The Potential of a New Linear Collider for the Measurement of the Trilinear Couplings among the Electroweak Vector Bosons

M. Bilenky\textsuperscript{a)}, J.-L. Kneur\textsuperscript{b)}, F. M. Renard\textsuperscript{b)}, D. Schildknecht\textsuperscript{a)}

\textsuperscript{a)} Department of Theoretical Physics, University of Bielefeld, 33501 Bielefeld, Germany
\textsuperscript{b)} Physique Mathématique et Théorique, CNRS-URA 768, Université Montpellier II, F-34095, Montpellier Cedex 5, France

Abstract

We study the accuracy to be obtained in measuring trilinear $Z^0W^+W^-$ and $\gamma W^+W^-$ couplings in the reaction $e^+e^- \rightarrow W^+W^-$ at "New Linear Collider" energies of 500GeV to 1000GeV. We derive simple scaling laws for the sensitivity in the measurement of these couplings. For most couplings the sensitivity increases as $\sqrt{L \cdot s}$, where $L$ denotes the integrated luminosity and $\sqrt{s}$ denotes the $e^+e^-$ center-of-mass energy. Detailed investigations based on various fits confirm these scaling laws and show that an accuracy of the order of the standard radiative corrections can be reached at the NLC for the design values of the luminosity.

\textsuperscript{†}Supported by Deutsche Forschungsgemeinschaft and the program PROCOPE of the DAAD for Franco-German scientific collaboration

\textsuperscript{*}Alexander von Humboldt Fellow. On leave of absence from Joint Institute for Nuclear Research, Dubna, Russia.
1 Introduction

Our present empirical knowledge on electroweak phenomena is largely confined to vector-boson-fermion interactions. Our knowledge on other properties of the weak vector bosons, apart from their masses, is rather limited. Indirectly, the electroweak precision data\[^1\] imply restrictions on non-standard bosonic self-interactions via model-independent bounds (see e.g. [2]) on radiative corrections. Direct investigations of these couplings at future colliders will be indispensable, however, for a full understanding of the electroweak interactions.

We will discuss the measurements of the trilinear couplings among vector bosons in the process $e^+e^- \rightarrow W^+W^-$ \[^1\]. The first experiments on this reaction will be carried out at LEP2 in the near future. The potential of LEP2 for measuring the trilinear couplings among the vector bosons was analysed recently \[^3\] in some detail (see also \[^7\] for earlier studies). At an $e^+e^-$ energy of about $190\,\text{GeV}$ and with an integrated luminosity of $500\,\text{pb}^{-1}$ an accuracy of order 0.1 for the determination of the trilinear couplings can be reached. This will improve present direct constraints on the $W^+W^-\gamma$ vertex from $p\bar{p}$ colliders \[^5\] by more than one order of magnitude. Indirect bounds on non-standard couplings estimated from loop corrections \[^6\], \[^11\] to electroweak precision data will be improved by factors ranging from about 2 to an order of magnitude. From these results, one will be able to rule out (or find) drastic deviations from standard-model predictions. Measurements at LEP2 will not be sufficient, however, as the order $10^{-3}$ to $10^{-5}$ \[^11\]. The question naturally arises, whether this level of sensitivity can be reached in the energy range and with the luminosities now envisaged for an $e^+e^-$ collider.

In the present work, our recent analysis for the LEP2 energy range \[^3\] is extended to the energies of $500\,\text{GeV}$ and $1000\,\text{GeV}$ of a future linear $e^+e^-$-collider (NLC) \[^12\], \[^13\].

2 Theoretical restrictions on non-standard couplings

For the present analysis, we disregard the possibility of CP-violating couplings\[^1\]. Assuming $C$– and $P$–invariant photon interactions, we can effectively describe \[^15\] the $\gamma W^+W^-$ and $Z^0W^+W^-$ couplings by the Lagrangian \[^1\]

\[
L = -ie[A_\mu(W^{-\mu\nu}W^\nu + W^{+\mu\nu}W^-_\nu) + F^{\mu\nu}W^\mu_\nu + W^-] - ie\gamma\tilde{F}^{\mu\nu}W^\mu_\nu
\]

\[
-ie(\text{ctg}\theta_W + \delta Z)[Z_\mu(W^{-\mu\nu}W^\nu + W^{+\mu\nu}W^-_\nu) + Z^{\mu\nu}W^\mu_\nu - W^-] - ieZ\tilde{Z}^{\mu\nu}W^\mu_\nu
\]

\[
+ie\frac{y_W^2}{M_W^2}F^{\mu\nu\lambda}W^\lambda_{\mu\nu} + ie\frac{y_Z^2}{M_Z^2}Z^{\mu\nu\lambda}W^\lambda_{\mu\nu}
\]

\[
+\frac{\epsilon\gamma Z}{M_W^2}\partial^\sigma\tilde{Z}_\rho(\partial^\rho W^\sigma - \partial^\sigma W^\rho) + \partial^\sigma W^\rho - \partial^\rho W^\sigma - \partial^\sigma W^\rho - \partial^\rho W^\sigma,
\]

