General stationary charged black holes as charged particle accelerators

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Abstract

We study the possibility of getting infinite energy in the center of mass frame of colliding charged particles in a general stationary charged black hole. For black holes with two-fold degenerate horizon, it is found that arbitrary high center-of-mass energy can be attained, provided that one of the particle has critical angular momentum or critical charge, and the remained parameters of particles and black holes satisfy certain restriction. For black holes with multiple-fold degenerate event horizons, the restriction is released. For non-degenerate black holes, the ultra-high center-of-mass is possible to be reached by invoking the multiple scattering mechanism. We obtain a condition for the existence of innermost stable circular orbit with critical angular momentum or charge on any-fold degenerate horizons, which is essential to get ultra-high center-of-mass energy without fine-tuning problem. We also discuss the proper time spending by the particle to reach the horizon and the duality between frame dragging effect and electromagnetic interaction. Some of these general results are applied to braneworld small black holes.

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I. INTRODUCTION

Recently, Bañados, Silk and West (BSW) [1] proposed a mechanism to obtain unlimited center-of-mass (CM) energy of two particles colliding at an extreme Kerr black hole (BH), which was henceforth asserted as a natural Planck-scale particle accelerator and the possible origins for the very highly energetic astrophysical phenomena, such as the gamma ray bursts and the active galactic nuclei. However, authors of [2] and [3] pointed out that the collision in fact takes an infinite proper time. Moreover, even the ultra-energetic collisions cannot occur in nature, because the astrophysically limited maximal spin prohibits the formation of real extreme Kerr BHs, meanwhile the back-reaction effects may reduce the allowed maximized CM energy below the Planck scale upon absorption of a first pair of colliding particles, and the gravitational radiation of the particle with fine-tuning critical angular momentum should be peaked at frequencies corresponding to marginally bound quasi-circular orbits.

To achieve arbitrary high CM energy under the limitation of maximal spin, the multiple scattering mechanism was taken into account in the nonextremal Kerr BH [4]. Another more direct method is to consider different extreme rotating BHs, such as Kerr-Newman BHs [5]. Actually, by making use of a special metric which is convenient to discuss the near-horizon geometry of general axially symmetric rotating BHs, Zaslavskii showed the unbound CM energy of colliding particles at the extreme horizon or nonextremal horizon by considering multiple scattering [6]. Bearing in mind the universal character, it was pointed out that the potential acceleration to large energies should be taken seriously both as manifestation of general properties of BHs and the effect relevant in astrophysics of high energies. Other concrete models were also studied and some new results which have not been included in the general frame appeared, such as the Kerr-(A)dS BH where three-fold degenerate horizons were considered [7]. In addition, it was pointed out that the existence of innermost stable circular orbit (ISCO) in Kerr BHs would avoid the artificial fine-tuning in an astrophysical context [8]. See more backgrounds like Sen BHs [9], Kaluza-Klein BHs [10], Kerr-Taub-NUT BHs [11], and naked singularities [12]. Noticing these alternative options for generating extremal black holes, the BSW mechanism has been further studied by calculating the escaping flux of massless particles for maximally rotating black holes, and it was suggested that the received spectrum should typically contain signatures of highly energetic products [13], see also the sequent numerical estimation [14].
In the aforementioned mechanisms for ultra-high CM energy, the frame dragging effect of rotating BHs is necessary. However, Zaslavskii [15] showed that a non-rotating but charged Reissner-Nordström (RN) BH can also serve as an accelerator with arbitrarily high CM energy of charged particles collided at the extreme horizon or nonextremal horizon considering multiple scattering. It was demonstrated that the upper bound of the electric charge of BHs after Schwinger emission is large enough to allow the ultra-high CM energy of charged colliding particles. The only restriction is that the BH should not be too light ($> 10^{20} g$). Moreover, noticing the correspondence between frame dragging effect and the electromagnetic interaction as well as the higher symmetry of RN BHs than Kerr BHs, an upper bound was suggested to exist for the total energy of colliding particles in the observable domain in the BSW process due to the gravity of the particles [16].

In this paper, we will investigate the BSW mechanism with the combined effect of frame dragging and electromagnetic interaction. Instead of restricting on a concrete charged BH, we will study a general background with charged test particles. Instead of the special metric adopted in [6], we will use a general stationary metric which can be reduced to Kerr metric using Boyer-Lindquist coordinates directly and hence can be related to observations (such as escape fraction) conveniently. We will not only discuss the universal existence of ultra-energetic collisions at usual two-fold degenerate horizons or nonextremal horizons considering multiple scattering, the associated proper time and the fine-tuning critical angular momentum or charge, but also discuss multiple-fold degenerate horizons and ISCO in general. Some new restrictions on parameters of particles and BHs, which are necessary for ultra-energetic collisions but have not been noticed before, will be revealed as well. Moreover, we will show the correspondence between frame dragging and electromagnetic interaction in BSW mechanism in more detail.

On the other hand, based on the well-known braneworld scenario with large or compact extra dimensions, the fundamental Planck scale is lowered to the order of magnitude around the TeV scale. A particularly exciting proposal is the possibility of creating mini BHs in super colliders, such as LHC, which can achieve the energy scale about 14 TeV and could be taken as the “factory” of small BHs [20]. This provides a potential terrestrial check of phenomenon around astrophysical BHs, including the BSW mechanism. As an application and in fact a strong motivation of our general frame, we will discuss the collision of charged test particles in the Arkani-Hamed, Dimopoulos and Dvali (ADD) braneworld Kerr-Newman (KNM) BH,
which was suggested recently in [17]. As it was pointed out, due to the strengthening of the gravitational field compared to the electromagnetic field in TeV gravity scenarios, the initial evaporation is not dominated by fast Schwinger discharge. So, the braneworld KNM BH is a general BH which could be formed after proton-proton collisions in LHC, providing the possible further signatures of BH events such as charge asymmetries. We will show that there is no degenerate horizon, but the ultra-energetic collisions can be produced by considering multiple scattering. To be complementary, we will discuss the BSW process near the degenerate horizon of the tidal charged BH based on Randall-Sundrum (RS) braneworld scenarios, which is similar to the 4-dimensional KNM BH, but the tidal charge could be large ($< 10^4 m^2$ in solar system tests) [18].

