Scheduling in Parallel Finite Buffer Systems: Optimal Decisions under Delayed Feedback

Anam Tahir, Bastian Alt, Amr Rizk, Heinz Koeppl

Abstract—Scheduling decisions in parallel queuing systems arise as a fundamental problem, underlying the dimensioning and operation of many computing and communication systems, such as job routing in data center clusters, multipath communication, and Big Data systems. In essence, the scheduler maps each arriving job to one of the possibly heterogeneous servers while aiming at an optimization goal such as load balancing, low average delay or low loss rate. One main difficulty in finding optimal scheduling decisions here is that the scheduler only partially observes the impact of its decisions, e.g., through the delayed acknowledgements of the served jobs. In this paper, we provide a partially observable (PO) model that captures the scheduling decisions in parallel queuing systems under limited information of delayed acknowledgements. We present a simulation model for this PO system to find a near-optimal scheduling policy in real-time using a scalable Monte Carlo tree search algorithm. We numerically show that the resulting policy outperforms other limited information scheduling strategies such as variants of Join-the-Most-Observations and has comparable performance to full information strategies like: Join-the-Shortest-Queue, Join-the-Shortest-Queue(d) and Shortest-Expected-Delay. Finally, we show how our approach can optimise the real-time parallel processing by using network data provided by Kaggle.

Index Terms—Parallel systems, job scheduling, partial observability.

I. INTRODUCTION

As the growth rate of single-machine computation speeds started to stagnate in recent years, parallelism seemed like an effective technique to aggregate the computation speeds of multiple machines. Since then, parallelism has become a main ingredient in compute cluster architectures [1], [2] as it incurs less processing and storage cost on any one individual server [3].

Beyond the aggregation of capacity, a major difficulty in the operation of parallel servers is optimizing for low latency and loss. The key to this optimization is the mapping of the input, that we denote as jobs, to the different and possibly heterogeneous serving machines, denoted servers, of time-varying capacity and finite memory (buffers). This mapping of input to servers is carried out by a scheduler.

Classical results prove the optimality of the Join-the-Shortest-Queue (JSQ) algorithm in terms of the expected job delay [4], [5] when the servers are homogeneous, have infinite buffer space and the job service times are independent, identically (iid) and exponentially distributed. For the case when the service times are exponentially distributed but with different service rates, Shortest-expected-delay (SED) has been shown to minimize the mean response time of jobs, especially in the case of heavy traffic limits [6], [7].

Note, however, that such types of algorithms assume that the scheduler has accurate, timely and synchronized information of all servers and their queues. In practice this assumption does not hold, e.g., in data center clusters it cannot be assumed that a job scheduler has timely and synchronized information of all available servers but rather observes some event or time-triggered server feedback. Fig. 1 shows a sketch of a modern application of such a system, i.e., cluster processing of a data stream where scenes are mapped to server clones for analysis.

The goal of this work is to model and optimize the scheduling decision-making over parallel and heterogeneous servers having finite buffers and each following first-in-first-out (FIFO) order, where these servers provide randomly delayed acknowledgments back to the scheduler. This model captures that the scheduler is only able to partially observe the server states at the decision time points. The scheduler does not directly observe the server queues but rather receives acknowledgments for completed jobs that are randomly delayed on the way back to the scheduler. Our contributions are:

- We present a model for controlled scheduling in parallel systems with randomly delayed acknowledgments.
- We find a control law by optimizing a predefined objective function subject to the stochastic dynamics induced by the model.
- We present POS - a Partial Observability Scheduler that estimates the unknown parameters of the parallel system at runtime and despite partial observations achieves a
job drop rate and response time comparable with full
information schedulers. The performance evaluation is
carried out in a simulation environment. Note that this
POS can be used for any number of servers and for any
kind of inter-arrival and service time distributions. In this
work we have tested a few of them.

The rest of this paper is organized as follows: In Section II
we first discuss the related work. Then, in Section III we
outline the system model and give some background on the
key topics of this work. In Section IV, we give our contributions
starting from the modelling of the partially observable queuing
system to our Monte Carlo approach for finding a near-optimal
scheduling policy. In Section V, we give our simulation results
and discuss the inference of unobserved system parameters in
Section V-D along with an experiment with real world data.

II. RELATED WORK

Dynamic scheduling for the performance optimization of par-
allel queuing systems has fueled numerous seminal algorithms
such as Join-the-shortest-queue (JSQ), and Shortest-expected-
delay (SED), and more generally Power-of-d policies [8]–[10].
JSQ provides optimal decisions, for minimizing the mean
response time of a job, when the servers are homogeneous and
the service times are independent and identically exponentially
distributed [4], [5]. A similar approach for heterogeneous
servers is the SED algorithm, which implicitly considers the
server rates and maps an incoming job to the server which
provides the smallest expected response time for the job at
hand. SED is known to perform well for heterogeneous servers,
especially in the case of heavy traffic [6], [7].

When the number of parallel systems \( N \) becomes large, the
assumption of knowing the state of each system before every
scheduling decision becomes too strong. For example, the state
may be the queue length or the required cumulative service
times for the waiting jobs at each system. Depending on the
type of system this information, e.g., the service times for
servers of random varying capacity, is not known in advance.

