Anisotropy of Electric Field Fluctuations Spectrum of Solar Wind Turbulence

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ABSTRACT
To investigate the power and spectral index anisotropy in the inertial range of solar wind turbulence, we use 70 intervals of electric field data accumulated by Cluster spacecraft in the free solar wind. We compute the electric field fluctuation power spectra using wavelet analysis technique and study its spectral index variation with the change in angle between the heliocentric radial direction and the local mean magnetic field. We find clear power and spectral index anisotropy in the frequency range 0.01 Hz ≪ 0.1 Hz, with more power in parallel fluctuations than perpendicular. We also report our study of anisotropy as a function of solar activity.

Key words: (Sun:) solar wind – (magnetohydrodynamics) MHD – turbulence – methods: data analysis

1 INTRODUCTION
The solar wind (SW) provides a natural laboratory for the study of fully developed plasma turbulence through in-situ measurements from spacecraft. Turbulence is believed to be one of the central characters responsible for the acceleration and heating of the SW. Despite being a weakly collisional plasma, solar wind turbulence (SWT) can be studied in the framework of magnetohydrodynamics (MHD) for the length scales superior to the ion gyroscale and ion inertial scale (~ 100 km). Similar to hydrodynamic turbulence, here too the energy is injected at some large length scale and finally decays due to viscosity at molecular diffusion length scale. Between those two scales, energy is neither added nor removed from the system but simply gets transferred to adjacent smaller length scales through local nonlinear interactions in k-space, often called the turbulent energy cascade. This intermediate range of scales, known as inertial zone, is usually characterized by a constant energy flux rate (ε) from one scale to the subsequent one. In practice, the turbulent fluctuations of the plasma fluid and the electromagnetic field are directly obtained in the form of a time series. Since both the spacecraft speed (~ 1 km/s) and the Alfvén speed (~ 50 km/s) of the solar wind at 1 AU are much lower than the solar wind speed (v ~ 600 km/s), one can use Taylor’s hypothesis (reference of thesis) to map length scales (ℓ) into corresponding time scales (τ) as τ ∝ ℓv or different frequencies (f) to equivalent wave numbers (k) as ω ∝ vk (Taylor 1938; Banerjee 2014). The power spectra for the magnetic field, velocity and density fluctuations in the solar wind have extensively been studied for many decades. In the MHD frequency range of 10⁻³ ~ 10⁻¹ Hz, where both the magnetic energy and density power spectrum are found to scale as f⁻⁵/₃ (or ∼ k⁻⁵/₃), velocity power spectrum is found to follow a shallower f⁻² spectrum (Podesta et al. 2006). Turbulence power spectrum for higher frequencies are also investigated systematically (Alexandrova et al. 2009; Sahraoui et al. 2009). It is only recently that the first measured power spectrum of electric fluctuations has been reported (Bale et al. 2005). The spectrum is shown to have a spectral exponent of ~ -5/3 and follows the magnetic field fluctuation spectrum until around 0.45 Hz (breakpoint) at the spacecraft measured frequencies. Beyond this breakpoint, the magnetic spectrum becomes steeper whereas the electric spectra becomes enhanced.

All these power spectra have been obtained under the assumption of isotropy. In MHD, unlike velocity, Galilean transformation cannot eliminate the effect of mean magnetic field (B₀) which leads to unequal deformation of the Alfvénic wave packets along and perpendicular to B₀. Due to a non-negligible B₀, anisotropy in SWT is normally expected in the turbulence power level as well as in the spectral indices. However, in the studies conducted using a global mean magnetic field (Tessein et al. 2009; Sari & Valley 1976; Chen et al. 2011), only power anisotropy and no spectral index anisotropy were reported. This puzzle can be addressed using several numerical studies of MHD turbulence showing that the deformation of wave packets of a specific length scale will be regulated by the mean field of comparable

