A modification of the Oersted experiment

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Abstract
The paper describes a simple set-up of the Oersted experiment. A planar coil of wires has been used to deflect vigorously the magnetic needle (more than 80 angular degrees) when a current of up to 1 A flows along it. Based on theoretical analysis the torque on the magnetic field is analytically expressed taking into account the inhomogeneity of the field and the needle shape. This modification of the Oersted experiment can be used as an easily-made and low-cost set-up or a laboratory workshop at the undergraduate level. Moreover, a procedure to measure the Earth’s magnetic component is implied and implemented and its magnitude has been estimated following the same steps.

1. Introduction
In 1820, Christian Oersted noticed that a compass needle deflected its initially aligned north–south direction in the presence of a current-carrying wire. This experiment was the first one to indicate that a current-carrying wire produces a magnetic field. However, the classical set-up employs a source of steady power supply and a current of 10–20 A, which makes the demonstration very difficult.

Hence, we propose a low-cost type of apparatus as a simple solution and practical modification of the Oersted experiment.

2. Theoretical analysis
2.1. The magnetic field of a finite wire
Let a straight finite wire of length \( L \) lie along the y-axis of the Cartesian coordinate system (figure 1). The wire is oriented symmetrically to the origin of the coordinate system. A steady
electric current, $I$, directed along the positive $y$-axis flows along the wire. The current carried by the wire creates a magnetic field around the wire.

At any point on the $z$-axis, the vector of magnetic induction, $\vec{B}$, at a distance $z$ from the origin of the coordinate system, is oriented to the positive direction of the $x$-axis (as shown in figure 1).

The magnitude of the magnetic induction vector is obtained by using the Biot–Savart law:

$$B_x = \frac{\mu_0 \cdot I}{4 \cdot \pi} \cdot \int_{-L/2}^{+L/2} \frac{z \cdot dy}{(y^2 + z^2)^{3/2}}.$$  

The solution to the integral is in [3]:

$$B_x = \frac{\mu_0 \cdot I}{2 \cdot \pi \cdot z} \cdot \frac{1}{\sqrt{1 + \left(\frac{2 \cdot z}{L}\right)^2}}.$$  

The first multiplier on the right of (2) is the field of a straight infinite wire [1]:

$$B_x(\infty) = \frac{\mu_0 \cdot I}{2 \cdot \pi \cdot z}.$$  

The second multiplier is a correction allowing for a wire of finite length. The same multiplier is a continuous function of $L/z$ to the value of 1 at $L/z \to \infty$. At $L/z = 10$, the multiplier has a value of 0.9806 while at $L/z = 20$, it has a value of 0.9950.

Hence, at $L/z \geq 20$, the field created by the wire is equal to that of an infinite wire to within 0.5%. For instance, if $z = 1$ cm, we can use a wire length of 20 cm.

### 2.2. A magnetic needle in the magnetic field of a wire

We would like to point out that the theoretical analysis is totally based on the interaction of a magnetic needle with the magnetic field of a straight infinite horizontal current-carrying wire.

![Figure 1. The magnetic field of a finite current-carrying wire.](image-url)
Figure 2. Mutual disposition of a magnetic needle and a wire.

Let a straight infinite metal current-carrying wire lie on the $y$-axis of the Cartesian coordinate system $K$ (figure 2). A steady current flows along the wire with a magnitude of $I$, having the same positive direction as the $y$-axis.

At a point on the $z$-axis, the suspension point lies at a distance of $z$ from the origin of the coordinate system of a magnetic needle of length $\Delta$. The pivot of the magnetic needle coincides with the $z$-axis while the magnetic needle lies and moves on a plane parallel to the $XOY$ plane. The actual position of the magnetic needle is characterized by the rotation angle of the needle $\theta$ with respect to the positive direction of the $y$-axis. The magnetic needle has a magnetic dipole moment of $m$.

The flowing current creates a magnetic field around the wire. The magnitude of the horizontal component $B_I$ of the magnetic induction vector in the positive direction of the $x$-axis can be obtained by the Biot–Savart law [1]:

$$B_I = \frac{\mu_0 \cdot I}{2 \cdot \pi} \cdot \frac{z}{\rho^2},$$  \hspace{1cm} (4)

where

$$\rho = \sqrt{z^2 + x^2}$$  \hspace{1cm} (5)

is the distance from the given point with coordinates of $(x, y, z)$ to the wire axis.

