Rotation, Statistical Dynamics and Kinematics of Globular Clusters

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Abstract. Evolution with mass segregation and the evolution of the rotation of cores are both discussed for self-similar core collapse. Evolution with $\Omega_0 \propto \rho_0^{1/2}$ is predicted. On the Dynamical Main Sequence of globular clusters the energy emission from binaries balances the energy expended in expanding the halo.

Newton’s exactly solved N-body problem is then given, along with recent generalisations, all of which have no violent relaxation, but a new type of statistical equilibrium is discussed.

Finally, we set the creation of streams in the Galaxy’s halo in the historical context of their discovery.

1. Introduction

When, over ten years ago, the main focus of my work moved away from stellar dynamics, three outstanding problems were left unsolved and my efforts to interest students in these have not borne fruit. In the hope that others may be stimulated these questions are posed here.

Q1. Is it possible to have self-similar core collapse when there is a continuous distribution of stellar masses?

Q2. For a weakly rotating cluster is there a self-similar collapse of the core and how does its rotation evolve as the core radius, $r_c$, decreases?

Of course question 2 can be tackled for equal mass stars and then one can make it more realistic by combining both questions together in Q1 + Q2.

Q3. It is now widely accepted that just as in stars the energy generated by nuclear reactions delays thermal evolution, so in star clusters the energy generated by binary stars delays core-collapse. Eddington’s theory of the Main Sequence led to many observational consequences. Can we make a theory of the Dynamical Main Sequence of globular clusters with real predictive power as to the form these clusters should take at equilibrium?

Section 2 outlines possible solutions but full solutions involve multidimensional problems in four or five dimensions (e.g., Energy, Angular Momentum, Time and Mass).

Some years ago Inagaki and I (1990) showed that such problems could be tackled analytically using trial functions and a Local Variational Principle. I still think that method should have a great future in these problems but those wishing
to use it should contact Takahashi (1992,3,5,6) who developed this approach numerically for 3 dimensional problems, but has since turned to another method.

Since few workers in N-body dynamics have even heard of Newton’s exact solution of an N-body problem for all N and all initial conditions, it is given in section 3 along with recent developments of this idea. These problems lead to some pretty dynamics which illustrate that there are exceptions with no violent relaxation and that thermodynamic equilibria can exist in a system undergoing a macroscopic radial pulsation or expansion.

Finally in section 4 we discuss the history of the discovery of tidal streams and give the methods recently invented for finding new ones. Once some globular cluster extensions or proper motions are reliably determined these methods will become far more powerful.

2. Core Collapse, Mass-Segregation, Rotation and the Dynamical Main Sequence

Much of globular cluster evolution follows from careful dimensional analysis, see Inagaki & Lynden-Bell (1983). Returning to our Question 1 we look for a self-similar solution with the density in the core and inner halo of the form

$$\rho(r,t) = \rho_0(t)\rho_\ast(r_\ast) \quad \text{where} \quad r_\ast = r/r_c(t)$$

Here $r_c(t)$ is the core’s radius.

Since the relaxation is far more rapid in the core than in the outer parts we may set $\partial/\partial t \approx 0$ for large $r$.

$$0 = (\partial\ln\rho/\partial t)_r = d\ln\rho_0/dt - (d\ln\rho_\ast/dr_\ast)d\ln r_\ast c/dt$$

Therefore for large $r$

$$d\ln\rho_0/d\ln r_\ast = d\ln\rho_\ast/d\ln r_\ast = -\alpha$$

Since the expression on the left is a function of $t$ alone, while that on the right is a function of $r_\ast$ alone, it follows that both are constant (at a value we have called $-\alpha$). Thus for all time

$$\rho_0 = Ar_c^{-\alpha}$$

(1)

and at large $r_\ast$, $\rho_\ast = r_\ast^{-\alpha}$.

The evolution of the core is due to the relaxation so

$$d\ln\rho_0/dt = K_1/T_c$$

(2)

where $K_1$ is a dimensionless constant and $T_c$ is the relaxation time in the core. A standard evaluation of $T_c$ gives for a velocity dispersion $\sigma$, Binney & Tremaine (1987)

$$T_c^{-1} = 3G^2 m \rho ln \Lambda /\sigma^3$$

(3)

and we would like to evaluate this for the core when there is a distribution of stellar masses. It would seem natural to take the distribution function to be an
equilibrium Maxwell-Boltzmann there proportional to $\exp \left[ -\beta m \left( \frac{1}{2} v^2 - \psi_0 \right) \right]$, but this cannot be true at energies close to the escape energy. The closeness of the escape energy is well emphasised by the following argument from the Virial Theorem.

