Reconstructing an $f(R)$ model from holographic dark energy: using the observational evidence

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Abstract

We investigate the correspondence relation between $f(R)$ gravity and an interacting holographic dark energy (HDE). By obtaining the conditions needed for some observational evidence such as positive acceleration expansion of the Universe, crossing the phantom divide line and validity of the thermodynamics second law in an interacting HDE model and corresponding it with the present Universe, we obtain the explicit evolutionary forms of the corresponding scalar field, potential and scale factor of the Universe.

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1. Introductions

Observational data [1, 2, 4–6] indicate that the current expansion of the Universe is accelerating. Several attempts have been made to solve the current accelerated expansion of the Universe [7–11]. One is the presentation of an unknown energy form, which is called dark energy (DE). An alternative approach is the modification of the gravitational theory, e.g. $f(R)$ gravity in which $f(R)$ is an arbitrary function of the scalar curvature $R$ [7, 12, 13]. Various recent observational data imply that the density of matter (ordinary matter + dark matter (DM) + radiation), $Ω_m = 0.27$, and the density of DE, $Ω_Λ = 0.73$, have potential value today (coincidence problem) [14–16]; besides this, based on recent data, the equation of state (EoS) parameter may evolve from $ω > −1$ (non-phantom phase) in the past to $ω < −1$ (phantom one) at the present epoch. One way to explain these data is to consider dynamical DE with proper interaction with matter [17].

In the quantum field theory, $ρ_Λ$ is regarded as zero-point energy density and is defined based on $L$, the size of the present Universe, (dubbed the holographic dark energy (HDE)) as follows:

$$ρ_Λ = 3c^2M_p^2L^{-2},$$

where $c^2$ is a numerical constant of order unity and $M_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass where $G$ is the Newtonian gravitational constant [18, 19]. Different choices may be adopted for the infrared (IR) cutoff of the Universe, e.g. the Hubble horizon, particle horizon and future event horizon [20]. In a non-interaction model, if we take the particle horizon as the IR cutoff, the accelerated expansion of the Universe cannot be explained [21], and if the Hubble horizon is chosen as the cutoff, then an appropriate EoS parameter for DM cannot be derived [22]. By taking, the future event horizon as the cutoff, the present expansion of the Universe may be explained but the coincidence problem remains unsolved. This problem may be alleviated by considering a suitable interaction between DM and HDE.

In this paper, we consider a flat Friedmann–Lemaître–Roberson–Walker (FLRW) Universe and assume that the universe is composed of two interacting perfect fluids, HDE and DM. We assume the IR cutoff to be a combination of the future and particle event horizons. After some general debate about the properties of the model, we discuss the conditions required to cross the phantom divide line in the $f(R)$ model. We show that this crossing imposes some relations between the parameters of the model.

In this paper, we will review an $f(R)$ model of gravity and establish a correspondence between the $f(R)$ model and
an interacting HDE model. By investigating the conditions needed for describing the present Universe, we can obtain a viable \( f(R) \) model of gravity.

### 2. Description and general properties of the model

The equation of motion for the \( f(R) \) model is

\[
R_{\mu\nu} f' - \frac{1}{2} f g_{\mu\nu} + \left( g_{\mu\nu} \Box - \nabla_{\mu} \nabla_{\nu} \right) f' = 8\pi G T_{\mu\nu},
\]

where the prime represents a derivative with respect to the curvature scalar \( R \) and \( \Box \) is the covariant D'Alembert operator \( (\Box \equiv \nabla^\alpha \nabla_\alpha) \). We will assume DE and cold DM perfect fluids with the stress–energy tensor given by

\[
T_{\mu\nu} = -g_{\mu\nu} p + (\rho + p) u_{\mu} u_{\nu},
\]

where \( \rho \) and \( p \) are the energy density and pressure of the fluid and \( u^\mu = (1, 0, 0, 0) \) is its normalized four-velocity in co-moving coordinates. The DE component has pressure \( p_\Lambda \) and energy density \( \rho_\Lambda \) and the cold DM component has zero pressure and energy density \( \rho_m \). The stress–energy tensor is covariantly conserved.