Power-of-d policies provide a remedy to the problem of
the scheduler not being able to know all system states at
decision time. Here, a number of servers, specifically \( d < N \), is
repeatedly polled at random at every decision instant and hence,
JSQ(d) or SED(d) is performed on this changing subset [11].
This policy is enhanced by a short term memory that keeps
knowledge of the least filled servers from the last decision
instant [12], [13]. Hence, instead of choosing \( d \) servers at
random for every job the decision is based on a combination
of newly randomly chosen \( d \) servers and the least filled servers
known from the last decision. For our evaluation purposes we
have chosen to compare our algorithm with SED, JSQ and
JSQ(d) since the comparison is with respect to the classes of
full and limited information at the scheduler.

The strong assumption behind the different variants of JSQ,
SED and the Power-of-d policies is that at every decision
instant the scheduler is aware of the current system state of all
or some of the servers. This includes for example the states of
the queues, the service times of the jobs waiting at the servers
or other required parameters. The main difference in this work
is that we hypothesize that this assumption of instant (and full)
knowledge is often not realistic, as due to the distributed nature
of the system the scheduler may only observe the impact of a
decision that maps a job to a server after some non-deterministic
feedback time. This feedback time may arise, e.g., due to
the propagation delay or simply that the scheduler receives
feedback only after the job has been processed. The impact
of this non-deterministic and heterogeneous feedback time on
the scheduling decision is significant, as the consequence of
a scheduling decision on performance metrics such as job
response times or job drop rates is only partially observed at
the decision instant.

Markov decision processes, MDP, have been used to achieve
optimal control in queuing systems under static and dynamic
environments [14]. In an MDP the current state, delayed or
not, is known to the agent [15]. The authors of [5] used an
MDP formulation together with a stochastic ordering argument
to show that JSQ maximizes the discounted number of jobs to
complete their service for a homogeneous server setting. The
authors of [16] study the problem of allocating customers to
parallel queues. They model this problem as an MDP with the
goal of minimizing the sojourn time for each customer and
produce a ‘separable rule’, which is a generalization of JSQ,
for queues with heterogeneous servers (in rates and numbers).
Note, however, that it is assumed that the queue filling is
known and available without delay to the scheduler. In [17]
the authors assume that the scheduler receives the exact queue
length information but with a delay of \( k \) steps. They formulate
their system as a Markov control model with perfect state
information by augmenting to the state space the last known
state (exact queue length) and all the actions taken until the
next known state. In their work, they solve the flow control
problem by controlling the arrival to a single server queue and
show for \( k = 1 \) that the optimal policy is of threshold type and
depends on the last action. In [18], along with the single server
flow control problem with similar results as [17], they also
showed that when \( k = 1 \), the optimal policy for minimizing the
discounted number of jobs in a system of two parallel queues
is join-the-shortest-expected-length. In [19], the scheduler at
time \( n \) knows the number of jobs that were present in each
(infinite) queue at time \( n-1 \), such that it takes decisions at a
deterministic delay of one time slot. The state space at any
time \( n \) is augmented and contains the actual queue filling at
time \( n-1 \), the action taken at time \( n \) and if there was an arrival
at time \( n \).

In our work, we assume that not only is the information
received by the scheduler randomly delayed, but it is also
not the state of the buffers, rather it is in the form of
acknowledgements of the number of jobs processed. This makes
the system partially observable and complex to solve, i.e., in
the sense of a partially observable Markov decision process
(POMDP) model. Several online and offline algorithms have
been introduced to solve POMDP [20], [21], but there is little
work on optimizing queuing systems as a POMDP. Standard
solutions, for POMDP, that do full-width planning [22], like
Value iteration and Policy iteration, perform poorly when the state space grows too large. This can easily be the case in queuing systems, due to the curse of dimensionality and the curse of history [23], [24]. For such large state problems, the POMCP algorithm [25], which uses Monte Carlo tree search (MCTS), is a fast and scalable algorithm for solving a POMDP. Our algorithm also makes use of the MCTS algorithm specifically designed to simulate a parallel queueing system with finite buffers. In addition, we also designed a Sequential Importance Resampling, SIR, particle filter to deal with the delayed feedback acknowledgements in the queueing system. We also use this MCTS approach for solving a Partially observable semi Markov decision process (POSDMDP) [26], which is needed when the time between decision epochs is no more exponential.

III. SYSTEM MODEL

We consider a queuing system with \( N \) parallel servers each having its own finite FIFO queue. The queue filling is denoted \( b_i \in \mathbb{B}, i = 1, \ldots, N \), where \( \mathbb{B} = \{0, \ldots, b_i\} \) and \( b_i \) is the buffer space for the \( i \)-th queue. We define the vector of queue sizes as \( \mathbf{b} = [b_1, \ldots, b_N]^\top \). In the following, we will use boldface letters to denote column vectors.

We consider the general case of heterogeneous servers where the service times \( V_i^{(1)}, V_i^{(2)} \ldots \) of consecutive jobs at the \( i \)-th server\(^1\) are independent and identically distributed (iid) according to a probability density \( f_i \). Jobs arrive to the scheduler, as depicted in Fig. 2, according to some renewal process described by the sequence \( (T^{(n)})_{n \in \mathbb{N}} \), where the job inter-arrival times \( U^{(i)} := T^{(j+1)} - T^{(j)} \) are drawn iid from a distribution \( F \) leading to an average arrival rate \( \lambda \). Each arriving job is mapped by the scheduler to exactly one server where we denote the average service rate of the \( i \)-th server as \( \mu_i \). If a job is mapped to a full buffer the job is lost. When a job leaves the system the server sends an acknowledgement back to the scheduler. The scheduler uses this feedback to estimate the buffer fillings, \( b_i \), of each queue.

