Effect of Near-field Distribution on Transmission Characteristics of Fiber-fed Fabry–Perot Etalons

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Received 2020 September 30; revised 2021 March 15; accepted 2021 March 17; published 2021 May 7

Abstract

Fiber-fed etalons are widely employed in advanced interferometric instruments such as gravitational-wave detectors, ultrastable lasers and calibration reference for high-precision spectrographs. We demonstrate that variation in near-field distribution of the feeding fiber would deteriorate the spectrum precision of the fiber-fed Fabry–Perot etalon, especially when precision at the order of $3 \times 10^{-10}$ or higher is required. The octagonal fiber reinforced with double scrambler could greatly improve the steadiness and uniformness of the near-field distribution. When building wavelength calibrators for sub-m s$^{-1}$ precision radial-velocity instruments, the double scrambler should be considered meticulously.

Unified Astronomy Thesaurus concepts: Fabry-Perot interferometers (524); Spectral energy distribution (2129); Radial velocity (1332)

1. Introduction

Fabry–Perot etalons (FPEs) are used in a variety of advanced applications, such as gravitational-wave detection (Miller et al. 2012), metrology (Chen et al. 2019a), and laser stabilization (Alnis et al. 2008). For astronomical observations, besides its long history in solar imaging and scanning spectroscopy, FPEs have emerged as a new type of economical and potentially reliable spectral reference for simultaneous calibration of high-precision astronomical spectrographs. Since the discovery of the first exoplanet orbiting around a solar-type star (Mayor & Queloz 1995), the radial-velocity (RV) method has become one of the most important techniques for exoplanet search and characterization. It measures the subtle change in Doppler shift induced by the exoplanet candidate on the host star spectrum along our line of sight. To detect an Earth-like exoplanet, a long-term precision of $10^{-11}$ cm s$^{-1}$ (corresponding to a fractional stability better than $3 \times 10^{-10}$) or higher would be required. This ever-increasing demand for high precision is pushing for new calibration techniques in the stead of traditionally used hollow-cathode lamps (HCLs; Lovis et al. 2006; Lovis & Pepe 2007) or gas absorption cells (Wang et al. 2019) in order to achieve sub-m s$^{-1}$ long-term stability.

Laser frequency combs (LFCs; Steinmetz et al. 2008; Wilken et al. 2010) have been demonstrated to be excellent calibration references for next-generation ultraprecise RV instruments. However, the high price, complex structure, and high-maintenance demand has made them hard to access for small-scale projects. In view of this, passively stabilized FPE illuminated with a broadband source has been developed as an economical alternative (Wildi et al. 2010). For which, stabilities of $10$ cm s$^{-1}$ during one night and $1$ m s$^{-1}$ over 60 days have been demonstrated on the HARPS and HARPS-N spectrographs (Wildi et al. 2012). Ideally, the transmitted peaks are equally spaced in the frequency domain and cover the entire spectral span of the calibrated instrument. The line width and spacing of the peaks are defined by the finesse and free spectral range (FSR) of the etalon. By anchoring the FPE peaks to atomic transition (Reiners et al. 2014; Schwab et al. 2015), it has been shown that a high tracking/locking-precision at the cm s$^{-1}$ level can be reached by this type of calibrator.

To achieve highly stable passive environmental control, high efficiency, and better compatibility with fiber-fed spectrographs, FPE-based calibrators are usually fed by a multimode fiber (MMF). Cersullo et al. (2017) discussed the influence of fiber diameter and centering on the spectral characteristics of FPE-based calibrators and showed that the line position and shape stability of the transmitted peaks are significantly influenced by the alignment condition of the input fiber. Yet, the influence of nonuniform near-field intensity distribution of the fiber output has not been fully addressed. Here, we analyze the effects of near-field distribution of the input fiber on the transmission characteristic of FPEs through numerical simulation. We then propose and experimentally demonstrate a mitigation method that combines the use of octagonal fiber and a double scrambler, an optical relay that exchanges the near and far fields (Hunter & Ramsey 1992; Halverson et al. 2015; Ye et al. 2020), by which both the near-field and far-field illumination patterns are stabilized.

