ANALYTICAL STUDY OF UNSTEADY SQUEEZED FLOW OF WATER BASE CNTS NANOFLUID WITH MAGNETIC FIELD AND VARIABLE THERMAL CONDUCTIVITY OVER A STRETCHING SURFACE

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ABSTRACT

This research paper explains the analytical solution unsteady squeezing flow of water based CNTs for both MWCNT and SWCNT in the presence of magnetic field and variable thermal conductivity. The given partial differential equation is converted to nonlinear ordinary differential equation by using the similarity transformation and solve by analytical method namely optimal homotopy asymptotic method (OHAM) to obtain analytical solution of the nonlinear problem which analyze the problem. The result of important parameter for both velocity and temperature profiles are plotted and discussed. The BVPPh 2.0 package is used to obtain the convergence of the problem up to 25 iteration. The skin friction coefficient and Nusselt number is explained in table form.

Keywords: Water based CNTs nanofluid, OHAM, squeezing flow, MHD, stretching surface.

1. INTRODUCTION

Due to the progress in fluid mechanics and its application in industrial, squeezing flow is interesting area for the researcher in the present time. The application of squeezing flow is polymer processing, food engineering, physical comparison, injection molding and biophysical. Kuzma et al. (1977) used parallel plate to discuss squeezing flow, Kuzma engineering, physical comparison, injection molding and biophysical. Gupta and Gupta (1977) used parallel plate to discuss squeezing flow, the researchers (Wang and Watson, 1979; Bhattacharyya and Pal, 1997; Xu et al., 2010; Ganji et al., 2014) discussed heat transfer involving squeezing flow. There are some important applications of the analysis of heat transfer, for example, energy production, heat conduction in tissues, astro physical flow, missiles physical flow and electrical power generation. Grubka and Bobba (1985) used stretching surface to discuss heat transfer phenomena. Dutta (1988) used stretching surface to investigate the hydro magnetic flow and heat transfer. The researcher (Rudraswamy et al., 2017; Rudraswamy and Gireesh, 2014; Kuiry and Bahadur, 2015; Rudraswamy et al., 2016; Ganesh et al., 2017) discussed the effect of thermal radiation and heat transfer. Chaim (1998) used stretching sheet to discuss the impact of variable thermal conductivity and heat transfer. Sharma and Singh (2009) used linearly stretching sheet to discuss MHD flow with variable thermal conductivity. Salahuddin et al. (2016) used Keller box method to investigate the effect of thermal conductivity and MHD flow, for this they use stretching cylinder. Malik et al. (2016) used stretching cylinder to investigate MHD flow of Sisko fluid.

The achievement of energy is not enough, but also to adjust the consumptions of energy and this is possible only to approve the development heat transmission liquids to mechanism the expenditures of energy and to improvement the most heat transmission which is the demand of the industry and other related scientific fields. Former to the application of nanotechnology, analysts and engineers have challenged such huge numbers of questions, recognizing with heat transmission fluids, still, with the development of nanometer sized particles and its uses in the heat transfer fluids have overall improved thermal conductivity. Rehman et al. (2019c) used stretching sheet to discuss the viscous dissipation of thin film unsteady flow of GO-EG/GO-W nanofluids. The most common lecture of carbon family is carbon nanotube (CNT). Carbon nanotube is used as a nanomaterial to rise heat transmission. Single wall carbon nanotube (SWCNT) and multi wall carbon nanotube (MWCNT) are two subclass of carbon nanotube (CNT). The key use of carbon nanotube (CNT) in engineering is for fluidization and heat conversation. Iijima was the first one to present carbon nanotube (CNT) in 1991 and also the first time present the multi wall carbon nanotube (MWCNT) by using Kraetschmer and Huddman method, see (Iijima, 1991; Ajayan and Iijima, 1993). Donald Bethune in 1993 studied multiwall carbon nanotube (MWCNT). They discuss the diameter range of multiwall carbon nanotube and show that the diameter range of multiwall carbon nanotube is 0.4 × 10−9m to 3 × 10−9m (To, 2006). In multiwall carbon nanotube there are 2 to 50 coaxial nanotube with diameter range from 3 × 10−9m to 30 × 10−9m (Dresselhaus et al., 1995).