$$2T = \sum_I m_I v_I^2 = -V = \frac{1}{2} \sum_I m_I \psi_I$$

(4)

Now $\psi_I = \frac{1}{2} v_e^2$ where $v_e$ is the velocity of escape to infinity, hence for any cluster obeying the virial theorem

$$\langle m v^2 \rangle = \frac{1}{4} \langle m v_e^2 \rangle .$$

(5)

So that the mass weighted rms velocity is HALF the mass weighted rms velocity of escape. At a thermodynamic equilibrium we have equipartition of $\frac{1}{2} m v^2$ so the above expression implies that stars with less than a quarter of the mean mass have an rms velocity equal to the escape velocity. This cannot occur in practice. Thus the equipartition of the Maxwell-Boltzmann distribution must be modified and as Michie (1963) first showed the lowered Maxwellian of the Michie-King models [evaluated in full by King (1966)] does this approximately viz

$$f = B(m) \left\{ \exp \left[ -\beta m \left( \frac{1}{2} v^2 - \psi \right) \right] - \exp (\beta m \psi_e) \right\}$$

(6)

Although in well developed core collapse such models have insufficient temperature contrast between the centre and the halo, Lynden-Bell & Wood (1968), Lynden-Bell & Eggleton (1980), nevertheless we shall adopt such a form for the low energy stars in the core (for which relaxation is the most rapid). If $F(m)dm$ gives the number density of stars at mass $m$ in the core, so that

$$\rho_0 = \int m F(m)dm$$

(7)

then we may re-express the relaxation rate in terms of the central ‘temperature’ $\beta^{-1} = \sigma^2/m$ and obtain from (3)

$$T_c^{-1} = 3 G^2 \beta^{3/2} \rho_0 \ell n \langle m^{7/2} \rangle / \langle m \rangle$$

(8)

where

$$\langle m^{7/2} \rangle = \int m^{7/2} F(m)dm / \int F(m)dm$$

(9)

Evidently the relaxation rate, $T_c^{-1}$, which determines the overall evolution of the cluster, depends on the mean seven halves power of the mass, evaluated over the core of the cluster. As evolution proceeds, the lighter stars in the core are preferentially expelled from it but simultaneously the heavy stars are gradually eliminated via stellar evolution. Before using (8) in (2) we need to re-express $\beta$ in terms of $\rho_0, r_c$ and $\langle m \rangle$. If $M_c = \frac{4}{3} \pi \rho_0 r_c^3$ is the core mass then

$$3 \sigma^2 = 3 (\beta \langle m \rangle)^{-1} = GM_c / r_c = \frac{4\pi}{3} G \rho_0 r_c^2$$

(10)
where we have chosen to define $r_c$ so that the constant of proportion is one.

$$T_c^{-1} = K_2G^{1/2} \rho_0^{-1/2} r_c^{-3} m_c = K_2G^{1/2} A^{-1/2} m_c(\rho_0/A)^{(6-\alpha)/(2\alpha)}$$  \hfill (11)

where we have used (1) to express $r_c$ in terms of $\rho_0$, $K_2$ is the dimensionless constant $(\frac{81}{8\pi/2}n\Lambda)$ and $m_c(t) = \langle m^{7/2} \rangle / \langle m \rangle^{5/2}$. To use expression (11) to solve (2) we must first evaluate the time dependence of $m_c$. This is caused by two separate effects, the progressive expulsion of lower mass stars from the core increases $m_c$ but it is mainly dependent on the high masses and they diminish steadily as stellar evolution and stellar death take their toll. With such a high power of the mass involved it is likely that stellar evolution plays the dominant role with $m_c$ behaving similarly to the age cut off. This suggests the approximate form

$$m_c = m_\odot (t/t_\odot)^{\delta-1} \text{ where } \delta \simeq 2/3, \ t_\odot \simeq 10^{10}\text{yr}, \ t > 10^6\text{yr}$$

and $t$ is time measured from the time the cluster was created. Inserting this into (11) and (11) into (2) the resulting equation for $\rho_0$ is readily solved to give

$$\rho_0 = C|t_0^\delta - t^\delta|^{-2\alpha/(6-\alpha)}$$  \hfill (12)

and therefore

$$r_c = (A/C)^{1/\alpha} |t_c^\delta - t^\delta|^{2/(6-\alpha)}$$  \hfill (13)

where $t_c$ is a constant of integration which gives the time of core collapse and the constant $C$ is

$$C = A \left[ \frac{6-\alpha}{2\alpha\delta} K_2G^{1/2} A^{-1/2} m_\odot t_\odot^{1-\delta} \right]^{-2\alpha/(6-\alpha)}.$$ 

As in Inagaki & Lynden-Bell (1983), we expect (12) and (13) to hold also after core collapse but then $\rho_0$ becomes a characteristic density rather than the central one which remains infinite if we ignore binaries and the giant gravothermal oscillations.

