By scaling the mesonic masses in the Walecka model, to the inverse of the hadronic effective mass in medium, we generate the Zimanyi-Moszkowski (ZM) models added to a surface term. For infinite nuclear matter, the surface term vanishes and this new model becomes equivalent to the usual ZM model. For finite nuclei calculations, the surface contribution changes the spin-orbit splitting in the right experimental direction. Calculations for some nuclei are presented.

I. INTRODUCTION

The relativistic linear $\sigma - \omega$ model (hereafter called Walecka model) \cite{1} satisfactorily explains many properties of nuclear matter and finite nuclei. A shortcoming of this model is, however, the prediction of a high value for the compression modulus $K = 550$ MeV. The introduction of nonlinear scalar self-coupling terms \cite{2} has brought $K$ to a reasonable value of 250 MeV in a theory with four free parameters. Modifications of this kind of model have been implemented by many authors \cite{3,4}. Zimanyi and Moszkowski (ZM) \cite{5} and Heide and Hudaz \cite{6}, aiming to keep only two free parameters have proposed nonlinear models, obtaining soft equations of state. The results for the compression modulus, $K = 224$ MeV, and nucleonic effective mass, $M^* = 797$ MeV, compare very well with Skyrme-type calculations \cite{7}. Many successful applications of ZM model have been done since its original proposal, regarding for example, quantum molecular dynamics approach \cite{8}, neutron stars \cite{9}, quark and gluon condensates in medium \cite{10}. In these applications, the softness of ZM model is essential for the obtaining of a desirable behavior at high density regimes. On the other hand, however, finite nuclei calculations showed that the spin-orbit interaction is too small to explain the observed spin-orbit splitting for finite nuclei \cite{11-14}.

Walecka and ZM models became therefore the extreme of the most simple quantum-hadron-dynamics models. The first with too much relativistic content while the second with too little. Both models are very simple and only differ in the coupling among the fields.

Recently Biró and Zimanyi \cite{15} proposed a new effective Lagrangian, adding to the usual ZM-Lagrangian a tensor coupling analogous to the one which leads to the anomalous gyromagnetic ratio. An additional free parameter in this term is suggested to be eliminated in favor of the improvement of the spin-orbit splitting for finite nuclei calculations. In this work, we intend to exhaust first the possibilities of the ZM model, still in a two-free parameters version, to improve the spin-orbit splitting for finite nuclei calculations. In Sec. II we show how we scale the mesonic masses in the Walecka model, in such a way that we generate the original ZM model added of a surface term, which does not contribute for infinite nuclear matter but changes the results for finite nuclei calculations, presented in Sec. III.

II. MODIFIED ZM MODEL

Let us start connecting the simple Walecka and ZM models, through an unified Model Lagrangian density \cite{16}

$$
\mathcal{L}_M = \bar{\psi} \left( \gamma^\mu (i\partial_\mu - g_\omega \omega_\mu) - f(\sigma) M \right) \psi - \frac{1}{4} \omega^{\mu\nu} \omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2),
$$

(1)
where the degrees of freedom are the baryon field $\psi$, the scalar meson field $\sigma$ and the vector meson field $\omega^\mu$. The real function $f(\sigma)$ is to be defined according with each model under consideration, with the condition that for zero density (it means, $\sigma$ going to zero) $f(\sigma)$ goes to one, and for higher densities the effective baryonic mass must approximate to zero asymptotically. Of course, $f(\sigma)$ also specifies the kind of scalar meson-nucleon coupling. The Dirac equation obtained from the Lagrangian density \( \mathcal{L}_Z \) gives $f(\sigma) = (1 + g_\sigma \sigma/M)^{-1}$. It is clear now that a connection between both models can be obtained if one redefines the scalar coupling constant in the Walecka model for short:

$$ \mathcal{L}_{ZM} = \mathcal{L}_{Walecka}(g_\sigma \rightarrow g_\sigma^*), $$

where $g_\sigma^*$ is now a function of $\sigma$, given by

$$ g_\sigma^* = g_\sigma f_{ZM}(\sigma) = g_\sigma m^* = g_\sigma (1 + g_\sigma \sigma/M)^{-1}. $$

As shown in Ref. [16], a modified version of the usual ZM model (called ZM3 in Ref. [17]) may be obtained from the Walecka model by performing a redefinition in both mesonic coupling constants,

$$ \mathcal{L}_{ZM3} = \mathcal{L}_{Walecka}(g_\sigma \rightarrow g_\sigma^*; g_\omega \rightarrow g_\omega^*), $$

where

$$ \frac{g_\sigma^*}{g_\sigma} = \frac{g_\omega^*}{g_\omega} = m^*. $$

Note that Eqs. (2) and (3) simplify the understanding of different kinds of ZM models since they can now be understood as directly coming from the Walecka model where the coupling constants become density dependent.

