Article

Multi-Rate Real-Time Simulation Method Based on the Norton Equivalent

Junjie Zhu*, Bingda Zhang

The Key Laboratory of Smart Grid of Ministry of Education, Tianjin University, Tianjin 300072, China; jzhu@tju.edu.cn (J.Z.); bdzhang@tju.edu.cn (B.Z.);
* Correspondence: jjzhu@tju.edu.cn; Tel.: +86-182-2206-9101

Abstract: For the problem of poor accuracy of the existing multi-rate simulation methods, this paper proposes a multi rate real-time simulation method based on the Norton equivalent, compared with multi-rate simulation method based on the ideal source equivalent. After the Norton equivalence of the fast subsystem and the slow subsystem, they are obtained simultaneously at the junction nodes. In order to reduce the amount of simulation calculation, the Norton equivalent circuit is obtained by incremental calculation. The data interface between the fast subsystem and the slow subsystem is realized by extrapolation method. For ensuring the real-time performance of the simulation, the method that the slow subsystem calculates ahead of the fast subsystem is given for the slow subsystem with a large amount of calculation. Finally, the AC/DC hybrid power system was simulated on the real-time simulation platform (FRTDS), and the simulation results were compared with the single-rate simulation, which verified the correctness and accuracy of the method.

Keywords: multi-rate real-time simulation; the ideal source equivalent; the Norton equivalent; increment; extrapolation method

1. Introduction

China has built the world’s largest AC/DC hybrid power system with the highest voltage level, and the regional power grids have been interconnected through Ultra-high voltage AC / DC transmission lines [1,2]. The control and protection system plays an important role in the safe and reliable operation of AC/DC hybrid power system, and the real-time simulation technology based on hardware-in-the-loop can be used to verify the correctness of the protection device action and the effectiveness of the control strategy [3,4].

Large-scale AC/DC hybrid power system contain a large number of power electronic devices. Compared with traditional AC power system, these devices have a small time scale, and their dynamic characteristics have a great impact on the safe and stable operation of the power system. In order to simulate power electronic equipment more accurately, the simulation step size is required to be smaller and smaller [5]. This is a challenge for the simulation of AC/DC hybrid power system. For improving the speed of simulation, two aspects are generally considered, one is to reduce the amount of simulation calculation, the other is to use parallel computing technology. Ref [6] proposed a multi-layer topology hybrid model for power electronic switching components, which allows a large number of single switches to be reduced to only two diodes and controlled voltage and current sources. This greatly reduces the number of switching combinations and the size of the generated code, and improves the speed of simulation. Ref [7] proposed a method of GPU based parallel matrix exponential algorithm for large scale power system electromagnetic transient simulation, which significantly improves the simulation calculation speed. However, the GPU cannot independently complete the process control and data scheduling in the simulation calculation process, so it needs to cooperate with the CPU to complete the simulation calculation [8]. The above methods are all single-rate electromagnetic transient simulation methods, ignoring the differences in the time scale of power equipment in different systems. The selection of simulation step size can only be based on the system
with the smallest time scale of power equipment, which will cause serious waste of computing resources. [9].

In view of the above problems, scholars at home and abroad have proposed the concept of parallel multi-rate electromagnetic transient computing, and different simulation steps are adopted for different systems according to the time scale of power equipment in the system [10–12]. Ref [13] proposed a multi-rate electromagnetic transient simulation method based on Latency technology, in which the external part of the subnet was modeled as an ideal source. As the ideal source model could not guarantee that the node signals (voltage and current) on both sides of the interface were equal, the simulation accuracy was reduced. Ref [14] proposed a node splitting interface (NSI) multi-rate parallel technology. It solves the state space equations of fast and slow subnets simultaneously, avoiding data prediction and signal delay, and improving the simulation accuracy, but the method cannot be used for real-time simulation. The above multi-rate simulations are all offline simulations and cannot be used for hardware-in-the-loop experiments. Ref [15] proposed a multi-rate real-time simulation method based on a joint simulation platform of real-time digital simulator (RTDS) and field programmable gate array (FPGA). It uses the asynchronous interaction method that FPGA data transmission is appropriately earlier than RTDS data transmission, which reduces a certain communication delay. But the communication delay problem of data interaction between different platforms still exists, which causes the reduction of simulation accuracy. In Ref [16] a FPGA based MR real-time simulation of MMC-HVDC grids is proposed. The MMC system is decoupled by a stub-line, and each MMC valve is implemented on one FPGA. The FPGAs can select different time steps to meet the requirement of the time constraints. However, using multiple FPGA blocks for simulation calculation will not only cause data synchronization problem and decrease the simulation accuracy, but also waste the hardware resources of FPGA.

