We present the design and first measurement results for an ultra-stable cryogenically cooled optical sapphire resonator system with a potential relative frequency stability better than $3 \times 10^{-17}$. This level of oscillator stability allows for more precise tests of Einstein’s theories of relativity and thus could help to find first hints of new physics. We will give some details on a projected experiment to test Lorentz invariance that will utilize these cavities.

1. Introduction

Time and frequency are the physical quantities which can be measured with by far the highest precision in modern physics and thus clock-comparison experiments are a class of exceptionally sensitive experiments that can be performed in the laboratory to test the foundations of modern physics, e.g., Lorentz invariance.

One particular type of ‘clock-comparison’ experiment uses optical cavities to search for possible Lorentz invariance violating anisotropies. These types of experiments are mostly referred to as modern Michelson-Morley experiments because of their similarity to the classic Michelson-Morley experiment performed in 1887. The basic idea is to compare the resonance frequencies $\nu = qc/(2nL)$ of two or more linear optical Fabry-Pérot resonators ($q$ is an integer, $c$ is the speed of light in vacuum, $n$ the index of refraction of the medium, if present, parallel to the resonators axis, and $L$ the length of the resonator) and to look for frequency changes $\delta \nu / \nu$ with respect to the orientation or velocity of the cavities in space, which would indicate a violation of rotation invariance or Lorentz boost invariance, respectively.

The comparison of the resonance frequencies is normally done by stabilizing lasers to the cavities using the Pound-Drever-Hall method and
taking a beat note measurement. Advantageous is the fact that in principle any type of Lorentz invariance violation that affects the isotropy of \( c, L, \) or \( n \) can be detected\(^4\text{–}6\) in a cavity experiment \((\delta \nu / \nu = \delta c / c - \delta L / L - \delta n / n)\) and thus a lot of Lorentz invariance violating coefficients of the SME\(^7\) can be determined that alter the Maxwell and Dirac equations of motion.

Today’s state-of-the-art modern Michelson-Morley experiments\(^8,9\) are limited by the performance of the optical resonators employed. Improving the stability of optical resonators would improve the sensitivity to possible signatures of ‘new physics.’ A promising path towards better stability performances is the development of cryogenic optical resonators.

2. Cryogenic optical resonators

Nowadays, the frequency stability of all optimized room-temperature optical reference cavities is limited by the displacement noise within the resonator substrates and mirror coatings due to thermal noise.\(^10\) For a reasonable cavity length (< 30 cm) the thermal noise limited relative frequency stability is restricted to \( \geq 10^{-16} \).

The rather straightforward method to reduce simply the influence of thermal noise by cooling down the resonators to cryogenic temperatures is not well applicable for most room-temperature cavities. This is simply because they are mostly based on glass ceramic materials whose mechanical and thermal properties change unfavorably with low temperatures. In contrast, crystalline materials like sapphire or silicon that are normally not utilized in room-temperature cavities due to their comparatively high coefficients of thermal expansion at room temperature offer excellent material properties at cryogenic temperatures.

Therefore, we worked out a special design for an ultra-stable sapphire optical cavity system operating at 4 Kelvin. Two opposing requirements need to be considered when designing a cryogenic optical resonator and its mounting structure: high thermal conductivity towards the cold bath and low sensitivity of the optical path length to vibrations. Our resonator design has been developed using FEM computations to reduce the influence of vertical and horizontal vibrations upon the optical path length while the resonator still offers a large thermal contact area for mounting (see Fig. 1).

The prospective thermal noise limited frequency stability should be better than \( 3 \times 10^{-17} \) (see Fig. 1), solely limited by the displacement noise originating from the high finesse mirror coatings based on Ta\(_2\)O\(_5\)/SiO\(_2\). The displacement noise of the spacer and mirror substrates would easily even generate a theoretical thermal noise floor well in the low \( 10^{-20} \) regime due
to the high stiffness (large Young modulus) and high mechanical Q-factor of sapphire. Furthermore, the thermal material properties (low CTE, high thermal conductivity) of sapphire at cryogenic temperatures make high performance active temperature stabilization feasible in order to minimize the influence of ambient temperature fluctuations. Thus, an exceptional high level of long term length stability and frequency stability, respectively, can also be expected.

In order to read out the ultra-stable eigenfrequencies of the cryogenic optical sapphire resonators, and thus making it available for other applications or precision tests of fundamental physics we use the Pound-Drever-Hall locking scheme, which stabilizes the frequency of a laser to one of the resonance frequencies of the cryogenic cavity. It has to be ensured that the electronics employed do not introduce additional noise on the frequency of the laser besides the fluctuations of the eigenfrequencies of the cryogenic optical resonator itself. Therefore, we will also implement techniques commonly used in ultra-stable resonator systems like residual-amplitude-modulation suppression and intensity-noise control.

3. Status

In total we had five 10 cm long sapphire cavities with a targeted finesse of 250,000 (corresponding to a resonance linewidth of $\sim 6$ kHz) custom-made. Upon inspection of the delivered cavities we noticed small debris on the highly reflective mirror coatings inside the cavity. Further investigation revealed that this pollution causes high losses in the fundamental modes of the cavities.

