A Dynamic and Incentive Policy for Selecting D2D Mobile Relays

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Abstract—

User-to-network relaying enabled via Device-to-Device (D2D) communications is a promising technique for improving the performance of cellular networks. Since in practice relays are in mobility, a dynamic relay selection scheme is unavoidable. In this paper, we propose a dynamic relay selection policy that maximizes the performance of cellular networks (e.g. throughput, reliability, coverage) under cost constraints (e.g. transmission power, power budget). We represent the relays’ dynamics as a Markov Decision Process (MDP) and assume that only the locations of the selected relays are observable. Therefore, the dynamic relay selection process is modeled as a Constrained Partially Observable Markov Decision Process (CPOMDP). Since the exact solution of such framework is intractable to find, we develop a point-based value iteration solution and evaluate its performance. In addition, we prove the submodularity property of both the reward and cost value functions and deduce a greedy solution which is scalable with the number of discovered relays. For the multi-user scenario, a distributed approach is introduced in order to reduce the complexity and the overhead of the proposed solution. We illustrate the numerical results of the scenario where throughput is maximized under energy constraint and evaluate the gain that the proposed relay selection policy achieves compared to a traditional cellular network.

Index Terms—D2D communications, relay selection, mobility, Constrained Partially Observable Markov Decision Process (CPOMDP)

1 INTRODUCTION

In traditional cellular networks, users access the network via single-hop communications. Introducing relays to cellular networks has become one of the major concern of cellular network planners that aim to improve the capacity and the coverage of their networks. The emergence of D2D communications (e.g. [1] and [2]) has revitalized the scenario of user-to-network relaying based on D2D communications between the User Equipment (UE) and the relays and traditional cellular communications between the relays and the Base Station (BS). The advantages of this scenario are mainly classified in two categories: (i) improving the performance of the network (e.g. capacity enhancement, coverage extension, transmission power reduction, load balancing, network offloading etc.) and enabling new services (e.g. data on demand).

The performance of D2D relaying has been investigated by the means of several mathematical tools. Stochastic geometry enables an analytic modeling and performance evaluation of cellular network with fix D2D relays in [3] and mobile D2D relays in [4]. Monte-Carlo simulations in [5] and [6] show how enabling D2D communications to carry relayed traffic can enhance the capacity and coverage of cellular networks. A system-level simulator in [7] was developed to evaluate the extension of the cellular coverage due to D2D relaying.

Several challenging issues require further investigation in order to achieve the performance gain promised by UE-to-Network relaying. One can ask how the relays should be strategically positioned in order to optimize the performance of the network. For example, work in [9] aims to minimize the overall energy consumption of a two-hop cellular network with mobile relays by identifying the optimal relay location from which this relay should start forwarding the data to the BS. Sharing the spectrum between D2D and cellular communications is one of the existing challenges. Using stochastic geometry modeling, authors in [9] studied underlay D2D enabled cellular networks. They analytically derived the tradeoff generated by the spectrum partition between D2D and cellular communications and they deduce the optimal spectrum partition that guarantees the fairness in the network. Several solutions of power and/or resource allocation in D2D relay assisted cellular networks have been studied in the recent literature. The work [10] proposes a distributed resource allocation for D2D relay-aided cellular networks using game theory tool and exposes a summary of the different existing centralized and distributed resource allocation schemes. In addition to resource and/or power allocation schemes, a mode and path selection algorithm was developed and simulated in [11]. Furthermore, Constrained Markov Decision Process (CMDP) problems were formulated in [12] and [13] to characterize the optimal packet scheduling decision for mobile relays.

One of the main challenges of D2D relay-aided cellular networks is to have a relay selection strategy that achieves the performance of cooperative relaying especially when relays are in mobility. Indeed, a relay selected at one position can be no longer helpful at another position. Therefore, a dynamic relay selection policy is necessary when relays’ mobility is assumed. Moreover, we limit the signaling overhead by considering that only the locations of a subset of relays are known, which makes the selection of the relays challenging. In addition, since the relaying functionality is costly for the relays (e.g. in terms of energy, data consumption etc), we assume a certain amount of charge for using the selected relays. This cost aims to encourage the terminals to behave as potential relays. Since the locations of the mobile
relays are partially observed and the relays’ reward and cost depend on the relays locations, we model the problem as a Constrained Partially Observable Markov Decision Process (CPOMDP) and propose a dynamic relay selection policy that maximizes some performance metric (e.g., throughput, coverage, reliability etc.) of cellular networks while satisfying some cost constraints (e.g., energy and/or data consumption, incentive budget etc.). In the numerical section, we apply our study to the case of cellular networks, however the results of this work can be generalized to any other relaying scenario.

1.1 Related Work

The existing rich literature on relay selection problems consider mainly fix relays. Some of these schemes are briefly presented in the sequel. Authors in [14] consider a stochastic formulation and propose a fully distributed single relay association scheme that aims to increase the spectral efficiency of the network. Assuming a delay at the level of the Channel State Information (CSI) feedback, authors in [15] study a joint resource allocation and relay selection scheme that aims to minimize the total transmit power. An energy efficient relay selection algorithm was proposed in [16] based on a Discrete Time Markov Chain (DTMC) modeling of the relay node with Discontinuous Reception (DRX) mechanism. In [17], authors use an iterative technique to propose a joint relay selection and power allocation scheme for relay-aided D2D underlying cellular networks. The work in [18] uses a queuing theory model to propose a single relay selection scheme that optimizes the network in terms of relay remaining battery life, end-to-end data rate and end-to-end delay criteria. For underlay D2D enabled cellular networks, interference is mitigated between cellular and D2D communications by considering a distributed relay selection algorithm in [19]. Moreover, authors in [20] assume a Markovian channel state model and address a relay selection policy that maximizes the long term transmission rate. This decision strategy is obtained by solving the formulated Partially Observable Markov Decision Process (POMDP) based on a dynamic programming-based algorithm.

The aforementioned works do not take user mobility into account but consider fix relays which limits their applicability in cellular mobile networks. Considering the mobility of the relays seems to be a challenging scenario. Nevertheless, a scenario with fix relays and mobile users have been the subject of the work in [21]. A dynamic relay selection scheme that aims to minimize the cost of relaying under performance constraint was proposed by solving a CMDP problem.

1.2 Contribution and Organization

The mobility of the relays is the main challenge for D2D relay selection decision in cellular networks. Compared to previous works, the particularity of this paper is the optimization of the cellular network performance (e.g., throughput and/or reliability and/or coverage etc.) under cost constraints (e.g., energy and/or data consumption, incentive budget etc.) in a scenario where relays are in mobility. In this paper, the particularity of our contributions are summarized as follows:

- The consideration of a realistic scenario where relays are in mobility, thus dynamic relay selection scheme is necessary.
- The cellular network performance is optimized under cost constraints.
- The relay selection is not limited to one relay, thus the performance gain is improved by increasing the cooperative diversity order.
- The relays’ mobility pattern is assumed acknowledged at the level of the agent decision maker (i.e. either UE or BS).
- Since not the locations of all the relays are observed but only the locations of the selected relays are observable, the sequential relay decision process is partially observable and is modeled by a CPOMDP. This assumption is considered in order to limit the signaling overhead require for acquiring such information.
- The complexity of the CPOMDP exact solution is discussed and an approximated solution is proposed. The existing trade-off between its complexity and precision is derived.
- The submodularity property of both cost and reward value functions is proven and a greedy solution that is scalable with the number of potential relays is deduced.
- Numerical results show the performance that cellular networks may gain by implementing the proposed relay selection policy.

The rest of this paper is organized as follows. Section 2 describes the system model for a single UE scenario. Section 3 formulates the optimization problem as a CPOMDP. Since the exact solutions of such problem are intractable to find, a low-complexity dynamic relay selection, called Constrained Point-Based Value Iteration (CPBVI), is proposed in section 4. The submodularity property is verified for this problem, thus a greedy form of this approximation, called Greedy Constrained Point-Based Value Iteration (GCPBVI), is deduced. These results are extended to a multi-UE scenario in section 5. For this scenario, a distributed approach is introduced in the aim of reducing the complexity of the centralized solution. Numerical results in section 6 corroborate our claims for a scenario where throughput is maximized under energy constraints. Section 7 concludes the paper whereas the proofs are provided in the appendices.

2 System Model

For the sake of clarity, we start by describing the system model for a single UE scenario. We will show in section 5 that extending the results to multi-UEs scenario is a straightforward process. Presenting the results for one UE first and then discussing the extension to multi-UEs is done for clarity purpose and for a better presentation of the concepts and methods proposed in this work. The following formulation of the relay selection problem is general and remains valid for any choice of the decision maker (i.e. whether BS, UE or any other entity in the network).
2.1 Network model

We start by considering one UE in a single cell scenario. In the aim of improving the performance of its cellular communications, UE is allowed to use D2D communications to access the network via mobile relays. UE discovers $K$ nearby potential relays by launching a discovery process of periodicity $T$. The time between two discovery processes is partitioned into decision epochs $t$ of constant duration (with $t \in \{1, 2, ..., T\}$). The goal of this work is to determine which relay selection decision should be made at each epoch $t$ in order to maximize its cumulative reward under cost constraints.

