Exploratory study of the temperature dependence of magnetic vertices in SU(2) Landau gauge Yang–Mills theory

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Vertices describe the interactions between the fundamental degrees of freedom, and are therefore of vital importance in many ab-initio descriptions of field theory, especially using functional methods. To this end, we present the first lattice study of the thermal behavior of (minimal) Landau-gauge SU(2) Yang–Mills three-points functions, i.e. three-gluon and ghost-gluon vertices. Focusing on the chromomagnetic sector, we find that the phase transition mainly affects the three-gluon vertex, while the ghost-gluon vertex is relatively inert.

PACS numbers: 11.10.Wx, 12.38.Gc, 12.38.Aw

I. INTRODUCTION

The study of heavy-ion collisions at RHIC and CERN have demonstrated that QCD is substantially more complex at and for some temperature range around the crossover than originally anticipated, see e. g. \[1\]. While lattice calculations have unraveled much of the static properties of the produced strongly-interacting matter \[2,3\], many of the dynamical properties are still not completely understood. These questions include, e. g. the fast thermalization or the collective behavior of soft pions \[4\]. Thus, ultimately a non-equilibrium ab-initio description will be necessary. This is, however, a challenging problem. One possible approach, for which real-time behavior becomes more and more accessible, are functional methods \[5,6\]. Still, these methods require approximations. To check and improve these approximations already static information is valuable. To obtain such information is the aim of the present paper.

This will be achieved by the study of the chromomagnetic three-point vertices. To understand their importance, recall that all information of any quantum field theory is encoded in its correlation functions. Since vertices describe the basic interactions between the fundamental degrees of freedom, they provide access to conceptual questions, bound states, and even phenomenological observables \[9,14\]. In this work we study, for the first time, and thus exploratory, the temperature-dependence of the three-point functions of SU(2) Yang–Mills theory in (minimal) Landau gauge, i.e. the three-gluon and ghost-gluon vertices, for a range of different kinematic configurations. The three-point functions are of particular importance, since in most cases the approximations in functional calculations rely on the ordering of correlation functions in the sense that only \(n\)-point functions up to a certain order are included. Until recently, even in the vacuum only the gluon and ghost propagators could be resolved fully, whereas approximations for \(n\)-point functions with \(n \geq 3\) have been made \[9,14\].

Already when including three-point vertices self-consistently, the level of complexity encountered in functional equations becomes much higher. As a consequence, vertex computations were so far mainly done at vanishing temperature \[15,16,23,25\], except for an incipient study on the thermal behavior of the ghost-gluon vertex \[15,16\]. The thermal behavior of the three-point functions presented here thus serves as an important step in finding appropriate truncations or, in addition, may even provide a guideline in the computation of vertices self-consistently in functional frameworks.

In addition, the thermal dependence of the vertices may help to resolve a pressing question concerning the deviation of finite temperature continuum results \[15,16,23,25\] from lattice simulations \[20,33\]: the (almost) independence of the ghost propagator with respect to temperatures from zero to well above the critical temperature \(T_c \approx 277\,\text{MeV}\) could not be recovered in functional computations yet. This may be due to an insufficient truncation of the temperature dependence of the three-point functions. As will be seen, the three-gluon vertex shows a dependence on the temperature while the ghost-gluon vertex is almost independent of temperature. These behaviors, if confirmed in a more systematic investigation, may provide a clue for the aforementioned insensitivity of the ghost propagator.

The main aim here is to get a first idea of interesting regions to set the stage for more detailed investigations. Hence, this study is limited to rather coarse and small lattices. Spatial volumes are \(N^3 = 20^3, 30^3\), whereas the temporal extent around the phase transition region is \(N_t = 4\). For the propagators there are significant lattice artifacts at these settings \[25,33\], so we do not expect yet quantitatively fully reliable results. However, for vertices so far the qualitative features in the vacuum were yet less sensitive to lattice artifacts than for the propagators \[34,37\].

