CONTACT DISCONTINUITIES IN MODELS OF CONTACT BINARIES UNDERGOING THERMAL RELAXATION OSCILLATION

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ABSTRACT

In this paper we pursue the suggestion by Shu, Lubow, & Anderson and Wang that contact discontinuity (DSC) may exist in the secondary in the expansion thermal relaxation oscillation (TRO) state. It is demonstrated that there is a mass exchange instability in some range of mass ratio for the two components. We show that the assumption of constant volume of the secondary should be relaxed in DSC model. For all mass ratios, the secondary always satisfies the condition that no mass flow returns to the primary through the inner Lagrangian point. The secondary will expand to equilibrate the interaction between the common convective envelope and the secondary. The DSC in contact binary undergoing thermal relaxation does not violate the second law of thermodynamics. The maintaining condition of DSC is derived in the time-dependent model. It is desired to improve the TRO model with the advanced DSC layer in future detailed calculations.

Key word: binaries: close

1. INTRODUCTION

W Ursae Majoris (W UMa) binary stars were first thought to be a particular binary population because of their abnormal mass-radius relationship, namely, the so-called Kuiper paradox, \( R_2/R_1 = (M_2/M_1)^{0.46} \) (Kuiper 1941). These particular binaries appear to consist of two main-sequence stars that possess photospheres exhibiting the almost same effective temperatures for the two components despite the fact that typical mass ratio in a system is 0.5. It was originally proposed that a common convective envelope (CCE) may be formed because of dynamic equilibrium (Osaki 1965), and mass and energy transfers would take place in the CCE to interpret the Kuiper paradox (Lucy 1968), although the specific mechanism of energy transfer for the circulation has not been fully understood (Robertson 1980; Sinjab, Robertson, & Smith 1990; Tassoul 1992). It is now firmly believed that W UMa stars are contact binaries in which both components have full Roche lobes, showing strong interactions (Mochancki 1981). Lucy's (1968) isentrope model as a zeroth-order one with thermal equilibrium, however, cannot explain the color-period diagram by Eggen (1967), which leads to the establishment of two parallel first-order theories of thermal relaxation oscillation (TRO) and contact discontinuity (DSC). The TRO model was advanced by Lucy (1976), Flannery (1976), and Robertson & Eggleton (1977), who suggest that a contact system cannot reach thermal equilibrium at a dynamical equilibrium configuration and may thus undergo TRO. DSC theory was proposed by Shu et al. (Shu, Lubow & Anderson 1976, 1979; Lubow & Shu 1977; also see Biermann & Thomas 1972; Vilhu 1973 for some earlier elements of the DSC model), who hypothesize that contact binaries can attain thermal and dynamical equilibrium but that there is a temperature inversion layer in the secondary. With great effort, the so-called Kuiper paradox and period-color diagram may be resolved by the two different hypotheses independently. However, both theories have some difficulties in explaining observations, such as the so-called W phenomenon, i.e., the smaller star is hotter than the massive component (Binnendijk 1970; see a concise summary of observations and theory by Smith 1984). There are especially great debates between the two in the nature in their simplest version (Kähler 1989). Observational studies continue and the theoretical controversy remains (Ruciński 1997). These imply that the first-order theories of contact binaries (i.e., TRO and DSC) should be improved.

The intensive disputes by the two contending schools (Lucy & Wilson 1979; Shu et al. 1979) lead to an intriguing suggestion by Shu et al. (1979), Shu (1980; from a theoretical viewpoint), and Wang (1995; from the analysis of observational data) that the TRO theory needs the DSC in some phases. Some observations seem to support TRO theory (Lucy & Wilson 1979; Hilditch, King, & Mcfarlance 1989; Samec et al. 1998). Although some criticisms about the DSC model exist (Shu, Lubow, & Anderson 1980), this theory is still attractive because it is successful in many aspects (Smith 1984). The DSC may be ironed out within the thermal timescale (Webbink 1977; Hazlehurst & Resfdal 1978; Papaloizou & Pringle 1979; Smith, Robertson, & Smith 1980) in a steady state; however, the existence of time-dependent DSC cannot be excluded because it does not violate the second law of thermodynamics (Papaloizou & Pringle 1979). However, a detailed analysis is needed for this. It is highly desirable to reconcile the two theories not only for removing the discrepancies but also for explaining more detailed observations (Shu 1980).

