Entropy of $N = 2$ black holes and their $M$-brane description

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Abstract

In this paper we discuss the $M$-brane description for a $N = 2$ black hole. This solution is a result of the compactification of $M$-5-brane configurations over a Calabi-Yau threefold with arbitrary intersection numbers $C_{ABC}$. In analogy to the $D$-brane description where one counts open string states we count here open $M$-2-branes which end on the $M$-5-brane.

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1. Introduction

It has been an open question for long time what kind of states are associated with the Bekenstein–Hawking entropy of black holes. In string theory we came a major step closer in answering this question. Many black holes can be embedded into type II string theory as intersections of $D$-branes. In this picture the horizon of the black hole becomes the surface of these branes. This opens the possibility to identify the microscopic states we are looking for with the open string states ending on the $D$-branes. This way it was possible to give a statistical interpretation for the Bekenstein-Hawking entropy of many types of black holes. For compactifications of type II string theories on $K3 \times T_2$ this has been done e.g. in [1] and the results for $T_6$ compactifications are given in [2], [3]. A microscopic interpretation of the entropy for $N=2$ black holes of an orientifold compactification was given in [4]. In this paper we are going to discuss a generic type II Calabi–Yau compactification in the limit of large vector multiplet moduli. The Bekenstein–Hawking entropy for the corresponding black holes was found recently in [5]. Explicit solutions have been given in [5] for the double extreme limit and in [6] for the general case.

Before we start we will fix our notation (see [5] and refs. therein). The $N=2$ supergravity includes one gravitational, $n_V$ vector and $n_H$ hyper multiplets. In what follows we will neglect the hyper multiplets, assuming that these fields are constant. The bosonic $N=2$ action is given by

$$S \sim \int d^4x \sqrt{G} \left\{ R - 2g_{AB} \partial z^A \partial \bar{z}^B + \frac{1}{4} (3N_{IJ} F^I \cdot F^J + \Re N_{IJ} F^I \cdot \star F^J) \right\},$$

(1)

where the gauge field part $F^I \cdot F^J \equiv F^I_{\mu \nu} F^J^{\mu \nu}$ and $I, J = 0, 1, \ldots, n_V$. The complex scalar fields of the vector multiplets $z^A (A = 1, \ldots, n_V)$ parameterize a special Kähler manifold with the metric $g_{AB} = \partial_A \partial_B K(z, \bar{z})$, where $K(z, \bar{z})$ is the Kähler potential. Both, the gauge field couplings $N_{IJ}$ and the Kähler potential $K$ are given in terms of the holomorphic prepotential $F(X)$ by

$$e^{-K} = i(\bar{X}^I F_I - X^I \tilde{F}_I),$$

$$N_{IJ} = \tilde{F}_{IJ} + 2i(\Re F_{IL} (\Re F_M J) X^L X^M) / (3F_{MN}) X^M X^N,$$

(2)

with $F_I = \frac{\partial F(X)}{\partial X^I}$ and $F_{MN} = \frac{\partial^2 F(X)}{\partial X^M \partial X^N}$ (note that these are not gauge field components). The scalar fields $z^A$ are defined by

$$z^A = \frac{X^A}{X^0},$$

(3)

and for the prepotential we take the cubic form

$$F(X) = \frac{1}{6} C_{ABC} X^A X^B X^C / X^0,$$

(4)

with general constant coefficients $C_{ABC}$. In type II compactification these are the classical intersection numbers of the Calabi–Yau three-fold. When there exists a dual heterotic model, then the cubic part of the prepotential contains both a classical piece, which is linear.
in the heterotic dilaton field and quantum corrections, that do not depend on the heterotic dilaton. In general there will be further corrections consisting (in the type II picture) of a constant part proportional to the Euler number of the Calabi Yau as well as worldsheet instanton corrections which are exponentially suppressed for large vector multiplet moduli. As explained in [5] these corrections are likewise suppressed in the Bekenstein–Hawking entropy formula for the black hole solutions that we are going to consider here.

