Research on Damage Tree Bayesian Network Simulation Metamodel Based on Fault Tree Analysis

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Abstract. It is difficult for the fault tree analysis method to fully adapt to the needs of damage tree analysis. Based on the original model of the damage tree, the corresponding Bayesian network simulation metamodel is constructed based on the internal relationship between the damage of equipment components reflected by the damage tree. The analysis of the damage tree from the Bayesian network perspective effectively solves the two key problems of the rapidity of damage location and the common cause of damage bottom event.

1. Introduction
In the modern battlefield environment, the fierce firepower strikes from both sides of the enemy make all kinds of weapons and equipment face severe challenges. In the course of operational use, the parts and components of weapons and equipment are seriously damaged or their functions are seriously degraded due to the attack of the enemy firepower, which affects their operational efficiency and causes serious equipment combat damage. Damage tree analysis technology mainly refers to fault tree analysis technology in reliability engineering. Its modeling process, qualitative and quantitative analysis method are similar to fault tree analysis (FTA), and even can directly map fault tree to damage tree. However, in the process of damage tree analysis, two key problems, the rapidity of damage location and the common cause of damage bottom events, need to be solved. Fault tree analysis lacks systematic analysis of these two key issues, which makes it difficult to fully meet the needs of damage tree analysis.

2. Principle of transforming damaged tree to bayesian network
From the point of view of construction and application, fault tree is very similar to Bayesian network. It also has the function of reasoning, and is also a kind of knowledge expression. When using fault tree for reasoning, if the top of the fault tree is analyzed downwards, it can find out which bottom events are related to the system failure, so as to find out the causes of the system failure comprehensively. If tracing back from the bottom of the tree, it can distinguish the way and degree of the impact of each bottom affair on the system, and then it can evaluate the causes of the failure of each component and the system reliability and security, which is the reasoning process of fault tree. Bayesian network not only has this reasoning function, but also can add the actual symptoms of the system into the network as evidence to determine $\lambda(x)$ (support for diagnosis) and $\pi(x)$ (support for prediction), so as to calculate the beliefs $Bel(x) = p(x|e)$, so as to quantitatively describe the reasoning elements through the way of probability. The two main components of Bayesian network are node and connection strength,
which correspond to events (top event, middle event, bottom event) and logic gates in fault tree respectively.

2.1. Mapping relationship between nodes and events
From the inference process and the description process of system state, the nodes in Bayesian network correspond to the events in fault tree, and the events in fault tree are also the complete set of nodes needed to build Bayesian network. However, Bayesian network is better than fault tree in describing system fault state. Since the fault tree assumes that all events have only normal and fault states, the state of the bottom event can be described as:

\[ x = \begin{cases} 
0, & \text{When the bottom event } i \text{ does not occur (normal)} \\
1, & \text{When the bottom event } i \text{ occurs (failure)} 
\end{cases} \]

And the structure function of top event \( T \) is \( \varphi(x) = \varphi(x_1, x_2, \ldots, x_n) \). In this formula:

\[ \varphi(x) = \begin{cases} 
0, & \text{When top event } T \text{ is in state 0} \\
1, & \text{When top event } T \text{ is in state 1} \\
\vdots & \\
n, & \text{When top event } T \text{ is in state } m 
\end{cases} \]

Therefore, events in the fault tree can be mapped directly to nodes in the Bayesian network, which have similarities in the topology.

2.2. Mapping relation between joint strength and logic gate
The logic gates in the fault tree describe the fault logic relationship between father and son events, and the two most critical logic gates are the AND gate and OR gate. For AND gate, output events occur only when all input events occur. For OR gate, output events occur when at least one input event occurs. Logic gates describe the connection relationship between father and son events, which corresponds to the concept of connection strength in Bayesian networks.

2.2.1. The AND gate is expressed by the joint strength. The AND gate structure is shown in Figure 1, and its structure function is \( T = x_1 \land x_2 \land \cdots \land x_n \). When the joint strength is used, the AND gate can be expressed as the conditional probability table shown in Table 1.

![Figure 1 AND gate structure fault tree](image)

Table 1 Connection Strength of AND gate (Conditional probability)

| \( x_1 \) | 1 | 0 |
| \( x_2 \) | 0 | 1 |
| \( \vdots \) | \( \vdots \) | \( \vdots \) |
| \( x_n \) | 0 | 1 |
| \( T \) | 0 | 0 |
| \( \vdots \) | \( \vdots \) | \( \vdots \) |

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From the conditional probability table of AND gate, we can see that the probability of output variable $T$ being in state 1 is 1 only when all input variables are in state 1. As long as the state of one variable is 0, the probability that the output variable $T$ is in state 1 becomes 0, which corresponds exactly to the structure function of the AND gate.

