Gravitational waves in braneworlds after multi-messenger events

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The GW170817 event opened a new window to test modifications to General Relativity with the aim of discard or impose strong constraints in extra dimension theories of gravity. In that regards, the Randall-Sundrum brane-world theory, a 4+1 spacetime model where its covariant Einstein field equations are composed with new extra terms that comes from extra dimensions, are begin tested in the multi-messenger astronomy field. In this vein, the aim of this paper is to impose new constrictions through the measurements of the time-delay between the gravitational and the electromagnetic signals: GW170817 and the recent S190425z, over the free cosmological parameters of this brane model and assuming that gravitational waves travelled along a shortcut in the extra dimension. In addition, we consider the standard ΛCDM model and perform a likelihood analysis in order to study effects like the $H_0$ tension, obtaining that it seems to be reduced at least $0.2\%$ in the low energy limit of the theory.

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1. INTRODUCTION

Currently, the GW170817 event \cite{1} and the recent S190425z detection \cite{2}, related with the multi-messenger gravitational waves (GW) astronomy and subsequently electromagnetic counterparts \cite{3}, have been useful to set constraints on viable gravitational theories. This is an interesting way to explore gravity with potential new applications to test the astrophysics of binary neutron stars merger \cite{4}, cosmic expansion \cite{5} and even modifications to General Relativity (GR) (see for instance \cite{6–10}) and its consequences.

In particular, theories that modify gravity have suffered a severe setback, due the constraint imposed by these astrophysical events. Being some of them like scalar-tensor theories and some dark energy (DE) models that now predict a different velocity of propagation for GW (see \cite{6–13} for details). Hence, several proposals have been focused in study the viability of models like Horndeski \cite{14}, beyond Horndeski \cite{15}, degenerated higher-order-scalar-tensor \cite{16, 17}, among others, discarding some of them or establishing strong constraints in their free parameters.

Observations from the gravitational wave that came from the binary neutron stars merger, which was detected by the LIGO-VIRGO collaboration, to the one followed-up by a short gamma ray burst (GRB170817A), seeing just $1.75 \pm 0.05$ s later by Fermi and the International Gamma-Ray Astrophysical Laboratory \cite{3}, have been used lately in order to understand the constraints over the cosmological parameters in a theory. Thanks to these observations now we can set a physical range of $0.86 - 2.26\,M_\odot$ to weight the neutron star, the measure of the luminosity distance $d_L = 40^{+31}_{-14}\,\text{Mpc}$, resulting from the bound $-3 \times 10^{-15} \leq c_g/c - 1 \leq 7 \times 10^{-16}$, which was found under the lowest limit for $d_L$ and a corrective 10 seconds delay between the gravitational wave and the gamma ray burst. Following the recipe of \cite{10}, we will consider the cutoff $|c_g - 1| \leq 5 \times 10^{-16}$, where $c = 1$ is in natural units. Moreover, the most recent detection of GW is given by the event so-called S190425z registered by LIGO-VIRGO detectors \cite{2}. This event is a high confidence (> 99\%) neutron stars merger with an electromagnetic counterpart caused by a weak gamma ray burst (GRB190425), being the only second event known in addition to GW170817. Both events, provide us with higher constrictions for GR or even for extensions or modifications to this fundamental theory.

From the theoretical point of view, brane world theories is an alternatively form to understand the nature of gravity and a feasible direction to understand puzzles like the dark matter and dark energy problems. In this vein,
the Randall-Sundrum models (RS) and its extensions are of great interest in the community [18, 19]. The idea of RS branes is focused in the proposition that the universe is conformed by a 4+1 dimensional space-time called the bulk, which is usually assumed anti-deSitter (AdS) and where it is immersed a four dimensional manifold called the brane with a metric that depends of what we are studying. The existence of extra dimensions in this framework and in particular, the modifications produced by braneworlds, can be constrained under the free parameter $\lambda$, which is known as brane tension. This latter being the parameter that relates the four dimensional structure with the five dimensional bulk. Several proposals have constrained the brane tension parameter as $\lambda \simeq 6.42h^2 \times 10^6 eV^4$ using a Joint analysis [20] of cosmological observations or $\lambda \simeq 138.59 \times 10^6 eV^4$ through Table-Top experiments [21] at astrophysical level. The results derived from them are considerable promising, constraining the brane tension of the order $\lambda \simeq 10^{32} eV^4$ [22–24]. Natural extensions to brane world theories are through extra degree of freedoms like the brane tension, which can depend of the time [21, 25, 26] and showing a better concordance with cosmological constraints (see e.g. [27]). Some attempts have been done in order to set a cutoff on the screening scale using gravitational and electromagnetic waves, e.g. for theories beyond 4 dimensions [28], where a 20 Mpc scale needs to be consider. Also, the addition of EM counterparts as the GRB170817A [29] was used in [6] to determine an upper bound of the radius of curvature Anti-deSitter (AdS$_5$).

