Large mechanical squeezing beyond 3dB of hybrid atom-optomechanical systems in highly unresolved sideband regime

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Abstract: We propose a scheme for the generation of strong mechanical squeezing beyond 3dB in hybrid atom-optomechanical systems in the highly unresolved sideband (HURSB) regime where the decay rate of cavity is much larger than the frequency of the mechanical oscillator. The system is formed by two two-level atomic ensembles and an optomechanical system with cavity driven by two lasers with different amplitudes. In the HURSB regime, the squeezing of the movable mirror can not be larger than 3dB if no atomic ensemble or only one atomic ensemble is put into the optomechanical system. However, if two atomic ensembles are put into the optomechanical system, the strong mechanical squeezing beyond 3dB is achieved even in the HURSB regime. Our scheme paves the way toward the implementation of strong mechanical squeezing beyond 3dB in hybrid atom-optomechanical systems in experiments.

1. Introduction

Quantum squeezing of quantum systems is a characteristic property of macroscopic quantum effects [1]. It can be used to detect weak forces and realize continuous variable quantum information processing [2]. Quantum squeezing can be achieved using the parametric interaction of a quantum system [3]. Unfortunately, quantum squeezing in this scheme can not be larger than 3dB, i.e., quantum noise of a system could not be reduced below half of the zero-point level [4]. Otherwise, the quantum system becomes unstable.

Up to now, several schemes have been proposed to overcome the 3dB limit by using continuous weak measurement and feedback [5–8], squeezed light [9, 10], quantum-reservoir engineering [11–14], strong intrinsic nonlinearity [15, 16], auxiliary cavities and atoms [17, 18], two driving lasers with different amplitudes [19–21], and frequency modulation [22].

For instance, strong mechanical squeezing beyond 3dB can be realized by injecting a squeezed light into an optomechanical system since the optical squeezing of the squeezed light can be transferred into the mechanical resonator due to the interactions between the cavity and mechanical resonator [9, 10]. Also, the authors of Ref. [16] have shown that the Duffing nonlinearity can be used to accomplish strong mechanical squeezing (beyond 3dB) in optomechanical systems when the nonlinear amplitude is large enough. In Ref. [19], arbitrarily large steady-state mechanical squeezing can be realized by applying two driving lasers with different amplitudes to a cavity in an optomechanical system. In 2015, the squeezing of mechanical mode is realized experimentally by the authors of Ref. [20] using the method proposed by Kronwald, Marquardt, and Clerk [19]. Note that in the unresolved sideband regime when the decay rate of the cavity \( \kappa \) is larger than the frequency of the mechanical oscillator \( \omega_m \), this scheme is no longer valid. Recently, it has been pointed out the beyond 3dB strong mechanical squeezing can be achieved with the help of frequency modulation [22].
Quantum squeezing beyond the 3dB limit has been realized in electromechanical systems in experiment [23]. In recent years, many efforts have been devoted to the study of optomechanical systems due to their wide applications [24–43]. In order to realize strong quantum squeezing beyond the 3dB limit in standard optomechanical systems, the decay rate of the cavity $\kappa$ must be much smaller than the frequency of the mechanical oscillator $\omega_m$ (resolved sideband regime with $\kappa \ll \omega_m$) [24, 25]. In experiments, the quality factor of optical cavities should be very high in order to satisfy the resolved sideband criterion. This limits the mass and size of mechanical resonator to be squeezed. The 3dB limit has not been overcome in optomechanical systems since it is difficult to enhance the quality of cavities in optomechanical systems with floppy mechanical elements [21] and the resolved sideband regime is difficult to achieved in experiments. Very recently, the authors of Ref. [21] have shown that quantum squeezing beyond the 3dB limit in the unresolved sideband regime ($\kappa \approx 30\omega_m$) can be realized in an optomechanical system with two auxiliary cavities and two lasers. It is worth noting that in order to realize mechanical squeezing the decay rates of two auxiliary cavities should be smaller than the frequency of the mechanical resonator [21]. In particular, the scheme is not valid in the HURSB regime with $\kappa \gg \omega_m$.

