Investigation of deformation at a centrifugal compressor rotor in process of interference on shaft

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Abstract. In this paper, according to the finite element method, we had implemented “master-slave” method of contact interaction in elastic deformable bodies, with consider of the friction in the contact zone. We had compiled the orientation of solving extremum problems with inequality restrictions, projection algorithm, which called “the closest point projection algorithm”. Finally, an example, had brought to show the calculation of the rotor nozzle centrifugal compressor on the shaft with interference.

1. Introduction
To calculate the stress-strain state interacting elements of three-dimensional structures, which are under complex loading commonly used techniques based on various numerical methods [1-9]. The most common calculations are based on the finite element discretization of the object [10-19]. Contact problem had investigated with the finite element implementation method in feature. To execution the contact conditions in finite element implementation, mainly had used method of Lagrange multipliers, penalty method, the augmented Lagrange method, direct solution of the variation problem inequality restrictions methods of quadratic programming. The common point of all these three algorithms is one algorithm which is to find the contact zone. Thus, became justified the orientation of solving extremum problems with inequality restrictions, projection algorithm, which called “the closest point projection algorithm”. Current algorithm allows to build the contact elements, based on access that calls master-slave. Formulation of variational form using considered approaches problem appears as nonlinear and solution reaches by iterative methods of solution. Mentioned methods of solution to most cases require the first derivative from total received system of nonlinear equations, that named coherent system of tangent matrix. This is required primarily to obtain quadratic rate of convergence and in some arguments even if exist linear velocity of convergence of iterative process. In solving the contact problem by finite element method the most adversity is in no-penetration mode, and also additional kinematic conditions in the case that problems with friction on general unknown border.

2. Appointment of contact interaction of two bodies problem
By considering the contact of two bodies A and B which have a surface with smoothness C\(^1\), i.e., like bodies, normal to the surface of which is continuous. We assume that if exist a common point C for two surfaces and the total normal both of bodies A and B are place in contact. Many of these points labeled as zones of contact \(S_C\). The contact area, as well as applied stress is unknown and our mission is to determine it from the solution. Base of the variational formulation of the contact problem is on...
satisfaction from non-penetration of bodies. To get that we consider the penetration functions as contacting bodies A and B. On the body B we choose a point C and project it on the surface of the body. Distance which is the shortest distance between the bodies, would determines the function of penetration. We define on the body A (Fig. 1) normal $n$.

Let:

$$ p = (r_c - r) \cdot n $$

given demonstration allows us to dispose the function of the contact body, if $p > 0$ there is not penetration (there is no contact between bodies), if $p \leq 0$ one body A will penetrate into next body, adequacy condition $p = 0$ become non-penetrate one body to another.

![Figure 1. Contact scheme.](image1)

![Figure 2. Finite element model.](image2)

The body B, in terms of which we will check the penetration modes, calls “slave” body and point – “slave” point S. The body A, on the surface of which we project “slave” point, calls the “master” body and its surface – “master” surface. Methods of solution in contact problems, established on the task “slave” points and determination of the value of penetration $p$ in the “master” surface, which called “master-slave” algorithm. Usually nodes in finite element act as “slave” points and “master” point C determines by the closest point projection algorithm.

3. The scheme for solving the problem

The global scheme for solving the contact problem based on geometrical and physical nonlinear will present as follows.

We show general variational equation as:

$$ \delta W + \delta W_c + \delta W_g + \delta W_p = 0, $$

where $\delta W$ refers to the entire structure, $\delta W_c$ describes contact interaction, $\delta W_g$ describes geometrically nonlinear deformation, $\delta W_p$ - physically nonlinear deformation.

Next is to perform the congregate of global stiffness matrix after congregation variational the equation takes the form:

$$ \{\delta q\}^T \cdot \{\{R\} + \{R_c\} + \{R_g\} + \{R_p\}\} = 0. $$

As result, we use Newton's method of the vector function $F$ to solve nonlinear equation.

$$ F = \{R\} + \{R_c\} + \{R_g\} + \{R_p\}, $$

where $F = F(\{q\})$, $\{q\}$ is the global vector of nodal displacements.
Accordingly, to specifying the initial approximation \( \{q^{(0)}\} \), we construct an iterative procedure:
\[
\{\Delta q\} = -F'(\{q\})^{-1} \cdot F(\{q\}), \quad \{q\}^{(n+1)} = \{q\}^{(n)} + \{\Delta q\}^{(n)}.
\]
The derivative functions of residuals give the tangent stiffness matrix:
\[
F'(u) = \frac{dF}{du} = [K] = [K_{ep}] + [K_c] + [K_g],
\]
where \([K]\) is the global stiffness matrix of structure, \([K_c] = [K_c^T] + [K_c^N]\) - is the global stiffness matrix contacts, gathered from local matrix on the contact elements, comprising part of the normal \([K_c^N]\) and tangential contact \([K_c^T]\), \([K_{ep}]\) - "elastic-plastic" stiffness matrix, \([K_g]\) - matrix of geometric stiffness. The transition to variational problem with algebraic discretization is performed by finite element method.

