Scattering of low-frequency radiation by a gyrating electron

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Accepted 2007 October 14. Received 2007 October 10; in original form 2007 August 27

ABSTRACT

The scattering of electromagnetic radiation by the particle gyrating in an external magnetic field is considered. Particular attention is paid to the low-frequency case, when the frequencies of incident radiation are much less than the electron gyrofrequency. The spectral and polarization features of the scattering cross-section are analysed in detail. It is found that the scattering transfers the low-frequency photons to high harmonics of the gyrofrequency, into the range of the synchrotron emission of the electron. The total scattering cross-section appears much larger than that for the particle at rest. The problem studied is directly applicable to the radio wave scattering in the magnetosphere of a pulsar. The particles acquire relativistic rotational energies as a result of resonant absorption of the high-frequency radio waves and concurrently scatter the low-frequency radio waves, which are still below the resonance. It is shown that the scattering can affect the radio intensity and polarization at the lowest frequencies and can compete with the resonant absorption in contributing to the low-frequency turnover in the pulsar spectrum. Moreover, the scattering can be an efficient mechanism of the pulsar high-energy emission, in addition to the synchrotron re-emission of the particles. Other astrophysical applications of the scattering by gyrating particles are pointed out as well.

Key words: radiation mechanisms: non-thermal – scattering – pulsars: general.

1 INTRODUCTION

The presence of an external magnetic field may substantially affect the process of photon scattering off an electron. The classical non-relativistic consideration of the magnetized Thompson scattering has been done in Canuto (1970), Canuto, Lodenquai & Ruderman (1971), Goldstein & Lenchek (1971), Blandford & Scharlemann (1976), Ventura (1979) and Börner & Mészáros (1979a,b). It has been found that the role of magnetic field is significant unless the photon frequency substantially exceeds the electron gyrofrequency, \( \omega \gg \omega_G \equiv eB/\mu c \), and the magnetized scattering cross-section is characterized by peculiar angular and frequency dependencies as well as specific polarization signatures. The fully relativistic treatment of the magnetic cross-section in terms of quantum electrodynamics has been developed in Herold (1979), Melrose & Parle (1983) and Daugherty & Harding (1986), and useful approximations have been given in Xia et al. (1985), Daugherty & Harding (1989) and Gonthier et al. (2000). The relativistic effects become important in extremely strong magnetic fields approaching the critical value \( B_c = 4.413 \times 10^{13} \text{G} \) defined as \( \hbar \omega_G(B_c) \equiv m c^2 \) and at frequencies roughly comparable with \( \omega_G(B_c) \). Then the magnetized scattering exhibits principally new features, such as resonances at high harmonics of the gyrofrequency and the possibility of electron excitation to higher Landau levels.

The relativistic regime may be applicable to close neighbourhoods of the neutron stars, whose surface magnetic fields are typically \( \sim 10^{13} \text{G} \) and in case of magnetars may be as large as \( \sim 10^{15} \text{G} \). The neutron stars have surface temperatures \( T_s \sim 10^5-10^6 \text{K} \) and the thermal X-ray photons are scattered off the primary particles, which are accelerated by the rotation-induced electric field of the neutron star to the Lorentz factors \( \gamma_p \sim 10^6-10^7 \). The resonant Compton scattering of the thermal radiation of the neutron star can be efficient (Blandford & Scharlemann 1976; Xia et al. 1985; Daugherty & Harding 1989) and is believed to have important implications. The upscattered photons are capable of producing the electron–positron pairs and may compete with the curvature emission of the primary particles in controlling the pair production cascade in the polar gap of a pulsar (Sturrock & Dermer 1994; Sturrock 1995; Luo 1996). Thus, the resonant Compton scattering may substantially affect the characteristics of the secondary pulsar plasma (Hibschman & Arons 2001a,b; Arendt & Eilek 2002; Harding & Muslimov 2002). This process can also account for the high-energy spectra of magnetars (Thompson, Lyutikov & Kulkarni 2002; Lyutikov & Gavriil 2006; Baring & Harding 2007; Beloborodov & Thompson 2007; Fernández & Thompson 2007; Rea et al. 2007). For a review of other applications of the resonant Compton scattering to pulsars, see e.g. Harding & Lai (2006).

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The scattering of the non-thermal radio emission of pulsars off the secondary plasma particles is an essentially distinct process and is also of interest. The secondary electron–positron plasma streams ultrarelativistically, with $\gamma \sim 10^2$, along the open magnetic lines of a pulsar and ultimately leaves the magnetosphere as a pulsar wind. The radio emission is believed to originate inside the plasma flow deep in the magnetosphere. Hence, on its way in the magnetosphere and beyond, pulsar radiation passes through the plasma and is subject to scattering off the plasma particles. Since for radio frequencies $hv/mc^2 \ll 1$, the quantum effects on the scattering are negligible and the classical treatment is appropriate. In the vicinity of the emission region, the photon frequency in the particle rest frame is much less than the electron gyrofrequency, $\omega \gamma (1 - \beta \cos \theta) \ll \omega_G$ (here $\beta$ is the particle velocity in units of $c$ and $\theta$ is the angle between the photon wavevector and the particle velocity), i.e., the magnetic field is strong enough to affect the scattering process. As the magnetic field strength rapidly decreases with distance from the neutron star, $B \propto R^{-3}$, in the outer magnetosphere the waves suffer cyclotron resonance, and in the pulsar wind the scattering is non-magnetic.

The scattering of pulsar radio emission in the magnetic and non-magnetic regimes has first been considered in Blandford & Scharlemann (1976) and Wilson & Rees (1978). Because of extremely high brightness temperatures of pulsar radiation, $T_B \sim 10^{25} - 10^{30}$ K, the induced scattering strongly dominates the spontaneous one and can be efficient in both regimes. Further studies (Lyubarskii & Petrova 1996; Petrova 2004a,b) have demonstrated that the magnetized induced scattering can lead to substantial redistribution of intensity in frequency and pulse longitude and thus can account for various phenomena characteristic of the observed radio pulses.

Close to the neutron star surface, the magnetic field is strong enough for any transverse momentum of the electrons to be almost immediately lost via synchrotron re-emission. However, it is not the case in the outer magnetosphere, where the radio waves meet the condition of cyclotron resonance. Correspondingly, in the resonance region the waves are subject to cyclotron absorption rather than resonant scattering. At the conditions relevant to pulsar magnetosphere, the process of cyclotron absorption not only affects the radio wave intensity (Blandford & Scharlemann 1976; Lyubarskii & Petrova 1998; Luo & Melrose 2001; Petrova 2002; Fussell, Luo & Melrose 2003), but also leads to a substantial increase of the transverse momenta of the absorbing particles (Blandford & Scharlemann 1976; Lyubarskii & Petrova 1998; Petrova 2002, 2003). Since the pulsar radiation is broad-band, $\nu \sim n \times 10^7 - n \times 10^{10}$ Hz, the resonance region is sufficiently extended. The particles entering the resonance region acquire relativistic gyration energies straight near its lower boundary, in the course of absorption of the waves with $\nu \gtrsim 10^{10}$ G (Petrova 2002). Then the lower frequency waves, $\nu \ll 10^{10}$ Hz, which are still below the resonance, $\nu \gamma (1 - \beta \cos \theta) \ll \nu_G$, are subject to the magnetized scattering off the relativistically gyrating particles. This process will be examined in detail in the present paper.

