We refer the reader to Part I for the notation and methods. This is an outline of the study:

(i) We define and measure BAO observables \( \hat{d}_a(z, z_c) \), \( \hat{d}_l(z, z_c) \), and \( \hat{d}_r(z, z_c) \) that do not depend on any cosmological parameter, see Part I [2]. From each of these observables we obtain the BAO correlation distance \( d_{\text{BAO}} \) with its respective dependence on cosmological parameters. These BAO observables are measured as a function of redshift \( z \) with the Sloan Digital Sky Survey (SDSS) data release DR12. From the BAO measurements alone, or together with the correlation angle \( \theta_{\text{MC}} \) of the Cosmic Microwave Background (CMB), we constrain the curvature parameter \( \Omega_k \) and the dark energy density \( \Omega_{\text{DE}}(a) \) as a function of the expansion parameter \( a \) in several scenarios. These observables are further constrained with external measurements of \( h \) and \( \Omega_b h^2 \). We find some tension between the data and a cosmology with flat space and constant dark energy density \( \Omega_{\text{DE}}(a) \).

(ii) We use the measured BAO distances, and the correlation angle \( \theta_{\text{MC}} \) of the Cosmic Microwave Background (CMB), as an uncalibrated standard ruler to constrain the correlated parameters \( \Omega_k \) and \( \Omega_{\text{DE}}(a) \). The cosmological parameters \( h \), \( \Omega_b h^2 \) and \( N_{\text{eff}} \) drop out of this analysis.

(iii) Finally we use the measured BAO distances and \( \theta_{\text{MC}} \) as a calibrated standard ruler to further constrain \( \Omega_k \) with external measurements of \( h \) and \( \Omega_b h^2 \).

The results of this analysis are compared with the final consensus results corresponding to the DR12 data [3].

**INTRODUCTION**

We present studies of baryon acoustic oscillations (BAO) with Sloan Digital Sky Survey (SDSS) data release DR12 [1]. Part I of this ongoing study is Ref. [2]. We combine the measurements of the two independent sub-samples presented in Table III of Part I [2] and Table III divided by \( \sqrt{2} \). This procedure obtains almost the same uncertainties as the method used for Table III of Part I [2] in spite of the fact that there are more galaxies in the northern galactic cap than in the southern galactic cap, i.e. we find larger background fluctuations in the north. The averages are summarized in Table III.

The two sets of measurements presented in Table III of Part I [2] and Table III have different background fluctuations due to galaxy clustering, have different fits, obtain uncertainties by different methods, and obtain essentially the same results and uncertainties. The root-mean-square of the differences of all entries in Table III of Part I [2] and Table III divided by \( \sqrt{2} \) is 0.00046 which is less than the total independent uncertainties assigned to each entry of each Table, so these measurements are consistent.

**BAO DISTANCES IN THE NORTHERN AND SOUTHERN GALACTIC CAPS**

To gain confidence in the identification of the BAO signal from among the background fluctuations, and to obtain the uncertainties by a different method, we repeat the measurements separately for galaxies in the northern and southern galactic caps. This time we analyze SDSS DR12 galaxies (passing the same quality selection flags and \( z_{\text{Err}} < 0.001 \)) with right ascension \( 110^\circ \) to \( 270^\circ \) and declination \( -5^\circ \) to \( 70^\circ \) with \( 17.0 < r_{35} < 27.0 \). The galactic plane separates this sample into two independent sub-samples defined by \( \text{dec} \gtrsim 27.0^\circ - 17.0^\circ \left[ (r - 185.0^\circ)/(260.0^\circ - 185.0^\circ) \right]^2 \). These sub-samples are referred to as northern (>) and southern (<) galactic caps. The results of measurements for G-G and G-C runs are presented in Tables II and III. Blank entries indicate that we were unable to reliably identify a BAO signal.

