Photon-assisted confinement-induced resonances for ultracold atoms

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We solve the two-particle s-wave scattering for an ultracold atom gas confined in a quasi-one-dimensional trapping potential which is periodically modulated. The interaction between the atoms is included in terms of Fermi’s pseudopotential. For a modulated isotropic transverse harmonic confinement, the atomic center of mass and relative degrees of freedom decouple and an exact solution is possible. We use the Floquet approach to show that additional photon-assisted resonant scattering channels open up due to the harmonic modulation. Applying the Bethe-Peierls boundary condition, we obtain the general scattering solution of the time-dependent Schrödinger equation which is universal at low energies. The binding energies and the effective one-dimensional scattering length can be controlled by the external driving.

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The ability to accurately control the effective atomic interactions in ultracold atom gases has opened the door to novel exciting physics in the recent years. The currently available experimental tools allow for a powerful implementation of analog quantum simulators realized by cold-atom assemblies\textsuperscript{[1] 2}. In the regime of strong atomic interactions, the quantum gas becomes scale-invariant and shows universal physical aspects quantified in terms of a few dimensionless coefficients. For instance, the three-dimensional (3D) s-wave scattering length characterizes the interatomic interactions and has to be compared to the mean interatomic distance which is of the order of the particles’ inverse momenta $k^{-1}$. It can be controlled over several orders of magnitude via the help of a magnetic field tuned across a Feshbach resonance\textsuperscript{[3]}. In 3D, the tunability of the scattering length, for instance, reveals the cross-over from the weakly interacting Bardeen-Cooper-Schrieffer superfluid state, where $1/(ka) \rightarrow -\infty$, to the strongly interacting Bose-Einstein condensate of dimer molecules, where $1/(ka) \rightarrow +\infty$\textsuperscript{[4]}.\textsuperscript{[1]}

In quasi-one-dimensional (1D) gases, another relevant length scale appears in form of the transverse confinement length $a_{\perp}$. Then, the scattering of two tightly confined quantum particles is known to induce universal low-energy features in form of confinement-induced resonances (CIRs)\textsuperscript{[4] 5}. At low energies, only the transverse ground state of the confining potential is significantly populated whereas the higher transverse states can be only virtually populated during the elastic collisions. In this regime, the remaining scattering processes in the longitudinal direction can be characterized by the effective 1D interaction strength $g_{1D}$. It is governed by a single parameter, being the ratio of $a$ and the zero-point-fluctuation length scale $a_{\perp}$, irrespective of the details of the confinement.

By tuning the confinement strength (or the 3D scattering length via a Feshbach resonance\textsuperscript{[3]}) across a CIR, it is possible to cross over from strongly repulsive to strongly attractive interactions. CIRs have been observed in a strongly confined 1D gas of fermionic K atoms\textsuperscript{[6] 7} and of bosonic Cs atoms\textsuperscript{[8]}. This has allowed to investigate the cross-over from a strongly repulsive Tonks-Girardeau gas to a strongly attractive Super-Tonks-Girardeau gas\textsuperscript{[8]}. CIRs have also been observed in a strongly interacting 2D Fermi gas\textsuperscript{[9] 10} and in mixed dimensions\textsuperscript{[10] 11}.

In close analogy to a Feshbach resonance, the CIR occurs when the continuum threshold for the lowest transverse state (the open channel) has the same energy as a bound state formed by two particles being in some transverse excited states (the closed channels)\textsuperscript{[5]}. Put differently, the transverse orbital degrees of freedom of the confined atoms play the same role as the internal atomic spin degrees of freedom for a Feshbach resonance\textsuperscript{[3]}.\textsuperscript{[5]}

When the transverse confinement of two equal atom species is purely harmonic, only one such bound state exists, leading to a single universal CIR\textsuperscript{[4] 6}. This feature can be traced back to the separability of the center of mass and relative coordinates\textsuperscript{[11]}. In turn, a multitude of CIRs appears when these degrees of freedom are no longer separable, i.e., for a mixture of different species\textsuperscript{[11] 12}, anisotropic\textsuperscript{[12] 13} and anharmonic confinement\textsuperscript{[11] 16}, and in mixed dimensions\textsuperscript{[10] 17}. Dipolar CIRs have been predicted to occur also in presence of long-range anisotropic interactions between different atomic angular momentum states\textsuperscript{[18]}. Coupled CIRs have also been predicted at higher energies when all partial scattering waves are taken into account\textsuperscript{[19]}.\textsuperscript{[20]}

