A New Mechanism to Solve the Small Scale Problem of Local Supersymmetry

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Abstract

We extend the Standard Model gauge group by a a gauged $U(1)_R$ R-Symmetry or a gauged $U(1)'$. The requirement of cancellation of anomalies is very constraining but can be achieved by adding three or four hidden-sector fields which are Standard Model singlets. The $U(1)_R$ or $U(1)'$ quantum numbers of these singlets are usually large producing a non-renormalisable superpotential with a high power in the singlet fields. We have minimized the supergravity scalar potential and have found solutions where the vacuum expectation values of all hidden-sector singlet fields are less than the Planck mass $<z_m> = O(M_{Pl}/10)$. This produces the small supersymmetry scale of order the weak scale from only the Planck scale. The mu problem is simultaneously solved in this manner.

One of the basic problems of particle physics is to understand how the electroweak scale is generated and why it is so small in comparison with the Planck scale associated with Newton’s constant. In the Standard Model the mass parameter of the Higgs field suffers from quadratic divergences. The physical mass parameter must then be tuned to be small, of order the weak-scale, order by order in perturbation theory. In supersymmetric theories, it is enough to tune the Higgs mass to be small at tree level. However, this does not answer
the question of how such a small scale arises in the first place. In locally supersymmetric theories, where gravity plays an important role, the only natural scale is the Planck mass. The purpose of this letter is to provide a mechanism which naturally generates a small scale of the order of $10^2 \text{GeV}$ out of the Planck mass $M_{Pl} = 2.43 \cdot 10^{18} \text{GeV}$ without any fine tuning.

The basic idea comes from our work on controlling lepton- and baryon-number violating operators by imposing an anomaly-free gauged $U(1)'$ or $U(1)_R$ symmetry \cite{1,2}, thus giving the leptons and quarks an additional gauge quantum number. We work in the framework of local supersymmetry which allows for gauging the R-symmetry\cite{3}. The most stringent condition on building such models is that of anomaly cancellation. The quantum numbers must also be rational thus guaranteeing that a superpotential can be used to break supersymmetry. It turns out that solutions are difficult to come by. They typically have a polynomial superpotential with the lowest term having a high power in the scalar fields. We then require that minimizing the supergravity scalar potential for an appropriate choice of the Kähler function produces minima for the hidden-sector scalar fields $z_m$ such that

$$<\kappa z_m> < 1, \forall z_m,$$ \hspace{1cm} (1)

where $\kappa = M_{Pl}^{-1}$. The actual values are model dependent. We shall see below that values of order $<\kappa z_m> = O(0.1)$ will be sufficient for our argument. We thus avoid all fine-tuning. For sufficiently large powers in $z_m$ we can look for solutions where

$$<\kappa^2 g(z_m)> = O(m_s).$$ \hspace{1cm} (2)

Here $m_s$ is the supersymmetry scale of order $10^2 \text{GeV}$. One advantage of this mechanism is that although the superpotential has an infinite number of terms, only the leading term(s) is (are) relevant.

We extend the minimal supersymmetric Standard Model (MSSM) by a $U(1)_R$ gauged R-symmetry or a $U(1)'$ gauge symmetry. The only chiral supermultiplets are the three families of quarks and leptons, $Q_i, \bar{D}_i, \bar{U}_i, L_i, \bar{E}_i$, two pairs of Higgs $SU(2)$-doublets, $H_1, H_2$, and Standard Model singlets $z_m$. They have the following gauge quantum numbers

$$L_i = (1, 2, -\frac{1}{2}, l_i), \quad \bar{E}_i = (1, 1, 1, e_i), \quad Q_i = (3, 2, \frac{1}{6}, q_i),$$

$$\bar{U}_i = (\bar{3}, 1, -\frac{2}{3}, u_i), \quad D_i = (\bar{3}, 1, \frac{1}{3}, d_i), \quad H_1 = (1, 2, -\frac{1}{2}, h_1),$$

$$H_2 = (1, 2, \frac{1}{2}, h_2), \quad z_m = (1, 1, 0, z_m),$$ \hspace{1cm} (3)

with respect to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$. $U(1)_X$ is either $U(1)_R$ or $U(1)'$. Above, for $U(1)_R$ the R-quantum numbers are taken for the fermions, $r_f^i$, which enter into the anomaly equations and which we use in the explicit model discussed below. When minimizing the potential, we shall use the bosonic $R$-charges $r_b^i = r_f^i + 1$, which are also the
charges of the superfields. The most general superpotential comprising of a hidden sector, \( z_m \), that breaks supersymmetry and an observable sector, \( S_i \), is

\[
g(z_m, S_i) = g_0(S_i) + g_1(z_m) + g_2(z_m, S_i),
\]

where

\[
g_0(S_i) = h_{ij}^i L_i H_1 E_j + h_{ij}^j Q_i H_1 \bar{D}_j + h_{ij}^{ij} Q_i H_2 \bar{U}_j.
\]

