SOFT SUPERSYMMETRY BREAKING FROM GAUGINO
CONDENSATION

B. de CARLOS\(^1\) and M. MORETTI\(^2\)

Department of Theoretical Physics, University of Oxford,
1 Keble Rd, Oxford OX1 3NP

Abstract

We study the structure of soft breaking terms in the context of a gaugino condensation
scenario. Assuming that the Supergravity Lagrangian is the correct quantum field theory
limit, at some momentum scale \(\mu_{UV}\), of a more fundamental one, we demonstrate that the
correct result is obtained simply by substituting, in the tree level Supergravity Lagrangian,
\(\lambda\lambda\) (the gaugino condensate) by its vacuum expectation value \(\Lambda^3\). In string inspired scenarios
this implies, in particular, that the scalar masses are vanishing at the string tree-level and
receive a contribution, at the one loop level, which is proportional to the Green Schwarz
coefficient \(\delta_{GS}\). Our results do not agree with the ones obtained in the effective Lagrangian
approach. We study in detail the origin of this discrepancy, and we argue that the use of
the supertrace anomaly to determine the effective theory for the condensate does not fix
its gravitational interactions, leaving the soft breaking terms and the vacua of the theory
unspecified.

OUTP–94–16P
July 1994

\(^1\)Work supported by a Spanish M.E.C. grant.
\(^2\)European Community Fellow, CHRX-CT93-0132 contract, Human Capital and Mobility program.
1 Introduction

Many extensions of the Standard Model embody supersymmetry (SUSY) as a fundamental ingredient. Since the low energy world is manifestly non-supersymmetric, these models are specified only when the source of SUSY breaking is defined.

The main motivation for low energy SUSY is linked to the absence of quadratic divergencies in the perturbative expansion, which is essential in providing a solution, at least at the technical level, to the hierarchy problem [1]. If one wants to keep this important property, the only allowed explicitly non-supersymmetric terms in the Lagrangian are the so-called soft breaking terms (SBT) [2], whose mass scale has to be the electroweak scale.

When SUSY is promoted to be a local symmetry of nature one obtains Supergravity theories (SUGRA) [3, 4, 5]. It is in this framework in which we hope to understand the origin of SBT and their typical scale. Furthermore, if some dynamical condensation occurs, SUSY is broken and the non-renormalizable terms of the Lagrangian origin the wanted SBT with a typical mass scale $m_{SB} = \Lambda^3/M_P^2$, $\Lambda$ being the scale of the condensate and $M_P$ the Plank mass. If $\Lambda \sim 10^{-5} M_P$ the required hierarchy is generated. Since in the SUGRA Lagrangian the only fundamental mass scale is $M_P$, one expects that $\Lambda$ is dynamically generated as the confinement scale of some non-abelian sector of the theory [3, 6] which is usually assumed to be decoupled, with the exception of non-renormalizable interaction terms, from the observable sector (the Standard Model one). A SUGRA Lagrangian is specified when three functions of the fields are given, namely the real-analytic Kähler potential $K$ and two holomorphic functions $P$ and $f$, the superpotential and the gauge kinetic function respectively. Since the theory is non-renormalizable, another essential ingredient is the ultraviolet cutoff.

In the context of a fundamental theory, it is possible to try to calculate the SBT, allowing for a non trivial test of it. Superstrings are the most promising candidates for such a theory [7]; in this framework there is only one fundamental parameter: the Planck scale ($M_P \sim 10^{18}$ GeV), and any other quantity is determined dynamically, that is, as the vev of a field. Furthermore the theory is finite, so it has to be anomaly free under any symmetry that it possesses. Such constraints allow in principle for an exceptional predictive power; so, under the assumption that SUGRA is indeed the correct field theory limit of superstrings, one can try to make use of the available information to limit the possible form of the SBT.

Given a particular compactification scheme, in principle, the $f$, $K$, and $P$ functional forms can be computed. In practice this has only been done for orbifold [8, 9] and large radius Calabi–Yau compactifications [10], for which we have expressions for $f$ and $K$ at the one loop level; concerning $P$, we must require that, at least, it contains the non-gauge standard model interactions.

Despite of all the progress done, there are still lots of problems to be solved in order to give string theories a complete predictive power; one of the most important is the com-
plete understanding of the SUSY breaking process. The most promising option is that of non-perturbative effects as the source of SUSY breaking \[11\], and, among these, gaugino condensation is the most appealing one \[8, 9, 13, 14\]. In SUGRA, as already discussed, non-renormalizable contributions usually appear in the expression of the auxiliary fields \(F^j\), and these terms can induce SUSY breaking in presence of a gaugino condensate (a non trivial vev for \(F^j\) indicates that SUSY is broken) \[13\]. Requiring that the effective potential for the gaugino condensate correctly reproduces the quantum behaviour under both anomalous and non anomalous R symmetries, and that it embodies modular invariance, an effective Lagrangian depending on the condensate, the dilaton and the moduli fields has been found \[13, 16\] (from now on, we shall refer to this as effective Lagrangian approach). The usual assumption which is made is that this non-perturbative contribution can be reproduced by a proper term in the effective superpotential. Under this, all non-perturbative effects, including other possible unknown contributions, can be accounted for assuming that the superpotential develops a non-zero vev triggering SUSY breaking \[15, 16, 17\].

Having specified the mechanism for SUSY breaking together with \(f\), \(K\) and \(P\), it is possible to examine the structure of the resulting SBT, trying to outline the peculiarities of the low energy spectrum. This has been done first in \[18\], and, more recently, in \[19, 20, 21\] the most remarkable feature being that a correlation between the gaugino masses and squark and lepton masses has been found.

In an attempt to study the dynamics of the condensation mechanism, it has been suggested in \[22\] that the strong binding effects in the hidden sector be parametrised by a four Fermi interaction along the lines of NJL model. Two major differences are found with respect to the alternative approach: a non trivial vacua even with one condensate only, and a different structure for the SBT. Studying gaugino condensation from a slightly different perspective from the usual one, we aim to investigate the origin of such discrepancies.

