Determination of the added-liquid masses in the problems of control of the motion and stealthiness of marine underwater objects

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Abstract. A numerical method for calculating the generalized additional masses of sea underwater objects with fins, performing both single and joint navigation of several objects, as well as with elastic vibrations of the hull, is presented. The flow is modeled using a continuous layer of sources and vortex frames that simulate the body and its fins. The method provides a strict account of the mutual influence of bodies and their elements and does not impose restrictions on their shape. Approbation of the method showed its reliability and the possibility of practical use in design practice, as well as in simulators and on-board complexes.

Introduction

In mathematical modeling of the movement of marine underwater objects (MUO) and in the problems of controlling their stealthiness by physical fields, it is required to determine the hydrodynamic forces and moments acting on the object. These reactions are necessary for calculating the propulsion and controllability of the MUO, including finding the parameters of the trajectory of its spatial motion. Knowledge of these parameters is also important for the stealthiness of the MUO, since they have a direct effect on its physical fields, first of all, on acoustic and hydrophysical ones. A separate task is to determine the vibration parameters of the MUO body, the knowledge of which makes it possible to predict its acoustic radiation.

Ensuring the required controllability of the MUO is achieved by choosing the shape of its hull and elements of the propulsion and steering complex, including rudders, stabilizers and a propeller, the parameters of which must be linked to the characteristics of the main power plant. The design process of such objects involves the consideration of a significant number of drawing and designing solutions, for each of which it is required to answer the question: "Will the requirements for the controllability of MUO be provided for this solution and to what extent?" To answer this question, it is necessary to determine the parameters of the object's motion under various control actions.

It is known that the calculation of these parameters requires the solution of a system of ordinary nonlinear differential equations of the first order. There are six such equations for an arbitrary spatial motion of a rigid body in a continuous medium. They are closed by a system of so-called equations of
kinematic connections between the projections of the translational and angular velocity of the body and the angles, as a rule, these are the Euler angles that determine its position in space. The solution of this system makes it possible to determine all the kinematic parameters of the body's motion, including its trajectory.

In the general case, it is not possible to obtain an analytical solution to this system, however, the solution can be obtained numerically on ordinary personal computers. In this case, the main problem is to determine the right-hand sides of the equations of motion, which are hydrodynamic reactions (forces and moments) acting on a moving body from the side of the fluid. They include hydrostatic and hydrodynamic reactions. The first ones, which include, for example, the force of support (Archimedes) are found quite simply using the known dependencies. However, their determination becomes somewhat more complicated if the MUO is located in seawater of heterogeneous density. The determination of the second - hydrodynamic reactions - is one of the most difficult problems of ship hydromechanics. Moreover, not only their very definition is difficult, but also the requirement that this procedure should be carried out in the loop of integration of the equations of motion, i.e., at each time step, the number of which can be tens of thousands.

Currently, there are two main approaches to the numerical simulation of the motion of bodies in a fluid: classical and modern. The second one is based on supercomputer technologies and makes it possible to determine all of the above reactions at each step of the integration of the equations of motion. Models and methods of this approach calculate pressures and shear stresses in a liquid, and their integration over the surface of an object is the desired reaction. Its advantages are obvious and in the future this approach will become the main one. But at present, its uncontested use is limited by the huge amount of computations required to calculate the power part of the problem being solved.

To calculate the reactions in this case, the numerical integration of nonlinear partial differential equations describing turbulent flows of a viscous fluid is performed. Such equations are the Navier-Stokes equations. The volume of calculations in their direct numerical simulation depends on the Reynolds number (Re), since the number of computational nodes with its growth increases according to the Re^{9/4} law. As the Reynolds number increases, the time step should also be refined. As a result, even the most high-performance modern supercomputers allow performing calculations of flows for which the Reynolds number does not exceed 10^4. The transition to averaged analogs of the Navier-Stokes equations will approach the full-scale values of this similarity criterion (~ 10^8-10^9). These are the Reynolds equations or the more modern equations of the large eddy method. But even in this case, the volume of calculations remains huge.