Now we pose the question whether there is another connection among these models through a rescaling of the mesonic masses. This question arises quite naturally once we know that for Walecka model as well as for ZM models what matters for the saturation of the infinite nuclear matter are the ratios $C_\sigma^2 = g_\sigma^2 M^2 / m_\sigma^2$ and $C_\omega^2 = g_\omega^2 M^2 / m_\omega^2$. To answer this question we start with the following Lagrangian density

$$ \mathcal{L}_W = \bar{\psi} \left\{ \gamma^\mu (i \partial_\mu - g_\omega \omega_\mu) - (M - g_\sigma \sigma) \right\} \psi - \frac{1}{4} \omega^{\mu\nu} \omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega^\mu + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m^* \sigma \sigma \right). $$

After performing the rescaling $\sigma \rightarrow h(\sigma)\sigma$ and imposing $m_\sigma^* = m_\sigma / h(\sigma)$, with $h(\sigma) = (1 - g_\sigma \sigma/M)$, we get

$$ \mathcal{L}_{MZW} = \bar{\psi} \left\{ \gamma^\mu (i \partial_\mu - g_\omega \omega_\mu) - M (1 + g_\sigma \sigma/M)^{-1} \right\} \psi - \frac{1}{4} \omega^{\mu\nu} \omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega^\mu - \frac{1}{2} m_\sigma \sigma \sigma^2 + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m^* \sigma \sigma \right). $$

We refer to this Lagrangian as $\mathcal{L}_{MZW}$, since at the level of Mean Field Approximation (MFA), where the derivative mesonic terms do not contribute, it becomes the usual ZM Lagrangian. Shortly, the connection between the Walecka model and this modified ZM model is

$$ \mathcal{L}_{MZW} = \mathcal{L}_{Walecka}(m_\sigma \rightarrow m_\sigma^*), $$

and

$$ \mathcal{L}_{MZW3} = \mathcal{L}_{Walecka}(m_\sigma \rightarrow m_\sigma^*; m_\omega \rightarrow m_\omega^*). $$
where
\[ m_\sigma^* = \frac{m_\sigma}{1 - g_\sigma \sigma / M}, \quad \text{(10)} \]
and
\[ m_\omega^* = \frac{m_\omega}{1 - \alpha g_\sigma \sigma / M}, \quad \text{(11)} \]
with \( \alpha = 0 \) for the MZM model and \( \alpha = 1 \) for the MZM3 model.

In this model the Euler-Lagrange equations can be written as
\[ \gamma^\mu (i\partial_\mu - g_\omega \omega_\mu) - (M - g_\sigma \sigma) \psi = 0, \quad \text{(12)} \]
\[ \partial_\mu \omega^{\mu\nu} + m_\sigma^2 \omega^{\nu} = g_\omega \bar{\psi} \gamma^\nu \psi, \quad \text{(13)} \]
\[ \partial_\mu \partial^\mu \sigma + m_\omega^3 \sigma = g_\sigma \bar{\psi} \psi + \frac{\alpha g_\sigma m_\omega^3}{M} \omega^{\mu} \omega^{\mu}. \quad \text{(14)} \]

Rescaling the mesonic fields in the form \( \sigma = (1 + g_\sigma \sigma'/M) \sigma' \) and \( \omega = (1 + \alpha g_\sigma \sigma'/M) \omega' \), with the scalar fields related through \( 1 + g_\sigma \sigma'/M = (1 - g_\sigma \sigma/M)^{-1} \) we obtain, at the MFA level, the equations of motion of the ZM models [14].

Eqs. (2) and (4) as well as Eqs. (8) and (9) indicate how to obtain the ZM models from the Walecka model within the MFA. The last set of relations, Eqs. (8) and (9), is particularly interesting by the following. First, the usual ZM model written in this form, becomes now clearly a particular case of the nonlinear Walecka model [2]. Indeed, we have expanded Eq. (10) up to order \( \sigma^2 \), what means to get a nonlinear Walecka model with scalar cubic and quartic terms, and observed that the changes in the nuclear matter bulk properties are not more than a few percent [18]. Second, the scaling exhibited by Eq. (10) and also by Eq. (11) points exactly to the inverse of the Brown-Rho scaling [19], obtained from chiral model Lagrangians. Brown-Rho scaling claims that in the medium the mesonic masses should scale as \( m^* \). ZM models, however, can be seen now as hadronic models where mesonic masses scale as \( 1/m^* \). Third, the connection is only true for the infinite nuclear matter where the last term of Eq. (3) is identically zero in the MFA. For finite nuclei surface effects are important, even in MFA, and this last term has to be considered anyway, changing the known results for ZM models.

