Statistics of acoustic emission in paper fracture: precursors and criticality

J Rosti, J Koivisto and M J Alava

Department of Applied Physics, Helsinki University of Technology, FIN-02015 HUT, Finland
E-mail: jro@fyslab.hut.fi, jko@fyslab.hut.fi and mjalava@cc.hut.fi

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Abstract. We present statistical analysis of acoustic emission (AE) data from tensile experiments on paper sheets, for loading mode I, with samples broken under strain control. The results are based on 100 experiments on unnotched samples and 70 samples with a long initial edge notch. First, AE energy release and AE event rates are considered for both cases, to test for the presence of ‘critical points’ in fracture. For AE energy, no clear signatures are found, whereas the main finding is that the event rate diverges when a sample-dependent ‘critical time’ of the maximum event rate is approached. This takes place after the maximum stress is reached. The results are compared with statistical fracture models of heterogeneous materials. We also discuss the dependence of the AE energy and event interval distributions on average event rates.

Keywords: fracture (experiment), heterogeneous materials (experiment)
1. Introduction

Fracture of heterogeneous material exhibits scaling properties familiar from statistical physics. Scaling laws of energy release and fluctuations in temporal statistics tempt one to interpret fracture via concepts of criticality and phase transitions. However, simple models have failed to convey the phenomenology, and the origins of observed scaling laws are lacking. In this work we have performed an extensive set of tensile experiments on paper sheets in order to consider the concept of criticality for mode I loading with imposed strain [1, 2].

The typical stress–strain curve for tensile fracture of paper samples exhibits some non-linearity before the maximum stress, $\sigma_c$. This is a result of both plastic, irreversible deformation and, to a much lesser degree, loss of elastic stiffness, or damage accumulation [3]. We study the damage accumulation by using 'acoustic emission' (AE). This is the release of elastic energy due to microcracking on various scales ranging from far below the fibre size to, perhaps, millimetres as in the individual advances made by a notch in a tensile test. AE studies have been done for a wide variety of materials in science and engineering [4]. Features common to most are the material failure process being complicated and disorder being present in the structure. Numerous studies have elucidated the statistical laws for describing AE. In general, these relate to usual loading conditions of mode I type or mode III type, and a common feature is that they exhibit scale-free features. The probability distribution function (pdf) of event energies usually follows a power law, with exponents in the range $\beta = 1-2$. The event intervals are, similarly, found to obey such fat-tailed pdfs [5]-[7], [2, 8].

Here we consider paper as a test material for the case of critical divergences during the fracture in the presence of structural randomness. This idea of criticality can be formulated in various ways, but essentially it implies the presence of a finite time singularity, so a quantity such as the AE energy release during the approach to failure...
becomes a function of \(1/(t_c - t)\) where \(t_c\) is the lifetime. In the following, we focus on the non-stationary AE signal from tensile experiments with imposed strain and study energy and event statistics of AE near failure. We study in detail local averages of e.g. released acoustic emission energy when \(t_c\), or the critical point, is approached. The relation between the critical point \(t_c\) and the time of maximum stress \(\sigma_c\) is discussed.

The total number of observed acoustic emission events in all experiments is 800,921. The high number of events in a single experiment (a few thousand events, comparable to rock and concrete fracture experiment cases [9,10]) combined with the large number of samples allows us to study the non-stationarity of the acoustic emission signal.

What kind of AE signature is produced by driving the fracture, e.g. by imposing a constant strain or creep stress, is not well understood. Thus, understanding the statistical properties of the acoustic emission and fracture precursors could provide a way to distinguish between different modes of loading. One possible application of this is in creating analogies with experimental findings in seismicity [11].

The main result of the analysis is that we can distinguish two fundamental cases: the dynamics of samples without a dominating notch and the dynamics in the presence of a large defect with slow, stable crack growth. In the former case, there is evidence of such a divergence, upon approach to a sample-dependent critical time. This takes place after the sample maximum stress is reached. A qualitative explanation is perhaps offered by the dynamics which stems from the nucleation and growth of a dominant microcrack.