$K = \{0, 1, 2, ..., K\}$ denotes the set of $K$ potential relays (from index 1 to $K$) as well as the direct cellular link (index 0). The relay selection policy consists of deciding whether the UE will have direct communication with the network or will pass by some mobile relays. In the latter case, this policy chooses the subset of relays that will be used for attaining the network. In general, D2D relaying can be applied to both Downlink (DL) and Uplink (UL) communications. Due to the practical consideration that mobile terminals do not support simultaneous signal transmission and reception, a two-phases transmission scheme is assumed. Therefore, for UL (resp. DL) relayed communication, the transmission protocol is divided into two phases: (i) in the first phase the relay receives the data from the UE (resp. BS) and (ii) in the second phase the relay transmits the received data to the BS (resp. UE).

2.2 Mobility Model

Relays’ locations in a service coverage area are quantized and represented by a set of regions $S = \{S_1, S_2, ..., S_{|S|}\}$. We assume that the relays remain in the same region during a decision epoch. We denote by $s_i(t)$ the location of relay $i$ at epoch $t$. In the next epoch, each relay changes its locations with a certain probability (i.e. by either staying in the same region or moving to another neighboring region). The mobility of relay $i$ is modeled by a transition matrix $P_i$, where each element $P_i(s_n, s_{n'})$ of this matrix denotes the probability that relay $i$ moves from region $s_n \in S$ to region $s_{n'} \in S$ in the next decision epoch. In this work, we assume that the decision maker is aware of the mobility pattern of each discovered relay (i.e. equivalent to the relay’s transition matrix). Similar mobility models can be found in [21] and references therein. The vector $s_i = (s_1(t), s_2(t), ..., s_K(t))$ denotes the location of the $K$ potential relays at epoch $t$.

We underline that each relay moves over time epochs based on its transition probability matrix and independently from the fact that this relay has been or not selected. At a given epoch $t$, we assume that the decision maker will be able to observe only the current localization region of the selected relays (i.e. not all the relays). This is practical assumption that aims to limit the overhead of signaling required for obtaining this information (i.e. locations of all the potential relays). On the other hand, we will show in the sequel how the decision maker will be able to acquire a certain belief of the locations of the non-selected relays.

2.3 Cost and Reward Model

We propose a relay selection policy that aims to optimize a cellular reward under cost constraints. The reward function, which depends on the relay’s location, represents the benefit of choosing a relay (e.g. in terms of throughput and/or reliability and/or coverage etc). The cost function, which depends on the relay’s location, defines the charge that each selected relay should receive (e.g. in terms of energy and/or incentive budget etc.). We respectively denote by $r_i(s_i)$ and $c_i(s_i)$ the reward and the cost of the $i$th relay in location $s_i \in S$. We assume that when a set of relays is selected then the total reward (resp. cost) is the sum of the rewards (resp. costs) of the relays belonging to the chosen set. We denote by $a = (a_1, a_2, ..., a_K)$ the vector of selected relays with $a_i = 1$ if relay $i$ is selected and 0 otherwise. Therefore, the total reward and cost at a given state $s$ are given by:

\[ R(s, a) = \sum_{i \in K} r_i(s_i) \mathbb{1}_{\{a_i = 1\}} \]
\[ C(s, a) = \sum_{i \in K} c_i(s_i) \mathbb{1}_{\{a_i = 1\}} \]

The considered reward model includes a large scope of reward metrics. In the following, we give few examples that indicate how the reward model can be applied:

- **Throughput criteria**: different packets are transmitted to the selected relays, thus the total throughput is the sum of the throughput of each selected link. Considering the example of Shannon capacity over a bandwidth of $W$ Hz, the total reward is given by

\[ R(s, a) = W \sum_{i \in K|a_i = 1} \log_2 \left(1 + \min \left\{ \text{SINR}_{\text{UE}}; \frac{\text{SINR}_{\text{Relay}}(s_i)}{\text{SINR}_{\text{BS}}(s_i)} \right\} \right) \]

- **Reliability criteria**: same packets are transmitted to the selected relays gives an error probability $q$ equals to the product of the error probability of each selected link $q_i$. Considering the example of bit error ratio in the case of Quadrature Phase-Shift Keying (QPSK) modulation and Additive White Gaussian Noise (AWGN) channel, we can express the reliability reward as $- \log$ of the error probability in order to have the overall reward as the sum of the reward of each selected relay, $R(s, a) = - \sum_{i \in K|a_i = 1} \log \left( \text{erfc} \left( \sqrt{\frac{\text{SINR}_{\text{Relay}}(s_i)}{\text{SINR}_{\text{UE}}; \frac{\text{SINR}_{\text{Relay}}(s_i)}{\text{SINR}_{\text{BS}}(s_i)}}} \right) \right)$.

The analysis above can be applied to any other error probability function (e.g. $e^{-\text{erf}(s)}$).

- **Coverage probability criteria**: same packets are transmitted to the selected relays, thus the overall outage probability is equal to the product of the outage probability of each selected link. Similarly to reliability criteria, we can express the coverage reward as $- \log$ of the outage probability in order to have the overall reward as the sum of the reward of each selected relay.

In addition, the considered cost model can be applied to a wide range of cost metrics. In the following, we give some examples to illustrate some applications of the cost model.

[2]
• Energy criteria: the total consumed energy is the sum of the energy consumed by each selected relay. Denoting by $P_i(s_i)$ the transmission power of relay $i$ when it is in state $s_i$ gives:

$$C(s, a) = \sum_{i \in K} P_i(s_i)$$

• Incentive criteria: the total charged cost is equal to the sum of the incentive budget required by each selected relay. Denoting by $L_i(s_i)$ the tokens used by relay $i$ in state $s_i$ gives:

$$C(s, a) = \sum_{i \in K} L_i(s_i)$$

# 3 Problem Formulation

Enabling user to network relaying functionality based on D2D communications leads to the following question: which mobile relays should be selected for ensuring an enhancement in cellular communications. This decision depends on the mobility of these potential relays. The main challenge of such relay selection procedure consists of choosing a sequence of mobile relays sets that optimizes a given reward function (3) through a sequence of selection at $1 \leq t \leq T$. For each decision, there exists a trade-off between exploitation and exploration phase that tends to achieve the highest expected reward and exploration phase that tends to get more information concerning the positions of other relays. In the sequel, we describe the CPOMDP formulation of this problem.

## 3.1 Dynamic Relay Selection as CPOMDP

The relay selection procedure consists of choosing a sequence of mobile relays sets that optimizes a given reward function (3) under cost constraints. However, the positions of the potential relays are partially known at the epoch of decision and turn to be more observable as time passes. We model this relay selection problem as a CPOMDP in which a subset of discovered relays should be chosen at each timeslot $t$. Formally, the dynamic relay selection problem is formulated as a finite-horizon CPOMDP which is characterized by the tuple $\langle S, A, T (\cdot), Z, O (\cdot), R, C, b_0, T, C_{th}, \gamma \rangle$ defined below:

• **State** $s = (s_1, s_2, \ldots, s_K)$ denotes the state vector or the location vector of the $K$ potential relays (where $s_i \in S$ for all $1 \leq i \leq K$). $S$ represents the set of all possible state vectors with $|S| = |S|^K$.

• **Action** $a = (a_1, a_2, \ldots, a_K) \in \{0, 1\}^K$ denotes the vector of the $K$ binary actions such that $a_i \in \{0, 1\}$ specifies whether relay $i$ is selected ($a_i = 1$) or not ($a_i = 0$) for all $1 \leq i \leq K$. $\hat{a} = \{i : a_i = 1\}$ represents the set of the selected relays’ indexes. $A$ denotes the set of all possible actions $|A| = \sum_{i=1}^{K} (K_i) = 2^K$ elements.

• **Transition function** $T(s, s') : S \times S \rightarrow [0, 1]$ represents the probability of transiting between states. $T(s, s')$ characterizes the probability of passing to state $s'$ in the next decision epoch knowing that the current state is $s$. Assuming that the relays’ mobility are independent and described by their transition probability matrix, then $T(s, s') = \prod_{i=1}^{K} P_i(s_i, s_i')$.

• **Observation** $z = (z_1, z_2, \ldots, z_K)$ denotes the observation vector of the $K$ potential relays. Selecting a relay leads to the observation of its state $z_i = s_i$, however when relay $i$ is not selected then $z_i = \emptyset$. The set of all observations is denoted by $Z$.

• **Conditional observation probability** $O(z', s', a) : Z \times S \times A \rightarrow [0, 1]$ represents the probability of receiving an observation $z' \in Z$ knowing that the decision action is $a \in A$ and that the state transits to $s' \in S$, we define:

$$O(z', s', a) = O(z' | s', a) = Pr(z_{t+1} = z' | s_{t+1} = s', a_t = a)$$

• **Reward** $R(s, a)$ is the reward achieved by taking action $a$ when the $K$ potential relays are in state $s$. We suppose $R_{min} \leq R(s, a) \leq R_{max} \forall s \in S$ and $a \in A$.

• **Cost** $C(s, a)$ is the cost charged by taking action $a$ when the $K$ potential relays are in state $s$. We suppose $C_{min} \leq C(s, a) \leq C_{max} \forall s \in S$ and $a \in A$.

• **Initial belief** $b_0$ is a vector of $S$ elements that denotes the initial distribution probability of being at each state $s \in S$.

• **Horizon** $T$ of the CPOMDP represents the total number of epochs of the relay selection policy.

• **$C_{th}$** the cost threshold.

• **$\gamma \in [0, 1]$** as a discount factor.

