Energy-Efficient Power Allocation for Secure Communications in Large-Scale MIMO Relaying Systems

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Abstract—In this paper, we address the problem of energy-efficient power allocation for secure communications in an amplify-and-forward (AF) large-scale multiple-input multiple-output (LS-MIMO) relaying system in presence of a passive eavesdropper. The benefits of an AF LS-MIMO relay are exploited to significantly improve the secrecy performance, especially the secrecy energy efficiency (bit per Joule). We first analyze the impact of transmit power at the relay on the secrecy outage capacity, and prove that the secrecy outage capacity is a concave function of transmit power under very practical assumptions, i.e. no eavesdropper channel state information (CSI) and imperfect legitimate CSI. Then, we propose an energy-efficient power allocation scheme to maximize the secrecy energy efficiency. Finally, simulation results validate the advantage of the proposed energy-efficient scheme compared to the capacity maximization scheme.

I. INTRODUCTION

The open nature of wireless medium is usually exploited to improve the performance through multiuser transmission, but also results in the information leakage to an unintended user. Traditionally, the problem of wireless security is addressed at the upper layers of the protocol stack by using sophisticated encryption techniques. Thanks to the seminal work of Wyner [1], it is found that secure communication could be realized only by physical layer techniques, namely physical layer security.

From an information-theoretic viewpoint, physical layer security is in essence to maximize the performance difference between the legitimate channel and the eavesdropper channel [2] [3]. Thus, it makes sense to impair the interception signal and to enhance the legitimate signal simultaneously. Motivated by this, a variety of advanced physical layer techniques are introduced to improve the secrecy performance. Wherein, MIMO relaying techniques have received considerable attentions [4] [5]. In [6] and [7], the optimal beamforming schemes at the relay for commonly used amplify-and-forward (AF) and decode-and-forward (DF) relaying protocols were presented in two-hop secure communications. Note that the above schemes require global channel state information (CSI) to design the transmit beams. In fact, the CSI, especially eavesdropper CSI, is difficult to be obtained, since the eavesdropper is usually passive and keeps silence. Then, a robust beamforming scheme was given in [8], assuming imperfect eavesdropper CSI at the relay. Furthermore, if there is no any eavesdropper CSI, a joint beamforming and jamming scheme was proposed in [9]. Through transmitting artificial noise in the null space of the legitimate channel, the interception signal is weakened, while there is no effect on the legitimate signal.

Note that the joint beamforming and jamming scheme consumes extra power to transmit artificial noise, resulting in a low energy efficiency. Moreover, if legitimate CSI is imperfect, the artificial noise will also affect the legitimate signal. Thus, it is necessary to introduce new MIMO relaying techniques to further enhance wireless security under very practical assumptions, i.e. no eavesdropper CSI and imperfect legitimate CSI. Recently, it is found that large-scale MIMO (LS-MIMO) can produce high-resolution spatial beam, so as to avoid the information leakage to the unintended user [10]. In [11], the secrecy performance of LS-MIMO relaying techniques in secure communications was analyzed. It was shown that even in very adverse environment, such as short-distance interception, the secrecy performance can be improved significantly. In secure communication, the transmit power has a complicate effect on the secrecy performance, especially in relaying systems. This is because the power will affect the legitimate and the eavesdropper signals simultaneously. Thus, it makes sense to choose an optimal power at the relay. Considering that energy efficiency is a pivotal metric in wireless communications, especially in secure communications [12] [13], this paper focuses on designing an energy-efficient power allocation scheme for secure communication in an AF LS-MIMO relaying system. The contributions of this paper are two-fold:

1) We derive the secrecy energy efficiency (bit per Joule) of an AF LS-MIMO relaying system under imperfect CSI, and reveal the impact of transmit power on the secrecy energy efficiency.
2) We propose an energy-efficient power allocation scheme by maximizing the secrecy energy efficiency.

