Understanding the neutrino mass constraints achievable by combining CMB lensing and spectroscopic galaxy surveys

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Abstract. We perform a thorough examination of the neutrino mass ($M_\nu$) constraints achievable by combining future spectroscopic galaxy surveys with cosmic microwave background (CMB) experiments, focusing on the contribution of CMB lensing. CMB lensing can help by breaking the $M_\nu$-curvature degeneracy when combined with baryon acoustic oscillation (BAO)-only measurements, but we demonstrate this combination wastes a great deal of constraining power, as the broadband shape of the power spectrum contributes significantly to constraints. We also expand on previous work to demonstrate how cosmology-independent constraints on $M_\nu$ can be extracted by combining measurements of the scale-dependence in the power spectrum caused by neutrino free-streaming with the full power of future CMB surveys. These free-streaming constraints are independent of the optical depth to the CMB ($\tau$) and generally give stronger constraints alone on $M_\nu$ than are given by the combination of BAOs and CMB lensing. Finally, we demonstrate that the effect of including the galaxy-CMB lensing cross power spectrum is negligible.

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1 Introduction

In a recent paper [1], we thoroughly deconstructed the constraints on the sum of neutrino masses, $M_\nu$, achievable with upcoming spectroscopic galaxy surveys. Our focus was to understand how sensitive forecasts were to cosmological assumptions (specifically about curvature and the dark energy equation of state), and to understand where the majority of the constraining power came from. We found that constraints could degrade significantly when moving beyond $\Lambda$CDM depending on the data that was used. For example, we showed that constraints derived from distance probes, such as baryon acoustic oscillations (BAOs), could become several factors weaker if curvature was allowed to be non-zero.

We determined that the most reliable probe of $M_\nu$, due to its robustness against changes in the underlying cosmology, was the distinctive scale-dependent free-streaming signature that massive neutrinos imprint on the underlying matter power spectrum and the structure growth rate, and provided a method of extracting isolated constraint forecasts from these effects alone. We demonstrated that, if measured, these signals could provide neutrino mass constraints that are insensitive to the assumed cosmology. Significantly, these constraints are also independent of $\tau$, the optical depth to the cosmic microwave background (CMB).

In this work, we extend our analysis considerably to include forecasts from future CMB experiments. In [1], we used a very conservative Planck prior, the compressed likelihood prior [2], which aims to provide constraints from effective observables only, and is therefore insensitive to assumptions about curvature and dark energy. Here we want to expand our calculations to include the full benefits provided by CMB surveys, including the temperature power spectrum from Planck [3] and forecasted polarisation anisotropy measurements and CMB lensing.

We perform calculations for three combinations. First, we examine the most optimistic case, analysing the advantages of combining CMB lensing measurements with full broadband galaxy power spectra from spectroscopic surveys. Second, we look at the combination of BAO measurements from galaxy surveys with CMB lensing, a combination that has been very
popular in existing forecasts [4–6]. Finally, we demonstrate that our cosmology-independent free-streaming constraints from [1] can be improved by combining with powerful CMB measurements, while still remaining cosmology-independent. We consider these constraints the most robust forecasts.

The primary motivation for combining the free-streaming constraints with CMB forecasts is that the CMB lensing power spectrum also contains a relative suppression on small scales caused by the effects of neutrino free-streaming, analogous to that in the matter power spectrum. While the galaxy power spectrum is a biased measurement of the matter power spectrum, CMB lensing probes the matter power spectrum directly. Additionally, while the galaxy power spectrum contains contributions from the baryon and cold dark matter transfer functions only, the matter power spectrum probed by CMB lensing includes all matter, including massive neutrinos. It seems reasonable the CMB lensing could provide a useful complement to these measurements.

Additionally, forecasted constraints on $M_\nu$ from upcoming galaxy surveys based on the full galaxy power spectrum will primarily be limited by weak constraints on $\tau$ (see [1] and also [6]). The correlation between the two parameters arises from both being strongly correlated with $A_s$ (see [1]). As shown here and also recently by [7], CMB lensing could help overcome this obstacle somewhat through its potential for constraining $A_s$.

We work exclusively in the linear regime in this work. Analysis of the effects of implementing the non-linear power spectrum will be left for future work. In future, it will of course be important to understand how non-linearities affect these constraints.

This paper is organised as follows. We outline our methodology in Section 2, and in Section 3 we provide a detailed breakdown of the effects of combining CMB lensing information with various types of galaxy survey forecasts, with some discussion. We conclude in Section 4.

