Energy Loss Versus Energy Gain of Heavy Quarks in a Hot Medium

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We study the energy loss and the energy gain of heavy quarks in a hot thermal medium. These include the study of the energy change due to the polarization and to the interaction with the thermal fluctuations of the medium. The dynamics of the heavy quarks with the medium is described by the Wong equations, that allow for the inclusion of both the backreaction on the heavy quarks due to the polarization of the medium, and of the interaction with the thermal fluctuations of the gluon field. Both the momentum as well as the temperature dependence of the energy loss and gain of charm and bottom quark are studied. We find that heavy quark energy gain dominate the energy loss at high-temperature domain achievable at the early stage of the high energy collisions. This finding supports the recently observed heavy quarks results in Glasma and will have a significant impact on heavy quark observables at RHIC and LHC energies.

I. INTRODUCTION

The medium consisting of quarks and gluons produced at various heavy-ion collision experimental facilities such as the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) provide a unique opportunity to explore the Quantum Chromodynamics (QCD) matter under extreme conditions of temperature and density. The bulk properties of such a state of matter, called Quark-Gluon Plasma (QGP) [1, 2], are governed by the light quarks and gluons. Though the small size and short-lived nature of the produced medium do not allow to observe it by the naked eyes and hence, we rely on the signatures observed at the detector end in the form of particle spectra.

Heavy quarks, namely Charm and Beauty, are considered as excellent probes of the QGP [3–10] and offers signatures of the production of the QGP itself. In fact, one of these signatures is the suppression of high $p_T$ heavy hadrons [11–13], that is understood as a result of the loss of energy of the high-energy charm and beauty quarks while they propagate through the dense matter formed after collisions. More generally, the energy change of charm and beauty in the QGP have two major contributions, namely the polarization of the medium, which leads to energy loss, and the interaction with the background thermal fluctuations fluctuations of the gluon field that is responsible of momentum diffusion. The polarization is responsible of energy loss [14, 15] while interaction with thermal fluctuations leads to energy gain and is effective in the low-velocity limit [16, 17], see also [14, 16, 18–53]. Heavy quarks can experience diffusion in the early stage of high energy nuclear collisions as well. In particular, recent studies suggest that due to the high energy density developed in the early stage, the motion of charm and beauty is dominated by field fluctuations that lead to a modest energy gain of the heavy probes and to a tilt in the spectrum [56–60], in qualitative agreement with previous studies on the propagation in a high temperature QGP medium [16, 17].

The purpose of the present study is to analyze the combined effect of energy gain and energy loss of heavy quarks in a high temperature QCD medium, analyzing the kinematic regime in which one of the two mechanisms dominates. In solving this problem, albeit using several approximations, we will show that even when energy gain and energy loss are considered consistently, the heavy quarks will experience a substantial energy gain if the temperature of the medium is large enough. We will address quantitatively the question which between energy gain or energy loss of heavy quarks is dominant in a given kinematic regime and at a given temperature. The conclusion is easy to imagine: if the temperature is quite larger than the kinetic energy of the heavy quark, then the medium will contribute substantially to increase the energy of the heavy probe as this propagates in the hot medium; energy loss will be important when the temperature of the medium is lower than the kinetic energy of the heavy quark. These qualitative statements need to be supported by quantitative findings, aiming to identify the kinematic regimes in which energy loss or energy gain dominate and thus giving a clearer understanding of the dynamics of heavy quarks in the QGP medium produced in collisions. This is what we want to study here.

In addition to this, our results offer a case study that supports the assumption of [56–58, 61] where heavy quarks propagate in the evolving Glasma fields, and in which energy loss has been ignored. In fact, although here we consider a thermalized medium while the evolving Glasma is out of equilibrium, the diffusion of heavy probes in Glasma resembles that in a thermal medium, at least when an average over the heavy quark spectrum is considered (see for example Fig. 7 of [58]); the energy density in the evolving Glasma is very large, implying that the effective temperature of the medium is also high and thus the loss of energy of low momentum quarks can be neglected.

The plan of the paper is as follows. In section II we shall discuss the polarization energy loss of heavy quarks moving in the hot QCD medium along with a brief de-
description of the change in energy of heavy quarks due to fluctuation. In section II we shall discuss the various results. Section IV is dedicated to the summary and future possibilities of the present work.

