Assessment of turbulence models for low turbulent natural convection heat transfer in rectangular enclosed cavity using OpenFOAM

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Abstract
This research aimed to study two-equation turbulence models, for the simulation of low turbulent natural convection heat transfer, which happened in a rectangular enclosure. These models include $k - \varepsilon$, $k - \omega$, RNG $k - \varepsilon$, $k - \omega$ SST and Realizable $k - \varepsilon$ which are two-equation turbulence models in OpenFOAM. The simulation was carried out to compare against experiment in literatures which gave a Rayleigh number (Ra) of $1.58 \times 10^9$. The 3D cell structure by the 50×50×100 cells for the rectangular enclosure dimensions of 0.75 m (width)×0.75 m (height)×1.50 m (length) was modeled. It gave the large $y+$ of 30 but was still in the initial sub-layer. It was found that the $k - \omega$ model provided the best prediction of turbulent natural convection heat transfer. The standard $k$ and $\omega$ wall functions were carried out to simulate well with the $k - \omega$ model. It was good modeling technique which the fine cells were not necessary to define nearly enclosure walls.

Keywords: Turbulence model, Natural convection, Heat transfer, Cavity

1. Introduction
The natural convection heat transfer in enclosures was widely carried out to study by many researchers. They mentioned both experimental and simulating approaches [1-4]. The experiment of convection motion was generated by the buoyancy force on fluid in rectangular enclosures which held two sides at different temperatures. This phenomenon was investigated in various engineering systems such as electronic cooling equipment, building, air conditioning application, solar energy collectors, etc. The numerical approach was carried out by the computational fluid dynamics (CFD). To govern the turbulent flow, the $k - \varepsilon$ turbulence model was the most simulation model which used for this phenomenon. Mao and Zhang [5] had evaluated turbulence models for the simulation of natural convection in a tall cavity. These models comprised $k - \varepsilon$, RNG $k - \varepsilon$ turbulence model and large eddy simulation (LES). They found that the $k - \varepsilon$ model had a high accuracy of velocity prediction while the LES model had good performance for temperature distribution. Rathore and Das [6] had studied the buoyancy-driven flow or
natural convection in a tall cavity, with an aspect ratio of 5. They applied the \( k-\varepsilon \), \( k-\omega \) and \( k-\omega \) SST turbulence model for simulating turbulent flow. The \( k-\varepsilon \) model based on the SIMPLEC algorithm for pressure-velocity coupling and \( y^+ \) less than 1 performed better.

The natural convection in a cavity, with or without an internal body, has been widely investigated. Square, circular and elliptic cylinders were the internal bodies [7]. The study of natural convection in a rectangular enclosure was extremely enhanced, whilst coupled with other effects. Cintolesi et al. [8] simulated natural convection with water evaporation and condensation in a cold enclosed cubic cavity, containing a hot rectangular plate. The internal surface of the cavity was covered by a thin water film. The LES was employed to simulate this phenomenon. Chen et al. [9] studied the natural convection in cavities with a horizontal fin on a hot sidewall. The zero-equation turbulence model was suitable for this study. The vortices at the upper two corners of the cavity, and near the upper surface of the fin edge, were varied by the fin positions. Debbi et al. [10] studied the thermal radiation influence on natural convection in enclosures. The LES was performed with and without wall-to-wall radiation modeling. Simulation results showed that the radiation was significant enough to increase the natural convection heat transfer. Iyi and Hasan [11] investigated moist air and heat transfer inside a rectangular cavity with different vertical heating walls by using CFD. The \( k-\varepsilon \) turbulence model was used with the moisture transport model for simulation. They found that the natural convection was sensitive to moisture content variation. Other shapes of enclosure - not square - were also investigated, in relation to natural convection heat transfer. The proper design of enclosures to achieve the highest heat transfer rate, was the objective for these studies [12]. Enayati et al. [13] studied natural convection heat transfer in cylindrical enclosures, shaped as a reactor. It contained a rack and 80 seeds mounted inside. The LES was used to simulate the heat transfer phenomenon. They found the important number of seeds, in relation to turbulence intensity. Previous literatures presented the importance of investigation for turbulent natural convection heat transfer in enclosures. The challenge being the numerical models for the prediction of turbulent buoyancy flow. Most researchers used commercial software having an expensive license cost for simulation. Consequently, the researchers without supporting funds for the software license, were obstructed. This research proposes using open source software, Open FOAM, to investigate the natural convection heat transfer in enclosed cavities. The RANS turbulence models, which were specifically the two-equation transport models, demonstrated accuracy for the simulation. The suitable model which is detected in this research can be used for applying natural convection in various engineering systems.

