Semileptonic $B_s \to D_{sJ}(2460)l\nu$ decay in QCD

T. M. Aliev *,†, K. Azizi ‡, A. Ozpineci §
Physics Department, Middle East Technical University, Turkey

Abstract

Using three point QCD sum rules method, the form factors relevant to the semileptonic $B_s \to D_{sJ}(2460)l\nu$ decay are calculated. The $q^2$ dependences of these form factors are evaluated and compared with the heavy quark effective theory predictions. The dependence of the asymmetry parameter $\alpha$, characterizing the polarization of $D_{sJ}$ meson, on $q^2$ is studied. The branching ratio of this decay is also estimated and is shown that it can be easily detected at LHC.
1 Introduction

Recently, very exciting experimental results have been obtained in charmed hadron spectroscopy. The observation of two narrow resonances with charm and strangeness, $D_{sJ}(2317)$ in the $D_s\pi^0$ invariant mass distribution [1, 2, 3, 4, 5, 6], and $D_{sJ}(2460)$ in the $D^*_s\pi^0$ and $D_s\gamma$ mass distribution [2, 3, 4, 6, 7, 8], has raised discussions about the nature of these states and their quark content [9, 10]. Analysis of the $D_{s0}(2317) \rightarrow D^*_s\gamma$, $D_{sJ}(2460) \rightarrow D^*_s\gamma$ and $D_{sJ}(2460) \rightarrow D_{s0}(2317)\gamma$ indicates that the quark content of these mesons are probably $\bar{c}s$ [11]. In [11] it is also shown that finite quark mass effects for the $c$-quark give non-negligible corrections.

When LHC begins operation, an abundant number of $B_s$ mesons will be produced creating a real possibility for studying the properties of $B_s$ meson and its various decay channels. One of the possible decay channels of $B_s$ meson is its semileptonic $B_s \rightarrow D_{sJ}(2460)l\nu$ decay. Analysis of this decay might yield useful information for understanding the structure of the $D_{sJ}(2460)$ meson.

It is well known that the semileptonic decays of heavy flavored mesons are very promising tools for the determination of the elements of the CKM matrix, leptonic decay constants as well as the origin of the CP violation. In semileptonic decays the long distance dynamics are parameterized by transition form factors, calculation of which is a central problem for these decays.

Obviously, for the calculation of the transition form factors, nonperturbative approaches are needed. Among the nonperturbative approaches, the QCD sum rules method [12] received special attention, because this method is based on the fundamental QCD Lagrangian. This method has been successfully applied to a wide variety of problems in hadron physics(for a review see [13]). The semileptonic decay $D \rightarrow \bar{K}^0l\nu$ is studied using the QCD
sum rules with three point correlation function in [14]. Then, the semileptonic decays $D^+ \rightarrow K^{0*} e^+ \nu e$ [15], $D \rightarrow \pi l \nu$ [16], $D \rightarrow \rho l \nu e$ [17] and $B \rightarrow D(D^*) l \nu e$ [18] are studied in the same framework. In the present work we study the semileptonic decay of $B_s$ meson to positive parity $D_{sJ}(2460)$ meson, i.e., $B_s \rightarrow D_{sJ}(2460) l \nu$, within QCD sum rules method. Note that, in [19], the decay $B_s \rightarrow D_{s0}(2317) l \nu$ has been studied using the QCD sum rules.

The paper is organized as follows: In section II the sum rules for the transition form factors are calculated; section III is devoted to the numerical analysis, discussion and our conclusions.

2 Sum rules for the $B_s \rightarrow D_{sJ}(2460) l \nu$ transition form factors

The $B_s \rightarrow D_{sJ}$ transition proceeds by the $b \rightarrow c$ transition at the quark level. The matrix element for the quark level process can be written as:

$$M_q = \frac{G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma_\mu (1 - \gamma_5) l \gamma_\mu (1 - \gamma_5) b$$  \hspace{1cm} (1)

In order to obtain the matrix elements for $B_s \rightarrow D_{sJ}(2460) l \nu$ decay, we need to sandwich Eq. (1) between initial and final meson states. So, the amplitude of the $B_s \rightarrow D_{sJ}(2460) l \nu$ decay can be written as:

$$M = \frac{G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma_\mu (1 - \gamma_5) l < D_{sJ} | \bar{c} \gamma_\mu (1 - \gamma_5) b | B_s >$$  \hspace{1cm} (2)

The main problem is the calculation of the matrix element $< D_{sJ} | \bar{c} \gamma_\mu (1 - \gamma_5) b | B_s >$ appearing in Eq. (2). Both vector and axial vector part of $\bar{c} \gamma_\mu (1 - \gamma_5) b$ contribute to the matrix element considered above. From Lorentz invariance and parity considerations, this matrix element can be
parameterized in terms of the form factors in the following way:

\[< D_{sJ}(p', \varepsilon) | \bar{c}\gamma_\mu \gamma_5 b | B_s(p)> = \frac{f_V(q^2)}{(m_{B_s} + m_{D_{sJ}})} \varepsilon_{\mu\nu\alpha\beta} \epsilon^* \nu P^{\alpha} p^{\beta} \]  

