Single-photon nonreciprocal transport in one-dimensional coupled-resonator waveguides

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We study the transport of a single photon in two coupled one-dimensional semi-infinite coupled-resonator waveguides (CRWs), in which both end sides are coupled to a dissipative cavity. We demonstrate that a single photon can transfer from one semi-infinite CRW to the other nonreciprocally. Based on such nonreciprocity, we further construct a three-port single-photon circulator by a T-shaped waveguide, in which three semi-infinite CRWs are pairwise mutually coupled to each other. The single-photon nonreciprocal transport is induced by the breaking of the time-reversal symmetry and the optimal conditions for these phenomena are obtained analytically. The CRWs with broken time-reversal symmetry will open up a kind of quantum devices with versatile applications in quantum networks.

I. INTRODUCTION

The realization of quantum networks, which are composed of many quantum nodes and quantum channels, is one of the main goals in quantum information science [1]. Quantum nodes are used to generate, process and store quantum information. Quantum channels, connected by quantum nodes, are used to transmit quantum states across the entire network. The coupled-resonator waveguide (CRW) with good scalability and integrability [2–5] provides an appropriate platform for studying quantum state transmission in quantum networks. Currently, single-photon transports in one-dimensional CRWs coupled to different kinds of quantum nodes are studied theoretically [6–19], and many important quantum devices are proposed, such as quantum switch [6–14], photon memory [15], single-photon router [16–18], and frequency converter [19]. In all these studies, the quantum devices based on CRWs are reciprocal. However, we know that unidirectional devices such as isolators and circulators are indispensable elements to realize quantum networks. It has already been shown that the breaking of Lorentz reciprocity is pivotal for isolators [20]. One class of nonreciprocal systems which can be used for isolators and circulators is based on the breaking of the time-reversal symmetry.

In general, the breaking of time-reversal symmetry in optical systems can be generated by two different ways: (i) using magneto-optical effects (e.g., Faraday rotation) [21–35] and (ii) non-magnetic strategies (e.g., employing optical non-linearity [36–50], dynamical modulation [51–75], etc). Especially, as a non-magnetic strategy, optical nonreciprocity in the coupled cavity modes with relative phase has drawn more and more attentions in recent years, and many different structures have been proposed theoretically [76–90] or demonstrated experimentally [91–93]. The isolators and circulators, made of three or four coupled cavity modes, can be viewed as a quantum node in the quantum network. However, the input and output fields in these nonreciprocal devices are usually treated by the input-output relations [94]. The detailed construction of the quantum channels coupled to the isolators and circulators are not considered seriously.

In this paper, we study single-photon nonreciprocal transport in CRWs. In Sec. II, we study the transport of a single photon in a waveguide consisting of two coupled semi-infinite CRWs, in which both end sides are coupled to a dissipative cavity, and show that a single photon can transfer from one semi-infinite CRW to the other nonreciprocally due to the breaking of the time-reversal symmetry. In Sec. III, a T-shaped waveguide is proposed by replacing the dissipative cavity discussed in Sec. II with another semi-infinite CRW, and we further demonstrate that the T-shaped waveguide can be used as a three-port single-photon circulator in the broken regime of the time-reversal symmetry. Finally, the main results of the paper are summarized in Sec. VI.

II. SINGLE-PHOTON NONRECIPROCITY IN TWO SEMI-INFINITE CRWS

A. Theoretical model and scattering matrix

As schematically shown in Fig. 1, a one-dimensional waveguide consists of two coupled semi-infinite CRWs, in which both end sides are coupled to a dissipative cavity. Each semi-infinite CRW is made of single-mode cavities, coupled to each other through coherent hopping of photons between neighbouring cavities. The CRWs are quantum channels for single-photon transmission and the three coupled cavities denoted by $a_0$, $b_0$ and $c_0$ are served as a quantum node for single-photon scattering. We assume that the two semi-infinite
CRWs have no dissipation. The Hamiltonian of the system can be described by

$$ H = \sum_{l=a,b,c} H_l + H_{\text{int}}. \quad (1) $$

The Hamiltonians for the two CRWs (quantum channels) are

$$ H_a = \omega_a \sum_{j=0}^{+\infty} a_j^+ a_j - \xi_a \sum_{j=0}^{+\infty} \left( a_{j+1}^+ H.c. \right), \quad (2) $$

