Morphology operators construction by adaptive elliptical structuring elements based on nonlinear structure tensor

Chunming Tang, Xinlei Liu, Yanjie Li and Hongbo Zhao

School of Electronics and Information Engineering, Tianjin Polytechnic University, Tianjin 300387, China

E-mail: Tangchunminga@hotmail.com

Abstract. As the linear structure tensor is tending to inaccurately or even wrong estimate the gradient direction of a gray-level image, we present a novel algorithm to construct adaptive elliptical structuring elements via estimating the local anisotropy of an image based on the nonlinear structure tensor. Erosion, dilation, opening, closing and Hit-or-Miss transform are redefined according to the presented structuring elements, which have been applied to some representative images. The processed results and the quantitative analysis show that the novel morphological operators have more advantages in structure adaptation, corner protection, filtering and targets extraction than the others.

1. Introduction

When traditional mathematical morphology is applied to image processing, the shape and size of a structuring element (SE) are fixed, which usually depend on the global feature of a gray-level image. The performance is usually unsatisfactory. Adaptive SEs are then appeared to improve it, whose size and shape can be adjusted depending on the local feature of an image. So far, a variety of methods are proposed. Curic and Landström[1] summarized the following three groups, based solely on connected homogeneous areas in the image, based on weighted combinations of gray-level and spatial information, and structure-based methods. The first one, such as general adaptive neighborhoods, was presented by Debayle and Pinoli[2, 3], which the SEs have no shape and size constraints, the only parameter \( m \in \mathbb{Z}^+ \) controls the homogeneity of the SEs. Hence, this approach has no relationship with the spatial relations of pixels. The second one presents the spatial constraints, taking the geometry of shapes into account, such as the morphological amoebas introduced by Lerrallut et al[4], which applied a weighted gray-level distance to restrict the shape of amoebas. But it is challenging for morphological amoebas to set an explicit weight parameter \( \lambda \). Amoebas is also sensitive to the small contrast changes that can effect the filtering results. Recently, Curic et al.[5, 6] followed the similar principle and proposed the salience adaptive structuring elements(SASEs) derived from a salience distance map of the original image. The SASEs can cross edges but can not handle the corners. The third one, such as the elliptical adaptive structuring elements (EASEs)[7] introduced by Landström, is based on the Linear Structure Tensor(LST). And the shapes of EASEs vary from lines to disks depending on the rate of anisotropy. However, the LST after Gaussian smoothing, has an important drawback, which is well known, blurring image details, especially the pixels closing to the boundary of two different objects.

Among these approaches, EASEs appear promising because of fully using the anisotropy. Therefore, this paper will present a novel method, which the adaptive elliptical SEs are defined based...
on the nonlinear structure tensor (NLST) [8] instead of the LST, as the NLST can obtain more accurate information about both local orientation and degree of the anisotropy. Meanwhile, different from reference [7], our novel method will apply the corner and edge strength defined by the eigenvalues of the NLST in order to distinguish the different parts of an image. In addition, it may improve the reference information about both local orientation and degree of the anisotropy. Meanwhile, different from nonlinear structure tensor (NLST) [8] instead of the LST, as the NLST can obtain more accurate anisotropic features of images. We apply two parameters [12]: corner strength and edge strength defined by the eigenvalues of the ST.

\[ \lambda_1 \geq \lambda_2, \] where \( \lambda_1 \) and \( \lambda_2 \) are a pair of orthogonal basis of ST. \( \theta_1 \) and \( \theta_2 \) are a pair of orthogonal basis of ST. \( \theta_1 \) represents the direction of the maximal gradient variation, while \( \theta_2 \) is orthogonal to \( \theta_1 \). \( g_1(\lambda_1) < g_2(\lambda_2) \) can ensure anisotropic diffusion, rewritten in the (3), (4).

\[ g_1(\lambda_1) = \frac{1}{(1 + \lambda_1 + \lambda_2)^{1/5}} \] (3)

\[ g_2(\lambda_2) = \frac{1}{(1 + \lambda_1 + \lambda_2)^{9/5}} \] (4)

