Nonadiabatic charge pumping across two superconductors connected through a normal metal region by periodically driven potentials

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Abstract
Periodically driven systems exhibit resonance when the difference between an excited state energy and the ground state energy is an integer multiple of \(\hbar\) times the driving frequency. On the other hand, when a superconducting phase difference is maintained between two superconductors, subgap states appear which carry a Josephson current. A driven Josephson junction therefore opens up an interesting avenue where the excitations due to applied driving affect the current flowing from one superconductor to the other. Motivated by this, we study charge transport in a superconductor–normal metal–superconductor junction where oscillating potentials are applied to the normal metal region. We find that for small amplitudes of the oscillating potential, driving at one site reverses the direction of current at the superconducting phase differences when difference between the subgap eigenenergies of the undriven Hamiltonian is integer multiple of \(\hbar\) times the driving frequency. For larger amplitudes of oscillating potential, driving at one site exhibits richer features. We show that even when the two superconductors are maintained at same superconducting phase, a current can be driven by applying oscillating potentials to two sites in the normal metal differing by a phase. We find that when there is a nonzero Josephson current in the undriven system, the local peaks and valleys in current of the system driven with an amplitude of oscillating potential smaller than the superconducting gap indicates sharp excitations in the system. In the adiabatic limit, we find that charge transferred in one time period diverges as a powerlaw with pumping frequency when a Josephson current flows in the undriven system. Our calculations are exact and can be applied to finite systems. We discuss possible experimental setups where our predictions can be tested.

Keywords: Floquet dynamics, quantum charge pumping, Josephson junction, Josephson effect

(Some figures may appear in colour only in the online journal)

1. Introduction
DC Josephson effect is a phenomenon of flow of current across two bulk superconductors maintained at a phase difference [1]. This current is directly proportional to the sine of the phase difference. A simple model to study this phenomenon consists of two semi-infinite one-dimensional superconducting channels connected by a suitable boundary condition that describes the junction. In such a model where the superconducting channels are described by Bogoliubov–de-Gennes mean-field Hamiltonian, the bulk states in the continuum band do not contribute to the Josephson current and the entire Josephson current is carried by the lower (at energy \(-E_0\) of the two quasiparticle bound

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states formed (at energies $\pm E_0$) within the superconducting gap [2].

The Josephson effect is important from the point of view of fundamental physics as well as applications. A few years after its experimental confirmation in 1963 [3], the value of $2e/h$ was experimentally measured [4]. The prediction by Josephson that a weaklink between two superconductors irradiated with microwave radiation can convert the frequency of the radiation into dc voltage was verified by Shapiro [5] and this eventually led to the development of voltage standard based on Josephson effect [6]. A Josephson junction embedded in a low temperature surrounding can sense the thermal noise through frequency modulations and this principle has been used to measure temperatures of the order of micro-milli Kelvin [7]. Nontrivial spin triplet superconductivity [2] embedded in a low temperature surrounding can sense the difference between a resonant energy and the Fermi energy is equal to $h$ times the frequency of the oscillating potential.

This is because interaction of an electron at the Fermi energy with the oscillating potential results in change in the electron’s energy by $\hbar \omega$ as depicted in reference [23]. This motivates us to study current in a Josephson junction with an oscillating potential in between the two superconductors. A simple question to ask would be—what is the effect of oscillating potential on the Josephson current, particularly when the difference between the two subgap bound state energies is equal to $h$ times the pumping frequency $\omega$? We show that for small amplitudes of oscillating potential, when the ratio $2E_b/\hbar \omega$ is a positive integer, the current deviates from Josephson current of the undriven system. For larger amplitudes of the oscillating potential, we find that the above said deviation of the current happens even when $2E_b/\hbar \omega$ is not an integer. We explain these deviations in the current.

A superconductor that is grounded acts as a reservoir for charge. A system described by lattice model with finite number of sites can be exactly studied by exact diagonalization of the Hamiltonian. Further, when an oscillating potential is applied the system is disturbed from the equilibrium ground state and the current is not periodic though the Hamiltonian is. Hence, current averaged over one time period is not a measure of pumped charge. Instead, current averaged over infinite time starting from the moment oscillating potential is switched on is a measure of pumped charge. We call this current ‘time averaged current’ and denote it by $I_{\text{av}}$. It was shown that by suitably choosing the basis, this current averaged over infinite time can be reduced to current averaged over one time period [24]. This is a numerical calculation and requires the Hilbert space dimension to be finite. Hence we choose the superconductors to have finite number of lattice sites.

