Unintegrated sea quark at small x
and vector boson production

F. Hautmann\textsuperscript{1}, M. Hentschinski\textsuperscript{2}, and H. Jung\textsuperscript{3}

\textsuperscript{1}University of Oxford, Oxford OX1 3NP, United Kingdom
\textsuperscript{2}Instituto de Física Teórica UAM/CSIC, Universidad Autónoma de Madrid, E-28049 Madrid, Spain
and
Physics Department, Brookhaven National Laboratory, Upton, NY, USA 11973
\textsuperscript{3}Deutsches Elektronensynchrotron, D-22603 Hamburg, Germany

Presented at the 47\textsuperscript{th} Rencontres de Moriond, QCD and High Energy Interactions
La Thuile, Italy
March 10 to 17, 2012

October 2012

Physics Department/Nuclear Theory Group/Office of Science
Brookhaven National Laboratory

U.S. Department of Energy
[DOE Program Office of Science, Nuclear Physics]

Notice: This manuscript has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy. The publisher by accepting the manuscript for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.

This preprint is intended for publication in a journal or proceedings. Since changes may be made before publication, it may not be cited or reproduced without the author's permission.
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party’s use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
We discuss recent work on the transverse momentum dependent sea quark density and its application to forward Drell-Yan production.

1 Introduction

Scattering processes with a single hard scale are well described in QCD within the framework of collinear factorization. The treatment of multi-scale processes, on the other hand, is more involved. In this case, generalized factorization formulas are needed to gain control over large logarithms in higher orders of perturbation theory. Such formulas typically involve transverse-momentum dependent (TMD), or “unintegrated”, parton distribution and parton decay functions. A broad class of multiple-scale events is given by small-\(x\) processes. These are one of the main sources of final states in the central region at the LHC, and lead to sizeable rates of forward jet production at the LHC\(^{2,3}\). At small \(x\), TMD parton distributions arise naturally as a consequence of high energy factorization and BFKL evolution\(^{4}\). \(k_T\)-factorization\(^{5,6}\) provides then the matching of these high energy factorized TMD distributions to collinear factorized distributions. For Monte Carlo applications a convenient description is given in terms of the CCFM evolution equation\(^{7}\) which interpolates for inclusive observables between DGLAP and BFKL evolution\(^{8}\). This therefore supplies a natural basis for a Monte-Carlo realization of \(k_T\)-factorization, such as that provided by the Monte Carlo event generator CASCADE\(^9\).

Computational tools based on TMD parton densities have so far been developed within a quenched approximation where only gluons and valence quarks are taken into account\(^{3,10}\). While this captures correctly the leading contributions at small \(x\), it is mandatory to go beyond this approximation in order to include preasymptotic effects and to treat final states associated with quark-initiated processes such as Drell-Yan production.

In this contribution we present work\(^{11}\) in this direction, and its application to forward Drell-Yan production. For further detail we refer to\(^{11,12}\).
2 Definition of a TMD sea quark distribution and off-shell $qq^* \rightarrow Z$ coefficient

The unintegrated sea-quark distribution is analyzed in \cite{11} to logarithmic accuracy $\alpha_s(\alpha_s \ln x)^n$ based on the off-shell TMD gluon-to-quark splitting function \cite{6}. This is obtained by generalizing the expansion in two-particle irreducible kernels of \cite{13} to finite transverse momenta, and reads

$$P_{qg} \left( z, \frac{k^2}{\Delta^2} \right) = \frac{\Delta^2}{\Delta^2 + z(1-z)k^2} \left[ (1-z)^2 + z^2 + 4z(1-z)^2 \frac{k^2}{\Delta^2} \right].$$

(1)

Here $\Delta = q - z \cdot k$ with $k$ and $q$ transverse momenta of the off-shell gluon and quark respectively, while $z$ is the fraction of the ‘minus’ light cone momentum of the gluon which is carried on by the $t$-channel quark. Although evaluated off-shell, the splitting probability is universal. Once combined with the gluon Green’s function, it takes into account the small $x$ enhanced transverse momentum dependence to all orders in the strong coupling. In this approach the transverse momentum of the sea quark arises as a consequence of subsequent branchings at small $x$, with no strong ordering in their transverse momenta.