Under the assumptions that the SUGRA Lagrangian is the correct quantum field theory limit, at some scale, of a more fundamental one, and that gaugino condensation is the only source of SUSY breaking, in section 2 we derive the expressions for the SBT as functions of the tree level parameters \(f\), \(K\) and \(P\), where by tree level we mean those which define the effective quantum field theory and are obtained after having integrated out all the heavy frequencies. In section 3 we apply our results to the specific case of string inspired SUGRA. Since the results do not agree with those obtained \[18, 19, 20, 21\] using the effective Lagrangian approach, in sections 4 and 5 we discuss the origin of the discrepancies and we point out that, in our opinion, the arguments relying on anomaly cancellation do not constrain the gravitational interactions of the condensate. This implies that all the terms of order \(\Lambda^6/M_P^2\) are completely arbitrary to the extent to which the anomaly is concerned. Different guesses for these terms lead to different results for the SBT. Finally we briefly discuss the issue of the cosmological constant, and we come to our conclusions.
2 Soft breaking terms

In the forthcoming sections we will closely follow ref. [4], which we will denote from now on as [C], and we will quote as (Cn) the (n) equation of paper [C].

Our aim is to examine gaugino condensation effects in the framework of SUGRA theories, relying on the minimum possible set of assumptions. Namely we restrict ourselves to three basic assumptions:

1) some ”fundamental” theory of nature admits an $\mathcal{N} = 1$ SUGRA effective quantum field theory limit at an effective ultraviolet cutoff scale $\mu_{UV}$.

2) it contains a sector (which in the following we will refer as hidden) which can be described as an infrared confining SUSY gauge theory.

3) gaugino condensation occurs, namely a non zero vev for $\lambda H \lambda H$ develops, where by $\lambda H$ we denote the fermionic partners of the ordinary gauge bosons.

It is clear that this is indeed the minimal possible set of assumptions under which the problem we want to study is not meaningless. We will demonstrate that this is sufficient to determine completely the contribution of gaugino condensation effects to the SBT as a function of $f$, $K$, $P$ and the condensate scale only. In summary, our starting point is a SUGRA theory with a hidden, non abelian, purely gauge sector (the extension to a hidden sector containing matter fields is straightforward and not essential to the point we want to make). The Lagrangian we are interested in, however, is not this but the one we obtain after we have integrated out the hidden sector modes. The resulting low energy theory, which in the following we will denote as $\mathcal{L}_{MSSM}$, will be the SUGRA (C4.17-4.20) one which is obtained setting to zero the hidden fields ($\mathcal{L}_{obs}$), plus, possibly, explicit SUSY breaking soft terms ($\mathcal{L}_{soft}$): $\mathcal{L}_{MSSM} = \mathcal{L}_{obs} + \mathcal{L}_{soft}$. The set of soft breaking terms is given by:

\[
\begin{align*}
\text{Trilinear piece} & : A_{ijk} h_{ijk} \varphi_i \varphi_j \varphi_k + \text{h.c.} \\
\text{Gaugino mass term} & : M_a \lambda_a \lambda_a \\
\text{Scalar mass term} & : m_i^2 |\varphi_i|^2
\end{align*}
\]

where the trilinear piece is derived only if a superpotential of the form: $W_Y = h_{ijk} \varphi_i \varphi_j \varphi_k$ is present, containing the Yukawa interactions between the observable matter fields ($\varphi_i$) (here $h_{ijk}$ are the Yukawa couplings). We have also another soft term, a bilinear coupling between the two Higgs fields. In general it will have the form: $B_\mu H_1 H_2 + \text{h.c.}$, with $\mu$ a mass dimensional quantity which will depend on the origin of this term. We will discuss later on the different possibilities proposed up to now.

In the following, for notational convenience, we will assume our ”Standard Model” to consist of only one matter chiral superfield $\Phi$ and one gauge sector. The scalar component of the superpotential is $P = h \varphi^3$ (where $\varphi$ is the scalar component of $\Phi$), and the fermionic partners of the gluons are collectively denoted by $\lambda_g$. The extension of this discussion to a more realistic Standard Model is straightforward.
Now that we have defined our notation, we are ready to discuss the structure of the SBT. As a first step we need to integrate out the hidden sector modes. We find more clear and enlightening to work in the explicit component notation $[C]$, in which the effective Lagrangian $L_{\text{MSSM}}$ is defined by:

$$e^\int L_{\text{MSSM}} = \langle e^\int (L_{\text{obs}} + L_{\text{OH}} + L_H) \rangle_H,$$

where $L_H$ is the SUGRA Lagrangian (C4.17-4.20) obtained setting to zero the observable fields, $L_{OH}$ denotes the whole set of gravitational interactions among hidden and observable fields, and $\langle \rangle_H$ denotes the functional integration over the hidden degrees of freedom. The relevant piece for the study of the soft breaking terms is:

$$e^\int L_{\text{MSSM}} = e^\int L_{\text{obs}} \left\langle \int \left[ \frac{\lambda_H \tilde{\lambda}_H}{M_P^2} (\tau \varphi^3 + 6 \lambda_9 \lambda_9) \right. + h.c. + S \frac{\lambda_H \tilde{\lambda}_H}{M_P^4} |\varphi|^2 \right. \left. \right] e^\int L_H \right\rangle_H + O \left( \left( \int \left[ \frac{\lambda_H \tilde{\lambda}_H}{M_P^2} |\varphi|^4 + \frac{\lambda_H \tilde{\lambda}_H}{M_P^4} |\varphi|^2 \varphi^3 + h.c. + \ldots \right] e^\int L_H \right\rangle_H, (3)$$

where $\tau$, $G$ and $S$ are the coefficients of the corresponding interactions in the Lagrangian and will be given later. Notice that the structure of $L_{OH}$ is, of course, richer than the one given in (3); however, by direct inspection of the SUGRA Lagrangian it turns out that, at the relevant order of expansion ($e^{L_{OH}} \simeq 1 + L_{OH}$), any other possible ”soft” contribution would lead to a disastrous breaking of Lorenz–Poincaré invariance.

