New Flavor $U(1)_F$ Symmetry for SUSY $SU(5)$

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Abstract

Within supersymmetric $SU(5)$ Grand Unified Theory, we present several new scenarios with anomaly free flavor symmetry $U(1)_F$. Within each scenario, a variety of cases offer many possibilities for phenomenologically interesting model building. We present three concrete and economical models with anomaly free $U(1)_F$ leading to natural understanding of observed hierarchies between charged fermion masses and CKM mixing angles.

1 Introduction

Noticeable hierarchies between charged fermion masses and mixings remain unexplained within the Standard Model and its minimal supersymmetric (SUSY) extension. Grand Unification (GUT) [1] gives some interesting asymptotic mass relations (like $m_b = m_{\tau}$ within $SU(5)$ GUT), but problem of flavor still remains unresolved. It may well be that the resolution of this puzzle has some physical origin, and a nice idea is existence of the flavor symmetry acting between different flavors of quarks and leptons. The simplest possibility is the Abelian $U(1)_F$ flavor symmetry [2], which has been extensively investigated with $U(1)_F$ being an anomalous symmetry [3] of a stringy origin [4]. Some attempts to find anomaly free setup with $U(1)_F$ symmetry, for explanation of fermion mass hierarchies, also exist in a literature [5, 6]. With anomaly free $U(1)_F$, without relying on some specific string construction, one can investigate a given scenario (within MSSM [5] or GUT [6]) based on conventional field theoretical arguments.

In this Letter within SUSY $SU(5)$ GUT, we suggest new way of finding non-anomalous $U(1)_F$ flavor symmetries. We present several scenarios with anomaly free $U(1)_F$ symmetries, which provide natural explanation of hierarchies between charged fermion masses and mixings.

The Letter is organized as follows. In the next section we pursue the way of finding non-anomalous $U(1)_F$ symmetries by possible embedding of $SU(5) \times U(1)_F$ into the anomaly free non-Abelian symmetry, and present our findings. In section 3, after listing requirements which we

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follow upon model building, classify various possibilities within scenarios we have found. Further, we present three concrete models which give natural understanding of hierarchies between charged fermion Yukawa couplings and CKM mixing angles. Within these models leptonic mixing angle $\theta_{\mu\tau}$ turns out to be naturally large, giving good background for building promising scenarios for neutrino masses and mixings. Our conclusions are given in Sect. 4, while in Appendix A we discuss the possibility of consistent $U(1)_F$ symmetry breaking, needed for realistic model building.

2 SUSY $SU(5)$ and Non-Anomalous Flavor $U(1)_F$

As already noted, we are working within the framework of SUSY $SU(5)$ GUT and looking for non-anomalous flavor $U(1)_F$ symmetry. Minimal chiral content for the fermion sector consists to $10 + \bar{5}$ multiplets per generation, whose SM matter composition and quantum numbers under $SU(3)_C \times SU(2)_L \times U(1)_Y \equiv G_{321}$ gauge group is

$$10 = q(3, 2, -\frac{1}{\sqrt{60}}) + u^c(\bar{3}, 1, \frac{4}{\sqrt{60}}) + e^c(1, 1, -\frac{6}{\sqrt{60}}), \quad \bar{5} = d^c(\bar{3}, 1, -\frac{2}{\sqrt{60}}) + l(1, 2, \frac{3}{\sqrt{60}}).$$

Last entries in the brackets represent hypercharge $Y$ with $SU(5)$ normalization (being generator of $SU(5)$, the $Y$ has the form $Y = \frac{1}{\sqrt{60}}$Diag$(2, 2, 2, -3, -3)$). Our aim is to find such family dependent $U(1)_F$ charge assignments which are anomaly free. Clearly, for one of the families, out of three, the simplest assignment $10_0 + \bar{5}_0$ (subscripts indicate $U(1)_F$ charges) is anomaly free. The non-zero charge assignment would require\(^2\) addition of new states which can be $SU(5)$ singlets charged under $U(1)_F$. We will look for extensions with minimal possible content. By minimal content we mean that non-trivial $SU(5)$ representations, which we introduce, will be just those of minimal SUSY $SU(5)$ GUT. These are three families of matter $(10 + \bar{5})$-supermultiplets, one pair of Higgs superfields $H(5) + \bar{H}(\bar{5})$ (including MSSM Higgs doublet superfields $h_u$ and $h_d$ respectively), and an adjoint $\Sigma(24)$ of $SU(5)$ (needed for symmetry breaking $SU(5) \to G_{321}$). We assume that $\Sigma$ has no $U(1)_F$ charge, $Q(\Sigma) = 0$, and thus does not contribute to anomalies. Thus $SU(5)$ states which may contribute to anomalies are three $10_i$-plets ($i = 1, 2, 3$), four $\bar{5}_k$-plets ($k = 1, 2, 3, 4$) and one 5-plet. With this set, the $SU(5)^3$ anomaly $A_{555} = 3A(10) + 4A(\bar{5}) + A(5) = 0$ vanishes because $A(10) = -A(\bar{5}) = A(5)$. As far as the anomalies $SU(5)^2 \cdot U(1)_F$, $(U(1)_F)^3$ and $(\text{Gravity})^2 \cdot U(1)_F$ are concerned, they should satisfy the following conditions:

$$SU(5)^2 \cdot U(1)_F : \quad A_{551} = \frac{3}{2} \sum_{i=1}^3 Q(10_i) + \frac{1}{2} \left( \sum_{k=1}^4 Q(\bar{5}_k) + Q(5) \right) = 0 ,$$

$$(U(1)_F)^3 : \quad A_{111} = 10 \sum_{i=1}^3 Q(10_i)^3 + 5 \left( \sum_{k=1}^4 Q(\bar{5}_k)^3 + Q(5)^3 \right) + \sum_s Q_s^3 = 0 ,$$

$$(\text{Gravity})^2 \cdot U(1)_F : \quad A_{GG1} = \text{Tr}Q = 10 \sum_{i=1}^3 Q(10_i) + 5 \left( \sum_{k=1}^4 Q(\bar{5}_k) + Q(5) \right) + \sum_s Q_s = 0 ,$$

where $Q_s$ denotes $U(1)_F$ charges of $SU(5)$ singlet states. Upon finding the anomaly free assignments we will limit ourself with scenarios involving small number of singlets. All other mixed anomalies vanish due to properties of $SU(5)$ generators.

\(^2\)Unless $U(1)_F$ charge assignments are such that anomalies coming from different families cancel each other.
Since $U(1)_F$ is Abelian symmetry, there is no overall normalization for the charges. However, in order to have realistic phenomenology, all charge ratios should be rational numbers; i.e. in the unit of one field’s charge, all remaining states’ charges should be rational numbers. To find such anomaly free charge assignment, leading to desirable phenomenology, one way is to find solution(s) of system of Eqs. (2)-(4) in a straightforward way \cite{5, 6}. Another way might be to extract $U(1)_F$ (as a subgroup) from anomaly free non-Abelian flavor symmetries \cite{7} (which are compatible with $SU(5)$ GUT). Different, and unexplored yet, way of finding is to embed $SU(5) \times U(1)_F$ (as a subgroup) in higher non-Abelian symmetries with anomaly free content. In this work, we follow the latter way in order to find anomaly free flavor $U(1)_F$ symmetries within SUSY $SU(5)$ GUT.

For this purpose, consider higher gauge symmetries containing $SU(5)$ as their subgroups plus $U(1)$ factors. Clearly, the rank of such non-Abelian groups should be $\geq 5$. Since all states will belong to non-Abelian groups, the condition $\text{Tr}Q = 0$ of Eq. (4) will be automatically satisfied. However, vanishing of other anomalies will require specific selection of the field content \cite{8}. One simple possibility emerges via $SO(10)$ group which has a maximal subgroup is $SU(5) \times U(1)'$. $SO(10)$’s spinorial representation - the 16-plet - decomposes under the $SU(5) \times U(1)'$ as \cite{9}

$$16 = 10_1 + \bar{5}_{-3} + 1_5 ,$$

where subscripts are $U(1)'$ charges \cite{3} which can be identified with $U(1)_F$ charges. In this way, $U(1)_F$ is anomaly free since all anomalies [$SU(5)^3$, $SU(5)^2 \cdot U(1)_F$, etc.] vanish. The $SU(5)$’s singlet $1_5$, charged under $U(1)_F$, plays important role for anomaly cancellation.

