Impossible Realization of Neutrino Resonance Oscillations 
Enhancement in Matter

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Abstract

In this paper, we study a photon, a massive charged particle, and 
a massive neutrino passing through matter. The hypothetical left-right 
symmetric weak interaction, which is used in Wolfenstein’s equation can 
generate the resonance enhancement of neutrino oscillations in matter, 
which disappears when neutrinos go out into vacuum from matter (the 
Sun). It is shown that since standard weak interactions cannot generate 
masses, the laws of conservation of the energy and the momentum of 
neutrino in matter will be fulfilled only if the energy $W$ of polarization 
of matter by the neutrino or the corresponding term in Wolfenstein’s 
equation, is zero. This result implies that neutrinos cannot generate 
permanent polarization of matter. This leads to the conclusion: reso-
nance enhancement of neutrino oscillations in matter does not exist. It 
is also shown that in standard weak interactions the Cherenkov radiation 
cannot exist.

PACS: 12.15 Ff-Quark and lepton masses and mixings. 
PACS: 96.40 Tv-Neutrinos and muons.

1 Introduction

In previous works [1], we studied the physical foundations of Wolfen-
stein’s equation [2], resulting in the conclusion: in Wolfenstein’s equa-
tion, a hypothetical left-right symmetrical weak interaction is used, but
not the standard (left-side) weak interaction. Therefore, the resonance enhancement of neutrino oscillations in matter obtained in Ref [2] have no connection with the weak interaction physics.

This work continues the discussion on the problem of neutrinos passing through matter.

2 Wolfenstein’s Equation for Neutrino in Matter and the Mechanism of Resonance Enhancement of Neutrino Oscillations in the Framework of a Hypothetical Weak Interaction

In the ultrarelativistic limit, the evolution equation for the neutrino wave function \( \nu_\Phi \) in matter has the form [2]

\[
i \frac{d \nu_\Phi}{dt} = (p \hat{I} + \frac{\hat{M}^2}{2p} + \hat{W}) \nu_\Phi,
\]

where \( p, \hat{M}^2, \hat{W} \) are, respectively, the momentum, the (nondiagonal) square mass matrix in vacuum, and the matrix taking into account neutrino interactions in matter,

\[
\nu_\Phi = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}, \quad \hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\]

\[
\hat{M}^2 = \begin{pmatrix} m^2_{\nu_e\nu_e} & m^2_{\nu_e\nu_\mu} \\ m^2_{\nu_\mu\nu_e} & m^2_{\nu_\mu\nu_\mu} \end{pmatrix}.
\]

As we can see from the form of Eq. (1), the above equation holds the left-right symmetric neutrinos wave function \( \Psi(x) = \Psi_L(x) + \Psi_R(x) \). This equation contains the term \( W \), which arises from the weak interaction (contribution of \( W \) boson) and contains only a left-side interaction of the neutrinos, and is substituted in the left-right symmetric equation (1) without indication of its left-side origin. Then we see that equation (1) is an equation that includes term \( W \) which arises not from the weak interaction but from a hypothetical left-right symmetric interaction (see
also works [1, 3]). Therefore this equation is not one for neutrinos passing through real matter. The problem of passing neutrinos through real matter will be discussed in the next section.

The matrix $\hat{M}^2$ is diagonalized by rotation through the angle $\theta$ ($\theta$ is the angle of vacuum oscillation):

$$\tan(2\theta) = \frac{2m_{\nu_e\nu\mu}^2}{|m_{\nu_e\nu\mu}^2 - m_{\nu_e\nu\mu}^2|}, \quad \hat{M}^2_{\text{diag}} = \begin{pmatrix} m_{1}^2 & 0 \\ 0 & m_{2}^2 \end{pmatrix}, \quad (2)$$

$$m_{1,2}^2 = \frac{1}{2}[(m_{\nu_e\nu\mu}^2 + m_{\nu_e\nu\mu}^2) \pm \sqrt{(m_{\nu_e\nu\mu}^2 - m_{\nu_e\nu\mu}^2)^2 + 4m_{\nu_e\nu\mu}^4}],$$

$$\Delta m^2 = \sqrt{(m_{\nu_e\nu\mu}^2 - m_{\nu_e\nu\mu}^2)^2 + 4m_{\nu_e\nu\mu}^4},$$

and the length of vacuum oscillation $L_0$ is

$$L_0 = \frac{4\pi p}{|m_{1}^2 - m_{2}^2|}, \quad E \cong pc. \quad (3)$$

Since $\hat{M}^2$ is a nondiagonal matrix, this vacuum oscillation of neutrinos will take place at any energies with the length of oscillation $L_0$.

The solution of equation (1), i.e. one for passing neutrinos through a ”matter” where they participate in a left-right symmetric hypothetical weak interaction, is considered in detail in [4] and here the main results will be shown in the reduced form.