An action is taken as function of the history of observations and actions that have been executed in the past. The result of [23] demonstrates that using the belief states for defining the optimal policy provides as much information as using the entire history of actions taken and observations received. Indeed, there is no need to explicitly save this history but having the current belief state is sufficient for deciding the upcoming actions. Using Bayes rule, all the
The action-value function

\[ V(s, \pi) = \sum_{z \in Z} \gamma^t \mathbb{E}[R(s, a, z) + \gamma \mathbb{E}[R(s', a', z') | s', a'] | z] \]

where \( R(s, a, z) \) is the reward of taking action \( a \) in state \( s \) with observation \( z \), \( R(s', a', z') \) is the reward of taking action \( a' \) in state \( s' \) with observation \( z' \), \( \gamma \) is the discount factor, and \( \mathbb{E} \) represents the expected value over all possible future states and actions.

The belief-based reward \( V(b_t) \) given initial belief \( b_t \) is defined as:

\[ V(b_t) = \sum_{a \in A} \mathbb{E}[R(s, a, z) + \gamma \mathbb{E}[R(s', a', z') | s', a'] | z] \]

where \( b_t = \{ b_t(s_1), b_t(s_2), ..., b_t(s_{|S|}) \} \) is the belief state at epoch \( t \). It describes the relays’ probability of being in all the possible locations (i.e., states). By analogy, we define \( b_{t-1} = \{ b_{t-1}(s_1), b_{t-1}(s_2), ..., b_{t-1}(s_{|S|}) \} \) as the belief state of the mobile relay \( i \) and its corresponding belief vector \( b_i \). Since the relays move independently, the belief state \( b_i \) of relay \( i \) at epoch \( t \) is recursively computed based on the previous belief state \( b_{i,t-1} \), previous action \( a_{i-1} \) and current observation \( z_t \), as follows:

\[ b_i = \begin{cases} b_{i,t-1}P_i & \text{if } a_i = 0 \\ P_i(z_t) & \text{if } a_i = 1 \end{cases} \]

where \( P_i = \{ P_i(S_j) = \{ P_i(S_{j_1}), P_i(S_{j_2}), ..., P_i(S_{j_{|S|}}) \} \} \). A policy \( \pi : b \to a \) is a function that determines the relay selection decision \( a \) to take at each belief state \( b \). For a given initial belief \( b_0 \), a policy \( \pi \) is characterized by a value function \( V^{r, \pi}(b) \) for the reward evaluation and \( V^{c, \pi}(b) \) for the cost evaluation:

\[ V^{r, \pi}(b_0) = \sum_{t=1}^T \gamma^t r_{t}(b_t, \pi(b_t)) = \mathbb{E} \left[ \sum_{t=1}^T \gamma^t R(s_t, a_t) | b_0, \pi \right] \]

\[ V^{c, \pi}(b_0) = \sum_{t=1}^T \gamma^t c_{t}(b_t, \pi(b_t)) = \mathbb{E} \left[ \sum_{t=1}^T \gamma^t C(s_t, a_t) | b_0, \pi \right] \]

The exact solution of the CPOMDP consists of representing the value functions \( V^{r, \pi} \) and \( V^{c, \pi} \) as PWLC functions over the infinite belief simplex \( \Delta \). Therefore, the reward and cost value functions \( V^{r, \pi} \) and \( V^{c, \pi} \) are represented as a finite set \( \mathcal{V} \) of \( \alpha \)-vectors pairs \( (\alpha^r, \alpha^c) \). Considering \( \mathcal{V} \) the set of \( \alpha \)-vectors pairs at epoch \( t \), then the set \( \mathcal{V}_{t+1} \) at the following epoch is constructed by applying the following dynamic programming update over all the action-observation pairs:

1) Compute the immediate reward and cost of taking action \( a \):

\[ \alpha^r \leftarrow \alpha^r_{t+1} + \gamma \sum_{z \in Z} \mathbb{E}[R(s, a, z) | z] \]

\[ \alpha^c \leftarrow \alpha^c_{t+1} + \gamma \sum_{z \in Z} \mathbb{E}[C(s, a, z) | z] \]

2) Generate the sets \( \Gamma^{r,z}_{t+1} \) and \( \Gamma^{c,z}_{t+1} \) for all \( a \in A \), all \( z \in Z \) and all pairs of \( \alpha \)-vectors \( (\alpha^r, \alpha^c) \) in \( \mathcal{V} \):

\[ \Gamma^{r,z}_{t+1} = \alpha^r_{t+1} + \gamma \sum_{s' \in S} T(s, s') O(z, s', a) \]

\[ \Gamma^{c,z}_{t+1} = \alpha^c_{t+1} + \gamma \sum_{s' \in S} T(s, s') O(z, s', a) \]

3) Generate the set of vectors for a given action \( a \) is:

\[ \Gamma^r = \Gamma^{r,z}_{t+1} + \gamma \sum_{z \in Z} \Gamma^{r,z}_{t+1} \]

\[ \Gamma^c = \Gamma^{c,z}_{t+1} + \gamma \sum_{z \in Z} \Gamma^{c,z}_{t+1} \]

4) Deduce the set \( \mathcal{V}_{t+1} \) as follows:

\[ \mathcal{V}_{t+1} = \{ \Gamma^r, \Gamma^c \} \]

At the worst case, each dynamic programming update will generate an exponentially increasing \( |\mathcal{V}_{t+1}| = |\mathcal{V}_t|^2 \) pairs of \( \alpha \)-vectors. However, some pairs of \( \alpha \)-vectors are never the optimal one in any region of the belief simplex (i.e., called useless vectors). Therefore, for mitigating this exponential explosion, different pruning algorithms were used.
developed in order to exclude these useless vectors. For CPOMDP a pruning operation for the value functions were proposed in [25] in order to generate the minimal set of pairs of \( \alpha \)-vectors at each iteration. The pruning function Prune that we can adopt for implementing this solution consists of keeping each pair \((\alpha^r, \alpha^b)\) in \(V_t\) that satisfies the cumulative cost constraint \(\alpha^r \cdot b \leq C_{th}\) and that has the higher cumulative reward \(\alpha^r \cdot b\) in some region of the belief simplex \(\Delta\). This can be decided by solving a Mixed-Integer Linear Program (MILP) for each pair of \(\alpha\)-vector (one can refer to equation (3) in [26]). Eliminating each pair of vectors \((\alpha^r, \alpha_b)\) that violates the cumulative cost constraint at a given iteration \(t\) may lead to a suboptimal policy. Therefore, randomized policies are the subject of further study to achieve the optimal solution of CPOMDP.

As presented in algorithm 1 the iteration \(t\) of the exact solution of CPOMDP follows this procedure: (i) exact dynamic programming to generate the pairs \(\alpha\)-vectors, (ii) pruning operation, based on a mixed integer linear program, that produces the minimal set of \(\alpha\)-vectors, (iii) deducing the optimal value function.

**Algorithm 1 Iteration of CPOMDP Exact Solution**

1. **Input:** \(\alpha\)-vector set \(V_t\), Actions \(A\), States \(S\), Observations \(Z\), Reward function \(R(\cdot, a)\), cost function \(C(\cdot, a)\), Cost threshold \(C_{th}\)
2. for \(a \in A\) do
3. \((\alpha^r, \alpha^b) \leftarrow (R(\cdot, a), C(\cdot, a))\)
4. for \(z \in Z\) do
5. for \((\alpha^r, \alpha^b) \in V_t\) do
6. \(\alpha^r,z(s) = \gamma \sum_{s' \in S} T(s, s') O(z, s', a) \alpha^r(s')\)
7. \(\alpha^c,z(s) = \gamma \sum_{s' \in S} T(s, s') O(z, s', a) \alpha^c(s')\)
8. \(\Gamma^r,z = \Gamma^r \oplus \alpha^r,z\) and \(\Gamma^c,z = \Gamma^c \oplus \alpha^c,z\)
9. end for
10. end for
11. \(V_{t+1} = V_t \cup \text{Prune}(\bigcup_{a} (\Gamma^r, \Gamma^c))\)
12. end for
13. Output: \(V_{t+1}\)

Due to PWLC propriety of the value function, the value iteration algorithm of CPOMDP is limited to find the set of \(\alpha\)-vector pairs \(V_{t+1}\) that represents the value functions \(V_{t+1}\) and \(V_{t+1}\) given the previous set \(V_t\). Constructing the set of hyperplans \(V_{t+1}\) by considering all the possible pairs of observations and actions over the previous set \(V_t\) has an exponential complexity of \(O(|A||V_t|^{|Z|})\) (i.e. considering the states it gives \(O(|S|^2|A||V_t|^{|0|})\)). Since many pairs of vectors in \(V_t\) are dominated by others, pruning algorithms are developed in order to eliminate useless vectors and find the smallest subset sufficient for representing the value functions. However, pruning techniques consists of resolving a MILP for each pair of \(\alpha\)-vector. Therefore, value iteration algorithm for CPOMDP remains computationally demanding to solve as the size of the problem increases. This requires the exploration of approximated solutions for finding the optimal solution of CPOMDP.