## 2 Methodology

Our calculations in this work focus on the combination of CMB and spectroscopic galaxy surveys. We refer the reader to our previous paper [1] for an overview of our Fisher matrix implementation for galaxy surveys, as well as our fiducial cosmology.

There are some minor changes to our method. The list of cosmological parameters we use and their fiducial values remain consistent with those in [1]. However, we also now add to the list $N_{\text{eff}}$ to account for the degeneracy between $N_{\text{eff}}$ and $M_\nu$ in CMB observables. This was not possible when using the Planck compressed likelihood, as the compressed likelihood priors are provided for a certain combination of free parameters, and we could not obtain one that also kept $N_{\text{eff}}$ free. Constraints in this paper are therefore marginalised over a total set of parameters: $\theta_s$, $A_s$, $N_{\text{eff}}$, $n_s$, $\omega_{\text{cdm}}$, $\omega_b$ and $\tau$ in all cases. For the extended models, the list may be extended to include some or all of $\Omega_k$, $w_0$ and $w_a$.

In this section, we discuss our Fisher matrix implementation for forecasting CMB constraints. The covariance matrix for a particular set of angular power spectra is given by

$$\langle \Delta C_\ell^{xy} \Delta C_\ell^{mn} \rangle = \frac{1}{(2l + 1) f_{\text{sky}}} C_\ell^{xy} C_\ell^{mn} + C_\ell^{xm} C_\ell^{yn}. \quad (2.1)$$

Here, $f_{\text{sky}}$ is the fraction of the sky observed. The $C_\ell$ values on the right-hand side of Equation 2.1 must include appropriate noise terms for auto-correlation power spectra. We propagate the forecasted $C_\ell$ measurement accuracies into constraints on our cosmological
parameters using the Fisher matrix formalism. We use the temperature anisotropy power spectrum and noise from the Planck Legacy Archive (2018 data release) [3]. At low $l$, the noise values are not symmetric. We take the larger values in each case. We use the temperature power spectrum in the range $2 \leq l \leq 2500$.

We forecast polarisation constraints for future surveys. We generate theoretical unlensed auto-correlation spectra $C_\ell^{EE}$ using the Boltzmann code CLASS [8]. The noise term for the polarisation auto-correlation spectra is calculated as:

$$N_\ell^{-1} = \sum_i \Delta P_i^{-2} \exp[-l(l + 1)\theta_i^2 / 8 \ln 2],$$  \hspace{1cm} (2.2)

where $i$ indexes the frequency band, $\Delta P_i$ is measured in $\mu$K-arcmin and $\theta_i$ is the FWHM beam size in arcmin. We calculate the polarisation power spectra for $l$ values of 30-2500.

Finally, we include the forecasted cross-correlation between the Planck temperature power spectrum and future polarisation power spectrum measurements ($C_\ell^{TE}$). We calculate the covariance using the existing temperature power spectrum and noise and the theoretical E-mode polarisation power spectrum and noise outlined above. We extract the noise for Planck for insertion into Equation 2.1 from the published variance values in the Planck Legacy Archive assuming $f_{\text{sky}} = 0.5$. In our calculations, we also include a prior of $\sigma(\tau) = 0.008$ (as quoted for TT,TE,EE+lowE in [9]).

When taking the derivatives of the theoretical CMB temperature and polarisation power spectrum with respect to the cosmological parameters, we do not include the effects of CMB lensing on the power spectrum, to avoid double-counting lensing information in the temperature and polarisation power spectra and CMB lensing power spectrum.

For lensing forecasts, we require the CMB lensing convergence power spectrum $C_\ell^{\kappa\kappa}$. When two surveys cover a shared area of the sky, we can also use the cross-correlation between galaxy positions and the convergence map as an additional information source. This requires the angular galaxy clustering power spectrum $C_\ell^{gg}$ and the cross power spectrum $C_\ell^{g\kappa}$ ($i$ indexes a specific redshift bin of the galaxy survey). All of these spectra can be derived from matter power spectra $P_{mn}(k, \bar{z})$ (see e.g. [10]) generated using CLASS.

