Relativistic three-body calculations of a $Y = 1, I = \frac{3}{2}, J^P = 2^+$

$\pi\Lambda N - \pi\Sigma N$ dibaryon

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Abstract

The $\pi\Lambda N - \pi\Sigma N$ coupled-channel system with quantum numbers $(Y, I, J^P) = (1, \frac{3}{2}, 2^+)$ is studied in a relativistic three-body model, using two-body separable interactions in the dominant $p$-wave pion-baryon and $^3S_1YN$ channels. Three-body equations are solved in the complex energy plane to search for quasibound-state and resonance poles, producing a robust narrow $\pi\Lambda N$ resonance about 10–20 MeV below the $\pi\Sigma N$ threshold. Viewed as a dibaryon, it is a $^5S_2$ quasibound state consisting of $\Sigma(1385)N$ and $\Delta(1232)Y$ components. Comparison is made between the present relativistic model calculation and a previous, outdated nonrelativistic calculation which resulted in a $\pi\Lambda N$ bound state. Effects of adding a $KNN$ channel are studied and found insignificant. Possible production and decay reactions of this $(Y, I, J^P) = (1, \frac{3}{2}, 2^+)$ dibaryon are discussed.

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I. INTRODUCTION

In recent work \[1, 2\] we have studied the \(\pi\Lambda N - \pi\Sigma N\) coupled channel system, in which the dominant two-body configurations are the pion-nucleon \(p\)-wave \(\Delta(1232)\) resonance with \(s\)-wave hyperon spectator, the pion-hyperon \(p\)-wave \(\Sigma(1385)\) resonance with \(s\)-wave nucleon spectator, and the \(YN\) \((Y \equiv \Lambda, \Sigma)\) \(^3\)S\(_1\) coupled channels with \(p\)-wave pion spectator. The contributions of these two-body configurations obviously maximize in the three-body channel with \((I, J^P) = (3/2, 2^+)\), where \(I, J, P\) denote the total isospin, total angular momentum and parity, respectively. Substantial attraction in this three-body configuration was found in a nonrelativistic three-body calculation, resulting in a possible \(\pi\Lambda N\) bound state. Having presented very recently a relativistic three-body Faddeev formalism appropriate for systems with \(p\)-wave two-body interactions \[3\], it is natural to apply it to the \(\pi\Lambda N - \pi\Sigma N\) coupled channels system with \(I = 3/2\) and \(J^P = 2^+\). The main consequence of adopting a relativistic formalism, as shown below, is that the \(\pi\Lambda N\) bound state dissolves, becoming a \(\pi\Lambda N\) resonance below the \(\pi\Sigma N\) threshold. We note that a relativistic three-body formalism equivalent to that of Ref. \[3\] was already applied in the context of searching for a \(\bar{K}NN\) \((I = 1/2, J^P = 0^-)\) quasibound state for which the dominant two-body configurations are all \(s\)-waves \[4\]. We have also studied the effect of adding to the \((3/2, 2^+)\) \(\piYN\) channels a \(\bar{K}NN\) channel, induced through a \(\Sigma(1385)\)-mediated two-body \(p\)-wave \(\bar{K}N - \piY\) coupling, and found it to be relatively insignificant. This is to be expected, observing that none of the Pauli-allowed \(s\)-wave \(NN\) configurations fits into a \((3/2, 2^+)\) \(\bar{K}NN\) channel with a \(p\)-wave meson spectator. For a recent overview of dibaryon candidates and related studies, see Refs. \[5–7\].

The paper is organized as follows: input two-body interactions are described in Sect. \[\text{II}\] and three-body equations are derived in Sect. \[\text{III}\]. Results are described in Sect. \[\text{IV}\] and discussed in Sect. \[\text{V}\]. Several production reactions by which to search for the present \((Y, I, J^P) = (1, 3/2, 2^+)\) dibaryon candidate are listed and briefly discussed in the Summary Sect. \[\text{VI}\].
II. TWO-BODY INTERACTIONS

As discussed in Ref. [1], the dominant two-body interactions are in the $p$-wave $\pi N (I, J^P) = (\frac{3}{2}, \frac{3}{2}^-) \Delta(1232)$ and $\pi \Lambda - \pi \Sigma (I, J^P) = (1, \frac{3}{2}^+) \Sigma(1385)$ channels, and in the $s$-wave $\Lambda N - \Sigma N (I = \frac{1}{2}, \frac{3}{2}S_1)$ channel. We note that these two-body interactions, taken here in separable form, are independent of energy whereas the resulting two-body amplitudes are obviously energy dependent, and even resonate in the $p$-wave channels. Since the introduction of two-body energy-dependent interactions geared to simulate additional energy-dependent background amplitudes poses problems within a relativistic kinematics treatment (see Ref. [8] for a recent discussion) we limit the two-body interaction input in the present three-body relativistic calculation to energy-independent separable forms described below. Our notational convention is to assign particle indices 1,2,3 to hyperons, nucleon and pion, respectively.

