Beyond Gauge Theories

John H. Schwarz
California Institute of Technology
Pasadena, CA 91125, USA

Abstract

Superstring theory, and a recent extension called M theory, are leading candidates for a quantum theory that unifies gravity with the other forces. As such, they are certainly not ordinary quantum field theories. However, recent duality conjectures suggest that a more complete definition of these theories can be provided by the large $N$ limits of suitably chosen $U(N)$ gauge theories associated to the asymptotic boundary of spacetime.

Presented at WEIN 98 in Santa Fe, NM (June 1998)

1Work supported in part by the U.S. Dept. of Energy under Grant No. DE-FG03-92-ER40701.
1 Introduction

The conference organizers suggested to me that I speak on the topic “Beyond Gauge Theories,” which is what I am doing. No doubt, they anticipated that my response would be that beyond gauge theory there is superstring theory and M theory, and (of course) it is. However, until recently we didn’t have a complete nonperturbative definition of what these are, even though we have learned much about them over the years. So the question that has been gradually coming into focus is “what underlies superstring theory and M theory?” The answer that has emerged recently is gauge theories. This means that certain gauge theories completely define consistent quantum gravity vacua! After 30 years, we’ve come full circle—but much has been learned in the process.

For those of you who haven’t been following this subject, or are new to the field, let me recall that string theory was developed in the period 1968–73 as a phenomenological theory of the strong interactions.\(^2\) String eigenmodes were identified as hadrons and rules for computing scattering amplitudes were developed. This program had some qualitative successes, such as incorporating Regge behavior and duality. There were serious problems, however, and the string approach to describing strong interactions was dropped (by almost everybody) in the mid-70’s for two main reasons: First, consistent string theories had numerous unrealistic features such as ten dimensions, massless particles (with \(J \leq 2\)), supersymmetry, no parton-like structures, etc. Second, QCD was developed and quickly recognized to be a compelling alternative.

In 1974, Joël Scherk and I proposed that since string theory incorporates general relativity as well as gauge theory and is ultraviolet finite, it should be considered a candidate for a unified quantum theory of gravity and all other forces.\(^3\) This meant that the characteristic string length scale \(\ell_s\) should be close to the Planck scale \(10^{-32}\text{cm}\) rather than the QCD scale \(10^{-13}\text{cm}\). Equivalently, the string mass scale \(m_s\) should be about \(10^{18}\text{ GeV}\) rather than 100 MeV.

Before giving you a progress report on the quarter-century-old quest of constructing a unified quantum theory of gravity and all other forces in terms of a unique underlying superstring theory that has no arbitrary dimensionless parameters, I would like to make a short digression. When we talk about this subject to our experimental friends, quite understandably they want to know whether our theory can be regarded as an extension of

\(^2\)For a collection of review articles from this period see \(^1\) and \(^2\).
the standard model, and if so, what experimentally testable predictions does it make. In fact, in the 13+ years since the pioneering paper of Candelas et al. [5], there have been many attempts to construct realistic superstring vacua. The conclusion of these studies is that it is possible to find ones that have many qualitatively correct features, but (at the present time) we cannot single out a particular one of them as a compelling candidate. The only general feature that they all seem to share is low-energy (i.e., electroweak scale) supersymmetry. So we are inclined to call supersymmetry a generic prediction. Suppose that the LHC rules this out. Will we still believe in this approach? I can only speak for myself, though I suspect that most others working in this field would agree. I believe that we have found the unique mathematical structure that consistently combines quantum mechanics and general relativity. So it must almost certainly be correct. For this reason, even though I do expect supersymmetry to be found, I would not abandon this theory if supersymmetry turns out to be absent. It is (remotely) conceivable that all supersymmetries are broken at some very high unification scale, and the gauge hierarchy problem is solved in some other way. Then the unification of the three gauge couplings in the minimal supersymmetric extension of the standard model would be just a bizarre coincidence.

Having pinned me down, you see that we have no absolute predictions at this point, except that we are working on the right theory. (We do have some solid postdictions, however: general relativity and gauge theory.) The problem is not our unwillingness to do phenomenology, but rather the strongly held belief that we need a better understanding of the mathematical structure that we are dealing with, before we can make reliable predictions. We are still discerning what the theory is, and in this I will report some significant progress. But even when that is settled, we will need to understand better how a realistic vacuum is chosen. In particular, it should have a vanishing cosmological constant (or at least one that is extremely small). This is easy when supersymmetry is unbroken, but when it is broken one tends to get a cosmological constant that is much too large. At the present time, no one has proposed a realistic superstring vacuum with a sufficiently small cosmological constant. There is no freedom to fine-tune it away, though that is not what we want to do, anyway. The point of this rather lengthy digression has been to explain why we continue to do abstruse mathematics without making any predictions. It’s not because we are perverse, or physicists gone astray. It’s what we believe is required to achieve our common goal.

