Effect of optimal uncoupling in enhancing synchronization stability in drive-response systems

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received 8 June 2020; accepted in final form 28 August 2020
published online 3 November 2020

PACS 05.45.Xt – Synchronization; coupled oscillators
PACS 05.45.-a – Nonlinear dynamics and chaos
PACS 05.45.Gg – Control of chaos, applications of chaos

Abstract – In this paper, we report the implementation of the method of optimal uncoupling and its effect in enhancing the stability of synchronization in certain coupled third-order chaotic systems. The clipping of phase space of the response system to a finite width having certain orientation about the coordinate axes representing the state variables of the response system insists that those state variables are coupled with their counterpart of the drive system. The stability of synchronization is studied through the master stability function (MSF). The optimal directions of implementing the clipping width to achieve stable synchronization is observed by studying the effectiveness of the clipping fraction and the sufficient range of orientation to identify the optimal directions is reported. The functional work steps for identifying the optimal directions are presented and the synchronization of the response system with the drive within the clipped region of phase space for different orientations of clipping width are studied. The stability of synchronization for different orientations of the clipping widths and the two-parameter bifurcation diagram indicating the negative valued MSF regions obtained for the optimal direction of clipping width are presented. The application of the method of optimal uncoupling in identifying the direction of the implication of the clipping width is discussed and the range of orientation over which the clipping width has to be varied is generalized.

Introduction. – The dynamical process of synchronization in coupled chaotic systems has greatly influenced researchers because of the sensitive dependence of chaos on initial conditions \([1,2]\). The high complexity and unpredictability prevailing in the dynamics of chaotic systems require a complete understanding on the synchronization dynamics of coupled systems as it has potential applications in secure transmission of information signals \([3–8]\). Several higher- and low-dimensional chaotic systems have been studied for synchronization and numerous electronic circuit systems have been analyzed for the application of chaos synchronization to secure communication \([4,6,9–15]\). The important requirement for signal transmission by chaos synchronization is that the coupled systems must exist in stable synchronized states over greater values of coupling strength. The existence of coupled chaotic systems in stable synchronized states is observed through the evaluation of the master stability function (MSF) \([16]\) and the negative valued regions of MSF become a necessary condition for the occurrence of synchronization. Recently, induced synchronization has been observed in coupled chaotic systems by Schröder et al. using the method of transient uncoupling \([17]\). This method induces synchronization in coupled systems and enhances the stability of synchronization to greater values of coupling strength \([18–20]\). Further, the effect of the size of the chaotic attractors with different Lyapunov dimension in enhancing synchronization stability is studied \([21]\). However, the direction dependence of the method transient uncoupling in clipping the phase space of chaotic attractors of the response system in a drive-response scenario is

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yet to be studied. This paper introduces a new approach to study the direction dependence of transient uncoupling, i.e., \textit{optimal uncoupling} and for the identification of optimal directions of implementing clipping widths to achieve greater stable synchronization in coupled chaotic systems. The following points are discussed in this article. The methods of transient and optimal uncoupling are briefly discussed followed by the implementation of the method of optimal uncoupling in enhancing synchronization stability in certain drive-response systems. This method has also been extended to study its applicability in coupled non-identical systems.

\textbf{Transient and optimal uncoupling.} – The method of \textit{transient uncoupling} involves the clipping of phase space of the response system along the coordinate axis to obtain the clipped region within which the coupling strength is active. The state equations of a $d$-dimensional chaotic system subjected to transient uncoupling driven by an identical chaotic drive system can be written as

$$\dot{x}_2 = F(x_2) + \epsilon \chi_A(x_2) G \times (x_1 - x_2),$$

(1) where $\epsilon$, $\chi_A$ represent the coupling strength and transient uncoupling factor and $G$ is the coupling matrix. The terms $x_1$, $x_2$ represent the state vectors of the drive and response systems and the transient uncoupling factor $\chi_A$ representing the region of the phase space $A$, where $A \subseteq \mathbb{R}^d$, is written as

$$\chi_A = \begin{cases} 1, & \text{if } x_2 \in A, \\ 0, & \text{if } x_2 \notin A. \end{cases}$$