Hone (2004) investigated that at room temperature the thermal conductivity of MWCNT is 3000 Wm−1K−1 and for SWCNT is 6600 Wm−1K−1. Haq et al. (2015) discussed the compression of engine base (CNT) and kerosene oil base (CNT) and show that the skin friction of water base (CNT) is greater than the kerosene oil base (CNT). Khan et al. (2014) used stretching plate to discuss the thermal conductivity and navier slip boundary condition for CNT. Kamali and Binesh (2010) discussed the impact of water base (CNT) and kerosene oil base (CNT) and show that the skin friction of water base (CNT) is greater than the kerosene oil base (CNT).
of magnetic field and viscous dissipation on the thin film unsteady flow of GO-EG/GO-W nanofluids.

Atashafrooz et al. (2020) investigated interacting infinities of Lorentz force and bleeding on the hydrothermal behaviors of nanofluid flow in a trapezoidal recess with help of second law of thermodynamics. Sheikholeslami (2019) used Al₂O₃ nanoparticles to discuss magnetic force and radiation influences by using permeable media. Sheikholeslami et al. (2014) investigated the influence of radiative heat transfer on thermal characteristics of nanofluid flow in the presence of magnetic field. Atashafrooz (2019) discussed the effect of buoyancy force on mixed convection heat transfer of MHD nanofluid flow and entropy generation in inclined duct by using Brownian motion. Sajjadi et al. (2019) used double MRT Lattice Boltzmann method to discuss the simulation of three dimensional MHD. Atashafrooz et al. (2019) investigated the interaction effects of an inclined magnetic field and nanofluid on forced convection heat transfer and flow irreversibility in a duct with and abrupt contraction. Sajjadi et al. (2018) investigated double MRT Lattice Boltzmann simulation of 3-D MHD natural convection in a cubic cavity with sinusoidal temperature distribution. Furthermore, Mahendesh and Shakanthala (2017) investigated the flow and heat transfer of carbon nanofluids over a vertical plate. Next, Astuti et al. (2019) studied the natural convection of nanofluids past an accelerated vertical plate with variable wall temperature in the presence of the radiation. Recently, Luay et al. (2019) studied the thermal and mechanical properties of fibre-glass multi-wall carbon nanotube/epoxy. They obtained several interesting results.

In this research paper, we explain the analytical solution of unsteady squeezing flow of water base CNTs for both MWCNT and SWCNT in the presence of magnetic field and variable thermal conductivity. The given partial differential equation is converted to nonlinear ordinary differential equation by using the similarity transformation and solve by analytical method, i.e., optimal homotopy asymptotic method (OHAM). Liao (2010) used this method for the solution of nonlinear problem and shows that this method is quickly convergent to the approximate solution. This method gives us series solution in the form of functions and all the physical parameters of the problem involved in this method. The result of important parameter for both velocity and temperature profiles are plotted and discussed.

2. MATHEMATICAL FORMULATION

Consider the unsteady boundary layer flow of water base CNTs. Figure 1 shows the closed squeezed channel with height \( h(t) \), supposed that \( h(t) \) is greater than that boundary layer thickness. \( L \) represent micro cantilever of length which is enclosed inside the channel. It is supposed that the squeezing stream is to be occurred from the tip to the surface. Moreover it is assumed that lower plat is in rest. The continuity and momentum equations are settled, see (Kumar et al., 2017):

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} + v(1-n) \frac{\partial^2 u}{\partial y^2} + \sqrt{2} \nu n f \left( \frac{\partial u}{\partial y} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \frac{u}{u} (2)
\]

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + v \frac{\partial U}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{\sigma B_0^2}{\rho} \frac{U}{U} \left( U - u \right) (3)
\]

