Determination initial approximations in solving the problem of numerical optimization of a large-sized space structure using linear extrapolation of optimal solutions

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Abstract. The parameters of a large-sized space structure are determined using numerical optimization using the gradient method. The complexity of the boundary of the constraints makes the optimal solution poorly conditioned relative to the initial point of the iterative process. The initial point is selected based on the optimal solutions found with an incomplete set of design variables. The initial point is found by linear extrapolation of these optimal solutions. In the problem under consideration, this approach made it possible to obtain a better locally optimal solution than was possible with the help of an intuitive choice of the starting point.

1. Introduction

The papers [1-3] discussed creating of a design layout for the design of the observation spacecraft using a diffraction lens instead of a mirror for focusing the input light flux. The configuration of the spacecraft’s diffraction optical system [4 - 7] was taken from the MOIRE project [8 - 10] (figure 1). In this project, the spacecraft has the diffraction lens with 10 m in diameter. The lens is at a distance of 60 m from the spacecraft body.

2. Requirements to the lens frame design

An element of an optical system has to be in a fixed position relative other elements with high precision. Therefore the lens frame design has to keep the relative position of optical system elements .

Obviously the construction has to be folded during orbital injection and has to be unfolded to operational position on orbit. To develop large, rigid, low weight and foldable construction is a difficult problem. Dimensions of the lens frame can change with a number of causes. These are plastic strains appeared during orbital injection, temperature strains and a vibration of the construction. Problems of plastic and temperature strains can be solved with construction material choice. Vibrations of spacecraft construction are excited with inertial forces when there is a setting of spacecraft orientation to an object of observation. If amplitude of the vibration of optics is sufficient large in order to distort the image then it is impossible to take a photograph before the vibration has decayed. If decay times of such the vibrations are sufficient long operation of the observation spacecraft is difficult. To prevent vibrations with large amplitude and long decay time it is necessary that the lens frame design has big enough stiffness. The traditional observation spacecrafts successfully operate with the lowest mode frequency from 1.0 to 2.5 Hz. If the lens frame design is enough rigid and light that a spacecraft consisting the frame has the lowest mode frequency in the same range we can hope that spacecraft operation will success.
3. The design layout of the lens frame
In the paper [1] the authors suggested a design layout of the lens frame that allows meeting the above requirements. Trusses connect the lens and the spacecraft body. Joining of the trusses by tensioned cables turns the number of single trusses into the joint design. The joint design has much greater stiffness than single trusses without the tensioned cables. The tensioned cables act on the trusses with transversal forces. It can produce large deformations of the trusses. To avoid that the trusses in the design are arc-shaped and longitudinal cables connect their ends (figures 2 and 3). The arc-shaped trusses have large transversal stiffness that allows stretching of transversal cables.

4. Finding parameters of the design layout
A necessary part of the formation of the design layout of the structure is the method of choosing the optimal values of its parameters. The parameters of the considered design layout can be divided into the parameters of cross sections of the structural elements and the parameters that determine the general geometry of the structure (geometric parameters). The parameters of the cross sections of the structural elements are cross-sectional areas of the truss rods and the cables. The main geometric parameters of the considered scheme are the spacecraft's diameter; diffraction lens diameter; distance between the lens and the body; geometric configuration of truss arcs. All these parameters except the geometric configuration of the truss arcs are determined for reasons not related to power work.

Figure 1. The project MOIRE of geostationary observation satellite with detractive optical system [1-3] (MOIRE - Membrane Optical Imager for Real-Time Exploitation).

Figure 2. The design of the diffraction lens frame.
In the work, a search is made for the best combination of values of this design parameters using numerical optimization. The gradient method of numerical optimization is used. The behavior of the structure is modeled using the finite element method. Calculations are executed in the MSC.Nastran system.

**Figure 3.** Joining of the trusses by tensioned cables: a) the internal forces in straight and arc-shaped truss due to the transversal cables stretching. Solid arrows represent internal forces in a part of a truss and thin arrows represent forces acting from adjacent parts of a truss; b) prevention of arc-shaped trusses straightening with connecting ends of the trusses by longitudinal cables.

