We introduce a measure $Q$ of the “quality” of a quantum which-way detector, which characterizes its intrinsic ability to extract which-way information in an asymmetric two-way interferometer. The “quality” $Q$ allows one to separate the contribution to the distinguishability of the ways arising from the quantum properties of the detector from the contribution stemming from a priori which-way knowledge available to the experimenter, which can be quantified by a predictability parameter $P$. We provide an inequality relating these two sources of which-way information to the value of the fringe visibility displayed by the interferometer. We show that this inequality is an expression of duality, allowing one to trace the loss of coherence to the two reservoirs of which-way information represented by $Q$ and $P$. Finally, we illustrate the formalism with the use of a quantum logic gate: the Symmetric Quanton-Detecton System (SQDS). The SQDS can be regarded as two qubits trying to acquire which way information about each other. The SQDS will provide an illustrating example of the reciprocal effects induced by duality between system and which-way detector.

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I. INTRODUCTION

The observation of an interference pattern and the acquisition of which-way information are mutually exclusive. This statement, which is often quoted as a definition of the duality principle [1], has driven the debate on the fundamentals of Quantum Mechanics since its foundation [2, 3]. A very elegant approach has been developed by Englert [1], which allows one to quantify the notion of wave-particle duality of a quantum system (the “Quanton”) in a two-way interferometer. He derives an inequality concerning duality, according to which the fringe visibility $V$ displayed at the output port of the interferometer sets an absolute upper bound on the amount of which-way information $D$ that is potentially stored in a generic which-way detector (WWD). Here $D$ is the distinguishability of the two ways defined in [1]. The inequality reads

$$D^2 + V^2 \leq 1,$$

(1)

encoding the extent to which partial which-way information and partial fringe visibility are compatible. In particular, the extreme situations characterized by perfect fringe visibility ($V = 1$) or full which-way information ($D = 1$) are mutually exclusive, so the bound in (1) can be interpreted as an expression of duality. Inequalities of this type involving Duality have attracted a great interest, both theoretically [4, 5] and experimentally [6, 7, 8, 9].

Nevertheless, two different sources of which-way information are represented in $D$. One is the predictability of the ways $P$, i.e., the a priori which-way knowledge that the experimenter has about the ways stemming from the preparation of the beam splitter and the initial state of the Quanton. This inherent lopsidedness of the system is simply a bias for one or the other way built into the initial state, as in a “loaded” coin toss. The second source of which-way information contained in $D$ is purely quantum mechanical, stemming from the WWD’s ability to correlate the two ways with two or more of its own final states, leading to the “storage” of some which-way information in the detector.

It is the purpose of this paper to investigate the case of interferometers characterized by $P \neq 0$ in order to sort out how much of the loss of fringe visibility originates in the predictability and how much results from the inherent quantum properties of the detector. To achieve this goal we introduce a measure $Q$ of the “quality” of the detector, which, roughly speaking, measures how good the WWD is. The parameter $Q$ characterizes the intrinsic ability of the detector to extract which-way information via quantum correlations. We then derive an inequality that treats the three quantities $P$, $Q$, and $V$ on an equal footing. This formalism allows one to trace the loss of coherence in asymmetric interferometers quantitatively to the two reservoirs of which-way information represented by $P$ and $Q$.

Asymmetric interferometers deserve attention because they represent the most general initial preparation of the Quanton: symmetric interferometers (with $P = 0$)...
are a particular case. Moreover, a number of proposed which-way experiments are essentially asymmetric: e.g., the Einstein recoupling slit in a Young double-slit interferometer [10], the quantum-optical Ramsey interferometer outlined in [11], and the recent experiments in [12], in which beam splitting is performed by the quantized cavity-mode of a high-finesse resonator. In all these cases beam splitting and which-way detection are provided by the same physical interaction. Thus, the asymmetry of the beam splitter in such devices—and in the present treatment—is directly coupled to the ability of the WWD to get entangled with the atom. Our formalism will prove useful in understanding the interplay between \( Q \) and \( D \) stemming from this coupling.

This paper is organized as follows. In section II, we describe the two-way interferometer setup and review Englert’s formalism [1]. In order to explain the insight gained by the introduction of a “quality” parameter, we first study the case of a classical WWD; this is done in Sec. III, where the WWD is described as a classical binary communication channel. Section IV furnishes the first study the case of a classical WWD; this is done in [11], and the recent experiments in [12], in which beam splitting is performed by the quantized cavity-mode of a high-finesse resonator. In all these cases beam splitting and which-way detection are provided by the same physical interaction. Thus, the asymmetry of the beam splitter in such devices—and in the present treatment—is directly coupled to the ability of the WWD to get entangled with the atom. Our formalism will prove useful in understanding the interplay between \( P \), \( Q \), and \( D \) stemming from this coupling.

This paper is organized as follows. In section II, we describe the two-way interferometer setup and review Englert’s formalism [1]. In order to explain the insight gained by the introduction of a “quality” parameter, we first study the case of a classical WWD; this is done in Sec. III, where the WWD is described as a classical binary communication channel. Section IV furnishes the definition of the quality \( Q \) of a quantum which-way detector and an analysis of its properties. In Sec. V we specialize our results to a simple illustrating example: the Symmetric Quanton-Detecton System. Finally, we end in Sec. VI with a summary and a discussion of the results.