Figure 1 shows a computer visualization of the calculation of the ascent of a body of the simplest form in a medium inhomogeneous in density at a relatively low Reynolds number of about 10^6. The calculation was carried out on a mini supercomputer with a performance of 1.3 Tflops (1.3 × 10^{12} operations per second) by an untimely deceased D.Eng.Sc. Tkachenko I.V. The actual ascent time was about 100 seconds, and the estimated time was more than a day.

From our experience and the experience of other researchers, we can conclude that such an approach to predicting the controlled movement of MUO can be used to a limited extent and only in the presence of high-performance computers.

The obtained estimate of the time for calculating the dynamics of the MUO when determining the forces based on the model of turbulent flows showed that there are projects in which this approach cannot be used even in the foreseeable future. We are talking about marine navigation simulators, automatic traffic control systems and other special systems, for example, control of the physical fields of MUO. One of the main elements of the software of such simulators is the module for calculating the dynamics of movement in the process of controlling a ship or other object. Moreover, obviously, this module should produce the result in real time, i.e. provide the same response of the simulator to the operator's control action as a real object.
Similar requirements are imposed on the above-mentioned control systems, which have already been introduced, for example, on the US nuclear submarine of the "Virginia" type. A unified submarine control and use system has been created for them, including systems of detection, radio countermeasures, navigation and weapon control.

For control systems for the physical fields of the MUO, it is necessary not only to issue recommendations on the parameters of movement, for example, the transition from one horizon to another, but also to determine the trajectory of controlled movement at this transition, since the levels of the created fields may depend on it.

![Fig. 1. Computer visualization of the process of the ascent of a solid body in a fluid inhomogeneous in density](image)

Thus, at the initial stages of design, when it is required to analyze the controllability of an object for a large number of options for its future appearance, as well as in simulators and in the aforementioned onboard systems, a more economical approach from a computational point of view should be used. This approach exists and differs from the first in the methods for calculating hydrodynamic reactions.

In this approach, these reactions are determined using the hypothesis that they are divided according to their physical nature into inertial, viscous, wave and a number of others, if required by the conditions of the problem. At the same time, an assumption is introduced about the possibility of separate determination of these reactions, moreover, by different methods. For example, viscous ones can be determined experimentally, and inertial ones - using calculation methods.
This approach is used, in particular, in the "Dynamics" software package. The Figure 2 shows an example of its application for calculating a complex maneuver - the ascent of the MUO through a jump in the density of seawater.

In this example, the ascent of the MUO from a 50 m horizon to a 15 m horizon in a stratified medium is considered in the absence of information about the change in its density. It turned out that at a speed of 3 knots, the maneuver time was 1800 s. With information on the hydrology of the sea, this time can be significantly reduced.

To calculate inertial reactions in the "Dynamics" complex, approximate calculation methods are used. The present work is devoted to the presentation of a universal numerical method for predicting these reactions, which, in the case of curvilinear motion of the MUO, make a significant contribution to the overall balance of forces.

Fig. 2. An example of calculating the dynamics of MUO in a heterogeneous marine environment

1. Generalized added masses and methods for their determination.

To calculate the kinetic energy of a liquid imparted to it by a moving object, and then the forces of inertial nature acting on it, special quantities \( \lambda_{ij} \) (i, j = 1,2 ... 6) are introduced, called generalized additional masses (GAM) [1]. It can be shown that for a body with 6 degrees of freedom there are 36 such quantities and they make up the following matrix

\[
\lambda_{ij} = \begin{bmatrix}
\lambda_{11} & \lambda_{12} & \ldots & \lambda_{16} \\
\lambda_{21} & \lambda_{22} & \ldots & \lambda_{26} \\
\ldots & \ldots & \ldots & \lambda_{11} \\
\lambda_{61} & \lambda_{62} & \ldots & \lambda_{66}
\end{bmatrix}
\]