Of course in the absence of stellar evolution $m_c$ will increase and the parameterisation

$$m_c = m_\odot (t/T_c)^\delta$$

might then be more appropriate but though such a model is soluble I shall not detail it here.

Expressions (12) and (13) can only be considered the solution to our problem once $\alpha$ is known. As Lynden-Bell and Eggleton showed $\alpha$ emerges as an eigen value in the full theory and for the equal mass case $\alpha = 2.22$. It is readily seen that $\alpha$ must lie between the equal mass isothermal sphere with $\alpha = 2$ and the limiting case with infinite core binding energy $\alpha = 5/2$. With the heavy masses more concentrated to the core one may expect $2.22 < \alpha < 2.5$ and perhaps toward the upper end of that range, on the other hand once most of the lighter stars have been ejected from the core one expects the evolution to revert toward the single mass case. Our best guess is therefore

$$\alpha = 2.3 \pm 0.08.$$  \hfill (14)
In discussing Question 2 we note that the flattening of a cluster is of second order in \( \Omega \) the angular velocity and may be neglected to first order. In any potential of the form

\[
\psi = \psi_0(r) + r^{-2}\psi_2(\theta)
\]

(15)

the expression \( I = \frac{1}{2}J^2 - \psi_2 \) is an exact integral of the motion where \( \mathbf{J} = (r \times \mathbf{v}) \). In practice any non-spherical part of \( \psi \) is second order in \( \Omega \) and the motion of a star is well approximated as lying in a precessing plane. The rate of precession can be worked out from the couple that the non-spherical potential exerts on the unperturbed rosette orbit that the star would have in the absence of asphericity. Thus in practice \( |\mathbf{J}| \) is almost an integral of the motion while \( J_z \) and the energy, \( \epsilon \), are exact ones in the absence of evolution. In practice it is convenient to use the radial action \( J_r = \frac{1}{2\pi} \int \sqrt{2\epsilon + \psi_0(r)} - J^2/r^2 dr \) in place of \( \epsilon \). In potentials of the form (15) it is an exact integral but for more general forms of \( \psi \) it is only approximate. A particular advantage of using \( |\mathbf{J}|, J_r, J_z \) is that they are adiabatically invariant for slow changes in the potential \( \psi \). Thus if we express the distribution function of a rotating cluster in the form \( f = f(|\mathbf{J}|, J_r, J_z, t) \) where \( f \) evolves little in one orbital time, then \( \partial f / \partial t \) is \( (\partial f / \partial t) \) encounters and there is no supplementary term due to the change of global potential induced by these encounters since for such changes the \( J \) (although not \( \epsilon \)) are adiabatically invariant. The \( J \) have dimensions of \( \sigma r_c \) and \( f \) has the dimensions of \( \rho_0 \sigma^{-3} \), so in self-similar evolution the evolving \( f \) will have the form

\[
f = \rho_0 \sigma^{-3} F_* (|\mathbf{J}|, J_r^*, J_z^*)
\]

(16)

where \( F_* \) is a dimensionless function of its dimensionless arguments \( |\mathbf{J}^*| = |\mathbf{J}|/\sigma r_c, J_r^* = J_r/(\sigma r_c), J_z^* = J_z/(\sigma r_c) \). As before \( \rho_0 = Avr_c^{-\alpha} \) so \( \sigma r_c \propto r_c^{2-\alpha/2} \). These same principles may be applied to discuss the way cluster core rotation evolves as the core contracts. Near the centre, relaxation is quite fast so we expect a rotating Maxwellian at energies little affected by escape

\[
f = B \exp -\sigma^{-2}(\epsilon - \Omega_0 J_z) .
\]

(17)

Our question is how does \( \Omega_0 \) evolve during core collapse. In self-similar evolution the quantities that are dimensionless do not evolve, so \( \Omega_0 r_c/\sigma = \text{const} \). Hence as the core shrinks we have our prime result

\[
\Omega_0 \propto r_c^{-\alpha/2} \quad \alpha \simeq 2.22 .
\]

