Inflation from periodic extra dimensions

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Abstract. We discuss a realization of a small field inflation based on string inspired supergravities. In theories accompanying extra dimensions, compactification of them with small radii is required for realistic situations. Since the extra dimension can have a periodicity, there will appear (quasi-)periodic functions under transformations of moduli of the extra dimensions in low energy scales. Such a periodic property can lead to a UV completion of so-called multi-natural inflation model where inflaton potential consists of a sum of multiple sinusoidal functions with a decay constant smaller than the Planck scale. As an illustration, we construct a SUSY breaking model, and then show that such an inflaton potential can be generated by a sum of world sheet instantons in intersecting brane models on extra dimensions containing orbifold. We show also predictions of cosmic observables by numerical analyzes.

Keywords: inflation, axions, extra dimensions

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1 Introduction

The cosmic microwave background (CMB) observation excellently supports the presence of cosmic inflation, which is an exponential expansion of space in the early universe [1–5]. The rapid expansion of the inflation scenario can solve the cosmological problems associated with two fine-tunings in the standard cosmology, i.e., the horizon problem and the flatness problem, simultaneously. Further, the fluctuation of the CMB temperature is explained by that of an inflaton scalar field, which drives the inflation. The scenario has been steadily tested in recent cosmic observations. In particular, slow-roll inflation scenario agrees with recent precise observations. The Planck observation shows the values of the spectral index and its running [6, 7],

\[
\begin{align*}
    n_s &= 0.9655 \pm 0.0062 \quad (68\% \text{ CL}), \\
    \alpha_s &= \frac{dn_s}{d\ln k} = -0.0033 \pm 0.0074 \quad (68\% \text{ CL}).
\end{align*}
\]

The observation also constrains the tensor-to-scalar ratio as

\[ r < 0.10 \quad (95\% \text{ CL}). \] (1.3)

These imply that the fluctuation of the CMB is almost scale invariant and the primordial gravitational wave caused by the inflation is not found yet at the current stage. Then, inflation models have been strictly classified by these facts.

An approximate shift symmetry of the inflaton

\[ \phi \rightarrow \phi + 2\pi f, \] (1.4)

often plays important roles in controlling the flatness of the scalar potential [8–12]. Here, \( f \) is a decay constant of an inflaton field \( \phi \). Natural inflation [8] has often been considered so far, and the inflaton potential is given by \( V = \Lambda^4 (1 - \cos (\phi/f)). \) This model is controlled...
by the above discrete shift symmetry of the inflaton. In natural inflation, it is known that natural inflation is consistent with the recent Planck results only for super-Planckian values of a decay constant, \( f \gtrsim 7M_P \) at 2\( \sigma \) level \([7]\), where \( M_P = 2.4 \times 10^{18} \) GeV is the Planck scale. (Hereafter we adopt the Planck unit \( M_P = 1 \) unless otherwise stated.) Some extensions of the natural inflation are proposed with a focus on the weak gravity conjecture \([13-16]\). Among them, one of extensions of natural inflation is to prepare multiple sinusoidal potentials:

\[
V = \sum_i \Lambda_i^4 \cos \left( \frac{\phi_i}{f_i} + \theta_i \right) + V_0. \tag{1.5}
\]

An overlapping of multiple sinusoidal potentials provides locally flatter regions where slow-roll inflation can successfully take place even though the decay constant takes a sub-Planckian value. Such models with multiple sinusoidal potentials are called multi-natural inflation \([17-19]\).

In this paper, we consider an origin of such multiple sinusoidal potentials and a relevant symmetry from the viewpoint of string inspired supergravity (SUGRA). In theories accompanying extra dimensions, compactification of them with small radii is required for realistic situations. Since the extra dimension can have a periodicity, there will appear (quasi-)periodic functions under transformations of moduli relevant to the extra dimensions in low energy scales. Such a periodic property can lead to a UV completion of so-called multi-natural inflation model. We show an illustrating example in type II superstring theories, i.e., intersecting/magnetized D-brane scenarios compactified on extra dimensions containing the \( T^2/\mathbb{Z}_2 \) orbifold. It is known that in toroidal compactification Yukawa couplings can be expressed by one of elliptic functions \([28, 29]\), which consist of multiple sinusoidal functions.\(^2\) Then, a Neveu-Schwartz (NS) axion plays an important role in driving cosmic inflation. Such an axion possesses the shift symmetry which originates from the original gauge symmetry for the NSNS two-form gauge field coupled to fundamental strings, and reflects on the periodicities of the extra dimensions after the compactification.\(^3\) This can be understood also by using string dualities. NS axions in type IIA are known to be mapped into (a part of) complex structures in type IIB via T-duality (mirror symmetry). In the F-theory, seven brane positions, which are coordinates on the (periodic) extra dimension, are treated as complex structures in an unified manner. Such axion shift symmetries can be related to periodicities or modular invariance in this sense.

This paper is organized as follows. In section 2, we review Jacobi’s theta functions which consist of multiple cosine functions. In section 3, we briefly review intersecting/magnetized D-brane scenarios in type II superstring theories, and construct an inflationary model by using a dynamical supersymmetry (SUSY) breaking model in hidden sector. In section 4, we show theoretical predictions of tensor-to-scalar ratio \( r \), spectral index \( n_s \) and its running \( \alpha_s \). Section 5 is devoted to conclusions and discussions.

## 2 Jacobi’s theta function

In this section, we review the Jacobi’s theta function and its mathematical properties. Subsequently, we discuss a relation between the Jacobi’s theta function and multi-natural inflation.

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1. In bottom-up approaches, phenomenological aspects of such multiple sinusoidal potentials for an axion dark matter have been investigated in refs. \([20-24]\). See also refs. \([25-27]\) for relaxion models.
2. It is also known that gauge coupling constants are written by the theta functions \([30]\).
3. A similar concept is studied also in ref. \([31]\) with a focus on modular invariance of the Dedekind eta function in a bottom-up approach.
The Jacobi’s theta function is known as one of elliptic functions which possesses quasi-periodicities, and its definition is given by

\[
\vartheta \left[ \begin{array} {c} a \\ b \end{array} \right] (\nu, \tau) = \sum_{l=-\infty}^{\infty} e^{\pi i (a+l)^2 \tau} e^{2\pi i (a+l)(\nu+b)}. \tag{2.1}
\]

Note that the Jacobi’s theta function has four kinds of arguments, \(a, b, \nu\) and \(\tau\). \(a \) and \(b\) take real values, and \(\nu\) and \(\tau\) are complex parameters. It is easily found that this function only converges if \(\text{Im} \ \tau > 0\), otherwise this function is not well-defined. Also, the Jacobi’s theta function is quasi-periodic under the transformations of \(a, b\) and \(\nu\),

\[
\vartheta \left[ \begin{array} {c} a+1 \\ b \end{array} \right] (\nu, \tau) = \vartheta \left[ \begin{array} {c} a \\ b \end{array} \right] (\nu, \tau), \quad \vartheta \left[ \begin{array} {c} a \\ b+1 \end{array} \right] (\nu, \tau) = e^{2\pi i a} \vartheta \left[ \begin{array} {c} a \\ b \end{array} \right] (\nu, \tau), \tag{2.2}
\]

\[
\vartheta \left[ \begin{array} {c} a \\ b \end{array} \right] (\nu+1, \tau) = e^{2\pi i a} \vartheta \left[ \begin{array} {c} a \\ b \end{array} \right] (\nu, \tau), \quad \vartheta \left[ \begin{array} {c} a \\ b \end{array} \right] (\nu+\tau, \tau) = e^{-2\pi i (b+\nu+\tau/2)} \vartheta \left[ \begin{array} {c} a \\ b \end{array} \right] (\nu, \tau). \tag{2.3}
\]