\( g_2(z_m, S_i) \) contains non-renormalisable interactions mixing the hidden-sector with the observable sector, where the only scale that is allowed to appear is the Planck mass. Imposing the condition that the gauged \( U(1)_R \) or \( U(1)' \) is anomaly-free determines the possible charges of the fields \( z_m \) and thus the form of the superpotential to be

\[
g_1(z_m) = \kappa^{-3} \sum_{t=1}^{\infty} a_t (\kappa z_1)^{n_t_1} (\kappa z_2)^{n_t_2} \ldots (\kappa z_p)^{n_t_p},
\]

for \( p \)-hidden sector fields. The sub-powers \( n_{ti} \) in the superpotential satisfy the gauge invariance constraint

\[
\sum_{i=1}^{p} r^b_i n_{ti} = \begin{cases} 
2, & \text{for } U(1)_R, \\
0, & \text{for } U(1)', 
\end{cases} \forall t,
\]

where for the \( U(1)_R \) case the \( r^b_i \) refer to the bosonic charges. We shall determine solutions where the minimum of the scalar potential satisfies Eq.\((1)\) \( < \kappa z_m > < 1, \ \forall m \). We shall mainly be interested in superpotentials where the total powers of each term

\[
N_t = \sum_{i=1}^{p} n_{ti},
\]

satisfy

\[
< \kappa z_m >^2 \ll < \kappa z_m >^N. \]

Thus the superpotential will be dominated by the first term.

The scalar potential for locally supersymmetric theories is given by

\[
V = \frac{1}{\kappa^4} e^G (G_b^{-1a} G_{\alpha a} G^{b} - 3) + \frac{1}{2\kappa^4} |\bar{g}_a G_{\alpha a} (T^a z)^a|^2,
\]

where \( G \) can be split into a Kähler function \( K \) and a superpotential \( g \)

\[
G = K(z^a, z_a) + \ln \frac{\kappa^6}{4} |g(z^a)|^2.
\]

The D-term of the potential has a Planck size cosmological constant for the gauged \( U(1)_R \) case. Therefore the analyses for the \( U(1)_R \) and the \( U(1)' \) case are not identical and we consider them separately.

1. Gauged \( U(1)_R \) Case

The choice of the Kähler function is only restricted by the physical requirement that at low energies (\( \kappa \rightarrow 0 \)) the kinetic energy of the scalar fields

\[
\frac{1}{\kappa^2} K_{\alpha a} D_\mu z^a D^\mu z_a.
\]
becomes canonical. The simplest possibility is to take a universal $K = K(u)$ where $u = z_a z^a$ (summation over $a$), which includes the case of minimal kinetic energy, $K(u) = \kappa^2 u$. The minimum of the scalar potential should satisfy the properties

$$V_{,a} = 0,$$  \hspace{1cm} (13)

$$V = 0,$$  \hspace{1cm} (14)

$$(D - \text{term}) = 0.$$  \hspace{1cm} (15)

One can show, however, that it is not possible to satisfy these properties simultaneously when the Kähler function has the universal form $K = K(u)$ and the hidden-sector potential is dominated by a single leading term. We thus consider the more general form for the Kähler function

$$K = \sum_a K^{(a)}(u^{(a)}), \quad u^{(a)} = z_a z^a.$$  \hspace{1cm} (16)

We now determine the minimum of the scalar potential for the condition (13). We thus consider only the leading term of the hidden-sector superpotential. Perturbative corrections to the minimum from higher terms will be small. Therefore the local minimum which we determine is stable. However, the question remains whether the complete potential has a different global minimum. At present, this potential is intractable and far beyond the scope of this letter. Given these assumptions, the potential for the hidden-sector simplifies to

$$V = \frac{\kappa^2}{4} |g|^2 e^{\sum K^{(a)}} \left[ \sum_a F(u^{(a)}) - 3 \right] + \frac{\bar{g}_R^2}{18\kappa^4} |(K'r)^{(a)} + 2|^2,$$  \hspace{1cm} (17)

where $(K'r)^{(a)} = K''^{(a)} + 3 u^{(a)}$ and the function $F$ for a given $u^{(a)}$ is given by

$$F(u) = \frac{u}{K' + u K''}.$$  \hspace{1cm} (18)