$$ H_b = \omega_b \sum_{j=0}^{+\infty} b_j^+ b_j - \xi_b \sum_{j=0}^{+\infty} \left( b_{j+1}^+ H.c. \right), \quad (3) $$

where $l_j$ ($l_j^\dagger$, $l = a, b$) is the bosonic annihilation (creation) operator of the $j$th cavity in the CRW-l with the same resonance frequency $\omega_l$, and the coupling strength between two nearest-neighbor cavities $\xi_l$ is also the same in the CRW-l. The Hamiltonian for the dissipative cavity is

$$ H_c = (\omega_c - i\gamma) c_0^+ c_0, \quad (4) $$

where $c_0$ ($c_0^+$) is the bosonic annihilation (creation) operator of the dissipative cavity with resonance frequency $\omega_c$ and damping rate $\gamma$. The interaction terms in the quantum node is

$$ H_{\text{int}} = J_{ab} e^{i\phi_{ab}} a_0^+ b_0 + e^{-i\phi_{ab}} a_0 b_0^+ + J_{bc} e^{i\phi_{bc}} b_0^+ c_0 + e^{-i\phi_{bc}} b_0 c_0^+ + J_{ca} e^{i\phi_{ca}} c_0^+ a_0 + e^{-i\phi_{ca}} c_0 a_0^+, \quad (5) $$

where $J_{ab} e^{i\phi_{ab}}$, $J_{bc} e^{i\phi_{bc}}$ and $J_{ca} e^{i\phi_{ca}}$ are the coupling constants between the cavities $a_0$, $b_0$ and $c_0$ with real strengths ($J_{ab}$, $J_{bc}$ and $J_{ca}$) and phases ($\phi_{ab}$, $\phi_{bc}$ and $\phi_{ca}$), and only the total phase $\phi = \phi_{ab} + \phi_{bc} + \phi_{ca}$ has physical effects. Without loss of generality, $\phi$ is only kept in the terms of $a_0^+ b_0$ and $a_0 b_0^+$ in Eq. (5) and the following derivation. It should be noted that the time-reversal symmetry of the whole system (without dissipation, i.e., $\gamma = 0$) is broken when we choose the phase $\phi \neq n\pi$ ($n$ is an integer). As we will show in the following, $\phi \neq n\pi$ is one of the crucial conditions to demonstrate single-photon nonreciprocity.

The stationary eigenstate of single-photon scattering in the waveguide is given by

$$ |E\rangle = \sum_{l=a,b} \sum_{j=0}^{+\infty} u_l (j) l_j^+ |0\rangle + u_c (0) c_0^+ |0\rangle \quad (6) $$

where $|0\rangle$ indicates the vacuum state of the whole system, $u_l (j)$ denotes the probability amplitude in the state with a single photon in the $j$th cavity of the CRW-l and $u_c (0)$ denotes the probability amplitude with a single photon in the dissipative cavity. The dispersion relation of the semi-infinite CRW-l is given by [16]

$$ E_l = \omega_l - 2\xi_l \cos k_l, \quad 0 < k_l < \pi, \quad (7) $$

where $E_l$ and $k_l$ are the energy and wave number of the single photon in the CRW-l with the bandwidth $4\xi_l$. Without loss of generality, we assume $\xi_l > 0$ in this paper. It should be pointed out that when the energy $E$ of the incident single photon is out of the energy band of the CRW-l, i.e. $E < \omega_l - 2\xi_l$ or $E > \omega_l + 2\xi_l$, then the wave number $k_l$ becomes a complex number and this single photon can not transport freely in the CRW-l.

