Deconfinement in the presence of a Fermi surface

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U(1) gauge theory of non-relativistic fermions interacting via compact U(1) gauge fields in the presence of a Fermi surface appears as an effective field theory in low dimensional quantum antiferromagnetism and heavy fermion liquids. We investigate deconfinement of fermions near the Fermi surface in the effective U(1) gauge theory. Our present analysis benchmarks the recent investigation of quantum electrodynamics in two space and one time dimensions (QED$_3$) by Hermele et al. [Phys. Rev. B 70, 214437 (2004)]. Utilizing a renormalization group analysis, we show that the effective U(1) gauge theory with a Fermi surface has a stable charged fixed point. Remarkably, the renormalization group equation for an internal charge $e$ (the coupling strength between non-relativistic fermions and U(1) gauge fields) reveals that the conductivity $\sigma$ of fermions near the Fermi surface plays the same role as the flavor number $N$ of massless Dirac fermions in QED$_3$. This leads us to the conclusion that if the conductivity of fermions is sufficiently large, instanton excitations of U(1) gauge fields can be suppressed owing to critical fluctuations of the non-relativistic fermions at the charged fixed point. As a result a critical field theory of non-relativistic fermions interacting via noncompact U(1) gauge fields is obtained at the charged fixed point.

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I. INTRODUCTION

Nature of quantum criticality is one of the central interests in modern condensed matter physics. Especially, deconfined quantum criticality has been proposed in various strongly correlated electron systems such as low dimensional quantum antiferromagnetism [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] and heavy fermion liquids [17, 18, 19, 20, 21]. At these quantum critical points off-critical elementary degrees of freedom such as magnons or electrons are proposed to break up into more elementary particles with fractional quantum numbers. In an opposite angle these off-critical elementary excitations can be considered to be composites of the fractionalized elementary excitations. One off-critical excitations in one phase can be smoothly connected to the other off-critical ones in the other phase via an appropriate fractionalization at the quantum critical point. Physics of these quantum critical points is usually called deconfined quantum criticality.

The main feature of deconfined quantum criticality is emergence of gauge symmetries associated with fractionalized excitations. Critical field theories describing deconfined quantum criticality are naturally given by gauge theories, where fractionalized elementary excitations interact via long range gauge fluctuations. In the present paper we focus our attention on U(1) gauge theories applicable to many proposed deconfined quantum critical points, as will be discussed in the main section. Although U(1) gauge theory formulation can explain non-Fermi liquid physics near quantum critical points [3, 12, 20], there exists one fundamental difficulty originating from the fact that U(1) gauge fields are basically compact. Compact U(1) gauge fields allow instanton excitations expressing tunnelling events between energetically degenerate but topologically inequivalent gauge vacua. In the U(1) gauge theory instantons are nothing but magnetic monopoles. It is well known that condensation of magnetic monopoles results in confinement of fractionalized excitations [22]. If confinement arises from monopole condensation, the effective U(1) gauge theory would be useless. This is because the resulting field theory should be written in terms of composites of fractionalized excitations. For the U(1) gauge theory to be meaningful or physically working as a critical field theory, the condensation of magnetic monopoles should be forbidden.

Recently, it was shown that the condensation of magnetic monopoles does not occur if the flavor number $N$ of massless Dirac fermions is sufficiently large [7]. A relativistic U(1) gauge theory in two space and one time dimensions usually dubbed ($2+1$)D quantum electrodynamics (QED$_3$) has a stable charged fixed point in the limit of large flavors [3, 5, 8, 9]. At the charged fixed point critical fluctuations of Dirac fermions sufficiently screen out the internal charge $e$ of Dirac fermions in the large $N$ limit and thus, the corresponding magnetic charge $e_m$ becomes very large owing to the electro-magnetic duality $ee_m = 1$. Excitations of magnetic monopoles are suppressed. The condensation of instantons (magnetic monopoles) is forbidden at the stable charged fixed point owing to critical fluctuations of Dirac fermions.

In the present paper we apply the analysis of the relativistic U(1) gauge theory by Hermele et al. [7] to a non-relativistic U(1) gauge theory with a Fermi surface. This U(1) gauge theory has been considered to be a critical field theory in the context of heavy fermion liquids [17, 18, 19, 20, 21] and frustrated quantum antiferromagnetism [14, 15]. Utilizing a renormalization group analysis, we show that the effective U(1) gauge theory has a stable charged fixed point as the QED$_3$. In a renormalization group equation for the internal charge $e$ of non-relativistic fermions we find that the conductivity...
\[ \sigma \text{ of fermions plays the same role as the flavor number } N \text{ of Dirac fermions. This leads us to the remarkable conclusion that if the conductivity is sufficiently large, instanton excitations can be suppressed at the charged fixed point. As a result a critical field theory of non-relativistic fermions interacting via noncompact } U(1) \text{ gauge fields is obtained at the charged critical point. In this critical field theory the coupling strength between fermions and gauge fields is given by } e/\sqrt{\sigma} \text{ as } e/\sqrt{N} \text{ in the } QED_3. \] 

This implies that at the charged fixed point correlation functions may be systematically calculated in the } 1/\sigma \text{ expansion as the } 1/N \text{ expansion in the } QED_3.

