Photonic Analogue of a Continuous Time Crystal

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Time crystals are an eagerly sought phase of matter in which time-translation symmetry is broken. Quantum time crystals with discretely broken time-translation symmetry have been demonstrated in trapped ions, atoms and spins while continuously broken time-translation symmetry has been observed in an atomic condensate inside an optical cavity. Here we report that a classical metamaterial nanostructure, a two-dimensional array of plasmonic metamolecules supported on flexible nanowires, can be driven to a state possessing all key features of a continuous time crystal: continuous coherent illumination by light resonant with the metamolecules’ plasmonic mode triggers a spontaneous phase transition to a state in which transmissivity oscillations result from a many-body interaction among the metamolecules, and which is characterized by long-range order in space and time. As the state can be manipulated optically, the phenomenon is of interest to topological and non-Hermitian physics and applications in frequency conversion, memory, modulation, nonreciprocity and amplification.

Isotropic homogeneous matter is invariant under space-translation symmetry while in crystals with periodic atomic lattices that symmetry is broken. In nature, the symmetry-breaking crystalline state, with long range order, is achieved spontaneously through a phase transition (e.g. from water to ice). In recent years, the physics community has been captivated by the newly described phase of matter known as a “time crystal”, with broken time-translation symmetry, analogous to conventional crystals in which space-translation symmetry is broken. A time crystal, as originally proposed by Wilczek\(^1\), is a quantum many-body system whose lowest-energy state is one in which the particles are in repetitive motion. Although it has been shown that such a system, breaking continuous time-translation symmetry by exhibiting oscillatory dynamics, is prohibited by nature\(^2\), a number of systems which show discrete time-translation symmetry-breaking imposed by an external modulated parametric drive have been realized on various platforms, including trapped atomic ions, spin impurities, ultracold atoms, condensates of magnons and quantum computers\(^3\)\(^\text{-}^{12}\).

Recently, a quantum time crystal that breaks time-translation symmetry continuously has been observed in an atomic Bose-Einstein condensate inside an optical cavity, manifested in the emergence of spontaneous oscillations of the intracavity photon number under optical pumping\(^13\). In this experiment, the pump laser was operated continuously, thereby respecting continuous time-translation symmetry while inducing a transition to the state wherein the
symmetry is broken. This atomic system realizes the spirit of Wilczek’s original proposal more closely than discrete time crystals and could be a platform for the observation of a large class of topological and non-Hermitian phenomena, and may be used in quantum computer memory.

Continuous time crystals are also potentially of great interest in photonics as they can support a variety of new wave propagation phenomena. However, to be of practical importance the material in which continuous time-translation symmetry is broken should be simple, accessible, broadband, free from the requirement of cryogenic cooling, and adaptable to different applications. Here we demonstrate that a classical nanophotonic structure, which can be manipulated and probed optically, can exhibit all the key features of a continuous time crystal.

A defining characteristic of a continuous time crystal is a spontaneous transition, in a many-body system, to a robust oscillatory state in reaction to a time-independent external stimulus. Here we show that a two-dimensional periodic lattice of 375 plasmonic metamolecules supported on 30 doubly-clamped nanowire beams cut from a semiconductor nano-membrane (Fig. 1) spontaneously transitions to a state characterized by persistent optical transmissivity oscillation when illuminated by coherent light that stimulates interaction among the metamolecules. Above a threshold level of incident optical power, the spectrally dispersed thermal fluctuations of the individual nanowires become spatially coherent synchronous oscillations over the entire array.

The experimental sample was fabricated by focused ion beam milling from a gold-coated silicon nitride nano-membrane. The metamolecules - Π-shaped arrangements of three gold plasmonic nanorods - are supported on pairs of neighboring nanowires in such a way that nanowires’ mutual displacement strongly affects the metamolecules’ resonant plasmonic scattering properties. Indeed, it has been shown previously that picometric thermomechanical fluctuations in such structures can measurably modulate scattered and transmitted light. This provides opportunity for detecting the state of the lattice optically by monitoring its transmissivity.

