Measuring the Stop Mixing Angle at the LHC

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Abstract

We present a method to determine the stop mixing angle and its CP-violating phase at the LHC. As an observable we use ratios of branching ratios for different decay modes of the light stop $\tilde{t}_1$ to charginos and neutralinos. These observables can have a very strong dependence on the parameters of the stop sector. We discuss in detail the origin of these effects. Using various combinations of the ratios of branching ratios we show that, depending on the scenario, one can achieve accuracies in the range of a few percent for determining the light stop mass, the mixing angle and the CP phase.
1 Introduction

Supersymmetry (SUSY) [1] is one of the best-motivated extensions of the Standard Model (SM). It allows one to stabilize the hierarchy between the electroweak (EW) scale and the Planck scale and to naturally explain electroweak symmetry breaking (EWSB) by a radiative mechanism. The naturalness of the scale of electroweak symmetry breaking and the Higgs mass places a rough upper bound on the superpartner masses of several TeV and the fits to the electroweak precision data point to a rather light SUSY spectrum [2]. Therefore a high potential for the discovery of SUSY is expected at the LHC [3].

Strongly interacting SUSY particles will be produced copiously at the LHC, with cross sections up to tens of pb for squarks and gluinos if their mass range is of a few hundreds GeV. Cross sections for direct production of scalar top quarks – the supersymmetric partners of top quarks – are expected to be smaller due to a different production mechanism, however still in a range of a few pb, e.g. for the SPS1a′ parameter point [3]. The other possible source of 3rd generation squarks, depending on the details of particle mass spectrum in the SUSY scenario, would be decays of other squarks and gluinos [5].

Stops are of a special interest since they play an important role in the mechanism of radiative electroweak symmetry breaking. Light stops together with CP-violating phases can also provide an attractive mechanism for electroweak baryogenesis by triggering a strong first-order electroweak phase transition [6]. Therefore a careful analysis of the stop sector can give an insight into the mechanism of EWSB and the origin of the matter-antimatter asymmetry. Finally, the stop sector has a large impact on the masses of the Higgs bosons [7], and in the presence of the CP-violating phases it triggers mixing between CP-odd and CP-even Higgs states [8]. Therefore a precise knowledge of the 3rd generation squark parameters would allow us to test the anatomy of the Higgs boson sector and electroweak symmetry breaking in the MSSM.

If stops are within the kinematic reach of the International Linear Collider or CLIC, production cross sections can be measured with a high accuracy [11, 12, 13]. Using polarized beams, this can provide information on the mixing angle and masses, and a precise determination of stop sector parameters can be foreseen [14, 15, 16]. In this paper, however, we concentrate on the measurement of the stop sector at the hadron collider. One of the advantages of hadron colliders, like the LHC, is the enhancement of cross section due to strong interactions [9, 10]. On the other hand, however, further challenges will become important due to the harsh experimental environment.

One of the possible sources of stops at the LHC will be decays of other supersymmetric particles, for example gluinos. The analysis of kinematical edges in the invariant mass distributions of the cascade decay chains provides an example of one measurement that
is often studied at the LHC. Taking the SPS1a’ scenario as an example, a large number of stops and sbottoms will appear in the gluino decay chain. Both, however, can give a similar experimental signature and consequently one has to do a simultaneous analysis of sbottom and stop sectors. This leads to good constraints for the sbottom sector but the constraints on the stop mixing angle are much weaker [5, 17]. Another possible observable is the polarization of top quarks in the decay $\tilde{t}_1 \rightarrow \tilde{\chi}^0_1 t$. The information on the stop mixing angle can be extracted here from the forward-backward asymmetries in leptonic and hadronic top decays [18].

In this paper we focus our attention on the decays of the light top squark to charginos and neutralinos that are possible in a wide range of scenarios of the Minimal Supersymmetric Standard Model (MSSM):

$$\tilde{t}_1 \rightarrow \tilde{\chi}^+_i b, \quad (1)$$

$$\tilde{t}_1 \rightarrow \tilde{\chi}^0_j t. \quad (2)$$

The stop and sbottom decays have already been analyzed in the literature in some detail, including radiative corrections [19, 20, 21, 22, 23]. In this paper we propose a method to measure the properties of the stop sector using simultaneously the decays Eqs. (1) and (2). We analyze three scenarios of the MSSM with different gaugino/higgsino composition of charginos and neutralinos. We show that the branching ratios for these decays can be a sensitive probe of the mixing angle in the stop sector and also of the CP-violating phase. We use a model-independent approach, i.e. without assuming a particular structure for the stop mass matrix, and parametrize the stop interactions in terms of the mixing parameters $\cos \theta_{\tilde{t}}$ and $\phi_{\tilde{t}}$. Since the absolute measurement of branching ratios is expected to be very difficult at the LHC we propose to exploit another set of observables – ratios of branching ratios, cf. Ref. [5, 17]. We argue that by looking at direct stop pair production and the following decays one can get a good accuracy for the determination of the mass and the mixing parameters of stops. We briefly discuss possible experimental issues for these processes. Finally, a $\chi^2$ fit is performed to give a range for the expected parameter determination precision.

The paper is organized as follows. In Section 2 we give a brief overview of the mixing and the couplings of the stop, chargino and neutralino sectors of the MSSM. In Section 3 we give analytic expressions for the decay widths of the light stop into charginos and neutralinos and analyze their dependence on the stop mixing parameters in chosen scenarios. Section 4 explains in detail how to determine the stop mixing parameters using stop decays at the LHC for our benchmark models. Finally, we summarize our findings in Section 5.
2 Spartice mixing and couplings

2.1 Stop sector of the MSSM

In the Minimal Supersymmetric Standard Model the stop sector is defined by the mass matrix $\mathcal{M}_t$ in the basis of gauge eigenstates $(\tilde{t}_L, \tilde{t}_R)$. The $2 \times 2$ mass matrix depends on the soft scalar masses $\tilde{M}_Q$ and $\tilde{M}_U$, the supersymmetric higgsino mass parameter $\mu$, and the soft SUSY-breaking trilinear coupling $A_t$. It is given as

$$\mathcal{M}_t^2 = \begin{pmatrix} m_t^2 + m_{LL}^2 & m_{LR}^* m_t \\ m_{LR} m_t & m_t^2 + m_{RR}^2 \end{pmatrix},$$

(3)

where

$$m_{LL}^2 = \tilde{M}_Q^2 + m_Z^2 \cos 2\beta \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right),$$

(4)

$$m_{RR}^2 = \tilde{M}_U^2 + \frac{2}{3} m_Z^2 \cos 2\beta \sin^2 \theta_W,$$

(5)

$$m_{LR} = A_t - \mu^* \cot \beta,$$

(6)

and $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of the two neutral Higgs fields which break the electroweak symmetry. From the above parameters only $\mu$ and $A_t$ can take complex values

$$A_t = |A_t| e^{i\phi_t}, \quad \mu = |\mu| e^{i\phi_\mu} \quad (0 \leq \phi_t, \phi_\mu < 2\pi),$$

(7)

thus yielding CP violation in the stop sector.