(3)

\[< D_{sJ}(p', \varepsilon) | \bar{c}\gamma_\mu b | B_s(p)> = i \left[ f_0(q^2)(m_{B_s} + m_{D_{sJ}}) \varepsilon_\mu^* + \frac{f_+(q^2)}{(m_{B_s} + m_{D_{sJ}})} (\varepsilon^* P_{\mu}) + \frac{f_-(q^2)}{(m_{B_s} + m_{D_{sJ}})} (\varepsilon^* q_{\mu}) \right] \]  

(4)

where \( f_V(q^2), f_0(q^2), f_+(q^2) \) and \( f_-(q^2) \) are the transition form factors and \( P_\mu = (p + p')_\mu, q_\mu = (p - p')_\mu \). In all following discussions, for customary, we will use following redefinitions:

\[ f_V'(q^2) = \frac{f_V(q^2)}{(m_{B_s} + m_{D_{sJ}})}, \quad f_0'(q^2) = f_0(q^2)(m_{B_s} + m_{D_{sJ}}) \]

\[ f_+'(q^2) = \frac{f_+(q^2)}{(m_{B_s} + m_{D_{sJ}})}, \quad f_-'(q^2) = f_-(q^2)(m_{B_s} + m_{D_{sJ}}) \]  

(5)

For the calculation of these form factors, QCD sum rules method will be employed. We start by considering the following correlator:

\[ \Pi^{V,A}_{\mu\nu}(p^2, p'^2, q^2) = i^2 \int d^4x d^4y e^{-ipx} e^{iq'y} < 0 | T[J_{\nu D_{sJ}}(y) J_{\mu}^{V,A}(0)] | B_s(x) > \]  

(6)

where \( J_{\nu D_{sJ}}(y) = \bar{c}\gamma_\nu \gamma_5 c, J_{\mu B_s}(x) = \bar{b}\gamma_\mu s, J_{\mu}^{V} = \bar{c}\gamma_\mu b \) and \( J_{\mu}^{A} = \bar{c}\gamma_\mu \gamma_5 b \) are the interpolating currents of the \( D_{sJ}, B_s, \) vector and axial vector currents respectively.

To calculate the phenomenological part of the correlator given in Eq. (6), two complete sets of intermediate states with the same quantum number as the currents \( J_{\nu D_{sJ}} \) and \( J_{\mu B_s} \) respectively are inserted. As a result of this procedure we get the following representation of the above-mentioned correlator:

\[ \Pi^{V,A}_{\mu\nu}(p^2, p'^2, q^2) = \]

\[ \frac{< 0 | J_{\nu D_{sJ}}^\prime | D_{sJ}(p')^\varepsilon > < D_{sJ}(p')^\varepsilon | J_{\mu}^{V,A} | B_s(p) > < B_s(p) | J_{\mu B_s} | 0 >}{(p'^2 - m_{D_{sJ}}^2)(p^2 - m_{B_s}^2)} + \ldots \]  

(7)
where \( \cdots \) represent contributions coming from higher states and continuum. The matrix elements in Eq. (7) are defined in the standard way as:

\[
\langle 0 | J_{\nu}^{D_sJ} | D_{sJ}(p') \rangle = f_{D_sJ} m_{D_sJ} \epsilon'_{\nu}, \quad \langle B_s(p) | J_{B_s} | 0 \rangle = -i f_{B_s} \frac{m_{B_s}^2}{m_b + m_s}
\]

where \( f_{D_sJ} \) and \( f_{B_s} \) are the leptonic decay constants of \( D_{sJ} \) and \( B_s \) mesons, respectively. Using Eq. (3), Eq. (4) and Eq. (8) and performing summation over the polarization of the \( D_{sJ} \) meson, Eq. (7) can be written as:

\[
\Pi^{V}_{\mu\nu}(p^2, p'^2, q^2) = -f_{B_s} m_{B_s}^2 \frac{f_{D_sJ} m_{D_sJ}}{(m_b + m_s)(p'^2 - m_{D_sJ}^2)(p^2 - m_{B_s}^2)} \times \left[f_0 g_{\mu\nu} + f_+ P_{\mu\nu} + f'_{-} q_{\mu} P_{\nu} + \text{excited states.} \right]
\]

\[
\Pi^{A}_{\mu\nu}(p^2, p'^2, q^2) = -i \epsilon_{\mu\nu\alpha\beta} p^\alpha p^\beta \frac{f_{B_s} m_{B_s}^2}{(m_b + m_s)(p'^2 - m_{D_sJ}^2)(p^2 - m_{B_s}^2)} \times \left[f_{D_sJ} m_{D_sJ} \right] + \text{excited states.}
\]

In accordance with the QCD sum rules philosophy, \( \Pi_{\mu\nu}(p^2, p'^2, q^2) \) can also be calculated from QCD side with the help of the operator product expansion (OPE) in the deep Euclidean region \( p^2 \ll (m_b + m_c)^2 \) and \( p'^2 \ll (m_c + m_s)^2 \). The theoretical part of the correlator is calculated by means of OPE, and up to operators having dimension \( d = 6 \), it is determined by the bare-loop and the power corrections from the operators with \( d = 3, < \bar{\psi} \psi >, \)

\[
d = 4, m_s < \bar{\psi} \psi >, \quad d = 5, m_0^2 < \bar{\psi} \psi > \quad \text{and} \quad d = 6, < \bar{\psi} \psi \bar{\psi} \psi >. \]