Under the \(\text{SL}(2, \mathbb{Z})\) transformation

\[
\tau \rightarrow \frac{a\tau + b}{c\tau + d}, \tag{2.4}
\]

where \(ad - bc = 1\), \(a, b, c, d \in \mathbb{Z}\), the theta function shows similar properties, for instance,

\[
\vartheta \left[ \begin{array} {c} a \\ b \end{array} \right] (\nu, \tau + 1) = e^{-\pi i (a^2-a)} \vartheta \left[ \begin{array} {c} a \\ a+b-\frac{1}{\tau} \end{array} \right] (\nu, \tau), \tag{2.5}
\]

\[
\vartheta \left[ \begin{array} {c} a \\ b \end{array} \right] \left( \frac{\nu}{\tau}, -\frac{1}{\tau} \right) = (-i\tau)^{1/2} e^{2\pi i (\nu^2/2\tau+ab)} \vartheta \left[ \begin{array} {c} a \\ -a \end{array} \right] (\nu, \tau). \tag{2.6}
\]

This is often called modular transformation of \(\tau\). The shift of \(\tau\) plays a role to a shift symmetry of an inflaton in our case.

As shown in eq. (2.1), the Jacobi’s theta function consists of infinite summation of sinusoidal functions weighted by a complex Gaussian factor \(e^{\pi i \tau}\). In ref. [32], the authors showed that the slow-roll inflation can take place with the potential given by

\[
V = \frac{\Lambda^4}{2} e^{-\pi i \tau} \left( \vartheta \left[ \begin{array} {c} 0 \\ 0 \end{array} \right] (0, \tau) - \vartheta \left[ \begin{array} {c} 0 \\ 0 \end{array} \right] \left( \frac{\phi}{2\pi f}, \tau \right) \right), \tag{2.7}
\]

where \(\Lambda\) denotes an inflation scale and \(\phi\) is an inflaton field. The shape of the inflaton potential is shown in figure 4 of ref. [32] and is symmetric under the shift of inflaton, \(\phi \rightarrow \phi + 2\pi f\). For large values of \(\text{Im} \ \tau\), natural inflation is realized, while for smaller values of \(\text{Im} \ \tau \ (\gtrsim 1)\), hilltop inflation model [33] is obtained. In the latter case, sub-leading cosine parts are enhanced and considerably contribute to the shape of the inflaton potential. As a result, one finds a quartic potential of the hilltop inflation, i.e.,

\[
V \simeq V_0 - \lambda \phi^4, \tag{2.8}
\]

where \(V_0\) and \(\lambda\) are constants. This potential predicts a small value of tensor-to-scalar ratio in a region consistent with the Planck 2015 results [7]. Since the above potential seems to be specific, there are other possibilities, for instance, that \(\text{Re} \ \tau\) becomes an inflaton candidate instead of \(\text{Re} \ \nu\). In the next section, we show an origin of the Jacobi’s theta function based on intersecting/magnetized D-brane scenarios.
3 Intersecting/magnetized D-branes

In this section, we briefly review the framework of intersecting/magnetized D-brane models compactified on tori, which are parts of the extra dimensions. After a short review, we concretely provide an inflaton potential including compactification moduli fields.

3.1 Bi-fundamental fields and Yukawa couplings

We suppose that the ten-dimensional spacetime consists of four-dimensional spacetime and six-dimensional extra dimensions and also that the latter is decomposed into a two-dimensional torus (or toroidal orbifold) and certain four-dimensional extra dimensions, e.g., $T^2 \times X_4$. Later, $T^2$ is replaced with toroidal orbifold $T^2/\mathbb{Z}_2$. In the following part, we assume that the compactification preserves the four-dimensional $\mathcal{N} = 1$ SUSY. We try to concretely construct an inflationary model only in the intersecting brane picture, since the intersecting brane model based on type IIA superstring theory is connected to the magnetized brane model based on type IIB superstring theory via T-duality \cite{28,29}. In intersecting brane models, there is a remarkable mechanism of generation of chiral matters which are bi-fundamental representations under the gauge groups realized on spacetime-filling D-branes. Intersections of two stacks of such D-branes can generate such matters, their degeneracies and Yukawa coupling constants among them.\footnote{See ref. \cite{34} for a review, and references therein.}

Hereafter, we will focus on spacetime-filling D6-branes. The zero-modes of openstring NS and Ramond (R) sectors (bosons and fermions) appear at points where a stack of $N_a$ D6$_a$-branes and a stack of $N_b$ D6$_b$-branes intersect each other. Such zero-modes correspond to supermultiplets transformed as bi-fundamental representation of $(N_a, \bar{N}_b)$ or $(\bar{N}_a, N_b)$ under SU($N_a$) × SU($N_b$) gauge group realized on the respective D-branes in the four-dimensional SUSY effective theory. In addition, the degeneracy of such bi-fundamental supermultiplets is given by the number of intersection points between D6$_a$-brane and D6$_b$-brane, $I_{ab}$. In the following part, we focus on a single $T^2$, on which D6-branes are wrapping on 1-cycles, for simple explanation. D6-branes are wrapping also on 2-cycles on $X_4$, such that $\mathcal{N} = 1$ SUSY is preserved and chirality is correctly generated.

Once any intersection numbers in the model are fixed, the number of bi-fundamental supermultiplets and their quantum numbers are also determined. Then, Yukawa couplings among such three bi-fundamental supermultiplets can be analytically written by the Jacobi’s theta function on $T^2$ \cite{28,29}, i.e.,

\[
y^{(T^2)}_{ijk} \sim \theta \left[ \begin{array}{c} I_{ij} \\ 0 \end{array} \right] (\varphi, \kappa),
\]

where we neglected an overall factor and three quantities dependent on compactification moduli fields are given by

\[
\delta_{ijk} = \frac{i}{I_{ab}} + \frac{j}{I_{bc}} + \frac{k}{I_{ca}} + \frac{s}{d},
\]

\[
\varphi = \frac{I_{ab}\theta_c + I_{ca}\theta_b + I_{bc}\theta_a}{d},
\]

\[
\kappa = \frac{|I_{ab}I_{bc}I_{ca}|}{d^2}(B + iA).
\]
This is a consequence of sums of world sheet instantons induced by windings of open strings stretching among intersecting D-branes. Here, $\theta_X (X = a, b, c)$ denote Wilson-line phase fields, $A$ is the torus area (the Kähler modulus) field and $B$ is so-called NSNS axion field. These compactification moduli fields are candidates of an inflaton, however, we will regard the $B$ axion as an inflaton as seen later. Flavor indices $i, j$ and $k$ label each of matter contents, e.g., $i = 0, 1, 2, \ldots, |I_{ab}| - 1$. Also, we define $d = \gcd (I_{ab}, I_{bc}, I_{ca})$, $s \equiv s(i, j, k) \in \mathbb{Z}$ is a linear function on integers $i, j$ and $k$. The form of the Yukawa couplings reflects on the modular invariance of the toroidal model under the transformation (2.4) with $\tau \equiv B + iA$ in a T-dualized picture of refs. [35, 36].