### 4 Approximated Relay Selection Policies

Since the value iteration algorithms for POMDP do not scale to highly sized real problems, an approximate POMDP planning solution called Point-Based Value Iteration (PBVI) was introduced in [27]. We recall that a given belief point describes the probability of being in all possible locations \(s \in S\) and the belief simplex corresponds to the set of all possible belief points. Most POMDP problems unlikely reach most of the points in the belief simplex \(\Delta\). Thus, it is preferable to focus the planning on the most probable belief points without considering all the possible belief points as exact algorithms do. Instead of considering the entire belief simplex, PBVI limits the value update to a representative small set of belief points \(B = \{b_0, b_1, ..., b_B\}\). An \(\alpha\)-vector is initialized for each belief point and then the value of this vector is iteratively updated. The PBVI algorithm can be simply adapted to solve CPOMDP problem (e.g. [26]).

#### 4.1 Constrained Point-Based Value Iteration Policy

We call Constrained Point-Based Value Iteration (CPBVI) the proposed suboptimal algorithm of relay selection inspired from PBVI. The corresponding algorithm Algo. 2 consists of modifying Algo. 1 in such a way that the value function update is restrictively done over a finite belief set \(B\) and then the pruning algorithm chooses the dominated pair of vectors for each belief state \(b \in B\). This allows the PBVI algorithm to achieve much better scalability. We respectively denote by \(V_{t}^{r,B}\) and \(V_{t}^{c,B}\) the reward and cost value functions generated by the CPBVI algorithm. At each iteration, CPBVI follows the following steps for computing the set of \(\alpha\)-vectors \(V_{t+1}\) at epoch \(t + 1\) given the previous one \(V_{t}^{B}\):

1. Generate the sets \(\Gamma_{t}^{r,z}\) and \(\Gamma_{t}^{c,z}\) for all \(a \in A\), all \(z \in Z\) and all pairs of \(\alpha\)-vectors \((\alpha^r_b, \alpha^b_c) \in V_{t}^{B}\).
2. Generate the sets \(\Gamma_{t}^{a,z}\) and \(\Gamma_{t}^{c,z}\) for all \(a \in A\):
3. \(\Gamma_{t}^{a,z} = R(\cdot, a) \oplus \bigoplus_{z \in Z} \Gamma_{t}^{r,z} \oplus \bigoplus_{z \in Z} \Gamma_{t}^{c,z}\)

Contrarily to PBVI algorithm used for approximately solving POMDP, we need the cross-summation overall the possible observations to find the sets \(\Gamma_{t}^{a,z}\) and \(\Gamma_{t}^{c,z}\). This cross-summation is mandatory in order to not impose the cost constraint on each action and observation pair while computing the best pair of \(\alpha\)-vector for each belief state \(b\). A local combination complexity \(|A||B||Z|\) is required.

3. Apply pruning operation. In our algorithm we propose to find one optimal pair of \(\alpha\)-vectors \((\alpha^r_b, \alpha^b_c)\) for each belief point \(b \in B\) as follows:

\[
(\alpha^r_b, \alpha^b_c) = \arg\max_{\alpha^r_c, \alpha^c_b \in \Gamma_{t}^{a,z}} \{\alpha^r_c, \alpha^c_b \leq C_{th}\}
\]

Note that the proposed deterministic policy ensures the satisfaction of the cumulative cost constraint \(\Sigma\) at each epoch. Thus, such deterministic policies can be sub-optimal for CPOMDP (by analogy to
CMDP). Ideally, randomized policies that consider convex combination of $\alpha$-vectors during the pruning operation can be applied to guarantee optimality.

4) Finally $V^B_{t+1} = \bigcup_{b \in B} (\alpha_r^b, \alpha_c^b)$.

**Algorithm 2 CPBVI Iteration**

1: **Input:** Pair of $\alpha$-vectors $V^B_t$, Actions $A$, States $S$, Observations $Z$, Rewards $R(s, a)$, costs $C(s, a)$, Belief subset $B$, thresholds $C_{th}$
2: for $a \in A$ do
3: for $z \in Z$ do
4: for $(\alpha_r^a, \alpha_c^a) \in V^B_t$ do
5: $\alpha_r^a(s) = \gamma \sum_{s' \in S} T(s, s') O(s', a, z) \alpha_r^a(s')$
6: $\alpha_c^a(s) = \gamma \sum_{s' \in S} T(s, s') O(s', a, z) \alpha_c^a(s')$
7: $\Gamma_r^c = \Gamma_{r,z}^c + \alpha_r^c$ and $\Gamma_c^r = \Gamma_{c,z}^r + \alpha_c^r$
8: end for
9: end for
10: $V^B_{t+1} = V^B_t + \bigcup (\alpha_r^b, \alpha_c^b)$
11: **Output:** $V^B_{t+1}$

**4.1.1 CPBVI Performance**

Evaluating the performance of the CPBVI algorithm is done by defining the belief set $B'$ which achieves an $\epsilon$-optimal policy (i.e. value function at distance at most $\epsilon$ from the exact solution). A belief set $B$ is characterized by a density $\epsilon^B$ which is the maximum distance from any point in the belief simplex $\Delta$ to the set $B$.

**Definition 4.1.** The density $\epsilon^B$ of a belief set $B$ is defined as:

$$\epsilon^B := \max_{b \in B} \min_{b' \in B} ||b - b'||_1$$

(17)

From theorem 1 of work [27], we know that for a belief set $B$ of density $\epsilon^B$, the errors $\eta_r^h$ and $\eta_c^h$ of the CPBVI algorithm at horizon $h$ have the following upper bounds:

$$\eta_r^h := ||V_r^c - V_r^c||_\infty \leq \frac{(R_{\text{max}} - R_{\text{min}})}{(1 - \gamma)^2} \epsilon^B$$

$$\eta_c^h := ||V_c^r - V_c^r||_\infty \leq \frac{(C_{\text{max}} - C_{\text{min}})}{(1 - \gamma)^2} \epsilon^B$$

(18)

One can remark that this result does not take into account the case where the discount factor is equal to 1. Hence, we extend the result to the case where $\gamma = 1$.

**Proposition 1.** At horizon $h$, the errors $\eta_r^h$ and $\eta_c^h$ of applying the CPBVI algorithm over a belief set $B$ of density $\epsilon^B$ are bounded as follows:

$$\eta_r^h \leq \sum_{t=1}^{h} t \ (R_{\text{max}} - R_{\text{min}}) \epsilon^B = \frac{h(h+1)}{2} (R_{\text{max}} - R_{\text{min}}) \epsilon^B$$

(19)

This result shows how the performance of the CPBVI algorithm depends on the selected belief set $B$. Indeed, these bounds are proportional to the density $\epsilon^B$ of the chosen belief set $B$. For this reason we define the belief set, denoted by $B'$, that limits the upper bounds $\eta_r^h$ and $\eta_c^h$ to a small $\epsilon$. It is clear that a trade-off exists between the size of the belief set and the precision of the value functions $\epsilon$. For constructing the belief set $B'$, we give the following definition.

**Definition 4.2.** We define the $h$-belief set $B(s, h)$ for an initial state $s$ and an horizon $h$ as follows:

$$B(s, h) = \{ \bigcup_{t=1}^{h} T^n (N_t (s), :) \ : n = 1, ..., h - t \}$$

(20)

with $T^n$ the transition matrix $T$ power to $n$ and $N_t (s)$ denotes the set of reachable states after passing $t$ epochs and knowing that the initial state is $s$. The size of a set $|B(s_0, h)|$ is bounded by

$$|B(s_0, h)| \leq \sum_{t=1}^{h} (h - t) N_1 (s_0)^{t-1}.$$

We define the horizon $h$ of the $h$-belief set $B(s, h)$ for which both $\eta_r^h$ and $\eta_c^h$ are bounded by $\epsilon$. To do so, we start by characterizing the density of the $h$-belief set $B(s, h)$.

**Theorem 4.3.** The density $\epsilon^B$ of an $h$-belief set $B(s, h)$ is bounded by:

$$\epsilon^B (B(s, h)) \leq \sum_{i=1}^{K} \frac{\lambda_i^{h}}{\pi_{i,\text{min}}}$$

(21)

where $\lambda_i^{h}$ is the highest eigenvalue of the transition matrix $P_i$ of relay $i$ and $\pi_{i,\text{min}} = \min_{s \in S} \pi_i (s)$ with $\pi_i$ the stationary distribution corresponding to $P_i$.

Proof. Please refer to appendix [A] \qed

We deduce, in the following theorem, the belief set $B'$ that leads to an $\epsilon$-optimal CPBVI policy.

**Theorem 4.4.** For an initial state $s_0$ and a problem of horizon $T$ and discount factor $\gamma < 1$, the belief set $B'$ that should be selected in order to achieve an $\epsilon$ performance (i.e. $\eta_r^{T} \leq \epsilon$ and $\eta_c^{T} \leq \epsilon$) is given by:

$$B' = B(s_0, \min \left( \frac{f_r (\epsilon) \log(\lambda^\gamma)}{\log(\lambda^\gamma)} \right), \frac{f_c (\epsilon) \log(\lambda^\gamma)}{\log(\lambda^\gamma)})$$

(22)

with

$$f_r (\epsilon) = \log \left( \frac{\epsilon \pi_{\text{min}} (1 - \gamma)^2}{2K (R_{\text{max}} - R_{\text{min}})^2} \right); f_c (\epsilon) = \log \left( \frac{\epsilon \pi_{\text{min}} (1 - \gamma)^2}{2K (C_{\text{max}} - C_{\text{min}})^2} \right);$$

$$\lambda^* = \max_{i \in K} \lambda_i^* \text{ and } \pi_{i,\text{min}}^* = \min_{i \in K} \pi_i^*$$

Proof. Please refer to appendix [B] for the proof of the theorem as well as its extension to $\gamma = 1$. \qed
4.1.2 CPBVI Complexity
In order to study the complexity of each iteration of the CPBVI algorithm, we detail the complexity of each step. Step 1 creates $|A||Z||B|$ pair of vectors because the previous set of $\alpha$-vector is limited to $|B|$ components. The cross sum in the second step generates $O\left(|A||B||Z|\right)$ operations. Since the size of the $\alpha$-vector set remains constant (equals to $|B|$), each CPBVI update takes only polynomial time to be executed. This complexity is linear with the number of possible actions $|A|$. However, in our settings, $|A| = 2^K$ leads to a poor scalability of our algorithm in the number of potential relays $K$. Therefore, in subsection 4.2 we propose a greedy version of the CPBVI algorithm.

