Chiral Dynamics and the $S_{11}(1535)$ Nucleon Resonance

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Abstract

The $SU(3)$ chiral effective lagrangian at next-to-leading order is applied to the S-wave meson-baryon interaction in the energy range around the $\eta N$ threshold. Potentials are derived from this lagrangian and used in a coupled channel calculation of the $\pi N$, $\eta N$, $K \Lambda$, $K \Sigma$ system in the isospin-1/2, $l = 0$ partial wave. Using the same parameters as obtained from a fit to the low energy $\bar{K}N$ data it is found that a quasi-bound $K \Sigma$-state is formed, with properties remarkably similar to the $S_{11}(1535)$ nucleon resonance. In particular, we find a large partial decay width into $\eta N$ consistent with the empirical data.

∗ Work supported in part by BMBF and GSI
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I. INTRODUCTION

The nucleon resonance $S_{11}(1535)$ has the outstanding property of a very strong $\eta N$ decay channel. The proper understanding of the low energy eta-nucleon interaction and threshold eta production consequently hinges on a correct description of the $S_{11}(1535)$ resonance. Recently precise eta photoproduction data off protons and nuclei close to threshold have become available. At MAMI (Mainz) very precise angular distributions for the reaction $\gamma p \rightarrow \eta p$ have been measured from threshold at 707 MeV up to 800 MeV photon lab energy. Together with an analogous experiment using virtual photons (electroproduction) performed at ELSA (Bonn) the data cover the whole range of the $S_{11}(1535)$ resonance. The measured cross sections clearly exhibit the strong $S_{11}(1535)$ dominance of near threshold $\eta$-production. Furthermore, the cross sections for the $\eta$-photoproduction off nuclei show an $A^{2/3}$-dependence on the mass number $A$, characteristic of strong, surface-dominated interactions. Certainly, these new data demand a closer look at this particular nucleon resonance.

The traditional picture of the $S_{11}(1535)$ is that of an excited three quark nucleon resonance, with one of three quarks orbiting in an $l = 1$ state around the other two. This approach has however difficulties in explaining the large ($30-55\%$) $\eta N$ branching ratio. In the tensor force from the hyperfine interactions due to one-gluon exchange can produce the required SU(6) mixing to cause a large coupling to the $\eta N$ channel for the $S_{11}(1535)$ and a near-zero coupling for the $S_{31}(1650)$. However, then there are problems in reproducing the observed total decay width.

At present a frequently used ansatz for incorporating the $S_{11}(1535)$ resonance into the $\pi N$, $\eta N$ (and $\gamma N$) system is to couple these channels directly to the $S_{11}(1535)$ via a phenomenological isobar model with background terms. In these models, the coupling constants $g_{\pi NN}$ and $g_{\eta NN}$ are treated as free parameters. Their values vary in the literature, but always $g_{\eta NN}$ is the larger of the two. The physical reason behind the large coupling of the $S_{11}(1535)$ to the $\eta N$ channel is not understood.

In this letter, we investigate the possibility that the $S_{11}(1535)$ is a quasi-bound meson-baryon S-wave resonance. The basis of our calculation is the $SU(3)$ effective chiral lagrangian, with explicit symmetry breaking due to the non-vanishing up, down and strange quark masses properly incorporated. This approach successfully describes the $\Lambda(1405)$ resonance as a quasi-bound $KN$ state. Using the same lagrangian parameters as determined from our $KN$ analysis, we extend the formalism to the energy range $1480$ MeV $< \sqrt{s} < 1600$ MeV to explore whether any $l = 0$, $I = 1/2$ resonances can be formed, and what their properties are. Indeed, we find a resonant state with a large $\eta N$ decay width as well as other characteristic properties of the $S_{11}(1535)$.

II. EFFECTIVE CHIRAL LAGRANGIAN AND PSEUDO-POTENTIAL APPROACH

The effective chiral lagrangian for meson-baryon interaction can be systematically expanded in powers of small external momenta.