It should be noted that all of elements in eq. (3.1) can not posses non-vanishing values. Whether the Yukawa elements are vanishing or not depends on coupling selection rules which are characterized by extra dimensional topologies in string theories,

$$i + j + k \equiv 0 \pmod{d}.$$  (3.5)

In toroidal compactifications, it is known that selection rules lead to discrete flavor symmetries [41]. After imposing selection rules, summing up all non-vanishing elements with respect to $s$ provides the general form of Yukawa couplings in intersecting D-brane models. In the next subsection, we will see that an inflaton potential appears through Yukawa couplings in the presence of strong dynamics. Indeed, such a situation can be realized in a model of dynamical SUSY breaking.

Finally, we comment on the orbifold extensions. In the following, we will treat the intersecting D-branes wrapping on the fixed points of the toroidal orbifold $T^2/\mathbb{Z}_2$. It is known that the D-branes wrapping on the rigid cycles can not move freely in toroidal compact space and opens string moduli are frozen. A merit in considering the rigid cycles is that the position moduli associated with the D-brane positions are stabilized. The same holds for the Wilson line phases. The allowed values of the Wilson line phases are summarized in the T-dual side [42]. For simplicity, we assume all the Wilson line phases are vanishing in the following parts. Since an extension to the non-vanishing cases is straightforward, we do not touch such cases in this paper. Thus, the orbifold model building is more useful in reducing the extra degree of freedom of light fields in the low energy effective theory. Note that the physical open string zero-modes on the rigid cycles can be expressed as linear combinations of the zero-modes appearing at the intersection points of the D-branes on the bulk of the original two-dimensional torus. Accordingly, the Yukawa couplings on the rigid cycles are also given by linear combinations, as shown in the next subsection.

### 3.2 Inflaton potential via IYIT mechanism

To obtain an inflationary potential energy, we use the Izawa-Yanagida-Intriligator-Thomas (IYIT) mechanism [43, 44]. It is known as a model of dynamical SUSY breaking. In the

\[A\] is normalized by string length, hence this is dimensionless.

\[\text{See refs.} \ [36-40] \text{for heterotic string cases.}\]

\[\text{If open string moduli were alive, they would tend to have a slightly steeper slope of the potential than that of } B \text{ in our case. When the potential consists of combinations of the theta function, we find that } \partial_B V \sim \partial_B \theta \sim \delta^2 \text{ and } \partial_B V \sim \partial_B \theta \sim \delta \text{ with } \delta < 1.\]
In this model, the gauge symmetry

We show an example of D-brane configuration in figure 1. In this figure, we take $(n_a, m_a) = (1, 0)$, $(n_b, m_b) = (1, -5)$ and $(n_c, m_c) = (1, 5)$. Then, one finds that

\[ I_{ab}^{(T^2)} = -5, \quad I_{bc}^{(T^2)} = 10 \quad \text{and} \quad I_{ca}^{(T^2)} = 5 \quad \text{only on} \ T^2, \]

where $I_{ab}^{(T^2)} = n_a m_b - m_a n_b$ and so on. To obtain Lorentz invariant Yukawa couplings, there must exist contributions of intersection numbers of $I_{ab}^{(X)} = \pm 1$, $I_{bc}^{(X)} = \pm 1$ and $I_{ca}^{(X)} = \mp 1$ also from $X_4$. Here, total intersection number is given by, e.g., $I_{ab} = I_{ab}^{(T^2)} I_{ab}^{(X_4)}$. Also, we may have other moduli contributions to Yukawa couplings from $X_4$, and such moduli are neglected in this paper with an assumption that they are stabilized by, e.g., fluxes on $X_4$ or there are no contributions.

\[ N_a = 2, \quad I_{ab} = |I_{ca}| = 5, \quad N_b = N_c = 1, \quad I_{bc} = 10. \] 

We show an example of D-brane configuration in figure 1. In this model, the gauge symmetry $U(2)_a \times U(1)_b \times U(1)_c \simeq SU(2)_a \times U(1)_b \times U(1)_c$ is realized by the stacks of such D-branes. The indices denote gauge groups on corresponding D-brane, i.e., $U(2)_a$ gauge theory is living on the $D_a$-brane. The $SU(2)_a$ gauge symmetry on the $D_a$-branes plays an important role in strong dynamics. $U(1)$ groups are neglected below since some of them will become anomalous symmetry. On the bulk of the original $T^2$, there are the five zero-modes for the $SU(2)_a$ doublets $|i\rangle_X$ $(i = 0, 1, \ldots, 4)$ and $|j\rangle_Y$ $(j = 0, 1, \ldots, 4)$. The former originates from intersection points between $D_a$- and $D_b$-branes, and the latter comes from ones between $D_c$- and $D_a$-branes. In addition, there exist also the ten zero-modes for the $SU(2)_a$ singlets $|k\rangle_{\Phi}$ $(k = 0, 1, \ldots, 9)$. These originate from intersection points between $D_c$- and $D_c$-branes. Here, $|0\rangle_X = |5\rangle_X$, $|0\rangle_Y = |5\rangle_Y$ and $|0\rangle_{\Phi} = |10\rangle_{\Phi}$. These zero-mode states transform under the $\mathbb{Z}_2$ orbifold projection as

\[ \mathbb{Z}_2 : |x\rangle \rightarrow |5 - x\rangle \quad \text{for} \ X, Y, \quad |y\rangle \rightarrow |10 - y\rangle \quad \text{for} \ \Phi. \]

Then, we can find the physical states on the rigid cycles as

\[ X_1 = \frac{1}{\sqrt{2}} (|1\rangle_X - |4\rangle_X), \quad X_2 = \frac{1}{\sqrt{2}} (|2\rangle_X - |3\rangle_X), \]

\[ Y_3 = \frac{1}{\sqrt{2}} (|1\rangle_Y - |4\rangle_Y), \quad Y_4 = \frac{1}{\sqrt{2}} (|2\rangle_Y - |3\rangle_Y), \]
In final, we exhibit the matter fields in the IYIT mechanism in for the SU(2) doublets and $K$ and where we define a shorthand notation, for the SU(2) singlets. The non-vanishing elements of the Yukawa couplings under consideration can be concretely written in ref. [46].