The remainder of this paper is organized as follows. The two-hop LS-MIMO relaying model and AF relaying protocol are introduced in Section II. In Section III, we propose an energy-efficient power allocation scheme. In Section IV, we present some simulation results to validate the effectiveness of
the proposed scheme. Finally, we conclude the whole paper in Section V.

II. SYSTEM MODEL

![Relaying System Diagram](image)

Fig. 1. The secure LS-MIMO relaying system.

In this section, we present the secure relaying system and the AF relaying protocol under consideration.

A. Secure Relaying System

We consider a time division duplex (TDD) two-hop LS-MIMO relaying system depicted in Fig. 1 where a single antenna source sends message to a single antenna destination with the aid of a relay equipped with $N_R$ antennas, while a single antenna passive eavesdropper intends to intercept the information. Note that $N_R$ is pretty large in such an LS-MIMO relaying system, i.e. $N_R = 100$ or even larger. Due to a long propagation distance between the source and the destination, we assume there is no direct transmission between them. In other words, any successful information transmission from the source to the destination must get the help of the relay. It is assumed that the eavesdropper is far away from the source and is close to the relay, since it thought the signal comes from the relay. Note that it is a common assumption in related literature [14], since it is difficult for the eavesdropper to overhear the signals from the source and the relay simultaneously. Then, the eavesdropper only monitors the transmission from the relay to the destination.

We use $\sqrt{\alpha_{i,j}}h_{i,j}$ to denote the channel from node $i$ to $j$, where $i \in \{S, R\}$, $j \in \{R, D, E\}$ with $S$, $R$, $D$ and $E$ representing the source, the relay, the destination and the eavesdropper, respectively. $\alpha_{i,j}$ is the distance-dependent path loss and $h_{i,j}$ is the small-scale channel fading vector with independent and identically distributed (i.i.d) zero mean and unit variance complex Gaussian entries. It is assumed that $\alpha_{i,j}$ remains unchanged during a relative long period and $h_{i,j}$ is block fading.

It takes two time slots to complete a whole transmission. In the first time slot, the source transmits the information to the relay. Thus, the received signal at the relay can be expressed as

$$y_R = \sqrt{P_S}h_{S,R} + n_R,$$

where $s$ is the normalized transmit signal, $P_S$ is the transmit power at the source, $n_R$ is the additive Gaussian white noise with unit variance at the relay. During the second time slot, the relay forwards the post-processed signal $r$ to the destination. Thus, the received signals at the destination and the eavesdropper can be expressed as

$$y_D = \sqrt{P_R}h_{R,D}^{H}r + n_D,$$

and

$$y_E = \sqrt{P_R}h_{R,E}^{H}r + n_E,$$

respectively, where $P_R$ is the transmit power at the relay, $n_D$ and $n_E$ are the additive Gaussian white noises with unit variance at the destination and the eavesdropper, respectively.

B. AF Relaying Protocol

The relay adopts an AF relaying protocol to forward the information. Then, the normalized signal to be transmitted at the relay is given by

$$r = Fy_R,$$

where $F$ is a transform matrix used at the relay.

We assume that the relay has perfect CSI $h_{S,R}$ by channel estimation and gets partial CSI $h_{R,D}$ due to channel reciprocity in TDD systems. The relation between the estimated CSI $h_{R,D}$ and the real CSI $h_{R,D}$ is given by

$$h_{R,D} = \sqrt{\rho}h_{R,D} + \sqrt{1 - \rho}e,$$

where $e$ is the estimation error noise vector with i.i.d. zero mean and unit variance complex Gaussian entries, and is independent of $h_{R,D}$. $\rho$, scaling from 0 to 1, is the correlation coefficient between $h_{R,D}$ and $h_{R,D}$. In addition, since the eavesdropper is usually passive and keeps silence, the CSI $h_{R,E}$ is unavailable at the relay. Therefore, $F$ is designed only based on $h_{S,R}$ and $h_{R,D}$, but is independent of $h_{R,E}$. Considering the low complexity and good performance in LS-MIMO systems, we combine maximum ratio combination (MRC) and maximum ratio transmission (MRT) at the relay to process the received signal. Thus, the transform matrix $F$ is given by

$$F = \frac{h_{R,D}}{\|h_{R,D}\|} \frac{1}{\sqrt{P_S}h_{S,R}\|h_{S,R}\|^2 + 1} h_{S,R}^{H}.$$