II. DUALITY IN TWO-WAY INTERFEROMETERS

Consider the schematic two-way interferometer depicted in Fig. 1. Following Englert [1], we describe the Quanton degree of freedom as a spin-\( \frac{1}{2} \) system. We prepare the Quanton in the pure state

\[
\rho_Q^{(0)} = \frac{1}{2} (1 + s_Q^{(0)} \cdot \sigma),
\]

(2)

where \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) are the usual Pauli spin operators. We have chosen an arbitrary initial pure state with polarization vector \( s_Q^{(0)} = (s_{Qx}^{(0)}, s_{Qy}^{(0)}, s_{Qz}^{(0)}) \), \( |s_Q^{(0)}| = 1 \). The pure state \( \rho_Q^{(0)} \) was parameterized in Ref. [1] as

\[
s_Q^{(0)} = (0, -\sin \theta, \cos \theta). \quad \text{The significance of choosing } s_Q^{(0)} \neq 0 \text{ is discussed below.}
\]

The detector is prepared in some initial state \( \rho_D^{(0)} \) so that the combined Quanton-plus-detector system is represented by

\[
\rho = \rho_Q^{(0)} \otimes \rho_D^{(0)}. \tag{3}
\]

As depicted in Fig. 1 the interferometer consists of a beam splitter, a phase shifter and WWD for the two ways, and a beam merger. The action of the beam splitter on the Quanton is described by the transformation

\[
\rho_Q \to \exp \left( -i \frac{\pi}{4} \sigma_y \right) \rho_Q \exp \left( i \frac{\pi}{4} \sigma_y \right), \tag{4}
\]

while the beam merger is described by the inverse transformation [13]. In the middle of the interferometer, the evolution of the detector degree of freedom after its interaction with the Quanton can be characterized by

\[
\rho_D \to U_D^\dagger \rho_D U_D \equiv \rho_D^{(\pm)} \quad \text{for the } \sigma_z = \pm 1 \text{ way,} \tag{5}
\]

where \( U_+ \) and \( U_- \) are unitary operators acting exclusively on the detector [14]. Thus, the unitary operator governing the evolution of the entire system in the “split-beam” section of the interferometer is

\[
\frac{1}{2} (1 + \sigma_z) e^{i \phi/2} U_+ + \frac{1}{2} (1 - \sigma_z) e^{-i \phi/2} U_- , \tag{6}
\]

where \( e^{\pm i \phi/2} \) are the relative phase factors induced by the phase shifters. The system’s final state is then given by the expression

\[
\rho^{(f)} = \frac{1}{4} (1 - s_Q^{(0)}) (1 - \sigma_z) U_D^\dagger \rho_D^{(0)} U_+ \\
+ \frac{1}{4} (1 + s_Q^{(0)}) (1 + \sigma_z) U_D^\dagger \rho_D^{(0)} U_- \\
+ \frac{1}{4} (s_{Qx}^{(0)} - i s_{Qy}^{(0)}) (\sigma_z + i \sigma_y) U_D^\dagger \rho_D^{(0)} U_- e^{-i \phi} \\
+ \frac{1}{4} (s_{Qx}^{(0)} + i s_{Qy}^{(0)}) (\sigma_z - i \sigma_y) U_D^\dagger \rho_D^{(0)} U_+ e^{i \phi}. \tag{7}
\]

After tracing over the detector degree of freedom, the final state of the Quanton is described in terms of a Bloch vector \( s_Q^{(f)} \) with

\[
\begin{align*}
\rho_Q^{(f)} &= \frac{1}{4} (1 - s_Q^{(0)}) (1 - \sigma_z) U_D^\dagger \rho_D^{(0)} U_+ \\
&+ \frac{1}{4} (1 + s_Q^{(0)}) (1 + \sigma_z) U_D^\dagger \rho_D^{(0)} U_- \\
&+ \frac{1}{4} (s_{Qx}^{(0)} - i s_{Qy}^{(0)}) (\sigma_z + i \sigma_y) U_D^\dagger \rho_D^{(0)} U_- e^{-i \phi} \\
&+ \frac{1}{4} (s_{Qx}^{(0)} + i s_{Qy}^{(0)}) (\sigma_z - i \sigma_y) U_D^\dagger \rho_D^{(0)} U_+ e^{i \phi},
\end{align*}
\]

(8)

where

\[
C = \text{tr}_D \left\{ U_D^\dagger \rho_D^{(0)} U_- \right\}. \tag{9}
\]

is a complex number—a contrast factor—characterizing the detector only.

To measure interference between the two ways after they are merged, the observable \( \sigma_z \) (or \( \sigma_y \)) is measured
at the output port of the interferometer. The probability of finding the value \( \pm 1 \) for given phase shift \( \phi \) is

\[
p_{\pm}(\phi) = \text{tr}_D \left\{ \frac{1 \pm \sigma_z}{2} \rho^{(f)} \right\} = \frac{1}{2} \left( 1 \pm s_{Qz}^{(f)} \right)
\]

\[
= \frac{1}{2} \left\{ 1 \pm \text{Re} \left[ (s_{Qy}^{(0)} - is_{Qy}^{(0)}) e^{-i\phi} \right] \right\}.
\]  

(10)

The magnitude of the term in square brackets is the observed fringe visibility \( V \). Now suppose the which-way detector were “turned off” \( (U_+ = U_-) \) or even removed \( (U_{\pm} = 1) \), in which cases \( C = 1 \). Equation (10) then implies that

\[
V_0 = \left| s_{Qz}^{(0)} - is_{Qy}^{(0)} \right| = \sqrt{(s_{Qy}^{(0)})^2 + (s_{Qz}^{(0)})^2}
\]  

(11)

is the \textit{a priori} visibility of the fringes displayed as the phase \( \phi \) is varied over repeated runs of the experiment. When the WWD is “turned on” \( (U_+ \neq U_-) \), the visibility becomes degraded by a factor \( |C| \):

\[
V = |C|V_0, \quad 0 \leq |C| \leq 1.
\]  

(12)

Therefore, any nontrivial measurement by a WWD invariably results in a loss of fringe visibility of the Quanton.