We calculate the galaxy power spectra for different redshift bins $i$ making the approximation that all galaxies are at the mean redshift $\bar{z}_i$ (i.e. assuming a Dirac delta-function distribution). The Limber approximation fails for thin redshift distributions, so we use the exact equation

$$C_\ell^{g\kappa} = \frac{2}{\pi} \int dk k^2 b_i^2 P_{cb}(\bar{z}_i, k) j_l^2[kd_A(\bar{z}_i)].$$  \hspace{1cm} (2.3)

In the previous article, we assumed a maximum wavenumber in each redshift bin of $k_{\text{max}} = 0.2 \ h \text{ Mpc}^{-1}$. For consistency, we calculate the angular galaxy power spectra in these calculations up to the corresponding appropriate $\ell_{\text{max}}$ value, by converting $k_{\text{max}}$ into units of $\text{Mpc}^{-1}$ and then multiplying by the comoving angular diameter distance at the given mean redshift, $d_A(z) = D_A(z) \times (1 + z)$. We calculate the appropriate $\ell_{\text{min}}$ value likewise, basing the value on the survey area.

The factor $b_i^2 P_{cb}(\bar{z}_i, k)$ corresponds to the three-dimensional galaxy power spectrum, $P_{gg}(\bar{z}_i, k)$. As in our previous work, we assume linear galaxy bias. The subscript $cb$ emphasises that massive neutrinos do not contribute to the galaxy power spectrum but only cold dark matter and baryons. $b_i$ is the fiducial linear galaxy bias and $j_l$ is the spherical Bessel function. For the convergence power spectrum, we use the Limber approximation [11]:

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\[ C_l^{\kappa \kappa} = \left( \frac{4 \pi G \rho_{m,0}}{c^2} \right)^2 2 \int_0^{z_*} dz (1+z)^2 \left( \frac{dA(z, z_*)}{dA(z_*)} \right)^2 P_{mm} \left[ k = \frac{l+1/2}{dA(z_*, z)} \right] H(z). \] (2.4)

\( \rho_{m,0} \) is the comoving matter density, \( z_* \) is the redshift of last scattering, and \( dA(z, z_*) \) represents the comoving angular diameter distance between the two redshifts. Accounting for curvature, this is calculated as [12]:

\[ d_A(z_1, z_2) = f_{k,2}(\chi) \left( 1 + \Omega_k \left( \frac{f_{k,1}(\chi)}{D_H} \right)^2 + f_{k,1}(\chi) \sqrt{1 + \Omega_k \left( \frac{f_{k,2}(\chi)}{D_H} \right)^2} \right), \] (2.5)

\[ f_k(\chi) = \frac{1}{\sqrt{k}} \sinh(\chi \sqrt{k}) \quad \Omega_k > 0, \]

\[ = \chi \quad \Omega_k = 0, \]

\[ = \frac{1}{\sqrt{k}} \sin(\chi \sqrt{k}) \quad \Omega_k < 0, \] (2.6)

where \( k = -\Omega_k (H_0/c)^2 \) and \( D_H = (c/H_0) \) is the Hubble distance.

In this case, the requirement to evaluate \( P_{mm}(z) \) for such a large number of redshifts presents somewhat of an inconvenience. As massive neutrinos change the shape of the matter power spectrum over time, it is not sufficient to simply multiply \( P_{mm}(0) \) by a scale-independent growth factor \( D(z)^2 \) at each instance. We output \( P_{mm}(k) \) at a large number of redshifts using CLASS, and interpolate the table at the necessary redshifts.

We also use the Limber approximation to calculate the galaxy-convergence cross-power spectrum [10]:

\[ C_l^{g \kappa} = \left( \frac{4 \pi G \rho_{m,0}}{c^2} \right) (1 + z_i) \frac{dA(z_i, z_*)}{dA(z_i) dA(z_*)} b_i P_{cb,m} \left[ k = \frac{l+1/2}{dA(z_i, z_*)}, z_i \right]. \] (2.7)

Here the factor \( b_i P_{cb,m} \) corresponds to the matter-galaxy cross-power spectrum.

In the case of the CMB lensing auto power spectrum, the corresponding noise is that associated with the reconstruction of the CMB lensing potential from CMB observations, which is calculated using the algorithm provided by Okamoto & Hu [13] by interfacing the Fortran module FUTURCMB (provided by [14]) with our code. This must be rescaled by a factor of \( \frac{5}{4} [((l+2)!/(l-2)!)] \) for use with the convergence \( \kappa \) (as opposed to the lensing potential \( \phi \)) power spectrum. For the galaxy power spectra, the shot noise term is given by the inverse of the surface density of the galaxies in the particular redshift bin in steradians, \( n_g^{-1} \).

