Weyl degree of freedom in the Nambu-Goto string through field transformation

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Abstract – We show how Weyl degrees of freedom can be introduced in the Nambu-Goto string in the path-integral formulation using the reparametrization invariant measure. We first identify Weyl degrees in conformal gauge using BFV formulation. Further we change the Nambu-Goto string action to the Polyakov action. The generating functional in light-cone gauge is then obtained from the generating functional corresponding to the Polyakov action in conformal gauge by using suitably constructed finite-field-dependent BRST transformation.

Introduction. – Bosonic strings are formulated using two alternative actions namely the Nambu-Goto (NG) action [1] and the Polyakov action [2]. In the Polyakov formulation, one uses the metric of the string worldsheet, \( g_{\alpha \beta} \), to manifest the Weyl and re-parametrization symmetries at the classical level. These symmetries are useful to eliminate the degrees of freedom of \( g_{\alpha \beta} \). Due to the presence of Weyl anomaly, the Weyl freedom may become dynamical at the quantum level [2]. The well-known factor, \((D - 26)\), appears naturally in the Polyakov path-integral formulation. On the other hand, in the NG string only string coordinates \( X^\mu (\mu = 0, \ldots, D - 1) \) are used as dynamical variables and hence no Weyl freedom is present from the beginning. Attempts have been made without much success to replace Weyl degrees of freedom by considering the quantum longitudinal mode in the covariant formulation of the NG string in the sub-critical dimension [3,4].

On the other hand, noncovariant gauges like the light-cone gauge are not easily incorporated in the path-integral formulation [5]. Hence a theoretical comparison of quantization in various gauges is restricted to a certain extent [6]. However this problem is overcome through Batalin-Fradkin-Vilkovisky (BFV) formulation [7] by considering the quantization using the most general gauge condition. The main idea of the BFV formulation is to convert the second-class constraints to first-class constraints [8], by introducing new canonical variables which are referred as the BF fields. We consider the NG string theory in BFV formulation at the sub-critical dimensions, where the BF fields appear as the conformal degrees of freedom [9,10]. The Weyl degrees of freedom are generally introduced based on the analysis of Fujikawa [11] using the re-parametrization invariant measure for the path integral. It has been observed [11] that for a re-parametrization invariant path-integral measure the string coordinates \( X^\mu \) must be scaled by \((- \det g_{\alpha \beta})^{1/4}\) so as to have the Weyl weight one.

BRST quantization [12] is an important and powerful technique to deal with a system with constraints [13]. It enlarges the phase space of a gauge theory and restores the symmetry of the gauge fixed action in the extended phase space keeping the physical contents of the theory unchanged. We indicate how various BRST invariant effective theories are interlinked by considering the finite-field-dependent version of the BRST(FFBRST) transformation, introduced by Joglekar and Mandal [14] about 24 years ago. FFBRST transformations are the generalization of the usual BRST transformation where the usual infinitesimal, anti-commuting constant transformation parameter is replaced by a field-dependent but global and anti-commuting parameter. Such generalized transformation protects the nilpotency and retains the symmetry of the gauge fixed effective actions. The remarkable property of such transformations is that they relate the generating functional corresponding to different effective actions. The nontrivial Jacobian of the path-integral measure under such a finite transformation is responsible for all the
new results. In virtue of this remarkable property, FF-BRST transformations have been investigated extensively and have found many applications in various gauge field theoretic systems [14–27]. A similar generalization of the BRST transformation with the same motivation and goal has also been carried out more recently in a slightly different manner [28] where a Jacobian for such transformation is calculated without using any ansatz.

In the present work, we considered the Polyakov action in the path-integral approach in the conformal gauge when Weyl degrees of freedom are naturally incorporated through the reparametrization invariance of the path-integral measure [11]. By constructing an appropriate finite-field–dependent BRST transformation we connect the generating functional in conformal gauge to that of in light-cone gauge in the Polyakov formulation of the NG string. Thus, Weyl degrees of freedom are incorporated in the NG string through the FFBRST transformation. We will closely follow the procedures followed by Igashishi and Kubo [29].

Now we present the plan of the paper. In the next section we will analyse the constraints for the NG action and we will use the BFV formulation [9]. In the third section we will transform the NG action to the Polyakov action. Then BRST invariant actions are written in both conformal and light-cone gauges. In the fourth section we will connect generating functionals in conformal and light-cone gauges using appropriately constructed FFBRST transformations. The last section is kept for concluding remarks.