Rest of the paper is structured as follows. Section 2 discusses the model. Section 3 discusses the details of calculation. Section 4 discusses the results and presents an analysis of the results. Finally we discuss the implications of our results, experimental setups where our results can be put to test and conclude in section 5.

2. Model

Each superconductor in the SNS junction consists of $L_g$ sites. The superconductor on the left has a phase $\phi_S/2$ and the superconductor on the right has a phase $-\phi_S/2$. We take two models for the normal metal in between. In the first model, the normal metal in between consists of only one site and an oscillating potential $V(t) = V_0 \cos(\omega t + \phi_0) \cdot \Theta(t)$ is applied on it. In the second model, the normal metal in between has two sites. An oscillating potential $V_1(t) = V_0 \cos(\omega t + \phi_0) \cdot \Theta(t)$ is applied to the first site and an oscillating potential $V_2(t) = V_0 \cos(\omega t + \phi_0 + \delta \phi) \cdot \Theta(t)$ differing by a phase factor of $\delta \phi$ is applied to the second site. In both the models, the oscillating potential is zero until $t = 0$ and for $t > 0$, it is sinusoidal with a frequency $\omega$. A static chemical potential $\mu$ is present
on both the superconductors and on the normal metal sites. The Hamiltonian for the system in the first model is given by $H = H_0 + H_1(t)$, where

$$H_0 = H_L + H_{LN} + H_N + H_{NR} + H_R,$$

$$H_L = -w \sum_{n=-1}^{L_g+1} (c_{n-1}^\dagger c_n + \text{h.c.}) + \sum_{n=-1}^{L_g} c_n^\dagger - \mu \tau_z$$

$$+ \Delta \cos \left( \frac{\phi_S}{2} \right) \tau_x + \Delta \sin \left( \frac{\phi_S}{2} \right) \tau_y c_n,$$

$$H_{LN} = -w' (c_{-1}^\dagger c_0 + \text{h.c.}),$$

$$H_N = -\mu c_0^\dagger c_0,$$

$$H_{NR} = -w' (c_1^\dagger c_0 + \text{h.c.}),$$

$$H_1(t) = V(t) c_0^\dagger c_0,$$  \( \tag{1} \)

$c_n = [c_n^\uparrow, -c_n^\downarrow, c_n^\uparrow, -c_n^\downarrow]^T$, where $c_{n,\sigma}$ annihilates an electron at site $n$ with spin $\sigma$. $\tau_{x,y,z}$ are the Pauli matrices acting in the particle–hole sector. The Hamiltonian for the second model which describes the phenomenon of pumping at two sites is given by $H = H_0 + H_1(t)$ resembling closely with equation (1) except for the following changes

$$H_S = -\mu c_{0A}^\dagger c_{0A} + c_{0B}^\dagger c_{0B} + \text{h.c.},$$

$$H_{LN} = -w' (c_{-1}^\dagger c_{0A} + \text{h.c.}),$$

$$H_{NR} = -w' (c_1^\dagger c_{0A} + \text{h.c.}),$$

$$H_1(t) = V_1(t) c_{0A}^\dagger c_{0A} + V_2(t) c_{0B}^\dagger c_{0B}.$$  \( \tag{2} \)

A schematic picture of the two models is shown in figure 1.

3. Details of calculation

To begin with, just a finite phase difference $\phi_S$ in absence of oscillating potential drives a Josephson current $I_J$ from one superconductor to the other. This Josephson current can be calculated by summing over expectation values of current operator at the bond between superconductor and the normal metal [(−1, 0) for the first model and (−1, 0A) for the second model] taken with respect to all eigenstates of $H_0$ which have energies less than zero. In the limit of large system size $(L_S \to \infty)$, only the (subgap) bound state with energy $-E_b$ less than zero contributes to the Josephson current. For a finite system, though the subgap state with energy $-E_b$ contributes the significantly to the Josephson current, the contribution from other occupied states below the Fermi energy cannot be neglected. Let $\{ |u_i\rangle, E_i, i = 1, 2, \ldots, N \}$ (where $N$ is the dimension of the Hilbert space) be the eigenstates and eigenenergies of $H_0$ arranged in the ascending order of the eigenenergies. For the first model which has one normal metal site in the middle, $N = 2(L_S + 1)$ and for the second model which has two normal metal sites in the middle, $N = 8(L_S + 1)$. For the first model, the current operator at bond (−1, 0) is

$$J = \frac{-i e w'}{\hbar} (c_{-1}^\dagger c_0 - c_0^\dagger c_{-1}).$$  \( \tag{3} \)

