High-performance coherent population trapping atomic clock with direct-modulation distributed Bragg reflector laser

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Abstract
The coherent population trapping (CPT) atomic clock is very promising for use in next-generation spaceborne applications owing to its compactness and high performance. In this paper, we propose and implement a CPT atomic clock based on the direct modulation of a large-modulation-bandwidth and narrow-linewidth distributed Bragg reflector laser, which replaces the usually used external bulk modulator in high-performance CPT clocks. Our method retains the high performance while significantly reducing the clock size. Using this highly compact bichromatic light source and simplest CPT configuration, in which a circularly polarized bichromatic laser interrogates the $^{87}$Rb atom system, a CPT signal of clock transition with a narrow linewidth and high contrast is observed. We then lock the local oscillator frequency to the CPT resonance and demonstrate an encouraging short-term frequency stability of $3.6 \times 10^{-13} \tau^{-1/2}$ ($4 \leq \tau \leq 200$ s). We attribute it to the ultralow laser frequency and intensity noise as well as to the high-quality-factor CPT signal. This study can pave the way for the development of compact high-performance CPT clocks based on our scheme.

Keywords: compact atomic clock, coherent population trapping, distributed Bragg reflector laser

(Some figures may appear in colour only in the online journal)

1. Introduction
The recent progress on high-performance coherent population trapping (CPT) [1, 2] atomic clocks with excellent short-term [3–6] and mid-term [7, 8] frequency stabilities paves the way for deep space exploration, next-generation global navigation satellite system, high-speed communication, etc. However, they need to be more compact and robust for broader applications.

In CPT atomic clocks, a coherent bichromatic laser is needed to couple two ground states to a common excited state and prepare the atom system to the CPT states.
2. Experimental setup and preparation

2.1. Experimental setup

Our setup is depicted in figure 1, which is composed roughly of three parts, the DBR laser, which generates a multi-chromatic light, reference cell for laser frequency locking, and clock $^{87}$Rb cell for local oscillator (LO) locking.

A DBR laser (Photodigm, PH795DBR) with a wavelength of 795 nm, linewidth of approximately 1 MHz, and output optical power up to 80 mW is used as a light source. Its temperature is regulated at 24 °C. We drive it with a current source (Viscent, D2-105) and tune its wavelength to the working point of the CPT resonance for the D$_1$ line of $^{87}$Rb with a current of approximately 100 mA. To generate a bichromatic light for the CPT experiment, a 3.417 GHz microwave signal is coupled to the laser diode current through a bias-tee. As demonstrated in our previous study [33], with a typical microwave power of 22–28 dBm, we can obtain the maximum efficiency (~50%) of laser power transfer to the ±1st sidebands, which form the desired bichromatic light for CPT interaction.

We then split part of the bichromatic light to implement a simple scheme [34] for locking the laser frequency, in which an AOM is used to compensate the buffer-gas-induced optical frequency shift in the CPT clock cell.

The remaining bichromatic light beam is right-circularly polarized and interacts with an atom ensemble in a buffer-gas vapor cell, to realize the simplest CPT scheme. The laser beam is expanded to a circular beam with a diameter of approximately 18 mm before passing through the vapor cell. The $^{87}$Rb isotope enriched cylindrical vapor cell (diameter: 20 mm, length: 50 mm) is filled with a mixture of buffer gases, Ar and N$_2$ in the pressure ratio 3:2, with a total pressure of 25 Torr; the cell temperature is stabilized to approximately 62 °C. Both are typical values as in reference [2]. Unless otherwise specified, a uniform magnetic field of $B_0 = 3.66 \mu$T is applied along the direction of the cell axis by means of a solenoid to remove the Zeeman degeneracy. The ensemble is surrounded by two magnetic shields to reject Earth and other stray magnetic fields.

2.2. Microwave phase noise

Before we describe the optical experiment, we briefly introduce our microwave source, which is crucial for the CPT clock. With the microwave chain, shown in the inset of figure 2, similar to that in reference [35], we built a 3.417 GHz microwave source with an ultralow phase noise. The single side-band phase noise power spectral density (PSD) of the 3.417 GHz signal is shown in figure 2, it reaches $-122.5$ dBc Hz$^{-1}$ at an offset frequency of 286 Hz, which is twice the atomic clock modulation frequency ($F_M$). Compared to the ideal frequency multiplication, it exhibits a deterioration of only 1 dB at 2$F_M$. The phase noise contribution to the CPT clock frequency stability is addressed in section 3.3.

