Chapline-Manton interaction vertices and Hamiltonian BRST cohomology

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Abstract
Consistent interactions between Yang-Mills gauge fields and an abelian 2-form are investigated by using a Hamiltonian cohomological procedure. It is shown that the deformation of the BRST charge and the BRST-invariant Hamiltonian of the uncoupled model generates the Yang-Mills Chern-Simons interaction term. The resulting interactions deform both the gauge transformations and their algebra, but not the reducibility relations.

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1 Introduction
The problem of consistent interactions that can be introduced among fields with gauge freedom in such a way to preserve the number of gauge symmetries has been reformulated as a deformation problem of the master equation in the context of the antifield-BRST formalism. This technique has been applied to Chern-Simons models, Yang-Mills theories and two-form gauge fields. Thus, the antifield BRST method was proved to be an elegant tool for analyzing the problem of consistent interactions.

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In this paper we study another interesting interaction, namely, the consistent interaction between the Yang-Mills vector potential and an abelian two-form, but from the Hamiltonian BRST point of view [10], [13]–[17]. Our procedure will lead to combined Yang-Mills-two-form system coupled through a Yang-Mills Chern-Simons term (the Chapline-Manton model) [18]–[21]. Chern-Simons couplings of a two-form to Yang-Mills theory play a major role in the Green-Schwarz anomaly cancellation mechanism [22], and hence are useful in string theory [23]. On the other hand, the Hamiltonian BRST approach appears to be a more natural setting for implementing the BRST symmetry in quantum mechanics [10] (chapter 14), as well as for establishing a proper connection with canonical quantization formalisms, like for instance the reduced phase-space or Dirac quantization procedures [24]. To our knowledge, the Hamiltonian approach to consistent interactions among fields with gauge freedom has not been investigated until now, so our paper establishes a new result.

The strategy to be developed is the following. Initially, we begin with the “free” model describing pure Yang-Mills theory and a free abelian two-form and determine its main Hamiltonian BRST ingredients, namely, the BRST charge and BRST-invariant Hamiltonian. The BRST symmetry of the uncoupled theory, \( s \), can be conveniently written like the sum between the Koszul-Tate differential and the exterior derivative along the gauge orbits, \( s = \delta + \gamma \). Subsequently, we pass to the deformation procedure along the lines of a cohomological approach. Thus, we start by writing down the general equations representing the core of the Hamiltonian deformation procedure, which describe the deformation of the BRST charge, respectively, of the BRST-invariant Hamiltonian of the “free” theory. Then, we proceed to solve the main equations in relation with the model under study taking into account the BRST cohomology of the “free” theory. In this way, we reach the BRST charge and BRST-invariant Hamiltonian underlying the deformed model. Further, we identify the Hamiltonian system behind the deformation procedure by analyzing its first-class constraints, first-class Hamiltonian and the accompanying gauge algebra. It turns out that the resulting system is nothing but the Yang-Mills theory coupled to the 2-form through the Yang-Mills Chern-Simons interaction term, also known as the Chapline-Manton model.
2 Hamiltonian BRST symmetry for the uncoupled theory

In this section we derive the Hamiltonian BRST symmetry for the "free" theory. In this respect, we begin with a Lagrangian action equal with the sum between the actions of Yang-Mills theory and a free 2-form

\[
S^L_0 \left[ A^a_\mu, B^\mu_\nu \right] = \int d^p x \left( -\frac{1}{4} F^a_\mu F^\mu_\nu - \frac{1}{12} F^\mu_\nu F^\nu_\rho \right), \tag{1}
\]

where

\[
F^a_\mu = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - f^{a}_{b c} A^b_\mu A^c_\nu, \tag{2}
\]

\[
F^\mu_\nu = \partial_\mu B^\nu_\rho + \partial_\rho B^\mu_\nu + \partial_\nu B^\rho_\mu \equiv \partial[A^\mu, B^\nu_\rho]. \tag{3}
\]