For the second model, the current operator at bond (−1, 0A) is

$$J = \frac{-i e w'}{\hbar} (c_{-1}^\dagger c_{0A} - c_{0A}^\dagger c_{-1}).$$  \( \tag{4} \)

Here, $e$ is electron charge. The Josephson current of the undriven system is sum of currents carried by the $N/2$ occupied states of the Hamiltonian $H_0$ and is given by

$$I_J = \sum_{i=1}^{N/2} \langle u_i | J | u_i \rangle.$$  \( \tag{5} \)

A time dependent potential switched on at time $t = 0$ takes the system away from equilibrium ground state. Though the Hamiltonian is now periodic in time, the current is not periodic. Hence, current averaged over one time period does not quantify the charge that is transferred from one superconductor to the other. On the other hand, current averaged over infinite time starting from $t = 0$ is a good measure of charge that is transferred from one superconductor to the other. It was shown that this infinite time average can be reduced to an average over one time period in the following way [24]. Let the time interval $[0, T]$ be divided into $M$ equal slices. Here, $T = 2\pi/\omega$ is the time period of the oscillating potential. Size of each slice is $\delta t = T/M$. Let $t_k$ be at the center of $k$th interval. Then, the unitary time evolution operator from $t = 0$ to $t = T$ under the discretized oscillating potential is

$$U(T, 0) = \prod_{k=1}^{M} \exp[-iH(t_k)\delta t],$$  \( \tag{6} \)

where $T$ is time ordering operator that moves the operator at earlier time to the right and the potential $V(t)$ is taken to be equal to $V(t_k)$ in the $k$th time interval. Let $|\psi_i\rangle$ be the eigenstate
of $U(T,0)$ with eigenvalue $e^{i\theta}$. Such eigenstates are called Floquet states. When $\{\theta_j, j = 1, 2, \ldots, N\}$ are nondegenerate, the time averaged current $I_{av}$ is given by

$$I_{av} = \sum_{j=1}^{N/2} \sum_{j=1}^{N} |c_{ij}|^2 (J_T)_{jj}$$

where $c_{ij} = \langle u_i | v_j \rangle$ and $(J_T)_{jj} = \frac{1}{T} \sum_{k=1}^{M} \langle v_j | U^j (t_k, 0) | J | U(t_k, 0) | v_j \rangle \mathrm{d}t$. \(7\)

Physically, one can see from equation \(7\) that current is carried by the Floquet states $|v_j\rangle$. The sum over $i = 1, 2, \ldots, N/2$ here means that contributions from all the initially occupied states $|u_i\rangle$ are counted. For later use, we shall define $I_{b,av} = \sum_{j=1}^{N} |c_{v0,j}|^2 (J_T)_{jj}$, the current contribution from the initially occupied subgap eigenstate of $H_0$ with eigenenergy $-E_b$. This is going to play a major role in our analysis as $I_{b,av}$ is substantially high in $I_{av}$.

