Extracting Traffic Primitives Directly from Naturalistically Logged Data for Self-Driving Applications

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Abstract—Developing an automated vehicle, that can handle the complicated driving scenarios and appropriately interact with other road users, requires the ability to semantically learn and understand the driving environment, oftentimes, based on the analysis of massive amount of naturalistic driving data. An important paradigm that allows automated vehicles to both learn from human drivers and develop deeper insights is understanding traffic primitives, representing principal compositions of the entire traffic. However, the exploding driving data growth presents a great challenge in extracting primitives from a long-term multidimensional time-series traffic scenario data with multiscale varieties of road users get involved. Therefore, automatic primitive extraction is becoming one of the cost-efficient ways to help autonomous vehicles understand and predict the complex traffic scenarios. In addition, the extracted primitives from raw data should 1) be appropriate for automated driving applications and also 2) be easily combined to generate new driving scenarios. Existing literature does not provide a method to automatically learn these primitives from large-scale traffic data. The contribution of this paper has two manifolds. One is that we proposed a new framework to generate new traffic scenarios from a handful of limited traffic data. The other one is that we introduce a nonparametric Bayesian learning method – a sticky hierarchical Dirichlet process hidden Markov model – that can automatically extract primitives from multidimensional driving data without prior knowledge of the primitive settings. The developed method is validated using one day of naturalistic driving data. Experiment results show that the nonparametric Bayesian learning method extracts primitives from traffic scenarios where both the binary and continuous events coexist.

Index Terms—Traffic scenario primitive, nonparametric Bayesian learning method, hierarchical Dirichlet process.

I. INTRODUCTION

Automated vehicles play a vitally important role in intelligent traffic systems, road safety, and driving workload reduction [1]. A lot of automated vehicle research has focused on how to learn end-to-end controllers [2], how to design and generate traffic scenarios for automated vehicle evaluation [3], how to understand the traffic scenes using naturalistic driving data based on deep learning and machine learning techniques, capable of offering supportive interventions to human drivers during a specific task. With this purpose, the automated vehicles need to fully understand how driving scenarios are changing and correctly predict what other road users surrounded will do. In such traffic scenarios where different road users are engaged, state changes of driving environment encompass the movements of both the automated vehicle and the other surrounding road users. When investigating traffic scenarios, researchers will manually and subjectively extract some specific scenarios they are interested from a handful of databases according to scenario definitions. These scenarios including car-following, lane-changing, overtaking behaviors, etc., however, are not able to cover the entire traffic case, and also not might be suitable to learn for algorithms. More specifically, these manually extracted scenarios are not flexible to be cascaded or combined to generate new traffic scenarios.

On the other hand, one of the greatest challenges also exists in manually extracting and reusing huge amounts of these scenarios because of dizzying databases and exploding data growth [4]. Advanced sensing technologies such as cameras, radars, lidars, and GPS can provide rich information for modern vehicles [5] (see APPENDIX A), enabling data-driven techniques to be one practicable way to deal with problematic issues in intelligent transportation systems [6]. Today, traffic in the real world is a vast and varied cyber-physical system, with thousands of kinds of drivers, vehicles and driving environment. The flood of data can overwhelm human insight and analysis because the size and complexity of traffic data sets are far larger and messier than a human being can manually cope with [7]. Many autonomous companies and researchers put great efforts into collecting high quality and useful data and then annotating it, which is a greatly both time- and resource-consuming procedure, thus limiting the fast development of automated vehicles. For example, DeepAI [8] spends 800 human hours to label data for every one hour recorded data using deep learning techniques.

To obtain valuable and useful information from large databases, researchers developed powerful new technologies such as learning-based approaches. For example, Bender et al. [9] developed an unsupervised method, Bayesian multivariate linear model, to segment a time-series inertial data into finite amounts of linear portions for inferring driver behaviors, but only for a two-dimensional data sequence of one vehicle. Taniguchi et al. [10] introduced a double articulation analyzer based on nonparametric Bayesian theory to predict driver behaviors with a six-dimensional data sequence, consisting of gas pedal position, brake pressure, steering angle, velocity, acceleration, and steering angle rate. They also developed an unsupervised approach to segment and predict driver’s upcoming behavior by detecting and learning contextual changing points [11]. Hamada et al. [12] applied a nonparametric Bayesian with linear dynamical sys-
tems to learn driver behavior primitives and predict drivers’ upcoming behaviors. Wang et al. [13] applied three different nonparametric Bayesian learning approaches to analyze human drivers’ car-following styles. Learning-based methods have been widely used to model and predict driver behavior; however, according to our knowledge, no literature presents an insight into traffic scenario primitive extraction with large-scale multi-vehicle involved and regenerates to new traffic scenarios for automated vehicles.