The canonical analysis of action (1) gives the first-class constraints

\[
G^{(1)}_a \equiv \pi^0_a \approx 0, \quad G^{(1)}_i \equiv \pi^0_i \approx 0, \tag{4}
\]

\[
G^{(2)}_a \equiv - \left( \partial_j \pi^j_a - f^b_{ac} \pi^j_b A^c_j \right) \approx 0, \quad G^{(2)}_i \equiv - 2 \partial^j \pi_{ji} \approx 0, \tag{5}
\]

and the first-class Hamiltonian

\[
H_0 = \int d^{p-1} x \left( \frac{1}{2} \pi_{a j} \pi^a_j + \frac{1}{4} F^a_i F^i_\mu + A^a_0 G^{(2)}_a - \right.
\]

\[
\left. \pi_{ij} \pi^ij + \frac{1}{12} F^i_\mu F^\mu_\nu + B^{0i} G^{(2)}_i \right). \tag{6}
\]

In (4–6), \( A^a_\mu \) and \( B^\mu_\nu \) denote the canonical momenta of \( A^a_\mu \), respectively, \( B^\mu_\nu \). The gauge algebra of the uncoupled model reads as

\[
\left[ G^{(1)}_a, G^{(1)}_b \right] = 0, \quad \left[ G^{(1)}_a, G^{(2)}_b \right] = 0, \quad \left[ G^{(2)}_a, G^{(2)}_b \right] = f^{c}_{ab} G^{(2)}_c, \tag{7}
\]

\[
\left[ G^{(1)}_i, G^{(1)}_j \right] = 0, \quad \left[ G^{(1)}_i, G^{(2)}_j \right] = 0, \quad \left[ G^{(2)}_i, G^{(2)}_j \right] = 0, \tag{8}
\]

\[
\left[ G^{(1)}_a, G^{(1)}_i \right] = 0, \quad \left[ G^{(1)}_a, G^{(2)}_i \right] = 0, \quad \left[ G^{(2)}_a, G^{(1)}_i \right] = 0, \quad \left[ G^{(2)}_a, G^{(2)}_i \right] = 0, \tag{9}
\]

\[
\left[ H_0, G^{(1)}_a \right] = G^{(2)}_a, \quad \left[ H_0, G^{(2)}_a \right] = - f^{c}_{ab} A^0_0 G^{(2)}_c, \tag{10}
\]

\[
\left[ H_0, G^{(1)}_i \right] = G^{(2)}_i, \quad \left[ H_0, G^{(2)}_i \right] = 0. \tag{11}
\]
In addition, the constraint functions $G_i^{(2)}$ are first-stage reducible, i.e.,

$$\partial^i G_i^{(2)} = 0. \quad (12)$$

On account of (7–12), the BRST charge and BRST-invariant Hamiltonian of the uncoupled theory are given by

$$\Omega_0 = \int d^{D-1}x \left( G_a^{(1)} \eta_1^a + G_a^{(2)} \eta_2^a + \frac{1}{2} f_{abc} \mathcal{P}_{2a} \eta_2^b \eta_2^c + G_i^{(1)} \eta_i^1 + G_i^{(2)} \eta_i^2 + \eta \partial^i \mathcal{P}_{2i} \right), \quad (13)$$

$$H_B = H_0 + \int d^{D-1}x \left( (\eta_1^a - f_{abc} \eta_2^b A_0^c) \mathcal{P}_{2a} + \eta_i^1 \mathcal{P}_{2i} \right). \quad (14)$$

In the above, $\eta_1^a$, $\eta_2^a$, $\eta_1^i$ and $\eta_2^i$ stand for the fermionic ghost number one Hamiltonian ghosts, $\eta$ denotes the bosonic ghost number two ghost for ghost, while the $\mathcal{P}$’s represent their corresponding canonical momenta (antighosts). The ghost number is defined like the difference between the pure ghost number ($pgh$) and the antighost number ($antigh$), with

$$pgh \left( z^A \right) = 0, \ pgh \left( \eta^\Gamma \right) = 1, \ pgh \left( \eta \right) = 2, \ pgh \left( \mathcal{P}_\Gamma \right) = 0, \ pgh \left( \mathcal{P} \right) = 0, \quad (15)$$

$$antigh \left( z^A \right) = 0, \ antigh \left( \eta^\Gamma \right) = 0, \ antigh \left( \eta \right) = 0, \ quad \ (16)$$

$$antigh \left( \mathcal{P}_\Gamma \right) = 1, \ antigh \left( \mathcal{P} \right) = 2, \quad (17)$$

where

$$z^A = (A^a, B^{\mu\nu}, \pi^\mu_a, \pi_{\mu\nu}), \ \eta^\Gamma = (\eta_1^a, \eta_2^a, \eta_1^i, \eta_2^i), \ \mathcal{P}_\Gamma = (\mathcal{P}_{1a}, \mathcal{P}_{2a}, \mathcal{P}_{1i}, \mathcal{P}_{2i}). \quad (18)$$

