Relativistic models for nuclear structure and low-energy QCD phenomenology

P Finelli
Physics Department, University of Bologna and INFN
Via Irnerio 46, 40126 Bologna (Italy)
E-mail: paolo.finelli@bo.infn.it
http://minimafisica.biodec.com

Abstract. In this contribution we will report about developments on a recently proposed relativistic nuclear energy density functional based on the characteristic features of low-energy QCD and its symmetry breaking pattern.

1. Introduction
The most complete and accurate description of structure phenomena in medium and heavy nuclei is currently provided by self-consistent non-relativistic and relativistic mean-field approaches. By employing phenomenological effective interactions, adjusted to empirical properties of symmetric and asymmetric nuclear matter, and to bulk properties of stable nuclei, self-consistent mean-field methods have achieved a high level of precision in describing ground states and properties of excited states in stable nuclei, exotic nuclei far from $\beta$-stability, and in nuclear systems at the nucleon drip-lines.

In the last years, a new approach has been proposed [1, 2, 3, 4] in which the effective interaction is constrained by two essential features of the low-energy (non-perturbative) sector of QCD. It is essentially based on the following conjectures:

(i) The nuclear ground state is characterized by strong scalar and vector mean fields which have short-range origin and can be satisfactorily interpreted as in-medium changes of the scalar quark condensate (the chiral condensate) and of the quark density [5].

(ii) Nuclear binding and saturation arise primarily from chiral (pionic) fluctuations (reminiscent of van der Waals forces) in combination with Pauli blocking effects, superimposed on the condensate background fields and calculated according to the rules of in-medium chiral perturbation theory [6].

2. Medium and long-range interaction: in-medium chiral dynamics
The starting point is the description of nuclear matter based on the chiral effective Lagrangian with pions and nucleons [6]. The relevant small scales are the Fermi momentum $k_f$, the pion mass $m_\pi$ and the $\Delta - N$ mass difference $\Delta \equiv M_\Delta - M_N \simeq 2.1 m_\pi$, all of which are well separated from the characteristic scale of spontaneous chiral symmetry breaking, $4\pi f_\pi \simeq 1.16$ GeV with the pion decay constant $f_\pi = 92.4$ MeV. The calculations have been performed to three-loop
order in the energy density. They incorporate the one-pion exchange Fock term, iterated one-pion exchange and irreducible two-pion exchange, including one or two intermediate Δ’s (see the insert in figure 2 for some relevant diagrams).

The only new ingredient in performing new calculations at finite nucleon density is the in-medium nucleon propagator. It expresses the fact that the ground-state of the system has changed from an empty vacuum to a filled Fermi sea of nucleons. As shown in figure 1 the full propagator splits additively into the vacuum nucleon propagator and a medium insertion. This allows to organize the diagrammatic calculation according to the number of medium insertions.

\[
G_N = (\not{p} + M_N)\left[\frac{i}{p^2 - M_N^2 + i\epsilon} - 2\pi\delta(p^2 - M_N^2)\theta(k_f - |p^2|)\theta(p_0)\right]
\]

**Figure 1.** The full nucleon propagator defined as the vacuum part plus the in-medium insertion.

The pion-nucleon interaction vertices are the pseudovector πNN-vertex and the Tomozawa-Weinberg-contact vertex of the form

\[
g_A \frac{\gamma_\mu q_a^\mu}{2f_\pi} \gamma_5 \tau_a \quad \text{and} \quad \frac{1}{4f_\pi^2} (\gamma_\mu q_b^\mu - \gamma_\mu q_b^\mu)\epsilon_{abc} \tau_c.
\]