A major challenge for deciding on the job routing arises when the acknowledgments are delayed. Here, we incorporate three main delay components, i.e., (i) the job waiting time in the queue it was assigned to, (ii) the job processing time, and (iii) the propagation delay of the acknowledgement back to the scheduler. The third component makes the decision problem particularly hard as a decision does not only impact the current state of the system but also future states due to its delayed acknowledgement feedback. Note that the scheduler makes the routing decision based only on these observed acknowledgments, thus making the system state partially observable, PO. In Fig. 2 we denote the number of acknowledgments, that are observed by the scheduler at the \( i \)-th server in one inter-arrival time as \( y_i \). Since the scheduler is PO, it does not directly observe the queue states \( b_i \), the job service times, or the delayed feedback \( x_i \), i.e., the number of acknowledgments that are on the way back to the scheduler but have not reached it yet.

Depending on the distribution type of the inter-arrival times, \( U \), and service times, \( V \), we model the decision-making process in this PO queuing system as a partially observable Markov decision process, POMDP, or a partially observable semi Markov decision process, POSMDP. A POMDP has an underlying Markov decision process (MDP), where the actual state of the system is not known to the agent, i.e., the scheduler in our case. This modelling can be used when both \( U \) and \( V \) are exponentially distributed, keeping the system Markovian. For non-exponential \( U \) and/or \( V \), POSMDP formulation can be used, where the underlying process is now semi-Markov and the actual state of the system is still not known to the scheduler.

Markov Decision Process with Partial Observability

An MDP [15] is defined as a tuple \( \langle \mathcal{S}, A, T, R \rangle \), where \( \mathcal{S} \) is the countable state space, \( A \) is the countable action space, \( T : \mathcal{S} \times \mathcal{S} \times A \rightarrow [0, 1] \) is the transition function, \( R : \mathcal{S} \times \mathcal{S} \times A \rightarrow \mathbb{R} \) is the reward function. Throughout this work we assume time homogeneity for the transition function \( T \), observation function \( O \) and reward function \( R \). An MDP with partial observations is also\(^2\) a controlled Markov process, where the exact state of the process is latent. It is in addition defined using: \( \mathcal{Z} \), the countable observation space, and \( O : \mathcal{Z} \times \mathcal{S} \times A \rightarrow [0, 1] \), the observation function. Consider the case of discrete epochs at time points \( t \in \mathbb{N}_0 \). Note that the clock given by \( t \) is an event clock and not a wall-clock time. If the decision epochs \( t \) are exponentially distributed then this PO process can be modelled as a POMDP [23], otherwise under the condition that the decision-making is done only at the epochs \( t \), it can be modelled as a POSMDP [26].

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\(^1\)We denote random variables by uppercase letters and their realizations as lowercase letters.

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![Fig. 2: A parallel queuing system with a scheduler that maps jobs to servers. The scheduler observes the inter-arrival times \( u \) and the feedback, i.e., the number of acknowledgements, from each server \( y_i \). The scheduler does not observe the queue states \( b_i \), the job service times, or the delayed feedback \( x_i \), i.e., the number of acknowledgements on the way back back.](image)
We consider a latent process \( S^{(t)} \in \mathcal{S} \) that can be controlled by actions \( A^{(t)} \in \mathcal{A} \). Since, the state is latent only observations \( Z^{(t)} \in \mathcal{Z} \) are available. The transition function \( T(s', s, a) \) is the conditional probability of moving from state \( s \) under action \( a \) to a new state \( s' \). The observation function \( O(z, s, a) \) denotes the conditional probability of observing \( z \) under the latent state \( s' \) and action \( a \). An agent receives a reward \( R^{(t)} = R(S^{(t+1)}, S^{(t)}, A^{(t)}) \), which it tries to maximize over time.

Since the current state is not directly accessible by the agent, it has to rely on the action-observation history sequence, \( H^{(t)} = \{A^{(0)}, Z^{(0)}, A^{(1)}, Z^{(1)}, \ldots, A^{(t)}, Z^{(t)}\} \), up to the current time point \( t \). A policy \( \pi(a, h) := \mathbb{P}(A^{(t)} = a | H^{(t)} = h) \) is the conditional probability of choosing action \( a \) under action-observation history \( h \). The solution then corresponds to a policy which maximizes an objective over a prediction horizon. The policy is defined as a function of the observation-action history of the agent, which makes it very challenging, since a naive planning algorithm requires to evaluate an exponentially increasing number of histories in the length of the considered time horizon. For this reason, different solution techniques are required.

Since keeping a record of the entire history, \( h \), is not feasible, one way is to represent this history in terms of the belief state, \( \rho^{(t)} \in \Delta[\mathcal{S}] \), where \( \Delta[\mathcal{S}] \) is an \( \mathcal{S} \) dimensional probability simplex, \( \rho^{(t)} = [\rho^{(t)}_1, \ldots, \rho^{(t)}_{|\mathcal{S}|}]^\top \) and the components \( \rho^{(t)}_s \) are the filtering distribution. If the state space is huge, this will be a very high dimensional vector. So, in order to break the curses of history and dimensionality, a certain number of particles can be used to represent the belief state \( \rho^{(t)} \) of the system at time \( t \). These particles represent the belief state \( \rho^{(t)} \) of the system and are updated using Monte Carlo simulations based on the action taken and observations received. This is the approach that we build upon in this paper. We consider an infinite horizon objective, where the optimal policy \( \pi^* \) is found by maximizing the expected total discounted future reward \( \pi^* = \arg \max_\pi \sum_{t=0}^{\infty} \mathbb{E}_\pi[\gamma^t R^{(t)}] \) with \( \gamma < 1 \).