We wish to point out that the procedure leading to (3) amounts to sum up all the contributions coming from loops of hidden particles only. For example, the scalar mass in (3) is equivalent to the ”calculation” of the scalar’s propagator allowing only hidden particles in the loops. As shown in figs. 1, 2 and 3 this is generically a sum of unknown functions and of unknown dimensionful numbers. This inspection turns out to be useful only because of the peculiarly simple structure of $L_{OH}$; indeed no coupling provides any contribution in

Figure 1: The squark propagator. $H_i$ denotes that only internal hidden fields, in a generic instanton background, are accounted for. The set of graphs denoted by $B$ do not contribute to wave function renormalization.
Figure 2: The contribution of hidden fields to the scalar’s wave function renormalization. Dashed lines denote scalars and the continuous one a generic hidden field.

fig. 2 and only the coupling \( S|\varphi|^2|\lambda_H\lambda_H|^2 \) contributes in fig. 3. The whole non perturbative contribution, as shown in fig. 4, can therefore be parametrized by a single unknown mass parameter \( \Lambda^3/M_P^2 \). To be more precise, the effect of the propagation of hidden particles is also to generate higher order (possibly non local) operators \( \sim \varphi^n \). These are, however, non renormalizable, and therefore we are forced to neglect them to obtain a predictive theory, hoping that the "final" theory will provide with a justification for this assumption.

Assuming factorization, we have:

\[
< |\lambda_H\lambda_H|^2 >_H = < \lambda_H\lambda_H >_H < \lambda_H\bar{\lambda}_H >_H
\]  

(4)

This has been demonstrated to hold for supersymmetric theories, even when SUSY is spontaneously broken [23], if \( \lambda_H\lambda_H \) is a gauge invariant quantity. In the case we are discussing this is not true, due to gravitational interactions; however, we expect that any deviation from (4) is suppressed at least by loop factors of order \( 1/(16\pi^2) \).

We obtain that the soft breaking terms of \( L_{MSSM} \) are given by:

\[
A = \tau \frac{\Lambda^3}{M_P^2} ; \quad M = G \frac{\Lambda^3}{M_P^2} ; \quad m_{\varphi}^2 = S \frac{\Lambda^6}{M_P^2} ,
\]  

(5)

Figure 3: Same as in fig. 2, but graphs contributing to the scalar’s mass renormalization only.
where $\Lambda^3 = \langle \lambda H \lambda^H e^{iL^H} \rangle_H$ and

$$\tau_h \varphi^3 = (P_j + K_j P)(K^{-1})^j_i \bar{f}^i$$

$$G = \frac{1}{32} \langle (K^{-1})^j_i f_j \bar{f}^i \rangle$$

$$S = \frac{1}{32} \left\langle \frac{\partial (K^{-1})^j_i}{\partial |\varphi|^2} f_j \bar{f}^i \right\rangle .$$

One should notice that steps (2) and (3) leading to (5) are only definitions. Nevertheless they prove to be extremely useful in showing that, at least in the computation of the SBT, the only information which is needed about the (non-perturbative) dynamics in the hidden sector is the value of $\Lambda^3$. Equations (2) and (3) are a very simple proof of the results which are obtained performing the "naive" substitution $\lambda^H \lambda H \rightarrow \Lambda^3$ in the SUGRA Lagrangian.

Let us make our point explicit also in the effective potential language. We go back to the component formulation [C], and using purely formal arguments, we want to study the effective potential of our theory. In particular, we need to work out a MSSM Lagrangian which contains explicitly the dependence on the $U$ composite field ($\mathcal{L}_{\text{MSSM}}$). In order to do so, we add an extra Gaussian integral to the path integral; we define

$$\int \mathcal{D}U e^{-\int \mathcal{L}_{\text{MSSM}}} = \frac{1}{N} e^{-\int \mathcal{L}_{\text{obs}}} \int \mathcal{D}\varphi_H \mathcal{D}U e^{-\int (\mathcal{L}_H + \mathcal{L}'_{\text{OH}} + |U - \frac{\lambda_H \lambda^H}{M^2_P}|^2)} ,$$

where $\varphi_H$ denotes a generic hidden field, $N$ is the normalization factor which is introduced to compensate for the Gaussian integral, and $\mathcal{L}'_{\text{OH}}$ contains all the relevant mixed terms between the hidden and the observable sectors:

$$\mathcal{L}'_{\text{OH}} = S|\varphi|^2|U|^2 + (\tau \varphi^3 + G \lambda^g \lambda^g) \frac{\lambda_H \lambda^H}{M^2_P} + \text{h.c.} + \cdots$$

From (7) and (8) we get the classical equation of motion for $U$:

$$U = \frac{\lambda_H \lambda^H}{M^2_P} \left( 1 - S \frac{|\varphi|^2}{M^2_P} \right) ,$$

Figure 4: The unique contribution to the scalar’s two point function in a SUGRA theory.
where we have neglected higher order terms in $|\varphi|^2$. This shows \textsuperscript{[24]} that, indeed, $\tilde{L}_{\text{MSSM}}$ is the Lagrangian we are looking for. From the expressions \textsuperscript{[2], 8} and \textsuperscript{[4]} it is clear that, after performing the integration over the hidden fields, we end up with

$$\tilde{L}_{\text{MSSM}} = L_{\text{obs}} + K \left( U - \tau \frac{\varphi^3}{M_P} - G \frac{\lambda_g \lambda_g}{M_P} \right) + \left( 1 + \mathcal{S} \frac{|\varphi|^2}{M_P^2} \right) |U|^2$$

$$+ V \left( U - \tau \frac{\varphi^3}{M_P} - G \frac{\lambda_g \lambda_g}{M_P} \right),$$

where $K(U)$ is a kinetic functional for $U$, and $V$ is the contribution to the effective potential, $V_{\text{eff}} = -|U|^2 - V(U)$, induced by the interaction of the other hidden fields with $U$. It is immediately seen, by inspecting \textsuperscript{[3]}, that $\tilde{L}_{\text{MSSM}}$ cannot be obtained, in the standard SUGRA framework, by a mere shift of the superpotential by a term accounting for gaugino condensation. From \textsuperscript{[4]}, minimizing $V_{\text{eff}}$ with respect to $U$, we get: $\langle \bar{U} \rangle = -\langle dV/dU \rangle |_0$.

