Analysis of Wide Box Gird’s Shear-lag Effect Based on Energy Method in Single-Cable-plane Cable Stayed Bridge

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Abstract: Box beam shear lag effect of single cable plane cable-stayed bridge under moment and axial force is analyzed. The Yellow River bridge in Jinan as an example, the formula of shear lag under dead load has been set up using the Energy variation method. It is convenient and practical. The theoretical calculation results are in good agreement with those obtained in the solid finite element model calculation. The results show that the shear lag effect of cable-stayed bridge of single cable plane with wide box girder is bigger. It should be highly valued in the design.

1. Introduction
The main girder of the cable-stayed bridge with single cable plane prestressed concrete wide box girder must adopt the closed thin-walled box girder section. Because of the wide top plate and the thin web of the box girder, the shear deformation of the flange is serious, which leads to the non-uniform distribution of the bending normal stress of the box girder in the transverse direction, that is, the phenomenon of "shear lag". Space finite element method has been widely used in the past [1] - [4]. In reference [5]-[6], the energy method is used to study the shear lag effect of the cracked continuous box girder. The shear lag effect of the continuous box girder does not consider the axial force, while the wide box girder of the single cable plane cable-stayed bridge bears a large axial force, so the shear lag effect of the axial force cannot be ignored. In this paper, a simple and practical calculation formula of shear lag coefficient is derived by using energy method, which is convenient for application in preliminary design. In addition, the normal stress of the cross section of the box girder of the cable-stayed bridge is not only the bending normal stress, but also the contribution of the axial pressure, so the influence of the axial force must be considered in the shear lag analysis.

2. Derivation of basic equation of shear lag coefficient based on energy method
The section form of wide box girder of single cable plane cable-stayed bridge is shown in Figure 1.

2.1 Bending normal stress under bending moment
The displacement functions of the box girder under the bending moment are established as follows
according to the web, cantilever plate, top plate and bottom plate respectively:

**Web**: $w(x) = w(x)$

$$u(x,y) = -[w(x) - \beta(x)] \cdot z$$

Among it $\beta(x) = \left( \frac{Q}{G_A} \right) x$

Cantilever plate: $z= h_3$

$$u_i(x,y) = h_3 \cdot \left( -\left[w'(x) - \beta(x)\right] + \frac{\kappa^2}{h_3^2} u_i(x) \right) \quad -h_3 \leq y \leq 0$$

Top plate: $(z=h_2)$

$$u_i(x,y) = h_2 \cdot \left( -\left[w'(x) - \beta(x)\right] + \frac{\kappa^2}{h_3^2} u_i(x) \right)$$

Bottom plate: $(z=h_1)$

$$u_i(x,y) = h_1 \cdot \left( -\left[w'(x) - \beta(x)\right] - \frac{\kappa^2}{h_3^2} u_i(x) \right)$$

Among it, $y = y - (h_1 + h_2 + h_3)$

The calculation formula of bending normal stress of box girder under bending moment is obtained by energy method.

Top plate:

$$\sigma_{g(x,y)} = \frac{Eh_2}{2} \left[-w'(x) - \beta(x)\right] + \left( 1 - \frac{\kappa^2}{h_3^2} \right) u_i(x) \quad (h_1 + h_2 \leq |y| \leq h_1 + h_2 + h_3)$$

Bottom plate:

$$\sigma_{g(x,y)} = \frac{Eh_1}{2} \left[-w'(x) - \beta(x)\right] + \left( 1 - \frac{\kappa^2}{h_3^2} \right) u_i(x) \quad (0 \leq |y| \leq h_3)$$

$$w(x) = \frac{M(x)}{EI} - \frac{3}{2} \sum_{i=1}^{4} \frac{E}{I} \mu_i \cdot \frac{1}{I} \beta(x)$$

$$u_i = \frac{5}{6} \beta(x) \left( 1 + \sum_{i=1}^{4} a_i \right) + \frac{5}{6} \frac{M(x)}{EI} \beta(x)$$

$$\beta(x) = \left( \frac{Q}{G_A} \right) x \quad a_i = \frac{1}{I} \quad (i=1,2,3,4)$$

$Q$ is the shear value at the section, $M(x)$ is the section moment, $G$ is the concrete shear modulus, $E$ is the concrete elastic modulus, $AW$ is the total area of the web, $I$ is the total section moment of inertia.

