TATOOINE’S FUTURE: THE ECCENTRIC RESPONSE OF KEPLER’S CIRCUMBINARY PLANETS TO COMMON-ENVELOPE EVOLUTION OF THEIR HOST STARS

VESelin B. KOSTOV1,6, KEvin MOORE2, DANiel TAMAYO3,4,5, RAY JAYAWARDHANA2, and STEphen A. RINEHART1
1 NASA Goddard Space Flight Center, Mail Code 665, Greenbelt, MD 20771, USA; veselin.b.kostov@nasa.gov
2 Faculty of Science, York University, 4700 Keele Street, Toronto, ON M3J1P3, Canada
3 Department of Physical & Environmental Sciences, University of Toronto at Scarborough, Toronto, Ontario M1C 1A4, Canada
4 Canadian Institute for Theoretical Astrophysics, 60 St. George St, University of Toronto, Toronto, Ontario M5S 3H8, Canada

ABSTRACT

Inspired by the recent Kepler discoveries of circumbinary planets orbiting nine close binary stars, we explore the fate of the former as the latter evolve off the main sequence. We combine binary star evolution models with dynamical simulations to study the orbital evolution of these planets as their hosts undergo common-envelope (CE) stages, losing in the process a tremendous amount of mass on dynamical timescales. Five of the systems experience at least one Roche-lobe overflow and CE stage (Kepler-1647 experiences three), and the binary stars either shrink to very short orbits or coalesce; two systems trigger a double-degenerate supernova explosion. Kepler’s circumbinary planets predominantly remain gravitationally bound at the end of the CE phase, migrate to larger orbits, and may gain significant eccentricity; their orbital expansion can be more than an order of magnitude and can occur over the course of a single planetary orbit. The orbits these planets can reach are qualitatively consistent with those of the currently known post-CE, eclipse-time variations circumbinary candidates. Our results also show that circumbinary planets can experience both modes of orbital expansion (adiabatic and nonadiabatic) if their host binaries undergo more than one CE stage; multiplanet circumbinary systems like Kepler-47 can experience both modes during the same CE stage. Additionally, unlike Mercury orbiting the Sun, a circumbinary planet with the same semimajor axis can survive the CE evolution of a close binary star with a total mass of 1 $M_\odot$.

Key words: binaries: close – binaries: eclipsing – methods: numerical – planetary systems – stars: individual (Kepler-47, -1647, NN Ser) – techniques: photometric

Supporting material: machine-readable table

1. INTRODUCTION

“Everything is in motion,” Plato quotes Heraclitus (Cratylus, Paragraph 402, section a, line 8), “and nothing remains still.” Despite the overwhelming timescales, from a human perspective, the natural life cycle of stars is a prime example of change on the cosmic stage. Following the laws of stellar astrophysics, from millions to billions of years, stars and stellar systems form, evolve, and eventually die (e.g., Kippenhahn & Wingert 1990 and references therein). And so do planetary systems, with their fate linked intimately to that of their stellar hosts.

The fate of planets orbiting single stars has been studied extensively, indicating that planets can survive their star’s evolution if they avoid engulfment or evaporation during the red giant branch (RGB) and the asymptotic giant branch (AGB) stages (e.g., Livio & Soker 1984; Rasio & Livio 1996; Duncan & Lissauer 1998; Villaver & Livio 2007; also see Veras et al. 2016). There is also accumulating evidence of planetary or asteroidal debris surrounding white dwarfs and polluting their atmospheres (see recent reviews by Farihi 2016 and Veras 2016). A planet’s survival depends both on the initial orbital separation and on its mass. For example, even a planet as massive as 15 $M_{\text{Jup}}$ around a 1 $M_\odot$ main-sequence star can be destroyed inside the stellar envelope during the AGB phase (Villaver & Livio 2007). An unpleasant prospect for the future of our own planet is that it might not survive the evolving Sun (e.g., Schroder & Connon Smith 2008). Despite the complications, theoretical considerations indicate that planets can indeed survive the evolution of single stars (Villaver & Livio 2007), and there is mounting observational evidence supporting this (e.g., Reffert et al. 2015; Wittenmyer et al. 2016, and references therein7).

Single stars, however, do not have a monopoly over planetary systems. Nearly half of solar-type stars are members of binary and higher order stellar systems (Raghavan et al. 2010), and an increasing number of planets have been discovered in such systems. While the presence of a distant stellar companion will have little effect on a planet around one (evolving) member of a wide binary stellar system (>100 au), a planet orbiting around both members of a close binary system (separation ∼10 au or less) will experience qualitatively and quantitatively different stellar evolution. Namely, where a single star loses mass and expands on timescales of millions to billions of years (Veras 2016) while on the main sequence (MS), the RGB, and the AGB—and its planets react accordingly—close binary stars can experience events of dramatic mass loss, orbital shrinkage on timescales of years, and common-envelope (CE; see Appendix for abbreviations and parameters) stages where the two stars share (and quickly expel) a common atmosphere (Paczynski 1976; Hilditch 2001). During the in-spiraling CE phase, the two stars can strongly interact with each other by transferring mass, smoothly coalesce or violently collide, or even explode as a supernova (SN). Both the amount of energy released during this stage and the timescale of the release are staggering: a close binary star

7 Also see https://www.lsw.uni-heidelberg.de/users/reffert/giantplanets.html for a list of known systems.
can lose an entire solar mass over the course of just a few months (e.g., Livio & Soker 1984; Rasio & Livio 1996; Ricker & Taam 2008, 2012; Passy et al. 2012; Ivanova et al. 2013; Nandez et al. 2014, but also see Sandquist et al. 1998 and De Marco et al. 2009 for longer timescales); in the extreme case of an SN, the binary will be completely disrupted. Overall, the evolution of close binary stars is much richer than single stars, as the separation, eccentricity, and metallicity of the binary add additional complications to an already complex astrophysical process. Interestingly, to date there are nine close MS binary systems harboring confirmed circumbinary planets (CBPs; Doyle et al. 2011; Orosz et al. 2012a, 2012b; Welsh et al. 2012, 2015; Kostov et al. 2013, 2014; Schwamb et al. 2013). It is reasonable to assume that the evolution of these planets to the violent CE phase of their host binary stars will be no less dramatic.

Previous studies of the dynamical response of planets in evolving multiple stellar systems have focused on post-CE (PCE) CBP candidates (e.g., Portegies Zwart 2013; Mustill et al. 2013; Volschow et al. 2014), on circumprimary planets in evolving wide binary systems (Kratter & Perets 2012), and on the survival prospects of CB planets (Veras et al. 2011; Veras & Tout 2012). In particular, Portegies Zwart (2013) and Mustill et al. (2013) start with the final outcome of close binary evolution—PCE systems with candidate CBP companions (H Aqu and NN Ser, respectively)—and reconstruct the initial configuration of their progenitors.

Inspired by recent discoveries of planets in multiple stellar systems, here we tackle the reverse problem: we start with the nine confirmed CBP Kepler systems with known initial binary configurations (using data from NASA’s Kepler mission), and we study their dynamical response as their stellar hosts evolve. We combine established, publicly available binary star evolution code (BSE; Hurley et al. 2002) with direct N-body integrations (using REBOUND, Rein & Liu 2012), allowing for stellar mass loss and orbital shrinkage on dynamical timescales. For simplicity, here we focus explicitly on dynamical interactions only (e.g., no stellar tides) and assume that the CBPs do not interact with the ejected stellar material.

We expand on the latter limitation in Section 5.1.

We note that Mustill et al. (2013) assume an adiabatic mass-loss regime (Hadjidemetriou 1963) during the CE stage for the evolution of NN Ser. In this case, the orbital period of the CB is much shorter than the CE mass-loss timescale (hereafter $T_{CE}$), and the planet’s orbit expands at constant eccentricity.8 Volschow et al. (2014) study the two analytic extremes of mass-loss events for the case of NN Ser: the adiabatic regime and the instantaneous regime where $T_{CE}$ is much shorter than the orbital period of the planet.9 The Kepler CBP systems, however, occupy a unique parameter space where the planets’ periods are comparable to the $T_{CE}$ used by Portegies Zwart (2013) and suggested by Ricker & Taam (2008, 2012) for binary systems similar to Kepler CBP hosts. Thus the adiabatic approximation may not be an adequate assumption when treating the CE phases of these systems. Specifically, the CE phase can occur on timescales as short as one year (Paczynski 1976; Passy et al. 2012), and during this year a binary can expel $\sim 2 M_{\odot}$ worth of mass (Ricker & Taam 2008, 2012; Iaconi et al. 2016). Interestingly, such timescales are well

8 In line with thermodynamics nomenclature, this is an isoecentric process.
9 We note that both the total amount of mass lost and the rate at which it is lost will affect the evolution of a CBP’s orbit during a CE phase.
### Table 4
BSE Evolution of Kepler-38 for $\alpha_{CE} = 0.5$

| Timea (Gyr) | $M_1$ ($M_\odot$) | $M_2$ ($M_\odot$) | Stell. Type | Stell. Type | $a_{bin}$ ($a_{Roche}$) | $e_{bin}$ | $e_{Roche}$ | $R_1/R_{Roche}$ | $R_2/R_{Roche}$ | Evol. Stage |
|-------------|------------------|------------------|-------------|-------------|------------------|--------|--------|---------------|---------------|-------------|
| Kepler-38b  | 0.0              | 0.95             | 0.25        | 1           | 0                | 31.6   | 0.10   | 0.05          | 0.03          | INITIAL     |
|             | 12.36            | 0.95             | 0.25        | 2           | 0                | 31.6   | 0.10   | 0.10          | 0.03          | KW CHNGE    |
|             | 13.01            | 0.95             | 0.25        | 3           | 0                | 31.61  | 0.10   | 0.14          | 0.03          | KW CHNGE    |
|             | 13.72            | 0.94             | 0.25        | 3           | 0                | 31.86  | 0.10   | 1.00          | 0.03          | BEG RCHIE   |
|             | 13.72            | 0.75             | 0.25        | 3           | 15               | 0.51   | 0.10   | 1.00          | 0.03          | COMENV      |
|             | 13.76            | 0.47             | 0.00        | 10          | 15               | 0.00   | −1.00 | 0.00          | −1.00         | KW CHNGE    |
|             | 15.00            | 0.47             | 0.00        | 10          | 15               | 0.00   | −1.00 | 0.00          | −1.00         | MAX TIME    |
| Kepler-38c  | 0.0              | 0.95             | 0.25        | 1           | 0                | 31.59  | 0.10   | 0.05          | 0.03          | INITIAL     |
|             | 12.36            | 0.95             | 0.25        | 2           | 0                | 31.60  | 0.10   | 0.10          | 0.03          | KW CHNGE    |
|             | 13.01            | 0.95             | 0.25        | 3           | 0                | 31.45  | 0.09   | 0.14          | 0.03          | KW CHNGE    |
|             | 13.70            | 0.94             | 0.25        | 3           | 0                | 24.58  | 0.00   | 1.00          | 0.04          | BEG RCHIE   |
|             | 13.70            | 0.91             | 0.25        | 3           | 15               | 0.37   | 0.00   | 1.00          | 0.04          | COMENV      |
|             | 13.75            | 0.66             | 0.00        | 4           | 15               | 0.00   | −1.00 | 0.00          | −1.00         | KW CHNGE    |
|             | 13.88            | 0.63             | 0.00        | 5           | 15               | 0.00   | −1.00 | 0.00          | −1.00         | KW CHNGE    |
|             | 13.89            | 0.55             | 0.00        | 6           | 15               | 0.00   | −1.00 | 0.00          | −1.00         | KW CHNGE    |
|             | 13.89            | 0.52             | 0.00        | 11          | 15               | 0.00   | −1.00 | 0.00          | −1.00         | KW CHNGE    |
|             | 15.00            | 0.52             | 0.00        | 11          | 15               | 0.00   | −1.00 | 0.00          | −1.00         | MAX TIME    |

**Notes.** The BSE results for our entire set of simulations for all Kepler systems are published in their entirety in an online, machine-readable format.

* Time in the online supplement is in Myr.
* NTCP.
* TCP.

(This table is available in its entirety in machine-readable form.)

---

**Figure 1.** Evolution of, from top to bottom, the stellar mass, the CBP semimajor axis, and eccentricity for a Kepler-38-like system, where the binary is replaced with a single star that loses mass according to the $\alpha_{CE} = 0.5$ BSE simulation (for the primary CE phase) for two mass-loss regimes: runaway (red line) and adiabatic (green). Here, $T_{mass-loss}$ is the same as the corresponding $T_{CE}$ for Kepler-38, $\alpha_{CE} = 0.5$. The black dotted line in the upper panel represents the critical mass loss that would result in the ejection of the planet. The dashed lines in the middle and lower panels indicate the runaway (red) and adiabatic (green) approximations. The numerical evolution of the planet’s semimajor axis and eccentricity is fully consistent with the theoretical expectation, validating the applicability of our simulations.
complications by directly integrating the equations of motion during the CE phases for each Kepler CBP system.

The transition from adiabatic to nonadiabatic regime can be characterized through a mass-loss index $\psi$. For a planet orbiting a single star, Veras & Tout (2012) define

$$\psi \equiv \frac{\dot{M}}{m u} = (2\pi)^{-1} \left( \frac{\dot{M}}{M \text{ yr}^{-1}} \right) \times \left( \frac{a_{p,0}}{1 \text{ au}} \right) \left( \frac{\mu}{1 M_\odot} \right)^{-\frac{1}{2}}.$$  \hspace{1cm} (1)

where $\dot{M}$ is the mass-loss rate, $a_{p,0}$ is the initial semimajor axis of the planet, and $\mu = M_\text{star} + M_\text{p}$ is the total mass of the system. If $\psi \ll 1$, then the evolution of the planet’s orbit is in the adiabatic regime and its semimajor axis grows at constant eccentricity.