In calculating the \( d = 6 \) operator, vacuum saturation approximation is used to set \( < \bar{\psi} \psi \bar{\psi} \psi > = < \bar{\psi} \psi >^2 \). In calculating the bare-loop contribution, we first write the double dispersion representation for the coefficients of corresponding Lorentz structures appearing in the correlation function as:

\[
\Pi_i^{per} = -\frac{1}{(2\pi)^2} \int ds ds' \frac{\rho_i(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}
\]

The spectral densities \( \rho_i(s, s', q^2) \) can be calculated from the usual Feynman integral with the help of Cutkosky rules, i.e. by replacing the quark prop-
agators with Dirac delta functions: 
\[
\frac{1}{p^2 - m^2} \rightarrow -2\pi\delta(p^2 - m^2),
\]
which implies that all quarks are real. After standard calculations for the corresponding spectral densities we obtain:

\[
\rho_V(s, s', q^2) = N_c I_0(s, s', q^2) [m_s + (m_s - m_b)B_1 + (m_s + m_c)B_2],
\]
\[
\rho_0(s, s', q^2) = N_c I_0(s, s', q^2) [8(m_b - m_s)A_1 - 4m_b m_c m_s
+ 4(m_s - m_b + m_c)m_s^2 - 2(m_s + m_c)(\Delta + m_s^2)
- 2(m_s - m_b)(\Delta' + m_s^2) + 2m_s u] \]
\[
\rho_+(s, s', q^2) = N_c I_0(s, s', q^2) [4(m_b - m_s)(A_2 + A_3) + 2(m_b - 3m_s)B_1
- 2(m_c + m_s)B_2 - 2m_s],
\]
\[
\rho_-(s, s', q^2) = N_c I_0(s, s', q^2) [4(m_b - m_s)(A_2 - A_3) - 2(m_b + m_s)B_1
+ 2(m_c + m_s)B_2 + 2m_s]
\]

where

\[
I_0(s, s', q^2) = \frac{1}{4\lambda^{1/2}(s, s', q^2)},
\]
\[
\lambda(s, s', q^2) = s^2 + s'^2 + q^4 - 2sq^2 - 2s'q^2 - 2ss',
\]
\[
\Delta' = (s' - m_c^2 + m_s^2),
\]
\[
\Delta = (s - m_b^2 + m_s^2),
\]
\[
u = s + s' - q^2,
\]
\[
B_1 = \frac{1}{\lambda(s, s', q^2)} [2s'\Delta - \Delta'u],
\]
\[
B_2 = \frac{1}{\lambda(s, s', q^2)} [2s\Delta' - \Delta u],
\]
\[
A_1 = \frac{1}{2\lambda(s, s', q^2)} [\Delta'^2 s + 2\Delta'm_s^2 s + m_s^4 s + \Delta^2 s' + 2\Delta m_s^2 s' + m_s^4 s' - 4m_s^2 ss' - \Delta\Delta'u - \Delta m_s^2 u - \Delta'm_s^2 u - m_s^4 u + m_s^2 u^2],
\]
\[
A_2 = \frac{1}{\lambda^2(s, s', q^2)} [2\Delta'^2 ss' + 4\Delta'm_s^2 ss' + 2m_s^4 ss' + 6\Delta^2 s'^2
\]
The subscripts V, 0 and ± correspond to the coefficients of the structures proportional to \( \varepsilon_{\mu\nu\alpha\beta}p^\alpha p^\beta \), \( g_{\mu\nu} \) and \( \frac{1}{2}(p_\mu p_\nu \pm p'_\mu p'_\nu) \) respectively. In Eq. (11) \( N_c = 3 \) is the number of colors.

The integration region for the perturbative contribution in Eq. (10) is determined from the condition that arguments of the three \( \delta \) functions must vanish simultaneously. The physical region in \( s \) and \( s' \) plane is described by the following inequalities:

\[
-1 \leq \frac{2ss' + (s + s' - q^2)(m_b^2 - s - m_s^2) + (m_s^2 - m_c^2)2s}{\lambda^{1/2}(m_b^2, s, m_s^2)\lambda^{1/2}(s, s', q^2)} \leq +1 \tag{13}
\]

For the contribution of power corrections, i.e. the contributions of operators with dimensions \( d = 3, 4 \) and 5, we obtain the following results:

\[
f_V^{(3)} + f_V^{(4)} + f_V^{(5)} = \frac{1}{rr'} < r s > - \frac{m_s}{2} < r s > \left[ \frac{-m_c}{rr'} + \frac{m_b}{rr'} \right]
+ \frac{m_s^2}{2} < r s > \left[ \frac{2m_c^2}{rr'} + \frac{m_b^2 + m_c^2 - q^2}{rr'} + \frac{2m_b^2}{rr'} \right]
- \frac{m_0^2}{6} < r s > \left[ \frac{3m_c^2}{rr'} + \frac{3m_b^2}{rr'} + \frac{2}{rr'} \right]
+ \frac{2m_b^2 + 2m_c^2 + m_b m_c - 2q^2}{rr'}
\]
\[ f'_0(3) + f'_0(4) + f'_0(5) = \frac{(m_b - m_c)^2 - q^2}{2rr'} < \bar{s}s > + \frac{m_s}{4} < \bar{s}s > \left[ \frac{-2m_b m'_c + m_c m'_b + m^3_c - m_c q^2}{rr'^2} - \frac{m_c + m_b}{rr'} + \frac{2m_b m^2_c - m^3_b - m_b m'_c + m_b q^2}{r'r'^2} \right] + \frac{m^2_s}{16} < \bar{s}s > \left\{ \frac{-16m_b m'_c + 8m^2_c m'_b + 8m^4_c - 8m^2_c q^2}{r'^3 r} + \frac{-16m^3_b m_c + 8m^4_b + 8m^2_c m'_b - 8m^2_b q^2}{r'r'^3} + \frac{4m^2_c - 8m_b m_c + 4m^2_b - 4q^2}{r'^2 r} + \frac{4m^2_c - 8m_b m_c + 4m^2_b - 4q^2}{r'^2 r} - \frac{8}{rr'} + \frac{1}{r'^2 r'^2} \left[ -8m^3_b m_c - 8m_b m'^3_c + 8m_b m_c q^2 + 4m^4_c + 8m^2_b m^2_c + 4m^4_c - 8m^2_b q^2 - 8m^2_c q^2 + 4q^4 \right] \right\} - \frac{m^2_b}{12} < \bar{s}s > \left[ \frac{3m^2_c (m^2_c + m^2_b - 2m_b m_c - q^2)}{rr'^2} + \frac{3m^2_b m'_c + m^3_b - 2m_b m_c - q^2}{rr'^3} + \frac{-3m_b m_c (m^2_c + m^2_b - q^2) + 2(m^2_c + m^2_b - q^2)^2 - 2m^3_b m_b}{rr'^2} + \frac{3m_c (m_c - m_b) + 2(m^2_b - q^2)}{rr'^2} + \frac{3m_b (-3m_c + m_b) + 4(m^2_c - q^2)}{rr'^2} - \frac{2}{rr'^2} \right] \]

\[ f'_+(3) + f'_+(4) + f'_+(5) = -\frac{1}{2rr'} < \bar{s}s > + \frac{m_s}{4} < \bar{s}s > \left[ \frac{-m_c}{rr'^2} + \frac{m_b}{rr'^2} \right] + \frac{m^2_s}{32} < \bar{s}s > \left[ -\frac{16m^2_c}{r'^3 r} - \frac{16m^2_b}{r'^3 r} + \frac{16}{r'^2 r^2} + \frac{-8m^2_b - 8m^2_c + 8q^2}{r^2 r'^2} \right] + \frac{m^2_b}{12} < \bar{s}s > \left[ \frac{-3m^2_c}{r'^3 r} + \frac{3m^2_b}{r'^3 r} - \frac{2}{r'^2 r^2} + \frac{2m^2_c + m_b m_c - 2q^2}{r'^2 r^2} \right] \]

\[ f'_-(3) + f'_-(4) + f'_-(5) = \frac{1}{2rr'} < \bar{s}s > - \frac{m_s}{4} < \bar{s}s > \left[ \frac{-m_c}{rr'^2} + \frac{m_b}{rr'^2} \right] \]
\[ m^2_s < \bar{s}s > = \frac{16m^2_c}{r'^6r^2} + \frac{16m^2_b}{r'^4r^2} + \frac{16}{r'^2r^2} + \frac{8m^2_b + 8m^2_c - 8q^2}{r'^2r^2} \]
\[-m^2_0 \frac{12}{12} < \bar{s}s > = \frac{3m^2_c}{r'^3r^2} + \frac{3m^2_b}{r'^3r^2} + \frac{6}{r'^2r^2} + \frac{2m^2_b + 2m^2_c + m_bm_c - 2q^2}{r'^2r^2} \]
\[ (14) \]
\[ \text{where } r = p^2 - m^2_b, r' = p'^2 - m^2_c. \]

We would like to note that the contributions of operators with \( d = 6 \) are also calculated. Numerically their contributions to the corresponding sum rules turned out to be very small and therefore we did not present their explicit expressions. Note also that, in the present work we neglect the \( \alpha_s \) corrections to the bare loop. For consistency, we also neglect \( \alpha_s \) corrections in determination of the leptonic decay constants \( f_{B_s} \) and \( f_{D_s^0} \).

The QCD sum rules for the form factors \( f'_V, f'_0, f'_+ \) and \( f'_- \) is obtained by equating the phenomenological expression given in Eq. (9) and the OPE expression given by Eqs. (11-14) and applying double Borel transformations with respect to the variables \( p^2 \) and \( p'^2 \) \((p^2 \rightarrow M^2_1, p'^2 \rightarrow M^2_2)\) in order to suppress the contributions of higher states and continuum:

\[ f'_i(q^2) = -\left(\frac{m_b + m_s}{f_{B_s}m_{B_s}}\right) \frac{1}{f_{D_s^+}m_{D_s^+}} e^{m^2_{B_s}/M^2_1 + m^2_{D_s^+}/M^2_2} \]
\[ \times \left[ -\frac{1}{(2\pi)^2} \int_{(m_b + m_s)^2}^{s_0} ds \int_{(m_c + m_s)^2}^{s'_0} ds' \rho_i(s, s', q^2) e^{-s/M^2_1 - s'/M^2_2} \right. \]
\[ \left. + \hat{B}\{f^3_i + f^4_i + f^5_i\} \right] \]
\[ (15) \]