Therefore, in this tenor, it is reasonable to propose new constraints in order to validate or refute theories based in the RS hypothesis. Hence, GW provides important constraints for RS theories and specifically after GW170817 and S190425z for the propagation velocity of GW in vacuum. Is with this idea, that we propose extra constrictions through GW both at the low and high field level as a complement of the previous studies presented in the literature.

The aim of this paper is to constraint RS branes using the covariant form of the modified Einstein field equations [30] in the weak field limit. In this scenario, the vacuum solution for the wave equation is not the traditional due that extra terms that come from Weyl terms will play an important role in the propagation and speed of GW. Previous research have been focused in weak field limit, studying the consequences of the quadratic part of the energy-momentum tensor in the dynamics of binary system (see for instance [31]) and also in extended theories of gravity [32, 33]. In order to study the low (Minkowski) and high (De Sitter) energy limits of the above landscape, we obtain the exact solutions for both cases and we adopt a Bayesian approach to combine the two N-S detections. We set the free cosmological parameters $\theta$ taking into account the $\Lambda$CDM as pivot model. Via the Bayes theorem, the probability is $p \propto (\theta, \kappa, d_L)$, where the former if the joint likelihood and the latter if the cosmological prior. As was pointed out in [6], we can consider an upper limit of $\ell < 0.535$Mpc in order to study the $H_0$ tension at low and high energy limits in this model.

This outline of the paper is as follows: in Sec. 2 we revisit the covariant form of the modified Einstein equations, identifying the source and geometrical terms in the field equations in vacuum. We study these equations in the weak field limit, showing that the canonical wave equation is now accompanied with the non-local term that comes from branes. Our results are compatible with the expected constrictions about the velocity propagation of gravitons and it is constrained with other observations. In Sec. 3 we discuss the brane world cosmic perturbations, focusing our attention in the Minkowski and De Sitter brane case. Moreover, Sec. 4 is dedicated to develop cosmological constraints with the braneworld GW propagation through the events GW170817 and S190425z. Finally, discussions and conclusions are presented in Sec. 5. Throughout the text, we will use natural units, unless we indicate otherwise.

2. BRANEWORLD GRAVITATIONAL WAVES IN WEAK FIELD LIMIT

The field equations without the presence of matter fields in the bulk can be written as $G_{\mu\nu} + \xi_{\mu\nu} = \kappa^2(\lambda) S_{\mu\nu}$, where $S_{\mu\nu} = T_{\mu\nu} + 6\Pi_{\mu\nu}/\lambda$. Here the l.h.s of the field equation contains the geometric part with $G_{\mu\nu}$ as the Einstein tensor and $\xi_{\mu\nu}$ related with the non-local Weyl tensor that comes from branes contributions. Hence, the r.h.s of this field equation contains the source of matter and energy with $T_{\mu\nu}$ as the energy-momentum tensor and $\Pi_{\mu\nu}$ the quadratic part of the energy-momentum tensor. In addition, $\kappa^2(\lambda) \equiv 8\pi G$ is the four dimensional coupling constant, $G$ is the gravitational Newton constant and $\lambda$ is the brane tension, which is the threshold between the traditional Einstein field equations and the corrections caused by the presence of branes. Notice that the brane tension will help to quantify the presence of extra dimensions in the various phenomena studied. We defined

$$\xi_{\mu\nu} \equiv C_{AEFB} n^F n^B g^A g^F,$$

$$\Pi_{\mu\nu} \equiv -\frac{1}{4} T_{\mu\alpha} T^{\alpha \nu} + \frac{1}{12} T_{\alpha \beta} T^{\alpha \beta} + \frac{1}{24} g_{\mu\nu}(3T_{\alpha \beta} T^{\alpha \beta} - T^2),$$
where $C_{AEBF}$ is the five dimensional Weyl tensor, $n^E$ is a unitary normal vector and $T \equiv T^{\alpha}_\alpha$. The symmetry of $\xi_{\mu\nu}$ imply that we can decompose it irreducibly as

$$\xi_{\mu\nu} = -\frac{6}{\kappa^{(4)}_6} \left[ U \left( u_{\mu} u_{\nu} + \frac{1}{3} \epsilon_{\mu\nu} \right) + P_{\mu\nu} \right],$$

where $U$ is the non-local energy density and $P_{\mu\nu}$ is the non-local anisotropic stress tensor, $u_{\mu}$ is the four-velocity, which also satisfy $g_{\mu\nu} u^\mu u^\nu = -1$, and $\epsilon_{\mu\nu} = g_{\mu\nu} + u_{\mu} u_{\nu}$ is orthogonal to $u_{\mu}$. Notice that

$$U \equiv -\frac{\kappa^{(4)}_6}{6} \xi_{\mu\nu} u^\mu u^\nu, \quad P_{\mu\nu} \equiv -\frac{\kappa^{(4)}_6}{6} \xi_{(\mu\nu)}.$$

