Relativistic Engine Based on a Permanent Magnet

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Abstract

Newton’s third law states that any action is countered by a reaction of equal magnitude but opposite direction. The total force in a system not affected by external forces is thus zero. However, according to the principles of relativity a signal cannot propagate at speeds exceeding the speed of light. Hence the action cannot be generated at the same time with the reaction due to the relativity of simultaneity, thus the total force cannot be null at a given time. The following is a continuation of a previous paper \cite{1} in which we analyzed the relativistic effects in a system of two current conducting loops. Here the analysis is repeated but one of the loops is replaced by a permanent magnet. It should be emphasized that although momentum can be created in the material part of the system as described in the following work momentum cannot be created in the physical system, hence for any momentum that is acquired by matter an opposite momentum is attributed to the electromagnetic field.

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1 Introduction

Among the major achievements of Sir Isaac Newton is the formulation of Newton’s third law stating that any action is countered by a reaction of equal magnitude but opposite direction \cite{2, 3}. The total force in a system not affected by external forces is thus zero. This law has numerous experimental
verifications and seems to be one of the corner stones of physics. However, by the middle of the nineteenth century Maxwell has formulated the laws of electromagnetism in his famous four partial differential equations [4, 5, 7] which were formulated in their current form by Oliver Heaviside [8]. One of the consequences of these equations is that an electromagnetic signal cannot travel at speeds exceeding that of light. This was later used by Albert Einstein [5, 7, 9] (among other things) to formulate his special theory of relativity which postulates that the speed of light is the maximal allowed velocity in nature. According to the principles of relativity no signal (even if not electromagnetic) can propagate at superluminal velocities. Hence an action and its reaction cannot be generated at the same time because of the relativity of simultaneity. Thus the total force cannot be null at a given time. In consequence, by not holding rigorously the simultaneity of action and reaction Newton’s third law cannot hold in exact form but only as an approximation. Moreover, the total force within a system that is not acted upon by an external force would not be rigorously null since the action and reactions are not able to balance each other and the total force on a system which is not affected by an external force in not null in an exact sense.

Most locomotive systems of today are based on open systems. A rocket sheds exhaust gas to propel itself, a speeding bullet generates recoil. A car pushes the road with the same force that is used to accelerate it, the same is true regarding the interaction of a plane with air and of a ship with water. However, the above relativistic considerations suggest’s a new type of motor in which the open system is not composed of two material bodies but of a material body and field. Ignoring the field a naive observer will see the material body gaining momentum created out of nothing, however, a knowledgeable observer will understand that the opposite amount of momentum is obtained by the field. Indeed Noether’s theorem dictates that any system possessing translational symmetry will conserve momentum and the total physical system containing matter and field is indeed symmetrical under translations, while every sub-system (either matter or field) is not.

In this paper we will use Jefimenkos equation [6] discuss the force between a current loop and a permanent magnet. In this respect the current paper differs from a previous one [1] which discussed the case of two current carrying coils. The current configuration may seem attractive since a permanent magnet does not require a power source.
2 The Magnetic Current Density

Let us begin by writing down Ampere’s law for the magnetic field $\vec{H}$:

$$\nabla \times \vec{H} = \vec{J} + \partial_t \vec{D}. \quad (1)$$

We use the standard notations of vector analysis, $\vec{J}$ is the electric current and $\vec{D}$ is the displacement field. In matter the magnetic field is related to the magnetic flux density $\vec{B}$ as:

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}. \quad (2)$$

In which $\vec{M}$ is the magnetization and $\mu_0$ is the vacuum permeability. Hence:

$$\nabla \times \left( \frac{\vec{B}}{\mu_0} \right) - \nabla \times \vec{M} = \vec{J} + \partial_t \vec{D}. \quad (3)$$

We now define the magnetic current as:

$$\vec{J}_M \equiv \nabla \times \vec{M}. \quad (4)$$

To obtain:

$$\nabla \times \vec{B} = \mu_0(\vec{J}_M + \vec{J} + \partial_t \vec{D}). \quad (5)$$

Thus, the magnetic current density $\vec{J}_M$ plays the same rule as the current density $\vec{J}$.

