Research on roundness error extraction method for precision machine tool spindle based on least square method

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Abstract. Based on the application requirement of online measurement and evaluation for roundness error of precision machine tool spindle, a specific method is proposed to fit the spindle roundness error based on the least square method according to the characteristics of the roundness error. Firstly, aiming to the radial vibration displacement signal of spindle, which collected by the sensor, a multi-harmonic signal with speed frequency of spindle as the fundamental frequency is constructed, and then the least square method is used to fit the amplitudes and phases of each harmonic signals. Finally, according to the fitting results, the rotation error and the interference of higher-order noise signals are eliminated, and the contour signal with only roundness error of the spindle is reconstructed, and then the roundness error of the spindle is evaluated by appropriate evaluation method. The simulation system of rotation and roundness errors separation for spindle system is developed, and the feasibility and accuracy of least square method to fit the spindle roundness error are verified by simulation analysis.

1. Introduction
Roundness error is an important quality index of the precision machine tool spindle, and it has a certain impact on the form-position error and surface quality of the machining workpiece. Based on the measurement and calculation of roundness error for precision spindle, the influence of roundness error on the machining accuracy of workpiece can be analysed, and the compensation control for roundness error can be implemented so as to improve the machining accuracy of workpiece [1]. Therefore, roundness error is a very important monitoring parameter in the production and application of precision machine tool spindle.

According to the measuring principle of the roundness error, the sensors are used to obtain the displacement signals of measuring points under the running condition of spindle, and then the appropriate signal processing methods are used to separate the roundness error signals. At present, with the development of detection technology, the sub-micron or even nano-scale measurement accuracy has been achieved for displacement signals. However, a large number of noise signals will be unavoidably mixed in the measurement signal along with the high-precision measurement. Therefore, how to accurately extract and evaluate the roundness error signal is the focus of related research. Based on the analysis of the four evaluation methods of roundness error, a least square method and the minimum area method was put forward by Liu Qingmin et al. [2]. Based on the dynamic characteristics of local least squares circle parameters, a highly robust evaluation method of workpiece roundness was designed by Cao Zhimin et al. [3]. The regular polygon search algorithm was used to evaluate roundness error by Wang Shenghuai et al., and the principle and calculation steps of the
roundness error evaluation method was discussed in detail [4]. Based on the geometric characteristics of roundness error, the roundness evaluation method of adaptive region search algorithm was proposed by Gong Yuling et al. Based on the evaluation results of roundness error, the expression guide of measurement uncertainty and Monte Carlo method were used to evaluate the uncertainty of roundness error by Wang Dongxia et al. [6].

Based on the radial and periodic properties of the roundness error, a least squares method is proposed to separate the roundness error for spindle, and the application of the method is discussed in this paper.

2. The separating method of roundness error

2.1. The geometric characteristics of roundness error

The radial and periodic properties are two main geometric characteristics of roundness error. The radial property means that the roundness error is reflected in the radius direction of the axis circumference. The periodic property means that the roundness error has the same rotating frequency with the spindle.

Based on the above characteristics, the periodic property of roundness error can be expressed by Fourier series, which is shown in equation (1):

\[ r(\omega t) = r_0 + \sum_{i=1}^{\infty} a_i \cos\omega_it + \sum_{i=1}^{\infty} b_i \sin\omega_it = r_0 + \sum_{i=1}^{\infty} c_i \sin(i\omega t + \alpha_i) \]  

And, \( c_i = \sqrt{a_i^2 + b_i^2} \), \( \alpha_i = \arctan \frac{a_i}{b_i} \)

In equation (1), \( r(\omega t) \) is the radius at the phase of \( \omega t \), \( r_0 \) is the theoretical radius of spindle, \( a_i \) and \( b_i \) are the Fourier coefficient, \( c_i \) and \( \alpha_i \) are the amplitude and phase of the i-th harmonic component respectively.