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots$$

(1)
where the superscript denotes the power of the meson momentum appearing in each term. In the heavy baryon mass formalism [11], the leading piece at order $q$ reads

$$\mathcal{L}^{(1)} = \text{tr}(B \nabla \cdot DB)$$

with the chiral covariant derivative $D^\mu B = \partial^\mu B + [\Gamma^\mu, B]$. As this part of the lagrangian incorporates all current-algebra results of the meson-baryon interaction, it is referred to as the Weinberg-Tomozawa or current algebra term. At next order in the expansion scheme, $q^2$, there is a host of new terms allowed by chiral symmetry [12].

In the heavy baryon formalism the most general form relevant to S-wave scattering is given by [9]

$$\mathcal{L}^{(2)} = \frac{1}{2M_0} \text{tr}(B( (v \cdot D)^2 - D^2 ) B)$$

$$+ b_D \text{tr}(B\{\chi_+, B\}) + b_F \text{tr}(B\{\chi_+, B\}) + b_0 \text{tr}(BB) \text{tr}(\chi_+$$

$$+ d_0 \text{tr}(BB) Tr(A^2 + (v \cdot A)^2)$$

$$+ d_1 \text{tr}(BA^\mu) \text{tr}(A^\mu B) + \text{tr}(B(v \cdot A)) \text{tr}((v \cdot A)B))$$

$$+ d_2 \text{tr}(BB) \text{tr}(A^2 + (v \cdot A)^2 + (v \cdot A)B(v \cdot A))) .$$

The first term above is a relativistic correction involving the baryon mass $M_0$ in the chiral limit. The parameters $b_D = 0.066 \text{ GeV}^{-1}$ and $b_F = -0.213 \text{ GeV}^{-1}$ are determined from the mass splittings in the baryon octet. The other six parameters have been determined in a fit to the low energy $KN$ experimental data [9], constrained by some $\pi N$ and $KN$ data.

In order to investigate the possibility of resonance formation, one needs a non-perturbative approach which resums a set of diagrams to all orders. Since this leads necessarily beyond the systematic expansion scheme of chiral perturbation theory, we use a potential model. A pseudo-potential is constructed such that in the Born approximation it has the same S-wave scattering length as the effective chiral lagrangian, at order $q^2$. We note that this approach is quite similar to the one used in [13] for the nucleon-nucleon interaction.

As in [9] we examine two ways of parameterizing the finite range of the potential while keeping the Born term the same: a local potential and one separable in the incoming and outgoing center-of-mass momenta. The local potential between channels i and j is chosen to have a Yukawa form

$$V_{ij}(\vec{r}) = \frac{C_{ij} \alpha_{ij}^2}{8\pi f^2} \frac{M_i M_j e^{-\alpha_{ij} r}}{s \omega_i \omega_j r}$$

where the indices $i, j \in \{1, 2, 3, 4\}$ label the four channels $\pi N$, $\eta N$, $K\Lambda$ and $K\Sigma$ respectively. $M_i$ and $\omega_i$ stand for the baryon mass and reduced meson-baryon energy in channel i, $s$ is the squared center-of-mass total energy and $f = 92.4 \pm 0.3$ MeV the pion decay constant [14]. The parameters $\alpha_{ij}$ can be interpreted as average ”effective masses” representing the spectrum of exchanged particles in the $t$-channel mediating the interaction. The relative interaction strengths $C_{ij}$ which follow directly from the effective chiral lagrangian are listed in the appendix.

The potential of Eq.(4) is then inserted into the coupled channel Schrödinger equation.
\[(\nabla^2 + k_i^2)\psi_i = 2\omega_i \sum_{j=1}^{4} V_{ij}\psi_j \tag{5}\]
to solve for the multi-channel S-matrix. For comparison we also examine a separable potential in momentum space, \(\tilde{V}_{ij}(k_i, k_j) = C_{ij}\sqrt{M_i M_j/\omega_i\omega_j}\sqrt{\alpha_i^2\alpha_j^2[4\pi^2f^2(\alpha^2_i + k^2_i)(\alpha^2_j + k^2_j)]^{-1}}\), which is iterated in a corresponding Lippmann-Schwinger equation as described in \[9\].