The analysis of the eigenvalues \( \eta_1, \eta_2 \) from iteration termination of NLST provides local anisotropic features of images. We apply two parameters [12]: corner strength \( m_c \) and edge strength \( m_e \), in equations (5) and (6), to describe the regions of interest (ROIs).

\[ m_c = \frac{4\eta_1\eta_2}{\eta_1 + \eta_2} \] (5)

\[ m_e = \eta_1 - \eta_2 \] (6)

Different ROIs can be distinguished by analyzing \( m_c \) and \( m_e \). It may be an edge when \( m_e >> 0 \) and \( m_c \approx 0 \), or be a homogeneous region when \( m_e \approx 0 \) and \( m_c \approx 0 \), or be a corner, when \( m_e \approx 0 \) and \( m_c > 0 \).

3. Adaptive elliptical structuring elements based on nonlinear structure tensor

We define an elliptical SE: \( SE(x, y) = [a(x, y), b(x, y), \varphi(x, y)] \), where \( a(x, y), b(x, y) \), and \( \varphi(x, y) \) are the semi-major axis, the semi-minor axis and the orientation of a SE respectively to a pixel \( P(x, y) \). The three parameters values should obey the following three constrains:

1. If pixel \( (x, y) \) is at a corner, \( a(x, y) \) and \( b(x, y) \) should be very small.
2). If pixel \((x, y)\) is at an edge, \(a(x, y)\) should be very large, and \(b(x, y)\) is very small. The shape of the elliptical SE is close to the line, which direction is perpendicular to the gradient direction.

3). If pixel \((x, y)\) is in a homogeneous region, \(a(x, y)\) and \(b(x, y)\) should be large. And the shape of the elliptical SE is close to a disk.

We redefine \(a(x, y)\) and \(b(x, y)\) in (7) and (8):

\[
a(x, y) = r \left\{ 1 - \exp\left(- \frac{C_m}{(m_c(x, y) / \beta_1)^{m}} \right) \right\}
\]

\[
b(x, y) = a(x, y) \cdot \left\{ 1 - \exp\left(- \frac{C_m}{(m_e(x, y) / \beta_2)^{m}} \right) \right\}
\]

Where \(r\) denotes the maximal allowed semi-major axis. \(\beta_1\) and \(\beta_2\) are the normalized parameters which are set to be 75\% of the maximum of \(m_c\) and \(m_e\), respectively. \(m\) and \(C_m\) are constrained via (9)[12].

\[
1 - \exp(-C_m)(1 + m C_m) = 0
\]

Here, we set \(m = 1.1\) and \(C_m = 0.1877\).

The orientation \(\phi(x, y)\) is defined by the eigenvector[7] of the NLSE in equation (10).

\[
\phi(x, y) = \begin{cases} \arctan\left( \frac{v_{2,1}(x, y)}{v_{2,0}(x, y)} \right), & v_{2,0}(x, y) \neq 0 \\ \pi / 2, & v_{2,0}(x, y) = 0 \end{cases}
\]

4. Resolution improvement
As the parameters of an elliptical SE being calculated by equations (7), (8) and (10) are continuous, the discretization should be followed. But the discretized SE has undesirable saw-toothed edge. So we have to improve the resolution of the discrete SE and the original image.

Discrete elliptical SE is defined by a set \(S\) which contains integer coordinates \(X(x_1, x_2), X \in S\), all \(X\) should satisfy the inequation (11):

\[
\left( \frac{x \cdot \cos \phi - y \cdot \sin \phi}{a} \right)^2 + \left( \frac{x \cdot \sin \phi - y \cdot \cos \phi}{b} \right)^2 \leq 1
\]

In order to improve the resolution of the discrete elliptical SE, we subdivide coordinate \(X(x_1, x_2)\) into \(X(d / 2^k, d / 2^k)\). Where \(d\) is an integer, and \(-r \times 2^k \leq d \leq r \times 2^k\).

