The gravitational dynamics of the primordial black holes cluster

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Abstract. We consider the numerical simulation of the $N$-body problem related to the primordial black holes (PBHs) cluster. To study the dynamics of the system we use modified NBODY6++ code aimed at the simulation of globular star clusters. Some qualitative results describing the evolution of the system are discussed. It is shown that the cluster successfully survives to the redshift $z \sim 20 \div 40$ (despite evaporation and coalescence of the PBHs). Starting from this moment an accretion begins to play significant role in speeding up the mass growth of the massive bodies.

Introduction

There are a number of the models describing the primordial black holes (PBH) birth in the early Universe [1,2]. In the current work we have focused on the models leading to the formation of a PBH cluster (e.g. gravitational collapse of closed domain walls [3]). These models may be related to a number of unsolved problems of cosmology and astrophysics [4,5]:

- existence of the supermassive black holes in galactic centres and the early quasars at $z > 6$ [6,9];
- binary BH mergers with masses $\gtrsim 10 M_\odot$ and low initial spins detected by LIGO/Virgo [10-13];
- dark matter nature [4,14];
- reionization problem [15-18];
- nature of the unidentified gamma-ray sources [19,20].

So wide problem coverage by the model emphasizes the necessity of a more detail study of the predicted clusters evolution. So far we have not satisfactory kinetic estimations of the cluster dynamics over time. Hence we should use the numerical simulation for our purposes.

$N$-body numerical simulation

The study of the motion of $N$ point-like masses with the initial positions and velocities $r_{i0}, v_{i0}$, $i = 1, 2, \ldots, N$ interacting through a pairwise force, is known as the $N$-body problem. The direct computation of $N \times 1$ pairwise force contributions yields to a $O(N^2)$ computational complexity. There are other well-known methods that allow reducing the computational complexity but they are designed for collisionless systems only. In our case the merging of
BH binaries play an important role, hence we have to use a direct integration codes. We have used NBODY6++ [21] code describing the evolution of globular clusters. It already contains many useful and accurate methods suitable in cases of both stars and BHs (which remains true while we treat them as point-like masses).

We have made some changes of the code to describe the behaviour of black holes correctly: all bodies have been deprived of their stellar characteristics (luminosity, types of star, metallicity, evolution ability, etc.), the merging mechanism has been added: the merging of two BHs takes place if the distance between them
\[ r_{ij} \leq \kappa (r_{gi} + r_{gj}), \]
where we put the parameter \( \kappa = 3 \). We define the objects’ «sizes» according to its masses
\[ r_g = 2GM/c^2, \]
where \( r_g \) is the gravitational radius).

Results

We made a set of simulations to trace the cluster gravitational evolution. The typical cluster parameters used in simulations are listed below:

- the number of PBHs \( N = 10^4 \);
- the simulation size \( R = 1 \) pc;
- the mass distribution \( \frac{dN}{dM} \propto \frac{1}{M^\alpha} \left( \frac{M}{M_\odot} \right)^{-\alpha} \);  
- the spatial distribution \( \rho(r) \propto r^{-\beta} \);
- the initial velocities were chosen according to the Maxwell distribution with the virial velocity \( v_0 \sim 1 \) km/s. The most massive black hole has zero initial velocity.

It is obtained that the central regions of the clusters can be approximated good enough by the parameters values \( \alpha = 2.4 \) and \( \beta = 2 \).

We limited the evolution time by 50 Myr. It is enough to track the most intensive cluster dynamics (see Fig. 1(b)). Moreover by this time an accretion on the massive BHs, which is out of the scope of the current code, is still negligible.

The results are shown in Fig. 1. One can see that a number of particles has been reduced by \( \sim 15\% \) due to evaporation and merging, the decrease of the cluster mass is the same. The final mass of the cluster is \( \sim 10^3 M_\odot \). The mass spectral index \( \alpha \) hasn’t noticeably changed. As it was said before, at the very beginning the number of bodies in the cluster changes rapidly. But...
at a later time $N$ changes much slower. So the cluster remains a gravitationally bound system by the moment $z \sim 20 \div 40$, when the accretion starts affecting considerably the growth of the most massive black holes \[22\].

**Conclusion**

We present the first efforts to trace the gravitational evolution of the PBHs cluster predicted by the model \[3\]. It is shown that the cluster reaches the moment $z \sim 20 \div 40$ without essential changes in the mass spectrum.

Note, that the clusters could be a «seeds» of early quasars. The quasars formation is possible in the case if their «seeds» had already existed with masses $10^{3-5} M_{\odot}$ by the moment $z \sim 20 \div 40$. For example, such mass is required to get the quasar with $M \sim 10^9 M_{\odot}$ observed at the redshift $z = 7.54$ \[22\].

**Acknowledgement**

The authors are grateful to K. M. Belotsky and S. G. Rubin for a lot of useful and fruitful discussions. The work was supported by the Ministry of Education and Science of the Russian Federation, MEPhI Academic Excellence Project (contract № 02.a03.21.0005, 27.08.2013). The work of A.A.K. is funded by the Russian Foundation for Basic Research, Project № 19-02-00930.

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