IV. Scheduling in Parallel Queuing Systems with Delayed Acknowledgments

In this section we explain how we model our partially observable (PO) queuing system and then give our proposed solution.

A. Modelling Scheduling in the Partially Observable Queuing System

In order to model the scheduling decision in a queuing system, with \( N \) parallel finite buffer servers (cf. Fig. 2), as a PO process, we define a state \( s \in \mathcal{S} \), using \( s = [s_1, \ldots, s_N]^\top \). Here, \( s_i \) is the augmented state of the \( i \)-th queue that is defined as \( s_i = [b_i, x_i, y_i]^\top \). The components of \( s_i \) are given as follows: (i) \( b_i \in \mathcal{B}_i \) denotes the current buffer filling at server \( i \), (ii) \( x_i \in \mathcal{B}_i \) denotes the number of delayed acknowledgements for executed jobs at server \( i \), hence, not observed by the scheduler in the current epoch\(^2\), and finally (iii) \( y_i \in \mathcal{B}_i \) denoting the number of acknowledgements observed by the scheduler in the current epoch. Hence, the state space is \( \mathcal{S} \subseteq \mathbb{N}_0^{3N} \) and an action \( a \in \mathcal{A} \), with \( |\mathcal{A}| = N \) corresponds to sending a job to the \( a \)-th server. An observation \( z \in \mathcal{Z} \) is the vector of observed acknowledgements at the scheduler, with the observation space being \( \mathcal{Z} \subseteq \mathbb{N}_0^N \).

B. The Dynamical Model

Next, we describe the dynamics of the underlying processes of the PO model. The corresponding probabilistic graphical model is depicted in Fig. 3. In case of the POMDP model, the time, \( t \) in Fig. 3 is exponentially distributed while for POSMDP it can be random (non-exponential). As we are using Monte Carlo simulations to solve the PO system, the transition probabilities do not have to be defined explicitly. Therefore, we define the transition function indirectly as a generative process.

We consider a scheduler that makes a mapping decision at each job arrival, where the inter-arrival times \( U^{(t)} \) are iid. In order to characterize the stochastic dynamics, we determine the random behavior of the number of jobs \( k_i \) that leave the \( i \)-th queue during an inter-arrival time. Note that the number of jobs leaving the queue is constrained by the current filling of the queue, hence we use \( k_i = \min(k_i, b_i) \), where \( k_i \) is the number of jobs that can be served, which is determined by the inter-arrival time and the service times of the \( i \)-th server. An important system constraint is that the buffer spaces are finite. Therefore, we define the generative model for the queuing dynamics as:

\[
b' = \min(\max(b - k, 0) + e_a, b),
\]

\(^2\)An epoch corresponds here to one inter-arrival time.
where \( \mathbf{b} \) denotes the queue size vector of the queuing system at some arrival time point and \( \mathbf{b}' \) is the queue size vector of the queuing system at the next arrival time point. By \( \mathbf{k} \) we denote the untruncated vector of number of jobs that can be served and \( \mathbf{e}_a \) is a vector of all zeros, but the \( a \)-th position is set to one to indicate a mapping of the incoming job to the \( a \)-th server. We use \( \mathbf{b} \) as the vector of maximum buffer sizes, and \( \min(\cdot, \cdot) \) and \( \max(\cdot, \cdot) \) denote the element wise minimum and maximum operation.

As the scheduler only observes the job acknowledgements, we update the augmented state space, \( \mathbf{s} \) (as defined in Section IV-A), using the following stochastic update equation:

\[
\begin{bmatrix}
\mathbf{b}' \\
\mathbf{x}'
\end{bmatrix} = \begin{bmatrix}
\min(\max(\mathbf{b} - \mathbf{k}, \mathbf{0}) + \mathbf{e}_a, \mathbf{b}) \\
\min(\mathbf{b}, \mathbf{k}) + \mathbf{x} - 1
\end{bmatrix} \mathbf{1},
\]

where \( \mathbf{1} \) is the vector containing the number of jobs which are observed by the scheduler for each queue at the current epoch. The number of unacknowledged jobs from the previous epoch is denoted \( \mathbf{x} \). The delay model according to which \( \mathbf{s} \) is sampled is given in the next section.

**C. Delay Model**

We assume that the number of jobs that can be served in one inter-arrival time is distributed as \( K_i \sim f_{K_i}(k_i) \) and we choose a delay model, where

\[
L_i \mid b_i, k_i, x_i \sim \text{Bin}(\min(b_i, k_i) + x_i, p_i).
\]

Here, \( \min(b_i, k_i) \) is the number of jobs leaving the \( i \)-th queue at the current epoch, \( x_i \) is the number of jobs from the \( i \)-th queue for which no acknowledgements have been previously observed by the scheduler, \( p_i \) is the probability that an acknowledgement is received by the scheduler in the current epoch and \( L_i \) is the distribution from which \( l_i \) is sampled for Eq.(2).

We have chosen the binomial distribution because it captures the fact that only a subset of the \( \min(b_i, k_i) + x_i \) jobs are observed in form of acknowledgements at the scheduler in the current epoch. We believe that this can reflect any kind of delay distribution in which some acknowledgements are delayed till the next epoch. The number of unacknowledged jobs from the previous epoch is denoted \( \mathbf{x} \). The delay model according to which \( \mathbf{s} \) is calculated is given in the next section.

\[
\begin{align*}
\mathbf{b}' &= \min(\max(\mathbf{b} - \mathbf{k}, \mathbf{0}) + \mathbf{e}_a, \mathbf{b}) \\
\mathbf{x}' &= \min(\mathbf{b}, \mathbf{k}) + \mathbf{x} - 1
\end{align*}
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 queues, as illustrated in Fig. 2.

