We analyze 26 archival Kepler transits of the exo-Neptune HAT-P-11b, supplemented by ground-based transits observed in the blue (B-band) and near-IR (J-band). Both the planet and host star are smaller than previously believed; our analysis yields $R_p = 4.31 R_{\oplus} \pm 0.06 R_{\oplus}$, and $R_* = 0.683 R_{\odot} \pm 0.009 R_{\odot}$, both about 3$\sigma$ smaller than the discovery values. Our ground-based transit data at wavelengths bracketing the Kepler bandpass serve to check the wavelength dependence of stellar limb darkening, and the J-band transit provides a precise and independent constraint on the transit duration. Both the limb darkening and transit duration from our ground-based data are consistent with the new Kepler values for the system parameters. Our smaller radius for the planet implies that its gaseous envelope can be less extensive than previously applied and a methodology to correct the planetary radius for the presence of both crossed and uncrossed star spots. Star spot crossings are concentrated at phases -0.002 and +0.006. This is consistent with inferences from Rossiter-McLaughlin measurements that the planet transits nearly perpendicular to the stellar equator. We identify the dominant phases of star spot crossings with active latitudes on the star, and we infer that the stellar rotational pole is inclined at about $12^\circ \pm 5^\circ$ to the plane of the sky. We point out that precise transit measurements over long durations could in principle allow us to construct a stellar Butterfly diagram, to probe the cyclic evolution of magnetic activity on this active K-dwarf star.

Subject headings: stars: planetary systems - transits - techniques: photometric

1. INTRODUCTION

The exo-Neptune HAT-P-11b (Bakos et al. 2010, hereafter B10) is prominent among extrasolar planets smaller than Saturn. HAT-P-11b transits a bright star (V=9.59) that lies in the Kepler field (Borucki et al. 2010). Based on its position in a mass-radius diagram (e.g., Figure 14 of B10), HAT-P-11b is likely to have a massive atmosphere. Moreover, B10 found good mass and radius agreement with metal-rich models for the planet (Baraffe et al. 2008), and the B10 spectroscopic analysis of the host star indicated that it was metal-rich. The planet’s atmosphere is therefore likely to exhibit a significant molecular absorption spectrum during transit and/or eclipse. It is a tempting target for future spectroscopic characterization, for example using precise ground-based spectrophotometry (e.g., Bean et al. 2010) in combination with HST (Pont et al. 2009), and/or Warm Spitzer (Desert et al. 2011). Prior to such efforts, it is important to improve our current knowledge of the system parameters and optical planetary radius by examining the Kepler data.

Based on photometric variations of the star, B10 concluded that star spots were common on the stellar photosphere. Rossiter-McLaughlin observations of the system (Winn et al. 2010; Hirano et al. 2011) indicate that the planet’s orbital angular momentum vector is nearly perpendicular to the orbital angular momentum vector of the star. Winn et al. (2010) predicted that this misalignment would produce a characteristic signature in the spot-crossing patterns seen during transit, and this should be quite evident in the Kepler data.

In this paper, we report an analysis of Q0-Q2 archival Kepler data for transits of HAT-P-11b, supplemented with new ground-based transit data at wavelengths bracketing the Kepler bandpass. The potential benefits of ground-based transit photometry as a complement to Kepler have been emphasized by Colon & Ford (2009).

2. OBSERVATIONS AND PHOTOMETRY

2.1. Ground-based Observations
We observed a transit of HAT-P-11b from Kitt Peak on UT date June 1, 2010, at two wavelengths approximately bracketing the Kepler bandpass. For the longer wavelength, we used the 2.1-meter reflector with the FLAMINGOS 2048x2048-pixel infrared imager ([Elston](#)), and a J-band (1.25 µm) filter, at 0.6 arc-sec per pixel scale. Following the conclusion of nightly public programs, we have access to the 0.5-meter telescope at the Kitt Peak Visitor Center (VCT). Simultaneous with the 2.1-meter J-band observations, we observed the same transit at short wavelength using the VCT and a 3072x2048 CCD camera at 0.45 arcsec per pixel. The VCT observations were acquired in an AstroDon B-band filter (390-510 nm). Observations at both telescopes used a defocus to about 10 arc-sec diameter to improve the photometric precision, and both used off-axis guiding to maintain pointing stability. Exposure times were 30-seconds at the 2.1-meter, and 60-seconds at the VCT. All of the optical CCD exposures at the VCT were binned 2x2 to facilitate rapid readout.

Flat-field observations were acquired using either twilight sky (VCT), or a series of night-sky FLAMINGOS exposures including pointing offsets to allow removal of stars via a median filter.

Our HAT-P-11 transits were observed on the 6th night of a 7-night contiguous run on the 2-meter telescope. At the outset of each observing run, we check the FLAMINGOS instrument clock to be certain that it is synced with GPS time signals. Analyzing the transit data from this May-June 2010 run, we have discovered large (~ 300 second) differences, exceeding the random errors, between transit times for other planets measured simultaneously at the VCT vs. the 2-meter. These anomalies occur during the last half of the observing run, and unfortunately the FLAMINGOS instrument clock was not re-checked at the end of the run. Although we have no direct evidence of clock errors, we conservatively regard our observed J-band or B-band transit times as suspect, and we omit these transit timings from the updated ephemeris described in Sec. 6. The photometric shapes of our ground-based transit curves are not affected by this issue.

### 2.2. Photometry on the Ground-based Data

Subsequent to dark current subtraction and division by a flat-field frame, we performed aperture photometry on the target star and comparison stars. The 20-arcmin field of FLAMINGOS provided 6 comparison stars of comparable IR brightness to HAT-P-11. We used a circular aperture 31-pixels in diameter (18.6 arc-sec diameter, 9.3 arc-sec in radius) to measure the stars. We varied the aperture size to optimize the precision in the ratio of HAT-P-11 to the ensemble of comparison stars. We subtracted sky background using an annulus having an inner radius of 18 pixels and an outer radius of 25 pixels. Normalizing HAT-P-11 to the comparison stars yielded a transit light curve with an observed scatter for the unbinned data that varied from 0.0016 before transit to 0.0008 after transit, due to the decreasing airmass during the observations. The latter precision (0.0008) is comparable to the current best ground-based photometry in the J-band (e.g., [Croll et al](#)). We estimated an error for each HAT-P-11 J-band photometric point as the standard deviation of the ratio to the individual comparison stars, divided by the square root of their number (error of the mean). All of our quoted error and precision values are given as linear ratios to the average intensity value, not as magnitudes.

