Precision measurements of $\sigma_{D}(e^+e^- \rightarrow K^0\bar{K}^{\pm}\pi^\mp)$ at center-of-mass energies between 3.8 and 4.6 GeV
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I. INTRODUCTION

The charmonium-like state \( Y(4260) \) was first observed in the initial state radiation (ISR) process, \( e^+e^- \rightarrow \gamma_{\text{ISR}} \pi^+\pi^- J/\psi \), by BABAR [1], and later confirmed by the CLEO [2] and Belle [3] experiments. A precise measurement of \( e^+e^- \rightarrow \pi^+\pi^- J/\psi \) cross sections at center-of-mass (c.m.) energies from 3.77 to 4.60 GeV [4], \( Y(4260) \) was found to have a mass of \( (4222.0 \pm 3.1 \pm 1.4) \) MeV/c\(^2\) and a width of \((44.1 \pm 4.3 \pm 2.0) \) MeV, in good agreement with the so called \( Y(4220) \) observed in \( e^+e^- \rightarrow \pi^+\pi^- h_c \) [5]. Since the \( Y(4260) \) state is produced through ISR in \( e^+e^- \) annihilation, its quantum numbers must be \( J^{PC} = 1^{--} \). However, \( Y(4260) \) seems to be rather different in properties compared with the known charmonium states with \( J^{PC} = 1^{--} \) in the same mass region, such as \( \psi(4040) \), \( \psi(4160) \) and \( \psi(4415) \) \([6–8]\). Although above \( D\bar{D} \) production threshold, the \( Y(4260) \) has strong coupling to the \( \pi^+\pi^- J/\psi \) final state, instead of the \( D^{(*)} \bar{D}^{(*)} \) final state [9]. Such a strong coupling to a hidden-charm final state suggests that the \( Y(4260) \) is a non-conventional \( \bar{c}c \) meson. Possible interpretations of this state are reviewed in Refs. [10–13], but all need to be validated with experimental data.

Most previous studies of the \( Y(4260) \) have utilized hadronic transitions. The CLEO experiment investigated 16 charmonium and light hadron decay modes based on 13.2 pb\(^{-1}\) of \( e^+e^- \) data collected at the c.m. energy of \( \sqrt{s} = 4.260 \) GeV, but only a few decay modes had significance greater than 3\( \sigma \) [14]. The BABAR collaboration has measured the cross section of \( e^+e^- \rightarrow K^0_S K^+\pi^+ \) [15] with the ISR process and found an excess around \( \sqrt{s} = 4.2 \) GeV, which is very close to the mass regions of \( \psi(4160) \) and \( Y(4220) \). Analyzing this process with a larger data sample will provide higher precision measurements and more information on \( Y(4260) \) (\( Y(4220) \)) decays to light hadrons.

In this paper, we report measurements of the Born cross section of \( e^+e^- \rightarrow K^0_S K^+\pi^- \), \( K_S^0 \rightarrow \pi^+\pi^- \) at c.m. energies from 3.8 to 4.6 GeV. The charge conjugate decays to \( K^0_S K^-\pi^+ \) are included in this analysis. The corresponding c.m. energies [16] and the integrated luminosities [17] of all the data samples used in this paper are summarized in Table I.

II. DETECTOR AND MONTE-CARLO SIMULATION

The BESIII detector [18] at the BEPCII collider [19] is a large solid-angle magnetic spectrometer with a geometrical acceptance of 93% of \( \pi^0 \). It has four main components: 1) A small-cell, helium-based (60% He, 40% C\(_3\)H\(_8\)) multilayer drift chamber (MDC) with 43 layers providing an average single-hit resolution of 135 \( \mu \)m, a charged-particle momentum resolution in a 1.0 T magnetic field of 0.5% at 1.0 GeV/c and a \( dE/dx \) resolution better than 6%; 2) A time-of-flight system (TOF) constructed of 5 cm thick plastic scintillator, with 176 detectors of 2.4 m length in two layers in the barrel and 96 fan-shaped detectors in the end-caps. The barrel (end-cap) time resolution of 80 ps (110 ps) provides a 2\( \sigma \) \( K/\pi \) separation for momenta up to \( \approx 1.0 \) GeV/c; 3) An electromagnetic calorimeter (EMC) consisting of 6240 CsI (T1) crystals in a cylindrical structure (barrel) and two end-caps. The energy and the position resolutions for 1.0 GeV photon are 2.5% (5%) and 6 mm (9 mm) in the barrel (end-caps), respectively; 4) A muon system (MUC) consisting of resistive plate chambers in nine barrel and eight end-cap layers, which provides a 2 cm position resolution.