III. RESULTS

The nuclear matter set of parameters \( C_\sigma^2 = g_\sigma^2 M^2 / m_\sigma^2 \), \( C_\omega^2 = g_\omega^2 M^2 / m_\omega^2 \) and \( C_\rho^2 = g_\rho^2 M^2 / m_\rho^2 \) for MZM and MZM3 are the same of ZM and ZM3 presented in Ref. [14]. Also the same as ZM and ZM3 are the incompressibility \( K \), the nucleonic effective mass \( m^* \), the scalar and vector potentials, \( S \) and \( V \), given in Table I. This happens because, for infinite nuclear matter, the last term of Eq. (3) does not contribute.

The calculations for finite nuclei follows the steps we have presented in Ref. [14]. Tables I-II show the results obtained for the static ground-state properties in \(^{16}\text{O}\) and \(^{208}\text{Pb}\) One feature to point out is that the modified versions predict a r.m.s. for the charge radius that is slightly smaller than the one calculated with the old version. For the binding energy this trend reverts itself. Consistent with a expected surface term effect, the last term of Eq. (3) is more relevant for \(^{16}\text{O}\) (from the shell model we know that most of the nucleons are in the surface), decreasing as the nuclei becomes heavier and giving a nearly nuclear matter behavior as, for example, in the \(^{208}\text{Pb}\) nucleus. The systematic behavior for the r.m.s. and the binding energy is due to the slightly deeper central potential presented in Fig. 1 for \(^{16}\text{O}\) and \(^{208}\text{Pb}\) nuclei. Regarding the spin-orbit splitting, the systematic may be understood through Fig. 2. In Tables III-IV we present the spectra for the \(^{16}\text{O}\) and \(^{208}\text{Pb}\) nuclei, for MZM and MZM3 compared with the old ZM and ZM3 versions. Note how the \( \Delta \varepsilon_{ls} \) for \(^{16}\text{O}\) differs more than that for \(^{208}\text{Pb}\), between the old and the new version models. The contributions for \(^{40}\text{Ca}\), \(^{48}\text{Ca}\) and \(^{90}\text{Zr}\) from the surface term in MZM and MZM3 lie in between the curves presented in Fig. 1-4.
We have presented two distinct ways to arrive at the ZM model starting from the Walecka model. The first, already presented in Ref. [14], consists in the rescaling of the mesonic coupling constants as described by Eqs. (2)-(5). In this case the nucleonic effective mass $m^*$ is defined as Eq. (3), in agreement with the definition of effective mass in the ZM model. In this case, the ZM model is reobtained obtained for infinite nuclear matter and for finite nuclei as well. The second one, in which consists the main contribution of this work, where ZM model is obtained from the Walecka model by rescaling the mesonic masses as given by Eqs. (8)-(11). The identification is clear only for infinite nuclear matter, since a surface term (the last term of Eq. (7)) remains for finite nuclei calculation. In this case the nucleonic effective mass is defined as $m^* = 1 - g_σσ/M$, in agreement with the definition of effective mass in the Walecka model. This effective local mass is shown in Fig. 3 for the $^{16}$O and $^{208}$Pb nuclei, where is clear the surface enhanced behavior of the modified models. The modified versions MZM and MZM3 change the spin-orbit splitting in the experimental direction, as we can see from the Tables I-II. In the particular case of $^{16}$O, where the fail of ZM and ZM3 was more visible (see Ref. [14]), the new version MZM3 brings the $p_{3/2} - p_{1/2}$ spin-orbit splitting to 4 MeV in a model with two free parameters. If one intends to improve the ZM models, following the interesting suggestion of Biró and Zimanyi [15], the presented modified versions MZM and MZM3 may be a better option for the starting point than the usual ZM models, since they provide already nearly experimental spectra for finite nuclei. Still thinking about a hadronic model with few parameters (three for example) one could use $\alpha$ in Eq. (11) as a free parameter to fit the spin-orbit splitting of $^{16}$O. The purpose of this work is however, by keeping the two varying parameters under control, i) to show the possibles ways from where the ZM models came from, ii) to attempt for the surfer contribution in finite nuclei, when mesonic scaling mass takes place and iii) to stats that ZM models incorporate a mesonic mass scaling to the inverse of the effective baryonic mass. By design this, we can understand simple models before to start increasing the number of free parameters to improve the observables, i.e., to have the major parameters under control.