This manuscript is structured as follows: In Section \[1\] we set the technical stage, supported by some details in
the Appendix A. The results are presented in the main part of the text in Section III. Some final conclusions are drawn in Section IV. A limited study of the volume-dependence of the results can be found in Appendix B. Since our volumes are much smaller than those employed for investigations of the propagators, we did not include results on the latter, but rather refer to the literature [26–28, 30–33].

II. VERTICES

In this section we briefly recall computations of three-point vertices using lattice gauge theory. In particular, we highlight the differences between computations at zero and non-zero temperature. Our approach is a straightforward extension of the analysis at zero temperature of [34, 35, 38], and therefore many of the details skipped here can be found in these references. The introduction of temperature, especially determining the actual temperature, is done in the same way as in [30]. See also Appendix A for further technical details.

Correlation functions are gauge-dependent quantities. Thus, one has to define a gauge condition. Here, we will use the standard minimal Landau gauge [12], to compare directly with results at vanishing temperature. As a consequence, the gluon propagator, $D_{\mu\nu}(p)$, is four-dimensionally transverse. Since higher $n$-point functions are connected via gluon or ghost propagators, the latter ones being Lorentz-scalar particles, transverse structures close among themselves. As a consequence, only the transverse parts of any $n$-point function contributes to the dynamics, if no extreme singularities are encountered [10]. In particular, on the lattice it is only possible to determine the transverse part of correlation functions [12]. In addition, Landau gauge is covariant, and thus O(4)-invariance allows at zero temperature for dependence on the modulus of the four-momentum only.

At non-zero temperature, however, the situation is more complicated. The heat bath, which we choose to be in temporal direction, permits to distinguish directions orthogonal and parallel to it. This generates two structural changes.

Firstly, Lorentz/O(4) symmetry is no longer manifest. As a consequence, the gluon propagator has to be spanned via two different tensor structures, the so-called (chromo-)electric and (chromo-)magnetic propagators, $G_{\mu\nu}^b(p)$ and $G_{\mu\nu}^A(p)$, respectively, in the three-dimensional spatial subspace. In contrast to the trivial tensor structure of the ghost propagator, $G_{\mu\nu}(p)$, is unaffected as it is a Lorentz scalar. Because the minimal Landau gauge does not constrain the global gauge degree of freedom, these propagators are color-diagonal, which has been confirmed explicitly by several calculations [12]. This also implies that for the gauge group SU(2) the three-point vertices’ color structure is completely determined by the structure constants.

Secondly, the presence of the heat bath triggers separate dependence on temporal and spatial components of the four-momenta: therefore, the dependence on a general four-momentum, $p$, is in fact a separate dependence on spatial momentum, $\vec{p}$, and Matsubara frequencies, $p_0$, which are discrete due to the compactification in (imaginary) time direction. For both gluons and ghosts the energies are given by $2\pi n T$, with $T$ the temperature, and $n$ integer. This distinguishes soft modes with $n = 0$ and hard modes with $n \neq 0$. The later show a rather trivial behavior for the propagators [27], and become irrelevant for the dynamics in the high-temperature limit. Anticipating a similar behavior for the vertices, we will concentrate here exclusively on the soft modes.

Turning to vertices now, the bases of possible tensors are richer. Here, we will only consider those vertex components proportional to the tree-level vertex. In contrast to the vacuum, where this restriction singles out exactly one tensor, at non-zero temperature the basis is enlarged because either magnetic or electric gluons may be attached to the vertices. However, because these vertices are proportional to the momenta, restricting to soft modes automatically singles out the chromomagnetic, i.e. purely spatial, part of the vertices. These are identical to the three-gluon vertex and ghost-gluon vertex of the dimensionally reduced theory in the infinite temperature limit [12]. Thus, by construction, we have to approach the known results for the three-dimensional vertices [34, 35] in this limit. In fact, already at roughly $2T$, the results are not too far away from the infinite-temperature-limit. This rapid approach is similar to what is found for the propagators.

