Distributed Sequential Hypothesis Testing with Dependent Sensor Observations

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Abstract—In this paper, we consider the problem of distributed sequential detection using wireless sensor networks (WSNs) in the presence of imperfect communication channels between the sensors and the fusion center (FC). We assume that sensor observations are spatially dependent. We propose a copula-based distributed sequential detection scheme that characterizes the spatial dependence. Specifically, each local sensor collects observations regarding the phenomenon of interest and forwards the information obtained to the FC over noisy channels. The FC fuses the received messages using a copula-based sequential test. Moreover, we show the asymptotic optimality of the proposed copula-based sequential test. Numerical experiments are conducted to demonstrate the effectiveness of our approach.

Index Terms—Wireless sensor network, distributed sequential detection, sequential probability ratio test, dependence modeling, copula theory

I. INTRODUCTION

Wireless sensor networks (WSNs) consist of a large number of spatially distributed sensors collaborating to solve inference problems, such as detection, estimation, and classification [1]. The local sensors collect noisy observations of the phenomenon of interest, and transmit the collected information to the fusion center (FC) which fuses the received data and produces a global decision. In this paper, we study the problem of distributed detection, where local sensors and the FC collaborate to detect the presence or absence of the target of interest.

The distributed detection problems in sensor networks with fixed-sample-size (FSS) have been studied extensively [2], [3], where the goal often is to minimize some cost at the FC based on a fixed number of observations. However, many detection problems are inherently sequential, where the observations are collected and processed sequentially [4]. In addition, sequential decision procedures provide additional advantage of faster decision making. In [5], Wald proposed the sequential probability ratio test (SPRT) for sequential binary hypothesis testing. SPRT was shown to be optimal in the sense that it takes minimum time on average to make a decision among all the tests which guarantee the same probabilities of error. Distributed versions of SPRT were first proposed in [6]–[8], where the proposed algorithms were shown to be (asymptotically) optimal under independence assumptions of the sensor observations. Moreover, in [6]–[8], the channels from the sensors to the FC were assumed to be noiseless. In this work, we propose a novel approach for sequential detection in a distributed sensor network which does not require the observations to be independent across sensors. Also, the channels from the sensors to the FC can be noisy.

Dependence across spatially distributed sensors is a critical issue in distributed detection problems. This dependence exists due to a variety of reasons such as sensing of the same phenomenon and corruption by correlated noise. Some works have studied the effect of dependence on the performance of detection systems [9]–[14]. However, in many systems of interest, this dependence is ignored and the sensor observations are generally assumed to be independent. Ignoring this underlying dependence degrades detection performance as shown in [15]–[17]. One way of combating this unknown dependence is by posing the sequential detection problem in a non-parametric framework [18], [19]. However, providing optimality guarantees for such problems is often not feasible. In this work, we use a copula-based approach to model the dependence and show its asymptotic optimality.

Copula-based approach [20] is a flexible parametric methodology that decomposes the joint distribution of sensor observations into arbitrary marginal distributions and a multivariate distribution (dependence) that is referred to as the copula distribution. It has been shown that copula-based fusion of multiple sensing observations can significantly improve inference performance [15]–[17], [21]. However, the work in [15]–[17], [21] were FSS based tests. In contrast, in this paper, we propose a novel copula-based distributed sequential hypothesis test with the main contributions of the work summarized below.

- We propose a copula-based sequential hypothesis testing approach to model the dependence across sensors with imperfect communication from the sensors to the FC.
- We show that the proposed test is not only asymptotically optimal, but also captures the unknown dependence across the sensors.
- Via simulations, we show that the proposed copula-based SPRT outperforms the SPRTs which usually ignore the underlying unknown dependence across sensors. Also, we show that the copula-based SPRT is less sensitive to low signal-to-noise ratios (SNRs).

The rest of the paper is organized as follows. In Section III we introduce the distributed sequential detection system.
In Section IV, we propose a copula-based SPRT. In Section V, we demonstrate the effectiveness of the proposed distributed sequential scheme through numerical examples. Finally, in Section VI, we conclude the paper.

II. COPULA THEORY BACKGROUND

A copula is a multivariate distribution with uniform marginal distributions, and it characterizes the dependence among multiple continuous variables. The unique correspondence between a multivariate copula and any multivariate distribution is stated in Sklar’s theorem [20] which is a fundamental theorem that forms the basis of copula theory.

