Students’ number sense on fraction problems

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Abstract. This study aims to reveal how students’ number sense is used for solving fraction problems using a descriptive qualitative study design. Nine seventh grade students in Palembang was involved in this study who were selected purposively. Data were collected using a constructed-response test and interview. In taking the test and interview, participants were accompanied by a teacher who taught at the school, considering that they did not hesitate to communicate with the teacher and participants had studied fractions with that teacher. The instrument in this study consisted of four number sense problems that were related to fractions. This study showed that all participants could only fulfill one number sense component, namely the number of operations. The students were capable of operating fractions, but they did not possess the conceptual understanding.

1. Introduction

Merdeka Belajar (Freedom to Learn) is a new policy on education in Indonesia. Merdeka Belajar demand that in 2021 National Examination will be replaced with the Minimum Competencies Assessment. One of the cognitive aspects of the Minimum Competencies Assessment is mathematical literacy. Mathematical literacy is the ability of the student to analyze and use numbers [1]. Moreover, it can also be interpreted as applying number concepts and arithmetic operations in everyday life [2]. Referring to the need for mathematical literacy for everyday life, students need to understand numbers and apply them to solve everyday problems. A person’s understanding of numbers and their ability to solve daily problems are related to the number sense [3].

Number sense is students’ general understanding of numbers and operations, which can flexibly solve mathematical problems [4]. It is not a calculation method but rather a kind of sensitivity to numbers [5]. It is also useful as the basis of all curricula of Mathematics, and it is also used to understand the concepts of measurement, algebra, geometry, and also data analysis [6]. Number sense is the key to understand mathematical concepts that influence students Mathematics learning [7], and it is also one of the most needed skills for successful Mathematics learning [8]–[9]. Therefore, number sense is considered essential and must be taught to students.

Number sense is not a new thing in Indonesia. Several studies on number sense conducted in Indonesia revealed that students’ number sense abilities were still low [10]–[13], but another study found that the students’ number sense abilities were sufficient [14]. These differences in research results related to students’ number sense in Indonesia is the reason why it is important to conduct a replication research related to number sense. Discussing number sense means that discussing understanding numbers.
Understanding numbers is not only knowing arithmetic but beyond that, such as understanding the numbers' properties and knowing the relations between numbers [5].

Students have studied numbers since the beginning of school and learning fractions in the third grade of elementary school. Although students have studied fractions, many students still did not understand fractional operation's standard procedures [5]. Therefore, the fraction is an essential subject in learning Mathematics because it is used in almost all Mathematics subject. The fraction is also an essential matter in everyday life because most of our daily lives cannot be separated from using fractions, such as cooking, carpentry, sewing, and others.

Studies related to number sense with fractions have been conducted in Indonesia [15]–[18]. The studies were intended to describe students' number sense in cognitive visualizer style [16], student's number sense from an introvert personality point of view [18], student's number sense in object imagery, spatial imagery and verbal [15], and students' improvement on fractions using number sense learning [17]. However, the previous studies did not focus on how students' number sense solved fraction problems based on number sense components. This study aims to reveal how students' number sense on fraction problems. This study will analyze how students' number sense in solving fraction problems based on number sense components by providing a constructed-response test with fraction content. This study can be used as a reference for the teacher to determine the appropriate learning on fractions to facilitate students' various number sense abilities, and it also can be used as a reference for other researchers in conducting a further or similar study.

2. Methods
This study used a descriptive qualitative study design. Nine seventh grade students in Palembang was involved in this study. The participants in this study were selected purposively. The participants' determination was with assistance from the teacher who taught at that school, considering that they wanted to communicate because they would do an interview. Also, the students have studied fractions. Then, the students suggested by the teacher to be the participants were asked about the students' willingness to became participants in this study. Finally, only nine students were willing to be participants in this study. This study was conducted to reveal how the students' number sense on fraction problems based on number sense components. Interviews and a constructed-response test were conducted to collect the data.

Moreover, this study's instrument consisted of four number sense questions related to fractions adapted from Menon [19] and Purnomo et al. [13]. During the data collection process, each participant was given 40 minutes to complete the test. The number sense components used as the framework in this study were adapted from McIntosh et al. [4]. The indicators of each number sense components can be seen in table 1.

### Table 1. The Indicators of Number Sense.

| Components of Number Sense | Indicators                                      |
|----------------------------|-------------------------------------------------|
| Sense of numbers orderliness (number meaning) | Capable of using fraction concept               |
| Multiple representations for numbers (number representation) | Capable of determining fraction representation |
| Sense of relative and absolute magnitude of numbers (number magnitude) | Capable of comparing fraction                   |
| Understanding mathematical properties (number of operations) | Capable of doing fraction operation flexibly    |

3. Result and Discussion
This study discussed students' number sense based on students' answers to the constructed-response test questions and the interviews. The result of the test indicated that students only fulfill one component of the number sense. Figure 1 demonstrates students' answers to a question on first component, Number Meaning.
How many different fractions between 2/5 and 3/5? Circle your answer and write the reason why you chose that answer.

a. None. Why?
b. Only one. What is it?
c. A few. Give two examples.
d. Many. Give three examples.

Student's answer:
Because after 2/5 is 3/5

Figure 1. Student's answer in number meaning question.