To discuss the divergence, we consider simple models of statistical fracture. We show that variants of a minimal quasi-static fracture (random fuse network, RFN) model do not exhibit such features of the data. It is well known that the RFN and other models do not reproduce quantitatively, e.g., the exponents that characterize AE statistical properties. It also turns out that the divergence is not found in such models; however a RFN variant with gradual failure gets slightly closer [12].

The structure of the rest of this paper is as follows. In section 2, we discuss the background and details of the experiments. Section 3 presents various aspects of the data analysis, and is completed with a subsection on modelling attempts. Section 4 presents discussion and a summary.

2. Experiments

Normal copy paper samples were tested in the crosswise direction on a mode I laboratory testing machine of type MTS 400/M. The deformation rate \(\dot{\epsilon}\) was 10 mm min\(^{-1}\). The AE system consisted of a piezoelectric receiver, a rectifying amplifier and continuous data acquisition. The data acquisition was free of dead time. The stress was measured simultaneously directly from the MTS 400/M using the same time resolution as for the AE signal; thus the accuracy of the time synchronization of the acoustic emission and stress-strain curves was below 10 \(\mu\)s.

During the experiment, we acquired bi-polar acoustic amplitudes simultaneously on two channels, using piezocrystal sensors, as a function of time. Two transducers were attached directly to the paper without a coupling agent. Each channel has 12-bit resolution and a sampling rate of 312,000 Hz. The time of transmission from the event origin to the sensors is of the order of 5 \(\mu\)s. The acoustic channels were first amplified and after that held using a sample-and-hold circuit.
70 samples had initial notches of size 10 mm, to achieve stable crack growth, and 100 samples were intact. The sample geometry was 150 × 50 mm². Samples were tested under standard conditions: temperature of 22°C and relative humidity of 30%.

The acoustic time series was post-processed after the measurement by detection of continuous and coherent events, and the calculation of an event energy $E_i$ is done using the integral of the squared amplitudes within the event: $E_i = \int A^2(t')\,dt'$. The event arrival time $t_i$ was taken from the instant when the amplitude rises above the threshold level. The dynamic range of the measurement device was 54 dB. The ensuing discrete set of events is characterized by a set of pairs: $\{(t_1, E_1), (t_2, E_2), \ldots\}$. Simultaneously, we measured the stress signal $\sigma(t)$ and, on the basis of that, we can define quantities also as a function of stress $\sigma(t)$.

Figure 1 shows the measurement set-up and a single AE event. Time versus stress curves for intact samples are shown in figure 2, which presents the sample to sample variations in different experiments. The curves are shown both after scaling the sample strength (at maximum stress) and the corresponding time to unity, and in the usual way (inset). Both modes illustrate the sample to sample variation, and also in proximity to the maximum stress. Figure 3 depicts time–stress curves with a notch. As is obvious, $\sigma(t)$ is of different shape and indicates stable crack growth.

3. Results

3.1. Temporal characteristics of the AE

We start our analysis from a time ordered set of time–event pairs from an experiment: $\{(t_1, E_1), (t_2, E_2), \ldots, (t_n, E_n)\}$. We divide the time interval $(t_1, t_n)$ into windows of length...
Figure 2. Time versus stress curves for all samples without a notch. The maximum stress is scaled to unity using $\sigma/\sigma_c$. This figure depicts a typical stress response and its variations as a function of time. The inset contains the same data without scaling of the stress axis.

Figure 3. Time versus stress curves for all samples with a notch. The maximum stress is scaled to unity using $\sigma/\sigma_c$. This figure depicts the typical stress response of notched samples. The average over all time–stress curves is shown.