Our photometric aperture for HAT-P-11 nominally excludes a faint companion (2MASS19505049+4805017) lying 10.7 arc-sec North of HAT-P-11. However, because of the defocus, some flux from the companion will fall within the HAT-P-11 aperture. Fortunately, the companion is 6.0 magnitudes fainter than HAT-P-11 at J, and 6.2 magnitudes fainter in the optical (Kepler magnitude). There is no information available as to possible variability of the companion, but we see no temporal anomalies in our ground-based data (nor in the Kepler data for HAT-P-11). The principal known effect of the companion is that the normalizing (out-of-transit) flux for our HAT-P-11 photometry may be overestimated by as much as 1.003. Fortunately, even the maximum contamination will have a negligible effect on our results. Consider an out of transit flux equal to 1 + , where is the contribution of the companion. Let the in-transit flux be 1 + − δ, where δ is the depth of the uncontaminated transit. The mis-normalized in-transit flux is therefore (1 + − δ)/(1 + ) ≈ 1 − δ + . In the case of HAT-P-11 we have δ ≈ 0.004, and possible contamination ≈ 0.003. The second-order term will affect no more than ±1 in the least significant digit of our results, even for our Kepler analysis (see below). We therefore ignore possible contamination from 2MASS19505049+4805017.

After normalizing to the comparison stars, the HAT-P-11 J-band photometry exhibited low-amplitude (0.0005) parabolic curvature in the out of transit baseline. It is probable that this baseline curvature is caused by differences in the effective wavelengths of the J-bandpass as a function of stellar color, in combination with the wavelength-variation in telluric water vapor absorption. This is a familiar effect that we have seen in other J-band photometry ([Sada et al](#)). We removed this baseline curvature using a 2nd-order polynomial fit, masking off the in-transit portion. Because the baseline intensity varies slowly with wavelength, it does not affect the point-to-point scatter in the transit curve. However, baseline uncertainty does represent a potential source of systematic error for the transit depth, as discussed in Sec. 8.

Photometry of the VCT B-band data used similar procedures as for the FLAMINGOS J-band data, except that only two comparison stars were usable, but no quadratic baseline correction was needed. Figure 1 shows our ground-based transit data for both bands, in comparison to transit curves calculated using the analytic relations of [Mandel & Agol](#) with the B10 system parameters, and limb-darkening coefficients from Kurucz model atmospheres (see Sec. 4).

### 3. Kepler Observations

We analyzed public archival data for HAT-P-11 in quarters Q0-Q2. Our analysis uses the short cadence ([Gilliland et al](#)) Pre-Search Data Conditioned (PDC) light curves from the MAST archive. We remove outlying photometric points from the light curves using a 4σ clip applied to the difference between the PDC light curve and a 5-cadence median of that same light curve. The brightness of HAT-P-11 varies slowly due to the rota-
tion of star spots with a 29-day period (B10). In principle, this brightness variation could be exploited to correct for the presence of those spots (Czesla et al. 2009). However, the particular circumstance of the HAT-P-11b transit motivates a better method to implement a star spot correction (explained below). We therefore remove and discard the stellar brightness variations that bracket each transit. We isolate a section of light curve spanning the center of each transit by ±2.3-hours. Masking off the transit, we fit a straight line to the stellar variation over that section, and ratio the transit light curve to that straight line.

We here verify that a straight line adequately represents the stellar rotational variability over the 4.6-hour duration of each transit event. As part of our investigation into the noise properties of the data (see below), we fit straight lines to 4.6-hour sections of the data centered on arbitrary planetary orbital phases where no transit or eclipse occurs. Because any residual curvature could vary from convex to concave, we average the absolute value of the deviations from each fitted straight line. We fit a parabola to the average of those absolute deviations. We repeated this test for 14 different phases in the planetary orbit, avoiding the transit itself. In the worst of the 14 cases, the maximum span of that parabola over 4.6-hours is less than 4 parts-per-million (ppm), and baseline excursions of that magnitude will have negligible impact on our analysis. The fact that the planetary orbit is not phased to the stellar rotation period implies that baseline effects from stellar rotation will be further reduced when stacking multiple transits. We conclude that straight line baselines are adequate for extracting transits of the planet over 4.6-hour sections of the Kepler data. Figure 2 shows an example of a transit with the linear baseline removed, as well as an additional transit illustrated at the stage prior to removal of the linear baseline.

3.1. Noise Properties of the Kepler Data

Our results for HAT-P-11 are heavily dependent on the Kepler data, so it is prudent to investigate the noise properties of these data, especially in the limit where they are averaged to achieve very high photometric precision. By noise properties, we mean not only the low amplitude artifacts inherent in the data themselves (Gilliland et al. 2010), but also the existence of fluctuations caused by stellar activity, such as low amplitude flares (Walkowicz et al. 2011). A conventional method to evaluate the noise properties of photometric data is to measure the standard deviation of binned data as a function of the number of points that are binned. For stationary white noise, we expect that the standard deviation of fluctuations about a mean value will decrease as the inverse square-root of the number of binned points. Standard deviations greater than this scaling law are attributed to so-called red noise (Pont et al. 2006).