2.3. Laser frequency and intensity noise

We use the sub-Doppler absorption to lock the laser frequency with the dual-frequency laser field method [34]. In this setup, we split a part of the multi-chromatic laser with an optical power of 1.8 mW and beam diameter of 3 mm to the reference cell, which is temperature-stabilized at 24 °C. The measured
Figure 2. Single side band phase noise power spectral density of the 3.417 GHz microwave signal; the inset shows the microwave chain.

Figure 3. Spectra of the $^{87}\text{Rb}$ D$_1$ line in the vacuum reference cell and clock cell recorded with the multi-chromatic laser. The distortion with a laser detuning lower than approximately $-3.7$ GHz is caused by the limited mode-hopping-free tuning ranges of the diode laser. (Inset) Atomic levels involved in the D$_1$ line of rubidium.

Spectra in figure 3 correspond to the D$_1$ line of the $^{87}\text{Rb}$ spectrum in the vacuum reference cell and clock cell recorded with the multi-chromatic laser. The optical transitions in the clock cell are broadened and shifted by collisions between the Rb atoms and buffer gas molecules. The frequency shift ($\sim222$ MHz) is already compensated by the AOM in this plot.

The laser-frequency servo-loop is performed with a sine-wave current modulation at 1.8 MHz, and demodulation of the reference cell output signal by a lock-in amplifier, as usual. With the narrow absorption peak and high signal-to-noise ratio, as shown in figure 3, it is simple to lock the laser frequency, without unlocking for days. In order to estimate the laser-frequency noise, we use the Doppler-free part of the resonance line $|5^2S_{1/2}, F = 1 & 2 \rangle \rightarrow |5^2P_{1/2}, F' = 2 \rangle$ as frequency discriminator. The laser frequency is tuned to the resonance, and we monitor that the error signal level is always in the linear range during the whole process. The error-signal noise (in V Hz$^{-0.5}$), recorded with an FFT spectrum analyzers (Stanford Research Systems, SR785), is converted into frequency noise by dividing it by the error signal slope (in V/Hz). The process is the same for unlocked and locked cases. The laser frequency noise suppression with this method compared to the free-running laser (figure 4(a)) is impressive. Its contribution to the CPT clock’s short-term frequency stability is negligible through the FM–AM effect, see table 1.

The laser RIN measured before passage through the resonance clock cell is also plotted in figure 4(b). The laser RIN is deteriorated by the laser frequency locking. The peak frequency is approximately 4 kHz to 5 kHz, equal to the feedback bandwidth of the laser frequency locking, as shown in figure 4(a). It is of interest to investigate the physics behind this notable phenomenon in future studies.

3. Experimental results

3.1. Spectroscopy studies

For the simplest CPT scheme, analog or digital modulation/demodulation methods could be applied to obtain the CPT signal and error signal, and then lock the LO. To optimize the CPT contrast and linewidth, we use the digital method with the time sequence shown in figure 5 to perform the CPT experiment.
We observed the CPT signals with this time sequence as shown in figure 6. There are three allowed CPT transitions between Zeeman sublevels of the $^{87}$Rb ground state in the $(\sigma^+, \sigma^-)$ CPT scheme with right-handed circularly polarized bichromatic light, i.e., $|F = 1, 2, m_F \leftrightarrow \mid F = 2, m_F - 1 \rangle$, with the magnetic quantum number $m_F = 0, \pm 1$, see inset of figure 6. Herein and after we denote them ($-1 \leftrightarrow -1$), ($0 \leftrightarrow 0$), ($+1 \leftrightarrow +1$), respectively. The transition ($0 \leftrightarrow 0$) is chosen as the clock transition due to its reduced sensitivity to the magnetic field, while the non-clock transitions ($\pm 1 \leftrightarrow \pm 1$) are more sensitive to the applied static field, thus are broadened and distorted by the magnetic-field inhomogeneity. The relative signal magnitudes of the transitions ($-1 \leftrightarrow -1$), ($0 \leftrightarrow 0$), ($+1 \leftrightarrow +1$), estimated from the area under each CPT peak, are 1, 0.42, 0.2, respectively. The amplitude of the clock transition is not the most pronounced, which is well known in the $(\sigma^-)$ CPT scheme due to optical pumping toward the extreme Zeeman sublevels.