$\Delta t$ and compute the event energy rate $\dot{E}(t)$ and event count rate $\dot{n}(t)$ for acoustic emission in each $\Delta t$ window defined according to equations (1) and (2):

$$\dot{E}(t) = \sum_i \frac{H(t - t_i)H(t_i - t + \Delta t)E_i}{\Delta t}$$  \hspace{1cm} (1)
Figure 4. Averaged cumulative event energies as a function of time from the beginning of the experiment. The data are averaged over 100 and 70 samples. The notched case presents a power law increase of the acoustic emission energy. The unnotched case highlights the onset of acoustic emission, when the plasticity starts.

\[
\dot{n}(t) = \sum_i \frac{H(t - t_i)H(t_i - t + \Delta t)}{\Delta t}
\]  

(2)

where \( H(x) \) is the Heaviside step function.

After energy and event count rates have been obtained, these quantities are averaged over all experiments. For the averaging, we have to use a similar choice for the time axis (or stress) and there is no natural choice. The time \( t = 0 \) corresponds to triggering from the beginning of the tensile experiment, which is unphysical for any averaging. Before computing the average, we define the origin of time and shift the signals \( \dot{n}(t) \) and \( \dot{E}(t) \) accordingly.

As a physical beginning of the experiment, the origin of time is defined as where we find the maximum slope of the time–stress curve (see figure 2) and the intersection of the time axis and a straight line fitted to the point of maximum slope. This choice removes non-idealities at the beginning of the time–strain curve which often arise from experimental limitations, e.g. a small amount of slack in the sample when it is inserted into the clamps. Figure 4 shows \( E(t) \), the integral of the event energy \( \dot{E}(t) \), averaged so that the time is shifted to the beginning of the experiments. The notched case shows stable crack growth. The unnotched energy integral depicts the onset of the acoustic emission when the plastic regime starts, but since the time to failure shows scatter from sample to sample, the meaning of the average becomes subtle. The energy integral in the unnotched case is similar to the result presented in [6]. The differences between the results in [6] and figure 4 are due to the fact that the latter does not include events after the stress has started to drop after the maximum has been reached, the larger scatter in failure times and the samples being more ductile. It is interesting to note that the data for
the notched samples show, over two orders of magnitude in integrated energy, a power law scaling \( E_{\text{cum}} \propto t^{1.6} \). Assuming stable crack growth with a constant FPZ size, this feature could be interpreted as the crack length time dependence. Note that there is roughly one order of magnitude of difference between the samples without and with notches in total energy release magnitude.

Next we test for apparent criticality, which would imply the divergence of the event energy rate near a critical point, that is, the finite lifetime of the sample (corresponding to maximum stress, or crack instability). Discussing this is meaningful only in the unnotched case, where one can see a rapid increase in the event energy near the maximum of the time–stress curve. Conventionally, the lifetime \( t_c \) is defined as the time at the maximum of stress \( \sigma(t) \), that is \( t_c |_{\max(\sigma(t))} \). However we discover that the time at maximum stress \( t_c \) is different from the time at the event of maximum energy and event count rate \( t'_c |_{\max(\dot{n}(t))} \). Schematically these differences are shown in figure 5. The sample to sample distribution of the quantity \( (t'_c - t_c) \) is seen in figure 6, which shows that there is a clear difference, with time at the maximum event rate being larger. In other words, \( t'_c \) is found without exception after the maximum stress.

When we average quantities over samples by shifting the critical time to the origin before the averaging, as shown in figure 7, we find that the event count rate \( \dot{n}(t_c - t) \) appears to follow a power law according to equation (3), with the lifetime given as \( t_c |_{\max(\dot{n}(t))} \):

\[
\dot{n}(t_c - t) \sim (t_c - t)^{-\Delta}
\]

where we find the value \( \Delta = 1.4 \pm 0.1 \). The error is defined using the min–max method.