Alternatively, if $\psi \gg 1$, then the planet sees an “instantaneous” mass loss, and its orbital evolution is in a “runaway” regime. In this scenario, the fate of the planet depends on the ratio between the final and initial mass of the system, that is, $\beta = \mu_{\text{final}}/\mu_{\text{init}}$, and on the orbital phase of the planet at the onset of the mass-loss event. For example, if a highly eccentric planet is at pericenter at the beginning of the CE stage, it becomes unbound. Overall, ejection occurs when the system loses sufficient mass by the end of the mass-loss event. To first order, the critical mass ratio for ejection $\beta_{\text{eject}}$ is given by Equation (39) of Veras et al. (2011), which we reproduce here for completeness:

$$\beta_{\text{eject}} = 0.5(1 + e_{p,0})$$  \hspace{1cm} (2)

where $e_{p,0}$ is the eccentricity of the planet at the beginning of the mass-loss event. A planet on a circular orbit or at pericenter is ejected from the system if $\beta < \beta_{\text{eject}}$. If a planet at apocenter remains bound even in the runaway regime, and its orbit expands to $a_{\text{runaway, apocenter}}$ and circularizes or reaches an eccentricity of $e_{\text{post-circ}}$. If a planet remains bound in the runaway regime, its semimajor axis is given by Equations (43) and (44) of Veras et al. (2011):

$$a_{\text{runaway}} = \frac{a_{p,0}(1 + e_{p,0})}{2 - \beta(1 + e_{p,0})},$$  \hspace{1cm} (3)

where the signs represent initial pericenter and apocenter, respectively.

According to the above prescription, the dynamical evolution of a CBP’s orbit for $\psi \sim 0.1$−1 lies between these two regimes. For both $\psi \sim 1$ and $\psi \gg 1$, the evolution is not adiabatic, and $e_{\text{CBP}}$ can vary (increase or decrease) as $a_{\text{CBP}}$ varies. Thus the planet may become unbound only if $\Psi \gg 1$, with the caveat that the transition is not clear-cut and the $\Psi \sim 0.1$−1 regime is more complex (Veras et al. 2011; Veras & Tout 2012; Portegies Zwart 2013; Veras 2016). As we show below, the fact that the central object in a CBP system is itself a binary instead of a single star adds yet another layer of complication to these theoretical considerations.

Based on Equation (1), and assuming a CE timescale of $T_{\text{CE}} \sim 10^5$ years, we would a priori expect that all Kepler CBPs remain bound to their hosts during their primary stars’ CE stage (e.g., see Figure 3, Veras & Tout 2012). However, for the rapid CE timescale of $T_{\text{CE}} \sim 1$ year suggested by recent results (e.g., Ricker & Taam 2008, 2012), some of the Kepler CBP systems are within a factor of 5−10 of the transition to the nonadiabatic regime during the primary or secondary CE (i.e., $\Psi \sim 0.1$−0.2), and within a factor of 2 of the transition during their respective secondary CE stage (i.e., $\Psi \sim 0.5$). We might therefore expect significant variations in the respective CBPs’ orbital eccentricities, as well as deviations from the final semimajor axes expected from adiabatic mass loss.

Because the transition between adiabatic and nonadiabatic regimes is difficult to study analytically, here we examine the dynamical evolution of all Kepler CBP systems numerically, using the adaptive-time-step, high-order integrator IAS15 (Rein & Spiegel 2015), which is available as part of the modular and open-source REBOUND package (Rein & Liu 2012). REBOUND is written in C99 and comes with an extensive Python interface.

This paper is organized as follows. In Section 2 we describe the algorithm we use for the evolution of the Kepler CBP...
Table 6
Binary Evolution from BSE

| $a_{CE}$ | Tides | $M_1$ ($M_{Sun}$) | $M_2$ ($M_{Sun}$) | $a_{min}$ ($R_\odot$) | $e_{min}$ | Time RLOP | Notes |
|----------|-------|------------------|------------------|----------------------|----------|-----------|-------|
| Kepler-34 | ... | 1.05 | 1.02 | 49.2 | 0.52 | 0 | ... |
| 0.5/1/3/5/10 | NTCP | ... | ... | ... | ... | 9.64(1.7) | DD SN |
| 0.5/1/3/5/10 | TCP | ... | ... | ... | ... | 9.66(1.7) | DD SN* |
| Kepler-38 | ... | 0.95 | 0.25 | 31.6 | 0.1 | 0 | ... |
| 0.5 | NTCP | 0.76 | ... | ... | ... | 13.72(0.6) | CE + Merger |
| 0.5 | TCP | 0.91 | ... | ... | ... | 13.70(0.6) | CE + Merger |
| 1/3/5/10 | NTCP | 0.27 | 0.25 | 1.0/2.8/4.5/7.8 | ... | 13.72(1.2) | CE |
| 1/3/5/10 | TCP | 0.25 | 0.25 | 0.7/2.0/3.2/5.7 | ... | 13.70(1.1) | CE* |
| Kepler-47 | ... | 0.96 | 0.34 | 17.5 | 0.02 | 0 | ... |
| 0.5 | NTCP | 1.08 | ... | ... | ... | 12.03(1.3) | CE + Merger |
| 0.5 | TCP | 1.14 | ... | ... | ... | 11.99(1.3) | CE + Merger |
| 1.0 | NTCP | 0.78 | ... | ... | ... | 12.03(1.5) | CE + Merger |
| 1.0 | TCP | 0.96 | ... | ... | ... | 11.99(1.5) | CE + Merger |
| 3/5/10 | NTCP | 0.23 | 0.34 | 1.7/2.6/4.5 | ... | 12.03(1.7) | CE |
| 3/5/10 | TCP | 0.21 | 0.34 | 1.2/1.9/3.4 | ... | 11.99(1.6) | CE |
| Kepler-64 | ... | 1.53 | 0.41 | 38.7 | 0.22 | 0 | ... |
| 0.5 | NTCP | 1.53 | ... | ... | ... | 3.02(0.3) | CE + Merger |
| 0.5 | TCP | 1.68 | ... | ... | ... | 3.02(0.3) | CE + Merger |
| 1.0 | NTCP | 0.8 | ... | ... | ... | 3.02(0.3) | CE + Merger |
| 1.0 | TCP | 1.35 | ... | ... | ... | 3.02(0.3) | CE + Merger |
| 3/5/10 | NTCP | 0.28 | 0.41 | 2.4/3.9/7.0 | ... | 3.02(0.3) | CE |
| 3/5/10 | TCP | 0.26 | 0.41 | 1.6/2.6/4.7 | ... | 3.02(0.3) | CE |
| 3 | TCP | 0.67 | ... | ... | ... | 6.20(0.4) | CE* + Merger |
| Kepler-1647 | ... | 1.22 | 0.97 | 27.4 | 0.15 | 0 | ... |
| 0.5 | NTCP | 1.81 | ... | ... | ... | 5.45(0.45) | CE + Merger |
| 0.5 | TCP | 1.87 | ... | ... | ... | 5.44(0.46) | CE + Merger |
| 1.0 | NTCP | 1.28 | ... | ... | ... | 5.45(0.44) | CE + Merger |
| 1.0 | TCP | 1.45 | ... | ... | ... | 5.44(0.43) | CE + Merger |
| 3/5/10 | NTCP | 0.25 | 0.97 | 4.0/6.2/10.1 | ... | 5.45(0.45) | CE |
| 3/5/10 | TCP | 0.24 | 0.97 | 3.6/5.4/8.9 | ... | 5.44(0.36) | CE |
| 3 | TCP | 1.16 | ... | ... | ... | 6.07(0.35) | CE* + Merger |
| 5 | NTCP | 0.25 | 0.17 | 0.6 | ... | 12.06(0.9) | CE* |
| 5 | NTCP | ... | ... | ... | ... | 12.93(1.0) | DD SN |
| 5 | TCP | 1.15 | ... | ... | ... | 9.85(0.5) | CE* + Merger |
| 10 | NTCP | 0.25 | 0.2 | 1.9 | ... | 12.27(1.1) | CE* |
| 10 | TCP | 0.24 | 0.16 | 0.7 | ... | 11.86(0.9) | CE* |
| 10 | TCP | ... | ... | ... | ... | 13.99(0.5) | DD SN |
| Kepler-35 | ... | 0.88 | 0.81 | 37.9 | 0.14 | 0 | ... |
| 0.5 | NTCP | 1.1 | ... | ... | ... | 12.99(1.44) | CE + Merger |
| 0.5 | TCP | 1.09 | ... | ... | ... | 12.99(1.44) | CE + Merger |
| 1/3/5/10 | NTCP | 0.29 | 0.81 | 3.5/8.8/12.7/19.1 | ... | 12.99(1.44) | CE |
| 1/3/5/10 | TCP | 0.28 | 0.81 | 3.1/7.9/11.5/17.4 | ... | 12.99(1.44) | CE |

Notes. The numbers quoted in parentheses for the time represent the corresponding range obtained from BSE for $Z \pm 3\sigma_z$ (see text for details). The typical range for $M_1$ and $M_2$ is $\sim 0.01-0.1 M_\odot$, and $\sim 0.1-1 R_\odot$ for $a_{min}$.

* Roche-lobe overflow.
* Double-degenerate supernova.
* Secondary CE.
* Second Roche lobe fill and merger shortly after first CE for $a_{CE} = 1$.
* Only lower range because BSE does not accept $Z > 0.3$ (the metallicity of Kepler-64).
Kepler-38

| $a_{\text{CE}}$ | Tides | $a_{\text{CE, PCE}}$ (au) | Mode | Range | $a_{\text{CE, PCE}}$ (au) | Mode | Range |
|----------------|-------|----------------|------|-------|----------------|------|-------|
| 0.5            | NTCP  | 0.6±0.01      | 0.8  | 0.58–1.7 | 0.14±0.04 | 0.26 | 0.05–0.67 |
| 0.5$^a$        | TCP   | 0.5±0.01      | 0.6  | 0.49–0.9 | 0.4±0.01 | 0.2  | 0.03–0.42 |
| 1/3/5/10       | NTCP  | 1.3±0.01      | 1.3  | 0.9–3.8  | 0.3±0.01 | 0.42 | 0.02–0.82 |
| 1/3/5/10       | TCP   | 1.4±0.01      | 1.4  | 1.1–1.7  | 0.46±0.01 | 0.42 | 0.28–0.56 |

Kepler-47

| $a_{\text{CE}}$ | Tides | $a_{\text{CE, PCE}}$ (au) | Mode | Range | $a_{\text{CE, PCE}}$ (au) | Mode | Range |
|----------------|-------|----------------|------|-------|----------------|------|-------|
| 0.5, P1$^b$   | NTCP  | 0.31±0.00     | 0.34 | 0.3–0.44 | 0.12±0.04 | 0.14 | 0.06–0.27 |
| 0.5, P2$^b$   | NTCP  | 0.66±0.02     | 0.88 | 0.61–1.67 | 0.14±0.05 | 0.22 | 0.06–0.56 |
| 0.5, P3$^b$   | NTCP  | 0.9±0.01      | 1.2  | 0.8–2.87 | 0.17±0.03 | 0.23 | 0.09–0.66 |
| 0.5, P1$^b$   | TCP   | 0.32±0.00     | 0.34 | 0.31–0.39 | 0.4±0.01 | 0.1  | 0.07–0.18 |
| 0.5, P2$^b$   | TCP   | 0.7±0.01      | 0.87 | 0.67–1.24 | 0.06±0.01 | 0.21 | 0.06–0.42 |
| 0.5, P3$^b$   | TCP   | 0.93±0.01     | 1.18 | 0.88–1.89 | 0.11±0.01 | 0.22 | 0.09–0.48 |
| 1.0, P1$^b$   | NTCP  | 0.49±0.01     | 0.49 | 0.47–0.5  | 0.08±0.01 | 0.08 | 0.04–0.13 |
| 1.0, P2$^b$   | NTCP  | 1.33±0.01     | 1.41 | 1.12–1.97 | 0.42±0.14 | 0.4  | 0.22–0.58 |
| 1.0, P3$^b$   | NTCP  | 1.65±0.03     | 2.13 | 1.52–4.15 | 0.5±0.14 | 0.51 | 0.29–0.75 |
| 1.0, P1$^b$   | TCP   | 0.38±0.00     | 0.39 | 0.37–0.41 | 0.11±0.01 | 0.07 | 0.01–0.12 |
| 1.0, P2$^b$   | TCP   | 0.91±0.01     | 1.03 | 0.89–1.22 | 0.36±0.12 | 0.26 | 0.14–0.38 |
| 1.0, P3$^b$   | TCP   | 1.23±0.01     | 1.46 | 1.2–1.83  | 0.44±0.01 | 0.32 | 0.17–0.45 |
| 3/5/10, P1$^d$| NTCP  | 0.67±0.01     | 0.67 | 0.66–0.68 | 0.11±0.01 | 0.11 | 0.04–0.17 |
| 3/5/10, P2$^d$| NTCP  | 3.23±0.00     | 3.4  | 2.23–7.95 | 0.71±0.06 | 0.73 | 0.57–0.89 |
| 3/5/10, P3$^d$| NTCP  | 7.64±2.71     | 9.97 | 4.48–22.52 | 0.97±0.02 | 0.89 | 0.74–0.95 |
| 3/5/10, P1$^d$| TCP   | 0.69±0.01     | 0.69 | 0.68–0.7  | 0.12±0.01 | 0.11 | 0.08–0.14 |
| 3/5/10, P2$^d$| TCP   | 3.68±0.01     | 4.18 | 3.34–5.62 | 0.75±0.01 | 0.78 | 0.72–0.83 |
| 3/5/10, P3$^d$| TCP   | 15.69±2.35    | 19.7 | 15.3–23.01 | 0.93±0.02 | 0.94 | 0.93–0.95 |

Notes. The modes are shown with their respective 68% range. We caution that most of the probability distribution functions (PDFs) are notably nonnormal, often with two dominant peaks at the range limits.

$^a$ Sinusoidal distributions in phase versus final $a_{\text{CE}}/a_{\text{PCE}}$ (see, e.g., Figure 5), typically with double-peaked PDFs.

$^b$ P1/P2/P3 have 40%/15%/75% probability to become dynamically unstable within 1 Myr after the CE phase.

$^c$ Given the large parameter space and the associated uncertainties, we consider $a_{\text{CE}} > 0.95$ as ejection.

$^d$ P1/P2/P3 have 60%/15%/85% probability to become dynamically unstable within 1 Myr after the CE phase.

Table 7

PCE CBP Semimajor Axis and Eccentricity for $a_{\text{CE}} = 1.0 M_{\text{Sun}}$ yr$^{-1}$ and for Primary CE

2. BINARY STAR EVOLUTION

The evolution of a gravitationally bound system of two stars depends primarily on its initial configuration. If the binary is wide enough, the stars will evolve in isolation, according to the prescription of single-star evolution theory. Alternatively, if the initial separation is sufficiently small, then the stars will influence each other’s evolution. The evolution of such systems, described as close (or interacting) binary stars (Hilditch 2001), is more complex and includes a number of additional processes, such as tidal interactions and mass transfer. The nine Kepler CBP-harboring binary systems fall in the latter category, and we use the established, open-source binary star evolution code (Hurley et al. 2002) to study their evolution. Briefly, the code works as follows.