For finding another assignment let us consider 27-plet of $E_6$ group (the rank six exceptional group). With $E_6 \rightarrow SO(10) \times U(1)'' \rightarrow SU(5) \times U(1)''$ decomposition we have \cite{9}

$$27 = 16_1 + 10_{-2} + 1'_4 = (10 + \bar{5} + 1)_1 + (5 + \bar{5}')_{-2} + 1'_4 ,$$

where subscripts denote $U(1)''$ charges. In this case $U(1)''$ can be identified with $U(1)_F$. We see that, in this case anomaly cancellation requires two $SU(5)$ singlets (charged under $U(1)_F$) and extra charged 5, 5 plets of $SU(5)$.

Each anomaly free content (5) and (6), we presented so far, includes one 10-plet of $SU(5)$. This happened because of simple and single anomaly free $SO(10)$ and $E_6$ representations 16 and 27-plets respectively. Another (higher) representations might give more 10-plets. Since those higher states would also involve extra exotic states, we do not consider such possibilities here.

As far as the unitary groups with rank greater than five, we start discussion with $SU(7)$. Lower group $SU(6)$ is subgroup of $E_6$ which was already considered above (detailed comment about this is given at the end of this section). As it will turn out, the $SU(7)$ group can give an interesting anomaly free field content. Consider $SU(7)$’s one particular set of chiral representations $35 + 2 \times 7$, which is anomaly free.\cite{4} Here 35 is three index antisymmetric representation and 7 is an anti-fundamental of $SU(7)$. Their decomposition via the chain $SU(7) \rightarrow SU(6) \times U(1)_7 \rightarrow SU(5) \times U(1)_6 \times U(1)_7$ is

$$35 = 20_3 + 15_{-4} = (10_{-3} + \bar{10}_3)_{3} + (10_2 + 5_{-4})_{-4} ,$$

$$7 = \bar{6}_{-1} + 1_6 = (\bar{5}_{-1} + 1_5)_{-1} + (1_0)_6 ,$$

\cite{7} We omit normalization factor, which is not essential here.

\cite{4} Other $SU(7)$’s anomaly free chiral sets like $21 + 3 \times 7$ and $21 + \bar{35} + 7$ etc., involve either too many $SU(5)$ singlets, or unwanted $SU(5)$ states and thus will not be considered here.
where inside and outside of parenthesis the \( U(1)_6 \) and \( U(1)_7 \) charges respectively are indicated as subscripts. Note that \( U(1)_6 \) and \( U(1)_7 \) are coming from \( SU(6) \) and \( SU(7) \) respectively. Their corresponding generators are
\[
Y_{U(1)_6} = \frac{1}{\sqrt{60}} \text{Diag} (1,1,1,1,1,−5) \quad \text{and} \quad Y_{U(1)_7} = \frac{1}{\sqrt{84}} \text{Diag} (1,1,1,1,1,−6)
\]
respectively. The normalization factors \( \frac{1}{\sqrt{60}} \) and \( \frac{1}{\sqrt{84}} \) are omitted in Eq. (7). Note that set of Eq. (7) includes \( SU(5) \)'s \( \overline{10} \)-plet, which we did not intend to introduce. However, there is one loophole which helps in this situation. Since consideration of \( SU(7) \) symmetry was just the way of finding the anomaly free \( U(1)_F \), we will consider \( SU(5) \times U(1)_F \) gauge symmetry, important is that \( SU(5)^3 \) and anomalies of Eqs. (2)-(4) vanish. So, if the pair of \((\overline{10} + 5)\)-plets is replaced by \((10 + 5)\) then \( SU(5)^3 \) anomaly will not be changed (i.e. will still vanish). With this substitution \( \overline{10} \rightarrow 10, 5 \rightarrow 5 \), without changing the \( U(1) \)-charges, the mixed and cubic anomalies (2)-(4) will remain intact. Therefore, we will consider the following content
\[
(10_{−3} + 10_3)_{−3} + (10_{−2} + 5_{−4})_{−4} + 2 \times [(5_{−1} + 1_5)_{−1} + (1_0)_6]
\]
which involves three pairs (three families!) of \((10 + 5)\)-plets.

Higher rank gauge groups \( SU(N > 7), SO(N > 10), E_7, E_8 \) etc. with corresponding anomaly free representations will give extra (unwanted) non-trivial representations of \( SU(5) \) and we do not consider them. Therefore, we will use the sets (5), (6) and (8) in our further studies for model building.

With Abelian symmetries \( U(1)' \), \( U(1)'' \), \( U(1)_6 \) and \( U(1)_7 \) various linear superpositions can be constructed. Starting with \( U(1)' \) and \( U(1)'' \), which respectively transform the sets given in Eqs. (5) and (6), let us consider the superposition
\[
Q_{\text{sup}} = aQ_{U(1)''} + bQ_{U(1)'}
\]
In order to construct such a superposition, the content of (5) should be extended with extra singlet \( 1' \) and \( 5, 5' \) plets, with \( U(1)' \) charges \( Q_{U(1)'}(1') = 0 \), \( Q_{U(1)'}(5) = 2q \), \( Q_{U(1)'}(5') = −2q \), where \( q \) is some number. \( Q_{\text{sup}} \) can be identified with \( U(1)_F = U(1)_{\text{sup}} \). In order that \( U(1)_F = U(1)_{\text{sup}} \) be anomaly free some constraints on \( a,b \) and \( q \) should be imposed. Simplest possibility, leading to realistic models, is to require cancellation of mixed anomalies \( U(1)' \cdot [U(1)'']^2 \) and \( [U(1)']^2 \cdot U(1)'' \). If these mixed anomalies will vanish, then \( U(1)_{\text{sup}} \) also will be anomaly free (because separately \( U(1)' \) and \( U(1)'' \) are anomaly free). One can easily make sure that with \( q = \pm 1 \) \( U(1)_{\text{sup}} \) is anomaly free for arbitrary values of \( a \) and \( b \). Note that \( q = −1 \) corresponds to \( SO(10) \) normalization, i.e. \( SU(5) \)'s multiplets \( 5 \) and \( 5' \) coming from the \( SO(10) \)'s fundamental 10-plet, should have charges \( −2 \) and \( 2 \) resp. Without loss of generality, we will choose \( q = −1 \). Thus, anomaly free field content is:
\[
10_{a+b} + 5_{a−3b} + 1_{a+5b} + 5_{−2a−2b} + 5'_{−2a+2b} + 1'_{a}
\]
Similarly, from \( U(1)_6 \) and \( U(1)_7 \) charges of the fields given in Eq. (8) we can build superposition
\[
\bar{Q}_{\text{sup}} = \bar{a}Q_{U(1)_6} + \bar{b}Q_{U(1)_7}
\]
Note that \( \bar{Q}_{\text{sup}} \) is automatically anomaly free for arbitrary \( \bar{a} \) and \( \bar{b} \), because the orthogonal generators \( Y_{U(1)_6} \) and \( Y_{U(1)_7} \) originate from single \( SU(7) \). Thus, using (8) we can write the anomaly free set
\[
10_{−3a+3b} + 10_{3a+3b} + 10_{2a−4b} + 5_{−4a−4b} + 2 \times (5_{−a−b} + 1_{5a−b} + 1'_{6b})
\]
where subscripts denote $\bar{Q}_\text{sup}$ charges. These charges could be identified with charges of flavor $U(1)_F$.

