Detection of spin injection into a double quantum dot: Violation of magnetic-field-inversion symmetry of nuclear polarization instabilities

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In mesoscopic systems with spin-orbit coupling, spin-injection into quantum dots at zero magnetic field is expected under a wide range of conditions. However, until now, a viable approach for experimentally identifying such injection has been lacking. We show that electron spin injection into a spin-blockaded double quantum dot is dramatically manifested in the breaking of magnetic-field-inversion symmetry of nuclear polarization instabilities. Over a wide range of parameters, the asymmetry between positive and negative instability fields is extremely sensitive to the injected electron spin polarization and allows for the detection of even very weak spin injection. This phenomenon may be used to investigate the mechanisms of spin transport, and may hold implications for spin-based information processing.

Time reversal symmetry is a fundamental law of nature, which places strong constraints on the types of behaviors which can occur in physical systems. Based on the Onsager concept of microscopic reversibility close to equilibrium, general transport coefficients exhibit particular symmetries under the reversal of direction of an applied magnetic field $B$. Such magnetic-field-inversion symmetry is robustly observed to high precision throughout a wide variety of experiments. Therefore it is quite remarkable to find examples of phenomena where this symmetry is violated. Moreover, such asymmetries can provide information about deviations from equilibrium.

Over the past several years, excitement about the prospect of spin-based information processing has led many authors to consider a variety of mechanisms for injecting and manipulating electron spins in nanoscale devices. Such works have shown that, through the spin-orbit interaction, significant spin injection can be produced even in the absence of an applied magnetic field. The only restrictions on spin injection are imposed by the action of time reversal symmetry together with unitarity, and they allow spin injection in systems with more than one outgoing channel. However, as shown in Ref.\textsuperscript{[5]}, coupling to an environment which breaks unitarity allows spin injection even into a single outgoing channel. Experimentally, spin injection through quantum point contacts has also been reported. Thus spin injection appears to be a generic phenomenon.

Although less extensively studied, analogous mechanisms should lead to spin injection of electrons into quantum dots in systems with spin-orbit coupling, at $B = 0$. For quantum dots coupled to source and drain electrodes, unitarity is broken due to the coupling to phonons and Fermi reservoirs, and there is no fundamental reason to expect a vanishing spin injection probability.

Although such spin injection is expected under a wide range of circumstances, until now a viable method for its detection has been missing. In this paper, we demonstrate that spin injection can be manifested in a dramatic violation of magnetic-field-inversion symmetry in dc transport through spin-blockaded double quantum dots. Experiments in this regime have demonstrated a variety of interesting nonlinear phenomena such as bistabilities and hysteresis, which are associated with the coupled dynamics of electron and nuclear spins. In particular, B-inversion asymmetry has apparently been recently observed by the Delft group. To date, the theoretical treatment of instabilities was based on the assumption of completely unpolarized injected electron spins. Here we use an extended version of the model of Ref.\textsuperscript{[15]} to show that spin injection breaks the magnetic-field-inversion symmetry of the dynamical instabilities. Furthermore, in the regime where hyperfine- and non-hyperfine-mediated decay rates are comparable, the degree of asymmetry is an extremely sensitive function of the injected electron spin polarization.

Why is magnetic-field-inversion symmetry violated for this system? The direction of the dc current flowing through the double dot breaks the time-reversal symmetry, even for very weak currents. Through spin-orbit coupling, this violation of time-reversal by the direction of the current is converted into spin injection into the dot. While the observation of spin injection \textit{per se} does not reveal its mechanism, by varying electrostatic gates which control the transport of electrons in the lead and/or barrier regions, the phenomenon can be used to investigate the nature of spin transport in the system.

Note that, in the presence of a large Zeeman splitting, electron spins may be injected with a high degree of polarization. However, such magnetically-induced spin injection is symmetric in the magnetic field $B$, and therefore \textit{does not} lead to magnetic-field-inversion asymmetry. Here we focus on the field-independent part of spin-injection which persists down to zero magnetic field. For simplicity, we further assume that magnetically-induced spin-injection is weak over the range of relevant fields. To illustrate the spin-injection induced magnetic-field-inversion-symmetry-breaking phenomenon most clearly, we start from the simple model of spin-blockaded transport proposed in Ref.\textsuperscript{[19]} and em-
polarization is encoded in the factors $N_I$ where clear polarization and feedback through the Overhauser shift accompanied by nuclear spin flips which lead to dynamical nutations between $|T_\pm\rangle$ and $|S\rangle$ are obtained from Fermi’s Golden Rule:

$$W_{HF}^\pm = \frac{2\pi}{\hbar} (1 \mp x) \mathcal{M}^2 f(\varepsilon_\pm), \quad f(\varepsilon) = \frac{\gamma/\pi}{\varepsilon^2 + \gamma^2}, \quad (2)$$