**Minimize queue lengths:** A first reward function aims to minimize the overall number of jobs waiting in the system. This is an intuitive objective that can be formalized as

\[ R(s', s, a) = -\sum_{i=1}^{N} b'_i \]  

(7)

as it takes the sum of all queue fillings. Similarly, one a polynomial or exponential reward functions, such as

\[ R(s', s, a) = -\sum_{i=1}^{N} \chi b'_i \]  

(8)

with \( \chi > 1 \), penalizes job mappings that lead to queue length imbalance. Intuitively, this objective tends to balance queue lengths. Using the variance amongst the current queue fillings also balances the load on queues. The reward function is then given as:

\[ R(s', s, a) = \text{Var}(b_1, \ldots, b_n) \]  

(9)

Note, however, that balancing queue lengths does not necessarily lead to lower delays if the servers are heterogeneous. Hence, proportional allocation provides more reward when jobs are mapped to the faster server as

\[ R(s', s, a) = -\sum_{i=1}^{N} \frac{b'_i}{\mu_i} \]  

(10)

**Minimize loss events:** To prevent job losses, we also formulate a reward function that penalizes actions that lead to fully filled queues, i.e.,

\[ R(s', s, a) = -\sum_{i=1}^{N} \mathbb{1}(b'_i = \bar{b}_i) \]  

(11)

The indicator function evaluates to one only when the corresponding queue is full.

**Minimize idle events:** A scheduler designer might also require that the parallel system remains work-conserving, i.e., no servers idling, as this essentially wastes capacity. Hence, in the simplest case we can formulate a reward function of the form

\[ R(s', s, a) = -\sum_{i=1}^{N} \mathbb{1}(b'_i = 0) \]  

(12)

Note that some of the reward functions above can be combined, e.g., in a weighted form. In the following, if not stated otherwise we utilize the following combined linear reward function for our scheduler

\[ R(s', s, a) = -\left[ \sum_{i=1}^{N} b'_i + \sum_{i=1}^{N} \kappa_1 \mathbb{1}(b'_i = 0) + \kappa_2 \mathbb{1}(b'_i = \bar{b}_i) \right] , \]  

(13)

where, \( b'_i \) is the new buffer state after taking action \( a \) and constants \( \kappa_1, \kappa_2 > 0 \) that enable tuning the scheduler.

**F. Partial Observability Scheduler: A Monte Carlo Approach for Delayed Acknowledgements**

In this section, we outline our approach to solve the partially observable system for the job routing problem in parallel queuing systems with delayed acknowledgements. Our solution is an alternate technique to Dynamic Programming and is based on a combination of the Monte Carlo Tree Search (MCTS) algorithm, [25], and Sequential Importance Resampling (SIR). One main contribution lies in the design of a simulator which incorporates the properties of the queuing model given in Section III into the algorithm.

The reason for choosing a MCTS algorithm is that queuing problems, like the one presented in this work, can span to very large state spaces. In these scenarios solution methods based on dynamic programming [28] often break due to the curse of dimensionality. MCTS solves this problem by using a sampling based heuristic approach to construct a search tree to represent different states of the system, the possible actions in those states and the expected value of taking each action. In recent years, these techniques have been shown to yield exceptional results in solving very large decision making problems [29] [30].

Our algorithm makes use of the simulator \( \mathcal{G} \), which is a generative model of the POMDP and POSMDP, to build the search tree. We designed this simulator based on the queuing model, depicted in Fig. 2, to incorporate the randomly delayed acknowledgements. The simulator \( \mathcal{G} \), provides the next state \( s^{(t+1)} \), the observation \( z^{(t+1)} \) and the reward \( R^{(t+1)} \), when given the current state \( s \) of the system and the taken action \( a \) as input,

\[ s^{(t+1)}, z^{(t+1)}, R^{(t+1)} \mid s^t, a^t \sim \mathcal{G}(s, a). \]  

(14)

This simulator \( \mathcal{G} \), is used in the MCTS algorithm to rollout simulations of different possible trajectories in the search tree. Each trajectory is a path in the search tree starting from the current belief state of the system and expanding (using our simulator) to a certain depth. While traversing through the search tree, the trajectories (actions) are chosen using the Upper Confidence bounds for Trees algorithm (UCT), which is an improvement over the greedy-action selection [31]. In UCT the upper confidence bounds guide the selection of the next action by trading off between exploiting the actions with the highest expected reward up till now and exploring the actions with unknown rewards. The search tree is then used to find the best action, \( a \), given a certain belief state of the system, \( \rho^{(t)} \), see Section III. The action \( a \) is choosen which has the highest expected reward and the trajectories for all other actions are then pruned from the search tree since they are no longer possible. This is done to avoid the tree from growing infinitely large.

Once an action \( a \) is taken and the job is allocated to a certain queue, the scheduler receives real observations, \( z \), from the system. These are the randomly delayed acknowledgements from the system. It then uses these observations as an input to the SIR particle filter, in order to update its belief of the state the system is in. The weights given to each particle (state), \( w(s_i) = p(z_i|s_i) \), while resampling in this SIR particle filter were designed to incorporate our queuing system and its delay model, which is given in Eq. (3). After applying the filter we have the new belief state of the system, \( \rho^{(t)} \), which is represented in terms of a certain number of particles. These
We set the buffer size, \( \lambda \). The evaluation box plots are based on all happen at each decision epoch, which is why it is possible to model the system as a POSMDP \([26]\) as well.