A. The $\pi N$ subsystem

The Lippmann-Schwinger equation for the pion-nucleon interaction is given by [3]:

\[
t_1(p_1, p'_1; \omega_0) = V_1(p_1, p'_1) + \int_0^\infty p''_1 dp''_1 \frac{1}{\omega_0 - \sqrt{m_N^2 + p''_1^2} - \sqrt{m_\pi^2 + p''_1^2} + i\epsilon} t_1(p''_1, p'_1; \omega_0),
\]

so that using the separable potential

\[
V_1(p_1, p'_1) = \gamma_1 g_1(p_1) g_1(p'_1),
\]

one gets

\[
t_1(p_1, p'_1; \omega_0) = g_1(p_1) \tau_1(\omega_0) g_1(p'_1),
\]

where

\[
[\tau_1(\omega_0)]^{-1} = \frac{1}{\gamma_1} - \int_0^\infty p_1^2 dp_1 \frac{g_1^2(p_1)}{\omega_0 - \sqrt{m_N^2 + p_1^2} - \sqrt{m_\pi^2 + p_1^2} + i\epsilon}.
\]

A fit to the $P_{33}$ phase shift and scattering volume using the form factor

\[
g_1(p_1) = p_1 [\exp(-p_1^2/\beta_1^2) + C p_1^2 \exp(-p_1^2/\alpha_1^2)],
\]

and a set of parameters listed in Table II row marked $P_{33}$, was shown and discussed in Ref. [3]. This form factor and parameters are used in the present calculations. Listed in the same
row are also r.m.s. radii values of momentum-space and coordinate-space representations of the $P_{33}$ form factor. These were discussed too in Ref. [3]; here we recall that $\tilde{g}_1(r)$, the coordinate-space Fourier transform of $g_1(p)$, is not necessarily a nodeless function at finite values of $r$, so that an appropriate measure of its spatial extension is provided by the value of its (single) zero $r_0^{(\pi N)}$, given by the last entry. This does not appear to present a problem in the case of the $\pi N P_{33}$ form factor, where the difference between the listed values of $\sqrt{<r^2>_{\tilde{g}_1}}$ and $r_0^{(\pi N)}$ is small, but it does present a problem in the case of the $\pi Y$ form factor where the squared radius $<r^2>_{\tilde{g}_1}$ assumes occasionally negative values. Returning to Table I listed in the row marked $P_{13}$ are parameters fitted to the $P_{13}$ phase shifts which are considerably smaller than the $P_{33}$ resonating phase shifts. This $\pi N P_{13}$ channel will act in the three-body calculation only together with a spectator $\Sigma$ hyperon, and its inclusion serves the purpose of estimating the role of $\pi B$ channels other than the resonating ones. For notational simplicity, and since the $\pi N P_{13}$ channel is excluded from most of the calculations reported here, it is suppressed in the derivation of the three-body equations below.

TABLE I: Fitted parameters of the $\pi N$ separable $p$-wave interaction (2) with form factor $g_1(p)$ (5). Values of the r.m.s. momentum $\sqrt{<p^2>_{g_1}}$ (fm$^{-1}$), r.m.s. radius $\sqrt{<r^2>_{\tilde{g}_1}}$ and zero $r_0^{(\pi N)}$ (both in fm) of the Fourier transform $\tilde{g}_1(r)$ are listed for the dominant $P_{33}$ channel.

| channel | $\gamma_1$ (fm$^4$) | $\alpha_1$ (fm$^{-1}$) | $\beta_1$ (fm$^{-1}$) | $C$ (fm$^2$) | $\sqrt{<p^2>_{g_1}}$ | $\sqrt{<r^2>_{\tilde{g}_1}}$ | $r_0^{(\pi N)}$ |
|---------|----------------------|------------------------|------------------------|--------------|----------------------|----------------------|-----------------|
| $P_{33}$ | -0.075869            | 2.3668                 | 1.04                   | 0.23         | 4.07                 | 1.47                 | 1.36            |
| $P_{13}$ | 0.033                | -                      | 1.325                  | 0.0          |                      |                      |                 |

The $\pi N$ $P_{33}$ amplitude in the three-body system can have either $\Lambda$ or $\Sigma$ hyperon as spectator and is given by

$$t^Y_1(p_1, p'_1; W_0, q_1) = g_1(p_1)\tau^Y_1(W_0, q_1)g_1(p'_1),$$

(6)

where $W_0$ is the invariant mass of the three-body system, $q_1$ is the relative momentum between the hyperon and the c.m. of the $\pi N$ subsystem and

$$[\tau^Y_1(W_0, q_1)]^{-1} = \frac{1}{\gamma_1} - \int_0^\infty p_1^2 dp_1 \frac{g_1^2(p_1)}{W_0 - \sqrt{\left(\sqrt{m_N^2 + p_1^2} + \sqrt{m_\pi^2 + p_1^2}\right)^2 + q_1^2 - \sqrt{m_Y^2 + q_1^2} + i\epsilon}},$$

(7)

where $Y$ is either $\Lambda$ or $\Sigma$. 