Equation (3) does not apply for the event energy release rate \( E(t_c - t) \). A power law behaviour for the event count rate but not for the event energy implies a change in the average event energy during the experiment. This is shown in figure 10: the average

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event energy as a function of normalized stress $\sigma/\sigma_{\text{max}}$ during the experiment. The result shows that there is rapid increase in the average event energy near $\sigma_c$. We can also look at event and energy rates as functions of normalized stress, $(\sigma_c - \sigma)/\sigma_c$, as shown in figure 8. The cumulative event count shows, over a range of rescaled stresses, an apparent power law dependence. The cumulative event energy behaves in a way similar to what was found by Garcimartin et al [13], but a power law dependence is not clear, when we have the cumulative count as the reference. The comparison is not straightforward due to the different loading modes. The connection is further explored in the discussion. As expected (on the basis of equation (3)), the event rate does not seem to scale here in any particular fashion.

3.2. Probability distributions of AE

Next we study statistical distributions of event energies $E_i$ and inter-event times. Inter-event times in an experiment are defined as sets of $\tau_i = t_{i+1} - t_i$. These exhibit almost without exception power law statistics such that one defines probability distributions $P(\tau)$ and $P(E)$ for the inter-event/waiting times and energies which are characterized by power law exponents $\alpha$ and $\beta$, respectively, according to equations (4) and (5):

$$P(E) \sim E^{-\beta}$$

$$P(\tau) \sim \tau^{-\alpha}.$$ 

We find values $\beta = 1.4 \pm 0.1$ and $\alpha = 1.3 \pm 0.2$ for event energies and inter-event times respectively. However, interpretation of these statistical distributions is difficult since the internal state of the material changes along the whole test duration. Statistical quantities
Figure 7. The event energy release rate $\dot{E}$ and the event rate $\dot{n}$ as a function of $t_c - t$. The energy release and event rates are computed in a window $\Delta t = 0.2$ s which is slid over the AE time series. The critical time $t_c$ is defined as a time where the energy release rate $\dot{E}$ reaches its maximum value. The event energy and event rates are averaged over 100 experiments using the AE from unnotched samples. The event rates decay as a power law, which is not true for event energies.

In figure 9 we show waiting time distributions from tensile experiments as a function of the event rate $\dot{n}$. The event rate $\dot{n}$ is computed in a window. Windows are divided into different classes on the basis of an event rate. The data sets in the figure represent waiting time distributions in the event rate class. The data set label indicates the averaged event rate in units of s$^{-1}$ in the event rate class. Event rate classes are defined such that each class contains approximately 20 000 events. The difference between panels lies in the algorithm which identifies events: the algorithm used for the lower figure allows overlapping of AE events while the one used for the upper figure does not. The implication is that the algorithmic details are not important, and differences that we see in distributions are not due to the clustering of events at larger event rates. From waiting time distributions, it appears that there is a complicated change of the waiting time distribution $P(\tau)$ as a function of event rate, though one must be careful as the window size is still (in the time domain) relatively large compared to $t_c$. Perhaps one can see signs of two different power laws. It is interesting to note that the purest power law behaviour is obtained at small event rates (small strains) while for larger ones it changes to a more complicated distribution.

The energy distributions are shown in figure 11. Due to the large variation in the average event size (see figure 10) near time at maximum stress, we studied changes in the probability density $P(E)$ close to $t_c$. The distribution in the figure 11 contains events up to the time $t_c - t'$. The time shift $t'$ is shown in the label of the data set. The number of such as $P(E)$ or $P(\tau)$ integrate over the whole history of a sample. Thus we study how these statistical distributions change e.g. in relation to $t_c$ or the event count rate $\dot{n}$.
Figure 8. Cumulative energy $E/E_{\text{tot}}$ and cumulative event count $n/n_{\text{tot}}$ as a function of $(\sigma_c - \sigma)/\sigma_c$. The event energy and event counts are averaged over 100 experiments using the AE from unnotched samples.

3.3. Theoretical models

In this section we compare experimental results to a random fuse network with a residual conductivity (RRFN). The model is analogous to the one proposed by Duxbury and Li [12]. It is a minimal quasi-static model for fracture, which includes stress enhancements and residual bonding, leading to apparent plasticity and crack arrest. The model is an electrical analogy of an elastic lattice. The main motivation for using the RRFN is that usual RFN simulations are much further away from experiments.