(2) The phase space of the response system is clipped normal to the axis of the coordinate variable ($x_2$), where $i = 1, 2, \ldots, d$, that couples the drive and response systems with respect to a point ($x_2^*$), to a width $\Delta$. The clipped region of the phase space is given as

$$A_\Delta = \{x_2 \in \mathbb{R}^d: |(x_2)_i - (x_2^*)_i| \leq \Delta\}.$$ 

(3) However, the clipping of the phase space of the response system has not to be always restricted to any of the coordinate axes and it can have orientations ($\theta$) with respect to the coordinate axes. The method of finding the optimal direction ($\theta^*$) for applying the clipping width to obtain stable synchronization leads to the evaluation of the effectiveness of the clipping fraction for which synchronization is observed in the coupled systems for a fixed value of the coupling strength [17] and is given as

$$S(\theta) = \int_0^1 s(f, \theta) df,$$

(4) where the synchrony indicator $s(f, \theta)$ is

$$s(f, \theta) = \begin{cases} 1, & \text{if } \lambda_{\max}^{+} < 0, \\ 0, & \text{if } \lambda_{\max}^{+} \geq 0 \end{cases}$$

(5) with $f$ being the temporal clipping fraction given as

$$f = \lim_{T \to \infty} \frac{1}{T} \int_0^T \chi_A(x_2(t)) dt.$$ 

(6) The \textit{master stability function} being the largest transverse Lyapunov exponent $\lambda_{\max}^{+}$ is obtained to identify the stability of synchronized states in coupled chaotic systems [16,22].

\textbf{Results and discussion.} – We present in this section, the effect of optimal uncoupling in enhancing the stability of synchronized states and explain the method of optimal uncoupling in identifying the optimal directions for applying the clipping widths. The orientation of the clipping width ($\theta$) in the phase space of the response system is considered to vary in the anticlockwise direction with respect to the $x$- or $z$-axis of the discussed corresponding attractor. The clipping width $\Delta$ oriented at an angle of $\theta$ radians, has components along both the coordinate axes. Hence, the region of the clipped phase space must include clipping along both the axes, which indirectly implies that both of the state variables representing the attractor in the phase space must be coupled to the respective variables of the drive system. The \textit{Rössler} and the \textit{Chua’s circuit} systems are studied in this paper using this new approach to identify the optimal directions of implementing clipping widths.

\textbf{Rössler system.} The state equations of coupled \textit{Rössler} systems [23] with the clipping of the phase space along a particular direction can be written as

$$\begin{align*}
\dot{x}_1 &= -y_1 - z_1, \\
\dot{y}_1 &= x_1 + ay_1, \\
\dot{z}_1 &= b + (x_1 - c)z_1, \\
\dot{x}_2 &= -y_2 - z_2 + \epsilon \chi_A(x_1 - x_2), \\
\dot{y}_2 &= x_2 + ay_2 + \epsilon \chi_A(y_1 - y_2), \\
\dot{z}_2 &= b + (x_2 - c)z_2,
\end{align*}$$

(7) where $x_{1,2}$, $y_{1,2}$, $z_{1,2}$ represent the state variables of the drive and response systems. Considering that the deviation of the attractor along the $z$-axis is minimum [17], the synchronization stability of the coupled \textit{Rössler} systems corresponding to the orientation of the clipping widths in the $x$-$y$ plane can be explored. Hence, an orientation of the clipping width in the $x$-$y$ plane must have its components along the corresponding coordinates axes leading to the coupling of the systems through the $x$ and $y$ state variables. Equations (7d) and (e) indicate that for clipping widths oriented with respect to the coordinate axes of the state space vectors ($x$ and $y$), i.e., for $\theta \neq 0, \pi$ and $\theta \neq \pi/2, 3\pi/2$, the systems are unidirectionally coupled through both the $x$ and $y$ variables by the factor $\epsilon \chi_A$. For clipping widths with orientations given by $\theta = 0, \pi$ or $\theta = \pi/2, 3\pi/2$, the systems are coupled through the state variables $x$ or $y$, respectively.
Fig. 1: Rössler system: clipping of phase space of the chaotic attractors corresponding to the response system (cyan) along with the attractor of the drive (green) in the $x_1,y_1,z_1$ phase planes obtained for the parameters $a = 0.2$, $b = 0.2$, $c = 5.7$ and $\epsilon = 0$. $O(x^*,y^*)$ is the center of the chaotic attractor with $(x^*,y^*) = (1.2,-1.5)$ and $OA = \Delta$ is the clipping width. Clipping is implemented along a particular direction “$\theta$” from the x-coordinate axis in the $x_2,y_2$ plane over a width of $2\Delta$. $\Delta_x = \Delta \cos(\theta)$ and $\Delta_y = \Delta \sin(\theta)$ represent the components of the vector $\Delta$ along the coordinate axes. The coupling strength is active only over the region of the phase space of the response system within the red-colored box indicating the intersection of the components $\Delta_x$ and $\Delta_y$ of the vector $\Delta$ along the corresponding axis.

