Cosmological Constraints from DES Y1 Cluster Abundances and SPT
Multi-wavelength data

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redshift has been used over the last two decades to place abundance of galaxy clusters as a function of mass and structures [see e.g. 1, 2, for reviews]. In particular, the galaxy clusters are a sensitive probe of the growth of Tracing the highest peaks of the matter density field, the BKG model. consistent between the two abundance and multi-wavelength follow-up data, is favored over the combination serves as a stricter test of our scatter models: the PRJ model, providing scaling relations including the SPT sample above the maximum redshift probed by the DES Y1 redMaPPer sample. Finally, we analyse the first combined optically-SZ selected cluster catalogue obtained by the exploitation of the DES cluster abundance data. To explore possible systematics related to the richness–mass relation of the optically selected redMaPPer catalog, enable the cosmological modeling of projection effects, we consider two calibrations of the observational scatter on rich-

We perform a joint analysis of the counts of redMaPPer clusters selected from the Dark Energy Survey (DES) Year 1 data and multi-wavelength follow-up data collected within the 2500 deg² South Pole Telescope (SPT) Sunyaev-Zel’dovich (SZ) survey. The SPT follow-up data, calibrating the richness–mass relation of the optically selected redMaPPer catalog, enable the cosmological exploitation of the DES cluster abundance data. To explore possible systematics related to the modeling of projection effects, we consider two calibrations of the observational scatter on richness estimates: a simple Gaussian model which accounts only for the background contamination (BKG), and a model which further includes contamination and incompleteness due to projection effects (PRJ). Assuming either a ΛCDM+Σ mν or wCDM+Σ mν cosmology, and for both scatter models, we derive cosmological constraints consistent with multiple cosmological probes of the low and high redshift Universe, and in particular with the SPT cluster abundance data. This result demonstrates that the DES Y1 and SPT cluster counts provide consistent cosmological constraints, if the same mass calibration data set is adopted. It thus supports the conclusion of the DES Y1 cluster cosmology analysis which interprets the tension observed with other cosmological probes in terms of systematics affecting the stacked weak lensing analysis of optically–selected low–richness clusters. Finally, we analyse the first combined optically-SZ selected cluster catalogue obtained by including the SPT sample above the maximum redshift probed by the DES Y1 redMaPPer sample (z = 0.65). Besides providing a mild improvement of the cosmological constraints, this data combination serves as a stricter test of our scatter models: the PRJ model, providing scaling relations consistent between the two abundance and multi-wavelength follow-up data, is favored over the BKG model.

I. INTRODUCTION

Tracing the highest peaks of the matter density field, galaxy clusters are a sensitive probe of the growth of structures [see e.g. 1, 2 for reviews]. In particular, the abundance of galaxy clusters as a function of mass and redshift has been used over the last two decades to place independent and competitive constraints on the density and amplitude of matter fluctuations, as well as dark energy and modified gravity models [e.g. 3–9]. Thanks to the increasing number of wide area surveys at different wavelengths — e.g. in the optical the Sloan Digi-
tal Sky Survey\textsuperscript{[1]} and the Dark Energy Survey\textsuperscript{[2]} (DES), in the microwave Planck\textsuperscript{[3]} South Pole Telescope\textsuperscript{[4]} (SPT) and Atacama Cosmology Telescope\textsuperscript{[5]} and in the X-ray eROSITA\textsuperscript{[6]} — cluster catalogs have grown in size by an order of magnitude compared to early studies, extending to lower mass systems and/or to higher redshifts. Despite this improved statistic, the constraining power of current cluster abundance studies is limited by the uncertainty in the calibration of the relation between cluster mass and the observable used as mass proxy [see e.g.\textsuperscript{[7] 13}]. In general, the observable–mass relation (or OMR) can be calibrated either using high-quality X-ray, weak lensing and/or spectroscopic follow-up data for a representative sub-sample of clusters [e.g.\textsuperscript{[5] 7 11}], or, if wide area imaging data are available, exploiting the noisier weak lensing signal measured for a large fraction of the detected clusters [e.g.\textsuperscript{[8] 9 12}]. Depending on the methodology adopted the mass estimates can be affected by different sources of systematics: e.g. violation of the hydrostatic or dynamical equilibrium when relying on X-ray or spectroscopic follow-up data, respectively, or shear and photometric biases in weak lensing analyses. The calibration of the scaling relation is further hampered by the cluster selection and correlations between observables, which, if not properly modeled, can lead to large biases in the inferred parameters. The recent analysis of the optical cluster catalog extracted from the DES year 1 data (Y1), which combines cluster abundance and stacked weak lensing data, exemplifies such limitations [9] hereafter DES20. The DES20 analysis results in cosmological posteriors in tension with multiple cosmological probes. The tension is driven by low richness systems, and has been interpreted in terms of an unmodeled systematic affecting the stacked weak lensing signal of optically selected clusters.

A possible route to improve our control over systematics relies on the combination of mass–proxies observed at different wavelengths, and thus not affected by the same sources of error. Even more advisable would be the combination of cluster catalogs selected at different wavelengths which would enable the full exploitation of the cosmological content of current and future cluster surveys. The DES and SPT data provides such an opportunity thanks to the large area shared between the two footprints and the high quality of the photometric and millimeter-wave data, respectively. Moreover, the X-ray and weak lensing follow-up data collected within the SPT survey provide an alternative data set to the stacked weak lensing signal adopted in DES20 to constrain the observable–mass scaling relations, that has already been extensively vetted [7 19]. The goal of this study is twofold: i) reanalyze the DES Y1 cluster abundance data adopting the SPT follow-up data to calibrate the observable–mass relation(s), and ii) provide a first case study for the joint analysis of cluster catalogs selected at different wavelengths. In turn, this serves as independent test of the conclusions drawn in DES20. Secondly, combining the abundance data of the two surveys, we explore the possible cosmological gain given by the joint analysis of the two catalogs and exploit the complementary mass and redshift range probed by the two surveys to test the internal consistency of the data sets. Concerning this last point, we consider two calibrations of the observational noise on richness estimates with the aim of assessing possible model systematics induced by a too simplistic modeling of the relation between richness and mass.

The paper is organized as follows: In section II we present the data sets employed in this work. Section III introduces the methodology used to analyze the data. We present our results and discuss their implication in section IV. Finally we draw our conclusions in section V.

\section{II. DATA}

In this work we combine cluster abundance data from the DES Y1 redMaPPer optical cluster catalog [DES Y1 RM;\textsuperscript{[9]} with multi-wavelength data collected within the 2500 deg\textsuperscript{2} SPT-SZ cluster survey [SPT-SZ;\textsuperscript{[7] 14}]. Exploiting the large overlap (∼ 1300 deg\textsuperscript{2}) of the DES Y1 and SPT-SZ survey footprints, we aim to use the SPT-SZ multi-wavelength data to calibrate the observable–mass relation of redMaPPer clusters, which in turn enables the derivation of cosmological constraints from the DES Y1 abundance data. Below we present a summary of the data sets employed in this work. To build our data vectors we follow the prescriptions adopted in DES20 and\textsuperscript{[7]} (hereafter B19) and refer the reader to the original

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\textsuperscript{1} https://www.sdss.org/  
\textsuperscript{2} https://www.darkenergysurvey.org  
\textsuperscript{3} https://www.cosmos.esa.int/web/planck  
\textsuperscript{4} https://pole.uchicago.edu/  
\textsuperscript{5} https://act.princeton.edu/  
\textsuperscript{6} https://www.mpe.mpg.de/eROSITA  

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works for further details.

A. DES Y1 redMaPPer Cluster Catalog

The DES Y1 redMaPPer clusters are extracted from the DES Y1 photometric galaxy catalog [15]. The latter is based on the photometric data collected by the DECam during the Year One (Y1) observational season (from August 31, 2013 to February 9, 2014) over ~1800 deg$^2$ of the southern sky in the g, r, i, z and Y bands. Galaxy clusters are selected through the redMaPPer photometric cluster finding algorithm that identifies galaxy clusters as overdensities of red-sequence galaxies [16,17]. redMaPPer uses a matched filter approach to estimate the membership probability of each red-sequence galaxy brighter than a specified luminosity threshold, $L_{\text{min}}(z)$, within an empirically calibrated cluster radius ($R_{\text{c}} = 1.0 \ h^{-1} \text{Mpc}(\lambda_{\text{bol}}/100)^{0.2}$). The sum of these membership probabilities is called richness, and is denoted as $\lambda_{\text{bol}}$. Along with the richness, redMaPPer estimates the photometric redshift of the identified galaxy clusters. Typical DES Y1 cluster photometric redshift uncertainties are $\sigma_{\text{phot}}/(1+z) \approx 0.006$ with negligible bias ($|\Delta z| \leq 0.003$). The photometric redshift errors are both redshift and richness dependent. To determine candidate central galaxies the redMaPPer algorithm iteratively self-trains a filter that relies on galaxy brightness, cluster richness, and local galaxy density. The algorithm centers the cluster on the most likely candidate central galaxy which is not necessarily the brightest cluster galaxy. [18] studied the centering efficiency of the redMaPPer algorithm using X-ray imaging and found that the fraction of correctly centered clusters is $f_{\text{cen}} = 0.75 \pm 0.08$ with no significant dependence on richness.