The rest of paper is arranged as follows. In section II, we will give the general gravitational background, the geodesic motion of charged particles under electromagnetic field, and the CM energy of colliding particles. In section III, we will analyze the effective potential to determine whether the particle can reach the horizon, and the CM energy will be studied for the nonextremal horizon considering the multiple scattering, the two-fold degenerate horizon, and the multiple-fold degenerate horizon, respectively. The proper time, possible ISCO, and the duality between frame dragging and coulomb interaction in BSW mechanism will also be investigated. In section IV, we will apply some obtained results in the general frame to the braneworld BH background. The final section is devoted to conclusion and discussion.

II. GEODESIC MOTION OF TWO CHARGED PARTICLES

Since the BSW mechanism relates to the geodesic motion of test particles in the gravitational background, we would like to analyze equations of their motion in detail. For simplification, we assume that the motion of particles occurs in the equatorial plane ($\theta = \pi/2$) of BHs (for the non-equatorial motion in the Kerr BH see [19]). For a general BH, the metric can be written as

$$ds^2 = \left[-g_1(r) + g_2(r)w^2(r)\right] dt^2 - 2g_2(r)w(r)dt d\phi + \frac{dr^2}{g_3(r)} + g_2(r)d\phi^2,$$

where $g_1(r), g_2(r), g_3(r)$, and $w(r)$ are arbitrary functions of the radial coordinate $r$. Compared with the metric adopted in [6] where $g_3(r) = 1$ and $g_1(r) = N^2$ (with the horizon
located at $N = 0$), the metric (1) is more convenient to compare with Kerr and RN cases
(with the horizon located at $g_3(r) = 0$) and can be directly imposed with asymptotically flat
which is important to compare with observations. One may worry about the complicated
near-horizon geometry with metric (1) which was simplified in the metric adopted in [6], but
we will show later that it still can be tackled when noticing the near-horizon behavior of $g_1(r)$
and $g_3(r)$. The electromagnetic potential of BHs are denoted as $A_t = B(r)$, $A_\phi = C(r)$,
which are treated as arbitrary functions of $r$. Other components of the electromagnetic
potential vanish due to the symmetry of background. Because the functions of metric are
independent to the coordinates $t$ and $\phi$, there are two Killing vectors for the BH, expressed
as $\xi^\mu = (1, 0, 0, 0)$ and $\eta^\mu = (0, 0, 0, 1)$. For a test particle with rest mass $m_0$ and charge $q$
per unit rest mass, it should has two conserved quantities

$$-\xi^\mu (u_\mu - q A_\mu) = E \quad \text{and} \quad \eta^\mu (u_\mu - q A_\mu) = L,$$

where $E$ and $L$ are the conserved parameters of the energy and angular momentum of the
test particle per unit rest mass, respectively. By combining these equations with the timelike
restriction of test particles $u \cdot u = -1$, the geodesic equations can easily be solved as

$$u^t = \frac{\Xi}{g_1},$$

$$u^r = -\sqrt{\frac{g_3}{g_1 g_2} \left[ g_2 \Xi^2 - g_1 (g_2 + \Lambda^2) \right]},$$

$$u^\phi = \frac{\Lambda g_1 + g_2 w_\Xi}{g_1 g_2},$$

where

$$\Lambda_i(r) = L_i + q_i C(r), \quad \Theta_i(r) = E_i - q_i B(r), \quad \Xi_i(r) = \Theta_i(r) - \Lambda_i(r) w(r),$$

and index $i = 1, 2$ denotes two different particles. Note that the minus sign in Eq. (3) means
ingoing particles that we are concerning, for the discussion on outgoing particles please see
[21].

From Eq. (3), we find that there is a particular interesting case in which $g_1(r)$ and $g_3(r)$
have to tend to zero with the same speed, otherwise the particle will have a vanishing or
divergent radial velocity on the horizon. This case is also required to be consistent with the
static vacuum background where $g_1(r) = g_3(r)$. Moreover, considering that the three-fold
degenerate horizons might take role in BSW mechanism [7], we will try to study what will
happen near general \( n \)-fold degenerate horizons (For convenience later, we will call \( n = 1 \) as non-degenerate horizons, \( n \geq 2 \) as any-fold degenerate horizons, and \( n \geq 3 \) as multiple-fold degenerate horizons.) For this aim, we take the replacement
\[
g_1(r) \rightarrow (r-r_H)^n g_6(r), \quad g_3(r) \rightarrow (r-r_H)^n g_5(r),
\]
where \( r_H \) denotes the location of horizon. Here we would like to stress that this replacement is a simple but key step to discuss the behavior near any-fold degenerate horizons. Furthermore, we note that, for the region outside the event horizon (while not the Cauchy horizon, see [22]), \( g_5(r_H) \) and \( g_2(r_H) \) are positive definite to preserve \( r \) and \( \phi \) as spatial coordinates. \( g_6(r_H) \) should also be positive definite, or there is no observer at fixed \( r \) and \( \theta \) will be permitted.

Now, let us discuss the CM energy of two particles with the same rest mass \( m_0 \) colliding in this background by calculating
\[
E_{c.m.} = \sqrt{2m_0 \sqrt{1 - u(1) \cdot u(2)}}.
\]
The result is
\[
\frac{E_{c.m.}^2}{2m_0^2} = 1 + \frac{1}{g_1 g_2} \left[ \frac{g_2 \Xi_1 \Xi_2 - g_1 \Lambda_1 \Lambda_2 - \sqrt{g_2 \Xi_1^2 - g_1 (\Lambda_1^2 + g_2)} \sqrt{g_2 \Xi_2^2 - g_1 (\Lambda_2^2 + g_2)}}{g_2 \Xi_1^2 - (r-r_H)^n g_6 (\Lambda_1^2 + g_2) \sqrt{g_2 \Xi_2^2 - (r-r_H)^n g_6 (\Lambda_2^2 + g_2)}} \right].
\]
One can find that the numerator and denominator of the fraction in the above equation are both vanishing on the horizon where \( g_1(r) = 0 \), if \( \Xi_i \geq 0 \) (We do not consider the case with \( \Xi_i < 0 \), because in the next section, we will show that the particles with \( \Xi_i < 0 \) can not fall into the horizon.). Thus, the fraction is underdetermined on the horizon. To analyze this underdetermination, one needs to consider the near-horizon behavior by the replacement (5).

