Electric-Field Control over Spin-Wave and Current Induced Domain Wall Motion and Magnonic Torques in Multiferroics

Iryna Kulagina and Jacob Linder
Department of Physics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway
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We discover that the way spin-waves exert magnetic torques in multiferroic materials can cause not only domain wall motion, but also magnetization dynamics for homogeneous magnetization textures. Interestingly, the domain wall motion can be controlled via purely electrical means with the spin-waves being generated by an ac electric field $E$ while the direction of the wall motion also is sensitive to an applied dc $E$ field. Moreover, we determine the interaction between spin-transfer torque from an electric current and a magnetic domain wall in multiferroics and show that the Walker breakdown threshold scales with the magnitude of a perpendicular electric field, offering a way to control the properties of domain wall propagation via electric gating.

Over the last decade, there has been a surge of interest in multiferroic materials [1–3] which displays simultaneously ferromagnetic, antiferromagnetic, ferroelectric, and/or ferroelastic order [4]. Besides the obvious allure of this multifunctionality from a practical viewpoint, such as usage for the purpose of magnetic random access memories [5, 6], the magnetoelectric cross-coupling between these orders is interesting from a fundamental physics perspective [7–13].

It is known that a wide variety of multiferroic materials host textured magnetic order parameter profiles [2] such as domain walls. Domain walls may be thought of as topological defects which interface different regions of a material and exhibit properties that differ from the ones in the homogeneous domains. Controlling the transport of magnetic domain wall structures is currently an active field of research [14] in spintronics as it offers an interesting way to transfer information from a fundamental physics perspective [7–13].

However, an analytical framework and understanding of how domain wall motion takes place in multiferroics when exposed to central driving forces in spintronics such as spin-polarized currents or spin-waves remains largely unexplored [17, 18]. In this paper, we answer the question: how does magnetic domain walls in multiferroics respond to the spin-transfer torque induced by electric currents and spin-waves?

We find that spin-waves generated in multiferroic materials are capable not only of causing domain wall motion, but also in inducing torques on homogeneous magnetization textures. This is different from conventional homogeneous ferromagnets where no such spin-wave torque exists. Moreover, we find that the domain wall motion can be controlled fully electrically: the spin-waves may be generated by an ac $E$ field and even the direction of motion of the wall is controlled via the application of a constant dc $E$ field. Again, this is different compared to the magnonic torque induced by spin-waves on domain walls in conventional ferromagnets where the wall was found to move toward the spin-wave source [19]. Finally, we show that the effect of a domain wall becoming distorted once exceeding a critical velocity, known as Walker breakdown, can be delayed by electric gating on magnetic domain walls in multiferroics under the influence of a current-induced spin-transfer torque.

Let us start by establishing the theoretical framework to be used in this work. To account for inhomogeneous magnetic textures the free energy under consideration includes the exchange interaction $F_{\text{exc}} = \int \text{d}r \frac{1}{2} (\nabla m)^2$ and anisotropy energy $F_{\text{an}} = \int \text{d}r (-\frac{K}{2} m^2 + \frac{K_{\perp}}{2} m^2)$, where the Zeeman-coupling due to an external magnetic field may also be included via $F_{\text{Z}} = -\int \text{d}r (M \cdot B_{\text{ext}})$. Above, $J$ is the exchange coefficient while $M = M_0 m$ is the magnetization with $M_0$ as its magnitude, $K > 0$ and $K_{\perp} > 0$ are constants of anisotropy for the easy and hard axis, while $B_{\text{ext}}$ is the external magnetic field. Since the system under consideration is not a conventional ferromagnet, but rather a multiferroic, we must include the cross-coupling term between the electrical and magnetic degrees of freedom $F_{\text{P}} = -\int \text{d}r E \cdot P$ where the polarization induced by the magnetic texture is given by [20] (considering a cubic lattice symmetry for concreteness) $P = \gamma_0 [\mathbf{M} (\nabla \cdot \mathbf{M}) - (\mathbf{M} \cdot \nabla) \mathbf{M}]$. The magnetoelastic coupling coefficient is denoted $\gamma_0$.