The paper is organized as follows. In the next section we describe a 4-dimensional black holes solution and show the relation to a 5-dimensional string, which is the intersection of three M5-branes. In section 3 we give a microscopic interpretation of the entropy. Finally, we summarize our results.

2. The black hole solution

In this section we give a black hole solution to the Lagrangian (1). It is an axion-free solution, i.e. the scalar fields $z^A$ and as consequence also the couplings $N_{IJ}$ are pure imaginary (adopting our conventions, which were specified above). Moreover we will consider a solution that carries $n_V + 1$ of the possible $2(n_V + 1)$ charges, which is not the most general axion-free solution [5]. The general solution (including axions and all eight charges possible for $n_V = 3$) for the case that only $C_{123}$ is non-trivial has been discussed in [7].

The solution we are going to analyze is given in terms of $n_V + 1$ harmonic functions $H^A$ and $H_0$ [6]

$$ds^2 = -e^{-2U}dt^2 + e^{2U}d\vec{x}d\vec{x} \quad , \quad e^{2U} = \sqrt{H_0 \frac{1}{6} C_{ABC} H^A H^B H^C}$$

$F^A_{m0} = \epsilon_{mnp} \partial_p H^A \quad , \quad F^A_{0m} = \partial_m (H_0)^{-1} \quad , \quad z^A = iH_0 H^A e^{-2U}$

(note that $F_{I\mu\nu} = N_{IJ} F^J_{\mu\nu}$). To be specific we choose for the harmonic functions

$$H^A = \sqrt{2}(h^A + \frac{p^A}{r}) \quad , \quad H_0 = \sqrt{2}(h_0 + \frac{q_0}{r})$$

where $h^A$, $h_0$ are constant and related to the scalar fields at infinity. The symplectic coordinates and the Kahler potential are given by

$$X^0 = e^U \quad , \quad X^A = iH^A H_0 e^{-U} \quad , \quad e^{-K} = 8(H_0)^2 e^{-2U}.$$  

The electric and magnetic charges are defined by integrals over the gauge fields at spatial infinity

$$q_I = \int_{S^2_{\infty}} N_{I,J}^* F^J = \int_{S^2_{\infty}} N_{I0}^* F^0,$$

$$p^I = \int_{S^2_{\infty}} F^J = \int_{S^2_{\infty}} F^A.$$

Thus, the black hole couples to $n_V$ magnetic gauge fields $F^A_{m0}$ and one electric gauge field $F^0_{0m}$ ($N_{A0} = 0$ for our solution). To get the mass we have to look on the asymptotic
geometry. First, in order to have asymptotically a Minkowski space we have the constraint 
\[ 4 h_0 \frac{1}{6} C_{ABC} h^A h^B h^C = 1. \]
Then
\[ e^{-2U} = 1 - \frac{2M}{r} \pm \cdots. \] (9)

Hence the mass is given by
\[ M = \frac{q_0}{4 h_0} + \frac{1}{2} p^A h_0 C_{ABC} h^B h^C. \] (10)

Using (7) and calculating the central charge \(|Z|\) we find that the black hole, as expected, saturates the BPS bound
\[ M^2 = |Z|_\infty^2 = e^K(\alpha X^0 - p^A F_A)_\infty^2, \] (11)
where the r.h.s. has to be calculated at spatial infinity \((e^{U}_\infty = 1)\).

On the other side if we approach the horizon all constants \(h^A\) and \(h_0\) drop out. The area of the horizon depends only on the conserved charges \(q_0, p^A\). Furthermore, if \(q_0 C_{ABC} p^A p^B p^C > 0\) the solution behaves smoothly on the horizon and we find for the area and entropy \(S_{BH}\)
\[ A = 4 S_{BH} = 4\pi \sqrt{4 q_0 \frac{1}{6} C_{ABC} p^A p^B p^C}. \] (12)

When comparing to reference [5] one has to take into account that we have replace \(F(X)\) by \(-F(X)\) and \(q_0\) by \(-q_0\) in this paper. Note also that the \(d_{ABC}\) used there is related to the intersection numbers \(C_{ABC}\) by \(d_{ABC} = -\frac{1}{6} C_{ABC}\).