where \( i = V, 0, \) and \( \pm \), and \( \hat{B} \) denotes the double Borel transformation operator. In Eq. (15), in order to subtract the contributions of the higher states and the continuum, quark-hadron duality assumption is used, i.e. it is assumed that

\[ \rho^{\text{higher states}}(s, s') = \rho^{\text{OPE}}(s, s')\theta(s - s_0)\theta(s - s'_0) \]
\[ (16) \]
In calculations the following rule for double Borel transformations is used:

\[
\hat{B} \frac{1}{r^m} \frac{1}{r^n} \rightarrow (-1)^{m+n} \frac{1}{\Gamma(m) \Gamma(n)} e^{-s/m^2} e^{-s'/m^2} \frac{1}{(M^2)^{m-1}(M^2)^{n-1}}. \tag{17}
\]

3 Numerical analysis

In this section we present our numerical analysis for the form factors \( f_V(q^2), f_0(q^2), f_+(q^2) \) and \( f_-(q^2) \). From sum rule expressions of these form factors we see that the condensates, leptoni
c decay constants of \( B_s \) and \( D_{sJ} \) mesons, continuum thresholds \( s_0 \) and \( s'_0 \) and Borel parameters \( M_1^2 \) and \( M_2^2 \) are the main input parameters. In further numerical analysis we choose the value of the condensates at a fixed renormalization scale of about 1 GeV. The values of the condensates are \[20\] :

\[
<\bar{\psi}\psi|_{\mu = 1\ GeV} = -240 \pm 10\ MeV^3, \quad <\bar{\psi}s >= (0.8 \pm 0.2) \quad <\bar{\psi}\psi > \quad m_0^2 = 0.8\ GeV^2.
\]

The quark masses are taken to be \( m_c(\mu = m_c) = 1.275 \pm 0.015\ GeV, m_s(1\ GeV) \sim 142\ MeV \[21\] \) and \( m_b = (4.7 \pm 0.1)\ GeV \[20\] \) also the mesons masses are taken to be \( m_{D_{sJ}} = 2.46\ GeV \) and \( m_{B_s} = 5.3\ GeV \). For the values of the leptonic decay constants of \( B_s \) and \( D_{sJ} \) mesons we use the results obtained from two-point QCD analysis:

\( f_{B_s} = 209 \pm 38\ MeV \[13\] \) and \( f_{D_{sJ}} = 225 \pm 25\ MeV \[11\] \). The threshold parameters \( s_0 \) and \( s'_0 \) are also determined from the two-point QCD sum rules:

\( s_0 = 35 \pm 2\ GeV^2 \[12\] \) and \( s'_0 = 9\ GeV^2 \[11\] \). The Borel parameters \( M_1^2 \) and \( M_2^2 \) are auxiliary quantities and therefore the results of physical quantities should not depend on them. In QCD sum rule method, OPE is truncated at some finite order, leaving a residual dependence on the Borel parameters. For this reason, working regions for the Borel parameters should be chosen such that in these regions form factors are practically independent of them. The working regions for the Borel parameters \( M_1^2 \) and \( M_2^2 \) can be determined by requiring that, on the one side, the continuum contribution should be small,
and on the other side, the contribution of the operator with the highest dimension should be small. As a result of the above-mentioned requirements, the working regions are determined to be $10 \text{GeV}^2 < M_1^2 < 20 \text{GeV}^2$ and $4 \text{GeV}^2 < M_2^2 < 10 \text{GeV}^2$.

In order to estimate the width of $B_s \to D_{sJ} l \nu$, it is necessary to know the $q^2$ dependence of the form factors $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$, and $f_-(q^2)$ in the whole physical region $m_l^2 \leq q^2 \leq (m_{B_s} - m_{D_{sJ}})^2$. The $q^2$ dependence of the form factors can be calculated from QCD sum rules [15, 16]. For extracting the $q^2$ dependence of the form factors from QCD sum rules we should consider a range $q^2$ where the correlator function can reliably be calculated. For this purpose we have to stay approximately $1 \text{GeV}^2$ below the perturbative cut, i.e., up to $q^2 = 8 \text{GeV}^2$. In order to extend our results to the full physical region, we look for parameterization of the form factors in such a way that in the region $0 \leq q^2 \leq 8 \text{GeV}^2$, this parameterization coincides with the sum rules prediction. The dependence of form factors $f_V(q^2)$, $f_0(q^2)$, $f_+(q^2)$ and $f_-(q^2)$ on $q^2$ are given in Figs.1, 2, 3 and 4, respectively. Our numerical calculations shows that the best parameterization of the form factors with respect to $q^2$ are as follows:

$$f_i(q^2) = \frac{f_i(0)}{1 + \tilde{\alpha} \tilde{q} + \tilde{\beta} \tilde{q}^2 + \tilde{\gamma} \tilde{q}^3 + \tilde{\lambda} \tilde{q}^4}$$

(18)

where $\tilde{q} = q^2 / m_{B_s}^2$. The values of the parameters $f_i(0), \tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$, and $\tilde{\lambda}$ are given in the Table 1.

