Analysis techniques for folded plate roofs and cellular bridges

general review and comparisons

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Abstract. Folded plates have attracted profound interest in recent years because of their economic advantages and architectural appearance. In particular, their basic structural response is indeed logical enough, explicit and simple although its required numerical computation procedure is, in a little bit, boring. This type of structures have gained increasing popularity and offers more advantages than more complex structures, such as cylindrical shells, arches and right folded plate frames. Similarly, the thin-walled cellular bridge decks can be treated as a multi-folded plates structure. This study produces an overall review of the historical development of the most popular methods utilized for analysis the folded plate structures which are offered with their applications and how these methods are developed gradually. Four common methods are chosen in this paper to show their highlights of references particularizing in analysis of the above mentioned types of structure; the folded plate elasticity method (FPEM), finite element method (FEM), finite strip method (FSM) and spline finite strip method (SFSM). This investigation covers the elastic behavior, and the experimental researches on the elastic reaction of folded plate structures.

1. Introduction
Folded plates, as a structural shape, first appeared fortuitously in central Europe in about 1929 and the first design method of the folded plate was published in 1930 by G. Ehlers and H. Craemer [1]. Then it was brought to the United States of America immediately after the second world war where it quickly became commonplace and was admitted as a new style of construction due to its capability of erection, confirmed implementation and its structural clearness in both analysis and design. Due to the increase in the use of computer programs in recent years and the great development of those programs and their diversity, which performs structural analysis of various types of buildings and engineering structures, It has become necessary for the engineer to choose the appropriate program for the structure to be analyzed so as to ensure accurate results in a less time and cost in addition to facilitating the input data and the identification of outgoing data so clearly that the designer can sense that data and know to what extent it is logical and true to reality.

2. Definition of Folded Plate Structures
Folded plates are flat plate assemblies connected together rigidly over their edges in a manner that the structural system is able of holding loads without the need for extra supporting beams along ridge edges. Some of them have constant thicknesses and the other ones have variable thicknesses according to the nature of the structure and the applied loads as shown in figure 1. The application of folded
structures is experienced in several types such as roofs, wall structures, steel sheet piles and floor structures …etc.

3. **Specific Statement of Thin-Walled Cellular Bridge Decks**

The thin-walled cellular bridge decks are made of box cross sections with thin walls created by combining large hollow openings through the slab depth. The box girder deck is the most common kind, with one or more rectangular or trapezoidal cells. Hollowed slab bridges, which have large diameter circular voids, may also be treated as a cellular bridge section. Figure 2 show some types of cellular bridge decks.

![Figure 1. Folded roofs (a) variable-thickness. (b) constant-thickness.](image)

![Figure 2. The cellular bridge decks](image)
4. General Description of Analysis Methods

4.1. Folded Plate Elasticity Method

4.1.1. General

This method uses the theory of elasticity plane stress and the theory of plate bending to define the in-plane stresses and bending moments in the members of folded plate system. This system involves a combination of longitudinal plate elements joining along their interior longitudinal edges (along which each pair of abutting strips meets), and simply supported ends without intermediate membranes. The thin-walled cellular bridge deck can be treated as an assembly of multi-folded plates connected together throw longitudinal edges so it can be analyzed like folded plates. The analysis of direct stiffness harmonic is utilized to get the solution of box-girder bridges with curved or simply supported conditions having a specific load case.

4.1.2. Application of the FPEM to Folded Plate Structures

In 1957, Goldberg and Leve [2] developed the method of elasticity for prismatic folded plate structures. Later in 1964, De Fries-Skene and Scordelis [3] programmed and applied an approach of the direct stiffness for folded plates and became quite popular because of its accuracy. However, the method suffers from being fairly complex, and is difficult to apply to orthotropic folded plate structures and to dynamic analysis.

In 1993, Kollar [4] detected the behavior of folded plate structures carrying partial horizontal and vertical loads statically. The triangular plate elements loaded vertically to their planes with plates subjected to concentrated forces.

