Estimation of the Crack Faces Interaction When Ice Beams are Destroyed by a Transvers Vertical Load

V V Knyazkov¹, N M Semenova¹, A S Sebin¹

¹“Shipbuilding and aviation technology” department, Nizhny Novgorod State Technical University n.a. R. E. Alekseev, 603950, Minin str., 24, Nizhny Novgorod, Russia

E-mail: ship@nntu.ru, ShaNaMix@yandex.ru, asebin@nntu.ru

Abstract. In the paper the mechanism of the conservation of the ice cover bearing capacity under the influence of transverse loads if there are cracks in it was considered. Experimental and theoretical study of the main aspects of this mechanism was performed on the ice beams obtained by sawing two parallel wide slits in the ice cover. At its ends, the beam was one with the entire ice plate. The obtained diagrams of beam failure made it possible to interpret cracks as hinges connecting separate blocks of ice cover in the lines of their axes. The finite element method for the subsequent evaluation of the proposed method of accounting for interaction on the crack faces was used. The research results can be used in the mathematical model making of the ice deformation and ice failure taking into account cracking, as well as in the development of simulation mode of ice cover.

1. Introduction

For a long time the failure criterion of ice cover by vertical load was considered to be the appearance of a crack in it from bending stresses [1, 2]. Observation the real situation of the ice failure under load [3] led to the formation of different failure mechanism. This mechanism in paper [4] was suggested. At the first stage when the load increases, the ice cover is divided by cracks into wedges coming from load application center. Further, as the load increases, the circumferential cracks appear cutting off the wedges from all ice cover. There is a peculiar construction from individual block caring even greater load due to the spacer forces appearing on the cracks faces. In building this phenomenon is called the “arch action” and was used as long ago as Ancient Rome when building bridge from individual stone blocks.

To demonstrate the arch action in the ice in paper [4] the experiment scheme with beams obtained by sawing two parallel wide slits in the ice cover was proposed. At its ends, the beam formed a single whole with the entire ice plate. The bending crack initiation in the center and at the ends of the beam proved to be not confined its resistance to increasing load. Later to interpret the cracks faces interaction a mathematical model of an elastic joint in paper [5] was proposed. This model is similar to the model with elastic-plastic joint used to describe the reinforced concrete deformation with cracks [9]. In this model, the relationship between the angle of mutual rotation of blocks and the ice plate characteristics was established.

Using this model in [6], it was shown that such a hinge model can explain the effect of the ring crack formation approaching the load.
In papers [4, 5] there are no sufficient complete experimental and theoretical investigations associated with crack faces interaction. This gap is filled by the paper [7]. New results of ice field failure research in laboratory environment are given. And then the results of beam failure research embedded in the ice cover are offered. The beam model embedded in the ice makes it possible to study the failure in a small pool [13].

2. Experimental investigation

2.1. Installation for execution of works

The experimental installation scheme of open ice pool is shown in the figure 1. The ice freezes naturally in circular pool 4.4 m in diameter and 0.8 m high. The load on the ice is created with a universal testing machine UTM-5 installed on the supporting truss 2. The frame 1 is connected to a fixed beam with two columns 3. The columns at the same time are guides for the walking beam 4. The weighting device is fixed to the screw 5. When loading the screw with its end rests on the ice. The weighting device consists of a body 6 and N. G. Tokarya dynamometer 7 with a displacement sensor DP-2SM 8. The sensor is connected to the vibration-measuring apparatus VI6-TN 11. The gear box of testing machine makes it possible to change the loading speed of ice cover from 2 to 100 mm per min. The weighting device is equipped with replaceable dies. To measure the ice cover deflection the PTP-11S potentiometer 12 is used. The thread 14 is thrown over the potentiometer block. One end of the thread is connected to the walking beam 4, and the other end is connected to the load 13. This load provides a constant thread tension when the walking beam is moved vertically during loading which is necessary for potentiometer block rotation. The sensor signals are recorded by a hardware-software complex. This system consists of a personal computer 15 with ADC 16. To record and process sensor signals the special software modules have been developed.