An alternative series of measurements on the output could determine which of the two ways the Quanton has taken on the average, e.g., by measuring the observable \( \sigma_x \) \( (\text{or} \sigma_z) \) by removing the beam merger. The probabilities for taking the \( \pm 1 \) are then given by

\[
w_{\pm} = \text{tr} \left\{ \frac{1 \pm \sigma_x}{2} \rho^{(f)} \right\} = \frac{1}{2} \left( 1 \pm s_{Qx}^{(f)} \right) = \frac{1}{2} \left( 1 \pm s_{Qz}^{(0)} \right),
\]  

(13)

respectively. The predictability of the ways is the magnitude of their difference,

\[
P = |w_+ - w_-| = |s_{Qx}^{(f)}| = |s_{Qz}^{(0)}|,
\]  

(14)

which is the same whether or not the detector or phase shifter is operating. The case \( P = 0 \) \( (s_{Qx}^{(0)} = 0) \) represents symmetric interferometers, where both ways are equally probable, while \( P = 1 \) \( (s_{Qx}^{(0)} = \pm 1) \) corresponds to the extreme asymmetric case of a single-way situation. In terms of which-way information, the predictability \( P \) represents the knowledge that the experimenter has about the ways before measuring \( \sigma_x \) or \( \sigma_y \) \( (\text{i.e., interference effects}) \) on the Quanton. \( P \) is \textit{a priori} which-way information stemming from the initial preparation of the state and the characteristics of the beam splitter chosen. For instance, the experimenter may opt for a maximally asymmetric beam splitter \( (\text{for instance by removing it}) \) so he or she knows in advance the way to be taken by an atom prepared in a certain state. In this case it is clear that \( P = 1 \) implies \( V = 0 \), as demanded by duality. In intermediate cases, the corresponding degradation of the \textit{a priori} fringe visibility induced by \( P \) is implicit in the constraint on the norm of the Bloch vector [1]:

\[
0 \leq |s_{Q}^{(0)}|^2 = P^2 + V_0^2 \leq 1.
\]  

(15)

Thus, \( \sqrt{1 - P^2} \) sets up an absolute upper bound on the value of \textit{a priori} fringe visibility that can be measured. Since \( V \leq V_0 \), the upper bound on the measured fringe visibility \( V \) is even less in the presence of a WWD.

Summing up, we can say that there are two different sources of which-way information resulting in degradation of the fringe visibility. One is the \textit{a priori} which-way knowledge characterized by \( P \). The second source of which-way information arises from the quantum properties of the detector, through the correlations established after its entanglement with the Quanton at the central stage of the interferometer. A quantitative measure of the total distinguishability of the ways as determined by the detector is afforded by the quantity [1]:

\[
D \equiv \text{tr}_D \left\{ \left| w_+ \rho_D^{(+)} - w_- \rho_D^{(-)} \right| \right\} \geq P,
\]  

(16)

where \( \rho_D^{(+)} \) \( (\text{or} \rho_D^{(-)}) \) are the detector’s two final states corresponding to each way. These compose the final detector state according to

\[
\rho_D^{(f)} = \text{tr}_Q \rho_D^{(f)} = w_+ \rho_D^{(+)} + w_- \rho_D^{(-)}.
\]  

(17)

As shown in Ref. [1], [2] and [16], always satisfy the inequality [1] quantifying the extent to which partial which-way information and partial fringe visibility are compatible. This statement of duality generalizes the inequality [15] to include the detector’s role in storing knowledge of the Quanton’s initial state.

### III. THE “QUALITY” OF A CLASSICAL BINARY CHANNEL

In order to explain the meaning of a quality parameter for a quantum WWD, we study first the case of a classical WWD, i.e., a WWD which can only establish classical correlations with the Quanton. The information gained by the WWD can be described as a problem of classical communication between a sender Quanton and a receiver WWD through a noisy channel [15]. A general diagram of the channel is shown in Fig. 2. Here \( q_{\pm} \) are the two possible states of the Quanton. Classical correlations are established with a binary WWD with two readouts, \( d_\pm \text{and} d_\mp \), through a noisy channel characterized by a probability of error \( \epsilon \). The joint probabilities defining the communication system are [16]:

\[
P_{QD}(q_+, d_+) = w_+(1 - \epsilon),
\]

\[
P_{QD}(q_+, d_-) = w_+ \epsilon,
\]

\[
P_{QD}(q_-, d_+) = w_- \epsilon,
\]

\[
P_{QD}(q_-, d_-) = w_-(1 - \epsilon).
\]  

(18)

Following the notation of the previous section, we have written the probabilities of each alternative in the sender as

\[
P_Q(q_{\pm}) = w_\pm.
\]  

(19)
The predictability of the ways, \( P = |w_+ - w_-| \) [Eq. (14)], gives the dispersion of the probability distribution. It may also be written
\[
P = \sqrt{2\langle P \rangle - 1},
\]
where \( \langle P \rangle = [P_Q(q_+)]^2 + [P_Q(q_-)]^2 \) is the mean value of the probability itself (\( \frac{1}{2} \leq \langle P \rangle \leq 1 \)).

We can therefore understand \( P \) as a measure of the information content of the probability distribution, and can further connect this information with a betting strategy of Englert [1]. Consider a measurement of the Quanton subsystem by the WWD. Before an outcome is registered, we can bet on which alternative is going to occur. The likelihood \( L \) of our guessing right is connected to the distinguishability of the results via
\[
L = \frac{1}{2}(1 + D).
\]
This means that when \( D = 1 \) the ways can be totally distinguished and we can win the bet 100% of the time, whereas \( D = 0 \) implies that there is no knowledge about the ways. Note that in the latter case (with \( L = \frac{1}{2} \)) we can still win 50% of the time, as in a coin toss, because even here we have some information about the system: we know in advance that the Quanton is a two-level system so we are right about the ways as often as not.

If we make a measurement of the “sender” Quanton directly, as characterized by Eq. (18), the best bet is to commit to the alternative occurring with the greatest probability. The likelihood of guessing correctly follows from (19):
\[
L = \text{Max}\{w_+, w_-\} = \frac{1}{2}(1 + P).
\]
Equation (20) reveals \( P \) as the classical a priori distinguishability of the ways: \( P = 1 \) indicates full a priori knowledge of the ways, while the maximum uncertainty occurs for \( P = 0 \). The alternatives are equally probable when \( P = 0 \) (\( w_+ = w_- \)) and the ways cannot be distinguished at all.

