Mixed modes in red-giant stars observed with CoRoT

B. Mosser1, C. Barban1, J. Montalbán2, P.G. Beck3, A. Miglio2,4, K. Belkacem5,6, M.J. Goupil7, S. Hekker4,6, J. De Ridder3, M.A Dupret8, Y. Elsworth4, A. Noels2, F. Baudin9, E. Michel1, R. Samadii, M. Auvergne1, A. Baglin1, and C. Catala1

1 LESIA, CNRS, Université Pierre et Marie Curie, Université Denis Diderot, Observatoire de Paris, 92195 Meudon cedex, France; e-mail: benoit.mosser@obspm.fr
2 Institut d’Astrophysique et de Géophysique, Université de Liège, Allée du 6 Août, 17 B-4000 Liège, Belgium
3 Instituut voor Sterrenkunde, K. U. Leuven, Celestijnenlaan 200D, 3001 Leuven, Belgium
4 School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom
5 Institut d’Astrophysique Spatiale, UMR 8617, Université Paris XI, Bâtiment 121, 91405 Orsay Cedex, France
6 Astronomical Institute ‘Anton Pannekoek’, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands

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ABSTRACT

Context. The CoRoT mission has provided thousands of red-giant light curves. The analysis of their solar-like oscillations allows us to characterize their stellar properties.

Aims. Up to now, the global seismic parameters of the pressure modes remain unable to distinguish red-clump giants from members of the red-giant branch. As recently done with Kepler red giants, we intend to analyze and use the so-called mixed modes to determine the evolutionary status of the red giants observed with CoRoT. We also aim at deriving different seismic characteristics depending on evolution.

Methods. The complete identification of the pressure eigenmodes provided by the red-giant universal oscillation pattern allows us to aim at the mixed modes surrounding the ℓ=1 expected eigenfrequencies. A dedicated method based on the envelope autocorrelation function is proposed to analyze their period separation.

Results. We have identified the mixed-mode signature separation thanks to their pattern compatible with the asymptotic law of gravity modes. We have shown that, independent of any modelling, the g-mode spacings help to distinguish the evolutionary status of a red-giant star. We then report different seismic and fundamental properties of the stars, depending on their evolutionary status. In particular, we show that high-mass stars of the secondary clump present very specific seismic properties. We emphasize that stars belonging to the clump were affected by significant mass loss. We also note significant population and/or evolution differences in the different fields observed by CoRoT.

Key words. Stars: oscillations - Stars: interiors - Stars: evolution - Stars: mass loss - Methods: data analysis

1. Introduction

The CoRoT and Kepler missions have revealed solar-like oscillation in thousands of red-giant stars. This gives us the opportunity to test this important phase of stellar evolution, and provides new information in stellar and galactic physics (Miglio et al. 2009; Bedding et al. 2011). Thanks to the dramatic increase of information recently available (De Ridder et al. 2009; Hekker et al. 2009; Mosser et al. 2010; Kallinger et al. 2010; Huber et al. 2010), we have now a precise view of pressure modes (p modes) corresponding to oscillations propagating essentially in the large convective envelopes. Gravity modes (g modes) may exist in all stars with radiative regions. They result from the trapping of gravity waves, with buoyancy as a restoring force. In red giants, gravity waves propagating in the core have high enough frequencies to be coupled to pressure waves propagating in the envelope (Dupret et al. 2009). The trapping of such waves with mixed pressure and gravity character gives the so-called mixed modes. The coupling insures non-negligible oscillation amplitudes in the stellar photosphere, hence the possible detection of these mixed modes.

Observationally, mixed modes were first identified in white-dwarf oscillation spectra (Winnet et al. 1991). They have also been observed in sub-giant stars, first with ground-based observations (Carrier et al. 2005; Bedding et al. 2007, 2010b), then recently in Kepler and CoRoT fields (Chaplin et al. 2010; Deheuvels et al. 2010). In red giants, they were first suspected by Bedding et al. (2010a). Their presence significantly complicates the fit of the p modes observed in CoRoT giants (Hekker et al. 2010; Barban et al. 2010) and they were identified as outliers to the universal red-giant oscillation spectrum (Mosser et al. 2011). Recent modelling of a giant star observed by the Kepler mission had to take their presence into account (Di Mauro et al. 2011). Finally, they have been firmly identified in Kepler data (Beck et al. 2011), making the difference clear between giants burning hydrogen in shell or helium in the core (Bedding et al. 2011).