3 Configuration

We will assume a uniform magnetization $\vec{M} = M_z(r, z)\hat{z}$ confined between the planes $z = -h_1$ and $z = -h$ such that $h < h_1$ and $h_1 - h = L$. The magnet has a cylindrical shape such that the magnetization is confined to a radius $r \leq R_2$. So that:

$$M_z = \begin{cases} M_0 & -h_1 \leq z \leq -h \quad \& \quad 0 \leq r \leq R_2, \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

In terms of the step function $u(x)$:

$$u(x) = \begin{cases} 1 & x \geq 0, \\ 0 & x < 0. \end{cases} \quad (7)$$
Figure 1: A cylindrical magnet (blue) and a current loop (red) above it. 
(three different sections)

the magnetization can be written as:

$$M_z = M_0 \left( u(z + h_1) - u(z + h) \right) u(R_2 - r).$$

(8)

The magnet and a current loop above it are depicted in figure

We can now calculate the magnetization current by evaluating equation

using cylindrical coordinates:

$$\vec{J}_M = \frac{1}{r} \left[ \partial_\theta(M_z) - \partial_z(rM_\theta) \right] \hat{r} + \left[ \partial_z(M_r) - \partial_r(M_z) \right] \hat{\theta}$$

$$+ \frac{1}{r} \left[ \partial_r(rM_\theta) - \partial_\theta(M_r) \right] \hat{z} = \frac{1}{r} \partial_\theta(M_z) \hat{r} - \partial_r(M_z) \hat{\theta}.$$  

(9)

Since $M_z = M_z(r, z)$ we obtain:

$$\vec{J}_M = -\partial_r M_z \hat{\theta}.$$  

(10)

Moreover, since the Dirac delta function $\delta(x)$ is related to the step function by the formulae:

$$\partial_x u(x) = \delta(x), \quad u(x) = \int_{-\infty}^{x} \delta(x') dx'.$$  

(11)
We obtain:
\[
\partial_r u(R^2 - r) = \frac{\partial u(R^2 - r)}{\partial(r(R^2 - r))} \frac{\partial (R^2 - r)}{\partial r} = -\delta(R^2 - r) \tag{12}
\]

Using equation (12) and equation (8) in equation (10) we obtain:
\[
\vec{J}_M = M_0(u(z + h_1) - u(z + h))\delta(R^2 - r)\hat{\theta} \tag{13}
\]

The above can also be written in the following form:
\[
\vec{J}_M = M_0(\hat{z} + h_1)\delta(z' + z)dz'\delta(r - R^2)\hat{\theta}. \tag{14}
\]

Now consider a change of variables:
\[
z'' = z' - z, \quad dz'' = dz'
\]

This will lead to:
\[
\vec{J}_M(r, z) = M_0 \int_{-h_1}^{h} \delta(z'' + z)dz''\delta(r - R^2)\hat{\theta}. \tag{16}
\]

Making an additional variable change:
\[
z''' = -z'', \quad dz''' = -dz''
\]

Results in:
\[
\vec{J}_M(r, z) = M_0 \int_{-h_1}^{-h} \delta(z - z''')dz'''\delta(r - R^2)\hat{\theta}. \tag{18}
\]

Hence we see that the magnetization current density is equivalent to a continuum of loop currents each with a current density:
\[
\vec{J}_M^{(1)}(r, z, z') \equiv M_0\delta(z - z')\delta(r - R^2)\hat{\theta}. \tag{19}
\]

We notice that the magnetization $M_0$ replaces the current $I$ appearing in the analog expression for charge current density. Hence we can write:
\[
\vec{J}_M(r, z) = \int_{-h_1}^{-h} \vec{J}_M^{(1)}(r, z, z')dz'. \tag{20}
\]
4 Force Calculations

Since in a previous paper [1] we have calculated the relativistic total force acting on a system of two coils (see equation (44) of [1]) we will take advantage of this calculation taking into account that the magnetic current density is static. For a single loop we have:

\[ \vec{F}_t = \frac{\mu_0}{8\pi} \left( \frac{h}{c} \right)^2 \vec{K}_{12,2}^{(1)} \dddot{I}_2 = \frac{\mu_0}{8\pi} \left( \frac{h}{c} \right)^2 \vec{K}_{12,2}^{(1)} M_0 \dddot{I}_1. \]  

In which \( \vec{K}_{12,2}^{(1)} \) is defined in equation (39) of [1] and we have replaced \( I_2 \) with \( M_0 \). The total relativistic force is an integration over the contribution of a continuum of current loops:

\[ \vec{F}_t = \int_{-h1}^{-h} \vec{F}_t dz' = \frac{\mu_0}{8\pi} \left( \frac{h}{c} \right)^2 M_0 \dddot{I}_1 \int_{-h1}^{-h} \vec{K}_{12,2}^{(1)} dz' = \frac{\mu_0}{8\pi} \left( \frac{h}{c} \right)^2 M_0 \dddot{I}_1 \vec{K}_{12,2}. \]  