We would like to point out that a stable charged fixed point in the non-relativistic } U(1) \text{ gauge theory with a Fermi surface was considered several years ago } [23, 24, 25]. \text{ However, it should be noted that the previous studies } [23, 24] \text{ are verified by the } 1/N \text{ expansion while the present study is justified by the } 1/\sigma \text{ expansion.}

### II. REVIEW OF QED$_3$

In this section we review the relativistic } U(1) \text{ gauge theory, } QED_3 \text{ in order to clarify the methodology utilized in the non-relativistic } U(1) \text{ gauge theory with a Fermi surface. We consider the following effective } QED_3 \text{ action}

\[ S = \int \! d^3x \left[ \bar{\psi}_\alpha \gamma_\mu (\partial_\mu - ia_\mu) \psi_\alpha + \frac{1}{2e^2} e_\mu \right]^2. \] 

Here } \psi_\alpha \text{ is a massless Dirac spinor with a flavor index } \alpha = 1, ..., N \text{ and } a_\mu \text{, a compact } U(1) \text{ gauge field. Eq. (1) was originally proposed to be an effective action in one possible quantum disordered paramagnetism of } SU(N) \text{ quantum antiferromagnets on two dimensional square lattices } [1, 2, 3]. \text{ Utilizing the fermion representation of the } SU(N) \text{ antiferromagnetic Heisenberg model and performing the Hubbard-Stratonovich transformation for appropriate interaction channels, one can obtain an effective one body action in terms of fermions and appropriate order parameter fields. In this effective action a stable mean field phase is known to be the } \pi \text{ flux state. In this mean field ground state low energy elementary excitations are given by massless Dirac fermions near nodal points showing gapless Dirac spectrum and } U(1) \text{ gauge fluctuations. This leads to Eq. (1) as a low energy effective field theory in one possible quantum disordered paramagnetism of the } SU(N) \text{ Heisenberg model. Recently, Eq. (1) was also considered to be an effective field theory in two dimensional geometrically frustrated antiferromagnets } [4, 5, 6].

It is well known that the } QED_3 \text{ with noncompact } U(1) \text{ gauge fields has a stable charged fixed point } [4, 5, 6, 7, 8, 9]. \text{ In order to see this fixed point we introduce the relation of } e_\mu^2 = \Lambda Z_a e_\mu^2, \text{ where } e_\mu^2(0) \text{ is the renormalized (bare) internal charge of Dirac fermions and } Z_a, \text{ the renormalization constant of the gauge field } a_\mu. \text{ This relation shows how the internal gauge charge evolves by varying the energy scale of our interest. The renormalization constant } Z_a \text{ can be obtained from singular corrections to the self-energy of the gauge field due to particle-hole polarizations of massless Dirac fermions, given by } Z_a = 1 - \lambda N e_\mu^2 \ln \Lambda. \text{ Here } \Lambda \text{ is a momentum cut-off and } \lambda \text{, a positive numerical constant, where its precise value is not important in the present consideration. Inserting the expression of } Z_a \text{ into the relation of internal gauge charges and performing the derivatives of } \frac{de_\mu^2}{d\ln \Lambda} = \frac{\Lambda}{2N} Z_a e_\mu^2 + \Lambda \frac{dZ_a}{d\ln \Lambda} e_\mu^2, \text{ we obtain a renormalization group equation for the internal charge}

\[ \frac{de_\mu^2}{d\ln \Lambda} = e^2 - \lambda N e^4, \] 

where the subscript } r \text{ in the renormalized charge } e_r \text{ is omitted. This renormalization group equation expresses a change of the internal charge } e_\mu^2 \text{ as a function of the momentum cut-off } \Lambda. \text{ The first term represents a bare scaling dimension of } e^2. \text{ In } (2+1)D \text{, } e^2 \text{ is relevant in contrast to the case of } (3+1)D, \text{ where it is marginal. The second term originates from the singular correction to the self-energy of the } U(1) \text{ gauge field by massless Dirac fermions. This renormalization group equation leads us to a stable charged fixed point of } e_\mu^2 = 1/N \Lambda \text{ in the } QED_3. \text{ A next question is if the charged fixed point remains stable after admitting instanton excitations. Using the electromagnetic duality, Hermele et. al obtained the following renormalization group equations of magnetic charge } g = 1/e^2 \text{ and instanton fugacity } y_m [7]. \text{ }

\[ \frac{dg}{d\ln \Lambda} = -g + \lambda N - \alpha g_m^2, \]

\[ \frac{dy_m}{d\ln \Lambda} = (3 - \beta g) y_m, \]

where } \alpha \text{ and } \beta \text{ are positive numerical constants. In the absence of massless Dirac fermions } (N = 0) \text{ Eq. (3) is reduced to the standard renormalization group equation for the sine-Gordon theory describing three dimensional Coulomb (monopole) gas } [22]. \text{ The last term in the first equation results from screening effects by monopole and anti-monopole pairs in the sine-Gordon theory. On the other hand, the second term } \lambda N \text{ is the contribution of massless Dirac fermions, originating from Eq. (2) via the electromagnetic duality } g = e^{-2}. \text{ This term leads a magnetic charge to have a large fixed point value proportional to } N, \text{ i.e., } g_c = \lambda N \text{ in the large } N \text{ limit. This large magnetic charge makes the instanton fugacity } y_m \text{ go to zero at the charged fixed point. This is the signal for suppression of instanton excitations. Although the above renormalization group equations are approximate, there exists a rather convincing argument } [7, 20]. \text{ An important basis for this argument is the existence of a charged critical point. At the scale invariant fixed point it is shown that the scaling dimension of an instanton insertion operator is proportional to the order of } N [20]. \text{ This leads one to the conclusion that in the large } N \text{ limit}
the internal flux changing operators are irrelevant at the critical fixed point, indicating the suppression of instanton excitations.

