Neutron stars in generalized f(R) gravity

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Abstract

Quartic gravity theory is considered with the Einstein-Hilbert Lagrangean \( R + aR^2 + bR_{\mu\nu}R^{\mu\nu} \), \( R_{\mu\nu} \) being Ricci’s tensor and \( R \) the curvature scalar. The parameters \( a \) and \( b \) are taken of order 1 km\(^2\). Arguments are give which suggest that the effective theory so obtained may be a plausible approximation of a viable theory. A numerical integration is performed of the field equations for a free neutron gas. The result is that the star mass increases with increasing central density until about 1 solar mass and then decreases. The baryon number increases monotonically, which suggests that the theory allows stars in equilibrium with arbitrary baryon number, no matter how large.

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1 Introduction

General relativity has passed all observation tests so far, but the real theory of gravity may well differ significantly from it in strong field regions. In fact conceptual difficulties in quantizing Einstein’s theory and astrophysical observations suggest that general relativity may require modifications. In recent years a great effort has been devoted to the study of extended gravity theories, mainly with the goal of finding physical explanations to the accelerated expansion of the universe and other astrophysical observations, like the flat rotation curves in galaxies [1], [2].

Compact stars are an ideal natural laboratory to look for possible modifications of Einsteins theory and their observational signatures. A rather
general class of alternative theories of gravity has been considered recently to study slowly rotating compact stars with the purpose of investigating constraints on alternative theories. Several studies of compact stars, in particular neutron stars, have been made within extended gravity theories. There are also theories which prevent the appearance of singularities like “gravastars” and Eddington inspired gravity.

The most popular modification of general relativity, since the early days of general relativity, derives from an extension of the Einstein-Hilbert action of the form

\[ S = \frac{1}{2k} \int d^4x \sqrt{-g} (R + F) + S_{\text{mat}}, \tag{1} \]

where \( k = \frac{8\pi}{c^2} \) is Newton’s constant and I use units \( c = 1 \) throughout, and \( F \) is a function of the scalars which may be obtained by combining the Riemann tensor, \( R_{\mu\nu\lambda\sigma} \), and its derivatives, with the metric tensor, \( g_{\mu\nu} \). In particular the theory derived from the choice \( F(R) \), where \( R \) is the Ricci scalar, has been extensively explored under the name of \( f(R) \)-gravity. More general is fourth order gravity, which derives from the choice

\[ F = F \left( R, R_{\mu\nu} R^{\mu\nu}, R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} \right), \tag{2} \]

\( R_{\mu\nu} \) being the Ricci tensor. A particular example of eq. (2) is the quadratic Lagrangian which may be written without loss of generality

\[ F = a R^2 + b R_{\mu\nu} R^{\mu\nu}. \tag{3} \]

(Riemann square does not appear because it may be eliminated using the Gauss-Bonnett combination

\[ R_{GB}^2 = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma}, \]

which does not contribute to the field equations in a quadratic Lagrangian.) The Newtonian limit of the field equations derived from the Lagrangian eq. (3) has been studied elsewhere.

In this paper I report on a calculation of neutron stars using the theory derived from eq. (3) with the particular choice of parameters \( b = -2a, \sqrt{a} = 1 \) km. That theory is apparently not viable for two reasons. Firstly, in order not contradicting Solar System and terrestrial observations the parameters \( a \) and \( b \) should be not greater than a few millimeters. Secondly the weak field
limit of the theory should not present ghosts\cite{16, 17}. A solution to both problems is to assume that eq.\eqref{3} is an approximation, valid for the strong fields appearing in neutron stars, of another function $F$ which is extremely small in the weak field limit. This would be the case, for instance, if $F$ has the form

$$F = aR^2 + bR_{\mu\nu}R^{\mu\nu} - c \log \left(1 + \frac{a}{c}R^2 + \frac{b}{c}R_{\mu\nu}R^{\mu\nu}\right).$$

with $a \simeq 10^6 m^2$, $b - 2a$, $c = 1/(10^{26} m^2)$. Thus eq.\eqref{4} may be approximated by

