LORENTZ VIOLATION AND TORSION

NEIL RUSSELL

Physics Department, Northern Michigan University,
Marquette, MI 49855, U.S.A.

In this proceedings, similarities between the structure of theories with Lorentz violation and theories with constant torsion in flat spacetime are exploited to place bounds on torsion components. An example is given showing the analysis leading to bounds on the axial-vector and mixed-symmetry components of torsion, based on a dual-maser experiment.

1. Introduction

In conventional Riemann-Cartan theory, torsion is minimally coupled to a fermion. Several nonminimal generalizations are possible, including non-minimal couplings to a fermion, nonminimal couplings involving a single particle of another species (e.g., the photon), and couplings involving more than one particle. We limit consideration to signals of torsion effects arising from nonminimal couplings of the torsion tensor to one or more fermions only. A more complete analysis is available elsewhere.

We focus on the analysis of effects relevant for laboratory experiments that can be, or have been, done. For simplicity, torsion is taken as a fixed background field in a Sun-centered inertial frame. To consider fluctuations, the Nambu-Goldstone and massive sectors have to be incorporated. Other approaches, which include ones seeking sensitivity to effects in dynamical torsion theories, are not considered here. The literature on torsion includes reviews by Hehl et al., Shapiro, and Hammond. Possible bounds from Hughes-Drever experiments have been discussed by Lämmerzahl.

Studies have shown that Lorentz violation and the associated CPT violation can arise in string theory. However, it is possible to describe all the effects at the level of effective field theory. A systematic framework that encompasses global Lorentz violation and local Lorentz violation exists, known as the Standard-Model Extension, or SME.

The similarities between lagrangian terms coupling fermions to torsion
and ones coupling fermions to Lorentz-violating backgrounds means that experimental sensitivity to Lorentz-violation effects cannot easily be decoupled from sensitivity to torsion effects. Here, we assume no Lorentz violation, and interpret experimental results entirely in terms of torsion.

2. Basics

We adopt the conventions of Ref. [13]. The general metric $g_{\mu\nu}$ has diagonal entries $(-1, 1, 1, 1)$ in the flat-space limit, and the antisymmetric tensor $\epsilon^{\mu\nu\alpha\beta}$ is defined so that $\epsilon^{0123} = -1$.

The Riemann-Cartan curvature tensor, denoted by $R_{\mu\nu\alpha\beta}$, consists of the usual Riemann curvature tensor $\tilde{R}_{\mu\nu\alpha\beta}$ and added terms involving the contortion. We are interested in the limit of spacetime with diagonal metric $(-1, 1, 1, 1)$, in which case the Christoffel symbols are zero and the usual curvature tensor $\tilde{R}_{\mu\nu\alpha\beta}$ vanishes. We refer to this as ‘flat spacetime.’ In flat spacetime, the Riemann-Cartan curvature tensor $R_{\mu\nu\alpha\beta}$ does not necessarily vanish.

The torsion tensor $T^{\alpha}_{\mu\nu}$ is antisymmetric in the second and third indices, and so has 24 independent components. We define the trace part $T^{\mu}_{\mu}$ and the antisymmetric part $A^{\nu}$ of the torsion tensor as follows:

$$T^{\mu}_{\mu\beta} = \frac{1}{3} (g_{\mu\alpha} T^{\alpha}_{\beta} - g_{\mu\beta} T^{\alpha}_{\alpha}) - \epsilon_{\alpha\beta\mu\nu} A^{\nu} + M_{\mu\alpha\beta}, \quad (1)$$

where $M_{\mu\alpha\beta}$ is unique and is called the mixed-symmetry component.

3. Fermions in flat spacetime with torsion

The lagrangian for an electron of mass $m$ in flat spacetime with all possible independent torsion couplings up to dimension five is [113]