4. Results and analysis

4.1. One-site pumping with small $V_0$

We first study one site pumping—the phenomenon of charge transport when oscillating potential is applied to only one site. We choose the amplitude of pumping potential $V_0$ to be small compared to the superconducting gap $\Delta$. For the system described by the Hamiltonian in equation \(1\), the following parameters are chosen: $L_S = 4$, $\mu = 0.01 w$, $\Delta = 0.5w$, $w' = 0.9w$, $V_0 = 0.05 w$, $\phi_{0V} = -0.5 \pi$ and $\hbar \omega = 0.2 w$. The time period $T = 2\pi/\omega$. The time interval $[0,T]$ is sliced into $M = 50$ slices. $V_0$ is chosen to be small so that the subgap states do not mix with the bulk states of the superconductor. $\Delta = 0.5 w'$ is chosen to be substantially large so that the wavefunction of the subgap energy state decays fast into the bulk of the superconductor so that we can work with a smaller superconductor. The zero energy wavefunction on last site for a system with $L_S = 4$ has a magnitude which is $1/e$ times its magnitude at the center. In figure 2, we plot the Josephson current and the time averaged current as a function of the superconducting phase difference $\phi_S$. The currents are antisymmetric about the phase difference $\phi_S = \pi$. At values $\phi_S/\pi = 0.3246, 0.5929, 0.8045, (2.0 - 0.8045), (2.0 - 0.5929), (2.0 - 0.3246)$, the time averaged current deviates the most from the Josephson current. At these values of phase difference, $I_{av}$ is very close to zero and $I_{b,av}$ is even more closer to zero. We explain the origin of these deviations in the following paragraph.

Since the subgap energy state at $-E_b$ is closest to the Fermi energy, this state participates the most in transport. As explained by the Floquet theory of pumping \(21\), $H_1(t)$ mixes the state at energy $E$ with the states at energies $E + n\hbar \omega$ where $n$ is any integer. As $\phi_S$ varies, the difference $E_0$ between the lowest unoccupied state of $H_0$ (at energy $E_0$) and the highest occupied state of $H_0$ (at energy $-E_b$) changes. When this difference is an integer multiple of $\hbar \omega$, the transitions mediated by $H_1(t)$ take place between the energy levels $\pm E_b$ and such a transition is the reason for the deviation of $I_{av}$ from $I_1$. In figure 3, we plot $2E_b/\hbar \omega$ versus $\phi_S$ and find that at the values of $\phi_S$ where the current $I_{av}$ saw substantially high deviation from $I_1$, $2E_b/\hbar \omega$ is an integer.

These deviations in the time averaged current $I_{av}$ from the Josephson current $I_1$ can be understood in another way. Let us focus on the current contribution $I_{b,av}$ due to the subgap state $|\psi_{0}/2\rangle$. Since $I_{b,av} = \sum_{j=1}^{N} |c_{\psi_{0}j}|^2 (J_T)_{jj}$, the current carried by the Floquet state $|\psi_{0}\rangle$ for which the overlap $|c_{\psi_{0}j}|^2$ is significant contributes the most to $I_{b,av}$. At any value of $\phi_S$ where the deviation of $I_{av}$ from $I_1$ is small, we find that the overlap $|c_{\psi_{0}j}|^2 \sim 1$. This means that the Floquet state $|\psi_{0}\rangle$ is very close to the subgap state $|\psi_{0}/2\rangle$ and hence the current it carries is almost equal to the Josephson current. At values of $\phi_S$ where the deviation of $I_{av}$ from $I_1$ is significant, we find that more than one Floquet state has significant overlap with the initial state $|\psi_{0}/2\rangle$. For the case of $\phi_S = 0.8045\pi$, $|c_{\psi_{0}/2j}|^2 = 0.50614$ and $|c_{\psi_{0}/2j}| = 0.49386$ meaning that there are two Floquet states
eigenstates of $\{v_1\}$ and $\{v_2\}$ which have high overlap with the initial subgap state. The currents carried by these Floquet states are $(J_f)_{11} = 0.004 472 e w / h$ and $(J_f)_{22} = -0.004 472 e w / h$ and these currents almost cancel out when calculating $I_{aw}$ due to the almost equal probabilities multiplying them. Therefore, $I_{sw} \approx 1.094 \times 10^{-8} e w / h$ (there is a factor of two multiplying the current here due to spin which we did not take into account while counting the states). Each of the two Floquet states that matters here has significant and almost equal overlap with the two subgap states $|u_{N/2}\rangle$ and $|u_{N/2+1}\rangle$ at energies $-E_b$ and $+E_b$. Hence the eventual Floquet states to which the ground state is driven into are (almost) equal superpositions of the ground state $|u_{N/2}\rangle$ and the excited state $|u_{N/2+1}\rangle$. Since the ground state and excited state are at energies $\mp E_b$, they are related by particle–hole symmetry. Hence they carry equal and opposite currents. Hence it can be heuristically said that the total current carried by the eventual nonequilibrium state which is an almost equal superposition of the ground state and the excited state is (almost) zero. Similarly, the deviation of $I_{aw}$ from $I_f$ at other values of the Josephson phase difference can be explained.