The total free energy is then represented by $F = F_{\text{exc}} + F_{\text{an}} + F_{\text{Z}} + F_{\text{P}}$ and we make use of the Landau-Lifshitz-Gilbert equation (LLG) [21] to investigate the dynamics of a domain wall in this multiferroic system. We will in this work consider both the influence of spin-waves induced torques and current-induced torques, commencing with the latter. In this case, the standard phenomenological equation of motion used to describe the spin-transfer torque effect of an electric current is (in normalized form):

\[
\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} - u \frac{\partial \mathbf{m}}{\partial c} + \beta u m \times \frac{\partial \mathbf{m}}{\partial c} \tag{1}
\]

where $\alpha$ is the Gilbert damping constant, $u$ is proportional to the current density, while $\beta$ is the non-adiabatic term whose origin, although subject to some controversy, mostly is believed to be spin-relaxation processes that cause the itinerant electron spins constituting the current to not follow the do-
main wall profile fully adiabatically [22]. Although the magnetization is allowed to take any direction, we consider only variation along one spatial dimension (denoted c above) in order to provide analytical results. In what follows, we will consider time $t$ in the unit of $(\gamma \mu_0 M_0)^{-1}$ where $\mu_0$ is the vacuum permeability, $\gamma$ is the gyromagnetic ratio, and use normalized length in the unit of $(J/M_0^2 \mu_0)^{1/2}$. Finally, we express the current density parameter $u$ in the unit of $\gamma \sqrt{J} \mu_0$, the anisotropy constants $K$ and $K_\perp$ in the unit of $M_0^2 \mu_0$, $E$ in units of $\gamma M_0 J_0 \mu_0$, and the magnetoelectric coupling constant $\gamma \mu_0$ in the unit of $(\gamma M_0^2 \mu_0)^{-1}$.

A key observation is that not all types of magnetic textures will provide a magnetoelectric polarization $P$: a net component of the magnetization along the direction of spatial variation is required, thus ruling out Bloch walls. For this reason, we will focus here on Neel (NDW) and head-to-head domain walls (HDW). To be concrete, we choose easy-axis of magnetic anisotropy along the $x$ direction and the hard axis along $y$ direction (see Fig. 1 for the schematic setup). Before we can explore the dynamics of multiferroic domain walls, one has to check whether an applied electric field alters the static domain wall profile itself. Some care must be exercised here, since we find that the validity of the usual Walker solution [23] for the domain wall profile depends on the orientation of the electric field relative the hard axis of anisotropy. For instance, the Walker profile is not valid for the NDW and HDW when the $E$ field is applied along the hard-axis direction. Thus, we consider the electrical field as $E = (0, 0, E_z)$ for NDW and $E = (0, E_y, 0)$ for HDW. Due to our choice for the coordinate axes, we can conventionally write the normalized magnetization in the same way for both types of domain walls: $m = (\sin(\theta) \cos(\phi), \sin(\theta) \sin(\phi), \sigma \cos(\theta))$, where $\theta(c) = 2 \arctan[\exp(-c-\lambda)/\lambda]$ where $c = y$ and $c = z$ for NDW and HDW correspondingly, $\lambda$ is the DW width, $\chi$ is the position of the DW center, and the topological charge of the domain wall is $\sigma$. The azimuthal angle for the static Walker profile $\phi = \pm \pi/2$ for both our geometries and we assume that $K \gg K_\perp$ to justify the description of the domain wall as a solitonic object described only by the degrees of freedom associated with its center position and tilt angle [14].

The equation of motion for the center-coordinate $\chi(t)$ and the angle $\phi(t)$ is for the NDW

$$\alpha \ddot{\chi} + \lambda \dot{\phi} = -\beta \dot{u} + \lambda B_x \sigma \chi - \alpha \lambda \dot{\phi} = -\beta \dot{u} - \frac{1}{2} \lambda K_\perp \sin(2\phi) + \sigma \lambda \pi \gamma_0 E_y \cos(\phi).$$

(2)

For the HDW, we have

$$\alpha \ddot{\chi} + \lambda \dot{\phi} = -\beta \dot{u} - \alpha \lambda \dot{\phi} = -\sigma u - \frac{1}{2} \lambda K_\perp \sin(2\phi) - \frac{B_x}{\pi} \cos(\phi) + 2 \sigma \lambda \pi \gamma_0 E_z \cos(\phi).$$

(3)

The behaviour of the domain wall is different in two regimes which are separated by Walker breakdown, defined by $\partial_t \phi \neq 0$. In the regime where there is no Walker breakdown, the following equations must be satisfied for NDW $\frac{u}{u_c} = \sin(2\phi) + b_{NDW} - \epsilon \cos(\phi)$ and for HDW $\frac{u}{u_c} = \sin(2\phi) + (b_{HDW} + \epsilon) \cos(\phi)$ where $\kappa_\perp = \frac{\sigma \alpha \lambda K_\perp}{2(\beta - \alpha)}$, $\lambda = B/B_0$, $\epsilon = E/E_0$, $E_0 = \frac{\pi K_\perp}{\beta \lambda}$, $B_0_{HDW} = \frac{\pi K_\perp}{2}$, $B_0_{NDW} = \frac{\alpha \lambda K_\perp}{2}$. This allows us to determine a quantity of central practical importance, namely the critical current density $u_c$ at which Walker breakdown takes place. We find:

\begin{equation}
\frac{u_{NDW,c}}{u_c} = \kappa_\perp [b + \sqrt{2f(\epsilon)/32}], \quad \frac{u_{HDW,c}}{u_c} = \sqrt{2\kappa_\perp} f'(\epsilon)/32, \quad f(x) = \left(3x + \sqrt{x^2 + 32}\right)[16 - x^2 + x\sqrt{x^2 + 32}]^{1/2}
\end{equation}

