Properties and challenges of hot-phonon physics in metals: MgB$_2$ and other compounds

Emmanuele Cappelluti$^a$, Fabio Caruso$^b$, Dino Novko$^{c,d}$

$^a$Istituto di Struttura della Materia, CNR (ISM-CNR), 34149 Trieste, Italy
$^b$Institut für Theoretische Physik und Astrophysik, Christian-Albrechts-Universität zu Kiel, D-24098 Kiel, Germany
$^c$Institute of Physics, 10000 Zagreb, Croatia
$^d$Donostia International Physics Center (DIPC), 20018 Donostia-San Sebastián, Spain

Abstract

The ultrafast dynamics of electrons and collective modes in systems out of equilibrium is crucially governed by the energy transfer from electronic degrees of freedom, where the energy of the pump source is usually absorbed, to lattice degrees of freedom. In conventional metals such process leads to an overall heating of the lattice, usually described by an effective lattice temperature $T_{ph}$, until final equilibrium with all the degrees of freedom is reached. In specific materials, however, few lattice modes provide a preferential channel for the energy transfer, leading to a non-thermal distribution of vibrations and to the onset of hot phonons, i.e., lattice modes with a much higher population than the other modes. Hot phonons are usually encountered in semiconductors or semimetal compounds, like graphene, where the preferential channel towards hot modes is dictated by the reduced electronic phase space. Following a different path, the possibility of obtaining hot-phonon physics also in metals has been however also recently prompted in literature, as a result of a strong anisotropy of the electron-phonon (el-ph) coupling. In the present paper, taking MgB$_2$ as a representative example, we review the physical conditions that allow a hot-phonon scenario in metals with anisotropic el-ph coupling, and we discuss the observable fingerprints of hot phonons. Novel perspectives towards the prediction and experimental observation of hot phonons in other metallic compounds are also discussed.

Contents

1 Introduction

2 Energy transfer and relaxation processes in time-resolved pump-probe experiments
| Section | Title                                           | Page |
|---------|-------------------------------------------------|------|
| 3       | Detecting hot phonons                           | 16   |
| 4       | Anisotropic electron-phonon material: MgB$_2$    | 21   |
| 5       | Hot-phonon physics in MgB$_2$                   | 25   |
|         | 5.1 Time-resolved Raman spectroscopy            | 27   |
|         | 5.2 Time-resolved correlated lattice dynamics    | 31   |
|         | 5.3 Time-resolved reflectivity                   | 33   |
| 6       | Wider view on other compounds and perspectives  | 37   |
| 7       | Concluding remarks                              | 41   |
1. Introduction

After years of steady formidable improvements and refinements, time-resolved spectroscopy, nowadays scaled down to femtosecond resolution, has become one of the major tools for investigating the physical properties and underlying mechanisms of condensed matter [1–6]. The increasing interest in this line of research is stimulated by relentlessly expanding possibilities to achieve control of quantum phenomena using light pulses. Typically, a pump-probe setup is employed, where a laser pump at \( t = 0 \) triggers electronic particle-hole excitations. The pump can also be calibrated to couple directly with lattice, magnon or other bosonic degrees of freedom. Alternatively, coherent modes (phonon, collective excitations, etc) can be also triggered in [7–16]. A later probe (optical, electronic, transport, diffraction) is thus used to assess the physical properties at a time delay \( t \). Within this context, pump-probe experiments have proven to be able to induce a variety of physical processes otherwise not allowed (or very slow) under steady conditions, as for instance insulator-metal transitions [17–23], light-induced superconductivity [24–29], chemical reactions [30–34], structural phase transitions [17, 35], local heat-transport, non-linear optics [14, 36, 37], exciton dynamics of relevance for light harvesting [38–40], detecting the effects of surface plasmons and/or other collective modes [41, 42], manipulation of magnetic degrees of freedom [43–46] etc. In accordance with the different physical processes under examination, a wide variety of time-resolved techniques have been developed, encompassing angle-resolved photoemission, electromagnetic response (including optics, magneto-optics, transport, Raman), electron and X-ray diffraction and other techniques. Insomuch laser pumping can be used to excite bound states, it can provide also a promising tool for encoding quantum information in selected quantum states [47]. The control and the possible tailoring of the decay processes of such excited states is thus essential for assessing the time scales over which the bound state persists, and for ruling the interactions with other degrees of freedom, and hence the possibility of reading out the information itself.

Among other degrees of freedom present in a real material, collective lattice vibrations (phonons) play a pivotal role in ultrafast pump-probe dynamics. The energy initially induced into the electronic states by laser pumping is typically transferred to the lattice within few picoseconds, mostly in the form of excitation of phonon modes [2, 48–63]. In most cases, the energy in the lattice sector is redistributed over all the lattice modes via phonon-phonon interactions, leading to a overall thermal heating which is usually detrimental for sustaining coherent states. As discussed in the next Section, however, in specific compounds and under specific conditions, energy transfer can occur between electrons and few selected phonon modes, which become hot, i.e., they acquire a larger (non-thermal) population than other lattice vibrations. This scenario is particularly appealing for several reasons. On the one hand, quenching the energy transfer from electrons to few phonon modes can contribute to limit the overall heating of the remnant lattice modes, and hence to sustain coherent states. On the other hand, hot-phonon modes are typically strongly coupled with electrons, so that
their presence can increase the electronic damping. Manipulating hot-phonon physics can help in optimizing high- and low-$T_c$ superconducting materials. Furthermore, hot-phonon modes, when associated with an additional degrees of freedom as phonon chirality [64–68], can represent a promising two-level state, useful for quantum-information purposes [69]. For these reasons a deep understanding of the processes and of the required physical conditions leading to the onset of hot-phonon modes is a crucial goal for a robust control of mechanism of ultrafast dynamics in pump-probe experiments.

For many years, the investigation of hot phonons has been focused on semiconductors and semimetals [13, 70–92], whereas standard metals have been considered as an unfavourable premise for sustaining hot phonons. In semiconductors and semimetals, indeed, hot phonons stem from the reduced phase space for electronic decays. Laser-induced particle-hole excitations are localized in few spots of the Brillouin zone close to the valleys (or Dirac points, in case of semimetal graphene). Electron-phonon scattering is thus operative only for the $q$-momenta connecting available valleys/Dirac cones. Only such few modes can profit of the energy transfer from the electronic degrees of freedom, at expenses of other lattice modes that remain cold (Fig. 1a).

Metals appear at a first glance at odds with such scenario. Large Fermi surfaces can trigger laser-induced particle-hole excitations in a vast part of the Brillouin zone. Electron-phonon scattering, in conventional metals, is also weakly $q$-momentum dependent, so that a plethora of lattice modes with different momenta $q$ can be effectively excited (Fig. 1b), leading to global heating of the phonon degrees of freedom without any preferential lattice mode [2, 48–63].

Midway between insulators and metals there are the so-called “bad metals”, characterized by finite but poor metallic properties. A particular mention among them goes to high-$T_c$ superconducting cuprates, layered materials that display strong anisotropy between the in-plane and out-of-plane features. Even restricting only to the in-plane physics, which is responsible for the Cooper pairing, the metallic character is highly non-trivial, having a very complex phase diagram, including an antiferromagnetic Mott insulator phase at extremely low doping up to a half-filling, a pseudogap phase in the underdoping region, where dynamical/short-range correlations appear in the coupled magnetic-electronic-lattice response, and a more conventional metallic phase in the overdoped regime. Such a rich phase diagram is thought to stem out from the underlying presence of one (or two) critical points associated with magnetic and/or lattice instabilities. Although the electron-phonon coupling was initially disregarded as a relevant source of scattering [102, 103], there are nowadays several evidences that it might play an important role in the physical properties of the normal and superconducting states. Quite crucial is thought to be the electron-phonon coupling associated with the changes of the distance/angle of the Cu-O-Cu bond in the CuO$_2$ layer that represents the relevant unit block [104]. The relevance of such electron-phonon interaction is shown in the observation of a remarkable kink in the angle-resolved-photoemission spectroscopy (ARPES) [105, 106]. The energy of such kink matches the energies of the optical modes associated with the changes in the Cu-O-Cu bond, providing thus a
Figure 1: Representative examples of electronic phase space available in semiconductors or semimetals (panel a) and in metals (panel b) upon particle-hole pump-driven excitations. For graphical reasons we show two paradigmatic two-dimensional systems: semimetal single-layer graphene, where the onset of hot phonons has been discussed extensively in Refs. [13, 79–92], and single-layer 1T-TaS$_2$, which is a metal suitable for pump-probe experiments in both the 1T and 1H phases [22, 93–99]. For simplicity, we show only the conduction, high-energy band where electrons are upgraded upon absorption of the photon energy. Top panels represent electronic dispersion and bottom panels momenta in the Brillouin zone where the electrons are excited. For a semiconductor (panel a) only electronic states with momenta close to the valleys are populated (blue areas). Electron-phonon coupling involves inelastic scattering with low-energy exchange, so that the initial electron can scatter only into empty states, i.e., not all possible phonons are exchanged but only phonons connecting intravalley and intervalley processes (red arrows). Only these selected modes can thus absorb energy from the electronic degrees of freedom. In metals (panel b) many electrons in the Fermi sea can be excited in different parts of the Brillouin zone (blue regions). Many different electron-phonon driven processes (red arrows) can occur, scattering an excited electron into available empty states, with very different momenta. These ground state properties were obtained by means of the density-functional-theory package QUANTUM ESPRESSO [100, 101] (in both cases 24 × 24 × 1 momentum grid is used in order to sample the Brillouin zone; unit cell parameters 2.461 Å and 3.333 Å are used for graphene and 1H-TaS$_2$, respectively.).

fingerprint of the key role of these modes [107]. The preferential coupling of the electronic degrees of freedom with these lattice modes has been as well investigated by means of time-resolved ultrafast spectroscopy [11, 43, 105, 111], that reveals a hot-phonon scenario where the energy transferred to the electronic sector by the pump laser is preferentially transferred in a shorter time to such optical modes, that become hot, and on a much longer time scale to the other
lattice modes. So far, however, because of the high sensitivity of the physical properties on extrinsic conditions (e.g. disorder, substrates, etc.), the evidence of hot phonons in cuprates has been used only as a possible way for characterization, rather than as a suitable tool for manipulating the electronic/lattice degrees of freedom.

Merging together the advantages of a good metallic character with suitable conditions for hot phonons, a novel path for sustaining hot-phonon physics in metals has been recently pointed out, as a result of a strong anisotropy of the electron-phonon coupling [112, 113]. The benchmark example of this scenario is magnesium diboride MgB$_2$, a metal compound that, besides supporting evidence of hot phonons, displays a significant superconductivity with critical temperature as high as $T_c \approx 39$ K [114]. The key ingredient responsible for hot phonons in MgB$_2$ is the peculiar property of having a large part of the electron-phonon coupling $\lambda$ concentrated in few phonon modes [115]. The possibility of sustaining hot phonons in a metal opens interesting perspectives in the field of time-resolved ultrafast dynamics, both on the application side as well as for understanding the underlying physical mechanisms. The electron-phonon coupling in metals plays indeed a crucial role in determining a variety of physical properties [116], resulting in a dynamical renormalization of single-particle excitations, as detected for instance in angle-resolved photoemission spectroscopy and in optics; governing the linewidths (and hence lifetimes) of electrons and of many collective modes (including phonons); ruling the transport properties, and; least but not last, in providing the fundamental glue for the superconducting Cooper pairing [117–119].

The investigation of possible ways of detecting hot-phonon modes and of the physical consequences of such scenario in MgB$_2$ and similar compounds is just in its early stages and the full potential is still to be explored. Furthermore, the physical conditions supporting hot-phonon physics in MgB$_2$ are not unique of this compound but they are expected to be realized in many other metals and/or charge-doped semiconductors. Aim of the present review article is to encompass a useful overview of the state-of-art knowledge and to provide a solid basis and guideline for future development in this field. The review is structured as follow: in Sec. 2 we summarize the physical mechanisms governing time-resolved dynamics in pump-probe setups, with a specific focus on the energy-transfer paths from electrons to lattice degrees of freedom, and on the conditions for sustaining hot-phonon physics; various state-of-the-art studies discussing hot-phonon physics along with the corresponding experimental techniques are reviewed in Sec. 3; in Sec. 4 we discuss the specific properties of MgB$_2$, governed by a strong anisotropy of the electron-phonon coupling; the consequences of such anisotropy of the electron-phonon coupling with respect to the onset and observation of hot-phonon physics in MgB$_2$ will be examined in Sec. 5 while in Sec. 6 we discuss how a similar scenario can be generalized in other materials.
2. Energy transfer and relaxation processes in time-resolved pump-probe experiments

Condensed-matter physics is in principle a forbidding complex topic where many degrees of freedom (electrons and ions) are coupled together through different sources of interactions. In spite of such complexity, we rely on a robust understanding of most of the physical properties of solids in terms of simplified but valid modelings. The Drude model for instance, as well as the Fermi-liquid theory at a more sophisticate level, provides a sufficient description of many conventional metals in terms of non-interacting (or weakly interacting) particles with few effective parameters, notwithstanding a large Coulomb-driven electron-electron interaction. The complex interaction between electron and ions can be also simplified thanks to the Born-Oppenheimer principle that permits a useful decoupling of the quantum nature of electrons and ions. The further quantum interaction between electrons and lattice vibrations (phonons) beyond the adiabatic Born-Oppenheimer approximation can be conveniently treated, thanks to Migdal’s theory, by means of an effective mean-field-like approach, the Eliashberg theory [116], which fulfills the Fermi-liquid picture in the normal state and provides a solid ground for a quantitative understanding of the superconducting properties in low-$T_c$ compounds. As a general rule, most of our “textbook” understanding of condensed matter applies to systems at thermodynamical equilibrium, where the different degrees of freedom are described by common thermodynamical parameters, more in particular by a unique temperature $T$.

Fundamental aim of pump-probe experiments is to perturb such initial thermodynamical state by bringing the systems under non-equilibrium conditions, and to investigate in real time the physical properties during the relaxation processes until the system reaches a new final equilibrium. Typical pumping lasers (or synchrotron light) are set at energies much higher than phonons or other collective modes, so that the incoming pump light mainly induces particle-hole excitations in the electronic degrees of freedom. Interesting refinements of the pump-probe setup also exploit laser light in a non-linear regime, as for instance when an impinging photon can trigger more complex excitations that a single particle-hole one [2], or when a sudden sharp pulse can induce coherent (macroscopical) phonon oscillations. It is worth to remark here that the hot-phonon physics (which in principle might occurs for any generic momentum $q$), as related to decay processes governed by the electron-phonon interaction, follows a basically different path than the generation of coherent phonons, which corresponds to a microscopical population of a $q = 0$ lattice mode as a consequence of an appropriate external field (see Fig. 2). In the present review we do not survey such scenario and we focus on hot-phonon physics restricting ourselves to the most common case where the external pump triggers linearly particle-hole excitations (i.e., electronic degrees of freedom) and where no coherent oscillations are induced.

In a schematic way, the relevant system at equilibrium before (and in the
absence of) pumping can be described by the standard Hamiltonian:

\[ H = \sum_{\mathbf{k},\sigma,\alpha} (\epsilon_{\mathbf{k}\alpha} - \mu) \hat{c}_{\mathbf{k},\sigma,\alpha}^\dagger \hat{c}_{\mathbf{k},\sigma,\alpha} + \sum_{\mathbf{q},\nu} \hbar \omega_{\mathbf{q},\nu} \hat{a}_{\mathbf{q},\nu}^\dagger \hat{a}_{\mathbf{q},\nu} + \frac{N_{\mathbf{k}}^{-1}}{2} \sum_{\mathbf{k},\mathbf{q},\sigma,\alpha,\beta} g_{\mathbf{k}\alpha,\mathbf{k}+\mathbf{q}\beta}^\nu \hat{c}_{\mathbf{k},\sigma,\alpha}^\dagger \hat{c}_{\mathbf{k},\sigma,\beta} (\hat{a}_{\mathbf{q},\nu} + \hat{a}_{-\mathbf{q},\nu}^\dagger) \]

\[ + H_{\text{anharm}} + H_{\text{e-e}}, \]

where \( \mu \) is the electron chemical potential, \( \hat{c}_{\mathbf{k},\sigma,\alpha}^\dagger \) creates an electron with momentum \( \mathbf{k} \) and spin \( \sigma \) in the band \( \alpha \) with energy \( \epsilon_{\mathbf{k}\alpha} \), \( \hat{a}_{\mathbf{q},\nu}^\dagger \) creates a phonon with momentum \( \mathbf{q} \) in the branch \( \nu \) with frequency \( \omega_{\mathbf{q},\nu} \), \( g_{\mathbf{k}\alpha,\mathbf{k}+\mathbf{q}\beta}^\nu \) is the matrix element of linear electron-phonon coupling scattering an electron from the state \((\mathbf{k},\beta)\) into the state \((\mathbf{k}+\mathbf{q},\alpha)\) with a phonon emission (adsorption) with momentum \( \mathbf{q} \) \((-\mathbf{q})\) in the branch \( \nu \). \( N_{\mathbf{k}} \) is the number of momenta sampling the Brillouin zone. The third line in Eq. (1) takes into account the phonon-phonon scattering due to the lattice anharmonicity and the Coulomb-driven electron-electron scattering. In most of the cases, the detailed description of these latter two processes is not relevant and they do not need to be specified in more detail. For the sake of simplicity, we consider in the following a model with density-density interaction: \( H_{\text{e-e}} = \sum_{\mathbf{q}} V(|\mathbf{q}|) \hat{\rho}_{\mathbf{q}} \hat{\rho}_{-\mathbf{q}} \), where \( \hat{\rho}_{\mathbf{q}} = \sum_{\mathbf{k},\sigma,\alpha} \hat{c}_{\mathbf{k}+\mathbf{q},\sigma,\alpha}^\dagger \hat{c}_{\mathbf{k},\alpha,\sigma} \). Other interaction terms, as disorder scattering in the electron-phonon spectra, can be added when relevant.

For a generic interacting system, one can define an electron distribution function \( f_{\mathbf{k},\alpha} = \langle \hat{c}_{\mathbf{k},\alpha}^\dagger \hat{c}_{\mathbf{k},\alpha} \rangle \) as well as phonon distribution function \( b_{\mathbf{q},\nu} = \langle \hat{a}_{\mathbf{q},\nu}^\dagger \hat{a}_{\mathbf{q},\nu} \rangle \). At thermodynamical equilibrium, both electron and phonon degrees of freedom are ruled by a thermal population described respectively by the Fermi-Dirac and Bose-Einstein distributions. In the non-interacting case, for
Figure 3: Representative sketch of the time evolution of the electronic distribution function in a pump-probe experiment. (a) Electronic system at $t < 0$ before the pump excitation. Electrons and holes obey a thermal distribution, here assumed to be ruled by a low temperature $T \approx 0$. (b) Pump laser photons induce at $t = 0$ particle-hole excitations, leading to a highly non-thermal distribution. (c) In a short time characterized by a time-scale $\tau_{ee}$, due to the electron-electron interaction, electrons and holes reach a thermal distribution characterized by a large electronic temperature $T_{ee}$. (d) On a time scale $\tau_{ep}$ governed by the electron-phonon coupling, electrons (and holes) further reduce their effective temperature by transferring energy to the lattice modes through emission of hot phonons (red wavy lines) and to a minor extent of cold phonons (green wavy lines). For simplicity we depict here the case of a typical semiconductor where, in the absence of recombination which occurs on longer times, electrons and holes obey separate distributions. For metal the scheme is similar where electrons and holes excitations stem out from a unique distribution function.

instance, we have for the electrons $f_{k,\alpha}^{(0)} = f_T[(\epsilon_{k,\alpha} - \mu)/k_B T]$, and for the phonons $b_{q,\nu}^{(0)} = b[\hbar \omega_{q,\nu}/k_B T]$, where $f_T[x] = 1/(e^x + 1)$, $b_T[x] = 1/(e^x - 1)$, and $k_B$ is the Boltzmann constant. In the interacting case, the energies $\epsilon_{k,\alpha}$, $\omega_{q,\nu}$ at a first qualitative approximation can be replaced by effective (renormalized) quantities $\tilde{\epsilon}_{k,\alpha}$, $\tilde{\omega}_{q,\nu}$.

Under the assumption of a linear coupling between the pump photon with energy $h\nu$ with particle-hole excitations, in a ideal case of a $\delta$-function pump profile in real time, we could describe at $t = 0$ the electronic system with a highly non-thermal distribution function as sketched in Fig. 3, with a distribution of particle excitations for $\epsilon_{k,\alpha} \geq \mu$ balanced by a distribution of hole excitations for $\epsilon_{k,\alpha} \leq \mu$.