**BRST for Nambu-Goto action.** – The Nambu-Goto action for the string coordinates $X^\mu$, $\mu = 0, 1, \ldots, D - 1$ on a two-dimensional worksheet parametrized by $x^a = (\tau, \sigma)$, $a = 0, 1$ is written as [1]

$$S_0 = \int d^2x(- \det G_{a\beta})^{\frac{1}{2}}, \quad (1)$$

where

$$G_{a\beta} = \partial_a X^\mu \partial_\beta X_\mu. \quad (2)$$

The momentum conjugate to $X^\mu$ for this theory is then written as

$$P_\mu = \sqrt{-\det G_{a\beta}} \partial_\mu X^a G^{a\alpha}, \quad (3)$$

where $G = \det G_{a\beta}$. The Hamiltonian corresponding to this system vanishes. This system has two primary constraints which generate two re-parameterizations of the string worksheet and are written as

$$\phi_\pm = \frac{1}{4}(P_\mu^2 + (\partial_\sigma X^\mu)^2) \pm \frac{1}{2} \partial_\sigma X^\mu P_\mu. \quad (4)$$

These constraints are first class at the classical level but appear as second class at the quantum level due to conformal anomaly. To convert them into first class we will introduce the new field $\theta$ and its momentum conjugate $\Pi_\theta$ in the action. The new effective constraints then take the form

$$\bar{\phi}_\pm = \phi_\pm + \frac{k}{\sqrt{2}}(\partial_\sigma \Pi_\theta \pm (\partial_\theta^2 \theta)) + (\Pi_\theta \pm \partial_\sigma \theta)^2, \quad (5)$$

where $k$ is a constant which is fixed as [30]

$$k = \frac{(25 - D)}{24\pi}. \quad (6)$$

We further extend the phase space by introducing the following pair of fields:

$$\{C^\pm, \bar{P}_\pm\}, \quad \{P^\pm, \bar{C}_\pm\}, \quad \{N^\pm, B_\pm\}. \quad (7)$$

Now, the action is written in the extended phase space as

$$S = \int d^2\sigma[\bar{X}^\mu P_\mu + \theta \Pi_\theta + \bar{C}^a \bar{P}_a + \{\psi, Q\}], \quad (8)$$

where the BRST charge $Q$ is given as

$$Q = \int d\sigma[C^\pm(\bar{\phi}_\pm + \bar{P}_\pm \partial_\sigma C^\pm) + B^\pm P_\pm]. \quad (9)$$

One can easily find out that $Q^2$ is nilpotent in nature and the gauge-fixing functional takes the form

$$\Psi = \int d\sigma(i\bar{C}_a \chi^a + \bar{P}_a N^a), \quad (10)$$

where $X^a$ does not depend on ghost, anti-ghost, $B$ and $N$ fields.

After eliminating all the nondynamical variables, the BRST transformation for the dynamical variables is written as [9,30]

$$\delta X^\mu = -\frac{1}{2}(C^a \partial_a X^\mu),$$

$$\delta \theta = -\frac{k}{2}(C^a \partial_a \theta + \frac{k}{2\sqrt{2}}(\partial_\tau C^+ - \partial_- C^-)), \quad \delta C^\pm = -\frac{1}{4} C^\pm \partial_\tau C^\pm, \quad \delta \bar{C}_\pm = -\frac{1}{4} \partial_\tau X^\mu \partial_\mu X_\mu \pm \bar{C}_\pm \partial_\tau C^\pm \pm \frac{1}{2} \partial_\tau \bar{C}_\pm C^\pm$$

$$\quad - \frac{1}{4} \partial_\tau \theta \partial_\tau \theta \pm \frac{k}{2\sqrt{2}} \partial_\tau \bar{C}_\pm \partial_\tau \theta, \quad (11)$$

which leaves the action in eq. (8) invariant.

Now, the total Lagrangian density has the form

$$\mathcal{L} = \mathcal{L}_x + \mathcal{L}_{gf} + \mathcal{L}_{gh}, \quad (12)$$

where $\mathcal{L}_x$ denotes the string part of the Lagrangian density and the gauge-fixing and ghost term are defined as

$$\lambda(\mathcal{L}_{gf} + \mathcal{L}_{gh}) = -i\delta(\bar{C}_a \chi^a), \quad (13)$$

where $\lambda$ is the infinitesimal Grassmann parameter. In the next section we are going to discuss the BRST symmetric Polyakov action in conformal as well as in light-cone gauge.

