Lifetime difference in $B_s$ mixing: Standard Model and beyond

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We present a calculation of $1/m_b^2$ corrections to the lifetime differences of $B_s$ mesons in the heavy-quark expansion. We find that they are small to significantly affect $\Delta \Gamma (B_s)$ and present the result for lifetime difference including non-perturbative $1/m_b$ and $1/m_b^2$ corrections. We also analyze the generic $\Delta B = 1$ New Physics contributions to the lifetime difference of $B_s$ mesons and provide several examples.

Mixing phenomena in heavy bosons system is considered as an important test of Standard Model(SM) and a probe for New Physics(NP) beyond it. Usually it is referred to the fact that such process occurs only at the one loop level in SM. This makes it sensitive to the effects of new particles running in the loop. These interactions induce non-diagonal elements in mass-matrix making flavor and mass eigenstates to be different. Analysis of mixing in charm, beauty systems led to positive signals which seem to be very well explained by Standard Model physics. The lifetime difference $\Delta \Gamma_s$ is generated by on-mass-shell intermediate states and seems to be one more test of Standard Model and heavy quark expansion. Yet some contribution of NP is still possible as an indirect probe of energy scales beyond currently accessible at experimental facilities. Further in paper we set up relevant formalism and discuss the need to compute $1/m_b^2$ corrections. After these corrections computed we consider impact of New Physics $\Delta b = 1$ interaction on the numerical value of $\Delta \Gamma_s$

I. FORMALISM

The width difference between mass eigenstates is then given by [1]

$$\Delta \Gamma_{B_s} \equiv \Gamma_L - \Gamma_H = -2\Gamma_{12} = -2\Gamma_{21},$$

where $\Gamma_{ij}$ are the elements of the decay-width matrix, $i,j = 1,2 \ (|1\rangle = |B_s\rangle, \ |2\rangle = |\bar{B}_s\rangle)$

Using optical theorem, off diagonal elements of mixing matrix can be related to the imaginary part of the forward scattering amplitude:

$$\Gamma_{21}(B_s) = \frac{1}{2M_{B_s}} \langle \bar{B}_s | T | B_s \rangle,$$

$$T = \text{Im} \int d^4x T \{ H_{\text{eff}}(x) H_{\text{eff}}(0) \},$$

where $H_{\text{eff}}$ is an effective weak hamiltonian defined as follows:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} \left( \sum_{r=1}^{6} C_r Q_r + C_8 Q_8 \right),$$

where four-quark operators are defined in the following way:

$$Q_1 = (\bar{b}_i c_j)_{V-A} (\bar{c}_j s_i)_{V-A},$$

$$Q_2 = (\bar{b}_i c_i)_{V-A} (\bar{c}_j s_j)_{V-A},$$

$$Q_3 = (\bar{b}_i s_i)_{V-A} (\bar{q}_j q_j)_{V-A},$$

$$Q_4 = (\bar{b}_i s_j)_{V-A} (\bar{q}_j q_i)_{V-A},$$

$$Q_5 = (\bar{b}_i s_i)_{V-A} (\bar{q}_j q_j)_{V+A},$$

$$Q_6 = (\bar{b}_i s_j)_{V-A} (\bar{q}_j q_i)_{V+A},$$

$$Q_8 = \frac{g_s}{8\pi^2 m_0} \bar{b}_i \sigma_{\mu\nu} (1 - \gamma_5) T_{ij}^a s_j G_{\mu\nu}^a.$$

In the heavy-quark limit the energy release is large and process is dominated by short-distance physics. An operator product expansion can be constructed which results in series of operators suppressed by pow-
The most recent calculations of $B_s$ lifetime difference and of QCD corrections to $\Delta \Gamma_s$ do not provide definitive theoretical prediction of its value. Heavy quark expansion corrections of order of 1 to be about 25% of leading order and QCD corrections are as big as 30%. We compute $1/m_b^2$ corrections in heavy quark expansion to directly check convergence of this series. In other words we compute matching coefficients of an effective $\Delta b = 2$ lagrangian. Computation of matrix elements of such operators is rather difficult task due to lack of results from Lattice QCD and Light cone QCD calculations. We used a factorization approach to estimate matrix elements of such operators.

