Research on Heat Conduction Model of Special Equipment Based on Difference Equation

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Abstract: Special equipment plays an important role in the life safety of special operators. With the development of economy and society, the performance requirements of special equipment are higher and higher. In this paper, the fire-proof clothing in special equipment is taken as an example to study the internal temperature change of the fire-proof clothing in high temperature environment and improve the design of the fire-proof clothing. Through Newton’s law of cooling and Fourier’s law, combined with the relevant physical parameters, we establish the heat conduction model of the fire suit, discretize the differential equation in the model, use the chasing and driving method to solve the tridiagonal matrix after discretization, and transform the unknown parameters (heat extraction coefficient) in the model into a two-parameter optimization problem, combining with the classical optimization algorithm—The genetic algorithm is solved. Experiments show that our model can better solve this kind of problems, and the fitting accuracy is increased by 10%. Through the fitting image of the experimental results, we can see the improvement of the accuracy directly. Through the improvement, our model can improve the fitting accuracy, which is of great significance for the design and research of special equipment Fire clothing, and also has certain reference value for solving similar problems.

Key words: Heat conduction model; Differential equation; Tridiagonal matrix ; Optimization

1. Introduction
The design of special equipment is an important and continuously concerned research. Among them, the research of thermal protective clothing is one of the important measures for national security and revitalization of textile industry, which plays an important role in chemical industry, national defense, fire protection and other fields. Fireproof clothing refers to the special work clothes worn in the high temperature environment, which can effectively slow down the heat transfer speed between the high temperature environment and the human skin, make the heat gather on the human skin as little as possible, and also has good flame retardant. At present, the performance evaluation of fire-proof clothing mainly depends on a large number of fire-proof clothing performance test experiments, a large number of high-temperature environment-based fire-proof clothing performance test experiments are expensive and most of them can not be reproduced, resulting in a large number of unnecessary waste. Therefore, it is necessary to establish the heat transfer model of the fireproof clothing in high temperature environment for the design of the fireproof clothing. Because it is difficult to accurately measure the temperature distribution inside the fireproof clothing in practical problems, which brings many difficulties to the design of the fireproof clothing. Later, the relevant researchers considered that through the easy to measure boundary temperature of the fireproof clothing and the physical parameters of the
relevant materials, combined with the mathematical model for numerical fitting analysis, the
temperature distribution inside the fireproof clothing was obtained, and then the design of the fireproof
clothing was guided. There are some errors in the numerical fitting, which will seriously affect the
performance of the designed fireproof clothing, so it is the key to accurately fit the temperature
distribution inside the fireproof clothing and reduce the fitting error as much as possible.

2. Relevant Research at Home and Abroad
The research of fireproof clothing began to rise in Europe at first. So far, a series of advanced and perfect
testing standards and methods of fireproof clothing have been formulated.
Torvi [1] proposed the heat transfer model of the outer fabric of the thermal protective clothing, and based
on this, Mell [2] and others further considered the heat conduction and heat radiation of the material, and
proposed the reflection model between the layers of the multilayer fabric. In recent years, with the
development of science and technology in our country, the research on fireproof clothing is also
increasing. In reference [3], the relevant influencing factors of the research on the fire-proof clothing are
analyzed, and the scheme of the optimization design of the fire-proof clothing is proposed, but the
specific mathematical model and method are not given.
Lu Wanying of Zhejiang University of science and technology [4], based on the existing heat transfer
model, added the moisture transfer equation, studied the heat and moisture transfer law in the fireproof
clothing, and obtained the distribution diagram of temperature and water vapor in each layer of the
fireproof clothing, which provided a certain theoretical basis for the design of the fireproof clothing in
the high temperature and humidity environment.
By taking into account the overall heat capacity in a confined space, Bian yifeng [5] obtained the
relationship between time and temperature difference. Yan ziqi [6] established the multivariable
optimization equation, based on the finite element method, combined with the idea of control variables,
established the iterative algorithm, and transformed the problem into the extremum problems of
functions of several variables, which achieved certain results.
Pan bin [7] studied the inverse problem of determining the parameters of thermal protective clothing;
Zhu Lingli and others [8][9][10] all used the difference equation to study the problem and made some
achievements; Li Jingyuan [11] and others used the query algorithm to study the solution; Wei Tian [12] and
others used the simulated annealing algorithm to solve the model to improve the accuracy to a certain
extent;
Wang Mingpei [13] et al. calculated the temperature distribution along the thickness direction with time
by using the idea of discretization of equations and progressive iteration of cellular automata algorithm.
Wang Bao [14] obtained the temperature distribution rule of multi-layer heat transfer based on fractional
partial differential equation and nonlinear optimization algorithm.
Guo Cheng [15] et al. Established the heat conduction equation with the help of heat transfer model,
analyzed the initial condition and boundary value, simulated the temperature distribution with
mathematical tools, and provided a feasible idea.

