FAST TRACK COMMUNICATION

Hunting for isocurvature modes in the cosmic microwave background non-Gaussianities

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Abstract

We investigate new shapes (in multipole space) of local primordial non-Gaussianities in the cosmic microwave background. Allowing for a primordial isocurvature mode along with the main adiabatic one, the angular bispectrum is in general a superposition of six distinct shapes: the usual adiabatic term, a purely isocurvature component and four additional components that arise from correlations between the adiabatic and the isocurvature modes. We present a class of early Universe models in which various hierarchies between these six components can be obtained, while satisfying the present upper bound on the isocurvature fraction in the power spectrum. Remarkably, even with this constraint, detectable non-Gaussianity could be produced by isocurvature modes. We finally discuss the prospects of detecting these new shapes with the Planck data, including polarization.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The very early Universe being a notoriously difficult territory to explore, it is crucial to extract as much information as possible from the measurement of cosmological perturbations, via the cosmic microwave background (CMB) and large-scale structure observations, as a way to test or constrain early Universe scenarios. This is why primordial non-Gaussianity has been the subject of intense study over the last few years (see, e.g., [1]).

So far, WMAP measurements of the CMB anisotropies [2] have set the present limit \(f_{\text{local}}^{\text{NL}} = 32 \pm 21\) (68% CL) and \(-10 < f_{\text{local}}^{\text{NL}} < 74\) (95% CL) on the parameter \(f_{\text{local}}^{\text{NL}}\) that characterizes the amplitude of the simplest type of non-Gaussianity, namely the local shape.
Although WMAP data hint at a possible deviation from Gaussianity, one must wait for the analysis of the Planck data in order to confirm or infirm this trend.

A detection of local primordial non-Gaussianity would have a tremendous impact on our view of the early Universe. This would indeed rule out all single-field inflation models, which generate only tiny local non-Gaussianity [3], and give strong support to scenarios with additional scalar fields, such as another inflaton, see, e.g., [4], a curvaton [5] or a modulaton [6], which can easily produce detectable local non-Gaussianity (this type of non-Gaussianity is also predicted in other scenarios, for instance the ekpyrotic model, see, e.g., [7]).

Interestingly, models with multiple scalar fields open up the possibility to generate, in addition to the usual adiabatic fluctuations, primordial isocurvature perturbations, corresponding to fluctuations in the relative particle number densities between cosmological fluids (whether isocurvature fluctuations can persist in the post-inflation era depends on the details of the thermal history of the Universe). The amplitude of such isocurvature modes, which could be correlated with the adiabatic one, is now severely constrained by observations of the CMB power spectrum [2].

In this work, we focus on local non-Gaussianity generated by these isocurvature fluctuations, previously studied in [8–12]. Improving on these earlier works, we show that the presence of an isocurvature mode, in addition to the usual adiabatic one, leads in general to an angular bispectrum that consists of the superposition of six elementary components: the well-known purely adiabatic bispectrum, a purely isocurvature bispectrum and four other bispectra that arise from the possible correlations between the adiabatic and the isocurvature modes. Because these six bispectra have different shapes in l-space, their amplitude can in principle be measured in the CMB and we have estimated, by computing the corresponding Fisher matrix analysis, what precision on these six parameters could be reached with the Planck data, including polarization.

A natural question is: whether realistic early Universe models could generate these new bispectra with detectable amplitude? To answer this question, we consider a simple class of models, which generates all six bispectra with various hierarchies between their relative amplitudes. Remarkably, some of these models produce detectable non-Gaussianity dominated by the isocurvature mode, while satisfying the present upper bound on the isocurvature fraction in the power spectrum. These considerations provide a strong motivation to look for these new shapes of non-Gaussianity in the CMB data.

2. Angular bispectra

Let us thus consider several primordial modes denoted collectively by $X^I$. We will later focus on the case of two primordial modes: the usual adiabatic mode, characterized by the total curvature perturbation $\zeta$ (on uniform total energy density hypersurfaces) and a single cold dark matter (CDM) isocurvature mode $S \equiv 3(\zeta_c - \zeta_r)$, with $c$ and $r$ denoting CDM and radiation, respectively.