Rigid cycles can be analytically calculated by taking the linear combinations of the Yukawa couplings $y_{ijk}$ on the bulk in eq. (3.1). For example, $y_{131}$ is written as

$$y_{131} = \frac{1}{2}(y_{110}^{(T^2)} - y_{410}^{(T^2)} - y_{140}^{(T^2)} + y_{440}^{(T^2)}).$$

In the right hand side, four elements of the Yukawa couplings on the bulk are obtained by plugging each of flavor indices $i, j$ and $k$ into eq. (3.1). Thus, by using the selection rules (3.5), non-vanishing elements of the Yukawa couplings under consideration can be concretely written as

$$y_{131} = \sqrt{2}y_{145} = \sqrt{2}y_{235} = -(\eta_{10} + \eta_{40} + \eta_{60} + \eta_{90} + \eta_{110}),$$

$$y_{241} = -\sqrt{2}y_{143} = -\sqrt{2}y_{233} = -(\eta_{20} + \eta_{30} + \eta_{70} + \eta_{80} + \eta_{120}),$$

$$y_{136} = \sqrt{2}y_{142} = \sqrt{2}y_{232} = -(\eta_{15} + \eta_{35} + \eta_{65} + \eta_{85} + \eta_{115}),$$

$$y_{246} = -\sqrt{2}y_{144} = -\sqrt{2}y_{234} = -(\eta_{5} + \eta_{45} + \eta_{55} + \eta_{95} + \eta_{105}),$$

where we define a shorthand notation,

$$\eta_N \equiv \begin{bmatrix} N/250 \\ 0 \end{bmatrix}(0, 10(B + iA)).$$

Table 1. IYIT matter fields induced by the intersection numbers (3.7). The representation under SU(2)$_a$ × U(1)$_a$ × U(1)$_b$ × U(1)$_c$ is shown.

| Intersection | Degeneracy | Representation | Field |
|--------------|------------|----------------|-------|
| $(a, b)$     | 2          | $2_{(1,-1,0)}$ | $X_I$ ($I = 1, 2$) |
| $(a, c)$     | 2          | $2_{(-1,0,1)}$ | $Y_J$ ($J = 3, 4$) |
| $(b, c)$     | 6          | $1_{(0,1,-1)}$ | $\Phi_K$ ($K = 1, 2, \ldots, 6$) |

9The detail of these physical states is written in ref. [46].
The other entries in the Yukawa couplings are all vanishing due to the selection rules in eq. (3.5).\textsuperscript{10} As noted already, our model will be modular invariant at the current stage \cite{35}, because no fields develop VEVs. At the energy scale below a dynamically generated scale \( \Lambda \), the low energy effective theory can be described by gauge-invariant meson fields \( V_{IJ} \sim X_I X_J \), and their effective superpotential is given by

\[ W_{\text{eff}} = y_{IJK} \Phi_K V_{IJ}, \]  

(3.21)

with

\[ \text{Pf} V_{ij} = \Lambda^4. \]  

(3.22)

The Pfaffian of the meson fields is expanded into the following form,

\[ \text{Pf} V_{ij} = V_{12} V_{34} - V_{13} V_{24} + V_{14} V_{23}. \]  

(3.23)

To evaluate the F-term scalar potential, we may choose a local minimum of three discrete local minima

\[ \text{Pf} V_{ij} = V_{14} V_{23} = \Lambda^4, \]  

(3.24)

with the other mesons’ VEVs vanishing. Choosing the above local minimum, we rewrite the effective superpotential (3.21) as

\[ W_{\text{eff}} = \Lambda^2 \sum_{K=1}^{6} (y_{14K} \Phi_K + y_{23K} \Phi_K). \]  

(3.25)

The lightest mode in the IYIT sector is SUSY breaking pseudo modulus, which is a linear combination of \( \Phi_K \), and obtains the mass of order \( \Lambda/4\pi \) at quantum level. Masses of other heavier modes are of order \( \Lambda \). After integrating out them, we consequently obtain the following F-term scalar potential,

\[ V \simeq \sum_{K=1}^{6} \left| \frac{\partial W_{\text{eff}}}{\partial \Phi_K} \right| = \Lambda^4 \sum_{K=1}^{6} |y_{14K} + y_{23K}|^2, \]  

(3.26)

where we set \( |V_{13}| = |V_{24}| = \Lambda^2 \), for simplicity. Also, we neglect the supergravity effect and the VEVs of the SU(2) singlets, \( \langle \Phi_K \rangle \ll \Lambda \). Note that the F-term scalar potential (3.26) depends on the area of the torus \( A \) and the NSNS axion \( B \) in addition to \( \Lambda \), because Yukawa couplings do on such moduli fields. (See eqs. (3.4) and (3.14) – (3.20).) Although the above-mentioned calculation has been based on the intersecting D-branes in the IIA superstring, we can eventually obtain the same scalar potential in T-dualized IIB superstring side since the calculation will be equivalent to that in the magnetized D-branes.

At this stage, modular invariance will be broken down because meson fields develop VEVs, if closed string moduli at the SU(2) dynamical scale \( \Lambda \) are stabilized. If not, there will exist modular invariance owing to a non-linear transformation of the moduli \cite{35}, however, inflation is not viable then because the closed string moduli disturb slow-roll condition. At any rate, we assume that the closed string moduli at \( \Lambda \) is stabilized during the inflation with flux compactifications \cite{47, 48}, and hence modular invariance is broken down in the presence of meson VEVs as a consequence. In other words, modular invariance of an original toroidal orbifold (in a T-dualized description) is broken down in the presence of D-branes. This is

\textsuperscript{10}See refs. \cite{45, 46} for details.
an analogue to an example that an original continuous shift symmetry of axion (inflaton) is broken down to discrete one in the presence of instantons for natural inflation models.

Since the $B$-field appears as a dimensionless field, we should normalize it as

$$B \rightarrow \frac{\phi}{f}$$

with an axion decay constant $f$. Here, $\phi$ is a canonically normalized axion. $f$ may be expected to be of order of a compactification scale, because the NSNS $B$-field is a kind of closed string modulus field of toroidal compactifications. Therefore, the scalar potential depends on $B$, $A$, $f$ and $\Lambda$.

### 3.3 Gauge threshold corrections

In the previous subsection, we obtained a scalar potential including an inflaton candidate of the $B$-field via Yukawa couplings with the IYIT mechanism. It is known that there is the moduli dependence also in gauge couplings [30, 49, 50]. Then, we include a gauge threshold correction to the dynamical scale $\Lambda$ in the SU(2) gauge theory.

Hereafter, we take account of a one-loop correction to a gauge kinetic function on the stacks of specific D-branes, e.g., the stacks of $N$ D7-branes and $n$ D3-branes [30].

$$f_{\text{one-loop}} \supset -\frac{n}{2\pi} \log \vartheta_1(\zeta, B + iA), \quad \text{where} \quad \vartheta_1(\zeta, B + iA) \equiv \vartheta \left[ \frac{1}{2} \right] \left( \zeta, B + iA \right). \quad (3.28)$$

Here, $\zeta$ is a brane position modulus. Further, there exists another correction depending on the Dedekind’s eta function $\eta(q)$

$$f_{\text{one-loop}} \supset -\frac{b}{2\pi} \log \eta(q), \quad (3.29)$$

where $b$ denotes a one-loop beta function coefficient in the $N=2$ SUSY sector and $\eta(q)$ is given by

$$\eta(q) \equiv q^{1/12} \prod_{n=1}^{\infty} (1 - q^{2n}), \quad \text{where} \quad q = \exp[i\pi(B + iA)]. \quad (3.30)$$

Along with these corrections, the dynamical scale of the SU(2) gauge theory is modified as

$$\Lambda^3 \rightarrow e^{-\frac{2b}{2\pi} f_{\text{gauge}}} \Lambda^3 [\vartheta_1(\zeta, q)]^{b/N} [\vartheta_1(\zeta, q)]^{n/N}. \quad (3.31)$$

Finally, we similarly recalculate the previous scalar potential (3.26), and then straightforwardly obtain a legitimate scalar potential,

$$V \simeq \Lambda^4 |\eta(q)|^{4b/3N} |\vartheta_1(\zeta, q)|^{4n/3N} \sum_{K=1}^{6} |y_{14K} + y_{23K}|^2. \quad (3.32)$$

We numerically checked that these corrections from gauge coupling do not essentially disturb the shape of the original potential (3.26) in parameter regions of our interest for $\zeta = O(1)$. Thus, we will neglect the factors of $\eta$ and $\vartheta_1$ hereafter. This is because the leading terms of $|\eta(q)| \simeq |q|^{1/12}$ and $|\vartheta_1(\zeta, B + iA)| \simeq |q|^{1/4}$ change just an overall scale, and sub-leading terms of $O(q^2)$ corrections in $\eta$ and $\vartheta_1$ do not drastically affect the inflaton potential.

