Kondo Physics in the Single Electron Transistor with ac Driving

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Using a time-dependent Anderson Hamiltonian, a quantum dot with an ac voltage applied to a nearby gate is investigated. A rich dependence of the linear response conductance on the external frequency and driving amplitude is demonstrated. At low frequencies the ac potential produces sidebands of the Kondo peak in the spectral density of the dot, resulting in a logarithmic decrease in conductance over several decades of frequency. At intermediate frequencies, the conductance of the dot displays an oscillatory behavior due to the appearance of Kondo resonances of the satellites of the dot level. At high frequencies, the conductance of the dot can vary rapidly due to the interplay between photon-assisted tunneling and the Kondo resonance.

PACS numbers: 72.15.Qm, 85.30.Vw, 73.50.Mx

It has been predicted that, at low temperatures, transport through a quantum dot should be governed by the same many-body phenomenon that enhances the resistivity of a metal containing magnetic impurities – namely the Kondo effect. The recent observation of the Kondo effect by Goldhaber et al. in a quantum dot operating as a single-electron transistor (SET) has fully verified these predictions. In contrast to bulk metals, where the Kondo effect corresponds to the screening of the free spins of a large number of magnetic impurities, there is only one free spin in the quantum-dot experiment. Moreover, a combination of bias and gate voltages allow the Kondo regime, mixed-valence regime, and empty-site regime all to be studied for the same quantum dot, both in and out of equilibrium.

Here we consider another opportunity presented by the observation of the Kondo effect in a quantum dot that is not available in bulk metals – the application of an unscreened ac potential. There is already a large literature concerning the experimental application of time-dependent fields to quantum dots. For a dot acting as a Kondo system, the ac voltage can be used to periodically modify the Kondo temperature or to alternate between the Kondo and mixed-valence regimes. Thus it is natural to ask what additional phenomena occur in a driven system which in steady state is dominated by the Kondo effect. Our results indicate a rich range of behavior with increasing ac frequency, from sidebands of the Kondo peak at low ac frequencies, to conductance oscillations at intermediate frequencies, and finally to photon-assisted tunneling at high ac frequencies.

The system of interest is a semiconductor quantum dot, as pictured schematically in Fig. 1. An electron can be constrained between two reservoirs by tunneling barriers leading to a virtual electronic level within the dot at energy $\sim \epsilon_{\text{dot}}$ (measured from the Fermi level) and width $\sim 2 \Gamma_{\text{dot}}$. We assume that both the charging energy $e^2/C$ and the level spacing in the dot are much larger than $\Gamma_{\text{dot}}$, so the dot will operate as a SET. In this work, we consider only the linear-response conductance between the two reservoirs. However, we will allow an oscillating gate voltage $V_{g}(t) = V_{0} + V_{ac} \cos \Omega t$ of arbitrary (angular) frequency $\Omega$ and arbitrary amplitude $V_{ac}$, which modulates the virtual-level energy $\epsilon_{\text{dot}}(t)$. Such a system may be described by a constrained ($U = \infty$) Anderson Hamiltonian

$$
\sum_{\sigma} \epsilon_{\text{dot}}(t) n_{\sigma} + \sum_{k \sigma} \left[ \epsilon_{k \sigma} n_{k \sigma} + (V_{k} c_{k \sigma}^\dagger c_{\sigma} + \text{H.c.}) \right].
$$

(1)