We do not include cross-spectra between redshift bins. Following [15], we assume that the covariance between \( P_{gg} \) and \( C_l^{g \kappa} \) can be neglected, and we can therefore do the Fisher matrix calculations for the two-dimensional and three-dimensional power spectra calculations separately and simply add the output Fisher matrices, i.e.:

\[ F = F(C_l^{TT}, C_l^{TE}, C_l^{EE}) + F(P_{gg}, k, z) + F(C_l^{g \kappa}) + F(C_l^{E \kappa}). \] (2.8)

Because we treat CMB lensing with a separate Fisher matrix for analysis purposes, the cross-correlation between the temperature and polarisation power spectra and the lensing power spectrum is not included. We also ran tests in which \( C_l^{TT} \) and \( C_l^{EE} \) were included, and found the change in the constraints to be less than 1% in all cases.
3 Results and Discussion

3.1 Survey Data

In our galaxy clustering paper, we focused on constraints from Euclid [16, 17]. Here we focus on forecasts from the combination of Euclid and Simons Observatory [18] (with existing information from Planck, in the forms of the CMB temperature power spectrum and a prior on $\tau$ of $\sigma(\tau) = 0.008$). The survey parameters assumed for Simons Observatory are taken from Table 1 of [18], with both the Small Aperture and Large Aperture Telescopes being included. For the galaxy-CMB lensing cross-correlation, we assume maximum overlap between Euclid and Simons Observatory. We present constraints for a wider range of survey combinations in Appendix A.

3.2 Results from the CMB Alone

Figure 1 shows the forecasted constraints on $M_\nu$ from Planck and Simons Observatory alone for various cosmologies. The constraints from temperature and polarisation alone are relatively weak but also quite insensitive to changes in curvature or the dark energy equation of state. Most of the information on $M_\nu$ comes from unlensed temperature anisotropy information, with the unlensed E-mode polarisation mostly improving constraints through tightening the constraints on other parameters rather than being directly sensitive to $M_\nu$.

Adding CMB lensing improves the constraints significantly. To determine how much the free-streaming effect on $C_\ell^{\kappa\kappa}$ contributes to the constraints, panel (c) shows the constraints from the shape of $C_\ell^{\kappa\kappa}$ alone (changes in the overall amplitude of the power spectrum in the derivatives are neglected). Panel (e) shows the constraints when the full $C_\ell^{\kappa\kappa}$ is used. Both the dark energy equation of state and curvature parameters add a scale-dependent effect that is somewhat degenerate with the neutrino effect in panel (c). However, because $\Omega_k$ is quite well constrained from temperature and polarisation data, and the dark energy equation of state parameters are not, freeing $w$ has a much more significant effect on the $M_\nu$ constraint. In panel (d), we see that fixing $\tau$ has very little effect, because the free-streaming effect is not degenerate with $\tau$.

When full lensing information is used in panel (e), the constraints on $M_\nu$ are much improved. However, here the curvature parameter degrades $M_\nu$ more than $w$. This is because a very small change in $\Omega_k$ produces a much larger change in the amplitude of $C_\ell^{\kappa\kappa}$ than $w$. The results are now strongly cosmology-dependent.

Adding CMB lensing makes $M_\nu$ correlated with $A_s$, and therefore with $\tau$ (as $A_s$ and $\tau$ are strongly degenerate in CMB measurements) unless we have a cosmic-variance-limited measurement of E-mode polarisation at $l \leq 30$. This is analogous to what happened when the CMB prior was combined with galaxy power spectrum measurements in our previous work [1]. However, in the case of Figure 1, our constraint on $\tau$ from Planck is already quite strong. Fixing $\tau$ causes an improvement of 20% in the $\Lambda$CDM case. For the other models, the constraints on the additional extension parameters remain the limiting factor.