In the following, we show numerical evaluation results for the proposed Partial Observability Scheduler (POS) under randomly delayed acknowledgements. Recall that if the acknowledgement is not observed in the current inter-arrival time, it is not incorporated into the scheduling decision for the next incoming job. In order to evaluate the impact of this delay, we consider in our simulations a probability of \( p_t = 0.6 \; \forall i \) in Eq. (3), if not stated otherwise. This means that an acknowledgement is delayed until the next epochs with probability: \( 1 - p_t = 0.4 \).

We set the buffer size, \( b_i \), for all queues to 10 jobs. Further, if not explicitly given, we use the combined linear reward function given in Eq. (13) with \( \kappa_1 = 0 \), since we want to focus more on avoiding job drops in the system.

We consider the system depicted in Fig. 2 for both cases of heterogeneous and homogeneous servers. In particular, we show numerical results comparing POS to different variants of scheduling strategies (with and without full system information) with respect to:

- the empirical job response time distribution (measured from the time a job enters the system until it completes service and leaves),
- the empirical distribution of the job drop rate (measured as the fraction of jobs that could not be accommodated into the system and were hence dropped),
- and the cumulative reward.

The evaluation box plots are based on 100 independent runs of \( 5 \cdot 10^3 \) jobs with whiskers at [0.5, 0.95] percentiles. The chosen inter-arrival and service time distributions are mentioned with the figures. The offered load ratio \( (\eta := \lambda / \sum_i \mu_i) \) is used to describe the ratio between arrival rate and the combined service rate. The higher the value of \( \eta \), the higher is the job load on the system. In order to be able to occupy all the queues, most of the experiments are done at the heavy traffic regime, i.e. nearly fully loaded systems, where \( \eta \approx 1 \).

### V. Simulation results

#### A. Compared Schedulers

We compare POS to the following scheduling strategies:

1. **Full information (FI) strategies:** These strategies have access to the exact buffer length of queues at the time of each job arrival, and also know the service rates of the servers.
   - JSQ-FI: Join-the-Shortest-Queue assigns the incoming job to the server with the smallest buffer filling.
   - DJSQ-FI: Join the shortest out of \( d \) randomly selected queues. If not stated otherwise, \( d = 2 \) has been used for our experiments.
   - SED-FI: Shortest-Expected-Delay assigns the incoming job to the server with the minimum fraction of the current buffer filling divided by the average service rate.

2. **Limited information (LI) strategies:** These strategies only have access to the randomly delayed acknowledgements.
   - JMO: Join-the-Most-Observations maps an incoming job to the server that has generated the most observations, i.e., received acknowledgements, in the last inter-arrival epoch. This might lead to servers becoming and remaining idle (stale).
   - JMO-E (with Exploration): explores stale servers, with a probability that we set to \( p_e = 0.2 \), by mapping a job to one of those servers uniformly at random.

For all strategies, ties are broken randomly.

#### B. Numerical Results

We first consider a system with \( N = 2 \) heterogeneous servers with exponentially distributed service times with rates \( \mu_1 = 4 \) and \( \mu_2 = 2 \). The inter-arrival times are also exponentially distributed with rate \( \lambda = 5 \). Fig. 4 shows the numerical comparison of POS with other schedulers. Observe that, even though POS does not have access to the exact state of parallel systems and also the acknowledgements from the different systems are randomly delayed, it still achieves comparable results to full information strategies, while it outperforms the

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Footnote:

Note that in POS receiving observations, action selection and belief update all happen at each decision epoch, which is why it is possible to model the system as a POSMDP \([26]\) as well.
other limited information strategies. The heatmap in, Fig. 4(d), represents the policy of POS at each buffer filling state. It can be seen that higher priority is given to the faster server, \( \mu_1 \), having buffer filling \( b_1 \). The light (dark) regions in the heatmap corresponds to the state where jobs are allocated to server 1(2). Note, however, that this policy is not present as such at POS, as it does not observe the buffer fillings \( b_i \).

Fig. 5 shows the performance of POS for \( N = 50 \) heterogeneous servers. The service rates of this setup are the same in Figure 4 extended to 50 queues with the offered load, \( (\eta \approx 1) \). This experiment shows that our scheduler is scalable to perform well for large number of queues. Note that the time taken by the experiments for \( N = 2 \) and \( N = 50 \) servers is the same.

Next, we remain with the case of \( N = 50 \) heterogeneous servers, however with inter-arrival times that are gamma distributed while the service times follow a heavy tailed Pareto distribution, with the offered load \( (\eta \approx 1) \). Fig. 6 shows that here too POS is able to outperform the LI strategies, while having a comparable job drop rate performance as two FI strategies. POS also outperforms the FI strategy, DJSQ-FI, in terms of job drops, which was the main focus of the reward function used in these experiments.

In Fig. 7 we have the setup with \( N = 100 \) homogenous servers with offered load \( \eta = 0.95 \). We consider gamma distributed inter-arrival times, with parameters: shape = rate = \( k\alpha \), and Pareto service times shape=\( \alpha \), while the rate is adjusted in order to achieve the desired offered load \( \eta = 0.95 \). The plot shows the response time performance of POS for \( k = \{1, 2, 4, 8, 16, 32\} \). As the \( k \) increases, while keeping the same load, the response time reduces.