As an optimization criterion, we take the minimization of the mass of the diffraction lens frame. The desired design must satisfy the following restrictions. Natural frequencies must not be lower than the preassigned value \([f]\). There is the case of loading containing the cable tension forces in combination with inertial loads at the fastest turn of the spacecraft in orbit is considered. In this case of loading the cable tension should not disappear, the strength and stability of the structure should be maintained.

**Figure 4.** Possible change in the geometry of the truss arc in the form of an arc.
Design variables are the cross-sectional areas of the structural elements. The geometric configuration of the truss arcs is taken in the form of an arc (see figure 4). The radius of this arc, $r$, is also a design variable.

The variable $r$ has a different nature than the parameters of the cross sections. Unlike other variables, it describes the general geometry of the structure. In the previous work [2], devoted to the solution of this problem, difficulties were observed in finding the optimal solution in the case when the design variables include both the parameters of the cross sections of the structure elements and the parameters that describe the general geometry of the structure. During the iterative process, only the cross sections of the structure elements changed, and the variables describing the geometry of the structure remained the initial values. In later studies, it was possible to achieve a change in geometric variables, but the optimal solutions for a complete set of variables were worse than for variable sections without geometric variables. Adding a new design variable should expand the search for better solutions. But in this problem, with a complete set of design variables, it was not possible to obtain even a solution found only for the cross section variables.

This is likely due to a complex configuration of constraints that generates many locally optimal solutions. Which of these locally optimal solutions will be found depends on the choice of the starting point for the iterative process. Adding variables describing the geometry of the structure complicates the configuration of the constraint surfaces in the space of design parameters. This leads to the fact that the optimization process stops in worse locally optimal solution than it was before adding the variable $r$ to design variables.

The assumption that the shape of the truss arcs is an arc leaves only one geometric variable, $r$, under consideration. This allows us to exclude this variable from the optimization calculation as follows. A series of optimization calculations is performed with the cross section variables. These calculations differ from each other in different values of the radius of the truss arcs. The optimal value of the radius of the truss arc is the one for which the optimization calculation has found the smallest mass. Thus, we are looking for the optimal design as follows:

$$
\{ \bar{s}^{\text{opt}} , r_2 \} = \arg \min \{ M(\bar{s}^k, r_k) | k = 1..n \};
$$

$$
\bar{s}^k = \arg \min \left\{ M(\bar{s}, r_k) \left| \begin{array}{c} s_i^{\min} \leq s_i \leq s_i^{\max}, \\
 f_j(\bar{s}, r_k) > [f], j = 7..12, \\
 \sigma_i^{\min} \leq \sigma_i(\bar{s}, r_k) \leq \sigma_i^{\max} \end{array} \right. \right\},
$$

where $M$ – mass of the considered spacecraft, kg; $\bar{s}$ – design variables vector (cross-sectional areas of the truss rods and the cables) $s^k$ – the optimal value of the vector of design variables with the value of the radius of the truss arcs $r_k$; $f_j$ – the j-th natural frequency of the spacecraft; $[f]$ – the lowest frequency value of the elastic forms of natural vibrations (for this spacecraft is equal to 1.0 Hz), $\sigma_i$ – stresses in tubes and cables; $n$ – the number of truss arcs radius values considered. Index $i$ runs over the numbers of structural elements, index $j$ - over the numbers of its natural frequencies.

5. Optimization Calculation Results

The result of a series of optimization calculations is the dependence of the optimal mass of the diffraction lens frame on the radius of the truss arcs (figure 5). Figure 5 shows that there are very different values of optimal mass for close radii. Obviously, optimal structures with close truss radii should be similar and their masses should not differ much. The reason for this instability may be as follows. All found optimal constructions are located in the extrema of the constraint surface. Which extremum of the constraint surface the iteration process falls into depends on the initial point. It turned out that some values of $r$ were unsuccessful in the sense that the iterative process fell into the extreme of constraints with a large mass of the structure. Apparently, there is also a solution at an extremum with a small mass, as at neighboring values of $r$, but it was not possible to reach it due to an unsuccessful starting point.
Let’s consider the case when there are two optimal solutions with close masses for the truss arc radii values $r_k$ and $r_{k-1}$, and for the value $r_{k+1}$ an optimal solution with larger mass is found. Suppose that the solutions for the radii values $r_k$ and $r_{k-1}$ are located near the same extremum of the boundary of constraints. Then the approximation of the solution for the radius value $r_{k+1}$ near the same extremum can be tried to find using linear extrapolation of solutions for the radii values $r_k$ and $r_{k-1}$ (see figure 6):