The theoretical analysis of these mixed modes in red giants has been performed prior to observations (Dziembowski et al. 2001; Christensen-Dalsgaard 2004; Dupret et al. 2009). Using a non-radial non-adiabatic pulsation code including a non-local
time-dependent treatment of convection and a stochastic excitation model, Dupret et al. (2009) have computed the eigenfrequencies and the mode heights in several red-giant models. They have shown that mixed modes have much larger mode inertias than p modes, hence present longer lifetimes and smaller linewidths. They were able to identify different regimes, depending on the location of the models on the red-giant branch. Recently, Montalbán et al. (2010) proposed to use the oscillation spectrum of dipole modes to discriminate between red-giant branch (RGB) and central-He burning (clump) evolutionary phase of red giants. This illustrates the fact that observing p- g-mixed modes and identifying their properties give us a unique opportunity to analyze the cores of red giants, since the g component is highly sensitive to the core condition (Dupret et al. 2009).

In this paper, we present and validate an alternative method to Bedding et al. (2011) to detect and identify mixed modes in red giants. We use it to analyze red-giant stars in two different fields observed by CoRoT and show that they present different properties: mixed modes do not only allow us to distinguish different evolutionary status, they can also show different population characteristics. We also assess clear observational differences between the fundamental and seismic properties of the red-giant stars, depending on their evolutionary status. We show for instance that the observation of mixed modes opens a new way to study the mass loss at the tip of the RGB. We have also clear indication that mixed modes in red giants are sensitive to chemical composition gradients in the deep interior, as they are in SPB and γ Doradus stars (Miglio et al. 2008). Their study will definitely boost both stellar and galactic physics.

2. Analysis

2.1. Mixed-mode pattern

According to Dupret et al. (2009), each \( \ell = 1 \) ridge is composed of a pressure mode surrounded by mixed modes with a pattern mostly dominated by the g-mode component. According to the asymptotic description (Tassoul 1980), both p modes and g modes present well organized patterns. For p modes, the organization is a comb-like structure in frequency, for g modes, regularity is observed in period. Mixed modes in giants are supposed to exhibit the well-organized pattern of g modes. In order to detect them in an automated way, we have searched for this asymptotic pattern:

\[
T_{n_\ell} = \frac{2\ell^2(n_\ell + \alpha_\ell)}{\sqrt{\ell(\ell + 1)}} \left[ \int_0^1 \frac{N_{\text{NV}} r}{N_{\text{core}}} \, dr \right]^{-1} \approx \frac{(n_\ell + \alpha_\ell) \Delta T_g}{\sqrt{\ell(\ell + 1)}} \tag{1}
\]

with \( n_\ell \) the gravity radial order, \( \alpha_\ell \) an unknown constant and \( N_{\text{NV}} \) the Brunt-Väisälä frequency. The period spacing \( \Delta T_g \) is the equivalent for g modes of the large separation for p modes.

2.2. Periodic spacings

We have first restricted our attention to \( \ell = 1 \) modes since mixed modes with \( \ell = 2 \) are not supposed to be as clearly visible (Dupret et al. 2009). Furthermore, the close proximity of \( \ell = 2 \) modes to radial modes complicates the analysis. For the \( \ell = 1 \) mixed modes, the periodic pattern simply becomes:

\[
T_{n_{\ell=1}} = (n_{\ell=1} + \alpha_1) \Delta T_1, \text{ with } \Delta T_1 = \Delta T_g / \sqrt{2}. \tag{2}
\]

The period spacing \( \Delta T_1 \), linked to the integral of the Brunt-Väisälä frequency by Eq. (1) can be deduced from the frequency spacing in the Fourier spectrum. To measure this spacing, we need to focus on the mixed modes. We are able to do this using the method introduced in Mosser et al. (2011) which allows for a complete and automated mode identification. We then use the envelope autocorrelation function (Mosser & Appourchaux 2009) to derive the period spacing. The method is described in the Appendix. As for the method presented by Bedding et al. (2011), it delimits a period \( \Delta T_{\text{obs}} \) less than \( \Delta T_1 \), due to the bumping of mixed modes. We have estimated that, for the 5-month long CoRoT time series, we measure \( \Delta T_{\text{obs}} \approx \Delta T_1/1.15 \) (see Appendix).