Here $c_{\sigma}^\dagger$ creates an electron of spin $\sigma$ in the quantum dot, while $n_{\sigma}$ is the corresponding number operator; $c_{k \sigma}^\dagger$ creates a corresponding reservoir electron; $k$ is shorthand for all other quantum numbers of the reservoir electrons, including the designation of left or right reservoir, while $V_{k}$ is the tunneling matrix element through the appropriate barrier. Because the charging energy to add a second electron, $U = e^2/C$, is assumed large, the Fock space in which the Hamiltonian operates is restricted to those elements with zero or one electron in the dot.
At low temperatures, the Anderson Hamiltonian \( \hat{H} \) gives rise to the Kondo effect when the level energy \( E_{\text{dot}} \) lies below the Fermi energy. In this regime, a single electron occupies the dot which, in effect, turns the dot into a magnetic impurity with a free spin. The temperature required to observe the Kondo effect in linear response is of order \( T_K \sim D \exp(-\pi |E_{\text{dot}}|/\Gamma_{\text{dot}}) \), where \( D \) is the energy difference between the Fermi level and the bottom of the band of states. For the temperature range that is likely to be experimentally accessible in a SET, \( T \sim T_K \) or higher, there exists a well tested and reliable approximation known as the non-crossing approximation (NCA) \( ^7 \). The NCA has been formally generalized to the full time-dependent nonequilibrium case \( ^8 \), and an exact method for the (numerical) solution implemented \( ^9 \). The time-dependent NCA has been applied to Kondo physics in charge transfer in hyperthermal ion scattering from metallic surfaces \( ^9,10 \) and to energy transfer \( ^9 \). The time-dependent NCA solution for a quantum dot over the full time-dependent nonequilibrium case \( ^8 \), and an exact method for the (numerical) solution implemented \( ^9 \). The non-driven case is also shown in the final panel. Throughout this letter, energies are in units of \( \Gamma_{\text{dot}} \).

In Fig. 1 we show the calculated \( \langle \rho_{\text{dot}}(\epsilon, t) \rangle \) as a function of energy \( \epsilon \) for a level with energy \( \epsilon_{\text{dot}}(t) = \epsilon_{\text{dot}} + \epsilon_{\text{ac}} \cos \Omega t \) at several different frequencies \( \Omega \). The corresponding conductance is shown by the curve labeled dot \( A, T = 0.005 \) in Fig. 1. For the lowest \( \Omega \), the response of the system is relatively adiabatic and the displayed spectral function resembles the spectral function that would have resulted if the system had been in perfect equilibrium for all the dot level positions over a period of oscillation of \( \epsilon_{\text{dot}}(t) \). The two broad peaks are the influence of the virtual level peaks at the two stationary points of this oscillation (here at \( \epsilon = -1 \) and \( \epsilon = -9 \)). As the frequency \( \Omega \) is increased, marked nonadiabatic effects result, the most obvious being the appearance of multiple satellites around the Kondo resonance \( ^4 \). These sidebands appear at energies equal to \( \hbar \) times multiples of the driving frequency \( \Omega \) \( ^13 \). As the frequency \( \Omega \) is increased, spectral weight is transferred from the main Kondo peak to these satellites. As the conductance is dominated by \( \langle \rho_{\text{dot}}(\epsilon, t) \rangle \) at the Fermi energy, this causes the slow logarithmic falloff of the conductance over two decades of frequency, as shown in Fig. 3.

As \( \hbar \Omega \) becomes larger than \( \Gamma_{\text{dot}} \), inspection of Fig. 2 shows that broad satellites also appear at energy separations \( n\hbar \Omega \) around the average virtual-level position \( \epsilon_{\text{dot}} \). These satellites of the virtual level are the analogues of those predicted in the noninteracting case \( ^{13,14} \), which decrease in magnitude as the order \( n \) of the Bessel function \( J_n \). Here, however, the virtual-level satellites have their

\[
\rho_{\text{dot}}(\epsilon, t) \]
own Kondo peaks; each of the latter gets strong when
the corresponding virtual-level satellite reaches a posi-
tion a little below the Fermi level, and then disappears
as the broad satellite crosses the Fermi level. This effect
produces the oscillations in the conductance that are
evident in the lower curves in Fig. 3. These oscillations are
very different from those that would occur in a noninter-
acting ($U = 0$) case: due to the Kondo peaks they are
substantially stronger, their maxima occur at different
frequencies and their magnitudes are temperature depen-
dent. As the last virtual-level satellite crosses the Fermi
level, $\hbar \Omega = |\epsilon_{\text{dot}}|$, the dot level energy begins to vary
too fast for the system to respond and the average spec-
tral function approaches (exactly as $\Omega \to \infty$) the equi-
librium spectral function for a dot level centered at the
average position $\epsilon_{\text{dot}}$. For the parameters of Fig. 3, the
high frequency region is uninteresting, because the tem-
perature is far above the Kondo temperature. Therefore,
the conductance shows little temperature or frequency
dependence at these high frequencies.