![Figure 1. The experimental installation scheme.](image-url)

2.2. Experiments on the beam failure

To check the fact that the cracking does not lead to a complete failure of ice cover bearing capacity and to study the mechanism of this phenomenon, a number of experiments on natural ice were performed. The tests on ice beams were carried out. The beams were obtained by sawing two parallel wide slits in the ice cover. It is the slits that prevented beam jamming when it is deformed. At its ends,
the beam formed a single whole with the entire ice plate. The load was applied in the beam center. The test circuit of ice beams is shown in figure 2.

![Figure 2. The test circuit of ice beams.](image)

For all experiments the deformation curve of ice cover $P=f(w)$ were obtained, i.e. the relation between the deflection under force and load was obtained. The charts are of the same type and have ascending and descending branches. The diagram is shown in figure 3.

![Figure 3. Diagram of ice beam failure: $l \times b \times h = 3800 \times 400 \times 26$ mm.](image)

When the ultimate tensile stresses are reached in the beam underlayer at the point of maximum bending moments under load, the first crack is formed. And this crack extends to the entire of the beam thickness (point I, see figure 3). However, the beam separated by the cracks exist in equilibrium due to contact compressive stresses on the cracks faces. The maximum compression and bearing stress of ice is known [9] to be several times higher than the maximum tensile stress.

Evidently the deflections of both faces at the crack are the same under joint deformation, and their rotation leads to an eccentric compression of the beam upper layers (figure 4). The pressure load can be replaced by an equivalent system of torque and compressive force along the zero line. The influence of this force on the bending elements is shown [2] can be ignored. Thus, the cracks faces interaction is reduced to the appearance of an additional bending moment preventing the beam deflection. Therefore, to further increase the deflection, it is necessary to increase the force value $P$.

When the force $P$ reaches the certain value in the areas of maximum bending moments in the upper layers at the beam ends, the next cracks were formed (point II, see figure 3). The cracks also spread to the entire beam thickness. In both cases, the central load drop corresponds to a moment of crack formation (point $I'$ и $II'$, see figure 3).
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Figure 4. Pressure load on the на crack faces:

φ – crack opening angel; w – beam deflection under force at the crack formation moment.

The obtained diagrams show that the ice cover behavior after the first cracks has an elastic character. It follows from the fact the subcritical part of the diagram (up to the point corresponding to the breaking load; point II, see figure 3) consist of linear sections with different slope angels to the X-axis. This diagram feature, i.e. the behavior of the ice cover with a crack under the load, makes it possible to use an approach similar to the one proposed in [5] and interpret the cracks as hinges connecting individual blocks of the ice cover along the lines of their axes.

3. Cracks simulation by the elastic joint

To determine additional bending moment acting on the crack faces, the Hertzian problem with mixed-boundary conditions depending on the crack opening angel is necessary to be solved. This task is rather difficult to solve. Therefore to simplify this problem the following assumption was introduced: the cracks faces interaction is elastic type and the relationship between the bending moment M and the crack opening angel φ is linear, i.e.

$$\phi = aM,$$ (1)

here a is proportionality constant.

Taking into account the thickness and elasticity of the ice the following formula can be obtained [5]:

$$a = \frac{A}{EI\alpha},$$ (2)

here A is nondimensional coefficient, characterizing the cracks faces interaction (0≤A<∞); EI – beam stiffness of a rectangular beam; E – ice elastic modulus; I – second moment.

The extreme cases for A=0 и A→∞ are corresponded to a beam without a crack and a free rotation of crack faces. The value of this coefficient can be determined using diagrams on the beam destruction.

The differential equation of the beam line on an elastic foundation is written as

$$EI \frac{d^4 w}{dx^4} + kw = q,$$ (3)

here q is the load intensity generally depending on the x coordinate; k = ρs g – foundation modulus (stiffness coefficient of the elastic foundation); ρs – density of water.