4.2 Greedy Constrained Point-Based Value Iteration Policy
The computational complexity of each iteration of the CPBVI algorithm is proportional to the size of the possible relay selection decisions $|A| = 2^K$ which increases exponentially with the number of potential relays $K$. Therefore, we propose a greedy CPBVI algorithm that exploits greedy maximization which consists of iteratively choosing the relay that should be selected in the aim of optimizing the value functions of the problem (i.e. without the need of considering all the possible actions $A$). We consider CPBVI algorithm in the previous section in order to enable the possibility of applying greedy maximization (i.e. CPBVI algorithms perform $\arg\max$ operations over a finite set of belief points which is essential for the application of the greedy approach, contrarily to exact methods that compute the value function over all the continuous belief simplex). Indeed, we replace $\arg\max$ in line 14 of algorithm 1 by $\text{greedy} - \arg\max$. The GCPBVI algorithm is basically deduced from the submodularity property of the $Q$-function. Please refer to [28] for more details on submodularity properties.

Submodularity of $Q$-function
We recall that both reward and cost $Q$-functions (respectively given by equations 13 and 14), have the following expression form:

$$Q^r_t(b,a) = \rho(b,a) + \sum_{z \in Z} P(r|z,a) V^r_{t+1}(b'_z)$$

$$= \rho(b,a) + \sum_{k=1}^{T} G^r_k(b',a')$$

where $G^r_k(b',a')$ is the expected immediate value under policy $\pi$ at epoch $k$ conditioned on the belief $b_t$ and the action $a_t$ at epoch $t$. Denoting by $z^{t:k}$ the vector of observations received in the interval $t$ to $k$ epochs gives:

$$G^r_k(b',a') = \gamma^k \sum_{z^{t:k}} P(z^{t:k}|b',a',\pi) \rho(b',a'|\pi)$$

**Theorem 4.5.** For all policies $\pi$, $Q^r, Q^c, Q^{r,\pi} \text{ (b, a)}$ and $Q^{c,\pi} \text{ (b, a)}$ are non-negatives, monotones and submodular in $a$.

**Proof.** Please refer to appendix □

**GCPBVI algorithm** Proving the submodularity of the reward and cost $Q$-functions leads to a greedy version of the CPBVI algorithm, i.e. called GCPBVI and given by algorithm 3. Greedy method has been used in [25] to solve discrete optimization problem of maximizing a submodular function subject to a submodular upper bound constraint, called submodular knapsack. The problem in this work is different (i.e. Markov Decision Process (MDP) and stochastic) and we show that greedy solution can be applied since we prove the submodularity of the reward and cost $Q$-functions. The objective of the GCPBVI algorithm is to avoid the iteration over all the possible actions $A$ of size $2^K$. The iteration of the GCPBVI algorithm shows that we limit the computation on considering each relay $i \in K$ aside without the need of introducing all possible actions $A$ at any level of the algorithm (i.e. avoiding by that the exponential complexity in $K$). Indeed, at each iteration, the GCPBVI follows the steps below for computing the set of $\alpha-$vectors at epoch $t + 1$ given the previous set $V^G_t$:

1) Generate the sets $\Gamma^{k,z_k}_r$ and $\Gamma^{k,z_k}_c$ for all the relays $k \in K$, all possible observations of each relay $z_k \in S$ and all pairs of $\alpha$-vectors $(\alpha^r, \alpha^c) \in V^G_t$:

$$\Gamma^{k,z_k}_r \leftarrow \alpha^{k,z_k}_r(s) = \gamma \sum_{s' \in S} T(s,s') O(z_k,s',k) \alpha^r(s')$$

$$\Gamma^{k,z_k}_c \leftarrow \alpha^{k,z_k}_c(s) = \gamma \sum_{s' \in S} T(s,s') O(z_k,s',c_k) \alpha^c(s')$$

2) Generate the sets $\Gamma^{k}_r$ and $\Gamma^{k}_c$ for each relay $k \in K$:

$$\Gamma^{k}_r = \gamma_t \left( . + \bigoplus_{z_k \in S} \Gamma^{k,z_k}_r \right)$$

$$\Gamma^{k}_c = \gamma_t \left( . + \bigoplus_{z_k \in S} \Gamma^{k,z_k}_c \right)$$

where $r_k = (r_k(s_1), r_k(s_2), ..., r_k(s_S))$ and $c_k = (c_k(s_1), c_k(s_2), ..., c_k(s_S))$. The computation of the sets $\Gamma^{k}_r$ and $\Gamma^{k}_c$ of each relay $k \in K$ has a complexity of $O\left(|S||B||S|\right)$.

3) Find the optimal action to take for each belief state $b \in B$. The greedy $\max$ optimization is applied:

$$\left(\alpha^r_b, \alpha^c_b\right) = \text{greedy} \arg\max_{\alpha_r, \alpha_c} \{\alpha_r, b : \alpha_c, b \leq C_t\}$$

The greedy optimization complexity is limited to $O\left(K^2\right)$.

4) Deduce $V^G_{t+1} = \bigcup_{b \in B} \{\alpha^r_b, \alpha^c_b\}$.

GCPBVI complexity and Performance Evaluation: The complexity of one GCPBVI iteration is equal to $O\left(K^2|S||B||S|\right)$. This algorithm enables much better scalability in the number of potential relays $K$. We respectively denote by $V^r,G$ and $V^c,G$ the reward and cost value function generated by the GCPBVI algorithm.

**Theorem 4.6.** At a given epoch $t$, the error in the reward value function due to greedy optimization is bounded by:

$$V^r,G(b) \geq \left(1 - \frac{1}{e}\right)^{2t} V^r,B(b)$$

**Proof.** Please refer to appendix □

5 Extension to Multi-user Scenario
The system model presented in section 2 is extended to the multi-user scenario where $N$ UEs aim to use D2D-aided relaying in order to improve the performance of their cellular communications. For the multi-user scenario, we generalized the single user notation as follows:
Algorithm 3 GCPBVI Iteration

**Input:** Pair of $\alpha$-vectors $Y_t^G$, Relays $K$, States $S$, Observations $Z$, Rewards $r_i$, Costs $c_i$ and Belief sub-set $B^*$, Threshold $C_{th}$.

**for** $k = 1 : K$ **do**

\[ (a_{k+,k}^b, a_{k}^b) \leftarrow (r_i, c_i) \]

**for** $z_k \in S$ **do**

**for** $(a_{k+1}^b, a_c) \in Y_t^G$ **do**

\[ a_{k+1,c}^b (s) = \sum_{s' \in S} T(s, s') O(z_k, z_k', s') x_{k+1}^b (s') \]

\[ a_{k+1,c}^b (s) = \sum_{s' \in S} T(s, s') O(z_k, z_k', s') x_{k+1}^b (s') \]

\[ \Gamma_{k+1}^{z_k} = \Gamma_{k+1}^{z_k} \cup a_{k+1,c}^b \]

**end for**

$\Gamma = \Gamma_{k+1}^{z_k}$

**end for**

**end for**

**end for**

Output: $Y_{t+1}^G$ and $\Gamma_{t+1}$

- $K_t$ of size $K_t$ elements denotes the set of candidate relays discovered by each UE $i$ with $i \in \{1, ..., N\}$. Thus, the set of potential relays is given by $K = \bigcap_{i=1}^N K_i$ with $|K| = |K_t|$.
- $x_{1,t}^b = (a_1, a_2, ..., a_N)$ denotes the matrix of all the actions $a_1$ taken by each UE $i$ with $i \in \{1, ..., N\}$. Thus, the size of the set of possible actions $|A| = 2^K$.
- $s = (s_1, s_2, ..., s_K)$ denotes the matrix of all the states $s_i$ of each relay $i$ with $i \in \{1, ..., K\}$. Thus, the size of the set of possible states $|S| = S^K$. We denote by $s_{K_i}$ the state vector of the candidate relays of UE $i$.
- The reward and cost model for a given action $\alpha = (a_1, a_2, ..., a_N)$ and a given state $s = (s_1, s_2, ..., s_K)$ is the following:

\[ R(s, \alpha) = \sum_{i=1}^N R_i(s_{K_i}, a_i) \]  

\[ C(s, \alpha) = \sum_{i=1}^N C_i(s_{K_i}, a_i) \]

where $R_i(s_{K_i}, a_i)$ of each UE $i$ is given by equation (26).

