Abstract

We derive the spectrum of the Dirac operator for the linear $\sigma$–model with quarks in the large $N_c$ approximation using renormalization group flow equations. For small eigenvalues, the Banks–Casher relation and the vanishing linear term are recovered. We calculate the coefficient of the next to leading term and investigate the spectrum beyond the low energy regime.
1 Introduction

The QCD Dirac operator contains all information about the quark dynamics in QCD. Therefore its spectrum is of high interest. In the literature one considers the Euclidean Dirac operator which reads $D = \partial + i g A$. We will use Euclidean space time throughout this work. Since the Dirac operator is antihermitean, its eigenvalues are purely imaginary. With the eigenvalue equation

$$D |\psi_k\rangle = i \lambda_k |\psi_k\rangle$$

one defines the spectral density

$$\rho(\lambda) = \left\langle \sum_k \delta (\lambda - \lambda_k) \right\rangle,$$

where the averaging is over the gluon background. The first discovery about the spectral density has been made by Banks and Casher [1] who connect the order parameter of spontaneous chiral symmetry breaking, i.e. the chiral condensate, to the QCD spectrum at zero eigenvalues. Since then, different approaches have led to a steady progress in this field. As the dynamics at low energies is mainly driven by chiral symmetry the behaviour of the spectral density in the vicinity of $\lambda = 0$ can be analysed using chiral perturbation theory. The first correction to the Banks–Casher relation was found by Smilga and Stern [2]

$$\rho(\lambda) = V_4 \left( \frac{\langle \bar{q}q \rangle}{\pi} + V_4 \frac{\langle \bar{q}q \rangle^2 (N_f^2 - 4)}{32\pi^2 N_f f_\pi^4} |\lambda| + O(\lambda^2) \right),$$

where $V_4$ denotes the Euclidean 4–volume. The constant term is the Banks–Casher result, whereas the linear term vanishes in the case $N_f = 2$.

Chiral random matrix theory considers the so called microscopic spectral density, in which all eigenvalues are rescaled according to the average spacing of small eigenvalues. It successfully describes the Dirac spectrum in the extreme infrared regime, where the spectral density is completely determined by the global symmetries of the Dirac operator (see e.g. [3]). Universal properties of the microscopic spectral density have been derived and identified in lattice calculations [4, 5]. Furthermore partially quenched chiral perturbation theory combines the results of chiral perturbation theory and chiral
random matrix theory \[3\].

The Dirac spectrum for small eigenvalues is driven by chiral symmetry which determines hadron dynamics for small momenta. Thus chiral perturbation theory and chiral random matrix theory can be successful in describing the Dirac spectrum.

The same holds for the Nambu–Jona-Lasinio (NJL) model which serves as an effective model for chiral symmetry. In this work we consider it in its bosonized form given by the linear $\sigma$-model. The QCD Dirac spectrum and the spectrum for the linear $\sigma$–model which we analyze in this paper should coincide in the regime, where the linear $\sigma$–model is a good effective description for QCD, that is for

$$\lambda < \Lambda.$$  \hspace{1cm} (4)

Here $\Lambda \approx 1 \text{ GeV}$ defines the momentum scale below which a hadronic description of QCD may be useful.

## 2 RG flow in the linear $\sigma$–model

The Lagrangian of the linear $\sigma$–model in Euclidean spacetime is

$$\mathcal{L} = \bar{q} \left[ Z_q \gamma^0 + g \left( \sigma + i \vec{\pi} \gamma_5 \right) \right] q + \frac{1}{2} Z_{\Phi} \left( (\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2 \right) + V(\sigma, \vec{\pi}, m_q).$$  \hspace{1cm} (5)