**Theorem 1 (Sklar’s Theorem):** For random variables \( x_1, \ldots, x_d \), their joint distribution function \( F \) can be cast as

\[
F(x_1, x_2, \ldots, x_d) = C(F_1(x_1), F_2(x_2), \ldots, F_d(x_d)),
\]

where \( F_1, F_2, \ldots, F_d \) are continuous marginal Cumulative Distribution Functions (CDFs) for all the random variables and \( C \) is a unique \( d \)-dimensional copula. Conversely, given a copula \( C \) and univariate CDFs \( F_1, \ldots, F_d \), \( F \) in (1) is a valid multivariate copula with marginals \( F_1, \ldots, F_d \).

For absolutely continuous distributions \( F \) and \( F_1, \ldots, F_d \), the joint Probability Density Function (PDF) of random variables \( x_1, \ldots, x_d \) can be obtained by differentiating both sides of (1):

\[
f(x_1, \ldots, x_d) = \prod_{m=1}^{d} f_m(x_m)c(F_1(x_1), \ldots, F_d(x_d)|\phi),
\]

where \( f_1, \ldots, f_d \) are the marginal densities and \( c \) is referred to as the density of the multivariate copula \( C \), that is given by

\[
c(F(x)|\phi) = \frac{\partial^d C(F(x)|\phi)}{\partial F_1 \cdots \partial F_d},
\]

where \( F(x) = [F_1(x_1), \ldots, F_d(x_d)] \). Moreover, \( \phi \) is the dependence parameter that characterizes the amount of dependence among \( d \) random variables. Typically, \( \phi \) is unknown and needs to be estimated, e.g., using Maximum Likelihood Estimation (MLE) or Kendall’s \( \tau \) [22]. Also, in general, \( \phi \) may be a scalar, a vector or a matrix.

Note that \( C(\cdot) \) is a valid CDF and \( c(\cdot) \) is a valid PDF for uniformly distributed random variables \( F_m, m = 1, 2, \ldots, d \). Since \( F_m \) represents the CDF of \( x_m \), the CDF of \( F_m \) naturally follows a uniform distribution over \([0, 1]\).

Various families of multivariate copula functions are described in [20], such as Elliptical and Archimedean copulas. Since different copula functions may model different types of dependence, selection of the optimal copula model to fit given data is a key problem. Moreover, note that the copula based measure of dependence is independent of marginals [23].

III. PROBLEM FORMULATION

Consider a sequential binary hypothesis testing problem for the sensor network shown in Fig. 1. The two hypotheses, denoted by \( H_1 \) and \( H_0 \), are associated with the random phenomenon of interest that is monitored by \( L \) sensors. Suppose that the \( l \)-th sensor acquires observations \( z_{li}, l = 1, 2, \ldots, L \) at each time instant \( i = \{1, 2, \ldots \} \), and forwards the observations over noisy channels to the FC that runs a sequential test and produces a global decision based upon its received messages from the sensors. At the FC, the sequential procedure has three possible outcomes: it may either 1) accept \( H_0 \) and stop the testing, or 2) accept \( H_1 \) and stop the testing or 3) make no decision and acquire a new observation. The FC repeats this process until a decision is reached, in which case the test stops. Let \( T \) denote the stopping time. The goal is to minimize the expected stopping time \( E_k[T] \) under hypothesis \( k = 0, 1 \) given that \( P_{F} \leq \alpha, P_{M} \leq \beta \), where \( P_{F} \) is the probability of false alarm with constraint \( \alpha \in (0, 1/2) \) and \( P_{M} \) is the probability of miss detection with constraint \( \beta \in (0, 1/2) \). We first make the following assumptions.

- **Sensor observations** \( z_{li}, l = 1, 2, \ldots, L, i = 1, 2, \ldots \) are continuous random variables and independent and identically distributed (i.i.d.) over time.

- **The channel links between the sensors and the FC are corrupted by additive noise** \( w_{li}, l = 1, 2, \ldots, L, i = 1, 2, \ldots \) Also, \( w_{li}, l = 1, 2, \ldots, L \) are assumed to be i.i.d. over time and independent of the messages sent by the local sensors.