Following [DES20], we use for the cluster count analysis the DES Y1 redMaPPer volume-limited catalog with $\lambda_{\text{bol}} \geq 20$, in the redshift interval $z \in [0.2, 0.65]$ , with a total of 6504 clusters. Galaxy clusters are included in the volume-limited catalog if the cluster redshift $z \leq z_{\text{max}}(\hat{n})$, where $z_{\text{max}}(\hat{n})$ is the maximum redshift at which galaxies at the luminosity threshold $L_{\text{min}}(z)$ are still detectable in the DES Y1 at 10$\sigma$. Figure 1 shows the cluster density in the two non-contiguous regions of the DES Y1 redMaPPer cluster survey considered in this work. The lower panel, dubbed the SPT region, corresponds to the ~1300 deg$^2$ overlapping area between the SPT-SZ and DES Y1 survey footprints.

Accordingly with the binning scheme adopted in [DES20], we split our cluster sample in four richness bins and three redshift bins as listed in Table 1. Moreover, we correct the cluster count data for miscentering effects following the prescription of [DES20]. Briefly, cluster miscentering tends to bias low the richness estimates and thus the abundance data, introducing covariance amongst neighboring richness bins. The correction and covariance matrix associated with this effect are estimated in [DES20] through Monte Carlo realizations of the miscentering model of [18]. The corrections derived for each richness/redshift bin are of the order of $\approx 3\%$ with an uncertainty of $\approx 1.0\%$ (see Table 1). In the figure, the data points represent the observed values for the two mass proxies with the corresponding observational errors. The solid lines correspond to the mean relations derived from the DES-NC+SPT-OMR analysis adopting either the BKG (red) or PRJ (dark cyan) calibration for $P(\lambda_{\text{bol}}|z)$ (see section III A). The dashed lines and bands represent, from the bottom to the top, the 0.13, 2.5, 16, 68, 97.5 and 99.87 percentile of the distributions for the BKG and PRJ models, respectively.

B. SPT-SZ 2500 Cluster Catalog and Follow-Up Data

Galaxy clusters are detected in the millimeter wavelength via the thermal Sunyaev-Zel’dovich signature [SZ, 19] which arises from the inverse Compton scattering of CMB photons with hot electrons in the intracluster medium (ICM). The SPT-SZ survey observed the millimeter sky in the 95, 150, and 220 GHz bands over a contiguous 2500 deg$^2$ area reaching a fiducial depth of $\lesssim 18 \mu \text{K-arcmin}$ in the 150 GHz band. Galaxy clusters are extracted from the SPT-SZ maps using a multiscale matched-filter approach [20] applied to the 95 and 150 GHz bands data as described in [14, 21, 22]. For each cluster candidate, corresponding to a peak in the matched-filtered maps, the SZ observable $\xi$ is defined as the maximum detection significance over twelve equally spaced filter scales ranging from 0.’25 to 3.’ [14]. The SPT-SZ cosmological sample consists of 365 candidates.

\footnote{The redMaPPer catalog can be found here: \url{https://des.ncsa.illinois.edu/releases/y1a1/key-catalogs/key-redmapper}}
TABLE I. Number of galaxy clusters in each richness and redshift bin for the DES Y1 redMaPPer catalog. Each entry takes the form $N(N) \pm \Delta N$ stat $\pm \Delta N$ sys. The first error bar is the statistical uncertainty in the number of galaxy clusters in that bin given by the sum of a Poisson and a sample variance term. The number between parenthesis and the second error bar correspond to the number counts corrected for the miscentering bias factors and the corresponding uncertainty (see section II A).

| $\lambda^{ob}$ | $z \in [0.2,0.35)$ | $z \in [0.35,0.5)$ | $z \in [0.5,0.65)$ |
|----------------|-----------------|-----------------|-----------------|
| [20, 30)       | 762 (785.1) $\pm$ 54.9 $\pm$ 8.2 1549 (1596.0) $\pm$ 68.2 $\pm$ 16.6 1612 (1660.9) $\pm$ 67.4 $\pm$ 17.3 |
| [30, 45)       | 376 (388.3) $\pm$ 32.1 $\pm$ 4.5 672 (694.0) $\pm$ 38.2 $\pm$ 8.0 687 (709.5) $\pm$ 36.9 $\pm$ 8.1 |
| [45, 60)       | 123 (127.2) $\pm$ 15.2 $\pm$ 1.6 187 (193.4) $\pm$ 17.8 $\pm$ 2.4 205 (212.0) $\pm$ 17.1 $\pm$ 2.7 |
| [60, $\infty$) | 91 (93.9) $\pm$ 14.0 $\pm$ 1.3 148 (151.7) $\pm$ 15.7 $\pm$ 2.2 92 (94.9) $\pm$ 14.2 $\pm$ 1.4 |

TABLE II. Summary of the SPT-SZ cluster data used in this analysis split in mass–calibration data (SPT-OMR), and abundance data (SPT-NC). For the SPT-OMR data we specify in the third column the number of clusters with a specific follow-up measurement (see section II B for details). Note that a cluster might have more than one follow-up measurement. 

| Data set   | Number of Clusters | Follow-up | $z$-cut |
|------------|-------------------|-----------|---------|
| SPT-OMR    | 187               | $\lambda$: 129 $0.25 < z < 0.65$ | $z > 0.25$ |
| SPT-NC     | 141               | X-ray: 89 $z > 0.25$ | $z > 0.65$ |

with $\xi > 5$ and redshift $z > 0.25$\(^8\) (blue circles in figure 1). Of these: 343 clusters are optically confirmed and have redshift measurements, 89 have X-ray follow-up measurements with Chandra\(^{[22]}\),\(^{[24]}\), 32 have weak lensing shear profile measurements from ground-based observations with Magellan/Megacam\(^{[19]}\) clusters and from space observations with the Hubble Space Telescope\(^{[13]}\) clusters.

Finally, to calibrate the redMaPPer richness–mass relation we assign richnesses to the SPT-SZ clusters by cross-matching the two catalogs. To mitigate the impact of the optical selection we consider for the matching procedure all the clusters with $\lambda^{ob} \geq 5$ in the DES Y1 redMaPPer volume-limited catalog. The match is performed following the criterion adopted in\(^{[27]}\); see also\(^{[28]}\) for an analogous study. Specifically: i) we sort the SPT-SZ and DES Y1 RM sample in descending order according to their selection observable, $\xi$ and $\lambda^{ob}$; ii) starting with the SPT-SZ cluster with the largest $\xi$, we match the system to the richest DES Y1 RM cluster within a projected radius of 1.5 Mpc and redshift interval $\delta z = 0.1$; iii) we remove the matched DES Y1 RM cluster from the list of possible counterparts and move to the next SPT-SZ system in the ranked list iterating step ii) until all the SPT-SZ clusters have been checked for a match.

We match all the 129 optically confirmed SPT-SZ clusters with $\xi > 5$ and $z > 0.25$ that are in the proper redshift range and that lie in the DES Y1 footprint. The remaining 214 non-matched systems reside either in masked regions of the DES Y1 footprint or at redshifts larger than the local maximum redshift $z_{\text{max}} (n)$ of the DES Y1 RM volume-limited catalog. Figure 2 shows the $\lambda^{ob}$ distribution of the matched sample as a function of the SZ detection significance. The median of the distribution is $\lambda^{ob} = 78$, while 68% and 95% of the matched sample resides above richness $\lambda^{ob} > 60$ and $\lambda^{ob} > 37$, respectively. To assess the probability of false association we repeat the matching procedure with 1000 randomized DES Y1 RM catalogs and compute the fraction of times that an SPT-SZ system is associated with a random redMaPPer cluster with $\lambda \geq \lambda^{ob}$. We find this probability to be less than 0.2% for all the SPT-SZ matched systems, and thus we neglect it for the rest of the analysis.

We also explore the possible cosmological gain given by the inclusion of the number count data from the SPT-SZ catalog. When included, we only consider SPT-SZ clusters above redshift 0.65 — the redshift cut adopted for the DES Y1 redMaPPer catalog — corresponding to 40% of the whole SPT-SZ sample. This redshift cut ensures the independence of DES Y1 RM and SPT-SZ abundance data, which allows a straightforward combination of the two data sets.

A summary of the SPT-SZ data employed in this analysis can be found in Table II.