2. Theory of the Fiber-fed Fabry–Perot Etalon

The FPE consists of two flat parallel mirrors coated with a highly reflective coating on the inner surfaces. Suppose an FPE is illuminated with a perfectly collimated beam. The frequency at peak transmission is an integer multiple of the FSR. In the frequency domain, the FSR is given by

$$\Delta \nu = \frac{c}{2nd \cos \theta},$$

where $d$ denotes the distance between the two mirrors, $n$ is the refractive index of the gap medium, and $\theta$ is the incident angle of the input beam. The finesse $\mathcal{F}$ of the etalon is defined as the ratio of FSR to the FWHM of the transmitted peak.
Assuming that the absorption and scattering losses are negligible, the transmission function of the FPE is given by

$$\tau(\lambda, \theta) = \frac{1}{1 + F \sin^2(\delta(\lambda, \theta)/2)}, \quad (2)$$

in which $\delta$ is the round-trip phase shift

$$\delta(\lambda, \theta) = \frac{4\pi}{\lambda} n d \cos \theta, \quad (3)$$

and $F$ is the finesse coefficient, defined as

$$F = \frac{4\rho}{(1 - \rho)^2}, \quad (4)$$

with $\rho$ as the reflectivity of the coatings. Assume achromatic reflective collimator with off-axis parabolic mirror is used to collimate the light beams. Figure 1 shows the diagram of the collimated light beam. Due to the fiber having an extended surface at the output instead of an ideal point, the off-axis light will incident onto the etalon with a small angle after collimating.

Suppose the intensity at a given point on the fiber cross-section is expressed as $p(r, \varphi)$ in polar coordinates, with the fiber center as the origin. According to the geometrical relation and properties of an off-axis parabolic mirror, the incident angle of the ray coming from this point after collimation is

$$\theta(r) = \arctan \frac{r}{f}, \quad (5)$$

where $f$ is the reflected focal length (RFL) of the reflective collimator. The effective transmission function at point $p(r, \varphi)$ can be expressed as

$$\tau(\lambda, r) = \frac{1}{1 + F \sin^2(2\pi d \cos(\arctan(r/f))/\lambda)}. \quad (6)$$

Thus, the effective transmission function $I_{\text{eff}}(\lambda)$ of a fiber-fed FPE is an integral over the entire fiber output surface:

$$I_{\text{eff}}(\lambda) = \int \int p(r, \varphi) \tau(\lambda, r) r dr d\varphi. \quad (7)$$

For simulation, an FPE and reflective collimator with properties as listed in Table 1 are employed. Figure 2 shows the theoretical transmission function of the FPE with a fiber of 100 μm diameter at normal incidence (solid line). For comparison, the case of ideal on-axis transmission (dashed line) is also shown. The spectral line generated by the extended source is broadened compared with the point source. An FWHM $\omega_0$ of $1.24 \times 10^{-3}$ nm is calculated for the ideal case, while FWHM $\omega_0$ of $3.29 \times 10^{-3}$ nm and an effective finesse of 10.94 are results in the 100 μm case, much lower than the specified value. The reason for such broadening can be explained as follows: the spectrum formed by the entire fiber surface is the superposition of segments formed by individual elements $dr$, as shown in Figure 3. An element further away from the fiber center results in a larger incident angle, thus contributing to the increase of blueshifted components of the integrated curve.

| Table 1 |
| Parameter | Symbol | Value |
|-----------|--------|-------|
| FPE gap   | $d$    | 5 mm  |
| FSR       | $\Delta \nu$ | 30 GHz (0.036 nm @ 600 nm) |
| Finesse   | $F$    | 37    |
| Reflectivity | $\rho$ | 0.92  |
| RFL       | $f$    | 15 mm |
| Refractive index | $n$ | 1 |
| Fiber core diameter | $d_f$ | 100 μm |

3. Effect of Fiber Near-field Distribution on the FPE Transmission

For radial-velocity measurements, the utilization of fiber input decouples the spectrograph from the telescope focus, significantly improving the instrumental profile stability. However, the near-field distribution of an optical fiber is sensitive to the injection conditions and external disturbances. Variance of illumination, movement, and temperature fluctuations of the feeding fiber will result in changes at the fiber output and image shifts on the detector, manifesting as false RV shifts (Avila & Singh 2008; Brown et al. 1991; Halverson et al. 2015). For coherent calibration light sources, such as LFCs, such a phenomenon is more prominent due to mode interference and speckle issues (Mahadevan et al. 2014). For FPE-based calibrators, an incoherent broadband light source can be employed; however, the influence of near-field distribution variation on the calibration precision is yet to be determined.