When \( i=1 \), it means that the base circle of spindle has eccentricity, which is shown as an eccentric circle with fixed radius in the polar coordinates. When \( i=2 \), it is shown as an ellipse in the polar coordinates. When \( i=3 \), it is shown as a triangular circle in the polar coordinates. The n-th harmonic wave is shown as an n-edged circle in the polar coordinates by analogy. According to the definition of roundness error, the influence of eccentricity, surface roughness (higher order harmonic component) and surface waviness on roundness error should be eliminated, that is, the roundness error function can be expressed as equation (2).

\[ \Delta r(\omega t) = \sum_{i=2}^{\infty} a_i \cos\omega_it + \sum_{i=2}^{\infty} b_i \sin\omega_it = \sum_{i=2}^{\infty} c_i \sin(i\omega t + \alpha_i) \]  

In equation (2), \( n \) denotes the highest order of the harmonics, and the signal, which the order is higher than \( n \), can be regarded as the noise and surface roughness signals.

2.2. The separating method for roundness error based on least square method

In practical application, the vibration displacement signal of the spindle can be expressed as a finite Fourier series, as shown in equation (3).

\[ r(\omega t) = r_0 + \sum_{i=1}^{n} a_i \cos\omega_it + \sum_{i=1}^{n} b_i \sin\omega_it \]  

The equation (4) can be gained by linearizing the equation (3).

\[ r(x) = r_0 + \sum_{k=1}^{2n} d_k x_k \]  

The equation (5) can be gained by comparing equation (3) with equation (4).
According to the principle of least square method, the square sum equation of error can be established and shown in equation (6).

\[
\phi(d_1, \ldots, d_{2n}, r_0) = \sum_{j=1}^{m} \left( r - r_j \right)^2
\]  

(6)

In equation (6), \(m\) is the sample length of the acquisition signal. The equation (7) can be gained by assuming the partial derivatives of equation (6) to \(d_1, \ldots, d_{2n}, r_0\) equal to zero.

\[
\frac{\partial \phi}{\partial d_k} = \sum_{j=1}^{m} \left( r - r_j \right) x_{kj} = 0
\]

\[
\frac{\partial \phi}{\partial r_0} = \sum_{j=1}^{m} 2 \left( r - r_j \right) = 0
\]

(7)

The \((2n+1) \times (2n+1)\) normal equations, which shown as equation (8), can be gained by simplifying the equation (7).

\[
\begin{bmatrix}
\sum_{j=1}^{m} x_{j1}^2 & \ldots & \sum_{j=1}^{m} x_{j2} x_{j1} & \ldots & \sum_{j=1}^{m} x_{jm} x_{j1} \\
\sum_{j=1}^{m} x_{j2} x_{j1} & \ldots & \sum_{j=1}^{m} x_{j2}^2 & \ldots & \sum_{j=1}^{m} x_{jm} x_{j2} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
\sum_{j=1}^{m} x_{jn} x_{j1} & \ldots & \sum_{j=1}^{m} x_{jn} x_{j2} & \ldots & \sum_{j=1}^{m} x_{jm} x_{jn} \\
\end{bmatrix}
\begin{bmatrix}
d_1 \\
d_2 \\
\vdots \\
d_{2n} \\
r_0
\end{bmatrix}
= \begin{bmatrix}
\sum_{j=1}^{m} x_{j1} r_j \\
\sum_{j=1}^{m} x_{j2} r_j \\
\vdots \\
\sum_{j=1}^{m} x_{jn} r_j \\
\sum_{j=1}^{m} r_j
\end{bmatrix}
\]  

(8)

The parameters of \(d_1, \ldots, d_{2n}, r_0\) can be calculated by column principal component Gauss elimination method, and then the amplitude and phase of each harmonic component in equation (1) can be calculated by equation (9).

\[
\begin{bmatrix}
c_i = \sqrt{d_{2i-1}^2 + d_{2i}^2} \\
\alpha_i = \arctan(d_{2i} / d_{2i-1})
\end{bmatrix}
\]  

(9)

According to the definition of roundness error, the eccentric component \((i=1)\) should be eliminated firstly, and then the amplitude and phase of each harmonic component of the spindle roundness error can be fitted. The sinusoidal function of each harmonic component can be constructed secondly according to the fitting amplitude and phase. Finally, the contour signal contains only the roundness error of the spindle can be reconstructed.

