Quark number scaling of hadronic $p_T$ spectra and constituent quark degree of freedom in p-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

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We show that the experimental data of $p_T$ spectra of identified hadrons released recently by ALICE collaboration for p-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV exhibit a distinct universal behavior — the quark number scaling. We further show that the scaling is a direct consequence of quark (re-)combination mechanism of hadronization and can be regarded as a strong indication of the existence of the underlying source with constituent quark degree of freedom for the production of hadrons in p-Pb collisions at such high energies. We make also predictions for production of other hadrons.

Introduction — The striking features observed recently by ALICE and CMS collaborations for high multiplicity events at Large Hadron Collider (LHC) such as long range angular correlations [1, 2], flow-like patterns [3], enhanced strangeness [4, 5] and baryon to mesons ratios at soft transverse momenta [6, 7] have attracted many discussions [8–17]. A core problem is whether Quark Gluon Plasma (QGP) is also formed in such small system in $pp$ and p-Pb collisions. At the same time, a series of measurements of transverse momentum $p_T$ spectra have also been carried out and high accuracy data have been obtained [4, 18, 19] even for decuplet hyperons such as $\Omega^-$, $\Xi^-$ and $\Sigma^-$ and vector mesons such as $\phi$ and $K^*$ in p-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. Because the decay influence is almost negligible, behaviors of such hadrons are usually believed as carrying more direct information from hadronization. It is thus of particular interest to see whether such data [4, 18, 19] show any regularities that may lead to deeper insights into reaction mechanism.

Quark number scaling of $p_T$ spectra in p-Pb collisions at LHC energies — For all the decuplet hyperons and vector mesons, we see in particular that $\Omega^-$ and $\phi$ are composed of only strange quarks (antiquarks). Besides the $s$-quark momentum distribution, their momentum spectra should be solely determined by the hadronization mechanism. Indeed, by looking at the midrapidity data on $p_T$ spectra of $\Omega^-$ and $\phi$ [4, 18], we see a very distinct feature. If we divide $p_T$, by the number $n_i$ of constituent quark(s) and/or antiquark(s), i.e. $p_T/3$ for $\Omega^-$ and $p_T/2$ for $\phi$, and compare the inverse quark number $1/n_i$ power of $p_T$ spectra, i.e. $f^{1/3}_\Omega$ and $f^{1/2}_\phi$, with each other, we see that they are parallel to each other. [Here, $f_b(p_T) = dN_b/dp_Tdy$ is the $p_T$ spectrum of $b$ at midrapidities.] This can be seen clearly in Fig. 1 where we re-normalize the data by a constant so that they just fall on one line. More precisely, we see that the data exhibit the following regularity

$$f^{1/3}_\Omega(p_T) = \kappa_{\phi,\Omega} f^{1/2}_\phi(2p_T),$$

where $\kappa_{\phi,\Omega}$ is a constant independent of $p_T$. In other words, both $f^{1/3}_\Omega$ and $f^{1/2}_\phi$ are given by a single $f_\phi(p_T)$ as

$$f_\phi(3p_T) = \kappa_\phi f_\phi^3(p_T),$$

where $\kappa_\phi$ and $\kappa_\Omega$ are constants, and $\kappa_{\phi,\Omega} = \kappa_\phi^{1/3}/\kappa_\phi^{1/2}$. We call this property the “quark number scaling” because it is similar to that of the elliptic flow of identified hadrons observed in relativistic heavy ion collisions [20–22].

Such a simple scaling behavior is consistent with that observed in AA [23] but is surprising for $pA$ collisions. We therefore continue to examine the data [18, 19] for other hadrons such as $\Xi^0$ and $K^{*0}$. Unfortunately, the results show that such simple scaling behavior is slightly violated. However, if we introduce a modification factor $r$ for quarks and antiquarks $\phi$ [3, 12] instead of $p_T/3$ for $\Xi^0$ and $p_T/2$ for $K^{*0}$, and divide that for $\Xi^0$ by that for $K^{*0}$, the result is again parallel to $f_\phi(p_T)$ obtained from $f^{1/3}_\Omega$ and $f^{1/2}_\phi$. More precisely, we obtain

$$f_{\Xi^0}((2+1) p_T) f_{K^{*0}}((1+1) p_T) = \kappa_{\phi,\Xi^0} f_\phi^{1/2}(2p_T),$$

where $\kappa_{\phi,\Xi^0}$ is a constant. In panels (b), (d), (e) of Fig. 1, we show results obtained this way as a function of $p_T$. We see that the scaling behavior is quite impressive.