As shown in figure 1, the answer illustrated that the student did not understand the equivalent concept of fractions. He/she considered fractions' concept similar to the whole number principle that three is followed by two. Hence, there was no fraction between 2/5 and 3/5. All of the participants in this study answered the same as that. When students were asked why they stated that three is after two, then 2/5 and 3/5 were consecutive fractions. Hence, there was no other fraction between 2/5 and 3/5. It can be seen that they did not understand the concept of fractions. This finding is in line with the findings of a study by Dyson et al. [20], which stated that students misplaced the concept of the whole number of principles to fractions. Figure 1 and the interview findings could be inferred that students did not comply with the number-meaning indicator because they did not use the fraction concept.

Figure 2 shows students' answers to the number representation question. As shown in figure 2, the student's answer specified that he did not understand the concept of fractions. He decided the improper fraction for the shaded area, which was 1/4 because there was an area shaded from the four parts. All students in this study noticed the fraction concept by counting all of the parts as the denominator, and the numerator was the shaded part without considering each part's size. They forgot that fractions should represent equal parts of a whole, but the picture had different size parts.

Moreover, based on the interview result, all the students answered: 1/4 since the shaded area was one out of four parts. They used an incorrect strategy to determine the fraction of the shaded area. These findings are supported by the findings of studies by Jordan et al. [7] and Newton et al. [21], stating that students had a limited understanding of the fraction concept. From figure 2 and the interview findings, it could be inferred that students did not comply with the number-representation indicator because they could not determine the fraction representation correctly.
What’s a fraction that represented the following figures and explained your answer

Student’s answer:
1/4 because there are four parts of that square, and the shaded area only one.

Figure 2. Students’ answer in number representation question.

The student’s answer on the third indicator, number magnitude, is represented in figure 3.

Which is the largest fraction? Circle your answer. Explain why you choose that answer.

Student’s answer:
Because usually, the fraction with a smaller numerator and denominator will be the larger fraction.
Student’s answers in figure 3 indicated that students did not understand fractions’ concept. From the interview results, students answered that they did not understand how to determine the greater fraction for the case in figure 3. This finding is supported by Rinne and Jordan [22] and Stafylidou and Vosniadou [23], stating that students had a limited understanding of the fraction concept. Referring to figure 3 and the interview result, students’ number magnitude could not be achieved because they did not understand fractions, resulting in the wrong answer.

The student’s answer in the next indicator, number of operations, is portrayed in figure 4.

![Image of the student's answer in figure 4]

**Translation:**

Which is larger fraction, \( A = \frac{1}{3} \times \frac{1}{5} \times \frac{1}{7} \) or \( B = \frac{1}{15} \times \frac{1}{28} \)? Explain your answer.

**Student’s answer:**

\[
A = \frac{1}{3} \times \frac{1}{5} \times \frac{1}{7} \\
= \frac{1}{3} \times \frac{1}{5} = \frac{1}{15} \\
= \frac{1}{4} \times \frac{1}{7} = \frac{1}{28}
\]

\[
B = \frac{1}{15} \times \frac{1}{28}
\]

jadi A = B karena sama \( \frac{1}{15} \times \frac{1}{28} \)

**Figure 4.** Student’ answer in number of operations question.

Figure 4 indicated that the student could operate fractions flexibly; The students provided a way and explained the answer. Based on an interview with a student, he could explain how to find the answer. An interview transcript between the student and the researcher is presented below.
Researcher: Okay, your answer is A = B. Why did you answer that? Why was the way like this? Why didn't you just multiply all the fractions on the A and B separately and compare the result?

Participant: Because I saw the fractions just like have the relation, just like here three times five is 15, then four times seven is 28, so A would be 1/15 times 1/28, it was similar to B. So, A = B. If I multiply all the fractions on the A and B separately and compare the result, it would take a long time. So, I think the answer could be seen from how I was finding the answer.

The interview results showed that the students could flexibly operate the fractions and explain his answers. It can be affirmed that the students fulfilled the number of operations indicators.

All participants could only fulfill a number sense component, namely the number of operations. They were able to operate fractions, but they did not have an adequate understanding to solve questions on fractions' concepts. This finding is supported by previous research, which found out that students could answer questions that demanded careful counting without requiring deep understanding [14]. Moreover, solving number sense requires not only using formulas or performing operations but also understanding the concept.

Based on the interviews with students, it can be inferred that they were not familiar with number sense questions, which required them to give reasons similar to those in the test. They were too embarrassed to ask questions during the learning process since it could be ridiculous for other students. Based on this finding, teachers are encouraged to regularly give more activities that promote students’ number sense. Teacher need to consider using learning models or methods that facilitate various students’ number sense and make students convenient during the learning so that they do not feel embarrassed to ask questions and contribute actively in class.

4. Conclusion
According to the analysis of the constructed-response test and interview results, the students only fulfill one number sense component, namely the number of operations. The students were capable of operating fractions, but they had difficulty conceptual understanding because students were not accustomed to solving problems, which required them to provide reasons.

However, this study did not analyze whether students did not fulfill some of the number sense components because of students' mathematical prior knowledge, unfamiliarity with the questions that demanded reasons, or inappropriate learning models/methods used in the class. Other researchers suggested analyzing whether students' mathematical prior knowledge or the learning models/methods used in the classroom affect students' number sense.

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Acknowledgments
The authors would like to deliver the most generous gratitude to Indonesia Endowment Fund for Education or Lembaga Pengelola Dana Pendidikan (LPDP) for the financial support during the first author’s master study program.