All the previous schemes [15–22] are not able to realize strong mechanical squeezing beyond the 3dB limit in the HURSB regime with $\kappa \gg \omega_m$. For instance, in Refs. [19, 20], large mechanical squeezing can be generated in electromechanical systems in the resolved-sideband regime. However, it is difficult to achieve the resolved-sideband regime for optomechanical systems. Therefore, the authors of Ref. [21] have suggested to generate strong mechanical squeezing in unresolved-sideband regime for optomechanical systems with the help of two auxiliary cavities. However, the quality factors of two auxiliary cavities must be high enough. In the present work, we propose a scheme to achieve strong mechanical squeezing beyond the 3dB limit in the HURSB regime with the help of two two-level atomic ensembles and two driving lasers with different amplitudes. It is feasible to couple atoms to photons of a cavity in experiments [44–50]. Particularly, the linewidth of atoms is very narrow and the decay rate of atoms could be much smaller than the frequency of the mechanical oscillator [48]. The cavity is driven by two lasers with different amplitudes. There is an optimal ratio of the driving strengths of the lasers. For realistic parameters, we can obtain mechanical squeezing beyond 3dB even in the HURSB regime.

### 2. Model and effective Hamiltonian

Here, we consider a hybrid atom-optomechanical system consisted of two two-level atomic ensembles within a single-mode cavity as shown in Fig.1. The cavity is driven by two lasers with different amplitudes $\Omega_\pm$. The Hamiltonian of the present model is (we set $\hbar = 1$)

$$H = \omega_c a^\dagger a + \omega_m b^\dagger b + \frac{\omega_1}{2} \sum_{j=1}^{N_1} \sigma_{z,1}^{(j)} + \frac{\omega_2}{2} \sum_{j=1}^{N_2} \sigma_{z,2}^{(j)} + ga^\dagger (b^\dagger + b) + [g_1 a \sum_{j=1}^{N_1} \sigma_{x,1}^{(j)} + g_2 a \sum_{j=1}^{N_2} \sigma_{x,2}^{(j)}$$

$$+(\Omega_+ e^{-i\omega_+ t} + \Omega_- e^{-i\omega_- t})a^\dagger + H.c.),$$

(1)

where $a^\dagger (a)$ and $b^\dagger (b)$ are the creation (annihilation) operators of the cavity field and mechanical resonator with frequencies $\omega_c$ and $\omega_m$, respectively. Here, $\sigma_{z,s}^{(j)} = |e\rangle_s^j \langle e| - |g\rangle_s^j \langle g|$, $\sigma_{x,s}^{(j)} = |e\rangle_s^j \langle g|$, and $\sigma_{z,s}^{(j)} = |g\rangle_s^j \langle e|$. $|e_j\rangle_s$ and $|g_j\rangle_s$ are the Pauli matrices of atom $j$ in ensemble $s$. The frequencies of ensemble 1 and ensemble 2 are $\omega_1$ and $\omega_2$, respectively. The numbers of atoms in ensemble 1 and ensemble 2 are $N_1$ and $N_2$, respectively. The coupling strength between ensemble 1 (ensemble 2) and the single-mode cavity is $g_1$ ($g_2$). The coupling strength between the cavity
The cavity is formed by a fixed mirror and a movable mirror. The movable mirror is perfectly reflecting while the fixed mirror is partially transmitting. Two ensembles formed by two-level atoms are put into a cavity with frequency $\omega_c$. The cavity is driven by two lasers with frequencies $\omega_{\pm} = \omega_c \pm \omega_m$ and amplitudes $\Omega_{\pm}$. In experiments, it is difficult to realize high quality factor cavities containing movable mechanical elements to satisfy the resolved sideband regime ($\kappa \ll \omega_m$). In the present work, we assume the decay rate of the cavity is much larger than the frequency of the mechanical resonator ($\kappa \gg \omega_m$).

