Weak Decays of Stable Doubly Heavy Tetraquark States

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In the light of the recent discovery of the Ξ++ by LHCb collaboration, we study the stable doubly heavy tetraquarks. These states are compact exotic hadrons which can be approximated as the diquark-anti-diquark correlations. In the flavor SU(3) symmetry, they form an SU(3) triplet or anti-sextet. The spectra of the stable doubly heavy tetraquark states are predicted by Sakharov-Zeldovich formula. We find that the $T^{+}_{ccar{u}ar{d}}(3)$ is about 16MeV below the $DD^*$ threshold, while $T^{-}_{bbar{u}ar{d}}(3)$ is about 73MeV below the $BB^*$ threshold. We then study the semileptonic and nonleptonic weak decays of the stable doubly heavy tetraquark states. The doubly heavy tetraquark decay amplitudes are parametrized in terms of flavor SU(3)-irreducible parts. Ratios between decay widths of different channels are also derived. At the end, we collect the Cabibbo allowed two-body and three-body decay channels, which are most promising to search for the stable doubly heavy tetraquark states at LHCb and Belle II experiments.

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I. INTRODUCTION

Up to now most hadrons found by experiments can be well established as quark-antiquark pair and triquarks configurations [1]. Based on the principle of color confining, the multiquark color singlet states such as $qqar{q}q$ (tetraquarks) and $qqqqar{q}$ (pentaquarks) can also exist. On the experimental aspect, many multiquark candidates have been observed even though their physical figures are still not established. The most aged of these exotic resonances is the neutral $X(3872)$ discovered in $B^\pm \rightarrow K^\pm X (X \rightarrow J/\psi \pi^+\pi^-)$ by Belle in 2003 [2]. Four years later, the Belle Collaboration observed a charged hidden charm tetraquark candidate, i.e. $Z^+(4430)$ [3]. In 2013, the BESIII Collaboration discovered $Z^+_c(3900)$ through the channel $Y(4260) \rightarrow \pi^-\pi^+J/\psi$ [4], which directly hadronic decays into $J/\psi\pi^+$, and then implies that it shall be a meson with quark contents $c\bar{c}u\bar{d}$. In 2015, the LHCb Collaboration discovered that two exotic baryons $P_c(4380)$ and $P_c(4450)$ hadronic decaying into $J/\psi p$, which are candidates for pentaquark states and shall be a baryon with quark contents $c\bar{c}u\bar{d}$.

The LHCb Collaboration have recently observed the doubly charm baryon $\Xi^{++}_{cc}$ in the $\Lambda^+_cK^-\pi^+\pi^+$ invariant mass spectrum, whose mass is measured to be $3621.40 \pm 0.72(stat.) \pm 0.27(syst.) \pm 0.14(\Lambda^+_c)$ MeV [5]. This discovery has attracted wide attentions from both the theoretical and experimental sides in high energy physics. From the diquark-based model, the doubly heavy quarks can provide a static color source as the attractive diquark in the color $\bar{3}$ representation. The attractive heavy diquark and the light quark in the color 3 representation then form a color singlet hadron. Thus it is natural to conceive the doubly heavy tetraquark states with attractive heavy diquark and attractive light diquark. From the basic principles of QCD, the long-distance interactions among light quarks and gluons has a characteristic scale of the order of 300MeV. When the two heavy quarks attract each other and their
separation is smaller enough than the separation to the light quark, the two heavy quarks interact with a perturbative one-gluon-exchange Coulomb-like potential. When the two heavy quarks have a large separation, the four quarks will form two weakly interacting mesons. It is an important issue to be discussed about whether or not the stable doubly heavy tetraquark states exist. When they steadily exist, it is another important issue on how to detect them.

On the theoretical aspect, the mass spectra of the doubly heavy tetraquark states have been studied in many literatures \[3\] \[21\]. Most of them supported the existence of the doubly heavy tetraquark states, however, they predicted differently the spectra of the doubly heavy tetraquark states in these works. The structures of the doubly heavy tetraquark states were also different in their descriptions. Unlike the \(Qq\bar{Q}q\) system which can be classified into four kinds of four quark structures \[21\], the structures of the \(QQ\bar{q}q\) system are relatively simple. Take the \(bb\bar{q}q\) for example, the four quark structures may be classified into two groups: one is treated as a bound state made of a loosely bound \(BB\) meson pair or two far separated and essentially weak interacting \(B\) mesons; the other one is treated as a bound state made of a heavy diquark with color anti-triplet and a light anti-diquark with color triplet.

Theoretical description of doubly heavy tetraquark states decays is few in current studies. Whether or not the QCD factorization is valid in the doubly heavy tetraquark states decays is an open question. An alternative and model-independent approach is to employ the flavor SU(3) symmetry, which has been successfully applied into the meson and the heavy baryon decays \[22–38\]. In this paper, we will investigate the amplitudes and decay widths of doubly heavy tetraquark states under the flavor SU(3) symmetry.

The paper will be presented as follows. In Sec. II we classify the doubly heavy tetraquark states into an SU(3) triplet or anti-sextet according to the decomposition of \(\bar{3} \otimes \bar{3} = 3 + 6\). Other related baryons and mesons are also listed in SU(3) flavor symmetry. In Sec. III we give the spectra of the doubly heavy tetraquark states. Their stability properties are essential for the discovery and will be discussed. In Sec. IV and V, we mainly study the semileptonic decays of the doubly heavy tetraquark. The decay amplitudes are explored with the SU(3) flavor symmetry. The ratios between the decay widths of different decay channels are predicted. We summarize and conclude in the end.

II. PARTICLE MULTIPLETS

Following the flavor SU(3) group, the doubly heavy tetraquark states and their decay products can be grouped into the particle multiplets.

In principle, doubly heavy tetraquark states with the \(QQ\bar{q}q\) are similar to the \(\bar{Q}\bar{Q}qq\). We are focusing on the \(QQ\bar{q}q\) states for simplification. The doubly heavy tetraquark \(QQ\bar{q}q\) can form an SU(3) triplet or anti-sextet by the decomposition of \(\bar{3} \otimes \bar{3} = 3 + 6\). The triplet has the expression

\[
T_{cc3} = \begin{pmatrix}
0 & T_{cc\bar{u}\bar{d}}^+ & T_{cc\bar{e}\bar{s}}^+ \\
-T_{cc\bar{u}\bar{d}}^+ & 0 & T_{cc\bar{e}\bar{s}}^+ \\
-T_{cc\bar{e}\bar{s}}^+ & -T_{cc\bar{u}\bar{d}}^+ & 0
\end{pmatrix}, \quad T_{bc3} = \begin{pmatrix}
0 & T_{bc\bar{c}u}^+ & T_{bc\bar{b}s}^+ \\
-T_{bc\bar{c}u}^- & 0 & T_{bc\bar{b}s}^- \\
-T_{bc\bar{b}s}^- & -T_{bc\bar{c}u}^- & 0
\end{pmatrix}, \quad T_{bb3} = \begin{pmatrix}
0 & T_{bb\bar{u}\bar{d}}^+ & T_{bb\bar{e}\bar{s}}^+ \\
-T_{bb\bar{u}\bar{d}}^- & 0 & T_{bb\bar{e}\bar{s}}^- \\
-T_{bb\bar{e}\bar{s}}^- & -T_{bb\bar{u}\bar{d}}^- & 0
\end{pmatrix}.
\]

While the doubly heavy tetraquark in anti-sextet can usually strong decay into the triplets and are not stable, then we will not consider them here.

When we study the weak decays of the doubly heavy tetraquarks under the flavor SU(3) symmetry, we should classify the products. The charmed bottom baryons can form a SU(3) triplet with \(F_{bc} = (\Xi_{bc}^-(bcu), \Xi_{bc}^0(bcd), \Omega_{bc}^0(bcs))\). The charmed anti-baryons are classified into a triplet and an anti-sextet

\[
F_{c3} = \begin{pmatrix}
0 & \Lambda_c^- & \Xi_c^- \\
-\Lambda_c^- & 0 & \Xi_c^0 \\
-\Xi_c^- & -\Xi_c^0 & 0
\end{pmatrix}, \quad F_{c6} = \begin{pmatrix}
\Sigma_c^- & \frac{1}{\sqrt{2}}\Sigma_c^- & \frac{1}{\sqrt{2}}\Sigma_c^- \\
\Sigma_c^+ & -\frac{1}{\sqrt{2}}\Sigma_c^+ & -\frac{1}{\sqrt{2}}\Sigma_c^+ \\
\frac{1}{\sqrt{2}}\Sigma_c^0 & \frac{1}{\sqrt{2}}\Sigma_c^0 & \frac{1}{\sqrt{2}}\Sigma_c^0
\end{pmatrix}.
\]
Similarly, the singly charmed baryons $F_{c3}$ and $F_{c6}$ can be described in the same way, whose explicit expressions can be found in Refs. \cite{34, 37}.

The anti-baryons with light quarks can be classified into an octet and an anti-decuplet. We write the octet as

\begin{equation}
F_8 = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^+ + \frac{1}{\sqrt{6}} \Xi^0 \\
\Sigma^- \\
\bar{p} \\
\bar{\Xi}^0 \\
\bar{\Sigma}^+ \\
\bar{\Sigma}^- \\
\bar{\Xi}^0 \\
\bar{\Xi}^0 \\
\end{pmatrix} = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^+ + \frac{1}{\sqrt{6}} \Xi^0 \\
\Sigma^- \\
\bar{p} \\
\bar{\Xi}^0 \\
\bar{\Sigma}^+ \\
\bar{\Sigma}^- \\
\bar{\Xi}^0 \\
\bar{\Xi}^0 \\
\end{pmatrix}.
\end{equation}
while the anti-decuplet can be written as

\[
\begin{align*}
(F_{10})^{111}_{\bar{1}} &= \bar{\Sigma}^{--}, \quad (F_{10})^{112}_{\bar{1}} = (F_{10})^{121}_{\bar{1}} = (F_{10})^{211}_{\bar{1}} = \frac{1}{\sqrt{3}} \Sigma^{-}, \\
(F_{10})^{222}_{\bar{1}} &= \bar{\Delta}^{+}, \quad (F_{10})^{122}_{\bar{1}} = (F_{10})^{212}_{\bar{1}} = (F_{10})^{221}_{\bar{1}} = \frac{1}{\sqrt{3}} \Delta^{+}, \\
(F_{10})^{113}_{\bar{1}} &= (F_{10})^{131}_{\bar{1}} = (F_{10})^{311}_{\bar{1}} = \frac{1}{\sqrt{3}} \bar{\Sigma}^{--}, \quad (F_{10})^{223}_{\bar{1}} = (F_{10})^{232}_{\bar{1}} = (F_{10})^{322}_{\bar{1}} = \frac{1}{\sqrt{3}} \bar{\Delta}^{+}, \\
(F_{10})^{123}_{\bar{1}} &= (F_{10})^{132}_{\bar{1}} = (F_{10})^{231}_{\bar{1}} = (F_{10})^{312}_{\bar{1}} = (F_{10})^{321}_{\bar{1}} = \frac{1}{\sqrt{6}} \bar{\Sigma}^{--}, \\
(F_{10})^{133}_{\bar{1}} &= (F_{10})^{313}_{\bar{1}} = (F_{10})^{331}_{\bar{1}} = \frac{1}{\sqrt{3}} \bar{\Sigma}^{--}, \quad (F_{10})^{233}_{\bar{1}} = (F_{10})^{323}_{\bar{1}} = (F_{10})^{332}_{\bar{1}} = \frac{1}{\sqrt{3}} \bar{\Delta}^{+}, \\
(F_{10})^{333}_{\bar{1}} &= \bar{\Pi}^{+}. \quad (4)
\end{align*}
\]

The light pseudo-scalar mesons belong to an octet and a singlet. The explicit expression of the octet can be found in Refs. [34, 37]. The flavor singlet η1 will not be considered here for simplicity. For the heavy flavor mesons, we can write them as \( B_1 = (B^{-}, B^{0}, \bar{B}^{0}) \) and \( B^+ = (B^+, B^{0}, B^0) \). Similarly, the charmed mesons can be written as \( D_i = (D^0, D^+, D^{+}) \) and \( D^3 = (\bar{D}_0, D^-, D^-_s) \). To a summary of the particle multiplets, we plot the flavor SU(3) weight diagrams for hadrons with different representations in Fig. 1 and Fig. 2.

### III. SPECTRA OF THE DOUBLY HEAVY TETRAQUARKS

The wave function of a tetraquark consists of four parts: space-coordinate, flavor, color, and spin subspaces,

\[
\Psi(Q, Q', q, q') = R(x_1, x_2, x_3, x_4) \otimes \chi_f(f_1, f_2, f_3, f_4) \\
\otimes \chi_G(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \otimes \chi_s(s_1, s_2, s_3, s_4),
\]

where \( R(x_i) \), \( \chi_f(f_i) \), \( \chi_G(\lambda_i) \), and \( \chi_s(s_i) \) denote the radial, flavor, color, and spin wave functions, respectively. The sub-labels 1, 2, 3, 4 in the above equation denote \( Q, Q', q, q' \), respectively.

For the two quark system, there are eight distinct diquark multiplets in flavor \( \otimes \) color \( \otimes \) spin space. According to the Pauli exclusion principle, the diquark-antidiquark configuration \([QQ'\bar{q}q']\) of doubly heavy tetraquark state only has four possible topologies \(^1\)

\[
\begin{align*}
|1_f(S), 3_f(A)\rangle &\otimes |\bar{3}_c(A), 3_c(A)\rangle \otimes |1_s(S), 0_s(A)\rangle, \\
|1_f(S), 3_f(A)\rangle &\otimes |6_c(S), \bar{6}_c(S)\rangle \otimes |0_s(A), 1_s(S)\rangle, \\
|1_f(S), 6_f(S)\rangle &\otimes |3_c(A), 3_c(A)\rangle \otimes |1_s(S), 1_s(S)\rangle, \\
|1_f(S), \bar{6}_f(S)\rangle &\otimes |6_c(S), \bar{6}_c(S)\rangle \otimes |0_s(A), 0_s(A)\rangle,
\end{align*}
\]

where the sub-label f, c, s denote the flavor, color, spin spaces, respectively. S and A denote the symmetric and antisymmetric properties. Each half bracket denotes the diquark configuration. For example, \([1_f(S), 3_f(A)\]) denotes the diquark \([QQ']\) is the singlet in flavor space and thus is symmetric (S), while the diquark \([\bar{q}q']\) is the triplet in flavor space and thus is antisymmetric (A). Consider that the color sextet diquarks have larger color electrostatic energy thus is not a well-favored configuration, and the odd parity diquark operators will vanish in the single mode

\(^1\) One should note that the two quarks and two antiquarks can be encoded by three different ways of color basis. One of them is the color basis of \([3_{12}, 3_{34}], [6_{12}, 3_{24}]\), and the other two are \([1_{12}, 1_{24}] ; [8_{13}, 8_{24}]\) and \([1_{14}, 1_{23}] ; [8_{14}, 8_{23}]\) respectively. Through decomposition, the singlet-singlet and octet-octet basis can be described by the triplet-antitriplet and antisextet-sextet base.
configuration. The diquarks $|3_c(A)\rangle \otimes |0_s(A)\rangle$ and $|\bar{3}_c(A)\rangle \otimes |1_s(S)\rangle$ in Eqs. 6a and 6c are the “scalar” and “axial-vector” diquarks respectively, as the “good” and “bad” diquarks named by Jaffe. Other configurations of the diquark are “worse” diquarks. For simplification, we will not consider these “worse” diquarks in the prediction of the spectra.

The constituent quark models have a robust power to predict hadron spectra, especially for the $S$-wave states. In these quark models, the hadrons are bound states composed of the constituent quarks. In Sakharov-Zeldovich formula, the hadron mass is given by

$$M_{\text{hadron}} = \sum_i m_i + \sum_{i<j} \left( -\frac{3}{8} \right) C^{ij}_{m_i,m_j} \bar{s}_i \cdot \gamma_i \bar{s}_j \cdot \gamma_j ,$$

where the overall strength can be given as $C^{ij} = v^{ij} \langle \delta (r_{ij}) \rangle$ with the coupling $v^{ij}$ and the strength of the radial wave function at zero separation $\langle \delta (r_{ij}) \rangle$ which is dependent on the hadron constituent quark flavors. The $\bar{s}_i$ is the Gell-Mann matrix for color SU(3) group, and $\bar{s}_i = \bar{s}_i / 2$ is the quark spin operator with the Pauli matrix $\bar{s}_i$.

The parameters in Eq. 7 can be fitted by the hadron spectra [41], which have been given in Tab. I. In Tab. I, the overall factor $C^{ij}/(m_i m_j)$ can be extracted from the hadron mass differences. The constituent quark masses are chosen as: $m_{u,d} = 305$ MeV, $m_s = 490$ MeV, $m_c = 1670$ MeV, and $m_b = 5008$ MeV for SET I [42-45]; $m_{u,d} = 330$ MeV, $m_s = 500$ MeV, $m_c = 1600$ MeV, and $m_b = 4950$ MeV for SET II.

The spectra of the triplet doubly charm tetraquark for SET I are determined as

$$m(T^+_{cc\bar{u}\bar{d}}(3)) = 3.86\text{GeV}, \quad J^P = 1^+ , \quad (8)$$
$$m(T^+_{cc\bar{u}\bar{d}}(3)) = m(T^{++}_{cc\bar{u}\bar{d}}(3)) = 4.10\text{GeV}, \quad J^P = 1^+ . \quad (9)$$

In above, $T^+_{cc\bar{u}\bar{d}}(3)$ lies the spectrum about 16MeV below the $DD^*$ threshold and about 120MeV above the $DD$ threshold. However, the $T^+_{cc\bar{u}\bar{d}}(3)$ is an axial-vector meson and can not directly hadronic decay to $DD$. Thus $T^+_{cc\bar{u}\bar{d}}(3)$
with spin-parity $1^+$ is a stable tetraquark, which shall be tested in experiment. $T_{cc\bar{u}d}^+(3)$ and $T_{cc\bar{d}s}^{++}(3)$ lie the spectrum about 124MeV above the $D_sD^*$ threshold and about 118MeV above the $D_s^*D$ threshold. These two states can hadronic decay thus are not stable.