(2.1)

where $F_{\mu\nu}$, $Z_{\mu\nu}$, $W^\pm_\nu$ are Abelian field-strength tensors for photon, $Z$ and $W^\pm$ bosons, respectively, and

\[
\tilde{Z}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}Z^{\sigma\rho}.
\]

\[^1\] We refer to, e.g., \[^3\] for a discussion of the quadrilinear couplings. Study of the self interactions of the vector bosons in other processes at future hadron colliders and in $e\gamma$ and $\gamma\gamma$ interactions can be found in \[^3\] and \[^7\], respectively.

\[^2\] A unique procedure to search for CP-violating $Z^0W^+W^-$ couplings via comparison of appropriate $W^-$ and $W^+$ spin-density-matrix elements was presented in \[^1\].
The parameter \( \delta_Z \) describes a deviation of the \( Z^0W^+W^- \) overall coupling from its standard value. Non-zero values of \( x_\gamma \) and \( x_Z \) parametrize potential deviations in the electromagnetic and weak dipole couplings from the standard model predictions, and \( y_\gamma \), \( y_Z \) denote the strengths of non-standard dimension-six quadrupole interactions of the \( W^\pm \). The coupling \( z_Z \) describes a \( CP \)-conserving, but \( C \)- and \( P \)-violating, so-called anapole coupling of the \( Z^0 \) to the \( W^\pm \).

The Lagrangian (2.1) contains the trilinear interactions of the standard model at tree level for the special case of

\[
\delta_Z = x_\gamma = x_Z = y_\gamma = y_Z = z_Z = 0. \tag{2.3}
\]

The symmetry and renormalizability requirements formulated in the standard electroweak theory lead to the restriction (2.3). In the most general phenomenological model-independent analysis of experimental data (assuming \( CP \) invariance) all six parameters in eq. (2.1) must be treated as independent ones. There are nevertheless theoretical as well as practical reasons to reduce the number of free trilinear (and quadrilinear) couplings by additional constraints based on \( SU(2) \)-symmetry requirements (refs. [16]-[20]), thus excluding the theoretically most disfavoured deviations from the standard electroweak theory.

The restrictions from \( SU(2) \) symmetry on the trilinear couplings in (2.1) can be simply reproduced \[ \] by performing a transformation from the \( \gamma Z^0 \) to the \( W^3B \) (or, alternatively, the \( \gamma W^3 \) current-mixing) base in the Lagrangian (2.1). Here, we briefly summarize the results and refer to refs. [6] and [16] to [20] for details.

Excluding intrinsic \( SU(2) \) violation, i.e., requiring restoration of \( SU(2) \) symmetry in the decoupling limit of the hypercharge \( (B_\mu) \) field, \( e = s_W = 0 \), implies the condition [16]

\[
x_Z = -\frac{s_W}{c_W} x_\gamma \tag{2.4}
\]

(where \( s_W = e/g_W \) denotes the sine of the weak mixing angle and \( c_W^2 \equiv 1 - s_W^2 \)), while allowing for

\[
x_\gamma \ , \ \delta_Z \ , \ y_\gamma \ , \ y_Z \neq 0. \tag{2.5}
\]

The number of free non-standard couplings is further reduced, if \( SU(2) \) symmetry is imposed on the quadrupole interaction, implying [17]

\[
y_Z = \frac{c_W}{s_W} y_\gamma. \tag{2.6}
\]

Further requirements, such as a fairly decent high-energy behaviour of the tree-level amplitudes for the scattering of vector bosons on each other, lead to additional constraints [18, 19]. The various constraints are collected in Table 1 taken from ref. [6].

