Non-local bias and the problem of large-scale power in the *Standard* Cold Dark Matter model

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Abstract. We study the effect of non-radial motions, originating from the gravitational interaction of the quadrupole moment of a protogalaxy with the tidal field of the matter of the neighboring protostructures, on the angular correlation function of galaxies. We calculate the angular correlation function using a *Standard* Cold Dark Matter (hereafter SCDM) model ($\Omega = 1$, $h=0.5$, $n = 1$) and we compare it with the angular correlation function of the APM galaxy survey (Maddox et al. 1990; Maddox et al. 1996). We find that taking account of non-radial motions in the calculation of the angular correlation function gives a better agreement of the theoretical prediction of the SCDM model to the observed estimates of large-scale power in the galaxy distribution.

Key words: cosmology: theory - cosmology: large scale structure of Universe - galaxies: formation

1. Introduction

The galaxy two-point correlation function $\xi_g(r)$ is a powerful discriminant between distinct models of structure formation in the universe. On scales $\geq 10h^{-1}\text{Mpc}$ correlations between galaxies are weak, namely $\xi_g(r) << 1$, so one may reasonably expect that $\xi_g(r)$ can be related to the fluctuations in the early universe by linear perturbation theory. The role of the two-points correlation function is particularly important in models that predict fluctuations that obey Gaussian statistics (see Bardeen et al. 1986) because in

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this case the field is completely specified, in a statistical sense, by a single function: the power spectrum $P(k)$ or by its Fourier transform, the autocorrelation function, $\xi(r)$. This means that a knowledge of $\xi_g(r)$ on large scales would give a powerful constraint on models of the early Universe, and if the fluctuations are Gaussian, we would obtain a complete description of large-scale structure. Analyses of galaxy surveys (APM, QDOT) have shown that excess correlations are found on scales larger than $10h^{-1}\text{Mpc}$ (Maddox et. al 1990; Efstathiou et al. 1990b; Saunders et al. 1991; Maddox et al. 1996) when observations are compared with the predictions of the standard CDM model and also radio galaxies are also strongly clustered on large scales (Peacock 1991; Peacock & Nicholson 1991). These data have hence been widely interpreted as ruling out SCDM model and new alternative models have been introduced in an effort to solve this and other problems of the model.

Several authors (Peebles 1984; Efstathiou et al. 1990a; Turner 1991) have lowered the matter density under the critical value ($\Omega_m < 1$) and have added a cosmological constant in order to retain a flat Universe ($\Omega_m + \Omega_{\Lambda} = 1$). Mixed dark matter models (MDM) (Shafi & Stecker 1984; Valdarnini & Bonometto 1985; Schaefer et al. 1989; Holtzman 1989; Schaefer 1991; Schaefer & Shafi 1993; Holtzman & Primack 1993) increase the large-scale power because neutrinos free-streaming damps the power on small scales. Alternatively, changing the primeval spectrum solves several problems of CDM (Cen et al. 1992). Finally it is possible to assume that the threshold for galaxy formation is not spatially invariant but weakly modulated (2% – 3% on scales $r > 10h^{-1}\text{Mpc}$) by large scale density fluctuations, with the result that the clustering on large-scale is significantly increased (Bower et al. 1993). This last alternative is part of those models attempting to rescue SCDM by invoking a form of bias that has different effects on clustering on different scales (Babul & White 1991; Bower et al. 1993). In fact as previously reported, large-scale clustering studies such as APM (Maddox et al. 1990; Maddox et al. 1996) and QDOT (Efstathiou et al. 1990b; Saunders et al. 1991; Peacock 1991) suggest a clustering amplitude which is larger than one would expect on the basis of the SCDM model. Moreover the level of temperature fluctuations seen by COBE are consistent with no large-scale bias, but when the CDM model is normalized to COBE results it has problems in accounting for small-scale structure. An obvious way to rescue the model, then, is a scale-dependent bias which can modify the slope of the correlation function so as to make it decay less steeply than the mass autocovariance function on large scales. As shown by Coles (1993) for Gaussian fields, a change of slope of this kind can be achieved by non-local biasing effects such as cooperative galaxy formation (Bower et al. 1993). In this last model Bower et al. (1993) adopt the assumption that the threshold level, $\delta_c$, depends on the mean mass density in the domain of influence, rather than being spa-
tially invariant. Galaxy formation is assumed to occur according to the prescriptions of the standard biased galaxy formation theory (Kaiser 1984; Bardeen et al. 1986) but is enhanced by the presence of nearby galaxies. This approach is able to produce enough additional clustering to fit the $\xi_g(r)$ of the APM galaxy survey. The main problem of this and of any theory of galaxy formation involving a bias of any kind is that they are not acceptable until the physical mechanisms producing the bias are elucidated. In some recent papers (Del Popolo & Gambera 1998a,b) we introduced a model that is able to reduce several of the SCDM model problems and also includes (differently from Bower et al. 1993) a clear explanation for the physical mechanisms that produce the bias.

