Optimal method of optimizing risk of portfolio

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Abstract. Diversification is a strategic method that investors use to optimize risk of portfolio. It is an opportunity by which investors move from micro-firm into macro-firm. The investors’ aim is to make an optimal choice that leads to minimization of risk and maximization of return, but the methods that lead to these objectives are not easily achieved. The purpose of this paper is to propose a method to minimize risk of portfolio. Firstly, this paper investigates the risk reduction strength of each asset and secondly, it explores the impact of each asset in minimizing risk of portfolio. The assets allocations divulge by Black Litterman model are used to estimate risk of both portfolios and assets. We explore DataStream (Yahoo finance) of Gold, Oil and Natural gas which spans from January, 2010 to September, 2016. It is observed that investing on Gold minimizes higher risk and achieve more benefits than other assets in the portfolio. In view of these facts, it means diversifying in gold acts as hedge/safe haven for investors during economic crisis.

Keywords. Portfolio, Diversification, Black Litterman, Investment, Asset, Risk, Return

1. Introduction

Diversification is investing in many assets in order to minimize risk or maximize return in the portfolio. It is an opportunity by which investors develop from his small firm into other market products (Badertscher, Shroff, & White, 2013). Study on diversification has caught the attention of many management scholars and is one of the vital areas of study in business. Among others, researchers have studied the antecedents of diversification and the financial performance (Elango, Ma, & Pope, 2008). Investors indeed would explore the benefit of diversification by investing on 10 to 15 securities as suggested by scholars of financial management. The benefit of investing in a large number of securities was visibly established in a more recent study (Fragkiskos et al., 2014)
Diversification is an approach by which firms multiply from its main business into other product markets (Hsu, Chen, & Cheng, 2013). Study reveals that corporate management strongly involves diversification activities and many scholars established this fact. Diversification advances debt capacity, reduce the chances of bankruptcy by introducing new products/markets (Higgins & Schall, 2016) and improves asset placement and productivity. A diversified firm can move funds from a cash surplus unit to a deficit unit without taxes or transaction costs. Diversified firms pool unsystematic risk and reduce the variability of operating cash flow enjoy comparative benefit in hiring because key employees may have a higher sense of job security (Nyaingiri & Ogollah, 2015).

Black Litterman model was developed by Fischer Black and Robert Litterman in Goldman Sachs (1990). It builds on the knowledge of two main theories of modern portfolio theory, the capital asset pricing model (CAPM) and Harry Markowitz mean-variance optimization theory (MPT).

Black Litterman model (BLM) is used in this research work to evaluate the risk and return of portfolio. BLM is a model that determines optimal asset allocation in a portfolio, it provides a clear way to specify investor’s views with prior information, BLM gives a quantitative framework for specifying the investor’s views, and a clear way to combine those investor’s views with an intuitive prior to arrive at a new combined distribution. The sample data is explored from monthly data of Gold, Copper and Oil from Yahoo finance DataStream. Augmented Dickey Fuller (ADF) test is adopted to transform non-stationary time series data to stationary time series data at first difference. The aim of this paper is to investigate the reduction strength of each asset and the effects of each asset in minimizing risk of portfolio. The remaining parts of this paper are organized as follow: section two reviews the literature, section three presents the methodology. Data analysis and results are considered in section four while section five concludes the paper.

2. Literature review

Modern Portfolio theory is a finance theory that attempts to minimize risk of the portfolio and maximize portfolio expected return. Harry Markowitz (1952) was the first to discover the theory of modern portfolio. His discovery was filled with insights and ideas that anticipated many of the subsequent growth in the field. He originated a portfolio problem as a choice of the mean variance portfolio of assets. He observed that risk encountering by investors was portfolio risk which would lead to a basic and important point that the risk of a stock should not only be estimated just by the variance of the stock but also by the covariance. Moreover, he also mentioned that the best (optimal) portfolio should consist of assets that are perfectly negatively correlated. He noted that there are many perfectly positively correlated assets in circulation. This observation gives rise to the theory of diversification (Markowitz, 1952, 1959).