Hence, Eq. (7) becomes to
\[
\frac{E_{c.m.}^2}{2m_0^2} = 1 + \frac{1}{(r-r_H)^n g_2 g_6} \left[ g_2 \Xi_1 \Xi_2 - g_6 \Lambda_1 \Lambda_2 (r-r_H)^n \right.
\]
\[
\left. - \sqrt{g_2 \Xi_1^2 - (r-r_H)^n g_6 (\Lambda_1^2 + g_2)} \sqrt{g_2 \Xi_2^2 - (r-r_H)^n g_6 (\Lambda_2^2 + g_2)} \right].
\]

III. BSW MECHANISM FOR GENERAL CASES

A. Critical angular momentum and critical charge

By making use of the l’Hospital’s rule to do \( n \)th-order differential on the numerator and denominator at the same time, the CM energy on the horizon can be obtained from Eq. (8).
In the denominator, the terms remaining with \( r - r_H \) after the \( n \)th-order differential will vanish while \( r \to r_H \), and the only term left is \( \sim g_2(r)g_6(r) \), no matter what the value of \( n \) is. The situation for the numerator is similar, as we will show below. One can rewrite the numerator of Eq. (8) as

\[
g_2\Xi_1\Xi_2 - g_6\Lambda_1\Lambda_2(r - r_H)^n - \frac{\sqrt{\left(g_2\Xi_1\Xi_2\right)^2 - g_2g_6[\Xi_1^2(\Lambda_2^2 + g_2) + \Xi_2^2(\Lambda_1^2 + g_2)](r - r_H)^n + g_6^2(\Lambda_1^2 + g_2)(\Lambda_2^2 + g_2)(r - r_H)^{2n}}.
\]

Respecting that \( (r - r_H)^n \) is a small quantity near the horizon, one can expand the last term of above equation as following:

\[
g_2\Xi_1\Xi_2 - \frac{g_2g_6[\Xi_1^2(\Lambda_2^2 + g_2) + \Xi_2^2(\Lambda_1^2 + g_2)](r - r_H)^n}{2g_2\Xi_1\Xi_2} + \mathcal{O}(r - r_H)^{2n}.
\]

The numerator hence can be simplified to

\[
\frac{(r - r_H)^n g_6 \left[ (\Lambda_2\Xi_1 - \Lambda_1\Xi_2)^2 + g_2(\Xi_1^2 + \Xi_2^2) \right]}{2\Xi_1\Xi_2}.
\]

Considering Eq. (9) as the form of a function multiplying \( (r - r_H)^n \) and using the Leibniz formula of high-order derivatives, one can calculate the \( n \)-th order differential of Eq. (9) with respect to \( r \) near horizons:

\[
n! \frac{g_6 \left[ (\Lambda_2\Xi_1 - \Lambda_1\Xi_2)^2 + g_2(\Xi_1^2 + \Xi_2^2) \right]}{2\Xi_1\Xi_2} \bigg|_{r \to r_H}.
\]

Finally, for arbitrary integer \( n \), the CM energy can be expressed as a very simple form

\[
\frac{E_{c.m.}^2}{2m_0^2} = 1 + \frac{(\Lambda_2\Xi_1 - \Lambda_1\Xi_2)^2 + g_2(\Xi_1^2 + \Xi_2^2)}{2g_2\Xi_1\Xi_2} \bigg|_{r \to r_H}.
\]

It is interesting to see the independence with \( n \).

The CM energy is infinite if one test particle has the critical angular momentum or critical charge, which can be solved by setting \( \Xi_i \) to zero, given by:

\[
L_c = \frac{E - qB - qCw}{w} \bigg|_{r \to r_H},
\]

and

\[
q_c = \frac{E - Lw}{B + Cw} \bigg|_{r \to r_H}.
\]

These two equations indicate that the critical angular momentum and critical charge of a test particle are entangled. In other words, for a test particle with large angular momentum, the infinite CM energy can be attained for the particle with small charge, and vice versa.
The critical angular momentum and charge can also be obtained by imposing $-\chi \cdot u = \Xi$ as zero at horizon. Here $\chi = \xi + w(r_H)\eta$ is the Killing vector generating the horizon. In [19], it was conjectured that the CM energy is unbound if and only if the ratio $\frac{\chi_1 \cdot p_1}{\chi_2 \cdot p_2}$ is zero or infinite at Killing horizon, where $p$ is the momentum. Now we find that the conjecture still holds if $p$ is replaced with $m_0 u$ in the presence of the electromagnetic interaction.

B. Effective potential

For the BSW mechanism, an important problem is whether a test particle with the critical angular momentum or critical charge can fall into the horizon. We would like to investigate this problem from the viewpoint of effective potential $V_{\text{eff}} = -\dot{r}^2/2$, where the dot denotes the derivative with respect to the proper time. After doing the replacement (5) and setting $m_0 = 1$ for simplicity, the effective potential is

$$V_{\text{eff}} = -\frac{g_5}{2g_2g_6} [g_2 \Xi^2 - (r - r_H)^n (\Lambda^2 + g_2)g_6].$$

(13)

For a non-degenerated horizon, the effective potential for the particle with critical angular momentum (11) can be expanded near the horizon

$$V_{\text{eff}}(L_c)|_{n=1} = \frac{g_5}{2} \left( 1 + \frac{\Theta^2}{g_2 w^2} \right) (r - r_H) + \mathcal{O} \left( (r - r_H)^2 \right).$$

(14)

Since $g_5(r_H)$ and $g_2(r_H)$ are both positive for event horizons, one has $V_{\text{eff}} > 0$ which violates the positiveness of $\dot{r}^2$. So this particle can not fall into the event horizon. For a usual two-fold degenerate horizon, the expansion of effective potential is

$$V_{\text{eff}}(L_c)|_{n=2} = \frac{g_5}{2g_2g_6w^2} \left[ g_6(\Theta^2 + g_2 w^2) - g_2 (q w \Phi + \Theta w')^2 \right] (r - r_H)^2 + \mathcal{O} \left( (r - r_H)^3 \right),$$

(15)

where $\Phi = B' + wC'$ and the prime denotes the derivative with respect to $r$. From this equation, we know that the particle with critical angular momentum can exist in the region near a two-fold degenerate horizon, if the coefficient of term $\sim (r - r_H)^2$ in Eq. (15) is negative. For the case of a multiple-fold degenerate horizon we can prove by following steps that $V_{\text{eff}}|_{n \geq 3}$ has the same form. First of all, we separate $V_{\text{eff}}$ into the terms with and without $(r - r_H)^n$, which reads

$$V_{\text{eff}} = \frac{(r - r_H)^n (\Lambda^2 + g_2) g_5}{2g_2} - \frac{g_5 \Xi}{2g_6}.$$  

(16)
Then, we substitute the value of $L$ into the second term and expand it near the horizon as
\[ -\frac{g_5\Xi(L_c)}{2g_6} = -\frac{g_5(qw\Phi + \Theta w')^2(r - r_H)^2}{2g_6w^2} + \mathcal{O}(r - r_H)^3. \] (17)

For $n \geq 3$ case, one can find that, comparing with Eq. (17), the first term of Eq. (16) is the higher order small quantity while $r \rightarrow r_H$. So when $n \geq 3$, the expansion of effective potential has the same form:
\[ V_{\text{eff}}(L_c)|_{n \geq 3} = -\frac{g_5(qw\Phi + \Theta w')^2(r - r_H)^2}{2g_6w^2} + \mathcal{O}(r - r_H)^3. \] (18)

Since the coefficient of term $\sim (r - r_H)^2$ is always negative for event horizons, we can conclude that the particle with $L_c$ can exist near the region of a multiple-fold degenerate horizon without any restriction that is needed for the case of two-fold degenerate horizons.