II. ENERGY CHANGE DUE TO POLARIZATION AND FLUCTUATION

In this section we discuss the theoretical setup on which we base our analysis. We treat charm and beauty as classical color sources that obey the Wong’s equations [62]: these equations describe the motion of classical colored particles interacting with a dynamical gluon field, $F_{a}^{\mu\nu}$, and in a Lorentz covariant form they are given by

\begin{align}
\frac{dx^{\mu}(\tau)}{d\tau} &= u^{\mu}(\tau), \quad (1) \\
\frac{dp^{\mu}(\tau)}{d\tau} &= gg^{a}(\tau)F_{a}^{\mu\nu}(x(\tau))u_{\nu}(\tau), \quad (2) \\
\frac{dq^{a}(\tau)}{d\tau} &= -gf^{abc}u_{\mu}(\tau)A_{b}^{c}(x(\tau))q_{c}(\tau); \quad (3)
\end{align}

in these equations, $q^{a}(\tau)$ is a classical charge (to not be confused with the fundamental, quantized color charge of the quark) that is introduced to describe the conservation of the color current in the classical theory, with $a = 1, 2, \ldots, N_{c}^{2} - 1$, $g$ is the coupling constant, $\tau, x^{\mu} \equiv X$, $u^{\mu} = \gamma(1, \nu)$ and $p^{\mu}(\tau)$ are the proper time, trajectory, 4-velocity and 4-momentum of the heavy quark, respectively. For $N_{c}$ fundamental colors of quarks there are $N_{c}^{2} - 1$ chromo-electric/magnetic fields, and $f^{abc}$ is the structure constant of $\text{SU}(N_{c})$ gauge group; finally, $A_{b}^{c}$ is the gauge potential. In solving these equations we assume the gauge condition $u_{\mu}A_{b}^{c}(X) = 0$ [23, 28], namely that the gauge potential vanishes on the trajectory of the particle and which implies that $q^{a}$ is independent of $\tau$; moreover, we assume that in the motion of the heavy quark in the thermal medium the magnitude of the velocity does not change much [25].

From the $\mu = 0$ component of Eq. (2) the energy change per unit time is

\begin{align}
\frac{dE}{dt} &= gq^{a}\nu \cdot E^{a}(X), \quad (4)
\end{align}

where here and in the following we use $E$ to denote the energy of the heavy quark and $E$ for the color-electric field, and $t = \gamma \tau$ is the time in the laboratory frame in which the heavy quark of mass $M$ moves with velocity $\nu = \frac{p}{\sqrt{p^{2} + M^{2}}}$. The color field consists of two terms,

\begin{align}
E^{a} = E_{\text{ind}}^{a} + E_{\text{fluct}}, \quad (5)
\end{align}

where $E_{\text{ind}}^{a}$ denotes the field induced by the motion of the heavy quark that polarizes medium (for this reason, this is also called the polarization contribution), hence representing an energy loss and its inclusion in the equation of motion amounts to consider the backreaction on the heavy quark, while $E_{\text{fluct}}^{a}$ denotes the color field induced by the thermal fluctuations in the gluon medium: the interaction of the heavy quark with $E_{\text{fluct}}^{a}$ can result in energy loss or energy gain depending on the temperature of the medium as well as on the heavy quark momentum, as we discuss later.

For the motion of the heavy quark in a thermal medium, the right hand side of Eq. (1) is replaced by its ensemble average,

\begin{align}
\frac{dE}{dt} &= gq^{a}(\nu(t) \cdot E^{a}(X(t))), \quad (6)
\end{align}

where the electric field is given by Eq. (5). The procedure to evaluate right hand side of the above equation is explained clearly in the literature, see for example [16], therefore we limit ourselves to quote the final result that is

\begin{align}
\frac{dE}{dt} &= \langle gq^{a}(\nu_{0} \cdot E^{a}) \rangle + g^{2}q^{a}q^{b} \int_{0}^{t} dt_{1} \langle E_{t}^{a}(t_{1}) \cdot E_{t}^{b}(t) \rangle \\
&\quad + g^{2}q^{a}q^{b} \int_{0}^{t} dt_{1} \int_{t}^{t} dt_{2} \langle \Sigma_{j}E_{t,j}^{a}(t_{2}) \rangle \langle \Sigma_{j}E_{t,j}^{b}(t) \rangle \\
&\quad \times \frac{\partial}{\partial \nu_{0j}}(\nu_{0} \cdot E_{t}^{a}(t)) \quad (7)
\end{align}