Monte-Carlo (MC) simulations are used to study the backgrounds and determine the detection efficiencies. The \textsc{geant4}-based [20] MC simulation package includes the geometric and material description of the BESIII detector, the detector response, and the digitization models, as well as the detector running conditions and performance. Signal MC samples of \( e^+e^- \rightarrow K^0_S K^+\pi^- \) are generated with phase space (PHSP) distributions with \textsc{evtgen} [21, 22], but correct energy and polar angle distributions are considered for ISR photons to properly include ISR effects [23, 24]. The \( \sigma_B(e^+e^- \rightarrow K^0_S K^+\pi^-) \) results from BABAR [15] are taken as the initial input, and the energy of the ISR photon is required to be less than 0.1 GeV since the events with large energy ISR photons will be rejected by the kinematic fit. An inclusive MC sample, corresponding to an equivalent integrated luminosity to that of data, includes open charm, low-mass
vector charmonium states produced by ISR, continuum light quark, Quantum Electrodynamics (QED) processes, etc. The known decay modes of the charmonium states are produced by EVTGEN [21, 22] by setting the branching fractions (BFs) to be the world average values from the Particle Data Group (PDG) [25], and the unknown decay modes are generated with the LUNDCHARM generator [26].

III. DATA ANALYSIS

For the process $e^+e^- \rightarrow K_{S}^{0}K^{+}\pi^{-}$, signal candidates are selected by requiring a $K_{S}^{0}$ and a kaon and pion pair with net charge zero.

The kaon and pion charged tracks, reconstructed using hits in the MDC, are required to be within the polar angle range $|\cos \theta| < 0.93$ and pass within a cylindrical region extending $\pm 10 \text{ cm}$ from the average interaction point (IP) of each run along the beam direction and with a 1 cm radius perpendicular to the beam direction. The time information from the TOF and the ionization measured in the MDC ($dE/dx$) are combined to calculate particle identification (PID) confidence levels (C.L.) for the $K$ and $\pi$ hypotheses, and the particle type with the highest C.L. is assigned to each track. An identified kaon and an identified pion with opposite sign of charge are required.

The $K_{S}^{0}$ candidates are reconstructed with all possible pairs of oppositely charged tracks, which are assumed to be pions. Their points of closest approach are required to be within 25 cm of the IP along the beam direction and 20 cm in the transverse plane. A vertex fit of the pion pair to a common vertex is performed. The $K_{S}^{0}$ candidates are required to have decay lengths, which are obtained from the secondary vertex fits [27], larger than twice their uncertainty and have $\pi^{+}\pi^{-}$ invariant mass $m_{\pi^{+}\pi^{-}}$ satisfying $m_{\pi^{+}\pi^{-}} - M_{K_{S}^{0}} < 0.020 \text{ GeV}/c^{2}$, where $M_{K_{S}^{0}}$ is the world average value of the $K_{S}^{0}$ mass [25]. The pions from the $K_{S}^{0}$ decay are further required to satisfy $E/P < 0.8$, where $E$ is the energy deposited in EMC and $P$ is the momentum measured in the MDC. If there are multiple $K_{S}^{0}$ candidates in an event, the one with the smallest $\chi^{2}$ of the secondary-vertex fit is selected.

A four constraint (4C) kinematic fit by imposing energy-momentum conservation under the $K_{S}^{0}K^{+}\pi^{-}$ hypothesis is performed to improve the momentum resolution and to suppress the background events, and $\chi^{2}_{4C} < 40$ is required.