Now we are going to discuss whether a particle can touch the horizon from infinity. It is usually judged by comparing the $L_c$ with the minimum and maximum angular momentum ($L_{\text{min}}$, $L_{\text{max}}$) at circular orbits $r_{\text{cir}}$ solved from $V_{\text{eff}} = 0$ and $\partial_r V_{\text{eff}} = 0$. Obviously, we can not follow this approach directly since $(r_{\text{cir}}, L_{\text{min}}, L_{\text{max}})$ can not be solved in a general background. However, it is interesting to see that we still can make the judgement in general.

Setting $V_{\text{eff}}(r_{\text{cir}}) = 0$, we get the relationship between the radial coordinate of these circular orbits and angular momentum:
\[ L(r_{\text{cir}}) = \frac{1}{g_2w^2 - (r - r_H)^n g_6} \left[ (r - r_H)^n qCg_6 + g_2w(\Theta - qCw) \right. \]
\[ \left. \pm \sqrt{(r - r_H)^n g_6g_2 \left[ \Theta^2 - (r - r_H)^n g_6 + g_2w^2 \right]} \right]_{r=r_{\text{cir}}}. \]

From this equation, it is easy to notice that $L(r_H) = L_c$, i.e., for a test particle with critical angular momentum, it has a possible circular orbit on the horizon. In order to ensure that this is a true circular orbit, $\partial_r V_{\text{eff}} = 0$ must also be satisfied. For a non-degenerated horizon, $\partial_r V_{\text{eff}}$ on the horizon is
\[ \partial_r V_{\text{eff}}(L_c)|_{n=1} = \frac{g_5}{2}(1 + \frac{\Theta^2}{g_2w^2})|_{r=r_H}. \]

It is nonvanishing, which means that there is no circular orbit for a test particle with critical angular momentum on the horizon. However, it is obvious that $\partial_r V_{\text{eff}} = 0$ on any-fold degenerate horizons, since it is at least the second order function of $r - r_H$ as just being seen in Eqs. (15) and (18). This indicates that $r_H$ is just the innermost circular orbit of the particle with $L_c$ for any-fold degenerate horizons. Furthermore, because the existence of
particles near a degenerate horizon requires that the coefficients of term \( (r - r_H)^2 \) in Eqs. (15) and (18) are negative, which means

\[
\partial^2_r V_{\text{eff}}(L_c)\big|_{n \geq 2, r = r_H} < 0,
\]

the potential just has the maximum on the horizon. Hence the particle with \( L_c \) can touch any-fold degenerate horizons from infinity (Rigorously, we also need to assume that the potential is so ordinary that the maximum is the global maximum, just as in the cases of Kerr and RN BHs where the effective potential has only one maximum. Note that the maximum at the horizon is obvious not the global maximum for the BH imbedded in AdS background where the potential is divergent at infinite [7]). For example, Eq. (19) is reduced to

\[
\partial^2_r V_{\text{eff}}(L_c)\big|_{n = 2, r = r_H} = 1 - 3E^2 < 0
\]

for Kerr BHs, which can be satisfied for \( E > \frac{1}{\sqrt{3}} \).

For charged and non-rotating BHs, \( L_c \) and \( 1/w \) are infinite. So the result gotten above is invalid and the above discussion process should be repeated by replacing the critical angular momentum with the critical charge. We briefly present the results as follows. Substituting Eq. (12) into Eq. (13), the effective potentials for different \( n \) are

\[
V_{\text{eff}}(q_c)\big|_{n = 1} = \frac{g_5}{2} \left( 1 + \frac{1}{g_2 \Omega^2} \right) (r - r_H) + \mathcal{O} (r - r_H)^2,
\]

\[
V_{\text{eff}}(q_c)\big|_{n = 2} = \frac{g_5}{2g_2g_6} \left[ g_6 (\Psi^2 + g_2 \Omega^2) - g_2 (\Gamma \Phi + \Psi w')^2 \right] (r - r_H)^2 + \mathcal{O} (r - r_H)^3,
\]

\[
V_{\text{eff}}(q_c)\big|_{n \geq 3} = -\frac{g_5}{2g_6 \Omega^2} (\Gamma \Phi + \Psi w')^2 (r - r_H)^2 + \mathcal{O} (r - r_H)^3,
\]

where

\[
\Psi_i = L_i B + E_i C, \quad \Omega = B + C w, \quad \Gamma_i = E_i - L_i w.
\]

One can conclude that the particle with critical charge can exist near two-fold degenerate horizons, and can exist near multiple-fold degenerate horizons. From Eqs. (20), (21) and (22), one can see that \( r_H \) is just the innermost circular orbit of the particle with \( q_c \) for any-fold degenerate horizons but not for non-degenerate horizons. If the following condition

\[
\partial^2_r V_{\text{eff}}(q_c)\big|_{n \geq 2, r = r_H} < 0
\]

is satisfied, the particle with \( q_c \) can touch any-fold degenerate horizons from infinity. For RN BHs as an instance, the condition is reduced to

\[
\partial^2_r V_{\text{eff}}(q_c)\big|_{n = 2, r = r_H} = 1 - E^2 + L^2 < 0.
\]
C. Multiple scattering

