Spinodal Enhancement of Light Nuclei Yield Ratio in Relativistic Heavy-Ion Collisions

KaiJia Sun (孙开佳)
kjsun@fudan.edu
Institute of Modern Physics
Fudan University, China

Ref.: K. J. Sun, W. H. Zhou, L. W. Chen, C. M. Ko, and F. Li, R. Wang, and J. Xu, arXiv:2205.11010(2022)
1. Motivation: Why light nuclei? Why $N_t N_p / N_d^2$ ($tp/d^2$)?

2. Spinodal enhancement of $tp/d^2$ from the first-order QCD phase transition

3. Summary and Outlook
1.1 QCD phase transition & light nuclei production

Critical Point: long-range correlation
First-order Phase Transition: Spinodal instability

X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017) A. Bzdak et al., Phys. Rept. 853, 1 (2020);
W. J. Fu, J. M. Pawlowski, F. Renneke, Phys. Rev. D101, 054032 (2020); LIGO & VIRGO, Phys. Rev. Lett. 119, 161101 (2017)
1.2 1st order QCD phase transition & light nuclei production (2)

P. Chomaz, M. Clonna, and J. Randrup, Phys. Rep. 389, 263 (2004)
Phase separation, spinodal decomposition (SD)

Phase separation, spinodal decomposition (SD)

Spinodal: \( \frac{\partial P}{\partial \rho_B} < 0 \)

\( \rho_B \)

Metastable

Density matrix formulation (coalescence)

\[
N_d \propto Tr[\hat{\rho}_s \hat{\rho}_d] \quad N_t \propto Tr[\hat{\rho}_s \hat{\rho}_t]
\]

\[
\frac{N_t N_p}{N_d^2} \propto \Delta \rho_n
\]

\( \Delta \rho_n = \frac{\int dx (\delta \rho(x))^2}{\int dx \rho(x)^2} \)
characterizes density inhomogeneity

See [PLB 816, 136258 (2021)] for critical effects on \( N_t N_p / N_d^2 \)
### 1.3 QCD phase transition & light nuclei production

#### 2012 - 2022

| Year | Other works | Our works |
|------|-------------|------------|
| 2012 | -           | -          |
| 2014 | -           | -          |
| 2015 | -           | -          |
| 2016 | -           | -          |
| 2017 | -           | -          |
| 2018 | -           | -          |
| 2019 | -           | -          |
| 2020 | -           | -          |
| 2021 | -           | -          |
| 2022 | -           | -          |

**First-order phase transition & composite particle production**

- J. Steinheimer et al. PRC 87, 054903 (2013)
- PRL 109, 212301 (2012) (Hydrodynamics)
- JHEP 12, 122 (2019) (Machine learning)

**Baryon clustering near the critical point**

- E. Shuryak, J.M. Torres-Rincon et al., PRC 100, 024903 (2019)
- PRC 101, 034914 (2020)
- EPJA 56, 241 (2020)
- PRC 104, 024908 (2021)

**Background effects**

- S. Wu et al., PRC 106, 034905 (2022)

**Probing QCD phase transition with light nuclei production**

- PLB 774, 103 (2017)
- K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu
  - \[ N_t N_p \approx \frac{1}{2\sqrt{3}} [1 + \Delta \rho_\pi] \]
- PLB 781, 499 (2018)
- PLB 816, 136258 (2021) (criticality)

**1st-order QCD phase transition**

- PRD 103, 014006 (2021)
- EPJA 57, 313 (2021) (Transport)
- arxiv:2205.11010 (Transport) (First-order PT in BES)
2. Spinodal enhancement of $tp/d^2$ from the first-order QCD phase transition
2.1 Equation of State (extended NJL model)

The eNJL provides a flexible equation of state (EoS) . The critical temperature can be easily changed by varying the strength of the scalar-vector interaction without affecting the vacuum properties.

Lagrangian density for eNJL

\[
\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - \hat{m})\psi + G_S \sum_{a=0} \left[ (\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\lambda^a\psi)^2 \right] \\
- K\left\{ \text{det}[\bar{\psi}(1 + \gamma_5)\psi] + \text{det}[\bar{\psi}(1 - \gamma_5)\psi] \right\} \\
+ G_{SV} \left\{ \sum_{a=1}^3 \left[ (\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\lambda^a\psi)^2 \right] \right\} \\
\times \left\{ \sum_{a=1}^3 \left[ (\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\lambda^a\psi)^2 \right] \right\},
\]

| $\Lambda$ [MeV] | 602.3 | $M_{u,d}$ [MeV] | 367.7 |
|----------------|-------|----------------|-------|
| $G\Lambda^2$ | 1.835 | $M_s$ [MeV]     | 549.5 |
| $K\Lambda^6$ | 12.36 | $\langle \bar{u}u \rangle^{1/3}$ [MeV] | -241.9 |
| $m_{u,d}$ [MeV] | 5.5 | $\langle \bar{s}s \rangle^{1/3}$ [MeV] | -257.7 |
| $m_s$ [MeV] | 140.7 | | |
2.2 Box Simulation