To simplify the Hamiltonian of the present model, we introduce the operators of atomic collective excitation modes of atomic ensembles

$$A_1 = \frac{1}{N_1} \sum_{j=1}^{N_1} \sigma_{1,j}^{(j)} \quad \text{and} \quad A_2 = \frac{1}{N_2} \sum_{j=1}^{N_2} \sigma_{2,j}^{(j)}.$$  

In the limit of low-excitation and large number of atoms ($N_1$ and $N_2$), we have the following commutation relations [51–57]

$$[A_1, A_1^\dagger] \approx [A_2, A_2^\dagger] \approx 1, [A_1, A_2] = [A_1, A_2^\dagger] = 0.$$  

The Hamiltonian can be expressed in terms of $a, b, A_1$, and $A_2$ as follows

$$H = \omega_c a^\dagger a + \omega_m b^\dagger b + \frac{\omega_1}{2} A_1^\dagger A_1 + \frac{\omega_2}{2} A_2^\dagger A_2 + ga^\dagger a(b^\dagger + b) + [a^\dagger(G_{A_1}A_1 + G_{A_2}A_2) + H.c.],$$  

equation (5)
we obtain the effective Hamiltonian

\[ H = \Delta_1 a_1^\dagger a_1 + \Delta_2 a_2^\dagger a_2 + [G_{A_1} a_1^\dagger a_1 + G_{A_2} a_2^\dagger a_2 + \delta a^\dagger (G_\omega b^\dagger + G_\eta b) + \delta a^\dagger (e^{-2i\omega_m t} G_\omega b + e^{2i\omega_m t} G_\eta b^\dagger) + H.c.], \]

where \( \Delta_{1,2} = \omega_{1,2} - \omega_c \) and \( G_\pm = g a_\pm^\dagger \) are the effective couplings between the cavity and mechanical oscillator. Without loss of generality, we assume \( G_\pm \) are real.
3. Quantum Langevin equations and solution

In this section, we derive the quantum Langevin equations of the present model using the effective Hamiltonian Eq.(12) of the previous section.

3.1. Quantum Langevin equations

The quantum Langevin equations of the system defined by $H_{\text{eff}}$ can be written as

\[
\delta \dot{a} = -\frac{k}{2} \delta a + i f_1(t) \delta b^\dagger + i f_2(t) \delta b - i G_{A_1} \delta a_1
\]
\[
- i G_{A_2} \delta a_2 + \sqrt{\kappa} a_{\text{in}},
\]
\[
(13)
\]
\[
\delta \dot{b} = -\frac{\gamma_m}{2} \delta b + i f_1(t) \delta a^\dagger + i f_3(t) \delta a + \sqrt{\gamma_m} b_{\text{in}},
\]
\[
(14)
\]
\[
\delta \dot{a}_1 = -\frac{\gamma_1}{2} + i \Delta_1 \delta a_1 - i G_{A_1} \delta a + \sqrt{\gamma_{A_1}} a_{\text{in}},
\]
\[
(15)
\]
\[
\delta \dot{a}_2 = -\frac{\gamma_2}{2} + i \Delta_2 \delta a_2 - i G_{A_2} \delta a + \sqrt{\gamma_{A_2}} a_{\text{in}},
\]
\[
(16)
\]
with $f_1(t) = -(G_+ + G_- e^{2i \omega t})$, $f_2(t) = -(G_+ + G_- e^{-2i \omega t})$, and $f_3(t) = -(G_+ + G_- e^{2i \omega t})$. Here, $a_{\text{in}}$, $b_{\text{in}}$, and $a_{j,\text{in}}$ are the noise operators of the cavity field, mechanical resonator, and atomic ensembles, respectively. They obey the following correlation functions

\[
\langle a_{\text{in}}(t) a_{\text{in}}^\dagger(t') \rangle = \delta(t - t'),
\]
\[
\langle a_{\text{in}}^\dagger(t) a_{\text{in}}(t') \rangle = 0,
\]
\[
\langle b_{\text{in}}(t) b_{\text{in}}^\dagger(t') \rangle = (n_{\text{th}} + 1) \delta(t - t'),
\]
\[
\langle b_{\text{in}}^\dagger(t) b_{\text{in}}(t') \rangle = n_{\text{th}} \delta(t - t'),
\]
\[
\langle a_{j,\text{in}}(t) a_{j,\text{in}}^\dagger(t') \rangle = \delta(t - t'),
\]
\[
\langle a_{j,\text{in}}^\dagger(t) a_{j,\text{in}}(t') \rangle = 0,
\]
\[
(17)
\]
where $n_{\text{th}}$ is the mean thermal excitation number of the mechanical oscillator.

