Saturated porous layers squeezed between parallel disks in enclosed cells

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Abstract. Theoretical and experimental evidences show that high lift forces can be generated when a porous layer imbibed with a fluid is subjected to compression by a rigid and impermeable component in normal (approaching) relative motion. If the porous layer is soft enough to neglect its solid structure reaction to compression then the pressure increase can be entirely attributed to the flow resistance of the porous structure when the fluid is squeezed out. The mechanism is highly dependent on the variation of permeability with porosity at its turn variable with the rate of compression. Such a mechanism can be used for impact damping but realistic applications need to consider an enclosed system which keeps the squeezed fluid inside and allows for re-imbibition. The paper presents a simple analytical model for the effects produced in highly compressible porous layers imbibed with Newtonian liquids, during compression between two parallel rigid disks placed in enclosed cells with variable volume buffer, similar to a hydro-pneumatic accumulator.

1. Introduction

Porous materials and squeeze-film systems have become increasingly popular for a variety of applications in engineering (e.g. shock absorbers and squeeze dampers). Various squeezing effects have been analysed with relatively rigid, porous materials, in different geometrical configurations (spherical, cylindrical or plane contacts) and loading conditions (constant speed, constant force or constant energy). All these applications were characterized by constant permeability. A more complex squeezing mechanism in the presence of highly deformable porous media has been relatively recently put into light [1]. It was named by professor M.D. Pascovici ex-poro-hidrodynamic (XPHD) lubrication. Its characteristic is the variation of permeability during the process, due to high compression rates that modifies (decrease) material porosity. It was shown, both theoretically and experimentally its potential in generating high lift forces produced by the resistance to flow through the porous structure [2,3]. One of the possible applications is in shock absorption.

This complex process involves the squeeze of fluid from the porous layer at different levels of permeability. XPHD mechanism is strongly dependent on porosity variation and describes the lift effects produced by the flow of fluid through a porous material. As the porous structure is highly deformable (very soft) the normal forces generated by elastic compression of the fibers comprising the solid phase are negligible compared to the pressure forces.

Similar lift effects have been extensively studied by Weinbaum [4], with special emphasis on sliding motion over highly porous and soft materials filled with air. Even if, apparently, the...
mechanism differs from that of normal motion, both show a lift force generated under rapid compaction of the soft material.

Until now the XPHD mechanism proved to be efficient for the different configurations (sphere/plane, cylinder/cylinder, cylinder/plane, disk/plane) in open squeeze system. It means that the relative pressure on the contact boundaries is null (atmospheric pressure). Pascovici and co-workers [5] analyzed the squeeze flow at the contact between a disk and a conjugated plane surface, in case of non-zero pressure, modelling the flow into an extended porous layer (porous layer diameter is greater than that of the squeezing disk).

However for practical applications the system should be able to operate continuously, that is after squeezing out, the fluid must be re-imbibed into the porous layer, able to support a new loading. As a consequence, the fluid expelled out must be collected into a variable volume buffer, able to collect the whole volume of the dislocated fluid. The buffer can be represented either by a volume with elastic limits (toroidal membrane similar to an elastic bladder) or by a variable volume accumulator separated by a differential piston from a gas buffer chamber (solution similar to hydro-pneumatic systems used in shock dampers). This arrangement is called closed-cell as there is no fluid flow to the environment. The model presented in this paper is focused on squeeze effects with variable pressure at the boundaries of the disks corresponding to the squeeze flow case in an encapsulated system when the fluid expelled out by the squeezing disks flows into a variable hydro-pneumatic buffer. The present work is originated from the necessity to investigate the damping capacity of the porous media imbibed with liquids, in cyclic loading conditions, a relatively new subject but which experiences an increasing interest due to its innovative approach and the variety of its possible applications.

