Measurement of Azimuthal Modulations in the Cross-Section of Di-Pion Pairs in Di-Jet Production from Electron-Positron Annihilation

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Abstract

We present an extraction of azimuthal correlations between two pairs of charged pions detected in opposite jets from electron-positron annihilation. These correlations may arise from the dependence of the di-pion fragmentation on the polarization of the parent quark in the process $e^+e^- \rightarrow q\bar{q}$. Due to the correlation of the quark polarizations, the cross-section of di-pion pair production, in which the pion pairs are detected in opposite jets in a dijet event, exhibits a modulation in the azimuthal angles of the planes containing the hadron pairs with respect to the production plane. The measurement of this modulation allows access to combinations of fragmentation functions that are sensitive to the quark’s transverse polarization and helicity. Within our uncertainties we do not observe a significant signal from the previously unmeasured helicity dependent fragmentation function $G_1^\perp$. This measurement uses a dataset of 938 fb$^{-1}$ collected by the Belle experiment at or near $\sqrt{s} \approx 10.58$ GeV.

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Fragmentation functions are fundamental QCD objects describing the hadronization of quarks into final-state hadrons. They cannot be computed using lattice QCD techniques due to the non-inclusive final-state and therefore have to be measured. In addition to studying properties of QCD, fragmentation functions are important tools to extract the partonic spin structure of the nucleon from semi-inclusive deep-inelastic scattering (SIDIS) and proton-proton (p+p) collision data [1, 2]. In recent years the role of polarization dependent fragmentation functions as quark polarimeters has become more important to access polarization dependent quark distributions in SIDIS and p+p [3].

Di-hadron fragmentation, e.g., \( q \rightarrow (h^+h^-) + X \), describes the hadronization of a quark into a final-state containing two distinguishable hadrons. Compared to single-hadron fragmentation, it offers access to transverse polarization of the fragmenting quark without involving either final-state polarimetry or transverse-momentum dependence of the fragmentation process. This is achieved through the additional degree of freedom provided by the relative momentum of the hadron pair. In case of the chiral-odd di-hadron fragmentation function \( H^\perp_1 \), the orientation of the plane spanned by this relative momentum and the total di-hadron momentum can be correlated with the transverse polarization of the fragmenting quark. Such correlation is reminiscent of the explicitly transverse-momentum dependent Collins fragmentation function \( H^\perp_1 \), where the transverse momentum of the outgoing hadron (or di-hadron) provides the plane correlated with the quark polarization. However, such correlation and thus \( H^\perp_1 \) does not survive simple integration over transverse momentum, which complicates the theoretical description. In contrast, the di-hadron plane does survive such integration, permitting application of the ordinary collinear framework.

The chiral-odd \( H^\perp_1 \) has already been subject to measurements in \( e^+e^- \) annihilation into di-hadrons [4] as well as in SIDIS and p+p. Its chiral-even partner, the helicity-dependent di-hadron fragmentation function \( G^\perp_1 \), has yet to be measured. Together with the chiral-odd fragmentation functions and the quark-polarization independent \( D_1 \), it completes the set of leading-twist di-hadron fragmentation functions. Unlike \( H^\perp_1 \), which has a counterpart in the single-hadron fragmentation with the Collins fragmentation function, for \( G^\perp_1 \) no such correspondence exist [5]. This makes it an elusive tool to probe quark helicity (e.g., in the nucleon) through fragmentation. In addition, it has been conjectured that the measurement of \( G^\perp_1 \) in \( e^+e^- \rightarrow (h_1h_2)(\bar{h}_1\bar{h}_2) + X \), i.e., production of two di-hadrons from both the quark and the anti-quark in \( e^+e^- \) annihilation, is related to jet-handedness correlations, which might receive contributions from CP violating effects [5].

The process of di-hadron production from both the quark and the anti-quark in \( e^+e^- \) annihilation is the cleanest way to access polarization-dependent di-hadron fragmentation. While in SIDIS and p+p the fragmentation functions are folded with partially unknown parton distributions, such “contamination” is missing in \( e^+e^- \) annihilation. Furthermore, the spin-correlation of the quark anti-quark pair in \( e^+e^- \) annihilation allows the study of polarization dependent fragmentation functions even in the case of an unpolarized initial state. For the study of \( G^\perp_1 \), \( e^+e^- \rightarrow (h_1h_2)(\bar{h}_1\bar{h}_2) + X \) has another advantage: formally \( G^\perp_1 \) is a transverse-momentum dependent fragmentation function and as such does not survive simple integration over the transverse momentum generated in the fragmentation process. However, the correlation between the two hadron pairs in this specific process introduces a particular weighting factor in the integration, such that the cross section itself survives integration over the observed transverse momentum. The fragmentation functions will thus enter as specific transverse-momentum moments in the cross section expressions. However, we shall adopt the simplified notation of \( G^\perp_1 (z, M^2) \) from Ref. [5], where \( z \) is the fractional
momentum carried by the hadron pair and $M$ its invariant mass.