4.2. One site pumping with larger $V_f$

Now, we make one change to the parameters compared to the previous subsection. We choose $V_0 = w$. This means the oscillating potential $V(t)$ helps to access higher energy eigenstates of $H_0$ starting from the ground state. Hence richer features are expected in the time averaged current. In figure 4 we plot the currents $I_{aw}$ and $I_f$ as a function of the phase difference $\phi_S$. Substantial deviation of $I_{aw}$ from $I_f$ is observed in the entire range $0 < \phi_S / \pi < 2$ except at $\phi_S = 0, \pi$. Furthermore, local peaks/valleys are observed at $\phi_S / \pi = 0.096, 0.1548, 0.2105, 0.2866, 0.2925, 0.5482, 0.8289, 0.9016, (2.0 – 0.9016), (2.0 – 0.8289), (2.0 – 0.5482), (2.0 – 0.2925), (2.0 – 0.2866), (2.0 – 0.2105), (2.0 – 0.1548), (2.0 – 0.096). This deviation in current is obviously due to mixing between eigenstates of $H_0$ caused by $H_1(t)$. Initially at time $t < 0$, the system is in the ground state with all negative energy states being occupied. After $t = 0$, $H_1(t)$ causes excitations. A measure of excitations is the deviation $\{-(E - E_0) / E_0\}$ in the time averaged energy $E$ relative to the ground state energy $E_0$, where

$$E = \sum_{i=1}^{N/2} \sum_{j=1}^{N} |c_{ij}|^2 \langle E_f \rangle j j, \ c_{ij} = \langle u_i | v_j \rangle ,$$

$$\langle E_f \rangle j j = \frac{1}{T} \sum_{k=1}^{M} \langle v_j | U(t_0) | \hat{H}_0 | U(t_0) | v_j \rangle \, dt,$$

and $E_0 = \sum_{i=1}^{N/2} |u_i | H_0 | u_i \rangle$.

A negative sign multiplies the numerator here since the denominator $E_0$ is negative. This relative deviation in energy versus the superconducting phase difference $\phi_S$ is plotted in figure 5. A comparison between figures 4 and 5 indicates that deviation in the current $I_{aw}$ from $I_f$ is a fingerprint of excitations in the Floquet system. Thus, a deviation in current $I_{aw}$ from $I_f$ can be seen as a sign of excitations but not as a direct quantitative measure. A local peak or a local valley in $I_{aw}$ is a sign of an excitation in the driven system.

4.3. Two site pumping with $\phi_S = 0$

We now turn to two site pumping—the phenomenon of charge transport when oscillating potentials are applied to two sites in the normal metal region. For the system described by the Hamiltonian in equation (2), the parameters are chosen to be $L_S = 4$, $\mu = 0.01 w$, $\Delta = 0.5 w$, $w' = 0.9$, $w'' = 0.3 w$, $V_0 = 0.05 w$, $\phi_{0V} = 0$, $\phi_S = 0$, $\omega = 2 E_b$, and the time interval of one time period $[0, 2 \pi / \omega]$ is divided into 50 intervals of equal size for calculating $U(T, 0)$. First we check whether there is charge transport purely due to oscillating potentials when the superconducting phase difference between the two superconductors is zero ($\phi_S = 0$). The pumping frequency is set equal to the energy difference between the lowest unoccupied energy level of $H_0 (E_b)$ and the highest occupied energy level of $H_0 (-E_b)$.
The pumped current which is time averaged current versus the phase difference \( \delta \phi V \) between the oscillating potentials is plotted in figure 6. The current can be pumped in either directions depending upon the difference in phases of the pumping potentials. However, none of the eigenstates of \( H_0 \) carry any nonzero current since the superconducting phase difference is zero. This means that even when there are excitations in the system, there may not be a change in the current. Hence, the pumped current \( I_w \) cannot be taken as a measure of excitations when \( \phi_S = 0 \). By the same logic, a nonzero pumped current does not signify excitations when \( \phi_S \) is an integer multiple of \( \pi \).