The spectra of the triplet charm-beauty tetraquark for SET I are determined as

$$m(T_{b\bar{u}d}^0(3)) = 7.20\text{GeV}, \quad J^P = 1^+, \quad (10)$$

$$m(T_{b\bar{d}s}^0(3)) = m(T_{b\bar{c}d}^+(3)) = 7.43\text{GeV}, \quad J^P = 1^+. \quad (11)$$

In this kind, $T_{b\bar{u}d}^0(3)$ lies the spectrum about 86MeV below the $BD^*$ threshold but about 5.8MeV above the $B^*D$ threshold. Thus $T_{b\bar{u}d}^0(3)$ can hadronic decay to $B^*D$ and has a large decay width. $T_{b\bar{c}d}^0(3)$ and $T_{b\bar{d}s}^+(3)$ lie the spectrum about 56MeV above the $B_sD^*$ threshold and about 137MeV above the $B^*D_s$ threshold. These two states are also not stable.

The spectra of the triplet doubly bottom tetraquark for SET I are determined as

$$m(T_{bb\bar{u}d}^-(3)) = 10.53\text{GeV}, \quad J^P = 1^+, \quad (12)$$

$$m(T_{bb\bar{d}s}^0(3)) = m(T_{bb\bar{c}d}^+(3)) = 10.77\text{GeV}, \quad J^P = 1^+. \quad (13)$$

For bottom sector, $T_{bb\bar{u}d}^-(3)$ lies the spectrum about 73MeV below the $BB^*$ threshold. Thus $T_{bb\bar{u}d}^-(3)$ with spin-parity $1^+$ is also a stable tetraquark, which shall be tested in experiment. $T_{bb\bar{c}d}^0(3)$ and $T_{bb\bar{d}s}^0(3)$ lie the spectrum about 78MeV above the $B_sB^*$ threshold and about 75MeV above the $B^*_sB$ threshold, which are not stable.

Considering the uncertainties from the constituent quark masses, we adopt another input for these parameters. When we adopt the constituent quark with SET II, we can easily get the tetraquark mass differences and also predict the doubly heavy quark tetraquark spectra. The mass of $T_{cc\bar{d}u}^+(3)$ will be reduced by 90MeV, while $T_{cc\bar{u}d}^+(3)$ and $T_{cc\bar{d}s}^{++}(3)$ will be reduced by 105MeV. The conclusions of the stability discussions of them are unchanged. For the triplet charm-beauty tetraquark with SET II, the mass of $T_{b\bar{c}d}^0(3)$ will be reduced by 78MeV, while $T_{b\bar{u}d}^0(3)$ and $T_{b\bar{d}s}^+(3)$ will be reduced by 93MeV. Then $T_{b\bar{c}d}^0(3)$ is about 159MeV below the $BD^*$ threshold and about 67.2MeV below the $B^*D$ threshold, which indicates that $T_{b\bar{c}d}^0(3)$ is a stable state. For the triplet doubly bottom tetraquark with SET II, the mass of $T_{bb\bar{u}d}^-(3)$ will be reduced by 66MeV, while $T_{bb\bar{c}d}^0(3)$ and $T_{bb\bar{d}s}^+(3)$ will be reduced by 81MeV. Thus $T_{bb\bar{u}d}^-(3)$ becomes more stable. $T_{bb\bar{c}d}^0(3)$ and $T_{bb\bar{d}s}^+(3)$ is about 3MeV below the $B_sB^*$ threshold and about 6MeV below the $B^*_sB$ threshold, which become stable. Future experiments shall tell us the answers to these different predictions between SET I and SET II.

To hunt for these possible doubly heavy tetraquarks, we will study their weak decays properties. Their semi-leptonic and nonleptonic decay amplitudes can be parametrized in terms of SU(3)-irreducible amplitudes. For completeness, we will investigate the weak two-body, three-body and four-body decays of the doubly heavy tetraquarks in flavor triplet.

IV. SEMI-LEPTONIC DECAYS

I. Semileptonic $T_{bb\bar{q}q}$ decays

1. Decays into mesons and $\ell\nu_{\ell}$

First we study the decays into mesons and $\ell\nu_{\ell}$, where the electro-weak Hamiltonian is

$$\mathcal{H}_{\text{eff}}^{\ell\nu_{\ell}} = \frac{G_F}{\sqrt{2}} \left[ V_{q'b'} q'^\mu (1 - \gamma_5) b' \bar{c} \gamma_\mu (1 - \gamma_5) \nu_{\ell} \right] + h.c., \quad (14)$$
FIG. 3: Feynman diagrams for semileptonic decays of doubly bottom tetraquark. Panels (a,b) correspond to the decays into a pair of mesons. In panel (c), there is only one meson in the final states. Panels (d,e) denote the decays into baryonic states. In panels (b,c,e), the two $b\bar{u}$ quarks in the initial state can annihilate, but such contributions are usually power suppressed.

with $q' = u, c$. The corresponding Feynman diagrams are given in Fig. 3. The transition of $b \to c\ell^-\bar{\nu}_\ell$ belongs to a SU(3) singlet, while the $b \to u\ell^-\bar{\nu}_\ell$ transition belongs to a SU(3) triplet and them can be described as $H_3$ which has the matrix elements $(H_3)^1 = V_{ub}$ and $(H_3)^2(3) = 0$ with the CKM matrix element $V_{ub}$. Except the two three-body decay channels shown in Fig. 3(c), i.e.,

$$T_{bb\bar{u}d}^- \to \bar{B}^0\ell^-\bar{\nu}_\ell, \quad T_{bb\bar{u}s}^- \to \bar{B}^0_s\ell^-\bar{\nu}_\ell,$$

most modes involve four particles in the final states. The semileptonic $T_{bb3}$ decays can be described through the hadron-level Hamiltonian which is written as

$$\mathcal{H} = a_3(T_{bb3})_{[ij]} \bar{B} \bar{D}_{ij}^f \bar{\ell}\nu_\ell + a_4(T_{bb3})_{[ij]} (H_3) \bar{B} \bar{D}_{ij}^f M_k^j \bar{\ell}\nu_\ell + a_5(T_{bb3})_{[ij]} (H_3)^k \bar{B} \bar{D}_{ij}^f M_k^j \bar{\ell}\nu_\ell.$$

(15)

Here the $a_i$s are the nonperturbative model-independent parameters. The $a_3$ and $a_5$ will be present in Fig. 3(b), and $a_4$ is related to the annihilation diagrams in Fig. 3(b). Through the Hamiltonian, we can obtain the decay amplitudes for different decay modes, which are given in Tab. III.

To satisfy the SU(3) flavor symmetry, one can ignore the effects of phase space when analyzing their decay widths. From Tab. III all six decay channels into a $B$ and a $D$ meson have the same decay widths. Besides, we can get the relations for the decays into a $B$ meson and a light meson:

$$\Gamma(T_{bb\bar{u}s}^- \to \bar{B}_s\pi^0\ell^-\bar{\nu}) = \frac{1}{2}\Gamma(T_{bb\bar{d}s}^0 \to \bar{B}^0 K^+\ell^-\bar{\nu}) = \frac{1}{2}\Gamma(T_{bb\bar{d}s}^0 \to \bar{B}_s\pi^+\ell^-\bar{\nu}),$$

FIG. 3: Feynman diagrams for semileptonic decays of doubly bottom tetraquark. Panels (a,b) correspond to the decays into a pair of mesons. In panel (c), there is only one meson in the final states. Panels (d,e) denote the decays into baryonic states. In panels (b,c,e), the two $b\bar{u}$ quarks in the initial state can annihilate, but such contributions are usually power suppressed.
TABLE III: Decay amplitudes for doubly bottom tetraquark $T_{bbq}$ semileptonic decays into mesons.

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| $T_{bbq}^{-} \to B^+ D_s^+ \ell^- \nu_l$ | $a_3 V_{cb}$ | $T_{bbq}^{-} \to B_s^0 D^- \ell^- \nu_l$ | $-a_3 V_{cb}$ |
| $T_{bbq}^{-} \to B^- D^+ \ell^- \nu_l$ | $a_3 V_{cb}$ | $T_{bbq}^{-} \to B_s^- D^- \ell^- \nu_l$ | $-a_3 V_{cb}$ |

2. Decays into a bottom baryon, a light anti-baryon and $\tau^\pm$

As shown in the last two panels in Fig. 3, the $T_{bbq}$ can transit into a bottom baryon and a light anti-baryon. Since the decuplet is anti-symmetric for light quarks in flavor space, the two spectator light quarks will not go into the decuplet. We write the Hamiltonian as:

$$
H = b_1(T_{bbq}|_{ij})\epsilon^{ijk}(F_8)^{i}_{a}b^c_q(H_{3})^k(F_{\pi})_{|kj|}\tilde{e}_l\ell_k + b_2(T_{bbq}|_{ij})\epsilon^{ijk}(F_8)^{i}_{a}b^c_q(H_{3})^k(F_{\pi})_{|kj|}\tilde{e}_l\ell_k \\
+ b_3(T_{bbq}|_{ij})\epsilon^{ijk}(F_8)^{i}_{a}b^c_q(H_{3})^k(F_{\pi})_{|kj|}\tilde{e}_l\ell_k + b_4(T_{bbq}|_{ij})\epsilon^{ijk}(F_8)^{i}_{a}b^c_q(H_{3})^k(F_{\pi})_{|kj|}\tilde{e}_l\ell_k \\
+ b_5(T_{bbq}|_{ij})\epsilon^{ijk}(F_8)^{i}_{a}b^c_q(H_{3})^k(F_{\pi})_{|kj|}\tilde{e}_l\ell_k + b_6(T_{bbq}|_{ij})\epsilon^{ijk}(F_8)^{i}_{a}b^c_q(H_{3})^k(F_{\pi})_{|kj|}\tilde{e}_l\ell_k \\
+ b_7(T_{bbq}|_{ij})\epsilon^{ijk}(F_8)^{i}_{a}b^c_q(H_{3})^k(F_{\pi})_{|kj|}\tilde{e}_l\ell_k + b_8(T_{bbq}|_{ij})\epsilon^{ijk}(F_8)^{i}_{a}b^c_q(H_{3})^k(F_{\pi})_{|kj|}\tilde{e}_l\ell_k.
$$

For convenience, we label the decay channels with different final states: class I for an octet baryon plus a heavy triplet baryon; class II for an octet baryon plus a heavy sextet baryon; class III for a decuplet light baryon plus a heavy sextet baryon. The last type of decays can occur only through the annihilation of $b \bar{u}$ shown in Fig. 3. The explicit amplitudes can be found in Tab. IV

From them, the relations for class I decays can be found:

$$
\Gamma(T_{bbq}^{-} \to \Sigma^0 \Lambda^0_b l^- \bar{\nu}) = \frac{1}{2} \Gamma(T_{bbq}^0 \to \Xi^+ \Xi_b^0 l^- \bar{\nu}) = \frac{1}{2} \Gamma(T_{bbq}^0 \to \Sigma^+ \Lambda_b^0 l^- \bar{\nu}),
$$

$$
\Gamma(T_{bbq}^{-} \to \Xi^0 \Xi^0_b l^- \bar{\nu}) = \frac{1}{2} \Gamma(T_{bbq}^0 \to \Xi^+ \Xi_b^- l^- \bar{\nu}) = \frac{1}{2} \Gamma(T_{bbq}^0 \to \Sigma^+ \Xi_b^0 l^- \bar{\nu}),
$$

The relations for class II decays become:

$$
\Gamma(T_{bbq}^{-} \to \Lambda^0 \Sigma^0_b l^- \bar{\nu}) = \frac{1}{6} \Gamma(T_{bbq}^0 \to \Xi^+ \Xi_b^0 l^- \bar{\nu}) = \frac{1}{2} \Gamma(T_{bbq}^0 \to \Lambda^0 \Sigma^0_b l^- \bar{\nu}) = \frac{1}{6} \Gamma(T_{bbq}^0 \to \Sigma^0 \Sigma^0_b l^- \bar{\nu}) = \frac{1}{6} \Gamma(T_{bbq}^0 \to \Sigma^0 \Sigma^0_b l^- \bar{\nu}),
$$

$$
\Gamma(T_{bbq}^{-} \to \Sigma^- \Sigma^0 b l^- \bar{\nu}) = 2 \Gamma(T_{bbq}^0 \to \Sigma^- \Sigma_b^0 l^- \bar{\nu}) = 2 \Gamma(T_{bbq}^0 \to \Sigma^- \Sigma_b^0 l^- \bar{\nu}) = \Gamma(T_{bbq}^{-} \to \Sigma^- \Sigma^0_b l^- \bar{\nu}),
$$

$$
\Gamma(T_{bbq}^{-} \to \Sigma^0 \Sigma^- b l^- \bar{\nu}) = 2 \Gamma(T_{bbq}^0 \to \Sigma^0 \Sigma_b^- l^- \bar{\nu}) = 4 \Gamma(T_{bbq}^0 \to \Sigma^0 \Sigma_b^- l^- \bar{\nu}) = 2 \Gamma(T_{bbq}^{-} \to \Sigma^0 \Sigma^- b l^- \bar{\nu}).
$$
### TABLE IV: Amplitudes for doubly bottom tetraquark $T_{bbq}q$ decays into a light anti-baryon octet plus a bottom baryon anti-triplet(class I) or sextet(class II), and a light anti-baryon decuplet plus a bottom baryon sextet (class III)

| Class | Amplitude | Class | Amplitude |
|-------|-----------|-------|-----------|
| $T_{bbq} \rightarrow \Sigma^{+} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell}$ | $\frac{2b_{q} + b_{b} + b_{c}}{2\sqrt{6}} V_{ub}$ | $\Gamma(T_{bbq} \rightarrow \Sigma^{+} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell})$ | $\frac{b_{q} V_{ub}}{\sqrt{6}}$ |
| $T_{bbq} \rightarrow \Sigma^{0} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell}$ | $\frac{2b_{q} + b_{b} - b_{c}}{\sqrt{3}} V_{ub}$ | $\Gamma(T_{bbq} \rightarrow \Sigma^{0} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell})$ | $\frac{b_{q} V_{ub}}{\sqrt{3}}$ |
| $T_{bbq} \rightarrow \Xi^{+} \Xi^{-}_{b} \ell^{-} \bar{\nu}_{\ell}$ | $- (b_{b} - 2b_{c}) V_{ub}$ | $\Gamma(T_{bbq} \rightarrow \Xi^{+} \Xi^{-}_{b} \ell^{-} \bar{\nu}_{\ell})$ | $\frac{b_{u} V_{ub}}{\sqrt{3}}$ |
| $T_{bbq} \rightarrow \Xi^{0} \Xi^{0}_{b} \ell^{-} \bar{\nu}_{\ell}$ | $- \frac{1}{2} (2b_{q} + 2b_{b} + b_{c}) V_{ub}$ | $\Gamma(T_{bbq} \rightarrow \Xi^{0} \Xi^{0}_{b} \ell^{-} \bar{\nu}_{\ell})$ | $\frac{b_{u} V_{ub}}{\sqrt{3}}$ |
| $T_{bbq} \rightarrow \Xi^{0} \Xi^{+}_{b} \ell^{-} \bar{\nu}_{\ell}$ | $- (b_{b} - 2b_{c}) V_{ub}$ | $\Gamma(T_{bbq} \rightarrow \Xi^{0} \Xi^{+}_{b} \ell^{-} \bar{\nu}_{\ell})$ | $\frac{b_{u} V_{ub}}{\sqrt{3}}$ |
| $T_{bbq} \rightarrow \Omega^{+} \Omega^{-}_{b} \ell^{-} \bar{\nu}_{\ell}$ | $- (b_{b} - 2b_{c}) V_{ub}$ | $\Gamma(T_{bbq} \rightarrow \Omega^{+} \Omega^{-}_{b} \ell^{-} \bar{\nu}_{\ell})$ | $\frac{b_{u} V_{ub}}{\sqrt{3}}$ |
| $T_{bbq} \rightarrow \bar{\Omega}^{0} \bar{\Omega}^{0}_{b} \ell^{-} \bar{\nu}_{\ell}$ | $- (b_{b} - 2b_{c}) V_{ub}$ | $\Gamma(T_{bbq} \rightarrow \bar{\Omega}^{0} \bar{\Omega}^{0}_{b} \ell^{-} \bar{\nu}_{\ell})$ | $\frac{b_{u} V_{ub}}{\sqrt{3}}$ |
| $T_{bbq} \rightarrow \bar{\Omega}^{+} \bar{\Omega}^{-}_{b} \ell^{-} \bar{\nu}_{\ell}$ | $- (b_{b} - 2b_{c}) V_{ub}$ | $\Gamma(T_{bbq} \rightarrow \bar{\Omega}^{+} \bar{\Omega}^{-}_{b} \ell^{-} \bar{\nu}_{\ell})$ | $\frac{b_{u} V_{ub}}{\sqrt{3}}$ |

The relations for class III decay widths are

$$
\Gamma(T_{bbq} \rightarrow \Sigma^{+} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell}) = \frac{1}{2} \left[ \Gamma(T_{bbq} \rightarrow \Sigma^{0} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell}) = \frac{1}{3} \Gamma(T_{bbq} \rightarrow \Omega^{+} \Omega^{-}_{b} \ell^{-} \bar{\nu}_{\ell}) = \frac{1}{2} \Gamma(T_{bbq} \rightarrow \bar{\Omega}^{0} \bar{\Omega}^{0}_{b} \ell^{-} \bar{\nu}_{\ell}) = \frac{1}{3} \Gamma(T_{bbq} \rightarrow \bar{\Omega}^{+} \bar{\Omega}^{-}_{b} \ell^{-} \bar{\nu}_{\ell})
\right]
$$

3. Decays into a charmed and bottomed baryon plus a light anti-baryon and $\ell \bar{\nu}_{\ell}$

$T_{bbq}$ can also decay into a light octet or anti-decoupel anti-baryon and a charmed and bottomed baryon for the $b \rightarrow c$ transition. After removing the forbidden constructions, the Hamiltonian becomes:

$$
\mathcal{H}_{eff} = b_{9}(T_{bbq})_{ij} e^{ij} (F_{8})_{ij}^{\frac{1}{2}} (\bar{F}_{bc})_{k} \bar{\ell} \bar{\nu}_{\ell} + b_{10}(T_{bbq})_{ij} e^{ik} (F_{8})_{ik}^{\frac{1}{2}} (\bar{F}_{bc})_{k} \bar{\ell} \bar{\nu}_{\ell}
$$