When neutrinos are passing through ”matter”, their influence (see equation (1)) leads to changes of the rotation angle $\theta$ for diagonalizing the mass matrix $\hat{M}^2$, if diagonal matrix $\hat{W}$, responsible for the difference between the interactions of the neutrinos ($\nu_e, \nu_\mu$), is added to the mass term $\hat{M}^2/2p$, and then $\theta$ becomes $\theta' (\theta' \neq \theta)$.

Thus, neutrino mixing in ”matter” is determined by $\sin^2(2\theta')$:

$$\sin^2(2\theta') = \sin^2(2\theta)/[(\cos(2\theta) - \frac{L_0}{L^0})^2 + \sin^2(2\theta)], \quad (4)$$

$$m_{1,2}^{\prime 2} = \frac{1}{2}[(m_{\nu_e\nu\mu}^{\prime 2} + m_{\nu_e\nu\mu}^{\prime 2}) \pm \sqrt{(m_{\nu_e\nu\mu}^{\prime 2} - m_{\nu_e\nu\mu}^{\prime 2}) + 4m_{\nu_e\nu\mu}^{4}}],$$

$$\Delta m^{\prime 2} = \sqrt{(m_{\nu_e\nu\mu}^{\prime 2} - m_{\nu_e\nu\mu}^{\prime 2})^2 + 4m_{\nu_e\nu\mu}^{4}},$$
\[ L_0' = \frac{L_0}{\sin(2\theta')} \]

where \( m_{\nu_e\nu_e}', m_{\nu_\mu\nu_\mu}' \) are masses of \( \nu_e, \nu_\mu \) in ”matter”, \( L^0 \) is a diffraction length (i.e., length of formation),

\[ L^0 = 2\pi m_p(2^{0.5}G_F\rho Y_e)^{-1} = 310^7(m)(\rho (g/cm^3)2Y_e)^{-1}, \quad (5) \]

\( Y_e \) is the number of electrons per nucleon in ”matter”.

In the common case \( \theta' \) depends on the difference of masses \( m_1, m_2 \), density \( \rho \) of ”matter” and the neutrino momentum.

At \( L_0' \approx L^0 \) the resonant neutrino oscillations take place, i.e., \( \sin^2(2\theta') \approx 1 \) or \( \theta' \approx \frac{\pi}{4} \).

But, since \( \Delta m'^2 \approx m_{\nu_e\nu_\mu}^2 \), the length of oscillations in the ”matter” \( L_0' \) is defined by \( m_{\nu_e\nu_\mu}^2 \)

\[ L_0' = \frac{4\pi p}{|m_{\nu_e\nu_\mu}^2|} \]

and increases relatively to the vacuum oscillations length \( L_0 \).

We remind that at resonance (if \( \nu_\mu(0) = 0 \) ) the oscillations are defined by the following expression:

\[ \nu_e(t) \approx \frac{1}{2}[\exp(-iE'_1t) + \exp(-iE'_2t)] \nu_e(0) \]

\[ \nu_\mu(t) \approx \frac{1}{2}[\exp(-iE'_1t) - \exp(-iE'_2t)] \nu_e(0), \quad (6) \]

or

\[ P(\nu_e \rightarrow \nu_e, t) = 1 - \sin^2(2\theta'(r)) \sin^2[\frac{\pi r}{L_0'(r)}], \quad r = ct, \quad (7) \]

(where \( E'_1, E'_2 \) are energies of \( \nu_1, \nu_2 \) neutrinos), there neutrino resonance mixings take place.

The equation (1) was obtained at supposition that the neutrino behavior in the ”matter” is analogous to a photon behavior in matter with refraction coefficient-\( n \).

The photon velocity \( c' \) in matter with refraction index \( n \) is

\[ c' = \frac{c}{n} \quad (8) \]
and depends on characteristics of matter.

The laws of conservation of the energy and the momentum of the photon in such matter have the following form:

\[ E_0 = E + E_{\text{matt}} = \frac{E_0}{n} + \frac{E_0(n-1)}{n} \]

\[ p_0 = p + p_{\text{matt}} = \frac{p_0}{n} + \frac{p_0(n-1)}{n} \]  \hspace{1cm} (9)

\[ E_0 = \hbar w_0, \quad w = \frac{w_0}{n}, \quad E = pc', \]

where \( E_0, p_0 \) are primary energy and momentum of the photon, \( E, p, E_{\text{matt}}, p_{\text{matt}} \) are, respectively, energy and momentum of the photon in matter and energy and momentum of the matter polarization of the passing photon (matter response).