The scattering by a gyrating electron differs substantially from that by an electron at rest. For the electron at rest, the scattering to high harmonics holds only within the framework of the relativistic treatment, in the magnetic fields close to the critical value and at high enough frequencies (Herold 1979; Melrose & Parle 1983; Daugherty & Harding 1986). (Note that the relativistic formalism of the scattering has been developed only for the case when the electron is initially at the ground Landau orbital.) For the gyrotronic electron, the high harmonic scattering is a purely classical effect and it may be efficient in arbitrary magnetic fields for the incident frequencies below the electron gyrofrequency. In application to pulsars, the scattering by the electrons at high Landau levels transfers the radio photons into the optical and X-ray ranges and thus provides a physical connection between the radio and high-energy emissions of a pulsar. Other astrophysical applications of the process, e.g. to synchrotron sources, are not excluded as well.

It should be noted that the pulsar radio emission is believed to be generated at frequencies of the order of the local Lorentz-shifted proper plasma frequency, $\omega \sim \omega_0 \sqrt{N}$, where $\omega_0 \equiv \sqrt{4\pi N e^2/m}$ and $N$ is the number density of the plasma particles. Therefore in the radio emission region and its close vicinity the scattering is a collective plasma process. The induced scatterings in the plasma are suggested as an appropriate. In the vicinity of the emission region, the photon frequency in the particle rest frame is much less than the electron gyrofrequency, $\omega \gamma (1 - \beta \cos \theta) \ll \omega_G$ (here $\beta$ is the particle velocity in units of $c$ and $\theta$ is the angle between the photon wavevector and the particle velocity), i.e., the magnetic field is strong enough to affect the scattering process. As the magnetic field strength rapidly decreases with distance from the neutron star, $B \propto R^{-3}$, in the outer magnetosphere the waves suffer cyclotron resonance, and in the pulsar wind the scattering is non-magnetic.

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A general formalism of the scattering by gyrating electrons in the magnetically active plasma has been developed in Melrose & Sy (1972). In the present paper, we introduce some corrections and simplifications to that treatment and obtain the scattering cross-section in the form suitable for the concrete applications. The plan of the paper is as follows. In Section 2 we derive the scattering cross-section for a gyrating electron and obtain its low-frequency approximation. The scattering of pulsar radio emission by relativistic gyrating particles is studied in Section 3. We investigate the validity of the formalism in application to pulsars in Section 3.1, examine the radio intensity suppression in Section 3.2 and consider the scattered power in Section 3.3. The results of the paper are discussed and summarized in Section 4. The induced scattering off the gyrating electrons will be considered in a separate paper.

2 SCATTERING CROSS-SECTION

2.1 General equations

Let us consider the scattering of transverse electromagnetic waves by a gyrating electron. For the sake of simplification we take that the component of the electron velocity along the external magnetic field is zero. In the classical formulation of the problem, the incident wave
fields perturb the motion of the electron, and it emits the secondary waves, which are interpreted as a scattered radiation. We proceed from the formalism developed in Landau & Lifshitz (1988) for the scattering by a free electron. The scattering cross-section is defined as the ratio of the average intensity of the waves scattered into the elementary solid angle $d \Omega'$ to the energy flux density of the incident radiation, and can be presented in the form

$$\frac{d \sigma}{d \Omega'} = \lim_{T \to \infty} \frac{P^2}{2 \pi} \int_{-\infty}^{\infty} |k' \times A_{\omega'}|^2 \, d \omega'/2\pi,$$

(1)

where $k'$ and $\omega'$ are the wavevector and frequency of the scattered waves, $A_{\omega'}$ and $E_{\omega'}$ are the Fourier components of the vector potential of the scattered waves and the electric field of the incident waves, and $R_0$ is the distance to an observer. The linearized vector potential of the scattered waves reads

$$A_{\omega'} = \frac{e^2}{c} R_0 \int_{-\infty}^{\infty} \epsilon(t) \, e^{i(k' \cdot r(t))} \, d\omega'/2\pi,$$

(2)

Here $v_0$ and $r_0$ are the velocity and coordinate of the unperturbed circular motion of the electron, $v_1$ and $r_1$ are the first-order perturbations of these quantities. In the coordinate system with the $z$-axis along the external magnetic field,

$$v_0 = v_0(\cos \Omega t, -\sin \Omega t, 0), \quad r_0 = \frac{v_0}{\Omega} (\sin \Omega t, \cos \Omega t, 0),$$

(3)

where $\Omega \equiv eB_0/(\gamma mc)$ and $\gamma_0 \equiv (1 - v_0^2/c^2)^{-1/2}$. The perturbed motion of the particle in the fields of the incident wave, $E_1$ and $B_1$, is described by the linearized equation of motion

$$m \gamma_0 \frac{d}{dt}(v_1 + \gamma_0 \frac{v_0 \cdot v_1}{c} v_0) = eE_1 + \frac{v_0 \times B_1}{c} + \frac{v_1 \times B_0}{c}.$$  

(4)

Hereafter the subscripts of the perturbed quantities will be omitted. It is convenient to project equation (4) on to the axes, one of which is along $v_0$ and another one along $B_0$:

$$e_1 = (\cos \Omega t, -\sin \Omega t, 0), \quad e_2 = (\sin \Omega t, \cos \Omega t, 0), \quad e_3 = (0, 0, 1).$$

Taking into account that $d e_1 / dt = -\Omega e_2$ and $d e_2 / dt = \Omega e_1$, one can write

$$\gamma_0^2 \frac{d}{dt}(v \cdot e_1) = F \cdot e_1,$$

$$\frac{d}{dt}(v \cdot e_2) - \Omega \gamma_0^2 \gamma_0^2 (v \cdot e_1) = F \cdot e_2,$$

$$\frac{d}{dt}(v \cdot e_3) = F \cdot e_3,$$

(5)

where $F \equiv e(E_1 + v_0 \times B_0)/m \gamma_0$ and $\beta_0 \equiv v_0/c$. Then we obtain the following solution:

$$v_x = f_1 \cos \Omega t + (f_2 + g) \sin \Omega t,$$

$$v_y = -f_1 \sin \Omega t + (f_2 + g) \cos \Omega t,$$

$$v_z = f_2,$$

(6)

where

$$f_1 = \frac{1}{\gamma_0^2} \int_0^t F \cdot e_1(t') \, dt',$$

$$f_2,3 = \int_0^t F \cdot e_{2,3}(t') \, dt',$$

$$g = \Omega \beta_0 \int_0^t \int_0^t F \cdot e_3(t''),$$

(7)

and the perturbed coordinate is given by $r = \int_0^t v(t') \, dt'$. Note that our equation of motion and its solution differ substantially from those given in Melrose & Sy (1972) (cf. equations 51–54 therein). First, in that paper, one of the terms of the linearized equation of motion is missing (namely the last term in equation 4 above). Secondly, the authors have not taken into account that $e(t) \, dp/dt \neq d(e \cdot p)/dt$. Consequently, the second term on the left-hand side of the second of equations (5) and the term $g$ in equation (6) are absent in their treatment.

