Accuracy Analysis of Geopotential Coefficients Recovered from In-situ Disturbing Potential by Energy Conservation Method

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Abstract  The characteristics of the normal equation created in recovering the Earth gravity model (EGM) by least-squares (LS) adjustment from the in-situ disturbing potential is discussed in detail. It can be concluded that the normal equation only depends on the orbit, and the choice of a priori gravity model has no effect on the LS solution. Therefore, the accuracy of the recovered gravity model can be accurately simulated. Starting from this point, four sets of disturbing potential along the orbit with different level of noise were simulated and were used to recover the EGM. The results show that on the current accuracy level of the accelerometer calibration, the accuracy of the EGM is not sufficient to reflect the time variability of the Earth’s gravity field, as the dynamic method revealed.

Keywords  energy conservation method; disturbing potential; least-squares

CLC number  P223.0

Introduction

The determination of EGM by the new generation dedicated satellite gravity missions, such as Challenging Minisatellite Payload (CHAMP) and Gravity Recovery and Climate Experiment (GRACE), has been studied for years in geodesy. There are two fundamental approaches to determine the EGM from the satellite tracking data, i.e. the dynamic method\cite{1-4} and the energy conservation approach\cite{5-8}. Generally speaking, the accuracy of the EGM computed by the dynamic method had been better than that by the energy conservation approach. Almost all the EGMs released by the main research groups in the world have been computed by the dynamic method. Compared with the dynamic method, the realization of the energy conservation approach is relatively simple: combining the accelerometer data and the precise orbit, disturbing potential along the orbit can be determined based on the Jacobi’s energy equation. Then, the gravitational coefficients of EGM can be recovered by the LS method. Because the energy conservation approach uses the accelerometer data to compute the in-situ potential, the preprocessing and calibration of the accelerometer data is very important.

The purpose of this study is to present a method to construct a relation between the accuracy of the EGM derived by the energy conservation approach and noise level of the in-situ disturbing potential. Then,
we determine the accuracy requirements of the data preprocessing according to the accuracy requirements from a specific problem on an EGM. In Section 1, the characteristic of the normal equation formed in this problem was analyzed in detail, and it can be concluded that the use of the reference field has no effect on the result. Based on this property, the scheme to test these ideas is given subsequently. In Section 2 four sets of simulated observations with different accuracy were used to verify the idea. The discussion and conclusion are in Section 3.

1 Characteristic of normal equation

Supposing that there exists the following observation equation:

\[ L = BX \]  

where \( L \) is the observation vector; \( X \) is the parameter vector; and \( B \) is the design matrix, which is dependent on \( X \) and has full column rank. According to the general indirect adjustment principle, the LS solution of Eq.(1) can be written:

\[ X^1 = (B^T PB)^{-1} B^T PL \]  

where \( P \) is the weight matrix of the observables.

If \( X_0 \) is chosen as the a priori value of the parameter vector, then correction \( \hat{x} \) to the parameter vector can be expressed as:

\[ \hat{x} = (B^T PB)^{-1} B^T P \]  

where \( l = L - BX_0 \).

If a priori value is chosen in the adjustment, the final solution \( X^2 \) reads:

\[ X^2 \equiv X_0 + \hat{x} = x_0 + (B^T PB)^{-1} B^T P(L - BX_0) = (B^T PB)^{-1} B^T PL = X^1 \]  

which indicates that the two results are equivalent theoretically.

It must be noted that solution \( X^1 \) is just a special case when setting \( X_0 = 0 \) in Eq.(3). Therefore, it can be concluded that if the observation equation is linear and the design matrix is independent on the parameters, the choice of the a priori values has no effect on the final result.