2. The model
The model for the closed-cell in hydro-pneumatic-like configuration is schematically presented in figure 1. Two disks of radius, \( R \), both perfectly rigid, separated by a soft porous layer, are placed into an enclosed cell. The porous layer of initial thickness, \( h_0 \), and initial porosity, \( \varepsilon_0 \), is completely imbibed with a Newtonian fluid of viscosity, \( \eta \), assumed constant during the squeeze process. The upper disk can move downwardly, squeezes the porous layer and the liquid expelled out is collected in a variable volume accumulator created by a differential piston that separates it from a buffer gas chamber. The initial volume of the gas chamber is \( V_0 \) and its pressure is \( p_0 \). On the other side of the piston, the initial volume of the accumulator is assumed zero. During the squeeze, the fluid expelled out radially from the compressed porous layer is collected into the accumulator with simultaneous displacement of the piston toward the gas chamber, compressing the gas. The new volume of the gas chamber is reduced with the volume of the dislocated fluid. Neglecting the piston weight and the friction force between the piston and cylinder wall, the pressure in the accumulator equals the pressure in the gas chamber.

![Figure 1. The working principle of an enclosed system.](image-url)
The expelling of fluid from the porous material generates high lift forces on the moving disk and compresses the gas from the buffer. For this hydro-pneumatic system it is customary to assume that the heat generated by compression dissipates into the environment and the temperature remains constant during the process; thus isothermal change of state according to Boyle-Mariotte law can be used.

So that the pressure, $p$, can be expressed function of the initial volume of the buffer, $V_0$, and of the porous layer thickness, $h$, as follows:

$$p = \frac{p_0 V_0}{V_0 - \pi R^2 (h_0 - h)} \quad (1)$$

Introducing the relative buffer initial volume, $\bar{V}_0 = \pi R^2 h_0 \varepsilon_0 / V_0$ and the dimensionless thickness, $H = h / h_0$, relation (1) can be rewritten in the following form:

$$p = \frac{p_0 \varepsilon_0}{\varepsilon_0 - \bar{V}_0 + \bar{V}_0 H} \quad (2)$$

The XPHD lubrication modelling is based on the following simplifying hypotheses:

- The fluid is Newtonian, the flow is laminar, isothermal and isoviscous;
- The contact surfaces are rigid and impermeable;
- The highly compressible porous layer (HCPL) is homogeneous, isotropic and relatively thin; as consequence the pressure is assumed constant across the thickness of the porous layer;
- During the squeeze process the rigid plates remain parallel so that the thickness of the porous layer is constant in space and varies only in time;
- The flow through the porous layer is modelled with Darcy equation;
- The elastic forces of the porous medium are negligible compared to the flow resistance forces.

Highly porous materials are mainly characterized by porosity (voids in the structure of the solid material) and by permeability, which is the ability of the material to allow the flow of fluid through it. Throughout this paper, we will define the porous layer using compactness which is the inverse of porosity, and defined by this equation:

$$\sigma = 1 - \varepsilon \quad (3)$$

During compression, the pore geometry changes, expelling the fluid from the material. In this process, the solid volume of the structure is compressed until it reaches zero porosity. Assuming that the porous layer does not change its circular area (does not expand during compression), the solid fraction conservation yields to the following equation:

$$\sigma h = \sigma_0 h_0 \quad (4)$$

The permeability of the porous layer is associated to the porosity/compactness according with Kozeny–Carman law [6], regularly used in previous studies ([1]-[3], [5], [8]-[12]):

$$\phi = \frac{D(1 - \sigma)^3}{\sigma^2} \quad (5)$$

where $D = d^2 / 16k$ is porous structure parameter, dependent on a characteristic dimension of the porous structure, $d$. Originally proposed for fluid flow through porous media consisting of spherical particles with relatively constant diameters, Kozeny–Carman equation proposes for the correction factor, $k$, correction values between 5 and 10. Later, Ghaddar [7] showed that equation 5 can be used with satisfactory results for fibrous-based porous structures, where $d$ is the average fiber diameter.
3. Analytical solution

The model described in previous section will be analysed in two loading conditions: constant velocity \((w = ct)\) and constant force \((F = ct)\), respectively. Each of them will be presented separately in the following chapters.

In both cases, as the squeezing surfaces remain parallel, the axisymmetry prevails and, assuming constant pressure across the thickness of the porous layer, the analysis becomes 1-D [13].