The hermitian matrix $\mathcal{M}_t^2$ is diagonalized by a unitary matrix $\mathcal{R}_t$ that rotates gauge eigenstates, $\tilde{t}_L$ and $\tilde{t}_R$, into the mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$:

$$\mathcal{R}_t \mathcal{M}_t^2 \mathcal{R}_t^\dagger = \begin{pmatrix} m_{t_1}^2 & 0 \\ 0 & m_{t_2}^2 \end{pmatrix},$$

(8)

where we choose the convention $m_{t_1}^2 < m_{t_2}^2$ for the masses of $\tilde{t}_1$ and $\tilde{t}_2$. The matrix $\mathcal{R}_t$ rotates the gauge eigenstates, $\tilde{t}_L$ and $\tilde{t}_R$, into the mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$ as follows

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \mathcal{R}_t \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} = \begin{pmatrix} \cos \theta_t & \sin \theta_t e^{-i\phi_t} \\ -\sin \theta_t e^{i\phi_t} & \cos \theta_t \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix},$$

(9)

where $\theta_t$ and $\phi_t$ are the mixing angle and the CP-violating phase of the stop sector, respectively. The masses are given by

$$m_{t_{1,2}} = \frac{1}{2} \left( 2m_t^2 + m_{LL}^2 + m_{RR}^2 \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4|m_{LR}|^2 m_t^2} \right),$$

(10)
whereas for the mixing angle and the CP phase we have
\begin{align}
\cos \theta_i &= \frac{-m_t |m_{LR}|}{\sqrt{m_t^2 |m_{LR}|^2 + (m_{\tilde{t}_1}^2 - m_{LL}^2)^2}}, \\
\sin \theta_i &= \frac{m_{LL}^2 - m_{\tilde{t}_1}^2}{\sqrt{m_t^2 |m_{LR}|^2 + (m_{\tilde{t}_1}^2 - m_{LL}^2)^2}}, \\
\phi_i &= \arg(A_t - \mu^* \cot \beta).
\end{align}

By convention we take $0 \leq \theta_i < \pi$ and $0 \leq \phi_i < 2\pi$. It must be noted that $\phi_i$ is an ‘effective’ phase and does not directly correspond to the phase of any MSSM parameter. Instead, the phase will have contributions from both $\phi_A$ and $\phi_\mu$. If $m_{LL} < m_{RR}$ then $\cos^2 \theta_i > \frac{1}{2}$ and $\tilde{t}_1$ has a predominantly left gauge character. On the other hand, if $m_{LL} > m_{RR}$ then $\cos^2 \theta_i < \frac{1}{2}$ and $\tilde{t}_1$ has a predominantly right gauge character.

### 2.2 Chargino mixing

In the MSSM, the mass matrix of the spin-1/2 partners of the charged gauge and charged Higgs bosons, $\tilde{W}^+$ and $\tilde{H}^+$, takes the form
\begin{equation}
\mathcal{M}_C = \begin{pmatrix}
M_2 & \sqrt{2}m_W \sin \beta \\
\sqrt{2}m_W \cos \beta & \mu
\end{pmatrix},
\end{equation}
where $M_2$ is the $SU(2)$ gaugino mass parameter. By reparametrization of the fields, $M_2$ can be taken real and positive, while the higgsino mass parameter $\mu$ can be complex, see Eq. (7). Since the chargino mass matrix $\mathcal{M}_C$ is not symmetric, two different unitary matrices are needed to diagonalize it
\begin{equation}
U^* \mathcal{M}_C V^\dagger = \begin{pmatrix}
m_{\tilde{\chi}_1^\pm} & 0 \\
0 & m_{\tilde{\chi}_2^\pm}
\end{pmatrix}, \quad \text{with} \quad m_{\tilde{\chi}_1^\pm} < m_{\tilde{\chi}_2^\pm}.
\end{equation}

$U$ and $V$ matrices act on the left- and right-chiral $\psi_{L,R} = (\tilde{W}, \tilde{H})_{L,R}$ two-component states
\begin{equation}
\tilde{\chi}_j^R = U_{jk}\psi_k^R, \quad \tilde{\chi}_j^L = V_{jk}\psi_k^L,
\end{equation}
giving two mass eigenstates $\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^\pm$.

### 2.3 Neutralino mixing

In the MSSM, the four neutralinos $\tilde{\chi}_i^0 (i = 1, 2, 3, 4)$ are mixtures of the neutral $U(1)$ and $SU(2)$ gauginos, $\tilde{B}$ and $\tilde{W}^3$, and the higgsinos, $\tilde{H}_1^0$ and $\tilde{H}_2^0$. The neutralino mass matrix in
the \((\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)\) basis,

\[
\mathcal{M}_N = \begin{pmatrix}
M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\
0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\
-m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\
m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0
\end{pmatrix}
\]

is built up by the fundamental SUSY parameters: the \(U(1)\) and \(SU(2)\) gaugino masses \(M_1\) and \(M_2\), the higgsino mass parameter \(\mu\), and \(\tan \beta = v_2/v_1\) \((c_\beta = \cos \beta, s_W = \sin \beta\) etc.). In addition to the \(\mu\) parameter, a non-trivial CP phase can also be attributed to the \(M_1\) parameter:

\[
M_1 = |M_1| e^{i\phi_1}, \quad (0 \leq \phi_1 < 2\pi).
\]

Since the complex matrix \(\mathcal{M}_N\) is symmetric, one unitary matrix \(N\) is sufficient to rotate the gauge eigenstate basis \((\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)\) to the mass eigenstate basis of the Majorana fields \(\tilde{\chi}_i^0\)

\[
\text{diag}(m_{\tilde{\chi}_1}, m_{\tilde{\chi}_2}, m_{\tilde{\chi}_3}, m_{\tilde{\chi}_4}) = N^* \mathcal{M}_N N^\dagger, \quad (m_{\tilde{\chi}_1} < m_{\tilde{\chi}_2} < m_{\tilde{\chi}_3} < m_{\tilde{\chi}_4}).
\]

The masses \(m_{\tilde{\chi}_i}^0\) \((i = 1, 2, 3, 4)\) can be chosen to be real and positive by a suitable definition of the unitary matrix \(N\).

### 2.4 Couplings of stops to charginos and neutralinos

We now give explicit formulae for the couplings relevant for decays Eqs. \((1)\) and \((2)\) in the convention of Ref. \([24]\). In terms of two-component (Weyl) gauge eigenstates the coupling between stop, top and neutral gauginos/higgsinos is given by

\[
\mathcal{L} = i\sqrt{2} \tilde{t}_L \left(g_2 T^3 \tilde{W}^3 + \frac{1}{6} g_1 \tilde{B}\right) t_L - \frac{2\sqrt{2}}{3} g_1 \tilde{t}_R^c \tilde{B}_R - Y_t \tilde{t}_R^c \tilde{H}_u^0 t_L - Y_t \tilde{t}_L^c \tilde{H}_u^0 t_R + \text{h.c.},
\]

where \(\epsilon = g_2 s_W = g_1 c_W\), \(T^3 = \frac{1}{2} I^3\) is the \(SU(2)\) generator and \(\tau^3\) is the Pauli matrix. After electroweak symmetry breaking for the mass eigenstates \(t\), \(\tilde{t}\) and \(\tilde{\chi}_j^0\) we get

\[
\mathcal{L}_{t\tilde{t}\tilde{\chi}_j^0} = \overline{\tilde{\chi}_j^0} \left(Q_{ij}^{0L} P_L + Q_{ij}^{0R} P_R\right) \tilde{t}_i^* + \text{h.c.},
\]