- **The signal received at the FC corresponding to sensor** \( l \) at time \( i \), after being corrupted by the imperfect channel, is \( y_{li} = z_{li} + w_{li} \).

- **The marginal PDFs** \( f_{k,l}(y_{li}) \) and CDFs \( F_{k,l}(y_{li}) \) with \( k \in \{0, 1\} \) under both hypotheses are assumed to be known. However, the manifestation of dependence among \( y_{l1}, \ldots, y_{Li} \), namely the copula density function \( c_k(\cdot|\phi_k), k = 0, 1 \) (see (3)), is not available a priori. This dependence may result from the dependent messages that local sensors sent, the dependent additive noise or both.

Before we proceed, we recall the definition of Kullback-Leibler (KL) divergence between two PDFs \( f(x) \) and \( g(x) \), denoted by \( D(f(x)||g(x)) \), and given as

\[
D(f(x)||g(x)) = \int f(x) \log \left( \frac{f(x)}{g(x)} \right) dx.
\]

Moreover, for any PDFs \( f(x) \) and \( g(x) \), \( D(f(x)||g(x)) \geq 0 \) with equality if and only if \( f(x) = g(x) \). Throughout this paper, we assume that

**A1:** For each sensor \( l \), \( D(f_{0,l}(\cdot)||f_{1,l}(\cdot)), D(c_{0}(\cdot|\phi_0)||c_{1}(\cdot|\phi_1)) \), \( D(f_{1,l}(\cdot)||f_{0,l}(\cdot)), D(c_{1}(\cdot|\phi_1)||c_{0}(\cdot|\phi_0)) \) are finite and positive.
Also, \( \int \left( \log \frac{f_{0,l}(\cdot)}{f_{1,l}(\cdot)} \right)^2 f_{0,l}(\cdot) d(\cdot) < \infty, \)
\( \int \left( \log \frac{f_{1,l}(\cdot)}{f_{0,l}(\cdot)} \right)^2 f_{1,l}(\cdot) d(\cdot) < \infty, \)
\( \int \left( \log \frac{c_{0,l}(\cdot|\phi_0)}{c_{1,l}(\cdot|\phi_1)} \right)^2 c_{0,l}(\cdot|\phi_0) d(\cdot) < \infty, \)
\( \int \left( \log \frac{c_{1,l}(\cdot|\phi_1)}{c_{0,l}(\cdot|\phi_0)} \right)^2 c_{1,l}(\cdot|\phi_1) d(\cdot) < \infty. \)

**Remark 1:** Note that the two conditions \( A_1 \) and \( A_2 \) guarantee that the two hypotheses are distinguishable, i.e., \( f_1(\cdot) \) is not equal to \( f_0(\cdot) \) almost everywhere.

### IV. Copula-Based SPRT

In this section, we propose a copula-based sequential test. The corrupted messages \( y_{li}, l = 1, 2, \ldots, L, i = 1, 2, \ldots \) are received at the FC sequentially. The FC performs a copula-based SPRT to make a final decision. Specifically, we solve the following binary hypothesis testing problem at the FC:

\[
H_0 : y_i \sim f_0(y_i), i = 1, 2, \ldots, \\
H_1 : y_i \sim f_1(y_i), i = 1, 2, \ldots,
\]
where \( y_i = [y_{i1}, \ldots, y_{iL}] \), \( f_1 \) and \( f_0 \) denote the joint PDFs under \( H_1 \) and \( H_0 \), respectively. Note that due to the existing complex dependence among the received observations, \( f_k(y_i) \neq \prod_{l=1}^L f_{k,l}(y_{li}), k = 0, 1 \). We take the dependence into account and design a copula-based SPRT at the FC.

Using Sklar’s theorem, i.e., Theorem 1, the joint PDFs in (4) for \( i = 1, 2, \ldots, n \) can be expressed in terms of the marginal distributions and the copula densities, \( c_1 \) and \( c_0 \), respectively, under \( H_1 \) and \( H_0 \) as