The multipole coefficients of the temperature anisotropies ($\Delta T/T = \sum_{lm} a_{lm} Y_{lm}$) are related to the primordial modes $X^I$ via the corresponding transfer functions $g_I^l(k)$ so that

$$a_{lm} = 4\pi (-i)^l \int \frac{d^3k}{(2\pi)^3} \left( \sum_I X^I(k) g_I^l(k) \right) Y^*_lm(\hat{k}).$$

(1)

Following [13], we substitute this expression into the angular bispectrum and obtain

$$B_{l_1 l_2 l_3}^{m_1 m_2 m_3} \equiv \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = d_{l_1 l_2 l_3}^{m_1 m_2 m_3} p_{l_1 l_2 l_3},$$

(2)

2
which is the product of the Gaunt integral
\[ G_{\ell m, m m}^{I} = \int d^3 \vec{n} Y_{\ell m}(\vec{n}) Y_{m m}(\vec{n}) Y_{I m m}(\vec{n}) \] (3)
and of the so-called reduced bispectrum
\[ b_{IJK} = \sum_{IJK} \left( \frac{2}{\pi} \right)^3 \int \left( \prod_{i=1}^{3} k_i^3 \delta \right) \delta_{IJK}^{(3)} \left( k_1, k_2, k_3 \right) \times B_{IJK}^{(1)}(k_1, k_2, k_3) \int_0^\infty r^2 dr j_i(k_1r) j_i(k_2r) j_i(k_3r) \] (4)
that depends on the bispectra of the primordial \( \mathcal{X}^I \):
\[ \langle \mathcal{X}^I(k_1) \mathcal{X}^J(k_2) \mathcal{X}^K(k_3) \rangle = (2\pi)^3 \delta(\Sigma_{123}) B_{IJK}^{(1)}(k_1, k_2, k_3). \] (5)
The reduced bispectrum (4) is here the sum of several contributions, thus generalizing the purely adiabatic expression given in [13].

In most inflationary models, the 'primordial' perturbations \( \mathcal{X}^I \) (defined during the standard radiation era) can be related to the fluctuations of light primordial fields \( \phi^a \), generated at Hubble crossing during inflation so that one can write, up to second order:
\[ \mathcal{X}^I = N^I_{\phi} \phi^a + \frac{1}{2} N^I_{\phi} \phi^a \phi^b + \cdots \] (6)
where \( \phi^a \) can usually be treated as independent quasi-Gaussian fluctuations, i.e. \( \langle \phi^a \phi^b(\mathbf{k}) \phi^d \phi^e(\mathbf{k}') \rangle = (2\pi)^3 \delta^{ab} \delta(k-k') \), with \( \delta(k) = 2\pi^2 k^3 H_0^2 / (2\pi)^2 \), a star denoting Hubble crossing time. Using Wick’s theorem, this implies that the bispectrum \( B_{IJK}^{(1)} \) is of the form
\[ B_{IJK}^{(1)}(k_1, k_2, k_3) = \lambda^{IJK} P_{\phi} (k_1) P_{\phi} (k_2) P_{\phi} (k_3) + \lambda^{IJ} P_{\phi} (k_1) P_{\phi} (k_2) P_{\phi} (k_3) + \lambda^{JK} P_{\phi} (k_1) P_{\phi} (k_2) P_{\phi} (k_3) \] (7)
with \( \lambda^{IJK} = \delta^{ab} N^a_{\phi} N^b_{\phi} N^c_{\phi} \) (the summation over scalar field indices is implicit). Let us note that the coefficients \( \lambda \) are symmetric under the interchange of the last two indices.

After substitution of (7) into (4), the reduced bispectrum can finally be written as
\[ b_{IJK} = \sum_{IJK} [\mathcal{B}^{(1)}_{IJK}] \phi_{IJK}^{(1)}, \] (8)
where each contribution is of the form
\[ b_{IJK}^{(1)} = 3 \int_0^\infty r^2 dr \alpha_I^{(1)}(r) \beta_{JK}^{(1)}(r) \beta_{IJK}^{(1)}(r), \] (9)
using \( \langle I J K \rangle = [I J K] + 5 \text{perms} / 3! \), with
\[ \alpha_I^{(1)}(r) = \frac{2}{\pi} \int k^2 dk j_i(kr) g_{\alpha}^{(1)}(k) \] (10)
\[ \beta_{JK}^{(1)}(r) = \frac{2}{\pi} \int k^2 dk j_i(kr) g_{\beta}^{(1)}(k) \] (11)
In \( \beta_I^{(1)} \), we use the adiabatic power spectrum: \( P_{\phi} = \langle \delta^{ab} N_{\phi}^a N_{\phi}^b \rangle \) \( P_{\phi} = A P_{\phi} \), which implies \( \beta_{JK}^{(1)} = \lambda^{IJK} / A^2 \). We also assume that the coefficients \( N_I \) are weakly time dependent so that the scale dependence of \( A^2 \) can be neglected. Note that the elementary bispectra (9) depend simply on the power spectrum and the transfer functions, whereas the main dependence on the early Universe model is embodied by \( \beta_{JK}^{(1)} \).

