Effect of Sensor Error on the Assessment of Seismic Building Damage

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Abstract—Natural disasters affect structural health of buildings, thus directly impacting public safety. Continuous structural monitoring can be achieved by deploying an Internet of things network of distributed sensors in buildings to capture floor movement. These sensors can be used to compute the displacements of each floor, which can then be employed to assess building damage after a seismic event. The peak relative floor displacement is computed, which is directly related to damage level according to the United States federal agencies standards. With this information, the building inventory can be classified into immediate occupancy, life safety, or collapse prevention categories. In this paper, we propose a zero velocity update technique to minimize displacement estimation error. Theoretical derivation and experimental validation are presented. In addition, we investigate modeling sensor error and interstory drift ratio distribution. Moreover, we discuss the impact of sensor error on the achieved building classification accuracy.

Index Terms—Earthquakes, interstory drift ratio (IDR), sensor networks, sensors, structural health monitoring (SHM), zero velocity update (ZUPT).

I. INTRODUCTION

MONITORING the structural health of buildings during and after natural disasters, such as earthquakes, provides the public and policy makers with a clear view of the state of critical infrastructure that affects the safety and well-being of the population. Previous research on building damage assessment generally falls into one of two main categories: remote sensing techniques and sensor-based technology. In the former, optical images are captured using spacecraft or aircraft, where before and after image comparisons are performed to assess the damage. This technique is effective in detecting partial to complete collapse of buildings; however, it cannot reliably detect incipient collapse because the resolution is too low [1]. On the other hand, sensor-based technology uses an Internet of things (IoT) network of preinstalled sensors to capture the movement of a building during an event, enabling distributed, accurate, and instantaneous monitoring of structures. In [2], examples of using sensors, such as vibration, temperature, tilt, and accelerometer sensors, for structural health monitoring (SHM) targeting bridges are presented, where sensors are interconnected in a wireless sensor network configuration to detect damage.

One of the well-known methodologies used for damage detection is tracking the shift in the natural frequency of a given structure, which can be measured by an accelerometer. For instance, long-term monitoring of Gabbia tower in Mantua showed effectiveness of detecting nonreversible structural damages based on frequency shifts [3]. In [4], low-cost microelectro mechanical system (MEMS) accelerometers were used to estimate the fundamental normal-mode frequency of steel buildings. A dynamic acceleration measurement system with an accelerometer integrated semipassive radio-frequency identification tag was presented in [5], where the network was used to remotely monitor the natural frequency of the structure. In [6], a low-cost distributed system for SHM based on MEMS accelerometers was developed. To enhance the accuracy and reduce the effect of noise and drifts, redundant accelerometer chips were used and online averaging as well as on-board filtering features were available. Data transmission after processing was achieved using a WiFi transceiver. The proposed system focused on real-time monitoring of building dynamic behavior represented by normal modes of building oscillation. Another similar wireless sensor node was demonstrated in [7]. The main node components were a low-cost accelerometer chip, microcontroller, wireless module, and a storage card. The measured parameters are mainly natural frequencies, damping ratios, and last floor modal displacement of a tower. In [8] and [9], SHM using a network of smart phones was presented. A mobile app was developed that utilizes the phones internal accelerometers to sense disaster vibration. Synchronization between devices was addressed using network time protocol.

The work presented in this paper is based on the fact that other than natural frequency, the interstory drift ratio (IDR) is an indicative feature that can be used for SHM as mentioned in [4]. Relative displacement of floors within a given building is used to calculate the IDRs for the building using (1). Documents released by the United States federal agencies and civil engineering societies such as Federal Emergency Management Agency (FEMA) and American Society of Civil Engineers relate IDR values to building damage level. Basically, these documents define two main critical thresholds of relative floor
displacement of a given building, such that the building can be classified into one of three categories: immediate occupancy (IO), life safety (LS), or collapse prevention (CP), which indicate that the building is either safe, needs further inspection, or unsafe, respectively. In other words, measuring the instantaneous relative floor displacement of a given building during an earthquake event is a good indicator of the structure state [10], [11]

\[
\text{IDR} = \frac{\text{Displacement}_{\text{Floor}} - \text{Displacement}_{\text{NextFloor}}}{\text{FloorHeight}}.
\]

The cost of an accelerometer depends on several parameters such as dynamic range, linearity, bandwidth, output data rate, output noise, and output type, i.e., analog or digital. Based on those specifications, cost ranges from a few dollars to a few thousand dollars. According to [12], sensor output noise is a major contributor to displacement measurement error, which is accentuated by double integration required to calculate displacement from acceleration.