3.2. Covariance matrix and solution

Now, we define the following quadrature operators $X_{O=a,b,a_1,a_2} = (\delta O^\dagger + \delta O)/\sqrt{2}$ and $Y_{O=a,b,a_1,a_2} = -i(\delta O^\dagger - \delta O)/\sqrt{2}$. The noise quadrature operators are defined as $X_{\text{in}}^{j,n}_{O=a,b,a_1,a_2} = (O_{\text{in}}^j + O_{\text{in}})/\sqrt{2}$ and $Y_{\text{in}}^{j,n}_{O=a,b,a_1,a_2} = -i(O_{\text{in}}^j - O_{\text{in}})/\sqrt{2}$. From the above quantum Langevin equations, we obtain

\[
\tilde{\dot{u}} = A\tilde{u} + \bar{n},
\]
\[
(18)
\]
where $\tilde{u} = (X_a, Y_a, X_b, Y_b, X_{a_1}, Y_{a_1}, X_{a_2}, Y_{a_2})^T$ and
\[ \tilde{n} = \begin{pmatrix} \sqrt{\kappa X_1}, \sqrt{\kappa Y_1} \end{pmatrix}, \]  
\[ A = \begin{pmatrix} -\frac{\Delta}{2} & 0 & -\mathcal{R}(f_{12}) & \mathcal{R}(f_{12}) & 0 & G_{A_1} & 0 & G_{A_2} \\ 0 & -\frac{\Delta}{2} & \mathcal{R}(f_{12}) & \mathcal{R}(f_{12}) & 0 & -G_{A_1} & 0 & -G_{A_2} \\ -\mathcal{R}(f_{13}) & \mathcal{R}(f_{13}) & -\frac{\gamma}{2} & 0 & 0 & 0 & 0 & 0 \\ \mathcal{R}(f_{13}) & \mathcal{R}(f_{13}) & 0 & -\frac{\gamma}{2} & 0 & 0 & 0 & 0 \\ 0 & G_{A_1} & 0 & 0 & -\frac{\gamma}{2} & \Delta_1 & 0 & 0 \\ 0 & G_{A_1} & 0 & 0 & -\Delta_1 & -\frac{\gamma}{2} & 0 & 0 \\ -G_{A_2} & 0 & 0 & 0 & 0 & 0 & -\Delta_2 & -\frac{\gamma}{2} \\ -G_{A_2} & 0 & 0 & 0 & 0 & 0 & -\Delta_2 & -\frac{\gamma}{2} \end{pmatrix}, \]

where \( \mathcal{R}(f) \) and \( \mathcal{I}(f) \) are the real and imaginary parts of a complex number \( f \), respectively, and \( f^+ = f(t) + f(t) \).

The dynamics of the present system can be completely described by a 8 \times 8 covariance matrix \( V \) with \( V_{jk} = \langle u_j u_k + u_k u_j \rangle / 2 \). Using the definitions of \( V, \tilde{n}, \) and Eq.(18), we get the evolution of the covariance matrix \( V \) as

\[ V = AV + VA^T + D, \]

with \( D \) being the noise correlation defined by \( D = \text{diag}[\frac{\gamma}{2}, \frac{\gamma}{2}, (2n_{th}+1), \frac{\gamma}{2}(2n_{th}+1), \frac{\gamma}{2}, \frac{\gamma}{2}, \frac{\gamma}{2}, \frac{\gamma}{2}] \).