2.3. Positive detection

The analysis has been performed on CoRoT red giants previously analyzed in Hekker et al. (2009) and Mosser et al. (2010). These stars were observed continuously during 5 months in 2 different fields respectively centered on the Galactic coordinates (37°, -07°45′) and (212°, -01°45′). Targets with a too low signal-to-noise ratio were excluded (see the Appendix). We therefore only considered stars with accurate global seismic parameters and labelled as the \( N_\ell \) set in Mosser et al. (2010).

Thanks to the method exposed in the Appendix, we have analyzed all red-giant spectra in an automated way. We have then checked all results individually. This allowed us to discard a few false positive results, and then to verify that the asymptotic expansion of the g modes (Eq. (2)) gives an accurate description of the mixed modes since it is able to reproduce their spacings. The irregularities of the spacings are discussed in Section 3.1 and in the Appendix. A few examples are given in Fig. 1. We have overplotted on red-giant oscillation spectra the expected location of the p-mode pattern derived from Mosser et al. (2011), and we have indicated the frequency separation of the mixed modes derived from the asymptotic description. Because of the mode bumping due to the coupling, the g-mode asymptotic expression is not able to derive the exact location of the mixed-mode eigenfrequencies, but we see that the spacings reproduce the observations. This agreement is certainly related to the fact that, according to Eq. (2), very high values of the gravity radial orders are measured, typically above 60 and up to 400 (Table 1). We were then able to provide a diagram representing the period separation \( \Delta T_{\text{obs}} \) of the mixed modes as a function of the large frequency separation \( \Delta \nu \) of the pressure modes (Fig. 2).

The 5-month long CoRoT time series provide a frequency resolution of about 0.08 µHz, accurate enough for detecting the mixed-modes in the red clump but limiting their possible detection at lower frequency. Therefore, we have taken care of possible artefacts. We have excluded the region of the \( \Delta \nu - \Delta T_{\text{obs}} \) diagram limited by the frequency resolution (Fig. 2). We have also taken care of the possible confusion with the small separation \( \delta \nu_{\odot} \), since in given frequency ranges its signature can mimic a g-mode spacing (dotted line in Fig. 2). Due to the low visibility of \( \ell = 3 \) modes, it appeared useless to test the influence of their pattern combined with radial or dipole modes.

The criterion of a positive detection of the mixed modes in at least two frequency ranges allowed us to discriminate regular period spacings from regular frequency spacings as caused by rotation. We also examined the possible confusion with the ragged, speckle-like, appearance of the structure due to the limited lifetime of the modes. Simulations have shown that the threshold level provided by the method (Mosser & Appourchaux 2009) combined with the detection in multiple frequency ranges excludes spurious signature.
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Fig. 1. Shaded regions of these bar code spectra reproduce the universal red-giant oscillation pattern and identify the location of the different harmonic degrees for three CoRoT targets (ID given on the right side); the more dense the background, the higher the probability to have the short-lived p mode realized there. Black dashes in the $\ell = 1$ ridges indicate the asymptotic spacing of the mixed modes as derived from Eq. 2; they do not give their exact eigenfrequencies, since the asymptotic relation does not account for the mode bumping of mixed modes.

Table 1. Typical parameters of mixed modes

| $\langle \Delta \nu \rangle$ | $\nu_{\text{max}}$ | $\delta \nu_{\text{env}}$ | $\Delta T_{\text{obs}}$ | $\Delta T_{1}$ proxy at $\nu_{\text{max}} - \delta \nu_{\text{env}}/2$ | $n_{g}; \Delta n_{g}$ | $n_{g}; \Delta n_{g}$ | $n_{g}; \Delta n_{g}$ | $T$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\mu$Hz         | $\mu$Hz         | s               | s               | s               | s               | s               | s               | day            |
| Red-clump stars |                 |                 |                 |                 |                 |                 |                 |                 |
| 4.0             | 32              | 13              | 250             | 287             | 137; 5          | 108; 3          | 89; 2           | 78             |
| 7.0             | 73              | 28              | 150             | 173             | 97; 3           | 78; 2           | 66; 1           | 25             |
| Red-giant branch stars |             |                 |                 |                 |                 |                 |                 |                 |
| 2.5             | 19              | 8               | 800             | 920             | 73; 3           | 57; 2           | 46; 1           | 70             |
| 4.0             | 32              | 13              | 80              | 92              | 429; 17         | 339; 10         | 281; 7          | 245            |
| 6.0             | 60              | 23              | 60              | 69              | 300; 9          | 241; 6          | 201; 4          | 93             |
| 9.0             | 102             | 38              | 60              | 69              | 174; 5          | 142; 3          | 119; 2          | 32             |