To proceed further we specify the characteristics of the incident and scattered waves. The wavevectors of the incident and scattered radiation can be written as

$$k = k(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta), \quad k' = k'(\sin \theta' \cos \phi', \sin \theta' \sin \phi', \cos \theta'),$$

(8)

where $\theta, \theta'$ and $\phi, \phi'$ are the polar and azimuthal angles in the spherical coordinate system with the polar axis along the external magnetic field. Since the plasma effects are neglected, the radiation presents transverse electromagnetic waves polarized either in the plane of the wavevector and the external magnetic field or perpendicularly to this plane. Then the electric fields of the waves are directed as

$$e_A = (\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta), \quad e_B = (\sin \phi, -\cos \phi, 0),$$

$$e_A' = (\cos \theta' \cos \phi', \cos \theta' \sin \phi', -\sin \theta'), \quad e_B' = (\sin \phi', -\cos \phi', 0),$$

and $B = k \times E$. 

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To find the vector potential (2) we use the Fourier representation of the monochromatic field, \( E \propto e^{-i\omega t-ikr(t)} \), in equation (7), and perform the expansion in Bessel functions

\[
e^{ikr(t)} = e^{i(\omega t/\Omega) \sin \theta} e^{i(\omega t/\Omega) \cos \phi} \sum_{n=-\infty}^{\infty} J_n \left( \frac{\omega \beta_0 \sin \theta}{\Omega} \right) e^{in\phi} e^{i\Omega t} e^{i\phi}
\]

and the analogous expansion for \( e^{-ikr(t)} \). With the commutation relations \( (J_{n+1} + J_{n-1})/2 = (n/\zeta)J_n \) and \( (J_{n+1} - J_{n-1})/2 = J'_n \), where \( \zeta \) is the argument of the Bessel function and the prime denotes the derivative with respect to \( \zeta \), one can obtain the useful relations

\[
\cos \Omega t + \phi e^{ikr(t)} = \sum_{n=-\infty}^{\infty} \frac{J_n(z)}{z} e^{i(\Omega t + \phi)}, \quad \sin \Omega t + \phi e^{ikr(t)} = -i \sum_{n=-\infty}^{\infty} J'_n(z) e^{i(\Omega t + \phi)}.
\]

where \( z = \omega \beta_0 \sin \theta/\Omega \) and \( z' = \omega \beta_0 \sin \theta'/\Omega \). Substituting this into equation (2) leads to the integral \( \int e^{i(\omega t - \omega t') - i\Omega t'} dt' \), which yields the \( \delta \)-function. Then, taking into account that \( |\delta(\omega)|^2 = (T/2\pi)\delta(\omega) \) and using this in equation (1), one can find the scattering cross-section.

We are interested in the four cross-sections corresponding to the cases when the incident and scattered waves have one of the two polarizations given by equation (9). The incident polarization enters \( \mathbf{F} \), whereas the scattered polarization can be included by projecting the magnetic field of the scattered radiation, \( \mathbf{B}_s = i \mathbf{k} \times \mathbf{A}_s \), on to the magnetic field directions in the normal waves, \( \mathbf{k} \times \mathbf{e}_{x',y'} \). Then the term \( |k' \times A_s|^2 \) is reduced to \( (A_{ij} \cdot e_{x',y'})^2 \). Routine calculations lead to the following cross-sections:

\[
\frac{d\sigma}{d\Omega} = \frac{r_e^2}{\gamma_0} \sum_{l=n}^{\infty} \omega^4 \sum_{l=-\infty}^{\infty} \frac{a_{il}}{(n\Omega - \omega)^2} e^{i\theta \phi - \phi'}^2,
\]

where \( \omega' = \omega + (l - n)\Omega \), \( r_e \) is the classical electron radius, the superscripts \( i, j \) denote the initial and final polarization states of the waves, and

\[
a^{AV} = i \cos \theta \cos \theta' \left[ (n\Omega - \omega)J'_n + \Omega(n/\zeta)J_n \right] \frac{(n\Omega - \omega)J'_n + \Omega(l/\zeta)J_l}{(n\Omega - \omega)^2 - \Omega^2}
\]

\[
+ i J_n \left[ (\beta_0 n/\zeta - \sin \theta)(\beta_0 l/\zeta - \sin \theta') + \cos \theta \cos \theta' n/\zeta l/\zeta \gamma_0^2 \right],
\]

\[
a^{BW} = \cos \theta \cos \theta' \left[ (n\Omega - \omega)J'_n + \Omega(n/\zeta)J_n \right] \frac{(n\Omega - \omega)(\beta_0 \sin \theta - l/\zeta)J_l - \Omega J'_l}{(n\Omega - \omega)^2 - \Omega^2}
\]

\[
- J_n \left[ n \cos \theta \gamma_0^2 + \beta_0 \cos \theta (\beta_0 n/\zeta - \sin \theta) \right],
\]

\[
a^{BA} = \cos \theta \cos \theta' \left[ \Omega J'_n - (n\Omega - \omega)(\beta_0 \sin \theta - n/\zeta)J_n \right] \frac{(n\Omega - \omega)(\beta_0 \sin \theta - l/\zeta)J_l - \Omega J'_l}{(n\Omega - \omega)^2 - \Omega^2}
\]

\[
+ J'_n \left[ l \cos \theta \gamma_0^2 + \beta_0 \cos \theta (\beta_0 l/\zeta - \sin \theta) \right],
\]

\[
a^{BB} = i \left[ (n\Omega - \omega)(\beta_0 \sin \theta - n/\zeta)J_n - \Omega J'_n \right] \frac{(n\Omega - \omega)(\beta_0 \sin \theta - l/\zeta)J_l - \Omega J'_l}{(n\Omega - \omega)^2 - \Omega^2}
\]

\[
+ i J'_n \left[ (1 + \beta_0 l/\zeta) \cos \theta \cos \theta' /\gamma_0^2 \right].
\]