3. The simulation analysis for roundness error extraction

According to the form of equation (1), the 10th harmonic (the roundness error is 10 edges circle) signal is taken as an example to simulate and analyse in the paper. The diameter of the spindle is set as 20 mm, the speed of spindle is set as 600 rpm, and the sampling frequency is set as 9000. The other simulation parameters are shown in Table 1.
Table 1. The simulation parameters of the 10th harmonic signal

| Order | 1    | 2    | 3    | 4    | 5    |
|-------|------|------|------|------|------|
| $c_i$ (mm) | 0.031 | 0.028 | 0.025 | 0.022 | 0.019 |
| $\alpha_i$ (°) | -44.4258 | 11.9148 | 18.3753 | 55.709 | 59.5547 |

| Order | 6    | 7    | 8    | 9    | 10   |
|-------|------|------|------|------|------|
| $c_i$ (mm) | 0.016 | 0.013 | 0.01 | 0.007 | 0.004 |
| $\alpha_i$ (°) | 16.5897 | 54.699 | 25.1319 | -48.108 | -15.5457 |

Based on the above parameters, the uniform white noise signal (the amplitude is 0.007mm) is added to the simulation signal, and the integer period sampling method is applied in the simulation signal. The five-cycle simulation signal and the outer contour curve of spindle are shown in Figure 1.

![Figure 1. The simulation signal of spindle shape and position error](image1)

(a) The simulated waveform of displacement signal  
(b) The outer contour curve in polar coordinates

Figure 1. The simulation signal of spindle shape and position error

The original simulation signal is fitted by the least square method, and the fitting results are shown in Table 2. The contour signal of roundness error is reconstructed after signal fitting as shown in Figure 2.

Table 2. The fitting amplitude and phase of the 10th harmonic signal by least square method.

| Order | 1    | 2    | 3    | 4    | 5    |
|-------|------|------|------|------|------|
| $c_i$ (mm) | 0.031056 | 0.02797 | 0.024991 | 0.022141 | 0.018844 |
| The relative error | 0.18% | 0.11% | 0.04% | 0.64% | 0.82% |
| $\alpha_i$ (°) | -44.6465 | 11.7129 | 18.4537 | 55.7769 | 60.3731 |
| The relative error | 0.5% | 1.69% | 0.43% | 0.12% | 1.37% |

| Order | 6    | 7    | 8    | 9    | 10   |
|-------|------|------|------|------|------|
| $c_i$ (mm) | 0.016067 | 0.012876 | 0.009808 | 0.0070065 | 0.003868 |
| The relative error | 0.42% | 0.78% | 1.92% | 0.09% | 3.3% |
| $\alpha_i$ (°) | 16.8016 | 54.7559 | 25.3447 | -48.7768 | -15.2336 |
| The relative error | 1.28% | 0.1% | 0.85% | 1.39% | 2% |

Notes: The relative error = | (theoretical value –fitting value) / theoretical value |

![Figure 2. The fitting and reconstructing signal of spindle shape and position error](image2)

(a) The Waveform of fitting and reconstructing signal  
(b) The outer contour curve in polar coordinates of fitting and reconstructing signal

Figure 2. The fitting and reconstructing signal of spindle shape and position error
Based on the definition of roundness error, the least square method is applied to evaluate the roundness error of the signal in Figure 2. The evaluation result is 0.14349 mm.

Comparing Table 1 with Table 2, it can be seen that the amplitude and phase of the fundamental frequency and roundness error signal of the simulation signal can be accurately fitted by the least square method.

Meanwhile, it can be seen that the rotation error and higher order noise signal can be eliminated after the amplitude and phase of the fundamental frequency and multiple harmonic roundness error signal has been fitted, and then the contour signal of the spindle with roundness error can be reconstructed.

4. Conclusions
(1) Based on the radial and periodic properties of the roundness error, the roundness error can be expressed as the superposition of the multi-harmonic signals whose fundamental frequency is the rotation frequency of spindle. Based on this principle, the least square method is proposed to fit the roundness error of spindle in this paper.

(2) The simulation results show that the amplitude and phase of fundamental and higher order harmonic signals can be accurately fitted by the proposed least squares method. The outline signal of the spindle with only roundness error can be reconstructed by combining harmonic signals.

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