Assume that the fiber is well coaxially aligned with the collimator. It has been previously demonstrated in the literature (Heacox 1986, 1987; Chen et al. 2019b) that the radial distribution...
The near-field intensity distribution of a circular MMF can be uneven while the azimuthal distribution is generally uniform under different injection conditions, as shown simulatively in Figure 4. Figure 4(a) is the ideal case where intensity variation is ignored. This ideal model served as the framework of the above FPE line-broadening analysis, but is unattainable in reality. The actual intensity distribution will be similar to Figure 4(b) (off-center injection) or Figure 4(c) (on-center injection).

These different intensity distributions are imported separately into the FPE simulation model, and the normalized transmission spectra are shown in Figure 5. As the radial intensity distribution is not uniform, light on the edge of the fiber surface contributes less to the final curve compared to the ideal case, shifting the peak to the red end. Due to this phenomenon, the symmetry of the peak is broken. The Gaussian fitted results of spectral drifts and equivalent RV shifts are shown in Table 2.

By comparing the fitting results of Figures 5(a)–(c), it can be seen that a very large spectral drift will occur when the near-field intensity distribution of the fiber goes through dramatic change.

The question remains: How stable does the near-field intensity distribution of the fiber need to be in order to achieve calibration stability at the 10 cm s\(^{-1}\) level? To quantify the effect of turbulence, two different types of slight intensity fluctuations of less than ±0.5% (as shown in Figure 6 inset) are introduced into the case of Figure 4(a). The spectral lines obtained by superimposing Figure 4(a) with these two different intensity fluctuations are then compared with the result from Figure 4(a), respectively. Consider a calibrator of a spectrograph that operates within the wavelength range of 500–700 nm; the spectral drifts of the FPE peaks within that operation range are calculated and plotted in Figure 6. The two different fluctuations result in different wavelength dependency of the induced drifts, with maximum variations of 3.5% and 4.3%, respectively, between the blue and red ends, both minute in nature. For direct

**Figure 3.** Anatomic diagram of the contribution of light from different near-field radius to the transmission spectrum. Left panel shows the individual rings \(dr\) for different areas, and the corresponding transmission spectrum is shown in the right panel. The integral spectrum is the superposition of curves formed by individual rings \(dr\).

**Figure 4.** Three simulated intensity distributions of the near-field (core diameter = 100 μm). Top panels show the diagram of the fiber cross-section. Bottom panel shows the 1D relative intensity distribution across the center of the fiber. Panel (a) is the ideal case, panel (b) may occur when the light source is injected off-center, and panel (c) may occur when it is injected on-center.

**Figure 5.** The intensity-normalized spectra resulting from different near-field distributions in Figure 4.

| FWHM          | \(\Delta \lambda / \lambda_{\text{peak}}\) | Equivalent RV |
|---------------|------------------------------------------|---------------|
| Figure 5(a)   | \(3.29 \times 10^{-3}\) nm               | 0             |
| Figure 5(b)   | \(2.28 \times 10^{-3}\) nm               | \(4.98 \times 10^{-7}\) 149.44 m \(s^{-1}\) |
| Figure 5(c)   | \(2.81 \times 10^{-3}\) nm               | \(6.86 \times 10^{-7}\) 205.90 m \(s^{-1}\) |

*The Astronomical Journal, 161:258 (6pp), 2021 June Hao et al.*
comparison, the amplitudes of induced spectral drifts around 600 nm are $1.91 \times 10^{-9}$ and $5.76 \times 10^{-9}$ (equal to 0.57 m s$^{-1}$ and 1.73 m s$^{-1}$ of RV shift), for the two different fluctuations. Errors at this level are unneglectable and could potentially affect the calibration precision, hence the need for special measures to ensure the system stability.

4. Experimental Demonstration of the Effect of Near-field Distribution Disturbance

In order to study the impact of the intensity jitter of the optical fiber, the experimental setup shown in Figure 7 is used to measure the near-field intensity distribution on the fiber output surface. The light of a broadband source (halogen lamp, 500–700 nm) is led into the setup by a 400 μm optical fiber, the endface of which is imaged at the 50 μm pinhole, then projects onto the test fiber. The near-field intensity distribution of the test fiber is then obtained by CCD with a microscope. Before experiment, the stability of setup should be demonstrated. The intensity distribution of the optical fiber output end is measured without any disturbance with a 1 s interval time over 80 s. As we can see from Figure 8, the fractional displacement ($\Delta \lambda / \lambda$) of the transmission peaks obtained by substituting the sets of measurements into the FPE simulation model pixel by pixel is approximately $6.54 \times 10^{-11}$ (equal to 1.96 cm s$^{-1}$ of RV shift).