We start the analysis in the weak-field limit imposing $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski tensor, $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$ and $|h_{\mu\nu}| \ll 1$ is a perturbation. Therefore in Einstein gauge (harmonic coordinates)\footnote{We set the Einstein gauge in the form $\tilde{h}_{\mu\nu}^\mu = h_{\mu\nu}^\mu - \frac{1}{2} h_{\nu} = 0$ where $h_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h$.} the field equations are reduced to

$$\Box h_{\mu\nu} = \frac{2}{\kappa^{(4)}_6} \left[ U (4u_{\mu} u_{\nu} + \eta_{\mu\nu}) + 3 P_{\mu\nu} \right] = \kappa^{(4)}_6 S^{(0)}_{\mu\nu},$$

therefore in the weak field limit the non-local term can be reduced to

$$\xi^{(0)}_{\mu\nu} = \left[ 2h_{\alpha\beta,\gamma} - \eta_{\alpha[\gamma} \Box h_{\delta]\beta] + \eta_{\beta[\gamma} \Box h_{\delta]\alpha} \right] n^\alpha n^\beta \eta^{\gamma\nu} + \mathcal{O}(5D),$$

where the d’Alambertian operator is $\Box \equiv \eta^{\mu\nu} \partial_{\mu} \partial_{\nu}$. In this case, we only consider the four-dimensional part of the Weyl tensor, i.e. $C_{AEBF} \rightarrow C_{\mu\nu\gamma\delta}$, under the assumption that it is the only part that can be detected by the LIGO and VIRGO interferometers. We also have that $S_{\mu\nu}$ and $\xi_{\mu\nu}$ are taken to lowest order in $h_{\mu\nu}$, denoted with the superscript $(0)$ and $\Box h = 0$ due that the Weyl tensor is traceless. The $S_{\mu\nu}$ tensor at lowest order will only contain the contributions to the energy-momentum tensor because it will have negligible contributions caused by $\Pi_{\mu\nu}$ since it contains contributions at higher order in the energy-momentum tensor as it is shown in Eq. (2), therefore $S^{(0)}_{\mu\nu} \rightarrow M^{(0)}_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\Lambda_\Lambda$.

In vacuum, the field equations do not depend of the energy-momentum tensor (and quadratic part) source i.e. $M^{(0)}_{\mu\nu} = 0$. However it depends of the non-local geometric contribution encoded in $\xi^{(0)}_{\mu\nu}$. It is notorious how the propagation of the GW depends of the presence of the r.h.s term shown in Eq. (7) in the form

$$\Box h_{\mu\nu} = \frac{2}{\kappa^{(4)}_6} \left[ U (4u_{\mu} u_{\nu} + \eta_{\mu\nu}) + 3 P_{\mu\nu} \right] = -\xi^{(0)}_{\mu\nu}.$$

Another form to express the previous equation is as follows: rearranging (7), with the use of Eq. (6) we have

$$\Box h_{\mu\nu} - (\eta_{\alpha[\gamma} \Box h_{\delta]\beta] - \eta_{\beta[\gamma} \Box h_{\delta]\alpha} - 2h_{\alpha\beta,\gamma}) n^\alpha n^\beta \eta^{\gamma\nu} + \mathcal{O}(5D) = 0.$$

In this case, the l.h.s is richer than the traditional GR, containing extra information provided by the Weyl tensor. Each term in Eq. (8) only has a geometric nature, remarking only the 4D characteristics, while 5D contributions are hidden from observations. Information about the propagation velocity is encoded in Eq. (6), being able to be constrained by the N-S events.

In this line of thought, and in order to extract information of Eq.(8), we adopt the ansatz $h_{\mu\nu} = h_{\mu\nu}(t - x)$.