To investigate the noise properties of the Kepler data, we performed both a conventional binning test, as well as a more specialized test. Both tests begin by extracting twenty six 4.6-hour sections of the data centered at an arbitrary phase, i.e., not centered on transit or secondary eclipse. We fit and remove a straight line to each of these data sections, and bin the residuals in two ways. First, we bin consecutive points in each section, compute the standard deviation of the binned data, and average the standard deviations over the 26 sections. Second, we bin the data non-consecutively, by combining points from different sections into a common phase bin whose width is approximately one cadence (60-seconds, 0.00015 in phase). Figure 3 shows the results from both binning procedures. The standard deviation for consecutive binning decreases as $N^{-0.34}$, indicating the presence of red noise in the data. However, the standard deviation for non-consecutive binning agrees very precisely with the $N^{-0.5}$ relation, indicating that noise components do not persist at a specific phase from one orbital period to another. Our procedure for combining transits (see below) shifts them to a common phase, i.e., we use non-consecutive binning. Although red noise in the data will increase the scatter for each individual transit, the noise in the phased and averaged transit decreases as $N^{-0.5}$, where $N$ is the number of points in each bin of the combined transit.

3.2. Removal of Transited Star Spots

Our analysis includes 26 transits of HAT-P-11 from the Kepler data and all of them show signs of star spot crossings, albeit less prominent than the Figure 2 example. We have corrected these HAT-P-11b transits for the presence of spots crossed by the planet as well as spots not crossed by the planet. We here describe the correction for crossed spots; the uncrossed spot correction we defer to Sec. 7.

Our correction methodology for crossed spots begins by combining the 26 transits, i.e., transforming them to a common time frame. In so doing, we must minimize interference by the crossed spots to our process of time-shifting the transits. We note that the effect of star spots will be greatly reduced at ingress and egress due to two favorable factors. First, spots during ingress and egress will be foreshortened by the limb-viewing geometry. Second, we expect few spots to occur at ingress and egress due to the nature of star spot distributions. The R-M results (Winn et al. 2010, Hirano et al. 2011) indicate a large angle between the stellar equator and the planetary orbital plane. Hence ingress and egress will probably occur near the poles of the star. We expect few star spots at the stellar poles, by analogy with our Sun, but we acknowledge that polar spots do occur on some active stars (Waite et al. 2011).

To exploit these favorable factors, we use the ingress and egress data to determine the center of temporal symmetry for each transit by constructing bisectors of the transit curve at six intensity levels: 0.9995, 0.9990, 0.9985, 0.9980, 0.9975 & 0.9970. We average those bisectors to obtain the temporal center of symmetry for each transit. We use those temporal centers to compute an updated ephemeris for the planet (Sec. 6). The deviations of individual transits from the new ephemeris is typically about 10-seconds, with no evidence for real timing variations. We therefore use the new ephemeris to combine the transits onto a common time frame. This combined transit is shown in Figure 4, with the abscissa expressed as orbital phase relative to transit center.

Active regions on our Sun are known to concentrate at ‘active latitudes’ (sometimes called ‘preferred latitudes’) located symmetrically about the solar equator. Figure 4 shows that the effect of HAT-P-11b’s star spot crossings
cluster at two orbital phases (-0.002 and 0.006), which we interpret as the planet crossing two active latitudes. This view is consistent with the expected distribution of star spots, as well as the R-M results \cite{Winn et al. 2010, Hirano et al. 2011} that indicate the planet is orbiting nearly perpendicular to the plane of the stellar rotational equator. Under the assumption that the active latitudes are located symmetrically about the stellar rotational equator, we find that the planet crosses the stellar equator at phase 0.002. This implies that the angle between the plane of the sky and the stellar rotational axis is approximately $12^\circ \pm 5^\circ$ degrees, where we estimate the precision from the phase-scatter of the two active latitude crossings.

To remove the effect of the active latitude crossings, we exploit gaps in the longitudinal distribution of the star spots. If spots completely covered the active latitudes on the star, the signatures of spot crossings would appear every time the planet crossed an active latitude. However, by analogy with our Sun, we expect significant gaps between spots on an active latitude. Thus, spot crossings only occur when the planet crosses an active latitude and the longitude of the planet and a star spot coincide. Moreover, Figure 4 implies that active latitudes are broad: the phase (hence, latitude) distribution of crossed spots in a given hemisphere is not single-valued. We therefore proceed to remove the effect of crossed spots as follows. First, we group the Figure 4 data using two bin sizes. During the transit (first to fourth contact), we use a bin width of 0.00015 in phase (1 minute, about the same as the Kepler short cadence time). Keeping the bin width short will eliminate significant distortion of the transit curve shape \cite{Kipping 2010}. Out of transit, we bin the stellar continuum with a bin width of 0.0006 in phase (4.2 minutes).

For each phase bin, we construct a histogram of relative intensities within that bin, and we fit a Gaussian to the peak of that histogram. The spot-corrected intensity for the transit curve at that phase is taken as the centroid of the Gaussian. To ensure that peaks in the wings of the histogram do not perturb the Gaussian fit to the main peak, we do the fitting iteratively. Following a preliminary Gaussian fit, we zero those portions of the histogram lying more than 2.5σ distant from the Gaussian center, then we re-fit to the main peak. The assumption underlying this method is that star spots are only a perturbation to the dominant intensity at a given phase, and that the most common intensity observed at that phase is not affected directly by star spots. If this assumption were not valid, then all methods that do not interpolate over the spot-crossing region from other phases would fail. In that sense our underlying assumption is no more restrictive than nature herself. Some of the low-lying points on Figure 4 could be due to bright active-region morphology, such as plage. Our methodology automatically corrects for plage perturbations as well as spots. Figure 5 shows an example of our histogram prior to zero-ing the wings, with the Gaussian fit to the peak.

Figure 6 shows the result of binning and spot-correcting the stacked transits from Figure 3. The effects of transited star spots are largely eliminated in the Figure 6 transit curve, except for a group of noisy points near phase $-0.0025$, and a single outlying point near phase $+0.0055$. The spot correction is imperfect in the sense that the scatter over much of the in-transit portion is about twice that expected from Kepler’s photometric precision. Nevertheless, this combined transit is much less affected by spot crossings than the individual transit curves. We note that our correction method is fundamentally statistical in nature, and will produce even better results as additional Kepler transits of this system become available.

4. LIMB DARKENING

Prior to fitting the ground-based and Kepler data to derive improved system parameters, it is necessary to consider the effect of stellar limb darkening. In fitting to Kepler data for TrES-2, \cite{Kipping & Bakos 2011} pointed out that adopting limb-darkening coefficients from model atmospheres can potentially make the transit results model-dependent to an undesirable degree. We have therefore used both approaches, adopting limb-darkening coefficients from a Kurucz model atmosphere for HAT-P-11, as well as fitting for them.