Expanding the operator product for small $z \sim 1/m_b$, the transition operator $T$ can be written, to leading order in the $1/m_b$ expansion, as and

$$ T = -\frac{G_F^2 m_b^2}{12\pi} (V_{cb}^* V_{cs})^2 \langle F(z) Q(\mu_2) + F_S(z) Q_S(\mu_2) \rangle,$$

which results in

$$ \Gamma_{21}(B_s) = -\frac{G_F^2 m_b^2}{12\pi (2M_{B_s})} \langle V_{cb}^* V_{cs} \rangle^2 \sqrt{1 - 4z} \times \left\{ [(1 - z) (2C_1 C_2 + N_c C_2^2) + (1 - 4z) C_2^2/2] \langle Q \rangle + (1 + 2z) (2C_1 C_2 + N_c C_2^2 - C_1^2) \langle Q_S \rangle \right\},$$

where $z = m_c^2/m_b^2$ and the $\Delta B = 2$ operators are as follows:

$$ Q = (\bar{b}_i s_i) V_{-A}(\bar{b}_j s_j) V_{-A},$$
$$ Q_S = (\bar{b}_i s_i) S_{-P}(\bar{b}_j s_j) S_{-P}. $$

Color re-arranged operators $\bar{Q} = (\bar{b}_i s_i) V_{-A}(\bar{b}_j s_j) V_{-A}$ and $\bar{Q}_S = (\bar{b}_i s_i) S_{-P}(\bar{b}_j s_j) S_{-P}$ that appear during calculations were eliminated using Fiertz identities and equation of motion.

The Wilson coefficients $F$ and $F_S$ are obtained by computing the matrix elements of $T$ in between quark states.

The coefficients in the transition operator at next-to-leading order, still neglecting the penguin sector, can be written as

$$ F(z) = F_{11}(z) C_2^2(\mu_1) + F_{12}(z) C_1(\mu_1) C_2(\mu_1) + F_{22}(z) C_4^2(\mu_1),$$

$$ F_{ij}(z) = F_{ij}^{(0)}(z) + \frac{\alpha_s(\mu_1)}{4\pi} F_{ij}^{(1)}(z),$$

$F_S(z)$ has similar structure. The leading order functions $F_{ij}^{(0)}$, $F_{S,ij}^{(0)}$ read explicitly

$$ F_{i1}^{(0)}(z) = 3\sqrt{1 - 4z} (1 - z),$$
$$ F_{S,i1}^{(0)}(z) = 3\sqrt{1 - 4z} (1 + 2z),$$
$$ F_{i2}^{(0)}(z) = 2\sqrt{1 - 4z} (1 - z),$$
$$ F_{S,i2}^{(0)}(z) = 2\sqrt{1 - 4z} (1 + 2z),$$
$$ F_{22}^{(0)}(z) = \frac{1}{2} (1 - 4z)^{3/2}.$$

The next-to-leading order (NLO) QCD expressions of $F_{ij}$, $F_{S,ij}$ and corrections to Eq. 13 arising from penguin diagram are given in Ref. 2.

II. $1/m_b^2$ CORRECTIONS

The general expression for lifetime difference of $B_s$ mesons can be presented in the following way:

$$ \Gamma_{21}(B_s) = -\frac{G_F^2 m_b^2}{12\pi (2M_{B_s})} \langle V_{cb}^* V_{cs} \rangle^2 \times \left\{ [\langle F(z) + P(z) \rangle Q + \langle F_S(z) + P_S(z) \rangle Q_S \right\} + \delta_{1/m} + \delta_{1/m^2},$$

where $\delta_{1/m}$ and $\delta_{1/m^2}$ denote contributions from operators suppressed as $1/m_b$ and $1/m_b^2$ respectively. These terms and their numerical values are computed further.

The matrix elements for $Q$ and $Q_S$ can be parametrized in the following way

$$ \langle \bar{B}_s | Q | B_s \rangle = f_{B_s}^2 M_{B_s}^2 \left( 1 + \frac{1}{N_c} \right) B, $$
$$ \langle \bar{B}_s | Q_S | B_s \rangle = -f_{B_s}^2 M_{B_s}^2 \left( M_{B_s}^2 (m_b + m_s)^2 \right) \left( 2 - \frac{1}{N_c} \right) B_s,$$

where $M_{B_s}$ and $f_{B_s}$ are the mass and decay constant of the $B_s$ meson and $N_c$ is the number of colors. $B$ and $B_S$ are defined such that $B = B_S = 1$ corresponds to the factorization (or ‘vacuum insertion’) approach, which can provide a first estimate. Their numerical values are known from Lattice QCD calculations.