Thus, we are concerned with two three-point vertices, the three-gluon and ghost-gluon vertices,

$$\Gamma^{(3), ab} = g_{A, \mu\nu}(p, q, r, T), \quad \Gamma^{(3), ab} = g_{ccA, \mu}(p, q, r),$$

where the subscript $A$ indicates that the external gluons are magnetic. In eq. (1) momentum conservation ensures that these vertices depend on two external momenta only, from which three independent kinematic variables can be formed in the vacuum: e.g. the moduli of the two four-momenta and the angle between them, respectively. At non-zero temperature, due to differentiation between spatial momenta and Matsubara frequencies, the vertices depend on five independent kinematic variables, two Matsubara frequencies and the moduli of and angle between spatial momenta. For the soft mode therefore only three parameters derived from the spatial momenta remain. In the following, we study momentum configurations which are directly compatible with the hyper-cubic symmetry of the spatial lattice. As detailed in [34, 35], due to their importance in continuum approaches we choose the orthogonal configuration, with two momenta being equal in magnitude and the angle between them being 90 degrees, the symmetric one, with all external momenta equal with an angle of 60 degrees between each pair of momenta, and a third configuration where one (gluon) momentum vanishes exactly, and the other two momenta are therefore at
180 degrees. Note that on a finite lattice it is not possible to study zero ghost momentum. Furthermore, because of the ghost-anti-ghost symmetry in Landau gauge, it is sufficient to study the dependence on ghost momentum. Furthermore, the Bose symmetry of the three-gluon vertex constrains its momentum dependence.

An important technical constrain is that, contrary to continuum approaches, on the lattice only full correlation functions are accessible but not vertex functions directly. The latter ones are parametrized by their dressing functions, $G^A^3(p, q)$ for the three-gluon vertex and $G^{c\bar{c}A}(p, q)$ for the ghost-gluon vertex, respectively. Schematically, we have

$$\Gamma^{(3), abc}_{\alpha\beta\gamma}(p, q, r) = G^{A^3}(p, q)\Gamma^{\mu
u\rho}_{\alpha\beta\gamma}(p, q, r) + \text{other tensor structures},$$

$$\Gamma^{(3), abc}_{c\bar{c}A, \mu}(p, q, r) = G^{c\bar{c}A}(p, q)\Gamma^{\mu
u\rho}_{c\bar{c}A, abc}(p, q, r)$$

where the $\Gamma^{\mu
u\rho}_{\alpha\beta\gamma}$ and $\Gamma^{\mu
u\rho}_{c\bar{c}A, abc}$ are the (color antisymmetric) classical tensor structures, including lattice artifacts. Access to the dressing functions is gained by amputation and projection via

$$G^{A^3} = \frac{\Gamma^{(3), ABC}_{ABG,DEF}G^{G,DEF}G^{G,DEF}G^{G,DEF}}{T\Gamma^{(2), ABC}_{ABG,DEF}G^{G,DEF}G^{G,DEF}G^{G,DEF}}.$$

III. RESULTS

In this section we present results on the thermal behavior of the magnetic three-point functions in comparison to their behavior at vanishing and infinite temperature. The behavior of the vertices is qualitatively similar on both spatial lattice volumes $N^3_x = 20^3$ and $N^3_x = 30^3$ studied here. Hence, we restrict the discussion in the main text to the larger lattice volume and defer the study of the volume dependence to Appendix B. We will study the two vertices separately in the following two subsections.

A. Ghost-gluon vertex

In the vacuum the SU(2) ghost-gluon vertex remains almost constant with deviations of the order of 30% from its tree-level part, in agreement between functional methods and lattice calculations.

The thermal dependence of the ghost-gluon vertex for temperatures $T \leq T_c$, is shown in FIG. 1. Within the statistical uncertainties, no pronounced temperature-dependence is observed, and the vertex remains essentially tree-level for the temperature range studied.