(18)

So as \( r_c \) shrinks, the core should rotate faster and faster. This result is very sensibly between the \( \Omega_0 r_c^2 = \text{const} \) that would follow if the heat flowed out of the core much more readily than angular momentum, and the \( \Omega = \text{const} \) that would follow if the opposite were true. Away from the centre it is plausible that the system remembers the rotation it had when that part of the inner halo was part of the core; in reality it gains a little more angular momentum as it leaves the remaining core, because the remaining core loses it. This suggests the behaviour \( \Omega \sim r^{-\alpha/2} \) but since \( \Omega^2 \) and \( G\rho^2 \) have the same dimensions a better prediction is that \( \Omega \) behaves approximately as \( \rho^{1/2} \); hence we conclude that cluster cores and inner haloes rotate as

\[
\Omega = \Omega_0 (\rho/\rho_0)^{1/2} = \Omega_0 \rho^{1/2}_c
\]

(19)
with $\Omega_0$ evolving according to (18). For $Q_1 + Q_2$ the same results will hold with $\alpha$ given by (14). Such predictions should be compared with the numerical work of Spurzem (2000) and the rapidly growing data from observations. A further consequence of (19) is that $\Omega_2/\pi G \rho$ which determines the flattening will be constant in the core and inner halo thus the isophotes there should have $b/a$ constant whereas if the core evolved with $\Omega_0 \propto r^{-2}$ the central isophotes would be more flattened. By contrast a uniformly rotating cluster has the greatest flattening in the outer isophotes. Turning now to Question 3 the theory of the Dynamical Main Sequence of Globular Clusters as yet awaits someone brave enough to make strong hypotheses such as ‘The only binaries that matter are in the core and the energy flux through the inner halo is constant and drives either the escape from the cluster or the expansion of its halo.’ Of course if core collapse ceases due to binaries then the steady core will relax to uniform rotation which will gradually extend from the core outwards.

3. Newton’s N-body Problem and its Generalisations

Let the force on body $I$ due to body $J$ be of the form

$$F_{IJ} = km_I m_J (r_J - r_I)$$

(20)

We sum over all $J$ to get the force on $I$ since the $I = J$ term is zero.

$$F_I = km_I \sum m_J r_J - Mr_I = km_I M (\bar{r} - r_I)$$

where $M$ is the total mass. Thus the total force is directed to the centre of mass $\bar{r}$ and is proportional to the distance from it. Newton’s (1687) general solution is that each particle moves in a central ellipse about the centre of mass which itself moves uniformly in a straight line. Newton’s system has a total potential energy

$$V + \frac{1}{2} KM^2 r^2$$

(21)

where

$$r^2 = M^{-1} \sum m_I (r_J - \bar{r})^2$$

(22)

The Virial theorem reads

$$\frac{1}{2} \dot{I} = 2T + n V$$

(23)

where $V \propto r^{-n}$ so $n = -2$ for the above system. To generalise, Newton’s result consider systems with total potential energies of the form

$$V = V(r)$$

with $r$ given by (22). This problem has a singular beauty if we consider the 3N dimensional space in which the $X_I = \sqrt{m_I}(r_I - \bar{r})$ are the coordinates. Letting the 3N vector $r = (x_1, x_2, \ldots, x_N)$ we note that $r^2$ is given by (22). The initial conditions in this space are the initial values of $r$ and $\dot{r}$. The accelerations in this space are central since $V = V(r)$ so the acceleration also lies in the plane defined by the initial $r$ and $\dot{r}$. Thus the motion continues in that plane. In fact
the whole problem now reduces to the planar orbit problem under the action of the central potential $V(r)$. Back in 3 space each particle feels the central force $V'(r)m_1M^{-1}(r_j - \bar{r})/r$ so each particle orbits in a plane. Its motion may be obtained by projecting the motion of the representative point in 3N down into the plane of the motion of particle I. As $r$ changes periodically around the planar orbit in 3N space it follows that $Mr^2$ vibrates forever so there is no violent relaxation to a Virial equilibrium. One may generalise this result still further by taking $V = V_0(r) + r^{-2}V_2(r/r)$ then the Virial theorem reads
\[ \frac{1}{2}Md^2r^2/dt^2 = 2T - rV'_0(r) + 2V_2 = 2E - 2V_0 - rV'_0 \]
since this last expression does not involve $V_2$ it follows that $r$ vibrates in the same way as it did when $V_2$ was 0. Hence once again there is no violent relaxation. Pretty N-body problems of this type are given by the forces
\[ F_{IJ} = Gm_im_j(r_J - r_I) \left[ \frac{1}{r^3} - \frac{k}{(r_J - r_I)^4} \right] \]
where $r$ is given by (22) or alternatively
\[ F_{IJ} = G'm_im_j(r_J - r_I) \left[ 1 - k(r_J - r_I)^{-4} \right] \]
both of these force laws have long range attractions and short range repulsions so they may produce liquid-like and solid-like phases. In each case the vibration in $r$ separates from the other motions so if lattices are possible, lattices with a breathing pulsation will be too. It is possible to do a statistical mechanics of the ‘angular’ motions only while $r$ remains breathing but for details the reader should refer to Lynden-Bell & Lynden-Bell (1999a). The 1999b paper solves Schrodinger’s equation for both spin 0 Bosons and spin $\frac{1}{2}$ Fermions giving the only known non-trivial three dimensional N-body solutions for interacting Bose or Fermi gases. The degenerate Fermi case gives a white-dwarf-like solution.