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B. The $\pi\Lambda - \pi\Sigma$ subsystem

Since we have in this case two coupled channels the corresponding Lippmann-Schwinger equation is

$$t_2^{YY'}(p_2, p'_2; \omega_0) = V_2^{YY'}(p_2, p'_2) + \sum_{Y''} \int_0^{\infty} p''_2 dp''_2$$

$$\times V_2^{YY''}(p_2, p''_2) \frac{1}{\omega_0 - \sqrt{m^2_\pi + p''_2^2} - \sqrt{m^2_{Y''} + p''_2^2} + i\epsilon} t_2^{YY''}(p''_2, p'_2; \omega_0).$$  \(8\)

Here we used the separable potential

$$V_2^{YY'}(p_2, p'_2) = \gamma_2 g_2^Y(p_2) g_2^{Y'}(p'_2),$$  \(9\)

so that the solution of the Lippmann-Schwinger equation is

$$t_2^{YY'}(p_2, p'_2; \omega_0) = g_2^Y(p_2) \tau_2(\omega_0) g_2^{Y'}(p'_2),$$  \(10\)

with

$$\tau_2^{-1}(\omega_0) = \frac{1}{\gamma_2} - \sum_Y \int_0^{\infty} p^2_d dp_d$$

$$\frac{[g_2^Y(p_d)]^2}{\omega_0 - \sqrt{m^2_\pi + p^2_d} - \sqrt{m^2_Y + p^2_d} + i\epsilon}.$$  \(11\)

The two-body amplitude in the three-body system with a nucleon as spectator is given by expressions analogous to \(6\) and \(7\). Following Ref. \[3\] we used the form factors

$$g_2^\Lambda(p_2) = p_2(1 + Ap_2^2) \exp(-p_2^2/\beta_2^2), \quad g_2^\Sigma(p_2) = Bg_2^\Lambda(p_2),$$  \(12\)

where the four parameters $\gamma_2$, $\beta_2$, $A$ and $B$ were fitted to the three pieces of data available, namely, the position and width of the $\Sigma(1385)$ resonance and the branching ratio for its two main decay modes. A family of such parameters is given in Table \[I\] for a range of $A$ values such that the spatial size (here $r_0^{(\pi Y)}$) associated with the resulting $\pi Y$ form factors is related physically to the spatial size $r_0^{(\pi N)}$ associated with the $P_{33} \pi N$ form factor of Table \[II\]. For more details and discussion, see Ref. \[3\].

C. The $YN$ subsystem

In the case of isospin \(\frac{1}{2}\) which corresponds to the coupled $\Lambda N - \Sigma N$ subsystem we have two coupled channels so that applying Eq. \(8\) to the separable potential

$$V_3^{YY'}(p_3, p'_3) = \gamma_3^{YY'} g_3^Y(p_3) g_3^{Y'}(p'_3)$$  \(13\)

We note that the superscripts $\Lambda$ and $\Sigma$ are erroneously interchanged in Eq. (7) of the published journal version where they appear as subscripts. None of the results in Ref. \[3\] is affected by this typo.
TABLE II: Fitted parameters of the $\pi\Lambda - \pi\Sigma$ $p$-wave separable interaction defined by Eqs. (9) and (12), for chosen values of the parameter $A$. Listed also are values of the r.m.s. momentum $\sqrt{<p^2>_{g_2}}$ (in fm$^{-1}$), the r.m.s. radius $\sqrt{<r^2>_{\tilde{g}_2}}$ (whenever real) and zero $r_0^{(\pi Y)}$ (both in fm) of the Fourier transform $\tilde{g}_2(r)$.

| $A$ (fm$^2$) | $\gamma_2$ (fm$^4$) | $\beta_2$ (fm$^{-1}$) | $B$ | $\sqrt{<p^2>_{g_2}}$ | $\sqrt{<r^2>_{\tilde{g}_2}}$ | $r_0^{(\pi Y)}$ |
|-------------|-----------------|-----------------|-----|-----------------|-----------------|-----------------|
| 0.25        | -0.0091851      | 2.5810          | 0.93671 | 4.30           | 0.33            | 1.36            |
| 0.30        | -0.0090934      | 2.4765          | 0.95132 | 4.13           | 0.23            | 1.41            |
| 0.35        | -0.0089513      | 2.3919          | 0.96559 | 4.00           | –              | 1.45            |
| 0.40        | -0.0087763      | 2.3216          | 0.97949 | 3.89           | –              | 1.48            |
| 0.45        | -0.0085787      | 2.2619          | 0.99298 | 3.80           | –              | 1.51            |

leads to

$$t_3^{YY'}(p_3, p_3'; \omega_0) = g_Y^3(p_3)\tau_3^{YY'}(\omega_0)g_Y^3(p_3'),$$

(14)

where $\tau_3^{YY'}(\omega_0)$ are easily obtained. We used Yamaguchi form factors

$$g_Y^3(p_3) = \frac{1}{1 + (p_3/\alpha_Y^3)^2},$$

(15)

so that there are five free parameters, three strengths and two ranges. These five parameters were fitted to the $\Lambda N S = 1$ scattering length $a_{\frac{3}{2}\frac{1}{2}} = 1.41$ fm and effective range $r_{\frac{3}{2}\frac{1}{2}} = 3.36$ fm, the real and imaginary parts of the $\Sigma N S = 1$ scattering length $a_{\frac{3}{2}\frac{1}{2}}' = 2.74 + i1.22$ fm, and the phase of the $\Lambda N - \Sigma N S = 1$ transition scattering length $\psi = 23.8^\circ$ obtained in the chiral quark model [9]. These parameters are given in Table III.