In 2002, Grigrenko and Rozhok [5] analyzed stress/strain state of variable thickness rectangular plates and constant weight. The laws of variation in the thickness were specified to depend on parameters so that the plate weight remained constant for any of their values. To solve the case, separated fourier series were used, which makes it possible to reduce a 2D problem to 1D.

In 2017, Bhamare, Bramhankar and Baviskar [6] have presented an analysis of radial folded plates and the behavior various parameters namely angle of inclined plate, radial angle, width to depth ratio, transverse moment developed at the joint, longitudinal moment at the mid-span, and deflection have been studied. There is a requirement to study the behavior of parameters for achieving economy in constructions. The analysis procedure of reinforced concrete and folded plates is based on elastic theory because they are subjected to large forces.

4.1.3. Application of the FPEM to Box-girder Bridges

The elasticity method was also utilized in the study of box girder bridges with simply supported boundary condition by Scordelis [7] in 1966 and by Chu and Dudnik [8] in 1969. Subsequently, the elasticity method was expanded to contain the curved box girder bridges analysis by Chu [9] in 1971 but the disadvantages outlined above existed also for this case.

The expansion of this method for intermediate diaphragms with and without external supports was explained by Scordelis [10] in 1971.

In 1971, Meyer and Scordelis [11] used the method to cellular structures, and for the same purpose Al-Rifaie and Evans, in 1979 [12], and in 1984, Evans [13] used that method.

Actually, the method is complicated and consumes time. So, the Canadian Code of Highway Bridge Design (CHBDC 2000) [14] limited this approach to bridges having support cases directly parallel to both ends support lines of the bridge.

Al-Hadithy, 1985 [15] analyzed the indeterminate bridges by using the orthotropic plate theory through the experimental tests on different kinds of bridge decks and various conditions of indeterminacy, thereafter checked the outcomes of displacements and bending moments in the two perpendicular directions, and the composite impacts of the shear forces torsion and moments over longitudinal axis, with Cusens and Pama's [16] closed geometry solution.

In 1990, Marsh and Taylor [7] made some developments on a technique that merges analysis of the classical folded plate for a combination of orthotropic and isotropic plates to shape box girders. The
assemblage consisted of beam elements, and under the impact of the available loads included the settlement action. A stiffness analysis was performed to establish the conformance of actions and displacements at element intersections.

In 2013, Alfred [18] utilized an analytical approach to the torsional-distortional behavior of box girder bridge decks with the thin walled using theory of Vlasov’s for thin-walled structures. Consideration was first given to Vlasov’s theory of thin-walled structures based on an energy formulation for the equilibrium equations were established using Vlasov’s relations of stress/strain and the expression through which the system of potential energy in equilibrium.

Finally, the programs of folded plate theory gave precise results for box structures made up of isotropic plates of uniform cross-section.

4.2. Finite Element Method
4.2.1. Definition
The well-known finite element method (FEM) is considered a great and multipurpose method for structural analysis. Theoretically, it is appropriate to analyze all structures. During the past three decades, it quickly became a very familiar method for the complicated problems solved by computer in the structural analysis. The approach may be considered an earlier expansion of established analytical methods, in which a structure is divided into separated members interconnected with a specific number of nodal lines.

The finite element method seemed to be used firstly in 1960 by Clough [19]. But, the connotation of finite elements was initially presented by Hubert at 1975 [20]. Then it started to firm after 1963 when Besselling, Melosh, Fraeijs, and others perceived that the FEM was a shape of the Ritz method and emphasized it as a general procedure to deal with elastic continuum cases.

In 1967, Zienkiewicz and Cheung [21] gave the finite element method more interpretation. The techniques of (FEM) concentrate on the displacement method, in which the real continuity is substituted by an accumulation of finite elements interrelated at knot points Rokey and Evans [22] and Holand and Bell [23].