For the problem of poor accuracy of the existing multi-rate simulation methods, this paper proposes a multi rate real-time simulation method with the Norton equivalent circuit based on multirate simulation method with the ideal source equivalent. After the Norton equivalence of the fast subsystem and the slow subsystem, they are obtained simultaneously at the junction nodes. In order to reduce the amount of simulation calculation, the Norton equivalent circuit is obtained by incremental calculation. The data interface between the fast subsystem and the slow subsystem is realized by extrapolation method. Finally, the AC/DC hybrid power system was simulated on the real-time simulation platform (FRTDS), which verified the correctness and accuracy of the method.

2. Multi-rate Simulation Calculation Timing

In the AC/DC hybrid power system, the power electronic equipment with fast control characteristics and the AC large-scale power grid are intertwined in different time scale processes, which makes the operation and control characteristics of the AC/DC hybrid power system more complicated [17]. Therefore, in the simulation of the AC/DC hybrid power system, various time scale processes in the AC/DC hybrid power grid should be considered. The simulation must be able to simulate the fast electromagnetic transient process (time scale of a few microseconds) of power electronic equipment such as converters and static synchronous compensators. It is also necessary to be able to simulate the switching process of the converter valve of the DC power system and the electromagnetic transient process of the AC power system (time scale of tens of microseconds to hundreds of microseconds) [18,19]. When using a single simulation step to simulate an AC/DC hybrid power system. If the simulation step is too small, the amount of calculation that needs to be completed within a single step is too large to complete. If the simulation step size is too large, the dynamic characteristics of the power electronic components cannot be simulated, which reduces the accuracy of the simulation. For balancing the accuracy of simulation and the scale of simulation, the entire system is decomposed into multiple subsystems. According to the time scale of the power equipment in the subsystem, different step sizes are used for simulation calculation, that is, multi-rate simulation.

There are two types of multi-rate simulation calculation timing: serial calculation timing and parallel calculation timing. Figure 1 shows the serial calculation timing and parallel calculation
timing of the two subsystems. Among them, Subsystem 1 uses a larger simulation step size $\Delta T$, and Subsystem 2 uses a smaller simulation step size $\Delta t$, where $\Delta T = n\Delta t$ ($n$ is a positive integer).

![Diagram of Subsystem 1 and Subsystem 2 timing](image)

**Figure 1.** Simulation calculation timing. (a) Serial calculation timing. (b) Parallel computing timing

In multi-rate simulation, the calculation tasks of serial calculation timing need to be completed in sequence. In Figure 1(a), (1) at the time of $(m-1)\Delta T$, subsystem 2 transmits the data to subsystem 1 through the interface; (2) Subsystem 1 uses the received data for simulation calculation; (3) Subsystem 1 transfers the data to Subsystem 2 through the interface; (4) Subsystem 2 uses the received data for simulation calculation. In serial calculation, the calculation tasks (1), (2), (3) and (4) need to be completed in sequence, which seriously affects the speed of simulation calculation. Therefore, the serial calculation sequence can generally only be used for offline simulation, not for real-time simulation calculation.

The task of parallel computing time sequence can be completed at the same time, which improves the computing speed. In Figure 1(b), (1) at the time of $(m-1)\Delta T$, subsystem 1 and Subsystem 2 conduct data interaction; (2) Subsystem 1 uses the received data for one simulation calculation, while subsystem 2 uses the received data for n times of simulation calculation. In Subsystem 2, the data required by the simulation calculation is not known and more prediction data is required, so the parallel multi-rate simulation accuracy is low. However, for ensuring the real-time nature of multi-rate simulation, only parallel calculation timing can be used.

