Bundling or Unbundling? Pricing Strategy for Complementary Products in a Green Supply Chain

Haiyan Shan 1,2,*, Chen Zhang 2 and Guo Wei 3

1 Institute of Climate Change and Public Policy, Nanjing University of Information Science & Technology, Nanjing 210044, Jiangsu, China
2 School of Management Science and Engineering, Nanjing University of Information Science & Technology, Nanjing 210044, Jiangsu, China; chenzhang@nuist.edu.cn
3 Mathematics & Computer Science, University of North Carolina at Pembroke, Pembroke, NC 28372, USA; guo.wei@uncp.edu
* Correspondence: hyshan@nuist.edu.cn; Tel.:+86-258731549

Received: 18 January 2020; Accepted: 10 February 2020; Published: 12 February 2020

Abstract: Retailers usually sell complementary products jointly with a discounted price to attract more consumers. However, the difference of complementary degree between products leads to the diversity of pricing. In parallel, with the development of green supply chains, the extra cost of manufacturers to conduct ecological product design makes the pricing of complementary products further complicated. Thus, it is important to clarify the pricing strategy for complementary products in a green supply chain. Based on the Stackelberg games between two manufacturers and a retailer, this paper constructs three pricing models to simultaneously analyze the changes in the optimal profits of supply chain members and the optimal green manufacturing degree of complementary products. The results demonstrate that: (i) In most cases, two manufacturers prefer the pure bundling pricing strategy, but the strategy preference of the retailer is complex. (ii) The green manufacturing is mutually beneficial for complementary manufacturers and worth advocating. (iii) The increasing sensitivity of consumers to the green manufacturing level of one product will also be detrimental to the improvement of the optimal green manufacturing level of its complementary products.

Keywords: complementary product; green supply chain; bundling strategy; green manufacturing degree

1. Introduction

1.1. Background

The rapid growth of urbanization and industrialization improves people’s living standards but also causes a series of negative issues, such as resource shortages, environmental degradation, and ecological crisis [1]. The severe reality has proven that minimizing environmental negative impact and maximizing resource efficiency throughout the life cycle of products, becomes increasingly important. Since the traditional production process, with higher consumption and lower efficiency cannot meet the requirements of social and economic development, many countries put green manufacturing into the national strategy, including “Industry 4.0” in Germany, “Made in China 2025” in China, and “Re-Industrialization” in the United States.

As an important branch of operations management, the supply chain has a significant effect on the environment [2]. In order to improve ecological benefits, the concept of green supply chain management has recently emerged, and this approach integrates environmental issues into the common supply chain activities [3], so as to mitigate the damage to the environment in the process of transforming raw materials into final products. Subsequently, the lean process, innovation and the green paradigms are gradually considered as policies for strengthening the competitiveness of supply chains [4].
In the market, sometimes consumers may have to purchase more than one product at the same time to gain the full utility of the products, and these are referred to as complementary products [5]. On the one hand, with more and more attention paid to green supply chains, traditional manufacturers can gain a competitive advantage by ecological transformation (i.e., the upgrading of equipment and technology), but also need to undertake the corresponding costs at the same time. Obviously, the ecological transformation of a manufacturer will lead to higher wholesale prices of its products. Meanwhile, due to the dependence of complementary products, this transformation also affects the market demands of corresponding complementary products. Thus, the market demands of products are different in green and non-green supply chains. On the other hand, since bundling sales can obtain more consumer surplus and reduce transaction costs through scale effect [6], retailers will usually adopt this strategy to sell two or more complementary products jointly at a discounted price [7]. However, such a strategy is not applicable to all complementary product portfolios. For instance, when the negative cross elasticity of demand needs to be considered, selling products separately may be a more appropriate way to improve the profit level [8]. Thus, it is important to investigate the optimal pricing strategy of complementary products in a green supply chain.

1.2. Literature Review

The relevant literature can be reviewed from three categories: the pricing in the green supply chain, the pricing for complementary products, and bundling strategies in product pricing.

1.2.1. The Pricing in the Green Supply Chain

With the increasingly severe environmental problems, the green supply chain has come up as a new research paradigm in operations management [9]. The green supply chain can be defined as a system that aims to prevent the production of waste, while increasing efficiencies in the use of energy, water, resources, and human capital [10]. For a long period of time, enterprises have mainly focused on cost, quality and other issues in the management of the classic supply chain, without fully considering the resource and environmental impact of the whole industrial chain. With the deepening of environmental degradation and the increase in environmental awareness, the implementation of green supply chain management (GSCM) is considered as a viable option to reduce the environmental impact of operations, while improving operational performance [11]. As a result, in the green supply chain, how to make the optimal pricing decisions which can not only minimize the pollution but also maximize the profit has been a hot topic concerning both enterprises and scholars.

Jamali and Rasti-Barzoki [12] deliberated the pricing of two substitute products (green and non-green) in the dual-channel supply chain and found that centralized pricing achieved a higher green degree. Yang et al. [13] studied the environmental performance of green manufacturers’ dual-channel structure strategy under fuzzy uncertainties; they suggested that the environmental responsibility degree hindered the greening of products. In parallel, some scholars are committed to the impact of government intervention on the pricing of the green supply chain. Hafezalkotob [14] proposed the optimal response strategies of manufacturers and retailers by developing price competition and cooperation models, and the results demonstrated that an appropriate tariff set by the government was helpful to achieve the expected goals. Xue et al. [15] concentrated on the pricing issues for energy-saving products in the green supply chain under government subsidies. In addition, extant studies indicate that consumer behavior also plays an important role in the pricing of the green supply chain. For example, Zhang et al. [16] integrated consumer environmental awareness into the pricing model of the green supply chain, and they thought the profit of the retailer would monotonically increase, while that of the manufacturer was convex. Hong et al. [17] verified that the reference behavior of consumers had a significant impact on green product design and pricing strategies.
1.2.2. The Pricing for Complementary Products

The research into optimal pricing decisions of complementary products has also been widely studied recently. Karray and Sigué [18] discussed the question of the optimal promotional partners in three firms that sold complementary or independent products, and the results indicated that the choice of promotional partners mainly depends on the complementary degree and demand of products. Dehghanbaghi and Sajadieh [19] proposed the joint optimization of pricing policies for complementary products in both centralized and decentralized supply chains. They emphasized that the profit of a centralized supply chain is more stable than that of a decentralized supply chain. Lately, the rapid development of electronic commerce has attracted the concern of scholars to distribution channels for complementary products. Ngendakuriyo and Taboubi [20] investigated the dynamic pricing of complementary products in a vertical channel structure by controlling transfer and retail prices. Wang et al. [21] focused on the pricing and service decisions of complementary products, and elaborated on the effectiveness of different supply chain structures and pricing forms. Zhao et al. [22] formulated four pricing models under different market power structures, and derived the corresponding optimal pricing strategies of complementary products. Considering the manufacturers’ cooperation or noncooperation strategies, Wei et al. [23] investigated the pricing problem of complementary products in the green supply chain.

1.2.3. Bundling Strategies in Product Pricing

As an attractive and profitable marketing strategy, the bundling sales strategy can be divided into mixed bundling sales and pure bundling sales [24]. So far, massive scholars have investigated bundling sales strategies from different perspectives, including the types of products [25–27], the marketing channel interactions [28–30], and the service quality [31,32].

Here, we are more concerned about the application of different bundling strategies in product pricing. Specifically, Giri et al. [33] probed the pricing for complementary products in a non-cooperative duopoly market under scenarios of separate sales and pure bundling sales, and the result showed that the profit of pure bundling sales is higher than that of separate sales. Pan and Zhou [34] investigated the bundling and pricing decisions of two complementary products in cases of decentralized decision. They thought that the optimal bundling decision was mainly determined by the complementarity of products. In order to identify the comprehensive impact of various factors on the bundling sales strategy, Kopczewski et al. [35] presented an integrated simulation model and explained the complex relationships between factors and bundling strategies. In an uncertain market of complementary products, Xie et al. [7] studied the effects of the stochastic demand and manufacturers’ decisions on the bundling sales strategies of retailers, and they found that the retailer would choose the no-bundling sales strategy due to the severe uncertainty.

1.3. The Main Aim and Originality of This Study

It must be noted that previous studies have provided us with plentiful results. However, for the problem of whether the green investment of manufacturers will affect the pricing strategy of complementary products, previous studies cannot offer a definite solution. Consequently, in order to fill this research gap, this paper aims to study the pricing strategy for complementary products in the green supply chain, and the following questions will be settled: (i) What is the optimal pricing strategy for complementary products in a green supply chain? (ii) How do the profits of green supply chain members change under a variety of bundling strategies? (iii) How will the green investment of manufacturers, the cross-price elasticity and the sensitivity of consumers to the green manufacturing level affect the green manufacturing levels of complementary products? We expect to find some valuable insights from the answers to these questions, so as to provide the theoretical basis for the pricing of complementary products in the green supply chain.
The main originality of this study is twofold. Firstly, pricing models of complementary products in a green supply chain are constructed by considering different sale strategies, i.e., individual pricing, pure bundling, and mixed bundling strategies, and then the optimal solutions of each pricing model are gained through the game-theoretical approach. This approach enriches the theoretical research into the pricing decisions of complementary products. Secondly, based on the numerical analysis, some interesting and insightful results are obtained to guide the practice for enterprises, as follows: (1) In most cases, two manufacturers would like the pure bundling pricing strategy, however the strategy preference of the retailer is complex. (2) Both complementary manufacturers will reach a win–win outcome if they make green investment actively and simultaneously. (3) The increasing sensitivity of consumers to the green manufacturing level of one product will be unfavorable to the improvement of the optimal green manufacturing level of its complementary products.

1.4. Approach to Answer the Three Questions

First, combining green manufacturing with bundling strategies, we investigate the pricing strategies for complementary products in the green supply chain, and obtain optimal solutions under the individual pricing, pure bundling, and mixed bundling strategies, respectively.

Second, the preferences of two manufacturers and the retailer for the three pricing strategies are clarified. In most cases, for manufacturers, the pure bundling pricing strategy will bring them higher profits, however the retailer needs to use an appropriate pricing strategy in specific circumstances.

Third, the dynamics of the optimal profits of supply chain members and the optimal green manufacturing levels of complementary products are investigated under different parameters. Generally, the optimal green manufacturing levels of two complementary products will decrease with the green input coefficient of the manufacturer under bundling strategies, and also with the cross-price elasticity under the individual pricing and mixed bundling strategies, respectively.

