Nonlinear Analysis of Factors Effecting on Different Reinforced Concrete Slabs

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Abstract. A nonlinear finite element (FEM) three-dimensional model introduced in this investigation well qualified for the examination of different reinforced concrete (RC) slabs till to failure. Three-dimensional isoperimetric quadratic elements with 20-node used to simulate the concrete, while an embedded within the concrete element of one-dimensional axial members applied to simulate the reinforcing bars. Carbon fiber reinforcement laminates were simulate as element externally attached to the brick element. During the loading process, there is an ideal bond connecting the concrete and reinforcing bars in the analysis. Hardening model of elasto-plastic work using to emulate the performance of concrete in compression then it followed by response with a completely plastic and finally is discontinue at the inception of crushing. Two crack patterns with (fixed orthogonal and a smeared crack model) with a tension-stiffening design has been adopted to simulate the concrete in tension with the incorporation of retained tensile stress of post-cracking models and due to cracking a shear retention design that remodel the rigidity of shear modulus. The Modified-Newton Raphson methods including an incremental-iterative technique used for solving the nonlinear equations of equilibrium. A force convergence criterion used to control the convergence of the solution. Integration rule with 27-Gaussian quadrature used as numerical integration in this analysis. The acquire results from the FE analysis are the slabs load-deflection response and stress distribution with numerous parametric investigations to examine the impact of some significant FE and material parameters. In overall, sufficient compromise between the FE solutions and the laboratory results has been achieved.

1. Introduction

RC slabs are shallow flat fundamental elements whose principal responsibility is to transmit loading acting normal to their plane. Slabs are employed as grounds and roofs of structures, as walls in containers and as bridges to forward moderately massive concentrated loading [1-3].

When dealing with RC slabs, the requirement for openings in slabs displays one of the traditional obstacles experience in construction planning. RC constructions customarily demand strengthening or adjustment at some location throughout their employ duration time [3-6]. The essential for strengthening can stand for an assortment of purposes, incorporate a necessity for rehabilitate the load-carrying ability, a requirement to perform modifications in the construction or insufficiency to determine obstacles that have taken place throughout the installation. Fiber-reinforced polymer (FRP) matrix composites can implement an adequate method for strengthening slabs. Preceding laboratory results reported that CFRP sheets externally-bonded can remarkably improve the flexural capacity of the scheme for those model incorporate in the positive moment section an opening [7, 8].

The FE techniques can be used to evaluate the deformational behavior during the loading history stresses and strains at any stage of loading at selected sampling points in the member. It is a valuable mechanism for examining obstacles with complicated geometry, material characteristics, and boundary condition. It is much cheaper than the full- scale experimental testing [3, 9]. It can be observed and verified interactively and the outcomes can be developed diagrammatically. The FE
method has been used to emulate the full range performance of several RC slabs during linear, for the time of nonlinear response and till the failure. The main objective was to investigate the overall structural behavior of RC slabs using FE models proper for the nonlinear analysis of RC slabs under monotonically improving load through a computer program which was originally developed by [10].

2. FE Techniques and Material Constitutive Relationships
Many researchers had studied the FE method to the analysis of RC members. The shape, size and amount of elements are related to the geometry of the member, material properties, loading conditions, accuracy desired, and computational effort. A full three-dimensional FE idealization has been applied to obtain an estimated solution for constructions including difficult shapes and loading composition. This idealization gives accurate simulation for geometry, type of failure and location of reinforcing bars. The 20-node isoparametric quadratic brick elements are adopted to represent the concrete as displayed in Figure 1. The primary intention of the isoperimetric FE expression is to manage the relationship connecting the element displacements at several points within the element and the element nodal point displacements straight within the use of the shape functions [3, 10, 12].

![Figure 1. Isoparametric brick element and Representation of reinforcement](image)

The principal origins of nonlinearity are tension cracking of the concrete, the inelastic compression behavior of concrete, concrete crushing in compression and the reinforcement yielding. Hardening design of elasto-plastic work managing to simulate the performance of concrete in compression then it succeeded by the response with an entirely plastic and eventually is discontinue at the initiation of crushing. In expressing the subsequent constituents, the model will be represented: crushing condition, hardening rule, yield criterion, and flow rule. However, before cracking a linear elastic model is managed to emulate the performance of tension in concrete. The ultimate major stress principle is the cause for the inception of cracking. Models of fixed orthogonal with a smeared crack is employed to describe the fractured concrete with the incorporation of models for reduced shear modulus and the retained post-cracking stress. Figures 2, 3 and 4 display the compression concrete uniaxial stress-strain drawing, post cracking model for RC and shear-retention model for RC [3, 10, and 12].