3Superstring theory is the only theory for which this is a problem, because it is the only quantum theory in which the cosmological constant can be computed.
2 The Superstring Revolutions

In the *first superstring revolution* (1984–85) it was established that there are five consistent string theories, each of which requires ten dimensions (nine space and one time) and supersymmetry. (For a pedagogical review see [6].) For four of them (the two *heterotic* string theories and the two *type II superstring* theories) the fundamental strings are oriented and unbreakable. For one of them (the *type I superstring* with gauge group SO(32)) the fundamental string is unoriented and breakable. The string coupling constant, $g_s$, is a dimensionless parameter determined as the value of a scalar field (the dilaton). It can be used as an expansion parameter in perturbation theory, in the usual fashion. Thus a scattering amplitude involving $N$ on-shell particles with 10-momenta $p_1^\mu, p_2^\mu, \ldots, p_N^\mu$ has an expansion

$$T(g_s; p_1, \ldots, p_N) = \sum_{n=0}^{\infty} (g_s)^{2n} T_n(p_1, \ldots, p_N).$$

As in quantum field theory, this is an asymptotic expansion. For the heterotic and type II theories there is a single $n$-loop Feynman diagram—a closed, orientable genus-$n$ Riemann surface with $N$ marked points. (Such a surface can be visualized as a sphere with $n$ handles.) Then the $n$-loop amplitude $T_n$ is given by a $(6n + 2N - 6)$-dimensional integral, which has no ultraviolet divergences. In the case of the type I theory the expansion is more complicated. In the subsequent decade (1985–94), much was learned about string theory, especially the possibilities for compactifying the extra dimensions. However, all these studies were within the context of perturbation theory.

The *second superstring revolution*, characterized by the discovery of nonperturbative properties of string theory, began around 1994. One fact that quickly became clear is that there is actually a *unique* underlying theory, with no arbitrary dimensionless parameters [7, 8, 9]. (For a review, see [10].) However, this theory admits many consistent solutions (or quantum vacua), so it does not uniquely determine the universe we observe. The five superstring theories that had been identified previously actually correspond to five limiting points in a very large continuous *moduli space* of consistent quantum vacua. Moreover, this moduli space has a sixth limiting point of high symmetry—namely, one with a flat 11-dimensional spacetime [11, 8]. The quantum theory with this vacuum is called *M theory*. At low energies, the leading approximation to M theory is 11-dimensional supergravity. This is a classical field theory that was discovered 20 years ago [12]. By itself, it does not define a consistent quantum theory, but the belief is that M theory is a well-defined quantum theory.
The challenge, of course, is to describe it precisely.

The key to recognizing that there is a unique underlying theory was the discovery of various dualities, which are labelled by the letters S, T and U. T dualities relate large compactification spaces to small ones [13]. For example, a circle of radius $R$ can be equivalent to one of radius $\ell_s^2/R$. In this way one relates the two type II theories and the two heterotic theories. S duality relates weak coupling and strong coupling. For example, one of the heterotic string theories with coupling constant $g_s$ is equivalent to the type I theory with coupling constant $g'_s = 1/g_s$. U dualities combine these notions by also relating compactification sizes to coupling strengths.

The eleven-dimensional M theory vacuum corresponds to the strong coupling limit of type IIA superstring theory. Let me sketch how this works. The claim is that the IIA theory, with string coupling $g_s$ and string mass scale $m_s$, actually has a circular 11th dimension that is invisible in perturbation theory. To see how this can happen, consider an eleven-dimensional theory with mass scale $m_p$ (the 11d Planck mass) and a circular spatial dimension of radius $R$. Then, it turns out that the proper identifications are provided by

$$m_s^2 = 2\pi R m_p^3,$$

and

$$g_s = 2\pi R m_s.$$

The significance of these formulas will be explained shortly. First, let us note that the perturbation expansion of the type IIA theory is an expansion in powers of $g_s$ for fixed $m_s$. The second relation shows that this is equivalent to an expansion about $R = 0$, which shows that the appearance of an eleventh dimension is nonperturbative. Decompactification ($R \to \infty$) is achieved in the strong coupling limit ($g_s \to \infty$).