The fundamental information in time-resolved physics concerns the different time scales within the system under such initial non-equilibrium state reaches a new final steady state at equilibrium with the all the degrees of freedom, and the channels employed to reach the new final equilibrium. In this regard, it is clear that the time dynamics is crucially based on many-body processes, and its investigation reveals the role of the different scattering sources. From a generic
point of view, neglecting particle-hole recombination, which occurs on much larger time scales than the range of interest here, all the theoretical approaches describing the time evolution of the coupled electron/phonon system, are fundamentally based on the equations of motion taking into account the collisional processes among and between the electronic and lattice degrees of freedom:

\[
\frac{df_{k,\alpha}}{dt} = df_{k,\alpha} + df_{k,\alpha} + I_k(t), \quad (2)
\]

\[
\frac{db_{q,\nu}}{dt} = \frac{df_{k,\alpha}}{dt} + df_{k,\alpha} + I_k(t), \quad (3)
\]

Here \( I_k(t) \) accounts for the perturbation induced by the pump pulse, the terms labeled “ee” represent scattering processes mediated by the Coulomb electron-electron scattering, the terms labeled “ep” describe electron-phonon mediated scattering processes driven by phonon anharmonicity. The arguments in curly brackets in the electron and phonon distribution functions, \( f_{k,\alpha} \), \( b_{q,\nu} \) reminds that these quantities have a mutual functional dependence. Considering, as an illustrative example, collision processes at the lowest order in a perturbation theory \([121]\), we can write for instance based on Eq. [1] (see \([120]\)):

\[
\frac{df_{k,\alpha}}{dt} = \frac{2\pi}{\hbar} \sum_{p,q}\langle q \rangle (2 - \delta_{\alpha,\beta}) \delta(\epsilon_{k,\alpha} + \epsilon_{p,\beta} - \epsilon_{k+q,\alpha} - \epsilon_{p-q,\beta})
\]

\[
\times \left\{ f_{k+q,\alpha} f_{p-q,\beta} (1 - f_{k,\alpha}) (1 - f_{p,\alpha}) - f_{k,\alpha} f_{p,\beta} (1 - f_{k+q,\alpha}) (1 - f_{p-q,\alpha}) \right\},
\]

\[
\text{ee}
\]

\[
\frac{df_{k,\alpha}}{dt} = \frac{2\pi}{\hbar} \sum_{p,q}\langle q \rangle (2 - \delta_{\alpha,\beta}) \delta(\epsilon_{k,\alpha} + \epsilon_{p,\beta} - \epsilon_{k+q,\alpha} - \epsilon_{p-q,\beta})
\]

\[
\times \left\{ f_{k+q,\alpha} f_{p-q,\beta} (1 - f_{k,\alpha}) (1 - f_{p,\alpha}) - f_{k,\alpha} f_{p,\beta} (1 - f_{k+q,\alpha}) (1 - f_{p-q,\alpha}) \right\},
\]

\[
\text{ep}
\]

\[
\frac{db_{q,\nu}}{dt} = \frac{2\pi}{\hbar} \sum_{p,q}\langle q \rangle (2 - \delta_{\alpha,\beta}) \delta(\epsilon_{k,\alpha} + \epsilon_{p,\beta} - \epsilon_{k+q,\alpha} - \epsilon_{p-q,\beta})
\]

\[
\times \left\{ f_{k+q,\alpha} f_{p-q,\beta} (1 - f_{k,\alpha}) (1 - f_{p,\alpha}) - f_{k,\alpha} f_{p,\beta} (1 - f_{k+q,\alpha}) (1 - f_{p-q,\alpha}) \right\},
\]

\[
\text{pp}
\]

More complex equations can be written down when the many-body interactions are treated at a higher order, using some kind of approximation, as for instance in a self-consistent Born scheme or using random-phase approximation (RPA). Similar coupled equations can be also equivalently derived within the framework of \textit{ab-initio} approach \([39, 60, 122]\). Such computation improvements are fundamental when quantitative predictions are needed, as for instance in assessing the effective dispersion and linewidths in time-dependent angle-resolved photoemission spectroscopy (ARPES) \([39]\), two-photon photoemission \([2]\) or in image-potential states \([60]\). On the other hand, the basic physics relevant for
the aims of the present paper, surveying the possible onset of hot phonons in MgB$_2$ and other metals, is already well captured by the lowest order scheme outlined in Eqs. (2)-(7).

On a macroscopic level, the dynamics of the electron and phonon distribution functions drives changes of the electronic and vibrational energies, which can be defined as

$$E_e = N_s \sum_{k,\alpha} \epsilon_{k,\alpha} f_{k,\alpha},$$  

$$E_p = \sum_{q,\nu} \hbar \omega_{q,\nu} b_{q,\nu},$$  

where $N_s = 2$ is the electron spin degeneracy.

The time evolution of the energies $E_e, E_p$ can be thus obtained by integrating Eqs. (2)-(7):

$$\frac{dE_e}{dt} = \frac{dE_e}{dt} \bigg|_{ep} + S(t),$$  

$$\frac{dE_p}{dt} = \frac{dE_p}{dt} \bigg|_{ep} = -\frac{dE_e}{dt},$$  

where

$$\frac{dE_p}{dt} \bigg|_{ep} = \frac{4\pi}{\hbar} \sum_{k,p,\alpha,\beta,\nu} \hbar \omega_{k-p,\nu} |g_{k\alpha,p,\nu}|^2 \delta(\epsilon_{k,\alpha} - \epsilon_{p,\beta} + \hbar \omega_{k-p,\nu})$$

$$\times \left( (f_{k,\alpha} - f_{p,\beta}) b_{k-p,\nu} - f_{p,\beta}(1-f_{k,\alpha}) \right),$$

and $S(t) = N_s \sum_{k,\alpha} \epsilon_{k,\alpha} I_k(t)$ accounts for rate of the energy pumping in the system [123]. Equations (9)-(10) do not rely on the assumption of a electron/phonon thermal distributions, but they are generally valid also for non-thermal distributions, once the solution of Eqs. (2)-(7) is provided. It is also worth noticing that the Coulomb electron-electron interaction, as well as the anharmonicity-driven phonon-phonon term, are not explicitly appearing in Eqs. (9)-(10). Indeed, while electron-electron scattering causes the redistribution of energy and momentum among electronic degrees of freedom, the conservation laws constrains the total energy and momentum to remain unchanged. Correspondingly, electron-electron interaction does not appear explicitly in Eqs. (9)-(10). Similar considerations apply for the influence of anharmonic phonon-phonon scattering process on vibrational degrees of freedom. Equations (9)-(10) underline thus the primary role of the electron-phonon interactions in governing energy transfer and relaxation among the degrees of freedom of the system.

Equations (2)-(7) [and hence Eqs. (9)-(10)] form a closed set of integro-differential equations which can be solved explicitly via conventional time-propagation algorithms to investigate the non-equilibrium dynamics of electronic and vibrational degrees of freedom [124][125]. In 3D metallic compounds, the solution of the Boltzmann equation is computationally very demanding...
due to the need of extremely dense meshes to sample phonon-assisted particle-hole transitions in a sufficiently wide region of reciprocal space (see Fig. 1). Approximate strategies can thus be employed to reduce the computational complexity of the problem and simplify numerical investigations of the coupled electron-phonon dynamics.

One of the widest scheme employed to this aim is the two-temperature (2T) model. Within this approach, electron and lattice degrees of freedom are assumed to obey a thermal distribution, \( f_{k,\alpha}[T_e] \) and \( b_{q,\nu}[T_p] \), ruled by two district effective temperatures, an electron \( T_e \) and a phonon one \( T_p \), respectively. Neglecting particle-hole recombination processes, which occur on a longer time scale, for metals the effective temperature \( T_e \) governs a normal thermal distribution about the Fermi sea, whereas in semiconductors/semimetals \( T_e \) governs two separate thermal distributions for electrons in the conduction band and for holes in the valence band, respectively.

Within the framework of the 2T model, the electronic and lattice total energies can be parametrized in terms of the electronic and phonon temperature \( T_e, T_p \), respectively. In particular, Eqs. (9)-(10) can be recast in terms of two coupled equations ruling the time dynamics of the effective temperatures:

\[
C_e \frac{dT_e}{dt} = -G(T_e - T_p) + S(t),
\]

\[
C_p \frac{dT_p}{dt} = G(T_e - T_p),
\]

where we introduced the specific heat capacity of electrons and phonons,

\[
C_e = N_s \sum_{k,\alpha} \epsilon_{k,\alpha} \frac{\partial f_{k,\alpha}[T_e]}{\partial T_e},
\]

\[
C_p = \sum_{q,\nu} \hbar \omega_{q,\nu} \frac{\partial b_{q,\nu}[T_p]}{\partial T_p},
\]

and the effective electron-phonon coupling constant \( G \), defined by:

\[
G = \frac{2\pi k_B}{\hbar} \sum_{k,\alpha,\beta} \sum_{q,\nu} |g_{k\alpha,k+q\beta}|^2 \omega_{q,\nu} \delta(\epsilon_{k,\alpha}) \delta(\epsilon_{k+q,\beta}).
\]

The dynamics of the electron and phonon temperatures is schematically illustrated in the top panel of Fig. 4 (see also corresponding panels in Fig. 3). The validity of such approximate description relies on the analysis of the relevant time scales. In full generality, the shortest one is designed to be the time-profile of the pump pulse, in an ideal case a \( \delta(t) \)-function. Due to the Heisenberg’s uncertainty principle, however a quasi-monochromatic laser cannot have a time confinement less than the inverse of its energy bandwidth, \( \Delta t \Delta W \approx \hbar/2 \). Typical pump-pulse have a duration of 30 – 50 fs, which provides the first intrinsic (not many-body determined) time scale.

The description in terms of a thermal electronic distributions further relies on the assumption that electrons reach a their thermal distribution in a time scale
Figure 4: Representative time dynamics of the characteristic temperatures in the 2T model (top panel) and in the 3T model (bottom panel). Readaptation from Ref. [127].

\( \tau_{ee} \) that is much faster than the characteristic time scales of electron-phonon scattering (Fig. 3c). Such fast electronic thermalization is conventionally attributed to the electron-electron Coulomb interaction \( V^C \) in Eq. (1), although the effective speed of such internal thermalization among the electron degrees of freedom can be also very different in semiconductors and metals. In the first case, due to the low metallic character – and hence due to the poor screening of the Coulomb interaction – the particle-particle interaction is very strong and the internal thermalization between the electronic degrees of freedom is thought to be reached in few tens of femtoseconds [128]. In conventional metals, on the other hand, since the Coulomb interaction is effectively screened by the strong metallic character, the thermalization of electronic degrees of freedom takes place on longer time scales, although a detailed description of such processes is nowadays still under debate [38, 59, 61, 62, 129, 130].

The reliability of the two-temperature model relies on the assumption that besides electrons also phonons have reached a thermal distribution before the energy transfer between electron and lattice degrees of freedom, as described by Eqs. (12)-(13), takes place. This is however a very delicate point. Phonon thermalization is a natural consequence of the anharmonic phonon-phonon scattering, which plays a similar role for the lattice degrees of freedom as the Coulomb interaction for the electrons. In most of the compounds of interest, however, anharmonic phonon-phonon coupling, although finite, represents the weakest interaction in Eq. (1), and hence is thought to take place over a time scale
$t_{pp}$ longer than the electron-phonon processes. The idea that phonons can be described by a thermal distribution during the time scale of the electron-phonon relaxation is thus, as a matter of fact, not supported by the assumption of a preliminary anharmonicity-driven phonon thermalization. In spite of this conundrum, the two-temperature model has provided a satisfactory description for the time-resolved dynamics of many known compounds, especially metals. The reason of such consistency is related to the available electronic phase space. As represented in Fig. 1b, in most of the materials the photoexcited electrons (as well as the photoexcited holes) are distributed all over the Brillouin zone with large patches in the momentum space. The subsequent electron-phonon scattered momenta sample the whole phonon Brillouin zone to a great extent in an isotropic way, providing an optimal ground for an easy and efficient thermalization. The link between the available electron and phonon phase spaces is sketched in Fig. 5 for standard metals as well as for other compounds.

Quite different can be the scenario for semiconductors and semimetals characterized by a valley degree of freedom, namely a band gap (ideally a zero or negative band gap, for semimetals) with degenerate band edges. Under such circumstances, the pump-driven excitations can be concentrated in a few regions nearby band valleys. A paradigmatic example of this scenario with a strong topical interest is graphene (see for instance Fig. 1a), a two-dimensional semimetal characterized by a linear Dirac dispersion extending for ±1 eV around the $K$, and $K'$ points of the Brillouin zone. The reduced phase space for particle-hole excitations implies a reduced phase space also for the electron-phonon scattering, where only few lattice modes ruling intra-valley processes (small phonon momenta) or inter-valley processes (large phonon momenta) are allowed, as schematically depicted in Fig. 5. Such selected modes (small-$q$ momenta; large-$q$ momenta) provide thus the main channels for energy transfer from the electronic degree of freedom to the lattice sector. Absorbing energy from the excited electrons, these modes become hot phonons, i.e., they acquire a non-thermal population much larger than the remnant lattice modes that stay cold, obeying an effective thermal distribution with a moderate temperature. Considered that anharmonic phonon-phonon mixing occurs on a much longer time scale than the dynamics relevant here, hot phonons can survive and be effectively detected on the scale of hundreds of femtoseconds or even few picoseconds.

In order to describe the time dynamics of a coupled electron-phonon system under hot-phonon conditions, a useful three-temperature (3T) model is often employed. In such extended version of the 2T model, the parameter $T_e$ still describes the effective temperature of the electron degrees of freedom which are assumed to be thermalized before the model applies. The majority of the lattice modes stay cold, and are thus described in an efficient way by a thermal distribution governed by the effective temperature $T_{\text{cold}}$. On the other hand hot modes, adsorbing energy directly from the electronic sector, and not redistributing to the other modes, acquire a phonon population $b_{\text{hot}}$ much larger than the cold modes. The basic (reasonable) assumption of the 3T model is that few hot modes are essentially degenerate (as for instance the
Figure 5: Sketch of electron and phonon phase spaces available for scattering for different families of materials. In metals, the available electronic phase space (dark blue) is spread in a uniform way all over the Brillouin zone. Consequently exchanged phonons are scattered in an isotropic way probing in an equal way the whole phonon phase space. In semiconductors and semimetals characterized by a valley degree of freedom (middle panel) pump-driven particle-hole excitations probe essentially only states close to the valleys (here represented by dark and light blue regions). Such situation brings to an intrinsic selection of phonon modes: only intravalley and intervalley modes, with small (red) and large (pink) momenta respectively, can effectively couple to electrons, and become hot. The available electronic phase space in metals with anisotropic electron-phonon coupling can be also divided in two main sectors: a core region (dark blue area) characterized by strong electron-phonon interaction (red), and a large part (light blue) with weak electron-phonon coupling (pink). In such anisotropic scenario strongly coupled modes can profit of energy transfer from the electrons becoming hot while the remnant modes remain cold.

optical $E_{2g}$ modes in graphene) displaying similar energies ($\omega_{\text{hot}}$) and a similar phonon population $b_{\text{hot}}$. The phonon population $b_{\text{hot}}$ can be hence conveniently expressed in terms of an effective temperature $T_{\text{hot}}$ for the hot modes:

$$b_{\text{hot}} = \frac{1}{\exp \left( \frac{\hbar \omega_{\text{hot}}}{k_{\text{B}} T_{\text{hot}}} \right) - 1}.$$  \hspace{1cm} (17)

Within the validity of the three-temperature model, Eqs. (12)-(13) can be
generalized as:

\[ C \frac{dT_e}{dt} = -G_{\text{hot}}(T_e - T_{\text{hot}}) - G_{\text{cold}}(T_e - T_{\text{cold}}) + S(t), \]  

(18)

\[ C_{\text{hot}} \frac{dT_{\text{hot}}}{dt} = G_{\text{hot}}(T_e - T_{\text{hot}}) - V_{\text{anharm}}(T_{\text{hot}} - T_{\text{cold}}), \]  

(19)

\[ C_{\text{cold}} \frac{dT_{\text{cold}}}{dt} = G_{\text{cold}}(T_e - T_{\text{cold}}) + V_{\text{anharm}}(T_{\text{hot}} - T_{\text{cold}}), \]  

(20)

where \( C_{\text{hot}}, C_{\text{cold}} \) account for the specific heat capacities of the hot and cold modes, respectively. In similar way \( G_{\text{hot}}, G_{\text{cold}} \) describe the electron-phonon relaxation rates related to hot and cold modes. In Eqs. (18)-(20) we have also explicitly included a term \( \propto V_{\text{anharm}} \) due to the anharmonic phonon-phonon coupling restoring (on a long time scale) the global equilibrium among the lattice degrees of freedom. This term is conventionally expressed in terms of the specific heat capacity of the hot modes by introducing a characteristic time scale \( \tau_{\text{anharm}}, V_{\text{anharm}} = C_{\text{hot}}/\tau_{\text{anharm}} \) [136]. A sketch of the time dynamics of the three representative temperatures is shown in the bottom panel of Fig. 4.

3. Detecting hot phonons

The concept of hot phonon refers to a scenario in which one (or few) vibrational modes of the lattice are characterized by a population \( b_{q,\nu} \) significantly larger than the majority of other modes. These latter (cold) modes obey a standard thermal distribution ruled by a background temperature \( T_{\text{cold}} \). In spite of such compelling definition, an active role of hot phonons in a material is often not probed directly and their presence is inferred in indirect ways. One of the first experimental evidences of the presence of hot phonons was prompted by the analysis of the electric-field dependence of transport properties in low-carrier-density doped semiconductors [72, 76, 137]. Within the framework of a Boltzmann approach, the charge transport is governed by an electronic distribution \( f_k(\vec{k},\vec{E}) \) in presence of an electric field \( \vec{E} \). The electric field in non-interacting systems would result in a constant shift in time of the electronic distribution, and hence to an infinite conductivity. A steady state is however obtained as a balance between such velocity drift and collision processes that scatter back the electrons, namely [138, 139]:

\[ -e\vec{E} \cdot \nabla_k f_{\vec{k},\vec{E}} = \left. \frac{df_k(\{f,b\})}{dt} \right|_{\text{ee}} + \left. \frac{df_k(\{f,b\})}{dt} \right|_{\text{ep}}, \]  

(21)

where the scattering terms \( df_k(\{f,b\})/dt|_{\text{ee}}, df_k(\{f,b\})/dt|_{\text{ep}} \) have been discussed in the previous Section. The net result can be regarded as an effective displacement of the Fermi volume along the direction of \( \vec{E} \) [138, 139], leading to a net finite electronic momentum. While the Boltzmann equations are often solved assuming an unperturbed phonon distribution function \( b_{q,\nu}^{(0)} \) (hence with a thermal profile), such assumption holds true only in the linear regime, valid for
small electric fields and for not too low temperatures where phonon drag effects can occur. In real systems (as well as in a variety of more refined theoretical modeling), the coupling between the electron and lattice degrees of freedom gives rise to a phonon distribution function $b_{q,\nu}(\{f, b\})$ which may differ significantly from the thermal unperturbed one $b_{q,\nu}^{(0)}$, affecting as well the resulting steady electronic function $f_k(\{f, b\})$. The role of a non-thermal $b_{q,\nu}(\{f, b\})$, initially regarded as “phonon disturbance” have been experimentally detected and theoretically investigated for a long time, especially in high electric-field regime, and it becomes more evident under hot-phonon conditions, where only few lattice modes are excited by the coupling with the electrons [70, 71, 73, 78, 137, 140–143].

It has also been shown that hot phonons and non-equilibrium phonons play a crucial role in tailoring the interband properties in narrow gap semiconductors, with respect to impact ionization processes and quantum laser cascades [71, 144–149], which provide in this class of materials another way to detect them.

The presence and an active role of hot phonons is also commonly revealed in the analysis of the time dependence of electronic properties upon non-equilibrium conditions. Given the plethora of ultrafast pump-probe experiments nowadays accessible, the physical properties under investigation can vary in a wide range, from optical probes (transmission [77, 84, 85, 87, 150, 151], reflectivity [13, 89, 110–112, 152–154], absorption [91, 155–158]) to non-linear optics [86], time-resolved photoelectron spectroscopy [105, 132, 133, 159–162], photoluminescence [83, 163], time-dependent Raman probes [164], ultrafast diffraction [6, 44, 109, 126, 165–171].

Similarly, hot-phonon effects have been traced down in electron properties and in time-resolved Raman spectroscopy by means of theoretical simulations [72, 73, 75].

Core of these analyses is the evidence of multiple relaxation time scales in the ultrafast dynamics, which can be roughly rationalized in terms of two different channels with different characteristic time scales (biexponential decay) as depicted in Fig. 6: one (faster) channel governed by the energy transfer between the electron degrees of freedom and the hot phonons; and a second slower one associated with the final thermalization of the remnant cold lattice modes. On the experimental ground these different time scales, and their relative weights, can be revealed by an appropriate fitting with two relaxation rates,

$$O_e(t) \approx \alpha_1 \exp\left(-\frac{t}{\tau_1}\right) + \alpha_2 \exp\left(-\frac{t}{\tau_2}\right),$$

where $O_e$ is a generic physical observable ruled by the electronic degrees of freedom.

A deeper quantitative insight can be gained if one is able to translate the time dependence of the electronic probe $O_e$ in the time dependence of a corresponding effective electronic temperature $T_e$. The multiple time-scale dependence of $T_e$ can be compared and fitted with the predictions of a three-temperature model as described in Eqs. (18)-(20) [131, 132, 136]. From this analysis fundamental information are hence extracted, as the factors $G_{\text{hot}}, G_{\text{cold}}$ which are
proportional to the electronic coupling with the hot and cold bosonic modes, respectively.

It is worth to pointing out that the above described ways of detecting hot phonons in a material rely on the analysis of hot-phonon effects on electronic properties. On the other hand, since a hot phonon is an object defined by its bosonic population which is a property logically independent of the electron-phonon coupling, it looks more natural devising and employing, when possible, techniques that are based on the analysis of purely lattice features. Along this perspective, considered that hot phonons are usually located at high-symmetry points of the Brillouin zone, and in particularly at the zone-center, a popular tool is the analysis, when feasible, of the relative intensity of the Stokes and anti-Stokes peaks in Raman spectroscopy \[81, 82, 88, 113, 162, 164, 172, 173\]. The Stokes and anti-Stokes intensity for a \( q = 0 \) Raman active mode \( \nu \), \( I_S \), \( I_{aS} \), respectively, are indeed known to be governed by the population of the Raman mode:

\[
I_S \approx 1 + n(\omega_\nu),
\]
\[
I_{aS} \approx n(\omega_\nu).
\]

The population \( n(\omega_\nu) \) can be thus simply estimated from the relation \[88, 113, 173\]:

\[
\frac{I_{aS}}{I_S} = \frac{\omega^4_{\nu,aS}}{\omega^4_{\nu,S}} \frac{n(\omega_\nu)}{1 + n(\omega_\nu)} \approx n(\omega_\nu),
\]

where \( \omega_{\nu,S}, \omega_{\nu,aS} \) are the frequencies of the Stokes and anti-Stokes resonances, respectively, and where the last relation holds true under standard conditions.
\( (\omega_{\nu,S} - \omega_{\nu,aS}) / (\omega_{\nu,S} + \omega_{\nu,aS}) \ll 1, n(\omega_{\nu} \ll 1) \). An effective temperature for this mode can be also promptly estimated from \[ 82 \]

\[ n(\omega_{\nu}) = \left[ \exp \left( \frac{\omega_{\nu}}{T_{\nu}} \right) - 1 \right]^{-1} \approx \exp \left( - \frac{\omega_{\nu}}{T_{\nu}} \right). \] (26)

The comparison between the time evolution of the effective temperature \( T_{\nu} \) of a candidate hot mode \( \nu \) with the electronic temperature \( T_e \) assessed by other means can thus provide a robust evidence of a hot-phonon scenario. Such evidence can be furthermore corroborated if the time evolution of another Raman-active mode, belonging to the cold-mode sector, can be experimentally resolved, providing a direct check of the different time dynamics of the two (hot and cold) lattice mode sectors.