Following traditional literature, it is possible to infer the only two non-negligeable functions as: $h_{22} = h_{22}(t - x)$ and $h_{23} = h_{23}(t - x)$, related with the polarisation states, notice that $h_{\mu\nu}(t - x)$ is equal to zero. Therefore, Eq. (8) has an extra terms only when $\mu = \delta = 2$ and $\nu = \beta = 3$, resulting in the expression (the other cases can be seen in Appendix A)

$$(\partial^2_x - \partial^2_x)(h_{23} + h_{22} n^3 n^2) + \mathcal{O}(5D) = 0.$$

If the normal unitary vectors are orthogonal to each other (as we expect) we recover the form $(\partial^2_x - \partial^2_x)h_{22} + \mathcal{O}(5D) = 0$. The other case is $(\partial^2_x - \partial^2_x)h_{22} + \mathcal{O}(5D) = 0$. Hence we obtain the traditional wave equation for GW propagating at
light velocity and cannot be ruled out by the N-S events. Additional information comes from 5D contributions and also has dependence of the coupling constant $\lambda$, which can be constrained. We notice that this parameter has not the capability to be detected by the LIGO antenna.

The l.h.s has the d’Alambertian operator associated to the perturbation and the r.h.s represent the double derivative of the perturbation $h_{\alpha\beta}$ which is associated with the Riemann tensor. From here, it is possible to observe that the propagation velocity must be equal to the light velocity in similarity with GR and of course, coinciding for e.g. with the GW170817 event.

As a complement, notice that the solution of Eq. (7), must be the retarded potential written as follows

$$h_{\mu\nu}(x, t) = \frac{1}{2\pi\kappa_4^2(4\lambda)} \int [4\mathcal{U}u_{\mu}u_{\nu} + 3\mathcal{P}_{\mu\nu} \times \left( \frac{(x', t - |x - x'|)}{|x - x'|} \right)] d^3 x',$$  \hspace{1cm} (10)

or in general with the presence of a energy momentum tensor

$$h_{\mu\nu}(x, t) = \frac{\kappa_4^2(4\lambda)}{2\pi} \int \frac{M_{\mu\nu}(x', t - |x - x'|)}{|x - x'|} d^3 x' - \frac{1}{2\pi\kappa_4^2(4\lambda)} \int [4\mathcal{U}u_{\mu}u_{\nu} + 3\mathcal{P}_{\mu\nu} \times \left( \frac{(x', t - |x - x'|)}{|x - x'|} \right)] d^3 x'. \hspace{1cm} (11)$$

This final result represent the gravitational waves in this framework that can be not only produced by the presence of the energy momentum tensor but also by the non local effects represented by the Weyl tensor in Eq. (10) or from Eq. (8) we write

$$h_{\mu\nu}(x, t) = \frac{1}{2\pi} \int \left[ h_{\alpha\delta}(x, t),_{\beta\gamma} - \frac{1}{4} (\eta_{\alpha\gamma}^\mu \eta_{\beta\delta}(x, t) - \eta_{\beta\gamma}^\mu \eta_{\alpha\delta}(x, t)) \right] \frac{n^\delta n^\mu n^\alpha g_{\beta\gamma}'(x', t - |x - x'|)}{|x - x'|} d^3 x' + O(5D), \hspace{1cm} (12)$$

which makes it an integro-differential equation. Therefore, in vacuum and under the Bianchi identities on the brane imply $\xi_{\mu,\nu} = 0$, and the Einstein gauge is fulfilled $\xi_{\mu,\nu} - \xi_{\nu,\mu}/2 = 0$ for a traceless Weyl tensor.

Notice that extra term in Eq. (11) is constrained for example by Astrophysics \cite{22,24} or Table Top experiments \cite{34}, giving a brane tension of the order $\lambda = 5 \times 10^{32} eV^4$ and $\lambda = 138.59 \times 10^{48} eV^4$ respectively resulting in correction of the order $\sim 2.88 \times 10^{23} eV^{-2}$ and $\sim 1.03 \times 10^{66} eV^{-2}$ respectively for the extra dimensions contributions.

### 3. BRANEWORLD COSMIC TENSOR PERTURBATIONS

Let us consider bulk Einstein equations $R_{\mu\nu} = 0$ (with $\Lambda = 0$). By Gauss relation we obtain the Ricci tensor

$$\bar{R}_{\mu\nu} = \bar{K}\bar{K}_{\mu\nu} - \bar{K}_\mu^\rho \bar{K}_{\nu\rho} \bar{\xi}_{\mu\nu}, \hspace{1cm} (13)$$

where $K_{\mu\nu} = -\bar{g}_\mu^\rho \nabla_\rho n_\nu$ is the extrinsic curvature tensor of the brane. $\xi \equiv C_{\mu\nu\rho\sigma} n^\mu n^\sigma$ is the contribution from Weyl where the bar denote curvature tensors pertaining to the brane metric $\bar{g}_{\alpha\beta}$. Using the junction conditions \cite{35} we obtain for the extrinsic curvature

$$K_{\alpha\beta} = \frac{4\pi G}{3} \left[ 3T_{\alpha\beta} + (T - \lambda)\bar{g}_{\alpha\beta} \right]. \hspace{1cm} (14)$$