Summarizing all possibilities discussed above, we can have the following options for flavor $U(1)_F$ charge assignments:

\begin{align*}
\text{A} & : \quad 10_0 + \bar{5}_0 , \\
\text{B} & : \quad 10_\alpha + 5_{-3\alpha} + 1_{5\alpha}, \quad (\alpha \neq 0) , \\
\text{C} & : \quad 10_{a+b} + \bar{5}_{a-3b} + 1_{a+5b} + 5_{-2a-2b} + \bar{5}'_{-2a+2b} + \bar{1}_a', \quad (a \neq 0, \ a \neq -5b) , \\
\text{D} & : \quad 10_{-3a+3b} + 10_{-3\bar{a}+3b} + 10_{2\bar{a}+4\bar{b}} + 5_{-4\bar{a}-4\bar{b}} + 2 \times (\bar{5}_{-\bar{a}-\bar{b}} + 1_{5\bar{a}-\bar{b}} + 1'_{6\bar{b}}), \quad (\bar{b} \neq 0).
\end{align*}

The conditions in brackets are imposed in order to avoid repetition of identical cases. For example, in case B, with $a = 0$ we recover case A with extra neutral $SU(5)$ singlet. Likewise, in case C, with $a = 0$ or $a = -5b$ we obtain case B augmented with extra vector-like states with opposite $U(1)_F$ charges. Also, condition $\bar{b} \neq 0$ for case D guarantees that we will not deal with case obtained from embedding of $SU(5) \times U(1)_F$ in $SU(6)$ group. Indeed, with $\bar{b} = 0$ together with states $10_{-3\bar{a}} + 10_{3\bar{a}} + 2 \times 1'_0$ (which do not contribute in $SU(5)^2 \cdot U(1)_F$, $(U(1)_F)^3$ and TrQ anomalies) we get set $10_{2\bar{a}} + 5_{-4\bar{a}} + 2 \times (\bar{5}_{-\bar{a}} + 1_{5\bar{a}})$. The latter field content can be obtained via $SU(6)$ embedding as follows. Consider $SU(6)$ field content $15 + 2 \times 6$ which is anomaly free. Decomposition of 15 and 6 under $SU(6) \to SU(5) \times U(1)_6$ is $15 = 10_2 + 5_{-4}$ and $6 = 5_{-1} + 1_5$, where for $U(1)_6$ charges the normalization factor $1/\sqrt{60}$ is neglected. Now making replacement $5_{-4} \to 5_{-4}$ and adding the pair $10_{-3} + 10_{3}$ the field content will remain anomaly free. Adding to these two neutral singlets ($1'_0$ taken two times) we will get the field content of D with $\bar{b} = 0$. Note that discussing embedding of $SU(5) \times U(1)_F$ in unitary groups, we skipped the $SU(6)$ group. The reason was that the case C, obtained from $E_6$ embedding, includes the case of $SU(6)$ embedding. This is not surprising since one of $E_6$’s maximal subgroup is $SU(6) \times SU(2)$ and 27 (of $E_6$) decomposition $E_6 \to SU(6) \times SU(2)$ is $27 = (15, 1) + (\bar{6}, 2)$. Taking in case C $a = 5/4, b = 3/4$ we will get states $10_{2a} + 5_{-4} + 2 \times (5_{-1} + 1_5)$. These are obtained by $SU(6) \to SU(5) \times U(1)_6$ decomposition of $15 + 2 \times 6$. That’s why consideration of unitary groups has been started from $SU(7)$.

Before closing this section, let us mention that for case D, in constructing the $\bar{Q}_\text{sup}$ charges, besides $Q_{U(1)_6}$ and $Q_{U(1)_7}$, one can also use another $U(1)$ - either charge of $U(1)'$ or $U(1)''$, or both together. However, one should make sure that superposition is such that all anomalies are zero. For example, use $U(1)'$ symmetry. Then instead of Eq. (11) we will have $\bar{Q}_\text{sup} = aQ_{U(1)_6} + \bar{b}Q_{U(1)_7} + \bar{c}Q_{U(1)''}$. To do this, we should pick up from set D $10 + 5 + 1 + 5 + 1 + 1$ and (according to last equation in (6)) assign $U(1)''$ charges $1, 1, 1, -2, -2, 4$ respectively to these states. Remaining two 10-plets can have $U(1)''$ charges $p$ and $-p$, while $U(1)'$ charges of remaining two singlets are $k$ and $-k$. Thus, the set with (one simple possible) $\bar{Q}_\text{sup}$ charge assignment will look:

\begin{align*}
10_{-3a+3b+p\bar{c}} + 10_{3a+3b-p\bar{c}} + 10_{2a-4b+\bar{c}} + \bar{5}_{-4a-4b-2\bar{c}} + \bar{5}_{-a-b+c} + \bar{5}_{-a-b-2\bar{c}} + 1_{5a-b+c} + 1'_{6b+k\bar{c}} + 1'_{6b-k\bar{c}} , \quad \text{with} \quad 30\bar{a}(3 + 2p) = \bar{c}(2k^2 + 10p^2 - 27) .
\end{align*}

Relations between $\bar{a}, \bar{c}, k$ and $p$ (imposed for $\bar{c} \neq 0$) given in (17) insures that all anomalies vanish. Clearly, with rational selection of $\bar{a}, k$ and $p$ the value of $\bar{c}$ also will be rational. The set given in Eq. (17) is one simple selection among several options and opens up many possibilities for model building with realistic phenomenology.
3 $SU(5) \times U(1)_F$ Models

For model building with $U(1)_F$ symmetry we list and discuss requirements which should be satisfied in order to obtain phenomenologically viable and economical setups.

(a) In total, we should have three 10-plets of $SU(5)$, four 5-plets and one 5-plet. Out of these multiplets three pairs of $(10 + \bar{5})$ are matter superfields (containing quark and lepton superfields, as given in Eq. (1)). The 5-plet and one remaining $\bar{5}$-plet are scalar superfields which will be denoted by $H$ and $\bar{H}$ respectively.

(b) In order to have top quark Yukawa coupling $\lambda_t \sim 1$, the $U(1)_F$ symmetry should allow coupling $10_3 \bar{10}_3 H$ at renormalizable level. At the same time, all 10-plets should have different $U(1)_F$ charges in order to generate adequately suppressed hierarchies of $\lambda_u/\lambda_t$ and $\lambda_c/\lambda_t$.

Since for $U(1)_F$ charge assignments we have options given in (13)-(16), for building three generation models with $U(1)_F$ flavor symmetry we can consider different combinations of these as signs. For example, one pair of 10, $\bar{5}$-plets can have $U(1)_F$ assignment A (of Eq. (13)), another pair of 10, $\bar{5}$-plets can have assignment B and third pair of 10, $\bar{5}$-plets can come from selection C. This collection can be refereed as ABC model. This model involves three 10-plets, four $\bar{5}$-plets and one 5-plet (satisfying requirement (a)). Other collections, such as AAB, BBB, etc., are also possible. However, selections like ACC, CCC, etc. are not allowed since they would involve extra 5-plet(s) (not satisfying requirement (a)). Note, considering, say, ABB model, for two sets of 10, $\bar{5}$-plets coming with $B$ charge assignments should be taken $\alpha$ and $\alpha' \neq \alpha$ (for satisfying requirement (b)). At the same time, for this selection extra pair of 5, $\bar{5}$-plets should be introduced with opposite $U(1)_F$ charges.

(c) Upon model building, one should make sure that only one $U(1)$ (identified with $U(1)_F$) emerges. For instance, if ABC model is considered, the parameters $\alpha, a, b$ should not be independent. They should be fixed as $\alpha = \frac{m_1}{n_1} \beta$, $a = \frac{m_2}{n_2} \beta$, $b = \frac{m_3}{n_3} \beta$ ($m_i, n_i$ are integers). This would avoid extra global $U(1)$ symmetries.

Summarizing, satisfying all these requirements, we will group models in following five classes:

| ABB | BBB | D |
|-----|-----|---|
| ABC | BBC |   |

(18)

Each of these includes several possibilities. Clarification of varieties of these possibilities is in order.

- Model ABB

In this case we combine sets given by Eqs. (13) and (14), and take: $10_0 + \bar{5}_0$, $10_\alpha + \bar{5}_{-3\alpha} + 1_{5\alpha}$ and $10_{\alpha'} + \bar{5}_{-3\alpha'} + 1_{5\alpha'}$. In addition, we introduce the pair $5_q + \bar{5}_{-q}$. Thus, for this class, the complete field content is:

$$
10_0 + \bar{5}_0 , \quad 10_\alpha + \bar{5}_{-3\alpha} + 1_{5\alpha}
$$

$$
10_{\alpha'} + \bar{5}_{-3\alpha'} + 1_{5\alpha'} , \quad 5_q + \bar{5}_{-q} .
$$

(19)

This selection is not unique. We can exchange 5-plet’s $U(1)_F$ charge with one of the 5-plets’ charge. With this, anomaly cancellation conditions are not changed. Thus, for $U(1)_F$ charge of the 5-plet, identified with Higgs superfield $H(5)$, we have three (qualitatively different) options $Q_H = 0, -3\alpha$.