Here \( J_n = J_n(z) \) and \( J_l = J_l(z) \). Equations (11)–(12) give the scattering cross-sections in case of a gyrating electron. In astrophysical applications, the particles generally perform helical motion, and the corresponding cross-sections can be obtained from equations (11)–(12) by means of relativistic transformations. In case of relativistic longitudinal motion of the electron, \( \gamma_1 = (1 - \beta_1^2)^{-1/2} \gg 1 \), it is convenient to involve the cross-section \( \Sigma \) defined as the ratio of the number of the scattered photons to the flux density of the photons flying against the electron. This quantity is a relativistic invariant and is related to the cross-section \( \sigma \) as \( (\omega \omega')/d\Omega = (1 - \beta_1) \cos \theta \) \( d\Sigma/d\Omega' \). Then making use of the transformations \( \omega_s = \omega \gamma_1 \eta, \omega'_s = \omega' \gamma_1 \eta' \), and \( d\Omega' = d\Omega/\gamma_0^2 \eta^2 \gamma_0^2 \eta'^2 \) (where \( \eta \equiv 1 - \beta_1 \cos \theta, \eta' \equiv 1 - \beta_1 \cos \theta' \), and the quantities of the guiding centre frame are denoted by the subscript \('c') \), one can obtain that

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega'} \left( \frac{\eta^2}{\gamma_0^2} \right)\frac{\eta'^2}{\gamma_0^2 \eta'^2}.
\]

where the quantities entering \( (d\sigma/d\Omega) \) should be expressed via the quantities of the laboratory frame.
It is worthy to examine the symmetry properties of the cross-sections (11)–(12). Keeping in mind that \( n\Omega - \omega = l\Omega - \omega' \), one can see that \( |s_{ij}| \) are symmetrical with respect to the simultaneous change \( \omega \leftrightarrow \omega' \), \( n \leftrightarrow l \) and \( i \leftrightarrow j \), and hence the cross-sections can be written in the form \( \frac{d\sigma}{d\Omega'} = \sum s_{ij}^{\nu',\nu} \), where \( s_{ij}^{\nu',\nu} \) are symmetrical in the above mentioned sense. This corresponds to the symmetry of each harmonic of the scattering probability with respect to the initial and final photon states. Indeed, the power supplied by the scattering electron can be presented as
\[
P = \int \frac{d\sigma}{d\Omega'} d\omega' w(N(k)) \frac{d^3k}{(2\pi)^3} d^3k' \frac{d^3k}{(2\pi)^3},
\]
where \( N(k) \) is the occupation number of the incident photons and \( w \) is the scattering probability. With equations (13)–(14) it is obvious that \( w \propto \sin \theta_0 \cos \eta \theta' \).

### 2.2 Useful approximations

The general form of the scattering cross-section given by equations (11)–(12) is so complicated that it can hardly be involved directly in concrete applications. To analyse the basic features of the scattering off a gyrating electron we turn to reasonable approximations. [Note that the approximate cross-sections obtained below refer to the guiding centre frame and it is necessary to apply the Lorentz transformation (13) in order to use them in any realistic calculations.] First of all, we consider the limiting case when \( \beta_0 \to 0 \). As \( z, \theta' \to 0 \), one can use the approximation of the Bessel function at small arguments,
\[
J_n(\zeta) \approx (\zeta/2)^n/n!, \quad \zeta \ll 1, n \geq 0,
\]
taking into account that \( J_{-n}(\zeta) = (-1)^n J_n(\zeta) \). Then only the zeroth harmonic, \( \nu = 0(\omega' = \omega) \), yields a non-zero contribution to the cross-sections, and they are reduced to the form
\[
\frac{d\sigma}{d\Omega'} = r_1^2 \sin \theta \sin \theta' + \cos \theta \cos \theta' \left[ \frac{\Omega^2 \sin \Delta\phi - \omega^2 \cos \Delta\phi}{\Omega^2 - \omega^2} \right]^2,
\]

\[
\frac{d\sigma}{d\Omega'} = \frac{r_2^2 \omega^2 \cos^2 \theta'}{(\Omega^2 - \omega^2)^2} \left[ \frac{\Omega^2}{\omega^2} \cos^2 \Delta\phi + \sin^2 \Delta\phi \right],
\]

\[
\frac{d\sigma}{d\Omega'} = \frac{r_3^2 \omega^2 \cos^2 \theta'}{(\Omega^2 - \omega^2)^2} \left[ \frac{\Omega^2 \delta}{\omega^2} \cos^2 \Delta\phi + \sin^2 \Delta\phi \right],
\]

\[
\frac{d\sigma}{d\Omega'} = \frac{r_4^2 \omega^4}{(\Omega^2 - \omega^2)^2} \left[ \frac{\Omega^2 \sin^2 \Delta\phi + \cos^2 \Delta\phi}{\omega^2} \right],
\]
where \( \Delta\phi = \phi - \phi' \). These equations coincide with the scattering cross-sections for the electron at rest (Canuto et al. 1971).

Below we dwell on the low-frequency approximation of equation (12), \( \omega \ll \Omega \), given arbitrary gyration velocities of the electron. (Note that our assumption does imply small frequencies rather than large gyrofrequencies, since in the latter case the electrons rapidly lose their gyration energies via synchrotron emission.) Then \( \zeta \) is still a small quantity, while \( \zeta = \omega' \beta_0 \sin \theta'/\Omega \) is small only at \( \nu = 0 \); at \( \nu \neq 0 \) \( \zeta \approx v \beta_0 \sin \theta'/\Omega \). The zeroth-harmonic cross-sections read
\[
\frac{d\sigma}{d\Omega'} = \frac{r_1^2}{\gamma_0^2} \sin^2 \theta \sin^2 \theta',
\]

\[
\frac{d\sigma}{d\Omega'} = \frac{r_2^2 \omega^2}{\gamma_0^2 \Omega^2} \left[ \cos \theta \cos \Delta\phi - \frac{\beta_0^2 \sin \theta \sin \theta' \cos \theta'}{2} \right]^2,
\]

\[
\frac{d\sigma}{d\Omega'} = \frac{r_3^2 \omega^2}{\gamma_0^2 \Omega^2} \left[ \cos \theta' \cos \Delta\phi - \frac{\beta_0^2 \sin \theta \sin \theta' \cos \theta'}{2} \right]^2,
\]

\[
\frac{d\sigma}{d\Omega'} = \frac{r_4^2 \omega^2}{\gamma_0^2 \Omega^2} \sin^2 \Delta\phi.
\]

Comparison of equation (17) with equation (16) at \( \omega/\Omega \ll 1 \) shows the following. For the scattering channel \( B \to B' \), the cross-sections are exactly the same, i.e. the scattering by the gyrating electron without a change in frequency nothing differs from that by the electron at rest. For the scattering channels with a change in polarization state, \( A \to B' \) and \( B \to A' \), the cross-sections (16) and (17) are somewhat different, though generally remain of the same order of magnitude. As for the \( A \to A' \) channel, the scattering by a gyrating electron is a factor of \( \gamma_0^{-2} \) weaker. Nevertheless, it may still dominate that in the other channels.

