Abstract

Pooling operations are a layer found in almost every modern neural network, which can be calculated at low cost and serves as a linear or nonlinear transfer function for data reduction. Many modern approaches have already dealt with replacing the common maximum value selection and mean value operations by others or even to provide a function that includes different functions which can be selected through changing parameters. Additional neural networks are used to estimate the parameters of these pooling functions. Therefore, these pooling layers need many additional parameters and increase the complexity of the whole model. In this work, we show that already one perceptron can be used very effectively as a pooling operation without increasing the complexity of the model. This kind of pooling allows to integrate multi-layer neural networks directly into a model as a pooling operation by restructuring the data and thus learning complex pooling operations. We compare our approach to tensor convolution with strides as a pooling operation and show that our approach is effective and reduces complexity. The restructuring of the data in combination with multiple perceptrons allows also to use our approach for upscaling, which is used for transposed convolutions in semantic segmentation.

1 Introduction

Convolutional neural networks are the successor in many visual recognition tasks Krizhevsky et al. (2012), Yuan et al. (2019) as well as graph classification Zhao and Wang (2019), Orsini et al. (2015) and time series annotation Palaz et al. (2015), Connor et al. (1994). The main focus of modern research on CNNs are architecture improvements He et al. (2016), Howard et al. (2017), optimizer enhancements Kingma and Ba (2014), Qian (1999), computational cost reduction Rastegari et al. (2016), training procedures Goodfellow et al. (2014), and also the building blocks like convolutions Long et al. (2015), graph kernels Yanardag and Vishwanathan (2015) or pooling operations Kobayashi (2019a,b), Eom and Choi (2018). The last mentioned pooling operations are used for data reduction, which reduces the calculation costs and makes the model robust against input variations.

The pooling operation itself is inspired by the biological viewpoint of the visual cortex which is based on a neuroscientific study Hubel and Wiesel (1962). Most works suggest therefore max pooling as the biologically considered best operator Riesenhuber and Poggio (1998, 1999), Serre and Poggio (2010). However, in practice, it turned out that average pooling also works for CNNs as well as combinatorial approaches of max and average pooling. Therefore, it can be said, that the optimal pooling operation is dependent on the model, the task and the data set used. To further improve the accuracy of CNNs, simple pooling operations (e.g. max and average) are replaced by other static functions as well as trainable operators.
The first group of operations is motivated by image scaling and uses wavelets Mallat [1989] in wavelet pooling Williams and Li [2018] or other image scaling techniques Weber et al. [2016] like in detailed-preserving pooling (DPP) Saeedan et al. [2018]. Another approach is the integration of formulas which can choose between several static pooling operations like max or average pooling. The first works in this area are mixed pooling and gated pooling Lee et al. [2016], Yu et al. [2014]. These selective methods have been extended with parameterizable functions that can map many different average and max pooling operations such as learned norm Gulcehre et al. [2014], alpha Simon et al. [2017], and alpha integration pooling Eom and Choi [2018]. This approach was further refined according to the maximum entropy principle Kobayashi [2019a], Lee et al. [2016] and, as with alpha integration pooling Eom and Choi [2018], provided with parameters that can be trained and optimized in an end-to-end fashion. The global-feature guided pooling Kobayashi [2019a] uses the input feature map to adapt the pooling parameters. Therefore, an additional CNN is used and jointly trained. In Lee et al. [2016] the authors proposed mixed max average pooling, gated max average pooling, and tree pooling.

In addition to the deterministic pooling operations already mentioned, other methods that introduce randomness were presented Zeiler and Fergus [2013]. The motivation of these pooling operations comes from drop out Srivastava et al. [2014] and variational drop out Kingma et al. [2015]. This approach can also be used in combination with all the other pooling operations. Another approach which does not formulate the combination of local neuron activations as a convex mapping or downsampling operation is gaussian based pooling Kobayashi [2019b]. The authors introduce a local gaussian probabilistic model with mean and standard deviation which are estimated using global feature guided pooling Kobayashi [2019a] and therefore, also requires an additional CNN model for parameter estimation.