\[
f_0(y) = \prod_{l=1}^n \prod_{i=1}^L f_{0,l}(y_{li}) \times c_0(F_0(y_i) | \phi_0),
\]
\[
f_1(y) = \prod_{l=1}^n \prod_{i=1}^L f_{1,l}(y_{li}) \times c_1(F_1(y_i) | \phi_1),
\]
where \( f_{k,l}(y_{li}) \), \( l = 1, \ldots, L \) are the marginal PDFs and \( F_k(y_{li}) = [F_{k1}(y_{i1}), \ldots, F_{kL}(y_{Li})] \) are the marginal CDFs for all the sensors at time instant \( i \) under hypothesis \( k, k = 0, 1 \). Moreover, \( \phi_0 \) and \( \phi_1 \) are the parameters of copula \( c_0 \) and \( c_1 \) at time instant \( i \), respectively. For known \( c_0(\cdot|\phi_0) \) and \( c_1(\cdot|\phi_1) \), the copula-based SPRT follows the following procedure: for \( n = 1, 2, \ldots \),

\[
\begin{cases}
\Lambda^n(y) \geq A, & \text{decide } H_1, \\
\Lambda^n(y) \leq -B, & \text{decide } H_0, \\
-B < \Lambda^n(y) < A, & \text{take another observation},
\end{cases}
\]
where \( A \) and \( B \) are the upper and lower thresholds, respectively, which are predetermined constants such that \( P_F \leq \alpha \) and \( P_M \leq \beta \). Also, \( \Lambda^n(y) \) is given as

\[
\Lambda^n(y) = \sum_{l=1}^n \sum_{i=1}^L \log f_{1,l}(y_{li}) f_{0,l}(y_{li}) + \sum_{l=1}^n \log c_1(F_1(y_i) | \phi_1) c_0(F_0(y_i) | \phi_0).
\]

In general, for given \( \alpha \) and \( \beta \), exact analytical expressions of the optimal thresholds \( A \) and \( -B \) are intractable. One may use the approximated thresholds obtained from Wald’s SPRT [5], which are given by

\[
A \approx \log \frac{1 - \beta}{\alpha}, \quad -B \approx \log \frac{\beta}{1 - \alpha},
\]
where if \( \alpha, \beta \in (0,1/2) \), we have \( -B < A \).

Let \( T \) be the stopping time for the proposed sequential scheme in (6). Since the messages received at the FC are i.i.d., we have \( P(T < \infty | H_k) = 1, k = 0, 1 \) under conditions \( A_1 \) and \( A_2 \) [4] Lemma 3.1.1. The goal for the above proposed copula-based SPRT is to minimize the average stopping time \( \mathbb{E}_k[T], k = 0, 1 \) such that \( P_F \leq \alpha, P_M \leq \beta \). In Theorem 2, we show its asymptotic optimality as \( A, B \to \infty \).

**Theorem 2:** For the proposed copula-based SPRT in (6), as \( A \to \infty \) and \( B \to \infty \), we have \( P_F \approx e^{-A} \) and \( P_M \approx e^{-B} \).

Moreover, the average stopping time under the two hypotheses is given by

\[
\mathbb{E}_{H_k}[T] \approx B \frac{D_0}{D_1}, \quad \mathbb{E}_{H_1}[T] \approx \frac{A}{D_1},
\]

as \( A, B \to \infty \), and where

\[
D_0 = \sum_{l=1}^L D(f_0) f_{0,l}(\cdot) + D(c_0(\cdot|\phi_0)) c_1(\cdot|\phi_1),
\]
and

\[
D_1 = \sum_{l=1}^L D(f_1) f_{1,l}(\cdot) + D(c_1(\cdot|\phi_1)) c_0(\cdot|\phi_0).
\]

**Proof:** The proof is omitted because of space limitations. \( \blacksquare \)

**Remark 2:** The asymptotic performance of (9) shows that the average detection time depends on the KL distance provided by each sensor and the KL distance due to the spatial dependence among \( L \) sensors. By including the spatial dependence in our analysis, we can reduce the detection time on an average.

Typically, the copula density function \( c_k(\cdot|\phi_k) \) and its corresponding parameter set \( \phi_k \) under hypothesis \( H_k, k = 0, 1 \) are not known and need to be estimated. Using maximum likelihood estimates in place of the true copula density functions and the true parameters, the copula-based SPRT in (6) becomes a generalized copula-based SPRT. In the following, we present the estimation of the best copula model.