$$F \approx \frac{1}{2c} \left(aR^2 + bR_{\mu\nu}R^{\mu\nu}\right)^2 \lesssim 10^{-32} R,$$  

for the Solar System and the relative error due to the terms neglected in going from eq.\eqref{4} to eq.\eqref{5} is smaller than $10^{-12}$. I have taken into account that $R^2 \sim R_{\mu\nu}R^{\mu\nu} \sim (k\rho)^2$ and that the typical density $\rho \sim 10^4$ kg/m$^3$. The inequality in \eqref{5} shows that the correction to GR due to the function $F$, eq.\eqref{4}, is negligible in Solar System or terrestrial calculations. Also the problem of the ghosts in the weak field limit disappears with that choice.

Indeed the theory is fine in the context of low-energy effective actions because the contribution of $R_{\mu\nu}R^{\mu\nu}$ is so small that it never dominates the dynamics of the background. On the other hand the $R^2$ term does not introduce extra graviton modes. In contrast in neutron stars, where $\rho \sim 10^{18}$ kg/m$^3$, the latter (logarithmic) term of eq.\eqref{4} is about $10^{-12}$ times the former terms and it may be ignored in the calculation.

2 Field equations

The tensor field equation derived from the functional eqs.\eqref{1} and the latter eq.\eqref{5} may be taken from the literature\cite{18, 19}. I shall write it in terms of the Einstein tensor, $G_{\mu\nu}$, rather than the Ricci tensor, $R_{\mu\nu}$, and in a form that looks like the standard Einstein equation of general relativity eq.\eqref{6}.

That is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = k \left(T_{\mu\nu}^{\text{mat}} + T_{\mu\nu}^{\text{ef}}\right),$$  

for the Solar System and the relative error due to the terms neglected in going from eq.\eqref{4} to eq.\eqref{5} is smaller than $10^{-12}$. I have taken into account that $R^2 \sim R_{\mu\nu}R^{\mu\nu} \sim (k\rho)^2$ and that the typical density $\rho \sim 10^4$ kg/m$^3$. The inequality in \eqref{5} shows that the correction to GR due to the function $F$, eq.\eqref{4}, is negligible in Solar System or terrestrial calculations. Also the problem of the ghosts in the weak field limit disappears with that choice. Indeed the theory is fine in the context of low-energy effective actions because the contribution of $R_{\mu\nu}R^{\mu\nu}$ is so small that it never dominates the dynamics of the background. On the other hand the $R^2$ term does not introduce extra graviton modes.

In contrast in neutron stars, where $\rho \sim 10^{18}$ kg/m$^3$, the latter (logarithmic) term of eq.\eqref{4} is about $10^{-12}$ times the former terms and it may be ignored in the calculation.
\[ kT_{\mu\nu}^{\text{ef}} \equiv -(2a + b) \left[ \nabla_{\mu} \nabla_{\nu} G - g_{\mu\nu} \Box G \right] - 2(a + b) \left[ -GG_{\mu\nu} + \frac{1}{4}g_{\mu\nu}G^2 \right] - b \left[ 2G_{\mu}^{\sigma}G_{\sigma\nu} - \frac{1}{2}g_{\mu\nu}G_{\lambda\sigma}G^{\lambda\sigma} - \nabla_{\sigma} \nabla_{\nu} G_{\mu}^{\sigma} - \nabla_{\sigma} \nabla_{\mu} G_{\nu}^{\sigma} + \Box G_{\mu\nu} \right]. \tag{7} \]

We are interested in static problems of spherical symmetry and will use the standard metric
\[ ds^2 = -\exp(\beta(r)) \, dt^2 + \exp(\alpha(r)) \, dr^2 + r^2 d\Omega^2. \tag{8} \]

Thus \( G_{\mu\nu}(r) \) and \( T_{\mu\nu}^{\text{mat}}(r) \) have 3 independent components each, so that including \( \alpha(r) \) and \( \beta(r) \) there are 8 variables. On the other hand there are 8 equations, namely 3 eqs\[7\], 3 more equations giving the independent components of \( G_{\mu\nu} \) in terms of \( \alpha \) and \( \beta \) and 2 equations of state relating the 3 independent components of \( T_{\mu\nu}^{\text{mat}} \). I shall assume local isotropy for matter so that one of the latter will be the equality \( T_{11}^{\text{mat}} = T_{22}^{\text{mat}} (= T_{33}^{\text{mat}} \text{ in spherical symmetry).} \) In principle the remaining 7 coupled non-linear equations may be solved exactly by numerical methods, as will be explained in Section 4.