For temperatures above the phase transition, $T > T_c$, the results are shown in FIG. 2. Again, within statistical accuracy almost no temperature dependence is observed. In this way, the ghost-gluon vertex shows a behavior quite similar to the soft mode of the ghost propagator itself, which is also essentially independent of temperature.

At very high temperature dimensional reduction forces the vertex to approach its three-dimensional limit. However, because of the almost-temperature-independence of the vertex, this is essentially true for all temperatures. This is emphasized in FIG. 3 where the vertex at the critical temperature is compared to the zero and infinite-temperature limit from. As is visible, within the comparatively large statistical uncertainties, the vertex is essentially the same. Note that for the comparison to the three-dimensional theory, the string tension in four and three dimension has been set to the same dimensionful quantity.

B. Three-gluon vertex

At vanishing temperature the three-gluon vertex is significantly suppressed at intermediate and low momenta compared to its tree-level value in both four and three
FIG. 1: Various kinematic configurations of the ghost-gluon vertex at temperatures below criticality. The lower right graph illustrates the vertex at the critical temperature as a function of ghost and gluon momentum.

The strong statistical fluctuations are also the bane of the present calculations, especially at high momenta. However, a very interesting observation can be made at low momenta below the phase transition, see FIG. [4]. Here, the vertex becomes substantially enhanced towards the infrared when approaching the phase transition. However, as in the case of the longitudinal propagator [30] [35], the maximum enhancement occurs not at the phase transition, but slightly below, around 0.95$T_c$.

This suppression is essentially immediately gone in the high-temperature phase, as is seen in FIG. [5]. In fact, already at about 1.5$T_c$, the original suppression is there, and even a sign-change can be observed. Thus, the behavior in the infinite-temperature-limit is recovered [35], as shown in FIG. [6]. This is, of course, a dramatic dependence on the temperature, inducing a qualitative change. Especially that the vertex stays positive is a genuine feature of the three-point correlation function, since the required normalization by the propagators in eq. [2] cannot alter the sign, just the magnitude.
FIG. 2: Ghost-gluon vertex as in figure 1 however, for temperatures above the transition. The lower-right plot is at the largest temperature.

IV. CONCLUSIONS

Summarizing, we have presented the first lattice investigation of the temperature-dependence of the (chromo-)magnetic, soft three-point vertices of SU(2) Yang–Mills theory in minimal Landau gauge. We find that the ghost-gluon vertex shows essentially no temperature-dependence, similar to the ghost propagator itself. However, the three-gluon vertex shows a pronounced temperature-dependence, with a strong positive infrared enhancement, in strong contrast to zero or infinite temperature, at a temperature slightly below the phase transition. In this, it follows the electric gluon propagator. Note however, that the lattice volumes are rather small, and the discretizations are rather coarse. Both facts having quite a large impact on the propagators \cite{28, 30–33}, and therefore these results should be taken with caution.

Ignoring this caution for one paragraph, and assuming that these results will hold in more detailed studies, the implications are quite significant. The absence of changes in the ghost-gluon vertex closes nicely with observations for propagators: within systematic errors due to volume and discretization effects they are compatible with temperature-insensitive magnetic and ghost propagators. This should indicate the right avenue to understand the situation in functional calculations \cite{15}. However, the strong changes of the three-gluon vertex indicate a subtle interplay in the gluonic sector at and around the phase transition. In fact, the infrared en-
hancement and absence of the sign change in the magnetic three-gluon vertex around the phase transition is quite unexpected.

Returning to a more adequate cautionary tone, the results are interesting, but surely require more careful study of lattice artifacts. On the other hand, the magnetic three-gluon vertex shows that the magnetic sector is perhaps not as unaffected by the dynamics of the phase transition as originally anticipated by the behavior of the propagators. It thus appears that further studies of the magnetic vertices would be very helpful, also for functional calculations. Of course, given the behavior of the electric propagator and the present results, this begs the question on the behavior of the electric and mixed vertices. This is certainly a demanding project, but nevertheless a necessary step towards an understanding of the dynamics around the phase transition in terms of the gluonic degrees of freedom.