We define Baryon Acoustic Oscillation (BAO) observables \( \hat{d}_a(z, z_c) \), \( \hat{d}_l(z, z_c) \), and \( \hat{d}_r(z, z_c) \) that do not depend on any cosmological parameter. From each of these observables we obtain the BAO correlation length \( d_{\text{BAO}} \) with its respective dependence on cosmological parameters. These BAO observables are measured as a function of redshift \( z \) with the Sloan Digital Sky Survey (SDSS) data release DR12. From the BAO measurements alone, or together with the correlation angle \( \theta_{\text{MC}} \) of the Cosmic Microwave Background (CMB), we constrain the curvature parameter \( \Omega_k \) and the dark energy density \( \Omega_{\text{DE}}(a) \) as a function of the expansion parameter \( a \) in several scenarios. These observables are further constrained with external measurements of \( h \) and \( \Omega_b h^2 \). We find some tension between the data and a cosmology with flat space and constant dark energy density \( \Omega_{\text{DE}}(a) \).
Let us consider corrections to the BAO distances due to peculiar velocities and peculiar displacements of galaxies towards their centers. A relative peculiar velocity $v_p$ towards the center causes a reduction of the BAO distances $d_{am}(z, z_c)$, $d_{bm}(z, z_c)$, and $d_{cm}(z, z_c)$ of order $0.5v_p/c$. In addition, the Doppler shift produces an apparent shortening of $d_{cm}(z, z_c)$ by $v_p/c$, and somewhat less for $d_{cm}(z, z_c)$.

We multiply the measured BAO distances $d_{am}(z, z_c)$, $d_{bm}(z, z_c)$, and $d_{cm}(z, z_c)$ by correction factors $f_{am}$, $f_{bm}$, and $f_{cm}$ respectively. Simulations in Ref. [4] obtain $f_{am} - 1 = 0.2283 \pm 0.0069%$ and $f_{bm} - 1 = 0.2661 \pm 0.0820%$ at $z = 0.3$, $f_{cm} - 1 = 0.1286 \pm 0.0425%$ and $f_{cm} - 1 = 0.1585 \pm 0.0611%$ at $z = 1$, and $f_{am} - 1 = 0.0435 \pm 0.0293%$ and $f_{cm} - 1 = 0.0582 \pm 0.0402%$ at $z = 3$. In the following

| $z$ | $z_{\text{min}}$ | $z_{\text{max}}$ | galaxies | centers | type | 100$d_{am}(z, z_c)$ | 100$d_{bm}(z, z_c)$ | 100$d_{cm}(z, z_c)$ |
|-----|----------------|----------------|----------|--------|------|----------------|----------------|----------------|
| 0.10| 0.0            | 0.2            | 184044   | 27485  | G-C  | 3.434 ± 0.008 | 3.340 ± 0.013 | 3.369 ± 0.012 |
| 0.25| 0.2            | 0.3            | 38672    | 38672  | G-C  | 3.284 ± 0.013 | 3.223 ± 0.013 | 3.334 ± 0.009 |
| 0.35| 0.3            | 0.4            | 53462    | 53462  | G-C  | 3.246 ± 0.016 | 3.210 ± 0.013 | 3.295 ± 0.009 |
| 0.46| 0.4            | 0.5            | 86528    | 86528  | G-C  | 3.324 ± 0.009 | 3.376 ± 0.015 | 3.288 ± 0.010 |
| 0.46| 0.4            | 0.5            | 86528    | 86528  | G-C  | 3.538 ± 0.009 | 3.487 ± 0.011 | 3.483 ± 0.023 |
| 0.54| 0.5            | 0.6            | 116301   | 116301 | G-C  | 3.487 ± 0.004 | 3.319 ± 0.011 | 3.458 ± 0.015 |
| 0.54| 0.5            | 0.6            | 116301   | 116301 | G-C  | 3.380 ± 0.010 | 3.486 ± 0.011 | 3.583 ± 0.013 |
| 0.67| 0.6            | 0.9            | 67772    | 67772  | G-C  | 3.360 ± 0.030 | 3.451 ± 0.013 | 3.492 ± 0.015 |
| 0.67| 0.6            | 0.9            | 67772    | 67772  | G-C  | 3.273 ± 0.012 | 3.492 ± 0.011 | 3.525 ± 0.011 |