![Figure 2. Uniaxial Stress-Strain curve for concrete in compression](image)

![Figure 3. Post cracking model for RC](image)

![Figure 4. Shear-retention model for RC](image)

In tension and compression, the reinforcement stress-strain performance would considered. In RC, steel reinforcement is commonly long and moderately slender, consequently, they will be frequently considered to be proficient in conveying axial forces individually. The uniaxial stress-strain reinforcement behavior is reproduced by an elastic perfectly plastic design as shown in Figure 5 with a yield stress of $f_y$. The use of elastic-perfectly plastic models leads to numerical convergence problems at stages close to the ultimate load strength. The modeling of reinforcing bars in connection with the FE analysis of RC members is much simpler than the modeling of concrete. The modeling of the
carbon fiber reinforced laminates act as externally attached reinforcement to the brick element [3, 10, and 12].

3. Numerical Examination of High (HS) and Normal Strength (NS) RC Slabs

The P3DNFEA is a program of 3-D nonlinear FE analysis that was originally developed by [10] were used in the analysis. This study is a verification study with the effect of different parameters from a part of experimental tests [11] carried out on simply supported RC slabs. The FE analysis has generally been accomplished utilizing the 27 integration rule, with force-based convergence criterion with a convergence tolerance of 5% and a work hardening plasticity pattern for the behavior in compression and Modified Newton-Raphson system KT2a in which the stiffens pattern is modernized at (iterations 2, 12, 22 of each increment of loading). The tensile strength used for the analysis of all slabs was (ft=0.1f'c), and the tension stiffening limitations were (α1=100, α2=0.65), however the selected shear retention limitations were (γ1, γ2, and γ3=10, 0.5 and 0.1). Seven high strength two-way simply supported RC slabs were investigated. Two other slabs were tested for normal strength concrete. A comparison was made between the HS concrete and NC concrete. These RC slabs had measurements of (1700*1700*120mm) and over all four sides were simply supported on round steel bars fixed to a steel plate to resemble roller support. The slabs were extended (50mm) beyond the centerline of the supports. They had variable openings size and locations as shown in Table (1). Steel bars used in the slabs were reinforced including a proportion of reinforcement (1.7%) in each direction. The load was applied at the center of the slabs. All slabs had the same reinforcement ratio. Figure 6 shows the measurements of a typical slab.
### Table 2

| Slab No. | Reinforcement ratio (%) | $f_c'$ | No. of opening | Distance from X-axis (mm) | Size of opening (mm) | Type of concrete |
|----------|-------------------------|--------|----------------|--------------------------|---------------------|-----------------|
| Slab A1  | 1.7                     | 66     | 2              | 100                      | 100 x 100           | Solid slab      |
| Slab A2  | 1.7                     | 66     | 2              | 200                      | 100 x 100           | Solid slab      |
| Slab A3  | .7                      | 66     | 2              | 400                      | 100 x 100           | Solid slab      |
| Slab A4  | 1.7                     | 66     | 2              | 100                      | 200 x 200           | Solid slab      |
| Slab A5  | 1.7                     | 66     | 0              | Solid slab               | Solid slab          | High strength concrete |
| Slab A6  | 1.7                     | 66     | 2              | 200                      | 200 x 200           | Normal strength concrete |
| Slab A7  | 1.7                     | 66     | 2              | 400                      | 200 x 200           | Normal strength concrete |
| Slab A8  | 1.7                     | 25     | 2              | 100                      | 200 x 200           | Normal strength concrete |
| Slab A9  | 1.7                     | 25     | 0              | Solid slab               | Solid slab          | Normal strength concrete |