The most important aspect of Markowitz model was his description of the impact on portfolio diversification by the number of securities within a portfolio and their covariance relationships (Mangram, 2013). They used data on sectoral level of employment and value added to generate new and robust evidence that economic growth through stages of diversification and that sectoral concentration follows a U-shaped pattern in relation to per capital income (Imbs & Wacziarg, 2003). It is observed that mean-variance (MV) optimization is still the best theory of portfolio optimization but it is difficult to implement in practice. The asset weights are extremely sensitive to inputs and the inputs are difficult to achieve. Furthermore through MV it is not possible for investors to express their opinions on relative asset performance and their confidence in their selected expected asset returns.

Black and Litterman improved on the original MV model by combining mean-variance optimization of Markowitz and CAPM (Black & Litterman, 1991). The original model was first developed in 1990 and a year later they elaborate on the strategic asset allocation that is embedded with investor’s views in a global sense. The model does not consider the assumption that expected returns are always at equilibrium with CAPM. Rather as expected returns deviate from the mean, imbalances in the markets will attempt to drive them back. Therefore, it is observed that investors...
would make more returns by combining their views about returns with the information in the equilibrium (Black & Litterman, 1991).

Moreover, additional vital feature of the BL structure is that investors should be willing to take risk according to their views and this should be done when they have strong evidence to support their views (Bevan & Winkelmann, 1998). BLM uses Bayesian approach to syndicate the views from the investor with respect to the expected returns of one or more assets with the market equilibrium vector of expected returns to provide a new, mixed estimate of expected returns. The new vector of returns results to intuitive portfolio gives a reasonable portfolio weight (Idzorek, 2005b). Hence, the model produces better stable result than classical mean-variance optimization.

Some researchers have tried to study the asset distribution and model simplification (Beach and Orlov, 2007; Meucci, 2006; Qian and Gorman, 2001; Satchell, 2000). Qian and Gorman (2001) made effort to extend the model by demystifying it; they applied the conditional distribution theory straight to the return vector. The authors amended both the return vector and the covariance matrix. The outcome is the mean vector returns that are the same to those of Black litterman, whereas the conditional covariance matrix is new. It also minimizes the sensitivity of the mean variance optimization to an investor’s volatility estimates. Chiarawongse et al. (2012) introduced new method for quantitative views, taking the form of linear inequalities attached to MV portfolio optimization. The authors evaluated the risk-adjusted measure (expected alpha) conditioned on qualitative views that in turn can be combined with a degree of confidence.

Mankert (2006) criticized all the previous studies on BLM and included behavioural finance in her study. She explained that an investor with home bias would have less or no confidence in the views about foreign assets than domestic assets. This would make the weights of assets closer to the benchmark weights when compared with the weights of the domestic assets. Bertsimas et al (2014) introduced another different method to measure the weight of the weights to the eigenvalues from the prior covariance matrix. Ganikhodjaev and Bayram (2016) criticized the Alternative Reference Model. Actually, it is a combination of opinion and fact that the only valid prior estimate is from statistical model. The authors arguments only based on Alternative reference model which is not relevant to the canonical Reference Model. The focus of their article is on basic statistical properties of time series. Schulmerich et al. (2015) stressed that the generated views may ooze from fundamental analysis, quantitative models or blind belief.

3. Methodology

The methodologies adopted are Mean-Variance Optimization and Black Litternan model.

Consider the following minimization constraint:

\[
\min \frac{1}{2} \sigma_p^2 = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}
\]

where \( w' = (w_1, w_2, ..., w_n) \) and \( \Omega \) is the \((n \times n)\) covariance matrix \(\{\sigma_{ij}\}\). The constraints are

\[
E(r_p) = \sum_{i=1}^{n} w_i E(r_i)
\]

\[
\sum_{i=1}^{n} w_i = 1
\]

For short selling, we use Langrange multipliers; \( \lambda_1 \ldots \lambda_n \) for the constraints

\[
\text{Min } L = \left( \frac{1}{2} \right) \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} - \lambda_1 \left( \sum_{i=1}^{n} w_i E(r_i) - E(r_p) \right) - \lambda_2 \left( \sum_{i=1}^{n} w_i - 1 \right)
\]
\[ L = \left( \frac{1}{2} \right) \left[ \sum w_i^2 \sigma_{11} + 2w_i w_j \sigma_{12} + 2w_i w_k \sigma_{13} + \sum w_i w_j \sigma_{22} + 2w_i w_j \sigma_{23} + w_i w_j \sigma_{33} \right] - \lambda_1 \left[ w_i \text{ER}_i + w_j \text{ER}_j + w_k \text{ER}_k - \text{ER}_p \right] - \lambda_2 \left[ w_i + w_j + w_k - 1 \right] \] 

