The Pion Light-Cone Wave Function $\Phi_\pi$ on the lattice: a partonic signal?

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We determine the conditions required to study the pion light-cone wave function $\Phi_\pi$ with a new method: a direct display of the partons constituting the pion. We present the preliminary results of a lattice computation of $\Phi_\pi$ following this direction. An auxiliary scalar-quark is introduced. The spectroscopy of its bound states is studied. We observe some indications of a partonic behavior of the system of this scalar-quark and the anti-quark.

INTRODUCTION

The light-cone wave function (LCWF), $\Phi_\pi(u)$, plays an important role in the study \cite{1} of pions with large momenta $P_\pi$. It is related to the probability of finding in the pion a quark and an anti-quark moving in the same direction than the pion with momenta $uP_\pi$ and $(1-u)P_\pi$ respectively. Within this picture, the two valence quarks are frozen in a transverse region of size $\Lambda_{\text{QCD}}$. To extract the partonic information contained in $\Phi_\pi(u)$ a “kick” is applied to the pion with a large transfer momentum $\vec{q}$. This is for example what has been done in the E791 experiment \cite{2} for the diffractive dissociation of a pion in two jets.

The LCWF $\Phi_\pi$ depends on a momentum scale which is typically $P_\pi$. In the large pion momentum frame, $\Phi_\pi$ describes the leading term of the $\Lambda_{\text{QCD}}^2/P_\pi^2$ expansion of the full pion wave function. It contributes only to the valence configuration $|q\bar{q}\rangle$ of the Fock space expansion. $\Phi_\pi$ contains all the terms of the $1/\log(P_\pi^2/\Lambda_{\text{QCD}}^2)$ expansion and is defined by an expression \textit{a la} Bethe-Salpeter involving the pion to vacuum matrix element of a non-local operator

\begin{equation}
\langle 0| \bar{d} (0) P \left[ \exp\left(i \int_x^0 d\tau \, A^\mu \right) \gamma_\mu \gamma_5 u(x) \right] |\pi(P_\pi)\rangle_{x^2=0} = -i P_\pi^\mu \int_0^1 du \, e^{-iuP_\pi \cdot x} \Phi_\pi(u)
\end{equation}

The Wilson string in the square bracket ensures the gauge invariance of the matrix element. $\Phi_\pi$ is a non-perturbative function, normalized by the condition $\int du \Phi_\pi(u) = 1$.

An important particular case is given by keeping only the dominant term in the expansion of $O(1/\log(P_\pi^2/\Lambda_{\text{QCD}}^2))$. It leads to the asymptotic form of $\Phi_\pi$, $\Phi_\pi^\infty(u) = 6u(1-u)$, which is fully computed by perturbative QCD \cite{1}.

On the lattice, the first method to study the LCWF is to examine the moments $M_n$ of $\Phi_\pi$ \cite{3},

\begin{equation}
M_n = \int_0^1 du \, u^n \Phi_\pi(u)
\end{equation}

This requires the computation of the matrix elements of local operators. For high $n$, $M_n$ involves operators with high order derivatives, which are increasingly difficult to discretize and renormalize. It was therefore proposed in \cite{4} to calculate $\Phi_\pi$ by studying directly the \textit{partonic} constituents of the pion. Following this idea, we have begun our work by stating the conditions needed to extract $\Phi_\pi$, and then achieved a first and preliminary attempt to isolate a partonic signal on the lattice \cite{5}.

1. FEASIBILITY CONDITIONS

The r.h.s of eq. (1) is computed \textit{via} a three-point Green function (see eq. (2) and fig. 3). The $f_\pi$ being the only non-perturbative quantity on this limit.
Wilson string needed to insure gauge invariance is replaced by a scalar coloured propagator $S(\vec{x}_t, t; 0)$ with the colour quantum numbers of a quark, but this choice is not unique.

With all these conditions verified, we compute the three-point Green function $F^\mu$:

$$F^\mu(p_\pi^\gamma, q; t) = \int d^3x d^3\bar{x} e^{-i\gamma \bar{x}} e^{-iq\cdot x} e^{E_s(t_\pi - t)}$$

$$\times \langle 0| P_\pi(x_\pi^\gamma, t_\pi) u(\bar{x}_t, t) S(\vec{x}_t, t; 0) \gamma^\mu \gamma_5 \bar{d}(0)| 0 \rangle$$

$$\propto p_\pi^\mu \sum_{u_i} e^{-(E_s + (1-u_i)E_\pi)t} \Phi_\pi(u_i)$$  \hspace{1cm} (2)

The matrix element now contains the interpolating field of the pion $P_\pi$ at the fixed time $t_\pi$ and the scalar propagator. We have also included the inverse pion propagator $e^{E_\pi(t_\pi - t)}$ and required the time interval $t_\pi - t$ to be large in order to have, at the small times $t$, an on shell pion. In the r.h.s of eq. (3) we have replaced the integral over $u$ by the sum because of the discretization constraint. It is important to note that the fraction which multiplies $\Phi_\pi$ can be understood within a partonic picture as the product of the inverse scalar propagator $e^{-E_s t}/2E_s$ times the inverse anti-quark propagator $e^{-(1-u_i)E_\pi t}$.