The rest of this study is organized as described below. Section 2 proposes the assumptions, develops three pricing models, and gives the corresponding optimal solutions. Section 3 performs the numerical simulation of some parameters, and Section 4 discusses the results. Finally, the main conclusions and limitation of this study are provided in Section 5.

2. Materials and Methods

2.1. Assumptions and Notations

This paper constructs a two-echelon green supply chain consisting of one retailer and two manufacturers (labeled $M_1$ and $M_2$), providing two complementary products. The basic assumptions are as follows:

(1) Considering the Stackelberg game between stakeholders of the supply chain, suppose that two manufacturers are leaders, and the retailer is a follower. Manufacturers can provide the retailer with respective wholesale prices, and the retailer can choose different sales strategies to pursue the optimal response. The backward induction method will be employed to analyze the sequential non-cooperative game between two manufacturers and a retailer.

(2) In order to follow the policy of the government for green development, manufacturers adopt the green manufacturing process. Due to the difference in technological level, the green manufacturers produce the products at a different unit manufacturing cost $c_i$ ($i = 1, 2$) and offer green products to the retailer at a different unit wholesale price $\omega_i$ ($i = 1, 2$). It is noteworthy that, to simplify the model, the retailer is regarded as a normal retailer rather than a green retailer [36], i.e., it has no green input.

(3) Since there exists a consumption dependence between two complementary products, consumers usually purchase them at the same time to obtain maximum utility. Therefore, it is assumed that the complementary products considered in this paper have the same potential market demand, self-price elasticity, and cross-price elasticity. Note that the parameters in the demand functions above are
symmetric in order to avoid some problems due to the asymmetry of parameter values [37,38], to facilitate the comparison of three pricing models and to obtain more managerial implications [8,39].

(4) The retailer can sell two complementary products to consumers with various pricing strategies, including individual pricing strategy, pure bundling strategy, and mixed bundling strategy. Note that the retailer will not sell two products separately in the pure bundling scenario.

(5) With the prevalence of “green consumption”, more and more consumers will compare products in different dimensions, such as environmentally friendly features, and some of them are even willing to pay a premium for green products [40]. Thus, suppose that consumers are relatively sensitive to the green manufacturing level of products, and the demand of consumers for a product has a positive linear relationship with the green manufacturing level of the product \( i \) \((\theta_i)\) [41].

(6) In order to develop green manufacturing and produce green products based on the original production process, two green manufacturers need to invest some extra cost. According to previous studies [42–44], the cost of green manufacturing is described with an increasing and convex cost structure that reflects how the green manufacturing level results in initial changes to products and processes. As a result, the green manufacturing cost can be defined as \( \eta \theta^2 \).

(7) All stakeholders in the green supply chain are rational, and they aim to maximize their own profits.

(8) The relevant parameters and decision variables used in the models are shown in Table 1. It is assumed that the supply chain is information-symmetric, which indicates these notations are common knowledge of all stakeholders.

| Parameters | Explanation |
|------------|-------------|
| \( a_0 \) | The potential market demand of product \( i \) in the individual pricing model and mixed bundling model |
| \( a_b \) | The potential market demand of bundled product in the pure bundling model and mixed bundling model |
| \( \alpha/\delta \) | The self-price elasticity of product \( i \) / bundled product |
| \( \beta \) | The cross-price elasticity of product \( i \) to the price of its complementary product |
| \( \mu_i \) | The cross-price elasticity of product \( i \) to the price of its substitutable bundled product |
| \( \lambda_i/\lambda_b \) | The sensitivity of consumers to the green manufacturing level of product \( i \) / bundled product |
| \( \eta_i \) | The green input coefficient of manufacturer \( i \) |
| \( c_i \) | Unit manufacturing cost of product \( i \) |
| \( D_i/D_b \) | The market demand of product \( i \) / bundled product |
| \( \Pi_M/\Pi_R/\Pi_{SC} \) | Profit of the manufacturer \( i \) / the retailer / the green supply chain |

| Decision variables | Explanation |
|-------------------|-------------|
| \( \omega_i \) | Unit wholesale price of product \( i \) |
| \( p_i/p_b \) | Unit retail price for product \( i \) / bundled product |
| \( \theta_i \) | Green manufacturing level of the product \( i \) |

\(^1\) \( i \) is equal to 1 and 2 unless otherwise specified.

Next, the three sales strategies of retailers will be modeled. In order to distinguish the formulas of three models obviously, we use superscripts \( (I) \), \( (II) \), and \( (III) \) to represent the individual pricing model, pure bundling model, and mixed bundling model, respectively. All optimal solutions are described with the superscript \( (\ast) \).

2.2. Individual Pricing Model

In this sub-section, the retailer will sell two eco-friendly complementary products to consumers at a respective unit retail price \( p_i (i = 1, 2) \), i.e., the individual pricing scenario. The structure of this model is shown in Figure 1.
Figure 1. Structure of the individual pricing model.

The demands of Product 1 and Product 2 are given as follows:

\[ D_1^I = a_0 - \alpha p_1 - \beta p_2 + \lambda_1 \theta_1, \]  
\[ D_2^I = a_0 - \alpha p_2 - \beta p_1 + \lambda_2 \theta_2, \]

where parameters \( a_0, \alpha, \) and \( \beta \) stand for the potential market demand, the self-price elasticity, and the cross-price elasticity, respectively. In general, the self-price elasticity is higher than the cross-price elasticity, i.e., \( \alpha > \beta > 0 \). Meanwhile, \( \lambda_i \theta_i \) reflects the dependence of demand on the green manufacturing level of products in a tractable form. Consequently, the demand functions regarding two complementary products in the green supply chain are shown in Equations (1) and (2).

Therefore, we focus on the correlation with respect to the green manufacturing level and the performances of supply chain members in different pricing strategies. In the profit functions, \( \eta \) represents the green input coefficient. Then, two manufacturers make green investments simultaneously, and the profit functions of supply chain members are expressed as shown below:

\[ \Pi_{M1}^I = (\omega_1 - c_1)D_1^I - \eta_1 \theta_1^2, \]  
\[ \Pi_{M2}^I = (\omega_2 - c_2)D_2^I - \eta_2 \theta_2^2, \]  
\[ \Pi_R^I = (p_1 - \alpha_1)D_1^I + (p_2 - \alpha_2)D_2^I, \]  
\[ \Pi_{SC}^I = \Pi_{M1}^I + \Pi_{M2}^I + \Pi_R^I. \]

Proposition 1. If conditions \( 4(\alpha^2 - \beta^2) > 0, 2\alpha_1 \eta_1 - \frac{\lambda_1^2}{4} > 0 \) and \( 2\alpha_2 \eta_2 - \frac{\lambda_2^2}{4} > 0 \) are satisfied concurrently, the optimal results are acquired in the individual pricing model, i.e.,

\[ \omega_{1^*}^I = \frac{\xi_1 \eta_1 \eta_2 + \zeta_1 \eta_1 + \xi_1 \eta_2 + \sigma_1}{\xi_0 \eta_1 \eta_2 + \zeta_0 \eta_1 + \zeta_0 \eta_2 + \sigma_0}, \omega_{2^*}^I = \frac{\xi_2 \eta_1 \eta_2 + \zeta_2 \eta_1 + \xi_2 \eta_2 + \sigma_2}{\xi_0 \eta_1 \eta_2 + \zeta_0 \eta_1 + \zeta_0 \eta_2 + \sigma_0}, \]  
\[ \theta_{1^*}^I = \frac{\zeta_3 \eta_2 + \sigma_3}{\xi_0 \eta_1 \eta_2 + \zeta_0 \eta_1 + \zeta_0 \eta_2 + \sigma_0}, \theta_{2^*}^I = \frac{\zeta_4 \eta_1 + \sigma_4}{\xi_0 \eta_1 \eta_2 + \zeta_0 \eta_1 + \zeta_0 \eta_2 + \sigma_0}, \]

\[ p_{1^*}^I = \frac{X_1 \eta_1 \eta_2 + Y_1 \eta_1 + Z_1 \eta_2 + A_1}{2(\xi_0 \eta_1 \eta_2 + \zeta_0 \eta_1 + \zeta_0 \eta_2 + \sigma_0)}, p_{2^*}^I = \frac{X_2 \eta_1 \eta_2 + Y_2 \eta_1 + Z_2 \eta_2 + A_2}{2(\xi_0 \eta_1 \eta_2 + \zeta_0 \eta_1 + \zeta_0 \eta_2 + \sigma_0)}. \]
The proof and the expressions of $\xi_0, \xi_2, \zeta_0, \zeta_3, \varsigma_0, \varsigma_3, \alpha_0, \sigma_4, X_1, X_2, Y_1, Y_2, Z_1, Z_2, \lambda_1$, and $\Delta_2$ can be found in Appendix A.

2.3. Pure Bundling Model

In this sub-section, two complementary products in the green supply chain are bundled into one product and sold at price $p_b$ by the retailer; this is called the pure bundling scenario. It should be noted that the price of the bundled product is lower than the sum of the individual retail prices, i.e., $p_b < p_1 + p_2$. Figure 2 illustrates the structure of the pure bundling model.

![Figure 2. Structure of the pure bundling model.](image)

Inspired by Pan and Zhou [34], a linear demand function of this bundled commodity is defined as follows:

$$D_b = a_b - \delta p_b + \lambda_b (\theta_1 + \theta_2),$$

where $a_b$ represents the potential market demand of the bundled product and $\delta$ measures the corresponding price elasticity. The demand function of the bundled product is also linearly related to its green manufacturing degree. The parameter $\lambda_b$ describes the sensitivity attached by consumers when they purchase the green bundled products. Analogously, the profit functions of all members can be calculated as shown below:

$$\Pi_{M1}^{II} = (\omega_1 - c_1)D_b - \eta_1 \theta_1^2,$$

$$\Pi_{M2}^{II} = (\omega_2 - c_2)D_b - \eta_2 \theta_2^2,$$

$$\Pi_{R}^{II} = (p_b - \omega_1 - \omega_2)D_b,$$

$$\Pi_{SC}^{II} = \Pi_{M1}^{II} + \Pi_{M2}^{II} + \Pi_{R}^{II}.$$

**Proposition 2.** When $2\alpha_1\eta_1 - \frac{\lambda_1^2}{\theta_1} > 0$ and $2\alpha_2\eta_2 - \frac{\lambda_2^2}{\theta_2} > 0$ are fulfilled simultaneously, the optimal decision variables in the pure bundling model are as follows

$$\alpha_1^{II*} = \frac{\xi_3\eta_1\eta_2 - c_1\lambda_1^2\eta_1 - c_1\lambda_2^2\eta_2}{12\delta\eta_1\eta_2 - \lambda_1^2\eta_1 - \lambda_2^2\eta_2},$$

$$\alpha_2^{II*} = \frac{\xi_4\eta_1\eta_2 - c_2\lambda_1^2\eta_1 - c_2\lambda_2^2\eta_2}{12\delta\eta_1\eta_2 - \lambda_1^2\eta_1 - \lambda_2^2\eta_2},$$

$$\theta_1^{II*} = \frac{\varsigma_4\eta_2}{12\delta\eta_1\eta_2 - \lambda_1^2\eta_1 - \lambda_2^2\eta_2},$$

$$\theta_2^{II*} = \frac{\varsigma_4\eta_1}{12\delta\eta_1\eta_2 - \lambda_1^2\eta_1 - \lambda_2^2\eta_2}.$$
\[ p_b^{II*} = \frac{\xi_5 \eta_1 \eta_2 + \zeta_4 \eta_1 + \zeta_4 \eta_2}{12 \delta \eta_1 \eta_2 - \lambda_2^2 \eta_1 - \lambda_2^2 \eta_2}. \]  

The proof and the expressions of \( \xi_3-\xi_5, \zeta_4, \) and \( \zeta_4 \) are given in Appendix A.