The difficulties of the pure TRO and DSC models in the explanation of the W-phenomenon motivate us to explore the possibility of developing a second-order theory. The interaction between the secondary and CCE is thought to be an important role in the W-phenomenon, Wang (1994) found that the W-phenomenon can be explained by the released gravitational energy of the secondary through its contraction corresponding to the TRO contracting phase in W-type contact systems. This is encouraging and leads to
the suggestion by Wang (1995) from a sample with 32 contact systems that the A-type systems may undergo TRO with DSC, whereas the contraction of secondary in W-type systems irons out this DSC.

The overriding virtue of a DSC is that it gives a clear mechanism for making the secondary physically larger, and the primary physically smaller, than their main-sequence single-star counterparts, as is needed to satisfy the Roche lobe filling requirements of the Kuiper paradox (Lubow & Shu 1977). On the other hand, if the system cannot be maintained in steady state by heat-carrying flows, then the capping of the radiative heat flow from the secondary by the hotter overlying common envelope should lead to an expansion of the secondary, with a resulting transfer of mass from the secondary to the primary. Such Roche lobe overflow, from a less massive star to a more massive one, is known to occur slowly, so the ultimate breaking of contact caused by the expansion of the binary orbit takes a relatively long time. Once contact has been broken, however, the common envelope disappears, the secondary is no longer capped, and it will begin to shrink toward its normal single-star size. Conversely, because the larger area of the common envelope is no longer available to carry away much of the primary’s interior luminosity, the primary can no longer be maintained at its suppressed contact size, and it will begin to overflow its Roche lobe. The transfer of mass from a more massive star to a less massive one is known to be unstable (e.g., Paczyński 1971), and the rapid shrinkage of the binary orbit causes the system to come into contact again. The reestablishment of the common envelope and the capping of the secondary results in its refilling its Roche lobe. Thus would the DSC hypothesis provide the physical mechanism for the TRO hypothesis, together with a justification why the duty cycle is long for the contact phase and short for the semidetached phase, as is required by the observational statistics. The rest of this paper attempts to establish the above ideas on a more quantitative basis.

This paper is organized as following: the instability of the Roche lobe and its operation in contact system are found in § 2, the surviving condition of DSC layer is derived from the thermodynamics in § 3, and the conclusions are drawn in § 4.

2. THE ROCHE LOBE INSTABILITY AND THE DSC MODEL

It is generally believed that the two components of W UMa stars share an optically thick CCE because of the dynamics equilibrium (Osaki 1965; Mochnacki 1981). The redistribution of the total luminosities (luminosities of each star added together), which takes place in CCE, deals with comprehensive fluid processes (Lucy 1968). Why and how to redistribute the luminosities is the main task of theoreticians. The debate of the existing theories of contact binaries attracted much consideration in the 1970s and 1980s (Lucy & Wilson 1979; Shu 1980; Shu & Lubow 1981). Shu (1980) clearly stated that the two superficially distinct theories are complementary, with the crucial theoretical issue to be resolved being the secular stability of the temperature inversion layer in his thought-provoking analysis. Here we argue that the DSC layer is a natural result of the TRO theory via the mechanism of Roche lobe instability, showing the presence of the DSC layer during the expansion TRO phase.