In a folded plate structures, the finite elements can contain elements of plate and cross or frame elements of longitudinal having a one-dimensional type or two-dimensional shell. Stiffness matrices of an approximate manner of the continuity are then improved for the finite elements depending on supposed displacement or stress modality. After that, it can proceed to determine displacements and internal stresses of nodal point. The precision of the obtained output data depends on the mesh fineness utilized in dividing the structure and the basic hypotheses applied to derive the stiffness matrices. Almost the obtained output data fulfill similarity, however, no real equilibrium exists in the entire continuum until the point that an adequately fine mesh is utilized.

4.2.2. Application of the FEM to Folded Plate Structures
In 1997, Samanta and Mukhopadhyay [24] reported, for the first time, an analysis of nonlinear geometric trapezoidal corrugated sheet. Besides 3D analysis, they proposed an orthotropic model equal to that included bending and extensional rigidities. To determinate the characteristics of free vibration for the corrugated sheet, the analysis was extended. They made comparisons of natural frequencies and displacements of folded plates to explain the multiplicity of the suggested technique.

In 2006, Peng, Kitipornchai and Liew [25] examined a method of free Galerkin mesh which was borrowed from the finite element method and founded on the theory of 1st order shear deformation utilized to analyze the stiffened and un-stiffened folded plate elastic bending problem subjected to various boundary conditions and loadings.

In 2008, Hernandez, and Hervella-Nieto [26] attempted a finite element technique of the folded plate free vibration. They considered Naghdi model, involving shear, bending and membrane actions for the folded plate.

In 2011, Haldar and Sheikh [27] presented a great accuracy composite plate bending member whose application to analyze the isotropic and compound folded plates was given to examine the behavior of the member. In that element, the effect of shear deformation was considered
In 2012, Nayak, Shenoi, and Blake [28] studied the performance of their finite element technique. They obtained new outcomes for the ephemeral analysis of originally stressed compound sandwich shape folded plate structures.

In 2014, Desai, Kewate and Hirkane [29] studied the material used for folded plate structures, and how to analyze them by the finite element strip.

4.2.3. Application of the FEM to Box-girder Bridges

In 1971, Chapman [30] performed an analysis by FEM on concrete and steel box-girder bridges to inspect the middle membranes action on the twisting stresses and deformation.

In 1971, Lim [31] improved a member that had an in-plane beam displacement field. The member had a shape of the trapezoid. Subsequently, it was utilized to analyze curved, right box-girder, or skew bridges having constant width and depth.

In 1972, William and Scordelis [32] studied the cellular structures elastic analysis having a specific form of constant depth in plan utilizing quadrilateral elements.

In 1976, Ramesh [33] developed the finite element method to be applicable to single and multicellular sections of uncoupled out-of-plane and in-plane forces with neglecting the shear distortion to present a curved member with six freedom degrees at every node.

In 1979, Sargious [34] explored the single-cell concrete box-girder bridge behavior with end diaphragms and openings supported by a central pillar.

In 1985, Dezi [35] inspected the effect of several income data on the distortion of beams with the curved single-cell box in transverse section versus straight type. These income data utilized the longitudinal and transverse positions under external load, the ratio of width to depth, span to the radius of the cell, and the number of transverse membranes.

In 1987, Chang and Zheng [36] inspected the negative effects of shear lag on cantilever box girders by utilizing the finite element method. They derived expressions to specify the zone of negative shear delay effect which was related with width and span parameters.

In 1992, Mishra [37] investigated how it closely using associated FDM to analyze the right box-girder bridges like a reasonable alternative to the FEM. In this approach, the overall energy of the bridge system was discretized into two types of energy owing to expansion and bending which resulted from twisting and shear participated by two rectangular member isolated sets created by an appropriate finite difference method.

In 1997, Abdelfattah [38] used the 3D model of finite element to examine the adequacy of various systems to brace the steel box-girders versus shear lag.

In 1998, Elbadry and Debaiky [39] produced a computer program and a numerical procedure to analyze distortions and stresses depending on the time created in concrete, curved, pre-stressed, cellular bridges because of shape changes, statically, and the load case during implantation. They based a procedure on the formulation of displacement by the FEM in which variable, multi-node, curved cross-section beam members were utilized for the model of prestressed concrete bridges with an arbitrary shape in plan.