Since the FC has no knowledge of the dependence structure of the received messages, we assume that the FC waits for \( N_0 \) messages before starting the copula-based SPRT. Note that \( N_0 \) can be determined by the goodness-of-fit tests for copula models [24]. Hence, the copula density functions and their corresponding parameters can be estimated. The estimation of optimal copula density functions under the two hypotheses is similar. Therefore, the hypothesis index \( k \) will be omitted for now to simplify notations. Note that the marginal CDFs need to be evaluated at each time instant.

Before determining the optimal copula, the copula parameter set \( \phi \) is obtained using MLE, which is given by

\[
\hat{\phi} = \arg \max_{\phi} \sum_{i=1}^{N_0} \log c(F_1(y_{i1}), \ldots, F_L(y_{Li}) | \phi).
\]
Once the copula parameter set is obtained, the best copula $c^\ast$ is selected from a predefined library of copulas, $C = \{c_m : m = 1, \ldots, M\}$ using the Akaike Information Criterion (AIC) [25] as the criterion, which is given as

$$\text{AIC}_m = -\sum_{i=1}^{N_0} \log c_m(F_1(y_{i1}), \ldots, F_L(y_{Li})); \Phi_m) + 2q(m),$$

where $q(m)$ is the number of parameters in the $m$th copula model.

The best copula $c^\ast$ is

$$c^\ast = \arg \min_{c_m \in C} \text{AIC}_m.$$  \hfill (12)

Now, we have the optimal copula density functions $c_1(\cdot; \hat{\Phi}_1)$ and $c_0(\cdot; \hat{\Phi}_0)$ under alternative and null hypotheses, respectively. The log-likelihood ratio test statistics $\Lambda^n(y)$ in (7) is

$$\Lambda^n(y) = \sum_{i=1}^{n} \log f_{1} (y_{li}) + \sum_{i=1}^{n} \log \frac{f_{1} (y_{i1})}{f_{0} (y_{i1})} \frac{c_0(\hat{\Phi}_0)}{c_0(\hat{\Phi}_0)}.$$  \hfill (11)

V. Numerical Results

In this section, we demonstrate the efficacy of our proposed copula-based SPRT through numerical examples. There are two hypotheses, where $H_1$ denotes the presence of a signal $s$ and $H_0$ indicates the absence of $s$. Also, $s$ is assumed to be a deterministic signal. We model the signals received at the sensors as:

$$H_1 : z_{li} = s + v_{li}, \quad l = 1, \ldots L; i = 1, 2, \ldots$$

$$H_0 : z_{li} = v_{li}, \quad l = 1, \ldots L; i = 1, 2, \ldots$$

where $v_{li} \sim N(0, \sigma^2_v)$ is the measurement noise at sensor $l$ and time instant $i$. The received signal $z_{li}$ is assumed to be temporally independent conditioned on either hypothesis. Also, the channel noise and the signals at the local sensors are mutually independent. However, the received messages $y_{i1}, y_{i2}, \ldots, y_{iL}$ are spatially dependent.

Unless specified, we assume that $L = 3$, $\sigma^2_w = \sqrt{3}$ and $\sigma^2_v = 1$. We use $N_0 = 100$ observations to estimate the copulas. The probability of false alarm and miss detection constraints are $\alpha = 0.01$ and $\beta = 0.01$, respectively. The signal spatial dependence is generated using multivariate Gaussian copula. Without loss of generality, we assume that the sensor observations under $H_0$ are independent. To exhibit the performance improvement by applying our proposed copula-based SPRT, we also evaluate the performance of product-based SPRT that ignores dependence of sensor observations.

In Table I and Table II we present the average $P_F$ and $P_M$ values as a function of $\alpha$ and $\beta$, respectively, by comparing the product-based scheme and the copula-based scheme for known copulas and different SNRs. As we can see, the average $P_M$ values obtained for the copula-based SPRT are satisfied given the constraints $\alpha$ and $\beta$ while those for the product-based SPRT are not satisfied. The average $P_F$ values are satisfied for both the copula-based SPRT and the product-based SPRT. This is because, under $H_0$, we assume that sensor observations are independent. Also, in Fig. 2 and Fig. 3 we show the corresponding expected stopping time $E[T]$ with varying $\alpha$ and $\beta$, respectively. As we observe, on average, the copula-based SPRT makes decisions faster than the product-based SPRT. Moreover, for lower SNRs, the product-based SPRT requires more time to complete the detection while the copula-based SPRT is less sensitive to low SNRs.