For this multi player scenario, the relay selection scheme aims to find the decision policy $(\alpha_1, ..., \alpha_T)$ that optimizes the following problem:

\[
\begin{align*}
\max_{\alpha} & \mathbb{E} \left[ \sum_{t=1}^T \gamma^t R(s_t, \alpha_t) \right] \\
\text{s.t.} & \mathbb{E} \left[ \sum_{t=1}^T \gamma^t C_i(s_t, \alpha_t) \right] \leq C_{th} \forall i \in \{1, ..., N\} 
\end{align*}
\]

**5.1 Centralized Relay Selection Policy**

The first strategy of relay selection is to make the decision by the BS in a centralized manner. Thus, the problem can be modeled, based on the single player scenario given in subsection 5.1, as a CPOMDP with the following characteristics: $s$ represents the state of all the potential relays; matrix $\vec{a}$ represents the taken action (with $\vec{a} (i,j) = 1$ if relay $i$ is selected by UE $j$); $T$ represents the transition matrix with $T(s, s')$ the probability of passing from a state $s$ to another $s'$; reward $R(s, \vec{a})$ and cost $C(s, \vec{a})$ models are respectively given by equations (26) and (27); $b_0$ represents the initial belief of being in each state $s \in S$; $T$ is the horizon of the problem and $C_{th}$ the cost threshold that should not be exceed by any UE. Considering a centralized approach leads to the assumption that the state of each relay $i$ is reported to the BS each time this relay $i$ is selected by one of the UEs. Therefore, the observation $z = (z_1, z_2, ..., z_K)$ corresponding to $z_i = s_i$ if relay $i$ is selected by any UE $(\exists j \in \{1, ..., N\}$ s.t. $\vec{a} (i,j) = 1$) and $z_i = 0$ otherwise $(\vec{a} (i,j) = 0 \forall j \in \{1, ..., N\})$. $O(z', s', \vec{a})$ represents the probability of observing $z'$ knowing that action $\vec{a}$ is taken and that the relays move to state $s'$.

The motivation of such centralized approach lies on the observation process described above. Since the BS is informed by the state of each selected relay, BS will have a global knowledge of all the selected relays’ state contrarily to the local observation of each UE limited by its own decision. Therefore, in this centralized approach, the BS profits from having a global state information for inducing a more efficient relay selection decision.

This CPOMDP is a straightforward generalization of the single user CPOMDP formulation. Thus, the results provided in the previous sections remain intact. This includes that the GCPBVI algorithm is still applicable for the case of multi-user scenario since the submodularity property of the $Q$-functions remains valid. However, this approach suffers from two challenges:

1. **Overhead:** This centralized approach requires a reporting of the state of each selected relay. This procedure generates an overhead of signaling.
2. **Complexity:** The state space for multi-UEs scenario will blowup because it will exponentially increase with the total number $N$ of UEs as well as the total number of candidates relays $K$. Indeed, the total number of states is $|S| = |S|^K$. 
5.2 Distributed Relay Selection Policy

To address the exponentially increasing in the size of the state space for multi-player scenario and to avoid the overhead of signaling, we propose a distributed variant for resolving the multi-UEs CPOMDP. We divide the multi-user problem into $N$ single-user problems (given by (3)), one for each UE, and solve them independently. For such distributed approach, each UE will not take advantage of the observation of other UEs because the states of the selected relays by each UE are not shared between each others. However, distributed designs remain interesting for escaping the large amount of signaling that is required to be exchanged as well as reducing the computational complexity at the BS level.

The distributed approach is equivalent to considering $N$ parallel and independent single UE problems. For each UE $n$ (with $n = \{1,...,N\}$), a relay selection problem is formulated as a CPOMDP as shown in section 3. Then, a relay selection strategy is launched for each UE based on the GCPBVI algorithm proposed in 4.2. Numerical results in section 6 show that this distributed approach reduces the complexity of the problem while achieving performance close to that of the centralized approach.

6 Numerical Results

We evaluate our claims by considering the following example: maximizing the users’ throughput under energy consumption constraints. We consider that the relays are moving within different rectangular regions. An example of partitioning the horizontal and vertical planes into 5 parts (25 different locations) is illustrated in figure 1, with $N = 3$ UEs and $K = 6$ relays. It is obvious that the preciseness and the performance of the results are improved by considering smaller partitioning granularity.

![D2D relaying scenario for numerical results](image)

We call $S_x$ (resp. $S_y$) the number of horizontal (resp. vertical) divisions. Each state $s \in S$ is defined by a $x$ and $y$ coordinates. We denote by $\epsilon_{fix}$ as the probability that the relay stays in the same region. This parameter is essential for defining the mobility pattern that is assumed in this numerical section. The transition matrix in the horizontal plane $P_x$ is constructed in such a way that each relay move left or right with an equal probability of $\frac{1}{2} (1 - \sqrt{\epsilon_{fix}})$. The transition matrix in the vertical plane $P_y$ is constructed in such a way that each relay move up or down with an equal probability of $\frac{1}{2} (1 - \sqrt{\epsilon_{fix}})$. Considering that the horizontal and vertical travels are independent, the transition matrix of each relay $i$ $P_i$ is deduced as follows:

$$P_i([x,y],(x',y')) = P_x(x,x')P_y(y,y')$$

One can remark that this structure of mobility matrix leads to a probability $\epsilon_{fix}$ of staying in the same location. The value of $\epsilon_{fix}$ is given in table 1.

The cost and reward of a relay depend on its position $s = (x,y)$ in the network. Closser the source and the destination nodes are higher is the reward and smaller is the cost of the corresponding source to destination link. In this section, we assume that the throughput is the reward criteria and the energy consumption is the cost metric. The maximum achieved throughput is $R_{max} = 500$ kbps/RB and the maximum transmitted power is $C_{max} = 250$ mW. We consider the following example of reward and cost functions were the reward’s (resp. cost’s) value of a given link is inversely proportional (resp. proportional) to the values of the horizontal and vertical divisions $d_x$ and $d_y$ between the source and destination nodes as follows:

$$r(d_x,d_y) = \frac{R_{max}}{d_x \times d_y}$$

$$c(d_x,d_y) = \frac{C_{max}}{(S_x - x + 1) + (S_y - y + 1)}$$

The cost threshold $C_{th}$ that the average cumulative average cost should not exceed is given in the table 1. Since throughput is the performance criteria, the reward of the UE-BS link passing through relay $i$ is equal to the half of the min of the throughput of both links UE-relay $i$ and relay $i$-BS. The cost of such link is equal to the transmission power of relay $i$. Beside computing the cumulative average reward and cost, we evaluate the average cumulative Energy Efficiency (EE) of the proposed algorithms. The average cumulative Energy Efficiency (EE) metric of UE $n$ is computed as follows:

$$EE_n = \mathbb{E} \left[ \frac{1}{T} \sum_{t=1}^{T} \frac{R_n(s_t,a_t)}{C_n(s_t,a_t)} \right]$$

The numerical settings that will commonly be used in the sequel are summarized in the table 1.

6.1 Single UE scenario

Before evaluating the performance of our relay selection policies, we show the motivation behind these approximations. We start by showing how the suggested algorithms highly reduce the computation complexity of the exact solution of the CPOMDP problem. We consider $S = 25$ possible

| Settings Parameter | Value |
|--------------------|-------|
| Immobile Probability $\epsilon_{fix}$ | 0.7 |
| Max. Reward $R_{max}$ | 500 kbps/RB |
| Max. Cost $C_{max}$ | 250 mW |
| Cost Threshold $C_{th}$ | 1000 mW |
| Reward and Cost Model | Given by (29) and (30) |
| Observation Horizon $T$ | 5 |
| Discount Factor $\gamma$ | 1 |
| Number of realizations | 100 |
| $\epsilon_{fix}^2$ of CPBVI algorithm | 0.01 |

TABLE 1: Numerical Settings
states. For a single UE scenario and \( K = 5 \) relays, we plot in figure 2 the \( \log_{10} \) of the complexity ratio between the proposed CPBVI solution (i.e. given in subsection 4.1) and the exact solution as function of the number of possible regions (i.e. called states). This figure shows that exact solution for our CPOMDP problem of realistic size is unfeasible and that the CPBVI solution highly reduce the complexity compared to the exact one.

![Figure 2: Complexity of CPBVI vs exact solution](image)

In addition, the exponential number of actions \( 2^K \) motivated us to propose a greedy design of the CPBVI algorithm that avoids the study of all the possible actions. We consider \( S = 25 \) possible states and we plot in figure 3 the complexity ratio between the CPBVI solution and its greedy alternative (i.e. given in subsection 4.2) as function of the number of the potential relays \( K \). This figure shows that, even for a small number of potential relays (e.g. \( K = 10 \)) we can reduce the complexity of a factor of 12.

![Figure 3: Complexity of GCPBVI vs CPBVI](image)

We evaluate the performance of the GCPBVI algorithm compared to the CPBVI scheme. Indeed, the comparison is done in terms of the average cumulative reward, cost and EE. The following simple scenario is assumed: single UE with \( K = 2 \) potential relays and \( S_x = S_y = 4 \) (i.e. 16 rectangular regions). Figure 4 verifies that the GCPBVI almost return the same average cumulative reward as the \( \epsilon \)-optimal policy CPBVI. Both solutions have an average cumulative cost that is lower than the cost threshold \( C_{th} \) given in table 1. Therefore, similar average cumulative energy efficiency is deduced for both algorithms.