Here $n$ is the power of $u$ in the leading term of the hidden-sector superpotential. At the minimum, we must then have

$$F'(u^{(a)}) = 0, \quad \forall u^{(a)},$$  \hspace{1cm} (19)

$$\sum_a F(u^{(a)}) = 3,$$  \hspace{1cm} (20)

$$\sum_a (K'r)^{(a)} = -2.$$  \hspace{1cm} (21)

It is difficult to find functions $K(u^{(a)})$ satisfying all these constraints and maintaining the positivity of the scalar kinetic energy. One appropriate Kähler function is a deformation of the no-scale function first used in [5]

$$K^{(a)}(u^{(a)}) = \frac{n_a}{\beta_a} \ln(1 + \alpha \sigma^2 n_a u^{(a)}),$$  \hspace{1cm} (22)
where $\sigma$ and $\beta_a$ are parameters, $\alpha = \pm 1$. We have inserted $\sigma^2$ since it is $\sigma$ which sets the scale of $<z_m>$ ($u^{(a)} = z_a^a$). The conditions at the minimum of the scalar potential are satisfied provided that

$$\sum_{a=1}^{p} n_a (1 + \beta_a) = \frac{3}{4}, \quad (23)$$

$$\sum_{a=1}^{p} \frac{r_a^b n_a}{1 + 2\beta_a} = -2, \quad (24)$$

$$u^{(a)} = \frac{\alpha}{\sigma^2 n_a} \frac{\beta_a}{1 + \beta_a}. \quad (25)$$

The kinetic energy of the scalar fields is positive if $\alpha/\beta_a > 0$. Since $u^{(a)} \geq 0$ we have two sets of solutions: if $\alpha = +1$ then $\beta_a > 0$, if $\alpha = -1$ then $-1 < \beta_a < 0$. It is straightforward to show that the only solution to these equations with these conditions is for all $\beta_a \in (-1, 0]$. The mass scale which is related to the value of the superpotential at the minimum is

$$m_s^2 = (\kappa^2 g')^2 = \frac{1}{\kappa^2} a_1^2 \cdot (\kappa^2 u_1)^{n_1} \cdot \cdots \cdot (\kappa^2 u_p)^{n_p}. \quad (26)$$

For illustration we give the following anomaly-free model based on our previous work [2] where the full anomaly-equations were given for the case of family-dependent charges. Consider a left-right symmetric model where the $U(1)_{R}$ charges of Eq.(3) satisfy:

$$e_i = l_i, \quad d_i = u_i = q_i.$$  

Furthermore, assume dominant third generation Yukawa couplings, so that only $h_{13}^{33}, h_{23}^{33}, h_{33}^{33} \neq 0$ in the superpotential at tree-level. A possible solution for the fermionic charges $r_i^f$ is

$$\{(l_1, l_2, l_3); (q_1, q_2, q_3); (h_1, h_2)\} = \{(-11/2, -8, 0); (-47/6, 28/3, 0); (-1, -1)\} \quad (27)$$

The remaining two anomaly equations containing the singlets are

$$\sum_{m} z_m^3 = \frac{23518}{8}, \quad \sum_{m} z_m = \frac{45}{2}. \quad (28)$$

For three singlets these equations have many solutions. The power of the leading hidden-sector superpotential term can vary from 3 to well over 100 with a continuous scatter. As an example, we consider the solution $(z_1, z_2, z_3) = (-\frac{75}{2}, \frac{57}{2}, \frac{63}{2})$. The lowest power of the superpotential is then $N_1 = 20$, with $(n_1, n_2, n_3) = (9, 9, 2)$. The next term has $N_2 = 26$ which is sufficiently suppressed. We have solved Eqs.(23)-(25) in terms of $\beta_3$ and we find a continuous set of solutions with $\beta_3 \in [-1, -0.83]$. A specific solution is

$$\beta_1 = -0.96, \quad \beta_2 = -0.97, \quad \beta_3 = -0.91. \quad (29)$$

We then obtain for the supersymmetry scale (26)

$$m_s = a_1 \sigma^{-20} \cdot (2.4 \cdot 10^{24}) GeV \quad (30)$$
which is of order the weak scale for $<z> \sim \frac{1}{\sigma} = 0.075 - 0.085$, $a_1 = \mathcal{O}(1)$. The mu-term $\mu H_1 H_2$ is prohibited in the superpotential at tree-level. The effective mu-term generated from $b_1 \kappa \prod (\kappa z)^{n_i} H_1 H_2$ has the same suppression as the leading term in the superpotential and therefore $\mu_{\text{eff}} = \mathcal{O}(m_s)$, for $b_1 = \mathcal{O}(1)$.

