Influence of information flow in the formation of economic cycles

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1 Introduction

Despite the fact that the Verhulst’s idea \cite{1} of regulated biological populations is 200 years old, it is still very useful since it allows to investigate features of various systems. Here an eight order logistic map is applied in modelling the influence of information flow delay onto the behaviour of an economic system.

The delay of information flow is an internal feature of all economic systems, because continuous monitoring of such systems both on macro and microeconomy scales is either extremely difficult or even impossible. The more so since the data is not easily available nor even reliable as it could be in physics laboratories. Macroeconomy parameters such as Gross Domestic Product, Gross National Product, inflation, demographic data etc. are announced in well defined time intervals (monthly, quarterly or annually). The same situation is observed in the case of various companies. They announce their financial statements about their economic results at specific dates and for given time intervals – according to internal or external rules (usually according to law regulations). Sometimes some “warning” is issued. However the tendency is that intervals between announcements are rather long, e.g. the value of a dividend is announced annually or at various trimester ends. It seems obvious that only very small companies are able to perform continuous monitoring. But even then, the process of collecting information from a significant (on a macroscopic scale) number of such companies inhibits or makes it impossible to perform continuous monitoring. In view of the data collecting procedure it is clear that every economic decision is based on some information describing a past situation. It is also important to notice that the time delays between information gathering, decision taking, policy implementation, and subsequent data gathering are not regularly spaced, nor is a fortiori a continuous variable, as that was considered in \cite{2}; indeed the information about the system is updated at the end of discrete time intervals.
Therefore econophysics-like modelling of such features encounters some
difficulty, surely at the testing level. Recently a microscopic-like approach has
been presented, through a model [3, 4, 5] including some measure of a company
fitness with respect to an external field, and a birth-death evolution, according
to some business plan, and the local company close range environment. The
information flow was however considered to occur instantaneously.

In order to investigate the discrete information flow time delay and its
effect, a model, hereby called the ACP model [3, 4, 5], has been modified
by splitting the information about the system into two parameters. One is
monitored continuously (is updated at every iteration step) and is known
to the system itself; the second, – like official statements of the system, is
announced at the end of discrete time intervals and is used by companies for
calculating their strategies. Therefore the strategy of a company depends on
the delay time information and the information itself. As it is shown in Sect.
3 the length of the time delay ($t_d$) influences quite strongly and in a nontrivial
way the behaviour of the overall system.

Detailed description of the ACP model is given in Sect. 2 and the prop-
erties of the system as a function of time delay and initial concentration are
investigated (Sect. 3) in the case of short, medium and long time delays.

2 ACP model

For the sake of clarity the basic ingredients of the ACP model are recalled
here below. The main problem was to simulate the behaviour of economic
systems in spatio-temporally changing environmental conditions, e. g. political
changes and destruction of economy barriers. The model was set in the form
of a Monte Carlo simulation. Notice that the ACP model [3, 4, 5] contains
among its variants an adaptation of the Bak – Sneppen model and was built
in order to answer economy questions³. The model consists of

1. **space** – a square symmetry lattice,
2. **companies**, which are initially randomly placed on the lattice, in an
3. **environment** characterised by a real field $F \in [0,1]$ and a selection
   pressure $sel$,
4. each company ($i$) is characterised by one real parameter $f_i \in [0,1]$ (so
called its **fitness**).

The following set of actions was allowed to companies:

1. companies survive with the probability

   $$ p_i = \exp(-sel|f_i - F|) $$  

   (1)

³Let us recall that the Bak- Sneppen model was originally built in order to
investigate the coevolution of populations [6]
2. companies may move on the lattice horizontally or vertically, one step at a time, if some space is available in the von Neuman neighbourhood.

3. if companies meet they may
   a) either merge with a probability $b$,
   b) or create a new company with the probability $1 - b$.

The ACP model may be described in a mean field approximation [7, 8] by introducing the distribution function of companies $N(t, f)$, which describes the number of companies having a given fitness $f$ at time $t$. The system is then additionally characterised by the concentration of companies $c(t)$.

The present report of our investigations is restricted to the case of the best adapted companies ($f = F$), so that the selection pressure has no influence on the survival of companies. So the only factor which could alter the number of companies is the strategy, i.e. the decision to merge or create a new entity. The ideas behind the mean field approximation [7, 8] is applied here and developed by introducing a strategy depending on the system state and the discrete time of the official announcement about the state of the system.