The three-free-parameter \( (\delta_Z, x_\gamma, y_\gamma) \) interaction Lagrangian with the constraints (2.4) and (2.6) on \( x_Z \) and \( y_Z \) may be incorporated [6, 22] into a Lagrangian which is invariant under local \( SU(2) \) transformations.\[3\] While this embedding of the interactions into such a framework is irrelevant for the (tree-level) phenomenology of the reaction \( e^+e^- \to W^+W^- \), it is of importance insofar as it provides an example of how non-standard couplings can coexist with LEP1 precision data: the linearly realized local \( SU(2) \) symmetry assures [4, 10] an at most logarithmic dependence on the cut-off in one-loop corrections to LEP1 observables and decoupling of “new physics” effects. In the case of the dimension-six quadrupole interactions, local \( SU(2) \) symmetry is simply obtained [17] by appropriate use of the non-Abelian field tensor for the \( W \)

\[3\]A complete list of such \( SU(2)_L \times U(1)_Y \)- operators can be found e.g. in ref. [21].
| Symmetry                                                                 | number of param. | Couplings and constraints                                                                 |
|-------------------------------------------------------------------------|------------------|------------------------------------------------------------------------------------------|
| Lorentz-invariance                                                     | 3                | \(\delta_Z, x_\gamma, x_Z\)                                                              |
| C-, P-invariance                                                        |                  |                                                                                          |
| Exclusion of intrinsic SU(2) violation; local \(SU(2)_L \times U(1)_Y\), by including dim.-6 Higgs interaction via \(L_{W\phi}, L_{B\phi}\) | 2                | \(\delta_Z, x_\gamma; x_Z = -\frac{s_W}{c_W} x_\gamma\)                                 |
| Exclusion of \(s^2\) terms in \(W^+W^-\), etc. scattering; Exclusion of \(\Lambda^4\)-divergence in \(\rho\); \(L_{W\phi}\) only | 1                | \(x_\gamma; \delta_Z = \frac{x_\gamma}{s_W c_W}, x_Z = -\frac{s_W}{c_W} x_\gamma\)      |
| \(L_{B\phi}\) only                                                    | 1                | \(x_\gamma; \delta_Z = 0, x_Z = -\frac{s_W}{c_W} x_\gamma\)                              |
| \(L_{W\phi} + L_{B\phi}\)                                              | 1                | \(x_\gamma; \delta_Z = \frac{x_\gamma}{s_W c_W}, x_Z = -\frac{s_W}{c_W} x_\gamma\)      |
| \(L_{W\phi} - L_{B\phi}\) or, alternatively, \(SU(2)_W \times SU(2)_V \times U(1)_Y\) | 1                | \(\delta_Z; x_\gamma = x_Z = 0\)                                                        |
| Lorentz-invariance, C-, P-invariance                                   | 5                | \(\delta_Z, x_\gamma, x_Z, y_\gamma, y_Z\)                                              |
| Exclusion of intrinsic \(SU(2)\) violation                             | 4                | \(\delta_Z, x_\gamma, y_\gamma, y_Z; x_Z = -\frac{s_W}{c_W} x_\gamma\)                 |
| Local \(SU(2)_L \times U(1)_Y\); \(L_{W\phi}, L_{B\phi}\) and quadrupole, \(L_W\) | 3                | \(\delta_Z, x_\gamma, y_\gamma; x_Z = -\frac{s_W}{c_W} x_\gamma, y_Z = \frac{c_W}{s_W} y_\gamma\) |
| Local \(SU(2)_L \times U(1)_Y\) \(L_{W\phi}\) and \(L_W\)             | 2                | \(x_\gamma, y_\gamma; \delta_Z = \frac{x_\gamma}{s_W c_W}, x_Z = -\frac{s_W}{c_W} x_\gamma, y_Z = \frac{c_W}{s_W} y_\gamma\) |
| Local \(SU(2)_L \times U(1)_Y\) \(L_W\) only                         | 1                | \(y_\gamma; y_Z = \frac{c_W}{s_W} y_\gamma, \delta_Z = x_\gamma = x_Z = 0\)            |
| Lorentz-invariance, C-, P-violation                                    | 6                | \(\delta_Z, x_\gamma, x_Z, y_\gamma, y_Z, z_Z\)                                          |