In contrast to the other approaches, we present the simple use of perceptrons Rosenblatt [1958] or neurons for use as pooling operator. To create a deeper network from these single neurons we describe a data restructuring, which also allows to scale the data up. This allows the pooling operation presented by us to be used not only in data reduction, but also in data expansion, which is used in semantic segmentation. By the simple use of neurons or multi-layer neural networks the parameters to be trained increase only minimally and the complexity of the pooling operation remains nearly the same. In comparison to other pooling operations presented, we also compare our approach with the strided tensor convolutions.

Our work contributes to the state of the art with regard to the following points:

1. We present an efficient usage of perceptrons as pooling operation and show a
2. Perceptron-based data upscaling.
3. We provide an efficient construction of multilayer neural networks with the proposed perceptron upscaling and perceptron pooling operations and
4. Provide CUDA implementations of the proposed approach for easy integration into research and application projects.

2  Method

Our fundamental idea to improve learnable pooling operations is to use one of the best known function approximators available today, i.e. the neural network which consists of single neurons (also called perceptrons) and is also known as multilayer perceptron (MLP). The main advantage of an MLP is that it can be easily integrated into deep neural networks (DNNs) since it consists of the same basic components as a DNN. This makes it easy to train it with the remaining layers and the same optimization methods.

Figure 1a) shows the basic concept of a pooling operation. Based on the input window, an output value is calculated, which differs depending on the selected pooling operation. Then the window is moved in the x and y dimension based on the stride parameter. If the pooling operation is average pooling, the weights (represented by the blue lines in Figure 1a) could be assigned the value 0.25. Starting from here, it is easy to replace the pooling operation with a perceptron, since the missing piece only the bias term (Figure 1b)). The calculation of the output is nearly identical to the average pooling with the constant 0.25 weights, which are multiplied by the corresponding input values. Afterwards, the sum is calculated together with the bias term and the activation function (ReLu Hahnloser et al. [2013]).
Figure 1: Visual explanation of our approach without activation functions. (a) is the average pooling operation where all weights are fixed to 0.25. (b) is the simplest form of our approach which is a perceptron as the pooling operator. (c) is a multilayer neural network where the first (hidden) layer has four perceptrons and the output layer has one perceptron.

The training of the perceptron or neural networks is the same as in any other layer of the superordinate neural network. As an additional memory requirement the generated error is added, as in any other layer, to store the backpropagated error, which is needed to calculate the gradient. The only difference to the other layers in the neural network is that the learning rate of the perceptrons for the weights and the bias term should be reduced ($10^{-1}$ in our experiments) as well as weight decay can be used with the same reduction but we disabled it due to slightly better results (Factor 0 in our experiments). It is of course also possible to train the perceptron or neural network for pooling at the same learning rate, but in the case of large input and output tensors, the training becomes unstable. This is due to the fact that the error of the entire tensor affects only a few weights, and therefore the weights vary greatly. For example, for the nets in Figure 2a) and c) it is possible to use the same learning rate without problems. In case of Figure 2b) and d), however, this can lead to an initially fluctuating training phase.

A further refinement for the effective use of perceptrons or neural networks as pooling operators is the initialization of the parameters. Normally, formula 16 from [Glorot and Bengio, 2010] is used for the random initialization of the parameters, which we have also used for all other layers. In the case of perceptron or neural networks, however, this can easily lead to failure, since these have only a few parameters and, in the case of an unfavorable initialization, may not be able to shift through the gradient to a good minima. A simple example would be the use of a perceptron for pooling with the average pooling parameter initialization. This means that each weight is set to 0.25 and the bias term to 0. The results after training of the entire model are in all of our evaluations significantly better compared to average pooling (see Table 1). In Table 1 it can also be seen that the ReLu has a considerable influence on the result but we will go into this in detail in the first Experiment 5.

