Pionic susceptibility for charged pions in asymmetric nuclei

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At low energies the particle-hole ($NN^{-1}$) part of the pionic susceptibility in isospin-symmetric nuclear matter is known to behave very differently from the susceptibility in finite nuclei due to the presence of an energy gap in the $NN^{-1}$ excitation spectrum. In this note we show that for charged pions in $N \neq Z$ nuclei the changes due to the gap are very similar to those in the symmetric case, except at very low momenta, where a qualitatively different behavior is found.

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1 In several applications [1, 2] the polarization function of a nuclear medium is needed for very low energy transfer $\omega \approx 0$. As an example, we will consider here the pionic susceptibility $\chi(\omega, k)$ [3], but our discussion applies likewise to other isovector channels. For finite nuclei, $\chi(\omega, k)$ is often calculated in a local-density approximation using results from the homogenous medium. In this case the particle-hole ($NN^{-1}$) contribution, which is dominant at low energies, is described by the Lindhard function. However, when applied to finite nuclei, this approximation is justified only if the energy $\omega$ is much larger than the lowest-lying $NN^{-1}$ excitation energy. The modifications due to this energy gap in the excitation spectrum were discussed in Ref. [1] for the isospin-symmetric case and are summarized below. In this note we will focus on the isospin-asymmetric case.

For charged pions in isospin-asymmetric matter the original Lindhard function [4, 5] cannot be used. Instead, it is convenient to split the Lindhard function into direct and crossed contributions. We define

$$\phi_{ab^{-1}}(\omega, k) = 2 \int \frac{d^3p}{(2\pi)^3} \frac{\theta(p_F^{(b)} - p)\theta(|p + k| - p_F^{(a)})}{\omega - k^2/2m_n + i\varepsilon},$$

where $a$ and $b$ indicate neutron ($n$) or proton ($p$), $p_F^{(a)}$ and $p_F^{(b)}$ denote the corresponding Fermi momenta, and $m_n$ is the nucleon mass. For completeness, the explicit expression for $\phi_{ab^{-1}}$ is given in the appendix. Using this definition, e.g., the non-interacting (Fermi-gas) susceptibility for a $\pi^+$ is given by

$$\chi^{(0)}_{\pi^+}(\omega, k) = 2f^2 \left[ \phi_{pn^{-1}}(\omega, k) + \phi_{np^{-1}}(-\omega, k) \right],$$

where $f$ denotes the $\pi NN$ coupling constant. The susceptibility for a $\pi^-$ can be obtained by replacing $\omega \leftrightarrow -\omega$. In realistic cases one has to include corrections from the short-range $NN$ correlations (Ericson-Ericson Lorentz-Lorenz effect [3]) which lead to an RPA resummation of $\chi^{(0)}_{\pi^+}$.

In symmetric matter the indices $a$ and $b$ can be omitted. In this case it is obvious from Eq. (1) that, for $\omega \neq 0$, the function $\phi$ fulfills $\phi(\omega \neq 0, k \rightarrow 0) = 0$ as a consequence of Pauli blocking, whereas for $\omega = 0$ one finds $\phi(\omega = 0, k \rightarrow 0) = -m_F/(2\pi^2)$. The reason for this behavior is that in the latter case both numerator and denominator of Eq. (1) vanish. However, as pointed out in Ref. [1], this results in a bad approximation for the susceptibility of a finite nucleus. Due to the finite excitation energy of the lowest-lying $NN^{-1}$ excitation, the denominator cannot vanish for $\omega = 0$. As a consequence the $NN^{-1}$ part of the susceptibility of a finite nucleus must go to zero as $k \rightarrow 0$. Following Ref. [1], this effect is easily included by adding an energy gap $\Delta$ to the $NN^{-1}$ excitation energy $(k^2 + 2k \cdot p)/(2m)$. Then the pionic susceptibility can be written as

$$\chi^{(0)}_{\pi^+}(\omega, k) = 2f^2 \frac{\Delta^2}{m^2} \left[ \phi(\omega - \Delta, k) + \phi(-\omega - \Delta, k) \right].$$

The drastic effect of this modification can be seen from the dashed lines in Fig. 1 which are in agreement with Fig. 2 of Ref. [1]. The short-dashed line corresponds to the usual Lindhard function, Eq. (2), whereas the long-dashed line represents the Lindhard function with a gap, Eq. (3).