![Figure 4: Comparison between the performance of following relay selection policies: (i) the CPBVI policy (see 4.1) and (ii) the GCPBVI policy (see 4.2)](image)

We show how implementing the proposed relay selection policy can enhance the throughput of cellular networks. We consider the single UE scenario with \( K = 3 \) relays, \( |S| = 16 \) regions and both scenarios: with and without D2D relaying. The relays’ velocity \( v \) of moving from a region to another is modeled by the transition matrix \( P^v \). For different speeds \( v \), we plot in figure 5 the histogram of the average cumulative reward of both scenarios with and without D2D relaying. In the scenario where D2D is enabled, we apply the proposed GCPBVI based relay selection algorithm. Figure 5 shows that we can gain up to 55% percent in terms of throughput by deploying our policy of relay selection in a cellular network. We note a slow decreasing in the throughput when the speed of the relays increases.
6.2 Multi-UE scenario

We consider the case of multiple UEs in the network. Both centralized and distributed relay selection policies were proposed. We consider $|S| = 16$ locations in the network and we study the complexity as well as the performance of both distributed and centralized relay selection solutions.

In figure 6, the complexity of the centralized solution for multi-UEs scenario (given in 5.1) is compared to that of the distributed relay selection solution (given in 5.2). The choice of developing a low complexity distributed approach for the multi-UEs scenario is illustrated in figure 6. Indeed, the complexity reduction that the distributed approach offers is proportional to both the number of UE $N$ and the number of relays $K$. As an indicative example, the distributed approach reduces the complexity up to 150 times compared to the centralized one in a realistic scenarios of $K = 10$ relays and $N = 10$ users.

The distributed approach reduces the computational complexity while satisfying a performance close to the centralized one. In figure 7, the performance of the distributed approach is compared to the centralized one. Similarly to the single UE case, the comparison is done in terms of the average cumulative reward, cost and EE. A simple scenario with $N = 5$ and $K = 4$ is considered. Figure 4 verifies that the low complexity distributed approach almost return the same average cumulative reward as the centralized one. Moreover, both centralized and distributed solutions verify the required cost constraints for each UE (i.e. average cumulative cost lower than the cost threshold $C_{th}$ given in table 1). In result, similar average cumulative EE is deduced for both algorithms. Indeed, applying distributed relay selection induces interesting performance enhancement of the network with a low computational complexity compared to the centralized approach.

6.2.1 D2D Relaying Performance

We show how implementing the proposed relay selection policy can improve the throughput of cellular networks. We consider the multiple UEs scenario with $N = 5$, $K = 4$ discovered relays and $|S| = 16$ regions. For different speeds of state changing $v$, we plot in figure 6-d the histogram of the average cumulative reward per UE for both scenarios with and without D2D relaying. In the scenario where D2D is enabled, we apply the proposed GCPBVI based relay selection algorithm. The speed $v$ in this figure illustrates the velocity of the relays in moving between the regions. Hence, a speed $v$ is modeled by considering a transition matrix of $P^v$. Each UE applies, in a distributed manner, the relay selection policy proposed in 4.2. Figure 6-d shows that, in
average, a UE can gain up to 30% percent of throughput by deploying our policy of relay selection in a cellular network. We note a slow decreasing in the throughput when the speed of the relays increases.

7 CONCLUSION
Finding the optimal relay selection policy is a challenging problem especially when relays are in mobility. In this paper, we have developed a dynamic relay selection strategy that maximizes a certain performance metric of the cellular networks while guaranteeing some cost constraints. A Constrained Partially Observable Markov Decision Process (CPOMDP) problem has been formulated and its complexity is discussed. Thus, a greedy Greedy Constrained Point-Based Value Iteration (GCPBVI) algorithm is addressed for achieving a low-complexity and close to optimal solution for the problem. Numerical results show the advantage of such approximation in reducing the problem complexity and the performance gain that such solution provides to mobile networks.

APPENDIX A
PROOF OF THEOREM 4.3
The following definition and lemma are used for proving theorem 4.3.

Definition A.1. The distance function of relay i with transition probability \( P_i \) and stationary distribution \( \pi_i \) is defined as follows:

\[
d_i(t) = \max_{s \in S} ||P_i^t(s, \cdot) - \pi_i||
\]

(32)

Lemma A.2. From [29], the distance function of relay i verifies the following properties:

- \( d_i(t) \leq \frac{\lambda_i^*}{\pi_i - \min} \) with \( \lambda_i^* \) the highest eigenvalue of the matrix \( P_i \) and \( \pi_i - \min \) the lowest component of the stationary distribution corresponding to \( P_i \).
- \( d_i(t + a) \leq d_i(t) \) \( \forall a \in \mathbb{N}^+ \)

The density \( \epsilon_B \) of a belief set \( B(s, h) \) is given by:

\[
\epsilon_B(B(s, h)) = \max_{b \in \Delta} \min_{b' \in B(s, h)} ||b - b'||_1
\]

\[
\leq \max_{b \in \Delta} \min_{b' \in B(s, h)} \left[ \sum_{i \in K} \sum_{s \in S} |b_i(s) - b_i'(s)| \right]
\]

For any reachable belief point \( \tilde{b}_i \) of relay i, there exists an integer \( n \) and a state \( S_i \in S \) such that \( \tilde{b}_i = P_i^n(s_i, \cdot) \). The construction of the belief set \( B(s, h) \) induces that \( \tilde{b}_i \) in the equation above can be written as \( \tilde{b}_i = P_i^n(s_i, 0); \cdot \) with the integer \( m \leq h \) and \( s_i(0) \) the initial state of relay i.

We will consider both cases:

- if \( n \leq h \), then the corresponding reachable point \( \tilde{b}_i \) has been taken into account in the belief set \( B(s, h) \), thus:
  \[
  \sum_{s \in S} |b_i(s) - \tilde{b}_i(s)| = 0
  \]
  (33)

- if \( n > h \), the expression \( \sum_{s \in S} |b_i(s) - \tilde{b}_i(s)| \) can be replaced by:
  \[
  \sum_{s \in S} |b_i(s) - \tilde{b}_i(s)| = \sum_{s \in S} |P_i^n(s_i(0), s) - P_i^n(s_i, s) |
  \]
  \[
  \leq \sum_{s \in S} |P_i^n(s_i(0), s) - \pi_i(s) + \sum_{s \in S} |P_i^n(s_i(0), s) - \pi_i(s) |
  \]
  \[
  \leq d_i(m) + d_i(n)
  \]

Based on lemma A.2 and since \( n > h \), then:

\[
\sum_{s \in S} |b_i(s) - \tilde{b}_i(s)| \leq 2d_i(h) \leq 2\frac{\lambda_i^h}{\pi_i,\min} (1 - \gamma)^h
\]

(34)

From equations (33) and (34) and summing over all the relays \( i \in K \) we prove the theorem 4.3.

APPENDIX B
PROOF OF THEOREM 4.4
The studied CPOMDP of horizon \( T \) and an initial state \( s_0 \) achieves an error bound of \( \epsilon \) when \( \eta_T^r \) and \( \eta_T^f \) (see equation (18)) are lower than \( \epsilon \). Limiting the belief set to \( B(s_0, h) \) generates the following errors on the value functions:

\[
\eta_T^r \leq (R_{\max} - R_{\min}) \frac{\sum_{i \in K} 2\lambda_i^h}{\pi_i,\min} \leq 2K \frac{(R_{\max} - R_{\min}) \lambda^h}{\pi_i,\min} (1 - \gamma)^h
\]

(35)

and

\[
\eta_T^f \leq 2K \frac{(C_{\max} - C_{\min}) \lambda^h}{\pi_i,\min} (1 - \gamma)^h
\]

(36)

with \( \lambda^* = \max_{i \in K} \lambda_i^* \) and \( \pi^* = \min_{i \in K} \pi_i,\min \).

Therefore, limiting \( \eta_T^r \) and \( \eta_T^f \) to \( \epsilon \) corresponds to choosing the parameter \( h \) of \( B(s_0, h) \) in such a way that the expressions in equations (35) and (36) are bounded by \( \epsilon \). Hence, theorem 4.4 is deduced.