Fig. 1. Photonic metamaterial analogue of a continuous time crystal - A two-dimensional array of plasmonic metamolecules is supported by nanowires cut from a semiconductor membrane. (a) Artistic impression of the basic building block of the crystal – a pair of silicon nitride nanowires decorated with plasmonic [gold] metamolecules. (b) Scanning electron microscope image of the entire 2D array. Illumination with coherent light [as indicated by the schematically overlaid laser spot] induces a transition to a state of persistent synchronized nanowire oscillations.
In such an array, the nanowires can be classically described by set of \( i \) coupled oscillators with the following equation of thermally-driven motion in the out-of-plane direction:

\[
\ddot{x}_i + \Gamma_\ell \dot{x}_i + \omega_{0i}^2 x_i + \alpha x_i^2 + \beta x_i^3 + \sum \xi_{ij}(I) (x_i - x_j) = \sqrt{2 \pi k_B T \Gamma_\ell / m_i} \eta(t) \tag{1}
\]

According to the Langevin treatment of thermally driven oscillators, the right-hand side of the equation is the time-dependent thermal force experienced by the oscillator. It depends on the dissipation factor \( \Gamma_\ell \) through the fluctuation-dissipation theorem. \( \eta(t) \) is a normalized white noise term, \( m_i \) is the effective mass of the nanowire, \( k_B \) is the Boltzmann constant, \( T \) is the temperature, and \( \omega_{0i} \) \((\gg \Gamma_\ell)\) is the nanowire’s natural angular frequency of oscillation. A nanowire clamped at both ends exhibits ‘geometric nonlinearity’ (terms \( \alpha x_i^2 \) and \( \beta x_i^3 \)) that becomes significant (non-negligible) when the amplitude of the oscillation approaches the thickness of the nanowire\(^{16}\). Due to fabrication imperfections and intrinsic variations of stress in the membrane, the fundamental frequencies of the nanowires \( \omega_{0i} \) are dispersed over a range \( \delta \omega = 2\pi \times 6 \text{ kHz} \) around a central frequency \( \omega_0 = 2\pi \times 0.94 \text{ MHz} \) and they have a typical resonance width \( \Gamma = 2\pi \times 0.5 \text{ kHz} \), as can be seen in transmission spectra.

It has been shown that in absence of noise, a pair of interacting nonlinear oscillators can be driven into the discrete time crystal state by parametric modulation of the oscillators’ natural frequencies\(^{17}\). Here we experimentally demonstrate a different route to the time crystal state: the combination of noise and light-induced interaction among oscillators drives a sharp, first-order dynamic phase transition to a state in which time-translation symmetry is broken continuously: When the lattice of plasmonic metamolecules is illuminated with continuous coherent light, an optically controllable interaction between nanowires, described by the term \( \xi_{ij}(I) x_j \) in the above equation of motion, emerges through dipole-dipole coupling of the induced plasmonic (gold nanorod) excitations. The coupling coefficient \( \xi_{ij}(I) \) is a function of intensity of light \( I \). The time-dependent stochastic thermal force on the right-hand side of the equation provides the seed noise needed for synchronization.

We observe that, above a threshold level of illumination intensity, the spectrally broadened ensemble of thermally driven nanowire oscillations synchronizes into a single mode. The array is illuminated with a single continuous beam of laser light at a wavelength of 1550 nm (close to the plasmonic absorption resonance of the metamolecules), which is used both to monitor the transmissivity of the array and to induce a phase transition to its synchronized oscillatory state. The sample is housed in a low vacuum optical cell with pressure maintained at \( 10^4 \) torr to suppress damping of nanowire movements. Light is normally incident on the array, polarized parallel to the nanowires, and focused to a spot of \( \sim 5 \mu \text{m} \) diameter.

Figure 2 shows the evolution of the array’s transmissivity spectrum with incident laser power. At low laser power, \(<110\mu\text{W}\), the spectrum contains several overlapping peaks of small amplitude at frequencies just below 1 MHz (Fig. 2a), corresponding to oscillations of the individual nanowires, seen on the background of the broad noise floor. This is typical of an ensemble of nanowire oscillators with uncorrelated thermomechanical oscillation dynamics of characteristic amplitude \( \langle \chi \rangle = \frac{1}{\omega_{0i}} \sqrt{\frac{k_B T}{m_i}} \sim 250 \text{ pm} \).