3. Multi-rate real-time simulation based on the Norton equivalence

3.1. Multi-rate Real-time simulation method based on the ideal source equivalence

In the multi-rate real-time simulation of the power system, the inductance and capacitor element can be equivalent to the ideal source and the system is decoupled into multiple independent subsystems by taking advantage of the fact that the inductance current and capacitance voltage in the circuit cannot be suddenly changed. For example, the circuit with inductance L is equivalent as shown in Fig. 2.
The relationship between current $i_2(t)$ and voltage $u_1(t), u_2(t)$ with inductance $L$ is as follows in Fig. 2(a).

\[ u(t) - u(t+\Delta t) = L \frac{di}{dt} \]  

(1)

Assuming that Subsystem 1 uses a larger simulation step size $\Delta T$, and Subsystem 2 uses a smaller simulation step size $\Delta t$, where $\Delta T = n \Delta t$ ($n$ is a positive integer). By using Euler method to differentiate equation 1, then we can find:

\[ i_2(t) = i_2(t - \Delta T) + \frac{\Delta T}{L} \left[ u_1(t) - \frac{1}{n} \sum_{i=1}^{n} u_2(t - \Delta T + k \Delta t) \right] \]  

(2)

At the time of $m \Delta T$, Subsystem 1 and Subsystem 2 exchange and synchronize information. The voltage $u_1(t)$ and $u_1(t - \Delta T)$ of Subsystem 1 is sent to Subsystem 2, and the voltage $u_2(t - \Delta T + k \Delta t)$ of Subsystem 2 is sent to Subsystem 1. Then perform data synchronization according to the substitution theorem, namely $i_2(t) = -i_2(t) = i_1(t)$. After that, the calculation can be divided into two parallel tasks. The voltage and current of the equivalent current source of Subsystem 1 and Subsystem 2 are obtained simultaneously.

For Subsystem 1, the current $i_1(t + \Delta T)$ of the equivalent current source can be represented as:

\[ i_1(t + \Delta T) = i_1(t) + \frac{\Delta T}{L} \left[ u_1(t) - u_1(t) \right] \]  

(3)

According to the current $i_1(t + \Delta T)$ of Subsystem 1, calculate the voltage $u_1(t + \Delta T)$ of Subsystem 1.

For subsystem 2, the current $i_2(t + k \Delta t)$ of the equivalent current source can be represented as:

\[ i_2(t + k \Delta t) = i_2(t + (k - 1) \Delta t) + \frac{\Delta T}{L} \left[ u_1(t + (k - 1) \Delta t) + u_1(t + (k - 1) \Delta t) \right] \]  

(4)

Where $u_1(t + (k - 1) \Delta t)$ and $i_2(t + (k - 1) \Delta t)$ are the voltage and current of the equivalent current source in Subsystem 2 and they are known. However, the current $u_1(t + (k - 1) \Delta t)$ in Subsystem 1 is unknown, and it can be estimated by extrapolation. Then the extrapolation formula is:

\[ u_1(t + (k - 1) \Delta t) = u_1(t) + \frac{k - 1}{n} \left[ u_1(t) - u_1(t - \Delta T) \right] \]  

(5)

According to the current $i_2(t + k \Delta t)$ of Subsystem 2, calculate the voltage $u_2(t + k \Delta t)$ of Subsystem 2.

After that, the simulation of the next moment will be continued. Similarly, the calculation process of multi-rate simulation method with capacitor C can be obtained. The multi-rate simulation method with the ideal source equivalence takes advantage of the fact that the state variables of the inductors and capacitors in the system can not be changed suddenly, and they are equivalent to the ideal source model. Then, the whole system is decoupled by substitution theorem. However, in the simulation calculation, the voltage and current of the ideal source are all predicted, so the accuracy of this method is very low.

### 3.2. Multi-rate Real-time Simulation Method Based on the Norton Equivalence

For the problem of the accuracy with the ideal source equivalence method, the idea of the Norton equivalent is used to replace the ideal source equivalent. After the Norton equivalence of the fast subsystem and the slow subsystem, they are obtained simultaneously at the junction nodes, and then the state variables in the sub system are obtained.
There are two linear sources-containing subsystems. A subsystem has a small time scale, represented by the subscript \( f \). The other subsystem has a larger time scale, denoted by subscript \( s \). The two subsystems are connected to each other through \( k \) nodes, as shown in Figure 3.