Before proceeding, a note about the signs convention is in order. As is well known different authors use different signs in the definition of the relevant quantities. Here I shall make a choice which essentially agrees with the one of Ref.\[11\]. It may be summarized as follows
\[ g_{00} = -\exp \beta, G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = kT_{\mu\nu}, T_0^0 = -\rho. \tag{9} \]

After that I shall write the three independent components of eq\[7\] using the notation
\[ T_0^0 = -\rho, T_1^1 = p, T_2^2 = q, T_{\mu}^{\mu} = T = p + 2q - \rho, \]
\[ (T_{\text{mat}})_0 = -\rho_{\text{mat}}, (T_{\text{mat}})_1^1 = (T_{\text{mat}})_2^2 = (T_{\text{mat}})_3^3 = p_{\text{mat}}. \tag{10} \]

In the following I will name \( \rho, p \) and \( q \) the total density, radial pressure and transverse pressure respectively, whilst \( \rho_{\text{mat}} \) and \( p_{\text{mat}} \) will be named matter density and pressure respectively (remember that we assume local isotropy for matter, that is the equality of radial and transverse matter pressures.) The differences \( \rho - \rho_{\text{mat}}, p - p_{\text{mat}} \) and \( q - p_{\text{mat}} \) will be named effective density, radial pressure and transverse pressure respectively.
After some algebra I get for the components of the tensor eq.(7)

\[- \rho_{\text{mat}} = -\rho + (2a + b)e^{-\alpha}\left[ -\frac{dT}{dr^2} - \left( \frac{2}{r} - \frac{1}{2} \alpha' \right) \frac{dT}{dr} \right] + (a + b)k(\frac{1}{2} T^2 + 2T\rho) \]

\[+ b \exp(-\alpha) \left[ -\frac{2\beta'}{r} (q - p) + \left( \frac{1}{2} \alpha' \beta' - \beta'' - \frac{2\beta'}{r} \right) (\rho + p) \right] \]

\[+ b \left[ -\Delta \rho + 2k \rho^2 - \frac{1}{2} k [\rho^2 + p^2 + 2q^2] \right], \] (11)

\[p_{\text{mat}} = p - (2a + b)e^{-\alpha}\left[ \frac{2}{r} + \frac{1}{2} \beta' \right] \frac{dT}{dr} + (a + b)k(\frac{1}{2} T^2 - 2Tp) \]

\[+ b \left[ \Delta p + 2kp^2 - \frac{1}{2} k [\rho^2 + p^2 + 2q^2] \right] \]

\[+ b \exp(-\alpha) \left[ \left( \frac{2\alpha'}{r} + \frac{4}{r^2} \right) (q - p) + \left( \frac{1}{2} \alpha' \beta' + \beta'' \right) (\rho + p) \right]. \] (12)

\[p_{\text{mat}} = q - (2a + b)e^{-\alpha}\left[ \frac{d^2 T}{dr^2} + \left( \frac{1}{r} + \frac{1}{2} \beta' - \frac{1}{2} \alpha' \right) \frac{dT}{dr} \right] \]

\[+(a + b)k(\frac{1}{2} T^2 - 2Tq) + b \left[ \Delta q + 2kq^2 - \frac{1}{2} k [\rho^2 + p^2 + 2q^2] \right] \]

\[+ b \exp(-\alpha) \left[ \left( -\frac{\alpha'}{r} + \frac{\beta'}{r} - \frac{2}{r^2} \right) (q - p) + \frac{\beta'}{r} (\rho + p) \right]. \] (13)