While the Raman spectroscopy can inspect in a direct experimental way the population of few (Raman-active) modes at the zone-center of the Brillouin zone, from a theoretical perspective the snapshot at given time \( t \) of the distribution of phonon population \( b_{q,\nu} \) for each branch \( \nu \) and momentum \( q \) can be obtained by the direct solution of Eqs. (2)-(7), permitting to reveal thus selective high populations for particular momenta. This approach has been indeed widely employed, ranging from the seminal work by Pötz \[ 72 \] to later works \[ 77–80 \] up to more recent refined analyses \[ 124, 126, 171 \].

Furthermore, direct information about the time evolution of the lattice dynamics under non-equilibrium conditions can be conveniently addressed by means of ultrafast diffraction measurements \[ 6, 44, 109, 126, 165–169, 171, 174–179 \]. At a first level, useful information can be gained by means of the time-resolved analysis of the intensity of the Bragg peaks, which can be related to the magnitude mean square lattice displacements as:

\[ \frac{I(Q,t)}{I(Q,0)} = \sum_i e^{-2W_i(Q)}, \] (27)

where \( Q \) denotes the wave-vector in the crystallographic reciprocal space, \( I(Q,0) \) is the intensity before the pump excitation and where \( W_i(Q) \) is the well-known Debye-Waller factor for a given atom \( i \) in the unit cell \[ 139 \]:

\[ W_i(Q) = \frac{1}{2} \text{ \langle } Q \cdot u_i \text{ \rangle}^2. \] (28)

A sudden increase of the Debye-Waller factor portends a sudden energy transfer from the electron to the lattice mode sector. Since the Debye-Waller factor results from the contribution of all the phonons in the Brillouin zone, it cannot reveal in a direct way the onset of hot-phonon physics. Hot modes and, in general, a non-thermal phonon distribution can however be inferred in an indirect way from the possible anisotropy of the estimated mean-square lattice displacements and from the observation of two-component decay rate \[ 6, 44, 109, 126, 165–169, 171 \].

Non-thermal phonon populations can be efficiently investigated by means of ultrafast electron diffuse scattering (UEDS), which provides a versatile diagnostic tool for detecting and investigating the emergence of hot phonons in
condensed matter [6, 167, 174, 176, 177]. In UEDS the probe is represented by an electron beam hitting the sample. The scattered electrons are detected with momentum resolution via an electron camera, as schematically depicted in Fig. 7a. While a fraction of electrons emerges from the sample unaffected (i.e., without undergoing scattering of any sort), additionally, electrons can undergo elastic and inelastic scattering processes with the lattice. The intensity of the scattered electrons can be Taylor-expanded as a sum of zero-phonon and one-phonon contributions [180, 181]:

$$I(Q) = I_0(Q) + I_1(Q) + \cdots .$$

(29)

The first term here accounts for Bragg (elastic) scattering, which can be expressed in the present formalism as:

$$I_0(Q) \propto \sum_{i,j} f_i(Q) f_j^*(Q) \cos [Q \cdot (\tau_i - \tau_j)]$$

$$\times e^{-W_i(Q)} e^{-W_j(Q)} \delta_{Q,G} .$$

(30)

where $\tau_i$ are the coordinates of the $i$-th atom in the unit cell, $W_i$ the corresponding Debye-Waller factor, and $f_i$ is the atomic scattering amplitude.

Further interesting information about the lattice properties are conveyed in the one-phonon term which can be expressed as:

$$I_1(Q) \propto \hbar \sum_{i,j} f_i(Q) f_j^*(Q)$$

$$\times \frac{e^{-W_i(Q)} e^{-W_j(Q)}}{\sqrt{M_i M_j}}$$

$$\times \sum_{\nu} \text{Re} \left[ Q \cdot e_{i,\nu}(Q) Q \cdot e_{j,\nu}^*(Q) e^{i Q \cdot [\tau_i - \tau_j]} \right] \frac{n_{Q\nu} + 1/2}{\omega_{Q\nu}^2} ,$$

(31)

where $e_{i,\nu}$ is the phonon eigenvectors for the phonon branch $\nu$, and $M_i$ is the atom mass for atom $i$. The linear dependence of $I_1$ on the phonon number $n_{Q\nu}$ provides a rationale for the relating transient changes in the scattered UEDS intensity [namely, $\Delta I(Q,t) \equiv I(Q,t) - I(Q,0)$] to underlying changes in the
phonon populations, and it provides thus a suitable experimental framework to explore the emergence of hot phonons [6].

UEDS is particularly well suited to detect momentum anisotropies established in the phonon population following photo-excitations [126, 168, 177, 178]. On the other hand, the determination of a mode-resolved phonon population is more challenging as it would require the knowledge of the scattered electron energy with the resolution of few meVs. To overcome this limitation, a novel procedure for recovering the phonon population dynamics with full mode resolution, based on the analysis of the transient scattering intensity, has recently been proposed by de Cotret et al., with a direct demonstration in graphite [178].

4. Anisotropic electron-phonon material: MgB$_2$

The crystal structure of MgB$_2$ is rather simple, with hexagonal graphene-like planes of B atoms spaced vertically by Mg atoms located in the center of the hexagons (Fig. 8a). MgB$_2$ has been on the shelves of chemistry laboratories for decades without attracting much attention. A considerable interest about this compound arose however in 2001 after superconductivity below $T_c \approx 39$ K was found [114]. Soon it was clear that the superconducting pairing in MgB$_2$ was driven by the electron-phonon coupling [115, 152, 185]. Discovered during the “gold rush” for new high-$T_c$ superconductors triggered by the report of superconductivity with $T_c \lesssim 100$ K in cuprates in 1986 and later years [187, 190], MgB$_2$ demonstrated that strong electronic correlations were not the unique path for high-$T_c$ superconductivity, but that electron-phonon coupling was not to rule out. Surely, it was also soon clear that the properties of the electronic structure and of the electron-phonon coupling in MgB$_2$ were quite peculiar with respect to conventional phonon-based low-$T_c$ superconductors. For instance, in the electronic band structure, two blocks of bands could be identified, as shown in Fig. 8b: a $\sigma$-band sector, with a strong two-dimensional character, built by the $p_x/p_y$ orbitals of boron, similarly as in graphene; and a $\pi$-band sector, with a strong three-dimensional character, resulting by the hybridization of Mg atoms with B-$p_z$ orbitals. It should be further noticed that, while the $\pi$-bands display large Fermi surfaces, denoting a robust metallic character, the $\sigma$-bands appear to be only slightly doped [183], resulting a thin tubular Fermi sheets (Fig. 8c). Because of the different properties of such two electron-band blocks, in-plane ($x-y$) and out-of-plane ($z$) electronic and optical properties results to be anisotropic [191, 194], revealing the multiband character of this material.

Like many metals, MgB$_2$ is not the only compound showing multiple different Fermi sheets. However, it can be considered without doubt, along with pnictides, the most evident superconducting material showing multiple different superconducting gaps, as assessed by a variety of experimental and theoretical checks [195, 200]. The logical shift from anisotropic multiband properties to anisotropic multigap properties should not be overlooked. Many conventional metals are characterized with multiple Fermi sheets having different electronic properties but rather homogeneous superconducting properties over all
the bands. The reason is that the electron-phonon scattering couples the electrons of one band with all the other bands, resulting in an effective averaging of the electronic properties relevant for the superconductivity. In order to sustain multiple gap features a superconducting material needs not only to display multiband properties, but also to possess an anisotropic electron-phonon coupling \[ \lambda_{ij} \] that allows to reflect the anisotropic multiband character in the superconducting properties.

Although MgB\(_2\) has two \(\sigma\)-bands and two \(\pi\)-bands at the Fermi level, it is a common practice to compact such electronic space in an unique \(\sigma\)-block and a unique \(\pi\)-block. In such reduced \(2 \times 2\) analysis, the total electron-phonon coupling described by the Eliashberg function \(\alpha^2 F(\omega)\) can be split in four components \(\alpha^2 F_{ij}(\omega)\) \((i, j = \sigma, \pi)\). One can as well compute the dimensionless electron-phonon coupling \(\lambda_{ij} = 2 \int d\omega \alpha^2 F_{ij}(\omega)/\omega\) constant that expresses in a
compact way the intraband and interband terms. The corresponding electron-phonon matrix presents a strongly anisotropic structure peaked in the $\sigma-\sigma$ sector [115, 183, 185, 186, 202, 203], with typical values [203]:

$$\hat{\lambda} = \begin{pmatrix} \lambda_{\sigma\sigma} & \lambda_{\sigma\pi} \\ \lambda_{\pi\sigma} & \lambda_{\pi\pi} \end{pmatrix} = \begin{pmatrix} 1.02 & 0.21 \\ 0.16 & 0.45 \end{pmatrix}. \quad (32)$$

The spectral analysis of the Eliashberg functions $\alpha^2 F_{ij}(\omega)$ shows that the predominance of the intraband $\sigma-\sigma$ coupling is associated with the appearance of a remarkable peak in the energy window $[550 : 650] \text{cm}^{-1}$ [115, 184, 185, 202, 204]. A deeper insight can be gained by investigating the momentum distribution of the electron-phonon coupling along the different branches of the phonon dispersion. A representative plot is shown in Fig. 8d where the size of the black circles reflects the magnitude of the electron-phonon coupling. One can notice that the electron-phonon coupling is strongly anisotropic also with respect to phonon momentum/branch, being almost exclusively concentrated at small momenta of the optical $E_{2g}$ branch with in-plane lattice displacements (Fig. 8d). Such modes are strongly coupled with the electronic $\sigma$-band and are essentially the only contributors to $\lambda_{\sigma\sigma}$. As a consequence of the two-dimensional character of the $\sigma$-bands, with a negligible dispersion along perpendicular momentum $k_z$, the electron-phonon coupling of these modes is also weakly dependent on the vertical component of the phonon momentum $q_z$, contributing in an equivalent way along the $\Gamma$-$A$ path. Note that the phonon band structure shows a remarkable depletion of the $E_{2g}$ dispersion in a close correspondence of the region with strong electron-phonon coupling. Such feature can be rationalized as an effect of a Kohn anomaly driven by the electron-phonon coupling of the $E_{2g}$ modes with the two-dimensional $\sigma$-bands, and following qualitatively the behavior of a two-dimensional Lindhard function. The role of the electron-phonon coupling is even more striking when comparing the phonon dispersion of MgB$_2$ with the similar isostructural compound AlB$_2$, which does not display such softening close to the $\Gamma$ point. AlB$_2$ presents strong similarities with MgB$_2$, with the only relevant difference of possessing one electron more per cell. Such additional charge leads to a complete filling of the $\sigma$-bands that in AlB$_2$ lay 1-1.5 eV below the Fermi level (see Fig. 9a), preventing thus a metallic screening due to the $\sigma$ bands.

The key role of the strongly-coupled zone-center $E_{2g}$ modes in governing the electron-phonon interaction, and hence the superconducting properties of MgB$_2$, has been extensively discussed in literature. The most striking feature soon after the discovery of superconductivity in MgB$_2$ was the huge broadening of the $E_{2g}$ peak observed in Raman spectroscopy, $\Delta \Omega_{E_{2g}} \approx 25 \text{meV}$, that can be compared with $\Delta \Omega_{E_{2g}} \approx 5 \text{meV}$ in AlB$_2$ (Fig. 9b). Anharmonicity was at the beginning suggested as responsible for such large broadening in MgB$_2$, supported by first-principle frozen-phonon calculations of the energy potential, showing a strong deviations from a quadratic behavior at large lattice displacements [184]. On the basis of such anharmonic potential the $\omega_{E_{2g}}$ phonon frequency was estimated to harden from $\omega_{E_{2g}} = 60.3 \text{meV}$ (harmonic limit) to $\omega_{E_{2g}} = 74.5$
Figure 9: (a) Electronic band structure of MgB$_2$ and AlB$_2$. Thick lines denote the $\sigma$-bands which are slightly hole-doped in MgB$_2$ whereas they are completely filled in AlB$_2$. From Ref. [205]. (b) A comparison of the $E_{2g}$ phonon spectrum in MgB$_2$ and AlB$_2$ as probed from Raman spectroscopy. (c) Theoretical phonon dispersion for MgB$_2$ and AlB$_2$. MgB$_2$ shows a remarkable softening of the $E_{2g}$ branch for small in-plane $q$ momenta (red area). Such softening results from the Kohn anomaly due to the metallic screening of the $\sigma$-bands, which are strongly coupled with the $E_{2g}$ modes. This feature is not present in AlB$_2$ where the $\sigma$-bands are completely filled and they do not provide any metallic screening. Panels (b)-(c) from Ref. [204].

meV (anharmonic calculation [184]. The actual relevance of anharmonicity was however later questioned by further experimental and theoretical investigations [206–209] showing that the anharmonic contribution due to the direct phonon-phonon scattering to the phonon linewidths and phonon shifts was marginal with respect to the observed ones [204, 210–216]. These controversial findings have been rationalized by noticing that the strong anharmonicity computed in Ref. [184] by frozen phonon calculations was not associated with a quartic term (fingerprint of a direct phonon-phonon coupling) but it was rather a byproduct of a strong conventional linear electron-phonon coupling in the presence of a dynamical Lifshitz transition induced by the effects of the $E_{2g}$ lattice displacement on the electronic structure [184, 205, 217, 218]. The Lifshitz transition appears to be physically accessible in MgB$_2$ due the small Fermi energy of the $\sigma$-bands ($E_F^\sigma \sim 0.4–0.5$ eV) and due to the strong lattice fluctuations triggered by the strong electron-phonon coupling of the $E_{2g}$ mode. Once again, such scenario
points towards the presence of a peculiarly strong electron-phonon coupling for the $E_{2g}$ modes, whereas at the same time the evidence of strong lattice fluctuations points out the relevance of nonadiabatic effects [217, 219–221]. A similar situation is encountered in a quantitative analysis of the Raman $E_{2g}$ phonon frequency and linewidth, where the experimental data has been successfully reproduced within the context of a many-body renormalized nonadiabatic theory for the phonon self-energy [219, 221].

5. Hot-phonon physics in MgB$_2$

The strong anisotropic multiband character of the electron-phonon coupling in MgB$_2$ has been widely discussed and assessed since long time, both from the experimental as from the theoretical points of view. It was however not recognized until recently that this scenario could naturally lead in MgB$_2$ also to hot-phonon physics [112]. Indirect experimental evidences of hot phonons in time-resolved optical probes were in that work also discussed.

Although prompting the seminal suggestion of hot phonons in MgB$_2$ that stem from the anisotropic electron-phonon coupling, the evidence in support of this scenario was in Ref. [112] quite indirect and no quantitative analysis of the energy transfer processes were given.

A full microscopical proof of a key role of hot phonons in MgB$_2$ was provided in Ref. [113] using ab-initio calculations. To this purpose, the electronic and phonon band structures were computed from density-functional theory using the QUANTUM ESPRESSO package [224]. The electron-phonon coupling constants $\lambda_{q,\nu}$ for each momentum $q$ and each branch $\nu$ were also computed in good agreement with Fig. 8d.

Equipped with all these theoretical inputs, the time dynamics of the each single electronic and lattice degree of freedom can be in principle numerically computed, as discussed in Sec. 2. As also discussed there, however, in MgB$_2$ as well as in other metals, due to large Fermi surfaces, this approach is not computationally affordable. The key advantage of MgB$_2$ is that the strong multiband character and the strong anisotropy of the electron-phonon coupling, provides a very efficient and compelling way for simplifying the system, namely for splitting in an unambiguous way the lattice degrees of freedom $(q, \nu)$ in two subsectors, the “hot” and “cold” modes. As evident from the previous discussion, only the $E_{2g}$ phonon branch, corresponding to in-plane out-of-phase lattice displacements of the two B atoms, is strongly coupled. More delicate is the criterion how to identify the $q$ vectors belonging to the hot-phonon sector. In this regard, we are advantaged by the remarkable Kohn anomaly visible in the phonon spectrum, that provides a fingerprint of the strong electron-phonon scattering with these modes. Such scenario was indeed analyzed in a quantitative way in Ref. [113]. Obeying to the softening due to the Kohn anomaly, the $E_{2g}$ branch was there essentially divided in two blocks (Fig. 10a): a strongly-coupled region (hot modes, marked as green shaded area), for small in-plane $q$ momenta and phonon frequencies $\omega_{q,E_{2g}} \sim 60 – 75$ meV, and a weakly-coupled region (cold modes), for large in-plane $q$ momenta and phonon
Figure 10: (a) Plot of the phonon dispersions of MgB$_2$ (solid lines) where the size of the black circles represents the electron-phonon coupling strengths $\lambda_{q\nu}$. Also shown are the experimental phonon energies of the $E_{2g}$ mode close to the $\bar{M}$ point and along the $\Gamma - \bar{A}$ path (red circles) [207], as well as along the $\bar{M} - \bar{\Gamma}$ cuts (purple empty squares) [222] as obtained by inelastic X-ray scattering. The Raman-active $B_{1g}$ mode is also denoted, with the zone-center frequency of $\omega_{B_{1g}} = 86$ meV and weak el-ph coupling, associated to out-of-plane lattice vibrations. (b) Corresponding phonon density of states $F(\omega)$ (dashed line) and the total Eliashberg function $\alpha^2F(\omega)$ (blue solid line). Green color shows the contribution to the Eliashberg function associated with the hot $E_{2g}$ modes around and along the $\Gamma - \bar{\Gamma}$ path, $\alpha^2F_{\text{hot}}(\omega)$. Panels (a) and (b) from Ref. [113]. (c) Time evolution of the characteristic effective temperatures $T_e$, $T_{\text{hot}}$, $T_{\text{cold}}$ in MgB$_2$ for the three-temperature model (top panel) and for a two-temperature model (bottom panel) where the electron-phonon coupling in MgB$_2$ is modeled to be isotropic. The grey dashed line shows the pulse profile, with the pulse duration of 45 fs and an absorbed fluence of 12 J/m$^2$. From Ref. [223].

frequencies $\omega_{q,E_{2g}} \sim 90$ meV. Due to the weak dispersion of the $E_{2g}$ modes along $q_z$, such splitting in the momentum space is reflected in a marked splitting in the frequency features of the Eliashberg function $\alpha^2F(\omega)$, as shown in Fig. 10b, where the contribution of the strong-coupled modes results in a pronounced peak in the window $\omega \in [60 : 75]$ meV, which is well detached by the a higher-frequency smaller peak at $\sim 90$ meV related to the weakly-coupled $E_{2g}$ modes at large momenta. Note that the structure of $\alpha^2F(\omega)$ at $\sim 90$ meV matches with a corresponding peak in the phonon density of states (dashed line in Fig. 10b), whereas the phonon density of states $F(\omega)$ is essentially flat in the frequency window $[60 : 75]$ meV pointing out how the peak of the Eliashberg function
at these energies is crucially driven by the strong electron-phonon coupling, selective for these modes.

Following this observation, the total Eliashberg function was divided in Ref. \[113\] as sum of two terms, $\alpha^2 F(\omega) = \alpha^2 F_{\text{hot}}(\omega) + \alpha^2 F_{\text{cold}}(\omega)$, where $\alpha^2 F_{\text{hot}}(\omega)$ contains the contribution of the hot $E_{2g}$ modes along and around the $\Gamma - \bar{\Lambda}$ path in the relevant energy range $\omega \in [60 : 75]$ meV, while $\alpha^2 F_{\text{cold}}(\omega)$ accounts for the weakly coupled cold modes in the remnant parts of the Brillouin zone. A similar splitting can be performed for the phonon density of states $F(\omega) = F_{\text{hot}}(\omega) + F_{\text{cold}}(\omega)$. The percentage of the $E_{2g}$ hot modes has been estimated to be roughly 5% of the total phonon modes \[223\], whereas the relative electron-phonon coupling strengths for the hot and cold modes were found to be $\lambda_{\text{hot}} = 0.26$ and $\lambda_{\text{cold}} = 0.34$, respectively \[113\].

The main relevant parameters ruling Eqs. (18)-(20) were thus also computed from the first-principles calculations in Refs. \[113, 223\]: $C_e = 90 \text{ J/m}^3\text{K}^2 \times T_e$, $C_{\text{hot}} = 0.13 \text{ J/m}^3\text{K}$, and $C_{\text{cold}} = 4.1 \text{ J/m}^3\text{K}$, $G_{\text{hot}} = 2.8 \times 10^{18} \text{ W/m}^3\text{K}$ and $G_{\text{cold}} = 3.6 \times 10^{18} \text{ W/m}^3\text{K}$. The relaxation time $\tau_{\text{anharm}} = 400$ fs has been estimated from the linewidth of the $E_{2g}$ modes at $\Gamma$, due to the direct anharmonic phonon-phonon scattering \[207\]. On this ground, within a three-temperature model for a hot-phonon material, the time evolution of characteristic temperatures $T_e, T_{\text{hot}}, T_{\text{cold}}$ was computed upon a pump pulse with a Gaussian profile of duration 45 fs, starting from an initial thermal equilibrium at room temperature. The ultrafast dynamics of $T_e, T_{\text{hot}}, T_{\text{cold}}$ (see Fig. 10c) shows a sharp increase of the electronic temperature for these parameters up to $T_e \approx 1850$ K. The energy pumping in the electronic sector is however efficiently compensated and reversed by the fast energy transfer from electronic excitations to hot phonons, whose effective temperature $T_{\text{hot}}$ increases up to $T_{\text{hot}}^{\text{max}} \approx 1250$ K at $t^* \approx 50$ fs, with a small delay compared to the time behavior of the electronic temperature $T_e$. The remnant cold phonon modes follow, on the other hand, a completely different behavior with a slow monotonic increase towards the final equilibrium state at $t \geq 0.3 - 0.4$ ps where all the degrees of freedom are thermalized with each other. Such time dynamics is quite different when the fundamental role of the hot phonons is neglected, as shown in the bottom panel of Fig. 10c where the electron-phonon coupling in MgB$_2$ is treated in an isotropic framework, i.e. where the energy from the electronic degrees of freedom is transferred in a equal way to all the lattice modes, without a preferential channel. In such case, where the hot-phonon bottleneck is absent, employing a two-temperature model the average phonon temperature $T_{\text{ph}}$ is predicted to show just a weak and smooth increase up to a final thermalization at about $T_{\text{ph}} = T_e = 420$ K \[223\].