| $z$ | $z_{\text{min}}$ | $z_{\text{max}}$ | galaxies | centers | type | 100$d_{am}(z, z_c)$ | 100$d_{bm}(z, z_c)$ | 100$d_{cm}(z, z_c)$ |
|-----|----------------|----------------|----------|--------|------|----------------|----------------|----------------|
| 0.10| 0.0            | 0.2            | 103008   | 103008 | G-G  | 3.262 ± 0.009 | 3.350 ± 0.013 | 3.351 ± 0.003 |
| 0.25| 0.2            | 0.3            | 21112    | 21112  | G-C  | 3.420 ± 0.019 | 3.307 ± 0.008 | 3.529 ± 0.010 |
| 0.35| 0.3            | 0.4            | 29739    | 29739  | G-G  | 3.473 ± 0.012 | 3.292 ± 0.010 | 3.504 ± 0.009 |
| 0.35| 0.3            | 0.4            | 29739    | 29739  | G-G  | 3.408 ± 0.032 | 3.347 ± 0.016 | 3.464 ± 0.031 |
| 0.46| 0.4            | 0.5            | 46447    | 46447  | G-G  | 3.388 ± 0.010 | 3.327 ± 0.010 | 3.463 ± 0.011 |
| 0.46| 0.4            | 0.5            | 46447    | 46447  | G-G  | 3.501 ± 0.015 | 3.280 ± 0.027 | 3.280 ± 0.027 |
| 0.54| 0.5            | 0.6            | 65217    | 65217  | G-G  | 3.302 ± 0.022 | 3.375 ± 0.027 | 3.416 ± 0.016 |
| 0.54| 0.5            | 0.6            | 65217    | 65217  | G-G  | 3.438 ± 0.014 | 3.399 ± 0.018 | 3.406 ± 0.009 |
TABLE III: Independent measured BAO distances $d_{\alpha}(z, z_c)$, $d_z(z, z_c)$, and $d_f(z, z_c)$ in units of $c/H_0$ with $z_c = 3.79$ (see Part I [2]) obtained by averaging measurements in the northern and southern galactic caps. Each BAO distance has an independent total uncertainty $\pm 0.00060$ dominated by systematics. No corrections have been applied.

| $z$ | $z_{\text{min}}$ | $z_{\text{max}}$ | $100d_{\alpha}(z, z_c)$ | $100d_z(z, z_c)$ | $100d_f(z, z_c)$ |
|-----|-----------------|-----------------|------------------|------------------|------------------|
| 0.10| 0.0             | 0.2             | 3.348            | 3.350            | 3.346            |
| 0.25| 0.2             | 0.3             | 3.365            | 3.261            | 3.434            |
| 0.35| 0.3             | 0.4             | 3.342            | 3.315            | 3.378            |
| 0.46| 0.4             | 0.5             | 3.507            | 3.319            | 3.375            |
| 0.54| 0.5             | 0.6             | 3.361            | 3.427            | 3.481            |
| 0.67| 0.6             | 0.9             | 3.377            | 3.435            |                  |

sections we present fits with the corrections

\[ f_a - 1 = 0.320% a^{1.35}, \]
\[ f_z - 1 = 0.381% a^{1.35}, \]
\[ f_f - 1 = 0.350% a^{1.35}. \]  

The effect of these corrections can be seen by comparing the first two fits in Table V below. Fits with corrections $\sim 15$ times larger, or no corrections at all, are presented in Part I [2]. An order-of-magnitude estimate of this correction can be obtained by calculating the r.m.s. $v_\perp$ corresponding to modes with $k \equiv 2\pi/\lambda < 2\pi/(4d_{\alpha}^{\text{BAO}})$ with Eq. (11) of Ref. [5] and normalizing the result to $\sigma_\perp$, i.e. to the r.m.s. density fluctuation in a volume $(8\text{Mpc/h})^3$.

COMPARISON WITH THE FINAL CONSENSUS DR12 ANALYSIS

We compare the measured BAO observables in Table III of Part I [2] and Table III with the final consensus “BAO+FS” analysis of the DR12 data set [3] which is summarized in Table IV. The notation of Ref. [3] is related to our notation as follows:

\[ D_M \frac{r_{d, \text{fid}}}{r_d} = \frac{c}{H_0} \chi(z) \frac{r_{d, \text{fid}}}{d_{\text{BAO}}} = \frac{c}{H_0} \frac{r_{d, \text{fid}}}{d_{\alpha}(z, z_c)} \exp(-z/z_c), \]

\[ H \frac{r_d}{r_{d, \text{fid}}} = H_0 E(z) \frac{d_{\text{BAO}}}{r_{d, \text{fid}}} = \frac{H_0}{r_{d, \text{fid}}} (1 - z/z_c) \exp(-z/z_c). \]  

where $r_{d, \text{fid}} = 147.78$ Mpc and $H_0 = 67.8 \pm 1.2$ km s$^{-1}$ Mpc$^{-1}$. We find agreement within the quoted uncertainties between our measurements in Tables III of Part I [2] or III and the final consensus measurements in Table IV.

