MASSES AND MAGNETIC MOMENTS OF CHARMED
BARYONS USING HYPER CENTRAL MODEL

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Abstract

Heavy flavour baryons containing one or two charm quarks with light flavour combinations are studied using the hyper central description of the three-body system. The confinement potential is assumed as hyper central coulomb plus power potential with power index $\nu$. The ground state masses and the magnetic moments of charmed, $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons are computed for different power index, $\nu$ starting from 0.5 to 2.0.

1 Introduction

The investigation of properties of hadrons containing heavy quarks is of great interest in understanding the dynamics of QCD at the hadronic scale. There is renewed interest both experimentally and theoretically in the static properties of heavy flavour baryons such as mass and magnetic moments.
Many of the narrow hadron resonances observed recently, brought up surprises in QCD spectroscopy. Recent years, experimental facilities at Belle, BABAR, DELPHI, CLEO, CDF etc have been successful in discovering heavy baryon states along with other heavy flavour mesonic states and many new states are expected in near future. Baryons are not only the interesting systems to study the quark dynamics and their properties but are also interesting from the point of view of simple systems to study three body problems. All these reasons make the study of heavy flavour spectroscopy extremely important and interesting. Here, we employ the hyper central approach to study the three-body problem, particularly the baryons constituting one or two charm quarks. In the present study, the magnetic moments of heavy flavour baryons are computed based on nonrelativistic quark model.

2 Hyper Central scheme for baryons

Quark model description of baryons is a simple three body system of interest. The Jacobi co-ordinates to describe baryon as a bound state of three constituent quarks are given by

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) ; \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$  \hspace{1cm} (1)

Further, defining the hyper spherical coordinates which are given by the angles

$$\Omega_{\rho} = (\theta_{\rho}, \phi_{\rho}) ; \Omega_{\lambda} = (\theta_{\lambda}, \phi_{\lambda})$$

together with the hyper radius, $x$ and hyper angle $\xi$ respectively as,

$$x = \sqrt{\rho^2 + \lambda^2} ; \xi = \arctan \left( \frac{\rho}{\lambda} \right)$$

the model Hamiltonian for baryons can be written as

$$H = \frac{P_{\rho}^2}{2 m_{\rho}} + \frac{P_{\lambda}^2}{2 m_{\lambda}} + V(\rho, \lambda) = \frac{P_x^2}{2 m} + V(x)$$  \hspace{1cm} (2)

Here the potential $V(x)$ is not purely a two body interaction but it contains three-body effects also. The reduced mass $m$ is defined as $m = \frac{2 m_{\rho} m_{\lambda}}{m_{\rho} + m_{\lambda}}$, where $m_{\rho} = \frac{m_1 m_2}{m_1 + m_2} ; m_{\lambda} = \frac{3 m_3 (m_1 + m_2)}{2 (m_1 + m_2 + m_3)}$ and $m_1$, $m_2$ and $m_3$ are masses of the three constituent quarks. For the present study, we consider the hyper central potential $V(x)$ as

$$V(x) = -\frac{Z}{x} + \beta x^\nu + \kappa + A e^{-\alpha x} \sum_{i \neq j} \sigma_i \cdot \sigma_j$$  \hspace{1cm} (3)
In this hyperspherical representation of the potential we consider \( \tau = \frac{2}{3} b \alpha_s \), 
\( \beta \approx m \tau, \kappa = (\sqrt{2} - 1) \alpha_s N_c N_f m \), where \( N_c = 3.0 \) while \( N_f = 4 \) for charmed baryons and other model parameters are given in [5]. The trial wave function is taken [1] as the hyper coulomb radial wave function. We study the baryons having different choice of the light flavour(q)combinations for qqc and ccq systems. The computed spin average mass with respect to the potential index, \( \nu \) are shown in Fig 1. The computed masses of the spin \( \frac{1}{2}, \frac{3}{2}, \) single charm and double charm baryons are given in Tables 1 and 2 respectively.