In the following discussions we assume that the total mass and angular momentum are conserved, neglecting the spin angular momentum of two components. These assumptions are basic and the same in TRO theory, but they are unnecessary in the DSC theory. In principle, the two assumptions put stronger constraints on the theoretical model. In the conserved systems there are mainly two other parameters: mass ratio \( q \), and the mass ratio changing rate due to mass exchange \( \dot{q} \), to determine the structure of the contact binaries in the TRO theory. The most serious shortcoming of TRO (mentioned in the previous section) is a strong indicator that we should relax some of assumptions in the TRO model. One possible way to remove this shortcoming is to supplement the interaction between CCE and the component. This inclusion may reconcile the two contending schools to each other (Wang 1995).

We first show that the instability of mass exchange may prevent the mass in the secondary from being pushed into the primary through the inner Lagrangian point \( L_1 \), because of the lid effects of CCE placed on the secondary (Shu et al. 1976). For a contact system with total angular momentum \( \mathcal{J} \) in a circular orbit and total mass \( M = M_1 + M_2 \), the separation between components reads

\[
\mathcal{D} = \left( \frac{\mathcal{J}^2}{G M} \right) \left( \frac{1 + q}{q} \right)^{1/3},
\]

where \( q = M_2/M_1 \) (for the convenience we take \( q \leq 1 \)) and \( G \) is the gravitation constant. The Roche lobe radius \( R_L \) of the secondary approximates for all mass ratios (Eggleton 1983),

\[
r_L = \frac{R_L}{\mathcal{D}} = \frac{0.49}{0.49 + q^{2/3} \ln (1 + q^{-1/3})}.
\]

The Roche lobe of the primary will be obtained when we replace \( q \) with \( 1/q \). It is important to note that the Roche lobe is changing because of the mass transfer between the two components. The variation rate of the Roche lobe due to mass transfer between the two components reads

\[
d \ln R_L \over dq = \frac{2r_L}{3q^{1/3}} \left[ \frac{1}{1 + q^{1/3}} - 2 \ln (1 + q^{-1/3}) \right] + \frac{2(q - 1)}{q(1 + q)},
\]

and then we have the timescale for this change with the help of \( d \ln R_L/dt = (d \ln R_L/dq)(dq/dt) \),

\[
t_{RL} = \left( \frac{d \ln R_L}{dt} \right)^{-1} = f(q)t_M,
\]

where \( t_M \) is the timescale of mass transfer defined as

\[
t_M = \frac{M_1}{\dot{M}_1}.
\]

Here the parameter \( \dot{M}_1 \) is the rate of mass transfer, and the function \( f(q) \) is

\[
f(q) = \left\{ \begin{array}{ll}
2(1/q - q) & \text{for } q < 1 \\
2r_L/3q^{1/3} & \text{for } q > 1
\end{array} \right.
\]

\[
\times \left[ \frac{1}{1 + q^{1/3}} - 2 \ln (1 + q^{-1/3}) \right] (1 + q)^{-1},
\]

which represents the ratio of the two timescales. We have calculated the function \( f(q) \) in Figure 1, showing its value for the range of \( q \) from 0.0 to 1.0. If \( f(q) > 0 \), then the Roche lobe will expand with increasing \( q \) or shrink with decreasing
of gaining mass from the primary until the mass ratio.
The expansion timescale is shorter than that
the Roche lobe of the secondary expands with the increas-
ment that the Roche lobe shrinks more rapidly than the
mass transfer is shorter than that of mass transfer. This
implies that the timescale of Roche lobe shrinkage due to
interaction with CCE; and

\[ t_{RL} \approx 3q t_M \]

(8)

We draw the line \( t_{RL} = 3q t_M \) in Figure 1. It is obvious that
the value of \( f(q) \) is always less than 3. This means that all
the cases satisfy the condition that no mass flow returns to
the primary even beyond the mass-exchange instability.
Therefore, the assumption of a constant volume of the
secondary should be relaxed in the advanced DSC model.
The DSC is time-dependent from this viewpoint at least,
coinciding with the theory that the DSC layer could be main-
tained in a time-dependent model (Papaloizou & Pringle
1979).

