A note on the possibility of roughness enhancement of adhesion in Persson’s theory

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Abstract

In an attempt to model the observed enhancement of adhesion in some classical experiments in the 1970-1980’s, Persson introduced in his theory of adhesion between rough solids a term which corresponds to an area increase due to roughness. In the old experiments, the adhesion enhancement was shown to be up to one order of magnitude, whereas the area increase could not be defined quantitatively because of possibly multiscale roughness. However, in more recent studies by Guduru and collaborators, this enhancement has been further explained with classical Linear Elastic Fracture Mechanics theory, the area enhancement has been shown to be negligible, and therefore the problem of adhesion of rough surfaces remains qualitatively and quantitatively unsolved by Persson’s theory.

Keywords:
Roughness, Adhesion, Persson and Tosatti’s theory, Fuller and Tabor’s theory

1. Introduction

In his elegant theory of adhesion of rough surfaces, Persson (2002) (see also Persson and Tosatti, 2001), in an attempt to justify some observations in Briggs & Briscoe (1977), and Fuller & Roberts (1981), postulate that an increase of adhesion may occur for the increase of surface area induced by roughness. This is clearly stated in Persson and Tosatti (2001) "for an (elastically) very soft solid the adhesion force may increase upon roughening the
substrate surface. This effect has been observed experimentally \cite{9}, and the present theory explains under exactly what conditions that will occur”, and in Persson (2002) “The increase in $\Delta \gamma_{eff}$ arises from the increase in the surface area”. Fuller & Roberts (1981) show that the adhesion enhancement can be up to one order of magnitude, and hence very significant indeed, and classical asperity models (Fuller and Tabor, 1975) were not able to capture this effect. However, in those experiments roughness was random and possibly multiscale and not characterized fully, and hence a good estimate of the area enhancement was not possible.

In general, Persson’s theory "is valid for surfaces with arbitrary random roughness”, and in Sec. 5 they use it with success with the case of Fuller and Tabor (1975) which correspond to an elastic sphere against a rough plate. In the case of self-affine fractals, the power spectrum as a function of wavevector $q$ has the form

$$C(q) = \begin{cases} \frac{H}{2\pi} \left( \frac{h_0}{\lambda_0} \right)^2 \left( \frac{q}{q_0} \right)^{-2(H+1)} & \text{for } q < q_0 \\ \text{for } q > q_0 \end{cases}$$

(1)

where $H = 3 - D_f$ (with $D_f$ being the fractal dimension of the surface comprised between 2 and 3), and $q_0$ is the lower cut-off wavevector which corresponds to the largest wavelength in the spectrum, and can be due to macroscopic shape.

They indeed obtain an "effective adhesion energy" which is "magnification-dependent”, where $\zeta = q/q_0$ is the magnification factor, which shows two competitive factors: i) and enhancement due to the area increase due to roughness, which for surface gradient $\nabla h(x) << 1$

$$A = A_0 + \frac{1}{2} \int \sqrt{1 + (\nabla h)^2} dS \simeq A_0 + \frac{1}{2} \int \left( 1 + \frac{1}{2} (\nabla h)^2 \right) dS$$

(2)

and ii) a decay due to the elastic deformation. This results in

$$\Delta \gamma_{eff} = \Delta \gamma \left( 1 + (q_0 h_0)^2 \left( \frac{g(H, \zeta)}{2} - \frac{1}{q_0^2} f(H, \zeta) \right) \right)$$

(3)

\footnote{Ref.9 is here Fuller & Roberts (1981).}
where $\delta = \frac{4\Delta}{E}$, $E^*$ is plane strain elastic modulus, and

$$g(H, \zeta) = \frac{H}{2(1-H)} (\zeta^{2(1-H)} - 1) \to \zeta^{2(1-H)}$$ \hspace{1cm} (4)

$$f(H, \zeta) = \frac{H}{1-2H} (\zeta^{1-2H} - 1) \to \zeta^{1-2H}$$ \hspace{1cm} (5)