Differing from previous research [9]–[12] focusing on individual driver’s behavior, we mainly concern how to generate an infinite number of new traffic scenarios from a handful of limited raw traffic data. To achieve this, we proposed a framework as shown in Fig. 1, which brings three key attributes – compositionality, causality, and learning to learn. The main tasks and challenges of achieving these are listed as follows:

1) Developing an algorithm that can automatically extract these primitives with less subjective intervention and without prior knowledge about the type and number of them.
2) Finding and clustering the analogous primitives, thus generating the object set or template.
3) Describing and modeling the topological relations between template sets, and then obtaining a causal structure for dynamic, stochastic relations.
4) Proposing a method to automatically generate infinite amounts of new scenarios being statistically equivalent to what the vehicle would have encountered in the real world.

We note that, the first two challenges are to parse a long-term multi-scale time-series driving data into primitives and cluster them. Therefore, we introduce a nonparametric Bayesian learning method to extract the traffic scenario primitives from a traffic data sequence. In the real traffic scenarios, however, two kinds of events primarily exist, one is described using binary states and the other one is described using continuous states. The binary state represents a new road user’s appearance/disappearance in current driving scenarios and the continuous state represents the state changes of current driving scenarios with a fixed number of other surrounding road users involved. Regarding to the two types of events, we demonstrate that the developed learning approach is still able to complete the primitive extraction work.

The remains of this paper are organized as follows. Section II introduces the developed nonparametric Bayesian learning method. Section III presents the experiment procedure and data collection. Section IV shows the experiment results and analysis. Finally, the conclusions are given in Section V.

II. METHODS

In this section, we will introduce a sticky HDP-HMM method, which has shown the powerful ability to model and predict driver behaviors in the case where the number of primitive driving patterns is not exactly known. In what follows, the theoretical basis is given, including hidden Markov model (HMM) and hierarchical Dirichlet processes (HDP).

A. HMM

When facing the uncertainty of driver behaviors in naturalistic settings, we can treat the entire driving process as a logic combination of primitives, and the dynamic process among primitives in driver behaviors as a probabilistic inferential process [15]. Here, driver behaviors were modeled as a dynamic process of primitives with the structure of HMM. The core of HMM consists of two layers: a layer of hidden
state and a layer of observation or emission, as shown in Fig. 2(a).

Given a time-series data sequence \( Y = \{y_t\}_{t=1}^T \) with \( Y \in \mathbb{R}^{d \times T} \) and a set of hidden state \( \mathcal{X} \), each hidden state \( x_t \) at time \( t \) will be subject to one entry of \( \mathcal{X} \), i.e., \( x_t = x_i \in \mathcal{X} \), where \( x_i \) is the \( i \)-th element in \( \mathcal{X} \). The transition probability from hidden state \( x_i \) to \( x_j \) is denoted as \( \pi_{i,j} \), and \( \pi_i = [\pi_{i,1}, \pi_{i,2}, \pi_{i,3}, \cdots] \). The observation \( y_t \) at time \( t \) given hidden state \( x_t \) is generated by \( y_t = F(y_t|x_t) \), called emission function. Therefore, the HMM can be described as

\[
\begin{align*}
x_t | x_{t-1} \sim & \pi_{x_{t-1}} \quad (1a) \\
y_t | x_t \sim & F(x_t) \quad (1b)
\end{align*}
\]

where \( F(\cdot) \) is the emission function and \( \theta_{x_t} \) is the emission parameter. Driver behavior, however, are changing and open-ended, so that the parameter space regarding hidden states in the model (i.e., the number of primitives, the size of transition matrix) becomes potentially infinite [15]. More specifically, the dimension of the set space of hidden states, \( |\mathcal{X}| \), is unknown. In such situations, we have to define a prior probability distribution on an infinite-dimensional space. A distribution on an infinite-dimensional space is a stochastic process with a specific path. Usually, the Dirichlet processes (DP) rapidly yield intractable computations. In what follows, we will introduce a hierarchical DP (HDP).