If the charges and parameters \(h^A\) are positive then the area of the horizon defines a lower bound for the mass. Minimizing the mass with respect to \(h^A\) and \(h_0\) gives us the area of the horizon [9]
\[ 4\pi M^2 \bigg|_{min.} = A. \] (13)
In this case
\[ h_0 = \frac{q_0}{c}, \quad h^A = \frac{p^A}{c}, \] (14)
where \(c^A = \frac{2}{3} q_0 C_{ABC} p^A p^B p^C\). For these values all scalars are constant, i.e. coincides with their value on the horizon \((z^A \equiv z^A|_{hor.})\). By this procedure we get the double extreme black holes [8]. Taking this limit, our solution [5] coincides with the solution given in [5].

We have discussed only the cubic part [4] of the prepotential and neglected the world-sheet instanton corrections. This approximation is justified as long as \(|z^A| \gg 1\), which holds whenever \(H_0\) is large, which means the black hole decompactifies to a string.

There are many ways to get the solution (3) by compactification of higher-dimensional configurations. On the type II side we have a Calabi–Yau compactification, e.g. of three \(D4\)-branes and a \(D0\)-brane for type IIA string theory. Alternatively we can see our solution as a compactification of an intersection of three \(M5\)-branes and a boost along the common string. Let us discuss the last possibility in more detail. If we have \(n_V = 3\) and if only
$C_{123}$ is non-vanishing, our solution (with 3 moduli $A = 1, 2, 3$) corresponds to the following intersection in 11 dimensions \[11]\n
\[ ds^2_{11} = \frac{1}{(H^1 H^2 H^3)^2} \left[ du dv + H_0 du^2 + H^1 H^2 H^3 d\vec{x}^2 + H^A \omega_A \right]. \quad (15) \]

The case of identical harmonic functions has been discussed before in \[12\]. This is a configuration where three $M$-5-branes intersect over a common string and each pair of $M$-5-branes intersects over a 3-brane. In going to 4 dimensions we first compactify over $H^A \omega_A$, with $\omega_A$ defining three 2-dimensional line elements. After this we are in 5 dimensions and have a string solution with momentum modes parametrized by $H_0$ ($H^A$ are parametrizing the $M$-5-branes). Generalizing this solution to a Calabi–Yau three–fold with generic intersection numbers $C_{ABC}$ we find for this 5-dimensional string solution

\[ ds^2 = \frac{1}{(\frac{1}{6} C_{ABC} H^A H^B H^C)^2} \left( dv du + H_0 du^2 + (\frac{1}{6} C_{ABC} H^A H^B H^C) d\vec{x}^2 \right). \quad (16) \]

Compactifying this string solution over $u$ yields our 4-dimensional black hole solution \[4\]. The electric gauge field results from Kaluza-Klein reduction from 5 to 4 dimensions. In 5 dimensions we have only magnetic gauge fields which are inherited by the $D = 4$ solution. In addition one of the 4-dimensional scalar fields is the compactification radius, which is related to $|H_0|$ and thus $|H_0| \gg 1$ gives us the decompactification limit, for which corrections due to the non–cubic terms in the prepotential are small.

For the generic case ($C_{ABC} p^A p^B p^C \neq 0$) this decompactified 5-dimensional string solution is non-singular and the asymptotic geometry near the horizon is given by $AdS_3 \times S^2$. Moreover every supersymmetric black hole (also the singular one) can be understood as a compactification of a non-singular configuration (e.g. self-dual 3- or 1-brane) with the asymptotic geometry $AdS_p \times S_q$ \[13\]. These geometries play an important role for supersymmetry restoration near the horizon (see also \[14\]).

