Frequency modulation, amplification and compression of microwave pulses in a system with helically corrugated waveguides as a dispersive elements

L A Yurovskiy, N S Ginzburg, I V Zotova, M N Vilkov, S V Samsonov and A S Sergeev

Institute of Applied Physics of RAS, 46 Ul’yanova st., 603950, Nizhny Novgorod, Russia

leo@ipfran.ru

Abstract. We consider the possible implementations of the Chirped Pulse Amplification (CPA) technique widely used in optics for the microwave frequency band. We propose using helically-corrugated waveguides for pulse stretching and compressing as their dispersion properties strongly depend on geometrical parameters. For stretched pulse amplification, a helical-waveguide gyro-TWT can be used as a broadband amplifier. Simulations with parameters of the experimentally realized 30 GHz gyro-TWT show that for a 300 W, 200 ps incident pulse, amplification up to 6 MW can be achieved in this system, while in the linear regime of the same gyro-TWT the output power is only about 250 kW.

1. Introduction

At present time the method of Chirped Pulse Amplification (CPA) introduced in optics in [1] is widely used for development of petawatt laser systems (see, for example, [2-4]). Typically, CPA setup comprises the three sections: stretcher - a dispersive line which elongates the initial femtosecond pulse by the several orders of magnitude and makes it frequency chirped; broadband amplifier – typically one or more lasers or parametric amplifiers with big aperture; and a compressor - a line with negative (opposite to the stretcher) dispersion that restores the pulse’s original form. Preliminary stretching allows for higher energies to be achieved in the output radiation, while direct amplification of the power ultra-short laser pulse can induce significant self-focusing and self-phase modulation, resulting in catastrophic optical damage.

Obviously, the CPA technique can be implemented in microwave electronics for the purpose of formation of ultrahigh (multimegawatt or multigigawatt) power pulses, which, depending of the frequency band, is of a considerable interest for different applications, including radars, plasma diagnostics, acceleration systems etc. Correspondingly, different types of amplifiers can be used in the microwave CPA system including traveling-wave tubes with Cherenkov or cyclotron interaction (Cherenkov TWT or gyro-TWT), free electron lasers (FEL), etc. In the millimeter waveband it is attractive to utilize helical-waveguide gyro-TWTs providing an extremely broad frequency bandwidth up to 10-15% [5-7]. For pulse stretching and compressing helically-corrugated waveguides can be also used, since their dispersion properties strongly vary depending on its geometrical parameters. For
example, effective compression of electromagnetic pulses in corrugated helical waveguides was demonstrated in [8,9].

This paper is devoted to theoretical study of a Ka-band three-section CPA scheme, in which electrodynamic systems of each section (stretcher, amplifier and compressor) are realized based on multi-fold helically-corrugated waveguides. The paper is organized as follows. In Section 2, we develop a time-domain self-consistent model, which allows us to simulate evolution of the incident ultrashort pulse in the scheme under consideration. In Section 3, within the frame of the developed model, we demonstrate applicability of the CPA technique for significant increase of output radiation peak power in the millimeter wave band.

2. Model and basic equations
In Fig.1 the principal scheme of considered 3-section microwave CPA system is shown. Here, an initial electromagnetic pulse, passed through the circulator, is inserted in the stretcher (section 1), represented by a multi-fold helically corrugated waveguide. Such scheme allows the pulse to pass forth and back through the stretcher, increasing its effective length by the factor two. After the stretcher the elongated pulse is amplified in the wide-band helical-waveguide gyro-TWT (section 2), driven by a rotating electron beam. Then, the signal is recompressed to the original duration in the helically-corrugated pulse compressor (section 3).