III. ANALYSIS METHOD

Operatively, we can split our data set in three subsamples and corresponding likelihoods: i) the DES Y1 RM abundance data (DES-NC), ii) the SPT-SZ multi-wavelength data (SPT-OMR) and iii) the SPT-SZ abundance data at $z > 0.65$ (SPT-NC). Our theoretical model for the DES Y1 RM number counts is the same as that described in detail in\(^{[8]}\) and\(^{[DE520]}\) while for the analysis of the SPT-SZ abundance and multi-wavelength data we rely on the model presented in\(^{[B19]}\). Here we only provide a brief summary of these methods and refer the reader to the original works for further details. Throughout the paper, all quantities labeled with “ob” denote quantities inferred from observation, while $P(Y|X)$ denotes the conditional probability of $Y$ given $X$. All masses are given

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\(^8\) Below $z=0.25$ the $\xi$-mass relation breaks due to confusion with the primary CMB fluctuations.
in units of $M_{\odot}/h$, where $h = H_0/100\ km\ s^{-1}\ Mpc^{-1}$, and refer to an overdensity of 500 with respect to the critical density. We use "log" and "ln" to refer to the logarithm with base 10 and $e$, respectively.

A. Observable-Mass Relations Likelihood

The SPT-SZ multi-wavelength data comprises four mass proxies: the SZ detection significance $\xi$, the richness $\lambda_{\text{obs}}$, the X-ray radial profile $Y_{\text{X, obs}}$, and the reduced tangential shear profile $g_{1}(\theta)$. The corresponding mean observable-mass relations for the intrinsic quantities $- \xi, \lambda, Y_{\text{X}}, M_{\text{WL}}$ – are parameterized as follows:

\[
\langle \ln \xi \rangle = \ln(\gamma_{\xi} A_{\text{SZ}}) + B_{\text{SZ}} \ln \left( \frac{M}{3 \times 10^{14} M_{\odot} h^{-1}} \right) + C_{\text{SZ}} \ln \left( \frac{E(z)}{E(0.6)} \right)
\]

\[
\langle \ln \lambda \rangle = \ln(A_{\lambda}) + B_{\lambda} \ln \left( \frac{M}{3 \times 10^{14} M_{\odot} h^{-1}} \right) + C_{\lambda} \ln \left( \frac{1 + z}{1 + 0.45} \right)
\]

\[
\langle \ln M_{\text{WL}} \rangle = \ln b_{\text{WL}} + \ln M,
\]

where $\gamma_{1}$ in equation 1 depends on the position of the SPT-SZ cluster and accounts for the variation of survey depth over the SPT footprint, while $E(z) = H(z)/H_0$. For each scaling relation we fit for the amplitude, slope, and redshift evolution (see Table 1), but for the weak lensing mass, $M_{\text{WL}}$, which we assume to be simply proportional to the true halo mass accordingly to the simulation-based calibration of B19.

We assume the logarithm of our four intrinsic observables, $\ln \mathcal{O}$, to follow a multivariate Gaussian distribution with intrinsic scatter parameters $D_{\mathcal{O}}$, and correlation coefficients $\rho(\mathcal{O}_i; \mathcal{O}_j)$:

\[
P(\ln \mathcal{O}| M, z) = \mathcal{N}(\langle \ln \mathcal{O} \rangle, \Sigma),
\]

where the covariance matrix elements read $G_{ij} = \rho(\mathcal{O}_i; \mathcal{O}_j) D_{\mathcal{O}_i} D_{\mathcal{O}_j}$ and $\rho(\mathcal{O}_i; \mathcal{O}_i) = 1$. All the intrinsic scatter is described by a single parameter $D_{\mathcal{O}}$ independent of mass and redshift, but the scatter on $\ln \lambda$ which includes a Poisson-like term $-\sigma_{\ln \lambda}^2 = D_{\lambda}^2 + (\langle \lambda(M) \rangle - 1)/\langle \lambda(M) \rangle^2$ – which does not correlate with the other scatter parameters. Finally, we set to zero the correlation coefficients between the $D_{Y_{\text{X}}}$ and the other scatter parameters. This approximation is justified by the fact that while the richness, SZ and weak lensing signal are sensitive to the projected density field along the line of sight of the system, the X-ray emission is mainly contributed by the inner region of the cluster. This approximation is also supported by the analysis of B19, which obtained unconstrained posteriors peaked around zero for the X-ray correlation coefficients. We explicitly verified that this approximation does not affect our results, while reducing noticeably the computational cost of the analysis.

To account for the observational uncertainties and/or biases, we consider the following conditional probabilities between the intrinsic cluster proxies and the actual observed quantities. For $\xi, Y_{\text{X}}$ and $\gamma_{1}(\theta)$ we follow the prescriptions outlined in B19, namely:

\[
P(\xi|\langle \xi \rangle) = \mathcal{N}(\sqrt{\xi^2 + 3}, 1)
\]

\[
P(Y_{\text{X}}|\langle Y_{\text{X}} \rangle) = \mathcal{N}(Y_{\text{X}}, \sigma_{Y_{\text{X}}}^2)
\]

where $\sigma_{Y_{\text{X}}}^2$ is the uncertainty associated with the X-ray measurements [see section 3.2.2 in B19 for further details]. The reduced tangential shear $g_{1}(\theta)$ is analytically related to the underlying halo mass $M_{\text{WL}}$, assuming a Navarro-Frenk-White (NFW) halo profile, a concentration-mass relation, and using the observed redshift distribution of source galaxies. Deviation from the NFW profile, large-scale structure along the line of sight, miscentering and uncertainties in the concentration-mass relation, introduce bias and/or scatter on the estimated weak lensing mass, $M_{\text{WL}}$. As introduced in equation 4 we assume $M_{\text{WL}}$ to be proportional to the true halo mass, and use the simulation-based calibration of $b_{\text{WL}}$ from B19 to account for such effects (see their Section 3.1.2 and Table 1 for further details). In total the weak lensing (WL) modeling introduces six free parameters which account for the uncertainties in the determination of the systematics associated to the weak lensing mass, $M_{\text{WL}}$, which we assume to be simply proportional to the true halo mass accordingly to the simulation-based calibration of B19.

As for the uncertainty on the richness, many studies already highlight the importance of projection effects on richness estimates [e.g. 30, 33]. In this context, projection effects denote the contamination from correlated and uncorrelated structures along the line of sight due to the limited resolution that a photometric cluster finding algorithm can achieve along the radial direction. In this study we consider two prescriptions based on the model presented in 35:

1. $P_{\text{bkg}}(\lambda_{\text{obs}}|\lambda) = \mathcal{N}(\lambda, \sigma_{\lambda}^{\text{bkg}})$, which accounts only for the "background subtraction" scatter, $\sigma_{\lambda}^{\text{bkg}}$, due to the misclassification of background galaxies.
as member galaxies and vice versa, labelled BKG throughout the paper.

2. \( P_{\text{prj}}(\lambda^\text{ob}|\lambda) \), defined in equation 15 of [35], which includes, besides the "background subtraction" noise, the scatter due to projection and masking effects (PRJ, hereafter).

The approximated BKG model is derived from \( P_{\text{prj}}(\lambda^\text{ob}|\lambda) \) by setting to zero the fraction of clusters affected by projection and masking effects and corresponds to the model often adopted in literature (e.g. [27, 28, 33]). PRJ is the model adopted in [DES20] and it has been calibrated by combining real data and simulated catalogs analysis. While being a more complete model which includes known systematics effects, its calibration, in part based on simulated catalogs, might be subject to biases. Comparisons of the results obtained with these two models are used to assess the capability of our simplest model (BKG) to absorb the impact of projection effects and, in turn, possible biases due to their incorrect calibration.

Putting all the above pieces together, the "observable–mass relation" likelihood for the SPT-SZ multturn, possible biases due to their incorrect calibration.

The cosmological and model parameters considered in this analysis are listed in Table III along with their priors. Our reference cosmological model is a flat \( \Lambda \)CDM

\[ \langle N(O^\text{ob}, z) \rangle = \int dM n(M, z) \Omega(z) \frac{dV}{dzd\Omega} \int dO P(O^\text{ob}|O) P(O|M, z), \]  

where \( \frac{dV}{dzd\Omega} \) is the comoving volume element per unit redshift and solid angle, whereas the conditional probabilities for the observed and intrinsic mass proxies are those described in the previous section.

The DES Y1 RM cluster abundance data are analyzed following the methodology adopted in [DES20] where the number counts likelihood takes the form:

\[ \mathcal{L}_{\text{NC}}(\mathbf{N}|\theta) = \frac{\exp \left[ -\frac{1}{2} (\mathbf{N}_\Delta - \langle \mathbf{N} \rangle)^T C^{-1} (\mathbf{N}_\Delta - \langle \mathbf{N} \rangle) \right]}{(2\pi)^{K/2} |C|^{1/2}}, \]  

where \( \mathbf{N}_\Delta \) and \( \langle \mathbf{N} \rangle \) are respectively the abundance data (see Table I), and the expected number counts in bins of richness and redshift obtained by integrating equation 10 over the relevant \( \lambda^\text{ob} \) and \( z \) intervals. The covariance matrix \( C \) is modeled as the sum of three distinct contributions: i) the Poisson noise, ii) a sample variance term due to density fluctuations within the survey area and iii) a miscentering component (see section II A). The Poisson and sample variance contributions are computed analytically at each step of the chain following the prescription outlined in Appendix A of [34]. At high richness, the Poisson term dominates the uncertainty, with sample variance becoming increasingly important at low richness [37]. Note that the large occupancy of all our bins — our least populated bin contains 91 galaxy clusters — justifies the Gaussian approximation adopted for the Poisson component.