The $1/m_b$ corrections are obtained expanding amplitude Eq. 2 in terms of light quark momentum and matching it to four-quark operators that contain derivatives.

The $\delta_{1/m}$ term can be written in the following form:

$$ \delta_{1/m} = \sqrt{1 - 4z} \left\{ (1 + 2z) [C_2^2 (R_2 + 2R_3)] - 2 (2C_1 C_2 + N_c C_2^2) (R_1 + 2R_2) \right\} \frac{z^2}{4}$$
$$ - \frac{12z^2}{1 - 4z} \left\{ (2C_1 C_2 + N_c C_2^2) (R_2 + 2R_3) + 2C_2^2 R_3 \right\}.$$
where additional operators that contain derivatives appear

\[
R_1 = \frac{m_b}{m_b} \tilde{b}_i \gamma^\mu (1-\gamma_5) s_i \tilde{b}_j \gamma_\mu (1+\gamma_5) s_j
\]

\[
R_2 = \frac{1}{m_b^2} \bar{b}_i D_\mu (1-\gamma_5) D^\mu s_i \gamma_\mu (1-\gamma_5) s_j
\]

\[
R_3 = \frac{1}{m_b^2} \bar{b}_i D_\mu (1-\gamma_5) D^\mu s_i \gamma_\mu (1-\gamma_5) s_j
\]

\[
R_4 = \frac{1}{m_b^3} \tilde{b}_i (1-\gamma_5) i D_\mu s_i \gamma_\mu (1-\gamma_5) s_j .
\]

Their matrix elements are

\[
\langle \vec{B}_s | R_1 | B_s \rangle = \left( 2 + \frac{1}{N_c} \right) \frac{m_s}{m_b} f_{B_s}^2 M_{B_s}^2 B_1^2
\]

\[
\langle \vec{B}_s | R_2 | B_s \rangle = \left( -1 + \frac{1}{N_c} \right) f_{B_s}^2 M_{B_s}^2 \left( \frac{M_{B_s}^2}{m_b^2} - 1 \right) B_2^2
\]

\[
\langle \vec{B}_s | R_3 | B_s \rangle = \left( 1 + \frac{1}{N_c} \right) f_{B_s}^2 M_{B_s}^2 \left( \frac{M_{B_s}^2}{m_b^2} - 1 \right) B_3^2
\]

\[
\langle \vec{B}_s | R_4 | B_s \rangle = -f_{B_s}^2 M_{B_s}^2 \left( \frac{M_{B_s}^2}{m_b^2} - 1 \right) B_4^2.
\]

Among these B-parameters, $B_1^2$ and $B_2^2$ are the most widely studied and well known in lattice and light cone QCD. In this paper we use the results of Ref. [3]. The rest of "bag" parameters is estimated using a vacuum insertion approximation. The color-rearranged operators $\bar{R}_i$, were eliminated using Fierz identities and the equations of motion as in Eq. (16).

As it was mentioned earlier $O(1/m_b)$ corrections are quite large [1,2]. Computing $O(1/m_b^2)$ we directly control convergence of $1/m$ expansion in lifetime difference calculation. At this order we get more operators that contribute to the $\Delta\Gamma(B_s)$. There are two different types of operators. One class of them involves operators computed by further expansion of Eq. (2) - they are called kinetic corrections. Another type arises from interaction of quarks with background gluon field.