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho C_p} \left( x(t) \frac{\partial T}{\partial y} \right) \frac{2}{\partial y^2} \left( \frac{\partial T}{\partial y} \right) (4)
\]

Here \( u \) and \( v \) are the velocity component with \( x \) and \( y \) direction. Kinematic viscosity is denoted by \( \nu \), the stream velocity is denoted by \( U \), the variable thermal conductivity is denoted by \( \alpha \), the power law index is denoted by \( n \), the time constant is denoted by \( C \), the electric charge density is denoted by \( \rho \), the fluid pressure is denoted by \( p \), and the temperature is denoted by \( T \). By eliminating the pressure term from equation (2) and equation (3), we obtain

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} + v(1-n) \frac{\partial^2 u}{\partial y^2} + \sqrt{2} \nu n f \left( \frac{\partial u}{\partial y} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho} \left( \frac{U}{U} - \frac{u}{u} \right) \frac{2}{\partial y^2} \left( \frac{\partial u}{\partial y} \right) \frac{2}{\partial y^2} \left( \frac{\partial u}{\partial y} \right) (5)
\]

with boundary condition

\[
u(x, 0, t), v(x, 0, t) = -v_0(t), -k \frac{\partial T(x, 0, t)}{\partial y} = q(x) \]

\[
u(x, \infty, t) = u(x, t), T(x, \infty, t) = T_{in} (6)
\]

The free stream temperature wall heat flux and free stream velocity is denoted by \( T_{in}, U(x, t) \) and \( q(x), \alpha = \alpha_0(1 + \epsilon \theta) \) represent the thermal conductivity. The similarity transformation is defined as

\[
U = \alpha x, \psi = x \sqrt{\omega f(\eta)} \cdot \eta = \frac{1}{s + bt}, u = ax f' \eta (7)
\]

\[
u = -f(\eta) \sqrt{\omega f, \eta} = \frac{T - T_{in}}{q_{in}} \sqrt{\frac{v}{v}} (7)
\]

where \( b \) is the squeezed flow index, \( s \) is arbitrary constant, \( a \) show strength squeezing flow parameter, heat flux is represented by \( q_0 \) and \( k \) is the thermal conductivity. Using the similarity transformation in equation (4) and equation (5) the partial differential equation is transform to ordinarily differential equation:

\[
f''' + (1 - \phi) f'' + \frac{1}{2} \left( 1 - \phi + \frac{\phi \rho_c}{\rho f} \right) \left( \frac{f + b n}{2} \right) f' - (f')^2 \]

\[
+ b b f' - 1 - n \nu a f''(\eta) \frac{f''(\eta)}{\eta} - M(1 - \phi) f' (1 - f') = 0 (8)
\]

\[
k_c f \theta'' + Pr \left[ (1 - \phi) + \frac{\phi \rho_c}{\rho f} \right] \left( \frac{f + b n}{2} \theta' + Pr \epsilon(\theta')^2 \right) - Pr \left[ f' + \frac{b n}{2} \theta \right] = 0 (9)
\]
\[
\begin{align*}
f(0) = -f_0, f'(0) = 0, \theta'(0) &= -1 \\
f'(\infty) &\rightarrow 1, \theta(\infty) \rightarrow 0
\end{align*}
\] (10)

where \( Pr = \frac{\nu}{\sigma} \) is Prandtl number, \( f_0 = \sqrt{\nu} \) is permeable velocity, \( \nu_e = \frac{\sqrt{\nu} \mu}{\sigma} \) and \( M = \frac{\sigma_0^2}{\sigma} \) is magnetic parameter and \( n \) is the power law index.