5.1. Time-resolved Raman spectroscopy

The onset of hot-phonon physics, as captured for instance by the three-temperature model shown in Fig. 10c, has important observable consequences on many physical properties. One of the most direct fingerprints of hot phonons can be provided by the analysis of the Raman features \[113\]. As discussed in Sec. 3 for Raman active modes (such as $E_{2g}$ in MgB$_2$) the evolution of the hot-phonon temperature $T_{\text{hot}}$
can be experimentally assessed from the time dependence of the intensities of the Stokes and anti-Stokes peaks, and their ratio. The analysis of Ref. [113] is shown in Fig. 11. Assuming a starting temperature $T = 300$ K, the intensity of the Stoke (anti-Stokes) peak is predict to increase at $T_{\text{hot}}$ up to a factor 2 (15
for anti-Stokes), leading to a corresponding increase of their relative intensity as large as $I_{\text{AS}}(T_{\text{max}})/I_{\text{S}}(T_{\text{max}}) \approx 7$. Such remarkable anomalies in the intensity of the $E_{2g}$ Raman resonances can be compared with predicted time behavior of the Raman active $B_{1g}$ mode which, as the $E_{2g}$ modes, is characterized by out-of-phase lattice displacements of the two B atoms, but along the *out-of-plane* direction. This mode has a frequency $\omega_{B_{1g}} \approx 86 \text{ meV}$ not far from the $E_{2g}$ peak but, due to symmetry, has a negligible linear electron-phonon coupling and belongs thus to the cold mode sector. As a consequence its Raman features obey the time dependence of $T_{\text{cold}}$, with negligible effect on the intensity of the Stokes peaks and a moderate increase for $I_{\text{AS}}$, with the time scales dictated by $T_{\text{cold}}$ (Fig. 11b). The comparison between the time dynamics of the Raman properties of the $E_{2g}$ and $B_{1g}$ resonances provides thus an unambiguous tool to reveal two different temperatures governing simultaneously different lattice modes.

Peculiar effects of the hot-phonon scenario can be traced not only by studying frequency-integrated quantities, as the Raman intensity, but also via spectral Raman properties. Theoretically, these features can be conveniently analyzed by computing the many-body phonon self-energy $\Pi(\omega; \{T\})$ of the $E_{2g}$ mode at $q \approx 0$ [223]. The phonon spectral function is thus evaluated as [200]:

$$B(\omega; \{T\}) = \frac{1}{\pi} \text{Im} \left[ \frac{2\omega_{E_{2g}}}{\omega^2 - \omega_{E_{2g}}^2 - 2\omega_{E_{2g}}\Pi(\omega; \{T\})} \right],$$

(33)

where $\omega_{E_{2g}} = 67 \text{ meV}$ is the harmonic adiabatic phonon frequency. For many conventional systems, the phonon self-energy can be reasonably computed by using non-interacting Green’s functions, with the additional inclusion of a phenomenological electronic damping. Such level of approximation is however insufficient in the case of MgB$_2$ where the electronic damping is crucially governed by the electron-phonon coupling itself [219, 221]. The interplay between the energy dependence of the electron and phonon damping gives rise thus to peculiar nonadiabatic effects that are properly taken into account in a nonadiabatic approach [221]. It should be stressed that in this framework, under non-equilibrium conditions, the phonon self-energy depends on the full set of electron and phonon temperatures $\{T\} = (T_e, T_{\text{hot}}, T_{\text{cold}})$.

The spectral properties of the $E_{2g}$ phonon mode are shown in Figs. 11c and 11d as a function of the time delay, for two representative values of the pump fluence. The peculiar features of these spectra can be parametrized in terms of few quantities, namely the renormalized phonon frequency $\Omega_{E_{2g}}^2 = \omega_{E_{2g}}^2 + 2\omega_{E_{2g}}\Pi(\Omega_{E_{2g}}; \{T\})$, and the many-body phonon linewidth $\Gamma_{E_{2g}} = -2\text{Im}\Pi(\Omega_{E_{2g}}; \{T\})$. The time behavior of these representative parameters is also shown in Figs. 11e and 11f. Fingerprints of the hot phonons can be detected in the time dependence of $\Omega_{E_{2g}}$ and $\Gamma_{E_{2g}}$. The latter one, in particular, appears quite striking since it reveals a counter-intuitive *reduction* of the phonon linewidth $\Gamma_{E_{2g}}$ right after the photo-excitation, followed by a subsequent increase due to the later thermalization with the cold phonon degrees of
freedom. As discussed in Ref. [113], such behavior is essentially driven by the contribution of the nonadiabatic intraband processes, which can be modeled as:

$$\Pi^{\text{intra,NA}}(\omega; \{T\}) = \frac{\omega \langle |g_{E_{2g}}|^2 \rangle_{T_e}}{\omega [1 + \lambda(\omega; \{T\})] + i\gamma(\omega; \{T\})}, \quad (34)$$

where $$\langle |g_{E_{2g}}|^2 \rangle_{T_e} = -\sum_{k\alpha} \left| g_{E_{2g}; k,\alpha} \right|^2 \frac{\partial f(c_{k,\alpha}; T_e)}{\partial c_{k,\alpha}}$$. The parameters $$\lambda(\omega; \{T\})$$, $$\gamma(\omega; \{T\})$$ play here the same role as the effective mass function $$m^*(\omega)$$ and the optical scattering rate $$\Gamma_{\text{opt}}(\omega) = \tau_{\text{opt}}^{-1}(\omega)$$ in the extended Drude model and a microscopic derivation can be provided in a similar way. A careful analysis has shown that $$\gamma(\omega; \{T\})$$ can be written as a sum of three independent contributions depending separately on each temperature $$T_e, T_{\text{hot}}, T_{\text{cold}}$$ [113]:

$$\gamma(\omega; \{T\}) = \gamma_e(\omega; T_e) + \gamma_{\text{hot}}(T_{\text{hot}}) + \gamma_{\text{cold}}(T_{\text{cold}}), \quad (35)$$

where

$$\gamma_e(\omega; T_e) = -\frac{\pi}{\omega} \int d\Omega \alpha^2 F(\Omega) \left[ \frac{\omega + \Omega}{2k_B T_e} - \frac{\omega - \Omega}{2k_B T_e} \right], \quad \gamma_{\text{hot}}(T_{\text{hot}}) = 2\pi \int d\Omega \alpha^2 F_{\text{hot}}(\Omega) \coth \frac{\Omega}{2k_B T_{\text{hot}}}, \quad \gamma_{\text{cold}}(T_{\text{cold}}) = 2\pi \int d\Omega \alpha^2 F_{\text{cold}}(\Omega) \coth \frac{\Omega}{2k_B T_{\text{cold}}}.$$

(36) (37) (38)

Note that, from this analysis, employing Kramers-Kronig transformations, the function $$\lambda(\omega; \{T\})$$ depends only on the quantity $$\gamma_e(\omega; T_e)$$, and thus only on the electronic temperature, i.e., $$\lambda(\omega; \{T\}) = \lambda_e(\omega; T_e)$$. As intuitive, an increase of the effective temperatures, especially $$T_{\text{hot}}$$, leads to an increase of the damping term $$\gamma(\Omega_{E_{2g}}; \{T\})$$. What is less intuitive, but easily understandable on the base of Eq. (34), is the effect of the $$\gamma(\Omega_{E_{2g}}; \{T\})$$ on the phonon linewidth $$\Gamma_{E_{2g}}$$. Under the room temperature equilibrium conditions, $$\gamma(\Omega_{E_{2g}}; T)$$ is usually smaller than the renormalized phonon frequency, $$\gamma(\Omega_{E_{2g}}; T) \ll \Omega_{E_{2g}} [1 + \lambda(\Omega_{E_{2g}}; T)]$$, so that the phonon damping scales linearly with $$\gamma(\omega; T)$$, i.e., $$\Gamma_{E_{2g}} \propto \gamma(\Omega_{E_{2g}}; T)$$. The strong increase of $$T_{\text{hot}}$$ in the hot-phonon scenario brings however the material in the unconventional regime where $$\gamma(\Omega_{E_{2g}}; \{T\}) \gg \Omega_{E_{2g}} [1 + \lambda(\Omega_{E_{2g}}; \{T\})]$$. As a consequence, easily noticeable in Eq. (34), the $$E_{2g}$$ phonon linewidth scales as $$\Gamma_{E_{2g}} \propto 1/\gamma(\Omega_{E_{2g}}; \{T\})$$. The increase of $$\gamma(\Omega_{E_{2g}}; \{T\})$$ in the hot-phonon regime results thus in the decrease of $$\Gamma_{E_{2g}}$$ as observed in Fig. [11]. Similar changes of regime occur in other physical contexts, as the crossover from an Elliott-Yafet to the Dyakonov-Perel spin-relaxation, or in the case of NMR motional narrowing [226, 228]. If only the nonadiabatic intraband term would be retained, the same process would lead to a moderate reduction of the renormalized $$E_{2g}$$ phonon frequency, as shown in Fig. [11b] (open orange squares). The real part of the phonon self-energy, determining the renormalized phonon frequency, is however affected also by
(adiabatic) interband transitions which bring to the additional blueshift (ruled uniquely by $T_e$), visible in Fig. 11e, that partially competes with the redshift induced by nonadiabatic intraband processes. It is worth stressing out that all these features have a remarkable dependence not only on time delay but also on the laser fluence. The combined analysis of these dependencies can bring a compelling experimental evidence of the presence of hot phonons in MgB$_2$ under non-equilibrium pump-probe conditions.

5.2. Time-resolved correlated lattice dynamics

As just discussed above, Raman spectroscopy is a powerful tool for studying the properties of the lattice dynamics, particularly suited for the $q = 0$ modes. For instance, an accurate determination of the intensity of the anti-Stokes peaks can access directly the population of the Raman mode under investigation. Despite these strengths, employing Raman spectroscopy in pump-probe experiments to completely disentangle the “hotness” of electron and lattice degrees of freedom can be challenging. In its core, Raman spectroscopy is a two-photon process which involves virtual electronic states, so that electron and lattice degrees of freedom are unavoidably mixed together. A direct detection of the hot-phonon properties that probes only the lattice dynamics not entangled with the electron degrees of freedom can provide thus a compelling smoking gun.

Along this perspective, an alternative and efficient way for detecting the $E_{2g}$ hot phonons in MgB$_2$ has been recently theoretically proposed in Ref. [223]. The basic idea is to exploit similar crystallographic information as the UEDS but, through a Fourier transform, with real-space resolution. Such analysis can determine the interatomic probability distribution function (PDF) $G(r)$, whose features conveys information about the crystallographic structure and about the mean-square displacements, not only of single-atom motion but also of the correlated interatomic motion [230]. Within this context $E_{2g}$ hot phonons can feasibly revealed in MgB$_2$ due to their intrinsic anticorrelated in-plane motion of the two B atoms.

The experimental assessment of the correlated interatomic motion in MgB$_2$ was demonstrated in Ref. [230] using neutron diffraction. The advantage of this technique is that, since local properties are under investigation, the needed data can be collected in powders. Although directional information is in this case lost, the lattice dynamics of a given interatomic bond can be unambiguously determined due to the specific distance $231\ [233]$. An example of the probability distribution function $G(r)$ in MgB$_2$ under equilibrium conditions is shown in Fig. 12a. Different peaks can be related (from the different interatomic distance) to different atom pairs $i-j$ (Fig. 12b), whereas the width of each peak provides an estimate to the mean-square displacement of the relative distance $r_i - r_j$. The basic information of this crystallographic approach in real space can be summarized in: (i) the mean-square lattice displacement for a given atom $i$ along a given direction $\alpha = x, y, z$: 

$$\sigma^2(i_\alpha) = \langle [\mathbf{u}_i \cdot \hat{r}_\alpha]^2 \rangle,$$

(39)
Figure 12: (a) Measured probability distribution function $G(r)$ at room temperature thermal equilibrium. From Ref. [230]. Labeled with A and B are the peaks corresponding to the nearest neighbor B-B and B-Mg pairs. (b) Lattice structure of MgB$_2$ where the B-B and the B-Mg bonds are marked (respectively, label A and B). Also shown is the lattice displacement eigenvector of the $E_{2g}$ mode. (c) Computed correlation factors $\rho_{B-B}$, $\rho_{B-Mg}$ as a function of time delay for a typical pump-probe experiment. Solid lines correspond to a three-temperature model accounting for hot-phonon physics, while the dashed lines represent a two-temperature model where phonons are assumed to obey a thermal distribution without hot phonons. From Ref. [223].

where $u_i$ is the lattice displacement of atom $i$ from its average position and $\hat{r}_\alpha$ is the unit vector pointing along the direction $\alpha = x, y, z$; (ii) the mean-square relative displacement of atomic pairs projected onto the vector joining the atom pairs [230, 232]:

$$\sigma^2_{ij} = \langle [(u_i - u_j) \cdot \hat{r}_{ij}]^2 \rangle,$$

where $\hat{r}_{ij}$ is the unit vector connecting atoms $i$ and $j$.

It is also convenient to introduce the dimensionless correlation factor $\rho_{ij}$ defined by the implicit relation

$$\sigma^2_{ij} = \sigma^2(i_j) + \sigma^2(j_i) - 2\sigma(i_j)\sigma(j_i)\rho_{ij},$$

where $\sigma^2(i_j) = \langle [(u_i \cdot \hat{r}_{ij})]^2 \rangle$. Positive values of correlation factor $\rho_{ij} > 0$ describe a situation where the couple of atoms $i, j$ move in phase, so that the resulting value of $\sigma^2_{ij}$ is smaller than for the uncorrelated case. On the other hand, a predominance of counter-phase atomic vibrations is expected to result in
a negative correlation factor $\rho_{ij} < 0$. Microscopically, the correlation factor $\rho_{ij}$ results from the contribution of all the phonons of the Brillouin zone, weighted with the appropriate projection factor. For systems at thermal equilibrium, $\rho_{ij}$ is usually slightly positive, reflecting a slight dominance of the acoustic (in-phase) modes in contributing to the lattice dynamics.

A key role in detecting hot phonons in MgB$_2$ is played by the nearest neighbor in-plane boron-boron lattice dynamics. The $E_{2g}$ modes, characterized by out-of-phase motion (Fig. 12b), have a strong anticorrelation character, but at thermal equilibrium their contribution is balanced by all the other modes. On the other hand, in the hot-phonon regime the strong increase of the phonon population of the $E_{2g}$ modes leads to a striking anomaly in $\rho_{B-B}$ [223], which shows a remarkable dip at $T_{hot}^{max}$ Fig. 12c. The experimental determination of the correlation factor $\rho_{B-B}$ could provide thus a robust fingerprint of the presence of the hot phonons in MgB$_2$, as well as a useful estimate of their effective temperature and of the time scale when all the lattice degrees of freedom reach their final thermalization [223].

It is worth mentioning that the real-space approach here described for MgB$_2$ for detecting hot phonons relies basically on the crucial properties that hot phonons have a well specific lattice displacement pattern dictated by the selection rules of the electron-phonon coupling and by the fact of laying at high-symmetry points. Hot phonons occurs naturally, because of the phase space considerations, at high-symmetry points of the Brillouin zone, with well-defined and well-known symmetry and atomic contents. Such analysis does not apply thus only to MgB$_2$ but it is quite general and it can be extended to a wide range of materials (semiconductors, metals or semimetals).

5.3. Time-resolved reflectivity

Hot phonons occur because of a preferential transfer of energy from the electronic degrees of freedom to few selected lattice modes, through the available electron-phonon coupling. Hot phonons are thus a result of the many-body electron-phonon interaction, and fingerprints of a hot-phonon regime have been since long time detected also in electronic properties, as discussed in Sec. 3.

As a matter of fact, the first experimental report of hot phonons in MgB$_2$ was deduced from the analysis of time-resolved optical properties [112], profiting of the specific peculiarities of this compound. Unlike previous investigations that have also used reflectivity measurements, and where hot phonons were inferred from the multi-exponential decay of the reflectivity, fitted with three- or four-temperature models including hot-phonon channels, the core of the analysis in Ref. [112] was the observation of an anomalous blueshift of the in-plane $\omega_{p,a}$ in the first time window ($\sim 170$ fs) after pumping, followed by a later blueshift of the out-of-plane plasma frequency $\omega_{p,c}$ as well, as shown in Fig. 13a.

Such blueshift, not observed before in other materials, is quite anomalous since in conventional materials the increase of the effective electron and lattice temperatures would lead to a natural reduction of the electronic kinetic energy, and hence of the plasma frequency. Other possible sources of anomalous shifts of the plasma frequency under nonequilibrium conditions (electron
heating, band structure renormalization due to the non-linear coupling with the ultrashort laser pulse) would predict as well a redshift [112] rather than the experimentally observed blueshift.

The increase of the in-plane plasma frequency $\omega_{p,a}$ (and of the out-of-plane one $\omega_{p,c}$ after a finite delay) has been rationalized as a result of the onset of
$E_{2g}$ hot modes in the presence of the strong particle-hole asymmetry of the $\sigma$ bands which, as discussed in Sec. 4, are characterized by a Fermi level very close to the top edge, and hence with a very low carrier density. The many-body electron-phonon coupling is known in this situation to be responsible not only for the usual finite electronic damping term $\Gamma(k, \omega) = -\text{Im}\Sigma(k, \omega)$, resulting from the imaginary part of the electronic self-energy, but also for a finite bandshift, related to the finite real part of the self-energy, $\chi(k, \omega) = \text{Re}\Sigma(k, \omega)$ [235-237]. While the consequences of the first effect in the transient optical properties have been discussed in detail [60, 238, 239], the role of the real part of the self-energy is usually neglected in most materials where no strong particle-hole asymmetry is present or when the particle- or hole-character of Fermi multiple sheets is washed out by an isotropic electron-phonon coupling averaging over all the electron and hole Fermi pockets. In this perspective MgB$_2$ is peculiar, in a similar fashion as iron-based pnictides, because the strong hole-character of a subsector of the electronic degree of freedom (the $\sigma$ bands) is highlighted by the anisotropic electron-phonon coupling, selecting the relevant scattering for the electron-phonon self-energy in the $\sigma$ sector, and hence enforcing an effective hole-character [235].

The role of hot phonons in this context can be captured in a simple modeling where only the dominant electron-phonon coupling in the $\sigma$ bands is taken into account, described in a very good approximation by two-dimensional parabolic bands, while the $\pi$ bands act as a charge reservoir [112]. The in-plane plasma frequency, dominated by the $\sigma$ bands, can be evaluated thus as:

$$\omega_{p,\sigma} = \sqrt{\frac{4\pi e^2 \tilde{n}_\sigma}{m}},$$

where $m$ is the effective mass of the parabolic $\sigma$ bands $k_F$ the Fermi momentum for the non-interacting system, and $\tilde{n}$ the many-body renormalized carrier density

$$\tilde{n} = \frac{k_F^2}{2\pi} = \frac{2m[E_T + \chi(T)]}{2\pi}.$$

A key role is thus played by the bandshift which, assuming different temperatures for the electrons and the hot-phonon modes, can be computed in a perturbation approach as [240]:

$$\chi(T) = \chi_0 + \chi_{el}(T_e) + \chi_{ph}(T_{hot}).$$
where

$$\chi_0 = -\lambda_{\text{hot}} \frac{\omega_{E_{2g}}}{2} \ln \left| \frac{E_T + \omega_{E_{2g}}}{E_B + \omega_{E_{2g}}} \right|,$$

$$\chi_{\text{el}}(T_e) = -\lambda_{\text{hot}} \frac{T_e}{2} \sum_n \frac{\omega_{E_{2g}}^2}{\omega_n^2 + \omega_{E_{2g}}^2} \ln \left| \frac{E_T^2 - \omega_{E_{2g}}^2}{E_B^2 - \omega_{E_{2g}}^2} \right| + \lambda_{\text{hot}} \frac{\omega_{E_{2g}}}{2} \ln \left| \frac{E_T + \omega_{E_{2g}}}{E_B - \omega_{E_{2g}}} \right|,$$

$$\chi_{\text{ph}}(T_{\text{hot}}) = -\lambda_{\text{hot}} \frac{\omega_{E_{2g}}}{2} f_{T_e}(\omega_{E_{2g}}) \ln \left| \frac{E_T^2 - \omega_{E_{2g}}^2}{E_B^2 - \omega_{E_{2g}}^2} \right| + \lambda_{\text{hot}} \frac{\omega_{E_{2g}}}{2} \ln \left| \frac{E_T + \omega_{E_{2g}}}{E_B - \omega_{E_{2g}}} \right|, \quad (46)$$

and where $\lambda_{\text{hot}}$ is the dimensionless electron-phonon coupling for the hot phonons, $E_{T/B}$ represents the band top/bottom edge, respectively, estimated with respect to the Fermi level, $f_{T_e}(x)$, $b_{T_{\text{hot}}}(x)$ are the Fermi and Bose distributions ruled by the electronic and hot-phonon temperatures respectively, and $\omega_n = \pi T_e (2n + 1)$ are fermionic Matsubara frequencies also governed by $T_e$. $\chi_0$ is a temperature-independent contribution, and it is identical at equilibrium or under non-equilibrium conditions, whereas $\chi_{\text{el}}(T_e)$, $\chi_{\text{ph}}(T_{\text{hot}})$ scale to zero with the corresponding temperatures. It is crucial to note here that, as discussed in Ref. [112] and shown in Fig. 13b, the bandshift $\chi\{T\}$ is dominated by the phonon temperature $T_{\text{hot}}$, whereas it shows a negligible dependence on the electronic temperature $T_e$. An effective increase of the phonon temperature leads thus in MgB$_2$ to a selective upwards bandshift of the $\sigma$ bands, resulting thus in an increase of the corresponding in-plane (hole-like) plasma frequency.

The overall experimental scenario has been explained in few time steps governed by the hot-phonon temperature [112], as depicted in Fig. 13. At $t = 0$ the pump pulse induces particle-hole excitations that soon thermalize towards an effective electronic temperature [241]. The preferential coupling with the $\sigma$-bands brings to a sudden increase of the hot-phonon temperature $T_{\text{hot}}$ which is accompanied by a many-body-driven upwards bandshift of the $\sigma$ bands. Charge transfer from the $\pi$ bands leads thus to a net increase of the $\sigma$-band carrier density and hence to a blueshift of the in-plane plasma frequency $\omega_{p,a}$ dominated by the $\sigma$ bands. At a later stage, the cooling down of the hot-phonon temperature $T_{\text{hot}}$, and the onset of phonon interband scattering reduces the bandshift of the $\sigma$ sector and charges flow back to the $\pi$ bands with strong three-dimensional character leading to the subsequent increase of the out-of-plane plasma frequency $\omega_{p,c}$.