The cross-section components at the non-zero harmonics can be reduced to
\[
\frac{d\sigma}{d\Omega'} = \frac{r_1^2 \omega^2 \nu^4}{\gamma_0^2 \alpha^2 \Omega^2} \sin^2 \theta \cos^2 \theta',
\]

\[
\frac{d\sigma}{d\Omega'} = \frac{r_2^2 \Omega^2 \nu^4}{\gamma_0^2 \alpha^2} \left[ \frac{\nu \sin \theta'}{\gamma_0^2 \alpha^2} \right]^2.
\]
One can expect that these particles subsequently scatter the low-frequency radio waves, which are still below the resonance, in the regime through the area covered by the pulsar radio beam, participate in cyclotron absorption and rapidly acquire relativistic rotational energies.

\[ \text{3 APPLICATION TO PULSARS} \]

A by larger gyration energies and larger \( \gamma \).

The cross-sections include the large parameters, \( \Omega^2/\omega^3 \) and \( \Omega^2/\omega^3 \), and strongly exceed those for the electron at rest, but it should be kept in mind that they cannot increase unrestrictedly with \( \Omega \), since strong enough magnetic field precludes the relativistic gyration of the electron. As can be seen from equation (21), the cross-sections for the same incident polarizations are of the same order of magnitude and in case of ultrarelativistic gyration, \( \beta_0 \approx 1 \), are related as \( \sigma^{A\beta}/\sigma^{A\alpha} = 3/13 \) and \( \sigma^{A\beta}/\sigma^{A\alpha} = 1/13 \), whereas \( (\sigma^{A\beta} + \sigma^{A\beta})/(\sigma^{A\alpha} + \sigma^{A\alpha}) = (16/9)\Omega^2/\omega^3 \). The latter quantity may be both small and large, so that any of the two polarizations may be scattered predominantly. Stronger scattering is favoured by larger gyration energies and larger \( \Omega/\omega \), and the scattering of \( B \)-polarization is more significantly affected by the former quantity, whereas the scattering of \( A \)-polarization by the latter one.

### 3 APPLICATION TO PULSARS

The scattering of low-frequency radiation by relativistic gyrating particles is believed to be the case in pulsars. It takes place in the outer magnetosphere, in the region of cyclotron resonance for the high-frequency radio waves. The particles of the secondary plasma, which pass through the area covered by the pulsar radio beam, participate in cyclotron absorption and rapidly acquire relativistic rotational energies. One can expect that these particles subsequently scatter the low-frequency radio waves, which are still below the resonance, in the regime considered in Section 2.2. Below we examine the validity of the scattering cross-section obtained above in application to pulsars, give the quantitative description of the consequences of the scattering process and compare them with the consequences of resonant absorption.

#### 3.1 Validity of the formalism

We start from examining the validity of the technique developed in Section 2.1. The linearization of the equation of the particle motion is appropriate on condition that

\[ v_1 < v_0 \sim c \quad \text{and} \quad r_1 < r_0 \sim c/\Omega. \]
In case of the B-polarized incident waves, when the incident electric field is perpendicular to the external magnetic field, from equation (4) one can obtain the following estimate: \( v_1 \max(\gamma \nu \omega, eB_0/c) \sim eB_1 \), which reduces to

\[
\frac{v_1}{c} \sim \left\{ \begin{array}{ll}
\frac{B_1}{B_0}, & \omega/\Omega \ll 1, \\
\frac{eB_1}{\gamma mc\nu}, & \omega/\Omega \gg 1.
\end{array} \right.
\] (23)

The estimate at \( \omega/\Omega \gg 1 \) is also true in the absence of the external magnetic field and coincides with the parameter \( f \) of the theory of synchro-Compton emission (Gunn & Ostriker 1971; Rees 1971; Blandford 1972). The restriction given by equation (22) implies that \( f \ll 1 \), i.e. the linearization technique is admissible only in the case when the secondary waves emitted by the electron present a customary Compton-scattered radiation rather than the synchrotron emission in the magnetic field of a strong low-frequency incident wave.

Our consideration is concerned with the regime of customary Compton scattering in a strong external magnetic field. As can be seen from equation (23), this regime is realized on condition \( B_1/B_0 \ll 1 \), i.e. if the energy density of the incident radiation is much less than the energy density of the external field. In case of pulsars, it is reasonable to analyse this condition directly in the laboratory frame, where the particles have the longitudinal component of the velocity as well. In this case, the general form of equation (4) remains the same, but the particle velocity and Lorentz factor, \( v_0 \) and \( \gamma_0 \), should be replaced by the total velocity \( v_1 \), which includes both the longitudinal and transverse components, \( \beta^2 = v^2/c^2 = \beta_1^2 + \beta_2^2 \), and the corresponding Lorentz factor \( \gamma = (1 - \beta^2)^{-1/2} \). Then for the quantities of the laboratory system we have: \( v_1/c \approx B_1/B_0 \), where \( v_1 \) is the frequency of the incident waves and \( \omega_0 = eB_0/mc \). The magnetic field of a pulsar has dipolar structure and its strength can be estimated as

\[
B_0 = 10^6 \frac{B_*}{10^{12} \text{G}} \left( \frac{R}{10^8 \text{cm}} \right)^{-3} \text{G},
\]

where \( B_* \) corresponds to the stellar surface, \( R \) is the distance from the star, and it is taken that the neutron star radius is \( 10^6 \text{ cm} \). Taking into account that \( B^2/8\pi = L/\nu \), where \( L \) is the radio luminosity, \( \nu \) is the area of the radio beam cross-section, \( \nu = \pi R^2 \chi^2 \), and \( \chi \) is the half-width of the beam, one can estimate the magnetic field strength in the waves as

\[
B_1 = 5 \chi^{-1} \left( \frac{L}{10^{32} \text{erg s}^{-1}} \right)^{1/2} \left( \frac{R}{10^8 \text{cm}} \right)^{-1} \text{G}.
\]

Hence,

\[
\frac{B_1}{B_0} = 5 \times 10^{-6} \chi^{-1} \left( \frac{L}{10^{37} \text{erg s}^{-1}} \right)^{1/2} \frac{10^{12} \text{G}}{B_*} \left( \frac{R}{10^8 \text{cm}} \right)^{-2/3}.
\] (24)

As is pointed out above, the scattering of low-frequency waves off the gyrating particles occurs in the region of cyclotron absorption of high-frequency waves, where the particles acquire substantial transverse energies. Its location in the magnetosphere can be estimated from the resonance condition for the high-frequency waves, \( \nu_0 \gamma_0 \nu''/2 = \omega_0 \omega/(2\pi) \), which follows from the invariance of the transverse momentum, \( p_\perp = \beta_1 \gamma_0 mc = \beta_0 \gamma_1 mc \), and assumed that \( \nu/\nu < \theta < 1 \):

\[
\frac{R}{10^8 \text{cm}} = 2\theta^{-2/3} \left( \frac{B_*}{10^{12} \text{G}} \frac{10^9 \text{ Hz}}{\nu_0} \right)^{1/3}.
\] (25)

Using equations (24)–(25) and taking into account that \( \chi \sim n \times 0.1 \), one can conclude that for any conceivable values of the parameters \( B_1/B_0 \) is less than unity and, correspondingly, the energy density of the incident radiation is much less than the energy density of the external magnetic field.