Following [31, 9], we assume a purely Poisson likelihood for the SPT-SZ abundance data [38]:

\[ \ln \mathcal{L}_{\text{Poisson}}(\mathbf{N}|\theta) = \sum_i \ln \langle N(\xi_i, z_i) \rangle - \int_{0.65} d(z) \int d\xi \langle N(\xi, z) \rangle, \]  

where the sum runs over all the SPT-SZ clusters above the redshift and SZ significance cuts (\( \xi_{\text{cut}} = 0.65 \), \( \xi_{\text{cut}} = 5 \)). Note that here we can safely neglect the sample variance contribution given large cluster masses (\( M \gtrsim 3 \times 10^{14} M_\odot h^{-1} \)) probed by the SPT-SZ survey (see [37, 39]).

C. Parameters Priors and Likelihood Sampling

The cosmological and model parameters considered in this analysis are listed in Table IIII along with their priors.
TABLE III. Cosmological and model parameter posteriors: a range indicates a top-hat prior, while \( \mathcal{N}(\mu, \sigma) \) stands for a Gaussian prior with mean \( \mu \) and variance \( \sigma^2 \).

| Parameter | Description | Prior |
|-----------|-------------|-------|
| \( \Omega_m \) | Mean matter density | [0.1, 0.9] |
| \( A_s \) | Amplitude of the primordial curvature perturbations | \( 10^{-10} - 10^{-3} \) |
| \( h \) | Hubble rate | \( [0.55, 0.9] \) |
| \( \Omega_b h^2 \) | Baryon density | \( [0.020, 0.024] \) |
| \( \Omega_c h^2 \) | Massive neutrinos energy density | \( [0.0096, 0.01] \) |
| \( n_s \) | Spectral index | \( [0.94, 1.0] \) |
| \( w \) | Dark energy equation of state | \( [-2.5, -0.33] \) |

SZ scaling relation

- \( A_{\text{SZ}} \) Amplitude
- \( B_{\text{SZ}} \) Power-law index mass dependence
- \( C_{\text{SZ}} \) Power-law index redshift evolution
- \( D_{\text{SZ}} \) Intrinsic scatter

Richness scaling relation

- \( A_{\lambda} \) Amplitude
- \( B_{\lambda} \) Power-law index mass dependence
- \( C_{\lambda} \) Power-law index redshift evolution
- \( D_{\lambda} \) Intrinsic scatter

X-ray \( Y_X \) scaling relation

- \( A_{Y_X} \) Amplitude
- \( B_{Y_X} \) Power-law index mass dependence
- \( C_{Y_X} \) Power-law index redshift evolution
- \( D_{Y_X} \) Intrinsic scatter

\( \frac{d \ln Y_X}{d \ln r} \) Radial slope \( Y_X \) profile

- \( \delta_{\text{WL,bias}} \) Uncertainty on WL bias
- \( \delta_{\text{bias}} \) HST/MegaCam
- \( \delta_{\text{scatter}} \) Uncertainty on intrinsic scatter

Correlation coefficients between scatterers

- \( \rho(\text{SZ}, \text{WL}) \) Correlation coefficient SZ-WL
- \( \rho(\text{SZ}, \lambda) \) Correlation coefficient SZ-\( \lambda \)
- \( \rho(\text{WL,} \lambda) \) Correlation coefficient WL-\( \lambda \)

Determinant OMC matrix (eq. [3]) \( \det[C] > 0 \)

IV. RESULTS

Table IV summarizes the results obtained for the different models and data combinations considered in this work. Along with the varied ones we also report posteriors for two derived parameters: the amplitude of the matter power spectrum on a \( 8h^{-1}\) Mpc scale, \( \sigma_8 \), and the cluster normalization condition, \( S_8 = \sigma_8(\Omega_m/0.3)^{0.5} \).

A. \( \Lambda \)CDM + \( \sum m_\nu \) cosmology

Figure 3 shows the parameter posteriors obtained from the four analyses carried out for the \( \Lambda \)CDM + \( \sum m_\nu \) model. We do not report posteriors for those parameters not constrained by our data or dominated by their priors. Also, to avoid overconstraining we omit from this figure the \( Y_X \) scaling relation parameters which can be found in appendix A along with the correlation matrix for a sub-set of parameters. The only two cosmological parameters constrained by our data are \( \Omega_m \) and \( \sigma_8 \). For all the other cosmological parameters — \( \Omega_b h^2 \), \( \Omega_c h^2 \) and \( n_s \) — we obtain almost flat posteriors, but for the Hubble model with three degenerate species of massive neutrinos (\( \Lambda \)CDM + \( \sum m_\nu \)), for a total of six cosmological parameters: \( \Omega_m \), \( A_s \), \( h \), \( \Omega_b h^2 \), \( \Omega_c h^2 \), \( n_s \). Being that our data set is insensitive to the optical depth to reionization, we fix \( \tau = 0.078 \). We also consider a \( \Lambda \)CDM + \( \sum m_\nu \) model where the dark energy equation of state parameter \( w \) is let free to vary in the range \([-2.5, -0.33]\). The four observable—mass scaling relations considered in this work comprise 19 model parameters. Besides those already introduced in section IIIA, the \( Y_X \) scaling relation has the additional parameter \( (d \ln Y_X/d \ln r) \) — the measured radial slope of the \( Y_X \) profile — which allows to re-scale and compare the measured and predicted \( Y_X \) profiles at a fixed fiducial redshift [see section 3.2.2 of [3] for additional details]. The parameters ranges and priors match those used in [19] apart from the richness—mass scaling relation parameters, which were not included in the [19] analysis, and for which we adopt flat uninformative priors. The parameter ranges for \( \Omega_b h^2 \) and \( n_s \) are chosen to roughly match the 5\( \sigma \) credibility interval of the Planck constraints [10], while the lower limit adopted for \( \Omega_c h^2 \) corresponds to the minimal total neutrino mass allowed by oscillation experiments, 0.056 eV [11].

We consider two different data combinations in this work. Our baseline data set is given by the combination of DES Y1 RM counts data and the SPT-SZ multi-wavelength data (DES-NC+SPT-OMR). Moreover, we explore the cosmological gain given by the further inclusion of the SPT-SZ abundance data (DES-NC+SPT-OMR,NC)]. The total log-likelihood is thus given by the sum of log-likelihoods corresponding to the data considered in each analysis. We remind here that the independence of the two abundance likelihoods is guaranteed by the redshift cut \( z > 0.65 \) adopted for the SPT-SZ number count data which ensures the absence of overlap between the volume probed by the two abundance data sets. The parameter posteriors are estimated within the cosmoSIS package [42] using the importance nested sampler algorithm MultiNest [43] with target error on evidence equal to 0.1 as convergence criterion. The matter power spectrum is computed at each step of the chain using the Boltzmann solver CAMB [44]. To keep the universality of the Tinker fitting formula in cosmologies with massive neutrinos we adopt the prescription of [45] neglecting the neutrino density component in the relation between scale and mass — i.e. \( M \propto (\rho_{\text{cdm}} + \rho_{\nu})R^3 \) — and using only the cold dark matter and baryon power spectrum components to compute the variance of the density field at a given scale, \( \sigma^2(R) \).
FIG. 3. Marginalized posterior distributions of the fitted parameters. The 2D contours correspond to the 68% and 95% confidence levels of the marginalized posterior distribution. The description of the model parameters along with their posteriors are listed in Table IV. Only parameters that are not prior dominated are shown in the plot.

parameter which is loosely constrained by the abundance data thanks to the mild sensitivity of the slope of the halo mass function and comoving volume element to variation of $h$.