As an alternative to the “orbital” Feshbach resonance to control the atomic scattering, a time-dependent modulation of internal atomic states generates an optical Feshbach resonance\textsuperscript{[20]}. It occurs when the optical radiation resonantly couples two atoms in their respective electronic ground state to a molecular state formed by electronically excited states, i.e., the relevant closed channels in this case. This mechanism has been also experimentally demonstrated\textsuperscript{[21]}.
We consider the atoms initially in the adiabatic transverse ground state and having a longitudinal momentum \( k \). They are described by the incoming wave function

\[
\psi_{in}(t) = \exp[-i\varepsilon t] \exp[i k z] u_0(x, t) u_0(y, t),
\]

where \( \varepsilon \) is the quasienergy \( \varepsilon = k^2/2\mu + \nu \) of the atoms. Here, the time-periodic functions \( u_0(x, t) \) are the Floquet eigenstates of the parametrically driven harmonic oscillator with quasienergy \( (n + 1/2)\nu \) \([23, 25]\). Notice that, when the driving is switched off adiabatically, \( \nu \to \omega_0 \) and the \( u_n(x) \) become the eigenstates of the harmonic oscillator.

Our goal is to compute the full solution \( \psi(t) \) which includes the scattered wave \( \psi_{out}(t) \) such that \( \psi(t) = \psi_{in}(t) + \psi_{out}(t) \). It is most convenient to introduce the time-periodic Floquet state \( \phi(t) = \exp[i\varepsilon t] \psi(t) \) as the solution of the eigenvalue problem \( H(r, t) \phi(r, t) = \varepsilon \phi(r, t) \), where \( H \equiv H - i\partial_t \) is the Floquet Hamiltonian \([24]\). With \( T = 2\pi/\omega_{ex} \) being the external modulation period, we can formally write

\[
\phi(r, t) = \phi_{in}(r, t) + \int_0^T dt' G(t, t; t') f(t') \frac{1}{2\mu},
\]

where \( \phi_{in}(t) = \exp[i\varepsilon t] \psi_{in}(t) \) and the second term on the r.h.s. represents the outgoing scattered wave function \( \phi_{out}(t) \). The integral kernel \( G(t, t; t') = (H_0 - \varepsilon - i\partial_t)^{-1} \) is the retarded Floquet-Green’s function with \( H_0 = H_0 - i\partial_t \). This wave function has to fulfill the Bethe-Peierls boundary condition

\[
\phi(r \to 0, t) \approx \left(1 - \frac{r}{a}\right) \frac{f(t)}{4\pi r},
\]

with the Bethe-Peierls amplitude \( f(t) \) yet to be determined. For large \( z \), the asymptotic scattered wave can be written as

\[
H(r, t) = H_0(t) + U(r), \quad H_0(r, t) = \frac{\mu^2}{2\mu} + \frac{1}{2} \mu \omega^2(t) r_\perp^2.
\]
be decomposed into partial waves as
\[
\phi_{\text{out}}(r) \approx \sum_{m=\text{open}} S^m_n \left( \frac{k}{k_{nm}} \right)^{1/2} e^{i k_{nm} |z|} u_{n_x}(x, t) u_{n_y}(y, t).
\]
(6)
Here, \( |S^m_n|^2 \) is the probability of the atoms to be excited into the transverse state with quantum number \( n = (n_x, n_y) \) after absorbing \( m > 0 \) photons from the field and thereby acquiring the momentum \( k_{nm} \), which is determined via
\[
\frac{k^2_{nm}}{2\mu} + (n + 1)\nu = \varepsilon + m\omega, \quad n = n_x + n_y.
\]
(7)