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The scalar superfield $\Sigma(24)$ (neutral under $U(1)_F$), needed for the symmetry breaking $SU(5) \to G_{321}$, is also assumed.
or \( q \). In counting these options, we took into account that the charge selection \( Q_H = -3\alpha' \) does not differ from selection \( Q_H = -3\alpha \) (former is obtained from the latter by substitution \( \alpha \to \alpha' \)). Also, the case with \( Q_H = -q \) is obtained from case \( Q_H = q \) by substitution \( q \to -q \). From the (remaining) four 5-plets one should be identified with the Higgs superfield \( H(5) \). For each given \( Q_H \), one should count how many qualitatively different charge assignments is possible for \( \tilde{H} \). One can make sure that for the pair \((Q_H, \tilde{Q}_H)\) eight different possibilities are allowed:

\[
(Q_H, \tilde{Q}_H)^{(i)} = \{(q, -q), (q, 0), (q, -3\alpha), (0, q), (0, -3\alpha), (-3\alpha, q), (-3\alpha, 0), (-3\alpha, -3\alpha')\} , \quad (20)
\]

where \( i = 1, 2, \cdots, 8 \) numerates (indicates) the options for the charge assignment for \( H \) and \( \tilde{H} \). Thus, the content \( \text{ABB} \) of Eq. (19) forms class with these different charge assignments. To make clear which particular \( U(1)_F \) charge assignment for \( H, \tilde{H} \) is considered, it is instructive to use notation \( \text{ABB}^{(i)} \). For instance, \( \text{ABB}^{(i=3)} \) would mean that we are taking \((Q_H, \tilde{Q}_H)^{(i=3)} = (q, -3\alpha) \) (see Eq. (20)).

- **Model ABC**
  
  In this case, we collect together sets of Eqs. (13), (14) and (15). Thus, the field content is:

\[
10_0 + \bar{5}_0 , \quad 10_\alpha + \bar{5}_{-3\alpha} + 1_{5\alpha} , \quad 10_{a+b} + \bar{5}_{a-3b} + 1_{a+5b} + 5_{-2a-2b} + \bar{5}'_{-2a+2b} + 1'_{4a} .
\]

(21)

Since (21) includes three 10-plets, four 5’s and one 5-plet, we do not need to introduce any additional vector-like states. Also in this case, we can exchange \( U(1)_F \) charge of 5-plet with one of the 5’s charge. It turns out that here we will have the following 20 possibilities for \((Q_H, \tilde{Q}_H)\) pair selection:

\[
(Q_H, \tilde{Q}_H)^{(i)} = \{-2a - 2b, 2b - 2a), (-2a - 2b, 0), (-2a - 2b, -3\alpha), (-2a - 2b, a - 3b), (0, -3\alpha), (0, a - 3b), (0, -2a - 2b), (0, 2b - 2a), (-3\alpha, 0), (-3\alpha, a - 3b), (-3\alpha, -2a - 2b), (-3\alpha, 2b - 2a), (a - 3b, 0), (a - 3b, -3\alpha), (a - 3b, -2a - 2b), (a - 3b, 2b - 2a), (2b - 2a, -3\alpha), (2b - 2a, a - 3b), (2b - 2a, -2a - 2b)\} . \quad (22)
\]

Thus, this \( \text{ABC}^{(i)} \) \((i = 1, 2, \cdots, 20)\) class unifies twenty possible charge assignments for the pair \((H, \tilde{H})\).

- **Model BBB**
  
  For constructing this case, we pick up the set of Eq. (14) three times (with corresponding charge assignments) and add the pair \( 5_q + 5_{-q} \). Thus, the complete content is:

\[
10_\alpha + \bar{5}_{-3\alpha} + 1_{5\alpha} , \quad 10_{\alpha'} + \bar{5}_{-3\alpha'} + 1_{5\alpha'} , \quad 10_{\alpha''} + \bar{5}_{-3\alpha''} + 1_{5\alpha''} , \quad 5_q + 5_{-q} .
\]

(23)

Here for \((Q_H, \tilde{Q}_H)\) pair selection we have four qualitatively different cases:

\[
(Q_H, \tilde{Q}_H)^{(i)} = \{(q, -q), (q, -3\alpha), (-3\alpha, -3\alpha'), (-3\alpha, q)\} . \quad (24)
\]

Therefore, this \( \text{BBB}^{(i)} \) \((i = 1, \cdots, 4)\) class unifies four options for the pair \((Q_H, \tilde{Q}_H)\).

- **Model BBC**
In general, scalar components of $X$ set of flavon pair $X$ breaking. The breaking of $U_\lambda$ malizable Yukawa coupling. Yukawa couplings discussion of possibility for $U$ in Appendix A. We introduce the notations obtained by $U$ in order to proceed with model building, first we give all acceptable up-type Yukawa textures. These textures will be referred as $U$ and $\epsilon$, respectively.

\begin{align}
10_a + 5_{-3a} + 1_{5a} , & \quad 10_a' + 5_{-3a'} + 1_{5a'} , \\
10_{a+b} + 5_{a-b} + 1_{a+5b} + 5_{-2a-2b} + 5'_{-2a+2b} + 1'_{4a} .
\end{align}

The list of possible $(Q_H, \bar{Q}_{\bar{H}})$ pairs is:

\begin{align}
(Q_H, \bar{Q}_{\bar{H}})^{(i)} = \{& (-2a - 2b, -3\alpha), (-2a - 2b, a - 3b), (-2a - 2b, 2b - 2a), (2b - 2a, -3\alpha), \\
& (2b - 2a, a - 3b), (2b - 2a, -2a - 2b), (-3\alpha, -3\alpha'), (-3\alpha, a - 3b), (-3\alpha, -2a - 2b), \\
& (-3\alpha, 2b - 2a), (a - 3b, -3\alpha), (a - 3b, -2a - 2b), (a - 3b, 2b - 2a)\},
\end{align}

giving thirteen possibilities unified in this $\mathbf{BBC}^{(i)} (i = 1, 2, \cdots, 13)$ class.

**Model D**

The field content of this model is given in (16). It includes three 10 and three 5-plets. So, we do not need to combine this content with other ones, but must add to it the pair $5_5 + 5_{-q}$. If the $U(1)_F$ charge assignments are just those given in (16), then for the pairs $(Q_H, \bar{Q}_{\bar{H}})$ we will have eight options. However, as already discussed, it is possible to build charge assignments utilizing additional $U(1)$-charges, as was done in the example given in Eq. (17). The latter case offers 13 distinct options for the pairs $(Q_H, \bar{Q}_{\bar{H}})$. These, open up varieties for the model building. One example from this $\mathbf{D}$-class of models is presented in Sect. 3.4.

### 3.1 Up-type Quark Yukawa Matrices

In order to proceed with model building, first we give all acceptable up-type Yukawa textures obtained by $U(1)_F$ symmetry. In our approach, among up-type quarks only top quark has renormalizable Yukawa coupling. Yukawa couplings $\lambda_u$ and $\lambda_c$ emerge after $U(1)_F$ flavor symmetry breaking. The breaking of $U(1)_F$ should be achieved by flavon superfields. Here we consider simple set of flavon pair $X + \bar{X}$ with $U(1)_F$ charges

\begin{equation}
Q(X) = -\beta , \quad Q(\bar{X}) = \beta .
\end{equation}

In general, scalar components of $X$ and $\bar{X}$ have different VEVs $\langle X \rangle$ and $\langle \bar{X} \rangle$ respectively. Detailed discussion of possibility for $U(1)_F$ symmetry breaking, giving fixed VEVs for $X$ and $\bar{X}$, is presented in Appendix A. We introduce the notations

\begin{equation}
\frac{|X|}{M_{Pl}} = \epsilon , \quad \frac{|\bar{X}|}{M_{Pl}} = \bar{\epsilon} ,
\end{equation}

where $M_{Pl} \approx 2.4 \cdot 10^{18}$ GeV is reduced Planck scale, which will be treated as natural cut off for all higher-dimensional non-renormalizable operators. Thus, the hierarchies between Yukawa couplings and CKM mixing angles will be expressed by powers of small parameters $\epsilon, \bar{\epsilon} \ll 1$.