In 2016, Wang, Xu, Luo, Wu and Yan [40] analyzed the dynamic response by using spatial finite element for thin-walled curved box girder bridges. By considering the torsional and flexural parameters of thin-walled curved box girder with the action of original curvature. Therefor the displacement of strain calculation interconnection was formed.

4.3. Finite Strip Method

4.3.1. Introductory Statement

For structures with simply supported situations and organized geometric plans, a whole analysis by using FEM is very overwhelmingly both excessive and needless, and at the same times may be impossible. So, the solutions cost can be very large by the magnitude order when a more refined, needs a higher dimensional analysis. Moreover, very often the problem size of an accurate analysis might be so great as to overtax whatever machines that are available to a designer or researcher, so that the problem either will have to be solved roughly, or some additional lengthy, time-consuming
subroutines written to lower the core requirements. Nevertheless, its effectiveness is limited owing to the need of a high-performance computer which is not always readily reachable.

According to the above, an economical and simple method, defined as the semi-analytical (Finite Strip Method), was effectively done in 1968 by Y. K. Cheung [41]. It is a procedure of distinguishing the form of the finite element procedure using the displacement approach and alternate methods which can decrease the calculations effort and core requirements, while at the same time holding to some amplitude the flexibility of the FEM, clearly popular for the aforementioned class of structures. This method has gained wide acceptance for regular structures like (folded plates, shallow shells, shear walls, box girder bridges, tunnels, and tall buildings, etc.).

In this approach, the structures are chopped into (2D) strips or (3-D) subdomains in which one of the sides (2-D) or one or more inverse pairs of faces (3-D) of the a subdomain which is in compliance with the structure boundaries. The shape of the structure is commonly constant over one or two axes in which the strip width or the layer or prism cross-section must not change from along the span. So, whilst voided slabs and box-girder bridges are appropriately spliced into prisms or strips, for isotropic, multi-layered, or thick plates and shells, splitting into layers would absolutely be more covenant.

Unlike the standard FEM, functions of polynomial displacement in all ways are used, the finite strip method requests for utilizing simple polynomials in some ways and continuously differentiable smooth series in the other directions, with the condition that such series should satisfy, as a priority, the boundary conditions at the ends of the strips or prisms.

4.3.2. Application of the FSM to Folded Plate Structures

In 1986, Sekulovic and Milasinovic [42] conducted an analysis of the folded plate and plate structures, taking into account the geometrical nonlinearities and the effects of creep, using the finite strip method. They made the assumption that only small deformations and large displacements and rotations exist. Their analysis did not neglect creep of concrete as it influences on some structures, especially when geometrical nonlinearities are taken into account.

In 1989, Golley and Grice [43] presented a step combining some good features of the finite strip method and the finite element method for the analysis of box girders and prismatic folded plate structures. Their procedure was based on displacement functions within strip-elements as combinations of polynomials used with conventional finite elements plus displacement functions used with simply supported finite strips, which are products of truncated trigonometric series and polynomials.

In 1990, Lavy, Yoseph and Rosenhouse [44] presented a new mixed-hybrid finite strip formulation for stress analysis of long folded plates combined with rectangular simply supported panels. The formulations evolved coupled in-plane and flexural stresses, while the model for the plate bending followed the Mindlin plate.

In 1992, Hinton and Rao [45] studied the computational tools developed for shape structure optimization of shells and folded plates in which the strain energy or the weight of the structure is minimized and subject to certain constraints. They considered the variability both thickness and shape to define the area of the structure. They carried out their analysis using curved variable depths finite strips.

In 2002, Ozakca, Taysi and Kolcumte [46] produced a structural form amendment of prismatic folded plates with considering the buckling load. That amendment procedure specified buckling loads using variable thickness, quadratic and cubic, linear, and C0 continuity finite strips. The overall structural optimization operation is conducted by integrating finite strip analysis, definition of cubic spline shape and depth, semi analytical sensitivity analysis and mathematical programming algorithm.