#### 4. FE Idealizations, Material Properties and Results of the Analysis

By holding into regard the influence of symmetry, one-quarter of the slab is utilized for the FE investigation. The selected quarter has been discretized into (58)-20 node brick elements as presented in Figures 7 and 8. This number of brick elements has been chosen according to a convergence study made to account for accuracy, the time consumed and the computer disk space consumed. All the slabs have the same FE mesh and the same boundary locations. Table 2 displays the material features and FE parameters. The center applied load was described by an equivalent assortment of nodal forces.

| Slab Designation | Slabs A(1-7) High strength concrete | Slabs A8, A9 Normal strength concrete |
|------------------|------------------------------------|-------------------------------------|
| Concrete         | Young's Modulus, $E_c$ (GPa)       | 30                                  | 19                                  |
|                  | Compressive Strength, $f_c'$ (MPa) | 66                                  | 25                                  |
|                  | Tensile Strength $f_t$ (MPa)       | 6.6                                 | 2.5                                 |
|                  | Poisson's ratio, $\nu$             | 0.2                                 | 0.2                                 |
|                  | Uniaxial Crushing Strain, $\varepsilon_{cu}$ | 0.003 | 0.003 |
|                  | Young's Modulus, $E_s$ (GPa)       | 200                                 | 200                                 |
|                  | Yld Stre $f_y$ Top                 | 450                                 | 450                                 |
|                  | Yld Stre $f_y$ Bottom              | 420                                 | 420                                 |
|                  | Area (mm$^2$) Top                  | 79                                  | 79                                  |
|                  | Area (mm$^2$) Bottom               | 201                                 | 201                                 |

The predicted mid-span load-deflection curve obtained for using the FE solutions are associated with the corresponding laboratory curves. The numerical load-deflection curves reveal perfect approval with the laboratory results for the whole of the entire stages of the loading; a slightly stiffer algebraic reaction will be obtained at post cracking stages of behavior. The numerical failure occurred by concrete crushing at the loaded region after yielding of steel reinforcement. The proportions of the predicted terminal load to the corresponding experimental ultimate load for all the analyzed slabs are arranged in Table 3 and Figure 9. Figure 10 shows a comparison of numerical and experimental results of load-deflection curves for every slabs.

1. It was observed that the reduction of the final load within the solid slab A5 and the opening slab A1 about (40%) is achieved.
2. It was observed that when the slab opening measurements enhanced from (50*100mm slab A1) to (100*200mm slab A4) a reduction of (17%) in the final load ability is achieved.
3. It was observed that an improvement in the final load within the slab A1 and the slab A2 when the opening location increases from (100 to 200mm) apart from the connected load about (8%) is achieved.
4. The difference between the HS concrete slab A4 and normal strength concrete slab A8 is about (42%) is achieved.
Figure 7. Isometric view for slabs.

Figure 8. FE mesh, equivalent nodal forces, symmetry and boundary conditions used for slabs. All dimensions are in mm.)
Figure 9. Effect of different openings size on the ultimate load slabs.

Table 3. Comparison of laboratory and analytical ultimate load results for slabs.

| Slab Designation | Exp. Ultimate load (kN) | F.E.M ultimate load (kN) | P_{F.E.M}/P_{Exp.} |
|------------------|--------------------------|--------------------------|---------------------|
| Slab A1          | 370                      | 380                      | 1.027               |
| Slab A2          | 380                      | 410                      | 1.078               |
| Slab A3          | 500                      | 507.9                    | 1.015               |
| Slab A4          | 300                      | 315                      | 1.050               |
| Slab A5          | 520                      | 530                      | 1.016               |
| Slab A6          | 370                      | 377                      | 1.018               |
| Slab A7          | 450                      | 457.5                    | 1.016               |
| Slab A8          | 180                      | 183                      | 1.016               |
| Slab A9          | 270                      | 266                      | 0.985               |

Figure 10. Comparison of numerical and experimental results of load-deflection curves for all slabs.

5. Parametric study

5.1 Meshing size
An essential action in FE modeling is the determination of mesh size. Better results are obtained when sufficient numbers of the element are used in the model. The RC slab (A4) was taken for the
convergence study. The number of elements chosen for modeling one-quarter of the slab was (58) and (48) quadratic isoperimetric brick elements, two response parameters were compared; these are the deflection and the ultimate load as shown in Figure 11. The mesh with (58) elements was found to be efficient and accurate. When the number of elements is decreased from (58 to 48) a reduction in the ultimate load by nearly (2%) is reached. Figure 13 shows the FE meshes used for the slab (A4).