Table IV also shows $\Omega_{\text{DE}}(z)$ extracted from $H$ with $\Omega_k = 0$ and $\Omega_m = 0.310 \pm 0.005$ [3]. These values of $\Omega_{\text{DE}}(z)$ are in agreement with our results in Fig. 2 below. The observed increase of $\Omega_{\text{DE}}(z)$ with $z$ was studied in Part I [2].

CONSTRAINTS ON $\Omega_k$ AND $\Omega_{\text{DE}}(a)$ FROM UNCALIBRATED BAO

Let us try to understand qualitatively how the BAO distance measurements presented in Table III constrain the cosmological parameters. In the limit $z \rightarrow 0$ we obtain $d_{\text{BAO}} = \hat{d}_{\alpha}(0, z_c) = \hat{d}_z(0, z_c) = \hat{d}_f(0, z_c)$, so the row with $z = 0.1$ in Table III approximately determines $d_{\text{BAO}}$. This $d_{\text{BAO}}$ and the measurement of, for example, $d_z(0.3, z_c)$ then constrains the derivative of $\Omega_m a^3 + \Omega_{\text{DE}} + \Omega_k a^2$ with respect to $a$ at $z \simeq 0.3$, i.e. constrains approximately $\Omega_{\text{DE}} + 0.5\Omega_k$ or equivalently $\Omega_{\text{DE}} - \Omega_m$. We need an additional constraint for Scenario 1. At small $a$, $E(a)$ is dominated by $\Omega_m$, so $\theta_{\text{MC}}$ plus $d_{\text{BAO}}$ approximately constrain $\Omega_m$, or equivalently $\Omega_{\text{DE}} + \Omega_k$, see Eq. (19) of Part I [2]. The additional BAO distance measurements in Table III then also constrain $\omega_0$ and $\omega_b$ or $\omega_1$.

We now constrain cosmological parameters with each of these three sets of independent BAO measurements: 18 BAO distances in Table III of Part I [2], 12 BAO distances in Table IV of Part I (rows with $0 < z < 0.4$ G-C, $0.4 < z < 0.5$ G-C, $0.5 < z < 0.6$ G-LC, and $0.6 < z < 0.9$ LG-LG), or 17 BAO distances in Table III.

In Table V we present the cosmological parameters obtained by minimizing the $\chi^2$ with 17 terms corresponding to the 17 BAO distance measurements in Table III for several scenarios. We find that the data is in agreement with the simplest cosmology with $\Omega_k = 0$ and $\Omega_{\text{DE}}(a)$ constant with $\chi^2$ per degree of freedom (d.f.) 15.4/15, so no additional parameter is needed to obtain a good fit to this data.

For the sets of 18, 12 or 17 BAO measurements we obtain, respectively, $\Omega_{\text{DE}} + 0.6\Omega_k = 0.620 \pm 0.030$, $0.652 \pm 0.041$, and $0.647 \pm 0.031$ for constant $\Omega_{\text{DE}}(a)$, or $0.638 \pm 0.058$, $0.585 \pm 0.063$, and $0.627 \pm 0.075$ if $\Omega_{\text{DE}}(a)$ is allowed to depend on $a$ as in Scenario 4. We present the variable $\Omega_{\text{DE}} + 0.6\Omega_k$ instead of $\Omega_{\text{DE}}$ because it has a smaller uncertainty. The constraints on $\Omega_k$ are weak.

