Retardation of Entanglement Decay of Two Spin Qubits by Quantum Measurements

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We study a system of two electrons spins each interacting with its small nuclear spin environment (NSE), which is a prototype system of two electron spin quantum dot (QD) qubits. We propose a way to counteract the decay of entanglement in two-electron spin subsystem (TESSS) by performing some manipulations on TESSS (the subsystem to which experimentalists have an access), e.g. repeatable quantum projective measurements of TESSS. Unlike in the quantum Zeno effect, the goal of the proposed manipulations is not to freeze TESSS in its initial state and to preclude any time evolution of the state by infinitely frequent quantum measurements. Instead of that, performing a few cycles of fast decay is the Fermi contact hyperfine interaction of electron spin with nuclear spins of atoms from which the nanostructure is built [6, 7]. The main factor of such a fast decay is the Fermi contact hyperfine interaction of electron spin with nuclear spins of atoms from which the nanostructure is built [6, 7].

There have already been proposed and implemented in the experiment a few strategies to enhance the decoherence times of electron spin such as: dynamical decoupling of spin qubits from their environment [8]; preparing an artificial state of environment (so-called narrowed state of nuclear spin bath) [9]; or simply making use of materials which are made of atoms with spinless nuclei, e.g. isotopically-purified $^{28}$Si [10]. All these strategies can be summarized as: avoiding as much as possible any interaction of the qubits with their environments.

In this paper we propose the opposite strategy to counteract the decoherence and induced by it decay of quantum correlations of two electron spin qubits. We explore the process of transfer of coherences and quantum correlations from a pair of entangled qubits to the environment combined with quantum measurements of the qubits’ subsystem. Provided that the environment is non-Markovian, e.g. preserves some memory of past interactions, it turns out that it, being in a quantum state obtained after a period of free evolution of the system, can dephase qubits with a lower rate. Using a simple model of a system of two electron spin QD qubits, presented in Sec. II, we investigate the effect of application of the manipulation procedure described in Sec. III on dynamics of entanglement decay. Results are discussed in Sec. IV, where it is shown that both parts of the procedure, namely free evolution of the system and quantum measurement of the qubits’ subsystem with subsequent postselection of the two-qubit quantum state, are equally important, and that only for specific combinations of durations of free evolution periods $\tau$ and number of cycles $n$, significant retardation of entanglement decay can be achieved.

We would like to stress that the proposed manipulation procedure is not a realization of the quantum Zeno effect [11]. Here, the goal is not to freeze qubits in their initial state and to preclude any time evolution of the state by infinitely frequent quantum measurements. Instead of that, we let qubits interact with their environments and transfer to them some coherences and quantum correlations during joint system evolution.

I. INTRODUCTION

Spin of an electron localized on a quantum dot (QD) in a semiconductor nanostructure is a promising physical realization of qubit as it can be reliably initialized, manipulated, and read out [1, 2]. To be a useful element of a quantum computer, such a qubit must fulfill, among others, the criterion of long decoherence times [3, 4]. Providing no manipulations aimed at mitigation of the influence exerted by the environment on the spin qubits have been applied, coherences as well as quantum correlations of a pair of electron spin QD qubits decay on a nanosecond timescale [5]. The main factor of such a fast decay is the Fermi contact hyperfine interaction of electron spin with nuclear spins of atoms from which the nanostructure is built [6, 7].

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II. THE MODEL OF ELECTRON SPIN QUANTUM DOT QUBITS

First, we describe the model of electron spin QD qubits, which we use to illustrate the proposed manipulation sequence. We consider a system of two semiconductor QDs (e.g. self-assembled InGaAs QDs or gated QDs created in GaAs-AlGaAs nanostructure), each of which has a localized electron on it. As such systems usually are operated at low temperatures, we suppose that electrons are in their ground states. In such a case one can exclude from further consideration the spatial part of the
electron’s wave functions and focus only on the spin parts of the wave functions.