The BRST differential $s \bullet = [\bullet, \Omega_0]$ of the uncoupled theory splits as

$$s = \delta + \gamma, \quad (19)$$

where $\delta$ is the Koszul-Tate differential, and $\gamma$ represents the exterior longitudinal derivative along the gauge orbits. These operators act like

$$\delta z^A = 0, \ \delta \eta^\Gamma = 0, \ \delta \eta = 0, \quad (20)$$

$$\delta \mathcal{P}_{1a} = -\pi^0_a, \ \delta \mathcal{P}_{2a} = \partial_j \pi^j_a - f^b_{ac} \pi^j_b A^c_j, \quad (21)$$
\[ \delta P_{1i} = -\pi_{0i}, \delta P_{2i} = 2 \partial^j \pi_{ji}, \delta P = -\partial^i P_{2i}, \] (22)

\[ \gamma A^a_0 = \eta^a_1, \gamma A^a_i = \partial_i \eta^a_2 + f^a_{bc} \eta^b_2 A^c_i, \gamma B^{0i} = \eta^i_1, \gamma B^{ij} = \partial^i \eta^j_2, \] (23)

\[ \gamma \pi^0_a = 0, \gamma \pi^i_a = f^b_{ac} \pi^i_b \eta^c_2, \gamma \pi_{0i} = 0, \gamma \pi_{ij} = 0, \] (24)

\[ \gamma \eta^a_1 = 0, \gamma \eta^a_2 = -\frac{1}{2} f^a_{bc} \eta^b_2 \eta^c_2, \gamma \eta^i_1 = 0, \gamma \eta^i_2 = \partial^i \eta, \gamma \eta = 0, \] (25)

\[ \gamma P_{1a} = 0, \gamma P_{2a} = f^c_{ab} P_{2c} \eta^b_2, \gamma P_{1i} = 0, \gamma P_{2i} = 0, \gamma P = 0. \] (26)

Formulas (20–26) will be employed in the next section in the framework of the deformation procedure.

3 Deformation of the “free” theory

In this section we deform the uncoupled model discussed above in the framework of the Hamiltonian BRST formalism. First, we write down the general equations underlying the deformation of the BRST charge and BRST-invariant Hamiltonian. Second, we solve these equations with respect to the model under study by using the cohomological technique. Finally, we identify the new gauge theory, which turns out to be nothing but the Chapline-Manton model.

3.1 Hamiltonian deformation problem

It is well-known that the solution to the master equation captures all the information on a given gauge theory at the level of the antifield BRST formalism. The gauge-fixed dynamics is generated by the gauge-fixed action, which is obtained from the solution to the master equation by using a certain gauge-fixing fermion. Moreover, it has been shown that the deformation of the solution to the master equation generates consistent interactions among fields with gauge freedom [5]. At the Hamiltonian level, the BRST charge \( \Omega_0 \) contains all the information on the structure of a first-class system. In this sense, the BRST charge plays a role similar to that of the solution to the master equation. However, in order to stipulate the correct dynamics, one needs a Hamiltonian, which is nothing but the gauge-fixed Hamiltonian \( H_K = H_B + [K, \Omega_0] \), where \( H_B \) stands for the BRST-invariant Hamiltonian.
and $K$ is the gauge-fixing fermion. Thus, we can conclude that the problem of deforming the master equation induces at the Hamiltonian level the deformation of the equation $[\Omega_0, \Omega_0] = 0$, as well as of the BRST-invariant Hamiltonian of the “free” theory.