Due to the pairing property \( \lambda_{ij} = \lambda_{ji} \), only 21 of them are independent by mass. Their introduction makes it possible to obtain the calculated dependences for inertial forces and moments and to write the
equations of motion of the MUO in the form given below (here, for brevity, only one of the six equations of motion is written)

\[
(m + \lambda_{11}) \frac{dV_{x1}}{dt} + (m + \lambda_{33}) \omega_{y1}V_{z1} - (m + \lambda_{22}) \omega_{z1}V_{y1} = R_{x1}
\]

where \(m\) is the mass of the MUO, \(\lambda_{ij}\) – its additional masses, \(V_{x1}\) and \(\omega_{y1}\) – projections on the associated coordinate axes of translational and angular velocity, \(R_{x1}\) – projection of non-inertial hydrodynamic forces acting on the object.

It is important to note that for a body of a given form in an infinite liquid, these additional masses have constant values, which means they can be determined once and for all, i.e. in these equations, they are the same constant as the mass of the object.

Theoretical, experimental, and numerical methods can be used to determine the GAM. Both theoretical and numerical methods use the general assumption that the fluid is non-viscous, and the induced flow is irrotational, or potential. When using the hypothesis of the independence of forces of different nature, this assumption is an advantage over experimental methods, since it excludes the effect of viscosity on inertial forces.

The advantage of theoretical methods is the form of presentation of the result convenient for calculations - in the form of an algebraic formula and, as a consequence, a high speed of calculation. However, they also have significant disadvantages. Exact dependences were obtained only for plane bodies (ellipses, polygons, etc.), as well as for ellipsoids. Therefore, to calculate the GAM of real objects, it is necessary to use approximate methods, for example, flat sections or an equivalent ellipsoid, which reduces the calculation accuracy.

Experimental methods have a large error and make it possible to find a very limited number of GAM. Therefore, now they are practically not used.

Numerical methods within the framework of the accepted assumptions practically have no drawbacks, and the advantages are obvious: the calculated objects can have an arbitrary shape of the body, a strict account of the mutual influence, for example, of the plumage and the body, is provided, as well as the possibility of calculating the GAM of several interacting bodies, each of which moves along its own trajectory. All this determined the choice of this direction for the creation of a universal method for calculating the GAM of marine objects and their groups.

For the numerical determination of these masses, the methods of finite elements and boundary integral equations (BIE) can be used. The main disadvantage of the former is that the solution of the three-dimensional Laplace equation for determining the potential must be carried out in the entire region occupied by the liquid. In a numerical solution, it is necessary to make this area finite and set some physically plausible conditions on the boundaries.

In the methods of BIE, the initially three-dimensional problem without loss of accuracy is reduced to a two-dimensional one, in which the sought potential is determined on the surface of the object and its elements under strict fulfillment of the condition of impermeability to them and the absence of induced disturbances at an infinite distance from the MUO. This not only improves the accuracy of calculating the GAM, but also significantly reduces the amount of calculations compared to the finite element method. In view of the obvious advantages for solving the task, the BIE method was chosen.

2. Formulation of the physical and mathematical problem of determining the GAM and the method for its solution.

Let an MUO move in an inviscid infinite fluid, which has a body of arbitrary shape with a plumage installed on it (Figure 3).
Let us replace its effect on the liquid by the effect of a continuous layer of sources distributed over the surface of the body $S$. Their intensity is not known in advance, but it can be found from the solution of the BIE obtained from the condition that this surface does not leak:

$$2\pi q(P) + \int_S q(Q) \frac{\hat{r} \cdot \hat{n}}{r^3} dS = \left( \hat{V}_0 + \hat{\omega} \times \hat{r}_0 - \hat{W}_f \right) \cdot \hat{n}(P)$$

where $\hat{V}_0$ and $\hat{\omega}$ - translational and angular velocity of MUO, $\hat{r}_0$ - radius vector, $\hat{n}$ - the unit vector of the outer normal to the surface $S$, $\hat{W}_f$ - speed caused by fins.