Combining with the bulk Einstein equations we obtain the Lie derivatives in the normal direction to the brane, which we need to linearize

$$D_n K_{\alpha\beta} = \bar{\xi}_{\alpha\beta} - K_{\alpha\beta} \bar{K}_\beta, \hspace{1cm} (15a)$$

$$D_n \bar{g}_{\alpha\beta} = -2 K_{\alpha\beta}, \hspace{1cm} (15b)$$

For the background metric

$$ds^2 = -n^2(\tau, \xi) dx^2 + a^2(\tau, \xi) \delta_{ij} dx^i dx^j + d\eta^2, \hspace{1cm} (16)$$

where $n$ and $a$ are functions to be found. Therefore the perturbed metric for AdS bulk is written in the form

$$ds^2 = \left( \frac{\ell}{s} \right)^2 (\eta_{\alpha\beta} + h_{\alpha\beta}) dx^\alpha dx^\beta, \hspace{1cm} (17)$$
where $z = \ell/a$ set a new locus of the brane. In addition, we choose the transverse-traceless TT gauge with $\eta^{\alpha\beta}h_{\alpha\beta} = 0$ and $\partial_n h^{\alpha\beta} = 0$. We require that the perturbed and background metric have the form
\begin{equation}
 ds^2 = -n^2(1 + \phi)dt^2 + 2anb dt dx^i + a^2(\delta_{ij} + h_{ij}) dx^i dx^j + d\eta^2,
\end{equation}
for the energy momentum tensor background we have $T_{\beta}^\alpha = \text{diag}(-\rho, p\delta_{ij})$ and for the perturbation
\begin{equation}
 \delta T_{\beta}^\alpha = \left( \begin{array}{c}
 -\delta \rho \\
 -a^{-1}n(\rho + p)(v_j + b_j)
 \end{array} \right),
\end{equation}
The variation of the spatial part is $\delta \bar{R}_{ij} = \delta R_{ij}^0 + \delta \bar{R}_{ij}$ and for the tensor part
\begin{equation}
 \delta \bar{R}_{ij} = \frac{a^2}{2n^2}h_{ij} + \frac{a^2}{2n^2} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) h_{ij} + \frac{a^2}{n^2} \left( \frac{\ddot{a}}{a} - \frac{\ddot{n}}{n} + \frac{2\dot{a}^2}{a^2} \right) h_{ij} + \frac{k^2}{2} h_{ij},
\end{equation}
where the dots represents derivatives with respect to the $\tau$ parameter. The Lie derivatives are written as:
\begin{align}
 K_{ij} & = \frac{1}{2} \frac{d}{dx} (a^2 h_{ij}), \\
 \delta K_{ij} & = \frac{a'}{a} \frac{d}{d\eta} (a^2 h_{ij}) - a'^2 h_{ij} - \frac{1}{2} \frac{d^2}{d\eta^2} (a^2 h_{ij}),
\end{align}
where the primes represents derivatives with respect to the $\eta$ parameter. We rewrite the perturbation of the Ricci tensor in the form:
\begin{equation}
 \delta \bar{R}_{ij} = 2a^2 h_{ij} + 1 \left( \frac{n'}{n} - \frac{a'}{a} \right) \frac{d}{d\eta} (a^2 h_{ij}) + \frac{1}{2} \frac{d^2}{d\eta^2} (a^2 h_{ij}).
\end{equation}
The spatial part of $R_{\mu\nu}$ from the background metric is
\begin{equation}
 \frac{\ddot{a}}{an^2} + 2 \frac{\dot{a}^2}{a^2 n^2} - \frac{\dot{a} \dot{n}}{an^3} - \frac{a''}{a} - \frac{a' n'}{an} - 2 \frac{a'^2}{a^2} = 0.
\end{equation}
From (20) and (22) and using the latter we obtain the RS graviton equation
\begin{equation}
 \ddot{h}_{ij} + \left( 3 \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) \dot{h}_{ij} + k^2 n^2 h_{ij} - n^2 \left( \frac{3 a'}{a} + \frac{n'}{n} \right) \dot{h}_{ij} - n^2 h^n_{ij} = 0,
\end{equation}
where $h_{ij} = h^x e^x_{ij} + h^x e^x_{ij}$ with two polarisation tensors $e^x_{ij}$.$^+$. From here we can related the sound speed of the graviton $c_s^2 = n^2$. The first GW detection impose $c_s^2 \approx 1$ which implies $n \approx 1$.