![Figure 1. Proposed scheme of a microwave CPA system](image)

As already noted, it is proposed to use a waveguides with multi-fold helical corrugation in all three sections. Such corrugation can be described in this way:

$$r(\varphi,z) = R + \bar{R}\cos(\bar{m}\varphi - \bar{h}z),$$

(1)

where $R$ is the waveguide mean radius; $\bar{R}, \bar{m}, \text{and } \bar{h} = 2\pi/d$ are the amplitude, azimuthal, and axial number of the corrugation; and $d$ is the corrugation period. Under Bragg resonance condition: $\bar{m} = m_A + m_B, \bar{h} \approx h_B$ where $m_{A,B}$ are azimuthal numbers of the waves, the corrugation provides the coupling of two counter rotating partial $TE_{mn}$ waves [5]. One of them is a near-cut-off mode $A$, which has a small axial wavenumber $h_A \to 0$, while the other partial wave, a travelling (far-from-cut-off) mode $B$, has a large axial wave number $h_B \to k$, where $k = \omega / c$. The electrical field of the partial waves can be presented in the form:
\[ E_A = \text{Re}\left\{ A(z,t) E_A^0(r) e^{i(\omega, t- m, \phi)} \right\}, \quad E_B = \text{Re}\left\{ B(z,t) E_B^0(r) e^{i(\omega, t-h_z-m, \phi)} \right\}, \] (2)

where \( A(z,t) \) and \( B(z,t) \) are the slowly varying amplitudes and the functions \( E_A^0(r) \) and \( E_B^0(r) \) describe the radial structures of the partial waves, which are the same as those of the regular waveguide modes, \( \omega_A \) is the cut-off frequency of the mode \( A \).

In the amplifier (section 2) selective excitation of the mentioned coupled modes by a rotating electron beam is provided at the cyclotron harmonic with number \( s = m_A \) under cyclotron resonance condition: \( \omega \approx \omega_A \approx \omega_H \), where \( \omega_H = eH/mc^2 \) is the gyrofrequency. In this case the electron-wave interaction can be described by the set of equations [7]:

\[
\frac{\partial^2 a}{\partial Z^2} - 2i \frac{\partial a}{\partial \tau} = 2\sigma b + i \frac{4eI_b}{\pi mc^3} \frac{1}{\beta_{ohm}} \frac{s^s}{2^s} \int_0^{2\pi} p^s d\theta_0, \tag{3a}
\]

\[
\left( \frac{\partial}{\partial Z} + \frac{1}{\beta_{gr}} \frac{\partial}{\partial \tau} \right) p_+ + i \frac{1}{2} \frac{p_+}{s} \left( \Delta + \left| p_+ \right|^2 - \left| p_{\perp,0} \right|^2 \right) = \frac{s^s}{2^s} \frac{1}{\beta_{ohm}} a \left( p_+ \right)^{s-1}. \tag{3c}
\]

Here the following dimensionless variables and parameters are used:

\[
\tau = \omega_A t, \quad Z = \kappa_A z, \quad a = \frac{eA\sqrt{N_A}}{mc^2\kappa_A}, \quad b = \frac{eB\sqrt{N_B}}{mc^2\kappa_B},
\]

\[
\sigma = \frac{\tilde{R}}{2R} \frac{\nu_B^2 - m_A m_B}{(\nu_A^2 - m_A^2)(\nu_B^2 - m_B^2)} \] is the coupling parameter [10], \( N_{A,B} = (\nu_{A,B}^2 - m_{A,B}^2)J_{m_A,B}^2(\nu_{A,B}) \) are dimensionless norms of the partial waves, \( J_{m_A}^2(\nu_A) = J_{m_B}^2(\nu_B) = 0, \quad \kappa_A = \omega_A / c, \quad \beta_{gr} = h_0 / \kappa_A \) is group velocity of the traveling wave \( B \) at the frequency \( \omega_A \), \( h_0 = h_B(\omega_A) \), \( p_+ = \left| p_+ \right|^2 \left| p_{\perp,0} \right|^2 \exp(-i\omega_A t + i(m_A - s)\phi) / mc \) is the normalized transverse electron momentum, \( V_{|0|} = \beta_{ohm} c \) is the initial longitudinal velocity of electrons, \( I_b \) is the beam current, \( \Delta = 2(\omega_A - s\omega_H) / \omega_A \) is the cyclotron resonance detuning, \( \beta_{ohm} = r_0 \delta_{scin}^{-1} (1 - m_A^2 / \nu_B^2) \) is the Ohmic quality factor, \( \delta_{scin} \) is the skin depth. The boundary conditions for the motion equation (3c) model a situation when the electrons at the input cross-section of the amplifier are uniformly distributed over the cyclotron rotation phases:

\[
\left. p_+ \right|_{\tau = 0} = p_{\perp,0} \exp(i\theta_0) \quad \theta_0 \in [0, 2\pi), \tag{4}
\]

and the beam has neither energy nor velocity spreads.

Due to the stretcher and compressor also have the form of multi-fold helical waveguides, the evolution of electromagnetic pulses in these sections can be described by the set of coupling equations (3), where the electron current should be set zero (\( I_b = 0 \)). Further we will assume that the transmission between system sections is carried out by means of a travelling wave \( B \), which can be described by corresponding boundary conditions:

\[
b^{\text{in}}_2(\tau) = b^{\text{out}}_2(\tau) S_{12} \exp(i\delta_2 \tau), \quad b^{\text{in}}_3(\tau) = b^{\text{out}}_3(\tau) S_{23} \exp(i\delta_3 \tau). \tag{6}
\]

Here \( \delta_2 = 2\beta_{12}^2(\omega_A - \omega_A) / \omega_A, \quad \delta_3 = 2\beta_{13}^2(\omega_A - \omega_A) / \omega_A \) are detunings of the carrier frequencies in the corresponding sections, \( S_{12} \) and \( S_{23} \) are transmission coefficients, which allow to take into
account the losses in intersectional mode converter, as well as the difference between the normalized parameters in the sequential sections. For the near cut-off waves $A$ at the edges of the interaction space in all three sections, the nonreflecting boundary conditions are applied [11] (here, for simplicity, the indexes of sections are omitted):

$$
\left( a \mp \frac{1}{\sqrt{2\pi i}} \int_{-\tau}^{\tau} \frac{1}{\sqrt{-\tau - \tau'}} \frac{\partial a(z, \tau')}{\partial Z} d\tau' \right)_{Z=Z_{in},Z_{out}} = 0.
$$

In further simulations the waveform of the incident pulse with the amplitude $b_0$ and duration $T$ is represented by the function: $b_{in}(\tau) = b_0 \sin^2(\pi \tau / 2T) \exp(i\delta_1 \tau)$, where $\delta_1 = 2\beta_{2,0}^2 (\omega_0 - \omega_A) / \omega_A$, $\omega_0$ is the carrier frequency.

Note, that optimal parameters of the CPA system sections can be determined based on the dispersion equation of the normal wave $W$ (see in detail, [10]):

$$
(2\Omega - \Gamma^2) \left( \frac{\Omega}{\beta_{gr}^2} - \Gamma - \frac{h - h_0}{\kappa_A} \right) = 2\sigma^2 \frac{\beta_{gr}^2}{\beta_{gr}^2}.
$$

Here $\Omega$ and $\Gamma$ are normalized frequency detuning and a longitudinal wavenumber. In the amplifier, the mutual arrangement of the $W$ wave dispersion curve and the beam line:

$$
\Omega = \beta_{gr}^2 \Gamma - \frac{\beta_{2,0}^2}{2} \Delta.
$$

is important. In fact the maximum amplification bandwidth is reached, when the grazing incidence between the beam line and the $W$ curve for close-to-zero longitudinal wave numbers realizes.

### 3. Results of simulations

Simulations were performed based on parameters of the experimentally realized 30 GHz gyro-TWT [7] with a 3-fold helically corrugated waveguide, which is coupling traveling $TE_{1,1}$ and quasi-critical $TE_{2,1}$ waves, and a driven electron beam with energy of 68 keV, current of 10 A and pitch factor 1.2. The amplification bandwidth achieves about 15%. In order to decrease the Ohmic losses, 5-fold helically-corrugated waveguides (coupling traveling $TE_{3,1}$ and quasi-critical $TE_{2,2}$ waves) with increased mean radii were proposed to use in the stretcher and compressor. Parameters of corrugations were chosen in such a way to produce the required dispersion properties in these sections. Further results correspond to the initial pulse with the central frequency close to 30 GHz, peak power of 300 W and pulse duration of 200 ps.

