Impedance-Sliding Mode Control with Force Constraints for Space Robots Capturing Non-cooperative Objects

Dong Tao1,2, Qiang Zhang2, Xiaoyu Chu2, Xiaodong Zhou2 and Liangyu Zhao1
1School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China
2Beijing Institute of Control Engineering, Beijing 100094, China

Corresponding author: Liangyu Zhao (e-mail: zhaoly@bit.edu.cn).
This work was supported by the National Key Research and Development Program of China (Grant No. 2018AAA0103004) and the National Natural Science Foundation of China (Grant No. 12072027, 11532002, 51805025).

ABSTRACT The safe realization of on-orbit capture remains a challenging task because of uncontrolled service objects, big impact forces, post-impact control, and so on. In this study, based on the relationships between the kinematics and forces established by impedance control, a force constraint function is developed to reduce the contact force during the collision process of capture. Additionally, to realize flexible contact and improve motion performance during the capture process, a control method combining impedance and a sliding mode is designed to make the end-effector exhibit mass-spring-damping behavior after contact with the target, decreasing movement overshoot and stabilization time. Finally, numerical simulations are carried out in a scenario where a planar two-link space robot is utilized to capture a non-cooperative target, and the effectiveness of the proposed method is demonstrated.

INDEX TERMS space robot, capture, non-cooperative target, impedance control, sliding mode.

I. INTRODUCTION

During the past decade, the use of space robots to achieve on-orbit services has been an active research in the field of aerospace. On-orbit service tasks include the on-orbit assembly of large spacecraft, maintenance, upgrading, and refueling, and towing of malfunctioning satellites to grave orbit[1], [2]. The precondition for realizing the above on-orbit service is to complete on-orbit capture. On-orbit capture operations are generally divided into four phases. In the first stage, the space robot uses a vision or laser sensor to detect the information of the target and carries out mission planning. In the second stage, the space robot approaches the target and simultaneously adjusts its attitude according to the planned path. The third stage is to perform the capture operation. The fourth stage is the stability control after capture, which is to realize the synchrony of the position and attitude between the space robot and target. When the space robot operates the capture, a collision between the end-effector and the target inevitably occurs. The contact force generated by the collision may disturb the base of the space robot and affect the stability of its system, but also may cause damage to the space robot and the target or push them away from each other. Therefore, on-orbit capture is a dangerous and significant technical challenge[3], [4]. At present, Japan and the United States have successively carried out on-orbit capture through verification experiments for space cooperation objectives, and Canada, China, and Germany have established ground-based in-orbit capture test systems[1], [5]. However, on-orbit capture technology for non-cooperative targets is still a long way to achieve.

To achieve safe capture, lots of scholars have conducted theoretical research on on-orbit capture technology. Aghili et al. [6] proposed a combined prediction and motion planning scheme for the robotic capture of a drifting and tumbling object with unknown dynamics. Based on visual feedback, it ensured that the relative velocity between the target and the end-effector of the robot converged to zero to avoid collision. Luo et al. [7] proposed an optimal path planning strategy for a space robot to capture a rolling target at zero relative velocity. It was realized by obtaining the path-independent workspace of the space robot and predicting the trajectory of the captured object to plan the path of the end-effector. However, owing to the measurement noise of sensors, machining errors of mechanical parts, and position tracking errors of controllers, it is practically difficult to realize
absolute zero relative velocity in space. Moreover, the inertial mass and stiffness of the space robot and the captured target are usually relatively large, meaning that even a small relative velocity during capture will produce a large contact force. In order to resist the rotation motion of non-cooperative targets, some scholars used eddy current or electrostatic technology to conduct racemization and achieve non-contact capture. However, the captured targets in these methods are usually small and it still takes more than a year to achieve successful capture. Therefore, they cannot meet the needs of on-orbit services [8], [9].

Once the end-effector captures the target at a nonzero relative velocity, the inevitable contact between them acts as disturbances to space robots. In view of this, some researchers studied control methods to suppress the disturbances caused by collision contact. In Ref. [10], the bias momentum was loaded into a space robot system in advance to offset the momentum change of the base of the space robot when capturing the target. There is an optimal capture time when the angle between the position vector and the relative velocity of the target capture point is zero or very small. Flores-Abad et al. took advantage of this principle to determine the time to capture a rolling target, and designed an optimal capture method to minimize the impact on the base of a space manipulator, simultaneously considering the random uncertainty of the initial and final boundary conditions [11]-[13]. Raina et al. [2] also developed an adaptive reactionless control strategy for uncontrolled dynamics and unknown target parameters during the collision process.