For completeness, let us discuss the heavy quark mass limit of form factors. In this limit form factors for the $B_s \to D_{sJ}(2460)$ transition is calculated in [21]. In order to perform the heavy quark mass limit and estimate
Table 1: Parameters appearing in the form factors of the $B_s \rightarrow D_{sJ}(2460)\ell\nu$ decay in a four-parameter fit, for $M_1^2 = 15 \text{ GeV}^2$, $M_2^2 = 6 \text{ GeV}^2$

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
 & $f(0)$ & $\alpha$ & $\beta$ & $\gamma$ & $\lambda$ \\
\hline
$f_V$ & 1.18 & -1.87 & -1.88 & -2.41 & 3.34 \\
$f_0$ & 0.076 & 1.85 & 0.89 & 19.0 & -79.3 \\
$f_+$ & 0.13 & -7.14 & 11.6 & 21.3 & -59.8 \\
$f_-$ & -0.26 & -4.11 & -3.27 & 15.2 & 18.6 \\
\hline
\end{tabular}
\end{table}

the dependence of form factors $f_V$, $f_0$, $f_+$ and $f_-$ on $y$ where

$$y = vv' = \frac{m_{B_s}^2 + m_{D_s^*}^2 - q^2}{2m_{B_s}m_{D_s^*}}$$  \hspace{1cm} (19)$$

we follow the procedure as proposed in [22] and evaluate the sum rules at $q^2 = 0$ by taking $m_b \to \infty$, with $m_c = m_b/\sqrt{z}$ where $z$ is fixed and is given by $\sqrt{z} = y + \sqrt{y^2 - 1}$ at $q^2 = 0$. Here, $v$ and $v'$ are the four velocities of $B_s$ and $D_{sJ}$ mesons, respectively. In the $m_b \to \infty$ limit the Borel parameters $M_1^2$ and $M_2^2$ take the form $M_1^2 = 2T_1m_b$ and $M_2^2 = 2T_2m_b/\sqrt{z}$, where $T_1$ and $T_2$ are the new Borel parameters. In this limit, the continuum thresholds $s_0$, and $s_0'$ become

$$s_0 = m_b^2 + m_b\nu_0, \quad s_0' = \frac{m_b^2}{z} + \nu_0\frac{m_b}{\sqrt{z}}$$  \hspace{1cm} (20)$$

and the new integration variables $\nu$ and $\nu'$ are defined as

$$s = m_b^2 + m_b\nu, \quad s' = \frac{m_b^2}{z} + \nu\frac{m_b}{\sqrt{z}}$$  \hspace{1cm} (21)$$

In the $m_b \to \infty$ limit leptonic decay constants $f_{B_s}$ and $f_{D_{sJ}}$ are rescaled as follows:

$$f_{B_s} = \frac{\hat{f}_{B_s}}{\sqrt{m_b}}, \quad f_{D_{sJ}} = \frac{\hat{f}_{D_{sJ}}}{\sqrt{m_c}}$$  \hspace{1cm} (22)$$
In deriving these results we take \( f_V, f_0, f_+ \) and \( f_- \) in the \( m_b \to \infty \) limit are given as

\[
f_V = -\frac{z^{1/4}(1 + \frac{1}{\sqrt{z}})}{\bar{f}_{D,s}\bar{f}_{B_s}} e^{(\Lambda + \bar{\Lambda}/T)} \left\{ -\frac{3}{z^2} \frac{z}{2} \int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu' (\nu - \nu') e^{-2z^{-1/2}} \right\}
\]

\[
\theta(2y\nu\nu' - \nu^2 - \nu'^2) + <\bar{\psi}\psi > [1 + \frac{m_0^2}{T^2}(\frac{1}{24} + \frac{1}{12\sqrt{z}} + \frac{\sqrt{z}}{12})]
\]

\[
f_0 = -\frac{z^{1/4}}{\bar{f}_{D,s}\bar{f}_{B_s}(1 + \frac{1}{\sqrt{z}})} e^{(\Lambda + \bar{\Lambda}/T)} \left\{ -\frac{3}{z^2} \frac{z}{2} \int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu' (\nu - \nu') e^{-2z^{-1/2}} \right\}
\]

\[
\theta(2y\nu\nu' - \nu^2 - \nu'^2) + <\bar{\psi}\psi > [\frac{1}{2} + \frac{1}{2z} - \frac{1}{\sqrt{z}} + \frac{m_0^2}{T^2}(\frac{1}{24} + \frac{1}{12\sqrt{z}} - \frac{\sqrt{z}}{24})]
\]

\[
f_+ = -\frac{z^{1/4}(1 + \frac{1}{\sqrt{z}})}{\bar{f}_{D,s}\bar{f}_{B_s}} e^{(\Lambda + \bar{\Lambda}/T)} \left\{ \frac{3}{z^2} \frac{z}{2} \int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu' (\nu - \nu') e^{-2z^{-1/2}} \right\}
\]

\[
\theta(2y\nu\nu' - \nu^2 - \nu'^2) + <\bar{\psi}\psi > \frac{m_0^2}{T^2}[\frac{1}{12} + \frac{1}{24\sqrt{z}} + \frac{\sqrt{z}}{24}]
\]

\[
f_- = -\frac{z^{1/4}(1 + \frac{1}{\sqrt{z}})}{\bar{f}_{D,s}\bar{f}_{B_s}} e^{(\Lambda + \bar{\Lambda}/T)} \left\{ \frac{3}{z^2} \frac{z}{2} \int_0^{\nu_0} d\nu \int_0^{\nu_0'} d\nu' (\nu - \nu') e^{-2z^{-1/2}} \right\}
\]

\[
\theta(2y\nu\nu' - \nu^2 - \nu'^2) - <\bar{\psi}\psi > \frac{m_0^2}{T^2}[\frac{1}{12} + \frac{1}{24\sqrt{z}} + \frac{\sqrt{z}}{24}]
\]