By studying the inclusive MC sample, the backgrounds are found to be mainly from processes with four charged tracks in the final state, e.g., $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$, due to particle misidentification between the kaon and pion, and $e^+e^- \rightarrow \gamma\gamma e^+e^-$, in which the radiative photon converts into an electron-positron pair and the electron and positron are misidentified as pions and kaon. The number of remaining background events, $N_{\text{bkg}}$, are evaluated using the events in the sideband regions, defined as $m_{\pi^{+}\pi^{-}} \in (0.435, 0.455)$ or $(0.545, 0.565)$ \text{ GeV}/c^{2}. The signal yields, $N_{\text{sig}}$, are obtained by counting the number of events in the mass region $[m_{\pi^{+}\pi^{-}} - M_{K_{S}^{0}}] < 0.015 \text{ GeV}/c^{2}$, and by subtracting the background events, where the invariant mass of $\pi^{+}\pi^{-}$ is the invariant mass of $\pi^{+}\pi^{-}$ after the kinematic fit. Table I summarizes the signal yields and the number of remaining background events in the different data sets.

Figure 1(a) shows the Dalitz plot of the surviving candidates for the data set with c.m. energy $\sqrt{s} = 4.226 \text{ GeV}$. Two vertical bands, corresponding to the neu-
tral $K^*(892)$ and $K_2^*(1430)$ decaying into $K^\pm \pi^\mp$, and a horizontal band, corresponding to the charged $K_2^*(1430)$ decaying into $K_0^0K^\pm$ are observed. There are also diagonal bands corresponding to the intermediate states, e.g. $a_2(1320)^\pm$ and excited $\rho^\pm$ with high mass, decaying into $K_0^0K^\pm$. In order to obtain the detection efficiencies, partial wave analyses (PWAs) are performed on the $K_0^0K\pi$ system at different c.m. energy points. The contributions of PHSP and possible intermediate states in the $K_0^0\pi$, $K\pi$ and $K_0^0K$ systems, including $K^*(892)$, $K_2^*(1430)$, $K_4^*(1780)$, $a_2(1320)$, $\rho(1700)$ and $\rho(2150)$, are taken into account. In the PWAs, these intermediate states are described with relativistic Breit-Wigner (BW) functions with masses and widths fixed to the world average values [25]. The amplitudes for the subsequent two body decays are constructed with the covariant helicity method [28, 29]. For a particle decaying into a two-body final state, i.e. $A(J, m) \rightarrow B(s, \lambda)C(\sigma, \nu)$, its helicity amplitude $F^J_{\lambda, \nu}$ [28, 29] is:

$$F^J_{\lambda, \nu} = \Sigma_{LS} \sqrt{\frac{2L+1}{2J+1}} g_{LS} \langle L|0|S\delta|J\delta\rangle \langle s\lambda\sigma - \nu|S\delta\rangle r_L B_L(r_0),$$

where $J$, $s$, and $\sigma$ are the spins of $A$, $B$, and $C$, respectively; $m$, $\lambda$, and $\nu$ are the helicities of $A$, $B$, and $C$, respectively; $L$ and $S$ are the total orbital angular momentum and spin of $AB$ system, respectively; $\delta = \lambda - \nu$, $g_{LS}$ is the coupling constant in the $L-S$ coupling scheme, the angular brackets denote Clebsch-Gordan coefficients, $r$ is the magnitude of the momentum difference between the two final state particles in their mother particle’s rest frame, $r_0$ corresponds to the momentum difference at the nominal mass of the resonance, and $B_L$ is the barrier factor [30]. The relative magnitudes and phases of complex coupling constants $g_{LS}$ are determined by an unbinned maximum likelihood fit to data with MINUIT [31], and the effect of backgrounds is subtracted from the likelihood as described in Ref. [32]. Figure 2 shows the amplitude fit results for the invariant mass distributions of $K\pi$, $K_0^0\pi$, and $K_0^0K$, together with the polar angle distributions of $\pi$, $K$, and $K_0^0$ at $\sqrt{s} = 4.226$ GeV, where good agreement with data is seen. Other data samples are similar. The detection efficiency $\epsilon$ is obtained by weighting the PHSP MC sample of $e^+e^- \rightarrow K_0^0K^+\pi^-$ according to the fitted PWA amplitude,

$$\epsilon = \frac{\Sigma_{i=1}^{N_{\text{MC}}^{\text{obs}}}|A_i|^2}{\Sigma_{i=1}^{N_{\text{MC}}}|A_i|^2},$$

where $N_{\text{MC}}^{\text{gen}}$ and $N_{\text{MC}}^{\text{obs}}$ are the numbers of generated MC events and those passing the event selection, respectively, and $A_i$ is the amplitude for event $i$ calculated with the fitted coupling constants.