A simple and effective method to improve the image resolution is interpolation. But some general methods, such as the bilinear interpolation, are easy to lose the texture feature. As random fractal interpolation[13] can fully express the statistical texture features of the image, we apply it to improve the resolution of images. Random fractal interpolation is an iterative process, which is ended till the desired spatial resolution is required. When the \(k\)th iteration, the gray value of the pixel \(P[2^k x - (2^k - 1), 2^k y - (2^k - 1)]\) in the interpolated image \(I_{II}\) is the value of the pixel \(P(x, y)\) in the original image \(I\), the rest pixels’ gray values in \(I_{II}\) are obtained by computation[13].

5. Adaptive morphology operators
5.1. Erosion, Dilation, Opening and Closing
The adaptive elliptical SE: \(SE(x, y) = [a(x, y), b(x, y), \phi(x, y)]\) can be calculated by equations (1)-(10) for any pixel \(P(x, y)\). Interpolated \(SE(x, y)\) is called ESE. Only the pixel \(P[2^k x - (2^k - 1), 2^k y - (2^k - 1)]\) in \(I_{II}\) has its ESE, while other pixels in \(I_{II}\) have no corresponding ESE. Thus, only pixels \(P^*\) in \(I_{II}\) will be processed when morphological operators are applied. Our novel adaptive erosion, dilation, opening and closing are defined in (12-15), respectively.
\[ IE(x, y) = (\delta(I_n))(i, j) = \bigwedge_{(p,q) \in ESE_{(i,j)}} I_n(p,q), \quad (p,q) \in D' \] (12)

\[ ID(x, y) = (\delta(I_n))(i, j) = \bigvee_{(p,q) \in ESE'_{(i,j)}} I_n(p,q), \quad (p,q) \in D' \] (13)

\[ IO(x, y) = \gamma(I_n)(i, j) = (\delta(\varepsilon))(I_n)(i, j) \] (14)

\[ IC(x, y) = \psi(I_n)(i, j) = (\varepsilon(\delta))(I_n)(i, j) \] (15)

Where \( D' \) is the domain of \( I_{f_i} \). \( D \) is the domain of \( I \). \( i = 2^x \times (2^x - 1), j = 2^y \times (2^y - 1), (x, y) \in D, (i, j) \in D' \). \( ESE'_{(i,j)} \) is the reflected neighborhood of \( ESE(i, j) \).

5.2. Hit-or-Miss Transform

Hit-or-Miss Transform (HMT)[14] can extract all pixels in an image matched by a given neighboring configuration, which is defined by two disjoint sets, called adaptive SE pair, denoted by \((B_{FG}, B_{BG})\). \( B_{FG} \) defines the set of pixels matching the foreground, while \( B_{BG} \), defines the set of pixels matching the background. However, the traditional HMT uses the fixed SE pair in the global image, and only extracts the object with the same size and shape as \( B_{FG} \). We present an novel adaptive SE pair to extract the objects having a similar geometry shape and different sizes in one image.

In order to extract suborbicular objects with different sizes in an image, the shape of adaptive SE is defined as circle, the size of it is adjusted adaptively according to the results of the chamfer distance transform (CDT) with \( 5 \times 5 \) template. The process of calculating the radius of the \( B_{FG} \) is as follows.

1). Applying the edge strength \( m_e \) (defined in equation (6)) to search the objects’ edges in \( I \), according to (16), where \( delta \) is an empirical threshold.

\[
\begin{align*}
  f(x, y) &= m_e(x, y), \quad \text{if } m_e(x, y) > delta \\
  f(x, y) &= 0, \quad \text{other}
\end{align*}
\] (16)

2). After computing non-maximal value suppression of the \( f(x, y) \) in (16), the pixels having the local maximal value are set to 1, having non-maximal value are set to 0. We then obtain an edge image \( f_{edge} \) with a single pixel width.