Due to the composition of the 10-plet given in Eq. (1) and taking into account that $H(5) \supset h_u$, the up-type quark masses emerge through the Yukawa couplings of the form $10 \cdot 10 \cdot H$, where family and $SU(5)$ indices are suppressed. As it turns out, within this setup, three acceptable Yukawa textures emerge for up-type quarks. These textures will be referred as $\mathbf{U1}, \mathbf{U2}$ and $\mathbf{U3}$. 

8
(i) Up Quark Yukawa Texture U1

The $U(1)_F$ charges of three 10-plets and the Higgs superfield $H$ are:

$$Q(10_1) = n\beta (n\beta + \beta) \ , \ Q(10_2) = n\beta - \beta \ , \ Q(10_3) = n\beta - 3\beta \ , \ Q(H) = 6\beta - 2n\beta \ . \ (29)$$

This selection provides the following Yukawa texture

$$U_1 : \begin{pmatrix} 10_1 & 10_2 & 10_3 \\ 10_1 & e^6(e^8) & e^5(e^6) & e^3(e^4) \\ 10_2 & e^5(e^6) & e^4 & e^2 \\ 10_3 & e^3(e^4) & e^2 & 1 \end{pmatrix} H \ , \ (30)$$

where dimensionless couplings (whose magnitudes are assumed to be $\sim 1/3 - 3$) are not displayed. With $\epsilon = 1/10 - 1/5$, the matrix (30) gives right hierarchies between up-type quark Yukawas.

(ii) Up Quark Yukawa Texture U2

In this case we use the following assignment

$$Q(10_1) = n\beta + 3\beta \ , \ Q(10_2) = n\beta \ , \ Q(10_3) = n\beta + \beta \ , \ Q(H) = -2n\beta - 2\beta \ , \ (31)$$

which gives the texture:

$$U_2 : \begin{pmatrix} 10_1 & 10_2 & 10_3 \\ 10_1 & \epsilon & \epsilon^2 \\ 10_2 & \epsilon & \epsilon^2 & \epsilon \\ 10_3 & \epsilon^2 & \epsilon & 1 \end{pmatrix} H \ . \ (32)$$

With selection $\tilde{\epsilon} = 1/10 - 1/20$, $\epsilon \sim (1/5 - 1/10) \cdot \tilde{\epsilon}^2$, the needed hierarchies for the ratios $\lambda_u/\lambda_c$, $\lambda_c/\lambda_t$ are generated.

(iii) Up Quark Yukawa Texture U3

Finally, with $U(1)_F$ charge selections

$$Q(10_1) = n\beta - \beta \ , \ Q(10_2) = n\beta \ , \ Q(10_3) = n\beta + \beta \ , \ Q(H) = -2n\beta - 2\beta \ , \ (33)$$

the up-type quark Yukawa couplings will be

$$U_3 : \begin{pmatrix} 10_1 & 10_2 & 10_3 \\ 10_1 & \tilde{\epsilon}^4 & \tilde{\epsilon}^3 & \tilde{\epsilon}^2 \\ 10_2 & \tilde{\epsilon}^3 & \tilde{\epsilon}^2 & \tilde{\epsilon} \\ 10_3 & \tilde{\epsilon}^2 & \tilde{\epsilon} & 1 \end{pmatrix} H \ , \ (34)$$

which for $\tilde{\epsilon} \sim 1/20 - 1/10$ gives successful explanation of hierarchies $\lambda_u/\lambda_c \sim \tilde{\epsilon}^2$ and $\lambda_c/\lambda_t \sim \tilde{\epsilon}^2$.

This classification of up-type Yukawa textures helps to build models emerging from classes of Eq. (18) (for each class, see discussion after Eq. (18)). As one can see, there are many possibilities to be considered in order to see which one gives phenomenologically viable model. Detailed investigation and complete list of acceptable scenarios will be presented in a longer paper [10]. Below we present three models with successful explanation of hierarchies between charged fermion masses and mixings.
Table 1: $U(1)_F$ charge assignment for $\text{ABC}^{(i=4)}$-$\text{U1}_{(n=1)}$ model.



| $Q_{U(1)_F}$ | $10_1$ | $10_2$ | $10_3$ | $\bar{5}_1$ | $\bar{5}_2$ | $\bar{5}_3$ | $H(5)$ | $\bar{H}(5)$ | $l_1$ | $l_2$ | $l_3$ |
|--------------|--------|--------|--------|-----------|-----------|-----------|-------|-----------|-------|-------|-------|
| $\beta$      | 0      | $-2\beta$ | 0      | $-3\beta$ | $-3\beta$ | 4$\beta$ | 5$\beta$ | $-\beta$ | 5$\beta$ | $-9\beta$ |

### 3.2 $\text{ABC}^{(i=4)}$-$\text{U1}_{(n=1)}$ Model

In this model, the content of Eq. (21) is considered and charges are matched in such a way as to obtain with up-type Yukawa texture $\text{U1}$ of Eq. (30). Here, selection $n = 1$ is made. Thus, according to Eq. (29), the charge of $H(5)$-plet is $Q_H = 4\beta$, while charges of 10-plets are $Q(10_i) = \{\beta, 0, -2\beta\}$. From the set (21) we will identify 10, 10 and 10+ with 1st, 2nd and 3rd families respectively, and $5_{-2a-2b}$ with $H$. Making the charge matching $\alpha = \beta$, $a + b = -2\beta$ and selection $a = -\beta/4$, we will have

$$\{\alpha, a, b\} = \{\beta, -\beta/4, -7\beta/4\}.$$  

Furthermore, since we are dealing with $\text{ABC}^{(i=4)}$ model, using Eqs. (22), (35) we have $(Q_H, Q_{\bar{H}})^{(i=4)} = (-2a - 2b, a - 3b) = (4\beta, 5\beta)$. The charges of remaining 5-plets: $5_0$, $\bar{5}_{-3\alpha}$ and $5_{-2a+2b}$, which we identify with 1st, 2nd and 3rd families of matter 5-plets respectively, will be $Q(5_i) = \{0, -3\beta, -3\beta\}$. The $U(1)_F$ charge assignment of all states of content (21) is summarized in Table 1. With this assignment, the Yukawa coupling matrices are determined as follows:

$$
\begin{pmatrix}
10_1 & 10_2 & 10_3 \\
10_2 & 10_3 & \\
10_3 & & \\
\end{pmatrix}
= 
\begin{pmatrix}
\epsilon^6 & \epsilon^5 & \epsilon^3 \\
\epsilon^5 & \epsilon^4 & \epsilon^2 \\
\epsilon^3 & \epsilon^2 & 1 \\
\end{pmatrix}
H

\begin{pmatrix}
\bar{5}_1 & \bar{5}_2 & \bar{5}_3 \\
\bar{5}_2 & \bar{5}_3 & \\
\bar{5}_3 & & \\
\end{pmatrix}
= 
\begin{pmatrix}
\epsilon^6 & \epsilon^3 & \epsilon^3 \\
\epsilon^5 & \epsilon^2 & \epsilon^2 \\
\epsilon^3 & 1 & 1 \\
\end{pmatrix}
\bar{H}.