(17)

The amplitudes are derived and given in Tab. [V]. From them, the relations of the related decay widths are:

$$
\Gamma(T_{bbq} \rightarrow \Sigma^{0} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell}) = \frac{1}{6} \Gamma(T_{bbq} \rightarrow \Sigma^{0} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell}) = \frac{1}{6} \Gamma(T_{bbq} \rightarrow \Sigma^{+} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell}) = \frac{1}{4} \Gamma(T_{bbq} \rightarrow \Sigma^{+} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell})
$$

$$
= \frac{1}{6} \Gamma(T_{bbq} \rightarrow \Sigma^{0} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell}) = \frac{1}{3} \Gamma(T_{bbq} \rightarrow \Sigma^{0} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell}) = \frac{1}{6} \Gamma(T_{bbq} \rightarrow \Sigma^{+} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell}) = \frac{1}{6} \Gamma(T_{bbq} \rightarrow \Sigma^{+} \Omega^{0}_{b} \ell^{-} \bar{\nu}_{\ell})
$$
TABLE V: Amplitudes for doubly bottom tetraquark $T_{bbq\bar{q}}$ decays into a light anti-baryon octet and a charmed and bottomed baryon.

| Channel | Amplitude | Channel | Amplitude |
|---------|-----------|---------|-----------|
| $T^+_{bbd\bar{s}} \rightarrow \bar{\Lambda} \Xi_0^{'+} \ell^- \bar{\nu}_\ell$ | $- \frac{G_F V_{cb}}{\sqrt{2}} | c \bar{d} |$ | $T^-_{bbd\bar{s}} \rightarrow \sum \Xi^{'+} \ell^- \bar{\nu}_\ell (-2b_9 - b_{10}) V_{cb}$ |
| $T^0_{bbd\bar{s}} \rightarrow \sum \Xi_0^{'+} \ell^- \bar{\nu}_\ell$ | $\frac{G_F V_{cb}}{\sqrt{2}} | s \bar{d} |$ | $T^0_{bbd\bar{s}} \rightarrow \sum \Xi_0^{'+} \ell^- \bar{\nu}_\ell (-2b_9 - b_{10}) V_{cb}$ |
| $T^+_{bbd\bar{s}} \rightarrow \sum \Xi^{'+} \ell^- \bar{\nu}_\ell$ | $\frac{G_F V_{cb}}{\sqrt{2}} | c \bar{d} |$ | $T^0_{bbd\bar{s}} \rightarrow \sum \Xi^{'+} \ell^- \bar{\nu}_\ell (-2b_9 + b_{10}) V_{cb}$ |
| $T^0_{bbd\bar{s}} \rightarrow \sum \Xi_0^{'+} \ell^- \bar{\nu}_\ell$ | $\frac{G_F V_{cb}}{\sqrt{2}} | s \bar{d} |$ | $T^0_{bbd\bar{s}} \rightarrow \sum \Xi_0^{'+} \ell^- \bar{\nu}_\ell (2b_9 + b_{10}) V_{cb}$ |
| $T^+_{bbd\bar{s}} \rightarrow \sum \Xi^{'+} \ell^- \bar{\nu}_\ell$ | $\frac{G_F V_{cb}}{\sqrt{2}} | c \bar{d} |$ | $T^0_{bbd\bar{s}} \rightarrow \sum \Xi^{'+} \ell^- \bar{\nu}_\ell (2b_9 + b_{10}) V_{cb}$ |

Fig. 4: Feynman diagrams for semileptonic decays of doubly charmed tetraquark. Panel (a,b) correspond to the decays into a pair of mesons, and in panel (c), there is only one meson in the final state. In panels (b,c), the two $c$ quarks in the initial state can annihilate, and usually such contributions are power suppressed.

$$= \Gamma(T^0_{bbd\bar{s}} \rightarrow \bar{\Lambda} \Xi_0^{'+} l^- \bar{\nu}_l) = \frac{1}{3} \Gamma(T^0_{bbd\bar{s}} \rightarrow \sum \Xi^{'+}_0 l^- \bar{\nu}_l) = \frac{1}{6} \Gamma(T^0_{bbd\bar{s}} \rightarrow \bar{\Xi}_0^{'+} l^- \bar{\nu}_l).$$

II. $T_{ccq\bar{q}}$ decays

The effective Hamiltonian from the charm semileptonic decays into a light quark is:

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left[V_{cq}^* \bar{q}_l \gamma^\mu (1 - \gamma_5) c_l \gamma_\mu (1 - \gamma_5) \ell^+_l \right] + h.c.,$$

(18)

where $q = d, s$. A SU(3) triplet denoted as $H_3$ with the elements $(H_3)^1 = 0$, $(H_3)^2 = V_{cd}^*$ and $(H_3)^3 = V_{cs}^*$ is introduced for the heavy-to-light quark operators. We plotted the corresponding Feynman diagrams in Fig. 11. The triplet state $T_{cc3}$ can decay to a charmed meson plus $l^+ \nu$:

$$T_{cc\bar{u}d}^+/T_{ccd\bar{s}}^+ \rightarrow D^0 l^+ \nu, \quad T_{cc\bar{d}s}^+/T_{ccd\bar{s}}^+ \rightarrow D^+ l^+ \nu, \quad T_{cc\bar{s}d}^+/T_{ccd\bar{s}}^+ \rightarrow D^*_s l^+ \nu,$$

and their Feynman diagram are given in Fig. 11). Thus we obtained:

$$\Gamma(T_{cc\bar{u}d}^+ \rightarrow D^0 l^+ \nu) = \Gamma(T_{cc\bar{d}s}^+ \rightarrow D^+ l^+ \nu), \Gamma(T_{cc\bar{d}s}^+ \rightarrow D^*_s l^+ \nu) = \Gamma(T_{cc\bar{u}d}^+ \rightarrow D^0 l^+ \nu).$$
The effective Hamiltonian for decays into a charmed meson and a light meson is written as:

$$\mathcal{H} = a_1 (T_{cc3})_{ij} (H_3)^{ij} (D)^k M_i \bar{u}_k \ell + a_2 (T_{cc3})_{ij} (H_3)^{ij} (D)^k M_k \bar{u}_k \ell. \quad (20)$$

The related Feynman diagrams are plotted in Fig. 8(a,b), and the related results for the decay width relations are given in Table VI. Thus we have

$$\Gamma(T_{cc3}^+ \rightarrow D^0 \pi^0 l^+ \nu) = \frac{1}{2} \Gamma(T_{cc3}^+ \rightarrow D^+ \pi^- l^+ \nu) = \frac{1}{2} \Gamma(T_{cc3}^+ \rightarrow D^0 \pi^+ l^+ \nu),$$

$$\Gamma(T_{cc3}^+ \rightarrow D^0 K^+ l^+ \nu) = \Gamma(T_{cc3}^+ \rightarrow D^0 K^- l^+ \nu),$$

$$\Gamma(T_{cc3}^+ \rightarrow D^0 \bar{K}^0 l^+ \nu) = \Gamma(T_{cc3}^+ \rightarrow D^+ \bar{K}^- l^+ \nu).$$

### III. Semileptonic $T_{bcq\bar{q}}$ decays

Both the bottom and charm quarks can decay in the semileptonic $T_{bcq\bar{q}}$ decays. For the bottom decay in $T_{bcq\bar{q}}$, one can easily get the decay amplitude from those for $T_{bq\bar{q}}$ decays with $T_{bq\bar{q}} \rightarrow T_{bcq\bar{q}}, B \rightarrow D$. For the charm decays in $T_{bcq\bar{q}}$, one can easily get them from those for $T_{cq\bar{q}}$ decays with the replacement of $T_{cq\bar{q}} \rightarrow T_{bcq\bar{q}}, D \rightarrow B$. Thus we do not need to give the tedious results here.

### V. NON-LEPTONIC $T_{bq\bar{q}}$ DECAYS

Next, we will study the non-leptonic decay amplitudes. For the bottom quark decay, there are four types:

$$b \rightarrow c\bar{d}/s, \quad b \rightarrow c\bar{u}/s, \quad b \rightarrow u\bar{d}/s, \quad b \rightarrow q_1 \bar{q}_2 q_3, \quad (21)$$

where $q_i$ with $i = 1, 2, 3$ denote the light flavors. We will discuss these decay modes one by one in the following.
FIG. 5: Feynman diagrams for nonleptonic decays of doubly heavy tetraquark. (a,b) are corresponding to the two mesons W-exchange process; (c,d) are corresponding to the baryon and anti-baryon process; (e,f,g.h,i,j) are corresponding to three mesons process ((e,f) match with $J/\psi$ plus B meson and light meson, (g,h) match with $B_c$ plus D and light meson, (i,j) match with B plus $\bar D$ and D).

TABLE VII: Amplitudes for the W-exchange $T_{b\bar bq\bar q}$ decays induced by the $b \to c\bar d/s$ transition. Note that these amplitudes have an additional identical CKM factor $V_{cb}$.  

| channel | amplitude ($/V_{cb}$) | channel | amplitude ($/V_{cb}$) |
|---------|------------------------|---------|------------------------|
| $T_{b\bar b\bar s} \to B^- J/\psi$ | $-f_1 V_{cs}^*$ | $T_{b\bar b\bar d}^0 \to \bar B_s^0 J/\psi$ | $-f_1 V_{cs}^*$ |
| $T_{b\bar b\bar s} \to \bar B_s^- J/\psi$ | $f_1 V_{cd}$ | $T_{b\bar b\bar d}^- \to B^- J/\psi$ | $-f_1 V_{cd}^*$ |
| $T_{b\bar b\bar s} \to D^0 B_c^-$ | $-f_2 V_{cs}^*$ | $T_{b\bar b\bar d}^0 \to D^+ B_c^-$ | $-f_2 V_{cs}^*$ |
| $T_{b\bar b\bar s} \to D_s^+ B_c^-$ | $f_2 V_{cd}$ | $T_{b\bar b\bar d}^- \to D^0 B_c^-$ | $-f_2 V_{cd}^*$ |

I. $b \to c\bar d/s$ transition

1. W-exchange Topology

The transition $b \to c$ or $\bar d/\bar s \to \bar c$ can be signed as W-exchange topology, and we plotted the corresponding Feynman diagrams in Fig. FIG. We gave the decay amplitudes in Tab. VII from which the relations of decay widths are:

$$\Gamma(T_{b\bar b\bar s} \to B^- J/\psi) = \Gamma(T_{b\bar b\bar d}^0 \to \bar B_s^0 J/\psi), \Gamma(T_{b\bar b\bar d}^- \to B^- J/\psi), \Gamma(T_{b\bar b\bar d}^- \to B^- J/\psi), \Gamma(T_{b\bar b\bar s} \to D^0 B_c^-), \Gamma(T_{b\bar b\bar s} \to D^0 B_c^-), \Gamma(T_{b\bar b\bar d}^0 \to B_s^0 J/\psi), \Gamma(T_{b\bar b\bar d}^- \to B^- J/\psi).$$

$$\Gamma(T_{b\bar b\bar s} \to D_s^+ B_c^-) = \Gamma(T_{b\bar b\bar d}^0 \to D_s^+ B_c^-), \Gamma(T_{b\bar b\bar d}^- \to D^0 B_c^-).$$

}
TABLE VIII: Doubly bottom tetraquark $T_{bbd}$ decays into a anti-charmed anti-baryon triplet(class I) or anti-sextet(class II) and a charmed bottom baryon.

| class I | amplitude\((/V_{cb})\) | class II | amplitude\((/V_{cb})\) |
|---------|----------------------|---------|----------------------|
| $T_{bbd} \rightarrow \Xi_{bc}^{0} \Lambda_{c}^{0}$ | $-a_{1}V_{cs}$ | $T_{bbd} \rightarrow \Xi_{bc}^{0} \Sigma_{c}^{++}$ | $-a_{3}V_{cs}$ |
| $T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}$ | $2a_{2}V_{cd}$ | $T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}$ | $-\frac{a_{1}}{\sqrt{2}}V_{cs}$ |
| $T_{bbd} \rightarrow \Omega_{bc}^{0} \Xi_{c}^{-} = (a_{1} - 2a_{2}) V_{cd}$ | | $T_{bbd} \rightarrow \Omega_{bc}^{0} \Xi_{c}^{-}$ | | $-\frac{a_{1}}{\sqrt{2}}V_{cs}$ |
| $T_{bbd} \rightarrow \Xi_{bc}^{0} \Lambda_{c}^{0}$ | $a_{1}V_{cs}$ | $T_{bbd} \rightarrow \Xi_{bc}^{0} \Sigma_{c}^{-}$ | | $-\frac{a_{1}}{\sqrt{2}}V_{cs}$ |
| $T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}$ | $-a_{1}V_{cd}$ | $T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}$ | | $-\frac{a_{1}}{\sqrt{2}}V_{cs}$ |
| $T_{bbd} \rightarrow \Omega_{bc}^{0} \Xi_{c}^{-} = (a_{1} - 2a_{2}) V_{cd}$ | | $T_{bbd} \rightarrow \Omega_{bc}^{0} \Xi_{c}^{-}$ | | $-\frac{a_{1}}{\sqrt{2}}V_{cs}$ |
| $T_{bbd} \rightarrow \Xi_{bc}^{0} \Lambda_{c}^{0}$ | $2a_{2}V_{cs}$ | $T_{bbd} \rightarrow \Xi_{bc}^{0} \Sigma_{c}^{-}$ | | $-\frac{a_{1}}{\sqrt{2}}V_{cs}$ |
| $T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}$ | $-a_{1}V_{cd}$ | $T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}$ | | $-\frac{a_{1}}{\sqrt{2}}V_{cs}$ |

2. Decays into an anti-charmed anti-baryon and a charmed bottom baryon

The transition $b \rightarrow c\bar{c}d/s$ can lead to the process of an anti-baryon plus a baryon, where the anti-charmed anti-baryons form a triplet or anti-sextet and the charmed bottom baryon form a SU(3) triplet. The effective Hamiltonian is described as:

$$\mathcal{H} = a_{1}(T_{bbd})_{ijj}(F_{c})_{ij}(H_{3})^{i}(\mathcal{T}_{bc})_{k} + a_{2}(T_{bbd})_{ijj}(F_{c})_{ij}(H_{3})^{k}(\mathcal{T}_{bc})_{k}$$

$$+ a_{3}(T_{bbd})_{ijj}(F_{c})^{ij}(H_{3})^{i}(\mathcal{T}_{bc})_{k}. \quad (23)$$

The related decay amplitudes are given in Tab.VIII in which class I represents triplet anti-baryon plus the charmed bottom baryon in the final states, and class II denotes the anti-sextet anti-charmed plus the charmed bottom baryon.

For class I, we have the relations:

$$\Gamma(T_{bbd} \rightarrow \Omega_{bc}^{0} \Xi_{c}^{-}) = \Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}) = \Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Lambda_{c}^{0}) = \Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}) = \Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}).$$

For class II, we have:

$$\Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Sigma_{c}^{-}) = 2\Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Sigma_{c}^{-}) = 2\Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Sigma_{c}^{-}) = 2\Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Sigma_{c}^{-})$$

$$= \Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Sigma_{c}^{-}) = 2\Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Sigma_{c}^{-}).$$

For class II, we have:

$$\Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}) = 1/2 \Gamma(T_{bbd} \rightarrow \Omega_{bc}^{0} \Xi_{c}^{-}) = 1/2 \Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Sigma_{c}^{-})$$

$$= \Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}) = \Gamma(T_{bbd} \rightarrow \Xi_{bc}^{0} \Xi_{c}^{-}).$$

3. Decays into three mesons

The transition $b \rightarrow c\bar{c}d/s$ leads to three body decays where the effective Hamiltonian becomes:

$$\mathcal{H} = a_{1}(T_{bbd})_{ijj}(H_{3})^{i} M_{i}^{(B)} J/\psi + a_{2}(T_{bbd})_{ijj}(H_{3})^{i} M_{i}^{(B)} J/\psi$$

$$+ a_{3}(T_{bbd})_{ijj}(H_{3})^{i} M_{i}^{(D)} J/\psi + a_{4}(T_{bbd})_{ijj}(H_{3})^{i} M_{i}^{(D)} B_{c}.$$
TABLE IX: Doubly bottom tetraquark $T_{bbsq}$ decays into a $J/\psi$, a bottom meson and a light meson.

| Channel | Amplitude ($\langle V_{cB} \rangle$) |
|---------|-------------------------------------|
| $T_{bbsq}^{-} \rightarrow B^{-} \pi^{0} J/\psi$ | $-\frac{a_{2}V_{cB}}{\sqrt{2}}$ |
| $T_{bbsq}^{-} \rightarrow B^{-} \eta J/\psi$ | $-\frac{a_{2}V_{cB}}{\sqrt{2}}$ |
| $T_{bbsq}^{-} \rightarrow \bar{T}_{s} \pi^{0} J/\psi$ | $-a_{2}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow B^{-} K^{0} J/\psi$ | $a_{1}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow B^{-} K^{0} J/\psi$ | $-a_{2}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow B^{-} K^{0} J/\psi$ | $a_{1}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow B^{-} K^{0} J/\psi$ | $-a_{2}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow B^{-} K^{0} J/\psi$ | $a_{1}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow B^{-} K^{0} J/\psi$ | $-a_{2}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow B^{-} K^{0} J/\psi$ | $a_{1}V_{cB}$ |

TABLE X: Doubly bottom tetraquark $T_{bbsq}$ decays into a charmed $B$ meson, a charmed meson and a light meson.

| Channel | Amplitude ($\langle V_{cB} \rangle$) |
|---------|-------------------------------------|
| $T_{bbsq}^{-} \rightarrow D^{0} \pi^{0} B_{c}^{-}$ | $-\frac{a_{2}V_{cB}}{\sqrt{2}}$ |
| $T_{bbsq}^{-} \rightarrow D^{0} \eta B_{c}^{-}$ | $-\frac{a_{2}V_{cB}}{\sqrt{2}}$ |
| $T_{bbsq}^{-} \rightarrow D^{0} \eta B_{c}^{-}$ | $-a_{2}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow D^{0} \eta B_{c}^{-}$ | $a_{1}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow D^{0} \eta B_{c}^{-}$ | $-a_{2}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow D^{0} \eta B_{c}^{-}$ | $a_{1}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow D^{0} \eta B_{c}^{-}$ | $-a_{2}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow D^{0} \eta B_{c}^{-}$ | $a_{1}V_{cB}$ |
| $T_{bbsq}^{-} \rightarrow D^{0} \eta B_{c}^{-}$ | $-a_{2}V_{cB}$ |

$+a_{5}(T_{bbsq})_{ij}(H_{3})^{i} D_{k}(\bar{D})^{j} \bar{B}^{k} + a_{6}(T_{bbsq})_{ij}(H_{3})^{k} D_{k}(\bar{D})^{i} \bar{B}^{j} + a_{7}(T_{bbsq})_{ij}(H_{3})^{i} D_{k}(\bar{D})^{k} \bar{B}^{j}$. (24)

The $T_{bbq}$ decays amplitudes into $J/\psi$ plus a bottom meson and a light meson are given in Tab. IX from which we have:

$\Gamma(T_{bbq}^{-} \rightarrow B^{-} \pi^{0} J/\psi) = \frac{1}{8} \Gamma(T_{bbq}^{-} \rightarrow B^{0} \pi^{-} J/\psi) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} \pi^{0} J/\psi) = \frac{1}{8} \Gamma(T_{bbq}^{-} \rightarrow B^{-} \pi^{-} J/\psi)$,

$\Gamma(T_{bbq}^{0} \rightarrow B^{-} K^{+} J/\psi) = \Gamma(T_{bbq}^{0} \rightarrow D^{0} K^{0} B_{c}^{-})$, $\Gamma(T_{bbq}^{0} \rightarrow D^{0} K^{0} B_{c}^{-}) = \Gamma(T_{bbq}^{0} \rightarrow B^{-} K^{0} B_{c}^{-})$,

$\Gamma(T_{bbq}^{-} \rightarrow B^{-} K^{0} J/\psi) = 2\Gamma(T_{bbq}^{-} \rightarrow B^{-} K^{0} J/\psi) = \Gamma(T_{bbq}^{-} \rightarrow B^{-} K^{0} J/\psi) = 2\Gamma(T_{bbq}^{-} \rightarrow B^{-} K^{0} J/\psi)$.

