Uncertain Hybrid Multi-Sensor Alliance Dynamic Control Problem Using an Uncertain Ideal Point Approach Under the $P_{EV}$ Principle

JIAHAO XIE, SHUCAI HUANG, AND DAOZHI WEI
Air and Missile Defense College, Air Force Engineering University, Xi’an 710051, China
Corresponding author: Jiahao Xie (18222517021@163.com)

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ABSTRACT
Multi-sensor alliance has been the main means of gathering intelligence information in the complex battlefield environment. This paper investigates uncertain multi-sensor alliance dynamic control problem, which is a major concern in sensor management. Unlike earlier studies, the explored model explored here is based on uncertain circumstances instead of certain circumstances, and constructed which is established under the constraints of sensor tracking ability in order to improve the sensor resource utilization and target tracking accuracy. Specially, this uncertainty model is converted to a certainty model by using an uncertain ideal point approach under the $P_{EV}$ principle instead of the traditional solution approach. Also, the prediction and re-prediction mechanism is proposed to realize the dynamic control of the sensor alliance updating. The proposed methodology possesses high solution quality and a low computational cost; it also reduces the tracking errors of the target in uncertain battlefield environments when. Finally, simulation results are provided to demonstrate the feasibility and effectiveness of the proposed approaches.

INDEX TERMS
Multi-sensor alliance, dynamic control, uncertain ideal point approach, $P_{EV}$ principle, target tracking error.

I. INTRODUCTION
Cooperative multi-sensor detection has a wide range of applications in intelligence information gathering, surveillance, reconnaissance, and many other fields [1]–[4], and it also plays a significant role in target detecting and, target tracking, among other activities. An essential cooperative pattern of multi-sensors is to build a multi-sensor alliance, and the cooperative pattern is a dynamic process. Moreover, the multi-sensor alliance dynamic control problem becomes an uncertain nonlinear system control problem [5]–[9] when considering the complexity and uncertainty of the battlefield environment, including shifting targets.

With regard to the uncertain dynamic control problem, three steps should be taken: the first is to establish an uncertain multi-sensor alliance model—in other words, introduction of an objective function obeying particular rules. The second is to obtain the optimal scheme from the objective function by traversing through all potential solutions or using optimization algorithms. The last step is alliance updating, using an alliance update mechanism after establishing the alliance. When modeling the objective functions, current existing approaches can be roughly classified into three types, namely, an approach using uncertain multi-objective programming, an approach establishing a multi-sensor alliance establishing, and an approach updating the multi-sensor alliance.

With regard to the first approach, the traditional solution approach [10]–[13] is to convert the original problem into a definite multi-objective programming problem, and then convert it into a definite single objective programming problem through the classical multi-objective programming method. In the second approach, most of the establishment of the multi-sensor alliance is based on serial structure [14]–[19]. The main problem of this structure is its low efficiency and the possibility of resource waste or excessive strain. Earlier work on multi-sensor alliance establishing problem has included studies of swarm intelligence algorithms [20]–[22], [22]–[26], represented by particle swarm optimization, and a series of other algorithms, such as a linear programming
method [27] and an auction algorithm [28]. One study [29] proposed particle swarm optimization (PSO) as a method to solve the alliance formation problem. Similarly, another study [30] used discrete particle swarm optimization (DPSO) to realize alliance formation, which partially confirmed the effectiveness of swarm intelligence algorithm. In the third approach, in the process of investigating multi-sensor alliance updating mechanism, one group proposed a mechanism for measure and update, another group proposed a mechanism for prediction and update, which could be better compared with the previous one.

After careful review of previous studies, it can be seen that three problems in the multi-sensor alliance dynamic control problem still need to be revolved. The first problem is that most of the cases using the traditional solution approach are applied for the none-related objective functions and the traditional approach ignores the uncertain nature of the uncertainty problems. The second problem is that swarm intelligence algorithms do not consider dynamic changes in the environment and the problem of consumption in the process of establishing the alliance. The third problem is that the existing alliance-updating mechanisms can lead to loss of the target because of rapid movement of the target in real-life applications; these mechanisms also do not take into account dynamic changes of the battlefield environment, which could increase the risk of losing targets in the updating process.

Based on the above analysis, the main contributions of this paper can be summarized as follows:

1) The uncertain multi-objective programming problem is transformed into the uncertain single objective programming problem by using an uncertain ideal point approach; and the uncertainty is the further realized under the $P_{EV}$ principle.

2) The fireworks algorithm based on the improved selection strategy is designed to acquire an effective solution to the process of establishing the alliance.

3) An updating mechanism of prediction and re-prediction is proposed to complete the updating task of the alliance and realize stable tracking of the target.

The structure of the rest of the paper is organized as follows. In Section II, the uncertain hybrid multi-sensor alliance model is provided, including a description and mathematical model of the target motion model and of the objective functions and constraint conditions. Section III introduces solution methodology under the $P_{EV}$ principle to transform the uncertain model to a certain model. In Section IV, we present a fireworks algorithm, based on an improved selection strategy (ISSFA), to establish the alliance, and we propose a prediction and re-prediction mechanism to update the alliance. Finally, we conduct simulations to verify the proposed model and algorithms, and present conclusions in Section V.