Table 1: Constraints on the \(\gamma W^+W^-\) and \(Z^0W^+W^-\) couplings in Lagrangian eq. (2.1). The second column shows the number of free parameters. The free parameters and the constraints defining the remaining parameters are displayed in the third column. In the upper part of the Table, dimension-six quadrupole terms are excluded by assumption \((y_\gamma = y_Z = 0)\), while in the lower part of the Table such terms are allowed.
field. For the dimension-four trilinear interactions, however, the introduction of non-standard Higgs interactions \([9, 10, 23, 22]\) is essential. The basic Lagrangian takes the form \([6, 22]\)

\[
L = \frac{2}{M_W^2} (x_\gamma - \delta_Z s_W c_W) L_{B\phi} + \frac{2}{M_W^2} \delta_Z s_W c_W L_{W\phi} + e - \frac{y_\gamma}{s_W M_W^2} L_W, \tag{2.7}
\]

with

\[
L_{B\phi} = i \frac{e}{2 c_W} B^{\mu\nu} (D_\mu \phi) \dagger (D_\nu \phi),
\]

\[
L_{W\phi} = i \frac{e}{2 s_W} \bar{w}^{\mu\nu} (D_\mu \phi) \dagger \bar{\tau} \cdot (D_\nu \phi), \tag{2.8}
\]

\[
L_W = \frac{1}{6} \bar{w}^{\mu\lambda} (\bar{w}_{\lambda\nu} \times \bar{w}^\mu),
\]

where \(D_\mu\) denotes the covariant derivative

\[
D_\mu = \partial_\mu + i \frac{e}{s_W} \tau_7 \bar{W}_\mu + i \frac{e}{c_W} B_\mu Y \tag{2.9}
\]

and \(\bar{w}_{\mu\nu}\) the non-Abelian field tensor

\[
\bar{w}_{\mu\nu} = \partial_\mu \bar{W}_\nu - \partial_\nu \bar{W}_\mu - \frac{e}{s_W} \bar{W}_\mu \times \bar{W}_\nu. \tag{2.10}
\]

Upon passing to the physical \(\gamma\) and \(Z^0\) fields in eq. (2.7), for the trilinear couplings, one recovers Lagrangian (2.1) with \((\delta_Z, x_\gamma, y_\gamma)\) as free parameters and the constraints (2.4) and (2.6) for \(x_Z\) and \(y_Z\), respectively.

Various specific cases of the Lagrangians (2.1), (2.7), corresponding to different constraints among the couplings, are collected in Table 1 taken from \([6]\).

### 3 Scaling laws for the bounds on non-standard couplings

The helicity amplitudes for the process \(e^+ e^- \rightarrow W^+ W^-\) corresponding to the general Lagrangian (2.1) were given in Table 3 of ref.\([3]\). Here we restrict ourselves to a brief discussion of the high-energy dependence of the cross section for the production of \(W^\pm\) bosons with correlated helicities. We will see that the sensitivity for the determination of non-standard couplings in the high-energy limit can be represented by a simple formula in terms of the \(e^+ e^-\) energy and the integrated \(e^+ e^-\) luminosity.

We consider the cross section, \(\sigma^{AA'}(s)\), where \((A, A') = (L, T)\), for the production of \(W^+ W^-\)-pairs of definite helicities. The indices \(T\) and \(L\) refer to longitudinal \((W^\pm\)-helicity 0) and transverse \((W^\pm\)-helicity \(\pm 1\)) polarizations, respectively. We assume that the non-standard couplings, \(\delta_Z, x_\gamma, x_Z\), etc., are sufficiently small to be treated in the linear approximation, i.e., all purely non-standard contributions (proportional to \(\delta_Z^2, x_\gamma^2, x_Z^2, \ldots\)) are neglected. The cross section \(\sigma^{AA'}(s)\) then becomes

\[
\sigma^{AA'}(s) = \sigma_0^{AA'}(s) + \sum_{i=1}^{6} x_i \Delta_i^{AA'}(s) + O(x_i^2), \tag{3.1}
\]

where
where \( \sigma_0^{AA'}(s) \) denotes the standard-model cross section, and the parameters \( x_i \) stand for the six non-standard couplings in eq. (2.1), \( (x_1, \ldots, x_6) \equiv (\delta_Z, x_\gamma, \ldots, z_Z) \). The asymptotic \( (s \gg 4M_W^2) \) energy dependence of the coefficients \( \Delta_i^{AA'}(s) \) in eq. (3.1) is easily obtained from the expressions for the helicity amplitudes explicitly represented in Tables 3 and 4 of ref. \([6]\). The result is displayed in Table 2\(^4\). Due to (energy-independent) selection rules for the dipole, quadrupole and anapole couplings, certain coefficients are vanishing in eq. (3.1) and, consequently, certain entries in Table 2 are absent.