TABLE III: Parameters of the spin-triplet $YN$ separable potentials defined by Eqs. (13) and (15) for isospin values $I_{YN} = \frac{1}{2}, \frac{3}{2}$.

| $I_{YN}$ | $\gamma_{3\Lambda}$ (fm$^2$) | $\gamma_{3\Sigma}$ (fm$^2$) | $\gamma_{3\Sigma}$ (fm$^2$) | $\alpha_3^\Lambda$ (fm$^{-1}$) | $\alpha_3^\Sigma$ (fm$^{-1}$) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\frac{1}{2}$ | -0.37704         | -0.047865       | -0.0059699      | 1.46            | 0.4            |
| $\frac{3}{2}$ | –               | –               | 0.36416         | –              | 1.491          |

The spin-triplet hyperon-nucleon subsystem with isospin $\frac{3}{2}$ corresponds to pure $\Sigma N$ scattering and it requires only two free parameters, one strength and one range. These two parameters were fitted to the $\Sigma N S = 1$ scattering length $a_{\frac{3}{2}\frac{1}{2}}' = -0.44$ fm and effective
range \( r_{21}' = -2.09 \text{ fm} \) obtained in the chiral quark model [9]. These parameters are also given in Table III.

### D. Compact form of the two-body amplitudes

The two-body amplitudes discussed above can be written in compact form as

\[
T_1^Y = \langle g_1 \pi N \rangle r_1 \langle g_1 \pi N \rangle, \quad Y = \Lambda, \Sigma, \tag{16}
\]

\[
t_2 = \begin{pmatrix} |g_2^\Lambda\rangle \\ |g_2^\Sigma\rangle \end{pmatrix} \tau_2 \left( \langle g_2^\Lambda | \langle g_2^\Sigma | \right), \tag{17}
\]

\[
t_3 = \begin{pmatrix} |g_3^\Lambda N \rangle r_3^{\Lambda N} \langle g_3^\Lambda N \rangle \, |g_3^\Lambda N \rangle r_3^{\Lambda N} \langle g_3^\Lambda N \rangle \\ |g_3^\Sigma N \rangle r_3^{\Sigma N} \langle g_3^\Lambda N \rangle \\ |g_3^\Sigma N \rangle r_3^{\Sigma N} \langle g_3^\Lambda N \rangle \end{pmatrix}. \tag{18}
\]

For applications wishing to extend the system of two-body \( \pi Y \) coupled channels into a system of \( \pi Y - KN \) channels, coupled through the \( \Sigma(1385) \) isobar, Eq. (17) is to be replaced by

\[
t_2 = \begin{pmatrix} |g_2^\Lambda\rangle \\ |g_2^\Sigma\rangle \\ |g_2^K N\rangle \end{pmatrix} \tau_2 \left( \langle g_2^\Lambda | \langle g_2^\Sigma | \langle g_2^K N | \right). \tag{19}
\]

### III. THREE-BODY EQUATIONS

Normally, the Faddeev amplitudes are labeled by the spectator particle which in general has the same label as the interacting pair. However, when there is particle conversion as in the present case one can have different interacting pairs for the same spectator or different spectators for the same interacting pair. For example, whereas \( \pi N \) is the interacting pair in the amplitude \( T_1 \) and the spectator is either \( \Lambda \) or \( \Sigma \), the interacting pair in the amplitude \( T_2 \) is either \( \pi \Lambda \) or \( \pi \Sigma \) and the spectator is a nucleon. Thus, we will label the corresponding Faddeev amplitudes either by the spectator or by the interacting pair as helpful as to make the notation clear. In this way, considering all possible transitions, one obtains the Faddeev equations

\[
T_1^Y = t_1^Y G_0(\pi Y N) T_2^Y + t_1^Y G_0(\pi Y N) T_3^Y N, \tag{20}
\]

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For applications wishing to extend the two-body $\pi Y$ coupled channels into a system of $\pi Y - \bar{K}N$ channels coupled through the $\Sigma(1385)$ isobar, the Faddeev amplitude (21) acquires the additional term $t_2^{\pi Y - \bar{K}N}G_0(\bar{K}NN){T_2^{\bar{K}N}}$ on the r.h.s., where

$$T_2^{\bar{K}N} = \sum_{Y'} t_2^{\bar{K}N - \pi Y}G_0(\pi Y')T_2^{\pi Y'} + \sum_{Y'} t_2^{\bar{K}N - \pi Y}G_0(\pi Y')T_2^{\pi Y'}.$$  

If we substitute Eq. (22) into Eqs. (20) and (21), using the expressions for the two-body amplitudes (16)–(18), we get that

$$T_1^Y = |g_1^{\pi N}|X_1^Y, \quad T_2^{\pi Y} = |g_2^{\pi Y}|X_2,$$

where the new amplitudes $X_1^Y$ and $X_2$ satisfy the equations

$$X_1^Y = \sum_{Y'} \tau_1^Y g_1^{\pi N}|g_2^{\pi Y'}X_2 + \int Y' Y'' \sum_{Y''} \tau_1^Y g_1^{\pi N}|g_3^{\pi Y'}|g_3^{\pi Y''} X_2 |g_2^{\pi Y''}|X_2 + \int Y' Y'' \sum_{Y''} \tau_1^Y g_1^{\pi N}|g_3^{\pi Y'}|g_3^{\pi Y''} X_2 |g_3^{\pi Y''}|X_2 |g_1^{\pi N}|X_2^{\pi Y''},$$

$$X_2 = \sum_{Y} \tau_2 g_2^{\pi Y}|g_1^{\pi N}X_1^{\pi Y} + \int Y' Y'' \sum_{Y''} \tau_2 g_2^{\pi Y}|g_3^{\pi Y'}|g_3^{\pi Y''} X_2 |g_2^{\pi Y''}|X_2 + \int Y' Y'' \sum_{Y''} \tau_2 g_2^{\pi Y}|g_3^{\pi Y'}|g_3^{\pi Y''} X_2 |g_3^{\pi Y''}|X_2 |g_1^{\pi N}|X_2^{\pi Y''}.$$  

As shown in Ref. [3], the one-dimensional integral equations corresponding to the Faddeev equations for the $\pi \Lambda N - \pi \Sigma N$ system can be read off from the AGS form Eqs. (25) and (26).