Summarizing, we have presented a new discussion on the ZM models. Our main conclusions are as follows:

The ZM models [1] have been presented in their original version as coming from the inclusion of a derivative coupling into the original Walecka model. Further, it was established that it is equivalent to a linear scaling of the coupling constants of the Walecka model with the effective nucleonic mass $m^* = 1/(1 + g_σσ/M)$ of the ZM model [16]. Here, in another kind of equivalence, ZM models may be seen as also coming from the rescaling of the mesonic masses in the Walecka model to the inverse of the nucleonic effective mass $m^* = 1 - g_σσ/M$.

This last equivalence exactly applies to nuclear matter, since in MFA the derivative mesonic terms do not contribute. However, for finite nuclei, surface terms become important and the usual ZM models lead quite naturally to be modified (MZM), without any new free parameter and having the same features of the nuclear matter ZM models.

We have performed calculations with MZM models for several finite nuclei and compared the results with the usual ZM models. We see that the spin-orbit splitting, usually a drawback in the ZM models, changes in the right experimental direction, when calculated with the MZM models. Consistent with the theory, we observe that the surface term contribution present in MZM models decreases as the size of the nuclei increases.

Recently, many authors [20] have addressed the question that since the nucleon is not a point object, but has structure, it should afford changes when inside the nuclear medium. In this context, it is strongly conjectured that the mesonic coupling constant should be density dependent. However, we believe that there is no especial reason why the mesonic masses could not also become effective, changing in the nuclear medium. Indeed, an effective density mesonic mass dependence has been conjectured in the analysis of the naturalness in the quark-meson coupling model [21]. In this sense, our work provides a contribution to a better understanding of the medium-dependent mesonic coupling and mass parameters of hadronic models.

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| Model | \( m^* \) (MeV) | \( \kappa \) (MeV) | \( S \) (MeV) | \( V \) (MeV) |
|-------|----------------|----------------|----------------|----------------|
| ZM    | 0.85           | 225            | -141           | 82             |
| ZM3   | 0.72           | 156            | -267           | 204            |

TABLE I. The incompressibility \( K \), the nucleonic effective mass \( m^* \), and the scalar and vector potentials, \( S \) and \( V \), in nuclear matter for the usual ZM models.

| \( \varepsilon \) (MeV) | ZM | MZM | ZM3 | MZM3 | Exp. |
|------------------------|----|-----|-----|------|------|
| \( \langle r^2 \rangle \) (fm\(^2\)) | 8.40 | 9.37 | 7.50 | 9.42 | 7.98 |
| \( \varepsilon_{1p1/2} - \varepsilon_{1p3/2} \) (MeV) | 1.4(1.4) | 1.6(1.6) | 2.9(2.9) | 4.0(4.0) | 6.1(6.3) |

TABLE II. Binding energy, mean squared charge radius and spin-orbit splitting for the \(^{16}\)O nucleus. Values between parenthesis are for protons, the others are for neutrons.

| \( \varepsilon \) (MeV) | ZM | MZM | ZM3 | MZM3 | Exp. |
|------------------------|----|-----|-----|------|------|
| \( \langle r^2 \rangle \) (fm\(^2\)) | 5.54 | 5.51 | 5.66 | 5.52 | 5.50 |
| \( \varepsilon_{2p1/2} - \varepsilon_{2p3/2} \) (MeV) | 0.2(0.2) | 0.2(0.2) | 0.4(0.5) | 0.5(0.6) | 0.5 |
| \( \varepsilon_{2f1/2} - \varepsilon_{2f5/2} \) (MeV) | 0.5 | 0.6 | 1.2 | 1.3 | 1.8 |
| \( \varepsilon_{3p1/2} - \varepsilon_{3p3/2} \) (MeV) | 0.2 | 0.2 | 0.4 | 0.5 | 0.9 |