A critical field theory in terms of massless Dirac fermions interacting via noncompact U(1) gauge fields is obtained at the charged fixed point

\[
S_c = \int d^3x \left[ \bar{\psi}_\alpha \gamma_{\mu} \left( \partial_{\mu} - i \frac{e}{\sqrt{N}} a_\mu \right) \psi_\alpha 
+ \frac{1}{16} (\partial \times a) \frac{1}{\sqrt{-\partial^2}} (\partial \times a) \right].
\]

(4)

At the tree level it can be easily checked that this effective action has scale invariance. Notice that the Maxwell kinetic energy of the gauge field was ignored. Because the scaling dimension of $|\partial \times a|^2$ is larger than 3, the Maxwell term is irrelevant at the charged critical point. The non-Maxwell kinetic energy of the gauge field arises from the contribution of critical Dirac fermions. We emphasize that integration over the Dirac fermions should be understood in the renormalization group sense. It is noted that the critical coupling constant between the Dirac fermions and gauge fields is given by $e_c/\sqrt{N}$ after replacing $a_\mu$ with $a_\mu/\sqrt{N}$. Thus, correlation functions can be systematically calculated in the $1/N$ expansion at the charged fixed point. In the following we discuss that the non-relativistic U(1) gauge theory with a Fermi surface has the similar structure with the QED$_3$.

III. DECONFINEMENT IN THE PRESENCE OF A FERMI SURFACE

A. Effective Field Theory

Now we consider the following U(1) gauge theory in terms of non-relativistic fermions interacting via compact U(1) gauge fields

\[
S_\chi = \int d^3x \left[ \chi_\alpha^\dagger \left( \partial_{\tau} - ia_{\tau} - \mu \right) \chi_\alpha 
+ \frac{1}{2m} (\bar{\chi} \cdot \bar{\nabla} - i\bar{a}) \chi_\alpha \right]^2
+ \frac{1}{2\epsilon^2} |\partial \times a|^2 \right].
\]

(5)

Here $\chi_\alpha$ represents a fermion field with spin $\alpha = \uparrow, \downarrow$ and $\mu$, its chemical potential. $a_\mu = (a_{\tau}, \bar{a})$ is a compact U(1) gauge field. The Maxwell kinetic energy of the gauge field can be considered to arise from the contribution of high energy excitations. Eq. (5) is proposed to be an effective field theory in various strongly correlated electron systems such as low dimensional geometrically frustrated quantum antiferromagnets[14, 15], heavy fermion liquids[17, 20, 21] and strange metals of high $T_c$ cuprates[23, 24, 25]. We briefly review how Eq. (5) appears to be an effective action in the geometrically frustrated quantum antiferromagnets on two dimensional triangular lattices[14, 15]. Utilizing the slave rotor representation of the Hubbard model[25] and performing the Hubbard-Stratonovich transformation for appropriate interaction channels, Lee and Lee obtained an effective one body action in terms of fermionic spinons and bosonic rotors coupled to hopping order parameters[15]. In the one body effective action the authors found that near a metal-insulator transition a stable mean field phase is a zero flux state. In this mean field ground state low energy elementary excitations are given by gapless non-relativistic spinons near a Fermi surface, gapped bosonic roters and compact U(1) gauge fields. Gapped bosons can be safely integrated out to produce the Maxwell kinetic energy for the U(1) gauge fields. As a result Eq. (5) is obtained to describe a spin liquid phase near the Mott critical point.

In the present paper we assume that bosonic excitations are gapped and thus, consider a spin liquid Mott insulator. This allows us to investigate the fermion-only theory Eq. (5). The role of gapped bosonic excitations is to generate the Maxwell kinetic energy for the U(1) gauge field.

B. Deconfinement in the presence of a Fermi surface

Now we examine the deconfinement of non-relativistic fermions near a Fermi surface. As mentioned in the introduction, our strategy is basically the same as that of Hermel[7]. We first check whether there exists a stable charged fixed point and then, investigate the stability of the charged fixed point against instanton excitations. Before doing this, we linearize the non-relativistic spectrum of $\chi_\alpha$ fermions near the Fermi surface

\[
S_\chi = \int d^3x \left[ \chi_\alpha^\dagger \left( [\partial_0 - ia_0] + \bar{v}_F \cdot (\bar{\nabla} + \bar{k}_F + \bar{a}) \right) \chi_\alpha
+ \frac{1}{2\epsilon^2} |\partial \times a|^2 \right],
\]

(6)

where $v_F$ is a Fermi velocity and $k_F$, a Fermi wave vector. In the absence of long range gauge interactions ($e^2 = 0$) the resulting field theory describes noninteracting fermions near the Fermi surface. This free fermion theory is a trivial critical field theory at the Fermi liquid fixed point[26], more accurately, Fermi gas fixed point. The Fermi liquid fixed point corresponds to the free Dirac liquid near the Fermi surface. This free fermion theory is unstable against long range U(1) gauge interactions. It is naturally expected that the Fermi liquid fixed point is also unstable against U(1) gauge fluctuations.