### III. DISCUSSION AND RESULTS

It is instructive first to discuss qualitative aspects of the calculation. The situation in the strangeness \(S = 0\) sector at energies near the \(K\Sigma\) threshold is similar to the \(S = -1\) case near the \(KN\) threshold. There the \(\Lambda(1405)\) resonance can be produced as a quasi-bound \(KN\) state resulting from the strong \(I = 0\) attraction between the anti-kaon and the nucleon, as well as between the pion and sigma hyperon. This attractive interaction comes at leading order \(q^2\) from the current algebra term, Eq.(2). For the quantum numbers \(S = 0, I = 1/2\) and \(l = 0\) there are several important features:

- There is a strong attraction between the kaon and sigma (see the large negative coefficient \(C_{44}\) in the appendix). Thus, as soon as the inverse range parameter \(\alpha\) exceeds a certain minimal value, a bound state will be necessarily formed below the \(K\Sigma\) threshold.

- The direct interaction between the \(\eta\) meson and the nucleon is very weak and there is a small direct coupling between the \(\pi N\) and \(\eta N\) channels (\(C_{22}\) and \(C_{12}\) are small).

- However, there is a strong coupling of both the \(\pi N\) and \(\eta N\) channels to the \(K\Sigma\) channel (\(C_{14}\) and \(C_{24}\) are sizable).

- Thus the resonance formed will strongly connect the \(\pi N\) and \(\eta N\) channels through the coupled channel dynamics.

Let us for the moment consider only the current algebra piece, \(i.e.\) all \(b\)- and \(d\)-parameters are zero and the \(1/M_0\) corrections are neglected. In this case \(C_{22} = C_{12} = 0\), but \(C_{23}\) and \(C_{24}\) are large. In Table 1 we show the resonance energy versus \(\alpha\) for both the local and separable potential using a common inverse range \(\alpha\) for all channels. The resonance position is identified when an eigenphase of the multi-channel S-matrix is equal to 90 degrees. Thus, if \(\alpha > 490\) MeV for the local potential (or 670 MeV for the separable one) a resonance is necessarily formed below the \(K\Sigma\) threshold from the current algebra piece alone. Experimentally, there are two \(S_{11}\) nucleon resonances in this energy range, at 1535 and 1650 MeV. Since only the \(S_{11}(1535)\) has a large \(\eta N\) branching ratio it is the main candidate for this dynamically generated resonance.

Next we include all order \(q^2\) terms using values of the \(b\)- and \(d\)-parameters as previously obtained from a fit to the low energy \(KN\) data and allow for a \(\pm 5\%\) uncertainty in the parameters. We note that they are similar for both potential forms \[4\]. Thus the only free parameters are the \(\alpha_{ij}\) in Eq.(4). Since the \(\pi N\) channel
is far above its threshold a satisfactory fit to all the data using only one common range for all channels could not possibly be expected. However, a good fit was found using only two range parameters: one for the $\pi N$ channel, and one common range for the other three. The off-diagonal ranges were taken to be $\alpha_{ij} = (\alpha_i + \alpha_j)/2$.

We performed a coupled channel calculation for the $\pi N S_{11}$ phase shift and inelasticity, as well as the $\pi^- p \rightarrow \eta n$ cross section. The results of the fit for both the local and separable potential forms are shown in Figs.1 and 2a,b. Here the range parameters are $\alpha_{\pi N} = 320$ MeV and $\alpha_{\eta N} = \alpha_{K \Lambda} = \alpha_{K \Sigma} = 530$ MeV for the local potential. For the separable potential the range parameters are $\alpha_{\pi N} = 573$ MeV and $\alpha_{\eta N} = \alpha_{K \Lambda} = \alpha_{K \Sigma} = 776$ MeV. The values of $b_0$ and the $d$ parameters, which only differ by 5% from [9], are listed in Table 2.