2.2.2. The OR gate is expressed by the joint strength. The OR gate structure is shown in Figure 2, and its structure function is $T=x_0 \lor x_1 \lor \cdots \lor x_n$. When the joint strength is used, the OR gate can be expressed as the conditional probability table shown in Table 2.

![Figure 2 OR gate structure fault tree](image)

| $x_0$ | 1 | 0 |
|------|---|---|
| $x_1$ | 1 | 0 | 1 | 0 |
| ...  | ... | ... | ... | ... |
| $x_n$ | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | ... | 0 | 1 | 0 |
| $T$ | 1 | 1 | 1 | 1 | ... | 1 | 1 | 1 | 1 | ... | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | ... | 0 | 0 | 0 | 0 | 0 | ... | 0 | 0 | 1 |

Table 2 Connection Strength of OR gate (Conditional probability)

From the conditional probability table of OR gate, we can see that the probability of output variable $T$ in state 1 is 0 only when all input variables are in state 0. As long as the state of one variable is 1, the probability that the output variable $T$ is in state 1 becomes 1, which corresponds exactly to the structure function of the OR gate.

The mapping relationship between logic gate and connection strength is established. Since the logic gate describes deterministic events, it is reflected in the conditional probability table. The probability of occurrence of output event $T$ is either 0 or 1. For uncertain events, the probability of occurrence of output event $T$ belongs to (0,1). Thus, logic gate is a special case of connection strength.

3. Common cause analysis of bottom events in damaged trees

3.1. Combination of damage bottom events
The damage tree directly transformed from fault tree is independent of each other, so it cannot reflect the common cause of equipment battlefield damage, which leads to the inconsistency between the structure of damage tree and the real equipment battlefield damage. For the bottom event with common cause damage, it can be merged into a bottom event group. The correlation coefficient analysis method is used to divide all the bottom events into several common cause event groups. The merging process is shown in Figure 3.
3.2. Damage correlation coefficient acquisition

Taking a certain equipment damaged by projectile fragments as an example, the number of hit fragments of each component of the equipment is used as the test index, and the correlation coefficient of damage is calculated according to the index. In order to determine the effective lethal distance of ammunition to equipment, the method of hypothesis test is used to calculate the effective lethal distance in statistical sense. Under the condition that the distance between the explosive point and the equipment is $L_j$, the hypothesis is tested under the significance level $\alpha$: $H_0: \mu_j = 0$ ($\mu_j$ indicates the total number of fragments hit by the equipment at the position of the $j$th explosive point, and $\mu_j \approx 0$ indicates that there is no fragments hit equipment).

According to the order of distance equipment from near to far, $m$ explosive points are set, and the distance between each explosive point and equipment is $L_j (j = 1, 2, ..., m)$. $n$ simulation tests are carried out at each explosive point. $X_{j1}, X_{j2}, ..., X_{jn}, X_j$ are obtained to indicate the total number of fragments hit by equipment in the $i$th simulation at the $j$th explosive point. Constructive variable

$$T_j = \frac{(X_j(n) - 0)\sqrt{n}}{S_j(n)},$$

which conforms to $t(n-1)$ distributions, of which

$$X_j(n) = \frac{1}{n} \sum_{i=1}^{n} X_{ji},$$

$$S_j^2(n) = \frac{1}{n-1} \sum_{i=1}^{n} (X_{ji} - X_j(n))^2.$$

For a given saliency level of $\alpha$, if $|X_j(n) - 0| \geq t_{\alpha/2} \frac{S_j(n)}{\sqrt{n}}$, then reject hypothesis $H_0$, that is to say, there is a significant difference between the total mean and 0, that is, the distance corresponding to the $j$th explosion point $L_j$ is the effective killing distance. if $|X_j(n) - 0| < t_{\alpha/2} \frac{S_j(n)}{\sqrt{n}}$, accept hypothesis $H_0$, that is, there is no difference between the total mean and 0, that is, the distance corresponding to the $j$th explosion point $L_j$ is the invalid killing distance. Among them, $t_{\alpha/2} (n-1)$ is the bilateral quantile of $t$ distribution $t(n-1)$.