3. THERMODYNAMICS OF THE DSC LAYER

By defining the thermal timescale as
\[ t_{Th} = \frac{M}{\dot{m} (4\pi r^2 \rho v_r)^{-1}} \]
Webbink (1977) first showed that the
thermal diffusion timescale in the common envelope is typi-
cally of the same order as the dynamical timescale (roughly
one orbital period). This makes the DSC disappear within
one orbital period. We call the thermal diffusive process as
interaction \( \epsilon \). Papaloizou & Pringle (1979) show that the
steady DSC violates the second law of thermodynamics, but
the time-dependent DSC may exist. However, in the time-
dependent model it is the interaction \( \epsilon \) that keeps the DSC
in the contact system undergoing TRO. The controversy of
inner structure may be removed by this kind of interaction
(Wang 1995). With the help of the conservation of mass and
momentum, we can rewrite the energy equation beyond the
energy generation region as

\[ \rho \frac{\partial}{\partial t} (\Psi + Ts) = -\rho T \nabla \cdot V S - V \cdot F + \epsilon \]

(9)

for the inviscid fluid (e.g., Webbink 1977), where \( t \) is time; \( \rho \),
the density; \( T \), the temperature; \( v \), the velocity; \( s \), the
specific entropy; \( F \), the energy flux radiated from the star; \( \epsilon \),
the energy density absorbed by the secondary in the unit time
because of the interaction with CCE; and \( \Psi \), the gravita-
tional energy per unit mass. Following the assumption by
Shu et al. (1980) that the specific entropy \( s \) can be decom-
poped in terms of a barotropic and a baroclinic one as
\( s = s_0(\Psi) + s_1(x, t) \), we integrate the above equation over
the volume enclosed by the equipotential surfaces \( C \) and \( D \)
and obtain

\[ \frac{dS}{dt} = s_0(\Psi) \frac{d(AM)}{dt} - \int_{CD} \rho \frac{\partial \Psi}{\partial t} dV + \frac{\epsilon}{T} \Delta V \\
- \int_{CD} \rho s_1(x, t) \nabla \cdot dA - \int_{CD} \frac{1}{T} V \cdot F dV \]

(10)
where $S = \iiint \rho sdV$, $\Delta M = \iiint \rho dV$, $\Delta V = \iiint dV$ is the volume enclosed by the two surfaces $C$ and $D$, $dA$ is the area of the surface of DSC, and $n$ is its normal vector. For the time-dependent case, we assume that the last two terms offset approximately as in the steady case (Shu et al. 1980), thus we have a more physically concise form of equation (10),

$$\frac{dS}{dt} = s_0(\Psi_d) \frac{d(\Delta M)}{dt} - \iiint_{CD} \rho \frac{\partial \Psi}{\partial t} dV + \frac{\epsilon}{T} \Delta V. \quad (11)$$

The first term of the right-hand side of equation (11) represents the entropy increase from mass exchange between the CCE and the secondary. The last term has the same meaning, but because of energy exchange, the second term contains the entropy decrease due to the Roche lobe expansion. The enclosed volume is an open system undergoing mass and energy exchanges with its surroundings rather than an isolated volume. This equation also tells us the resulting expansion because of the interaction $\epsilon$ if the DSC layer survives. (1) If there is only an exchange of energy by thermal diffusion, namely, $\Delta M = \text{constant}$, we have

$$\iiint_{CD} \rho \frac{\partial \Psi}{\partial t} dV \geq \epsilon \Delta V. \quad (12)$$

This clearly states that the surviving of DSC must be provided by the expansion. Detailed calculations should be done in the future. (2) There is a mass exchange between the CCE and the secondary accompanying the energy interaction, i.e., $\frac{d(\Delta M)}{dt} > 0$, so the expansion is at least

$$\iiint_{CD} \rho \frac{\partial \Psi}{\partial t} dV \geq \epsilon \Delta V + s_0(\Psi_d)T \frac{d(\Delta M)}{dt}. \quad (13)$$