The kinetic corrections can be written as:

\[
\frac{\delta_{1/m^2}}{m^2} = \frac{24\pi^2}{(1-4z)^2} (3-10z) [C_1^2 W_3 + (2 C_1 C_2 + N_c C_2^2)(W_1 + W_2/2)]
- \frac{12z}{1-4z} \frac{m_s^2}{m_b^2} [C_1^2 Q_S - (2 C_1 C_2 + N_c C_2^2)(Q_{S} + Q_{2}/2)]
+ \frac{24z}{1-4z} \frac{m_s^2}{m_b^2} [2C_1^2 W_4 - 2(2 C_1 C_2 + N_c C_2^2)(W_1 + W_2/2)]
- \frac{1 - 2z}{m_b^2} \left(C_1^2 + 2 C_1 C_2 + N_c C_2^2\right) Q_R.
\]

The operators in Eq. (26) are defined as

\[
Q_R = (\bar{b}_i s_i)_{S+P} (\bar{b}_j s_j)_{S+P}.
\]

\[
W_1 = \frac{m_s}{m_b} \bar{b}_i \gamma^\mu (1-\gamma_5) \gamma_\mu (1+\gamma_5) s_j
\]

\[
W_2 = \frac{1}{m_b^2} \bar{b}_i \gamma^\mu (1-\gamma_5) \gamma_\mu (1+\gamma_5) s_i \gamma_\mu (1+\gamma_5) s_j
\]

\[
W_3 = \frac{1}{m_b^2} \bar{b}_i \gamma^\mu (1-\gamma_5) \gamma_\mu (1+\gamma_5) s_i \gamma_\mu (1+\gamma_5) s_j
\]

\[
W_4 = \frac{1}{m_b^3} \bar{b}_i \gamma^\mu (1-\gamma_5) i D_\mu s_i \gamma_\mu (1+\gamma_5) s_j.
\]

where, as before, we have eliminated the color-rearranged operators $\bar{W}_i$ in favor of the operators $W_i$. Due to absence of results from lattice and light cone QCD, the parametrization of the matrix elements of these operators is given. In the pure factorization approach all the bag parameters $\alpha_i$ should be set to 1:

\[
\langle \vec{B}_s | Q_R | B_s \rangle = -f_{B_s}^2 M_{B_s}^2 \left( \frac{M_{B_s}^2}{m_b^2} - 1 \right) \alpha_1,
\]

\[
\langle \vec{B}_s | W_1 | B_s \rangle = \left( 1 + \frac{1}{N_c} \right) f_{B_s}^2 M_{B_s}^2 \left( \frac{M_{B_s}^2}{m_b^2} - 1 \right) \alpha_2,
\]

\[
\langle \vec{B}_s | W_2 | B_s \rangle = \frac{1}{2} \left( -1 + \frac{1}{N_c} \right) f_{B_s}^2 M_{B_s}^2 \left( \frac{M_{B_s}^2}{m_b^2} - 1 \right)^2 \alpha_3,
\]

\[
\langle \vec{B}_s | W_3 | B_s \rangle = \frac{1}{2} \left( 1 + \frac{1}{N_c} \right) f_{B_s}^2 M_{B_s}^2 \left( \frac{M_{B_s}^2}{m_b^2} - 1 \right)^2 \alpha_4,
\]

\[
\langle \vec{B}_s | W_4 | B_s \rangle = -\frac{1}{2} f_{B_s}^2 M_{B_s}^2 \left( \frac{M_{B_s}^2}{m_b^2} - 1 \right)^2 \alpha_5.
\]

In addition to the set of kinetic corrections considered above, the effects of the interactions of the intermediate quarks with background gluon fields should also be included at this order. The contribution of those operators can be computed from the diagram on Fig. 1 resulting in

\[
T_{spec,G} = \frac{G_F^2 (\gamma_{V_p} V_{c*})^2}{4\pi \sqrt{1 - 4z}} \left( C_1^2 \right) -(1 - 4z) P_1 P_2 +
\]

FIG. 1: Diagrams contributing to corrections due to interaction with background gluon field.
The local four-quark operators contributing to this correction are given in Eq. (29).

\[
P_1 = \bar{b} \gamma^\mu (1 - \gamma_5) s_i \bar{b} \gamma^\nu (1 - \gamma_5) \bar{G}_{\mu\nu}^a s_i \tag{29}
\]

\[
P_2 = \bar{b} \gamma^\mu (1 - \gamma_5) t_k^a \bar{G}_{\mu}^a s_i \bar{b} \gamma^\nu (1 - \gamma_5) s_i,
\]

\[
P_3 = \frac{1}{m_b^2} \bar{b} \gamma^\mu \bar{D}^\nu \bar{D}^\alpha (1 - \gamma_5) s_i \bar{b} \gamma^\nu (1 - \gamma_5) \bar{G}_{\mu\nu}^a s_i,
\]