The trace of equation (2) gives an equation of motion for the new scalar degree of freedom (compared to Einsteinian general relativity) [23–25],

\[
3 \Box f' = 8\pi G T^2 + 2f - R f',
\]

where \( T \) is the trace of the stress–energy tensor. It is helpful to redefine the scalar degree of freedom through

\[
\phi = f' - 1.
\]

Then equation (4) can be reexpressed as an equation of motion for a canonical dimensionless scalar field \( \phi \) with a force term \( \mathcal{F} \) and potential \( V \),

\[
\Box \phi = V'(\phi) - \mathcal{F},
\]

\[
3V'(\phi) = 2f - R f',
\]

where the force term that drives the scalar field \( \phi \) is proportional to the trace of the stress–energy tensor, \( \mathcal{F} = -8\pi G T^2/3 \).

Now we consider a homogeneous and spatially flat space-time with the FLRW line element

\[
d s^2 = d t^2 - a^2(t)(d x^2 + d y^2 + d z^2),
\]

where \( a(t) \) is the scale factor. The \( tt \) component of the gravitational equations (2), for the metric (8), can be simplified to

\[
H^2 + \frac{1}{H} \frac{d}{d t} \ln f' - \frac{1}{6} \left( \frac{f - R f'}{f'} \right) = \frac{8\pi G}{3 f'} \rho_m.
\]

The Friedmann equation, (6), can be written in a somewhat more conventional form as

\[
H^2 = \frac{8\pi G}{3(1 + \phi)} (\rho_m + \rho_\Lambda),
\]

where we assume that the new scalar degree of freedom behaves like DE with DE density

\[
\rho_\Lambda = \frac{3(1 + \phi)}{8\pi G} \left[ H \frac{d(\ln f')}{d t} - \frac{1}{6} \left( \frac{f - R f'}{f'} \right) \right].
\]

Also, one can write the Friedmann equation as

\[
\Omega_m + \Omega_\Lambda = 1,
\]

where the density parameters \( \Omega_m = \rho_m/\rho_c \), \( \Omega_\Lambda = \rho_\Lambda/\rho_c \), and the critical energy density is

\[
\rho_c = \frac{3H^2(1 + \phi)}{8\pi G}.
\]

From the Friedmann equation (10) and conservation of the stress–energy tensor, we have

\[
H = \frac{4\pi G (\rho_\Lambda + \rho_m)}{1 + \phi} - \frac{\phi H}{2(1 + \phi)},
\]

where \( \rho_\Lambda = \rho_m + \rho_\Lambda \). The vanishing of the covariant divergence of the stress–energy tensor for the whole system gives the conservation equation in the metric (8),

\[
\dot{\rho}_\Lambda + 3H (\rho_\Lambda + \rho_m) = 0.
\]

But, because of interactions between the two components, each individual component is not necessarily conserved. So, one can write [3]

\[
\dot{\rho}_\Lambda + 3H (\rho_\Lambda + \rho_m) = -Q,
\]

\[
\dot{\rho}_m + 3H \rho_m = Q.
\]

We consider different forms of \( Q \) below. A number of different models have been proposed for DE. Here we want to investigate the HDE model and see if it can be related to the \( f(R) \) model. HDE is described in terms of an IR cutoff length, \( L \), and the energy density is defined as

\[
\rho_\Lambda = \frac{3c^2 M_p^2}{L^2},
\]

where \( c^2 \) is a constant of the order of unity and \( M_p \) is the Planck mass. This is motivated by quantum theory of gravity considerations, in particular the holographic principle [21, 22, 26]. It was shown in [27] that in quantum field theory the ultraviolet (UV) cutoff \( \Lambda \) should be related to the IR cutoff \( L \) due to a limit set by forming a black hole with the Schwarzschild radius \( L \). If \( \rho_\Lambda = \Lambda^4 \) is the vacuum energy density of the UV cutoff scale, the total energy in volume \( L^3 \) should not exceed the mass of the system-size black hole. This means that \( L^3 \rho_\Lambda \leq M_p^2 L^2 \). So, for the largest cutoff \( L \), one can define the HDE as (18). From equations (18) and (13), the density parameter of HDE can be written as

\[
\Omega_\Lambda = \frac{c^2}{(1 + \phi)} \frac{H^2 L^2}{L^2}.
\]