![Figure 1: Variation of spin average masses with potential index \( \nu \) for single charm baryons [a] and double charm baryons [b].](image)

### 3 Effective quark mass and Magnetic moments of heavy baryons

We define an effective mass for the bound quarks within the baryon as

\[
m_i^{eff} = m_i \left( 1 + \frac{<H>}{\sum_i m_i} \right)
\]

and we express the magnetic moment of baryons in terms of its constituent quarks as

\[
\mu_B = \sum_i \left\langle \phi_{sf} \left| \frac{e_i}{2m_i^{eff}} \sigma_i \right| \phi_{sf} \right\rangle
\]
Table 1: Single charm baryon masses (masses are in MeV)

| Baryon | P.I.($\nu$) | $J^P = \frac{1}{2}^+$ | Others | $J^P = \frac{3}{2}^+$ | Others |
|--------|-------------|------------------------|--------|------------------------|--------|
| $\Sigma_c^{++}$ | 0.5 | 2550 | 2453 $^{\pm 3}$ | 2618 | – |
| (uuc) | 0.7 | 2473 | 2454±0.18 $^{\pm 3}$ | 2538 | 2518±0.6 $^{\pm 3}$ |
| 1.0 | 2443 | 2460±80 $^{\pm 3}$ | 2506 | 2440±70 $^{\pm 3}$ |
| 1.5 | 2436 | 2499 | | |
| 2.0 | 2436 | 2498 | | |
| $\Sigma_c^+$ | 0.5 | 2568 | 2451 $^{\pm 3}$ | 2638 | – |
| (udc) | 0.7 | 2491 | 2439 $^{\pm 2}$ | 2557 | 2518 $^{\pm 2}$ |
| 1.0 | 2460 | 2453 $^{\pm 2}$ | 2525 | 2520 $^{\pm 2}$ |
| 1.5 | 2454 | 2452 $^{\pm 10}$ | 2518 | 2538 $^{\pm 10}$ |
| 2.0 | 2453 | 2448 $^{\pm 17}$ | 2517 | 2505 $^{\pm 17}$ |
| $\Sigma_c^0$ | 0.5 | 2586 | 2452 $^{\pm 3}$ | 2658 | – |
| (ddc) | 0.7 | 2508 | 2454±0.18 $^{\pm 3}$ | 2577 | 2518±0.5 $^{\pm 3}$ |
| 1.0 | 2477 | 2454 | 2544 | |
| 1.5 | 2471 | 2537 | | |
| 2.0 | 2470 | 2537 | | |
| $\Xi_c^+$ | 0.5 | 2642 | 2466 $^{\pm 3}$ | 2720 | – |
| (usc) | 0.7 | 2561 | 2481 $^{\pm 2}$ | 2636 | 2654 $^{\pm 2}$ |
| 1.0 | 2530 | 2468 $^{\pm 12}$ | 2603 | 2650 $^{\pm 12}$ |
| 1.5 | 2523 | 2473 $^{\pm 10}$ | 2596 | 2680 $^{\pm 10}$ |
| 2.0 | 2523 | 2496 $^{\pm 17}$ | 2595 | 2633 $^{\pm 17}$ |
| $\Xi_c^0$ | 0.5 | 2653 | 2472 $^{\pm 3}$ | 2734 | – |
| (dsc) | 0.7 | 2579 | 2471±0.4 $^{\pm 3}$ | 2656 | 2646±1.2 $^{\pm 3}$ |
| 1.0 | 2548 | 2471±0.4 $^{\pm 3}$ | 2623 | | |
| 1.5 | 2541 | 2616 | | |
| 2.0 | 2541 | 2615 | | |
| $\Omega_c^0$ | 0.5 | 2720 | 2698 $^{\pm 3}$ | 2810 | – |
| (ssc) | 0.7 | 2652 | 2698 $^{\pm 2}$ | 2739 | 2768 $^{\pm 2}$ |
| 1.0 | 2620 | 2710 $^{\pm 2}$ | 2704 | 2770 $^{\pm 2}$ |
| 1.5 | 2613 | 2678 $^{\pm 10}$ | 2697 | 2752 $^{\pm 10}$ |
| 2.0 | 2613 | 2701 $^{\pm 17}$ | 2697 | 2759 $^{\pm 17}$ |
| | | 2680±70 $^{\pm 17}$ | 2697 | 2660±80 $^{\pm 17}$ |
Table 2: Doubly charm baryon masses (masses are in MeV)