For the sake of clarity, we also assume that during periods of free evolution of the system there is no any inter-QD interaction, which could create some entanglement between the two QDs and, especially, between electron spins (e.g. electrons are strongly localized on QDs because of a high enough inter-QD potential barrier or relatively long distance between the QDs and the electron’s wave functions hardly overlap, so no interaction between the two electron spins occurs). It is worth to mention that we will be analyzing the behaviour of entanglement on the time scale from 0 to a few $T_2^*$. For such short times none realistic part of the interaction Hamiltonian, apart from the Fermi contact hyperfine interaction of electron spin with nuclear spins from its environment, is essential, because it does not manifest at short times (the energies of dipolar or quadrupolar interaction of nuclear spins are orders of magnitude lower than energy of hyperfine interaction), whereas the Fermi contact hyperfine interaction leads to the fast complete decay of initially present in the electron spins subsystem entanglement in any finite magnetic field [7].

Thus, the Hamiltonian of the system of two QDs has the form:

$$\hat{H} = \hat{H}^{(1)} \otimes \mathbb{1} + \mathbb{1} \otimes \hat{H}^{(2)}. \quad (1)$$

The Hamiltonian $\hat{H}^{(i)}$ of a single QD contains the following terms

$$\hat{H}^{(i)} = \hat{H}_{\text{el.}}^{(i)} + \hat{H}_{\text{nucl. env}}^{(i)} + \hat{H}_{\text{int}}^{(i)}. \quad (2)$$

The first and the second terms of $\hat{H}^{(i)}$ are the Zeeman energies of electron and nuclear spins, respectively:

$$\hat{H}_{\text{el.}}^{(i)} = \Omega \hat{S}_z \otimes \mathbb{1} \otimes \mathbb{1}, \quad (3)$$

where $\Omega = g_{\text{eff}} \mu_B B_z$ is the Zeeman splitting of electron spin, $g_{\text{eff}}$ is effective g-factor of electron spin, $\mu_B$ is the Bohr magneton, $B_z$ is z-component of magnetic field, $\hat{S}_z$ is the operator of the z-component of electron spin, $N$ is the number of nuclear spins interacting with electron spin. For the sake of simplicity, we have also adopted the assumption that all nuclear spins are of the same type $J$, so the identity operator $\mathbb{1}$ used above is of dimension $2J + 1$.

$$\hat{H}_{\text{nucl. env}}^{(i)} = \sum_{n=1}^{N} \omega^{(n)} \mathbb{1} \otimes (\mathbb{1} \otimes \hat{j}_z^{(n)} \otimes \mathbb{1} \otimes \mathbb{1}, \quad (4)$$

where $\omega^{(n)} = g^{(n)} \mu_N B_z$ is the Zeeman splitting of nth nuclear spin, $g^{(n)}$ is the nuclear g-factor of nth nuclear spin, $\mu_N$ is the nuclear magneton.

The last term of the Hamiltonian $\hat{H}^{(i)}$ is the hyperfine interaction between electron spin and nuclear spins:

$$\hat{H}_{\text{int}}^{(i)} = \sum_{n=1}^{N} A_n \hat{S} \otimes \mathbb{1} \otimes (\mathbb{1} \otimes (\mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1}, \quad (5)$$

where $\hat{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$ is the electron spin operator, $\hat{j}^{(n)} = (\hat{j}_x^{(n)}, \hat{j}_y^{(n)}, \hat{j}_z^{(n)})$ is the nth nuclear spin operator and $A_n$ is the hyperfine coupling between electron spin and nth nuclear spin.