To relate this parton splitting kernel to forward vector boson production, we analyze the flavor exchange process $g^*q \rightarrow Zq$, see Fig. [1]. At high (partonic) center of mass energy, this process can be treated according to the “reggeized quark” calculus\cite{14,15}. The latter extends the effective action formalism\cite{16}, currently explored at NLO\cite{17}, to amplitudes with quark exchange in terms of effective degrees of freedom, the so-called reggeized quarks\cite{18,19}. The use of the effective vertices\cite{14,15} ensures gauge invariance of the coefficients relevant to perform the high-energy factorization\cite{5,6,11} for vector boson production, despite the off-shell parton.

If taken literally, the reggeized quark calculus leads for the $g^*q \rightarrow Zq$ process to a rather crude approximation to the $g^* \rightarrow q^*$ splitting function, associated with the lightcone momentum ordering condition which sets the ‘plus’ momenta of the off-shell quark for the $g^* \rightarrow q^*$ splitting to zero. For Eq. [1] this corresponds to the limit $z \rightarrow 0$. It is however possible\cite{11} to relax this kinematic restriction and to keep $z$ finite, while maintaining the gauge invariance properties of the original vertex. For the $g^* \rightarrow q^*$ splitting this yields then precisely the splitting function Eq. [1].

On the other hand, in the $qq^* \rightarrow Z$ coefficient the high energy limit sets the ‘minus’ component of the quark momentum to zero. It proves to be possible to relax the ordering prescription also in this case. It is thus interesting to investigate the effect of these kinematic corrections, which are subleading in the collinear and high energy limits. In \cite{11} we express the off-shell
coefficient for the $Z$-boson cross section as

$$\hat{\sigma}_{qq^* \rightarrow Z} = \sqrt{2} G_F M_Z^2 (V_q^2 + A_q^2) \frac{\pi}{N_c} \delta(z x_1 x_2 s + T - M_Z^2).$$

(2)

Here the variable $T$ parametrizes the off-shellness of the $t$-channel quark. In the collinear limit $T \rightarrow 0$ so that Eq. (2) agrees with the lowest order $qq \rightarrow Z$ coefficient. For the general off-shell case, $T$ interpolates between the squared transverse momentum of the off-shell quark, if strong minus momentum ordering is fulfilled, and modulus of the four-momentum transfer, if this condition is relaxed. Correspondingly, the $qq^* \rightarrow qZ$ cross section is expressed in terms of convolutions in transverse momentum and four momentum transfer respectively.

3 Numerical analysis

Fig. 2 shows a numerical comparison of the factorized formulas discussed above with the $qq^* \rightarrow qZ$ matrix element result and with an expression which uses only the collinear splitting function. For small $|\Delta|$, the differences between $t$ and $k_T$-factorized expressions are numerically small, and both expressions are close to the full result; as $|\Delta|$ increases, we find that the deviations due to the kinematic contributions by which the two expressions differ become non-negligible, and that the $t$-factorized expression gives a better approximation to the full result.

Future extensions of the above results concern large-$x$ contributions, parton shower Monte Carlo implementations, inclusion of full quark emissions in the evolution, see for related work in the context of next-to-leading order BFKL evolution.

Acknowledgments

We thank the organizers for the opportunity to present this work at the meeting. M. H. is grateful for financial support from the German Academic Exchange Service (DAAD), the MICINN under grant FPA2010-17747, the Research Executive Agency (REA) of the European Union under the Grant Agreement number PITN-GA-2010-264564 (LHCPhenoNet), the Helmholtz Terascale Analysis Center, the U.S. Department of Energy under contract number DE-AC02-98CH10886 and a BNL Laboratory Directed Research and Development grant (LDRD 12-034).
References