1. Models and data combinations comparison

The left panels of Figure 4 compare the abundances of the DES Y1 RM clusters (boxes) with the corresponding mean model predictions (markers). The right panels show the residuals between the data and the model ex-


| Data | DES-NC+SPT-OMR | DES-NC+SPT-OMR,NC | DES-NC+SPT-OMR,NC | DES-NC+SPT-OMR,NC | DES-NC+SPT-OMR,NC |
|------|----------------|--------------------|--------------------|--------------------|--------------------|
| $P(\lambda;\lambda^{\text{true}})$ model | ACDM + $\sum m_{\nu}$ | wACDM + $\sum m_{\nu}$ | ACDM + $\sum m_{\nu}$ | wACDM + $\sum m_{\nu}$ | ACDM + $\sum m_{\nu}$ |
| $\Omega_m$ | 0.322$^{+0.079}_{-0.067}$ | 0.264$^{+0.047}_{-0.033}$ | 0.420$^{+0.057}_{-0.046}$ | 0.372$^{+0.064}_{-0.046}$ | 0.308$^{+0.041}_{-0.054}$ |
| $10^8 A_s$ | 2.38$^{+0.42}_{-0.13}$ | 4.25$^{+0.04}_{-0.20}$ | 1.21$^{+0.23}_{-0.92}$ | 1.28$^{+0.038}_{-0.092}$ | 1.64$^{+0.12}_{-0.82}$ |
| $h$ | 0.715$^{+0.075}_{-0.091}$ | 0.677$^{+0.011}_{-0.020}$ | 0.720$^{+0.075}_{-0.044}$ | 0.644$^{+0.076}_{-0.004}$ | 0.765$^{+0.048}_{-0.048}$ |
| $\sigma_8$ | 0.790$^{+0.038}_{-0.063}$ | 0.795$^{+0.045}_{-0.059}$ | 0.725$^{+0.030}_{-0.040}$ | 0.719$^{+0.027}_{-0.042}$ | 0.808$^{+0.041}_{-0.044}$ |
| $S_8$ | 0.808$^{+0.062}_{-0.049}$ | 0.736$^{+0.049}_{-0.043}$ | 0.854$^{+0.043}_{-0.038}$ | 0.796$^{+0.048}_{-0.038}$ | 0.813$^{+0.049}_{-0.044}$ |
| $w$ | −1 | −1 | −1 | −1 | −1 |
| $\Lambda_{SZ}$ | 5.18$^{+0.74}_{-0.95}$ | 5.36$^{+0.75}_{-1.0}$ | 4.84$^{+0.72}_{-1.0}$ | 5.34$^{+0.79}_{-1.0}$ | 4.16$^{+0.60}_{-0.97}$ |
| $B_{SZ}$ | 1.59$^{+0.14}_{-0.14}$ | 1.53$^{+0.12}_{-0.14}$ | 1.80$^{+0.11}_{-0.14}$ | 1.69$^{+0.11}_{-0.10}$ | 1.67$^{+0.14}_{-0.14}$ |
| $\Delta_{SZ}$ | 0.91$^{+0.74}_{-0.42}$ | 0.68$^{+0.52}_{-0.05}$ | 0.87$^{+0.32}_{-0.24}$ | 0.82$^{+0.41}_{-0.24}$ | 1.05$^{+0.42}_{-0.42}$ |
| $D_{SZ}$ | 0.193$^{+0.072}_{-0.040}$ | 0.172$^{+0.005}_{-0.070}$ | 0.182$^{+0.008}_{-0.074}$ | 0.163$^{+0.009}_{-0.074}$ | 0.193$^{+0.002}_{-0.043}$ |
| $\Lambda_{\lambda}$ | 76.3$^{+6.9}_{-8.6}$ | 72.0$^{+3.8}_{-7.7}$ | 75.6$^{+7.9}_{-7.9}$ | 72.4$^{+5.1}_{-7.9}$ | 66.1$^{+4.1}_{-8.6}$ |
| $B_{\lambda}$ | 0.957$^{+0.059}_{-0.047}$ | 0.850$^{+0.040}_{-0.031}$ | 1.028$^{+0.043}_{-0.031}$ | 0.935$^{+0.045}_{-0.031}$ | 1.015$^{+0.048}_{-0.037}$ |
| $C_{\lambda}$ | 0.48$^{+0.45}_{-0.35}$ | 0.45$^{+0.35}_{-0.25}$ | 0.95$^{+0.30}_{-0.25}$ | 0.51$^{+0.35}_{-0.25}$ | 0.67$^{+0.34}_{-0.25}$ |
| $D_{\lambda}$ | 0.217$^{+0.051}_{-0.058}$ | 0.183$^{+0.064}_{-0.048}$ | 0.254$^{+0.050}_{-0.075}$ | 0.207$^{+0.061}_{-0.045}$ | 0.219$^{+0.058}_{-0.045}$ |
| $A_Y$ | 6.91$^{+0.88}_{-0.60}$ | 6.41$^{+0.76}_{-0.91}$ | 7.22$^{+0.72}_{-0.72}$ | 6.82$^{+0.72}_{-0.91}$ | 6.42$^{+0.65}_{-0.84}$ |
| $B_Y$ | 0.499$^{+0.036}_{-0.040}$ | 0.519$^{+0.044}_{-0.047}$ | 0.452$^{+0.033}_{-0.038}$ | 0.479$^{+0.030}_{-0.038}$ | 0.485$^{+0.036}_{-0.046}$ |
| $C_Y$ | 0.40$^{+0.20}_{-0.20}$ | 0.20$^{+0.24}_{-0.20}$ | 0.40$^{+0.14}_{-0.11}$ | 0.11$^{+0.12}_{-0.11}$ | 0.05$^{+0.12}_{-0.11}$ |
| $D_Y$ | 0.147$^{+0.070}_{-0.064}$ | 0.168$^{+0.093}_{-0.064}$ | 0.152$^{+0.093}_{-0.078}$ | 0.171$^{+0.099}_{-0.058}$ | 0.151$^{+0.084}_{-0.073}$ |

TABLE IV. Cosmological and model parameter constraints obtained for the different models and data combinations considered in this work. For all the parameters we report the mean of the 1-d marginalized posterior along with the 1-$\sigma$ errors. We omit from this table parameters whose posteriors are equal to or strongly dominated by their priors. DES-NC, SPT-OMR and SPT NC stand for the different data set considered in the analyses, respectively; cluster counts from DES Y1 RM, multi-wavelength data from SPT-SZ, and abundance from the SPT-SZ cluster catalog above $z > 0.65$. BKG and PRJ refer to the model adopted to describe the observational noise on the richness estimate (see section III A).
FIG. 4. Observed (shaded areas) and mean model predictions (markers) for the DES Y1 RM cluster number counts as a function of richness for each of our three redshift bins. The $y$ extent of the data boxes is given by the square root of the diagonal terms of the covariance matrix. The right panels show the residual between the data and the mean model predictions. The error bars on the predicted number counts represent one and two standard deviations of the distribution derived sampling the corresponding chain. All points have been slightly displaced along the richness axis to avoid overcrowding.

As for the correlation coefficients between scatters in all the four cases analyzed the posteriors are prior dominated. We note, however, that while the posteriors of the correlation coefficients between SZ and WL and WL and $\lambda$ peak around zero, the $\rho(SZ,\lambda)$ posterior always has its maximum at $\sim -0.2$, suggesting an anti-correlation between the two observables (see figure 12 in appendix A).

2. Goodness of fit

The four analyses perform similarly well in fitting the DES Y1 abundance data. The model predictions are all consistent within 2$\sigma$ with the data but for the highest richness/redshift bin, where all the models over-predict the number counts by $\sim 35\%$ (see right panels of figure 4). Notably, while the SPT-OMR data is only available for clusters above $\lambda > 40$, the scaling relation extrapolated at low richness provides a good fit to the DES Y1 abundance data. Our composite likelihood model and parameter degeneracies do not allow us to apply a $\chi^2$ statistic to assess the goodness of the fit. The same tensions between predictions and DES Y1 RM abundance data was observed in [DES20], where the authors verified that dropping the highest-$\lambda/z$ bin from the data does not affect their results, but improve the goodness of the fit. Here we use the posterior predictive distribution to assess the likelihood of observing the highest-$\lambda/z$ data point given our models [see e.g. 46, section 6.3]. The method consists of drawing simulated values from the posterior predictive distribution of replicated data and comparing these mock samples to the observed data. The posterior
predictive distribution is defined as:

\[
P(y^{\text{rep}}|y) = \int d\theta P(y^{\text{rep}}|\theta) P(\theta|y)
\]  

(13)

where \( y \) is the observed data vector, \( y^{\text{rep}} \) the replicated one, and \( \theta \) the model parameters. In practice, we generate our replicated data for the highest-\( \lambda/\bar{z} \) by sampling the posterior distribution, \( P(\theta|y) \), and drawing for each sampled \( \theta \) a value from the multivariate normal distribution defined by equation (10) and covariance matrix \( C \). We draw 500 samples for each of the four analyses, and fit the distributions with a Gaussian to easily quantify the likelihood of the observed data point. As can been seen in figure 6 for the two models and data combinations considered here the observed data lies within the 3\( \sigma \) region (dashed and dotted vertical lines), thus we conclude that the highest-\( \lambda/\bar{z} \) data point is not a strong outlier of the predicted distribution and our model suffices to describe it.