**Proposition 3.** The green manufacturing level \( \theta_i \) decreases with \( \eta_i (i = 1, 2) \), i.e., \( \frac{\partial \theta_{II*}}{\partial \eta_i} \leq 0 \).

The proof is shown in Appendix A.

Proposition 3 demonstrates that, in the pure bundling scenario, when the green input coefficient of one manufacturer increases, the optimal green manufacturing degree of its product will decrease, as well as the optimal green manufacturing level of another product.

### 2.4. Mixed Bundling Model

The individual pricing scenario offers the complementary products separately and the pure bundling scenario offers only the bundled products, whereas the mixed bundling strategy provides both the bundled product and the component products individually. The framework is described in Figure 3.

![Figure 3](image)

**Figure 3.** Structure of the mixed bundling model.

Since the retailer will sell two individual products and a bundled product at the same time, substitutability, as well as complementarity, is shown in the demand functions [45]. We describe the cross-price elasticity by the substitutability of products with the parameter \( \mu \). Consequently, the demand functions of two complementary products and a bundled product are defined as follows:

\[ D_{i1}^{III} = a_0 - \alpha p_1 - \beta_1 p_2 + \mu_1 p_b + \lambda_1 \theta_1, \]  
\[ D_{i2}^{III} = a_0 - \alpha p_2 - \beta_1 p_1 + \mu_2 p_b + \lambda_2 \theta_2, \]  
\[ D_{b}^{III} = a_0 - \delta p_b + \mu_1 p_1 + \mu_2 p_2 + \lambda_b (\theta_1 + \theta_2). \]  

According to the demands and the cost of green investment, the following objective profit functions of all supply chain members are formulated correspondingly.

\[ \Pi_{\text{M1}}^{III} = (\omega_1 - c_1)(D_{11}^{III} + D_{b}^{III}) - \eta_1 \theta_1^2, \]  
\[ \Pi_{\text{M2}}^{III} = (\omega_2 - c_2)(D_{12}^{III} + D_{b}^{III}) - \eta_2 \theta_2^2, \]  
\[ \Pi_{\text{R}}^{III} = (\omega_1 - c_1)(D_{11}^{III} + D_{b}^{III}) - \eta_1 \theta_1^2, \]  
\[ \Pi_{\text{C}}^{III} = (\omega_2 - c_2)(D_{12}^{III} + D_{b}^{III}) - \eta_2 \theta_2^2. \]
\[ \Pi^{III}_{M_2} = (\omega_2 - c_2)(D_2^{III} + D_b^{III}) - \eta_2 \theta_2^2, \]  
\[ \Pi^{III}_R = (p_1 - \omega_1) D_1^{III} + (p_2 - \omega_2) D_2^{III} + (p_b - \omega_b - \omega_2) D_b^{III}, \]  
\[ \Pi^{III}_{SC} = \Pi^{III}_{M_1} + \Pi^{III}_{M_2} + \Pi^{III}_R. \]  

**Proposition 4.** There exist optimal solutions in the mixed bundling model if these parameters satisfy \(4(\alpha^2 - \beta^2) > 0, 8\alpha(\mu_2^2 + \mu_1^2) = 80(\alpha^2 - \beta^2) - 16\beta\mu_1\mu_2 < 0, (2(\alpha + \delta - 2\mu_1)\eta_1 - (\lambda_1 \lambda_2) / 4 > 0, \) as well as \(2(\alpha + \delta - 2\mu_2)\eta_2 - (\lambda_2 + \lambda_3)^2 > 0, \)

\[ \omega_1^{III} = \frac{\xi_7 \eta_1 \eta_2 + \xi_6 \eta_1 + \xi_6 \eta_2 + \sigma_6}{\xi_6 \eta_1 \eta_2 + \xi_6 \eta_1 + \xi_6 \eta_2 + \sigma_5}, \quad \omega_2^{III} = \frac{\xi_8 \eta_1 \eta_2 + \xi_7 \eta_1 + \xi_7 \eta_2 + \sigma_7}{\xi_6 \eta_1 \eta_2 + \xi_6 \eta_1 + \xi_6 \eta_2 + \sigma_5}, \]  
\[ \theta_1^{III} = \frac{\xi_6 \eta_1 \eta_2 + \xi_6 \eta_1 + \xi_6 \eta_2 + \sigma_6}{\xi_6 \eta_1 \eta_2 + \xi_6 \eta_1 + \xi_6 \eta_2 + \sigma_5}, \quad \theta_2^{III} = \frac{\xi_8 \eta_1 \eta_2 + \xi_8 \eta_1 + \xi_8 \eta_2 + \sigma_8}{\xi_6 \eta_1 \eta_2 + \xi_6 \eta_1 + \xi_6 \eta_2 + \sigma_5}, \]  
\[ p_1^{III} = \frac{X_3 \eta_1 \eta_2 + Y_3 \eta_1 + Z_3 \eta_1 + \Delta_3}{2(\xi_6 \eta_1 \eta_2 + \xi_6 \eta_1 + \xi_6 \eta_2 + \sigma_5)}, \]  
\[ p_2^{III} = \frac{X_4 \eta_1 \eta_2 + Y_4 \eta_1 + Z_4 \eta_1 + \Delta_4}{2(\xi_6 \eta_1 \eta_2 + \xi_6 \eta_1 + \xi_6 \eta_2 + \sigma_5)}, \]  
\[ p_b^{III} = \frac{X_5 \eta_1 \eta_2 + Y_5 \eta_1 + Z_5 \eta_1 + \Delta_5}{2(\xi_6 \eta_1 \eta_2 + \xi_6 \eta_1 + \xi_6 \eta_2 + \sigma_5)} \]

The proof and the expressions of \(\xi_6, \xi_7, \xi_8, \eta_5 - \eta_8, \sigma_5 - \sigma_8, \eta_5 - \eta_8, X_3 - X_5, Y_3 - Y_5, Z_3 - Z_5, \) and \(\Delta_3 - \Delta_5 \) can be found in Appendix A.

The optimal solutions of three pricing models are summarized in Table 2.

| Table 2. Optimal solutions of three pricing models. |
|-----------------------------------------------|
| **Model I** | **Model II** | **Model III** |
| \(\alpha_1^I\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) | \(\xi_6 \eta_1 \eta_2 - c_1 \lambda_1 \eta_1 + c_2 \lambda_2 \eta_2\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) |
| \(\alpha_2^I\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) | \(\xi_6 \eta_1 \eta_2 - c_1 \lambda_1 \eta_2 + c_2 \lambda_2 \eta_1\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) |
| \(\theta_1^I\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) | \(\xi_6 \eta_1 \eta_2 - c_1 \lambda_1 \eta_1 + c_2 \lambda_2 \eta_2\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) |
| \(\theta_2^I\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) | \(\xi_6 \eta_1 \eta_2 - c_1 \lambda_1 \eta_2 + c_2 \lambda_2 \eta_1\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) |
| \(p_1^I\) | \(\xi_2 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) | \(\xi_6 \eta_1 \eta_2 - c_1 \lambda_1 \eta_2 + c_2 \lambda_2 \eta_1\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) |
| \(p_2^I\) | \(\xi_2 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) | \(\xi_6 \eta_1 \eta_2 - c_1 \lambda_1 \eta_1 + c_2 \lambda_2 \eta_2\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) |
| \(p_b^I\) | \(\xi_2 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) | \(\xi_6 \eta_1 \eta_2 - c_1 \lambda_1 \eta_1 + c_2 \lambda_2 \eta_2\) | \(\xi_6 \eta_1 \eta_2 + c_1 \eta_1 + c_2 \eta_2 + \omega)\) |

### 3. Numerical Analysis

This section performs a numerical analysis to explain the theoretical solutions, with a focus on the impact of some parameters on the profits of all supply chain members and the optimal green manufacturing degree of the product. As we mainly study complementary products and the green supply chain, three parameters will be chosen as follows: the green input coefficient of Manufacturer 1 (\(\eta_1\)), the cross-price elasticity (\(\beta\)), and the sensitivity of consumers to the green manufacturing level of products (\(\lambda_1\)). It should be emphasized that when we analyze the impact of one of the above three parameters, the values of the other parameters are fixed.

Note that the values of parameters, including \(\alpha, \beta, \delta, \mu_1, \mu_2, \lambda_1, \lambda_2, \lambda_b, \eta_1\) and \(\eta_2\) need to satisfy the assumptions and the conditions of each proposition in the previous sections first. Then, we refer to the assignment of the literature [8,39], combine our model with the actual situation and attempt to assign several different sets of values for \(a_0, a_b, c_1, c_2\), as well as the above parameters when the
basic assumptions and conditions are met. Finally, through a series of numerical simulations, without
the loss of generality, the benchmarks of the parameters used in models are set as follows: $c_1 = 30,$
$c_2 = 25, \alpha = 1.5, \beta = 0.6, \delta = 3, \mu_1 = 0.5, \mu_2 = 0.3, \lambda_1 = 0.8, \lambda_2 = 0.5, \lambda_b = 1, \eta_1 = 0.4,$ and $\eta_2 = 0.2.$
According to Assumption (3), the potential market demand of two complementary products should
be the same in the three pricing strategies. Here, we suppose the potential market demand in each
model is equal to 600. Therefore, $a_0$ is equal to 300 in the individual pricing model, and $a_b$ is equal to
600 in the pure bundling model. As for the mixed bundling model, let $a_0$ and $a_b$ be equal to 150 and
300, respectively.