As shown by Barrow & Silk (1981) and Szalay & Silk (1983) the gravitational interaction of the irregular mass distribution of a test proto-structure with the neighbouring ones gives rise to non-radial motions, within the test proto-structure, which are expected to slow the rate of growth of the density contrast and to delay or suppress the collapse. According to Davis & Peebles (1977), Villumsen & Davis (1986) and Peebles (1990) the kinetic energy of the resulting non-radial motions at the epoch of maximum expansion increases so much as to oppose the recollapse of the proto-structure. As shown by Del Popolo & Gambera (1998a,b), within high-density environments, such as rich clusters of galaxies, non-radial motions slow down the collapse of low-$\nu$ peaks thus producing an observable variation in the time of collapse of the shell and, as a consequence, a reduction in the mass bound to the collapsed perturbation. Moreover, the delay of the collapse produces a tendency for less dense regions to accrete less mass, with respect to a classical spherical model, inducing a biasing of over-dense regions toward higher mass. Non-radial motions change the energetics of the collapse model by introducing another potential energy term in the equation of collapse, leading to a change of the turn around epoch, $t_m$, and consequently the critical threshold, $\delta_c$, for collapse. The change of $\delta_c$ is in the same sense as that described by Bower et al. (1993).

In this paper we apply the quoted model to galaxies showing how non-radial motions modify the galaxies correlations. The plan of the paper is the following: in Sect. 2 we introduce the model; in Sect. 3 show the results and in Sect. 4 we draw our conclusions.

2. The model

In the standard high-peak galaxy model, galaxies form from mass located near high peaks of the linear density field. The density contrast at early times, $\delta(x) = \frac{\rho(x) - \rho_b}{\rho_b}$, is assumed to be Gaussian, and the field $\delta(x)$ is smoothed by convolving it with a spherical symmetric window function $W(r, R_g)$, where the characteristic scale $R_g$ is chosen so that the enclosed mass matches the halo of a bright galaxy. Galaxy formation sites are identified with peaks rising above a threshold, $\delta > \delta_c$. The value of $\delta_c$ is quite dependent on the
choice of smoothing window used to obtain the dispersion (Lacey & Cole 1994). Using a
top-hat window function $\delta = 1.7 \pm 0.1$, while for a Gaussian window the threshold is sig-
nificantly lower. In non-spherical situation things are more complicated (Monaco 1995).
In any case in the standard biased galaxy formation the threshold is not scale-dependent
and is taken to be universal.
Several studies have shown that there is no convincing justification for this choice (Cen
& Ostriker 1992; Bower et al. 1993; Coles 1993; Del Popolo & Gambera 1998a,b,c; Kauff-
mann et al. 1998; Willmer et al. 1998; Governato et al. 1998; Peacock 1998).
Some authors (see Barrow & Silk 1981; Szalay & Silk 1983 and Peebles 1990) have pro-
posed that non-radial motions would be expected within a developing proto-galaxy due
to the tidal interaction of the irregular mass distribution around them, typical of hier-
archical clustering models, with the neighbouring proto-galaxies. The kinetic energy of
these non-radial motions prevents the collapse of the proto-structure, enabling the same
to reach statistical equilibrium before the final collapse (the so-called previrialization
conjecture by Davis & Peebles 1977, Peebles 1990). In other words one expects that
non-radial motions change the characteristics of the collapse and in particular the turn
around epoch, $t_m$, and consequently the critical threshold, $\delta_c$, for collapse.