The stability of synchronization of the coupled Rössler systems can be analyzed by observing the MSF to identify the optimal direction of implementing the clipping width with respect to a particular coordinate axis. Figure 2(a), shows the variation of MSF as a function of the orientation of the clipping width ($\theta$) and the clipping fraction ($\Delta'$). The orientation of the clipping width is varied from 0 to \( \pi \) radians with respect to the x-coordinate axis of the response system. The 3D plot shown in fig. 2(a) shows the existence of a certain range of optimal directions over which the coupled system confines to stable synchronized states. Figure 2(b), showing the variation of the effectiveness of the clipping fraction $S(\theta)$ as a function of the orientation angle $\theta$ in the range $0 \leq \theta \leq \pi$, leads to some interesting results. Firstly, the curve is symmetrical on both sides about the orientation angle $\theta = \pi/2$. Hence, the clipping width can be sufficiently varied through the angle $0 \leq \theta \leq \pi/2$ to study the effectiveness of the clipping orientation in coupled systems. Secondly, from fig. 2(b), it can be observed that the optimal direction $\theta^*$ over which the effectiveness of the clipping fraction is observed and greater stable synchronized states are promised exists over a range of orientations of clipping widths. For the Rössler system, the optimal directions for stable synchronization are observed in the ranges $0.1667\pi \leq \theta^* \leq 0.3444\pi$ and $0.6556\pi \leq \theta^* \leq 0.8333\pi$, respectively. Figure 3 shows the variation of MSF ($\lambda_{\text{max}}^\perp$) with $\Delta'$ for different orientations of the clipping width for a common value of the coupling strength $\epsilon = 10$. Figure 3(a) and (b), showing the MSF variation with $\Delta'$ for $\theta = 0$ (x-coupling) and $\theta = \pi/2$ (y-coupling), indicates stable synchronized states in the range of clipping fractions $0.1145 \leq \Delta' \leq 0.7739$ to achieve stable synchronization is summarized as follows:

1. Fix the center of the chaotic attractor of the response system $(x_2^*,y_2^*)$ along the coordinate axis of the state variable coupled to the drive system.
2. For a fixed value of $\theta$ and clipping width $\Delta$ identify the region of phase space within which the coupling strength is active by resolving the horizontal ($\Delta_x = \Delta \cos(\theta)$) and vertical component ($\Delta_y = \Delta \sin(\theta)$) of the vector $OA$ ($OA = \Delta$) and estimate $\lambda_{\text{max}}^\perp$.
3. Vary the clipping fraction $\Delta'$ ($\Delta'_x = 2\Delta_x/\Omega_x$, $\Delta'_y = 2\Delta_y/\Omega_y$) in the range $0 \leq \Delta' \leq 1$ in steps, identify the active phase-phase region of the coupling strength to estimate $\lambda_{\text{max}}^\perp$ in each step and evaluate the effectiveness of the clipping fraction $S(\theta)$ using the synchrony indicator $s(\theta)$ obtained for each step of the clipping fraction.
4. Evaluate $S(\theta)$ for each value of $\theta$ by varying $\theta$ in steps in the range $0 \leq \theta \leq \pi$ by repeating steps 2) and 3).
5. Plot $S(\theta)$ obtained for the corresponding value of $\theta$ to find the optimal directions $\theta^*$. The functional work steps involved in identifying the optimal directions of applying clipping widths