The Lagrangian deformation implies that the BRST charge of the uncoupled theory is deformed as

$$\Omega_0 \to \Omega = \Omega_0 + g \int d^{D-1} \omega_1 + g^2 \int d^{D-1} \omega_2 + O\left(g^3\right) = \Omega_0 + g \Omega_1 + g^2 \Omega_2 + O\left(g^3\right),$$

where $\Omega$ should satisfy the equation

$$[\Omega, \Omega] = 0.$$

Equation (28) splits accordingly the deformation parameter as

$$[\Omega_0, \Omega_0] = 0,$$

$$2 [\Omega_0, \Omega_1] = 0,$$

$$2 [\Omega_0, \Omega_2] + [\Omega_1, \Omega_1] = 0,$$

$$\vdots$$

Obviously, equation (29) is automatically satisfied. From the remaining equations we deduce the pieces $(\Omega_k)_{k>0}$ on account of the “free” BRST differential. With the deformed BRST charge at hand, we then deform the BRST-invariant Hamiltonian of the “free” theory

$$H_B \to \tilde{H}_B = H_B + g \int d^{D-1} h_1 + g^2 \int d^{D-1} h_2 + O\left(g^3\right) = H_B + g H_1 + g^2 H_2 + O\left(g^3\right),$$

and require that

$$[\tilde{H}_B, \Omega] = 0.$$
\[ [H_B, \Omega_1] + [H_1, \Omega_0] = 0, \quad (35) \]
\[ [H_B, \Omega_2] + [H_1, \Omega_1] + [H_2, \Omega_0] = 0, \quad (36) \]

Clearly, equation (34) is again fulfilled, while from the other equations one can determine the components \((H_k)_{k \geq 1}\) relying on the BRST symmetry of the “free” model.

### 3.2 Deformation of BRST charge

Here, we solve the equations (30–31) in the context of the uncoupled model under discussion taking into account that the “free” BRST differential splits as in (19). Equation (34) holds if and only if \(\omega_1\) is a s-co-cycle modulo \(\tilde{d} = dx^i \partial_i\), i.e.,

\[ s \omega_1 = \partial_j j^k, \quad (37) \]

for some \(j^k\). In order to solve equation (37) we expand \(\omega_1\) according to the antighost number

\[ \omega_1 = (0) + (1) + \cdots + (J), \quad \text{antigh} \left( \omega_1 \right) = I, \quad (38) \]

where the last term in (38) can be assumed to be annihilated by \(\gamma\). Since \(\text{antigh} \left( \omega_1 \right) = J\) and \(\text{gh} \left( \omega_1 \right) = 1\), it follows that \(\text{pgh} \left( \omega_1 \right) = J + 1\). On the other hand, we have that

\[ \rho = \frac{1}{3} f_{abc} \eta^a \eta^b \eta^c, \quad (39) \]

and the ghost for ghost \(\eta\) are \(\gamma\)-invariant, hence we can represent \(\omega_1\) as

\[ \omega_1 = \mu_J \sum_{N,M} (\rho)^N (\eta)^M, \quad (40) \]

where \(N, M\) are some nonnegative integers with \(3N + 2M = J + 1\). With this choice, it is easy to check that the \(\gamma\)-invariant coefficient \(\mu_J\) belongs to \(H_J \left( \delta | \tilde{d} \right)\). Using the results from [25] adapted to the Hamiltonian treatment, it follows that \(H_J \left( \delta | \tilde{d} \right) = 0\) for \(J > 2\) in the case of our uncoupled model.
This means that the last term in (38) corresponds to $J = 2$, which then leads to $3N + 2M = 3$. As a consequence, we have that $N = 1$, $M = 0$ (the ghost for ghost brings no contribution), such that (38) takes the form

$$\omega_1 = \omega_1^{(0)} + \omega_1^{(1)} + \omega_1^{(2)},$$

(41)

where

$$\omega_1^{(2)} = 3\mu_2 f_{abc} \eta_2^a \eta_2^b \eta_2^c,$$

(42)

and $\mu_2$ from $H_2(\delta|\tilde{d})$, therefore solution to the equation

$$\delta \mu_2 + \partial_k v^k = 0,$$

(43)

for some $v^k$. From the last relation in (22) we find that $\mu_2 = \mathcal{P}$, so

$$\omega_1^{(2)} = \frac{1}{3} f_{abc} \mathcal{P} \eta_2^a \eta_2^b \eta_2^c.$$

(44)

At antighost number one, equation (37) takes the form

$$\delta \omega_1^{(2)} + \gamma \omega_1^{(1)} = \partial_k v^k.$$

(45)

Starting from

$$\delta \omega_1^{(2)} = \frac{1}{3} f_{abc} \partial_i \mathcal{P}_{2i} \eta_2^a \eta_2^b \eta_2^c,$$