### 4. Strong mechanical squeezing in the HURSB regime

Fig. 2. The mechanical squeezing \( S_{dB} \) are plotted as functions of the ratio \( G_+/G_- \) for \( \gamma = 0.001\omega_m \) (red line), \( \gamma = 0.005\omega_m \) (green line), and \( \gamma = 0.01\omega_m \) (blue line) with \( \kappa = 1000\omega_m, \gamma_m = 10^{-5}\omega_m, G_{A_1} = G_{A_2} = 10\omega_m, G_- = \omega_m, n_{th} = 0, \Delta_1 = 2\omega_m, \) and \( \Delta_2 = -2\omega_m \). The decay rate of cavity is much larger than the frequency of the mechanical oscillator (\( \kappa \gg \omega_m \)). The mechanical squeezing can be larger than 3dB even in the HURSB regime in the present model as one can see clearly from this figure.
There is an optimal ratio $G$ values of decay rate of cavity and frequency of mechanical oscillator $\kappa/\omega_m$ for $G_{A_1} = 0, G_{A_2} = 10\omega_m$ (green line) and $G_{A_1} = 10\omega_m, G_{A_2} = 10\omega_m$ (blue line) with $G_+ = \omega_m, \gamma = 0.001\omega_m$, $\gamma_m = 10^{-5}\omega_m$, $n_{th} = 0$, $\Delta_1 = 2\omega_m$, and $\Delta_2 = -2\omega_m$. The red line is the 3dB limit. All the points in the figure have been optimized over $G_+ / G_-$. If no atomic ensemble or only one atomic ensemble is put into the cavity, the 3dB limit cannot be overcome. However, if two atomic ensembles are put into the cavity, the beyond 3dB mechanical squeezing can be realized even in the HURSB regime.

The mechanical squeezing is defined as (in units of dB) [21]

$$S_{dB} = -10 \log_{10}(\langle \Delta X^2 \rangle / \langle \Delta X^2 \rangle_{ZPF})$$

$$= -10 \log_{10}(2 \langle \Delta X^2 \rangle_b),$$

where $\langle \Delta X^2 \rangle_{ZPF} = 0.5$ is the zero-point fluctuations. The mechanical squeezing can be calculated from Eq.(21) of the previous section.

In Fig. 2, we plot the mechanical squeezing (in units of dB) as functions of $G_+/G_-$ for different values of decay rate $\gamma$ (we set $\gamma_1 = \gamma_2 = \gamma$) with $\Delta_1 = 2\omega_m$ and $\Delta_2 = -2\omega_m$. Here, we assume the system is in the HURSB regime with $\kappa = 1000\omega_m$. Clearly, the mechanical squeezing of the present work can overcome the 3dB limit even in the HURSB regime. The mechanical squeezing $S_{dB}$ first increases with the increase of the ratio $G_+ / G_-$ and then decreases with ratio $G_+ / G_-$. There is an optimal ratio $G_+ / G_-$. If the blue-detuned laser is not applied ($G_+ = 0$), the 3dB limit cannot be surpassed since $S_{dB} < 3$ as one can clearly see from this figure. Comparing the lines of the figure, we find the optimal ratio $G_+ / G_- |_{opt}$ increases with the decrease of the decay rates of the atomic ensembles.

The influence of the blue-detuned laser can be explained as follows. In the rotating wave approximation, the direct interactions between the optical cavity and mechanical resonator is represented by the term $\delta a^\dagger (G_+ \delta b^\dagger + G_- \delta b) + H.c.$ in Eq.(12). Using the standard squeezing transformation [16], this term can be rewritten as $G_{eff} (\delta a^\dagger \delta B + H.c.)$ with $G_{eff} = \sqrt{G_+^2 - G_-^2}$, $\delta B = \cosh r \delta b + \sinh r \delta b^\dagger$ being the Bogoliubov mode, and $r = \ln[(G_- + G_+)/(G_- - G_+)]/2$ being the squeezing parameter. If the blue-detuned laser is not applied ($G_+ = 0$), the squeezing parameter $r$ is zero and there is no mechanical squeezing. In fact, the mechanical squeezing is determined by two competing effects. One is the squeezing parameter $r$. The other is the effective direct coupling between the optical cavity and mechanical resonator denoted by $G_{eff}$. The squeezing parameter $r$ increases with the increase of the ratio $G_+ / G_-$. However,
$G_{\text{eff}}$ decreases with the increase of the ratio $G_+/G_-$. Consequently, the maximal mechanical squeezing is a tradeoff between these two competing effects.