QCM and constituent quark degree of freedom — We show that the scaling behavior given by Eqs. (1-4) and demonstrated in Fig. 1 is a direct consequence of the quark (re-)combination mechanism (QCM) [22, 24–33] for quarks and antiquarks with independent momentum distributions.

The situation for hadrons composed of quark(s) and/or antiquark(s) of only one flavor such as $\Xi^0$ and $\phi$ discussed above is very simple. For consistency, we start with the general formulae. As formulated explicitly in e.g. [30], in general, in QCM, for a baryon $B_j$ composed of $q_1q_2q_3$ and a meson $M_j$ composed of $q_1\bar{q}_2$, we have

$$f_{B_j}(p_B) = \int dp_1 dp_2 dp_3 R_{B_j}(p_1, p_2, p_3; p_B) \times f_{q_1\bar{q}_2q_3}(p_1, p_2, p_3),$$

$$f_{M_j}(p_M) = \int dp_1 dp_2 R_{M_j}(p_1, p_2; p_M) f_{q_1\bar{q}_2}(p_1, p_2),$$

where $f_{q_1\bar{q}_2q_3}(p_1, p_2, p_3)$ is the joint momentum distribution for $q_1$, $q_2$, and $q_3$; and $R_{B_j}(p_1, p_2, p_3; p_B)$ is the combination
the hadron, i.e., $p$-p and $r$ distributions of quarks and antiquarks, we have

$$f_q(\vec{p}_T) = f_{q_1}(p_1)p_{q_1}/\kappa_{q_1},$$

and $(b)$

$\Omega^-$, $\phi$, $\Xi^0$, and $K^0$ in $p$-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The data of $\Xi^0$ and $K^0$ in 0-20% multiplicity class are compared with those of $\Omega^-$ and $\phi$ in 5-10% multiplicity classes in panel (b), and others are in the same multiplicity classes. $r = 2/3$ and $\kappa_{q,\bar{q}}$ in six classes are (0.4, 0.425, 0.425, 0.425, 0.435, 0.465) and $\kappa_{q,q',\Xi^0}$ in three classes (0.08, 0.089, 0.09). Insets show the data of $p_T$ spectra of these hadrons taken from [4, 18, 19].

Suppose the combination takes place mainly for quark and/or antiquark that takes a given fraction of momentum of the hadron, i.e.,

$$R_B(p_1, p_2, p_3; p_B) = \frac{1}{2}\sum_{i=1}^{3} \delta(p_i - x_ip_B),$$

and

$$R_M(p_1, p_2; p_M) = \frac{1}{2}\sum_{i=1}^{2} \delta(p_i - x_ip_M).$$

we obtain

$$f_B(p_B) = \kappa_B f_q(x_1p_B)f_q(x_2p_B)f_q(x_3p_B),$$

and

$$f_M(p_M) = \kappa_M f_q(x_1p_M)f_q(x_2p_M).$$

Now, for the simplest case, i.e., for hadrons such as $\Omega^-$ and $\phi$ that are composed of only strange quark(s) and/or antiquark(s), we obtain immediately the results given by Eqs. (2) and (3) from Eqs. (11) and (12) respectively if we take $f_s(p_T) = f_s(p_T)$. The combination takes place for three quarks with the same $p_{r_s}/3$ to form a $\Omega^-$ with $p_{r_s}$ and $s$ with $p_{r_s}/2$ to form a $\phi$ with $p_{r_s}$.