B10 inferred stellar parameters for HAT-P-11 of 4500/4.5/0.3 in Teff/log$_{10}$g/[Fe/H]. We used a Kurucz model atmosphere with parameters 4500/4.5/0.3 because the Kurucz grid is tabulated at 0.5-increments in metallicity. We verified, by comparison with a solar metallicity model, that the difference between metallicity $+0.3$ and $+0.5$ will be negligible for limb-darkening. At each of 17 disk positions ($\mu$-values), we weight the model atmosphere intensities by the filter or Kepler bandpass functions, and integrate over wavelength. This defined the limb darkening for each bandpass.

We fit linear and quadratic coefficients for the standard expressions to the results of our model atmosphere integrations \cite{Claret et al. 1995, Sing 2010} has calculated limb darkening coefficients for the Kepler bandpass, and we compared our results to his tabulation. For 4500/4.5/0.5 we find (linear, quadratic) coefficients of (0.6136, 0.1062) vs. (0.6206, 0.1057) for \cite{Sing 2010}. We performed similar calculations for the J- and B-band filter functions. Comparing our quadratic coefficients to \cite{Claret et al. 1995} for the J-band filter and 4500/4.5/0.5, we find reasonable agreement: (0.290, 0.244) for us, and (0.267, 0.255) from \cite{Claret et al. 1995}. For the B-band filter, we find that the Kurucz limb darkening can be well approximated using linear limb darkening, i.e., with the quadratic coefficient set to zero.

We conclude that our calculations of limb-darkening coefficients accurately reflect the output of Kurucz model atmospheres. But whether the actual star conforms to those calculations must be determined by comparison to our transit data.

5. TRANSIT-FITTING METHODOLOGY

5.1. Priors, and Transit Curves

The orbit of HAT-P-11 has non-zero eccentricity ($e = 0.198, B10$). In principle, precise transit light curves may reveal some direct signature of a non-zero eccentricity such as acceleration between ingress and egress. However, detection of the secondary eclipse (e.g., using Spitzer
Transits of HAT-P-11b

photometry) is potentially the most sensitive constraint on the eccentricity of the orbit (Charbonneau et al. 2005; Deming et al. 2003). Spitzer observations of HAT-P-11 spanning the time of secondary eclipse were obtained by R. Barry (Spitzer program 60063), but those data are still under analysis. Indeed, it is possible that the secondary eclipse of this relatively cool planet will prove too weak to be detectable in the Spitzer data. Therefore, for all of our transit fits, we fix the orbital eccentricity and argument of periastron at the values derived by B10, except that we alter \( \omega \) by 180-degrees, as per the difference between \( \omega \) for radial velocity observations (orbit of the star) and \( \omega \) for transits (orbit of the planet).

We compute transit curves using a new version of the Mandel & Agol (2002) algorithms. This new version includes the effects of a non-circular orbit, calculating the sky-projected distance of the planet from the center of the stellar disk by solving the elliptical geometry. We also specifically verified that our code can reproduce the observed radial velocities for this system (B10 & Hirano et al. 2011; Winn et al. 2010). The new code is also faster than previous versions.

5.2. Markov Chain Monte Carlo Code

We fit theoretical transit curves to the transit data using a Markov Chain Monte Carlo (MCMC) algorithm (Ford 2005). We use the Metropolis-Hastings algorithm with Gibbs sampling. We adjust the step size for each variable so as to obtain acceptance rates between 30% and 60%, and we run our chains for \( 10^6 \) samples. Prior to starting each MCMC chain, we re-scale the error bars for the data to ensure that the best reduced \( \chi^2 \) will be close to unity. This helps to insure that the errors calculated from the posterior distributions are realistic. The re-scaling factor was approximately 2 for the binned Kepler data. For the ground-based data, we adopt error bars equal to the observed scatter in the data, so no re-scaling is necessary.

We discard the first 20% of each chain when tabulating the posterior distributions. To verify convergence, we compare the posterior distributions from four chains that have different starting values and slightly different step sizes. Since our code is new, we tested it in several ways. These tests began with simple numerical problems such as fitting to an average of a series of numbers, and fitting to data that scatter around a straight line. Our final test was to generate synthetic transit data by adding noise to theoretical transit curves based on the Mandel & Agol (2002) formulae, and fitting to those synthetic transits to verify that we recover the system parameters that were used to generate the synthetic data.

5.3. Kepler Transit Fits

Ideally, we would like fit to all of the individual transit curves simultaneously, allowing the central transit times to be free parameters in the fit. However, the spot crossings that contaminate individual transits force us to fit to the binned and cleaned transit (Figure 6). One caveat to this procedure is that imperfections in the mutual phasings of the transits could potentially broaden and distort the binned transit. To check our result, we combined the transits two ways. First, we use our new ephemeris (Sec. 6) to shift each transit to a common phase. Second, we use the raw individual central transit times from the bisector analyses to phase the transits. We performed all of our MCMC fits to binned transits constructed using both methods, and found agreement within the random errors. We are confident that phasing errors do not contaminate our results to a significant degree. We report the fit results from the ephemeris phasing, because it employs the constraint that the transit times should be strictly periodic in the absence of planetary perturbations.

Our MCMC fits to the binned and cleaned Kepler data (Figure 6) include 6 variables in the fit: \( a/R_s \), \( R_p/R_s \), orbital inclination, quadratic and linear limb darkening coefficients, and a correction to transit center time. The latter is expected to be zero because the stacking and binning procedures aligned the individual Kepler transits to the common transit-centered time frame. Within the errors, the MCMC fits retrieved a central time correction consistent with zero. We performed a second independent set of MCMC fits to the Kepler data by fixing the limb-darkening coefficients at their model-atmosphere values.