In this paper, we adopt the secrecy outage capacity $C_{SOC}$ to evaluate wireless security, since it is impossible to provide a steady secrecy rate over fading channels if there is no eavesdropper CSI at the relay. Secrecy outage capacity is defined as the maximum available rate under the condition that the outage probability of the real transmission rate being greater than secrecy capacity is equal to a given value. Mathematically, it is given by

$$P_r(C_{SOC} > C_D - C_E) = \varepsilon,$$

where $C_D$ and $C_E$ are the legitimate and eavesdropper channel capacities, respectively. $\varepsilon$ is an outage probability associated to a secrecy outage capacity $C_{soc}$. 
III. ENERGY-EFFICIENT POWER ALLOCATION

In this section, we first present the secrecy outage capacity for an AF LS-MIMO relaying system with imperfect CSI, analyze the impact of transmit power at the relay on the secrecy outage capacity, and finally derive a power allocation scheme by maximizing the secrecy energy efficiency, namely a ratio of secrecy outage capacity over total power consumption.

Based on the received signals in (2) and (3), the legitimate and eavesdropper channel capacities are given by

\[ C_D = W \log_2(1 + \gamma_D), \]
and

\[ C_E = W \log_2(1 + \gamma_E), \]
respectively, where \( W \) is the spectral bandwidth. \( \gamma_D \) and \( \gamma_E \) are the signal-to-noise ratios (SNR) at the destination and the eavesdropper, which can be expressed as

\[ \gamma_D = \frac{P_S P_{ROS,ROS,R,D} |h_{R,D}^H h_{R,D}|^2 |h_{S,R}|^2}{P_R |h_{R,D}^H h_{R,D}|^2 + |h_{R,D}|^2 (P_S |h_{S,R}|^2 + 1)} \]
and

\[ \gamma_E = \frac{P_S P_{ROS,ROS,E,R,E} |h_{R,D}^H h_{R,D}|^2 |h_{S,R}|^2}{P_R |h_{R,D}^H h_{R,D}|^2 + |h_{R,D}|^2 (P_S |h_{S,R}|^2 + 1)} \]

Thus, for the secrecy outage capacity, we have the following lemma:

**Lemma 1:** For a given outage probability by \( \varepsilon \), the secrecy outage capacity of an AF LS-MIMO relaying system with imperfect CSI can be expressed as

\[ C_{soc}(P_R) = W \log_2 \left( 1 + \frac{P_S^{P_{ROS,ROS,R,D} h_{R,D}^H h_{R,D} |h_{S,R}|^2}}{P_R |h_{R,D}^H h_{R,D}|^2 + |h_{R,D}|^2 (P_S |h_{S,R}|^2 + 1)} \right) - W \log_2 \left( 1 + \frac{P_S^{P_{ROS,ROS,R,D} h_{R,D}^H h_{R,D} |h_{S,R}|^2}}{P_R^{P_{ROS,ROS,R,D} h_{R,D}^H h_{R,D} |h_{S,R}|^2} + |h_{R,D}|^2 (P_S |h_{S,R}|^2 + 1)} \right) \]

Proof: The secrecy outage capacity can be obtained based on (7) by making use of the property of channel hardening in LS-MIMO systems \[15\]. We omit the proof due to space limitation, and the detail can be referred to our previous work \[11\].

