Asteroseismology of 36 Kepler subgiants – I. Oscillation frequencies, linewidths, and amplitudes

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ABSTRACT

The presence of mixed modes makes subgiants excellent targets for asteroseismology, providing a probe for the internal structure of stars. Here we study 36 Kepler subgiants with solar-like oscillations and report their oscillation mode parameters. We performed a so-called peakbagging exercise, i.e. estimating oscillation mode frequencies, linewidths, and amplitudes with a power spectrum model, fitted in the Bayesian framework and sampled with a Markov chain Monte Carlo algorithm. The uncertainties of the mode frequencies have a median value of 0.180 μHz. We obtained seismic parameters from the peakbagging, analysed their correlation with stellar parameters, and examined against scaling relations. The behaviour of seismic parameters (e.g. Δν, Vmax, εp) is in general consistent with theoretical predictions. We presented the observational p–g diagrams, namely γ1–Δν for early subgiants and ΔΩ1–Δν for late subgiants, and demonstrate their capability to estimate stellar mass. We also found a log g dependence on the linewidths and a mass dependence on the oscillation amplitudes and the widths of oscillation excess. This sample will be valuable constraints for modelling stars and studying mode physics such as excitation and damping.

Key words: stars: low-mass – stars: solar-type – stars: oscillations.

1 INTRODUCTION

Some of the first detections of solar-like oscillations, using ground-based spectroscopy, were in subgiant stars such as η Boo (Christensen-Dalsgaard, Bedding & Kjeldsen 1995; Kjeldsen et al. 1995, 2003; Guenther & Demarque 1996; Di Mauro et al. 2003a; Guenther 2004; Carrier, Eggenberger & Bouchy 2005), β Hyi (Bedding et al. 2001, 2007; Carrier et al. 2001; Fernandes & Monteiro 2003; Di Mauro, Christensen-Dalsgaard & Paternò 2003b; Brandão et al. 2011), ν Ind (Bedding et al. 2006; Carrier et al. 2007), and μ Her (Bonanno et al. 2008; Pinheiro & Fernandes 2010; Grundahl et al. 2017). Observations by the space telescopes CoRoT and Kepler enabled high-quality photometry with high duty cycle. These include subgiants such as HD 49385 (Deheuvels & Michel 2010; Paxton et al. 2013), HD 169392A (Mathur et al. 2013), KIC 11026764 (‘Gemma’; Metcalfe et al. 2010), KIC 11395018 (‘Boogie’; Mathur et al. 2011), KIC 10920273 and KIC 10273246 (‘Scully’ and ‘Mulder’; Campante et al. 2011), the α-enhanced star KIC 7976303 (Ge et al. 2015), and the binary twin system KIC 7107778 (Li et al. 2018).

Beginning with η Boo, it was realized that subgiants show mixed modes (Christensen-Dalsgaard et al. 1995), which are now recognized as a feature in all evolved stars. Main-sequence (MS) dwarfs with solar-like oscillations host a convective envelope, which excites and propagates pressure (p) modes. In subgiants, the so-called mixed modes that result from coupling between the pressure mode cavity and gravity (g) mode cavity carry information from the core and have observable amplitudes on the surface.

The mixed modes have long been realized to have strong diagnostic potential because the frequencies of g modes, which we denote by γ, evolve quite rapidly (Christensen-Dalsgaard et al. 1995). Bedding (2014) suggested a new asteroseismic diagram, the p–g diagram, which plots γ versus the large separation of p modes Δν. By comparing observed values of γ and Δν with stellar models on this diagram, a stellar mass can be estimated. Benomar et al. (2012) showed that the coupling strength (see Section 3) is a strong indicator of evolutionary stage in subgiants. These suggestions of the diagnostic power of mixed modes were confirmed with...
modelling. When modelling Gemma (KIC 11026764), Metcalfe et al. (2010) showed that the asteroseismic age could be constrained to a precision of 15 per cent, which was set by the choice of input physics. Li et al. (2019) modelled υ Her and demonstrated that the asteroseismic age did not significantly change as a function of the mixing-length parameter, or the initial helium abundance. In addition to global parameters, the mixed modes also shed light on the interior physics of stars, for example, constraining the buoyancy frequency profile (Li et al. 2019), internal angular momentum transport (Deheuvels et al. 2014; Eggenberger et al. 2019), and convective core overshooting (Deheuvels et al. 2016).

A complete analysis of oscillation modes has been made for Kepler observations of 35 planet-host stars (Davies et al. 2016) and 66 MS stars (Lund et al. 2017), the so-called LEGACY sample. However, similar analyses for subgiants have only been done on a subset of Kepler data (e.g. Appourchaux et al. 2012). Therefore, it is valuable to gather and analyse subgiants with the full set of Kepler observations, given the great promise of mixed modes. In this paper, we fit the oscillation modes and analyse the fitted frequencies, linewidths, and amplitudes from the fit. In a companion paper (Li et al. submitted, hereafter Paper II), we perform a detailed modelling to these stars with the constraints of frequencies obtained here.

2 OBSERVATIONS

We considered Kepler targets observed in a short-cadence mode, which samples at an integration time Δt = 58.89 s, or frequency f_s = 1/Δt = 16 980.8 µHz. There are about 5000 stars observed in this mode, and we detected about 50 subgiants. In this context, we define subgiants as stars showing oscillations of mixed modes, recognized as mode bumping (see Section 3.1 for a detailed discussion), which starts around the time of hydrogen exhaustion in the core. However, we required that the density of the mixed modes should be low enough, such that ΔΠ > Δν/ν², where ΔΠ is the period spacing of dipolar g modes. This occurs around the subgiant phase. Only subgiants with high signal-to-noise ratios (S/Ns) were selected to avoid any ambiguity of the correct assignment of the spherical degree l of each mode (see Section 3.1). Further, we required the observation duration, denoted by t_obs, to be at least two months. This resulted in a total of 36 subgiants that we analyse here.

The atmospheric parameters (T_eff and [Fe/H]) were adopted from the KIC DR25 release (Mathur et al. 2017). Table 1 lists the 36 stars in our sample along with t_obs, nicknames used in previous papers and references to any previous studies. Fig. 1 shows the targets in the ∆ν–T_eff plane. They extend from the MS turn-off phase to the bottom of red giant branch (RGB). The MS stars in the Kepler LEGACY sample (Lund et al. 2017) are also shown.

We used light curves measured from simple aperture photometry (SAP) by the Kepler Science Center.¹ We corrected instrumental effects with the KASOC FILTER.² For a full description of this reduction procedure, see García et al. (2011) and Handberg & Lund (2014). Briefly, it constructs moving-median high-pass filters, one with t_long = 1/2 d, and another with t_short = 1/24 d. Trends longer than 1/t_long ∼ 23 µHz are removed, and any sharp feature that could produce drastically different signals for the two filters are eliminated. The light curves were converted into relative flux in ppm (parts per million) and we calculated power spectra by a Lomb–Scargle Periodogram algorithm (Lomb 1976; Scargle 1982).

