WIMP dark matter in gauge-mediated SUSY breaking models and its phenomenology

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Abstract

We propose an extended version of the gauge-mediated SUSY breaking models where extra $SU(2)_L$ doublets and singlet field are introduced. These fields are assumed to be parity-odd under an additional matter parity. In this model, the lightest parity-odd particle among them would be dark matter in the Universe. In this paper, we discuss direct detection of the dark matter and the collider signatures of the model.
1 Introduction

Supersymmetry (SUSY) is one of the best-motivated physics beyond the standard model (SM) since it naturally solves the gauge-hierarchy problem. SUSY must be spontaneously broken, however, in order for SUSY particles to obtain sizable masses. Among proposed SUSY breaking models, the gauge-mediated SUSY breaking (GMSB) models [1] are interesting, since the SUSY flavor problem does not arise in GMSB models [2].

Although low-energy phenomenology of GMSB models seems to be quite successful, it has non-trivial aspects from a cosmological point of view. In GMSB models the lightest SUSY particle (LSP) is the gravitino, superpartner of the graviton. The relic abundance of the gravitino in the Universe depends on the reheating temperature after inflation, and a stringent upper bound on the reheating temperature is obtained so that the gravitino does not exceed the present dark matter (DM) abundance [3]. In particular, this implies that GMSB models are not compatible with the thermal leptogenesis scenario [4] for most range of the gravitino mass. An exception is the low-energy GMSB models where the gravitino is lighter than about 10 eV and no upper bound on the reheating temperature is imposed [3, 5, 6]. In this case, no candidate for DM exists in the minimal setup of GMSB [1].

One may consider that the QCD axion [7] can play a role of DM. However, cosmology of SUSY axion models is quite non-trivial taking into account the existence of axino, fermionic superpartner of the axion, and saxion, scalar partner of the axion. The axino is produced thermally in the early Universe with a significant amount, and hence the reheating temperature is more severely restricted [8, 9, 10]. The saxion coherent oscillation and its subsequent decay also gives catastrophic cosmological effects unless the reheating temperature is sufficiently low [11, 12]. Thus to make the axion the dominant component of DM in GMSB model requires careful considerations.

In this paper we extend the GMSB models to include a candidate for WIMP DM. The minimum extension might be to add chiral supermultiplets with fundamental representation of $SU(2)_L (H', \bar{H}')$. Those fields are parity-odd under an additional $Z_2$-parity assigned. If all other minimal SUSY standard model (MSSM) fields are even under the $Z_2$-parity, $H'$ and $\bar{H}'$ can be stable and a candidate for WIMP DM. Since they are weakly-interacting, their relic abundance falls into a correct range in the thermal history of the Universe. However, this kind of extension is already excluded since it predicts too large direct detection rates in the DM search experiments through the coherent vector coupling to nucleons by $Z$-boson exchange. In order to avoid the direct detection bounds, we further introduce a singlet chiral multiplet $S'$ which is also parity-odd. Then WIMP DM becomes a mixture of $S'$, $H'$ and $\bar{H}'$, and the lightest parity-odd particle is real scalar boson or Majorana fermion so that the vector coupling of DM-nucleon is forbidden. Sizable

\[ \text{In the low-energy GMSB scenario, a model in which a baryonic bound state of strongly interacting messenger particles becomes cold DM is proposed [13, 14]. In this case DM has a mass of $\mathcal{O}(100)$ TeV, and cannot be detected by accelerator searches and direct detection experiments.} \]

\[ \text{The strong CP problem may be solved in the low-energy GMSB models through the Nelson-Barr mechanism [15, 16]. When the SUSY breaking sector is decoupled with the spontaneous CP-violating sector, the radiative correction to the QCD-theta term is suppressed due to the non-renormalization theorem. Thus the axion is not necessarily needed in this case.} \]
Table 1: Particle contents of the model.

|                | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $Z_2$  |
|----------------|-----------|-----------|-----------|-------|
| $H^0$          | 1         | 2         | $-1/2$   | even  |
| $H^+$          | 1         | 2         | $+1/2$   | even  |
| $H'^0$         | 1         | 2         | $-1/2$   | odd   |
| $H'^+$         | 1         | 2         | $+1/2$   | odd   |
| $H'_c^0$       | 3         | 1         | $+1/3$   | odd   |
| $H'_c^+$       | 3         | 1         | $-1/3$   | odd   |
| $S'$           | 1         | 1         | 0         | odd   |

Interactions still exist through the Higgs exchange process, which is within the reach of on-going/future direct detection experiments.

This paper is organized as follows. In Sec. 2 we define our model and study the properties of the DM particle. In Sec. 3 the DM direct detection rate is evaluated and it is shown that it is within the reach of current/future direct detection experiments. We discuss the LHC signature of this setup in Sec. 4. Finally in Sec. 5 we give conclusions.