\[
P_4 = \frac{1}{m_b^2} \bar{b} \gamma^\mu \bar{D}^\nu \bar{D}^\alpha (1 - \gamma_5) s_i \bar{b} \gamma^\nu \gamma^\alpha (1 - \gamma_5) s_i,
\]

\[
P_5 = \frac{1}{m_b^2} \bar{b} \gamma^\mu \bar{D}^\nu \bar{D}^\alpha (1 - \gamma_5) s_i \bar{t}_k^a \bar{G}_{\mu\nu}^a \bar{b} \gamma^\nu (1 - \gamma_5) s_i,
\]

\[
P_6 = \frac{1}{m_b^2} \bar{b} \gamma^\mu \bar{D}^\nu \bar{D}^\alpha (1 - \gamma_5) s_i \bar{t}_k^a \bar{G}_{\mu\nu}^a \bar{b} \gamma^\nu (1 - \gamma_5) s_i,
\]

\[
P_7 = \frac{1}{m_b^2} \bar{b} \gamma^\mu \bar{D}^\nu \bar{D}^\alpha (1 - \gamma_5) s_i \bar{t}_k^a \bar{G}_{\mu\nu}^a \bar{b} \gamma^\nu (1 - \gamma_5) s_i,
\]

\[
P_8 = \frac{1}{m_b^2} \bar{b} \gamma^\mu \bar{D}^\nu \bar{D}^\alpha (1 - \gamma_5) s_i \bar{t}_k^a \bar{G}_{\mu\nu}^a \bar{b} \gamma^\nu (1 - \gamma_5) s_i.
\]

Following [6] these operators are parametrized the following way:

\[
\langle B_s | P_i | B_s \rangle = \frac{1}{4} f_B^2 M_{B_s}^2 \left( \frac{M_{B_s}^2}{m_b^2} - 1 \right)^2 \beta_i. \tag{30}
\]

It is hard to obtain precise prediction for lifetime difference with so many operators contributing. Nevertheless contribution from \( \delta_{1/m} \) and \( \delta_{1/m^2} \) can be evaluated. In our numerical calculations we assume the pole mass of b-quark to be \( m_b = 4.8 \pm 0.2 \text{GeV} \) and \( f_B = 230 \pm 25 \text{MeV} \). In order to see the effect of \( O(1/m_b^2) \) corrections we fix all perturbative parameters in the middle of their allowed ranges to show dependence of \( \Delta \Gamma_{B_s} \) on non perturbative parameters \( B_t, \alpha_1, \beta_1 \) defined in Eq. [22, 25, 28, 30].

\[
\Delta \Gamma_{B_s} = 0.0005 B + 0.1732 B_s + 0.0024 B_1
-0.0372 B_2 - 0.0024 B_3 - 0.0436 B_4
+2 \times 10^{-5} \alpha_1 + 4 \times 10^{-5} \alpha_2 + 4 \times 10^{-5} \alpha_3
+0.0005 \alpha_4 - 0.0007 \alpha_5
+0.0002 \beta_1 - 0.0002 \beta_2 + 6 \times 10^{-5} \beta_3
-6 \times 10^{-5} \beta_4 - 1 \times 10^{-5} \beta_5
-1 \times 10^{-5} \beta_6 + 1 \times 10^{-5} \beta_7 + 1 \times 10^{-5} \beta_8 \] \tag{31}

It is obvious that \( O(1/m_b^2) \) corrections provide minor effect on calculation of \( B_s - \bar{B}_s \) lifetime difference. Contribution from interaction with background gluon field is essentially negligible.

To obtain full SM prediction of \( \Delta \Gamma_{B_s} \) we vary values of parameters of matrix elements. We generate 100000-point probability distribution of the lifetime difference obtained by randomly varying our parameters within \( \pm 30\% \) range around their factorization value or within \( \pm 1\sigma \) for parameters known from experimental data or lattice QCD calculations. The resulting distribution is presented on the Fig. 2. There is no theoretically consistent way to treat this diagram that it is not expected for theoretical predictions to have Gaussian distribution. Nevertheless we can give a numerical prediction estimating position of peak as the most probable value and the peak width at half of height as theoretical uncertainty.

\[
\frac{\Delta \Gamma_{B_s}}{\Gamma_{B_s}} = 0.104 \pm 0.049 \tag{32}
\]

where in latter result we added theoretical error for our calculation of \( \Delta \Gamma_{B_s} \) and experimental error from determination of \( \Gamma_{B_s} \) in quadrature. Additional improvement in lattice or QCD sum rules determination of “bag” parameters would make this prediction even more solid.