For combinations of quark(s) and/or antiquark(s) with different flavors such as $\Xi^0$ and $K^*$, the result given by Eq. (4) is actually a direct consequence of combination of equal transverse velocity. We recall that the velocity $v = p/E = p/\gamma m$. 
Equal velocity implies \( p_i = \gamma m_i \propto m_i \) that leads to

\[
x_i = m_i / \sum_{r} m_r.
\]

We denote \( x_u / x_d = x_s / x_s = m_u / m_s = r \) and we obtain

\[
f_{2e} = f_2^{(2 + r)p_T} = \kappa_e f_2^{(p_T)} f_2(r p_T),
\]

\[
f_{K} = f_2^{(1 + r)p_T} = \kappa_K f_2^{(p_T)} f_2(r p_T).
\]

This leads immediately to Eq. (4) and \( r \approx 2 / 3 \) if we take \( m_s = 500 \text{MeV} \) and \( m_u = m_d = 330 \text{MeV} \). Here, we take \( f_2(p_T) = f_2^{(p_T)} = f_2^{(2 p_T)} = f_2^{(3 p_T)} \) for the midrapidity region at LHC.

We see clearly that the quark number scaling exhibited by the hadronic \( p_T \)-spectra in \( p-\text{Pb} \) collisions is a direct consequence of QCM of quarks and antiquarks with independent momentum distributions. The combination takes place among quarks and/or antiquarks with the same transverse velocity. Furthermore, we obtain also the following direct results.

1. We see that \( f_2^{(p_T)} \) in Eq. (2) or (3) is nothing else but the \( p_T \) spectrum of the strange quarks and antiquarks. We can easily extract the \( p_T \) spectra of the constituent quarks and antiquarks at hadronization from the data [4, 18, 19].

Inspired by the Lévy-Tsallis function [34] for \( p_T \) spectra of hadrons, we use the following form to parameterize the \( p_T \)-distribution for quarks

\[
f_q^{(0)}(p_T) = N_q \sqrt{p_T} \left[ 1 + \frac{1}{n_q \epsilon_q} \left( \sqrt{p_T^2 + m_q^2} - m_q \right) \right]^{-n_q},
\]

where \( N_q \) is the normalization constant and we use a superscript \((n)\) to denote the normalized \( p_T \)-distribution. By using the data of \( \phi \) [18] and Eq. (3), we fix the parameters \( n_s \) and \( \epsilon_s \) for strange quarks. For \( u \) and \( d \) quarks, we use data of \( K^{(0)} \) [18] and Eq. (15). The obtained results for these parameters in different multiplicity classes are shown in Table I [35]. We see in particular that \( n_q \) decreases with decreasing centrality and \( n_s \) is larger than \( n_u \). As an example, we plot \( f_q^{(n)}(p_T) \) and \( f_s^{(n)}(p_T) \) in 20-40% multiplicity class in Fig. 2. We see that the obtained \( p_T \)-spectrum for strange quarks is harder than that for \( u \) or \( d \) quarks for \( p_T \) less than 3 GeV. We also plot the ratio between them where we see it raises with \( p_T \) and seems to reach the maximum at \( p_T \) around 3 GeV. These behaviors are similar to those obtained in \( AA \) collisions at RHIC and LHC energies [23, 36, 37].

2. By applying Eqs. (11) and (12) to other hadrons such as \( \Lambda \) and \( \rho \), we obtain e.g.,

\[
f^{1/3}_\Lambda(3 p_T) = \kappa_{\rho, \Lambda} f^{1/2}_\rho(2 p_T) = \kappa_{\rho, \Lambda} f^{1/2}_\rho(2 p_T),
\]

and other similar results that can be tested by future experiments.

3. We note that the constants \( \kappa_B \) and \( \kappa_M \) can be obtained from Eqs. (11) and (12) as

\[
\kappa_B = N_{q_i,q_j} \langle N_{B_i} \rangle / \langle N_{q_i} \rangle \langle N_{q_j} \rangle \langle N_{q_i} \rangle,
\]

\[
\kappa_M = N_{q_i,q_j} \langle N_{M_i} \rangle / \langle N_{q_i} \rangle \langle N_{q_j} \rangle,
\]

where \( \langle N_h \rangle \) is the average yield of \( h \), \( \langle N_{q_i} \rangle \) is the average number of \( q_i \) and \( N_{q_i,q_j} \) and \( N_{q_i,q_j} \) are determined by the normalization conditions

\[
N_{q_i,q_j} \int dp_T \prod_{i=1}^{3} f_q^{(0)}(x_i p_T) = 1,
\]

\[
N_{q_i,q_j,q_k} \int dp_T f_{q_i}^{(0)}(x_1 p_T) f_{q_j}^{(0)}(x_2 p_T) = 1,
\]

respectively. We see that besides the normalization constant that depends on the shape of \( f_q^{(n)}(p_T) \), the constant \( \kappa_q \) is determined by the average yield of hadron and average numbers of quarks and/or antiquarks.