**Figure 3.** The diagram of power system. The input of the slow subsystem and the fast subsystem consists of two parts, the independent current source inside the subsystem, and the voltage or current at the interface. Where \( \mathbf{i}_x \) and \( \mathbf{i}_s \) are the current vectors at the subsystem interface; \( \mathbf{u} \) is the voltage vector at the subsystem interface; \( \mathbf{i}_{\text{fast}} \) and \( \mathbf{i}_{\text{slow}} \) are the independent current source vectors inside the subsystem.

The Norton equivalent is applied to the slow subsystem and the fast subsystem, and the simulation diagram based on the Norton equivalent is as shown in Figure 4.

**Figure 4.** Simulation diagram based on the Norton equivalent circuit.

Assuming that the slow subsystem uses a larger simulation step size \( \Delta T \), and the fast subsystem uses a smaller simulation step size \( \Delta t \), where \( \Delta T = n \Delta t \) (\( n \) is a positive integer). The Norton equivalent circuit of the slow subsystem and the fast subsystem can be expressed as:

\[
\mathbf{i}_x(t) = \mathbf{G}_x \mathbf{u}(t) + \mathbf{I}_x(t)
\]

(6)

\[
\mathbf{i}_s(t) = \mathbf{G}_s \mathbf{u}(t) + \mathbf{I}_s(t)
\]

(7)

Where, \( \mathbf{i}_x(t) \) and \( \mathbf{i}_s(t) \) are the current vectors at the Norton equivalent circuit interface; \( \mathbf{u}(t) \) are the voltage vector at the Norton equivalent circuit interface; \( \mathbf{G}_x \) and \( \mathbf{G}_s \) are the admittance matrix of the Norton equivalent circuit, which are related to the state of dynamic elements in the slow subsystem and the fast subsystem. \( \mathbf{I}_x(t) \) and \( \mathbf{I}_s(t) \) are the current sources of the Norton equivalent circuit, and they are linear combinations of individual current sources and historical current sources.

So the current sources of the Norton equivalent circuit \( \mathbf{I}_s(t) \) and \( \mathbf{I}_f(t) \) can be represented as:

\[
\mathbf{I}_s(t) = \mathbf{C}_x \mathbf{x}_s(t - \Delta T) + \mathbf{D}_s \mathbf{i}_{\text{fast}}(t)
\]

(8)

\[
\mathbf{I}_f(t) = \mathbf{C}_f \mathbf{x}_f(t - \Delta t) + \mathbf{D}_f \mathbf{i}_{\text{fast}}(t)
\]

(9)

Where, \( \mathbf{x}_s(t - \Delta T) \) and \( \mathbf{x}_f(t - \Delta t) \) are the state variables at the previous moment in the slow subsystem and the fast subsystem. \( \mathbf{i}_{\text{fast}}(t) \) and \( \mathbf{i}_{\text{fast}}(t) \) are the independent current sources at the moment in the slow subsystem and the fast subsystem. \( \mathbf{C}_x, \mathbf{D}_s, \mathbf{C}_f \) and \( \mathbf{D}_f \) are the parameter matrix.

On the interface between the slow subsystem and the fast subsystem, there is:

\[
\mathbf{i}_i(t) = -\mathbf{i}_f(t)
\]

(10)

From formula (6), (7) and (10), we can find:

\[
(\mathbf{G}_x + \mathbf{G}_f) \mathbf{u}(t) = -[\mathbf{I}_s(t) + \mathbf{I}_f(t)]
\]

(11)

Where equation (11) is called the voltage equation of the interface node. From equation (11), the voltage vector of the interface between the slow subsystem and the fast subsystem is obtained. After that, the state variables \( \mathbf{x}_s \) and \( \mathbf{x}_f \) in the slow subsystem and the fast subsystem can be calculated.
When \( t = m \Delta T \) (\( m \) is a positive integer), the state of each dynamic element in the two subsystems and the state variables \( x_i(m \Delta T - \Delta T) \) and \( x_i(m \Delta T - \Delta t) \) at the previous moment of the two subsystems are known. Then the Norton equivalent is applied to the two subsystems, and calculate \( G_f, I_s(t), G_s \) and \( I_s(t) \). The voltage of the interface node can be obtained by the equation (11). Finally, the state variables of the whole network is obtained.