The fermion fields describe constituent quarks which appear in two flavours and three colours. The bosonic potential

$$V(\sigma, \vec{\pi}, m_q) = U(\Phi^2) - c \sigma$$  \hspace{1cm} (6)

contains a chiral symmetric part $U(\Phi^2)$ depending on the $O(4)$-symmetric representation $\Phi = (\sigma, \vec{\pi})$ and a linear symmetry breaking term $c \sigma$. For later purposes we explicitly stress the dependence of the meson potential $V(\sigma, \vec{\pi}, m_q)$ on the current quark mass $m_q$. At the ultraviolet scale, bosonization of the NJL model fixes the form of the meson potential $V_\Lambda(\sigma, \vec{\pi}, m_q)$ to be

$$V_\Lambda(\sigma, \vec{\pi}, m_q) = \frac{m_\Lambda^2}{2} \Phi^2 - c \sigma.$$  \hspace{1cm} (7)
The linear symmetry breaking term $c$ is related to the current quark mass $m_q$ by

$$c = \frac{m_q^2}{g_\Lambda}.$$  
(8)

We use renormalization group (RG) flow equations for the linear $\sigma$-model with quarks in the Schwinger proper time formalism. These flow equations have been discussed in various papers. For a general review on RG flow equations see [7] and for the Schwinger proper time approach used here see [8]. The evolution equations provide a framework to study nonperturbative theories by integrating out smaller and smaller momentum shells above a limiting infrared cutoff $k$. This cutoff enters in the form of a smooth regulator function $f_k(\tau)$ in the Schwinger proper time representation of the logarithm containing one-loop fluctuations. We consider the set of regulator functions given by

$$f^{(n)}(\tau k^2) = \sum_{j=0}^{n} \frac{(\tau k^2)^j}{j!} e^{-\tau k^2}.$$  
(9)

The flow equations which we present in this paper are obtained with $n = 2$. In order to check the cutoff dependence of our results we use regulator functions with different $n$. Generalization of the equations is straightforward.

Note, the parameter $c$ is scale independent, since a linear symmetry breaking term does not evolve under RG flow [9]. Therefore only the chiral symmetric part $U_k(\Phi^2)$ of the meson potential $V_k(\sigma, \vec{\pi}, m_q) = U_k(\Phi^2) - c\sigma$ evolves under flow of the IR cutoff scale $k$.

In this paper we restrict ourselves to the flow equations in the large $N_c$ approximation, which have been considered recently [10] and whose results have proven equivalent to the standard selfconsistent NJL large $N_c$ approach [11, 12]. They are particularly simple and read

$$k \frac{\partial U_k}{\partial k} = -\frac{N_f N_c}{8\pi^2} \frac{k^6}{k^2 + \Phi^2},$$  
(10)

$$k \frac{\partial Z_{\Phi_k}}{\partial k} = -\frac{N_f N_c}{4\pi^2} \frac{k^6}{(k^2 + \Phi^2)^3}.$$  
(11)

The couplings $Z_q$ and $g$ in eq. (3) do not evolve in large $N_c$ and have been set to 1 for convenience.
An additional flow equation which determines the chiral condensate can be obtained by differentiation of the partition function $Z$ with respect to an appropriately introduced source term $\Delta \bar{q}q$ which probes the chiral condensate

$$\langle \bar{q}q \rangle = \left. \frac{\partial}{\partial \Delta} \log Z(\Delta) \right|_{\Delta=0}.$$

(12)

This flow equation is derived analogously to the other flow equations by introduction of the Schwinger proper time cutoff in $Z$ and reads

$$k \frac{\partial \langle \bar{q}q \rangle_k}{\partial k} = \frac{N_f N_c}{4\pi^2} \frac{k^6}{(k^2 + \Phi^2)^2}.$$

(13)

In large $N_c$ all flow equations can be solved analytically. The initial conditions and solutions are discussed in [10]. Denoting $\Phi = (\sigma, \vec{\pi})$ one finds for the potential (at $k = 0$)

$$V(\sigma, \vec{\pi}, m_q) = V_\Lambda(\sigma, \vec{\pi}, m_q) + \frac{N_c}{8\pi^2} \left( \Phi^4 \log \left( \frac{\Lambda^2 + \Phi^2}{\Phi^2} \right) - \Phi^2 \Lambda^2 \right).$$

(14)

The minimum of this potential yields the vacuum expectation value $\Phi_0$ of the bosonic field which is nonvanishing due to the spontaneous and explicit breaking of chiral symmetry. The physical value for the quark condensate is obtained as $\langle \bar{q}q \rangle(\Phi_0)$. Its flow is shown in Fig. 1.