Even for $n = 1$ case where a test particle with $L_c$ can not fall into the horizon, there is still a possible mechanism to achieve ultra-high CM energy. Consider a particle with angular momentum close to the critical value, i.e. $L = L_c(1 - \delta)$, where $\delta$ is an arbitrary positive number. At the horizon, its effective potential is

$$V_{\text{eff}}|_{r=r_H} = -\frac{\delta^2 g_5 (L_c w)^2}{2g_6}.$$ 

For $g_5(r_H)/g_6(r_H) > 0$, the effective potential will be negative, which means a test particle with angular momentum $L = L_c(1 - \delta)$ can exist in the region close to the event horizon. The value of this angular momentum may be too large such that the test particle can not fall into the horizon far from the horizon directly. However, it is possible that a test particle with small angular momentum is ingoing to the horizon and interacts with other particles on the accretion disc or decays to be more light particles so that it gets larger angular momentum. This is the so-called multiple scattering mechanism proposed in Ref. [4]. The corresponding CM energy can be calculated as

$$\frac{E_{c.m.}^2}{2m_0^2} = 1 - \frac{\Theta_1 \Theta_2}{g_2 w^2} + \frac{\Xi_2 (\Theta_1^2 + g_2 w^2)}{2g_2 w^3 L_c} \frac{1}{\delta} + O(\delta)^1. \tag{23}$$

On the other hand, the charge of the particle can be amplified to $q = q_c(1 - \delta)$ by the multiple scattering mechanism, since the pairs of electron and positron could be created by the collision of the high energy photons and massive atoms. For this particle, its effective potential is

$$V_{\text{eff}}|_{r=r_H} = -\frac{\delta^2 g_5 (E - Lw)^2}{2g_6}.$$ 

Similar to the former case, this test particle can exist in the region close to the horizon, and the CM energy is

$$\frac{E_{c.m.}^2}{2m_0^2} = 1 - \frac{\Psi_1 \Psi_2}{g_2 \Omega^2} + \frac{\Xi_2 (\Psi_1^2 + g_2 \Omega^2)}{2g_2 \Gamma_1 \Omega^2} \frac{1}{\delta} + O(\delta)^1. \tag{24}$$

When $\delta \to 0$, both CM energy (23) and (24) will be arbitrary high.
D. Proper time problem

Now, we would like to have a glance on the proper time required for a test particle to reach the horizon. It can be obtained from

$$\tau = \int_{r_i}^{r_f} \left( \frac{1}{\sqrt{-2V_{\text{eff}}(r)}} \right) dr.$$  \hspace{1cm} (25)

Since the effective potentials are the first order functions of \((r - r_H)\) in Eqs. (13) and (21), the proper time for the particle with critical angular momentum or charge falling into the non-degenerate horizon will be finite. On the contrary, a test particle with critical angular momentum or charge takes infinite proper time to fall into any-fold degenerate horizons, because the effective potentials (15), (18), (21), and (22) are the second order functions of \((r - r_H)\), which means that the proper time is logarithmic divergent.

E. ISCO for the test particle with critical angular momentum or charge

To obtain an arbitrary high CM energy, the angular momentum and charge of a test particle must be fine-tuned. The existence of ISCO has been realized as a possibility to solve this problem [8]. Now we will extend the discussion of the ISCO in Kerr BHs [8] to a general case. We point out that the key point to get arbitrary high CM energy without the fine-tuning problem is to require the ISCO with critical angular momentum or charge exactly located on the horizon. In other words, the CM energy of one particle collided with another one moving along the ISCO is finite, unless the ISCO is located on the horizon and the particle along the ISCO has the critical parameters. For a two-fold degenerate horizon, by setting \(\partial^2 V_{\text{eff}} = 0\) where \(V_{\text{eff}}\) is given by (15), we obtain the condition to require the ISCO with critical angular momentum just on the horizon, which reads

$$g_6(\Theta^2 + g_2w^2) - g_2(qw\Phi + \Theta w')^2|_{r=r_H} = 0.$$  \hspace{1cm} (26)

For \(n \geq 3\) cases, from Eq. (18), we know that the condition for ISCO on the horizon is

$$qw\Phi + \Theta w'|_{r=r_H} = 0,$$

which can be rewritten clearly as

$$E = -\frac{q(wB' + w^2C' - Bw')}{w'}|_{r=r_H}.$$  \hspace{1cm} (26)
The case of a test particle with critical charge has similar results. We can obtain
\[ g_6 \left( \Psi^2 + g_2 \Omega^2 \right) - g_2 \left( \Gamma \Phi + \Psi w' \right)^2 \bigg|_{r=r_H} = 0 \]
for \( n = 2 \), and
\[ \Gamma \Phi + \Psi w' = 0 \]
for \( n \geq 3 \) cases, which can be recast as
\[ E = \left. \frac{L \left( wB' + w^2 C - Bw' \right)}{B' + wC' + Cw'} \right|_{r=r_H}. \] (27)

From Eqs. (26) and (27), it is interesting to note that both frame dragging effect and electromagnetic interaction are necessary for ISCO on the multiple-fold degenerate horizons, provided that the energy is nonvanishing.

F. Duality between frame dragging effect and electromagnetic interaction

In Ref. [16], an upper limit was found to exist for the total energy of colliding shells in the observable domain in the BSW process due to the gravity of the shells. Although this result is obtained in the RN background, since RN BHs are easily to be tackled based on their higher symmetry than Kerr BHs, it has been suspected that an upper limit might also exist for the Kerr background, noticing the similarity of BSW mechanism in Kerr and RN BHs.

Here we would like to clarify the corresponding relationship between the frame dragging effect and the electromagnetic interaction in the BSW mechanism from the viewpoint of critical angular momentum and charge, effective potential and CM energy, which are three essential factors in the BSW mechanism.

In Eq. (12) with \( w(r) = 0 \), by doing the transformation
\[ B(r) \rightarrow w(r) \quad \text{and} \quad q \rightarrow L, \] (28)
Eq. (12) will equate to Eq. (11) with \( q = 0 \). Therefore, from the viewpoint of critical angular momentum and critical charge, there exists an exact duality. Then, we are interested in the duality of effective potential. We can expand the effective potential for a rotating non-charged BH background by setting \( q = 0 \), \( B(r) = 0 \) and \( C(r) = 0 \), which reads
\[ \frac{g_3(r)}{2} - \frac{E^2 g_3(r)}{2g_1(r)} + \frac{L^2 g_3(r)}{2g_2(r)} + \frac{ELg_3(r)w(r)}{g_1(r)} - \frac{L^2 g_3(r)w^2(r)}{2g_1(r)}. \] (29)
We also expand it for a static charged BH background by setting \( L = 0 \) and \( w(r) = 0 \), which gives
\[
\frac{g_3(r)}{2} - \frac{E^2 g_3(r)}{2g_1(r)} + \frac{qEB(r)g_3(r)}{g_1(r)} - \frac{q^2B(r)^2g_3(r)}{2g_1(r)}.
\]
(30)