For applications wishing to extend the description of the $\Sigma(1385)$ isobar in terms of $\pi Y$ coupled channels into $\pi Y - \bar{K}N$ coupled channels, the definition of $X_2$ in Eq. (24) is generalized to

$$\begin{pmatrix} T_2^{\pi Y} \\ T_2^{\bar{K}N} \end{pmatrix} = \begin{pmatrix} |g_2^{\pi Y}| \\ |g_2^{\bar{K}N}| \end{pmatrix} X_2,$$

with Eq. (26) modified by adding on its r.h.s. the term $\tau_2 g_2^{\pi N}|g_0(\bar{K}NN)|g_2^{\bar{K}N}X_2$. 

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IV. RESULTS

We started by searching for \( (I = 3/2, J^P = 2^+) \) \( \pi \Lambda N - \pi \Sigma N \) bound-state poles, i.e. considering real values of \( W_0 < m_\pi + m_\Lambda + m_N \) for which there are no three-body singularities. The one-dimensional integral equations which follow from the coupled-amplitude AGS equations (25) and (26) were solved. Unlike the nonrelativistic cases studied in [1] and [2] we found no pole which would correspond to a bound state. In order to artificially generate such a pole we multiplied the strengths \( \gamma_1 \) and \( \gamma_2 \) by factors \( f_1 > 1 \) and \( f_2 > 1 \) which exactly produce a bound state pole at the \( \pi \Lambda N \) threshold \( W_0 = m_\pi + m_\Lambda + m_N \). We then rotated the integration contour into the complex plane as described in [3], i.e., \( q_i \to q_i \exp(-i\phi) \) which allowed us to reduce slowly the factors \( f_i \) and follow the bound state pole into the complex plane to its final position once \( f_1 = f_2 = 1 \). Finally, we checked that the position of the pole is independent of the value of \( \phi \).

| \( A \) (fm\(^2\)) | \( r_0^{(\pi Y)} \) (fm) | \( E \) (MeV) |
|----------------|----------------|-------------|
| 0.25           | 1.36           | \(-19.8 - i2.6\) |
| 0.30           | 1.41           | \(-17.6 - i2.9\) |
| 0.35           | 1.45           | \(-15.6 - i3.2\) |
| 0.40           | 1.48           | \(-13.7 - i3.5\) |
| 0.45           | 1.51           | \(-11.9 - i3.8\) |

In Table IV we list the energy eigenvalues, measured with respect to the \( \pi \Sigma N \) threshold, as calculated using the \( P_{33} \) \( \pi N \) form factor from Table I and the family of \( \pi Y \) form factors recorded in Table II. The sensitivity of the calculated pole energy to the parametrization of the \( \pi Y \) form factor amounts to less than 10 MeV. In all cases the eigenvalue lies above the \( \pi \Lambda N \) threshold, but below the \( \pi \Sigma N \) threshold. If we neglect the \( YN \) interaction, the real part of the pole energy rises approximately 10 MeV while the imaginary part remains almost the same. Finally, in order to check the effect of other non-resonating partial waves, we repeated the calculation of the first row in Table IV adding the \( \pi N \) \( P_{13} \) partial wave from the second row of Table II. The energy changed then from \( E = -19.755 - i2.611 \) MeV.
to $E = -19.734 - i2.613$ MeV, demonstrating that this effect is quite negligible.

V. DISCUSSION

In this section we discuss two aspects of the present relativistic three-body calculation, (i) relativistic vs nonrelativistic and (ii) the inclusion of a $\bar{K}NN$ channel.

A. Relativistic vs Nonrelativistic

As observed in the previous section the effects of a relativistic treatment are quite important for the $\pi\Lambda N - \pi\Sigma N$ system, removing the $\pi\Lambda N$ bound-state solution obtained in the nonrelativistic (NR) model [1, 2].

In order to understand the origin of the discrepancy between the relativistic and NR results we have repeated the calculation of the $\pi\Lambda N$ problem [1] for the simple case where there is no coupling to the $\pi\Sigma N$ channel and one neglects the $YN$ interaction. In this case, the Faddeev equations of the $\pi\Lambda N$ bound-state problem are

$$X_{\pi N} = \tau_{\pi N} \langle g_{\pi N} | G_0(\pi\Lambda N) | g_{\pi\Lambda} \rangle X_{\pi\Lambda},$$

$$X_{\pi\Lambda} = \tau_{\pi\Lambda} \langle g_{\pi\Lambda} | G_0(\pi\Lambda N) | g_{\pi N} \rangle X_{\pi N},$$

where $\tau_{\pi i}$ with $i=\Lambda, N$ are the isobar propagators of the $\pi i$ subsystems and $
\langle g_{\pi i} | G_0(\pi\Lambda N) | g_{\pi j} \rangle$ are the one-pion-exchange diagrams. The $\pi N$ and $\pi\Lambda$ separable potentials used in [1] are of the form