We recall that, in QCM, roughly speaking, if we take the approximation that the probability for a \( q \bar{q} \) to form a meson or a \( q q q \) to form a baryon is flavor independent, the relative average yields of hadrons are well determined with a few parameters. This was formulated explicitly in e.g. [30], where we found [38]

\[
\langle N_{M_i} \rangle = C_M p_{q_i} p_{\bar{q}_i} \langle N_{M_i} \rangle,
\]

\[
\langle N_{B_i} \rangle = C_B N_{iter} p_{q_i} p_{\bar{q}_i} p_{q_i} \langle N_{B_i} \rangle,
\]

where \( C_M \) is the probability for a produced meson to be \( M_j \) if it is of flavor \( q_i \bar{q}_j \) and similar for \( C_B \), they are determined by the vector to pseudo-scalar meson ratio \( R_{V/P} \) and the decuplet to octet baryon ratio \( R_{D/O} \) respectively; \( N_{iter} \) is number of iteration for \( q_i \bar{q}_j \); \( \langle N_{M_i} \rangle \) and \( \langle N_{B_i} \rangle \) are average numbers of mesons and baryons and \( \langle N_{B}/\langle N_{M} \rangle \approx 1/12 \) in QCM; \( p_{q_i} \) is the probability for a quark \( q \) to take flavor \( q_i, p_{\bar{q}_i} : p_{q_i} : p_{\bar{q}_i} = 1 : 1 : \lambda \) and \( \lambda \) is the strangeness suppression
By inserting Eqs. (22-23) into (18-19), we obtain [38]
\[ \kappa_{B_i} \approx N_{q_{1|q_0}} C_{B_i} N_{\text{dec}} / 15 \langle N_q \rangle^2, \]
\[ \kappa_{M_j} \approx 4N_{q_{1|q_0}} C_{M_j} / 5 \langle N_q \rangle, \]
where \( \langle N_q \rangle = \sum_q \langle N_{q_i} \rangle \) is total quark number. Hence, extracting \( \langle N_q \rangle^{\text{cut}}(p_T) \) from the data on \( \phi \) and \( K^* \) [18, 19], we can not only calculate the shapes but also the relative heights of \( p_T \)-spectra of other hadrons by using Eqs. (11-12) and (24-25). Taking also the decay contributions into account, we calculate the \( p_T \)-spectra for hadrons where data are available [4, 7, 18, 19]. The results are shown in Fig. 3. The fitted values of \( \langle N_q \rangle \) and \( \lambda \) are given in Table I; \( R_{\psi/p} \) and \( R_{p/p} \) are taken as 0.45 and 0.4 respectively. We see that they are in good agreement with the data [4, 7, 18, 19].

**Summary and discussions** — We show that the LHC data on \( p_T \)-spectra in \( p+Pb \) collisions exhibit an explicit quark number scaling. The scaling behavior is a direct consequence of quark combination mechanism of quarks and antiquarks with independent momentum distribution under “equal velocity combination”. This result is a strong evidence that constituent quark degree of freedom plays an important role in hadronization also in such “small system” and may be considered as a signature of formation of the deconfined system in \( p+Pb \) collisions at such high energy. The mechanism provides not only a simple way to extract quark \( p_T \) spectra from data but also a simple way to calculate \( p_T \) spectra for different hadrons.

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For event classes shown in Fig.1 where data on $\phi$ and $K^0$ are available, we fix the parameters by fitting these data as described here. For other event classes where data on $\phi$ and $K^0$ are not available, we fix the parameters by fitting of the data on $p$, $\Lambda$ and $\Xi$ as given in Fig.3 after taking the decay contributions into account.

For very small $\langle N_q \rangle$, the result is influenced very much by the fluctuation of quark number in particular for hyperons with high strangeness. See e.g. F. L. Shao, R. Q. Wang, H. H. Li, and J. Song, Phys. Rev. C 95, 064911 (2017), for detailed discussions.