As the excessive contact force during on-orbit capture may not only lead to the failure of capture missions, but also damage the space robots, it is necessary to study how to reduce contact forces during the capture process. Some studies have demonstrated that the impedance control method can be applied to solve this issue. The core concept of the impedance control method is to achieve flexible contact between the manipulator and the environment based on the dynamic relationship between the manipulator position and contact forces [14]. Assume that the driving force on a robot base can be precisely controlled, Moosavian et al. [15] designed an impedance control law for a multi-arm space robot system, and the entire system could exhibit the expected contact behavior when contacting a captured object. Improving on this method, Nakaniishi et al. [16] realized flexible contact between an end-effector and target in an inertial coordinate system without precise control of the base. Abiko et al. [17] treated the contact force during the capture as an interference force on the end-effector and proposed an impedance control method that was independent of any inertial parameters of the capture target. Yoshida et al. [18] used impedance control to minimize the base deviation of a space robot during the pre-capture and post-capture phases for a tumbling target. In Refs. [19], [20], a method combining proportional-derivative control and impedance control was employed for non-cooperative target capture. In this way, the end-effector behaves like a mass-spring-damping system, regardless of the reaction motion of the base. Uyama et al. [21] proposed a contact control method with reference to the recovery coefficient to adjust the impedance parameters. It dealt with the problems of undesired contact force and relative motion after collision between a service system and the captured target. In Ref. [22], a contact dynamics model was reestablished by considering the friction force between the end-effector of a space robot and the target grasping handle, and an admittance controller for the decomposed motion was designed. Flores-Abad et al. [23] used precise inverse dynamics to observe collision contact forces and proposed an impedance control method based on a disturbance observer to achieve flexible contact during capture. The Ref. [24] presented a comprehensive control scheme based on impedance control to achieve racemization and contact with non-cooperative targets. However, the contact force control methods discussed above merely focus on flexible behavior and do not consider how to reduce the contact forces during the capture process, which may cause the instability of the controller and damage the space manipulator and target.

In this paper, an impedance-sliding mode control method for space robots capturing non-cooperative targets considering force constraints is proposed. By designing a force constraint function with a scaling factor, the contact force, which served as input for the impedance control, is modified to reduce the contact force during the capture process. To solve the motion problems caused by the force constraint function, impedance control and sliding mode control are further combined to realize rapid stability by adjusting the parameters of the controller. The stability of the proposed method is proven using the Lyapunov direct method. The main contributions of this study can be summarized as follows: (1) Based on hyperbolic tangent function with adjustable limit, the force constraint strategy is developed to effectively reduce the contact force during the capture process and prevent the instability resulting from the excessive contact force. By introducing a scaling factor, the slope of the force constraint function is modified to achieve a greater adjustment while the contact force is small. (2) An impedance-sliding mode controller is developed to improve the motion performance during the contact process. By adjusting the parameters of this controller, the overshoot is reduced and the stabilization time is shortened.

The remainder of this paper is organized as follows. In Section 2, the kinematics and dynamics of a space robot and the dynamics model of a target are discussed. In Section 3, the contact dynamics modeling is presented. In Section 4, the design of force constraints, impedance control, and sliding control is described in detail, and the stability analysis of the controller is presented. In Section 5, the study of different cases to verify the effectiveness of the proposed controller is
performed. Finally, the conclusions are summarized in Section 6.

II. SYSTEM DYNAMICS MODEL
A. ASSUMPTIONS
To facilitate analysis without loss of generality, kinematics and dynamics models of space robots are established based on the following assumptions.

i. Both the space robot and target are rigid bodies, and the position and attitude of the space robot are not controlled.

ii. The influence of gravity gradients, solar wind, and magnetic fields on the space robot are ignored.

iii. The motion states of both the space robot and target are known.

B. KINEMATICS AND DYNAMICS OF SPACE ROBOT
As shown in Figure 1, the space robot consists of a satellite base and a series manipulator with \( n \) degrees of freedom, where \( \Sigma_i \) represents the inertial coordinate system, \( \Sigma_b \) denotes the base spacecraft body coordinate system whose origin is at the centroid of the base, \( \Sigma_i \) represents the target body coordinate system, and \( \Sigma_i \perp \Sigma_b \) are joint 1 to joint \( n \) coordinate systems; \( r_i \in \mathbb{R}^3 \) is the position vector of the centroid of link \( i \); \( \gamma_i \in \mathbb{R}^3 \) is the position vector of the end-effector; \( a_i \in \mathbb{R}^3 \) is the position vector from joint \( i \) to the centroid of link \( i \); and \( b_j \in \mathbb{R}^3 \) is the position vector from the centroid of link \( i \) to joint \( i+1 \). \( \rho_i \in \mathbb{R}^3 \) is the position vector of joint \( i \), \( \theta \in \mathbb{R}^n \) is the joint angle vector, meaning \( \theta = [\theta_1 \cdots \theta_n] \), \( \theta_i \) is the rotation angle of joint \( i \), \( \omega_i \) is the angular velocity vector of link \( i \), \( \omega_0 \) is the linear velocity and angular velocity vectors of the base, respectively. \( v_i \) and \( \omega_i \) are the linear and angular velocity vectors of the end-effector, respectively, where \( z_i \in \mathbb{R}^3 \) is the unit vector of the rotation direction of each link.

![FIGURE 1. Multi-body dynamics model of space robot and target.](image)

The relationship between the linear and angular velocities of the end-effector of the space robot, linear and angular velocities of the base, and angular velocity of the manipulator joint can be written as follows [25],[26]

\[
\begin{bmatrix}
    v_g \\
    \omega_g
\end{bmatrix} = J_b \begin{bmatrix} v_e \\ \omega_e \end{bmatrix} + J_\theta \dot{\theta}
\]

(1)

where, \( J_b \) is the Jacobian matrix between the base and end-effector, \( J_\theta \) is the Jacobian matrix between the space manipulator and end-effector.