| Table I: Known-copula: Estimated $P_F$ and $P_M$ with $\alpha = 0.01$. |
|-----------------------------------------------|
| $SNR = -6$ dB | $SNR = -9$ dB |
| Product-based SPRT | Copula-based SPRT | Product-based SPRT | Copula-based SPRT |
| $\beta$ | $P_F$ | $P_M$ | $P_F$ | $P_M$ | $P_F$ | $P_M$ |
| 0.3 | 0.0078 | 0.4341 | 0.0052 | 0.0321 | 0.0089 | 0.5106 | 0.0052 | 0.0299 |
| 0.2 | 0.0075 | 0.3786 | 0.0039 | 0.0186 | 0.0080 | 0.4366 | 0.0046 | 0.0205 |
| 0.1 | 0.0056 | 0.2849 | 0.0036 | 0.0101 | 0.0079 | 0.3362 | 0.0039 | 0.0099 |
| 0.01 | 0.0061 | 0.1245 | 0.0030 | 0.0010 | 0.0077 | 0.1416 | 0.0037 | 8.8000e-4 |
| 0.001 | 0.0060 | 0.0548 | 0.0032 | 1.1500e-4 | 0.0076 | 0.0617 | 0.0037 | 8.4000e-5 |

| Table II: Known-copula: Estimated $P_F$ and $P_M$ with $\beta = 0.01$. |
|-----------------------------------------------|
| $SNR = -6$ dB | $SNR = -9$ dB |
| Product based SPRT | Copula based SPRT | Product based SPRT | Copula based SPRT |
| $\alpha$ | $P_F$ | $P_M$ | $P_F$ | $P_M$ | $P_F$ | $P_M$ |
| 0.3 | 0.1803 | 0.0918 | 0.1093 | 0.0018 | 0.2337 | 0.0914 | 0.1207 | 0.0010 |
| 0.2 | 0.1178 | 0.0968 | 0.0702 | 6.000e-4 | 0.1568 | 0.1027 | 0.0825 | 0.0011 |
| 0.1 | 0.0573 | 0.1019 | 0.0366 | 0.0010 | 0.0771 | 0.1151 | 0.0368 | 0.0012 |
| 0.01 | 0.0061 | 0.1245 | 0.0030 | 0.0010 | 0.0077 | 0.1416 | 0.0037 | 8.8000e-4 |
| 0.001 | 5.1000e-4 | 0.1352 | 3.1100e-4 | 9.7100e-4 | 8.2200e-4 | 0.1563 | 3.6900e-4 | 8.3000e-3 |

| Table III: Unknown-copula: Estimated $P_F$ and $P_M$ with $\alpha = \beta = 0.01$ and $SNR = -6$ dB. |
|-----------------------------------------------|
| $P_F$ | $P_M$ | $E[T]$ |
| Case 1: Product-based SPRT | 0.0061 | 0.1254 | 12.518 |
| Case 1: Copula-based SPRT | 0.0063 | 0.0038 | 2.625 |
| Case 2: Product-based SPRT | 0.0246 | 0.1254 | 12.317 |
| Case 2: Copula-based SPRT | 0.0093 | 0.0012 | 2.847 |
In Table III, we present the average $P_F$ and $P_M$ values for unknown copula models for two cases. The first case is that only the sensor observations under $H_1$ are dependent, while the second case is that sensor observations under both $H_1$ and $H_0$ are dependent. As we can see, for the second case, the average $P_F$ and $P_M$ values for the copula-based SPRT are satisfied given the constraints $\alpha$ and $\beta$, respectively, while those for the product-based SPRT are not satisfied. For the first case, the average $P_F$ values are satisfied for the product-based SPRT since under $H_0$, sensor observations are independent.

**VI. Conclusion**

In this paper, we proposed a copula-based sequential scheme for the problem of distributed hypothesis testing, where the sensor observations are assumed to be spatially dependent. Moreover, the imperfect communication from the local sensors to the FC was addressed. We have shown the asymptotic optimality of the proposed copula-based SPRT. Via simulations, we have shown that our proposed copula-based SPRT can efficiently capture the unknown dependence, and outperform the product-based SPRT which ignores the underlying dependence. Moreover, we have shown that the copula-based SPRT is less sensitive to low SNRs.

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