While other technologies such as global positioning system (GPS) are widely used for localization and position estimation, accuracy becomes a major limiting factor in their suitability for SHM. As will be discussed later in this paper, to be useful, IDR values need to be estimated with an accuracy that is within a few centimeters from ground truth. This degree of accuracy is not possible using GPS alone, unless high-end GPS receiver is used, which is much more expensive than a standard GPS receiver [13], [14]. Furthermore, GPS signals are not available indoors, which mandates outdoor installation for the sensing devices. Another approach to estimate position is to use vision-based displacement estimation techniques as mentioned in [15]. Although this approach does not suffer from error accumulation, it faces other challenges such as, measurement error due to heat haze and ground motion, in addition to errors due to dim lighting and optical noise.

Therefore, in this paper, we focused on studying the limits of using accelerometers to estimate structural displacement for a number of reasons: 1) earthquake event time is relatively short (∼20–30 s) which results in bounded accumulated error that can be quantified; 2) accelerometers can work indoors which is not the case of GPS; and 3) accelerometers are not affected by ambient light conditions as compared to cameras.

As mentioned earlier, accelerometer inherent noise is one of the main challenges in displacement estimation. However, noise cancellation can be achieved if some of the disaster signal characteristics are known, such as the fact that a disaster vibration intensity fades gradually and eventually stops at zero velocity and acceleration. In this case, the measured velocity at the end of shaking (EOS) reflects the accumulated error in the preceding samples, which can be used to minimize the estimation error. This technique is known as zero velocity update (ZUPT) [16], [17].

The main contributions of this paper can be summarized as follows.

1) Derive how ZUPT can be applied to minimize displacement estimation error; a theoretical derivation is presented and validated by shake table experiments.

2) Study how displacement measurement error affects the accuracy of building damage classification based on its maximum IDRs.

3) Present how different system parameters such as sensor noise and IDR can be accurately modeled.

4) Apply the derived methodology on a number of commercially available sensors to relate the probability of error versus duration of observation.

The rest of this paper is organized as follows. In Section II, ZUPT algorithm is derived. Section III describes the classification methodology and derives the probability of classification error, in addition to accelerometer noise and IDR distribution modeling. System overall probability of error and sensor selection charts are presented in Section IV. Finally, the conclusion is drawn in Section V.

II. NOISE CANCELLATION

An earthquake signal is characterized by stopping at zero acceleration and zero velocity. The EOS instant can be detected when the absolute acceleration is below a certain threshold δ within a specified window of time W, as illustrated in Fig. 1. The selection of W is arbitrary, whereas δ is dependent on the sensor noise. If the sensor noise standard deviation (STD) is σ, then we believe selecting δ = 3σ is a reasonable assumption, which indicates that the noise is below that threshold most of the time. In this region, the sensor has true zero velocity. Any nonzero velocity measured at this time is due to the sensor noise, and is correlated with the noise at shaking time. As mentioned before, using such characteristic in noise cancellation is known in the literature as ZUPT [16], [17].

ZUPT has been used in inertial navigation systems, specifically pedestrian ones [18]. In such systems, navigation devices are mounted on a pedestrian’s foot, which is known to be stationary on the ground once every step. The goal of applying ZUPT in that case is to reset the velocity and prevent
further error accumulation, which in turn reduces the error in upcoming velocity samples and consequently reduces the error in displacement estimation as well. However, in this paper, we are only concerned in correcting displacement estimation for the time window prior to the EOS instant, since that window contains the peak relative displacement which reflects the damage state.

A. Noise Cancellation Using ZUPT

1) Accelerometer Model: We use a linear model for accelerometer which has a scale factor and a constant bias. In addition, the model parameter values drift with temperature as mentioned in [19]. In our application, the measurement time window is short and temperature can be assumed to be constant throughout the measurement. Hence, the temperature-dependent bias is estimated right before or after each measurement using a narrow-band low-pass filter to be subtracted from the readings and hence, the temperature effect on bias is minimized. On the other hand, the scale factor error in acceleration results in the same scale error in displacement. Usually, scale factor variation versus temperature is constrained by the manufacturer to be small and reported in the device datasheet. Therefore, the error in displacement due to scale factor temperature dependence is negligible compared to other sources of error such as double integration of additive noise.

2) Misalignment: In this paper, we are concerned with measuring floors’ horizontal displacement. Hence, we assume that accelerometers will be oriented to measure only horizontal motion, i.e., gravity will not affect the reading. However, to account for miss-orientation, we consider a constant bias $g \times \sin(\theta)$ is added to the measurement and a scale factor $\cos(\theta)$ is multiplied by the measurement, where $g$ is the gravitational constant and $\theta$ is the misalignment angle, i.e., ideally $\theta = 0$. Assuming linear motion, $\theta$ is constant throughout the measurement; hence, $g \times \sin(\theta)$ is constant and be removed with the sensor temperature varying bias as explained in Section II-A1. With respect to the scale factor error, assuming $\theta$ is small enough, then similar to the temperature effect on scale factor, the scale factor error due to misalignment is negligible compared to other sources of error. More information about orientation and misalignment is found in [20]. However, in the case of curvilinear motion, $\theta$ is not constant, and removing the gravity component in this case is more complex and can be addressed by using techniques described in [21]. Curvilinear motion is out of the scope of this paper and will be investigated in future work.