Just as the case of QED$_3$, we introduce the relation of $e^2 = \Lambda Z_\alpha e^2$ between the renormalized and bare internal charges, $e_\tau$ and $e_\alpha$, respectively, where $Z_\alpha$ is the renormalization constant of the gauge field $a_\mu$. Remember that singular corrections to the self-energy of the gauge field due to particle-hole excitations of fermions near the Fermi surface contribute to the renormalization constant $Z_\alpha$. Integrating over the fermions near...
the Fermi surface, we obtain the following expression for an effective action, \( S_a = -T \text{Tr} \ln \left( \left[ \partial_0 - i a_0 \right] + \vec{v}_F \cdot \left[ \nabla + \vec{k}_F + \vec{a} \right] \right) \). Expanding the logarithmic term to quadratic order for the U(1) gauge field \( a_\mu \), we obtain \( S_a = \frac{1}{2} \sum_{q, \omega} a_\mu(q, i \omega) \Omega_{\mu\nu}(q, i \omega) a_\nu(-q, -i \omega) \), where \( \Omega_{\mu\nu}(q, i \omega) \) is the density-density \((\mu = \nu = \tau)\) or current-current \((\mu \neq \nu = x, y)\) correlation function of gapless fermions. In this expression the time and space components of the gauge field decouple. Since the time component is screened by density fluctuations \((\Pi_{ij})\) of gapless fermions and gives rise to only a short-range interaction, it’s sufficient to consider the spatial components (labeled \( i, j = x, y \)) only. The current-current correlation function is given by \( \Pi_{ij}(x, \tau) = -\langle T_F (J_F_i(x, \tau) J_F_j(0, 0) - \delta_{ij} \rho_F \delta(x) \delta(\tau)) \rangle \), where \( J_F_i = v_F \chi_\alpha \chi_\alpha \) and \( \rho_F = \chi_\alpha \chi_\alpha \) are the current and density operators of fermions, respectively. It is convenient to choose the Coulomb gauge \( \nabla \cdot \vec{a} = 0 \), in which case the spatial part of the gauge field is purely transverse. It should be noted that since the density term in the current-current correlation function originates from the \( \alpha^2 \) term in Eq. (5), it does not arise from Eq. (6) owing to the linearization of a fermion dispersion near the Fermi surface. For the gauge field to be transverse, this term should be taken into account explicitly. It is well known that the transverse current-current correlation function is given by \[ \Pi_{ij}(q, i \omega) = \left( \delta_{ij} - \frac{q_i q_j}{Q^2} \right) \Pi(q, i \omega), \] \[ \Pi(q, i \omega) = \sigma |\omega| + \chi q^2, \] \( \sigma \) and \( \chi \) are the conductivity and diamagnetic susceptibility of fermions near the Fermi surface. In appendix A we show this derivation. As a result we obtain the effective gauge action in the Coulomb gauge \[ S_{eff} = \frac{1}{2} \sum_{q, \omega} \left[ \left( \frac{1}{e^2} + \chi \right) q^2 + \sigma |\omega| \right] \left( \delta_{ij} - \frac{q_i q_j}{Q^2} \right) a_i(q, i \omega) a_j(-q, -i \omega), \] The above expression can be easily expected. The transverseness is naturally understood in the respect that U(1) gauge symmetry restricts the resulting dynamics of gauge fields. Since the fermions are gapless excitations, singular corrections are expected to arise, renormalizing the internal charge \( e \) as the case of QED3. In the non-Maxwell kinetic energy of the gauge field a new feature is emergence of the conductivity \( \sigma \) of fermions. This is reflection of the Fermi surface. Note that the non-Maxwell kinetic energy depends on the absolute value of frequency. This indicates dissipative dynamics of the gauge field. In the present paper we consider the case of Ohmic dissipation, where the conductivity \( \sigma \) is given by a constant value depending on the density of states and mean free time of fermions near the Fermi surface. We would like to comment that in Eq. (8) the conductivity \( \sigma \) lies in the same place as the flavor number \( N \) of Dirac fermions in the QED3 Eq. (4). This leads us to expect that the conductivity plays the same role as the flavor number. If so, the expansion of the logarithmic term to the second order for the U(1) gauge field can be justified in the 1/\( \sigma \) expansion as the 1/\( N \) expansion in the QED3. The diamagnetic susceptibility is given by \( \chi \sim m^{-1} \) in a Fermi liquid with \( m \), a mass of fermions. In order to justify Eq. (8) we expand the resulting logarithmic term to higher order and write down the effective gauge action for transverse gauge fields \( a_i \) in a highly schematic form \[ S_a = \sigma \int d^3 q |\omega| |a_i|^2 + \int (d^3 q)^3 b_4(f(q))|a_i|^4 + O(a_i^5), \] where \( f(q) \) is a function of momentum and frequency. The coefficient \( b_4 \) of nonlinear gauge interactions is given by \[ b_4 \sim \int (d \Omega)^2 k G^4(k, \Omega) \] with the single particle green function \( G(k, \Omega) = [\Omega - \epsilon_k + i \eta]^{-1} \) under \( \Omega \rightarrow \Omega + i \eta \). The main point is whether the coefficient \( b_4 \) is proportional to \( \sigma^2 \), square of the conductivity of fermions near the Fermi surface. In this case the 1/\( \sigma \) expansion would be broken. If one utilizes the linearized spectrum near the Fermi surface, i.e., \( \epsilon_k = v_F \cdot (\vec{k} - \vec{k}_F) \), the integral over momentum in the expression of \( b_4 \) would vanish owing to its multiple pole structure. Other coefficients in higher order terms are also given by some constants. Now we can see how the 1/\( \sigma \) expansion works. For the gaussian gauge action to be finite in the large \( \sigma \) limit, fluctuations of gauge fields should follow \( a_i \sim 1/\sqrt{\sigma} \). Then, the nonlinear terms are apparently higher order in the 1/\( \sigma \) expansion than the leading gaussian term. This justifies the 1/\( \sigma \) expansion for the non-Maxwell kinetic energy of the gauge field \( a_\mu \). A similar argument for the 1/\( N \) expansion in the QED3 can be found in Ref. 33.