3. Model
Firstly, the model is reasonably simplified. Since the air convection and the volume of human body are
not considered, we can simplify the problem to a one-dimensional space only related to the thickness,
and use $m_i$ to represent the critical interface of the medium.
The one-dimensional heat conduction model of the fire suit system is established by the micro element
method, and the temperature at the time $t$ of the system $x$ is represented by $u(x,t)$. The Fourier law
points out that the heat $Q$ transferred by the micro element $\Delta s$ in unit time is directly proportional to
the temperature change rate perpendicular to the section, but the heat transfer direction is opposite to the
temperature rise direction. The micro element formula of heat can be obtained as follows:

$$dQ = -\lambda \frac{\partial u}{\partial n} dsdt$$ (1)
Where $\lambda$ is the thermal conductivity and $u$ is the derivative of temperature along $n$ direction. Due to the conservation of energy, the heat flowing into the medium should be equal to the heat absorbed by the temperature rise in the medium, so the following energy conservation equation can be established, the following heat conduction equation can be obtained:

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$$

(2)

Among $k^2 = \frac{\lambda}{c\rho}$, $\lambda$ is the thermal conductivity of the medium, $c$ is the specific heat of the medium, and $\rho$ is the density of the medium.

According to the Fourier law of heat conduction, in the process of heat conduction, the heat flux density at the interface between two adjacent media is the same, so there are:

$$\lambda_i \frac{\partial u_i}{\partial x} = \lambda_{i+1} \frac{\partial u_{i+1}}{\partial x}, x = m_i$$

(3)

At the same time, according to the same temperature at the critical interface of two media at any time, the following relationship can be obtained:

$$u_i = u_{i+1}, x = m_i$$

(4)

Determination of initial value condition and boundary condition

Determine initial value conditions.

1 At time $t = 0$, the temperature in the medium is the same as the constant temperature outside the skin of the dummy, that is, both are $37^\circ C$. Therefore, the initial conditions can be given:

$$u_i(x, 0) = 37$$

(5)

2 Determine the boundary value conditions.

For the left boundary, the left side of the medium is in contact with the outside world. According to Newton’s law of cooling, the heat flowing into the medium per unit time should be: $h_1(75 - u_i)$, Meanwhile, according to Fourier law, the left boundary condition of the equation can be obtained as follows:

$$-\lambda_4 \frac{\partial u}{\partial n} = h_1(75 - u_i)$$

(6)

For the right boundary, the heat exchange between the right boundary of the medium and the dummy skin is conducted, and the right boundary conditions can be obtained by using Newton’s law of cooling:

$$-\lambda_4 \frac{\partial u}{\partial n} = h_2(u_b - \bar{u})$$

(7)

Among them, $u_b$ represents the temperature of the medium close to the skin of the dummy, $h_1$ is heat extraction coefficient between air and medium, $h_2$ is heat extraction coefficient between the medium and the skin, $\bar{u}$ is the constant temperature inside the dummy, that is $37^\circ C$.

3.1 Implicit Backward Difference:

The finite difference method is usually used to solve the definite solutions of partial differential
equations, take the time step to be \( \Delta t = \frac{T}{n} \), write the space step of the i-th layer medium as \( \Delta x_i = \frac{L_i}{M_i} \). 
\( L_i \) is the thickness of the i-th layer, \( M_i \) is equal to the fraction of the i-th layer medium, \( u_{ji} \) is the temperature at \( x_i \) at time \( t_j \), which is \( u_{ji} = u(x_i, t_j) \).

The heat conduction equation \( \frac{\partial u}{\partial t} \) uses the first order backward difference quotient:

\[
\frac{\partial u}{\partial t}(x_i, t_j) = \frac{1}{\Delta t}[u(x_i, t_j) - u(x_i, t_{j-1})]
\]  
(8)

For the heat conduction equation \( \frac{\partial^2 u}{\partial x^2} \), the second order central difference quotient is used:

\[
\frac{\partial^2 u}{\partial x^2}(x_i, t_j) = \frac{1}{(\Delta x_i^2)}[u(x_{i-1}, t_j) - 2u(x_i, t_j) + u(x_{i+1}, t_j)]
\]  
(9)