2. Natural Convection Flow Models

2.1 Governing equation

The turbulent natural convection flow of compressible fluid in a square enclosure is governed by the conservative equations as follows:

\[
\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = 0
\]

\[
\frac{\partial (\rho u_j)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = - \frac{\partial P}{\partial x_j} + \rho g_i + \frac{\partial}{\partial x_j} \left( (\mu + \mu_t) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \frac{\partial (\rho u_i u_j)}{\partial x_j} \right)
\]

\[
\frac{\partial (\rho h)}{\partial t} + \frac{\partial (\rho u_i h)}{\partial x_j} + \frac{\partial (\rho K)}{\partial t} + \frac{\partial (\rho u_i K)}{\partial x_j} = \frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{P_r} + \frac{k_e}{C_p} \right) \frac{\partial h}{\partial x_j} + \rho u_i g_i
\]

\[
\frac{\partial (\rho e)}{\partial t} + \frac{\partial (\rho u_i e)}{\partial x_j} + \frac{\partial (\rho K)}{\partial t} + \frac{\partial (\rho u_i K)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{P_r} + \frac{k_e}{C_p} \right) \frac{\partial e}{\partial x_j} + \rho u_i g_i
\]

2.2 The two equation turbulence model

The number of equations of the RANS turbulence is counted by using the transport equation number of turbulence parameters for estimating the Reynolds stress term. Consequently, the two-equation
turbulence models in OpenFOAM consist of $k - \varepsilon, k - \omega, RNG k - \varepsilon, k - \omega SST$ and Realizable $k - \varepsilon$.

2.2.1 The $k - \varepsilon$ model
The $k - \varepsilon$ model composes the turbulent kinetic energy ($k$) and rate of viscous dissipation ($\varepsilon$) transport equations [14, 15]. These equations can be written as follows:

$$\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \mu \frac{\partial k}{\partial x_i} \right) + 2 \mu_t \varepsilon S_{ij} \cdot S_{ij} - \rho \varepsilon \quad (5)$$

$$\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho \varepsilon U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \mu_t \frac{\partial \varepsilon}{\partial x_i} \right) + C_{1\varepsilon} \frac{\varepsilon}{k} 2 \mu_t S_{ij} \cdot S_{ij} - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} \quad (6)$$

$$\mu_t = \rho \frac{\varepsilon^2}{\omega} S_{ij} \quad (7)$$

where $C_{1\mu} = 0.09, \sigma_k = 1.00, \sigma_\varepsilon = 1.30, C_{1\varepsilon} = 1.44, C_{2\varepsilon} = 1.92$.

2.2.2 The $k - \omega$ model
The turbulence frequency ($\omega$) had been developed for the length scale determining variable of two-equation turbulence models. The $k - \omega$ model composes the turbulent kinetic energy ($k$) and turbulence frequency ($\omega$) transport equations [16]. These equations can be written as follows:

$$\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \mu \frac{\partial k}{\partial x_i} \right) + 2 \mu_t S_{ij} \cdot S_{ij} - \frac{2}{3} \rho k \frac{\partial U_i}{\partial x_j} \delta_{ij} - \beta^* \rho k \omega \quad (8)$$

$$\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho \omega U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \mu_t \frac{\partial \omega}{\partial x_i} \right) + \gamma_1 \left( 2 \rho \omega S_{ij} \cdot S_{ij} - \frac{2}{3} \rho \omega \frac{\partial U_i}{\partial x_j} \delta_{ij} \right) - \beta_1 \rho \omega^2 \quad (9)$$

where $\mu_t = \rho \frac{k^2}{\omega} \sigma_k = 2.00, \sigma_\omega = 2.00, \gamma_1 = 0.553, \beta^* = 0.09, \beta_1 = 0.075$.