where $\varepsilon_\pm = \varepsilon \pm E_Z$ (see Fig. 1b), and we assume a Lorentzian lineshape of width $\gamma$ for the decaying singlet state. Here $E_Z = -\mu_e B + A x$ is the effective Zeeman energy including the Overhauser shift $A x$, where $\mu_e = -g^* \mu_B$ is electron magnetic moment with $g^*$ the effective $g$-factor of the material ($g^* \approx -0.4$ in GaAs), $\mu_B$ is the Bohr magneton, $B = |B|$ is the magnitude of the external magnetic field, and $A$ is the hyperfine coupling strength. The matrix element $\mathcal{M} \sim A/\sqrt{N}$ for electron-nuclear spin exchange is set by the typical scale of the random transverse hyperfine field. The factor $\frac{1}{2}(1 \mp x)$ counts the available phase space for finding a properly oriented (down or up) nuclear spin to flip.

We now seek the steady state values of nuclear polarization, obtained by setting $\dot{x} = 0$ in Eq. (1). Transforming to the set of dimensionless parameters $\bar{\varepsilon} = \varepsilon / A$, $\bar{B} = \mu_e B / A$, $\bar{\gamma} = \gamma / A$, $\bar{\eta} = \mathcal{M}^2 / (\hbar W^m A)$, and the spin-injection coefficient $\eta = (P_+ - P_-)/(P_+ + P_-)$, the steady state values of the nuclear polarization are given by the third-order algebraic equation

$$F(x) \equiv a x^3 + b x^2 + c x + d = 0, \quad (3)$$

with

$$a = 1, \quad b = \eta(2\bar{\varepsilon} + \bar{\gamma}\bar{m} - 1) - 2\bar{B},$$
$$c = \bar{\varepsilon}^2 + \bar{B}^2 + \bar{\gamma}^2 - 2\bar{\varepsilon}(1 + \bar{\eta} \bar{B}) + 2\bar{B}\bar{\eta},$$
$$d = 2\bar{B}\bar{\varepsilon} - \eta(\bar{B}^2 + \bar{\varepsilon}^2 + \bar{\gamma}^2 + \bar{\gamma}\bar{m}). \quad (4)$$

Typically, $A \approx 100 \mu eV$, while the singlet-triplet splitting $\varepsilon$ and level width $\gamma$ can be on the $\mu eV$ scale or less. Therefore, below we take $\bar{\varepsilon}, \bar{\gamma} < 1$. As mentioned above, we disregard Zeeman-splitting-induced spin injection, which would produce an effect even $B$. Thus we consider $\eta$ as field independent.

A cubic equation with real coefficients, such as that in Eq. (3), may have either one or three real solutions, depending on the values of the coefficients. Each such solution, which corresponds to a steady state of Eq. (1), can be stable or unstable, depending on whether the flow $\dot{x}$ tends to restore or amplify small deviations from the steady state. In parameter regimes where Eq. (1) possesses two stable fixed points $\dot{x} = 0$, the system is bistable and will typically exhibit hysteresis and/or possible switching. As a parameter such as the magnetic field $B$ is varied, bistability disappears at bifurcation points, where two real roots of Eq. (3) annihilate and become a complex-conjugate pair.
Steady-State Polarization

A typical pattern of fixed points for systems with $\eta = 0$ is illustrated in the instability diagram in Fig. 2, where we plot the roots of $\Delta(F(x))$, as a function of magnetic field $\tilde{B}$. Solid (dotted) lines indicate stable (unstable) fixed points. Note that in absence of spin injection, i.e. for $\eta = 0$, the solutions are symmetric with respect to $B$-inversion. The system exhibits bistability over a wide range of magnetic field strengths, with bifurcation points near $|\tilde{B}| = 0.7$ where bistability disappears.

To investigate the pattern of instabilities in more detail, we examine the discriminant of Eq. (3), which we denote by $\Delta[F(x)]$. For a general polynomial, the discriminant $\Delta = \prod_{i<j} (x_i - x_j)^2$ is a symmetric function of the polynomial’s roots $\{x_i\}$. Each complex-conjugate pair of roots contributes a factor of $-1$ to $\Delta$. Therefore the bifurcation points, where two real solutions merge and turn into a complex conjugate pair, correspond to the zeros (sign-changing points) of the discriminant.

Because the discriminant $\Delta[F(x)]$ is a symmetric function of the roots of $F(x)$, it can be expressed directly as a polynomial in the coefficients of $F(x)$. For a cubic polynomial of the form $\sum_{i=0}^{3} x_i^i a_i$, the discriminant is given by [21]

$$\Delta[F(x)] = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2.$$ \hspace{1cm} (5)

Thus the problem of mapping out the bifurcations of the fixed points of the flow $\dot{x}$ in Eq. (1) is reduced to the problem of solving for the roots of $\Delta[F(x)]$ in Eq. (5), with $a, b, c,$ and $d$ taken from Eq. (4).