It is remarkable that such a good fit to the $\pi N S_{11}$ phase shift and the $\eta$-production cross section is obtained with only two free parameters. Clearly, one cannot expect the $S_{11}$ inelasticity to be accurate since the $\pi\pi N$ channel is neglected here. Nevertheless, this picture of the $S_{11}(1535)$ as a dynamic resonance based on the effective chiral lagrangian reproduces many of its properties. For example we obtain a resonance mass $M^* = 1557$ MeV and a full width $\Gamma_{tot} = 179$ MeV. These values agree favorably with existing empirical determinations [14], [1]. As byproduct we extract the $\eta N$ S-wave scattering length to be $a_{\eta N} = (0.68 + i 0.24)$ fm. This number is close to values found from other analyses [13, 8, 16].

In Fig.3 we display the $K\Sigma$ and $K\Lambda$ components of the bound state wave function at resonance. The root mean square radii are 0.70 fm and 0.88 fm for the $K\Sigma$ and $K\Lambda$ components.

### IV. RESONANCE AND BACKGROUND EFFECTS

If there are only two reaction channels, it is often useful to parameterize the $S$-matrix in terms of its two eigenphases and a mixing angle $\epsilon$. In the case of a pure Breit-Wigner resonance the $T$-matrix has the following energy dependence (on $W = \sqrt{s}$):

$$T(W) = \frac{1}{2(M^* - W) - i\Gamma(W)} \left( \begin{array}{cc} \gamma_1 & \sqrt{\gamma_1 \gamma_2} \\ \sqrt{\gamma_1 \gamma_2} & \gamma_2 \end{array} \right),$$

with $\det T(W) = 0$. The constant $M^*$ is the resonance mass and $\Gamma(W)$ the (energy dependent) width. For a S-wave resonance which decays into two-particle final states unitarity requires the energy dependence of the width to be $\Gamma(W) = \gamma_1 k_1(W) + \gamma_2 k_2(W)$. Here, $k_i(W)$ is the center-of-mass momentum in channel $i$ and the constants $\gamma_i$ are related to the partial decay widths $\gamma_i k_i(M^*)$. For a pure Breit-Wigner resonance, one eigenphase of the $S$-matrix (background) is zero and the other one (resonant) has the energy dependence

$$\tan \delta_{res}(W) = \frac{\Gamma(W)}{2(M^* - W)}.$$

Even though we do not have a pure Breit-Wigner resonance we find a resonant eigenphase (see Fig.4) which is very close to a Breit-Wigner form. In this figure, we plot both the resonant and non-resonant eigenphases versus pion lab kinetic energy for our
calculation. The dots correspond to a Breit-Wigner form with parameters $M^* = 1557$ MeV, $\gamma_\pi = 0.26$ and $\gamma_\eta = 0.25$. These numbers result in partial decay widths $\Gamma_\pi = 124$ MeV and $\Gamma_\eta = 55$ MeV. The branching ratio $b_\eta = 0.31$ is still compatible with the existing analysis [14] whereas $b_\pi = 0.69$ is somewhat too large, presumably due to the neglect of the $\pi\pi N$ channel and our way of extracting $\gamma_i$. Here, the $\gamma_i$ are determined from the energy dependence of the resonant eigenphase. We note furthermore that at the $\eta N$-threshold ($W_{th} = M_N + m_\eta$) the $\pi N$ $S_{11}$ phase shift reads according to Eq.(7)

$$\tan \delta_{11}(W_{th}) = \frac{\gamma_\pi k_\pi(W_{th})}{2(M^* - W_{th})}$$

within the two-channel calculation, since the background phase is zero at $W_{th}$. Therefore a good knowledge of this particular phase constrains the resonance mass and the $\pi N$ partial decay width.

From the T-matrix for a pure Breit-Wigner resonance in Eq.(6), the ratio of cross sections for scattering from channel $1 \to 2$ divided by that for elastic scattering ($1 \to 1$) is $\sigma_{21}(W)/\sigma_{11}(W) = \gamma_2 k_2(W)/\gamma_1 k_1(W)$. Therefore, the ratio $R_{BW}$ defined as

$$R_{BW} = \frac{\gamma_1 k_1(W) \sigma_{21}(W)}{\gamma_2 k_2(W) \sigma_{11}(W)}$$

is exactly one for a pure Breit-Wigner resonance. Any deviation from unity originates from the background eigenphase, assuming that the resonant eigenphase has well determined partial widths. In Fig. 5 we plot the quantity