where

\[
Q_{ij}^{0L} = -\frac{\epsilon}{\sqrt{2} s_W c_W} \mathcal{R}_{i1}^l \left( \frac{1}{3} s_W N_{j1}^* + c_W N_{j2}^* \right) - Y_t \mathcal{R}_{i2}^l N_{j4}^*,
\]

\[
Q_{ij}^{0R} = \frac{2\sqrt{2} \epsilon}{3c_W} \mathcal{R}_{i2}^l N_{j1} - Y_t \mathcal{R}_{i1}^l N_{j4},
\]

with the top Yukawa coupling given by

\[
Y_t = \frac{\epsilon m_t}{\sqrt{2} m_W s_W \sin \beta}.
\]
We now see that the right squark couples only to the bino and the higgsino components of the neutralino. If the $\mu$ parameter is much larger than the gaugino mass parameters then the light chargino $\tilde{\chi}_1^\pm$ and light neutralinos $\tilde{\chi}_1^0, \tilde{\chi}_2^0$ are gauginos with small higgsino components. In this case the Yukawa term in Eq. (23) is negligible for stop decays into these states. On the other hand, as can be seen in Eqs. (22) and (23), left squarks couple both to the bino and the wino, however with the bino coupling suppressed by a factor $1/3$ due to hypercharge. Therefore, having a prior knowledge on the composition of neutralinos we can infer the structure of the stop sector by comparing strength of the stop coupling to different neutralino states.

Let us turn now to the coupling between chargino, stop and bottom quark. The interaction Lagrangian in terms of gauge eigenstates reads in Weyl notation

$$\mathcal{L} = ig_2 \bar{t}_L \tilde{W}^+ b_L + Y_t \bar{t}_R \tilde{H}^+_u b_L + Y_b \bar{b}_L \tilde{H}^+_d b_R + \text{h.c.}$$

After electroweak symmetry breaking and rotation of fields to their mass eigenstates we get

$$\mathcal{L}_{b \tilde{t} \tilde{\chi}} = \overline{\tilde{\chi}_j^+} \left( Q_{ij}^{\pm, L} P_L + Q_{ij}^{\pm, R} P_R \right) b \tilde{t}_i^* + \text{h.c.},$$

where

$$Q_{ij}^{\pm, L} = -\frac{e}{s_W} R_{ij}^L V_{j1}^* V_{i1} + Y_t R_{ij}^L V_{j2}^*,$$  

$$Q_{ij}^{\pm, R} = Y_b R_{ij}^R U_{j2},$$

with the bottom Yukawa coupling given by

$$Y_b = \frac{e m_b}{\sqrt{2} m_W s_W \cos \beta}.$$  

The right stop couples only to the higgsino component of the chargino via the Yukawa term in Eq. (28), whereas the left stop couples both to the higgsino and the wino. When the light chargino is mainly wino-like the higgsino couplings are small and only the stop-bottom-wino term becomes relevant. Therefore, similarly as for interactions with neutralinos, measurement of coupling strength to the light chargino can probe the left-right composition of the light stop.

### 3 Stop decays to charginos and neutralinos

#### 3.1 Analytical formulae

We start this section by giving formulae for the decay widths of the top squarks into charginos and neutralinos [15, 23]. Using couplings defined in Sec. 2.4 the tree-level width for the decay
Eq. (11) can be written as
\[
\Gamma(\tilde{t}_i \to \tilde{\chi}^+_j b) = \frac{\kappa(m^2_{\tilde{t}_i}, m^2_{\tilde{b}}, m^2_{\tilde{\chi}^+_j})}{16\pi m^2_{\tilde{t}_i}} \left( (|Q^L_{ij}|^2 + |Q^R_{ij}|^2) \left( m^2_{\tilde{t}_i} - m^2_{\tilde{b}} - m^2_{\tilde{\chi}^+_j} \right) - 4 \text{Re} \left[ Q^L_{ij}Q^R_{ij} \right] m_{\tilde{b}} m_{\tilde{\chi}^+_j} \right),
\]
(30)
with the kinematic triangle function
\[
\kappa(x, y, z) = \sqrt{(x - y - z)^2 - 4yz},
\]
(31)
and the couplings $Q^L_{ij}$ given by Eqs. (27), (28). Substituting the explicit matrix elements of Eq. (9) we can make the following expansion in terms of the stop mixing angle and the phase
\[
|Q^L_{ij}|^2 + |Q^R_{ij}|^2 = \cos^2 \theta_t \left( Y_t^2 |U_{j2}|^2 + \frac{e^2}{s_W} |V_{j2}|^2 \right) + \sin^2 \theta_t Y_t^2 |V_{j2}|^2 - 2 \sin \theta_t \cos \theta_t \frac{e}{s_W} Y_t \text{Re} \left[ e^{-i\phi_t} V_{j1} V_{j2}^* \right],
\]
(32)
\[
\text{Re} \left[ Q^L_{ij}Q^R_{ij} \right] = - \cos^2 \theta_t \frac{e}{s_W} Y_t \left[ U_{j2}^* V_{j1}^* \right] + \sin \theta_t \cos \theta_t Y_t Y_t \text{Re} \left[ e^{-i\phi_t} U_{j2}^* V_{j2}^* \right].
\]
(33)
We see explicitly that the dependence of the phase $\phi_t$ appears only if there is a significant higgsino component ($U_{j2}$ or $V_{j2}$) in the chargino $\tilde{\chi}_j^+$ we are interested in.

Analogously, for decays to neutralinos we have [15, 23]
\[
\Gamma(\tilde{t}_i \to \tilde{\chi}^0_j t) = \frac{\kappa(m^2_{\tilde{t}_i}, m^2_{\tilde{b}}, m^2_{\tilde{\chi}^0_j})}{16\pi m^2_{\tilde{t}_i}} \left( (|Q^L_{ij}|^2 + |Q^R_{ij}|^2) \left( m^2_{\tilde{t}_i} - m^2_{\tilde{b}} - m^2_{\tilde{\chi}^0_j} \right) - 4 \text{Re} \left[ Q^L_{ij}Q^R_{ij} \right] m_{\tilde{b}} m_{\tilde{\chi}^0_j} \right),
\]
(34)
with $\kappa(x, y, z)$ given by Eq. (31) and couplings $Q^L_{ij}$ by Eqs. (22), (23). Similarly we obtain
\[
|Q^L_{ij}|^2 + |Q^R_{ij}|^2 = \cos^2 \theta_t \left( \frac{e^2}{2s_Wc_W} \left[ \frac{1}{3} s_W N_{j1} + c_W N_{j2} \right]^2 + Y_t^2 |N_{j4}|^2 \right) + \sin^2 \theta_t \left( \frac{8e^2}{9c_W} |N_{j1}|^2 + Y_t^2 |N_{j4}|^2 \right) + 2 \sin \theta_t \cos \theta_t Y_t \left( \frac{e}{\sqrt{2} s_W c_W} \text{Re} \left[ e^{i\phi_t} \left( \frac{1}{3} s_W N_{j1}^* + c_W N_{j2}^* \right) N_{j4} \right] - \frac{2\sqrt{2} e}{3c_W} \text{Re} \left[ e^{-i\phi_t} N_{j1} N_{j4}^* \right] \right),
\]
(35)
\[
\text{Re} \left[ Q^L_{ij}Q^R_{ij} \right] = \cos^2 \theta_t \frac{e}{\sqrt{2} s_W c_W} Y_t \left[ \text{Re} \left[ \left( \frac{1}{3} s_W N_{j1}^* + c_W N_{j2}^* \right) N_{j4} \right] + \sin^2 \theta_t \frac{2\sqrt{2} e}{3c_W} Y_t \text{Re} \left[ N_{j4}^* N_{j1} \right] + \sin \theta_t \cos \theta_t Y_t \left( 2 e^2 \left( \frac{1}{3} s_W N_{j1}^* + c_W N_{j2}^* \right) N_{j4} \right) - \frac{2 e^2}{3 s_W c_W} \text{Re} \left[ e^{i\phi_t} \left( \frac{1}{3} s_W N_{j1}^* + c_W N_{j2}^* \right) N_{j4} \right] \right),
\]
(36)
An interesting feature of Eqs. (30) and (34) is the relative importance of the squared $|Q_{ij}^L|^2 + |Q_{ij}^R|^2$ terms and the left-right interference $\text{Re} \left[ Q_{ij}^L Q_{ij}^{R*} \right]$ terms. As they are multiplied by mass factors, it is going to be sensitive to the mass splitting between stop and $\tilde{\chi}_i^0$ pairs. If the given decay mode is close to its kinematic threshold (which will be the case for heavier neutralinos) the second term will become dominant, whereas far from the threshold the first term will usually be much larger.