Addition of these 3 equations gives the trace equation, that is

\[T_{\text{mat}} \equiv 3p_{\text{mat}} - \rho_{\text{mat}} = T - (6a + 2b) \Delta T, \] (14)

where \(\Delta\) is the Laplacean operator in curved space-time

\[\Delta \equiv \exp(-\alpha) \left[ \frac{d^2}{dr^2} + \left( \frac{2}{r} + \frac{1}{2} \beta' - \frac{1}{2} \alpha' \right) \frac{d}{dr} \right]. \] (15)

The quantities \(G_{\mu}^\nu\) are related to the metric coefficients \(\alpha\) and \(\beta\) and their derivatives (see e.g. [20]), hence to \(\rho, p\) and \(q\), that is

\[\exp(-\alpha) = 1 - \frac{2m}{r}, \quad \frac{\alpha'}{2} = \frac{m - 4\pi \rho r^3}{r^2 - 2mr}, \quad \beta' = \frac{2m + 4\pi \rho r^3 p}{r^2 - 2mr}, \]

\[\beta'' = \frac{8\pi r^2 (\rho + rp + 3p')}{r^2 - 2mr} - \frac{4 (m + 4\pi r^3 p)(r - m - 4\pi r^3 p)}{(r^2 - 2mr)^2}. \] (16)
where I have used units $k = 8\pi$, $c = 1$ and the radial derivative of $\alpha (\beta')$ is labelled $\alpha' (\beta'')$. The mass parameter $m$ is defined by

$$m = \int_0^r 4\pi x^2 \rho(x) dx.$$  \hfill (17)

The condition that Einstein tensor, $G_{\mu\nu}$, is divergence free leads to the hydrostatic equilibrium equation, that is

$$\frac{dp}{dr} = \frac{2(q - p)}{r} - \frac{1}{2} \beta' (\rho + p).$$  \hfill (18)

### 3 Application to neutron stars

For neutron stars, when are quadratic gravity corrections relevant? In order to answer this question we should estimate the conditions where $T_{\mu\nu}^{ef}$, eq.(7), is comparable to $T_{\mu\nu}^{mat}$. Terms with derivatives are of order

$$a \Box G \sim \left( a / R_0^2 \right) G,$$

$R_0$ being the radius of the hypothetical star. Thus these terms are relevant if the dimensionless quantity $a / R_0^2$ is of order unity, which implies that $a$ and $b$ should be of order the star radius, that is a few kilometers. Terms without derivatives are of order

$$a G^2 \sim \left( a k \rho / c^2 \right) G,$$

similar to those with derivatives.

In order to solve eqs.(11) to (18) we need an equation of state (eos), that is a relation between $p_{mat}$ and $\rho_{mat}$, appropriate for a system of neutrons. For the calculation here reported I shall choose the eos of a free (non-interacting) neutron gas. In order to make easier the rather involved numerical integration of the equations, I will simplify the said eos writing

$$\rho_{mat} = 3 p_{mat} + C p_{mat}^{3/5}, \quad C = 2.34,$$  \hfill (19)

where $\rho_{mat}$ and $p_{mat}$ are in units of $7.2 \times 10^{18}$ kg m$^{-3}$. This equation is correct in the limit of high density, where $\rho_{mat} \simeq 3p_{mat}$, and has the same dependence $p_{mat} \propto \rho_{mat}^{5/3}$ as the eos of the free neutron gas in the nonrelativistic limit of low density. The constant $C$ is so chosen that we get the same result as
Oppenheimer and Volkoff\cite{Oppenheimer1939} for the maximum mass stable star in their
general relativistic calculation.

A relevant quantity is the baryon number of the star, \( N \), which may be calculated from the baryon number density \( n(r) \) via

\[
N = \int_0^R \frac{n(r)}{\sqrt{1 - 2m(r)/r}} 4\pi r^2 dr,
\]

in our units. A relation between the number density and the matter density (or pressure) may be got from the solution of the equation

\[
p_{\text{mat}} = n \frac{d\rho_{\text{mat}}}{dn} - \rho_{\text{mat}},
\]

which follows from the first law of thermodynamics. Inserting eq.\textcolor{red}{(19)} here we get a differential equation which may be easily solved with the condition \( \rho_{\text{mat}}/n \to \mu \) for \( \rho \to 0 \), \( \mu \) being the neutron mass. I get

\[
n = C^{5/8} \rho_{\text{mat}}^{3/5} \left( 4 \rho_{\text{mat}}^{2/5} + C \right)^{3/8},
\]

where the unit of baryon number density is \( \mu^{-1} \times 10^{18} \) kg m\(^{-3}\).