2 Model

In this section we define our model and discuss the properties of the DM particles, such as their mass, spin and interactions, in the model. We show parity-odd chiral multiplets newly introduced and MSSM Higgs doublets in Table 1. The lightest particle among mixtures of parity-odd fields is stable and can be a WIMP DM candidate. In order to maintain the unification of the gauge couplings, we also introduce $SU(3)_C$ triplets $(H'_c, \bar{H}'_c)$ which compose 5 and 5 of $SU(5)$ with $H'$ and $\bar{H}'$. The most general renormalizable superpotential is

$$L = -\int d^2 \theta \left( \mu_H \bar{H}H^0 + \mu' H'\bar{H}' + \mu'_c \bar{H}'_c H'_c + \lambda_1 H' \bar{H} S' + \lambda_2 H \bar{H}' S' + \frac{1}{2} M_S S'^2 \right) + \text{h.c.} \quad (1)$$

$SU(2)_L$ products are defined as $H \bar{H} = H^0 \bar{H}^0 - H^- \bar{H}^+$. Five real parameters and one CP violating phase, $\theta = \arg(\mu' M_S \lambda_1^* \lambda_2^*)$, are introduced. For simplicity, we take $\theta = 0$, so that all parameters including $\mu'$ are real and positive.\(^3\)

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\(^3\) The electric dipole moments (EDMs) are induced by electroweak two-loop diagrams in our model. In Ref. [17] the EDMs are discussed in a model similar to ours, where $SU(2)_L$ doublet and singlet fermions are introduced in the standard model. It is found from their result that when the CP violating phase and the couplings are $O(1)$, the electron EDM induced by parity-odd fermions reaches current experimental bound for $\mu' \sim M_S \lesssim 1 \text{ TeV}$. When $M_S \lesssim \mu'$, the constraints are more loosened. Thus, the constraints are not severe at present, though future EDM searches might give severer constraints on the model or find the signature.
The $\mu$ parameter in Eq. (1) is of the electroweak scale from the naturalness argument. In addition, the mass parameters $\mu'$ and $M_S$ are also expected to be of the electroweak scale so that the DM relic abundance explains the WMAP measurement. Those mass parameters should have a common origin. In the low-energy GMSB models, an extra dynamical sector is introduced to generate a constant term in the superpotential for the cosmological constant to vanish. This sector can also produce the dimensional couplings in Eq. (1) with the magnitude of $\mathcal{O}(100)$ GeV \cite{13}.

Soft SUSY-breaking terms for the parity-odd fields are generically given as
\begin{equation}
V_{\text{soft}} = m_{H'}^2 |H'|^2 + m_{H'}^2 |H|^2 + m_{S'}^2 |S'|^2
+ (B'\mu H'\bar{H}' + \frac{1}{2}B'S^2 + A'_1 H'\bar{H}'S' + A'_2 H\bar{H}'S' + \text{h.c.}).
\end{equation}

We study particle spectrum based on the minimal gauge-mediation model (MGM) \cite{19} throughout this work, in which the messenger sector is composed of vector-like $5 + \bar{5}$ representations of $SU(5)$. In the MGM, $A$- and $B$-terms vanish at the messenger scale $M_{\text{mess}}$. In this model, the scalar squared mass of the singlet is also zero at $M_{\text{mess}}$. The only non-vanishing terms at $M_{\text{mess}}$ in Eq. (2) are
\begin{equation}
m_{H'}^2 = m_{\bar{H}'}^2 = 2N_{\text{mess}} \left[ \frac{1}{4} \left( \frac{\alpha_Y}{4\pi} \right)^2 + \frac{3}{4} \left( \frac{\alpha_2}{4\pi} \right)^2 \right] \Lambda^2 f \left( \frac{\Lambda}{M_{\text{mess}}} \right),
\end{equation}
where $N_{\text{mess}}$ represents the number of $SU(5)$ representations introduced as messenger fields, $\Lambda = \langle F \rangle / M_{\text{mess}}$, and $\langle F \rangle$ is the vacuum expectation value of the $F$-term which couples to messenger fields. The loop function $f(x)$ is given in Ref. \cite{2}.