If cosmological perturbations depend on a single scalar field, only the adiabatic mode \( \zeta \) exists in (6), and (8) reduces to the familiar adiabatic bispectrum with \( f_{\phi}^{(1)} = (6/5) f_{\phi}^{IJK} = N_{\phi}^a / (N_{\phi}^0)^2 \). In slow-roll inflation, this coefficient is a combination of the usual slow-roll parameters and is thus too small to be detectable.
However, if perturbations are generated by at least two light scalar fields, an isocurvature mode can later coexist with the adiabatic mode. The crucial property, then, is that the adiabatic and isocurvature transfer functions, which enter into (10) and (11), are very different (as illustrated by the respective $C_l = \langle a_{lm} a_{lm} \rangle$ plotted in figure 2). Consequently, the angular bispectrum is now the sum of six distinct terms (as illustrated in figure 1), with respective weights $f_\zeta\zeta\/NL$, $f_\zeta\zeta\/NL$, $f_\zeta\zeta\/NL$, $f_\zeta\zeta\/NL$, $f_\zeta\zeta\/NL$ and $f_\zeta\zeta\/NL$.

In the particular case where the adiabatic and isocurvature modes depend on two disjoint subsets of scalar-field fluctuations, the angular bispectrum contains only a purely adiabatic contribution and a purely isocurvature one. But, in general, the two modes can depend on common scalar field(s), which leads to correlations between the adiabatic and the isocurvature modes. The four mixed contributions to the angular bispectrum must then be taken into account.

### 3. Example: a curvaton model

To illustrate this general situation, we consider a simple class of models based on the presence of a spectator light scalar field during inflation, dubbed curvaton [5]. This curvaton acquires nearly scale-invariant fluctuations during inflation and, later, behaves as a pressureless fluid when it oscillates at the bottom of its potential, before decaying.

Here, we allow the curvaton $\sigma$ to decay into both radiation and CDM with the respective branching ratios $\gamma_r$ and $\gamma_c$. Since, in general, CDM can already be present before the decay, we define the fraction of CDM created by the decay as $f_c = \gamma_c \Omega_r / (\Omega_r + \gamma_c \Omega_\sigma)$, where $\Omega$'s represent the relative abundance just before the decay.
As shown in [14], the ‘primordial’ adiabatic and isocurvature perturbations, i.e. defined after the curvaton decay, can be written in the form (6), with

\[
\begin{align*}
N^S_\sigma &= \frac{2r}{3\sigma_\phi}, & N^I_\sigma &= \frac{2r}{3\sigma^2_\phi}, \\
N^S_\alpha &= \frac{2}{\alpha_e}(f_c - r), & N^I_\alpha &= \frac{2}{\sigma^2_e}[f_c(1 - 2f_c) - r],
\end{align*}
\]

where \( r \equiv 3 \gamma \Omega_\sigma / [(4 - \Omega_\sigma)(1 - (1 - \gamma_r)\Omega_r)] \) is assumed to be small, since significant non-Gaussianities arise only if \( r \ll 1 \).