4. Numerical example
To describe rotor of a centrifugal compressor here used to apply different types of connection the wheels to shaft. The key seat in a shaft, splined, connection through interference and combinations are the most common element in this field. In this paper we consider the connection with interference. Goal of this work is to build the estimated compound volume modelled on interference, with consider to possible geometric variations on the contact surfaces, and evaluation while the rotor is at deformed state. To ensure the immobility connection between the impeller shaft in the design lays the necessary diametric interference. The average (nominal) contacts pressure, given by the attenuation interference in centrifugal forces, should be as when the frictional force exceeding external shear forces. Such torque and axial force appears in determination of external shear for centrifugal wheel. In stress-strain state in the hub of wheel it’s necessary to calculate the optimal value of the diametrical interference, in which had provided by a fixed connection to the shaft of the wheel, and appearance of contact stress does not exceed the permissible limits. In the hub part we apply minimum yield strength of the shaft and permissible limit stress for the wheel part. As rule, the wheel has made from material with higher strength and mechanical properties. So if we use steel for the manufacture of wheels we provide amount of yield strength 835 MPa, the yield strength amount for steel to be used for the manufacture of shafts, does not exceed 450 MPa. Because of economic reasons if aim of manufacture of steel shaft is not to be much rigid so we can use from of less strength properties one. The mechanical properties of the shaft material are selected to become strength and durability condition at the area of key seat compound from the applied torque. Compression stresses in contact zone of the shaft with the wheel is not criteria. Under the influence of centrifugal forces occur weakening interference in the connection. Due to complex geometry of the wheel and its variable rigidity in the radial direction, as rule a significant decrease in interference occurs on the belt on the opposite side to the neck. To ensure the immobility of the connection at wheel with the shaft at the operating amount of rotation, it is necessary to increase the diametrical interference. In this case, calculations show that a fixed connection contact stresses can reach until the yield strength of the wheel material, i.e. so at contact zone on the shaft, occurs plastic deformation. It is considered, that, it’s confirmed in practice, that the increased level of the stress state at the shaft in the contact zone does not affect the performance of the rotor entirely. This is due to the massiveness interfaced details and redistribution of stresses. Concerns to deformed state of the rotor, the assumption of uniform distribution of contact pressure in the circumferential direction in axisymmetric calculations show that will occur bending at shaft. Arise cases when after nozzle of wheels on the shaft change happen actually at geometry of rotor (increased values of the shaft radial beats at the control points). One possible reason for this situation may be compression stress tension at the zone of landing surface on the shaft. As part of this computational model is considered when the contact stress has a different amount in size in the circumferential direction, i.e., was considered irregular compression of the shaft from interference by making the contact surfaces with the maximum possible deviations indicated in construction documentation.
In practice, the conjunction surfaces are never one hundred percent. The best result is achieved with a conical under landing pressure after numerous compression and overflow on checked interfaced surfaces after each removing and with further elaboration of them. In cylindrical parts, check and modify of usage in the manufacture of rotors of centrifugal compressors is so difficult, so we don’t conduct these operations in practice. For a particular test task to simulate wheel, in which observe irregular compression shaft in the circumferential direction was modeled as follows: interference was set for two sectors in each zone, located at an angle of 180° relative to each other.

Thus, it was modeled theoretical situation when interface surfaces of disc and the shaft on each circle is 50%, and where there is no interference coupling is completely absent. In this arrangement the surfaces which interference in that had chosen from maximum possible impact condition on shaft bending. The calculation was based on the method which has presented in [20-23]. The results of calculation have shown in Figure 1. As seen in Figure 1, the assumption is that bending may occur due to irregular compression of the shaft. Result in sample the maximum shaft deflection was 0.01 mm.

![Figure 3. Contour stress intensity in connection with a disc shaft at the diameter 120 mm interference](image)

![Figure 4. Radial stress (MPa) at a variable interference (250-300 micrometers) rotor](image)

In practical terms, insignificant bending of the shaft after appearance of nozzle on the wheel, should not cause serious concerns from the normal operation of the rotor entirely. In diametric dimensions in the denomination situation by leaving a small stock on the shaft, and then having deducted nozzle on wheel, this problem solves. However, this solution is valid only for a fixed rotor. As mentioned above, during the operation there is a weakening interference until the opening of one of the girdles, with the result that will happen redistribution of contact stresses, which can again cause the rotor bending and lead to increase in vibration. As a specific example to illustrate the effect of variable interference, considered for blower rotor, during which production was set by this fact to change the geometry of the shaft after nozzle occurs on the wheel. Material properties: yield strength - 835 MPa, modulus of elasticity - 200000 MPa, density - 7800 kg/m$^3$. Shaft Material: material yield strength - 735 MPa, modulus of elasticity - 200000 MPa, density - 7800 kg/m$^3$. The range of diametrical interference within tolerance - (250-300) microns. It should be noted that tolerance at cylindrical shape at shaft 0.005 mm set just only in the areas which compression located. On the landing surfaces of shafts and wheels these parameters are not requirements. In this example, we assume that the connection has made with dimensions that can have a variable interference in the circumferential direction in the range of 250 to 300 microns. Finite element mesh model is shown in figure 3. Maximum interference of 300 microns and minimum interference 250 microns were set to sectors that provide the highest possible bending moment on the shaft. Figure 2 shows the radial displacement of the shaft with a variable interference (250-300) microns. As had seen from the calculation of the maximum shaft deflection is 0.0013 mm at the permissible value for the drawing - 0.01 mm.
5. Conclusion

Notice that in calculations used to be conducted various degree of sampling design consideration, committed similar results, indicate that the accuracy and stability of the solutions. From obtained results, we can conclude that finite element method is effective solutions for contact problems. It allows a single software implementation considers a fairly extensive category of problems with different conditions of contact interactions. The method is indifferent with respect to the geometry of the contacting parts, complicated boundary conditions and volume load. Material properties would be chosen nonhomogeneous and anisotropic, and the relationship between stress and deformation is nonlinear. All these factors insignificantly complicate the algorithms and problem solving. Comparison of the results at several problems with accurate and approximate methods, shows the reliability and accuracy of the results obtained at a relatively low complexity, and is far to become complete in possibilities of program. Valuable is the fact that as result of the calculation not only to obtain the contact pressure, but also considered stress-strained state parts in entire volume.

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