### 3. Microscopic interpretation of the entropy

In this section we will propose a microscopic interpretation for the entropy \[12\]. Adapting the procedure of \[1, 2\] we have to count the states of the 5-dimensional magnetic string and to identify them as black hole states yielding the Bekenstein-Hawking entropy. This procedure is known as $D$-brane counting for the $N = 4$ embedding where open string states are counted which end on the $D$-brane\footnote{For an equivalent counting of the NS-NS states see [15].}. Here, however, the starting point is 11-dimensional supergravity where we have 2-branes and 5-branes, which are called $M$-branes. In analogy to type II open strings and $D$-branes, open $M$-2-branes can end on $M$-5-branes \[16\]. Therefore the states of the 5-dimensional string are related to open membrane states attached to the $M$-5-branes. Since one could in principle compactify to $D = 10$ first, there is a direct relation to open string states on $D$-4-branes.
A counting procedure for configurations of $M$-5-branes with triple-intersections along strings has been proposed in [17]. We will adapt it to the case at hand. The first thing to notice is that we are dealing with a BPS saturated solution (see (11)), which means that the momentum in the 5-dimensional string is purely left-moving (see figure 1). For a non-extreme solution, which does not satisfy the BPS bound, there would be both left- and right-moving modes. Since the 5-th dimension is compact, the momentum is quantized. The integer quantum number $N_L$ has the interpretation of an electric charge in four dimensions: $N_L = q_0$. The statistical entropy of left-moving states is given by

$$S_{\text{stat}} = \log d(N_L) = 2\pi \sqrt{\frac{1}{6} c_{\text{eff}} N_L} , \quad (17)$$

where $c_{\text{eff}}$ is the effective central charge associated to the 5-dimensional string. Recall that in order to put momentum on the 5-dimensional string one has to excite some internal degree of freedom. This is so because the spacelike coordinate of the string world-sheet has been identified with the 5-th coordinate of the target space. Therefore translational invariance (as well as transversality of physical states) forbids oscillating modes in the 5-th direction. One can however excite some internal degrees of freedom transversal to the string and let them move in left direction around the string as described in figure 1. These internal excitations have to be massless, because they are purely left-moving. The entropy is found by counting in how many ways one can distribute $q_0$ quanta of momentum among these massless internal modes. One now assumes that the massless internal degrees of freedom are described by a supersymmetric conformal sigma model with central charge $c_{\text{eff}} = \frac{3}{2} D_{\text{eff}}$. The effective target space dimension $D_{\text{eff}}$ is the number of massless excitations and for each dimension there is a bosonic and a fermionic world-sheet field, which contribute $c = 1$ and $c = \frac{1}{2}$ to the effective central charge, respectively. The counting of states available with momentum $q_0$ is equivalent to counting the number of left-moving oscillator states at level $N_L = q_0$ in a conformal field theory with central charge $c_{\text{eff}}$. For large $q_0$, i.e. for $q_0 \gg c_{\text{eff}}$ the corresponding statistical entropy is given by [17].
The construction described so far is universal in the sense that it applies to all 4-
dimensional (and 5–dimensional) black holes with finite horizon that can be constructed out
of a 5–dimensional (6–dimensional) string. We now have to determine \( c_{\text{eff}} \) by specifying
the massless modes of the internal sector. We will follow here the proposal made in [17] for
the counting of massless internal excitations of strings arising from triple–intersections of
\( M \)-5-branes. To explain this we take first some configurations of \( M \)-5-branes which mutually
intersect transversely along 3-branes and have triple–intersections along strings. Excitations
of such a configuration are described by open membranes which have boundaries on different
\( M \)-5-branes. In a generic situation no \( M \)-5-branes are sitting on top of each other. Therefore
the \( M \)-2-branes are stretched and there are no massless states associated with them. There
are however triple–intersections along the strings and a \( M \)-2-brane sitting at an intersection
describes the massless excitations of this string. One now has to assume that the relevant
\( M \)-2-branes are those with three boundaries, one sitting on each of the three intersecting
\( M \)-5-branes. Branes with less than 3 boundaries are giving a subleading contribution, since
the number of states grows with number of boundaries. On the other side the number of
branes with more than 3 boundaries would grow too fast.