Similarly for the SPT-SZ abundance data, the models retrieved from the posteriors of the DES-NC+SPT-[OMR,NC] and DES-NC+SPT-[OMR,NC]+PRJ analyses provide a good fit to the SPT number counts but for the highest \( \xi \) bin, where the model predictions lie at the edge of the \( \sim 2\sigma \) region (see lower panel of figure 5).

As for the SPT-OMR data we inspect the goodness of the fit of the derived \( P(\lambda^{\text{obs}}|\xi) \) distributions against the cross-matched sample. Specifically, we verified that all the data points lie within the 3\( \sigma \) region of the posterior predictive distributions independently from the data combination and model assumed for the observational scatter on \( \lambda^{\text{obs}} \) (see figure 2).

To determine whether our data sets prefer one of the two models adopted for \( P(\lambda^{\text{obs}}|\lambda) \) — BKG and PRJ — we rely on the Deviance Information Criterion [hereafter DIC; 17]. Specifically, for a given model \( M \) the DIC is computed from the mean \( \chi^2 \) over the posterior volume and the maximum posterior \( \chi^2 \) as:

\[
\text{DIC}(M) = 2\langle \chi^2 \rangle_M - \chi^2_{\text{MaxP}}(M).
\]  

(14)

The model with the lower DIC value either fits better the data — lower \( \langle \chi^2 \rangle \) — or has a lower level of complexity — lower \( \chi^2_{\text{MaxP}} \). For the data combination DES-NC+SPT-OMR we obtain \( \Delta \text{DIC} = \text{DIC(P RJ)} - \text{DIC(BKG)} = 3.5 \), while for the full data set \( \Delta \text{DIC} = -3.8 \). Adopting the Jeffreys’ scale to interpret the DIC values, the DES-NC+SPT-OMR data combination has a “positive” (\( |\Delta \text{DIC}| \in [2,5] \)) — even though not “strong” (\( |\Delta \text{DIC}| \in [-5,10] \)) or “definitive” (\( |\Delta \text{DIC}| > 10 \)) — preference for the BKG model, while the full data combination has a “positive” preference for the PRJ model. Additional follow-up data extending to lower richness — as the one soon available from the combination of DES Y3 and Y6 data with the full SPT surveys or eROSITA — will help to identify the model which better describes the data.

3. Comparison with other cosmological probes

Figure 7 compares the \( \sigma_8-\Omega_m \) posteriors derived in this work for a \( \Lambda \text{CDM} + \sum m_\nu \) cosmology including (lower panels) or excluding (upper panels) the PRJ calibration, to other results from the literature. To assess the consistency of two data sets \( A \) and \( B \) in the \( \sigma_8-\Omega_m \) plane we test the hypothesis \( p_A - p_B = 0 \) [see method ‘3’ in 49], where \( p_A \) and \( p_B \) are the \( \sigma_8-\Omega_m \) posterior distributions as constrained by data sets \( A \) and \( B \), respectively.

Starting with the simpler scatter model (BKG, upper panels), our baseline data combination (DES-NC+SPT-OMR) is consistent within 2\( \sigma \) with all the probes considered here. The largest tension (1.7\( \sigma \)) is found with the results from DES20 [DES-[NC,\( M_{\nu\text{WL}} \)] in figure 7 which combine DES Y1 RM abundances and mass estimates from the stacked weak lensing signal around DES Y1 RM clusters [50]. The tension with DES-[NC,\( M_{\nu\text{WL}} \)] results is not surprising and reflects the different richness–mass scaling relation preferred by the DES Y1 weak len-
The inclusion of SPT-NC data (DES-NC+SPT-[OMR,NC]) worsens the consistency with the other low-redshift probes considered here by shifting the $\Omega_m/\sigma_8$ posteriors towards higher/lower values. In particular, the agreement is degraded with the DES 3x2pt and WtG results, with which the tension in the $\sigma_8$-$\Omega_m$ plane raises to 1.8$\sigma$ and 1.9$\sigma$, respectively. Notably, the full data combination is at 1.3$\sigma$ tension also with results from SPT-SZ 2500 with which it shares part of the abundance data (SPT-SZ counts above $z = 0.65$) and the follow-up data. The fact that the DES-NC+SPT-[OMR,NC] posteriors do not lie in the intersection of the DES-NC+SPT-OMR and SPT-SZ 2500 contours suggests the presence of some — yet not statistically significant — tension between the DES-NC, SPT-OMR and SPT-NC data, possibly driven by an imperfect modeling of the scaling relations.

On the other hand, by turning the $\sigma_8$-$\Omega_m$ degeneracy direction, the inclusion of the PRJ model (lower panel) improves the agreement of the DES-NC+SPT-OMR posteriors with the SPT-SZ 2500 results (from 1$\sigma$ to 0.5$\sigma$ tension), at the expense of larger, yet not significant (1.3$\sigma$), tension with CMB data (red contours). Also the tension with the [DES20] results decreases (0.7$\sigma$) as a consequence of the improved consistency between the richness–mass scaling relations (see section [VA 4]). Similarly, when considering the full data combination, the PRJ model shifts the cosmological posteriors in the intersection of the DES-NC+SPT-OMR and SPT-SZ 2500 contours, solving the above mentioned tension between the three data set. We will go back to this point in the next section.

4. The mass–richness relation

Being constrained by the SPT multi-wavelength data both the SZ and $Y_X$ scaling relations derived from the DES-NC+SPT-OMR analysis are perfectly consistent with those obtained in [B19]. The inclusion of SPT-NC data in our analysis shifts the slope of the SZ relation, $B_{SZ}$, by 1.5$\sigma$ towards steeper values to compensate for the larger $\Omega_m$ value preferred by the full data combination. As mentioned before, the shift of the cosmological posteriors along the $S_8$ direction suggests the presence of some inconsistencies between the scaling relations preferred by the different data sets: DES-NC, SPT-OMR and SPT-NC. To pinpoint the source of tension we re-analyze the abundance and multi-wavelengths data independently using as cosmological priors the product of the posterior distributions obtained from the DES-NC+SPT-OMR

Note that at odds with the [B19] analysis, here we show results for the SPT-SZ 2500 analysis obtained assuming 3 degenerate massive neutrino species and adopting the massive neutrino prescription for the halo mass function presented in [13], consistently with our analysis. The different massive neutrino scheme and the inclusion of this prescription lowers the $\sigma_8$ posterior by 0.024 (corresponding to $\sim 0.5\sigma$) compared to original results of [B19].

To exclude the possibility that the tension is driven by SPT-SZ abundance data at low redshift we re-analyze the SPT-SZ 2500 catalog excluding the cluster counts data below $z = 0.65$ — i.e. analysing the data combination SPT-[OMR,NC] — finding posteriors fully consistent with SPT-SZ 2500 results [see also figure 16 in [7].

FIG. 6. Posterior predictive distributions for the highest-$\lambda/z$ data point derived from the four analyses considered in section [IV A]. The solid black line correspond to the observed cluster abundance in that bin, while the four dashed and dot-dashed lines mark the $3\sigma$ limit of the corresponding posterior predictive distribution. Although residing in the tail of the distributions, in none of the four analyses the observed data point lies outside the $3\sigma$ region.
FIG. 7. Upper panels: Comparison of the 68% and 95% confidence contours in the $\sigma_8$-$\Omega_m$ plane derived in this work adopting the BKG scatter model (black and orange contours) with other constraints from the literature: DES Y1 cluster counts and weak lensing mass calibration [DES20, dot–dashed magenta contours]; DES-Y1 3x2 from [48, dark violet contours]; Planck CMB from [40, brown contours]; cluster number counts and follow-up data from the SPT-SZ 2500 survey [B19, dot-dashed pink contours]; cluster abundance analysis of Weighing the Giants [5, WtG, dashed dark blue contours]. Lower panels: Same as left panel but considering the projection effect model (PRJ) for the scatter between true and observed richness (see section III A).

OMR and SPT-[OMR,NC] analyses (roughly the intersection between the black and pink contours in the upper right panel of figure 7). This test will allow us to understand why that region of the $\sigma_8$-$\Omega_m$ plane is disfavored by the full data combination.

As can be seen in figure 8 the tension between DES-NC+SPT-OMR, SPT-NC and DES-NC+SPT-[OMR,NC] arises from the different amplitude of the richness and SZ scaling relation preferred by the abundance (blue contours) and SPT-OMR data (orange contours). The PRJ model, lowering the $A_\lambda$ value preferred by the abundance data (black dot-dashed contours), but leaving almost unaffected the SPT-OMR posteriors (green dot-dashed contours), largely alleviates the tension between
data sets. Once we let the cosmological parameters free to vary, the tight correlation between the SZ and richness scaling relation parameters introduced by the SPT-OMR data, along with the different posteriors for the amplitudes preferred by the latter, moves the $\Omega_m$ posterior of the full data combination towards larger values. The larger shift with respect to the DES-NC+SPT-OMR data combination observed for the BKG analysis can be understood in terms of the larger tension between multi-wavelength and abundance data displayed in figure 8. Despite the better agreement of the $A_{\text{SZ}}-A_{\text{SZ}}$ posteriors derived assuming the PRJ calibration, the DIC suggests a mild preference for this model only for the full data combination (see section IV A 1).