When \( t = m \Delta T + k \Delta t \) (\( k=1,2,\ldots,n-1 \)), the state of each dynamic element in the two subsystems and the state variables \( x_i(m \Delta T + (k-1)\Delta t) \) at the previous moment of the fast subsystem are known. However, the state variables \( x_i(m \Delta T - \Delta T + k \Delta t) \) of the slow subsystem are unknown. We can calculate it as followed by extrapolation.

\[
x_i(m \Delta T - \Delta T + k \Delta t) = x_i(m \Delta T) + \frac{k}{n}(x_i(m \Delta T) - x_i(m \Delta T - \Delta T))
\]  

(12)

For the slow subsystem, the parameter matrix \( G_s \) corresponds to the state of the dynamic element at time of \((m-1)\Delta T + k \Delta t \) to \( m \Delta T + k \Delta t \), which spans two periods. For simplifying the calculation, the dynamic element state at time \( m \Delta T \) is used to solve the Norton equivalent of the slow subsystem, namely \( G_s(m \Delta T + k \Delta t) = G_s(m \Delta T) \). When solving the current sources of the Norton equivalent circuit, for reducing the amount of calculation, the current sources \( I_s(m \Delta T + k \Delta t) \) is obtained by interpolation through \( i_{\text{int}}(m \Delta T) \) and \( i_{\text{int}}(m \Delta T + \Delta T) \), instead of \( i_{\text{int}}(m \Delta T + k \Delta t) \). The current sources \( I_s(m \Delta T + k \Delta t) \) can be expressed as:

\[
I_s(m \Delta T + k \Delta t) = I_s(m \Delta T) + k I_s(m \Delta T)
\]

(13)

Where the increment of equivalent current source \( k I_s(m \Delta T) \) can be represented as:

\[
\Delta I_s(m \Delta T) = \frac{1}{n}[C_i[x_i(m \Delta T - \Delta T) - x_i(m \Delta T - 2\Delta T)] + D_i[i_{\text{int}}(m \Delta T + \Delta T) - i_{\text{int}}(m \Delta T)]]
\]

(14)

It is obvious that the current source of the Norton equivalent circuit only need be calculated once in a large simulation step size, and the remaining current source can be obtained by the increment of equivalent current source. Therefore, does not need to use parameter matrix frequently calculate them, which greatly reduces the amount of calculation.

It can be seen from the previous analysis that the solution of the voltage equation with the interface node requires the Norton equivalent of the fast subsystem and the slow subsystem. Therefore, the conductance and current source with the Norton equivalent of the slow subsystem need to be sent to the fast subsystem to solve the node voltage of the interface. After the node voltage of the interface is solved, it needs to be sent to the slow subsystem to solve the internal state variables. When \( t = m \Delta T \), summarizing the above solution process, the process of the multi-rate real-time simulation method based on the Norton equivalent is as follows:

1. Calculate the Norton equivalent circuit of the slow subsystem and the fast subsystem at the kth (\( k=0,1,2,\ldots,n-1 \)) period.

\[
\begin{align*}
I_s(t + k \Delta t) &= C_s x_s(t - \Delta T) + D_s i_{\text{int}}(t) \quad k = 0 \\
I_s(t + k \Delta t) &= I_s(t) + k I_s(t) \quad k \neq 0
\end{align*}
\]

(15)

2. According to \( G_f, G_s, I_s(t + k \Delta t) \) and \( I_s(t + k \Delta t) \), write the voltage equation of the interface node.

\[
(G_f + G_s)u(t + k \Delta t) = [I_s(t + k \Delta t) + I_s(t + k \Delta t)]
\]

(17)

3. Solve the voltage equation of the interface node to get the voltage of the interface node \( u(t + k \Delta t) \).

4. According to \( u(t + k \Delta t) \), the state variable \( x_i(t + k \Delta t) \) of the fast subsystem can be obtained, and the state variable \( x_i(t) \) of the slow subsystem also can be obtained.

3.3. The data interaction of multi-rate simulation.

For hardware-in-the-loop simulation, all calculations of a step size must be completed within the specified time. Otherwise, the real-time performance of the simulation cannot be perfect. In the
multi-rate real-time simulation method based on the Norton equivalent, the Norton equivalent circuit parameters of the slow subsystem and the fast subsystem are used to solve the voltage of interface node. However, compared with the fast subsystem, there are many nodes slow subsystem. The calculation amount of solving the Norton equivalent circuit is very large, and it is difficult to complete in a small step size. Since the slow subsystem is an AC power system, it is not very important for the moment of state change of dynamic elements. In order not to delay the solution of the interface node voltage, the calculation of the slow subsystem can be earlier than the fast subsystem. For the time $\Delta S$ of the earlier solving the slow subsystem, it can be assumed that $\Delta S = 0.5 \Delta T$. If the calculation of the slow subsystem cannot be completed, we should gradually increase $\Delta S$. Otherwise, gradually decrease until the optimal $\Delta S$ is found.

