Roles of Direct Break-up Reaction on Neutrino Scattering off $^{12}$C near Nucleon Threshold Region

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Neutrino (antineutrino) scattering off $^{12}$C is one of various important key reactions for $\nu$-process in the nucleosynthesis of light nuclei. Most of neutrino-nucleus scattering are considered through indirect processes within the energy range from a few to tens of MeV. Target nuclei are excited by incident neutrino (antineutrino) through various transitions, and subsequently decay into other nuclei with emitting particles. But, direct processes are also feasible, in which incident neutrino (antineutrino) strips directly one nucleon from target nuclei. Consequently, direct processes may affect abundances of $^{11}$C and $^{11}$B additionally to indirect processes. We investigate direct neutrino (antineutrino) quasi-elastic scattering off $^{12}$C around the energy region liberating one nucleon and discuss implications of direct processes in the nucleosynthesis. The direct processes might be comparable to the indirect processes if the final state interaction is taken into account.

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Neutrino ($\nu$) (antineutrino ($\bar{\nu}$)) scattering with a complex nucleus plays important roles of studying $\nu$ properties, such as $\nu$ oscillation and masses, as well as nuclear structure probed by weak interaction $[1, 2, 3]$. Hence, a lot of interests for neutrino (antineutrino)-nucleus ($\nu(\bar{\nu}) - A$) scattering have been increased to the nuclear astrophysics, for instance, $\nu$-process in the formation of a core collapsing supernova, because cross sections for the $\nu(\bar{\nu}) - A$ scattering are one of the most important input data for a network calculation estimating light nuclei abundance like $^{7}$Li and $^{11}$B. The abundance ratio turns out to be sensitive to the $\nu$ oscillation parameters, mass hierarchy, and mixing angle $\theta_{13}$ $[2]$. Incident

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\( \nu(\bar{\nu}) \) energies exploited in these calculations \[1, 2\] are focused on the energy range from a few to a few tens of MeV, because relevant \( \nu(\bar{\nu}) \) energy spectra emitted from a proto-neutron star are presumed to be mostly peaked around the energy region.

Most of calculations for the \( \nu(\bar{\nu}) - A \) scattering are performed by considering indirect processes. Incident \( \nu(\bar{\nu}) \) leads target nuclei to some excited states through various transitions \( i.e., \) super allowed Fermi \( (J^\pi = 0^+) \), allowed Gamow Teller \( (J^\pi = 1^+) \), spin dipole \( (J^\pi = 0^-, 1^-, 2^-) \), and other higher multipole transitions. The excited nuclei subsequently decays into other nuclei with emitting particles such as proton, neutron, alpha, \( \gamma \), and so on \[2, 3\].

Since weak interaction is mediated by \( Z^0 \) and \( W^\pm \) bosons, there are two kinds of reactions, charge current (CC) and neutral current (NC) reactions. In the NC reaction, the incident \( \nu(\bar{\nu}) \) excites target nuclei, and then the excited target nuclei are subsequently decayed into other nuclei by emitting some particles incoherently,

\[
A(\nu(\bar{\nu}), \nu'(\bar{\nu}')) A^* \rightarrow B + \text{outgoing particles},
\]

\( ^{12}\text{C}(\nu, \nu')^{12}\text{C}^* \rightarrow ^{11}\text{B} + p \) (or \( ^{11}\text{C} + n \) reaction) is one of the indirect processes for \( \nu-^{12}\text{C} \) scattering. But, the direct knocked-out processes are also possible \[4\], in which a nucleon inside nuclei is stripped from target nuclei without any excitation of target nuclei. For instance, \( ^{12}\text{C}(\nu, \nu'p)^{11}\text{B} \) or \( ^{12}\text{C}(\nu, \nu'n)^{11}\text{C} \) reactions should be differentiated from the indirect processes.