\[
\begin{array}{|c|c|c|c|c|}
\hline
& \text{TT} & \text{TT} & \text{LL} & \text{LT} \\
\tau = \tau' & \tau = -\tau' & & & \\
\hline
\sigma_0^{AA'}(s) & s^{-3} & s^{-1} & s^{-1} & s^{-2} \\
\Delta_{\delta Z}^{AA'}(s) & s^{-2} & \text{const.} & & s^{-1} \\
\Delta^{AA'}_{x_\gamma, x_Z}(s) & \text{const.} & & & s^{-1} \\
\Delta^{AA'}_{y_\gamma, y_Z}(s) & s^{-1} & & & s^{-1} \\
\Delta_{zz}^{AA'}(s) & & & \text{const} & \\
\hline
\end{array}
\]

Table 2: The asymptotic \((s \gg 4M_W^2)\) energy dependence of the standard model cross section (first row) and of the various non-standard contributions \(\Delta_i^{AA'}(s)\) in eq. (3.1). If both vector bosons, \(W^+\) and \(W^-\), have transverse polarization (TT), the two cases of equal \((\tau = \tau' = \pm 1)\) and opposite \((\tau = -\tau' = \pm 1)\) helicities have different high-energy behaviour, as indicated.

Using Table 2, the energy dependence of the sensitivity for the measurement of the non-standard couplings is easily derived. Restricting ourselves to small non-standard contributions to \(\sigma^{AA'}(s)\), an assumption already introduced in eq. (3.1), the statistical error of a measurement of the cross-section \(\sigma^{AA'}(s)\) may be approximated by the error corresponding to the number of events calculated from the standard cross section, \(\sigma_0^{AA'}(s)\), via

\[
\delta N^{AA'} = \text{const.} \sqrt{L \cdot \sigma_0^{AA'}(s)}.
\]

(3.2)

In (3.2), \( L \) denotes the integrated \(e^+e^-\) luminosity. An energy-independent proportionality constant, substantially larger than unity\(^5\), appears in (3.2), since in actual experiments the

\(^4\)We thank Dr. A. Pankov for a useful discussion related to the content of Table 2.

\(^5\)Actually, the proportionality constant depends on \(A, A'\). Its absolute value is irrelevant, however, for the relative values of the sensitivity (as a function of energy and luminosity) under consideration in the present section.
helicity information can only be extracted by an analysis of the $W^\pm$ decay distributions (in their respective rest frames, compare ref. [1] and section 4 of the present paper). Equating the statistical error eq. (3.2) with the number of non-standard excess events predicted by eq. (3.1),

$$\sqrt{L_0^{AA'}(s)} = \text{const.} L \cdot | \sum_i x_i \Delta_i^{AA'}(s) |,$$

(3.3)

allows one to determine (the energy dependence of) the sensitivity of a measurement of the couplings $x_i$. Asymptotically, for given helicities, $A, A'$, according to Table 2, those non-standard couplings are dominant in (3.3) which are associated with either a constant (in case of $\delta_Z, x_\gamma, x_Z$ and $z_Z$) or else an $s^{-1}$ (in case of $y_\gamma, y_Z$) energy dependence. Accordingly, from eq. (3.3) the sensitivity for the measurement of $x_i$ (defined by the inverse of the magnitude of $x_i$) is determined by the proportionality\footnote{When deriving (3.4) from (3.3), in general a sum of several $x_i$ with energy-independent coefficients will appear in the numerator of (3.4). The proportionality (3.4) then follows immediately.}

$$\frac{1}{|x_i|} \sim \Delta_i^{AA'}(s) \sqrt{\frac{L}{\sigma_0^{AA'}(s)}},$$

(3.4)

where for a definite choice of $x_i$, the right-hand side has to be evaluated for those helicities $A, A'$, for which, according to Table 2, the cross section $\sigma^{AA'}(s)$ is dominated by the contribution proportional to $x_i$, i.e., we have to use the following correspondence when evaluating eq. (3.4):

$$(y_\gamma, y_Z) \rightarrow TT(\tau = \tau'),$$

$$(\delta_\gamma, x_\gamma, x_Z) \rightarrow LL,$$

$$z_Z \rightarrow LT.$$

(3.5)

Explicitly, from eqs. (3.4) and (3.5), using Table 2, we find the results displayed in Table 3. According to this Table, the sensitivity for the anapole interaction increases as $s\sqrt{L}$, while for all other couplings it increases as $\sqrt{s \cdot L}$. We draw attention to the fact that the simple behaviour (3.4), (3.3) leading to Table 3, according to (3.3), is based on the linear approximation in $x_i$. The neglect of terms of order $x_i^2 \cdot (s/M_W^2)$ will break down at sufficiently large values of $s$, even for small values of $x_i$. We will see, however, that the sensitivity is well described by Table 3 in the range of energies and luminosities to be considered explicitly below. Finally, scaling

| coupling       | sensitivity        |
|----------------|-------------------|
| $\delta_Z, x_\gamma, x_Z, y_\gamma, y_Z$ | $\sqrt{s \cdot L}$ |
| $z_Z$          | $s\sqrt{L}$       |

Table 3: Sensitivity for the determination of the non-standard couplings as a function of the integrated $e^+e^-$ luminosity, $L$, and the square of the center-of-mass energy, $s$. 
laws can be different if the full $W^{\pm}$ helicity information is not taken into account, as helicity information explicitly enters our error analysis (e.g., in (3.2)).