As shown by Del Popolo & Gambera (1998a,b,c), if non-radial motions are taken into
account, the threshold $\delta_c$ is not constant but is function of mass, $M$, (Del Popolo &
Gambera 1998a,b):

$$
\delta_c(\nu) = \delta_{co} \left[ 1 + \frac{8G^2}{\Omega_0^2 H_0 r_i^1 \bar{\delta} (1 + \bar{\delta})^3 \int_{a_{min}}^{a_{max}} \frac{L^2 \cdot da}{a^3} \right]
$$

(1)

where $\delta_{co} = 1.68$ is the critical threshold for a spherical model, $r_i$ is the initial radius,
$L$ the angular momentum, $H_0$ and $\Omega_0$ the Hubble constant and the density parameter
at the current epoch, respectively, $a$ the expansion parameter and $\bar{\delta}$ the mean fractional
density excess inside a shell of given radius. The mass dependence of the threshold
parameter, $\delta_c(\nu)$, and the total specific angular momentum, $h(r, \nu) = L(r, \nu)/M_{sh}$,
acquired during expansion, was obtained in the same way as described in Del Popolo &
Gambera (1998b) and is displayed in Fig. [Fig. 1].

In order to find the galaxy correlation function, we combine Kaiser’s (1984) analysis
with the theory of gravitational clustering (Press & Schecter 1974). In this last theory
non-linear clumps are identified as overdensities in the filtered linear density field. When
these overdensities exceed a critical threshold, $\delta_c$, they will be incorporated in a non-
linear object of mass $M \propto R_g^3$ or greater. Since the linear density field is assumed to be
Fig. 1. The specific angular momentum for three values of the parameter $\nu$ ($\nu = 2$ solid line, $\nu = 3$ dotted line, $\nu = 4$ dashed line).

Gaussian, the probability that on scale $M$ one would find a density contrast between $\delta$ and $\delta + d\delta$ would be:

$$p(\delta)d\delta = \frac{1}{\sqrt{2\pi}\sigma(M)} \exp\left[-\frac{\delta^2}{2\sigma(M)^2}\right]d\delta$$

and the Press-Schechter ansatz leads to the following fraction of mass incorporated in objects of mass $> M$ (or the probability that an object of mass $M$ has turned around at any time in the past):

$$P_M = \int_{\delta_c}^{\infty} p(\delta)d\delta = \frac{1}{2} \text{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma}\right)$$

And in a similar way, the probability density for finding simultaneously, on scale $M$, the density contrast $\delta_1 = \delta(x_1)$ and $\delta_2 = \delta(x_2)$ of two field points separated by $r = |x_1 - x_2|$ is:

$$p(\delta_1, \delta_2, \xi) = \frac{1}{2\pi\sqrt{\xi^2(0) - \xi^2}} \times \exp\left[-\frac{\xi(0)\delta_1^2 + \xi(0)\delta_2^2 - 2\xi\delta_1\delta_2}{2(\xi^2(0) - \xi^2)}\right]$$
where
\[ \xi(r) = \frac{1}{2\pi^2} \int_0^\infty P(k)k^2 \sin(kr) \frac{1}{kr} \exp(-k^2 R_g^2) \] (5)
is the mass autocorrelation function of the primeval density distribution, \( \delta_1 \) and \( \delta_2 \) are used to denote the density contrast at the positions \( x_1 \) and \( x_2 \) of the two field points and \( \sigma \) denotes the r.m.s. of \( \delta \). The power spectrum used has the form given by Bardeen et al. (1986):
\[ P(k) = Ak^{-1} \ln (1 + 4.164k)^2 \cdot (192.9 + 1340k + 1.599 \cdot 10^5 k^2 + 1.78 \cdot 10^5 k^3 + 3.995 \cdot 10^6 k^4)^{-1/2} \] (6)
and \( A \) is the normalizing constant, which gives the amplitude of the power spectrum.