The dynamics equation for the space robot and the end of the manipulator under the effect of the external forces/moments is given by [24],[27]

\[
\begin{bmatrix}
    H_b \\
    H_m
\end{bmatrix} \ddot{\theta} + \begin{bmatrix} c_b \\ c_m \end{bmatrix} = \begin{bmatrix} f_b \\ f_m \end{bmatrix} + \begin{bmatrix} J_b^T \\ J_m^T \end{bmatrix} \tau
\]

(2)

where \( H_b \in \mathbb{R}^{36} \) is the inertial matrix of the base, \( H_m \in \mathbb{R}^{6\times6} \) is the inertial matrix of the link of the manipulator, \( J_b \in \mathbb{R}^{36 \times \theta} \) is the coupling inertial matrix of the base and link of the manipulator, \( c_b \in \mathbb{R}^{36 \times \theta} \) is the velocity dependent nonlinear term associated with the base, \( c_m \in \mathbb{R}^{6\times6 \times \theta} \) is the velocity dependent nonlinear term associated with the manipulator; \( f_b \in \mathbb{R}^{36} \) is the vector of moments and forces exerted on the centroid of base; \( \dot{\theta} \in \mathbb{R}^{6\times6} \) denotes the linear and angular accelerations of the base of the space robot; \( \tau \in \mathbb{R}^{6\times6} \) is the angular acceleration of the joint of the manipulator; \( \tau \in \mathbb{R}^{6\times6} \) is the vector of manipulator joint torques; and \( f_m \in \mathbb{R}^{6\times6} \) denotes the external forces and moments exerted on the end-effector of the manipulator.

Because the base position and attitude of a free-floating space robot are not controlled, it is assumed that \( f_b = 0 \). By substituting this value into Eq. (2) and eliminating \( \dot{x}_0 \), the dynamics equation for a free-floating space robot can be written as follows:

\[
\ddot{\theta} = \tau + J_\theta^T \dot{\theta}
\]

(3)

where, \( \ddot{\theta} = \dot{H}_m - \dot{H}_m \dot{H}_m^T \dot{H}_m \), \( \ddot{c}_m = c_m - \dot{H}_m \dot{H}_m^T c_m \), and \( J_\theta = \dot{J}_m \dot{H}_m \).

C. DYNAMICS MODEL OF A TARGET
The target is assumed to be a free-floating rigid body, and its dynamics equation is given by [28]

\[
\begin{bmatrix}
    m_E \ddot{r}_e \\
    I_e \ddot{\omega}_e
\end{bmatrix} + \begin{bmatrix} m_E \omega_e \times r_e \\ I_e \omega_e \times \omega_e \end{bmatrix} = J_e \tau
\]

(4)

where \( m \) is mass of the target; \( E \in \mathbb{R}^{3\times3} \) is identity matrix, \( I_e \in \mathbb{R}^{3\times3} \) is inertia tensor of the target, \( \dot{r}_e \in \mathbb{R}^3 \) is vector of linear accelerations of the target, \( \omega_e \in \mathbb{R}^{3\times3} \) is the vector of angular velocity of the target, \( \dot{\omega}_e \in \mathbb{R}^3 \) is the vector of angular acceleration of the target. The grasping matrix \( J_e \) can be expressed as
where $J_u$ is the contact stiffness matrix:

$$J_u = \begin{bmatrix} R' & -R' h_u \times \\ 0 & R' \end{bmatrix}$$

where $R' \in \mathbb{R}^{3\times3}$ is rotation matrix from target body-fixed coordinate system to end-effector’s coordinate system, and $h_u \in \mathbb{R}^{3\times1}$ is the position vector of the grasping point with respect to the mass center of the target.

### III. CONTACT DYNAMICS MODELING

When a space robot captures a target, its end-effector will inevitably collide with the target. Excessive impact forces will not only cause great disturbance to the base of the space robot, but also affect the stability of the space robot system, even causing damage to the space robot and target. It is essential to construct a collision dynamics model to research the reduction of the contact forces during the capture process.

According to the Hertz contact theory\[29\], it is assumed that the contact between the end-effector of the space robot and the target is an elastic collision. The contact impact force is divided into elastic force and damping force. The magnitude of the force is related to factors such as initial velocity, contact area, Young's modulus of the material, Poisson's ratio, and penetration depth. This force can be expressed as follows [22]

$$f_c = K_c\delta^c + D_c\dot{\delta}$$

where $\delta$ represents the relative depth of compression between two contacting bodies at the contact point, and $\dot{\delta}$ is their relative velocity. $\mathbf{\lambda} \in \mathbb{R}^{3\times1}$ is the unit vector along the common normal of the contacting surfaces at the contact point, and $\ell$ is the force index, which is determined by the materials and geometric characteristics of the contact area. For the metal contact, the value of $\ell$ is 1.5. $D_c$ is the damping coefficient and $K_c$ is the contact stiffness coefficient, which is defined as:

$$K_c = \frac{4}{3} \frac{E_1 E_2 \sqrt{R}}{E_1 + E_2 - E_1 \gamma_1 - E_2 \gamma_2}$$

where $E_1$ and $E_2$ are the Young’s moduli of the two bodies, $\gamma_1$ and $\gamma_2$ are the Poisson’s ratios of the two bodies, and $R$ is the effective radius of the contact surface, which can be defined as:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

where $R_1$ and $R_2$ are the radii of the gripper end of the manipulator and the target body, respectively.