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11 See also refs. [51, 52].

12 Note that the one-loop corrections to the dynamical scale are vanishing for the case of $\zeta = 0$ for instance. This stems from the appearance of new massless modes. However, as long as the effective theories are viable, such a situation does not take place.

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4 Numerical analysis

In this section, we show the shape of the scalar potential in eq. (3.26) and inflationary predictions. As explained already, our model looks similar to hilltop inflation with a tuning of the torus area $A$. Here and hereafter, we assume the moduli stabilization mechanism such that the area modulus $A$ is stabilized at a proper value of $A = O(1)$ and is heavier than the Hubble scale $\sim \Lambda^2$ during inflation. This is expected due to the fact that the leading Kähler potential depends only on the modulus $A$ \cite{53, 54}, for instance,

\begin{align}
K &\sim - \log(T + \bar{T}), \\
T &\equiv A - iB.
\end{align}

In type IIB superstring side, the modulus $T$ is connected to the complex structure modulus via T-duality. Lots of complex structure modulus can be appropriately stabilized in the framework of flux compactifications and SUSY breaking mechanisms \cite{55–57}. Our scenario would be available if flux does not fix a complex structure in the absence of a specific direction of the flux and SUSY breaking fixes just a (real) part of it \cite{32, 58, 59}. However, that is beyond the scope of this paper, and we leave a detail of moduli stabilizations for future work.

Now that the scalar potential in eq. (3.26) is assumed to be a function of a dynamical $B$ with parameters of $A$, $f$ and $\Lambda$. With the Kähler potential of eq. (4.1), the axion decay constant $f$ has a connection with the torus area $A$ \cite{54}

\begin{equation}
f = \frac{1}{\sqrt{2A}}.
\end{equation}

We canonically normalize the $B$ axion as $B \to \phi/f$ with this decay constant. Also, a combination of $A$ and $\Lambda$ is fixed to fit the adiabatic density perturbation observed by the Planck satellite. After all, we have only a single parameter, roughly speaking $A$, in the scalar potential with the dynamical $B$.

4.1 The shape of an inflaton potential

First, the shape of the inflaton potential (3.26) is discussed. In figure 2, we show a schematic picture of the inflaton potential with varying values of the area of the torus $A$. The blue, yellow and green lines correspond to the values of $A = 1.0, 1.5$ and $2.0$, respectively. Figure 2 shows that there is a hilltop-like plateaux in the $B$-field direction around $B \simeq 7.5$ for $A \simeq 1.2$ (and we have the same structure around $B \simeq 17.5$). Similar studies are done also in refs. \cite{31, 32}. Figure 3 is an enlarged view of the hilltop region in figure 2. In the figure, the two bumps disappear for $A = 1.2$ or more larger values. From this figure, there seems to exist a periodicity of $B \equiv B + 25$. It is expected that D-branes wrapping on the torus change the original periodicity of $B \equiv B + 1$ on the torus without D-branes.

4.2 Relation to hilltop inflaton potential

In this subsection, we explain how our model with theta functions can be interpreted as hilltop inflaton for $A \simeq 1.2$. We expand the inflaton potential (3.26) around the hilltop $B \simeq 7.5$, then the inflaton potential can be symbolically written by plugging $B = 7.5 - \phi/f$ as

\begin{equation}
\frac{V}{\Lambda^4} = \frac{V_0}{\Lambda^4} + c_1(A)\frac{\phi}{f} + c_2(A)\left(\frac{\phi}{f}\right)^2 + c_3(A)\left(\frac{\phi}{f}\right)^3 + c_4(A)\left(\frac{\phi}{f}\right)^4 + \ldots,
\end{equation}

where
Figure 2. Schematic pictures of the inflaton potential along the NSNS $B$-field direction. Reading from top to down, three (blue-, yellow- and green-colored) lines denote the potential for $A = 1.0, 1.5$ and 2.0, respectively. The periodicity seems to exist as $B \equiv B + 25$.

Figure 3. The hilltop region of the inflaton potential. From top to down, six lines correspond to $A = 1.0, 1.1, 1.2, 1.3, 1.4$ and 1.5.

where $V_0$ denotes a constant and $c_n$’s ($n = 1, 2, 3, 4$) are coefficients of the $\phi^n$-term depending on the modulus $A$. The $A$-dependence of $c_n$’s are shown in figure 4. We find that $c_1 \simeq c_2 \simeq c_3 \simeq 0$ and $c_4 < 0$ for $A \simeq 1.2$. Then, the inflaton potential for $A \simeq 1.2$ can be expressed as

$$V \simeq V_0 - \lambda \phi^4,$$

with $\lambda \propto c_4(A \simeq 1.2)$. Note that this is not mere hilltop inflation but hilltop inflation with small linear- and cubic-terms. In refs. [18, 60, 61], it is pointed out that such terms play an important role to give inflationary predictions different from hilltop inflation.

4.3 Predictions of an inflation with $B$-field modulus

In this subsection, we numerically analyze predictions on spectral index $n_s$, running of spectral index $\alpha_s$ and tensor-to-scalar ratio $r$. They are strictly constrained by the Planck 2015 results [7], and directly related to the slow-roll parameters defined by the scalar potential and
Figure 4. $A$-dependence of $c_i$'s. We find $c_1 \simeq c_2 \simeq c_3 \simeq 0$ and $c_4 < 0$ around $A \simeq 1.2$.

its derivatives as below. We find that these observables are consistent with the constraint in many choices of the decay constant $f$ smaller than the Planck scale in our model.\(^{13}\)

In a slow-roll inflation scenario with a single inflaton, $n_s$, $\alpha_s$ and $r$ are given by

\begin{align*}
    n_s &= 1 + 2\eta - 6\epsilon, \quad (4.6) \\
    \alpha_s &= -24\epsilon^2 + 16\epsilon\eta - 2\xi, \quad (4.7) \\
    r &= 16\epsilon. \quad (4.8)
\end{align*}

Here, the slow-roll parameters are defined by

\begin{align*}
    \epsilon &= \frac{1}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = \frac{V''}{V}, \quad \xi = \frac{V'V'''}{V^2}. \quad (4.9)
\end{align*}

A symbol ($'$) denotes a derivative with respect to $\phi$.