Similarly to Bower et al. (1993), since our model, like the original high-peak model, calculates \( \xi_g \) from \( \xi/\sigma^2 \), the amplitude of the power spectrum drops out of our analysis.

The probability of finding two objects of masses \( M \) separated by \( r \) that have turned around in any time between the actual time and \( t = 0 \) is:
\[ P_{MM} = \int_{\delta_c}^\infty \int_{\delta_c}^\infty p(\delta_1, \delta_2, \xi/\sigma^2)d\delta_1d\delta_2 = \]
\[ \frac{1}{2\sigma \sqrt{2\pi}} \int_{\delta_c}^\infty \exp(-\frac{\delta_1^2}{2\sigma^2}) \text{erfc} \left[ \frac{\nu - \xi(0)\delta_1}{\sqrt{2} (1 - \xi^2/\xi^2(0))} \right] d\delta_1 \] (7)
Following Kaiser (1984), the correlation function of peaks above a given threshold, on scales larger than \( R_g \), can be approximated by that of points above the same threshold. According to Kaiser’s (1984) definition we have:
\[ \xi_g(r) = \frac{P_{MM}}{P_M^2} - 1 \] (8)
giving the fractional excess probability that two points at separation \( r \) are both above the threshold. Eq. (8) shows that the \( \xi_g(r) \) is a function of \( \xi(r)/\sigma^2 \) and of \( \delta_c \). In the limit \( \delta_c \to \infty, \xi \to 0 \) we have:
\[ \xi_g(r) \simeq \left( \frac{\delta_c}{\sigma} \right)^2 \xi(r) \] (9)
(Kaiser 1984). This approximation is not so accurate on the scales we are considering, we thus prefer to evaluate \( \xi_g(r) \) numerically after having reduced the dimensionality of the integrals involved in the calculation, as Bower et al. (1993) have done, by means of:
\[ \xi_g(r) = \xi(r) \int_0^1 \left[ \xi(0)^2 - s^2 \xi(r)^2 \right]^{-1/2} \exp \left[ - \frac{\delta_c^2 \xi(0)}{\xi(0) + s \xi(r)} \right] ds \times \left[ \int_{\delta_c}^\infty \exp(-u^2/2)du \right]^{-2} \] (10)
In order to compare our model predictions of large-scale power in galaxy distribution with its estimates from the APM survey (Maddox et al. 1990) we have to calculate the angular
two points autocorrelation function, \( w(\theta) \). This last is related to the spatial correlation function, \( \xi(r) \), through Limber’s (1954) equation (see also Peebles 1980, Peacock 1991):

\[
\frac{w(\theta)}{y^4 \phi^2} = \frac{\xi(\sqrt{x^2 + y^2 \theta^2})}{\int_0^\infty y^2 \phi dy} \left( \int_0^\infty y^4 \phi dy \right)^{-1} 
\]

(11)

where the luminosity function, \( \phi(y) \), is that recommended by Maddox et al. (1990) and where:

\[
\phi(y)dy = \phi^* y^\alpha \exp(-y)dy
\]

(12)

being

\[
y = 10^{0.4(M_\xi(z) - M)}
\]

(13)

and

\[
\phi^* = 1.3 \times 10^{-2}h^3\text{Mpc}^{-3}
\]

(14)

with

\[
M_\xi(z) = M_\xi^* + M_\tau^* z
\]

\[
\alpha(z) = \alpha_0 + \alpha_1 z
\]

\[
M_\xi^* = -19.8
\]

\[
M_\tau^* = 1
\]

\[
\alpha_0 = -1
\]

\[
\alpha_1 = -2
\]

In order to calculated Eq. (11) we need the spatial counterpart \( \xi(r) \) for all \( r \). We have used an approach similar to that by Maddox et al. (1990) and Bower et al. (1993), namely we calculated correlation functions according to what we have previously described in this section but on small scales (\( r \leq 5.7h^{-1}\text{Mpc} \)) we extrapolated our model correlation functions by using \( \xi(r) = \left( \frac{5.7h^{-1}\text{Mpc}}{r} \right)^{1.7} \).