Equation (7) corresponds to the full energy change of the heavy quark: the first addendum on the right hand side is the energy loss due to the work against the induced field, see Eq. (9) [14, 19, 23, 63]. The procedure to evaluate right hand side of the above equation is explained clearly in the literature, see for example [16], therefore we limit ourselves to quote the final result that is

\begin{align}
\frac{dE}{dt} &= \langle gq^{a}(\nu_{0} \cdot E^{a}) \rangle + g^{2}q^{a}q^{b} \int_{0}^{t} dt_{1} \langle E_{t}^{a}(t_{1}) \cdot E_{t}^{b}(t) \rangle \\
&\quad + g^{2}q^{a}q^{b} \int_{0}^{t} dt_{1} \int_{t}^{t} dt_{2} \langle \Sigma_{j}E_{t,j}^{a}(t_{2}) \rangle \langle \Sigma_{j}E_{t,j}^{b}(t) \rangle \\
&\quad \times \frac{\partial}{\partial \nu_{0j}}(\nu_{0} \cdot E_{t}^{a}(t)) \quad (7)
\end{align}

and we discuss the two terms separately below.

A. Energy loss due to the induced field

Firstly we analyze the energy loss due to the work against the induced field, see Eq. [14, 19, 23, 63].
The induced field can be obtained by solving the Yang-Mills equations for a thermalized gluon system with the source given by the color current carried by the heavy quark, namely [15],
\[
E_{\text{ind}}^a(X) = -i \frac{g q^a}{\pi} \int d\omega d^3k \frac{1}{\omega k^2} \left[ k \cdot (k \cdot v) \left( \epsilon_L^1 - 1 \right) + \left( k^2 v - k \cdot (k \cdot v) \right) \left( \epsilon_T - k^2 \right)^{-1} \right] \frac{1}{\omega^2} e^{i(k \cdot x - \omega t)}.
\]

performing the \( \omega \) integration in Eq. (11) and substituting in Eq. (9) we get
\[
\frac{dE}{dt} = \frac{C_F \alpha_s}{2\pi^2|v|} \int_{k_0}^{k_{\text{max}}} d^3k \frac{\omega}{k^2} \left\{ \left( k^2 |v|^2 - \omega^2 \right) \right\} \frac{1}{\omega^2} e^{i(k \cdot x - \omega t)}.
\]

where, \( k^\mu = (\omega, k) \) with \(|k| = k\) and \( \alpha_s \) is the QCD coupling; moreover, \( C_F = 4/3 \) is the Casimir invariant in the fundamental representation of the SU(\( N_c \)), \( \epsilon_L \) and \( \epsilon_T \) are the longitudinal and transverse components of the medium dielectric permittivity, that have been computed using the semi-classical transport theory approach in [15].

B. Interaction with the fluctuating field

The energy change due to the interaction of the heavy quark with the fluctuating field can be written as [16],
\[
\frac{dE}{dt} = \frac{C_F \alpha_s}{8\pi^2 E_0^2 v^2} \int_0^{k_{\text{max}} v} d\omega \coth \frac{\beta \omega}{2} F(\omega, k = \omega/v) + \frac{C_F \alpha_s}{8\pi^2 E_0^2 v^2} \int_0^{k_{\text{max}}} dkk \int_0^k d\omega \coth \frac{\beta \omega}{2} G(\omega, k),
\]

where
\[
F(\omega, k) = 8\pi \omega^2 \frac{\text{Im}[\epsilon_L]}{|\epsilon_L|^2},
\]
\[
G(\omega, k) = 16\pi \frac{\text{Im}[\epsilon_T]}{\epsilon_T - k^2/\omega^2}. \tag{14}
\]

In Eq. (13) we have put \( E_0 = \sqrt{p^2 + M^2} \) and introduced an ultraviolet cutoff, \( k_{\text{max}} \), which is of the order of the Debye screening mass [16] [46]; in the following we will consider two representative values of this cutoff, namely \( k_{\text{max}} = m_D \) and \( k_{\text{max}} = 2m_D \); while the specific value of \( k_{\text{max}} \) affects the results quantitatively, the qualitative picture is almost unaffected by this choice.