For our initialization, we have therefore also used random values, but have made sure that they are

| Pooling method                | Run 1  | Run 2  | Run 3  | Run 4  | Run 5  |
|-------------------------------|--------|--------|--------|--------|--------|
| Average                       | 84.82  | 84.61  | 84.96  | 85.01  | 84.58  |
| (ours) Perceptron (ReLu)      | 84.92  | 84.53  | 84.82  | 85.07  | 84.41  |
| (ours) Perceptron             | 85.94  | 85.76  | 85.92  | 86.16  | 86.06  |
symmetrical or follow rotated/mirrored patterns based on the sign of the values. This idea comes from the manually created filters originating from classical image processing, such as the edge filters and also the weighted average pooling. In the case of a single perceptron this means, either all positive or negative and vice versa, a diagonal negative and the rest positive, or that the transition between positive and negative is along the x or y axis. For several perceptrons in the same layer, we calculated a random pattern and rotated it or mirrored it along the x or y axis, making sure that the pattern was not repeated. This was repeated until all perceptrons in the layer had an initialization.

**Additional parameters for a perceptron:** Each perceptron or neuron has a input window size of $W \times H$ and a bias term $b$. Therefore, we have $W \times H + 1$ additional parameters for a perceptron.

**Additional parameters for the multilayer neural network:** The amount of neurons in the first layer is $p_1$ and in the last layer $p_L$. For the first layer we would have $p_1 \times (W_1 \times H_1 + 1)$ additional parameters. Each following layer has $p_l \times (W_l \times H_l + 1)$ parameters. Thus, the total number of parameters can be specified as $\sum_{l=1}^{L} p_l \times (W_l \times H_l + 1)$.

**Complexity of the perceptron:** We perform per index value one multiplication and one addition. For the bias term we need one additional addition. Therefore, the complexity is $O(\frac{2(W \times H) n}{\text{stride}_1^2} + \frac{n}{\text{stride}_1^2})$ which is theoretical $O(n)$ as for the standard pooling operations.

**Complexity of the multi layer neural network:** The amount of neurons in the first layer is $p_1$ and in the last layer $p_L$. Furthermore, we have $n_l$ input values at layer $l$. Therefore, the first layer needs $O(p_1 \times W_1 \times H_1 \times n_1 + \frac{n_1}{\text{stride}_1^2})$ operations. The following layers need $O(\frac{L}{\text{stride}_l^2} \times \frac{n_l}{\text{stride}_l^2} + \frac{n_l}{\text{stride}_l^2})$ operations. Since the amount of perceptrons or neurons per layer is independent of $n$ we still have a theoretical complexity of $O(n)$. With $\text{stride}_l^2$ we expect the same shift in each x and y dimension of the input tensor at layer $l$.

3 Neural Network Models

Figure 2 shows all architectures we used in our experiments. Figure 2(a) shows a small neural network we adapted from [Eom and Choi 2018] and is employed in Experiment 1 to compare different pooling operations as well as spatial pooling with fields and tensors of neurons on the CIFAR10 data set [Krizhevsky et al. 2009]. The network in Figure 2(b) was taken over from [Kobayashi 2019b] and is used for comparison with the state-of-the-art on the CIFAR100 data set [Krizhevsky et al. 2009] as shown in Experiment 2. The third model (Figure 2(c)) is used in Experiment 3 and does not include batch normalization. This model was employed to compare the pooling operations with the same random initialization and the same batches during training. The last model in Figure 2(d) is a fully convolutional neural network [Long et al. 2015] with the U-Net connections [Ronneberger et al. 2015]. It is used to compare the pooling operations and the high scaling for semantic segmentation. We implemented our approach into DLIB [King 2009] and also used it for all evaluations and comparisons.

4 Datasets

In this section we present all datasets used in our experiments and describe their training parameters. We also define the batch size as well as the optimizer and its parameters. In the case of data augmentation we have kept the number of datasets to a minimum for reproduction purposes, which is also described in detail in the following.