Now we quantify the classical which-way information that can be acquired when we turn the “receiver” WWD on. The information about the occurrence of \( q_\pm \) acquired by reading an outcome \( d_\pm \) is given by the conditional probabilities
\[
P_{D/Q}(d_+/q_+) = P_{D/Q}(d_-/q_-) = (1 - \epsilon),
P_{D/Q}(d_+/q_-) = P_{D/Q}(d_-/q_+) = \epsilon.
\]

The distance between the two conditional probabilities leading to the same outcome in the WWD tells us how noisy the channel is. This motivates the following definition of the quality \( Q \) of the channel:
\[
Q \equiv |P_{D/Q}(d_+/q_+) - P_{D/Q}(d_-/q_-)| = 1 - 2\epsilon
\]
The case \( \epsilon = 0 \) characterizes a noiseless channel, for which the subsystems Quanton and WWD are perfectly correlated and the quality is maximal (\( Q = 1 \)). In this case, the alternatives of the Quanton are totally distinguishable by reading the outcomes of the WWD. In the opposite extreme, \( \epsilon = \frac{1}{2} \) represents a maximally noisy channel of the poorest quality (\( Q = 0 \)) and the outcomes in the sender and receiver are totally uncorrelated. A reading of the WWD cannot increase our knowledge of the alternatives of the Quanton at all. From this it follows that \( Q \) must contribute to the distinguishability of the ways, once the WWD has been turned on.

In order to quantify the contribution of \( Q \) to the distinguishability, let us measure some property \( f \) of the WWD. The mean value of that property is
\[
\langle f \rangle = f(d_+) P_D(d_+) + f(d_-) P_D(d_-).
\]
The probabilities \( P_D(d_\pm) \) for each outcome in WWD can be calculated by summing the two contributions leading to each possible outcome \( d_\pm \). From Eqs. (18) we obtain
\[
P_D(d_+) = w_+(1 - \epsilon) + w_- \epsilon,
P_D(d_-) = w_+ \epsilon + w_-(1 - \epsilon).
\]
After an outcome \( d_\pm \) in WWD is obtained, we can bet on the Quanton’s alternative that contributes most to the probability \( P_D(d_\pm) \). As in (22), after many repetitions this betting procedure yields the likelihood for guessing right:
\[
L = \text{Max}\{w_+(1 - \epsilon), w_- \epsilon\} + \text{Max}\{w_+ \epsilon, w_-(1 - \epsilon)\}.
\]
Comparing Eqs. (20) and (21), and using the identity
\[
\text{Max}\{x, y\} = \frac{1}{2}(x + y) + \frac{1}{2}|x - y|, \hspace{1cm} \forall x, y,
\]
we obtain the result
\[
D = \text{Max}\{P, Q\}.
\]
Summarizing, we can say that there are two sources of which-way information in the communication system. One is the a-priori distinguishability $\mathcal{P}$ inherent in the preparation of the Quanton system previous to the interaction with the WWD. The other is the distinguishability $Q$ provided by the WWD. As stated in Eq. (28), the total distinguishability $D$ is given by the greater of the two. The situation turns out to be far more complicated when we quantize the WWD, as will be shown in the following section.

IV. THE “QUALITY” OF A QUANTUM WWD

Consider that the Quanton interferometer is equipped with a quantum WWD, in the fashion described in Section I. In order to isolate the contribution to the distinguishability arising solely from the quantum properties of the detector, we define the “quality” of the quantum WWD to be

$$Q \equiv \frac{1}{2} \text{tr}_D \{ |\rho_D^+ - \rho_D^-| \}. \quad (29)$$

Thus, $Q$ coincides with $D$ in the case of symmetric interferometers ($\mathcal{P} = 0, w_{\pm} = \frac{1}{2}$). On the other hand, in contrast to (10), $Q$ does not involve the a priori probabilities of the ways represented by $w_{\pm}$, so both quantities may differ substantially in the case of asymmetric interferometers. Note that, as in the previous section, $Q$ is the distance between two conditional probabilities, $\rho_D^{(+)}$ and $\rho_D^{(-)}$, in the trace-class norm, and thus it is a quantitative measure of the detector’s intrinsic ability to distinguish between the ways. The detector cannot distinguish between the ways at all if $Q = 0$ and, conversely, full which-way information can be extracted by the detector when $Q = 1$. The states $\rho_D^{(+)}$ can be prepared by means of measuring the actual way taken by the atom. Thus, the value of $Q$ can be experimentally checked along the lines described in [1] for measuring $D$.

To establish a relation between (10) and (29), we first consider the detector to be prepared in a pure state, so that the equality holds in (1). Then (29) is a distance between projectors, which can be easily calculated to yield

$$Q^2 + |C|^2 = 1. \quad (30)$$

Comparing (30) with (1) and using (12) we obtain

$$Q^2 = \frac{D^2 - P^2}{1 - P^2} \leq D^2 \leq 1. \quad (31)$$

Thus, for pure state preparation, the different distinguishability measures satisfy $D \geq Q$, $D \geq \mathcal{P}$.

Consider next the general case in which we allow the detector to be in a mixed state. In analogy to (1), the three quantities $Q$, $\mathcal{P}$ and $\mathcal{V}$ should be related by an inequality that is an expression of duality. We can obtain this inequality in a straightforward fashion by noticing that the derivation of (1) presented in [1] also applies to our case under the replacements

$$\mathcal{D} \rightarrow Q, \quad \mathcal{V} \rightarrow |C|, \quad (32)$$

which transform (1) into

$$Q^2 + |C|^2 \leq 1. \quad (33)$$

Then inserting (12) and (15) into (33) gives

$$(1 - P^2) Q^2 + P^2 + V^2 \leq 1. \quad (34)$$

This equation, which is a quantitative statement about duality, constitutes the central result of this paper.