Acknowledgments — We thank Jan M. Pawlowski for discussions. LF is supported by the European Research Council under the Advanced Investigator Grant ERC-AD-267258 and Science Foundation Ireland grant 11-RFP.1-PHY3193. AM is supported by the DFG under grant number MA 3935/5-1 and MA 3935/8-1 (Heisenberg program) and thanks the NUI Maynooth for hospitality. This work has been supported by Agence Nationale de la Recherche project 11-BS04-015-01. The ROOT framework [41] has been used in this project.

Appendix A: Simulation setup

The results in this work have been obtained as a generalization of [27, 34, 35]. We use a standard Wilson action for $SU(2)$ Yang–Mills theory. The configurations were generated using hybrid-over-relaxation (HOR) updates by alternating five over-relaxation with one heat-bath sweeps, the latter ones being done via a mixed Creutz and Kennedy-Pendleton algorithm. Before the first gauge-fixing $N_{\text{init}}$ HOR updates were discarded for thermalization, with $N_{\text{init}} = 2(10 N_x + 300)$, and between two consecutive measurements $N_{\text{init}}/10$ HOR updates were performed. Gauge fixing to the minimal Landau gauge has been performed with an adaptive stochastic over-relaxation algorithm using a global stopping criterion [34]. We use asymmetric lattices $N_t \times N_x$ and vary $\beta$ to scan the temperature around the phase transition. To map to physical temperatures we obtain the string tension as a function of $\beta$ by interpolating the results of [42], and setting the string tension to $(440 \text{ MeV})^2$, to be compatible with previous works, especially [39]. The particular lattice settings employed are listed in table I.

Appendix B: Volume artifacts

Simulations were done on two different lattices with spatial volumes of $N_x^3 = 20^3, 30^3$, see table I but fixed lattice spacing. Thus, we can only study the volume dependence of our results, but not yet the discretization dependence. The results for selected temperatures for the two volumes for the ghost-gluon vertex, shown in FIG. 7, shows essentially no dependence on the volume within the statistical error. The same applies to all other temperatures. Thus, within our systematic reach, the ghost-gluon vertex appears volume-independent.

In case for the three-gluon vertex, shown in FIG. 8, we do see some volume-dependence. However, increasing the volume further amplifies the enhancement observed.
FIG. 4: Various kinematic configurations of the three-gluon vertex at temperatures below criticality. The lower right graph illustrates the vertex at the highest temperature in the figure as a function of two independent momenta. Only data with relative error smaller than 1 are shown.

in the main text, especially at the point of maximum enhancement. Also, at high temperatures, the suppression becomes stronger with increasing volume. Thus, the volume-dependence favors the behavior described in the main text. However, here a word of caution is mandatory: For the gluon propagator at zero temperature, the volume-dependence on volumes much larger than the present ones is actually qualitatively different than in the case of the present volumes (see [12] for a review). Hence, it cannot be excluded that also for the vertex a qualitative change may occur at much larger volumes. Given the amount of statistical fluctuations, this will require substantially more computing power than employed in the present calculation, and is therefore left until such resources become available.

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FIG. 5: Three-gluon vertex as in FIG. 3, however, for temperatures above criticality.

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Three-gluon vertex, one momentum vanishing

\[ G'(p,p,0) \]

Three-gluon vertex, all momenta equal

\[ G'(p,p,p) \]

FIG. 6: The three-gluon vertex at the point of maximum enhancement at 0.96\( T_c \) and at 1.55\( T_c \), compared to the results from zero and infinite temperature from [33]. Only data with relative error smaller than 1 are shown.

