The evaporation and Greybody factor of the new EUP-corrected Schwarzschild black hole

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Based on the minimum measurable momentum concepts associated with the quantum gravity effects acting on the large-scale dynamics of the universe. We use the new extended uncertainty principle (EUP) to study the Hawking temperature and black hole evaporation. The results show the new EUP quantum correction may shorten the lifetime of the massive black hole. To verify the new EUP on the black hole stability, we further investigate the quasinormal frequencies in terms of the 3rd order WKB approximation. The corrected Schwarzschild black hole is still stable under the uncertainty parameter \( \alpha \) is considered to be a small value. Furthermore, we analyze the transmission and reflection coefficients and discuss the absorption cross section with graphical methods in detail. It shows that the uncertainty parameter \( \alpha \) has a non-negligible effect on the scattering amplitudes of the corrected black hole.

I. INTRODUCTION

In 1915, Einstein proposed the general theory of relativity where gravity is assumed to be a geometric property of the space and time [1]. Since then, this theory is regarded as the best description of the gravity. In general relativity, one set of solutions to Einstein’s field equations indicates the existence of black holes. Black hole is one of the mysterious facts of the universe which produces an effective area that even light cannot escape. Basically, it is believed that black holes with characteristics of mass, charge, and angular momentum are formed in the final stage of the gravitational collapse of stars [2, 3]. Black holes are considered a laboratory for investigating gravity because they can emit gravitational radiation associated with their own oscillations. Such radiation is assumed to be very important since it would carry internal information of the black hole’s characteristics, such as its mass and charge [4, 5]. Nowadays, quantum phenomena is assumed to have a vital role in the black hole radiation, thus the quantum theory of gravity. Recently, the LIGO-VIRGO cooperation proposed a method to directly detect the gravitational waves from mergers of black holes and neutron stars [6, 7].

In 1970, Vishveshwar introduced a type of spacetime perturbation that depends on particular conditions, near the black hole event [8]. According to his paper for a set of complex-valued frequencies, it is possible to consider outgoing and incoming waves in the spatial infinity and in the vicinity of the event horizon, respectively. Soon afterward these frequencies are called the quasinormal (Q-N) modes [9] and have been extensively examined [10, 11]. In those studies, it is shown that the real part of a Q-N frequency determines the oscillation frequency while its imaginary part means the damping rate [12]. The Wentzel-Kramers-Brillouin (WKB) approximation is one of the mathematical tools used for analyzing Q-N modes. Schutz and Will were the pioneers and they employed this approach up to first-order [13]. Then, the method is extended up to the second-order in [14] and lately up to the sixth-order [15]. As an alternative method to WKB, Leaver used numerical methods to calculate the gravitational Q-N modes of stationary and rotating black holes [16, 17]. A brief summary of further and recent methods that produce accurate results in the determination of Q-N modes can be found in [18, 19].

In general relativity, the existence of singularities corresponding to the solutions of Einstein’s field equations associated with black holes will promote the breakdowns of general relativity theory. In order to eliminate the existing singularity problems, a lot of ideas have been proposed. According to one of them, a quantum deformation can be used for the Schwarzschild black hole to obtain the solution of the regular black hole which was given by Bardeen in 1968 [20]. To this end, Kazakov and Solodukhin employed a deformation in the Schwarzschild black hole solution and showed that the singularity at \( r = 0 \) is shifted to a finite radius value [21]. In [22], the authors explored the Hawking radiation in the framework of the tunneling process for the quantum corrected Schwarzschild black hole. Furthermore, some authors studied the thermodynamics properties and weak deflection angle of the different black holes according to the modified uncertainty principles in [23, 24]. Besides, by considering a Lorentzian distribution [25] to introduce the noncommutativity, the authors demonstrated the thermodynamics of the black hole in the noncommutative background [26, 27]. Anacleto et al. explored
the scattering problem for the noncommutative corrected Schwarzschild black hole in [43].