Figure 5 shows results of observables ($n_s$, $r$ and $\alpha_s$) with numerical plots in the range of $1.17 \leq A \leq 1.23$. The yellow (blue) curved lines denote the predictions for the e-foldings $N = 60$ ($N = 50$). For a value of $A$ in this range, we obtain a sub-Planckian decay constant, $f \sim 0.58 < 1$, which leads to predictions consistent with the Planck 2015 observation:

\begin{align*}
    r \sim 10^{-5}, \quad \alpha_s \sim -0.001. \quad (4.10)
\end{align*}

Then, the inflation scale and inflaton mass $m_\phi$ are given by

\begin{align*}
    V_{\text{inf}}^{1/4} &= 1.8 \times 10^{15} \text{ GeV} \cdot \left( \frac{r}{10^{-5}} \right)^{1/4} \sim \Lambda, \quad (4.11) \\
    m_\phi &\sim \frac{\Lambda^2}{f} \sim 2.3 \times 10^{12} \text{ GeV} \cdot \left( \frac{r}{10^{-5}} \right)^{1/2}. \quad (4.12)
\end{align*}

Since $\partial_\phi W \sim \Lambda^2$ is the SUSY breaking scale,\(^{14}\) the gravitino mass is estimated as

\begin{align*}
    m_{3/2} \gtrsim \frac{\Lambda^2}{f} \sim 1.3 \times 10^{12} \text{ GeV} \cdot \left( \frac{r}{10^{-5}} \right)^{1/2}. \quad (4.13)
\end{align*}

\(^{13}\)See refs. [32, 62] for models where a sizable $\alpha_s$ is realized in smaller $A$ cases.

\(^{14}\)This IYIT sector may contribute to the vacuum energy as uplifting sector for obtaining Minkowski/de Sitter vacuum from AdS one in SUGRA [48, 56, 63–66], if $\Lambda^2$ is the main source of the SUSY breaking.
Figure 5. Predictions on spectral index $n_s$, its running $\alpha_s$ and tensor-to-scalar ratio $r$. The yellow (blue) solid lines denote the e-foldings $N = 60$ ($N = 50$).

Here, we take into account the possibility that there may be other sources of the SUSY breaking. Note that the Hubble scale during the inflation $H_{\text{inf}}$ is given by $V_{\text{inf}}^{1/2} \sim H_{\text{inf}} \sim \Lambda^2$. If reheating process takes place by the inflaton decay through the axion-like interaction between $\phi$ and the Standard Model gauge bosons

$$c \frac{g_{\text{SM}}^2}{32\pi^2} \frac{\phi}{f} \epsilon^{\mu
u\rho\sigma} F_{\mu
u} F_{\rho\sigma}, \quad (4.14)$$

the reheating temperature is estimated as

$$T_R \sim 10^6 \text{ GeV} \cdot c \cdot \left( \frac{r}{10^{-5}} \right)^{3/4}. \quad (4.15)$$

Here, we used $4\pi/g_{\text{SM}}^2 \sim 25$ for the Standard Model gauge coupling. Several phenomenological consequences are similar to those shown in ref. [18].

5 Conclusions and discussions

In this paper, we have utilized the Jacobi’s theta function to construct an inflationary model and investigated its predictions. To this end, we have constructed a SUSY breaking model on $T^2/\mathbb{Z}_2$ and focused on the $B$-field (inflaton) dependence in the Yukawa couplings. As a result, it is found that a hilltop-type inflation can be realized for a certain area of the torus ($A \simeq 1.2$).
We comment on corrections to the inflaton potential, and then the required conditions in our setup is discussed. First, if there exist light modes with mass $\lesssim H_{\text{inf}}$ during the inflation, the slow-roll inflation fails via mixings between the inflaton and extra fields. To evade this, the Hubble scale $H_{\text{inf}} \sim \Lambda^2$ should be smaller than mass scales of possible extra modes. For instance, there is the pseudo modulus with mass $m_\Phi \sim \Lambda/4\pi$ and the other heavy modes with masses $\sim \Lambda$ in the SUSY breaking sector. Against these, the condition is naturally satisfied for $\Lambda < 1$. Further, it is noted that string moduli stabilization can distort our inflaton potential generically, and vice versa. If there exist moduli whose masses are smaller $H_{\text{inf}}$ during the inflation, vacuum of such moduli can be destabilized by the inflation energy of $3H_{\text{inf}}^2$ and run away to decompactification vacuum, thus slow-roll inflation fails. Hence, string moduli should be heavier than $H_{\text{inf}}$. Then, vacuum of string moduli can be stable even during the inflation. However, an inflaton potential can generally be made much steeper by large corrections from a scalar potential making moduli sufficiently heavy [67, 68] (when the stabilization potential depends on the inflaton). A way to elude this issue is to decouple potential energy of heavy moduli from inflaton sector in a supersymmetric manner during the inflation. This can be done by so-called strong moduli stabilization [47, 69], i.e., $W(\Phi) = \partial_\Phi W = 0$ in the (inflationary) vacuum. Here, $W(\Phi)$ is a superpotential of heavy moduli $\Phi$. Then, our inflation model becomes viable, although such a stabilization may require a tuning of parameters in $W(\Phi)$ to cancel a steep part of the inflaton potential.\footnote{Additional conditions of moduli stabilization will be required not to prevent a slow-roll inflation in the presence of the mixing between the inflaton and $\Phi$ in $W(\Phi)$ [67, 68].} So far, we ignored SUGRA corrections. Supposing the VEV of the pseudo modulus in the SUSY breaking sector $\langle \Phi_K \rangle \sim \Lambda^2$ [70], the inflaton potential is to be modified as $V_{\text{SUGRA}} \simeq V + O(\Lambda^6)$ owing to the shift symmetry of $B$. Our analysis is quantitatively valid because of $\Lambda^2 \sim 10^{-6}$. Finally, we discuss (non-)perturbative quantum corrections via shift symmetry breaking [59, 71], which will induce the inflaton mass, and we parametrize it as $\Delta V \sim (c/16\pi^2)\Lambda^4\phi^2$, where a coefficient constant $c$ can contain the Yukawa coupling squared and unknown quantum gravity effects. For $c \lesssim 1$, the inflationary predictions do not drastically change (see figure 4) [32].

Before closing this section, we would like to comment on the CP violation. It is pointed out that the CP symmetry among the quarks in the Standard Model is broken by non-vanishing values of an imaginary part of the complex structure modulus $\tau$ in type IIB side [72]. Since the complex structure modulus corresponds to the NSNS $B$-field via T-duality, the $B$-field axion is likely to play an important role in the CP violation. Indeed, if the inflaton develops its non-vanishing VEV after inflation, the CP symmetry is broken down and the value of a CP-violating phase is determined by the VEV, the intersecting numbers of D-branes in the quark sector and so forth. The construction of a concrete model in the visible sector containing the SM quarks and leptons is attractive from the phenomenological point of view. Such a study is left for our future work.

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References

[1] A.H. Guth, *The inflationary universe: a possible solution to the horizon and flatness problems*, Phys. Rev. D 23 (1981) 347 [arXiv:1303.5082] [SPIRE].

[2] K. Sato, *First order phase transition of a vacuum and expansion of the universe*, Mon. Not. Roy. Astron. Soc. 195 (1981) 467 [SPIRE].

[3] R. Brout, F. Englert and E. Gunzig, *The creation of the universe as a quantum phenomenon*, Annals Phys. 115 (1978) 78 [SPIRE].