After doing the transformation (28), one can find that Eq. (30) is the same as Eq. (29), up to only one term \( \frac{L^2g_3(r)}{2g_2(r)} \). We also notice that this term will be vanished on the horizon, where \( g_3(r) \to 0 \). The last step is to consider the CM energy. Eq. (10) for static charged BH background is
\[
\frac{E_{c.m.}^2}{2m_0^2} = 1 + \frac{\Theta_2(r_H)}{2\Theta_1(r_H)} + \frac{\Theta_1(r_H)}{2\Theta_2(r_H)}.
\]
(31)

After doing the transformation (28), Eq. (10) for the rotating non-charged BH can be written as
\[
\frac{E_{c.m.}^2}{2m_0^2} = 1 + \frac{\Theta_2(r_H)}{2\Theta_1(r_H)} + \frac{\Theta_1(r_H)}{2\Theta_2(r_H)} + \frac{(E_2q_1 - E_1q_2)^2}{2\Theta_1(r_H)\Theta_2(r_H)g_2(r_H)}.
\]
(32)

One can find that the difference between Eqs. (31) and (32) is the last term of (32). Since the charge energy ratio of two particles should not be the same for gaining ultra-high CM energy, this term is nonvanishing, and it will diverge as well as all other terms in (32) when one of the colliding particle has the critical charge. Thus, we can conclude that the duality between frame dragging effect and electromagnetic interaction is not exact, but if one is only interested in the properties on the horizon, this duality is qualitatively effective.

IV. BRANEWORLD BLACK HOLES

In this section, we will apply some obtained general results to braneworld BHs. It is very interesting since braneworld BHs not only could exist as astrophysical BHs, but also could be produced at LHC, which hence provides a possibility to check the BSW mechanism terrestrially.

In braneworld theory, the standard model particles are confined on the brane, with only gravity propagating in the bulk. We will assume that the test particle moves on the equatorial plane of braneworld BHs.
A. ADD KNM BHs

The ADD model has $d$ flat, compact extra dimensions. Assuming the 3-brane located at $\theta_i = \pi/2$ ($i = 1, \cdots, d$), the metric of the equatorial plane of KNM BHs in this model is \[ ds^2 = -\frac{\Delta - a^2}{\Sigma} dt^2 - \frac{2a(a^2 + r^2 - \Delta)^2}{\Sigma} dt d\phi + \frac{(a^2 + r^2)^2 - a^2 \Delta}{\Sigma} d\phi^2 + \frac{r^2}{\Delta} dr^2, \] (33) where 
\[ \Delta = r^2 + a^2 + Q^2 - \frac{\mu}{r^{d-1}}, \quad \Sigma = r^2, \]
$Q$ is the charge of the BH, $\mu$ is the mass constant and will be set as 2 for convention.

Comparing this metric with Eq. (1), we obtain the expression of those functions in Eq. (1):
\[ g_1(r) = \frac{\Delta \Sigma}{(a^2 + r^2)^2 - \Delta a^2}, \quad g_2(r) = \frac{(a^2 + r^2)^2 - a^2 \Delta}{\Sigma}, \]
(34)
\[ g_3(r) = \frac{\Delta}{\Sigma}, \quad w(r) = \frac{a(a^2 + r^2 - \Delta)}{(a^2 + r^2)^2 - \Delta a^2}. \]
(35)
Correspondingly, the electromagnetic potential is
\[ A_t = B(r) = -\frac{Qr}{\Sigma}, \quad A_\phi = C(r) = \frac{Qar}{\Sigma}. \]

When $d = 0$, the metric (33) reduces to the KNM metric in four-dimensional spacetime. To investigate the property of the horizon of ADD KNM BHs, let us set $\partial_r \Delta = 0$, this leads to
\[ r = (1 - d)^{\frac{1}{d+1}}. \]
This equation is important since it indicates that there is no degenerated horizon for ADD KNM BHs, when $d > 0$.

Now, let us use the six-dimensional ADD KNM BH as an example to show the BSW mechanism in the ADD model (since the five-dimensional ADD model has been ruled out). We assume the angular momentum $a \leq 0.998$ and charge $Q \leq \frac{4}{3} \sqrt{1/137} = 0.113$ [17]. For a six-dimensional charged ADD BH with $a = 0.998$ and $Q = 0.100 (< 0.113)$, the location of horizon becomes to
\[ r_H = \frac{2^{1/3} (a^2 + Q^2)}{\left[54 + \sqrt{2916 + 108(a^2 + Q^2)}\right]^{1/3}} + \frac{54 + \sqrt{2916 + 108(a^2 + Q^2)^3}}{3 \times 2^{1/3}} = 0.999, \]
and the critical angular momentum (11) and charge (12) are $L_c = 2.00E - 0.100q$ and $q_c = 20.0E - 10.0L$, respectively.
We note that it is difficult to solve \( V_{\text{eff}} = 0 \) and \( \partial_r V_{\text{eff}} = 0 \) analytically to determine whether the particle with \( L_c \) or \( q_c \) can fall into the BH. However, from Eqs. (14) and (20), and \( g_5(r_H) = 4.02 > 0 \), one immediately recognizes that the answer is negative.

The multiple scattering process is necessary to achieve ultra-high CM energy in this case. For the particle with \( L_1 = L_c(1 - \delta) \) or \( q_1 = q_c(1 - \delta) \) colliding with another particle on the horizon, the CM energy (23) and (24) are

\[
\frac{E_{\text{c.m.}}^2}{2m_0^2} = 1 + \frac{0.501(0.999 + E_1^2 + 0.200E_1q_1 + 0.010q_1^2)(E_2 - 0.501L_2 + 0.050q_2)}{E_1 - 0.050q_1} \frac{1}{\delta} + O(\delta)
\]

and

\[
\frac{E_{\text{c.m.}}^2}{2m_0^2} = 1 + 50.0(0.010 + 0.010E_1^2 - 0.020E_1L_1 + 0.010L_1^2)(E_2 - 0.501L_2 + 0.050q_2) \frac{1}{\delta} + O(\delta).
\]

We note that \( E_1 - 0.050q_1 \sim L_c \) and \( E_1 - 0.501L_1 \sim q_c \) can not be vanishing, and the divergent degree (index of \( \delta \)) is not influenced by \( q_1 \) in Eq. (36) and \( L_1 \) in Eq. (37).

**B. RS tidal charge BHs**

After having investigating the ADD KNM BH in which the horizon is non-degenerated, we would like to study the degenerated horizon of an extreme RS tidal charged BH.