The frequency $\nu_{\text{max}}$ corresponds to the maximum oscillation signal; $\delta \nu_{\text{env}}$ indicates the full width at half-maximum of the observed excess oscillation power (Mosser et al. 2010). The proxies of the period spacing $\Delta T_{1}$ are estimated as $1.15 \Delta T_{\text{obs}}$. The values of $n_{g}$ are derived from Eq. 2 with the parameter $\alpha_{\ell,1} = 0$. $\Delta n_{g}$ corresponds to the maximum number of observable mixed modes in a 0.25 $\Delta \nu$-wide interval around each pure p mode. $T$ indicates the observational length required to measure the g-mode spacing according to the Shannon criterion.

Finally, we have identified an $\ell = 1$ mixed-modes pattern in about 42% of the targets (387 out of a total of 929), and for more than 75% of the stars with a R magnitude brighter than 13. The few remaining bright stars do not present a reliable signature, either because their spectrum is free of mixed modes, or because the signature was not found reliable according to the threshold detection level. When detected, the pattern follows closely both the asymptotic expression of gravity modes (Tassoul 1980) and the description made by Dupret et al. (2009). The peaks generating these spacings are visible in a frequency range of about 0.25 $\Delta \nu$ around the expected pure p-mode eigenfrequencies $\nu_{n,1}$. This represents typically, at the peak of the red clump distribution ($\Delta \nu \approx 4 \mu$Hz and $\nu_{\text{max}} \approx 32 \mu$Hz), from two to ten mixed modes (Table 1). Due to the frequency dependence derived from Eq. 2, the number of observable modes varies very rapidly, as observed by Beck et al. (2011) and made explicit in Table 1.

According to Dupret et al. (2009), $\ell = 2$ mixed modes are expected to be much more damped than $\ell = 1$ modes. However, their presence is clearly identified for about a third of the targets (Fig. 3). The agreement of their mean separation $\Delta T_{2} \approx \Delta T_{1}/\sqrt{3}$ (Eq. 1) is clear enough to be disentangled from the speckle-like aspect of short-lived modes. Their observation will give strong constraints on the inner envelope, where g modes are coupled to p modes, the coupling being highly sensitive to the respective location of the Brunt-Väisälä and $\ell = 2$ Lamb frequency. The high density of mixed modes, due to the high values
Fig. 2. Period separation of mixed modes $\Delta T_{\text{obs}}$ as a function of the mean frequency separation of the pressure modes ($\Delta \nu$). Blue (purple) crosses represent the targets observed in the center (anticenter) direction. The dashed line indicates the frontier below which the frequency resolution is not fine enough for deriving $\Delta T_{\text{obs}}$ and the dotted line represents the location of the spurious signature that would correspond to the small separation $\delta \nu_{02}$ (Mosser et al. 2011). The two separate domains correspond to the location of $\Delta T_1$ expected by Montalbán et al. (2011) and observed by Bedding et al. (2011). The vertical thick line indicates the mean 1-$\sigma$ error bars (the error bar on the large separation $\Delta \nu$ is in fact very small).

of the radial gravity orders observed, makes it possible to achieve this high-resolution analysis. We observe in most cases groups of only two $\ell = 2$ mixed modes, rarely three, spread in a frequency range of about 0.04 $\Delta \nu$. A further consequence of the observation of $\ell = 2$ mixed modes is a correction to the asymptotic laws reported for the small separation $\delta \nu_{02}$ by Huber et al. (2010) and Mosser et al. (2011); they are correct, but only indicative of the barycenter of the $\ell = 2$ modes, if mixed.

3. Discussion

3.1. Period spacing

The measurement of mixed-mode spacings varying as $1/\nu^2$ (Eq. A.2) validates the use of the asymptotic law of g modes, even if a detailed view of the mixed modes indicates irregularities in their spacings (Fig. 3). These shifts may be interpreted as a modulation due to the structure of the core with a sharp density contrast compared to the envelope, as observed in white dwarfs. The determination of $\Delta T_1$ will allow modelers to define the size of the radiative core region. The direct measurement of the individual shifts $\Delta T_1(n_g)$ will give access to the core stratification (Miglio et al. 2008), as well as the observation (or not) of $\ell = 2$ mixed modes that test a different cavity.