Extreme situations characterized by perfect fringe visibility or perfect which-way information are mutually exclusive. Thus, duality demands

$$\mathcal{V} = 1 \quad \Rightarrow \quad D = \mathcal{P} = Q = 0, \quad (35a)$$

$$\mathcal{P} = 1 \quad \Rightarrow \quad \mathcal{V} = 0, \quad (35b)$$

$$D = 1 \quad \Rightarrow \quad \mathcal{V} = 0, \quad (35c)$$

$$Q = 1 \quad \Rightarrow \quad \mathcal{V} = 0. \quad (35d)$$

It is easy to check that the extreme situations described in (35a)–(35d) devolve from Eq. (24). In particular, the condition $\mathcal{P} = 1$ exhausts the amount of which-way information that can be available about the Quanton so $Q$ has to be satisfied whatever the value of $Q$ is. Conversely, (35a), (35c), and (35d) has to be satisfied for arbitrary values of $\mathcal{P}$. This feature is contained in the structure of the left hand side of Eq. (34), which does not involve the value of $Q$ in the case that $\mathcal{P}$ becomes maximum, and vice-versa. Note, moreover, that Eq. (24) is invariant under the permutation $Q \leftrightarrow \mathcal{P}$, which is clear from its alternative form

$$V^2 \leq (1 - P^2)(1 - Q^2). \quad (36)$$

Consequently, as far as the degradation of the Quanton’s fringe visibility is concerned, both sources of which-way information stand on an equal footing in situations where $Q$ is satisfied with the equal sign. This symmetry can be appreciated in Fig. 3 where $D^2$ and $V^2$ are plotted as a function of $\mathcal{P}$ and $Q$ in the pure state preparation case.

In the case where we also allow the Quanton to be initially in a mixed state, we get the totally general inequality

$$\left( |s_Q^{(0)}|^2 - P^2 \right) Q^2 + |s_Q^{(f)}|^2 \leq |s_Q^{(0)}|^2, \quad (37)$$

where $|s_Q^{(f)}|$ is the norm of the final Quanton’s Bloch vector, $s_Q^{(f)} = \text{tr}_{QD} \{ \rho^{(f)} s_Q \}$. Which-way information is stored in the detector due to its entanglement with the Quanton. The expression (37) offers us a link relating $Q$ to the degree of purity left in the Quanton on account of this entanglement. The magnitude of the entanglement
can be measured by the norm of the Quanton’s Bloch vector, which satisfies

$$|s_Q|^2 = p^2 + v^2 = 1 + 2 \text{tr} \{\rho_Q^2 - \rho_Q\} \leq 1,$$

where $\rho_Q = \text{tr}_D \rho$. The first equality in (38) follows from Eqs. 33, 11 and 14, the second equality follows trivially from 2 and the properties of the Pauli matrices. The decrease in the norm of the Bloch vector measures the degree of deviation of the Quanton from a pure state. In fact, the bounds in 12 guarantee that the entropy-like quantity 17 always increases when the Quanton’s fringes degrade as a result of its entanglement with the detector. More explicitly, inserting 12 and 33 into 39 we find

$$\Delta G_Q \equiv G_Q - G_Q^{(0)} = \frac{1}{2} (v_Q^2 - v_Q^2).$$

The inequality given in 33 can now be recast in terms of the “linear entropy” 40 as

$$Q^2 \leq \frac{2\Delta G_Q}{v_Q^2} \leq 1.$$  (41)

The implications to be drawn from Eq. 11 are straightforward. First, we see that $Q = 0$ is obtained for $G_Q^{(0)} = G_Q$, i.e., degradation of the purity of the state of the Quanton is a necessary condition for the extraction of quantum WWI about the Quanton alternatives ($Q \neq 0$). Conversely, according to Eq. 11, maximal $Q = 1$ can only occur when $G_Q$ is maximized. In this case $v = 0$, so that $|s_Q|$ for the Quanton in Eq. 33 has been maximally degraded from $|s_Q^{(0)}|$. Moreover, we can say that $Q = 1$ indicates optimal which-way detection at the expense of the total destruction of the fringe pattern ($v = c = 0$). In intermediate situations, the amount of which-way information that can be extracted by the detector is bounded by the degree of purity lost by the Quanton.

V. THE SYMMETRIC QUANTON-DETECTON SYSTEM

We consider in this section a particular model for the WWI. A 2-state detector, or Detecton, is the simplest possible quantum device that can probe WWI about the Quanton. The Detecton, as the Quanton itself, is a 2-ways interferometer, likewise describable by a predictability $P_D$, and a fringe visibility $V_D$. Its initial state can also be described by a Bloch vector $s_D^{(0)}$, which can be subjected to the transformations described in Section II i.e., BS, phase shift $\phi_D$ and BM, as the original Quanton. Both interferometers are assumed to interact at their central stages, where they become entangled according to Eq. 6. Since both interferometers can play the role of System or WWI of each other, two parameters $Q_D$, $Q_Q$ have to be introduced to measure Detectons and Quanton’s "qualities" as which way detectors, respectively. Actually, we have designed the device so as to the system becomes entirely symmetric between the labels Q and D, as can be seen in Fig. 4. Hence, we will call it the Symmetric Quanton-Detecton System (SQDS) 18.

The SQDS is a good illustrating example. First, it allows for a considerable simplification of the formalism. Second, it provides further insight into the reciprocal effects of Duality between Quanton and Detecton. Third, it is an interesting system by itself. Actually, a pair of entangled two-level system (or qubits) form the fundamental brick in the construction of quantum logic gates. The SQDS is a quantum logic gate where each qubit, system and control, play the role of Quanton or WWI of the other.