3). \( f_{edge} \) is processed by CDT, the result is called \( R_{HMT}(x, y) \). The distance of each pixel in \( R_{HMT} \) is the radius of \( B_{FG} \), which is defined by (17) [15].

\[ B_{FG}(x, y) = \{(x, y) - (p, q) < R_{HMT}(x, y), (p, q) \in D \} \] (17)

In addition, \( B_{FG} \) and \( B_{BG} \) should share an origin, and \( B_{FG} \cap B_{BG} = \emptyset \). Thus \( B_{BG} \) is defined by (18)[15]:

\[ B_{BG}(x, y) = \left\{ \frac{\text{size of uncertainty regions}}{2} + |(x, y) - (p, q)| < R_{HMT}(x, y) + u, (p, q) \in D \right\} \] (18)

Where \( u \) is the size of the uncertainty regions[15] and \( b \) is the radius of \( B_{BG} \). \( u \) and \( b \) are fixed for all pixels in the image \( I \).

6. Adaptive morphology operators

6.1. Structure Adaptation

Figure 1(a) is an original image. Figure 1(b-d) are the erosion results by EASEs[7] with \( r_w = 4, M = \{4, 8, 16\} \). Figure 1(e-g) are the erosion results by EASEs[7] with \( r_w = 16, M = \{4, 8, 16\} \). The erosion results in Figure 1(h-j) are applied by our presented algorithm, we select \( r = \{1, 2, 4\} \). From Figure 1(b-g), we can see the width of the ring with the same gray value is changed in one eroded image. The reason is that \( r_w \) needs to keep balanced with \( M \) to capture of interested features. Our presented algorithm can make width same of the rings in each erosion operation.
6.2. Corner Protection

We concern a synthetic image with fifteen different size, intensity and fuzzy edge diamonds, which is shown in Figure 2(a). Figure 2(b) and (c) are the images after dilation by the SASEs and the EASEs correspondingly. Figure 2(d)-(f) are the erosion, dilation and opening results by our presented method. From the Figure 2(b), (c) and (e), we can see that neither SASEs nor EASEs can protect the diamonds’ corners well. But ours perform better than these two. If we define the intersection of a diamond’s two adjacent edges as a corner, to the bottom right diamond, the number of missing pixels of one corner in Figure 2(b), (c) and (e) is 13, 7 and 4, respectively, which can prove that our presented method can protect corner best. Additionally, the opening result of our presented method in Figure 2(f) is almost the same as the original image.

6.3. Image Filtering

6.3.1. Gaussian Noise Suppression. We consider an original image in Figure 3(a), which is added the Gaussian white noise with $\sigma = 20$ artificially, shown in Figure 3(b). Figure 3(c)-(e) are the images after opening operations by the fixed SE, the EASEs and our presented method respectively. From these results, we can see that Figure 3(c) appears some blocking artifacts, Figure 3(d) blurs the edges of the star, while Figure 3(e) has a much better denoising result compared with them. We have computed the PSNR and RMSE of each filtering result, being listed in Table 1, which can verify our presented opening operation has much better performance than the others in Gaussian noises suppression.

6.3.2. White Artifacts Suppression. Two metallurgic grain boundaries images are shown in Figure 4(a) and(d). The goal is to filter out the white artifacts in the inner part of the grain boundaries. After analysis to the image, we can see that the gradient between the artifacts and the background is the
same as the one between the grain boundaries and the background, and the connected region of the white artifacts is much smaller than that of the grain boundaries. Thus, an adaptive maximal allowed

![Figure 3](image)

**Figure 3.** Comparison for fixed SE, EASEs in [7] and our presented method for Gaussian denoising. (a) original image (178×178 pixels); (b) Noise images with standard deviation $\sigma = 20$. (c) opening by fixed disk SE of radius 3. (d) opening by EASEs for $M = 2$. (e) opening by our presented method for $r = 1$.