**CIFAR10** [Krizhevsky et al. 2009] consists of 60,000 $32 \times 32$ colour images. The dataset has ten classes. For training, 50,000 images are provided with 5,000 examples in each class. For validation, 10,000 images are provided (1,000 examples for each class). The task in this dataset is to classify a given image to one of the ten categories.

**Training:** We used a batch size of 50 with a balanced amount of classes per batch and an initial learning rate of $10^{-3}$. As optimizer, we used ADAM [Kingma and Ba 2014] with weight decay of $5 \times 10^{-5}$, momentum one with 0.9 and momentum two with 0.999. For data augmentation, we cropped a $32 \times 32$ region from a $40 \times 40$ image, where the original image was centered on the $40 \times 40$ image and the border on each side are 4 pixels set to zero. The training itself was conducted for 300 epochs, whereby the learning rate was decreased by $10^{-3}$ after each 50 epochs. The images are
Figure 2: All used architectures in our experimental evaluation. The orange blocks are replaced with different pooling operations or in case of the transposed convolutions, the upscaling is replaced with our approach. (a) represents a small neural network model with batch normalization taken from Eom and Choi [2018]. (b) is a 14 layer architecture taken from Kobayashi [2019b]. (c) is a small model without batch normalization. (d) is a residual network using the interconnections from U-Net Ronneberger et al. [2015] for semantic image segmentation.

preprocessed by mean substraction (mean-red 122.782, mean-green 117.001, mean-blue 104.298) and division by 256.0.

CIFAR100 [Krizhevsky et al., 2009] is similar to CIFAR10 and consists of 32x32 color images, which must be assigned to one out of 100 classes. For training, 500 examples of each class are provided. The validation set consists of 100 examples for each class. Thus, CIFAR100 has the same size as CIFAR10, with 50,000 images in the training set and 10,000 images in the validation set, respectively. 

Training: We used a batch size of 100 and an initial learning rate of $10^{-1}$. As optimizer we used SGD with momentum [Qian, 1999] (0.9) and a weight decay of $(5 \times 10^{-4})$. For data augmentation, we normalized the images to zero mean and one standard deviation and cropped a $32 \times 32$ region from a $40 \times 40$ image, where the original image was centered on the $40 \times 40$ image and the border on each side are 4 pixels set to zero. The training itself was conducted for 160 epochs, whereby after the 80th and 120th epoch the learning rate was decreased by $10^{-1}$. This is the same procedure as specified in Kobayashi [2019b].

VOC2012 [Everingham et al.] is a detection, classification and semantic segmentation dataset. We only used the semantic segmentations in our evaluation, which contains 20 classes. The task for semantic segmentation is to give a pixelwise classification of a given image. Each image can contain multiple objects of the same class. Furthermore, and not all classes are present in each image. In addition to the object segmentation, the background has to be correctly segmented as no class to. Therefore, the amount of classes increases to 21. For training, 1,464 images are provided with a total of 3,507 segmented objects. For validation another 1,449 images are given with a total of 3,422 segmented objects on it. In this dataset the amount of objects is unbalanced, which increases the challenge additionally. In addition to the segmented images of the training and validation set, a third set without segmentations is given and contains 2,913 images with 6,929 objects. For our training we did not use the third dataset.
Training: We used a batch size of 10 and an initial learning rate of $10^{-1}$. As optimizer we used SGD with momentum $\mu$ \cite{Qian1999} (0.9) and weight decay ($1 \times 10^{-2}$). For data augmentation, we used random cropping of $227 \times 227$ regions with a random color offset and left right flipping of the image. The training itself was conducted for 800 epochs, whereby after each 200 epochs the learning rate was decreased by $10^{-1}$. The images are preprocessed by mean substraction (mean-red 122.782, mean-green 117.001, mean-blue 104.298) and division by 256.0.