$$V_{\pi i}(p, p') = \gamma_{\pi i} g_{\pi i}(p) g_{\pi i}(p'),$$

with

$$g_{\pi i}(p) = p(1 + p^2) \exp(-p^2/\alpha_{\pi i}^2),$$

where the parameters $\gamma_{\pi i}$ and $\alpha_{\pi i}$ were fitted to the position and width of the resonances as given by the Particle Data Group [10]. We list these parameters in Table V as well as the corresponding ones obtained using the relativistic formulation in Ref. 3. Using the parameters listed in the table, the NR model predicts a bound state at about $-110$ MeV while in the case of the relativistic model there is no bound state. If in the relativistic model we replace the one-pion-exchange diagrams by their NR versions we obtain almost the same
results for the Fredholm determinant and consequently no bound state. On the other hand, if we replace the isobar propagators by their NR versions, the Fredholm determinant changes radically giving rise to even deeper bound state. Thus, the problem with the NR model lies in the definition of the isobar propagators.

### TABLE V: Parameters of the $\pi N$ and $\pi \Lambda$ separable potentials Eqs. (30) and (31) for the nonrelativistic (NR) and relativistic (R) models as well as the corresponding isobar propagators evaluated at $W_0 = m_\pi + m_\Lambda + m_N$ and $q_i = 0$.

| Model | $\gamma_{\pi N}$ (fm$^2$) | $\alpha_{\pi N}$ (fm$^{-1}$) | $\tau_{\pi N}(W_0; q_i)$ (fm$^2$) | $\gamma_{\pi \Lambda}$ (fm$^2$) | $\alpha_{\pi \Lambda}$ (fm$^{-1}$) | $\tau_{\pi \Lambda}(W_0; q_i)$ (fm$^2$) |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| NR    | $-0.02116$      | $2.02135$       | $-0.091220$     | $-0.00564$      | $2.523999$      | $-0.042807$     |
| R     | $-0.01463$      | $1.85836$       | $-0.035758$     | $-0.00471$      | $2.236443$      | $-0.016387$     |

The isobar propagators of the relativistic model are given by Eq. (7) of this paper, while the NR ones are given by

$$[\tau_{\pi i}(W_0, q_i)]^{-1} = \frac{1}{\gamma_{\pi i}} - \int_0^\infty p_i^2 dp_i \frac{g^2_{\pi i}(p_i)}{W_0 - m_\pi - m_\Lambda - m_N - p_i^2/2\eta_i - q_i^2/2\nu_i + i\epsilon},$$

(32)

where $\eta_i$ and $\nu_i$ are the usual reduced masses. We give in the table the value of the isobar propagators of the NR and relativistic models for $W_0 = m_\pi + m_\Lambda + m_N$ and $q_i = 0$. As one sees, the NR isobar propagators are about three times larger than the relativistic ones. In addition, from Eqs. (7) and (32) one sees that $\tau_{\pi i}(W_0, q_i) \to \gamma_{\pi i}$ when $q_i \to \infty$, so that from the values of Table V one sees that also in this limit the NR isobar propagators are larger than the relativistic ones and hence artificially boost the attraction, thereby giving rise to the appearance of bound states in the case of a NR theory.

The large differences between the nonrelativistic and relativistic isobar propagators can be understood by observing that the $\pi N$ $\Delta(1232)$ resonance is 154 MeV above the $\pi N$ threshold and the $\pi \Lambda \Sigma(1385)$ resonance is 131 MeV above the $\pi \Lambda$ threshold, i.e., the excitation energies are approximately equal to the mass of the pion and therefore the use of nonrelativistic kinematics is not appropriate.

In Ref. [1] we also presented results based in the relativistic on-mass-shell spectator formalism [11–13] which produced similar bound states as the nonrelativistic formalism. We checked that the problem here lies again in the isobar propagators even though the kinematics is relativistic. The problem, as we pointed out in [1], is that solutions that fit
the experimental data exist only if one puts the light particle (in this case the pion) on the mass shell while physically one expects that rather the heavy particle ($N$ or $Λ$) should be the one staying on the mass shell.

B. Including $\bar{K}NN$

Here we study the effects of expanding the three-body model space from $πΛN−πΣN$ coupled channels to $πΛN−πΣN−\bar{K}NN$ coupled channels. The primary reason to exclude the $\bar{K}NN$ channel from the very beginning was that the three-body quantum numbers $I = \frac{3}{2}, J^P = 2^+$ are compatible only with a Pauli forbidden $I_{NN} = 1, J^P = 1^+$ leading $NN$ configuration. A secondary reason was that although SU(3) predicts a natural-size coupling between the $KN$ and $πY$ two-body channels through the $Σ(1385)$ $p$-wave resonance, there is ample empirical evidence that this coupling is quite weak [14–16]. To extend the relativistic $πΛN−πΣN$ coupled channels calculation, we generalized the $πY$ form factors (12) to include also a coupled $\bar{K}N$ form factor as follows:

$$g_2^Λ(p_2) = p_2(1 + A p_2^2) \exp(-p_2^2/\beta_2^2), \quad g_2^Σ(p_2) = B g_2^Λ(p_2), \quad g_2^KN(p_2) = C g_2^Λ(p_2),$$

(33)

with an overall strength parameter $γ_2$. The fitted parameters, starting with the parameters in the first row of Table II for $C = 0$ and varying $C$ between 0 to 1, are listed in Table VI together with the pole energy with respect to the $πΣN$ threshold as obtained by solving the one-dimensional integral equations corresponding to the Faddeev equations in the AGS form given by Eqs. (25) and (26), with the modification indicated at the end of section III.