The external disturbing potential \( T \) can be expanded into solid harmonics as follow\[^9\]:

\[ T(r, \varphi, \lambda) = \frac{R}{\mu} \sum_{n=2}^{N_{\text{max}}} \left( \sum_{m=0}^{n} (\mathcal{C}_{nm} \cos m\lambda + \mathcal{S}_{nm} \sin m\lambda) P_{nm}(\sin \varphi) \right) \]  

where \( (r, \varphi, \lambda) \) are the spherical coordinates of point outside sphere, denoting geocentric distance, latitude and longitude, respectively; \( \mu \) is the gravitational constant of the Earth; \( R \) is the mean radius of the Earth; \( \mathcal{C}_{nm} \) and \( \mathcal{S}_{nm} \) are the normalized coefficients; and \( P_{nm}(\sin \varphi) \) is the associated Legendre polynomials with degree \( n \) and order \( m \). The model is truncated at degree \( N_{\text{max}} \).

Eq.(5) is the observation equation in the problem of determining EGM by the energy conservation approach, and the design matrix can be computed as:

\[ \frac{\partial T}{\partial (\mathcal{C}_{nm}, \mathcal{S}_{nm})} = \frac{\mu R}{r} \left( \sum_{n=2}^{N_{\text{max}}} \sum_{m=0}^{n} \mathcal{C}_{nm} \cos m\lambda \right) \times \left( \sin m\lambda, \cos m\lambda \right) \]  

From Eq.(5) and Eq.(6), it is obvious that the observation equation is linear and independent on the unknowns, which had been the starting point of the ideas in this paper. Considering that the accuracy of disturbing potential is mainly affected by the accelerometer data and orbit states, it is reasonable to hypothesize that the in-situ disturbing potential has the same accuracy, i.e. weights of the observables are the same (\( P = I \) where \( I \) is identity matrix). Thus, the normal equation \( (N \equiv B^T PB = B^T B) \) reads:

\[ (N)_{ij} = \frac{R}{\mu} \left( \sum_{k=1}^{t_i} \frac{R^2}{r_k} \right) (\cos m(j) \lambda_k)^+ (\sin m(j) \lambda_k)^+ \times \sum_{l=1}^{n} \sum_{m(i) \neq n(l)} \frac{P_{n(i)m(i)}(\sin \varphi_k)^+ P_{n(l)m(l)}(\sin \varphi_k)^+}{(\cos m(i) \lambda_k^+ \sin m(i) \lambda_k^+)(\cos m(j) \lambda_k^+ \sin m(j) \lambda_k^+)} \]  

where \( k \) is the index of the observable; \( n_r \) is the total number of the observables; \( (r_k, \varphi_k, \lambda_k) \) are coordinates of the station \( k \); and \( n(i), m(i) \) are the corresponding degree and order of geopotential coefficient which is the \( i \)th parameter in vector \( X \). \( n(j) \) and \( m(j) \) have the same meaning. After the degree \( n \) and order \( m \) of the coefficient had been determined by its index in the parameter vector \( X \), there existed the difference between \( \mathcal{C}_{nm} \) and \( \mathcal{S}_{nm} \). So, the notation 1 in Eq.(7) had been used and only one term can be
chosen from the four combinations:
\[
\{ \cos (i) \lambda_k \cos (j) \lambda_k, \cos (i) \lambda_k \sin (j) \lambda_k, \\
\sin (i) \lambda_k \cos (j) \lambda_k, \sin (i) \lambda_k \sin (j) \lambda_k \}
\]
From Eq.(7), it is clear that when the weights of the observables are equal, the normal equation depends only on the satellite orbit.

2 Numerical results and analysis

The orbit data of GRACE B from DOY (213) to DOY (243) in 2003 were used; the in-situ disturbing potential was computed with EGM96 as the true gravity field. For the statistics of the simulated disturbing potential, see Table 1.

| Table 1  Disturbing potential along GRACE orbit implied by EGM96/ m²·s⁻² |
|-----------------|-----|-----|-----|-----|
| Total           | Min | Max | Mean | Std. |
| 43 203          | −553.56 | 717.57 | 5.02 | 207.20 |

The accuracy of POD can achieve 5 cm\[10\]. Therefore, the effects of the orbit error on the normal equation were neglected in this study, while the effects from the accelerometer data have been the focus. To overcome the difficulty that there is no absolute standard which can be used to assess the recovered EGM, the following scheme had to be adopted:

1) Whether using reference field or not almost has no effects on the result. The numerical simulation shows that the differences are on the order of 10⁻¹⁸. Therefore, the numerical simulation supports the conclusion given in Section 1. The coefficients can be recovered completely from the clean data.