In summary, once the sensitivity is known for a specific energy (sufficiently above threshold, $s \gg 4M_W^2$), and luminosity, the dependence from Table 3 may be used to predict the sensitivity for any other (asymptotic) energy. Explicit numerical results for the increase in sensitivity from

|      | $L [pb^{-1}]$ | $\delta Z, x_{\gamma}, x_{Z}, y_{\gamma}, y_{Z}$ | $zz$ |
|------|---------------|-----------------------------------------------|------|
| LEP 200 | 500           | $\sim 11$                                   | $\sim 28$ |
| NLC 500 | 10000         | $\sim 4$                                    | $\sim 8$ |
| NLC 1000 | 44000         |                                              |      |

Table 4: Increase in sensitivity according to (3.4), (3.5) and Table 3.. The assumed integrated luminosities are listed in the second column.

LEP2 to the NLC\(^7\), obtained by evaluating the formulae of Table 3, are presented in Table 4. In section 4, these results will be compared with the results of numerical simulations performed for the cases listed in Table 4. As expected from the above derivations, we will find perfect agreement with the simple scaling laws of Tables 3 and 4.

4 Simulation of the experimental determination of the trilinear couplings.

In the numerical analysis of the precision to be expected for the measurements of the trilinear couplings, we only consider events of the type

$$e^+e^- \to W^+W^- \to \begin{cases} e^+\nu_e & + 2 \text{ jets}, \\ \mu^+\nu_\mu & + 2 \text{ jets}. \end{cases}$$

(4.1)

For these events identification of the charge of the $W$-bosons will be simple. We will not consider here the possibility of electron (positron) beam polarization (see e.g. \cite{24}). With the luminosities of Table 4, when assuming standard tree-level amplitudes for (on-shell) $W^{\pm}$ production in the angular range of

$$-0.98 \leq \cos \vartheta \leq 0.98,$$

(4.2)

one obtains

$$\sim 2900 \text{ events } (E_{e^+e^-} = 190GeV),$$

$$\sim 13200 \text{ events } (E_{e^+e^-} = 500GeV),$$

$$\sim 14000 \text{ events } (E_{e^+e^-} = 1000GeV).$$

\(^7\)The integrated luminosity of $10fb^{-1}$ at $500GeV$ corresponds to $\sim 10^7$ sec. of operation for the Palmer F design of a linear collider \cite{12}. This option of a linear collider has a narrow energy distribution around $500GeV$. 

7
Following ref. [6], we simulate the two-step procedure for the analysis of the data suggested therein.

According to this procedure, in a first step, all observables, i.e., the differential cross section as well as the various single-particle spin-density-matrix elements and the $W^+W^-$ spin correlations, are to be determined. Only the well known $V-A$ charged-current weak interaction enters, when extracting the spin properties of the produced $W^\pm$ bosons from the measured $W^\pm$ decay distributions. Consequently, as far as the $W^\pm$ production process is concerned, this first step in the data analysis is entirely model independent. Once actual data will become available, a comparison with the results of the model-independent analysis with (standard and/or non-standard) theoretical predictions is to be carried out. In the subsequent second step the trilinear couplings are to be determined by a fitting procedure, using the differential cross section and the spin-density matrix elements as the empirical input.

In our simulation of the first step of the data analysis we generate "data" for the three-fold differential cross sections

$$
\frac{d\sigma}{d\cos \vartheta d\cos \theta_1 d\phi_1}, \frac{d\sigma}{d\cos \vartheta d\cos \theta_2 d\phi_2}, \frac{d\sigma}{d\cos \vartheta d\cos \theta_1 d\cos \theta_2}
$$

in accordance with the standard model, assuming the integrated luminosities given in Table 4 and the corresponding event numbers (4.3). In eq. (4.4), $\vartheta$ denotes the $W$-production angle and $\theta_i, \phi_i (i = 1, 2)$ denote the polar and azimuthal $W^\pm$ decay angles. As a result of the fit, we obtained "data" in the form of standard-model values with statistical errors for all above-mentioned observables.