III. RESULTS

In this section we summarize our results: firstly we focus on charm, then we turn on beauty. We use the set of parameters \( N_c = 3, N_f = 2 \) and \( \alpha_s = 0.3 \). In all the figures below we show the energy change per unit length since the latter is the most used in the literature: this can be obtained easily from the change of energy per unit time that we have computed in the previous section,
\[
\frac{dE}{dx} = \frac{1}{|v|} \frac{dE}{dt}. \tag{15}
\]

where \( v \) is the velocity of the heavy quark. Moreover, to uniform to the existing literature we plot \(-dE/dx\) since this quantity is been mostly used to quantify the energy loss and is therefore positive.

A. Charm

In Fig. 1 we plot \(-dE/dx\) versus temperature for three values of the heavy quark momentum. In the figure, upper and lower panels correspond to \( k_{\text{max}} = m_D \) and \( k_{\text{max}} = 2m_D \) respectively. As anticipated, the backreaction represented by the interaction of the heavy quark
Figure 2. Energy change of charm due to the fluctuation of the hot medium, \((-\frac{dE}{dx})_{\text{fluct}}\), versus temperature, for three values of the initial charm quark momentum. Upper and lower panels correspond to \(k_{\text{max}} = m_D\) and \(k_{\text{max}} = 2m_D\) respectively.

with the induced field results in an energy loss. This can be understood easily since the motion of the heavy quark in the thermal medium results in the polarization of the medium itself, and for this process to happen energy has to be transferred from the quark to the medium. For example, for a charm quark with initial momentum \(p = 10\) GeV, at a temperature \(T = 1\) GeV we find \((-\frac{dE}{dx})_{\text{ind}} \approx 0.1\) GeV/fm for \(k_{\text{max}} = m_D\) and \((-\frac{dE}{dx})_{\text{ind}} \approx 0.5\) GeV/fm for \(k_{\text{max}} = 2m_D\).

In Fig. 2 we plot \((-\frac{dE}{dx})_{\text{fluct}}\) versus temperature for three values of the initial heavy quark momentum; upper and lower panels correspond to \(k_{\text{max}} = m_D\) and \(k_{\text{max}} = 2m_D\) respectively. Differently from the cases shown in Fig. 1 we find that the interaction with the thermalized gluon field leads to energy gain rather than energy loss. For example, considering again \(p = 10\) GeV and \(T = 1\) GeV we find \((-\frac{dE}{dx})_{\text{fluct}} \approx -0.02\) GeV/fm for \(k_{\text{max}} = m_D\) and \((-\frac{dE}{dx})_{\text{fluct}} \approx -0.4\) GeV/fm for \(k_{\text{max}} = 2m_D\). The results shown in Figg. 1 and 2 agree qualitatively with those obtained within a purely classical model for the diffusion and the energy loss in a Brownian motion [60], in which the backreaction as the source of the energy loss and the interaction with the thermal fluctuations as resulting in energy gain and momentum broadening appear clearly. In addition to this, comparing the results shown in Figg. 1 and 2 we notice that for \(p/T \gg 1\) the energy loss due to polarization of the medium is larger than the energy gain, but this situation changes when \(p/T \lesssim 1\).

In Fig. 3 we plot the total energy change per unit length of the charm quark versus temperature, for three values of the initial momentum \(p\); this is obtained by adding the results shown in Figg. 2 and 1. In Fig. 3 the upper and lower panels correspond to \(k_{\text{max}} = m_D\) and \(k_{\text{max}} = 2m_D\) respectively. We notice that for \(p = 1\) GeV, in the full range of temperature considered the sum of the polarization and the fluctuation contributions results in an energy gain of the quark. For the other two representative values of \(p\), namely for \(p = 5\) GeV and \(p = 10\) GeV, we find that up to \(T \approx 1\) GeV the charm losses energy by polarization of the medium, while for higher temperatures it gains energy from the medium itself; the exception that we find is that if the initial momentum is very large, see \(p = 10\) GeV in the figure, and \(k_{\text{max}} = m_D\) then energy loss dominates over energy gain over the whole range of temperature studied.