A second crucial ingredient (the dualities are the first one) in understanding nonperturbative string theory is the identification and understanding of various dynamical objects called $p$-branes [14]. These are objects whose energy is concentrated on a spatial surface of $p$ dimensions. The energy density, which is usually a constant, is called the tension of the $p$-brane. In this nomenclature, point particles are 0-branes (tension is just mass in this case), and fundamental strings are 1-branes. These objects can be studied with good mathematical control when they preserve some of the supersymmetry of the ambient theory.

In most cases of interest\(^4\) the $p$-branes are sources for generalized gauge fields $A_{\mu_1 \mu_2 \ldots \mu_n}$.

\(^4\)Type I strings are an exception, which is why they are breakable.
These fields are antisymmetric in their Lorentz indices, and can be regarded as generalizations of the Maxwell field, which has \( n = 1 \). An \( n \)-index gauge field can have an electric source with \( p = n - 1 \) dimensions or a magnetic source with \( p = D - n - 3 \) dimensions. In the case of Maxwell theory \( (n = 1, D = 4) \), electric and magnetic sources both have \( p = 0 \) (point particles). M theory has a 3-index potential, and therefore in this case \( n = 3 \) and \( D = 11 \). The electric source, which has \( p = 2 \), is called the \textit{M2-brane}, and the magnetic source with \( p = 5 \) is called the \textit{M5-brane}. Type IIB superstring theory contains various gauge fields, including one with four indices. Therefore in this case \( n = 4 \) and \( D = 10 \). The corresponding brane has \( p = 3 \), and it is simultaneously electric and magnetic (self-dual). It is called the \textit{D3-brane}.

Some of the \( p \)-branes that exist in the type IIA theory can be deduced from the relation between M theory and the type IIA theory discussed above. Specifically, the M5-brane can give type IIA \( p \)-branes with either \( p = 4 \) or \( p = 5 \) depending on whether or not one of its dimensions is wrapped around the circular 11th dimension. These particular \( p \)-branes are called the NS5-brane and the D4-brane. Similarly, the M2-brane can give \( p = 1 \) or \( p = 2 \). The \( p = 1 \) case is the fundamental string. This means that the type IIA superstring is actually a wrapped membrane! This fact is consistent with eq. (2), which can be interpreted as relating the fundamental string tension \( (2\pi m_s^2) \) to the M2-brane tension \( (2\pi m_p^3) \). The \( p = 2 \) case is called the D2-brane.

The type IIA theory also contains states that correspond to Kaluza–Klein excitations of the circular 11th dimension. The momentum on the circle is quantized \( (p_{11} = N/R) \). The 10d mass is \( M_{10} = \sqrt{M_{11}^2 + p_{11}^2} \), which for an 11d supergraviton \( (M_{11} = 0) \) gives \( M_{10} = N/R \). The \( N = 1 \) mode has mass

\[
M = \frac{1}{R} = 2\pi \frac{m_s}{g_s},
\]

where we have used eq. (3). This particle is identified as the D0-brane of the IIA theory. Note that its mass diverges at weak coupling, which means that it is a nonperturbative excitation.

A specific proposal for a nonperturbative formulation of eleven-dimensional M theory, called \textit{Matrix Theory}, was put forward by Banks \textit{et al.} in 1996 \[15\]. It is a supersymmetric gauge theory in which the coordinates of \( N \) D0-branes are represented by \( N \times N \) matrices. The idea, roughly, is to go to the infinite momentum frame by letting \( N \to \infty \) and \( R \to \infty \) at the same time. An interpretation of the finite \( N \) theory has also been proposed \[16\]. This
approach has many features in common with the old parton approach to QCD. However, thanks to supersymmetry, it seems to be somewhat better defined. Indeed, many checks of this proposal have been made by comparing Matrix Theory calculations to graviton scattering amplitudes in 11d supergravity. For a while there appeared to be some discrepancies, but now these have all been resolved, so the conjecture seems to be correct. Even though this represents important progress, it is not the last word. Not only is Matrix Theory awkward to use, but (more fundamentally) it seems to be applicable to only a limited class of quantum vacua. In particular, it does not seem able to describe realistic vacua in which all but four dimensions are compactified.