The function $g$ is of the order of 100-500 in the original plots of Persson and Tosatti (2001), and strongly depends on magnification $\zeta$, but clearly these numbers do not have much sense, since the equation they use is obtained in the limit $\nabla h(x) << 1$, and at $\nabla h(x) = 1$ obviously we only have a mere 41% increment, which is indeed the maximum enhancement which Persson shows in his FIG. 2 of Persson, (2002). Beyond this point, we would be in areas where finite deformations, and many other deviations from the usual approximations would happen. Notice that the function $f$ decreases if $H > 0.5$ (fractal dimension $D < 2.5$, which is the common case), indicating that the effective adhesion energy tends to return to the original value without roughness — a result that is however not as clear as this analysis is limited by the strong assumption of full contact which is so far uncontrolled.

Even when both functions grow, $g$ grows much faster than $f$, and the authors do not suggest where we should stop. However, the point is not this, but that the area increase is completely unrelated to the adhesion enhancement of Briggs & Briscoe (1977), and Fuller & Roberts (1981).

Indeed, after Persson’s theory has appeared, in very interesting experiments using a single scale axisymmetric roughness between gelatin and Perspex flat rough plates, by Guduru and his group (Guduru (2007), Guduru & Bull (2007), Waters et al (2009)), the adhesion enhancement has been studied in details, and shown to be of an order of magnitude even when the surface area increase is (as we easily estimate below) of much less than 0.1%.

Waters et al (2009) have a good summary of Guduru’s group theory and experiments. They have a surface defined as

$$f(r) = \frac{r^2}{2R} + A \left( 1 - \cos \frac{2\pi r}{\lambda} \right)$$ \hspace{1cm} (6)

where $A$ can be both positive in the case of a central convex asperity, and negative, for a central concave trough.
The enhancement is shown to occur when complete contact occurs and the contact area is simply connected. In the earlier paper (Guduru (2007)), conditions were derived for the gap to be monotonically increasing with radius, but this condition is overly restrictive, as it is well known even from Persson’s energy balance concept, that adhesion permits a wavy surface to spontaneously achieve full contact. The analysis follows conveniently introducing two parameters

\[
\alpha = \frac{AR}{\lambda^2}, \quad \beta = \frac{\lambda^3E^*}{2\pi\Delta\gamma R^2}
\]  

(7)

The physical meaning of \( \alpha \) is that obviously it represents the degree of surface waviness. There are only two scales really in the process, one represented by the radius of the sphere \( R \) (no specific reference to the amplitude and waviness), and the other by amplitude and wavelength of roughness: \( \alpha \) is also obviously the ratio between the radii of the sphere and that of the asperities. Large \( \alpha \) correspond to surfaces with high amplitude, short wavelength waviness. The parameter \( \beta \) is instead a measure of the relative stiffness of the material to the surface energy. The adhesion amplification is seen in a clear map in Fig.5 of Waters et al (2009) for the JKR regime. It is seen to reach values over 4 (in terms of pull-off, but equivalently in terms of \( \Delta\gamma_{\text{eff}} \) in Persson’s theory notation), for values of \( \alpha < 0.25 \).

Guduru and Bull (2007) demonstrated the actual validity of these predictions, with experiments with soft gelatin, with waviness amplitude \( A = 1.2\mu m \) and wavelength \( \lambda = 0.2mm \). This corresponds to an estimate increase of area of \( 1 + \left(\frac{\pi A}{\lambda}\right)^2 = 1 + \left(\frac{\pi 1.2}{0.2}\times10^{-3}\right)^2 \simeq 1.0001 \), whereas the pull-off increase was a factor about 2. Even worse the comparison with waviness amplitude \( A = 5.5\mu m \) and wavelength \( \lambda = 0.43mm \). Here, \( 1 + \left(\frac{\pi 5.5}{0.43}\times10^{-3}\right)^2 \approx 1.0004 \), while the amplification factor was about 6. These examples illustrate the very different nature of increases in adhesive strength resulting from the presence of shallow waviness on soft elastic surfaces.

