Electrically tunable topological transport of moiré polaritons

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**Abstract**

Moiré interlayer exciton in transition metal dichalcogenide heterobilayer features a permanent electric dipole that enables the electrostatic control of its flow, and optical dipole that is spatially varying in the moiré landscape. We show the hybridization of moiré interlayer exciton with photons in a planar 2D cavity leads to two types of moiré polaritons that exhibit distinct forms of topological transport phenomena including the spin/valley Hall and polarization Hall effects, which feature remarkable electrical tunability through the control of exciton-cavity detuning by the interlayer bias.

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**1. Introduction**

Exciton-polaritons are half-light, half-matter quantum particles formed through the hybridization of photon with exciton in semiconductors, which are of key scientific and technological interest [1,2]. Atomically thin transition metal dichalcogenides (TMDs) have become a new area to explore polariton physics with excitons in the truly 2D limit [3,4], exploiting the strong light coupling and the exotic physics of exciton’s valley pseudospin [5–18].

Topological properties of polariton arise from its internal quantum degrees of freedom. The dependence of photon’s polarization vectors on momentum introduces a non-Abelian gauge structure that leads to a polarization-dependent anomalous velocity upon passing through medium of inhomogeneous refractive index, i.e., the Hall effect of light [19,20]. With optical selection rule setting correspondence between spin or valley pseudospin of exciton to the polarization of photon in their hybridization [21,22], polaritons can inherit this gauge structure, and the resultant polariton spin/-valley Hall effects are discovered in III-V semiconductors and monolayer TMDs in microcavities [14,23–26]. With the absence of electric coupling, temperature or density gradient can be exploited to drive the flow of the neutral polariton in these systems [27]. For optoelectronic applications, however, it is desirable to have electrical control on the spin/valley transport of these light-matter hybrid particles.

Interlayer exciton (IX) in TMDs heterobilayers features a permanent electric dipole that enables the desired electrostatic control to direct its flow [28–31]. This electric dipole further enables the gate tunability of IX detuning from the cavity mode for engineering the polariton dispersions, as well as stronger exciton-exciton interactions [32]. Remarkably, the formation of moiré pattern due to the lattice mismatch and twisting of the heterobilayers leads to a superlattice landscape that endows IX an extra pseudospin, characterizing its trapping at different locales with distinct optical dipoles in a moiré supercell [33,34]. Various signatures of these moiré IX are recently observed in experiments [31,35–40]. Numerical calculations and experimental observations both show that the IX optical dipole is about one order of magnitude smaller than that of the monolayer ones [33,40–43]. From the measured $\sim$50 meV Rabi splitting values of monolayer exciton-polaritons [5–17], we estimate that the coupling strength between the IX and the microcavity photon can reach several meV, comparable to the IX linewidth in hBN encapsulated heterobilayers [31]. With the possibility to reach strong coupling of IX with microcavity photon, the remarkable properties of moiré IX point to a desirable new polariton system.

Here we analyze the fine structures of polaritons formed by the hybridization of moiré IXs in a TMD heterobilayer with photons in a planar 2D cavity. We show that in a moiré superlattice potential there are three IX eigenmodes per valley that feature left-handed in-plane, right-handed in-plane, and out-of-plane optical dipoles respectively, which can be selectively coupled to cavity TE and TM modes with the conjugating mode spatial profiles and polariza-
tions. These lead to the possibility to engineer two types of moiré polaritons that inherit gauge structures in different ways from the photon constituents. These moiré polaritons exhibit distinct forms of topological transport phenomena including the spin/valley Hall and polarization Hall effects, which feature remarkable electrical tunability through the control of IX-cavity detuning by the interlayer bias.