3.2 Discussion of typical mixing scenarios

In order to analyze the dependence of the stop mixing angle on the decay widths and the branching ratios we consider three benchmark points of the MSSM. The first scenario is the well known mSUGRA inspired SPS1a′ parameter point [4] – in the following we will refer to it as Scenario A. A feature of mSUGRA scenarios is that the charginos and the neutralinos are to a large extent pure gaugino/higgsino states: the lightest neutralino is bino-like, the light chargino and the second neutralino are winos, and the heavy chargino and the heavy neutralinos are higgsino-like. Scenarios B and C are adopted from Ref. [25]. In Scenario B the wino mass parameter $M_2$ and the higgsino mass parameter $\mu$ are of a similar order, giving strong mixing between the wino and the higgsino components of the charginos and the neutralinos. This makes the determination of $\theta_{\tilde{t}}$ more difficult since both left and right couplings of Eqs. (21) and (26) come into play for any value of $\cos \theta_{\tilde{t}}$. On the other hand this gives the possibility to study the dependence on the CP-violating phase $\phi_{\tilde{t}}$, thanks to the last terms of Eqs. (32), (33), (35) and (36). Finally, Scenario C features the wino mass parameter two times larger than the $\mu$ parameter. In this case higgsino-like states will be lighter than winos with rather small mixing. In both cases, Scenarios B and C, the lightest supersymmetric particle $\tilde{\chi}_1^0$ is bino-like. In order to study the possible dependence of branching ratios on the CP-violating phase in the last two scenarios we introduce a CP phase for the stop trilinear coupling $A_{\tilde{t}}$. For all three scenarios we keep the values of other parameters (i.e. slepton and squark sectors) as in the SPS1a′ scenario. The values of the gaugino, higgsino and stop sector parameters are collected in Tab. 1 and the nominal values of masses, mixing angles and branching ratios are listed in Tables 2 and 3.

We now discuss the behaviour of the decay widths and the branching ratios with respect to the stop mixing angle and the CP phase in each of the scenarios.

Scenario A – mSUGRA

According to the discussion in Sec. 2.4 for Scenario A we expect that if the $\tilde{t}_1$ is mainly a left stop (i.e. for $\cos \theta_{\tilde{t}} = \pm 1$) then it will dominantly couple to $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^0$ (which are both winos), whereas the coupling to the bino-like $\tilde{\chi}_1^0$ is suppressed. On the other hand,
Table 1: MSSM parameters of Scenarios A, B and C relevant in the present study. Mass parameters and trilinear coupling are given in GeV.

|       | $M_1$ | $M_2$ | $\mu$ | $\tan \beta$ | $A_t$ | $M_Q$ | $M_U$ |
|-------|-------|-------|-------|--------------|-------|-------|-------|
| Scenario A | 103.3 | 193.2 | 396.0 | 10 | $-565.1$ | 471.4 | 387.5 |
| Scenario B | 109.0 | 240.0 | 230.0 | 10 | $-610 \, e^{i \pi/2}$ | 511.0 | 460.0 |
| Scenario C | 105.0 | 400.0 | $-190.0$ | 20 | $-610 \, e^{i \pi/4}$ | 511.0 | 460.0 |

Table 2: Masses (in GeV) of stops, charginos and neutralinos in Scenarios A, B and C calculated by SPheno 2.2.3 [26].

|       | $m_{\tilde{t}_1}$ | $m_{\tilde{t}_2}$ | $m_{\tilde{\chi}^\pm_1}$ | $m_{\tilde{\chi}^\pm_2}$ | $m_{\tilde{\chi}^0_1}$ | $m_{\tilde{\chi}^0_2}$ | $m_{\tilde{\chi}^0_3}$ | $m_{\tilde{\chi}^0_4}$ |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Scenario A | 366.5 | 585.5 | 183.7 | 415.7 | 97.7 | 183.9 | 400.5 | 413.9 |
| Scenario B | 395.5 | 609.0 | 178.0 | 302.9 | 101.4 | 237.8 | 303.2 |
| Scenario C | 396.9 | 608.0 | 182.9 | 419.0 | 99.0 | 186.6 | 199.5 | 418.9 |

Table 3: Nominal values of mixing angles in the stop sector and branching ratios for stop decays calculated from Eqs. (30) and (34) for Scenarios A, B and C. In the last row, cross sections for stop pair production at the LHC with $\sqrt{s} = 14$ TeV at NLO from Prospino 2.1 [10, 27].

| Parameter | Scenario A | Scenario B | Scenario C |
|-----------|------------|------------|------------|
| $\cos \theta_{\tilde{t}}$ | 0.56 | 0.62 | 0.62 |
| $\phi_{\tilde{t}}$ | 0 | 1.53 | 0.80 |
| $\Gamma(\tilde{t}_1)$ [GeV] | 1.45 | 3.25 | 6.36 |
| $BR(\tilde{t}_1 \to \tilde{\chi}_1^+ b)$ | 73.5% | 60.7% | 63.7% |
| $BR(\tilde{t}_1 \to \tilde{\chi}_2^0 b)$ | 17.6% | — | — |
| $BR(\tilde{t}_1 \to \tilde{\chi}_1^0 t)$ | 20.1% | 8.8% | 6.4% |
| $BR(\tilde{t}_1 \to \tilde{\chi}_2^0 t)$ | 6.4% | 12.9% | 8.5% |
| $BR(\tilde{t}_1 \to \tilde{\chi}_3^0 t)$ | — | — | 21.4% |
| $\sigma(pp \to \tilde{t}_1\tilde{t}_1^*)$ [pb] | 3.44 | 2.27 | 2.27 |

if the $\tilde{t}_1$ is predominantly a right stop (i.e. for $\cos \theta_{\tilde{t}} = 0$) we should observe enhancement in the coupling to the LSP and suppression for the decay to the light chargino and second neutralino. This general feature can be seen in the upper left panel of Fig. 1, where we show the dependence of the decay width on the stop mixing angle $\cos \theta_{\tilde{t}}$. The minima for decays to chargino and $\tilde{\chi}^0_2$ are somewhat shifted which is the result of the higgsino Yukawa
Figure 1: Decay widths (left column) and branching ratios (right column) for $\tilde{t}_1$ in Scenario A as a function of the stop mixing angle $\cos \theta_{\tilde{t}}$ (upper row) and the stop CP phase $\phi_{\tilde{t}}$ (lower row). Black, red and blue lines are for $\tilde{\chi}_1^+ b$, $\tilde{\chi}_0^0 t$ and $\tilde{\chi}_0^0 \tilde{t}$ final states, respectively.

contributions from Eqs. (22), (23), (27) and (28). On top of that, the decay $\tilde{t}_1 \rightarrow \tilde{\chi}_2^0 t$ is further suppressed by the phase space, since $m_{\tilde{\chi}_2} + m_t = 355$ GeV is only slightly lower than the light stop mass. As one can see, the decay widths change by an order of magnitude or more. Therefore they are a sensitive probe of the mixing between left and right stop states. The upper right panel of Fig. 1 shows the dependence of the branching ratios on $\cos \theta_{\tilde{t}}$ that exhibit a similar behaviour as the decay widths.