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length scale rather than that of the largest scale (Cho & Vishniac 2000; Milano et al. 2001). For solar wind, it is found that the minimum variance direction of the inertial range fluctuations closely follows the scale dependent local mean magnetic field direction (Horbury et al. 1995). Recently, using wavelet method, prominent anisotropies both in the power levels and the spectral indices of magnetic power spectra were observed for polar and ecliptic solar winds (Horbury et al. 2008; Podesta 2009). The parallel power is found to be at least 5 times less than the perpendicular power and the spectral index varies from around -2 to -5/3 as the angle between the local mean magnetic field and the flow direction changes from 0° to 90°, thereby being consistent with the critically balanced MHD turbulence theory by Goldreich & Sridhar (1995). Clear signatures of anisotropy for (i) magnetic power spectra at MHD and sub-ion scales and (ii) inertial range velocity spectra were also found for near ecliptic solar wind using both single and multi spacecraft data (Chen et al. 2010, 2011; Wicks et al. 2011).

Unlike velocity and magnetic field, the anisotropy of electric field fluctuations are studied occasionally. The first (and probably the only) study of power and spectral index anisotropy in the electric field power spectra was carried out by projecting the field along and perpendicular to the global mean magnetic field (Mozer & Chen 2013). After following stringent selection criteria, they could find only three suitable intervals from the THEMIS and Cluster data. From the analysis, it was found that the parallel power spectrum magnitudes were comparable to or greater than the perpendicular powers. Both the spectra had similar shape and inertial range power-law slopes of about -5/3. But the results obtained could not be validated over more intervals due to lack of data. In this paper, following the similar technique of (Horbury et al. 2008), we investigate the power and spectral index anisotropy of electric field fluctuations in the ordinary MHD scale. To measure the anisotropy, we study how the spacecraft-frame spectrum of electric fluctuations varies with the angle between average flow direction and local mean magnetic field. Following the wavelet transform technique as implemented in Horbury et al. (2008) for magnetic field data, here we can decompose a time series of electric field data into wavelet coefficients those are localized in both time and frequency (or wavelet scale). In order to capture the local mean magnetic field, we use Gaussian windows of different standard deviations thereby giving a measure of the corresponding length scale. In this paper, We also study the effect of solar activity on the anisotropy of electric field fluctuations.

The paper is organized as follows. In §2 we discuss the selection criteria for the intervals chosen and mention the data used. In §3 we present the data analysis methods used for the study, in §4 we present the results and in §5 we discuss our findings.

2 DATA INTERVALS
Cluster is the first multi-spacecraft mission (2000, ESA) of four spacecraft flying in tetrahedral configuration. The simultaneous in-situ measurements of these four spacecraft are used to investigate the small-scale structures and dynamics in solar wind in three-dimensions. This makes Cluster, an excellent choice to study various physical processes occurring in the solar wind (Escoubet et al. 2001). To study the anisotropy in SWT and the effect of sun’s activity on the distribution of energy in space around earth, we select the data obtained from Cluster during the solar cycle 23, which extends from 1996 to 2008, reaching its maximum in November 2001. We use the data intervals when the spacecraft is in the pristine fast solar wind in the ecliptic plane, outside the earth’s magnetic environment at a distance of 1 AU. In this study, we analyze 70 such intervals after searching through 5 years of data from January to May each year from 2001-04 and 2007. The selected intervals are from parts of the orbit when the spacecrafts are at geocentric distances of between $15R_E$ and $20R_E$. The time intervals studied here contain no data gaps. The time series were visually ensured to be approximately time-stationary and we tried to find the streams of longest durations possible with sustained high speeds.

We choose a few physical quantities for the study of anisotropy of turbulent solar wind flow. Electric field measurements are made by the electric field and wave (EFW) experiment (Gustafsson et al. 1997) and magnetic field is measured by fluxgate magnetometer (FGM) instrument (Balogh et al. 1997) on board the cluster spacecraft. We use the magnetic field data with a resolution of 22 vectors per second and electric field with a resolution of 25 vectors per second, measured in geocentric solar ecliptic (GSE) coordinate system. Ion moments (velocity, density and temperature) of the solar wind are obtained from cluster ion spectrometry (CIS) experiment (Reme et al. 1997). The electric field and magnetic field data are taken from Cluster 4 (C4) and plasma data are taken from Cluster 1 (C1) spacecraft. As mentioned above, we use 70 intervals for our study whose mean parameters are as follows : (i) Solar wind velocity - (-479.9, 40.5, 5.6)km$s^{-1}$ (ii) Ion Number Density - 6.5cm$^{-3}$ (iii) Ion perpendicular temperature - 27.9eV. In the table 1, we mention parameters for two of the longest duration streams that we could find, one each from the period of sun’s maximum and minimum activity. For the study of anisotropy in the period of solar maximum, we have used the data interval from the year 2002 when spacecraft C1 and C4 both were at a distance of $18.25R_E$ from the earth. For the investigation during the sun’s minimum activity, we choose the data interval from the year 2007 when the spacecrafts were at a distance of $19R_E$ from the earth, in the free solar wind. Rest all the intervals were analyzed to test the statistical robustness of the results obtained. In the table’s last column, S.I. stands for spectral index obtained from fast fourier transform (FFT) scheme.