After solving it, the potential and speeds caused by the body can be found. If we solve this equation six times, separately for translational and rotational motion along each of the coordinate axes, then we can find the sought-for attached GAM using the following dependence

$$\lambda_{ij} = -\rho \int_S \frac{\partial \phi_j}{\nabla} dS$$

The stabilizers and rudders of the object are wings of small thickness and finite elongation, which makes it possible to replace them with thin plates of the same shape in the plan, and these plates are a system of triangular vortex frames with unknown circulation $\Gamma$ (Figure 4). Let the MUO have $N$ fin elements, each of which is modeled by $K_n$ vortex frames. The potential caused by the $n$-th element is determined by the summation of all the potentials of the frames of this surface, and the total tail potential is determined by the summation of the potentials of these surfaces.

![Fig. 3. To the statement of the problem of determining the GAM](image)

![Fig. 4. Triangular vortex frame](image)
In this case, the potential from each such frame is determined by the formulas written below [2]

\[ \phi_{vf}(P) = \frac{\Gamma}{4\pi} \Omega_f \text{sign} \mu; \quad \Phi_k = \sum_{m=1}^{N} \frac{\Gamma_m}{4\pi} \Omega_m \text{sign}(\mu_m), \]

where \( \Omega_f = 4\arctg \sqrt{\frac{\beta - \alpha_1}{2} \sqrt{\frac{\beta - \alpha_2}{2} \sqrt{\frac{\beta - \alpha_3}{2}}} - \text{spatial angle}, \quad \beta = \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{2}, \)

and the intensity is from the solution of a system of algebraic equations expressing the condition of impermeability at one control point of each triangular element:

\[ \sum_{n=1}^{N} \sum_{k=1}^{K_n} \Gamma_{nk} \cdot \tilde{w}_{BPnk}(K_{nk}, M_{lm}) \cdot \tilde{n}(M_{lm}) = \left[ \tilde{V}_f(M_{lm}) - \tilde{U}_{\text{body}}(M_{lm}) \right] \cdot \tilde{n}(M_{lm}) \]

We emphasize that the right side of this system includes the velocity \( \tilde{U}_{\text{body}} \) caused by the MUO body, and in the right part of the BIE recorded above, the speed from the plumage is included, which ensures a strict account of their mutual influence.

To approximate the surface of the MUO body when solving the BIE, its parametric continuous description was applied, which removed some restrictions on the shape of the surfaces under consideration. For the approximation along the coordinate lines, a cubic spline was used [3].

The main computational complexity of the problem posed is the solution of the BIE, in which the integrand function has a weak singularity, but the integral itself is convergent. The integral for calculating the velocity exists only in the sense of the principal value. To calculate them, a special method was proposed based on a change of variables with a symmetric arrangement of the integration nodes when using the Gauss method in the vicinity of a singular point. The equation itself is solved by the iterative method.

Based on the described method, an algorithm was developed and a computer program was compiled. A wide computational experiment was carried out in order to test them and determine the design parameters, for example, the size of the computational grid on the body.

Comparison of the obtained GAM values for ellipsoids of revolution with their exact analytical values showed a high degree of convergence of the results - the discrepancy did not exceed 1%.

Fig. 5. Comparison of the values of the transverse additional mass of the plate (on the left) and the attached moment of inertia obtained by the proposed method and other authors using approximate methods.

The Figure 5 shows a comparison of the obtained values of the GAM of plates of various elongations with the results of other authors. In this case, the discrepancy is more significant, but the
approximation of other models makes it possible to speak about the satisfactory accuracy of the proposed numerical method.

The method was also used to calculate the GAM of an object of real geometry. In this case, preliminary comparisons were made with the attached prototype masses, which showed satisfactory convergence. A significant discrepancy was noted only for the GAM $\lambda_{26}$, $\lambda_{34}$, and $\lambda_{35}$, which, in our opinion, is associated with very approximate estimates of these quantities in other methods.