### III. NEW PHYSICS CONTRIBUTIONS TO LIFETIME DIFFERENCE

In the previous section it was shown that \( O(1/m_b^2) \) corrections to the lifetime difference of \( B_s \) and \( \bar{B}_s \) mesons are small. Since we have reliable prediction of \( \Delta \Gamma_{B_s} \) it might be interesting to consider possible effects of physics beyond the Standard Model on the lifetime difference in \( B_s \) system.

As was pointed out long time ago [7, 8], CP-violating contributions to \( M_{12} \) must reduce the lifetime difference in \( B_s \)-system, as

\[
\Delta \Gamma_s = \Delta \Gamma_s^{SM} \cos^2 2\theta_s, \tag{33}
\]
where $\theta_5$ is a CP-violating phase of $M_{12}$, which is
thought to be dominated by some $\Delta B = 2$ New
Physics. On other hand, CP conserving $\Delta B = 1$
NP amplitude can interfere with SM contribution
constructively or destructively, depending on the NP
model.

There was no spectacular NP phases observed in $B_s$
mixing, thus it is important to estimate the CP-
conserving contribution to $\Delta \Gamma_s$. We shall consider it
using the generic set of effective operators, and then
apply our results to popular extensions of the SM.

Using the completeness relation the NP contribu-
tion to the $B_s^0 \to \pi^0 \pi^0$ lifetime difference becomes
\begin{equation}
    y = \frac{2}{M_{B_s} \Gamma_{B_s}} \langle \bar{B}_s | \text{Im} T | B_s \rangle ,
\end{equation}
\begin{equation}
    T = i \int d^4 x T (\mathcal{H}_{SM}^{\Delta b = -1} (x) \mathcal{H}_{NP}^{\Delta b = -1} (0)) .
\end{equation}

We represent the generic NP $\Delta b = 1$ hamiltonian as
\begin{equation}
    \mathcal{H}_{NP}^{\Delta b = -1} = \sum_{i,q} D_{qq'} [ \mathcal{C}_1 (\mu) Q_1 + \mathcal{C}_2 (\mu) Q_2 ] \tag{35}
\end{equation}
\begin{equation}
    Q_1 = \bar{b}_i \Gamma_{q_1} q' \Gamma_{2} s_1 , \quad Q_2 = \bar{b}_i \Gamma_{q_1} q' \Gamma_2 \Gamma_2 s_2 ,
\end{equation}
where $\Gamma_{q_1,2}$ are arbitrary combinations of Dirac
matrices and $\bar{C}_{1,2} (\mu)$ are Wilson coefficients evaluated
at energy scale $\mu$. This gives us the following contribu-
tion to lifetime difference:
\begin{equation}
    \Delta \Gamma_{NP} = \frac{4 G_F \sqrt{\frac{\Sigma}{M_{B_s}}} \sum_{qq'} D_{qq'} V_{qs}^* V_{q's}}{M_{B_s}} \sum_{qq'} D_{qq'} V_{qs}^* V_{q's} \tag{36}
\end{equation}
\begin{equation}
    \times (K_1 \delta_{i g} \delta_{k \gamma} + K_2 \delta_{i g} \delta_{k \gamma} ) \sum_{j=1}^5 I_j (x, x') \langle \mathcal{T} | O_{j}^{\beta} \mathcal{T} | B_s \rangle \tag{37}
\end{equation}