In 2011, Jiang and Au a general [47] developed a finite strip model for the static and vibration analyses of folded plate structures. They modelled the geometric constraints of the folded plates, like the situations at the end and mid supports, by very stiff translational and rotational springs as appropriate.
In 2017, Milasinovic, Majstorovic and Vukomanovic [48] analyzed the uniformly compressed folded plate structures, made of homogenous materials. They analyzed two provenances of non-linearity, the first source due to high deflection which involving geometrical non-linearity, and the other involving material non-linearity due to inelastic behavior, by making a full-energy finite strip method (FSM).

4.3.3. Application of the FSM to Box-girder Bridges

In 1970, Cheung, Cheung and Ghali [49] used the finite strip method for the analysis of simply-supported and continuous slab girder bridge decks of constant widths. They presented new features including the stiffness matrix derivation of a beam and its coupling with strips, the stiffness matrix derivation of a strip with longitudinal variation in thickness, and the analysis of bridges with discrete internal column supports.

In 1973, Buragohain and Agrawal [50] produced an approach on a curved box-girder bridge by analyzing it harmonically in the peripheral direction and FDM in the cross section. They turned the total potential energy into two types: extended shear twisting and bending which formed two-member matrices.

In 1974, Cusens and Loo [51] studied a general FSM to multi and single-span box-bridges with respect to prestressing forces analysis. Kabir and Scordelis [52], in the same year, evolved a FSM computer package to make analysis to the continuous curved span of cellular bridge decks having planar frame bents supported by interior radial diaphragms.

In 1978, Cheung and Chan [53] utilized the FSM to calculate the compression flange active width of straight multi-spine and multi-cell box-girder bridges.

In 1984, Cheung [54] developed a numerical approach established from the FSM to analyze curved continuous multi-cell box-girder bridges.

In 1989, Cheung and Li [55] expanded the application range of FSM to analyze continuous variable depth web box-girder bridges with haunch shape strips.

In 1990, Abdullah and Abdul-Razzak [56] analyzed a prestressed concrete box girder bridge by applying the (FSM) based on an auxiliary nodal line technique and higher order bending in-plane strips.

In 1991, Shimizu and Yoshida [57] utilized the FSM to estimate the response forces in order to design the load firmness membranes at the internal support condition of two span curved continuous box girder bridges.

In 1992, Bradford and Wong [58] studied the local the straight composite concrete deck buckling for box section of steel in negative bending by using the FSM.

In 1992, Hinton and Rao [59] studied the linear elastic analysis of prismatic folded plate and shell structures which is supporting on two opposite edges diaphragms with the other two edges randomly restrained. They performed their analysis by variable thickness, utilizing curved, Mindlin-Reissner finite strips. They presented the family of (C0) strips by theoretical formulation, then they tested the precision and relative representation of the strips for curved cases.

In 1995, Lounis and Cohn [60] used the nonlinear program for optimum design based on the FSM and FDM to design the prestressed concrete cellular single and two-cell box bridge decks or perforated slab systems. They suggested that the analyzing of approximate live load moment will determine the sensitivities of the moment to change in flange thickness and the depth of deck.

In 2003, Ozakca and Taysi [61] studied the analysis and finding the optimum shapes of curved box-girder bridges by using the practical development and effective computational tools. In this method the weight or the strain energy of the structure was minimized subject to particular constrains. They determined the stresses and displacements by using the FSM depending on Mindlin-Reissner shell theory.

In compare with the FEM, the FSM considered the saver in both computer effort and time, due to just little unknown number are generally needed in the analysis. Nevertheless, the FSM is limited to simply support prismatic systems having simply supported end (CHBDC 2000) [14].
4.4. Spline Finite Strip Method

4.4.1. Theme

The "conventional" finite strip method has experienced difficulties in analyzing structures with multiple spans, concentrated forces, mixed boundary conditions, and discrete column-supports at strip ends, etc. So to overcome these shortcomings in the classical method, a developed spline finite strip method by Fan, S. C. in 1982 [62] successfully solved many problems of regular structures such as plates with complex boundary conditions, folded plate structures, shells and concrete box-girder bridges. Nowadays, both the finite strip method and the spline function have been amply applied not only to the analysis of structures (including linear and nonlinear, elastic, viscoelastic and plastic, static and dynamic analysis, etc.) but also to many other fields such as thermal conduction, geotechnics, the propagation of stress waves and even cracking problems. Hundreds of papers in the field of finite strip method have been published. Nevertheless, in most cases, both the "conventional" and the SFM have been confined to analyze regular domains. Since there are many irregularly shaped problems in the modern construction and industries, economical and efficient methods for the analysis of them are required, in view of the advantages of the finite strip method.