We then studied the CPT clock transition’s contrast ($C$), linewidth (FWHM), and quality factor $q = C/\text{FWHM}$ as functions of the laser power ($P_L$) and microwave power ($P_{\mu w}$), plotted in figures 7 and 8, respectively. In these measurements, $T_C$ and $t_w$ are already optimized for maximizing contrast. With the guide of these optimal parameters, we obtained a CPT signal and error signal (figure 9) with FWHM = 156 Hz and contrast of 4.3%, comparable to the values obtained in reference [5], which provides a good base for clock applications.

### 3.2. Measured short-term frequency stability

With the observed CPT error signal, we can lock our LO to the CPT clock transition frequency. Figure 10 shows the Allan standard deviations (ADEVs) of the unlocked and locked LO frequencies, measured against our hydrogen maser (VREMYA-CH, VCH-1003M), which is a good reference with excellent short- and mid-term frequency stability $\leq 8 \times 10^{-14} \tau^{-1/2}$ ($1 \leq \tau \leq 10000\ s$).

The locked LO frequency stability is $3.6 \times 10^{-13} \tau^{-1/2}$ with averaging time from 4 s to 200 s, and it reaches $1.8 \times 10^{-14}$ at 200 s. This encouraging stability paves the way for a compact high-performance CPT clock.

### 3.3. Short-term frequency stability analysis

The various contributions to the overall Allan deviation are discussed below, and summarized in table 1. In a first approach, we consider that they are uncorrelated and can be independently summed. Similar to the estimations methods in reference [5], the total Allan variance can be computed as

$$\sigma_y^2(\tau) = \sum_i \sigma_{x_i}^2(\tau) + \sigma_{y_i}^2(\tau) + \sigma_{i_s}^2(\tau),$$

where $\sigma_{x_i}^2(\tau)$ is the Allan variance of the clock frequency induced by the fluctuations of the parameter $p_i$, $\sigma_{y_i}^2(\tau)$ is the contribution due to the phase noise of the LO, and $\sigma_{i_s}^2(\tau)$ is the light-shifts via the instability of laser intensity and frequency, as well as microwave power.
Figure 6. Zeeman spectrum. The center peak is the (0 ↔ 0) clock transition. The working parameters are \( T_c = 3.5 \text{ ms}, t_w = 1 \text{ ms}, \) laser power \( P_L = 0.654 \text{ mW}, \) microwave power \( P_{\mu w} = 26.9 \text{ dBm}, \) and \( B_0 = 146.4 \text{ nT}. \) The inset shows the atomic levels involved in the \((\sigma^-, \sigma^-)\) CPT scheme, where the population distributions is just for an illustration.

Figure 7. Contrast \((C), \) linewidth (full width at half maximum (FWHM)), and quality factor \( q = C/\text{FWHM} \) versus the laser power. The working parameters are \( T_c = 3.5 \text{ ms}, t_w = 1 \text{ ms}, \) and \( P_{\mu w} = 26.9 \text{ dBm}. \)

The first term on the right side of equation (1) can be estimated from following expression [5]:

\[
\sigma^2_{\chi_{\phi_1}}(\tau) = \frac{1}{f_c^2} S_{\phi_1}(F_M) \frac{C_{\phi_1}^2 T_c}{S_{\phi_1}^2} \frac{T_c}{2T_w} \tau, \tag{2}
\]

where \( f_c, S_{\phi_1}, T_c, t_w, \) and \( \tau \) are clock frequency, slope of the frequency discriminator, cycle time, detection window duration and averaging time, respectively. \( S_{\phi_1}(F_M) \) is the value of the PSD of \( \phi_1 \) at the Fourier frequency \( F_M \) (assuming a white frequency noise around \( F_M \)). \( V_{wp} \) is the detected signal value at the interrogating frequency, which is the clock resonance frequency plus or minus the modulation depth, its sensitivity (or conversion coefficient) to \( \phi_1 \) is \( C_{\phi_1} = \delta V_{wp}/\delta \phi_1. \) In the following estimation, the working parameters of our clock are \( S_{\phi_1} = 0.268 \text{ mV Hz}^{-1}, \) \( V_{wp} = 314.352 \text{ mV}, T_c = 3.5 \text{ ms}, \) \( t_w = 1 \text{ ms}. \)