We obtain an infrared value $\langle \bar{q}q \rangle_{RG} = 253.6$ MeV, which compares well to the value obtained from the Gell-Mann–Oakes–Renner relation $\langle \bar{q}q \rangle_{GOR} = 255.2$ MeV. This result is not surprising (cf. [12]) since the initial conditions of the flow equations have been adjusted to reproduce $f_\pi = 93$ MeV and $M_q = 320$ MeV at $k = 0$ using a current quark mass $m_q = 7$ MeV at the ultraviolet scale.

3 Extracting the Dirac operator spectrum from the flow equations

While the QCD–Dirac operator $D$ is antihermitean, the Dirac operator in the linear $\sigma$–model

$$\tilde{D} = \tilde{\theta} + g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)$$

(15)
Figure 1: The evolution of the chiral quark condensate as a function of the renormalization scale $k$ with an explicit symmetry breaking due to a current quark mass of $m_q = 7$ MeV.

can have complex eigenvalues. In the flow equations we have neglected the imaginary part of the fermion determinant, which is related to anomalous processes [12]. In this approximation the eigenvalues of the Dirac operator in the linear $\sigma$-model are purely imaginary in coincidence with the QCD spectrum. For general complex eigenvalues $z = x+iy$ it is useful to consider the two dimensional spectral density [13, 14]

$$P(x, y) = \left\langle \sum_k \delta(x - x_k) \delta(y - y_k) \right\rangle_{\Phi},$$  \hspace{1cm} (16)

which can be obtained from the complex resolvent [13]

$$G(z) \equiv \left\langle Tr \frac{1}{z - \tilde{D}} \right\rangle_{\Phi} = \left\langle Tr \frac{\partial}{\partial z} \log (z - \tilde{D}) \right\rangle_{\Phi} = \left\langle \frac{\partial}{\partial z} \log \text{Det} (z - \tilde{D}) \right\rangle_{\Phi}$$

$$= \left\langle \frac{\partial}{\partial z} \log \int Dq D\bar{q} \exp \left[ - \int d^4x \left( \bar{q} \left( z - \tilde{D} \right) q \right) \right] \right\rangle_{\Phi}. \hspace{1cm} (17)$$
The complex eigenvalue $z$ is the generator in this resolvent, which in our low energy effective theory involves an average over the meson fields denoted by $\langle \ldots \rangle_\Phi$. In large $N_c$ approximation there are no meson fluctuations. Therefore the averaging over the meson fields can be interchanged with the logarithm in the last line and one finds

$$G(z) = -V_4 \langle \bar{q}q \rangle(z). \quad (18)$$

The connection between $G(z)$ and $P(x, y)$ can be derived with eq. (16)

$$G(z) = \left\langle \frac{1}{z - D} \right\rangle_\Phi = \left\langle \int dx' dy' \sum_k \frac{\delta(x' - x_k)\delta(y' - y_k)}{x - x' + i(y - y')} \right\rangle_\Phi = \int dx' dy' \frac{P(x', y')}{x - x' + i(y - y')} . \quad (19)$$

Combining eqs. (18) and (19) yields

$$G(x, y) = -V_4 \langle \bar{q}q \rangle = \int dx' \int dy' \frac{P(x', y')}{x - x' + i(y - y')} . \quad (20)$$

This relation can be inverted [13] and gives

$$P(x, y) = -\frac{V_4}{\pi} \frac{\partial}{\partial z^*} \langle \bar{q}q \rangle(z), \quad (21)$$

where

$$\frac{\partial}{\partial z^*} = \frac{1}{2} \left( \frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right) . \quad (22)$$

The complex eigenvalue $z$ enters in the resolvent eq. (17) just as an explicit quark mass. Therefore no new flow equations have to be evaluated. After bosonization $z$ appears in a linear symmetry breaking term in the meson potential via eq. (8) with $m_q = z$.