The IR cutoff, \( L \), is presumably determined by the available length scale. To retain generality, we assume that it is a linear
combination of the particle horizon, \( R_p \), and the future event horizon, \( R_f \); that is, we choose \( L \) to be
\[
L = \alpha R_f + \beta R_p,
\]
(20)
where
\[
R_f = a(t) \int_{t_m}^{\infty} \frac{dt}{a(t)}, \quad R_p = a(t) \int_{t_m}^{t} \frac{dt}{a(t)},
\]
(21)
in which \( t_m \) is the time when the particle was created, and \( 0 < \alpha, \beta \leq 1 \) and \( \alpha + \beta = 1 \). For \( \alpha = 1, \beta = 0 \), we obtain \( L = R_f \), whereas \( \alpha = 0, \beta = 1 \) gives \( L = R_p \).

Taking the time derivative of (18), and using (16), one can obtain the EoS parameter \( \omega_d = p_d/\rho_d \),
\[
\omega_d = -\frac{1}{3} \left[ 1 + 3b^2 - \frac{2(\beta - \alpha)}{\gamma(1 + \phi)} \right].
\]
(22)
To proceed, we have to specify the interaction term \( Q \). A generic form of \( Q \) is not available. Three forms which are often discussed in the literature are \( Q = 3b^2 H \rho_d, 3b^2 H \rho_n, 3b^2 H \rho_H \), where \( b^2 \) is a constant that has to be positive, because following the second law of thermodynamics, energy transfer can only be from DE to cold DM. These three forms of interaction give almost the same result, so for definiteness we choose
\[
Q = 3b^2 H \rho_d.
\]
(23)
Using (19), (22) and (23), we find that
\[
\omega_d = -\frac{1}{3} \left[ 1 + 3b^2 - \frac{2(\beta - \alpha)}{\gamma(1 + \phi)} \right].
\]
(24)
As mentioned in the introduction, observational data indicate that the current cosmological expansion is accelerating. In the fluid model this accelerated expansion requires that \( \omega_d < -1/3 \). This constraint results in
\[
\frac{2(\beta - \alpha)}{\gamma} \sqrt{1 + \phi} \Omega_d < 3b^2.
\]
(25)
Defining a positive constant \( 0 < k_0 < 1 \), we can rewrite (25) as
\[
\frac{2(\beta - \alpha)}{\gamma} \sqrt{1 + \phi} \Omega_d = k_0 b^2.
\]
(26)
The second law of thermodynamics requires that the entropy \( S \) is increasing with time and \( S > 0 \). We assume that \( S \) is the entropy attributed to the surface area \( A = 4\pi L^2 \), where \( L \) is the IR cutoff length appearing in (18). Also, making use of the Noether charge method, one can obtain the entropy in the \( f(R) \) model of gravity for a horizon with surface \( A = 4\pi L^2 \) as [28]
\[
S = \frac{A f'(R)}{4} = \pi L^2 (1 + \phi).
\]
(27)
Then, considering the thermodynamics second law, the time derivative of entropy, \( \dot{S} \), should be
\[
\frac{\dot{S}}{\pi L^2} = (1 + \phi) H \left[ 2 + \frac{\phi}{H(1 + \phi)} + \frac{2(\beta - \alpha)}{HL} \right] \geq 0.
\]
(28)
By accepting \( (1 + \phi) H > 0 \), one can set equation (26) as
\[
2H + \frac{\dot{\phi}}{1 + \phi} + \frac{2(\beta - \alpha)}{HL} = S_0(t) > 0,
\]
(29)
where \( 0 \leq S_0(t) \) for all time; this means that it is not necessary that \( S_0 \) be a constant. \( S_0 = 0 \) when the accelerating expansion of the horizon of the Universe is adiabatic. Here we assume that \( S_0 > 0 \) and then
\[
\frac{2(\beta - \alpha)}{HL} \geq -\frac{\dot{\phi}}{H(1 + \phi)} - 2.
\]
(30)
On the other hand, based on recent data, the DE component seems to have an EoS parameter \( \omega_d < -1 \) at the present epoch, while \( \omega_d > -1 \) in the past [29]. Therefore, we expect the EoS parameter to cross the phantom divide line; then when \( \omega = -1 \), the crossing is allowed. So, by applying the phantom crossing line constraint on \( \omega_d \), (24), we have
\[
\frac{2(\beta - \alpha)}{HL} = 3b^2 - 2.
\]
(31)
From (26) and (31) we have
\[
0 < k_0 = 1 - \frac{2}{3b^2} < 1;
\]
(32)
this shows that \( 0 < 2/3b^2 < 1 \). This means that one of the constants can be omitted. Moreover, to cross \( \omega_d = -1, \dot{\omega}_d \) must be negative at the transition time, \( \omega_d = -1 \). So by making use of (30) and (31), we have
\[
\frac{\dot{\phi}}{H(1 + \phi)} = \theta_0.
\]
(33)
Here \( \theta_0 = -3\xi_0 b^2 \), where \( 0 < \xi_0 < 1 \). So, by using (31) and the time derivative of (24), we have
\[
\dot{\omega} = -\left( \frac{H}{H^2 + \frac{3}{2}b^2} \right) \left( b^2 - \frac{2}{3} \right) < 0.
\]
(34)
From (32), we have \( b^2 > 2/3 \), so that relation (34) is satisfied when
\[
\frac{H}{H^2} > -\frac{3}{2} b^2.
\]
(35)
So, solving (35) gives
\[
H = \frac{h_0}{1 + h_0 \gamma t},
\]
(36)
where \( \gamma = 3\xi_0 b^2 / 2 = \xi_0 / \gamma_0, \xi_0 \) is an arbitrary constant that satisfies the \( 0 < \xi_0 < 1 \) condition. We assume that \( H_0h_0 \sim 1 \) (\( H_0 \) and \( h_0 \) are the Hubble parameter and the present time, respectively); then \( h_0 = H_0 / (1 - \gamma) \). By making use of (36) and (33), we can easily find that the scale factor of the Universe and the scalar field \( \phi \) are
\[
a(t) = a_0 (1 + h_0 \gamma t)^\frac{1}{\gamma},
\]
(37)
\[
\phi(t) = -1 + \phi_0 (1 + h_0 \gamma t)^\frac{2a_0}{\gamma},
\]
(38)
where \( a_0 = (1 - \gamma)^{1/\gamma} \) (we assume that the scale factor at the present time is equal to 1, \( a(t_0) = 1 \)) and \( \phi_0 \) are the integration
constants. It is clearly seen that, from (37), at the early time, \( a(0) \neq 0 \), and then we have bouncing in the beginning of the Universe. It is well known that the Ricci scalar in the flat FLRW is given as