Next, a string inside a \( M \)-5-brane has four transversal directions and therefore carries
\( c = 6 \). To get the effective central charge one simply has to multiply this with the number
of intersections: \( c_{\text{eff}} = 6 \cdot \sharp (\text{intersections}) \). In order to construct a 4-dimensional black hole
one now has to compactify from 11 to 5 dimensions while wrapping 4 dimensions of each
\( M \)-5-brane in such a way that all the strings resulting from triple–intersections are put on
top of each other. Then one wraps the resulting string around the 5-th dimension.

The authors of [17] took a configuration of \( p^1 \) parallel \( M \)-5-branes which intersect with
further \( p^2 \) parallel \( M \)-5-branes in 3-branes. The number of intersections was \( p^1 p^2 \) (see figure
2). Including the third set of \( p^3 \) parallel \( M \)-5-branes they got for the total number of
intersections \( p^1 p^2 p^3 \). The black hole was then constructed by toroidal compactification and
appropriate wrapping. The entropy of this configuration was \( S_{\text{stat}} = 2\pi \sqrt{q_0 p^1 p^2 p^3} \) which
agreed with the Bekenstein–Hawking entropy of the corresponding \( D = 4 \) black hole solution
[17].

We will now generalize this to the case of a Calabi–Yau compactification. Let us first
consider the case where the only non–vanishing intersection number of the Calabi–Yau is
\( C_{123} \). We first take \( p^1 \) \( M \)-5-branes and wrap them around \( p^1 \) homologous but distinct
primitive 4-cycles. This generalizes the case of parallel, but non–coinciding \( M \)-5-branes
discussed before. The reason why we insist on wrapping the branes around different 4-cycles
is the same as in the flat space situation discussed in [17]: If \( M \)-5-branes coincide, then the
\( M \)-2-branes connecting them are no longer stretched and can become massless. For \( M \)-2-
branes with three boundaries this would give a contribution \( \sim (p^1)^3 \) to the entropy. But
according the Bekenstein–Hawking formula (12) such contributions should be associated
with transversal triple–intersections and not with coinciding \( M \)-branes. Especially there
should be no contribution if the self–intersection number \( C_{111} \) is zero, which we assume here
(the case \( C_{111} \neq 0 \) will be discussed below). Thus in order to interprete (12) one should
take a configuration of non–coinciding 4-cycles that has triple–intersections in a discrete set
of points.
Fig. 2 Wrapping a brane $p^1$ times around a 4-cycle gives $p^1$ branes lying on top of each other. An equivalent point of view is to assume that one has $p^1$ parallel branes wrapping once around a 4-cycle. Intersecting two such 4-cycles yields $p^1 p^2$ branes that lying on the common intersection.

Let us now proceed in giving an interpretation to (12) for the case that only $C_{123}$ is non–vanishing. We next wrap $p^2$ $M$-5-branes around cycles in the second and $p^3$ $M$-5-branes around cycles in the third primitive homology class. Each triple of cycles with one cycle chosen from a different homology classes intersects in $C_{123}$ points. The total number of intersections is $C_{123} p^1 p^2 p^3$ (see also figure 2). This can be written as $\frac{1}{6} C_{ABC} p^A p^B p^C$ using that $C_{ABC}$ is symmetric. The resulting entropy is

$$S_{\text{stat}} = 2\pi \sqrt{\frac{1}{6} C_{ABC} p^A p^B p^C}$$

which coincides with the Bekenstein–Hawking entropy found in [7] and recovered in (12).