$$s^{k+1}_0 = \frac{\left(s^k - s^{k-1}\right)}{\left(r_k - r_{k-1}\right)} \cdot \left(r_{k+1} - r_{k-1}\right)$$

If the value $r_{k+1}$ is close to $r_k$ and $r_{k-1}$, then we can hope that the approximation obtained using (2) will be near the same extremum. Starting the optimization procedure from the point $s^{k+1}_0$ will allow to find for the value of the radius $r_{k+1}$ the same successful optimal solution that has already been found for the values of the radii $r_k$ and $r_{k-1}$. The obtained new successful optimal solution can be used to find the initial approximations for next value of $r$.

The proximity of the optimal mass values for the radii $r_k$ and $r_{k-1}$ does not mean that both solutions lie near the same extremum. If these points are at different extrema, then a linear extrapolation of the optimal solutions will not give a good approximation (for example, in figure 7).

It is possible to get an idea of whether neighboring optimal points are located in the same extremum by the position of three neighboring optimal points. If the angle between the vector $\Delta s^k = (s^k - s^{k-1})$ and the vector $\Delta s^{k+1} = (s^{k+1} - s^k)$ is small, then the points are probably in the same extremum (angle $\beta$ in figure 6). A large angle $\beta$ indicates that extrapolation from them will not make sense (as in figure 7).

![Figure 6](image6.png)

**Figure 6.** The position of the optimal points for neighbor values of $r$ at the boundary of constraints. Neighboring optimal points are located in one extreme of the boundary of constraints.
Figure 7. The position of the optimal points for neighbor values of $r$ at the boundary of constraints. Neighboring optimal points are located in different extrema of the boundary of constraints.

In figures 6 and 7 show the constraint surfaces in the case of two design variables (one of them is $r$). In the considered problem there are variables eight and angle $\beta$ is in eight-dimensional space. You can find the value of the angle $\beta$ using the scalar product of vectors, as in three-dimensional space:

$$\cos \beta = \frac{\Delta s^{k+1} \cdot \Delta s^k}{|\Delta s^{k+1}| |\Delta s^k|}.$$  \hspace{1cm} (3)

Figure 8 shows the dependence of the angle $\beta$ on radius of the truss arcs, $r$. We see that low $\beta$ values are observed at the following $r$ values: 56 m, 80 m, 110 m, 125 m, 162 m. The optimal point for $r = 80$ m is located in an extremum with a large mass of the structure and is not of interest. Using the extrapolation, the dependences of optimal solutions on $r$ were found from the remaining points. Such a dependence for $r = 110$ m is shown in figure 9.

The optimal point for $r = 65$ m is one of the best solutions found in terms of the mass of the structure. The large value of the angle $\beta$ for this point indicates the impossibility of looking for successful solutions near this point using extrapolation. If you find a second point close to the point $r=65$ m, you can use the approach described above. For this, the optimization procedure for $r=66$ m was started from the optimal solution for $r = 65$ m.

The found series of optimal solutions found by successive extrapolation from points with $r=56$ m, 65 m, 110 m, 125 m and 162 m are shown in figure 10. For each value of $r$, as the optimal solution, we select the solution with the minimum mass among all the series of solutions. For different values of $r$, the minimum mass of the structure is observed either in one series or in another. Therefore, in the graphs of values of design variables for the best designs (with the minimum mass), gaps are observed in places where another series of solutions becomes the best (figure 11). These gaps indicate that
although the masses of solutions from different series are close, the values of the design variables are different. Thus, the series of solutions found is not one solution, but several competing solutions of similar masses.