As a first application, nuclear matter has been extensively studied [6]: the equation of state for symmetric and asymmetric systems, real and imaginary single particle potentials, the liquid-gas phase transition, the role of Δ-isobar degree of freedom, pairing gaps, the Landau-Migdal parameters and the chiral condensate dynamics. In figure 2 we show only a small selection of the results obtained in this framework, and in particular the very important role of Δ(1232) excitations in stabilizing the calculations is emphasized by blue thick lines. In the small insert we have written the nuclear matter equation of state as an expansion in powers of the Fermi momentum \(k_f\). The expansion coefficients are functions of \(k_f/m_\pi\) and \(\Delta/m_\pi\), the dimensionless ratios of the relevant small scales. Divergent momentum space loop integrals are regularized by introducing subtraction constants in the spectral representations of these terms. The (few) subtraction constants are the only parameters in this approach. They equivalently correspond to two- and three-nucleon contact interactions (and derivatives thereof), encoding short-distance dynamics not resolved in detail at the characteristic momentum scale \(k_f \ll 4\pi f_\pi\). The finite parts of the energy density, written in closed form as functions of \(k_f/m_\pi\) and \(\Delta/m_\pi\), represent long and intermediate range (chiral) dynamics with input fixed entirely in the \(\pi N\) sector. The low-energy constants (contact terms or subtraction constants) are adjusted to reproduce basic properties of symmetric and asymmetric nuclear matter at the saturation point. The same framework has also been used for other purposes: deriving a (non-relativistic) energy density functional [7], extracting the nuclear spin-orbit interaction [8] and the description of hyperons in a nuclear medium [9]. Concerning the spin-orbit interaction, and in particular its microscopic origin, it is worthy mentioning the new remarkable interpretation presented in Ref. [10]: large spin-orbit splittings in ordinary nuclei and almost vanishing spin-orbit splittings in Λ hypernuclei can naturally arise as different balancing mechanisms between short- and long-range components of the nuclear force without the introduction of any ad-hoc hypothesis.
\[ E(k_f) = \frac{E(k_f)}{A} = \sum_n F_n\left(\frac{k_f}{m_\pi} k_f/m_\Delta\right) \]
\[ = \frac{3k_f^2}{10M_N} + c_3 \frac{k_f^2}{M_N^2} + c_4 \frac{k_f^4}{M_N^3} + c_5 \frac{k_f^5}{M_N^4} + c_6 \frac{k_f^6}{M_N^5} + \ldots \]

**Figure 2.** Some results obtained with in-medium chiral perturbation theory. For details about calculations and figures see Ref. [6].
3. Short-distance dynamics: in-medium QCD sum rules

The QCD ground state is characterized by condensates of quark-antiquark pairs and gluons, an entirely non-perturbative phenomenon. The quark condensate \( \langle \bar{q}q \rangle \), i.e. the ground state expectation value of the scalar quark density, plays a particularly important role as an order parameter of spontaneously broken chiral symmetry. At a renormalization scale of about 1 GeV (with up and down quark masses \( m_u + m_d \simeq 12 \text{ MeV} \)) the strength of the chiral vacuum condensate is \( \langle \bar{q}q \rangle_0 \simeq -(240 \text{ MeV})^3 \simeq -1.8 \text{ fm}^{-3} \), more than an order of magnitude larger than the nuclear matter density at saturation \( \rho_0 \simeq 0.16 \text{ fm}^{-3} \). Hadrons, as well as nuclei, are excitations built on this condensed ground state. The strength of the chiral condensate at normal nuclear densities is reduced by about one third from its vacuum value. The density-dependent changes of the condensate structure in the presence of baryonic matter are a source of strong scalar and vector fields experienced by nucleons (and hyperons). In-medium QCD sum rules relate the leading changes of the scalar quark condensate and quark density at finite baryon density, with the scalar and vector self-energies of a nucleon in the nuclear medium.

The nuclear ground state is characterized by strong scalar \( U_S \) and vector \( U_V \) mean fields which have their origin in the in-medium changes of the scalar quark condensate (the chiral condensate) and of the quark density (more precisely, nucleons in the medium acquire large scalar and vector self-energies). They can be calculated by QCD sum rules techniques [5] to obtain, at leading order, an approximation valid at nucleon densities below and around saturated nuclear matter,

\[
U_S = -\frac{\sigma_N M_N}{m_\pi^2 f_\pi^2} \rho_S \quad \text{and} \quad U_V = \frac{4(m_u + m_d)M_N}{m_\pi^2 f_\pi^2} \frac{\rho}{\rho_S} \tag{1}
\]