### B. HDP

We assume that the number of latent states in (1) is priorly unknown and these modes of HMM is subject to a specific distribution defined over a measure space. The Dirichlet process (DP) is a measure on measures, denoted by \( DP(\gamma, H) \), and provides a distribution over discrete probability measures with an infinite collection of atoms

\[
\begin{align*}
G_0 = & \sum_{i=1}^{\infty} \beta_i \delta_{\theta_i}, \quad \theta \sim H \quad (2a) \\
\beta_i = & \nu_i \prod_{\ell=1}^{i-1} (1 - \nu_\ell), \quad \nu_i \sim \text{Beta}(1, \gamma) \quad (2b)
\end{align*}
\]

on a parameter space \( \Theta \) that is endowed with a base measure \( H \). Here, the weights \( \beta_i \) sampled by a stick-breaking construction and we denote \( \beta \sim \text{GEM}(\gamma) \), with \( \beta = [\beta_1, \beta_2, \beta_3, \cdots] \) and \( \sum_i \beta_i = 1 \).

According to the above discussion, an HDP can be used to define a prior on the set of HMM transition probability measures \( G_{j,i} \)

\[
G_{j,i} = \sum_{i=1}^{I} \pi_{j,i} \delta_{\theta_i} \quad (3)
\]

where \( \delta_0 \) is a mass concentrated at \( \theta \). Assuming that each discrete measure \( G_j \) is a variation on a global discrete measure \( G_0 \), thus the Bayesian hierarchical specification takes \( G_j \sim \text{DP}(\alpha, G_0) \), where \( G_0 \) is draw from \( \text{DP}(\gamma, H) \):

\[
\begin{align*}
G_0 = & \sum_{i=1}^{\infty} \beta_i \delta_{\theta_i}, \quad \beta|\gamma \sim \text{GEM}(\gamma) \quad (4a) \\
G_j = & \sum_{i=1}^{\infty} \pi_{j,i} \delta_{\theta_i}, \quad \pi_j | \alpha, \beta \sim \text{DP}(\alpha, \beta) \quad (4b) \\
\theta_i | H \sim & H \quad (4c)
\end{align*}
\]

### C. Sticky HDP-HMM

For the sticky HDP-HMM(\( \gamma, \alpha, H \)), by adding an extra parameter \( \kappa > 0 \) that biases the process toward self-transition in (4b), increasing the expected probability of self-transition by an amount proportional to \( \kappa \), we can obtain

\[
\begin{align*}
\beta|\gamma \sim & \text{GEM}(\gamma) \quad (5a) \\
\pi_j | \alpha, \beta, \kappa \sim & \text{DP}(\alpha + \kappa, \alpha \beta + \kappa \delta_i), \quad i = 1, 2, \cdots \quad (5b) \\
x_t | x_{t-1} \sim & \pi_{x_{t-1}}, \quad t = 1, 2, \cdots , T \quad (5c) \\
y_t | x_t, \theta_{x_t} \sim & F(\theta_{x_t}), \quad t = 1, 2, \cdots , T \quad (5d) \\
\theta_i | H \sim & H, \quad i = 1, 2, \cdots \quad (5e)
\end{align*}
\]

where \( T \) is the data length. Note that when \( \kappa = 0 \) in (5b), the original HDP-HMM is obtained.

### D. Emission Model

The observation model is determined by the type of function \( F(\theta_i) \), which can be Gaussian emissions [16] or switch linear dynamic models (SLDSs) [17] (e.g., vector autoregressive). One main challenge with non-parametric approaches is that one must derive all the necessary expressions to properly perform inference [18]. Here, to make our algorithm tractable, we assume that observations are drawn from a Gaussian distribution like in [16]. The \( \theta_i \) is set as \( \theta_i = [\mu_i, \Sigma_i] \). Therefore, if the priors for observations and transition distributions are learned correctly, the full-conditional posteriors can be computed using Gibbs sampling method. Johnson and Willsky [19] present further details of the inference method using Gibbs sampling methods.

### III. Experiment and Data Collection

#### A. Experiment Procedure

Driving data used in this paper are extracted from the Safety Pilot Model Deployment (SPMD) database logged in Ann Arbor, Michigan. We use the equipped vehicles to run experiments and collected on-road data. The experiment vehicles are equipped with data acquisition systems and Mobileye. The road information (e.g., lane width, lane curvature) and the surrounding vehicle’s information (e.g., relative distance, relative speed) are recorded by Mobileye. The subject vehicle information such as speed, steering angle, acceleration/brake pedal position is extracted from CAN-bus signal [20]. All of the data are recorded at 10 Hz.

Drivers had an opportunity to become accustomed to the equipped vehicles. They performed casual daily trips for several months without any restrictions on or requirements for their trips, the duration of the trips, or their driving style.
Fig. 3. An example of equipped vehicles in our experiment. (a) Equipped vehicle; (b) Mobileye; (c) data collection systems.

The data processing and recording equipment were hidden from the drivers, thus avoiding the influence of recorded data on driver behavior.