Although Scenario A does not contain CP phases, we include them here to analyze the sensitivity of the decay widths and the branching ratios. The respective plots can be seen in the lower row of Fig. 1. The most significant change is for the decay to a chargino and a bottom quark. This results from the third term of Eq. (32) that changes sign when varying
Figure 2: Decay widths for $\tilde{t}_1$ in Scenario B as a function of the stop mixing angle $\cos \theta_{\tilde{t}}$ (left panel) and the stop CP phase $\phi_{\tilde{t}}$ (right panel). Black, red, blue and green lines are for $\tilde{\chi}_1^+ b$, $\tilde{\chi}_1^0 t$, $\tilde{\chi}_2^0 t$ and $\tilde{\chi}_2^+ b$ final states, respectively.

$\phi_{\tilde{t}}$ from $0$ to $\pi$ giving destructive interference. Although the dependence on $\phi_{\tilde{t}}$ is clearly visible the constraints on this parameter, as we will see it later, will be rather weak.

**Scenario B – mixed gaugino/higgsino $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^0$**

The situation changes significantly in Scenario B. The second chargino $\tilde{\chi}_2^+$ is now lighter than the stop $\tilde{t}_1$ so there is a new decay channel open. Both charginos and the neutralino $\tilde{\chi}_2^0$ now have a significant higgsino component. The dependence on the stop mixing angle of all the decay widths is much flatter now, as can be seen in the left panel of Fig. 2. We note a well pronounced enhancement of the decay widths for right stops (around $\cos \theta_{\tilde{t}} = 0$). For charginos it can be understood by looking at Eq. (25) where the coupling of $\tilde{t}_R$ is proportional to the large top Yukawa coupling, whereas the coupling of $\tilde{t}_L$ is proportional to the smaller bottom Yukawa coupling. For the decay to $\tilde{\chi}_2^0 t$ the enhancement is due to the left-right interference term, whereas for $\tilde{\chi}_1^0 t$ it is due to the quadratic term in Eq. (34).

Thanks to the presence of the higgsino component in charginos and neutralinos, we now become more sensitive to the phase $\phi_{\tilde{t}}$, see the right panel of Fig. 2. Apart from the decay to $\tilde{\chi}_2^0 t$, all other decay widths can change by up to an order of magnitude depending on the CP phase. The former remains almost unchanged due to the accidental cancellations between two terms of Eq. (34). For the decays to charginos $\tilde{\chi}_1^+$ ($\tilde{\chi}_2^+$) the suppression (enhancement) of the decay width with the phase arises due to change of the sign of $\cos \phi_{\tilde{t}}$ when $\phi_{\tilde{t}} \to \pi$, cf. Eqs. (32) and (33). The difference for $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^+$ is the result of the sign difference between $V_{11} V_{12}^*$ and $V_{21} V_{22}^*$. 
Figure 3: Decay widths for $\tilde{t}_1$ in Scenario C as a function of the stop mixing angle $\cos \theta_{\tilde{t}}$ (left panel) and the stop CP phase $\phi_{\tilde{t}}$ (right panel). Black, red, blue and green lines are for $\tilde{t}_1 b$, $\tilde{t}_1 t$, $\tilde{t}_2 t$ and $\tilde{t}_3 t$ final states, respectively.

**Scenario C – higgsino-like $\tilde{\chi}_1^+$, $\tilde{\chi}_2^0$ and $\tilde{\chi}_3^0$**

The last discussed scenario features the hierarchy $M_1 < \mu < M_2$. Therefore the light chargino and neutralinos $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$ are higgsino-like with small mass differences between them. The lightest neutralino is bino-like as in the previous scenarios. The dependence of the decay widths on the stop mixing angle has been shown in the left panel of Fig. 3. The difference in the decay to the chargino for left and right stops is a consequence of the $Y_t$ coupling for right states in Eq. (25). A similar effect was seen in Scenario B, however it is now more pronounced due to the higgsino nature of the light chargino $\tilde{\chi}_1^+$. We also observe the interesting exchange of the decay widths to heavier neutralinos when the sign of $\cos \theta_{\tilde{t}}$ changes. This feature arises due to the $Y_t^2 \Re [N_{j4}^2]$ term in the second line of Eq. (36) that is enhanced both by the large top Yukawa coupling and the higgsino nature of the two neutralinos. Since neutralinos $\tilde{\chi}_2^0$ and $\tilde{\chi}_3^0$ have opposite intrinsic CP parities, cf. Ref. [28], the entries in the neutralino mixing matrix that correspond to $\tilde{\chi}_3^0$ are purely imaginary. Therefore the contribution has an opposite sign in the decay width and hence different behaviour with respect to the sign of $\cos \theta_{\tilde{t}}$.

A similar dependence of the decay widths $\tilde{\chi}_2^0 t$ and $\tilde{\chi}_3^0 t$ on the sign of $\cos \phi_{\tilde{t}}$ can be seen in the right panel of Fig. 3. Its origin is the same as in the above discussed case for $\cos \theta_{\tilde{t}}$. As before the change in the width of the decay to $\tilde{\chi}_1^+ b$ is caused by change in the sign of the last term of Eq. (32) with $\cos \phi_{\tilde{t}}$, as $\phi_{\tilde{t}}$ is varied from 0 to $\pi$. It is interesting to note that now the branching ratio for the decay to chargino $\tilde{\chi}_1^+$ does not show a strong dependence on the phase $\phi_{\tilde{t}}$ (as opposite to Scenario B). However the dependence of the branching ratios
4 Analysis at the LHC

4.1 Ratios of branching ratios

As one can see in Figs. 1, 2 and 3, the decay widths can change by up to a few orders of magnitude depending on the stop mixing angle and the CP phase. In addition, the branching ratios are also very sensitive to these parameters. However since the measurement of decay widths and branching ratios will be difficult at the LHC we propose to analyze the ratios of branching ratios. That means comparing the number of stops decaying to one final state with the number of stops decaying to another final state. Having three decay modes possible we can define the following ratios of branching ratios for each of the Scenarios A, B and C

\[ R_{1t}^{lb} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_1^+ b)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_1^0 t)}, \quad R_{2t}^{lb} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_2^+ b)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_2^0 t)}, \quad R_{2t}^{1t} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_1^0 t)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_2^0 t)}. \]  

(37)