Substituting the stationary eigenstate in Eq. (6) and the Hamiltonian in Eq. (1) into the eigenequation $H |E\rangle = E |E\rangle$, we can obtain the coupled equations for the probability amplitudes in the quantum channels as

$$ (\omega_a - E) u_a (0) - \xi_a u_a (1) + J_{ab} e^{i\phi_{ab}} u_b (0) + J_{ca} u_c (0) = 0, \quad (8) $$

$$ (\omega_b - E) u_b (0) - \xi_b u_b (1) + J_{ab} e^{-i\phi_{ab}} u_a (0) + J_{bc} u_c (0) = 0, \quad (9) $$

$$ (\omega_c - E - i\gamma) u_c (0) + J_{ca} u_a (0) + J_{bc} u_b (0) = 0, \quad (10) $$

and the coupled equations for the probability amplitudes in the quantum channels as

$$ \omega_l u_l (j) - \xi_l u_l (j + 1) - \xi_l u_l (j - 1) = E u_l (j) \quad (11) $$

with $j > 0$ and $l = a, b$.

If a single photon with energy $E$ is incident from the infinity side of CRW-l, the interactions among cavities $a_0$, $b_0$ and $c_0$ in the quantum node will result in photon scattering between different CRWs or absorbed by the dissipative cavity. The general expressions of the probability amplitudes in the CRWs ($j \geq 0$) are given by

$$ u_l (j) = e^{-ik_l j} + s_l e^{ik_l j}, \quad (12) $$

$$ u_l' (j) = s_{l'} e^{ik_{l'} j}, \quad (13) $$

where $s_{l'}$ denotes the scattering amplitude from the CRW-l to the CRW-l' ($l', l = a, b$). Substituting Eqs. (12) and (13) into Eqs. (8)-(11) then we obtain the scattering matrix as

$$ S = \begin{pmatrix} s_{aa} & s_{ab} \\ s_{ba} & s_{bb} \end{pmatrix}, \quad (14) $$
where

\[
\begin{align*}
\mathbf{s}_{aa} &= D^{-1} \left[ (J_{ab} e^{-i\phi} + J_{ba,\text{eff}}) (J_{ab} e^{i\phi} + J_{ba,\text{eff}}) \\
&\quad - (\xi_a e^{ik_a} + J_{aa,\text{eff}}) (\xi_b e^{-ik_b} + J_{bb,\text{eff}}) \right], \\
\mathbf{s}_{ab} &= D^{-1} \left[ (\xi_a e^{ik_a} - \xi_a e^{-ik_a}) (J_{ab} e^{-i\phi} + J_{ba,\text{eff}}) \right], \\
\mathbf{s}_{ba} &= D^{-1} \left[ (\xi_b e^{ik_b} - \xi_b e^{-ik_b}) (J_{ab} e^{i\phi} + J_{ba,\text{eff}}) \right], \\
\mathbf{s}_{bb} &= D^{-1} \left[ (J_{ab} e^{-i\phi} + J_{ba,\text{eff}}) (J_{ab} e^{i\phi} + J_{ba,\text{eff}}) \\
&\quad - (\xi_a e^{-ik_a} + J_{aa,\text{eff}}) (\xi_b e^{ik_b} + J_{bb,\text{eff}}) \right], \\
D &= (\xi_a e^{-ik_a} + J_{aa,\text{eff}}) (\xi_b e^{ik_b} + J_{bb,\text{eff}}) \\
&\quad - (J_{ab} e^{-i\phi} + J_{ba,\text{eff}}) (J_{ab} e^{i\phi} + J_{ba,\text{eff}}),
\end{align*}
\]

with the effective coupling strengths \( J_{\text{eff},ba} \) induced by the dissipative cavity defined by

\[
\begin{align*}
J_{ba,\text{eff}} &= \frac{J_{bc} J_{ca}}{E - \omega_c + i\gamma}, \\
J_{aa,\text{eff}} &= \frac{J_{2a}^c}{E - \omega_c + i\gamma}, \\
J_{bb,\text{eff}} &= \frac{J_{2b}^c}{E - \omega_c + i\gamma}.
\end{align*}
\]

By setting the incident flow as unit, we define the scattering flows of the single photons from CRW-\( l \) to the CRW-\( l' \) as the square modulus of the scattering amplitudes \( s_{l'l} \) multiplying the group velocity rates in the corresponding CRWs as [19]

\[
I_{l'l} = |s_{l'l}|^2 \frac{2\pi}{\xi} \sin k_{l'} \xi \sin k_l.
\]

In our model, \( I_{ba} \neq I_{ab} \) represents the appearance of nonreciprocal response, and the perfect nonreciprocity is obtained when \( I_{ba} = 1 \) and \( I_{ab} = 0 \), or \( I_{ba} = 0 \) and \( I_{ab} = 1 \).

