Heavy quark production
in the semihard QCD approach at THERA

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Abstract
In the framework of the semihard ($k_T$ factorization) QCD approach, we consider the photoproduction of $D^{*\pm}$ mesons associated with two hadron jets and the $D^{*\pm}$ production in DIS at THERA conditions with the emphasis on the BFKL and CCFM dynamics of gluon distributions. In the photoproduction of $D^{*\pm}$ mesons the attention is focused on the variable $x_\gamma$, which is the fraction of the photon momentum contributed to a pair of jets with largest $p_T$. We show that our theoretical results are sensitive to the BFKL type dynamics which may be investigated at THERA energies. We also discuss possible effect of $J/\psi$ meson spin alignment, which is thought to be a vivid manifestation of gluon off-shellness.

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1. Introduction

The experimental results on heavy flavour production processes obtained by the H1 and ZEUS collaborations at HERA provide a strong impetus for further theoretical and experimental studies in a new energy region at THERA conditions.

In due time, the experimental data have been compared with next-to-leading order (NLO) perturbative QCD calculations using the ‘massive’ and ‘massless’ schemes. The measured cross sections generally lie above the predicted level, and an agreement between the theoretical and experimental results can only be achieved using some extreme parameter values. In particular, the production rates of $D^{*\pm}$ mesons in the NLO massive scheme require as low quark mass as $m_c = 1.2$ GeV and as sharp charm fragmentation function as $\epsilon = 0.02$ (in the Peterson parametrization). However, even within this set of parameters, the shapes of the $D^{*\pm}$ transverse momentum and rapidity distributions cannot be said well reproduced. A good agreement between the massless scheme and the measured $p_T(D^*)$ (though not $\eta(D^*)$) spectrum was achieved upon introducing an additional charm excitation contribution assuming an incredibly large charm content in the photon structure functions: $c(x) \approx u(x)$.

The so called $k_T$ factorization, or the semihard approach (SHA), on one hand may give a reasonable solution for some of the above problems. On the other hand the significance of $k_T$ factorization (semihard) approach becomes more and more commonly recognized. Its applications to a variety of photo-, lepto- and hadroproduction processes are widely discussed in the literature. In many cases a remarkable agreement is found between the data and the theoretical calculations regarding the photo- and electroproduction of $D^{*\pm}$ mesons, forward jets, as well as for specific kinematical correlations observed in the associated $D^{*\pm}$ + jets photoproduction at HERA and also the hadroproduction of beauty, $\chi_c$ and $J/\psi$ at Tevatron. The theoretical predictions made in ref. have triggered a dedicated experimental analysis of the $J/\psi$ polarization (i.e., spin alignment) at HERA energies.

To some extent, the SHA based on the BFKL gluon dynamics includes the relevant effects of higher order contributions. It has been also demonstrated in that the SHA effectively imitates the anomalous coupling of the resolved photon and an ad hoc contribution from the resolved photon is no longer needed at least for the description of the $x_\gamma$ distribution in $D^{*\pm}$ photoproduction at HERA energies.

In the present paper we use the semihard QCD approach to predict some features of the $D^{*\pm}$ and $J/\psi$ production processes in the new energy region of THERA collider.

2. The semihard QCD approach

The production of $J/\psi$ mesons and open-flavoured $c\bar{c}$ pairs is described in terms of the photon-gluon fusion mechanism. A generalization of the usual parton model to the $k_T$-factorization approach implies two essential steps. These are the introduction of unintegrated gluon distributions and the modification of the gluon spin density matrix in the parton-level matrix elements.

At first we consider the relevant partonic subprocesses. Let $k_1$, $k_2$, $k_3$ and $p_\psi$ be the momenta of the initial state photon, the initial state gluon, the final state gluon and the final state $J/\psi$, respectively, $\epsilon_1$, $\epsilon_2$, $\epsilon_3$ and $\epsilon_\psi$ the polarization vectors, and $k = k_1 + k_2$. The photon-gluon fusion matrix elements then read:

$$
\mathcal{M}(\gamma g \rightarrow \psi g) = tr\{  f_1 (p_c - k_1 + m_c) f_2 (\epsilon) f_3 (p_c - k_3 + m_c) f_3 J(\epsilon) \}
\times [k_1^2 - 2(p_c k_1)]^{-1} [k_3^2 + 2(p_c k_3)]^{-1} + \text{permutations}
$$