The relationship between the damping coefficient and the amount of compression can be expressed as:

$$D_c = \lambda \delta^c$$

where $\lambda$ is a hysteretic damping factor defined as:

$$\lambda = \frac{3K_c (1 - C^c)}{4\delta^{c-1}}$$

where $C^c$ and $\delta^{c-1}$ are the coefficients of restitution and the initial impact velocity, respectively.

### IV. CONTROLLER DESIGN

It is known that the contact force can be avoided when two objects meet at a zero relative velocity. However, it is difficult to achieve such an interaction because of factors such as sensor noise, mechanical errors, and position tracking errors. When a basketball player receives a pass, the arm subconsciously moves backward to reduce the impact of the contact force. Similarly, a space robot can control its manipulator arm to produce backward motion to reduce the impact force when it captures a flying target. It is well known that impedance control establishes the relationship between forces and kinematics. However, when a space robot captures an approaching target from the opposite direction, can the force method achieve active backward motion? Based on this problem, a force constraint strategy is considered. When the space manipulator contacts the target, the contact force is modified using the force constraint function. According to the admittance relationship between the force and the kinematics established by impedance control, the desired backward acceleration, which is larger than that of the actual contact force, can be obtained. The backward acceleration will lead to the backward action of the end-effector of the space manipulator, so that reduce the contact force during the collision process.

After contacting the target, the space manipulator is locked into a combination with the target. However, the force constraint function increases the range of motion of the space manipulator to stabilize to the desired position, generates an overshoot, and increases the stabilization time. In order to solve the above problems, the sliding mode control, which has the characteristics of fast response and insensitivity to interference, is employed, and the combination of impedance control and sliding mode control is applied to improve the motion performance and achieve fast stability. The principal block diagram of impedance-sliding mode control with force constraints is shown in FIGURE 2. The contact force between the space robot and the target serves as the input of the force constraint function to obtain the modified contact force. Based on this modified force, the desired acceleration of the joint can be obtained through the calculation of the impedance and kinematics. By the input of the desired joint acceleration and angular velocity error into the sliding mode controller, the joint position tracking can be realized, and furthermore, the position adjustment of the end-effector of the space robot can be achieved.
A. FORCE CONSTRAINTS
In order to generate active retrograde acceleration, the force constraint function must be continuous and play a larger role in amplification when the contact force is small. Meanwhile, considering the limitations of the joint motor, driver, and controller, the force constraint function must be bounded. Based on the above considerations, a hyperbolic positive force constraint function is designed, and its variation rule is shown in FIGURE 3. To ensure that the modified contact force does not exceed the capacity of the space robot, the bound of the force constraint is defined as $F_{\text{max}}$. Simultaneously, a scaling factor is introduced to change the slope of the force constraint function in order to achieve a large modified effect when the contact force is small. The force constraint function is expressed as:

$$f_{\text{c},i} = F_{\text{max},i} \tanh \left( \frac{f_{\text{c},i}}{\mu} \right) = F_{\text{max},i} \frac{e^{f_{\text{c},i}/\mu} - e^{-f_{\text{c},i}/\mu}}{e^{f_{\text{c},i}/\mu} + e^{-f_{\text{c},i}/\mu}}$$

(11)

where $F_{\text{max}} \in \mathbb{R}^{3 \times 1}$ is the known upper-bound vector corresponding to the actuator capacity and $\mu \in \mathbb{R}^{3 \times 1}$ is the scaling factor vector; $f_{\text{c},i}$ is the external force exert on the end-effector of the manipulator, $i = 1, 2, 3$; $f_{\text{c},i}^*$ is the modified contact force with respect to $f_{\text{c},i}, i = 1, 2, 3$.

As shown in FIGURE 3 and Eq.(11), the hyperbolic tangent force constraint function has the following characteristics: The force constraint function is nonlinear, continuous, and monotonically increasing. If $f_{\text{c},i} > 0$, the first derivative decreases monotonically; if $f_{\text{c},i} < 0$, then the first derivative increases monotonically. In addition, the value of the first derivative can be changed by adjusting $\mu, 0 < \mu \leq \frac{F_{\text{max}}}{2}, i = 1, 2, 3$.

According to Eq.(11), $f_{\text{c},i}^*$ is bounded and can be described as:

$$| f_{\text{c},i}^* | = F_{\text{max},i} \tanh \left( \frac{f_{\text{c},i}}{\mu} \right) \leq F_{\text{max},i}$$

(12)

B. IMPEDANCE CONTROL
To modify the stiffness of the system to have a specified mechanical impedance when the end-effector of space robot contacts a target, the impedance relationship between the input contact force of the controller and the end-effector should meet the following requirement:

$$M_t \Delta \dot{r}_t + C_t \Delta \dot{r}_t + K_t \Delta r_t = \dot{F}_{\text{c},i}$$

(13)

where $M_t, C_t$, and $K_t$ represent the inertial matrix, damping matrix, and stiffness matrix, respectively, which are all positive definite skew-symmetric matrices. Here, $\Delta r_t$ and $\Delta \dot{r}_t$ can be expressed as follows:

$$\Delta r_t = r_{\text{e},i} - r_{\text{t},i}$$

$$\Delta \dot{r}_t = \dot{r}_{\text{e},i} - \dot{r}_{\text{t},i}$$

(14)

where $r_{\text{e},i}$ and $\dot{r}_{\text{e},i}$ are the expected position, velocity of the end-effector, respectively, $r_{\text{t},i}$ and $\dot{r}_{\text{t},i}$ are the actual position and velocity of the end-effector, respectively.