2.2 Bending stress under axial force

Similar to the analysis method under the action of bending moment, the quadratic parabola is still used. At the same time, the roof and floor are assumed to be different displacement functions.

Under the action of the axial force, the generalized function $X(x)$ and $u(x,y)$ are introduced to represent the longitudinal displacement function of the center of the upper wing plate of the box girder and the displacement function of any point respectively, which are the same as the assumption under the action of the bending moment.

1. The transverse bending deformation of the plate can be neglected;
2. The shear deformation out of plane $\gamma x z, \gamma y z$ the vertical extrusion $\xi x$ and transverse strain of the upper and lower wing plates, the transverse bending and strain can be ignored;
3. It is assumed that the longitudinal displacement of the plate is a quadratic parabola distribution.
The general form of normal stress of top and bottom plate under axial force obtained by energy variation method is as follows:

\[
\sigma_{\text{top}}(x,y) = \frac{1}{2A} \left[ N(x) + \left[ n_B K_B(B - \frac{2A y^2}{b^2}) \right] \left[ a, chK, x + a, shK, x - \frac{N'(x)}{K_i} \right] \right]
\]

\[
\sigma_{\text{bottom}}(x,y) = \frac{1}{2A} \left[ a, chK, x + a, shK, x - \frac{N'(x)}{K_i} \right] + \frac{1}{2A} N(x) \left[ a, chK, x + a, shK, x - \frac{N'(x)}{K_i} \right]
\]

It should be noted that if \( N(x) \) is a piecewise function, then the boundary condition and the continuous condition of displacement need to be reused at each piecewise point. At this time, \( u(x,y) \) becomes a piecewise function. Then the values of \( a_1 \) and \( a_2 \) are different in each segment. However, the displacement function \( u(x,y) \) and the coefficients \( a_1 \) and \( a_2 \) obtained above are all the cases where the axial force changes continuously from \( x = 0 \) to \( x = l \) by default.

When \( N(x) \) varies according to parabola and linearity, the coefficients \( a_1 \) and \( a_2 \) are arranged as shown in Table 1.

| Change of axial force | Coefficient \( a_1 \) | Coefficient \( a_2 \) |
|-----------------------|------------------------|------------------------|
| \( N(x) = A_1 x + B_1 \) | \( a_1 = \frac{A_1 - A_2 chK_1 l}{K_1 shK_1 l} \) | \( a_2 = \frac{A_1}{K_1^2} \) |
| \( N(x) = A_2 x^2 + B_2 x + C_0 \) | \( a_1 = \frac{2A_1 l + B_2 - B_1 chK_1 l}{K_1^2 shK_1 l} \) | \( a_2 = \frac{B_2}{K_1^2} \) |

Among it:

\[
K = \frac{1}{b} \sqrt{\frac{4ADG}{3E(4AC - B^2)}} \quad n = \frac{1}{4AC - B^2}
\]

\[
A = \frac{1}{2} A_1 + A_2 + (1 - \epsilon_i) A_i + (1 + \epsilon_i) A_i \quad t, b + \epsilon_i \quad t, b
\]

\[
B = 2\epsilon_i A_1 + \frac{2}{3} (1 - \epsilon_i) A_i + \frac{2}{3} (1 + \epsilon_i) A_i + \left( \frac{2\epsilon_i^2 + 4\epsilon_i}{3} \right) t, b + \left( \frac{\epsilon_i^2 + 8\epsilon_i}{15} \right) t, b
\]

\[
C = \epsilon_i A_1 + \frac{1}{5} (1 - \epsilon_i) A_i + \frac{1}{5} (1 + \epsilon_i) A_i + \left( \frac{\epsilon_i^2 + 8\epsilon_i}{15} \right) t, b
\]

\[
D = (1 + \epsilon_i) t, b + \frac{t, b}{\epsilon_i}
\]