4. Streams 1950-2000

Streams about the Galaxy have much in common with the meteor streams left behind by comets in the solar system which were discovered earlier. However the galactic orbits are rosettes rather than ellipses and their planes precess about the galactic pole due to the aspherical potential. Bertil Lindblad (1958)(1961) was one of the first to consider the theory of streams spreading from a common origin but already Eggen (1959) (1989) was on the march finding moving groups in the velocity space among both disc and halo stars. As data were refined Eggen changed his mind about the central velocities of some groups and this led to consequent changes in their membership which only increased scepticism among his critics. However Eggen was convinced he was onto something and claimed that it was much easier for others to criticise the inclusion of a star when new data showed its membership to be unlikely, than to discover the group to start with.

The subject of streams really come to life with the discovery of the Magellanic Stream stretching more than 120° around the galaxy. But this discovery
was the culmination of a sequence. In 1965 Nan Dieter discovered a high velocity cloud at the South Galactic Pole. Soon afterward Hulsbosch & Raimond (1966) published their survey of such objects and showed that cloud to be strongly elongated. It was the technological innovation of Wrixon’s low noise diodes that allowed Wannier & Wrixon (1972) to see fainter 21 cm emission. Their great arc of 21 cm emission through Dieter’s cloud showed a systematic variation of radial velocity by several hundred km/sec across the sky. At Herstmonceux Kalnajs (1972) excitedly drew my attention to their paper claiming that it must be of Galactic scale. A day or two later he discovered that both the line of the arc and the radial velocity extrapolated to the Magellanic Clouds. Mathewson too was very excited by this discovery and no sooner had he got back to Australia than he, Cleary & Murray (1974) started delineating how the stream joined the Magellanic Clouds. A few years later during a non-photometric night at Sutherland I was imagining how the Magellanic Stream lay across the sky when it struck me that other satellites of the Milky Way might be members of streams and might likewise have high velocity clouds associated with them. I rapidly found that Draco & Ursa Minor are almost antipodal to the LMC and the SMC in the galactocentric sky. This meant that they too might be long lost members of the Magellanic Stream, Lynden-Bell (1976). Kunkel & Demers (1977) noticed the dwarf spheroidals were distributed non-randomly over the sky but their compromise plane was some 30° from the Magellanic Stream. In their attempt to explain the antipody of Draco & Ursa Minor, Hunter & Tremaine (1977) noticed that their elongations lay along the stream while I searched for further streams using elongations as a supplementary guide, Lynden-Bell (1982); however it was not until 1995 that this developed into a systematic method. Since tidal tearing occurs essentially in the plane of orbit, one needs to find objects scattered along a great circle in the galactocentric sky. Such great circles are hard to see in any of the projections of the sky so we need a good method of finding them. An object may be associated with the set of all great circles that pass through it. The poles of these great circles lie on another great circle which we call a polar path. (The pole of that great circle lies of course at the object we started with). The great circle through any two objects has its pole at the intersection of their polar paths. If 3 objects lie on a great circle all 3 polar paths will go through the same point. Thus we may find objects with common great circles by plotting their polar paths (in any equal area projection) and looking for multiple intersections. Figure 1a which plots 45 polar paths for dwarf spheroidals and outer globular clusters shows a result that more closely resembles a ball of wool than a useful scientific plot. More information is needed to get any definitive groupings. A known proper motion would reduce the polar path of an object to a very short segment determined by the directional error of the proper motion. If we are prepared to assume that the elongation of an object is along its orbit, as appears to be true for Draco & Ursa Minor and might be true for the outer extensions recently found around globulars (e.g., Grillmair et. al., Meylan this volume) then likewise the pole must lie in only a small sector of the polar path (see Figure 1b). But great circles are not enough. Radial velocities provide a discriminant. If all the objects in a stream follow the orbit of the progenitor then they will share the same specific energy $E$ and specific angular momentum $h$. 
Equal area projection of the sky b > -12 deg. Polar paths of all 45 objects do not by themselves show obvious streams. 