According to Eq. (13), the admittance relation is obtained:

$$\Delta \dot{r}_t = M_t^{-1} \left( f_{\text{c},i} - C_t \Delta \dot{r}_t + K_t \Delta r_t \right)$$

(15)

C. CONTROL LAW
Considering the fast response, insensitivity to disturbance and strong robustness of sliding mode control, a sliding mode control method combined with impedance control is designed to realize the rapid stability of the end-effector of the space robot after contact with the target. To realize this method, a suitable sliding surface is designed. The sliding surface is defined as follows:

$$s(z) = \sigma z + \dot{z}$$  \hspace{1cm} (16)

where $z = \theta_0 \cdot \theta$ , $\theta_0 \in \mathbb{R}^*$ is the expected angle vector, $\theta \in \mathbb{R}^*$ is the current angle vector, $\dot{z} = \dot{\theta}_0 \cdot \theta$ , $\dot{\theta}_0 \in \mathbb{R}^*$ is the expected angular velocity vector, $\theta \in \mathbb{R}^*$ is the current angular velocity vector, and $\sigma$ is a positive definite matrix.

Taking the derivative of Eq. (16) with respect to time gives:

$$\dot{s} = \sigma \dot{z} + \dot{\dot{z}} = \sigma \dot{\theta}_0 \cdot \theta + \dot{\sigma} z + \dot{\dot{z}}$$  \hspace{1cm} (17)

where $\dot{\theta}_0 \cdot \theta$ , $\dot{\theta}_0 \in \mathbb{R}^*$ is the expected angular acceleration vector, and $\dot{\sigma} z + \dot{\dot{z}}$ is the current angular acceleration vector.

The exponential rate of approach is given by

$$\dot{s} = \dot{\sigma} s - \dot{\dot{z}}$$  \hspace{1cm} (18)

By substituting Eq. (18) into Eq.(17), the following is obtained:

$$\tau = \ddot{H}_a(\dot{\sigma} z + \dot{\dot{z}} + \dot{\sigma} z + \dot{\dot{z}})$$  \hspace{1cm} (19)

The revised expected acceleration of the end-effector of the space robot $\ddot{r}_e$ is obtained from the admittance controller, as follow:

$$\ddot{r}_e = \ddot{r}_a - M_f^{-1}(f_e - C_e \dot{\theta}_e - K_e \Delta \theta_e)$$  \hspace{1cm} (20)

By taking the derivative of Eq. (16) with respect to time, the revised expected acceleration can be expressed as:

$$\ddot{r}_a = \dot{v}_e = J_{b_v} \begin{bmatrix} v_0 \\ \dot{\theta}_0 \end{bmatrix} + J_{b_v} \begin{bmatrix} v_0 \\ \dot{\theta}_0 \end{bmatrix} + J_{a_v} \dot{\theta}_a + \ddot{J}_{a_v} \dot{\theta}_a$$  \hspace{1cm} (21)

By substituting Eq. (20) into Eq. (21) and rearranging the results, we obtain the following:

$$\dot{\dot{\theta}}_a = J_{a_v}^{-1}(\ddot{r}_a - M_f^{-1}(f_e - C_e \dot{\theta}_e - K_e \Delta \theta_e))$$  \hspace{1cm} (22)

By substituting Eq. (22) into Eq.(19), the following is obtained:

$$\tau = \ddot{H}_a(\dot{\sigma} z + J_{a_v}^{-1}(\ddot{r}_a - M_f^{-1}(f_e - C_e \dot{\theta}_e - K_e \Delta \theta_e)))$$  \hspace{1cm} (23)

To prove the stability, the Lyapunov functions are given by

$$V = \frac{1}{2} s^2$$  \hspace{1cm} (24)

By taking the derivative of Eq. (24) and substituting it into Eq.(18), the following is obtained:

$$\dot{V} = s \dot{s} = -\sigma s |s|^2$$  \hspace{1cm} (25)

Therefore, $\dot{V} \leq 0$ for all $t$, the global asymptotic convergence of the tracking error can be obtained. Accordingly, as $t \rightarrow \infty$, we can derive $\theta \rightarrow \theta_0$ and $\dot{r}_a \rightarrow \ddot{r}_a$.