4 Neutron stars in extended gravity

In order to derive the structure of neutron stars in generalized f(R) gravity
theory, as defined by eqs.\textcolor{red}{(7)}, we should solve the coupled eqs.\textcolor{red}{(11)} to \textcolor{red}{(20)} plus the hydrostatic equilibrium eq.\textcolor{red}{(18)} with our choice of the parameters \( a \) and \( b \), namely \( b = -2a \), \( \sqrt{a} = 0.96 \) km. This choose of \( a \) and \( b \) makes the calculation specially simple.

We need just 3 amongst the 4 eqs.\textcolor{red}{(11)} to \textcolor{red}{(14)}, because only 3 are independent. I choose eqs.\textcolor{red}{(14)}, the difference eq.\textcolor{red}{(13)} minus eq.\textcolor{red}{(12)}, and eq.\textcolor{red}{(12)}, which may be rewritten

\[
\frac{dT}{dr} = T', \quad \frac{dT'}{dr} = - \left( \frac{2}{r} + \frac{1}{2} \beta' - \frac{1}{2} \alpha' \right) \frac{dT}{dr} + \frac{T - T_{\text{mat}}}{2a},
\]
\[
\frac{dh}{dr} = h', h \equiv q - p,
\]
\[
\frac{dh'}{dr} = -\left(\frac{2}{r} + \frac{1}{2} \beta' - \frac{1}{2} \alpha'\right) h' + \exp(\alpha) \left[\frac{h}{2a} + kTh - 2k(h + 2p)h\right]
- \left[\left(-\frac{3\alpha'}{r} + \frac{\beta'}{r} - \frac{6}{r^2}\right) h + \left(\frac{\beta'}{r} - \beta'' + \frac{1}{2} \alpha' \beta'\right)(\rho + p)\right],
\]

(23)

\[
p_{\text{mat}} = p + ak(2T p - \frac{1}{2} T^2 - 3p^2 + \rho^2 + 2q^2)
- 2a \exp(-\alpha) \left[\Delta p + \left(\frac{2\alpha'}{r} + \frac{4}{r^2}\right) h + \left(\beta'' - \frac{1}{2} \alpha' \beta'\right)(\rho + p)\right],
\]

(24)

where the Laplacean operator \(\Delta\) was defined in eq.(15). Finally we need the hydrostatic equilibrium eq.(18).

The numerical calculation goes as follows. From the values of all variables at a given radial coordinate \(r\), integration of the linear differential eqs.(22), (23) and (18), taking eq.(17) into account, provides the values of \(m, T, T', h, h'\) and \(p\) at \(r + dr\). Hence the relation (see eq.(10))

\[
\rho = p + 2q - T = 2h + 3p - T,
\]
gives \(\rho (r + dr)\), whence eq.(18) gives \(p' (r + dr)\) which allows obtaining \(\rho' (r + dr)\).

After that we have all quantities needed to get \(d^2p/dr^2\) from the derivative of eq.(18), that is

\[
\frac{d^2p}{dr^2} = \frac{2h'}{r} - \frac{2h}{r^2} - \frac{1}{2} \beta'' (\rho + p) - \frac{1}{2} \beta' (\rho' + p').
\]

Hence we get \(p_{\text{mat}}\) from eq.(24) taking eqs.(15) and (16) into account, which allows obtaining \(\rho_{\text{mat}}\) via the eos eq.(19), whence \(T_{\text{mat}} = 3p_{\text{mat}} - \rho_{\text{mat}}\) follows (remember that we assume local isotropy for matter, that is \(p_{\text{mat}} = q_{\text{mat}}\).) In this way we obtain all the quantities at \(r + dr\) and the process may be repeated in order to get the quantities at \(r + 2dr\), and so on. This shows that our equations form a consistent system.