(5)

Differentiating equation (5) with respect to \( w_i, \lambda_1 \) gives the following first order conditions (FOC)

\[
\frac{\partial L}{\partial w_i} = \left( 2w_i \sigma_{11} + 2w_j \sigma_{12} + 2w_k \sigma_{13} \right) - \lambda_1 \text{ER}_i - \lambda_2 = 0 
\]

(6)

\[
\frac{\partial L}{\partial w_j} = \left( 2w_j \sigma_{22} + 2w_j \sigma_{23} + 2w_j \sigma_{33} \right) - \lambda_1 \text{ER}_j - \lambda_2 = 0 
\]

(7)

\[
\frac{\partial L}{\partial w_k} = \left( 2w_k \sigma_{33} + 2w_k \sigma_{13} + 2w_k \sigma_{23} \right) - \lambda_1 \text{ER}_k - \lambda_2 = 0 
\]

(8)

\[
\frac{\partial L}{\partial \lambda_1} = \sum_{i=1}^{4} w_i \text{ER}_i - \text{ER}_p = 0 
\]

(9)

\[
\frac{\partial L}{\partial \lambda_2} = \sum_{i=1}^{4} w_i - 1 = 0 
\]

(10)

For minimum variance portfolio

From equations (9) and (10) and generalized to \( n \)-assets case can be written as

\[
\sum_{i=1}^{n} w_i \text{ER}_i = \text{ER}_p 
\]

(11)

\[
\sum_{i=1}^{n} w_i = 1 
\]

(12)

where \( \Omega = \sigma_{ij} = \left( n \times n \right) \) covariance matrix, \( \text{ER} \) is \( \left( n \times 1 \right) \), \( \lambda_i \) are scalars, \( w_i \) are weights of assets, \( \sigma_{ii} \) are variances, and \( \sigma_{ij} \) equation (10) arbitrarily set \( \text{ER}_p \) to any fixed value, we have \( \left( n + 2 \right) \) linear equation and \( \left( n + 2 \right) \) unknowns, the \( w_i \) and \( \lambda_i \). These linear equations are easily solved using Microsoft Excel (spread sheet) to give the optimal weights for one point on the minimum variance portfolio. We estimate expected returns \( \text{ER}_i \), standard deviations and covariances \( \sigma_{ij} \). Having obtained the optimal weight \( w_i \left( i = 1, 2, ..., n \right) \) these substituted in \( \sigma_p^2 = w^T \Omega w \) and \( \text{ER}_p = w^T \text{ER} \) to give one point on the efficient frontier.

A portfolio of \( n \) assets is denoted by a vector \( \text{x} \in R^n \) with \( \sum_{i=1}^{n} x_i = 1 \). Let the returns of an asset \( i \) be denoted by \( \text{R}_i \) and expected return of asset \( i \) be \( E(\text{R}_i) \). Then the expected return vector is \( E(\text{R}) = \text{col}(E(\text{R}_i)) \in R^n \), \( (i=1, 2, ..., n) \). The covariance matrix is denoted by \( \sum \in R^{n \times n} \). The covariance of assets \( i \) and \( j \) is given as \( \sigma_{ij} \) (Horasanli & Fidan, 2007). The return \( \text{R}_p \) of portfolio is estimated by
The variance of return of the portfolio can be computed as:

$$\sigma_p^2 = \sum_{i=1}^{n} \sigma_i^2 \left( \sum_{i=1}^{n} x_i \right)$$  \hfill (13)

The expected return of equilibrium portfolio as:

$$\Pi = \delta \sum x_{ndr}$$  \hfill (15)

where $\Pi$ is the expected return of market equilibrium, $\delta$ is the risk aversion, $x_{ndr}$ is the market weight.

The improvement in the BLM allows the investors to combine their views directly in the model in an intuitive way. The views can be relative. The views have to be in the same format with constraints. The investors should be able to fix a level of confidence in his views. This requirement may be as follows:

$$P, E(\Omega) = Q + \epsilon$$  \hfill (16)

where $P$ is the vector that describes the assets concerned by the views, $Q$ is the vector of their performances and $\epsilon$ is the random normal vector of error terms, $\epsilon \sim N(0, \Omega)$ with diagonal variance matrix $\Omega$. It is assumed that the market is rotating around an equilibrium point and the same with investors’ portfolio in respect to CAPM hypothesis (Hidalgo & Desportes, 2014).