(36)

Taking into account Eq. (1) and $H \supset h_u$, $\bar{H} \supset h_d$, Eq. (36) yield:

$$\lambda_u : \lambda_c : \lambda_t \sim \epsilon^6 : \epsilon^4 : 1, \quad \lambda_t \sim 1,$$

$$\lambda_e : \lambda_\mu : \lambda_\tau \sim \epsilon^6 : \epsilon^2 : 1, \quad \lambda_\mu : \lambda_\nu \sim \epsilon^6 : \epsilon^2 : 1.$$  

(37)

Assuming that in (36) there are dimensionless Yukawa couplings with natural values - in a range $\sim 1/3 - 3$, with selection $\epsilon \approx 0.2$, the hierarchies in (37) can fit well with the experimental data. Notice that $\lambda_{b,\tau} \sim 1$, which means that in this scenario $\tan\beta \approx 55 - 60$. As far as the CKM mixing angles are concerned, from (36) one can obtain:

$$|V_{us}| \sim \epsilon, \quad |V_{cb}| \sim \epsilon^2, \quad |V_{ub}| \sim \epsilon^3.$$  

(38)

These are also of right magnitudes (with $\epsilon \approx 0.2$). Because of the charge equality $Q(\bar{5}_2) = Q(\bar{5}_3)$, corresponding entries in 2nd and 3rd columns of the second matrix of Eq. (36) have comparable sizes. Taking into account that $\bar{5} \supset l$, this leads to the naturally large mixing between $l_2$ and $l_3$ lepton flavors:

$$\tan\theta_{\mu\tau} \sim 1.$$  

(39)
providing good explanation for large $\nu_\mu - \nu_\tau$ neutrino oscillations. To demonstrate this, we also discuss neutrino sector in some extent. Let us work in a basis where the matrix responsible for the charged lepton masses (2nd matrix in Eq. (36)) is diagonal. Thus, the mixing matrix emerging from the neutrino sector will coincide with lepton mixing matrix. We will apply the singlet state $1_1$ (see Tab. 1) as a right-handed neutrino. The relevant couplings are $\lambda_3(5_3 + \bar{t}5_2)1_1H + \bar{M}\bar{t}^21_{11}$, with $\lambda_3$, $t$ being dimensionless couplings and $\bar{M}$ some scale. Moreover, we also include higher order operators $\lambda_1\epsilon^5\bar{5}_1\bar{5}_2HH/M'$ and $\lambda_2\epsilon^2\bar{5}_2\bar{5}_2HH/M''$. Integration out of the state $1_1$, together with latter operators, give the neutrino mass matrix:

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & t^2 & t \\ 0 & t & 1 \end{pmatrix} m + \begin{pmatrix} 0 & 1 & 0 \\ 1 & \delta & 0 \\ 0 & 0 & 0 \end{pmatrix} \overline{m}, \quad (40)$$

with $m = \lambda_3^2\epsilon^2/\bar{M}^2$, $\overline{m} = \lambda_3\epsilon^2\bar{M}/\bar{m}$ and $\delta = \lambda_3\bar{M}/\lambda_1\bar{M}^2$. The first matrix at r.h.s. of (40) (emerged by integrating out the $1_1$ state) is mostly responsible for the mass $m_{\nu_3}$ and leptonic $\theta_{23}$ mixing. Indeed, in the limit $\overline{m} \to 0$, we get $\tan\theta_{23} = |t|$. This, for $|t| \sim 1$ (natural value), gives $\theta_{23} \approx 45^\circ$. Inclusion of the $\overline{m}$ terms are responsible for mixing angles $\theta_{12}, \theta_{13}$ and masses $m_{\nu_{1,2}}$. With a selection $m = 0.029$ eV, $\overline{m} = 0.0116$ eV, $t = 0.78$, $\delta = 0.8$ we obtain $\Delta m^2_{\text{atm}} = m^2_{\nu_3} - m^2_{\nu_2} \approx 2.6 \cdot 10^{-3}$ eV$^2$, $\Delta m^2_{\text{sol}} = m^2_{\nu_2} - m^2_{\nu_1} \approx 7.2 \cdot 10^{-5}$ eV$^2$, $\theta_{12} = 34^\circ$, $\theta_{23} = 45.3^\circ$, $\theta_{13} = 9^\circ$. These agree well with a recent data [11].

In this considered case neutrinos are hierarchical in mass: $m_{\nu_1} = (0.00688, 0.01093, 0.052)$ eV. The values of these scales remain unexplained within this scenario, we have showed that the model can be compatible with neutrino sector. More detailed study of this and related issues will be presented in [10].

As in minimal SU(5) GUT, some care is needed to cure the problem of $M_D - M_E$ mass degeneracy. For fixing this problem one can use either an extension by scalar 45-supersmultiplets [12], or include powers of adjoint 24-plet in the Yukawa couplings [13], or utilize extra heavy matter supermultiplets [14]. Study of this problem is beyond the scope of this Letter.

Before closing this subsection, let us mention that within this scenario the splitting between masses of doublets and triplets (coming from $H, \bar{H}$) should be obtained via fine tuning (as in minimal SUSY SU(5)) of the model parameters. However, one should make sure that this is possible to achieve. Due to the $U(1)_F$ symmetry, renormalizable superpotential couplings $(M_H + \lambda_H \Sigma)H\bar{H}$ are forbidden. However, in this scenario we have extra SU(5) singlet states charged under $U(1)_F$ (see Table 1). For instance, picking up the states $1_2$ and $1_3$ and announcing them as scalar superfields (with positive matter $R$-parity), the relevant lowest superpotential couplings (including them) will be $M^2_{12}\epsilon^51_2 + \bar{M}^2\epsilon^91_3 + M_{12}\epsilon^41_21_3$, where dimensionless couplings have been neglected (assuming that they are of the order of unity). One can check that vanishing of the $F$-terms $F_{12} = F_{13} = 0$ lead to the induced VEVs $\langle 1_2 \rangle \sim M_{12}\epsilon^5$ and $\langle 1_3 \rangle \sim M_{12}\epsilon^4/\epsilon^4$. With selection $\epsilon \sim 0.25$ we will have $\langle 1_2 \rangle \sim 10^{-3}M_{13}$, $\langle 1_3 \rangle \sim 0.1M_{13}$ without affecting anything in the discussion above. However, the couplings $1_3(\lambda_H + \lambda'_H \Sigma_{M_{13}})H\bar{H}$ with $\langle \Sigma \rangle = V \cdot \text{Diag}(2, 2, 2, -3, -3)$ and tuning condition $\lambda_H = 3\lambda_H V/M_{13}$ (satisfied with $\lambda_H \sim 0.1$, $\lambda'_H \sim 2 - 8$, rendering theory self consistent) lead to the massless doublets ($M_{H_2} = 0$) and colored triplets with masses $M_{H_3} = 0.3\lambda_H \langle 1_3 \rangle \sim \text{few} \cdot M_{\text{GUT}}$. 

11
and with up-type Yukawa texture $U_3$.

Within the $BBB$ model with content (23), one successful scenario is obtained with $i = 2$ (in Eq. (24)) and with up-type Yukawa texture $U_3$ with $n = -2/5$ (see Eqs. (33), (34)). For 10-plets charges we will make the following matching $Q(10_i) = \{n\beta - \beta, n\beta, n\beta + \beta\} = \{\alpha''', \alpha', \alpha\}$. With this, using Eqs. (24), (33) we will have $(Q_H, Q_R)^{(i=2)} = (q, -3\alpha) = (-2n\beta - 2\beta, -3n\beta - 3\beta)$. Remaining 5-plets, $5_{3\alpha''}$, $5_{3\alpha'}$ and $5_{-q}$ will be identified as 1st, 2nd and 3rd families respectively of the matter 5 states. Therefore, $Q(5_i) = \{3\beta - 3n\beta, -3n\beta, 2\beta + 2n\beta\}$. With selection $n = -2/5$, the $U(1)_F$ charges of all states from the content (23) are given in Table 2. With these assignments, couplings responsible for up-type quark Yukawas are given in Eq. (34), while couplings generating charged lepton and down quark masses are:

$$
\begin{pmatrix}
\bar{5}_1 \\
\bar{5}_2 \\
\bar{5}_3
\end{pmatrix}
\begin{pmatrix}
\epsilon \\
\bar{\epsilon} \\
\epsilon
\end{pmatrix}
\begin{pmatrix}
\bar{H}
\end{pmatrix}
$$

These (with $\epsilon < \bar{\epsilon}$) give

$$
\lambda_{\mu} : \lambda_{\alpha} : \lambda_{\tau} \sim \epsilon : \bar{\epsilon}: 1 ,
\lambda_{\mu} : \lambda_{\alpha} : \lambda_{\tau} \sim \epsilon : \bar{\epsilon}: 1 .
$$

Taking $\bar{\epsilon} \sim 1/20 - 1/10$ and $\epsilon \sim 3 \cdot 10^{-4}$, the pattern (42) describe well hierarchies between charged fermion Yukawa couplings. Also, the CKM matrix elements are properly suppressed: $|V_{us}| \sim \bar{\epsilon}, |V_{ub}| \sim \epsilon^2, |V_{cb}| \sim \bar{\epsilon}$. Because of the large mixing between $\bar{5}_2$ and $\bar{5}_3$ states, also in this case for leptonic mixing we expect tan $\theta_{\mu\tau} \sim 1$, providing large $\nu_\mu - \nu_\tau$ oscillations. Demonstration of this can be done in a same way as for the model presented in Sect. 3.2.