Similarly, for the production of an open-flavoured $c\bar{c}$ pair (see fig. 1b):

$$
\mathcal{M}(\gamma g \rightarrow c\bar{c}) = \bar{u}(p_c)\{ f_1 (p_c - k_1 + m_c) f_2 [k_1^2 - 2k_1 p_c]^{-1}
+ f_2 (p_c - k_2 + m_c) f_2 [k_2^2 - 2k_2 p_c]^{-1} \} u(p_c)
$$
In the expression (1), the projection operator \( J(\epsilon_\psi) = k_\psi (p_\psi + m_c) / \sqrt{m_\psi} \) guarantees the proper spin structure of the \( c\bar{c} \) state, and the charmed quarks are assumed to each carry one half of the \( J/\psi \) momentum, \( p_c = p_\psi / 2, m_c = m_\psi / 2 \). The formation of a meson from the \( c\bar{c} \) pair is a nonperturbative process. Within the nonrelativistic approximation we are using, this probability reduces to a single parameter related to the meson wave function at the origin \( |\Psi(0)|^2 \), which is known for \( J/\psi \) and \( \Upsilon \) families from the measured leptonic decay widths.

The evaluation of traces in (1)-(3) is straightforward and is done using the algebraic manipulation system FORM [30]. The complete set of matrix elements has been tested for gauge invariance by substituting the gluons momenta for their polarization vectors.

The multiparticle phase space \( \prod d^3p_i / (2E_i) \delta^4(\sum p_{in} - \sum p_{out}) \) is parametrized in terms of transverse momenta, rapidities and azimuthal angles: \( \frac{d^2p_i}{2E_i} = \frac{dp_{iT}}{2\pi} dy_i \frac{d\phi_i}{2\pi} \). Let \( \phi_1 \) and \( \phi_2 \) be the azimuthal angles of the initial photon and gluon, \( \phi_3, \phi_\psi, \phi_c \) and \( \phi_\bar{c} \) the azimuthal angles of the partonic subprocess products (i.e., of a \( J/\psi \) and a gluon, or a charmed quark and an antiquark, respectively) and \( y_3, y_\psi, y_c \) and \( y_\bar{c} \) their rapidities. Then, the fully differential cross sections read:

\[
\begin{align*}
\frac{d\sigma(ep \to \psi gX)}{4 x_2 s^2} &= \frac{1}{8} \sum_{\text{spins}} \sum_{\text{colours}} |M(\gamma g \rightarrow \psi g)|^2 \\
&\times \mathcal{F}_g(x_2, k_{2T}^2, \mu^2) \frac{d^2k_{1T}}{2\pi} \frac{d^2k_{2T}}{2\pi} \frac{dp_{1\psi}}{2\pi} \frac{dp_{2\psi}}{2\pi} dy_3 dy_\psi \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{d\phi_\psi}{2\pi}, \\
\frac{d\sigma(ep \to c\bar{c}X)}{16\pi x_2 s^2} &= \frac{1}{4} \sum_{\text{spins}} \sum_{\text{colours}} |M(\gamma g \rightarrow c\bar{c})|^2 \\
&\times \mathcal{F}_g(x_2, k_{2T}^2, \mu^2) \frac{d^2k_{1T}}{2\pi} \frac{d^2k_{2T}}{2\pi} \frac{dp_{1c}}{2\pi} \frac{dp_{2c}}{2\pi} dy_c dy_\bar{c} \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} \frac{d\phi_c}{2\pi}.
\end{align*}
\]

The phase space physical boundary is determined by the inequality

\[
G(s, t, k_1^2, k_2^2, k_3^2, k_4^2) \leq 0,
\]

where \( k_3 \) and \( k_4 \) denote the partonic subprocess final state momenta, \( s = (k_1 + k_2)^2, \ t = (k_1 - k_3)^2 \), and \( G \) is the standard kinematical function [31].

