Influence of Slip and Heat and Mass Transfer Effects on Peristaltic motion of Power-law fluid Prone to the Tube

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Abstract. Present study deals with the study of peristaltic motion of a power-law fluid with nanoparticles in a tube with permeable walls. Heat and mass transfer effects and slip effect are studied in this investigation. Axial velocity, pressure gradient and frictional force are expressed analytically and investigated various parameter effects on these flow variables. The present model revealed that, heat transfer coefficient and mass transfer coefficients increases in the region [-1, 0] and decreases in the region [0, 1] with the increase of thermophoresis parameter and shows an opposite behavior with the increase of Brownian motion parameter. Pressure drop increases with the increase of slip parameter. Frictional force decreases with the increase of slip parameter and converges to 1.

1. Introduction

Peristalsis is very important phenomena in the human body. This phenomenon has many biological and industrial applications. Many researchers have done investigations in the peristaltic transport. (Brasseur et al. (1987), Valanis and Sun (1969), Mishra and Ramachandra Rao (2003), K. M. Prasad (2009), Hayat et al. (2014), Chandra and Pandey (2018)).

“Power-law law fluid is a fluid in which the shear stress at any point is proportional to the shear rate at that point raised to some power”. The problems based on non-Newtonian fluids have many applications and hence good number of researchers started working in this area. Ostwald-de Waele model is widely used model for non-Newtonian fluids focusing on power-law rheology. Power-law fluids are classified into three different types of fluids as given below:

| n   | Type of Fluid            |
|-----|--------------------------|
| <1  | Shear-Thinning Fluids    |
| =1  | Newtonian Fluid          |
| >1  | Shear-Thickening Fluids  |

Many researchers done their research in this field (El Naby and El Shamy (2007), Hayat et al. (2006), Shukla and Gupta (1982)).

Nanofluids have many biomedical and industrial applications. New techniques are used using nanofluids for cancer treatments and for safer surgery for the delivery of drugs. A good amount of
research has been done by researchers because of its applications. (S. U.S. Choi (1995), Buongiorno
(2005), Akbar and Nadeem (2013), Ellahi (2018) and Narayanan and Rakesh (2018).
Most of the researchers have done their study using no slip boundary condition at the walls of the
vessels. But the blood vessel walls may be movable, flexible and permeable in nature. (Chu and Fang
(2000), El Naby and El Shamy (2007), Tasawar Hayat et al. (2014).
Motivated by all the above studies, influence of slip and heat and mass transfer effects on peristaltic
motion of a power-law fluid in a tube with permeable walls have been studied. Expressions for axial
velocity, pressure drop and frictional force are derived and graphs have been drawn for various
parameters.

2. Mathematical Formulation
An incompressible power-law fluid with nanoparticles in a uniform tube with permeable walls have been
considered. The geometry of the wall surface is described by the following figure.

The wall deformation is explained by the equation

\[ R = H(z,t) = a_1 + b_1 \sin \frac{2\pi}{\lambda_1} (z - c_1 t_1) \]  

(1)

here radius of the tube is represented by \( a_1 \), amplitude is represented by \( b_1 \), wave speed is represented by \( c_1 \) and wave length is represented by \( \lambda_1 \).

Applying the transformations below to transform from fixed frame of reference to moving frame of
reference

\[ z = Z - c_1 t_1, \quad r = R, \quad \theta = \theta, \quad w_1 = W - c_1, \quad u = U \]

and also applying the non-dimensional quantities; low Reynolds number and long wave length
approximations, the constituent equations are as follows:

\[ \frac{\partial p}{\partial z} = -\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\partial w_1}{\partial r} \right)^n + G_r \theta_1 + B_r \sigma_1 \]  

(2)

\[ \frac{\partial p}{\partial r} = 0 \]  

(3)

\[ 0 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta_1}{\partial r} \right) + N_b \left( \frac{\partial \sigma_1}{\partial r} \right) + N_t \left( \frac{\partial \theta_1}{\partial r} \right)^2 \]  