On the other hand, we observe that quantum deformation usage is not limited to these studies, papers that examine Q-N modes in the presence of the quantum deformation becomes popular recently. For example, last year Konoplya studied the Q-N modes of a quantum-modified Schwarzschild black hole [43]. Based on the existence of the minimal length, Kempf proposed the generalized uncertainty principle (GUP) in terms of the Heisenberg Algebra [15]. Anacleto et al. also studied the Q-N modes and shadow of the Schwarzschild black hole, and calculated the Q-N frequencies of the black hole up to the sixth-order WKB method [36]. Similar work can be found in Ref. [47–50]. In this work, our motivation is to investigate the new EUP-corrected evaporation rate to explore the stability of black holes. To this end, we further study the Q-N frequencies of the black hole so as to verify the stability of the black hole. In addition, we also analyze the impact of the new EUP on the black hole scattering problem to enrich the content. The paper is organized as follows: In Sec. 2, we investigate the evaporation rate of the new EUP corrected Schwarzschild black hole. In Sec. 3, we investigate the Q-N modes of the Schwarzschild black hole by using the sixth-order WKB approximation, then discuss the influence of the EUP parameter \( \alpha \) on the absorption cross section. The final comments are in Sec.4.

II. THE EVAPORATION RATE OF THE NEW EUP CORRECTED SCHWARZSCHILD BLACK HOLE

In this section, we start by considering the following 1 dimensional deformed Heisenberg algebra (EUP) that leads to measure a minimum momentum value [51]:

\[
[X, P] \geq \frac{ih}{1 - \alpha |X|}, \tag{1}
\]

here, \( \alpha \) is the deformation parameter which takes value in the range of \( 0 \leq \alpha \leq 1 \). Following this deformation, we find

\[
\Delta X \Delta P \geq \frac{h}{2} \left( \frac{1}{1 - \alpha |X|} \right)
\]

\[
\geq \frac{h}{2} \left[ 1 + \alpha \langle |X| \rangle + \alpha^2 |X|^2 + q^3 |X|^2 + \ldots \right]
\]

\[
\geq \frac{h}{2} \left[ -\alpha (\Delta X) + \alpha (\Delta X) + 1 + \alpha \langle |X| \rangle + \ldots \right]
\]

\[
\geq \frac{h}{2} \left[ -\alpha (\Delta X) + \frac{1}{1 - \alpha (\Delta X)} \right]. \tag{2}
\]

Then, we express the momentum uncertainty in terms of position uncertainty. We arrive at

\[
\Delta P \geq \frac{h}{2\Delta X} \left[ -\alpha (\Delta X) + \frac{1}{1 - \alpha (\Delta X)} \right]. \tag{3}
\]

It can be found that the new EUP gives the minimal momentum \( \Delta P \geq \frac{\alpha}{2} \). If we take uncertainty parameter \( \alpha = 0 \), the equation (3) recovers the Heisenberg uncertainty principle \( \Delta X \Delta P \geq \frac{1}{\hbar} \). The Hawking temperature corresponding to any massless quantum particle near the Schwarzschild black hole event horizon is given by (the units \( \hbar = c = G = 1 \))

\[
T = \frac{\Delta P}{\kappa}, \tag{4}
\]

here \( \kappa \) is the Boltzmann constant. Based on this, we can find that the lower bound of the black hole temperature reads

\[
T \geq T_{\text{min}} = \frac{3}{2} \alpha \tag{5}
\]

Based on the properties of the uncorrected horizon radius \( r_h \) and uncertain position \( \Delta X \) in Ref. [52–54]

\[
\Delta X \simeq 2\pi r_h = 4\pi M. \tag{6}
\]

In this case, we can calculate the general total of the mass-temperature

\[
M = \frac{1}{8\pi T} \left[ 1 - \left( 1 - \frac{4\alpha}{\alpha + 2\pi T} \right)^{\frac{1}{2}} \right]. \tag{7}
\]

For a small EUP parameter \( \alpha \), the new EUP-corrected Hawking temperature can be expressed as

\[
T_{\text{EUP}} = \frac{4\pi \alpha M - 1 - 16\pi^2 \alpha^2 M^2}{8\pi \kappa M(4\pi \alpha M - 1)}. \tag{8}
\]

If we consider the new EUP parameter \( \alpha \) at the limit value \( \alpha = 0 \), the uncorrected Hawking temperature of the Schwarzschild black hole is given by

\[
T = \frac{1}{8\pi Mr^2}, \tag{9}
\]

which is consistent with the previously obtained result for the usual form of the Hawking temperature in Ref. [29]. In addition, the area of horizon \( A \) can be given by

\[
A = 4\pi r_h^2 = 16\pi M^2. \tag{10}
\]

Based on the Stefan-Boltzmann law [55–56], the radiated power reads

\[
\frac{dE}{dt} = \sigma AT^4, \tag{11}
\]

here \( \sigma = \pi^2 / 60 \) is the Stefan-Boltzmann constant. Furthermore, we assume the lost mass of the black hole is completely converted to energy:

\[
dE = dM. \tag{12}
\]