We tabulate best-fit values for the system parameters by averaging over the last 500,000 samples for four independent MCMC chains. As a check on those best-fit values, we implemented an independent \( \chi^2 \) minimization solution using a Levenberg-Marquardt algorithm. Our \( \pm1\sigma \) error limits equal the values where high and low-side tails of the MCMC posterior distributions contained 15.9% of the total samples. The error limits were close to symmetric on each side of the distributions (high- and low-side errors typically agree within 10%). We conservatively adopt the greater value as the symmetric error for each fitted parameter. The best-fit values and errors are listed in Table 1, except for the quadratic limb darkening coefficient that has little impact on our analysis. The best-fit transit curve is plotted over the binned transit data in Figure 6, together with the curve expected from the B10 discovery parameters.

5.4. J- and B-band Transit Fits

The ground-based data give us an opportunity to check parameters such as limb darkening over an extended wavelength range. Moreover, the greatly reduced limb darkening in the J-band results in sharp ingress and egress times, and that sharp definition of the transit duration will prove to be useful, as we discuss in Sec. 8, and as Colon & Ford (2009) predicted. However, because the ground-based data do not have photometric precision comparable to the Kepler data, we find it prudent to restrict the ground-based fits to extract fewer parameters. For the J-band, we set the quadratic limb-darkening coefficient to equal the model atmosphere prediction (0.244), and we set the B-band quadratic coefficient to zero as noted above. Anticipating our results, we find rough agreement between the fits and the model atmosphere limb-darkening predictions. The quadratic portion of the limb-darkening has a minor effect compared to the linear coefficient, especially in B-band where the strong limb darkening is well approximated by the linear law. Note that Southworth (2008) found linear limb darkening to be adequate for the analysis of high quality ground-based transit observations. We judge that we lose little information by our adopted restrictions, but the subsequent restriction in the fitted parameters helps
to increase the usefulness of the ground-based data.

In the case of the the B-band transit data, we are primarily interested in the consistency between the retrieved linear limb-darkening coefficient and the model atmosphere prediction. So in that case, we fix both the orbital inclination and the value of $a/R_*$ to their best-fit Kepler values, and we fit only for the linear limb-darkening coefficient, as well as $R_p/R_*$ and central transit time. As regards the linear limb darkening coefficient, we impose the restriction that the MCMC chains cannot step to values exceeding unity, since those values produce unphysical (negative) disk intensities.

Our first exploratory MCMC chains for the J-band fit showed a strong degeneracy between orbital inclination and $a/R_*$. This is not surprising, since we have previously highlighted this degeneracy for small planets (Sada et al. 2010). In the limit of a small planet transiting a uniform stellar disk, the transit curve approaches an inverse square-wave function where the duration of the transit measures only the total length of the chord across the stellar disk. In that case, the impact parameter (i.e., orbital inclination) and stellar radius can trade-off freely. Hence, in the J-band we fix the orbital inclination at the Kepler value (89.41°, Table 1), and we solve for $a/R_*$.

Results from the J- and B-band fit procedures are included in Table 1, and best-fit transit curves are overplotted on Figure 1.

6. UPDATED TRANSIT EPHEMERIS

A useful by-product of our transit analyses is that we can update the transit ephemeris for this system. We include transits at the two epochs reported by B10, as well as transits from Hirano et al. (2011) and Dittmann et al. (2008), and the Kepler transits. Table 2 gives the central transit times and errors for the Kepler transits, using our bisector method. The precision of the updated ephemeris is dominated by the Kepler transits, that each have a timing precision of order 10 seconds. An error-weighted linear least-squares solution for the ephemeris yields $T_0 = 2454605.89155 \pm 0.00013$, in a barycentric TDB frame (Eastman et al. 2010), and $P = 4.8878018 \pm 1.6 \times 10^{-6}$ days. We consider this to be a provisional update, because many additional values for transit center times will be possible with future Kepler data. Figure 7 shows residuals for the times of individual transits, after removing the best-fit ephemeris. As noted in Sec. 2.1, we omit our 2010 J- and B-band transits from the ephemeris solution, but we include their residuals on Figure 7 where they individually lie off the best-fit ephemeris, but agree with it on average.

7. CORRECTION FOR UNCROSSED STAR SPOTS

In the case where multiple high quality transit light curves are available for a planet that transits at an oblique angle to the stellar equator, we can correct the derived planetary radius for the effect of star spots that are not crossed by the planet during transit. The method we describe here assumes that the distribution of star spots is not correlated with the transits. We argue that this method has advantages over inferences based on the rotational light curve of the star (Czesla et al. 2009), because star spot effects in the rotational light curve can be reduced when multiple spots are distributed uniformly over longitude.

The formalism of our star spot correction has two broad steps. First, we integrate over the path crossed by the planet, and average over all observed transits, to calculate the average flux deficit due to star spots on the path of the planet. The orbit of HAT-P-11b is essentially perpendicular (within the errors) to the stellar equator (Winn et al. 2010; Hirano et al. 2011). We find (Table 1) that the orbital inclination is very close to 90-degrees, and therefore the impact parameter is near zero. Moreover, the stellar active latitudes are not far from the stellar equator. Therefore the flux deficit that we calculate in this first step will be characteristic of regions near the center of the stellar disk. The second broad step will extend the flux deficit calculated over the transit path of the planet, to estimate the total spot coverage over the entire Earth-facing hemisphere of the star.

Figure 8 shows a cartoon of the transit geometry. Near disk center, the planet subtends an approximately constant range of longitude as it transits (indicated by blue meridians of longitude on Figure 8). This approximation of course breaks down near the poles because meridians of longitude converge, but we expect few if any star spots at the poles. Hence, by integrating over the path of the planet we are essentially defining the spot coverage in the range of longitude defined by the angular extent of the planet.

Consider a highly simplified situation where the planet transits a very small star spot present on a star without limb darkening. Let $F_0$ be the flux from the unspotted star, and let $\delta F$ be the stellar flux deficit caused by the small star spot. In transit, before the star spot is crossed, the observed flux $F_{\text{obs}}$ is $F_0 - F_0(R_p^2/R_0^2) - \delta F$, where $R_p$ and $R_0$ are the radii of the planet and star respectively. When the planet crosses the star spot, the $\delta F$ term vanishes, so the flux becomes $F_{\text{obs}} = F_0(1 - R_p^2/R_0^2)$, which is the usual expression for the in-transit flux of a star neglecting limb darkening. The effect of the star spot will be seen in the transit curve as an inverted square wave of amplitude $\delta F$, lasting for a crossing time (measured in phase units) $t_\phi$. (Note that we use orbital phase as a time variable, not an angle.)