Assuming that \( r_1 \sim v_1/\omega \), one can find that

\[
\frac{r_1}{r_0} \sim \frac{B_1}{B_0} \frac{\Omega}{\omega} \quad \omega/\Omega \ll 1.
\] (26)

Thus, \( r_1/r_0 \ll 1 \) appears a more restrictive condition than \( v_1/v_0 \ll 1 \) because of the large parameter \( \Omega/\omega \). However, it should be kept in mind that in our case the cyclotron frequency should be small enough to provide the resonance of the high-frequency radio waves, so that \( \Omega/\omega \sim \nu_0 \beta_0 \gamma_0 v_1 \). Since pulsar radio emission spans the frequency range \( n \times 10^7 - n \times 10^{10} \text{ Hz} \), \( \nu_0 \nu_1/\nu \sim 10^2 - 10^4 \), and the condition \( r_1/r_0 \ll 1 \) is also fulfilled.

The linearization of the vector potential given by equation (2) is valid under the condition

\[
k \cdot r_1 \sim \omega' \frac{B_1}{\omega B_0} \ll 1, \quad \omega/\Omega \ll 1,
\] (27)

which is still more restrictive, since the waves are predominantly scattered into the frequency range \( \omega' \sim \gamma_0^3 \Omega \).

Given that the incident waves have A-polarization, the electric field and the perturbed velocity are almost aligned with \( B_0 \), and the restrictions take the form

\[
\frac{v_1}{v_0} \sim \frac{B_1}{B_0} \frac{\Omega}{\omega}, \\
\frac{r_1}{r_0} \sim \frac{B_1}{B_0} \frac{\Omega^2}{\omega^2}, \\
k' r_1 \sim \frac{B_1}{B_0} \frac{\Omega^2}{\omega^2} r_0^2.
\] (28)
Based on the above consideration one can conclude that our treatment is valid for $\gamma_0 < 10$. Such rotational energies are indeed typical of the particles of the secondary plasma in the magnetosphere of a pulsar (see e.g. Petrova 2002, 2003).

In the guiding centre frame, the scattered power can be estimated as $P^s = L \sigma / S$, where $J$ denotes the polarization state of the incident waves. It is reasonable to compare $P^s$ with the synchrotron power of the electron, $P_{\text{syn}} \sim e^2 \omega^2_{\text{e}} \gamma^2 / c$. Making use of the cross-sections (21) at $\beta_0 \approx 1$ and taking into account that $L / Sc = B^2 / 8 \pi$, one can obtain

$$\frac{P^A}{P_{\text{syn}}} \sim \frac{B^2_i \Omega^2_i}{B^2_0 \omega^2 \gamma^6_0},$$

$$\frac{P^B}{P_{\text{syn}}} \sim \frac{B^2_i \Omega^2_i}{B^2_0 \omega^2 \gamma^6_0}.$$

(29)

Keeping in mind the restrictions (23), (26)–(28), one can see that the scattered power is always less than the synchrotron power. As $k' r_1$ approaches unity, $P^B / P_{\text{syn}} \sim 1$ and $P^A / P_{\text{syn}} \sim \gamma_0^{-2}$.

### 3.2 Intensity transfer

As is shown in Section 2.2, the low-frequency waves, $\omega_c \ll \Omega$, are predominantly scattered into the range of high harmonics of the gyrofrequency, $\omega_c' \sim \gamma_i' / \Omega$. In the laboratory frame $\omega_c' \gamma_i' \eta' = \omega_0 \eta_0$, where $\eta' \sim 1 / \gamma_i^2$ because of relativistic beaming effect. Taking into account that $\omega_0 / \gamma_0 = 2 \pi \nu_0 / \gamma_0$, one can estimate the characteristic frequency of the scattered radiation as

$$\nu' \sim 10^{17} \theta^2, \quad \nu_0 \frac{\nu_0}{10^6 \text{Hz}} \left( \frac{\gamma_i}{10} \right)^2 \left( \frac{\nu_0}{10} \right)^3 \text{Hz}.$$  

(30)

It strongly depends on the particle rotational energy $\gamma_0$ and other parameters and is expected to fall into the optical or soft X-ray range. Note that the scattering of the low-frequency radiation into the high-energy band strongly dominates the inverse process, since the radio emission of a pulsar is much more intense.

The scattering depth can be written as

$$\Gamma^\iota = \int \Sigma \eta N_c dR = \int \frac{\eta \sigma_i^\iota \eta N_c}{\omega_0 \eta} dR,$$

(31)

where $N_c$ is the number density of the scattering particles and $\sigma_i^\iota$ stands for the scattering cross-sections (21) expressed in terms of the quantities of the laboratory frame and summed over the final polarization states:

$$\sigma^A = 32 \pi \gamma_0^6 (\nu_0 / \nu_1)^3 / 3 \theta^3 \gamma_0^2,$$

$$\sigma^B = 28 \pi \gamma_0^6 (\nu_0 / \nu_1)^2 / 3.$$  

(32)

Here it is taken that $\beta_0 \approx 1$, $1 / \gamma_1 < \theta < 1$, $\Omega = 2 \pi \nu_0 / \gamma_1 \eta$ and $\omega_c = 2 \pi \nu_1 / \gamma_1 \eta$. The number density of pulsar plasma can be presented in terms of the Goldreich–Julian number density, $N_c = \kappa B_0 / 2 \pi R_L e$ (where $\kappa$ is the plasma multiplicity factor and $R_L = 5 \times 10^8 P$ cm is the light cylinder radius), and estimated as

$$N_c = 6.4 \times 10^7 \frac{\kappa}{10^7} \frac{B_0}{10^{12} G} \frac{1}{P} \left( \frac{R}{10^8 \text{cm}} \right)^{-3} \text{cm}^{-3}.$$

(33)