**B. Single-photon nonreciprocity**

Let us now study the optimal conditions for single-photon nonreciprocity analytically. For simplicity, we assume that the two CRWs have the same parameters (i.e., \( \omega_a = \omega_b, \xi \equiv \xi_a = \xi_b, \) and \( k \equiv k_a = k_b \)) and they are symmetrically coupled to the dissipative cavity \( (J_{c} \equiv J_{ca} = J_{bc}) \). From Eqs. (16) and (17), we can see that \( s_{ab} \neq s_{ba} \) (i.e., \( I_{ab} \neq I_{ba} \)) in the case \( \phi \neq n\pi \) (\( n \) is an integer). The conditions for perfect nonreciprocity \( (I_{ba} = 1 \) and \( I_{ab} = 0 \), or \( I_{ba} = 0 \) and \( I_{ab} = 1 \)) are obtained from Eqs. (16) and (17) as

\[
J_{ab} = \xi,
\]

\[
k = \begin{cases} 
\phi, & 0 < \phi < \pi, \\
2\pi - \phi, & \pi < \phi < 2\pi,
\end{cases}
\]

\[
\Delta = \xi^{-1} \left( J_{c}^2 - 2\xi^2 \right) \cos \phi,
\]

where \( \Delta \equiv \omega_c - \omega_a = \omega_c - \omega_b \) is the frequency detuning between the dissipative cavity and the cavities in the CRWs. To ensure that \( \phi \) from Eq. (26) is real, we should choose the parameters \( \gamma \xi \leq J_{c}^2 \).

Scattering flows \( I_{ab} \) (black solid curve) and \( I_{ba} \) (red dashed curves) as functions of the wave number \( k/\pi \) for different phases: (a) \( \phi = \pi/2 \), (b) \( \phi = 3\pi/2 \), (d) \( \phi = \pi/3 \) and (e) \( \phi = 5\pi/3 \). Scattering flows \( I_{ab} \) (black solid curves) and \( I_{ba} \) (red dashed curves) as functions of the phase \( \phi/\pi \) for (c) \( k = \pi/2 \) and (f) \( k = \pi/3 \). The other parameters are \( J_c = \gamma = \xi \) for (a)-(c); \( J_c = \sqrt{2} \xi \) and \( \gamma = \sqrt{3} \xi \) for (d)-(f). The other parameters are \( \Delta = 0 \) and \( J_{ba} = \xi \).
In Figs. 3(a)-(c), we show the scattering flows \( I_{ab} \) (black solid curves) and \( I_{ba} \) (red dashed curves) as functions of the wave number \( k/\pi \) when \( J_c \) and \( \gamma \) are taken three different sets of parameters under the condition in Eq. (28). The scattering flows \( I_{ab} \) (black solid curves) and \( I_{ba} \) (red dashed curves) exhibit very different behaviors except the point \( k = \phi \) for given conditions. The ratio \( I_{ab}/I_{ba} \) for scattering flows \( I_{ab} \) and \( I_{ba} \) is shown in Fig. 3(d). In the regime for \( k \neq \phi \) (\( 0 < \phi < \pi \)), \( I_{ab}/I_{ba} \) goes down with the increase of \( J_c \) and \( \gamma \). So the bandwidth of the region of nonreciprocity can be effectively broadened by increasing \( J_c \) and \( \gamma \) [satisfying Eq. (28)] simultaneously.

According to Eq. (27), when \( \Delta \neq 0 \), i.e., the dissipative cavity is not resonant with the cavities in the CRWs, we can still obtain perfect nonreciprocity at the point \( k = \phi \) (\( 0 < \phi < \pi \)) or \( k = 2\pi - \phi \) (\( \pi < \phi < 2\pi \)) when the parameters satisfy

\[
\gamma = \xi^{-1}J_c^2 \sin \phi. \tag{30}
\]

