Agent-based analysis of risk contagion in stock market from perspective of econophysics

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Abstract. This work discusses the risk contagion in the stock market from perspective of econophysics. Through the agent-based analysis, agents in stock market are supposed to have four types: susceptible agents, infected agents, contagious agents and immune agents. As different types of agents are set to have different contagious probabilities, grapevine news is likely to be transmitted among different agents. The results show that, according to the conditions imposed, the experiment stops at time 292. At the point, the most agents are immune agents and only the few are susceptible agents and contagious agents. In addition, infected agents are nonexistence at the end of experiment. In the respect of curve characters, the curve of susceptible agents shows a gradually increasing trend, and the curve of immune agents presents a gradually decreasing trend. In particular, the curves of infected agents and contagious agents are a similar feature, which is an increasing trend at first and then a decreasing trend.

1. Introduction
Risk contagion is a common phenomenon in the social economy. The earliest theory of risk contagion was put forwarded by Krugman, who regarded economic expansion and exchange rate fluctuation as main reasons of risk contagion in international markets [1]. The model of currency crisis proposed by Obstfeld, showed that devaluation expectations had a great effect on risk contagion, and caused currency speculation in financial crisis [2]. After analyzing American sub-prime crisis and European debt crisis, the primary reasons of risk contagion are summarized as credit expansion of financial intermediary and inflated asset bubble. Although macroscopic analysis methods find the reasons of risk contagion, the internal mechanisms of risk contagion is different to be revealed by the traditional research approaches. So the researches on risk contagion gradually shift their eyes to investors in financial markets. Behavioral finance theory suggests that market traders have heterogeneous investment behaviors, such as overconfidence complex, loss aversion, herd mentality, repentance and prudence. Their investment behaviors could make asset price change and form risk contagion. But investor behavior is difficult to be determined under the influences of series of mental features. As a new interdisciplinary, econophysics is more suitable for explaining risk contagion.

In this research, risk contagion is discussed through agent modeling and simulation experiment from the perspective of econophysics. The reminder of this paper is organized as follows. Section 2 shows the recent achievements of econophysics in studying agent behavior in stock market. Subsequently, section 3 designs an agent-based model to analyze risk contagion in stock market. Next, the simulation experiment is in section 4. Finally, section 5 is the conclusion of this paper.
2. Related studies
Econophysics mainly applies physics theories and models to economic and finance fields for the sake of better analysis of uncertainty problem and nonlinear problem in reality. Because of the different risk preferences and investment goals, investors have characteristic of heterogeneity in financial market. In order to reveal the interactions among investors, the agent-based model can be built. For example, Huang built a heterogeneous agent model for studying the relationship between expected price and moving average value in Taiwanese stock market [3]. Based on multi-agent models, a hybrid recommendation system could be used for simulating and predicting the price in stock market with a higher accuracy in Ref. [4]. In addition, artificial financial market was built to simulate the decision-making process of agents. For instance, Bertella et al. found the excessive volatility of asset price in the established artificial market with an increase in heterogeneity of the strategies chosen by agents from Ref. [5]. As discussed by Bertella et al [5], Yim et al [6] further developed the agent-based model to reveal the nonlinear interaction among heterogeneous agents, which could be regarded as “stylized facts” occurred in financial market. In this model, agents included three types of participants: optimist, pessimist and fundamentalist. Every type of agents was likely to become another type or keep ordinal type on the next trading day, according to their herding and relative payoff strategies. By means of complex network theory, the heterogeneous of interaction among multi-agents easily resulted in “stylized facts”. A novel heterogeneous model was presented in Ref. [7], agents were supposed to two new types, that is zero-intelligence agents and less intelligence agents. After adjusting evolution speed of model, investment strategy and intelligence degree of agents, the simulation models were built based on Chinese stock market and American stock market.

In the framework of econophysics, agent-based models are suitable for simulating the interactions of autonomous agents because the heterogeneous investors generally exist in financial market [8]. Risk contagion essentially results from the nonlinear interaction among heterogeneous agents, and therefore, the approach of econophysics is more suitable for the study of risk contagion [9].