In the model we connect a set of fuses to a square lattice with a size $L \times L$. Fuses are labelled with index $j$ and associated with a constant conductivity $g_j = 1.0$ and with randomly distributed critical currents $i_{c,j}$. The distribution is uniform and characterized by its width: $i_{c,j} \in [1 - W, 1 + W]$. For the breaking process we apply a slowly increasing voltage across the lattice until the hottest fuse approaches its critical current $i_{c,j}$. The important ingredient, as compared with the usual RFN, is that fuses break in two phases. In the first phase the conductivity drops from $g_j = 1.0$ to a residual conductivity $g_j = r$. The fuse remains unbroken in the network with a residual conductivity until its current threshold is exceeded again and then its conductivity drops to zero, which finally corresponds to breaking the fuse and generating an event. We apply this rule to the conductivity of the hottest fuse and start increasing the voltage from zero again. The process of applying conductivity drops and re-evaluating the hottest fuse is repeated until
the conductivity of the network goes to zero and burned fuses form a percolating path across the lattice.

When the residual conductance approaches unity, the model is identical to the perfectly brittle random fuse network. When the residual conductance is close to zero, stress enhancements create very tough fuses which are capable of arresting cracks and leading to plasticity. The width of the disorder parameter $W$ leads to competing effects of the disorder and stress enhancements.

By finding the $V$–$I$ characteristic of the whole network, we obtain an entity which corresponds to experimental stress–strain curves. We record a voltage–current pair $(V'_i, I'_i)$ when the conductivity of a fuse drops to zero. We apply voltage control for the ordered set $\{(V'_i, I'_i), \ldots\}$ by requiring that when the voltage $V'_i$ overcomes the highest value previously

Figure 9. Waiting time distributions for tensile experiments. The AE time series are divided into $\Delta t = 0.2$ s time windows and waiting times $\tau_i$ and the event rate $\dot{n}$ is computed in a window. Windows are divided into different classes on the basis of an event rate in the window and the data set in the figure indicates the waiting time distribution in the event rate class. The label gives the averaged event rate in the event rate class.
Statistics of acoustic emission in paper fracture: precursors and criticality

**Figure 10.** Average event energy, $\dot{E}/\dot{n}$, as a function of $\sigma/\sigma_c$. This result shows that the event energy is not constant during the experiment.

**Figure 11.** The figure depicts the evolution in the probability density for event energies when the time at the maximum stress $t_c$ is approached. The distribution contains events which are cumulated up to the time $t_c - t'$. The time shift $t'$ is shown in the label. The distribution becomes broader when the maximum event rate is approached. The solid straight line is a power law fit: $P(E) \sim E^{-1.4}$.

recorded we get one point for the $V-I$ curve $(V_k, I_k)$, where $k$ identifies an ‘acoustic emission’ event. In figure 12 we show all voltage–current pairs $V'_i, V'_i$ from one numerical experiment and the envelope corresponds to the $V-I$ curve $\{(V_k, I_k), \ldots\}$. In figure 13 we show voltage-controlled $V-I$ curves from 99 simulations for the system size 124.

The event count rate is defined similarly to that in the experiments. We divide the voltage axis into windows of size $\Delta V$ and compute the number of events in a window

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Figure 12. Voltage–current pairs from a single RRFN simulation. The system size is $L = 124$, the disorder parameter is $W = 0.8$ and the residual conductivity is $r = 0.2$. Filled squares indicate $V$–$I$ pairs when a fuse is burnt—that is, its conductivity drops to zero. The straight line corresponds to linearly elastic behaviour. The envelope curve corresponds to the stress–strain curve of the numerical experiment.