While the binary is in a detached state, the code evolves each star individually using a single-star evolutionary code (Hurley et al. 2000), which includes tidal and braking mechanisms and wind accretion. When the stars begin to interact, BSE uses a suite of binary-specific features, such as mass transfer and accretion, CE evolution, collisions and mergers, and angular momentum loss mechanisms. The evolution algorithm allows the specific processes to be turned on and off, or even modified based on custom requirements. By default, all stars are assumed to be initially on the zero-age main sequence (ZAMS), but any possible evolutionary state, such as corotation with the orbit, can be the starting point of the simulation. BSE uses an evolution time step small enough to prevent the stellar mass and radius from changing too much (not more than 1% in mass and 10% in radius), which allows identification of the time when, and if, a star first fills its Roche lobe.

At the onset of Roche-lobe overflow (ROLOF), the two stars either come into contact and coalesce or initiate a CE stage. CE evolution is a complex mechanism that typically occurs when an evolved, giant star transfers mass to an MS star on a dynamical timescale.12 When the giant star overfills the Roche lobe of both stars, its core and the MS star share a single envelope. This envelope rotates slower than the orbiting “cores” within due to expansion, and the resulting friction causes in-spiraling and energy transfer to the envelope, described by an efficiency parameter $a_{\text{CE}}$. The outcome of this phase is either ejection of the envelope (assumed to be isotropic) if neither core fills its Roche lobe, leaving a close white dwarf–MS pair (assumed to be in corotation with the

12 A collision between a dense core and a star can also trigger a CE phase.
orbit), or coalescence of the cores. A challenging process to study numerically, BSE recognizes the CE phase—beginning with RLOF—and forces the system through it on an instantaneous timescale, allowing the evolution to continue according to the appropriate PCE parameters. The code resolves the outcome of the CE phase based on the initial binding energy of the envelope and on the initial orbital energy of the two cores.\textsuperscript{13}

On input, BSE requires a number of parameters. Namely, the CE efficiency parameter \(\alpha_{\text{CE}}\) (in the range of 0.5–10), the masses and stellar types of both the primary and secondary stars (in the range of 0.1–100 \(M_{\odot}\)), the binary orbital period (in days), the eccentricity (0.0–1.0), the maximum time for the evolution (here we choose 15 Gyr, roughly the age of the universe, unless a particular Kepler CBP system stops evolving—or the stars coalesce—much sooner, at which point we stop the simulations), and the metallicity (0.0001–0.03) of the system. The code also checks whether tidal evolution is included, noted as “on” (hereafter TCP for tidal circularization path) or “off” (NTCP for no tidal circularization path). Given the still-uncertain mechanism of the CE stage, the parameters describing the efficiency of envelope ejection and the treatment of tidal decay—and their associated uncertainties—are likely the dominant source of error in our BSE results.

Depending on \(\alpha_{\text{CE}}\), a binary system may exit a CE stage as a merged star, as a very tight PCE binary star, or may even trigger a supernova explosion. For example, a higher \(\alpha_{\text{CE}}\) accounts for energy sources other than orbital energy, and the system then requires less energy to dissipate the envelope. The BSE code also allows an alternate CE model to be used, the de Kool CE evolution model (de Kool 1990), which first introduced the CE evolution binding energy factor \(\lambda\).

Likewise, tides are integral to the evolution of close binary stars. The strength of tidal dissipation will affect the orbital separation and eccentricity when mass transfer begins and hence the future evolution under RLOF or a CE stage. BSE achieves orbital circularization mainly through tides, but also by accounting for more mass accretion at pericenter than apocenter of an eccentric orbit. Another effect is the exchange of orbital angular momentum with the component spins due to tidal synchronization, thus causing the evolution to proceed in a similar way to closer binary systems if tides were ignored. In addition, primary spin–orbit corotation can be forced within the code to avoid unstable RLOF, which gives two possible outcomes for each system. To account for the effect of tides on binary evolution, we use both the tides “on” (TCP) or “off” (NTCP) options in BSE. As a result, the binary starts the first

\textsuperscript{13} BSE also accounts for collisions that do not proceed through CE evolution, and for the possibility of core sinking depending on the stellar types (which may lead to rejuvenation).
Table 9
Same as Table 7 but for $\alpha_M = 0.1 \, M_{\odot} \, \text{yr}^{-1}$

| $\alpha_{\text{CE}}$ | Tides | $a_{\text{CBP-PCCE}}$ (au) | Range | Mode | $e_{\text{CBP-PCCE}}$ | Range | Notes |
|---------------------|-------|-----------------------------|-------|------|------------------------|-------|-------|
| Kepler-38           |       |                             |       |      |                        |       |       |
| 0.5                 | NTCP  | 0.73                        | 0.73  | ...  | 0.12                   | 0.12  | ...   | $[a_{\text{CBP,0}} = 0.46]$ |
| 0.5                 | TCP   | 0.6                         | 0.6   | ...  | 0.03                   | 0.03  | ...   | |
| 1/3/5/10            | NTCP  | 1.06                        | 1.06  | ...  | 0.13                   | 0.13  | ...   | |
| 1/3/5/10            | TCP   | 1.1                         | 1.11  | ...  | 0.02                   | 0.02  | ...   | |
| Kepler-47           |       |                             |       |      |                        |       |       |
| 0.5, P1             | NTCP  | 0.34                        | 0.34  | ...  | 0.04                   | 0.04  | ...   | $[a_{\text{CBP,0}} = 0.29]$ |
| 0.5, P2             | NTCP  | 0.83                        | 0.83  | ...  | 0.04                   | 0.04  | ...   | $[a_{\text{CBP,0}} = 0.70]$ |
| 0.5, P3             | NTCP  | 1.13                        | 1.14  | ...  | 0.06                   | 0.06  | ...   | $[a_{\text{CBP,0}} = 0.96]$ |
| 0.5, P1             | TCP   | 0.34                        | 0.34  | ...  | 0.01                   | 0.01  | ...   | |
| 0.5, P2             | TCP   | 0.8                        | 0.83  | ...  | 0.02                   | 0.02  | ...   | |
| 0.5, P3             | TCP   | 1.13                        | 1.14  | ...  | 0.03                   | 0.03  | ...   | |
| 1.0, P1             | NTCP  | 0.48                        | 0.48  | ...  | 0.03                   | 0.03  | ...   | |
| 1.0, P2             | NTCP  | 1.18                        | 1.18  | ...  | 0.05                   | 0.05  | ...   | |
| 1.0, P3             | NTCP  | 1.6                         | 1.61  | ...  | 0.06                   | 0.06  | ...   | |
| 1.0, P1             | TCP   | 0.39                        | 0.39  | ...  | 0.01                   | 0.01  | ...   | |
| 1.0, P2             | TCP   | 0.96                        | 0.96  | ...  | 0.02                   | 0.02  | ...   | |
| 1.0, P3             | TCP   | 1.3                         | 1.31  | ...  | 0.03                   | 0.03  | ...   | |
| 3/5/10, P1          | NTCP  | 0.66                        | 0.66  | ...  | 0.04                   | 0.04  | ...   | |
| 3/5/10, P2          | NTCP  | 1.61                        | 1.61  | ...  | 0.07                   | 0.07  | ...   | |
| 3/5/10, P3          | NTCP  | 2.2                         | 2.2   | ...  | 0.08                   | 0.08  | ...   | |
| 3/5/10, P1          | TCP   | 0.69                        | 0.69  | ...  | 0.01                   | 0.01  | ...   | |
| 3/5/10, P2          | TCP   | 1.67                        | 1.67  | ...  | 0.03                   | 0.03  | ...   | |
| 3/5/10, P3          | TCP   | 2.28                        | 2.28  | ...  | 0.04                   | 0.04  | ...   | |

Notes. Kepler-38b, Kepler-47bcd, and Kepler-64b evolve adiabatically. Unless indicated otherwise, the spread of their semimajor axis and eccentricity range is practically zero and for simplicity omitted.

(a) Sinusoidal distributions in phase versus final $a_{\text{CBP}}/e_{\text{CBP}}$ (see Figure 5), typically with double-peaked PDFs.

(b) Given the large parameter space and the associated uncertainties, we consider $e > 0.95$ as ejection.

RLOF at either $e_{\text{bin,RLOF}} = 0$ and $a_{\text{bin,RLOF}} < a_{\text{bin,0}}$ (for TCP), or at $e_{\text{bin,RLOF}} = e_{\text{bin,0}}$ and $a_{\text{bin,RLOF}} = a_{\text{bin,0}}$ (for NTCP).

Additionally, while the binary periods of the Kepler CBP systems are known to very high precision, their stellar masses and metallicities have nonnegligible uncertainties. For example, the primary masses for Kepler-38 and Kepler-64 have 1σ errors of 0.05 $M_{\odot}$ and 0.1 $M_{\odot}$, respectively, and the 1σ errors on the metallicities of half of the systems are as large as, or even larger than, the measured values. To better characterize the impact of these uncertainties on the evolution of the Kepler CBP systems, we obtain BSE results for both the best-fit mass and metallicity values and also for their corresponding 3σ range.

The entire parameter space we explored, shown in Table 1, includes a range of CE parameters as well and consists of 22,680 BSE simulations. Our default configurations are denoted in boldface in the table; for the rest of the input parameters, we use the default values from model A of Hurley et al. (2002), described as the most favorable and effective. Model A uses all default wind-related parameters and a Reimers mass-loss coefficient of $\eta = 0.5$.

As BSE evolves the stars, their types change, and each type is checked throughout the code to ensure the correct evolutionary tracks are used for both the individual stars and the system as a whole. These numerical values, as well as what they represent, are listed in Table 2. At each significant event, such as at the beginning and end of the RLOF and of the CE phase, when the stars coalesce or collide, the code also produces a “type” label. These labels, along with their meaning, are shown in Table 3.

An example of the BSE output for Kepler-38, $\alpha_{\text{CE}} = 0.5$, NTCP, and TCP is shown in Table 4. The BSE results for all Kepler systems can be found in an online supplement to Table 4.

In the following section we describe how we use the initial and BSE-generated final binary masses and orbital separations for each Kepler CBP system to explore the dynamical response of the planets during the respective CE stages.

3. DYNAMICAL SIMULATIONS

While the concept of CE evolution for binary stars was proposed 40 years ago (Paczynski 1976), detailed understanding of this important stage is a challenging task that requires computationally intensive hydrodynamical calculations. The outcome of this complex phase has been studied both numerically and analytically (see Table 1 of Iaconi et al. 2016 for a list of references). For example, hydrodynamical simulations by Ricker & Taam (2012) for a 1.05 $M_{\odot}$+0.36 $M_{\odot}$ binary star with an initial orbital period of 44 days (similar to Kepler CBP hosts) show that in the later stages of the CE the system loses mass at a rate of 2 $M_{\odot}$ yr$^{-1}$, the binary orbital separation decreases by a factor of 7 after ~56 days, and ~90% of the outflow is contained within 30 degrees of the binary’s orbital plane.
Table 10
Same as Table 9

| α<sub>CE</sub> | Tides | α<sub>CBP,PCE (au)</sub> | e<sub>CBP,PCE</sub> | Notes |
|--------------|-------|-----------------|-----------------|-------|
|               | Mode  | Median | Range | Mode  | Median | Range |
| Kepler-64    |       |        |       |       |        |       |
| 0.5          | NTCP  | 0.82   | 0.82  | ...   | 0.25   | 0.25  | 0.22-0.27 | \[α_{CBP,0} = 0.65\] |
| 0.5<sup>f</sup> | TCP   | 0.72   | 0.72  | ...   | 0.02   | 0.02  | ... |
| 1.0          | NTCP  | 1.58   | 1.58  | ...   | 0.31   | 0.33  | 0.31-0.35 |
| 1.0<sup>f</sup> | TCP   | 0.91   | 0.91  | ...   | 0.01   | 0.01  | ... |
| 3/5/10       | NTCP  | 1.83   | 1.83  | ...   | 0.3    | 0.35  | 0.31-0.39 |
| 3/5/10       | TCP   | 1.9    | 1.9   | ...   | 0.02   | 0.02  | ... |
| Kepler-1647<sup>b</sup> |       |        |       |       |        |       |
| 0.5          | NTCP  | 2.52<sup>±</sup>1.53<sup>±0.01</sup> | 3.55 | 2.33-40.61 | 0.7<sup>±</sup>0.02 | 0.71 | 0.62-0.95<sup>a</sup> | 27% ejected |
| 0.5<sup>f</sup> | TCP   | 3.1<sup>±</sup>0.19<sup>±0.01</sup> | 3.19 | 3.08-3.3 | 0.11<sup>±</sup>0.08 | 0.07 | 0.02-0.12 |
| 1.0          | NTCP  | 3.91<sup>±</sup>0.67<sup>±0.01</sup> | 4.45 | 3.83-20.27 | 0.72<sup>±</sup>0.22 | 0.81 | 0.71-0.95 | 27% ejected |
| 1.0<sup>f</sup> | TCP   | 4.1    | 4.1   | ...   | 0.07   | 0.07  | ... |
| 3/5/10       | NTCP  | 4.7<sup>±</sup>0.63<sup>±0.01</sup> | 4.93 | 4.29-19.14 | 0.7<sup>±</sup>0.10 | 0.76 | 0.57-0.95 | 4% ejected |
| 3/5/10       | TCP   | 4.93   | 4.93  | ...   | 0.08   | 0.08  | ... |
| Kepler-1647<sup>c</sup> |       |        |       |       |        |       |
| 5.0<sup>c</sup> | NTCP  | 7.9<sup>±</sup>5.3<sup>±0.01</sup> | 9.9  | 7.2-55.9 | 0.48<sup>±</sup>0.9<sup>±0.1</sup> | 0.55 | 0.2-0.95 | 82% ejected |
| 10.0<sup>c</sup> | TCP   | 7.7<sup>±</sup>0.38<sup>±0.01</sup> | 9.2  | 6.9-31.1 | 0.32<sup>±</sup>0.05<sup>±0.01</sup> | 0.54 | 0.24-0.95 | 81% ejected |
| 10.0<sup>c</sup> | TCP   | ...    | ...   | ...   | ...    | ...   | ... |
| Kepler-35<sup>d</sup> |       |        |       |       |        |       |
| 0.5          | NTCP  | 0.94   | 0.94  | ...   | 0.37   | 0.37  | 0.33-0.41 | \[α_{CBP,0} = 0.60\] |
| 0.5          | TCP   | 0.95   | 0.95  | ...   | 0.03   | 0.03  | ... |
| 1/3/5/10     | NTCP  | 0.94   | 0.94  | ...   | 0.27   | 0.27  | 0.18-0.39 |
| 1/3/5/10     | TCP   | 0.95   | 0.95  | ...   | 0.02   | 0.02  | ... |

Notes.