A special class of $p$-branes in type II superstring theories, a few of which we have already encountered, are called $D$-branes \[17\]. Here, $D$ stands for Dirichlet, because these are branes on which fundamental strings can end, which is represented by Dirichlet boundary conditions. This class of $p$-branes has a number of distinctive properties. For one thing they couple to gauge fields in the Ramond–Ramond sector, which means that they can be represented as bispinors. Secondly, their tensions are given by

$$T_{Dp} = 2\pi m^{p+1}_{s}/g_{s}. \quad (5)$$

This dependence on the coupling constant is intermediate between that of fundamental strings (whose tension is independent of $g_{s}$) and that of ordinary solitons, such as the NS5-brane, whose tension is proportional to $(g_{s})^{-2}$.

Another important fact about type II $Dp$-branes comes into play when $N$ of them are parallel and (nearly) coincident. In this case the $(p + 1)$-dimensional world volume of the branes contains excitations confined to the vicinity of the branes. Then there is an effective world-volume theory which is approximated at low energies by $U(N)$ gauge theory in $p + 1$ dimensions with maximal supersymmetry (16 conserved supercharges). The $N^2$ gauge fields and their supersymmetry partners arise as the ground-state excitations of fundamental strings connecting pairs of D-branes.

Generalizations of this basic construction of supersymmetric gauge theories using D-branes turn out to have some very important applications. It can be generalized to more complicated brane configurations in which other $p$-branes are also involved and some of the supersymmetry is broken. One area of application has been to the study of black holes \[18\]. The basic idea is that for small $g_{s}$ one can carry out controlled perturbative string theory calculations of the microphysics including an enumeration of degrees of freedom. For large
$g_s$, on the other hand, one has strong gravitational fields, an event horizon, and a black hole that can be described by general relativity. However, in cases with supersymmetry one can argue that the degrees of freedom cannot change in continuing from small $g_s$ to large $g_s$. Thus one can count states (i.e., compute the statistical mechanical entropy) for small $g_s$ and compare to the area of the event horizon for large $g_s$. One finds precise agreement with the Bekenstein–Hawking formula for a wide variety of examples. Despite this remarkable achievement, a general understanding of why the microscopic entropy of a black hole should be $1/4$ the area of the event horizon has not yet been achieved.

A second area of application of brane configurations has been to the study of exact non-perturbative properties of supersymmetric gauge theories [19]. Many deep results, including the classic discoveries of Seiberg and Witten [20], can be understood quite simply in this way. This is also a very active subject.

3 AdS/CFT Duality

Let me now turn to the latest development in this field, which goes by the name of AdS/CFT duality. Here, AdS stands for anti de Sitter space and CFT stands for conformal field theory. This is a new duality, quite different from all previous ones, which is a very hot topic at the present time. AdS/CFT duality was proposed by Maldacena in November 1997 [21]. As is usually the case with such developments, there were important prior [22] and subsequent [23] contributions by many others. In the remainder of this talk, I will sketch the basic ideas.

A $p$-brane, or collection of $p$-branes, gives rise to a certain space-time geometry and gauge field configuration, which can be analyzed using the appropriate supergravity field equations. In a number of cases one finds that the geometry has an event horizon, giving a higher-dimensional analog of black holes. In some of these cases the geometry near the horizon is approximated by $AdS_{p+2} \times S^{D-p-2}$. This means that the AdS space has $p + 2$ dimensions and the remainder of the $D$ dimensions form a sphere of $D - p - 2$ dimensions. There are three basic examples that have maximal supersymmetry (32 conserved supercharges). A stack of D3 branes in type IIB superstring theory has near horizon geometry $AdS_5 \times S^5$, a stack of M2-branes in M theory gives $AdS_4 \times S^7$, and a stack of M5-branes in M theory gives $AdS_7 \times S^4$. These solutions to type IIB and 11d supergravity were discovered in the mid 1980’s [5] but were not pursued in the context of superstring/M theory until recently.