5.2 Thickness of slab
To examine the impact of increasing the thickness of the slab on the overall performance of RC slabs, slab (A2) was numerically analyzed with three separate measures of slab thickness (100, 120 and 140mm). The load-deflection curves received from the FE examination, mutually with the laboratory examination conclusion are displayed in Figure 12. The FE conclusion reports that a significant improvement in the final load potential has been reached by increasing the slab thickness. It was found that when the slab thickness is increased from (120 to 140mm) an improvement of (22%) in the final load capacity is performed. While during the slab thickness is modified from (120 to 100mm) a reduction of (29%) in the final load capability is achieved.

5.3 Brick Element
Slab (A4) has been chosen to study the accuracy of using a various type of brick elements. The slab has been analyzed using the 8 and 20 node brick elements as shown in Figures 14 and 16. The 20 node brick element produced in a predicted response within reach of the laboratory performance. A moderately stiffer response has been achieved for 8 node brick elements. When the type of brick elements is changed from 20-node brick elements to 8-node brick elements an improvement of approximately (14%) in the terminal load occurs.

5.4 Integration Rules
The slab (A4) has been chosen to examine the efficiency of multiple integration rules. The slab has been examined utilizing the 15A, 15B, 14, 8 and 27-point integration rules as shown in Figure 15. The integration rule with a 27-point produced in a predicted response within reach of the laboratory
A moderately stiffer response has been received for 15A, 15B and 14, 8 point rules while the good response is given by 27- integration rule.

Figure 14. Effect of Type of brick elements on the Load-Deflection of RC slabs (A4).

Figure 15. Effect of integration rules on the load-deflection of RC slab (A4).

Figure 16. Type of brick elements used for Slab (A4).

5.5 Convergence tolerance

The convergence of force principle has been utilized in the current study. Slab (A2) was used to investigate the impact of using multiple values for the convergence tolerance of 2, 5 and 8%. The load-deflection curves obtained using a tolerance of 8% is stiffer than the experimental curves and it can be noted that the processing time performed by the computer for a tolerance of 2% was greater than that required for the 5% tolerance. Consequently, the numerical analysis have achieved utilizing a convergence tolerance of 5%. Another analysis was used to study the effect of using displacement and force convergence tolerance. When the convergence tolerance is changed from (2 to 5%) a decreased about (0.7%) is achieved. While when the convergence tolerance is changed from (5 to 8%) an increased about (1.2%) in the final load capacity is obtained. When the force convergence tolerance is changed to a displacement convergence tolerance an increased in the final load capacity around (29%) is performed, as shown in Figures 17 and 18.

Figure 17. Effect of force and displacement convergence tolerance on the Load-Deflection of RC slab (A2).

Figure 18. Effect of convergence tolerance on the Load-Deflection of RC slab (A2).

6. Stress Distribution for slab A4
In the FE analysis, each integration point can be considered as a gauge for measuring six components of stresses and strains at any loading stage. Figure 19 presents the normal stress distribution ($\sigma_x, \sigma_z$) at the bottom face of slab A4 obtained from FE examination at the ultimate converged increment of loading. The figure indicated a maximum compression stress of about (1 MPa) for stress in the x-direction, (1 MPa) for stress in Z-direction, the maximum tensile stress of about (5 MPa) at x-direction and the maximum tensile stress about (5 MPa) in the z-direction. Figure 20 displays the stress arrangement at the top surface of the ($\sigma_x, \sigma_z$) of slab slab4. The figure indicated a maximum compression stress of about (16 MPa) for stress in the x-direction, (13 MPa) for stress in Z-direction, the maximum tensile stress of about (4 MPa) at x-direction and the maximum tensile stress about (1 MPa) in the z-direction.