5 Experiment 1: Spatial Invariant vs Spatial Pooling

Table 2: Results of different pooling operations for model a) from Figure 2 on the CIFAR10 data set. Our approaches are highlighted in italics. Perceptron means only a single perceptron for pooling. NN-4-1 is a multilayer neural network with 4 neurons in the first layer and 1 output neuron. NN-Z corresponds to one perceptron for pooling per layer of the input tensor. In NN-Field we used one perceptron per pooling region in the x,y plane and with NN-Tensor we used for each pooling region in the input tensor a separate perceptron. ReLu is here the abbreviation for rectifier linear unit.

| Pooling method                        | Accuracy on CIFAR10 | Additional Parameters |
|---------------------------------------|---------------------|-----------------------|
| Average                               | 85.04               | 0                     |
| Max                                   | 84.43               | 0                     |
| Strided tensor convolution (ReLu)     | 87.70               | 82,112                |
| Strided tensor convolution            | 86.78               | 82,112                |
| (ours) Perceptron (ReLu)              | 85.22               | 10                    |
| (ours) Perceptron                     | **87.71**           | 10                    |
| (ours) Perceptron no bias (ReLu)      | 84.71               | 8                     |
| (ours) Perceptron no bias             | 85.11               | 8                     |
| (ours) NN-4-ReLu-I-ReLu               | 85.45               | 50                    |
| (ours) NN-4-ReLu-I                   | 86.40               | 50                    |
| (ours) NN-4-I                        | 87.29               | 50                    |
| (ours) NN-Z (ReLu)                   | 83.87               | 770                   |
| (ours) NN-Z                          | 84.37               | 770                   |
| (ours) NN-Field (ReLu)               | 84.23               | 1,600                 |
| (ours) NN-Field                      | 85.28               | 1,600                 |
| (ours) NN-Tensor (ReLu)              | 81.04               | 122,880               |
| (ours) NN-Tensor                     | 80.93               | 122,880               |

Table 2 shows the comparison of different pooling operations on the CIFAR10 data set. The model chosen was a) from Figure 2. Each pooling operation was trained a total of ten times with random initialization and of all ten runs, the best result was entered in Table 2. First, Table 2 shows that a single perceptron as a pooling operation is as good as a tensor convolution with stride. Also, one can see that a multi-layer neural network (NN-4-1) performs slightly worse. The single perceptron was also trained and evaluated without bias term and as can be seen, it is only slightly better than average pooling. Thus, it can be assumed that the bias term has a significant influence on this model and this data set.

What can also be clearly seen in this evaluation is that the ReLu (Rectifier Linear Unit) has a strongly limiting influence on the classification accuracy. Our idea why this is so is that we use the neural network like a function embedded in a larger network. By restricting it, we reduce the amount of functions that can be learned. Similar to a directly used neural network, the outputs are not limited. Since the tiny neural networks with ReLu score significantly worse in all evaluations we do not use the ReLu in the following experiment. For the strided tensor convolution as pooling operation we continued with the ReLu due to the better results.

As in \cite{Lee2016} we have additionally evaluated spatially separated placements of neurons (NN-Z, NN-Field, and NN-Tensor). NN-Z is a separate perceptron for each channel of the input tensor. For the NN-Field, we assigned a single perceptron to all pooling windows in the x,y plane and moved them along the channels. In the last evaluated spatial arrangement NN-Tensor, we assigned a single perceptron to each pooling region in the input sensor. As can be seen in Table 2, the accuracy of all of them is significantly worse than the standard max and average pooling operations. The worst is NN-Tensor and requires more parameters than the strided tensor convolution. Thus, we can also
confirm for the perceptrons that a spatial arrangement does not provide any improvement, as the authors in [Lee et al., 2016] have done for their approach.