Now we discuss the properties of DM particles. The mass matrix for parity-odd fermions in this model is
\begin{equation}
-\mathcal{L} = \frac{1}{2} \bar{\psi}^T M_F \psi + \text{h.c.}, \quad \psi = \begin{pmatrix} \tilde{S}' \\ \tilde{H}^{0} \\ \bar{H}^{0} \end{pmatrix},
\end{equation}
\begin{equation}
M_F = \begin{pmatrix} -M_S & -\lambda_1 \bar{v} & -\lambda_2 \bar{v} \\ -\lambda_1 \bar{v} & 0 & -\mu' \\ -\lambda_2 \bar{v} & -\mu' & 0 \end{pmatrix},
\end{equation}
where $v = \langle H^0 \rangle$, $\bar{v} = \langle \bar{H}^{0} \rangle$, and $v^2 + \bar{v}^2 = 2m_Z^2/(g^2 + g'^2)$. Similarly, the squared mass matrices for parity-odd bosonic states are
\begin{equation}
-\mathcal{L} = \frac{1}{2} \bar{\varphi}_R^T M_B^{(+)} \varphi_R + \frac{1}{2} \bar{\varphi}_I^T M_B^{(-)} \varphi_I, \quad \varphi_R = \begin{pmatrix} S'_R \\ H^{0}_R \\ \bar{H}^{0}_R \end{pmatrix}, \quad \varphi_I = \begin{pmatrix} S'_I \\ H^{0}_I \\ \bar{H}^{0}_I \end{pmatrix},
\end{equation}
\footnote{Fortunately, the messengers do not generate the $B$-terms in this mechanism at the leading order as mentioned in text, and we do not need to care about the so-called $\mu/B\mu$ problem. When the Higgs multiplets have (direct or indirect) couplings to SUSY-breaking field to generate the $\mu$ term, the $B\mu$ term would be too large.}
\[
M_{\mu}^{(\pm)^2} = \begin{pmatrix}
\lambda_1^2 v^2 + \lambda_2^2 v^2 + M_S^2 + m_{\mu}^2 & \frac{\lambda_2 \mu + \lambda_1 M_S \bar{v} \pm (\lambda_1 \mu + A^\dagger_1 \bar{v})}{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})} & \frac{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})}{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})} \\
\lambda_2 \mu + \lambda_1 M_S \bar{v} \pm (\lambda_1 \mu + A^\dagger_1 \bar{v}) & \frac{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})}{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})} & \frac{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})}{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})} \\
\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v}) & \frac{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})}{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})} & \frac{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})}{\lambda_1 \mu + \lambda_2 M_S \bar{v} \pm (\lambda_2 \mu + A^\dagger_2 \bar{v})}
\end{pmatrix}
\]

\[\Delta = \frac{1}{2} m_Z^2 \cos 2\beta, \quad \bar{\Delta} = -\Delta,\]

where \(\tan \beta = \bar{v}/v\). The fields with subscripts \(R\) and \(I\) are CP-even and odd states, respectively, and canonically normalized as \(\phi = \frac{1}{\sqrt{2}} (\phi_R + i \phi_I)\).

We show mass, spin and type of the lightest parity-odd particle in Fig. 1. In the numerical calculation, we assume MGM with \(5 + 5\) messengers, and we use the result of [20]. Here we took \(\lambda_1 = \lambda_2 = 0.3, \tan \beta = 42, \mu = 660\) GeV, \(N_{\text{mess}} = 1\), the gluino mass \(1\) TeV, and \(\Lambda/M_{\text{mess}} = 0.5\). \(\mu\) and \(\tan \beta\) are fixed from electroweak symmetry breaking conditions and a condition that \(B_\mu = 0\) at the messenger scale. We set these value as reference point throughout this work.

If \(\mu' \gg M_S\), DM is singlet-like CP-even boson. The singlet bosons get no SUSY breaking mass terms in the GMSB model at the leading order. The bosonic and fermionic states are degenerate in masses. In our set up, the \(F\)-components of \(H\) and \(\tilde{H}\) generate mass splitting between CP-even and odd states. Thus, one of the bosonic states tends to...
be lighter than fermionic one. (If we take $\mu'$ negative and $|\mu'| \gg M_S$, DM is singlet-like CP-odd boson. In this case, mass splitting source of CP-odd boson is larger than that of CP-even boson as expected from Eq. (7).) If $M_S$ is too small, the lightest bosonic state becomes tachyonic and the $Z_2$-parity is broken spontaneously. If $\mu' \sim M_S$, DM is a fermion which is mixture of singlet and doublets. In the region of $\mu' \ll M_S$, DM is doublet-like fermion. The doublet-like bosonic states are heavier than fermionic one due to the GMSB effect.

Now we see that this model predicts a correct relic DM abundance measured by WMAP [21] with appropriate parameters. Since our model includes particles with mass close to the DM mass, we should take into account the coannihilation effect for calculating the relic DM abundance.

First, consider the case where DM is singlet-like CP-even boson ($S'_R$). In this case we consider three coannihilating states ($S'_R, S'_I, \tilde{S}'$). The main annihilation processes in a non-relativistic limit are $S'_RS'_R \to h^0h^0$ and $S'_IS'_I \to h^0h^0$, where $h^0$ stands for the SM-like Higgs boson, as far as they are kinematically allowed. Here we can neglect other processes like $\tilde{S}'h^0$, $S'_Rh^0$, and $S'_Rh^0A^0$ where $A^0$ is CP-odd Higgs boson, and so on, because $H^0$ and $A^0$ are assumed to be heavy. The reason for omitting the $\tilde{S}'$ annihilation is as follows. Since $\tilde{S}'$ is a Majorana fermion, in the non-relativistic limit, a pair of them is in CP-odd state. Then the opening channel from the viewpoint of CP conservation is the pair annihilation of $\tilde{S}'$ into a pair of $h^0$ in $p$-wave state. This channel, however, is forbidden by the angular momentum conservation.