| Baryon | P.I.($\nu$) | $J^P = \frac{1}{2}^+$ Others | $J^P = \frac{3}{2}^+$ Others |
|--------|-------------|-------------------------------|-------------------------------|
| $\Xi^{++}_{cc}$ (ccu) | 0.5 | 3838 3612+17 (19) 3915 3706+21 (19) | 3915 3706+21 (19) |
| | 0.7 | 3760 3620 (2) 3833 3727 (2) | 3833 3727 (2) |
| | 1.0 | 3730 3480 (20) 3800 3610 (20) | 3800 3610 (20) |
| | 1.5 | 3723 3740 (21) 3792 3860 (21) | 3792 3860 (21) |
| | 2.0 | 3723 3478 (21) 3792 3610 (21) | 3792 3610 (21) |
| $\Xi^+_{cc}$ (ccd) | 0.5 | 3862 3605±23 (22) 3945 3685±23 (22) | 3945 3685±23 (22) |
| | 0.7 | 3786 3620 (2) 3862 3727 (2) | 3862 3727 (2) |
| | 1.0 | 3755 3480 (20) 3828 3610 (20) | 3828 3610 (20) |
| | 1.5 | 3748 3740 (21) 3821 3860 (21) | 3821 3860 (21) |
| | 2.0 | 3748 3478 (21) 3820 3610 (21) | 3820 3610 (21) |
| $\Omega^+_{cc}$ (ccs) | 0.5 | 3962 3702±41 (19) 4056 3783±22 (19) | 4056 3783±22 (19) |
| | 0.7 | 3889 3778 (2) 3978 3872 (2) | 3978 3872 (2) |
| | 1.0 | 3857 3590 (20) 3944 3690 (20) | 3944 3690 (20) |
| | 1.5 | 3850 3760 (21) 3936 3900 (21) | 3936 3900 (21) |
| | 2.0 | 3850 3590 (21) 3936 3690 (21) | 3936 3690 (21) |

Here $e_i$ and $\sigma_i$ represents the charge and the spin of the quark constituting the baryonic state and $|\phi_{sf}\rangle$ represents the spin-flavour wave function of the respective baryonic state. Extending the SU(2)$_S \times SU(3)_f$ spin flavour structure of the light flavour sector in SU(2)$_S \times SU(4)_f$ spin flavour structure with charm, we compute the magnetic moments of the spin $\frac{1}{2}$ and spin $\frac{3}{2}$ charmed baryons. Our results are listed in Tables 3 and 4 respectively.

4 Results and Discussion

We have employed the hyper central model with hyperspherical potential of the coulomb plus power potential to study the masses and magnetic moments of baryons containing one or two charm flavour quarks. It is important to see that the baryon mass do not change appreciably for the potential power index $\nu > 1.0$ (See Fig 1). The model parameters are fixed for this saturated value.
Table 3: Magnetic moments of single charm baryons in terms of $\mu_N$

| Baryon | 0.5 | 0.7 | 1.0 | 1.5 | 2.0 | RQM | NRQM |
|--------|-----|-----|-----|-----|-----|-----|-----|
| $\Sigma_c^{++}$ | 1.8809 | 1.9394 | 1.9635 | 1.9688 | 1.9692 | 1.76 | 1.86 |
| $\Sigma_c^{+}$ | 3.2806 | 3.3837 | 3.4272 | 3.4373 | 3.4379 | – | – |
| $\Sigma_c^+$ | 0.3959 | 0.4082 | 0.4133 | 0.4144 | 0.4144 | 0.36 | 0.37 |
| $\Xi_c^{++}$ | 1.1092 | 1.1441 | 1.1588 | 1.1622 | 1.1624 | – | – |
| $\Xi_c^{+}$ | 0.4542 | 0.4684 | 0.4742 | 0.4755 | 0.4755 | 0.41 | 0.37 |
| $\Omega_c^0$ | 1.1893 | 1.2269 | 1.2425 | 1.2460 | 1.2463 | – | – |
| $\Omega_c^*$ | -0.9612 | -0.9860 | -0.9981 | -1.0006 | -1.0008 | -0.85 | -0.85 |
| $\Xi_c^0$ | -0.8703 | -0.8931 | -0.9044 | -0.9068 | -0.9070 | – | – |
| $\Xi_c^*$ | -1.0854 | -1.1193 | -1.1332 | -1.1362 | -1.1364 | -1.04 | -1.11 |
| $\Xi_c^0$ | -1.0540 | -1.0873 | -1.1012 | -1.1044 | -1.1046 | – | – |
| $\Xi_c^*$ | -1.0231 | -1.0526 | -1.0655 | -1.0683 | -1.0685 | -0.95 | -0.98 |
| $\Xi_c^0$ | -0.9618 | -0.9898 | -1.0024 | -1.0052 | -1.0054 | – | – |

