Generalized Parton Distributions, Deeply Virtual Compton Scattering and TMDs

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Abstract. Parton distributions in impact parameter space, which are obtained by Fourier transforming GPDs, exhibit a significant deviation from axial symmetry when target and/or quark are transversely polarized. In combination with the final state interactions, this transverse deformation provides a natural mechanism for naive-T odd transverse single-spin asymmetries in semi-inclusive DIS. The deformation of PDFs in impact parameter space can also be related to the transverse force acting on the active quark in polarized DIS at higher twist.

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DISTRIBUTION OF QUARKS IN THE TRANSVERSE PLANE

In the case of transversely polarized quarks and/or nucleons, parton distributions in impact parameter space show a significant transverse deformation. In the case of unpolarized quarks in a nucleon polarized in the $+\hat{x}$ direction, this deformation is described by the $\perp$ gradient of the Fourier transform of the GPD $E^q$ [1]

$$q_{q/p}(x, b_\perp) = \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta} E^q(x, 0, -\Delta^2) = \frac{1}{2M} \partial_y \int \frac{d^2x_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta} E^q(x, 0, -\Delta^2),$$

for quarks of flavor $q$. Since $E^q(x, 0, t)$ also arises in the decomposition of the Pauli form factor $F_2^q = \int_{-1}^1 dx E^q(x, 0, t)$ for quarks with flavor $q$ (here it is always understood that charge factors have been taken out) w.r.t. $x$, this allows to relate the $\perp$ flavor dipole moment to the contribution from quarks with flavor $q$ to the nucleon anomalous magnetic moment (here it is always understood that charge factors have been taken out)

$$d^q \equiv \int d^2b_\perp q_{+\hat{x}}, b_\perp) b_y = \frac{1}{2M} F_2^q(0) = \frac{1}{2M} \kappa_{q/p}. \quad (2)$$

Here $e_q \kappa_{q/p}$ is the contribution from flavor $q$ to the anomalous magnetic moment of the proton. Neglecting the contribution from heavier quarks to the nucleon anomalous magnetic moment, one can use the proton and neutron anomalous magnetic moment to solve for the contributions from $q = u, d$, yielding $\kappa_{u/p} \approx 1.67$ and $\kappa_{d/p} \approx -2.03$. The resulting significant deformation ($|d_q| \sim 0.1$fm) of impact parameter dependent PDFs in the transverse direction (fig. [1]), which is in opposite directions for $u$ and $d$ quarks, should have observable consequences in other experiments as well as will be discussed in the following.
FIGURE 1. Distribution of the $j^+$ density for $u$ and $d$ quarks in the $\perp$ plane ($x = 0.3$ is fixed) for a proton that is polarized in the $x$ direction in the model from Ref. [1]. For other values of $x$ the distortion looks similar.

TRANSVERSE SINGLE-SPIN ASYMMETRIES

In a DIS experiment on a transversely polarized target, the (on average attractive) final state interactions (FSI) should cause a transverse momentum asymmetry that is opposite to the transverse position space asymmetry. This simple argument leads to the prediction [2] that the Sivers function $f_{1T}^q(x, k_T^2)$ parameterizing the transverse momentum asymmetry [3, 4]

$$f_{q/p}(x, k_T) = f_1^q(x, k_T^2) - f_{1T}^q(x, k_T^2) \frac{(\hat{P} \times k_T) \cdot S}{M},$$

has the opposite sign as $\kappa_{q/p}$. This prediction was confirmed in the HERMES experiment [5]. It is also consistent with a vanishing Sivers function for a deuterium target [6].

TRANSVERSE FORCE ON QUARKS IN DIS

The polarized twist three parton distribution $g_2(x)$ can be expressed as a sum of a piece that is entirely determined in terms of $g_1(x)$ plus an interaction dependent twist-3 part $\bar{g}_2(x)$ that involves quark gluon correlations [7]

$$g_2(x) = g_2^{WW}(x) + \bar{g}_2(x) \quad \text{with} \quad g_2^{WW}(x) = -g_1(x) + \int_y^1 \frac{dy}{y} g_1(y).$$