The general integral of equation (3), if q is an algebraic function of power no higher than the cube, can be written as follows [11]:
\[ w = \frac{q}{k} + C_1 F_1(\alpha x) + C_2 F_2(\alpha x) + C_3 F_3(\alpha x) + C_4 F_4(\alpha x), \]  

(4)

here \( F_i(\alpha x) \) are combinations of hyperbolic and trigonometric functions providing the transition of one function when differentiating into another and equality to zero for a zero argument of all functions (or their derivatives of the same order) expect one:  

\[ F_1(\alpha x) = \cosh \alpha x \cos \alpha x; \]
\[ F_2(\alpha x) = \sinh \alpha x \cos \alpha x - \cosh \alpha x \sin \alpha x; \]
\[ F_3(\alpha x) = -2 \sinh \alpha x \sin \alpha x; \]
\[ F_4(\alpha x) = -2 (\sinh \alpha x \cos \alpha x + \cosh \alpha x \sin \alpha x); \]

\[ \alpha = \left( \frac{k}{4EI} \right)^{1/4} \] – elastic foundation argument, \( l/m \).

The arbitrary constant of equation (4) \( C_i \) are defined from the boundary conditions.

The expression for the maximum deflection of beam with clamped ends and loaded with concentrated force in the middle of the beam span, according to [11] takes the following form:

\[ w_0 = \frac{P}{192EI} \varphi_2(u), \]  

(5)

here \( u = \frac{l}{2} \left( \frac{k}{4EI} \right)^{1/4} \) – dimensionless argument of the elastic foundation; \( \varphi_2(u) \) – I.G. Bubnov function.

For further transformations, the expression (5) is written as follows:

\[ w_0 = \frac{P \alpha (ad)^3}{2k 24} \varphi_2(u). \]  

(6)

The problem of beam deformation with a single central crack is solved. The expression for deflections of the right half of the beam is represented as

\[ w = C_1 F_1(\alpha x) + C_2 F_2(\alpha x) + C_3 F_3(\alpha x) + C_4 F_4(\alpha x). \]  

(7)

The constant \( C_i \) in the expression (7) is defined from the boundary conditions.

If \( \alpha x = 0 \): \( EIw'' = \frac{P}{2} \); \( 2w' = aM \), and if \( \alpha x = \frac{ad}{2} \): \( w = w' = 0 \).

Taking into account (1) and (2) it is not difficult to obtain that \( 2w' = aM \). And consequently \( 2w' = \frac{A}{E I \alpha} w'' \).

After inserting the values of functions \( F_i(\alpha x) \) and their derivatives into the boundary conditions, a combined equation for determining \( C_i \) is obtained:

\[
\begin{align*}
C_1 F_1(\frac{ad}{2}) + C_2 F_2(\frac{ad}{2}) + C_3 F_3(\frac{ad}{2}) + C_4 F_4(\frac{ad}{2}) &= 0; \\
C_1 \alpha F_1(\frac{ad}{2}) + C_2 \alpha F_2(\frac{ad}{2}) + C_3 \alpha F_3(\frac{ad}{2}) + C_4 \alpha F_4(\frac{ad}{2}) &= 0; \\
EI[C_1 \alpha^2 F_1(0) - C_4 4 \alpha^4 F_2(0) - C_4 4 \alpha^4 F_3(0)] &= \frac{P}{2}; \\
2[C_1 \alpha F_1(0) + C_2 \alpha F_2(0) + C_3 \alpha F_3(0) - C_4 4 \alpha F_4(0)] &= \frac{A}{E I \alpha} \left[ C_1 \alpha^2 F_1(0) + C_2 \alpha^2 F_2(0) - C_3 4 \alpha^2 F_3(0) - C_4 4 \alpha^2 F_4(0) \right].
\end{align*}
\]  

(8)

After solving the combined equation (8) the formula for deflection under load \( (\alpha x = 0) \) can be obtained:
\[ w_0 = \frac{P\alpha}{2k} \left[ \frac{F_2}{F_1} - \frac{(F_2^2 - F_1 F_3)(F_3 + A F_4)}{(F_3 F_2 - F_4 F_1) + \frac{A}{2} (F_4 F_2 + 4F_1^2)} \right] \]  

(9)

here is \( F_i = F_i \left( \frac{a}{d} \right) \).