One can notice that the present scenario depends strongly on the intraband character of the electron-phonon coupling with the hot phonons. A similar situation might be encountered in iron-based pnictides or similar materials, where spin-fluctuations play the role of the exchanged hot bosons. In those compounds however spin-fluctuations mediate interband exchange between band with different particle/hole character. In such context the situation can be reversed and hot-phonon driven bandshift might result in a net redshift of the
plasma frequency. So far, no experimental check of this scenario in time-resolved spectroscopy has been carried out at our knowledge.

6. Wider view on other compounds and perspectives

As widely discussed above, good metals are usually characterized by large Fermi surfaces. As a consequence, electron-phonon scattering is spread over a wide phonon momentum space, and no hot phonons are sustained. At odds with this scenario, MgB$_2$ provides a bright example that a different path for achieving hot phonons is possible. The key element for this alternative context to semiconductors and semimetals is a strong anisotropy of the electron-phonon coupling, where a sizable amount of the electron-phonon coupling is concentrated in a few phonon modes, along with a weak residual electron-phonon interaction widely spread over all the other modes. Such framework leads to an effective separation of the energy transfer from electrons to lattice into two parallel channels, where the small specific heat capacity of the strongly-coupled modes leads to a fast and large increase of the quantum population of the corresponding phonon states that get easily hot.

While the above mechanism is fulfilled in the most striking way in MgB$_2$, which represents a benchmark example, the physical conditions for this scenario are quite general and provide a guideline for predicting hot phonons in other metallic compounds.

A promising candidate along this direction is hole-doped diamond, which was discovered to become superconductor with $T_c \approx 4$ K upon 2.8% of boron doping. Baring interesting similarities with MgB$_2$, boron-doped diamond has indeed regarded as “three-dimensional” MgB$_2$. From the electronic point of view, such high level of boron doping, achieved thanks to the small size of boron, leads indeed to a slight hole doping of the three-dimensional bands of diamond, in similar way as in the $\sigma$-bands of MgB$_2$. Hole-doping in diamond is also accompanied by a downshift and broadening of the only Raman-active mode. This scenario can be also explained, just as in MgB$_2$, as result of a Kohn anomaly with proper differences in diamond dictated by the three-dimensional character. Fano-like asymmetries of the Raman features, pointing out a relevant electron-phonon coupling, have been also reported in boron-doped diamond, although Fano asymmetry appears to be absent in N-doped samples. Based on such a strong similarity of the physical properties, we can predict an effective hot-phonon scenario for boron-doped diamond.

Such prediction can be sustained at a quantitative level by the analysis of the momentum-resolved electron-phonon coupling $\lambda_{q,\nu}$, shown in Fig. 14, for a 2.7% of boron doping. Just as in MgB$_2$, a sizable electron-phonon coupling (here $\lambda \approx 0.21$) is concentrated in a few optical modes. The relevance of the electron-phonon coupling is shown, like for MgB$_2$, by the comparison between the frequency dependence of Eliashberg function $\alpha^2 F(\omega)$ and of the phonon density of states $F(\omega)$: the peak structure in $\alpha^2 F(\omega)$ at $\omega \approx 130 - 150$ meV, together with the knowledge of $\lambda_{q,\nu}$, identifies in an unambiguous way the hot-phonon modes, whereas the frequency structure of $F(\omega)$, spread over a wide
Figure 14: Phonon band structure and electron-phonon coupling of (a) 2.6 % hole-doped diamond and (b) 1T-TiSe$_2$ single layer. The strength of the momentum- and band-resolved electron-phonon coupling $\lambda_{q\nu}$ is represented with the size of the red dots. The corresponding phonon density of states $F(\omega)$ and Eliashberg functions $\alpha^2 F(\omega)$ are depicted with the blue and red lines. The ground state electronic properties were obtained by means of the density-functional-theory package QUANTUM ESPRESSO [100, 101]. For obtaining phonon properties and electron-phonon coupling, the combination of density functional perturbation theory [250] and wannierization procedure [251, 252] is used with the help of EPW code [253]. Eliashberg functions $\alpha^2 F(\omega)$ are obtained from the electron-phonon coupling constants $\lambda_{q\nu}$ calculated on the $k = 40 \times 40 \times 40$ and $q = 40 \times 40 \times 40$ momentum grids in the case of hole-doped diamond and $k = 600 \times 600 \times 1$ and $q = 120 \times 120 \times 1$ in the case of 1T-TiSe$_2$.

range of frequencies, with no correspondence in $\alpha^2 F(\omega)$, characterizes the cold modes. The very weak coupling associated with the cold modes makes boron-doped diamond an extreme case of anisotropic electron-phonon system where the linear coupling of electrons with the cold lattice sector is almost negligible. This situation might be even more favourable than MgB$_2$ for detecting hot phonons since hot phonons are expected to show longer decay rate to the cold modes. At the same time, these conditions appear appealing also from the electronic point of view, since the reduced coupling with the remnant lattice modes would imply a slower heating of the sample, which is usually harmful for optoelectronics.

Possible candidates for hot phonons appear also among related compounds, like hole-doped Si and Ge, where for small doping the amount of strongly-coupled modes is a few percentage [243], just like in doped diamond. Although the deformation potential in these materials has been estimated to be significantly lower than in diamond, this is not necessarily negative for sustaining hot phonons. Indeed, it should be remarked that, as long the coupling with the cold modes is negligible, the largest hot-phonon temperature $T_{\text{hot}}^{\text{max}}$ achievable in the time dynamics is ruled uniquely by the ratio of the specific heat capacities $C_i$, regardless of the electron-phonon relaxation parameters $G_i$. Within this context, the smaller values of $G_i$ in hole-doped Si (or Ge) with respect to hole-doped diamond would not imply a smaller $T_{\text{hot}}^{\text{max}}$, but rather a longer time to reach it.
Furthermore, several boron-related materials with electronic and vibrational band-structures resembling bulk MgB$_2$ have been theoretically discussed as a promising high-T$_c$ materials, such as hole-doped LiBC and its different allotropes [254–258] as well as lithium boride (LiB) [259, 260]. Each of these compounds are characterized by hexagonal layered structure, where boron and carbon atoms occupy hexagonal sites, while lithium is placed between hexagonal layers (with the similar role as Mg in MgB$_2$). Here also the B-C or B-B bond-stretching phonon mode at the center of the BZ contribute predominantly to the overall electron-phonon coupling strength, which makes these materials ideal for exploring hot phonons. Realization of the efficient hot phonon physics could also possible by extended to MgB$_2$-related monolayers, such as MgB$_2$ monolayer [261], hydrogenated monolayer MgB$_2$ [262], and LiBC monolayer [263], where in each case the significant strength of electron-phonon coupling is concentrated in only a few modes. Another boron-carbon-based candidate for hot-phonon is SrRbB$_6$C$_6$, which is predicted to sustain high-T$_c$ superconductivity and where a sizable amount of the electron-phonon coupling in concentrated in few selected lattice modes in a energy window [40:50] meV [264, 265].

Some common trends can be pointed out in the above materials. In hole-doped diamond for instance, as in MgB$_2$, the strong coupling of selected modes candidates for hot-phonon physics is accompanied by a marked Kohn anomaly which is revealed by a large phonon softening in the phonon dispersion. Kohn anomalies restricted to few $q$ momenta, on the other hand, are commonly observed in systems close to a charge-density-wave (CDW) or to a structural transition. Within this context, the approaching of a second-order structural transition is signalized by a softening of a given phonon mode $\omega_o(q)$ which vanishes $\omega_o(q) \rightarrow 0$ at the quantum critical point (Peierls instability). Such softening is accompanied by a formal divergence of the electron-phonon coupling mediated by this mode, as a result of the relation $\lambda = 2 \int d\omega \alpha^2 F(\omega)/\omega$. According to this picture, one can naively argue that compounds at the verge of a second-order structural transition would be the best candidates for hot phonons. However, although the basic argument is valid, one should also take into account that the relaxation rates $G_i$ governing the energy transfer from the electrons to the lattice modes can be qualitatively estimated as $G_i \propto \lambda_i \langle \omega_i^2 \rangle$ [19, 63]. This implies that softening modes for $\omega_o(q) \rightarrow 0$ do not actually contribute at the energy transfer from the electronic degrees of freedom. The most favourable conditions for hot phonons, balancing all the effects, are thus encountered in systems approaching but not too close to a lattice instability. A representative example of a possible candidate is 1T-TiSe$_2$, a layered transition-metal dichalcogenide with a semimetal character at room temperature. Such compound undergoes a charge-density-wave transition with a $2 \times 2 \times 2$ periodicity below $T_{CDW} \approx 200$ K [266]. This structural transition, which persists also in few-layer and single-layer samples [267], is preceded by the sizable softening of a phonon mode at the M and L points of the three-dimensional Brillouin zone [268], suggesting an electron-phonon driven mechanism, although electronic models based on an excitonic condensate have been also proposed [269]. Recent UEDS studies reveal
under pump-probe conditions an unconventional behavior of the intensity of the diffuse scattering at the M/L points, resulting from the competing increasing of the phonon population and the reduced electronic screening [179]. The possibility of hot-phonon physics is pointed out by an ab-initio investigation in the normal state of the momentum-resolved electron-phonon coupling in single-layer TiSe$_2$, as shown in Fig. 14, where the red dots on top of the phonon dispersion represent the strength of the electron-phonon coupling. We note that a strong electron-phonon coupling is concentrated in few lattice modes at the M point which shows at the same time a strong Peierls-like softening. The relevance of these modes with respect to a hot-phonon scenario is underlined by the comparison in the right panel of Fig. 14 between the phonon density of states $F(\omega)$ and the Eliashberg function $\alpha^2 F(\omega)$. Just like in MgB$_2$, the Kohn anomaly is reflected in a strong peak ($\sim 10$ meV) in the Eliashberg function $\alpha^2 F(\omega)$ which is not expected from the phonon density of states but it is uniquely driven by the electron-phonon coupling. A direct evidence of hot phonons in 1T-TiSe$_2$ is one of the future experimental challenges in this field [170].

The closeness to a charge-density-wave phase is a promising path for tailoring the onset of hot phonons in metals which is not restricted to TiSe$_2$, but is just a representative example. Other systems close to a charge-density-wave instability can be investigated as possible candidates, not only in the family of layered transition-metal dichalcogenides $MX_2$ ($M =$Nb, Ta; $X =$S, Se), but also in three-dimensional compounds, as A15 (Nb$_3$Sn, V$_3$Si or barium-bismuthate perovskites [270]) Recent experimental studies based on time-resolved ARPES and time-resolved X-ray diffraction show a critical role of strongly coupled phonons and mode-selective electron-phonon coupling in 1T-TaSe$_2$ [271, 272], 1T-VSe$_2$ [135], and several 3D systems with the CDW order [134, 273–275]. Quite remarkable, one of these interesting examples shows that a nonthermal phonon distribution in the form of hot phonons can lead to the CDW melting and to a later CDW recovery which is markedly different from the equilibrium case [134].

A further promising development is suggested by the possibility of exciting hot-phonon physics in surface phonons. This scenario bares additional interest in MgB$_2$ due to the topological character of phonons, which is particularly revealed at the surfaces [276, 277]. Recent experimental techniques have been shown to be efficient in investigating large electron-phonon coupling constants and lattice instabilities on the surfaces [278, 279].

A last mention should be devoted in this framework to iron-based pnictides, superconducting materials with bulk critical temperatures up to $T_c \approx 55$ K [252] and above 100 K in FeSe single-layer on doped SrTiO$_3$ [283]. Like MgB$_2$, these compounds display a multigap superconductivity which, as discussed before, reveals an anisotropic scattering between the different Fermi sheets (see Fig. 15 for a typical band structure and multigap Fermi properties). However, there is nowadays a wide consensus that superconductivity in iron-based pnictides, unlike MgB$_2$, is not related to the electron-phonon coupling (although present) but it is driven by spin-fluctuation exchange. Spin-fluctuations represent a particularly strong channel for interband scattering, connecting hole-like bands
Figure 15: Multiband/multigap properties for iron-based pnictides. (a) Band structure of LaFePO. Hole-like bands at the Γ point are marked in red, while electron-like bands at M point are marked in green. From Ref. [280]. (b) Multigap structure of Ba$_{0.6}$K$_{0.4}$Fe$_2$As$_2$. Bottom plane represents ARPES intensity at the Fermi level. Upper plane represents the value of the superconducting gap measured by ARPES on different Fermi sheets. In the inset: temperature evolution of the different superconducting gap on different Fermi sheets. From Ref. [281].

close to the Γ point with electron-like bands at the M point [284], while intraband coupling is relatively weak. The strong anisotropy of the coupling provides a similar robust platform as MgB$_2$ for hot-mode physics, where spin-fluctuations at (±π, ±π) represent the preferential bosonic channel for energy transfer from the electron sector. Future work in this direction can assess the validity of this scenario.

7. Concluding remarks

In this paper we have reviewed the recent progresses regarding a novel path for observing hot phonons in metals as a consequence of a strong anisotropic electron-phonon coupling. Based on a summary of the fundamental physics involved in pump-probe out-of-equilibrium experiments, we have discussed the physical conditions that allow sustaining non-thermal populations of few selected modes in metals.

A special focus has been devoted to MgB$_2$ that represents maybe the best prototype of metallic compounds fulfilling such conditions. Physical observables of the hot-phonon scenario in MgB$_2$ have been discussed, both on the experimental and theoretical grounds. Besides MgB$_2$, other materials have been outlined as promising candidates for realization of hot phonons. Among them, low-carrier doped semiconductors, as for instance boron-doped diamond, and compounds with soft modes at the verge of a structural transition.

The feasibility of controlling the onset and the properties of hot phonons, not only in semiconductors but also in metals, opens the way to the possibility of
including metal components in ideal optoelectronic devices where a hot-phonon bottleneck could be desirable for controlling the electronic relaxation decay rates and the effective lattice heating. The possibility of exciting hot phonons with a chiral character might open also new interesting perspectives in the field of quantum information where a basic quantum bit could be engraved in the chiral state of a long-lived chiral phonon.

The continuous and fast development of refined experimental techniques and theoretical tools provides a promising scenario where hot-phonon physics in metals could be a common tool on the advanced research workbench.

Acknowledgments

E.C. owes special thanks to L. Benfatto, F. Carbone, E. Baldini, G. Campi, D. Fausti and A. Tomadin for many fruitful discussions. F.C. acknowledges funding by the Deutsche Forschungsgemeinschaft (DFG Projektnummer 443988403) and PRACE for awarding access to Prometheus, CYFRONET, Poland. D.N. acknowledges financial support from the Croatian Science Foundation (Grant no. UIP-2019-04-6869) and from the European Regional Development Fund for the “Center of Excellence for Advanced Materials and Sensing Devices” (Grant No. KK.01.1.1.01.0001).

References

[1] C. Giannetti, M. Capone, D. Fausti, M. Fabrizio, F. Parmigiani, D. Mihailovic, Ultrafast optical spectroscopy of strongly correlated materials and high-temperature superconductors: a non-equilibrium approach, Adv. Phys. 65 (2016) 58.
[2] H. Petek, S. Ogawa, Femtosecond time-resolved two-photon photoemission studies of electron dynamics in metals, Prog. Surf. Sci. 56 (1997) 239.
[3] W. E. King, G. H. Campbell, A. Frank, B. Reed, J. F. Schmerge, B. J. Siwick, B. C. Stuart, P. M. Weber, Ultrafast electron microscopy in materials science, biology, and chemistry, J. Appl. Phys. 97 (2005) 111101.
[4] R. Ulbricht, E. Hendry, J. Shan, T. F. Heinz, M. Bonn, Carrier dynamics in semiconductors studied with time-resolved terahertz spectroscopy, Rev. Mod. Phys. 83 (2011) 543.
[5] M. Bauer, A. Marienfeld, M. Aeschlimann, Hot electron lifetimes in metals probed by time-resolved two-photon photoemission, Prog. Surf. Sci. 90 (2015) 319.
[6] L. Waldecker, R. Bertoni, R. Ernstorfer, J. Vorberger, Electron-Phonon Coupling and Energy Flow in a Simple Metal beyond the Two-Temperature Approximation, Phys. Rev. X 6 (2016) 021003.
[7] A. V. Kuznetsov, C. J. Stanton, Theory of Coherent Phonon Oscillations in Semiconductors, Phys. Rev. Lett. 73 (1994) 3243.

[8] R. Merlin, Generating coherent THz phonons with light pulses, Solid State Comm. 102 (1997) 207.

[9] A. V. Kuznetsov, C. J. Stanton, Theory of Coherent Phonon Oscillations in Bulk GaAs, in: K. T. Tsen (Ed.), Ultrafast Phenomena in Semiconductors, Springer, New York, NY, 2001, p. 353.

[10] T. E. Stevens, J. Kuhl, R. Merlin, Coherent phonon generation and the two stimulated Raman tensors, Phys. Rev. B 65 (2002) 144304.

[11] K. Ishioka, M. Hase, M. Kitajima, H. Petek, Coherent optical phonons in diamond, Appl. Phys. Lett. 89 (2006) 231916.

[12] U. Bovensiepen, Ultra-fast dynamics of coherent lattice and spin excitations at the Gd(0001) surface, Appl. Phys. A: Mater. Sci. Process. 82 (2006) 395.

[13] K. Ishioka, M. Hase, M. Kitajima, L. Wirtz, A. Rubio, H. Petek, Ultrafast electron-phonon decoupling in graphite, Phys. Rev. B 77 (2008) 121402.

[14] M. Först, C. Manzoni, S. Kaiser, Y. Tomioka, Y. Tokura, R. Merlin, A. Cavalleri, Nonlinear phononics as an ultrafast route to lattice control, Nat. Phys. 7.

[15] D. Boschetto, L. Malard, C. H. Lui, K. F. Mak, Z. Li, H. Yan, T. F. Heinz, Real-Time Observation of Interlayer Vibrations in Bilayer and Few-Layer Graphene, Nano Lett. 13 (2013) 4620.

[16] M. Udina, T. Cea, L. Benfatto, Theory of coherent-oscillations generation in terahertz pump-probe spectroscopy: From phonons to electronic collective modes, Phys. Rev. B 100 (2019) 165131.

[17] A. Cavalleri, C. Tóth, C. W. Siders, J. A. Squier, F. Ráksi, P. Forget, J. C. Kieffer, Femtosecond Structural Dynamics in VO$_2$ during an Ultrafast Solid-Solid Phase Transition, Phys. Rev. Lett. 87 (2001) 237401.

[18] J. Demsar, L. Forró, H. Berger, D. Mihailovic, Femtosecond snapshots of gap-forming charge-density-wave correlations in quasi-two-dimensional dichalcogenides 1T-TaS$_2$ and 2H-TaSe$_2$, Phys. Rev. B 66 (2002) 041101.

[19] S. Iwai, M. Ono, A. Maeda, H. Matsuzaki, H. Kishida, H. Okamoto, Y. Tokura, Ultrafast Optical Switching to a Metallic State by Photoinduced Mott Transition in a Halogen-Bridged Nickel-Chain Compound, Phys. Rev. Lett. 91 (2003) 057401.

[20] M. Rini, R. Tobey, N. Dean, J. Itatani, Y. Tomioka, Y. Tokura, R. Schoenlein, A. Cavalleri, Control of the electronic phase of a manganite by mode-selective vibrational excitation, Nature 449 (2007) 72.
[21] L. Perfetti, P. Loukakos, M. Lisowski, U. Bovensiepen, M. Wolf, Time-resolved photoemission of an insulator-metal transition, in: Corkum, P. and Jonas, D and Miller, RJD and Weiner, AM (Ed.), Ultrafast Phenomena XV, Vol. 88 of Springer Series in Chemical Physics, 2007.

[22] L. Perfetti, P. A. Loukakos, M. Lisowski, U. Bovensiepen, H. Berger, S. Biemann, P. S. Cornaglia, A. Georges, M. Wolf, Time Evolution of the Electronic Structure of 1T-TaS2 through the Insulator-Metal Transition, Phys. Rev. Lett. 97 (2006) 067402.

[23] Y. Kawakami, S. Iwai, T. Fukatsu, M. Miura, N. Yoneyama, T. Sasaki, N. Kobayashi, Optical Modulation of Effective On-Site Coulomb Energy for the Mott Transition in an Organic Dimer Insulator, Phys. Rev. Lett. 103 (2009) 066403.

[24] D. Fausti, R. Tobey, N. Dean, S. Kaiser, A. Dienst, M. Hoffmann, S. Pyon, T. Takayama, H. Takagi, A. Cavalleri, Light-Induced Superconductivity in a Stripe-Ordered Cuprate, Science 331 (2011) 189.

[25] S. Kaiser, C. R. Hunt, D. Nicoletti, W. Hu, I. Gierz, H. Y. Liu, M. Le Tacon, T. Loew, D. Haug, B. Keimer, A. Cavalleri, Optically induced coherent transport far above Tc in underdoped YBa2Cu3O6+δ, Phys. Rev. B 89 (2014) 184516.

[26] M. Mitrano, A. Cantaluppi, D. Nicoletti, S. Kaiser, A. Perucchi, S. Lupi, P. Di Pietro, D. Pontiroli, M. Riccoò, S. Clark, D. Jaksch, A. Cavalleri, Possible light-induced superconductivity in K3C60 at high temperature, Nature 530 (2016) 461.

[27] A. Cantaluppi, M. Buzzi, G. Jotzu, D. Nicoletti, M. Mitrano, D. Pontiroli, M. Ricco, A. Perucchi, P. Di Pietro, A. Cavalleri, Pressure tuning of light-induced superconductivity in K3C60, Nat. Phys. 14 (2018) 837.

[28] T. Suzuki, T. Someya, T. Hashimoto, S. Michimae, M. Watanabe, M. Fujisawa, T. Kanai, N. Ishii, J. Itatani, S. Kasahara, Y. Matsuda, T. Shibatachi, K. Okazaki, S. Shin, Photoinduced possible superconducting state with long-lived disproportionate band filling in FeSe, Commun. Phys. 2 (2019) 115.