If we suppose that the neutrinos in ”matter” behave in analogy with the photon in matter and the neutrino refraction indices are defined by the expression

\[ n_i = 1 + \frac{2\pi N}{p^2}\Delta f_i(0) , \]  \hspace{1cm} (10)

(where \( i \) is type of neutrinos (\( e, \mu, \tau \)), \( N \) is density of ”matter”, \( f_i(0) \) are a real part of the forward scattering amplitude), then the velocity of neutrinos in ”matter” is determined by \( n_i \).

The electron neutrino (\( \nu_e \)) in ”matter” interacts via \( W^\pm, Z^0 \) bosons and \( \nu_\mu, \nu_\tau \) interact only via \( Z^0 \) boson. These differences in interactions lead to the following differences on the refraction coefficients of \( \nu_e \) and \( \nu_\mu, \nu_\tau \)

\[ \Delta n = \frac{2\pi N}{p^2}\Delta f(0) , \]  \hspace{1cm} (11)

\[ \Delta f(0) = -\sqrt{2}\frac{G_F}{2\pi} , \]

where \( G_F \) is Fermi constant.

Therefore the velocities of \( \nu_e \) and \( \nu_\mu, \nu_\tau \) in ”matter” are different. And at the suitable density of ”matter” this difference can lead to a resonance enhancement of neutrino oscillations in ”matter” [4, 1].
Since this considered hypothetical weak interaction is left-right symmetric one, it is clear that we can use some analogy with the Electrodynamics. Then elastic and inelastic interactions can take place in matter. Here we are interested only in elastic interactions, namely potential interactions of the charged particles in matter. These interactions lead to polarization of the matter, in the result of it a definite part of energy and momentum of the massive charged particles go for polarization of matter. The laws of conservation of the energy and the momentum of the charged particle in matter have the following form:

\[ E_0 = E + E_{\text{matt}} \]
\[ p_0 = p + p_{\text{matt}}, \]  

(12)

where \( E_0, p_0 \) are primary energy and momentum of the charged particle; \( E, p, E_{\text{matt}}, p_{\text{matt}} \) are, respectively, energy and momentum of the charged particle in matter and energy and momentum of the matter polarization.

It is clear that the matter polarization moves with the velocity which is equal to the velocity of the charged particle in matter, \( v \), if this velocity \( v \) is less than the velocity of light in matter \( c' \). If \( v \) is equal or larger than \( c' \), the energy and the momentum of polarization will go for the Cherenkov radiation [5], i.e., the energy and the momentum losses will take place.

It is interesting to know distributions of the energy and momentum between the charged particle and the matter polarization or which part of the energy goes for mass alteration of the charged massive particle. To solve this problem, it is necessary to do a detailed computing of this interaction (connection between \( m_0, m \) can be obtained using equations (12)). Since it is out of our interest, we do not do this computing. Our interest is connected with the problem of resonance neutrino oscillations in the hypothetical left-right symmetrical interaction which is used in Wolfenstein’s equation.

So, from Wolfenstein’s equation (1) with the hypothetical left-right symmetric weak interaction we obtain the resonance neutrino oscillation enhancement in "matter".

One needs to notice, from the analogy of the electrodynamics, that the considered process is an elastic one, therefore and if even the neutrino
resonance enhancement arises inside the ”matter” (the Sun), when these neutrinos go out from ”matter” (the Sun) into vacuum, this enhancement disappears without leaving a trace and the oscillations transit to vacuum neutrino oscillations (in vacuum the masses, energies and momenta of the neutrinos are restored). The same result is obtained from Eq. (6) since in vacuum \( \frac{L_0}{L'} \to 0 \) and \( \sin^2(2\theta') \to \sin^2(2\theta) \). Hence, we come to conclusion that behavior of neutrinos in the ”matter” is like behavior of photons or charged particles (effective masses and momenta are changed) in matter but not the behavior of vector polarization of photons (i.e., rotation of photon vector polarization) in matter as it is supposed in [4].

It is very interesting to notice that in the considered case (the ”matter” with the hypothetical left-right symmetrical weak interaction) if

\[ n_i - 1 > 0, \]  

then, in analogy with the electrodynamics, when \( v_i > \frac{c}{n_i} \), the Cherenkov radiation will take place.

Let us come to consideration of neutrinos passing through real matter (i.e., in the case when only right components of neutrinos participate in the weak interaction).

### 3 Real Neutrinos in Matter or Impossibility to Realize the Mechanism of Resonance Enhancement of Neutrino Oscillations in Matter in the Framework of the Standard Weak Interaction

There is a question: can real neutrinos in matter behave as the photon or massive charged particle in matter? Then the resonance enhancement of neutrino oscillations in matter will take place.

Let us pass to a more detailed discussion of the problem of resonance enhancement of neutrino oscillations in matter through the standard weak interaction.