**Polyakov action.** – Following the technique in ref. [29] we convert the NG action to a Polyakov action as

$$\mathcal{L}_x = -\frac{1}{2}\bar{\phi}^{ab} \partial_\mu X^a \partial_\mu X^b - \frac{1}{2} \bar{\phi}^{ab} \partial_\mu \theta \partial_\mu \theta. \quad (14)$$
Here the $\theta$-dependent term in the above Lagrangian density brings extra degrees of freedom in the system. We need a reparametrization invariant measure in the path-integral formulation to construct the BRST symmetry of this theory. Using the methods described in [31,32] we construct the BRST transformation as

$$\delta X^\mu = -\frac{1}{2}(C^a \partial_\mu X^a),$$

$$\delta \theta = -\frac{1}{2}(C^a \partial_\mu \theta) + \frac{k}{2\sqrt{2}}(\partial_\mu C^0 - \partial_1 C^1),$$

$$\delta C^a = -\frac{1}{4} C^a \cdot \partial_\mu C^a,$$

$$\delta C_a = \frac{1}{4} \partial_a X^\mu \partial_\mu X_a \pm C_a \partial_\mu C^a \pm \frac{1}{2} \partial_a \overline{\partial}_\mu C^a$$

$$\delta C^{ab} = \partial_a C^a \overline{\partial}_b C^b + \partial_b C^b \overline{\partial}_a C^a - \partial_\mu (C^a \overline{\partial}_b C^{ab}).$$

We define the generating functional in the path-integral formulation as

$$Z = \int D\phi \exp \left(i \int d^2x (L_x + L_{gf} + L_{gh}) \right),$$

where the Lagrangian density is given by eqs. (14) and (13) and $D\phi$ is the generic notation for the path-integral measure. Transformation in eq. (15) leaves the effective action invariant. Now we fix the gauge more specifically and discuss BRST invariant effective theories in conformal as well as light-cone gauges.

**Conformal gauge.** Conformal gauge has been used extensively in the discussion of various problems. It has been used to study strings, gravity etc. in path-integral and covariant operator formalism. This gauge is very useful to remove conformal anomaly, to introduce Weyl symmetry and in renormalizing the theory [11,30,31].

The conformal gauge condition is expressed as $\overline{g}^{ab} = \eta^{ab}$ [10,31] and is incorporated into the following gauge-fixing and FP ghost term in a BRST invariant manner:

$$L_{gf} = \lambda (L_{gf} + L_{gh}) = -i \delta B (\overline{C}^a \overline{g}^{ab} + \overline{C}^1 \overline{g}^{a1}).$$

Here $\overline{C}^a$ and $\overline{C}^1$ are anti-ghost fields.

**Light-cone gauge.** On the other hand, light-cone gauge is used to eliminate unphysical degrees of freedom and also in decoupling ghost fields. Light-cone gauge has also been used in the Kaku-Kikkawa string field theory, in showing the ultraviolet finiteness of $N = 4$ supersymmetric Yang-Mills theory, in dimensional regularization, in gravity, supergravity, string and superstrings theories [33].

The light-cone gauge condition $(X^+ = f(\sigma), \overline{g}^{ab} = 0)$ [10,32] is incorporated into the following gauge-fixing and ghost term in a BRST invariant manner:

$$L_{lc} = \lambda (L_{gf} + L_{gh}) = -i \delta B (\overline{C}^0 \overline{g}^{a0} + \overline{C}^1 (X^+ - f(\sigma))).$$

Here $f(\sigma)$ is an arbitrary function of $\sigma^0$ and $\sigma^1$.

Now we proceed to use FFBRST to address the Weyl degrees of freedom in NG string formulation.