6 Experiment 2: Comparison to the state-of-the-art

Table 3: Results of different pooling operations for model b) form Figure 2 on the CIFAR100 data set. Our approaches are highlighted in italics. Perceptron means only a single perceptron for pooling. NN-4-1 is a multilayer neural network with 4 neurons in the first layer and 1 output neuron. The same notation was used for NN-16-1 with 16 neurons in the first layer. In the last entry we also replaced the GAP layer with a perceptron.

| Pooling method                              | Accuracy on CIFAR100 | Additional Parameters |
|---------------------------------------------|----------------------|-----------------------|
| Average                                     | 75.40                | 0                     |
| Max                                         | 75.36                | 0                     |
| Strided tensor convolution (ReLU)            | 77.53                | 184,608               |
| Stochastic [Zeiler and Fergus, 2013]         | 75.66                | 0                     |
| Mixed [Lee et al., 2016]                    | 75.90                | 2                     |
| DPP [Saedan et al., 2018]                   | 75.56                | 4                     |
| Gated [Lee et al., 2016]                    | 76.03                | 18                    |
| GFPG [Kobayashi, 2019a]                     | 75.81                | 46,080                |
| Half-Gauss [Kobayashi, 2019b]               | 76.74                | 69,840                |
| iSP-Gauss [Kobayashi, 2019b]                | 76.85                | 69,840                |
| (ours) Perceptron                           | 76.06                | 10                    |
| (ours) NN-4-1                               | 76.21                | 50                    |
| (ours) NN-16-1                              | 77.14                | 194                   |
| (ours) Perceptron & GAP                     | 76.37                | 75                    |

Table 3 shows the comparison of our approach with the state-of-the-art on the CIFAR100 data set. As in [Kobayashi, 2019b], we have trained each model three times with random initialization. In the end, we entered the best results in Table 2. As can be seen, the strided tensor convolution has achieved the best results, but it also requires the most additional parameters (184,608). The second best results are obtained with the NN-16-1 neural network (97 additional parameters), the iSP-Gauss [Kobayashi, 2019b] (69,840 additional parameters) and the Half-Gauss [Kobayashi, 2019b] (69,840 additional parameters). This is followed by our two smaller models with a single perceptron and tiny neural network which both require significantly less additional parameters, i.e., only 50, compared to the above mentioned Gaussian-based approaches. If the global average pooling (GAP) is replaced by a perceptron, the number of parameters increases by 65 and the result improves by 0.34%. To perform training with the perceptron as a GAP replacement, we have set the learning rate factor (bias and weights) for this perceptron to $10^{-3}$. At this point it must also be mentioned that our approach can be calculated in O(n) and we have only evaluated very small neural networks. It is of course also possible to use deeper and wider nets as pooling operation.

7 Experiment 3: Equal Randomness and Batch Data Comparison

Table 4: Results of different pooling operations for model c) form Figure 2 on the CIFAR10 data set. Our approaches are highlighted in italics. Perceptron mean only a single perceptron for pooling. NN-4-1 is a multilayer neural network with 4 neurons in the first layer and 1 output neuron. Each convolution and fully connected layer had the same random initialization as well as all models saw the same batches during training.

| Pooling method                                   | Run 1     | Run 2     | Run 3     | Run 4     | Additional Parameters |
|--------------------------------------------------|-----------|-----------|-----------|-----------|-----------------------|
| Average                                          | 84.12     | 84.13     | 84.23     | 84.47     | 0                     |
| Max                                              | 85.63     | 85.77     | 85.36     | 86.01     | 0                     |
| Strided tensor convolution (ReLU)                | **86.95** | **87.68** | 87.11     | 87.84     | 344,512               |
| (ours) Perceptron                                | 85.73     | 86.13     | 85.95     | 85.18     | 15                    |
| (ours) NN-4-1                                    | 86.37     | 87.15     | **87.21** | **87.89** | 75                    |
Table 4 shows an evaluation of different pooling operations, where the initial parameters of the convolution layers and the fully connected layers are set the same for all. The data set used is CIFAR10 and the model is c) from Figure 2. Of course, this does not apply to the parameters of the pooling operations, since these are also of different sizes. Also, the individual batches and the sequence of the batches were the same for all models. With this evaluation, we want to show a comparison between the pooling operations under the same conditions. As can be seen in Table 4, the overall best result was achieved by the NN-4-1 in the fourth evaluation. Comparing the NN-4-1 with the tensor convolution, the results are always similar, whereas the tensor convolution is much more stable in the range of values. A closer look at the standard pooling operations max and average pooling reveals that max pooling is always much better than average pooling for this data set with the model c) from Figure 2. If we compare the individual perceptron with max and average pooling, it outperforms both in three from four runs for the model c) from Figure 2 and the CIFAR10 data set.