The method developed in this paper presents many interesting characteristics compared to the method used by Bedding et al. (2011). First, it is directly applicable in the Fourier spectrum and does not require the power spectrum to be expressed in period. The method is fully automated, since it is coupled to the identification of the spectrum based on the universal pattern, and it includes a systematic search of periodic spacings that are not related to the p-mode pattern. Based on an $H_0$ test, it intrinsically includes a statistical test of reliability.

This test defines a threshold level that makes the method efficient even at low signal-to-noise ratio. Its most interesting property certainly consists in its ability to derive the measurement of the variation of $\Delta T_1$ with frequency, as the EACF method used for p modes (Mosser & Appourchaux 2009; Mosser 2010). In fact, the method was developed in parallel to the one mainly presented in Bedding et al. (2011), and gave similar results that confirmed the detection of mixed modes in Kepler giants.

3.2. Red-clump and red-giant branch stars

Red-clump stars have been characterized in previous work (e.g. Fig. 5 of Mosser et al. 2010). They contribute to a distribution in $\Delta \nu$ with a pronounced accumulation near 4 $\mu$Hz (equivalent to the accumulation near $\nu_{\text{max}} \approx 32$ $\mu$Hz). However, population analysis made by Miglio et al. (2009) has shown that this population of stars with $\Delta \nu \approx 4$ $\mu$Hz effectively dominated by red-
clump stars also contains a non-negligible fraction (≈ 30%) of RGB stars. None of the global parameters of p mode (neither \( \Delta \nu \) nor \( \nu_{\text{max}} \)) is able to discriminate between RGB and red-clump stars. However, as shown by Bedding et al. (2011) with Kepler data, the examination of the relation between the p-mode and g-mode spacings shows two regimes (Fig. 2). The signature of the red-clump stars piling up around \( \langle \Delta \nu \rangle \approx 4 \mu \text{Hz} \) is visible with \( \Delta T_{\text{obs}} \approx 250 \text{s} \). Another group of stars has \( \Delta T_{\text{obs}} \) values lower than 100 s. Regardless of any modelling, this gives a clear signature of the difference between red-clump stars that burn helium in their core and stars of the red-giant branch that burn hydrogen in a shell (Montalbán et al. 2010). The agreement with theoretical values is promising (Montalbán et al. 2011).

The contribution of the red-clump stars in the \( \Delta \nu-\Delta T_{\text{obs}} \) diagram presented in Fig. 1 is unambiguous. Stars with large separation larger than the clump value (about 4 \( \mu \text{Hz} \)) are located on the ascending red-giant branch or members of the secondary clump (Girardi 1998; Miglio et al. 2009). At lower frequency than the clump, using mixed modes to disentangle the evolutionary status is more difficult. According to the identification of the clump provided by the distributions of the large separation (Mosser et al. 2010), we assume that the giants with \( \Delta \nu \lesssim 3.5 \mu \text{Hz} \) and \( \Delta T_{\text{obs}} \gtrsim 200 \text{s} \) belong to the RGB. The following analysis shows further consistent indications.

We finally note that the detection of RGB stars having a large separation similar to the peak of the clump stars (≈ 4 \( \mu \text{Hz} \)) is only marginally possible, due to insufficient frequency resolution. According to Table 1 more than 200 days are necessary to resolve the mixed modes, while CoRoT runs are limited to about 150 days.

### 3.3. Mode lifetimes and heights

We have remarked that, as predicted by Dupret et al. (2009), the lifetimes of the mixed modes trapped in the core are much larger than the lifetimes of radial modes (Fig. 1). In most cases, the mixed modes are not resolved. When the mixed modes are identified, they correspond in most cases to a comb-like pattern of thin peaks, without a larger and broader peak that could correspond to the pure \( \ell = 1 \) pressure mode. However, since the detection of mixed modes is only achieved for a limited set of stars, one cannot exclude that some red giants only show pure \( \ell = 1 \) pressure modes.

### 3.4. Mass-radius relation

The possibility to distinguish the evolutionary status allows us to refine the ensemble asteroseismic analysis made on CoRoT red giants, especially the mass and radius distributions (Fig. 12 and 13 of Mosser et al. 2010). Benefitting from the same calibration of the asteroseismic mass and radius determination as done in this work, we have investigated the mass-radius relation, having in mind that without information of the evolutionary status, there is no clear information (Hekker et al. 2011).