A very small star spot would create a simple square-wave type signature in the transit curve, but real star spots are comparable to the size of the planet itself, their intensity varies from the outer penumbral to their umbral core, and they often occur in groups. Therefore their signature in transit light curves can be complex, not a simple square wave. Nevertheless, we can derive the total star spot flux deficit crossed by the planet during a transit, $\delta F_I$, as:

$$\delta F_I = \int \delta F(\phi) d\phi / t_\phi$$

where $\delta F(\phi)$ is the amplitude of the deviation seen in the transit curve at phase $\phi$, and the integral is taken over the path of the planet, literally over the transit curve. We need not explicitly consider the intensity gradient across a star spot, we can derive the total flux deficit from the above integral, independent of star spot morphology. Moreover, we can evaluate the integral directly from the observed transit curves; we use the stacked transit curves (Figure 4). We
also calculate that \( t_0 = 0.785D/vP = 0.000997 \), where \( v \) is the tangential velocity of the planet in its orbit, \( P \) is orbital period, and \( D \) is the planet’s diameter. The factor of 0.785 allows for the fact that the average chord across a circular planet is less than the diameter. The numerical integration of the Figure 4 transits yields \( \delta F_t = 0.001037 \) in flux units where \( F_{\text{obs}} = 1 \). This value applies to the flux deficit over a range of longitude defined by the angular extent of the planet at disk center (between the two blue meridians on Figure 8). Over that longitude range, the total flux deficit due to star spots is (on average), about 0.1%. Because \( 2R_p/R_s \approx 0.118 \), the 0.1% applies to a range of about 0.118 radians. There are 26 such wedges of longitude on the entire Earth-facing hemisphere of the star. Neglecting limb effects, the total star spot flux deficit could be as large as 2.7%. Our second broad step assumes that the average size and abundance of star spots is independent of disk position, but that the projected area - hence the flux deficit - of star spots decreases as \( \cos \theta \), where \( \theta \) is the angular longitude distance from disk center. Our second step will therefore integrate over longitude (\( \theta \)) to obtain the total flux deficit for the entire Earth-facing hemisphere of the star. Thus:

\[
F_0 = F_{\text{obs}} + \int_{-\pi/2}^{\pi/2} \delta F_t \theta_p^{-1} \cos(\theta) d\theta = F_{\text{obs}} + 2\delta F_t/\theta_p
\]

where \( \theta_p \) is the longitude interval covered by the planet during transit. This integration yields \( F_0 = 1.0176F_{\text{obs}} \). So we calculate that the star spots on HAT-P-11 cause the observed stellar flux to be, on average, lower by 1.76% compared to an unspotted star of the same radius and spectral type. The peak-to-peak variations seen in the stellar rotational light curve are about 1.5% in the Kepler data. This is somewhat smaller than 1.76% because the broad distribution of the spots in longitude reduces their signature in the rotational light curve. We note that B10 found a significantly smaller peak-to-peak rotational light curve amplitude (0.6%), but their photometry was obtained 1- to 2-years earlier than our Kepler data.

The methodology described above makes approximations that depend on the fortuitous geometry wherein HAT-P-11b crosses nearly perpendicular to the stellar equator. Moreover, we also retained the approximation of neglecting stellar limb darkening. We believe that a more general formalism could be developed along the same line of reasoning, that could be applied to less strongly inclined planets, and could include realistic limb darkening. That generalization is beyond the scope of this paper, and we will utilize our current estimate of the total spot flux deficit of HAT-P-11 when interpreting our results in the next Section.

8. RESULTS AND DISCUSSION

Our MCMC fits produce excellent agreement with the Kepler data (Figure 6). The retrieved parameters agree closely whether we solve for limb darkening, or fix the coefficients at their Kurucz model atmosphere values. Moreover, the retrieved linear coefficient in the former case \((u = 0.626 \pm 0.014, \text{Table 1})\) is in excellent agreement with the model atmosphere value (0.6179). We conclude that the HAT-P-11 system parameters derived from the Kepler MCMC fits are robust, and not model-dependent via limb darkening. Knutson et al. (2007) reached a similar conclusion in their analysis of HST observations of HD 209458b.

The \( \chi^2 \) value for the best fit solution shown on Figure 6 is 417 for the in-transit points, for 112 degrees of freedom. Thus, the error bars based purely on Kepler photometric precision must be increased by a factor of 2 to account for the imperfect precision of star spot removal. That factor was applied in the MCMC fits, as noted in Sec. 5.2.

Both the J-band and B-band transits imply linear limb darkening coefficients (\( u \) in Table 1) that are in reasonable agreement with model atmosphere values, but they do hint that limb darkening for the real star could vary more strongly with wavelength than the model atmosphere predicts. The MCMC posterior distribution for \( u \) in the B-band (not illustrated) peaks at unity, and values exceeding unity are unphysical. The 1-sided Gaussian distribution with \( \sigma = 0.080 \), so our result (Table 1, \( u = 1.000 \pm 0.080 \)) falls above the model atmosphere value (0.862) by 1.8\( \sigma \). The J-band result (\( u = 0.086 \pm 0.065 \)) similarly falls below the model atmosphere value (0.244) by 2.4\( \sigma \). If discrepancies in model atmosphere predictions for \( u \) do vary with wavelength in this fashion (stronger at short-\( \lambda \), weaker at long-\( \lambda \)), we would not necessarily expect a significant effect in the Kepler band, because it lies intermediate in wavelength between our B- and J-band data. Moreover, any such systematic variation should be confirmed using transits of larger planets, exhibiting deeper transits, where greater precision in derived limb darkening can be achieved. We note that there is observational precedent for limb darkening at short wavelengths to be stronger than model atmosphere predictions (see HJK). We plan additional simultaneous B- and J-band observations, of giant planet transits.