4.4.2. Application of the SFM to Folded Plate Structures

In 1982, Mizusawa and Fuji [63] utilized SFM to analyze typical plates and shells in both transverse and longitudinal directions.

In 1988, Wah-yuk [64] presented a new technique to analyze the arbitrarily shaped structures by merging the SFM with the range transformation concept. The author in this method extended the applicability of this technique considerably.

In 1990, Chen, Gutkowski and Puckett [65] derived a B-spline column element in three dimension space combining with the B-spline compound strip method to analyze the folded plate structures with intermediate supports.

In 2007, Eccher, Rasmussen and Zandonini [66] analyzed geometric cribriform folded plate structures nonlinearly by using the isoperimetric SFM. The general theory of this method described the kinematics, strain–displacements and constituent hypotheses which are then applied to the SFM. They derived the stiffness matrices and the tangential secant by utilizing the equilibrium condition and its gradual form.

In 2012, Loja, Soares and Barbosa [67] analyzed the free vibration and static manner of functionally graded sandwich plate shape structures, utilizing B-SFSM on many models depending on various shear deformation theories.

In 2015, Viswanathan, Navaneethakrishnan and Aziz [68] analyzed the buckling of rectangular plates with variable-thickness lean on elastic foundation through utilizing a 5th degree spline approximation approach. They assumed that the plate thickness was varying linearly in the direction of one edge and to be sinusoidal and exponential. The plate is loaded in two opposite edges by in-plane load.

4.4.3. Application of the SFM to Box-girder Bridges

In 1983, Cheung and Fan [69] presented an easy way for immediate and comparatively precise statically analyzing of the right box-girder bridge decks having various interior conditions and end supports through utilizing the SFM.

An auxiliary nodal line method (ANL) was employed by Abdul-Razzak [70] in 1987, who used the 6th and 3rd order strips for plate bending and plane stress impacts, respectively for analyzing box-girder bridge decks having both continuous and simply supported conditions.

The SFM was employed by Chang and Gang [71] in 1990, to make an analysis of the box-girder bridge with a cantilever deck having a single-cell. They took the deformation actions of a thin-walled box section in the calculations by assuming a horizontal spring support distributed over the cantilever deck, which was treated as a cantilever slab.

In 1991, Cheung and Li [72] expanded the SFM for free vibration analysis of curved box-girder bridges. Later, in 1992, Cheung and Jaeger [73] exercised the SFM to the same bridge arrangement.
In 1992, Cheung and Au [74] analyzed a right box girder bridge by using SFSM with considering calculated form functions in the cross way. Their technique led to a comparatively tight range matrix which needed just little work of computations to solve.

In 1993, Ng and Chen [75] examined the spline finite strip method to analyze the arbitrary Mindlin plates. Firstly, they charted the plate into a square range using the natural coordinate plane, by utilizing the cubic shell shape function. Then they divided the charted plate into a number of strips. Finally, they used the interpolation functions principle to describe the displacements of each strip which was given as a product of B3-spline functions and piecewise polynomials.

In 1995, Au and Cheung [76] employed the isoparametric spline finite strips for shells to give the solution for static and free vibration cases of the bridges with variable depth which has arbitrary alignments. Firstly, they divided the continuous bridge into substructures by utilizing this method. Each substructure was represented as an assemblage of isoparametric spline finite strips.

In 1996, Senthilvasan [77] evolved the matrices of mass and stiffness for curved multicellular and single bridges through merging the SFSM with a folded plate model (horizontally curved) of the bridge.