3) ZUPT Algorithm: Using rectangular numerical integration method [22], true velocity is expressed by (2), where $a_{true}[k]$ is the ground truth $k$th horizontal acceleration sample and $\Delta t$ is the sampling time. True displacement is obtained by (3) which can be simplified as shown in Appendix V to be calculated by (4), and the vector form is expressed by (5), where $A_{true}$ is the true acceleration vector and $P = [i - 1/2, i - 3/2, \ldots, 1/2]^T$

\[
v_{true}[i] = \sum_{k=0}^{i-1} a_{true}[k] \Delta t \quad (2)
\]

\[s_{true}[i] = s_{true}[i - 1] + v_{true}[i - 1] \Delta t + \frac{1}{2} a_{true}[i - 1] \Delta t^2 \quad (3)
\]

\[= \sum_{k=0}^{i-1} \left( i - k - \frac{1}{2} \right) a_{true}[k] \Delta t^2 \quad (4)
\]

\[= P^T A_{true} [0 : i - 1] \Delta t^2. \quad (5)
\]

Measurement noise $z[k]$ is considered additive with zero mean, since constant bias is estimated by long term averaging and then subtracted from the measurement [12]. Hence, measured acceleration is expressed by (6). Consequently, measured displacement is shown by (7). As a result, displacement error is shown by (8), where $Z$ is the noise vector

\[a[i] = a_{true}[i] + z[i] \quad (6)
\]

\[s[i] = \sum_{k=0}^{i-1} \left( i - k - \frac{1}{2} \right) a[k] \Delta t^2 \quad \text{and then adding the result to the displacement measurement.}
\]

\[c_i QT Z[0 : i - 1] \Delta t^2. \quad (8)
\]

Let the shaking window length be $n$ samples, where the shaking window is defined as the time window starting from the beginning of earthquake shaking to the EOS instant. $v[n]$ is the measured velocity at the EOS instant, which is equal to the accumulated noise since the true velocity at that instant is 0. As shown in (9), ZUPT is applied at any given sample $i$, by multiplying $v[n]$ by coefficient $c_i$ and then adding the result to the displacement measurement.

Note that in traditional inertial systems, ZUPT is triggered when zero velocity is detected to cancel noise at this time instant in order to decrease the error in the next velocity/displacement samples. However, in this paper, we focus on using the zero velocity information to minimize the error in the preceding samples to the zero velocity moment rather than the next ones. We use the fact that measuring the velocity at the zero velocity instant reflects the accumulation of noise of all the acceleration samples that precede the zero velocity instant. Hence, the displacement error at any time instant is correlated with the velocity measured at the EOS instant.

Thus, the modified displacement error is calculated by (10), where $Q$ is an $n \times 1$ vector of ones

\[s_{ZUPT}[i] = P^T A[0 : i - 1] \Delta t^2 + c_i v[n] \Delta t \quad (9)
\]

\[e_{ZUPT}[i] = P^T Z[0 : i - 1] \Delta t^2 + c_i \sum_{k=0}^{n-1} z[k] \Delta t^2 \quad (10)
\]
Let $\sigma^2_S[i]$ and $\sigma^2_{S,ZUPT}[i]$ be the mean-squared error in displacement at sample $i$ without and with applying ZUPT, respectively, as shown in the following equations:

$$
\begin{align*}
\sigma^2_S[i] &= E[e^2] = P^T E[Z[0 : i - 1]Z[0 : i - 1]^T]P \Delta t^4 \\
&= P^T R_{ii} P \Delta t^4 \\
\sigma^2_{S,ZUPT}[i] &= E[e^2_{ZUPT}] \\
&= P^T E[Z[0 : i - 1]Z[0 : i - 1]^T]P \Delta t^4 \\
&+ c_i^2 Q^T E[Z[0 : n - i]Z[0 : n - i]^T]Q \Delta t^4 \\
&+ 2c_i P^T E[Z[0 : i - 1]Z[0 : n - i - 1]^T]Q \Delta t^4 \\
&= P^T R_{ii} P \Delta t^4 + c_i^2 Q^T R_{nn} Q \Delta t^4 \\
&+ 2c_i P^T R_{in} Q \Delta t^4
\end{align*}
$$

where $E[.]$ denotes the expectation operator and the noise covariance $R_{in}$ and $R_{nn}$ are given in the following equations, respectively. The value of $c_i$ is calculated such that $\sigma^2_{S,ZUPT}$ is minimized as shown in (15) and (16).