Let us first discuss linear perturbations. It is useful to introduce the curvaton contribution to the total adiabatic power spectrum \( \Xi \equiv (N^S_\phi)^2 / [(N^S_\phi)^2 + (N^I_\phi)^2] \), where \( N^S_\phi = H/\phi \) is associated with the inflaton fluctuation, and \( N^I_\phi = 0 \). \( \Xi \) is directly related to the correlation \( \mathcal{C} \equiv P_{c,S}/\sqrt{P_{c}P_{S}} \equiv \sqrt{\Xi} \) sgn(\( f_c - r \)). The isocurvature–adiabatic ratio, given by

\[
\alpha \equiv \frac{P_S}{P_c} = \frac{(N^S_\phi)^2}{(N^S_\phi)^2 + (N^I_\phi)^2} = \left(1 - \frac{f_c}{r}\right)^2 \Xi,
\]

is constrained by CMB observations [2] to be small (typically \( \alpha \lesssim 0.07 \)), which requires either \( f_c \simeq r \) or \( \Xi \ll 1 \).

Let us now turn to non-Gaussianities. Using (12), one finds \( f_{6NL}^{LS} = 3 \Xi^2 / (2r) \). This is the dominant contribution in the regime \( f_c \simeq r \), the other components being suppressed. We thus concentrate on the more interesting case \( \Xi \ll 1 \) to discuss the size of the various components in terms of \( f_c \) and \( r \), considered as free parameters in our phenomenological approach.

In the regime \( f_c \ll r \ll 1 \), the purely adiabatic coefficient is the smallest one. The other ones are enhanced by powers of \( -3 \) (since \( N^S_\sigma / N^S_\phi = N^S_\alpha / N^S_\phi = -3 \)):

\[
\begin{align*}
\tilde{f}_{6NL}^{IJK} &= (-3)^p \tilde{f}_{6NL}^{\xi^2\xi}, & \tilde{f}_{6NL}^{\xi^2\xi} &= \frac{\alpha^2}{54r},
\end{align*}
\]

where \( p \) is the number of ‘S’ in the triplet \( [I,J,K] \). In particular, the purely isocurvature coefficient is enhanced by a factor 27, but with the opposite sign: \( f_{6NL}^{SS} = -\alpha^2 / (2r) \). All coefficients can be significant if \( r \) is sufficiently smaller than \( \alpha^2 \).

In the opposite regime \( r \ll f_c \ll 1 \), the purely adiabatic coefficient is, once again, the smallest one. All the coefficients are now positive and enhanced by factors \( (3f_c/r)^p \), where \( p \) is again the number of ‘S’ indices:

\[
\begin{align*}
\tilde{f}_{6NL}^{IJK} &= \left(\frac{3f_c}{r}\right)^p \tilde{f}_{6NL}^{\xi^2\xi}, & \tilde{f}_{6NL}^{\xi^2\xi} &= \frac{\alpha^2r^3}{54f_c^3},
\end{align*}
\]

Note that the enhancement factor is much bigger than in the previous case (15). The purely isocurvature coefficient, which dominates, is \( f_{6NL}^{SS} = \alpha^2 / (2f_c) \) and can be large if \( f_c \) is sufficiently small, while the relative size of the other coefficients depends on the ratio \( r/f_c \).

The above results show that a small isocurvature fraction in the power spectrum is compatible with a dominantly isocurvature bispectrum detectable by Planck (e.g. \( \alpha \simeq 10^{-2} \) and \( r \ll f_c \simeq 10^{-8} \) yields \( f_{6NL}^{SS} \simeq 5 \times 10^3 \)). Of course, relations (15) or (16), are specific to the models considered here and would be a priori different in other models. It is therefore important to try to measure these six coefficients separately, in order to obtain model-independent constraints from observations.
4. Observational prospects

To estimate these six parameters, which we now denote $f^{(i)}$, the usual procedure is to minimize

$$
\chi^2 = \left( \sum_i \left( B^{\text{obs}} - \tilde{f}^{(i)} B^{(i)} \right) \right) \left( \sum_i \tilde{f}^{(i)} B^{(i)} \right),
$$

where $B$ is the angle-averaged bispectrum $B_{l_1l_2l_3} \equiv \sum_{m_1m_2m_3} \left( l_1 m_1, l_2 m_2, l_3 m_3 \right) B_{l_1l_2l_3}^{m_1m_2m_3}$, the matrix $\tilde{f}$ denoting the Wigner-3$j$ symbol. The above scalar product is defined by $\langle (B, B') \rangle \equiv \sum_{l_1 \leq l_2 \leq l_3} B_{l_1l_2l_3}^{(i)} B_{l_1l_2l_3}^{(i)'}$, where the variance $\sigma_{l_1l_2l_3} \equiv \langle B_{l_1l_2l_3}^{(i)} \rangle^2 = \Delta l_1 l_2 l_3$, with $n$ being the number of identical indices among $l_1, l_2, l_3$. For a real experiment, the noise power spectrum is added to $C_l$ in this expression. The best estimates for the parameters are thus obtained by solving