The doublet-triplet splitting within this scenario can be achieved in the same manner as was discussed at the end of the Sect. 3.2 for $ABC^{(i=4)}-U1_{(n=1)}$ model. Without going in this discussion, let us proceed to consider another scenario.

### 3.3 $BBB^{(i=2)}-U3_{(n=-2/5)}$ Model

Within the $BBB$ model with content (23), one successful scenario is obtained with $i = 2$ (in Eq. (24)) and with up-type Yukawa texture $U_3$ with $n = -2/5$ (see Eqs. (33), (34)). For 10-plets charges we will make the following matching $Q(10_i) = \{n\beta - \beta, n\beta, n\beta + \beta\} = \{\alpha''', \alpha', \alpha\}$. With this, using Eqs. (24), (33) we will have $(Q_H, Q_R)^{(i=2)} = (q, -3\alpha) = (-2n\beta - 2\beta, -3n\beta - 3\beta)$. Remaining 5-plets, $5_{3\alpha''}$, $5_{3\alpha'}$ and $5_{-q}$ will be identified as 1st, 2nd and 3rd families respectively of the matter 5 states. Therefore, $Q(5_i) = \{3\beta - 3n\beta, -3n\beta, 2\beta + 2n\beta\}$. With selection $n = -2/5$, the $U(1)_F$ charges of all states from the content (23) are given in Table 2. With these assignments, couplings responsible for up-type quark Yukawas are given in Eq. (34), while couplings generating charged lepton and down quark masses are:

$$
\begin{pmatrix}
\bar{5}_1 \\
\bar{5}_2 \\
\bar{5}_3
\end{pmatrix}
\begin{pmatrix}
\epsilon \\
\bar{\epsilon} \\
\epsilon
\end{pmatrix}
\begin{pmatrix}
\bar{H}
\end{pmatrix}
$$

These (with $\epsilon < \bar{\epsilon}$) give

$$
\lambda_{\mu} : \lambda_{\alpha} : \lambda_{\tau} \sim \epsilon : \bar{\epsilon}: 1 ,
\lambda_{\mu} : \lambda_{\alpha} : \lambda_{\tau} \sim \epsilon : \bar{\epsilon}: 1 .
$$

Taking $\bar{\epsilon} \sim 1/20 - 1/10$ and $\epsilon \sim 3 \cdot 10^{-4}$, the pattern (42) describe well hierarchies between charged fermion Yukawa couplings. Also, the CKM matrix elements are properly suppressed: $|V_{us}| \sim \bar{\epsilon}, |V_{ub}| \sim \epsilon^2, |V_{cb}| \sim \bar{\epsilon}$. Because of the large mixing between $\bar{5}_2$ and $\bar{5}_3$ states, also in this case for leptonic mixing we expect tan $\theta_{\mu\tau} \sim 1$, providing large $\nu_\mu - \nu_\tau$ oscillations. Demonstration of this can be done in a same way as for the model presented in Sect. 3.2.

The doublet-triplet splitting within this scenario can be achieved in the same manner as was discussed at the end of the Sect. 3.2 for $ABC^{(i=4)}-U1_{(n=1)}$ model. Without going in this discussion, let us proceed to consider another scenario.

### 3.4 D-U3_{(n=1)} Model: Content of Eq. (17)

The field content of this model is given in Eq. (17) augmented with Higgs superfields $H(5)$ and $\bar{H}(\bar{5})$ of $U(1)_F$ charges $q$ and $-q$ respectively. We will match charges of the 10-plets with assignments of $U_3$ texture (see Eq. (33)) as follows $Q(10_i) = \{n\beta - \beta, n\beta, n\beta + \beta\} = \{2\alpha - 4\beta + \bar{\alpha}, 3\alpha + 3\beta - p\beta, -3\alpha + 3\beta + p\beta\}$. Therefore, $q = -2n\beta - 2\beta$. A phenomenologically viable model is obtained with the selection $p = k = -8/3$. This, with the matching given above and condition in Eq. (17), give $(\bar{a}, \bar{b}, \bar{c}, n) = (2\beta, \frac{1}{2}\beta, -3\beta, 1)$. Furthermore, we make the identification of flavors of 5-plets
Table 3: $U(1)_F$ charge assignment for $D$-$U3_{(n=1)}$ model with content of Eq. (17). For parameters the following selection is made $(\bar{a}, \bar{b}, \bar{c}) = (\frac{5}{2} \beta, \frac{1}{2} \beta, -3 \beta)$, $p = k = -8/3$.

| $Q_{U(1)_F}$ | 10₁ | 10₂ | 10₃ | 5₁ | 5₂ | 5₃ | $H(5)$ | $\tilde{H}(5)$ | 1₁ | 1₂ | 1₃ | 1₄ |
|--------------|-----|-----|-----|-----|-----|-----|--------|----------|-----|-----|-----|-----|
| 10₁          | 0   | 10₁ | 10₂ | 10₃ | 5₁  | 5₂  | 5₃  | $H(5)$ | $\tilde{H}(5)$ | 1₁ | 1₂ | 1₃ | 1₄ |

as: $(5₁, 5₂, 5₃) = (5_{-\bar{a}-\bar{b}-2\bar{c}}, 5_{-4\bar{a}-4\bar{b}-2\bar{c}}, 5_{-\bar{a}-\bar{b}+\bar{c}})$. With these selections and parameters determined above, all $U(1)_F$ charges get fixed (in the unit of $\beta$). In Table 3 we summarize the charges of all states. With these assignments, the Yukawa couplings are:

$$
\begin{pmatrix}
10₁ & 10₂ & 10₃ \\
10₂ & 10₁ & 10₃ \\
10₃ & 10₁ & 10₂
\end{pmatrix}
H,
\begin{pmatrix}
\bar{5}_₁ & \bar{5}_₂ & \bar{5}_₃ \\
\bar{5}_₂ & \bar{5}_₁ & \bar{5}_₃ \\
\bar{5}_₃ & \bar{5}_₁ & \bar{5}_₂
\end{pmatrix}
\hat{H}.
$$

These textures lead to:

$$
\lambda_u : \lambda_c : \lambda_t \sim \bar{\epsilon}^4 : \bar{\epsilon}^2 : 1,
\lambda_t \sim 1,
\lambda_c : \lambda_{\mu} : \lambda_{\tau} \sim \epsilon^7 : \epsilon : 1,
\lambda_{\mu} : \lambda_u : \lambda_{b} \sim \epsilon^7 : \epsilon : 1.
$$

With $\bar{\epsilon} \sim 1/20 - 1/10$ and $\epsilon \approx 0.3$, the ratios in Eq. (44) describe well observed hierarchies between charged fermion masses. Also, the CKM mixing angles have adequately suppressed values: $|V_{us}| \sim \bar{\epsilon}, |V_{ub}| \sim \bar{\epsilon}^3, |V_{tb}| \sim \bar{\epsilon}$, while for the leptonic mixing angle $\theta_{\mu\tau}$ one expects $\tan \theta_{\mu\tau} \sim 1$ (as was demonstrated for the model presented in Sect. 3.2).

Since in this scenario the superfields $H$ and $\hat{H}$ have opposite $U(1)_F$ charges, the doublet-triplet splitting can be obtained in the same way (by fine tuning) as within minimal SUSY $SU(5)$. Thus, no additional effort is needed, unlike the scenarios considered in Sections 3.2 and 3.3.