- <sup>a</sup> Given the large parameter space and the associated uncertainties, we consider \( e > 0.95 \) as ejection.
- <sup>b</sup> Primary RLOF and CE.
- <sup>c</sup> Secondary RLOF and CE.
- <sup>d</sup> Experiences CE for \( Z - 1\sigma_z \).
- <sup>e</sup> Nonnormal PDF with multiple peaks of comparable strength, with mode not well defined.

To numerically account for the dynamical impact on Kepler CBPs from such drastic orbital reconfigurations and tremendous, short-timescale mass-loss phases, we use REBOUNDx, a library for incorporating additional effects beyond point-mass gravity in REBOUND simulations (https://github.com/dtamayo/reboundx). In particular, we shrink the binary orbits with the REBOUNDx modify_orbits_forces routine, which adds to the equations of motion a simple drag force for opposite particles’ velocity vectors (Papaloizou & Larwood 2000). When orbit-averaged over a Keplerian orbit, this approach results in an exponential damping of the semimajor axis where the e-folding timescale is set through the particle parameter \( \tau_a \). Hydrodynamical simulations by Passy et al. (2012) and Iaconi et al. (2016) exhibit similar orbital decay during the CE stage (e.g., see their respective Figures 5 and 14), validating our approach.

Given the complexities and uncertainties of the mass-loss mechanism during the CE phases, we adopt a simple analytical parameterization similar to the method of Portegies Zwart (2013), who tackled the problem by using a constant mass-loss rate. Here we choose to instead use the same functional forms for the CE-driven changes in \( M_{\text{star}} \) as we do for \( \dot{M}_{\text{bin}} \) (utilizing the REBOUNDx modify_mass routine), that is, an exponential mass loss for particles on their assigned e-folding timescales \( \tau_M \). Namely, for a given CE stage where one of the stars transitions from an initial mass \( M_0 \) to a final mass \( M_f \) (both provided by BSE), the mass loss occurs over a time \( T_{\text{CE}} = (M_f - M_0)/\dot{M}_M \). We note that an exponential mass loss has been explored by Adams et al. (2013) and Adams & Bloch (2013) as well, but for the case of single-star systems.

To ensure a smooth mass evolution (and avoid possible numerical artifacts) that asymptotes at \( M_f \) we linearly evolved REBOUNDx’s exponential mass-loss rate so that it vanished at time \( T_{\text{CE}} \). In particular, in terms of the e-folding timescale \( \tau_M \),

\[
\frac{M}{\tau_M}, \quad \tau_M = \frac{\tau_{M,0}}{(1-1/T_{CE})},
\]

This differential equation can be solved analytically, yielding

\[
M(t) = M_f \left( \frac{M_0}{M_f} \right)^{1-(1/T_{CE})}
\]

as long as \( \tau_{M,0} \) in Equation (4) is chosen as

\[
\tau_{M,0} = \frac{T_{CE}}{2 \ln \left( \frac{M_0}{M_f} \right)}.
\]

As the star’s mass decreases from \( M_0 \) to \( M_f \) during the respective CE stage, the binary orbit must shrink from \( \theta_{\text{bin},0} \) to \( \theta_{\text{bin,PCE}} \) (as provided by BSE) as well, and both must occur over a time \( T_{\text{CE}} \). We apply the same prescription for evolving \( \theta_{\text{bin}} \) as that for the mass loss (i.e., Equations (4)–(6) with \( \tau_a \).
instead of $\tau_M$, but we caution that while this would evolve $a_{bin,0}$ from $a_{bin,0}$ to $a_{bin,PCE}$ in isolation, the mass loss is simultaneously increasing $a_{bin}$. To correct for this and achieve the desired $a_{bin,PCE}$, we manually adjust the corresponding $\tau_{a,0}$ parameter.

While tides are not directly accounted for in our dynamical simulations as the particles involved are treated as point masses, we capture their effects in a simple parameterized way. Specifically, we impose the same exponential decay on the evolution of $e_{bin}$ as we do for the binary’s semimajor axis shrinking and stellar mass loss (i.e., Equations (4)–(6) but with $\tau_e$). As a result, $e_{bin}$ undergoes dampened oscillations during the CE phase, at the end of which it settles within a few percent of zero, as required by BSE.

As mentioned in the previous section, while the BSE code yields initial and final values for the stellar masses and $a_{bin}$ at the beginning and end of the CE phases, it does not resolve the extremely rapid mass-loss phase. We bridge this gap by exploring two regimes of mass loss, namely $\alpha_{M} = 1.0 M_\odot \, \text{yr}^{-1}$ and $\alpha_{0.1} = 0.1 M_\odot \, \text{yr}^{-1}$, denoted by their respective subscripts. The former is based on the results of Ricker & Taam (2008, 2012) and Portegies Zwart (2013) for systems similar to the Kepler CBP hosts. Values of $\alpha_{0.1}$ (and smaller) typically guarantee that the orbital evolution of these CBPs is in the adiabatic regime (see Equation (1)).

To test the applicability of our numerical simulations to the stated problem, we examine the evolution of a mock Kepler-38-like system where the binary is replaced with a single star of the same total initial and final mass according to the BSE prescription for Kepler-38 (see Table 4) and using the same initial orbit of the planet. Here we test two mass-loss regimes: runaway (for $\alpha_M = 100.0 M_\odot \, \text{yr}^{-1}$) and adiabatic (for $\alpha_M = 0.1 M_\odot \, \text{yr}^{-1}$). The results are shown in Figure 1, where the three panels represent (from upper to

Figure 2. Orbital reconfiguration of the Kepler-38 system during the primary CE stage. The binary orbits are shown on the left panels (blue lines for the primary star, magenta lines for the secondary) and the CBP orbits on the right (green lines for $a_{0.1}$, red lines for $a_1$, where dashed red corresponds to the largest and the solid red to the smallest achieved orbit). The binary merges for $\alpha_{CE} = 0.5$. The square symbols indicate the initial positions of the two stars and the planet; the circle symbols in the left panels indicate the final positions of the primary (blue) and secondary (black) star. The right panels show the evolution of $a_{CBP}$ from different initial conditions, as indicated by the respective squares.
During the primary RLOF and CE for a CE phase for different orbiting a single star losing mass, where the planet
the subsequent lower panels represent the respective adiabatic and runaway expansions.

16 For the planet to be ejected, the mass lost is smaller than the critical mass limit for ejection,
should remain bound even in the runaway mass-loss regime as indicated by the dotted line in the upper panel of the
during a CE phase for different $\Delta \theta_0$, the initial true anomaly difference between the binary star and the CBP at the onset of the CE ($\Delta \theta_0 \equiv \theta_{\text{CBP},0} - \theta_{\text{bin},0} - \omega_{\text{bin}}$). Unlike the case of a planet orbiting a single star losing mass, where the planet’s orbit follows a single evolutionary track, a CBP orbiting a binary star can experience different evolutionary tracks during a CE phase for different $\Delta \theta_0$. The individual evolutionary tracks for $a_{\text{CBP}}$ and $e_{\text{CBP}}$ are condensed into hatched regions on the third and fourth panels (and in the subsequent figures) for clarity. The dotted black line in the upper panel represents the critical mass for ejection of the CBP, and the dashed lines in the middle and lower panels represent the respective adiabatic and runaway expansions.

Figure 3. Evolution of, from top to bottom, the mass of the Kepler-38 primary star and the binary semimajor axis (blue), $a_{\text{CBP}}$, and $e_{\text{CBP}}$ (red for $\alpha_{\text{CE}}$, green for $\alpha_0$) during the primary RLOF and CE for $\alpha_{\text{CE}} = 0.5$. The binary merges at the end of the CE, at $t = T_{\text{CE}}$. The second panel shows the dependence of the evolution of $a_{\text{CBP}}$ on $\Delta \theta_0$, the initial true anomaly difference between the binary star and the CBP at the onset of the CE ($\Delta \theta_0 \equiv \theta_{\text{CBP},0} - \theta_{\text{bin},0} - \omega_{\text{bin}}$). Unlike the case of a planet orbiting a single star losing mass, where the planet’s orbit follows a single evolutionary track, a CBP orbiting a binary star can experience different evolutionary tracks during a CE phase for different $\Delta \theta_0$. The individual evolutionary tracks for $a_{\text{CBP}}$ and $e_{\text{CBP}}$ are condensed into hatched regions on the third and fourth panels (and in the subsequent figures) for clarity. The dotted black line in the upper panel represents the critical mass for ejection of the CBP, and the dashed lines in the middle and lower panels represent the respective adiabatic and runaway expansions.

lower, respectively) the evolution of the stellar mass, of the planet’s semimajor axis, and of the planet’s eccentricity as a function of time (such that mass loss starts at $t = 0$ and ends at $t = T_{\text{mass-loss}}$). According to the analytic prescription, the planet should remain bound even in the runaway mass-loss regime as the mass lost is smaller than the critical mass limit for ejection, as indicated by the dotted line in the upper panel of the figure.16 The horizontal dashed lines in the middle and upper panels represent the theoretical adiabatic (green) and runaway (red) approximations. As seen from these two panels, the numerical simulations are fully consistent with the theoretical approximations, demonstrating the validity and applicability of our simulations.

In order to interpret the results from numerical integrations, it is useful to define a critical mass-loss rate, $\alpha_{\text{crit}}$, corresponding to $\Psi = 1$ (see Equation (1)). CBPs in systems with $\alpha_{\text{M}} \gg \alpha_{\text{crit}}$ will experience “instantaneous” mass loss, while the orbits of those in systems with $\alpha_{\text{M}} \ll \alpha_{\text{crit}}$ should evolve adiabatically. A CBP in a system where $\alpha_{\text{M}}$ becomes comparable to or larger than $\alpha_{\text{crit}}$ at any point during the binary evolution risks becoming unbound. Secondary CE stages will have two different $\alpha_{\text{crit}}$ values as the respective CBPs will have two different values for $a_{\text{CBP}}$ and $e_{\text{CBP}}$ (but the same $\mu$) at the end of the preceding CE stages (see Equation (1)): one for $\alpha_1$ and another for $\alpha_0$. We denote these critical values as $\alpha_{\text{crit}}(\alpha_1)$ and $\alpha_{\text{crit}}(\alpha_0)$. In the subsections below, for each CBP Kepler system, we compare $\alpha_{\text{crit}}$ to $\alpha_1$ and $\alpha_{\text{crit}}(\alpha_0)$ for each CE phase. Where the adopted $\alpha_1$ is within a factor of 10 of the respective $\alpha_{\text{crit}}$, we pay special attention to the CBP orbit and test its evolution for 0.1, 0.2, $\alpha_1$, and 0.5 $\alpha_1$ as well. Unless indicated otherwise, the final fate of the CBP for these cases is similar to the case for $\alpha_1$.

The description of a three-body system containing an evolving binary star is inherently multidimensional, and minor changes in one parameter can cascade into major changes in the rest. In addition to the importance of $\alpha_{\text{CE}}$, tides, and $\alpha_{\text{M}}$, another key issue—as shown in the next section—is the phase offset between the binary star and the CBP at the onset of the CE phase, $\Delta \theta_0 \equiv \theta_{\text{CBP},0} - \theta_{\text{bin},0} - \omega_{\text{bin}}$ (i.e., the difference between their true anomalies, taking into account the binary’s argument of the pericenter).

Thus for each system we test four initial binary phases corresponding to the binary orbit turning points, that is, eastern and western elongations (EE and WE), and superior and inferior conjunctions (SC and IC), and 50 initial CBP phases (between 0 and 1, with a step of 0.02) for a total of 200 initial conditions. We explore each of these initial conditions for
\[ \alpha_{\text{CE}} = (0.5, 1.0, 3.0, 5.0, 10.0), \] for TCP/NTCP, and for \( \alpha_1 \) and \( \alpha_{0.1} \).

Finally, we note that the post-CE semimajor axis of a CBP \( (a_{\text{CBP,PCE}}) \) that was initially on an eccentric orbit (i.e., \( e_{\text{CBP,0}} > 0 \)) depends on its orbital phase at the onset of the CE stage. For those Kepler systems that undergo a secondary CE phase with mass loss, we start the respective integrations at the modes of \( a_{\text{CBP,PCE}} \) and \( e_{\text{CBP,PCE}} \) from the preceding CE stage.

4. RESULTS

Here we describe the results of our simulations for those Kepler CBP systems that experience at least one CE phase. Unless specified otherwise, the evolution of each system is for the default CE parameters listed in Table 1. The initial orbital parameters for each system are listed in Table 5. We note that our \( N \)-body integrations only cover the CE phases.

Below we outline the major binary evolution stages (as produced by BSE), along with the dynamical responses of the CBPs; the main results are presented in Tables 6–10. All reported times are in Gyr since ZAMS. For simplicity, we assume that the current Kepler CBP systems are at ZAMS and that all CBPs start on initially circular, coplanar orbits.

Overall, five of the nine Kepler CBP systems experience at least one CE phase for their default metallicities (see Table 1) over the duration of the BSE simulations (15 Gyr): Kepler-34, -38, -47, -64, and -1647. Kepler-35 experiences a CE for \( Z - 1 \sigma_Z \), and Kepler-453 for \( Z - 2 \sigma_Z \).

4.1. Kepler-34

The binary consists of 1.05 \( M_\odot \) and 1.02 \( M_\odot \) stars on a highly eccentric (\( e_{\text{bin,0}} = 0.52 \)), 28 day orbit that changes little in the first \( \sim 9 \) Gyr. The CBP has an initial orbit of 1.09 au.

The primary star fills its Roche lobe at \( \sim 9.6 \) Gyr, and the binary enters a CE stage and triggers a double-degenerate supernova event, disrupting the system.

4.2. Kepler-38

The binary consists of 0.95 \( M_\odot \) and 0.25 \( M_\odot \) stars on a slightly eccentric (\( e_{\text{bin,0}} = 0.1 \)), 19 day orbit that changes little in the first \( \sim 13 \) Gyr. The CBP has an initial orbit of 0.46 au. The main evolution stages of the binary star and their effect on the CBP are as follows.