Moving to the mass–richness relation, figure 9 compares the scaling relations derived in this work (hatched bands) with other results from the literature. The scaling relation from DES20 originally derived for $M_{200,m}$ has been converted to $\langle M|\lambda_{\text{ob}},z \rangle$ imposing the condition $n(M_{500,c})dM_{500,c} = n(M_{200,m})dM_{200,m}$ to the Tinker halo mass function. The mean mass-richness relation and its uncertainty are computed from the $\lambda$-mass parameter posterior through Bayes’ theorem as follows:

$$\langle M|\lambda_{\text{ob}},z \rangle \propto \int dM n(M,z) P(\lambda_{\text{ob}}|\lambda,z)P(\lambda|M,z).$$

(15)

Fitting the $\langle M|\lambda_{\text{ob}},z \rangle$ relation derived from DES-NC+SPT-OMR to a power law model similar to the one assumed in 60, we get:

$$\langle M_{500,c}|\lambda_{\text{ob}},z \rangle = 10^{14.29\pm 0.03} \left( \frac{\lambda_{\text{ob}}}{60} \right)^{1.11\pm 0.06} \frac{1+z}{1+0.35}^{-0.55\pm 0.75}.$$  

(16)

The DES-NC+SPT-OMR and DES-NC+SPT-OMR,NC analyses provide mass–richness relations consistent with each other within one standard deviation (gray and hatched orange bands). These results are also consistent with a similar analysis performed by 27 who calibrate the $\lambda$-mass relation combining cluster counts from both SPT-SZ and SPTpol Extended Cluster Survey, richnesses obtained by matching the SZ sample with the redMaPPer DES Year 3 catalog, and assuming the fiducial cosmology $\sigma_8 = 0.8$ and $\Omega_m = 0.3$ (magenta band). Also here, the slightly steeper $M-\lambda_{\text{ob}}$ relation preferred by our data is due to the different cosmologies preferred by the DES and SPT abundance data. Indeed, when we include the SPT-NC data in our analysis, the $\langle M|\lambda_{\text{ob}},z \rangle$ relation totally overlaps with the results from 27 (hatched orange and magenta bands). Similarly, 51 derived a richness–mass relation consistent with ours ($A_\lambda = 83.3 \pm 11.2$ and $B_\lambda = 1.03 \pm 0.10$) analysing the same redMaPPer-SPT matched sample and adopting as priors the results of 111 $B_\lambda = 0.96^{+0.06}_{-0.07} \pm 0.04$, who calibrate the richness-mass relation of a X-ray selected, optically confirmed cluster sample through galaxy dynamics. However, a direct interpretation of their results in the context of this analysis is not possible due to the different assumptions on the X-ray scaling relation and the scatter of the richness–mass relation, made in that work.

A larger than 1σ tension below $\lambda_{\text{ob}} \approx 60$ is found with the DES20 results which base their mass calibration on the stacked weak lensing analysis of 50 (cyan band in figure 9). As noted in DES20 the weak lensing mass estimates for $\lambda < 30$ are responsible for the low values derived for the slope and amplitude of the richness–mass relation compared to the ones preferred by the SPT multi-wavelength data. We stress again here that the SPT-OMR data can actually constrain the richness–mass relation only at $\lambda_{\text{ob}} \gtrsim 50$ and the constraints at low richness follow from the power law model assumed for the $\langle \lambda|M \rangle$ relation.

The inclusion of the PRJ calibration, increasing the fraction of low mass clusters boosted to large richnesses, lowers the mean cluster masses compared to the BKG model up to ~ 25% at $\lambda_{\text{ob}} \lesssim 60$ (compare green and yellow with gray and orange bands in figure 9) respectively. Specifically, from the DES-NC+SPT-OMR+PRJ analysis we obtain:

$$\langle M_{500,c}|\lambda_{\text{ob}},z \rangle = 10^{14.22\pm 0.03} \left( \frac{\lambda_{\text{ob}}}{60} \right)^{1.21\pm 0.05} \frac{1+z}{1+0.35}^{-0.50\pm 0.65}.$$  

(17)

The improved consistency between the scaling relations derived from the analyses adopting the PRJ calibration and DES20 reflects the improved agreement between the corresponding cosmological posteriors due to the lower $\Omega_m$ value preferred by the former (see figure 7). The fact that the mass-richness relations derived from the two $P(\lambda_{\text{ob}}|\lambda_{\text{true}})$ models display a larger than 1σ tension below $\lambda_{\text{ob}} \lesssim 50$, but perform equally well in fitting the data (see section IV A 1), is due to the lack of multi-wavelength data at low richness. Additional follow-up data at $\lambda_{\text{ob}} \lesssim 40$ will be fundamental to clearly reject one of the models and thus enable the full exploitation of the cosmological information carried by photometrically selected cluster catalogs.

It is worth noting that at odds with other studies which rely on stacked weak lensing measurements to calibrate the mean scaling relation [e.g. 8, 9], the SPT-OMR data, allowing a cluster by cluster analysis (see equation 8), enable to constrain also the scatter of the richness–mass relation. This is particularly relevant for the analysis of optically–selected cluster samples for which reliable
The larger tension seen in figure 9 between the scaling relations motivating this work and [50] (filled boxes). We also include the weak lensing mass estimates employed in the DES Y1 cluster analysis (hatched boxes) which adopt an updated calibration of the selection bias based on the simulation analysis of [52] [see also appendix D of 9]. Both weak lensing mass estimates and mean mass predictions have been derived assuming $\Omega_m = 0.3$, $h = 0.7$ and $\sigma_8 = 0.8$[12]. The mean mass predictions for the DES-NC+SPT-OMR analysis are in tension with both weak lensing mass estimates. In particular, in the lowest richness bins, $\lambda^{ob} \in [20, 30]$, the mean mass predictions are 25% to 40% higher than the weak lensing mass estimates, while they are consistent within 1 $\sigma$ with the lensing masses at $\lambda^{ob} > 30$. The inclusion of the PRJ model, lowering the mean mass predictions, largely reduces the tension at low richness with both weak lensing mass estimates, while at $\lambda^{ob} > 30$ the model predictions are consistent within 1 $\sigma$ with the weak lensing masses derived adopting the selection effect bias calibration of [52]. These results are consistent with those of [DES20] for the DES Y1 cluster cosmology analysis to be consistent with other probes the weak lensing mass estimates of $\lambda^{ob} < 30$ systems need to be boosted. Or conversely, the weak lensing mass estimates of $\lambda^{ob} < 30$ systems are biased low compared to the mean masses predicted by DES Y1 abundance data alone assuming a cosmology consistent with other probes. As discussed in [DES20], this tension might be due to an overestimate of the selection effect correction at low richness, or to another systematic not captured by the current synthetic cluster catalogs. The good agreement of the PRJ mass predictions with the weak lensing masses adopted in [DES20] reflects the consistency of our cosmological priors with those derived in DES Y1 cluster analysis (see the lower left panel of figure 7). The same conclusions last also for the full data combination analyses (not shown in figure 10), which provide model predictions fully consistent with those obtained from the combination of DES abundances and SPT multi-wavelength data.

**B. $w$CDM + $\sum m_\nu$ cosmology**

We consider an extension to the vanilla $\Lambda$CDM model by allowing the dark energy equation of state parameter $w$ to vary in the range $[-2.5, -0.33]$. Here we are interested in the capability of the DES-NC+SPT-OMR data to constrain the equation of state parameter $w$, and the possible cosmological gain given by the inclusion of the high redshift SPT abundance data. For this reason we report here only results for the BKG scatter model. Nevertheless, we explicitly verified that the PRJ model provides for both data combinations posteriors on $w$ fully consistent with those obtained assuming the BKG model. In figure 11 and table IV we show constraints for the DES-NC+SPT-OMR and DES-NC+SPT-[OMR,NC] data sets. Both data sets prefer a $w$ value smaller than -1 at more than one $\sigma$ ($w = -1.76^{+0.48}_{-0.33}$ and $w = -1.95^{+0.48}_{-0.19}$), even though consistent within $2\sigma$ with a cosmological constant. Despite that the inclusion of the SPT-NC data increases the redshift range probed by the abundance data up to $z \simeq 1.75$, the constraints on $w$ improve only by 15%. This again is due to the fact that the analysis is

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[12] The larger tension seen in figure 9 between the scaling relations derived in this work and [50] is due to the different cosmology preferred by the two analyses.
limited by the uncertainty in the calibration of the scaling relations with which the $w$ parameter is degenerate. For the DES-NC+SPT-OMR analysis the model extension minimally affects the cosmological posteriors on $\sigma_8$ and $\Omega_m$ compared to the $\Lambda$CDM model despite the mild anti-/correlation of the two parameters with $w$ ($\rho \sim \pm 0.25$) and the preference for $w < -1$. Interestingly in this case, the inclusion of the SPT-NC data does not cause the large $\sigma_8$-$\Omega_m$ shift observed in the $\Lambda$CDM scenario, and the DES-NC+SPT-|$\Omega_m$| posteriors almost completely overlap with those derived in the DES-NC+SPT-OMR analysis. This difference with the $\Lambda$CDM results is explained by the degeneracy of the equation of state parameter $w$ with the SZ and $\lambda$-$M$ scaling relation parameters. In particular for the DES-NC+SPT-OMR analysis, the preference for $w < -1$ and the anti-/correlation of $w$ with the slope and amplitude parameters of the richness–mass relation shifts the corresponding posteriors into the same region of the parameter space preferred by the full data combination (see figure 13 in appendix C). Despite the modest ($\sim 0.5 - 1.0\sigma$) shift of the $\lambda$-$M$ posteriors observed for the $w$CDM model, the resulting mass-richness
As mentioned above, an improved calibration of the scaling relations and their evolution will be paramount for future cluster surveys aimed to disentangle a cosmological constant from a $w_{\Lambda}$CDM model [e.g. 54].