Let us now consider the case with generic $C_{ABC}$. A Calabi–Yau manifold has $b_4 = n_V$ different primitive 4-cycles which can have various mutual as well as self–intersections. The general configuration is given by $p^A$ $M$-5-branes wrapped around distinct 4-cycles in the $A$-th primitive homology class, where $A = 1, \ldots, n_V$. The only point that deserves a further comment is the case where self–intersections occur, i.e. if $C_{AAA} \neq 0$ or $C_{AAB} \neq 0$. Let’s consider the case $C_{111} \neq 0$, which means that three generic 4-cycles chosen from the first primitive homology class triple-intersect in $C_{111}$ points. Then the total number of intersection points is $\frac{1}{6} C_{111} p^1 (p^1 - 1)(p^1 - 2)$. The factor of $\frac{1}{6} = \frac{1}{3!}$ is needed to avoid overcounting, because now all the branes are in the same homology class. (When counting intersections between branes in different classes, a factor $\frac{1}{6}$ was introduced for different reasons, namely to compensate for permutations arising from the summation over the indices. ) Moreover intersections occur only between different cycles so that we have to replace $(p^1)^3$ by $p^1 (p^1 - 1)(p^1 - 2)$. Note that this number is divisible by 6. In the limit of large charges the number of intersections is dominated by the cubic piece and we get $\frac{1}{6} C_{111} (p^1)^3$. The discussion of non–vanishing intersection numbers $C_{AAB}$ is similar and one finds $\frac{1}{6} C_{AAB} p^A (p^A - 1)p^B \sim \frac{1}{6} (p^A)^2 p^B$ intersection points. As a result the microscopic entropy formula for a Calabi–Yau compactification with generic intersection numbers is given by (18) and coincides with the Bekenstein–Hawking entropy (12).
4. Discussion

In this paper we have given a microscopic interpretation for the Bekenstein-Hawking entropy of a $N = 2$ black hole. Our solution has been obtained by a $M$-theory compactification over a Calabi–Yau three–fold with arbitrary self–intersection numbers $C_{ABC}$. In 11 dimensions this solution corresponds to a configuration of $M$-5-branes with triple–intersections along strings. We have counted states on the $M$-5-branes, however not open string states as in the $D$-brane technique but open membrane states. Following [17] we assumed that the leading contribution to the entropy comes from massless open $M$-2-branes with three boundaries, each sitting on a different $M$-5-brane. Since such $M$-2-branes can only become massless in the vicinity of a triple–intersection of $M$-5-branes we considered a configuration of $M$-5-branes with triple–intersections along strings. Compactifying this configuration over the Calabi–Yau threefold, i.e. wrapping the $M$-5-branes around 4-cycles, we got in 5 dimensions a magnetic strings with momentum modes corresponding to the open membrane states. The magnetic charges counts how many times we had wrapped a $M$-5-brane around a 4-cycle. Or, from a different point of view, the magnetic charge counts the number of parallel $M$-5-branes that are wrapped once around the 4-cycle. The electric charge gives then the number of momentum modes. The intersection pattern of the $M$-5-branes was governed by the intersection form $C_{ABC}$ of the Calabi–Yau. The only freedom consisted off choosing the numbers $p^A$ of $M$-5-branes that we wrapped around 4-cylces in a given primitive homology class. The number of intersection points was found to be $\frac{1}{6}C_{ABC}p^Ap^Bp^C$, which is precisely the number one needs to reproduce the Bekenstein–Hawking entropy by state counting. We also note here that the structure of the entropy formula (18) is very similar to the one of the 5–dimensional black hole discussed by Strominger and Vafa[1]. Both solutions are related by replacing $D$-branes by $M$-branes, the $K^3$ surface by a Calabi–Yau space and internal 2-cycles by 4-cycles. It would be very interesting to analyse these parallels in more detail. This could help to develop the world–volume theory of curved $M$–branes. In analogy to the findings of [1] one expects this to be a partially topologically twisted theory, which should encode geometrical properties of the wrapping cycles inside the Calabi–Yau space.