Detector noise. The square root of the PSD of the photodetector (Thorlabs, PDA36A) measured in the dark is shown in figure 11. It is \( \sqrt{S_{\phi_1}(F_M)} = 36.8 \text{ nV Hz}^{-0.5} \) in 1 Hz bandwidth at the Fourier frequency 143 Hz. According to equation (2), the contribution of the detector noise to the Allan deviation at 1 s is \( 0.266 \times 10^{-13}. \)

Shot noise. With the trans-impedance gain \( G_R = 1.5 \times 10^4 \text{ V A}^{-1} \) and the detector current \( I = V_{wp}/G_R = 21.0 \mu \text{A}, \) the shot noise is \( \sqrt{S_{SN}(F_M)} = \sqrt{2eIG_R} = 38.8 \text{ nV Hz}^{-0.5}, \) where \( e \) is the electron charge. Its contribution to the Allan deviation at 1 s is \( 0.281 \times 10^{-13}. \)

Laser frequency-modulation to amplitude-modulation (FM–AM) [4, 6, 36, 37] noise: it is the amplitude noise induced by the laser carrier frequency noise through the atomic optical resonance. The slope of the signal \( V_{wp} \) with respect to the laser frequency \( f_L \) measured around the optical resonance is \( C_{\phi_1} = 71.5 \mu \text{ V MHz}^{-1}. \) According to equation (2), with the laser frequency noise data from figure 4(a) at 143 Hz: \( \sqrt{S_{SN}(F_M)} = 19.0 \text{ Hz Hz}^{-0.5}, \) we get an Allan deviation of \( 0.010 \times 10^{-13} \) at 1 s.

Laser AM–AM noise. This noise is defined as the amplitude noise induced by the laser intensity noise. The measured signal sensitivity to the laser power is \( C_{\phi_1} = 1.06 \text{ mV \mu W}^{-1} \) at \( P_L = 387 \mu \text{W}, \) combined with the laser RIN shown in figure 4(b), it leads to the amplitude noise \( C_{\phi_1} \times P_L \times \text{RIN}(143 \text{ Hz}) \)
Figure 8. $C$, FWHM, and $q$ versus the 3.417 GHz microwave power. The working parameters are $T_c = 3.5$ ms, $t_w = 1$ ms, and $P_L = 0.387$ mW.

Figure 9. Typical CPT and error signal of the clock transition. The working parameters are $T_c = 3.5$ ms, $t_w = 1$ ms, $P_L = 0.387$ mW, and $P_{\mu w} = 26.9$ dBm.

Figure 10. Allan standard deviations (ADEVs) of the LO frequency stability: free running (black squares), and locked (red dots) on the atomic resonance. The slope of the green dashed guide line is $3.6 \times 10^{-13} \tau^{-1/2}$.

$= 125.3$ nV Hz$^{-0.5}$, and an Allan deviation of $0.905 \times 10^{-13}$ at 1 s.

LO phase noise. The phase noise contribution to the CPT clock frequency stability via the intermodulation effect [38] can be estimated by $\sigma_{\phi,\text{LO}}(1 \text{ s}) \sim F_M \sqrt{S_\phi(2F_M)/f_c}$. With the phase noise value of 3.417 GHz signal from figure 2, the contribution to the Allan deviation is $0.442 \times 10^{-13}$ at 1 s.