When calculating the spin average of the matrix element squared, we substitute the full lepton tensor for the photon polarization matrix:

\[
\begin{align*}
\epsilon_1^\mu \epsilon_2^{\mu*} &= 4\pi \alpha [p_\psi^{\mu} p_\psi^{\nu} - 4(p, k_1) g^{\mu\nu}] / (k_1^2)^2 \\
&= \frac{\epsilon_1^{\mu} \epsilon_2^{\mu*}}{|k_{2T}|^2}.
\end{align*}
\]

The formula converges to the usual \( \sum \epsilon^\mu \epsilon^{\mu*} = -g^{\mu\nu} \) when \( k_{2T} \rightarrow 0 \). The final state gluon in (1) is assumed on-shell, \( \sum \epsilon_3^{\mu} \epsilon_3^{\mu*} = -g^{\mu\nu} \). The \( J/\psi \) polarization vector \( \epsilon_\psi \) is defined as an explicit four-vector. In the frame with \( z \)-axis along the \( J/\psi \) momentum, \( p_\psi = (0, 0, |p_\psi|, E_\psi) \), it reads for different helicity states:

\[
\epsilon_{\psi(h=\pm 1)} = (1, \pm i, 0, 0) / \sqrt{2}, \quad \epsilon_{\psi(h=0)} = (0, 0, E_\psi, |p_\psi|) / m_\psi.
\]

The initial photon and gluon momentum fractions \( x_1 \) and \( x_2 \) are calculated from the energy-momentum conservation laws in the light cone projections:

\[
\begin{align*}
(k_1 + k_2)_{E+p_{||}} &= x_1 \sqrt{s} = m_{3T} \text{exp}(y_3) + |m_{4T}| \text{exp}(y_4), \\
(k_1 + k_2)_{E-p_{||}} &= x_2 \sqrt{s} = m_{3T} \text{exp}(-y_3) + |m_{4T}| \text{exp}(-y_4).
\end{align*}
\]
The multidimensional integration in (3), (4) has been performed by means of Monte-Carlo technique, using the routine VEGAS [32].

Another important ingredient of the semihard approach is the so called unintegrated gluon distribution \( F(x,k^2_T,\mu^2) \), which determines the probability to find a gluon carrying the longitudinal momentum fraction \( x \) and transverse momentum \( k_T \). In calculations we use two different sets of unintegrated gluon distributions. One of them is based on the approach of ref. [33] and is constructed as a leading-order perturbative solution of the BFKL equations. Technically, the unintegrated gluon density \( F_g(x,k^2_T,\mu^2) \) is calculated as a convolution of the ordinary gluon density \( G(x,\mu^2) \) with universal weight factors:

\[
F_g(x,k^2_T,\mu^2) = \int_0^1 G(\eta,k^2_T,\mu^2) \frac{x}{\eta} G\left(\frac{x}{\eta},\mu^2\right) d\eta,
\]

\[
G(\eta,k^2_T,\mu^2) = \frac{\hat{\alpha}_s}{\eta k^2_T} J_0\left(2\sqrt{\hat{\alpha}_s \ln(1/\eta) \ln(\mu^2/k^2_T)}\right), \quad k^2_T < \mu^2,
\]

\[
G(\eta,k^2_T,\mu^2) = \frac{\hat{\alpha}_s}{\eta k^2_T} I_0\left(2\sqrt{\hat{\alpha}_s \ln(1/\eta) \ln(k^2_T/\mu^2)}\right), \quad k^2_T > \mu^2,
\]

where \( J_0 \) and \( I_0 \) stand for Bessel functions (of real and imaginary arguments, respectively), and \( \hat{\alpha}_s = \alpha_s/3\pi \). The LO GRV set [34] was used here for the boundary conditions. Another set of unintegrated gluon densities was extracted from a numerical simulation of the CCFM equations [35] and then tabulated in the form of a FORTRAN code [18]. Finally, the charm quarks were converted into \( D^{*\pm} \) mesons using the Peterson fragmentation function [36].

In the paper [16] we used the standard GRV parametrization [34] for the collinear gluon density, from which the unintegrated gluon distribution was developed according to eqs. (10)-(12). Some other essential parameters were chosen as follows: the charm quark mass \( m_c = 1.5 \text{ GeV} \), the Peterson fragmentation parameter \( \epsilon = 0.06 \), the overall \( c \rightarrow D^* \) fragmentation probability 0.26. The Pomeron intercept \( \Delta \) was regarded as free parameter, and then the value \( \Delta = 0.35 \) has been extracted from a fit to the experimental \( p_T(D^*) \) spectrum measured by the ZEUS collaboration [2]. Close estimates for \( \Delta \) have also been obtained by many other authors, see, e.g. [37, 38]. Since the agreement with the data achieved within this set of parameters was really good [16, 17], we continue using it in the present calculations.