![Diagram](image)

**Figure 5.** The time of input and output and the logical relationship of calculation tasks with multi-rate real-time simulation method based on the Norton equivalent.

It can be seen from Figure 5 that within a smaller simulation step size, the Norton equivalent circuit of the fast subsystem, the voltage of the interface node, and the state variables of the fast subsystem must be completed. The Norton equivalent circuit of the slow subsystem, the voltage of the interface node, and the state variables of the slow subsystem must be completed within a larger simulation step size. In each smaller simulation step size (except the node of the larger simulation step size), it is necessary to predict the Norton equivalent circuit parameters of the slow subsystem for the solution of the voltage of the interface node.

### 3. Case Study

#### 3.1 Real-time simulation platform FRTDS
FRTDS (FPGA based Real-Time Digital Solver) is an electromagnetic transient real-time simulation device developed by the Key Laboratory of the Ministry of Education for Smart Grid of Tianjin University. It encapsulates commonly calculation formulas and functions in the microprocessor core, and convert simulation scripts into instructions that control the operation of the microprocessor core by self-developed compilation software. In order to improve the efficiency of writing simulation scripts, a graphical modeling tool for real-time simulation of power systems has been developed. The power system database can be generated by using the graphical modeling tool, and then the FTRDS simulation script can be further generated, which further reduces the workload of the researchers. Users only need to generate power system database with graphical modeling tools, and do not need the capabilities of FPGA programming.

The UDP communication protocol is used between FRTDS and the simulated industrial computer, the IEC61850 communication protocol is used between FRTDS and the real digital protection device, and the Aurora communication protocol of Xilinx is used between FRTDS and the independently developed experimental controller. The FRTDS real-time simulation platform selects Xilinx’s Virtex-7 FPGA VC709 as the solver development board. The working frequency is 200MHz. The compilation system, the script generation system and the operation monitoring system in the industrial PC are all developed by the QT C++ platform. The hardware-in-the-loop experimental platform based on FRTDS is shown in Figure 6.

![Figure 6. Structure of hardware in the loop simulation system based on FRTDS](image1)

### 3.2 Simulation System and Result

The four-machine AC/DC hybrid system shown in Figure 7 is selected as the simulation example. The larger simulation step size of 50μs is selected for AC system, including generator, transformer, line and load, etc. The smaller simulation step size of 10μs is selected for DC system, including converter transformer, MMC converter station, DC line, DC filter, reactive power compensation capacitor, etc.

![Figure 7. Four-machine AC/DC hybrid system](image2)

In the AC/DC hybrid system, the converter is MMC with 77 level. The MMC is modulated by the nearest level modulation (NLM). The MMC1 is controlled by the constant active power and the constant reactive power. The MMC2 is controlled by the constant DC voltage and the constant reactive power. The relevant parameters of AC/DC hybrid system are shown in Table 1.

| name         | parameter |
|--------------|-----------|

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For verifying the correctness and effectiveness of the multi-rate real-time simulation method, single-rate simulation method with the simulation step size of 10μs (method 1) and two multi-rate simulation methods are carried out for AC/DC hybrid system. Multi-rate real-time simulation method based on the ideal source equivalence is method 2 and multi-rate real-time simulation method based on the Norton equivalence is method 3. For single-rate simulation, FRTDS provides 2000 maximum instruction clocks. For multi-rate simulation, the memory space is opened with the simulation step size of the slow subsystem. FRTDS provides 10000 maximum instruction clocks. The software provided by FRTDS is used to write simulation scripts of single-rate and multi-rate methods respectively, and then the tasks are assigned. The list of instruction space occupied by the calculation task is shown in Table 2.

| Method | Number of Maximum Instruction Clocks | Number of Task Instruction Clocks |
|--------|--------------------------------------|-----------------------------------|
| Method 1 | 2000                                 | 2810                              |
| Method 2 | 10000                                | 9080                              |
| Method 3 | 10000                                | 9530                              |

Table 2. The comparison of single-rate and multi-rate instruction clocks.