As the resulting meson potential $V_k(\sigma, \vec{\pi}, z)$ becomes complex, we must comment on the meaning of $\Phi_0, k(z)$. Instead of restricting ourselves to positive
real values $\Phi_{0,k}$, we now have to allow also complex fields $\Phi_{0,k}(z)$. Instead of finding a real minimum of the potential $V_k(\sigma, \vec{\pi}, m_q)$ we have to determine a complex saddle point, which minimizes the real part of the complex meson potential $V_k(\sigma, \vec{\pi}, z)$.

From eq. (21) it is obvious, that the two dimensional spectral density is zero if $\langle \bar{q}q \rangle(z)$ is analytic in $z$. In presence of the complex quark mass $z$ the chiral condensate at $k = 0$ becomes in the large $N_c$ limit with eq. (13)

$$\langle \bar{q}q \rangle(z) = \frac{N_c}{4\pi^2} \left[ \frac{-\Phi_0(z)\Lambda^2(\Lambda^2 + 2\Phi_0(z)^2)}{\Lambda^2 + \Phi_0(z)^2} + 2\Phi_0(z)^3 \log \left(1 + \frac{\Lambda^2}{\Phi_0(z)^2}\right) \right]. \tag{23}$$

This expression is analytic if $\Phi_0(z)$ is analytic and $\text{Re}(\Phi_0(z)) \neq 0$. As $\Phi_0(z)$ is the bare pion decay constant in presence of $z$ and therefore minimizes $\text{Re}(V(\sigma, \vec{\pi}, z))$, the second condition $\text{Re}(\Phi_0(z)) \neq 0$ is always true. But the global minimum of $\text{Re}(V(\sigma, \vec{\pi}, z))$ can switch if $z$ changes infinitesimally. This is exactly what happens and leads to a discontinuity in $\Phi_0(z)$. Fig 3 demonstrates this behaviour.

For $z = 0$ it follows from eq. (14) in the chiral limit that with $\Phi_0$ as a global minimum of $\text{Re}(V(\sigma, \vec{\pi}, z))$ also $-\Phi_0$, $\Phi_0^*$ and $-\Phi_0^*$ are global minima of $\text{Re}(V(\sigma, \vec{\pi}, z))$. Additionally they are saddle points of $V(\sigma, \vec{\pi}, z)$. For either $z = x$ or $z = iy$ the global minimum is still twice degenerate. In case of an infinitesimal $x$ the sign of $x$ chooses the minimum, i.e. $\text{Re} (\Phi_0)$ changes its sign under the transformation from $x = +\epsilon$ to $x = -\epsilon$. The same holds for infinitesimal $y$ and for $\text{Im}(\Phi_0)$, but in this case an infinitesimal $y$ only gives rise to an infinitesimal $\text{Im}(\Phi)$, consequently no discontinuity arises in $\Phi(z)$. We are thus left with the result, that $P(x, y)$ can only be nonzero for $x = 0$. The position of our Dirac spectrum thus coincides with the QCD spectrum.

Using the ansatz $P(x, y) = \delta(x)\rho(y)$ we find from eqs. (20), (21) and (23)

$$\rho(y) = \frac{V_4}{2\pi} \lim_{\epsilon \to 0} \left[ \langle \bar{q}q \rangle(iy - \epsilon) - \langle \bar{q}q \rangle(iy + \epsilon) \right]. \tag{24}$$

Now let us consider the flow equations for the potential and the chiral condensate. In the large $N_c$ limit they reveal a simple connection, namely

$$\partial_k \langle \bar{q}q \rangle_k(\Phi(z)) = \partial_k \partial_{\Phi} U_k(\Phi(z)). \tag{25}$$