7. Numerical Example Two Way Simply Supported Slab with Central Opening

Three two-way simply supported RC slabs were examined to study the applicability and efficiency of the selected FE model using the same procedure in the previous analysis but with the effect of FRP. These slabs were selected from the experimental work carried out by [8]. Table 4 reviews the general measurements of the two-way RC slabs. Three models are formed of a standard slab with no opening (SQ1) and a slab with a centrally positioned square opening (SQ2), were tested to failure. Slab (SQ3) had an opening similar to that of SQ2 strength with CFRP plies implemented to the tension face adjoining to the individual side of the opening. Test outcomes confirmed that CFRP laminates externally bonded remarkably enhanced jointly the on the whole flexural capacity and stiffness of slabs with a central opening. A reinforcement ratio of ($\rho=41\%$) of steel bars were used to reinforced RC slabs. Slabs laid down on beam with a length of 1067.5mm and width of 152mm at 762.5mm from the corners. The slabs have the same in reinforcement ratio. Figure 21 shows the slab geometry and reinforcement details. The four symmetrical loads were applied at the center every side of the opening of slabs as shown in Figure 22.

Table 4. Full measurements of the three two-way RC slabs.

| Slab Dimensions (mm) | Slab thickness (mm) | Cutout dimensions (mm) | CFRP strengthening along each side of the opening |
|----------------------|---------------------|------------------------|-----------------------------------------------|
| SQ1 3660 x 3660      | 140                 | Solid                  |-------                                        |
| SQ2 3660 x 3660      | 140                 | 1220 x 1220            |-------                                        |
| SQ3 3660 x 3660      | 140                 | 1220 x 1200            | 3 plies                                       |
8. Strengthened Carbon Fiber Laminates (FRP)
By employing 3 layers per strip of unidirectional carbon FRP laminate the slab SQ3 was strengthened, utilized by standard lay-up. Particular ply was 533 mm wide and 3354 mm length. The quantity of CFRP applied to strengthen slab SQ3 was estimated following the assumption that the lack of steel reinforcement affected by the discontinuity will be substituted by a comparable quantity of FRP related to the subsequent simplistic analogy:

\[ \frac{E_s}{A_{lost}} = 1 \]

The quantity of the lost steel reinforcement is comparable to:

\[ A_s \, lost = N \times A_s = 568 \, \text{mm}^2 \]

where \( N=8 \) is the amount of steel bars that have been cut. By exchange eq. (2) into eq. (1) one can estimate the corresponding area of CFRP as:

\[ A_f = \frac{E_s}{E_f} \times A_s \, lost = 500 \, \text{mm}^2 \]

Considering individual ply has a formal width \( W_f=533 \, \text{mm} \) the required comprehensive CFRP laminate thickness is provided by:

\[ t_{tot} = \frac{A_f}{W_f} \]

Presented that thickness of one plies, \( t =0.165 \, \text{mm} \), the complete quantity of plies demanded is:

\[ n' = \frac{t_{total}}{t'} = 5.7 \]

plies in each directions

By using the influence of symmetry, one-quarter of the slab has been employed for the FE analysis. The selected portion has been discretized into (50) brick elements for slab SQ1 and (64) brick
elements for slabs SQ2 and SQ3 as shown in Figure 23. Table 5 shows material properties and FE parameters. Loads were designated by an equivalent assortment of nodal forces. The strengthened position with FE mesh for SQ3 is shown in Figure 24.

![Figure 23. FE element mesh, equivalent nodal forces, symmetry and boundary conditions used for slabs SQ1 and SQ2, SQ3.](image)

![Figure 24. FE mesh and CFRP laminates details used for slab (SQ3).](image)

| SLAB DESIGNATION | SQ1 | SQ2 | SQ3 |
|------------------|-----|-----|-----|
| **CONCRETE**     |     |     |     |
| Young’s Modulus, $E_c$ (GPa) | 20.56 | 21.22 | 22.16 |
| Compressive Strength, $f_c$ (MPa) | 31 | 33 | 36 |
| Tensile Strength $f_t$ (MPa) | 3.1 | 3.3 | 3.6 |
| Poisson's ratio, $\nu$ | 0.2 | 0.2 | 0.2 |
| Uniaxial Crushing Strain, $\varepsilon_{cu}$ | 0.007 | 0.007 | 0.007 |
| **STEEL**        |     |     |     |
| Young’s Modulus, $E_s$ (GPa) | 200 | 200 | 228 |
| Yield Stress $f_y$ (MPa) | Top | 413 | 413 |
|                  | Bottom | 413 | 413 |
| Bar arrangement | Top | $10 \phi 152.5$ | $10 \phi 152.5$ | $10 \phi 152.5$ |
|                 | Bottom | $10 \phi 152.5$ | $10 \phi 152.5$ | $10 \phi 152.5$ |