$$\mathcal{L}^{T, 5} \approx \frac{1}{2} i \bar{\psi} \gamma^{\mu} \overset{\leftrightarrow}{\partial_\mu} \psi - m \bar{\psi} \psi + \xi_{1}^{(4)} T^{\mu}_{\mu} \bar{\psi} \gamma^{\nu} \gamma^{\mu} \psi + \xi_{2}^{(4)} T^{\mu}_{\nu} \bar{\psi} \gamma_{5} \gamma^{\mu} \psi + \xi_{3}^{(4)} A_{\mu} \bar{\psi} \gamma^{\nu} \psi + \xi_{4}^{(4)} A_{\mu} \bar{\psi} \gamma_{5} \gamma^{\mu} \psi + \frac{1}{2} \xi_{5}^{(5)} T^{\mu}_{\nu} \bar{\psi} \overset{\leftrightarrow}{\partial_\mu} \psi + \xi_{6}^{(5)} T^{\mu}_{\nu} \bar{\psi} \gamma_{5} \overset{\leftrightarrow}{\partial_\mu} \psi + \frac{1}{2} \xi_{7}^{(5)} A_{\mu} \bar{\psi} \gamma_{5} \overset{\leftrightarrow}{\partial_\mu} \psi + \frac{1}{2} \xi_{8}^{(5)} T^{\lambda\mu\nu} A_{\lambda\mu} \bar{\psi} \overset{\leftrightarrow}{\partial_\nu} \sigma_{\mu\nu} \psi + \frac{1}{2} \xi_{9}^{(5)} T^{\lambda\mu\nu} T^{\lambda}_{\mu\nu} \bar{\psi} \overset{\leftrightarrow}{\partial_\mu} \sigma_{\mu\nu} \psi + \frac{1}{2} \xi_{9}^{(5)} \epsilon^{\lambda\kappa\mu\nu} A_{\lambda\mu} \bar{\psi} \overset{\leftrightarrow}{\partial_\kappa} \sigma_{\mu\nu} \psi. \quad (2)$$
This expression includes four coupling constants with dimension $m^{0}$: $\xi^{(4)}_{1}, \ldots, \xi^{(4)}_{4}$, and nine with dimension $m^{-1}$: $\xi^{(5)}_{1}, \ldots, \xi^{(5)}_{9}$. These terms can be arranged so as to match the Minkowski-spacetime limit of the fermion-sector Lorentz-violating lagrangian as given in Ref. 13. In making this match, we assume the torsion components are constants, as are the SME coefficients. Using Eqs. (12) to (14) of Ref. 13, and assuming zero torsion and no electromagnetic field, we have:

$$\mathcal{L}_{SME} = \frac{1}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi - a_{\mu} \bar{\psi} \gamma_5 \gamma^\mu \psi - \frac{1}{2} H_{\mu\nu} \bar{\psi} \sigma^{\mu\nu} \psi - \frac{1}{2} i e_{\mu} \bar{\psi} \gamma^{5} \gamma^\mu \partial^\nu \psi - \frac{1}{2} i d_{\mu\nu} \bar{\psi} \gamma_5 \gamma^\mu \partial^\nu \psi - \frac{1}{2} i g_{\mu\nu\lambda} \bar{\psi} \sigma^{\mu\nu} \partial^\lambda \psi.$$ (3)

The SME coefficient $g_{\lambda\mu\nu}$ appearing in the last term can be decomposed in the same way as in Eq. (1):

$$g_{\mu\nu} = \frac{1}{3} (g^{(T)}_{\mu} \delta^\lambda_\nu - g^{(T)}_{\nu} \delta^\lambda_\mu) - \epsilon_{\mu\nu} \lambda^\kappa g_{\kappa}^{(A)} + g_{\mu\nu}^{(M)} \lambda,$$ (4)

where $g^{(T)}_{\mu}$, $g^{(A)}_{\mu}$ and $g^{(M)}_{\lambda\mu\nu}$ are suitably-defined trace, axial-vector, and mixed-symmetry components. If we make this substitution and match Eqs. (2) and (3) term by term, a number of identities result, including, for example:

$$b_{\mu} = -\xi^{(4)}_{2} T_{\mu} - \xi^{(4)}_{4} A_{\mu},$$ (5)

$$g_{\kappa}^{(A)} = -2(\xi^{(5)}_{8} T_{\kappa} + \xi^{(5)}_{9} A_{\kappa}),$$ (6)

$$g_{\mu\nu}^{(M)} = -2\xi^{(5)}_{5} M_{\lambda\mu\nu}.$$ (7)

A variety of experiments are sensitive to Lorentz-violation coefficients, including $b_{\mu}$ and $g_{\lambda\mu\nu}$, and these equations show they must also be sensitive to torsion effects.

### 4. Connecting with experiments

Experimental sensitivities in the case of ordinary matter are to 40 tilde coefficients, defined in Appendix B of Ref. 15. As an example, we’ll consider two of these, $\tilde{b}_{X}$ and $\tilde{g}_{DX}$. Using the decomposition in Eq. (1), they can
be expressed in terms of the irreducible components of $g_{\lambda \mu \nu}$:

\[
\tilde{b}_X = b_X - m g^{\lambda (A)}_X + m g^{(M)}_{YZT}, \\
\tilde{g}_{DX} = -b_X + m g^{(A)}_X + 2 m g^{(M)}_{YZ T}.
\] (8)

If we now use Eqs. (5), (6), and (7), we obtain relationships between experimental tilde observables in the SME and irreducible components of the torsion tensor:

\[
\tilde{b}_X = -(\xi_4^{(4)} - 2 m \xi_5^{(5)}) T_X - (\xi_4^{(4)} - 2 m \xi_9^{(5)}) A_X - 2 m \xi_5^{(5)} M_{TYZ}, \\
\tilde{g}_{DX} = +(\xi_2^{(4)} - 2 m \xi_8^{(5)}) T_X + (\xi_4^{(4)} - 2 m \xi_9^{(5)}) A_X - 4 m \xi_5^{(5)} M_{TYZ}.
\] (10)

It follows that any experiment with sensitivity to $\tilde{b}_X$ or $\tilde{g}_{DX}$ is also sensitive to the torsion components $T_X, A_X, M_{TYZ}$. To investigate the expected sensitivity, we next consider a specific experiment.