For the purposes of this Section, The SQDS can be regarded as two interferometers trying to acquire WWI about each other. In fact, the SQDS is in essence a pair of two-ways interferometers coupled at their central stage by a dispersive interaction. We take the Detecton phase
shifter to depend on the ways in the form
\[
U_{PS}^\pm = \exp \left[ \frac{i}{2} (\phi_D \pm \Phi) \sigma_{Dz} \right], \\
= \frac{1}{2} (1 + \sigma_{Dz}) e^{i\phi_D/2} e^{\pm i\Phi/2} \\
+ \frac{1}{2} (1 - \sigma_{Dz}) e^{-i\phi_D/2} e^{\mp i\Phi/2}.
\] (42)

The phase shifts \((\phi_Q, \phi_D, \Phi)\) represent three arbitrary parameters externally controlling the SQDS. However, due to the simplicity of the system, entanglement is controlled in the SQDS by a single parameter, the entangling phase \(\Phi\). For \(\Phi = 0 \mod \pi\) both systems are disentangled, \(\rho = \rho_Q \otimes \rho_D\). In fact, Eq. (42) can be regarded as a quantization of the phase shifter, since the direction of rotation of the angle \(\Phi\) performed by one subsystem depends on the way chosen by the other subsystem. This dispersive interaction provides the conditional dynamics that lies at the heart of any quantum logic gate.

In order to calculate \(Q_D\), we first take the initial state of the Detecton as
\[
\rho^{(0)}_D = \frac{1}{2} \left( 1 + s^{(0)}_D \cdot \sigma_D \right), \quad s^{(0)}_D = (s^{(0)}_{Dx}, 0, s^{(0)}_{Dz}),
\] (43)
where \(s^{(0)}_{Dx} = \rho_D, V^0_D = |s^{(0)}_{Dz}| \leq 0 \leq |s_D| \leq 1\). For the sake of simplicity, we have taken \(s^{(0)}_{Dy}\) to vanish. As a consequence of the \(Q \leftrightarrow D\) symmetry, the BS and BM for the Detecton have the same form as that of their Quanton counterparts given in (4). Thus, the action of the BS on \(\rho^{(0)}_D\) generates the state
\[
\rho^{BS}_D = \frac{1}{2} \left( 1 + s^{(0)}_D \cdot \sigma_D \right).
\] (44)

Now, inserting Eq. (42) and (44) into (1), the Detecton evolution can be calculated as
\[
\rho^\pm_D = \frac{1}{2} \left( 1 + s^\pm_D \cdot \sigma_D \right),
\] (45)
with
\[
s^\pm_D = \left( s^{(0)}_{Dz} \cos \varphi \pm, s^{(0)}_{Dz} \sin \varphi, \mp s^{(0)}_{Dx} \right),
\] (46)
where we have defined the auxiliary variable \(\varphi = \Phi \pm \Phi\). With the help of (45), and taking into account the unitary character of (12), we write the Quality in (2) as
\[
Q = \frac{1}{2} \left| t^{(0)}_{Dz} \left( s^+_D - s^+_D \sigma_D \right) \right| \\
= \frac{1}{2} \left| s^+_D - s^+_D \right|.
\] (47)

Inserting (46) into (47), we arrive at the result
\[
Q_D = V^0_D |\sin \Phi|.
\] (48)
We see that in order to get maximum quality of the WWD two conditions are required. First, we need maximum entanglement (\(\Phi = \frac{\pi}{2} \mod (\pi)\)). Second, maximum initial visibility of the Detecton interferometer \(V^0_D = 1\). As will be shown later, the Detecton acquires WWI about the Quanton by degrading its visibility. Thus, maximum storage of WWI requires maximum initial visibility \(V^0_D\). Thus, Detectons in a mixed state cannot act as perfect-quality which-way detectors. On the other hand, as \(P^2_D + V^2_D \leq 1\), any amount of predictability in the Detecton limits its quality. Another interesting feature in (45) is that \(Q_D\) does not depend on the predictability nor on the initial state of the Quanton. Full WWI can be stored in the state of the Detecton for any qubit state sent through the Quanton interferometer. This feature makes the SQDS an interesting candidate for a quantum logic gate as will be shown in a forthcoming publication.

In order to compute Englert’s distinguishability \(D_Q\), we define first the operator
\[
\Delta \equiv \omega_+ \rho_D^+ - \omega_- \rho_D^- \\
= \sqrt{P_Q^2 P_D^2 + V_D^2} \left( \sin^2 \phi + P_Q^2 \cos^2 \phi \right).
\] (51)
Eq. (51) is very illustrative. First, note that for \(P_Q = 0\), \(D_Q = R_Q = Q_D\). In this case, there is only one source for Quanton’s WWI, so the distinguishability is directly given by the Quality of the WWD. As we show in this paper, the introduction of a non-vanishing predictability \(P_Q\) turns out the analysis of distinguishability much more involved, as can be seen by comparing Eq. (48) to (10). Second, it can be seen that Eq. (51) assures the inequality \(D_Q \geq P_Q\) given in (10). Third, for a decoupled systems (i.e., \(\Phi = 0\)), Eq. (51) reduces to \(R_Q = P_Q |s^+_D| \leq P_Q\). Therefore we recover in this case the result \(D_Q = P_Q\): there is no WWD available to increase distinguishability above the a-priori WWI. Consider now the case of maximal coupling (\(\Phi = \frac{\pi}{2} \mod (\pi)\)). In this case \(R_Q = \sqrt{P_Q^2 P_D^2 + V_D^2}\). Thus, \(V_D = 1\) is required in order to increase \(D_Q\) from \(P_Q\) to 1. Thus, as commented before after calculation of \(Q_D\), the existence of a non-vanishing \(P_D\) acts as a limiting factor for WWI storage capability. Fourth, notice that Eq. (51) can be rewritten as
\[
R_Q^2 = P_Q^2 |s^+_D|^2 + Q_D^2 (1 - P_Q^2).
\] (52)
Therefore, for \(|s^+_D| = 1\) we obtain \(R_Q \geq P_Q\) and \(D_Q = R_Q\) in this case. Thus, \(R_Q\) gives the total distinguishability for initial pure state preparation of the Detecton. Also, since \(Q_D \leq |s^+_D|\), Eq. (52) implies \(R_Q \leq |s^+_D|\), i.e., \(R_Q\) is bounded by the initial purity
of the Detecton. On the other hand, it is easy to show from Eq. (52) that $R_Q \geq Q_D$. Thus, in the SQDS, the following inequalities are satisfied with generality