(4)

where $\epsilon' = b + \epsilon$. From this, the maximum domain wall velocity $V_{DW,c}$ that is attainable before deformation sets in is computed via $V_{DW,c} = -\beta u_c/\alpha$. The angle $\phi_c$ corresponding to the constant tilt angle of the domain wall is $\phi_c = -\arcsin(\frac{\lambda}{\pi}((b - \sqrt{\epsilon^2 + 32}))$ and $\phi_c = \pi - \arcsin(\lambda/\pi((b - \sqrt{\epsilon^2 + 32}))$ for NDW and HDW, respectively. We here included the presence of a magnetic field for generality, and in the limit without any spin-transfer torque effect our expressions are consistent with Ref. [18]. Setting $B = 0$ in order to focus on the spin-transfer torque effect, it is seen from the above equations that the critical current in both the NDW and HDW case is the same and increases with $E$. This could be of practical importance since it offers a way to delay Walker breakdown induced by electric current, and increase the velocity of the domain wall transport, via a gate voltage. In Fig. [11(c)], we plot the normalized maximum domain wall velocity and Walker breakdown current density as a function of the applied electric field to illustrate this effect.

We now turn our attention to the question of how spin-waves interact dynamically with both homogeneous magne-
ization textures and domain wall structures in multiferroic materials. As it turns out, these two situations are inseparable and must be considered together. The reason for this is that we find that spin-waves induce a torque even on a homogeneous magnetization due to the magnetoelectric coupling. To illustrate this effect analytically, consider a thin-film ferromagnet with propagating magnons where the magnetization lies in-plane (say, xz-plane). Writing out the effective field explicitly, we then have:

\[
H_{\text{eff}} = J\partial_z^2 m + K m \hat{x} + 2\gamma_0 \partial_z m (E_x \hat{x} + E_y \hat{y}) - 2\gamma_0 \hat{z} (E_x \partial_z m + E_y \partial_y m_y).
\]

To describe spin-wave propagation and its influence on the magnetic order parameter, we write the total normalized magnetization as \(m = (\sigma_0, \delta m_y + s_y, \delta m_z + s_z)\) where \(\sigma_0 = \pm 1\) describes the equilibrium macrospin orientation, taking into account the possibility of ordering along both \(\pm \hat{x}\) for the sake of generality. Moreover, \(\delta m_1\) and \(s_j\) describe the change in the magnetic order parameter and the spin-wave excitations, respectively, and are assumed to be small compared to \(\sigma_0\) which allows for a perturbation treatment. With the above effective field, we insert \(m\) into the LLG equation and average over one spin-wave oscillation period. Discarding higher order terms, we are left with the following equations:

\[
J\partial_z^2 m_z = H_k m_z + \gamma_0 E_y \partial_z m_y + \gamma_0 \sigma_0 E_x (s_y \partial_z s_z),
\]

\[
J\partial_z m_y = H_k m_y + \gamma_0 E_y \partial_z m_y + \gamma_0 \sigma_0 E_x (s_y \partial_y s_z).
\]

We also obtain a set of equations for the spin-wave amplitudes \(s_j\) to leading order:

\[
\sigma_0 \partial_t s_y + \alpha \partial_t s_z = \gamma J\partial_z^2 s_z - \gamma s_z H_k + \gamma_0 E_y \partial_z s_y,
\]

\[
\sigma_0 \partial_t s_z - \alpha \partial_t s_y = -\gamma A \partial_z^2 s_y + \gamma s_y H_k - \gamma_0 E_y \partial_y s_z.
\]

The underlying assumption here is that the spin waves vary on a much shorter time scale than the magnetization texture, as is reasonable. Consider first the case with an electric field only along the x-direction of the film, such that \(E_y = 0\). Remarkably, the above equations then become formally equivalent to the equations of motion for spin-waves and subsequent change in magnetization due to the spin-waves as occurring in both topological insulators [24] and ferromagnets with Dzyaloshinskii-Moryia interaction [25]. We may thus immediately conclude that there is a spin-wave induced magnetoelectric torque acting even homogeneous magnetization textures in multiferroic materials. This effect vanishes completely if one sets the magnetoelectric coupling \(\gamma_0\) to zero. What is more, however, the present case appears to offer additional physics compared to the aforementioned scenarios: if we allow for an out-of-plane component for the electric field, \(E_y \neq 0\), an extra term proportional to \(\partial_y s_j\) and \(\partial_z \delta m_j\) appear in Eqs. (6) and (7). This term influences the magnonic torque and offers an additional way to control it which differs from the influence of the in-plane electric field component. The influence of the new term \(\propto E_y\) complicates the analytical solution, and so we choose to proceed via a numerical route in order to also investigate the influence of magnons on inhomogeneous spin-textures in multiferroics.