3.1. Analysis for Profits of All Supply Chain Members

3.1.1. The Green Input Coefficient

In order to probe the impact of the green input coefficient on the optimal profits of each stakeholder
within the supply chain, we increase $\eta_1$ from 0.4 to 1.4 at 0.1 intervals. The simulation results are
shown in Figure 4.

![Figure 4](image-url)

Figure 4. Impact of the green input coefficient ($\eta_1$) on optimal profits of each stakeholder. (a) Supply
chain; (b) retailer; (c) Manufacturer 1; (d) Manufacturer 2.
Apparently, Figure 4a,c illustrate that the optimal profits of supply chain and Manufacturer 1 have similar downward trends under all sales strategies. In contrast, the optimal profit of Manufacturer 2 under the individual pricing strategy shows an upward trend (see Figure 4d). However, in terms of the level of optimal profits and the supply chain, Manufacturer 1 and Manufacturer 2 are accordant, i.e., the optimal profit in the pure bundling model is highest, and the optimal profit in the individual pricing model is second highest.

It is noteworthy that the results of the retailer’s profits are interesting. Figure 4b shows that, if the green input coefficient is small, mixed bundling can be regarded as the optimal strategy, while pure bundling is no longer competitive in this case. However, when $\eta_1$ increases consistently, the individual pricing strategy has more advantages, and the mixed bundling strategy will gradually lose its positive impact on the retailer’s profit. Thus, the retailer can adjust sales strategies according to different green input coefficients to maximize its profit.

3.1.2. The Cross-Price Elasticity

In order to probe the impact of the cross-price elasticity on the optimal profits of each stakeholder within the supply chain, we increase $\beta$ from 0 to 1 at 0.1 intervals. Figure 5 shows the simulation results.

![Figure 5](image-url)

**Figure 5.** Impact of the cross-price elasticity ($\beta$) on optimal profits of each stakeholder. (a) Supply chain; (b) retailer; (c) Manufacturer 1; (d) Manufacturer 2.
As shown in Figure 5, except for the pure bundling strategy scenario, the more the cross-price elasticity, the lower the profits of the supply chain and its members. That is, once the retailer uses the individual pricing or mixed bundling strategies, the profits will negatively correlate with the cross-price elasticity. In addition, for the profits of the supply chain and two manufacturers, there is a huge gap between the individual pricing strategy and the mixed bundle pricing strategy, but this gap seems to be narrowing with the growth of the cross-price elasticity (see Figure 5a,c,d). Generally, when $\beta$ is large, the pure bundling strategy shows an obvious advantage in the performance of all stakeholders. Of course, if the value of $\beta$ is small, the individual pricing strategy is better for the supply chain and manufacturers, while the mixed bundling strategy is dominant for the retailer.

3.1.3. The Sensitivity of Consumers to the Green Manufacturing Level of Products

To explore the impact of the consumers’ sensitivity to the green manufacturing level of products on the optimal profits of each stakeholder within the supply chain, we increase $\lambda_1$ from 0 to 1 at 0.1 intervals. The results are illustrated in Figure 6.

![Figure 6](image_url)

**Figure 6.** Impact of the sensitivity of consumers to the green manufacturing level of products ($\lambda_1$) on optimal profits of each stakeholder. (a) Supply chain; (b) retailer; (c) Manufacturer 1; (d) Manufacturer 2.

According to Figure 6, under the strategies of individual pricing and mixed pricing, the profits of the supply chain, retailer, and Manufacturer 1 have a positive relationship with the sensitivity of...
consumers to the green manufacturing level of products, while those of Manufacturer 2 show the opposite trajectory.

From the profit level of the three pricing strategies, the situation of the supply chain and two manufacturers are concise and clear, i.e., no matter how the sensitivity of consumers to the green manufacturing level of products changes, their optimal profits under the pure bundling pricing strategy are always dominant, which is much higher than the optimal profits under the mixed bundling strategy. In contrast, the situation of the retailer is more complicated. Only if the sensitivity of consumers to the green manufacturing level of products is less than 0.5, will the retailer be willing to adapt the pure bundling strategy; otherwise, for the retailer, the mixed bundling strategy is a more appropriate choice.

3.2. Analysis for Green Manufacturing Degree of Product

3.2.1. The Green Input Coefficient

To investigate the impact of the green input coefficient on the optimal green manufacturing degree of products within the supply chain, we increase $\eta$ from 0.4 to 1.4 at 0.1 intervals. The results are shown in Figure 7.

Figure 7a demonstrates that there is a negative relationship between the optimal green manufacturing degree of Product 1 and the green input coefficient in the three strategy models, which also verifies the correctness of Proposition 3. In addition, the optimal green manufacturing degree of Product 1 is highest in the mixed bundling strategy, and that of the pure bundling pricing strategy ranks second.

Obviously, the trends of the three curves in Figure 7b are relatively smoother than those in Figure 7a. Interestingly, in two bundling strategies, the optimal green manufacturing degree of Product 2 decreases with the increase in the green input coefficient, whereas in the individual pricing strategy, it illustrates an upward trend. In contrast to Figure 7a, $\theta_2$ is supreme in the pure bundling strategy. Therefore, the gaps among the three pricing strategies are quite conspicuous.

![Figure 7a](image1.png)  ![Figure 7b](image2.png)

**Figure 7.** Impact of the green input coefficient ($\eta_1$) on optimal green manufacturing level of products. (a) Manufacturer 1; (b) Manufacturer 2.

3.2.2. The Cross-Price Elasticity

To reveal the impact of the cross-price elasticity on the optimal green manufacturing degree of products within the supply chain, we increase $\beta$ from 0 to 1 at 0.1 intervals. The simulation results are shown in Figure 8.

![Figure 8](image3.png)
As can be seen from Figure 8a,b the optimal green manufacturing degree of products (including \( \theta_1 \) and \( \theta_2 \)) under the individual pricing and the mixed bundling pricing scenarios declines with the growth of the cross-price elasticity, while that of products under the pure pricing scenario is unchanged. For Product 1, no matter whether cross-price elasticity is low or high, the mixed bundling strategy is always the best option. Regarding Product 2, it is not difficult to see that the mixed bundling and pure bundling strategies are dominant at low and high cross-price elasticities, respectively.

![Figure 8](image1.png)

**Figure 8.** Impact of the cross-price elasticity (\( \beta \)) on optimal green manufacturing level of products. (a) Manufacturer 1; (b) Manufacturer 2.

### 3.2.3. The Sensitivity of Consumers to the Green Manufacturing Level of Products

To elaborate the impact of the sensitivity of consumers to the green manufacturing level of products on the optimal green manufacturing degree of products within the supply chain, we increase \( \lambda_1 \) from 0 to 1 at 0.1 intervals. Figure 9 illustrates the simulation results.

![Figure 9](image2.png)

**Figure 9.** Impact of the sensitivity of consumers to the green manufacturing level of products (\( \lambda_1 \)) on optimal green manufacturing level of products. (a) Manufacturer 1; (b) Manufacturer 2.

As shown in Figure 9a, when the sensitivity of consumers to the green manufacturing level of products increases continuously, the optimal green manufacturing degree of products will also rise correspondingly in the individual pricing and mixed bundling models. However, Figure 9b illustrates a disparate situation. Specifically, in the individual pricing model, the higher the sensitivity of consumers to the green manufacturing level of products, the lower the optimal green manufacturing degree of products.
Product 2 and, in the mixed bundling model, the curve is inverted and U-shaped. In addition, the variations in all the optimal green manufacturing degrees of Product 2 are slight.

4. Discussion

From the theoretical analysis and numerical simulation, it can be determined that several parameters have a considerable impact on the pricing for complementary products in the green supply chain.

Regarding the green input coefficient, on the one hand, the variation in the green input coefficient will not change the preference of two manufacturers for the optimal pricing strategy of complementary products—i.e., they can always obtain maximum profits under the pure bundling strategy. Since two manufacturers take actions simultaneously in the model, they can make full use of their bargaining power to maximize wholesale prices and profits. As two complementary products will only be sold at the same time in one transaction under the pure bundling strategy, this strategy has the highest benefits for manufacturers compared with the other two strategies, on account of the increasing amounts that the retailer wholesales two complementary products for to both manufacturers.

On the other hand, for the retailer, they will adjust their sales strategies according to the green input coefficient. Theoretically, bundling is a favorable strategy to promote new products and stimulate consumption, since the retail prices of bundling are more attractive than individual pricing for consumers. However, the simulation results demonstrate that such a viewpoint is not always correct (see Figure 4b). The main reason is because, with the growth of the green input coefficient, the cost of green manufacturing will certainly increase. According to Assumption (6), manufacturers will continually raise wholesale prices to transfer the influence of incremental green investment. In parallel, the market for complementary products is also expanded gradually. This indicates that the retailer no longer needs to attract consumers by using low prices. Thus, at this point, the individual pricing strategy will bring the most profits to the retailers.

Regarding the sensitivity of consumers to the green manufacturing level of products, it should be highlighted that we only consider the sensitivity of consumers to the green manufacturing level of Product 1 and, due to the symmetry of the model, the situation of Product 2 is similar. Under the individual pricing strategy, with the increase in the sensitivity of consumers to the green manufacturing level of Product 1, the changes in the optimal profit and the optimal green manufacturing level of the product of Manufacturer 2 are anomalous (see Figure 6d, Figure 9b). The main reason is that green manufacturing will inevitably bring extra costs, but manufacturers are not always willing to bear such costs. The increase in the sensitivity of consumers to the green manufacturing level of Product 1 will impel Manufacturer 1 to improve its green production technology. Then, due to the complementarity of products, Manufacturer 2 can reduce their investment in green manufacturing by adopting the “free-riding” behavior. Therefore, the green manufacturing level of Product 2 also decreases correspondingly. In addition, such a decrease will directly affect the market demand of Product 2 (see Equation (2)), and this can also explain why the profit of Manufacturer 2 has a negative relationship with the sensitivity of consumers to the green manufacturing level of Product 1 in the individual pricing model.