3 Shear lag coefficient of energy method

The bending stress of elementary beam theoretical box girder under the action of moment and axial force is as follows:

\[
\sigma_{\text{primary beam}} = \frac{N(x)}{A} \pm \frac{M(x) \cdot \gamma}{RJ}
\]

Shear lag coefficient:

\[
\lambda = \frac{\sigma_{\text{energy method}}}{\sigma_{\text{primary beam}}}
\]

The shear lag coefficient of energy method is calculated by taking Figure 1 of Jianbang Yellow River Highway Bridge in Jinan as an example.

The longitudinal distribution of cable tension from SC’1 to SC’7 is shown in Table 2 (take the position of Auxiliary Pier as point \( x = 0 \)).
### Tab.2 Location of the cable and axial force

| Cable number | Cable force (kN) | Angle (°) | Position x(m) | N(x) (kN) |
|--------------|------------------|-----------|----------------|-----------|
| SC'7         | 3639.70          | 37.51     | 6.00           | 2887.22   |
| SC'6         | 3492.90          | 39.82     | 12.00          | 5569.90   |
| SC'5         | 3243.20          | 42.79     | 18.00          | 7950.11   |
| SC'4         | 3090.30          | 46.43     | 24.00          | 10080.26  |
| SC'3         | 2825.00          | 51.11     | 30.00          | 11853.80  |
| SC'2         | 2553.70          | 57.00     | 36.00          | 13244.61  |
| SC'1         | 2590.95          | 65.24     | 42.00          | 14329.87  |

Fit according to the linear and parabolic changes respectively, and the polynomials and coefficients obtained are shown in Table 3.

### Tab.3 Values of different coefficient

| Fitting form | Polynomial form of fitting | Coefficient a1 | Coefficient a2 |
|--------------|----------------------------|----------------|----------------|
| linear       | \( y = 318.9x + 1762.1 \) | -12934.77      | 12937.64       |
| parabola     | \( y = -4.513x^2 + 535.592x + 187.5175 \) | -21727.31      | 21727.42       |

According to the different distribution forms in the above table, calculate the normal stress at the selected position of the mid span top and bottom slab, and add the obtained stress with the calculated stress under the action of bending moment, as shown in Table 4.

### Tab.4 Stress of different position using energy variation method

| Selected location | core (\( y = 0 \)) | Side rib (\( y = 4.5 \)) | Inclined web (\( y = 11.25 \)) |
|-------------------|----------------------|--------------------------|-------------------------------|
| roof (MPa)        | 4.089                | 1.245                    | 2.426                         |
| floor (MPa)       | -23.36               | -21.033                  |                               |

The stress of the top and bottom plate of the midspan section calculated by the elementary beam theory is shown in Table 5.

### Tab.5 Stress of different position using elementary beam theory

| Position | roof | floor |
|----------|------|-------|
| Stress value (MPa) | -3.762 | -20.763 |

The shear lag coefficients at different positions of the roof and floor are calculated, as shown in Table 6.

### Tab.6 Shear-lag coefficient using energy variation method

| Selected location | core (\( y = 0 \)) | Side rib (\( y = 4.5 \)) | Inclined web (\( y = 11.25 \)) |
|-------------------|----------------------|--------------------------|-------------------------------|
| roof \( \lambda \) | 1.087                | 1.162                    | 0.645                         |
| floor \( \lambda \) | 1.125                | 1.043                    |                               |

The shear lag phenomenon exists at the mid web of the mid span floor and the top slab, but the bottom slab is obvious, about 3.5% higher than the corresponding position of the top slab; the shear lag effect at the junction of the top slab and the side web is more obvious, but the corresponding position of the bottom slab is not very different; and the negative shear lag effect appears at the junction of the top slab and the inclined web.
4 Solution of shear lag coefficient by spatial finite element method

4.1 Establishment of finite element model

The spatial finite element model is established by the finite element analysis program ANSYS. Solid65 is used to simulate the beam. A total of 40 meters concrete box girder with 5 stay cable segments on the side of side tower is selected to build the model. The segment model generates 94901 nodes, which are divided into 70512 solid elements. The cable force of the stay cable is applied according to the surface force load under the anchor plate. The boundary condition is consolidation at the bridge tower and unconstrained at the other end. The finite element model is shown in Figure 2.