This paper reports on the first measurement of the $G^+_1$-induced correlation between two di-pions in the process $e^+e^- \rightarrow (\pi^+\pi^-)(\pi^+\pi^-) + X$, where the two di-pions are from two separate and opposing jets.

The Belle experiment [6] at the KEKB storage ring [7] recorded about 1 ab$^{-1}$ of $e^+e^-$ annihilation data. The data was taken mainly at the $\Upsilon(4S)$ resonance at $\sqrt{s} = 10.58$ GeV, but also at $\Upsilon(1S)$ to $\Upsilon(5S)$ resonances and at $\sqrt{s} = 10.52$ GeV. The Belle instrumentation used in this analysis includes a central drift chamber and a silicon vertex detector, which provides precision tracking for tracks in 17.0° < $\theta_{\text{Lab}}$ < 150.0°, and electromagnetic calorimeters (ECL) covering the same region. Particle identification is performed using information on $dE/dx$ in the CDC, a time-of-flight system in the barrel, aerogel Cherenkov counters in the barrel and the forward endcap, as well as a muon and $K_L$ identification system outside the superconducting solenoid, which provides a 1.5 T magnetic field.

Using the event shape variable thrust, a sample of light and charm quark ($u,d,s,c$) fragmentation can be selected [4,8]. The thrust is defined as

$$T = \max_{|\vec{p}_i|} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}.$$  

Here the $\vec{p}_i$ are all particles in the event. We use a thrust cut of $T > 0.8$ which reduces the contribution of $B$ mesons to less than 1%. Since the polarization-dependence of the fragmentation would average out in unpolarized $e^+e^-$ annihilation into one di-pion, a measurement has to use the correlation between the polarizations of the produced quark-antiquark pairs. This then leads to a correlation of hadron-pairs detected in opposite hemispheres [3]. The correlation is expressed as a modulation of the di-hadron pair cross-section in the angles $\Phi_{R'_1}$ and $\Phi_{R'_2}$. They are defined in Ref. [5] as the azimuthal angles of the planes containing the hadron pairs with respect to the event plane spanned by the beam axis and the momentum sum vector $\vec{P}_h = \vec{h}_1 + \vec{h}_2$ of one hadron pair in the Center-of-Mass System (CMS), where $h_1$, $h_2$ are the momenta of the hadrons forming the pair. It should be noted, that the reference frame defined here, is asymmetric with respect to the hadron pairs, both angles $\Phi_{R'_1}$ and $\Phi_{R'_2}$ are measured around $\vec{P}_h$. With this definition, the di-pion fragmentation function $G^+_1$ can be accessed in the moment

$$\langle \cos(2(\Phi_{R'_1} - \Phi_{R'_2})) \rangle \propto \sum_{q\bar{q}} e_q^2 G^+_1(z,M^2)G^{1-\bar{q}}(\bar{z},\bar{M}^2).$$

Here $q$, $\bar{q}$ are the quark and antiquark flavors, $e_q$ the corresponding electric charges and $\bar{z}$, $\bar{M}$ the fractional energy and invariant mass of the hadron pair produced in the antiquark fragmentation.

Instead of using the asymmetric frame defined above, this analysis uses a symmetric frame in which the angles are computed using the jet axes of di-jet events. The jet axes are reconstructed using the anti-$k_T$ jet algorithm [9]. They serve here as proxies of the directions of the fragmenting (anti)quarks. A similar approach had been employed in the previous measurements of $H^+_1$ and the Collins fragmentation function [4,8] in a symmetric frame using the thrust axis. The coordinate system is depicted in Fig. 1.

The angles $\Phi_{R_i}$ can be expressed as:

$$\Phi_{R_i} = \text{sgn} \left( \vec{j}_i \cdot \left( (\vec{e} \times \vec{j}_i) \times (\vec{\bar{j}}_i \times \vec{R}_i) \right) \right) \text{arccos} \left( \frac{\vec{e} \times \vec{j}_i \cdot \vec{\bar{j}}_i \times \vec{R}_i}{|\vec{e} \times \vec{j}_i| \cdot |\vec{\bar{j}}_i \times \vec{R}_i|} \right).$$
FIG. 1: The coordinate system used for this measurement. For convenience, the jets are depicted to lie in the event plane which in turn is spanned by the beam axis and the jets.