The number of available open channels depends on \( m \). The S-matrix elements \( S^m_n \) are determined by the Bethe-Peierls boundary condition Eq. (5) (for technical details, see Ref. [25]) as
\[
S^m_n = \frac{i}{2\sqrt{k/k_{nm} T}} \int_0^T dt' e^{i m \omega_{\text{ex}} t} u^*_{n_x}(0, t') u^*_{n_y}(0, t') f(t').
\]
(8)
Next, we insert Eq. (12) into Eq. (5) and define a scalar product \( \langle f | g \rangle = T^{-1} \int_0^T dt f^*(t) g(t) \) on the Hilbert space of time-periodic functions. Then, we can derive an inhomogeneous linear equation for the Bethe-Peierls amplitude \( f(t) \) as
\[
\left( \tilde{\varepsilon} + \frac{a_\perp}{a} \right) |f\rangle = -4\pi N^{-1/2} |\text{in}\rangle.
\]
(9)
Here, we have introduced the regularized integral kernel
\[
\langle t | \tilde{\varepsilon} | t' \rangle = \frac{2\pi a_\perp}{\mu} \left[ G_z(r, 0; t, t') - \delta(t - t') \frac{T\mu}{2\pi r} \right] r \to 0
\]
(10)
and the normalized vector \( |\text{in}\rangle \) via \( \langle t | \text{in}\rangle = N^{1/2} a_\perp \phi_\text{in}(0, t) \). For small initial momenta, \( k \ll k_{nm} \), the scattering is dominated by elastic collisions. In this regime, it is convenient to divide the kernel \( \tilde{\varepsilon} \) into a smooth part \( \tilde{\varepsilon}_s \), that can be evaluated for \( k = 0 \), and the contribution of the channel of the incoming atoms, where the \( k \)-dependence is retained. This yields [25]
\[
|f\rangle = -4\pi N^{-1/2} \frac{k a_{1D}}{k_{1D} - i} \left( \tilde{\varepsilon}_s + \frac{a_\perp}{a} \right)^{-1} |\text{in}\rangle.
\]
(11)
Here, it is convenient to introduce the effective 1D scattering length \( a_{1D} \) for the longitudinal scattering according to
\[
\frac{a_\perp}{a_{1D}} = -2\pi N^{-1} \left( |\text{in}\rangle \langle \tilde{\varepsilon}_s + \frac{a_\perp}{a} | |\text{in}\rangle \right)^{-1}.
\]
(12)
Since the operator \( \tilde{\varepsilon}_s \) is not Hermitian, it acquires an imaginary part whose meaning is discussed further below. Using Eq. (1) in Eq. (8), we obtain the S-matrix element
\[
S^0_{00} = -\frac{i}{k_{1D} - i}.
\]
(13)
This is the probability amplitude for the reflection of a 1D particle due to the effective scattering potential \( U_{1D} = g_{1D} \delta(z) \) with complex interaction strength \( g_{1D} = -1/(\mu a_{1D}) \). Its imaginary part refers to the loss of atoms in the excited transverse states which occurs due to inelastic scattering processes into other channels provided by the modulation. From this, we obtain the elastic cross section, which is the probability of an elastic scattering event, as \( \sigma_\ell = |S^0_{00}|^2 \) and its inelastic counterpart as \( \sigma_r = 1 - \sigma_\ell - |1 + S^0_{00}|^2 \), respectively. From Eq. (13), we find
\[
\sigma_\ell = \left( 1 + k^2 |a_{1D}|^2 - 2k \text{Im } a_{1D} \right)^{-1},
\]
\[
\sigma_r = -2k \text{Im } a_{1D},
\]
(14)
(Im \( a_{1D} < 0 \)). The effective quasi-1D scattering cross sections (for the case \( |\text{Im } a_{1D}|/|a_{1D}| = 0.15 \)) are shown in Fig. 2. The probability of an elastic scattering event tends to one for small relative momenta of the scattering atoms, \( k \ll k_{nm} \). On the other hand, the scattering is dominated by inelastic scattering events for comparatively larger momenta, \( k \gg |\text{Im } a_{1D}|^{-1} \). Hence, when \( |a_{1D}| \) becomes smaller than the typical longitudinal de Broglie wavelength of the atoms a scattering resonance results.

Formally, the 1D scattering length \( a_{1D} \) can be expressed in terms of the spectrum \{\( \lambda_m \)\} and the right and left eigenvalues \( |v^R_m| \) and \( |v^L_m| \) of the kernel \( \tilde{\varepsilon}_s \),
\[
\frac{a_\perp}{a_{1D}} = -2\pi N^{-1} \sum_m \frac{|\langle \text{in}| v^R_m \rangle |\langle \text{in}| v^L_m \rangle |}{\lambda_m + a_\perp/a}.
\]
(15)
Hence, if several eigenvectors significantly overlap with the vector \( |\text{in}\rangle \) of the incoming wave, more than one scattering resonances may occur. The resonances in \( |a_\perp/a_{1D}| \) are well resolved Lorentzian peaks with their center determined by the resonance condition \( a_\perp/a = \text{Re } \lambda_m \) and with their line width given by \( \text{Im } \lambda_m \), if their mutual distance exceeds the corresponding widths.

The kernel \( \tilde{\varepsilon}_s \) can be computed fully analytically only for the trivial case \( F = 0 \), see Supplemental Information.

FIG. 2. Effective scattering cross sections in quasi-1D: total (\( \sigma \)) and elastic (\( \sigma_\ell \)) cross section as a function of the relative longitudinal momentum \( k \) for \( |\text{Im } a_{1D}|/|a_{1D}| = 0.15 \).
In this case, the left and right eigenvectors, \( |v^L_m\rangle \) and \( |v^R_m\rangle \), are plane waves, \( \langle t |v^R_m|^\ast = \langle v^R_t|t = \exp[i\omega_{\text{ex}}t] \), and the incoming wave is given by \( |\text{in}\rangle = |v^R_0\rangle \). Thus, the overlap is zero for \( m \neq 0 \) and we recover the standard result: there is only one CIR for \( a_{\perp}/a = -\lambda_0 = -\zeta(1/2,1) \) (\( \zeta(x,y) \) is the Hurwitz zeta function) when the energy of the virtual bound state formed by the closed channels, coincides with the continuum threshold \( \omega_0 \).