[29] K. Isoyama, N. Yoshikawa, K. Katsumi, J. Wong, N. Shikama, Y. Sakishita, F. Nabeshima, A. Maeda, R. Shimano, Light-induced enhancement of superconductivity in iron-based superconductor FeSe0.5Te0.5, Commun. Phys. 4 (2021) 160.

[30] R. R. Cavanagh, D. S. King, J. C. Stephenson, T. F. Heinz, Dynamics of nonthermal reactions: femtosecond surface chemistry, J. Phys. Chem. 97 (1993) 786.

[31] W. Ho, Reactions at Metal Surfaces Induced by Femtosecond Lasers, Tunneling Electrons, and Heating, J. Phys. Chem. 100 (31) (1996) 13050.
[32] Kühn, O. and Wöste, L. (eds), Analysis and Control of Ultrafast Photoinduced Reactions, Vol. 87 of Chem. Phys., Springer-Verlag Berlin Heidelberg, 2007.

[33] Y. Matsumoto, Photochemistry and Photo-Induced Ultrafast Dynamics at Metal Surfaces, Bull. Chem. Soc. Jpn 80 (2007) 842.

[34] H. Petek, J. Zhao, Ultrafast Interfacial Proton-Coupled Electron Transfer, Chem. Rev. 110 (2010) 7082.

[35] D. Fausti, O. V. Misochko, P. H. M. van Loosdrecht, Ultrafast photoinduced structure phase transition in antimony single crystals, Phys. Rev. B 80 (2009) 161207.

[36] R. Mankowsky, A. Subedi, M. Foerst, S. O. Mariager, M. Chollet, H. T. Lemke, J. S. Robinson, J. M. Glownia, M. P. Minitti, A. Frano, M. Fechner, N. A. Spaldin, T. Loew, B. Keimer, A. Georges, A. Cavalleri, Nonlinear lattice dynamics as a basis for enhanced superconductivity in YBa2Cu3O6.5, Nature 516.

[37] A. Subedi, A. Cavalleri, A. Georges, Theory of nonlinear phononics for coherent light control of solids, Phys. Rev. B 89 (2014) 220301.

[38] X. Cui, C. Wang, A. Argondizzo, S. Garrett-Roe, B. Gumhalter, H. Petek, Transient excitons at metal surfaces, Nat. Phys. 10 (2014) 505.

[39] E. Perfetto, D. Sangalli, A. Marini, G. Stefanucci, First-principles approach to excitons in time-resolved and angle-resolved photoemission spectra, Phys. Rev. B 94 (2016) 245303.

[40] C. Trovatello, F. Katsch, N. J. Borys, M. Selig, K. Yao, R. Borregonavarillas, F. Scotognella, I. Kriegel, A. Yan, A. Zettl, P. J. Schuck, A. Knorr, G. Cerullo, S. Dal Conte, The ultrafast onset of exciton formation in 2D semiconductors, Nat. Comm. 11 (2020) 5277.

[41] R. Sundararaman, P. Narang, A. S. Jermyn, W. A. Goddard, III, H. A. Atwater, Theoretical predictions for hot-carrier generation from surface plasmon decay, Nat. Commun. 5 (2014) 5788.

[42] A. Kogar, A. Zong, P. E. Dolgirev, X. Shen, J. Straquadine, Y.-Q. Bie, X. Wang, T. Rohwer, I.-C. Tung, Y. Yang, R. Li, J. Yang, S. Weathersby, S. Park, M. E. Kozina, E. J. Sie, H. Wen, P. Jarillo-Herrero, I. R. Fisher, X. Wang, N. Gedik, Light-induced charge density wave in LaTe3, Nat. Phys. 16 (2020) 159.

[43] K. W. Kim, A. Pashkin, H. Schaefer, M. Beyer, M. Porer, T. Wolf, C. Bernhard, J. Demsar, R. Huber, A. Leitenstorfer, Ultrafast transient generation of spin-density-wave order in the normal state of BaFe2As2 driven by coherent lattice vibrations, Nat. Mater. 11 (2012) 497.
[44] J. A. Johnson, T. Kubacka, M. C. Hoffmann, C. Vicario, S. de Jong, P. Beaud, S. Grübel, S.-W. Huang, L. Huber, Y. W. Windsor, E. M. Bothschafter, L. Rettig, M. Ramakrishnan, A. Alberca, L. Patthey, Y.-D. Chuang, J. J. Turner, G. L. Dakovski, W.-S. Lee, M. P. Minitti, W. Schlotter, R. G. Moore, C. P. Hauri, S. M. Koochpayeh, V. Scagnoli, G. Ingold, S. L. Johnson, U. Staub, Magnetic order dynamics in optically excited multiferroic TbMnO$_3$, Phys. Rev. B 92 (2015) 184429.

[45] M. Hudl, M. d’Aquino, M. Pancaldi, S.-H. Yang, M. G. Samant, S. S. P. Parkin, H. A. Dür, C. Serpico, M. C. Hoffmann, S. Bonetti, Nonlinear Magnetization Dynamics Driven by Strong Terahertz Fields, Phys. Rev. Lett. 123 (2019) 197204.

[46] R. M. Geilhufe, V. Juričić, S. Bonetti, J.-X. Zhu, A. V. Balatsky, Dynamically induced magnetism in KTaO$_3$, Phys. Rev. Res. 3 (2021) L022011.

[47] J. Yuen-Zhou, J. J. Krich, I. Kassal, A. S. Johnson, A. Aspuru-Guzik, Ultrafast Spectroscopy, 2053-2563, IOP Publishing, 2014.

[48] S. Anisimov, B. Kapeliovich, T. Perel’man, Electron emission from metal surfaces exposed to ultrashort laser pulses, Sov. Phys. JETP 39 (1974) 375.

[49] P. B. Allen, Theory of thermal relaxation of electrons in metals, Phys. Rev. Lett. 59 (1987) 1460.

[50] H. E. Elsayed-Ali, T. B. Norris, M. A. Pessot, G. A. Mourou, Time-resolved observation of electron-phonon relaxation in copper, Phys. Rev. Lett. 58 (1987) 1212.

[51] R. W. Schoenlein, W. Z. Lin, J. G. Fujimoto, G. L. Eesley, Femtosecond studies of nonequilibrium electronic processes in metals, Phys. Rev. Lett. 58 (1987) 1680.

[52] D. W. Snoke, W. W. Rühle, Y.-C. Lu, E. Bauser, Evolution of a non-thermal electron energy distribution in GaAs, Phys. Rev. B 45 (1992) 10979.

[53] W. S. Fann, R. Storz, H. W. K. Tom, J. Bokor, Electron thermalization in gold, Phys. Rev. B 46 (1992) 13592.

[54] W. S. Fann, R. Storz, H. W. K. Tom, J. Bokor, Direct measurement of nonequilibrium electron-energy distributions in subpicosecond laser-heated gold films, Phys. Rev. Lett. 68 (1992) 2834.

[55] C.-K. Sun, F. Vallée, L. H. Acioli, E. P. Ippen, J. G. Fujimoto, Femtosecond-tunable measurement of electron thermalization in gold, Phys. Rev. B 50 (1994) 15337.
[56] R. H. M. Groeneveld, R. Sprik, A. Lagendijk, Femtosecond spectroscopy of electron-electron and electron-phonon energy relaxation in Ag and Au, Phys. Rev. B 51 (1995) 11433.

[57] V. E. Gusev, O. B. Wright, Ultrafast nonequilibrium dynamics of electrons in metals, Phys. Rev. B 57 (1998) 2878.

[58] J. Hohlfeld, S.-S. Wellershoff, J. Güdde, U. Conrad, V. Jähnke, E. Matthias, Electron and lattice dynamics following optical excitation of metals, Chem. Phys. 251 (2000) 237.

[59] N. Del Fatti, C. Voisin, M. Achermann, S. Tzortzakis, D. Christofilos, F. Vallée, Nonequilibrium electron dynamics in noble metals, Phys. Rev. B 61 (2000) 16956.

[60] P. Echenique, J. Pitarke, E. Chulkov, A. Rubio, Theory of inelastic lifetimes of low-energy electrons in metals, Chem. Phys. 251 (2000) 1.

[61] B. Rethfeld, A. Kaiser, M. Vicanek, G. Simon, Ultrafast dynamics of nonequilibrium electrons in metals under femtosecond laser irradiation, Phys. Rev. B 65 (2002) 214303.

[62] L. Pietanza, G. Colonna, S. Longo, M. Capitelli, Non-equilibrium electron and phonon dynamics in metals under femtosecond laser pulses, Eur. Phys. J. D 45 (2007) 369.

[63] Z. Lin, L. V. Zhigilei, V. Celli, Electron-phonon coupling and electron heat capacity of metals under conditions of strong electron-phonon nonequilibrium, Phys. Rev. B 77 (2008) 075133.

[64] L. Zhang, Q. Niu, Angular Momentum of Phonons and the Einstein–de Haas Effect, Phys. Rev. Lett. 112 (2014) 085503.

[65] L. Zhang, Q. Niu, Chiral Phonons at High-Symmetry Points in Monolayer Hexagonal Lattices, Phys. Rev. Lett. 115 (2015) 115502.

[66] H. Zhu, J. Yi, M.-Y. Li, J. Xiao, L. Zhang, C.-W. Yang, R. A. Kaindl, L.-J. Li, Y. Wang, X. Zhang, Observation of chiral phonons, Science 359 (2018) 579.

[67] H. Chen, W. Zhang, Q. Niu, L. Zhang, Chiral phonons in two-dimensional materials, 2D Mater. 6 (2019) 012002.

[68] H. Komiyama, S. Murakami, Universal features of canonical phonon angular momentum without time-reversal symmetry, Phys. Rev. B 103 (2021) 214302.

[69] F. L. Semiao, K. Furuya, G. J. Milburn, Vibration-enhanced quantum transport, New J. Phys. 12 (2010) 083033.
[70] P. Kocevar, Non-ohmic transport and phonon amplification in polar semiconductors, J. Phys. C: Solid State Phys. 5 (1972) 3349.

[71] P. Kocevar, Hot phonon dynamics, Physica B+C 134 (1985) 155.

[72] W. Pütz, Hot-phonon effects in bulk GaAs, Phys. Rev. B 36 (1987) 5016.

[73] R. P. Joshi, D. K. Ferry, Hot-phonon effects and interband relaxation processes in photoexcited GaAs quantum wells, Phys. Rev. B 39 (1989) 1180.

[74] R. Mickevičius, A. Reklaitis, Hot phonon effects on impact ionization dynamics in InSb, Solid State Comm. 73 (1990) 145.

[75] D.-s. Kim, P. Y. Yu, Hot-electron relaxations and hot phonons in GaAs studied by subpicosecond Raman scattering, Phys. Rev. B 43 (1991) 4158.

[76] R. Mickevičius, V. Mitin, G. Paulavičius, V. Kochelap, M. A. Stroscio, G. J. Iafriate, Hot-phonon effects on electron transport in quantum wires, J. Appl. Phys. 80 (1996) 5145.

[77] P. Langot, N. Del Fatti, D. Christofilos, R. Tommasi, F. Vallée, Femtosecond investigation of the hot-phonon effect in GaAs at room temperature, Phys. Rev. B 54 (1996) 14487.

[78] M. Lazzeri, F. Mauri, Coupled dynamics of electrons and phonons in metallic nanotubes: Current saturation from hot-phonon generation, Phys. Rev. B 73 (2006) 165419.

[79] S. Butscher, F. Milde, M. Hirtschulz, E. Malić, A. Knorr, Hot electron relaxation and phonon dynamics in graphene, Appl. Phys. Lett. 91 (2007) 203103.

[80] M. Richter, A. Carmele, S. Butscher, N. Bücking, F. Milde, P. Kratzer, M. Scheffler, A. Knorr, Two-dimensional electron gases: Theory of ultrafast dynamics of electron-phonon interactions in graphene, surfaces, and quantum wells, J. Appl. Phys. 105 (2009) 122409.

[81] H. Yan, D. Song, K. F. Mak, I. Chatzakis, J. Maultzsch, T. F. Heinz, Time-resolved Raman spectroscopy of optical phonons in graphite: Phonon anharmonic coupling and anomalous stiffening, Phys. Rev. B 80 (2009) 121403.

[82] S. Berciaud, M. Y. Han, K. F. Mak, L. E. Brus, P. Kim, T. F. Heinz, Electron and Optical Phonon Temperatures in Electrically Biased Graphene, Phys. Rev. Lett. 104 (2010) 227401.

[83] C. H. Lui, K. F. Mak, J. Shan, T. F. Heinz, Ultrafast Photoluminescence from Graphene, Phys. Rev. Lett. 105 (2010) 127404.
[84] H. Wang, J. H. Strait, P. A. George, S. Shivaraman, V. B. Shields, M. Chandrashekhar, J. Hwang, F. Rana, M. G. Spencer, C. S. Ruiz-Vargas, J. Park, Ultrafast relaxation dynamics of hot optical phonons in graphene, Appl. Phys. Lett. 96 (2010) 081917.

[85] M. Breusing, S. Kuehn, T. Winzer, E. Malić, F. Milde, N. Severin, J. P. Rabe, C. Ropers, A. Knorr, T. Elsaesser, Ultrafast nonequilibrium carrier dynamics in a single graphene layer, Phys. Rev. B 83 (2011) 153410.

[86] M. Scheuch, T. Kampfrath, M. Wolf, K. von Volkman, C. Frischkorn, L. Perfetti, Temperature dependence of ultrafast phonon dynamics in graphite, Appl. Phys. Lett. 99 (2011) 211908.

[87] L. Huang, B. Gao, G. Hartland, M. Kelly, H. Xing, Ultrafast relaxation of hot optical phonons in monolayer and multilayer graphene on different substrates, Surf. Sci. 605 (2011) 1657.

[88] S. Wu, W.-T. Liu, X. Liang, P. J. Schuck, F. Wang, Y. R. Shen, M. Salmeron, Hot Phonon Dynamics in Graphene, Nano Letters 12 (2012) 5495.

[89] D. Golla, A. Brasington, B. J. LeRoy, A. Sandhu, Ultrafast relaxation of hot phonons in graphene-hBN heterostructures, APL Materials 5 (2017) 056101.

[90] J. Koivistoinen, P. Myllyperkiö, M. Pettersson, Time-Resolved Coherent Anti-Stokes Raman Scattering of Graphene: Dephasing Dynamics of Optical Phonon, J. Phys. Chem. Lett. 8 (2017) 4108.

[91] D. Novko, M. Kralj, Phonon-assisted processes in the ultraviolet-transient optical response of graphene, NPJ 2D Mater. Appl. 3 (2019) 48.

[92] T. P. H. Sidiropoulos, N. Di Palo, D. E. Rivas, S. Severino, M. Reduzzi, B. Nandy, B. Bauerhonne, S. Krylow, T. Vasilieiadis, T. Danz, P. Elliott, S. Sharma, K. Dewhurst, C. Ropers, Y. Joly, K. M. E. Garcia, M. Wolf, R. Ernstorfer, J. Biegert, Probing the energy conversion pathways between light, carriers, and lattice in real time with attosecond core-level spectroscopy, Phys. Rev. X 11 (2021) 041060. doi: 10.1103/PhysRevX.11.041060. URL https://link.aps.org/doi/10.1103/PhysRevX.11.041060

[93] L. Perfetti, P. A. Loukakos, M. Lisowski, U. Bovensiepen, M. Wolf, H. Berger, S. Biermann, A. Georges, Femtosecond dynamics of electronic states in the Mott insulator 1T-TaS2 by time resolved photoelectron spectroscopy, New J. Phys. 10 (2008) 053019.

[94] N. Dean, J. C. Petersen, D. Fausti, R. I. Tobey, S. Kaiser, L. V. Gasparov, H. Berger, A. Cavalleri, Polaronic Conductivity in the Photoinduced Phase of 1T-TaS2, Phys. Rev. Lett. 106 (2011) 016401.
[95] J. C. Petersen, S. Kaiser, N. Dean, A. Simoncig, H. Y. Liu, A. L. Cavalleri, C. Cacho, I. C. E. Turcu, E. Springate, F. Frassetto, L. Poletto, S. S. Dhesi, H. Berger, A. Cavalleri, Clocking the Melting Transition of Charge and Lattice Order in $1T-\text{TaS}_2$ with Ultrafast Extreme-Ultraviolet Angle-Resolved Photoemission Spectroscopy, Phys. Rev. Lett. 107 (2011) 177402.

[96] S. Hellmann, T. Rohwer, M. Kallaene, K. Hanff, C. Sohrt, A. Stange, A. Carr, M. M. Murnane, H. C. Kapteyn, L. Kipp, M. Bauer, K. Rossnagel, Time-domain classification of charge-density-wave insulators, Nat. Comm. 3 (2012) 1069.

[97] F. Andreatta, H. Rostami, A. G. Čabo, M. Bianchi, C. E. Sanders, D. Biswas, C. Cacho, A. J. H. Jones, R. T. Chapman, E. Springate, P. D. C. King, J. A. Miwa, A. Balatsky, S. Ulstrup, P. Hofmann, Transient hot electron dynamics in single-layer TaS$_2$, Phys. Rev. B 99 (2019) 165421.

[98] Y. D. Wang, W. L. Yao, Z. M. Xin, T. T. Han, Z. G. Wang, L. Chen, C. Cai, Y. Li, Y. Zhang, Band insulator to Mott insulator transition in $1T$-TaS$_2$, Nat. Comm. 11 (2020) 4215.

[99] A. Simoncig, M. Stupar, B. Ressel, T. Saha, P. Rebernik Ribic, G. De Ninno, Dissecting Mott and charge-density wave dynamics in the photoinduced phase of $1T-\text{TaS}_2$, Phys. Rev. B 103 (2021) 155120.

[100] P. Giannozzi, S. Baroni, N. Bonini, M. Calandra, R. Car, C. Cavazzoni, D. Ceresoli, G. L. Chiarotti, M. Cococcioni, I. Dabo, A. Dal Corso, S. de Gironcoli, S. Fabris, G. Fratesi, R. Gebauer, U. Gerstmann, C. Gougoussis, A. Kokalj, M. Lazzeri, L. Martin-Samos, N. Marzari, F. Mauri, R. Mazzarello, S. Paolini, A. Pasquarello, L. Paulatto, C. Sbraccia, S. Scandolo, G. Schlueter, A. P. Seitsonen, A. Smogunov, P. Umari, R. M. Wentzcovitch, QUANTUM ESPRESSO: a modular and open-source software project for quantum simulations of materials, J. Phys.: Condens. Matter. 21 (2009) 395502.

[101] P. Giannozzi, O. Andreussi, T. Brumme, O. Bunau, M. B. Nardelli, M. Calandra, R. Car, C. Cavazzoni, D. Ceresoli, M. Cococcioni, N. Colonna, I. Carnimeo, A. Dal Corso, S. de Gironcoli, P. Delugas, R. A. DiStasio, Jr., A. Ferretti, A. Floris, G. Fratesi, G. Fugallo, R. Gebauer, U. Gerstmann, F. Giustino, T. Gorni, J. Jia, M. Kawamura, H.-Y. Ko, A. Kokalj, E. Kucukbenli, M. Lazzeri, M. Marsili, N. Marzari, F. Mauri, N. L. Nguyen, H.-V. Nguyen, A. Otero-de-la Roza, L. Paulatto, S. Ponce, D. Rocca, R. Sabatini, B. Santra, M. Schlipf, A. P. Seitsonen, A. Smogunov, I. Timrov, T. Thonhauser, P. Umari, N. Vast, X. Wu, S. Baroni, Advanced capabilities for materials modelling with QUANTUM ESPRESSO, J. Phys.: Condens. Matter 29 (2017) 465901.
[102] P. Anderson, *The theory of superconductivity in the high-$T_c$ cuprates*, Princeton, N.J.: Princeton University Press, 1997.

[103] E. Dagotto, *Correlated electrons in high-temperature superconductors*, Rev. Mod. Phys. 66 (1994) 763.

[104] E. Pavarini, I. Dasgupta, T. Saha-Dasgupta, O. Jepsen, O. K. Andersen, *Band-Structure Trend in Hole-Doped Cuprates and Correlation with $T_c^{\text{max}}$*, Phys. Rev. Lett. 87 (2001) 047003.

[105] A. Lanzara, P. Bogdanov, X. Zhou, S. Kellar, D. Feng, E. Lu, T. Yoshida, H. Eisaki, A. Fujimori, K. Kishio, J. Shimoyama, T. Noda, S. Uchida, Z. Hussain, Z. Shen, *Evidence for ubiquitous strong electron-phonon coupling in high-temperature superconductors*, Nature 412 (2001) 510.

[106] X. Zhou, T. Yoshida, A. Lanzara, P. Bogdanov, S. Kellar, K. Shen, W. Yang, F. Romming, T. Sasagawa, T. Kakeshita, T. Noda, H. Eisaki, S. Uchida, C. Lin, F. Zhou, J. Xiong, W. Ti, Z. Zhao, A. Fujimori, Z. Hussain, Z. Shen, *Universal nodal Fermi velocity*, Nature 423 (2003) 398.

[107] Z. Li, M. Wu, Y.-H. Chan, S. G. Louie, *Unmasking the Origin of Kinks in the Photoemission Spectra of Cuprate Superconductors*, Phys. Rev. Lett. 126 (2021) 146401.

[108] L. Perfetti, P. A. Loukakos, M. Lisowski, U. Bovensiepen, H. Eisaki, M. Wolf, *Ultrafast Electron Relaxation in Superconducting Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ by Time-Resolved Photoelectron Spectroscopy*, Phys. Rev. Lett. 99 (2007) 197001.

[109] F. Carbone, D.-S. Yang, E. Giannini, A. H. Zewail, *Direct role of structural dynamics in electron-lattice coupling of superconducting cuprates*, Proc. Nat. Ac. of Sci. 105 (2008) 20161.

[110] B. Mansart, D. Boschetto, A. Savoia, F. Rullier-Albenque, F. Bouquet, E. Papalazarou, A. Forget, D. Colson, A. Rousse, M. Marsi, *Ultrafast transient response and electron-phonon coupling in the iron-pnictide superconductor Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$*, Phys. Rev. B 82 (2010) 024513.

[111] S. Dal Conte, C. Giannetti, G. Coslovich, F. Cilento, D. Bossini, T. Abebaw, F. Banfi, G. Ferrini, H. Eisaki, M. Greven, A. Damascelli, D. van der Marel, F. Parmigiani, *Disentangling the Electronic and Phononic Glue in a High-Tc Superconductor*, Science 335 (2012) 1600.