![Figure 1](image-url)  
*Figure 1: The differential cross section \( d\sigma/dx_{\gamma} \) for \( Q^2 < 1 \text{ GeV}^2 \) with BFKL and CCFM unintegrated gluon distributions at THERA.*
3. Numerical results

3.1 $D^{*\pm}$ and dijet associated photoproduction at THERA

The ZEUS collaboration has measured the associated charm and dijet production [2] as a further test of the underlying parton dynamics. In these measurements, the quantity of interest is the fraction of the photon momentum contributing to the production of two jets with highest $E_T$, which is experimentally defined as

$$x_\gamma = \frac{E_{1T} \exp(-\eta_1) + E_{2T} \exp(-\eta_2)}{(2E_e y)}$$

with $E_{iT}$ and $\eta_i$ being the transverse energy and rapidity of these hardest jets.

In the ref. [17] the theoretical calculations have been made within the SHA with different unintegrated gluon distributions at HERA energies. In Fig. 1 we present the results of similar theoretical calculations made within the semihard approach with BFKL and CCFM unintegrated gluon distributions at THERA energies. The simulation procedure consists in generating a photon-gluon fusion event using the off-shell matrix elements and the unintegrated gluon distribution functions described in Section 2.

The basic $2 \rightarrow 2$ partonic subprocess gives rise to two high-energy quarks, which can further evolve into hadron jets. Actually, as the matter of some reasonable approximation, the calculations were restricted to parton level, and so the produced quarks (with their known kinematical parameters) were taken to play the role of the final jets: $E_T(jet_{1,2}) = E_T(q, \bar{q})$.

The two quarks are accompanied by a number of gluons radiated in the course of the gluon evolution. It has been mentioned already that, on the average, the gluon transverse momentum decreases from the hard interaction block towards the proton. As an approximation, we assume that the gluon closest to the quark block with its momentum $k'_T$ compensates the whole transverse momentum of the virtual gluon participating in the hard interaction: $\vec{k}'_T \approx -\vec{k}_T$, while all the other emitted gluons are collected together in the ‘proton remnant’, which is assumed to carry only a negligible transverse momentum compared to $\vec{k}_T$. This gluon gives rise to a third hadron jet with $E_T = |\vec{k}'_T|$. From among the three hadron jets represented by the quark, antiquark and gluon we choose the two carrying the largest transverse energies, and then calculate the quantity $x_\gamma$ according to its definition given by equ. (13).

In a significant fraction of events, the gluon radiated from the BFKL cascade appears to be harder than one or even both of the quarks produced in hard parton interaction [19]. In fact, the specified events are responsible for the wide plateau seen in the $x_\gamma$ distribution in Fig. 1.

3.2 Deep inelastic $D^{*\pm}$ production at THERA

The process of deep inelastic $D^{*\pm}$ production at HERA is truly semihard because of the presence of two large scales: the virtuality of the exchanged photon ($Q^2$) and the charm mass ($m_c^2$), both being much larger than $\Lambda_{QCD}$ but much smaller than $s$. Therefore, experimental data concerning the $D^{*\pm}$ production in DIS at THERA provide a strong impetus for further theoretical studies of this process.

In Fig. 2 the theoretical predictions on the differential cross sections of deep inelastic $D^{*\pm}$ production are shown for the THERA kinematical region: $1 < Q^2 < 1000$ GeV$^2$, $1.5 < p_T(D^{*\pm}) < 15$ GeV and $|\eta(D^{*\pm})| < 1.5$. Different curves in Fig. 2 correspond to the BFKL and CCFM unintegrated gluon distributions. At HERA energies the SHA calculations with BFKL unintegrated gluon distribution have shown [17] some shift to negative $\eta(D^*)$ with respect to the ZEUS data. When we have used the CCFM unintegrated gluon density from MC generator CASCADE [18] with JETSET based fragmentation function [3]

\footnote{The full FORTRAN code is available from the authors on request.}
Figure 2: Differential cross sections for deep inelastic $D^{\pm}$ production with BFKL and CCFM unintegrated gluon distributions at THERA as functions of: (a) $\log_{10} Q^2$, (b) $\log_{10} x$, (c) $W$, (d) $p_T(D^*)$, (e) $\eta(D^*)$ and (f) $z(D^*)$.

implemented in, we obtain good agreement between our theoretical results and the ZEUS experimental data \cite{3} also for $d\sigma/d\eta(D^*)$ \cite{17}.