5. Conclusions

This study addresses the bundling and pricing strategies for complementary products in a green supply chain. First, we established three pricing models, including individual pricing, pure bundling pricing, and mixed bundling pricing models. Secondly, with the game-theoretic approach, the corresponding equilibrium solutions in three models are derived, respectively. Finally, the impact of several parameters on the optimal profits of all supply chain members and the optimal green manufacturing levels of products are performed. The main findings and managerial implications with respect to our research questions are summarized in Table 3.
| Research Questions | Findings | Managerial Implications |
|--------------------|----------|------------------------|
| **What is the optimal pricing strategy for complementary products in a green supply chain?** | (i) When the green input coefficient of Manufacturer 1 and the sensitivity of consumers to the green manufacturing level of Product 1 increase gradually, the preference of two manufacturers for pricing strategy is constant, i.e., they can always obtain the highest profits under the pure bundling strategy. (ii) However, for the retailer, in the changing process of these two parameters, its optimal pricing strategies will shift from mixed bundling strategy to individual pricing strategy, and from pure bundling strategy to mixed bundling strategy, respectively. (iii) When the cross-price elasticity is very large, the pure bundling strategy is beneficial for all stakeholders. | (i) When consumers are not very sensitive to the green manufacturing level of one product, it is wise to sell two complementary products with the pure bundling strategy, which can maximize the profits of enterprises. (ii) With the gradual increase in green investment and consumer environmental awareness, it is a compromise proposal for both green manufacturers and the retailer that the individual pricing strategy is desirable. (iii) As for highly complementary products in a green supply chain (e.g., new energy vehicles and green tires), adopting the pure bundling strategy can also bring more profit for all members. |
| **How do the profits of green supply chain members change under a variety of bundling strategies?** | (i) With regard to the retailer and Manufacturer 1, their optimal profits decrease with the growth of the green input coefficient of Manufacturer 1 under the three pricing strategies, while they increase with the sensitivity of consumers to the green manufacturing level of Product 1, except for the pure bundling strategy scenario. (ii) As for Manufacturer 2, the variation in its optimal profit is opposite compared with that of the retailer and Manufacturer 1 under the individual pricing strategy scenario. (iii) Moreover, with the increase of the cross-price elasticity, all supply chain members’ optimal profits decrease under the individual pricing and mixed bundling strategies, while the profits under the pure bundling strategy are unchanged. | (i) Consumer environmental awareness is a driving force for enterprises in the green development and, as a result, it is necessary to enhance the sensitivity of consumers to green products. Under the individual pricing and mixed bundling strategies, enterprises should deepen the connection between their brand and environmental protection, integrate the environmental concept into the product design, improve the quality of green products, and attach importance to propagation. (ii) Although higher green investment will cause greater cost pressure for manufacturers, the profits of their complementary manufacturers will increase under the individual pricing strategy. In other words, green manufacturing is mutually beneficial for complementary manufacturers and is worth advocating. |
| **How will the green investment of manufacturers, the cross-price elasticity and the sensitivity of consumers to the green manufacturing level affect the green manufacturing levels of complementary products?** | (i) The optimal green manufacturing levels of two complementary products have a negative relationship with the green input coefficient of Manufacturer 1 under bundling strategies and the cross-price elasticity under the individual pricing and the mixed bundling strategies, respectively. (ii) The optimal green manufacturing level of Manufacturer 1 will rapidly increase with its greenness sensitivity under the individual pricing and mixed bundling strategies, while the result of its complement is opposite. | These findings indicate that one of the complements’ manufacturers wants to benefit from free riding with the interdependent characteristic of complements, so as to reduce the investment in green manufacturing; however, such a behavior is disadvantageous from the perspective of profit. Thus, both complementary manufacturers will reach a win–win outcome if they make green investment actively and simultaneously. |
Of course, there are some limitations in this study. Firstly, the interaction within the green supply chain has not been considered. In fact, competition and cooperation usually exist among green supply chain members, and their relationships will change as time goes on. Secondly, in this paper, the three pricing models are built under the deterministic demand environment that is mainly reflected by the retail prices and green manufacturing levels of complementary products. Thus, for uncertain requirements, our models may not be fully appropriate. In the future, we intend to develop a more comprehensive model to overcome these two limitations, which will be an interesting and challenging work.

**Author Contributions:** Conceptualization, H.S. and C.Z.; methodology, H.S. and C.Z.; software, H.S. and C.Z.; validation, C.Z. and H.S.; formal analysis, H.S.; investigation, H.S.; resources, H.S.; data curation, C.Z.; writing—original draft preparation, C.Z. and H.S.; writing—review and editing, H.S., C.Z. and G.W.; visualization, C.Z.; supervision, H.S.; project administration, H.S.; funding acquisition, H.S. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the National Natural Science Foundation of China (71503135) and 2018th Opening Foundation of the Institute of Climate Change & Public Policy (18QHA002).

**Conflicts of Interest:** The authors declare no conflict of interest.

**Appendix A**

**Proof of Proposition 1.** In order to ensure the retailer’s profit maximal in the individual model, we need to check the concavity of the retailer profit function (see Equation (5)) at first. The Hessian matrix of $\Pi^R_M$ is given as follows:

$$H_1 = \begin{pmatrix} \frac{\partial^2 \Pi^R_M}{\partial p_1^2} & \frac{\partial^2 \Pi^R_M}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \Pi^R_M}{\partial p_2 \partial p_1} & \frac{\partial^2 \Pi^R_M}{\partial p_2^2} \end{pmatrix} = \begin{pmatrix} -2\alpha & -2\beta \\ -2\beta & -2\alpha \end{pmatrix}. \quad (A1)$$

The retailer profit function would be concave if $\alpha$ and $\beta$ satisfy the condition $4(\alpha^2 - \beta^2) > 0$.

Then, setting $\frac{\partial \Pi^R_M}{\partial p_1}$ and $\frac{\partial \Pi^R_M}{\partial p_2}$ equal to 0 as follows, we can obtain retail prices $p_1$ and $p_2$ which maximize the retailer’s profit:

$$\frac{\partial \Pi^R_M}{\partial p_1} = -2\alpha p_1 - 2\beta p_2 + \alpha \omega_1 + \beta \omega_2 + \lambda_1 \theta_1 + a_0 = 0, \quad (A2)$$

$$\frac{\partial \Pi^R_M}{\partial p_2} = -2\alpha p_2 - 2\beta p_1 + \alpha \omega_2 + \beta \omega_1 + \lambda_2 \theta_2 + a_0 = 0. \quad (A3)$$

Combining Equations (A2) and (A3), the solutions are

$$p_1 = \frac{\alpha \omega_1}{2} + \kappa_1 \theta_1 + \tau_1 \theta_2 + \chi_1, \quad (A4)$$

$$p_2 = \frac{\alpha \omega_2}{2} + \kappa_2 \theta_1 + \tau_2 \theta_2 + \chi_1. \quad (A5)$$

Putting Equations (A4) and (A5) into Equations (3) and (4), we can derive that:

$$\Pi^R_{M_1} = -\frac{\alpha \omega_1^2}{2} + \frac{(a_0 + a c_1 + \lambda_1 \theta_1) \omega_1}{2} - \frac{\beta \omega_1 \omega_2}{2} - \frac{\beta c_1 \omega_2}{2} - \eta_1 \theta_1 - \frac{c_1 \lambda_1 \theta_1}{2} - \frac{a_0 c_1}{2}, \quad (A6)$$

$$\Pi^R_{M_2} = -\frac{\alpha \omega_2^2}{2} + \frac{(a_0 + a c_2 + \lambda_2 \theta_2) \omega_2}{2} - \frac{\beta \omega_1 \omega_2}{2} - \frac{\beta c_2 \omega_1}{2} - \eta_2 \theta_2 - \frac{c_2 \lambda_2 \theta_2}{2} - \frac{a_0 c_2}{2}. \quad (A7)$$
Similarly, we should check the concavity of Manufacturer 1 and 2’s profit functions, i.e., Equations (3) and (4), respectively, to maximize two manufacturers’ profits. Accordingly, the Hessian matrices of $\Pi_{M_1}$ and $\Pi_{M_2}$ are as follows:

\[
H_2 = \begin{pmatrix}
\frac{\partial^2 \Pi_{M_1}}{\partial \omega_1^2} & \frac{\partial^2 \Pi_{M_1}}{\partial \omega_1 \partial \omega_2} \\
\frac{\partial^2 \Pi_{M_1}}{\partial \omega_2 \partial \omega_1} & \frac{\partial^2 \Pi_{M_1}}{\partial \omega_2^2}
\end{pmatrix} = \left(\begin{array}{cc}
-\alpha & \frac{\lambda_1}{2} \\
\frac{\lambda_1}{2} & -2\eta_1
\end{array}\right),
\]

(A8)

\[
H_3 = \begin{pmatrix}
\frac{\partial^2 \Pi_{M_2}}{\partial \omega_1^2} & \frac{\partial^2 \Pi_{M_2}}{\partial \omega_1 \partial \omega_2} \\
\frac{\partial^2 \Pi_{M_2}}{\partial \omega_2 \partial \omega_1} & \frac{\partial^2 \Pi_{M_2}}{\partial \omega_2^2}
\end{pmatrix} = \left(\begin{array}{cc}
-\alpha & \frac{\lambda_2}{2} \\
\frac{\lambda_2}{2} & -2\eta_2
\end{array}\right).
\]

(A9)

If the condition $2\alpha \eta_1 - \frac{\lambda_1^2}{4} > 0$ is satisfied, the profit function of Manufacturer 1 is concave. Likewise, the profit of Manufacturer 2 is a concave function when $\alpha, \eta_2$ and $\lambda_2$ satisfy $2\alpha \eta_2 - \frac{\lambda_2^2}{4} > 0$.