¹https://archive.stsci.edu/kepler/
²https://github.com/tasoc/corrections.

Figure 1. Asteroseismic H-R diagram showing ∆ν versus T_eff for the 36 subgiants shown as black circles colour-coded by metallicity. For comparison, the LEGACY sample (Lund et al. 2017) is together shown in triangles labelled as dwarfs. The Sun is marked by the usual symbol. The theoretical evolutionary tracks with solar metallicity (Stello et al. 2013) are shown before (dashed lines) and after (solid lines) the exhaustion of central hydrogen, with each track labelled with mass in solar units.

Figure 2. Power spectrum density of Gemma (KIC 11026764). The original (grey), smoothed (black), and fitted power spectra (green) are shown. The peaks around 4500 µHz are artifacts (Gilliland et al. 2010). The inset shows the normalized spectral window.

equivalent to a sine-wave fitting, as implemented in ASTROPY (Astropy Collaboration 2013, 2018). The power density spectra were constructed by multiplying the power by the total observing time (Kjeldsen & Bedding 1995). Fig. 2 uses Gemma (KIC 11026764) as an example to illustrate the power density spectrum and the spectral window. The former is comprised of a background and a group...
of peaks with a Gaussian-like envelope centred on $v_{\text{max}}$. Although there are small gaps in the original data, they have a negligible influence on the shape of oscillation modes. This can be seen from the fact that the spectral window has sidelobes with very low power.

## 3 PARAMETER ESTIMATION

### 3.1 Mode identification

Each oscillation mode is associated with three quantum numbers $(n, l, m)$. The first step in mode identification is to assign an $l$-degree to each peak. In the asymptotic regime ($n \gg l$) of $p$ modes (Tassoul 1980; Gough 1986), this is straightforward because modes follow regular patterns. The frequencies are approximately equally spaced by $\Delta v$:

$$v_p(n_p, l) \approx \Delta v \left(n_p + \frac{l}{2} + \epsilon_p\right) - \delta v_{\text{bg}},$$

where $\Delta v$ is the separation between adjacent radial modes, $\epsilon_p$ is the offset of the pattern, and the small separation $\delta v_{\text{bg}}$ defines the frequency differences between $l = 0$ and $l$.

The $g$ modes, which we do not observe directly, are approximately equally spaced in period in the asymptotic regime:

$$\Pi_g(n_g, l) = v_{\text{bg}}^{-1}(n_g, l) \approx \Delta \Pi_l\left(n_g + \epsilon_g\right),$$

where $\Delta \Pi_l$ and $\epsilon_g$ are the period spacing and offset for $g$ modes, respectively. Shibahashi (1979) derived an asymptotic relation for mixed modes. Specifically, for dipolar modes ($l = 1$), the mixed-mode frequencies $v_{\text{bg}}$ satisfy

$$\tan \theta_p = q \tan \theta_g,$$

where

$$\theta_p = \frac{\pi}{\Delta v} (v_m - v_p),$$

and

$$\theta_g = \frac{\pi}{\Delta \Pi_l} \left(\frac{1}{v_m} - \frac{1}{v_g}\right),$$

and $q$ is a coupling factor determined by the width of the evanescent zone (Mosser et al. 2015). Less-evolved subgiants have large period spacings, with one $g$ mode often coupling with several $p$ modes. The coupled modes can be bumped very far away from their regular $p$-mode spacings. By replicating echelle diagrams

### References.

(a) Appourchaux et al. (2012); (b) Deheuvels et al. (2014); (c) Tian et al. (2015); (d) Appourchaux et al. (2014); (e) Davies et al. (2016); (f) Campante et al. (2011); (g) Mathur et al. (2011).
3.2 Power spectrum model

We now describe the model used to fit the power spectrum. First, we consider the background caused by granulation, modelled as a sum of Lorentzian profiles:

$$B(v) = \sum_{i=1}^{n} \frac{2\sqrt{2}}{\pi} \frac{a_{i}^2}{1 + (v/b_{i})^2}. $$

The number of power profiles was set to be $n = 1$ or $2$ (Harvey 1985; Kallinger et al. 2014), subject to the goodness of fit, which is sufficient to account for the background near $v_{\text{max}}$. Secondly, we describe the signal of oscillations. For a mode with quantum number $(n, l, m)$, we adopted a power spectrum model

$$L_{nlm}(v) = \frac{\mathcal{E}_{lm}(i^*)2A_{nl}^2/(\pi\Gamma_{nl})}{1 + 4(v - \nu_{nl} + m\nu_{n,l})^2/\Gamma_{nl}^2}, $$

to account for oscillations, where the frequency $\nu_{nl}$, linewidth $\Gamma_{nl}$, amplitude $A_{nl}$, and rotational splitting $\nu_{n,l}$ vary with $n$ and $l$ (Anderson, Duvall & Jefferies 1990). The relative height within a multiplet depends on the inclination angle $i^*$ of the star and is modulated by a visibility function:

$$\mathcal{E}_{lm}(i^*) = \frac{(l - |m|)!}{(l + |m|)!} \left[ P_{l}^{(m)}(\cos i^*) \right]^2, $$

where $P_{l}^{m}$ are Legendre functions (Gizon & Solanki 2003). The final model to fit the whole power spectrum was

$$M(v) = \eta^2(v) \left[ \sum_{n=\text{max}}^{\text{min}} \sum_{l=0}^{3} \sum_{m=-l}^{l} L_{nlm}(v) + B(v) \right] + W. $$

We used a frequency-independent constant $W$ to account for white/photon noise. The apodization of the signal due to the integration time $\Delta t = 58.89$ s was

$$\eta^2(v) = \frac{\sin^2(\pi v \Delta t)}{(\pi v \Delta t)^2}. $$

The free parameters to fit the power spectrum were $\theta = \{ \nu_{nl}, A_{nl}, \Gamma_{nl}, \nu_{n,l}, i^*, \alpha_{i}, b_{i}, c_{i}, W \}$.

3.3 Fitting method

We estimated the model parameters in the Bayesian framework (Handberg & Campante 2011). Given data $D$, model $M$, and prior information $I$, the parameters $\theta$ were estimated via posterior probability using Bayes’ theorem:

$$p(\theta | D, M, I) = \frac{p(D | \theta, M, I)p(\theta | M, I)}{p(D | M, I)}. $$

The posterior probability can be seen as a product of the prior and the likelihood function, divided by the Bayesian evidence, which is the marginalization of that product over all parameter space.