2.2.3 The RNG $k - \varepsilon$ model
The RNG $k - \varepsilon$ model expresses effect of small scale removing by the large scale motion term and modification of viscosity. This equation can be written as follows:

$$\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \left( \alpha_k \frac{k^2}{\varepsilon} \right) \frac{\partial k}{\partial x_i} \right) + \gamma_2 (k \cdot \tau)_{ij} - \rho \varepsilon \quad (10)$$

$$\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho \varepsilon U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{\varepsilon}{k} \frac{\partial \varepsilon}{\partial x_i} \right) + C_{2\varepsilon} \frac{\varepsilon}{k} \gamma_2 \left( k \cdot \tau ight)_{ij} - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} \quad (11)$$

where $C_{1\mu} = 0.0845, \alpha_k = 1.39, \alpha_\varepsilon = 1.39, C_{1\varepsilon} = 1.42, C_{2\varepsilon} = 1.68$.

2.2.4 The $k - \omega SST$ model
The rate of viscous dissipation in $k - \varepsilon$ model is equal to $k \omega$. Consequently, the $k - \omega SST$ model can be written by the following equations.

$$\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \left( \alpha_k \frac{k^2}{\varepsilon} \right) \frac{\partial k}{\partial x_i} \right) + \gamma_2 (k \cdot \tau)_{ij} - \rho \varepsilon \quad (12)$$

$$\frac{\partial (\rho \omega)}{\partial t} + \frac{\partial (\rho \omega U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{\varepsilon}{k} \frac{\partial \varepsilon}{\partial x_i} \right) + \gamma_1 \left( 2 \rho \omega S_{ij} \cdot S_{ij} - \frac{2}{3} \rho \omega \frac{\partial U_i}{\partial x_j} \delta_{ij} \right) - \beta_1 \rho \omega^2 + \frac{2 \rho}{\sigma_{\omega} \omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i} \quad (13)$$

$$\mu_t = \frac{\rho}{\max \left( \alpha_k, \omega_1 \sqrt{2 \gamma_2 S_{ij} S_{ij}} \right)} \quad (14)$$

where $\sigma_k = 1.00, \sigma_{\omega_1} = 2.00, \sigma_{\omega_2} = 1.17, \gamma_1 = 0.44, \beta^* = 0.09, \beta_1 = 0.083, F_2$ is the blending function.

2.2.5 The Realizable $k - \varepsilon$ model
The rate of viscous dissipation transport equations and realizable eddy viscosity formulation are proposed to improve the $k - \varepsilon$ model. The Realizable $k - \varepsilon$ model composes the following equations.
\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{\mu_k}{\sigma_k} \frac{\partial k}{\partial x_i} \right) + 2\mu_s S_{ij} \cdot S_{ij} - \rho \varepsilon \tag{15}
\]
\[
\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho \varepsilon U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{\mu\varepsilon}{\sigma_k} \frac{\partial \varepsilon}{\partial x_i} \right) + C_1 \rho \varepsilon \frac{2S_{ij} \cdot S_{ij} - C_2 \rho}{k + \sqrt{\varepsilon}} \tag{16}
\]
\[
C_i = \max \left\{ 0.43, \frac{\eta}{5 + \eta} \right\}, \eta = 5k \varepsilon, C_\mu = \frac{1}{A_0 + A_s U^* \sqrt{\frac{k}{\varepsilon}}}, U^* = \sqrt{S_{ij} \cdot S_{ij} + \Omega_{ij} \cdot \Omega_{ij}}, A_s = \sqrt{6} \cos \phi \tag{17}
\]

where \( \sigma_k = 1.00, \sigma_\varepsilon = 1.20, C_2 = 1.9, A_0 = 4.0. \)

2.3 Boundary conditions

The number of equations of the RANS turbulence is counted by using the transport equation number. The wall function has been developed to satisfy the physic of flow in near wall region. It bridges the inner region between the wall and the turbulence fully developed region. In the viscous sub-layer (\( y^+ < 5 \)), the \( k, \varepsilon \) and \( \omega \) wall functions [17] are written as follows:

\[
k = \tau_w \frac{C_{\mu}^{-1/2}}{\rho}, \varepsilon = C_{\mu}^{3/4} \frac{k^{3/2}}{\rho \gamma}, \omega = \frac{6\tau_w}{\beta_1 \mu (y^+)^2} \tag{18}
\]

In the logarithmic area (\( 30 < y^+ < 200 \)), the turbulent variations are dominated by the following equations.

\[
k = \frac{u_{\tau}^2}{\sqrt{C_{\mu}}}, \varepsilon = \frac{u_{\tau}^3}{\rho \gamma}, \omega = \frac{u_{\tau}^3}{\nu \sqrt{C_{\mu} y^+}} \tag{19}
\]

where \( u_{\tau} \) is the friction velocity and \( \nu \) is the kinematic viscosity.