\[ X = \sqrt{1 - \sin b} \cos \ell, \quad Y = \sqrt{1 - \sin b} \sin \ell \quad \text{where} \quad (\ell, b) \quad \text{are galactocentric galactic coordinates.} \]

Figure 1. a & b

As Figure 1a but with only distant objects plotted and those portions of the polar paths perpendicular to the major axes of the objects emphasised.

Figure 2. The radial energy plotted against \( r_g^{-2} \). The gradient fives \(-h^2/2\) where \( h \) is the specific angular momentum of the stream. From this the predicted proper motions are deduced (see L-B) 1995.
Taking sets of objects that share a common polar intersection and therefore lie on a galactocentric great circle one may plot \( E_r = E - \frac{1}{2}h^2r^{-2} = \frac{1}{2}v_r^2 - \psi(r) \) against \( r^{-2} \) the inverse square of their galactocentric distances. This should be a straight line so \( E \) and \( h^2 \) can be deduced as the intercept and gradient of such a line. In practice the observed radial velocity is not the galactocentric one so \( v_r \) can be deduced by estimating an initial \( h \) and then iterating to find a consistent gradient of \( -\frac{1}{2}h^2 \). In practice this method converges rapidly but needless to say some of the objects do not lie on straight lines and are interlopers rather than stream members. Since the method gives \( h^2 \) from the observed radial velocities, and since the poles of the great circle involved are known, we have used these methods not only to produce possible streams among the dwarf spheroidals and outer globulars but also to predict their proper motions. The method works well for the Magellanic Stream and for the Sagittarius Stream which was discovered by Ibata, Gilmore & Irwin (1995). There is now great hope that we can identify orbital streams associated with the later merging events emphasised by Searle and Zinn. Further work may take us back to the orbits of objects that took part in the initial formation itself. The ELS (1962) picture had the bulk of the metal poor globulars formed in the fragmentation of the initial collapse, but following Ambartsumian we know that most of the stars would have been formed in looser aggregates and stellar associations which subsequently broke up and spread around the Galaxy. The idea that ELS pictured a perfectly smooth collapse is not in their paper but has been added by others who have not read it. Eggen (1959) had already worked on streams in the halo, the fragmentation of collapsing systems was treated by Hunter (1962), and I was working on the Large Scale Instabilities of shape that would make the collapse non-axis symmetric, Lynden-Bell (1964). The non-linear behaviour of these instabilities was later treated in the simple non-rotating case by Lin, Mestel & Shu (1965). Those who want to read for themselves what was thought at that time would do well to read my paper to IAU Symposium No. 31 on the Formation of the Galaxy, Lynden-Bell (1967). Streams from the initial collapse are less likely to be preserved than the younger streams from late additions, but finding correlated stellar motions from the initial creation of the Galaxy was what ELS was all about and their method of using abundances and adiabatic invariants as fossilised evidence on how the Galaxy formed is probably its most lasting legacy to astronomy. The merging of galaxies was suggested as important in the very percipient paper on bridges and streams by the Toomre's (1972) and White's (1976) demonstration of fragmentation of a uniform overdensity showed that even within the free fall time of the whole much fragmentation and subsequent merging of those fragments occurs.

Returning to streams there is too much modern work to detail all of it, but work by Johnston (1998) holds out the prospect of more accurate determination of the Galaxy’s potential using streams of stars just escaping from a satellite galaxy. By contrast, Helmi & White (1999) take such stars to be emitted with a not inconsiderable velocity dispersion so such streams would be harder to detect and would define the potential less accurately. It is not yet clear to me which view is more realistic. Lastly, Putman et. al. have at last detected a forward stream from the Magellanic Clouds, albeit somewhat skew of the main great circle. Currently the most plausible explanation is that it was gas around the
SMC that was torn but much of the forward stream ran into the LMC and the rest was diverted by it, however other interpretations are possible.

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