3 METHOD
As discussed in the introduction, we can use Taylor’s hypothesis to study SWT near earth and hence the time series data, measured by the spacecraft, correspond to a simple one-dimensional spatial sample (Taylor 1938). In this study, we measure the reduced spectrum which is defined as (Fredricks & Coroniti 1976):

$$P(f) = \int d^3k \, P(k) \, \delta(2\pi f - \mathbf{k} \cdot \mathbf{V}) \tag{1}$$

Anisotropy of electric field fluctuation spectrum with respect to the mean magnetic field can be measured by studying how
Sample text about wavelet analysis and magnetic field analysis.
It provides the time- and frequency- localized mean field direction. We now find the inclination ($\theta_{vb}$) of this mean field with respect to the average flow direction $\mathbf{V}_{sw}$ (sampling direction) of solar wind (sw) using:

$$\cos (\theta_{vb}) = \frac{\mathbf{V}_{sw} \cdot \mathbf{b}}{|\mathbf{V}_{sw}| \cdot |\mathbf{b}|}. \quad (7)$$

For the inertial range turbulence that is axisymmetric about the magnetic field direction, the power spectra are independent of the azimuthal angle $\phi$ and hence we consider the power values averaged over all $\phi$ (Horbury et al. 2008). After obtaining the directions of the local mean magnetic field, we now bin the square amplitude of the wavelet coefficients at a given scale according to the scale-dependent field to find the power distribution in different directions.

In this study, we obtain different angle sets for different intervals, such as $20^\circ$ to $180^\circ$, for the distribution of bin counts shown in Fig. 2(a) and $0^\circ$ to $140^\circ$ in Fig. 2(b). We construct bins of equal spacing of $10^\circ$, say, for the 2002 interval we get 16 bins. The distributions of bin counts was qualitatively and quantitatively similar at all frequencies from 2.7 mHz to 11 Hz. For each bin and for a given frequency $f_l$, we average the trace power values corresponding to all the angles that lie in the bin. By doing this, we obtain the average trace power value $P(f_l, \theta_{vb})$ corresponding to the frequency $f_l$ for that particular angle bin. As a result, we obtain the electric power spectrum when field is pointing in that particular direction. The number of times the field points in a particular direction gives us the measurements of the field. Many bins have thousands of such measurements. A reasonable statistical sample is the one where we have sufficient number of measurements, so we reject those angle bins which have less than 200 contributing power levels.

4 RESULTS

4.1 Anisotropy of Inertial Range Turbulence

Here we present the results of our study on anisotropy of electric fluctuations spectrum with respect to the local mean magnetic field direction. We analyzed 30 data intervals from 2001, 2002 and 2004 to achieve statistical robustness of our results. Fig. 3 is obtained by conducting analysis on the data from one of the 30 intervals, that is, from 1400 hrs of February 2, 2002 to 0006 hrs of February 3, 2002. We choose to present this interval as it is found to be the only one among the 30 which has the angle spread from $90^\circ$ to $180^\circ$ covering both parallel and perpendicular directions to the local mean magnetic field. For most of the selected intervals the spread was found to be between $40^\circ$ and $90^\circ$. The figure shows the trace power spectral density $P(f_l, \theta_{vb})$ plotted with frequency $f_l$ for two angle bins, $90^\circ$ to $100^\circ$ (blue) and $170^\circ$ to $180^\circ$ (orange). The power values have been computed using Morlet Wavelet scheme for a total of 25 different frequencies ranging from 2.7 mHz to 11 Hz.