3.3 Full Galaxy Power Spectra

Figure 2 presents forecasted constraints from the Euclid full galaxy power spectrum and Planck/Simons Observatory both with and without CMB lensing information included for a range of cosmologies. Note that the $x$-axis scale is reduced by a factor of 10 compared to that in Figure 1.
We first examine the ΛCDM case. As emphasised in previous work [1], for a powerful galaxy survey like Euclid, if the full galaxy power spectrum is used with CMB information, the constraints on \( M_\nu \) come to be limited by the weak constraints on \( \tau \). The cause of this is that the effects of \( M_\nu \) and \( A_s \) on the galaxy power spectrum are strongly degenerate, and \( A_s \) and \( \tau \) are measured in combination from the CMB. CMB lensing does not provide any additional direct information on \( \tau \), but does help constrain \( A_s \) better. This leads to the modest improvement in \( \sigma(M_\nu) \) seen in Figure 2 when comparing panels (a) and (c). Because of our strong \( \tau \) prior, the relative gain from adding CMB lensing is relatively small (about 5%). In panel (b), on the other hand, nothing is gained by adding information from the shape of the convergence power spectrum only. This is further evidence that the improvement in panel (c) is a result of the improved constraints on \( A_s \), and not the result of the scale-dependence of \( M_\nu \) in the power spectrum being included. Panel (d) shows that the gain when adding
Figure 2: Forecasted constraints on $M_\nu$ for a combination of Simons Observatory/Planck and Euclid. The full (broadband) galaxy power spectra are used to generate the Euclid Fisher matrix. Panel (a) shows the constraints without any lensing information, panel (b) adds shape information from the CMB convergence spectrum, panel (c) replaces this with full CMB lensing information and panel (d) further adds the cross-correlation between galaxy positions in Euclid and the CMB lensing map (assuming maximum overlap between the two surveys). The final two panels show the impact on the constraints without lensing and with full CMB lensing and galaxy-CMB lensing when $\tau$ is fixed.

cross-correlation information between galaxy positions and the lensing map is also negligible.

The final two panels show what can be achieved if $\tau$ is perfectly constrained. Comparing panels (e) and (f), it is clear that adding CMB lensing in this case does not help. This is to be expected, as constraining $\tau$ to this level would also significantly alter constraints on $A_s$, making what CMB lensing provides redundant.

It is significant to note that in all cases, the constraints on $M_\nu$ still depend quite heavily on the cosmological model assumed. CMB lensing does contribute to tightening constraints on curvature [19], but the dark energy equation of state can degrade constraints on $M_\nu$ considerably, particularly when the constraint on $\tau$ reaches its limit.

The relationship between the $\Lambda$CDM constraints in all of the panels in Figure 2 can be understood completely in terms of the degeneracy between $M_\nu$, $A_s$ and $\tau$. Imposing a particular relative improvement in the constraints on one of these parameters leads to an
Figure 3: The relationship between the constraints on $M_\nu$, $A_s$, and $\tau$ from Euclid (marginalised over the other parameters in the text) with and without CMB lensing for various $\tau$ priors. The errors on these three parameters are very strongly correlated, to the extent that a change in the error on one of these parameters leads to an equal relative error change on the other two parameters. CMB lensing can help improve these constraints when $\tau$ is weakly constrained, but has much less impact when a $\tau$ prior from Planck is included.

almost equal relative improvement in the constraints on the other two. Figure 3 shows this relationship in contour form. While CMB lensing is useful for improving the constraints on these three parameters, it is less powerful than the Planck $\tau$ prior we include.

3.4 BAO-Only Information

The combination of BAO and CMB lensing data is a common focus in forecasts [4–6]. As can be seen in Figure 4, the constraints from Euclid from BAOs alone are much weaker than those from the full power spectrum (note the $x$ axis scale has been increased by a factor of 10 relative to Figure 2), but a better relative improvement is obtained by combining with CMB lensing information, particularly for the more complex models. As was highlighted in [1], there is a strong degeneracy between $M_\nu$ and $\Omega_k$ in their effects on cosmological distance parameters, as is clear from panel (a). In our previous work, there was very little interaction between $M_\nu$ and the dark energy equation of state parameters in the BAO-only case. However, we can see from panel (a) that there is a significant degeneracy between $w_0/w_a$ and $M_\nu$ here. This is a
result of using the Planck temperature power spectrum and Simons Observatory polarisation forecasts instead of the compressed likelihood prior used in [1].

We first examine the case without CMB lensing. BAO information does not constrain $A_s$ or $\tau$, but CMB lensing does. Because of the lack of information on $A_s$, the degeneracy between $M_\nu$ and $A_s$ (and therefore $\tau$) that arises in the combined case does not arise here. Therefore, by comparing panels (a) and (e) in Figure 4, one can see that fixing $\tau$ makes effectively no difference to the neutrino mass constraint.