5. Dual-maser experiment

As an example, consider the result for a combination of coefficients\textsuperscript{16} given in the second line of Table II of Ref.\textsuperscript{17} which reports on a recent He-Xe dual-maser experiment:

\[-\tilde{b}_X + 0.0034 \tilde{d}_X - 0.0034 \tilde{g}_{DX} < (2.2 \pm 7.9) \times 10^{-32} \text{ GeV}.\] (12)

We note by inspection of Eqs. (2) and (3) that the $\tilde{d}_X$ coefficient is not relevant for torsion bounds, and may therefore be taken as zero. To introduce the experimental result, we substitute Eqs. (10) and (11), with the mass taken as that for the neutron, $m_n \simeq 0.938 \text{ GeV}$. This gives one particular torsion combination bounded by this experiment. Some simplification occurs because the substitution involves combinations of the same torsion terms, and so we may neglect the third term since the factor of 0.0034 is small. We extract a bound with a confidence level of about 90% by doubling the one-sigma uncertainty:

\[|\left(\xi_4^{(4)} - 2 m \xi_8^{(5)}\right) T_X + \left(\xi_4^{(4)} - 2 m \xi_9^{(5)}\right) A_X + 2 m \xi_5^{(5)} M_{TYZ}| < 1.6 \times 10^{-31} \text{ GeV}.\] (13)

The case of minimally-coupled torsion is recovered when $\xi_4^{(4)} = 3/4$ and all the other couplings are zero. Then, Eq. (13) yields the result

\[|A_X| < 2.1 \times 10^{-31} \text{ GeV}.\] (14)

To extract additional torsion results, we look at each term in Eq. (13) under the assumption that the other terms vanish. We find, for example, that

\[|\xi_5^{(5)} M_{TYZ}| < 10^{-31}.\] (15)
Additional bounds on components of the torsion tensor can be extracted by considering the other terms in Eq. (13), and by considering the other bounds reported in this particular experiment. More generally, the method adopted here can be used to seek torsion bounds from other experiments.

References
1. V.A. Kostelecký, N. Russell and J.D. Tasson, Phys. Rev. Lett. to appear, [arXiv:0712.4393].
2. R. Bluhm and V.A. Kostelecký, Phys. Rev. D 71, 065008 (2005); R. Bluhm et al., Phys. Rev. D, in press, [arXiv:0712.4119]; B. Altschul and V.A. Kostelecký, Phys. Lett. B 628, 106 (2005); V.A. Kostelecký and R. Potting, Gen. Rel. Grav. 37, 1675 (2005).
3. S.M. Carroll and G.B. Field, Phys. Rev. D 50, 3867 (1994).
4. A.S. Belyaev, I.L. Shapiro, and M.A.B. do Vale, Phys. Rev. D 75, 034014 (2007).
5. F.W. Hehl et al., Rev. Mod. Phys. 48, 393 (1974).
6. I.L. Shapiro, Phys. Rep. 357, 113 (2002).
7. R.T. Hammond, Rep. Prog. Phys. 65 599 (2002).
8. C. Lämmerzahl, Phys. Lett. A 228, 223 (1997).
9. V.A. Kostelecký and S. Samuel, Phys. Rev. D 39, 683 (1989); Phys. Rev. D 40, 1886 (1989); Phys. Rev. Lett. 63, 224 (1989).
10. V.A. Kostelecký and R. Potting, Nucl. Phys. B 359, 545 (1991).
11. V.A. Kostelecký and R. Potting, Phys. Rev. D 51, 3923 (1995).
12. D. Colladay and V.A. Kostelecký, Phys. Rev. D 55, 6760 (1997); Phys. Rev. D 58, 116002 (1998); V.A. Kostelecký and R. Lehnert, Phys. Rev. D 63, 065008 (2001).
13. V.A. Kostelecký, Phys. Rev. D 69, 105009 (2004).
14. For a tabulation, see V.A. Kostelecký and N. Russell, [arXiv:0801.0287].
15. R. Bluhm et al., Phys. Rev. D 68, 125008 (2003).
16. V.A. Kostelecký and C.D. Lane, Phys. Rev. D 60, 116010 (1999).
17. F. Canè et al., Phys. Rev. Lett. 93, 230801 (2004).