The soft breaking terms are obtained expanding the effective potential $V_{\text{eff}}$ around the minimum $U_0$. In this case,

$$-V_{\text{eff}} = |U_0|^2 \left( 1 + \mathcal{S} \frac{|\varphi|^2}{M_P^2} \right) + \frac{1}{M_P} \bar{U}_0 (\tau \varphi^3 + G \lambda_g \lambda_g) + \text{h.c.} + \cdots$$

Eq. (11) is consistent with expressions (3). We stress that the above equation is obtained with the only underlying assumption that the standard SUGRA Lagrangian is an effective field theory below the compactification scale. Notice that, in the above expression, we automatically obtain the properties previously referred as decoupling (4). This is a consequence of the fact that in the definition which we have given of $\tilde{L}_{\text{MSSM}}$ (see eq. (7)) no integration over the $U$ field is assumed and, therefore, all the effects of its propagation are neglected.

3 Application to a string inspired scenario

In order to give explicit expressions for the soft breaking terms, we need to specify the form of the functions which define our SUGRA Lagrangian in four dimensions, namely the Kähler potential, $K$, the gauge kinetic function, $f$, and the superpotential, $P$, all of them depending on the chiral superfields ($K$ is real–analytic, while $f$ and $P$ are holomorphic in these fields).

We are interested in effective field theories derived from a higher dimensional string theory, for which $K$, $f$ and $P$ are completely determined given a compactification scheme, although in practice they are only sufficiently known for orbifold compactifications \textsuperscript{[8, 9]} (and for some Calabi–Yau spaces also \textsuperscript{[10]}). In this framework we have:

\begin{align*}
    f^a & = k^a S + \epsilon^a \log(\eta^2(T)) \\
    K & = -\log(Y) - 3 \log(T + \bar{T}) + K_i^i(T, \bar{T}) |\varphi_i|^2 \\
    P & = h_{ijk}(T) \varphi_i \varphi_j \varphi_k
\end{align*}

7
where \( e^a = b^a - k^a \delta^{GS} \), \( Y = S + \bar{S} + \delta^{GS} \log(T + \bar{T}) \), \( S \) and \( T \) are the dilaton and modulus fields respectively, \( k^a \) are the Kac–Moody levels and \( b^a \) the one–loop beta function coefficients associated to the gauge groups \( G^a \), \( n_{\varphi_i} \) are the modular weights associated to the formerly defined matter fields \( \varphi_i \), \( \delta^{GS} = \delta^{GS}/4\pi^2 \), where \( \delta^{GS} \) is the usual Green–Schwarz coefficient \([23]\) and \( \eta(T) \) is the Dedekind function \( \eta(T) = e^{-\frac{3\pi}{4} \prod_{n=0}^{\infty}(1 - e^{-2\pi n T})} \). Finally, in order to simplify the notation, we will take a generic matter field \( \varphi \) and assume for the superpotential the following simplified expression: \( P = h(T) \varphi^3 \), where \( h(T) \) is a Yukawa coupling.

From now on we will work only with one gauge group \( G \) in the hidden sector and the corresponding Kac–Moody level \( k = 1 \), being the generalization of our results for more than one gauge group and non trivial levels completely straightforward. Also the dependence of the Kähler potential \( K \) in the matter fields is known as a series expansion \([25]\). In what concerns to our calculation, it is enough to keep terms up to \( K \) and \( \eta(T) \) is the Dedekind function \( \eta(T) = e^{-\frac{3\pi}{4} \prod_{n=0}^{\infty}(1 - e^{-2\pi n T})} \). Finally, in order to simplify the notation, we will take a generic matter field \( \varphi \) and assume for the superpotential the following simplified expression: \( P = h(T) \varphi^3 \), where \( h(T) \) is a Yukawa coupling.

Now, from the expression of the SUGRA Lagrangian \([C]\), and taking into account the factorization property \([4]\) and eq. \([3]\), we can derive the soft breaking terms for this particular case:

\[
M = \frac{1}{32 \text{Ref}} \left[ \frac{Y^2}{(3Y + \delta^{GS})} \left| \delta^{GS} - 2\epsilon(T + \bar{T}) \frac{\eta'(T)}{\eta(T)} \right|^2 \right] \frac{\Lambda^3}{M_P^6}
\]

\[
A = \frac{-Y^{1/2}}{4(T + \bar{T})^{3(n_{\varphi} + 1)/2}} \left[ 1 - \frac{1}{3Y + \delta^{GS}} (3(n_{\varphi} + 1) - (T + \bar{T}) \frac{h_T(T)}{h(T)}) \right]
\times \left( \delta^{GS} - 2(T + \bar{T}) \frac{\eta'(T)}{\eta(T)} \right) \frac{\Lambda^3}{M_P^6}
\]

\[
m^2_{\varphi} = \frac{-n_{\varphi}}{32} \frac{Y^2}{(3Y + \delta^{GS})^2} \left| \delta^{GS} - 2(T + \bar{T}) \frac{\eta'(T)}{\eta(T)} \right|^2 \frac{\Lambda^6}{M_P^6}
\]

where \( h_T = \partial h/\partial T \) and \( \eta' = \partial \eta/\partial T \). Here we have included the normalization for matter fields and gauginos due to non canonical kinetic terms.

The most remarkable comment which is in order is that scalar masses vanish in the limit in which no one–loop corrections to \( K \) and \( f \) are taken into account (that is, when \( \delta^{GS} = 0 \)). Notice that, to have scalar masses of the same order of magnitude of the gaugino ones, a large \( \delta^{GS} \) coefficient and/or a large modular weight for the field \( \varphi \) is needed.

We now turn out to discuss the \( \mu \) and \( B \) terms; it is known that a coupling of the form \( \mu H_1 H_2 \) has to be present in the matter superpotential in order to generate the correct \( SU(2)_L \times U(1) \) breaking; furthermore, \( \mu \) should be of the order of the SUSY breaking scale, and the soft term which generates in the Lagrangian is of the form: \( B \mu H_1 H_2 + \text{h.c.} \). Its
calculation is analogous to that of the trilinear term:

\[ B\mu = -\frac{Y^{1/2}}{4(T + \bar{T})^{(3+n_1+n_2)/2}} \left[ \mu - \frac{1}{3Y + \delta_{GS}} ((3 + n_1 + n_2)\mu - (T + \bar{T})\mu_T) \right] \times \left( \delta_{GS} - 2(T + \bar{T})\epsilon \right)_{\eta(T)} \frac{\Lambda^3}{M_P^2} \]  

(14)

where \( \mu_T = \partial \mu / \partial T \).