### Equations for $R_{in}$ and $R_{nn}$

$$
R_{in} = \begin{bmatrix}
    r_0 & r_1 & \ldots & r_{i-1} & \ldots & r_{n-1} \\
    r_1 & r_0 & \ldots & r_{i-2} & \ldots & r_{n-2} \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    r_{i-1} & r_{i-2} & \ldots & r_0 & \ldots & r_{n-i}
\end{bmatrix}
$$

$$
R_{nn} = \begin{bmatrix}
    r_0 & r_1 & \ldots & r_{i-1} \\
    r_1 & r_0 & \ldots & r_{i-2} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{i-1} & r_{i-2} & \ldots & r_0
\end{bmatrix}
$$

### Derivative of $\sigma^2_{S,ZUPT}[i]$ with respect to $c_i$

$$
\frac{\partial \sigma^2_{S,ZUPT}[i]}{\partial c_i} = 0 \\
0 = 2c_i Q^T R_{nn} Q + 2P^T R_{in} Q
$$

$$
\Rightarrow c_i = -\frac{P^T R_{in} Q}{Q^T R_{nn} Q}
$$

### Noise Covariance $R[0 : i - 1]$ and $R[0 : n - i - 1]$ for White Noise

Assuming that the noise can be modeled as a stationary process as will be illustrated in Section III-A1, then as shown in Appendix V, for noise processes that are characterized by having decaying covariance coefficients, i.e., $r_0 \gg r_1 \gg \ldots RQ \approx Q\eta$, where $\eta$ is calculated by (17). Hence, (16) can be simplified as shown in (18). The resulting mean-squared error $\sigma^2_{S,ZUPT}$ is expressed by (19).

$$
\eta = \sum_{k=-\infty}^{\infty} r_k
$$

$$
c_i = -\frac{P^T Q[0 : i - 1]Q}{Q^T Q\eta} \approx -\frac{i^2}{2n}
$$

$$
\sigma^2_{S,ZUPT}[i] = \left( \left( P^T R_{ii} P + i^4 \frac{Q^T R_{nn} Q}{4n^2} \Delta t^4 \right) - \frac{i^2}{n} P^T R_{in} Q \right) \Delta t^4.
$$

It is clear that the resulting mean-squared error is a function of $R$ which depends on the noise characteristics. For example, in the case of white noise, $R = \sigma^2 I$ and $\eta = \sigma^2$, where $I$ is the identity matrix and $\sigma^2$ is the noise variance. By substituting in (11), the mean-squared error without applying ZUPT is calculated by (20) and can be simplified by (21) for sufficiently large $i$.

### Equations for $\sigma^2_{S,ZUPT}[i]$ for White Noise

$$
\sigma^2_{S,ZUPT}[i]_{\text{white}} = \sigma^2 P^T I \Delta t^4 = \sigma^2 P^T I \Delta t^4 = \sigma^2 \Delta t^4 \sum_{k=0}^{i-1} \left( k + \frac{1}{2} \right)^2
$$

$$
= \sigma^2 \Delta t^4 \sum_{k=0}^{i-1} k^2 + k + \frac{1}{4}
$$

$$
= \sigma^2 \Delta t^4 \left( \frac{(i-1)(2i-1)}{6} + \frac{(i-1)i}{2} + \frac{i}{4} \right)
$$

$$
\approx \sigma^2 \Delta t^4 \left( \frac{i^3}{3} \right).
$$

Similarly, by substituting in (19), the mean-squared error with applying ZUPT is calculated by (22), where $G$ is an $i \times (n-i)$ matrix of zeros. For sufficiently large $i$, (22) can be simplified as shown in (23).

$$
\sigma^2_{S,ZUPT}[i]_{\text{white}} = \sigma^2 P^T I \Delta t^4 = \sigma^2 P^T I \Delta t^4 = \sigma^2 \Delta t^4 \sum_{k=0}^{i-1} \left( k + \frac{1}{2} \right)^2
$$

$$
= \sigma^2 \Delta t^4 \sum_{k=0}^{i-1} k^2 + k + \frac{1}{4}
$$

$$
= \sigma^2 \Delta t^4 \left( \frac{(i-1)(2i-1)}{6} + \frac{(i-1)i}{2} + \frac{i}{4} \right)
$$

$$
\approx \sigma^2 \Delta t^4 \left( \frac{i^3}{3} \right).
$$

At the EOS instant, i.e., at $i = n$, the mean-squared error without and with applying ZUPT are expressed by (24) and (25), respectively. Comparing both (24) and (25), it is concluded that using ZUPT reduces the mean-squared error by 75% at the EOS instant, i.e., $\sigma_{S,ZUPT}$ is smaller than $\sigma_S$ by 50%. For the rest of this paper, we will refer to $\sigma_{S,ZUPT}$ as $\sigma_S$.