These results include an estimate of the numerical load mid-span deflection curves with corresponding laboratory curves. Figure 25 reveals that the numerical load-deflection curves are in perfect approval with the laboratory results at linear stages of loading which shows the same behavior. However, at the post cracking stages, a moderately stiffer numerical response has been achieved. The numerical failure occurred by yielding steel reinforcement followed by the crushing of concrete at loaded regions. The ratios of FE ultimate load to the corresponding experimental ultimate load for all the analyzed slabs are listed in Table 6.

| Slab Designation | Experimental ultimate load (kN) | F.E.M ultimate load (kN) | $P_{F.E.M}/P_{Exp.}$ |
|------------------|-------------------------------|--------------------------|----------------------|
| Slab SQ1         | 489                           | 498                      | 1.018                |
| Slab SQ2         | 336                           | 350                      | 1.0416               |
| Slab SQ3         | 676                           | 638                      | 0.943                |

Table 5. Material properties and FE parameters used for slabs.

Table 6. Comparison of laboratory and analytical ultimate loads results for slab.
9. Conclusions

1. The 3D FE pattern applied in the current work is proficient to reproduce the performance of RC slabs. The general behavior obtained in this research work using the FE models represented by anticipate load-deflection performance and the failure loads are in perfect approval with the laboratory results. The main conclusion drowns from this study, is that the non-linear FE analysis is a powerful tool and it can provide the engineer with important information that cannot be supplied by the experimental tests.

2. For the tension stiffening model applied in the current research, the tension stiffening parameter $\alpha_1$, influences the post-cracking response at early stages after cracking and has a slight effect on the predicted ultimate load. While, the parameter $\alpha_2$, has a significant effect on the load deflection behavior after cracking and the value of the terminate load. The inclusion of a tension-stiffening model is important for avoiding numerical problems in connection with crack formation and propagation.

3. For the shear retention model used in the present study, the shear retention parameter $\gamma_1$, has a slightly effect on the predicted load. The shear retention parameter $\gamma_2$, which has significant effect on the predicted load. While the shear retention parameter $\gamma_3$, has a slightly effect on the load deflection response.

4. It was found that the thickness of the slab affects load-deflection behavior. When the slab thickness is increased from (120 to 140mm) an improvement of (22%) in the final load capacity is obtained. While during the slab thickness is decreased from (120 to 100mm) a reduction of (29%) in the final load capacity is occurred for (slab A2).

5. By comparing the FE results for the solid slab (A5) and those obtained for the slab with an opening (A4) with the opening of dimension (100*200mm), it was observed that the final load capacity is reduced by (40%) because of the presence of the opening. When the measurement of the opening is developed from (50*100mm), slab (A1) to (100*200mm), slab (A4), a modification in the final load of (17%) occurs. It was found that as the distance between the opening and the position of the subjected load increases, the ultimate load is increased, the FE investigation explains that when the position of the opening is transfer from (100mm) Slab (A1) to (200mm) (SlabA2) apart from the location of the concerned load an improvement of (8%) in the final load is performed.

6. From the numerical tests, carried out on the effect of integration rules represented by the accuracy of results, it was found that the type of integration scheme has an inconsequential impact on the behavior. The load-deflection achieved adopting the rule with 27-point has been correlated with those regarded utilizing the (15a, 15b, 14 and 8 point rules). The FE solutions show that the 27-point integration rule results in good results. While the stiffer response has been obtained for the other rules.

7. According to the FE analysis conducted in this work, it was found that the force convergence criterion is more powerful than the displacement criteria. When the force convergence tolerance is changed to a displacement convergence an increased in the final load approximately (29%) is achieved for the slab (A2).
8. Decrease of the ultimate load between the solid slab SQ1 and the slab with opening SQ2 is about (30%).
9. During the opening slab, SQ2 is reinforced with carbon fiber strengthening laminates SQ3 an advance of (82%) in the final load capacity is performed.

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