V. NUMERICAL SIMULATIONS

A. SIMULATION EXAMPLE

A free-floating satellite equipped with a planar two-link manipulator is used as an example to verify the effectiveness of the proposed method. Its configuration is shown in and its parameters are listed in Table 1. The initial state of the space robot is $\theta_0 = 0$ rad, $\theta_1 = \pi / 6$ rad, $\theta_2 = \pi / 6$ rad, and $r_e = [1.8809 \ 1.3809] \ m$. As a space target generally has translational and rotational movement, the space robot should adjust its manipulator to the desired position and attitude through path planning in the early stages. Therefore, the simulation scenario is set up with a free-floating state for the space robot, and the inertial system is established on the centroid of the space robot’s base, while the capture target has translational and rotational movement.

| Case | Base (Link 0) | Link 1 | Link 2 | Target |
|------|---------------|--------|--------|--------|
| Case 1 | 0.5 | 0.25 | 25 | 2.5 |
| Case 2 | 0.5 | 0.25 | 25 | 2.5 |

The parameters used to calculate the contact force are listed in Table 2.

### Table 2. Material and contact parameters of the robot gripper and target.

| Parameter | Symbol | Case 1 and Case 3 | Value |
|-----------|--------|-------------------|-------|
| Young’s modulus of the end-effector | $E_1$ | 7.3×10^9 | 7.3×10^9 |
| Young’s modulus of the target | $E_2$ | 7.3×10^9 | 7.3×10^9 |
| Poisson’s ratio of the end-effector | $\nu_1$ | 0.3 | 0.3 |
| Poisson’s ratio of the target | $\nu_2$ | 0.3 | 0.3 |
| Radius of the robot end-effector’s contact surface | $R_1$ | 0.01 m | 0.01 m |
| Radius of the target’s contact surface | $R_2$ | 0.5 m | 0.5 m |
| Coefficient of restitution | $C_r$ | 0.1 | 0.1 |

The initial states of the centroid of the target are $\mathbf{r}_t = [2.41 \ 1.3838] \ m$ and $\theta_t = 0$ rad, respectively. The position vector of the grasping point with respect to the centroid of the target is $\mathbf{h}_g = [-0.5 \ 0] \ m$. The position and velocity of the grasping point are calculated using the
following formula: \[ r_{tg} = r_c + R_{tg}h_{tg} \], \[ \dot{r}_{tg} = \dot{r}_c + R_{tg}(\dot{\theta} \times h_{tg}) \], where \( R \) is the rotation matrix of the target centroid coordinate system relative to the inertial coordinate system with an initial value of \( R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \). When the end-effector of the space robot is in contact with the target, the rotation matrix from the centroid of the target to the end-effector is \( R^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \).

The impedance parameters of the end-effector of the space robot are given by, \( M = \begin{bmatrix} 25 & 0 \\ 0 & 50 \end{bmatrix} \), \( C = \begin{bmatrix} 500 & 0 \\ 0 & 500 \end{bmatrix} \), and \( K = \begin{bmatrix} 9000 & 0 \\ 0 & 9000 \end{bmatrix} \).

To verify the effectiveness of the proposed method, simulation Case 1 is designed. To demonstrate that the method proposed in this paper is independent of the contact material and initial relative velocity, simulation Case 2 and 3 are designed.

**FIGURE 4.** Simulation system.

**B. SIMULATION RESULTS AND DISCUSSION**

Case 1: The linear and angular velocities of the target centroid are \( \dot{r}_c = [0.05 \ 0] \) m/s and \( \dot{\theta}_c = 0.01 \) rad/s, respectively. The parameters used to calculate the collision contact forces are listed in Table 2. Based on the method proposed in this paper, the control parameters are given by \( \sigma = [100 \ 100]^T \) and \( [300 \ 300]^T \), with \( \zeta = 0.001 \) and \( \kappa = 5 \). The parameters of the force constraint function are given by \( F_{\text{max}} = [1500 \ 1500]^T \) and \( \mu = [300 \ 300]^T \). The simulation results are presented in FIGURE 5.
When the force constraint function is not used, the contact force of the end-effector of the space manipulator in the X and Y directions of inertial coordinate system are indicated by the blue solid line in FIGURE 5 (a) and (b), respectively, and the moment exerted on the target is shown as the blue solid line in FIGURE 5 (c). When the force constraint function is used, the maximum contact force of the end-effector of the space manipulator in the X direction decreases to 2/3 of that in the absence of the force constraint, as shown by the cyan dotted line in FIGURE 5 (b) and (c).

Additionally, when the force constraint function is not used, the positions of the end-effector of the space manipulator and the grasping handle of the target in the X and Y directions are shown by the blue and green solid lines in FIGURE 5 (d) and (e), respectively, and the trajectory in the X-Y plane is shown by the blue solid line in FIGURE 5 (f). When the force constraint function is used, the contact forces between the end-effector of the space manipulator and the target are decreased, their motion amplitudes are increased, and overshoots are generated, as shown by the red and cyan dotted lines in FIGURE 5 (d) and (e). These larger motion amplitudes prove that the proposed force constraint function effectively realizes the desired backward movement when capturing the target, which is more intuitively represented by the cyan dotted line in FIGURE 5 (f). By adjusting the parameters of the impedance-sliding mode controller, the motion range of the end-effector is reduced, overshoot is eliminated, and the stabilization time is shortened, as shown in FIGURE 5 (d), (e) and (f). Meanwhile, the contact forces between the end-effector of the manipulator and the target do not increase, nor does the moment exerted on the target, as shown by the magenta dotted line in FIGURE 5 (a), (b), and (c). The simulation results demonstrate that the proposed method is effective.