The DM effective cross section [22] of these processes is given by

$$\langle \sigma_{\text{eff}} v \rangle = \left( \frac{1}{1 + w} \right)^2 \langle \sigma v \rangle_{S'_RS'_R \to h^0h^0} + \left( \frac{w}{1 + w} \right)^2 \langle \sigma v \rangle_{S'_IS'_I \to h^0h^0},$$

where

$$\langle \sigma v \rangle_{S'_RS'_R \to h^0h^0} = \frac{1}{64\pi} \frac{1}{m_{S'_R}^2} \sqrt{1 - \left( \frac{m_{h^0}}{m_{S'_R}} \right)^2} \left( \lambda^2 - \frac{(M_S\lambda_1 + A_1^2)}{m_{H_1}^2 + m_{S'_R}^2 - m_{h^0}^2} - \frac{(\mu'\lambda_1 + \mu\lambda_2)^2}{m_{H_2}^2 + m_{S'_R}^2 - m_{h^0}^2} \right),$$

$$\langle \sigma v \rangle_{S'_IS'_I \to h^0h^0} = \frac{1}{64\pi} \frac{1}{m_{S'_I}^2} \sqrt{1 - \left( \frac{m_{h^0}}{m_{S'_I}} \right)^2} \left( \lambda^2 - \frac{(M_S\lambda_1 - A_1^2)}{m_{H_1}^2 + m_{S'_I}^2 - m_{h^0}^2} - \frac{(\mu'\lambda_1 - \mu\lambda_2)^2}{m_{H_2}^2 + m_{S'_I}^2 - m_{h^0}^2} \right),$$

$$w = \left( \frac{m_{S'_I}}{m_{S'_R}} \right)^2 \exp \left( -x \frac{m_{S'_I}^2 - m_{S'_R}^2}{m_{S'_R}^2} \right),$$

$$x = \frac{m_{S'_R}}{T_{\text{freeze out}}}.$$
The gray region in Fig. 1 (bottom one) shows the parameter space in which the DM relic abundance is consistent with WMAP observations within 2σ level \[21\]. Thus we can see that the relic abundance falls into the correct range measured by WMAP for certain choice of parameters. We note that the coannihilation effect can change the relic abundance only 10% or so in the correct range for DM abundance.

In the case of doublet-like fermion DM, we consider four coannihilating states (\(\widetilde{H}^0, \widetilde{H}^{0*}, \widetilde{H}^{-}, \widetilde{H}^{+}\)). The cross section and its relic abundance are basically the same as those of MSSM Higgsino-like DM whose mass is heavier than \(W\)-boson mass. If the coannihilation effect is efficient, the DM effective cross section and its relic abundance are given in Ref. \[23\]:

\[
\langle \sigma v \rangle_{\text{eff}} = \frac{g^4}{512\pi\mu'^2} \left( 21 + 3\tan^2\theta_W + 11\tan^4\theta_W \right),
\]

\[
\Omega_{\widetilde{H}^0}h^2 = 0.10 \left( \frac{\mu'}{1\text{ TeV}} \right)^2,
\]

where four states are taken to be degenerate in mass. The effective cross section in Eq. (15) drops final states including ordinary MSSM superpartners, because their total contribution is at most 10% in our reference point comparing with the SM final state contribution. The correct DM relic abundance consistent with WMAP observations within 2σ level is obtained in the gray region in Fig. 1 (top one).

Finally, we discuss about \(H'_c\) and \(\bar{H}'_c\) in Eq. (1), which are introduced in order to maintain the successful gauge coupling unification. They have color and electric charges, and their masses are also expected to be order of electroweak scale. In the model, they are stable at the renormalizable level and unstable due to the higher dimensional operators suppressed by relevant physics scale \(M_{\text{phys}}\) such as

\[
\frac{1}{M_{\text{phys}}}(5\bar{5}_{\text{SM}})(5\bar{5}_{\text{SM}}).
\]

In the GMSB model, triplet fermion is lighter than bosonic one. When DM is doublet-like fermion, the lifetime is \(O(1) - O(10^3)\) sec for masses 100 – 1000 GeV in the case that \(M_{\text{phys}} = M_{\text{GUT}} \approx 10^{16}\) GeV. Here, it is assumed that the triplet fermion can decay into the MSSM SUSY particles directly. When DM is singlet-like boson, the decay of the triplet fermion is suppressed by the mixing between the singlet and doublet states. The feature of these long-lived colored particles are constrained from the viewpoint of cosmology. Their relic abundance would be determined by geometrical cross sections of order of 10 mb effectively at temperatures below the QCD phase transition, and they annihilate efficiently before the Big Bang nucleosynthesis (BBN) if their masses are lighter than about \(10^{11}\) GeV \[24\]. Their relic abundance after this annihilation is estimated as

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5 The \(t\bar{t}\) final state via stop exchange is ignored in Ref. \[23\], and our case does not include such a process.

6 If \(\mu'_c\) is equal to \(\mu'\) at the GUT scale, \(\mu'_c\) is about twice larger than \(\mu'\) at the electroweak scale.
$10^{-8}n_b(\mu^c/\text{TeV})^{1/2}$, where $n_b$ is the baryon number density. The future collider experiments may prove existence of the colored particles. We discuss its possibility in Sec. 4.

### 3 Direct detection rates

In this section we evaluate the DM-nucleon scattering cross section in our model. DM particles in the model have sizable interactions with nucleons and hence they may be detected through on-going or future direct detection experiments.