2. IX in a moiré superlattice and its coupling to light

The optical response of monolayer TMDs is dominated by the Wannier-type intralayer excitons with the momentum space distribution localized near the hexagonal Brillouin zone corners, namely the $\pm k$ valleys. The two valleys are related by a time reversal and have spin-valley locking due to the large spin-orbit splitting ($\sim 30$ meV for the conduction band and 100–500 meV for the valence band) [44]. Van der Waals heterobilayers of TMDs can be formed from two different monolayers. Its band structure has a type-II alignment, i.e., conduction and valence bands edges are formed from two different monolayers. Its band structure has a type-II alignment, i.e., conduction and valence bands edges are formed from two different monolayers. The atomic registry of a local region much smaller than the size of the light cone, see Fig. 1 b. Note that an IX with spin-triplet has a similar structure at a different energy [34]. With the spin-valley locking in 2D TMDs [44], the IX with $\tau = 0$ has spin up (down), and will be denoted hereafter as $X_{\tau = 0} = X_{\uparrow} = X_{\downarrow} = X_{\uparrow \downarrow}$. The spin-up (down) bright IXs at the three main light cones are then $X_{\uparrow k} = X_{\uparrow \downarrow k} = X_{\uparrow \downarrow k}$ and $X_{\downarrow k} = X_{\uparrow \downarrow k}$ with $k < E_h/\hbar c \sim 10^{-3}$ nm$^{-1}$ that is the size of the light cone, see Fig. 1b. Note that an IX with $Q = 0$ are not $C_3$-symmetric since it has a center-of-mass velocity $\propto Q$, in general its photon emission should be elliptically polarized. Meanwhile the three main light cones are related by the $C_3$-rotation, the optical dipole of $\alpha$, $\beta$, and $\gamma$ then have the forms [41]

$$D_{\alpha} = D_{\alpha e} = D_{\alpha e} + D_{\alpha e}$$

$$D_{\beta} = D_{\beta e} + e^{-i2\pi D_{\beta e}} + e^{-i2\pi D_{\beta e}}$$

$$D_{\gamma} = D_{\gamma e} + e^{-i2\pi D_{\gamma e}} + e^{-i2\pi D_{\gamma e}}$$

Here $e_{\pm} = (e_{\pm} \pm i e_{\pm})/\sqrt{2}$ with $e_+ e_-$ and $e_\pm$ the three Cartesian unit vectors. $e_\pm$ and $e_\mp$ components of the optical dipole couple to the in-plane left-right-handed circular ($\sigma_\pm$) and vertical linear ($\sigma_z$) polarized photon, respectively. These with spin-down are the time reversal of Eq. (2).

The IX also feels a moiré induced local-to-local variation of the band structure, i.e., $V_{\text{inter}}(r_i, r_j)$. Since the spatial scale of this variation (determined by the moiré period, usually in the range between several to several tens nm) is much larger than the IX Bohr radius, the effect of $V_{\text{inter}}(r_i, r_j)$ can be approximated by a periodic potential $V(R)$ applied on the IX center-of-mass degree of freedom, with $R = m_0 e_\pm e_\mp m_0 e_\pm$ the center-of-mass coordinate and $m_0 \equiv m_0 + m_0$ the exciton mass. $V(R)$ corresponds to the local gap between the conduction and valence bands, and has been obtained using first-principles calculations and analytical fits [33] (Fig. 1c). Such a periodic potential can be expanded into the Fourier series. Keeping only the leading Fourier components, $V(R) = V_0 e_{\sigma_0 R} e_{\sigma_0 R} e_{\sigma_0 R} + V_0'(e_{-\sigma_0 R} + e_{-\sigma_0 R} + e_{-\sigma_0 R})$, with $V_0, V_0'$ the complex Fourier amplitudes $b_{\sigma_0}$, the primitive reciprocal lattice vectors of the moiré, and $b_{\sigma_0} = -b_{\sigma_0} - b_{\sigma_0}$. $V(R)$ then couples a main light cone to its six nearest-neighbor light cones with a
strength $V_0$ (Fig. 1b). The Hamiltonian of moiré IXs at the main light cones becomes

$$
\hat{H}_K = \left( E_K + \frac{\hbar^2 K_m^2}{2M_e} \right) \sum_{\sigma=\uparrow,\downarrow} \left( |\sigma\rangle \langle \sigma| + |\beta_{\sigma,k}\rangle \langle \beta_{\sigma,k}| + |\gamma_{\sigma,k}\rangle \langle \gamma_{\sigma,k}| \right) + V_0 \sum_{\sigma=\uparrow,\downarrow} \left( |\beta_{\sigma,k}\rangle \langle \beta_{\sigma,k}| + |\gamma_{\sigma,k}\rangle \langle \gamma_{\sigma,k}| \right) + h.c.,
$$

(3)

where $E_K$ can be dramatically tuned by a vertical electric field. For R-type MoSe$_2$/WSe$_2$ heterobilayers, $E_K \sim 1.4$ eV at zero field, and 

$$ab\ ini\to$$
calculations give $V_0 = (-2.0 - 8.9)i$ meV [33]. For twist angles $\gg 3$, the energy difference between the Umklapp and main light cones is $\gg 3 \sqrt{2} k_F \delta$ $\geq 50$ meV (Fig. 1d), much larger than $|V_0|$, so the Umklapp light cones can be well neglected.