As is known (see Appendix and also [1, 3]), this interaction cannot generate the masses. Therefore, when the massive neutrino is passing through matter, its mass does not change.
The laws of conservation of the neutrino energy and the momentum have the following form (we do not take into account inelastic processes):

\begin{align}
\text{a)} & \quad E_0 = E + W, \\
\text{b)} & \quad p_0 = p + W\beta,
\end{align}

where $E_0, p_0, E, p$ are, respectively, energy and momentum neutrino in vacuum and in matter, $W$ is elastic energy of neutrino interactions in matter, $\beta = \frac{v}{c}$.

It is obvious that the response of matter moves with the velocity $c(\beta = 1)$ since the weak interaction cannot generate a mass (the mass of system does not change). If we put expression b) into expression a), we obtain the following expression:

$$\sqrt{p_0^2 + m_0^2} = \sqrt{(p_0 - W\beta)^2 + m_0^2} + W,$$

which is solved only if:

$$m_0 = 0,$$

or

$$W = 0.$$  \hfill (16)

The requirement of conservations of energy and momentum for the weak interacting particle in matter leads to a conclusion that or

$$m_0 = 0 \quad \text{then} \quad W \neq 0,$$

or if

$$m_0 \neq 0 \quad \text{then} \quad W = 0,$$

it means that the energy of matter polarization by neutrinos (or the energy of the matter response $W$) must be zero $W = 0$, i.e., only elastic and inelastic interactions of the neutrinos in matter take place and there is no permanent response of matter (in contrast to the Electrodynamics).

As soon as the neutrinos are massive particles in Wolfenstein’s equation, $W$ must be equal to zero, and, this is why there are no any changes of neutrino oscillations in matter.

In the reverse cases (when $m_0 = 0$) the $W$ can differ from zero, but in this case, as it is well known, the vacuum oscillation of neutrinos cannot take place.
So, we come to a conclusion: no resonance enhancement of neutrino oscillations in matter arises through the standard weak interaction.

It is interesting to remark that, when the neutrinos are passing through matter, there is no permanent polarization \((n_i = 1)\) of the matter, that is why the Cherenkov radiation cannot arise there.

## 4 Conclusion

In this paper, we studied a photon, a massive charged particle, and a massive neutrino passing through matter. The hypothetical left-right symmetric weak interaction, which is used in Wolfenstein’s equations can generate the resonance enhancement of neutrino oscillations in matter, which disappears when neutrinos go out into vacuum from matter (the Sun).

It was shown that since standard weak interactions cannot generate masses, the laws of conservation of the energy and the momentum of neutrino in matter will be fulfilled only if the energy \(W\) of polarization of matter by the neutrino or the corresponding term in Wolfenstein’s equation, is zero. This result implies that neutrinos cannot generate permanent polarization of matter, this leads to the conclusion: resonance enhancement of neutrino oscillations in matter does not exist.

It was also shown that in standard weak interactions the Cherenkov radiation cannot exist either.

The nonresonance mechanism of the neutrino oscillations enhancement in matter through the quasi-elastic weak interactions was suggested in [6].

In conclusion we would like to stress that in the experimental data from [7] there is no visible change in the spectrum of the \(B^8\) Sun neutrinos. The measured spectrum of neutrinos lies lower than the computed spectrum of the \(B^8\) neutrinos [8]. In the case of realization of the resonance enhancement mechanism this spectrum must be distorted.

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5 Appendix

We will operate in the framework of the quantum theory.

Let \( \hat{M} \) be a mass operator and \( \Psi \) a wave function or state function of a fermion. Then mass of the fermion is:

\[
M = \langle \Psi | \hat{M} | \Psi \rangle .
\] (A.1)

If \( \Psi \) is separated on the left and right states \( \Psi = \Psi_L + \Psi_R (\bar{\Psi} = \bar{\Psi}_L + \bar{\Psi}_R) \) then one can rewrite (A.1) in the following form:

\[
M = \langle \Psi_L | \hat{M} | \Psi_R \rangle + \langle \Psi_R | \hat{M} | \Psi_L \rangle .
\] (A.2)

Since the \( \Psi_R (\bar{\Psi}_R) \) do not take part in weak interactions of \( W \) bosons, i.e. \( \Psi_R = \bar{\Psi}_R \equiv 0 \), then one obtains:

\[
M_w = \langle \Psi_L | \hat{M} | 0 \rangle + \langle 0 | \hat{M} | \Psi_L \rangle \equiv 0.
\] (A.3)

Let \( \hat{M}^2 \) is the squared mass operator. Then the squared fermion mass is

\[
M^2 = \langle \Psi_L | \hat{M}^2 | \Psi_R \rangle + \langle \Psi_R | \hat{M}^2 | \Psi_L \rangle .
\] (A.4)

Since the \( \Psi_R (\bar{\Psi}_R) \) do not take part in weak interactions of \( W \) bosons, i.e. \( \Psi_R = \bar{\Psi}_R \equiv 0 \), then one obtains:

\[
M_{w}^2 \equiv 0.
\] (A.5)

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