Thus, the measured fineses of the cavities lay between 13,000 ($\sim 115$ kHz) and 52,000 ($\sim 29$ kHz), which would set higher demands on the Pound-Drever-Hall locking electronics in order to not be influenced too much by parasitic resonance frequencies in the laser system. However, in the meantime the origin of the contamination within the fabrication process was identified by the vendor, who is currently cleaning and repairing two of the cavities.

We are in the process of setting up the final cryogenic cavity system, which has to provide a quiet environment for the sapphire resonators in order not to disturb them too much. At present, we are using a 20 year old cryostat system for pre-characterizations of the cavities and the laser lock and control circuits. In this setup, the laser beam is guided to the cavities through windows in the cryostat instead of optical fibers as it is planned for the final system.
Unfortunately, the old cryostat lost mechanical stiffness over the years due to transportation across Germany and a previous vacuum accident. This leads to a rather big differential movement of the sapphire resonators inside the cryostat and the laser beam system outside the cryostat. The differential movement causes comparatively large Doppler shifts of the resonance frequency of the cavities with respect to the incoming laser beam frequency, which in turn is tracked by the Pound-Drever-Hall lock. Thus, when comparing the eigenfrequencies of two of the cryogenic optical sapphire cavities by measuring the beat frequency of lasers locked to these cavities, we are at the moment mostly measuring the motion of the cavities rather than their intrinsic noise (see Fig. 1).

Therefore, we cannot yet confirm the prospected stability performance of the cryogenic cavities. Nevertheless, the electronic part of the laser stabilization system performs as intended and once the final cryogenic cavity system is set up we expect to be able to measure the predicted relative stability of $< 3 \times 10^{-17}$. 
4. Intended tests of fundamental physics

Once the stability of the ultra-stable cryogenic optical sapphire resonators can be measured accurately with lasers using the Pound-Drever-Hall locking scheme, we plan to perform a modern Michelson-Morley experiment. Preparations for the needed ultra-stable rotating setup are already being done in our laboratories in parallel to the work on the cryogenic cavity system.

The measurement sensitivity of a modern Michelson-Morley experiment is limited by the provided frequency stability \( \sigma(\tau) \) of the eigenfrequencies of the resonators at the integration time \( \tau = T/2 \), where \( T \) is the rotation period of the experimental setup. Assuming a gaussian distribution for single measurements, the measurement sensitivity can be enhanced by a factor of \( 1/\sqrt{N} \) by integrating over \( N \) individual measurements.

Accordingly, the optimal reachable measurement sensitivity \( S_{\text{max}} \) of a modern Michelson-Morley-type experiment spanning some time period \( T_{\text{MM}} \) and rotated at a rotation period of \( T \) can be estimated by \( S_{\text{max}} = \sigma(T/2) / \sqrt{T_{\text{MM}}/T} \). Thus, the expected relative frequency stability of below \( 1 \times 10^{-16} \) of the cryogenic optical sapphire resonators with a reasonable rotation period between 10 s and 100 s would yield a measurement sensitivity of \( 10^{-19} \) to \( 10^{-20} \) for signals of Lorentz invariance violations in a one year measurement campaign.

Actually, we plan to perform a more advanced version in which we will compare the eigenfrequencies of two rotating optical linear Fabry-Pérot resonators and of two rotating microwave whispering-gallery resonators in one setup. We will set up and perform this co-rotating experiment together with the group of Prof. M. Tobar from the University of Western Australia.\(^{11,12}\)

The co-rotating setup allows testing different possible aspects of Lorentz invariance violations simultaneously by comparing optical frequencies with microwave frequencies, electromagnetic propagation in vacuum with propagation in matter, and linear modes with whispering-gallery modes. All these different aspects are considered in the full SME\(^{13,14}\) and hence we can set new limits on a variety of coefficients including some with no bounds so far.

The usual right angle between the axes of the resonators in a Michelson-Morley-type experiment is insensitive\(^{14}\) to some higher-order terms of the full SME. Therefore, we are either going to deviate from the usual right angle and set the resonators up in a different angle or we might compare the eigenfrequencies of the rotating resonators with stationary ones of similar performance.
5. Summary and outlook

We are developing ultra-stable cryogenic optical resonators with a prospected relative frequency stability $< 3 \times 10^{-17}$. This unprecedented stability will allow more precise tests of fundamental physics. We are planning to use these ultra-stable resonators in an advanced modern Michelson-Morley experiment in conjunction with ultra-stable cryogenic microwave whispering-gallery resonators operated by the University of Western Australia. This co-rotating experiment will allow us to set new or even first limits on several coefficients of the full SME with a sensitivity between $10^{-19}$ and $10^{-20}$.

Moreover, we are currently investigating crystalline high finesse coatings based on $\text{Al}_{1-x}\text{Ga}_x\text{As}$. These novel coatings could reduce the influence of thermal noise within an optical cavity by more than one order of magnitude. Hence, using these coatings could boost the performance of our ultra-stable cryogenic optical sapphire resonators to a relative frequency stability around $10^{-18}$. In turn, this would allow us to reach a sensitivity for Lorentz invariance violating signals in a modern Michelson-Morley-type experiment within the $10^{-21}$ regime.

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