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Abstract. Rotary screw machine (or a machine on the screw) has been widely used in Russia in 1960-1970. In contrast to vehicles equipped with conventional types of propulsion, the dynamics of screw machines is poor. The uniqueness of calculation of screw machines in the geometric linear movement of the screw is considered.

1. Introduction

Previous screw-propelled vehicles were designed and built with a rigid or semi-rigid suspension system. Within the framework of the investigation the design of a screw-propelled vehicle has been proposed with a novel visco-elastic suspension capable of decreasing the dynamic loads on the vehicle’s body, which arise due to the unbalance of the screw rotors and the bearing surface. The author’s investigation in this field have been carried out since 1996 resulting in the articles [3, 4]; for the last 20 years the mathematical theories obtained have been improved and put into practice.

2. Screw geometry

An analysis of displacements of the screw-propelled vehicle permits one to obtain the scheme of interaction between the vehicle and the environment. The mathematical model displays a geometric line depending on the work of front and rear suspensions. This effect is unique only for vehicles with rotor propulsion devices. The linearity of the contact of the bearing surface and the rotors is shown in Figure 1. (Geometric parameters of the propulsion device of the screw-propelled vehicle when overcoming an obstacle.)

The vertical forces arising from irregularities of the pathway are transmitted to the vehicle’s body only through springing elements and dampers, as shown in Figure 2 (three-mass equivalent system of dynamics of the screw-propelled vehicle).

The coordinates describing position of the sprung and unsprung masses under vibration are chosen depending on the problem under consideration. When studying vibrations of the vehicle’s body, it is appropriate to choose coordinates $z_0, \varphi, x_0, y_0, \beta$ i.e., the displacements of the center of gravity of the sprung part and the angles of its rotation. It is also necessary to consider:

- $z_1, z_2, z_3, z_4$ - coordinates of displacement of the body’s points above the axis of the front or back mountings of the rotor propulsion devices;
- $x_1, x_2, x_3, x_4$ - coordinates of horizontal lengthwise displacements of the body’s points of the front or back mountings of the rotor propulsion devices;
- $y_1, y_2, y_3, y_4$ - coordinates of horizontal lateral displacements of the body’s points of the front or back mountings of the rotor propulsion devices.

![Figure 1. The geometric dependence of the rotor’s parameters](image)

Investigating the vibration system shown in Fig. 2, we infer the dependencies between the parallel displacement vectors $\vec{z}_1, \vec{z}_2, \vec{z}_3, \vec{z}_4$ and the system’s resultant $\vec{Z}_0$ Figure 3:

$$Z_0 = \frac{\sum l_i \vec{z}_i}{\sum l_i} \tag{1}$$

Using the above-mentioned formula, we obtain the system of dependencies between the geometric parameters of the sprung mass of the vehicle:
\[
\begin{align*}
\dot{Z}_0 &= \frac{1}{2t}[(\ddot{x}_l - \ddot{Z}_1 l_1) + (\ddot{x}_4 - \ddot{Z}_4 l_2)] \\
\varphi &= \frac{1}{b}[(\ddot{x}_4 - \ddot{x}_3) + (\ddot{x}_3 - \ddot{x}_2)] + \frac{1}{l}[(\ddot{y}_4 - \ddot{y}_2) + (\ddot{y}_2 - \ddot{y}_3)] \\
\ddot{X}_0 &= \frac{1}{b}[(\ddot{y}_4 b_2 - \ddot{x}_1 b_1) + (\ddot{x}_3 b_2 - \ddot{x}_2 b_1)] \\
\alpha &= \frac{1}{b}[(\ddot{z}_1 - \ddot{Z}_4) + (\ddot{z}_2 - \ddot{Z}_3)] \\
\ddot{Y}_0 &= \frac{1}{l}[(\ddot{y}_1 l_1 - \ddot{y}_2 l_2) + (\ddot{y}_3 l_1 - \ddot{y}_3 l_2)] \\
\beta &= \frac{1}{l}[(\ddot{z}_2 - \ddot{Z}_1) + (\ddot{z}_3 - \ddot{Z}_4)]
\end{align*}
\]

(2)

Vibrations of the unsprung masses of the vehicle (rotors) are given by the elements of displacements \( \xi_1, \xi_2, \xi_3, \xi_4 \).

The uniqueness of this vibration system is that the displacements of the end points of the rotors are linearly dependent mutually. When colliding with an obstacle, not only the front suspension is actuated but also the force is transmitted to the back suspension by the rotor’s body, therefore, the quantities \( \xi_2, \xi_3 \) influence the quantities of displacements \( \xi_1, \xi_4 \).