First let us consider the case where the DM particle is fermionic (denoted by $\tilde{\chi}$). DM interacts with nucleons through the $Z$-boson and Higgs-boson exchange diagrams. The former yields a spin-dependent (SD) and the latter yields a spin-independent (SI) effective interactions. The effective Lagrangian for these interactions is written at the parton level as

$$\mathcal{L}_{\text{eff}} = \sum_{q=u,d,s} d_q \tilde{\chi} \gamma^\mu \gamma_5 \tilde{\chi} \gamma_\mu q + \sum_{q=u,d,s,c,b,t} f_q \tilde{\chi} \tilde{\chi} q,$$

where coupling constants are given by

$$d_q = \frac{g^2 T_{3q}}{8m_W^2} \left( (O_F)_{12}^2 - (O_F)_{13}^2 \right),$$

and

$$f_q = m_q \frac{g}{2m_W} \left( c_{h\tilde{\chi}\tilde{\chi}} c_{q\tilde{q}} + c_{H\tilde{\chi}\tilde{\chi}} c_{q\tilde{q}} \right),$$

with

$$c_{huu} = \frac{\cos \alpha}{\sin \beta}, \quad c_{Hu\bar{u}} = \frac{\sin \alpha}{\sin \beta},$$

$$c_{hdd} = -\frac{\sin \alpha}{\cos \beta}, \quad c_{Hdd} = \frac{\cos \alpha}{\cos \beta},$$

$$c_{\tilde{h}\tilde{\chi}} = \frac{1}{\sqrt{2}} \left( \lambda_1(O_F)_{11}(O_F)_{12} \cos \alpha - \lambda_2(O_F)_{11}(O_F)_{13} \sin \alpha \right),$$

$$c_{H\tilde{\chi}} = \frac{1}{\sqrt{2}} \left( \lambda_1(O_F)_{11}(O_F)_{12} \sin \alpha + \lambda_2(O_F)_{11}(O_F)_{13} \cos \alpha \right).$$

Here $O_F$ is a $3 \times 3$ mass diagonalizing matrix, which is obtained from Eq. [3].

Using these couplings, the SI scattering cross section between DM and nucleus with mass $m_T$ is expressed as [26, 27]

$$\sigma_{\text{SI}} = \frac{4}{\pi} \left( \frac{m_{\tilde{\chi}} m_T}{m_{\tilde{\chi}} + m_T} \right)^2 \left( n_p f_p + n_n f_n \right)^2,$$

$^7$ It is also argued in Ref. [25] that even such a small abundance may affect the BBN and a lifetime of the colored particles should be shorter than about 200 sec. This would give a lower bound on the triplet fermion mass, depending on $M_{\text{phys}}$ and the main decay mode, though we do not include this constraint in this paper.
and the SD cross section is given by
\[ \sigma_{SD} = \frac{4}{\pi} \left( \frac{m_{\tilde{\chi}} m_T}{m_{\tilde{\chi}} + m_T} \right)^2 \left[ 4 J + 1 \right] \left( a_p \langle S_p \rangle + a_n \langle S_n \rangle \right)^2. \] (26)

Here \( n_p(n_n) \) is the number of proton (neutron) in the target nucleus. \( J \) is the total nuclear spin, and \( \langle S_{p(n)} \rangle = \langle A|S_{p(n)}|A \rangle \) are the expectation values of the spin content of the proton and neutron groups within the nucleus \( A \) [28], and
\[ a_N = \sum_{q=u,d,s} d_q \Delta q_N, \] (27)
\[ 2s_\mu \Delta q_N = \langle N|\bar{q}_\mu q_\gamma|N \rangle. \] (28)

where \( s_\mu \) is the nucleon’s spin and \( N = n,p \). The DM-nucleon effective coupling is constructed from the DM-quark effective coupling as follows [29],
\[ f_{Tq} = 1 - \sum_{q=u,d,s} f_q^{(N)}. \] (30)

For the nucleon mass matrix elements, we take \( f_{T_u}^{(p)} = 0.023 \), \( f_{T_d}^{(p)} = 0.034 \), \( f_{T_u}^{(n)} = 0.019 \), \( f_{T_d}^{(n)} = 0.041 \) [30] [31] and \( f_{T_s}^{(p)} = f_{T_s}^{(n)} = 0.025 \) [32].

Next let us consider the case of bosonic DM (\( \chi \)). The effective Lagrangian through the Higgs boson exchange diagram is written as
\[ \mathcal{L}_{\text{eff}} = \sum_{q=u,d,s,c,b,t} f_q \chi^2 \bar{q}q. \] (31)