Let \( \rho_{D,R} N_R = A, -\rho_{E,R} N_R = A \cdot \gamma_1, P_S |h_{S,R}|^2 N_R = B \), where \( \gamma_1 = \frac{-\alpha_{E,R} N_R}{\alpha_{R,D} N_R} \) is defined as the relative distance-dependent path loss. Then, the secrecy outage capacity can be rewritten as

\[ C_{soc}(P_R) = W \log_2 \left( 1 + \frac{P_R A B}{P_R A + B + 1} \right) - W \log_2 \left( 1 + \frac{P_R A B}{P_R A + B + 1} \right). \]

Examining (12), it is found that if and only if \( 0 < \gamma_1 < 1 \), the secrecy outage capacity is positive. Obviously, only when \( C_{soc} \) is positive, the problem of energy efficiency makes sense. In what follows, we only consider the case of \( 0 < \gamma_1 < 1 \).

Prior to designing an energy-efficient power allocation scheme, we first investigate the total power consumption in such a secure relaying system. Herein, we model the total power \( D \) as the sum of powers at the source and the relay, which is given by

\[ D(P_R) = \frac{1}{2} P_S + \frac{1}{2} P_R + P_C, \]

where \( P_C \) is the constant circuit power consumption at both the relay and the source, including the power dissipations in the transmitter filter, mixer, frequency synthesizer, digital-to-analog converter and so on, which are independent of the actual transmit signals. The factor \( 1/2 \) before \( P_S \) and \( P_R \) appears since the source and the relay only send message in one slot, respectively. We further assume that the transmit power \( P_S \) at the source is fixed, and focus on power allocation at the relay.

Hence, the secrecy energy efficiency for such a secure relaying system is defined as the average number of bit per Joule securely delivered to the destination. In this paper, we aim to maximize the secrecy energy efficiency by distributing the transmit power at the relay. Mathematically, the power allocation at the relay is equivalent to the following optimization problem

\[ J_1: \max_{P_R} \frac{C_{soc}(P_R)}{D(P_R)} \quad \text{s.t.} \quad P_R \leq P_T. \]

where \( P_T \) is the transmit power constraint at the relay. \(14\) is the so called secrecy energy efficiency. \(15\) is the transmit power constraint at the relay. To solve the above optimization problem, it is better to know the monotonicity of \( C_{soc}(P_R) \). For the secrecy outage capacity, we have the following property:

**Proposition 1:** The secrecy outage capacity of the LS-MIMO relaying system increases when \( P_R \in \left(0, \sqrt{-\frac{P_{ROS,ROS,R,D} h_{R,D}^H h_{R,D} |h_{S,R}|^2}{\alpha_{R,D} N_R}} \right) \), and decreases when \( P_R \in \left(\sqrt{-\frac{P_{ROS,ROS,R,D} h_{R,D}^H h_{R,D} |h_{S,R}|^2}{\alpha_{R,D} N_R}}, \infty\right) \).

**Proof:** Please refer to Appendix I.

From Proposition 1, it is known that the maximum secrecy energy efficiency must appear when \( P_R \in \left(0, \sqrt{-\frac{P_{ROS,ROS,R,D} h_{R,D}^H h_{R,D} |h_{S,R}|^2}{\alpha_{R,D} N_R}} \right) \). This is because the secrecy outage capacity is a decreasing function when \( P_R \) belongs to \( \left(\sqrt{-\frac{P_{ROS,ROS,R,D} h_{R,D}^H h_{R,D} |h_{S,R}|^2}{\alpha_{R,D} N_R}}, \infty\right) \). Then, as \( P_R \) increases, the secrecy energy efficiency decreases. Hence, the optimization problem \( J_1 \) can be rewritten as

\[ J_2: \max_{P_R} \frac{C_{soc}(P_R)}{P_S + P_R + 2P_C} \quad \text{s.t.} \quad P_R \leq P_{min}. \]