In 1999, Ayad A Slaby [78] produced the analysis of curved box girder bridge decks by the spline finite strip method, in which he derived the SFSM depending on Mindlin's thick shell theory to shape the isoparametric finite shell strip. He utilized the idea of isoparametric FEM simultaneously with the spline finite strip method.

In 2000, Lau Cheung and Cheng [79] presented a numerical analysis solution for the 3D flutter analysis of bridges depending on the SFSM. For the first time then, the SFSM has been spread over the area of bridge aerodynamics in wind engineering. The SFSM in this application, was utilized to model the bridge girder.

In 2002, Choi Kim and Hong [80] performed an analysis of prestressed box-girder bridges by the spline finite strip method utilizing the non-periodic interpolation of B-spline. When they analyzed the prestressed box-girder bridges, the rapid prestressing losses were taken into account and every force of the tendon at the tendon point was rated by summing up the neighboring segment forces.

In 2015, Yarah [81] analyzed isotropic bridge decks through utilizing the method of spline finite strip. He studied the suitability and precision of this method by analyzing the bending action on ten plates bending examples comprising a wide diversity of boundary conditions and loading. He confirmed that the SFSM was a very strong technique especially when dealing with bending actions of rectangular plates having various compounds of boundary conditions producing a high range of reliability and validity.

In 2016, Al-Hadithy and Ali [82] studied the effective simplification on the advantage of the SFSM in the analysis of orthotropic and ribbed bridge decks by introducing two stages of identity. Firstly, they transformed the real ribbed bridge deck into similar orthotropic plate, and then they subdivided it to perpendicular finite strips related with plate bending impact just through using the B3-spline technique to indicate the deflection function in the longitudinal way of strips.

5. Application Example
The concrete variable-thickness folded plate structure shown in figure 3 has a simply supported single span. The Details of loads and section for this structure showed in figure 4. The properties of material are $E=20\times10^9$ N/m² and $\nu=0.15$. Originally, the dimensions of this structure took from Cheung [41] while loading took from investigational recommendations. The analytical model of SFSM used six strips and six sections per nodal line as shown in figure 5 Al-Masoudy [83]. Figures 6, 7, 8, and 9 shows, respectively, the results of deflections, transverse stress, transverse moments and longitudinal moments at mid-section for total load which are obtained from SFSM, FSM and FEM. The FEM results are acquired by using a computer program named Variable-thickness Finite Strip Auxiliary Nodal Line (VFSANL), which is constructed by Fortran77 and developed by MATLAB Khalis [84] in which an analytical model of finite strip 6+3 with 10 strips and 99 terms of fourier series was used. The FEM solution obtained by an engineering analysis tool named (JL Analyzer), which is a family of
Finite Element Analysis tools integrated into a GUI Windows environment Khalis [84]. Applications are static and dynamic analysis (buckling, frequency analysis) and to electric and thermal analysis.

![Figure 3. Folded Plate Elevation and Loading Cheung [41].](image)

Total Load = 6500 N/m²

![Figure 4. Folded Plate Sectional Dimensions and Loading](image)

![Figure 5. Spline Finite Strip Simulation.](image)
Table 1. Vertical Deflection in Folded Plate at Mid-Span Cross Section.

| Distance (m) | (W) cm | (W) cm | (W) cm |
|--------------|--------|--------|--------|
| X-Direction  | (FSM)  | (SFSM)| (FEM) |
| 0            | -4.30E-02 | -3.52E-02 | -4.20E-02 |
| 8.66         | -2.00E-02 | -1.44E-02 | -2.50E-02 |
| 18.5         | 1.00E-02  | 2.34E-02  | 8.00E-03  |
| 28.36        | -2.00E-02 | -1.44E-02 | -2.50E-02 |
| 37           | -4.30E-02 | -3.52E-02 | -4.20E-02 |

Figure 6. Vertical Deflection in Slab Bridge at Mid-Span Cross Section.