Notice that these results occur in a situation where the large amplitude of roughness is in partial contact, and the roughness scale is in a full contact. Persson’s theory takes into account of the possibility of partial contact in later parts of the papers, but only to derive further reduction of adhesion, and certainly not increase. Hence, the partial contact correction can only make the comparison worse. In Fig. 5 of Guduru and Bull (2007), it
is shown that beyond a critical $\beta$, there is a region where the "enhanced" strength occurs only if the contact is loaded first sufficiently to cause full contact in the roughness scale. Beyond an even greater $\beta$, finally a reduction in pull-off force for the wavy surface compared to the flat surface occurs, when locally the contact is one of two isolated spherical asperities with a much reduced equivalent radius. However, even in this range, an increase of $\alpha$ leads to an increase of adhesion. Therefore, not even this is the regime usually indicated in asperity models like Fuller and Tabor (1975) as roughness destroying adhesion. There are other aspects of Guduru’s enhancement which are not considered in Persson’s theory, and a remarkable one is the absence of irreversible processes leading to an increase of toughness. There is a qualitative discussion at the end of par.4 of Persson and Tosatti (2001) about this aspect, but none of these effects is included in the theory.

2. Discussion

The Guduru enhancement of adhesion could be even stronger for multiscale roughness, and the limitations will be (i) that some adhesion enhancements will be load-dependent; and (ii) that Guduru’s analysis considers separation originating at the periphery, which may be increasingly a strong assumption when multiscale roughness is included. We have recently attempted to consider the possibility of separation to occur at the local minima of the surface waves, where tensile interface stresses will be highest for a Gaussian random roughness (Ciavarella, 2016). The analysis shows that a very simple approximate solution is possible: we consider the full contact solution which is known in closed form, and consider the condition for the gaps in regions of tensile stresses to remain open or close. This leads to a solution very similar to Persson’s solution in contact mechanics without adhesion (Persson, 2001), namely that

$$\frac{A_c(0)}{A_0} = \text{erf} \left( \frac{\sqrt{2}(\overline{p} + p_{\text{min}})}{E^* \sqrt{V}} \right)$$

which is valid for positive mean pressure $\overline{p} > 0$ only, and where $V$ is the variance of full contact pressure variations. Also,

$$\frac{p_{\text{min}}}{\sqrt{V}} \sim \zeta^{2/5(2H-1)}$$

and therefore increases without limit with magnification for $H > 0.5$ or for fractal dimension of the surface $D < 2.5$. It wasn’t noticed in (Ciavarella,
2016) that this model show therefore that under zero load the ratio of the area of contact to the nominal area would be

\[ \frac{A_c(0)}{A_0} = \text{erf} \left( \frac{\sqrt{2} p_{\text{min}}}{2E^* \sqrt{V}} \right) \tag{10} \]

and therefore a propensity of reaching full contact, but never obviously the exact full contact. However, notice the analogy with Persson’s function \( f \) which decreases if \( H > 0.5 \). Here, we obtain this result about the contact area directly considering partial contact, and only with some small approximations to obtain the closed form results (for details, see Ciavarella, 2016). We do obtain that for \( D < 2.5 \), the common case, the contact area tends to be complete. We would be tempted to say that this seems to give some meaning of ”effective adhesion energy”: here it takes the sense of the energy available when we start the process of unloading, but very little can be said about the reversible and irreversible processes that start upon unloading, nor the maxima we could reach of pull-off force. Persson seems to take another more meaning, of effective energy assuming full contact, which has to be corrected considering partial contact, resulting in a circular definition.

The situation of unloading unfortunately cannot be treated with this model, as cannot be treated in Persson’s theory of adhesion, and therefore, the problem remains largely unsolved.

Finally, another limitation of the Guduru effect will be at small scales, in that the Waters et al (2009) show the enhancement to be limited to the JKR regime (Johnson et al., 1971), whereas the small scales are essentially in the DMT regime.

3. Conclusions

The large increments of adhesion measured in soft solids cannot be captured by Persson’s model of adhesion. Therefore, the competition between these adhesion enhancement with multiscale roughness has not yet been understood.

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