II. UNCERTAIN HYBRID MULTI-SENSOR ALLIANCE MODEL

A. TARGET MOTION MODEL

The uncertain hybrid multi-sensor alliance assumes that each sensor has the ability of independent decision-making.
The formula of Kalman filter is:
\[
\hat{X}(k+1|k+1) = \hat{X}(k+1|k) + \sum_{i=1}^{m} K(k+1)Z_i(k+1) - H(k+1)\hat{X}(k+1|k)
\]  
(1)
\[
\hat{X}(k+1|k) = F(k+1, k)\hat{X}(k|k)
\]  
(2)
\[
K(k+1) = P(k+1|k)H^T(k+1)\left[ \left( H(k+1)P(k+1|k)H^T(k+1) + R_i \right)^{-1} \right] \sum_{i=1}^{m} Q_{k+1}
\]  
(3)
\[
P(k+1|k+1) = \left( P_{k+1}^{-1} + \sum_{i=1}^{m} (H_i^T(k+1)R_i(k+1))^{-1} \right) \left( H_i(k+1) \right)^{-1}
\]  
(4)

B. ESTABLISHING THE MODEL

Suppose that under the uncertain battlefield environment mentioned above, we deploy M sensors \( \{s_1, s_2, \ldots, s_M\} \) to detect N targets \( \{t_1, t_2, \ldots, t_N\} \). The definitions of the parameter in the uncertain hybrid multi-sensor alliance are the following:

\[ \xi = (\xi_1, \xi_2, \ldots, \xi_m) \]

is an uncertainty vector, in which each component is an independent uncertain variable, obeying regular and uncertain distributions \( \Phi_1, \Phi_2, \ldots, \Phi_m \) respectively, where all uncertainties are defined in uncertain space \( (\Gamma, \mathcal{L}, \mathcal{M}) \).

The uncertain objective function \( f_j(t, \xi) \) and uncertain constraint conditions \( g_j(t, \xi) \) are the functions under consideration when forming the alliance, where the objective function is the binary function of the time and the uncertain variable.

Because the process of forming an uncertain hybrid multi-sensor alliance will be affected by many non-determinants, it can be analyzed based on the expert reliability of the actual frequency of the event. The performance indicator of sensor performance in a complex dynamic environment is based on the relative energy-consumption value of the sensor, the detection range of the sensor, and the threat level of the sensor, where the three kinds of uncertainty are defined as the uncertain variable in the uncertain space \( (\Gamma, \mathcal{L}, \mathcal{M}) \), and then the expert reliability is given by the expert.

C. OBJECTIVE FUNCTIONS AND CONSTRAINT CONDITIONS

The objective function of an uncertain hybrid multi-sensor alliance model is as follows:

1) THE SHORTEST TIME REQUIRED TO ESTABLISH AN ALLIANCE

The minimum time it takes to form a sensor alliance to perform a task, indicating that each task is implemented by the recent sensor alliance, greatly improving the allocation of sensor resources and completion of the task. Based on expert reliability analysis, the sensor’s relative energy value and the threat level detected by the sensor will have a large impact on the time required for forming an alliance, and the following target functions can be established as follows, where \( t_{ij} \) represents the time every formed alliance used.

\[ T(t, \xi) = \sum_{i=1}^{n} \sum_{j=1}^{M} \left( \frac{\xi_1 \cdot t_{ij}}{\xi_1 + \xi_3 - t_{ij}} \right) \]  
(6)

2) THE LEAST CONSUMPTION REQUIRED TO ESTABLISH AN ALLIANCE

The consumption of hybrid multi-sensor alliance to complete the task mainly includes the consumption of sensor itself and the consumption of each sensor. Assume that every sensor’s consumption is \( c_j (j = 1, \ldots, M) \), and that every sensor alliance is marked as \( A_{M \times N} \), where \( a_{ij} = 1 \) represents the sensors tracks the target and \( a_{ij} = 0 \) represents the sensors don’t tracks the target. The sensor’s detection distance and the threat of the sensor have a large impact on the formation of a dynamic alliance, which can be established as follows:

\[ C(t, \xi) = \sum_{i=1}^{n} \sum_{j=1}^{M} \left( \ln \frac{\xi_2}{\xi_3 + 1} \cdot c_j(t) \cdot a_{ij} \right) \]  
(7)

3) THE HIGHEST TRACKING ACCURACY REQUIRED TO ESTABLISH AN ALLIANCE

Assuming that \( \{p_1, p_2, \ldots, p_M\} \) is the tracking accuracy factor corresponding to each sensor, the tracking accuracy factor is defined as follows:

\[ p_i = \frac{tr(P_i)}{\sum_{j=1}^{M} tr(P_j)} \]  
(8)

where \( P_i (i = 1, 2, \ldots, n) \) is the error matrix of least squares estimation of the target state based on the \( i \)th sensor, and \( tr (\cdot) \) is the trace of the matrix.

Based on expert reliability analysis, it can be concluded that the degree of threat level and the relative energy consumption value of the sensor have a large t influence on the formation of the alliance, with the objective function defined as follows:

\[ P(t, \xi) = \prod_{i=1}^{N} \left( \frac{\ln \xi_3}{\xi_3 + 1} p_i(t) \right) \]  
(9)

The constraint conditions are as follows:

1) Whether the sensors join the hybrid multi-sensor alliance to realize the target tracking process, where \( a_{ij} = 1 \) represents the situation where sensors track the target and \( a_{ij} = 0 \) represent the situation where the sensors do not track the target.

2) Each sensor alliance can undertake a maximum number of tasks, where \( N_{\text{max}} \) represents the maximum alliance number, \( M_{\text{max}} \) represents the maximum task number.

3) The number of targets that each sensor can track is limited and cannot exceed the maximum allowable range \( \sigma_M \).
The mathematical expression of the constraints is shown in Eq. (10).

\[
\begin{align*}
\min_{t} & \quad T(t, \xi), C(t, \xi), P(t, \xi) \\
\text{subject to:} & \\
& a_{ij} \in [0, 1] \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m \\
& N_{j} \leq N_{\text{max}} \leq M_{\text{max}} \\
& \sum_{i=1}^{N} s_{1i} \leq \sigma_{1}, \sum_{i=1}^{N} s_{2i} \leq \sigma_{2}, \ldots, \sum_{i=1}^{N} s_{Ni} \leq \sigma_{M} 
\end{align*}
\]

(10)

III. SOLUTION METHODOLOGY UNDER PEV PRINCIPLE

The uncertain multi-objective programming model can be expressed as the equation below.

\[
\begin{align*}
\min_{x \in R} f(x, \xi) &= [f_{1}(x, \xi), f_{2}(x, \xi), \ldots, f_{p}(x, \xi)] \\
\text{subject to} & \\
& g_{i}(x, \eta) \leq 0, \quad i = 1, 2, \ldots, m 
\end{align*}
\]

where \( x \in R \) is the decision variable, suppose each objective function has the same uncertain variable, \( \xi_{1}, \xi_{2}, \ldots, \xi_{m} \) is an uncertain variable, in which each component is an independent uncertain variable, and all uncertainties are defined in uncertain space.