The proposed method was extended to the case of joint motion of two or more bodies in a liquid. For two bodies, the total number of degrees of freedom of the system is 12, and the formula for calculating the kinetic energy of a liquid can be written as follows [1]

$$2T = \sum_{i=1}^{6} \sum_{j=1}^{6} v_{ij} \cdot v_{jk} \cdot A_{ij} + \sum_{i=1}^{6} \sum_{j=1}^{6} v_{ij} \cdot v_{jk} \cdot D_{ij} + 2 \sum_{i=1}^{6} \sum_{j=1}^{6} v_{ij} \cdot v_{jk} \cdot B_{ij}$$

where $v_{ij}$ and $v_{jk}$ – generalized speeds of motion of the first (k = 1) and second (k = 2) bodies, $A_{ij}$, $D_{ij}$ and $B_{ij}$ – generalized associated masses, the number of which in this case is 144, and the number of independent masses is 78.

These GAMs are also expressed in terms of unit potentials, for example $A_{ij} = -\rho \int S_1 \frac{\partial \phi_{ij}}{\partial n_1} dS$, where $S_1$ – first body surface.

In contrast to the case of motion of a single body in an infinite medium for two or more bodies, these masses are not constant, but depend on the relative position of interacting objects. To calculate the GAM in this case, it is required to perform the solution of the system of two BIEs 12 times for the motion of each of the bodies with one of six unit speeds.

A practical example of the use of the developed method for calculating the GAM for two bodies was the solution of the problem of separating the payload from the carrier. A torpedo was considered as a load (Figure 6). For this task, it is relevant to ensure the shockless output of the product.

![Fig. 6. Separation of the payload from the carrier](image)

In figure 6 on the right is a diagram of the considered movement for a number of positions of the load, and on the right - the values of some of its added masses. It can be seen that as the movement
proceeds, which is characterized by the dx coordinate (the length of the part of the load that has gone beyond the carrier), the GAM radically change their values.

When solving a number of problems, for example, when calculating the oscillation (vibrations) of the body of an object, it is also necessary to determine the GAM for calculating the kinetic energy of the fluid involved in motion by vibrations. An example of such a problem is the computational forecasting of the primary hydro-acoustic field of the MUO in the control systems of its physical fields.

And in this case, assuming the same assumptions about the absence of fluid viscosity and the potential nature of the induced flow, the described method can be used to determine the additional masses. To do this, it is necessary to solve the above BIE, but with a new form of the right side, which is still determined from the non-leakage condition. If the function of vibrations of the body surface is given, then the right-hand side of this equation will have the form \( F = \frac{\partial Z}{\partial t} \), where \( Z(t) \) – displacement of the body surface along the normal to it. For elastic vibrations, it is expedient to set this function in the form \( Z(s,t) = \Psi(s) \cdot \tau(t) \), where \( \Psi(s) \) – a function that characterizes the change in displacements along the surface, \( \tau(t) \) – function describing the dependence of displacement on time. This method is an alternative to the currently used, which imposes significant restrictions on the geometry of the body. As a rule, they consider individual sections of it (flat plates, cylindrical shells of finite or infinite length, etc.), and the vibrations are considered small.

**Conclusion**

The developed method of numerical prediction of generalized additional masses is universal and makes it possible to calculate them for MUOs of real geometry, equipped with plumage, for systems of bodies performing joint maneuvering, as well as for elastic vibrations of the body. Test calculations have shown a sufficient degree of agreement between the obtained GAM values with the data of other authors.

The economical algorithm of the method allows performing calculations on personal computers in a short time, which ensures its practical application at various stages of design for an operational comparison of various design solutions, as well as a part of simulators and on-board complexes for modeling the dynamics of MUO and ensuring their secrecy by physical fields.

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