Let $\frac{\partial \Pi_{M_1}}{\partial \omega_1}, \frac{\partial \Pi_{M_1}}{\partial \theta_1}, \frac{\partial \Pi_{M_2}}{\partial \omega_2}$ and $\frac{\partial \Pi_{M_2}}{\partial \theta_2}$ be equal to 0 as shown below and we can find optimal wholesale prices $\omega_1^{0*}, \omega_2^{0*}$ and green manufacturing levels $\theta_1^{0*}, \theta_2^{0*}$.

\[
\frac{\partial \Pi_{M_1}}{\partial \omega_1} = -\alpha \omega_1 - \frac{\beta \omega_2}{2} + \frac{\lambda_1 \theta_1}{2} + \frac{a_0 + ac_1}{2} = 0,
\]

(A10)

\[
\frac{\partial \Pi_{M_1}}{\partial \theta_1} = \frac{\lambda_1 \omega_1}{2} - 2\eta_1 \theta_1 - \frac{c_1 \lambda_1}{2} = 0,
\]

(A11)

\[
\frac{\partial \Pi_{M_2}}{\partial \omega_2} = -\alpha \omega_2 - \frac{\beta \omega_1}{2} + \frac{\lambda_2 \theta_2}{2} + \frac{a_0 + ac_2}{2} = 0,
\]

(A12)

\[
\frac{\partial \Pi_{M_2}}{\partial \theta_2} = \frac{\lambda_2 \omega_2}{2} - 2\eta_2 \theta_2 - \frac{c_2 \lambda_2}{2} = 0.
\]

(A13)

Solving Equations (A10), (A11), (A12), and (A13), we get that

\[
\omega_1^{0*} = \frac{\xi_1 \eta_1 \eta_2 + \xi_1 \eta_1 + \xi_1 \eta_2 + \sigma_1}{\xi_0 \eta_1 \eta_2 + \xi_0 \eta_1 + \xi_0 \eta_2 + \sigma_0}, \quad \omega_2^{0*} = \frac{\xi_2 \eta_1 \eta_2 + \xi_2 \eta_1 + \xi_2 \eta_2 + \sigma_2}{\xi_0 \eta_1 \eta_2 + \xi_0 \eta_1 + \xi_0 \eta_2 + \sigma_0},
\]

(A14)

\[
\theta_1^{0*} = \frac{\xi_3 \eta_1 + \sigma_3}{\xi_0 \eta_1 \eta_2 + \xi_0 \eta_1 + \xi_0 \eta_2 + \sigma_0}, \quad \theta_2^{0*} = \frac{\xi_3 \eta_1 + \sigma_4}{\xi_0 \eta_1 \eta_2 + \xi_0 \eta_1 + \xi_0 \eta_2 + \sigma_0}.
\]

(A15)

Substituting Equations (A14) and (A15) into Equations (A4) and (A5), the optimal retail prices $p_1^{0*}$ and $p_2^{0*}$ can be obtained.

\[
p_1^{0*} = \frac{X_1 \eta_1 \eta_2 + Y_1 \eta_1 + Z_1 \eta_2 + \Delta_1}{2(\xi_0 \eta_1 \eta_2 + \xi_0 \eta_1 + \xi_0 \eta_2 + \sigma_0)},
\]

(A16)

\[
p_2^{0*} = \frac{X_2 \eta_1 \eta_2 + Y_2 \eta_1 + Z_2 \eta_2 + \Delta_2}{2(\xi_0 \eta_1 \eta_2 + \xi_0 \eta_1 + \xi_0 \eta_2 + \sigma_0)}.
\]

(A17)

where

\[
\kappa_1 = \frac{a \lambda_1}{2(a^2 + \beta_1^2)}, \quad \kappa_2 = \frac{-\beta \lambda_1}{2(a^2 + \beta_1^2)}, \quad \tau_1 = \frac{-\beta \lambda_2}{2(a^2 + \beta_1^2)}, \quad \tau_2 = \frac{a \lambda_2}{2(a^2 + \beta_1^2)}, \quad \lambda_1 = \frac{a_0}{2(a^2 + \beta_1^2)},
\]

\[
\xi_0 = 64a^2 - 16\beta_1^2, \quad \xi_1 = 32a^2c_1 - 16a\beta_1c_2 + 32a_0 - 16a_0, \quad \xi_2 = 32a^2c_2 - 16a\beta_1c_1 + 32a_0 - 16a_0,
\]

\[
\zeta_0 = -8a^2c_1^2, \quad \zeta_1 = 4(\beta c_1 - a c_1 - a_0)c_1^2, \quad \zeta_2 = -8a^2c_2^2, \quad \zeta_3 = (8a^2c_2 - 4a\beta c_1 + 4\beta^2 c_2 + 8a_0 - 4a c_0)c_2^2,
\]

\[
\zeta_4 = 8a^2c_1^2 - 4a\beta c_2 + 4\beta^2 c_1 + 8a_0 - 4a c_0)\lambda_1, \quad \sigma_0 = \lambda_1^2 + \lambda_2^2, \quad \sigma_1 = c_1 \lambda_1^2, \quad \sigma_2 = c_1 \lambda_2^2.
\]
Proof of Proposition 2. First, we are required to judge the concavity of \( \Pi^H_R \) and the second-order derivative of \( \Pi^H_R \) is given

\[
\frac{\partial^2 \Pi^H_R}{\partial p^2_R} = -2\delta. \tag{A18}
\]

Since Equation (A18) is negative, \( \Pi^H_R \) has the maximum solution. If we give the following equation

\[
\frac{\partial \Pi^H_R}{\partial p_b} = 0
\]

and solve it, then we can have the retail price \( p_b \) which maximizes the profits of the retailer.

\[
\frac{\partial \Pi^H_R}{\partial p_b} = -2\delta p_b + \delta \omega_1 + \delta \omega_2 + \lambda_b \theta_1 + \lambda_b \theta_2 + a_b = 0, \tag{A19}
\]

\[
p_b = \frac{\lambda_1 + \lambda_2}{2} + k_3(\theta_1 + \theta_2) + \chi_2. \tag{A20}
\]

Substituting Equation (A20) into Equations (11) and (12), we obtain that:

\[
\Pi^H_{M_1} = -\frac{\delta \omega_1^2}{2} + \left[ c_b + c_1 \delta + \lambda_b(\theta_1 + \theta_2)\right] \omega_1 + \frac{c_1 \delta \omega_2}{2} - \frac{c_1 \lambda_b(\theta_1 + \theta_2)}{2} - \eta_1 \theta_1^2 - \frac{a_b c_1}{2}, \tag{A21}
\]

\[
\Pi^H_{M_2} = -\frac{\delta \omega_2^2}{2} + \left[ c_b + c_2 \delta + \lambda_b(\theta_1 + \theta_2)\right] \omega_2 + \frac{c_2 \delta \omega_1}{2} - \frac{c_2 \lambda_b(\theta_1 + \theta_2)}{2} - \eta_2 \theta_2^2 - \frac{a_b c_2}{2}. \tag{A22}
\]

After that, we are required to judge the concavity of two manufacturers’ profit functions (i.e., Equations (11) and (12), respectively) to make sure both of them own maximum profits. The Hessian matrices of \( \Pi^H_{M_1} \) and \( \Pi^H_{M_2} \) are presented as follows.

\[
H_4 = \begin{pmatrix}
\frac{\partial^2 \Pi^H_{M_1}}{\partial \omega_1^2} & \frac{\partial^2 \Pi^H_{M_1}}{\partial \omega_1 \partial \omega_2} \\
\frac{\partial^2 \Pi^H_{M_1}}{\partial \omega_2 \partial \omega_1} & \frac{\partial^2 \Pi^H_{M_1}}{\partial \omega_2^2}
\end{pmatrix} = \begin{pmatrix}
-\delta & \frac{\lambda_b}{2} \\
-\frac{\lambda_b}{2} & -2\eta_1
\end{pmatrix}, \tag{A23}
\]

\[
H_5 = \begin{pmatrix}
\frac{\partial^2 \Pi^H_{M_2}}{\partial \omega_1^2} & \frac{\partial^2 \Pi^H_{M_2}}{\partial \omega_1 \partial \omega_2} \\
\frac{\partial^2 \Pi^H_{M_2}}{\partial \omega_2 \partial \omega_1} & \frac{\partial^2 \Pi^H_{M_2}}{\partial \omega_2^2}
\end{pmatrix} = \begin{pmatrix}
-\delta & \frac{\lambda_b}{2} \\
-\frac{\lambda_b}{2} & -2\eta_2
\end{pmatrix}. \tag{A24}
\]

As the condition \( 2a_\eta_1 - \frac{\lambda_b^2}{4} > 0 \) is fulfilled, \( \Pi^H_{M_1} \) will achieve maximum. In the same way, \( \Pi^H_{M_2} \) will also be maximal if \( 2a_\eta_2 - \frac{\lambda_b^2}{4} > 0 \).

Consequently, make \( \frac{\partial \Pi^H_{M_1}}{\partial \omega_1} = 0 \), \( \frac{\partial \Pi^H_{M_1}}{\partial \omega_2} = 0 \), and \( \frac{\partial \Pi^H_{M_2}}{\partial \omega_1} = 0 \) and \( \frac{\partial \Pi^H_{M_2}}{\partial \omega_2} = 0 \) equal to 0 and optimal wholesale prices \( \omega^{H_1}_{\eta_1} \), \( \omega^{H_2}_{\eta_2} \) and green manufacturing levels \( \theta^{H_1}_{\eta_1} \), \( \theta^{H_2}_{\eta_2} \) can be derived.

\[
\frac{\partial \Pi^H_{M_1}}{\partial \omega_1} = -\delta \omega_1 - \frac{\delta \omega_2}{2} + \frac{\lambda_b(\theta_1 + \theta_2)}{2} + \frac{c_1 \delta}{2} = 0, \tag{A25}
\]

\[
\frac{\partial \Pi^H_{M_1}}{\partial \omega_2} = \frac{\lambda_b \omega_2}{2} - 2\eta_1 \theta_1 - \frac{c_1 \lambda_b}{2} = 0, \tag{A26}
\]
\[ \frac{\partial \Pi^l_{M_1}}{\partial \omega_2} = -\delta \omega_2 - \frac{\delta \omega_1}{2} + \frac{\lambda_2 \alpha_1 + \theta_2}{2} + \frac{a_b + c_2 \delta}{2} = 0, \]  
(A27)

\[ \frac{\partial \Pi^l_{M_2}}{\partial \theta_2} = \frac{\lambda_3 \omega_2}{2} - 2\eta_2 \theta_2 - \frac{c_2 \lambda_3}{2} = 0. \]  
(A28)

Combine and solve Equations (A25), (A26), (A27), and (A28), then we can acquire:

\[ a_1^{1r} = \frac{\xi_3 \eta_1 \eta_2 - c_1 \lambda^2_1 \eta_1 - c_1 \lambda^2_2 \eta_2}{12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2}, \quad a_2^{1r} = \frac{\xi_4 \eta_1 \eta_2 - c_2 \lambda^2_1 \eta_1 - c_2 \lambda^2_2 \eta_2}{12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2} \]  
(A29)

\[ \theta_1^{1r} = \frac{\zeta_4 \eta_1}{12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2}, \quad \theta_2^{1r} = \frac{\zeta_4 \eta_1}{12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2} \]  
(A30)

When Equations (A29) and (A30) are substituted into Equation (A20), we get the optimal retail price \( p_b^{1r} \).

\[ p_b^{1r} = \frac{\xi_5 \eta_1 \eta_2 + \zeta_4 \eta_1 + \zeta_4 \eta_2}{12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2}. \]  
(A31)