RQM [3], NRQM [3]

to the experimental spin average mass of the $\Sigma_c^*(2518) - \Sigma_c(2454)(udc)$ system [15] and the spin hyperfine parameter is fixed to yield their mass difference of 64 MeV [15]. All other baryonic masses are predicted without changing any of these parameters. It is interesting to note that our results are in good agreement with existing experimental as well as other theoretical model predictions. The result of single charm baryons are in accordance with the lattice results as well as with other model predictions. The mass variations of the single charm baryons with respect to $\nu$ from 0.5 to 2.0 are found to be around 100 MeV only.

Our predictions on the doubly charm baryons are compared with other theoretical model predictions in Table 2. Since there are larger disagreement among the different model predictions, only the future experiments on these doubly charm baryons would be able to test the validity of the theoretical model predictions. However, the hyperfine splitting of 76.6 MeV for $\Xi_{cc}^* - \Xi_{cc}$ obtained from Lattice predictions [22] is very close to our calculations of 73 MeV. It is important to note that the predictions of the magnetic moment of all the heavy hadrons studied here are with no free parameters. Our results for the magnetic moment of single charm baryons with spin $\frac{1}{2}$ are in accordance with the predictions of the full treatment of relativistic quark model as well as
Table 4: Magnetic moments of doubly charm baryons in terms of $\mu_N$

| Baryon      | Potential index $\nu$ | 0.5 | 0.7 | 1.0 | 1.5 | 2.0 | NRQM | AL1 |
|-------------|------------------------|-----|-----|-----|-----|-----|------|-----|
| $\Xi^{++}_{cc}$ |                        | -0.0151 | -0.0154 | -0.0156 | -0.0156 | -0.0156 | -0.01 | -0.208 |
| $\Xi^{*++}_{cc}$ |                        | 2.1934 | 2.2406 | 2.2602 | 2.2646 | 2.2649 | –     | 2.670 |
| $\Omega^{+}_{cc}$ |                        | 0.6910 | 0.7040 | 0.7098 | 0.7110 | 0.7110 | 0.67  | 0.635 |
| $\Omega^{++}_{cc}$ |                        | 0.0908 | 0.0926 | 0.0934 | 0.0936 | 0.0936 | –     | 0.139 |
| $\Xi^{+}_{cc}$ |                        | 0.7279 | 0.7426 | 0.7487 | 0.7500 | 0.7501 | 0.74  | 0.785 |
| $\Xi^{*+}_{cc}$ |                        | 0.0031 | 0.0031 | 0.0032 | 0.0032 | 0.0032 | –     | -0.311 |

NRQM [3], AL1 [19]

with the nonrelativistic approximation reported in [3]. In the case of doubly charm baryons, our predictions for both $J = \frac{1}{2}$ and $\frac{3}{2}$ baryons are found to be in good accordance with the recent predictions based on a potential model, AL1 [19] and NRQM [3] results [See Table 4].

We conclude that the three body interaction assumed in our model plays a significant role in the description of heavy flavour baryonic properties in particular their masses and magnetic moments. The behavior of the masses against the potential index $\nu$ as shown Fig. 1 indicates saturation of the basic quark-quark interactions within the heavy baryons as the potential index $\nu > 1.0$. We hope that, the predicted many of the baryonic ground state will be observed in the future high luminosity heavy flavour experiments.

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