In the second step, the differential cross sections and the density-matrix elements (with their errors) serve as the input for the determination of the trilinear couplings. Fits were carried out for the different parametrizations presented in Table 1. The conclusion of ref. [6] that helicity information is particularly important in multi-parameter fits and that it frequently improves bounds by factors of the order of 2, or by even larger factors, was found to remain valid at NLC energies.

The resulting bounds on the non-standard couplings are collected in Table 5, and contour plots are presented for two different two-parameter fits in figs. 1a,b and for a three-parameter fit in figs. 2a,b,c.

According to Table 5, the absolute values of the bounds (for one-parameter cases) at 500 GeV reach the order of magnitude of the standard radiative corrections, which, when represented in terms of the couplings $\delta_Z, x_\gamma, x_Z$, etc. [11], are of the order of 0.01 to 0.001. For the beam energy 1000 GeV such an accuracy can even be reached in certain multi-parameter cases. The strong bound on the anapole coupling, $z_Z$, even in the presence of all other non-standard terms, is related to the very particular helicity dependence of the anapole interaction (see Table 2).

The explicit numerical results in Table 5 are in very good agreement with the predictions of the scaling law given by eqs. (3.4), (3.5) and explicitly evaluated in Tables 3 and 4. Even though the scaling laws are based on the high-energy limit, $s \gg 4M_W^2$, the numerical analysis shows their validity even when the LEP2 results at 190 GeV are used as starting point.

The disappearance of $(x_\gamma, y_\gamma)$-correlations with increasing energy in fig. 1b is related to the fact that the contribution to the transverse-transverse cross section of $\delta_Z = x_\gamma / s_W c_W$ (according to Table 2) decreases asymptotically as $1 / s^2$ and becomes negligible, thus allowing for a clear separation of $x_\gamma$ and $y_\gamma$, as in this limit these parameters contribute to different helicity

8Statistical errors only are taken into account. A discussion of systematic errors is beyond the scope of the present investigation. Some studies of systematical uncertainties can be found in [25].

8
For each set of free parameters, we present the results of the fits carried out at various choices of free parameters and constraints correspond to the cases listed in Table 1. Table 5: The 95% C.L. bounds on the non-standard trilinear couplings obtained from fits. For each set of free parameters, we present the results of the fits carried out at $\sqrt{s} = 190\,\text{GeV}$, 500\,\text{GeV} and $1000\,\text{GeV}$ for the respective luminosities in Table 4. Note that the various choices of free parameters and constraints correspond to the cases listed in Table 1.
amplitudes only. In contrast, as seen from fig. 1a, the correlations between the dimension-4 couplings, $\delta_Z, x_\gamma, x_Z$, become stronger at high energies.

5 Conclusions

In the present work, we have extended our LEP 2 analysis on $e^+e^- \rightarrow W^+W^-$ to the energy range of a future $e^+e^-$ linear collider working at 500GeV to 1000GeV. For integrated luminosities of $10fb^{-1}$ and $44fb^{-1}$ at 500GeV and 1000GeV, respectively, we found that the bounds on non-standard couplings will be of the order of $10^{-2}$ to $10^{-3}$. With respect to measurements at LEP2, this will be an improvement by at least one order of magnitude. In fact, the bounds will reach the magnitude of (standard) radiative corrections.

We have derived simple scaling laws for the sensitivity of the reaction $e^+e^- \rightarrow W^+W^-$ for the determination of non-standard couplings. According to the scaling laws the sensitivity increases as $s\sqrt{L}$ for the anapole and as $\sqrt{sL}$ for all other couplings. This agrees with what we have found in the detailed simulation of the analysis of future data.

Acknowledgement

We would like to thank S. Katsanevas for the useful discussions.
References

[1] Plenary Talk by J. Lefrançois at the European International Conference on High Energy Physics, Marseille (July 1993).

[2] M. Bilenky, K. Kolodziej, M. Kuroda and D. Schildknecht, Bielefeld preprint BI-TP 93/46, to appear in Phys.Lett. B

[3] G. Belanger and F. Boudjema, Phys. Lett. B288 (1992) 201;
C. Grosse-Knetter and D. Schildknecht, Phys. Lett. B302 (1993) 309.