Due to the fact that the uncertainty factors remain constant in the process of building the uncertainty hybrid multi-sensor alliance, the uncertain vectors are the same, that is, the objective functions are related, where the uncertain variables could have only two possible effects on the objective functions. One effect is a monotonic increase at the same time, and the other is monotonic decrease at the same time; for example, the detection range of the sensor change will lead to the consumption and time of sensors establishing; the change in threat level of the sensors will affect the tracking accuracy and the time of formation of the sensor alliance.

In practical engineering applications, the specific eigenvalues of different objective functions have different meanings; thus, it is necessary to determine the specific significance of the specific eigenvalues of different objective functions through several order-relation criteria. Because the uncertain objective function itself is also an uncertain variable, this paper uses the symbols < and \( \preceq \) to define the relationship between the uncertain variables. The order relation principle usually includes an expectation principle and an expectation variance principle, which are defined as follows:

**Definition 1 (Expectation Principle, PE Principle):** Suppose \( \xi \) and \( \eta \) are two uncertain variables, if and only if \( E[\xi] \leq E[\eta] \) or \( E[\xi] \leq E[\eta] \), there is \( \xi \preceq \eta \) or \( \xi \preceq \eta \).

**Definition 2 (Expectation-Variance Principle, PEV Principle):** Suppose \( \xi \) and \( \eta \) are two uncertain variables, if and only if \( E[\xi] \leq E[\eta] \) or \( E[\xi] \leq E[\eta] \) and \( V[\xi] \leq V[\eta] \) or \( V[\xi] \leq V[\eta] \), there is \( \xi \preceq \eta \) or \( \xi \preceq \eta \).

Because the PEV principle has a wide range of practical engineering applications, this study has incorporated the PEV principle in designing a solution to the uncertain hybrid multi-sensor alliance problem.

However, uncertain programming problems often have inherent uncertainty and the objective functions are not completely independent of each other. Therefore, we have used the uncertain ideal point approach to solve this kind of problem.

The uncertain ideal point approach is used to transform the uncertain multi-objective programming problem to a certain single objective programming problem by optimizing the distance from each objective function point to the optimal ideal point. Applying the approach of uncertain ideal point, we obtain:

\[
\begin{align*}
\min_{x \in \mathbb{R}} U(x, \xi) &= \sqrt{\sum_{j=1}^{p} [f_{j}(x, \xi) - f_{j}^{0}]^{2}} \\
\text{subject to} & \\
& M(g_{i}(x, \xi) \leq 0) \geq a_{i}, \quad i = 1, 2, \ldots, m
\end{align*}
\]

(12)

In which uncertain constraint conditions \( g_{i}(x, \xi) \) can be transformed into certain constraint conditions by uncertain measurement and expert reliability, and \( f_{j}^{0} \) is the lower boundary when the objective function \( f_{j}(x, \xi) \) does not consider other objective functions.

Under the PEV principle, the definition of the effective solution to the uncertain multi-objective programming problem can be expressed as theorem 1.

**Theorem 1:** The optimal solution \( x^{*} \) to uncertain single-objective programming problem under PEV principle is the PEV effective solution to the uncertain multi-objective programming problem.

**Proof:** Suppose that \( x^{*} \) is not the effective solution to the uncertain multi-objective programming problem. There must exist feasible solution to make \( f_{j}(\bar{x}, \xi) \preceq f_{j}(x^{*}, \xi), \ (j = 1, 2, \ldots, p) \) and \( f_{j}(\bar{x}, \xi) \preceq f_{j}(x^{*}, \xi), \ (j = 1, 2, \ldots, p) \) hold.

We can obtain that

\[
[f_{j}(\bar{x}, \xi) - f_{j}^{0}]^{q} \preceq [f_{j}(x^{*}, \xi) - f_{j}^{0}]^{q},
\]

(13)

When \( j \neq j_{0} \), we can obtain that

\[
[f_{j}(\bar{x}, \xi) - f_{j}^{0}]^{q} \preceq [f_{j}(x^{*}, \xi) - f_{j}^{0}]^{q}
\]

(14)

Then, we find that

\[
\sqrt{\sum_{j=1}^{p} \lambda_{j} [f_{j}(\bar{x}, \xi) - f_{j}^{0}]^{q}} \preceq \sqrt{\sum_{j=1}^{p} \lambda_{j} [f_{j}(x^{*}, \xi) - f_{j}^{0}]^{q}}
\]

(15)

where,

\[
\lambda \in \Lambda^{++} = \left\{ \lambda = (\lambda_{1}, \lambda_{2}, \ldots, \lambda_{p})^{T} \mid \lambda_{j} > 0, \sum_{j=1}^{p} \lambda_{j} = 1 \right\}, q
\]

is an integer greater than one. That is to say, \( U(\bar{x}, \xi) \prec U(x^{*}, \xi) \), which is contradicts with previous hypothesis,
The solution process of the model.

so \( x^* \) is the effective solution to uncertain multi-objective programming problem.

From the proof above, we can obtain the solution process of the uncertain hybrid multi-sensor alliance in Figure 2.