The separate parts of the magnetic needle at a rotation angle of $\theta$ are at different distances from the wire axis, so the magnitude of the magnetic field (4) exerted on them is different. Due to the inhomogeneity of the magnetic field round the wire, the torque of the magnetic needle $M_I$ is obtained by the integration of the infinite small torques created by the different infinite small parts of the magnetic needle.

Each infinite small part of the needle of a length of $dl$ is characterized by an infinite small dipole moment of $dm$ having the same direction as that of the needle. The interaction of this part of the needle with the magnetic field of the wire leads to a torque of $dM_I$ [1]:

$$dM_I = -\frac{\mu_0 \cdot I}{2 \cdot \pi} \cdot \frac{z \cdot \cos \theta}{\rho^2} \cdot dm.$$  \hspace{1cm} (6)

The torque of the whole needle, $M_I$, is

$$M_I = -\frac{\mu_0 \cdot I}{2 \cdot \pi} \cdot \frac{m \cdot \cos \theta}{z} \cdot \int_{0}^{\Delta} \frac{z^2}{\rho^2} \cdot \frac{dm}{m} = -\frac{\mu_0 \cdot I}{2 \cdot \pi} \cdot \frac{m \cdot \cos \theta \cdot f(\delta)}{z}.$$  \hspace{1cm} (7)
The integral on the right side of (7) is a dimensionless function \( f(\delta) \) of the parameter \( \delta \) (which is also dimensionless):

\[
\delta = \frac{\Delta \cdot \sin \theta}{2 \cdot z}.
\]

The three-dimensional configuration in figure 2 represents a torque having the same direction as that of the magnetic field of the wire. Obviously, this can be done by adjusting the north–south direction of the needle to the positive \( x \)-axis using the torque.

The functional dependence of \( f(\delta) \) is determined by the shape of the needle. Generally, the function of \( f(\delta) \) is even and decreases with an increase of \( \delta \) due to the needle symmetry which takes the pivot into account. At zero, the function of \( f(\delta) \) has a value of 1.

For example, if the needle has a rectangular shape, the solution to the integral is

\[
f(\delta) = \frac{\arctan(\delta)}{\delta} \approx 1 - \frac{\delta^2}{3} + \frac{\delta^4}{5} - \frac{\delta^6}{7} + \cdots.
\]

The function of \( f(\delta) \) is an important factor and it should be taken into consideration for comparable \( \Delta \) and \( z \). However, there is a great variety of magnetic needle shapes manufactured by different companies. Therefore, the dependence of \( f(\delta) \) cannot be considered as universally valid and its theoretical values are a result of complex mathematical expressions. A tabulation of the function obtained experimentally should be used in such cases.

2.3. A magnetic needle in the magnetic field of the Earth

The Earth has a constant magnetic field; therefore, if the needle was placed only in the Earth’s magnetic field, it would orient itself to point to the magnetic meridian of the geographical point, and that direction is to be nominated as north.

The geomagnetic field is a homogeneous one. The horizontal component of the induction of the earth’s magnetic field orients the positive direction of the \( y \)-axis (figure 2) and has a magnitude of \( B_e \). The interaction of the needle with it results in a torque of \( M_e \):

\[
M_e = -m \cdot B_e \cdot \sin \theta.
\]

2.4. A magnetic needle in the Earth’s and the wire’s magnetic fields

We assume that the magnetic needle has a moment of inertia, \( J \), towards its pivot. Taking into consideration (7) and (10), the simultaneous action of both the Earth’s magnetic field and that created by the wire can be expressed by the following equation of twisting motion:

\[
J \cdot \frac{d^2 \theta}{dt^2} = -m \cdot B_e \cdot \sin \theta - m \cdot \frac{\mu_0 \cdot I}{2 \cdot \pi \cdot z} \cdot \cos \theta \cdot f(\delta).
\]

The above equation has the following stationary solutions.

(i) At zero current along the wire and applying (11), we obtain \( \theta = 0^\circ \), and the needle orients itself along the Earth’s magnetic field.