In Table V we present the cosmological parameters obtained by minimizing the $\chi^2$ with 19 terms corresponding to the 18 BAO distance measurements listed in Table III of Part I [2] plus the correlation angle $\theta_{\text{MC}} = 0.010413 \pm 0.000006$ of the CMB [6]. We present the variable $\Omega_{\text{DE}} + 2\Omega_k$ instead of $\Omega_{\text{DE}}$ because it has a smaller uncertainty. The corresponding fits for the 17 BAO measurements of Table III plus $\theta_{\text{MC}}$ are presented in Table VII.

From the fits for $\theta_{\text{MC}}$ plus the set of 18, or 12 or
TABLE V: Cosmological parameters obtained from the 17 BAO measurements in Table III in several scenarios. Corrections for peculiar motions are given by Eq. (1) except, for comparison, the fit "1*" which has no correction. Scenario 1 has $\Omega_{DE}(a)$ constant. Scenario 3 has $w = w_0$. Scenario 4 has $\Omega_{DE}(a) = \Omega_{DE}[1 + w_1(1 - a)]$.

| $\Omega_{DE}$ | Scenario 1* | Scenario 1 | Scenario 1 | Scenario 3 | Scenario 4 | Scenario 4 |
|---------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\Omega_k$    | 0 fixed     | 0 fixed     | -0.413 ± 0.234 | 0 fixed     | 0 fixed     | -0.377 ± 0.263 |
| $w_0$ or $w_1$| n.a.        | n.a.        | n.a.        | 1.346 ± 0.349 | n.a.        | n.a.        |
| $\chi^2$/d.f. | 3.37 ± 0.03 | 3.38 ± 0.03 | 3.40 ± 0.03 | 3.44 ± 0.07  | 3.44 ± 0.07  | 3.42 ± 0.07  |

17 BAO measurements we obtain, respectively, $\Omega_{DE} + 2\Omega_k = 0.718 ± 0.006$, $0.749 ± 0.024$, or $0.717 ± 0.007$ when $\Omega_{DE}(a)$ is allowed to vary as in Scenario 4. The constraints on $\Omega_k$ are, respectively, $0.037 ± 0.011$, $0.043 ± 0.015$, or $0.022 ± 0.012$ for constant $\Omega_{DE}(a)$, or $0.044 ± 0.041$, $0.116 ± 0.057$, or $0.060 ± 0.052$ when $\Omega_{DE}(a)$ is allowed to vary as in Scenario 4. The constraints on $w_1$ are respectively $0.74 ± 0.24$, $1.00 ± 0.40$, or $0.38 ± 0.23$ for $\Omega_k = 0$.

Note that the BAO plus $\theta_{MC}$ data is consistent with $\Omega_k = 0$ or with constant $\Omega_{DE}(a)$, i.e. $w_1 = 0$, but there is some tension when both constraints are applied. A summary of tensions is presented in Table VIII. The tension is not statistically significant for the 17 BAO plus $\theta_{MC}$ data (in part because the 17 BAO set has no measurement of $\hat{d}(0.67, z_c)$, see Figures 1 and 2 below).

MEASUREMENT OF $\Omega_{DE}(a)$

We obtain $\Omega_{DE}(a)$ from the 5 independent measurements of $\hat{d}(z, z_c)$ in Table III and Eqs. (17) and (2) of Part I [2], for the case $\Omega_k = 0$. The values of $d_{BAO}$ and $\Omega_m = 1 - \Omega_{DE} - \Omega_k$ are obtained from the fit for Scenario 4 in Table VII. The results are presented in Fig. 4. To guide the eye, we also show the straight line corresponding to the central values of $\Omega_{DE}$ and $w_1$ of the fit for Scenario 4.

To check the robustness of $\Omega_{DE}(a)$ in Fig. 4 we add the 6 measurements of $\hat{d}(z, z_c)$ in Table III of Part I [2] and the 17 measurements of $\hat{d}(z, z_c)$ in Table IV of Part I and obtain Fig. 2. Note that these measurements of $\hat{d}(z, z_c)$ are partially correlated. Note that there is tension with a constant $\Omega_{DE}(a)$ for $0 < a < 0.6$. Note also that the final consensus measurements of DR12 [3] in the last column of Table IV are in agreement with Fig. 2 and also show the tension with $\Omega_k = 0$ and constant $\Omega_{DE}(a)$.