[112] E. Baldini, A. Mann, L. Benfatto, E. Cappelluti, A. Acocella, V. M. Silkin, S. V. Ereemev, A. B. Kuzmenko, S. Borroni, T. Tan, X. X. Xi, F. Zerbetto, R. Merlin, F. Carbone, *Real-Time Observation of Phonon-Mediated $\sigma-\pi$ Interband Scattering in MgB$_2$*, Phys. Rev. Lett. 119 (2017) 097002.
[113] D. Novko, F. Caruso, C. Draxl, E. Cappelluti, Ultrafast Hot Phonon Dynamics in MgB$_2$ Driven by Anisotropic Electron-Phonon Coupling, Phys. Rev. Lett. 124 (2020) 077001.

[114] J. Nagamatsu, N. Nakagawa, T. Muranaka, Y. Zenitani, J. Akimitsu, Superconductivity at 39 K in magnesium diboride, Nature 410 (2001) 63.

[115] Y. Kong, O. Dolgov, O. Jepsen, O. Andersen, Electron-phonon interaction in the normal and superconducting states of MgB$_2$, Phys. Rev. B 64 (2001) 020501.

[116] G. Grimvall, The electron-phonon interaction in metals, North-Holland, Amsterdam ; New York, 1981.

[117] D. Scalapino, The Electron-Phonon Interaction and Strong-Coupling Superconductors, in: P. R.D. (Ed.), Superconductivity, Dekker, New Yprk, 1969.

[118] P. B. Allen, B. Mitrović, Theory of Superconducting $T_c$, Vol. 37 of Solid State Phys., Academic Press, 1983, p. 1.

[119] J. P. Carbotte, Properties of boson-exchange superconductors, Rev. Mod. Phys. 62 (1990) 1027.

[120] For sake of simplicity, we consider here only the case of spin degeneracy. The spin index will be thus omitted when not necessary, although it will be properly taken into account in the correct counting of physical processes (i.e. diagrams).

[121] Such scheme is equivalently named in different contexts as Bloch-Boltzmann-Peierls formulas, lowest order Born approximation, or Born-Markov approximation.

[122] A. Molina-Sánchez, D. Sangalli, L. Wirtz, A. Marini, Ab Initio Calculations of Ultrashort Carrier Dynamics in Two-Dimensional Materials: Valley Depolarization in Single-Layer WSe$_2$, Nano Lett. 17 (2017) 4549.

[123] In more realistic descriptions, the pump-induced energy adsorption can be modelled as varying within the sample along the $z$-axis perpendicular to the surface. A term taking into account the energy transfer along the $z$-direction due to the thermal conductivity is in that case also included.

[124] F. Caruso, Nonequilibrium Lattice Dynamics in Monolayer MoS$_2$, J. Phys. Chem. Lett. 12 (2021) 1734.

[125] X. Tong, M. Bernardi, Toward precise simulations of the coupled ultrafast dynamics of electrons and atomic vibrations in materials, Phys. Rev. Res. 3 (2021) 023072.
[126] H. Seiler, D. Zahn, M. Zacharias, P.-N. Hildebrandt, T. Vasilieiadis, Y. W. Windsor, Y. Qi, C. Carbogno, C. Draxl, R. Ernstorfer, F. Caruso, Accessing the Anisotropic Nonthermal Phonon Populations in Black Phosphorus, Nano Lett. 21 (2021) 6171.

[127] S. L. Johnson, M. Savoini, P. Beaud, G. Ingold, U. Staub, F. Carbone, L. Castiglioni, M. Hengsberger, J. Osterwalder, Watching ultrafast responses of structure and magnetism in condensed matter with momentum-resolved probes, Struct. Dyn. 4 (2017) 061506.

[128] S. Tan, A. Argondizzo, C. Wang, X. Cui, H. Petek, Ultrafast Multiphoton Thermionic Photoemission from Graphite, Phys. Rev. X 7 (2017) 011004.

[129] A. Li, M. Reutzel, Z. Wang, D. Novko, B. Gumhalter, H. Petek, Plasmonic Photoemission from Single-Crystalline Silver, ACS Photon. 8 (2021) 247.

[130] D. Novko, V. Despoja, M. Reutzel, A. Li, H. Petek, B. Gumhalter, Plasmonically assisted channels of photoemission from metals, Phys. Rev. B 103 (2021) 205401.

[131] J. C. Johannsen, S. Ulstrup, F. Cilento, A. Crepaldi, M. Zacchigna, C. Cacho, I. C. E. Turcu, E. Springate, F. Fromm, C. Raidel, T. Seyller, F. Parmigiani, M. Grioni, P. Hofmann, Direct View of Hot Carrier Dynamics in Graphene, Phys. Rev. Lett. 111 (2013) 027403.

[132] A. Stange, C. Sohrt, L. X. Yang, G. Rohde, K. Janssen, P. Hein, L.-P. Oloff, K. Hanff, K. Rossnagel, M. Bauer, Hot electron cooling in graphite: Supercollision versus hot phonon decay, Phys. Rev. B 92 (2015) 184303.

[133] F. Caruso, D. Novko, C. Draxl, Photoemission signatures of nonequilibrium carrier dynamics from first principles, Phys. Rev. B 101 (2020) 035128.

[134] J. Maklar, Y. W. Windsor, C. W. Nicholson, M. Puppin, P. Walmsley, V. Esposito, M. Porer, J. Rittmann, D. Leuenberger, M. Kubl, M. Savoini, E. Abreu, S. L. Johnson, P. Beaud, G. Ingold, U. Staub, L. R. Fisher, R. Ernstorfer, M. Wolf, L. Rettig, Nonequilibrium charge-density-wave order beyond the thermal limit, Nat. Comm. 12 (2021) 2499.

[135] P. Majchrzak, S. Pakdel, D. Biswas, A. J. H. Jones, K. Volckaert, I. Marković, F. Andreatta, R. Sankar, C. Jozwiak, E. Rotenberg, A. Bostwick, C. E. Sanders, Y. Zhang, G. Karras, R. T. Chapman, A. Wyatt, E. Springate, J. A. Miwa, P. Hofmann, P. D. C. King, N. Lanatà, Y. J. Chang, S. Ulstrup, Switching of the electron-phonon interaction in $1T-\text{VSe}_2$ assisted by hot carriers, Phys. Rev. B 103 (2021) L241108.

[136] L. Perfetti, P. A. Loukakos, M. Lisowski, U. Bovensiepen, H. Eisaki, M. Wolf, Ultrafast Electron Relaxation in Superconducting $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ by Time-Resolved Photoelectron Spectroscopy, Phys. Rev. Lett. 99 (2007) 197001.
[137] J. Shah, A. Pinczuk, H. L. Störmer, A. C. Gossard, W. Wiegmann, Electric field induced heating of high mobility electrons in modulation-doped GaAs-AlGaAs heterostructures, Appl. Phys. Lett. 42 (1983) 55.

[138] G. D. Mahan, Many Particle Physics, Third Edition, Plenum, 2000.

[139] J. M. Ziman, Principles of the Theory of Solids, 2nd Edition, Cambridge University Press, 1972.

[140] P. J. Price, Hot phonon effects in heterolayers, Superlattices Microstr. 1 (1985) 255.

[141] P. J. Price, Hot phonon effects in heterolayers, Physica B+C 134 (1985) 164.

[142] P. Lugli, Hot phonon dynamics, Solid-State Electron. 31 (1988) 667.

[143] Q. Weng, L. Yang, Z. An, P. Chen, A. Tzalenchu, W. Lu, S. Komiyama, Quasadiabatic electron transport in room temperature nanoelectronic devices induced by hot-phonon bottleneck, Nat. Comm. 12 (2021) 4752.

[144] R. Mickevičius, A. Reklaitis, Hot intervalley phonons in GaAs, J. Phys.: Condens. Matter. 2 (1990) 7883.

[145] G. Paulavičius, V. V. Mitin, N. A. Bannov, Coupled electron and nonequilibrium optical phonon transport in a GaAs quantum well, J. Appl. Phys. 82 (1997) 5580.

[146] G. Paulavičius, R. Mickevičius, V. Mitin, M. A. Stroscio, Hot-phonon effects on electron runaway from GaAs quantum wires, J. Appl. Phys. 82 (1997) 3392.

[147] G. Paulavičius, V. Mitin, M. A. Stroscio, Hot-optical-phonon effects on electron relaxation in an AlGaAs/GaAs quantum cascade laser structure, J. Appl. Phys. 84 (1998) 3459.

[148] R. C. Iotti, F. Rossi, M. S. Vitiello, G. Scamarcio, L. Mahler, A. Tredicucci, Impact of nonequilibrium phonons on the electron dynamics in terahertz quantum cascade lasers, Appl. Phys. Lett. 97 (2010) 033110.

[149] Y. B. Shi, I. Knezevic, Nonequilibrium phonon effects in midinfrared quantum cascade lasers, J. Appl. Phys. 116 (2014) 123105.

[150] E. A. A. Pogna, X. Jia, A. Principi, A. Block, L. Banszerus, J. Zhang, X. Liu, T. Sohier, S. Forti, K. Soundarapandian, B. Terrés, J. D. Meheu, C. Trovatello, C. Coletti, F. H. L. Koppens, M. Bonn, H. I. Wang, N. van Hulst, M. J. Verstraete, H. Peng, Z. Liu, C. Stampfer, G. Cerullo, K.-J. Tielrooij, Hot-Carrier Cooling in High-Quality Graphene Is Intrinsically Limited by Optical Phonons, ACS Nano 15 (2021) 11285.
[151] C. C. S. Chan, K. Fan, H. Wang, Z. Huang, D. Novko, K. Yan, J. Xu, W. C. H. Choy, I. Lončarić, K. S. Wong, Uncovering the Electron-Phonon Interplay and Dynamical Energy-Dissipation Mechanisms of Hot Carriers in Hybrid Lead Halide Perovskites, Adv. Energy Mater. 11 (2021) 2003071.

[152] C. Gadermaier, A. S. Alexandrov, V. V. Kabanov, P. Kusar, T. Mertelj, X. Yao, C. Manzoni, D. Brida, G. Cerullo, D. Mihailovic, Electron-Phonon Coupling in High-Temperature Cuprate Superconductors Determined from Electron Relaxation Rates, Phys. Rev. Lett. 105 (2010) 257001.

[153] S. Dal Conte, L. Vidmar, D. Golež, M. Mierzejewski, G. Soavi, S. Peli, F. Banfi, G. Ferrini, R. Comin, B. M. Ludbrook, L. Chauviere, N. D. Zhitigadlo, H. Eisaki, M. Greven, S. Lupi, A. Damascelli, D. Brida, M. Capone, J. Bonča, G. Cerullo, C. Giannetti, Snapshots of the retarded interaction of charge carriers with ultrafast fluctuations in cuprates, Nat. Phys. 11 (2015) 421.

[154] M. B. Price, J. Butkus, T. C. Jellicoe, A. Sadhanala, A. Briane, J. E. Halpert, K. Broch, J. M. Hodgkiss, R. H. Friend, F. Deschler, Hot-carrier cooling and photoinduced refractive index changes in organic-inorganic lead halide perovskites, Nat. Comm. 6 (2015) 8420.

[155] Y. Yang, D. Ostrowski, R. France, K. Zhu, J. van de Lagemaat, J. Luther, M. Beard, Observation of a hot-phonon bottleneck in lead-iodide perovskites, Nat. Photon. 10 (2016) 53.

[156] J. Yang, X. Wen, H. Xia, R. Sheng, Q. Ma, J. Kim, P. Tapping, T. Harada, T. W. Kee, F. Huang, Y.-B. Cheng, M. Green, A. Ho-Bailie, S. Huang, S. Shrestha, R. Patterson, G. Conibeer, Acoustic-optical phonon upconversion and hot-phonon bottleneck in lead-halide perovskites, Nat. Comm. 8 (2017) 14120.

[157] J. Fu, Q. Xu, G. Han, B. Wu, C. H. A. Huan, M. L. Leek, T. C. Sum, Hot carrier cooling mechanisms in halide perovskites, Nat. Comm. 8 (2017) 1300.

[158] D. Novko, First-principles study of ultrafast dynamics of Dirac plasmon in graphene, New Jour. Phys. 23 (2021) 043023.

[159] J. C. Johanssen, S. Ulstrup, F. Cilento, A. Crepaldi, M. Zacchigna, C. Cacho, I. C. E. Turcu, E. Springate, F. Fromm, C. Raidel, T. Seyller, F. Parmigiani, M. Grioni, P. Hofmann, Direct View of Hot Carrier Dynamics in Graphene, Phys. Rev. Lett. 111 (2013) 027403.

[160] L. Rettig, R. Cortés, H. S. Jeevan, P. Gegenwart, T. Wolf, J. Fink, U. Bovensiepen, Electron–phonon coupling in 122 Fe pnictides analyzed by femtosecond time-resolved photoemission, New J. Phys. 15 (2013) 083023.
[161] I. Avigo, R. Cortés, L. Rettig, S. Thirupathaiah, H. S. Jeevan, P. Gegenwart, T. Wolf, M. Ligges, M. Wolf, J. Fink, U. Bovensiepen, Coherent excitations and electron–phonon coupling in Ba/EuFe$_2$As$_2$ compounds investigated by femtosecond time- and angle-resolved photoemission spectroscopy, J. Phys.: Condens. Matter 25 (2013) 094003.

[162] J.-A. Yang, S. Parham, D. Dessau, D. Reznik, Novel Electron-Phonon Relaxation Pathway in Graphite Revealed by Time-Resolved Raman Scattering and Angle-Resolved Photoemission Spectroscopy, Sci. Rep. 7 (2017) 40876.

[163] F. Sekiguchi, H. Hirori, G. Yumoto, A. Shimazaki, T. Nakamura, A. Wakamiya, Y. Kanemitsu, Enhancing the Hot-Phonon Bottleneck Effect in a Metal Halide Perovskite by Terahertz Phonon Excitation, Phys. Rev. Lett. 126 (2021) 077401.

[164] D. C. Hannah, K. E. Brown, R. M. Young, M. R. Wasielewski, G. C. Schatz, D. T. Co, R. D. Schaller, Direct Measurement of Lattice Dynamics and Optical Phonon Excitation in Semiconductor Nanocrystals Using Femtosecond Stimulated Raman Spectroscopy, Phys. Rev. Lett. 111 (2013) 107401.

[165] F. Carbone, N. Gedik, J. Lorenzana, A. H. Zewail, Real-Time Observation of Cuprates Structural Dynamics by Ultrafast Electron Crystallography, Adv. Condens. Matter Phys. 2010 (2010) 958618.

[166] B. Mansart, M. J. G. Cottet, G. F. Mancini, T. Jarlborg, S. B. Dugdale, S. L. Johnson, S. O. Mariager, C. J. Milne, P. Beaud, S. Grübel, J. A. Johnson, T. Kubacka, G. Ingold, K. Prsa, H. M. Rønnow, K. Conder, E. Pomjakushina, M. Chergui, F. Carbone, Temperature-dependent electron-phonon coupling in La$_{2-x}$Sr$_x$CuO$_4$ probed by femtosecond x-ray diffraction, Phys. Rev. B 88 (2013) 054507.

[167] M. Harb, H. Enquist, A. Jurgilaitis, F. T. Tuyakova, A. N. Obraztsov, J. Larsson, Phonon-phonon interactions in photoexcited graphite studied by ultrafast electron diffraction, Phys. Rev. B 93 (2016) 104104.

[168] L. Waldecker, R. Bertoni, H. Hübener, T. Brumme, T. Vasiileiadis, D. Zahn, A. Rubio, R. Ernstorfer, Momentum-Resolved View of Electron-Phonon Coupling in Multilayer WSe$_2$, Phys. Rev. Lett. 119 (2017) 036803.

[169] T. Konstantinova, J. D. Rameau, A. H. Reid, O. Abdurazakov, L. Wu, R. Li, X. Shen, G. Gu, Y. Huang, L. Rettig, I. Avigo, M. Ligges, J. K. Freericks, A. F. Kemper, H. A. Duerr, U. Bovensiepen, P. D. Johnson, X. Wang, Y. Zhu, Nonequilibrium electron and lattice dynamics of strongly correlated Bi$_2$Sr$_2$CaCu$_2$O$_{8+δ}$ single crystals, Sci. Adv. 4 (2018) eaap7427.
[170] T. E. Karam, J. Hu, G. A. Blake, Strongly Coupled Electron–Phonon Dynamics in Few-Layer TiSe2 Exfoliates, ACS Photon. 5 (2018) 1228.

[171] D. Zahn, P.-N. Hildebrandt, T. Vasileiadis, Y. W. Windsor, Y. Qi, H. Seiler, R. Ernstorfer, Anisotropic Nonequilibrium Lattice Dynamics of Black Phosphorus, Nano Lett. 20 (2020) 3728.

[172] K. Kang, D. Abdula, D. G. Cahill, M. Shim, Lifetimes of optical phonons in graphene and graphite by time-resolved incoherent anti-Stokes Raman scattering, Phys. Rev. B 81 (2010) 165405.

[173] N. Pellatz, S. Roy, J.-W. Lee, J. Schad, H. Kandel, N. Arndt, C. Eom, D. Reznik, Relaxation timescales and electron-phonon coupling in optically-pumped YBa$_2$Cu$_3$O$_{6+x}$ revealed by time-resolved Raman scattering, arXiv:2010.15958v3.

[174] M. Trigo, J. Chen, V. H. Vishwanath, Y. M. Sheu, T. Graber, R. Henning, D. A. Reis, Imaging nonequilibrium atomic vibrations with x-ray diffuse scattering, Phys. Rev. B 82 (2010) 235205.

[175] R. P. Chatelain, V. R. Morrison, B. L. M. Klarenna, B. J. Siwick, Coherent and Incoherent Electron-Phonon Coupling in Graphite Observed with Radio-Frequency Compressed Ultrafast Electron Diffraction, Phys. Rev. Lett. 113 (2014) 235502.

[176] T. Chase, M. Trigo, A. H. Reid, R. Li, T. Vecchione, X. Shen, S. Weathersby, R. Coffee, N. Hartmann, D. A. Reis, X. J. Wang, H. A. Dürr, Ultrafast electron diffraction from non-equilibrium phonons in femtosecond laser heated Au films, Appl. Phys. Lett. 108 (2016) 041909.

[177] M. J. Stern, L. P. René de Cotret, M. R. Otto, R. P. Chatelain, J.-P. Boisvert, M. Sutton, B. J. Siwick, Mapping momentum-dependent electron-phonon coupling and nonequilibrium phonon dynamics with ultrafast electron diffuse scattering, Phys. Rev. B 97 (2018) 165416.

[178] L. P. René de Cotret, J.-H. Pöhls, M. J. Stern, M. R. Otto, M. Sutton, B. J. Siwick, Time- and momentum-resolved phonon population dynamics with ultrafast electron diffuse scattering, Phys. Rev. B 100 (2019) 214115.

[179] M. R. Otto, J.-H. Pöhls, L. P. R. de Cotret, M. J. Stern, M. Sutton, B. J. Siwick, Mechanisms of electron-phonon coupling unraveled in momentum and time: The case of soft phonons in TiSe$_2$, Sci. Adv. 7 (20) (2021) eabf2810.

[180] M. Zacharias, H. Seiler, F. Caruso, D. Zahn, F. Giustino, P. C. Kelires, R. Ernstorfer, Multiphonon diffuse scattering in solids from first principles: Application to layered crystals and two-dimensional materials, Phys. Rev. B 104 (2021) 205109.
We neglect here higher-order contributions. For a detailed discussion see Ref. [285].

J. Kortus, I. I. Mazin, K. D. Belashchenko, V. P. Antropov, L. L. Boyer, Superconductivity of Metallic Boron in MgB$_2$, Phys. Rev. Lett. 86 (2001) 4656.

J. M. An, W. E. Pickett, Superconductivity of MgB$_2$: Covalent Bonds Driven Metallic, Phys. Rev. Lett. 86 (2001) 4366.

T. Yildirim, O. Gülseren, J. Lynn, C. Brown, T. Udovic, Q. Huang, N. Rogado, K. Regan, M. Hayward, J. Slusky, T. He, M. Haas, P. Khalfah, K. Inumaru, R. Cava, Giant Anharmonicity and Nonlinear Electron-Phonon Coupling in MgB$_2$: A Combined First-Principles Calculation and Neutron Scattering Study, Phys. Rev. Lett. 87 (2001) 037001.

A. Y. Liu, I. I. Mazin, J. Kortus, Beyond Eliashberg Superconductivity in MgB$_2$: Anharmonicity, Two-Phonon Scattering, and Multiple Gaps, Phys. Rev. Lett. 87 (2001) 087005.

H. J. Choi, D. Roundy, H. Sun, M. L. Cohen, S. G. Louie, The origin of the anomalous superconducting properties of MgB$_2$, Nature 418 (2002) 758.

J. Bednorz, K. Müller, Possible high $T_c$ superconductivity in the Ba-La-Cu-O system, Z. Phys. B 64 (1986) 189.

M. K. Wu, J. R. Ashburn, C. J. Torng, P. H. Hor, R. L. Meng, L. Gao, Z. J. Huang, Y. Q. Wang, C. W. Chu, Superconductivity at 93 K in a new mixed-phase Y-Ba-Cu-O compound system at ambient pressure, Phys. Rev. Lett. 58 (1987) 908.

R. J. Cava, B. Batlogg, R. B. van Dover, D. W. Murphy, S. Sunshine, T. Siegrist, J. P. Remeika, E. A. Rietman, S. Zahurak, G. P. Espinosa, Bulk superconductivity at 91 K in single-phase oxygen-deficient perovskite Ba$_2$YCu$_3$O$_9$–$\delta$, Phys. Rev. Lett. 58 (1987) 1676.

R. M. Hazen, C. T. Prewitt, R. J. Angel, N. L. Ross, L. W. Finger, C. G. Hadidiacos, D. R. Veblen, P. J. Heaney, P. H. Hor, R. L. Meng, Y. Y. Sun, Y. Q. Wang, Y. Y. Xue, Z. J. Huang, L. Gao, J. Bechtold, C. W. Chu, Superconductivity in the high-$T_c$ Bi-Ca-Sr-Cu-O system: Phase identification, Phys. Rev. Lett. 60 (1988) 1174.