Then substituting Equations (A29), (A30), and (A31) into Equations (11), (12), and (13), we acquire that

\[ \Pi^l_{M_1} = \frac{(c_1 \delta + c_2 \delta - a_b)^2 (8 \delta \eta_1 - \lambda^2_1) \eta_1 \eta_2}{(12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2)^2}, \]  
(A32)

\[ \Pi^l_{M_2} = \frac{(c_1 \delta + c_2 \delta - a_b)^2 (8 \delta \eta_2 - \lambda^2_2) \eta^2 \eta_2}{(12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2)^2}, \]  
(A33)

\[ \Pi^l_{S_2} = \frac{4(c_1 \delta + c_2 \delta - a_b)^2 \delta \eta_1 \eta_2}{(12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2)^2}. \]  
(A34)

Finally, we can easily derive the optimal performance of supply chain

\[ \Pi^l_{S_2} = \frac{(c_1 \delta + c_2 \delta - a_b)^2 (20 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2) \eta_1 \eta_2}{(12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2)^2}. \]  
(A35)

where

\[ \kappa_3 = \frac{\lambda_3}{2 \delta}, \quad \lambda_2 = \frac{a_b}{2 \delta}, \quad \xi_3 = 8 c_1 \delta - 4 c_2 \delta + 4 a_b, \quad \xi_4 = 8 c_2 \delta - 4 c_1 \delta + 4 a_b, \quad \xi_5 = 2 c_1 \delta + 2 c_2 \delta + 10 a_b, \quad \zeta_4 = - (c_1 + c_2) \lambda_2, \quad \text{and} \quad \zeta_4 = [a_b - \delta (c_1 + c_2)] \lambda_2. \]  

**Proof of Proposition 3.** The first-order derivative of \( \theta_1^{1r} \) in Equation (16) with \( \eta_1 \) and \( \eta_2 \) are shown as the following

\[ \frac{\partial \theta_1^{1r}}{\partial \eta_1} = - \frac{\zeta_4 \eta_2 (12 \delta \eta_2 - \lambda^2_2)}{(12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2)^2}, \]  
(A36)

\[ \frac{\partial \theta_1^{1r}}{\partial \eta_2} = - \frac{\zeta_4 \eta_1 \lambda^2_2}{(12 \delta \eta_1 \eta_2 - \lambda^2_1 \eta_1 - \lambda^2_2 \eta_2)^2}. \]  
(A37)

Recall that \( \eta_1 > 0, \eta_2 > 0, \lambda_2 > 0, 2 a_\eta - \lambda^2_2 > 0, 2 a_\eta - \lambda^2_2 > 0, \) \( \text{and} \quad \zeta_4 = [a_b - \delta (c_1 + c_2)] \lambda_2 > 0. \)

Thus, we have \( \frac{\partial \theta_1^{1r}}{\partial \eta_1} \leq 0 \) and \( \frac{\partial \theta_1^{1r}}{\partial \eta_2} \leq 0. \)

The same Theorem proves that \( \frac{\partial \theta_1^{1r}}{\partial \eta_1} \leq 0 \) and \( \frac{\partial \theta_1^{1r}}{\partial \eta_2} \leq 0. \) \( \square \)
Proof of Proposition 4. In the mixed bundling model, first of all we should assure the profit of retailer is a concave function. The Hessian matrix of $\Pi^R_{\alpha}$ is formulated as:

$$H_6 = \begin{pmatrix}
\frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_1^2} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_1 \rho_2} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_1 \rho_3} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_1 \rho_4} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_1 \rho_5} \\
\frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_2 \rho_1} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_2^2} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_2 \rho_3} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_2 \rho_4} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_2 \rho_5} \\
\frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_3 \rho_1} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_3 \rho_2} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_3^2} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_3 \rho_4} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_3 \rho_5} \\
\frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_4 \rho_1} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_4 \rho_2} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_4 \rho_3} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_4^2} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_4 \rho_5} \\
\frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_5 \rho_1} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_5 \rho_2} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_5 \rho_3} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_5 \rho_4} & \frac{\partial^2 \Pi^R_{\alpha}}{\partial \rho_5^2}
\end{pmatrix} = \begin{pmatrix}
-2\alpha & -2\beta & 2\mu_1 \\
-2\beta & -2\alpha & 2\mu_2 \\
2\mu_1 & 2\mu_2 & -2\delta
\end{pmatrix}.$$

(A38)

When $4(\alpha^2 - \beta^2) > 0$ and $8\alpha(\mu_1^2 + \mu_2^2) - 8\delta(\alpha^2 - \beta^2) - 16\beta\mu_1\mu_2 < 0$, it can be easily found that the retailer profit function is concave.

Let $\frac{\partial \Pi^R_{III}}{\partial \rho_1}$, $\frac{\partial \Pi^R_{III}}{\partial \rho_2}$, and $\frac{\partial \Pi^R_{III}}{\partial \rho_5}$ equal to 0 as the following, then we will get retail prices $p_1$, $p_2$, and $p_b$ which make the retailer’s profit maximum.

$$\frac{\partial \Pi^R_{III}}{\partial \rho_1} = -2\alpha p_1 - 2\beta p_2 + 2\mu_1 p_b + (\alpha - \mu_1)\omega_1 + (\beta - \mu_1)\omega_2 + \lambda_1 \theta_1 + a_0 = 0,$$  

(A39)

$$\frac{\partial \Pi^R_{III}}{\partial \rho_2} = -2\alpha p_2 - 2\beta p_1 + 2\mu_2 p_b + (\alpha - \mu_2)\omega_1 + (\beta - \mu_2)\omega_2 + \lambda_2 \theta_2 + a_0 = 0,$$  

(A40)

$$\frac{\partial \Pi^R_{III}}{\partial \rho_b} = 2\mu_1 p_1 + 2\mu_2 p_2 - 2\delta p_b + (\delta - \mu_1)\omega_1 + (\delta - \mu_2)\omega_2 + \lambda_b (\theta_1 + \theta_2) + a_b = 0.$$  

(A41)

Solving the simultaneous Equations (A39), (A40), and (A41), we obtain that

$$p_1 = \frac{\alpha_1}{2} + \kappa_4 \theta_1 + \tau_3 \theta_2 + \chi_5,$$  

(A42)

$$p_2 = \frac{\alpha_2}{2} + \kappa_5 \theta_1 + \tau_4 \theta_2 + \chi_4,$$  

(A43)

$$p_b = \frac{\alpha_1 + \alpha_2}{2} + \kappa_6 \theta_1 + \tau_5 \theta_2 + \chi_5.$$  

(A44)

Then, substituting Equations (A42), (A43), and (A44) into Equations (21) and (22), we obtain that:

$$\Pi^R_{\alpha_1} = \frac{(2\lambda_1 - \alpha - \delta)\omega_1^2}{(\alpha + \beta - \mu_1 - \mu_2)\omega_1 - \alpha_1} + \frac{a_0 + a_3 + \alpha_1 c_1 (\beta - \mu_1) + \lambda_1 \theta_1 + \lambda_2 \theta_2 + \lambda_3 \theta_3 + \lambda_4 \theta_4 + \lambda_5 \theta_5}{\omega_1} \eta_1 \theta_1^2 + \frac{c_1 \lambda_1 \theta_1 - c_1 \lambda_2 \theta_2}{2} - \frac{(\alpha + \beta) c_1}{2},$$

(A45)

$$\Pi^R_{\alpha_2} = \frac{(2\lambda_2 - \alpha - \delta)\omega_2^2}{(\alpha + \beta - \mu_1 - \mu_2)\omega_2 - \alpha_1} + \frac{a_0 + a_3 + \alpha_2 c_2 (\beta - \mu_2) + \lambda_2 \theta_2 + \lambda_3 \theta_3 + \lambda_4 \theta_4 + \lambda_5 \theta_5}{\omega_2} \eta_2 \theta_2^2 + \frac{c_2 \lambda_2 \theta_2 - c_2 \lambda_1 \theta_1}{2} - \frac{(\alpha + \beta) c_2}{2}.$$  

(A46)

All the optimal decisions of two manufacturers can be attained only if their profit functions are concave. Hence, we give the Hessian matrices of $\Pi^R_{\alpha_1}$ and $\Pi^R_{\alpha_2}$ respectively.

$$H_7 = \begin{pmatrix}
\frac{\partial^2 \Pi^R_{\alpha_1}}{\partial \omega_1^2} & \frac{\partial^2 \Pi^R_{\alpha_1}}{\partial \omega_1 \theta_1} & \frac{\partial^2 \Pi^R_{\alpha_1}}{\partial \omega_1 \theta_2} \\
\frac{\partial^2 \Pi^R_{\alpha_1}}{\partial \omega_1 \theta_1} & \frac{\partial^2 \Pi^R_{\alpha_1}}{\partial \theta_1^2} & \frac{\partial^2 \Pi^R_{\alpha_1}}{\partial \theta_1 \theta_2} \\
\frac{\partial^2 \Pi^R_{\alpha_1}}{\partial \omega_1 \theta_2} & \frac{\partial^2 \Pi^R_{\alpha_1}}{\partial \theta_1 \theta_2} & \frac{\partial^2 \Pi^R_{\alpha_1}}{\partial \theta_2^2}
\end{pmatrix} = \begin{pmatrix}
-\alpha - \delta + 2\mu_1 & \lambda_3 + \lambda_4 & \lambda_3 + \lambda_4 \\
\lambda_3 + \lambda_4 & -2\eta_1 & \lambda_3 + \lambda_4 \\
\lambda_3 + \lambda_4 & \lambda_3 + \lambda_4 & -2\eta_2
\end{pmatrix},$$  

(A47)

$$H_8 = \begin{pmatrix}
\frac{\partial^2 \Pi^R_{\alpha_2}}{\partial \omega_2^2} & \frac{\partial^2 \Pi^R_{\alpha_2}}{\partial \omega_2 \theta_1} & \frac{\partial^2 \Pi^R_{\alpha_2}}{\partial \omega_2 \theta_2} \\
\frac{\partial^2 \Pi^R_{\alpha_2}}{\partial \omega_2 \theta_1} & \frac{\partial^2 \Pi^R_{\alpha_2}}{\partial \theta_2^2} & \frac{\partial^2 \Pi^R_{\alpha_2}}{\partial \theta_2 \theta_1} \\
\frac{\partial^2 \Pi^R_{\alpha_2}}{\partial \omega_2 \theta_2} & \frac{\partial^2 \Pi^R_{\alpha_2}}{\partial \theta_2 \theta_1} & \frac{\partial^2 \Pi^R_{\alpha_2}}{\partial \theta_1^2}
\end{pmatrix} = \begin{pmatrix}
-\alpha - \delta + 2\mu_2 & \lambda_3 + \lambda_4 & \lambda_3 + \lambda_4 \\
\lambda_3 + \lambda_4 & -2\eta_2 & \lambda_3 + \lambda_4 \\
\lambda_3 + \lambda_4 & \lambda_3 + \lambda_4 & -2\eta_1
\end{pmatrix}.$$

(A48)
If $2(\alpha + \delta - 2\mu_1)\eta_1 - \frac{(\lambda_1 + \lambda_2)^2}{4} > 0$, Manufacturer 1 will achieve the maximum profit. Similarly, the maximum of Manufacturer 2’s profit will appear when $2(\alpha + \delta - 2\mu_2)\eta_2 - \frac{(\lambda_2 + \lambda_3)^2}{4} > 0$.