The 1/\( \sigma \) expansion may be understood physically in the following way. In the 1/\( N \) expansion the flavor number \( N \) of Dirac fermions can be considered to be the number of screening channels for gauge interactions. In a similar way the conductivity is associated with the screening channels. In the case of Ohmic dissipation the conductivity is given by \( \sigma = ne^2 \tau \), where \( n, e, \tau \), and \( m \) are the density, charge, transport time, and mass of fermions. It should be noted that the density is involved with the conductivity. In this respect the conductivity may be considered to be the number of screening channels for gauge interactions. The resulting non-Maxwell kinetic energy (Landau damping term) has the same scaling as that in the QED3 Eq. (4). This singular correction in Eq. (8) leads to the following renormalization constant \( Z_{\alpha} = 1 - \gamma e^2 \ln \Lambda \), where \( \Lambda \) is a momentum cut-off and \( \gamma \), a positive numerical constant. Its precise value is not important in
the present consideration. Inserting this expression of $Z_0$ into the relation of internal charges and performing derivatives with respect to $\ln \Lambda$ as the case of $QED_3$, we reach a renormalization group equation for the internal charge $\sigma$

$$\frac{d\sigma^2}{\ln \Lambda} = e^2 - \gamma \sigma e^4,$$

(10)

where the subscript $r$ in the renormalized charge $e_r^2$ is omitted. The first term represents a bare scaling dimension of $e^2$ in $(2 + 1)D$, and the second term originates from the singular correction to the self-energy of the $U(1)$ gauge field by non-relativistic fermions. Remarkably, this renormalization group equation has the essentially same structure as Eq. (2) in the $QED_3$ if the flavor number $N$ is replaced with the conductivity $\sigma$. The Fermi liquid fixed point of $e^2 = 0$ is unstable against a nonzero value of internal charge. A finite internal charge drives a renormalization group flow away from the Fermi liquid fixed point, terminating at the stable charged fixed point of $e_c^2 = 1/(\gamma \sigma)$. The effective $U(1)$ gauge theory $S_\alpha$ in Eq. (5) has a stable charged fixed point as the $QED_3$ Eq. (1).

A next job is to examine the stability of the charged critical point against instanton excitations. Using the electromagnetic duality, we first obtain a renormalization group equation for magnetic charge $g = 1/e^2$

$$\frac{dg}{\ln \Lambda} = -g + \gamma \sigma - \bar{\alpha} g_m g^3,$$

(11)

where $\bar{\alpha}$ is a positive numerical constant. The first and second terms in right hand side originate from Eq. (10) via the electromagnetic duality $g = e^{-2}$. The last term results from the screening effect of monopole and anti-monopole pairs in a non-relativistic sine-Gordon theory, $S_{sG} = \int [\frac{d^2 k}{(2\pi)^2}] \frac{1}{2} (k^2 + \sigma^{-1} g |\omega| k^2) |\varphi(k, \omega)|^2 - \int d^2 x dr m_c \cos \varphi(x, r) \overline{\varphi}(x, r),$ where $\varphi$ is a magnetic potential field mediating interactions between magnetic monopoles and $m_c$, monopole (instanton) fugacity. We note that owing to the dissipative dynamics of the gauge field in Eq. (8) the above sine-Gordon action has nontrivial momentum and frequency dependencies in the kinetic energy of the $\varphi$ fields in contrast to the standard sine-Gordon action, $S_{sG} = \int [\frac{d^2 k}{(2\pi)^2}] \frac{1}{2} (k^2 + \omega^2) |\varphi(k, \omega)|^2 - \int d^2 x dr m \cos \varphi(x, r).$ The non-relativistic sine-Gordon action leads to the following renormalization group equation for the monopole fugacity $m_c$

$$\frac{dm_c}{\ln \Lambda} = 2 - \bar{\beta} \sigma \ln(1 + \bar{\alpha} \sigma^{-1} g) m_c,$$