Meanwhile, in the CC reaction, we have another direct processes, \( ^{12}\text{C}(\nu_e, e^-p)^{11}\text{C} \) and \( ^{12}\text{C}(\bar{\nu}_e, e^+n)^{11}\text{B} \), in addition to the indirect processes

\[
A(\nu_l(\bar{\nu}_l), l(\bar{l})) B^* \rightarrow C + \text{outgoing particles},
\]

which are \( ^{12}\text{C}(\nu_e, e^-) \rightarrow ^{12}\text{N}^* \rightarrow ^{11}\text{C} + p \) and \( ^{12}\text{C}(\bar{\nu}_e, e^+) \rightarrow ^{12}\text{B}^* \rightarrow ^{11}\text{B} + n \). Therefore, it is possible for the direct processes to influence the abundances of redundant nuclei in the network calculation initiated from \( ^{12}\text{C} \) by the \( \nu \)-process \[2\].

A few experimental data for the \( \nu(\bar{\nu})-^{12}\text{C} \) reaction through the indirect processes have been reported as flux averaged out total cross sections since 1990. Detailed references are summarized at Refs. \[3, 5\]. The data for inclusive reaction such as \( ^{12}\text{C}(\nu_e, e^-)^{12}\text{N}^* \) show about \( 4.3 \sim 5.7 \), while the data for exclusive reaction like \( ^{12}\text{C}(\nu_e, e^-)^{12}\text{N}_{g.s.} \) are restricted to \( 8.9 \sim 10.5 \) in the \( 10^{-42} \text{ cm}^2 \) unit. All these data are measured from accelerated-based data. Future \( \nu \) factory for intense and pure \( \nu (\bar{\nu}) \) beam, so called as beta beam \[6\], could yield more fruitful data for the \( \nu(\bar{\nu}) - A \) scattering.
Many theoretical calculations [3, 4, 5, 7, 8, 9] have been reported for the $\nu(\bar{\nu}) - ^{12}\text{C}$ scattering since the pioneering work on weak interactions on $^{12}\text{C}$ by J. S. Cornell et al. [10]. Of course, all of theoretical results only assume indirect processes. Results of most shell model (SM) calculations [3, 7] converge more or less to the experimental data although they depend on the particle model space and the given Hamiltonian. But the results by the random phase approximation (RPA) (or Continuum RPA) [5, 7, 8] and Quasi-particle RPA (QRPA) [4, 7, 9] calculations overestimated the data by a factor of $4 \sim 5$, in specific, for the exclusive reaction. Since the energy weighted sum rule is satisfied in the RPA approach [7], these differences seem to be inescapable and are claimed to come from the small model space basis used in the SM calculation [3, 7].

In this report, we presume that the $\nu(\bar{\nu}) - ^{12}\text{C}$ scattering can be also proceeded via direct processes, and present results for the direct CC reactions, $^{12}\text{C}(\nu_e, e^-)$ and $^{12}\text{C}(\bar{\nu}_e, e^+)$, and results for the direct NC reaction, $^{12}\text{C}(\nu, \nu')$, where ground states of $^{11}\text{C}$ and $^{11}\text{B}$ are taken as final nuclei with all summation of possible knocked-out nucleon states. In the Born approximation, the CC reactions, $^{12}\text{C}(\nu_e, e^-)$ and $^{12}\text{C}(\bar{\nu}_e, e^+)$, are calculated by integrating the kinetic energy and all the possible states of knocked-out nucleon on the direct processes, $^{12}\text{C}(\nu_e, e^- p)$ and $^{12}\text{C}(\bar{\nu}_e, e^+ n)$, respectively. Likewise, the NC reactions, $^{12}\text{C}(\nu, \nu')$ and $^{12}\text{C}(\bar{\nu}, \bar{\nu}')$, are given by the integration of the kinetic energy and all the possible states of outgoing nucleon on the direct processes, $^{12}\text{C}(\nu, \nu' N)$ and $^{12}\text{C}(\bar{\nu}, \bar{\nu}' N)$, respectively. Since we consider direct processes, the excited states of final nuclei are not considered.

The direct processes considered here just correspond to a low energy tail of quasi-elastic scattering peak. And to calculate this procedure we use the distorted wave Born approximation (DWBA) formalism which has been successfully applied to the quasi-elastic electron scattering for a long time.