To ensure that \( J_c \) is real, the detuning \( \Delta \) should be in the the regime \( \Delta > -2\xi \cos \phi \) (\( 0 < \phi < \pi \)) or \( \Delta < -2\xi \cos \phi \) (\( \pi < \phi < 2\pi \)). In Figs. 3(e)-(g), we show the scattering flows \( I_{ab} \) (black solid curves) and \( I_{ba} \) (red dashed curves) as functions of the wave number \( k/\pi \) for different detunings \( \Delta \) with phase \( \phi = \pi/3 \), where \( J_c \) and \( \gamma \) are chosen according to Eqs. (29) and (30). The scattering flows \( I_{ab} \) and \( I_{ba} \) exhibit very different behaviors except the point \( k = \phi \) for the different detunings \( \Delta \). The ratio \( I_{ab}/I_{ba} \) of scattering flows \( I_{ab} \) and \( I_{ba} \) is shown in Fig. 3(h). In the regime of \( k \neq \phi \) (\( 0 < \phi < \pi \)), \( I_{ab}/I_{ba} \) also goes down (i.e., the bandwidth of the region of nonreciprocity becomes broader) with the increase of detuning \( \Delta \) (\( J_c \) and \( \gamma \) increase accordingly).

### III. SINGLE-PHOTON CIRCULATOR IN T-SHAPED WAVEGUIDE

#### A. Theoretical model and scattering matrix

Based on the single-photon nonreciprocity discussed in Sec. II, we now study a three-port circulator for single photons in a T-shaped waveguide, which is constructed by replacing the dissipative cavity shown in Fig. 1 with a semi-infinite CRW (quantum channel \( c \)), as schematically shown in Fig. 4. The T-shaped waveguide can be described by the Hamiltonian of Eq. (1) with \( H_c = (\omega_c - i\gamma)\hat{c}_c^\dagger\hat{c}_c \) in Eq. (4) replaced by

\[
H_c = \omega_c \sum_{j=0}^{+\infty} c_j^\dagger c_j - \xi_c \sum_{j=0}^{+\infty} (c_j^\dagger c_{j+1} + \text{H.c.}), \tag{31}
\]
FIG. 5: (Color online) Scattering flows $I_{l'l}(l',l = a, b, c)$ as functions of the wave number $k/\pi$ for different phases: (a)-(c) $\phi = \pi/2$, (d)-(f) $\phi = 2\pi/3$, and (g)-(i) $\phi = 3\pi/2$. The other parameters are $\xi = J_c = J_{ab} = \xi$ and $k = k_c$.

where $c_j (c_j^\dagger)$ is the bosonic annihilation (creation) operator of the $j$th cavity in the CRW-c with the same resonance frequency $\omega_c$ and the same coupling strength $\xi$ between two nearest-neighbor cavities.

The stationary eigenstate of single-photon scattering in the T-shaped waveguide is given by

$$|E\rangle = \sum_{l=a,b,c} \sum_{j=0}^{\infty} u_l(j) l_j^\dagger |0\rangle$$

where $|0\rangle$ is the vacuum state of the T-shaped waveguide, $l_j^\dagger$ denotes the creation operator of the $j$th cavity and $u_l(j)$ denotes the probability amplitude in the state with a single photon in the $j$th cavity of the CRW-$l$. The dispersion relation of the CRW-c is given by [16]

$$E_c = \omega_c - 2\xi_c \cos k_c, \quad 0 < k_c < \pi,$$

where $E_c$ is the energy of the single-photon and $k_c$ is the wave number of the single-photon in the CRW-c. Substituting the stationary eigenstate and the Hamiltonian into the eigenvalue equation $H |E\rangle = E |E\rangle$, we can obtain the coupled equations for the probability amplitudes in the quantum node and quantum channels as in Eqs. (8)-(11) with Eq. (10) replaced by

$$\left(\omega_c - E\right) u_c(0) - \xi_c u_c(1) + J_{bc} u_b(0) + J_{ca} u_a(0) = 0,$$

and the subscript $l = a, b$ in Eq. (11) replaced by $l = a, b, c$.