The RS model consists of a single, positive tension brane in an infinite extra dimension. The role of extra dimension is played by a tidal charge \( Q \). The effective metric of a RS BH can be expressed like [23]

\[
d s^2 = -\left(1 - \frac{2Mr - Q}{\Sigma}\right)dt^2 - 2\frac{a(2Mr - Q)}{\Sigma}dt d\phi + \frac{\Sigma}{\Delta}d\phi^2 + \left(r^2 + a^2 + \frac{2Mr - Q}{\Sigma}a^2\right)d\phi^2,
\]

where

\[
\Delta = r^2 - 2Mr + a^2 + Q, \quad \Sigma = r^2.
\]

Note that the mass \( M \) of BHs will be set as 1 later. By comparing with Eq. (23), one can find

\[
g_1(r) = \frac{\Delta \Sigma}{(a^2 + r^2)^2 - \Delta a^2}, \quad g_2(r) = \frac{(a^2 + r^2)^2 - \Delta a^2}{\Sigma},
\]

\[
g_3(r) = \frac{\Delta}{\Sigma}, \quad w(r) = \frac{a(a^2 + r^2 - \Delta)}{(a^2 + r^2)^2 - \Delta a^2}.
\]
If $a^2 + Q = 1$, this BH has a two-fold degenerate horizon located at $r_H = 1$. Under the limit of the solar system on the tidal charge $Q \leq 8 \times 10^4 m^2 = 0.037$ for a BH with one sun mass [18], the horizon can be degenerated if $a \geq 0.981$. From the discussion about two-fold degenerated horizons in the preceding section, we know that, for the extreme RS tidal charge BH with $a = \sqrt{1 - Q}$, the CM energy of two particles will be divergent, provided that one particle’s angular momentum equates to the critical value [11]

$$L = E \frac{2 - Q}{\sqrt{1 - Q}}, \quad (38)$$

and the condition [19]

$$1 + \left( \frac{1}{1 - Q} - 4 \right) E^2 < 0$$

is satisfied. This inequality is saturated when

$$E = \sqrt{\frac{1 - Q}{3 - 4Q}}, \quad (39)$$

which is one of the conditions for ISCO on the horizon that can be invoked to avoid the fine-turning problem.

An important problem of the BSW mechanism in braneworld BHs is about the infinite proper time spent by a particle with $L_c$ on falling into the degenerated horizon of BH. One may worry about whether the ultra-high energy can be achieved rapidly in small braneworld BHs, since in four-dimension case, the lifetime of BH decreases with its mass $M$ rapidly: $\tau \approx 8.3 \times 10^{-26} M^3 s$ [25]. Considering the particles colliding at $r_f = 1 + \delta'$ near the horizon, and expanding Eq. [8] near the horizon $r_H = 1$, we have

$$\frac{E_{c.m.}^2}{2m_0^2} = 1 + \frac{\alpha}{\delta'} + O(\delta')^0, \quad (40)$$

where

$$\alpha = \left( E_2 \frac{2 - Q}{\sqrt{1 - Q}} - L_2 \right) \left( 2E_1 - \sqrt{E_1^2 - \frac{3 - 4Q}{1 - Q} - 1} \right).$$

From Eq. [25], one can obtain the proper time cost for achieving the energy expressed in [40] as

$$\tau \approx - \frac{\sqrt{1 - Q}}{\sqrt{(4E_1^2 - 1)(1 - Q) - E_1^2}} \log \delta'$$

$$= \frac{\sqrt{1 - Q}}{\sqrt{E_1^2(3 - 4Q) + Q - 1}} \left[ \log \left( \frac{E_{c.m.}^2}{2m_0^2} - 1 \right) - \log \alpha \right], \quad (41)$$
where we have omitted the effect from a finite $r_i$. As an example, we will study the proper time spent by a particle on falling into a micro RS BH with mass $M$. For this BH, we would like to consider how long the BSW process would take to get 1TeV energy, which is a significant scale denoting both the quantum gravity scale in extra-dimensional theory and the mass of BHs. From Eq. (41), we obtain $\tau \approx 10^{-59} \frac{M}{1\text{TeV}} \text{s}$, assuming $m_0$ as the electron mass (noting that other parameters are not important). This value is larger than the lifetime of four dimensional TeV BHs $\sim 10^{-88} \text{s}$, but it is smaller than the typical lifetime of small braneworld BHs, which is about $10^{-26} \text{s}$ for ADD BHs [20], and can even reach up to $10^9 \text{s}$ for RS cases [24].

One can solve the ISCO for a RS BH from the equations

$$V_{\text{eff}} = 0, \quad \partial_r V_{\text{eff}} = 0 \quad \text{and} \quad \partial_r^2 V_{\text{eff}} = 0.$$ (42)

Setting $a = 1$ and $Q = 0$, the particle on the ISCO satisfies

$$L = \frac{2}{\sqrt{3}} , \quad E = \frac{1}{\sqrt{3}} , \quad r = 1,$$

consistent with the result for Kerr BHs. For $Q \neq 0$ case, Eq. (42) is difficult to solve directly. However, based on the fact that the ISCO should be on the horizon, we have obtained the parameters of ISCO, which are given by Eqs. (38) and (39). One can substitute the value of $L$ and $E$ into $\partial_r^2 V_{\text{eff}}$ and check that it is zero indeed. It is also easy to notice that Eqs. (38) and (39) will go back to the ISCO of Kerr BHs with $Q = 0$. To solve the fine-tuning problem, one should consider ISCO on $r = 1 + \delta'$ near the horizon of the RS BH with $a = \sqrt{1-Q}(1-\delta)$. The event horizon of this BH locates at $r_H = 1 + \sqrt{(1-Q)(2-\delta)}$. We can also solve the $E(r)$ and $L(r)$ of circular orbits for this case. Then we will solve $\partial_r E(r) = 0$ and $\partial_r L(r) = 0$, instead of calculating $\partial_r^2 V_{\text{eff}} = 0$ directly, which leads to

$$\delta = \frac{(1-2Q)\delta'^2}{4(1-Q)^2} + O(\delta')^4.$$ (43)