Figure 4 shows the relationship between the average number of shrapnel hitting and the distance of the equipment at each explosion point in a certain direction. Through zero hypothesis test, it is considered that the lethal effect of the ammunition on the equipment can be neglected after the position of the No.13 explosion point. That is to say, in the damage simulation test, the explosion point should be set within the range of $L_{13}$ to ensure the acquisition of equipment damage data with correlation analysis value.
3.3. Damage correlation analysis

Assuming that the equipment consists of \( l \) components, the average number of shrapnel hit by each component at \( h \) explosion points can be obtained by damage simulation, which is \( Y_{jk} \). \( j (j = 1,2,\ldots,h) \) denotes the ordinal number of the position of the explosion point. \( k (k = 1,2,\ldots,l) \) represents the serial number of components. Thus, the correlation coefficient \( r_{pq} \) between the number of fragments hit by component \( p \) and component \( q \) can be calculated:

\[
r_{pq} = \frac{\sum_{j=1}^{h} (Y_{jp} - \bar{Y}_{p})(Y_{jq} - \bar{Y}_{q})}{\sqrt{\sum_{j=1}^{h} (Y_{jp} - \bar{Y}_{p})^2} \sqrt{\sum_{k=1}^{m} (Y_{kp} - \bar{Y}_{k})^2}} (p,q = 1,2,\ldots,m)
\]

Where, \( \bar{Y}_{p} \) and \( \bar{Y}_{q} \) represent the average number of fragments hit by the \( p \)th and \( q \)th part at \( h \) explosion points, respectively. It can be expressed as:

\[
\bar{Y}_{p} = \frac{\sum_{j=1}^{h} Y_{jp}}{h}
\]

Finally, the correlation coefficient matrix of the number of shrapnel hit by each component can be obtained. Obviously, there are \( r_{pp} = 1, r_{pq} = r_{qp} \).

The smaller \( |r_{pq}| \) is, the worse the linear regression effect is, which indicates that the linear correlation between the damage of two components is not obvious. The bigger \( |r_{pq}| \) is, the better the regression effect is, the more significant the linear correlation between the damage of two components is. Therefore, \( |r_{pq}| \) test can judge whether the linear correlation is significant. The critical value \( r_{\alpha} \) of \( r_{pq} \) test can be found in the critical value table of correlation coefficient, and the corresponding \( r_{\alpha} \) values can be found according to the significance level \( \alpha \) and sample size. When \( r > r_{\alpha} \), the regression effect is significant and the correlation is strong. At \( r \leq r_{\alpha} \), the regression effect is not significant and the correlation is weak. According to this principle, if the damage correlation coefficients between some components in the damage tree exceed the critical value, the corresponding damage base events of these components can be merged into a common cause base event group, and the problem that the common cause of the damage base event is neglected can be solved.

4. An example of transforming damage tree to bayesian network simulation metamodel

The damage tree of an artillery equipment is shown in Figure 5 (a). After damage simulation, the correlation coefficient between bottom event \( X_2 \) and bottom event \( X_3 \) corresponding components damage exceeds the critical value, and they are merged into a bottom event group \( X_{2,3} \), as shown in
Figure 5(b). The relationship between the number of shrapnel hit by two components is shown in Figure 6, which shows a strong correlation. The damage tree shown in Figure 5 (b) can be transformed into the corresponding Bayesian network as shown in Figure 5 (c).

![Damage Tree and Bayesian Network Conversion](image)

(a) Initial damage tree (b) Damage tree after merging bottom events (c) Bayesian networks

Figure 5 Bottom event merging and bayesian network conversion in damage tree

![Relation between Number of Shrapnel Hit](image)

Figure 6 Relation between the number of fragments hit by components

As shown in Table 3, the probability of occurrence of $X_1$, $X_{2,3}$ and $X_4$ is 75.68%, 50.81% and 46.27% respectively after the initial information $T$ is fed into the network. According to the ranking of probability, the damage of $X_1$ part is detected firstly, and the detection shows that $X_1$ does happen, that is, the damage of $X_1$ part. Then the damaged parts are repaired to restore their normal state and re-entered into the network as evidence. If $T$ still occurs, then $X_{2,3}$ must occur. The order of damage location is determined by the probability of occurrence of events, so that the rapidity of damage location can be realized.

| Damage location process | (1) Initial information | (2) Damage location | (3) Repair $X_1$ |
|-------------------------|------------------------|--------------------|------------------|
| Input evidence          | $T$ occurrence         | Detection of $X_1$ | $X_1$ occurs.    |
| $X_1$ Occurrence probability | 75.68%            | 100%                | 0                |
| $X_{2,3}$ Occurrence probability | 50.81%             | 35%                 | 100%             |
| $X_4$ Occurrence probability | 46.27%             | 29%                 | 100%             |

Table 3 Damage location analysis process

5. Conclusion

Damage tree is actually a special case of Bayesian network. It can only analyze two-state (normal, damage) events and deterministic damage logic relationship (with gate, or gate), while Bayesian
network can analyze polymorphic events and Uncertain Causal logic relationship (conditional probability). Based on the original model of DMA, the corresponding Bayesian network simulation meta-model is constructed according to the intrinsic relationship between the damage of equipment components reflected by DTA. The analysis of damage tree from the perspective of Bayesian network can effectively solve the problems of rapid damage location and common cause of damage bottom events. This method not only extends damage tree analysis, but also applies to fault tree analysis.

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