$\hat{H}_K$ has three branches of eigenstates (Fig. 2a):

$$
|A_{\sigma,k}\rangle = \frac{1}{\sqrt{3}} \left( |\sigma\rangle \langle \sigma| + |\beta_{\sigma,k}\rangle \langle \beta_{\sigma,k}| + |\gamma_{\sigma,k}\rangle \langle \gamma_{\sigma,k}| \right),
$$

$$
|B_{\sigma,k}\rangle = \frac{1}{\sqrt{3}} \left( |\sigma\rangle \langle \sigma| + e^{i2\pi/3} |\beta_{\sigma,k}\rangle \langle \beta_{\sigma,k}| + e^{i4\pi/3} |\gamma_{\sigma,k}\rangle \langle \gamma_{\sigma,k}| \right),
$$

$$
|C_{\sigma,k}\rangle = \frac{1}{\sqrt{3}} \left( |\sigma\rangle \langle \sigma| + e^{i2\pi/3} |\beta_{\sigma,k}\rangle \langle \beta_{\sigma,k}| + e^{i4\pi/3} |\gamma_{\sigma,k}\rangle \langle \gamma_{\sigma,k}| \right),
$$

(4)

with well separated energies: $E_A = E_X + \frac{\hbar^2 K_m^2}{2M_e} + V_0 + V_0'$, $E_B = E_X + \frac{\hbar^2 K_m^2}{2M_e} + V_0 e^{-i\pi/3} + V_0 e^{i\pi/3}$, $E_C = E_X + \frac{\hbar^2 K_m^2}{2M_e} + V_0 e^{-i\pi/3} + V_0 e^{i\pi/3}$.

$\sigma = +1/ -1$ correspond to $\uparrow / \downarrow$. For $V_0 = (-2.0 - 8.9)i$ meV, $E_A - E_B \approx 9.4$ meV and $E_C - E_A \approx 21.4$ meV, see Fig. 2a. It is straightforward to get their optical dipoles from Eq. (2):

$$
D_A = \sqrt{3} D_{\gamma}, D_B = \sqrt{3} D_{\beta}, D_C = \sqrt{3} D_{\gamma},
$$

(5)

$A$, $B$, and $C$ couple to photons with in-plane circular polarizations $\sigma_+$ and $\sigma_-$, respectively. Whereas $C$ couples to a photon with z polarization (Fig. 2a). In these eigenmodes in the moiré potential, the IX probability distribution is a periodic function of $R$ maximized at $A$, $B$ and $C$ locales in Fig. 1c respectively.

We emphasize that the three distinct IX eigenmodes $A_\sigma$, $B_\sigma$, and $C_\sigma$ originate from the twisting and/or lattice-mismatch induced displacement between $K$ and $K$. Such a displacement results in the three different main light cones $\tau K_m$, $\tau C \hat{K_m}$ and $\tau C' \hat{K_m}$ ($\hat{K_m} \parallel K - K$) for each valley, as well as the emergence of a moiré superlattice. The distinct polarization selection rules of the three eigenmodes come from the local $C_3$ symmetry at $A$, $B$ and $C$ in the moiré supercell [33]. On the other hand, in a lattice-matched heterobilayer with $K = K$ all light cones merge into a single one at $Q = 0$, thus there is only one bright eigenmode with zero kinetic energy. In contrast to $A_\sigma$, $B_\sigma$ and $C_\sigma$ eigenmodes of the moiré super-
lattice, the optical dipole in a lattice-matched heterobilayer depends sensitively on the interlayer registry, and generally contains all $e_x$, $e_y$, and $e_z$ components.