In which \( \vec{K}_{12,2} \) is an integration the K factor of all magnetization current loops. Now we wish to calculate \( \vec{K}_{12,2}^{(1)} \) for this we use equation (39) of [1] which is:

\[ \vec{K}_{12,2}^{(1)}(z') = -\frac{1}{h^2} \oint \oint \vec{R} d\vec{l}_1 \cdot d\vec{l}_2. \]

Notice that \( \vec{R} = \vec{X}_1 - \vec{X}_2 \) is a difference vector between the location vectors of current elements on the charge current and magnetization current loops. Hence using suitable variables:

\[ \vec{X}_1 = (R_1 \cos \theta_1, R_1 \sin \theta_1, 0), \quad \vec{X}_2 = (R_2 \cos \theta_2, R_2 \sin \theta_2, z'). \]

We obtain:

\[ \vec{R} = \vec{X}_1 - \vec{X}_2 = (R_1 \cos \theta_1 - R_2 \cos \theta_2, R_1 \sin \theta_1 - R_2 \sin \theta_2, -z'). \]

And thus \( R \) is:

\[ R = \sqrt{(R_1 \cos \theta_1 - R_2 \cos \theta_2)^2 + (R_1 \sin \theta_1 - R_2 \sin \theta_2)^2 + z'^2} = \sqrt{R_1^2 + R_2^2 - 2R_1 R_2 \cos (\theta_1 - \theta_2) + z'^2}. \]

Now we can calculate the vector line elements:

\[ d\vec{l}_1 = R_1 d\theta_1 \hat{\theta}_1 = R_1 d\theta_1 (-\sin \theta_1, \cos \theta_1, 0), \]
\[ d\vec{l}_2 = R_2 d\theta_2 \hat{\theta}_2 = R_2 d\theta_2 (-\sin \theta_2, \cos \theta_2, 0). \]
The scalar product of those line elements is:
\[ d\vec{l}_1 \cdot d\vec{l}_2 = R_1 R_2 d\theta_1 d\theta_2 \cos(\theta_1 - \theta_2). \] (28)

Combining the above results we can write the K integral of equation (23) as:
\[ \vec{K}^{(1)}_{12,2}(z') = -\frac{R_1 R_2}{h^2} \int_0^{2\pi} d\theta_2 \int_0^{2\pi} d\theta_1 \cos (\theta_1 - \theta_2) \hat{R}(\theta_1, \theta_2, z'). \] (29)

We now make a change in variables:
\[ \theta' = \theta_1 - \theta_2, \quad d\theta' = d\theta_1. \] (30)

The second above equation is correct since \( \theta_2 \) is constant for the \( \theta_1 \) integral. Hence:
\[ \vec{K}^{(1)}_{12,2}(z') = -\frac{R_1 R_2}{h^2} \int_0^{2\pi} d\theta_2 \int_{-\theta_2}^{2\pi-\theta_2} d\theta' \cos (\theta') \hat{R}(\theta' + \theta_2, \theta_2, z'). \] (31)

In the above:
\[ \hat{R} = \frac{\vec{R}}{R} = \frac{(R_1 \cos (\theta' + \theta_2) - R_2 \cos \theta_2, R_1 \sin (\theta' + \theta_2) - R_2 \sin \theta_2, -z')}{\sqrt{R_1^2 + R_2^2 - 2R_1 R_2 \cos \theta + z^2}}. \] (32)

Now we notice that all the function contained in the integrand of equation (31) are periodic in \( \theta' \) with a period of \( 2\pi \) hence we can replace \( \int_{-\theta_2}^{2\pi-\theta_2} d\theta' \rightarrow \int_0^{2\pi} d\theta' \). The following step would to change the order of integrals performing the \( \theta_2 \) integral first and noticing that all the functions which are periodic in \( \theta_2 \) have a null contribution to the integral. Hence we obtain:
\[ \vec{K}^{(1)}_{12,2}(z') = \frac{2\pi R_1 R_2}{h^2} \int_0^{2\pi} d\theta' \frac{d\theta' \cos \theta' z'}{\sqrt{\alpha^2 + h^2 - 2R_1 R_2 \cos \theta' + z^2}}. \] (33)