where \( \sigma_N = \langle N | \bar{q}q | N \rangle \) is the nucleon sigma term \((\simeq 50 \text{ MeV})\), \( m_\pi \) the pion mass \((138 \text{ MeV})\), \( M_N \) the nucleon mass \((939 \text{ MeV})\), \( m_u,d \) the quark masses \((\simeq 5 \text{ MeV})\), \( f_\pi \) the pion decay constant and \( \rho \) and \( \rho_S \) are the baryon and the scalar density, respectively. The resulting \( U_S \) and \( U_V \) are individually of the order of \( 300 - 400 \text{ MeV} \) in magnitude. \( G_S^{(0)} \) and \( G_V^{(0)} \) could be interpreted as coupling strengths for an effective contact nucleon-nucleon interaction. It is remarkable that the ratio

\[
\frac{U_S}{U_V} = -\frac{\sigma_N}{4(m_u + m_d)} \frac{\rho_S}{\rho} \tag{2}
\]

is close to \(-1\). As a result, in the single-nucleon Dirac equation there is an almost complete cancellation in the central potential \((\sim U_V + U_S)\) (a feature characteristic of relativistic mean-field phenomenology), giving a negligible contribution to the binding of the system, but, at the same time, a large contribution to the spin-orbit potential

\[
V_{LS} \sim \frac{1}{2M_N^2} \frac{1}{r} \left( \frac{\partial}{\partial r} (U_V - U_S) \right) \mathbf{l} \cdot \mathbf{s}. \tag{3}
\]

Of course, the constraints implied by Eqs. (1) are not very accurate on a quantitative level. Corrections from condensates of higher dimension and uncertainties in the values of \( \sigma_N \) and \( m_u + m_d \) lead to an estimated error for the ratio \( \Sigma_S^{(0)} / \Sigma_V^{(0)} \simeq -1 \) of about 20%. As a consequence QCD sum rules calculations should be used as theoretical estimates for a posteriori comparison with the fitted values for \( G_S^{(0)} \) and \( G_V^{(0)} \).

4. The model

The relativistic density functional describing the ground-state energy of the system can be written as a sum of four distinct terms:

\[
E_0[\hat{\rho}] = E_{\text{free}}[\hat{\rho}] + E_H[\hat{\rho}] + E_{\text{coul}}[\hat{\rho}] + E_\pi[\hat{\rho}] \tag{4}
\]
with

\[ E_{\text{free}}[\hat{\rho}] = \int d^3r \left\langle \phi_0 | \hat{\psi} [-i \gamma \cdot \nabla + M_N] \psi | \phi_0 \right\rangle , \]  
\[ E_{\text{H}}[\hat{\rho}] = \frac{1}{2} \int d^3r \left\{ \langle \phi_0 | G_S^{(0)} (\hat{\psi} \psi)^2 | \phi_0 \rangle + \langle \phi_0 | G_V^{(0)} (\bar{\psi} \gamma_\mu \psi)^2 | \phi_0 \rangle \right\} , \]  
\[ E_{\pi}[\hat{\rho}] = \frac{1}{2} \int d^3r \left\{ \langle \phi_0 | G_S^{(\pi)} (\hat{\rho})(\bar{\psi} \psi)^2 | \phi_0 \rangle + \langle \phi_0 | G_V^{(\pi)} (\bar{\psi} \gamma_\mu \psi)^2 | \phi_0 \rangle + \langle \phi_0 | G_{T_S}^{(\pi)} (\bar{\psi} \tau \psi)^2 | \phi_0 \rangle + \langle \phi_0 | G_{T_V}^{(\pi)} (\bar{\psi} \gamma_\mu \tau \psi)^2 | \phi_0 \rangle \right\} , \]  
\[ E_{\text{cont}}[\hat{\rho}] = \frac{1}{2} \int d^3r \left\langle \phi_0 | A^0 e^\rho \frac{\gamma_3}{2} (\bar{\psi} \gamma_\mu \psi) | \phi_0 \right\rangle , \]  

where |\phi_0\rangle denotes the nuclear ground state. Here \( E_{\text{free}} \) is the energy of the free (relativistic) nucleons including their rest mass. \( E_{\text{H}} \) is a Hartree-type contribution representing strong scalar and vector mean fields, in connection with the leading terms of the corresponding nucleon self-energies deduced from in-medium QCD sum rules. Furthermore, \( E_{\pi} \) and \( E_{\text{cont}} \) are the part of the energy generated by chiral \( \pi N \Delta \)-dynamics, including a derivative (surface) term. For more details we refer the reader to Ref. [3].