Based on this, the evaporation rate of the new EUP corrected Schwarzschild black hole can expressed as

\[
\frac{dM}{dt} \simeq \frac{4\pi^3 M^2}{15} T_{\text{EUP}}^4 \simeq \frac{4\pi^3 M^2}{15} \left( \frac{4\pi \alpha M - 1 - 16\pi^2 \alpha^2 M^2}{8\pi \kappa M(4\pi \alpha M - 1)} \right)^4. \tag{13}
\]
From the behavior shown in FIG. 1, we can see that the uncorrected Schwarzschild black hole has a maximum evaporation rate at a lower mass. As the mass increases, the black hole tends to be more stable due to the remnant long-lived and owns a larger time to decay. However, the new EUP parameter $\alpha$ has a non-negligible effect on the evaporation rate in the massive black holes, and it is concluded that quantum correction associated with the new EUP may drastically shorten the lifespan of the Schwarzschild black holes.

III. Q-N MODES AND ABSORPTION CROSS SECTION OF THE NEW EUP CORRECTED SCHWARZSCHILD BLACK HOLE.

In this section, we will give the line element of the EUP-corrected black hole so that the Q-N modes can be studied. Therefore, we consider a massless particle and assume $\Delta P \sim P \sim E$, and find the following bound:

$$E \Delta X \geq \frac{1}{2}. \quad (14)$$

which leads to the new EUP in equation (3) to be written in the form of

$$\varepsilon \geq E \left[ -\alpha(\Delta X) + \frac{1}{1 - \alpha(\Delta X)} \right] \geq E \left[ -\alpha r_h + \frac{1}{1 - \alpha r_h} \right]. \quad (15)$$

Here, $\varepsilon$ denotes the black hole corrected energy due to the EUP. We follow the arguments given in [53, 54] and consider the position uncertainty nearly equal to the horizon radius, $\Delta X \sim 2r_h$. Then, we rewrite equation (3) in terms of the mass by assuming $E \sim M$, $\varepsilon = M_{EUP}$, and $r_h = 2M$. We find

$$M_{EUP} \geq M \left[ -\alpha r_h + \frac{1}{1 - \alpha r_h} \right] = M \left[ -2\alpha M + \frac{1}{1 - 2\alpha M} \right], \quad (16)$$

where, $M_{EUP}$ is the EUP-corrected mass function of the black hole. Then, we derive the following relation for the event horizon out of equation (16)

$$r_{hEUP} = 2M_{EUP} = M \left[ -4\alpha M + \frac{2}{1 - 2\alpha M} \right]. \quad (17)$$

Based on these findings, we modify the Schwarzschild black hole metric, the EUP-corrected line element reads

$$ds^2 = \left( 1 - \frac{2M_{EUP}}{r} \right) dt^2 - \left( 1 - \frac{2M_{EUP}}{r} \right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (18)$$

Next, we use the WKB approximation method to calculate the Q-N frequency. On this basis, we choose the case of massless scalar fields. To describe scattered waves, we have to deal with

$$\frac{1}{\sqrt{-g}} \partial_r \left( \sqrt{-g} g^{\mu\nu} \partial_r \right) \Psi = 0, \quad (19)$$

with the background given in equation (18). After the separation of variables is applied, we find the wave function in the form of

$$\Psi_{lm}(r, t) = R_{\omega l}(r) Y_{lm}(\theta, \phi) e^{-i \omega t}, \quad (20)$$

where $Y_{lm}(\theta, \phi)$ denotes the spherical harmonics and $\omega$ indicates the frequency. Then, we obtain the radial equation in the following form

$$\frac{d^2 R_{\omega l}(r_s)}{dr_s^2} + \left[ \omega^2 - V_{eff}(r) \right] R_{\omega l}(r_s) = 0. \quad (21)$$

Here, $r_s$ is the so-called tortoise coordinate and defined with

$$dr_s = \frac{1}{1 - \frac{2M_{EUP}}{r}} dr, \quad (22)$$

while the effective potential, $V_{eff}(r)$, is

$$V_{eff}(r) = \left( 1 - \frac{M_{EUP}}{r} \right) \left[ \frac{l(l + 1)}{r^2} + \frac{1}{r} \left( 1 - \frac{M_{EUP}}{r} \right) \right]. \quad (23)$$