The introduction of the strategy depending on the state of the system reflects the idea of Verhulst [1], when replacing the constant Malthus grow rate by the function $1 - x$, which introduced a limit for the system to grow. In the present investigation it is assumed that the strategy should depend on the state of the system. Moreover the company board takes its decision knowing informations announced about its environment. The generation of new entities is more likely in the case of a low concentration of companies than when this concentration is high. The merging parameter describes the reversed dependency, i.e. merging is more likely to occur in the case of a high density of companies than if the density is low. The simplest function which fulfils this condition is $1 - c$, the same as in Verhulst original work [1].

The additional ingredients to the ACP model are thus
1. the merging parameter $b$ is replaced by a strategy $(1 - c)$,
2. the companies know the value of the concentration $c$ according to official statements announced after the time delay $t_d$.

The evolution equation of the system with companies, using the state dependent strategy is:

$$c_t = c_{t-1} + \frac{1}{2} c_{t-1}(1 - c_{t-1}^8)(1 - (1 - c_{t-1})^8)(2ST(c(g(t))) - 1),$$

where $ST(c) = 1 - c$, $g(t) = k[t]$ and $\lfloor \cdot \rfloor$ denotes the procedure of taking a natural number not larger than the one given in the brackets. The time is measured in iteration steps $IS$.

**3 Results**

Numerical methods were used in order to investigate properties of the system. Because the coevolution equation (2) is given as an iteration equation the time
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is discrete and counted in iteration steps (IS). The following features of the system were examined:

1. the **coevolution** of \( c(t) \) as a function of the initial concentration,
2. the **stability time** defined as the time required to achieve a unique stable solution; because of numerical reasons the criterium applied here is \(|c_{n+1} - c_n| < 10^{-10}\),
3. the **crash time** – \( t_c \), such that \( c_{t_c} < 0 \) (it is understood as a time when all companies are wiped out from the system),
4. the **stability intervals** – the intervals of initial values for which the evolution of the system is longer than a given time \( t_s \)
5. the complex Lyapunov exponent

\[
\lambda = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( \frac{dx_{n+1}}{dx_n} \right).
\] (3)

The Lyapunov exponent calculated in its complex form (3) gives also some information about the oscillations of the system. Using the properties of logarithm:

\[
a < 0 \Rightarrow \log(a) = \log(-1 \cdot |a|) = \log(-1) + \log(|a|).
\] (4)

the imaginary part of \( \log_2 \left( \frac{dx_{n+1}}{dx_n} \right) \) gives some information on whether the distances between consecutive iterations are monotonic.

The numerical iterations were performed for the initial concentration in the interval \( c_0 \in (0, 1) \), at consecutive values distant of 0.02. Therefore 500 histories of evolution were investigated.

There are possible to observe three types of coevolution – a unique, a periodic and a chaotic solution. In the case of unique solution the system may approach this solution in the form "damped" coevolution or "damped oscillation". The damped coevolution is if \( \forall t > 0 \ c(\infty) - c(t) > 0 \) or \( \forall t > 0 \ c(\infty) - c(t) < 0 \) and \( |c(t-1) - c(t)| \geq |c(t) - c(t+1)| \), where \( c(\infty) \) is the asymptotic state of the system. This means that the distance between concentration and asymptotic concentration is decreasing in every iteration step and either the concentration is smaller or bigger than the asymptotic concentration. The damped oscillations are observed if \( |c(t-1) - c(t)| \geq |c(t) - c(t+1)| \) and \( \exists t_0 \) such that \( \forall t > t_0 \ c(t) > c(\infty) \) and \( c(t+1) < c(\infty) \), this means the distance between consecutive concentrations of companies is decreasing. In the case of a periodic solution for \( t > t_0 \) there exists a n-tuple of concentrations which is repeated for \( t > t_0 \), where \( t_0 \) is the time required by the system to reach the stable or periodic solution. The length of the n-tuple is defined as the period of oscillations. The system is chaotic if the real part of the Lyapunov exponent \( Re(\lambda) > 0 \).

The coevolution of the system is presented either as a function of time (Fig. 1, 4, 7, 10, 13, 15, where the coevolution is plotted for chosen initial concentrations) or as a function of initial concentration (Fig. 2, 5, 8, 11, 14, 16, where the coevolution of the system is plotted in one vertical line so the plot is a set of coevolutions for 500 different initial concentrations.)
3.1 Stability window

The short time delay \( t_d \) is defined as a \( 2 IS \leq t_d \leq 4 IS \). In this case the system evolves to the unique stable solution \( c = 0.5 \). Within this time delay the Lyapunov exponent is equal to zero; no chaotic behaviour is seen.

\[ t_d = 2 IS \]

The time delay \( t_d = 2 IS \) means that the information about the system is updated every two iteration steps. The evolution of the system is presented in Fig. 2 and is plotted as a function of initial iteration. For every 500 initial concentrations 103 iteration steps have been used. The history examples are presented in Fig. 1 as a function of concentration in time. In the case of the shortest time delay considered here the system has a unique solution \( c = 0.5 \).