(12)

where $\bar{\beta}$ and $\bar{\alpha}$ are positive numerical constants. A detailed derivation of Eq. (12) can be found in Eq. (B10) of Ref. 34. Eq. (11) and Eq. (12) yield that instanton (monopole) excitations can be suppressed at the charged critical point in the large $\sigma$ limit. Eq. (11) shows that the magnetic charge $g$ can have a large fixed point value proportional to $\sigma$, i.e., $g_c = \gamma \sigma$ in the large $\sigma$ limit. Inserting this fixed point value into Eq. (12), one can easily find that the monopole fugacity goes to zero in the large $\sigma$ limit. This is in contrast to the result of Ref. 36. The reason why there is no phase transition in Ref. 36 lies in the introduction of a $\hat{a}_\omega$ parameter. However, the presence of the $\hat{a}_\omega$ parameter destroys the charged critical point even in the absence of instanton excitations. In this respect we think that introduction of the $\hat{a}_\omega$ parameter is not fully justified. If this parameter is ignored, Kosterlitz-Thouless ($KT$) like phase transition is expected as a confinement-deconfinement transition. This possibility is distinct from our scenario since the structure of Eq. (11) is essentially different from that of the renormalization group equation in the $KT$ transition. Although the present result seems to be consistent with the previous analytical study 32 arguing the existence of a finite critical conductivity for the confinement-deconfinement transition, the nature of the transition would be different. We would like to point out a report of Monte Carlo simulation claiming deconfinement of non-relativistic particles. In the study the authors investigated an effective nonlocal gaussian gauge action. From their Monte Carlo simulation they argued that deconfinement of non-relativistic particles always occurs. This is not contrast to the present result in the sense that the present analysis can be applied in the large $\sigma$ limit.

It should be noted that the above renormalization group equations, Eq. (11) and Eq. (12) are approximate since they are obtained in the gaussian approximation for the $U(1)$ gauge fields. In order to overcome this level of approximation it is necessary to apply the methodology of Ref. 27 in the relativistic $U(1)$ gauge theory to the non-relativistic one. Remember that the important basis of this nonperturbative argument is the existence of a scale invariant critical point. In this respect we expect that scaling dimensions of instanton insertion operators may be given by the order of $\sigma$ as the order of $N$ in the $QED_3$. This important issue should be addressed near future.

A critical field theory in terms of non-relativistic fermions interacting via noncompact $U(1)$ gauge fields is obtained at the charged fixed point in the Coulomb gauge

$$S_c = \int d^4 x \left[ \chi_\alpha \left( \partial_\alpha + v_F \cdot i \vec{\nabla} + \frac{e_c q}{\sqrt{\sigma} a} \right) \chi_\alpha \right] + \frac{1}{2} \sum_{q, \omega} \left( \frac{1}{q^2} + \frac{\omega^2}{q^2} \right) f_{xy}^2(q, \omega),$$

(13)

where the field strength tensor $f_{xy}$ is given by $f_{xy} = \partial_x a_y - \partial_y a_x$ in real space. In the non-relativistic case the Maxwell kinetic energy of the gauge field cannot be ignored since it is not irrelevant at the charged critical point. If we assign the scaling dimensions of $a$ and $\chi_\alpha$ as $[\hat{a}] = [\hat{q}]$ and $[\chi_\alpha] = [\hat{q}]^{1/2}$ under $[\omega] = [\hat{q}]^0$ with $[\hat{q}]$, the scaling dimension of the variable $O$, the above effective
action has scale invariance at the tree level except the time derivative term for the $\chi_\alpha$ fermions. This can be resolved by the self-energy correction of fermions via dissipative gauge interactions. Performing the standard one loop calculation, we can easily find $[\Sigma] = [\omega]^{1/2}$, where $\Sigma$ is the self-energy of the $\chi_\alpha$ fermions. Then, the resulting effective action including the self-energy correction has scaling invariance\cite{23,24}. We note that the Maxwell kinetic energy is higher order than the singular non-Maxwell kinetic energy in the $1/\sigma$ expansion. Remember that nonlinear interactions between gauge fields are the order of $1/\sigma^2$. In this respect it is consistent to keep the Maxwell term in the $1/\sigma$ expansion. We would like to point out that the critical coupling constant between the non-relativistic fermions and gauge fields is given by $e_c/\sqrt{\sigma}$ after replacing $\vec{a}$ with $\frac{\vec{a}}{\sqrt{\sigma}}$. This clarifies the fact that the conductivity $\sigma$ plays the same role as the flavor number $N$ of Dirac fermions at the charged critical point. This implies that correlation functions may be systematically evaluated in the $1/\sigma$ expansion at the charged fixed point as the $1/N$ expansion in the $QED_3$.