Minimization of the ground-state energy, represented in terms of a set of auxiliary Dirac spinors \( \psi_k \), leads to the relativistic analogue of the Kohn-Sham equations in atomic physics. These single-nucleon Dirac equations are solved self-consistently in the “no-sea” approximation.

In the mean-field approximation, the single-nucleon Dirac equation is found by minimization with respect to \( \psi_k \):

\[ [-i \gamma \cdot \nabla + M_N + \gamma_0 \Sigma_V + \gamma_0 \tau_3 \Sigma_{TV} + \gamma_0 \Sigma_R + \Sigma_S + \tau_3 \Sigma_{T_S}] \psi_k = \epsilon_k \psi_k , \]  

where \( \Sigma_i \) are the self-energies in the mean field approximation (see Ref. [3])

\[ \Sigma_V = [G^{(0)}_V + G^{(\pi)}_V (\rho)] \rho + \epsilon A^0 \frac{1 + \tau_3}{2} , \]  
\[ \Sigma_{TV} = G^{(\pi)}_{TV} (\rho) \rho_3 , \]  
\[ \Sigma_S = [G^{(0)}_S + G^{(\pi)}_S (\rho)] \rho_S + D^{(\pi)}_S \nabla^2 \rho_S , \]  
\[ \Sigma_{TS} = G^{(\pi)}_{T_S} (\rho) \rho_{S3} , \]  
\[ \Sigma_R = \frac{1}{2} \left\{ \frac{\partial G^{(\pi)}_V (\rho)}{\partial \rho} \rho^2 + \frac{\partial G^{(\pi)}_S (\rho)}{\partial \rho} \rho_S^2 + \frac{\partial G^{(\pi)}_{TV} (\rho)}{\partial \rho} \rho_3 + \frac{\partial G^{(\pi)}_{T_S} (\rho)}{\partial \rho} \rho_{S3} \right\} , \]  

\( A^0(r) = \frac{\epsilon}{4\pi} \int d^3r' \frac{\rho(r')}{|r-r'|} \) is the Coulomb potential and \( \epsilon_k \) denotes the single-nucleon energy for the \( k \) particle. \( \Sigma_R \) is the rearrangement term. As usual, densities are defined as (all spatial components are vanishing for spin saturated systems)

\[ \rho = \left\langle \phi_0 | \bar{\psi} \gamma_0 \psi | \phi_0 \right\rangle = \rho^p + \rho^n , \]  
\[ \rho_3 = \left\langle \phi_0 | \bar{\psi} \tau_3 \gamma_0 \psi | \phi_0 \right\rangle = \rho^p - \rho^n , \]  
\[ \rho_S = \left\langle \phi_0 | \bar{\psi} \tau_2 \psi | \phi_0 \right\rangle = \rho_S^p + \rho_S^n , \]  
\[ \rho_{S3} = \left\langle \phi_0 | \bar{\psi} \tau_3 \tau_2 \psi | \phi_0 \right\rangle = \rho_{S3}^p - \rho_{S3}^n , \]  
\[ \rho_{ch} = \left\langle \phi_0 | \bar{\psi} \gamma_0 \frac{1 + \tau_3}{2} \psi | \phi_0 \right\rangle = \rho^p . \]
The density dependent couplings $G_i(\hat{\rho})$ ($i = S, V, TS, TV$) can be written as a sum of two distinct terms:

$$G_i(\hat{\rho}) = G_i^{(0)}(\hat{\rho}) + G_i^{(\pi)}(\hat{\rho}) \quad \text{(for } i = S, V \text{)}$$