We note here that the mass distribution is almost uniform in the RGB, contrary to clump stars (Fig. 4). The number of high-mass stars in the RGB, above 1.8 \( M_\odot \), as defined in Bedding et al. (2011), is about half of that in the clump, consistent with their expected more rapid evolution. Similarly, stars with masses below 1 \( M_\odot \) are significantly rarer (by a factor of six to one) in the RGB by comparison with the clump. If we assume that the scaling relations, valid along the whole evolution from the main sequence to the giant class, remain valid after the tip of the RGB, this comparative study proves that low-mass stars present in the clump but absent in the branch have lost a substantial fraction of their mass due to stellar winds when ascending up to the tip of the RGB. Therefore, after the helium flash, these stars show lower mass. The quantitative study of this mass loss will require a careful unbiased analysis, out of the scope of this paper. On the contrary, high-mass stars, even if they lost mass too, can be observed as clump stars since they spend a much longer time in the core-helium burning phase than while ascending the RGB. Most of these stars belong to the secondary red clump (Girardi 1999; Bedding et al. 2011). We note that the secondary-clump stars present a larger spread in the mass-radius distribution, certainly due to the fact that those stars that have not undergone the helium flash present different interior structures. On the other side, the low-mass stars of the clump present a clear mass-radius relation: the helium flash shall have made their core largely homologous.
The number of detections of solar-like oscillations toward the anticenter direction is lower than towards the center, due to a lower red-giant density in the outer galactic regions and to dimmer magnitudes (Mosser et al. 2010). Hence, the number of reliable measurements of $\Delta T_{\text{obs}}$ is lower too, but with the same proportion of 40%. The sample is large enough to make sure that the difference between the distributions is significant (Fig. 2).

We have verified that the mean large period separation of the mixed modes gives a clear indication of the evolution of the star. Assuming the validity of the asteroseismic scalings laws for the stellar mass and radius, we have shown that the mass distribution of the RGB is much more uniform than in the red clump. Asteroseismology confirms that these red-clump stars have undergone a significant mass loss. Furthermore, red-giant low-mass stars after the helium flash do present homologous interiors and a clear mass-radius relation, contrary to stars in the secondary clump. Longer time series recorded with Kepler will allow us to investigate this relation in more detail. Benefitting from the fact that CoRoT provides observations in two fields, towards the Galactic center and in the opposite direction, we have now a performing indicator for making the population study more precise. This will be done in future work.

These data confirm the power of red-giant asteroseismology: we have access to the direct measure of the radiative central regions. Even if observations only deliver a proxy of the large period separation, comparison with modelling will undoubtedly be very fruitful. In a next step, the dedicated analysis of the best signal-to-noise ratio targets will allow us to sound in detail the red-giant core. Modulation of the large period spacing, observed in many targets, will give a precise view of the core layers.

### Appendix A: Method

Identifying $\ell = 1$ mixed modes first requires to aim at them precisely. This first step is achieved by the method presented in Mosser et al. (2011), able to mitigate the major sources of noise that perturb the measurement of the large separation $\Delta \nu$ and then to derive the complete identification of the $p$-mode oscillation pattern. Complete identification means that all eigenfrequencies, their radial order and their degree, are unambiguously identified, as for instance the expected frequencies of the pure pressure $\ell = 1$ modes:

$$v_{n,\ell=1} = [n + 1/2 + \varepsilon(\Delta \nu) - d_{01}] \Delta \nu,$$

(A.1)

with $\varepsilon(\Delta \nu)$ representing the surface term and $d_{01}$ accounting for the small separation of $\ell = 1$ pure $p$ modes.

The frequency spacings of the mixed modes are then analyzed with the envelope autocorrelation function (EACF) based on narrow filters centered on the frequencies $v_{n,1}$ in the

![Fig. 6. Proxy of the number of observable modes, given by the ratio $\delta v_{\text{env}}/\Delta \nu$, as a function of $v_{\text{max}}$. Same color code as in Fig. 4. The rectangle indicates the mean 1-$\sigma$ error bars.](image-url)
victory of the frequency \( v_{\text{max}} \) of maximum oscillation amplitude (Mosser & Appourchaux 2009). We deliberately chose the EACF method since it has proved to be efficient at very low signal-to-noise ratio (Mosser et al. 2009; Gaulme et al. 2010; Hekker et al. 2011a), thanks to a statistical test of reliability based on the null hypothesis.