With increasing laser power (Fig. 2b-d), we observe red shifting of the spectral peaks, due to thermal expansion of the nanowires induced by laser illumination, as described in Ref \(^{18}\). At around 113 \( \mu\text{W} \), the onset of synchronization is observed in the narrowing of the spectrum (Fig. 2c), and further increase of laser power leads to spontaneous synchronization of nanowires’ thermal oscillations is manifested in the emergence of a single large peak in the transmissivity oscillation spectrum (Fig. 2d). The resonant spectral density of transmissivity oscillations in this synchronized regime, when all nanowires contribute coherently to the
modulation of transmitted light, increases by more than four orders of magnitude as compared to peak amplitudes at low power (i.e. beyond the synchronization threshold). Complete

Fig. 2. Phase transition to a synchronized oscillatory regime of thermal “superradiance”. (a-d) Spectra of the metamolecule array’s thermal transmissivity fluctuations at a wavelength of 1.55 µm, at a selection of incident laser power levels [as labelled] showing spectral narrowing and gigantic increase in peak amplitude above ~115 µW [note the orders-of-magnitude change in vertical axis scales between panels (a, b) and (c, d)]. (e) Amplitude of the dominant spectral component of transmissivity oscillation as a function of incident laser power, showing the abrupt [first-order] step in amplitude.
synchronization is seen at 126.5 μW (Fig. 2d), where the spectral line shape becomes close to Lorentzian and line width becomes comparable to the typical width of the individual nanowire resonances, Γ = 2π ×0.5 kHz. Figure 3e shows the transmissivity spectra peak amplitude as a function of incident light intensity, clearly revealing a first order phase transition from the unsynchronized to the synchronized regime.

One of the main features of a continuous time crystal is that, once initiated by an external stimulus, oscillations persist over arbitrarily long times, and this is true of our experimental system: Fig. 3a shows how the spectrum of the array’s synchronized transmissivity oscillations is invariant as a function of time, for a constant level of incident optical power. Here we can also see that the periodic modulation of transmissivity is weakly anharmonic - the transmitted light has a spectral component of modulation at the second harmonic of the main out-of-plane mode of the structure. This is explained by the geometric nonlinearity of nanowire motion: for nanowires of asymmetric cross-section (semiconductor beams decorated on one side with plasmonic nanorods), both quadratic and cubic terms are present in the equation of motion.

The observed oscillations of transmissivity are persistent and stable against a range of perturbations. Indeed, Fig. 3b shows slow modulation of the transmissivity oscillation frequency driven by thermal expansion/contraction of the nanowires with modulation of incident power (and thereby nanowire temperature). Here, the incident light intensity is modulated by 5% at a frequency of 260 Hz, and the temperature of the nanowires adiabatically increases/decreases laser power as their cooling time of 85 μs is much shorter than the 3.85 ms period of intensity modulation.

When the incident optical power is more strongly modulated (20%), periodic suppression and recovery (bursts) of transmissivity modulation are observed, as illustrated in Fig. 3c and 3d. Figure 3e shows a typical time domain trace of such a burst. The dominant frequency of spontaneous oscillation varies by about ±2.5% from burst to burst as synchronization starts spontaneously each time at a slightly different power level. To illustrate that the oscillations have arbitrary phase each time a burst is generated, as is characteristic of a continuous time crystal 19, we analyzed the phase distribution of the bursts: We find that the phase of oscillation at the dominant frequency, relative to the cycle of incident light modulation, is indeed randomly distributed over the [0, 2π] range, as illustrated in Fig 3f where the real and imaginary parts of the amplitude of the dominant spectral components are presented for 155 consecutive bursts.