5The $AdS_4 \times S^7$ case is introduced in [24], the $AdS_7 \times S^4$ case in [25], and the $AdS_5 \times S^5$ case in [26].
Let me briefly describe some of the salient features of anti de Sitter space. $AdS_{n+1}$ is a maximally symmetric spacetime with a negative cosmological constant.\footnote{Nobody claims that this realistic, only that it is instructive to study.} It can be described as a hypersurface in flat space by the equation

$$u_1^2 + u_2^2 - v_1^2 - v_2^2 - \ldots - v_n^2 = R^2,$$

where $R$ is called the AdS radius. This spacetime has Lorentzian signature and reduces to Minkowski spacetime in $n + 1$ dimensions in the limit $R \to \infty$. Just as an $n + 1$-dimensional sphere ($S^{n+1}$) has $SO(n + 2)$ symmetry, the symmetry of this spacetime is $SO(2, n)$, a noncompact version of the rotation group in $n + 2$ dimensions. This contracts to the Poincaré group (consisting of the Lorentz group $SO(1, n)$ and translations) in the $R \to \infty$ limit. An intrinsic description of $AdS_{n+1}$ is given by the metric

$$ds^2 = \frac{R^2}{z^2}(dz^2 + dx^\mu dx_\mu), \quad z > 0,$$

where

$$dx^\mu dx_\mu = dx_1^2 + \ldots + dx_{n-1}^2 - dt^2.$$ 

Note that the $z = 0$ boundary of $AdS_{n+1}$ is an $n$-dimensional Minkowski spacetime, aside from a divergent factor. What matters is the conformal structure, which is not sensitive to this divergent factor. Thus, even though the boundary is infinitely far from any point in the interior, it is useful to take it seriously.

The $SO(2, n)$ isometries of the $(n + 1)$-dimensional anti de Sitter space induce the group of conformal transformations on its $n$-dimensional Minkowski boundary. (Strictly speaking, the boundary should be compactified by adding a point at infinity.) The conformal group is therefore also $SO(2, n)$. Let me illustrate how this works with a couple of examples. The $SO(1, n-1)$ subgroup of $SO(2, n)$ given by Lorentz transformations of the $x^\mu$ corresponds to the Lorentz group of the boundary. The important point is that these transformations map $z = 0$ to $z = 0$, so that they are well-defined on the boundary. Another example is the isometry $x^\mu \to \lambda x^\mu, z \to \lambda z$ where $\lambda$ is a positive scale factor. This clearly leaves the AdS metric in eq. (6) invariant and preserves the boundary. Thus the corresponding conformal transformations of the boundary are scale transformations $x^\mu \to \lambda x^\mu$.

The basic idea of AdS/CFT duality is to identify a conformally invariant field theory (CFT) on the $n$-dimensional boundary with a suitable quantum gravity theory in the $(n+1)$-
dimensional AdS bulk. To be specific, from now on I will focus on the \( AdS_5 \times S^5 \) solution of the IIB superstring theory.

As we discussed, the IIB theory contains a four-index field \( A_{\mu \nu \rho \lambda} \) for which the D3-brane is a source. It has a field strength \( F_{\mu \nu \rho \lambda \sigma} \), which is self-dual (in ten dimensions). In the \( AdS_5 \times S^5 \) solution of the theory, the field \( F \) has a quantized flux on the sphere. Schematically,

\[
\int_{S^5} F = N, \tag{9}
\]

where \( N \) is a positive integer. This integer determines the radius \( R \) of the \( AdS_5 \) and of the \( S^5 \), which are the same. Aside from a constant numerical factor, one finds that

\[
R = (g_s N)^{1/4} \ell_s, \tag{10}
\]

Thus the curvatures (which are proportional to \( R^{-2} \)) are small compared to the string scale for \( g_s N \gg 1 \) and small compared to the Planck scale for \( N \gg 1 \). The first limit suppresses stringy corrections to supergravity, whereas the latter suppresses quantum corrections to classical string theory.

Maldacena’s duality conjecture is that type IIB superstring theory on \( AdS_5 \times S^5 \) with \( N \) units of \( F \) flux is equivalent to \( N = 4 \), \( D = 3 + 1 \) \( U(N) \) Yang–Mills theory with \( g_{YM}^2 = g_s \). For this conjecture to be plausible, it is a crucial fact the \( \mathcal{N} = 4 \) super Yang–Mills theory [27] is a CFT with vanishing beta function, a fact that was proved in the early 1980s [28]. As should be clear from our presentation, this conjecture arose from considering the near-horizon geometry of a stack of \( N \) D3-branes, in the limit \( N \to \infty \). This duality—if true—implies an amazing fact: the 4d gauge theory, for large \( N \), is actually a 10d string theory! Well, it is not yet “proved,” but the evidence is mounting rapidly.