2. Gauged $U(1)'$ Case

The advantage of the $U(1)'$ case is that one can make use of a universal Kähler function

$$ K = K(u), \quad u = z_a z^a. \tag{31} $$

As before, we look for solutions where the superpotential is dominated by the first term. The scalar potential then reduces to

$$ V = \frac{\kappa^2}{4} e^K |g|^2 \left[ \frac{1}{K'} \sum \frac{n_a^2}{u_a} + F(u) \right] + \frac{1}{8\kappa^4} \left( \kappa^2 + g' |g| q_a u_a \right)^2, \tag{32} $$

where $u_a = z_a z^a$ (no summation), $u = \sum u_a$ and now

$$ F(u) = \frac{K'}{K' + uK''}(K'^2 u - N^2 K'' K' + 2NK') - 3K'. \tag{33} $$

Here $N = \sum n_a$. Imposing the conditions that the $D$-term vanishes at the minimum we find that the total potential vanishes at the minimum. The remaining conditions are

$$ u_a = \frac{n_a}{N} u, \tag{34} $$

$$ F = -\frac{N^2}{u}, \tag{35} $$

$$ F' = \frac{N^2}{u^2}. \tag{36} $$

We choose the same form for the Kähler function $K$ as in Eq.(22)

$$ K = -\frac{\lambda}{c} \ln(1 - cu). \tag{37} $$

The equations (33)-(37) can then be reduced to one linear equation in $u$

$$ u \left( \frac{1}{N^2 c^2} \cdot A \left\{ \frac{1}{N^2 c^2} (\lambda - Nc)^2 \cdot A + B \right\} + 2N^2 - \frac{1}{c^2} (\lambda - Nc)^2 \right) = \frac{N^2}{u^2} \left( \frac{1}{N^2 c^2} (\lambda - Nc)^2 \cdot A + B \right) \tag{38} $$

where

$$ A = (2N^2 c + 3\lambda c - \lambda^2), \quad B = (2N\lambda - N^2 c - 3\lambda). \tag{39} $$

The low-energy mass scale is generated by the expectation value of the hidden-sector superpotential

$$ m_s^2 = (\kappa^2 g_1)^2 = a_1^2 (n_1)^{n_1} (n_2)^{n_2} \ldots (n_p)^{n_p} \left( \frac{1}{N} \right)^N u^N \left( \frac{1}{\kappa^2} \right). \tag{40} $$
For illustration we give the following model. We consider family-independent \( U(1)' \) charges, \( i.e. \ l_1 = l_2 = l_3 = l \), etc. The anomaly-equations are given in our notation in \([1]\). We employ the Green Schwarz mechanism \([7]\) to cancel the anomalies and thus include additional constants \( c_1, c_2, c_3 \) in the anomaly-equations as for example in \([2]\). As a possible solution, we find

\[
(l, e, q, u, d, h_1, h_2) = (1, -1, -10, -14, 10, 0, 24),
\]

\[
(c_1, c_2, c_3) = (-\frac{81}{2}, -\frac{63}{2}, -36).
\]

The remaining anomaly equations are

\[
\sum_m z_m^3 = 6045, \quad \sum_m z_m = 165.
\]

For four singlets we have found only two solutions. The solution with the higher leading superpotential power is \((z_1, z_2, z_3, z_4) = (-193, 131, 168, 59)\), with \(N_1 = 14\) and \((n_1, n_2, n_3, n_4) = (5, 3, 2, 4)\). \(N_2 = 17\). Inserting \(N_1 = 14\) into Eq(38) we obtain a set of solutions for \(u\). As an example solution, we present

\[
\lambda = 7.8, \quad c = -2.15, \quad \Rightarrow \quad u = 0.012.
\]

The supersymmetry scale \([40]\) is of order the weak scale for \(a_1 = 10\). The \(\mu\)-term is again prohibited at tree-level. The lowest power term \(b_1\kappa \Pi(\kappa z_i)^{m_i} H_1 H_2\) generating an effective term \(\mu H_1 H_2\) has a power \(\sum m_i = 12\) and \((m_i) = (3, 1, 0, 8)\). The mass parameter \(\mu\) is then of order the weak scale for \(b_1 = 1/20\).

In conclusion, within the framework of local supersymmetry, we have demonstrated a mechanism which dynamically breaks supersymmetry and generates a supersymmetry scale of order the weak scale from the Planck scale alone, without any fine-tuning. We have minimally extended the supersymmetric standard model by an anomaly-free \(U(1)\) gauge symmetry which can be an \(R\)-symmetry as well as three to four Standard Model singlets. The \(\mu\)-problem is simultaneously solved by the same mechanism.

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