$$R_{BW} = \frac{\gamma_\pi k_\pi \sigma(\pi N \to \eta N, S_{11})}{\gamma_\eta k_\eta \sigma(\pi N \to \pi N, S_{11})}$$

which involves $S_{11}$ partial wave cross sections only, versus the pion lab kinetic energy. The solid line corresponds to the potential model used here. Since the $\eta$-production near threshold is strongly S-wave dominated, one can identify $\sigma(\pi N \to \eta N, S_{11})$ with $\frac{1}{2} \sigma(\pi^- p \to \eta n)$. The $S_{11}$ component of the elastic $\pi N$ cross section can be constructed from the partial wave analysis of [14,18]. Using these inputs together with $\gamma_\pi/\gamma_\eta = 1.04$ as determined from the shape of our resonant eigenphase, we display the result for this ratio. Since presently the branching ratios $b_\pi, b_\eta$ have large uncertainties, we choose for reasons of comparison the values obtained here. The error bars in Fig.5 reflect only those of $\eta$-production data. It is visible that $R_{BW}$ deviates from unity as required for a pure Breit-Wigner resonance. This is an indication that background effects (corresponding to a non-resonant eigenphase) are not negligible. In [8] a similar phenomenon was observed in so far as the coupling constants of the resonance depended strongly on the treatment of the background (nonresonant) amplitude. Also in [19] the coupling constants of the $S_{11}(1535)$ had to be varied up to 20% from the values obtained from the widths in order to obtain a good fit to the scattering data. This all points towards the presence of some background amplitude. We remark however that inclusion of the $\pi\pi N$ channel in our analysis would change the branching ratios and could considerably lower the ratio $R_{BW}$ from the value shown in Fig.5. If possible an experimental determination of this ratio would be very valuable.
In summary, we have used the effective chiral lagrangian at next-to-leading order to investigate the possibility that the \( S_{11}(1535) \) resonance is a quasi-bound \( K\Sigma-K\Lambda \) state. Using the same parameters as obtained from fitting the low energy \( \pi N \) data and two free finite range parameters, a resonance can be formed at 1557 MeV with the characteristic properties of the \( S_{11}(1535) \). The \( \pi N \) phase shifts and inelasticities as well as the \( \eta N \)-production cross section are remarkably well reproduced. The dynamically generated resonance has a full width of 179 MeV and branching ratios extracted from the shape of the resonance of 69\% into \( \pi N \) and 31\% into \( \eta N \) final states. Furthermore, we elaborated on the background effects in the reactions dominated by the \( S_{11}(1535) \) and proposed as a measure for it the ratio \( R_{BW} \) in Eq.(10). The coupled channel approach presented here can also be used in calculations of the \( \eta \)-photoproduction process, and we hope to report on this topic in the near future.

**Appendix**

Here we list the expressions of the relative coupling strengths \( C_{ij} \) in the \( I = 1/2 \) basis entering the potential of Eq.(4) in terms of the chiral lagrangian parameters. The indices 1, 2, 3, 4 refer to the \( \pi N \), \( \eta N \), \( K\Lambda \), \( K\Sigma \) channel respectively and the \( \eta \)-particle is identified with the \( SU(3) \)-octet state \( \eta_8 \). Furthermore, \( E \) denotes the center-of-mass meson energy and \( M_0 \approx 0.91 \) GeV is the octet baryon mass in the chiral limit.