The power law exponents found are $-1.49$ and $-1.64$ for perpendicular and parallel direction respectively, calculated by performing linear least-squares fitting in the same frequency range as used for the calculation from fourier spectrum of the electric fluctuations for the same interval in Fig. 1, that is, 0.01 Hz to 0.1 Hz. The wavelet electric power spectrum in the direction perpendicular ($90^\circ$-$100^\circ$) to the local mean magnetic field is observed to have less power than that in the parallel direction ($170^\circ$ $- 180^\circ$) in the inertial range but similar values otherwise. The outliers in the spectra are probably because of the inadequate count in the angle bin at those scales.

In Fig. 4, we present the spectral indices plotted with respect to $\theta_{vb}$ fitted over the inertial range 0.01 Hz to 0.1 Hz. The power law exponents have been averaged over the 30 intervals and plotted for outward and inward sectors separately in Fig. 4(a) and (b), respectively. It illustrates the distribution of spectral exponents with the increase in angle between the flow direction and the local mean magnetic field. We observe that the power law indices are confined in the range $-1.5 \pm 0.1$ for both the sectors and found to be approximately the mirror image of each other.

4.2 Effect of Solar Activity on Anisotropy

In this section, we try to find the dependence of anisotropy on solar activity, if any. We aim to observe if there is any variation in anisotropy during solar maximum or minimum. Solar cycle 23 lasted from 1996 to 2008 allowing the cluster spacecraft to make measurements during both the solar maximum and solar minimum period. Sun’s activity peaked in late 2001. After searching through 4 years of Cluster data from the years 2001-03 (near solar maximum) and 2007 (near solar minimum), we found 60 intervals of pristine solar wind data, with 30 intervals from each maximum and minimum period.

For each interval, we computed the electric field fluctuations spectrum using wavelet analysis for all the possible angle ranges for the particular interval and measured their spectral indices between the spacecraft frame frequency ranging from 0.01 Hz to 0.1 Hz. We then plotted the histograms to observe the distribution of power-law exponents calculated for parallel ($0^\circ$-$30^\circ$) and perpendicular ($70^\circ$ to $90^\circ$) directions with respect to the local mean magnetic field during the solar maximum and minimum period. As stated above, we could find very few intervals with angle range spreading from $0^\circ$ to $90^\circ$ upon searching through 5 years of Cluster data. So, for parallel and perpendicular directions, few angle bins were clubbed so as to obtain sufficient number of bin counts from 30 intervals each during sun’s maximum and minimum activity. From the histograms (see fig. 5), it is clearly found that the spectral indices for near perpendicular electric power stays approximately $-1.5$ irrespective of the solar activity whereas for small angle ($0^\circ$ to $30^\circ$) power spectra, the index is having a mode near $-1.7$ during solar activity and $-1.4$ during solar calm.

5 DISCUSSION

In this paper, we investigate the spectral index anisotropy of solar wind turbulence in the inertial range with respect to the local mean magnetic field. We present observational results from wavelet analysis of Cluster electric field fluctuations. Wavelet analysis enabled us to calculate the scale-dependent field which in turn allowed us to study scale-dependent anisotropy. The frequency power spectra of electric field fluctuations were calculated as a function of
angles between the heliocentric radial direction and the local mean magnetic field. Our study clearly shows that electric power undergoes prominent power and spectral index anisotropies within the frequency range $10^{-2} - 10^{-1}$ Hz. In contrast with the magnetic power spectra, parallel electric power slightly overcomes the perpendicular power. During the maximum solar activity period, the perpendicular power spectrum is less steeper (-1.5) than the parallel one (-1.7) which is opposite to the case during minimum solar activity where the parallel spectra is slightly less steeper (-1.4) than the perpendicular power spectra (-1.5).

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Figure 4. Power-law exponents of the angle-dependent trace power spectrums vs $\theta_{vb}$ fitted over 0.01 Hz to 0.1 Hz for (a) outward and (b) inward sectors.

Figure 5. Distribution of spectral indices in parallel (0°-30°) and perpendicular directions (70°-90°) during both minimum and maximum activity of sun.

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