Because we are primarily interested in the magnetic-field-inversion symmetry/asymmetry of the system, we focus on the $B$-dependence of the discriminant $\Delta[F(x)]$. With all other parameters fixed, the equation $\Delta[F(x)] = 0$ yields a fifth-order polynomial in $\tilde{B}$, whose roots determine the bifurcations of the fixed points of the flow $\dot{x}$. The full expression for $\Delta[F(x)]$ is quite cumbersome, and we do not reproduce it here. The expansion of $\Delta$ in the regime $\tilde{\varepsilon}, \tilde{\gamma}, \tilde{m}, \eta \ll 1$, up to third order in all parameters, reads as $\Delta \approx \Delta^{(2)} + \Delta^{(3)}$, with $\Delta^{(2)} = -4(\tilde{\gamma}^2 + \tilde{\varepsilon}^2)\tilde{B}^4 + 4\tilde{\varepsilon}^2\tilde{B}^2$ and $\Delta^{(3)} = -4\eta(\tilde{m}\tilde{\gamma})\tilde{B}^5 + 4\eta(\tilde{m}\tilde{\gamma} + 4\tilde{\gamma}^2 + 2\varepsilon^2)\tilde{B}^3 - 4\eta\tilde{m}\tilde{\gamma} + 4\tilde{\gamma}^2 + 2\varepsilon^2)\tilde{B}^2 - 8\eta\varepsilon^2\tilde{B} + 32\varepsilon^3$. Note that $\eta$ first appears in $\Delta^{(3)}$, which is linear in $\eta$. There, $\eta$ multiplies each odd power of $\tilde{B}$, ensuring that the discriminant is invariant under $\eta \rightarrow -\eta, \tilde{B} \rightarrow -\tilde{B}$. This is a fundamental symmetry of the system, and holds to all orders.

As demonstrated in Fig. 3 for $\eta \neq 0$, i.e. when the incident current carries spin-polarization, the zeros of $\Delta$ can be highly asymmetric in $\tilde{B}$. To explore the degree of magnetic-field-inversion symmetry breaking in more detail, in Fig. 4 we plot the instability asymmetry parameter $B^* = \frac{1}{2}(B^*_{+} + B^*_{-})$ as a function of the spin injection coefficient $\eta$, and the parameter $\tilde{m}$ which describes the competition between the hyperfine transition rates $W_{\pm}^{HF}$ and the nuclear-spin-independent decay rate $W^{\text{in}}$. Here $B^*_{+}$ and $B^*_{-}$ are the upper and lower bifurcation points, as indicated by the dashed lines in Fig. 4. For $g^* < 0$, as is typical for GaAs and InAs, two other roots of the fifth-order equation $\Delta = 0$ are complex, while the fifth root is unphysical because it corresponds to a nuclear spin polarization of greater than 100%.

For weak spin injection, i.e. for small $\eta$, the instability asymmetry $B^*$ grows monotonically with $\eta$. The rate at which $B^*$ grows with $\eta$ is controlled by the competition
Asymmetry, $\bar{B}$

Competition Factor, $\log_{10} \tilde{m}$

and the system is stable, being partially polarized, for all boundary beyond which all bifurcation points disappear, $(\tilde{m} = 1, \text{solid line})$. Note that for very large $\eta$, we find a boundary for which all bifurcation points disappear, and the system is stable, being partially polarized, for all values of $\tilde{B}$.

Within the quantum dot, the levels $|T_+\rangle$ and $|T_-\rangle$ are defined with respect to a quantization axis which is directed near the external field $B$. However, the polarization axis of electron spin injection is mostly determined by the spin-orbit interaction along the path between the source and the dot. In the discussion above, we have implicitly assumed that injected spins were polarized along $B$, in which case spin injection directly leads to an imbalance of the probabilities $P_+$ and $P_-$ to load the $|T_+\rangle$ and $|T_-\rangle$ states. Suppose instead that the magnetic field is oriented perpendicular to the axis of electron spin injection. In this case, the system will on average have no preference for loading either $|T_+\rangle$ or $|T_-\rangle$, and therefore we would find $\eta = 0$. Thus we expect that, within the simplest model of Zeeman-field-independent spin-injection, the spin-injection coefficient $\eta$ should vary like the cosine of the angle between the external field and the spin-injection axis.

In conclusion, spin-orbit coupling results in spin polarization of the electrons injected from nonmagnetic electrodes into a quantum dot even in the absence of an external magnetic field $B$. He have shown that $B$-inversion asymmetry of the nuclear polarization instabilities can serve as a highly sensitive tool for detecting this polarization.

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