution of lower energy neutrinos is larger in the effective signal than in the produced signal.

For gallium and chlorine experiments the change \( S_{SSM} \rightarrow S_{LUNA} \) results in a signal reduction \( \Delta R(Ga) = 2 \) SNU and \( \Delta R(Cl) = 0.1 \) SNU. This alleviates the slight tension between the chlorine result and the LMA+SSM prediction noticed in [4] and [3].

Complementary to neutrinos, helioseismic observations provide us with a detailed view of the solar interior. By means of helioseismology one can reconstruct the sound speed profile inside the sun. Recent SSMs, all including gravitational settling and elemental diffusion, are well in agreement with helioseismic data, see e.g. [19]. Since the efficiency of the CNO cycle is important for energy generation in the innermost part of the sun, in this region one expects that the sound speed is sensitive to \( S \).

In Fig. 5 we compare solar models computed with different \( S \) values. We find that all models with \( S < S_{SSM} \) cannot be distinguished from the SSM within the uncertainties of the helioseismic observables. On the other hand, models with \( S > 5 \cdot S_{SSM} \) are excluded at 3\sigma or more. In other words, the helioseismic limit to the energy generation rate by the CNO cycle in the sun is \( L(CNO)/L_0 < 7.5\% \), comparable to the bound obtained in [3] from solar neutrino experiments.

### C. Globular clusters and \( ^{14}N(p,\gamma)^{15}O \)

The age of globular clusters, the oldest objects in the Galaxy, is extremely important for understanding the galactic evolution and to give a firm lower limit to the Galaxy formation epoch.

The effect of the \( ^{14}N(p,\gamma)^{15}O \) cross section on the evolutionary characteristics of population II stars has been analysed in [20]. In order to assess the impact of the LUNA result, we repeat and extend the calculations of [20] by means of a FRANEC version containing updated physical inputs, see [22,23], and including microscopic diffusion of He and heavy elements. The external convection is treated within a mixing length approach and its efficiency is calibrated so as to reproduce the red giant branch (RGB) color of globular clusters with different chemical composition. The resulting models reproduce the color-magnitude diagram of well observed globular clusters as M68, M3 and M13 [23].

The present calculations have been made for two chemical compositions (Z=0.002, Y=0.230 and Z=0.001, Y=0.232) which are well representative of the globular cluster population.

As well known, the main parameter marking the age of a stellar cluster is the luminosity at Turn-Off, \( L_{TO} \), in the Hertzsprung-Russell (HR) diagram, see e.g. [24]. Stellar evolution theory can predict the behaviour of \( L_{TO} \) as a function of cluster age \( t \). Clearly, the relationship between \( L_{TO} \) and \( t \) depends on the physical inputs adopted in the calculations and in particular on the assumed value of \( S \).

In Fig. 6 we show the 12 Gyr isochrones obtained for different values of the \( ^{14}N(p,\gamma)^{15}O \) astrophysical factor, with the same chemical composition (Z=0.001, Y=0.232). By decreasing \( S \), \( L_{TO} \) increases. In fact the Turn-Off (TO) marks the onset of the CNO hydrogen
burning in shells. If the CNO efficiency is reduced, this onset is delayed and TO occurs at a later time and with a larger luminosity [20].

The dependence is shown more quantitatively in Fig.7 for the higher metallicity composition (the low metallicity case looks similar). By halving $S$ the same value of $L_{TO}$ corresponds to an age increase $3 \Delta t \simeq 0.7$ Gyr.

This approach assumes that $L_{TO}$ can be fixed from observations, independently of $S$. Actually the determination of $L_{TO}$ requires the knowledge of the cluster distance modulus, which is often obtained by using RR Lyrae stars in the Horizontal Branch (HB) $^4$ as standard candles. In this case the relevant observable is the ratio of $L_{TO}$ to the HB luminosity, $L_{HB}$, which is independent of the cluster distance. A frequently used variable for determining the cluster age is defined as:

$$\log L_{HB-TO} = \log(L_{HB}/L_{TO}) \quad .$$

(7)

As discussed in [20], variations of $S$ also affect the HB stars, which are powered by He burning in the core and by H burning in a surrounding shell, mainly through the CNO cycle.

A decrease of $S$ has two competing effects: it decreases the CNO cycle efficiency, driving a decrease of $L_{HB}$, and at the same time it produces an increase of the helium core mass at He ignition, which reflects into an increase of $L_{HB}$. The net effect depends on the cluster metallicity. For low metallicities ($Z = 0.0002$) we find that a decrease of $S$ by a factor two leads to an increase $\Delta \log L_{HB} \sim 0.01$. For moderately metal-rich HB stars ($Z = 0.001$), where CNO burning is more important, the same variation produces instead a decrease $\Delta \log L_{HB}$ of about the same amount. This means that when using $\log L_{HB-TO}$ as age indicator the LUNA result leads to an increase of the estimated age which depends on the cluster metallicity: we obtain $\Delta t \sim 0.5$ Gyr for $Z = 0.0002$ and $\Delta t \sim 1$ Gyr for $Z = 0.001$.

The determination of globular clusters ages is presently affected by several uncertainties, resulting from the chemical composition, from the adopted physical inputs and from the efficiency of various physical mechanisms (e.g. microscopic diffusion). Additional uncertainties arise from the comparison between theoretical and observed luminosities, see e.g. [25–28].