1. J.C. Collins, *Foundations of perturbative QCD*, CUP 2011.
2. M. Deak et al., JHEP **0909** (2009) 121; [arXiv:0908.1870 [hep-ph]]; F. Hautmann, [arXiv:0909.1250 [hep-ph]]; PoS ICHEP2010 (2010) 108; [arXiv:1101.2656 [hep-ph]].
3. M. Deak et al., Eur. Phys. J. C **72** (2012) 1982; [arXiv:1206.7090 [hep-ph]]; [arXiv:1112.6386 [hep-ph]]; [arXiv:1012.6037 [hep-ph]].
4. V. S. Fadin, E. A. Kuraev and L. N. Lipatov, Phys. Lett. B **60** (1975) 50, I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822 [Yad. Fiz. **28** (1978) 1597].
5. M. Deak et al., Eur. Phys. J. C **72** (2012) 1982; [arXiv:1206.7090 [hep-ph]]; [arXiv:1112.6386 [hep-ph]]; [arXiv:1112.6379 [hep-ph]].
6. S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. B **307** (1993) 147; Nucl. Phys. B **366** (1991) 135; Phys. Lett. B **242** (1990) 97.
7. F. Hautmann and H. Jung, Nucl. Phys. B Proc. Suppl. **184** (2008) 64; [arXiv:0808.0873 [hep-ph]]; F. Hautmann, Acta Phys. Polon. B **40** (2009) 2139; PoS ICHEP2010 (2010) 150.
8. H. Jung et al., Eur. Phys. J. C **70** (2010) 1237.
9. F. Hautmann and H. Jung, JHEP **0810** (2008) 113; [arXiv:0804.1746 [hep-ph]].
10. F. Hautmann and H. Jung, Nucl. Phys. B **427** (1994) 475; Phys. Lett. B **315** (1993) 157.
11. M. Ciafaloni, Nucl. Phys. B **296** (1988) 49, S. Catani, F. Fiorani and G. Marchesini, Nucl. Phys. B **336** (1990) 822 [Yad. Fiz. **28** (1978) 1597].
12. G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B **175** (1980) 27.
13. L. N. Lipatov and M. I. Vyazovsky, Nucl. Phys. B **597** (2001) 399.
14. F. Hautmann and H. Jung, Nucl. Phys. B **427** (1994) 475; Phys. Lett. B **315** (1993) 157.
15. M. Ciafaloni, Nucl. Phys. B **296** (1988) 49, S. Catani, F. Fiorani and G. Marchesini, Nucl. Phys. B **336** (1990) 822 [Yad. Fiz. **28** (1978) 1597].
16. G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B **175** (1980) 27.
17. L. N. Lipatov and M. I. Vyazovsky, Nucl. Phys. B **597** (2001) 399.
18. V. S. Fadin and V. E. Sherman, Zh. Eksp. Teor. Fiz. 72 (1977) 1640; Pisma Zh. Eksp. Teor. Fiz. 23 (1976) 599.
19. B.A. Kniehl, V. A. Saleev, A.V. Shipilova and E.V. Yatsenko, Phys. Rev. D **84** (2011) 074017; B.A. Kniehl, V. A. Saleev and A.V. Shipilova, Phys. Rev. D **79** (2009) 034007; V. A. Saleev, Phys. Rev. D **80** (2009) 114016; Phys. Rev. D **78** (2008) 114031.
20. M. Garcia-Echevarria, A. Idilbi and I. Scimemi, [arXiv:1111.4996 [hep-ph]]; Phys. Rev. D **84** (2011) 011502; A. Idilbi and I. Scimemi, [arXiv:1012.4419]; Phys. Lett. B **695** (2011) 463.
21. F.A. Ceccopieri, Mod. Phys. Lett. A **24** (2009) 3025; [arXiv:1006.4731 [hep-ph]]; F.A. Ceccopieri and L. Trentadue, Phys. Lett. B **660** (2008) 43; Phys. Lett. B **636** (2006) 310.
22. I.O. Cherednikov and N.G. Stefanis, Nucl. Phys. B **802** (2008) 146; Phys. Rev. D **77** (2008) 094001; Phys. Rev. D **80** (2009) 054008; I.O. Cherednikov, A.I. Karanikas and N.G. Stefanis, Nucl. Phys. B **840** (2010) 379; I.O. Cherednikov, [arXiv:1206.4212 [hep-ph]].
23. A. Jain, M. Procura and W.J. Waalewijn, JHEP **1204** (2012) 132.
24. J.-Y. Chiu, A. Jain, D. Neill and I.Z. Rothstein, Phys. Rev. Lett. 108 (2012) 151601; JHEP **1205** (2012) 084.
25. J.C. Collins and F. Hautmann, JHEP **0103** (2001) 016; Phys. Lett. B **472** (2000) 129; F. Hautmann, Nucl. Phys. B **604** (2001) 391; Phys. Lett. B **655** (2007) 26; [arXiv:0708.1319 [hep-ph]].
26. M. Hentschinski, [arXiv:1112.6339 [hep-ph]]; G. Chachamis, M. Hentschinski, A. Sabio Vera and C. Salas, [arXiv:0911.2662 [hep-ph]]; M. Hentschinski, A. S. Vera and C. Salas, [arXiv:1209.1353 [hep-ph]], in preparation.