\[
C_{11} = -E_\pi + \frac{1}{2M_0}(m_\pi^2 - E_\pi^2) + 2m_\pi^2(b_D + b_F + 2b_0) - E_\pi^2(d_D + d_F + 2d_0) \\
C_{12} = 2m_\pi^2(b_D + b_F) + E_\pi E_\eta(d_2 - d_D - d_F) \\
C_{13} = \frac{3}{8}(E_\pi + E_K) + \frac{3}{16M_0}(E_\pi^2 - m_\pi^2 + E_K^2 - m_K^2) - \frac{1}{2}(m_K^2 + m_\pi^2)(b_D + 3b_F) + \frac{E_\pi E_K}{2}(d_D + 3d_F - d_2) \\
C_{14} = -\frac{1}{8}(E_\pi + E_K) - \frac{1}{16M_0}(E_\pi^2 - m_\pi^2 + E_K^2 - m_K^2) + \frac{1}{2}(b_F - b_D)(m_\pi^2 + m_K^2) + \frac{E_\pi E_K}{2}(d_D - d_F - 2d_1 - 3d_2) \\
C_{22} = \frac{16}{3}m_K^2(b_D - b_F + b_0) + 2m_\pi^2(\frac{5}{3}b_F - b_D - \frac{2}{3}b_0) + E_\eta^2(d_F - \frac{5}{3}d_D - 2d_0 + \frac{2}{3}d_2) \quad (11) \\
C_{23} = \frac{3}{8}(E_\eta + E_K) + \frac{3}{16M_0}(E_K^2 - m_K^2 + E_\eta^2 - m_\eta^2) + (b_D + 3b_F)(\frac{5}{6}m_K^2 - \frac{1}{2}m_\pi^2) \\
- E_\eta E_K(\frac{d_F}{2} + \frac{d_D}{6} + d_1 + \frac{5d_2}{6}) \\
C_{24} = \frac{3}{8}(E_\eta + E_K) + \frac{3}{16M_0}(E_K^2 - m_K^2 + E_\eta^2 - m_\eta^2) + (\frac{5}{2}m_K^2 - \frac{3}{2}m_\pi^2)(b_F - b_D) + \frac{E_\eta E_K}{2}(d_D - d_F - d_2) \\
C_{33} = (\frac{10}{3}b_D + 4b_0)m_K^2 + E_K^2(\frac{2d_2}{3} - 2d_0 - \frac{5d_D}{3}) \\
C_{34} = 2m_K^2b_D + E_K^2(d_2 - d_D) \\
C_{44} = -E_K - \frac{1}{2M_0}(E_K^2 - m_K^2) + 2m_K^2(b_D - 2b_F + 2b_0) + E_K^2(2d_F - d_D - 2d_0) 
\]
Table 1. The energy of the $K\Sigma-K\Lambda$ ($I = 1/2$) quasi-bound state produced from the current algebra (Weinberg-Tomozawa) term alone as a function of the range parameter $\alpha$ for both the local and the separable potential. The range parameter $\alpha$ is the same for all channels.

| Potential   | $b_0$ | $d_0$ | $d_D$ | $d_F$ | $d_1$ | $d_2$ | $\alpha_{\pi N}$ | $\alpha_{K\Sigma}$ |
|-------------|-------|-------|-------|-------|-------|-------|-------------------|---------------------|
| Local       | -0.517 | -0.68 | -0.02 | -0.28 | +0.22 | -0.41 | 0.32              | 0.53                |
| Separable   | -0.279 | -0.42 | -0.23 | -0.41 | +0.27 | -0.65 | 0.57              | 0.77                |

Table 2. Values of the Lagrange parameter entering at order $q^2$ in units of GeV$^{-1}$. The inverse ranges $\alpha$ are given in GeV.

Figure Captions

Fig.1 The cross section $\sigma(\pi^-p \to \eta n)$ versus the pion lab kinetic energy $T_\pi$. The selected data are taken from [20]. The solid/dashed line corresponds to the local/separable potential form.

Fig.2a The pion-nucleon $S_{11}$ phase shift as a function of the pion lab kinetic energy. The triangles/circles are from the phase shift analysis of [17]/[18]. The full/dashed curve corresponds to a calculation using a local/separable potential form.

Fig.2b The pion-nucleon $S_{11}$ inelasticity as a function of the pion lab kinetic energy. The notation is the same as in Fig.2a.

Fig.3 The two-component bound state wave function at resonance versus the meson baryon distance $r$.

Fig.4 The eigenphases of the multi-channel $S$-matrix below the $K\Lambda$-threshold. The heavy dots correspond to a Breit-Wigner fit of the resonant phase.

Fig.5 The ratio $R_{WB}$ defined in Eq.(10). The notation is the same as in Fig.2a. The error bars reflect only those of the $\eta$-production cross sections.
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$\frac{q_p}{q_h} = 1.04$