| Performance parameters | Fixed disk SE | EASEs | Our presented method |
|------------------------|---------------|-------|----------------------|
| RMSE                   | 27.5988       | 18.0771 | 10.7628              |
| PSNR                   | 44.4699       | 52.9323 | 63.3033              |

semi-major axis $r$ from the following three steps is proposed: 1) The original image is classified into three feature regions, which are white artifacts, grain boundaries and background. The orientation $\varphi$ of the elliptical SEs is calculated firstly by equation (10). 2) To any pixel $(x_1, y_1)$ in the grain boundaries or background, $r$ is defined as the number of pixels that belongs to one connected region in $\varphi(x_1, y_1)$ direction. 3) To any pixel $(x_2, y_2)$ in the white artifacts region, $r$ is defined as the maximum of the pixel numbers that belongs to one connected region between in $\varphi(x_2, y_2)$ direction and orthogonal to $(x_2, y_2)$.

Figure 4(b) and (e) are the erosion results of Figure 4(a) and (d) by [7], we can see that the boundaries are blurred and the artifacts are not filtered out completely. The results based on our adaptive $r$ are shown in Figure 4(c) and (f), the grain boundaries are kept better than [7] and the white artifacts are filtered out very completely.

![Figure 4](image)

**Figure 4.** (a,d) Two real metallurgic grain boundaries images (178×178 pixels); (b, e) Erosion by EASEs in [7] with $M=r_w=2$; (c, f) Erosion by our presented method with adaptive radius $r$.

6.4. Targets Extraction
Figure 5(a) and Figure 6(a) have been conducted to be tested our adaptive HMT. Figure 5(a) is a synthesized image. We compare our proposed HMT results with the classical HMT, shown in Figure 5(b), and the adaptive HMT [15], shown in Figure 5(c). Only the origins of the targets with the same size as the SE have been extracted in (b). The authors of [15] draw a conclusion that this extraction result is relevant to the contrast nor the size, therefore some low contrast disks have not been extracted in (c). Inspiringly, all of the disks have been extracted by our proposed adaptive HMT in (d).

![Figure 5](image1.png)

**Figure 5.** (a,d) Two real metallurgic grain boundaries images(178×178 pixels); (b, e) Erosion by EASEs in [7] with $M=r_w=2$; (c, f) Erosion by our presented method with adaptive radius $r$.

Figure 6(a) is a graphite cast iron image with complex background. In order to detect the graphite, we let $u$ be different values while keeping $b=1$, $b$ is a radius of $B_{BG}$. Figure 6(b)-(f) are the HMT results of $u\in\{2, 3, 4, 5, 6\}$. These different results are statistic out in Table 2. It is obvious that with the increases of the size of the uncertainly region $u$, more similar-circle graphite has been extracted. The main reason is when $u$ is small, in Figure 7(a), the dark region denotes the target, and the white point denotes its origin, the $B_{BG}$ can not match the background. When $u$ is big, in Figure 7(b), the $B_{BG}$ can match the background. However, $u$ is not the bigger, the better. When $u$ is set a rather big, the $B_{BG}$ may match an adjacent target, rather than the background, which leads to some targets can not be extracted.

![Figure 6](image2.png)

**Figure 6.** Adaptive HMT for different $u$ and constant $b=1$. (a) original image(305×305 pixels); (b-f) Adaptive HMT with $u = 2, 3, 4, 5, 6$.

**Table 2.** Statistical results of objects extraction with Figure 6

| Values | Rate of extraction |
|--------|--------------------|
| 2      | 29.90%             |
| 3      | 60.82%             |
| 4      | 83.51%             |
| 5      | 86.60%             |
| 6      | 84.54%             |
Figure 7. (a) $B_{FG}$ and $B_{BG}$ are used for the extraction of objects with smaller value of $u$; (b) $B_{FG}$ and $B_{BG}$ are used for the extraction of objects with larger value of $u$.

7. Conclusion
We present a novel method to construct the adaptive elliptical SEs via estimating the local anisotropy feature of the image based on the NLST. Erosion, dilation, opening, closing and HMT are redefined according to the presented adaptive SEs. After some images are processed by our presented morphology operators, the performance is much better in structure adaption, corner protection, filtering and objects extraction than the other advanced operators.

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