C. Discussion: Effect of Disorder on Deconfined Quantum Criticality

In this section we discuss effects of nonmagnetic disorders on deconfined fermions near the Fermi surface at the charged fixed point. Recently, the role of nonmagnetic impurities in the relativistic critical field theory Eq. (4) was investigated by the present author\cite{8}. In contrast to the free Dirac theory in two space dimensions\cite{39,40} long range gauge interactions reduce strength of disorders and induce a delocalized state at zero temperature\cite{8}. The presence of nonmagnetic disorders destabilizes the free Dirac fixed point. The renormalization group flow goes away from the fixed point, indicating localization\cite{39,40}. On the other hand, the charged fixed point in the $QED_3$ remains stable at least against weak randomness\cite{8}. A new unstable fixed point separating delocalized and localized phases is found\cite{8}. The renormalization group flow shows that the effect of random potentials vanishes if we start from sufficiently weak disorders. In the present critical theory Eq. (13) a similar result is expected. Deconfined fermions near the Fermi surface would remain delocalized at least against weak randomness. However, it should be considered that nonmagnetic disorders reduce the fermion conductivity $\sigma$. Thus, even if the charged fixed point can be stable against weak disorders in the case of noncompact $U(1)$ gauge fields, the fixed point can be unstable against instanton excitations owing to reduction of the conductivity. If so, the fermions would be confined owing to the presence of disorders. This may be experimentally verified. If nonmagnetic impurities like Zn are doped in the strange metal phase of high $T_c$ cuprates\cite{23,24,25} in the quantum critical regime of Kondo systems\cite{11,20,21}, or in the spin liquid Mott insulator of geometrically frustrated quantum antiferromagnetism\cite{14,15}, the confinement of fermions can break quantum criticality, detected in measurements of conductivity or magnetic susceptibility. This important issue should be addressed in more quantitative level near future.

IV. SUMMARY

In the present paper we investigated deconfinement of non-relativistic fermions near a Fermi surface. The main findings are the existence of the charged critical point and its stability against instanton excitations. This leads us to the critical field theory Eq. (13), where the critical coupling constant between the fermions and noncompact $U(1)$ gauge fields is given by $e_c/\sqrt{\sigma}$. This coupling strength makes it possible to calculate correlation functions in the $1/\sigma$ expansion at the charged critical point.

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APPENDIX A

In appendix A we sketch the derivation of a fermion polarization function Eq. (7). We rewrite the current-current correlation function

$$\Pi_{ij}(x,\tau) = -\langle T_r [\bar{J}_{F_i}(x,\tau)J_{F_j}(0,0) - \delta_{ij}\rho_{F}\delta(x)\delta(\tau)] \rangle. \quad (A1)$$

Here $\vec{J}_F = i\chi_\alpha^0(\vec{\nabla}/2m)\chi_\alpha + h.c.$ is the fermion current operator and $\rho_F = \chi_\alpha^\dagger \chi_\alpha$, its density operator. The fermion current operator is reduced to $\vec{J}_F = \vec{v}_F\chi_\alpha^0$ with $\vec{v}_F = \vec{v}/m$ at the Fermi energy, consistent with the expression in section (III-B). Inserting these operators into Eq. (A1) and performing some algebra, we obtain the following expression in energy-momentum space

$$\Pi_{ij}(q,\omega) = -\int \frac{d^2k}{(2\pi)^2} \frac{1}{m^2} \frac{1}{2}(k_i + \frac{q_i}{2})(k_j + \frac{q_j}{2})$$