As can be seen from Table 2, the instruction space occupied by the calculation task of method 1 has exceeded the allowable space, which cannot meet the requirements of real-time simulation. Both of the two multi-rate simulation methods can satisfy the real-time performance of simulation.

For proving the accuracy of multi-rate real-time simulation method based on the Norton equivalent, it is compared with single-rate off-line simulation and multi-rate real-time simulation based on ideal source equivalent.

(1) Three phase ground fault in converter bus 3

When t = 2s, three phase metal grounding fault of converter bus 3 is set, and the fault is cleared after 0.2s. The simulation result of DC voltage and AC current under single rate and two multi-rate methods is shown in Figure 8.

Figure 8. Three phase ground fault in converter bus 3. (a) AC current. (b) DC voltage

(2) The ground fault in DC line

When t = 2s, the ground fault in DC line is set, and the fault is cleared after 0.2s. The simulation result of DC voltage and AC current under single rate and two multi-rate methods is showed in Figure 9.
The error between the simulation results of the two multi-rate simulation methods and the single-rate simulation results is shown in Figure 10. It can be seen that the error of multi-rate simulation method based on the Norton equivalent is less than 0.8%, and the maximum error of multi-rate real-time simulation method based on the ideal source equivalence is 2%. The simulation results of the two multi rate methods meet the simulation correctness, but multi-rate real-time simulation method based on the Norton equivalence has higher accuracy.

Figure 10. Error of two multi-rate methods

It can be concluded that the accuracy of multi-rate real-time simulation method based on the Norton equivalence is higher than that based on the ideal source equivalence through the above pictures. Multi-rate real-time simulation method based on the ideal source equivalence take the advantage of the characteristic that the inductance current and capacitance voltage in the circuit cannot be suddenly changed. And the whole system is decoupled by the method of substitution theorem. The equivalent source of the other system is predicted when the fast and slow system is calculated, so the accuracy is slightly worse. The conductance of the Norton equivalence circuit is the true value. It is only necessary to predict the current source of the Norton equivalent circuit of the slow system during the calculation of the fast system, so the accuracy is higher than multi-rate real-time simulation method based on the ideal source equivalent.

4. Conclusions

With the widespread application of MMC-based power electronic equipment in power systems, multi-rate real-time simulation of AC/DC hybrid power system has a broad application prospect in the field of power system simulation. However, the existing multi-rate simulation methods are not perfect and have poor accuracy.

This paper proposes a multi rate real-time simulation method based on the Norton equivalent, compared with multi-rate simulation method based on the ideal source equivalent. After the Norton equivalence of the fast subsystem and the slow subsystem, they are obtained simultaneously at the junction nodes. In order to reduce the amount of simulation calculation, the Norton equivalent circuit is obtained by incremental calculation. The data interface between the fast subsystem and the slow subsystem is realized by extrapolation method. For ensuring the real-time performance of the simulation, the method that the slow subsystem calculates ahead of the fast subsystem is given for...
the slow subsystem with a large amount of calculation. Compared with the traditional multi-rate simulation methods, the paper proposed method improves the simulation accuracy without losing the simulation scale, and has important theoretical and practical significance to research on the real-time simulation of the AC/DC hybrid power system.

**Author Contributions:** conceptualization, J.Z.; methodology, J.Z.; software, J.Z.; validation, J.Z.; formal analysis, J.Z.; investigation, J.Z.; resources, J.Z.; data curation, J.Z.; writing — original draft preparation, J.Z.; writing — review and editing, J.Z., B.Z.; visualization, J.Z.; supervision, B.Z.; project administration, B.Z.; funding acquisition, B.Z.

**Funding:** This research was funded by the National Natural Science Foundation of China, grant number 51477114.

**Conflicts of Interest:** The authors declare no conflict of interest.

**References**

1. Liu, H.B.; Bian, D.; Sun, L.; Yun, Z.J.; Li, Y. Research on Electromechanical-Electromagnetic Transient Hybrid Simulation of AC/DC Hybrid System. *Power System Protection and Control*. **2019**, *47*, 39-47.