where $i, \beta, g, \gamma$ stand for color indices, operators $O_j^{\beta}$
are the following:
\begin{equation}
    O_1^{\beta k \gamma} = (\bar{b}_i \Gamma^\nu \gamma^\rho \Gamma_2 s_\gamma) (\bar{b}_k \Gamma_\nu \Gamma_\rho \Gamma_2 s_\gamma) \tag{38}
\end{equation}
\begin{equation}
    O_2^{\beta k \gamma} = (\bar{b}_i \Gamma^\nu \gamma^\rho \Gamma_2 s_\gamma) (\bar{b}_k \Gamma_\nu \Gamma_\rho \Gamma_2 s_\gamma) \tag{39}
\end{equation}
\begin{equation}
    O_3^{\beta k \gamma} = (\bar{b}_i \Gamma^\nu \Gamma_2 s_\gamma) (\bar{b}_k \Gamma_\nu \Gamma_\rho \Gamma_2 s_\gamma) \tag{40}
\end{equation}
\begin{equation}
    O_4^{\beta k \gamma} = (\bar{b}_i \Gamma^\nu \Gamma_2 s_\gamma) (\bar{b}_k \Gamma_\nu \Gamma_\rho \Gamma_2 s_\gamma) \tag{41}
\end{equation}

with $p$ being a $b$-quark momentum, and $K_1$ are the
following combinations of Wilson coefficients
\begin{equation}
    K_1 = (C_2 \bar{C}_2 N_e + (C_2 \bar{C}_1 + C_1 \bar{C}_2))
\end{equation}
\begin{equation}
    K_2 = C_1 \bar{C}_1 \tag{42}
\end{equation}

with the number of colors $N_e = 3$.

Defining $z = m_q^2 / m_B^2$ and $z' = m_{q'}^2 / m_B^2$
coefficients $I_j (z, z')$ can be written as follows:
\begin{equation}
    I_1 (z, z') = -\frac{\Phi m_e}{48 \pi} [1 + (z + z') - (z + z')^2]
\end{equation}
\begin{equation}
    I_2 (z, z') = -\frac{\Phi}{24 m_e \pi} [1 + (z + z') - 2(z - z')^2]
\end{equation}
\begin{equation}
    I_3 (z, z') = \frac{\Phi}{8 \pi} [1 + (z + z') - (z - z')^2]
\end{equation}
\begin{equation}
    I_4 (z, z') = \Phi \frac{1}{8 \pi} [1 + (z + z') - (z - z')^2]
\end{equation}
\begin{equation}
    I_5 (z, z') = \Phi \frac{m_e}{4 \pi} [1 + (z + z') - (z - z')^2]
\end{equation}

A. Multi-Higgs model

One of possible realizations of New Physics is a
charged Higgs doublet model proposed in [10]. This
model provides new flavor changing interaction medi-
ated by charged Higgs bosons. It leads to the following
four-fermion interaction:
\begin{equation}
    \mathcal{H}_{CHH}^{\Delta b = 1} = -\frac{\sqrt{G_F}}{M_H^2} \bar{b}_1 q'_1 \bar{q}_1 \Gamma_2 s_j \tag{40}
\end{equation}
\begin{equation}
    \bar{b}_i \Gamma_1 q'_1 \bar{q}' \Gamma_2 s_j ,
\end{equation}

This leads to three operators with various coefficients,
matrix elements of which contribute to the $y_{CHH}$:
\begin{equation}
    y_{CHH} = \frac{8 G_F^2 m_e^2}{M_H^4} \langle V_{us}(V_{us})^\dagger \rangle^2 \times
\end{equation}
\begin{equation}
    [ (Q_1) (4 K_2 x_s I_1 cot \beta L - m_b V_{ts} \tan \beta R) +
    \times (Q_2) (2 K_1 x_s I_1 cot \beta + (cot^2 \beta m_b^2 x_s I_2 - m_b x_s I_1)(K_2 - K_1)) +
    \times (Q_3) (K_1 + K_2) (x^2 tan^2 \beta I_5 - m_b x_s I_3) \tag{42}
\end{equation}
\begin{equation}
    I_1 \text{ and } I_2 \text{ were defined above. } x = m_e / m_b \text{ and } x_s = m_s / m_b . \text{ (Q)i are as follows:}
\end{equation}
\begin{equation}
    Q_1 = (\bar{b}_1)_{L} (s_l)_{R} (\bar{b}_k)_{R} (s_k)_{L} \tag{43}
\end{equation}
\begin{equation}
\begin{aligned}
    \langle Q_1 \rangle &= -\frac{1}{4} f_B^2 M_B^2 \frac{M_B^2}{(m_b + m_s)^2} \left( \frac{2}{N_e} \right)
\end{aligned}
\end{equation}
\begin{equation}
    Q_2 = (\bar{b}_1)_{R} \gamma^\nu (s_l)_{R} (\bar{b}_k)_{L} \gamma^\nu (s_k)_{L} \tag{44}
\end{equation}
\begin{equation}
\begin{aligned}
    \langle Q_2 \rangle &= -\frac{1}{4} f_B^2 M_B^2 \left( \frac{1}{N_e} \right)
\end{aligned}
\end{equation}
\begin{equation}
    Q_3 = (\bar{b}_1)_{L} \gamma^\nu (s_l)_{L} (\bar{b}_k)_{L} \gamma^\nu (s_k)_{L} \tag{45}
\end{equation}
\begin{equation}
\begin{aligned}
    \langle Q_3 \rangle &= \frac{1}{4} f_B^2 M_B^2 \left( \frac{1}{N_e} \right)
\end{aligned}
\end{equation}