The Born cross sections are calculated with

$$\sigma_B = L \times B \times \epsilon \times (1 + \delta_{\text{ISR}}) \times \frac{1}{|1-H|^2},$$

where $N_{\text{sig}}^{\text{MC}}$ is the signal yield, $L$ is the integrated luminosity, $B$ is the BF of the decay $K_0^0 \rightarrow \pi^+\pi^-$, $\epsilon$ is the signal detection efficiency obtained by incorporating the PWA results as described above, $(1 + \delta_{\text{ISR}})$ is the ISR correction factor, and $\frac{1}{|1-H|^2}$ is the vacuum polarization factor, which is taken from Ref. [33]. The ISR correction factor is obtained with an iterative procedure starting with the input initial cross section from Ref. [15] and continuing until the cross section converges. The measured Born cross sections for the individual c.m. energy points are summarized in Table 1, as well as other quantities used to calculated the Born cross section. A comparison of the Born cross sections between our measurement and BABAR’s results in the c.m. energy region $\sqrt{s} = 3.800 \sim 4.660$ GeV is shown in Fig. 3. The measured cross sections agree with but are of much higher precision than those obtained by BABAR [15].
FIG. 2. (Color online) Comparisons between data and MC simulation at $\sqrt{s} = 4.226$ GeV. The plots (a)-(c) are the invariant mass of $K\pi$, $K^0_S\pi$ and $K^0_S K$, and the plots (d)-(f) are the polar angle distributions of $\pi$, $K$ and $K^0_S$, respectively. Dots with error bars are data, and the red histograms are the MC projection according to the amplitude analysis results.

FIG. 3. (Color online) The energy dependent Born cross sections of $e^+e^- \rightarrow K^0_S K^+\pi^-$. Red dots are results from this work with statistical and systematic uncertainties, and the blue triangles are from the BABAR experiment [15].
The Born cross sections of $e^+e^- \rightarrow K^0_S K^+ \pi^-$ from this work are fitted with a simple $1/s^n$ dependence. BABAR’s [15] results have large uncertainty in the energy region above 3.8 GeV, so they are not included. In additional, the data point at around 3.8 GeV is not used in the fit. Since an attempt to fit the cross section around this energy should consider the contribution from $\psi(3770)$, and there is just only one data point close to the $\psi(3770)$ peak, which is not enough to constrain the parameters associated with $\psi(3770)$ with sufficient accuracy. The correlations among different data points are considered in the fit, with the chisquare function constructed as Eq. (4), which is minimized by MINUIT [31].

$$\chi^2 = \sum_i \left( \frac{(\sigma_{B_i} - h \cdot \sigma_{B_i}^{\text{fit}})^2}{\delta_i^2} + (h - 1)^2 \right). \tag{4}$$

Here, $\sigma_{B_i}$ and $\sigma_{B_i}^{\text{fit}}$ are the measured and fitted Born cross sections of the $i$th energy point, respectively; $\delta_i$ is the independent part of the total uncertainty, which includes the statistical uncertainty and the uncorrelated part of the systematic uncertainty (the details in Sec. IV); $\delta_c$ is the correlated part of the systematic uncertainty, i.e., the uncertainties associated with the tracking, PID, $K^0_S$ reconstruction efficiency, signal model, integrated luminosity, and BF, etc.; $h$ is a free parameter introduced to take into account the correlations. The fit result to the continuum only is shown in Fig. 4(a), with the goodness-of-the-fit of $\chi^2/\text{NDF} = 13.9/12 \approx 1.2$, which shows that the Born cross sections cannot be explained well with the continuum process only.