In Fig. 4 we plot the energy change due to polarization (upper panel) and fluctuations (middle panel) of charm quarks versus the initial momentum, for two representative values of temperature and for \(k_{\text{max}} = 2m_D\) (results for \(k_{\text{max}} = m_D\) are similar to those shown here). At relatively low temperature the energy loss dominates over energy gain for \(p \gtrsim 2\) GeV, while for higher temperatures the energy gain due to the interaction with the fluctuat-
Figure 4. Energy change of charm quark versus the initial momentum, at $T = 0.5$ GeV (red lines) and $T = 2$ GeV (blue lines). Upper and middle panels correspond to the polarization and fluctuations contributions respectively, while the lower panel corresponds to the sum of the two contributions. Results correspond to $k_{\text{max}} = 2m_D$.

Figure 5. Energy change beauty due to polarization (upper panel) and fluctuations (middle panel), as well as the combination of the two (lower panel). Results correspond to $k_{\text{max}} = 2m_D$.

B. Beauty

In this subsection we report on the analysis of energy loss and gain of beauty quarks in the hot medium; since the qualitative picture is unchanged with respect to that of the charm quark, here we limit ourselves to present only a few representative results. In Fig. 5 we plot energy change induced by polarization (upper panel), interaction with fluctuating medium (middle panel) and total (lower panel) versus temperature, for three values of the initial beauty quark momentum; the results correspond to $k_{\text{max}} = 2m_D$. Clearly, there is some quantitative difference between charm and beauty, due to the different masses of the two quarks, e.g., for the given values of parameters, beauty quark loses less energy in the case of polarization and also gains less energy in the case of fluctuation.
IV. CONCLUSIONS

We have studied, within linear response theory, the energy change of heavy quarks in a hot thermalized QCD medium, analyzing the combined effect of energy loss due to the polarization of the medium, and energy gain due to interaction with the thermal fluctuations of the gluon field of the medium. We have considered the effects on both charm and beauty quarks. This study has been inspired by a series of works on the propagation of heavy probes in the early stage of the high energy nuclear collisions, in which the energy gain due to the diffusion in the evolving Glasma is crucial to bend the initial pQCD spectrum of the heavy quarks before the formation of the quark-gluon plasma. Although we do not consider the Glasma in the present study, we think that the results found here support at least qualitatively the diffusion-dominated scenario found in [50, 55]: in fact, despite the fact that the evolving Glasma is a system out of thermal equilibrium, the diffusion of heavy color probes (see also Ref. [64]) in it is not very different from the diffusion in a Brownian motion, at least when an average over the full heavy quark spectrum is taken: because of this similarity, it is likely that the results on diffusion in a fluctuating medium studied here can be applied qualitatively to the diffusion in the evolving Glasma as well.

We have found that in the kinematic regime \( p/T \lesssim 1 \), where \( p \) is the initial heavy quark momentum and \( T \) the temperature of the medium, energy gain dominates of the energy loss, and the situation inverts in the complementary regime \( p/T \gtrsim 1 \). These results are consistent with previous literature [15, 16]. If we applied these conclusions to the early stages of high energy nuclear collisions, our findings would suggest a diffusion dominated propagation for \( p \lesssim 2 \) GeV while energy loss would be substantial for \( p \gtrsim 10 \) GeV, while in between there would be a balance between the two.

The results may have a significant impact on the experimental observables like the nuclear suppression factor and elliptic flow \( \varepsilon_{2} \) of heavy mesons produced at RHIC and LHC energies both for the nucleus-nucleus and p-nucleus collisions. Also a thorough understanding of the initial stage dynamics is a timely fundamental task and may affect observables like the triggered \( D \bar{D} \) angular correlation [65] and the heavy quark directed flow \( v_{1} \) [66]. Apart from this, as it is well known that the thermal systems have comparatively weaker fluctuations than the non equilibrated systems. Therefore, incorporating the momentum anisotropy (which remains inevitable throughout the medium evolution) and also viscosity while modelling the medium [67, 68] in the current study will bring us much closer to the real picture of the high energy nuclear collision. Hence, it will be an immediate future extension to the current work.

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