![Fig.2 Space finite element model of tower root extension to 40 m segment](image)

4.2 Analysis of shear lag coefficient

According to the ANSYS spatial finite element model, the normal stress at the web interface of the top and bottom plate of the midspan section is shown in Table 7.

| Selected location | core \( y = 0 \) | Side rib \( y = 4.5 \) | Inclined web \( y = 11.25 \) |
|-------------------|-----------------|-----------------|------------------|
| roof (MPa)        | -3.848          | -4.029          | -2.344           |
| floor (MPa)       | 25.33           | 24.313          |                  |

According to the calculation formula of the shear lag coefficient, the normal stress values of the mid-span and bottom plates obtained in the ANSYS model are compared with the corresponding normal stress values of the mid-span calculated from the simple beam theory to obtain the shear forces across the mid-span and bottom plates. The hysteresis coefficient is shown in Table 8.

| Selected location | core \( y = 0 \) | Side rib \( y = 4.5 \) | Inclined web \( y = 11.25 \) |
|-------------------|-----------------|-----------------|------------------|
| roof \( \lambda \) | 1.023           | 1.071           | 0.623            |
| floor \( \lambda \) | 1.219           | 1.171           |                  |

Both the bottom plate and the top plate have obvious positive shear lag effect at the mid-span web. The position is not obvious, there is a negative shear lag effect at the inclined web, and the bottom plate is about 9.4% higher than the top plate at the same position.

5 Comparison and analysis of calculation results of two methods

In this paper, the calculation method of the shear lag coefficient derived from the energy method is compared with the finite element method to calculate the shear lag coefficient. The results are shown in Table 9.
Tab.9 Comparison of the shear lag coefficient

| Selected location          | core \((y = 0)\) | Side rib \((y = 4.5)\) | Inclined web \((y = 11.25)\) |
|---------------------------|-----------------|-------------------------|-------------------------------|
| Finite Element Method     | 1.023           | 1.071                   | 0.623                         |
| roof \(\lambda\)         |                 |                         |                               |
| Variational method roof   | 1.087           | 1.162                   | 0.645                         |
| \(\lambda\)              |                 |                         |                               |
| Finite Element Method     | 1.219           |                         | 1.171                         |
| floor \(\lambda\)        |                 |                         |                               |
| Variational method floor  | 1.125           |                         | 1.043                         |

The error between the shear lag coefficient calculated by the energy method and the shear lag coefficient obtained by the space finite element method is relatively small. The center height of the top plate is 6.3%, the boundary of the ribs is 8.5%, and the boundary of the inclined webs is 3.5%. The center of the floor is 8.4%, and the web junction is 12.3%. Because the forces at the web junction are complex, the displacement function assumed in the energy method is difficult to describe the displacement completely.

6 Conclusions

The following conclusions can be drawn:

1. In this paper, the energy method is used to analyze the shear lag effect of a single cable plane wide box girder, and the formula for calculating the shear lag coefficient of a single cable plane wide box girder is obtained by considering both the bending moment and the axial force.

2. The energy method used in this paper to calculate the shear lag coefficient of a single-cable wide box girder is in good agreement with the calculated value using the finite element method. Therefore, the calculation method of the shear lag coefficient of a single-cable wide box girder can meet the engineering requirements. The calculation formula is simple and reliable.

3. The wide box girder of a single cable plane cable-stayed bridge has a large shear lag effect, which should be given high attention during design.

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