We utilized three kinds of prior functions. The first was a uniform prior, which sets the prior probability as a constant over a parameter range:

$$p(\theta | M, I) = \begin{cases} \frac{1}{\theta_{\text{max}} - \theta_{\text{min}}}, & \theta_{\text{min}} < \theta < \theta_{\text{max}} \\ 0, & \text{otherwise} \end{cases}. $$

We used uniform priors for most parameters. For example, the priors on the frequencies, $\nu_{nl}$, were set to be $3\mu$Hz ranges centred around their initial guessed values. The inclination angle was assigned a uniform prior across $[-\pi/2, \pi]$ and folded to $[0, \pi/2]$ after sampling. We also tested an isotropic prior for the inclination, but found no
significant bias on mode parameters. The second is a modified Jeffreys prior, which was set for the linewidths and amplitudes:

$$p(\theta | M, I) = \begin{cases} \frac{1}{\theta U_{\text{max}}} \log((\theta U_{\text{max}})^{1/2}) \theta_{\text{min}}, & 0 < \theta < \theta_{\text{max}} \\ 0, & \text{otherwise} \end{cases} , \quad (13)$$

We tested the impact of adopting uniform priors for these two particular parameters. The amplitudes do not present obvious bias while the linewidths are typically overestimated using the uniform priors. Despite the present bias, more than 95 per cent modes in our sample agree within 1σ. For the splitting frequency, $v_{s,n}$, we used the third prior, a flat prior with a half-Gaussian:

$$p(\theta | M, I) = \begin{cases} 0, & \theta < S \\ C, & S \leq \theta < U \\ C \cdot \exp\left(-\frac{(\theta - U)^2}{2\sigma^2}\right), & \theta \geq U \end{cases} , \quad (14)$$

where $C = (U - S + \sqrt{\pi/2})^{-1}$. We applied this prior to $v_{s,n}$ with $S = 0$, $U = 1$, and $\sigma = 0.5 \mu$Hz (Deheuvels et al. 2014).

Gaussian noise in the time domain translates into a $\chi^2$ distribution with 2 degrees of freedom in the frequency domain (in power). Assuming all frequency bins, indexed by $i$ in the power spectrum, are statistically independent, the logarithm of the likelihood function is

$$\ln p(D | \theta, M, I) = -\sum_i \left[ \ln M_i(\theta) + \frac{D_i}{M_i(\theta)} \right] . \quad (15)$$

We used an H1 (odds ratio) approach to test whether a frequency range contains a mode (Appourchaux et al. 2012; Corsaro & De Ridder 2014; Davies et al. 2016). H1 is the hypothesis that a range of a power spectrum contains the mode, while H0 is the null hypothesis. In short, we computed the Bayes factor $\ln K = \ln p(D | M_1, I) - \ln p(D | M_0, I)$ and assessed it based on the Kass & Raftery (1995) scale:

$$\ln K = \begin{cases} < 0 & \text{favours H0} \\ 0–1 & \text{not worth more than a bare mention} \\ 1–3 & \text{positive} \\ 3–5 & \text{strong} \\ > 5 & \text{very strong} \end{cases} . \quad (16)$$

3.4 Fitting details

Based on the methods mentioned in the previous sections, we fitted the data as follows:

(i) The power spectrum was initially fitted with equation (9) but the sum of Lorentzians was replaced by a single Gaussian profile centred around $v_{\text{max}}$ (see Fig. 2). The background spectrum was determined from this fit. By dividing the signal by the background, we obtained the S/N spectrum, which fluctuates around 1.

(ii) We selected seven modes with the highest amplitudes to fit simultaneously and to determine the inclination angle by maximizing the posterior probability.

(iii) The power spectrum was divided into segments and modes were fitted in each segment separately with a fixed inclination determined in step (ii). The sizes of the segments were based on the proximity of consecutive mode frequencies. Any two modes closer than 10 μHz were grouped into the same segment. There was a minimum of one and a maximum of five modes per segment.

(iv) The Bayes factor was calculated for each mode to evaluate its significance, by comparing the Bayesian evidence with or without that mode in the model. Fig. 4 shows the fit of the three modes in a segment. All of them have very strong detections.

We developed PYTHON software called SOLARLIKEPEAKBAGGING,3 providing a wrapper for the Markov chain Monte Carlo (MCMC) sampling algorithm implemented in EMCEE (Goodman & Weare 2010; Foreman-Mackey et al. 2013), also featuring customizable Bayesian statistics, models, and other fitting algorithms. For the above fits, the sampler was chosen either to be an affine-invariant ensemble sampler (step ii) or a parallel-tempering sampler with 20 temperatures (step iii), depending on whether the Bayes factor was calculated. We initialized 500 walkers with values adopted from a least-squares fit, then burned-in for 1000 steps and iterated for 2000 steps. After each run, we checked convergence and the goodness of sampling by several metrics, including auto-correlation time, acceptance fraction, the evolution of model parameters, and the shapes of the marginal probability distributions.

3.5 Fitting results

We present mode frequencies, amplitudes, and linewidths for $m = 0$ modes in Table 2. The rotational splittings will be comprehensively studied in a future paper. Fig. 5 shows the power spectrum of Gemma.

Figure 4. Significance tests for three modes of Gemma (KIC 11026764). Each panel presents the fits of the mode marked by the symbol, under H1 and H0 hypotheses separately. All three modes show very strong detections according to the Kass & Raftery (1995) scale.

The estimation of each parameter was obtained by marginalizing the posterior probability and calculating median and 68 per cent credible limit values.

3https://github.com/parallelpro/SolarlikePeakbagging.
Table 2. Mode parameters obtained from the peakbagging.

| KIC   | l | $\nu_0$ (μHz) | $\Gamma_0$ (μHz) | $A_{\nu}$ (ppm) | $\ln K$   |
|-------|---|--------------|-----------------|-----------------|-----------|
| 2991448 | 0 | 889.34 ± 0.312 | 0.79 ± 0.96 | 5.72 ± 1.53 | 2.16 ± 0.75 |
| 2991448 | 0 | 949.46 ± 1.111 | 8.02 ± 2.37 | 12.95 ± 1.61 | 3.61 ± 0.98 |
| 2991448 | 0 | 1011.17 ± 0.202 | 1.15 ± 0.46 | 9.47 ± 1.23 | 20.49 ± 4.32 |
| 2991448 | 0 | 1072.78 ± 0.999 | 0.76 ± 0.24 | 14.78 ± 1.49 | 39.42 ± 2.54 |
| 2991448 | 0 | 1134.18 ± 0.113 | 0.89 ± 0.28 | 14.21 ± 1.38 | 47.86 ± 2.81 |
| 2991448 | 0 | 1195.89 ± 0.547 | 3.29 ± 1.72 | 13.37 ± 2.38 | 5.40 ± 2.13 |
| 2991448 | 0 | 1257.81 ± 0.969 | 3.53 ± 2.63 | 10.04 ± 2.96 | 2.68 ± 1.52 |
| 2991448 | 0 | 1319.52 ± 1.294 | 3.00 ± 3.56 | 5.09 ± 3.29 | 0.18 ± 1.20 |
| 2991448 | 1 | 857.60 ± 0.384 | 0.97 ± 0.79 | 5.88 ± 1.62 | -0.21 ± 0.69 |
| 2991448 | 1 | 915.60 ± 0.153 | 0.81 ± 0.32 | 9.62 ± 1.27 | 19.19 ± 1.16 |

Notes. These are $m = 0$ modes. Only the first 10 rows are shown. The full table is available online.