(4)

\[ 0 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \sigma_1}{\partial r} \right) + N_b \left( \frac{\partial \theta_1}{\partial r} \right)^2 + N_t \left( \frac{\partial \sigma_1}{\partial r} \right) \frac{\partial \theta_1}{\partial r} \]  

(5)

Here axial velocity is represented by \( w_1 \), temperature profile by \( \theta_1 \), nanoparticle phenomena by \( \sigma_1 \),
Brownian motion parameter by \( N_b \), thermophoresis parameter by \( N_t \), local temperature Grashof number
by \( G_r \) and local nanoparticle Grashof number by \( B_r \).

Non-dimensional boundary conditions are as follows:

\[ \frac{\partial w_1}{\partial r} = 0, \quad \frac{\partial \theta_1}{\partial r} = 0, \quad \frac{\partial \sigma_1}{\partial r} = 0 \quad \text{at} \quad r = 0 \]  

(6)

\[ w_1 = -kn \frac{\partial w_1}{\partial r}, \quad \theta_1 = 0, \quad \sigma_1 = 0 \quad \text{at} \quad r = h(x) \]  

(7)
Here, $k$ is slip parameter and $n$ represents power-law index.

3. Solution of the problem

The coupled equations (4) and (5) of temperature profile and nanoparticle phenomena are solved by using Homotopy Perturbation Method with the initial conditions $\theta_0(r,z) = \frac{r^2-h^2}{4}$ and $\sigma_0(r,z) = -\left(\frac{r^2-h^2}{4}\right)$, then the solutions for above said parameters is given by

$$\theta_1 = \frac{r^4-h^4}{64} \left( N_b - N_t \right)$$

$$\sigma_1 = -\left(\frac{r^2-h^2}{4}\right) \left( N \right)$$  

(8)

Using equations (8) and (9) in equation (2) and applying boundary conditions equations (6) and (7), the expression for axial velocity is given by

$$w_1 = \frac{r^{n+1}}{n+1} \frac{n}{1+n} \left( \frac{dp}{dz} \right)^\frac{1}{n} - n \left( \frac{h^n}{2} \right)^\frac{1}{n} \left( k + \frac{h}{n+1} \right) \left( \frac{dp}{dz} \right)^\frac{1}{n}$$

$$- \left( \frac{G_r}{64} (N_b - N_t) \right)^\frac{1}{n} \left( \frac{w}{n} \right)^\frac{2}{n} \left( 2 - 2 + \frac{4}{n} \right)^\frac{1}{n}$$

$$\left( \frac{3h^3r + r^3}{3h^2} \right)^\frac{1}{n} HypergeometricPFQRegularized \left[ \left\{ \left( \frac{1}{4} \right)^\left( \frac{1}{n} \right), -1 \right\}, \left\{ \left( \frac{1}{4} \right)^\left( \frac{1}{n} \right), \frac{1}{3} \right\} \right]$$

$$\left( \frac{h_3}{4} \right)^\frac{1}{n} \left( \frac{w}{n} \right)^\frac{2}{n} \left( 2 - 2 + \frac{4}{n} \right)^\frac{1}{n}$$

$$\left( \frac{3h^3r + r^3}{3h^2} \right)^\frac{1}{n} HypergeometricPFQRegularized \left[ \left\{ \left( \frac{1}{4} \right)^\left( \frac{1}{n} \right), -1 \right\}, \left\{ \left( \frac{1}{4} \right)^\left( \frac{1}{n} \right), \frac{1}{3} \right\} \right]$$

(9)

The dimension less flux $q$ in the moving frame is given by

$$q = q^h 2 r w_1 d r$$

(10)

The expression for $\frac{dp}{dz}$ is calculated by substituting equation (11) into equation (10).