Through the approach of uncertain ideal point, the multi-objective programming problem can be transformed into the single objective programming problem. The specific form is as follows:

\[
\min_{t \in \mathbb{R}} U [t, \xi]
= \sqrt{T(\xi) - T_0^2} + [C(\xi) - C_0]^2 + [P(\xi) - P_0]^2 \tag{17}
\]

where, \( T_0, C_0 \), and \( P_0 \) are the lower boundary of \( T(t, \xi), C(t, \xi) \) and \( P(t, \xi) \) on the sequence of the effective solution sets.

The uncertain single objective programming problem can be further transformed into the corresponding equivalent deterministic model under PEV principle.

\[
\min_{t \in \mathbb{R}} E \{U(t, \xi)\}
= E \left\{ \sqrt{T(\xi) - T_0^2} + [C(\xi) - C_0]^2 + [P(\xi) - P_0]^2 \right\} \tag{18}
\]

Suppose that the three uncertainty variables \( \xi_1, \xi_2, \xi_3 \) obey the linear uncertain distribution, zigzag uncertain distribution and normal probability distribution, \( \xi_1 \sim L(1.5, 18) \), \( \xi_2 \sim Z(0.8, 1.2, 1.6) \), \( \xi_3 \sim N(1.5, 3) \), which are marked as \( \Phi_1(\alpha), \Phi_2(\alpha), \Phi_3(\alpha) \).

Taking \( \xi_1 \) as an example, after inviting experts to evaluate the impact of the relative energy consumption of sensors on the formation of multi-sensor alliances, they think that with 100% reliability, the impact of the relative energy consumption of sensors on formation of alliance will be less than 18. At the same time, experts believe that with 0% reliability, the relative energy consumption of the sensor will have less than 1.5 impact on formation of the alliance. It is assumed that the degree of influence of relative energy consumption on the sensor is linear, with expert reliability \( t \) within the numerical range, and the specific uncertainty distribution as follows:

\[
\Phi_1(\alpha) = \begin{cases} 0 & \alpha \leq 1.5 \\ \frac{\alpha - 1.5}{16.5} & 1.5 \leq \alpha \leq 18 \\ 1 & \alpha \geq 18 \end{cases}
\]

The same can be obtained:

\[
\xi_2 \sim Z(0.8, 1.2, 1.6) \quad \text{and} \quad \xi_3 \sim N(1.5, 3).
\]

In order to describe it clearly, we denote:

\[
T^{-1}(\alpha) = T \left[ t, \Phi_1^{-1}(\alpha), \Phi_2^{-1}(1-\alpha), \Phi_3^{-1}(\alpha) \right] \\
C^{-1}(\alpha) = C \left[ t, \Phi_1^{-1}(\alpha), \Phi_2^{-1}(1-\alpha), \Phi_3^{-1}(\alpha) \right] \\
P^{-1}(\alpha) = P \left[ t, \Phi_1^{-1}(\alpha), \Phi_2^{-1}(1-\alpha), \Phi_3^{-1}(\alpha) \right]
\]

Eq. (17) can be transformed into a problem of determining single objective planning under the PEV principle:

\[
\min_{t \in \mathbb{R}} E \{U(t, \xi)\}
= \int_0^1 \sqrt{T(\xi) - T_0^2} + [C(\xi) - C_0]^2 + [P(\xi) - P_0]^2 \, d\alpha \tag{19}
\]

According to the three objective functions, \( T(t, \xi), C(t, \xi), P(t, \xi) \) can increase strictly monotonically with respect to \( \xi_2 \) and \( \xi_3 \), and decrease with respect to \( \xi_3 \). Thus, we can plug the lower bound of \( \xi_2 \) and \( \xi_3 \), and the upper bound of \( \xi_1 \) into three objective functions.

IV. DYNAMIC CONTROL PROCESS OF HYBRID MULTI-SENSOR ALLIANCE MODEL

A. ALLIANCE ESTABLISHING PROCESS

When the original problem is transformed into a deterministic hybrid multi-sensor alliance formation problem, the fireworks algorithm based on an improved selection strategy (ISSFA) is designed to solve the problem.

The fireworks algorithm [33]–[35] is a mathematical model based on the abstraction of the phenomenon of fireworks explosion in nature. In this algorithm, the fireworks lack of information exchange mechanism between fireworks and insufficient use of the guidance information of optimal fireworks, but the improved algorithm ensures that high quality sparks are selected for the next generation of fireworks and the diversity of fireworks. The fireworks chosen under this strategy are the most marginal sparks, and can continuously explore out to the edge.

Suppose \( X(t) = [x_1, x_2, \ldots, x_i, \ldots, x_N] \) is the initial fireworks set of the first iteration, where \( N \) is the number of the fireworks; \( x_i \in R^D \) is the information of the first firework in the solution space, and its fitness is \( f(x_i) \). Each firework particle \( x_i \) is exploded to produce a spark particle set \( Y_i(t) = [y_{i,j}, y_{i,2}, \ldots, y_{i,N}] \) with the number of \( s_j \). \( y_{i,j} \in R^D \) has the same dimension as \( x_i \), the formation process of spark particle \( y_{i,j} \) can be expressed by Eq. (20):

\[
y_{i,j} = y_{i,j}^d \max + \left| y_{i,j}^d \right| \% (y_{i,j}^d - y_{i,j}^d) \quad \text{min} \tag{20}
\]
where $B$ is a $1 \times D$-dimensional random matrix, the value of matrix elements is 0 or 1, and $A_i$ is the explosion amplitude of firework particle $x_i$, which can be expressed as:

$$A_i = \hat{A} \sum_{k=1}^{N} \left[ f(x_k) - \min_{1 \leq k \leq N} f(x_k) \right] + \varepsilon$$

(21)

where $\hat{A}$ is a constant to restrict the biggest explosion amplitude of firework particle, $\varepsilon$ is a minimal constant to avoid zero. The sparks number $s_i$ produced by each firework $x_i$ is decided by Eq. (22) as follows,

$$s_i = m - \max_{1 \leq k \leq N} \left[ f(x_k) - f(x_i) \right] + \varepsilon$$

(22)

where $m$ is a constant to restrict the total number of the sparks in order to avoid too much or too few sparks from the explosion.

$$\tilde{s}_i = \begin{cases} \text{round}[am], & s_i < am; \\ \text{round}[bm], & s_i > bm, a < b < 1; \\ \text{round}[s_i], & \text{otherwise} \end{cases}$$

(23)

where $s_i$ is the number of sparks that the first fireworks would eventually produce, \text{round}[ ] is the rounding bracket function, $a$ and $b$ are given constants.