Figure 5. Power spectrum of Gemma (KIC 11026764). The fitted power spectrum (black) is overlaid on the original power spectrum (light grey).

(KIC 11026764) with fitted mode frequencies overlaid. We present the plots of the other stars in Appendix A.

In Fig. 6, we show the histograms of uncertainties. The median value of frequency uncertainties is 0.180 μHz. As expected, the uncertainty is a function of S/N. The typical uncertainty is 0.1 μHz for S/N = 3. The median uncertainties of linewidths are 31.7 per cent and amplitudes are 10.1 per cent.

As a check, we can consider the classical maximum likelihood estimator (MLE; Libbrecht 1992; Toutain & Appourchaux 1994; Ballot et al. 2008). This calculates uncertainties by inverting the Hessian matrix whose elements are the second derivatives of the likelihood function to the parameters. The Cramér–Rao bound states that an unbiased MLE reaches the lowest variance bound, so any other unbiased estimators are expected to obtain a larger variance. Libbrecht (1992) derived an analytical form of the uncertainty. We found the estimated uncertainties derived from the MCMC-based posterior distributions were typically larger than those obtained from the above analytical forms, by factors of 1.15 (frequency), 1.18 (linewidth), and 1.41 (height), respectively. Thus, the uncertainties are safe to use for modelling.

4 MODE FREQUENCIES

We fitted the radial mode frequencies with equation (1) to estimate $\Delta \nu$ and $\epsilon_\nu$. The errors were considered within the Bayesian framework; thus, they were propagated from the priors and likelihoods. The likelihood function was selected to reflect the residuals between data and the models, weighted by uncorrelated uncertainties from both dependent and independent variables, as normal distributions.
The p-mode offset $\epsilon_p$ is sensitive to the lower and upper turning points of a mode (e.g. Gough 1986). It has been demonstrated to vary with $T_{\text{eff}}$ and $\Delta \nu$ (White et al. 2011b; Hon, Stello & Yu 2018), and was used to resolve ambiguous mode identification problems found in F-type dwarfs (White et al. 2012) and to discriminate between RGB and helium-core-burning (HeB) stars (Kallinger et al. 2012). The upper turning point lies at the surface, where $T_{\text{eff}}$ and log $g$ are relevant quantities, so it may be conjectured that $\epsilon_p$ is related to both of them. In Fig. 7, we present $\epsilon_p$ as a function of $T_{\text{eff}}$. As Ong & Basu (2019) pointed out, $\epsilon_p$ depends on the method of measurement, so we recalculated the $\epsilon_p$ of the MS dwarfs from Lund et al. (2017) using our approach. As the figure shows, the general trend of $\epsilon_p$ in subgiants follows a similar pattern as for dwarfs. We also identify a dependence on log $g$: stars with similar $T_{\text{eff}}$ but smaller log $g$ have lower $\epsilon_p$. The dwarf stars, which have higher log $g$, sit above the subgiants on the $\epsilon_p - T_{\text{eff}}$ diagram. In the bottom panel of Fig. 8, we present $\epsilon_p$ as a function of $\Delta \nu$. The obvious offset between

**Figure 6.** Histograms of the uncertainties for measured mode frequencies (left-hand panel), linewidths (middle panel), and amplitudes (right-hand panel). The linewidths and amplitudes are shown in relative uncertainty.

**Figure 7.** $\epsilon_p$ versus $T_{\text{eff}}$ with colour-coded log $g$. The measurement of red giants were adopted from Huber et al. (2010). The theoretical evolutionary tracks (White et al. 2011b) are labelled with mass in solar units. The crosses mark the zero-age MS.

**Figure 8.** Top and middle panels: C–D diagrams, which show the small separation $\delta \nu_{02}$ as a function of $\Delta \nu$, with colour-coded log $g$. The solid line is a linear fit to the subgiants, and the dashed line is a similar fit to low-luminosity red giants (Bedding et al. 2010). Bottom panel: $\epsilon_p$ versus $\Delta \nu$. The measurement of dwarfs and red giants were adopted from Lund et al. (2017) and Huber et al. (2010), respectively. The theoretical evolutionary tracks (White et al. 2011b) are labelled with mass in solar units with crosses marking the zero-age MS.
observations and stellar models stems from the improper modelling of near-surface layers (Christensen-Dalsgaard, Dappen & Lebreton 1988).

We show the small separations $\delta \nu_{02}$ versus $\Delta \nu$ in Fig. 8, the so-called C–D diagram (Christensen-Dalsgaard 1984; Mazumdar 2005). The evolutionary tracks with different masses are well separated on the MS. By comparing to models, the diagram is useful for estimating mass and age when $\delta \nu_{02}$ and $\Delta \nu$ are known. We note that the $\epsilon_p - \Delta \nu$ diagram could be used for a similar purpose, provided the surface effect is well accounted for. However, the tracks become more degenerate in the subgiant phase (White et al. 2011a, b). We performed a linear fit to the subgiants:

$$\delta \nu_{02} = (0.088 \pm 0.001) \Delta \nu + (0.070 \pm 0.059) \muHz.$$  

We point out that some obvious outliers from the fitted line are present. They are affected by bumped $l = 2$ modes, which lift the average distance of a quadrupolar mode to a radial mode. Bedding et al. (2010) fitted the same relation to a sample of red giants, which have a steeper slope (as shown by the dashed line in Fig. 8). This difference is also visible from the calculation of stellar models. Similarly, we fitted $\delta \nu_{03}$ using ten stars that have detected $l = 3$ modes:

$$\delta \nu_{03} = (0.152 \pm 0.002) \Delta \nu + (2.578 \pm 0.102) \muHz.$$  

Finally, we use the radial modes to determine $\nu_{max}$ because they are purely acoustic, since there are no radial g modes for them to couple with. By fitting the amplitudes versus frequencies of radial modes with a Gaussian function, $\nu_{max}$ was obtained as the Gaussian’s centre. The height of the Gaussian, $A_{max}$, together with the width, are analysed in Section 7. We fitted the well-established $\Delta\nu - \nu_{max}$ relation (e.g. Stello et al. 2009; Huber et al. 2009) to our subgiants,