In contrast, we can relate the friction term from the above gravitational wave equation which can denote how the GW amplitude decreases in the propagation across cosmological distances, from the source to the observer. In particular, for inspiraling binaries this factor combines with other factors coming from the transformation of masses and frequency from the source frame to the detector frame. To produce the usual dependence of the GW amplitude we write the luminosity distance as
\begin{equation}
 \bar{h}_{ij}(\eta, \vec{k}) \propto \frac{1}{d_L(z)},
\end{equation}
this friction term would modify the speed of propagation of GWs and $\eta$ represents the conformal time related with $\tau$. We can now excluded this coefficient at the level $|c_{gw} - c|/c < O(10^{-15})$, by the observation of GW170817 and S190425z is a likely binary neutron star (BNS) merger at $d_L = 156 \pm 41$ Mpc. These kind of constrains have ruled out a large class of scalar-tensor and vector-tensor modifications of GR. To eliminate the friction term, we introduce $\bar{h}_{ij}(\eta, \vec{k})$ from
\begin{equation}
 \tilde{h}_{ij}(\eta, \vec{k}) = \frac{1}{\dot{a}(\eta)} \bar{h}_{ij}(\eta, \vec{k}),
\end{equation}
where
\begin{equation}
 \frac{\dot{a}'}{\dot{a}} = \mathcal{H} [1 - \delta(\eta)],
\end{equation}
where we perform the change to conformal time and \( \delta(\eta) \) is a function that parameterize the deviation from GR. We can integrate the latter to obtain

\[
d_{L}^{gw}(z) = d_{L}^{em}(z) \exp \left\{ -\int_{0}^{z} \frac{dz'}{1+z'} \delta(z') \right\},
\]

then, we see that we must in distinguish between the usual luminosity distance appropriate for electromagnetic signal, \( d_{L}^{em}(z) \) and a GW luminosity distance \( d_{L}^{gw}(z) \).

In order to analyse low (Minkowski) and high (De Sitter) energy regime we consider the following two exact solutions:

### 3.1. Minkowski brane case

Here the proposed solution is given by \( a(\tau, \eta) = n(\tau, \eta) = \exp(-\eta/\ell) \), where \( \ell \) is the curvature radius of AdS\(_5\) (or also known as the length scale). According to this proposal, the axis normal to the brane is constrained through GW170817 with \( \eta \approx 0 \). From \([24]\) we obtain

\[
\ddot{h} + k^2 h = e^{-2\eta/\ell} \left( h'' - \frac{4}{\ell} h' \right),
\]

and by separation of variables \( h(\tau, \eta) = A(\tau) B(\eta) \), we have the following pair of equations

\[
\begin{align*}
\dot{A} + (k^2 + m^2) A &= 0, \quad \rightarrow \quad A \propto e^{\sqrt{k^2 + m^2} \tau}, \\
B'' - 4\ell^{-1} B' + m^2 e^{2\eta/\ell} B &= 0, \quad \rightarrow \quad B \propto e^{2\eta/\ell} Z(m e^{\eta/\ell}).
\end{align*}
\]

From the numerical solution in Figure 1 we notice the GW decays faster at large conformal times.

### 3.2. De Sitter brane case

This case is started assuming the condition\(^2\) \( \rho_{\text{matter}} \approx \text{const.} \), therefore the proposed solution is given by \( a(\tau, \eta) = n(\tau, \eta) = a_0(\tau) F(\eta) \), where

\[
F(\eta) = e^{-\eta/\ell} + \frac{\rho}{2\lambda} (e^{\eta/\ell} - e^{-\eta/\ell}).
\]

\(^2\) Recent paper is posted on the pre-print repository, studying the De Sitter case [36], constraining the curvature radius as \( \ell \gtrsim 7.5 \times 10^2 \text{Tpc} \).
Again, to constraint through GW170817 event we require $F(\eta) \approx 1$, therefore $\rho \approx 2\lambda(1 + e^{\eta/\ell})^{-1}$. From Eq. (24) we obtain

$$\ddot{h} + 2\frac{\dot{a}_0}{a_0} \dot{h} + k^2 h = a_0^2 F^2 h'' + 4a_0^2 FF'h', \quad (33)$$

and by separation of variables $h(\tau, \eta) = A(\tau) B(\eta)$, it is possible to write:

$$\ddot{A} + 2\frac{\dot{a}_0}{a_0} \dot{A} + (k^2 + m^2 a_0^2) A = 0, \quad \rightarrow \quad A \propto \tau^{3/2 \pm \alpha}, \quad (34)$$

$$B'' + 4\frac{F'}{F} B' + \frac{m^2}{F^2} B = 0, \quad \rightarrow \quad B_{m=0} \propto e^{-4\eta/\ell}. \quad (35)$$

From Figure 2 we notice that this is an appropriate description away from the source where gravitational waves have been emitted in comparison to the Minkowski case. Also, the fluctuations considered provides a consistent truncation of the fluctuated RS-dimensional theory, meaning that our numerical solutions will also be linear solutions to a D-dimensional theory with all metric fluctuations.