3. Results and discussion

The result of our model is directly compared (see Fig. 3) with the angular correlation function estimate by Maddox et al. (1990) from the APM survey, in order to find whether it can match observed estimates of large-scale power in the galaxy distribution. The data (kindly provided by W. Sutherland) plot the angular correlation function, \( w(\theta) \), for six disjoint apparent magnitude slices between \( 17 \leq b_j \leq 20.5 \), all scaled to the magnitude limit of the Lick catalogue (Groth & Peebles 1977), \( b_j = 18.4 \). At small angles the angular correlation function is a power law:

\[
w(\theta) = B\theta^{1-\gamma}
\]

(15)
For $0.01^\circ < \theta < 1^\circ$ the values of $\gamma$ and $B$ are:

$$\gamma = 1.699 \pm 0.032$$
$$B = 0.0284 \pm 0.0029$$

(Maddox et al. 1996). At larger angles the angular correlation function steepens and lies below the extrapolation of the power law. At magnitude $b_j = 20$ the steepening occurs at $\simeq 2^\circ$.

The dotted line shows the angular correlation function calculated using the SCDM model, then assuming a uniform biasing threshold. Fig. 2 clearly shows the well SCDM known problem of lack of large-scale power, namely the two-point angular correlation function, when fit to the observations on the 0.03-0.3 degree scale, is significantly below the observations on scales greater than 1 degree, when these are scaled to the depth of the Lick survey. This provides strong evidence for large-scale power in the galaxy distribution that cannot be reconciled with the SCDM model. The same conclusion is achieved by analysis of three-dimensional data (Vogeley et al. 1992) and from an independent redshift survey of a subset of the APM-Stromlo galaxies (Loveday et al. 1992). An important point to stress is that this problem is normalization independent, it cannot be solved by changing the bias strength, (Maddox et al. 1990; Bartlett & Silk 1993; Ostriker 1993) because the theoretical $w(\theta)$ has the wrong shape.

The solid line shows the angular correlation function obtained in our model (CDM with $\Omega = 1, h = 0.5$ and taking account of non-radial motions). Our $w(\theta)$ is less steep, at large angles, than that expected from the SCDM model and is in better agreement with observations. The relatively enhanced power on large scales, with respect to smaller scales, is insensitive to the amplitude of the power spectrum: the better agreement with observations is due to the non uniform biasing threshold in our model as given by Eq. (1).

Similarly to what is shown in Del Popolo & Gambera (1998a,b), Fig. 3 shows that the threshold, $\delta_c$, is a decreasing function of the mass, $M$. This means that peaks in more dense regions must have a lower value of the threshold, $\delta_c$, with respect to those of under-dense regions, in order to form structure. In fact, as clearly shown in Fig. 1, the angular momentum acquired by a shell centered on a peak in the CDM density distribution is anti-correlated with density: high-density peaks acquire less angular momentum than low-density peaks (Hoffman 1986; Ryden 1988). A greater amount of angular momentum acquired by low-density peaks (with respect to the high-density ones) implies that these peaks can more easily resist gravitational collapse and consequently it is more difficult for them to form structure. This results in a tendency for less dense regions to accrete less mass, with respect to a classical spherical model, inducing a biasing of over-dense regions toward higher mass. This also explains why the value of $\delta_c$, that a peak must rise above in order to form a structure, is larger for low-mass peaks than high density
Fig. 2. Angular correlation function for galaxies for the APM survey and in CDM models. The data (kindly provided by W. Sutherland) represents estimates of \( w(\theta) \) for six disjoint magnitude slices in the range \( 17 \leq b_j \leq 20.5 \) scaled to the magnitude limit of the Lick catalogue, \( b_j = 18.4 \), published in Maddox et al. 1996. The dashed line shows correlations in the SCDM model while the solid line shows correlations in our model.