[4] D. Zeppenfeld and S. Willenbrock, Phys. Rev. D37 (1988) 1775;
U. Baur and D. Zeppenfeld, uNucl. Phys B308 (1988) 127;
G.L. Kane, J. Vidal and C.P. Yuan, Phys. Rev. D39 (1989) 2617;
A.F. Falk, M. Luke and E.H. Simmons, Nucl. Phys. B365 (1991) 523;
G. Gounaris and F.M. Renard, Z. Phys. C59 (1993) 143; G. Gounaris, J. Layssac and F.M. Renard, Montpellier preprint PM93/26

[5] S.Y. Choi and F. Schrempp, Phys. Lett. B272 (1991) 149;
E. Yehudai Phys. Rev. D44 (1991) 3434.

[6] M. Bilenky, J.L. Kneur, F.M. Renard and D. Schildknecht, Nucl. Phys. B409 (1993) 22.

[7] M. Davier and D. Treille in Proc. of the ECFA Workshop on LEP200, A. Böhm, W. Hoogland eds. CERN 87-08 (1987).

[8] UA2-Collaboration, Phys. Lett B277 (1992) 194.

[9] A. De Rujula, M. B. Gavela, P. Hernandez and E. Masso, Nucl. Phys. B384 (1992) 3;
P. Hernandez and F. J. Vegas, Phys. Lett. B307 (1993) 116.

[10] K. Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld, Phys. Lett.B283 (1992) 353;
Phys. Rev. D48 (1993) 2182.

[11] J. Fleischer, J. L. Kneur, K. Kolodziej, M. Kuroda and D. Schildknecht, Nucl. Phys. B378(1992) 443.

[12] Proc. of the Workshop on $e^+e^-$ Collisions at 500 GeV: the Physics Potential, DESY 92-123B, 1992, ed. P. Zerwas.

[13] Proc. of the Workshop on Physics and Experiments with Linear Colliders, Saariselkä, Finland, 1991 (World Scientific 1992), eds. P. Eerola et al.

[14] G. Gounaris, D. Schildknecht and F. M. Renard,Phys. Lett. B263 (1991) 291

[15] K. Gaemers and G. Gounaris, Z. Phys. C1 (1979) 259;
K. Hagiwara, R. Peccci, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B282 (1987) 253.

[16] J. Maalampi, D. Schildknecht and K. H. Schwarzer, Phys. Lett. B166 (1986) 361;
M. Kuroda, J. Maalampi, K. H. Schwarzer and D. Schildknecht, Nucl. Phys.B284 (1987) 271; Phys. Lett.B190 (1987) 217.
[17] M. Kuroda, F. M. Renard and D. Schildknecht, *Phys. Lett.* **B183** (1987) 366.

[18] H. Neufeld, J. D. Stroughair and D. Schildknecht, *Phys. Lett.* **B198** (1987) 563.

[19] C. Bilchak, M. Kuroda, D. Schildknecht, *Nucl. Phys.* **B299** (1988) 7.

[20] D. Schildknecht in *Proc. 6th Topical Workshop on p$ar{p}$-Collider Physics, Aachen, ed. K. Eggert, H. Faissner and E. Rademacher (World Scientific, 1986)* p. 125.

[21] W. Buchmüller and D. Wyler, *Nucl. Phys.* **B268** (1986) 621

[22] C. Grosse-Knetter, I. Kuss and D. Schildknecht, *Z. Phys.* **C60** (1993) 375

[23] G. Gounaris and F. M. Renard, *Z. Phys.* **C59** (1993) 133

[24] T.L. Barklow, in [13].

[25] M. Frank, P. Mättig, R. Settles and W. Zeuner in [12], p. 223.

**Figure captions**

**Figure 1:** Contour plots (95% C.L.) obtained in the two-parameter fits of
a) the parameters $\delta_Z$ and $x_\gamma$ with the constraints $x_Z = -\frac{s_W}{c_W}x_\gamma$, $y_\gamma = y_Z = 0$,
b) the parameters $x_\gamma$ and $y_\gamma$ with the constraints
$$\delta_Z = \frac{x_\gamma}{s_W c_W}, \quad x_Z = -\frac{s_W}{c_W}x_\gamma, \quad y_Z = \frac{c_W}{s_W}y_\gamma.$$  

**Figure 2:** Contour plots (95% C.L.) for the three-free-parameter case, $(\delta_Z, x_\gamma, y_\gamma)$,
with the constraints $x_Z = -\frac{s_W}{c_W}x_\gamma$, $y_Z = \frac{c_W}{s_W}y_\gamma$
 in
a) the $(x_\gamma, \delta_Z)$ plane,
b) the $(y_\gamma, \delta_Z)$ plane,
c) the $(x_\gamma, y_\gamma)$ plane.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9312202v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9312202v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9312202v1