T. Masui, S. Tajima, Normal state transport properties of MgB$_2$, Physica C: Superconductivity 385 (2003) 91.

Y. Fudamoto, S. Lee, Anisotropic electrodynamics of MgB$_2$ detected by optical reflectance, Phys. Rev. B 68 (2003) 184514.
[193] V. Guritanu, A. B. Kuzmenko, D. van der Marel, S. M. Kazakov, N. D. Zhigadlo, J. Karpinski, Anisotropic optical conductivity and two colors of MgB$_2$, Phys. Rev. B 73 (2006) 104509.

[194] X. X. Xi, Two-band superconductor magnesium diboride, Rep. Progr. Phys. 71 (2008) 116501.

[195] F. Giubileo, D. Roditchev, W. Sacks, R. Lamy, D. Thanh, J. Klein, S. Miraglia, D. Fruchart, J. Marcus, P. Monod, Two-Gap State Density in MgB$_2$: A True Bulk Property Or A Proximity Effect?, Phys. Rev. Lett. 87 (2001) 177008.

[196] S. Tsuda, T. Yokoya, T. Kiss, Y. Takano, K. Togano, H. Kito, H. Ihara, S. Shin, Evidence for a Multiple Superconducting Gap in MgB$_2$ from High-Resolution Photoemission Spectroscopy, Phys. Rev. Lett. 87 (2001) 17706.

[197] X. Chen, M. Konstantinović, J. Irwin, D. Lawrie, J. P. Franck, Evidence for Two Superconducting Gaps in MgB$_2$, Phys. Rev. Lett. 87 (2001) 157002.

[198] R. Gonnelli, D. Daghero, G. Ummarino, V. Stepanov, J. Jun, S. Kazakov, J. Karpinski, Direct Evidence for Two-Band Superconductivity in MgB$_2$ Single Crystals from Directional Point-Contact Spectroscopy in Magnetic Fields, Phys. Rev. Lett. 89 (2002) 247004.

[199] D. Mou, R. Jiang, V. Taufour, S. Bud’ko, P. Canfield, A. Kaminski, Momentum dependence of the superconducting gap and in-gap states in MgB$_2$ multiband superconductor, Phys. Rev. B 91 (2015) 214519.

[200] F. Giustino, Electron-phonon interactions from first principles, Rev. Mod. Phys. 89 (2017) 015003.

[201] The concept of anisotropy is meant here relatively to the band index space $\alpha$, so that electronic/superconducting anisotropic properties mean that they are strongly varying upon $\alpha$.

[202] H. J. Choi, D. Roundy, H. Sun, M. L. Cohen, S. G. Louie, First-principles calculation of the superconducting transition in MgB$_2$ within the anisotropic Eliashberg formalism, Phys. Rev. B 66 (2002) 020513.

[203] A. Golubov, J. Kortus, O. Dolgov, O. Jepsen, Y. Kong, O. Andersen, B. Gibson, K. Ahn, R. Kremer, Specific heat of MgB$_2$ in a one- and a two-band model from first-principles calculations, J. Phys.: Condens. Matter 14 (2002) 1353.

[204] K.-P. Bohnen, R. Heid, B. Renker, Phonon Dispersion and Electron-Phonon Coupling in MgB$_2$ and AlB$_2$, Phys. Rev. Lett. 86 (2001) 5771.
[205] L. Boeri, G. Bachelet, E. Cappelluti, L. Pietronero, Small Fermi energy and phonon anharmonicity in MgB$_2$ and related compounds, Phys. Rev. B 65 (2002) 214501.

[206] P. Rafailov, M. Dworzak, C. Thomsen, Luminescence and Raman spectroscopy on MgB$_2$, Solid State Comm. 122 (2002) 455.

[207] A. Shukla, M. Calandra, M. d’Astuto, M. Lazzeri, F. Mauri, C. Bellin, M. Krisch, J. Karpinski, S. M. Kazakov, J. Jun, D. Daghero, K. Parlinski, Phonon Dispersion and Lifetimes in MgB$_2$, Phys. Rev. Lett. 90 (2003) 095506.

[208] M. Lazzeri, M. Calandra, F. Mauri, Anharmonic phonon frequency shift in MgB$_2$, Phys. Rev. B 68 (2003) 220509.

[209] M. d’Astuto, M. Calandra, S. Reich, A. Shukla, M. Lazzeri, F. Mauri, J. Karpinski, N. D. Zhigadlo, A. Bossak, M. Krisch, Weak anharmonic effects in MgB$_2$: A comparative inelastic x-ray scattering and Raman study, Phys. Rev. B 75 (2007) 174508.

[210] A. Goncharov, V. Struzhkin, E. Gregoryanz, J. Hu, R. Hemley, H. k. Mao, G. Lapertot, S. Bud’ko, P. Canfield, Raman spectrum and lattice parameters of MgB$_2$ as a function of pressure, Phys. Rev. B 64 (2001) 100509.

[211] J. Hlinka, I. Gregora, J. Pokorný, A. Plecenik, P. Kúš, L. Satrapinsky, Š. Beinačka, Phonons in MgB$_2$ by polarized Raman scattering on single crystals, Phys. Rev. B 64 (2001) 140503.

[212] P. Postorino, A. Congeduti, P. Dore, A. Nucara, A. Bianconi, D. Di Castro, S. De Negri, A. Saccone, Effect of the Al content on the optical phonon spectrum in Mg$_{1-x}$Al$_x$B$_2$, Phys. Rev. B 65 (2001) 020507.

[213] J. W. Quilty, S. Lee, A. Yamamoto, S. Tajima, Superconducting Gap in MgB$_2$: Electronic Raman Scattering Measurements of Single Crystals, Phys. Rev. Lett. 88 (2002) 087001.

[214] J. W. Quilty, S. Lee, S. Tajima, A. Yamanaka, c-Axis Raman Scattering Spectra of MgB$_2$: Observation of a Dirty-Limit Gap in the π Bands, Phys. Rev. Lett. 90 (2003) 207006.

[215] H. Martinho, C. Rettori, P. Pagliuso, A. Martin, N. Moreno, J. Sarrao, Role of the $E_{2g}$ phonon in the superconductivity of MgB$_2$: a Raman scattering study, Solid State Comm. 125 (2003) 499.

[216] B. Renker, K. Bohnen, R. Heid, D. Ernst, H. Schober, M. Koza, P. Adelmann, P. Schweiss, T. Wolf, Strong Renormalization of Phonon Frequencies in Mg$_{1-x}$Al$_x$B$_2$, Phys. Rev. Lett. 88 (2002) 067001.

[217] L. Boeri, E. Cappelluti, L. Pietronero, Small Fermi energy, zero-point fluctuations, and nonadiabaticity in MgB$_2$, Phys. Rev. B 71 (2005) 012501.
[218] A. Bianconi, T. Jarlborg, Lifshitz transitions and zero point lattice fluctuations in sulfur hydride showing near room temperature superconductivity, Nov. Supercond. Mater. 1 (2015) 37.

[219] E. Cappelluti, Electron-phonon effects on the Raman spectrum in MgB2, Phys. Rev. B 73 (2006) 140505.

[220] E. Cappelluti, L. Pietronero, Electron–phonon interaction and breakdown of the adiabatic principle in fullerides and MgB2, J. Phys. Chem. Solids 67 (2006) 1941.

[221] D. Novko, Nonadiabatic coupling effects in MgB2 reexamined, Phys. Rev. B 98 (2018) 041112.

[222] A. Q. R. Baron, H. Uchiyama, Y. Tanaka, S. Tsutsui, D. Ishikawa, S. Lee, R. Heid, K.-P. Bohnen, S. Tajima, T. Ishikawa, Kohn Anomaly in MgB2 by Inelastic X-Ray Scattering, Phys. Rev. Lett. 92 (2004) 197004.

[223] E. Cappelluti, D. Novko, Fingerprints of hot-phonon physics in time-resolved correlated quantum lattice dynamics, arXiv:2110.13274.

[224] P. Giannozzi, S. Baroni, N. Bonini, M. Calandra, R. Car, C. Cavazzoni, D. Ceresoli, G. L. Chiarotti, M. Cococcioni, I. Dabo, et al., QUANTUM ESPRESSO: a modular and open-source software project for quantum simulations of materials, J. Phys: Condens. Matter 21 (2009) 395502.

[225] M. Lazzeri, F. Mauri, Nonadiabatic Kohn Anomaly in a Doped Graphene Monolayer, Phys. Rev. Lett. 97 (2006) 266407.

[226] P. Boross, B. Dora, A. Kiss, F. Simon, A unified theory of spin-relaxation due to spin-orbit coupling in metals and semiconductors, Sci. Rep. 3 (2013) 3233.

[227] L. Szolnoki, A. Kiss, B. Dora, F. Simon, Spin-relaxation time in materials with broken inversion symmetry and large spin-orbit coupling, Sci. Rep. 7 (2017) 9949.

[228] M. Mehring, High Resolution NMR Spectroscopy in Solids, Vol. 11 of NMR Basic Principles and Progress, Springer-Verlag Berlin Heidelberg, 2076.

[229] A. Rigamonti, F. Borsa, P. Carretta, Basic aspects and main results of NMR-NQR spectroscopies in high-temperature superconductors, Rep. Prog. Phys. 61 (1998) 1367.

[230] G. Campi, E. Cappelluti, T. Proffen, X. Qiu, E. Bozin, S. Billinge, S. Agrestini, N. Saini, A. Bianconi, Study of temperature dependent atomic correlations in MgB2, Eur. Phys. J. B 52 (2006) 15.
[231] I.-K. Jeong, T. Proffen, F. Mohiuddin-Jacobs, S. J. L. Billinge, Measuring Correlated Atomic Motion Using X-ray Diffraction, J. Phys. Chem. A 103 (1999) 921.

[232] I.-K. Jeong, R. H. Heffner, M. J. Graf, S. J. L. Billinge, Lattice dynamics and correlated atomic motion from the atomic pair distribution function, Phys. Rev. B 67 (2003) 104301.

[233] M. Thorpe, V. Levashov, M. Lei, S. Billinge, in: S. Billinge, M. Thorpe (Eds.), From semiconductors to proteins: beyond the average structure, Kluwer/Plenum, New York, 2002, 2002, p. 105.

[234] These relations hold true in the present form in the ideal clean case. In the analysis of experimental data of real materials, the mean-square displacements should be considered at the net of disorder contribution [230].

[235] L. Ortenzi, E. Cappelluti, L. Benfatto, L. Pietronero, Fermi-Surface Shrinking and Interband Coupling in Iron-Based Pnictides, Phys. Rev. Lett. 103 (2009) 046404.

[236] L. Benfatto, E. Cappelluti, C. Castellani, Spectroscopic and thermodynamic properties in a four-band model for pnictides, Phys. Rev. B 80 (2009) 214522.

[237] L. Benfatto, E. Cappelluti, Effects of the Fermi-surface shrinking on the optical sum rule in pnictides, Phys. Rev. B 83 (2011) 104516.

[238] P. Echenique, J. Pitarke, E. Chulkov, V. Silkin, Image-potential-induced states at metal surfaces, J. Electron. Spectros. Relat. Phenomena 126 (2002) 163.

[239] M. Hase, M. Kitajima, A. Constantinescu, H. Petek, The birth of a quasiparticle in silicon observed in time-frequency space, Nature 426 (2003) 51.

[240] The different contributions to the total bandshift are here expressed in a slightly different but more intuitive way than in Ref. [112].

[241] Assumption of fast electronic thermalization guided by the Coulomb electron-electron interaction is not actually required in the present scenario, as shown in Ref. [113]. Its employment here is just for sake of simplicity in the later discussion.

[242] E. Ekimov, V. Sidorov, E. Bauer, N. Mel’nik, N. Curro, J. Thompson, S. Stishov, Superconductivity in diamond, Nature 428 (2004) 542.

[243] L. Boeri, J. Kortus, O. K. Andersen, Three-Dimensional MgB2-Type Superconductivity in Hole-Doped Diamond, Phys. Rev. Lett. 93 (2004) 237002.
[244] C. Thomsen, M. Cardona, Physical Properties of High-Temperature Superconductors, in: T. Ginsberg (Ed.), Raman Scattering in High-$T_c$ Superconductors, World Scientific, Singapore, 1989, p. 409.

[245] C. Thomsen, Light scattering in high-$T_c$ superconductors, in: M. Cardona, G. Guntherodt (Eds.), Light Scattering in Solids VI, Springer, Berlin, 1991, p. 285.

[246] Y. G. Wang, S. P. Lau, B. K. Tay, X. H. Zhang, Resonant Raman scattering studies of Fano-type interference in boron doped diamond, J. Appl. Phys. 92 (2002) 7253.

[247] P. Szirmai, T. Pichler, O. A. Williams, S. Mandal, C. Bäuerle, F. Simon, A detailed analysis of the Raman spectra in superconducting boron doped nanocrystalline diamond, Phys. Status Solidi B 249 (2012) 2656.

[248] V. Mortet, Z. V. Žíková, A. Taylor, M. Davydová, O. Frank, P. Hubík, J. Lorincik, M. Aleshin, Determination of atomic boron concentration in heavily boron-doped diamond by Raman spectroscopy, Diam. Relat. Mater. 93 (2019) 54.

[249] V. Mortet, I. Gregora, A. Taylor, N. Lambert, P. Ashcheulov, Z. Gedeonova, P. Hubík, New perspectives for heavily boron-doped diamond Raman spectrum analysis, Carbon 168 (2020) 319.

[250] S. Baroni, S. de Gironcoli, A. Dal Corso, P. Giannozzi, Phonons and related crystal properties from density-functional perturbation theory, Rev. Mod. Phys. 73 (2001) 515.

[251] N. Marzari, A. A. Mostofi, J. R. Yates, I. Souza, D. Vanderbilt, Maximally localized Wannier functions: Theory and applications, Rev. Mod. Phys. 84 (2012) 1419.

[252] F. Giustino, M. L. Cohen, S. G. Louie, Electron-phonon interaction using Wannier functions, Phys. Rev. B 76 (2007) 165108.

[253] S. Poncé, E. Margine, C. Verdi, F. Giustino, EPW: Electron–phonon coupling, transport and superconducting properties using maximally localized Wannier functions, Comput. Phys. Commun. 209 (2016) 116.

[254] H. Rosner, A. Kitaigorodsky, W. E. Pickett, Prediction of High $T_c$ Superconductivity in Hole-Doped LiBC, Phys. Rev. Lett. 88 (2002) 127001.

[255] J. M. An, S. Y. Savrasov, H. Rosner, W. E. Pickett, Extreme electron-phonon coupling in boron-based layered superconductors, Phys. Rev. B 66 (2002) 220502.

[256] J. K. Dewhurst, S. Sharma, C. Ambrosch-Draxl, B. Johansson, First-principles calculation of superconductivity in hole-doped LiBC: $T_c = 65$K, Phys. Rev. B 68 (2003) 020504.
[257] Q.-Z. Li, X.-W. Yan, M. Gao, J. Wang, Electron-phonon coupling and superconductivity in LiB1+xCl1-x, Europhys. Lett. 122 (2018) 47001.

[258] M. Gao, Z.-Y. Lu, T. Xiang, Prediction of phonon-mediated high-temperature superconductivity in Li3B4C2, Phys. Rev. B 91 (2015) 045132.

[259] A. N. Kolmogorov, S. Curtarolo, Prediction of different crystal structure phases in metal borides: A lithium monoboride analog to MgB2, Phys. Rev. B 73 (2006) 180501.

[260] M. Calandra, A. N. Kolmogorov, S. Curtarolo, Search for high Tc in layered structures: The case of LiB, Phys. Rev. B 75 (2007) 144506.

[261] J. Bekaert, A. Aperis, B. Partoens, P. M. Oppeneer, M. V. Milošević, Evolution of multigap superconductivity in the atomically thin limit: Strain-enhanced three-gap superconductivity in monolayer MgB2, Phys. Rev. B 96 (2017) 094510.

[262] J. Bekaert, M. Petrov, A. Aperis, P. M. Oppeneer, M. V. Milošević, Hydrogen-Induced High-Temperature Superconductivity in Two-Dimensional Materials: The Example of Hydrogenated Monolayer MgB2, Phys. Rev. Lett. 123 (2019) 077001.

[263] P. Modak, A. K. Verma, A. K. Mishra, Prediction of superconductivity at 70 K in a pristine monolayer of LiBC, Phys. Rev. B 104 (2021) 054504.

[264] S. di Cataldo, S. Qulaghasi, G. Bachelet, L. Boeri, High-Tc Superconductivity in doped boron-carbon clathrates, arXiv:2110.05333v1.

[265] S. di Cataldo and L. Boeri, private communication (2021).

[266] F. J. Di Salvo, D. E. Moncton, J. V. Waszczak, Electronic properties and superlattice formation in the semimetal TiSe2, Phys. Rev. B 14 (1976) 4321.

[267] M. D. Watson, A. Rajan, T. Antonelli, K. Underwood, I. Markovic, F. Mazzaola, O. J. Clark, G.-R. Siemann, D. Biswas, A. Hunter, S. Jandura, J. Reichstetter, M. McLaren, P. Le Fevre, G. Vinai, P. D. C. King, Strong-coupling charge density wave in monolayer TiSe2, 2D Mater. 8 (2021) 015004.

[268] M. Holt, P. Zschack, H. Hong, M. Y. Chou, T.-C. Chiang, X-Ray Studies of Phonon Softening in TiSe2, Phys. Rev. Lett. 86 (2001) 3799.

[269] C. Monney, C. Battaglia, H. Cercellier, P. Aebi, H. Beck, Exciton condensation driving the periodic lattice distortion of 1t–tise2, Phys. Rev. Lett. 106 (2011) 106404.

[270] V. Meregalli, S. Y. Savrasov, Electron-phonon coupling and properties of doped BaBiO3, Phys. Rev. B 57 (1998) 14453.
[271] X. Shi, W. You, Y. Zhang, Z. Tao, P. M. Oppeneer, X. Wu, R. Thomale, K. Rossnagel, M. Bauer, H. Kapteyn, M. Murnane, Ultrafast electron calorimetry uncovers a new long-lived metastable state in 1T-TaSe$_2$ mediated by mode-selective electron-phonon coupling, Sci. Adv. 5 (2019) eaav4449.

[272] Y. Zhang, X. Shi, W. You, Z. Tao, Y. Zhong, F. Cheenicode Kabeer, P. Maldonado, P. M. Oppeneer, M. Bauer, K. Rossnagel, H. Kapteyn, M. Murnane, Coherent modulation of the electron temperature and electron–phonon couplings in a 2D material, Proc. Nat. Ac. Sci. 117 (2020) 8788.

[273] Z. Tao, T.-R. T. Han, C.-Y. Ruan, Anisotropic electron-phonon coupling investigated by ultrafast electron crystallography: Three-temperature model, Phys. Rev. B 87 (2013) 235124.

[274] G. Storeck, J. G. Horstmann, T. Diekmann, S. Vogelgesang, G. von Witte, S. V. Yalunin, K. Rossnagel, C. Ropers, Structural dynamics of incommensurate charge-density waves tracked by ultrafast low-energy electron diffraction, Struct. Dyn. 7 (3) (2020) 034304.

[275] P. E. Dolgirev, A. V. Rozhkov, A. Zong, A. Kogar, N. Gedik, B. V. Fine, Amplitude dynamics of the charge density wave in LaTe$_3$: Theoretical description of pump-probe experiments, Phys. Rev. B 101 (2020) 054203.

[276] K.-H. Jin, H. Huang, J.-W. Mei, Z. Liu, L.-K. Lim, F. Liu, Topological superconducting phase in high-Tc superconductor MgB$_2$ with Dirac–nodal-line fermions, NPJ Comput. Mater. 5 (1) (2019) 57.

[277] J. Li, Q. Xie, J. Liu, R. Li, M. Liu, L. Wang, D. Li, Y. Li, X.-Q. Chen, Phononic Weyl nodal straight lines in MgB$_2$, Phys. Rev. B 101 (2020) 024301.

[278] G. Benedek, S. Miret-Artés, J. R. Manson, A. Ruckhofer, W. E. Ernst, A. Tamtögl, Origin of the Electron–Phonon Interaction of Topological Semimetal Surfaces Measured with Helium Atom Scattering, J. Phys. Chem. Lett. 11 (2020) 1927.

[279] G. Benedek, J. R. Manson, S. Miret-Artés, The Electron–Phonon Interaction of Low-Dimensional and Multi-Dimensional Materials from He Atom Scattering, Adv. Mater. 32 (25) (2020) 2002072.

[280] S. Lebègue, Electronic structure and properties of the Fermi surface of the superconductor LaOFeP, Phys. Rev. B 75 (2007) 035110.

[281] H. Ding, P. Richard, K. Nakayama, K. Sugawara, T. Arakane, Y. Sekiba, A. Takayama, S. Souma, T. Sato, T. Takahashi, Z. Wang, X. Dai, Z. Fang, G. F. Chen, J. L. Luo, N. L. Wang, Observation of Fermi-surface-dependent nodeless superconducting gaps in Ba$_{0.6}$K$_{0.4}$Fe$_2$As$_2$, Europhys. Lett. 83 (2008) 47001.
[282] R. Zhi-An, L. Wei, Y. Jie, Y. Wei, S. Xiao-Li, L. Zheng-Cai, C. Guang-Can, D. Xiao-Li, S. Li-Ling, Z. Fang, Z. Zhong-Xian, Superconductivity at 55 K in Iron-Based F-Doped Layered Quaternary Compound Sm[O_{1-x}F_x]FeAs, Chin. Phys. Lett. 25 (2008) 2215.

[283] J.-F. Ge, Z.-L. Liu, C. Liu, C.-L. Gao, D. Qian, Q.-K. Xue, Y. Liu, J.-F. Jia, Superconductivity above 100 K in single-layer FeSe films on doped SrTiO_3, Nat. Mater. 14 (2015) 285.

[284] Labelling of the high-symmetry points of the Brillouin zone can be different in literature according to the choice of the unit cell, including one or two iron atoms. In the present notation the electron-like bands are located at the M points at (∓π, ∓π).

[285] M. Zacharias, H. Seiler, F. Caruso, D. Zahn, F. Giustino, P. C. Kelires, R. Ernstorfer, Efficient First-Principles Methodology for the Calculation of the All-Phonon Inelastic Scattering in Solids, Phys. Rev. Lett. 127 (2021) 207401.