In order to further improve the diversity of spark population, the Gauss mutation process is introduced in the process of solving the fireworks algorithm. $p(0 < p \leq N)$ sparks are randomly selected from the $X(t)$ set, and the Gauss mutation operation is carried out according to Eq.(24) to generate the Gauss mutation spark particle set.

$$Z(t) = \left[ z_1, z_2, \ldots, z_h, \ldots, z_p \right]$$

$$z_h = x_h B g, \quad 1 \leq h \leq p.$$  

(24)

where $x_h$ is the fireworks set randomly selected from the $X(t)$ set; $B$ is a $1 \times D$-dimensional random matrix, the value of matrix elements is 0 or 1; $g$ is a random number obeying $g \sim N(0, 1)$.

In order to prevent the newly generated spark particles from exceeding the search range, the fireworks algorithm uses the mapping rule of modular operation to pull the sparks beyond the feasible range. When the sparks particle $y_{i,j}$ exceeds the feasible range, Eq.(25) is used to calculate.

$$y_{d,i,j} = y_{d,\text{min}} + \left( \frac{y_{d,i,j} - y_{d,\text{min}}}{y_{d,\text{max}} - y_{d,\text{min}}} \right) \delta_i$$

(25)

where $y_{d,i,j}$ is the position of spark particle $y_{i,j}$ on the $d$-dimension, $y_{d,\text{min}}$ and $y_{d,\text{max}}$ are the upper and lower search boundaries of the $d$-dimension, and $\delta_i$ is the modular operation.

At the end of each iteration, the fireworks algorithm selects $N$ particles from the set $W(t) = \{ X(t) \cup Y(t) \cup Z(T) \}$ as the initial fireworks of the next iteration. The best individual in the particle swarm is retained and the other $N - 1$ fireworks are selected by roulette gambling. The probability of individual $\omega_i \in W(t)$ being selected is based on distance. The calculated equations are as follows.

$$L(\omega_i) = \sum_{j=1}^{W(t)} \| \omega_i - \omega_j \|$$

(26)

$$P(\omega_i) = \frac{L(\omega_i)}{\sum_{j \in W(t)} L(\omega_j)}$$

(27)

where $L(\omega_i)$ is the sum of the distance between individual $\omega_i$ and other individuals, using a Euclidean distance measure; and $P(\omega_i)$ is the selected probability of individual $\omega_i$.

Assuming that $f_i(i = 1, 2, \ldots, n)$ denotes the fitness value of the $x_i$ spark, and makes it normalization.

$$f_i' = 1 - \frac{f_i - f_{\text{min}}}{f_{\text{max}} - f_{\text{min}}}$$

(28)

where $f_i'$ is the transferred meaning fitness value; as in a minimize optimization problem, the larger the transferred meaning fitness value, the better the sparks.

The definition of the distances among the sparks rely on the transferred meaning fitness value as follows.

$$\delta_i = \begin{cases} \min_{d} d(x_i, x_j), & f_i' \neq 1 \\ \max_{d} \delta_i, & f_i' = 1 \end{cases}$$

(29)

Defining the sparks distance normalization and calculating the product of the transferred meaning fitness value and

### Table 1. Pseudo code of ISSFA.

| Algorithm: Improved selection strategy of fireworks algorithm (ISSFA) |
|---|
| 1: Begin |
| 2: Randomly initialize a swarm $X$ of $N$ fireworks and a swarm $Y$ of $N$ sparks; |
| 3: Initialize algorithm parameters $\hat{A}$ (a constant to restrict the biggest explosion amplitude of firework particle), $m$ (a constant to restrict the total number of the sparks); |
| 4: While ($t < \text{max generation}$) or (stop criterion) |
| 5: for each firework $x_i \in X$ |
| 6: Compute explosion amplitude $A_i$ of firework particle $x_i$; |
| 7: Compute the number of sparks that the first fireworks would eventually produce $s_i$ for $x_i$; |
| 8: for $i=1$ to $N$ and $j=1$ to $s_i$ |
| 9: Compute the formation process of spark particle $y_{i,j}$; |
| 10: end |
| 11: end |
| 12: Randomly select a set $Z$ of $p$ fireworks from $X$; |
| 13: Compute the transferred meaning fitness value; |
| 14: Compute the sparks distance; |
| 15: Compute the product; |
| 16: Select $N-1$ peak sparks according to the sequence from big to small of $r_i$; |
| 17: Select the exploratory sparks; |
| 18: Make the next generation’s $N$ fireworks; |
| 19: end while |
| 20: End |
TABLE 2. Pseudo code of prediction and re-prediction mechanism.

Algorithm2: Prediction and re-prediction mechanism(P&R-P)

1: Begin
2: Choose members to form alliance;
3: Observe the target and get the observation value $Z_i(k), Z_j(k), ..., Z_s(k)$;
4: Prediction of the target information at $k+1$ time based on the target information at $k$ time obtained by EKF algorithm;
5: if $k+1$ time meets the tracking accuracy requirements then
6: Alliance updating;
7: else
8: Dichotomy to reduce sampling period;
9: end if
10: End

FIGURE 3. Sensors and targets deployment information.

We can call the sparks as the exploratory sparks when they follow the Eq.(33).

$$x_e = \arg \max \sum_{j=1}^{n} d(x_i, x_j)$$ (32)

According to Eq.(32-33), the improved selected strategy can select the next generation’s $N$ fireworks. In these fireworks, the best fitness value of previous generation is selected, and the area where more sparks exist and has the lowest fitness value could be selected, avoiding selecting the same similar spark; due to the existence of the exploratory spark, the sparks with better global search ability will also be selected.