4.2.1. The Binary

The primary star fills its Roche lobe at \( t \sim 13.7 \) Gyr, and the binary enters a primary CE stage. During this stage, \( \Psi (\alpha_1) \approx 0.04 \) and \( \alpha_{\text{crit}} \approx 26 \; M_\odot \; \text{yr}^{-1} \), much larger than \( \alpha_1 \) and \( \alpha_{0.1} \). As a result of the CE, the binary merges as a first giant branch star and loses \( \sim 25-40\% \) of its mass for \( \alpha_{\text{CE}} = 0.5 \); here \( \beta > \beta_{\text{eject}} \) (see Equation (3)), and the planet should remain bound in a runaway regime (i.e., \( \Psi \gg 1 \)) even at pericenter (see Equations (37)–(44), Veras et al. 2011). The system continues to slowly lose mass by the end of the BSE simulations (15 Gyr), so \( a_{\text{CBP,PCE}} \) adiabatically expands by up to a factor of 2.
For $\alpha_{CE} = 1.0, 3.0, 5.0, \text{ and } 10.0$, the binary orbit shrinks by a factor of 4–30 and its mass decreases by ~60%; here $\beta < \beta_{sect}$, that is, the planet should be ejected in a runaway regime. For $\alpha_{CE} = 1$ and TCP, the system experiences a secondary RLOF at $t \sim 13.74$ Gyr and coalesces into a first giant branch star without mass loss (thus with no changes in $a_{\text{CBP, PCE}}$ or $e_{\text{CBP, PCE}}$). The binary does not lose additional mass by the end of the BSE simulations.

Overall, by 15 Gyr the binary evolves into either a merged helium or carbon/oxygen white dwarf (HeWD, COWD) or a very close HeWD–MS star binary.

The orbital reconfiguration of the binary system during the primary CE stage is shown on the left panels of Figure 2 for $\alpha_{CE} = 0.5$ (upper panels) and $\alpha_{CE} = 1.0, 3.0, 5.0, \text{ and } 10.0$ (lower panels). The observer is looking along the $y$ axis, from below the figure. The maximum (red dashed line; binary initially at EE, CBP initially at phase 0.73) and minimum (red solid line; binary at EE, CBP at phase 0.08) orbital expansion of the CBP is shown on the right panels of the figure for $\alpha_{1}$ (red), as well as the maximum expansion for $\alpha_{0.1}$ (green solid line). To test the dynamical stability of the CBP at maximum expansion (dashed red line), we integrated the system for a thousand planetary orbits. The orbit precesses but does not exhibit chaotic behavior for the duration of the integration. We leave examination of the long-term behavior and stability of the system for future work.

4.2.2. The CBP

In Figure 3 we show the $\alpha_{CE} = 0.5$ CE evolution of $M_{1}$, of $a_{\text{bin}}$ (blue lines, upper panel), and of $a_{\text{CBP}}$ (middle panels) and $e_{\text{CBP}}$ (lower panel) for $\alpha_{1}$ (red) and $\alpha_{0.1}$ (green); on Figure 4 we show the corresponding evolution for $\alpha_{CE} = 1, 3, 5, \text{ and } 10$. Unlike the case of a planet orbiting a single star, where the evolution of the planet’s orbit follows a single path for each $\alpha_{M}$ (see Figure 1), the dynamical response of a CBP to a binary undergoing a CE stage depends on the

Figure 5. Dependence of $a_{\text{CBP, PCE}}$ (upper) and $e_{\text{CBP, PCE}}$ (lower) on the initial phase difference between the binary and the CBP $\Delta \theta_{0}$ and on $\alpha_{M}$ for Kepler-38, and for $\alpha_{CE} = 0.5$. The different colors for NTCP (blue, red, green, cyan) correspond to different initial binary phases: eastern and western elongation (EE and WE) and superior and inferior conjunction (SC, IC). The case of TCP is represented by a single color (magenta) as the binary is circular at the onset of the RLOF and thus has a single initial condition (set to inferior conjunction for simplicity). The dashed black lines in the upper panels represent the corresponding adiabatic expansion approximation. As seen from the upper right panel, the results from our numerical simulations for $a_{\text{CBP, PCE}}$ and for $\alpha_{0.1}$ are fully consistent with the adiabatic approximation. For $\alpha_{0.1}$, the modes of the $a_{\text{CBP, PCE}}$ and $e_{\text{CBP, PCE}}$ distributions are 0.5/0.6 au and 0.4/0.5 for TCP/NTCP, respectively, and the planet can reach $a_{\text{CBP}}$ of up to 1.7 au and $e_{\text{CBP}}$ of up to 0.67.

13
The Astrophysical Journal, 832:183 (30pp), 2016 December 1  
Kostov et al.

initial configuration of the system, that is, the phase difference $\Delta \theta_0 \equiv \theta_{\text{CBP},0} - \theta_{\text{bin},0} - \omega_{\text{bin}}$, where $\theta_{\text{CBP},0}$ and $\theta_{\text{bin},0}$ are the initial true anomaly of the CBP and the binary, respectively, and $\omega_{\text{bin}}$ is the binary’s argument of pericenter. This is reminiscent of the importance of the initial true anomaly of a planet orbiting a single star on an eccentric orbit (see Veras et al. 2011). The dependence is shown in the second panel of Figure 3, where each curve represents the evolution of $a_{\text{CBP}}$ for $\Delta \theta_0$ varying between 0 and 1 (for clarity, the curves here represent every tenth phase where in the simulations the phase difference varies with a step of 0.02).

Since our simulations encompass a discrete set of initial conditions for $\alpha_{\text{CE}}$, tides (NTCP or TCP for minimum to maximum strength, respectively), and $\Delta \theta_0$, the true evolution of $a_{\text{CBP}}$ and $e_{\text{CBP}}$ would be continuous. We represent this by the hatched regions on the lower panels of Figures 3 and 4, where the different hatches correspond to TCP or NTCP. For clarity, we show individual $a_{\text{CBP}}$ evolutionary tracks only in Figure 3; these are replaced with hatched regions in all subsequent figures. We note that the distribution of $a_{\text{CBP},\text{PCE}}$ and $e_{\text{CBP},\text{PCE}}$ (i.e., $a_{\text{CBP}}$ and $e_{\text{CBP}}$ at the end of the CE phase) is neither uniform nor necessarily normal, as seen from the second panel on Figure 3, but instead depends on $\Delta \theta_0$. This dependence is demonstrated on Figure 5 for $\alpha_{\text{CE}} = 0.5$ and on Figure 6 for $\alpha_{\text{CE}} = 1, 3, 5,$ and 10, and the corresponding probability distribution functions (PDFs) are shown in Figures 7 and 8. The PDFs can have a prominent peak with a long one-sided tail (e.g., upper left panel on the first figure), or be double-peaked with the peaks near the edges (e.g., lower left panel on the first figure); this is a natural consequence of the vaguely sinusoidal dependencies seen in Figures 5 and 6. Given the prominent diversity of the PDFs, the distributions are better represented by their modes than their medians. Thus for completeness we list the mode, median, and min/max range for each $a_{\text{CBP},\text{PCE}}$ and $e_{\text{CBP},\text{PCE}}$ in Tables 7 through 10, but we cite the 68%-range upper and lower bounds on the modes only because the medians may not be appropriate in some cases. Unless otherwise noted, we refer to the modes of $a_{\text{CBP},\text{PCE}}$ and $e_{\text{CBP},\text{PCE}}$ as the default results.

It is important to note that the osculating orbital elements of a CBP do not describe a closed ellipse during the CE phase, and its orbital motion is not Keplerian. Instead, the orbit spirals outward with time and, as seen from Figures 3 and 4, while the binary is losing mass, $a_{\text{CBP}}$ and $e_{\text{CBP}}$ oscillate together with...
the oscillations in $e_{\text{bin}}$ (see inset figure, lower panel). Similar behavior was observed in numerical simulations of planets around single stars experiencing mass loss (Veras et al. 2011; Adams et al. 2013), where the planet’s orbital elements oscillate on the planet’s period due to the different effects of mass loss at pericenter and apocenter. This is reproduced in our simulations for the binary star itself; the orbital elements of the secondary star oscillate on the binary period as seen from the inset figure in the lower panel of Figure 3. The CBP, initially on a circular orbit, quickly gains eccentricity and responds to the binary’s oscillations.

Overall, the orbital expansion of the Kepler-38 CBP for $a_{1}$ is fully consistent with the adiabatic approximation, and the planet gains slight eccentricity (see Table 9). The outcome for the case of $a_{1}$ is diverse: $a_{\text{CBP,PCE}}$ ranges from below the corresponding adiabatic approximation up to 3.8 au, and $e_{\text{CBP,PCE}}$ can reach up to 0.8 (see Table 7).

4.3. Kepler-47

The binary consists of 0.96 $M_{\odot}$ and 0.34 $M_{\odot}$ stars on a nearly circular ($e_{\text{bin,0}} = 0.02$), 7.45 day orbit that changes little in the first ~12 Gyr. The three CBPs have initial orbits of 0.3, 0.72, and 0.99 au and masses of 2 $M_{\oplus}$, 19 $M_{\oplus}$, and 3 $M_{\oplus}$ (J. Orosz 2016, private communication), from inner to outer, respectively. For completeness, we integrate the system for the appropriate CBP masses to take into account planet–planet interactions. The main evolutionary stages are as follows.

**4.3.1. The Binary**

The primary star experiences an RLOF at $t \sim 12$ Gyr, and the binary enters a CE stage. During this stage, $\Psi(a_{1}) \approx 0.02/0.06/0.1$ and $a_{\text{crit}} \approx 60/16/10$ $M_{\odot}$ yr$^{-1}$ for planets 1, 2, and 3, respectively. For $a_{1}$ the time of CE mass loss ($T_{\text{CE}}$) is comparable to the orbital periods of planets 2 and 3, that is, $T_{\text{CE}}/P_{\text{CBP}} \sim 0.5$–2, indicating that these two CBPs evolve in the transition regime.

As a result of the primary CE, for $a_{\text{CE}} = 0.5/1$ the binary merges as a first giant branch star and loses ~15–40% of its mass; $\beta < \beta_{\text{eject}}$, and the CBPs should remain bound even in a runaway regime. From the end of the CE to the end of the BSE simulations, the system slowly loses 50% of its mass, and $a_{\text{CBP,PCE}}$ of those planets that remain long-term stable after the CE (discussed below) expand adiabatically by a factor of 2.
For $\alpha_{\mathrm{CE}} = 3/5/10$, the binary evolves into a HeWD–MS star pair and loses 56% of its mass, and $a_{\mathrm{bin}}$ decreases by a factor of 4–10; here $\beta > \beta_{\mathrm{crit}}$, and the CBPs should be ejected in a runaway regime. The system does not experience further mass loss by the end of the BSE simulations.

4.3.2. The CBPs

In Figures 9 and 10 we show the evolution of $a_{\mathrm{CBP}}$ and $e_{\mathrm{CBP}}$ for the three CBPs (and for $\alpha_1$ and $\alpha_{0.1}$) caused by the primary RLOF and CE. For simplicity here and for the rest of the Kepler systems, we do not show the evolution of $M_1$ and $a_{\mathrm{bin}}$ since it is qualitatively very similar to the case of Kepler-38. We note that, unlike the corresponding figures for Kepler-38, here the colors represent the three planets (magenta, green, and red for planets 1, 2, and 3, respectively), and the different panels represent the evolution of $a_{\mathrm{CBP}}$ and $e_{\mathrm{CBP}}$ for $\alpha_1$ (upper and middle panels) and for $\alpha_{0.1}$ (lower panels).

As seen from the lower panels in Figures 9 and 10, the orbital expansion of all three of Kepler-47’s CBPs is fully consistent with the adiabatic approximation for $\alpha_{0.1}$, and the planets gain slight eccentricity (not shown in the figures, but listed in Table 9). The orbital evolution of planets 2 and 3 for the primary RLOF is much richer for $\alpha_1$, as shown on the upper and middle panels of Figures 9 and 10 (see also Table 7). The planets gain high eccentricities and for $\alpha_{\mathrm{CE}} = 3/5/10$ may even become ejected during the CE stage, where, given the complexity, uncertainties, and large parameter space of the evolution of the binary and of the CBP, here and throughout the paper we define ejection as $e_{\mathrm{CBP}} > 0.95$. Specifically, planet 3 becomes unbound in 46% of the NTCP simulations and in 81% of the TCP simulations.

We note that, during the primary CE stage, for the case of $\alpha_1$, the orbit of the inner CBP grows adiabatically, while those of the outer two do not (see upper panel of Figure 10). This is because the orbital period of the inner planet is much shorter than the duration of $T_{\mathrm{CE}}$, while those of the outer two are comparable. Thus a multiplanet CB system can experience two regimes of planetary orbital evolution during the same CE stage.

The dependence of $a_{\mathrm{CBP},\mathrm{PCE}}$ and $e_{\mathrm{CBP},\mathrm{PCE}}$ of planet 2 on $\Delta\theta_0$ is shown in Figures 11 and 12. The planet’s orbit evolves adiabatically for $\alpha_{0.1}$ (upper right panels), nonadiabatically for $\alpha_1$, and remains bound during the CE in both cases, regardless of $\Delta\theta_0$.

Even if an ejection does not occur during the primary CE phase, $a_{\mathrm{CBP},\mathrm{PCE}}$ and $e_{\mathrm{CBP},\mathrm{PCE}}$ of all three CBPs for $\alpha_1$ are such that the planets will continue to interact with each other after
the end of the CE in such a way that further ejections or collisions are possible. To study the long-term evolution of the system, we integrated the equations of motion for all five bodies (two stars and three planets) for 1 Myr, using the planets’ mode($a_{\text{CBP,PCE}}$) and mode($e_{\text{CBP,PCE}}$), and randomizing their initial arguments of pericenter and true anomalies. Any planet that achieves $e_{\text{CBP}} > 0.95$ is marked as ejected and removed from the simulations, and collision is defined as a planet–planet separation smaller than $d_{\text{min}} = R_{\text{P,i}} + R_{\text{P,j}}$.

Overall, planet 2 dominates the 1 Myr dynamical evolution and has the highest probability of remaining bound to the system. For $\alpha_{\text{CE}} = 0.5/1$ the modes of $a_{\text{CBP,1/2/3}}$ have 40%/15%/75% ejection probability in 1 Myr. Where planet 2 remains bound, its orbital elements remain mostly within the respective 68% range of mode($a_{\text{CBP,PCE,2}}$) and mode($e_{\text{CBP,PCE,2}}$).\(^{17}\) Here the modes of $a_{\text{CBP,1/2/3}}$ have 60%/15%/85% ejection probability in 1 Myr, and the orbital elements of planet 2 again remain mostly within the 68% range of mode($a_{\text{CBP,PCE,2}}$) and mode($e_{\text{CBP,PCE,2}}$).