With the fiber near-field being kept on catch, the change of a single test parameter (incident point shift or fiber body movement) is introduced. It is carefully ensured that the time elapsed of the whole process does not exceed 80 s. In such a way, the measurements contain information regarding both the stability of setup and the changed variable. In Figure 9, the results of circular fiber (CF; 5 m in length) with different disturbances are shown. The incident point and state of fiber are
To maintain high calibration performance and increased stability of output of near-field intensity distribution in the near-field of the CF is sensitive to environmental disturbances. Therefore, to maintain high calibration precision, the fiber needs to be placed in an absolutely stable condition, which is not always feasible in practical environments. A number of studies have shown that double scrambler can improve the stability of near-field intensity distribution (Hunter & Ramsey 1992; Halverson et al. 2015). Additionally, it has been shown that octagonal fibers demonstrate more uniform and stable near-field output compared to CFs (Chazelas et al. 2010). Here, a combination of double scrambler and octagonal fiber (Ye et al. 2020) as a solution to meet the above discussed precision requirement is proposed and discussed.

Switching to an octagonal fiber (3 m in length), we perform similar tests as done for the CF. The results are shown in Figure 10. In which the way of intensity redistribution in the near-field after disturbance may vary across different fiber configurations, resulting in a different sign of the fractional displacement ($\Delta \lambda / \lambda$), but the effect is equivalent. When the point of incidence shifts, it causes a fractional drift of $\Delta \lambda / \lambda \approx 5.43 \times 10^{-10}$ (RV $\approx 16.30$ cm s$^{-1}$) in the case of an octagonal fiber, a significant improvement compared to the similar case in CF. By adding a double scrambler operating during the shift (the fiber length before and after the double scrambler is 3 m and 3 m, respectively), the fractional drift is further reduced to $1.23 \times 10^{-10}$ (RV $\approx 3.70$ cm s$^{-1}$). Similarly, when the fiber body is moved to a different position, a drift of $\Delta \lambda / \lambda \approx 3.15 \times 10^{-10}$ (RV $\approx 9.45$ cm s$^{-1}$) is observed. The employment of the double scrambler drops the value to $8.95 \times 10^{-11}$ (RV $\approx 2.70$ cm s$^{-1}$), almost the limit of setup. The octagonal fiber performs satisfactorily, and the clamor-ment of a double scrambler excellently improves the performance. The experimental results show that the octagonal fiber along with a double scrambler can significantly reduce the fractional drift caused by near-field intensity distribution in a fiber-fed FPE, making it less sensitive to environmental disturbances and more suitable for use as a high-precision calibrator.

5. Summary
The intensity distribution variation in the near-field of the CF output will result in displacement and distortion of the transmitted spectral peaks of the FPE. This in turn leads to errors in applications where extreme high precision is required. Even a mere $\pm 0.5\%$ of intensity fluctuation can cause a corresponding $\sim 3 \times 10^{-9}$ (1 m s$^{-1}$) fractional shift of the transmission peaks in an FPE-based calibrator, a considerable error in the study of exoplanet through the RV method, thus putting rigorous requirements on the operation environment. This work proposes and proves via numerical simulations that octagonal fiber with double scrambler can substantially increase the stability of output of near-field intensity distribution. The spectral peak deviation caused by intensity fluctuation can be reduced to $<1.5 \times 10^{-10}$ (5 cm s$^{-1}$), demonstrating outstanding resistance to environmental disturbances.

This work was partially supported by the National Natural Science Foundation of China (grant Nos. 11773044, 11727806, 11673046, 11873071, and 11903060) and the Operation, Maintenance and Upgrading Fund for Astronomical Telescopes and Facility Instruments, budgeted from the Ministry of Finance of China (MOF) and administrated by the Chinese Academy of Sciences (CAS). L.T. acknowledges support from the China Postdoctoral Science Foundation (2020M671638) and Jiangsu Planned Projects for Postdoctoral Research Funds.

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Figure 10. Four sets of near-field measurements of different octagonal fiber (OF) configurations. (a) The incident point is shifted off-center; (b) the fiber body is moved to a different position; (c) the incident point is shifted off-center while a double scrambler (DS) is used; (d) the fiber body is moved to a different position, while a double scrambler is used.