The coefficient of skin friction and Nusselt number is defined as

\[
\sqrt{Re}C_f = \left[ (1-n)f''(\eta) + \frac{n}{2} \nu_e f'(\eta)^2 \right]_{\eta=0} \quad \text{and} \quad Nu_x \sqrt{Re_x} = -\theta'(\eta)_{\eta=0}
\] (11)

**Method of Solution**

The analytical method (OHAM) is used to find the approximate analytical solution of the nonlinear ordinary differential equation. The BVPh 2.0 package function of Mathematica is used to obtain the numerical results, the result of important parameters. The given equations (7) and (8) are solved analytically by analytical method, i.e., optimal homotopy asymptotic method (OHAM) which is given below

\[
L(u(x)) + N(u(x)) + g(x) = 0, B(u(x))
\] (10)

where \( L \) is linear operator, \( x \) is independent variable, \( g(x) \) is the unknown function, \( N \) is the nonlinear operator and \( B(u) \) is a boundary operator. By using this method, we first find a family of equations

\[
H(\phi(x), p) = (1 - p) [L(\phi(x), p) + g(x)] - HP(1) [L(\phi(x), p) + g(x) + N(\phi(x), p)] = 0
\]

where \( p \) is an embedding or inserting parameter and lies in \([0,1] \). \( H(p) \) is the nonzero auxiliary function for \( p \neq 0 \) and \( H(0) = 0 \) and \( \phi(x, p) \) is an unknown function. Using the initial guessed values and auxiliary linear operators from equations (8-9)

\[
f_0(\eta) = \frac{(b - m)}{24} \eta^4 + \frac{c_2 \eta^2}{2} - f_0 = 0
\] (12)

\[
L_f = \frac{d^4 f}{d \eta^4}, L_0 = \frac{d^2 \theta}{d \eta^2}
\] (13)

When \( p = 0 \), equation (11) become as

\[
L(\phi(x, p)) + g(x) = 0.
\] (14)

Let the solution of equation (14) be

\[
v_0(x).
\] (15)

When \( p = 1 \), equation (14) become as

\[
L(\phi(x, p)) + g(x) + N(\phi(x, p)) = 0,
\] (16)

which has a solution, let the solution is

\[
v(x).
\] (17)

Thus by increasing \( p \) from 0 to 1, the solution change form \( v_0(x) \) to \( v(x) \). Choose the supplementary function as

\[
H(P) = PC_1 + P^2C_2 + P^3C_3 + \ldots
\] (18)

where \( C_1, C_2, C_3, \ldots \) are constants to be determined. Expanding \( \phi(x, p) \) in a series with respect to \( p \), we have

\[
\phi(x, p, C_i) = v_0(x) + \sum_{k=1}^{\infty} v_k(x, C_i)p^k
\] (19)

for \( i = 1, 2, 3 \ldots \). Put equation (19) into equation (11) and comparing the coefficient of different power of \( p \), we get differential equations with boundary condition. By solving this equation, we find a series solution in the form of functions \( v_0(x), v_1(x, C_1), v_2(x, C_2), \ldots \) and the solution of equation (10) can be written as

\[
v(x, C_i) = v_0(x) + \sum_{k=1}^{\infty} v_k(x, C_i).
\] (20)

The residual is as follow

\[
R(x, C_i) = L(v(x, C_i)) + N(v(x, C_i)) + g(x)
\] (21)

If in equation (21), \( (x, C_i) = 0 \), then equation (20) has exact solution but for nonlinear differential equation is not possible. We try to minimize the \( R(x, C_i) \) to get the best approximate analytical solution of equation (10).

3. RESULTS AND DISCUSSION

**Table 1** Comparison of the skin friction for the two nanofluids when \( Pr = 5.6, \nu_e = 0.1, \epsilon = 1 \).

| \( b \) | \( M \) | MWCNT | SWCNT |
|-------|-------|-------|-------|
| 0.1   | 0.2   | 0.729705 | 0.99471 |
| 0.3   | 0.2   | 0.712164 | 0.979547 |
| 0.5   | 0.2   | 0.711474 | 0.97264 |
| 0.4   | 0.2   | 0.7100877 | 0.96694 |
| 0.6   | 0.2   | 0.70608 | 0.93724 |
| 0.7   | 0.2   | 0.698432 | 0.9246 |
| 0.8   | 0.2   | 0.65818 | 0.91641 |