**3. Moiré IX-polaritons and their topological properties**

Here we analyze the coupling between the moiré IX and the cavity photon. Considering the distinct optical dipoles of $A_\sigma$, $B_\sigma$ and $C_\sigma$, we need to analyze the electric field profile of the cavity mode first. A 2D planar cavity hosts two types of modes with the mode profiles (Fig. 2b, also see the Supplementary materials for details)

\[
\begin{align*}
\mathbf{u}_{\sigma,\mathbf{k}} &= \sqrt{2} \sin(kz)\left(-\sin \theta_k \mathbf{e}_x + \cos \theta_k \mathbf{e}_y\right), \\
\mathbf{u}_{\sigma,\mathbf{k}} &= \sqrt{2} \left(\frac{k_x \sin(kz)}{(k_x^2 + k_z^2)^{1/2}} \cos \theta_k \mathbf{e}_x + \frac{k_y \sin(kz)}{(k_x^2 + k_z^2)^{1/2}} \sin \theta_k \mathbf{e}_y - \frac{i k \cos(kz)}{(k_x^2 + k_z^2)^{1/2}} \mathbf{e}_z\right).
\end{align*}
\]

$k = k(\cos \theta_k, \sin \theta_k)$ is the in-plane wave vector. TE mode couples to $A_\sigma$ and $B_\sigma$. IX eigenmodes of the in-plane optical dipoles, TM mode can couple to both the $A_\sigma$, $B_\sigma$ IX eigenmodes and the $C_\sigma$ eigenmode with the out-of-plane dipole. Keeping up to the linear term of $k$, it is convenient to use the basis $|\mathbf{s}_k\rangle = \frac{\hbar \mathbf{k}}{\sqrt{2}} e^{i \theta_k}$, where the moiré IX-cavity coupling is

\[
\mathcal{H}_{X-C} = \sin(kz) \left( g_A |A_{\uparrow,\mathbf{k}}\rangle + g_B |B_{\downarrow,\mathbf{k}}\rangle |+k\rangle \right.

+ \sin(kz) \left( g_A |A_{\downarrow,\mathbf{k}}\rangle + g_B |B_{\uparrow,\mathbf{k}}\rangle |-k\rangle \right)

+ \cos(kz) \left( h \mathbf{k} v \left( |C_{\downarrow,\mathbf{k}}\rangle + |C_{\uparrow,\mathbf{k}}\rangle \right) \frac{e^{i \theta_k} |+k\rangle + e^{-i \theta_k} |-k\rangle}{\sqrt{2}} + \text{h.c.} \right)\]
When $\cos(k_z) = 0$, $C_p$ mode of IX is decoupled from the cavity. The Hamiltonian of such AB-polaritons is

$$
\hat{H}_{C-Pol} = \hat{H}_C [C_{A,k} | A_{k} | C_{A,k}] + \hat{H}_D [C_{B,k} | B_{k} | C_{B,k}] + \hbar \omega_k (|+k\rangle \langle +k|)
\quad + \langle g_a | A_{k} | g_B | B_{k} \rangle (|+k\rangle \langle +k| + |E_i | A_{k} | A_{k} |)
\quad + \langle E_B | B_{k} | B_{k} | + \hbar \omega_k (|-k\rangle \langle -k|)
\quad + \langle g_a | A_{k} | g_B | B_{k} \rangle (|-k\rangle \langle -k|),
$$

which can be separated into two decoupled subspaces $|+k, A_{k}, B_{k}\rangle$ and $|-k, A_{k}, B_{k}\rangle$ related by a time reversal.

Fig. 3c, d show the calculated AB-polariton dispersions with the three well separated branches. Each branch is two-fold degenerate.