3.3. $J/\psi$ photoproduction at THERA

The role of the gluon virtuality may be seen in Fig. 3, where we show the results of calculations for $J/\psi$ photoproduction made with the BFKL unintegrated gluon distribution. The results correspond to THERA conditions, i.e. electron proton collisions at the energy $\sqrt{s} = 1000$ GeV, where no other cuts were applied except the photoproduction limit $Q^2 < 1$ GeV$^2$ and the inelasticity requirement $0.4 < z < 0.9$.

The effects of initial gluon off-shellness may be, best of all, seen in the transverse momentum spectra \cite{24}, because the gluon virtuality is proportional to its transverse momentum: $m^2 = -k_T^2/(1-x)$. In contrast with the conventional (massless) parton model, the SHA shows that the fraction of $J/\psi$ mesons in the helicity zero state increases with their transverse momentum $p_T$. A deviation from the parton model behaviour becomes well pronounced already from $p_T > 3$ GeV at HERA energies \cite{24}, and at $p_T > 6$ GeV the helicity zero polarization tends to be even dominant (Fig. 3c).

Qualitatively, the difference between the model predictions refers to different origins of the $J/\psi$ transverse momentum. In the case of conventional parton model $J/\psi$ meson obtains its transverse momentum from the hard photon gluon interaction, while in the SHA there is also a large the contribution from the initial gluon transverse momentum.

The degree of spin alignment can be measured experimentally since the different polarization states of $J/\psi$ result in significantly different angular distributions of the $J/\psi \to l^+l^-$ decay leptons:

$$d\Gamma_{h=\pm}/d\cos\Theta = 1 - \cos^2\Theta, \quad d\Gamma_{h=0}/d\cos\Theta = 1 + \cos^2\Theta$$

(14)
Figure 3: Differential cross sections for inelastic $J/\Psi$ photoproduction with BFKL unintegrated gluon distributions at THERA: (a) Inclusive $J/\Psi$ transverse momentum distribution, (b) the same, but for $J/\Psi$ zero helicity states only, (c) the fraction of $J/\Psi$ mesons in helicity zero state (degree of spin alignment).

Here $\Theta$ is the angle between the lepton and $J/\psi$ directions, measured in the $J/\psi$ meson rest frame. Evidently the most informative regions relate to $\cos \Theta = \pm 1$.

4. Conclusions

In the framework of semihard QCD approach, we obtained some predictions for the cross sections of inclusive $D^{*\pm}$ meson production at THERA conditions using different unintegrated gluon distributions driven by the BFKL and CCFM evolution equations.

We have considered the photoproduction of $D^{*\pm}$ mesons associated with two hadron jets and also $D^{*\pm}$ production in DIS at THERA conditions, which may be a sensitive indicator of the underlying parton dynamics. The results of the simulations show that theoretical results are very sensitive to BFKL type dynamics, in particular, to the unintegrated gluon distribution in the proton.

We have considered also the effects of initial gluon off-shellness in SHA for $J/\psi$ meson photoproduction at THERA energies. Gluon virtuality connected with its transverse momentum is one of the inherent properties of noncollinear (BFKL) parton evolution theory. Compared to traditional (collinear) parton model, gluons are characterized by a different spin density matrix. The latter is found to affect the polarization of $J/\psi$ mesons produced in $ep$ collisions via photon gluon fusion subprocess. The effect is best pronounced at large $J/\psi$ transverse momenta and can be detected experimentally by measuring the $J/\psi \rightarrow l^+l^-$ decay lepton angular distributions. We recommend the above process as a direct probe of the gluon virtuality, which can eventually testify for the validity of BFKL gluon evolution.

Thus the experimental and theoretical investigations in the new kinematic region of THERA collider will provide additional tests of the semihard ($k_T$ factorization) approach and, in particular, of the “universality” of unintegrated gluon distribution.

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