### III. MANIPULATION PROCEDURE WITH QUANTUM MEASUREMENTS AND POSTSELECTION OF TWO-QUBIT STATE

Motivated by experimentalists’ capabilities to initialize localized in QDs electrons in singlet state and to perform projective measurements onto singlet state [12], we consider a quantum measurement of two-electron spin subsystem (TESSS), specifically, the measurement answering whether TESSS is in a certain two-qubit state or not. In general, such a quantum measurement can be described by measurement operators $\hat{M}_1$ (“yes” result) and $\hat{M}_2$ (“no” result):

$$\hat{M}_1 = \sqrt{k} \hat{\Pi}_{2q} \otimes \mathbb{I}_{2\text{env}} + \sqrt{1-k} \left( \mathbb{I} - \hat{\Pi}_{2q} \otimes \mathbb{I}_{2\text{env}} \right), \quad (6)$$

$$\hat{M}_2 = \sqrt{1-k} \hat{\Pi}_{2q} \otimes \mathbb{I}_{2\text{env}} + \sqrt{k} \left( \mathbb{1} - \hat{\Pi}_{2q} \otimes \mathbb{I}_{2\text{env}} \right). \quad (7)$$

where $\hat{\Pi}_{2q}$ is a projector in TESSS subspace onto a chosen two-qubit state, parameter $k \in [\frac{1}{2}, 1]$ is a strength of measurement, $\mathbb{I}$ is the identity operator of dimension of the system state space, and $\mathbb{I}_{2\text{env}}$ is the identity operator of dimension of the two NSEs subsystem state space. The extreme values of the quantum measurement strength have clear physical meanings: $k = 1$ corresponds to the case of measurements of the highest strength, i.e. the projective measurement,

$$\hat{M}_1 = \hat{\Pi}_{2q} \otimes \mathbb{I}_{2\text{env}}, \quad (8)$$

$$\hat{M}_2 = \mathbb{I} - \hat{\Pi}_{2q} \otimes \mathbb{I}_{2\text{env}}. \quad (9)$$

and $k = \frac{1}{2}$ corresponds to the case of completely ineffective measurement,

$$\hat{M}_1 = \hat{M}_2 = \frac{1}{\sqrt{2}} \mathbb{I}. \quad (10)$$

The intermediate values of strength $k$ correspond to such quantum measurements that give the outcomes which are the probabilistic mixture of the outcomes of projective operators $\hat{\Pi}_{2q} \otimes \mathbb{I}_{2\text{env}}$ and $\mathbb{I} - \hat{\Pi}_{2q} \otimes \mathbb{I}_{2\text{env}}$, i.e. the fidelity of the outcomes, compared with these of projective measurement, is determined by the measurement strength and is equal to $k^2$. By construction, the measurement operators $\hat{M}_1$, $\hat{M}_2$ fulfill the completeness relation $\sum_{i=1}^{2} M_i^\dagger M_i = \mathbb{1}$ for any $k$ from its range.

The manipulation procedure consists of initialization of the system in its initial state and a few manipulation cycles. The manipulation cycle, in turn, has two parts:
We would like to note that it is crucial in the simulations to keep the density matrix of the whole system, \( \hat{\rho}_n(t) \), and not to reduce it to the two-qubit density matrix \( \hat{\rho}_{2q}(t) \) by tracing out NSEs. Having at hand the full density matrix, one can investigate the transfer of coherences and quantum correlations in the system to the greatest degree. As dimension of the system state space grows exponentially with the number of nuclear spins, our capabilities to simulate application of the proposed manipulation procedure are limited to small systems, so we present here the results obtained for the system of two QDs with homonuclear \((J = \frac{1}{2})\) NSEs of the same size \( N_1 = N_2 = 5 \).

In simulations, as an initial TESSS state we have used singlet state, \( \hat{\rho}_{2q}(0) = |\psi-\rangle\langle\psi-|, \) where \( |\psi-\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \). The projector operator \( \Pi_{2q} \) have also been chose to be a projector onto singlet state, \( \Pi_{2q} = |\psi-\rangle\langle\psi-| \).