$$
\sigma_S^2 \text{ white & EOS} = \sigma^2 \Delta t^4 \left( \frac{n^3}{3} \right)
$$

$$
\sigma_{S,ZUPT}^2 \text{ white & EOS} = \sigma^2 \Delta t^4 \left( \frac{n^3}{12} \right).
$$

### B. Experimental Validation

In order to validate the developed algorithm, shake table experiments have been performed. We have used different amplitudes of sinusoidal, frequency sweep, and random vibration profiles for a duration of 20 s. The sensing device is...
a smart phone that captures acceleration using its internal accelerometer and transmits the data to a PC. The phone internal accelerometer chip is InvenSense MPU6500 which is a 6-axis inertial module that contains 3-axis accelerometer and 3-axis gyroscope sensors, and is widely used in commercial devices [23]. Fig. 2 shows the experimental setup. MPU6500 chip noise is measured and characterized such that a complete noise model is developed that includes not only white noise, but also bias instability (BI) and rate random walk (RRW), which is described in detail in Section III-A1. That noise model is used to apply ZUPT method.

The motion starts and ends by zero velocity to mimic a seismic event. For each experiment, a vibration profile is selected. Then, the whole experiment is repeated for 20 times to generate repeated acceleration measurements for the same vibration profile. Without applying ZUPT, for each trial, the displacement is calculated by double summation of acceleration. At each time sample, the error is calculated by evaluating the standard deviation of the calculated displacements for all the trials of the same experiment. Then, ZUPT is applied to get better displacement estimate from acceleration, and similarly, the error is calculated by evaluating the standard deviation of the calculated displacements. Table I summarizes the experimental results and reports the error of each experiment at $t = 20$ s, and Fig. 3 shows the STD of theoretical and measured error in displacement $\sigma_s$ versus time. It is clear that the measured error follows the theoretical one with and without applying the ZUPT algorithm. It is worth noting that the white noise approximation leads to pessimistic results in terms of error reduction performance. As shown in Fig. 3, when using the full noise model, the noise reduction exceeds the expected 50% reduction achievable by only white noise assumption mentioned in Section II-A3. It is also worth noting that there is a slight difference between the measured and the theoretical model results; the measured error is slightly larger. This is attributed to other sources of error such as time jitter and nonlinearity that have not been included in the model.

### III. BUILDING CLASSIFICATION

According to government documents, buildings are classified according to their damage state as IO, LS, or CP buildings. For instance, Table II lists the IDR limits for steel moment frame buildings which are stated in [10] and [11], and the corresponding physical tag used to signal the buildings’ postevent condition. Hence, a building’s performance can be assessed by comparing its peak IDR to the predefined thresholds. Knowing the floor height, which is 4 m in typical U.S. construction,
thresholds in IDR corresponds to certain thresholds in relative floor displacement that we denote by $d_0$ and $d_1$.

Let the true displacement of the two floors be denoted by $S_1$ and $S_2$, then the relative displacement $D$ is expressed by (26).

$$D = S_2 - S_1$$

Since each displacement measurement has its own error, then the measured relative displacement $D_e$ is calculated by (27), where $e_1$ and $e_2$ are the measurement error for $S_1$ and $S_2$, respectively

$$D_e = S_2 + e_2 - (S_1 + e_1)$$

$$D_e = S_2 - S_1 + (e_2 - e_1)$$

let $X = e_2 - e_1$ then

$$D_e = D + X$$

(29)

and knowing that the errors in both measurements are not correlated, then the mean-squared error in relative displacement measurement is expressed by (30). If identical sensors are used, then $\sigma_{D_e}^2 = \sigma_{S_1}^2 = \sigma_S^2$ and (30) reduces to (31)

$$\sigma_X^2 = \sigma_{S_2}^2 + \sigma_{S_1}^2$$

$$\sigma_X^2 = 2\sigma_S^2$$

(30)

(31)

To evaluate classification accuracy, let $B$ and $B_{\text{true}}$ be the building’s estimated and true states, respectively. Equation (29) shows the measured relative displacement of two consecutive floors. The accuracy of the true classification of a building is obtained by evaluating the conditional probability $P(B|B_{\text{true}})$ as shown in the following equation:

$$P(B|B_{\text{true}}) = \frac{P(B \cap B_{\text{true}})}{P(B_{\text{true}})}$$

(32)

where $P(B \cap B_{\text{true}})$ and $P(B)$ are expressed as follows:

$$P(B \cap B_{\text{true}}) = \int \int fx, D(x, d) \, dd \, dx$$

(33)

$$P(B_{\text{true}}) = \int f_D(d) \, dd$$

(34)

where $fx, D(x, d)$ is the joint probability density function (PDF) of $X$ and $D$, and $f_D(d)$ is the marginal PDF of $D$. The integral in (33) is done over the area shown in Fig. 4. Besides, limits of the integral in (34) is given in Table III.