3. An agent-based model of risk contagion
In a real stock market, investors show different investment behaviors under the influences of heterogeneity [10]. In other words, every investor is special and distinctive considering investors’ different natural endowments, preferences, information, etc. These characteristics of investors can be also explained from a behavioral finance theory perspective [11]. Drawing on behavioral finance theories, the heterogeneous characteristics of investors in stock market mainly includes four parts as follows: heterogeneous constraints, heterogeneous incomes, heterogeneous preferences and preference heterogeneous beliefs [12]. In order to simplify the problem of risk contagion, investors can be assumed as agents that are classified as four kinds: susceptible agents, infected agents, contagious agents and immune agents in this paper. Let’s Sa, Fa, Ca and Ma be the above four kinds of agents.

Concretely speaking, Sa as a susceptible agent, is an investor that may receive some grapevine news. Next, Fa as an infected agent, is an investor who receives the grapevine news, has not decided whether to spread it. If Fa spreads the news to others, it will be called as contagious agent, Ca for short. Conversely, if Fa doesn’t spread the news to others, it will be called as immune agent, Ma for short.

Let us suppose that there are the fours contagious probabilities: \( p_1 \), \( p_2 \), \( p_3 \) and \( p_4 \). When the grapevine news appears in stock market, the above different kinds of agents have different behavior characteristics. If a part of susceptible agents choose to believe the grapevine news, they will transform into infected agents. Let \( p_1 \) be the transition probability from Sa to Fa. And for infected agents, they may transform into contagious agents or immune agents with a probability of \( p_2 \) or \( p_3 \), respectively. Over a period of time, contagious agents may lose interest in spreading grapevine news, and stop spreading this news. At this point, contagious agents will transform into immune agents with a probability of \( p_4 \). In this paper, we assume that the percentages of Sa, Fa, Ca and Ma are in turn \( sa(t) \), \( fa(t) \), \( ca(t) \) and \( ma(t) \) at time t. The relationship of different percentages can be described as expression (1).
\[
    sa(t) + fa(t) + ca(t) + ma(t) = 1
\]  

(1)

As an epidemic outbreak can rapidly spread in a very short time, this propagating process of epidemic diseases is similar to the risk contagion in stock market. According to the Kermack-McKendrick model for an epidemic outbreak in Ref. [13], the non-linear dynamic characteristics of agents in stock market are shown by expression (2).

\[
\begin{align*}
    \frac{d{sa}(t)}{dt} &= -{p_1} \cdot {sa}(t) \cdot {ca}(t) \\
    \frac{d{fa}(t)}{dt} &= {p_1} \cdot {sa}(t) \cdot {ca}(t) - {p_2} \cdot {fa}(t) - {p_3} \cdot {fa}(t) \\
    \frac{d{ca}(t)}{dt} &= {p_2} \cdot {fa}(t) - {p_4} \cdot {ca}(t) \\
    \frac{d{ma}(t)}{dt} &= {p_3} \cdot {fa}(t) + {p_4} \cdot {ca}(t)
\end{align*}
\]  

(2)

In order to reveal the evolutionary mechanism of risk contagion among different agents, a simulation experiment is design to develop further discussion in the next part.

4. The simulation experiment

In the simulation experiment, the amount of agents is assumed to be 5000. Suppose that the amount of contagious agents is equal to 1 at the initial stage of the experiment. At this point, others are susceptible agents. And the amount of infected agents and immune agents is equal to 0, respectively. Considering the effects of contagious agents on susceptible agents are obvious, the total amount of susceptible agents has a decreasing trend. Meanwhile, susceptible agents are likely to transform into contagious agents with time. But the amount change of infected agents and immune agents is complex. To better reveal the dynamic characteristics of different agents, we set the value of \( p_1, p_2, p_3 \) and \( p_4 \) in the table 1.

| Table 1. The contagious probability in the simulation experiment. |
|---------------------------------------------------------------|
| Contagious probability | \( p_1 \) | \( p_2 \) | \( p_3 \) | \( p_4 \) |
| Probability value     | 0.0001 | 0.15 | 0.20 | 0.02 |

Figure 1 shows the dynamic characteristics of agents, according to the probability values in table 1. We suppose if the amount of agent is low than or equal to 0.0005 in the simulation experiment, this experiment will stop. In other words, at the initial stage the amount of infected agents and immune agents is set to 0. And in the final stage, if the amount of any kind of agents is low than or equal to 0.0005, this agent is assumed to be nonexistent. In figure 1, the dynamic characteristics of agents can be shown from the four curves: the Sa curve, the Fa curve, the Ca curve and the Ma curve.

