Quark–diquark model for $p(\bar{p})–p$ elastic scattering at high energies

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1 Introduction

Recently the TOTEM Collaboration reported the first experimental data on the $pp$ elastic cross-section at the total energy in the center of mass system $\sqrt{s} = 7$ TeV (everywhere the Planck constant, $\hbar$, and the speed of light, $c$, are assumed to be unit, $\hbar = c = 1$) [1]. Therefore there is opportunity to describe one in the more wide area of energy using the early data (see, for example, [2]). In general there are a number of models of the elastic $pp$ scattering description [2, 3].

We discuss in this note the quark–diquark ($qQ$-) model in which baryons are considered as bound states of quark and diquark (a quasi-particle state of two quarks). This model appeared at the end sixties [4] and was used for description of different problems: baryon spectroscopy [5, 6], multiparticle production [7, 8], deep-inelastic processes [9] and others. The $qQ$-model was proposed by V.A. Tsarev [10] in 1979 to describe the characteristics of proton–proton elastic scattering and to explain the absence of second dip in the proton–proton differential cross-section, $d\sigma_{el}/dt$, at $-t \sim 1.3$ GeV$^2$. The model [11] was applied in [12] to describe at the same level of accuracy the TOTEM data [1] for $d\sigma_{el}/dt$ at $\sqrt{s} = 7$ TeV.

We recall below the main features of the $qQ$-model [10] suitable for numerical calculations and provide comparison with the experimental data on the $pp$ elastic differential cross-section in the region of high energies, $\sqrt{s} \geq 546$ GeV.

2 Quark–diquark model for $p(\bar{p})–p$ elastic scattering

The proton–proton differential elastic cross-section can be expressed in terms of the scattering amplitude $F(s, t)$:

$$
\frac{d\sigma_{el}}{dt} = \frac{\pi}{p^2} |F(s, t)|^2,
$$

(1)

where $p$ is the proton momentum in the center of mass system.

The model [10] limits the consideration of the scattering amplitude by contributions from one- and two-pomeron exchanges between quark–quark (1–1), diquark–diquark (2–2) and quark–diquark (1–2). In this approximation $F(s, t)$ can be expressed as

$$
F(s, t) = F_1(s, t) - F_2(s, t) - F_3(s, t),
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(2)

where $F_1(s, t)$ is the scattering amplitude with one-pomeron exchange, while $F_2(s, t)$ corresponds to two-pomeron exchanges between the proton constituents, quark and diquark, and $F_3(s, t)$ corresponds to two-pomeron exchanges between the quark (or diquark) of one proton and the quark and the diquark of another proton at the same time. The amplitude $F_1(s, t)$ reads

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\[ F_1(s, t) = \frac{i \sigma_{\text{tot}}(s)}{4\pi} \left[ B_1 \exp(A_{11} t) + B_2 \exp(A_{22} t) + 2\sqrt{B_1 B_2} \exp(A_{12} t) \right], \]

where \( \sigma_{\text{tot}}(s) \) is the total proton–proton cross-section. The coefficients \( B_1 \) and \( B_2 \) parametrize the quark–quark, \( \sigma_{11} \), and the diquark–diquark, \( \sigma_{22} \), cross-sections, respectively:

\[ \sigma_{11} = B_1 \sigma_{\text{tot}}(s), \quad \sigma_{22} = B_2 \sigma_{\text{tot}}(s). \]

The model assumes the quark–diquark cross-section, \( \sigma_{12} = \sqrt{\sigma_{11} \sigma_{22}} \).

The coefficients \( A_{jk} \), \( (j, k = 1, 2) \) are derived taking into account the Gauss distribution of quark and diquark in proton together with the standard pomeron parametrization. They read \( (s_0 = 1 \text{ GeV}^2) \)

\[
A_{jk} = \frac{r_1^2 + r_2^2}{16} + \tilde{a} \left[ \ln \frac{s}{s_0} - \frac{i\pi}{2} \right] + \lambda \left[ \left( \frac{m - m_1}{m} \right)^2 + \left( \frac{m - m_2}{m} \right)^2 \right].
\]

Here \( r_1, m_1 \) are the quark radius and mass, and \( r_2, m_2 \) are the diquark radius and mass, respectively; \( \tilde{a} = 0.15 \text{ GeV}^{-2} \) is the pomeron trajectory slope, and \( \lambda = r^2/4 \), where \( r \) is the proton radius. It is assumed in the model that \( m_1 = m/3 \) and \( m_2 = 2m/3 \), where \( m \) is the proton mass. The radii \( r_1 \) and \( r_2 \) were found by the fitting of experimental data: \( r_1 = 0.173r_2 \), \( r_2 = 0.316r_2 \).