TABLE I: The lattice setups employed. The temperature is given in units of the critical temperature \( T/T_c \) with \( T_c \approx 277 \text{ MeV} \), the lattice coupling \( \beta \) and the number of lattice sites \( N_t \) \( N_x \) in temporal (spatial) direction. The average of the three-gluon (ghost-gluon) vertex is taken over \( N_{\text{conf}} \) \( N_{\text{conf}} \) configurations, which were generated in \( N_{\text{runs}} \) \( N_{\text{runs}} \) individual runs.

| \( T/T_c \) | \( \beta \) | \( N_t \) | \( N_x \) | \( N_{\text{conf}} \) \( (N_{\text{runs}}) \) | \( N_{\text{conf}} \) \( (N_{\text{runs}}) \) | \( T/T_c \) | \( \beta \) | \( N_t \) | \( N_x \) | \( N_{\text{conf}} \) \( (N_{\text{runs}}) \) | \( N_{\text{conf}} \) \( (N_{\text{runs}}) \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.52 | 2.200 | 6 | 20 | 718 (120) | 228 (40) | 0.52 | 2.200 | 6 | 30 | 1176 (290) | 164 (40) |
| 0.67 | 2.299 | 6 | 20 | 744 (120) | 252 (40) | 0.67 | 2.299 | 6 | 30 | 1152 (290) | 156 (40) |
| 0.90 | 2.260 | 4 | 20 | 747 (120) | 244 (40) | 0.90 | 2.260 | 4 | 30 | 1143 (284) | 154 (40) |
| 0.92 | 2.268 | 4 | 20 | 718 (120) | 235 (40) | 0.92 | 2.268 | 4 | 30 | 1141 (289) | 180 (40) |
| 0.94 | 2.276 | 4 | 20 | 731 (120) | 226 (40) | 0.94 | 2.276 | 4 | 30 | 1151 (290) | 171 (40) |
| 0.96 | 2.284 | 4 | 20 | 735 (120) | 240 (40) | 0.96 | 2.284 | 4 | 30 | 1140 (285) | 163 (40) |
| 0.98 | 2.292 | 4 | 20 | 707 (120) | 244 (40) | 0.98 | 2.292 | 4 | 30 | 1142 (286) | 165 (40) |
| 1.00 | 2.299 | 4 | 20 | 718 (120) | 248 (40) | 1.00 | 2.299 | 4 | 30 | 1140 (258) | 160 (40) |
| 1.02 | 2.306 | 4 | 20 | 708 (120) | 217 (40) | 1.02 | 2.306 | 4 | 30 | 1585 (212) | 246 (40) |
| 1.04 | 2.312 | 4 | 20 | 998 (120) | 345 (40) | 1.04 | 2.312 | 4 | 30 | 1952 (244) | 320 (40) |
| 1.06 | 2.319 | 4 | 20 | 1283 (120) | 456 (40) | 1.06 | 2.319 | 4 | 30 | 1952 (244) | 320 (40) |
| 1.08 | 2.326 | 4 | 20 | 1440 (120) | 480 (40) | 1.08 | 2.326 | 4 | 30 | 1952 (244) | 320 (40) |
| 1.10 | 2.332 | 4 | 20 | 1440 (120) | 480 (40) | 1.10 | 2.332 | 4 | 30 | 1952 (244) | 320 (40) |
| 1.55 | 2.200 | 2 | 20 | 1440 (120) | 480 (40) | 1.55 | 2.200 | 2 | 30 | 2200 (275) | 320 (40) |
| 2.00 | 2.299 | 2 | 20 | 1440 (120) | 480 (40) | 2.00 | 2.299 | 2 | 30 | 2216 (277) | 320 (40) |

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FIG. 7: Volume dependence of ghost-gluon vertex. Filled (empty) symbols denote the spatial lattice of size $N_s^3 = 30^3$ ($20^3$).

FIG. 8: Volume dependence of three-gluon vertex as in FIG. 7. Only data with relative error smaller than 1 are shown.

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