$$\Delta \nu = a(\nu_{max}/\muHz)^b \muHz,$$  

and determined the best-fitting parameters as $a = 0.158 \pm 0.036$ and $b = 0.847 \pm 0.034$. Huber et al. (2011) determined $a = 0.22$ and $b = 0.797$ from a sample consisting of both MS stars and red giants. We plot the two relations in Fig. 9. The most noticeable effect is that the subgiants lie above the dwarfs, which can be understood from simple scaling arguments. The asteroseismic scaling relations (Brown et al. 1991; Kjeldsen & Bedding 1995) state that $\Delta \nu \propto \sqrt{\rho} \propto M^{1/2} R^{-3/2}$, and $\nu_{max} \propto g/\sqrt{T_{eff}} \propto M R^{-2} T_{eff}^{-1/2}$. Dividing the two relations, we obtain $\Delta\nu/\nu_{max} \propto M^{-1/2} R^{1/2} T_{eff}^{1/2}$. The subgiants and dwarfs have similar $M$ and $T_{eff}$, but subgiants are more inflated, which ultimately leads to higher $\Delta\nu/\nu_{max}$. Indeed, this spread is the signal upon which the widely used scaling relations for mass and radius are based (Stello et al. 2008; Kallinger et al. 2010).

### 5 THE P–G DIAGRAM

We now focus on the frequencies of the underlying g modes, which we denote by $\gamma$ (Aizenman, Smeers & Weigert 1977). Adjusting the model parameters to fit the observed frequencies can be difficult and time-consuming. As pointed out by Bedding (2014), the information contained in the mixed modes can be used more elegantly, by considering the frequencies of the avoided crossings themselves. The suggestion was that, before fitting models to the observed mixed modes, one can first consider the underlying g modes. These are not directly detected but their frequencies can be inferred from the locations of the avoided crossings.

This discussion suggests a new asteroseismic diagram (Bedding 2014), inspired by the classical C–D diagram, in which the frequencies of the avoided crossings $\gamma$ are plotted against the large separation of the p modes. This p–g diagram, so named because it plots g-mode frequencies versus p-mode frequencies, could prove to be an instructive way to display results of many stars and to make a first comparison with theoretical models (Campante et al. 2011; Bedding 2014).

For example, the echelle diagram in Fig. 3 shows three avoided crossings of dipole modes, at about 950, 750, and 650 $\muHz$. We recognize these as the frequencies of the $\gamma$ modes, that is, the pure g modes that would exist in the core cavity if there was no coupling to the p modes in the envelope (Aizenman et al. 1977). Much of the diagnostic information contained in the mixed modes can be captured in this way. This is because the overall pattern of the mixed modes is determined by the mode bumping at each avoided crossing, and these patterns are determined by the g modes trapped in the core. For related discussion on this point, see Deheuvels & Michel (2010) and Benomar et al. (2013).

We determined the g-mode frequencies, $\gamma$, from fitting $l = 1$ mixed-mode frequencies $\nu_{n,l-1}$. Specifically, Lorentzian profiles were fitted to $\nu_{n+1,l-1} - \nu_{n,l-1}$ versus $(\nu_{n+1,l-1} + \nu_{n,l-1})/2$. A dip is present wherever there is an avoided crossing. This is because the presence of mixed modes makes two adjacent modes closer in frequency. We identified the centres of the Lorentzian profiles as the g-mode frequencies, $\gamma$. The errors were estimated following a Monte Carlo procedure by adding Gaussian errors to mode frequencies, repeating the fitting for 1000 times, and adopting the standard deviation of $\gamma$ from the distribution. If multiple avoided crossings (at least three) are present, we further estimated $\Delta \Pi_1$ by calculating the average differences between $1/\gamma$. The results are shown in Table 3. Some stars do not have reported $\gamma_1$ or $\Delta \Pi_1$ because they tend to have uncertain identifications of the first g mode, and they also present a small number of avoided crossings.

For less-evolved subgiants in which the first ($n_g = 1$) g-mode frequency, $\gamma_1$, is present, we made the $\gamma_1 - \Delta \nu$ diagram in Fig. 10. This provides an indicative measurement of stellar mass when evolutionary tracks are simultaneously plotted, unlike the C–D diagram where stellar tracks are degenerate. The stars evolve diagonally, so $\gamma_1$ and $\Delta \nu$ can determine not only mass, but age as well.

Another important diagnostic diagram is to plot the period spacings of g-mode $\Delta \Pi_1$ in equation (2) versus $\Delta \nu$ (another p-
The theoretical evolutionary tracks (Paper II) are labelled with mass in solar units, with the black arrow pointing in the direction of evolution. For subgiants, stellar models predict these are lower order g modes so we are not in the asymptotic regime. For subgiants, stellar models predict ΔΠ1 is rather independent of Δν, especially for higher mass subgiants, such that they evolve almost vertically (Benomar et al. 2013; Gai et al. 2017). Finally, we note that using the two diagrams to infer mass and age provides a method independent of asteroseismic scaling relations. This is discussed in more detail in Paper II.
6 MODE LINEWIDTHS

6.1 Linewidths as a function of frequency

The modes of solar-like oscillations are damped by convection. For these observations, the modes have lifetimes much shorter than the length of time series, which results in a broadened Lorentzian shape in the power spectrum. The modes are hence ‘resolved’. The linewidth [full width at half-maximum (FWHM)] of that shape can be translated to the lifetime through \( \tau = (\pi \Gamma)^{-1} \), or damping rate \( \eta = \pi \Gamma \). This is important to constrain the physics of the superadiabatic layers near the surface, including the mechanism of how the kinetic energy of oscillations is dissipated into turbulent convection. The answer requires a detailed treatment of convection, but it is difficult to formulate with current convection theories (Gough 1980; Balmforth 1992a; Gabriel 1996; Houdek et al. 1999; Grigahcène et al. 2005; Xiong, Deng & Zhang 2015). However, the theories are able to reproduce the frequency dependence of the linewidths seen in the observations, provided the free parameters in the modelling are carefully calibrated, in red giants (Aarslev et al. 2018) and in MS stars (Houdek et al. 2019). 3D hydrodynamical convection simulations may offer a promising path, by eliminating the degrees of freedom in the theory and providing a numerical solution (e.g. Belkacem et al. 2019; Zhou, Asplund & Collet 2019).

