ISAR imaging and cross-range scaling of maneuvering targets using the scaled transform

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Abstract. In this paper, a novel method based on the scaled transform is proposed for the ISAR imaging and cross-range scaling of a maneuvering target with uniform rotating acceleration. Firstly, the cross-range signals in each range cell can be modeled as multiple chirp signals. The scaled transform is utilized to eliminate the coupling with the slow time and lag time in the time-dependent autocorrelation function of multiple chirp signals. Then the 2-D accumulation plane can be obtained by the 2-D FFT and the cross-range profile can be obtained by a projection from the 2-D accumulation plane to the azimuth dimension. Meanwhile, the initial frequency and chirp rate of each scatterer can be estimated by the corresponding peak in the 2-D accumulation plane. Finally, the rotating motion parameters are estimated by the least square method and the cross-range scaling is performed. Simulation results prove the effectiveness of the proposed method.

1. Introduction
Inverse synthetic aperture radar (ISAR) imaging is a type technique to generate high-resolution image of a moving target. The range-Doppler (RD) algorithm, which is the most popular ISAR imaging technique, is suitable for the target with uniform velocity [1]. However, for a maneuvering target with uniform rotating acceleration, the RD algorithm cannot be used to generate the well-focused ISAR image because the Doppler frequency of the scatterer is time-varying.

The current ISAR imaging techniques for the maneuvering target with uniform rotating acceleration can be divided into two classes: non-parametric method and parametric method. The former method obtains the range-instantaneous Doppler image by utilizing time-frequency transform. This method includes the joint time frequency method (JTF) [2], Wigner-Ville distribution (WVD) method [3, 4] and smoothed pseudo Wigner-Ville distribution (SPWVD) method [5], etc. However, these methods are prone to frequency resolution losses and cross-terms. The latter method obtains the image by estimating the parameters of the echo. The methods such as Radon-Wigner transform (RWT) method [6], fractional Fourier transform (FRFT) method [7], and modified Keystone transform (MKT) [1] are used to analyze the LFM signal. However, these methods require the parameter estimation, which leads to a heavy computational burden.

This paper proposes a novel method for the ISAR imaging and cross-range scaling of a maneuvering target based on the scaled transform. Section 2 introduces the model of the maneuvering targets. Section 3 describes the proposed imaging method. Simulation results are presented in Section 4 to prove the effectiveness of the proposed method. Section 5 summarizes the conclusions.
2. Signal model of the maneuvering targets

It is assumed that the translational compensation has been ideally implemented and the target can be considered as a target rotating around a reference point. Meanwhile, the migration through resolution cells of all the scatterers can be ignored. Figure 1 shows the geometry of a rotating target. The received signal in the $y$th range bin can be expressed as follows:

$$s(t) = \sum_{i=1}^{N} \sigma_i \exp \left( -j \frac{4\pi}{\lambda} \left( x_i \sin \theta(t) + y_i \cos \theta(t) \right) \right)$$

where $N$ is the number of the scatterers in the $y$th range bin, $\sigma_i$ denotes the amplitude of the $i$th scatterer, $x_i$ and $y_i$ are the coordinates of the $i$th scatterer, $\theta(t)$ denotes the rotation angle, $\lambda$ is the wavelength. When the target has not severely maneuver, the rotation angle is assumed to be approximated as a second-order polynomial and expressed as:

$$\theta(t) = \omega t + \frac{1}{2} \Omega t^2$$

where $\omega$ is the initial angular velocity, $\Omega$ is the initial angular acceleration.

With limited aspect angle change, we have $\sin \theta(t) \approx \theta(t) = \omega t + \frac{1}{2} \Omega t^2$ and $\cos \theta(t) \approx 1 - \frac{1}{2} \omega^2 t^2$.

Then (1), the received signal can be expressed as:

$$s(t) = \sum_{i=1}^{N} \sigma_i \exp \left( -j \frac{4\pi}{\lambda} \left( x_i \left( \omega t + \frac{1}{2} \Omega t^2 \right) + y_i \left( 1 - \frac{1}{2} \omega^2 t^2 \right) \right) \right)$$

$$= \sum_{i=1}^{N} \sigma_i \exp \left( -j \frac{4\pi}{\lambda} y_i \right) \exp \left( j2\pi \left( f_i t + \frac{1}{2} k_i t^2 \right) \right)$$

where $f_i = -2x_i \omega / \lambda$, $k_i = -2 \left( x_i \Omega - y_i \omega^2 \right) / \lambda$.