(ii) At non-zero current along the wire and applying (11), we obtain

\[
\tan \frac{\theta}{f(\delta)} = -\frac{\mu_0}{2 \cdot \pi \cdot z \cdot B_e} \cdot I = -\frac{I}{I_e},
\]

where \( I_e \) is equivalent to the current of the Earth’s magnetic field when \( z \) is given:

\[
I_e = \frac{2 \cdot \pi \cdot z \cdot B_e}{\mu_0}.
\]
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Equation (12) expresses the dependence of the deflection angle of the needle on the wire current. Hence, this dependence can be used as a method for measuring the current magnitude along a wire. It could be called a tangent galvanometer, and this is what the vertical current coils are called[4].

The minus (−) sign in (12) means that when the current flows along the positive direction of the $OY$ axis, the needle deviates to the right.

3. Experimental set-up

3.1. A planar coil of wires

We constructed a planar coil of wires with overall dimensions of 20 cm length and 40 cm width to wind the wire in the middle section as many times as possible and create a bundle of windings there.

The magnetic field created by the side wires in the middle of the bundle of wires is compensated because areas $AB$ and $GH$, $BC$ and $FG$, $CD$ and $EF$ mutually neutralize their fields due to the symmetrical location of the areas and to the fact that the current flows along them symmetrically. Thus, the field created by the bundle of wires in the middle of the coil is what should be noted.

Figure 3(a) shows an uneven number of windings in the bundle while in figure 3(b) the number of windings in the bundle is even. In the set-up, we preferred the option with the even number of windings taking into consideration that the current input and output were located on one side of the plane framework.

We used a dielectric sheet (plastic sheet in our set-up), and 8 mm holes were drilled into it. The location of the holes is shown in figure 4.

The wire was wound onto the sheet while the holes were used to fasten the framework. The bundle of windings was located under the dielectric sheet. During the experiment, the planar coil was placed horizontally and the compass lay on the dielectric sheet in its geometrical centre. It was the sheet gauge that determined the minimum distance between the compass and the bundle windings.

1 http://en.wikipedia.org/wiki/Galvanometer.
3.2. The horizontal frame as a source of magnetic field

In the set-up, we used the planar coil with an even number of windings. The total number of windings in the bundle was 24. A steady current was supplied by a dc power supply rectifier with an output voltage of up to 15 V and a maximum current of 2 A. A resistance of 15 Ω and a power of 20 W were connected in serial into the power supply and the planar coil which was to be used as a ballast.

During the experiment, the planar coil was put on a horizontal table and the compass lay on the dielectric sheet in its geometrical centre (figure 5). The bunch windings were oriented along the north–south direction determined by the compass as the current was switched off along the frame. The compass scale was set to zero when the coil was oriented that way and the current was switched off, the magnetic needle pointing north, i.e. $\theta = 0^\circ$.

Thus, the magnetic field created by the bunch of windings $B_I$ had a direction perpendicular to the direction of the horizontal component of the Earth’s magnetic field, $B_e$.

When the current was switched on along the coil, the magnetic needle deflected left or right at the angle of $\theta$ with regards to the current direction (see figure 4). The angle of deflection on the left has a positive sign.

A current of $I$ flowed along each winding. The range of change of $I$ was from $-0.87$ A to $0.87$ A at a maximum deflection of $\theta = 84^\circ$.

Figure 6 represents the measurements of the deflection angle of $\theta$ as a multiplication of the current $I$ along each wire by the number of windings, $N$.

4. Consideration of results

4.1. The planar coil as a practical modification of the Oersted experiment

The planar coil framework shown in figure 4 is one of many examples which could be used when we do not want to use metal fastening elements or glue. Certainly, there are many other options as well.

The planar coil in the set-up makes the needle deflect vigorously (more than 80 angular degrees) when the current flow is up to 1 A.
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Figure 5. The bunch windings oriented along the north–south direction by the compass.

The magnetic needle is strongly deflected if a small battery of 1.5 A is directly connected to the coil as a voltage source (without any ballast resistance). Thus, the Oersted experiment can be conducted in an easy and simple set-up.

If the planar coil of wires has $L/z < 10$, the same results are obtained: the current-carrying wire creates a magnetic field and the field direction is dependent on the current direction. However, precise quantitative measurements will show serious deviations from the Biot–Savart law ranging from 10 to 15%.