The stability time as a function of initial concentration is shown in Fig. 3. For a very low initial concentration \( 0 < c_0 \leq 0.01 \) a long time \( t_s \geq 47 IS \) is needed in order to achieve the stable state. It is also illustrated in Fig. 1, where in the case of low initial concentrations \( c_0 = 0.002 \) the stability time is quite long time (about 100 IS). However except for very small initial concentrations \( (c_0 > 0.1) \) the stability time is short \( t_s \in (10 IS, 20 IS) \).

![Fig. 1. Coevolution of the system for chosen initial concentrations; delay time \( t_d = 2 IS \)](image-url)
Fig. 2. Coevolution of the system as a function of initial concentration. The coevolution of a system is represented by a vertical series of dots; delay time $t_d = 2 IS$.

Fig. 3. The time required for the system to achieve a stable concentration as a function of initial concentration; delay time $t_d = 2 IS$. 
Extending the time delay by one, up to three iteration steps, induces important changes in the system. In the evolution of the system, damped oscillations become observable, e.g., for $c_0 = 0.002$ damped oscillations are observed for $t \in (140 IS, 155 IS)$ (Fig. 5 and Fig. 4). The maximum time required for the system to achieve a stable state extends to $t_s \geq 220 IS$ as compared with $t_s \geq 47 IS$ for $t_d = 2 IS$. For most initial concentrations ($c_0 > 0.05$) the stability time is in the interval $t_s \in (70 IS, 100 IS)$. Therefore the system requires a longer time to achieve a stable state. However there are some “stability points” for which the system achieves a stable state markedly faster. These can be found on Fig. 6; these points are: $c_0 = 0.074, t_s = 76 IS$; $c_0 = 0.136, t_s = 73 IS$; $c_0 = 0.284, t_s = 61 IS$; $c_0 = 0.5, t_s = 1 IS$; $c_0 = 0.826, t_s = 58 IS$; $c_0 = 0.952, t_s = 67 IS$.

Comparing the results obtained in the case of $t_d = 2 IS$ and $t_d = 3 IS$ it can be noticed that the stability times is significantly extended and new features become visible (damped oscillations). Therefore we can conclude that the system is very sensitive to the flow of information and extension by only one IS step of the time delay changes the behaviour of the system quite significantly.

**Fig. 4.** Coevolution of the system for given initial concentrations; delay time $t_d = 3 IS$
Fig. 5. Coevolution of the system as a function of initial concentration. The coevolution of a system is represented by a vertical series of dots; delay time $t_d = 3 IS$.

Fig. 6. The time required for the system to achieve stable concentrations as a function of its initial concentration; delay time $t_d = 3 IS$. 
$t_d = 4\ IS$

For a time delay $t_d = 4\ IS$, the lately seen features (damped oscillations) are also present as it can be observed on both figures showing the coevolution for the considered initial concentrations and for chosen histories presenting explicitly time evolution of the system – Fig.5 and Fig.8 respectively. It is worth noticing that the damping of oscillations is much weaker than in the case $t_d = 4$ (compare Fig.5 and Fig.8). The oscillation amplitude is decreasing significantly more slowly for the case $t_d = 4\ IS$ than for $t_d = 3\ IS$. However in all considered cases $t_d = 2\ IS$, $t_d = 3\ IS$, $t_d = 4\ IS$, the system has one stable solution, but the stability time depends on the delay time; it is the longest in the case $t_d = 4\ IS$ ($3200\ IS \leq t_s \leq 4200\ IS$). The time required for the system to achieve stable state is presented in Fig. 9. As in the previous case $t_d = 3\ IS$ there are initial concentrations for which the system reaches the stable state significantly quicker, e.g. $c_0 = 0.23$.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Fig7.png}
\caption{Evolution of the system for given initial concentrations; delay time $t_d = 4\ IS$}
\end{figure}

3.2 Medium time delay

$t_d = 5\ IS$ and $t_d = 6\ IS$

The five iteration step delay time ($t_d = 5\ IS$) is very interesting, because this is the shortest time for which cycles of concentration can be observed. For
Fig. 8. Coevolution of the system as a function of initial concentration. The coevolution of a system is represented by a vertical series of dots; delay time $t_d = 4 IS$.