If a single photon with energy $E$ is incident from infinity side of CRW-$l$, the quantum node with the interactions among cavities $a_0, b_0$ and $c_0$ will result in photon scattering between different quantum channels. The general expressions for the probability amplitudes in the three quantum channels ($l = a, b, c$) are given by ($j \geq 0$)

$$u_l(j) = e^{-ik_lj} + s_{l'l} e^{ik_{l'}j},$$

$$u_{l'}(j) = s_{l'l} e^{ik_{l'}j},$$

where $s_{l'l}$ denotes the scattering amplitude from the CRW-$l$ to the CRW-$l'$ (CRW-$l''$). Substituting Eqs. (35)-(37) into the coupled equations for the probability amplitudes then we obtain the scattering matrix as

$$S = M^{-1} N.$$
the coupled cavities in the T-shaped waveguide have the same equal to zero. For the sake of simplicity, we assume that all the assumptions, the scattering amplitudes are obtained analytically as shown in Appendix A. The optimal conditions for perfect circulator are summarized as follows. If the coupling strengths $J_c, \xi_c$, and $\xi$ are equal, i.e.,

$$J_c = \xi_c = \xi,$$  

(42)

then the perfect circulator is obtained for

$$k = k_c = \begin{cases} \phi & 0 < \phi < \pi \\ 2\pi - \phi & \pi < \phi < 2\pi \end{cases}. $$

(43)

However, if $J_c \neq \xi_c \neq \xi$, then the perfect circulator can only be obtained at

$$k = k_c = \frac{\pi}{2},$$

(44)

$$\phi = \frac{\pi}{2} \text{ or } \frac{3\pi}{2},$$

(45)

with the coupling strengths $J_c, \xi_c$, and $\xi$ satisfying the following condition

$$J_c^2 = \xi_c^2. $$

(46)

This condition is consistent with the condition for single-photon nonreciprocity in the case of two CRWs with a dissipative cavity as given in Eq. (28).

In Fig. 5, the scattering flows $I_{l'l'}(a, o' = a, b, c)$ are plotted as functions of the wave number $k/\pi$ for different phases with equal coupling strengths $\xi_c = J_c = \xi$. As shown in Figs. 5(a)-(f), when $0 < \phi < \pi$, we have $I_{ba} = I_{cb} = I_{ac} = 1$ with the other scattering flows which are equal to zero for the wave number $k = \phi$. As shown in Figs. 5(g)-(i), when $\pi < \phi < 2\pi$, we have $I_{ab} = I_{bc} = I_{ca} = 1$ with the other scattering flows which are equal to zero for the wave number $k = 2\pi - \phi$. In other words, when $0 < \phi < \pi$, the signal is transferred from one CRW to another clockwise ($a \rightarrow b \rightarrow c \rightarrow a$) for $k = \phi$. On the contrary, when $\pi < \phi < 2\pi$, the signal is transferred from one CRW to another counterclockwise ($a \rightarrow c \rightarrow b \rightarrow a$) for $k = 2\pi - \phi$.

When $\xi_c \neq \xi$, the band widths of CRW-$a$ and CRW-$b$ are different from the band width of CRW-$c$, and nonreciprocity ($I_{l'l'} \neq I_{l'l}$) can only be obtained in the overlap band regime among the three CRWs. In this case, we can have perfect circulator ($I_{ba} = I_{cb} = I_{ac} = 1$ with the other zero scattering flows) when $k = \pi/2$ for $0 < \phi < \pi$ or $3\pi/2$. In Fig. 6, the scattering flows $I_{l'l'}(l' = a, b, c)$ are plotted as functions of the wave number $k/\pi$ with different coupling strengths $\xi_c \neq J_c = \xi$. As shown in Figs. 6(a)-(c), when $\xi_c = \xi/2$ and $J_c = \xi/\sqrt{2}$, the scattering flows with $I_{l'l'} \neq I_{l'l}$ appear in the regime $\pi/3 < k_a < k_b < 2\pi/3$ and $0 < k_c < \pi$, and we have $I_{ba} = I_{cb} = I_{ac} = 1$ with all the other scattering flows equal to zero. For the sake of simplicity, we assume that all the coupled cavities in the T-shaped waveguide have the same resonant frequency ($\omega_a = \omega_b = \omega_c$), the CRW-$a$ and CRW-$b$ have the same parameters (i.e. $\xi \equiv \xi_a = \xi_b$, $k \equiv k_a = k_b$) with the coupling strength $J_{ab} = \xi$, and they are symmetrically coupled to CRW-$c$ (i.e. $J_c = J_{bc} = J_{ca}$). On these assumptions, the scattering amplitudes are obtained analytically as shown in Appendix A. The optimal conditions for perfect circulator are summarized as follows. If the coupling strengths $J_c, \xi_c$, and $\xi$ are equal, i.e.,

$$J_c = \xi_c = \xi,$$  

(42)

then the perfect circulator is obtained for

$$k = k_c = \begin{cases} \phi & 0 < \phi < \pi \\ 2\pi - \phi & \pi < \phi < 2\pi \end{cases}. $$