On the other hand, it has been suggested an alternative origin for this coupling [27], namely the presence in the Kähler potential of terms like:

\[ K' = K + \nu(S, T)H_1H_2 + \text{h.c.} \]  

(15)

In this case the soft term is induced in the same way as the scalar masses do, so that we end up with a \( B \) term of the form:

\[ B = -\frac{1}{32\mu (3Y + \delta_{GS})^2} (T + \bar{T})^{2-(n_1+n_2)} \left[ \nu_T^2(T + \bar{T}) - \nu_T(n_1 + n_2) \right] \times \left( \delta_{GS} - 2(T + \bar{T})\epsilon \right)_{\eta(T)} \frac{\Lambda^6}{M_P^4} , \]  

(16)

where \( \nu_T(\bar{T}) = \partial \nu / \partial T(\bar{T}) \) (we have only considered a possible \( T \)-dependence of \( \nu \)).

4 Gravitational interactions of the condensate

Since the expressions we have got for the soft breaking terms disagree with the ones computed using the effective Lagrangian approach [18, 19, 21], it is useful to study in detail the results obtained in this latter framework, and try to understand the origin of the discrepancy. We shall closely follow the approach of ref. [16], which in the following we will always quote as [B], and we will refer as (Bn) the equation (n) of [B].

In the notation of [B] the SUGRA Lagrangian (B3.7), (B3.10), (B3.11) is given by

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{YM} + \mathcal{L}_{pot} \]

where:

\[ \mathcal{L}_0 = -3 \int d^2\Theta \mathcal{E}\mathcal{R} + \text{h.c.} \]

\[ \mathcal{L}_{YM} = \frac{1}{4g^2} \int d^2\Theta \mathcal{E} f(\Phi)U + \text{h.c.} \]  

(17)

\[ \mathcal{L}_{pot} = \int d^2\Theta \mathcal{E} e^{K/2} P(\Phi) + \text{h.c.} \]

where \( U = \frac{1}{4} W^\alpha W_\alpha \), and \( W_\alpha \) is the chiral gauge supermultiplet (for the notation, see [B]).
Using these expressions we obtain for the component Lagrangian (\( \mathcal{L}^c_s \)), which contains only scalar interactions, the following equation:

\[
\mathcal{L}^c_s = F_j (G^{-1})^j_i \bar{P}^i - 3e^G,
\]

(18)

where \( G = K + \log |P|^2 \) and

\[
F_j = -e^{K/2}[K_j P + P_j] + \frac{1}{4} f_j U.
\]

(19)

This generates three different types of terms:

\[i)\quad e^K[(K_j P + P_j)(K^{-1})^j_i (K^i \bar{P} + \bar{P}^i) - 3|P|^2]
\]

\[ii)\quad -\frac{1}{4} e^{K/2}(K_j P + P_j)(K^{-1})^j_i \bar{f} \bar{U} + \text{h.c.}
\]

(20)

\[iii)\quad \frac{1}{32} f_j (K^{-1})^j_i \bar{f} U \bar{U}.
\]

In our particular case, we are assuming gaugino condensation as the only source of SUSY breaking, so we take \(< P > = 0\), and a superstring inspired scenario, in which both \( f \) and \( K \) are going to depend on \( S \) and \( T \). So we end up with \( iii) \) in eq. (20) as the only surviving interaction, with \( i, j = S, T \).

According to [B], we can rewrite (17) by making the following shift:

\[
\mathcal{L}'_{YM} = 0, \quad \mathcal{L}'_{pot} = \int d^2 \Theta e^{K/2} P' (\Phi) + \text{h.c.}
\]

\[
P' = P + \frac{1}{4} e^{-K/2} f U.
\]

(21)

As discussed in [B], this is going to introduce a non holomorphic piece in the superpotential \( P' \). To recast it in an holomorphic way, in [B] a new superfield \( H \) is introduced with the proper Kähler transformation: \( U = e^{K/2} \rho(S) H^3 \) (see eq. (B4.16)). In terms of this \( H \) field the superpotential \( P' \) reads:

\[
P' = P + \frac{1}{4} \rho(S) H^3
\]

(22)

(see eq. (B4.21), with \( \rho \) instead of \( f \) and \( f \) instead of \( S \)).

Let us note that all these formal steps are perfectly consistent with the required Kähler invariance of the theory. However, the previous field redefinition does not correspond only to a simple reparametrization of the field \( U \) in terms of \( H \); it is clear from (B4.15-4.17) that \( H \) and \( U \) have different conformal weights, as explicitly noticed in [B] and, therefore, they are expected to have different gravitational interactions.
Now that the effects of gaugino condensation have been included in the superpotential, we go back to the component notation and look for the scalar interactions in this new language. The F term is now:

\[ F'_{j} = -e^{K/2} \left[ K_{j}P + P_{j} - \frac{1}{4}(K_{j}f\rho(S) + f_{j}\rho(S) + f\rho_{j}(S))H^{3} \right]. \] (23)

The crucial difference between (19) and (23) is given by the appearance of the \( e^{K/2} \) factor in front of the \( H \) field in this latter equation, which is a consequence of considering this field as a fundamental dynamical variable.\(^3\)

Given the former expression for \( F'_{j} \), the part of the Lagrangian which contains scalar interactions, that is (18), will consist now of a term analogous to \( i \) in eq. (20), with \( P' \) instead of \( P \):

\[ \mathcal{L}_{s}^c = e^{K} \left[ \frac{1}{32}(K_{j}\rho(S)f + \rho_{j}(S)f + \rho(S)f_{j})(K^{-1})^{j}_{i}(K^{i}_{k}\bar{\rho}(\bar{S})\bar{f} + \bar{\rho}(\bar{S})\bar{f} + \bar{\rho}(\bar{S})\bar{f})|H^{3}|^{2} \right. \\
- \left. 3|\rho(S)fH^{3}|^{2} \right] , \quad k, l = S, T , \] (24)

where we again stress the presence of the global \( e^{K} \) factor multiplying the condensate.