### 4.4. Two site pumping with \( \phi_S = 0.9 \pi \)

Now, we switch on a superconducting phase difference \( \phi_S = 0.9 \pi \) between the two superconductors which drives a Josephson current \( I_j = -0.08333eV/\hbar \). Setting \( \phi_{0V} = 0.5 \pi \), \( \omega = 0.2w \) and keeping other parameters same as before, we calculate the time averaged current as a function of the phase difference between the oscillating potentials \( \delta \phi V \) and plot it in figure 7. We have chosen \( \omega \) to be close to \( 2E_b = 0.21823w \). The current \( I_w \) which is the sum of the Josephson current and the pumped current deviates from the Josephson current substantially for nonzero \( \delta \phi V \) while the deviation for zero \( \delta \phi V \) is minimal due to small \( V_0 \). Since the Josephson current carried by different eigenstates of \( H_0 \) is substantially different for this choice of superconducting phase difference \( \phi_S \), the time averaged current \( I_w \) can be expected to signify the excitations of the system. In figure 8, we plot the excitation energy of the driven system relative to the ground state energy of the undriven system versus the difference in phases of the oscillating potentials for the same set of parameters. We find that excitation energy shows the same dependence on the phase difference between the oscillating potentials as the deviation of current \( I_w \) of the driven system from the Josephson current \( I_j \) of the undriven system. Thus, for a value of superconducting phase difference 

\[
\phi_S \neq 0, \text{ the deviation of current } I_w \text{ from the Josephson current } I_j \text{ is a sign of excitations of the system. We find that such a correlation between the current } I_w \text{ and the excitations of the driven system holds good for small amplitude of oscillating potential } (V_0 < \Delta).
\]

### 4.5. Approaching the adiabatic limit

Now we turn our attention to the adiabatic limit. We study two site pumping since this can transfer nonzero pumped charge in the adiabatic limit when the superconducting phase difference is zero. We take the adiabatic limit by taking the pumping frequency \( \omega \to 0 \). Since this implies that the time period \( T \to \infty \), we maintain the size of the time slicing to be the same which means that the number of slices is proportional to the time period, given by \( M = 5[T \Delta / \hbar] \). Then, we calculate the charge transferred in one time period (since the current \( I_w \) may be zero
Figure 9. Logarithm of charge transferred per time period $T$ in units of $e$ versus logarithm of pumping frequency $\omega$ for the choice of parameters: $L_S = 4$, $\mu = 0.01w$, $\Delta = 0.5w$, $\omega' = 0.3w$, $V_0 = 0.05w$, $\phi_{\delta} = 0.8\pi$. Left panel: $\delta \phi_V = 0.4\pi$, right panel: $\delta \phi_V = 0$. A linear fit of the form $y = mx + c$ to the curves with $(m, c, \text{error}) = (1.0004, 0.5513, 0.0004)$ for the left panel and $(m, c, \text{error}) = (1.0003, 0.5532, 0.0003)$ for the right panel show that in both the cases, the charge transferred diverges in the limit $\omega \to 0$ as a powerlaw.

5. Discussion and conclusion

A closely related phenomenon is inverse ac Josephson effect [38] in which a microwave electromagnetic radiation shined on the Josephson junction results in a time-varying voltage whose dc component is quantized. However, a sinusoidally varying current is induced in this effect which results in variation of Josephson phase difference with time in contrast to our work where the Josephson phase difference remains fixed. The time dependent potentials in our work create an excited Floquet state for the hybrid device but not a potential difference between one superconductor and the other.

It is known that periodically driven interacting systems approach a periodic steady state [32, 33]. So, due to weak interactions in the realistic system, the system approaches a periodic steady state described by the Floquet states after a long time. The current averaged over one time period in such a state will be $I_{av}$. And the excitation energy averaged over one time period will be $E$ given by equation (8). Hence, $I_{av}$ can be measured in a realistic system. Our prediction that deviation of time averaged current $I_{av}$ of the driven system from the Josephson current $I_J$ of the undriven system is a signature of excitations of the driven system (when $I_J \neq 0$) can be applied to realistic systems. Identification of a correlation between physical quantities of interest in Floquet systems to a measurable physical quantity such as current is an important step that can improve our understanding of periodically driven systems.