(46)

we deduce

$$\omega_1^{(1)} = -f_{abc} \mathcal{P}_{2i} \eta_2^a \eta_2^b A^c_i,$$

(47)

such that

$$\delta \omega_1^{(2)} + \gamma \omega_1^{(1)} = \partial_i \left( \frac{1}{3} f_{abc} \mathcal{P}_{2i} \eta_2^a \eta_2^b \eta_2^c \right).$$

(48)

At antighost number zero, equation (37) is given by

$$\delta \omega_1^{(1)} + \gamma \omega_1^{(0)} = \partial_k w^k.$$

(49)

On account of (17), it results that

$$\delta \omega_1^{(1)} = 2f_{abc} \eta_2^a \eta_2^b A_i^c \partial_j \pi^{ji},$$

(50)
which further leads to
\[ \omega_1^{(0)} = 4\pi^{ji} (\partial_j A_{ai}) \eta^a_2, \] (51)
such that
\[ D^{(1)} \omega_1^{(0)} + \gamma \omega_1^{(0)} = \partial_j \left( 2f_{abc} \eta^a_2 \eta^b_2 A^c_i \pi^{ji} \right). \] (52)
Thus, we have generated the first-order deformation of the BRST charge under the form
\[ \Omega_1 = \int d^{D-1}x \left( 4\pi^{ji} (\partial_j A_{ai}) \eta^a_2 - f_{abc} P_{2i} \eta^a_2 \eta^b_2 A^c_i + \frac{1}{3} f_{abc} P \eta^a_2 \eta^b_2 \eta^c_2 \right). \] (53)
The deformation is consistent also to order $g^2$ if and only if $[\Omega_1, \Omega_1]$ is $s$-exact (see (31)). It is easy to see that $[\Omega_1, \Omega_1] = 0$, so $\Omega_2 = 0$. The higher-order equations are then satisfied with $\Omega_3 = \Omega_4 = \cdots = 0$. In this way, we inferred that $\Omega = \Omega_0 + g \Omega_1$ is a complete solution for the equation (28) that describes the deformation of the BRST charge.

3.3 Deformation of BRST-invariant Hamiltonian

Next we pass to determine the deformation of the BRST-invariant Hamiltonian (14). Initially, we compute $H_1$ as solution to the equation (35). Simple calculations lead to the expression of the first term in (35) of the type
\[ [H_B, \Omega_1] = \int d^{D-1}x \left( f_{abc} P_{2i} \eta^a_2 \eta^b_2 (\pi^{ci} - \partial_i A^c_i - f_{de} A^d_0 A^e_i) - 4 (\pi^i_a - f_{abc} A^b_0 A^c_i) \partial_j (\pi_{ji} \eta^a_2) - 2 (\partial_i F^{ijk}) (\partial_j A_{ai}) \eta^a_2 - \left( \eta^c_i - f_{de} A^c_0 \eta^d_2 \right) \left( f_{abc} P \eta^a_2 \eta^b_2 + 4\pi^{ji} \partial_j A_{ci} + 2f_{abc} P_{2i} A^{ci} \eta^b_2 \right) \right) = \int d^{D-1}x \lambda. \] (54)
In consequence, (54) gives
\[ sh_1 + \lambda = \partial_k \alpha^k, \] (55)
for some $\alpha^k$. Then, we further obtain
\[ h_1 = 4 \left( A^0_a \partial_i A^a_j - \pi_{ai} A^a_j \right) \pi^{ij} - \frac{1}{3} F^{ijk} \left( f_{abc} A^a_i A^b_j A^c_k + A^a_i F_{ajk} \right) + 2 \left( \pi^i_a + f_{abc} A^b_0 A^{ib} \right) \eta^a_2 P_{2i} + f_{abc} A^0_i P \eta^a_2 \eta^b_2, \] (56)
such that
\[ sh_1 + \lambda = \partial_k \left( \left( f_{abc} A^c_k \eta^b_2 - 2 F^{ijk} \partial_i A_{aj} \right) \eta^a_2 \right). \]  

(57)

With \( h_1 \) at hand, we pass to solve equation (36). The first term in (36) vanishes \((\Omega_2 = 0)\), while the second term is given by

\[
[H_1, \Omega_1] = \int d^{D-1}x \left( 4 \left( f_{abc} \pi^a_j \eta^b_2 - 4 \partial_j (\pi^j \eta_{2c}) A^c_{ik} \right) + 2 \left( \partial^i \left( f_{abc} A^a_i A^b_j A^c_k + A^a_i F_{ajk} \right) \right) \left( \partial^{ij} A^k_d \right) \eta^d_2 \right) = \int d^{D-1}x \nu. \quad (58)
\]