Now we sum up contributions to the K factor from all loops:
\[ \vec{K}_{12,2} = \int_{-h_1}^{-h} dz' \vec{K}^{(1)}_{12,2}(z') = \frac{2\pi R_1 R_2}{h^2} \int_0^{2\pi} d\theta' \cos \theta' \int_{-h_1}^{-h} dz' \frac{z'}{\sqrt{\alpha^2 + h^2}}, \] (34)
in which we define: \( \alpha^2 = R_1^2 + R_2^2 - 2R_1 R_2 \cos \theta' \). A simple integration leads to:
\[ \vec{K}_{12,2} = \frac{2\pi R_1 R_2}{h^2} \int_0^{2\pi} d\theta' \cos \theta' (\sqrt{\alpha^2 + h^2} - \sqrt{\alpha^2 + h_1^2}) \dot{\hat{z}}. \] (35)
Now we can write:

$$\sqrt{\alpha^2 + h^2} = \sqrt{h^2 + R_1^2 + R_2^2 - 2R_1R_2 \cos \theta'}$$

$$= \sqrt{R_1R_2} \sqrt{\frac{h^2}{R_1R_2} + \frac{R_1}{R_2} + \frac{R_2}{R_1} - 2 \cos \theta'}.$$  \hspace{1cm} (36)

And define:

$$b \equiv \frac{h^2}{R_1R_2} + \frac{R_1}{R_2} + \frac{R_2}{R_1}, \quad b_1 \equiv \frac{h_1^2}{R_1R_2} + \frac{R_2}{R_1} + \frac{R_1}{R_2}.$$  \hspace{1cm} (37)

Hence:

$$\sqrt{\alpha^2 + h^2} = \sqrt{R_1R_2} \sqrt{b - 2 \cos \theta'}, \quad \sqrt{\alpha^2 + h_1^2} = \sqrt{R_1R_2} \sqrt{b_1 - 2 \cos \theta'}.$$  \hspace{1cm} (38)

In terms of the above definitions

$$\vec{K}_{12,2} = \frac{2\pi(R_1R_2)^{1.5}}{h^2}(g(b) - g(b_1))z.$$  \hspace{1cm} (39)

In which:

$$g(b) \equiv \int_0^{2\pi} d\theta' \cos \theta' \sqrt{b - 2 \cos \theta'}.$$ \hspace{1cm} (40)

It is obvious that when $b \gg 2$ the respective part of the integral vanishes (the same is true for $b_1$) this is evident since the integral is performed over a constant time a cosine function of period $2\pi$. It also clear that $b$ is a sum of a factor dependent on the magnet vertical dimensions and a factor dependent the ratio between $R_1$ and $R_2$ which we denote $s \equiv \frac{R_2}{R_1}$. The second factor is given by:

$$f(s) = s + \frac{1}{s}.$$ \hspace{1cm} (41)

It is obvious that $f(\infty) = \infty$ and $f(0) = \infty$. Moreover, $f'(s) = 1 - \frac{1}{s^2}$ and for $f'(s) = 0$ we obtain the minimum value $s = 1$ which indicates an equal radii to the magnet and the current loop. For this case $b = 2 + \frac{h^2}{R_1^2}$ and for the case that the current loop is put on the magnet $b = 2$ and $g(2) = -\frac{8}{3}$. For a "thick" magnet $h_1 \to \infty$ and thus also $b_1 \to \infty$ so that $g(b_1) \to -\frac{8}{\sqrt{b_1}}$ which is unfortunately a rather slow decrease. Usually the magnet does not have to be too thick only enough to make $|g(b_1)| < |g(b)|$. The function $g(b)$ can be written explicitly in terms of the elliptic functions $Ee(m)$ and
Figure 2: The function $g(b)$.

$Ke(m)$ as:

$$g(b) = \frac{1}{3} \{ \sqrt{b - 2} [-b Ee(\frac{4}{2-b}) + (b + 2) Ke(\frac{4}{2-b})] + \sqrt{b + 2} [-b Ee(\frac{4}{2+b}) + (b - 2) Ke(\frac{4}{2+b})] \}$$

$Ee(m) \equiv \int_0^{\frac{\pi}{2}} d\theta \sqrt{1 - m \sin^2 \theta}$

$Ke(m) \equiv \int_0^{\frac{\pi}{2}} d\theta \frac{1}{\sqrt{1 - m \sin^2 \theta}}$ (42)

A graph of $g(b)$ is given in figure 2.