When $\sin(k_z) = 0$, or when $A_0$ and $B_0$ modes are far red detuned from the cavity (Fig. 2c), we can just focus on the coupling of $C_0$ mode with the cavity photon. The Hamiltonian for such C-polaritons is

$$
\hat{H}_{C-Pol} = \hat{H}_C [C_{A,k} | C_{A,k}] + \hat{H}_D [C_{B,k} | C_{B,k}] + \hbar \omega_k (|+k\rangle \langle +k|)
\quad + \langle g_a | C_{A,k} | g_B | C_{B,k} \rangle (|+k\rangle \langle +k| + |E_i | C_{A,k} | C_{A,k} |)
\quad + \langle E_B | C_{B,k} | C_{B,k} | + \hbar \omega_k (|-k\rangle \langle -k|)
\quad + \langle g_a | C_{A,k} | g_B | C_{B,k} \rangle (|-k\rangle \langle -k|).
$$

Here $|C_{B,k}\rangle \equiv (|C_{A,k}\rangle + |C_{A,k}\rangle) / \sqrt{2}$ couples to the cavity TM mode, whereas $|C_{B,k}\rangle \equiv (|C_{A,k}\rangle - |C_{A,k}\rangle) / \sqrt{2}$ is dark. The resulted four branches of eigenstates are the cavity TE photon (TE) and dark IX (D), as well as the lower-polariton (LP) and upper-polariton (UP) that are the hybridizations of the cavity TM photon with the bright IX (Fig. 4a).

When the IX is subject to an in-plane external force $F$, e.g., the gradient of an interlayer bias introduced by a split gate (see Fig. 3a), the polariton's momentum is then driven: $h \frac{d}{dt} \mathbf{k}(t) = \mathbf{F}$. Here $\mathbf{F} = \rho_{A,k} \mathbf{F}$, $\rho_{A,k}$ being IX's weighting in the nth polariton branch. As the polariton inherits both the IX spin and photon polarization, the driving force can introduce spin and/or polarization dependent transport, originating from the gauge structures of the photonic component (Fig. 3b).

For the AB-polariton, we write the two degenerate eigenstates of the nth branch as $|n_{\pm,k}\rangle (n = 1, 2, 3$, see Fig. 3c, d). In each AB-polariton branch, the dependence of the internal structure of polariton, i.e., polarization vector of the photonic component, on the momentum $\mathbf{k}$, leads to an Abelian Berry curvature $\Omega_{nk}^{\mathbf{k}} = i |n_{\pm,k}\rangle \times |n_{\pm,k}\rangle / (e^2 \epsilon' k^2)$, which quantifies the spin Hall response at $\mathbf{k}$.

Fig. 3. (Color online) The dispersion and polarization/spin Hall conductivity of the AB-polariton. (a) The gradient of an interlayer bias introduced by a split gate can apply an in-plane force $F$ on the IX. (b) Illustration of the polarization Hall effect driven by $F$. (c, d) The AB-polariton dispersions for two different IX-cavity energy detunings, with coupling strengths $g_A = 5 \text{ meV}, g_B = 3 \text{ meV}$. The green dashed curves are the cavity mode and $A_0, B_0$ IX dispersions without coupling. The color scale indicates the brightness (photon fraction) of the polariton. (e-f) $\Omega_{nk}^{\mathbf{k}}$, which quantifies the photon polarization Hall response at $\mathbf{k}$, for the three polariton branches $n = 1, 2, 3$. (g, h) $\Omega_{nk}^{\mathbf{k}}$, which quantifies the spin Hall response at $\mathbf{k}$. 

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for a certain polariton branch can be widely tuned by varying the polarization Hall currents quantified by $X$ and its constituent, but its strength is several orders of magnitude materials). The polariton can also inherit the Berry curvature from the IX constituent, and its strength is several orders of magnitude smaller than that from the photon.

The driving force $\mathcal{F}$ then introduces an anomalous velocity $\pm \Omega_{\mathbf{k}}^{(s)} \times \mathcal{F}$. Both the IX spin $S_{\mathbf{k}} = |A_{\mathbf{k}}|A_{\mathbf{k}} - |A_{\mathbf{k}}|A_{\mathbf{k}} + |B_{\mathbf{k}}|B_{\mathbf{k}} - |B_{\mathbf{k}}|B_{\mathbf{k}}$, and photon polarization $\sigma_{\mathbf{k}} = |+\mathbf{k}|(+\mathbf{k}) - |-\mathbf{k}|(-\mathbf{k})$ have opposite expectation values in the doubly degenerate $|\pm\mathbf{k}\rangle$, so the anomalous velocity gives rise to the spin and polarization Hall currents quantified by $\Omega_{\mathbf{k}}^{(s)} = \rho_{n\mathbf{k}}(S_{\mathbf{k}}^z)\mathcal{F}_{\mathbf{k}}$, and $\Omega_{\mathbf{k}}^{(\sigma)} = \rho_{n\mathbf{k}}(\mathbf{n}_{\mathbf{k}}\sigma_{\mathbf{k}})\mathcal{F}_{\mathbf{k}}$, respectively. We show the calculated polariton dispersions in Fig. 3c, d and the corresponding $\Omega_{\mathbf{k}}^{(s/p)}$ under different IX-cavity detuning in Fig. 3e–h, which indicates that the spin and polarization Hall conductivities for a certain polariton branch can be widely tuned by varying the IX-cavity detuning.