[4] D. Kazanas, *Dynamics of the universe and spontaneous symmetry breaking*, Astrophys. J. 241 (1980) L59 [SPIRE].

[5] A.A. Starobinsky, *A new type of isotropic cosmological models without singularity*, Phys. Lett. B 91 (1980) 99 [SPIRE].

[6] Planck collaboration, P.A.R. Ade et al., *Planck 2013 results XXII. Constraints on inflation*, Astron. Astrophys. 571 (2014) A22 [arXiv:1303.5082] [SPIRE].

[7] Planck collaboration, P.A.R. Ade et al., *Planck 2015 results XX. Constraints on inflation*, Astron. Astrophys. 594 (2016) A20 [arXiv:1502.02114] [SPIRE].

[8] K. Freese, J.A. Frieman and A.V. Olinto, *Natural inflation with pseudo-Nambu-Goldstone bosons*, Phys. Rev. Lett. 65 (1990) 3233 [SPIRE].

[9] M. Kawasaki, M. Yamaguchi and T. Yanagida, *Natural chaotic inflation in supergravity*, Phys. Rev. Lett. 85 (2000) 3572 [hep-ph/0004243] [SPIRE].

[10] E. Silverstein and A. Westphal, *Monodromy in the CMB: gravity waves and string inflation*, Phys. Rev. D 78 (2008) 106003 [arXiv:0803.3085] [SPIRE].

[11] L. McAllister, E. Silverstein and A. Westphal, *Gravity waves and linear inflation from axion monodromy*, Phys. Rev. D 82 (2010) 046003 [arXiv:0808.0706] [SPIRE].

[12] N. Kaloper and L. Sorbo, *A natural framework for chaotic inflation*, Phys. Rev. Lett. 102 (2009) 121301 [arXiv:0811.1989] [SPIRE].

[13] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, *The string landscape, black holes and gravity as the weakest force*, JHEP 06 (2007) 060 [hep-th/0601001] [SPIRE].

[14] J. Brown, W. Cottrell, G. Shiu and P. Soler, *Fencing in the swampland: quantum gravity constraints on large field inflation*, JHEP 10 (2015) 023 [arXiv:1503.04783] [SPIRE].

[15] K. Choi and H. Kim, *Aligned natural inflation with modulations*, Phys. Lett. B 759 (2016) 520 [arXiv:1511.07201] [SPIRE].

[16] R. Kappl, H.P. Nilles and M.W. Winkler, *Modulated natural inflation*, Phys. Lett. B 753 (2016) 653 [arXiv:1511.05560] [SPIRE].

[17] M. Czerny and F. Takahashi, *Multi-natural inflation*, Phys. Lett. B 733 (2014) 241 [arXiv:1401.5212] [SPIRE].

[18] M. Czerny, T. Higaki and F. Takahashi, *Multi-natural inflation in supergravity*, JHEP 05 (2014) 144 [arXiv:1403.0410] [SPIRE].

[19] M. Czerny, T. Higaki and F. Takahashi, *Multi-natural inflation in supergravity and BICEP2*, Phys. Lett. B 734 (2014) 167 [arXiv:1403.5883] [SPIRE].

[20] T. Higaki, K.S. Jeong, N. Kitajima and F. Takahashi, *Quality of the Peccei-Quinn symmetry in the aligned QCD axion and cosmological implications*, JHEP 06 (2016) 150 [arXiv:1603.02090] [SPIRE].

[21] G. D’Amico, T. Hamill and N. Kaloper, *Quantum field theory of interacting dark matter/dark energy: dark monodromies*, Phys. Rev. D 94 (2016) 103526 [arXiv:1605.00996] [SPIRE].
[22] J. Jaeckel, V.M. Mehta and L.T. Witkowski, Monodromy dark matter, *JCAP* 01 (2017) 036 [arXiv:1605.01367] [InSPIRE].

[23] T. Higaki, K.S. Jeong, N. Kitajima, T. Sekiguchi and F. Takahashi, Topological defects and nano-Hz gravitational waves in aligned axion models, *JHEP* 08 (2016) 044 [arXiv:1606.05552] [InSPIRE].

[24] R. Daido, T. Kobayashi and F. Takahashi, Dark matter in axion landscape, *Phys. Lett.* B 765 (2017) 293 [arXiv:1608.04092] [InSPIRE].

[25] P.W. Graham, D.E. Kaplan and S. Rajendran, Cosmological relaxation of the electroweak scale, *Phys. Rev. Lett.* 115 (2015) 221801 [arXiv:1504.07551] [InSPIRE].

[26] K. Choi and S.H. Im, Realizing the relaxion from multiple axions and its UV completion with high scale supersymmetry, *JHEP* 01 (2016) 149 [arXiv:1511.00132] [InSPIRE].

[27] D.E. Kaplan and R. Rattazzi, Large field excursions and approximate discrete symmetries from a clockwork axion, *Phys. Rev.* D 93 (2016) 085007 [arXiv:1511.01827] [InSPIRE].

[28] D. Cremades, L.E. Ibáñez and F. Marchesano, Yukawa couplings in intersecting D-brane models, *JHEP* 07 (2003) 038 [hep-th/0302105] [InSPIRE].

[29] D. Cremades, L.E. Ibáñez and F. Marchesano, Computing Yukawa couplings from magnetized extra dimensions, *JHEP* 05 (2004) 079 [hep-th/0404229] [InSPIRE].

[30] M. Berg, M. Haack and B. Körs, Loop corrections to volume moduli and inflation in string theory, *Phys. Rev.* D 71 (2005) 026005 [hep-th/0404087] [InSPIRE].

[31] T. Kobayashi, D. Nitta and Y. Urakawa, Modular invariant inflation, *JCAP* 08 (2016) 014 [arXiv:1604.02995] [InSPIRE].

[32] T. Higaki and F. Takahashi, Elliptic inflation: interpolating from natural inflation to $R^2$-inflation, *JHEP* 03 (2015) 129 [arXiv:1501.02354] [InSPIRE].

[33] L. Boubekeur and D. Lyth, Hilltop inflation, *JCAP* 07 (2005) 010 [hep-ph/0502047] [InSPIRE].

[34] L.E. Ibanez and A.M. Uranga, String theory and particle physics: an introduction to string phenomenology, Cambridge University Press, Cambridge U.K., (2012) [InSPIRE].

[35] T. Kobayashi, S. Nagamoto and S. Uemura, Modular symmetry in magnetized/intersecting D-brane models, *PTEP* 2017 (2017) 023B02 [arXiv:1608.06129] [InSPIRE].

[36] S. Ferrara, N. Magnoli, T.R. Taylor and G. Veneziano, Duality and supersymmetry breaking in string theory, *Phys. Lett.* B 245 (1990) 409 [InSPIRE].

[37] M. Cvetic, A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, Target space duality, supersymmetry breaking and the stability of classical string vacua, *Nucl. Phys.* B 361 (1991) 194 [InSPIRE].

[38] J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, On loop corrections to string effective field theories: field dependent gauge couplings and $\sigma$-model anomalies, *Nucl. Phys.* B 372 (1992) 145 [InSPIRE].

[39] G. Lopes Cardoso and B.A. Ovrut, A Green-Schwarz mechanism for $D = 4, N = 1$ supergravity anomalies, *Nucl. Phys.* B 369 (1992) 351 [InSPIRE].