Since the framework of the DWBA is focused on a nucleon inside nuclei, main ingredients are wave functions of bound and continuum nucleons, and a weak transition current operator. Detailed descriptions are given in our previous papers [11, 12, 13, 14], which satisfactorily described the quasi-elastic $\nu - A$ [11, 12] as well as the electron-nucleus scattering [13, 14].

For obtaining the nucleon bound state wave functions, the Dirac equation is solved in the presence of the strong vector and scalar potentials based on $\sigma - \omega$ model [15]. The wave functions of the continuum nucleons are the solution of the Dirac equation with a relativistic phenomenological optical potential generated by Ohio State University group [16].
We choose the nucleus fixed frame where target nucleus is seated at the origin of the coordinate system. Four-momenta of incident and outgoing $\nu(\bar{\nu})$ are labelled $p_i^\mu = (E_i, p_i)$, $p_f^\mu = (E_f, p_f)$. $p_A^\mu$, $p_{A-1}^\mu$, and $p^\mu$ represent four-momenta of target nucleus, residual nucleus, and final nucleon, respectively. In the laboratory frame, the differential cross section is given by the contraction between the lepton tensor and the hadron tensor \[11\]

\[ \frac{d\sigma}{dT_p} = 4\pi^2 \frac{M_NM_{A-1}}{(2\pi)^3 M_A} \int \sin\theta_l d\theta_l \int \sin\theta_p d\theta_p p_f^{-1} f_{rec} \sigma_M^Z [v_L R_L + v_T R_T + h v'_T R'_T], \tag{3} \]

where $M_N$ is the nucleon mass, $\theta_l$ denotes the scattering angle of the lepton, and $h = -1$ ($h = +1$) corresponds to the helicity of the incident $\nu(\bar{\nu})$. $\theta_p$ and $T_p$ represent the polar angle and the kinetic energy of the knocked-out nucleons, respectively. For the NC reaction, $\sigma_M^Z$ is defined by

\[ \sigma_M^Z = \left( \frac{G_F \cos(\theta_l/2) E_f M^2_Z}{\sqrt{2\pi} (Q^2 + M^2_Z)} \right)^2, \tag{4} \]

and for the CC reaction,

\[ \sigma_M^{W\pm} = \sqrt{1 - \frac{M^2_Z}{E_f}} \left( \frac{G_F \cos(\theta_C) E_f M^2_W}{2\pi (Q^2 + M^2_W)} \right)^2, \tag{5} \]

where $G_F$ is the Fermi constant given by $G_F \simeq 1.16639 \times 10^{-11}$ MeV$^{-2}$, and $M_Z$ ($M_W$) is the rest mass of $Z$ ($W$)-boson. $\theta_C$ denotes the Cabibbo angle given by $\cos^2 \theta_C \simeq 0.9749$. Detailed forms for recoil factor $f_{rec}$, kinematical coefficients $v$, and the corresponding response functions $R$ are given in Ref. [11].

The nucleon current $J$ represents the Fourier transform of the nucleon current density written as

\[ J^\mu = \int \bar{\psi}_p \hat{J}^\mu \psi_b e^{i\mathbf{q}\cdot\mathbf{r}} d^3r, \tag{6} \]

where $\hat{J}^\mu$ is a free nucleon current operator, and $\psi_p$ and $\psi_b$ are the wave functions of the knocked-out and the bound state nucleons, respectively. Total cross section is given as the integration of Eq.(3) to the kinetic energy of the knocked-out nucleon:

\[ \sigma = \int \frac{d\sigma}{dT_p} dT_p. \tag{7} \]

In Figs. 1 - 3, we show total cross sections for the NC and CC reactions by the direct processes in terms of the incident $\nu(\bar{\nu})$ energy, i.e., $^{12}$C($\nu$, $\nu'$), and $^{12}$C($\nu_e$, $e^-$) and $^{12}$C($\bar{\nu}_e$, $e^+$), respectively. Since the threshold energy for liberating nucleon is just the binding energy of the nucleon inside nuclei, our results are presented from the averaged binding energy.
Emitting muon in the CC reaction is energetically forbidden on the energy region considered here.