Appourchaux et al. (2014) first parametrized the \( \Gamma - \nu \) relation in a given star and Lund et al. (2017) improved the fitting details. Here, we followed the definition in Lund et al. (2017) to fit the linewidths of radial modes, as follows:

\[
\ln(\Gamma(\nu; \theta)) = \alpha \ln(\nu/\nu_{\text{max}}) + \ln(\Gamma_0) + \frac{\ln(\Delta \Gamma_{\text{dip}})}{1 + 2(\ln(\nu/\nu_{\text{max}}))},
\]

where the free parameters are \( \theta = (\alpha, \Gamma_0, \Delta \Gamma_{\text{dip}}, \nu_{\text{dip}}, W_{\text{dip}}) \). This \( \Gamma - \nu \) relation follows a power-law but saturates near \( \nu_{\text{max}} \) to form a plateau (or a dip) centred on \( \nu_{\text{dip}} \) with width \( W_{\text{dip}} \) and height \( \Delta \Gamma_{\text{dip}} \). The dip could originate from a resonance of the thermal time-scale in the superadiabatic layer and mode frequency (Balmforth 1992a; Belkacem et al. 2011). To fit the parameters, we optimized the likelihood function:

\[
\ln L \propto \sum \left( \ln \Gamma_i - \ln(\Gamma(\nu_i; \theta)) \right)^2 / \sigma_{\ln \Gamma_i}^2.
\]

The modes used in this fit were radial modes satisfying Bayes factors \( \ln K > 1 \). The priors of \( \Delta \Gamma_{\text{dip}} \) and \( W_{\text{dip}} \) deserve a further mention. As suggested by Lund et al. (2017), the values of \( \Delta \Gamma_{\text{dip}} \) were bound to be between 0 and 1 to reduce correlations between the parameters. \( W_{\text{dip}} \) can have two solutions: one larger than \( \nu_{\text{max}} \) and one smaller. We kept the convention from Lund et al. (2017) to use the larger one. We estimated the FWHM of the dip through \( \text{FWHM} = \nu_{\text{dip}} \sqrt{W_{\text{dip}}/\nu_{\text{max}} - W_{\text{dip}}/\nu_{\text{dip}}} \) (Appourchaux et al. 2016), and the amplitude of the dip as \( A_{\text{dip}} = \exp(\ln(\Delta \Gamma_{\text{dip}})) \) (Lund et al. 2017). Fig. 12 shows the fit for Gemma (KIC 11026764), which shows a dip around \( \nu_{\text{max}} \). Note that some stars do not present the evidence of a dip. Their linewidths increase monotonically with frequencies. We only fitted those stars with the first two terms of equation (20). Fig. 12 also shows an example, KIC 9512063, where the large random errors obscure the presence of either a dip or a plateau. Table 4 lists the fitted parameters for all stars.

The dipole modes have a more complicated \( \Gamma - \nu \) relation. The dipole modes with more g-mode characteristics have larger inertia and are less affected by damping from the surface than are radial modes, hence resulting in smaller linewidths. Benomar et al. (2014) suggested using linewidths to measure mode inertias, which could also be obtained from the asymptotic formalism of mixed modes (Shibahashi 1979; Goupil et al. 2013). Mosser et al. (2018) introduced a stretch function \( \zeta \), which is the degree of mode trapping characterized by the ratio of mode inertia inside the g-mode cavity to that throughout the star. The linewidths of mixed modes, \( \Gamma_1 \), relate to those of radial modes, \( \Gamma_0 \), as follows (Mosser et al. 2011, 2015, 2018; Vrard et al. 2016; Hekker & Christensen-Dalsgaard 2017):

\[
\Gamma_1 = \Gamma_0(1 - \zeta).
\]

where

\[
\zeta(\nu) = \left[ 1 + \frac{q}{N(\nu)} q^2 \cos^2 \theta_p(\nu) + \sin^2 \theta_p(\nu) \right]^{-1},
\]

\[
N(\nu) = \frac{\Delta \nu}{v^2 \Delta \Pi_1},
\]

and

\[
\theta_p(\nu) = \pi \left\{ \frac{\nu}{\Delta \nu} - \left[ n_p + \frac{l}{2} + \epsilon_p - d_{01} + \frac{\alpha_p}{2} \left( n_p - \frac{\nu_{\text{max}}}{\Delta \nu} \right)^2 \right] \right\}.
\]

Using the above definition of \( \theta_p \), together with equations (3) and 5, we fitted the asymptotic parameters \{\( \epsilon_p \), \( \alpha_p \), \( d_{01} \), \( \Delta \nu \), \( \Delta \Pi_1 \), \( \epsilon_g \)\} to the dipole mode frequencies estimated from the peakbagging. The value of \( \zeta \) was then straightforwardly obtained. In Fig. 13, we compare \( \Gamma_1 \) with \( \Gamma_0(1 - \zeta) \) and find good agreement. That is, the widths of the dipole modes are smaller than those of the radial modes, as expected. This suggests that the asymptotic theory agrees well with the observation.
Table 4. Fit parameters of the $\Gamma_{v}$–$v$ relation.