where \( P_{min} = \min \left(P_T, \sqrt{-\frac{P_{ROS,ROS,R,D} h_{R,D}^H h_{R,D} |h_{S,R}|^2}{\alpha_{R,D} N_R}} \right) \). The objective function \(16\) in a nonlinear fractional manner is usually non-convex. In general, by making use of the property of fractional programming \(16\), the objective function is equivalent to \( C_{soc}(P_R) - q^* (P_S + P_R + 2P_C) \), where \( q^* \) is the maximum secrecy energy efficiency, namely \( q^* = \max_{P_R \leq P_{min}} \frac{C_{soc}(P_R)}{P_S + P_R + 2P_C} \).
Thus, the optimization problem $J_3$ is transformed as

$$J_3 : \max_{P_R} C_{soc}(P_R) - q^*(P_S + P_R + 2P_C)$$

s.t. $-P_R + P_{min} \geq 0.$

Checking the convexity of $J_3$, we have the following proposition:

Proposition 2: $C_{soc}(P_R) - q^*(P_S + P_R + 2P_C)$ is a concave function of $P_R$, $\forall$ $P_R \in (0, \sqrt{\frac{P_{soc}\alpha_{R,E}N_R+1}{\alpha_{S,R}\alpha_{R,E}D}\rho_{R,E}\ln\epsilon}).$

Proof: Please refer to Appendix II.

According to Proposition 2, $J_3$ can be solved by the Lagrange multiplier method. Firstly, the Lagrange dual function can be written as

$$\mathcal{L}(P_R, \theta) = C_{soc}(P_R) - q^*(P_S + P_R + 2P_C) - \theta(P_R - P_{min}),$$

(20)

where $\theta$ is the Lagrange multiplier corresponding to the constraint (19). Therefore, the dual problem of $J_3$ is given by

$$\min_{\theta} \max_{P_R} \mathcal{L}(P_R, \theta).$$

(21)

For a given $\theta$, the optimal transmit power $P_R^*$ can be derived by solving the following KKT condition

$$\frac{\partial \mathcal{L}(P_R, \theta)}{\partial P_R} = \frac{\partial C_{soc}(P_R)}{\partial P_R} - q^* - \theta = 0.$$

(22)

In addition, $\theta$ can be updated by the gradient method, which is given by

$$\theta(n+1) = [\theta(n) - \triangle \theta(-P_R + P_{min})]^+, \quad \text{(23)}$$

where $n$ is the iteration index, and $\triangle \theta$ is the positive iteration step. Above all, we propose an iteration algorithm for energy-efficient power allocation at the relay as follows:

Algorithm 1: Energy-Efficient Power Allocation.

1) Initialize the maximum number of iterations $L_{max}$ and the maximum tolerance $\delta$.
2) Set a maximum energy efficiency $q = 0$ and iteration index $n = 0$.
3) Figure out the solution $P'$ of (22) for a given $q$.
4) Update $\theta$ according to (23) and let $n = n + 1$.
5) If $C_{soc}(P_R) - q(P_S + P' + 2P_C) < \delta$, then return $P_R^* = P'$ and $q^* = \frac{C_{soc}(P_R)}{C_{soc}(P_R)}$. Otherwise, if $n < L_{max}$ go to 3) with $q = \frac{C_{soc}(P_R)}{C_{soc}(P_R)} + \frac{P_R + P' + 2P_C}{P_S + P' + 2P_C}$.

IV. SIMULATION RESULTS

To examine the effectiveness of the proposed energy-efficient power allocation scheme for an AF LS-MIMO relaying system, we present several simulation results in the following scenarios: we set $N_R = 100$, $W = 10 \text{ KHz}$, $\rho = 0.9$, $P_C = 5 \text{ dB}$, $P_T = 10 \text{ dB}$ and $\varepsilon = 0.05$. We assume that the relay is in the middle of the source and the destination. For the sake of calculational simplicity, we normalize the path loss as $\alpha_{S,R} = \alpha_{R,D} = 1$ and use $\alpha_{S,E}$ to denote the relative path loss. Note that $\alpha_{R,E} > 1$ means the eavesdropper is closer to the relay than the destination. We set $\alpha_{R,E} = 1.5$ without extra explanation. In the following results, “number of iterations” refers to the number of iterations in Algorithm 1.