In addition we notice a more subtle difference arising between the two formulations. The fact that the SUGRA Lagrangian is not positive definite is due to a factor \(-3|P|^{2}\), which appears in the scalar potential (eq. (20) \( i \)) after substituting the auxiliary field \( \omega \) of the Weyl compensator superfield, introduced in order to give the right conformal weight to the fields, as can be checked by looking at (C4.3-4.5) (with \( u \) instead of \( \omega \)). Given that the \( U \) and \( H^{3} \) fields have different conformal weights, the dependence of \( \omega \) on \( H^{3} \) and \( U \) is completely different; in particular, again from a direct inspection of the component Lagrangian, one can check that using the \( H \) language a factor \(-3|\rho(S)fH^{3}|^{2}\) is present, which has no corresponding piece in the \( U \) language.

While it is not clear to us how all these differences affect the problem of minimizing the potential, they completely change the structure of the SBT. We conclude that the usual assumption that all the non–perturbative effects can be taken into account shifting the tree–level potential by an amount which depends only on the composite field leads to an incorrect conclusion.

Notice that, since all the steps done in [B] to go from the \( U \) to the \( H \) formulation always lead to a Lagrangian with the correct Kähler transformation, it should be true also for the inverted one. Given that the two formulations lead to different conclusions, we guess that the arguments relying on anomaly cancellation to inspect the possible behaviour of the effective potential suffer from some ambiguities. Indeed it seems that physically inequivalent

\(^{3}\)In fact, we could formally rewrite (23) with \( P' \) given in terms of the \( U \) field as defined in eq. (21): the \( e^{K/2} \) factor would disappear and we would recover expression (19). Notice that this is not the case for the theory in terms of the \( H \) field: substituting back \( U \) by \( H \) in (19) does not give the same result as (23).
potentials have the same properties under both the anomalous transformation (B4.3), (B4.4) and the nonanomalous one (B4.7-B4.9). We notice that, motivated by a different purpose, Binétruy and Gaillard reached essentially the same conclusion in [28] and a somewhat related discussion was developped in [29]. Finally we think that a more extreme option could be possible: namely it might be that higher order effects cannot be incorporated in the standard SUGRA Lagrangian without including higher order derivatives.

5 Cutoff dependence of the effective theory

The effective theory we are looking for depends critically on its ultraviolet cutoff which, as any other quantity, is generically a field dependent one. In string theory the cutoff is twofold: i) at the string scale $M_S$ the particles of the theory begin to feel their non-pointlike structure; ii) at the compactification scale $M_C$, the low energy degrees of freedom feel the interactions with the infinite tower of heavy Kaluza Klein modes, generated by the compactification of the extra space dimensions.

Clearly the effects induced by $M_C$ can lead to some field dependence. Indeed, it may very well be that $M_C$ is a dynamical variable, and that the compactification to four space time dimensions arises because it is energetically favoured [30]. Moreover, all the interactions of the light degrees of freedom with the heavy modes induce a coupling dependent effective cutoff $\mu_{UV}$, which consequently implies its field dependence. In string theory the field dependence of the cutoff is somewhat understood [B], however it is not clear to us whether it is controlled with the level of accuracy which is needed for the purpose here discussed.

We believe that this possible ambiguity is not reflected in the study of the soft breaking terms. Considering it from the low energy theory point of view, the effective cutoff has to be a pure number, which means that it has to be the vev of some background non-propagating field like the $S$ and $T$ fields. What might happen is that the same interactions which give rise to the cutoff actually modify the couplings of the low energy effective theory. To the extent that this is a SUGRA theory, the only possible modification affecting the low energy modes is through a modification of $f$, $K$ and $P$. Notice that, in fact, this is what happens in the cases where high energy modes’ effects are known, namely these induce a modification both of $f$ and $K$, leaving $P$ invariant according to the non–renormalization theorem. The $S - T$ mixing term, which is the origin of the scalar masses we found in (13), is indeed a manifestation of this mechanism: the same interactions which act as an effective cutoff for the theory originate an effective $|\phi|^2|\lambda_H \lambda_H|^2$ coupling which is not present at the string tree level. What it is not completely clear to us is if the threshold effects computed up to now [25, 31] are general enough. In quantum field theory it is well known that Yukawa type interactions do not affect (at one loop) the renormalization of gauge couplings, however
they induce a non trivial wave function renormalization. If this case has some parallel in superstrings, the Kähler potential might be affected inducing an extra dependence on \(|\varphi|^2\) in \(K_{\bar{S}}\) which would modify the structure of \(m_{\varphi}^2\).

Finally, let us notice an important point related to the cutoff dependence of the effective potential. We have stated that we regard this problem as a serious one for any attempts of studying the minima of the potential relying on the effective Lagrangian approach. In addition to a possible arbitrary dependence of \(\mu_{UV}\) on \(S\) and \(T\), which perhaps can be somehow controlled using symmetry arguments, in our opinion a more serious problem arises. Notice that all the discrepancies among the approach we are suggesting and the effective Lagrangian one are of order \(\delta^2 = \Lambda^2 / M_{Pl}^2\), and that to study the minima in the \(S\) and \(T\) directions, using the effective Lagrangian, these terms are essential. Now, we believe that, in doing so, one should pay attention to the fact that the one loop super trace anomalies, which are the building blocks for the effective Lagrangian approach, are computed in the field theoretical limit, namely in the limit of infinite cutoff. Consistently, order \(\delta^2\) terms should be neglected or, alternatively, one should spell out the supertrace anomalies including \(1 / \mu_{UV}^2\) corrections, which does not seem easy without having a detailed understanding of how the effective cutoff works.

Let us explain this point more in detail. In the effective Lagrangian approach one constructs an effective Lagrangian \(L_{\text{eff}}\) for the condensate \(\hat{\phi} = \beta(g) / (2g) \lambda \lambda\) which reads as follows \[32\]

\[
L_{\text{eff}} = \alpha^{-1}(\phi^* \phi)^{-2/3} \partial_{\mu} \phi^* \partial_{\mu} \phi - \frac{1}{9} \alpha (\phi^* \phi)^{2/3} \left| \log \frac{\phi}{\mu_{UV}^2} \right|^2 ,
\]

which, properly rescaling the field \(\phi\), reads as

\[
L_{\text{eff}} = \partial_{\mu} \hat{\phi}^* \partial_{\mu} \hat{\phi} - \frac{\alpha^2}{81} \left| \hat{\phi}^2 \log \frac{\sqrt{\alpha \hat{\phi}}}{3 \mu_{UV}} \right|^2.
\]

with \(\hat{\phi} = (3 / \sqrt{\alpha}) \phi^{1/3}\), and \(\alpha\) a constant.