The pressure drop over a wave length $\Delta p_x$ is given by

$$\Delta p_x = \int_0^1 \frac{dp}{dz} dz$$

(11)

By using the expression for $\frac{dp}{dz}$ in equation (12), the expression for $\Delta p_x$ is

$$\Delta p_x = q^h L_1 + L_2$$

(12)

where $L_1 = \int_0^1 \frac{1}{n} dz$ and

$$L_2 = -\left( \frac{G_r}{64} (N_b - N_t) \right) \left( \frac{n}{1+n} \right)^n \left( \frac{1}{\Gamma \left( \frac{1}{n} \right)} \right)^n \left( \frac{1}{\Gamma \left( \frac{1}{4} \cdot \left( \frac{5}{1} + \frac{1}{n} \right) \right)} \right)^n$$

$$\int_0^1 \frac{h^{3n+5}}{A^n} \left( \frac{3}{2} + \frac{1}{n} \right) HypergeometricPFQRegularized \left[ \left\{ \left( \frac{1}{1} \right)^\left( \frac{1}{n} \right), -1 \right\}, \left\{ \left( \frac{1}{1} \right)^\left( \frac{1}{n} \right), \frac{1}{3} \right\} \right]$$

$$+ \Gamma \left( \frac{1}{4} \cdot \left( \frac{1}{n} \right) \right) HypergeometricPFQRegularized \left[ \left\{ \left( \frac{1}{4} \cdot \left( \frac{1}{n} \right) \right), -1 \right\}, \left\{ \left( \frac{1}{4} \cdot \left( \frac{1}{n} \right) \right), \frac{1}{3} \right\} \right]$$

$$\int_0^1 \frac{G_r}{64} (N_b - N_t) \left( \frac{n}{1+n} \right)^n \left( \frac{3}{2} + \frac{1}{n} \right) HypergeometricPFQRegularized \left[ \left\{ \left( \frac{1}{1} \right)^\left( \frac{1}{n} \right), -1 \right\}, \left\{ \left( \frac{1}{1} \right)^\left( \frac{1}{n} \right), \frac{1}{3} \right\} \right]$$

$$\int_0^1 \frac{h^{3n+5}}{A^n} \left( \frac{3}{2} + \frac{1}{n} \right) HypergeometricPFQRegularized \left[ \left\{ \left( \frac{1}{1} \right)^\left( \frac{1}{n} \right), -1 \right\}, \left\{ \left( \frac{1}{1} \right)^\left( \frac{1}{n} \right), \frac{1}{3} \right\} \right]$$

$$\int_0^1 \frac{h^{3n+5}}{A^n} \left( \frac{3}{2} + \frac{1}{n} \right) HypergeometricPFQRegularized \left[ \left\{ \left( \frac{1}{1} \right)^\left( \frac{1}{n} \right), -1 \right\}, \left\{ \left( \frac{1}{1} \right)^\left( \frac{1}{n} \right), \frac{1}{3} \right\} \right]$$

(13)
\begin{equation}
\frac{-B_r}{4} \left( \frac{N_t}{N_b} \right) \int_0^1 h^3+2n \ A^n \ n^n \ \left( \frac{k^n}{3} + \left( \frac{h}{1+n} \right)^{n+1} \right) \ \text{Hypergeometric2F1} \left[ \frac{-1}{n}, \frac{1+n}{2n}, \frac{1}{2} \left( 3 + \frac{1}{n} \right), \frac{1}{2} \right] \ dz
\end{equation}

\begin{equation}
A = \frac{-n^2}{(n+1)(3n+1)} 2^{n-1} n^n \ h^3 \ n + 2^{-1/n} h^{2n+1/n} \left( k + \frac{h}{n+1} \right)
\end{equation}

The same procedure is adopted as done by Shapiro et al. (1969), the time averaged flux over one period in laboratory frame $\bar{Q}$ is given by

\begin{equation}
\bar{Q} = 1 + \frac{\epsilon^2}{2} + q
\end{equation}

The dimension less frictional force at the wall is expressed as

\begin{equation}
\bar{F} = \int_0^1 h^2 \left( \frac{-dp}{dz} \right) \ dz
\end{equation}