First, we illustrate the convergence speed of the proposed iterative algorithm with different transmit powers at the source $P_S$. As observed in Fig. 2, the iterative algorithm converges to the maximum energy efficiency within no more than 8 iterations, so the algorithm is reliable and efficient. Moreover, it is found that the energy efficiency is not a monotone increasing function of source transmit power $P_S$, since the maximum energy efficiency with $P_S = 0 \text{ dB}$ is bigger than that with $P_S = 10 \text{ dB}$. Then, it makes sense to choose an optimal source transmit power. We will analyze the optimal source transmit power in future work.

Then, we show the energy efficiency gain of the proposed energy-efficient power allocation scheme compared to the secrecy outage capacity maximization scheme with $P_S = 10 \text{ dB}$. As seen in Fig. 3, the proposed energy-efficient scheme obviously performs better than the capacity maximization scheme, especially at small $\alpha_{R,E}$ regime. For example, at $\alpha_{R,E} = 0.1$, there is about 1.2 Kb/J gain. As $\alpha_{R,E}$ increases, the gain becomes smaller. The reasons are two-fold. On the one hand, as $\alpha_{R,E}$ increases, the secrecy outage capacity decreases accordingly. On the other hand, the feasible set of $P_R$ is also reduced.

Next, we examine the effect of $P_S$ on the maximum energy efficiency. As seen in Fig. 4 the maximum energy efficiency is a concave function of $P_S$. As $P_S$ tends to zero, the maximum energy efficiencies with different $\alpha_{R,E}$ asymptotically approach zero. This is because the secrecy outage capacity is approximately zero and the total power assumption is nonzero under such a condition. At high $P_S$ region, the maximum energy efficiency also approaches zero, since the secrecy...
outage capacity will saturated if \( P_S \) is sufficiently large and the total power consumption is quite large. Hence, there is an optimal source transmit power in the sense of maximizing the secrecy energy efficiency.

Finally, we investigate the function of the number of antennas at the relay on the energy efficiency of the proposed scheme with \( P_S = 10 \) dB. As shown in Fig. 5 with the increase of \( \alpha_{R,E} \), all the maximum energy efficiencies with different numbers of relay antennas decrease. However, for a given \( \alpha_{R,E} \), as \( N_R \) adds, the maximum energy efficiency increases significantly. Hence, we can increase the secrecy energy efficiency by simply adding the antennas at the relay, which is a main advantage of LS-MIMO relaying systems.

**V. Conclusion**

This paper proposed an energy-efficient power allocation scheme for an AF LS-MIMO relaying system with imperfect CSI. The energy-efficiency scheme has a fast convergence characteristics and obviously outperforms better than the capacity maximization scheme. More importantly, it is feasible to obtain a high secrecy energy efficiency even in the case of short-distance interception by adding the number of antennas at the relay.

**Appendix A**

**Proof of Proposition 1**

At first, we take derivative of (12) with respect to \( P_R \), which is given by (24) at the top of the next page. Let \( C'_{soc}(P_R) = 0 \), we get two solutions

\[
P_R = \frac{1}{A_{R_1}} \sqrt{r_1(B + 1)},
\]

and

\[
P_R = -\frac{1}{A_{R_1}} \sqrt{r_1(B + 1)}.
\]

Considering \( P_R > 0 \), (25) is the unique optimal solution in this case. What’s more, when \( P_R < \frac{1}{A_{R_1}} \sqrt{r_1(B + 1)} \), we have \( C'_{soc}(P_R) > 0 \). Otherwise, if \( P_R > \frac{1}{A_{R_1}} \sqrt{r_1(B + 1)} \), we have \( C'_{soc}(P_R) < 0 \). Specifically, \( C_{soc}(P_R) \) improves as \( P_R \) increases in the region from 0 to \( \frac{1}{A_{R_1}} \sqrt{r_1(B + 1)} \), while \( C_{soc}(P_R) \) decreases as \( P_R \) increases in the region from \( \frac{1}{A_{R_1}} \sqrt{r_1(B + 1)} \) to infinity. Hence, we get the Proposition 1.