\footnote{To be correct we should have written $\partial_k \langle \bar{q}q \rangle_k(\sigma(z), \vec{\pi}) |_{\vec{\pi} = 0} = \partial_k \partial_{\sigma} U_k(\sigma(z), \vec{\pi}) |_{\vec{\pi} = 0}$. As the expectation value of the pion field vanishes the notation above is unambiguous.}
With the initial conditions $\langle \bar{q} q \rangle_\Lambda = 0$ and $U_\Lambda = \frac{m^2_\Lambda}{2} \Phi^2$ at $k = \Lambda$ this equation can be integrated out and yields

$$\langle \bar{q} q \rangle (\Phi(z)) = \partial_\Phi U (\Phi(z)) - m^2_\Lambda \Phi(z) .$$

Eq. (24) gives the spectral density in terms of the discontinuity of the generalized chiral condensate $\langle \bar{q} q \rangle(z)$ across the real axis, which can be calculated from the saddle point $\Phi_0(z)$ of $V(\Phi, z)$ fulfilling $\partial_\Phi V(\Phi, z)|_{\Phi=\Phi_0} = 0$. With eq. (26) we find in the chiral limit

$$\rho(y) = \frac{V_4}{2\pi} \lim_{\epsilon \to 0} m^2_\Lambda [\Phi(iy + \epsilon) - \Phi(iy - \epsilon)]$$

$$= \frac{m^2_\Lambda V_4}{\pi} \text{Re} (\Phi(iy + \epsilon)) .$$

The result is plotted in fig. (1). Next we derive the behaviour of the spectral density at small eigenvalues.
Therefore we have to evaluate $\Phi_0(iy)$ for small $y$. The value $\Phi_0(iy) := \Phi_0(0) + \alpha(y) + i\beta(y)$ is determined by the saddle point equation

$$\partial_\Phi V(\Phi, y)|_{\Phi = \Phi_0} = \partial_\Phi U(\Phi)|_{\Phi = \Phi_0 + \alpha(y) + i\beta(y)} - im^2 y = 0 . \quad (28)$$

$\alpha(y)$ and $\beta(y)$ can be expanded in a Taylor series. The coefficients are obtained from

$$\frac{d^{(n)}}{dy^{(n)}} \left[ \partial_\Phi U(\Phi)|_{\Phi = \Phi_0 + \alpha(y) + i\beta(y)} - im^2 y \right]_{y=0} = 0 , \quad n = 0, 1, 2 . \quad (29)$$

We find

$$\Phi_0(y) = \Phi_0(0) + i \frac{m^2}{\partial_\Phi U(\Phi)|_{\Phi = \Phi_0}} y + \frac{1}{2} \frac{m^4}{\left[ \partial_\Phi^3 U(\Phi)|_{\Phi = \Phi_0} \right]^3} y^2 + ... \quad (30)$$
Figure 4: Behaviour of the spectral density at small eigenvalues, which exhibits a quadratic rise due to the absence of a linear contribution in the case $N_f=2$.

With eq. (27) the spectral density

$$
\rho(y) = \frac{m^2 V_4}{\pi} \left[ \text{Re} [\Phi_0(0)] + \frac{1}{2} \frac{m^4 \partial^2 U(\Phi)|_{\Phi=\Phi_0(0)}}{2\pi^2} y^2 + O(y^3) \right]^{(31)}
$$

can be calculated explicitly for small eigenvalues, see eq. (14) and (26)

$$
V_4^{-1} \rho(y) = \frac{\langle \bar{q}q \rangle}{\pi} + \frac{2\pi^3 m^6_\Lambda}{\Phi_0(0)} + 12\pi^2 m^2_\Lambda (\Lambda^2 + \Phi_0(0)^2)^6 - N_c \Lambda^6 (3\Lambda^2 - \Phi_0(0)^2) (\Lambda^2 + \Phi_0(0)^2)^3 y^2 + O(y^3)
$$

The constant coefficient is the Banks–Casher result [1]. Since our computation is done for $N_f=2$ there is no linear term in agreement with the findings
of Smilga and Stern for $N_f = 2$\cite{2}. Numerically, the quadratic coefficient is $87$ MeV for the choice of parameters given in \cite{10}. The quadratic term in $y$ can be related to the correlation function of three scalar $\sigma$–quanta. In the numerator of eq. (31) one sees the effective $\sigma^3$ coupling constant contained in our potential $U(\Phi)$ multiplied by the cube of the $\sigma$–propagator $\frac{1}{m_3^2}$ at zero momenta. This result can also be found in the paper of Smilga and Stern \cite{2}.