The inclusion of all possible self-intersections of the 4-cycles reveals some new features. For many Calabi–Yau compactification we do not need anymore four independent charges to have a black hole with a non-singular horizon. A simple cubic term, like $C_{333}$, in the prepotential makes already the horizon non-singular. In the discussion of black holes that are non-singular only due to self-intersections of 4-cycles we have, however, to keep in mind that this procedure can only be trusted as long as all charges are large, which is related to a large black hole in 4 dimensions. If the charges become small (or even vanish) there are corrections to be taken into account, which can be parametrized by additional terms to the prepotential (4).

Throughout the paper we have restricted ourselves to the cubic part of the prepotential, which for type II A strings is valid in the limit of large Kähler moduli. Moreover the radius of the 5-th direction has to be taken even larger then the size of the Calabi–Yau manifold. In terms of charges this means $q_0 \gg c_{\text{eff}} \sim C_{ABC}p^Ap^Bp^C$, i.e. that we just began to explore a large moduli space starting from a particular corner. Since the type II
dilaton sits in a hyper multiplet, the prepotential is not expected to get perturbative or non–perturbative quantum corrections. There are however $\alpha'$-correction, both perturbative and non–perturbative. The perturbative corrections yield a constant term proportional to the Euler number of the Calabi–Yau, whereas the non–perturbative corrections result in an infinite series of world–sheet instantons. One therefore has to expect that the entropy depends on the Euler number as well as on the world–sheet instanton numbers that count the rational holomorphic curves inside the Calabi–Yau. Indeed, as discussed in \cite{5} the constant term enters the Bekenstein–Hawking entropy for some type of black hole solution. But this term comes with a transcendental prefactor $\zeta(3)$ which is hard to reproduce by state counting\textsuperscript{d}. Therefore further contributions to the entropy from world–sheet instantons are needed in order make a statistic interpretation possible. Note that it is natural to include the constant term in the world–sheet instanton series as its zero mode. To see this recall that the correction to the cubic term of the prepotential has the form \cite{19}

$$-\chi^2 \zeta(3) + \sum_{m_i} n_{m_i} Li_3(e^{-m_it_i}),$$

where $\chi$ is the Euler number, $m_i \neq 0$ is a multi–index that labels world–sheet instantons, $n_{m_i}$ is the number of world–sheet instantons of type $m_i$ and $t_i$ are the moduli. Since $\zeta(3) = Li_3(1)$ one can include the constant term in the sum by defining $n_0 = -\frac{\chi^2}{2}$.

We would also like to mention that there is a class of $N = 2$ string models, where the cubic part of the prepotential is exact. These models are very similar to $N = 4$ models in that there are no perturbative quantum corrections. (We are using here the heterotic string picture, where the world–sheet instanton corrections of the type II picture are mapped to quantum loop corrections.) This class contains the II A orientifold, for which the microscopic entropy was studied in \cite{4}. In \cite{18} the Calabi–Yau threefold corresponding to this model was described. It is a self–mirror, implying that the Euler number as well as all world–sheet instanton corrections must vanish. This model might be good laboratory for a deeper study of the geometric structure behind the $M$-brane picture discussed here. Note also that the vanishing of the constant and of the instanton correction to the prepotential are enforced in once. This supports our speculation above about the close relationship of these terms.

Finally we would like to recall that our black hole solution was based on the prepotential, which only takes into account the minimal terms in the effective action, i.e. those with the minimal number of derivatives. Since it is well known that string–effective action contain an infinite series of higher derivative terms, it is interesting to ask how these will modify the picture of stringy black holes that we have today.

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