We repeat these two figures for the fit for Scenario 1 in Table VII see Figs. 3 and 2. For these figures $\Omega_k = 0.0218$. The excesses of $\Omega_{DE}(a)$ for $0.6 < a < 0.67$ are not understood.

CONSTRAINTS ON $\Omega_k$ AND $\Omega_{DE}(a)$ FROM CALIBRATED BAO

Up to this point we have used the BAO distance $d_{BAO}$ as an uncalibrated standard ruler. The cosmological parameters $h$, $\Omega_b h^2$ and $N_{\text{eff}}$ drop out of such an analysis. In this Section we consider the BAO distance $d_{BAO} = r_S$ as a calibrated standard ruler and use independently measured $h$ and $\Omega_b h^2$, while keeping $N_{\text{eff}} = 3.36$ fixed, to further constrain the cosmological parameters.

The sound horizon is calculated from first principles [7] as follows:

$$r_S = \int_0^{a_{\text{dec}}} \frac{c_s da}{a} = \int_0^{a_{\text{dec}}} \frac{c_s da}{H_0 a^2 E(a)},$$

where the speed of sound is

$$c_s = \frac{c}{\sqrt{3(1 + 3\rho_0 a^2/4\rho_0)}}.$$
TABLE VI: Cosmological parameters obtained from the 18 BAO measurements in Table III of Part I [2] plus $\theta_{MC}$ in several scenarios. Corrections for peculiar motions are given by Eq. (1). Scenario 1 has $\Omega_{DE}(a)$ constant. Scenario 2 has $w(a) = w_0 + w_a(1-a)$. Scenario 3 has $w = w_0$. Scenario 4 has $\Omega_{DE}(a) = \Omega_{DE}[1 + w_0(1-a)]$.

| Scenario 1 | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 4 |
|------------|------------|------------|------------|------------|------------|
| $\Omega_k$ | 0 fixed    | 0 fixed    | 0 fixed    | 0 fixed    | 0 fixed    |
| $\Omega_{DE} + 20k$ | 0.735 ± 0.004 | 0.718 ± 0.006 | 0.780 ± 0.082 | 0.726 ± 0.005 | 0.720 ± 0.006 |
| $w_0$ | n.a. | n.a. | n.a. | n.a. | n.a. |
| $w_a$ or $w_1$ | n.a. | n.a. | n.a. | n.a. | n.a. |
| $10^3\sigma_{BAO}$ | 3.47 ± 0.02 | 3.39 ± 0.03 | 3.40 ± 0.05 | 3.36 ± 0.04 | 3.35 ± 0.04 |
| $\chi^2/\text{d.f.}$ | 36.7/17 | 23.6/16 | 23.0/15 | 24.0/16 | 24.8/16 |

TABLE VII: Cosmological parameters obtained from the 17 BAO measurements in Table III plus $\theta_{MC}$ in several scenarios. Corrections for peculiar motions are given by Eq. (1). Scenario 1 has $\Omega_{DE}(a)$ constant. Scenario 2 has $w(a) = w_0 + w_a(1-a)$. Scenario 3 has $w = w_0$. Scenario 4 has $\Omega_{DE}(a) = \Omega_{DE}[1 + w_0(1-a)]$.

| Scenario 1 | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 4 |
|------------|------------|------------|------------|------------|------------|
| $\Omega_k$ | 0 fixed    | 0.022 ± 0.012 | 0 fixed    | 0 fixed    | 0 fixed    |
| $\Omega_{DE} + 20k$ | 0.727 ± 0.004 | 0.716 ± 0.007 | 0.767 ± 0.071 | 0.721 ± 0.005 | 0.719 ± 0.006 |
| $w_0$ | n.a. | n.a. | n.a. | n.a. | n.a. |
| $w_a$ or $w_1$ | n.a. | n.a. | n.a. | n.a. | n.a. |
| $10^3\sigma_{BAO}$ | 3.43 ± 0.02 | 3.38 ± 0.03 | 3.41 ± 0.05 | 3.37 ± 0.04 | 3.37 ± 0.04 |
| $\chi^2/\text{d.f.}$ | 19.5/16 | 15.7/15 | 15.0/14 | 16.1/15 | 16.4/15 |