$$D_Q \geq R_Q \geq Q_D, \quad D_Q \geq P_Q,$$

setting a hierarchy for the different distinguishability measures. This leads trivially to the following hierarchy of duality inequalities

$$Q_D^2 + V_Q^2 \leq R_Q^2 + V_Q^2 \leq D_Q^2 + V_Q^2 \leq 1,$$

$$P_Q^2 + V_Q^2 \leq 1,$$

where we have used Eq. (11) in (53) and Eqs. (14) and (58) as

Now, we are in conditions to show an interesting feature. In the SQDS, the inequality (53) is more stringent than the Englerts inequality (1). In order to show this, we define the auxiliary quantity

$$f_Q \equiv \frac{(1 - P_Q^2)(1 - Q_D^2)}{(1 - D_Q^2)}.$$ (55)

For $|s_D^o| = 1$, $f_Q = 1$ and both inequalities are equivalent. Also, $f_Q = 1$ follows trivially from $P_Q = 0$, or $Q_D = 0$, or $Q_Q = 1$. In the general case, we show in the appendix that $f_Q \leq 1$, showing the validity of our stringency statement. A typical behavior of $f_Q$ is given in Fig. 5 where Eq. (55) is plotted for $|s_Q^o| = 0.882$ versus different values of $P_Q$ and $Q_D \leq |s_Q^o|$. As can be seen in this plot, there are two behaviors, corresponding to two different regions of the parameter space where $R_Q > P_Q$ (right) or $R_Q < P_Q$ (left).

In order to see how all these parameters are related to fringe degradation, we compute now $V_Q$ as given in (12). Inserting (55) into (12), we obtain

$$C = \cos \Phi + i s_D^o \sin \Phi.$$ (56)

Therefore

$$V_Q = V_Q^o \sqrt{\cos^2 \Phi + P_Q^2 \sin^2 \Phi}.$$ (57)

Now, we are in condition to illustrate several inequalities involving duality that appeared in the previous section. For instance, for pure state preparation we compute Eq. (30) as

$$Q_D^2 + |C|^2 = (1 - P_D^2) \sin^2 \Phi + \cos^2 \Phi + P_D^2 \sin^2 \Phi = \cos^2 \Phi + \sin^2 \Phi = 1.$$ (58)

As can be seen from the above equation, the introduction of $P_D$ decreases the Quality of the WWD and increases the contrast factor by the same amount, so the sum remains constant. For the general case we compute

$$Q_D^2 + |C|^2 = |s_D^o|^2 \sin^2 \Phi + \cos^2 \Phi \leq 1,$$

so we see that mixed state preparation ($|s_D^o| < 1$) forces (55) to be satisfied as an inequality.

Finally, we analyze the increase of linear entropy in the SQDS. Inserting (57) into (10) we have

$$\Delta G_Q = \frac{1}{2} V_Q^o (1 - P_D^2) \sin^2 \Phi.$$ (60)

Using Eq. (45) and particularizing Eq. (33) for the Detecton, we rewrite Eq. (60) as

$$\frac{2 \Delta G_Q}{V_Q^o} = 1 - |s_D^o|^2 + Q_D^2,$$

satisfying the general inequality given in (11). As commented below Eq. (11), degradation of the purity of the Quanton state is a necessary condition for the extraction of quantum WWI ($Q_D \neq 0$). However, it is not a sufficient condition. This can be seen from Eq. (33) where $\Delta G_Q$ can be nonvanishing for $Q_D = 0$, provided the Detecton is prepared in a mixed state ($|s_D^o| < 1$).

In the case of pure state preparation of the Detecton, Eq. (61) simplifies to the expression

$$\frac{2 \Delta G_Q}{V_Q^o} = Q_D^2.$$ (62)

Thus, in this case, the Quality of the detector directly gives the increase in linear entropy of the Quanton, once normalized to its initial visibility. The implications of (62) are straightforward. For $Q_D = 0$ there is no degradation of the purity of the Quanton. For $Q_D = 1$, the linear entropy increase to its maximum $\Delta G_Q = 1/2 V_Q^o$. Maximum WWI is extracted at the expense of the total degradation of the fringe pattern of the Quanton, i.e., $V_Q = 0$, as demanded by duality (see Eq. (55)). Moreover, the Detecton interferometer also degrades its visibility pattern. In order to see this, we calculate $V_D$ just by applying the Q↔D symmetry to the labels of Eq. (12) to obtain

$$V_D = V_D^o \sqrt{\cos^2 \Phi + P_Q^2 \sin^2 \Phi}.$$ (63)
Combining Eq. (63) and (67), we obtain the result

\[
(1 - P^2_Q) \frac{\Delta V^2_Q}{V^2_Q} = (1 - P^2_D) \frac{\Delta V^2_D}{V^2_D},
\]

where \( \Delta V^2 \equiv V^2 - V_0^2 \). Equation (64) is valid for arbitrary initial preparation of Quanton and Detecton. In the case of initial pure state preparation for both sub-systems, Eq. (64) reduces to the simple form

\[
\Delta V^2_Q = \Delta V^2_D.
\]

The above equation provides a clear illustration on the reciprocal effects of Duality in the SQDS. When WWI is extracted on one system, both Quanton and Detecton degrade their interference pattern by the same amount. In the more general case of mixed state preparation, the balance in the reciprocal degradation is weighted by the factors appearing in Eq. (64). For instance, consider the balance in the reciprocal degradation is weighted by the factors appearing in Eq. (64). For instance, consider the case in which we prepare the Quanton as the total unpolarized state \( |\phi_D\rangle = P_Q = V_Q^0 = 0 \), and Detecton in a pure state with \( V_D^0 = 1 \). Since in the SQDS, \( Q_D \) independent on the state of the Quanton, full WWI \( (Q_D = 1) \) can be extracted by the Detecton at the expense of totally degrading its fringe visibility from \( V_D^0 = 1 \) to \( V_D = 0 \). When both systems are prepared initially as a classical mixture, then \( Q_D = Q_Q = 0 \), and no WWI can be extracted no matter the value of the entanglement between both interferometers. The reciprocity between fringe degradation in Quanton and Detecton Systems can be explained by the reciprocity between their "quality" measures, since both of them are connected by duality. In fact, applying the Q↔D symmetry to Eq. (68) we have

\[
\frac{Q_D}{V_D^0} = \frac{Q_Q}{V_Q^0}.
\]

According to the above equation, any potential acquisition of WWI is mutual. In other words, if the Detecton may acquire WWI about the Quanton, the Quanton may acquire WWI about the Detecton. This explains the simultaneous degradation of the fringe pattern of both systems as one of them extracts WWI about the other.