Here we have neglected $m_q$ for simplicity. For example, its $x^2$ moment yields [8, 9]

$$\int dxx^2 \bar{g}_2(x) = \frac{d_2}{3}$$

with

$$g \langle P, S | \bar{q}(0) G^+(0) \gamma^+ q(0) | P, S \rangle = 2MP^+ P^+ S^x d_2.$$
In the limit where $Q^2$ is so low that the virtual photon wavelength is larger than the nucleon size, the electro-magnetic field associated with the two virtual photons appearing in the forward Compton amplitude corresponding to the structure function is nearly homogenous across the nucleon and the spin-dependent structure function $g_2(x, Q^2)$ can be related to spin-dependent polarizabilities. In contradistinction, in the Bjorken limit, the matrix elements describing the moments of $g_2(x, Q^2)$ are given by local correlation functions, such as (6). Nevertheless, because of the abovementioned low $Q^2$ interpretation of $g_2$, the local matrix elements appearing in (6)

$$\chi_E 2M^2 S = \langle P, S | q \gamma^\mu \bar{q} E q | P, S \rangle$$

$$\chi_B 2M^2 S = \langle P, S | q \gamma^\mu \bar{q} B q | P, S \rangle,$$

where

$$d_2 = \frac{1}{4} (\chi_E - 2\chi_M),$$

(note that $\sqrt{2} G^{+\gamma} = B^x - E^y$) are sometimes called color electric and magnetic polarizabilities [10]. In the following we will discuss why, at high $Q^2$, a better semi-classical interpretation for these matrix elements is that of a ‘force’.

In electro-magnetism, the $\hat{y}$-component of the Lorentz force $F^y$ acting on a particle with charge $e$ moving, with the speed of light along the $-\hat{z}$ direction, reads

$$F^y = e \left[ \vec{E} + \bar{v} \times \vec{B} \right] = e (E^y - B^x) = -e \sqrt{2} F^{+\gamma},$$

which involves the same linear combination of Lorentz components that also appears in the gluon field strength tensor in (6). Therefore (6) implies a relation between $d_2$ and the color Lorentz force on a quark that moves (in a DIS experiment) with $\bar{v} \approx (0, 0, -1)$ [11]

$$F^y(0) \equiv -\frac{\sqrt{2}}{2P^+} \langle P, S | \bar{q}(0) G^{+\gamma}(0) \gamma^+ q(0) | P, S \rangle = -\sqrt{2} MP^+ S^x d_2 = -M^2 d_2,$$

where the last equality holds only in the rest frame ($P^+ = \frac{1}{\sqrt{2}} M$) and for $S^x = 1$, can be interpreted as the averaged transverse force acting on the active quark in the instant right after it has been struck by the virtual photon.

For a nucleon polarized in the $+\hat{x}$ direction, the $\gamma^+$-distribution for $u$ ($d$) is shifted towards the $\pm \hat{y}$ direction. This observation suggests that these quarks also ‘feel’ a nonzero color-electric force pointing on average in the $\mp \hat{y}$ direction, i.e. one would expect that $d_2$ is positive (negative) for $u$ ($d$) quarks. This would also be consistent with the observed signs of the corresponding Sivers functions [5, 6].

A measurement of the $x^2$-moment $f_2$ of the twist-4 distribution $g_3(x)$ [12] allows determination of the expectation value of a different linear combination of Lorentz/Dirac components of the quark-gluon correlator appearing in (6) [13]

$$f_2 M^2 S^\mu = \frac{1}{2} \langle p, S | \bar{q} G^\mu\nu \gamma^\nu q | p, S \rangle.$$

Using rotational invariance, to relate various Lorentz components one thus finds a linear combination of the matrix elements of electric and magnetic quark-gluon correlators (7)

$$f_2 = \chi_E - \chi_M,$$

(12)
that differs from that in (8). In combination with (6) this allows a decomposition of the force into electric and magnetic components $F^y = F_E^y + F_M^y$.

A relation similar to (10) can be derived for the $x^2$ moment of interaction dependent twist-3 part $\bar{e}(x)$ of the scalar PDF $e(x)$. The average transverse force at $t = 0$ (right after being struck) on a quark with transversity in the $+\hat{x}$ direction reads

$$F^y(0) = \frac{1}{2\sqrt{2}p^+} g \langle p | \bar{q} \sigma^{+\gamma} G^{+\gamma} q | p \rangle = \frac{1}{\sqrt{2}} MP^+ S^y e_2 = \frac{M^2}{2} e_2 \equiv \frac{M^2}{2} \int_0^1 dx \bar{e}(x)x^2 \quad (13)$$

(in the rest frame of the target nucleon and for $S^y = 1$).

The impact parameter distribution for quarks with transversity in the $+\hat{x}$ direction was found to be shifted in the $+\hat{y}$ direction \cite{14, 15, 16}. The chromodynamic lensing model \cite{2} thus implies a force in the negative $-\hat{y}$ direction for these quarks and one thus expects $e_2 < 0$ for both $u$ and $d$ quarks. Furthermore, since $|\kappa_\perp| > |\kappa'|$, one expects $|e_2| > |d_2|$.

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