The space dependence of the threshold implies also a scale dependence of the bias parameter, \( b \), because the two parameters are connected (see Borgani 1990; Mo & White 1996; Del Popolo & Gambera 1998a,b,c).

Another way of describing the differences between our model and the standard biased galaxy formation is the following. In order to describe the distribution of objects we consider the biased field:

\[
\rho_{\delta_c,R_y}(x) = t[\delta_R(x) - \delta_c]
\]

(16)

where \( t(y) \), the threshold function, relates the biased field \( \rho_{\delta_c,R_y}(x) \) to the background field, \( \rho_{R_y}(x) \). The threshold function, \( 0 \leq t(y) \leq 1 \), gives the probability that a fluctuation of a given amplitude turns out to be revealed as an object, while the threshold level \( \delta_c \) is defined as the value of the density contrast at which a fluctuation has a probability of 1/2 of giving rise to an object. The simplest case is that of the standard biased galaxy
formation in which the selection function, is a Heaviside function \( t(y) = \theta(y) \) and \( \delta_c \) is the limiting height of the selected fluctuations. In this scheme, fluctuations below \( \delta_c \) have zero probability of developing an observable object and fluctuations above \( \delta_c \) have zero probability not to develop an object. As we showed in a previous paper (Del Popolo & Gambera 1998a) one of the effects of non-radial motions is that the threshold function differs from a Heaviside function (sharp threshold), (see Fig. 7 of Del Popolo & Gambera 1998a). This means that objects can also be formed from fluctuations below \( \delta_c \) and that there is a non-zero probability for fluctuations above \( \delta_c \) to be sterile.

The model is similar to the cooperative galaxy formation theory but now there is a simple explanation for the mass dependence of the threshold, \( \delta_c \): it is due to non-radial motions. As we said previously non-radial motions arise from the gravitational interaction of the quadrupole moment of the system with the tidal field of the matter of the neighbouring proto-galaxies (Barrow & Silk 1981; Szalay & Silk 1983; Peebles 1990). The energy connected to these motions enters the equation of spherical collapse thus changing the turnaround epoch and \( \delta_c \). Being Eq. 9 dependent on the threshold, \( \delta_c \),

**Fig. 3.** The threshold \( \delta_c \) as a function of the mass \( M \), for a CDM spectrum \((\Omega_0 = 1, h = 1/2)\), taking account of non-radial motions.
the galaxy correlation function is changed as well. The final result is an increase of the galaxy correlations, $\xi_g$, predicted on large scales.

4. Conclusion

The galaxy two-point correlation function $\xi_g(r)$ has a special place amongst statistics of the galaxy distribution and is a powerful discriminant between distinct models of structure formation in the universe. Several studies (Maddox et al. 1990, Efstathiou et al. 1990; Saunders et al. 1991; Loveday et al. 1992; Maddox et al. 1996) have shown that the $\xi_g(r)$ obtained from an SCDM model, independently from normalization, is difficult to reconcile with the observed $\xi_g(r)$ if the bias is scale independent. In this paper we showed how a scale dependent bias, due to non-radial motions, can reduce the problem of large-scale lack of power in the SCDM model. We calculated the mass dependence of the threshold parameter, $\delta_c$, (due to non-radial motions) and then used it to find the two-points correlation function following Press & Schecter (1974) and Kaiser (1984). This was used to find the angular correlation function, $w(\theta)$, through Limber's (1954) equation. The $w(\theta)$ found in this way was compared with data of the APM survey (Maddox et al. 1990; Maddox et al. 1996). We found a less steep $w(\theta)$ in good agreement with estimates of large-scale power in the galaxy distribution.

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