Light-shifts. Here we address effects of the light-frequency- and-intensity instabilities on the clock frequency, i.e. the (FM–FM) conversion noise, the (AM–FM) conversion noise, and effect of microwave power ($P_{\mu w}$) noise. As a matter of fact, microwave power fluctuations induce light power redistributions among the various sidebands of the DBR laser diode emitted multicolor laser, thus leading to intensity light shift fluctuations. Microwave power fluctuations can also produce laser frequency variations through a thermal effect in the DBR laser diode, leading to frequency light shift fluctuations. The (FM–FM) sensitivity coefficient is $5.5 \times 10^{-13}$ MHz in relative unit, measured by shifting the laser frequency with the AOM while the clock is kept locked. The (AM–FM) sensitivity coefficient, $5.4 \times 10^{-9}$ mW, is measured by scanning the laser power around the working value with the clock locked. The $P_{\mu w}$ sensitivity, $6.6 \times 10^{-9}$ W, is measured in the same way. The various contributions to the overall Allan deviation are computed using equation (2) for Allan deviations at 1 s with the fluctuations of laser carrier frequency, laser power, and microwave power: $\sim 50$ Hz, $1.3$ nW, $31.9$ $\mu$W, respectively. (FM–FM, AM–FM) noise contributions are negligible; see table 1, while microwave power noise is the dominant effect at $2.1 \times 10^{-13}$.

The laser intensity noise after the interaction with the atomic vapor is shown in figure 11. It shows the different contributions to the amplitude noise, i.e., the detector noise, shot noise, laser FM–AM, and laser AM–AM noise. The noise spectral density is $140.8$ nV Hz$^{-0.5}$ at the Fourier frequency of 143 Hz, i.e., the microwave modulation frequency, which leads to an Allan deviation of $1.02 \times 10^{-13}$ at 1 s. According to table 1, this value is very close to the quadratic sum of the first four individual noise contributions, i.e., $0.98 \times 10^{-13}$, which supports our analysis.

The predicted short-term stability is close to the measured value considering that some contributions can be correlated.
Figure 11. Square root of the PSD of light intensity noise after the clock cell (blue, upper curve), detector noise (red, middle curve) and analyzer noise floor (black, bottom curve). The working parameters are the same as in figure 9.

Nevertheless, this analysis allows us to identify the main limitations. The largest contribution is the microwave-power fluctuations, which could be reduced by microwave power stabilization or choosing a certain laser power, referred to as ‘magic power point’ [39, 40]. The next dominant contribution is the laser RIN, which could be reduced by active laser power locking, or to a lesser extent by overcoming the RIN degradation induced by the frequency servo-loop observed in section 2.3.

As we focused on the high-quality-factor CPT signal and potential for a high-performance CPT clock, the mid-term and long-term frequency stabilities were not investigated yet. We aim to evaluate them after integration of the table experiment setup into a compact clock.

4. Outlook and summary

To the best of our knowledge, this is the first report on a CPT clock based on a direct-modulation diode laser to reach a short-term frequency stability of $3.6 \times 10^{-13} \pm 12\% (4 \leq \tau \leq 200 \text{ s})$, which is attributed mainly to three factors: (1) the low laser frequency and intensity noise DBR diode used in our scheme; (2) the laser frequency stabilization to the very narrow and strong dual-frequency sub-Doppler spectrum [34] that contributes to the very low FM noise shown in figure 4(a); (3) a CPT signal in the D1 line that is known to provide better CPT signals than the D2 line does as used in [11, 41], especially the narrow linewidth and high-quality-factor CPT signal observed in our experiment.

This progress may provide three opportunities for CPT clocks. First, our scheme could be applied to a CPT configuration with even higher contrast and quality factor, such as push-pull CPT, lin/lin CPT, DM CPT, and Ramsey-CPT, which could improve the CPT clock frequency stability to an even higher level. Second, the microwave power consumption in our scheme is comparable to that of the external modulator method [5]. However, with the DBR diode technology development, optimization of the impedance match, and cavity-enhanced modulation efficiency method [42, 43], we could be able to remarkably reduce the microwave power. This point is crucial to implement a compact high-performance CPT atomic clock. Third, the bulk AOM used in our laser locking setup could be omitted with a paraffin-coated clock cell [44, 45] or even the laser locking setup can be abandoned by using the clock cell. The degradation of the laser FM noise in this case is mitigated by the narrow linewidth diode laser, as well as the low FM–FM and FM–AM conversion coefficients. We can lock the laser frequency as usually implemented in CSAC. Combining the second and third suggestions, the laser could be adapted in CSAC and boost its performance.

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