3.1. Constant velocity squeeze \((w = ct)\)

The flow rate conservation equation at any radius, \(r\), for a given normal velocity, \(w\), can be written by equating the fluid dislocated during compression with the fluid flowing radially across the circular boundary, according to Darcy law:

\[
\pi r^2 w = -2\pi r \frac{\phi h dp}{\eta dr} \tag{6}
\]

Combining equations (4), (5) and (6) and rearranging the terms, the pressure gradient can be expressed as:

\[
\frac{dp}{dr} = -\frac{\eta \omega \sigma_0^2 r}{2Dh_0(H - \sigma_0)^3} \tag{7}
\]

Integrating equation (7) for variable pressure on the outer edge of the porous layer \(p = p(H) - \text{equation (2)}\) at \(r = R\), we get the parabolic pressure distribution on the disk:

\[
p = \frac{p_0(1 - \sigma_0)}{V_0 H - V_0 - \sigma_0 + 1} + \frac{\eta \omega \sigma_0^2}{4Dh_0(H - \sigma_0)^3}(R^2 - r^2) \tag{8}
\]

Integrating the pressure distribution on the surface of contact, we obtain the lift force at a given compression, expressed in terms of dimensionless layer thickness, \(H\):  

\[
F = 2\pi \int_0^R rpdr = \frac{\pi R^2 p_0(1 - \sigma_0)}{V_0 H - V_0 - \sigma_0 + 1} + \frac{\pi \eta \omega \sigma_0^2 R^4}{8Dh_0(H - \sigma_0)^3} \tag{9}
\]

Finally, using a complex parameter, \(P = \eta wR^2/Dh_0p_0\) and dimensionless lift force, \(\bar{F} = F/\pi R^2p_0\), equation (9) takes the form:

\[
\bar{F} = \frac{1 - \sigma_0}{V_0 H - V_0 - \sigma_0 + 1} + \frac{P \sigma_0^2}{8(H - \sigma_0)^3} \tag{10}
\]

From equation (10) one can observe that the dimensionless lift force generated at a certain compression level, \(H\), depends on three parameters: the initial compactness, \(\sigma_0\), the relative buffer initial volume, \(V_0\) and the complex parameter, \(P\).

For a parametric analysis we selected typical values used in preliminary tests where a disk of radius \(R = 25\) mm moves downwardly compressing with constant velocity \((w = 5\) mm/s\) onto a 4 mm thickness fiber-based textile layer imbibed with water \((\eta = 10^{-3}\) Pa s\) placed in an enclosed cell whose internal pressure is 1 bar. According to microscope image measurements, the fibers diameters range in between 20 and 50 \(\mu m\) and thus, the complex permeability parameter ranges between \(3 \cdot 10^{-12}\) and \(3 \cdot 10^{-10}\) \(m^2\). Correspondingly, the complex parameter is close to \(P = 1\) and two other possible values have been also selected for the numerical applications \(P = 0.1\) and \(P = 10\).

Figure 2 plots the evolution of the lift force in function of thickness, \(H\) for a value of initial compactness, \(\sigma_0 = 0.2\) in three cases: \(P = 10\), \(P = 1\) and \(P = 0.1\). The force variation is different for different values of the initial gas volume, noting that the lift force increases with increasing of the pressure, \(p_0\) and thus with decreasing of the volume, \(V_0\). The case of \(V_0 = 0\) corresponds to the hypothetical case of an infinite volume buffer.
The parametric analysis of the lift force shows that the higher is the initial compactness, $\sigma_0$, the higher is the force generated by solid structure of the porous material (figure 2.c).

![Figure 2](image-url)

**Figure 2.** Variation of the dimensionless lift force, $\bar{F}$ versus dimensionless HCPL thickness, $H$ for different values of complex parameter, $P = 10$ (a), $P = 1$ (b, c), respectively $P = 0.1$ (d).

It can be seen that the lift force increases with the compression of the material, due to the porosity that drops down to zero, a behaviour also observed in the case of null pressure at the boundaries.

### 3.2. Constant force squeeze ($F = ct$)

The solution of the model for the constant force loading is based on the fact that the upper disk velocity is variable:

$$w = -\frac{dh}{dt} \quad (11)$$

Combining equations (9) and (11) and using a new variable, $X = H - \sigma_0$, the following differential equation with separable variables is obtained:
\[
\psi(V_0X + \bar{V}_0\sigma_0 - \bar{V}_0 - \sigma_0 + 1) \frac{dX}{X^3[\psi(V_0X + \bar{V}_0\sigma_0 - \bar{V}_0 - \sigma_0 + 1) - 1 + \sigma_0]} = -\frac{8}{\pi\sigma_0^2} d\tau
\]

where \(\tau = \frac{FDt}{\eta R^2} \) is the dimensionless time and \(\psi = \frac{F}{\pi R^2 \bar{p}_0} \) is the dimensionless constant load.