TABLE VIII: There is some tension between the data and fits with $\Omega_k = 0$ and $\Omega_{DE}(a)$ constant, i.e. $w_1 = 0$ in Scenario 4. Shown is the reduction of the $\chi^2$ of the fits when either $\Omega_k$ or $w_1$ is released. The BAO measurements correspond to Tables III or IV of Part I [2], or Table III Entries with * have no significant tension with $\Omega_k = 0$ and constant $\Omega_{DE}(a)$.

| Reduction of $\chi^2$ by releasing | $\Omega_k$ | $w_1$ |
|-----------------------------------|------------|--------|
| Data:                            |            |        |
| 18 BAO + $\theta_{MC}$           | 13.1       | 11.9   |
| 12 BAO + $\theta_{MC}$           | 10.8       | 8.9    |
| 17 BAO + $\theta_{MC}$ *         | 3.8        | 3.1    |
| 18 BAO + $\theta_{MC}$ + $A = 1.000 \pm 0.022$ | 12.7       | 10.4   |
| 12 BAO + $\theta_{MC}$ + $A = 1.000 \pm 0.022$ | 9.8        | 6.9    |
| 17 BAO + $\theta_{MC}$ + $A = 1.000 \pm 0.022$ * | 4.0        | 2.3    |
| 18 BAO + $\theta_{MC}$ + $A = 0.968 \pm 0.012$ | 4.9        | 16.3   |
| 12 BAO + $\theta_{MC}$ + $A = 0.968 \pm 0.012$ | 2.2        | 16.7   |
| 17 BAO + $\theta_{MC}$ + $A = 0.968 \pm 0.012$ | 0.6        | 5.7    |

We can write the result for our purposes as

$$ r_S = 0.03389 \times A \times \left( \frac{0.30}{\Omega_m} \right)^{0.255} $$

where

$$ A = \left( \frac{h}{0.72} \right)^{0.489} \left( \frac{0.023}{\Omega_{DE} h^2} \right)^{0.098} \left( \frac{3.36}{N_{\text{eff}}} \right)^{0.245} $$

(we have neglected the dependence of $\Omega_{\text{dec}} = 1090.2 \pm 0.7$ on the cosmological parameters). We take $\Omega_h h^2 = 0.023 \pm 0.002$ from Big-Bang nucleosynthesis [9]. With the latest direct measurement $h = 0.720 \pm 0.030$ by the HUBBLE satellite [8] we obtain $A = 1.000 \pm 0.022$. The alternative value $h = 0.673 \pm 0.012$ is obtained from PLANK + WP + high L [6] assuming $\Omega_k = 0$ and constant $\Omega_{DE}(a)$. For this $h$ we obtain $A = 0.968 \pm 0.012$. The cosmological parameters that minimize the $\chi^2$ with 19 terms (17 BAO measurements from Table III plus $\theta_{MC}$ plus $A$) are presented in Table IX.

From the fits to $\theta_{MC}$ plus $A$ plus each of the sets of 18, 12 or 17 BAO measurements we obtain, for free $\Omega_{DE}(a)$ as in Scenario 4, $\Omega_k = 0.027 \pm 0.018$, 0.031 ± 0.019, and 0.023 ± 0.019 for $A = 1.000 \pm 0.22$, and 0.001 ± 0.010, 0.006 ± 0.012, and $-0.004 \pm 0.010$ for $A = 0.968 \pm 0.012$. Note that the external constraint from $A$ reduces the uncertainty on $\Omega_k$.

**CONCLUSIONS**

The results of these studies are:

(i) We define and measure BAO observables $\hat{d}_a(z, z_c)$, $\hat{d}_z(z, z_c)$, and $\hat{d}_{\perp}(z, z_c)$ that do not depend on any cosmological parameter. From each of these observables we obtain the BAO correlation distance $d_{\text{BAO}}$ in units of $c/H_0$ with its respective dependence on the cosmological parameters. It is difficult to distinguish the BAO signal from the background fluctuations due to the clustering of galaxies. To gain confidence in the results we repeat the measurements many times with different galaxy selections to obtain different background fluctuations. The measured BAO observables in Tables III and IV of Part I [2] and Table III are the main result of these studies. These measurements in combination with independent
TABLE IX: Cosmological parameters obtained from the 17 BAO measurements in Table III plus $\theta_{MC}$ plus $A$ in several scenarios. Corrections for peculiar motions are given by Eq. (1). Scenario 1 has $\Omega_{DE}(a)$ constant. Scenario 4 has $\Omega_{DE}(a) = \Omega_{DE}[1 + \omega_1(1 - a)]$.