This yields the following SI scattering cross section,
\[ \sigma_{SI} = \frac{1}{\pi m_{\tilde{\chi}}^2} \left( \frac{m_{\tilde{\chi}} m_T}{m_{\tilde{\chi}} + m_T} \right)^2 \left( n_p f_p + n_n f_n \right)^2, \] (32)

where \( f_p(f_n) \) is given by Eq. [29], with \( c_{h\chi\chi}(c_{H\chi\chi}) \) in \( f_q \) (Eq. [20]) is replaced by the following,
\[ c_{h\chi\chi} = \frac{1}{2\sqrt{2}} \left( (O_B Y_1 O_B^T)_{11} \sin \alpha - (O_B Y_2 O_B^T)_{11} \cos \alpha \right) , \] (33)
\[ c_{H\chi\chi} = \frac{1}{2\sqrt{2}} \left( -(O_B Y_1 O_B^T)_{11} \cos \alpha - (O_B Y_2 O_B^T)_{11} \sin \alpha \right) , \] (34)
\[ Y_1 = \frac{\partial}{\partial v} M_B^2, \quad Y_2 = \frac{\partial}{\partial \bar{v}} M_B^2. \] (35)
Figure 2: Contours of SI (top) and SD (bottom) cross sections with a proton on $\mu'-M_S$ plane. Broken lines are boundaries between fermionic and bosonic DM.
Here $O_B$ is a $3 \times 3$ mass diagonalizing matrix for the mass matrix $M_B^2(\equiv M_B^{(+)}2)$, which is given in Eq. (7).

Fig. 2 shows contours of the SI and SD interactions with a proton on $\mu^'-M_S$ plane. Parameters are set to be the same as those in Fig. 1. The most stringent bound on the SI cross section comes from CDMS-II results [33, 34]. The bound reads $\sigma_{SI}/m_\chi \lesssim 3 \times 10^{-46}$ cm$^2$/GeV for $m_\chi \gtrsim 100$ GeV. The observed two DM-like events at CDMS-II [34] can be explained for appropriate parameters, if we take the two events seriously and assume they are caused by DM-nucleon scatterings. The predicted SI cross section is within the reach of future or on-going direct detection experiments.

Among the direct detection experiments, the best bound on the SD cross section comes from the XENON experiment [35] ($\sigma_{SD}/m_\chi \lesssim 3 \times 10^{-40}$ cm$^2$/GeV for $m_\chi \gtrsim 100$ GeV). In addition, the SD cross section is also constrained by observations of energetic neutrinos from the Sun produced by annihilations of DM particles captured by the Sun [36, 37, 38, 39]. Super-Kamiokande [40], AMANDA [41] and IceCube with 22 strings give the stringent limits [42]. The fermionic DM in the present model mainly annihilates into $W^+W^-$, and their subsequent decay modes $W \rightarrow e\nu$ produce high-energy neutrinos which may be detectable at IceCube. The current IceCube bound for mass at 250 GeV is $\sigma_{SD} \lesssim 3 \times 10^{-40}$ cm$^2$, assuming the $W^+W^-$ final state.

The bound on the SD cross section is still far from the prediction. However, the experiments sensitive to it would be important to determine spin of the DM particle after the DM is discovered. Furthermore, we might reconstruct the model by using results of the direct DM searches sensitive to SD and SI cross sections under assumption of the thermal relic scenario, when the DM is fermionic. This is because the model has only four input parameters, $\mu^'$, $M_S$ and $\lambda_{1/2}$.

### 4 Collider signatures

One of the handles to confirm this model is discovery for the signatures of the extra particles at the LHC. In the confirmation, the key issue is the selection of missing energy events of this model under the backgrounds of ordinary GMSB models. In this section, we estimate the discovery reach of extra parity-odd particles, and discuss the feasibilities for the selection assuming that the GMSB model has been already experimentally established as the SUSY breaking model.

First, we discuss the case that the lightest parity-odd particle is doublet-like neutral fermion, $\tilde{H}^0_L$. The mass difference between $\tilde{H}^0_L$ and heavier doublet-like neutral (charged) fermion, $\tilde{H}^0_H$ ($\tilde{H}^{\pm}$), is less than 40 GeV when $\mu' > 94$ GeV within the parameters of Fig. 1. Accordingly, the two-body decays of $\tilde{H}^0_H$ and $\tilde{H}^{\pm}$ are inaccessible.

Clean multilepton events, $pp \rightarrow \tilde{H}^0_H\tilde{H}^{\pm} \rightarrow ul\nu_l\tilde{H}^0_L + l:\nu_l\tilde{H}^0_L$, offer a promising way for the selection. In the low-energy GMSB models the SUSY events accompany energetic tau leptons or photons since the next-lightest SUSY particle is typically stau or Bino-like neutralino. Thus, an observation of the missing energy events accompanying trileptons clearly point to $\tilde{H}^0_H\tilde{H}^{\pm}$ production as its source and would be the evidence of underlying
Figure 3: Cross sections for $pp \rightarrow H^0 H'^\pm$ and $pp \rightarrow \tilde{H}_H^0 \tilde{H}'^\pm$. Here $H^0$ and $H'^\pm$ stand for neutral CP-even (CP-odd) and charged extra boson, which are components of common extra doublet. $\tilde{H}_H^0$ and $\tilde{H}'^\pm$ stand for superpartners of neutral and charged extra doublet boson. We took only direct pair production as their production processes, and included all combinations of final state pair in each result.

Figure 4: Production cross sections for extra triplet boson $H'_c$ and its superpartner $\tilde{H}'_c$ as a function of their mass. We took only direct pair production as their production processes.
physics responsible for the extra particles.