| KIC   | $\Gamma(v_{\text{max}})$ | $\alpha$ | $\Gamma_{a}$ (µHz) | $\Delta \Gamma_{dp}$ (µHz) | $v_{dp}$ (µHz) | $W_{dp}$ (µHz) | FWHM (µHz) | $A_{dp}$ (µHz$^{-1}$) |
|-------|--------------------------|----------|-------------------|---------------------------|---------------|--------------|-------------|---------------------|
| 2991448 | 0.73 ± 0.22 | 1.35 ± 1.38 | 56.96 ± 55.33 | 0.01 ± 0.01 | 1477 ± 93 | 311 ± 72 | 80 ± 87 |
| 3852594 | 4.42 ± 1.18 | 1.35 ± 0.64 | 5.95 ± 0.40 | – | – | – | – |
| 5607242 | 0.43 ± 0.16 | 4.24 ± 0.71 | 2.35 ± 1.17 | 0.12 ± 0.04 | 698 ± 7 | 898 ± 76 | 191 ± 60 | 8 ± 3 |
| 5689820 | 0.13 ± 0.05 | 1.71 ± 1.65 | 12.70 ± 30.15 | 0.12 ± 0.03 | 731 ± 67 | 2534 ± 1580 | 1029 ± 565 | 69 ± 233 |
| 5951522 | 1.67 ± 0.41 | 3.17 ± 0.94 | 11.83 ± 28.40 | 0.10 ± 0.12 | 874 ± 37 | 1729 ± 554 | 621 ± 298 | 10 ± 12 |
| 6064910 | 5.13 ± 1.33 | 0.16 ± 0.17 | 6.59 ± 0.38 | – | – | – | – |
| 6442183 | 0.64 ± 0.13 | 2.93 ± 0.41 | 2.79 ± 1.44 | 0.20 ± 0.07 | 1166 ± 12 | 1572 ± 181 | 350 ± 136 | 5 ± 2 |
| 6603861 | 0.59 ± 0.46 | 5.39 ± 2.75 | 42.11 ± 58.02 | 0.01 ± 0.03 | 838 ± 39 | 1529 ± 337 | 532 ± 195 | 71 ± 174 |
| 7174707 | 0.42 ± 0.22 | 1.70 ± 1.41 | 18.61 ± 39.44 | 0.02 ± 0.07 | 806 ± 27 | 1723 ± 473 | 613 ± 238 | 54 ± 211 |
| 7193997 | 1.15 ± 0.26 | 2.23 ± 0.34 | 1.41 ± 0.07 | – | – | – | – |
| 7747078 | 1.04 ± 0.16 | 3.15 ± 1.56 | 10.96 ± 34.76 | 0.09 ± 0.18 | 958 ± 66 | 2006 ± 751 | 753 ± 389 | 11 ± 20 |
| 7976303 | 1.76 ± 0.26 | 2.11 ± 1.09 | 8.63 ± 34.46 | 0.18 ± 0.28 | 933 ± 121 | 2790 ± 2583 | 1155 ± 1027 | 6 ± 9 |
| 8524425 | 0.60 ± 0.65 | 4.64 ± 1.13 | 2.97 ± 3.81 | 0.21 ± 0.13 | 1096 ± 35 | 1603 ± 284 | 447 ± 199 | 5 ± 3 |
| 8702606 | 0.31 ± 0.09 | 3.14 ± 0.95 | 12.17 ± 32.95 | 0.04 ± 0.10 | 660 ± 26 | 1547 ± 512 | 572 ± 239 | 23 ± 55 |
| 9512063 | 1.21 ± 0.63 | 1.75 ± 0.92 | 1.48 ± 0.18 | – | – | – | – |
| 10018963 | 2.37 ± 0.42 | 2.60 ± 0.22 | 2.37 ± 0.09 | – | – | – | – |
| 10147635 | 2.79 ± 0.73 | 3.71 ± 0.43 | 2.43 ± 0.16 | – | – | – | – |
| 10273246 | 1.68 ± 0.46 | 5.07 ± 1.08 | 8.51 ± 21.03 | 0.23 ± 0.31 | 1359 ± 695 | 5690 ± 2432 | 2919 ± 1719 | 4 ± 6 |
| 10593351 | 2.03 ± 0.64 | 3.04 ± 0.44 | 2.29 ± 0.17 | – | – | – | – |
| 10873176 | 3.26 ± 1.59 | 3.67 ± 2.10 | 3.42 ± 0.76 | – | – | – | – |
| 10920273 | 0.57 ± 0.25 | 8.48 ± 3.66 | 21.88 ± 44.10 | 0.03 ± 0.07 | 1082 ± 35 | 1577 ± 443 | 459 ± 311 | 33 ± 81 |
| 10972873 | 0.67 ± 0.14 | 3.69 ± 1.40 | 13.36 ± 38.59 | 0.05 ± 0.14 | 1062 ± 52 | 2204 ± 952 | 823 ± 483 | 18 ± 46 |
| 11026764 | 0.73 ± 0.17 | 2.17 ± 0.43 | 1.99 ± 0.36 | 0.31 ± 0.06 | 882 ± 10 | 1065 ± 51 | 183 ± 43 | 3 ± 1 |
| 11137075 | 0.53 ± 0.22 | 10.02 ± 1.98 | 49.24 ± 55.66 | 0.01 ± 0.01 | 1224 ± 25 | 1912 ± 173 | 601 ± 115 | 110 ± 173 |
| 11193861 | 0.86 ± 0.12 | 3.06 ± 0.37 | 1.20 ± 0.06 | – | – | – | – |
| 11395016 | 0.67 ± 0.15 | 1.72 ± 1.56 | 35.73 ± 51.69 | 0.02 ± 0.03 | 822 ± 21 | 1497 ± 200 | 469 ± 115 | 45 ± 70 |
| 11414712 | 0.69 ± 0.16 | 2.66 ± 0.26 | 0.94 ± 0.04 | – | – | – | – |
| 11771760 | 0.63 ± 0.17 | 0.79 ± 0.52 | 1.05 ± 0.07 | – | – | – | – |
| 12508433 | 0.27 ± 0.06 | 3.92 ± 1.66 | 26.15 ± 44.41 | 0.01 ± 0.03 | 796 ± 56 | 2294 ± 556 | 890 ± 230 | 73 ± 154 |

6.2 Scaling relations for linewidths

Scaling relations for linewidths were studied only recently, focusing on the linewidths around $v_{\text{max}}$, denoted by $\Gamma(v_{\text{max}})$. Chaplin et al. (2009) used non-adiabatic pulsation computations and ground-based observations to study the linewidths across different stages of evolution. They showed that the linewidth scale with fundamental stellar parameters, primarily $T_{\text{eff}}$. This dependence on surface properties is expected because damping mainly happens in the superadiabatic regions near the surface (Goldreich & Kumar 1991).

In Fig. 14, we show $\Gamma(v_{\text{max}})$ for our subgiant sample, together with MS stars from Lund et al. (2017), and RGB and HeB stars from Vrard et al. (2018). Note that Vrard et al. (2018) calculated $\Gamma(v_{\text{max}})$ for the giants differently to how it was done for subgiants and MS stars by averaging linewidths using three modes near $v_{\text{max}}$. Fig. 14 shows that $\Gamma(v_{\text{max}})$ mainly depends on $T_{\text{eff}}$, and weakly correlates with log $g$. To parametrize this relation, we fitted $\Gamma(v_{\text{max}})$ using

$$\log(\Gamma_{0}) = c_{1}\log(T_{\text{eff}}/5777 \text{K}) + c_{2}\log(g/274 \text{ms}^{-1}). \quad (26)$$

We optimized the likelihood function:

$$\ln L \propto \ln \Gamma - \ln \Gamma(T_{\text{eff}}, g; \theta)^{2}/(2\sigma_{\ln \Gamma}^{2}). \quad (27)$$
Some previous works (Belkacem et al. 2012; Samadi, Belkacem & Sonoi 2015) pointed out that $\Gamma(v_{\text{max}})$ depends on $T_{\text{eff}}$ and $\log g$ differently in each evolutionary stage, which motivates us to fit them separately as well.