The amplitudes \( F_2(s, t) \) and \( F_3(s, t) \) are

\[
F_2(s, t) = \frac{i \sigma_{\text{tot}}(s)}{8\pi s_0 \Re \left( A_{12} + 4\lambda/9 \right)} \times \left[ \exp \left( \frac{A_{11} A_{22} - (4\lambda/9) t}{2(A_{12} + 4\lambda/9)} \right) + \exp \left( \frac{A_{12} - 4\lambda/9}{2} \right) \right],
\]

and

\[
F_3(s, t) = \frac{i \sigma_{\text{tot}}(s)}{4\pi} \left[ \frac{B_1}{A_{11} + A_{12} - 4\lambda/9} \times \exp \left( \frac{A_{11} A_{12} - (2\lambda/9) t}{A_{11} + A_{12} - 4\lambda/9} \right) + \frac{B_2}{A_{12} + A_{22} + 2\lambda/9} \times \exp \left( \frac{A_{12} A_{22} - (\lambda/9) t}{A_{12} + A_{22} + 2\lambda/9} \right) \right].
\]

respectively. The quark–quark cross-section, \( \sigma_{11} \) and the proton radius \( r \) are the free parameters defining (together with \( \sigma_{\text{tot}}(s) \)) the \( s \)-dependence of the \( d\sigma_{el}/dt \). The diquark–diquark cross-section and the parameter \( B_2 \) are derived from the optical theorem, which results in the following equation:

\[
\sigma_{\text{tot}}(s) b_1 B_1 B_2 + \sigma_{\text{tot}} \sqrt{B_1 B_2} (b_2 B_1 + b_3 B_2) = B_1 + B_2 + 2\sqrt{B_1 B_2} - 1,
\]

where

\[
b_1 = \frac{1}{4\pi} \Re \left[ \frac{1}{A_{12} + 4\lambda/9} \right],
\]

\[
b_2 = \frac{1}{4\pi} \Re \left[ \frac{1}{A_{11} + A_{22} - 4\lambda/9} \right],
\]

\[
b_3 = \frac{1}{4\pi} \Re \left[ \frac{1}{A_{12} + A_{22} + 2\lambda/9} \right].
\]

Equation (7) is the third-order equation relative to \( \sqrt{B_2} \). For \( 0 < B_1 < 1 \), it has rational root \( 0 < \sqrt{B_2} < 1 \) which is used in the model.

### 3 Comparison with experimental data

Figures 1, 2, and 3 show the antiproton–proton differential elastic cross-section versus \( |t| \) at \( \sqrt{s} = 546 \text{ GeV} \), \( \sqrt{s} = 1960 \text{ GeV} \), and the proton–proton differential elastic cross section at \( \sqrt{s} = 7 \text{ TeV} \), respectively. The curves are the predictions of our model. We see that the proposed \( qQ \)-model describes reasonably the differential elastic cross sections of the antiproton–proton and proton–proton scattering in a wide region of energy.

The results shown in Figs. 1, 2 and 3 correspond to the \( s \)-dependence of the model free parameters which are shown (open circles are the experimental data [13, 14])
Fig. 2 The antiproton–proton differential elastic cross-section versus $|t|$ at $\sqrt{s} = 1960$ GeV. The curve is the prediction of our model. The open circles are the experimental data [15].

Fig. 3 The proton–proton differential elastic cross-section versus $|t|$ at $\sqrt{s} = 7$ TeV. The curve is the prediction of our model. The open and closed circles are the LHC TOTEM experimental data from [1], in Figs. 4 and 5. The parametrization of the proton radius reads

$$\ln(r \cdot \text{GeV}) = 1.72 + 0.004 \ln \frac{s}{s_0}.$$ 

The model results in increase of the proton radius and the quark–quark cross-section with growth of $s$.

4 Discussion and summary

We have considered the $qQ$-model of the $pp$-elastic scattering at high energies. It was obtained reasonable description of the differential cross section of elastic $pp$ scattering in a wide region of energies. The position of the $d\sigma_{el}/dt$-minimum is in the satisfactory agreement with experimental data, while the value of dip is overestimated, though less than in the model [11]. The reason is that the model [10] has nonzero real part of the scattering amplitude coming from the pomeron parametrization. However, the value of the real part is not enough for correct description of the dip value.

The model tuning with more sophisticated form-factors, investigation of additional sources for the scattering amplitude real part, and broader comparison of the model with experimental data are current plans.

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