\(^{17}\) Except in one simulations where planet 1 is ejected, planet 2 migrates to a 5.67 au, 0.71-eccentricity orbit, and planet 3 migrates to a 0.55 au, 0.66-eccentricity orbit.

4.4. Kepler-64

The binary consists of 1.53 $M_\odot$ and 0.41 $M_\odot$ stars on a slightly eccentric ($e_{\text{bin}} = 0.22$), 20 day orbit that changes little in the first ~3 Gyr; the CBP has an initial orbit of 0.65 au. The system evolves as follows.

4.4.1. The Binary

The primary star fills its Roche lobe at $t \sim 3.02$ Gyr, and the binary enters a primary CE stage. During this stage, $\Psi$ ($\alpha_{\text{1}} \approx 0.03$ and $\alpha_{\text{crit}} \approx 32 M_\odot$ yr\(^{-1}\), much larger than $\alpha_{\text{1}}$ and $\alpha_{\text{0,1}}$. However, for $\alpha_{\text{1}}$ the time of mass loss ($T_{\text{CE}}$) is comparable to the orbital period of the CBP, that is, $T_{\text{CE}}/P_{\text{CBP}} \sim 1.1–3.4$, depending on $\alpha_{\text{CE}}$. Thus while the CBP is theoretically in the adiabatic regime, the comparable timescales result in a nonadiabatic dynamical response.

The binary merges as a first giant branch star and loses ~20–58% of its mass for $\alpha_{\text{CE}} = 0.5/1$. In this regime, $\beta > \beta_{\text{ej ect}}$ for $\alpha_{\text{CE}} = 0.5$, and for $\alpha_{\text{CE}} = 1$, TCP and the planet should remain bound in a runaway regime even at pericenter; $\beta < \beta_{\text{ej ect}}$ for $\alpha_{\text{CE}} = 1.0$ and NTCP, indicating a potential ejection of the CBP in a runaway regime. For $\alpha_{\text{CE}} = 0.5$ and 1, the system continues to slowly lose mass by
15% transition regime for binary and the CBP (the end of the BSE simulations). Figure 10. Same as Figure 9 but for $\alpha_{\text{CE}} = 3/5/10$. The orbital periods of planets 2 and 3 are comparable to the CE mass-loss timescale $T_{\text{CE}}$ and evolve in the transition regime for $\alpha_3$. Planet 3 is ejected during the CE in 46%/85% of our simulations for NTCP/TCP, respectively. For $\alpha_3$, planets 1/2/3 are ejected in 60%/15%/85% after 1 Myr. For $\alpha_0$, their orbits evolve adiabatically and do not experience ejections. The CBP should be ejected in a runaway regime. For $\alpha_3$, the CBP is fully consistent with the adiabatic approximation for $\alpha_0$, and again the planet gains slight eccentricity (Table 10). The outcome for the case of $\alpha_3$ is quite diverse. Depending on $\Delta \theta_0$, the planet can gain very high eccentricity and even become ejected ($e_{\text{CBP}} > 0.95$). This is shown as missing points in Figures 14 and 15. The Kepler-64 CBP can reach such eccentricities in $\sim 4\%$ of our simulations for $\alpha_1$ and $\alpha_{\text{CE}} = 3/5/10$. Interestingly, for $\alpha_{\text{CE}} = 0.5$ and TCP, $a_{\text{CBP}}$ can decrease during the CE, and mode($a_{\text{CBP,PCE}}$) = 0.6–0.8 au is smaller than the initial orbit $a_{\text{CBP,0}} = 0.65$ au, though the mode’s 68% range is significant. On the other side of the spectrum, $a_{\text{CBP}}$ can reach up to $\sim 20$ au for $\alpha_{\text{CE}} = 3/5/10$ and NTCP.

### 4.5. Kepler-1647

The initial binary consists of 1.22 $M_\odot$ and 0.97 $M_\odot$ stars on a slightly eccentric ($e_{\text{bin,0}} = 0.15$), 11 day orbit that changes little in the first $\sim 5$ Gyr. The CBP has an initial orbit of 2.7 au. The main stages of the evolution of the system are as follows.

#### 4.5.1. The Binary

The primary star fills its Roche lobe at $t \sim 5.4$ Gyr, and the binary enters a primary CE stage. During this stage, $\Psi (\alpha_1) \approx 0.2$ and $\alpha_{\text{crit}} \approx 4.6 M_\odot$ yr$^{-1}$, comparable to $\alpha_1$. Additionally, for NTCP and $\alpha_{\text{crit}}$, the time of mass loss ($T_{\text{CE}}$) is comparable to the orbital period of the CBP, that is, $T_{\text{CE}} / P_{\text{CBP}} \sim 1.3/3.0$ for $\alpha_{\text{CE}} = 0.5/1.0$, respectively. Thus the orbit of the CBP evolves in the transition regime.
Compared to the other Kepler CBP systems, the binary star of Kepler-1647 experiences the richest evolution. For $\alpha_{CE} = 0.5/1$, the binary merges as a first giant branch star and loses $\sim 15$–40% of its mass. In this regime, $\beta > \beta_{eject}$ and the planet should remain bound in a runaway regime even at pericenter. The system continues to slowly lose mass by the end of the BSE simulations (15 Gyr), so a CBP,PCE further expands (adiabatically) by up to a factor of 3.

For $\alpha_{CE} = 3/5/10$, the binary shrinks by a factor of 2–7 into a HeWD–MS star binary, and its mass decreases by $\sim 45\%$, close to the critical mass loss for runaway ejection (0.5). Here, $\beta < \beta_{eject}$ and the CBP should remain bound even in a runaway regime.

The binary experiences a secondary RLOF and CE for $\alpha_{CE} = 3/5/10$ (see Table 6 for details), and its subsequent evolution can follow several paths. By the end of the BSE simulations, the binary (1) merges without mass loss (i.e., no changes in $a_{CBP,PCE}$ and $e_{CBP,PCE}$) into a first giant branch for $\alpha_{CE} = 3/5$ and TCP, which by 15 Gyr evolves into a COWD; and (2) evolves into a PCE WD–WD binary (with mass loss, thus $a_{CBP,PCE}$ and $e_{CBP,PCE}$ change) for $\alpha_{CE} = 5$ (and for NTCP) and for $\alpha_{CE} = 10$ (for both TCP and NTCP), which can experience a third and final RLOF, triggering an SN explosion (for $\alpha_{CE} = 5/10$ and TCP).

4.5.2. The CBP

The corresponding evolution of $a_{CBP}$ and $e_{CBP}$ for the primary RLOF and for $\alpha_{1}$ and $\alpha_{0.1}$ is shown in Figure 16. Unlike the previous systems, the $\alpha_{0.1}$ case for Kepler-1647 represents an adiabatic evolution only for the TCP. For $\alpha_{0.1}$ and NTCP, where $T_{CE}$ and $P_{CBP}$ are comparable for all $\alpha_{CE}$, the CBP’s orbit evolves in the transition regime (as it does for $\alpha_{1}$).

Specifically, for $\alpha_{0.1}$ and NTCP, the CBP reaches sufficiently high eccentricities ($e_{CBP} > 0.95$) in $\sim 5$–25% of the simulations to be ejected from the system; for $\alpha_{1}$ the planet is ejected in $\sim 45$–55% of the simulations. In addition, for $\alpha_{CE} = 0.5/1$ and NTCP, and for $\alpha_{CE} = 3/5/10$ and NTCP, $a_{CBP}$ can decrease during the primary CE phase, and the corresponding mode of $a_{CBP,PCE}$ (1.6–2.3 au, depending on $\alpha_{CE}$ with a wide 68% range) is smaller than the initial semimajor axis of the CBP (2.71 au). Some of the simulations produce the opposite result, namely orbital expansion by up to a factor of $\sim 20$ such
that $a_{\text{CBP,PCE}}$ reaches $\sim50$ au. The corresponding dependence of $a_{\text{CBP,PCE}}$ and $e_{\text{CBP,PCE}}$ on $\Delta\theta_0$ is shown in Figures 17 and 18, where the missing $\Delta\theta_0$ phase coverage in all panels represents planet ejection.

Using the respective $a_{\text{bin,PCE}}$ and $e_{\text{bin,PCE}}$ and the modes of $a_{\text{CBP,PCE}}$ and $e_{\text{CBP,PCE}}$ from the end of the preceding primary RLOF as initial conditions, we further explore the evolution of the CBP for the secondary RLOF and CE stage for the three cases where the binary evolves into a WD–WD pair with mass loss: (1) $\alpha_{\text{CE}} = 5$, NTCP; (2) $\alpha_{\text{CE}} = 10$, TCP; and (3) $\alpha_{\text{CE}} = 10$, NTCP. The results are as follows.

During this secondary CE phase, for (1) and (2) $\Psi(\alpha_1) \approx 0.4$ and $\alpha_{\text{crit}} \approx 2.4 \, M_\odot \, \text{yr}^{-1}$, comparable to $\alpha_1$, indicating that the CBP evolves in the transition regime. For (3) $\Psi(\alpha_1) \approx 3.6$, $\alpha_{\text{crit}} \approx 0.3 \, M_\odot \, \text{yr}^{-1}$, and the evolution of the planet’s orbit is in the runaway regime. The time of mass loss ($T_{\text{CE}}$) is close to the orbital period of the CBP, that is, $T_{\text{CE}}/P_{\text{CBP}} \approx 0.3$–2.5. The binary shrinks by a factor of $\sim5$–10 into a WD–WD pair and loses $\sim60$–70% of its mass. As a result, $\beta < \beta_{\text{eject}}$ and the CBP should become unbound in a runaway regime.

The evolution of $a_{\text{CBP}}$ and $e_{\text{CBP}}$ during the secondary CE phase is shown in Figure 19, where the upper panel represents case (1) ($\alpha_{\text{CE}} = 5$, NTCP), the middle panel case (2) ($\alpha_{\text{CE}} = 10$, NTCP), and the lower panel case (3) ($\alpha_{\text{CE}} = 10$, TCP). The CBP is ejected in more than $\sim80\%$ of the simulations. For $a_1$, the CBP’s $a_{\text{CBP}}$ expands to a mode value of $\sim7.95$ au (depending on the scenario described above), with a maximum of up to 50–100 au; the mode of $e_{\text{CBP}}$ is in the range of 0.5–0.75, reaching a maximum of 0.95. For $a_{0.1}$, the CBP’s orbit can expand up to $\sim30$–50 au.

The system does not experience a CE evolution for $Z_{\text{nominal}}$, but the primary undergoes an RLOF and CE for $Z = 1\sigma_Z$. Given the sensitivity of the binary evolution to the uncertainties in $Z$, here we outline the results for this system as well.
two panels: adiabatic and runaway expansions. For the case of
by the end of the BSE simulations
Figure 13.

The Astrophysical Journal, 832:183 (30pp), 2016 December 1
Kostov et al.

4.6.1. The Binary

The binary consists of 0.88 $M_\odot$ and 0.81 $M_\odot$ stars on a
slightly eccentric ($e_{\text{bin,0}} = 0.15$), 21 day orbit that changes little
in the first $\sim 13$ Gyr. The CBP has an initial orbit of 0.6 au.

The primary star fills its Roche lobe at $t \approx 12.99$ Gyr, and
the binary enters a primary CE stage. During this stage, $\Psi$
($\alpha_1$) $\approx 0.03$ and $\alpha_{\text{crit}} \approx 29.6 M_\odot$ yr$^{-1}$, much larger than $\alpha_1$.
As a result of the CE, the binary merges as a first giant branch star
and loses $\sim 35\%$ of its mass for $\alpha_{\text{CE}} = 0.5$; here
$\beta > \beta_{\text{eject}}$, and the planet should remain bound in a runaway
regime. The system does not experience significant mass loss
by the end of the BSE simulations (15 Gyr), so there are no
major changes in $a_{\text{CBP}}$.

For $\alpha_{\text{CE}} = 1/3/5/10$ the binary orbit shrinks by a factor of
$\sim 2–10$, loses $\sim 35\%$ of its initial mass, and evolves into a He–
WD pair; here $\beta > \beta_{\text{eject}}$, and the planet should remain bound
in a runaway regime.

4.6.2. The CBP

Overall, for $\alpha_{0.1}$ the orbital evolution of the CBP is
consistent with the adiabatic approximation. For $\alpha_{0.2}$, mode
($a_{\text{CBP,PCE}} \sim 0.8–1$ au, and the planet gains an eccentricity of
$0.2–0.4$; the min/max ranges can be significant. In particular,
for $\alpha_{\text{CE}} = 0.5$, $\alpha_2$, and NTCP, the CBP reaches sufficiently
high eccentricities ($e_{\text{CBP}} > 0.95$) in $\sim 4\%$ of the simulations to
be ejected from the system; the CBP remains bound in all other
cases. For $\alpha_1$ and NTCP, $a_{\text{CBP,PCE}}$ can reach up to 7–10 au,
with $e_{\text{CBP,PCE}} > 0.9$.

5. DISCUSSION AND CONCLUSIONS

Several key results emerge from our study:
(1) Five of the nine Kepler CBP hosts experience at least
one RLOF and a CE phase for their default parameters (masses,
metallicities) by the end of our BSE simulations (15 Gyr);
Kepler-35 experiences a primary RLOF for $Z = 1\sigma_Z$. Depending
on the treatment of the CE stage, the binaries either merge after
a primary or secondary CE (typically for $\alpha_{\text{CE}} = 0.5/1$) or evolve
into very short-period WD–MS pairs (for $\alpha_{\text{CE}} = 1/3/5/10$)$^{18}$. Kepler-1647 evolves into a WD–WD pair after a
secondary RLOF and CE for $\alpha_{\text{CE}} = 5/10$. Two systems
trigger a double-degenerate supernova explosion: Kepler-34
for primary RLOF, and Kepler-1647 for a third RLOF and
$\alpha_{\text{CE}} = 5$, TCP, and $\alpha_{\text{CE}} = 10$, NTCP. The binary systems
lose a tremendous amount of mass—up to $\sim 60–70\%$—during
the CE stages and can shrink to sub-$R_\odot$ orbital separations.
(2) From the dynamical perspective presented here,
Kepler CBPs predominantly remain bound to their host

$^{18}$ Which can also merge for a secondary-triggered CE phase.
binaries after the respective CE phases even for mass-loss rates as high as 1.0 \( \dot{M} \) yr\(^{-1}\). There are only four scenarios where the respective CBP has a 100% probability of becoming unbound: (a) for Kepler-34 where the SN explosion occurs with the first CE; (b) for the secondary CE phase of Kepler-1647 for \( \alpha_{CE} = 0.5 \); (c) for the SN caused by the third RLOF of Kepler-1647, \( \alpha_{CE} = 5 \); and (d) SN caused by the third RLOF of Kepler-1647, \( \alpha_{CE} = 10 \). In all other scenarios (106 total, not accounting for the initial phase differences \( \Delta \theta_0 \)), the CBPs remain bound in the majority of cases (only Kepler-1647 has nonnegligible ejection probability, again highly dependent on \( \Delta \theta_0 \)). For mass-loss rates of 0.1 \( \dot{M} \) yr\(^{-1}\), the orbits of the CBPs evolve adiabatically—well reproduced by REBOUNDx—except for Kepler-1647. The orbital expansion of some of the Kepler CBPs is consistent with the adiabatic approximation even for mass-loss rates of 1.0 \( \dot{M} \) yr\(^{-1}\) (e.g., inner planet of Kepler-47, Kepler-64); in other cases, the mode of \( a_{CBP, PCE} \) is smaller than the adiabatic approximation (e.g., Kepler-1647).