The fitted parameters are shown in Table 5. From the table, we conclude that $\Gamma(v_{\text{max}})$ primarily depends on $T_{\text{eff}}$ but with significantly different power-law indices ($c_1$) for red giants and non-red giants. This could be attributed to different dominant damping mechanisms, as some works suggested (e.g. Baudin et al. 2011). However, it could also be an artefact while selecting modes to compute $\Gamma(v_{\text{max}})$ for red giants (Belkacem et al. 2012). The power-law index for $\log g$ ($c_2$) in subgiants has a reversed sign compared to MS stars and red giants. This effect could be slightly spotted in Fig. 14.

### Table 5. Fit parameters of the $\Gamma(v_{\text{max}})$ scaling relation.

| Phase  | $\Gamma_0$  | $c_1$      | $c_2$     |
|--------|-------------|------------|-----------|
| MS     | 1.10 ± 0.01 | 13.41 ± 0.10 | 0.05 ± 0.02 |
| Subgiants | 0.50 ± 0.04 | 13.71 ± 0.19 | −0.65 ± 0.06 |
| RGB    | 0.25 ± 0.00 | 2.78 ± 0.09  | 0.09 ± 0.00  |
| HeB    | 0.47 ± 0.02 | 1.70 ± 0.13  | 0.19 ± 0.01  |

Some previous works (Belkacem et al. 2012; Samadi, Belkacem & Sonoi 2015) pointed out that $\Gamma(v_{\text{max}})$ depends on $T_{\text{eff}}$ and $\log g$ differently in each evolutionary stage, which motivates us to fit them separately as well.

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### 7 MODE AMPLITUDES

#### 7.1 Amplitudes as a function of frequency

The amplitudes of radial modes follow a roughly Gaussian distribution centred on $v_{\text{max}}$, as we analysed in Section 4. The dipole mixed modes have an added dependence on mode inertia, but are still predictable using the stretch function: $A_{2}^2 = (1 - \xi) A_{0}^2$, with $A_0$ being the radial-mode amplitudes (Benomar et al. 2014; Belkacem et al. 2015; Mosser et al. 2018). Considering $\Gamma_1 = (1 - \xi) \Gamma_0$, we obtained $H_1 = H_0$, suggesting similar mode heights. In Fig. 15, we show $H_1$ against $H_0$. The points deviate from the 1:1 relation because the height of dipole modes are influenced by a geometric effect. Dividing $H_1$ by $H_0$, we get the visibility factor $V_{2}^2$ with a median value equal to 1.5, similar to the one calculated using stellar atmosphere models (Ballot, Barban & van’t Veer-Menneret 2011), which is also around 1.5.

#### 7.2 Scaling relations for amplitudes

Kjeldsen & Bedding (1995) first proposed a scaling relation for the amplitudes of solar-like oscillations to scale from the Sun to other stars, relating both radial velocity and photometric amplitude to fundamental parameters $L, M$ and $T_{\text{eff}}$. It received numerous applications, e.g. for optimizing target selections (Chaplin et al. 2011b; Schofield et al. 2019), and understanding how modes are excited with different treatment for convection and excitation processes (e.g. Goldreich & Keeley 1977; Balmforth 1992b; Samadi & Goupil 2001; Chaplin et al. 2005; Zhou et al. 2019). As we mentioned in Section 4, while estimating $v_{\text{max}}$, we also measured the maximum photometric amplitude expressed by $A_{\text{max}}$ and the width of the Gaussian-like envelopes. To compare $A_{\text{max}}$ with scaling relations, we used two results from Huber et al. (2011):
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8 CONCLUSIONS

In this paper, we presented oscillation frequencies, linewidths and amplitudes for 36 subgiants observed by the 4-yr Kepler mission. We derived those parameters with uncertainties from MCMC fitting, using an open-source software package SOLARLIKEPEAKBAGGING. Significance tests and visual inspections were applied to ensure a robust identification. Our main results are summarized as follows:

(i) With the long baseline of Kepler observations, the median value for the frequency uncertainties of the subgiants is 0.180 μHz. For modes with S/N = 3, the typical uncertainty is 0.1 μHz, providing strong constraint on stellar models with these data.

(ii) The asymptotic parameters $\epsilon_p$, $\Delta \nu$, $v_{\text{max}}$, and $\delta \nu_{\text{dip}}$ were derived using the mode parameters. We identified a $T_{\text{eff}}$ and $\log g$ dependence for $\epsilon_p$. We also revisited the C – D diagram and the $\Delta \nu - v_{\text{max}}$ diagram with a focus on subgiants. The subgiants deviate slightly from the general trend, but could be explained using simple scaling arguments or stellar models.

(iii) We presented two $p$ – $g$ diagrams, the $\gamma_1 - \Delta \nu$ diagram for less-evolved subgiants, and the $\Delta \Pi_1 - \Delta \nu$ diagram for more evolved subgiants. Both diagrams were populated with Kepler observations and can be used as a first simple estimation of stellar mass.

(iv) The linewidths of radial modes were analysed as functions of frequencies, and the linewidths of dipolar mixed modes agree well with asymptotic predictions. We verified the dependence of $\Gamma(v_{\text{max}})$ on $T_{\text{eff}}$ and also observed a weak $\log g$ effect.

(v) The amplitudes of dipolar mixed modes also agree with asymptotic predictions. The mass dependence of $A_{\text{max}}$ and the width is present both in subgiants and MS stars.

The unprecedented quality of asteroseismic data presented by this sample is valuable for the study of stellar physics. The mode frequencies are further used as modelling input in Paper II. The linewidths and amplitudes derived in this work can be used to study mode excitation and damping. In the observation data, one missing piece that remains unexplored in this work is the rotational splitting, which provides a golden opportunity to study the angular momentum transport in subgiants. We will continue to study this topic in this series of papers.
ACKNOWLEDGEMENTS

We thank Mikkel Lund and Jørgen Christensen-Dalsgaard for very helpful discussions. We also appreciate efforts from the referee to improve the quality of this paper, especially during a pandemic outbreak. The Kepler Discovery mission is funded by NASA's Science Mission Directorate. We acknowledge the Joint Research Fund in Astronomy (U1631236) under cooperative agreement between the National Natural Science Foundation of China (NSFC) and Chinese Academy of Sciences (CAS) (NSFC 11273007 and 10933002), and the Fundamental Research Funds for the Central Universities. We also acknowledge funding from the Australian Research Council.

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SUPPORTING INFORMATION

Supplementary data are available at MNRAS online.

Table 2. Mode parameters obtained from the peakbagging.

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APPENDIX A: POWER SPECTRA OF 36 SUBGIANTS

Left-hand panels: echelle diagrams. The blue circles ($l = 0$), red upward triangles ($l = 1$), green squares ($l = 2$), and purple downward triangles ($l = 3$) mark the extracted frequencies. Right-hand panels: power spectra. The fitted power spectra (black) are overlaid on the original power spectra (light grey) and the 1.0-μHz smoothed spectra (grey).
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