In (3), the first exponential term is a constant and has no effect on Doppler profile. The second exponential term can be approximated to multiple chirp signals with different initial frequency $f_i$ and chirp rate $k_i$. The chirp rate $k_i$ has severe effect on the Doppler profile.

**Figure 1.** Geometry of target motion.
3. Imaging algorithm

3.1. Imaging by the scaled transform

For the sake of simple presentation, only the exponential term with respect to $t$ of the $i$th scatterer is reserved for analysis. Thus, (3) can be expressed by:

$$s_i(t) = \exp \left( j2\pi \left( ft + \frac{1}{2}kt^2 \right) \right)$$

(4)

The time-dependent autocorrelation function of $s_i(t)$ can be expressed by:

$$R_i(t, \tau) = s_i(t + \frac{\tau + 1}{2})s_i^*(t - \frac{\tau + 1}{2})$$

$$= \exp(j2\pi f_i(\tau + 1))\exp(j2\pi k_i(\tau + 1)t)$$

where $\tau$ denotes the lag variable. Taking the Fourier transform on (5) with respect to $\tau$, we have:

$$R_i(t, f_a) = \int_{-\infty}^{\infty} R_i(t, \tau) \exp(-j2\pi f_a \tau) d\tau$$

$$= T_a \exp(j2\pi(f + k_it))\text{sinc}[\pi T_a(f_a - (f + k_it))]$$

(6)

where $T_a$ denotes the imaging time.

If the coupling between $t$ and $\tau$ is removed, $R_i(t, f_a)$ will be a beeline parallel to the slow time axis in the time-frequency plane. Thus, the cross-range imaging of the signal $s_i(t)$ can be performed by using the horizontal line integral in the time-frequency plane. In order to remove the coupling, we rescale the slow time axis by the scaled transform as follows:

$$(\tau + 1)t = t'$$

(7)

where $t'$ is the new slow time variable.

By substituting (7) into (5), (6) can be rewritten as:

$$R_i(t', \tau) = \exp(j2\pi f_i(\tau + 1))\exp(j2\pi k_it')$$

(8)

In (8), it is worth noting that the coupling between $t$ and $\tau$ is removed by the scaled transform. By taking the Fourier transform on (8) with respect to $\tau$, we have:

$$R_i(t', f_a) = T_a^2 \exp(j2\pi f_i)\text{sinc}[\pi T_a(f_a - f_i)]\exp(j2\pi k_it')$$

(9)

It is worth noting that the signal distribution becomes a beeline parallel to the new slow time axis. Therefore, by taking the Fourier transform on (9) with respect to $t'$, we obtain:

$$R_i(f_a, f_a) = T_a^2 \exp(j2\pi f_i)\text{sinc}[\pi T_a(f_a - f_i)]\text{sinc}[\pi T_a(f_a - k_i)]$$

(10)

It can be seen from (10) that the $f_a$-axis corresponding to the frequency domain and the $f_b$-axis corresponding to the pseudo frequency domain contains the initial frequency and the chirp rate information, respectively. When the perimeter of $R_i(f_b, f_a)$ locates at $(f_b', f_a')$, the initial frequency $f_i$ and chirp rate $k_i$ can be estimated by $f_i = f_i'$ and $k_i = k_i'$.

In fact, there are several scatterers in the range cell and the cross-terms should be considered. Assume that $s(t)$ can be expressed as:

$$s(t) = s_i(t) + s_j(t)$$

(11)

where $s_i(t)$ and $s_j(t)$ are chirp signals and can be expressed as:

\[
\begin{align*}
    s_i(t) &= \exp(j2\pi(f_i t + \frac{1}{2}r_i t^2)) \\
    s_j(t) &= \exp(j2\pi(f_j t + \frac{1}{2}r_j t^2))
\end{align*}
\]
After applying the scaled transform, we have:

\[
R(t', f_a) = T_u \exp(j2\pi f_s) \text{sinc}[\pi T_u (f_a - f_s)] \exp(j2\pi r t')
+ T_u \exp(j2\pi f_s) \text{sinc}[\pi T_u (f_a - f_s)] \exp(j2\pi r t') + R_y(t', f_a) + R_m(t', f_a)
\]