4 The Dirac operator in the strong coupling regime of QCD

We have presented a method to obtain the spectral density of the Dirac operator for the linear $\sigma$-model in the large $N_c$ limit. Because of asymptotic freedom in QCD, we expect $\rho(\lambda) \propto \lambda^3$ for asymptotically large eigenvalues, since for a free theory the spectral density is only determined by the phase space. This situation of a non interacting theory can be tested within our approach in the limit $m_\Lambda \to \infty$. In this case the NJL four fermion interaction $G \propto 1/m_\Lambda^2$\cite{10} vanishes. As shown in fig. (3) the resulting spectral density becomes the density of a free theory. For $\lambda \geq \Lambda$ the phase space is cut off sharply.

If the interaction strength $G$ is turned on by a finite $m_\Lambda$ one sees that quark-quark interactions lead to a level repulsion increasing the density for small $\lambda$ and pushing some strength above $1$ GeV. Varying $n$ and thereby the shape of the cutoff functions $f_n^{(n)}$ in eq. (9) we adjust the cutoff parameters $m_\Lambda$ and $\Lambda$ in such a way that $f_n$ and $M_q$ are kept constant. This way we can test the sensitivity of the spectral density to the cutoff function. We see that below $1$ GeV the choice of the cutoff function leads to stable results within $10\%$.

In the realistic case of QCD we expect that gluonic interactions above $\Lambda$ influence the spectrum nearby and below. Therefore, the decrease of the spectral density $\rho(\lambda)$ above $\lambda \approx 0.8$ GeV is probably an artifact of the model space, which is restricted by the soft cutoff.

The interactions in the strong coupling regime of QCD induce level repulsion. Higher modes are strongly suppressed, while the increase of the average eigenvalue density for smaller eigenvalues drives the chiral condensate which appears on the low energy end of the spectrum. It seems that level repulsion
and condensation are the dominating effects for the Dirac operator spectral density in the strong coupling regime of QCD.

5 Conclusions

In this paper we have presented a new method to obtain the Dirac spectrum with renormalization group flow equations. We have used proper time flow equations for the linear $\sigma$-model with quarks of two flavors. In the large $N_c$ approximation we have found a simple connection between the meson potential $U_k(\Phi)$ and the chiral condensate which allowed us to calculate the Dirac spectrum directly from the full effective meson potential $V(\Phi, z)$. We have recovered the Banks–Casher relation for small eigenvalues. The next to leading order coefficient is 87 MeV.
Figure 6: The spectral density for different regulator functions $f_k^{(n)}$. The uppermost curve corresponds to $n=2$. The others are respectively $n = 3, 5, 10, 15$, where the last two are already nearly equal.

The calculation of $\rho(\lambda)$ should be valid up to several hundred MeV. Thus our approach allows to enter a new regime in the description of the Dirac spectrum which may complement other results from chiral perturbation theory and chiral random matrix theory. Compared to a free Dirac theory we have found the eigenvalue density suppressed for large eigenvalues and increased for small eigenvalues. All results are obtained in the infinite volume limit. Since the Schwinger proper time cutoff functions are simple to handle, the same calculation can be done for finite Euclidean space time volumes, where the evolution of the flow equations enters decisively. Such a calculation could then be compared to QCD lattice computations \[15\]. Further we plan to extend the calculation beyond the large $N_c$ approximation. The renormalization group flow equations of the linear $\sigma$-model with quarks provide a framework to include meson loops in the NJL-model without an additional new cutoff \[16\].
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