Case 2: The Young's moduli of the end-effector of the space robot and target capture point are both $7.3 \times 10^7$ N/m$^2$, and the other parameters are the same as those in Case 1. The simulation results of the proposed method are shown in FIGURE 6.
When Young’s modulus is changed, that is, the stiffness is changed, the method proposed in this paper can still achieve smaller contact forces and smaller moment than previous methods. By adjusting the parameters of the impedance-sliding mode controller, the motion range of the end-effector is reduced, the overshoot is eliminated, and the stabilization time is shortened. The positions of the end-effector of the manipulator and grasping handle of the target in the X and Y directions are shown by the magenta and black dotted lines in FIGURE 6 (d) and (e), respectively, and the trajectory of the end-effector in the X-Y plane is shown by the red dotted line in FIGURE 6 (f). Although the maximum contact force and maximum moment on the target are increased slightly after adjusting the parameters of the impedance-sliding mode controller, they are still less than the contact force and moment generated without force constraint, as shown by the magenta dotted line in FIGURE 6 (a), (b), and (c). The simulation results verify the effectiveness of the proposed method when the stiffness is different.

Case 3: The linear velocity of the target centroid is \[ \dot{r}_T = 0.02 \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ m/s}, \] and the other parameters are the same as those in Case 1. The simulation results of the proposed method are shown in FIGURE 7.
When the initial velocity of the target is changed, the method proposed in this study can still achieve smaller contact forces and smaller moment than previous methods. The contact forces in the X and Y directions are indicated by the cyan dotted lines in FIGURE 7 (a) and (b), respectively, and the moment on the target is indicated by the cyan dotted line in FIGURE 7 (c). By adjusting the parameters of the impedance-sliding mode controller, the motion range of the end-effector is reduced, the overshoot is eliminated, and the
stabilization time is shortened, as shown in FIGURE 7 (d), (e), and 7 (f). In other words, adjusting the parameters of the impedance-sliding mode controller does not increase the contact force and moment. The simulation results verify the effectiveness of the proposed method when the initial velocity is different.

In summary, based on the simulation results of the three cases discussed above, the following conclusion can be drawn: the method proposed in this paper can reduce contact force and improve the stability during the capture process effectively, which is not affected by material characteristics or the initial relative velocity between the space robot and target. Overall, the simulations verified the effectiveness of the proposed method.

VII. CONCLUSION
With the goal of reducing the contact force between the end-effector and the target when a space robot performs an on-orbit capture mission, an impedance-sliding mode control method is proposed for a space robot to capture a non-cooperative object considering force constraints. A hyperbolic tangent force constraint function is firstly designed to effectively reduce the contact force during the collision process. To realize rapid adjustment when the contact force is small, a scaling factor is introduced to adjust the slope of the hyperbolic tangent force constraint function.

Simultaneously, considering the limitations of the motor and driver, the bound of the force constraint is defined to avoid the instability of the controller caused by the excessive contact force. To improve the motion performance of the end-effector during the capture process, an impedance-sliding mode controller is further designed to suppress overshoot and reduce the stabilization time. Three simulation cases demonstrated the effectiveness of the proposed method on reducing the contact force and rapidly achieving stability with different material properties and relative velocities. Future work will focus on addressing the fact that the data from contact force sensors are inaccurate, as well as the compliance control of multi-arm space robots with force constraints. Solving these two issues will enhance the practicability of the proposed method.

ACKNOWLEDGMENT
Dong Tao gratefully acknowledges discussions with Professor Qian Zhao.