**Table 2** Comparison of the Nusselt number \( \frac{1}{Nu_x} \) for the two nanofluids when \( f_0 = 0.1, M = 0.2, b = 0.5 \).

| \( Pr \) | \( \nu_e \) | MWCNT | SWCNT |
|-------|-------|-------|-------|
| 1.5   | 0.1   | 0.61231 | 0.49077 |
| 2.5   | 0.1   | 0.59341 | 0.48237 |
| 3.5   | 0.1   | 0.57451 | 0.47397 |
| 0.2   | 0.55614 | 0.45647 |
| 0.3   | 0.53776 | 0.44897 |
| 0.4   | 0.52795 | 0.43121 |
| 0.5   | 0.55021 | 0.41346 |

**Table 3** Convergence of the method for SWCNT when \( Pr = 6.7, M = 0.1, f_0 = 0.1, \phi = 0.01, \epsilon = 1 \).

| \( m \) | \( \epsilon_m \) | SWCNT | \( \epsilon_m \) | SWCNT |
|-------|-------|-------|-------|-------|
| 5     | 1.55438 × 10^{-1} | 2.86775 × 10^{-1} |
| 10    | 7.69094 × 10^{-2} | 1.48738 × 10^{-2} |
| 15    | 2.20944 × 10^{-7} | 1.07298 × 10^{-4} |
| 20    | 3.37298 × 10^{-9} | 8.54131 × 10^{-5} |
| 25    | 3.37387 × 10^{-11} | 7.94423 × 10^{-6} |
Table 4. The convergence of method for MWCNT when \( Pr = 6.7, M = 0.1, w_e = 0.1, \phi = 0.01, \varepsilon = 1 \).

| \( m \) | \( \varepsilon^f_m \) MWCNT | \( \varepsilon^B_m \) MWCNT |
|---|---|---|
| 5  | \( 1.07991 \times 10^{-1} \) | \( 2.88574 \times 10^{-1} \) |
| 10 | \( 5.65266 \times 10^{-2} \) | \( 1.0759 \times 10^{-3} \) |
| 15 | \( 4.12383 \times 10^{-3} \) | \( 1.0759 \times 10^{-5} \) |
| 20 | \( 3.4616 \times 10^{-4} \) | \( 8.55721 \times 10^{-7} \) |
| 25 | \( 3.133 \times 10^{-5} \) | \( 8.006632 \times 10^{-9} \) |
The main objective of this section is to study the effect of various model factors like $M$, $f_0$, $Pr$, $We$, $\epsilon$, $n$, $b$ (magnetic parameter, permeable velocity, Prandtl number, Weissenberg number, small quantity, power law index, and squeezed flow index) on the skin friction coefficient and Nusselt number. In Tables 1 and 2, the numeric results illustrate the impacts of different model factors on skin friction coefficient and temperature profiles. The obtained outputs are deliberated as follows: It is observed that:

- Increasing Weissenberg number, decreasing the velocity field.
- Increasing the squeezed flow index, decreasing the velocity field.
- Increasing the values of the magnetic field, decreasing the velocity field.
- By increasing the Prandtl number $Pr$, decreasing the temperature profile.
- By increasing the $\epsilon$, increasing the temperature field.

### 4. CONCLUSIONS

This paper explained the unsteady squeezing flow of water based nanofluid CNTs for both MWCNT and SWCNT in the presence of magnetic field and variable thermal conductivity. The model is solved by analytical method namely optimal homotopy asymptotic method (OHAM) to obtain analytical solution of the nonlinear problem which analyze the problem. The result of important parameter for both velocity and temperature profiles are plotted and discussed. The skin friction coefficient and Nusselt number is explained in table form. The obtained outputs are deliberated as follows. It is observed that:

- Increasing Weissenberg number, decreasing the velocity field.
- Increasing the squeezed flow index, decreasing the velocity field.
- Increasing the values of the magnetic field, decreasing the velocity field.
- By increasing the Prandtl number $Pr$, decreasing the temperature profile.
- By increasing the $\epsilon$, increasing the temperature field.

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