2) In the real data processing of computed in-situ disturbing geopotential by the conservation approach, the calibration of accelerometer had been difficult, which is one of the main error sources in the disturbing geopotential. Therefore, white noise had to be added into the simulated observations and used to assess the accuracy of EGM. Adding noise with various variances (Table 2) into the simulated observations, and comparing the recovered EGM with the true field, the accuracy of the recovered EGM can be determined.

The difference between the recovered EGM and the true field had been taken as the external check. Considering that the energy spectrum of geoid is mainly on the low and medium degree, the maximum degree 60 had been chosen to comprise the computation efforts. The relative error of coefficients can therefore be evaluated as:

\[
E_r = \frac{C_t - C_r}{C_t}
\]  

(8)

where \( C_t \) is true value, which is the model value of EGM96 in this study; and \( C_r \) is the recovered value.

The basic rules that have been adopted here are that the accuracy of the recovered coefficients must be at least one order higher than the coefficients themselves, and the absolute error should be lesser than the time-variable part, which had been from the difference between the results derived by the dynamic method for DOY (213)-DOY (243) and DOY (182)-DOY (212), respectively. From Fig.1 it is obvious that the time variability of the coefficients computed by the dynamic method are distributed between 10⁻⁹-10⁻¹⁰, therefore, this domain had been chosen as the threshold. For the simulated results, as seen in Fig.2, the left is for the absolute error and the number of coefficients whose errors in a given domain have been present, the right is for the relative accuracy implied by Eq.(8), and the percentage of the coefficients whose relative errors in a given domain have also been present.

When the accuracy of disturbing geopotential is at the level of 0.01 m²/s², it can be concluded from Fig.2(a) that the absolute error of the recovered coefficients are mainly between 10⁻⁹-10⁻¹⁰ and their relative errors are all less than 1, most of them are even less than 0.1. Fig.2(b) shows that the absolute accuracy of the recovered coefficients are also very small when the noise is at the level of 0.1 m²/s², while their relative error had deteriorated seriously compared with that of case 1. From Fig.2(c) and Fig.2(d), it can be seen that the accuracy of recovered coefficients had been corrupted. It is clear that only when the noise is at the level of 0.01 m²/s² is the accuracy
of the coefficients recovered by the energy conservation approach close to that of the dynamic method. For any other cases, the accuracy had been very limited. After careful preprocessing, the accuracy of the disturbing potential had been at the level of 0.1-1.08 m²/s², therefore, the accuracy of energy conservation approach had not been as good as that of the dynamic method. To improve the accuracy, some new accelerometer calibration techniques should be developed to make the calibration results more stable and reliable.

Fig.1  Absolute (left) and relative (right) monthly variability from CSR

Fig.2  Distribution of the absolute error (left) and relative error (right) of the coefficients recovered from contaminated observations
3 Conclusions

In this study, the feature of the normal equation in recovering of geopotential coefficients by the energy conservation approach has been discussed in detail. It can be concluded that the normal equation depends only on the orbit when the observations have equal weights. Based on the four sets of simulated observations with different noise level, the accuracy of the recovered coefficients were evaluated, and the simulation demonstrates that the accuracy of the disturbing geopotential computed from the hl-SST data had not been enough to recover an EGM with the accuracy as that by the dynamic method given the limitation of the accelerometer calibration.

Acknowledgement

The authors are grateful to Prof. CHAO Dingbo for his critical comments and also thank Dr. Dadzie very much for his proof-reading.

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