Quantum Communication Complexity: A Survey

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Abstract

This is a survey talk on the topic of Quantum Communication Complexity. A full survey paper by the author is available in Foundations of Physics.

1. Summary

Different resources can be used for the purpose of processing information, and sometimes they can be traded for one another, but this does not always come at a cheap price. In this talk, we shall consider two apparently very dissimilar resources: classical communication and quantum entanglement. The latter, which was derisively dubbed *spukhafte Fernwirkungen* by Einstein, is perhaps the most remarkable nonclassical manifestation of quantum mechanics. It has come to be the soul of most novel phenomena that make Quantum Information Processing so fascinating.

On one hand, we know that entanglement by itself cannot be used to communicate, for otherwise faster-than-light signalling would result. It is therefore natural to assume that under no circumstances could entanglement take the place of classical communication. Nevertheless, there are distributed computational tasks that would require communication in a classical world, yet they can be carried out without any communication whatsoever provided prior entanglement is available. This allows two individuals to react in a co-ordinated manner to their local stimuli in a way that could not be explained classically unless each had instantaneous access to the other’s stimulus. We call this phenomenon *Spooky Communication*—in honour of Einstein—or *Pseudo-Telepathy*.

On the other hand, one must sometimes pay a prohibitive price in order to replace entanglement with classical communication. We exhibit tasks that can be performed with very little entanglement, yet they would require a large amount of classical communication in the absence of entanglement. Moreover, the availability of entanglement can greatly enhance the value of classical communication: some distributed computational tasks can be accomplished at an exponentially smaller classical communication cost in the presence of entanglement, compared to a purely classical scenario. This is known as *Entanglement-Assisted Communication*. Thus we see that there is virtue in making classical and quantum information work together towards common goals.

Can we trade classical communication for quantum entanglement and vice versa? The answer is: sometimes... provided we are willing to pay the price.

In this introductory-level survey talk, we shall not assume any prior knowledge of quantum mechanics, in particular concerning the notion of entanglement. A full written survey on the topic, including Yao’s earlier idea of using a quantum channel to save on the amount of communication required to solve certain distributed computational tasks, is available in Foundations of Physics [1].

Acknowledgements

My modest contribution to this saga would not have been possible without my fruitful collaboration with Richard Cleve and Alain Tapp.

Reference

[1] G. Brassard. Quantum communication complexity. *Foundations of Physics*, 33(11):1593–1616, November 2003.
Preliminary version available online at URL: http://arxiv.org/abs/quant-ph/0101005.