Substituting equations (32)–(33) into equation (31) and taking into account that $\eta \nu / \omega_0 \eta' \sim (\nu_0 / \nu_1) \gamma_0^{-3}$, we find

$$\Gamma^A = 10^{-6} \gamma_0^3 \left( \frac{\nu_0 / \nu_1}{10^2} \right)^3 \left( \frac{10^2 \text{10}^6 \text{cm}}{\gamma_0} \right)^2 \kappa \frac{B_0}{10^7 G} \frac{1}{P} \left( \frac{R}{10^8 \text{cm}} \right)^{-3} \text{cm}^{-3} \theta^2,$$

$$\Gamma^B = 10^{-6} \gamma_0^3 \left( \frac{\nu_0 / \nu_1}{10^2} \right)^3 \kappa \frac{B_0}{10^7 G} \frac{1}{P} \left( \frac{10^9 \text{cm}^2}{\gamma_0} \right)^2 \theta^2.$$

(34)

One can see that the intensity suppression can be efficient only marginally, at $\gamma_0 \sim 10$ and $\nu_0 / \nu_1 \sim 10^3$. Note that the scattering depths for the two polarizations have distinct dependencies on the parameters and may differ substantially. Hence, if efficient, the scattering should affect polarization of outgoing radiation, since the latter is an incoherent mixture of the waves with the two polarization states.

It is interesting to compare the above scattering efficiencies with the optical depth to resonant absorption. The latter quantity is given by

$$\Gamma_\epsilon = 2 \frac{\kappa}{10} \left( \frac{B_0}{10^{12} G} \right)^{3/5} \left( \frac{10^7 \text{Hz}}{\nu} \right)^{2/5} \left( \frac{1}{P} \right)^{9/5} \sin^{4/3} \xi,$$

(35)

where $\xi$ is the angle between the rotational and magnetic axes of a pulsar (see e.g. equation 2.8 in Lyubarskii & Petrova 1998). Given that the plasma effects are ignored, the absorption depth is the same for the waves of the two polarizations. As is obvious from equation (35), resonant absorption can markedly affect the intensity of pulsar radio emission, especially at low enough frequencies. Note also that the scattering efficiencies depend on the frequency stronger than the absorption depth, so that the scattering can noticeably contribute to intensity suppression at the lowest radio frequencies.
3.3 Power of the scattered radiation

For a single electron, the scattered power can be written as

\[ P^j \approx \int \sigma_\omega^j \frac{n_j^2}{n_\omega} I_\omega(\omega, \theta, \phi) d\omega dO, \tag{36} \]

where \( n_j^2/n_\omega \) is the spectral intensity of the incident radiation, \( j \) is the final polarization state of the scattered radiation, and \( \sigma_\omega^j \) stands for the sum over the two initial polarizations. The radio emission of a pulsar is generated by the plasma and because of relativistic beaming is concentrated into a narrow cone of the opening angle \( \sim \gamma / \beta_\gamma \). Therefore one can assume that the angular distribution of the incident radiation is characterized by the \( \theta \)-function, which peaks at the angle \( \theta \) with respect to the external magnetic field \( (\gamma / \beta_\gamma \ll \theta < 1) \) and at an arbitrary azimuth. Pulsar spectra are generally described by the power law,

\[ I_\omega = I_{\omega_0} \left( \frac{\omega}{\omega_0} \right)^{-\alpha}, \tag{37} \]

where the spectral index \( \alpha \) ranges from 1 to 3 and

\[ I_{\omega_0} \approx \frac{L}{2\pi v_0 S} \tag{38} \]

with \( v_0 \) corresponding to the low-frequency turnover in the spectrum, \( v_0 \sim 10^8 \) Hz.

Since the scattering cross-sections are also decreasing functions of frequency, the main contribution to the integral in equation (36) comes from the lowest frequencies, and we have

\[ P^j \sim \frac{L}{S} \sigma_\omega^j(v_0) \gamma_0^2 \eta^2. \tag{39} \]

In our case it is reasonable to take that \( v_0 \gamma_0 / \gamma_0 \gg 1 \). Then the scattering of the A-polarization dominates and we obtain

\[ P^A = 10^{-6} \gamma_0^4 \left( \frac{v_0}{v_0} \right)^4 \frac{L}{10^{27} \text{erg s}^{-1}} \left( \frac{R}{10^8 \text{cm}} \right)^{-2} \eta^2 \theta^2 \chi^2 \text{erg s}^{-1}, \]

\[ P^W = \frac{13}{3} P^A. \tag{40} \]

The total power provided by the system of the scattering particles, \( P \sim (P^A + P^B) N_s SR \), is estimated as

\[ P = 4 \times 10^{26} \gamma_0^6 \left( \frac{v_0}{v_0} \right)^4 \frac{L}{10^{27} \text{erg s}^{-1}} \left( \frac{R}{10^8 \text{cm}} \right)^{-2} \chi \frac{B_s}{10^6 \text{T}} \frac{1}{P} \text{erg s}^{-1}. \tag{41} \]

Although the scattered power does not exceed the synchrotron power of the particles (see the end of Section 3.1), it may be large enough to be observable. Thus, the scattering of low-frequency radiation by gyrating particles may be an additional mechanism of the pulsar high-energy emission.

As has been demonstrated in Petrova (2002, 2003), the evolution of the particle distribution function is mainly determined by the resonant absorption, whereas the contribution of the spontaneous synchrotron re-emission is typically insignificant. Hence, the influence of the scattering on the particle momenta is all the more weak. Note, however, that the induced scattering off the gyrating particles, which is believed to be efficient because of extremely high brightness temperatures of pulsar radiation, may somewhat contribute to the increase of the particle rotational energies; this point will be examined in detail elsewhere.

4 DISCUSSION AND CONCLUSIONS

We have examined the magnetized scattering off a gyrating electron and have particularly concentrated on the low-frequency case, when the incident waves are well below the cyclotron resonance. The electron gyration changes the character of the scattering essentially. The scattered radiation presents a series of the harmonics of the gyrofrequency, \( \omega' = \omega + n \Omega \), \( n = 0, 1, 2, \ldots \). At \( n = 0 \), the scattering cross-sections for different polarization channels resemble those in case of the electron at rest, but do not coincide exactly, except for the channel \( B \rightarrow B' \). Furthermore, the scattering cross-sections appear to peak at high harmonics, \( n \sim \gamma_0^2 \), so that the scattered radiation concentrates in the same range as the synchrotron emission of the electron, \( \omega' \sim \gamma_0^2 \Omega \). The total scattering cross-section summed over the harmonics greatly exceeds the magnetized cross-section for the electron at rest, and the scattering process has distinct polarization signatures. In case of the electron at rest, the scattering in the channel \( A \rightarrow A' \) strongly dominates, whereas the cross-sections for the other channels are of the same order, \( \sim \omega_0^2 / \Omega^2 \) less, and differ from each other by geometrical factors. For the relativistic gyrating electron, the cross-sections for the two incident polarizations are related as \( \sigma^{A} / \sigma^{B} = 16 \Omega^2 / 9 \omega_0^2 \gamma_0^2 \), this ratio can be small and large, and \( \sigma^{A} / \sigma^{B} = 3 / 13 \), \( \sigma^{A} / \sigma^{B} = 1 / 13 \).