Figure 4 shows the above ratios of branching ratios in Scenario A as functions of \( \cos \theta_t \) and the CP-violating phase \( \phi_t \). For Scenario B we have three additional combinations due to the decay \( \tilde{t}_1 \rightarrow \tilde{\chi}_2^+ b \) being open

\[ R_{2b}^{lb} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_1^+ b)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_2^+ b)}, \quad R_{1t}^{lb} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_1^0 t)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_2^0 t)}, \quad R_{2t}^{2t} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_2^0 t)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_2^+ b)}. \]  

(38)

For Scenario C due to the decay \( \tilde{t}_1 \rightarrow \tilde{\chi}_3^0 t \) being allowed we have

\[ R_{3t}^{lb} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_1^+ b)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_3^0 t)}, \quad R_{3t}^{2t} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_1^0 t)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_3^0 t)}, \quad R_{3t}^{2t} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_3^0 t)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_3^0 t)}. \]  

(39)

Because of the higgsino nature of neutralinos \( \tilde{\chi}_2^0 \) and \( \tilde{\chi}_3^0 \) they are very close in mass and it might turn out that they are impossible to disentangle at the LHC. Therefore we define two additional ratios by combining the decay modes to \( \tilde{\chi}_2^0 t \) and \( \tilde{\chi}_3^0 t \)

\[ R_{23t}^{lb} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_1^+ b)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_2^0 t) + BR(\tilde{t}_1 \rightarrow \tilde{\chi}_3^0 t)}, \quad R_{23t}^{1t} = \frac{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_1^0 t)}{BR(\tilde{t}_1 \rightarrow \tilde{\chi}_2^0 t) + BR(\tilde{t}_1 \rightarrow \tilde{\chi}_3^0 t)}. \]  

(40)

In our analysis we focus on direct stop production \( pp \rightarrow \tilde{t}_1 \tilde{\chi}_1^0 \) in order to have better control over the number of observed stops and to reduce the background due to bottom squarks. In the SPS1a’ scenario the cross section for this process amounts to 3.44 pb at the next-to-leading order \([10, 27]\), whereas the total SUSY cross section is 60 pb. The cross sections for stop pair production in Scenarios B and C are given in Tab. 3. Due to mass splitting between stop states the cross section for \( pp \rightarrow \tilde{t}_2 \tilde{\chi}_2^0 \) is much smaller with a value of 0.26 pb. Similarly for sbottoms we get \( \sigma(pp \rightarrow \tilde{b}_1 \tilde{b}_1^0) = 0.6 \) pb and \( \sigma(pp \rightarrow \tilde{b}_2 \tilde{b}_2^0) = 0.4 \) pb. This
Possible final states are as follows:

\begin{align*}
    pp &\rightarrow \tilde{t}_1\tilde{t}_1^* \rightarrow t\tilde{\chi}_2^0 + \tilde{t}\tilde{\chi}_2^0 \\
    pp &\rightarrow \tilde{t}_1\tilde{t}_1^* \rightarrow t\tilde{\chi}_2^0 + \tilde{t}\tilde{\chi}_2^0 \rightarrow t\ell^+\ell^-\tilde{\chi}_1^0 + \tilde{t}\ell^+\ell^-\tilde{\chi}_1^0 \rightarrow 4\ell 4j 2b + E_{\text{miss}}, \\
    pp &\rightarrow \tilde{t}_1\tilde{t}_1^* \rightarrow t\tilde{\chi}_2^0 + \tilde{t}\tilde{\chi}_2^0 \rightarrow t\ell^+\ell^-\tilde{\chi}_1^0 + \tilde{t}\ell^+\ell^-\tilde{\chi}_1^0 \rightarrow 2\ell 4j 2b + E_{\text{miss}}, \\
    pp &\rightarrow \tilde{t}_1\tilde{t}_1^* \rightarrow t\tilde{\chi}_2^0 + b\tilde{\chi}_1^+ \rightarrow t\ell^+\ell^-\tilde{\chi}_1^0 + b\ell^+\nu\tilde{\chi}_1^0 \rightarrow 3\ell 2j 2b + E_{\text{miss}}, \\
    pp &\rightarrow \tilde{t}_1\tilde{t}_1^* \rightarrow t\tilde{\chi}_2^0 + \tilde{t}\tilde{\chi}_2^0 \rightarrow 4j 2b + E_{\text{miss}}, \\
    pp &\rightarrow \tilde{t}_1\tilde{t}_1^* \rightarrow t\tilde{\chi}_2^0 + b\tilde{\chi}_1^+ \rightarrow t\tilde{\chi}_1^0 + b\ell^+\nu\tilde{\chi}_1^0 \rightarrow \ell 2j 2b + E_{\text{miss}}, \\
    pp &\rightarrow \tilde{t}_1\tilde{t}_1^* \rightarrow b\tilde{\chi}_1^- + b\tilde{\chi}_1^+ \rightarrow b\ell^+\nu\tilde{\chi}_1^0 + b\ell^+\nu\tilde{\chi}_1^0 \rightarrow 2\ell 2b + E_{\text{miss}}.
\end{align*}

The production process itself can be tagged using a clean decay mode for one of the stops, for instance the decay to $\tilde{\chi}_1^0 t$ followed by a leptonic neutralino decay and hadronic top decay. For an integrated luminosity of $L = 100$ fb$^{-1}$ we would have more than 300 000 stop pair production events. Assuming that on average 10\% of charginos and neutralinos decay to leptons in our scenarios \cite{26}, taking into account the hadronic top branching ratio and a selection efficiency of 5\%, cf. Ref. \cite{18}, one can expect more than 1000 stop events to be observed. Therefore in our further analysis we will assume that 1000 events have been correctly identified and show that even with this amount of experimental data one can still get strong constraints on the stop mixing angle and the mass.

The other important point we wish to emphasize are the branching ratios for decays of the chargino $\tilde{\chi}_1^\pm$ and the neutralino $\tilde{\chi}_2^0$ into leptons. Although one may expect that the related uncertainty will cancel out to some extent in the ratio $R_{2t}^{1b}$ (as in our scenarios

Figure 4: Ratios of branching ratios as a function of stop mixing angle $\cos\theta_t$ and CP phase $\phi_t$ in Scenario A, left and right panel, respectively. Black, red and blue lines are for ratios $R_{1t}^{1b}, R_{2t}^{1b}$ and $R_{2t}^{2t}$ (see Eq. (37)), respectively.
\( \tilde{\chi}_1^\pm \) and \( \tilde{\chi}_2^0 \) have similar gaugino/higgsino composition), this is not true for the other ratios involving decays to the LSP. Because our focus here is on the stop sector we will assume that the leptonic branching ratios of the charginos and neutralinos are known. However as this would require better knowledge of the structure of the gaugino/higgsino sectors it is possible that the measurements from the LHC would have to be supplemented by the linear collider experiment, where charginos and neutralinos can be measured with a high precision. This would be an interesting example of LHC/ILC interplay [17], in particular for the scenarios where direct stop production is beyond the kinematical reach of the ILC.

A large number of SUSY and SM backgrounds are expected for stop production at the LHC. The most severe Standard Model background, especially for the channels Eqs. (44)–(46), will be \( t\bar{t} \) production. As shown in Ref. [18], for the process Eq. (44) this background can be effectively removed by using appropriate cuts. In any case, the key feature to distinguish the signal from SM background will be missing transverse energy, which is much larger for stop production due to large energy carried by the LSPs.