$$\frac{1}{\beta} \sum_\nu G_0(k,\nu)G_0(k + q,\nu + \omega), \quad (A2)$$

where $G_0(k,\nu) = [\nu - \epsilon_k]^{-1}$ is a fermion propagator with its bare dispersion $\epsilon_k = k^2/2m$. Here the spin index $\alpha$ is not taken into account. Performing the sum of
Matsubara frequencies of the fermions, we obtain
\[
\Pi_{ij}(q, i\omega) = -\frac{1}{2m} \int \frac{d^2k}{(2\pi)^2 m^2} f(k) \frac{1}{i\omega - \epsilon_k - \epsilon_q/2m} \frac{\partial f(\epsilon_k)}{\partial \epsilon_k} (\epsilon_k - \epsilon_{q/2}) \approx -\frac{m}{k_F} \delta(k - k_F).
\]
where \(f(\epsilon_k)\) is the Fermi-Dirac distribution function. Shifting \(k \rightarrow k - \vec{q}/2\), expanding \(\delta\) for \(|\vec{q}| \ll k_F\), and using \(\epsilon_{k+q/2} \approx \epsilon_k + \frac{\vec{q} \cdot \vec{q}}{2m}\), one can find
\[
f(\epsilon_{k+q/2}) - f(\epsilon_{k-q/2}) \approx \frac{\partial f(\epsilon_k)}{\partial \epsilon_k} \frac{\vec{q} \cdot \vec{q}}{m}.
\]
At zero temperature one obtains
\[
\frac{\partial f(\epsilon_k)}{\partial \epsilon_k} = -\delta(\epsilon_k - \epsilon_F) = -\delta\left(\frac{k^2 - k_F^2}{2m}\right) \approx -\frac{m}{k_F} \delta(k - k_F).
\]
Inserting Eq. (A4) and Eq. (A5) into Eq. (A3), Eq. (A7) reads
\[
\Pi_{ij}(q, i\omega) = \frac{1}{m^2} \int \frac{d^2k}{(2\pi)^2} \left(\frac{\vec{q} \cdot \vec{q}}{m} \frac{m}{k_F} \delta(k - k_F)\right)
\]
Performing the momentum integration with care of \(k_i k_j\) and \(\vec{q} \cdot \vec{q}\), one obtains the following expression for the transverse current-current correlation function
\[
\Pi_{ij}(q, i\omega) = \frac{\delta(q_i q_j)}{q^2} \Pi(q, i\omega),
\]
where \(\sigma\) and \(\chi\) are the conductivity and diamagnetic susceptibility. This form is quite reasonable because in the \(q \rightarrow 0\) limit the conductivity is reduced to \(\sigma = (1/\omega)\Pi(\omega + i\epsilon)\) with the Wick rotation \(i\omega \rightarrow \omega + i\epsilon\), and in the \(\omega \rightarrow 0\) limit only the diamagnetic contribution proportional to \(q^2\) survives, both of which are well known. Furthermore, this expression shows that in the \(q, \omega \rightarrow 0\) limit the paramagnetic contribution (the first term in Eq. (A1)) cancels the diamagnetic one (the second term in Eq. (A1)) exactly in a normal Fermi liquid. In this respect the present \(U(1)\) gauge action may be applied to various shapes of Fermi surface. This statement is justified by the fact that the expression Eq. (A7) can be derived from the Maxwell equation, as well shown in page 113 of Ref. [30]. In a free fermion gas the conductivity is given by \(\sim q^{-1}\), resulting in the familiar Landau damping term. However, in this paper we consider the transport time \(\tau_\epsilon\) due to scattering mechanism such as disorder. In the case of \(q < (\nu_F \tau_\epsilon)^{-1}\) the conductivity is given by \(\sim \epsilon^{-1}\), corresponding to the Ohm’s law. In this paper we consider the Ohmic dissipation instead of the familiar Landau damping. The diamagnetic susceptibility is given by \(\chi \sim m^{-1}\) from \(\partial^2 S/\partial a_i^2\), where \(S\) is the action defined in Eq. (5). For more details, see Refs. [27, 30].

\[\text{References}\]

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In Ref. [36] the following non-relativistic sine-Gordon action is investigated

\[ S_{SG} = \int \frac{d^2k\omega}{(2\pi)^3} \left( \frac{|\omega|k^2}{l} + a_k\omega^2 \right) |\varphi(k,\omega)|^2 + \int dx^2 dy_m \cos \varphi(x,\tau). \]

Here \( l \) and \( a_k \) correspond to \( \sigma \) and \( 1/g \) in the present consideration, respectively. On the other hand, \( a_\omega \) is newly introduced in Ref. [32]. Based on this sine-Gordon theory the authors derived the following renormalization group equations

\[
\begin{align*}
\frac{d\hat{a}_k}{d\ln b} &= \frac{\hat{y}_m^2}{2\pi\hat{a}_k} \\
\frac{d\hat{a}_\omega}{d\ln b} &= 2\hat{a}_\omega + \frac{\hat{y}_m^2}{4\hat{a}_k} \\
\frac{d\hat{y}_m}{d\ln b} &= \left( 2 - \frac{1}{8\pi\sqrt{\hat{a}_k\hat{a}_\omega - (1/2l)^2}} \right) \hat{y}_m
\end{align*}
\]

with the redefined variables, \( \hat{a}_k = a_k/\Omega, \hat{a}_\omega = a_\omega\Omega/\Lambda^2 \), and \( \hat{y}_m = y/\Lambda^2\Omega \), where \( \Omega \) and \( \Lambda \) are energy and momentum cut-offs, respectively. \( b \) is a scaling parameter. Notice that these renormalization group equations result in \( \hat{y}_m \to \infty \) owing to the newly introduced parameter \( \hat{a}_\omega \), indicating condensation of magnetic monopoles (instantons). The main problem in these renormalization group equations is that even in the absence of instanton excitations, i.e., \( \hat{y}_m \equiv 0 \) the remaining gaussian action in the above sine-Gordon action \( S_{SG} \) is not a critical theory owing to the \( \hat{a}_\omega \) parameter. See the second renormalization group equation for \( \hat{a}_\omega \) showing the relevance of \( \hat{a}_\omega \) in the case of \( \hat{y}_m \equiv 0 \). This is in contrast to the previous studies [23, 24, 25] and the present result claiming the existence of a scale invariant critical point in the case of noncompact U(1) gauge fields. In this respect we set the parameter \( \hat{a}_\omega \) to be identically zero. In this case the resulting renormalization group equations are given by

\[
\begin{align*}
\frac{d\hat{a}_k}{d\ln b} &= \frac{\hat{y}_m^2}{2\pi\hat{a}_k} \\
\frac{d\hat{y}_m}{d\ln b} &= \left( 2 - \frac{l}{4\pi^2} \ln \left( 1 + \frac{1}{l\hat{a}_k} \right) \right) \hat{y}_m.
\end{align*}
\]

These renormalization group equations have a fixed line, \( \hat{y}_m = 0 \) and \( \hat{a}_k \leq \left( \left( e^{\pi^2/4l} - 1 \right) \right)^{-1} \) as the KT transition. This deconfinement transition differs from our scenario in the respect that the deconfinement in the present paper can occur at the charged critical point instead of a critical line.