To quantify the amount of entanglement of TESSS state \( \hat{\rho} \), we use concurrence [13], which is defined as

\[
C(\hat{\rho}) = \max \{ 0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 \},
\]

where \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \) are the square roots of the eigenvalues of matrix \( \hat{\rho}_{\lambda} \) where \( \hat{\rho} = (\hat{\sigma}_y \otimes \hat{\sigma}_y)\hat{\rho}^* (\hat{\sigma}_y \otimes \hat{\sigma}_y) \). Here \( \hat{\rho}^* \) denotes the operation of complex conjugation of each element of \( \hat{\rho} \). The concurrence ranges from \( C = 0 \) for separable states to \( C = 1 \) for maximally entangled states. We also use negativity [14] to estimate the level of entanglement between two parts of the system. We show below that of particular interest is the negativity between TESSS and NSEs.

While considering the quantum correlation dynamics, it is convenient for further analysis to express time in units of two-qubit \( T_2^* \) defined as follows

\[
\frac{1}{(T_2^*)^2} = \frac{1}{(T_2^{*\text{(1)}})^2} + \frac{1}{(T_2^{*\text{(2)}})^2},
\]

where \( T_2^{*\text{(i)}} \) is the single-qubit dephasing time, \( T_2^{*\text{(i)}} = \hbar/\sqrt{3N_i}/\sum_{n=1}^{N_i} A_n^{(i)} \). The decay of entanglement of two electron spin qubits plotted using such a time unit is independent of the system size and the absolute value of the hyperfine interaction [7].

The results of simulations, which are shown in figures [2][2], have been obtained for the system being in moderate magnetic field \( \Omega = 5 \frac{\hbar}{T_2} \). In Fig. [2] the time dependencies of concurrence of TESSS state (top panel) and negativity (bottom panel) are shown for a few different numbers \( n \) of performed cycles. As it can be seen from the top panel of Fig. [2] the time evolution of the state is completely lost after time \( t \approx 1.5T_2^* \), but application of just a single manipulation cycle causes a significantly rise of the level at all times and noticeably retards its decay. With increasing number \( n \) of performed
cycles, level of entanglement systematically grows, reaching almost its maximal value. Along with the decay of entanglement in TESSS, we see appearance of entanglement between initially uncorrelated parts of the system, namely between TESSS and their NSEs (see bottom panel of Fig. 2).

In order to estimate the effect of retardation of entanglement decay produced by application of the manipulation procedure, we monitor the level of concurrence calculated for $t = 2T_2^*$, which is shown in Fig. 3 as a function of number $n$ of performed projective measurements and time $\tau$ between them. Using this map, one can find the optimal values of the parameters $n$ and $\tau$, which maximize the retardation of entanglement decay. On one hand, increasing the number of manipulation cycles almost always enhances the effect, on the other hand, it turns out that there exists the most optimal duration $\tau$ of the free evolution periods ($\tau_{\text{opt}} \approx 2T_2^*$ for the simulated system).

The probability to obtain the desired state $\hat{\rho}_n(0)$, which is shown in left panel of Fig. 3, decreases monotonically with number $n$ of performed cycles due to the fact that in each cycle the probability to obtain the post-selected TESSS state which is the same as the initial one is strongly less than one. It is also worth noting that probability of $\hat{\rho}_n(0)$ decreases sub-exponentially with increasing $n$, so it drops relatively slow and it is at the level a few percent after execution of about $n = 20$ cycles.

For $\tau > T_2^*$, probability of $\hat{\rho}_n(0)$, which is shown in right panel of Fig. 3, becomes a weakly dependent function of $\tau$ for a fixed parameter $n$ and oscillates around the corresponding mean value.

For practical purposes, one should find the optimal combination of procedure parameters $n$ and $\tau$ such that maximizes simultaneously the effect of retardation of entanglement decay (see Fig. 3) and the probability to obtain such a state (see map in Fig. 3).

In Fig. 3 the dependence of intensity of the effect on strength $k$ of quantum measurement used in the manipulation procedure is shown. It turns out that with increasing parameter $n$ concurrence of TESSS state, as a function of $k$, gradually develops a plateau at nearly maximal possible for a given value of $n$ level. The plateau is situated between $k = 1$ and some lower value of $k$, and for increasing parameter $n$, progressively reaches surprisingly low values of $k$. Thus, a long sequences of manipulation cycles lower requirement for the strength $k$ of quantum measurement with practically no loss in the end effect.