according to equation (6) ($V_{\text{max}}$ is the voltage when the current is at its maximum; the ‘acoustic emission’ event in this model is associated with the second conductivity drop of the fuse, i.e. when the conductivity drops to zero; and calculations for creating an event from all conductivity drops were also carried out, but this did not lead to an acceleration of the event rate):

$$\dot{n}(V/V_{\text{max}}) = \frac{\sum_k H(V - V_k)H(V_k - V + \Delta V)}{\Delta V}.$$  

We performed sets of simulations from system sizes $30 \times 30$ up to $124 \times 124$. The combination of the disorder parameter $W \approx 0.8$ and the residual conductivity $r \approx 0.2$ led to a $V$–$I$ characteristic similar to that from the experiments: activity started after the experiment approached half of the critical voltage and we observed activity after the maximum voltage was approached (figure 13). This implies that even large cracks are arrested before a dominating crack is formed.

Figure 14 shows the event count rate as a function of $V/V_{\text{max}}$ averaged over 99 experiments. The event count rate starts to increase after 50% of the maximum voltage is approached. The event rate is at its maximum near $V_{\text{max}}$. There is a small tail in the event count rate after the maximum stress, which becomes steeper when the system size increases. The increase in the event count rate is neither a power law nor critical.

In figure 15 there is the histogram of the maximum event rate a function of $V/V_{\text{max}}$. This depicts the difference between experiments and simulations: the maximum event rate occurs before or at the maximum current in simulations, while in experiments it is observed after the maximum stress. However, there are still a few occurrences of the
maximum event rate after the maximum current in simulations. We conclude that the time of maximum event rate and the time of maximum current are not identical. However, the relation is inverse to the one found in experiments.

4. Discussion

We have compared fracture precursors using different ‘critical points’ $t_c|\max(\sigma_c)$ and $t'_c|\max(\dot{n})$ in mode I loading and imposing a constant strain rate for copy paper samples. A divergence is observed in samples without a notch, using time as a control parameter and looking at the event rate when approaching the time at the maximum event rate. We have shown that the characteristic behaviour of the event rate $\dot{E}$ is different from that of the event rate $\dot{n}$. The behaviour of the event rate $\dot{n}(t_c - t) \sim t^{-\Delta}$ might be taken to imply criticality, when $t_c$ is chosen to be the time at the maximum of the event rate. We note that the maximum of the event energy and of the event rate corresponds to a time which is observed after the maximum of the stress $\sigma_c$.

A plausible argument for explaining the causal nature of the AE observations (without a notch) is as follows. At the maximum stress it follows that $\partial_t \sigma = E_c \epsilon + E$ becomes negative. Thus $E_c$ is negative, which could be taken to indicate that one of the microcracks dominates and has started to grow. The event rate increases with the crack growth velocity, until crack instability arises. The divergence of the event rate could then arise from a divergence of crack velocity. The relation of the rate divergence and the concomitant increase of the event energy release presents a complicated problem. The rate of energy release should (recall that we assume here a single propagating stable crack) be proportional to the new fracture process zone (FPZ) area created (if the AE event energy is related to the new FPZ area multiplied by the fracture toughness). However, we have
shown that the diverging quantity seems to be the event rate and not the event energy in such strain imposed loading.

Earlier experimental studies have shown indications of an event energy divergence when one imposes a constant pressure rate on a heterogeneous material [14, 13, 15]. The critical exponent $\gamma$ in $E \sim (\bar{p} - p_c)/p_c^\gamma$ was found to be $\gamma = 1.4$ in [14], $\gamma = 1.27$ in [13] and $\gamma = 1.0$ in [15]. In the case of an imposed constant strain rate there was not found to be any critical divergence of the energy release rate [16]. The analysis however was not along the same lines, and excluded the mechanism suggested above. Finally it was suggested that the real control parameter is time, since imposing constant stress or imposing cyclic stress seems to indicate a critical divergence of the event energy [17]. Our results include a somewhat similar behaviour for the event energies as the maximum stress is approached.