$$
\sum_i \langle B^{(i)} \rangle = \langle B^{(i)} B^{\text{obs}} \rangle, \tag{18}
$$

while the statistical error on the parameters is deduced from the second-order derivatives of $\chi^2$, which define the Fisher matrix, given in our case by $F_{ij} \equiv \langle (B^{(i)}, B^{(j)} \rangle$.

We have computed this Fisher matrix by extending the numerical code described in [16] to include isocurvature modes and E-polarization. This treatment takes into account the pure TTT and EEE bispectra, as well as all correlations such as TTE and TEE (for more details on the inclusion of polarization see, e.g., [17]). We have taken into account the noise characteristics of the Planck satellite [15], using only the 100, 143 and 217 GHz channels, combined in quadrature. Our computation goes up to $l_{\text{max}} = 2500$ and uses the WMAP-only 7 year best-fit cosmological parameters [2].

From the Fisher matrix given in table 1, one finds that the 68\% error on the parameters $\tilde{f}^{(i)}$ is given by

$$
\Delta \tilde{f}^{(i)} = \sqrt{(F^{-1})_{ii}} = [10, 7, 141, 146, 163, 124]. \tag{19}
$$

The first two uncertainties are much smaller than the last four. This is due to the severe suppression of the isocurvature transfer function at high $l$, which leads to a saturation of the signal-to-noise ratio for the last four parameters. By contrast, the large $l$ behaviour of the first two bispectra is governed by the adiabatic transfer function (less severely suppressed), and the error on the first two parameters is thus further reduced at higher $l$, as shown in figure 2. Including polarization has significantly improved (between two and six times) the precision on the last four parameters.

It is also instructive to compare (19) with the naive uncertainties $\Delta \tilde{f}^{(i)} = 1/\sqrt{F_{ii}} = [5, 4, 55, 82, 44, 64]$ obtained by ignoring the correlations, or, equivalently by assuming that only one parameter is nonzero. In particular, the contamination of the purely adiabatic signal by the other shapes increases the uncertainty, but only by a factor 2, which is rather moderate (a two-parameter analysis with $f^{(1)}$ and $f^{(6)}$, assuming uncorrelated adiabatic and isocurvature
Figure 2. The two thick lines are the adiabatic and isocurvature power spectra, more precisely $10^{16} \times l(l+1)C_l/2\pi$. The thin lines represent the uncertainties for the six non-Gaussianity coefficients, as functions of the cut-off $l_{\text{max}}$. At $l = 100$ the eight curves are, from top bottom: the adiabatic and isocurvature power spectra, the $S, \zeta, S, \zeta, S, \zeta, \zeta, SS, S, \zeta, SS, \zeta, SS, \zeta, \zeta$ uncertainties.

5. Conclusion

In summary, we have introduced a novel analysis of the CMB data, including polarization, which relies on the decomposition of a generic local adiabatic–isocurvature bispectrum into elementary bispectra. The corresponding amplitudes are independent of the details of the early Universe scenario and can be extracted separately from the data (the correlations between these parameters and the errors on their measurement are contained in the Fisher matrix, which we have computed explicitly for the Planck experiment). The example of the early Universe scenario discussed here shows that a detectable CMB bispectrum dominated by isocurvature modes should be considered seriously. Looking for isocurvature non-Gaussianity is complementary to the search for an isocurvature component in the power spectrum, which is now standard routine, and it would be highly desirable to conduct systematically both types of analysis for the future CMB data.

This study could be extended in several directions. From the observational point of view, it would be useful to investigate the contamination by foreground emissions or secondary effects. It would also be interesting to see how the search for isocurvature non-Gaussianities could be improved by combining CMB and large-scale structure observations, or even taking into account the angular trispectrum [18, 19]. From the theoretical point of view, it would be
natural to generalize our study to other types of isocurvature modes [20]. Moreover, it would be worth undertaking a systematic investigation of the possible isocurvature non-Gaussianities predicted by various high-energy physics scenarios to take advantage of this new observational window.

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