If the driving is weak and far away from any parametric resonance

\[
F \ll \omega_{\text{ex}}^2, \quad |2\omega_0 - m\omega_d| \gg \omega_d (4F/\omega_d^2)^m, \quad (16)
\]

we expect the eigenvalues \( \{\lambda_m\} \) of the kernel \( \zeta \), to only smoothly deviate from their values for \( F = 0 \). To obtain a specific result, we evaluate and diagonalize the kernel \( \zeta \) with a semi-analytical procedure outlined in the Supplementary Material \[24\]. The standard CIR for a weak non-resonant driving is shown in Figs. 3 (a) and (b). The zero-photon resonance is clearly broadened by the inelastic collisions. In Fig. 3 (c) we show that the width \( \text{Im}\lambda_0 \propto F^2 \).

For a finite driving there is no selection rule preventing the remaining eigenvectors of the kernel \( \zeta \) to yield a finite contribution in the r.h.s of Eq. \[15\] to \( a_{\perp,1D} \). For weak non-resonant driving, we can label the eigenvalues \( \lambda_m \) with the number \( m \) of radiofrequency photons which are virtually absorbed (emitted for \( m < 0 \)) and later re-emitted (re-absorbed) during an elastic collision. From Eq. \[15\] we see that the contribution to \( a_{1D} \) from the processes where the photons are virtually absorbed is largest for \( a_{\perp}/a = -\text{Re}\lambda_m \). In the Supplemental material, we show that, for the case \( m > 0 \), this occurs when the \( m \)-photon transition from the continuum threshold \( h\omega_0 \) to the virtual bound state with energy \( E_B(m) \) formed by the transverse channels which are still closed after the absorption of \( m \) photons [with transverse energy \( E > E_M = \text{Int}[m\omega_{\text{ex}} + \omega_0] \) is resonant, \( E_B(m) = \omega_0 + m\omega_{\text{ex}} \). We do not expect that these processes lead to a scattering resonance because the corresponding bound states leak very quickly into the open channels. In fact, the imaginary part \( \text{Im}\lambda_m \) can be shown to be finite even for \( F \to 0, \text{Im}\lambda_m \gtrsim 1 \) \[25\]. On the other hand, the contribution to \( a_{1D} \) from processes where the photons are first emitted (\( m < 0 \)) is largest for

\[
\frac{a_{\perp}}{a} = -\text{Re}\lambda_m \approx -\zeta(1/2, |m\omega_{\text{ex}}/2\omega_0|) \quad (17)
\]

Notice that the energy \( E_B \) of the molecular bound state (for \( F = 0 \)) is given by \( \frac{a_{\perp}}{a} = -\zeta(1/2, (E_B - 1)/2\omega_0) \) \[5\]. Hence, the processes where \( |m| \)-photons are virtually emitted leads to the largest enhancement of scattering when the molecular recombination accompanied by the emission of \( m \) photons is resonant, \( \omega_0 - |m\omega_{\text{ex}} \approx E_B \). Since \( \lim_{F \to 0} \text{Im}\lambda_m = 0 \) \[25\] (the molecular bound state can only dissociate because of the driving), these processes lead to sharp CIRs. The resonance in the case of resonant emission of a single photon (\( m = -1 \)) is shown in Fig. 4 for different values of \( F \). Notice that, the scattering resonances investigated here involve the true molecular bound state and not a virtual bound state formed by the closed channels. Therefore, they are not Feshbach-type resonances but rather the dynamical equivalent of a shape resonance (a scattering resonance that occurs when a generic potential has a bound state closed to the continuum threshold) \[29\].
Conclusions - We have shown that the s-wave scattering of two atoms confined in a tight quasi-1D trap can be coherently controlled by a RF modulation of the transverse confinement. The scattering in the atom cloud in the adiabatic transverse ground state can be efficiently described as atoms in a 1D waveguide interacting via a short range interaction. In this case, the coupling constant $g_{1D}$ acquires an imaginary part and incorporates inelastic scattering into the transverse excited states. This mechanism generates a new kind of photon-assisted or dynamical confinement-induced resonances. The results are universally valid in the regime of low energies. The dynamical confinement-induced resonances are another example in which photon-assisted processes carry signatures of atomic interparticle interactions which have also been seen in the photon-assisted tunneling in a Bose-Einstein condensate [27]. Such processes could be used for avoided-level-crossing spectroscopy in strongly interacting quantum gases [28].

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