The dispersion curve of the normal wave in the stretcher is shown in Fig.2a and corresponds to the mean waveguide radius $R_1 = 8.91$ mm, the corrugation period $d_1 = 8.92$ mm and the corrugation amplitude $R_1' = 2.04$ mm. Simulations show that after passing through the 2 meter length stretcher (1 meter of the real waveguide), the initial pulse is elongated to 1.47 ns (48 times).

After that, the stretched pulse passes through a circulator and a $TE_{3,1} \rightarrow TE_{1,1}$ converter, and then it enters the 30 cm long amplifier with following parameters: the waveguide radius $R_2 = 3.56$ mm, the corrugation period $d_2 = 7.36$ mm and the corrugation amplitude $R_2' = 1.47$ mm. With the guiding magnetic field $H_0 = 6.1$ kOe, electron-wave interaction realizes at the second cyclotron harmonic near the grazing incidence with the normal wave (Fig. 3a). After the amplification the pulse peak power increases up to 240 kW and its duration further elongated up to 14 ns (Fig. 3b). The increase in the pulse length and the distortion of its shape is related to the nonlinear regime of operation in the amplifier, when the maximum amplification gain is reached.

Signal from the amplifier through a $TE_{1,1} \rightarrow TE_{3,1}$ converter and circulator transmits to the entrance
of compressor. The parameters of the compressor (the mean waveguide radius $R_3 = 8.4 \text{ mm}$, the corrugation period $d_3 = 6.81 \text{ mm}$ and the corrugation amplitude $\bar{R}_3 = 4.03 \text{ mm}$) were selected in such a way that the dispersion curve of the normal wave in the compressor (Fig.4a) has a good match with the stretcher dispersion characteristics. After double pass through the 1 meter length compressor the pulse original form is restored, and peak power of 6 MW with pulse duration of 300 ps is achieved (Fig.4b). It should be noted that the output peak power $P$ exceeds the power of driving electron beam $P_{\text{beam}}$ in the amplifying section in 9 times ($P/P_{\text{beam}} \approx 9$), while without preliminary stretching, the peak power of the amplified pulse doesn’t exceed 250 kW (conversion coefficient: $P/P_{\text{beam}} \approx 0.4$).

![Figure 2](image1.png)

**Figure 2.** (a) The dispersion diagram in the stretcher. (b) Profiles of the incident ultrashort pulse (left scale) and the elongated signal after the stretcher (right scale).

![Figure 3](image2.png)

**Figure 3.** (a) The dispersion diagram of the normal wave and the beam line (dashed curve) in the amplifier. (b) Profile of the amplified signal.

![Figure 4](image3.png)

**Figure 4.** (a) The dispersion diagram in the compressor. (b) Profiles of the compressed pulse.
4. Conclusions

Thus, in this paper we have demonstrated the possibility of the CPA technique application in the microwave frequency band with multi-fold helically corrugated waveguides in all sections. For experimentally realized parameters, the peak power of Ka-band ultrashort pulse in the CPA scheme can be in ~20 times higher than in the case of the same helical gyro-TWT without stretcher and compressor. It should be noted that, in principal, smooth waveguides and axisymmetric Bragg structures can be implemented as dispersive lines for the purpose of stretching and compressing microwave pulses. However, in order to achieve high group velocity dispersion we are in need to work close to the cut-off frequency of such waveguides, meaning that some energy will be reflected back to the amplifier resulting in its possible parasitic self-oscillations. Note in conclusion, that proposed scheme can be safely implemented for use with different types of amplifiers, allowing us to produce electromagnetic pulses with ultrahigh power.

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