The possibility to significantly retard entanglement decay by performing the manipulations with quantum measurements originates from the non-Markovian dynamics of the system [13, 14]. During free evolution, electron spins, initially being in an entangled state, transfer through the hyperfine interaction some of their quantum correlations to their nuclear spin environments. Execution of the quantum measurement of two-electron spin subsystem with subsequent postselection of two-electron spin states...
spin state restores its quantum correlations. The state of nuclear spin subsystems, conditioned on past dynamics, in turn, preserves the previously obtained from electron spins quantum correlations, and thus, the flow rate of quantum correlations from electron spins to nuclear spin environments in following periods of the system evolution may be reduced, which is manifested as the retardation of electron spin entanglement decay.

V. CONCLUSIONS

In contrast to the fast decay of TESSS entanglement on a timescale of the order of $T_2^*$ (shown in Fig. 1 of [6] or [7] and here in Fig. 3), performing a few cycles of evolution of initially entangled two electron spin qubits interacting with their NSEs followed by quantum measurement performed on TESSS gradually builds up coherences in the entire system and the rest decay of quantum correlations of TESSS may be significantly slowed down for specific cycle durations $\tau$ and numbers $n$ of the performed cycles.

The disadvantage of such a way of counteracting the decoherence is the necessity to postselect the proper two-qubit state after each quantum measurement and the associated with that decreasing probability of success. On the other hand, the probability to obtain the desired state $\hat{\rho}_n(t)$ decreases sub-exponentially with $n$.

The strong (projective) measurements produce maximal effect of retardation of entanglement decay, but the effect can be also achieved in the case of weak measurements. The more cycles have been performed (the larger $n$), the weaker quantum measurements can be used to achieve a nearly maximal effect.

Since the proposed procedure of retardation of entanglement decay requires only the execution of quantum measurements of two-electron subsystem, its practical realization seems to be much easier than execution of dy-

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**FIG. 4.** Map: Probability to obtain the state $\hat{\rho}_n(0)$ as a function of number $n$ of performed projective measurements and time $\tau$ between them. Graphs (cross sections of the map): Probability to obtain the state $\hat{\rho}_n(0)$ as a function of number $n$ of performed projective measurements (left panel) and as a function of time $\tau$ between projective measurements (right panel). NSEs consist of $N_1 = N_2 = 5$ uniformly coupled spins $\frac{1}{2}$. The system is in moderate magnetic field $\Omega = 5 \left\{ \frac{\tau}{T_2^*} \right\}$. 

**FIG. 5.** Map: Concurrence of two-qubit state calculated at $t = 2T_2^*$ as a function of strength $k$ of quantum measurements (QM) performed with period $\tau = 2T_2^*$. Graph (cross section of the map): Concurrence of two-qubit state calculated at $t = 2T_2^*$ as a function of strength $k$ of quantum measurements performed with period $\tau = 2T_2^*$. NSEs consist of $N_1 = N_2 = 5$ spins $\frac{1}{2}$. The system is in moderate magnetic field $\Omega = 5 \left\{ \frac{\tau}{T_2^*} \right\}$. 
namical decoupling of qubits from their environments or preparation of a narrowed nuclear spin bath state, but due to the indeterminicity involved in the manipulation procedure, only a fraction of executed runs will give the desired state $\hat{\rho}_n(t)$, and therefore it is not the most convenient way to counteract the decoherence. On the other hand, simulations show that when one applies the manipulation procedure with a number of cycles $n \geq 10$, the quantum measurement need not to be of projective type ($k = 1$) anymore, it can be of moderate strength ($k \approx 0.8$), and the probability to obtain the desired state $\hat{\rho}_n(t)$, which will exhibit a slower decay of entanglement, is pretty large (about 10%). Thus, it may be viewed of fundamental interest to implement such a manipulation procedure in currently existing systems of two-electron spin QD qubits in order to check experimentally whether predicted effect of retardation of entanglement decay is achievable in real systems.

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