The linear dependence of displacements of the end points of the rotors is represented in the system of equations 3.

\[
\begin{align*}
\xi_1 &= \sin \beta \cdot L - \xi_2 \\
\xi_2 &= \sin \beta \cdot L - \xi_1 \\
\xi_3 &= \sin \beta \cdot L - \xi_4 \\
\xi_4 &= \sin \beta \cdot L - \xi_3
\end{align*}
\]

(3)

To derive the dynamical equations, one should apply the forces \( Z_n, X_n, Y_n \) acting on the masses of the vehicle (Fig. 2). The force \( Z_n \) transmitted through the suspension consists of two terms: \( Z_p \) - from the springing element and \( Z_a \) - from the damper. The forces \( Z_n, X_n \) and \( Y_n \) replace the suspension’s quantities and are interdependent.

We obtain a system of equations (4) describing the dependencies of dynamic forces.

\[
\begin{align*}
Z_{n1} &= 2C_{p1}(z_1 - \xi_1) + 2k_1(\dot{z}_1 - \dot{\xi}_1); \\
Z_{n2} &= 2C_{p2}(z_2 - \xi_2) + 2k_2(\dot{z}_2 - \dot{\xi}_2); \\
Z_{n3} &= 2C_{p3}(z_3 - \xi_3) + 2k_1(\dot{z}_3 - \dot{\xi}_3); \\
Z_{n4} &= 2C_{p3}(z_3 - \xi_3) + 2k_3(\dot{z}_3 - \dot{\xi}_3); \\
X_{n1} &= Z_{n1} \tan \beta; \\
X_{n2} &= Z_{n2} \tan \beta; \\
Y_{n1} &= Z_{n1} \tan \alpha; \\
Y_{n2} &= Z_{n2} \tan \alpha; \\
X_{n3} &= Z_{n3} \tan \beta; \\
X_{n4} &= Z_{n4} \tan \beta; \\
Y_{n3} &= Z_{n3} \tan \alpha; \\
Y_{n4} &= Z_{n4} \tan \alpha.
\end{align*}
\]

(4)

3. Differential equation of oscillations

Differential equations of vibrations are obtained by using Lagrange’s equations.

For sprung and unsprung masses \( M \) and \( m_{1,2} \) the following systems of equations of equilibrium are derived.

\[
\begin{align*}
(m\ddot{\xi}_1 - 2C_{p1}[z_1 - \xi_1] - 2k_1[\dot{z}_1 - \dot{\xi}_1]) + \\
(m\ddot{\xi}_2 - 2C_{p2}[z_2 - \xi_2] - 2k_2[\dot{z}_2 - \dot{\xi}_2]) = H_z(t); \\
(m\ddot{\xi}_4 - 2C_{p4}[z_4 - \xi_4] - 2k_4[\dot{z}_4 - \dot{\xi}_4]) + \\
(m\ddot{\xi}_3 - 2C_{p3}[z_3 - \xi_3] - 2k_3[\dot{z}_3 - \dot{\xi}_3]) = H_z(t)
\end{align*}
\]

(5)

The equations of motion for the coordinate systems (Figures 1 and 2) are derived using the formulas of the systems 3 and 4 and the expressions for \( Z_n \) which are written in terms of coordinates \( z_1, z_2, z_3, z_4 \).