As initial conditions for the differential equations we need the values of the following variables at the origin: \(T(0), h(0), p(0), T'(0), h'(0)\). The latter 2 should be taken equal to zero in order that there is no singularity, and
\( h(0) = 0 \) because there is no distinction between radial, \( p \), and transverse pressure, \( q \) at the origin. We are left with just two free parameters, namely \( p(0) \) and \( T(0) \), but there is a constraint, that is the condition that \( T \to 0 \) for \( r \to \infty \). Indeed the matter density and pressure are positive within the star, so that \( p_{\text{mat}}(r) = 0 \) for any \( r > R \), \( R \) being the star radius (incidentally, there is some contribution to the star mass outside the star surface due to the effective density.) For \( r > R(\tau) \) the quantity \( T(\tau) \) (and the density \( \rho(\tau) \)) should decrease rapidly with increasing \( r \). As a consequence only the value of \( p(0) \) may be chosen at will, whilst the value of \( T(0) \) should be so chosen as to guarantee the rapid decrease of \( T(r) \) for \( r > R \). Consequently I have been lead to perform the integration several times for each choice of \( p(0) \), with a different value of \( T(0) \) each time, until I got a value of \( T(r) \) sufficiently small for large \( r \) (that is greater than the star radius). This procedure presents the practical difficulty that requires a fine tuning of \( T(0) \) due to the fact that for large \( r > R \) the solution of eqs. \( (22) \) is approximately of the form

\[
T(0) \sim \frac{A}{r} \exp \left( \frac{r}{\sqrt{2a}} \right) + \frac{B}{r} \exp \left( -\frac{r}{\sqrt{2a}} \right).
\]

Thus the parameter \( A \) should be very accurately nil in order that the first term does not surpass the second one at large \( r \). This is specially so if the parameter \( a \) is small, and this is why I have chosen to study the case of \( a \) a relatively large value of \( a \). Also in order to alleviate the problem I have substituted a differential equation for a new variable \( f \) for the eqs. \( (22) \) where

\[
T = \frac{f}{r} \exp \left( -\frac{r}{\sqrt{2a}} \right).
\]

Thus the condition that \( f \) remains bounded for \( r \to \infty \) replaces the stronger condition that \( T \to 0 \) and the numerical procedure is less unstable.

In summary we obtain a one-parameter family of equilibrium stars, one for each value of the central total pressure \( p(0) \). Table 1 reports the results of the calculation. As in the standard (GR) theory of neutron stars \(^{21}\) the radius decreases with increasing central density, whilst the mass increases until a maximum value and then decreases. Therefore our theory also predicts a maximum mass for equilibrium neutron stars. However there is a dramatic difference in the behaviour of the baryon number, which here is always increasing with increasing central density. Of course in stars with very large central density, matter will not be in the form of neutrons but
will consists of a mixture of different particles but I will assume that the total baryon number is well defined. Although I have not made a rigorous proof, the results of the calculation suggest that there may be equilibrium configurations of neutron stars for any baryon number no matter how large. A consequence of the strong increase of the baryon number with a decrease of the mass implies that the binding energy becomes very large, about 90% of the mass in the stars with the highest central density amongst those studied here.

Table 1 also shows that both the baryon number density, $n$, and the matter density, $\rho_{\text{mat}}$, become very large for moderately large central total density. This implies that the effective density, $\rho_{\text{eff}} = \rho - \rho_{\text{mat}}$, is negative in the central region of the star although becoming positive near and beyond the surface. However neither $\rho_{\text{eff}}$ nor $\rho_{\text{mat}}$ have a real physical meaning, only the total density $\rho$ being meaningful, and it remains always positive. A similar thing happens with the pressure. The surface relative red shift is higher in our theory than in the standard (GR) theory, but the difference is not dramatic.