In order to only select mixed modes, the full width at half-maximum of the filter is fixed to \( \Delta \nu/2 \) (Fig. A.1). From the differentiation of the relation between period and frequency, the regular spacing of Eq. (2) in period translates into spacings varying with frequency as:

\[
\delta \nu_{g,1} = \nu_{g,1} \Delta T_L = \nu_{g,1}^2 \Delta T_1.
\]  

(A.2)

For an individual measure centered on a given \( p \) mode, the frequency spacings selected by the narrow filter can be considered as uniform, since the ratio \( \Delta n_g/n_g \) that represents the relative variation of the gravity radial order within the filter is less than about 1/25 (Table 1). Then, when comparing the different measures around different pressure radial orders, obtaining mean frequency spacings varying as \( \nu_{g,1}^2 \) validates the hypothesis of a Tassoul-like g-mode pattern (Fig. A.2).

We have checked that the method can operate with a filter narrow enough to isolate the mixed modes. This is clearly a limit, since the performance of the EACF varies directly with the width of the filter (Mosser & Appourchaux 2009). We have also checked that such a filter is able to derive the signature of \( \ell = 1 \) mixed modes in a frequency range as wide as possible, without significant perturbation of possible \( \ell = 2 \) mixed modes. As filters with a narrower width focus too much on the region where, due to the vicinity of the \( \ell = 1 \) pure pressure mode, bumped mixed modes present a narrower period spacing than that expected from Eq. (2) we consider the \( \Delta \nu/2 \) width to be the best compromise.

Since measurements can be verified in different frequency ranges (Fig. A.1 and A.2), we have chosen a threshold value corresponding to the rejection of the H0 hypothesis at the 10% level. For the characteristics of mixed modes, different to the characteristics of the \( p \) modes considered in Mosser & Appourchaux (2009), this corresponds to a normalized EACF of about 4.5 at \( v_{\text{max}} \). In practice, stellar time series with a low signal-to-noise ratio are excluded by this threshold value. In case of reliable detection, error bars can be derived following Eq. A.8 of Mosser & Appourchaux (2009).

The measurement of the g-mode frequency spacing \( \delta \nu_{g,1} \) is not only validated by a correlation signal larger than the threshold value. We only selected results with \( \delta \nu_{g,1} \) measured in at least two frequency ranges and verifying Eq. (2) within 20%. This flexibility allows us to account for possible discrepancy to the exact asymptotic relation (Eq. (2)), as produced by the avoided crossings resulting from coupling of the \( p \) mode in the stellar envelope to the \( g \) modes in the core. It also accounts for the possible modulation of the period due to a composition gradient in the core (Miglio et al. 2008).

Deriving an estimate \( \Delta T_{\text{obs}} \) of \( \Delta T_1 \) from the spacings \( \delta \nu_{g,1} \) is then direct. Due to avoided crossings, \( \Delta T_{\text{obs}} \) is close to but less than \( \Delta T_1 \) (Althaus et al. 2010, Beck et al. 2011, Bedding et al. 2011). This is called mode bumping and results from the fact that the mixed modes around \( v_{g,1} \) present necessarily smaller spacings than pure \( g \) modes since the mixing of the \( g \) modes with one \( p \) mode gives one supernumerary mixed mode per \( \Delta \nu \) frequency interval, as shown in Beck et al. (2011). The ratio \( \Delta T_1/\Delta T_{\text{obs}} \approx 1.15 \) is derived from the examination, whether possible, of the g-mode spacing far from the expected \( p \) mode, assuming as in Bedding et al. (2011) that this spacing is unperturbed by the mode bumping and corresponds to the asymptotic g-mode spacing. The ratio differs from the value obtained with the Kepler data, since the frequency resolutions and the analysis methods are different. First, the frequency resolution is 2.5 times less fine for CoRoT data; as a consequence, the influence of the
mode bumping is more smooth. Second, and more importantly, the envelope autocorrelation method is able to derive a mean value of the spacing in a larger frequency range than the method exposed in [Bedding et al. (2011)] thanks to the $\Delta \nu/2$-broad filter used to select the mixed modes. This helps to enhance the contribution of non-bumped mixed modes.

When relaxing the condition expressed by Eq. A.2 the method can be applied, with ultra-narrow filters centered on individual mixed modes, to search for rotational splittings. This search was unfortunately negative for CoRoT red-giant spectra.

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