In Fig. 3, we plot the mechanical squeezing $S_{dB}$ as functions of $\kappa/\omega_m$ for different coupling strengths $G_A_1$ and $G_A_2$, with $\gamma = 0.001\omega_m$, $\Delta_1 = 2\omega_m$, and $\Delta_2 = -2\omega_m$. The 3dB limit is plotted as red line. From this figure, one can find that if only one atomic ensemble is put into the cavity ($G_A_1 = 0$, green line), then the 3dB limit can not been overcome since $S_{dB} < 3$. In fact, we find that if no atomic ensemble is put into the cavity, the mechanical squeezing $S_{dB}$ is less than −30 in the case of $\kappa \gg \omega_m$ (not shown in this figure). This implies that the mechanical squeezing beyond 3dB can not been achieved in the HURSB regime without the atomic ensemble. The situation is totally different when the two atomic ensembles are put into the cavity as one can clearly see from the blue line of this figure. In this case, the mechanical squeezing $S_{dB}$ can be larger than the 3dB limit even in the case of $\kappa = 1000\omega_m$. The mechanical squeezing $S_{dB}$ decreases with the increase of the ratio $\kappa/\omega_m$. As one can see from Fig. 2, we can decrease the decay rates of atomic ensembles $\gamma_1$ and $\gamma_2$ to realize mechanical squeezing beyond the 3dB limit in the HURSB regime.

The two atomic ensembles play an important role in the present scheme. The reason is as follows. In Ref. [21], two auxiliary high-Q cavities are introduced in order to modulate the optical density of states in the cavity. As a result, the the damaging effects of the counter-rotating terms can be suppressed [21]. In order to overcome the 3dB limit, the decay rates the two auxiliary cavities must be smaller than the frequency of the mechanical oscillator. In the present scheme, the two atomic ensembles can also adjust the optical density of states in the cavity similar to [21]. In fact, the two atomic ensembles and the cavity can be considered as an engineered reservoir for the mechanical oscillator [19, 21]. In particular, there are two main advantages of the present scheme. First, it is feasible to couple atoms to photons of a cavity field in experiments [44–50]. Second, the key requirement of the present scheme is that the decay rate of atoms must be much smaller than the frequency of the mechanical oscillator which can be satisfied [48]. Thus, the scheme proposed here is feasible in experiments.

5. Conclusion

In the present work, we have proposed a scheme to realize strong mechanical squeezing beyond the 3dB limit in the HURSB regime in hybrid atom-optomechanical systems. Two two-level atomic ensembles were put into the cavity which was driven by two lasers. The amplitudes of two lasers were assumed to be unequal. In the limit of low-excitation and large number of atoms, the atomic ensembles can be expressed in terms of bosonic operators. First, we derived an effective Hamiltonian of the present model in the interaction picture. In the resolved sideband case with $\kappa \ll \omega_m$, the counter-rotating terms of the effective Hamiltonian can be neglected. However, in the HURSB regime with $\kappa \gg \omega_m$, the influence of the counter-rotating terms can not been ignored. The dynamics of the present system can be described by a covariance matrix $V$. Then, we solved the equation of motion numerically and plotted the mechanical squeezing as functions of the ratio $G_+/G_-$ or $\kappa/\omega_m$. We found that the 3dB limit of the mechanical squeezing can be overcome even in the HURSB regime. The mechanical squeezing $S_{dB}$ first increases with the ratio $\kappa/\omega_m$ and then decreases with the ratio $\kappa/\omega_m$. In particular, the 3dB limit can not be surpassed when the blue-detuned laser is not applied. In addition, if no atomic ensemble or only one atomic ensemble is put into the optomechanical system, the squeezing of the movable mirror can not be larger than 3dB. However, if we put two atomic ensembles into the cavity, the mechanical squeezing beyond 3dB is achieved in the HURSB regime. Our scheme paves the way toward the realization of large mechanical squeezing beyond the 3dB limit in hybrid atom-optomechanical systems in the HURSB regime.
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**Disclosures**

The authors declare no conflicts of interest.

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