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The stability of synchronization of the coupled Rössler systems can be analyzed by observing the MSF to identify the optimal direction of implementing the clipping width with respect to a particular coordinate axis. Figure 2(a), shows the variation of MSF as a function of the orientation of the clipping width ($\theta$) and the clipping fraction ($\Delta'$). The orientation of the clipping width is varied from 0 to \( \pi \) radians with respect to the x coordinate axis of the response system. The 3D plot shown in fig. 2(a) shows the existence of a certain range of optimal directions over which the coupled system confines to stable synchronized states. Figure 2(b), showing the variation of the effectiveness of the clipping fraction $S(\theta)$ as a function of the orientation angle $\theta$ in the range $0 \leq \theta \leq \pi$, leads to some interesting results. Firstly, the curve is symmetrical on both sides about the orientation angle $\theta = \pi/2$. Hence, the clipping width can be sufficiently varied through the angle $0 \leq \theta \leq \pi/2$ to study the effectiveness of the clipping orientation in coupled systems. Secondly, from fig. 2(b), it can be observed that the optimal direction $\theta^*$ over which the effectiveness of the clipping fraction is observed and greater stable synchronized states are promised exists over a range of orientations of clipping widths. For the Rössler system, the optimal directions for stable synchronization are observed in the ranges $0.1667\pi \leq \theta^* \leq 0.3444\pi$ and $0.6556\pi \leq \theta^* \leq 0.8333\pi$, respectively. Figure 3 shows the variation of MSF ($\lambda_{\text{max}}^\perp$) with $\Delta'$ for different orientations of the clipping width for a common value of the coupling strength $\epsilon = 10$. Figure 3(a) and (b), showing the MSF variation with $\Delta'$ for $\theta = 0$ (x-coupling) and $\theta = \pi/2$ (y-coupling), indicates stable synchronized states in the range of clipping fractions $0.1145 \leq \Delta' \leq 0.7739$.
and \( \Delta' \geq 0.1023 \), respectively. Figure 3(c) shows the variation of \( \lambda_{\text{max}}' \) with \( \Delta' \) for the optimal direction \( \theta^* = \pi/4 \) indicating larger negative values of \( \lambda_{\text{max}}' \) for the region \( \Delta' \geq 0.163 \). The parameter regions in the \( \Delta'-\epsilon \) plane indicating the negative valued regions of \( \lambda_{\text{max}}' \) for the optimal direction \( \theta^* = \pi/4 \) are shown in fig. 3(d). The synchronization of the response system with drive within the clipped region of the phase space obtained for certain values of \( \theta \) is shown in fig. 4. Figure 4(a) and (b) shows the phase-portraits in the \( x_1, x_2, y_1, 2 \) planes indicating the synchronization of drive (red) and response (yellow) systems within the clipped region of the phase space (black dotted box), over which the coupling strength \( \epsilon \) is active, for the parameters \( \theta = 0.1\pi, \epsilon = 10, \Delta' = 0.6 \) and \( \theta = 0.25\pi, \epsilon = 10, \Delta' = 0.6 \), respectively.

The method of optimal uncoupling presented above can be validated through its application to another chaotic system namely, the Chua’s circuit system.

**Chua’s circuit system.** The dynamical equations of the Chua’s circuit system \([4, 24, 25]\) with the drive and response systems coupled through the \( y \) and \( z \) variables is written as

\[
\begin{align*}
\dot{x}_1 &= \alpha(y_1 - x_1 + f(x_1)), \\
\dot{y}_1 &= x_1 - y_1 + z_1, \\
\dot{z}_1 &= -\beta y_1 - \gamma z_1, \\
\dot{x}_2 &= \alpha(y_2 - x_2 + f(x_2)), \\
\dot{y}_2 &= x_2 - y_2 + z_2 + \epsilon \chi_A(y_1 - y_2), \\
\dot{z}_2 &= -\beta y_2 - \gamma z_2 + \epsilon \chi_A(z_1 - z_2),
\end{align*}
\]

where \( f(x_1) \), \( f(x_2) \) represent the three-segmented piecewise-linear functions of the drive and response systems.
Chua’s circuit system: Clipping of the phase space of the chaotic attractors corresponding to the response system (cyan) along with the attractor of the drive (green) in the $z_1, z_2$ plane for $\epsilon = 0$. The clipping width $\Delta = \Delta$ is the center of the chaotic attractor with $(z^*, y^*) = (-0.116, -0.002)$ and $OA = \Delta$ is the clipping width. Clipping is implemented along a particular direction $\theta$ from the $z$-coordinate axis in the $z_2$-$y_2$ plane over a width of $2\Delta_c$. $\Delta_c = \Delta \cos \theta$ and $\Delta_y = \Delta \sin \theta$ represent the components of the vector $\Delta$ along the coordinate axes. The coupling strength is active only over the region of the phase space of the response system within the red-colored box indicating the intersection of the components $\Delta_c$ and $\Delta_y$ of the vector $\Delta$ along the corresponding axis.