According to the analytical prescription, some systems should retain their CBPs even in a runaway mass-loss regime (\( \Psi \gg 1 \)). Our code fully reproduces the runaway approximation for single-star systems.

(3) The transition regime (\( \Psi \sim 0.1-1 \))—where the evolution of some of Kepler CBPs falls for \( \alpha_{CE} \)—is complex, and the final eccentricities and semimajor axes of the planets depend both on the treatment of the CE stage (i.e., \( \alpha_{CE} \), treatment of tides) and on the initial configuration of the system (i.e., \( \Delta \theta_0 \)). We find that the orbits of Kepler CBPs can expand by more than an order of magnitude over the course of a single CBP year, reaching \( a_{CBP, PCE} \) of tens of astronomical units (during the secondary CE of Kepler-1647, \( a_{CBP} \) can expand from \( \sim 10 \) au to \( \sim 100 \) au; see Table 8).

Multiplanet CBP systems add yet another level of complexity because they can experience both regimes simultaneously. For example, where the orbit of the inner Kepler-47 CBP expands adiabatically, the middle and outer CBP orbits grow nonadiabatically—all during the primary CE phase.

Overall, the CE-induced orbital evolution of CBPs is a dynamically rich and complex process in which a planet can migrate adiabatically during one CE phase and nonadiabatically...
during another. As a result, a CBP cannot experience the same CE twice.

(4) If CE mass-loss rates are indeed high (e.g., ∼1.0 M_⊙ yr^{-1})—as suggested by theoretical work—and the orbits of Kepler-like CBPs evolve (and survive) according to our simulations, we should expect to detect potentially highly eccentric planets orbiting PCE systems on very large orbits. Alternatively, if Ψ ≈ 1, then we should find PCE CBPs on low-eccentricity orbits. Interestingly, recent observational efforts have produced a number of PCE eclipse-time variations (ETV) CBP candidates on wide, notably eccentric orbits (e.g., Zorotovic & Schreiber 2013 and references therein), suggesting a possible connection with PCE Kepler-like CBPs.

In Figure 20 we show a_{CBP,PCE} versus e_{CBP,PCE} of Kepler’s CBPs and compare them to those of the PCE ETV CBP candidates. We caution that the comparison is not direct as the CE stages of the different Kepler systems do not occur all at the same time, and the PCE ETV CBPs do not have identical ages. Instead, given the handful of known targets—Kepler’s nine compared to estimated millions of similar CBPs (Welsh et al. 2012) and a dozen PCE ETV CBPs—the lower two panels on the figure portray a potential distribution of an underlying PCE CBP population with a range of ages and evolutionary stages.

Overall, the PCE orbital configurations of Kepler’s CBPs are qualitatively consistent with those of the observed population of the currently known PCE ETV CBP candidates. Thus our results assist in both interpreting the nature of these candidates and in guiding future observational efforts to discover new systems. Such discoveries will provide a deeper understanding of the evolution of planets in a binary star system and also much-needed observational constraints on the stellar astrophysics of the complex CE phase, as “CE is one of the most important unsolved problems in stellar evolution,” according to Ivanova et al. (2013), “and is arguably the most significant and

\footnote{19 For Kepler-47 we only show the middle planet (planet 2) as it has the highest probability to survive both the CE phase and subsequent planet–planet interactions.}

\footnote{20 For those candidates that do not have published eccentricities, we set the respective e_{CBP} to zero.}

\footnote{21 With the caveat that the latter are much too massive.}

Figure 15. Same as Figure 14 but for α_{CR} = 3/5/10. Here mode(a_{CBP,PCE}) = 2.4/1.6 au and mode(e_{CBP,PCE}) = 0.49/0.8 for NTCP/TCP, max(a_{CBP,PCE}) = 19.6 au and max(e_{CBP,PCE}) = 0.95. The planet is ejected (e_{CBP} > 0.95) in 4% of the simulations, indicated by the missing Δθ coverage for the green and red colors in the left-hand panels.
A CB body gravitationally perturbs its host binary star, and if the latter is eclipsing, these perturbations can be manifested as deviations from linear ephemeris in the measured stellar eclipse times. The cause of these variations can be either dynamical, where the third body’s influence changes the orbital elements of the binary star, or a light travel time effect, where the tertiary object and the binary revolve around a common center of mass. ETVs are indeed a powerful and highly productive method of discovering and studying stellar triple and higher order systems (e.g., Borkovits et al. 2016; Orosz 2015, and references therein), and they have recently been used to study CBPs as well. In particular, the former effect has been measured for several of the Kepler CBP systems and used to constrain the respective planetary masses (Doyle et al. 2011; Welsh et al. 2012; Kostov et al. 2016). The latter effect has been suggested as the cause for measured ETVs for a number of PCE EB systems with proposed CB companions (as mentioned above; also see Volschow et al. (2016) and references therein).

Assuming that the orbits of the Kepler CBPs studied here evolve only dynamically and their masses remain constant (i.e., no changes due to, for example, interactions with a CE-triggered CB disk), it is informative to evaluate the respective planets’ ETV-based detectability after the CE stages of the relevant systems. To calculate the expected amplitudes of the two effects discussed above, \( A_{\text{dyn}} \) and \( A_{\text{LTTE}} \), we use the formalism of Borkovits et al. (2012, 2015).

The respective amplitudes and periods for Kepler-35 \( (M_p = 0.13 M_{\text{Jup}}) \) and Kepler-1647 \( (M_p = 1.5 M_{\text{Jup}}) \)—the only two systems with well-constrained CBP masses—are listed in Table 11 (for the respective range of binary and CBP parameters listed in Tables 8) and compared to the detected signals for the two PCE CBP candidates NN Ser and HW Vir. As seen from Table 11, detection of such ETVs is feasible in terms of both the amplitudes and periods of the expected signal, which are qualitatively similar to the ETV signals of the two PCE CBP candidates.

Another option for the detection of a PCE CBP is to directly image the planet. The benefits for such detection are twofold: (a) the contrast ratio between the binary and the CBP decreases after each CE phase as the primary or secondary evolves from a least-well-constrained major process in binary evolution” (but also see Taam & Sandquist 2000; Taam & Ricker 2010 and Webbink 2008 for alternative reviews).

Figure 16. Evolution of Kepler-1647 CBP’s \( a_{\text{CBP}} \) and \( e_{\text{CBP}} \) during the primary RLOF and CE for \( \alpha_{\text{CE}} = 0.5/1 \) (upper two panels) and for \( \alpha_{\text{CE}} = 3/5/10 \) (lower two panels), where red indicates \( a_1 \), and green indicates \( a_0.1 \). The binary merges at the end of the CE (at the end of the CE for \( \alpha_{\text{CE}} = 0.5/1 \). The dashed lines represent the corresponding adiabatic and runaway expansions. The CBP is ejected \( (e_{\text{CBP}} > 0.95) \) in \( \sim 5-50\% \) of the simulations depending on \( \alpha_{\text{CE}} \) and the tidal evolution, for both \( a_1 \) and \( a_0.1 \). The latter case is nonadiabatic as \( T_{\text{CE}} \) is comparable to the period of the CBP at the start of the simulations \( (P_{\text{CBP,0}}) \) and the CBP can expand to large orbits, gaining very high eccentricities.

The Astrophysical Journal, 832:183 (30pp), 2016 December 1

Kostov et al.
luminous MS star to a much fainter WD; and (b) the angular separation between the binary and the CBP increases\textsuperscript{22} (e.g., Hardy et al. \textsuperscript{2015}). There is potentially a third benefit, where the CBP can accrete mass after the CE stage (e.g., Zorotovic \& Schreiber \textsuperscript{2013}; Bear \& Soker \textsuperscript{2014}) and becomes brighter, thus further decreasing the contrast ratio. Recently, Hardy et al. (\textsuperscript{2015}) used VLT/SPHERE to observe the V471 Tau system—an eclipsing binary star with measured ETVs—with the goal to directly image a substellar-mass CB candidate suspected to be the cause of the ETVs. Given the known age of the system and the masses of the binary and the CB candidate, direct detection of such a tertiary body should have been well within the capabilities of the instrument. Hardy et al. (\textsuperscript{2015}), however, report a null detection, casting doubt on the interpretation of the measured ETV signal (but see Vaccaro et al. \textsuperscript{2015} for an alternative explanation). Nevertheless, based on our results for the dynamical survivability of CBPs around evolving binary stars, and on estimates of the occurrence rates of planets orbiting short-period MS binary stars (e.g., Welsh et al. \textsuperscript{2012}), we encourage the continuation of such direct imaging efforts, aimed at both PCE, very short period binaries and PCE, WD-coalesced binaries where the contrast ratio is even more favorable.

\textsuperscript{22} Such that the minimum increase is typically for adiabatic orbital expansion.
binary star itself, the CE phase may in fact promote planet survivability.

(7) With the exception of Kepler-1647, the Kepler CBPs currently orbit their binary star hosts within a factor of 2 of the critical limit for dynamical stability (Holman & Wiegert 1999, HW hereafter), that is, \( a_{\text{crit},0} = a_{\text{bin},0}(1 - e_{\text{CBP},0}) \). However, as the binaries shrink and lose mass during the CE and the planets migrate to larger, eccentric orbits, this ratio will change. To calculate \( a_{\text{crit},\text{PCE}} \) — the PCE critical limits (for those scenarios where the binaries do not coalesce, i.e., \( \alpha_{\text{CE}} = 3/5/10 \)) — we use Equation (3) from HW using a binary mass ratio \( \mu = M_{A,\text{PCE}}/(M_{A,\text{PCE}} + M_{B,\text{PCE}}) \).\(^{23}\) As the CE phases result in a variety of \( \alpha_{\text{bin}}, \alpha_{\text{CBP, PCE}}, \) and \( e_{\text{CBP, PCE}} \) — here we quote only the most constraining critical limits, where the respective separation between \( \alpha_{\text{bin}} \) and \( a_{\text{CBP, PCE}}(1 - e_{\text{CBP, PCE}}) \) is at a minimum. For \( \alpha_{\text{bin}} \), the limits for Kepler-38, Kepler-47, Kepler-64, Kepler-1647 (NTCP), and Kepler-1647 (TCP) in terms of \( a_{\text{CBP, PCE}}(1 - e_{\text{CBP, PCE}})/a_{\text{inel, PCE}} \) are 3.3, 7.9, 5.3, 15.9, 14.7, and 29.5, respectively, indicating that the CBPs remain in a dynamically stable regime.

While it is beyond the scope of this study, we note that tidal evolution of the binary orbit prior to the CE phase may affect the CBP orbits. Specifically, as most of the planets are currently orbiting close to their host binaries, a pre-CE decay in \( a_{\text{bin}} \) means that strong mean motion resonances (MMR) may sweep over the planet, leading to eccentricity excitation and potential destabilization. As an example, \( a_{\text{bin}} \) of Kepler-38 decreases from 31.6 \( R_{\oplus} \) to 24.6 \( R_{\oplus} \) prior to the CE phase for TCP (see Table 4), indicating that the CBP’s orbit crosses 6:1, 7:1, and 8:1 MMR. The MMR crossings are (a) 7:1, 8:1, and 9:1 for Kepler-47 (planet 1); (b) 7:1 through 11:1 for Kepler-64; and (c) 99:1 through 116:1 for Kepler-1647.

(8) “...It is interesting to consider the situation at a much earlier time in the past when the primary was near the zero-age main sequence...” note Orosz et al. (2012a) for the case of Kepler-38b: “The primary’s luminosity would have a factor \( \approx 3 \) smaller....” While Kepler-38b is currently closer to its binary host than the inner edge of the habitable zone (HZ) for the system, four of the Kepler CBPs (Kepler-16, Kepler-47c,
Kepler-453, Kepler-1647) reside in the HZ. As their host binaries evolve, however, the location of the HZ will change. Unlike the case for single stars where both the HZ and a planet’s orbit will expand during the RGB and AGB stages (Villaver & Livio 2007), as a close binary star evolves through a CE stage the HZ can shrink while its CBP migrates outward. The shrinking is caused by the luminosity drop of the central binary as one of its stars rapidly transitions from the MS to a compact object. Thus a CBP residing in the HZ during the pre-CE binary will leave the zone after the CE phase (e.g., Kepler-1647), whereas a CBP that is initially interior to the HZ may migrate to a PCE orbit coinciding with the PCE HZ.

Consequently, it would be equally interesting to consider the configuration of a CBP system at a much later time in the binary evolution. For example, the current orbital configuration of Kepler-35b is such that the planet’s orbit is internal to the HZ (Welsh et al. 2012). However, as its host binary evolves through the CE phase, for $\alpha_{CE} = 1/3/5/10$ the primary star evolves into a HeWD (while the secondary remains on the MS as a 0.81 $M_\odot$ star), and the CBP migrates to mode($a_{CBP,PCE}$) = 0.84–1.0 au, mode($e_{CBP,PCE}$) = 0.0–0.4 (see Tables 8). This migration places the planet near the conservative HZ of 0.98–1.77 au (for 1 $M_\odot$; Kopparapu et al. 2014), using the luminosity, 0.4 $L_\odot$, and temperature.