Once CMB lensing is added, the $\tau$ degeneracy is re-established (compare panels (d) and (f)). In panel (b), we see that the shape of the CMB lensing power spectrum actually makes a contribution here, unlike in the combined case (Figure 2), because the initial constraints are weaker. CMB lensing also tightens the constraints on curvature significantly, breaking the $M_\nu-\Omega_k$ degeneracy in panel (a). On the other hand, CMB lensing contributes little to the constraints on the dark energy equation of state parameters.

### 3.5 Free-Streaming Information

Figure 5 shows how CMB lensing affects our ‘free-streaming’ constraints. Massive neutrinos suppress the growth of structure on small scales to a degree that is primarily dependent on
Figure 5: As for Figure 2, but with the constraining power from Euclid only deriving from the scale-dependence of the power spectrum and structure growth rate, using the method developed in [1].

the total neutrino mass. This creates a small but distinctive scale-dependent signature in both the matter power spectrum and in the structure growth rate $f$ (which can be measured independently using redshift-space distortions). If measured, the magnitude of the suppression can be used to obtain a cosmology-independent and $\tau$-independent probe of the neutrino mass, as discussed extensively in [1]. Our ‘free-streaming’ constraint forecasts are calculated by isolating the scale-dependence in the matter power spectrum as the observable in our Fisher matrix, and marginalising over the overall amplitude, and doing likewise for the structure growth rate.

This is the first time that we present the free-streaming constraints in combination with a full CMB temperature and polarisation forecast. We see that the gains are significant over the Planck compressed-likelihood prior used previously, and that the final constraints remain effectively cosmology-independent (see panel (a)).

The gains from CMB lensing here are small. In panel (b), we see that using the suppression in the convergence power spectrum alone and neglecting the amplitude maintains the cosmology-independence but improves constraints by less than 5% in the best cases. Although we showed in Section 3.2 that the free-streaming signature in the CMB lensing power spectrum can provide meaningful constraints, the constraints from the galaxy power spec-
trum are much stronger, so the relative improvement when adding CMB lensing is small. The improved constraint on $A_s$ provided by CMB lensing is also not particularly helpful (see panel (c)), because the free-streaming measurement neglects all amplitude information (only the relative suppression is measured), so the only degeneracy between $A_s$ and $M_\nu$ is that inherent in the full CMB lensing measurement itself. This also explains why fixing $\tau$ in panel (e) has very little effect.

These constraints could be further improved, while remaining cosmology-independent, by improving constraints on $N_{\text{eff}}$, due to the anti-correlation between these two parameters. Our calculations show that fixing $N_{\text{eff}}$ can improve the constraints by 20-25%. Further information on $N_{\text{eff}}$ from big bang nucleosynthesis theory or from the BAO shift parameter \cite{20, 21} could be added to help with this. For now, we leave this for future work.

3.6 Comparisons with Previous Work

It is difficult to do direct comparisons between various forecasts of this type in the literature because of the many different assumptions that can be made, from survey choice to error management to cut-off scales. However, comparisons with some recent works can provide some reinforcement for the results provided here, particularly to manage scepticism about the Fisher matrix methodology, and can also provide some interesting insights. Here we examine how our full galaxy clustering constraints compare to those in the literature.

The authors of \cite{22} provide some MCMC forecasts for Euclid galaxy clustering assuming $\Lambda$CDM+$M_\nu + N_{\text{eff}}$. They take Planck as their CMB survey and also include cosmic shear, but their results are very close to ours (28 meV in our case vs. 24 meV or 27 meV in their realistic and conservative cases, respectively). This could further support our conclusion here that galaxy clustering information really is dominant over CMB and lensing information (in this case, cosmic shear).

In \cite{23}, table 4 gives an uncertainty on $M_\nu$ of 17 meV for CMB-S4 and Euclid in the $\Lambda$CDM+$M_\nu + N_{\text{eff}}$ case. Our corresponding value is 20 meV. Their final constraint on $\tau$ is also 20% stronger than ours, so correcting for that leaves our constraint at 16 meV. These values are very close considering we assume different CMB surveys, which may suggest that beyond improved constraints on $\tau$, ever-stronger CMB surveys have little to offer neutrino mass constraints.