Let me briefly mention some of the supporting evidence for AdS/CFT duality. (I will only discuss the \( AdS_5 \times S^5 \) example described above, but there are similar stories for other examples.) First, the symmetries match: the two dual theories have the same symmetry supergroup \( SU(2, 2|4) \). This supergroup incorporates 32 fermionic symmetries and a bosonic subgroup \( SU(2, 2) \times SU(4) \). \( SU(2, 2) \) is the double cover of \( SO(2, 4) \) the isometry group of \( AdS_5 \) and the conformal symmetry group of the 4d gauge theory. \( SU(4) \) is the double cover of \( SO(6) \), the isometry group of \( S^5 \), and it is the \( R \)-symmetry group of an \( \mathcal{N} = 4 \) gauge theory in four dimensions.

The AdS/CFT duality conjecture has been made more precise in [23]. These papers have proposed a mapping between the bulk string theory and the boundary gauge theory. It gives
a one-to-one correspondence between on-shell particles of the bulk theory and gauge-invariant operators of the boundary theory. Moreover, correlation functions of these gauge-invariant operators are related to the response of the type IIB theory to boundary conditions for the associated fields. These correspondences have been partially verified. For example, there is a perfect correspondence between particles belonging to short representations of the AdS supersymmetry algebra and chiral primary operators of the gauge theory.

The large $N$ limit of $SU(N)$ gauge theories for fixed $\lambda = g^2_{YM} N$ was studied by 't Hooft in 1974 [29]. He showed that only Feynman diagrams of planar topology contribute in this limit. Moreover, he conjectured that the theory should exhibit a stringy behavior in this limit. Now, this suggestion has been made precise. In principle, the complete $\lambda$ dependence of $\mathcal{N} = 4$ gauge theory in the 't Hooft limit should be given by classical type IIB superstring theory on $AdS_5 \times S^5$. Many people are currently studying this.

Finite temperature gauge theory is described by Euclideanizing the time coordinate and taking it to be periodic. Witten has shown that Euclideanized AdS space, which has this structure on its boundary, contains a black hole at the same temperature [30]. I think that much more remains to be learned from studying this correspondence.

An important concept that has emerged in recent years, called the holographic principle [31], is incorporated by AdS/CFT duality. This concerns the number and location of degrees of freedom in a theory. In a local quantum field theory, the locality implies that the number of degrees of freedom in a spatial region is proportional to its volume. However, this cannot be correct for a quantum gravity theory, where the maximum entropy in a region is proportional to its surface area. (This bound is saturated in the case of a black hole.) So the idea of the holographic principle is that the physics in a region of space can be encoded holographically on a surface that surrounds it. This is what happens in the case of AdS/CFT duality. The physics of the AdS bulk (given by superstring theory) is not a local QFT; rather, it is projected onto the boundary theory, which is a local QFT.

The subject of AdS/CFT duality is still in its early days and developing rapidly. Whatever else I say is likely to be outdated by the time this is published. Suffice it to say that people are exploring all sorts of generalizations. These include breaking supersymmetries and conformal symmetries and constructing analogous systems for $SO(N)$ and $Sp(N)$ gauge theories. So far, AdS/CFT duality has taught us more about gauge theories than it has about string theory. This has the curious consequence that it may have some useful spinoffs.
for the study of gauge theories such as QCD. This would not require constructing a quantum vacuum that gives a realistic description of all forces, only that it incorporates a reasonably good description of the gauge theory one wants to study. This approach to the study of gauge theories might turn out to be a useful alternative to lattice gauge theory, though that remains to be demonstrated. I’m not sure whether it is more accurate to say that the second superstring revolution is still going strong or that the third one has begun.

4 Conclusion

Beyond gauge theories there is superstring theory and M theory, but beyond superstring theory and M theory there are gauge theories.

References

[1] Dual Theory, ed. M. Jacob, Phys. Reports reprint volume (North-Holland 1974).

[2] J. Scherk, Rev. Mod. Phys. 47 (1975) 123.

[3] T. Yoneya, Prog. Theor. Phys. 51 (1974) 1907.

[4] J. Scherk and J.H. Schwarz, Nucl. Phys. B81 (1974) 118.

[5] P. Candelas, G.T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. B258 (1985) 46.

[6] M.B. Green, J.H. Schwarz, and E. Witten, Superstring Theory, 2 vols., (Cambridge U. Press 1987).