It is seen that the $Y = 1, I = \frac{3}{2}, J^P = 2^+$ resonance energy goes up monotonically upon boosting the $\bar{K}N−πY$ coupling via increasing the parameter $C$. For weak coupling the resonance energy is still below the $πΣN$ threshold, but for strong coupling ($C \geq 0.5$) it is above this threshold. Altogether, the variation of the real part of the energy amounts to about 50 MeV upward shift for $C$ between 0 to 1. This is accompanied by a substantial increase of the width from about 5 to 40 MeV. We estimate $C < \sim 0.2$ from studies of $Σ(1385)$ impact on low-energy and subthreshold $\bar{K}$-nucleon [14, 15] and $\bar{K}$-nucleus [16] phenomenology. Hence, it is fair to conclude that the effect of including explicitly a weakly coupled $\bar{K}NN$ channel in the present $πΛN−πΣN$ coupled channels calculation is rather insignificant.
TABLE VI: Fitted parameters of the $\pi Y - \bar{K}N$ form factors \cite{33}, for $A = 0.25$ and a sequence of values $C = 0 \cdots 1$, together with pole energies with respect to the $\pi \Sigma N$ threshold obtained by solving the three-body equations.

| $C$ | $\beta_2$ (fm$^{-1}$) | $\gamma_2$ (fm$^4$) | $B$ | $E$ (MeV) |
|-----|-----------------|-----------------|-----|--------|
| 0.0 | 2.5810          | -0.0091851      | 0.9367 | -19.8−i2.6 |
| 0.1 | 2.5774          | -0.0092150      | 0.9364 | -18.7−i2.8 |
| 0.2 | 2.5668          | -0.0093005      | 0.9356 | -15.6−i3.2 |
| 0.3 | 2.5497          | -0.0094420      | 0.9342 | -10.9−i4.0 |
| 0.4 | 2.5264          | -0.0096400      | 0.9323 | -5.0−i5.2  |
| 0.5 | 2.4978          | -0.0098901      | 0.9299 | +1.8−i6.9 |
| 0.6 | 2.4646          | -0.0101955      | 0.9269 | +8.8−i9.1 |
| 0.7 | 2.4276          | -0.0105512      | 0.9236 | +15.7−i11.1 |
| 0.8 | 2.3876          | -0.0109590      | 0.9197 | +22.2−i14.6 |
| 0.9 | 2.3452          | -0.0114181      | 0.9155 | +27.9−i17.8 |
| 1.0 | 2.3011          | -0.0119291      | 0.9108 | +33.0−i21.2 |

VI. SUMMARY AND CONCLUSIONS

In this work we have formulated and solved a set of relativistic three-body Faddeev equations for $\pi \Lambda N - \pi \Sigma N$ coupled channels in search for a bound state or a resonance with quantum numbers $I = 3/2, J^P = 2^+$. The leading two-body attractive interactions were $p$-wave interactions in the $\pi N$ and $\pi \Lambda - \pi \Sigma$ channels dominated by the $\Delta(1232)$ and $\Sigma(1385)$ resonances, respectively, and to a lesser extent the $^3S_1 YN$ s-wave interactions. These interactions were fitted by energy-independent separable forms constrained by available data. In particular, the $\Delta(1232)$ and $\Sigma(1385)$ members of the SU(3) baryon decuplet were generated dynamically as $p$-wave meson-baryon resonances without recourse to their intrinsic quark structure. A robust $\pi \Lambda N$ resonance some 10–20 MeV below the $\pi \Sigma N$ threshold was found upon solving the relativistic three-body coupled channels equations. This prediction outdates our earlier prediction of a $\pi \Lambda N$ bound state \cite{1,2}, which was based on a nonrelativistic formulation shown here to be inappropriate. Also discussed in the present work was the effect of coupling a $\bar{K}NN$ channel to the $\pi \Lambda N - \pi \Sigma N$ driving channels, which turned out
to be a secondary effect.

We conjecture that the \((I = 3/2, J^P = 2^+)\) \(\pi\Lambda N\) resonance calculated in the present work provides the lowest-mass strangeness \(S = -1\) s-wave dibaryon which we denote \(\mathcal{Y}\). It may be viewed as a \(^5S_2\) \(\Sigma(1385)N - \Delta(1232)Y\) quasibound state with mass \(M(\mathcal{Y})\) over 50 MeV below the lowest threshold \((\Sigma(1385)N)\) and over 150 MeV below the \((I = 1/2, J^P = 2^+)\) \(\Sigma(1385)N - \Delta(1232)\Sigma\) dibaryon configuration which provides the lowest \(S = -1\) dibaryon predicted in quark-gluon dynamics \([17]\). In the present underlying meson-baryon dynamics, with pion assisted dibaryons, the \((I = 1/2, J^P = 2^+)\) \(\Sigma(1385)N - \Delta(1232)\Sigma\) configuration is realized as a three-body \(\pi\Lambda N\) resonance at \(E = 90 - i52\) MeV with respect to the \(\pi\Sigma N\) threshold, for the same two-body interactions that produce the \((I = 3/2, J^P = 2^+)\) \(\pi\Lambda N\) resonance at \(E = -20 - i2.6\) MeV (first row, Table \[IV\]). This difference of about 100 MeV arises because the \(p\)-wave \(\pi B\) interactions in the \(I = 1/2\) three-body configuration are no longer completely exhausted by the resonating \(\Delta(1232)\) and \(\Sigma(1385)\) isobars.