Noting that \(\alpha = (1 - \bar{V}_0)(1 - \sigma_0)\) and \(\beta = (1 - \sigma_0)/\psi\), the above equation becomes:

\[
\frac{\bar{V}_0X + \alpha}{X^3(\bar{V}_0X + \alpha - \beta)} dX = -\frac{8}{\pi\sigma_0^2} d\tau
\]

The variation in time of the dimensionless thickness of porous layer, \(H\), by squeezing with a constant load, results from the differential equation (13) solved with the initial condition \(h = h_0\) (\(H = 1\)) at \(t = 0\):

\[
\tau = \frac{\pi\sigma_0^2}{8(\alpha - \beta)} \left\{ \frac{\beta\bar{V}_0^2}{(\alpha - \beta)^2} \ln \left[ \frac{1 - \beta - \bar{V}_0(1 - H)}{(H - \sigma_0)(1 - \beta)} \right] + \right. \\
+ \left. \frac{1 - H}{(1 - \sigma_0)(H - \sigma_0)} \left[ \frac{\alpha(1 - 2\sigma_0 + H)}{2(1 - \sigma_0)(H - \sigma_0)} - \frac{\beta\bar{V}_0}{\alpha - \beta} \right] \right\}
\]

Figure 3 plots the evolution of the squeeze duration function of the thickness, \(H\) for initial compactness, \(\sigma_0 = 0.2\) in the case of dimensionless constant load, \(\psi = 0.1\). As can be seen, the longer time for the impacting disk to reach zero porosity corresponds to the lower buffer volume (\(\bar{V}_0 = 1\)).

**Figure 3.** Variation of the dimensionless HCPL thickness, \(H\) in time for the case \(\psi = 0.1\).

### 4. Concluding remarks

Analytical solutions for squeeze flow at the contact between a disk and a conjugated porous surface were developed in the case that pressure on the outer edge of the porous layer is variable (enclosed systems). Throughout this paper, the squeeze process of HCPL imbibed with fluids was analyzed for two loading cases: constant speed squeeze \((w = ct)\) and constant force squeeze \((F = ct)\).

The results show that for porosities greater than 0.6 the enclosed cell generates significant greater forces than the classical open squeeze system. Noted that the lift force increases with increasing of the buffer pressure. In addition, the lower is the buffer volume the higher is the lifting force generated by solid structure of the porous material during squeeze process.
Same as in the case of null pressure at the boundaries, it can be observed that the initial value of the lift force is not zero due to a reaction of the porous material. These theoretical studies prove the lift effect generation and therefore the undeniable applicability in impact damping applications. For the future, parametric squeeze models that were studied theoretically for the variable pressure case can be used for model validation.

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Nomenclature

**Latin letters**
- d – characteristic dimension of the porous structure [m];
- D – porous structural parameter [m²];
- F – lift force [N];
- \( \bar{F} \) – dimensionless lift force, \( \bar{F} = F / \pi R^2 p_0 \);
- \( h_0 \) – thickness of the porous material [m];
- H – dimensionless thickness, \( H = h / h_0 \);
- k – constant in Kozeny–Carman equation;
- p – pressure [Pa];
- \( p_0 \) – initial pressure in the buffer [Pa];
- P – complex parameter, \( P = \eta w R^2 / D h_0 p_0 \);
- r – radial coordinate [m];
- R – disk radius [m];
- t – time [s];
- \( V_0 \) – initial volume of the buffer [m³];
- \( \bar{V}_0 \) – relative buffer initial volume, \( \bar{V}_0 = \pi R^2 h_0 \varepsilon_0 / V_0 \);
- w – squeeze velocity [m/s];

**Greek letters**
- \( \varepsilon \) – porosity [-];
- \( \eta \) – viscosity of the liquid [Pa·s];
- \( \sigma \) – compactness [-];
- \( \tau \) – dimensionless time (for constant force squeeze), \( \tau = F D t / \eta R^4 \);
- \( \phi \) – permeability of the porous layer [m²];
- \( \psi \) – dimensionless constant load, \( \psi = F / \pi R^2 p_0 \).

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