C. Sensitivity Analysis

Next, we discuss the impact of the limited observations on POS under different acknowledgements delays, \( p_i \). Recall, that POS is not able to observe the buffer fillings, but rather receives the randomly delayed acknowledgements of the served jobs. These delayed acknowledgements enable inferring the queue states and, hence, calculating an optimal mapping of incoming jobs to servers. Fig. 8 depicts sample runs showing the evolution of the job queue states (red solid line) and the belief state that POS has on each queue state at each time step, under different acknowledgement delays. Observe that increasing delays (i.e., lower \( p_i \)) increases the uncertainty in belief of each state. Note that this increase in uncertainty is not a sampling issue, rather it is the inherent uncertainty (partial observability) of the scheduler about the state of the system, which increases with the increase in delay. Also observe how POS is still able to track the system state for different delay distribution parameters for each server. Having different delays in acknowledgements from each server reflects a distributed system, where network conditions may be different for each server and may lead to different delays in acknowledgements from different servers.
This is done to show that even though POS has high job drop and response, it outperforms the LI strategies and the \( \eta \) an exponentially tailed distribution which changes with lower offered load, the response time distribution resembles \( \eta \) distribution as the offered load reaches \( \eta \) = 1.

It can be seen that POS has almost no job losses up to a load \( \eta \) under varying offered loads ranging from \( \eta \) = 0 to \( \eta \) = 1. POS is able to track the real state of the system, as long as the delay is not too high.

In Fig. 9 we analyse the performance of the POS scheduler under varying offered loads ranging from \( \eta \) = 0.2 to \( \eta \) = 1.2. It can be seen that POS has almost no job losses up to a load of \( \eta \) = 1. Note the qualitative change of the response time distribution as the offered load reaches \( \eta \) = 1 and beyond. For lower offered load, the response time distribution resembles an exponentially tailed distribution which changes with \( \eta \) = 1 and beyond.

In Fig. 10 we show the performance comparison of different LI and FI strategies for the high offered load case of \( \eta \) = 1.2. This is done to show that even though POS has high job drops and response, it outperforms the LI strategies and the FI strategy DJSQ-FI, while having comparable performance to the other FI strategies.

Another sensitivity analysis we performed is given in Fig. 11, where the number of homogeneous servers \( N \) was increased from \( N = 10 \) to \( N = 90 \) with increments of 20, while keeping the offered load the same, \( \eta = 0.99 \). The inter-arrival times and service times were exponentially distributed. It can be seen that as the number of servers increase, the response time decreases for the same number of jobs, which is the expected trend. The packet drop performance was the same for all from \( N = 30 \) onwards, since the offered load was the same. While, \( N = 10 \) is very less and the system can get overloaded, due to high offered load, resulting in some packet being dropped.

For the sake of completeness, Fig. 12 compares the performance of POS using different reward functions from subsection IV-E, while keeping all the other parameters the same. The

Fig. 8: \( N = 10 \) homogenous servers from which 5 were chosen at random to see their sample run for 50 time steps. The delay \( p \) for the acknowledgements from each of the server was also allocated randomly, ranging from 0.1 to 1.0. In each subplot is given the queue/server number and its delay, \( p \). The solid line trajectory is the sample path for each queue (not known to POS), while the shaded region around it is the belief probability of \( \eta \) for each state at each time step. Exponentially distributed inter-arrival and service times were used, with the delay \( p \) at random to see their sample run for \( N \) time steps. The delay \( p \) is given the queue/server number and its delay, \( p \).

Fig. 9: Varying offered load for a setup as in Fig. 5. As long as the offered load is \( \eta \leq 1 \), POS has almost no packet losses. For loads \( \eta < 1 \) we observe a load dependent exponential tail of the response time distribution.

Fig. 10: For the setup from Fig. 9 with an offered load \( \eta = 1.2 \): POS shows a comparable performance to full information (FI) strategies, while outperforming limited information (LI) ones.

Fig. 11: Homogeneous servers with exponentially distributed inter-arrival and service times. The number of servers are increased in intervals of 10 up to 100 servers, while keeping the offered load always (\( \eta = 0.99 \)). The performance in terms of packet drops is same for \( N = 30 \) onwards, since the offered load is kept constant, while the effect of increasing the servers can be seen more clearly by the reduction in the response time.
job drop rates were the same for all and hence have been omitted. Hence, based on the information available and the system requirements, any reward function can be designed and used.

In the next section, we discuss the scenario when some system parameters are not known and need to be inferred from the available data.

D. Inferring arrival flow and system parameters

In system deployments where the distribution of the inter-arrival times and the service times is unknown, the following method can be used to infer them. For POS to be deployed in an unknown environment, we require an estimate of the inter-arrival density as well as of the service rates. We resort here to a Bayesian estimation approach to infer the densities \( f_U(u) \) and \( f_V(v) \). We select a likelihood model \( f(D | \theta) \) for the data generation process and a prior \( f(\theta) \), with model parameters \( \Theta \). We assume we have access to data \( D = \{d^{(1)}, d^{(2)}, \ldots, d^{(n)}\} \), where \( d^{(j)} \) is inter-arrival times between the \( j \)-th and \( j + 1 \)-st job arrival event that is observed by the scheduler or the service times for each server. For inference-based scheduling in POS we use the inferred distribution of the inter-arrival times as in the posterior predictive \( f(d^* | D) = \int f(\theta | D) f(d^* | \theta) d\theta \), of a new data point \( d^* \), which is used in the simulator, see Eq. (6). Same is done with the data for the service times.