The total current in the Josephson junction with an applied oscillating potential can be seen as a sum of the Josephson current and a pumped current. The pumped current can be nonzero when spatial inversion symmetry is broken (by a nonzero superconducting phase difference) and a time dependent potential is applied to one site. We see from figures 2 and 4 that this is indeed the case. Moreover, at some values of the superconducting phase difference the pumped current is not only in opposite direction to the Josephson current but the former is greater in magnitude than the latter. This is analogous to the current reversal by shining microwave radiation on Josephson junction proposed by Gorelik et al [39]. Further, we have shown that by applying oscillating potentials at two sites differing by a nonzero phase, charge can be pumped between two superconductors maintained at same superconducting phase. In the adiabatic limit, we find that charge gets transferred in one time period when a Josephson current flows from one superconductor to another in the undriven system. This charge diverges as a powerlaw with the pumping frequency in the adiabatic limit. The charge transferred so is predominantly carried by the Josephson current of the undriven system but with a correction. However, the charge transferred in one time period purely due to pumping cannot be said to be zero.

We have described a way of calculating pumped current exactly for a nonadiabatically driven Josephson junction. To test our predictions experimentally, we propose to connect superconducting leads to the electrostatically defined semiconductor quantum dots where the gate voltages can be used to apply time dependent potentials. Charge pumping has been achieved in quantum dots synthesized from GaAs–AlGaAs heterostructures (except that the reservoirs are normal metal
leads [18]. Further, superconductivity can be induced in GaAs heterostructures [40]. Hence, gate tunable quantum dots can be connected to superconducting lead. Another direction to experimental realization comes from the fact that carbon nanotubes can be employed as quantum dots and certain superconductors can be connected to carbon nanotubes [41]. Apart from these, there are many more platforms where quantum dots have been coupled to superconductors [42]. In these systems, the quantum dots can be gate tuned. AC voltage source need to be connected to the gate electrodes to induce time dependent potentials in the normal metal region. The task in engineering the system we studied is to maintain a tunable Josephson phase difference between the two finite superconductors connected on either sides of the quantum dot. We envisage that such periodically driven Josephson junctions will be realized in near future and our predictions can be tested.