In deriving these results we take \( T_1 = T_2 = T \) and the parameters \( \Lambda \) and \( \bar{\Lambda} \) are obtained from two-point sum rules that predicts \( \Lambda = 0.62 \text{ GeV} \) [23] and \( \bar{\Lambda} = 0.86 \text{ GeV} \) [24]. Numerical analysis of the above sum rules gives:

\[
f_V(y_{max}) = 0.4, \quad f_0(y_{max}) = 0.031, \quad f_+(y_{max}) = 0.15, \quad f_-(y_{max}) = -0.27
\]

(27)

Where \( y_{max} = 1.30931 \), which corresponds to \( q^2 = 0 \). When we compare these results with the ones given in Eq. [18] at \( q^2 = 0 \), we see that finite mass
corrections are essential for \( f_V(q^2) \) and \( f_0(q^2) \). Here we note that HQET limit for the transition form factors for \( B \) decays is discussed in [25].

For \( B_s \to D_{sJ}(2460) \nu \) decay it is also possible to determine the polarization of the \( D_{sJ}(2460) \) meson. For this aim we determine the asymmetry parameter \( \alpha \), characterizing the polarization of the \( D_{sJ}(2460) \) meson, as

\[
\alpha = 2 \frac{d\Gamma_L/dq^2}{d\Gamma_T/dq^2} - 1 \tag{28}
\]

where \( d\Gamma_L/dq^2 \) and \( d\Gamma_T/dq^2 \) are differential widths of the decay to the states with longitudinal and transversal polarized \( D_{sJ}(2460) \) meson. After some calculations for differential decay rates \( d\Gamma_L/dq^2 \) and \( d\Gamma_T/dq^2 \) we get

\[
\frac{d\Gamma_T}{dq^2} = \frac{1}{8\pi^2m_{B_s}^2} |\vec{p}'| |G_F|^2 |V_{cb}|^2 \left\{ (2A + Bq^2) \left[ |f'_V|^2 (4m_{B_s}^2 |\vec{p}'|^2 + |f'_0|^2) \right] + |f'_0|^2 \right\} \tag{29}
\]

\[
\frac{d\Gamma_L}{dq^2} = \frac{1}{16\pi^4m_{B_s}^2} |\vec{p}'| |G_F|^2 |V_{cb}|^2 \left\{ (2A + Bq^2) \left[ |f'_V|^2 (4m_{B_s}^2 |\vec{p}'|^2 - |f'_0|^2) \right] + m_{B_s}^2 \frac{|\vec{p}'|^2}{m_{D_{sJ}}^2} (m_{B_s}^2 - m_{D_{sJ}}^2 - q^2) + |f'_0|^2 \right\} - \frac{m_{B_s}^2}{m_{D_{sJ}}^2} \left[ \frac{|\vec{p}'|^2}{2} (2m_{B_s}^2 + 2m_{D_{sJ}}^2 - q^2) - |f'_-|^2 \left| \frac{m_{B_s}^2}{m_{D_{sJ}}^2} |\vec{p}'|^2 \right| q^2 \right.
\]

\[
-2 \frac{m_{B_s}^2}{m_{D_{sJ}}^2} \left[ \left( \text{Re}(f'_0 f'_+^* + f'_0 f'_-^*) + (m_{B_s}^2 - m_{D_{sJ}}^2) f'_+ f'_-^* \right) \right]
\]

\[
-2B \frac{m_{B_s}^2}{m_{D_{sJ}}^2} \left[ |f'_0|^2 + (m_{B_s}^2 - m_{D_{sJ}}^2)^2 |f'_+|^2 + q^4 |f'_-|^2 + 2(m_{B_s}^2 - m_{D_{sJ}}^2) \text{Re}(f'_+ f'_-^*) \right] \right\} \tag{30}
\]

where

\[
|\vec{p}'| = \frac{\lambda^{1/2}(m_{B_s}^2, m_{D_{sJ}}^2, q^2)}{2m_{B_s}}
\]
\begin{align*}
A &= \frac{1}{12q^2}(q^2 - m_l^2)^2 I_0 \\
B &= \frac{1}{6q^4}(q^2 - m_l^2)(q^2 + 2m_l^2)I_0 \\
I_0 &= \frac{\pi}{2}(1 - \frac{m_l^2}{q^2})
\end{align*}

(31)

The dependence of the asymmetry parameter $\alpha$ on $q^2$ is shown in Fig. 5. From this figure we see that asymmetry parameter $\alpha$ varies between -0.3 and 0.3 when $q^2$ lies in the region $m_l^2 \leq q^2 \leq 6 \text{ GeV}^2$. An interesting observation is that around $q^2 = 5.2 \text{ GeV}^2$ the asymmetry parameter changes sign. Therefore measurement of the polarization asymmetry parameter $\alpha$ at fixed values of $q^2$ and determination of its sign can give unambiguous information about quark structure of $D_{sJ}$ meson.