We are now in a position to determine how spin-waves interact with a domain wall texture, which thus also requires their interaction with the homogeneous part of the domains to be taken into account according to the above results. This is different from previous works on magnon-induced domain wall motion in ferromagnets, where no such homogeneous torque is present. We have thus solved the full LLG equation without any perturbative approximations where the initial profile at \(t = 0\) consists of a magnetic domain wall center around \(z = 0\). Anti-reflection boundary conditions were implemented near the edges of the system in order to remove spin-wave backscattering, modelled by allowing the Gilbert damping \(\alpha\) to rise rapidly very close to the edges. As a consistency check against previous works for the same parameter regime in the absence of magnetoelectric coupling [19], we verified that spin-wave generation via an ac external magnetic field \(B(t)\), applied locally in a small region of one of the domains, induced motion in the opposite direction of the magnon flow.

Turning to the present multiferroic system, we now demonstrate that the presence of the magnetoelectric coupling in the effective field offers a new result compared to previous work on spin-wave induced domain wall motion. Since a gradient in the magnetization couples to the electric field, one could envision that not only an ac magnetic field could drive spin-wave induced domain wall motion, but that the same could take place via an ac electric field. An important aspect of realizing such an effect is that the electric field would have to be applied in a region where there was a magnetization gradient, in effect not too deep inside the domains with fixed magnetization direction. To determine if electric-field induced domain wall motion via magnons is possible, we applied an ac electric field \(E(t)\) locally near the domain wall region and the result is shown in Fig. 2(a) (see figure caption for parameters used). As seen, the spin-waves emanating from this proce-
dure indeed trigger domain wall motion and thus demonstrates the possibility to achieve electric control over magnon-induced magnetization texture transport. Remarkably, we find that even the direction of motion of the domain wall can be controlled by applying an additional constant gate-field: whereas any such dc field, it moves away from the domain wall in its presence. This finding suggests that the magnetoelectric coupling alters the effective potential felt by the spin-waves as they propagate through the domain wall, causing it to deviate from the reflectionless potential which is experienced by spin-waves in conventional ferromagnets with a belonging phase shift after passing through the wall [19]. In fact, the physical mechanism behind this effect is suggested by closer inspection of the curves in Fig. 2. When $E_{dc} = 0$, the spin-waves pass through the domain wall and the wall moves toward the spin-wave source due to conservation of angular momentum. However, when $E_{dc} \neq 0$, it is seen that no spin-waves emanate on the other side of the domain wall: instead, they are reflected and the domain wall moves away from the spin-wave source due to a transfer of linear momentum $p$. In this way, the direction of the domain wall motion is controllable by a gate voltage effect.

We have investigated the interaction between the spin-waves and the domain wall over a range of magnitudes for $E_{dc}$ and find that the electric field alters the amount of spin-wave reflection: Fig. 2(b) shows a scenario where the reflection is almost complete. An analytical description of this effect has proven elusive to us so far, due to the complicating factor of the spin-wave torque acting even on the homogeneous domains of the magnetization profile, although this is work in progress. We note that linear-momentum transfer of spin-waves to domain walls in ferromagnets have also been investigated in Ref. [26], and been shown to be possible at special resonance frequencies of the applied dc $B$ field. In our treatment of the current-induced case, the dominant effect of the applied current is the spin-transfer torque effect described by the two last terms in Eq. [1] and not the associated electric field along the structure which accompanies such a current, an approximation which should be better the higher conductivity the multiferroic is (in order to reduce the required voltage-drop, and thus field $E$, that generates the current). Candidate materials for the effects predicted in this work include epitaxial iron garnet films, which when grown on (210) and (110) gadolinium-gallium garnet substrates generates a Neel component of the domain wall structure due to anisotropy and hence activates the magnetoelectric coupling [15]. We also note that very recently, domain wall motion via electric field was observed in a hybrid multiferroic consisting of ferromagnetic-ferroelectric heterostructure [27].

Concluding, we have here demonstrated that domain wall motion in multiferroic materials hosts a wealth of interesting effects which are distinct from conventional ferromagnets in terms of its response to spin-wave and current-induced torques, including the possibility to control the direction of the domain wall motion via a gate voltage, and hope that these findings may stimulate further investigations.

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