V. SUMMARY AND CONCLUSION

In this study, we derive cosmological and scaling relation constraints from the combination of DES Y1 cluster abundance data (DES-NC) and SPT follow-up data (SPT-OMR). The former contains $\sim 6500$ clusters above richness 20 in the redshift range $0.2 < z < 0.65$, the latter consists of high-quality X-ray data from Chandra and imaging data from HST and Megacam for 121 clusters collected within the SPT-SZ 2500deg$^2$ survey, along with richness estimates for 129 systems cross matched with the DES Y1 redMaPPer catalog. The SPT multi-wavelength data allows us to constrain the richness–mass scaling relation, enabling the cosmological exploitation of the DES cluster counts data. Mass proxies based on photometric data are prone to contaminations from structures along the line of sight — i.e. projection effects — which hamper the calibration of the scaling relations. To explore possible model systematics related to the latter we consider two calibrations of the observational scatter on richness estimates: i) a simple Gaussian model which accounts only for the noise due to misclassification of background and member galaxies, and ii) the model developed in [35] which includes also the scatter on $\lambda_{\text{obs}}$ introduced by projection effects (labelled respectively BKG and PRJ throughout the paper).

Independently from the model adopted for the scatter on the observed richness, we derive cosmological constraints for a ΛCDM model consistent with CMB data and low redshift probes, including other cluster abundance studies. Our results are in contrast with the findings of [DES20] which obtained cosmological constraints in tension with multiple cosmological probes analysing the same DES abundance data but calibrating the $\lambda - M$ relation with mass estimates derived from stacked weak lensing data. Our results thus support the conclusion of [DES20] which suggests that the tension is due to the presence of systematics in the modeling of the stacked weak lensing signal of low richness clusters ($\lambda_{\text{obs}} \lesssim 30$). Indeed, the mass–richness relations derived in this work adopting the BKG and PRJ models are in tension with that derived in [DES20] below $\lambda_{\text{obs}} \sim 60$ and $\lambda_{\text{obs}} \sim 40$, respectively. We stress however that the SPT-OMR data are mainly available for $\lambda_{\text{obs}} \gtrsim 40$ systems and thus we need to extrapolate the $\lambda_{\text{obs}} - M$ relation when fitting the DES abundance data at lower richness. Nevertheless, both scatter models perform well in fitting the DES cluster abundance at all richnesses, supporting the goodness of the relation extrapolated at low richness.

We further consider the combination of the DES-NC and SPT-OMR data with the SPT number counts data above redshift $z = 0.65$ (SPT-NC), to assess possible cosmological gains given by the analysis of the joint abun-
dance catalog. This also serves as a test of the consistency of the three combined data sets. When included in the analysis the SPT-NC data reduces the $\sigma_8$ and $\Omega_m$ uncertainties by 30% and 20% respectively, while shifting their posteriors along the $S_8$ degeneracy direction, increasing the tension with other cosmological probes, and especially with the SPT-SZ 2500 results, with which it shares the SPT abundance at $z > 0.65$ and follow-up data. The shift is due to the tension between the scaling relation parameters preferred by the DES and SPT abundance data and the SPT follow-up data at the "fiducial" cosmology $\sigma_8 \sim 0.75$, $\Omega_m \sim 0.3$. This tension is largely solved once we consider the PRJ model. Compared to the BKG results, it provides cosmological posteriors for the full data combination in better agreement with all the other probes considered here. Adopting the DIC for the model selection, we find a "positive" preference for the BKG model for the DES-NC+SPT-OMR data combination, and a "positive" preference for the PRJ model for the full data combination. Additional follow-up data, especially at low richness will be necessary to clearly identify which scatter model for $\lambda^{ab}$ is best suited to describe the data. In this respect, the upcoming SZ and X-ray surveys SPT-3G and eROSITA are expected to provide valuable follow-up data by lowering the limiting mass of the detected clusters to $\sim 10^{14} M_\odot$ [see e.g. 52].

Finally we consider a $wCDM$ model and derive cosmological constraints for the DES-NC+SPT-OMR and DES-NC+SPT-OMR,NC] data combinations assuming the BKG model. We find in both cases a preference at more than $1 \sigma$ for $w$ values lower than $-1$, but consistent with a cosmological constant. The inclusion of the SPT-NC does not substantially improve the $w$ constraints despite the larger redshift leverage provided by the SPT abundance data, indicating that also in this case we are limited by the uncertainty in the calibration of the scaling relations and their evolution. According to the DIC the $wCDM$ model is "strongly" preferred over the $\Lambda CDM$ one, thanks to the improved fit to the DES-NC data provided by the extended model. However, given the strong degeneracy between $w$ and the scaling relation parameters we cannot exclude that this preference is due to a flawed modeling of the scaling relations which is absorbed by $w$. Again, an improved calibration of the scaling relations and their evolution, will be necessary for future cluster surveys aimed to constrain the dark energy equation of state parameter. Future optical survey such as Euclid and LSST, in combination with data from the forthcoming eROSITA and SPT-3G surveys, will provide the necessary high-redshift multi-wavelength data to break such degeneracies and thus constrain parameters affecting the growth rate of cosmic structures [see e.g. 52].

The results of this work highlight the capability of multi-wavelength cluster data to improve our understanding of the systematics affecting the observable-mass scaling relations, and the potential power that a joint analysis of cluster catalogs detected at different wave-lengths will have in future cosmological studies with galaxy clusters.

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**Appendix B: Observed richness–mass scaling relations**

To ease the comparison and use of our results we report here the mean observed richness–mass scaling relations derived from the DES-NC+SPT-OMR data combination for the two scatter models adopted. The mean relations and uncertainties are derived from the appropriate model for $P(\lambda|\lambda,|M, z) = \int d\lambda P(\lambda|\lambda,|M, z)$ sampling the posterior distributions of the richness–mass relation. Fitting the mean relation to a power law model we obtain for the BKG model:

$$\langle \lambda^{ob}|M_{500,c}, z \rangle = 79.8 \pm 5.0 \left( \frac{M_{500,c}}{3 \times 10^{14} M_\odot h^{-1}} \right)^{0.93 \pm 0.03} \left( 1 + z \right)^{-0.49 \pm 0.71} (1 + 0.45)$$

while for the PRJ model we obtain:

$$\langle \lambda^{ob}|M_{500,c}, z \rangle = 80.1 \pm 4.1 \left( \frac{M_{500,c}}{3 \times 10^{14} M_\odot h^{-1}} \right)^{0.88 \pm 0.03} \left( 1 + z \right)^{-0.06 \pm 0.6} (1 + 0.45)$$

**Appendix C: wCDM results: scaling relations and correlation coefficients**

As for the ACDM analysis we report in figure 13 the posteriors obtained for the wCDM model including the scaling relation parameters and the correlation coefficients omitted in the main text. The inset plot in figure shows the correlation matrix for a subset of the varied parameters obtained from the DES-NC+SPT-OMR analysis.
FIG. 12. Marginalized posterior distributions for the \( \Lambda \text{CDM}+\sum m_{\nu} \) model for a subset of the fitted parameters. The 2D contours correspond to the 68% and 95% confidence levels of the marginalized posterior distribution. The description of the model parameters along with their posteriors are listed in Table III. Inset panel: Correlation matrix for the scaling relations and cosmological parameters derived from the DES-NC+SPT-OMR analysis.
FIG. 13. Marginalized posterior distributions for the $\nu$CDM+$\sum m_\nu$ model. The 2D contours correspond to the 68% and 95% confidence levels of the marginalized posterior distribution. The description of the model parameters along with their posteriors are listed in Table III. Inset panel: Correlation matrix for the scaling relations and cosmological parameters derived from the DES-NC+SPT-OMR analysis.