Our results are separately presented with and without an optical potential, which is introduced to take final state interaction (FSI) of outgoing nucleon with residual nuclei into account. With the FSI, cross sections are generally reduced by a factor of 2 compared with those without the FSI. This reduction also appears on other calculations \cite{12,17}. In specific, the FSI affects in the whole energy region. Therefore, the FSI of outgoing nucleon could be one of vital important ingredients even in the indirect processes on the low energy region.

Results of the indirect processes symbolized as data points in the all figures are taken from the SM calculation tabulated in Ref. \cite{2}. They present two theoretical calculations based on two different Hamiltonian, SFO and PSDMK2 \cite{3}. No remarkable difference between the two results can be seen in the log scale cross sections.

Comparison of our results i.e., direct processes including the FSI, to those by the indirect processes reveals that the cross sections of the direct processes are smaller by a factor of 2 \(~3\) for \(^{12}\text{C}(\nu,\nu')\) and \(^{12}\text{C}(\bar{\nu}_e,e^+)\), and by a factor of 3 \(~4\) for \(^{12}\text{C}(\nu_e,e^-)\) rather than those of the indirect processes (see differences between data points and solid curves at figures). It means that the contributions of the direct processes to the abundance of light nuclei could be small by a factor of 2 \(~4\) compared to those by the indirect processes.

However, it should be noted that most of calculations for the indirect processes did not take into account of the FSI, and the FSI due to the strong interaction of outgoing nucleons with residual nuclei could lower the cross sections in the low energy \(\nu(\bar{\nu})\), even if outgoing particles emit from compound nuclei. As shown in Figs. 1 - 3, the reduction of cross sections by the optical potential at the nucleon threshold energy may support such a conjecture. Therefore, the relatively small present contribution of the direct processes could be comparable or even larger to those of the indirect processes if the FSI effects could be taken into account in the indirect processes.

In order to compare with forthcoming experimental data, we should need to consider flux averaged (folded) cross section, although there are still no data for the direct processes. It needs to know the neutrino energy spectrum which inevitably depends on a given temperature, just like the Fermi distribution usually adopted in most calculation. Detailed studies of cross sections by direct processes to the given temperature and their effects on the abundances in the network calculation of nucleosynthesis are in progress.
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FIG. 1: (Color online) NC reaction for $\nu$ by direct process, $^{12}\text{C}(\nu, \nu')$, obtained by integrating the kinetic energy and summing all possible knocked-out nucleon states for $^{12}\text{C}(\nu, \nu'N)$ reaction [11]. Data points for indirect processes, which is a sum of two cross sections, $^{12}\text{C}(\nu, \nu')^{12}\text{C}^* \rightarrow ^{11}\text{B} + p$ and $^{11}\text{C} + n$, come from the SM calculation [2]. SFO and PSDMK2 mean two different Hamiltonian exploited in the calculation.
FIG. 2: (Color online) CC reaction for $\nu_e$ by direct process, $^{12}\text{C}(\nu_e, e^-)$, obtained by integrating the kinetic energy and summing all possible knocked-out proton states in the reaction, $^{12}\text{C}(\nu_e, e^- p)$. Data points for indirect processes come from the SM calculation for $^{12}\text{C}(\nu_e, e^-) \rightarrow ^{12}\text{N}^* \rightarrow ^{11}\text{C} + p$ \cite{2}. Others are same as Fig.1.
FIG. 3: (Color online) CC reaction for $\bar{\nu}_e$ by direct process, $^{12}\text{C}(\bar{\nu}_e,e^+)$, obtained by integrating the kinetic energy and summing all possible bound neutron states in the reaction, $^{12}\text{C}(\bar{\nu}_e,e^+n)$. Data points for indirect processes come from the SM calculation for $^{12}\text{C}(\bar{\nu}_e,e^+) \rightarrow ^{12}\text{B}^* \rightarrow ^{11}\text{B}+n$ \cite{2}. Others are same as Fig.1.