B. Data Collection

In this work, we consider the driving scenarios where the ego vehicle can sense vehicles in front of it using Mobileye, as shown in Fig. 4. In order to describe the data sequence easily, we define a channel, $C_k^T$, to record the data sequence over time $t = 1, 2, \ldots, T$ from a single target car $k$, where $k = 1, \ldots, K$. Here, we set $K = 5$ since the maximum amount of target cars that Mobileye can detect in front of the ego car is 5. In each channel, the data at time $t$ recorded from each target car consists of three variables:

- $\Delta d^t_x$, relative distance (range);
- $\Delta v^t$, relative range rate (i.e., relative speed);
- $\Delta d^t_y$, lateral displacement of target cars with respect to lane boundary.

For each channel, we have $C_k^T = [\Delta d^1_x; \Delta v^1; \Delta d^1_y]^\top \in \mathbb{R}^{3 \times T}$. For all channels, we initially set all variables to zeros. When the Mobileye detect the appearance of a target car in front of the ego car, the corresponding channel was activated and then recorded data. If the target car disappears in the detection region of Mobileye, the data in this channel was set to zero again.

Fig. 4. Driving scenarios consisting of surrounding vehicles for data collection.

Note that the collected data sequence contains two types of information or events – binary and continuous:

1) For the binary event, it records the appearance and disappearance of a target car in front of the ego car. The value in $C_k$ will usually be in form of step signal, which means that the cut-in or cut-out behavior of target cars in front of the ego car can be detected.

2) In the continuous event, it records the target cars’ states, the ego car’s states (e.g., longitudinal speed, $v_x$; and acceleration, $a_x$) and their relative dynamic states (i.e., $\Delta d_x$, $\Delta v$, $\Delta d_y$) when no target car cut-in or cut-out.

In addition, we also use an additional channel, $C_0$, to record the ego vehicle’s states, with $C_0^T = [v_x^T; v_x^T] \in \mathbb{R}^{2 \times T}$. It is obvious that the channel $C_0^T$ will always record the continuous events. Totally, a data sequence $C^T$ with dimension $3K + 2$ is recorded from $K$ target cars and one ego car, i.e., $C^T = [C_0^T, \{C_k^T\}_{k=1}^K] \in \mathbb{R}^{(3K+2) \times T}$. The developed model in this paper should be able to extract primitives from not only the binary events but also the continuous events.

C. Training Procedure

We develop and test the developed models based on Johnson and Willsky’s [19] as well as Fox’s [21] previous work using Python. The hyperparameters are determined following rules:

1) We place a Gamma($\alpha, \beta$) prior on the hyperparameters $\gamma, \alpha, \kappa$ as shown in Table I, where $d$ is the dimension.

\[
\begin{array}{|c|c|c|}
\hline
\text{parameter} & \text{description} & \text{value} \\
\hline
(\alpha_a, \beta_a) & \alpha \text{ gamma prior} & (1.1) \\
(\alpha_v, \beta_v) & \gamma \text{ gamma prior} & (1.1) \\
(\alpha_x, \beta_x) & \kappa \text{ gamma prior} & (100, 1) \\
\gamma_0 & \text{IW prior degree of freedom} & d + 2 \\
\bar{S}_0 & \text{IW prior scale} & 0.75 \Sigma \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{iteration step} & \text{log-likelihood} \\
\hline
0 & 500 \\
50 & 500 \\
100 & 500 \\
\hline
\end{array}
\]

Fig. 5. The log-likelihood values with respect to the iteration steps for dealing with (a) binary events and (b) continuous events.
Fig. 6. Example of experiment results for daily driving data with five channels, $T = 100$ s, and 12 kinds of primitives. Green line: channel #0, $C^0_T$; red line: channel #1, $C^1_T$; black dot line: channel #2, $C^2_T$; and blue line: channel #3, $C^3_T$; signals in channel #4 and channel #5 are always being zero since no target car appeared in this channel for this example. Channels $C^k_T$, $k = 1, 2, 3$ consist of three variables, i.e., $\Delta d_x$, $\Delta v_x$, $\Delta d_y$. The step signal means that a target car appeared or disappeared in Mobileye’s sensing region.

In this work, the observation variables are generated from a Gaussian model and we set $d = 3K + 2$ in this work.

2) The Inverse-Wishart (IW) prior is conjugate to the Gaussian distributions, thus the hyperparameters for $\theta_i$ are taken to be from an IW with a hyper-parameter $\gamma$, i.e.,

$$\Sigma_i | n_0, S_0 \sim \text{IW}(n_0, S_0)$$

where $n_0$ is IW prior degree of freedom and $S_0 = 0.75\Sigma$ is the IW prior scale with the covariance ($\Sigma$) of the observed data.