Note that for a discount factor \( \gamma = 1 \), the sum over the horizon \( T \) gives the following upper bounds of the errors \( \eta_T^r \) and \( \eta_T^f \):

\[
\eta_T^r \leq 2KT \frac{(R_{\max} - R_{\min}) \lambda^h}{\pi_i,\min}
\]

and

\[
\eta_T^f \leq 2KT \frac{(C_{\max} - C_{\min}) \lambda^h}{\pi_i,\min}
\]

Thus, for \( \gamma = 1 \), the expressions of \( f_r(\epsilon) \) and \( f_c(\epsilon) \) in equation (22) of theorem 4.3 are given by:

\[
f_r(\epsilon) = \log \left( \frac{\epsilon \pi_i,\min}{2KT (R_{\max} - R_{\min})} \right)
\]

and

\[
f_c(\epsilon) = \log \left( \frac{\epsilon \pi_i,\min}{2KT (C_{\max} - C_{\min})} \right)
\]
Appendix C

Proof of Theorem 4.5

Please note that we consider the reward Q-function as an example. By analogy, we can deduce the same property for the cost Q-function. In the following, we define the discrete derivative of a relay $e \in K$ knowing that an action set $a_M$ (corresponding to action $a_M$) is taken:

$$\Delta Q^*_e (e|a_M) := Q^*_e (b^t, a_M \cup e) - Q^*_e (b^t, a_M)$$

$$+ \sum_{k=t+1}^{T} \left[ G^*_k (b^t, a_M \cup e) - G^*_k (b^t, a_M) \right]$$

Q^*_e (b, a) is non-negative, monotone and submodular in $a$ if the discrete derivative of the Q-function $\Delta Q^*_e$ verifies the following:

$$\Delta Q^*_e (e|a_M) \geq 0$$

and

$$\Delta Q^*_e (e|a_M) \geq \Delta Q^*_e (e|a_M \cup \hat{a}_N)$$

Considering the reward model $1$, the discrete derivative $\Delta Q^*_e (e|a_M)$ is computed as follows:

$$\Delta Q^*_e (e|a_M) = \rho (b^t, a_M \cup e) - \rho (b, a_M)$$

$$\sum_{k=t+1}^{T} \left[ G^*_k (b^t, a_M \cup e) - G^*_k (b^t, a_M) \right]$$

$$= \sum_{s \in S} b^*_s (s) r_e (s) + \sum_{k=t+1}^{T} \left[ G^*_k (b^t, a_M \cup e) - G^*_k (b^t, a_M) \right]$$

$$= \sum_{s \in S} b^*_s (s) r_e (s)$$

$$= \sum_{s \in S} \gamma^k \sum_{z^{t:k}} \left[ P \left(z^{t:k} | b^t, a_M \cup e, \pi \right) \rho \left(b^*_z, a^\pi \right) \right]$$

$$- \sum_{k=t+1}^{T} \gamma^k \sum_{z^{t:k}} \left[ P \left(z^{t:k} | b^t, a_M \cup e, \pi \right) \rho \left(b^*_z, a^\pi \right) \right]$$

Since the total reward model, given by equation $1$, is equal to the sum of the reward of each selected relay, then the difference $G^*_k (b, a_M \cup e) - G^*_k (b, a_M)$ will be limited to the reward of relay $e$ as follows:

$$\Delta Q^*_e (e|a_M) = \sum_{s \in S} b^*_s (s) r_e (s) + \sum_{k=t+1}^{T} \gamma^k \sum_{z^{t:k}} \left[ P \left(z^{t:k} | b^t, a_M \cup e, \pi \right) \rho \left(b^*_z, a^\pi \right) \right]$$

$$- \sum_{k=t+1}^{T} \gamma^k \sum_{z^{t:k}} \left[ P \left(z^{t:k} | b^t, a_M \cup e, \pi \right) \rho \left(b^*_z, a^\pi \right) \right]$$

where $z^{t:k}$ is the observation of the state of relay $e$ between $t$ and $k$ epochs, $b^*_e$ the belief vector of relay $e$ at epoch $t$, $b^*_e$ the belief vector of relay $e$ after an observation $z^{t:k}$ between $t$ and $k$ epochs. In order to prove the properties $38$ and $39$ and deduce by that the submodularity of the Q-function, we compute $\Delta Q^*_e (e|a_M)$ for the following two cases:

- Relay $e$ has not been chosen by the policy $\pi$ between the epochs $t + 1$ and $T$; in this case, the belief vector $b^*_e$ of relay $e$ (for $n = t + 1, \ldots, T$) is the same in both expressions $G^*_k (b, a_M)$ and $G^*_k (b, a_M \cup e)$. Hence, $G^*_k (b, a_M \cup e) - G^*_k (b, a_M) = 0$ for all $k = t + 1, \ldots, T$.

Thus, in this case the discrete derivative of the Q-function verifies both properties $38$ and $39$.

- Relay $e$ has been chosen by the policy $\pi$ at epoch $t^*$; in this case, the decision maker observes the state of relay $e$: $z^{t^*}$. Thus,

$$\Delta Q^*_e (e|a_M) = \sum_{s \in S} b^*_s (s) r_e (s)$$

$$+ \sum_{k=t+1}^{T} \gamma^k \left[ G^*_k (b, a_M \cup e) - G^*_k (b, a_M) \right]$$

$$= \gamma^{t^*+1} \left[ G^*_k (b, a_M \cup e) - G^*_k (b, a_M) \right]$$

The second term $\sum_{k=t^*+2}^{T} \gamma^k \left[ G^*_k (b, a_M \cup e) - G^*_k (b, a_M) \right] = 0$ since relay $e$ has not been chosen between $t + 1$ and $t^*$ epochs. In addition, the fourth term $\sum_{k=t^*+2}^{T} \gamma^k \left[ G^*_k (b, a_M \cup e) - G^*_k (b, a_M) \right] = 0$ since relay $e$ has been chosen at epoch $t^*$. Therefore, the policy $\pi$ will have the same value for both $G^*_k (b, a_M \cup e)$ and $G^*_k (b, a_M)$. Thus,

$$\Delta Q^*_e (e|a_M) = \sum_{s \in S} b^*_s (s) r_e (s)$$

$$= \gamma^{t^*+1} \left[ G^*_k (b, a_M \cup e) - G^*_k (b, a_M) \right]$$

$$= \gamma^{t^*+1} \left[ \sum_{s \in S} \rho \left(z^{t:k} | b^t, a_M \cup e, \pi \right) \rho \left(b^*_z, a^\pi \right) \right]$$

$$= \gamma^{t^*+1} \left[ \sum_{s \in S} \rho \left(z^{t:k} | b^t, a_M \cup e, \pi \right) \rho \left(b^*_z, a^\pi \right) \right]$$

$$= \gamma^{t^*+1} \left[ \sum_{s \in S} \rho \left(z^{t:k} | b^t, a_M \cup e, \pi \right) \rho \left(b^*_z, a^\pi \right) \right]$$

$$= \gamma^{t^*+1} \left[ \sum_{s \in S} \rho \left(z^{t:k} | b^t, a_M \cup e, \pi \right) \rho \left(b^*_z, a^\pi \right) \right]$$

$$= \gamma^{t^*+1} \left[ \sum_{s \in S} \rho \left(z^{t:k} | b^t, a_M \cup e, \pi \right) \rho \left(b^*_z, a^\pi \right) \right]$$

Thus,

$$\Delta Q^*_e (e|a_M) = \sum_{s \in S} r_e (s) \left[ b^*_s (s) + \gamma^{t^*+1} \left( P^*_{t^*+3} b^*_s (s) - \gamma^{t^*+1} (P^*_{t^*+2} b^*_s (s)) \right) \right]$$

Lemma C.1. For a relay $e$, the two following statements are verified by induction:

- if $P^*_n b^*_s (s) \geq b^*_s (s)$ then $P^*_n b^*_s (s) \geq \left( P^*_{n+1} b^*_s (s) \right) \forall n \in \mathbb{Z} \geq 2$
Based on the lemma \[21\], we deduce that, in this case, the discrete derivative $\Delta r(G|a_M)$ verifies the properties (38) and (39) for the two possible cases studied above; therefore, theorem 4.3 is deduced.

\section*{Appendix D}
\textbf{Proof of theorem 4.6}

The proof is done by induction. We start by verifying theorem 4.5 is deduced.

The proof is done by induction. We start by verifying theorem 4.5 is deduced.

We use the following notations where all the maximization are done under cost constraint:

- $a_{Q^G}^G = \text{greedy} - \arg \max_a Q_t^G(b, a)$
- $a_{Q^G}^G = \text{argmax} Q_t^G(b, a)$
- $a_{Q^G}^B = \text{greedy} - \arg \max_a Q_t^B(b, a)$
- $a_{Q^G}^B = \text{argmax} Q_t^B(b, a)$

Thus,

$V_t^rG(b) = \text{greedy} - \arg \max_a Q_t^G(b, a) = Q_t^rG(b, a_{Q^G}^G)$

Given that $Q_t^G$ function is submodular and constrained to a modular $Q_t^G$, then results shown in \[28\] gives:

$V_t^rG(b) = Q_t^G(b, a_{Q^G}^G) \geq \left(1 - \frac{1}{e}\right) Q_t^G(b, a_{Q^G}^B)$

From the definition of $a_{Q^G}^G$ we know that $Q_t^G(b, a_{Q^G}^G) \geq Q_t^rG(b, a)$ for all $a \in \mathcal{A}$ including $a_{Q^G}^B$. Therefore,

$V_t^rG(b) \geq \left(1 - \frac{1}{e}\right) Q_t^G(b, a_{Q^G}^B)$

For equation \[40\], $Q_t^G(b, a_{Q^G}^B) \geq \left(1 - \frac{1}{e}\right) Q_t^rG(b, a_{Q^G}^B)$. Thus:

$V_t^rG(b) \geq \left(1 - \frac{1}{e}\right) Q_t^rG(b, a_{Q^G}^B) \geq \left(1 - \frac{1}{e}\right) Q_t^rG(b, a_{Q^G}^B)$

Given that $Q_t^rG$ function is submodular and constrained to a modular $Q_t^rG$, then as shown in \[28\]:

$Q_t^rG(b, a_{Q^G}^B) \geq \left(1 - \frac{1}{e}\right) Q_t^rG(b, a_{Q^G}^B)$

We deduce that:

$V_t^rG(b) \geq \left(1 - \frac{1}{e}\right) Q_t^rG(b, a_{Q^G}^B)$

Therefore, theorem 4.6 is deduced by induction.

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