At the end of this section we would like to present the value of the branching ratio of this decay. Taking into account the $q^2$ dependence of the form factors and performing integration over $q^2$ in the limit $m_l^2 \leq q^2 \leq (m_{B_s} - m_{D_{sJ}})^2$ and using the total life-time $\tau_{B_s} = 1.46 \times 10^{-12} \text{s}$ [26] we get for the branching ratio

$$B(B_s \rightarrow D_{sJ}(2460)\ell\nu) \simeq 4.9 \times 10^{-3}$$

(32)

which can be easily measurable at LHC.

In conclusion, the semileptonic $B_s \rightarrow D_{sJ}(2460)\ell\nu$ decay is investigated in QCD sum rule method. The $q^2$ dependence of the transition form factors are evaluated. The dependence of the asymmetry parameter $\alpha$ on $q^2$ is investigated and the branching ratio is estimated to be measurably large at LHC.
4 Acknowledgment

One of the authors (A.O) would like to thank TUBA-GEBIP for their financial support.
References

[1] B. Aubert et. al, BaBar, Collaboration, Phys. Rev. Lett. 90, 242001(2003).

[2] D. Besson et. al, CLEO Collaboration, Phys. Rev. D68, 032002(2003).

[3] Y.Nikami et. al. Belle Collaboration, Phys. Rev. Lett. 92, 012002(2004).

[4] P. Krokovny et. al, Belle Collaboration, Phys. Rev. Lett. 91, 262002(2003).

[5] A. Drutskoy et. al. Belle Collaboration, Phys. Rev. Lett. 96, 061802(2005).

[6] B. Aubut et. al. Babar Collaboration, Phys. Rev. Lett. 93, 181807(2004).

[7] B. Aubut et. al. Babar Collaboration, Phys. Rev. D. 69, 031101(2004).

[8] B. Aubut et. al. Babar Collaboration, hep-ex/0408067

[9] P.Colangelo, F. De Fazio and R. Ferrandes, Mod. Phys. Lett. A 19, 2083(2004)

[10] E. S. Swanson, Phys. Rept. 429, 243(2006).

[11] P. Colangelo, F. De Fazio, and A. Ozpineci, Phys. Rev. D72, 074004 (2005).

[12] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147, 385 (1979).

[13] P. Colangelo and A. Khodjamirian, in At the Frontier of Particle Physics/Handbook of QCD, edited by M. Shifman (World Scientific, Singapore, 2001), Vol. 3, p. 1495.
[14] T. M. Aliev, V. L. Eletsky, and Ya. I. Kogan, Sov. J. Nucl. Phys. 40, 527 (1984).

[15] P. Ball, V. M. Braun, and H. G. Dosch, Phys. Rev. D44, 3567 (1991).

[16] P. Ball, Phys. Rev. D 48, 3190 (1993).

[17] A. A. Ovchinnikov and V. A. Slobodenyuk, Z. Phys. C44, 433 (1989); V. N. Baier and A. Grozin, Z. Phys. C47, 669 (1990).

[18] A. A. Ovchinnikov, Sov. J. Nucl. Phys. 50, 519 (1989).

[19] T. M. Aliev, M. Savci, Phys. Rev. D73, 114070(2006)

[20] B. L. Ioffe, Prog. Part. Nucl. Phys. 56, 232 (2006).

[21] Ming Qiu Huang, Phys. Rev. D69, 114015 (2004).

[22] M. Neubert, Phys. Rep. 245, 259 (1994).

[23] T. Huang and C.W. Luo, Phys. Rev. D50, 5775 (1994).

[24] Y. B. Dai, C. S. Huang, C. Liu, and S. L. Zhu, Phys. Rev. D68, 114011 (2003).

[25] Adam K. Leibovich, Zoltan Ligeti, Iain W. Stewart, Mark B. Wise Phys.Rev.Lett. 78 3995-3998 (1997)

[26] S. Eidelman et. al. (Particle Data Group), Phys. Lett. B592, 1 (2004).
Figure 1: The dependence of $f_V$ on $q^2$ at $M_1^2 = 15 \, GeV^2$, $M_2^2 = 6 \, GeV^2$, $s_0 = 35 \, GeV^2$ and $s_0' = 9 \, GeV^2$.

Figure 2: The dependence of $f_0$ on $q^2$ at $M_1^2 = 15 \, GeV^2$, $M_2^2 = 6 \, GeV^2$, $s_0 = 35 \, GeV^2$ and $s_0' = 9 \, GeV^2$.
Figure 3: The dependence of $f_+$ on $q^2$ at $M_1^2 = 15 \text{ GeV}^2$, $M_2^2 = 6 \text{ GeV}^2$, $s_0 = 35 \text{ GeV}^2$ and $s'_0 = 9 \text{ GeV}^2$.

Figure 4: The dependence of $f_-$ on $q^2$ at $M_1^2 = 15 \text{ GeV}^2$, $M_2^2 = 6 \text{ GeV}^2$, $s_0 = 35 \text{ GeV}^2$ and $s'_0 = 9 \text{ GeV}^2$. 
Figure 5: The dependence of $\alpha$ on $q^2$. 