Fractal patterns in turbulent flow for laden particles

M Farhan1, F C G A Nicolleau1, A F Nowakowski1 and J-R. Angilella2
1Sheffield Fluid Mechanics Group - Mechanical Engineering, University of Sheffield, United Kingdom,
2Laboratoire Environnement, Géomécanique et Ouvrages, Nancy-Université, Vandoeuvre-les-Nancy, France
E-mail: m.farhan@sheffield.ac.uk

Abstract. We use Kinematic Simulation as a particular kind of synthetic turbulence model to study the preferential accumulation of laden particles with inertia and gravity. Particles are released as a uniform cloud in the periodic simulation box. We allow particles to settle in synthetic flow and after some times particles concentrate in a particular sub-domain. We study the dimensional properties of these attractors as functions of drift parameter and Stokes number. The attractor’s topology varies from curve(D=1) to fractal plane.

1. Synthetic turbulence and Kinematic Simulation models

Synthetic turbulence models can be regarded as any technique where the Eulerian velocity field is postulated (or synthesized). Historically such approach was limited to Lagrangian prediction though at present Eulerian applications such as initial condition field or subgrid model can be found [1]. Kinematic Simulation (KS) is probably the most popular of such models. In this case the predetermined Eulerian field \( u(x,t) \) based on [2] for incompressible isotropic turbulence is reduced to a truncated Fourier series, sum of \( N_k \) random Fourier modes:

\[
    u(x,t) = \sum_{n=0}^{N_k} a_n \cos(k_n \cdot x + \omega_n t) + b_n \sin(k_n \cdot x + \omega_n t)
\]

where \( N_k \) is the total number of modes included, \( a_n \) and \( b_n \) are decomposition coefficients corresponding to the wave vector \( k_n \), and \( \omega_n \) is the unsteadiness frequency.

For the present study we choose a periodic box. Rather than random values of wave numbers, the flow has an arithmetic distribution of Fourier modes and wave numbers. In this paper, We use the turbulence model with constant energy spectra and the unsteadiness parameter is set to zero.

2. Heavy particles

Heavy particles have significant effects on the turbulence. Concentration of inertial particles may change the process all together. Equations of motion of heavy particles have been discussed in
previous literature [3, 4] and may contain terms other than gravity and inertia. In this paper, we consider the simplified equation of motion for the heavy particle that can be written as:

$$\frac{dV}{dt} = \frac{1}{\tau_a} (u - V + V_d)$$

(2)

where $\tau_a$ is the aerodynamics response time and

$$V_d = \tau_a g$$

(3)

the Stokes terminal fall velocity in still fluid or particle drift velocity. For the sake of convenience we introduce two non-dimensional parameters, the drift parameter defined as the ratio of the particle’s drift velocity to the turbulence velocity fluctuation rms value $u'$:

$$\gamma = \frac{V_d}{u'} = \frac{\tau_a g}{u'}$$

(4)

and the Stokes number defined as the ratio of the particle’s inertial time to the turbulence characteristic time:

$$St = \frac{\tau_a u'}{L}$$

(5)

where $L$ is the turbulence integral length-scale.

3. Methodology

Initially, the uniform cloud of particles is released at $t = 0$ in the box under the action of gravity. It is then let to evolve in the KS periodic box, when a particle leaves the box it is re-injected in consistency with the periodic KS field. Depending upon the Stokes number and drift parameter, particles settle in particular domains in the flow. We analyse the coalescence of particles with different drift parameters $\gamma$ and the Stokes number $St$ constant. One of the important factors is time given to particles to be settled in the flow. We see that particles accumulated in preferential regions after a certain time. The clustering effects are important in practical applications like in hydrocyclone [5].

Figure 1. Evolution of the particles cloud for $St = 0.413$ (a): $\gamma = 0.575$, and (b): $\gamma = 0.621$. 
4. Results

We study the shape of the particles accumulation for different pairs of \((St, \gamma)\) in order to characterise the coalescence of the heavy particles. We analyse two sets of data. First set exhibits the variation in pattern of laden particles with change in \(\gamma\) and \(St\) is kept constant. Figures 1 and 2 show how the particles attractor changes from a linear to a layered one with different value of \(\gamma\).

\[\text{Figure 2. Evolution of the particles cloud for } St = 0.413 \ (a): \ \gamma = 0.804, \ (b): \ \gamma = 5.754.\]

In Figure 1 (a) the cloud evolves to a one-dimensional structure. As the value \(\gamma\) is increased the structure changes to thickening one as shown in Figure 1 (b). Further increase in \(\gamma\) with the same \(St\) disturbs pattern into no clear shape Figure 2 (a). For high value of \(\gamma = 5.745\) with same Stokes number \(St = 0.413\), pattern of particles accumulation will shape into layered structure Figure 2 (b).

In the second case we analyse the particles preferential coalescence by increasing both non dimensional parameters \(St\) and \(\gamma\). We note that for relatively small values of \(St\) and \(\gamma\), the particles attractor remains one-dimensional Figure 3 (a). As \(St\) and \(\gamma\) are increased, the structure is changed from no clear to layered one Figure 3 (b) and Figure 4 (a). For very high values of \(St = 41.331\) and \(\gamma = 57.455\), it is again not a clear structure Figure 4 (b).

\[\text{Figure 3. Asymptotic location of the particles cloud (a): for } St = 0.207 \text{ and } \gamma = 0.287, \ (b): \text{ for } St = 0.455 \text{ and } \gamma = 0.632.\]
Figure 4. Evolution of the particles cloud (a): for $St = 8.266$ and $\gamma = 11.491$, (b): for $St = 41.331$ and $\gamma = 57.455$.

5. Conclusion

On the basis of above results, we can conclude that for relatively low values of drift parameter and Stokes number, particles accumulation pattern is one dimensional and it changes to thickening structure. For few pairs of $St$ and $\gamma$, the structure is not very clear. It is also observed that for moderate values we have a layered pattern which is likely to be a fractal plane.

In this paper we limit the flow field with the unsteadiness parameter set to zero and we choose the Kolmogrov energy spectra. Further work is being carried out and necessary for different energy spectra and some values of unsteadiness parameter using kinematic simulation.

References

[1] Nicolleau F, Cambon C, Redondo J M, Vassilicos J, Reeks M and Nowakowski A (eds) 2011 New approaches in modelling multiphase flows and dispersion in turbulence, fractal methods and synthetic turbulence (Ercoftac Series vol (In Press)) (Springer Science)
[2] Fung J, Hunt J, Malik N and Perkins R 1992 J. Fluid Mech. 236 281–317
[3] Martin R Maxey and James J. Riley 1983 Phys. Fluids Vol. 26 No.4
[4] George Haller and Themistoklis Sapsis 2007 Physica D 237 (2008) 573-583
[5] Kraipech, W., Chen, W., Dyakowski, T and Nowakowski, A. The performance of the empirical models on industrial hydrocyclone design 2006 International journal of Mineral Processing 80(2-4)pp. 100-115