\[
R = 6 \left[ \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right],
\]  
(39)

and as

\[
R = \frac{R_0}{(1 + h_0yt)^2},
\]  
(40)

where \( R_0(\xi_0, \xi_0, b) = 6\hbar^2(1 - 2\gamma) \). So, using (5), (39) and (40), one can obtain a viable \( f(R) \), which allows the crossing from \( \omega = -1 \), as

\[
f(R) = f_0 + C_0(\xi_0, \xi_0, b)\bar{R}^{1 - \frac{\alpha}{4}},
\]  
(41)

where \( C_0 = \xi_0 \phi_0 R_0^{1/2} / \xi_0 \), and \( f_0 \) is the constant of integration which can be as well as the cosmological constant, \( \Lambda \). By making use of the chameleon mechanism this kind of \( f(R) \) model has been studied in [30] and these authors show that this kind of \( f(R) \) model is viable and satisfy the observational constraints of the solar system. Also, using (38) and (41), one can rewrite (7) as

\[
V(\phi) = V_0 + V_1 \phi + \frac{V_2}{(1 + \phi)^2}/\rho_0,
\]  
(42)

where \( V_0 \) is a constant of integration, \( V_1 = 3 f_0/2 \) and

\[
V_2 = \frac{\xi_0(\xi_0 + \xi_0)\phi_0^2}{(\xi_0 - \xi_0)(2\xi_0 - \xi_0)} R_0.
\]