Therefore, equation (36) implies

\[ sh_2 + \nu = \partial^i \beta_i. \]  

(59)

The solution to (59) reads as

\[ h_2 = -8 A^a A^b \pi_{ik} \pi^{ij} + \frac{1}{3} \left( f_{abc} A^a_i A^b_j A^c_k + A^a_i F_{ajk} \right)^2 + 8 A^a \pi_{ij} \eta^a \eta^b \]  

(60)

so we find that

\[ sh_2 + \nu = \partial^i \left( 2 \left( f_{abc} A^a_i A^b_j A^c_k + A^a_i F_{ajk} \right) \left( \partial^{ij} A^k_d \right) \eta^d_2 \right). \]  

(61)

In this manner, we inferred also the order \( g^2 \) deformation of the BRST-invariant Hamiltonian. The equation describing the order \( g^3 \) deformation is clearly satisfied for \( h_3 = 0 \) because all the terms that do not involve \( h_3 \) vanish. The higher-order deformation equations are then fulfilled for \( h_4 = h_5 = \cdots = 0 \). In conclusion, \( \tilde{H}_B = H_B + g \int d^{D-1}x h_1 + g^2 \int d^{D-1}x h_2 \), with \( h_1 \) and \( h_2 \) expressed by (56), respectively, (60), is solution to the deformation problem of the BRST-invariant Hamiltonian.

### 3.4 Identification of the new gauge theory

Putting together the results deduced in the previous two subsections, we remark that the complete solutions to the deformation problems related to the BRST charge and BRST-invariant Hamiltonian are pictured by

\[
\Omega = \int d^{D-1}x \left( - \left( \partial_j \pi^j - f^{ab} \pi^b \frac{\partial_i \pi_i}{A_{aj}} \right) \eta^a_2 + \pi^a_0 \eta^a_1 + \pi_{0i} \eta^a_1 + \eta \partial^i \mathcal{P}_{2i} - 2 \left( \partial^i \pi^j_{ji} \right) \eta^a_2 + \frac{1}{2} f^{ac} \left( \mathcal{P}_{2c} + 2 g A^c_i \mathcal{P}_{2i} \right) \eta^b_2 \eta^b_2 + \frac{g}{3} \eta^a_2 \eta^b_2 \eta^b_2 \right),
\]

(62)
respectively,

\[
\hat{H}_B = \int d^{D-1}x \left( -A^a_0 \left( \partial_j \pi^j_a - f^b_{ac} \pi^j_b A^c_j - 4g \pi^{ij} \partial_i A_{aj} \right) - 2B^{bi} \partial^j \pi_{ji} -\right.
\]

\[
\frac{1}{2} \left( \pi^a_i + 4g A^{ak} \pi_{ik} \right) \left( \pi^i_a + 4g A_{aj} \pi^{ij} \right) + \frac{1}{4} F^a_{ij} F^a_{ij} + \frac{1}{12} H_{ijk} H^{ijk} -
\]

\[
\pi_{ij} \pi^{ij} + \left( \eta^a_i + 2g \left( f_{abc} A^b_i + \pi^i_a + 4g A_{aj} \pi^{ij} \right) \eta^a_j \right) \mathcal{P}_{2i} +
\]

\[
\left( \eta^a_i - f^a_{bc} A^c_0 \eta^b_j \right) \mathcal{P}_{2a} + gf_{abc} \pi^a_i \eta^b_j \mathcal{P},
\]

(63)

where

\[
H_{ijk} = F_{ijk} - 2g \left( f_{abc} A^b_i A^c_j A^a_k + A^a_0 F_{ajk} \right).
\]

(64)

From the antighost-independent terms in (62) we observe that the deformation of the BRST charge implies the deformed first-class constraints

\[
\tilde{G}^{(2)}_a \equiv - \left( \partial_j \pi^j_a - f^b_{ac} \pi^j_b A^c_j - 4g \pi^{ij} \partial_i A_{aj} \right) \approx 0,
\]