\[ \langle \rho_{\text{dot}}(\epsilon, t) \rangle \approx \frac{1}{T_{\text{eff}}} \left[ J_1(\epsilon_{\text{ac}}/\hbar \Omega) \right]^2 \Gamma_{\text{dot}}(\epsilon_{\text{dot}} + \hbar \Omega). \]

The above rate carries with it an energy uncertainty,
which we speculate has roughly the same effect on the
Kondo peak as the energy smearing due to a finite tem-
perature. We can test this conjecture by calculating the
equilibrium conductance at an effective temperature $T_{\text{eff}}$
given by $T_{\text{eff}} = T + \Gamma_{\text{decay}}$. The results of such a cal-
culation are shown in Fig. 3 (PAT curves), where they
compare very favorably with our results for the conduc-
tance in the ac-driven system.

Returning to the behavior at low frequencies, we find
that it can be best understood in terms of the Kondo
Hamiltonian, which, with respect to properties near the
Fermi level, is equivalent to the Anderson Hamiltonian
$H$ in the extreme Kondo region $-\epsilon_{\text{dot}} \gg \Gamma_{\text{dot}}$ [17]. In
this limit the dot can be replaced simply by a dynami-
ical Heisenberg spin $\hat{S}$ ($S^2 = \frac{3}{4}$), which scatters electrons
both within and between reservoirs. The Kondo Hamil-
tonian corresponding to the Anderson model $H$ is

\[ \sum_{kk'\sigma\sigma'} J_{kk'}(t) \left( \hat{S} \cdot \delta_{\sigma\sigma'} + \frac{1}{2} \delta_{\sigma\sigma'} \right) c_{k\sigma}^\dagger c_{k'\sigma'}, \]

where the components of $\hat{\sigma}$ are the Pauli spin matri-
ces. For near Fermi level properties we can suppress the
detailed $k$ dependence of $J$ and $V$ and introduce a large
energy cutoff $D$ [18], in which case the relation-
ship between the Kondo and Anderson Hamiltonians [17]
is $J(t) = |V|^2/\epsilon_{\text{dot}}(t)$ for our $U = \infty$ case. If we let $w_{\text{leads}}(\epsilon)/\hbar$ be the total rate at which lead electrons of
energy $\epsilon$ undergo intralead and interlead scattering by
the dot, then $w_{\text{leads}}(\epsilon)$ will have a Kondo peak for $\epsilon$ near the Fermi level. Furthermore, if $J$ is modulated
as $J(t) = \langle J(1 + \alpha \cos \Omega t) \rangle$, then an electron scattered by
the dot will be able to absorb or emit multiple quanta
of energy $\hbar \Omega$, leading to satellites of the Kondo peak in
$\langle \rho_{\text{dot}}(\epsilon, t) \rangle$ through the exact Anderson model relation

\[ \Gamma_{\text{decay}} \approx |J_1(\epsilon_{\text{ac}}/\hbar \Omega)|^2 \Gamma_{\text{dot}}(\epsilon_{\text{dot}} + \hbar \Omega). \]
where \( \rho \) is the density of the dot. At high frequencies, a cutoff of the Kondo peak due to photon-assisted tunneling processes accounts for the reduction of conductance. We hope that our work will inspire experimental investigation of these phenomena and other ramifications of ac driving applied to Kondo systems.

The work was supported in part by NSF grants DMR 95-21444 (Rice) and DMR 97-08499 (Rutgers), and by US-Israeli Binaitional Science Foundation grant 94-00277/1 (BGU).

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