The SQDS highlights the role of the initial visibility of both interferometers in duality exchanges of WWI. As a matter of fact, combining Eqs. (63) and (67), setting \( P_D = P_Q = 0 \), and applying the Q↔D symmetry between labels, we obtain

\[
\frac{D_Q^2}{V_Q^0} + \frac{V^2_Q}{V_Q^0} = \frac{D_D^2}{V_D^0} + \frac{V^2_D}{V_D^0} = \cos^2 \Phi + \sin^2 \Phi = 1.
\]

The above equation can be written as

\[
D_Q^2 + \frac{V^2}{V_Q^0} \frac{V^2}{V_D^0} = V_D^0.
\]

The duality implications of Eq. (68) are straightforward. Since \( V^2 \leq V^0 \leq 1 \), \( D_Q = 1 \) implies both \( V_D^0 = 1 \) and \( V_Q^0 = 0 \) as demanded by duality. Maximum Detecton initial visibility is required to totally degrade the Quanton's interference pattern. On the other hand, \( V_Q = 1 \) in Eq. (68) forces \( D_Q = 0 \). Thus, in contrast to the inequality given in Eq. (1), Eq. (68) is an equality. It is an equality involving duality valid even for mixed state preparation of Quanton and Detecton.

VI. SUMMARY

We have introduced in this paper a quality measure \( Q \), which characterizes how good a quantum detector is by quantifying the maximum which-way information that it can acquire when placed in a two-way interferometer. In this way, we are able to separate the contribution to the distinguishability arising from the quantum properties of the detector, given by \( Q \), from that stemming from the \( a \ priori \) which-way knowledge involved in the preparation of the interferometer. The latter is given by the predictability \( P \) characterizing how asymmetrically the two-way interferometer is constructed.

In the spirit of (1), we have derived an inequality relating \( Q \) and \( P \) to the value of the fringe visibility \( V \) displayed at the output port of the interferometer. This inequality allows us to quantify the degradation of the fringe visibility \( V \) involved in the availability of the two kinds of which-way information. For instance, it shows that maximum which-way information available can be stored in the WWD, even for strongly asymmetric interferometers. In addition we have shown that, in the case where both systems are prepared in a pure state, both kinds of which-way information represented by \( Q \) and \( P \) stand on an equal footing concerning loss of coherence.

In the case of a classical WWD, \( Q \) can be regarded as the quality of a noisy communication channel which is independent of the state of the sender. Here, \( Q \) and \( P \) are clearly separable as distinguishabilities stemming from uncoupled sources of information; the total distinguishability is just the maximum of the two. The situation turns out to be much more complicated for a quantum WWD. Due to the entanglement between Quanton and Detecton, the value of the quantum Quality depends in general on the state of both systems.

We have applied our formalism to a quantum logic gate: the Symmetric Quantum-Detecton system (SQDS). This system can be regarded as two coupled interferometers trying to acquire WWI about each other. In the SQDS both interferometers are coupled by a dispersive interaction. There is no energy transfer between Quanton and Detecton, just an information transfer established via quantum correlations. We have shown that in the SQDS the inequality involving duality in terms of the distinguishabilities \( Q \) and \( P \) is more stringent than the Englert's inequality given in terms of the total distinguishability \( D \). Also, our formalism has shown useful in order to characterize the reciprocal effects induced by duality on both systems. Finally, we have shown an equality.
involving duality for the SQDS. This equality highlights the role of the initial visibility of both systems as limiting factors in the mutual transfer of WWI between both qubits.

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APPENDIX

In this appendix, we show that the quantity

\[ f_Q = \frac{(1 - P_Q^2)(1 - Q_D^2)}{1 - D_Q^2}, \tag{A.1} \]

satisfies \( 0 \leq f_Q \leq 1 \). Since \( D_Q = \max \{P_Q, R_Q\} \), we consider two cases separately. First, take \( P_Q \geq R_Q \). Here \( D_Q = P_Q \) and \( f_Q = (1 - Q_D^2) \leq 1 \). Else, for \( P_Q < R_Q \) we have

\[ f_Q = \frac{(1 - P_Q^2)(1 - Q_D^2)}{1 - P_Q^2|s_D^2|^2 - Q_D^2(1 - P_Q^2)}, \tag{A.2} \]

where we have used Eq. (52). Let us define the auxiliary quantities

\[ \xi = (1 - P_Q^2)(1 - Q_D^2) \geq 0, \]
\[ g = 1 - R_Q^2 - \xi = 2P_Q^2Q_D^2 + P_Q^2(1 - |s_D^2|^2) \geq (A.3) \]

In terms of \( g \), Eq. (A.1) can be written as

\[ f_Q = \frac{\xi}{g + \xi}. \tag{A.4} \]

Therefore, since \( g \geq 0 \) we conclude \( 0 \leq f_Q \leq 1 \).

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[19] \( D \) in (50) is the distinguishability of the ways of the Quantum interferometer. Hence the label \( D_Q \). On the other hand, the Quality \( Q_s \) \( (Q_Q, Q_D) \) of each interferometer measures their properties as detectors of WWI about the other interferometer. Thus, according to our notation, \( Q_D \) contributes to \( D_Q \).