We now return to the asymmetric case. To be specific, we will discuss the case of neutron-rich matter, i.e., $p_F^{(n)} > p_F^{(p)}$. In this case $\phi_{pm^{-1}}(\omega, k)$ is non-zero for all values of $\omega$ and $k$, since Pauli blocking can never be complete. This is in contrast to $\phi_{np^{-1}}(\omega, k)$, which is identically zero due to Pauli blocking for all momenta $k \leq p_F^{(n)} - p_F^{(p)}$. It should also be noted that $\phi_{pm^{-1}}(\omega = 0, k)$ and thus also $\chi^{(0)}_{\pi^+}(\omega = 0, k)$ are complex for $k < 2p_F^{(n)}$.

The previous discussion is valid for infinite matter which is adequate e.g., for the interior of a neutron star. In view of the strong modifications in the symmetric case due to the energy gap in finite nuclei, it is interesting to include similar corrections also in the asymmetric case. We will consider two modifications:

1. A neutron star is stabilized against $\beta$ decay by the Fermi energy of the electrons, $\epsilon_F^{(e)} = \epsilon_F^{(n)} - \epsilon_F^{(p)}$.  

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In fact, over a wide range of momenta it seems to be a very accurate approximation to neglect the asymmetry completely. However, at momenta below $\approx p_F^{(a)} - p_F^{(p)}$ the susceptibility stays almost constant in the asymmetric case (solid line), while it goes to zero for $k \to 0$ in the symmetric case (long-dashed line). As mentioned above, this qualitative difference stems from the fact that in the asymmetric case Pauli blocking can never be complete.

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APPENDIX: EXPLICIT EXPRESSIONS

In the explicit expression for the function $\phi_{ab-1}(\omega, k)$ defined in Eq. (1) four different cases must be distinguished:

(a) $k \geq p_F^{(a)} + p_F^{(b)}$ (no Pauli blocking):

$$\phi_{ab-1}(\omega, k) = f_{ab-1}^{(a)}(\omega, k, -p_F^{(a)}, -p_F^{(b)}) , \quad (A.1)$$

(b) $|p_F^{(a)} - p_F^{(b)}| \leq k < p_F^{(a)} + p_F^{(b)}$:

$$\phi_{ab-1}(\omega, k) = f_{ab-1}^{(a)}(\omega, k, -p_F^{(a)}, -p_F^{(b)}) + f_{ab-1}^{(b)}(\omega, k, p_F^{(a)}, -k) , \quad (A.2)$$

(c) $k < |p_F^{(a)} - p_F^{(b)}|$ and $p_F^{(a)} \leq p_F^{(b)}$:

$$\phi_{ab-1}(\omega, k) = f_{ab-1}^{(a)}(\omega, k, -p_F^{(a)}, -p_F^{(a)} - k) + f_{ab-1}^{(r)}(\omega, k, -p_F^{(a)} - k, p_F^{(b)} - k) + f_{ab-1}^{(r)}(\omega, k, p_F^{(a)} - k, p_F^{(b)}) , \quad (A.3)$$

(d) $k < |p_F^{(a)} - p_F^{(b)}|$ and $p_F^{(a)} > p_F^{(b)}$ (complete Pauli blocking):

$$\phi_{ab-1}(\omega, k) = 0 . \quad (A.4)$$

In these equations $f_{ab-1}^{(a)}$ and $f_{ab-1}^{(r)}$ denote the integrals over $p_{\|}$ and $p_{\perp}$ in the regions of $p_{\|}$ where the integration over $p_{\perp}$ is unrestricted or restricted by Pauli blocking, respectively:

$$f_{ab-1}^{(a)}(\omega, k, p_1, p_2) = \frac{m}{16\pi^2k^3} \times [2k(p_1 - p_2)(k^2 - k(p_1 + p_2) - 2m\omega) + (4k^2p_F^{(b)} - (k^2 - 2m\omega)^2) \ln x] , \quad (A.5)$$
\[ f^{(r)}_{ab-1}(\omega, k, p_1, p_2) = \frac{m}{4\pi^2k} \left[ 2k(p_1 - p_2) \right. \\
+ \left. (p^{(b)}_F)^2 - (p^{(a)}_F)^2 + 2m\omega \right] \ln x, \tag{A.6} \]

with
\[ x = \frac{k^2 + 2kp_1 - 2m\omega - i\varepsilon}{k^2 + 2kp_2 - 2m\omega - i\varepsilon}. \tag{A.7} \]

If one is only interested in the real part of Eq. (2), the Pauli-blocking effects in the two terms cancel, and it is possible to use the simple formula (A.1) for all cases, see Eqs. (10.6) to (10.8) of Ref. [7]. However, for Eqs. (3) and (4), the distinction of the four cases (a) to (d) is necessary.

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