4. USING GW170817 AND S190425Z AS STANDARD SIRENS FOR THE BRANEWORLD

Using the energy limits on GW propagation of the brane model, the next question that arise is that if we can distinguish these cases from the $\Lambda$CDM model. Therefore, to obtain an answer of the latter, we compare our exact solutions in both energy limits on GW propagation that can already be obtained by the standard sirens provided by the detection of the GWs from the neutron star binary GW170817 and from S190425z detection. Then, we select the modifications of the perturbation equations over the cosmological background. From the identification of the electromagnetic counterpart the redshift of the source can be constrained by $z = 0.01$, therefore in both cases we are at very small redshift. Since we have two concrete cases, we can specific the luminosity distances for both components and consider the Bayesian estimation for a $\Lambda$CDM model using CMB+BAO+SNeIa samplers. Other effect that should be taken into account is the Shapiro gravitational delay which affects our results from the GW. For this reason we present in Appendix B a short revision of this effect discussing the consequences in our results.

To perform the Bayesian analyses it is useful to understand in physical terms who to compare these cases with the standard $\Lambda$CDM model. To this goal, we start from a generic $w$CDM model, with a fixed value of $w_0$, and set the measurements of the luminosity distances with standard sirens that could help in discriminating it from $\Lambda$CDM. Let us consider

$$d_L(z; H_0, \Omega_m, w_0) = \frac{1 + z}{H_0} \times \int_0^z \frac{dz}{\sqrt{\Omega_m(1 + z)^3 + (1 - \Omega_m)(1 + z)^3(1 + w_0)}}, \quad (36)$$

where we have written explicitly the dependence of $d_L$ on the cosmological parameters and we consider them with the samplers mentioned: $H_0 = 67.64$ and $\Omega_m = 0.309$ as fiducial values for $\Lambda$CDM. After this, we can compute

$$\frac{\Delta d_L}{d_L} = \frac{d_L(z; H_0, \Omega_m, w_0) - d_L^{\text{CDM}}(z; H_0, \Omega_m)}{d_L^{\text{CDM}}(z; H_0, \Omega_m)}. \quad (37)$$
This, for example, can represent the difference between the luminosity distance in $w$CDM with a given value of $w_0$, and the luminosity distance in $\Lambda$CDM (where $w_0 = -1$), at fixed $\Omega_m, H_0$. Similarly, we can compute this quantity for each brane limit model by considering $d_L$ in (37) of the pivot model now as $d_L^{gw}$ as in (28). We construct the 2D joint contours plots of the cosmological parameters in comparison to $\Lambda$CDM model with both GW detections with a Markov Chain Monte Carlo (MCMC) simulation. For our exact solutions, we have the posteriors respectively showed in Figures 3 and 4. We notice that the $H_0$ tension seems to be relaxed at least $0.2\%$ in the Minkowski case in comparison to the De Sitter case. The region contour plot, $\Omega_m - \Omega_\Lambda$ is the primary constraint on dark energy. As a geometry probe the GW detections drive most of the statistical information and the constraint is much better in the De Sitter case. Interesting enough these results can drive an even more drop of the tension if we consider future GRB and WL detections.

5. DISCUSSION AND CONCLUSIONS

In this paper we study a brane world theory based in RS model in order to analyse if the theory is consistent with the constraints obtained from GW170817 and S190425z events. Through the weak field limit for braneworlds, our results shown that the propagation of the GW in this scenario is the light velocity in concordance with GR theory. However, extra components that comes from five dimensional physics are present in Eqs. (10)-(11), where the terms that make up the Weyl tensor plays an important role. From Eq. (11) we notice that the brane tension is an essential ingredient and represents the difference with the standard knowledge of a gravitational wave in GR, indeed, using previous constraints we found the most restrictive as $\sim 1.03 \times 10^6 \text{eV}^{-2}$ for Table Top experiments that set a cutoff over the value of $\lambda$. Even more, we showed the cosmic tensor perturbations in this scenario, dividing our studies in Minkowski (low energies) and De Sitter (higher energies) cases, presenting the evolution of the GW through the conformal time. The Minkowski case presents a behaviour that mimic a damped oscillator while De Sitter case presents a subtle increase of oscillations due to the topology involved in this case. As a final study, we perform statistical studies from standard sirens definitions and obtaining a relaxation of the tension presented in the values of $H_0$ at least $0.2\%$ in the Minkowski case in comparison with the De Sitter case. This result is an incentive to keep exploring theories that involves extra dimensions and wait for more data in order to increase the statistics involved.
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Appendix A: Other cases in the weak field limit