In contrast, the C-polariton has a vanishing Berry curvature and thus zero anomalous velocity in each branch (see the Supplementary materials). Nevertheless, due to the $k$-dependence of the TE/IM in-plane polarizations, the momentum-space motion driven by the force can coherently mix TE and TM, leading to a $k$-dependent circular polarization $\langle \tilde{\sigma}_z \rangle$. Unlike the anomalous Hall velocity of the AB-polariton, the topological nature of C-polariton manifest in this non-equilibrium polarization $\langle \tilde{\sigma}_z \rangle$, which, combined with the group velocity $\frac{d\omega_{\mathbf{k}}}{dk}$, results in a polarization current. Analysis shows that the finite Berry curvature of the AB-polariton originates from the $\mathbf{k}$-dependence of the vertical electric field for the cavity TM mode (see Eq. (7), whereas $\langle \tilde{\sigma}_z \rangle$ of the C-polariton originates from the $\mathbf{k}$-dependence of in-plane directions of TE/IM polarizations (see the Supplementary materials for details).

For the C-polariton UP/LP branch, we find the driving force introduces a photon polarization $\langle \tilde{\sigma}_z \rangle_{UP/LP} = \frac{\mathbf{E}_{\mathbf{UP/LP}} \cdot \mathbf{B}_{\mathbf{UP/LP}}}{\mathbf{E}_{\mathbf{UP/LP}} \cdot \mathbf{B}_{\mathbf{UP/LP}}}$ up to the linear order in $\mathcal{F}$ (see the Supplementary materials). Note that both $\langle \tilde{\sigma}_z \rangle_{UP}$ and $\langle \tilde{\sigma}_z \rangle_{LP}$ are proportional to $\rho_{LP/UP}$ which is the product of the IX and photon fractions. This implies that those strongly hybridized polaritons contribute most significantly to polarization current, as the driving force only affects IX component whereas the photon structure comes from the photonic component. In Fig. 4b, c, we show $\langle \tilde{\sigma}_z \rangle_{UP}/\mathcal{F}$ and $\langle \tilde{\sigma}_z \rangle_{LP}/\mathcal{F}$ as functions of $\mathbf{k}$ when $\mathcal{F}$ is along the $y$-direction. Note that the above result does not apply to $k \to 0$ or very large $k$ value. For these two limits either $\hbar \omega_{\mathbf{k}} - E_{UP} \to 0$ or $E_{UP} - \hbar \omega_{\mathbf{k}} \to 0$, and our perturbative treatment breaks down.

The total polarization current depends on the polariton $\mathbf{k}$-space occupation. If the occupation is isotropic, then the current direc-
tion is perpendicular to the driving force, which corresponds to a polarization Hall effect. On the other hand, one can selectively excite a polariton branch at given \( \mathbf{k} \) by choosing the frequency and incident direction of the excitation, and the resultant polarization current can be non-perpendicular to \( \mathbf{F} \).

The above polarization Hall effect for the C-polariton can also be understood from a spin model under the effect of a time-dependent Zeeman field. Introducing the spin-1 Pauli matrices 

\[
\sigma_i = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \sigma_x = \frac{1}{2} \left( \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \sigma_y \right), \quad \sigma_y = \frac{1}{2} \left( \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)
\]

with \( \sigma = \sigma_x \sigma_y \). Generally, the polarization generated by Rayleigh scattering can also be understood from the instantaneous direction of the field but only changes the momentum of polariton [23,24]. A polariton can also cause effects. Combined with the electrical tunability from IX’s permanent electric dipole, the Hall conductivities and the current direction of the moiré polaritons feature remarkable electrostatic control through an interlayer bias.