\( u_{ji} \) represents the temperature of the k-th layer medium at the node \( (x_i, t_j), k = 1, 2, 3, 4 \), the coupling condition \( \lambda_1 \frac{\partial u}{\partial x} = \lambda_{i+1} \frac{\partial u_{i+1}}{\partial x} \) of the medium boundary is differentiated:

\[
\lambda_k \frac{u_{k,m+1}^{i+1} - u_{k,m}^{i+1}}{\Delta x_k} = \lambda_{k+1} \times \frac{u_{k+1,m+2}^{i+1} - u_{k+1,m+1}^{i+1}}{\Delta x_{k+1}}, \quad k = 1, 2, 3
\]  
(10)

Right hand value condition \(- \lambda_4 \frac{\partial u}{\partial n_4} = h_4(u_{n_4} - \overline{u}) \) first order backward difference:

\[
- \lambda_4 \frac{u_{m_4}^{i+1} - u_{m_4}^{i+1}}{\Delta x_4} = h_4(u_{m_4}^{i+1} - 37)
\]  
(11)

Left hand value condition \(- \lambda_4 \frac{\partial u}{\partial n_4} = h_4(75 - u_1) \) first order forward difference:

\[
- \lambda_4 \frac{u_{m_4}^{i+1} - u_{m_4}^{i+1}}{\Delta x_1} = h_4(u_{m_4}^{i+1} - 37)
\]  
(12)

Equations (10) - (14) combine with the initial conditions to form the implicit difference scheme of partial differential equations, it's three pairs of linear simultaneous equations of \( u \), \( Au = b \), and we're going to use the chasing and driving method to solve them, so it takes less memory, and we're going to solve them quickly.

3.2 Determination of Conversion Factors

Determination of we two unknown parameters in the model, namely the left boundary and the right boundary of heat extraction coefficient \( h_1 \) and \( h_2 \), these two parameters will directly affect the results of the numerical fitting, we can only according to these two parameters are provided in the title of the data and the model of temperature distribution in solving the numerical fitting results of solving, the essence of which is a double variable optimization problem, which we require to \( h_1 \), \( h_2 \) numerical precision, the result of numerical simulation and the temperature distribution of error are given in the
title as small as possible, to solve the optimization problem of algorithm, genetic algorithm is stable, not easy to fall into local optimal solution, the characteristics of We consider using the genetic algorithm to solve the problem. The key to the genetic algorithm is to design the loss function. The loss function is designed as follows:

$$f(h_1, h_2) = \min \sum_{i=1}^{500} (u_i^* - u_i)$$

(13)

In the formula, $u_i^*$ represents the numerical values at different times of skin surface temperature obtained by numerical fitting, and $u_i$ represents the skin surface temperature at different times given in the problem. By minimizing the total fitting error, the optimal heat extraction coefficient is obtained.

4. Experiment

Through Matlab programming, the temperature distribution of fire protection clothing at different times and positions was solved, and the approximate and accurate values of heat extraction coefficient were obtained. The fitting data were obtained under the conditions of $h_1=90.8384$ $W/(m^2 \cdot ^\circ C)$, $h_2=8.3585$ $W/(m^2 \cdot ^\circ C)$, and the comparison data was obtained under heat extraction coefficient in reference. The changes of temperature at the interface of medium and the left and right boundary with time are shown as follows:

![Comparison between fitting data and actual data](image)

Figure 1. Comparison diagram between fitting data and actual data.

Figure 1 compares and analyzes the differences between the actual data and the fitting data. It can be found that the fitting data obtained by us is more accurate than the fitting data obtained by reference [1] under heat extraction coefficient.
Figure 2. Overall distribution of temperature.
Figure 2 shows the overall distribution of temperature. It can be found that the rising speed of temperature gradually decreases and the final temperature tends to be stable, which conforms to the objective law.

Figure 3. Temperature changes of each critical surface.
Figure 3 further analyzes the temperature change of each critical interface, \( u \) represents the temperature, \( t \) represents the time. It is found that the temperature of each critical interface increases slowly and tends to be stable value. From the outside to the inside, the final stable temperature of each critical interface gradually decreases, and the temperature difference is obvious, which indicates that the fireproof clothing plays a good protective role. The genetic algorithm is \( h_1 = 90.8384 \, W / (m^2 \cdot ^\circ C) \), \( h_2 = 8.3585 \, W / (m^2 \cdot ^\circ C) \).

5. Conclusion
Through the experimental results, we can see that the temperature distribution of each layer has the same trend, tends to a fixed value after the temperature rises for a period of time, and at the same time, the temperature of the fire protection clothing gradually decreases from inside to outside, indicating that the fire protection clothing design does have a certain insulation function. Numerical fitting temperature almost completely coincide with the actual temperature, and is superior to contrast data fitting results, shows the model has good practicability, also explains the parameters of high precision, demonstrates our parameters by solving optimization direction is correct, the parameters of the model fitting out can be used to guide the actual design of the fire service, similar problems can refer to the thinking model.
of the design.

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