8 Experiment 4: Usage in Semantic Segmentation

Table 5: Average pixel classification accuracy on Pascal VOC2012 semantic segmentation dataset with model d) from Figure 2. Our approaches are highlighted in italics. Perceptron is the downscaling operation (One single perceptron) and NN-4/16-UP are four/sixteen neurons for upscaling. The sixteen neurons are in the last layer before the output.

| Pooling method                  | Pixel accuracy on VOC2012 | Additional Parameters |
|---------------------------------|---------------------------|-----------------------|
| As in Figure 2 (d)              | 85.15                     | 0                     |
| (ours) Perceptron & Transpose   | 86.36                     | 32                    |
| (ours) Perceptron & NN-4/16-UP  | **87.62**                 | 172                   |

Table 5 shows the result of the U-Net from Figure 2(d) on the VOC2012 data set. Each net was initialized and trained with random values. For Perceptron & Transpose we replaced only the pooling operations with a perceptron. For Perceptron & NN-4/16-UP we replaced the pooling and upscaling operations with perceptrons. As can be seen, our approach improves the results both as a pooling operation and for upsampling. Since VOC2012 is a very hard data set and semantic segmentation is a difficult task, we see this as a significant improvement of the results.

9 Limitations

Despite the above presented parameter reduction, our methods still has some disadvantages compared to the classical maximum value selection or the mean value pooling. One disadvantage is that we still a few additional parameters to calculate the perceptron or the neural network. Additionally, this means that we have to provide memory for back-propagating the error, as it is the case for each learning layer in a neural networks. Of course, this also affects the optimizer, which also needs additional memory for the moments. The use of neural networks as pooling operators also extends the search space for model finding and thus their complexity and computing requirements. However, in general, our approach does not increase the complexity of the calculation of a pooling operation in the case of the perceptron, but it does improve the accuracy of the model. In the case of using a multilayer neural network for the pooling operation, our approach naturally increases the number of computations, but compared to a tensor convolution as pooling operation, this increase of our approach is only minimal, whereas the tensor convolution increases the complexity by the output tensor depth. As a general remark it must also be said that in case of unstable training it has always been successful for us to reduce the learning rate of the perceptron or small neural network.

10 Conclusion

In this paper we have shown that single perceptrons can be used effectively as pooling operators without increasing the complexity of the model. We have also shown that neural networks can be formed as pooling operators by simply restructuring the output data of several perceptrons. These increase the complexity and number of parameters of the model only minimally compared to tensor convolutions as pooling operator and are almost as effective. These multi-layer neural networks and the presented restructuring can also be used to learn a scaling that can be effectively used for
transposed convolutions. Here it is also possible to learn the scaling via tensors but or two dimensional matrices which would be an extension of our approach. In addition to the evaluated models it is of course also possible to train deeper nets as pooling operators or to equip individual layers with more perceptrons. In this way the results can be further improved and we leave this open for future research. The approach presented by us is easy to integrate into modern architectures and can be learned together with all other parameters without creating parallel branches in a model. Thus, the approach can also be effectively computed on a GPU.

Broader Impact

Since we use the already proven and widely used concept of perceptrons, which in our approach are only applied to an input sensor in a simplified way, our approach helps to reduce network parameters significantly without loss in accuracy. Also the high scaling of the data consists of a simple restructuring. Thus our approach can be easily integrated into any neural network. In addition to this paper, we publish code for CUDA based NVIDIA GPUs, such that our approach can be easily reimplemented and integrated to various applications.

a) Everybody who uses neural networks with pooling operations.

b) The parameter space for architecture search is increased.

c) Another pooling operation would be chosen.

d) Not applicable.

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