[7] C. Hull and P. Townsend, Nucl. Phys. B438 (1995) 109, hep-th/9410167.

[8] E. Witten, Nucl. Phys. B443 (1995) 85, hep-th/9503124.

[9] P. Hořava and E. Witten, Nucl. Phys. B460 (1996) 506, hep-th/9510209.

[10] A. Sen, hep-th/9802051.

[11] P.K. Townsend, Phys. Lett. B350 (1995) 184, hep-th/9501068.

[12] E. Cremmer, B. Julia, and J. Scherk, Phys. Lett. 76B (1978) 409.
[13] For a review see A. Giveon, M. Porrati, and E. Rabinovici, *Phys. Rept.* **244** (1994) 77, hep-th/9401139.

[14] For a review see M.J. Duff, R.R. Khuri, and J.X. Lu, *Phys. Rept.* **259** (1995) 213, hep-th/9412184.

[15] T. Banks, W. Fischler, S. Shenker, and L. Susskind, *Phys. Rev.* **D55** (1997) 112, hep-th/9610043. For reviews see T. Banks, hep-th/9710237; D. Bigatti and L. Susskind, hep-th/9712072.

[16] L. Susskind, hep-th/9704080.

[17] J. Polchinski, *Phys. Rev. Lett.* **75** (1995) 4724, hep-th/9510017; p. 293 in *Fields, Strings, and Duality* (TASI 96), eds. C. Eftimiou and B. Greene, World Scientific 1997, hep-th/9611050.

[18] For reviews see J. Maldacena, hep-th/9607233; D. Youm, hep-th/9710046; A. Peet, hep-th/9712253.

[19] For a review see A. Giveon, and D. Kutasov, *Nucl. Phys. Proc. Suppl.* **68** (1998) 310, hep-th/9802067.

[20] N. Seiberg and E. Witten, *Nucl. Phys.* **B426** (1994) 19, hep-th/9407087; *Nucl. Phys.* **B431** (1994) 484, hep-th/9408099.

[21] J.M. Maldacena, hep-th/9711200.

[22] I.R. Klebanov, *Nucl. Phys.* **B496** (1997) 231, hep-th/9702070; S.S. Gubser, I.R. Klebanov, and A.A. Tseytlin, *Nucl. Phys.* **B499** (1997) 217, hep-th/9703040; S.S. Gubser and I.R. Klebanov, *Phys. Lett.* **B413** (1997) 41, hep-th/9708003; A.M. Polyakov, hep-th/9711002.

[23] S.S. Gubser, I.R. Klebanov, and A.M. Polyakov, *Phys. Lett.* **B428** (1998) 105, hep-th/9802109; E. Witten, hep-th/9802150.

[24] P. Freund and M. Rubin, *Phys. Lett.* **97B** (1980) 233; for a review see M.J. Duff, B.E.W. Nilsson, and C.N. Pope, *Phys. Rept.* **130** (1986) 1.

[25] K. Pilch, P. van Nieuwenhuizen, and P.K. Townsend, *Nucl. Phys.* **B242** (1984) 377.
[26] H.J. Kim, L.J. Romans, and P. van Nieuwenhuizen, *Phys. Rev.* D32 (1985) 389; M. Günaydin and N. Marcus, *Class. Quant. Grav.* 2 (1985) L11.

[27] L. Brink, J.H. Schwarz, and J. Scherk, *Nucl. Phys.* B121 (1977) 77; F. Gliozzi, J. Scherk, and D. Olive, *Nucl. Phys.* B122 (1977) 253.

[28] S. Mandelstam, *Nucl. Phys.* B213 (1983) 149; L. Brink, O. Lindgren, and B.E.W. Nilsson, *Phys. Lett.* 123B (1983) 323; P.S. Howe, K.S. Stelle, and P.C. West, *Phys. Lett.* 124B (1983) 55; P.S. Howe, K.S. Stelle, and P.K. Townsend, *Nucl. Phys.* B236 (1984) 125.

[29] G. ’t Hooft, *Nucl. Phys.* B72 (1974) 461.

[30] E. Witten, [hep-th/9803131](http://arxiv.org/abs/hep-th/9803131).

[31] G. ’t Hooft, [gr-qc/9310026](http://arxiv.org/abs/gr-qc/9310026); L. Susskind, *J. Math. Phys.* 36 (1995) 6377, [hep-th/9409089](http://arxiv.org/abs/hep-th/9409089).