Here, $l$ indicates the multipole number of the spherical harmonics. To present the effect of the deformation on the effective potential, we present FIG. 2. Regarding the influence of the EUP parameter, we observe that the effective potential does not alter sensibly, since the deformation parameter takes values between zero and one, i.e. $\alpha = 0, 0.05, and 0.1$. Therefore, we conclude that Q-N mode frequencies might not present large differences according to the considered quantum correction. Contrary, according to FIG. 2 we observe that the multipole number has a decisive influence on the effective potential. The QNMs can be computed by solving the wave
The characteristic frequencies $\omega$ are complex. If the effective potential, given in equation (22), initially makes a peak and then falls to a constant in the asymptotic region, one can compute the Q-N mode frequencies by employing the WKB approximation as done by Schutz et al. in [15][17]. In this manuscript, we use 3rd order WKB formula that is given in [15]

$$\omega_n = \sqrt{(V_0 + \Delta) - i \left( n + \frac{1}{2} \right) \sqrt{-2V_0'/(1 + \Omega)}}.$$  

TABLE I. Fundamental quasinormal mode of the New EUP as a function of $\alpha$ calculated by the 3rd order WKB approach.
\[ \Delta = \frac{1}{8} \left( \frac{V_0^{(4)}}{V_0'} \right) \left( \frac{1}{4} + \gamma^2 \right) - \frac{1}{288} \left( \frac{V_0''}{V_0'} \right)^2 (7 + 60\gamma^2) \] (26)

and

\[ \Omega = - \frac{1}{2V_0'} \left\{ \frac{5}{6912} \left( \frac{V_0''}{V_0'} \right)^4 (77 + 188\gamma^2) \right. \\
- \frac{1}{384} \left[ \left( \frac{V_0''}{V_0'} \right)^2 \left( \frac{V_0^{(4)}}{V_0''^2} \right) \right] (51 + 100\gamma^2) \\
+ \frac{1}{2304} \left( \frac{V_0^{(4)}}{V_0''} \right)^2 (65 + 68\gamma^2) \right. \\
+ \frac{1}{288} \left( \frac{V_0''}{V_0''} \right)^2 (19 + 28\gamma^2) \right. \\
- \frac{1}{288} \left( \frac{V_0^{(5)}}{V_0''^3} \right) (5 + 4\gamma^2) \right\} . \] (27)

We have \( \gamma = n + 1/2 \) and \( V_0^{(n)} \) means the \( n \)-order derivative of the effective potential on the maximum point \( r_+ \). Lately, the Wentzel-Kramers-Brillouin (WKB) approximation is generalized by Konoplya and Zhidenko to the sixth order [18]. It is worth noting that the WKB approach is extended to higher orders in [57], and there it is shown that it achieves a higher accuracy if the Padé approximant \([55][64]\) is used. The corrections \( A_k(K^2) \) to the eikonal formula are of the order \( k \) and polynomials in \( K^2 \) with rational coefficients. The corrections \( A_k(K^2) \) depend on the values of the higher-order derivative of the effective potential \( V(r) \). Note that, the sixth order WKB method with \( \tilde{m} = 5 \) (where \( \tilde{m} \) is given in the same form of [55][59]). However, the third-order WKB method has better accuracy when considering \( n \leq l \) case in small overtones number \( n \). Based on this, by considering the EUP-corrected Schwarzschild black hole, the results for the Q-N modes with different parameters \( (n, \alpha) \) are presented in TABLE I. When we set mass \( M = 1 \), we found a decrease in Q-N frequency (the real and imaginary part) with the increase of the EUP parameter \( \alpha \). This indicates that the quadratic part of the Q-N frequency in uncertainty momentum contributes to the decrease of the Q-N frequency. Besides, we observe that the imaginary part of the frequency associated with the damping time scale \( \omega_I \) is negative, which implies that the EUP-corrected Schwarzschild black hole is still stable. However, when we consider the new EUP-corrected massive Schwarzschild black hole \( (M = 6) \), with the increase of the new EUP parameter \( (\alpha = 0.09 \text{ and } 0.1) \), the stability of the black hole is destroyed. This indirectly verifies the faster evaporation rate of the black hole when the EUP parameter is larger for the massive black hole. Similarly, the concept based on the minimum observable length, Anacleto etc. investigated the Q-N modes of the GUP-corrected Schwarzschild black hole [16], the influence of GUP parameters \( \beta \) on Q-N modes is similar to the EUP parameter \( \alpha \) in our work. In quantum theory, coordinates and momentum are the two most basic operators, which indirectly indicates the correctness of our work.