In this work, the observation variables are generated from a Gaussian model and we set $\mu_i = 0$ according to [12]. For the case of binary events, we test and evaluate the method performance in the daily traffic scenarios where the appearance/disappearance of target cars will be involved. For the case of continuous events, we evaluate the method performance in the primitives extracted from daily traffic scenarios. Fig. 5 gives the log-likelihood of learning results with respect to the iteration steps for dealing with binary and continuous events.

IV. RESULTS AND ANALYSIS

For the developed method, we will evaluate its utility based on the ability to extract primitive from binary and continuous events in time-series sequences.

A. Binary Event

Regarding the binary event, we evaluate the utility by checking whether the proposed method can detect the appearance and disappearance of heading target cars. The ground truth can be obtained from the changes of target cars’ label. Fig. 6 shows an example of the learning primitive extraction
results. The fact that the signal step points (i.e., appearance/disappearance of target cars) are detected indicates that the sticky HDP-HMM extracts traffic scenario primitives from a high dimensional (in this paper, the dimension of data sequence is $d = 17$) with different types of variables, though few points are not extracted such as at time $t = 31.6$ s and $t = 70.6$ s. Also, the developed approach can cluster the primitives possessing the same attributes, that is, primitives with the same color have been assigned to the same label. For the $100s$-data sequence, we finally obtained 12 primitives.

B. Continuous Event

There is no ground truth for the primitives extracted from continuous variables, that is, we would not subjectively and manually define the length of each primitive. Take speed for example, we would not empirically set a subjective threshold to segment speed profiles due to the variances in the speed profile among drivers [22]. Here, we learn the primitives using the sticky HDP-HMM, which can automatically find the primitive edges and then assign labels to each primitive. Fig. 7 presents an example of primitive extraction results for multidimensional continuous variables without step signal. We note that the sticky HDP-HMM can automatically learn primitives and assign the primitives with similar attributes to the same cluster, labeled with the same color.

C. Statistical Results for Primitives

In order to show the utility of the sticky HDP-HMM, we also give the statistical results for primitives. Primitives with the same color have been labeled to the same ID, called primitive ID. Fig. 8 presents the statistical results of learned primitives for one driver, with 353 primitives in total and 22 kinds of primitives. The horizontal axis is the primitive ID and the vertical axis is the percentage of each primitive. We note that the sticky HDP-HMM can automatically extract traffic scenario primitives from multiscale traffic database and then assign primitives endowing with the same attributes to one cluster. Each primitive ID indicates a primitive set which consists of a varying number of primitives. Table II lists the experiment results of one day driving data for ten drivers with a high dimension at $d = 14$ and $d = 17$. The experiment results demonstrate that the introduced nonparametric Bayesian learning method can be applied to high-dimension and large time-scale data sequences.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed a new framework to generate an infinite number of new traffic scenarios with a handful of limited raw traffic data, consisting of four steps: primitive extraction, learning primitive sets, topology modeling between primitive sets, and generate new traffic scenarios using primitives. To achieve this, we introduced a nonparametric Bayesian learning method to deal with the challenges in the first two steps, i.e., extracting primitives from multiscale traffic scenarios, where the binary and continuous events are both involved, and obtain the object sets. The experiment results show that the introduced method can automatically obtain the primitives for binary events that encompass distinct primitive edges and also segment continuous events being without recognizable primitive edges. Also, this approach
can also automatically cluster the primitives. The introduced nonparametric Bayesian learning approach enables one to extract primitives from a huge amount of multiscale time-series traffic data in a low cost of time and resources.

This paper shows approaches to deal with the challenges in the first two steps for generating an infinite number of new traffic scenarios. Also, the primitive extraction can be used to analyze, model, and predict driver behaviors [12], [13]. The propose framework in Fig. 1 can also be applied to robotics or human behavior analysis. In our future work, we will focus on how to model the dynamic topology among primitives and how to generate new traffic scenarios using primitives.

**APPENDIX A**

Ten released databases are listed as follows:

- KITTI Vision Benchmark Suite\(^1\)
- Vision for Intelligent Vehicles and Applications\(^2\)
- Oxford RobotCar Dataset\(^3\)
- The University of Michigan North Campus Long-Term Vision and LI DAR Datasets\(^4\)
- DIPL EC S Autonomous Driving Datasets\(^5\)
- V elodyne SLAM Dataset\(^6\)
- SY NTHIA Dataset\(^7\)
- Daimler Urban Segmentation Dataset\(^8\)
- MIT Age Lab\(^9\)
- MOLP dataset\(^10\)
- The UAH-DriveSet\(^11\)

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