The measurement error only depends on the accelerometer itself and its inherent sources of noise, which is not related to the excitation signal. Hence, noise distribution is considered independent of IDR distribution. As a result, the joint PDF of $X$ and $D$ is expressed as follows:

$$fx, D(x, d) = fx(x) f_D(d)$$

(35)

A. Modeling

1) Sensor Noise: Acceleration measured by an accelerometer sensor is contaminated by several sources of noise. Referring to [12], noise can be modeled as: constant bias, angle random walk (ARW) or velocity random walk), BI, and RRW (or acceleration random walk), where each of these is considered an independent Gaussian noise source with certain power spectral density (PSD). Since we are only concerned with relatively short durations, higher order noise sources such as drift rate ramp (DRR) are ignored and removed with the constant bias.

According to [24], different noise sources can be modeled as white Gaussian noise shaped with a shaping finite impulse response (FIR) function $H_f(z)$. Since the input of the FIR filter is white Gaussian noise, i.e., wide sense stationary (WSS) noise process, then the generated noise is also WSS.

Case Study: For instance, we consider the Invensense MPU6500, which is a 6-axis inertial module that contains 3-axis accelerometer and 3-axis gyroscope. MPU6500 is widely used in commercial devices such as smartphones. To characterize the noise profile of the sensor, the output of the chip was recorded for 12 h without motion. Using methods described in [25], noise can be modeled as ARW,
Bi, and RRW, and the overall noise covariance matrix $R$ is calculated. Fig. 5 shows the real (measured) and modeled noise density of MPU6500 accelerometer sensor. It is clear that the noise model matches the real one at low frequency, whereas there is some discrepancy at high frequency, which is due to the fact that the sensor has a low-pass filter in the output. However, since in our application the data are double integrated, the low-frequency content is the main contributor to the displacement error. Therefore, the discrepancy at high frequency is irrelevant.

In order to provide a reference for comparison, we selected a number of sensors with different noise characteristics as summarized in Table IV. Fig. 6 shows the noise spectral density of the selected sensors based on their data sheets. From Fig. 6, it is clear that some of the sensors noise can be approximated as white noise such as MTI100 and AXO215 sensors, whereas for other sensors higher order noise sources as BI and ARW should be considered.

2) IDR Distribution: Simulation of building response is conducted in order to model the IDR distribution as a result of earthquake excitation. We consider four- and eight-story buildings designed by NIST [30] in Seattle to be representative of steel frame buildings. The buildings have 42.7 m x 30.5 m plans as shown in Fig. 7(a). Three-bay perimeter steel special moment frames (SMFs) on each side of the building are used for the lateral load resisting system. The SMFs are designed with reduced beam sections. With respect to the type of soil, we consider site class $D$ which includes mixtures of dense clays, silts, and sands, which is the most common site class throughout the United States [31]. The seismic design category is $D_{\text{max}}$, i.e., structures are expected to suffer from considerable rotational loads during strong earthquakes [32]. As shown in Fig. 7(b), finite-element models of the SMFs are created using HyperMesh [33] and analyzed using the commercial code LS-DYNA [34]. The steel is ASTM-A992 and its engineering stress–strain properties are converted into true stress–strain data then assigned to the finite elements as done in [35]. Gravity loads from the tributary area shown in Fig. 7(a) are directly applied to the frame and the remainder of the gravity loads are applied to a leaning column connected to the SMF by truss members. Mass weighted damping of 2.5% is assumed at the first mode period of the SMFs. Additional modeling details can be found in [35].

The distributions of peak relative displacement are computed for three seismic hazard levels: 2% probability of exceedance in 50 years, 10% in 50 years, and 50% in 50 years. Eleven seismic records are selected from the far-field ground motion record set in FEMA [36] and scaled to the three specified hazard levels at the first period of each building, resulting in 33 records for each building. The first period spectral accelerations corresponding to the three hazard levels are 0.55, 0.26, and 0.07 g for the four-story building and 0.41, 0.17, and 0.04 g for the eight-story building, respectively. Each building is then subjected to the scaled seismic records for each hazard level and the peak relative displacement is
computed. The histogram of peak relative displacement is shown in Fig. 8(a)–(c). The distribution can be approximated as Gaussian with mean $\mu_D$ and variance $\sigma_D^2$ that depend on the hazard level, with slight variation depending on the building type.