This equation predicts the secular change of orbital period due to the shift of mass ratio. (3) If the secondary keeps constant volume, as originally suggested by Shu et al. (1976) and the term $\frac{d\Psi}{dt} = 0$, we always have $\frac{dS}{dt} > 0$, which means that the discontinuity will be ironed out with the thermal diffusive timescale. The only possible way to relax this condition is the inclusion of a change in the Roche lobe. This will permit us to unify the twocontending hypotheses. According to the simplest version of star structure, equations (12) and (13) will provide the expansion velocity

$$v_{\text{int}} \geq R_2 \left( \frac{\epsilon (\Delta V)}{V_2} + T s_0(\Psi_d) g_2^{-1} \frac{d(\ln \Delta M)}{dt} \right), \quad (14)$$

where $\epsilon = (GM_2 \Delta M/R_2)/V_2$ is the mean density of the gravitational energy between the secondary and the CCE, $g_2 = GM_2/R_2^2$ the gravitational acceleration, and $V_2$ is the volume of the secondary. One should note that in the above estimation we neglect the down-directed propagation of energy to the interior with a dynamical timescale. Therefore, the present estimation is somewhat higher than the actual. Both the mass exchange and energy interaction result in the expansion of the secondary; we thus have the minimum velocity

$$V = \max (v_{\text{int}}, v_{DSC}), \quad (15)$$

where $v_{DSC} = dR_1/dt$ is the expanding velocity of the Roche lobe. This is the condition maintaining DSC. From the viewpoint of total energy (by the nuclear) conservation, the exhausted energy (i.e., $\epsilon$) to expand the secondary lowers the reradiating efficiency of the transferred energy from the primary. The lower temperature of the secondary than the primary may be an indicator of the presence of DSC.

4. Conclusions and Discussion

Introducing a discontinuity of temperature by Shu et al. (1976, 1979), the thermal instability of a binary (Lucy 1976; Flannery 1976) can be suppressed, but its maintenance of the DSC layer opens. In this paper we try to construct the physical scenario of a time-dependent model of contact binary. Only two assumptions, that total mass and angular momentum of the contact system are conserved, are employed in this paper. It is found that the mass exchange results in the instability of Roche lobe in some ranges of mass ratio. We show that this instability always satisfies the condition that keeps the mean density of the secondary $d \ln \rho/dt \leq 0$. Therefore, it ensures that no mass flow returns to the primary through the inner Lagrangain point $L_1$. The second-order theory predicts that the contact binary may have oscillations that take place about a state with a DSC. The temperature differences of DSC layers across the interface are determined by the expansion velocity.

The existing TRO and DSC theories (Lucy 1976; Flannery 1976; Robertson & Eggleton 1977; Shu et al. 1976, 1979) neglect the effects of interaction $\epsilon$ between the CCE and the secondary. Here we argue that it is the DSC layer that results in the interaction between the CCE and the secondary, and meanwhile, it is the interaction that maintains the DSC. We find that this interaction $\epsilon$ can result in some interesting issues. First, it is the reason why the temperature of the secondary in A types is lower than that of the primary. Second, the maintenance of DSC needs faster mass transfer, which breaks down the deep contact. Thus, the shortcomings of the TRO theory are removed. It is highly desired that the unification of the TRO theory and the DSC model should be calculated to discover the nature of the contact binaries.

In the present work we do not specify the mechanism of thermal diffusion process. Although we have not performed the time-dependent model unifying the TRO and DSC hypotheses, this time-dependent unified theory might give some predictions. First, the secular behavior of the period change of A-type systems is more violent than that in W-type systems, to survive the existence of DSC. Second, the maintenance of DSC may lead to the radial oscillation of the secondary with period from a few to several 10 minutes. The interaction between the CCE and the secondary drives such an oscillation, similar to the $\kappa$-mechanism working in other types of stars. It is thus expected to find the light variation during the primary eclipse as another probe of DSC in A-type systems.

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