Furthermore, when the features and consequences of a new model of DE are represented, one should test the viability of the new DE model. For this purpose, we test the stability of our model against perturbations. On the basis of on [31], a key quantity for investigating stability is the squared speed of sound \( c_s^2 = d\rho_\phi / d\rho_d \). The sign of \( c_s^2 \) plays a crucial role in determining the stability of the background evolution. If \( c_s^2 > 0 \), it means the classical stability (instability) of a given perturbation. So, using equations (16), (19), (24) and (33), we obtain

\[
c_s^2 = \frac{\ddot{\rho}_d}{\dot{\rho}_d} = \omega_0 + \frac{\rho_0}{\dot{\rho}_d} = \frac{1}{3} (1 + \beta \dot{\omega}_0) \left[ 1 + \omega_0 \right] + \omega_0 \left[ \frac{\left( \left( \frac{\omega_0}{3} + \frac{3}{2} \omega_1 \sqrt{1 + \phi} \right) \Omega_d \right]}{2 + 3\omega_1 \sqrt{1 + \phi} \Omega_d},
\]  
(43)

where

\[
\omega_0 = \frac{3\omega_1 \sqrt{1 + \phi} \Omega_d}{2 + 3\omega_1 \sqrt{1 + \phi} \Omega_d},
\]

and \( \omega_1 = 2(\beta - \alpha)/3c \). In order to gain better insight, we choose a set of parameters as \( \xi_0 = 0.1, b = 0.25, \phi_0 = c = 1 \), in the future horizon and at present time, namely \( \Omega_d = 0.74 \). Our calculations for this set of parameters show that \( c_s^2 = 0.58 \) and \( \omega_0 = -1.18 \). This shows that with a suitable choice of parameters, one can find \( c_s^2 > 0 \), and \( \omega_0 < -1 \). This means that by fine-tuning the free parameters, this model can be stable in phantom phase. Note that in almost every cosmological model, fine-tuning of parameters is necessary and our model is no exception.

### 3. The coincidence problem

We also consider a spatially flat FLRW universe dominated by pressureless (dark) matter and DE, in which the DE component comes from the effect of \( f(R) \) gravity. We define \( r = \frac{\rho_d}{\rho_d} \). We recall that \( r \) determines the ratio of energy density DM to energy density DE and is of the order of unity in the present epoch. The time derivative of \( r \) is

\[
\dot{r} = \frac{Q}{\rho_d} (r + 1) + 3H\omega_0 r.
\]  
(44)

According to equation (10), we have

\[
\rho_d = \frac{3H^2(1 + \phi)}{8\pi G(1 + r)},
\]  
(45)

and by substituting equation (45) into (44), we have

\[
\dot{r} = 3Hr \left[ \omega_0 + \frac{8\pi G Q(1 + r)^2}{9H^3(1 + \phi)} r \right].
\]  
(46)

Obviously, one can integrate equation (46) for an explicit function of \( Q \) in terms of \( r \) and \( H \). Moreover, note that \( r \) is a decreasing function of time and \( \dot{r} \) must be negative. So, to have \( \dot{r} < 0 \) the following relation must to be satisfied:

\[
|\omega_0| > \frac{8\pi G Q(1 + r)^2}{9H^3(1 + \phi)}.
\]  
(47)

We can write \( \dot{r} \) as

\[
\dot{r} = \frac{H}{dH},
\]  
(48)

where

\[
H = -\frac{H^2}{2} \left[ \frac{(3 + 3\omega_0 + \beta_0) + (3 + \beta_0) r}{1 + r} \right],
\]  
(49)

and to obtain equation (49) we have used equations (14) and (33). Since for \( t < t_0 \) the Hubble parameter, \( H \), satisfies \( H(t) > H(t_0) \), so \( H \) has to be negative and the relationship

\[
3|\omega_0| < (3 + \beta_0)(1 + r)
\]  
(50)

must be fulfilled. Therefore, according to equations (47) and (50), we have

\[
\frac{8\pi G Q(1 + r)^2}{3H^3(1 + \phi) r} < 3|\omega_0| < (3 + \beta_0)(1 + r).
\]  
(51)

One can rewrite equation (48) as follows:

\[
\frac{dr}{dH} = \frac{R}{H},
\]  
(52)

where

\[
R = -6 \left[ \frac{r + 1}{(3 + 3\omega_0 + \beta_0)(3 + \beta_0) r} \right] \left[ \omega_0 r + \frac{8\pi G Q(1 + r)^2}{9H^3(1 + \phi)} r \right].
\]  
(53)