**Appendix B**

**Proof of Proposition 2**

Obviously, \( C_{soc}(P_R) - q^*(P_S + P_R + 2P_C) \) has the same convexity or concavity as \( C_{soc}(P_R) \), since \( q^*(P_S + P_R + 2P_C) \) is affine. The proof that \( C_{soc}(P_R) \) is concave is given as follows:

\[
C_{soc}(P_R) = W \log_2 \left( 1 + \frac{P_{R}AB}{P_R A + B + 1} \right)
- W \log_2 \left( 1 + \frac{P_{R}AB}{P_R A + B + 1} \right)
= W \log_2 \left( 1 + \frac{P_{R}AB}{P_{R}AB + 1} \right).
\]
Let $g_0(P_R) = \frac{1 + \frac{P_R AB}{P_R A + B + 1}}{1 + \frac{P_R AB}{P_R A + B + 1}}$, then we have

$$g_0(P_R) = \frac{1 + \frac{P_R AB}{P_R A + B + 1}}{1 + \frac{P_R AB}{P_R A + B + 1}} = \frac{P_R AB + P_R A + B + 1}{P_R A + B + 1} \times \frac{P_R A + B + 1}{(P_R A + B + 1)(P_R A + B + 1)} = \frac{P_R A + B + 1}{P_R A + B + 1} \times \frac{P_R A + B + 1}{P_R A + B + 1} = 1 + \frac{P_R A + B + 1}{P_R A + B + 1} \times \frac{P_R A + B + 1}{P_R A + B + 1}.$$ 

For the sake of notational and computational simplicity, let $P_1 = P_R A$, then $g_1(P_R)$ can be written as $g_2(P_1) = \frac{1}{P_1 r_1 + 1} - \frac{1}{P_1 A + B + 1}$. Because of $A > 0$, $g_2(P_1)$ preserves the convexity or concavity of $g_1(P_R)$, which has the same convexity or concavity as $g_0(P_R)$. In other words, $g_2(P_1)$ and $g_0(P_R)$ have the same convexity or concavity. By some simplification, $g_2(P_1)$ can be rewritten as

$$g_2(P_1) = \frac{(1 - r_1) P_1}{(P_1 r_1 + 1)(P_1 + B + 1)}.$$ 

Now, we prove the convexity of $g_3(P_1)$. First, we take second derivative of $g_2(P_1)$ with respect to $P_1$, which is given by

$$g_3''(P_1) = \frac{P_1 r_1 (P_1^2 r_1^2 - 3 B - 3 - 2 P_1 r_1 - 2 B r_1 - r_1 - B - 1)}{(P_1 r_1 + 1)(P_1 + B + 1)} \times (1 - r_1).$$ 

Because of $B > 0$, $P_1 > 0$ and $0 < r_1 < 1$, $-2 B r_1 - r_1 - B - 1$ must be negative, and $P_1 r_1 (P_1^2 r_1^2 - 3 B - 3 - 2 P_1 r_1 - 2 B r_1 - r_1 - B - 1)$ must be positive. Then, once $P_1 \in \left(0, \frac{\sqrt{3(B+1)}}{r_1}\right)$, $g_3''(P_1)$ is negative. In other words, $g_2(P_1)$ is concave when $P_1 \in \left(0, \frac{\sqrt{3(B+1)}}{r_1}\right)$. Therefore, $g_0(P_R)$ is also concave when $P_R \in \left(0, \frac{\sqrt{3(B+1)}}{r_1}\right)$. Thus, $W \log_2(g_0(P_R))$, namely $C_{soc}(P_R)$, is a concave function due to $C_{soc}(P_R) = -W g_0(P_R) g_0'(P_R) g_0''(P_R) \leq 0$. Above all, $C_{soc}(P_R) - q' (P_S + P_R + 2 P_C)$ is concave. Hence, we get the Proposition 2.