REFERENCES
[1] A. Flores-Abad, O. Ma, K. Pham, and S. Ulrich, “A review of space robotics technologies for on-orbit servicing”, Prog. Aerosp. Sci., vol. 68, pp. 1-26, Jul. 2014.
[2] D. Raina, S. Gora, D. Maheshwari, and S. V. Shah, “Impact modeling and reactionless control for post-capturing and maneuvering of orbiting objects using a multi-arm space robot”, Acta Astronaut, vol 182, pp. 21-36, May 2021.
[3] G. Li and P. Xu, “Design and analysis of a deployable grasping mechanism for capturing non-cooperative space targets”, Aero. Sci. Technol., vol 106, pp. 171-180, Nov. 2020.
[4] L. Yan, W Xu, Z Hu, and B Liang, “Multi-objective configuration optimization for coordinated capture of dual-arm space robot”, Acta Astronaut, vol 167, pp. 189-200, Feb. 2020.
[5] J. Virgili-Llop, J. V. Drew, R. Zappulla II, and M. Romano, “Laboratory experiments of resident space object capture by a spacecraft-manipulator system”, Aero. Sci. Technol., vol 71, pp. 530-545, Dec. 2017.
[6] F. Aghili, “A prediction and motion-planning scheme for visually guided robotic capturing of free-floating tumbling objects with uncertain dynamics”, IEEE Trans. Robot., vol 28, no. 3, pp. 634-649, Jun. 2012.
[7] J. Luo, L. Zong, M. Wang, and J. Yuan, “Optimal capture occasion determination and trajectory generation for space robot grabbing tumbling objects”, Acta Astronaut, vol 136, pp. 380-386, Jul. 2017.
[8] T. Bennett, D. Stevenson, E. Hogan, and H. Schaub, “Prospects and challenges of touchless electrostatic detumbling of small bodies”, Adv. Space Res., vol 56, no. 3, pp. 557-568, Aug. 2015.
[9] D. Han, P. Huang, X. Liu, and Y. Yang, “Combined spacecraft stabilization control after multiple impacts during the capture of a tumbling target by a space robot”, Acta Astronaut, vol 176, pp. 24-32, Nov. 2020.
[10] D. N. Dimitrov and K. Yoshida, “Momentum distribution in a space manipulator for facilitating the post-impact control”, Proceeding of 2004 IEEE/RSJ International Conference on Intelligent Robots and Systems, Sendai, Japan, 2004, pp. 3345-3350.
[11] A. Flores-Abad, L. Zhang, Z. Wei, and O. Ma, “Optimal Capture of a Tumbling Object in Orbit Using a Space Manipulator”, Journal of Intelligent & Robotic Systems, vol 86, pp. 199-211, May. 2017.
[12] A. Flores-Abad, Z. Wei, O. Ma, and K. Pham, “Optimal control of a space robot to approach a tumbling object for capture with uncertainties in the boundary conditions”, AIAA Guid. Navig. Control Conf., Boston, MA, USA, 2013, pp. 1-13.
[13] A. Flores-Abad, Z. Wei, O. Ma, and K. Pham, “Optimal control of space robots for capturing a tumbling object with uncertainties”, J. Guid. Contr. Dynam., vol 37, no. 6, pp. 2014-2017, Oct. 2014.
[14] N. Hogan, “Impedance control: an approach to manipulation: parts i-iii”, J Dyn. Syst., Meas. Control, vol 107, no. 1, pp. 1-24, 1985.
[15] S.A.A. Moosavian, R. Rastegari, and E. Papadopoulos, “Multiple impedance control for space free-flying robots”, J. Guid. Contr. Dynam., vol 28, no. 5, pp. 939-947, Sep. 2005.
[16] H. Nakanishi, and K. Yoshida, “Impedance control for free-flying space robots basic equations and applications”, Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems, Beijing, China, 2006, pp. 3137-3142.
[17] S. Abiko, R. Lampariello, and G. Hirzinger, “Impedance control for a free-floating robot in the grasping of a tumbling target with parameter uncertainty”, Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems, Beijing, China, 2006, pp. 1020-1025.
[18] K. Yoshida, D. Dimitrov, and H. Nakanishi, “On the capture of tumbling satellite by a space robot”, Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems, Beijing, China, 2006, pp. 4127-4132.
[19] A. Stolfi, P. Gasbarri, and M. Sabatini, “A combined impedance-PD approach for controlling a dual-arm space manipulator in the capture of a non-cooperative target”, Acta Astronaut, vol 139, pp. 243-253, Oct. 2017.
[20] A. Stolfi, P. Gasbarri, and M. Sabatini, “A parametric analysis of a controlled deployable space manipulator for capturing a non-cooperative flexible satellite”, Acta Astronaut, vol 148, pp. 317-326, Jul. 2018.
[21] N. Uyama, D. Hirano, H. Nakanishi, K. Nagaoaka, and K. Yoshida, “Impedance-based contact control of a free-flying space robot with respect to coefficient of restitution”, 2011 IEEE/SICE International Symposium on System Integration (SII), Kyoto, Japan, 2011, pp. 1196-1201.
[22] S. Wu, F. Mou, Q. Liu, and J. Cheng, “Contact dynamics and control of a space robot capturing a tumbling object”, Acta Astronaut, vol 151, pp. 532-542, Oct. 2018.
[23] A. Flores-Abad, A. Crain, M. Nandayapa, M.A. Garcia-Teran, and S. Ulrich, “Disturbance observer-based impedance control for a compliance capture of an object in space”, 2018 AIAA Guidance, Navigation, and Control Conference, Kissimmee, Florida, USA, 2018.

[24] M. Wang, J. Luo, J. Yuan, and U. Walter, “An integrated control scheme for space robot after capturing non-cooperative target”, Acta Astronaut, vol 147, pp. 350-363, 2018.

[25] D. Meng, W. Lu, W. Xu, Y. She, X. Wang, B. Liang, and B. Yuan, “Vibration suppression control of free-floating space robots with flexible appendages for autonomous target capturing”, Acta Astronaut, vol. 151, pp. 904-918, Oct, 2018.

[26] T. Rybus, K. Seweryn, and J. Z. Sasiadek, “Control system for free-floating space manipulator based on nonlinear model predictive control (NMPC)”, J. Intell. Robotic Syst., vol. 85, pp. 491-509, Jul, 2016.

[27] X. Zhang and J. Liu, “Effective motion planning strategy for space robot capturing targets under consideration of the berth position”, Acta Astronaut, vol. 148, pp.403-416, Jul, 2018.

[28] M. Wang, J. Luo, J. Yuan, and U. Walter, “Detumbling strategy and coordination control of kinematically redundant space robot after capturing a tumbling target”, Nonlinear Dyn., vol. 92, no. 3, pp. 1023-1043, Feb. 2018.

[29] Z. F. Bai, J. Chen, and Y. Sun, “Effects of contact force model on dynamics characteristics of mechanical system with revolute clearance joints”, IJST, Transactions of Mechanical Engineering, vol. 38, no. M2, pp. 375-388, Autumn, 2014.