In order to complement the results for the gravitational wave equation, we present the other cases for the subscripts $\mu$, $\nu$, $\beta$ and $\delta$. Hence, for $\mu, \nu = 2, 2$ we have

\[ \Box h_{22} - (\eta_{22} \Box h_{22} - \eta_{22} \Box h_{22} - 2h_{22,22}) n^\beta n^\delta + O(5D) = 0, \quad (A1) \]

\[ \Box h_{22} - \frac{1}{2} (\eta_{22} \Box h_{22} - \eta_{22} \Box h_{22} - \eta_{22} \Box h_{22} - 2h_{22,22}) n^\beta n^\delta + O(5D) = 0. \quad (A2) \]

From here we can explore several cases depending the values of $\beta$, $\delta$ and $\mu$:

- When $\beta = \delta = 2$

  \[ \Box h_{22} - \frac{1}{2} (\eta_{22} \Box h_{22} - \eta_{22} \Box h_{22} - \eta_{22} \Box h_{22} - 2h_{22,22}) n^2 n^2 + O(5D) = 0, \quad (A3) \]
  \[ \Box h_{22} + O(5D) = 0. \quad (A4) \]

- When $\beta = 2, \delta = 3$

  \[ \Box h_{22} - \frac{1}{2} (\eta_{22} \Box h_{32} - \eta_{22} \Box h_{32} - \eta_{22} \Box h_{32} + \eta_{23} \Box h_{22} - 2h_{23,22}) n^2 n^3 + O(5D) = 0, \quad (A5) \]
  \[ \Box h_{22} + O(5D) = 0. \quad (A6) \]
When $\beta = 3$, $\delta = 2$ with $\mu = 2$ and $\nu = 3$

$$\Box h_{22} - \frac{1}{2} (\eta_{22} \Box h_{23} - \eta_{22} \Box h_{23} - \eta_{32} \Box h_{22} + \eta_{23} \Box h_{22} - 2 h_{22,32}) n^3 n^2 + O(5D) = 0,$$

(A7)

$$\Box h_{23} + \left[ \frac{1}{2} (\eta_{\beta \delta} \Box h_{\delta 2} - \eta_{\beta \delta} \Box h_{32}) + 2 h_{28,\beta 3} \right] n^\beta n^\delta + O(5D) = 0.$$  

(A8)

When $\beta = 2 = \delta$

$$\Box h_{23} + \left[ \frac{1}{2} (\eta_{23} \Box h_{22} - \eta_{22} \Box h_{32} + 2 h_{22,23}) \right] n^2 n^2 + O(5D) = 0,$$

(A9)

$$\Box h_{23} - \frac{1}{2} \Box h_{32} n^2 n^2 + O(5D) = 0.$$  

(A10)

When $\beta = 2 \ y = 3$

$$\Box h_{23} + \left[ \frac{1}{2} (\eta_{33} \Box h_{22} - \eta_{22} \Box h_{32} + 2 h_{28,23}) \right] n^3 n^\delta + O(5D) = 0.$$  

(A11)

When $\beta = 3 \ y = 2$

$$\Box h_{23} + \frac{1}{2} \Box h_{22} n^3 n^2 + O(5D) = 0.$$  

(A12)

Appendix B: Shapiro delay new constrains

To extend the physics behind the constrains found, we extend the description of the the Shapiro delay in our scenario. First, we write the proper time on the bulk for a null geodesic ($ds^2 = 0$) in the form [5]

$$d\tau^2 = V(r)^{-2} dr^2 + V(r)^{-1} r^2 d\Omega^2 + V(r)^{-1} f(R)^{-2} dR^2,$$

(B1)

where $V(r) = 1 - 2M/r$, being $M$ a near point mass in Schwarzschild coordinates, $f(R) = (R/\ell)^2$ is related with the $\text{AdS}_5$ scale factor and $\ell$ is the curvature radius of the $\text{AdS}_5$. The first two terms corresponds to the standard Shapiro delay well known in four-dimensional physics, while the last terms is the correction provided by five dimensional physics. Notice, how the curvature radius is one of the main parameters that give us information about the presence of corrections by extra dimensions. Is in this sense that [6] generate a constraint of the order $\ell < 0.535$ Mpc, being not competitive in comparison with the constraint produced by black holes X-ray binaries as $\ell \lesssim 10^{-2}$mm [37], stressing that corrections produced in this scenario in the Shapiro effect can be considered as negligible.

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