2.4 The wall heat flux

The wall heat transfer coefficient is evaluated using the Nusselt number as follows:

\[
Nu = \frac{QL}{k(T_h - T_c)} \tag{20}
\]

where \( Q \) is the wall heat flux, \( L \) is the width of the cavity, \( k \) is the thermal conductivity, \( T_h \) is the hot wall temperature and \( T_c \) is the cold wall temperature.

3. Computational Fluid Dynamics Model

The experiment of low turbulent natural convection flow of air in a rectangular enclosure had been performed by Tian and Karayiannis [18, 19]. The dimensions of enclosure were 0.75 m \( \times \) 0.75 m \( \times \) 1.50 m. The hot and cold walls of the enclosure were isothermal at 50°C and 10°C, respectively. The experiment was studied in the 2D plane at the mid-length of the enclosure. In order to achieve CFD domain as same as the experimental enclosure, the rectangular cavity has been created as shown in figure 1. The wall function was controlled in the inertial sub-layer then the \( y^+ \) was chosen at 30. Subsequently, the cell structure was assembled by the hexahedron cells with the only one dimension that was 0.015×
0.015×0.015 m. This cell structure was built by the total cells of 250,000 cells. The wall of domain at the left hand (X = 0 m) is the hot wall. It had been defined the constant temperature of 50ºC. The adverse wall was the cold wall (X = 0.75 m) with the constant temperature of 10ºC. The other walls of enclosure were defined the adiabatic wall. The air inside the CFD domain was assigned to be the compressible fluid. At the initial time, the air temperature inside the domain was 30 ºC. The initial pressure inside domain was 1.0×10^5 Pa. The pressure gradient was the pressure boundary on enclosure walls. To activate the buoyant flow, the acceleration gravity was specified carefully in the –Y direction. The no slip condition was assigned on the enclosure walls for the air velocity. The two-equation turbulence models composed of \( k-\varepsilon \), \( k-\omega \), RNG \( k-\varepsilon \), \( k-\omega \) SST and Realizable \( k-\varepsilon \) were assigned to govern the low turbulent flow; therefore, the \( k \), \( \varepsilon \) and \( \omega \) wall functions were defined on the enclosure walls following the turbulence models. The wall functions would activate the turbulent flow nearly the enclosure walls; therefore, it was not concerned for the grid independence of enclosure domain.

The solutions had been obtained by solving the transport equations. The upwind and central-differencing scheme were used to discretize the convective and diffusive terms, respectively. The velocity-pressure coupling was performed using PIMPLE algorithm [17]. All of the low turbulent natural convection flow cases, the personal computer with the i7-8700K processor and 8 GB DDR RAM memory was intended to calculate the effect of wall functions together with the two-equation turbulence models of OpenFOAM.

4. Results and Discussion

4.1. The temperature distribution

Figure 2a shows an example of the temperature distribution in the mid-length plane (Z = 0.75 m) of enclosure by using RNG \( k-\varepsilon \) model. The color contour was indicated the air temperature. The hot air was red while the cold air was blue. The red color started buoyance up at the bottom edge of hot wall. The hot air buoyant to impinge with the top wall of enclosure. The cold air was displaced by the hot air; therefore, the boundary layer started at the top edge of the cold wall. It flow down to the bottom of enclosure nearly the cold wall. The flow region near the walls was turbulent flow caused of the viscosity of air on these walls. The temperature distribution by the two-equation turbulence models at the height of 0.225 m is compared with the experiment as shown in figure 2b. The simulation points inside CFD domain along the X-axis accorded the experimental data. The temperature changed across the enclosure was steep nearly both isothermal walls. It shot at X = 0.0263 and 0.7238 m, respectively. The comparison
Figure 3. (a) Velocity distribution on the mid-length plan of enclosure by $k-\varepsilon$ model and (b) The $v$-velocity distribution across the mid-length plane of enclosure at the height of 0.525 m.

had been performed every 10% of the enclosure height. It was found that the $RNG\;k-\varepsilon$ showed the temperature distribution in the most agreement with the experiment. The $k-\varepsilon$ and $k-\omega$ turbulence models were the second and third accuracy of results, respectively. The average error of the $RNG\;k-\varepsilon,\;k-\varepsilon$ and $k-\omega$ turbulence models was 19.48%, 19.58% and 19.60%, respectively. In addition, the temperature profile was linear nearly the hot and cold walls.