Our results for the J-band transit are inconsistent with the Kepler results as regards the planetary radius. In J-band, we find that \( R_p/R_s = 0.0627 \) vs. 0.0589, and the difference is more than 5 times the precision of the J-band measurement. We regard the Kepler result as definitive, so we consider how this discrepancy can be explained.

One potential explanation of a discrepant radius in J-band is that it reflects a true variation of the planetary radius with wavelength, due to atmospheric opacity. However, the difference here seems implausibly large, and we prefer a more mundane explanation. The transit of HAT-P-11b is relatively long in duration (2.3 hours), and shallow (0.004). Ground-based infrared photometry can be subject to baseline fluctuations caused by telluric water vapor absorption at the edges of the JHK bandpasses. The longer the duration of a transit event, the more sensitive it is to baseline effects, because the adopted baseline has to span a longer interval. Also, a given baseline error will have a greater relative effect for shallow transits. Hence, HAT-P-11 is particularly prone to baseline errors, and we regard the \( R_p/R_s \) value from our J-band MCMC fits as unreliable at the level of accuracy needed for meaningful comparison with Kepler results. We note that the discrepancy would be even larger without the baseline correction that we applied in Sec. 2.2; evidently our correction underestimated the telluric effects.
Although $R_p/R_*$ from our J-band data is questionable compared to the Kepler value, we believe that $a/R_*$ is reliable, and a useful complement to the Kepler results. As noted in Sec. 5.4, the J-band transit is sensitive to the total chord length. Already we have seen (Figure 1), that the duration of the J-band transit predicted from the B10 discovery results (blue curve on Figure 1) does not fit the transit duration seen in the J-band data. Unlike the situation with transit depth, the transit duration observed in J-band is insensitive to the telluric atmosphere. At this wavelength, the ingress and egress are sharp and well defined, and rapid changes of this type are much less likely to be caused by the telluric atmosphere. Fixing the orbital inclination at the Kepler value, our J-band fits give $a/R_*$ = 16.454 ± 0.131, in excellent agreement with the Kepler value of 16.549 ± 0.230. Transit determination of $a/R_*$ can be used to derive the stellar density (Seager & Mallen-Ornelas 2003), for comparison to asteroseismology results. However, the Kepler asteroseismology results for HAT-P-11 were regarded as preliminary by Christensen-Dalsgaard et al. (2010), so the comparison is premature.

We adopt our Kepler-1 solution (see Table 1) for the system parameters, and note that the stellar mass ($0.809 M_\odot$) is well-determined from the Hipparcos parallax and the isochrone fits performed by B10. This yields $R_*=0.683 R_\odot ± 0.009 R_\odot$, a 3.3$\sigma$ revision from the B10 value (0.752$R_\odot ± 0.021 R_\odot$). Using our $R_p/R_*$ = 0.05892 ± 0.00027, yields $R_p = 4.39 R_\odot ± 0.06 R_\odot$, a 2.1$\sigma$ revision from B10 ($R = 4.73 ± 0.16 R_\odot$). However, the presence of unseen star spots will cause the planet to appear larger in the transit solutions by 1.76\%, as discussed in Sec. 7. We thus correct our radius for the planet downward to $R_p = 4.31 R_\odot ± 0.06 R_\odot$. Hence we find that both the planet and star are smaller than the B10 discovery values, at about the 3$\sigma$ level.

We have been able to improve the system parameters of HAT-P-11 over the very thorough analysis by B10, in large part due to the numerous precise Kepler transits. These data have allowed us to remove the effect of star spot crossings, resulting in a deeper transit. Although the deeper transit would tend to produce a larger planet, we also find a smaller star and the decrease in stellar size, combined with our correction for uncrossed star spots, results in a smaller planet compared to B10. Also in comparison to B10, we find a longer transit duration, and a smaller impact parameter since our orbital inclination is closer to 90 degrees.

Our revised radius for HAT-P-11b has noteworthy implications for its internal structure. A mass-radius diagram for planets similar to HAT-P-11b is illustrated by Lissauer et al. (2011) (their Fig. 5). The H-He envelope mass implied by the previous radius of HAT-P-11b is close to 20\%; our new radius reduces the required envelope mass to about 14\%, very close to the envelopes implied for GJ 436b and Kepler-4b.

The radius of HAT-P-11b is now known with sufficient precision to contemplate useful comparisons with other space-borne transit observations, such as HST and Spitzer could acquire. That comparison could potentially reveal an atmospheric signature of HAT-P-11b. Moreover, we point out that long-term monitoring of the system by Kepler, potentially supported by precise ground-based photometry after Kepler has ended, could reveal fundamental information on the magnetic cycle of the star. Because the planet transits nearly perpendicular to stellar active latitudes, monitoring of the number and orbital phase of star spot crossings could allow us to construct a stellar ‘Butterfly diagram’ depicting the cyclic evolution of magnetic eruptions at the stellar photosphere of this active K-dwarf star.

While this paper was under review, two additional analyses of HAT-P-11b based on the Kepler data were announced. Southworth (2011) obtains system parameters that agree with our Table 1. He does not explicitly correct for the effect of star spots, instead treating the spot perturbations as correlated errors. An analysis by Sanchis-Ojeda & Winn (2011) also finds similar system parameters as per our Table 1. These authors correct for crossed spots ‘by hand’, but do not correct the planetary radius for the effect of uncrossed spots. Interestingly, Sanchis-Ojeda & Winn (2011) also discuss a second possible orientation for the system in which the star is viewed nearly pole-on. In that orientation, the two preferred phases of star spot crossings could be caused by one active band of spots encircling the stellar pole. While we cannot rigorously exclude that geometry, we note that our correction for uncrossed spots (Sec. 7) - based on viewing the star as per the Figure 8 geometry - predicts a peak-to-peak variation in the stellar rotational light curve of ≤ 1.76\%, in good agreement with the variability seen in the Kepler data (∼ 1.5\%). That agreement would have to be accidental if the star is being viewed pole-on, so we believe that our Figure 8 geometry is correct, and the stellar pole is nearly in the plane of the sky.