Heat transfer coefficient at the wall is

\begin{equation}
Z_\theta (r, z) = \left( \frac{\partial h}{\partial z} \right) \left( \frac{\partial \theta_1}{\partial r} \right)
\end{equation}

Mass transfer coefficient at the wall is

\begin{equation}
Z_\sigma (r, z) = \left( \frac{\partial h}{\partial z} \right) \left( \frac{\partial \sigma_1}{\partial r} \right)
\end{equation}

4. Results and Discussions

Equations for axial velocity, pressure drop, time averaged flux, frictional force, heat, and mass transfer coefficients are expressed in the above section. Graphs on these flow variables have been drawn using Mathematica 11.0 software.

It is noticed from Figs. 2.1 to 2.6 that, pressure drop $\Delta p_\lambda$ and frictional force $\bar{F}$ both increases with the increase of Brownian motion parameter $N_b$. Pressure drop $\Delta p_{\lambda}$ increases with the increase of local temperature Grashof number $G_r$, and decreases with the increase of local nanoparticle Grashof number $B_r$, thermophoresis parameter $N_t$. But frictional force $\bar{F}$ shows opposite behaviour with the increase of above parameters. Especially pressure drop $\Delta p_{\lambda}$ increases with the increase of slip parameter $k$, whereas frictional force $\bar{F}$ decreases with the increase of slip parameter $k$ and converges to 1. Further pressure drop and frictional force decreases in the case of $n = 1$ i.e. for Newtonian fluids and parallel to axis for non-Newtonian fluids i.e. for $n > 1$.

The present model also revealed that, heat transfer coefficient and mass transfer coefficient increases in the region $[-1, 0]$ and decreases in the region $[0, 1]$ with the increase of thermophoresis parameter and shows an opposite behavior with the increase of Brownian motion parameter.

Fig. 2.1 Plots for the Pressure Drop $\Delta p_{\lambda}$ showing the effects of changing local nanoparticle Grashof number $B_r$ and local temperature Grashof number $G_r$. 
Fig. 2.2 Plots for the Pressure Drop $\Delta p_{\lambda}$ showing the effects of changing Brownian motion parameter $N_B$.

Fig. 2.3 Plots for the Pressure Drop $\Delta p_{\lambda}$ showing the effects of changing slip parameter.

Fig. 2.4 Plots for the Frictional force $F$ showing the effects of changing local nanoparticle Grashof number $B_r$ and local temperature Grashof number $G_r$.

Fig. 2.5 Plots for the Frictional force $F$ showing the effects of changing local nanoparticle Grashof number $B_r$ and
5. Conclusions
The study of peristaltic motion of a power-law fluid with nanoparticles in a tube with permeable walls. Heat and mass transfer effects and slip effect are studied in this investigation. Axial velocity, pressure gradient and frictional force are expressed analytically.

The main points observed are
a. Pressure drop $\Delta p_\lambda$ increases with the increase of local temperature Grashof number $G_r$, and decreases with the increase of local nanoparticle Grashof number $B_r$, thermophoresis parameter $N_t$.
But frictional force $F_\lambda$ shows opposite behaviour with the increase of above parameters.
b. Especially pressure drop $\Delta p_\lambda$ increases with the increase of slip parameter $k$, whereas frictional force $F_\lambda$ decreases with the increase of slip parameter $k$ and converges to 1.
c. Further pressure drop and frictional force decreases in the case of \( n = 1 \) i.e. for Newtonian fluids and parallel to axis for non-Newtonian fluids i.e. for \( n > 1 \).

d. The present model also revealed that, heat transfer coefficient and mass transfer coefficient increases in the region \([-1, 0]\) and decreases in the region \([0, 1]\) with the increase of thermophoresis parameter and shows an opposite behavior with the increase of Brownian motion parameter.

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