**Connection between generating functionals in conformal and light-cone gauges.** Before going for the connection between the two effective theories we briefly discuss the ideas of FFBRST developed in ref. [14]. The BRST transformations are generated from BRST charge using the relation $\delta \phi = -[Q, \phi] \delta \Lambda$, where $\delta \Lambda$ is an infinitesimal anti-commuting global parameter. Following their technique the anti-commuting BRST parameter $\delta \Lambda$ is generalized to be finite-field–dependent instead of the infinitesimal but space-time–dependent parameter $\Theta[\phi]$. Since the parameter is finite in nature unlike the usual case the path-integral measure is not invariant under such finite transformation. The Jacobian for these transformations for a certain $\Theta[\phi]$ can be calculated in the following way:

$$D\phi = J(k) D\phi' (k)$$

$$= J(k + dk) D\phi'(k + dk),$$

where a numerical parameter $k$ ($0 \leq k \leq 1$), has been introduced to execute the finite transformation in a mathematically convenient way. All the fields are taken to be $k$-dependent in such a fashion that $\phi(x, 0) = \phi(x)$ and $\phi(x, k = 1) = \phi'(x)$. $S_{eff}$ is invariant under FFBRST which is constructed by considering successive infinitesimal BRST transformations ($\phi(k) \rightarrow \phi(k + dk)$). The non-trivial Jacobian $J(k)$ is then written as local functional of fields and will be replaced as $e^{i S_1[\phi(k), k]}$ if the condition

$$\int D\phi(k) \left[ \frac{1}{J(k)} \frac{dJ(k)}{dk} - i \frac{dS_1}{dk} \right] e^{i S_1[S_1, S_{eff}]} = 0$$

holds [13]. Here $\frac{d}{dk}$ is a total derivative of $S_1$ with respect to $k$ in which the dependence on $\phi(k)$ is also differentiated. The change in the Jacobian is calculated as

$$\frac{J(k)}{J(k + dk)} = \Sigma \pm \frac{\delta \phi(x, k + dk)}{\delta \phi(x, k)}$$

$$= 1 - \frac{1}{J(k)} \frac{dJ(k)}{dk} dk,$$

where ± is for bosonic and fermionic fields, respectively.

In this section, we construct the FFBBRST transformation with an appropriate finite parameter to obtain the generating functional corresponding to $L_{gf}$ from that corresponding to $L_{fc}$. We calculate the Jacobian corresponding to such a FFBBRST transformation following the method outlined in ref. [14] and show that it is a local functional of fields and accounts for the differences of the two FP effective actions.

The generating functional corresponding to the FP effective action $S_{eff}$ is written as

$$Z_{gf} = \int D\phi \exp(i S_{gf}[\phi]),$$

where $S_{gf}$ is given by

$$S_{gf} = \int d^2 x (L_x + L_{gf}).$$
Now, to obtain the generating functional corresponding to $S_{lc}$, we apply the FFFBRST transformation with a finite parameter $\Theta[\phi]$ which is obtained from the infinitesimal but field-dependent parameter, $\Theta'[\phi(k)]$, through
\[
\int_0^1 \Theta'[\phi(k)]dk; \text{ we construct } \Theta'[\phi(k)] \text{ as }
\]
\[
\Theta'[\phi] = i \int d^2x [\gamma C_1 \{(X^+ - f(\sigma)) - \delta_{\phi} \}]. \quad (24)
\]
Here $\gamma$ is an arbitrary constant parameter and all the fields depend on the parameter $k$. The infinitesimal change in the Jacobian corresponding to this FFFBRST transformation is calculated using eq. (21)

\[
\frac{1}{J(k)} \frac{dJ(k)}{dk} = -i \int d^2x [\delta^B(C_1) \{(X^+ - f(\sigma)) - \delta_{\phi} \} - (C^a \partial_a X^+) \delta_{\phi} - \delta g_{\phi} - C_1]. \quad (25)
\]

To express the Jacobian contribution in terms of a local functional of fields, we make an ansatz for $S_1$ by considering all the possible terms that could arise from such a transformation as

\[
S_1[\phi(k), k] = \int d^2x \left[ -\xi_1 \delta^B(C_1)(X^+ - f(\sigma)) - \xi_2 \delta^B(C_1)\delta_{\phi} - \xi_3 (C^a \partial_a X^+) C_1 + \xi_4 \delta g_{\phi} - C_1 - \xi_5 \delta^B(C_0) \delta_{\phi} + \xi_6 \delta g_{\phi} - \delta_{\phi} \right], \quad (26)
\]

where all the fields are considered to be $k$-dependent and we have introduced arbitrary $k$-dependent parameters $\xi_n(\xi_n(k) \text{ for } n = 1, 2, \ldots, 6)$ with the initial condition $\xi_n(0) = 0$. It is straightforward to calculate