We argue that the underlying dynamics of the observed light-assisted phase transition to a persistent, high amplitude transmissivity oscillation regime analogous to a time crystal, can be interpreted as resulting from the synchronization of the ensemble of noisy thermal optomechanical nanowire oscillators via light-induced coupling ξij(I). Indeed, we are dealing here with the classical problem of synchronization among an ensemble of oscillators with randomly distributed frequencies, introduced by Kuramoto20: when the interaction between the oscillators is sufficiently strong, most will synchronize their dynamics to a single frequency (which may differ from the natural frequency of any one of the synchronized oscillators). Indeed, it can be shown for the present case that at the laser power threshold for synchronization, the additional energy of dipole-dipole interactions between metamolecule components on neighboring nanowires is sufficient to shift the out-of-plane oscillation frequencies of the nanowires to compensate for their natural distribution.

The ensemble of nanowire oscillators exhibiting thermal motion and interacting with help of an external light field can be seen as a classical analogue of an ensemble of quantum emitters interacting with a common light field and exhibiting Dicke superradiance21: If the wavelength of the light is much greater than the separation of the emitters, then the emitters interact with the light collectively and coherently emit light with intensity proportional to N^2
(superradiance), drastically different from the intensity proportional to $N$ for a group of independent atoms. Indeed, the relationship between the Dicke superradiant state and discrete time crystal ordering was recently theoretically generalized in terms of the Landau theory of phase transitions\textsuperscript{22} and was also noted in regard to the recently demonstrated quantum continuous time crystal\textsuperscript{19}.

In summary we have demonstrate an artificial photonic material - a two-dimensional array of plasmonic metamolecules supported on flexible nanowires - that can be driven to a state exhibiting all the defining characteristics of a continuous time crystal - a novel state of matter that continuously breaks time-translation symmetry: i) Time-independent excitation, in the form of illumination with coherent light that is resonant with the plasmonic modes of the metamolecules, spontaneously triggers strong periodic oscillation of the array’s optical

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**Fig. 3. Synchronized oscillations show the main features of the time continuous crystal state.** (a-d) Show evolution of the spectrum of transmissivity oscillations over time under different regimes of perturbation [spectral density on the logarithmic color scale shown to the right of (b)]: (a) with time-invariant incident optical power [here $P = 124.3 \, \mu W$] the frequency of synchronized motion does not change. The fundamental spectral component of transmissivity modulation dominates. (b) Transmissivity oscillations are stable against small parametric perturbations: the oscillation frequency changes adiabatically when the intensity of incident light slowly sinusoidally modulated [here with 5\% depth from a maximum of $P = 124.3 \, \mu W$ at a frequency of 260 Hz]. (c) When the depth of incident optical power modulation is increased to 20\%, synchronization is periodically destroyed and recovered, creating ‘bursts’ of synchronized oscillation, as shown in closer detail in panel (d). (e) Time domain trace of a single transmissivity oscillation burst [red line] overlayed with a trace showing the time dependence of incident pump power [in the 20\% modulation regime, dashed black line]. f) Imaginary vs. real part of signal amplitude at the fundamental oscillation frequency for 155 consecutive burst of synchronous transmissivity oscillation. [Figures (e-f) are presented for an incident light intensity modulation at 560 Hz.]
transmissivity; ii) The oscillations result from many-body interactions among a large number of metamolecules coupled by electromagnetic dipole-dipole interactions (induced by the incident light), whereby the individual thermal oscillations of the nanowires are replaced by coherent, superradiant-like thermal motion of the entire array; iii) The transition to the coherent oscillatory regime has the nature of a first-order phase transition; iv) The transmissivity oscillations are robust to small perturbations and the phase of the oscillation is random for different realizations; v) The state exhibits long range order in space, as manifested in synchronization of the whole array; and a long range order in time, as seen in the robust indefinite persistence of synchronous oscillation. Such media, in which nontrivial temporal dynamics of tailored material dispersion, anisotropy, nonlinearity, etc. can be optically controlled offer a wealth of novel opportunities for wave manipulation and control, for example in frequency shifting, active beam steering and magnet-free nonreciprocity, and luminal amplification, compression and localization, and are of interest to topological and non-Hermitian physics.

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