The \((I = 3/2, J^P = 2^+)\) \(\pi\Lambda N\) resonance found in this work is rather narrow. Its ‘fall-apart’ width is seen from Table \[V\] to increase from a few MeV to over 40 MeV as the resonance energy goes up by about 50 MeV. Extrapolating \(\text{Im} \ E\) as a function of \(\text{Re} \ E\), a width of 113 MeV is obtained for \(\text{Re} \ E = 76\) MeV, this latter value providing the excitation energy of \(\Delta(1232)\) with respect to the two-body \(\pi N\) system, assuming the \(\Lambda\) hyperon is at rest. This width is (perhaps fortuitously) close to the free-space \(\Delta\) width of 110 MeV deduced from the input \(P_{33}\) phase shifts.

The small ‘fall-apart’ width does not include the effect of true pion absorption into a \(d\)-wave \(\Sigma N\) lower channel which was disregarded in the present work. Further calculations are necessary to clarify the effect of incorporating this pionless channel in our three-body formulation, but its inclusion is unlikely to disrupt the existence of the \(\pi\Lambda N\) resonance explored here. We note that a \(d\)-wave \(\Sigma N\) configuration is connected by a strong one-pion exchange (OPE) tensor potential to the \(^5S_2\) \(\Sigma(1385)N\) and \(\Delta(1232)Y\) components of the dibaryon \(\mathcal{Y}\). Such OPE tensor transition potential could give rise to a pionless decay width of \(\mathcal{Y}\) in the range of few tens of MeV, employing estimates similar to those made for the width of quasibound \(\Sigma\) hyperon nuclear states arising from the \(\Sigma N(3S_1) \rightarrow \Lambda N(3D_1)\) OPE tensor transition potential \([18]\).

The structure of \(\mathcal{Y}\) is reminiscent of the \(S = 0\) \(I = 0, ^7S_3\) s-wave \(\Delta\Delta\) dibaryon candidate recently observed in double-pion production reactions in \(NN\) collisions \([19]\). The \(\mathcal{Y}\) dibaryon
could also be searched in \( pp \) collisions, say in
\[
p + p \rightarrow \Upsilon^{++} + K^0
\]
\[
\leftarrow \Sigma^+ + p \quad (34)
\]
at energies above the \( \Sigma(1385) \) production threshold. Here, owing to the doubly-positive charge \( Q = +2 \), the decay \( \Upsilon^{++} \rightarrow \Sigma^+ p \) offers a unique decay channel. The production and decay (34) are analogous to those conjectured for the \((Y = 1, I = 1/2, J^P = 0^-) \bar{K}NN\) quasibound state \( K \) in the recent DISTO re-analysis at \( T_p = 2.85 \) GeV [20]:
\[
p + p \rightarrow \mathcal{K}^+ + K^+
\]
\[
\leftarrow \Lambda + p. \quad (35)
\]
Of course, \( \Upsilon \) may also be studied in \( pp \) collisions with outgoing \( K^+ \) meson, but the decay \( \Upsilon^+ \rightarrow (\Sigma^+ n, \Sigma^0 p) \) may not be easily distinguished from the decay \( \mathcal{K}^+ \rightarrow (\Sigma^+ n, \Sigma^0 p) \).
The production of \( \Sigma(1385) \) charge states in \( pp \) collisions with outgoing \( K^+ \) meson has been studied recently in great detail by the HADES Collaboration at GSI [21].

Other possible production reactions are
\[
K^- + d \rightarrow \Upsilon^- + \pi^+
\]
\[
\leftarrow \Sigma^- + n, \quad (36)
\]
\[
\pi^- + d \rightarrow \Upsilon^- + K^+
\]
\[
\leftarrow \Sigma^- + n, \quad (37)
\]
\[
\pi^+ + d \rightarrow \Upsilon^{++} + K^0
\]
\[
\leftarrow \Sigma^+ + p, \quad (38)
\]
or
\[
\pi^+ + d \rightarrow \Upsilon^+ + K^+
\]
\[
\leftarrow \Sigma^+ + n, \Sigma^0 + p, \quad (39)
\]
similar to the E27 experiment scheduled at J-PARC [22]:
\[
\pi^+ + d \rightarrow \mathcal{K}^+ + K^+
\]
\[
\leftarrow \Lambda + p. \quad (40)
\]
This structural similarity between production and decay schemes of $K$ and of $Y$ helps to realize that the proposed ($I = 3/2, J^P = 2^+$) $Y$ dibaryon is related to a dominant $\Sigma(1385)N$ configuration much the same as the ($I = 1/2, J^P = 0^-$) $K$ dibaryon is related to a dominant $\Lambda(1405)N$ configuration. For both dibaryons, pionic three-body decay modes, $K \to \pi \Sigma N$ and $Y \to \pi \Lambda N$ may also provide useful experimental signature, provided they are energetically allowed.

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