Fig. 9. The time required for the system to achieve stable concentrations; delay time $t_d = 4 IS$. 
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this time delay the system has several solutions. Despite the fact that the real part of the Lyapunov exponent is still equal to zero, its imaginary part is not. According to Eq. (4) this shows that the system has a periodic solution. These solutions can be seen in Fig. 11 and Fig. 10. In the case of Fig. 11 the evolution is shown as a function of its initial concentration, whereas Fig. 10 presents the system evolution as a function of time for chosen initial concentrations.

Oscillating solutions can be also found in the case $t_d = 6 IS$; the imaginary part of the Lyapunov exponent, as in the previous case ($t_d = 5 IS$) is negative (Fig. 12).

3.3 Long time delay

$t_d \geq 7 IS$

Extending the delay time above six iteration steps leads to a possible collapse of the system. For $t_d \geq 7 IS$ the system may crash. The crash is defined when the concentration of companies becomes negative or zero. Examples of such evolutions which lead to a crash are presented in the case of $t_d = 12 IS$ and $t_d = 15 IS$ on Fig. 14 and Fig. 16 respectively. The crash of the system is presented in such plots as a white band containing very few points in the
Fig. 11. Coevolution of the system as a function of initial concentration. The coevolution of a system is represented by a vertical series of dots; delay time $t_d = 5 IS$.

Fig. 12. Lyapunov exponent for $t_d = 6 IS$. 


vertical direction. It is also seen in Fig. 17, where for several intervals on the initial concentration axis, e.g. \( t_d \in (0.15; 0.2) \cup (0.34; 0.36) \cup (0.53; 0.61) \) the crash of the system occurs very quickly. However there are some initial concentrations for which the evolution of the system before crash time is quite long (up to 400 IS). Additionally in the case of \( t_d = 15 \), the system may evolve toward a stable state, with a full occupation of the environment by companies. Examples of such an evolution as a function of time for given initial concentrations are presented in Fig. 13 and Fig. 15 for the cases \( t_d = 12 \) and \( t_d = 15 \) respectively.

![Fig. 13. Coevolution of the system as a function of time for chosen initial concentrations; \( t_d = 12 IS \)](image)

**4 Conclusions**

Economic cycle causes and occurrences are fascinating and relevant subjects of interest in many economy questions [9]-[10]. The problem has been studied also by means of sociology technics [11], showing that changes of opinions about recession or prosperity undergo drastic changes from one equilibrium to another, both having fluctuations in stochastically resonant systems. In the present investigation, an information flow, typical of economy systems, has been incorporated into the ACP model [3, 4, 5]. This has led to observe
Fig. 14. Coevolution of the system as a function of its initial concentration. The coevolution of a system is represented by a vertical series of dots; $t_d = 12 IS$.

Fig. 15. Coevolution of the system as a function of time for chosen initial concentrations; $t_d = 15 IS$. 

$c_0 = 0.002$  
$c_0 = 0.01$  
$c_0 = 0.02$  
$c_0 = 0.03$
Fig. 16. Coevolution of the system as a function of its initial concentration. The coevolution of a system is represented by a vertical series of dots; $t_d = 15 IS$.

Fig. 17. The crash time of the system as a function of its initial concentration; $t_d = 15 IS$. 
different forms of so called cycles, through concentration oscillations. In the case of short delay time $t_d \in (2IS, 4IS)$, between data acquisition and policy implementation by a company, the system evolves toward a unique stable equilibrium state. This situation can be highly welcomed in some economy systems. Indeed this indicates that, through an information control, a system can insure the existence of a high number of companies, whence not threatening the system of a collapse.

In the case of medium size delay times $t_d = 5IS$ or $t_d = 6IS$, the system undergoes oscillations: stable concentration cycles appear in the system. This form of evolution is often observed in economy, e.g. agricultural markets, where without external control the level of agricultural production oscillates between over- and underproduction. Since the enlarging of the delay time leads to the possibility of the system to crash, such a system may require some external (governmental) control, for its stability. In reality, the delay of information flow and policy implementation may also fluctuate. For long information flow delay times, $t_d \geq 7$, the systems may crash for most initial concentrations. However, despite the frequent possibility of the system to crash the situation is not hopeless because the crash time in many cases is long enough to allow for some particular control and to avoid the collapse of the company concentration. It is also possible to observe a "economy resonance" where despite a long delay time the system evolves for a long time or can even reach a stable state, which insures its existence. This latest observation is especially interesting for market control purposes, because it points to the existence of initial conditions for which the system may evolve during a very long time, which is vital for the possibility of creating and applying some control procedures.

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