Effective mass:

\[
\begin{align*}
M_u &= m_u - 4G_S \phi_u + 2K \phi_d \phi_s \\
-2G_{SV} (\rho_u + \rho_d)^2 (\phi_u + \phi_d), \\
M_d &= m_d - 4G_S \phi_d + 2K \phi_u \phi_s \\
-2G_{SV} (\rho_u + \rho_d)^2 (\phi_u + \phi_d), \\
M_s &= m_s - 4G_S \phi_s + 2K \phi_u \phi_d
\end{align*}
\]

\[
\phi_i = -2N_c \int_0^\Lambda \frac{d^3 p}{(2\pi \hbar)^3} \frac{M_i}{E_i} (1 - f_i - \bar{f}_i)
\]

\[
\rho_i = 2N_c \int_0^\Lambda \frac{d^3 p}{(2\pi \hbar)^3} (f_i - \bar{f}_i)
\]

Test -particle method: J. Xu, arXiv:1904.00131 (2019)

\[
\frac{d\mathbf{r}}{dt} = \mathbf{v},
\]

\[
\frac{d\mathbf{p}}{dt} = -\frac{M}{E^*} \nabla_x M \pm \mathbf{E} \pm \mathbf{v} \times \mathbf{B}
\]

M. Buballa, Phys. Rept. 407, 205 (2005)
K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)
2.3 Relativistic Heavy-Ion Collisions

- **Initialization**
- **Parton evolution**
  - Mean field (eNJL) + scattering
  - Exhibits dynamical chiral phase transition
- **ART** (A Relativistic Transport model for hadrons)
  - Hadronization: Quark coalescence
  - Nucleon coalescence
- **Light nuclei**
Phase trajectories of central cells in the phase diagram

\[ \rho_N = \frac{\int d\mathbf{x} \rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x} \rho(\mathbf{x})} \]

\[ y_2 = \frac{\left[ \int d\mathbf{x} \rho(\mathbf{x}) \right] \left[ \int d\mathbf{x} \rho^3(\mathbf{x}) \right]}{\left[ \int d\mathbf{x} \rho^2(\mathbf{x}) \right]^2} \]
2.5 Survival of density fluctuation in an expanding fireball (8)

Off-equilibrium effects

Density moment:

\[
\overline{\rho^N} = \frac{\int dx \rho^{(N+1)}(x)}{\int dx \rho(x)}
\]

\[
y_2 = \frac{[\int dx \rho(x)][\int dx \rho^3(x)]}{[\int dx \rho^2(x)]^2}
\]

If the expansion is self-similar or scale invariant

\[
\rho(\lambda(t)x, t) = \alpha(t)\rho(x, t_h)
\]

then \(y_2(t) = y_2(t_h)\), i.e., remains a constant

‘Memory effects’: Large density inhomogeneity survives to kinetic freezeout
2.7 Collision energy dependence

1. Without a critical point:
The energy dependence of $tp/d^2$ is almost flat.

2. With a first-order phase transition:
The spinodal instability induced enhancement of $tp/d^2$ during the first-order phase transition increases as increasing the critical temperature.

STAR, arXiv:2209.08058(2022)
Hui Liu (STAR), QM2022
T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).
2.7 Collision energy dependence

1. Without a critical point:
The energy dependence of $tp/d^2$ is almost flat.

2. With a first-order phase transition:
The spinodal instability induced enhancement of $tp/d^2$ during the first-order phase transition increases as increasing the critical temperature.
The spinodal enhancement of $tp/d^2$ subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.
The spinodal enhancement of $t_p/d^2$ subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.

The slope with EoS-I is 5 times smaller.
2.9 Possible critical effects
2.9 Possible critical effects

With long-range correlation:

\[
\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}} \left[ 1 + \Delta \rho_n + \frac{\lambda}{\sigma} G \left( \frac{\xi}{\sigma} \right) \right]
\]

\[
G(z) = \frac{\sqrt{2}}{\pi} \frac{1}{z} e^{\frac{1}{2z}} \text{erfc} \left( \frac{1}{\sqrt{2z}} \right)
\]

Further investigations are needed.
3. Summary and Outlook

Main findings:
1. With scans of the collision energy and centrality as well as the equation of state using a novel transport model, we find that large density inhomogeneities generated by the spinodal instability during the first-order QCD phase transition can survive the fast expansion of the subsequent hadronic matter and lead to an enhanced $tp/d^2$ in central Au+Au collisions at $\sqrt{s_{NN}} = 3 – 5$ GeV for $T_c \geq 80$ MeV, which is in accordance with the STAR measurements.
2. We also find that the spinodal enhancement of $tp/d^2$ subsides with increasing collision centrality because of the shortening of fireball lifetime, and this effect results an almost flat centrality dependence of $tp/d^2$ at $\sqrt{s_{NN}} = 3$ GeV, which can also be used as a signal for the occurrence of a first-order phase transition.

Future developments:
1. Incorporation of Polyakov loop
2. Inclusion of long-range correlation