Let the mean $E(\Omega) = \Pi$, the covariance, assumed to be proportional to $\Sigma$, with factor of uncertainty $\tau$, $E(\Omega) \sim N(\Pi, \tau \Sigma)$.

The equation below is known as the Black Litterman equation and represents the expected return vectors that is produced from a Bayesian mixing of the implied equilibrium excess return vector ($\Pi$) and the vector of investor views ($Q$).

$$E(\Omega) = [(\tau \Sigma)^{-1} + \Pi \Omega^{-1} P]^{-1} [(\tau \Sigma)^{-1} \Pi + \Pi \Omega^{-1} Q]$$  \hfill (17)

4. Data
The sample data was explored from monthly data of Gold, Natural Gas and Oil from Yahoo finance DataStream. The data spans from January, 2010 to September, 2016 with a total of 83 observations, the actual data for this study are non-stationary. In view of this, the non-stationary data need to be transformed to stationary by first differencing.
5. Result and discussion
The assets allocations results divulge from Black Litterman model is used for estimation of portfolios’ risk and assets. As stated above that the aim of this paper is to investigate assets’ reduction strength and their effects in minimizing portfolio risk. Three assets are used in this study; Gold, Oil and Natural gas. Table 1 presents risk reduction strength of the three assets; gold possesses 8.7% strength, oil contains 8.37% strength and Natural gas has 0.47% strength. This shows that gold possesses higher risk reduction strength followed by oil. The second aim of this paper is to investigate the effects of these assets strengths in minimizing risk of portfolio. Table 2 presents portfolios with their assets and corresponding risk; Benchmark portfolio contains all the three assets used in the paper with 0.0038 risk, hence these assets are partitioned into three portfolios. Portfolio 1 contains gold and oil with 0.0085 risk, portfolio 2 comprises of gold and natural gas with 0.0875 risk while portfolio 3 consists of oil and natural gas with 0.0908 risk. It is shown vividly that portfolio 1 divulges minimum risk compared with other portfolios, reason is that the two assets in the portfolio possessed higher risk reduction strength. Portfolio 3 with higher risk proved the absence of gold with highest risk reduction strength. Figure 1, clearly shows the summary of the study that if asset with higher risk reduction strength is included in a portfolio, it minimizes the portfolio’s risk.

Table 1: Assets’ Risk Reduction strength

|                | Gold | Oil  | Natural gas |
|----------------|------|------|--------------|
| Risk Reduction strength | 0.0870 | 0.0837 | 0.0047       |

Table 2: Assets’ portfolios with corresponding risk

|                  | Benchmark Portfolio | Portfolio1 | Portfolio2 | Portfolio3 |
|------------------|---------------------|------------|------------|------------|
| Gold             | Gold                | Gold       | Oil        | Oil        |
| Oil              | Oil                 | Oil        | Natural gas| Natural gas|
| Natural gas      |                     |            |            |            |
| Portfolio risk   | 0.0038              | 0.0085     | 0.0875     | 0.0908     |

Figure 1: Graph of Risk in portfolio
6. Conclusion
This paper proposed a method to minimize risk of portfolio. In view of this, investigation of risk reduction strength of each asset was carried out and the effects of each asset in minimizing risk of portfolio. Diversification is a strategic approach for minimizing risk of portfolio but if not done according to optimal method, it may not fulfil its purpose. In this study BLM was used for assets allocations being the best assets allocation model in finance at present (Idzorek, 2005a). The results of BLM were used to estimate both risk exhibits by portfolios and assets. Hence, it is observed that portfolio1 is the best portfolio to invest for rational investors.

Moreover, it is observed that portfolio1 has the lowest risk, this is as a result of the presence of gold and absence of natural gas in the portfolio. According to this study, in order to minimize risk of portfolios, there is need for investors to; first estimate the risk of each asset to know the strength of risk reduction of all the assets, second to compute risk of portfolios in order to indentify the one with minimum risk to decide on the best portfolio. Moreover, investors should endeavour to add high risk reduction assets like gold to portfolios and get rid of less reduction asset like natural gas in order to minimize portfolio risk. In view of this fact, we wish to state that gold is of great strength in risk reduction that can serve as hedge and safe haven during financial crisis.

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