Decay amplitudes for $T_{bbq}$ decays into $B_{c}$ meson plus a charmed meson and a light meson are given in Tab. IX. The decay width relations are:

$\Gamma(T_{bbq}^{-} \rightarrow D^{0} \pi^{0} B_{c}^{-}) = \frac{1}{2} \Gamma(T_{bbq}^{-} \rightarrow D^{+} \pi^{-} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{+} \pi^{-} B_{c}^{-}) = \frac{1}{8} \Gamma(T_{bbq}^{-} \rightarrow D^{0} \pi^{+} B_{c}^{-})$,

$\Gamma(T_{bbq}^{0} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{0} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{0} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{0} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$. 

$\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$. 

$\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$. 

$\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$. 

$\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$, $\Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-}) = \Gamma(T_{bbq}^{-} \rightarrow D^{0} K^{+} B_{c}^{-})$.
TABLE XI: Doubly bottom tetraquark \( T_{bbd\bar{s}} \) decays into a B meson, a charmed meson and an anti-charmed meson.

| channel                        | amplitude\((/V_{cb})\) |
|--------------------------------|-------------------------|
| \( T_{bb\bar{s}} \rightarrow D^+D^-B^- \) | \((a_5 - a_T) V_{cs}^*\) |
| \( T_{bb\bar{s}} \rightarrow D^0D^-B_s^- \) | \(a_6 V_{cd}^*\) |
| \( T_{bb\bar{s}} \rightarrow D_s^+D^-B^- \) | \(-a_T V_{cs}^*\) |
| \( T_{bb\bar{s}} \rightarrow D_s^0D^-B_s^- \) | \((a_5 + a_6) V_{cs}^*\) |

The \( T_{bbd\bar{s}} \) decays amplitudes into a bottom meson plus a charmed meson and an anti-charmed meson are given in Tab. XI. The decay widths become:

\[
\Gamma(T_{bb\bar{s}}^- \rightarrow D^0\pi^-B_-^-) = \Gamma(T_{bb\bar{s}}^- \rightarrow D^0K^-B_-^-) = 2\Gamma(T_{bb\bar{s}}^- \rightarrow D_s^+\pi^-B_-^-),
\]

\[
\Gamma(T_{bb\bar{s}}^- \rightarrow D^0\pi^0B_-^-) = \frac{1}{2}\Gamma(T_{bb\bar{s}}^- \rightarrow D^0K^0B_-^-) = \frac{1}{2}\Gamma(T_{bb\bar{s}}^- \rightarrow D^+\pi^-B_-^-).
\]

II. \( b \rightarrow c\bar{d}/s \) transition

1. Decays into a bottom meson and a charmed meson by W-exchange process

For the bottom quark decays to a charm quark, the effective Hamiltonian is given by:

\[
\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ts}^* \left[ C_1 Q_{11}^{cs} + C_2 Q_{12}^{cs} \right] + h.c.,
\]  

(25)

From the above Hamiltonian, the light quarks reduce an octet where the nonzero component is \((H_8)_{12}^2 = V_{ts}^*\) for the \( b \rightarrow c\bar{d} \) or \( b\bar{d} \rightarrow c\bar{u} \) transition, and \((H_8)_{23}^2 = V_{ts}^*\) for the \( b \rightarrow c\bar{s} \) or \( b\bar{s} \rightarrow c\bar{u} \) transition. We then obtain the
FIG. 6: Feynman diagrams for nonleptonic decays of doubly heavy tetraquark. (a,b) are corresponding with two mesons W-exchange process; (c,d) are corresponding with the baryon and anti-baryon process; (e,f,g,h,i,j) are corresponding with the B plus D and light meson process.

The hadron-level effective Hamiltonian

\[ \mathcal{H} = f_3(T_{bb\bar{s}})_{ij} (\overline{T})^j (H_S)^i_k (\overline{T})^k + f_4(T_{bb\bar{s}})_{ij} (\overline{T})^k (H_S)^i_k (\overline{T})^j. \]  

Decay amplitudes are collected in Tab. XII from which the relations of decay widths become:

\[ \frac{\Gamma(T_{bb\bar{s}}^0 \rightarrow B^- D_s^+)}{\Gamma(T_{bb\bar{s}}^0 \rightarrow B^- D_s^-)} = \frac{\Gamma(T_{bbd\bar{s}}^0 \rightarrow B^- D_s^0)}{\Gamma(T_{bbd\bar{s}}^0 \rightarrow B^- D_s^0)} = \frac{\Gamma(T_{bb\bar{s}}^- \rightarrow B^- D_s^0)}{\Gamma(T_{bb\bar{s}}^- \rightarrow B^- D_s^0)} = \frac{|V_{ud}^*|^2}{|V_{us}|^2}. \]

2. Decays into a light anti-baryon and a charmed bottom baryon

There are two kinds of multiplet for the final states, which lead to the Hamiltonian

\[ \mathcal{H} = a_4(T_{bb\bar{s}})_{ij} \epsilon^{ijkl}(F_S)_{ij}^k (H_S)_{j}^l (\overline{T})_{il} + a_5(T_{bb\bar{s}})_{ij} \epsilon^{ijkl}(F_S)_{ij}^k (H_S)_{j}^l (\overline{T})_{il} + a_6(T_{bbd\bar{s}})_{ij} \epsilon^{ijkl}(F_S)_{ij}^k (H_S)_{j}^l (\overline{T})_{il} + a_7(T_{bbd\bar{s}})_{ij} \epsilon^{ijkl}(F_S)_{ij}^k (H_S)_{j}^l (\overline{T})_{il} + a_8(T_{bb\bar{s}})_{ij} \epsilon^{ijkl}(F_S)_{ij}^k (H_S)_{j}^l (\overline{T})_{il} + a_9(T_{bbd\bar{s}})_{ij} \epsilon^{ijkl}(F_S)_{ij}^k (H_S)_{j}^l (\overline{T})_{il}. \]  

Decay amplitudes are presented in Tab. XIII, where different final states are labeled with class I or II. Note that the factor 2a7 + a7 always appear in the results, thus we remove the a7 in the final results. For class I, we have the relations:

\[ \Gamma(T_{bb\bar{s}}^- \rightarrow \Sigma^- \Omega_{bc}^0) = 2\Gamma(T_{bb\bar{s}}^0 \rightarrow \Sigma^0 \Omega_{bc}^0). \]

For class II, we have the relations:

\[ \Gamma(T_{bb\bar{s}}^- \rightarrow \overline{\Sigma}^- \Xi_{bc}^0) = 3\Gamma(T_{bb\bar{s}}^- \rightarrow \overline{\Sigma}^- \Xi_{bc}^0) = 3\Gamma(T_{bb\bar{s}}^- \rightarrow \Sigma^- \Omega_{bc}^0) = 3\Gamma(T_{bb\bar{s}}^- \rightarrow \overline{\Sigma}^0 \Xi_{bc}^0) \]
TABLE XIII: Doubly bottom tetraquark $T_{b\bar{b}q\bar{q}}$ decays into a light anti-baryon octet(class I) or anti-decuplet(class II) and a charmed bottom baryon.

| class I amplitude $(/V_{cb})$ | class II amplitude $(/V_{cb})$ |
|--------------------------------|--------------------------------|
| $T_{b\bar{b}\bar{u}\bar{s}} \rightarrow \Sigma^{-} \Xi_{bc}^{0}$ | $-2a_{7}V_{ud}$ |
| $T_{b\bar{b}\bar{s}\bar{u}} \rightarrow \Sigma^{+} \Xi_{bc}^{0}$ | $T_{b\bar{b}\bar{u}\bar{s}} \rightarrow \Sigma^{-} \Xi_{bc}^{0}$ |
| $T_{b\bar{b}\bar{s}\bar{u}} \rightarrow \Sigma^{+} \Xi_{bc}^{0}$ | $-2a_{7}V_{ud}$ |
| $T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{-} \Xi_{bc}^{0}$ | $-2a_{7}V_{ud}$ |
| $T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{+} \Xi_{bc}^{0}$ | $-2a_{7}V_{ud}$ |
| $T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{-} \Xi_{bc}^{0}$ | $-2a_{7}V_{ud}$ |
| $T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{+} \Xi_{bc}^{0}$ | $-2a_{7}V_{ud}$ |
| $T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{-} \Xi_{bc}^{0}$ | $-2a_{7}V_{ud}$ |
| $T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{+} \Xi_{bc}^{0}$ | $-2a_{7}V_{ud}$ |
| $T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{-} \Xi_{bc}^{0}$ | $-2a_{7}V_{ud}$ |
| $T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{+} \Xi_{bc}^{0}$ | $-2a_{7}V_{ud}$ |

$\Gamma(T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{-} \Xi_{bc}^{0}) = 2\Gamma(T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{+} \Xi_{bc}^{0}) = \frac{1}{3}\Gamma(T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{+} \Xi_{bc}^{0}) = \frac{1}{3}\Gamma(T_{b\bar{b}\bar{d}\bar{s}} \rightarrow \Sigma^{+} \Xi_{bc}^{0}).$

3. Decays into a bottom meson, a charmed meson and a light meson.

The effective Hamiltonian for decays into a bottom meson, a charmed meson and a light meson is

$$\mathcal{H} = a_{5}(T_{b\bar{b}d})_{ij} \bar{B}^{i}(D)^{j} M_{i}^{k} (H_{8})_{k}^{j} + a_{6}(T_{b\bar{b}d})_{ij} \bar{B}^{i}(D)^{j} M_{i}^{k} (H_{8})_{k}^{j} + a_{7}(T_{b\bar{b}d})_{ij} \bar{B}^{i}(D)^{j} M_{i}^{k} (H_{8})_{k}^{j} + a_{8}(T_{b\bar{b}d})_{ij} \bar{B}^{i}(D)^{j} M_{i}^{k} (H_{8})_{k}^{j} + a_{9}(T_{b\bar{b}d})_{ij} \bar{B}^{i}(D)^{j} M_{i}^{k} (H_{8})_{k}^{j} + a_{10}(T_{b\bar{b}d})_{ij} \bar{B}^{i}(D)^{j} M_{i}^{k} (H_{8})_{k}^{j} + a_{11}(T_{b\bar{b}d})_{ij} \bar{B}^{i}(D)^{j} M_{i}^{k} (H_{8})_{k}^{j}. \tag{28}$$

Decay amplitudes are collected in Tab. XIV where no relation for decay widths is found.

III. $b \rightarrow u\bar{c}d/s$ transition

1. Decays into two mesons by $W$-exchange process

We write the effective Hamiltonian for the anti-charm quark production as

$$\mathcal{H}_{eff} = \frac{G_{F}}{\sqrt{2}} V_{ub} V_{cb}^{*} \left[ C_{1} O_{1}^{uc} + C_{2} O_{2}^{uc} \right] + h.c. \tag{29}$$

According to the flavor SU(3) group, the $H_{8}^{0}$ is anti-symmetric while the $H_{6}$ is symmetric. The nonzero components are $(H_{8}^{0})_{13}^{31} = -(H_{3}^{0})_{31} = V_{cs}$ and $(H_{6})_{31}^{31} = (H_{6})_{31}^{31} = V_{cs}$, for the $b \rightarrow u\bar{c}d$ transition. When interchange of 2 ↔ 3 and $s ↔ d$, we get the nonzero components for the transition $b \rightarrow u\bar{c}d$. 


TABLE XIV: Doubly bottom tetraquark $T_{bbqg}$ decays into a bottom meson, a charmed meson and a light meson.

| Channel | Amplitude ($\langle V_{cb} \rangle$) |
|---------|-------------------------------------|
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \pi^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \eta$ | $(a_{b} - 2a_{c} - a_{b} - a_{c} - a_{b} - a_{c} 2a_{11} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \pi^{-}$ | $(a_{b} - a_{c} V_{u^u})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} K^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \eta$ | $(a_{b} - 2a_{c} - a_{b} - a_{c} - a_{b} - a_{c} 2a_{11} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \pi^{-}$ | $-(a_{b} + a_{10}) V_{u^u}$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} K^{-}$ | $-(a_{b} + a_{7} - a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c}) V_{ub}^\pi$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \eta$ | $-(a_{b} + a_{10}) V_{u^u}$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \pi^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} K^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \eta$ | $-(a_{b} + a_{10}) V_{u^u}$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \pi^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} K^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \eta$ | $-(a_{b} + a_{10}) V_{u^u}$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \pi^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} K^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \eta$ | $-(a_{b} + a_{10}) V_{u^u}$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \pi^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} K^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \eta$ | $-(a_{b} + a_{10}) V_{u^u}$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \pi^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} K^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \eta$ | $-(a_{b} + a_{10}) V_{u^u}$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} \pi^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |
| $T_{bbqg}^{-} \rightarrow B^{-} D^{0} K^{-}$ | $(a_{b} - a_{c} - a_{b} - a_{c} - a_{b} - a_{c} V_{ub})$ |

FIG. 7: Feynman diagrams for nonleptonic decays of doubly bottom tetraquark. (a,b) are corresponding with B plus $\overline{T}$ or $B_c$ plus light meson W-exchange process respectively. (c,d) are corresponding with the baryon and anti-baryon process, (e,f,i,j) math with B plus $\overline{T}$ and light meson. (g,h) match with $B_c$ plus two light mesons.
We get the effective Hamiltonian
\[
\mathcal{H} = f_5(T_{bb\bar{s}})_{ij}\left(\bar{B}^j k (H_0')^j (D)_k + f_6(T_{bb\bar{s}})_{ij}\left(\bar{B}^j k (H_3')^j (D)_k + f_7(T_{bb\bar{s}})_{ij}\left(H_0'^{||}(ik) (D)_k + f_8(T_{bb\bar{s}})_{ij}\left(H_3'^{||}(ik) M_B^f K c\right)
\right.
+ f_9(T_{bb\bar{s}})_{ij}\left( |V_{cb}|^2 / |V_{td}|^2 \right).
\]

For a bottom meson and an anti-charmed meson produced, the amplitudes are given in Tab. XVI. And we have:
\[
\frac{\Gamma(T_{bb\bar{s}} \rightarrow B^- D^-)}{\Gamma(T_{bb\bar{d}} \rightarrow B^- D^-)} = \frac{\Gamma(T_{bb\bar{s}} \rightarrow B^- D^-)}{\Gamma(T_{bb\bar{d}} \rightarrow B^- D^-)} = \frac{\Gamma(T_{bb\bar{d}} \rightarrow B^- D^-)}{\Gamma(T_{bb\bar{d}} \rightarrow B^- D^-)} = \frac{|V_{cb}|^2}{|V_{td}|^2}.
\]

\(T_{bb\bar{s}}\) decays amplitudes into a charmed bottom meson and a light meson for different channels are given in Tab. XVI. And we have
\[
\Gamma(T_{bb\bar{s}} \rightarrow B^- \pi^0) = 2\Gamma(T_{bb\bar{s}} \rightarrow B^- \pi^0).\]

2. Decays into an anti-charmed anti-baryon and a bottom baryon

The effective Hamiltonian for decays into an anti-charmed anti-baryon and a bottom baryon is
\[
\mathcal{H}_{eff} = b_1(T_{bb\bar{s}})_{ij}(F_{3\bar{d}})^j (H_0')^j (F_{b\bar{b}})_{ijkl} + b_2(T_{bb\bar{s}})_{ij}(F_{3\bar{d}})^j (H_3')^j (F_{b\bar{b}})_{ijkl} + b_3(T_{bb\bar{s}})_{ij}(F_{3\bar{d}})^j (H_0'^{||}) (F_{b\bar{b}})_{ijkl} + b_4(T_{bb\bar{s}})_{ij}(F_{3\bar{d}})^j (H_3'^{||}) (F_{b\bar{b}})_{ijkl} + b_5(T_{bb\bar{s}})_{ij}(F_{3\bar{d}})^j (H_0'^{||}) (F_{b\bar{b}})_{ijkl} + b_6(T_{bb\bar{s}})_{ij}(F_{3\bar{d}})^j (H_3'^{||}) (F_{b\bar{b}})_{ijkl} + b_7(T_{bb\bar{s}})_{ij}(F_{3\bar{d}})^j (H_0'^{||}) (F_{b\bar{b}})_{ijkl} + b_8(T_{bb\bar{s}})_{ij}(F_{3\bar{d}})^j (H_3'^{||}) (F_{b\bar{b}})_{ijkl} + b_9(T_{bb\bar{s}})_{ij}(F_{3\bar{d}})^j (H_0'^{||}) (F_{b\bar{b}})_{ijkl} + b_{10}(T_{bb\bar{s}})_{ij}(F_{3\bar{d}})^j (H_3'^{||}) (F_{b\bar{b}})_{ijkl} + b_{11}(T_{bb\bar{d}})_{ij}(F_{3\bar{d}})^j (H_0'^{||}) (F_{b\bar{b}})_{ijkl} + b_{12}(T_{bb\bar{d}})_{ij}(F_{3\bar{d}})^j (H_3'^{||}) (F_{b\bar{b}})_{ijkl}.
\]
TABLE XVII: Doubly bottom tetraquark $T_{bbqq}$ decays into an anti-charmed anti-baryon triplet and a bottom baryon anti-triplet (class I) or sextet (class II), an anti-charmed anti-baryon anti-sextet and a bottom baryon anti-triplet (class III) or sextet (class IV).