As mentioned earlier, there are different forms of \( Q \). In this paper we consider \( Q = 3b^2 H \rho_d \), which has come
from (23). Therefore we substitute this form of \( Q \) into equations (46), (53) and (51) and obtain
\[
\dot{r} = 3H \left[ b^2 + (b^2 + \omega_0) r \right],
\]
(54)
\[
\mathcal{R} = -2 \left[ b^2 + (b^2 + \omega_0) r \right] \frac{r + 1}{(3 + 3\omega_0 + \theta_0) + (3 + \theta_0) r},
\]
(55)
and
\[
\frac{3b^2(1+r)}{r} < 3|\omega_0| < (3 + \theta_0)(1 + r).
\]
(56)
Using \( H = -dz/(1+z)dt \) and integrating (54) with respect to \( z \), we can find \( r \) as a function of \( z \) as
\[
r(z) = \frac{1}{b^2 + \omega_0} \text{Re} \left[ \left( b^2 + \omega_0 \right) r_0 + b^2 \left( 1 + z \right)^{b^2 + \omega_0} \right] - \frac{b^2}{b^2 + \omega_0},
\]
(57)
where \( \text{Re} \) specifies the real part of the corresponding quantity. Also, by using (55) and integrating equation (52) with respect to \( H \), we can find the Hubble parameter as a function of \( r \) as
\[
H = H_0 \exp \left[ \mathcal{I}(r) - \mathcal{I}(r_0) \right],
\]
(58)
where \( r_0 \) and \( H_0 \) can be constants of integration, and \( \mathcal{I} \) is the real part of \( \mathcal{I} \), which is given by
\[
\mathcal{I} = \frac{1}{2} \left[ \ln(1+r) - \left( \frac{3 + 3\omega_0 + \theta_0 + 3b^2}{b^2 + \omega_0} \right) \ln \left( b^2 + (b^2 + \omega_0) r \right) \right].
\]
(59)
It is seen that for \( \theta_0 = \phi = 0 \), the results of this section are exactly the same as the results obtained in [15]. We plot equation (58) in figure 1. This figure indicates \( H/H_0 \) versus \( r = \rho_{de}/\rho_0 \) as given by equation (58) for \( \omega_0 = -0.95 \) (quintessence-solid) and \( \omega_0 = -1.1 \) (phantom-dashed). The general behavior in this figure is very similar to the results of [15]. From this figure one can understand that, with the passage of time, the Hubble parameter as a function of time is decreases and the decreasing of it in the quintessence phase is faster than that in the phantom phase and also the final value of \( H(t) \) in the quintessence phase is smaller than that in the phantom phase.

4. Conclusion

The HDE model is an attempt at probing the nature of DE within the framework of quantum gravity [32]. In this work, we used the HDE model, which is in interaction with DM in the flat FLRW universe. We established a correspondence between the interacting HDE model and the \( f(R) \) model of gravity in the flat FLRW universe. These correspondences are important to understand how different models that have been a candidate for explaining the present Universe are mutually related to each other. However, by taking into account an IR cutoff as a combination of the particle and future event horizons and using the HDE energy density, we obtained the EoS parameter for the interacting HDE. Using equations derived for the EoS parameter of HDE and its time derivative, the condition required for crossing the phantom divide line was derived. Also the condition for validity of the thermodynamics second law for the IR cutoff was obtained. Thus we studied the evolving behavior of the interacting HDE and, implying some observational evidence such as the, positive acceleration expansion of the Universe \( (\omega < -1/3 \) and \( q < 0) \), crossing the phantom divide line \( (\omega < -1) \) and the validity of the second law of thermodynamics for an interacting model of HDE, we reconstructed the \( f(R) \) model which describes the accelerated expansion of the Universe. We also obtained the explicit evolutionary forms of the corresponding scalar fields, potential and scale factor of the Universe.

Finally, we study the stability and coincidence problem of the model. We show that with a suitable choice of free parameters, one can find a stable model that is in good agreement with observational evidence. Note that in almost every cosmological model, fine-tuning of parameters is necessary and our model is no exception.

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