Then we fit the Born cross sections with the coherent sum of the continuum and the $\psi(4160)$ or $Y(4220)$ amplitude. The fit function used is

$$\sigma = \sqrt{\sigma_{\text{con}} + e^{i\phi} \sqrt{\frac{12\pi \Gamma_{e^+e^-} B_{K^0_S K^+ \pi^-}}{s - M^2 + iM\Gamma}}}, \tag{5}$$

where $s_{\text{con}}$ and $n$ are the fit parameters for the continuum process, $\phi$ the relative phase between the continuum and resonant amplitudes, $\Gamma$ and $\Gamma_{\text{e}^+\text{e}^-}$ the width and partial width to $e^+e^-$, respectively, $B_{K^0_S K^+ \pi^-}$ the BF of the resonance decays into $K^0_S K^+ \pi^-$, and $M$ the mass of resonance. The masses and total widths of $\psi(4160)$ and $Y(4220)$ are fixed to Refs. [25, 34]. Two solutions with the same minimum value of $\chi^2$ are found with different interference between the two amplitudes. The fit results are shown in Figs. 4(b) and (c) and summarized in Table II (the lineshapes of the two solutions are identical, so we just plot that from solution I). The corresponding significance for $\psi(4160)$ is 2.8σ and for $Y(4220)$ 2.6σ.

**IV. SYSTEMATIC UNCERTAINTIES**

Various sources of systematic uncertainties are investigated for the cross section measurements of $e^+e^- \rightarrow K^0_S K^+ \pi^-$. All the systematic uncertainties are summarized in Table III.

The systematic uncertainties associated with tracking and PID have been studied using control samples of $J/\psi \rightarrow \pi^+\pi^- p\overline{p}$ and $J/\psi \rightarrow K^0_S K^+\pi^-$ with $K^0_S \rightarrow \pi^+\pi^-$ [35], the kaon and pion tracking and PID efficiencies for data agree with those of MC simulation within 1%, so the total tracking and PID uncertainties are both determined to be 2% (1.0% per track).

The uncertainty associated with $K^0_S$ reconstruction is
studied with the processes $J/\psi \rightarrow K^{*\pm}K^{\mp}$ and $J/\psi \rightarrow \phi K_{S}^{0}K^{\mp}\pi^{\mp}$ [36]. The difference of the reconstruction efficiency between data and MC simulation is found to be 1.2%, which is taken as the systematic uncertainty.

The systematic uncertainty due to the kinematic fit is estimated by correcting the track helix parameters of charged tracks and the corresponding covariance matrix for the signal MC sample to improve the agreement between data and MC simulation. The detailed method can be found in Ref. [37]. The resulting change of the detection efficiency with respect to the one, obtained without the corrections, is taken as the systematic uncertainty.

In the measurement of cross section for $e^{+}e^{-} \rightarrow K_{S}^{0}K^{\mp}\pi^{-}$, the detection efficiency is estimated with the weighted PHSP MC samples, where the weights are obtained according to the PWA results. To estimate the corresponding systematic uncertainty associated with the signal MC model, we repeat the PWA by changing the resonant parameters of the intermediate states by one standard deviation [25] and by excluding the intermediate state with the least significance in the fit. The PWA results obtained are used to recalculate the detection efficiency, and the resulting differences are taken as the systematic uncertainties. Assuming the uncertainties of two scenarios are uncorrelated, the overall uncertainty associated with the signal MC model is the sum of above individual values in quadrature. To minimize the effect on the limited statistics of data, this uncertainty is only studied for the data sample at $\sqrt{s} = 4.226$ GeV, which has the largest integrated luminosity of all the samples, and the obtained value, 2.0%, is assigned to all c.m. energy points.

For the systematic uncertainties associated with the signal yields, we redetermine the signal yields by changing the mass interval of $M_{\pi^{+}\pi^{-}}$ from 0.03 to 0.04 GeV/c², and by changing the $K_{S}^{0}$ sideband regions to $m_{\pi^{+}\pi^{-}} \in (0.43, 0.45) \cup (0.45, 0.47)$ GeV/c². The largest change of the signal yields with respect to the nominal value among all c.m. energy points, 1.8%, is taken as the systematic uncertainty.

The uncertainty associated with the vacuum polarization factor [33] is negligible compared with the other uncertainties. For the ISR correction factors, the iteration procedure is carried out until the measured Born cross section converges. The convergence criterion, 1.0%, is taken as the systematic uncertainty.