TABLE III. Binding energy, mean squared charge radius and spin-orbit splitting for the \(^{208}\)Pb nucleus. Values between parenthesis are for protons, the others are for neutrons.
| Level   | ZM       | MZM      | ZM3      | MZM3     | Exp.    |
|---------|----------|----------|----------|----------|---------|
| 1s_{1/2} | 35.4(31.2) | 37.7(34.4) | 36.2(32.1) | 43.2(38.7) | 47.0(40±8) |
| 1p_{3/2} | 19.6(15.6) | 21.1(17.0) | 19.4(15.6) | 23.3(19.2) | 21.8(18.4) |
| 1p_{1/2} | 18.2(14.2) | 19.5(15.4) | 16.5(12.7) | 19.4(15.2) | 15.7(12.1) |

**TABLE IV.** Energy spectra for the $^{16}$O nucleus. Energies are in MeV. Values between parenthesis are for protons, the others are for neutrons.

| Level   | ZM       | MZM      | ZM3      | MZM3     | Exp.    |
|---------|----------|----------|----------|----------|---------|
| 1s_{1/2} | 45.1(35.6) | 45.4(36.2) | 50.6(40.0) | 52.3(42.0) |         |
| 1p_{3/2} | 41.0(31.8) | 41.4(32.6) | 45.3(35.2) | 47.3(37.5) |         |
| 1p_{1/2} | 40.7(31.6) | 41.3(32.5) | 45.1(34.9) | 47.0(37.2) |         |
| 1d_{5/2} | 35.8(26.9) | 36.5(27.9) | 39.0(29.4) | 41.1(31.8) |         |
| 1d_{3/2} | 35.6(26.6) | 36.2(27.6) | 38.4(28.7) | 40.5(31.1) |         |
| 1f_{7/2} | 29.0(21.2) | 30.6(22.2) | 31.9(22.7) | 34.0(25.0) |         |
| 1f_{5/2} | 29.4(20.6) | 30.1(21.6) | 30.9(21.5) | 32.9(23.8) |         |
| 1g_{9/2} | 23.2(14.6) | 23.9(15.6) | 24.3(15.4) | 26.2(17.5) |         |
| 1g_{7/2} | 22.4(13.8) | 23.1(14.8) | 22.6(13.6) | 24.4(15.6) | (11.4)  |
| 1h_{11/2} | 15.9(7.41) | 16.6(8.27) | 16.2(7.65) | 17.7(9.24) | (9.4)   |
| 1h_{9/2} | 14.8 | 15.4 | 13.9 | 15.1 | 10.8 |
| 1i_{13/2} | 8.18 | 8.65 | 7.98 | 8.84 | 9.0 |
| 2s_{1/2} | 33.3(23.7) | 33.7(24.4) | 36.2(26.1) | 38.0(27.8) |         |
| 2p_{3/2} | 25.8(16.5) | 26.1(17.0) | 27.8(18.1) | 28.9(19.1) |         |
| 2p_{1/2} | 25.6(16.3) | 25.9(16.8) | 27.4(17.6) | 28.3(18.5) |         |
| 2d_{5/2} | 18.1(8.87) | 18.1(8.98) | 19.4(9.92) | 19.5(10.0) | (9.7)   |
| 2d_{3/2} | 17.7(8.50) | 17.7(8.57) | 18.5(9.06) | 18.6(9.06) | (8.4)   |
| 2f_{7/2} | 10.2 | 9.80 | 11.0 | 10.3 | 9.7 |
| 2f_{5/2} | 9.69 | 9.25 | 9.85 | 8.99 | 7.9 |
| 3s_{1/2} | 16.3(6.75) | 16.1(6.63) | 17.6(7.81) | 17.4(7.49) | (8.0)   |
| 3p_{3/2} | 7.88 | 7.31 | 8.83 | 7.77 | 8.3 |
| 3p_{1/2} | 7.69 | 7.11 | 8.38 | 7.27 | 7.4 |

**TABLE V.** Energy spectra for the $^{208}$Pb nucleus. Energies are in MeV. Values between parenthesis are for protons, the others are for neutrons.
FIG. 1. The central potential $V_0$ for $^{16}\text{O}$ and $^{208}\text{Pb}$ nuclei. Besides the curves for the models MZM-MZM3 presented in this work, the usual ZM-ZM3 ones are shown.
FIG. 2. The spin-orbit potential $V_{ls}$ for $^{16}$O and $^{208}$Pb nuclei. Besides the curves for the models MZM-MZM3 presented in this work, the usual ZM-ZM3 ones are shown.
FIG. 3. The local effective masses $M^*$ for $^{16}$O and $^{208}$Pb nuclei. Besides the curves for the models MZM-MZM3 presented in this work, the usual ZM-ZM3 ones are shown.