Table 2. Transverse Stresses ($\sigma_x$) at Mid-span Cross-section

| Distance (m) | ($\sigma_x$) N/m$^2$ | ($\sigma_x$) N/m$^2$ | ($\sigma_x$) N/m$^2$ |
|--------------|-----------------|-----------------|-----------------|
| X-Direction  | (FSM)           | (SFSM)          | (FEM)           |
| 0            | 2.50E+06        | 1.85E+06        | 1.60E+06        |
| 8.66         | -2.60E+06       | -2.93E+06       | -3.80E+06       |
| 18.5         | -3.60E+06       | -3.92E+06       | -4.80E+06       |
| 28.36        | -2.60E+06       | -2.93E+06       | -3.80E+06       |
| 37           | 2.50E+06        | 1.85E+06        | 1.60E+06        |
**Figure 7.** Transverse Stresses ($\sigma_x$) at Mid-span Cross-section.

**Table 3.** Longitudinal Moment (My) at Mid-span Cross-section

| Distance (m) X-Direction | (My) N.m/m (FSM) | (My) N.m/m (SFSM) | (My) N.m/m (FEM) |
|--------------------------|------------------|-------------------|------------------|
| 0                        | 1.60E+04         | 1.60E+04          | 1.80E+04         |
| 8.66                     | -2.50E+03        | -2.58E+03         | -2.40E+03        |
| 18.5                     | -7.50E+03        | -8.13E+03         | -1.20E+04        |
| 28.36                    | -2.50E+03        | -2.58E+03         | -2.40E+03        |
| 37                       | 1.60E+04         | 1.60E+04          | 1.80E+04         |
Figure 8. Longitudinal Moment (My) at Mid-span Cross-section.

Table 4. Transverse Moment (Mx) at Mid-span Cross-section.

| Distance (m) X-Direction | (Mx) N.m/m (FSM) | (Mx) N.m/m (SFSM) | (Mx) N.m/m (FEM) |
|--------------------------|------------------|-------------------|------------------|
| 0                        | 1.50E+04         | 1.07E+04          | 1.80E+04         |
| 8.66                     | 1.00E+02         | -1.72E-03         | 1.00E+02         |
| 18.5                     | -8.00E+03        | -5.42E+03         | -1.20E+04        |
| 28.36                    | 1.00E+02         | -1.72E-03         | 1.00E+02         |
| 37                       | 1.50E+04         | 1.07E+04          | 1.80E+04         |
6. Conclusions
Many researchers have made different works to make the analysis and design of folded plate structures effectively. The methods proposed never produced a total and universal design and analysis of those structures types. However, more researches need to be done. These methods contain the techniques that are randomly directed with less calculation and ease of processing, whose results are easy and all-inclusive while realizing and description of the loads and consideration reactions, for instance; time saving, safety, economy and reliability. The technique to be anticipated must be multiple and associating realistic computer simulation and results of the experimental investigation in order to achieve a sufficient level of design, predicting safety and economy for folded plate structures.

According to the above mentioned methods of analysis, the below conclusions are drawn from this investigation paper:

1. The finite element method among the redefined methods, is likely the most interested and time exhaustion. Nevertheless, it is yet the most common and overall approach for dynamic and static analysis, containing all portions acting the structural reaction. The other approaches confirmed to be adequate but restricted in applicability and scope.

2. Getting evaluated, the semi-analytic technique called “finite strip method” in its original state adopting fourier series for the prime longitudinal displacement variation has quantitatively and qualitatively lifted the accuracy, efficiency and feasibility of the analysis of longitudinally regular structures (into which folded plate roofs and box-girder bridge decks are categorized).

3. When utilizing B3-spline function in the folded plate strip formulation, it can be got a simple method that transacts the substituting of variable-thickness plates for all boundary conditions with different elastic rigidities and subjected to any probable load case.

4. Submitting a new approach for analyzing the variable-thickness folded plate systems with a regular geometrical section based on dividing the structure into longitudinal strips and even number of transverse sections utilizing simplified terms for elastic rigidities in this object has confirmed to be active and powerful.

Figure 9. Transverse Moment (Mx) at Mid-span Cross-section.
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