Here $\vec{e}$ is the $e^-$-beam direction, $\vec{j}_i$ the jet axes and $\vec{R}_i = \vec{h}_i^1 - \vec{h}_i^2$, with $\vec{h}_i^1$ the momentum of the positive pion and $\vec{h}_i^2$ the momentum of the negative pion in jet $i$. All quantities are in the CMS. Note that one of the jet-axis directions is flipped such that in an ideal back-to-back event, both jet axes point in the same direction. Compared to the asymmetric frame, the theory for the symmetric frame has not been worked out completely, though we believe it to be just a modification of Eq. (2). Compared to [4] and [8], where the thrust axis of the event was used as a proxy for the quark direction, the jet method will be more robust against final-state gluon radiation, which is usually described by Sudakov factors or evolution equations.

We use jets reconstructed with the anti-$k_T$ algorithm as implemented in the FastJet 3.0.5 package [10, 11] with a jet radius of $R = 1.0$ and a minimum energy of 3.75 GeV for each jet. This cut on the jet energy also effectively removes $e^+e^- \rightarrow \tau\bar{\tau}$ events and leaves only dijet events. We restrict the allowed range of the polar angle of the jets in the CMS to $1.38 \text{ rad} < \theta_{\text{CMS}} < 1.75 \text{ rad}$ to minimize systematic effects at the edges of the barrel acceptance. Jets are reconstructed from all charged tracks that originate from a region around the reconstructed interaction point (IP) as well as photons with energies larger than 50 MeV in the barrel, 75 MeV in the forward endcap and 100 MeV in the backward endcap. The IP is required to fulfill $|dr| < 2 \text{ cm}$ and $|dz| < 4 \text{ cm}$, where $dr$ and $dz$ are the distance of closest approach to the interaction point in the plane perpendicular to the beam direction and along the direction of the beams. We select charged pion pairs by using all charged tracks, which have not been identified as kaons, protons, electrons, or muons [12] and requiring a minimum fractional energy $z$ of the individual tracks of 0.1.

We perform a two-dimensional $\chi^2$ fit to the normalized di-pion pair yields $\frac{N(\Phi_R^1, \Phi_R^2)}{\langle N \rangle}$.
FIG. 2: Results for $A^{\cos(2(\Phi_{R_1}-\Phi_{R_2}))}$ binned in $M$ and $z$. The black error bars are statistical and the green bands show the systematic uncertainty.

using the truncated Fourier expansion

$$1 + A^{\cos(\Phi_{R_1}+\Phi_{R_2})}\cos(\Phi_{R_1} + \Phi_{R_2}) + A^{\cos(2(\Phi_{R_1}-\Phi_{R_2}))}\cos(2(\Phi_{R_1} - \Phi_{R_2})).$$

(4)

The amplitude $A^{\cos(2(\Phi_{R_1}-\Phi_{R_2}))}$ corresponds then to the quantities in Eq. (2), whereas the amplitude $A^{\cos(\Phi_{R_1}+\Phi_{R_2})}$ corresponds to the previously published measurement of $H_1^{\perp}$ [4]. Results binned in $M$ and $z$ of one hadron pair are shown in Fig. 2 integrated over $z$ and $M$ of the other pair.

The purity of the sample is above 90% for di-pion pairs, with the remaining 10% coming from an admixture of kaons. The leading systematic effects are the contributions from detector smearing and fiducial cut edge effects to the measured asymmetries. These are estimated from simulations using Pythia [13] and EvtGen [14] for various physics processes not including the polarization dependent ones of Eq. (2), and GEANT3 [15] for the detector effects. In addition to the light ($u,d,s$) quark fragmentation, our results also contain significant contributions from charm quark production. The charm contribution is 18% on average and approximately constant over the invariant mass bins. In $z$, it drops monotonically from 21% to 5%.

Within our statistical and systematic uncertainties, we do not observe a deviation from zero for $A^{\cos(2(\Phi_{R_1}-\Phi_{R_2}))}$. Statistical uncertainties are symmetric and about $10^{-3}$ in magnitude, whereas our systematic uncertainty is asymmetric and varies between $2 \cdot 10^{-3}$ at low $M$ and low $z$ and $5 \cdot 10^{-3}$ at high $M$ and $z$. It has been conjectured that the $G_1^{\perp}$ fragmentation function receives non-zero contributions mainly from $p-p$ wave interference in the partial-wave expansion of $G_1^{\perp}$. We note that an enhancement of such contribution by restricting $\theta_{\text{decay}}$, the decay angle of the positive hadron in the hadron-pair CMS used in such expansion, to values for which $\cos(\theta_{\text{decay}})$ is positive does not result in a significantly non-zero amplitude either.

In summary, we show first results for azimuthal modulations in the cross-section of di-pion pairs in di-jet production from electron-positron annihilation. The amplitude of the $\cos(2(\Phi_{R_1}-\Phi_{R_2}))$ modulation, which is sensitive to the helicity dependent fragmentation function $G_1^{\perp}$, is consistent with zero within our statistical and systematic uncertainties.

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