Having done this, one incorporates \[13, 16\] \(L_{\text{eff}}\) in a SUGRA formalism, adding to the superpotential a term accounting for \[25\] and its supersymmetric counterpart, and adding to the Kähler potential a term generating the kinetic piece. After expanding the SUGRA Lagrangian, one obtains again \[23\] plus additional gravitational interactions that, with respect to \[26\], are suppressed by a factor \(\delta^2\). The leading contribution to the expression for the gaugino condensate is obtained by setting

\[
\left| \hat{\phi}^2 \log \frac{\sqrt{\alpha \hat{\phi}}}{3 \mu_{UV}} \right|^2 = 0 .
\]

Now comes the problem: to study the minima in the \(T\) and \(S\) directions or the SBT, one has to rely on the subleading contributions since the leading one is vanishing. At this level
we believe that (26) is likely to be not accurate enough. Indeed, we think that

$$L'_{\text{eff}} = \partial_{\mu} \hat{\phi}^* \partial_{\mu} \hat{\phi} - \frac{\alpha^3}{81} \left( \hat{\phi}^2 \log \frac{\sqrt{\alpha \hat{\phi}}}{3 \mu_{\text{UV}}} + \epsilon_1 \frac{\hat{\phi}^3}{\mu_{\text{UV}}} \right)^2$$

(28)
is indistinguishable from $L_{\text{eff}}$ to the accuracy at which the divergence of the anomalous current is known. If we derive the superpotential from (28) the resulting SUGRA Lagrangian contains an extra contribution, proportional to $\epsilon_1^2$. Terms of this order are essential when studying the minimum equations for $S$ and $T$ and computing the SBT. This observation, together with the discussion of the previous section, leads us to conclude that the effective Lagrangian approach, as it stands now, is not adequate to fix the gravitational interactions of the gaugino condensate. Indeed the "anomaly driven" effective Lagrangian is accurate only up to terms of order $1/M_P^2$, which are completely arbitrary to the extent that the anomaly is concerned.

Having stated the problem we can now stress that the SBT are indeed the first known information about the gravitational interaction of the condensate. The effective Lagrangian should reproduce the results we obtained in (3). The problem of whether these constraints (and, possibly, others following from analogous considerations) are sufficient to fix the ambiguities we were pointing out is still an open question. If this approach should work the outcome would be extremely interesting: one, in fact, would get a non trivial insight about the dependence of the theory on the physical cutoff. We plan to come back to this point in a separate publication.

If this is the case, one has to go back and find the origin of the discrepancies between the SBT in (13) and the ones [19, 20] obtained using the $F_{S,T}$ dominance hypothesis. We guess that the answer has to be that $F_\psi$ becomes as important as $F_{S,T}$, and contributes to the SBT in a peculiar way which is not parametrizable using $< F_{S,T} >$ only, as suggested in [20].

Up to now we have neglected the issue of the cosmological constant. We now briefly comment about this point. From eq. (11) it is clear that the natural order of magnitude for the cosmological constant $V_C$ is $m_{SB}^2 M_P^2$, $m_{SB}$ being the scale of SUSY breaking in the observable sector. There are several possibilities which cannot be discarded without a "theory" for $V_C$ which is presently unavailable:

i) The cosmological constant is unrelated to the value of the effective potential at the minimum. In this case it is conceivable that, if gaugino condensation is the only mechanism for SUSY breaking, the discussion of the former sections is unchanged.

ii) Due to some unknown effect the effective potential of (13) is vanishingly small at the minimum. Again the discussion of the previous sections is unchanged.

iii) The extra term needed to cancel $V_C$ comes from a non vanishing VEV ($P_O$) for the superpotential. In this case new contributions have to be added to the SBT.
6 Conclusions

In this paper we have addressed the issue of computing the soft breaking terms (SBT) for a generic Supergravity Lagrangian assuming that gaugino condensation is the origin of supersymmetry breaking. This is usually done under the assumption that the effects of gaugino condensation can be described by a proper superpotential for which an ansatz has been proposed [13, 16]. We have tried to obtain our results avoiding assumptions and ansatze.

Our starting point was a SUGRA effective quantum field theory with ultraviolet cutoff $\mu_{UV}$ and a hidden sector where strong binding effects lead to gaugino condensation. Working in the component formulation and integrating, in a purely formal way, over the hidden degrees of freedom, we have firstly found expressions for the SBT, which explicitly break SUSY in the observable sector of the theory, in terms of $f$, $K$, $P$ and the vev of the gaugino condensate ($\Lambda$), the only dynamical (non perturbative) information about the hidden sector which is needed. We have also applied our procedure of integrating out the hidden sector modes (which we stress again is purely formal, no particular estimation of non perturbative effects is claimed) to work out an “effective potential” for the condensate which, in spite of being defined in terms of an unknown function, allows us to compute SBT consistent with the ones we already obtained.

Secondly, we have discussed the form of these expressions for the case of string inspired SUGRA, i.e., assuming that we are dealing with an effective field theory derived from a higher dimensional string theory. This allowed us to use the known expressions of $f$ and $K$ ($P$ containing just the Standard Model interactions) for orbifold compactifications, to obtain gaugino and scalar masses and trilinear and bilinear couplings. The results show that, in the absence of threshold corrections to $f$ and $K$, scalar masses vanish. These expressions are different from those obtained in the effective Lagrangian approach and, therefore, we turned out to examine both formulations in order to find the source of the discrepancy. We point out that the usual procedure of incorporating the gaugino bilinear in the superpotential forces its reparametrization in a way that changes its gravitational interactions, which are going to be crucial for the computation of the SBT, possibly originating the previously mentioned differences between the two formalisms. This makes us think that having a Lagrangian with the correct Kähler transformation and the right properties with respect to the anomalous and non–anomalous symmetries, is not enough to determine it unambiguously.