Evidently, at the moment of the central crack formation (points I и I; figure. 3) formulas (6) и (9) are equivalent, i.e.

\[ \frac{P_1 \alpha (a d)^3}{2k} \varphi_2(u) = \frac{P_{i1} \alpha}{2k} \left[ \frac{F_2}{F_1} - \frac{(F_2^2 - F_1 F_3)(F_3 + A F_4)}{(F_3 F_2 - F_4 F_1) + \frac{A}{2} (F_4 F_2 + 4F_1^2)} \right] \]  

(10)

By converting (10) the expression for the coefficient determining characterizing the cracks interaction was obtained

\[ A = \frac{\left[ \left( \frac{a}{d} - \tan \frac{a}{d} \right) - \frac{P_1 (a d)^3}{P_{i1}} \varphi_2(u) \right] (\sin a d + \sinh a d) + \left( \frac{\sin^2 a d}{2} + \sin^2 \frac{a d}{2} \right) \left( \frac{a}{d} + \tan \frac{a}{d} \right) \left( \frac{a}{d} + \frac{a}{d} \right)}{\left( \frac{a}{d} - \tan \frac{a}{d} \right) - \left( \frac{a}{d} - \tan \frac{a}{d} \right) \left( \frac{\cos^2 a d}{2} + \cosh^2 \frac{a d}{2} \right) - \left( \frac{\sin^2 a d}{2} + \sin^2 \frac{a d}{2} \right) \left( \frac{a}{d} + \tan \frac{a}{d} \right) \left( \frac{a}{d} + \frac{a}{d} \right)} \]  

(11)

Calculations for beams using the dependence (11) and experimentally obtained diagrams were performed. Coefficient A appeared to characterize sufficient stability and change in the range of 1.1-1.5. It is accord with the date [5] obtained by trial-and-error method to agree with the experiment. The stability and independence of the coefficient A from the size parameters and characteristics of ice beams indicate that this coefficient can be used to estimate the flexibility of the ice cover with crack during its failure.

4. Finite element modulation of ice beam failure

For an approximate numerical solution of the problem of ice beams deformation the FEA COSMOS package is used. This method was used to solve the problem of the ice wedge bending after the initial crack formation [8] and a rigidly restrained circular ice plate [12] and allowed to start the problem bending beams embedded in the ice.

In order to further evaluate the proposed method for taking into account the interaction on the crack faces, a beam with built ends and loaded with loaded with concentrated force in the midspan was considered. For such beam, the solution is known (5). To decrease analysis time only the model quarter is considered, i.e. the boundary symmetry conditions are accepted (figure 5). Due to the rather simple body geometry, the standard grid is used. For this grid the global size and tolerance is determined.
The main calculations results (the deflection plot, the stress values in the rigid support and deflection under force) characterizing the behavior of a solid beam and beam separated by a crack are shown in figure 6.

The beam at the moment of the first crack under load (figure 6, a) and one half of the beam after its appearance (figure 6, b, c) are shown in a deformed state. Taking into account the compression on the crack faces (replacing the second half of the beam with compressive forces) leads to a change in the mode of deformation of the beam. Deflection under the force and stress in the anchorage become less. Therefore in order to create a second crack and increase the beam deflection corresponding to the beam break, a further load increment is required. For greater clarity, the scale for deflections is different, and the deflection values under force and the bending stress values are shown in the figures.

5. Main conclusions
1. A satisfactory agreement between the calculation of deflection of beam (figure 6) and the failure diagram of full-scale beam in the experiment (figure 3)
2. The obtained solutions led to the conclusion that when evaluating of the ice cover deformation separated by cracks, at the initial stage they can be interpreted by elastic joint. Taking this into account the ice block deformation up to point II (figure 3) can be described. To interpret the part III, further studies should be directed to account for the crumpling and breaking up the contacting parts of the beams. It will allow to obtain the value of the breaking load theoretically.

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