and

$$G_i(\hat{\rho}) = G_i^{(\pi)}(\hat{\rho}) \quad \text{(for } i = TS, TV \text{)},$$

i.e. density-independent parts $G_i^{(0)}$, which arise from strong isoscalar scalar and vector background fields, and density-dependent parts $G_i^{(\pi)}(\hat{\rho})$ generated by (regularized) one- and two-pion exchange dynamics (see Ref. [3] for a complete explanation). It is assumed that only pionic processes contribute to the isovector channels. There are 7 free parameters ($G_S^{(0)}, G_V^{(0)}$, two isoscalar and two isovector contact terms in the contact couplings $G_i^{(\pi)}(\hat{\rho})$ and the surface term $D_S^{(\pi)}$) that have to be adjusted in order to reproduce ground-state properties of closed shell nuclei [3].

5. Results

To show the quality of present approach and the capability to make predictions about the structure of finite nuclei, we will show some (unpublished) results for the Isovector Giant Dipole Resonances (IVGDR) in closed shell nuclei. In order to perform calculations of these collective states, we worked in the Random Phase Approximation (RPA), i.e. we solved the following set of coupled equations [12]

$$\omega \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) - \left( \begin{array}{cc} A & B \\ B^* & A^* \end{array} \right) \left( \begin{array}{c} X \\ Y \end{array} \right) = 0.$$  \hspace{1cm} (21)

The $A$ and $B$ matrices are defined as

$$A = \left( (\epsilon_p - \epsilon_h)\delta_{pp'}\delta_{hh'} + (\epsilon_a - \epsilon_h)\delta_{aa'}\delta_{hh'} \right) + \left( V_{ph'h'} V_{ph'a'} \right),$$

$$B = \left( V_{pp'h'} V_{pa'h'} V_{pa'h'} V_{aa'h'}^* \right),$$  \hspace{1cm} (22)

where $p$ labels particle states, $a$ antiparticles and $h$ holes. $X$ and $Y$ amplitudes are defined as follows

$$X = \left( \frac{\delta p_{ph}}{\delta p_{ah}} \right) \quad \text{and} \quad Y = \left( \frac{\delta p_{hp}}{\delta p_{ha}} \right),$$  \hspace{1cm} (24)

in terms of the transition densities. For a derivation of the RPA equations and a complete discussion about their properties we refer the reader to the well known literature [13]. Multipole transitions are described by the reduced transition probability

$$B^T(EJ, J_i \rightarrow J_f) = \frac{1}{2J_i + 1} |\langle f||\hat{Q}^T_j||i\rangle|^2,$$  \hspace{1cm} (25)

where $\hat{Q}^T_j$ corresponds to the electric multipole transition. In case of IVGDR we have

$$\hat{Q}^T_{1M} = e \sum_{i=1}^{A} \left( \frac{\tau}{2A} - \frac{N - Z}{2A} \right) r_i Y_{1M}(\hat{r}_i),$$  \hspace{1cm} (26)

where the center-of-mass motion has been conveniently subtracted. $B(E)$ is usually employed to calculate

$$R(E) = \sum_i B(E_i) \frac{\Gamma^2}{4(E - E_i)^2 - \Gamma^2},$$  \hspace{1cm} (27)
where $\Gamma$ is a phenomenological width (= 1 MeV). In figure 3 we show calculations for some closed-shell nuclei. The agreement with experimental data [14] is fairly good, in particular for heavy systems. Phenomenological approaches give very similar results [12].

6. Conclusions
This short contribution is able to demonstrate that chiral effective field theory provides a consistent microscopic framework in which both the isoscalar and isovector channels of a universal nuclear energy density functional can be formulated. We believe that the present

---

|            | $^{16}$O  | $^{40}$Ca | $^{90}$Zr | $^{208}$Pb |
|------------|----------|----------|-----------|------------|
| Exp.       | 23.8±0.5 | 19.8±0.5 | 16.5±0.2  | 13.5±0.2   |
| FKVW       | 20.1     | 17.7     | 16.7      | 13.1       |

Figure 3. Isovector giant dipole resonances for $^{16}$O, $^{40}$Ca, $^{90}$Zr and $^{208}$Pb. Experimental values [14] are denoted with red arrows. In case of lead we also show the (total, neutron and proton) transition densities at the peak energy.
approach to nuclear DFT establishes a fundamental link between low-energy QCD and ground-state properties of finite nuclei.