**Table 1.** Our calculation. Central total pressure, $p(0)$, central total density, $\rho(0)$, and central matter density, $\rho_{\text{mat}}(0)$, are in units $\rho_c \equiv 7.2 \times 10^{18}$ kg/m$^3$. Central baryon number density, $n(0)$, in units $\rho_c/\mu$, $\mu$ being the neutron mass. Star radius, $R$, is in kilometers, mass, $M$, in solar masses and baryon number, $N$, in units of solar baryon number. I report also the dimensionless fractional surface red shift, $\Delta\lambda/\lambda = 1/\sqrt{1 - 2M/R} - 1$, and percent binding energy, $BE = 100(N - M)/N$. An expressions like $1.6E2$ means $1.6 \times 10^2$.

|       | $p(0)$ | $\rho(0)$ | $\rho_{\text{mat}}(0)$ | $n(0)$ | $R$ | $M$ | $N$ | $BE$ | $\Delta\lambda/\lambda$ |
|-------|--------|-----------|-------------------------|--------|-----|-----|-----|------|-------------------------|
|       | 0.01   | 0.1       | 1.0                     | 4.5    | 4.0 | 0.67  | 0.73  | 8.9% | 0.106                   |
|       | 0.18   | 0.82      | 4.5                     | 34     | 2.7 | 0.39  | 0.94  | 15%  | 0.22                    |
|       | 1.6E2  | 2.5E3     | 4.3E4                   | 7.8E5  | 3.1E2 | 0.33  | 1.03  | 41%  | 0.34                    |
|       | 56     | 4.5E2     | 3.7E3                   | 3.3E4  | 3.2E5 | 0.264 | 1.13  | 65%  | 0.31                    |
|       | 10.7   | 6.7       | 4.0                     | 2.7    | 2.1  | 0.264 | 1.44  | 82%  | 0.23                    |
|       | 0.67   | 0.80      | 0.60                    | 0.39   | 2.2  | 0.264 | 2.00  | 87%  | 0.23                    |
|       | 0.73   | 0.94      | 1.03                    | 1.34   | 2.2  | 0.264 | 2.00  | 89%  | 0.23                    |
|       | 8.9%   | 15%       | 41%                     | 65%    | 82%  | 87%   | 89%   | 89%  | 0.26                    |
5 Discussion

The calculations of this paper show that, if there are corrections to general relativity of the form of eqs.1 and 3, then the structure of neutron stars would be dramatically different from the one predicted by the general relativity. In particular a new scenario would emerge for the evolution of the central region of massive white dwarfs stars after the supernova explosion. Indeed the said central region might contract strongly by emitting an amount of energy corresponding to a very large fraction of the total mass. The final result will be a neutron star with a mass maybe surpassing the believed (Oppenheimer-Volkoff) limit. It is not possible to know how large is the new mass limit until calculations with more realistic equations of state are performed. In addition the predictions of the theory here considered may be quite different for other choices of the parameters a and b.

6 Appendix. Neutron stars in general relativity

For the sake of comparison with the results of our calculation using eqs.11 to 21, I summarize in Table 2 the results of a calculation similar to the one performed by the Oppenheimer and Volkoff calculation21 using general relativity. It corresponds to taking a = b = 0 in eqs.11 to 14, so that ρ = ρmat, p = pmat, and I use the eos eq.19. I have extended the calculation to quite high values of the central pressure because for those values the corrections to GR in our generalized f(R) gravity theory are most relevant.

Table 2. General relativistic calculation. Central pressure, p(0), and central density, ρ(0), are in units 7.2 × 10^{18} kg m^{-3}, star radius, R, in kilometers, mass, M, in solar masses and baryon number, N, in units of solar baryon number. I report also the percent binding energy, BE, defined by the ratio 100(N − M)/N and the fractional surface red shift, Δλ/λ = 1/√1 − 2M/R − 1.
Table 2 shows that both the mass, $M$, and the baryon number, $N$, increase with increasing central density until $\rho(0) \simeq 0.46$, where $M \simeq 0.72 M_\odot$ (the OV mass limit), and both $M$ and $N$ decrease after that. There are no equilibrium configurations, either stable or unstable, with baryon number above $N \simeq 0.74$. Actually for every baryon number $N < 0.74$ there are two equilibrium configurations, one of them with $\rho(0) < 0.46$ and another one with $\rho(0) > 0.46$, the latter having higher mass than the former. Furthermore, as is shown in Table 2, stars with large central density have a negative binding energy and therefore cannot be stable.

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