(13)

where

\[
R_y(t', f_a) = \int \{ \exp[j2\pi(f_r - f_f)t'/\tau + j\pi(f_r - f_f)\tau] + j\pi(r_r - r_f)(t')^2 / \tau + j\pi((r_r - r_f)t') / \tau + j\pi(r_r - r_f)\tau (t') / \tau + j\pi(r_r - r_f)\tau (t') / \tau \} \exp(-j2\pi f_s t') dt'
\]

(14)

and

\[
R_m(t', f_a) = \int \{ \exp[j2\pi(f_r - f_f)t'/\tau + j\pi(f_r - f_f)\tau] + j\pi(r_r - r_f)(t')^2 / \tau + j\pi((r_r - r_f)t') / \tau + j\pi(r_r - r_f)\tau (t') / \tau + j\pi(r_r - r_f)\tau (t') / \tau \} \exp(-j2\pi f_s t') dt'
\]

(15)

It is worth noting from (13) that the time-frequency distribution of \(s_f(t)\) and \(s_f(t)\) becomes a beeline parallel to the new slow time axis and that of the cross-terms spreads into different frequency cells due to quadratic and higher order phase terms. Thus, we can realise the coherent integration of each chirp signals and reduce the energy of the cross-terms by the FFT. Therefore, by taking the Fourier transform on (15) with respect to \(t'\), we have:

\[
R(f_b, f_a) = R_y(f_b, f_a) + R_y(f_b, f_a) + \int R_y(t', f_a) \exp(-j2\pi f_s t') dt' + \int R_y(t', f_a) \exp(-j2\pi f_s t') dt'
\]

(16)

where

\[
R_y(f_b, f_a) = T_u^2 \exp(j2\pi f_s) \text{sinc}[\pi T_u (f_a - f_s)] \text{sinc}[\pi T_u (f_b - f_r)]
\]

(17)

and

\[
R_y(f_b, f_a) = T_u^2 \exp(j2\pi f_s) \text{sinc}[\pi T_u (f_a - f_s)] \text{sinc}[\pi T_u (f_b - f_r)]
\]

(18)

In the above discussion, it is worth noting that scatterers corresponding to different initial frequency can be distinguished in the frequency-pseudo frequency domain. Therefore, the Doppler profile of all scatterers in each range cell can be obtained by the projection of the perk of every scatterer onto the \(f_a\)-axis. Thus, the Doppler profile in each range cell can be expressed by

\[
\text{Dopplerprofile}(m) = \max_{-M/2 \leq n < M/2} |R(n,m)|
\]

(19)

where \(R(n,m)\) denotes the discrete form of \(R(f_b, f_a)\), \(M\) is the Doppler profile sampling size, \(m \in [-M/2, M/2 - 1]\). After repeating the above Doppler profile processing in all range cells, the well-focused ISAR image of the target can be obtained.

3.2. Cross-range scaling

Suppose that the initial frequency \(f = (f_1, f_2, \ldots, f_p)^T\) and chirp rate \(k = (k_1, k_2, \ldots, k_p)^T\) of \(P\) dominant scatterers are estimated by the scaled transform in several range cells. One of the \(P\) dominant scatterers is selected as the reference scatterer \(q\) and its scatterer position is \(y_q\). Then, the \(p\)th component’s scatterer position can be expressed as follows:

\[
y_p = y_q + \Delta y_p
\]

(20)

where \(p = 1, 2, \ldots, P\), \(\Delta y_p = n_{pq}/\Delta r\) denotes the range difference from the \(p\)th component’s scatterer to the reference scatterer, \(n_{pq}\) is the range cell difference and \(\Delta r\) is the range resolution.
The difference of the estimated initial frequency and chirp rate among the \( P \) dominant scatterers and the reference scatterer point \( q \) can be expressed as:

\[
\Delta f_p = -2\Delta x_p \omega / \lambda \\
\Delta k_p = -2\left(\Delta x_p \Omega - \Delta y_p \omega^2\right) / \lambda
\]  

(21)

(22)

where \( \Delta x_p = x_p - x_q \).