In order to optimize the trileptons events, one should reduce the $W^\pm Z$ background and $tt$ background with suitable cuts. Those have been discussed in works [43, 44, 45], which focus on trileptons events from chargino-neutralino production, and after the cuts they find a total SM background of 19.6 fb. Here cuts designed in Ref [45] are as follows:

1. 3 leptons with $p_T > 10$ GeV.

2. At least one Opposite Sign Same Flavor (OSSF) dilepton with $M_{OSSF} > 20$ GeV to suppress low-mass $\gamma^*, J/\Psi, \Upsilon,$ and conversion backgrounds.

3. Lepton track isolation: $p_{T,tTk}^{0,2} < 1$ GeV for muon and < 2 GeV for electron, where $p_{T,tTk}^{0,2}$ is the maximum $p_T$ of any additional track within a cone $R = 0.2$ around the lepton.

4. $E_T^{\text{miss}} > 30$ GeV.

5. No OSSF dilepton has invariant mass in the range $81.2$ GeV < $M_{OSSF}$ < $102.2$ GeV.

6. No jet with $p_T > 20$ GeV.

In addition to the SM background, we also comment on the background coming from ordinary superpartners. The GMSB model with stau NLSP may produce the trilepton events, e.g., $\tilde{\chi}_1^+ \tilde{\chi}_2^0 \rightarrow \tilde{\tau} \nu_\tau + \tilde{\tau} \tilde{\tau} \rightarrow \tilde{G} \tau \nu_\tau + \tilde{G} \tau \bar{\tau} \rightarrow E_T^{\text{miss}} + \text{trilepton},$ and so on. Those MSSM backgrounds include tiny branching ratio $\text{BR}(3\tau \rightarrow l + \text{OSSF dilepton} + E_T^{\text{miss}})$, and hence the MSSM backgrounds could be reduced enough.

Red dash-dotted and purple dotted lines in Fig. 3 show the cross sections for the direct production $pp \rightarrow \tilde{H}_H^0 \tilde{H}_H^\pm$ and $pp \rightarrow \tilde{H}_H^0 \tilde{H}_H^-$, respectively. They are calculated with the CalcHEP [46] implementing the Lagrangian Eq. (1) and the CTEQ6L code [47] as a parton distribution function. In the calculation, we took $m_{\tilde{H}_H^0} = m_{\tilde{H}_H^\pm}$, and set the center of mass energy to be $\sqrt{s} = 14$ TeV. From the numerical result, production cross section can be parametrized as $\sigma(pp \rightarrow \tilde{H}_H^0 \tilde{H}_H^{\pm}) = 2.47 \times 10^{-4}(\text{TeV}/m_{\tilde{H}})^{4.1}$ pb, where $m_{\tilde{H}_H^0} = m_{\tilde{H}_H^\pm}(\equiv m_{\tilde{H}_H})$.

Since their decay modes into two bodies are kinematically inaccessible, they decay into dileptons and $\tilde{H}_L^0$ via off-shell weak gauge boson. Their branching ratios into dileptons are, therefore, uniquely determined; $\text{BR}(\tilde{H}_H^0 \rightarrow l\bar{l} \tilde{H}_L^0) \simeq 6.73\%$ and $\text{BR}(\tilde{H}_H^{\pm} \rightarrow l' \nu \tilde{H}_L^0) \simeq 21.32\%$. Here results have summed over electron and muon. Thus the $5\sigma$ discovery reach for the trilepton signals is estimated as follows,

$$N_{\text{signal}} = \sigma(pp \rightarrow \tilde{H}_H^0 \tilde{H}_H^{\pm}) \times \mathcal{L} \times \text{BR}(\tilde{H}_H^0 \rightarrow l\bar{l} \tilde{H}_L^0) \times \text{BR}(\tilde{H}_H^{\pm} \rightarrow l' \nu \tilde{H}_L^0) \approx 0.354 \times \left(\frac{\text{TeV}}{m_{\tilde{H}}^{\pm}}\right)^{4.1} \left(\frac{\mathcal{L}}{100 \text{ fb}^{-1}}\right) > 5\sqrt{N_{\text{BG}}}.$$

Here $\mathcal{L}$ stands for an integrated luminosity, and $N_{\text{signal}} (N_{\text{BG}})$ stands for the number of multilepton events (SM background events). Thus, assuming an integrated luminosity 100
fb$^{-1}$ and demanding the SM background 19.6 fb, the superpartners of extra doublet for $m_H' = m_H^\pm \lesssim 205$ GeV would be discovered at the 5$\sigma$ level. Indeed, for the discussion of discovery reach, we should mention the acceptance of detector. However it needs precise simulation of signal events, and is beyond the scope of this work.

Next, we discuss the case that the lightest extra particle is singlet-like boson, $S'$. They are mainly produced by the decay of heavier doublet-like bosons accompanying the SM-like Higgs boson, $h^0$. Accordingly, the signal events contain $b\bar{b}$ and the missing energy, and hence provide a distinguishable signature from the ordinary GMSB ones.

The dominant background to the two $b$-jets plus large missing energy events presumably comes from $t\bar{t}$ production. It can be reduced by the following cuts:

1. $E_T^{\text{miss}} > 100$ GeV.
2. $b$-jets with $p_T > 50$ GeV.
3. $E_T^{\text{miss}} + \sum E_T^j > 1500$ GeV.