Finally, let us note that by proper shift of $U(1)_F$ charges of the states of Table 3, one can obtain the charge assignments of model $BBB^{(n=2)}$-$U3_{(n=2/5)}$ given in Tab. 2. However, the latter’s assignment leads to different phenomenology (such as the different couplings required for the doublet-triplet splitting etc.). That’s why, as a different model, this scenario has been presented separately.

Since within considered scenarios matter superfields ($f_i$) have family dependent $U(1)_F$ charges $Q_{f_i}$, there is potentially new source for sfermion mass non-universality. In particular, as given at the end of Appendix, after SUSY breaking $D_{U(1)_F}$-term becomes $2(m_X^2 - m_{\tilde{X}}^2)/g^2$, where $g$ is $U(1)_F$’s coupling constant and $m_X^2$ and $m_{\tilde{X}}^2$ are soft mass$^2$s of the scalar components of the flavon superfields $X$ and $\tilde{X}$ respectively. Non-zero $D_{U(1)_F}$-term give non-universal contribution to the sfermion masses of the form $\Delta m_{\tilde{F}}^2 = Q_{f_i}(m_X^2 - m_{\tilde{X}}^2)/2$ raising new source for FCNC. However, within minimal $N = 1$ SUGRA [15], due to $m_{\tilde{X}}^2 = m_X^2$ universality, this contribution vanish and we have no additional source for flavor violation. Note that the relation $m_{\tilde{X}}^2 = m_X^2$ is quite stable against radiative

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6Since with this assignment $1₂$’s $U(1)_F$ charge is zero and it does not contribute to anomalies, there is no need for introducing state $1₂$. However, presence of four singlets (including $1₂$) is required with more general assignment of Eq. (17).
corrections. Only couplings which might affect this relation, via loop corrections, are couplings of $X$ and $\bar{X}$ states with matter. However, this kind of couplings, appearing at high-dimensional operator level, are strongly suppressed. This insures stability of the relation $m_X^2 = m_{\bar{X}}^2$. Note also that below the $U(1)_F$ symmetry breaking scale the $D_{U(1)_F}$-term is not renormalized. Therefore, we conclude that in order to avoid new contributions to the FCNC (which is common problem within generic SUGRA) one should work within framework (such as minimal SUGRA) giving universality of soft masses.

4 Conclusions

In this Letter we have presented new examples of non-anomalous flavor $U(1)_F$ symmetries within SUSY $SU(5)$ GUT. Our way of finding of such $U(1)_F$s was to embed the $SU(5) \times U(1)_F$ in non-Abelian group with anomaly free content. Our selection was based on the requirement that non-trivial $SU(5)$ states should be just those of minimal SUSY $SU(5)$, while the number of additional singlet states should not be large. The latter, within concrete scenario, can be exploited for model building with realistic phenomenology. For demonstrative purposes we have presented three models which nicely explain hierarchies between charged fermion masses and mixings. We have not addressed the problem of wrong asymptotic mass relations $M_D = M_{\bar{E}}$, common also for minimal $SU(5)$ GUT. Solution of this problem can be achieved either by inclusion of scalar 45 supermultiplets [12], or appropriate powers of the Higgs supermultiplet of 24 (adjoint) in the Yukawa interactions [13], or specific extension of the matter sector [14] can be considered. Within the models, we have found, many varieties of possibilities emerge which require detailed investigation. Complete study of these, together with neutrino sector (some of the singlets, involved in the considered models, can serve as right-handed neutrinos) and other phenomenological issues will be presented in forthcoming publication [10].

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A Breaking of $U(1)_F$

In this appendix we discuss the breaking of $U(1)_F$ gauge symmetry and show that desired VEVs for the flavon fields can be generated. As was mentioned in the text of the paper, the minimal setup of the charged flavon superfields, which we consider is $X$ and $\bar{X}$ with $U(1)_F$ charges given in Eq. (27). Since we are dealing with Abelian flavor symmetry, in general the Fayet-Iliopoulos (FI) term is allowed and we will include it in our consideration. It has the form $\xi \int d^4 \theta V_{U(1)_F}$, where $\xi$ is parameter with dimension of mass square. This FI term together with standard $D$-term Lagrangian
couplings, for $V_{U(1)_F}$’s auxiliary component give:

$$D_{U(1)_F} = \xi - \beta |X|^2 + \beta |\bar{X}|^2 .$$  \hspace{1cm} (A.1)

Moreover, in order to fix all VEVs we need to have some superpotential couplings. For this purpose we introduce the superfield $S$ which is neutral $(Q(S) = 0)$ under $U(1)_F$. The most general renormalizable superpotential involving $X$, $\bar{X}$ and $S$ will have the form

$$W = \lambda S(X\bar{X} - \mu^2) + \frac{1}{2} m_S S^2 + \frac{1}{3} \sigma S^3 ,$$  \hspace{1cm} (A.2)

where $\mu$ and $m_S$ are some mass parameters, while $\lambda$ and $\sigma$ are dimensionless couplings. From (A.2), for $F$-components we derive

$$-F^*_S = \lambda(X\bar{X} - \mu^2) + m_S S + \sigma S^2 , \quad F^*_X = -\lambda S \bar{X} , \quad F^*_\bar{X} = -\lambda S X .$$  \hspace{1cm} (A.3)

In the unbroken SUSY limit $D$ and $F$-terms should satisfy $F_S = F_X = F_{\bar{X}} = D_{U(1)_F} = 0$, which using (A.1) and (A.3) gives

$$|X|^2 - |\bar{X}|^2 = \xi/\beta , \quad X\bar{X} = \mu^2 , \quad S = 0 .$$  \hspace{1cm} (A.4)

These give non-zero VEVs for $X$ and $\bar{X}$ fields:

$$|X| = \sqrt{\frac{\xi}{\beta} + \sqrt{\frac{\xi^2}{\beta^2} + 4|\mu|^2}}^{1/2} , \quad |\bar{X}| = \sqrt{2}|\mu|^2 \left(\frac{\xi}{\beta} + \sqrt{\frac{\xi^2}{\beta^2} + 4|\mu|^2}\right)^{-1/2} .$$  \hspace{1cm} (A.5)

From (A.5) we see that $|X|$ and $|\bar{X}|$ have different values. It is interesting to consider two limiting cases:

a) $\xi/\beta < 0$, $|\mu|^2 \ll -\xi/\beta$,

$$|X| \approx \frac{|\mu|^2}{\sqrt{-\xi/\beta}} , \quad |\bar{X}| \approx \sqrt{-\xi/\beta} , \quad |X| \ll |\bar{X}| ,$$

b) $\xi/\beta > 0$, $|\mu|^2 \ll \xi/\beta$,

$$|X| \approx \sqrt{\xi/\beta} , \quad |\bar{X}| \approx \frac{|\mu|^2}{\sqrt{\xi/\beta}} , \quad |X| \gg |\bar{X}| .$$  \hspace{1cm} (A.6)

Thus, with notations of Eq. (28), case a) gives $\epsilon \ll \bar{\epsilon}$, while in case b) we have $\epsilon \gg \bar{\epsilon}$. When the scales satisfy relation $\frac{\xi}{\beta} \sim |\mu|^2$, Eq. (A.5) gives $\epsilon \sim \bar{\epsilon}$. Note that with solution (A.5) and $\langle S \rangle = 0$, all states coming from the superfields $X$, $\bar{X}$ and $S$ get masses.

Including soft SUSY breaking terms in the potential, VEVs of the fields will be slightly shifted. In particular, with soft mass squares $m^2_X$ and $m^2_{\bar{X}}$ for the fields $X$ and $\bar{X}$ respectively, one can readily check that their VEVs are shifted in such a way that $D_{U(1)_F} \approx 2(m^2_X - m^2_{\bar{X}})/\tilde{g}^2$ ($\tilde{g}$ is $U(1)_F$’s coupling constant). As discussed in the end of Sect. 3, this would have impact on flavor violating processes. On the other hand, within minimal SUGRA scenario, the universality $m^2_X = m^2_{\bar{X}}$ insures that $D_{U(1)_F} = 0$. 

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