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However, if $J_c \neq \xi_c \neq \xi$, then the perfect circulator can only be obtained at

$$k = k_c = \frac{\pi}{2},$$

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$$\phi = \frac{\pi}{2} \text{ or } \frac{3\pi}{2},$$

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with the coupling strengths $J_c, \xi_c$, and $\xi$ satisfying the following condition

$$J_c^2 = \xi_c^2. $$

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This condition is consistent with the condition for single-photon nonreciprocity in the case of two CRWs with a dissipative cavity as given in Eq. (28).

In Fig. 5, the scattering flows $I_{l'l'}(o, o' = a, b, c)$ are plotted as functions of the wave number $k/\pi$ for different phases with equal coupling strengths $\xi_c = J_c = \xi$. As shown in Figs. 5(a)-(f), when $0 < \phi < \pi$, we have $I_{ba} = I_{cb} = I_{ac} = 1$ with the other scattering flows which are equal to zero for the wave number $k = \phi$. As shown in Figs. 5(g)-(i), when $\pi < \phi < 2\pi$, we have $I_{ab} = I_{bc} = I_{ca} = 1$ with the other scattering flows which are equal to zero for the wave number $k = 2\pi - \phi$. In other words, when $0 < \phi < \pi$, the signal is transferred from one CRW to another clockwise ($a \rightarrow b \rightarrow c \rightarrow a$) for $k = \phi$. On the contrary, when $\pi < \phi < 2\pi$, the signal is transferred from one CRW to another counterclockwise ($a \rightarrow c \rightarrow b \rightarrow a$) for $k = 2\pi - \phi$.

When $\xi_c \neq \xi$, the band widths of CRW-$a$ and CRW-$b$ are different from the band width of CRW-$c$, and nonreciprocity ($I_{l'l'} \neq I_{l'l}$) can only be obtained in the overlap band regime among the three CRWs. In this case, we can have perfect circulator ($I_{ba} = I_{cb} = I_{ac} = 1$ with the other zero scattering flows) when $k = \pi/2$ for $0 < \phi < \pi$ or $3\pi/2$. In Fig. 6, the scattering flows $I_{l'l'}(l' = a, b, c)$ are plotted as functions of the wave number $k/\pi$ with different coupling strengths $\xi_c \neq J_c = \xi$. As shown in Figs. 6(a)-(c), when $\xi_c = \xi/2$ and $J_c = \xi/\sqrt{2}$, the scattering flows with $I_{l'l'} \neq I_{l'l}$ appear in the regime $\pi/3 < k_a < k_b < 2\pi/3$ and $0 < k_c < \pi$, and we have $I_{ba} = I_{cb} = I_{ac} = 1$ with all the other scattering flows equal to zero. For the sake of simplicity, we assume that all the coupled cavities in the T-shaped waveguide have the same resonant frequency ($\omega_a = \omega_b = \omega_c$), the CRW-$a$ and CRW-$b$ have the same parameters (i.e. $\xi \equiv \xi_a = \xi_b$, $k \equiv k_a = k_b$) with the coupling strength $J_{ab} = \xi$, and they are symmetrically coupled to CRW-$c$ (i.e. $J_c = J_{bc} = J_{ca}$). On these assumptions, the scattering amplitudes are obtained analytically as shown in Appendix A. The optimal conditions for perfect circulator are summarized as follows. If the coupling strengths $J_c, \xi_c$, and $\xi$ are equal, i.e.,

$$J_c = \xi_c = \xi,$$  

(42)

then the perfect circulator is obtained for

$$k = k_c = \begin{cases} \phi & 0 < \phi < \pi \\ 2\pi - \phi & \pi < \phi < 2\pi \end{cases}. $$