**Figure 19.** Evolution of Kepler-1674 CBP’s $a_{CBP}$ and $e_{CBP}$ during the secondary RLOF and CE for $\alpha_{CE} = 5$, NTCP (upper panel); $\alpha_{CE} = 10$, NTCP (middle panel); and $\alpha_{CE} = 10$, TCP (lower panel) for $a_1$ (red) and $a_0.1$ (green). The planet is ejected in ~80–100% of the simulations.
of the secondary star, 5200 K (Welsh et al. 2012), and ignoring the flux contribution of the WD.

5.1. Limitations

The results presented here are based on the assumptions that the CBPs do not interact with the material ejected from their binary star. However, numerical studies have indicated that this material is neither lost isotropically from the binary during the CE phase, nor does it all become unbound. Instead, 1–10% of the ejecta may fall back into a CB disk according to Kashi & Soker (2011), and Passy et al. (2012) suggest that ~80% of the ejected material may remain gravitationally bound to the binary (also see Kuiper 1941; Shu et al. 1979; and Pejcha et al. 2016 for mass loss outflows through the \( L_2 \) Lagrange point\(^ {24} \)). Either of these scenarios would significantly complicate the dynamical evolution of the system as the CBP could accrete material and gain mass and also experience migration similar to that during planetary formation.\(^ {25} \) Such accretion of material of a different specific angular momentum will change the orbital

\( \text{Figure 20. Upper six panels: } a_{\text{CBP,PCE}} \text{ vs. } e_{\text{CBP,PCE}} \text{ for Kepler’s CBPs for various CE treatments (green, blue, and red symbols) and initial conditions (} \Delta \theta_0 = 0 \div 1\text{), along with the respective ejection probabilities. Lower two panels: comparison between Kepler’s } a_{\text{CBP,PCE}} \text{ and } e_{\text{CBP,PCE}} \text{ and those of the currently known PCE ETV } \text{CBP candidates (black symbols) at the end of the respective Kepler CE (left) and at the end of our BSE simulations (15 Gyr, right). Given the associated observational and numerical uncertainties, our forward-evolution results are qualitatively consistent with the observed } a_{\text{CBP,PCE}} \text{ vs. } e_{\text{CBP,PCE}} \text{ distributions. See text for details.} \)

\( \text{24 Mass loss through } L_2 \text{ results in several possible outcomes, e.g., isotropic or equatorial wind, CB disk; for details, see Table 1 and Figures 12 and 13 of Pejcha et al. (2016).} \)

\( \text{25 There are, however, two potential benefits of the former in terms of detection: a more luminous planet would be more amenable to direct imaging efforts, and a more massive planet would cause stronger ETVs.} \)
The Astrophysical Journal, 832:183 (30pp), 2016 December 1

KOSTOV ET AL.

Table 11
Expected Dynamical and Light Travel Time ETV Amplitudes and Periods after the Respective CE Stages for Kepler CBPs with Known Masses (Kepler-1647 and Kepler-35)

| \( \alpha_M \) (\( M_\odot \) yr\(^{-1} \)) | \( A_{\text{dyn}} \) (min/max) | \( A_{\text{LTT}} \) (min/max) | \( P \) (min/max) | Notes |
|---|---|---|---|---|
| Kepler-1647b\(^a\) | 1.0 | 0.1/13.4\(^b\) | 68/1682 | 2.4/292 | ... |
| 0.1 | 154/680 | 8.1/75 | ... |
| Kepler-1647b\(^c\) | 1.0 | 0.1/0.2 | 736/9680 | 31/1490 | ... |
| 0.1 | 668/5420 | 27/624 | ... |
| Kepler-35b\(^a\) | 1.0 | 0.1/56.5 | 4.3/36.9 | 0.7/17 | ... |
| 0.1 | 0.2/44.2 | 5.0/5.1 | 0.9/0.9 | ... |
| NN Ser bc | ~5/30\(^d\) | ~5/30 | ~7/15 | ... |
| HW Vir bc | ~55/550\(^d\) | ~50/550 | ~13/55 | ... |

Notes. For comparison, we also list the detected ETV amplitudes and periods for the CBP candidates in NN Ser and HW Vir.

\(^a\) After primary CE.

\(^b\) Min/max for the range of parameters from Table 8.

\(^c\) After secondary CE.

\(^d\) Showing the combined effect of inner/outer CBPs (instead of min/max).

We have presented numerical studies of the dynamical evolution of the nine Kepler CBP systems as their host binary stars undergo CE phases. Five of the systems undergo at least one RLOF and CE phases; Kepler-1647 experiences three RLOFs. Two systems trigger a double-degenerate SN explosion. Depending on the treatment of the CE phase (i.e., efficiency and tidal evolution), the binaries either coalesce or shrink into very close WD–MS or WD–WD pairs of stars. Despite the dramatic reconfiguration of the binary stars during these violent evolutionary phases, the planets predominantly survive the CE stage even for mass-loss rates of 1 \( M_\odot \) yr\(^{-1} \). The PCE orbital configurations of these CBPs depend on the rate at which the binary loses mass. For mass-loss rates of 1 \( M_\odot \) yr\(^{-1} \), the CBPs migrate nonadiabatically outward to highly eccentric orbits, while for slower mass-loss rates (0.1 \( M_\odot \) yr\(^{-1} \)) our simulations reproduce the expected adiabatic orbital expansion. Overall, the PCE semimajor axes and eccentricities of Kepler CBPs are qualitatively consistent with those of the currently known PCE CBP candidates. The mode of orbital expansion depends on the particular configuration of the system, such that a CBP can migrate adiabatically during the primary CE stage but nonadiabatically during the secondary CE stage. Multiplanet CBP systems can experience both modes simultaneously: an inner planet can migrate adiabatically while an outer does so nonadiabatically, both occurring during the same CE stage. Our results also indicate that planets are more likely to survive around evolving close binary stars than around evolving single stars, thus improving the discovery prospects for CBPs in PCE systems.

We thank the anonymous referee for the insightful comments that helped us improve this paper. The authors are grateful to Jarrod Hurley and Marten van Kerkwijk for valuable discussions. VBK gratefully acknowledges support by an appointment to the NASA Postdoctoral Program at the Goddard Space Flight Center. DT was supported by a postdoctoral fellowship from the Centre for Planetary Sciences at the University of Toronto at Scarborough and is grateful for additional support from the Jeffrey L. Bishop Fellowship. This work was supported in part by NSERC grants to RJ. We acknowledge conversations with Daniel Fabrycky, Nader Haghighipour, Kaiitlin Kratter, Boyana Lilian, Jerome Orosz, and William Welsh.

APPENDIX

LIST OF ABBREVIATIONS AND PARAMETERS

| BSE: Binary star evolution code |
| CE: Common envelope |
| CBP: Circumbinary planet |
| EB: Eclipsing binary |
| NTCP: No tidal circularization path (tides “OFF” in BSE) |
| PCE: Postcommon envelope |
| RLOF: Roche-lobe overflow |
| TCP: Tidal circularization path (tides “ON” in BSE) |

\( \alpha_M \): Common-envelope mass-loss rate

\( \alpha : 1 \ M_\odot \) yr\(^{-1} \) mass-loss rate

\( \alpha_{0.1} : 0.1 \ M_\odot \) yr\(^{-1} \) mass-loss rate

\( \alpha_{\text{crit}} \): Critical common-envelope mass-loss rate
α_{\text{CE}}: Common-envelope efficiency parameter

α_{\text{CBP,PCE}}: Initial semimajor axis of the circumbinary planets

β: Ratio between initial and final mass of the system

β_{\text{psect}}: Runaway ejection ratio between initial and final mass of the system

e_{\text{CBP,PCE}}: Initial eccentricity of the circumbinary planets

e_{\text{CBP,PCE}}: Eccentricity of the circumbinary planets at the end of the common-envelope phase

ψ: Common-envelope mass-loss index

T_{\text{CE}}: Common-envelope mass-loss timescale

REFERENCES

Adams, F. C., Anderson, K. R., & Bloch, A. M. 2013, MNRAS, 431, 438

Adams, F. C., & Bloch, A. M. 2013, ApJ, 777, 30

Armitage, P. J., & Hansen, B. M. S. 1999, Natur, 402, 633

Bear, E., & Soker, N. 2016, arXiv:160608149B

Bear, E., & Soker, N. 2014, MNRAS, 444, 1698

Borkovits, T., Derekas, A., Kiss, L. L., et al. 2016, MNRAS, 455, 4136

Borkovits, T., Hajdu, T., Szakovicz, J., et al. 2015, MNRAS, 448, 1656

Borkovits, T., Rappaport, S., Hajdu, T., & Szakovicz, J. 2015, MNRAS, 448, 948

de Kool, M. 1990, ApJ, 358, 189

De Marco, O., Farihi, J., & Nordhaus, J. 2009, JPhCS, 172, 012031

Doyle, L. R., Carter, J. A., Fabrycky, D. C., et al. 2011, Sci, 333, 6049

Duncan, M. J., & Lissauer, J. J. 1998, Icar, 134, 303

Farihi, J. 2016, NewAR, 71, 9

Hardy, A., Schreiber, M. R., Parsons, S. G., et al. 2016, arXiv:1604.05808

Hardy, A., Schreiber, M. R., Parsons, S. G., et al. 2015, ApJ, 800, 24

Hilditch, R. W. (ed.) 2001, An Introduction to Close Binary Stars (Cambridge: Cambridge Univ. Press), 392

Holman, M. J., & Wiegert, P. A. 1999, AJ, 117, 621

Hurley, J. R., Pols, O. R., & Tout, C. A. 2000, MNRAS, 315, 543

Hurley, J. R., Tout, C. A., & Pols, O. R. 2002, MNRAS, 329, 897

Iaconi, R., Reichardt, T., Staff, J., et al. 2016, arXiv:160301953

Ivanova, N., Justham, S., Chen, X., et al. 2013, A&ARv, 21, 59

Jones, D. 2015, EAS, 71, 113

Kashi, A., & Soker, N. 2011, MNRAS, 417, 1466

Kippenhahn, R., & Wiegert, A. 1990, Stellar Structure and Evolution (Berlin: Springer)

Kopparapu, R. K., Ramirez, R. M., SchottelKotte, J., et al. 2014, ApJ, 787, 2

Kostov, V. B., McCullough, P. R., Carter, J. A., et al. 2014, ApJ, 784, 14

Kostov, V. B., McCullough, P. R., Hinse, T. C., et al. 2013, ApJ, 770, 52

Kostov, V. B., Orosz, J., Welsh, W., et al. 2016, ApJ, 827, 86

Kratter, K., & Perets, H. B. 2012, ApJ, 753, 91

Kuiper, G. P. 1941, ApJ, 93, 133

Livio, M., & Soker, N. 1984, MNRAS, 208, 763

Mustill, A., Marshall, J. P., Villaver, E., et al. 2013, MNRAS, 436, 2515

Nandez, J. L. A., Ivanova, N., & Lombardi, J. C., Jr. 2014, ApJ, 786, 39

Orosz, J. A. 2015, ASPC, 496, 55

Orosz, J. A., Welsh, W. F., Carter, J. A., et al. 2012a, Sci, 337, 1511

Orosz, J. A., Welsh, W. F., Carter, J. A., et al. 2012b, ApJ, 758, 87

Paczyński, B. 1976, BAAS, 8, 442

Papaloizou, J. C. B., & Larwood, J. D. 2000, MNRAS, 315, 823

Passy, J.-C., De Marco, O., Fryer, C. L., et al. 2012, ApJ, 744, 52

Pejcha, O., Metzger, B. D., & Tomida, K. 2016, MNRAS, 461, 3

Perets, H. 2010, arXiv:1001.0581

Portegies Zwart, S. 2013, MNRAS, 429, 45

Raghavan, D., McAlister, H. A., Henry, T. J., et al. 2010, ApJS, 191, 1

Rasio, F., & Livio, M. 1996, ApJ, 471, 366

Reffert, S., Bergmann, C., Quirrenbach, A., et al. 2015, A&A, 574, 116

Rein, H., & Liu, S.-F. 2012, A&A, 537, 128

Rein, H., & Spiegel, D. S. 2015, MNRAS, 448, 58

Ricker, P. M., & Taam, R. E. 2008, ApJ, 672, 41

Ricker, P. M., & Taam, R. E. 2012, ApJ, 746, 74

Sandquist, E. L., Taam, R. E., Chen, X., et al. 1998, ApJ, 500, 909

Schleicher, D. R. G., & Dreizler, S. 2014, A&A, 563, 61

Schleicher, D. R. G., Dreizler, S., Volschow, M., et al. 2015, AN, 336, 458

Schröder, K.-P., & Connon Smith, R. 2008, MNRAS, 386, 15

Schwamb, M. E., Orosz, J. A., Carter, J. A., et al. 2013, ApJ, 768, 127

Shu, F. H., Anderson, L., & Lubow, S. H. 1979, ApJ, 229, 223

Taam, R. E., & Ricker, P. M. 2010, NewAR, 54, 65

Taam, R. E., & Sandquist, E. L. 2000, ARA&A, 38, 113

Tutukov, A. V., & Fedorova, A. V. 2012, ARep, 56, 305

Vaccaro, T. R., Wilson, R. E., Van Hamme, W., & Terrell, D. 2015, ApJ, 810, 157

Veras, D., Georgakarakos, N., Dobbs-Dixon, I., & Gaensicke, B. T. 2016, arXiv:1600.05307V

Veras, D., & Tout, C. A. 2012, MNRAS, 422, 1648

Veras, D., Wyatt, M. C., Mustill, A. J., et al. 2011, MNRAS, 417, 2104

Veras, D. 2016, RSOS, 3.0571

Verhoelst, T., van Aarle, E., & Acke, B. 2007, A&A, 470, 21

Villaver, E., & Livio, M. 2007, ApJ, 661, 1192

Volschow, M., Banerjee, R, & Hessman, F. V. 2014, A&A, 562, 19

Volschow, M., Schleicher, D. R. G., Perets, H., & Banerjee, R. 2016, A&A, 587, 34

Webbink, R. F. 2008, ASSL, 352, 233

Welsh, W. F., Orosz, J. A., Carter, J. A., et al. 2012, Natur, 481, 475

Welsh, W. F., Orosz, J. A., Short, D. R., et al. 2015, ApJ, 809, 26

Wittenmyer, R. A., Johnson, J. A., Butler, R. P., et al. 2016, ApJ, 818, 35

Wolszczan, A., & Frail, D. A. 1992, Natur, 355, 145

Zijlstra, A. A. 2014, RMxAA, 51, 221

Zorotovic, M., & Schreiber, M. R. 2013, A&A, 549, 95