For values of $M_H = 85 GeV$ and cot $\beta = 0.05$ it
gives $y_{CHH} \approx 0.0034$. This is about 10% of Standard
Model value. Dependence of $y_{CHH}$ on mass of Higgs
boson is given on Fig.3.
B. Left-Right Models

One of the possible extensions of the SM is a Left-Right Symmetric Model (LRSM) which assumes the extended $SU(2)_L \times SU(2)_R$ symmetry of the theory. In this model additional flavor changing interaction is provided by mediating right-handed $W^{(R)}$-bosons. In this case flavor mixing is described by right-handed CKM matrix $V^{(R)}_{ik}$ and

\[ \bar{\Gamma}_{1,2} = \gamma^\mu P_R \tag{46} \]

\[ D_{qq'} = V^{(R)}_{cb} V^{(R)}_{cs} \frac{G^F_{W^{(R)}}}{\sqrt{2}} \tag{47} \]

where $\frac{G^F_{W^{(R)}}}{\sqrt{2}} = g^2_R / 8 M^2_{W^{(R)}}$ and for future calculations we take $g_L = k g_R$.

Such model gives us the following prediction for value of $y$:

\[ y_{LR} = -V^{(R)}_{cb} V^{(R)}_{cs} G^F_{W^{(R)}} \frac{G^F_{W^{(R)}}}{\sqrt{2}} \]  
\[ \times \left( C_1 \langle Q_2 \rangle + C_2 \langle \bar{Q}_2 \rangle \right) \tag{48} \]

One of possible realizations of such scenario which gives the biggest numerical value of $y_{LR}$ is a "Non-manifest LR" ($V_{ij}^{(R)} \approx 1$) with $M_{W^{(R)}} = 1$ TeV value of $y_{LR} \approx -0.015$ was obtained. In case of "manifest LR" ($V_{ij}^{(R)} = V_{ij}$) contribution from this model is less. Dependence of $y_{LR}$ on mass of $W^{(R)}$ boson is given on Fig. 4.

IV. CONCLUSIONS

We computed the subleading $O(1/m_b^2)$ corrections to the lifetime difference of $B_s$ mesons. The corrections depend on 13 non-perturbative parameters $\alpha_i$ and $\beta_i$. We generated probability distribution of lifetime difference by varying parameters $\pm 30\%$ around their "factorization" values or within $1\sigma$ for parameters know from Lattice QCD. The results are presented on Fig.2. Translating this diagram into numerical prediction for $\Delta \Gamma_{B_s}/\Gamma_{B_s}$ we obtained the most precise available today theoretical prediction for lifetime difference:
\[ \Delta \Gamma_{B_s} = 0.072 \pm 0.034 \text{ps}^{-1} \]
\[ \frac{\Delta \Gamma_{B_s}}{\Gamma_{B_s}} = 0.104 \pm 0.049 \]  \hspace{1cm} (49)

The effect of \(1/m_b^2\) corrections to the lifetime difference is small.

The generic \(\Delta B = 1\) New Physics contribution to the lifetime difference in \(B_s\) system is considered. We considered four-fermion effective Hamiltonian of the generic Standard Model extension and computed its contribution to the \(\Delta \Gamma_{B_s}\). It can reduce or increase the SM contribution depending on particular choice of the model. Two models of physics beyond the Standard Model considered. Contribution of charged Higgses to the lifetime difference is negligible. LRSM contribution is significant and parameters of this model can be constrained based on \(\Delta \Gamma_{B_s}\) measurements.

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