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Fig. 1.— Transit of HAT-P-11 observed in the J-band (upper panel) and B-band (lower panel) from Kitt Peak on June 1, 2010. A quadratic baseline has been removed from the J-band photometry, as noted in the text. The error bars per point are indicated by the inset points; for the J-band observations the error bars decreased after transit due to decreasing air mass. The overplotted blue curves are the nominal transit shapes expected using the system paramaters from [Bakos et al. 2010] and limb-darkening from a Kurucz model atmosphere. Note that the observed duration of transit for the J-band is greater than expected from the blue curve. The red curves are based on our best-fit parameters from Table 1.
Fig. 2.—Upper panel: Detrended and normalized transit of HAT-P-11 observed by Kepler on May 15, 2009. The vertical dashed line marks a prominent effect due to a star spot crossing. Lower panel: the next successive transit observed by Kepler following the May 15 transit, shown before normalization and trend removal. The red line is the variability of the star, taken to be linear over the span of the transit.
Fig. 3.— Noise properties of the HAT-P-11 short cadence data, showing standard deviation (ordinate, in parts-per-million) of binned data versus bin size (abscissa). The open squares are for binning consecutive data points, and lie above the theoretical relation (dashed line) due to correlated red noise of low amplitude. The solid points show noise amplitudes from bins at a given phase, constructed from $N$ 4.6-hour sections of data, versus $N$. These points lie almost exactly on the theoretical relation (dashed line), showing that noise is uncorrelated from one orbit of HAT-P-11 to another.
Fig. 4.— Stack of 26 detrended and normalized transits of HAT-P-11 from the Kepler Q0-Q2 data. Note the tendency for perturbations by star spot crossings to concentrate at orbital phases near $-0.002$ & $0.006$. We attribute this to crossings of active latitudes by the planet in a highly inclined orbit ([Winn et al. 2010; Hirano et al. 2011]).
Fig. 5.— Example of star spot removal at one particular phase in the stacked HAT-P-11 transits of Figure 4. The dashed line is a fit of a Gaussian to the histogram of relative intensities at (in this example) phase $-0.00098$. The center of the Gaussian fit at intensity 0.99567 is the value of the transit curve at this phase with star spot crossings removed.
Fig. 6.— Average Kepler transit curve for HAT-P-11, based on removing star spot crossings and averaging the Figure 4 data. The red line is our best-fit MCMC solution (Kepler-1 line in Table 1). Error bars for these Kepler data are plotted, but are difficult to discern, being comparable to the size of the plot symbols. The square points near phase -0.0025 and the single point at +0.0055 were zero-weighted in the fit. The blue curve is the transit expected using the B10 system parameters, with the Kepler-1 limb darkening coefficients (Table 1).
Fig. 7.— Transit time residuals for HAT-P-11 after removing the best-fit ephemeris described in Sec. 6. Top panel shows all data; the two right-most points are from our ground-based data, and they are not included in the ephemeris solution as noted in Sec. 2.1. The cluster of points near $BJD = 2455000$ are the Kepler transits; they dominate the ephemeris solution, and that time interval is expanded on the lower panel.
Fig. 8.— Cartoon of the HAT-P-11 transit geometry, illustrating that the planet transits nearly perpendicular to the stellar equator, along the sub-Earth longitude of the star. The two blue meridians illustrate that star spot regions crossed by the planet are approximately bounded within a range of longitude (but not to exact scale on this cartoon). For clarity of the Sec. 7 discussion, the path of the planet is here illustrated to be perpendicular to the stellar equator. While a perpendicular path is within the error limits of the R-M results, we note that the sky-projected path is nominally tilted by $103^\circ$ degrees to the stellar equator ([Winn et al.] 2010).
Table 1

| Band and Methodology | $u$   | $i$  | $R_p/R_s$ | $a/R_s$ |
|----------------------|-------|------|-----------|---------|
| Kepler-1             | 0.626 | 89.41| 0.05892   | 16.549  |
| Kepler-2             | 0.6179| 89.58| 0.05865   | 16.574  |
| J-band               | 0.086 | 89.41| 0.06274   | 16.454  |
| B-band               | 1.000 | 89.41| 0.0596    | 16.549  |

Note: Kepler-1 and -2 refer to different treatment of limb darkening. The Kepler-1 line is our preferred solution; it fits to the linear ($u$) and quadratic limb darkening coefficients, whereas the Kepler-2 solution holds both $u$ and the quadratic coefficient fixed at their Kurucz model atmosphere values (see text). Both the J-band and B-band solutions hold the inclination fixed at the Kepler-1 value, and the B-band solution also holds $a/R_s$ fixed at the Kepler-1 value.

Table 2

| N | $T_c$ (BJD-2450000) | Error (1σ) |
|---|---------------------|------------|
| 72| 4957.81318          | 0.00007    |
| 73| 4962.70114          | 0.00010    |
| 74| 4967.58897          | 0.00007    |
| 75| 4972.47667          | 0.00013    |
| 76| 4977.36448          | 0.00007    |
| 77| 4982.25215          | 0.00010    |
| 78| 4987.1406           | 0.00011    |
| 79| 4992.02769          | 0.00008    |
| 80| 4996.91569          | 0.00008    |
| 82| 5006.69137          | 0.00012    |
| 83| 5011.57889          | 0.00015    |
| 85| 5021.35458          | 0.00012    |
| 86| 5026.24279          | 0.00011    |
| 87| 5031.13042          | 0.00014    |
| 89| 5040.90599          | 0.00012    |
| 90| 5045.79376          | 0.00007    |
| 91| 5050.68135          | 0.00007    |
| 92| 5055.56929          | 0.00009    |
| 93| 5060.45709          | 0.00012    |
| 94| 5065.34506          | 0.00011    |
| 95| 5070.23280          | 0.00007    |
| 96| 5075.12061          | 0.00007    |
| 97| 5080.00832          | 0.00007    |
| 98| 5084.89609          | 0.00007    |
| 99| 5089.78421          | 0.00014    |

Note: Times are barycentric TDB (Eastman et al. 2010), and our best-fit ephemeris, with the ground based transits included, is $T_c = 2454605.89155 + 4.8878018N$, as given in Sec. 6.