To estimate the portion of the initial radiation near the event horizon, we calculate the grey-body factors. In other words, we aim to give the reflection and transmission coefficients. This is going to pave the way for our next work, the scattering problem. In fact, in the scattering problem, we consider the wave equation under boundary conditions, which allow waves to be incident from infinity. The scattering boundary conditions are given by [13][16]

\[ \Psi = e^{-i\omega r} + Re^{i\omega r}, \quad r_s \to +\infty, \]
\[ \Psi = T e^{-i\omega r}, \quad r_s \to -\infty, \] (28)

where \( R \) and \( T \) represent the reflection coefficient and transmission coefficient, respectively. It is well known that the effective potential exists in the form of a potential barrier that decreases monotonically at both points of infinity. The WKB method can be used to determine the reflection, \( R \), and transmission, \( T \), coefficients. \( \omega^2 \) is real, so that by considering the boundary conditions, given in equation (28), we can relate \( K \) to the reflection and transmission coefficients as given in [16]

\[ |R|^2 = \frac{|A_{out}|^2}{|A_{int}|^2} = \frac{1}{1 + e^{-2\pi K\omega}}, \quad 0 < |R|^2 < 1, \] (29)

and

\[ |T|^2 = \frac{|A_{tr}|^2}{|A_{int}|^2} = \frac{1}{1 + e^{-2\pi K\omega}} = 1 - |R|^2. \] (30)

Here, \( K \) can be found from the following equation:

\[ K - i \frac{(\omega^2 - V_0)}{\sqrt{-2V_0'}} - \sum_{i=2}^{i=6} \Lambda_i(K) = 0. \] (31)

In FIG.3, we plot the transmission and reflection coefficients versus the frequency. In the left panel, we consider three different multipole numbers namely \( l = 1, l = 2, \) and \( l = 3 \) from left to right, and three EUP parameters as \( \alpha = 0, \alpha = 0.05, \) and \( \alpha = 0.1 \). We observe that the increase of the EUP parameter affects the transmission and reflection coefficients.

Based on the transmission coefficient \( T \), we define the partial absorption cross section, \( \sigma_t \), and total absorption cross section, \( \sigma \), respectively [21]

\[ \sigma_t = \frac{\pi (2l + 1)}{\omega^2} |T_l(\omega)|^2, \] (32)

and

\[ \sigma = \sum_l \frac{\pi (2l + 1)}{\omega^2} |T_l(\omega)|^2. \] (33)
In FIG. 4, we plot the contributions of partial and all absorption for modes $l = 1, 2, 3$. When the EUP parameter $\alpha$ is set to 0.05, the absorption cross section is almost coincident with the Schwarzschild black hole ($\alpha = 0$). The amplitude of the absorption cross section increases with the increase of $\alpha$. Besides, we find that GUP and EUP parameters have a similar influence on the absorption cross section of black hole [47].

**IV. CONCLUSIONS**

In this work, we employed the new EUP corrected Schwarzschild black hole to investigate the Hawking temperature and evaporation rate. Based on the Stefan-Boltzmann law properties, the evaporation rate of the black hole is derived. To analyze the effect of the new EUP parameter $\alpha$ on the evaporation rate, we depicted the evaporation rate as the function of $M$ for the different EUP parameter values ($\alpha = 0, 0.05, 0.1$). We can observe that new EUP may drastically shorten the lifespan of the massive Schwarzschild black holes. To verify this conclusion, we continue to study the Q-N modes of the black hole. First, we derived the line element of the quantum-corrected Schwarzschild black hole and analyzed the influence of the deformation parameter on the effective potential. Then, we determined the Q-N modes numerically by the third-order WKB approximation. The result showed that the corrected-black hole does not lose its stability when the uncertainty parameter $\alpha$ is small, and quasinormal frequency decreases in the real and imaginary part with the increment of the EUP parameter. However, when we take the massive black hole ($M = 6$), with the increase of the new EUP parameter ($\alpha = 0.09$ and 0.1), the stability of the black hole is destroyed. This also indirectly proves the result of the black hole evaporation rate. In addition, we studied the partial and all absorption cross section. It can be found that for quantum-corrected Schwarzschild black hole, either from the minimal measurable length perspective or the minimal observable momentum, the modified uncertainty principle has similar effects on the Q-N modes and
scattering problems of the Schwarzschild black hole.

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