The algorithm pseudo code is shown in Table 1 below.

According to the pseudo code of the ISSFA above, it is necessary to analyze the complexity of the algorithm.

According to the pseudo code of the ISSFA above, it is necessary to analyze the complexity of the algorithm.

Assuming that the number of the sparks is $m$, and the complexity of step 13, 15, 17 is $O(m)$, step 14 should estimate the fitness value first, the complexity is $O(m \log m)$, next is to calculate the distance among the sparks, after calculating, the complexity now is $O(m^2)$, so the final complexity of step 14 is $O(m^2)$. Because the sequence has already been arranged, the time to select the peak spark only costs a few constant times. To sum up, the complexity of ISSFA is $O(m^2)$.

TABLE 3. Parameters settings of the deployed sensors.

| Parameter                                      | Value   |
|-----------------------------------------------|---------|
| Initial fireworks number                     | 5       |
| The maximum explosion amplitude              | 90      |
| Constraint of the total number of sparks     | 10      |
| A minimal constant to avoid zero             | $10^4$  |
| Maximum iterations                           | 300     |
| Bounds                                        | [-10 10]|

TABLE 4. ISSFA parameter settings.

| Parameter                                      | Value   |
|-----------------------------------------------|---------|
| Initial fireworks number                      | 5       |
| The maximum explosion amplitude              | 90      |
| Constraint of the total number of sparks     | 10      |
| A minimal constant to avoid zero             | $10^4$  |
| Maximum iterations                           | 300     |
| Bounds                                        | [-10 10]|

B. ALLIANCE UPDATING PROCESS

After the establishment of the uncertain hybrid multi-sensor alliance, each sensor sometimes can not respond to the operational requirements in time because of the large coverage of the early-warning detection network composed of multi-sensors. At the same time, estimation of the state of the target is changing constantly in a dynamic environment. When the actual tracking accuracy of the time target can not reach the predicted accuracy, it is easy to lose the target in the process of alliance updating. The prediction is made by filtering.
algorithm, and the moving state of the target at any time will appear in the prediction range, which is unable to track the target. Therefore, we propose a mechanism, called prediction and re-prediction, to reduce the probability of losing the target and better to achieve updating of the alliance, that is, at moment k, because the state estimation of sensor and target is dynamic. When target tracking cannot be achieved within the prediction range, the prediction fails, thus, the dichotomy approach should be used to reduce sampling time, and predictions should be repeated until tracking accuracy is sufficient to meet the conditions. The updating algorithm of the hybrid multi-sensor alliance is shown in Table 2.

V. SIMULATION

A. PARAMETERS SETTING

Figure 3 (below) shows the deployment information of the three targets and eight sensors, and Figure 4 shows the performance indicators of each sensor and target. Table 3 shows the remaining parameters of the deployed sensors.

The initial parameters of the EKF algorithm are set as follows:

Covariance matrix of observation noise

\[
R_k = \begin{bmatrix}
R_x & 0 & 0 \\
0 & R_y & 0 \\
0 & 0 & R_z
\end{bmatrix}
\]

Covariance matrix of system noise

\[
Q_k = 0.001 \begin{bmatrix}
T^5/20 & T^4/8 & T^3/6 & 0 & 0 & 0 \\
T^4/8 & T^3/6 & T^2/2 & 0 & 0 & 0 \\
T^3/6 & T^2/2 & T & 0 & 0 & 0 \\
0 & 0 & 0 & T^5/20 & T^4/8 & T^3/6 \\
0 & 0 & 0 & T^4/8 & T^3/6 & T^2/2 \\
0 & 0 & 0 & T^3/6 & T^2/2 & T
\end{bmatrix}
\]

Parameters settings of the deployed sensors are shown in Table 3.

Parameters settings of the improved selection strategy of fireworks algorithm (ISSFA) are shown in Table 4.
B. SIMULATION OF ALLIANCE ESTABLISHING

In order to verify the effectiveness of the proposed algorithm, we used the basic fireworks algorithm [34] (FWA) and discrete dynamic particle swarm optimization algorithm [35] (DDPSO) for comparative analysis and discussion. Applying the parameter settings of the improved selection strategy of fireworks algorithm, the solutions are shown in Figure 5.

As shown in Figure 5, the three algorithms have the same downward trend, and finally converge to 3.09596 min, 1.20661 and 0.54308 respectively. But the convergence rate of these three algorithms is different for each. The ISSFA algorithm converges to the optimal value quickly, but the FWA algorithm converges slowly, the optimization effect is poor, and it is easy to fall into the local optimal solution. Like the ISSFA algorithm, the DDPSO algorithm can find the global optimal solution, but its convergence speed is not as fast as that of the ISSFA algorithm. The results show that the ability of ISSFA to prevent falling into a local optimum is better than that of DDPSO and FWA, and thus more conducive to solving complex optimization problems.

After the solution of the three parameters is obtained, the three parameter values can be used to calculate the formation scheme of multi-sensor alliance under the PEV principle.

Figure 6 shows the sensor response sequence number within the multi-sensor alliance using the traditional approach. Figure 7 shows the sensor response number in a multi-sensor alliance using the uncertain ideal point approach.

It can be seen from Figures 6 and 7 that both the traditional solution approach and the uncertain ideal point approach can generate hybrid multi-sensor alliances, and the sensors play some role in the entire process of detection. In comparison, the hybrid multi-sensor alliance scheme obtained by the uncertain ideal point approach is better than that obtained by the traditional solution approach. For each alliance scheme, there are almost no idle eight sensors, and the sensor utilization rate in the scheme is higher, which effectively avoids the waste of sensor resources; therefore, the established alliance will greatly improve the tracking accuracy of the target, and that the detection effect will be higher in the subsequent detection process. For the same uncertain problem, it can also be seen that different ideal point approaches yield different results, mostly because each approach has a different processing order for the uncertain problem; under the PEV principle, however, all of these are effective multi-sensor alliance schemes.