The most important SUSY background process is going to be gluino production with subsequent decays to stops or sbottoms. One important difference between the signal and these backgrounds is the number of \( b \)-jets. The signal event always results in exactly 2 \( b \)-jets, whereas SUSY backgrounds will typically have 4 \( b \)-jets and this feature can be used to suppress them.

Finally, we note that the signal process with leptonic top decay, e.g. \( \tilde{\chi}_1^0 t \to b\ell + E_{\text{miss}} \), can give the same final state as the decay mode with charginos, i.e. \( \tilde{\chi}_1^+ b \to b\ell + E_{\text{miss}} \). However, we note that this complication does not affect the result of the fit since it does not introduce any new unknown parameters. The fitted observables would be a linear combination of the original ones, Eq. (37), and the fit would rely on the same set of information. Hence, one can combine the above channels and actually enhance the signal.

An important note is that it will not be sufficient to simply remove as much background as possible using the relevant cuts. We will also need to understand with a high degree of accuracy how each individual signal channel will be affected by the backgrounds. Understanding the background well is required, as for each channel we study, the number of background events contaminating the sample will be different. Therefore the pollution due to backgrounds will affect our ratios of branching ratios. The reconstruction efficiency, cuts and triggers will also have a different effect on each channel and will have to be well understood for our measurements to be accurate. We leave detailed analysis of these effects for our different final states and the additional uncertainties they may induce for future work.
4.2 Determination of stop mass and mixing angle

In order to show the possible advantages of using ratios of branching ratios for the analysis of the stop sector we first define the normalized ratios

\[ \hat{R}_{ij} = R_{ij} - R_{ij}^{\text{nominal}} \quad \text{with} \quad R_{ij}^{\text{nominal}} = R_{ij}(\cos \theta_{\tilde{t}}^{\text{nominal}}), \]  

(47)

where \( \theta_{\tilde{t}}^{\text{nominal}} \) is the actual mixing angle in the given scenario and \( i, j \) run over all possible channels, i.e. 1b, 1t, 2t etc., cf. Eqs. (37)–(40). According to this definition \( \hat{R}_{i}(\cos \theta_{\tilde{t}}^{\text{nominal}}) \equiv 0 \). Furthermore, we assume that we have \( n = 1000 \) of well identified events of stop \( \tilde{t}_1 \tilde{t}_1^* \) pair production. We now take the expected number of events in each decay mode \( n_i = n \times BR_i \).

The statistical error for \( n_i \) is \( \Delta_{\text{stat}} n_i = \sqrt{n_i} \). The resulting error for ratios of branching ratios is given by

\[ \Delta R_{ij} = \sqrt{\left( \frac{\Delta n_i}{n_j} \right)^2 + \left( \frac{n_i \Delta n_j}{n_j^2} \right)^2} \quad \text{with} \quad R_{ij} = \frac{n_i}{n_j}. \]  

(48)

Before analyzing the expected accuracy of determination of stop sector parameters let us study the possible influence of the gaugino/higgsino sector parameters, taking as an example Scenario A. The precise knowledge of the LSP mass and the mixing angles of the charginos and the neutralinos may only be accessible after the results from a linear \( e^+e^- \) collider are available. In Fig. 5 we show the dependence of the normalized ratios Eq. (37) on the gaugino mass parameter \( M_2 \) and the mass of the LSP, \( m_{\text{LSP}} \equiv m_{\tilde{\chi}_1^0} \). In the first case we keep the mass differences \( m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} \) and \( m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} \) fixed as these are expected to be measured with high precision at the LHC. As can be seen, the value of \( \hat{R}_{1b}^{1t} \) is very stable in both cases, whilst \( \hat{R}_{2t}^{1b} \) and \( \hat{R}_{2t}^{1t} \) exhibit an increase for larger values of \( M_2 \) and \( m_{\text{LSP}} \). This is because both \( \hat{R}_{2t}^{1b} \) and \( \hat{R}_{2t}^{1t} \) include a branching ratio for the decay to \( \tilde{\chi}_1^0 t \) that is close to its kinematic threshold. Therefore, for an increasing \( m_{\text{LSP}} \) or \( M_2 \) (note that \( M_2 \simeq m_{\tilde{\chi}_2^0} \) in Scenario A) we approach the point where this decay becomes impossible. High sensitivity of the decay width near the threshold means that to use such a decay mode to determine the mixing angle, one would have to know the masses extremely precisely. In this case the ratio of branching ratios is no longer a good observable. Moreover the branching ratio for such a decay usually becomes very small.

In order to analyze the possible accuracy in extracting the mixing parameters of the stop sector we start with the example of Scenario A. In Fig. 6 we show the behaviour of the normalized ratios of branching ratios, Eq. (47), near the nominal value of the mixing angle \( \cos \theta_{\tilde{t}}^{\text{nominal}} \). Using only one of three possible ratios, the smallest error and hence the best estimate we get is using the ratio \( \hat{R}_{1t}^{1b} \), which depends on the dominant decay modes \( \tilde{\chi}_1^0 t \) and
Figure 5: Dependence of the normalized ratios of branching ratios \( \hat{R}_i \), Eq. (47), in stop decays on the mass of the lightest neutralino (left panel) and the gaugino mass parameter \( M_2 \) (right panel) in Scenario A. The values of the other parameters and the mass difference between the LSP and \( \tilde{\chi}_0^2 \), \( \tilde{\chi}_1^\pm \) for the left plot are fixed to their nominal values. Red lines are for \( \hat{R}_{1b} \), black lines for \( \hat{R}_{2b} \), and blue lines for \( \hat{R}_{1t} \), see Eq. (37). The upper and lower lines show the band due to 1-\( \sigma \) uncertainty with respect to the central value as defined in Eq. (48).

Figure 6: Normalized ratios \( \hat{R}_{ij} \), Eq. (47), near the nominal value of \( \cos \theta_{\tilde{t}} = 0.56 \) for Scenario A. Red lines are for \( \hat{R}_{1l} \), black lines for \( \hat{R}_{2l} \), and blue lines for \( \hat{R}_{1t} \), see Eq. (37). The upper and lower lines show the 1-\( \sigma \) uncertainty as defined in the text. The region allowed by all three ratios is shaded in grey.

For the ratio \( \hat{R}_{2t} \) the impact of the error is slightly larger due to the limited statistics. On the other hand the ratio \( \hat{R}_{2b} \) gives the weakest constraints because both \( \tilde{\chi}_1^+ \) and \( \tilde{\chi}_2^0 \) are winos, hence their couplings to \( \tilde{t}_1 \) follow a similar pattern. We assume here that the values of the other SUSY parameters, including the \( \tilde{t}_1 \) mass, are known. Using only the information
Figure 7: A $\chi^2$ fit of ratios $R_{1t}^{1b}$, $R_{2t}^{1b}$ and $R_{2t}^{1t}$, Eq. (37), to the stop mass and the mixing angle $\cos \theta_{\tilde t}$ (left panel), and the stop mixing angle and the CP phase $\phi_{\tilde t}$ (right panel) in Scenario A for $n = 1000$ events. Bold, normal and dashed lines are for 1-, 2- and 3-$\sigma$ contours, respectively.

from the ratio $\hat{R}_{1t}^{1b}$ we get the estimate

$$0.53 < \cos \theta_{\tilde t} < 0.59$$  \hspace{1cm} (49)$$

at the 1-$\sigma$ level, cf. Fig. 6.