| class I  | amplitude ($/V_{ub}$) | class IV | amplitude ($/V_{ub}$) |
|----------|------------------------|----------|------------------------|
| $T_{bb\bar{q}d} \rightarrow \Lambda^+_c A^0_b$ | $(4b + b_2 - b_4) \ V_{cs}$ | $T^{0}_{bb\bar{d}} \rightarrow \Sigma^0_c \Xi^0_b$ | $-\frac{1}{3}b_{11}V_{cd}$ |
| $T_{bb\bar{q}d} \rightarrow \Xi^-_c A^0_b$ | $(b_2 + 4b_3 + b_4) \ V_{cs}$ | $T^{+}_{bb\bar{d}} \rightarrow \Sigma^+_c \Xi^0_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T_{bb\bar{d}} \rightarrow \Sigma^-_c \Xi^+_b$ | $(b_2 + 4b_3 - b_4) \ V_{cs}$ | $T^{0}_{bb\bar{d}} \rightarrow \Sigma^0_c \Xi^+_b$ | $\frac{1}{3}b_{11}V_{cd}$ |
| $T^{+}_{bb\bar{d}} \rightarrow \Xi^-_c \Xi^0_b$ | $2(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{0}_{bb\bar{d}} \rightarrow \Sigma^-_c \Xi^0_b$ | $(b_2 + 4b_3 - b_4) \ V_{cs}$ | $T^{0}_{bb\bar{d}} \rightarrow \Sigma^0_c \Xi^+_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{0}_{bb\bar{d}} \rightarrow \Sigma^-_c \Xi^0_b$ | $(b_2 + 4b_3 - b_4) \ V_{cs}$ | $T^{0}_{bb\bar{d}} \rightarrow \Sigma^0_c \Xi^+_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{+}_{bb\bar{d}} \rightarrow \Xi^0_c \Xi^-_b$ | $(b_2 + 4b_3 + b_4) \ V_{cs}$ | $T^{+}_{bb\bar{d}} \rightarrow \Sigma^+_c \Xi^0_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{+}_{bb\bar{d}} \rightarrow \Xi^0_c \Xi^-_b$ | $(b_2 + 4b_3 - b_4) \ V_{cs}$ | $T^{+}_{bb\bar{d}} \rightarrow \Sigma^+_c \Xi^0_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{0}_{bb\bar{d}} \rightarrow \Sigma^-_c \Xi^0_b$ | $(b_2 + 4b_3 - b_4) \ V_{cs}$ | $T^{0}_{bb\bar{d}} \rightarrow \Sigma^0_c \Xi^+_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{0}_{bb\bar{d}} \rightarrow \Sigma^-_c \Xi^0_b$ | $(b_2 + 4b_3 - b_4) \ V_{cs}$ | $T^{0}_{bb\bar{d}} \rightarrow \Sigma^0_c \Xi^+_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{+}_{bb\bar{d}} \rightarrow \Xi^0_c \Xi^-_b$ | $(b_2 + 4b_3 + b_4) \ V_{cs}$ | $T^{+}_{bb\bar{d}} \rightarrow \Sigma^+_c \Xi^0_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{+}_{bb\bar{d}} \rightarrow \Xi^0_c \Xi^-_b$ | $(b_2 + 4b_3 - b_4) \ V_{cs}$ | $T^{+}_{bb\bar{d}} \rightarrow \Sigma^+_c \Xi^0_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{0}_{bb\bar{d}} \rightarrow \Sigma^-_c \Xi^0_b$ | $(b_2 + 4b_3 - b_4) \ V_{cs}$ | $T^{0}_{bb\bar{d}} \rightarrow \Sigma^0_c \Xi^+_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{0}_{bb\bar{d}} \rightarrow \Sigma^-_c \Xi^0_b$ | $(b_2 + 4b_3 - b_4) \ V_{cs}$ | $T^{0}_{bb\bar{d}} \rightarrow \Sigma^0_c \Xi^+_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{+}_{bb\bar{d}} \rightarrow \Xi^0_c \Xi^-_b$ | $(b_2 + 4b_3 + b_4) \ V_{cs}$ | $T^{+}_{bb\bar{d}} \rightarrow \Sigma^+_c \Xi^0_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |
| $T^{+}_{bb\bar{d}} \rightarrow \Xi^0_c \Xi^-_b$ | $(b_2 + 4b_3 - b_4) \ V_{cs}$ | $T^{+}_{bb\bar{d}} \rightarrow \Sigma^+_c \Xi^0_b$ | $(2b_{10} + b_{11}) \ V_{cd}$ |

Different decay channel amplitudes are given in Tab. XVII where class I corresponds with triplet anti-baryon plus anti-triplet baryon; class II corresponds with triplet anti-baryon plus sextet baryon; class III corresponds with anti-sextet anti-baryon plus anti-triplet baryon; class IV corresponds with anti-sextet anti-baryon plus sextet baryon.

For class I, we obtain the relations of decay widths:

$$\frac{\Gamma(T_{bb\bar{d}} \rightarrow \Lambda^-_c A^0_b)}{\Gamma(T_{bb\bar{d}} \rightarrow \Xi^-_c \Xi^0_b)} = \frac{\Gamma(T_{bb\bar{d}} \rightarrow \Xi^-_c \Xi^0_b)}{\Gamma(T_{bb\bar{d}} \rightarrow \Lambda^-_c A^0_b)} = \frac{\Gamma(T_{bb\bar{d}} \rightarrow \Lambda^-_c \Xi^0_b)}{\Gamma(T_{bb\bar{d}} \rightarrow \Xi^-_c A^0_b)}$$
For class IV, the results are:

\[ \Gamma(T_{bb\bar{s}} \to s\bar{c}_b) = 2\Gamma(T_{bb\bar{u}} \to s\bar{c}_b), \Gamma(T_{bb\bar{s}} \to \Xi^0\Xi^-) = \frac{1}{2} \Gamma(T_{bb\bar{u}} \to \Xi^0\Omega^-), \]
\[ \Gamma(T_{bb\bar{u}} \to \Sigma^-\Lambda^0) = 2\Gamma(T_{bb\bar{d}} \to \Lambda^-\Sigma^0), \Gamma(T_{bb\bar{s}} \to \Xi^-\Sigma^+_b) = 2\Gamma(T_{bb\bar{u}} \to \Xi^-\Xi^0_b). \]

For class III, we obtain the relations of decay widths:

\[ \Gamma(T_{bb\bar{s}} \to s\bar{c}_b) = 2\Gamma(T_{bb\bar{u}} \to s\bar{c}_b), \Gamma(T_{bb\bar{s}} \to \Xi^0\Xi^-) = \frac{1}{2} \Gamma(T_{bb\bar{u}} \to \Xi^0\Omega^-), \]
\[ \Gamma(T_{bb\bar{u}} \to \Sigma^-\Lambda^0) = 2\Gamma(T_{bb\bar{d}} \to \Lambda^-\Sigma^0), \Gamma(T_{bb\bar{s}} \to \Xi^-\Sigma^+_b) = 2\Gamma(T_{bb\bar{u}} \to \Xi^-\Xi^0_b). \]

For class IV, the results are:

\[ \Gamma(T_{bb\bar{s}} \to s\bar{c}_b) = \Gamma(T_{bb\bar{u}} \to s\bar{c}_b) = \frac{1}{2} \Gamma(T_{bb\bar{s}} \to \Xi^0\Xi^-) = \frac{1}{2} \Gamma(T_{bb\bar{s}} \to \Xi^0\Xi^-) = \]
\[ \Gamma(T_{bb\bar{d}} \to \Sigma^-\Lambda^0) = \Gamma(T_{bb\bar{u}} \to \Sigma^-\Lambda^0) = \Gamma(T_{bb\bar{s}} \to \Xi^-\Sigma^+_b) = \Gamma(T_{bb\bar{u}} \to \Xi^-\Sigma^+_b). \]

3. Decays into three mesons

The effective Hamiltonian is written as

\[ \mathcal{H} = b_1(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_2(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_3(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_4(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_5(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_6(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_7(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_8(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_9(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_{10}(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_{11}(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_{12}(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} + b_{13}(T_{bb\bar{s}})_{ijkl} |\mathcal{B}| J_iD_iM_i^H(H^i_3)_{ijkl} \]

The decay amplitudes into a bottom meson plus an anti-charmed meson and a light meson are given in Tab. XVIII

\[ \Gamma(T_{bb\bar{d}} \to \Xi^0\Xi^-) = 2\Gamma(T_{bb\bar{s}} \to \Xi^0\Xi^-) \]
\[ \Gamma(T_{bb\bar{d}} \to B^-D^-) = 2\Gamma(T_{bb\bar{s}} \to \Xi^-\Xi^0_b) \]

Deriving the formulae, we get:

\[ \Gamma(T_{bb\bar{d}} \to \Xi^0\Xi^-) = 2\Gamma(T_{bb\bar{s}} \to \Xi^0\Xi^-), \Gamma(T_{bb\bar{d}} \to B^-D^-) = 2\Gamma(T_{bb\bar{s}} \to \Xi^-\Xi^0_b). \]
TABLE XVIII: Doubly bottom tetraquark $T_{bbq\bar{q}}$ decays into a bottom meson, an anti-charmed meson and a light meson.

| Channel | Amplitude ($\langle V_{ab} \rangle$) |
|---------|------------------|
| $T_{bbq\bar{q}} \rightarrow B^- T^0 D^+ K^0$ | $(-b_2 + b_5 + b_7 + b_9) V_{cs}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- T^0 D^- K^0$ | $-$ \[b_1 - 2 b_3 + b_6 - b_8 - b_9 \] $V_{cs}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- T^0 D^- \pi^0$ | $-$ \[b_1 - 2 b_3 + b_6 - b_8 - b_9 \] $V_{cs}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- T^0 D^- \eta$ | $-$ \[b_1 - 2 b_3 + b_6 - b_8 - b_9 \] $V_{cs}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- D^+ K^0$ | $(-b_4 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- D^0 K^0$ | $(-b_4 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- D^+ \pi^0$ | $(-b_4 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- D^+ \eta$ | $(-b_4 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- D^0 K^0$ | $(-b_4 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- D^+ \pi^0$ | $(-b_4 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- D^+ \eta$ | $(-b_4 + b_5 + b_8 + b_9) V_{cd}^*$ |

TABLE XIX: Doubly bottom tetraquark $T_{bbq\bar{q}}$ decays into an anti-charmed B meson and two light mesons.

| Channel | Amplitude ($\langle V_{ab} \rangle$) |
|---------|------------------|
| $T_{bbq\bar{q}} \rightarrow B^- K^- \pi^0$ | $(-b_1 + 2 b_2 + b_5 + b_6 + b_7 + b_9) V_{cs}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^0$ | $(-b_1 + 2 b_2 + b_5 + b_6 + b_7 + b_9) V_{cs}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^+$ | $2 (b_2 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^+$ | $2 (b_2 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^+$ | $2 (b_2 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^+$ | $2 (b_2 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^+$ | $2 (b_2 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^+$ | $2 (b_2 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^+$ | $2 (b_2 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^+$ | $2 (b_2 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^+$ | $2 (b_2 + b_5 + b_8 + b_9) V_{cd}^*$ |
| $T_{bbq\bar{q}} \rightarrow B^- K^- K^+$ | $2 (b_2 + b_5 + b_8 + b_9) V_{cd}^*$ |

\[ \frac{1}{2} \] 2015-01-01
The related decay amplitudes of an anti-charmed bottom meson plus two light mesons are given in Tab. XXII. Thus we obtain the relations as follows:

\[
\Gamma(T_{bb\bar{s}} \to B_c^+\pi^-) = \frac{1}{2}\Gamma(T_{bb\bar{s}} \to B_c^-\pi^+\pi^-), \Gamma(T_{bb\bar{d}} \to B_c^-\pi^0\pi^0) = \frac{1}{2}\Gamma(T_{bb\bar{d}} \to B_c^-\pi^+\pi^-),
\]

\[
\Gamma(T_{bb\bar{s}} \to B_c^-\pi^0K^0) = 3\Gamma(T_{bb\bar{s}} \to B_c^-K^0) = \frac{1}{2}\Gamma(T_{bb\bar{s}} \to B_c^-\pi^-K^+),
\]

\[
\Gamma(T_{bb\bar{d}} \to B_c^-\pi^+K^-) = 6\Gamma(T_{bb\bar{d}} \to B_c^-\pi^0K^-) = 2\Gamma(T_{bb\bar{d}} \to B_c^-\pi^0K^0),
\]

\[
\Gamma(T_{bb\bar{s}} \to B_c^-\pi^0\pi^0) = \frac{3}{2}\Gamma(T_{bb\bar{s}} \to B_c^-\pi^+\pi^-), \Gamma(T_{bb\bar{d}} \to B_c^-\pi^+\pi^-) = \frac{3}{2}\Gamma(T_{bb\bar{d}} \to B_c^-\pi^0\pi^0).
\]
IV. Charmless $b \to q_1q_2q_3$ transition

1. Decays into a bottom meson and a light meson by $W$-exchange process

The bottom to light quark transition leads to the effective Hamiltonian:

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ut}^* \left[ C_1 O_1^{uu} + C_2 O_2^{uu} \right] - V_{tb} V_{td}^* \left[ \sum_{i=3}^{10} C_i O_i \right] \right\} + \text{h.c.,}
\]

(33)

where $O_i$ is the weak four-fermion effective operators. The tree operators are described as a vector $H_3$, a tensor $H_7$, and a tensor $H_15$. The penguin operators are described as another vector $H_3$. The nonzero components of these operators are

\[
(H_3)^2 = 1, \quad (H_7)_1^{12} = -(H_7)_1^{23} = -(H_7)_3^{23} = 1,
\]

\[
2(H_15)_1^{12} = 2(H_15)_1^{23} = -3(H_15)_2^{23} = -6(H_15)_3^{23} = 6,
\]

(34)

for the non-strange decays. After doing the exchange of $2 \leftrightarrow 3$, we will get the formulae for the $\Delta S = 1(b \to s)$ decays. We get the effective hadron-level Hamiltonian for decays into the bottom anti-triplet

\[
\mathcal{H}_{\text{eff}} = f_{10}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + f_{11}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + f_{12}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + f_{13}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + f_{14}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + f_{15}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + f_{16}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right)
\]

(35)

The amplitudes are given in Tab. XXX for the $\Delta S = 0(b \to s)$ decays and Tab. XXXI for the $\Delta S = 1(b \to s)$ decays.

2. Decays into a light anti-baryon and a bottom baryon

There are four kinds of different final states which are light octet or anti-decuplet anti-baryon plus anti-triplet or sextet baryon respectively. Thus the Hamiltonian become

\[
\mathcal{H}_{\text{eff}} = c_{1}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + c_{2}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + c_{3}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + c_{4}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + c_{5}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + c_{6}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + c_{7}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + c_{8}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + c_{9}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right) + c_{10}(T_{bb3})_{ij} \left( \overline{B}^k (H_3)^i_3 (H_3)^j_3 (F_{bb3})^k_{33} \right)
\]

(36)
Decay amplitudes are given in Tab. XXII for the transition $b \to d$, Tab. XXXIII for the transition $b \to s$. We remove the similar contributions in the amplitudes, such as $c_1 - 2c_1, 2c_2 - c_3, c_2 - 2c_3 + c_5 + c_7 - c_8, c_2 + c_5 + c_9 + c_7 - c_8, 2d_2 + d_7, d_1 - d_2 + d_3 + d_9 - d_7 + d_2 + d_4$.