With (21) and (22), removing the \( \Delta x_p \), we have:

\[
\Delta k = \eta_1 \Delta f + \eta_2 \Delta y
\]

\[
= X \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}
\]  

(23)

where \( \Delta k = (\Delta k_1, \Delta k_2, \ldots, \Delta k_p)^T \), \( \Delta f = (\Delta f_1, \Delta f_2, \ldots, \Delta f_p)^T \), \( \Delta y = (\Delta y_1, \Delta y_2, \ldots, \Delta y_p)^T \), \( X = (\Delta f, \Delta y) \). \( \eta_1 \) and \( \eta_2 \) can be expressed as \( \eta_1 = \frac{\Omega}{\omega} \) and \( \eta_2 = \frac{2\omega^2}{\lambda} \).

In (23), the parameters can be estimated by the linear least squares method, then we have:

\[
\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = (X^T X)^{-1} X^T \Delta K
\]  

(24)

Therefore, if \( \eta_1 \) and \( \eta_2 \) are estimated, \( \omega \) and \( \Omega \) can be determined by \( \eta_1 = \frac{\Omega}{\omega} \) combined with \( \eta_2 = \frac{2\omega^2}{\lambda} \). Then, the ISAR image can be cross-range scaled.

**Figure 2.** ISAR images of the Boeing 727 aircraft data: (a) RD method; (b) WVD method; (c) STFT method; (d) The proposed method.
4. Simulation results
In this section, the simulated data of a Boeing 727 aircraft has been used to prove the effectiveness of the proposed method [8]. The radar data has been simulated with a stepped-frequency signal. The carrier frequency is 9 GHz and the bandwidth is 150 MHz. The data has 256 pulses.

Figure 2(a) shows the ISAR image obtained by the range-Doppler method which is blurred in the cross range. Figure 2(b) presents the ISAR image obtained by the WVD method. In Figure 2(b), the image quality has seriously degraded by the cross-term interference. Figure 2(c) shows the ISAR image obtained by the STFT method. From Figure 2(c), it can be noted that the STFT method can effectively suppress the cross-term interference. Figure 2(d) shows the ISAR image obtained by the proposed method, from which it can be noted that image is well focused. These results indicate that the proposed method can effectively image the uniformly accelerated target.

The angular velocity and angular acceleration are estimated to be 0.022 rad/s and 0.055 rad/s², respectively. The cross-range resolution is 0.457 m. With this estimation, the ISAR image in Figure 2(d) can be cross-range rescaled as in Figure 3.

![Rescaled ISAR image](image.png)

5. Conclusions
This paper proposed a novel method based on the scaled transform for the ISAR imaging and cross-range scaling of the maneuvering target. The scaled transform is used to eliminate the couple between the slow time and the lag time in the time-dependent autocorrelation function of multiple chirp signals. Then, the cross-range profile can be obtained by the projection from the 2-D accumulated plane to the cross-range domain. Finally, the rotating velocity is estimated by the linear square method and the cross-range scaling is performed by the cross-range resolution. Simulation results show that the proposed method can obtain the focused ISAR image and scale the ISAR image.

References
[1] Ruan Hang, Wu Yanhong, Jia Xin and Ye Wei 2014 Novel ISAR Imaging Algorithm for Maneuvering Targets Based on a Modified Keystone Transform *IEEE Geoscience and Remote Sensing Letters* 11(1) 128-132
[2] Wang Yuanxun, Ling Hao and Chen Victor C 1998 ISAR motion compensation via adaptive joint time-frequency technique *IEEE Transactions on Aerospace and Electronic Systems* 34(2) 670-677
[3] Chen Victor C and Shie Qian 1998 Joint time-frequency transform for radar range-Doppler imaging *IEEE Transactions on Aerospace and Electronic Systems* 34(2) 486-499
[4] Lv Xiaolei, Xing Mengdao, Wan Chunru and Zhang Shouhong 2010 ISAR imaging of maneuvering targets based on the range centroid doppler technique *IEEE Transactions on Image Processing* 19(1) 141-153
[5] Chen Victor C 2002 Time-frequency transforms for radar imaging and signal analysis *Artech House Radar Library*

[6] Li Wenchen, Wang Xuesong and Wang Guoyu 2010 Scaled Radon-Wigner Transform Imaging and Scaling of Maneuvering Target *IEEE Transactions on Aerospace and Electronic Systems* **46**(4) 2043-2051

[7] Du Liping and Su Guangchuan 2005 Adaptive inverse synthetic aperture radar imaging for nonuniformly moving targets *IEEE Geoscience and Remote Sensing Letters* **2**(3) 247-249

[8] Chen Victor C http:\\airborne.nrl.navy.mil\\vchen\tftsa.html