[40] L.E. Ibáñez and D. Lüst, Duality anomaly cancellation, minimal string unification and the effective low-energy Lagrangian of 4D strings, *Nucl. Phys.* B 382 (1992) 305 [hep-th/9202046] [InSPIRE].

[41] H. Abe, K.-S. Choi, T. Kobayashi and H. Ohki, Non-Abelian discrete flavor symmetries from magnetized/intersecting brane models, *Nucl. Phys.* B 820 (2009) 317 [arXiv:0904.2631] [InSPIRE].

[42] T.-H. Abe, Y. Fujimoto, T. Kobayashi, T. Miura, K. Nishiwaki and M. Sakamoto, $Z_N$ twisted orbifold models with magnetic flux, *JHEP* 01 (2014) 065 [arXiv:1309.4925] [InSPIRE].
[43] K.-I. Izawa and T. Yanagida, *Dynamical supersymmetry breaking in vector-like gauge theories*, *Prog. Theor. Phys.* 95 (1996) 829 [hep-th/9602180] [SPIRE].

[44] K.A. Intriligator and S.D. Thomas, *Dynamical supersymmetry breaking on quantum moduli spaces*, *Nucl. Phys. B* 473 (1996) 121 [hep-th/9603158] [SPIRE].

[45] H. Abe, K.-S. Choi, T. Kobayashi and H. Ohki, *Three generation magnetized orbifold models*, *Nucl. Phys. B* 814 (2009) 265 [arXiv:0812.3534] [SPIRE].

[46] T.-H. Abe et al., *Classification of three-generation models on magnetized or bifolds*, *Nucl. Phys. B* 894 (2015) 374 [arXiv:1501.02787] [SPIRE].

[47] R. Kallosh and A.D. Linde, *Landscape, the scale of SUSY breaking and inflation*, *JHEP* 12 (2004) 004 [hep-th/0411011] [SPIRE].

[48] H. Abe, T. Higaki and T. Kobayashi, *More about F-term uplifting*, *Phys. Rev. D* 76 (2007) 105003 [arXiv:0707.2671] [SPIRE].

[49] H. Jockers and J. Louis, *The effective action of $D$7-branes in $N = 1$ Calabi-Yau orientifolds*, *Nucl. Phys. B* 705 (2005) 167 [hep-th/0409098] [SPIRE].

[50] P. Corvilain, T.W. Grimm and D. Regalado, *Shift-symmetries and gauge coupling functions in orientifolds and F-theory*, *JHEP* 05 (2017) 059 [arXiv:1607.03897] [SPIRE].

[51] N. Akerblom, R. Blumenhagen, D. Lüst and M. Schmidt-Sommerfeld, *Instantons and holomorphic couplings in intersecting D-brane models*, *JHEP* 08 (2007) 044 [arXiv:0705.2366] [SPIRE].

[52] N. Akerblom, R. Blumenhagen, D. Lüst and M. Schmidt-Sommerfeld, *Thresholds for intersecting D-branes revisited*, *Phys. Lett. B* 652 (2007) 53 [arXiv:0705.2150] [SPIRE].

[53] R. Blumenhagen, B. Körts, D. Lüst and S. Stieberger, *Four-dimensional string compactifications with D-branes, orientifolds and fluxes*, *Phys. Rept.* 445 (2007) 1 [hep-th/0610327] [SPIRE].

[54] D. Baumann and L. McAllister, *Inflation and string theory*, Cambridge University Press, Cambridge U.K., (2015).

[55] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev. D* 66 (2002) 106006 [hep-th/0105097] [SPIRE].

[56] S. Kachru, R. Kallosh, A.D. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev. D* 68 (2003) 046005 [hep-th/0301240] [SPIRE].

[57] M. Berg, M. Haack and B. Körts, *On volume stabilization by quantum corrections*, *Phys. Rev. Lett.* 96 (2006) 021601 [hep-th/0508171] [SPIRE].

[58] A. Hebecker, P. Mangat, F. Rompineve and L.T. Witkowski, *Winding out of the swamp: evading the weak gravity conjecture with F-term winding inflation?*, *Phys. Lett. B* 748 (2015) 455 [arXiv:1503.07912] [SPIRE].

[59] T. Kobayashi, A. Oikawa and H. Otsuka, *New potentials for string axion inflation*, *Phys. Rev. D* 93 (2016) 083508 [arXiv:1510.08768] [SPIRE].

[60] F. Takahashi, *New inflation in supergravity after Planck and LHC*, *Phys. Lett. B* 727 (2013) 21 [arXiv:1308.4212] [SPIRE].

[61] K. Harigaya, M. Ibe and T.T. Yanagida, *Lower bound on the gravitino mass $m_{3/2} < O(100)$ TeV in R-symmetry breaking new inflation*, *Phys. Rev. D* 89 (2014) 055014 [arXiv:1311.1898] [SPIRE].

[62] H. Abe, T. Kobayashi and H. Otsuka, *Natural inflation with and without modulations in type IIB string theory*, *JHEP* 04 (2015) 160 [arXiv:1411.4768] [SPIRE].

[63] O. Lebedev, H.P. Nilles and M. Ratz, *De Sitter vacua from matter superpotentials*, *Phys. Lett. B* 636 (2006) 126 [hep-th/0603047] [SPIRE].
[64] E. Dudas, C. Papineau and S. Pokorski, *Moduli stabilization and uplifting with dynamically generated F-terms*, JHEP 02 (2007) 028 [hep-th/0610297] [inSPIRE].

[65] H. Abe, T. Higaki, T. Kobayashi and Y. Omura, *Moduli stabilization, F-term uplifting and soft supersymmetry breaking terms*, Phys. Rev. D 75 (2007) 025019 [hep-th/0611024] [inSPIRE].

[66] R. Kallosh and A.D. Linde, *O’KKLT*, JHEP 02 (2007) 002 [hep-th/0611183] [inSPIRE].

[67] S. Kachru, R. Kallosh, A.D. Linde, J.M. Maldacena, L.P. McAllister and S.P. Trivedi, *Towards inflation in string theory*, JCAP 10 (2003) 013 [hep-th/0308055] [inSPIRE].

[68] D. Baumann, A. Dymarsky, I.R. Klebanov, L. McAllister and P.J. Steinhardt, *A delicate universe*, Phys. Rev. Lett. 99 (2007) 141601 [arXiv:0705.3837] [inSPIRE].

[69] E. Dudas, A. Linde, Y. Mambrini, A. Mustafayev and K.A. Olive, *Strong moduli stabilization and phenomenology*, Eur. Phys. J. C 73 (2013) 2268 [arXiv:1209.0499] [inSPIRE].

[70] R. Kitano, *Gravitational gauge mediation*, Phys. Lett. B 641 (2006) 203 [hep-ph/0607090] [inSPIRE].

[71] M. Berg, M. Haack and B. Körs, *String loop corrections to Kähler potentials in orientifolds*, JHEP 11 (2005) 030 [hep-th/0508043] [inSPIRE].

[72] T. Kobayashi, K. Nishiwaki and Y. Tatsuta, *CP-violating phase on magnetized toroidal orbifolds*, JHEP 04 (2017) 080 [arXiv:1609.08608] [inSPIRE].