There is no relation of decay widths for class I. The relations of decay widths for class II become:

$$\Gamma(T_{bb\bar{b}s}^0 \to \Sigma^+ \Sigma^0_b) = 2\Gamma(T_{bb\bar{b}s}^0 \to \Sigma^- \Xi^0_b).$$

The relations of decay widths for class III become:

$$\Gamma(T_{bb\bar{b}s}^{-} \to \Sigma^- \Lambda_b^0) = \Gamma(T_{bb\bar{b}s}^{-} \to \Xi^- \Xi^0_b), \quad \Gamma(T_{bb\bar{b}s}^{-} \to \Sigma^- \Lambda_b^0) = 2\Gamma(T_{bb\bar{b}s}^{-} \to \Sigma^- \Xi^0_b),$$

$$\Gamma(T_{bb\bar{b}s}^0 \to \Sigma^0 \Lambda_b^0) = \frac{1}{2}\Gamma(T_{bb\bar{b}s}^0 \to \Xi^0 \Xi^0_b), \quad \Gamma(T_{bb\bar{b}s}^0 \to \Sigma^0 \Lambda_b^0) = \frac{1}{2}\Gamma(T_{bb\bar{b}s}^0 \to \Sigma^0 \Xi^0_b).$$
TABLE XXIII: Doubly bottom tetraquark $T_{bbq}$ decays into a light anti-baryon octet and a bottom baryon anti-triplet (class I) or sextet (class II), a light anti-baryon anti-decuplet and a bottom baryon anti-triplet (class III) or sextet (class IV) induced by the charmless $b \to s$ transition.

| Class | Amplitude |
|-------|-----------|
| $T_{bbq} \to \Lambda^+ \Xi_0^-$ | $c_1 + 2c_2 + 3c_3 - 4c_4 + 5c_5 - 6c_6 + 7c_7 - 8c_8$ |
| $T_{bbq} \to \Sigma^0 \Xi_0^-$ | $-c_1 - 2c_2 + 3c_3 - 4c_4 + 5c_5 - 6c_6 + 7c_7 - 8c_8$ |
| $T_{bbq} \to \Sigma^0 \Xi_0^-$ | $c_1 - 2c_2 - 3c_3 + 4c_4 + 5c_5 + 6c_6 - 7c_7 - 8c_8$ |
| $T_{bbq} \to \Sigma^0 \Xi_0^-$ | $-c_1 - 2c_2 - 3c_3 + 4c_4 + 5c_5 + 6c_6 - 7c_7 - 8c_8$ |
| $T_{bbq} \to \Sigma^0 \Xi_0^-$ | $c_1 - 2c_2 + 3c_3 - 4c_4 + 5c_5 - 6c_6 + 7c_7 - 8c_8$ |
| $T_{bbq} \to \Sigma^0 \Xi_0^-$ | $-c_1 - 2c_2 + 3c_3 - 4c_4 + 5c_5 - 6c_6 + 7c_7 - 8c_8$ |
| $T_{bbq} \to \Sigma^0 \Xi_0^-$ | $c_1 - 2c_2 - 3c_3 + 4c_4 + 5c_5 + 6c_6 - 7c_7 - 8c_8$ |
| $T_{bbq} \to \Sigma^0 \Xi_0^-$ | $-c_1 - 2c_2 - 3c_3 + 4c_4 + 5c_5 + 6c_6 - 7c_7 - 8c_8$ |

The relations of decay widths for class IV become:

$$\Gamma(T_{bbq}^- \to \Sigma^- \Xi^-_0^-) = \Gamma(T_{bbq}^0 \to \Xi^0 \Xi^-_0^-), \Gamma(T_{bbq}^- \to \Xi^0 \Xi^-_0^-) = \frac{1}{2} \Gamma(T_{bbq}^0 \to \Xi^0 \Xi^-_0^-).$$

The relations of decay widths for class I become:

$$\Gamma(T_{bbq}^- \to \Sigma^- \Xi^-_0^+), \Gamma(T_{bbq}^- \to \Sigma^- \Xi^-_0^+) = \Gamma(T_{bbq}^- \to \Xi^- \Xi^-_0^+), \Gamma(T_{bbq}^- \to \Xi^- \Xi^-_0^+) = \frac{1}{2} \Gamma(T_{bbq}^- \to \Xi^- \Xi^-_0^+).$$

The relations of decay widths for class II become:

$$\Gamma(T_{bbq}^- \to \Sigma^+ \Xi^-_0^-) = \frac{1}{2} \Gamma(T_{bbq}^- \to \Xi^+ \Xi^-_0^-).$$
The relations of decay widths for class III are:

\[ \Gamma(T_{bb\bar{u}\bar{s}} \to \Xi^0 \Lambda_b^0) = 2\Gamma(T_{bb\bar{d}s} \to \Xi^0 \Sigma_b^0), \Gamma(T_{bb\bar{d}s} \to \Xi^0 \Lambda_b^0) = 2\Gamma(T_{bb\bar{d}s} \to \Xi^0 \Sigma_b^0), \Gamma(T_{bb\bar{u}d} \to \Xi^0 \Xi_b^0), \Gamma(T_{bb\bar{u}d} \to \Xi^0 \Xi_b^0) = \Gamma(T_{bb\bar{u}d} \to \Xi^0 \Xi_b^0), \Gamma(T_{bb\bar{u}d} \to \Xi^0 \Xi_b^0). \]

The relations of decay widths for class IV become:

\[ \Gamma(T_{bb\bar{d}s} \to \Xi^0 \Lambda_b^0) = 2\Gamma(T_{bb\bar{d}s} \to \Xi^0 \Sigma_b^0), \Gamma(T_{bb\bar{d}s} \to \Xi^0 \Lambda_b^0) = 2\Gamma(T_{bb\bar{d}s} \to \Xi^0 \Sigma_b^0), \Gamma(T_{bb\bar{u}d} \to \Xi^0 \Xi_b^0), \Gamma(T_{bb\bar{u}d} \to \Xi^0 \Xi_b^0) = \Gamma(T_{bb\bar{u}d} \to \Xi^0 \Xi_b^0), \Gamma(T_{bb\bar{u}d} \to \Xi^0 \Xi_b^0). \]

3. Decays into a bottom meson and two light mesons

The Hamiltonian for decays into a bottom meson and two light mesons is

\[ \mathcal{H}_{eff} = c_1(T_{bb\bar{s}})_{ij} (\mathcal{B})^j M^i_k M^k_j (H_3)^j + c_2(T_{bb\bar{s}})_{ij} (\mathcal{B})^j M^i_k M^k_j (H_3)^j + c_3(T_{bb\bar{s}})_{ij} (\mathcal{B})^j M^i_k M^k_j (H_3)^j + c_4(T_{bb\bar{s}})_{ij} (\mathcal{B})^j M^i_k M^k_j (H_3)^j + c_5(T_{bb\bar{s}})_{ij} (\mathcal{B})^m M^i_k M^k_j (H_m)^ijk + c_6(T_{bb\bar{s}})_{ij} (\mathcal{B})^j M^i_k M^k_j (H_3)^j + c_7(T_{bb\bar{s}})_{ij} (\mathcal{B})^m M^i_k M^k_j (H_m)^ijk + c_8(T_{bb\bar{s}})_{ij} (\mathcal{B})^j M^i_k M^k_j (H_3)^j + c_9(T_{bb\bar{s}})_{ij} (\mathcal{B})^m M^i_k M^k_j (H_m)^ijk + c_{10}(T_{bb\bar{s}})_{ij} (\mathcal{B})^j M^i_k M^k_j (H_3)^j + c_{11}(T_{bb\bar{s}})_{ij} (\mathcal{B})^m M^i_k M^k_j (H_m)^ijk + c_{12}(T_{bb\bar{s}})_{ij} (\mathcal{B})^j M^i_k M^k_j (H_3)^j + c_{13}(T_{bb\bar{s}})_{ij} (\mathcal{B})^m M^i_k M^k_j (H_m)^ijk + c_{14}(T_{bb\bar{s}})_{ij} (\mathcal{B})^j M^i_k M^k_j (H_3)^j + c_{15}(T_{bb\bar{s}})_{ij} (\mathcal{B})^m M^i_k M^k_j (H_m)^ijk + c_{16}(T_{bb\bar{s}})_{ij} (\mathcal{B})^j M^i_k M^k_j (H_3)^j + c_{17}(T_{bb\bar{s}})_{ij} (\mathcal{B})^m M^i_k M^k_j (H_m)^ijk \]

The \( c_{11} \) and \( c_{11} \) terms give the same contribution which always contain the factor \( c_{11} - c_{11} \). We remove the \( c_{11} \) term in the expanded amplitudes. The amplitudes are given in Tab. [XXXIV] for the transition \( b \to d \) and Tab. [XXXV] for the transition \( b \to s \). The relations are:

\[ \Gamma(T_{bb\bar{u}s} \to B^- \pi^0 K^0) = 3\Gamma(T_{bb\bar{u}s} \to B^- K^0 \eta), \Gamma(T_{bb\bar{d}s} \to B^0 \pi^0 K^0) = 3\Gamma(T_{bb\bar{d}s} \to B^0 K^0 \eta). \]

\[ \Gamma(T_{bb\bar{d}s} \to B^- \pi^+ \pi^0) = 3\Gamma(T_{bb\bar{u}s} \to B^- \pi^0 \pi^-), \Gamma(T_{bb\bar{d}s} \to B^0 \pi^0 \pi^-) = 3\Gamma(T_{bb\bar{d}s} \to B^0 \pi^+ \pi^-). \]

VI. NON-LEPTONIC \( T_{bcq\bar{q}} \) DECAYS

The charm decays or (and) bottom decays can be present in the decays of \( T_{bcq\bar{q}} \). For the bottom decay, there is a new decay channel in which two heavy quarks of tetraquark interact by a virtual \( W \)-boson. The others can be obtained from those for \( T_{bcq\bar{q}} \) decays with \( B \to D \) and \( B_c \to J/\psi \). For the charm decays, the decay amplitudes can be obtained from those for \( T_{ccq\bar{q}} \) decays with the replacement of \( D \to B \) and \( J/\psi \to B_c \). Thus we do not present those results again.
The transitions $bc \rightarrow cd/s$ can lead to the tetraquark to decays to two mesons, whose corresponding Feynman diagrams are given in Fig. (S1). The Hamiltonian becomes

$$H_{\text{eff}} = f_1(T_{bc3}\langle ij\rangle(D)^kM^k_l(H_3)^{ij} + f_2(T_{bc3}\langle ij\rangle(D)^kM^k_l(H_3)^{ij}$$

$$+ f_3(T_{bc3}\langle ij\rangle M^k_lM^k_l(H_3)^{ij} + f_4(T_{bc3}\langle ij\rangle M^k_lM^k_l(H_3)^{ij})$$

$$+ f_5(T_{bc3}\langle ij\rangle M^k_lM^k_l(H_3)^{ij} + f_6(T_{bc3}\langle ij\rangle M^k_lM^k_l(H_3)^{ij})\right).$$

(38)

### 1. $bc \rightarrow ud/s$ or $bc \rightarrow cd/s$ transition

#### 1. decays into two mesons
TABLE XXV: Doubly bottom tetraquark $T_{bbgq}$ decays into a bottom meson and two light mesons induced by the charmless $b \to s$ transition.

| channel | amplitude |
|---------|-----------|
| $T_{bbgq}^0 \to B^- \pi^+ \pi^-$ | $2c_1 - c_3 - c_6 + c_7 + 4c_9 + 2c_{10} + 2c_{14} + 3c_{15} - c_{16}$ |
| $T_{bbgq}^0 \to B^- \pi^0 \pi^0$ | $2c_1 - c_3 - c_6 + c_7 - 2c_9 + 4c_9 + 2c_{10} + 2c_{13} + 3c_{14} + 4c_{16}$ |
| $T_{bbgq}^\pm \to B^- \pi^0 \eta$ | $- c_1 + c_2 + 2c_3 + c_6 + c_7 - 2c_{10} + 2c_{13} + 3c_{14} - 4c_{15} + 6c_{16}$ |
| $T_{bbgq}^\pm \to B^- K^+ K^-$ | $2c_1 + c_2 - c_3 - c_4 - 2c_5 - 2c_6 - c_8 + 4c_9 + 2c_{10} - c_{11} + 3c_{12} + c_{13} - 2c_{15} - 3c_{16}$ |
| $T_{bbgq}^0 \to B^- K^0 K^0$ | $2c_1 + c_2 - c_6 + 4c_9 + c_{11} - c_{12} - 3c_{13} - 3c_{14}$ |
| $T_{bbgq}^\pm \to B^- \pi^0 \eta$ | $6c_1 + 4c_2 - c_3 - 2c_4 + 4c_6 + 3c_7 + 12c_9 + 2c_{10} - 12c_{12} - 6c_{13} - 9c_{14} + 9c_{15} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- \pi^0 \pi^0$ | $-\sqrt{2} (c_7 + c_8 - 2 (c_{15} + c_{16}))$ |
| $T_{bbgq}^0 \to B^- \pi^0 \eta$ | $-\sqrt{2} (c_7 + c_8 - 2 (c_{15} + c_{16}))$ |
| $T_{bbgq}^0 \to B^- K^0 K^0$ | $-c_3 - c_4 - 2c_5 - 2c_6 + 6c_8 - 3c_{10} + 3c_{14} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- \pi^0 \eta$ | $-c_3 - c_4 + 2c_5 + c_6 + c_8 + 2c_{10} - c_{14} + 4c_{15} + 3c_{16}$ |
| $T_{bbgq}^0 \to B^- K^0 K^0$ | $-c_3 - c_4 - 2c_5 - 2c_6 + 6c_8 - 3c_{10} + 3c_{14} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- \pi^0 \eta$ | $-c_3 - c_4 - 2c_5 - 2c_6 + 6c_8 - 3c_{10} + 3c_{14} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- K^0 K^0$ | $2c_1 - c_3 + c_6 - c_7 - 4c_9 - 2c_{10} - 2c_{13} + 4c_{14} - 4c_{15} - 4c_{16}$ |
| $T_{bbgq}^0 \to B^- \pi^0 \eta$ | $2c_1 + c_2 + c_4 + c_8 + c_{10} + 3c_{13} + 2c_{14} + 3c_{15} + 2c_{16}$ |
| $T_{bbgq}^0 \to B^- K^0 K^0$ | $2c_1 + c_2 + c_6 - 4c_8 - c_{11} + 3c_{13} - c_{14} + c_{15} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- \pi^0 \eta$ | $2c_1 + c_2 + c_6 - 4c_8 - c_{11} + 3c_{13} - c_{14} + c_{15} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- K^0 K^0$ | $2c_1 + c_2 + c_6 - 4c_8 - c_{11} + 3c_{13} - c_{14} + c_{15} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- \pi^0 \eta$ | $2c_1 + c_2 + c_6 - 4c_8 - c_{11} + 3c_{13} - c_{14} + c_{15} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- K^0 K^0$ | $2c_1 + c_2 + c_6 - 4c_8 - c_{11} + 3c_{13} - c_{14} + c_{15} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- \pi^0 \eta$ | $2c_1 + c_2 + c_6 - 4c_8 - c_{11} + 3c_{13} - c_{14} + c_{15} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- K^0 K^0$ | $2c_1 + c_2 + c_6 - 4c_8 - c_{11} + 3c_{13} - c_{14} + c_{15} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- \pi^0 \eta$ | $2c_1 + c_2 + c_6 - 4c_8 - c_{11} + 3c_{13} - c_{14} + c_{15} + 4c_{16}$ |
| $T_{bbgq}^0 \to B^- K^0 K^0$ | $2c_1 + c_2 + c_6 - 4c_8 - c_{11} + 3c_{13} - c_{14} + c_{15} + 4c_{16}$ |

The decay amplitudes for $T_{bc3}$ decays into a charmed meson and a light meson are given in Tab. XXVI. We have the relations:

$$
\Gamma(T^0_{bc3} \to D^0 \pi^0) = \frac{1}{2} \Gamma(T^0_{bc3} \to D^+ \pi^-) = \frac{1}{2} \Gamma(T^+_{bc3} \to D^0 \pi^0) = \Gamma(T^+_{bc3} \to D^+ \pi^0),
$$

$$
\Gamma(T^0_{bc3} \to D^0 K^0) = \Gamma(T^0_{bc3} \to D^0 K^-), \Gamma(T^0_{bc3} \to D^+ K^0) = \Gamma(T^0_{bc3} \to D^+ K^-),
$$

$$
\Gamma(T^0_{bc3} \to D^0 K^0) = \Gamma(T^0_{bc3} \to D^+ K^-), \Gamma(T^0_{bc3} \to D^0 \eta) = \Gamma(T^0_{bc3} \to D^+ \eta),
$$

$$
\Gamma(T^0_{bc3} \to D^+ \pi^-) = \Gamma(T^0_{bc3} \to D^0 K^0) = 2\Gamma(T^0_{bc3} \to D^+ \pi^0),
$$

$$
\Gamma(T^0_{bc3} \to D^0 \pi^0) = \frac{1}{2} \Gamma(T^+_{bc3} \to D^+ K^0) = \frac{1}{2} \Gamma(T^0_{bc3} \to D^+ \pi^-).
$$
The decay amplitudes for $T_{bcar{q}}$ are:

$$\Gamma(T_{bc\bar{q}} \rightarrow D^0 \pi^0) = \frac{1}{2} \Gamma(T_{bc\bar{q}} \rightarrow \pi^+ K^-) = 3 \Gamma(T_{bc\bar{q}} \rightarrow \eta K^0),$$

$$\Gamma(T_{bc\bar{q}} \rightarrow \pi^0 K^+) = \frac{1}{2} \Gamma(T_{bc\bar{q}} \rightarrow \pi^+ K^0) = 3 \Gamma(T_{bc\bar{q}} \rightarrow \eta K^+),$$

$$\Gamma(T_{bc\bar{q}} \rightarrow \eta K^0) = \frac{3}{2} \Gamma(T_{bc\bar{q}} \rightarrow \pi^+ \eta) = 3 \Gamma(T_{bc\bar{q}} \rightarrow \pi^0 \eta),$$

$$\Gamma(T_{bc\bar{q}} \rightarrow \pi^0 \pi^0) = \frac{1}{2} \Gamma(T_{bc\bar{q}} \rightarrow \pi^+ \pi^-(\eta K^0)) = 3 \Gamma(T_{bc\bar{q}} \rightarrow \eta K^0),$$

The relations of decay widths are:

The decay amplitudes for $T_{bc\bar{q}}$ decays into two light mesons are given in Tab. XXVII. The relations of decay widths are:

| channel | amplitude/$\langle V_{cb} \rangle$ | channel | amplitude/$\langle V_{cb} \rangle$ |
|---------|-----------------------------------|---------|-----------------------------------|
| $T_{bc\bar{q}} \rightarrow D^0 \pi^0$ | $-\frac{f_2}{2}V_{cb}$ | $T_{bc\bar{q}} \rightarrow D^0 K^0$ | $f_1V_{cb}$ |
| $T_{bc\bar{q}} \rightarrow D^0 \eta$ | $-\frac{f_2}{2}V_{cb}$ | $T_{bc\bar{q}} \rightarrow D^+ \pi^-$ | $-f_2V_{cb}$ |
| $T_{bc\bar{q}} \rightarrow D^+ \pi^-$ | $-f_2V_{cb}$ | $T_{bc\bar{q}} \rightarrow D^+ K^-$ | $(f_1 + f_2)V_{cb}$ |
| $T_{bc\bar{q}} \rightarrow D^+ K^+$ | $-f_2V_{cb}$ | $T_{bc\bar{q}} \rightarrow D^0 \pi^0$ | $(f_1 + f_2)V_{cb}$ |
| $T_{bc\bar{q}} \rightarrow D^0 \pi^0$ | $-\frac{f_2}{2}V_{cb}$ | $T_{bc\bar{q}} \rightarrow D^0 K^0$ | $f_1V_{cb}$ |
| $T_{bc\bar{q}} \rightarrow D^0 \eta$ | $-\frac{f_2}{2}V_{cb}$ | $T_{bc\bar{q}} \rightarrow D^+ \pi^-$ | $-f_2V_{cb}$ |
| $T_{bc\bar{q}} \rightarrow D^+ K^-$ | $-f_1V_{cb}$ | $T_{bc\bar{q}} \rightarrow D^+ K^-$ | $-f_2V_{cb}$ |