(43)

However, if $J_c \neq \xi_c \neq \xi$, then the perfect circulator can only be obtained at

$$k = k_c = \frac{\pi}{2},$$

(44)

$$\phi = \frac{\pi}{2} \text{ or } \frac{3\pi}{2},$$

(45)

with the coupling strengths $J_c, \xi_c$, and $\xi$ satisfying the following condition

$$J_c^2 = \xi_c^2. $$

(46)

This condition is consistent with the condition for single-photon nonreciprocity in the case of two CRWs with a dissipative cavity as given in Eq. (28).
In summary, we have shown that a single photon can non-reciprocally transport between two coupled one-dimensional semi-infinite CRWs, in which both end sides are coupled to a dissipative cavity. Moreover, we have shown that a T-shaped waveguide consisting of three pairwise mutually coupled semi-infinite CRWs can be used as a single-photon circulator. The optimal conditions for the single-photon nonreciprocity and the single-photon circulator are given analytically. The CRWs connected by quantum node with broken time-reversal symmetry will open up a kind of quantum devices with versatile applications in the quantum networks.

Finally, let us provide some remarks on the experimental feasibility of our proposal. In our model, the key element is the quantum node of the three coupled cavity modes with relative phase, which have already been realized experimentally with superconducting Josephson junctions [95] and optomechanical (electromechanical) interactions [96] recently. Moreover, the coupled cavity modes also can be replaced by the coupling qubits [97], because they are equivalent at the single-photon level. The quantum channels (CRWs) can be realized by using coupled superconducting transmission line resonators [2], superconducting quantum interference device arrays [3], or defect resonators in two-dimensional photonic crystals [4]. Thus, if the quantum nodes can be connected by quantum channels, then the single-photon nonreciprocity and circulator may be demonstrated in the proposed system, and this may be an important step forward in the realization of the quantum networks.

Note added
After finishing this work, a preprint studying nonreciprocal propagation in linear time-invariant waveguides appears on arXiv [98].

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Appendix A: Scattering probability amplitudes for T-shaped waveguide

On the assumptions, $\omega_a = \omega_b = \omega_c$, $\xi \equiv \xi_a = \xi_b = J_{ab}$, $k \equiv k_a = k_b$, $J_c \equiv J_{bc} = J_{ca}$, the scattering amplitudes can be given analytically as follows,

\begin{align}
    s_{aa} &= -\xi J_c^2 (e^{i\phi} + e^{-i\phi} - e^{-ik} - e^{ik}) D^{-1}, \\
    s_{ba} &= \xi e^{-ikc} (\xi_c e^{-i\phi} - J_c^2 e^{ikc}) (e^{ika} - e^{-ika}) D^{-1}, \\
    s_{ca} &= \xi^2 J_c (e^{-ik} - e^{ik}) (e^{-i\phi} - e^{i\phi}) D^{-1}, \\
    s_{ab} &= \xi e^{-ikc} (\xi_c e^{i\phi} - J_c^2 e^{-ikc}) (e^{ik} - e^{-ik}) D^{-1}, \\
    s_{bb} &= -\xi J_c^2 (e^{i\phi} + e^{-i\phi} - e^{ik} - e^{-ik}) D^{-1}, \\
    s_{cb} &= \xi^2 J_c (e^{-ik} - e^{ik}) (e^{-i\phi} - e^{i\phi}) D^{-1}, \\
    s_{ac} &= \xi J_a \xi_c (e^{-ikc} - e^{ikc}) (e^{i\phi} - e^{-i\phi}) D^{-1}, \\
    s_{bc} &= \xi J_a \xi_c (e^{-ikc} - e^{ikc}) (e^{-i\phi} - e^{i\phi}) D^{-1}, \\
    s_{ec} &= [\xi J_c^2 (2e^{-ik} - e^{i\phi} - e^{-i\phi}) + \xi^2 \xi_c e^{ikc} (1 - e^{-2ik})] D^{-1}, \\
    D &= \xi J_c^2 (e^{i\phi} + e^{-i\phi} - 2e^{-ik}) + \xi^2 \xi_c e^{ikc} (e^{-2ik} - 1).
\end{align}
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