We discuss, as well, the role of the ultraviolet cutoff and its possible field dependence, and conclude that, although a field dependent cutoff may modify the study of the vacum structure of our theory in an unknown way, it can affect the structure of the SBT only through a modification of $f$ and $K$, namely through threshold effects, as happens for orbifold compactifications. An important point in this discussion is that the supertrace anomalies are only known up to $\Lambda/M_P$ corrections. Higher order effects of this type would affect in
an essential way the ansatz for the effective SUGRA Lagrangian, opening perhaps a way of matching the results obtained in the two approaches. We finally comment very briefly about the possible modification of our results when the cosmological constant issue is taken into account.

Acknowledgements

We are indebted to G.G. Ross for suggesting us to start this investigation and for many enlightening discussions. We also thank P. Binétruy for a very useful discussion. M.M. wishes to thank the Italian INFN for financial support in the very early stage of this work.

References

[1] G. ’t Hooft, in Recent Developments in Gauge Theories, ed. by ’t Hooft et al., Plenum Press, New York (1981);
M. Veltman, Acta Phys. Pol. B12 (1981) 437;
L. Maiani, Proceedings of the Summer School of Gif-sur-Yvette (Paris, 1980);
E. Witten, Nucl. Phys. B188 (1981) 513.
[2] L. Girardello and M.T. Grisaru, Nucl. Phys, B194 (1982) 65.
[3] R. Barbieri, S. Ferrara and C. Savoy, Phys. Lett. B119 (1982) 343;
A. Chamseddine, R. Arnowitt and P. Nath, Phys. Lett. B49 (1982) 970;
E. Cremmer, P. Fayet and L. Girardello, Phys. Lett. B122 (1983) 41;
L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D27 (1983) 2359;
S.K. Soni and H.A. Weldon, Phys. Lett. B126 (1983) 215.
[4] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. B212 (1983) 413.
[5] J. Bagger, Nucl. Phys. B211 (1983) 302;
H.P. Nilles, Phys. Rep. 110 (1984) 1.
[6] H.P. Nilles, Phys. Lett. B115 (1982) 193; Nucl. Phys. B217 (1983) 366.
[7] See, for example, M.B. Green, J. Schwarz and E. Witten, Superstring Theory, Cambridge University Press (1986).
[8] L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. B261 (1985) 651;
L.E. Ibáñez, H.P. Nilles and F. Quevedo, Phys. Lett. B187 (1987) 25;
K. Narain, M. Sarmadi and C. Vafa, Nucl. Phys. B288 (1987) 951;
L.E. Ibáñez, J. Mas, H.P. Nilles and F. Quevedo, Nucl. Phys. B301 (1988) 157.

[9] J.A. Casas, E. Katehou and C. Muñoz, Nucl. Phys. B317 (1989) 171;
J.A. Casas and C. Muñoz, Phys. Lett. B214 (1988) 63.

[10] P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258 (1985) 46.
[11] T. Banks and L. Dixon, Nucl. Phys. B307 (1988) 93.
[12] S. Ferrara, L. Girardello and H.P. Nilles, Phys. Lett. B125 (1983) 457.
[13] J.P. Derendinger, L.E. Ibáñez and H.P. Nilles, Phys. Lett. B155 (1985) 65.

[14] M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B156 (1985) 55;
C. Kounnas and M. Porrati, Phys. Lett. B191 (1987) 91.
[15] S. Ferrara, N. Magnoli, T. R. Taylor and G. Veneziano, Phys. Lett. B245 (1990) 409.
[16] P. Binétruy and M. K. Gaillard, Phys. Lett 232B (1989); Nucl. Phys. B358 (1991) 121.
[17] A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, Phys. Lett. B245 (1990) 401.
[18] M. Cvetič, A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, Nucl. Phys. B361 (1991) 194;
L.E. Ibáñez and D. Lüst, Nucl. Phys. B382 (1992) 305.

[19] V. Kaplunovsky and J. Louis, Phys. Lett. B306 (1993) 269;
R. Barbieri, J. Louis and M. Moretti, Phys. Lett. B312 (1993) 451;
J.L. López, D.V. Nanopoulos and A. Zichichi, Phys. Lett. B319 (1993) 451.
[20] A. Brignole, L.E. Ibáñez and C. Muñoz, preprint FTUAM-26/93 (1993).
[21] B. de Carlos, J.A. Casas and C. Muñoz, Phys. Lett. B299 (1993) 234.
[22] A. de la Macorra and G.G. Ross, Nucl. Phys. B404 (1993) 321; preprint OUTP-31P (1994).

[23] A.I. Vainshtein, B. Zakharov and M.A. Shifman, Yad. Fiz. 42 (1985) 554.
[24] D. Gross and A. Neveu, Phys. Rev. D10 (1974) 3235.
[25] L. Dixon, V. Klapunovsky and J. Louis, Nucl. Phys. B355 (1991) 649;
J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B372 (1992) 145.

[26] J.E. Kim and H.P. Nilles, Phys. Lett. B138 (1984) 150;
J.E. Kim and H.P. Nilles, Phys. Lett. B263 (1991) 79;
E.J. Chun, J.E. Kim and H.P. Nilles, Nucl. Phys. B370 (1992) 105.
[27] G.F. Giudice and A. Masiero, Phys. Lett. B206 (1988) 480; J.A. Casas and C. Muñoz, Phys. Lett. B306 (1993) 288.

[28] P. Binétruy and M. K. Gaillard, Phys. Lett. B253 (1991) 119.

[29] H.P. Nilles and M. Olechowski, Phys. Lett. B248 (1990) 268.

[30] T. R. Taylor and G. Veneziano, Phys. Lett. B212 (1988) 147.

[31] V. Kaplunovsky, Nucl. Phys. B307 (1988) 145; G. Lopes Cardoso and B. Ovrut, Nucl. Phys. B369 (1992) 351; preprint UPR-0481T (1991); I. Antoniadis, K. S. Narain and T. R. Taylor Phys. Lett. B267 (1991) 37; I. Antoniadis, E. Gava and K. S. Narain, Nucl. Phys. B383 (1992) 93; Phys. Lett. B283 (1992) 209.

[32] G. Veneziano and S. Yankielowicz. Phys. Lett. B113 (1982) 231.