EVEN SETS OF NODES ARE BUNDLE SYMMETRIC

G. CASNATI & F. CATANESE

0. Introduction

Let $k$ be an algebraically closed field of characteristic $p \neq 2$, and let $F := \{ f = 0 \} \subseteq \mathbb{P}_k^3$ be a normal surface of degree $d$. Let $\pi: \widetilde{F} \to F$ be a minimal resolution of singularities. We denote by $H \subseteq F$ a general plane section of $F$ defined by a general linear form $h$. Assume, for simplicity, that $F$ is a nodal surface (i.e., its singularities are only ordinary quadratic, nodes for short).

Let $\Delta$ be a subset of the set of nodes of $F$, and let $\widetilde{\Delta} := \pi^{-1}(\Delta)$. $\Delta$ is said to be a $\delta/2$–even set of nodes, $\delta = 0, 1$, if the class of $\Delta + \delta \pi^*H$ in Pic($\widetilde{F}$) is 2–divisible (when $\delta = 0$ we shall simply say that $\Delta$ is even).

The condition that $\Delta$ is $\delta/2$–even is equivalent to the existence of a double cover $\tilde{p}: \tilde{S} \to \tilde{F}$ branched exactly along $\tilde{\Delta} + \delta \pi^*H$ and (cf. [6, 2.11, 2.13]) it is possible to blow down $\tilde{p}^{-1}(\tilde{\Delta})$ getting a commutative diagram

$$
\begin{array}{ccc}
\tilde{S} & \xrightarrow{\tilde{\pi}} & S \\
\downarrow \tilde{\rho} & & \downarrow p \\
F & \xrightarrow{\pi} & F
\end{array}
$$

(0.1)

where $S$ is a nodal surface and $p$ is finite of degree 2 branched exactly on $\Delta$ when $\delta = 0$ (respectively on $\Delta$ and $H$ when $\delta = 1$; in this case $d$ has to be even). The surface $S$ is then endowed with a natural involution $i$ such that $F \cong S/i$ and $p$ is the quotient map. Thus we have an $\mathcal{O}_F$–linear map $i^\#: p_*\mathcal{O}_S \to p_*\mathcal{O}_S$ giving rise to a splitting of $\mathcal{O}_F$–modules $p_*\mathcal{O}_S \cong \mathcal{O}_F \oplus \mathcal{F}$ where $\mathcal{O}_F$ and $\mathcal{F}$ are the $+1$ and $-1$ eigenspaces of $i^\#$. 

Received February 13, 1996, and, in revised form, May 23, 1997.