2. decays into three mesons

The effective Hamiltonian which leads to the tetraquark to decays to three mesons is

$$\mathcal{H}_{e,f} = d_1(T_{bc\bar{q}})_{ij} \langle D^0 \rangle^j M^k \bar{M}^k (H_3)^i + d_2(T_{bc\bar{q}})_{ij} \langle D^0 \rangle^j M^k \bar{M}^k (H_3)^i + d_3(T_{bc\bar{q}})_{ij} \langle D^0 \rangle^j M^k \bar{M}^k (H_3)^i + d_4(T_{bc\bar{q}})_{ij} \langle D^0 \rangle^j M^k \bar{M}^k (H_3)^i + d_5(T_{bc\bar{q}})_{ij} M^k \bar{M}^m (H_3)^{ljm} + d_6(T_{bc\bar{q}})_{ij} M^k \bar{M}^m (H_3)^{ljm}$$
TABLE XXVII: Doubly heavy tetraquark $T_{bcqg}$ decays into two light mesons.

| Channel | Amplitude ($\langle V_{bc} \rangle$) | Channel | Amplitude ($\langle V_{bc} \rangle$) |
|---------|---------------------------------|---------|---------------------------------|
| $T_{bcuq}^0 \to \pi^+ \pi^-$ | $(-f_3 + 4f_5 + f_6) V_{bc}^*$ | $T_{bcuq}^0 \to \pi^0 \pi^0$ | $(-f_3 + 4f_5 + f_6) V_{bc}^*$ |
| $T_{bcuq}^0 \to \pi^0 K^0$ | $(f_3 + 2f_4 - f_5)V_{bc}^*$ | $T_{bcuq}^0 \to \pi^0 \eta$ | $(f_3 - 2f_4 + f_5)V_{bc}^*$ |
| $T_{bcuq}^0 \to \pi^- K^+$ | $-(f_3 + 2f_4 + f_6)V_{bc}^*$ | $T_{bcuq}^0 \to K^+ K^-$ | $-2(f_3 + f_4 - 2f_5)V_{bc}^*$ |
| $T_{bcuq}^0 \to K^+ K^0$ | $-(f_3 - 4f_5 + f_6)V_{bc}^*$ | $T_{bcuq}^0 \to K^0 \eta$ | $(f_3 + 2f_4 + f_5)V_{bc}^*$ |
| $T_{bcuq}^0 \to \eta \eta$ | $\frac{\pi}{3} (5f_3 + 4f_4 + 3(f_6 - 4f_5)) V_{bc}^*$ | $T_{bcuq}^0 \to \pi^+ \pi^0$ | $(f_3 + 2f_4 - f_6)V_{bc}^*$ |
| $T_{bcoq}^+ \to \pi^+ \eta$ | $\sqrt{\frac{2}{3}} (-f_3 - 2f_4 + f_6)V_{bc}^*$ | $T_{bcoq}^+ \to \pi^0 K^-$ | $\sqrt{\frac{2}{3}} (-f_3 + 2f_4 - f_6)V_{bc}^*$ |
| $T_{bcoq}^+ \to K^+ K^0$ | $(f_3 - 2f_4 + f_6)V_{bc}^*$ | $T_{bcoq}^+ \to \pi^+ \pi^0$ | $(f_3 + 2f_4 - f_6)V_{bc}^*$ |
| $T_{bcoq}^+ \to \pi^0 \pi^-$ | $-2(f_3 + f_4 - 2f_5)V_{bc}^*$ | $T_{bcoq}^+ \to \pi^- K^+$ | $-2(f_3 + f_4 - 2f_5)V_{bc}^*$ |
| $T_{bcoq}^+ \to \pi^0 \pi^0$ | $-2(f_3 + f_4 - 2f_5)V_{bc}^*$ | $T_{bcoq}^+ \to \pi^- K^+$ | $-2(f_3 + f_4 - 2f_5)V_{bc}^*$ |
| $T_{bcoq}^+ \to K^+ K^0$ | $-(f_3 + 4f_5 + f_6)V_{bc}^*$ | $T_{bcoq}^+ \to \pi^+ \pi^0$ | $(f_3 + 2f_4 + f_5)V_{bc}^*$ |

The corresponding Feynman diagrams are given in Fig. 31b,c,d. We give the amplitudes in Tab. XXVIII for a D meson plus two light mesons, and Tab. XXIX for three light mesons. Then the relations of decay widths for a D meson plus two light mesons are as follows:

$$
\begin{align*}
\Gamma(T_{bcuq}^0 \to D^0 \pi^0 \pi^0) &= \frac{1}{2} \Gamma(T_{bcuq}^0 \to D^0 \pi^+ \pi^-) = \frac{1}{2} \Gamma(T_{bcuq}^+ \to D^+ \pi^0 \pi^0), \\
\Gamma(T_{bcuq}^0 \to D^0 \pi^0 K^0) &= 3 \Gamma(T_{bcuq}^0 \to D^0 K^0 \eta) = \frac{3}{4} \Gamma(T_{bcuq}^0 \to D^+ \pi^- \eta) = \frac{1}{2} \Gamma(T_{bcuq}^0 \to D^+ K^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^+ K^0 K^-) &= 3 \Gamma(T_{bcuq}^0 \to D^0 \pi^0 K^-) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^0 \pi^- \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ K^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-) &= 3 \Gamma(T_{bcuq}^0 \to D^+ K^0 \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ \pi^0 \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ K^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^0 \pi^0 K^-) &= 6 \Gamma(T_{bcuq}^0 \to D^0 K^0 \eta) = 2 \Gamma(T_{bcuq}^0 \to D^0 \pi^- \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-) &= 6 \Gamma(T_{bcuq}^0 \to D^0 \pi^0 K^-) = 2 \Gamma(T_{bcuq}^0 \to D^0 \pi^- \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-) &= 6 \Gamma(T_{bcuq}^0 \to D^0 \pi^0 K^-) = 2 \Gamma(T_{bcuq}^0 \to D^0 \pi^- \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^0 K^0 \eta) &= 3 \Gamma(T_{bcuq}^0 \to D^0 K^0 \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^0 \pi^- \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^+ K^0 K^-) &= 6 \Gamma(T_{bcuq}^0 \to D^0 \pi^0 K^-) = 2 \Gamma(T_{bcuq}^0 \to D^0 \pi^- \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-) &= 6 \Gamma(T_{bcuq}^0 \to D^0 \pi^0 K^-) = 2 \Gamma(T_{bcuq}^0 \to D^0 \pi^- \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-) &= 6 \Gamma(T_{bcuq}^0 \to D^0 \pi^0 K^-) = 2 \Gamma(T_{bcuq}^0 \to D^0 \pi^- \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^0 K^0 \eta) &= \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^0 \pi^- \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-), \\
\Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-) &= \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^0 \pi^- \eta) = \frac{3}{2} \Gamma(T_{bcuq}^0 \to D^+ \pi^0 K^-). 
\end{align*}
$$

(39)
TABLE XXVIII: Doubly heavy tetraquark $T_{bc\bar{c}q}$ decays into a charmed meson and two light mesons.

| Channel | Amplitude $(V_{ch})$ | Channel | Amplitude $(V_{ch})$ |
|---------|----------------------|---------|----------------------|
| $T_{bc\bar{c}u}^0 \to D^0 \pi^+ \pi^-$ | $(2d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 \pi^0 \pi^0$ | $(d_2 + d_4) V_{cs}$ |
| $T_{bc\bar{c}u}^0 \to D^0 \pi^0 K^0$ | $(d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 \pi^0 \eta$ | $(d_2 + d_4) V_{cs}$ |
| $T_{bc\bar{c}u}^0 \to D^0 \pi^- K^+$ | $(d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 \pi^0 K^-$ | $(d_2 + d_4) V_{cs}$ |
| $T_{bc\bar{c}u}^0 \to D^0 K^+ K^-$ | $(d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 K^- K^-$ | $(d_2 + d_4) V_{cs}$ |
| $T_{bc\bar{c}u}^0 \to D^0 K^+ \bar{K}^-$ | $(d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 K^0 \bar{K}^0$ | $(d_2 + d_4) V_{cs}$ |
| $T_{bc\bar{c}u}^0 \to D^0 K^0 \bar{K}^0$ | $(d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 \pi^0 \eta$ | $(d_2 + d_4) V_{cs}$ |
| $T_{bc\bar{c}u}^0 \to D^0 K^+ \eta$ | $(d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 \pi^- \eta$ | $(d_2 + d_4) V_{cs}$ |
| $T_{bc\bar{c}u}^0 \to D^0 \pi^0 \eta$ | $(d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 \pi^- \eta$ | $(d_2 + d_4) V_{cs}$ |
| $T_{bc\bar{c}u}^0 \to D^0 K^- \eta$ | $(d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 \pi^0 \eta$ | $(d_2 + d_4) V_{cs}$ |
| $T_{bc\bar{c}u}^0 \to D^0 \pi^- \eta$ | $(d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 \pi^- \eta$ | $(d_2 + d_4) V_{cs}$ |
| $T_{bc\bar{c}u}^0 \to D^0 \pi^- \eta$ | $(d_2 + d_4) V_{cs}$ | $T_{bc\bar{c}u}^0 \to D^0 \pi^- \eta$ | $(d_2 + d_4) V_{cs}$ |

$\Gamma(T_{bc\bar{c}u}^0 \to D^+ K^0 \bar{K}^0) = \Gamma(T_{bc\bar{c}u}^0 \to D^0 K^+ K^-), \Gamma(T_{bc\bar{c}u}^0 \to D^0 K^0 \bar{K}^0) = \Gamma(T_{bc\bar{c}u}^0 \to D^+ K^+ K^-),$ 

The relations of decay widths for three light mesons can be written as:

\[
2\Gamma(T_{bc\bar{c}u}^0 \to \pi^+ \pi^0 \pi^-) = \frac{4}{3} \Gamma(T_{bc\bar{c}u}^0 \to \pi^0 \pi^0 \pi^0) = \frac{1}{2} \Gamma(T_{bc\bar{c}u}^0 \to \pi^+ \pi^+ \pi^-) = 2\Gamma(T_{bc\bar{c}u}^0 \to \pi^+ \pi^0 \pi^0) = \Gamma(T_{bc\bar{c}u}^0 \to \pi^+ \pi^+ \pi^-),
\]

\[
\Gamma(T_{bc\bar{c}u}^0 \to \pi^0 \pi^- K^+) = \frac{3}{2} \Gamma(T_{bc\bar{c}u}^0 \to \pi^0 K^0 \eta) = 3\Gamma(T_{bc\bar{c}u}^0 \to \pi^- K^+ \eta) = \Gamma(T_{bc\bar{c}u}^0 \to \pi^0 K^0 \eta) = \frac{3}{2} \Gamma(T_{bc\bar{c}u}^0 \to \pi^- K^+ \eta),
\]

\[
\Gamma(T_{bc\bar{c}u}^0 \to \pi^0 K^+ K^-) = 3\Gamma(T_{bc\bar{c}u}^0 \to K^+ K^- \bar{K}^0) = \frac{3}{4} \Gamma(T_{bc\bar{c}u}^0 \to \pi^+ K^- \eta) = \frac{3}{2} \Gamma(T_{bc\bar{c}u}^0 \to \pi^+ \eta K^0),
\]

\[
\Gamma(T_{bc\bar{c}u}^0 \to \pi^+ \pi^- K^+) = \frac{1}{2} \Gamma(T_{bc\bar{c}u}^0 \to K^+ K^+ K^+), \Gamma(T_{bc\bar{c}u}^0 \to \pi^+ \pi^- K^+) = \frac{1}{2} \Gamma(T_{bc\bar{c}u}^0 \to K^0 K^0 \bar{K}^0),
\]
TABLE XXIX: Doubly heavy tetraquark $T_{bcqq}$ decays into three light mesons.

| Channel | Amplitude ($V_{cb}$) |
|---------|----------------------|
| $T^{0}_{bcūs} \rightarrow π^+π^0π^−$ | $(-d_0 - 2d_7 + d_{10} + 2d_{11})V_{cb}$ |
| $T^{0}_{bcūs} \rightarrow π^+π^−0$ | $(-d_0 - 2d_7 + d_{10} + 2d_{11})V_{cb}$ |
| $T^{0}_{bcūs} \rightarrow π^+π^0π^0$ | $3(d_0 - 2d_7 + d_{10} + 2d_{11})V_{cb}$ |
| $T^{0}_{bcūs} \rightarrow π^+0π^−0$ | $(-2d_8 + 2d_7 + 2d_6 + 2d_0 + d_{10} - 2d_{11})V_{cb}$ |
| $T^{0}_{bcūs} \rightarrow π^+K^+K^−$ | $(-d_3 - 2d_6 + 2d_7 + 2d_0 + d_{10} - 2d_{11})V_{cb}$ |
| $T^{0}_{bcūs} \rightarrow π^+K^0K^0$ | $(-d_0 - 2d_7 - 2d_5 + d_{11} + 2d_{11})V_{cb}$ |

VII. NON-LEPTONIC $T_{ccqq}$ DECAYS

For the charm quark decays, we classified them into: Cabibbo allowed, singly Cabibbo suppressed, and doubly Cabibbo suppressed, i.e.

$c \rightarrow sū$,  $c \rightarrow uūd/sūs$,  $c \rightarrow dū$.  

(40)
For the Cabibbo allowed decays, the nonzero components are:

\[(H_\bar{C})^{31}_{12} = -(H_\bar{C})^{13}_{12} = 1, \quad (H_{15})^{31}_{12} = (H_{15})^{13}_{12} = 1.\]

(41)

For the singly Cabibbo-suppressed decays, the nonzero components become:

\[(H_\bar{C})^{31}_{13} = -(H_\bar{C})^{13}_{12} = -(H_\bar{C})^{21}_{12} = \sin(\theta_C), \quad (H_{15})^{31}_{13} = -(H_{15})^{13}_{12} = -(H_{15})^{21}_{12} = \sin(\theta_C).\]

(42)

At last, for the doubly Cabibbo suppressed decays, the nonzero formulae become:

\[(H_\bar{C})^{21}_{13} = -(H_\bar{C})^{12}_{13} = \sin^2(\theta_C), \quad (H_{15})^{21}_{13} = (H_{15})^{12}_{13} = \sin^2(\theta_C).\]

(43)

1. Decays into a charmed meson and a light meson by W-exchange process

Following the above formulae, the effective Hamiltonian for decays involving a charmed meson and a light meson is

\[H_{\text{eff}} = f_1(T_{cc3})_{ij} (D)^j M^k_i (H_{\bar{C}})_{kj}^{[ij]} + f_2(T_{cc3})_{ij} (D)^j M^k_i (H_{\bar{C}})_{kj}^{[ij]} + f_3(T_{cc3})_{ij} (D)^k M^j_i (H_{15})_{kj}^{[ij]} + f_4(T_{cc3})_{ij} (D)^k M^j_i (H_{15})_{kj}^{[ij]}.\]

(44)

The corresponding Feynman diagrams are shown in Fig. 9(a,b). We expand the Hamiltonian to obtain the decay amplitudes which are given in Tab. XXX. The relations of decay widths become:

\[\Gamma(T_{cc\bar{u}}^+ \rightarrow D^+ K^0) = \Gamma(T_{cc\bar{d}}^+ \rightarrow D^+ \bar{K}^0), \quad \Gamma(T_{cc\bar{u}}^{++} \rightarrow D_s^+ \pi^0) = \Gamma(T_{cc\bar{d}}^{++} \rightarrow D_s^+ \pi^0), \quad \Gamma(T_{cc\bar{u}}^{++} \rightarrow D^0 \pi^+).\]

\[\Gamma(T_{cc\bar{u}}^{++} \rightarrow D^0 K^+) = \Gamma(T_{cc\bar{u}}^{++} \rightarrow D^0 \bar{K}^+).\]
TABLE XXX: Dually charmed tetraquark $T_{ccqar{q}}$ decays into a charmed meson and a light meson. $sC$ is the abbreviation of $\sin(\theta_C)$.