The systems given as

$$f(x_{1,2}) = \begin{cases} -bx_{1,2} + (a - b), & \text{if } x_{1,2} > 1, \\ -ax_{1,2}, & \text{if } |x_{1,2}| < 1, \\ -bx_{1,2} - (a - b), & \text{if } x_{1,2} < -1, \end{cases}$$

where $x_{1,2}, y_{1,2}, z_{1,2}$ represent the state variables of the drive and response systems. Figure 5 shows the implementation of the clipping width for a finite orientation $\theta$ about the $z$-axis in the phase space of the response system. The point $O(z^*, y^*)$ is the center of the chaotic attractor and the clipping of the phase space is performed to a width of $2\Delta_c$. The intersection of the components of the clipping width $\Delta_c$ and $\Delta_y$ on both sides of the center leading to a total width of $2\Delta$. The optimal directions of the clipping width $\Delta_c$ and $\Delta_y$ of the response system, as indicated by the red-colored dashed box in fig. 5, within which the coupling strength is valid. The clipping fraction is $\Delta' = 2\Delta_c\Delta_y/\Omega_2\Omega_y$, where $\Omega_z$ and $\Omega_y$ represent the width of the chaotic attractor along the $z$ and $y$ axes, respectively. The chaotic attractor shown in fig. 5 is obtained for the system parameters $\alpha = 10, \beta = 14.87, \gamma = 0, a = -1.55, b = -0.68$ and has a Lyapunov dimension $L_d = 2.1192$.

The stability of synchronization of the coupled Chua systems can be analyzed similar to the Rössler system to identify the optimal directions for implementing the clipping width. Figure 6(a) shows the variation of MSF as functions of the orientation of the clipping width ($\theta$) and the clipping fraction ($\Delta'$). Figure 6(b) showing the variation of the effectiveness of the clipping fraction $S(\theta)$ with orientation of the clipping width $\theta$ indicating symmetry of the curve about the angle $\theta = \pi/2$. The optimal directions of clipping exists in the ranges $0.1667\pi \leq \theta^* \leq 0.3111\pi$ and $0.6889\pi \leq \theta^* \leq 0.8333\pi$, respectively.

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Figure 5: Chua’s circuit system: (a) 3D plot indicating the variation of $\lambda^{\perp}_{\text{max}}$ with the orientation of the clipping width $\theta$ and clipping fraction $\Delta'$; (b) variation of the effectiveness of clipping fraction $S(\theta)$ with the orientation of the clipping width $\theta$ indicating symmetry of the curve about the angle $\theta = \pi/2$. The optimal directions of clipping exists in the ranges $0.1667\pi \leq \theta^* \leq 0.3111\pi$ and $0.6889\pi \leq \theta^* \leq 0.8333\pi$, respectively.

Figure 6(b) showing the variation of the effectiveness of the clipping fraction $S(\theta)$ with orientation $\theta$ in the range $0 \leq \theta \leq \pi$ indicates optimal directions ($\theta^*$) for stable synchronization in the ranges $0.1667\pi \leq \theta^* \leq 0.3111\pi$ and $0.6889\pi \leq \theta^* \leq 0.8333\pi$, respectively. Further, the curve is symmetrical on both sides about the angle $\theta = \pi/2$. Hence, it is confirmed that optimal directions of the clipping width can be obtained by studying the effectiveness of the clipping fraction over the range $0 \leq \theta \leq \pi/2$. Figure 7 shows the variation of $\lambda^{\perp}_{\text{max}}$ with $\Delta'$ for different orientations of the clipping width for the coupling strength $\epsilon = 5$. Figures 7(a) and (b) showing the MSF variation with $\Delta'$ for $\theta = 0$ ($z$-coupling) and $\theta = \pi/2$ ($y$-coupling) indicate stable synchronized states in the ranges $0.6766 \leq \Delta' \leq 0.931$ and $\Delta' \geq 0.4468$, respectively. Figure 7(c) shows the variation of $\lambda^{\perp}_{\text{max}}$ with $\Delta'$ for the optimal direction $\theta^* = \pi/4$ indicating larger negative values of $\lambda^{\perp}_{\text{max}}$ for $\Delta' \geq 0.3295$. The parameter regions
Fig. 7: Chua’s circuit system: variation of $\lambda^*_{\text{max}}$ with the clipping fraction ($\Delta'$) for different orientations of the clipping width $\theta$ with the coupling strength fixed at $\epsilon = 5$. The variation of $\lambda^*_{\text{max}}$ with $\Delta'$ for (a) $\theta = 0$, i.e., $z$-coupling, indicates stable synchronization in the range $0.6760 \leq \Delta' \leq 0.931$; (b) $\theta = \pi/2$, i.e., $y$-coupling, indicates stable synchronization for $\Delta' \geq 0.4468$; (c) the optimal direction of clipping width $\theta^* = \pi/4$ indicates stable synchronization for $\Delta' \geq 0.3295$; (d) parameter regions for stable synchronization (gray) in the $\Delta'$-$\epsilon$ plane.