Note that one must impose $1-2Q \gg 0$ to preserve the effective approximation in Eq. (43). Thus, we get the expression of $E$ and $L$ for the ISCO on $r = 1 + \delta'$, which reads

$$E = \sqrt{\frac{1-Q}{3-4Q}} + \frac{3(1-2Q)(1-Q)^{1/2}}{(3-4Q)^{3/2}} \delta' + O(\delta')^2, (44)$$

$$L = \frac{2-Q}{\sqrt{3-4Q}} + \frac{3(2-5Q+2Q^2)}{(3-4Q)^{3/2}} \delta' + O(\delta')^2.$$ (45)
Obviously, the value of $E$ and $L$ is the same as Eqs. (38) and (39) when $\delta' = 0$. We consider the collision with one particle on this ISCO and another particle with $E = 1$ and $L = 0$ for simplicity. The CM energy is

\[
\frac{E_{\text{c.m.}}^2}{2m_0^2} = 1 + \frac{\sqrt{2}(2 - Q)(1 - Q)}{\sqrt{(3 - 4Q)(1 - 2Q)}} \frac{1}{\delta'^{3/2}} = \frac{3(1 - 2Q)^{3/2}(2 - Q)}{\sqrt{2}(3 - 4Q)^{3/2}} \frac{1}{\delta'^{3/2}} + \mathcal{O}(\delta')^{1/2}.
\]  

(46)

We notice that the CM energy can be arbitrary high when the value of $\delta'$ is arbitrary small. Also, the divergent degree is not influenced by the tidal charge.

V. CONCLUSION AND DISCUSSION

BSW mechanism provides a remarkable possibility that Kerr BHs might act as Plank-scale particle accelerators. Some extended works showed that the frame-dragging effect is important to achieve unbound CM energy. It was further confirmed in [6], where the arbitrary high CM energy was found as a general property in a general axially symmetric rotating BH. After that, however, there were still some works on complicated rotating backgrounds, partially because the metric adapted in [6] and the consequent results, such as the expression of CM energy, can not be compared with those concrete backgrounds directly. Moreover, the ISCO in Kerr BHs, which was introduced to avoid the fine-turning problem [8], has not been applied to a general background. On the other hand, it was found that the collision of charged particles in RN BHs has the similar mechanism for arbitrary high CM energy, but the general charged background has not been considered either. In particular, the combined effect of frame dragging and electromagnetic interaction is very worth to be studied, because theoretically, it could provide a better understanding of these two kinds of effects in BSW mechanism; and practically, the ADD KNM BH is a general BH that could be formed after proton-proton collisions in LHC [17].

Our work addressed these problems mentioned above. We investigated the CM energy of two charged particles colliding in the background of a general stationary charged BH, adapting a metric which is convenient to compare with observations. It is shown that the CM energy can be arbitrarily high, provided that the following three conditions are satisfied. First, the collision should occur near the horizon of BHs. Second, only one particle has the entangled (near) critical angular momentum $L_c$ and critical charge $q_c$. The last condition depends on the degenerate degree of horizons. Concretely, for two-fold degenerate horizons,
there are some restrictions on the parameters of particles from the requirement that the effective potential with $L_c$ or $q_c$ should be negative near horizons. Since the negative potential near horizons just imposes the maximum of potential at horizons, the particle with $L_c$ or $q_c$ under the restrictions can reach the horizons even from infinity. For multiple-fold degenerate event horizons, there is no such restriction. The ultra-high CM energy can also be gained for the particle collision near non-degenerate horizons by invoking the multiple scattering mechanism to amplify the angular momentum or charge of the falling particle from infinity.

We derived the general formula of CM energy for non-degenerate and any-fold degenerate horizons when one particle has $L_c$ or $q_c$.

Furthermore, we obtained the condition for the existence of ISCO with $L_c$ or $q_c$ on degenerate horizons, and pointed out that it is essential to get arbitrary high CM energy without the fine-tuning problem. It is interesting to see that both frame dragging effect and electromagnetic interaction are necessary for the existence of ISCO on multiple-fold degenerate horizons with nonvanishing energy parameter $E$. Moreover, we showed that the proper time taken for achieving infinite CM energy is finite for the particle collision at non-degenerate horizons but is logarithmic divergent for the collision at degenerate horizons. We also clarified that there is a qualitatively effective duality between frame dragging effect and electromagnetic interaction for the properties of the BSW mechanism on the horizon, which could be helpful to investigate whether the CM energy of colliding particles around rotating BHs has a similar upper limit which was found by studying the acceleration of colliding shells around a RN black hole [16].

It should be pointed out that we have ignored the back-reaction and gravitational radiation of the colliding particles, which may have important effect on the CM energy but could not be analyzed in the present general frame.

We then applied some general results to the cases of ADD and RS braneworld BHs. It was shown that there is no degenerate horizon in ADD KNM BHs, so we calculated the CM energy with near critical angular momentum or near critical charge obtained by the multiple scattering. It was found that the divergent degree is not influenced by the charge or angular momentum of the particle. For RS BHs, we found that the proper time spending by the particles to arrive at 1TeV CM energy is smaller than the typical lifetime of braneworld BHs. We also evaluated the CM energy of one particle colliding with another particle on the ISCO when the horizon is near degenerate. Also, the divergent degree is not influenced.
by the tidal charge.

At last, we expect that the BSW mechanism could be checked in LHC. We have noted that the lifetime of these small BHs is long enough to afford the BSW process to get the ultra-high energy. Thus, we can take the small BHs, which are assumed to have been produced by proton-proton collisions in LHC, as the background when the other two particles fall into and collide near the small BHs. In other words, the braneworld small BHs in LHC plays the role of the astrophysical black holes in the BSW mechanism. Although the small BH could be distorted more easily than astrophysical BHs under the back reaction of falling particles and the ultra-high CM energy could not be attained, one still can expect that the CM energy of particles collided in the background with small BHs would be apparently larger than the background without BHs. Consider the ingoing particles being static at infinite and the mass of small RS BHs is about 1TeV. We can estimate the maximized CM energy after counting the back-reaction effect, which can be implement by considering the absorption of the first pair of colliding particles \cite{2}. From Eq. (46) and $\delta' \sim (m_0/M)^{1/3}$, we have $E_{c.m.} \lesssim 6.5 \left(\frac{m_0}{\text{1 GeV}}\right)^{3/4} \left(\frac{M}{\text{1 TeV}}\right)^{1/4}\text{GeV}$. This result means that two particles colliding in an LHC experiment can reach higher energy due to interaction with the small BHs, which have been produced by proton-proton collisions before the BSW collision occurs. In particular, the CM energy would be maximized when the angular momentum or charge of one particle approaches the critical parameter $L_c$ or $q_c$, and decrease when another particle also has the critical parameters. We expect that LHC could check these unique properties of BSW mechanism. Nevertheless, it should be noticed that our estimation is very rough since it has neglected the underlying details of experiments.

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