A precise determination of the overall uncertainty is thus difficult. The cluster age as determined from the absolute value of $L_{TO}$ (i.e. assuming that the cluster distance is known in an independent way) is affected by an error of $\sim 1.5$ Gyr [22,28,25]. If $\log L_{HB-TO}$ is used as an age indicator, the uncertainty is $\sim 2.0$ Gyr, see [22,28]. The increase of the globular cluster ages following from LUNA result is thus within the error bar of the present determinations. Nevertheless the new and more precise value of $S$ will be important when better astrophysical inputs will be available.

We conclude this section by discussing the effect on another interesting evolutionary feature of globular clusters: the so called RGB bump, a region of the HR cluster diagram with higher star density. The RGB bump corresponds the momentaneous decrease of the stellar

\[\text{A similar conclusion has been obtained by O. Straniero et al. in [21].}\]

\[\text{For the sake of precision, our candles are provided by the HB lower envelope (Zero Age Horizontal Branch, ZAHB) in the RR Lyrae region.}\]
luminosity in RGB which marks the encounter of the CNO H-burning shell with the chemical discontinuity produced by the first dredge-up, see e.g. [32–34]. We find that a reduction of $S$ by a factor two leads to an increase of the bump luminosity of about $\Delta \log L_{\text{bump}} \sim 0.02$, which is well within the estimated theoretical and observational uncertainties on this quantity, see e.g. [29–31].

D. Concluding remarks

We summarize here the main points of this paper:

- The LUNA result on the astrophysical $S$-factor for $^{14}N(p, \gamma)^{15}O$ implies that SSM+LMA predictions for the gallium and chlorine experiments are reduced by 2 and 0.1 SNU respectively. This alleviates the slight tension between theory and chlorine result.
- The new $S$ value does not change significantly helioseismic observables.
- On the other hand, helioseismology excludes a CNO contribution to solar luminosity larger than 7.5%.
- The age of globular clusters is increased by a quantity 0.5–1 Gyr, depending on the method for determining the Turn-Off luminosity and on the cluster metallicity.

We are extremely grateful to C. Broggini, V. Castellani, H.P. Trautvetter and C. Rolfs for useful discussions.

This work was performed within the Astroparticle Physics project financed by MIUR as PRIN-2002.
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TABLES

TABLE I. Slope of the flux dependence on $S$.

| Source | $a_i$  |
|--------|--------|
| pp     | -0.013 |
| pep    | -0.018 |
| Be     | -0.003 |
| B      | +0.018 |
| N      | +0.875 |
| O      | +1.008 |

TABLE II. Produced fluxes and signals with $S_{SSM} = 3.2$ keVb. All other inputs as in [18].

| Source | Flux $[10^9 \text{cm}^{-2}\text{s}^{-1}]$ | Cl [SNU] | Ga [SNU] |
|--------|----------------------------------------|-----------|-----------|
| pp     | 59.99                                  | 0         | 70.3      |
| pep    | 0.142                                  | 0.227     | 2.89      |
| Be     | 4.52                                   | 1.08      | 32.4      |
| B      | $5.21 \times 10^{-3}$                  | 5.94      | 12.5      |
| N      | 0.515                                  | 0.0875    | 3.13      |
| O      | 0.437                                  | 0.297     | 4.97      |
| total  |                                       | 7.64      | 126.3     |

TABLE III. Produced fluxes and signals when $S_{SSM} \rightarrow S_{LUNA}$

| Source | Flux $[10^9 \text{cm}^{-2}\text{s}^{-1}]$ | Cl [SNU] | Ga [SNU] |
|--------|----------------------------------------|-----------|-----------|
| pp     | 60.33                                  | 0         | 70.7      |
| pep    | 0.143                                  | 0.229     | 2.92      |
| Be     | 4.53                                   | 1.09      | 32.5      |
| B      | $5.17 \times 10^{-3}$                  | 5.90      | 12.4      |
| N      | 0.305                                  | 0.0518    | 1.84      |
| O      | 0.226                                  | 0.154     | 2.57      |
| total  |                                       | 7.43      | 123.2     |
FIG. 1. The CNO bi-cycle.
FIG. 2. Produced O, N and pp fluxes. Crosses denote solar model results and the straight lines are linear fits. We use $S_{SSM} = 3.2$ keVb. The arrow corresponds to the new value $S_{LUNA} = 1.7$ keVb.
FIG. 3. Produced B, pep and Be fluxes. Same notation as in Fig. 2.
FIG. 4. Produced signals in radiochemical experiments. Same notation as in Fig. 2.
FIG. 5. Relative change (model-SSM)/SSM of the squared isothermal sound speed $u = P/\rho$ as a function of the radial coordinate, for the indicated values of $S/S_{SSM}$. The dark (light) shaded area corresponds to the $1\sigma$ ($3\sigma$) uncertainty on helioseismic determination [19].
FIG. 6. Isochrone dependence on $S$. The calculated luminosity $L$, in units of the solar luminosity $L_0$, is presented as a function of the effective temperature $T_e$ in Kelvin.
FIG. 7. Luminosity at Turn-Off $L_{TO}$ as a function of the cluster age $t$. Points are the results of evolutionary calculations, continuous curves correspond to linear fits in the log $L_{TO} - \log t$ plane.