| $A$ | Scenario 1 | Scenario 1 | Scenario 4 | Scenario 4 | Scenario 4 | Scenario 4 |
|-----|------------|------------|------------|------------|------------|------------|
| $\Omega_k$ | 0.000 ± 0.022 | 0.068 ± 0.012 | 0.000 ± 0.022 | 0.068 ± 0.012 | 0.068 ± 0.012 | 0.068 ± 0.012 |
| $\Omega_{DE} + 2\Omega_k$ | 0.727 ± 0.004 | 0.726 ± 0.004 | 0.721 ± 0.006 | 0.716 ± 0.007 | 0.717 ± 0.006 | 0.719 ± 0.007 |
| $\Omega_{DE}$ | n.a. | n.a. | 0.313 ± 0.213 | -0.049 ± 0.377 | 0.457 ± 0.201 | 0.488 ± 0.212 |
| $1000d_{BAO}$ | 3.44 ± 0.02 | 3.43 ± 0.02 | 3.38 ± 0.04 | 3.39 ± 0.04 | 3.35 ± 0.04 | 3.36 ± 0.04 |
| $\chi^2$/d.f. | 19.7/17 | 22.6/17 | 17.4/16 | 15.7/15 | 16.9/16 | 16.7/15 |

(iii) From the BAO measurements alone we obtain the constraint $\Omega_{DE} + 0.6\Omega_k = 0.647 \pm 0.031$ for constant $\Omega_{DE}(a)$, or $0.627 \pm 0.075$ when $\Omega_{DE}(a)$ is allowed to depend on $a$ as in Scenario 4. See Table VII for fits in several scenarios.

(iv) From the BAO measurements plus $\theta_{MC}$ from the CMB we obtain the constraints $\Omega_{DE} + 2\Omega_k = 0.717 \pm 0.007$ and $\Omega_k = 0.060 \pm 0.052$ when $\Omega_{DE}(a)$ is allowed to vary as in Scenario 4. See Tables VI and VII for fits in several scenarios. The cosmological parameters $h$, $\Omega_0 h^2$ and $N_{eff}$ drop out of this analysis.

(v) The data is consistent with the constraint $\Omega_k = 0$ or the constraint $\Omega_{DE}(a)$ constant, but there is some tension when both constraints are required. These tensions are presented in Table VIII and in Fig. 2. The measured excess of $\Omega_{DE}(a)$ for $0.6 < a < 0.67$ is not understood.

(vi) We note that the final consensus results of DR12 data 3 also show this tension, see last column in Table IV which is in agreement with Fig. 2. Finally we note that the BAO measurements in this study are in agreement with 3; compare Table V with Tables III or IV of Part I 2 or Table III. The two studies are complementory.
FIG. 3: Measurements of $\Omega_{DE}(a)$ obtained from the 5 $d_z(z, z_c)$ in Table III for $\Omega_k = 0.0218$, and the corresponding $d_{BAO}$ and $\Omega_{DE}$ from the fit for Scenario 1 in Table VII. The straight line is $\Omega_{DE}(a) = 0.6723$ constant from the central value of this fit. The uncertainties correspond only to the total uncertainties of $d_z(z, z_c)$. For clarity some offsets in $a$ have been applied. We present results for $(d_{BAO}, \Omega_{DE}) = (0.03383 - 0.00033, 0.6723)$ (squares), $(0.03383 + 0.00033, 0.6723)$ (triangles), $(0.03383, 0.6723 - 0.0068)$ (inverted triangles), and $(0.03383, 0.6723 + 0.0068)$ (circles).

FIG. 4: Same as Figure 3 with the addition of the 6 measurements of $d_z(z, z_c)$ in Table III of Part I [2], and the 17 measurements of $d_z(z, z_c)$ in Table IV of Part I. These measurements are partially correlated. We present results for $(d_{BAO}, \Omega_{DE}) = (0.03383, 0.6723)$.

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