4 Conclusions

This paper represents a continuation of our work in \cite{1}. While there we focused on spectroscopic galaxy surveys alone, here we have expanded our analysis to include the full power of planned CMB experiments. We consider three possible methods of constraining $M_\nu$ from galaxy surveys and the effects of CMB lensing on each: the use of the full redshift-space galaxy power spectrum, the use of BAOs alone to infer distance constraints, and the use of the signatures of neutrino free-streaming only (a method developed in \cite{1}).

Overall, we have shown that CMB lensing measurements are a much less powerful probe of the neutrino mass than large-scale structure surveys for the scales considered. When combined with the full galaxy power spectrum from a spectroscopic galaxy survey like Euclid, CMB lensing contributes to constraints on $M_\nu$ primarily by tightening constraints on $A_s$, as these two parameters are very strongly correlated in the measured galaxy power spectrum. This correlation is also the source of the $M_\nu$-$\tau$ degeneracy that limits $M_\nu$ constraints.
when CMB and galaxy clustering measurements are combined. However, if $\tau$ is already well constrained, the primary gain from adding CMB lensing becomes redundant.

CMB lensing primarily contributes to BAO-only constraints on $M_\nu$ by improving constraints on curvature. As we emphasised previously, $M_\nu$ and $\Omega_k$ are highly degenerate in their effects on distance parameters at low redshifts. This means allowing for a very small non-zero curvature can degrade the constraints on $M_\nu$ by several factors.

Finally, we look at the cosmology-independent free-streaming-only constraining method we developed in [1]. Combining the free-streaming constraints from a survey like Euclid with a full CMB temperature/polarisation forecast improves the constraints significantly while still keeping them cosmology-independent. The gains from including CMB lensing are small (see Figure 5). This is to be expected. The free-streaming constraints do not suffer from the degeneracy with $A_s$ that applies in the full galaxy power spectrum case, so CMB lensing cannot contribute much in this regard. The gain provided from the direct effects of $M_\nu$ on the lensing power spectrum are relatively small. Although the free-streaming signal in the CMB lensing power spectrum can significantly improve CMB-only constraints, it is much weaker than the corresponding signal in the galaxy power spectrum.

In combinations of galaxy clustering and CMB lensing measurements, the galaxy power spectrum is a much more powerful probe of $M_\nu$. However, as $\tau$ is better constrained, the information provided by CMB lensing will become redundant in neutrino mass constraints. The constraints provided by the full galaxy power spectrum will also become increasingly cosmology-dependent as $\tau$ becomes better known. BAO-only constraints become more robust when combined with CMB lensing but waste a lot of valuable information. The constraints extracted through the effects of free-streaming on the power spectrum, on the other hand, are cosmology-independent and independent of $\tau$. They are also stronger than those from BAO measurements and CMB lensing combined for all models apart from the simplest ($\Lambda$CDM+$M_\nu$), in which case they are just slightly weaker with our current calculation method. Using a slightly more conservative prior on $\tau$ (we use $\sigma(\tau) = 0.008$), the free-streaming constraints can become stronger than those from BAOs and CMB lensing for all models. Using the current Planck temperature power spectrum, the forecasted Simons Observatory E-mode polarisation spectrum and the free-streaming signals extracted from the Euclid galaxy power spectrum, reliable 1-$\sigma$ constraints on $M_\nu$ of approximately 0.06 eV can be achieved.

Throughout this work, we have assumed a maximum scale of $k = 0.2 \ h \ Mpc^{-1}$ and a linear bias model. In reality, reaching these small scales with future surveys may require us to have non-linear bias effects well under control. Possible degeneracies between the effects of massive neutrinos and non-linear bias on the power spectrum will be a focus of our upcoming work.

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A Other survey combinations

Here we present extended constraints for additional galaxy surveys in combination with CMB lensing from Simons Observatory. For the galaxy survey parameters used, we refer the reader to [1]. For the CMB survey parameters used, see Section 3.1.
| Galaxy Survey | Combined Constraint | Free-Streaming Constraint |
|---------------|---------------------|---------------------------|
| DESI (ELG only) | 0.034 | 0.071 |
| PFS | 0.054 | 0.1 |
| WFIRST | 0.045 | 0.09 |
| HETDEX | 0.1 | 0.16 |

Table 1: Forecasted 1σ constraints on $M_\nu$ in eV for various measurement methods and galaxy surveys, combined with lensing from Simons Observatory. All include full CMB forecasts, including CMB lensing (but not galaxy-CMB lensing, which we have shown has little effect). $\Lambda\text{CDM} + M_\nu$ is assumed in all cases.

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