Fig. 8: Chua’s circuit system: phase-portraits of the drive (red) and response (yellow) systems in $x_1$-$y_1$, $y_1$-$x_2$ planes indicating synchronization of the coupled systems within the clipped region (black dotted box) for the parameters (a) $\theta = 0.1\pi$, $\epsilon = 5$, $\Delta' = 0.8$ and (b) $\theta = 0.25\pi$, $\epsilon = 5$, $\Delta' = 0.8$, respectively.

in the $\Delta'$-$\epsilon$ plane indicating the negative valued regions of $\lambda^*_{\text{max}}$ for the optimal direction $\theta^* = \pi/4$ are shown in fig. 7(d). The synchronization of the drive (red) and response (yellow) systems within the clipped region of the phase space (black dotted box) obtained in the $x_1$-$x_2$, $y_1$-$x_2$ phase planes for the parameters $\theta = 0.1\pi$, $\epsilon = 5$, $\Delta' = 0.8$ and $\theta = 0.25\pi$, $\epsilon = 5$, $\Delta' = 0.8$ is as shown in fig. 8(a) and (b), respectively. Hence, the response system synchronizes with the drive within the clipped region of the phase space for suitable values of $\theta$ and $\Delta'$.

Non-identical Rössler systems. The method of optimal uncoupling can also be extended to analyze the stability of synchronized states in coupled non-identical chaotic systems. The evaluation of the MSF in coupled non-identical chaotic systems has been studied recently [26–28]. The state equations of unidirectionally coupled, non-identical Rössler systems [27,28] are written as

\[
\begin{align*}
\dot{x}_1 &= -\omega_1 y_1 - z_1, \\
\dot{y}_1 &= \omega_1 x_1 + ay_1, \\
\dot{z}_1 &= b + (x_1 - c)z_1, \\
\dot{x}_2 &= -\omega_2 y_2 - z_2 + \epsilon x_A(x_1 - x_2), \\
\dot{y}_2 &= \omega_2 x_2 + ay_2 + \epsilon x_A(y_1 - y_2), \\
\dot{z}_2 &= b + (x_2 - c)z_2,
\end{align*}
\]

where $\omega_1 = 1.07$ and $\omega_2 = 1.05$ represent the frequency of oscillation of the coupled systems and the system parameters take the values $a = 0.2$, $b = 0.2$, $c = 5.7$. The response system is coupled to the drive through the state variables $x$ and $y$ and the transient uncoupling factor is introduced in the response system. Figure 9(a) shows the variation of $\lambda^*_{\text{max}}$ in the $\theta$-$\Delta'$ plane and the variation of $S(\theta)$ with $\theta$ indicating narrow regions of optimal directions for stable
synchronization is shown in fig. 9(b). Figure 9(c) shows that synchronization vanishes in the $x$-coupled system for $\epsilon \geq 2.0089$ in the absence of optimal uncoupling and the existence of stable synchronized states in the presence of optimal uncoupling for different orientations of clipping width is shown in fig. 9(d)–(f).

**Conclusion.** – In this paper, we have reported the implementation of optimal uncoupling in enhancing the stability of synchronization in coupled identical and non-identical chaotic systems. The orientation of the clipping width in the phase space of the attractor leads to the coupling of the systems through both of the state variables representing the phase space of the attractor. The optimal directions for implementing the clipping width to achieve stable synchronization are observed over certain ranges of orientation and the functional work steps for identifying the optimal directions are presented. The method presented in the paper reveals the sufficient directions of orientation that have to be studied to identify the optimal directions, which has been confirmed through studies of the Rössler and Chua’s circuit systems. Further, we have also reported the applicability of this method in enhancing synchronization stability in coupled non-identical Rössler systems. The present study leads to the implementation of the method of transient uncoupling to enhance the synchronization stability in coupled chaotic systems over any directions in the phase space of the attractors.

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