C. SIMULATION OF ALLIANCE UPDATING

In order to verify the effectiveness of the “prediction and re-prediction” mechanism used by the sensor alliance in the updating process, the mechanism proposed in this paper is compared with the two updating mechanisms—the “measurement and re-update (M&R-P)” and “prediction is update (P&U)” mechanisms. The sampling number is set to 200. The simulation results are shown in Figure 8. The position root mean square error (RMSE) of target tracking is shown in Table 5. The running time of different update mechanisms is shown in Figure 9.

Figures 8-9 and Table 5 demonstrate that the proposed updating mechanism has smaller error and better convergence than the other two handover mechanisms. The hybrid alliance established by the state estimation of the target by the sensor in the dynamic environment of the target can provide accurate
 measurement values, thus greatly reducing the tracking error and running time.

VI. CONCLUSION

In this paper, the uncertain hybrid multi-sensor alliance control problem is solved by using an uncertain ideal point approach under the $P_{EV}$ principle. First, after the definition of the relationship between uncertain variables is given, the ideal point method of uncertainty is applied to the model, and the uncertain multi-objective programming problem is transformed into the definite single objective programming problem. Second, the fireworks algorithm based on the improved selection strategy is proposed to obtain the effective solution of the model. The solution to the algorithm is obtained by comparing FWA and DDPSO, proving that the algorithm can avoid selecting particles with similar performance and poor quality for iteration. Next, the scheme of multi-sensor hybrid alliance is obtained by calculation. Third, compared with the traditional approach and the uncertain ideal point approach, the uncertain ideal point approach can solve the problem related to objective function, so that the final alliance scheme is superior to the traditional approach overall. Finally, by comparing and analyzing the tracking errors of three targets and the running time of the algorithm, the validity of the prediction and re-prediction mechanism is verified.

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REFERENCES

[1] Y. Tao and H. Chongzhao, “Sensor management for multi-target detection and tracking based on PCRLB,” in Proc. 20th Int. Conf. Inf. Fusion, Xi’an, China, Jul. 2017, pp. 1–6.
[2] A. K. Gostar, R. Hoseinezhad, and A. Bab-Hadiashar, “Multi-Bernoulli sensor-selection for multi-target tracking with unknown clutter and detection profiles.” Signal Process., vol. 119, pp. 28–42, Feb. 2016.
[3] C. Pang, G. L. Shan, X. S. Duan, and G. G. Xu, “A multi-mode sensor management approach in the missions of target detecting and tracking,” Electronics, vol. 71, no. 8, pp. 1–18, 2019.
[4] G. G. Xu, C. Pang, X. S. Duan, and G. L. Shan, “Multi-sensor optimization scheduling for target tracking based on PCRLB and a novel intercept probability factor,” Electronics, vol. 140, no. 8, pp. 1–22, 2019.
[5] L. Ma, X. Huo, X. Zhao, and G. Zong, “Adaptive fuzzy tracking control for a class of uncertain switched nonlinear systems with multiple constraints: A small-gain approach,” Int. J. Fuzzy Syst., vol. 21, no. 8, pp. 2699–2624, Nov. 2019.
[6] Y. Chang, Y. Wang, F. E. Alsaadi, and G. Zong, “Adaptive fuzzy output-feedback tracking control for switched stochastic pure-feedback nonlinear systems,” Int. J. Adapt. Control Signal Process., vol. 33, no. 10, pp. 1567–1582, Oct. 2019.
[7] L. Ma, N. Xu, X. Huo, and X. Zhao, “Adaptive finite-time output-feedback control design for switched pure-feedback nonlinear systems with average dwell time,” Nonlinear Anal. Hybrid Syst., vol. 37, Aug. 2020, Art. no. 100908.
[8] C. Xiao-Heng, X. Jun, and J. H. Park, “Estimation for a class of parameter-controlled tunnel diode circuits,” IEEE Trans. Syst., Man, Cyber. Syst., early access, Aug. 13, 2018, doi: 10.1109/TCSC.2018.2859933.
[9] X. H. Chang and G. H. Yang, “Nonfragile H∞ filtering of continuous-time fuzzy systems,” IEEE Trans. Signal Process., vol. 59, no. 4, pp. 1528–1538, Apr. 2011.
[10] B. D. Liu, Uncertainty Theory, vol. 2, 1st ed. Berlin, Germany: Springer-Verlag, 2004, pp. 15–64.
[11] B. D. Liu, “Uncertain set theory and uncertain inference rule with application to uncertain control,” J. Uncert Syst., vol. 4, no. 2, pp. 83–98, 2010.
[12] Y. G. Zha, “Functions of uncertain variables and uncertain programming,” J. Uncertain Syst., vol. 6, no. 4, pp. 278–288, 2012.
[13] Y. H. Liu, “Uncertain random programming with applications,” Fuzzy Optim. Decis. Making, vol. 12, no. 2, pp. 53–169, 2013.
[14] M. L. Fu, H. Wang, B. F. Fang, and X. L. Huang, “Task allocation for distributed self-interested agents,” PatternRecognit. Artif. Intell., vol. 31, no. 12, pp. 1061–1073, 2018.
[15] M. Marcos, A. N. Vasconcellos, and M. Urbashi, “Optimal sensor management strategies in networked estimation,” in Proc. IEEE 56th Annu. Conf. Decis. Control, Melbourne, VIC, Australia, Dec. 2017, pp. 5378–5385.
[16] O. M. Bushnaq, A. Chaaban, S. P. Chepuri, G. Leus, and T. Y. Al-Naffouri, “Sensor placement and resource allocation for energy harvesting IoT networks,” Digit. Signal Process., pp. 1–21, Jun. 2019.
[17] G. Joel, R. Luís, and C. Noélia, “Resource allocation model for sensor clouds under the sensing as a service paradigm,” Computers, vol. 3, no. 18, 2019, Art. no. e8010018.
[18] T. Ishak, J. Aleksandar, Y. N. Shimon, and E. Yael, “A modified distributed bees algorithm for multi-sensor task allocation,” Sensors, vol. 18, no. 759, 2018, Art. no. s18030759.
[19] W. Jie, Q. Jianhui, M. Qichao, K. Yu, and F. Xinxin, “Optimal sensor management for two linear dynamical systems under limited resources in sensor networks,” Neurocomputing, vol. 273, no. 17, pp. 101–110, 2018.
[20] J. Y. Sang, P. S. Anish, M. Seo, C. H. Han, P. Minho, and K. E. Lee, “Joint spectrum sensing and resource allocation optimization using genetic algorithm for frequency hopping–based cognitive radio networks,” Int. J. Commun. Syst., vol. 31, no. 13, p. 3733, 2018.
[21] C.-F. Wang and K. Liu, “A novel particle swarm optimization algorithm for global optimization,” Comput. Intell. Neurosci., vol. 2016, pp. 1–9, Jan. 2016.
[22] X. T. Yang, J. F. Feng, and Y. Feng, “A multi-sensor-target assignment algorithm based on genetic particle swarm optimization,” Electr. Optim. Control, vol. 18, pp. 5–8, 2011.
[23] S. Chen, Y. Liu, L. Wei, and B. Guan, “PS-FW: A hybrid algorithm based on particle swarm and fireworks for global optimization,” Comput. Intell. Neurosci., vol. 2018, pp. 1–27, Feb. 2018.
[24] A. G. Fei, L. Y. Zhang, and G. Liu, “The technique for air-to-air missile guidance superiority handover based on particle swarm hybrid algorithm,” J. Astro, vol. 34, no. 3, pp. 340–346, 2013.