\[
\frac{dS_1}{dk} = \int d^2x \left[ \xi_1 \delta^B(C_1)(X^+ - f(\sigma)) - \xi_2 \delta^B(C_1)\delta_{\phi} - \xi_3 (C^a \partial_a X^+) C_1 + \xi_4 \delta g_{\phi} - C_1 - \xi_5 \delta^B(C_0) \delta_{\phi} + \xi_6 \delta g_{\phi} - \delta_{\phi} \right], \quad (27)
\]

where $\xi_n = \frac{d\xi_n}{dk}$. Now we will use the condition of eq. (20):

\[
\int D\phi \exp[i(S_{cf} + S_1[\phi(k), k])] \int d^2x [(\gamma - \xi_1) \delta^B(C_1) \times (X^+ - f(\sigma)) - (\gamma + \xi_1) \delta^B(C_1)\delta_{\phi} - (\gamma + \xi_1) \delta g_{\phi} - C_1 - \xi_5 \delta^B(C_0) \delta_{\phi} + \xi_6 \delta g_{\phi} - \delta_{\phi} \times \Theta' \{(\xi_1 - \xi_3) C^a \partial_a X^+ - \delta^B(C_1)(\xi_2 + \xi_4) \delta g_{\phi} - \delta^B(C_1) + (\xi_5 + \xi_6) \delta g_{\phi} - \delta^B(C_0) \} = 0. \quad (28)
\]

The terms proportional to $\Theta'$, which are nonlocal due to $\Theta'$, vanish independently if

\[
\begin{align*}
\xi_1 - \xi_3 &= 0, \\
\xi_2 + \xi_4 &= 0, \\
-\xi_5 + \xi_6 &= 0. \quad (29)
\end{align*}
\]

To make the remaining local terms in eq. (28) vanish, we need the following conditions:

\[
\begin{align*}
\gamma - \xi_1 &= 0, \\
\gamma + \xi_1 &= 0, \\
\gamma - \xi_3 &= 0, \\
\gamma - \xi_4 &= 0, \\
\xi_2 &= 0, \\
\xi_6 &= 0. \quad (30)
\end{align*}
\]

The differential equations for $\xi_n(k)$ can be solved with the initial conditions $\xi_n(0) = 0$ to obtain the solutions

\[
\xi_1 = \gamma k, \quad \xi_2 = -\gamma k, \quad \xi_3 = \gamma k, \quad \xi_4 = \gamma k, \quad \xi_5 = \xi_6 = 0. \quad (31)
\]

Putting the values of these parameters in the expression of $S_1$, and choosing the arbitrary parameter $\gamma = -1$ we obtain

\[
S_1[\phi(1), 1] = \int d^2x [\delta^B(C_1)(X^+ - f(\sigma)) - \delta^B C_1 \delta_{\phi} - (C^a \partial_a X^+) C_1 - \delta g_{\phi} - C_1]. \quad (32)
\]

Thus, the FFFBRST transformation with the finite parameter $\Theta$ that is defined by eq. (24) changes the generating functional $Z_{cf}$ as

\[
\begin{align*}
Z_{cf} &= \int D\phi \exp(iS_{cf}[\phi]) \\
&= \int D\phi' \exp[i(S_{cf}[\phi'] + S_1[\phi', 1])] \\
&\quad = \int D\phi \exp[i(S_{cf}[\phi] + S_1[\phi, 1])] \\
&\quad = \int D\phi \exp(iS_{cf}[\phi] + S_1[\phi, 1]) \equiv Z_{lc}. \quad (33)
\end{align*}
\]

Here $S_{lc}$ is defined as

\[
S_{lc} = \int d^2x (L + L_{lc}). \quad (34)
\]

In this way the FFFBRST transformation with the finite-field-dependent parameter in eq. (24) connects the generating functional for the Polyakov action in conformal gauge to that of in the light-cone gauge.

**Conclusion.** – In this present work we have demonstrated how Weyl degrees of freedom are incorporated in the formulation of NG string through a certain field transformation. Weyl degrees of freedom are first identified in conformal gauge using BFV formulation. Then we have established the connection between conformal gauge to light-cone in a Polyakov-type action for NG string using the technique of FFFBRST transformation, which connects various theories through the nontrivial Jacobian of the path-integral measure. The nonlocal BRST transformation by Igarashi et al. in [29] is nothing but a particular type of FFFBRST transformation. The parameter $\lambda$ in
the nonlocal transformation in [29] is identified with the FFBRST parameter \( \Theta' \).

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