Having at hand three possible decay modes we can constrain not only the mixing angle $\cos \theta_{\tilde t}$ but also the mass of the light stop quark and the CP-violating phase $\phi_{\tilde t}$. This can be done using a $\chi^2$ fit defined as follows

$$\chi^2 = \sum_{\{i,j\}} \left( \frac{R_j^i (\cos \theta_{\tilde t}) - R_j^{i_{\text{nominal}}}}{\Delta R_j^i} \right)^2 ,$$  \hspace{1cm} (50)$$

where the error is defined by Eq. (18) and the sum runs over the respective ratios for each of the scenarios, e.g. $\{i, j\} = \{\{1b, 1t\}, \{1b, 2t\}, \{1t, 2t\}\}$ in Scenario A, cf. Eqs (37)–(40). The results of fitting the stop mass $m_{\tilde t_1}$ and the mixing angle $\cos \theta_{\tilde t}$ in Scenario A are shown in the left panel of Fig. 7. We find two minima of $\chi^2$ that fit the input data well. In order to resolve the two-fold ambiguity, additional observables will be needed. Assuming that we can pin down the correct solution we get the following 1-$\sigma$ estimate of the two parameters

$$m_{\tilde t_1} = 366^{+3}_{-2}, \quad \cos \theta_{\tilde t} = 0.56 \pm 0.04, \quad \theta_{\tilde t} = 0.98 \pm 0.05$$  \hspace{1cm} (51)$$

for 1000 events. The better lower bound for the measured mass is a consequence of the earlier discussed small difference between $m_{\tilde t_1}$ and $m_{\chi_2^0} + m_t$. In the right panel of Fig. 7
we show the results of the $\chi^2$ fit to the mixing angle and the CP phase $\phi_\tilde{t}$. As expected, the sensitivity to the CP phase is poor and taking into account the possible ambiguity in the mixing angle $\cos \theta_\tilde{t}$, the full range of phases remains allowed.

The situation changes for Scenarios B and C. We now have 6 possible ratios in each case, for Scenario B: $R_{1t}^{1b}$, $R_{2t}^{1b}$, $R_{2t}^{1t}$, $R_{2t}^{1b}$, $R_{2t}^{1b}$, and $R_{2t}^{2b}$, and for Scenario C: $R_{1t}^{1b}$, $R_{2t}^{1b}$, $R_{2t}^{1t}$, $R_{3t}^{1b}$, $R_{3t}^{1t}$, $R_{3t}^{2t}$. The results of the fit for $n = 1000$ events have been shown in Figs. 8 and 9. We again consider two cases: fitting of the mass $m_\tilde{t}$ together with the mixing angle $\cos \theta_\tilde{t}$ and fitting of the mixing angle together with the CP-violating phase. In each case we assume that the value of the third parameter is known. Charginos and neutralinos now have a significant higgsino component and, as we saw in Figs. 2 and 3, the dependence on the mixing angle is much weaker. Therefore the constraints for the mixing angle and the mass that we get are not as good as in the case of Scenario A. It is interesting to note that in general the results of the fit are better in Scenarios A and C (gaugino and higgsino, respectively) than in Scenario B (mixed case). Consequently we conclude that the scenario with strong mixing between gauginos and higgsinos would be the most difficult to resolve.

Analyzing both the mixing angle and the phase, we obtain four allowed regions. Nevertheless only small regions are allowed for the CP phase as our observables are more sensitive to it than in Scenario A. Branching ratios are CP-even observables, therefore they cannot resolve ambiguities for the CP phase. This shows that for precise measurements in the stop
sector one has to use other CP-sensitive observables, like triple products of momenta [25, 29]. Only such a combined analysis of CP-even and CP-odd observables can give an unambiguous determination of the stop sector parameters.

Finally we discuss the results in Scenario C when combining decay modes $\tilde{\chi}_0^2 t$ and $\tilde{\chi}_3^0 t$, as the two close-in-mass higgsino-like neutralinos may be difficult to disentangle at the LHC. The fit is now performed to 5 ratios of branching ratios: $R_{11}^{b}$, $R_{21}^{b}$, $R_{21}^{t}$, $R_{23}^{b}$ and $R_{23}^{t}$, Eqs. (37) and (40). In Fig. 10 we show the fit to the stop mass and the mixing angle (left panel), and to the mixing angle and the CP phase (right panel). It turns out that we lose sensitivity to the elements of the stop mixing matrix. In such a case additional input, for example from linear collider, would be needed in order to resolve properties of the stop sector.

5 Conclusions

In this paper we have analyzed the stop sector of the Minimal Supersymmetric Standard Model. In particular, we have looked at the couplings and the decays of the supersymmetric top partners to the charginos and the neutralinos. As stops play an important role in the MSSM, it is crucial to measure their couplings and masses at future colliders in order to understand the underlying model. Therefore we have proposed a promising method for the determination of the stop sector parameters at the LHC.
Figure 10: A \( \chi^2 \) fit of ratios \( R_{11}, R_{21}, R_{23}, R_{23} \), and \( R_{23} \), Eq. (37) and Eq. (40), to the stop mass and the mixing angle \( \cos \theta_{\tilde{t}} \) (left panel) and the stop mixing angle and the CP phase \( \phi_{\tilde{t}} \) (right panel) in Scenario C for \( n = 1000 \) events. Bold, normal and dashed lines are for 1-, 2- and 3-\( \sigma \) contours, respectively.

A careful analysis of the couplings of scalar tops to electroweak gauginos and higgsinos shows a strong dependence on the mixing angle and the CP-violating phase of the stop sector. This effect arises due to the structure of the electroweak gauge couplings and the Yukawa couplings of left and right stop states. We have analyzed three benchmark scenarios with different structures for the gaugino and higgsino sectors, where the light charginos and neutralinos had gaugino-like, higgsino-like or mixed composition. Analysis of the decay widths and the branching ratios has shown a strong relation between the stop mixing parameters and the decay pattern in each of the scenarios.

Next, we have discussed a possible approach to determine the light stop mass, the mixing angle and the CP-violating phase at the Large Hadron Collider. Since stops will be produced in large numbers at this machine one can hope to learn the stop properties from their decay pattern. As the branching ratios are going to be difficult to be measured at the LHC we propose to analyze the ratios of branching ratios for different decay modes. These observables inherit a strong dependence on the mixing angle from stop decay widths and therefore can be a sensitive probe of the stop sector. Since they rely only on the relative numbers of stops decaying via various channels, many experimental uncertainties will cancel. In particular, one does not need to control all of the possible decay modes. In fact, as we have shown for the SPS1a parameter point, using only two decay modes can give good constraints on the stop mixing angle. Finally we have performed \( \chi^2 \) fits to show that the ratios of branching
ratios can give strong bounds on the parameters of the stop sector: the mass of \( \tilde{t}_1 \), the stop mixing angle \( \cos \theta_{\tilde{t}} \) and the CP-violating phase \( \phi_{\tilde{t}} \). The expected accuracy depends upon the scenario studied but looks the most promising for mSUGRA models.

Application of this method will require the study of many possible final states. Therefore a good control of detector effects, like fake rates for leptons and \( b \)-jets, and SM as well as SUSY backgrounds will be needed. It is clear that more detailed experimental studies are needed to assess the validity of the method and its possible accuracy. However, taking into account the importance of the stop sector for our understanding of the supersymmetric model, we think that such a study deserves further attention.

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