For polaritons subject to a periodic potential, earlier works have also shown the possibility to engineer polariton bands of nontrivial topological number exploiting an applied Zeeman field and a large TM-TE splitting of cavity mode [50,51], which leads to protected chiral edge channels. In contrast, the topological transports addressed here are the bulk effects naturally endowed by the moiré pattern, where the moiré modulations of exciton energy and optical dipole result in the unique exciton dispersions and pseudospin splitting for hybridizing with the cavity photon. Compared to the small gaps for the exploitation of the chiral polariton edge channel [50,51], the energy scale for exploring the bulk topological transport is characterized by the O(10) meV splitting between the polariton branches. This, combined with the large excitonic binding energies in TMDs, implies that the effect can be potentially explored for room temperature operation.

The formation of the moiré polariton requires strong coupling between the IX and the cavity photon, i.e., the coupling strength (estimated to be several meV in MoSe$_2$/WSe$_2$) should be larger than the IX linewidth. The IX optical dipole can be further enhanced through reducing the interlayer distance with pressure or use heterobilayers with strong hybridization between inter- and intralayer excitons [38]. For the linewidth, in contrast to the ~2 meV value for intralayer excitons in ultraclean monolayer samples [52,53], the IX linewidth is found to be ~5 meV [31] which should be limited by the inhomogeneous broadening from spatial variation. Better heterobilayer sample qualities can help to further reduce the IX linewidth.

We estimate that the moiré polariton discussed here can be explored over a range of twist angle \( \delta \theta \) from \( \sim 3 \) to \( \sim 10 \) (with a moiré period from 6 to 2 nm). The upper limit is based on the consideration of having a well-defined moiré superlattice (for \( \delta \theta \) larger than 10$^\circ$, the moiré starts to lose the periodicity). The lower limit is from the following considerations. Note that the energy difference \( \Delta E \) between the Umklapp and main light cones is \( \propto \delta \theta \) (see Fig. 1b, d). For \( \delta \theta \lesssim 3^\circ, \Delta E \gtrsim 50 \) meV, much larger than the moiré potential induced coupling strength \( |V_0| \approx 9 \) meV between the Umklapp and main light cones. So the three main light cones (with large optical dipoles) are effectively decoupled from the Umklapp ones (with nearly zero optical dipoles). The main light cones hybridize to form the three \( A_\theta \), \( B_\theta \) and \( C_\theta \) IX eigenmodes, all with large optical dipoles. For smaller \( \delta \theta \), the coupling \( V_\theta \) can effectively mix the main and Umklapp light cones, leading to a large number of split eigenmodes with small optical dipoles (each mode having a small fraction of the main light cones). This will complicate the analysis of polariton and may hinder the realization of a strong coupling regime. In the regime \( \delta \theta \lesssim 3^\circ \), the energies of \( A_\theta \), \( B_\theta \) and \( C_\theta \) IX eigenmodes are 

\[ E_{A_\theta} = E_{\text{X}} + \frac{\delta \theta}{2 \Delta K} + V_0 + V_\theta, \quad E_{B_\theta} = E_{\text{X}} + \frac{\delta \theta}{2 \Delta K} + V_0 - V_\theta, \quad E_{C_\theta} = E_{\text{X}} + \frac{\delta \theta}{2 \Delta K} - V_0 \]

and 

\[ E_{\text{X}} = E_{\text{X}} + \frac{\delta \theta}{2 \Delta K} \]

Their energy splitting is determined by \( V_\theta \), insensitive to the twist angle. The change of twist angle \( \delta \theta \) can result in an overall energy shift through the common \( \frac{\delta \theta}{2 \Delta K} \) term for all three branches, but does not change their splitting.

### Conflict of interest

The authors declare that they have no conflict of interest.

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Author contributions

Hongyi Yu and Wang Yao conceived and designed the research. Hongyi Yu performed the calculations. Hongyi Yu and Wang Yao wrote the manuscript.

Appendix A. Supplementary materials

Supplementary materials to this article can be found online at https://doi.org/10.1016/j.sciib.2020.05.030.

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