The low-frequency scattering off the particles performing ultrarelativistic helical motion is directly applicable to pulsars. Close to the neutron star surface, the magnetic field is so strong that the particle rotational energies are almost immediately lost via synchrotron emission. However, in the outer magnetosphere, where the synchrotron losses are much less, the particles can acquire substantial transverse energies as a result of resonant absorption of the pulsar radio emission. The scattering in the regime under consideration is believed to take place at the bottom of the resonance region of radio waves, where the high-energy waves meet the condition of cyclotron resonance, whereas the low-frequency ones are still well below the resonance.
It is known that in pulsar case the spontaneous scattering by the rectilinearly moving particles affects the radio intensities negligibly. Although the scattering cross-section for the gyrating particles is much larger, the scattering efficiency still remains small over the radio frequency range, except for the lowest frequencies. It should be noted that the resonant absorption can noticeably suppress the radio intensities, especially at low frequencies, and can account for the low-frequency turnovers in pulsar spectra (Luo & Melrose 2001; Fussell et al. 2003). Since the scattering efficiencies for both polarization states are stronger functions of frequency than the absorption depth, one can expect that the scattering may contribute to intensity suppression beyond the spectral turnover. Note that in contrast to the resonant absorption the scattering affects the polarization of outgoing radiation. The scattering signatures in pulsar radio emission are yet to be studied observationally. At present, the range beyond the low-frequency turnover is accessible only for a few radio telescopes (Konovalenko, Lecacheux & Rosolen 2000; Braude, Konovalenko & Még 2002) and is difficult to investigate. The recent progress in the observational low-frequency radio astronomy, in particular, construction of the LOFAR telescope, seems very promising as to the thorough studies of pulsar radio emission at the lowest radio frequencies (Stappers et al. 2007).

The characteristic frequencies of the scattered radiation are the same as those of the synchrotron re-emission of the particle and fall into the optical or soft X-ray band. The scattering can noticeably contribute to the pulsar high-energy emission. Generally speaking, the mechanism of pulsar high-energy emission is still a matter of debate (see e.g. Harding 2005, for a review). It should be noted that the distinctive feature of the mechanisms based on synchrotron re-emission of the particles participating in resonant absorption of the radio emission (Petrova 2003; Harding, Usov & Muslimov 2005) is a physical connection between the radio and high-energy emissions of a pulsar. An evidence of such a connection has recently been found in observations (Lommen et al. 2007). The low-frequency scattering off the gyrating particles is the additional mechanism of the pulsar high-energy emission, which also implies a connection with the lowest radio frequencies. Thus, the scattering process studied in the present paper may have important observational consequences.

The scattering cross-section derived in Section 2.2 is believed to allow a number of other astrophysical applications. The spontaneous scattering considered above is expected to be accompanied with the induced one. Besides that, in the magnetosphere of a pulsar, there may be other scattering sites, e.g. in the region of closed magnetic field lines. The scattering regime examined may also be applicable to synchrotron sources. In the classical formulation of the problem on the Compton losses in a synchrotron source, the frequencies of synchrotron emission greatly exceed the particle gyrofrequency and the assumption of the scattering by the rectilinearly moving particles is well justified. However, in case of a substantially broad distribution function of the particles and/or significant magnetic field gradients the values of the gyrofrequency lie over a wide range and the condition of the low-frequency scattering may also be satisfied. Since this process is much more efficient, it may have important implications.

ACKNOWLEDGMENT

I am grateful to the anonymous referee for useful comments.

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© 2007 The Author. Journal compilation © 2007 RAS, MNRAS 383, 1413–1424
Performing term-by-term differentiation of the series and finding the first- and second-order derivatives, one can obtain

\[
\sum_{\nu} \frac{v^4 J_{\nu}(2\nu \epsilon)}{(2\nu \epsilon)^2} \equiv \frac{\epsilon^2 (1 + \epsilon^2)}{2(1 - \epsilon^2)^2}. \quad (A1)
\]

Performing term-by-term differentiation of the series and finding the first- and second-order derivatives, one can obtain

\[
4(1 - 1/\epsilon^2) \sum_{\nu} v^4 J_{\nu}(2\nu \epsilon) = x''(\epsilon) + x'(\epsilon)/\epsilon. \quad (A2)
\]

Above we have taken into account the Bessel equation, \(J'_0(z) + J'_1(z)/z + J_1(1 - \nu^2 z^2) = 0\). Using equations (A1)–(A2) yields

\[
\sum_{\nu} v^4 J_{\nu}(2\nu \epsilon) = \frac{\epsilon^2 (1 + 14\epsilon^2 + 21\epsilon^4 + 4\epsilon^6)}{2(1 - \epsilon^2)^2}. \quad (A3)
\]

Then making use of the integral (Gradshteyn & Ryzhik 1980)

\[
\int_{0}^{\pi/2} J_{\nu}(2\nu x \sin \theta) d\theta = \frac{\pi}{2} J_{\nu}^2(vx), \quad (A4)
\]

one can write

\[
\sum_{\nu} v^4 J_{\nu}^2(vx) = \frac{1}{\pi} \int_{0}^{\pi/2} x^2 \sin^2 \theta \frac{1 + 14x^2 \sin^2 \theta + 21x^4 \sin^4 \theta + 4x^6 \sin^6 \theta}{(1 - x^2 \sin^2 \theta)^3} d\theta. \quad (A5)
\]

Performing routine integration, we find finally

\[
\sum_{\nu} v^4 J_{\nu}^2(vx) = \frac{x^2(64 + 592x^2 + 472x^4 + 27x^6)}{256(1 - x^2)^{3/2}}. \quad (A6)
\]

To get the sum of the analogous series \(\sum_{\nu} v^4 J_{\nu}^2(vx)\) we proceed from the well-known formula of the theory of synchrotron emission

\[
x(x) \equiv \sum_{\nu} v^4 J_{\nu}^2(vx) = \frac{x^2(4 + x^2)}{16(1 - x^2)^{3/2}}. \quad (A7)
\]
and differentiate it twice. This yields
\[ 2 \sum_{\nu=1}^{\infty} \nu^4 J_\nu^2(\nu x) - 2(1 - 1/x^2) \sum_{\nu=1}^{\infty} \nu^2 J_\nu^4(\nu x) = s''(x) + s'(x)/x, \quad (A8) \]
which can be reduced to
\[ \sum_{\nu=1}^{\infty} \nu^4 J_\nu^2(\nu x) = \frac{64 + 624x^2 + 632x^4 + 45x^6}{256(1 - x^2)^{3/2}}. \quad (A9) \]

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