Here $\sum E_T^j$ indicates the transverse energy sum over untagged jets \[48\]. The most promising event for the discrimination, therefore, would be missing energy events accompanying $b\bar{b}$ and energetic jet, $pp \rightarrow H^0H^\prime \rightarrow h^0S' + W^\pm S' \rightarrow b\bar{b}S' + \text{jet}S'$. In both GMSB models with stau NLSP and Bino-like neutralino NLSP, there would exists no background events of ordinary superpartners under the cut conditions.

Gray dashed and red solid lines in Fig. 3 show the cross sections for the direct production $pp \rightarrow H^0H^\prime$ and $pp \rightarrow H^0H^\prime$, respectively. The production cross section is parametrized as

$$\sigma(pp \rightarrow H^0H^\prime) = 1.027 \times 10^{-4} \left(\frac{\text{TeV}}{m_{H^\prime}}\right)^{4.2} \text{pb},$$

where $m_{H^0} = m_{H^\pm}$ ($\equiv m_{H^\prime}$). Since the branching ratio for $H^0 \rightarrow b\bar{b}S'$ has a complicated dependency on model parameters, we take it as a free parameter. When charged Higgs boson is much heavier than $W^\pm$, $H^\prime$ dominantly decays into $W^\pm$ and $S'$, and hence the branching ratio of $H^\prime$ into jets and $S'$ is $\text{BR}(H^\prime \rightarrow \text{jets} + S') \simeq 67.60\%$. Thus 5$\sigma$ discovery reach for them is estimated as follows,

$$N_{\text{signal}} = \sigma(pp \rightarrow H^0H^\prime) \times \mathcal{L} \times \text{BR}(H^0 \rightarrow b\bar{b}S') \times \text{BR}(H^\prime \rightarrow \text{jet} + S') \times (b\text{-tag efficiency}) \approx 0.174 \left(\frac{\text{TeV}}{m_{H^\prime}}\right)^{4.2} \left(\frac{\mathcal{L}}{100 \text{ fb}^{-1}}\right) \left(\frac{\text{BR}(H^0 \rightarrow b\bar{b}S')}{10\%}\right) \left(\frac{b\text{-tag efficiency}}{50\% \times 50\%}\right) \> 5\sqrt{N_{\text{BG}}} \> .$$

(37)

The cross section of the $t\bar{t}$ background for these events is given by \[48\], $\sigma_{t\bar{t}} = 0.89$ fb ($\sigma_{t\bar{t}} = 0.72$ fb) for the 2% (1%) $b$ mistagging probability. Thus assuming $\mathcal{L} = 100 \text{ fb}^{-1}$, $\text{BR}(H^0 \rightarrow b\bar{b}S') = 10\%$, $b$-tag efficiency $= 50\% \times 50\%$, and demanding the $t\bar{t}$ background 1 fb conservatively, the extra doublets for $m_{H^0} = m_{H^\pm} \lesssim 260$ GeV would be discovered at the 5$\sigma$ level.

Finally, we discuss the collider signature of the color-triplet states, introduced in order to maintain the successful gauge coupling unification, which is another key ingredient for the selection at the LHC.
Those colored extra particles are produced, and then they hadronize in the detector materials. The hadronized particles would be electrically either neutral or charged, and they can reverse the sign of its charge through the scattering in the detector materials [49, 50, 51]. If a hadronized particle is electrically neutral and would not undergo the charge reversal, it leaves detectors. The resultant large missing energy without energetic tau lepton or photon would be the mark for the selection from ordinary GMSB model. On the other hand, some of the hadronized particles are charged, and they lose their kinetic energy through ionization with the detector materials. The ionization energy loss is a function of $\beta \gamma$ and the electric charge of penetrating particle [52]. When the energy loss and momentum of penetrating particle is measured, $\beta \gamma$ can be obtained, and hence its mass is determined. In addition, since massive long-lived charged particles produce a track in detectors, we can speculate its production rate. The estimated discovery potential with this method, however, is necessarily dependent on the scattering model, which of charge reversal predictions [49]. Thus the discussion of feasibility for the discovery requires model dependent full analysis, and we leave it for future work.

5 Conclusions

In this paper we proposed an extension of the GMSB model by adding extra doublets and singlet with parity odd under an additional $Z_2$ parity, while ordinary MSSM fields are parity-even. In this class of model, a natural candidate for DM appears as the lightest linear combination of additional $Z_2$-odd fields. It has sizable interactions with nucleons and will be detected future/on-going direct detection experiments. We have also discussed typical collider signatures of this model and found that they may be discovered at 5 sigma level with an integrated luminosity of 100 fb$^{-1}$, depending on model parameters.

The calculation of the relic abundance of bosonic-singlet DM indicates DM mass is around 250 GeV. If this is realized, this model could be confirmed by both the LHC experiment and future DM direct detection experiments. Furthermore, these DM can reproduce the DM-like events at CDMS-II. On the other hand, fermionic-doublet DM mass is around 1 TeV as shown in Fig. 1. In this case, it is very hard to discover these extra particles at the LHC, and DM direct detection experiments would not observe its signal.

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