3) Earthquake Strong Motion Duration: Damage prone buildings will suffer damage during the strong shaking part of the seismic event. As mentioned in [37], there are several definitions for the strong motion duration, which is calculated based on acceleration magnitude or cumulative energy obtained by integrating squared acceleration. In [37], strong motion duration of 140 earthquake records were evaluated, and Fig. 9 shows the cumulative density function (CDF) of strong motion duration.

**B. Probability of Classification Error**

As mentioned in Section III-A, relative displacement measurement error can be modeled as zero mean Gaussian of variance $\sigma_X^2$, and relative displacement distribution can be modeled as Gaussian of mean $\mu_D$ and variance $\sigma_D^2$ which varies according to the hazard level. As a result, substituting into (35), the joint probability distribution $f_{X,D}(x,d)$ can be expressed as follows:

$$f_{X,D}(x,d) = N(0, \sigma_X^2)N(\mu_D, \sigma_D^2)$$

(36)

where $N(\mu, \sigma^2)$ is a Gaussian distribution with mean $\mu$ and variance $\sigma^2$. Fig. 10 shows a sketch of Gaussian peak relative displacement distribution. Classification boundaries are highlighted, where error is expected to occur.

Substituting (36) in (32) and (33), conditional probabilities can be calculated. For instance, for IO buildings, the probability of correct classification is defined as $P(IO|IO)$, whereas probability of error is defined as $P(IO|\neg IO)$, i.e., $P(LS \cup CP|IO)$. Similarly, for LS buildings, the probability of correct classification is defined as $P(LS|LS)$, whereas the probability of error is defined as $P(IO \cup CP|LS)$, and with respect to CP buildings, the probability of correct classification is defined as $P(CP|CP)$, whereas the probability of error is defined as $P(IO \cup LS|CP)$. The probability of error of the system is calculated by the following equation, and will be used later in
Section IV for sensor selection:

\[
p_e = \frac{(P(\text{LS}|\text{IO}) + P(\text{CP}|\text{IO}))P(\text{IO})}{P(\text{IO})} + \frac{(P(\text{IO}|\text{LS}) + P(\text{CP}|\text{LS}))P(\text{LS})}{P(\text{LS})} + \frac{(P(\text{IO}|\text{CP}) + P(\text{LS}|\text{CP}))P(\text{CP})}{P(\text{CP})}.
\]  

(37)

IV. SENSOR SELECTION

In Section III, we showed that the probability of classification error is a function of displacement measurement accuracy, hazard level, and strong motion duration. In this section, we demonstrate how a sensor can be selected based on the acceptable probability of error \( p_e \) which is calculated by (37). The probability of classification error is calculated for each of the sensors mentioned in Section III-A1.

Fig. 11(a) shows the probability of error in buildings classification as a function of strong motion duration in the case of 50% in 50 years hazard level. As expected, it is clear that the high-accuracy seismic sensors such as Mistras1030 and KB12VD have very small probability of error, and the probability of error increases as sensor accuracy decreases. In Fig. 11(a), we also compare between the simple white noise model and the more complex model that takes into account other noise components as explained in Section III-A1. It is worth noting that for high-accuracy sensors, using white noise model results in negligible probability of error which is not plotted in Fig. 11. Hence, only the more complex noise model is plotted for the two high-accuracy sensors Mistras1030 and KB12VD. However, for MTI-100 and AXO215, using only the simple white noise model results in probability of error slightly smaller but comparable to the complex model. With respect to MPU6500, there is a larger discrepancy between white noise model and complex noise model results. Intuitively, that result was expected, as mentioned in Section III-A1 by comparing the noise density curves shown earlier in Fig. 6, it is clear that only MTI-100 and AXO215 noise can be approximated as flat white noise. Similarly, Fig. 11(b) and (c) shows the probability of error in the case of 10% in 50 years and 2% in 50 years hazard levels, respectively. Depending on the acceptable probability of error, the curves presented in Fig. 11(a)–(c) can be used to evaluate the maximum accepted noise density; hence, an appropriate sensor can be selected.

V. CONCLUSION

Monitoring structural health of buildings during and after natural disasters is crucial, and directly impacts public safety. Buildings can be added to an IoT network by deploying inertial sensors in civil infrastructure, which facilitates postdisaster identification of structurally unsound buildings. In this paper, we illustrated how accelerometer sensors can be employed to identify buildings damage state. We presented a theoretical derivation of a ZUPT algorithm that is used to increase displacement measurement accuracy, and consequently increase buildings classification accuracy. The developed algorithm has been validated experimentally using shake table experiments. We investigated the effect of sensors inherent noise on the overall building classification accuracy. The probability of error was calculated as a function of sensor noise density, earthquake duration time, and IDR distribution.