Substituting these expressions into the differential equations of equilibrium we obtain a system of differential equations 5 and 6 which represent the most complete and accurate calculation of (linear and angular) displacements of points of the sprung and unsprung masses of the screw-propelled vehicle.
\[
M \ddot{z}_0 + (2k_1 [\dot{z}_1 - \dot{\xi}_1] + 2C_{p1} [z_1 - \xi_1]) + \\
(2k_2 [\dot{z}_2 - \dot{\xi}_2] + 2C_{p2} [z_2 - \xi_2]) + (2k_3 [\dot{z}_3 - \dot{\xi}_3] + 2C_{p3} [z_3 - \xi_3]) + \\
+ (2k_4 [\dot{z}_4 - \dot{\xi}_4] + 2C_{p4} [z_4 - \xi_4]) = H_z(t);
\]
\[
M \ddot{\phi} + \tan \alpha (2k_1 [\dot{z}_1 - \dot{\xi}_1] + 2C_{p1} [z_1 - \xi_1]) + \\
+ \tan \beta (2k_2 [\dot{z}_2 - \dot{\xi}_2] + 2C_{p2} [z_2 - \xi_2]) + \\
+ \tan \beta (2k_3 [\dot{z}_3 - \dot{\xi}_3] + 2C_{p3} [z_3 - \xi_3]) + \\
+ \tan \beta (2k_4 [\dot{z}_4 - \dot{\xi}_4] + 2C_{p4} [z_4 - \xi_4]) = H_y(t);
\]
\[
M \ddot{\psi} + \tan \beta + \left(\frac{2C_{p1} b_1 [z_1 - \xi_1] + 2k_1 b_1 [\dot{z}_1 - \dot{\xi}_1] + \\
+ 2C_{p4} b_1 [z_4 - \xi_4] + 2k_4 b_1 [\dot{z}_4 - \dot{\xi}_4] - \\
- 2C_{p2} b_2 [z_2 - \xi_2] - 2k_2 b_2 [\dot{z}_2 - \dot{\xi}_2] - \\
- 2C_{p3} b_3 [z_3 - \xi_3] - 2k_3 b_3 [\dot{z}_3 - \dot{\xi}_3]}{2C_{p1} b_1 [z_1 - \xi_1] + 2k_1 b_1 [\dot{z}_1 - \dot{\xi}_1] + \\
+ 2C_{p4} b_1 [z_4 - \xi_4] + 2k_4 b_1 [\dot{z}_4 - \dot{\xi}_4] - \\
- 2C_{p2} b_2 [z_2 - \xi_2] - 2k_2 b_2 [\dot{z}_2 - \dot{\xi}_2] - \\
- 2C_{p3} b_3 [z_3 - \xi_3] - 2k_3 b_3 [\dot{z}_3 - \dot{\xi}_3]}\right)\tan \alpha = M_{\phi}(t)
\]
\[
M \ddot{\varphi} + 2C_{p1} b_1 [z_1 - \xi_1] + 2k_1 b_1 [\dot{z}_1 - \dot{\xi}_1] + 2C_{p2} b_2 [z_2 - \xi_2] + \\
+ 2k_2 b_2 [\dot{z}_2 - \dot{\xi}_2] - 2C_{p4} b_2 [z_4 - \xi_4] - 2k_4 b_2 [\dot{z}_4 - \dot{\xi}_4] - \\
- 2C_{p3} b_3 [z_3 - \xi_3] - 2k_3 b_3 [\dot{z}_3 - \dot{\xi}_3] = M_{\varphi}(t)
\]
\[
M \ddot{\psi} + 2C_{p1} b_1 [z_1 - \xi_1] + 2k_1 b_1 [\dot{z}_1 - \dot{\xi}_1] + 2C_{p4} b_4 [z_4 - \xi_4] + \\
+ 2k_4 b_4 [\dot{z}_4 - \dot{\xi}_4] - 2C_{p2} b_2 [z_2 - \xi_2] - 2k_2 b_2 [\dot{z}_2 - \dot{\xi}_2] - \\
- 2C_{p3} b_3 [z_3 - \xi_3] - 2k_3 b_3 [\dot{z}_3 - \dot{\xi}_3] = M_{\psi}(t)
\]

The system of equations (6) shows calculation of forces and describes the dynamics of the actuated mechanisms of visco-elastic suspensions. The forces horizontal toX and Y, are then reduced to the vertical forces via Z.

To solve systems of equations (5) and (6) by numerical methods is possible if we know vibration values, i.e., if the boundary values of the quantities are \( z_1, z_2, z_3, z_4, Z_0, X_0, \phi, \alpha, \beta, \xi_1, \xi_2, \xi_3, \xi_4 \).

The vehicle’s parameters (LandB) obtained experimentally have been specified in advance, and one should determine the drag coefficients of the dampers \( k_1, k_2, k_3, k_4 \) and the spring rate for elements \( C_{p1}, C_{p2}, C_{p3}, C_{p4} \).

Having an exact solution to the system of equations (6), we can find numerically \( C_p \).

Assuming that the screw-propelled vehicle is geometrically symmetric and the characteristics of the visco-elastic suspension are completely of the same type, performing mathematical operations allows to reduce system of equations (5) and (6) to the form of systems of differential equations (7) and (8).

Thus, the generalized systems of differential equations take the form for the unsprung masses:
Fig. 3 shows the amplitude-frequency characteristic for the visco-elastic suspension of the screw-propelled vehicle.

4. **Machine design solutions**

To ensure the best possible contact between the propulsion devices and the bearing surface, a new design of the screw-propelled vehicle (Fig. 3) is developed within the framework of this study. This design comprises a body (1), a rotor propulsion device (5), a visco-elastic suspension (12) of rotors with springs and dampers, wherein the dampers (13) and springs (14) of the visco-elastic suspension are aligned and rigidly attached to the vehicle’s frame and coupling elements (15) installed on a fixed spindle (6) bearing electric motors (7) and harmonic drive units (8), the latter are rigidly connected with the rotor (5) through the bushes (9) to which trapezoidal rod dings (11, 16) hinged to the frame (10) elements are attached.
This vehicle has not two but four rotor propulsion devices, which, by virtue of the visco-elastic suspension, provide the greatest tractive force due to an increased contact point between the rotors and the bearing surface.

For increasing the vehicle’s vibroprotection and comfort of the driver, quite a number of design concepts for hydraulic vibratory bearings [1, 2] have been proposed. Evaluation of the quantities of vibration displacements of rotor propulsion devices in the bearings by the method of contactless measurement becomes possible applying an ultrasonic phase vibration transducer [7]. The design concept was awarded the bronze medal in Seoul.

Thus, significant practical and theoretical experience has been gained in investigating and adjusting parameters of the visco-elastic suspension of vehicles having a linear contact between the propulsion devices and the bearing surface.

![Design of the screw-propelled vehicle with four propulsion devices](image)

**Figure 4.** Design of the screw-propelled vehicle with four propulsion devices

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