According to the results, the simulation experiment finally stops after 292 steps. In other words, when the experiment time is equal to 292, the entire experiment will be completed. In figure 1, all curves are divided into two types, according to the changing trend of different agents in the simulation experiment. The first type includes the Sa curve and the Ma curve, which decrease and increase gradually with the experiment time, respectively. More specifically, the Sa curve decreases from 4999 to 0.1, and the Ma curve increases from 0 to 4978.8. The changing rates of the above two curves are slow at the initial stage (0<time<40) and the last stage (80<time<292), then high in the middle stage (40<time<80). The second type contains the Fa curve and the Ca curve, which have the trend of
increase at first and decrease late. When time is 62, the Fa curve shows a maximum that is 509.0375. And when time is 77, the Ca curve shows a maximum which is 1401.1.

Figure 1. The amount of agent in simulation experiment.

In table 2, we show the number and proportion of each agent at the end of the simulation (i.e., time = 292) presented in figure 1. The two indexes are shown in table 2, which are amount and proportion. At the end of the experiment, almost the whole agents are immune agents, and the proportion is 99.576%. However, susceptible agents and contagious agents are very few at this point. The proportion of Sa and Ca is 0.002% and 0.422%, respectively. For Fa, there is non-existent and the proportion is 0.000%.

Table 2. The quantity structure of agents at the end of simulation experiment.

| The different kinds of agents | Sa   | Fa   | Ca   | Ma   |
|-------------------------------|------|------|------|------|
| Amount                       | 0.1  | 0.0005 | 21.1 | 4978.8 |
| Proportion                   | 0.002% | 0.000% | 0.422% | 99.576% |

As the amount of infected agents in the simulation is equal to 0.0005 at time 292, the simulation stops according to the hypothesis in this paper. When the amount of any kind of agents is less than or equal to 0.0005, this kind of agent is regarded as nonexistence. Thus, the corresponding proportion is 0.000%.

5. Conclusions

This paper discusses the risk contagion in stock market from perspective of econophysics. Firstly, we assume there are four different kinds of agents: susceptible agents, infected agents, contagious agents and immune agents. Secondly, the mechanism of risk contagion among agents is designed in stock market. Concretely, a non-linear differential equation group is used to analyze the non-linear dynamic characteristics of agents based on Kermack-McKendrick model. Thirdly, the simulation experiment is performed so as to reveal the changing trend of agents.

Under the circumstance of experiment in this paper, the results show that (1) the amount of susceptible agents decreases from 4999 to 0.1 with the slow speed at first (0<time<40), then with the rapid speed (40<time<80), and finally with the slow speed (80<time<292); and (2) the amount of
infected agents increases firstly (0<time<62) and then decreases (62<time<292), from 0.00 to 509.0375 to 0.005; then (3) the amount of contagious agents also increases firstly (0<time<77) and then decreases (77<time<292), from 1 to 1401.1 to 21.1; at last (4) the amount of immune agents increases from 0 to 4978.8 with the slow speed at first (0<time<40), then with the rapid speed (40<time<80), and finally with the slow speed (80<time<292).

In this paper, risk contagion happens among the four different kinds of agents. With the appearance of grapevine news, a contagious agent can transmit the news to susceptible agents rapidly. Then susceptible agents transform into infected agents who decide whether to transmit this news to other agents. If the transmission happens, infected agents will transform into contagious agents. Conversely, infected agents will transform into immune agents. Finally, if the contagious agents lose their interests in the news, they will transform into immune agents. As more and more agents lose their interests in the news in the process of risk contagion, almost all of agents are immune agents at the end of experiment.

Acknowledgments
Thanks to the reviewers for their useful comments and the Fundamental Research Funds for the Central Universities of China (No. GK201803093).

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