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References

[1] Josephson B D 1962 Possible new effects in superconductive tunneling Phys. Lett. 1 251
[2] Furusaki A 1999 Josephson current carried by Andreev levels in superconducting quantum point contacts Superlattices Microstruct. 25 809–18
[3] Anderson P W and Rowell J M 1963 Probable observation of the Josephson superconducting tunneling effect Phys. Rev. Lett. 10 230–2
[4] Parker W H, Taylor B N and Langenberg D N 1967 Measurement of 2e/h using the ac Josephson effect and its implications for quantum electronics Phys. Rev. Lett. 18 287–91
[5] Shapiro S 1963 Josephson currents in superconducting tunneling: the effect of microwaves and other observations Phys. Rev. Lett. 11 80–2
[6] Levinsen M T, Chiao R Y, Feldman M J and Tucker B A 1977 An inverse ac Josephson effect voltage standard Appl. Phys. Lett. 31 776–8
[7] Kamper T S and Zimmerman J E 1971 Noise thermometer with the Josephson effect J. Appl. Phys. 42 132–6
[8] Khaire T S, Khasawneh M A, Pratt W P and Birge N O 2010 Observation of spin-triplet superconductivity in co-based Josephson junctions Phys. Rev. Lett. 104 137002
[9] Heersche H B, Jarillo-Herrero P, Oostinga J B, Vanderven L M K and Morpurgo A F 2007 Bipolar supercurrent in graphene Nature 446 56–9
[10] Williams J R, Bestwick A J, Gallagher P, Hong S S, Cui Y, Bleich A S, Analysis J G, Fisher I R and Goldhaber-Gordon D 2012 Unconventional Josephson effect in hybrid superconductor-topological insulator devices Phys. Rev. Lett. 109 056803
[11] Rokhinson L P, Liu X and Furdyna J K 2012 The fractional ac Josephson effect in a semiconductor-superconductor nanowire as a signature of Majorana particles Nat. Phys. 8 795
[12] Laroche D et al 2019 Observation of the 4π-periodic Josephson effect in indium arsenide nanowires Nat. Commun. 10 245
[13] Kleiner R 2007 Filling the terahertz gap Science 318 1254–5
[14] Wendin G and Shumeiko V S 2007 Quantum bits with Josephson junctions Low Temp. Phys. 33 724–44
[15] Clarke J and Wilhelm F K 2008 Superconducting quantum bits Nature 453 1031–42
[16] Thouless D J 1983 Quantization of particle transport Phys. Rev. B 27 6083–7
[17] Brouwer P W 1998 Scattering approach to parametric pumping Phys. Rev. B 58 R1035–8
[18] Switkes M, Marcus C M, Campman K and Gossard A C 1999 An adiabatic quantum electron pump Science 283 1905–8
[19] Brouwer P W 2001 Rectification of displacement currents in an adiabatic electron pump Phys. Rev. B 63 121303
[20] Avron J E, Elgart A, Graf G M and Sadun L 2001 Optimal quantum pumps Phys. Rev. Lett. 87 236601
[21] Moskalets M and Büttiker M 2002 Floquet scattering theory of quantum pumps Phys. Rev. B 66 205320
[22] Agarwal A and Sen D 2007 Equation of motion approach to non-adiabatic quantum charge pumping J. Phys.: Condens. Matter 19 046205
[23] Agarwal A and Sen D 2007 Nonadiabatic charge pumping in a one-dimensional system of noninteracting electrons by an oscillating potential Phys. Rev. B 76 235316
[24] Soori A and Sen D 2010 Nonadiabatic charge pumping by oscillating potentials in one dimension: results for infinite system and finite ring Phys. Rev. B 82 115432
[25] Marra P, Citro R and Orlita C 2015 Fractional quantization of the topological charge pumping in a one-dimensional superlattice Phys. Rev. B 91 125411
[26] Citro R and Romeo F 2006 Pumping in a mesoscopic ring with Aharonov–Casher effect Phys. Rev. B 73 233304
[27] Paul G C and Saha A 2017 Quantum charge pumping through resonant crossed Andreev reflection in a superconducting hybrid junction of silicene Phys. Rev. B 95 045420
[28] Tripathi K M, Rao S and Das S 2019 Quantum charge pumping through Majorana bound states Phys. Rev. B 99 085435
[29] Blaabjerg M 2002 Charge pumping in mesoscopic systems coupled to a superconducting lead Phys. Rev. B 65 235318
[30] Governale M, Taddei F, Fazio R and Hekking F W J 2005 Adiabatic pumping in a superconductor–normal–superconductor weak link Phys. Rev. Lett. 95 256801
[31] Grifoni M and Hänggi P 1998 Driven quantum tunneling Phys. Rep. 304 229–354
[32] Russomanno A, Silva A and Santoro G E 2012 Periodic steady regime and interference in a periodically driven quantum system Phys. Rev. Lett. 109 257201
[33] Lazarides A, Das A and Moessner R 2014 Periodic thermodynamics of isolated quantum systems Phys. Rev. Lett. 112 150401
[34] Lazarides A, Das A and Moessner R 2014 Equilibrium states of generic quantum systems subject to periodic driving Phys. Rev. E 90 012110
[35] Moessner R and Sandhii S L 2014 Periodic Floquet matter Nat. Phys. 10 424–8
[36] Oka T and Kitamura S 2019 Floquet engineering of quantum materials Annu. Rev. Condens. Matter Phys. 10 387–408
[37] Giovannini U and Hübener H 2019 Floquet analysis of excitations in materials J. Phys. Mater. 3 012001
[38] Langenberg D N, Scalapino D J, Taylor B N and Eck R E 1966 Microwave-induced dc voltages across Josephson junctions Phys. Lett. 20 563–5
[39] Gorelik L Y, Shumeiko V S, Shekhter R I, Wendin G and Johnson M 1995 Microwave-induced ‘somersault effect’ in flow of
Josephson current through a quantum constriction *Phys. Rev. Lett.* **75** 1162–5

[40] Wan Z, Kazakov A, Manfra M J, Pfeiffer L N, West K W and Rokholson L P 2015 Induced superconductivity in high-mobility two-dimensional electron gas in gallium arsenide heterostructures *Nat. Commun.* **6** 7426

[41] Cleuziou J-P, Wernsdorfer W, Bouchiat V, Ondarçuhu T and Monthioux M 2006 Carbon nanotube superconducting quantum interference device *Nat. Nanotechnol.* **1** 53–9

[42] De Franceschi S, Kouwenhoven L, Schönberger C and Wernsdorfer W 2010 Hybrid superconductor–quantum dot devices *Nat. Nanotechnol.* **5** 703–11