(65)

the remaining constraints in (4–5) being undeformed. Moreover, the term \(gf_{abc} A^c_i \mathcal{P}_{2a} \eta^b_j \mathcal{P}\) shows that the Poisson brackets among the new constraint functions \(\tilde{G}^{(2)}_a\) are also deformed like

\[
[\tilde{G}^{(2)}_a, \tilde{G}^{(2)}_b] = f^c_{ab} \left( \tilde{G}^{(2)}_c + 2g A^i_c \tilde{G}^{(2)}_i \right),
\]

(66)

so the first-class constraint algebra becomes open. On the other hand, the antighost-independent piece in (63)

\[
\hat{H} = \int d^{D-1}x \left( -A^a_0 \left( \partial_j \pi^j_a - f^b_{ac} \pi^j_b A^c_j - 4g \pi^{ij} \partial_i A_{aj} \right) -
\]

\[
2B^{bi} \partial^j \pi_{ji} -\right.
\]

\[
\frac{1}{2} \left( \pi^a_i + 4g A^{ak} \pi_{ik} \right) \left( \pi^i_a + 4g A_{aj} \pi^{ij} \right) -
\]

\[
\pi_{ij} \pi^{ij} + \frac{1}{4} F^a_{ij} F^a_{ij} + \frac{1}{12} H_{ijk} H^{ijk},
\]

(67)

is nothing but the first-class Hamiltonian of the deformed theory. The components linear in the antighost number one antighosts from (63) emphasize that the Poisson brackets among the new first-class Hamiltonian and new first-class constraint functions \(\tilde{G}^{(2)}_a\) are modified as

\[
[\hat{H}, \tilde{G}^{(2)}_a] = -f^c_{ab} A^i_0 \left( \tilde{G}^{(2)}_c + 2g A^i_c \tilde{G}^{(2)}_i \right) + 2g \left( \pi^j_i + 4g A_{aj} \pi^{ij} \right) \tilde{G}^{(2)}_i,
\]

(68)
the others being kept unchanged with respect to the uncoupled model. The resulting first-class Hamiltonian and gauge algebra describe the Yang-Mills Chern-Simons couplings among a Yang-Mills-2-form system, known as the Chapline-Manton model. As the first-class constraints generate gauge transformations, from the deformations \( \Delta A, \Delta B \) we can conclude that the added interactions involved with \( \Delta C \) modify both the gauge transformations and their gauge algebra. However, our procedure does not affect in any way the reducibility relations of the uncoupled theory.

The Lagrangian version of the resulting deformed model can be derived as usually, via employing the extended and total formalisms, which then produce nothing but the well-known Lagrangian action \([18]–[21]\)

\[
\tilde{S}_0^L \left[ A^a_\mu, B_{\mu\nu} \right] = \int d^D x \left( -\frac{1}{4} F^{a}_{\mu\nu} F_{\mu\nu}^a - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right),
\]

subject to the gauge transformations

\[
\delta_\epsilon A^a_\mu = (D_\mu)^a_b \epsilon^b \quad \delta_\epsilon B_{\mu\nu} = \partial_{[\mu} \epsilon_{\nu]} + 2g \epsilon_a \partial_{[\mu} A^a_{\nu]},
\]

where

\[
H_{\mu\nu\rho} = F_{\mu\nu\rho} - 2g \left( f_{abc} A^a_{\mu} A^b_{\nu} A^c_{\rho} + A^a_{[\mu} F_{a_{\nu}\rho]} \right),
\]

and \((D_\mu)^a_b = \delta^a_b \partial_{\mu} + f_{abc} A^c_{\mu}\) is the covariant derivative. It is precisely the piece linear in the deformation parameter from \([33]\) that leads to the second term in the Lagrangian gauge transformations of \(B_{\mu\nu}\).

### 4 Conclusion

To conclude with, in this paper we have derived the consistent interactions that can be introduced among Yang-Mills gauge fields and an abelian two-form. Beginning with the BRST differential for the uncoupled model, we have initially deduced the first-order deformation of the BRST charge by expanding the co-cycles accordingly the antighost number. Subsequently, we have shown that this deformation is consistent also at higher-orders. In the next step we have determined a deformed BRST-invariant Hamiltonian, that is quadratic in the deformation parameter. In this manner, we have generated precisely the combined Yang-Mills-two-form system coupled through Yang-Mills Chern-Simons term. The added interactions deform both the gauge transformations and gauge algebra, but not the reducibility relations.
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