Next, we describe models of different complexities for the data generation. Since, most models do not admit a closed form solution, we resort to a Monte Carlo sampling approach to sample from the posterior predictive.

1) Inference for exponential inter-arrival times: Here, we briefly show the calculation for the posterior distribution and posterior predictive distribution for renewal job arrivals with exponentially distributed inter-arrival times. For the likelihood model we assume

\[
D^{(j)} | m \sim \text{Exp}(m), \quad j = 1, \ldots, n
\]

where \( m \) is the rate parameter of the exponential distribution. We use a conjugate Gamma prior \( M \sim \text{Gam}(\alpha_0, \beta_0) \). Hence, the posterior distribution is

\[
M | D \sim \text{Gam}(\alpha_0 + n, \beta_0 + \sum_{j=1}^{n} d^{(j)})
\]

And the posterior predictive distribution is found as

\[
D^* | D \sim \text{TP}(\alpha_0 + n, \beta_0 + \sum_{j=1}^{n} d^{(j)}),
\]

where TP(\( \alpha, \beta \)) denotes the translated Pareto distribution.

2) Inference for Gamma distributed inter-arrival times: In case of a gamma likelihood of the form

\[
D^{(j)} | a, b \sim \text{Gam}(a, b), \quad j = 1, \ldots, n
\]

we use independent Gamma priors for the shape and the rate, with \( A \sim \text{Gam}(\alpha_0, \beta_0) \) and \( B \sim \text{Gam}(a_1, b_1) \). Finally, we sample from the posterior predictive using Hamiltonian Monte Carlo (HMC) [32], which can be implemented using a probabilistic programming language, e.g. using PyMC3 [33].

3) Service times distributed as an infinite Gamma mixture: Here, we present a framework to non-parametrically infer the
We assume that the servers have a finite buffer of size $B = 10$ and a delay in acknowledgements of $p = 0.6$. Fig. 15 shows that even with limited information, POS has the lowest average packets drops. However, due to the limited information available to POS and the reward function it is using, its average response time is as high as the full information strategy DJSQ-FI. POS is able to allocate more packets (due to fewer drops), which comes at a cost of higher response time for some jobs. Note that the reward function we used, (13) with $\kappa_1 = 0$, focuses on avoiding packet drops and not on minimizing the response time.

VI. DISCUSSION & CONCLUSION

In this work, we analyzed online algorithms for mapping incoming jobs to parallel and heterogeneous processing systems under partial observability constraints. This partial observability is rooted in the assumption that the entity controlling this mapping, denoted scheduler, takes decisions only based on randomly delayed feedback of the parallel systems. Unlike classical models that assume full knowledge of the parallel systems, e.g., knowing the queue lengths (Join-the-Shortest Queue - JSQ) or additionally the job service times (Shortest Expected Delay - SED) this model is particularly suited for large distributed processing systems that only provide an acknowledgement-based feedback.

In addition to presenting a partially observable (semi-)Markov decision process model that captures the scheduling decisions in this parallel queuing system under delayed acknowledgements, we provide an online scheduler - denoted Partial Observable Scheduler (POS) - to find near-optimal solutions in real-time. A particular strength of POS is that it allows to define the objectives of the system and lets it find the appropriate scheduling policy instead of manually defining a fixed one. It can also be used for any kinds of inter-arrival and service time distributions and is scalable to a large number of queues. We numerically show that the resulting scheduler is especially powerful under server heterogeneity. We also show that the POS scheduling policies obtained under partial observability are comparable to fixed scheduling policies such as JSQ, JSQ(d) and SED having full information. This is the case although POS receives less, and in addition randomly delayed, informative feedback.

In a possible extension to this work, one may explore finite horizon objectives for time dependent transition dynamics.

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**E. Experiments with trace data**

For the results of this section, we make use of Labeled Network Traffic Flow data, provided by Kaggle in 2019 [35], [36]. We used the above frameworks to infer the underlying distributions of the inter-arrival and service times provided in this data set. The inferred distribution based on data of the inter-arrival times of the chosen source is given in Fig. 13, it can be seen to follow an exponential distribution with high arrival rate. We then selected 20 heterogeneous servers from the available data such that they all followed the Gamma Mixture distribution. Gamma Mixture was selected to show the performance of POS with yet another type of service time distributions. The empirical distribution as a histogram as well as the posterior mean estimate for some of these selected servers is given in Fig. 14. The hyperparameters used are: $c = 3$, $a_i, b_i = 1$, $m = 1$. POS makes use of the samples generated using the posterior predictions.

We assume that the servers have a finite buffer of size $B = 10$ and a delay in acknowledgements of $p = 0.6$. Fig. 15 shows that even with limited information, POS has the lowest average packets drops. However, due to the limited information available to POS and the reward function it is using, its average response time is as high as the full information strategy DJSQ-FI. POS is able to allocate more packets (due to fewer drops), which comes at a cost of higher response time for some jobs. Note that the reward function we used, (13) with $\kappa_1 = 0$, focuses on avoiding packet drops and not on minimizing the response time.

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In a possible extension to this work, one may explore finite horizon objectives for time dependent transition dynamics.
Another direction for extending this work is to include the memory effect of the last served job in the model simulator, e.g., through state space extension. This work can also be extended to more than one scheduler, working in parallel in a multiagent manner.

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