Acknowledgements
I would like to thank MIUR and INFN for financial support and my collaborators: S. Fritsch, N. Kaiser and W. Weise of the Technical University of Munich (Germany) and D. Vretenar, T. Nikšić and Nils Paar of the University of Zagreb (Croatia).

References
[1] Finelli P, Kaiser N, Vretenar D and Weise W, 2002 Eur. Phys. J. A 17 573
[2] Finelli P, Kaiser N, Vretenar D and Weise W, 2004 Nucl. Phys. A 735 449
[3] Finelli P, Kaiser N, Vretenar D and Weise W, 2006 Nucl. Phys. A 770 1
[4] Finelli P, Kaiser N, Vretenar D and Weise W, 2007 Nucl. Phys. A 791 57
[5] Cohen T D, Furnstahl R J and Griegel D K 1991 Phys. Rev. Lett. 67 961; Furnstahl R J, Griegel D K and Cohen T D 1992 Phys. Rev. C 46 1507; Cohen T D, Furnstahl R J and Griegel D K 1995 Prog. Part. Nucl. Phys. 35 221; Drukarev E G and Levin E M 1990 Nucl. Phys. A 511 679; Drukarev E G and Levin E M 1991 Prog. Part. Nucl. Phys. 27 77
[6] Kaiser N, Fritsch S and Weise W 2002 Nucl. Phys. A 697 255; Kaiser N, Fritsch S and Weise W 2002 Nucl. Phys. A 700 343; Fritsch S, Kaiser N and Weise W 2002 Phys. Lett. B 545 73; Fritsch S and Kaiser N 2003 Eur. Phys. J. A 17 11; Fritsch S and Kaiser N 2004 Eur. Phys. J. A 21 117; Fritsch S, Kaiser N and Weise W 2005 Nucl. Phys. A 750 259; Kaiser N, Nikšić T and Vretenar D 2005 Eur. Phys. J. A 25 257; Kaiser N 2006 Nucl. Phys. A 768 99; Kaiser N, de Homont P and Weise W 2008 Phys. Rev. C 77 052504; Kaiser N and Weise W 2009 Phys. Lett. B 671 25;
[7] Kaiser N, Fritsch S and Weise W 2003 Nucl. Phys. A 724 47; Kaiser N 2003 Phys. Rev. C 68 014323;
[8] Kaiser N 2002 Nucl. Phys. A 709 251; Kaiser N 2003 Nucl. Phys. A 720 157; Kaiser N 2003 Phys. Rev. C 68 054001; Kaiser N 2004 Phys. Rev. C 70 034307;
[9] Kaiser N and Weise W 2005 Phys. Rev. C 71 015203; Finelli P, Kaiser N, Vretenar D and Weise W 2007 Phys. Lett. B 658 90; Finelli P, Kaiser N, Vretenar D and Weise W 2009 submitted to Nucl. Phys. A
[10] Kaiser N and Weise W 2008 Nucl. Phys. A 804 60;
[11] Ioffe B L 2003 Phys. At. Nucl. (Yad. Fiz.) 66 30 and references therein
[12] Ring P, Ma Z Y, Van Giai N, Vretenar D, Wandelt A and Cao L G (2001) Nucl. Phys. A 694 249; Ma Z Y, Wandelt A, Van Giai N, Vretenar D, Ring P and Cao L G (2002) Nucl. Phys. A 703 222; Paar N, Ring P, Nikšić T and Vretenar D (2003) Phys. Rev. C 67 034312;
[13] Ring P and Schuck P 1980 The Nuclear Many Body Problem (Springer-Verlag)
[14] Gleissl P, Brack M, Meyer J and Quentin P 1990 Ann. Phys. (N.Y.) 197 205