4.2. The velocity distribution

The air velocity was investigated in X and Y directions. The color contour also used to interpret the air velocity of the simulation results and the arrows were added to investigate the flow direction. The high velocity was red while low velocity was blue. Figure 3a shows color contour of the velocity distribution on the mid-length plane of enclosure by $k-\varepsilon$ model. The velocity values in front of the central region of the hot and cold walls were higher than the other regions. Nearly the hot wall, the arrows of air velocity pointed upward. It confirmed the buoyant flow of air happened nearly the hot wall. On the other hand, the air flow down nearly the cold wall. The vortex of air flow happened distinctly in front of the hot and cold walls in the clockwise direction. It confirmed that the turbulent flow phenomenon was happened. The circulation of air flow happened distinctly near the top and bottom walls. It was caused by the flow past upper right angle between hot and top walls and lower right angle between cold and bottom walls. The great difference of air velocity between nearly isothermal wall and inside enclosure

Figure 4. The u-velocity distribution across the enclosure at the height of 0.375 m.

Figure 5. Comparison of the local Nusselt number.
buoyance velocity was equal to 1. The comparison also performed every 10% of the enclosure height. It was found that the $k-\varepsilon$ showed the vertical velocity distributed in the most agreement with the was the other cause that the turbulent flow happened. The $v$-velocities of simulation on the mid-length plane at the height of 0.525 m is compared with the experiment as shown in figure 3b. $V_0$ was the experimental data. The RNG $k-\varepsilon$ and $k-\omega$ turbulence models were the second and third accuracy, respectively. The average error of the $k-\varepsilon$, RNG $k-\varepsilon$, and $k-\omega$ turbulence models was 12.55%, 13.12% and 13.20%, respectively.

Figure 4 shows the comparison graphs of u-velocity between simulation and experiment at the height of 0.375 m. The $k-\omega$ showed the horizontal velocity distributed in the most agreement with the experimental data. The RNG $k-\varepsilon$ and $k-\omega$ SST turbulence models were the second and third accuracy, respectively. The average error of the $k-\omega$, RNG $k-\varepsilon$, and $k-\omega$ SST turbulence models was 14.57%, 19.53% and 24.50%, respectively. The horizontal velocity value was one order lower than the vertical velocity. It was significantly distributed at different height; therefore, the error of CFD happened distinctly. Figure 5 shows the comparison of Nusselt number between experiment and simulation. The $k-\omega$ model was good agreement with experiment. The average error was 21.84% showed the accuracy of this model. The Ra in this study was $1.58\times10^9$ which depended on the different temperature between hot and cold wall and then the Nusselt number was increased following the Ra.

5. Conclusion
The temperature and velocity changed sharply near the isothermal walls, and conducted the turbulence natural convection in an air-filled square cavity. A 3D cell structure was built to study the temperature and velocity distribution inside the enclosure. The cell size was large and uniform to control the $y^+$ as 30. The standard wall functions include the $k$, $\varepsilon$ and $\omega$ wall functions were applied on the enclosure walls. The two-equation turbulence models of Open FOAM including $k-\varepsilon$, $k-\omega$, RNG $k-\varepsilon$, $k-\omega$ SST and Realizable $k-\varepsilon$ were employed to simulate the turbulent buoyance flow. It was found that the $k-\omega$ model was the most accurate turbulence model, to represent the low turbulent natural convection heat transfer, in rectangular enclosed cavities. Particularly, the $k$ and $\omega$ wall functions suitably matched with this turbulence model. Instead, fine cells could be used to control the small $y^+$ less than 1. Consequently, simulation time was not consumed. The average error of this model was less than 15.79% when the temperature, v-velocity and u-velocity across the mid-length plane of enclosure every 10% of the height were investigated. This study depicts a good choice of turbulence model in Open FOAM software, for applying natural convection in various engineering systems.

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