On the lattice we have tried to fulfill as much as possible the restrictive conditions just presented. This seriously constrains the simulation and turn the search of a signal into a delicate task.

2. LATTICE RESULTS

The simulation was performed on a $16^3 \times 40$ lattice ($\beta = 6.0$, 100 configurations) in the quenched approximation. We selected light parton masses ($\kappa_q \in \{0.1333, 0.1339\}$ for the quarks and $\kappa_s \in \{0.1428, 0.1430, 0.1431\}$ for the scalar) and the following values for $P_\pi$, $P_\pi \in \{(0, 0, 0); (1, 0, 0); (1, 1, 0); (2, 0, 0)\}$ in units of $2\pi/L_x$. A large set of transfer momenta $\vec{q}$ was chosen.

As we have introduced in eq. (3) a non physical object, the scalar-quark, the first step of the analysis was to test its hadronic behavior. We measured therefore the two-point Green functions for the $\pi$ and the scalar-scalar states. These two-point functions show the characteristic exponential signal of hadrons and verify the Einstein spectral law. Interestingly, the spectroscopy of the scalar reveals that these non-conventional bound states behave as real hadrons.

We will now consider the three-point Green
function $F^0(\vec{p}_\pi, \vec{q}; t)$, eq. (2). As already justified, we use the momentum $\vec{p}_\pi = (2, 0, 0) \times 2\pi/L_x$ in order to have $u = 1/2$ although large momenta bring up some noise in a lattice signal.

We will try to give a partonic interpretation of the data in the short time region by building the following ratios

$$R_p = \frac{F^0(\vec{p}_\pi, \vec{q}; t)}{[e^{-E_\pi t}/2E_\pi]} \Phi_\pi(1/2),$$

$$R_h = \frac{F^0(\vec{p}_\pi, \vec{q}; t)}{[e^{-E_\pi t}]}$$

We will look for plateaus in time of these quantities for the whole set of momenta $\vec{q}$. A plateau for $R_p$ ($R_h$) would be a sign of a partonic (hadronic) behavior of the antiquark–scalar system. In fig. 2, we plot the slopes of the ratios $R_p$ and $R_h$ in the short time region $t \in [0, 4]$, for our set of momenta $\vec{q}$. A value of the slope close to zero suggests a plateau in time. By comparing both plots, we observe that the partonic slopes are closer to zero than the hadronic ones. According to our data, the partonic interpretation is somehow favoured: $\chi^2/d.o.f = 1.9$ for a vanishing slope compared to 4.1 in the hadronic case. Nevertheless, we are clearly not yet in a position to give a fully convincing claim.

As a cross-check of the previous result, we have examined the product $F^0(\vec{p}_\pi, (2, 0, 0), \vec{q}; t = 0) \times E_\pi(1/2)$. Within the partonic picture, we expect this product to be independent of the transfer momentum $\vec{q}$ (see eq. (2)). This is indeed what we find when we plot in fig. 3 this product for our full set of momenta $\vec{q}$. The data show the expected constancy around the average represented by the solid horizontal line.

Figure 2. Slopes of the partonic ratio $R_p$ and of the hadronic ratio $R_h$ in the small time $t$ interval $[0, 4]$. The horizontal axis displays the set of momenta $\vec{q}$ labelled with an arbitrary index.

Figure 3. Values of $F^0(\vec{p}_\pi, (2, 0, 0), \vec{q}; t = 0) \times E_\pi(1/2)$ for our full set of momenta $\vec{q}$.

CONCLUSION

We wanted to check the feasibility on the lattice of a direct method to determine non-perturbative objects like the pion light-cone wave function $\Phi_\pi$. We have stated the conditions for this study and we have seen that they induce restrictive constraints and hierarchies which turn the search of a clear lattice signal into a difficult task. We have found an interesting spectroscopy of bound states involving a scalar-quark, and we have a hint of a partonic behavior of the system of the scalar and the anti-quark by considering the three-point Green function related to $\Phi_\pi$. This signal will continue to be tested in a further analysis. Work supported by the European Community’s Human potential programme under HPRN-CT-2000-00145 Hadrons/LatticeQCD.

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