The integrated luminosities at each c.m. energy point are measured using large angle Bhabha scattering events with an uncertainty of 1.0% [17]. The uncertainty on the BF of the decay $K_{S}^{0} \rightarrow \pi^{+}\pi^{-}$ is from the PDG [25].

Assuming all sources of systematic uncertainties are uncorrelated, the total systematic uncertainty is obtained by adding the individual values in quadrature and are summarized in Table III.

### Table II. The fit results to $\sigma_{B}(e^{+}e^{-} \rightarrow K_{S}^{0}K^{\mp}\pi^{-})$. Shown in the table are the product of the $e^{+}e^{-}$ partial width and the BF to $K_{S}^{0}K^{\mp}\pi^{-}$ final state $\Gamma_{e^{+}e^{-} \rightarrow B_{K_{S}^{0}K^{\mp}\pi^{-}}}$, the relative phase between the different amplitudes $\phi$, and the corresponding significance of $\psi(4160)$ and $Y(4220)$. The uncertainties of the parameters are from fits.

| Source                  | Relative uncertainty (%) | Solution I       | Solution II      | Solution I       | Solution II      |
|------------------------|--------------------------|-------------------|-------------------|-------------------|-------------------|
| Tracking               | 2.0                      | 2.71±0.13         | 0.0118±0.0098     | 2.03±0.05         | 0.0038±0.0029     |
| PID                    | 2.0                      | -1.60±0.03        | 1.71±0.38         | -1.61±0.02        | 2.11±0.43         |
| Kinematic fit          | 0.5                      |                   |                   |                   |                   |
| Signal model           | 2.0                      |                   |                   |                   |                   |
| $K_{S}^{0}$ mass window| 1.8                      |                   |                   |                   |                   |
| Vacuum polarization    | -                        |                   |                   |                   |                   |
| ISR factor             | 1.0                      |                   |                   |                   |                   |
| Integrated luminosity  | 1.0                      |                   |                   |                   |                   |
| BF                     | -                        |                   |                   |                   |                   |
| Total                  | 4.4                      |                   |                   |                   |                   |

### Table III. Systematic uncertainties of the measurements of $\sigma(e^{+}e^{-} \rightarrow K_{S}^{0}K^{\mp}\pi^{-})$.

| Source                  | Relative uncertainty (%) | $\psi(4160)$       | $Y(4220)$        |
|------------------------|--------------------------|--------------------|------------------|
| Source                  | Relative uncertainty (%) | Solution I       | Solution II      | Solution I       | Solution II      |
| Tracking               | 2.0                      | 2.71±0.13         | 0.0118±0.0098     | 2.03±0.05         | 0.0038±0.0029     |
| PID                    | 2.0                      | -1.60±0.03        | 1.71±0.38         | -1.61±0.02        | 2.11±0.43         |
| Kinematic fit          | 0.5                      |                   |                   |                   |                   |
| Signal model           | 2.0                      |                   |                   |                   |                   |
| $K_{S}^{0}$ mass window| 1.8                      |                   |                   |                   |                   |
| Vacuum polarization    | -                        |                   |                   |                   |                   |
| ISR factor             | 1.0                      |                   |                   |                   |                   |
| Integrated luminosity  | 1.0                      |                   |                   |                   |                   |
| BF                     | -                        |                   |                   |                   |                   |
| Total                  | 4.4                      |                   |                   |                   |                   |

V. SUMMARY

Measurements of the $e^{+}e^{-} \rightarrow K_{S}^{0}K^{\mp}\pi^{-}$ cross section $\sigma_{B}(e^{+}e^{-} \rightarrow K_{S}^{0}K^{\mp}\pi^{-})$ have been performed by BESIII in the c.m. energy region from 3.8 to 4.6 GeV and are shown in Fig. 3 and summarized in Table I. The cross sections agree with the BABAR’s results [15], but with significantly improved precision. The line shape of the Born cross sections cannot be well described with only the continuum process. The fit to the Born cross sections from this work, with $\psi(4160)$ ($Y(4220)$) added, is performed. Only evidence for the $\psi(4160)$ ($Y(4220)$) is observed and the corresponding significance is 2.8σ (2.6σ). Further study of this channel with more energy points and large statistics will be essential for a deeper understanding of the possible structures in the line shape and contributions from charmonium and charmonium-like states.
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