2. Li Y.L.; Zhang X., Li, Y.J.; Chen, Z.J.; Wu, M.Q. Current Situation and Challenges of Simulation Technology for AC/DC Hybrid Power Grid. *Electric Power Construction*. **2015**, *36*, 1-8.

3. He, Y.Y.; Zheng, X.D.; Tai, N.L.; Hou, J.X.; Xu, J.; Huang, W.T. Overview of Modeling Methods of LCC-HVDC Converter in AC-DC Hybrid Power Grid. *Proceedings of the CSEE*. **2019**, *39*, 3119-3128.

4. Dong, X.Z.; Tang, Y.; Bu G.Q. Confronting Problem and Challenge of Large Scale AC-DC Hybrid Power Grid Operation. *Proceedings of the CSEE*. **2019**, *39*, 3107-3119.

5. Zhang, M.X. Research on real time multi rate joint simulation technology for power electronic system. Master’s Degree, Beijing Institute of Technology, Beijing, 2016.

6. Jost, A.; Niklaus, F.; Min, L. High Fidelity Real-Time Simulation of Multi-Level Converters. 2018 International Power Electronics Conference, Niigata, Japan, May 20-24, 2018, 2199-2203.

7. Zhao, J.L.; Liu, J.T.; Li, P. GPU Based Parallel Matrix Exponential Algorithm for Large Scale Power System Electromagnetic Transient Simulation. 2016 IEEE Innovative Smart Grid Technologies-Asia, Melbourne, Australia November 28- December 1, 110-114.

8. Lu, F.S.; Song, J.Q.; Yin, F.K.; Zhang, L.L. Overview of CPU/GPU collaborative parallel computing research. *Computer Science*. **2011**, *38*, 5-9.

9. Han, J.; Dong, Y.F.; Miao, S.H.; Liu, Y.L.; Liu, Z.W. MATE-based multi-rate electromagnetic transient parallel simulation method for power system sub-network. *High Voltage Engineering*. **2019**, *45*, 1857-1865.

10. CROW, M.L.; CHEN, J.G. The method for simulation of power system dynamics. *IEEE Trans on Power Systems*. **1994**, *9*, 1684-1890.

11. CHEN, J.G.; CROW, M.L. A variable partitioning strategy for the multi-rate method in power system. *IEEE Trans on Power Systems*. **2008**, *23*, 259-266.

12. Tang, Y. New Development of Research on Simulation and Modeling of Multi-time Scale Whole Process of AC/DC Power System. *Power System Technology*. **2009**, *33*, 1-8.

13. Moreira, F.A.; Marti, J.R. Latency techniques for time-domain power system transients simulation. *IEEE Transactions on Power Systems*. **2005**, *20*, 246-253.

14. Mu, Q.; Liang, J.; Zhou, X.X.; Li, G.; Zhang, X. A Node Splitting Interface Algorithm for Multi-rate Parallel Simulation of DC Grids. *CSEE Journal of Power and Energy Systems*. **2018**, *4*, 388-397.

15. Wang, X.; Zhang, B.D.; Chen, M. Multi-rate time Simulation Method Based on RTDS and FPGA Co-simulation Platform. *Automation of Electric Power Systems*. **2016**, *40*, 144-150.

16. Zhai, X.B.; Lin, C.; Gregoire, L.A. Multi-rate Real-time Simulation of Modular Multilevel Converter for HVDC Grids Application. IECON 2017 - 43rd Annual Conference of the IEEE Industrial Electronics Society., Beijing, China, October 29-November, 1, 2017, 1325-1330.

17. Ou, K.J.; Li, P.F.; Guan, L.; Chai, Z.X.; Zhang, Y.J. Design and Research of Multi-time Scale Hybrid Real-Time Simulation System for AC Power Grid. *Southern Power System Technology*. **2017**, *11*, 53-58+64.

18. Zhang, F.; Huang, W.C.; Li, C.D. MMC generalized fast simulation model suitable for multiple sub-module topologies. *Electric Power Automation Equipment*. **2019**, *39*, 129-136+143.

19. Jost, A.; Niklaus, F. Sub-Cycle Average Models with Integrated Diodes for Real-Time Simulation of Power Converters. 2017 IEEE Southern Power Electronics Conference, Puerto Varas, Chile, December 4-7, 2017, 6-11."