![Figure 9](image1.png)

**Figure 9.** Dependence of the mass of optimal solutions on the radius of truss arcs $r$. A series of solutions obtained by extrapolation from the point $r = 110 \text{ m}$. Designations: M - initial dependence; Me are the initial points obtained by extrapolation; $M'$ - the mass of optimal solutions found from these initial points.

![Figure 10](image2.png)

**Figure 10.** Dependences of the optimal mass on the radius of the truss arcs in different series of solutions. Designations: M - initial dependence; numbers are values of $r$ of the initial points from which extrapolation was produced.

![Figure 11](image3.png)

**Figure 11.** Dependences of the optimal values of the cross-sectional areas of some structural elements on the radius of the truss arcs. Optimal solutions were found from the initial points obtained by extrapolation.
6. Conclusion
The work performed numerical optimization of the large-sized space structure. In the process of solving, poor conditionality of the optimal solution with respect to the initial approximation was observed. To find successful locally optimal solutions, an approach was taken to determine successful initial approximations. The initial approximation is found by linear extrapolation of the optimal solutions obtained for an incomplete set of variables. With using this approach the optimal design the large-sized space structure was found. The optimization was performed in space of parameters of cross sections of the structural elements with a constant value of the geometry parameter of this structure (radius of the truss arch, \( r \)). This approach made it possible to find low mass solutions for the entire range of values considered of the variable \( r \).

7. References
[1] Soyfer V A, Salmin V V, Chetverikov A S, Peresypkin K V and Tkachenko I S 2017 Insertion into Geostationary Orbit of a Spacecraft with Diffractive Orbit by Low Torque Electric Propulsion Procedia Engineering 185 332-337.
[2] Salmin V V, Peresypkin K V, Chetverikov A S and Tkachenko I S 2017 Modeling control over large space structure on geostationary orbit CEUR Workshop Proceedings 1904 168-173 DOI: 10.18287/1742-6596/1368/4/042063.
[3] Anshakov G P, Salmin V V, Peresypkin K V, Chetverikov A S and Tkachenko I S 2019 Numerical optimization of geometric configuration of large space structure Journal of Physics: Conference Series 1368 042063 DOI:10.1088/1742-6596/1368/4/042063.
[4] Skidanov R, Strelkov Y, Volotovsky S, Blank V, Ganchevskaya S, Podlipnov V, Ivliev N and Kazanskiy N 2020 Compact Imaging Systems Based on Annular Harmonic Lenses Sensors 20(14) 3914 DOI: 10.3390/s20143914.
[5] Kazanskiy N, Ivliev N, Podlipnov V and Skidanov R 2020 An Airborne Offner Imaging Hyperspectrometer with Radially-Fastened Primary Elements Sensors 20(12) 3411 DOI: 10.3390/s20143914.
[6] Nikonorov A V, Petrov M V, Bibikov S A, Yakimov P Y, Kutikova V V, Yuzifovich Y V, Morozov A A, Skidanov R V and Kazanskiy N L 2018 Toward Ultralightweight Remote Sensing With Harmonic Lenses and Convolutional Neural Networks IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing 11(9) 3338-3348 DOI: 10.1109/JSTARS.2018.2856538.
[7] Nikonorov A V, Petrov M V, Bibikov S A, Kutikova V V, Morozov A A and Kazanskiy N L 2017 Image restoration in diffractive optical systems using deep learning and deconvolution Computer Optics 41(6) 875-887 DOI: 10.18287/2412-6179-2017-41-6-875-887.
[8] Early J, Hyde R and Baron R 2004 Twenty meter space telescope based on diffractive Fresnel lens Proceedings of SPIE - The International Society for Optical Engineering 5166 148-156.
[9] Atcheson P, Stewart C, Domber J, Whiteaker K, Cole J, Spuhler P, Seltzer A and Smith L 2012 MOIRE - Initial demonstration of a transmissive diffractive membrane optic for large lightweight optical telescopes Proceedings of SPIE - The International Society for Optical Engineering 8442 844221.
[10] Atcheson P, Domber J, Whiteaker K, Britten A, Dixit S N and Farmer B 2014 MOIRE - Ground demonstration of a large aperture diffractive transmissive telescope Proceedings of SPIE - The International Society for Optical Engineering 9143 91431W.