Figure 4 represents a planar coil of wires of $L/z > 20$. The magnetic field created by the bundle of windings and of accuracy better than 1% adjusted the magnetic field of a single straight infinite wire carrying a current of $I \times N$. Consequently, it can be used as a modification of the Oersted experiment.

4.2. The planar coil of wires as a tangent galvanometer

The presence of a source of a magnetic field adjusting the magnetic field of a single straight infinite rectilinear current-carrying wire can be used and implemented in many other set-ups with practical application.

The above-mentioned theoretical assumption (12) to use the planar coil of wires as a tangent galvanometer could become reality if an accurate graduated curve was to be drawn to take nonlinearities into account.

4.3. Measurement of the horizontal component of the Earth’s magnetic field

We would like to point out the following procedure for measuring the horizontal component of the earth’s magnetic field. It is based on the above-mentioned theoretical analysis and on the possibility for the planar coil of wires to be used as a tangent galvanometer.
First, we transform (12) by using (8) into

\[
-\frac{\tan \theta}{I \cdot N} = \frac{1}{I_e} \cdot f\left(\frac{L \cdot \sin \theta}{2 \cdot z}\right).
\]

(14)

Then, the experimental results shown in figure 6 are processed by MS Excel. Figure 7 represents the left side of equation (14) as a function of \(\sin \theta\), i.e. the dynamics of argument \(\delta\) (8). The inhomogeneity of the magnetic field of the wire leads to the dynamic character of \(f(\delta)\).

Next, the graphic dependence of the experimental data obtained is automatically approximated by using the Trendline MS Excel function which conducts a polynomial approximation of the experimental points.

The Trendline MS Excel automatically performs the following substitution (abscissa and ordinate in the graph):

\[
y = -\frac{\tan \theta}{I \cdot N},
\]

(15)

\[
x = \sin \theta.
\]

(16)

At \(\sin \theta = 0\), the value of the approximating function (shown in figure 7) is as follows:

\[
y(0) = 0.6294.
\]

Taking into account \(f(0) = 1\) and comparing (14) and (15), we obtain

\[
1/I_e = 0.63 \pm 0.02A^{-1}.
\]

(17)
The variations of each experimental point around the approximating function (valued at ±0.02 A⁻¹) shown in figure 6 are taken into account in (17). These variations are dependent on the measured accuracy of the deflection angle of the magnetic needle, θ.

In the set-up we measured the distance, z, from the axis of the bundle of windings to the magnetic needle and it was \( z = 11.9 \) mm. Using the measured value of \( z \), the results from (17) together with (13) led to

\[
B_e = (26.7 \pm 0.8) \cdot 10^{-6} \, T.
\]  

Hence, the magnitude of the horizontal component of the Earth’s magnetic field in the area (in the laboratory of physics) of the Sliven Engineering and Pedagogical Faculty was estimated.

Owing to a lack of measuring results by a precise magnetometer, the result obtained (18) can be compared with the model value of USA National Geophysical Data Center². For the town of Sliven (coordinates 42° 41’ N, 26° 20’ E), the model gives for the horizontal component of the Earth’s magnetic field the value of \( B_e = 23.82 \pm 0.15 \, \mu T \). The match of the two values is very close. The reason why the value is off the limit of the above-cited error is due to local inhomogeneity of the urban environment and the laboratory inside.

² http://www.ngdc.noaa.gov/geomag/magfield.shtml.
5. Conclusion

First, the proposed type of apparatus for the Oersted experiment is very simple. The planar coil of wires makes the magnetic field deflect vigorously (more than 80 angular degrees) when a current of up to 1 A flows along it. Furthermore, a considerable deflection can be observed by using a small battery of 1.5 V as a power supply, connected with a planar coil of wires without ballast.

Second, the theoretical analysis points out the dependence of the magnetic needle torque on the shape of the magnetic needle taking into account the inhomogeneity of the magnetic field around the current-carrying wire. Furthermore, it outlines that the planar coil of wires can be used as a tangent galvanometer.

Finally, a measuring procedure for the horizontal component of the Earth’s magnetic field is implied and implemented and its magnitude is estimated.

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