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| $T_{ccuar{s}} \rightarrow D^0\pi^+$ | $f_1 - 2f_2 + f_4$ | $T_{ccuar{s}} \rightarrow D^0K^+$ | $(f_1 - 2f_2 + f_4) sC$ |
| $T_{ccuar{s}} \rightarrow D^+\pi^0$ | $\frac{2f_2 - f_3 - f_4}{\sqrt{2}}$ | $T_{ccuar{s}} \rightarrow D^+K^0$ | $(-2f_2 + f_3 - f_5) sC$ |
| $T_{ccuar{s}} \rightarrow D^+\eta$ | $\frac{-f_2 + f_3 - 3f_5}{\sqrt{2}}$ | $T_{ccuar{s}} \rightarrow D^+\pi^0$ | $(f_5 - f_2 - f_3 + f_5) sC$ |
| $T_{ccdar{s}} \rightarrow D_s^+K^0$ | $f_1 - 2f_2 - f_4$ | $T_{ccdar{s}} \rightarrow D_s^+\eta$ | $(-3f_1 + 4f_2 + f_3 - 3f_5) sC$ |
| $T_{ccdar{s}} \rightarrow D_s^+\pi^+$ | $f_1 - f_3 + f_4 - f_5$ | $T_{ccdar{s}} \rightarrow D_s^+K^+$ | $(f_1 - f_3 + f_4 - f_5) sC$ |
| $T_{ccdar{s}} \rightarrow D_s^+K^0$ | $f_1 - f_3 - f_4 + f_5$ | $T_{ccdar{s}} \rightarrow D_s^+\pi^+$ | $(f_1 - f_3 + f_4 - f_5) sC$ |
| $T_{ccdar{s}} \rightarrow D_s^+\eta$ | $\sqrt{2}f_5 sC^2$ | $T_{ccdar{s}} \rightarrow D_s^+K^0$ | $(f_1 - 2f_2 + f_3) sC$ |
| $T_{ccdar{s}} \rightarrow D_s^+\pi^0$ | $-\sqrt{2}f_5 sC^2$ | $T_{ccdar{s}} \rightarrow D_s^+\eta$ | $(-3f_1 + 2f_2 + f_3 + 3f_5) sC$ |

2. Decays into a charmed meson and two light mesons

For decays involving a charmed meson and two light mesons, the effective Hamiltonian is

$$\mathcal{H}_{eff} = b_1(T_{cc3})_{[ij]}(\overline{D})^iM_{i}^mM_{k}^m(H^2_{ik})^{[jk]} + b_2(T_{cc3})_{[ij]}(\overline{D})^mM_{k}^iM_{l}^i(H^2_{kl})^{[ijkl]} + b_4(T_{cc3})_{[ij]}(\overline{D})^iM_{i}^mM_{k}^m(H^2_{ik})^{[jkl]} + b_6(T_{cc3})_{[ij]}(\overline{D})^mM_{k}^iM_{l}^i(H^2_{kl})^{[ijkl]} + b_8(T_{cc3})_{[ij]}(\overline{D})^mM_{k}^iM_{l}^i(H^2_{kl})^{[ijkl]} + b_{10}(T_{cc3})_{[ij]}(\overline{D})^mM_{k}^iM_{l}^i(H^2_{kl})^{[ijkl]} + b_{12}(T_{cc3})_{[ij]}(\overline{D})^mM_{k}^iM_{l}^i(H^2_{kl})^{[ijkl]} + b_{14}(T_{cc3})_{[ij]}(\overline{D})^mM_{k}^iM_{l}^i(H^2_{kl})^{[ijkl]}.$$

(45)

These related Feynman diagrams with the three final states are shown in Fig. 3. For the production of two light mesons, some terms contain one QCD coupling while the others contain two QCD couplings. We found that $b_1$ and $\overline{b}_1$ terms give the same contribution which always contain the factor $b_1 - \overline{b}_1$. Then $\overline{b}_1$ terms are removed in the final results. The decay amplitudes are given in Tab. 3 and Tab. 4. Based on the expanded amplitudes, the relations become:

$$\Gamma(T_{ccuar{s}} \rightarrow D^0\pi^+K^0) = \Gamma(T_{ccuar{s}} \rightarrow D^0K^+\overline{K}^0), \Gamma(T_{ccuar{s}} \rightarrow D^+\pi^-K^+) = \Gamma(T_{ccuar{s}} \rightarrow D_s^+\pi^-K^-),$$

$$\Gamma(T_{ccuar{s}} \rightarrow D^+\pi^+\pi^-) = \Gamma(T_{ccuar{s}} \rightarrow D^+K^-\overline{K}^-), \Gamma(T_{ccuar{s}} \rightarrow D^+\pi^-K^0) = \Gamma(T_{ccuar{s}} \rightarrow D_s^+K^-\overline{K}^0),$$

$$\Gamma(T_{ccuar{s}} \rightarrow D^+\pi^+\pi^-) = \Gamma(T_{ccuar{s}} \rightarrow D_s^+K^-\overline{K}^-), \Gamma(T_{ccuar{s}} \rightarrow D_s^+\pi^-K^0) = \frac{1}{2}\Gamma(T_{ccuar{s}} \rightarrow D_s^+\pi^-\pi^-),$$

$$\Gamma(T_{ccuar{s}} \rightarrow D^+\pi^+\pi^+) = 4\Gamma(T_{ccuar{s}} \rightarrow D^0\pi^+\pi^+), \Gamma(T_{ccuar{s}} \rightarrow D_s^+\pi^+\pi^+) = 4\Gamma(T_{ccuar{s}} \rightarrow D_s^+\pi^+\pi^+),$$

$$\Gamma(T_{ccuar{s}} \rightarrow D^+K^0\overline{K}^0) = \Gamma(T_{ccuar{s}} \rightarrow D_s^+K^0\overline{K}^0).$$
TABLE XXXI: Doubly charmed tetraquark $T_{cqcq}$ decays into a charmed meson and two light mesons. $sC$ is the abbreviation of $\sin(\theta_c)$.

| channel | amplitude | channel | amplitude |
|---------|-----------|---------|-----------|
| $T_{ccqs}^+ \rightarrow D^0 \pi^+ \pi^0$ | $b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q$ | $T_{ccqs}^+ \rightarrow D^0 K^+ K^0$ | $(b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q) sC^2$ |
| $T_{ccqs}^+ \rightarrow D^0 \pi^+ \eta$ | $2b_4 + b_4 + b_4 + 2b_q + b_q + b_q + b_q + b_q$ | $T_{ccqs}^+ \rightarrow D^0 K^0 K^0$ | $2(b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q) sC^2$ |
| $T_{ccqs}^+ \rightarrow D^0 K^+ \eta$ | $b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q$ | $T_{ccqs}^+ \rightarrow D^0 \pi^+ K^0$ | $2(b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q) sC^2$ |
| $T_{ccqs}^+ \rightarrow D^+ \pi^+ \pi^+$ | $b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q$ | $T_{ccqs}^+ \rightarrow D^+ \pi^+ K^+$ | $(b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q) sC^2$ |
| $T_{ccqs}^+ \rightarrow D^0 K^+ K^-$ | $2(b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q)$ | $T_{ccqs}^+ \rightarrow D^0 K^+ K^0$ | $2(b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q) sC^2$ |
| $T_{ccqs}^+ \rightarrow D^+ K^+ K^-$ | $b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q$ | $T_{ccqs}^+ \rightarrow D^+ K^+ K^0$ | $2(b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q) sC^2$ |
| $T_{ccqs}^+ \rightarrow D^0 \pi^+ \eta$ | $b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q$ | $T_{ccqs}^+ \rightarrow D^0 \pi^+ K^0$ | $(b_4 + b_4 + b_4 + b_4 + b_q + b_q + b_q + b_q + b_q) sC^2$ |

VIII. GOLDEN DECAY CHANNELS

In order to hunt for the doubly heavy tetraquarks, the golden decay channels are very useful. So we list them and give an estimation of the decay branching fractions in this section. In the following lists, a hadron is a general particle and can be replaced by the states with the identical quark contents. The light pseudoscalar may replace by the light vector meson such as replacing the $K^0$ by $\overline{K}^0$ which decays into $K^- \pi^+$. The modes involving the neutral mesons $\pi^0, \eta, \rho^0, \omega$ are removed because a neutral meson is difficult to be reconstructed at hadron-hadron colliders.

I. $T_{ccqq}$

For the $T_{ccqq}$ decays, we collected Cabibbo allowed decays in Tab. XXXIII. From the data of the D meson decays, we conclude that these Cabibbo allowed decay channels in Tab. XXXIII may lead to branching fractions at a few percent level. Note that to reconstruct the final charm meson, another factor of $10^{-3}$ is required.

II. $T_{ccqq}$

To reconstruct the $T_{ccqq}$ tetraquark, lists of possible modes are given in Tab. XXXIV. The decay width is dominant by the charm quark decay from the estimation of the magnitudes of CKM matrix elements. For the charm quark decay, the typical branching fractions in Tab. XXXIV are estimated to be a few percents. If the bottom quark decays,
TABLE XXXII: Doubly charmed tetraquark $T_{ccqar{q}}$ decays into a charmed meson and two light mesons. $sC$ is the abbreviation of $sin(\theta_C)$.

| channel | amplitude |
|---------|-----------|
| $T_{ccqar{q}} \rightarrow D^0 \pi^+ K^0$ | $(b_1 - b_4 - 2b_7 - b_8 + b_9 + b_{11}) sC$ |
| $T_{ccqar{q}}^+ \rightarrow D^0 \pi^+ K^+$ | $(b_1 + b_4 + 2b_7 + 2b_8 + b_9 + b_{11}) sC$ |
| $T_{ccqar{q}}^- \rightarrow D^0 \pi^+ \eta$ | $-(b_1 + b_4 + 2b_7 + 2b_8 + b_9 + b_{11}) sC$ |
| $T_{ccqar{q}}^+ \rightarrow D^0 \pi^0 K^+$ | $(b_1 + b_4 + 2b_7 + 2b_8 + b_9 + b_{11}) sC$ |
| $T_{ccqar{q}}^- \rightarrow D^+ \pi^- K^+$ | $(2b_2 + b_3 - b_5 + 2b_7 - b_{10} - b_{12}) sC$ |
| $T_{ccqar{q}}^+ \rightarrow D^+ \pi^- \eta$ | $-(b_1 + b_3 - 4b_6 + b_8 + b_9 + b_{10}) sC$ |
| $T_{ccqar{q}}^- \rightarrow D^+ \pi^0 \eta$ | $(b_1 - b_3 + 4b_6 + b_8 - b_{10}) sC$ |
| $T_{ccqar{q}}^+ \rightarrow D_1^+ \pi^+ K^+$ | $(b_1 + 2b_2 + 2b_3 + b_4 - 4b_6 + 2b_7 - b_8 - b_9 + b_{12}) sC$ |
| $T_{ccqar{q}}^- \rightarrow D_1^+ \pi^+ \eta$ | $-(b_1 + 2b_2 + 2b_3 + b_4 - 4b_6 + 2b_7 - b_8 - b_9 + b_{12}) sC$ |
| $T_{ccqar{q}}^+ \rightarrow D_1^+ \pi^0 K^+$ | $(b_1 + 2b_2 + 2b_3 + b_4 - 4b_6 + 2b_7 - b_8 + b_{11} + b_{12}) sC$ |
| $T_{ccqar{q}}^- \rightarrow D_1^+ \pi^0 \eta$ | $-(b_1 + 2b_2 + 2b_3 + b_4 - 4b_6 + 2b_7 - b_8 + b_{11} + b_{12}) sC$ |
| $T_{ccqar{q}}^+ \rightarrow D_2^+ \pi^+ K^0 \bar{K}^+$ | $-(b_1 + b_4 + 2b_7 - b_8 + b_9 + b_{11} + b_{12}) sC$ |
| $T_{ccqar{q}}^- \rightarrow D_2^+ \pi^+ \eta$ | $-(b_1 + b_4 + 2b_7 - b_8 + b_9 + b_{11} + b_{12}) sC$ |
| $T_{ccqar{q}}^+ \rightarrow D_2^+ \pi^0 K^+$ | $(b_1 + 2b_2 + 2b_3 + b_4 - 4b_6 - 2b_7 + b_8 - b_9 - b_{11} + b_{12}) sC$ |
| $T_{ccqar{q}}^- \rightarrow D_2^+ \pi^0 \eta$ | $(b_1 + 2b_2 + 2b_3 + b_4 - 4b_6 - 2b_7 + b_8 + b_{11} + b_{12}) sC$ |
| $T_{ccqar{q}}^+ \rightarrow D^3_s \pi^+ \bar{K}^0$ | $(b_1 + b_2 + b_4 + b_5 - 4b_6 - 2b_7 + b_8 - b_9 - b_{11} + b_{12}) sC$ |
| $T_{ccqar{q}}^- \rightarrow D^3_s \pi^+ \eta$ | $(b_1 + b_2 + b_4 + b_5 - 4b_6 - 2b_7 + b_8 + b_{11} + b_{12}) sC$ |

TABLE XXXIII: Cabibbo allowed $T_{ccqar{q}}$ decays. $\bar{K}^0$ can be replaced by vector meson $\bar{K}^{*0}$.

Two body decays

| Two body decays |
|-----------------|
| $T_{ccqar{q}} \rightarrow D^0 \pi^+ \nu$ | $T_{ccqar{q}} \rightarrow D^0 \pi^- \nu$ |
| $T_{ccqar{q}}^+ \rightarrow D^+ \pi^+ \nu$ | $T_{ccqar{q}}^+ \rightarrow D^+ \pi^- \nu$ |
| $T_{ccqar{q}}^- \rightarrow D^0 \pi^+ \bar{\nu}$ | $T_{ccqar{q}}^- \rightarrow D^0 \pi^- \bar{\nu}$ |
| $T_{ccqar{q}}^+ \rightarrow D^+ \pi^+ \bar{\nu}$ | $T_{ccqar{q}}^+ \rightarrow D^+ \pi^- \bar{\nu}$ |

Three body decays

| Three body decays |
|-------------------|
| $T_{ccqar{q}} \rightarrow D^0 \pi^- \nu$ | $T_{ccqar{q}} \rightarrow D^0 \pi^- \nu$ |
| $T_{ccqar{q}}^+ \rightarrow D^+ \pi^- K^0 \bar{\nu}$ | $T_{ccqar{q}}^+ \rightarrow D^+ \pi^- \bar{\nu}$ |
| $T_{ccqar{q}}^- \rightarrow D^0 \pi^- \bar{\nu}$ | $T_{ccqar{q}}^- \rightarrow D^0 \pi^- \bar{\nu}$ |
| $T_{ccqar{q}}^+ \rightarrow D^+ \pi^- K^0 \bar{\nu}$ | $T_{ccqar{q}}^+ \rightarrow D^+ \pi^- \bar{\nu}$ |
| $T_{ccqar{q}}^- \rightarrow D^0 \pi^- \bar{\nu}$ | $T_{ccqar{q}}^- \rightarrow D^0 \pi^- \bar{\nu}$ |
### TABLE XXXIV: Cabibbo allowed $T_{bcar{q}}$ decays. $\bar{K}^0$ can be replaced by vector meson $K^+\pi^-$.  

| Two body decays (charm decays) |  |
|---------------------------------|--|
| $T_{bc\bar{u}\bar{s}}^0 \rightarrow B^+\pi^-\bar{T}$  | $T_{bc\bar{u}\bar{s}}^0 \rightarrow B^+\pi^-\overline{B}$  |
| $T_{bc\bar{d}\bar{s}}^0 \rightarrow B^+\pi^-\bar{T}$  | $T_{bc\bar{d}\bar{s}}^0 \rightarrow B^+\pi^-\overline{B}$  |
| $T_{bc\bar{d}\bar{s}}^0 \rightarrow B^+\pi^-\bar{T}$  | $T_{bc\bar{d}\bar{s}}^0 \rightarrow B^+\pi^-\overline{B}$  |

| Three body decays (charm decays) |  |
|---------------------------------|--|
| $T_{bc\bar{d}\bar{s}}^0 \rightarrow B^+\pi^-\bar{T}$  | $T_{bc\bar{d}\bar{s}}^0 \rightarrow B^+\pi^-\overline{B}$  |
| $T_{bc\bar{d}\bar{s}}^0 \rightarrow B^+\pi^-\bar{T}$  | $T_{bc\bar{d}\bar{s}}^0 \rightarrow B^+\pi^-\overline{B}$  |
| $T_{bc\bar{d}\bar{s}}^0 \rightarrow B^+\pi^-\bar{T}$  | $T_{bc\bar{d}\bar{s}}^0 \rightarrow B^+\pi^-\overline{B}$  |

| Two body decays (bottom decays) |  |
|---------------------------------|--|
| $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+J/\psi$  | $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+\bar{B}^0$  |
| $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+J/\psi$  | $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+\bar{B}^0$  |
| $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+J/\psi$  | $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+\bar{B}^0$  |

| Three body decays (bottom decays) |  |
|---------------------------------|--|
| $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+\pi^-\bar{T}$  | $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+\pi^-\overline{B}$  |
| $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+\pi^-\bar{T}$  | $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+\pi^-\overline{B}$  |
| $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+\pi^-\bar{T}$  | $T_{bc\bar{u}\bar{s}}^0 \rightarrow D^+\pi^-\overline{B}$  |

The branching fraction might be smaller than $10^{-3}$. Note that to reconstruct the final charm or bottom meson, another factor of $10^{-3}$ is also required.
TABLE XXXV: Cabibbo allowed $T_{bar{q}q}$ decays. $\bar{K}^0$ can be replaced by vector meson $\bar{K}^{*0}$.

| Two body decays |
|------------------|
| $T_{bb\bar{s}} \rightarrow B^- J/\psi$ | $T_{bb\bar{a}} \rightarrow B^0 B^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- B^0 D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- B^0 D^- \bar{c}$ |
| $T_{bb\bar{a}} \rightarrow \Xi^- \Lambda_c^0$ | $T_{bb\bar{a}} \rightarrow \Xi^- \Lambda_c^0$ | $T_{bb\bar{a}} \rightarrow \Xi^- \Lambda_c^0$ | $T_{bb\bar{a}} \rightarrow \Xi^- \Lambda_c^0$ | $T_{bb\bar{a}} \rightarrow \Xi^- \Lambda_c^0$ |
| $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ |
| $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ |
| $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ |
| $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ |
| $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ |
| $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ | $T_{bb\bar{a}} \rightarrow \Xi^0_1 \bar{c}^- \bar{s}$ |

| Three body decays |
|-------------------|
| $T_{bb\bar{a}} \rightarrow B^- J/\psi$ | $T_{bb\bar{a}} \rightarrow B^- J/\psi$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ |
| $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ |
| $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ |
| $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ |
| $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ | $T_{bb\bar{a}} \rightarrow B^- D^- \bar{c}$ |

III. $T_{b\bar{q}q}$

To reconstruct the $T_{b\bar{q}q}$, we listed the gold decay channels in Tab. XXXX. The branching fractions are estimated at the order $10^{-3}$. In these decay channels, the reconstruction of the final bottom meson or bottom baryon state requires another factor of $10^{-3}$. The reconstruction of $J/\psi$ or $D$ or charmed baryons, the corresponding factor becomes to be $10^{-2}$. Thus to reconstruct the doubly bottom tetraquark, the channel $T_{bb\bar{a}d} \rightarrow \bar{p}\Sigma^0 \rightarrow p\Sigma^- + B^-$ is a wonderful tool and whose branching fraction is the order of $10^{-6}$.

IX. CONCLUSIONS

Most theoretical works including the Lattice QCD simulations supported the possibility of the stable doubly heavy tetraquarks. In this work, we have given the spectra of the doubly heavy tetraquarks by Sakharov-Zeldovich formula.
We found that the stable $T_{\text{cc}}^{+}(3)$ is below the $DD^*$ threshold, and $T_{\text{bb}}^{-}(3)$ is below the $BB^*$ threshold. In order to hunting for these stable doubly heavy tetraquarks, we investigated systematically the semileptonic and nonleptonic weak decay amplitudes of the stable doubly heavy tetraquarks under the flavor SU(3) symmetry, which is a powerful tool to analyze the general decay properties. The ratios between decay widths of different channels were also given. We have given the Cabibbo allowed two-body and three-body decay channels of the stable doubly heavy tetraquarks, which have large branching ratios and shall be employed as the “discovery channels” in the reconstructions at LHCb and Bell II experiments.

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