[25] H. Bao, B. Zhang, C. Li, and Z. Yao, “Mobile anchor assisted particle swarm optimization (PSO) based localization algorithms for wireless sensor networks,” Wireless Commun. Mobile Comput., vol. 12, no. 15, pp. 1313–1325, Oct. 2012.

[26] Y. Q. Zhou, X. R. Zhao, Q. F. Luo, and C. M. Wen, “Sensor deployment scheme based on social spider optimization algorithm for wireless sensor networks,” Neural Process. Lett., vol. 48, pp. 71–94, Sep. 2018.

[27] S. G. Ying, F. C. Sun, Y. H. Hu, H. P. Liu, and X. J. Zhang, “Multi-objective dynamic programming algorithm for aircraft arrival sequencing and runway scheduling,” Control Theory Appl., vol. 27, no. 7, pp. 827–835, 2010.

[28] L. Zhong, S. Huang, F. Wu, and G. Chen, “TRADE: A truthful online combinatorial auction for spectrum allocation in cognitive radio networks,” Wireless Commun. Mobile Comput., vol. 15, no. 9, pp. 1320–1330, Jun. 2015.

[29] A. K. Gostar, R. Hoseinnezhad, T. Rathnayake, X. Wang, and A. Bab-Hadiashar, “Constrained sensor control for labeled multi-Bernoulli filter using Cauchy-Schwarz divergence,” IEEE Signal Process. Lett., vol. 24, no. 9, pp. 1313–1317, Sep. 2017.

[30] Y. Liu, H. Wang, and C. Hou, “UKF based nonlinear filtering using minimum entropy criterion,” IEEE Trans. Signal Process., vol. 61, no. 20, pp. 4988–4999, Oct. 2013.

[31] H. Fan, S. C. Huang, M. F. Gao, and D. Z. Wei, “Research on technique of multi-target detection using multi-sensor cross-cueing based on dynamic alliance,” J. Astro, vol. 32, no. 11, pp. 2380–2386, 2011.

[32] C. Pang, S. C. Huang, Y. C. Liu, W. Zhao, and D. Z. Wei, “Application of multi-sensor cross cueing technology in sensor alliance,” J. Xi’an Jiaotong Univ., vol. 51, no. 7, pp. 148–155, 2017.

[33] Y. Tan and Y. Zhu, “Fireworks algorithm for optimization,” in Proc. Int. Conf. Swarm Intell. Springer-Verlag, 2010.

[34] Q. B. Zhu, Z. Y. Wang, and M. Huang, “Fireworks algorithm with gravitational search operator. Control and Decision,” Control Decis., vol. 31, no. 10, pp. 1853–1859, 2016.

[35] L. P. Fang, J. W. Wang, and J. F. Qiu, “Dynamic search fireworks algorithm with learning factor,” J. Frontiers Comput. Sci. Technol., vol. 11, no. 3, pp. 501–791, 2017.

JIAHAO XIE was born in 1996. He received the bachelor’s degree from the School of Mechanical Engineering, Hebei University of Technology, Tianjin, China, in 2017, and the master’s degree from the Air and Missile Defense College, Air Force Engineering University, Xi’an, Shaanxi, China, in 2020. He is currently pursuing the Ph.D. degree with the Air and Missile Defense College. His research interests include multiobjective tracking and information fusion.

SHUCAI HUANG was born in 1966. He received the bachelor’s, master’s, and Ph.D. degrees from the Air and Missile Defense College, Air Force Engineering University, Xi’an, Shaanxi, China. He is currently a Professor with the Air and Missile Defense College. His research interests include multiobjective tracking, sensor management, and digital signal processing.

DAOZHI WEI was born in 1977. He received the bachelor’s, master’s and Ph.D. degrees from the Air and Missile Defense College, Air Force Engineering University, Xi’an, Shaanxi, China. He is currently a Vice Professor with the Air and Missile Defense College. His research interests include multiobjective tracking and multisensor cross cueing technology.