While the focus of this paper is accelerometers, we believe that hybrid systems that combine multiple modalities (e.g., accelerometers + GPS + Camera) will provide enhanced accuracy over a single modality. The tradeoffs involved in these systems will be the subject of future work.

APPENDIX A

STATIONARY PROCESS COVARIANCE MATRIX ROW SUMMATION

In this appendix, we derive an approximate summation formula of rows of a covariance matrix of a stationary random process that is characterized by having decaying covariance coefficients, i.e., \( r_0 \gg r_1 \gg \cdots \). As mentioned in Section II-A, stationary process covariance matrix is expressed by (14). Hence, the summation of covariance matrix rows is calculated using (38). Let the summation of row \( k \) be denoted...
Fig. 11. Probability of error in classification $p_e$ versus strong motion duration time $T$ for several sensors. Sensors noise is modeled according to their data sheets except for MPU6500 we used the noise model mentioned in Section III-A1. (a) 50% in 50 years hazard level. (b) 10% in 50 years hazard level. (c) 2% in 50 years hazard level.

by $\eta_k$ as shown by (39)

$$RQ = \begin{bmatrix} r_0 & r_1 & \ldots & r_{n-2} \\ r_1 & r_0 & \ldots & r_{n-3} \\ \vdots & \vdots & \ddots & \vdots \\ r_k & r_{k-1} & \ldots & r_{n-k} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n-1} & r_{n-2} & \ldots & r_0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \sum_{j=0}^{n-1} r_j + \sum_{j=0}^{n-2} r_j = \sum_{j=0}^{n-1} r_j$$

$$\eta_k = \sum_{j=1}^{k} r_j + \sum_{j=0}^{n-k-1} r_j.$$

If the stationary noise process is characterized by having decaying covariance coefficients values, i.e., $r_0 \gg r_1 \gg r_2 \gg \cdots$, then the maximum value at any row is $r_0$, and the values decay and can be neglected compared to $r_0$, $r_1$ and the first few coefficients. Hence, approximately $\eta_k$ for rows away from the boundaries is given by (40), and because of the covariance symmetry, $\eta_k$ can be expressed by (41) which does not depend on $k$. Hence, by neglecting the error at the boundary rows, $RQ$ is given by (42)

$$\eta_k \approx \eta = \sum_{j=1}^{n} r_j + \sum_{j=0}^{n} r_j$$

**Appendix B
Displacement Calculation**

In this appendix, we derive the discrete time displacement as a function of acceleration, assuming that acceleration is considered constant in the duration between any two successive time samples. Discrete time velocity is calculated by (43), whereas displacement is calculated by (44). Equations (45) and (46) are the Z-transforms of (43) and (44), respectively. Hence, $S(z)$ is obtained by (47)

$$v[i] = \sum_{k=0}^{i-1} a[k] \Delta t$$

$$s[i] = s[i-1] + v[i-1] \Delta t + \frac{1}{2} a[i-1] \Delta t^2$$

$$V(z) = \frac{z^{-1}}{1 - z^{-1}} A(z) \Delta t$$

$$S(z) = z^{-1} S(z) + z^{-1} V(z) \Delta t + \frac{1}{2} z^{-1} A(z) \Delta t^2$$

$$= z^{-1} S(z) + z^{-1} \left( \frac{z^{-1}}{1 - z^{-1}} A(z) \Delta t^2 + \frac{1}{2} z^{-1} A(z) \Delta t^2 \right)$$

$$= \frac{1}{1 - z^{-1}} \left( z^{-1} A(z) \Delta t^2 + \frac{1}{2} z^{-1} A(z) \Delta t^2 \right)$$

$$= \frac{z^{-2}}{(1 - z^{-1})^2} A(z) \Delta t^2 + \frac{z^{-1}}{2 (1 - z^{-1})} A(z) \Delta t^2.$$
By using the inverse Z transform, \( s[i] \) is expressed by (48) knowing that \( a[i] \), \( b[j] \), and \( s[i] \) are zeros \( \forall i < 0 \)

\[
s[i] = \sum_{k=0}^{i-1} \sum_{j=0}^{k-1} a[j] \Delta t^2 + \frac{1}{2} \sum_{k=0}^{i-1} a[k] \Delta t^2
\]

\[
= \sum_{k=0}^{i-1} (i - k - 1)a[k] \Delta t^2 + \frac{1}{2} \sum_{k=0}^{i-1} a[k] \Delta t^2
\]

\[
= \sum_{k=0}^{i-1} (i - k - \frac{1}{2}) a[k] \Delta t^2. \quad (48)
\]

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