When $\frac{\partial \Pi_{M1}^{III}}{\partial \omega_1}, \frac{\partial \Pi_{M1}^{III}}{\partial \alpha_1}, \frac{\partial \Pi_{M2}^{III}}{\partial \omega_2}$, and $\frac{\partial \Pi_{M2}^{III}}{\partial \alpha_2}$ are equal to 0 as follows, the optimal wholesale prices $\omega_1^{III}, \omega_2^{III}$ and green manufacturing levels $\alpha_1^{III}, \alpha_2^{III}$ can be figured out.

\[
\begin{align*}
\frac{\partial \Pi_{M1}^{III}}{\partial \omega_1} &= \varphi_1 \alpha_1 + \delta_1 \alpha_2 + \kappa_7 \theta_1 + \tau_6 \theta_2 + \chi_6 = 0, \\
\frac{\partial \Pi_{M1}^{III}}{\partial \alpha_1} &= \kappa_7 \omega_1 - 2\eta_1 \theta_1 + \chi_8 = 0, \\
\frac{\partial \Pi_{M2}^{III}}{\partial \omega_2} &= \delta_1 \omega_1 + \varphi_2 \omega_2 + \tau_6 \theta_1 + \kappa_9 \theta_2 + \chi_7 = 0, \\
\frac{\partial \Pi_{M2}^{III}}{\partial \alpha_2} &= \kappa_9 \omega_2 - 2\eta_2 \theta_2 + \chi_9 = 0.
\end{align*}
\]

We can acquire the optimal results by solving simultaneous Equations (A49), (A50), (A51), and (A52).

\[
\begin{align*}
\omega_1^{III} &= \frac{\zeta_6 \eta_1 \omega_2 + \zeta_5 \theta_1 + \varsigma \sigma}{\xi_6 \eta_1 \omega_2 + \varsigma \sigma}, \\
\omega_2^{III} &= \frac{\zeta_6 \theta_1 + \varsigma \sigma}{\zeta_6 \eta_1 \omega_2 + \varsigma \sigma}, \\
\alpha_1^{III} &= \frac{\zeta_8 \eta_1 + \varsigma \sigma}{\xi_6 \eta_1 \omega_2 + \varsigma \sigma}, \\
\alpha_2^{III} &= \frac{\zeta_8 \eta_1 + \varsigma \sigma}{\xi_6 \eta_1 \omega_2 + \varsigma \sigma}.
\end{align*}
\]

The optimal retail prices can be derived when we substitute Equations (A53) and (A54) into Equations (A42), (A43), and (A44).

\[
\begin{align*}
\rho_1^{III} &= \frac{X_3 \eta_1 \omega_2 + Y_3 \eta_1 + Z_3 \eta_2 + \Delta_3}{2(\xi_6 \eta_1 \omega_2 + \varsigma \sigma)}, \\
\rho_2^{III} &= \frac{X_4 \eta_1 \omega_2 + Y_4 \eta_1 + Z_4 \eta_2 + \Delta_4}{2(\xi_6 \eta_1 \omega_2 + \varsigma \sigma)}, \\
\rho_b^{III} &= \frac{X_5 \eta_1 \omega_2 + Y_5 \eta_1 + Z_5 \eta_2 + \Delta_5}{2(\xi_6 \eta_1 \omega_2 + \varsigma \sigma)}.
\end{align*}
\]

where

\[
\begin{align*}
\kappa_4 &= \frac{(a-b)^2 \alpha_1 + (a+b-\beta_1) \beta_1}{2[(a-b)^2 \beta_1 - (b^2 + \beta_1^2) \alpha_2 + 2b \mu_1 \mu_2]}, \quad \kappa_5 = \frac{(a-b)^2 \beta_1 + (a+b-\beta_1) \alpha_1}{2[(a-b)^2 \beta_1 - (b^2 + \beta_1^2) \alpha_2 + 2b \mu_1 \mu_2]}, \\
\kappa_6 &= \frac{(a+b-\beta_1) \beta_1 + (a-b)^2 \alpha_1}{2[(a-b)^2 \beta_1 - (b^2 + \beta_1^2) \alpha_2 + 2b \mu_1 \mu_2]}, \quad \kappa_7 = \frac{\lambda_1 + \lambda_2}{2}, \quad \kappa_8 = \frac{\lambda_2 + \lambda_3}{2}, \\
\tau_3 &= \frac{(a-b)^2 \beta_1 + (a+b-\beta_1) \lambda_1}{2[(a-b)^2 \beta_1 - (b^2 + \beta_1^2) \alpha_2 + 2b \mu_1 \mu_2]}, \quad \tau_4 = \frac{(a-b)^2 \alpha_1 + (a+b-\beta_1) \lambda_1}{2[(a-b)^2 \beta_1 - (b^2 + \beta_1^2) \alpha_2 + 2b \mu_1 \mu_2]}, \\
\tau_5 &= \frac{(a+b-\beta_1) \beta_1 + (a-b)^2 \lambda_1}{2[(a-b)^2 \beta_1 - (b^2 + \beta_1^2) \alpha_2 + 2b \mu_1 \mu_2]}, \quad \tau_6 = \frac{\lambda_3}{2}, \quad \varphi_1 = -\alpha - \beta + 2 \mu_1, \quad \varphi_2 = -\alpha - \beta + 2 \mu_2, \\
\delta_1 &= \frac{(a-b)^2 \beta_1 + (a+b-\beta_1) \mu_2 + (a-b) \mu_2}{2[(a-b)^2 \beta_1 - (b^2 + \beta_1^2) \alpha_2 + 2b \mu_1 \mu_2]}, \\
\chi_3 &= \frac{(a-b)^2 \beta_1 + (a+b-\beta_1) \mu_2 + (a-b) \mu_2}{2[(a-b)^2 \beta_1 - (b^2 + \beta_1^2) \alpha_2 + 2b \mu_1 \mu_2]}, \\
\chi_4 &= \frac{(a-b)^2 \beta_1 + (a+b-\beta_1) \mu_2 + (a-b) \mu_2}{2[(a-b)^2 \beta_1 - (b^2 + \beta_1^2) \alpha_2 + 2b \mu_1 \mu_2]}, \quad \chi_5 = \frac{(a-b)^2 \beta_1 + (a+b-\beta_1) \mu_2 + (a-b) \mu_2}{2[(a-b)^2 \beta_1 - (b^2 + \beta_1^2) \alpha_2 + 2b \mu_1 \mu_2]}, \\
\chi_6 &= \frac{c_1 (a+b-\beta_1) + (a-b) \mu_2}{2}, \quad \chi_7 = \frac{c_1 (a+b-\beta_1) + (a-b) \mu_2}{2}, \quad \chi_8 = \frac{c_1 (a+b-\beta_1) + (a-b) \mu_2}{2}, \quad \chi_9 = -c_1 (a+b-\beta_1), \\
\zeta_4 &= 4 \delta_1^2 - 4 \varphi_1 \varphi_2, \quad \zeta_5 = 4 \chi_6 \varphi_2 - 4 \chi_7 \varphi_1, \quad \zeta_6 = 4 \chi_6 \varphi_2 - 4 \chi_7 \varphi_1, \quad \zeta_7 = 4 \chi_6 \varphi_2 - 4 \chi_7 \varphi_1, \quad \zeta_8 = 4 \chi_6 \varphi_2 - 4 \chi_7 \varphi_1, \quad \zeta_9 = 4 \chi_6 \varphi_2 - 4 \chi_7 \varphi_1.
\end{align*}
\]
\[ 
\begin{align*}
\zeta_7 &= 2\chi_k \kappa_7^2 - 2\chi_k \kappa_7 \tau_6 - 2\chi_k \kappa_7 \delta_1 + 2\chi_8 \kappa_7 \varphi_1, \\
\zeta_8 &= 2\chi_k \kappa_7 \delta_1 - 2\chi_8 \kappa_7 \varphi_1 + 2\chi_8 \delta_1, \\
\sigma_5 &= \kappa_7 \kappa_8 \kappa_6 \tau_6 - \kappa_7 \kappa_6 \tau_8, \\
\sigma_6 &= \chi_8 \kappa_7 \kappa_8 \kappa_6 \tau_6 - \chi_8 \kappa_7 \kappa_6 \tau_8 - \chi_8 \kappa_7 \varphi_1 - \chi_8 \delta_1 + \chi_8 \kappa_7 \tau_6 \varphi_2, \\
\sigma_9 &= \chi_8 \kappa_7 \kappa_8 \kappa_6 \tau_6 - \chi_8 \kappa_7 \kappa_6 \tau_8 - \chi_8 \kappa_7 \varphi_1 - \chi_8 \delta_1 + \chi_8 \kappa_7 \tau_6 \varphi_2, \\
X_3 &= 2\chi_k \xi_6 + \xi_7, \\
X_4 &= 2\chi_k \xi_6 + \xi_8, \\
X_5 &= 2\chi_k \xi_6 + \xi_7 + \xi_8, \\
Y_3 &= 2\chi_k \xi_5 + 2\kappa_8 \xi_4 + \xi_6 + \chi_7, \\
Y_4 &= 2\chi_k \xi_5 + 2\kappa_8 \xi_4 + \xi_6 + \chi_7, \\
Z_4 &= 2\chi_k \xi_5 + 2\kappa_8 \xi_4 + \xi_6 + \chi_7, \\
\Delta_3 &= 2\chi_5 \xi_5 + 2\kappa_8 \xi_4 + \xi_6 + \chi_7, \\
\Delta_4 &= 2\chi_5 \xi_5 + 2\kappa_8 \xi_4 + 2\varphi_4 + \sigma_7, \\
\Delta_5 &= 2\chi_5 \xi_5 + 2\kappa_8 \xi_4 + 2\varphi_4 + \sigma_6 + \sigma_7. 
\end{align*} 

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