Statistical distributions of event energies $P(E)$ and inter-event times $P(\tau)$ were measured earlier several times for paper. Power laws have been found with exponents $\beta = 1.4 \pm 0.1$ and $\alpha = 1.3 \pm 0.2$ for event energies and inter-event times, respectively. We may contrast the current result to those from earlier work by Salminen et al where acoustic emission has been measured for paper in the machine direction [6]. The energy exponent $\beta$ is here slightly larger, compared to $\beta = 1.25 \pm 0.10$, which might be attributed to the much more ductile nature of paper as a material when stressed in the crosswise machine direction. The waiting time exponent $\alpha$ is almost identical when it is integrated over the whole experiment.

In any case, in general most of the statistical signals that have been explored experimentally are still awaiting theoretical explanations. For $P(E)$, for example, brittle
fracture models allow one to derive scaling forms that are like power laws, but with exponents $\beta$ that are in general far too large. To see whether qualitative agreement could be found, we compared the event rate from experimental data to the residual random fuse network. The event rate was found to increase when the voltage at maximum current was approached—contrary to the case for perfectly brittle ordinary RFN. The result was not a power law divergence as for the experimental data. Note that introducing fracture toughness (residual fuses) to the model caused a change in the behaviour of the form of the event count rate: rapid increase of the event energy at $V_{\text{max}}$ was not found in the perfectly brittle RFN model. Further studies of the RRFN would seem to be of interest. Note that Amitrano and Helmstetter [18] have proposed a numerical model in order to model time-dependent damage and deformation of rocks under creep. In the model, the 2D finite element method is used and separate time-dependent and time-independent damage progressions and laws for time to failure are introduced. Time-independent damage progression for an element is introduced as a gradual drop of an elastic modulus if the stress threshold is exceeded: $E_i(n+1) = E_i(n)(1 - D_0)$ where $E_i(n)$ is the elastic modulus of an element after $n$ damage events and $D_0$ is a constant damage parameter. Quasi-static stress redistribution may induce an avalanche of damage events during a single loading step. The damage progression law and quasi-static stress redistribution are similar to the conductivity drop and re-evaluation of the voltage in the RRFN. The difference lies in the damage law of an element. The result of the model is a power law acceleration of the strain rate $\dot{\varepsilon}$ and the rate of damage events $\dot{n}$ near the failure time $t_c$. However, if we look at number of events as a function of strain, $n/\dot{\varepsilon} \sim 1/(t_c - t)^\gamma$, the rate for damage events decreases when $t_c$ is approached. The result is qualitatively in agreement with what is observed for the residual random fuse network when it is assumed
that every conductivity drop creates an acoustic emission event. The gradual failure does
not explain the event rate acceleration near failure in the experiments.

Equation (3) is in essence identical to Omori’s law of aftershock rates. In the
stationary paper peeling experiment, one observes Omori’s law with the exponent close
to $\Delta = 1.4$ and it is also comparable to e.g. tectonic seismicity. For that case the
generic temporal properties are further discussed in [11]. Approaching a ‘main shock’ in
a stationary experiment might be identical to approaching time at the maximum event
rate in strain-driven non-stationary fracture experiments when we consider the behaviour
of the event rate $\dot{n}$. The fundamental differences of non-stationary and stationary AE
signals are the different forms of the inter-event time distributions and the increase in
the average event energy before failure (or main shock). However, this acceleration of the
event rate in the stationary experiment is significant when averaging over a large number
of sequences, and the similarity may be coincidental.

The limitations of the experimental set-up are the limited choice of loading modes
and lack of feedback control of the imposed strain or the stress. For example, driving
material based on the acoustic emission event rate, as is done for rock fracture in [19],
we were able to obtain an experimental test for the criticality and diverging quantities of
the paper fracture; it would be interesting to see whether it is possible to alter the event
rate divergence near $t_c$ by controlling the strain on the basis of AE feedback during an
experiment.

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Statistics of acoustic emission in paper fracture: precursors and criticality

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