5 Specific Examples

Let us look at the case in which the radii of the magnet and the current loop are equal and the current loop is placed on top of a thick magnet. In this case:

$$K_{12,2} = -\frac{16\pi R_1^3}{3 h^2 \hat{z}}.$$ (43)

Inserting equation (43) into equation (22) will result in:

$$\vec{F}_t = -\frac{2}{3} \mu_0 M_0 \hat{f}_1 R_1^3 \hat{z}.$$ (44)
However, since the residual magnetic flux density is related to the magnetization as \( M_0 = \frac{B_r}{\mu_0} \) we may write:

\[
\vec{F}_t = -\frac{2}{3} B_r \dot{I}_1 \frac{R_1^3}{c^2} \hat{z}.
\] (45)

We notice that for strong magnets of the NdFeB type the residual magnetic flux density \( B_r \approx 1 - 1.4 \text{ Tesla} \). Hence the pre factor \( \frac{2}{3} B_r \) is of order 1.

Let us now look at the more general case of a thick magnet with a current loop on top but now the radii of the cylindrical magnet and the current loop are not equal hence \( h = 0, s \neq 1 \) and the force formula takes the form:

\[
\vec{F}_t = -\frac{1}{4} B_r \ddot{I}_1 \frac{R_1^3}{c^2} s^{1.5} g(s + \frac{1}{s}) \hat{z}.
\] (46)

Since:

\[
\lim_{s \to \infty} -s^{1.5} g(s + \frac{1}{s}) = \pi s
\] (47)

We arrive at the approximate force equation in the case that the radius of the magnet cylinder is much bigger than the radius of the loop current:

\[
\vec{F}_t = \pi \frac{1}{4} B_r \ddot{I}_1 \frac{R_2^2}{\tau^2} \hat{z}.
\] (48)

If we take the second derivative \( \ddot{I}_1 \) to be of the order \( \ddot{I}_1 \approx \frac{I_p}{\tau^2} \) we obtain:

\[
\vec{F}_t \approx \pi \frac{1}{4} B_r I_p \left( \frac{R_2^2}{\tau^2} \right) R_2 \hat{z}.
\] (49)

we see that the decisive factor is the ration of the current rising time and the time it will take a light signal to travel across the current loop.

Finally we would like to address the question of the possibility of the device to lift from the ground for this the force generated by the device should be larger or equal to the gravitational force

\[
F_g = gm = g \rho V = g \rho L \pi R_2^2
\] (50)

In the above \( g \) is the gravitational acceleration on earth (\( \sim 9.8 \frac{m}{s^2} \) not to be confused with the function \( g(b) \)), \( m \) is the mass of the magnet, \( \rho \) is the mass density of the magnet and \( V \) is the volume of the magnet. The ratio of relativistic and gravitational forces is given by:

\[
\frac{F_t}{F_g} = \frac{-\frac{1}{4} B_r \ddot{I}_1 \frac{R_1^3}{c^2} s^{1.5} g(s + \frac{1}{s})}{g \rho L \pi s^2 R_1^2} = \frac{B_r \ddot{I}_1 \frac{R_2}{\tau^2}}{4 g \rho L \pi} \left( -g(s + \frac{1}{s}) \right)
\] (51)
If the magnet is large with respect to the current loop we can take the limit:

\[
\lim_{s \to \infty} \left( \frac{-g(s + \frac{1}{s})}{\sqrt{s^3}} \right) = \frac{\pi}{s^2}
\]

(52)

Which leads to the force ratio:

\[
\frac{F_t}{F_g} \approx \frac{B_r \iota_1 \frac{R_2}{R_1}^2}{4g \rho L s^2}.
\]

(53)

Now since the current loops are small we may consider to put \( N \) loops on the magnet the maximum number in a single layer would be: \( N = \frac{R_2^2}{R_1^2} = s^2 \)

hence the total force ratio is:

\[
\frac{N F_t}{F_g} \approx \frac{B_r \iota_1 \frac{R_2}{R_1}^2}{4g \rho L}.
\]

(54)

For a magnet of radius of 1 meter and thickens of 1 meter we obtain for a NdFeB magnet the density is \( \rho = 7500 \frac{kg}{m^3} \) hence we need at least a \( \iota_1 \sim 1.9 * 10^{22} \frac{A}{s} \) for this flying saucer to fly. This type of acceleration it typical to microwave currents of frequency 10 GHz and current amplitude of about 4.8 Ampere.

6 Conclusion

We have shown in this paper that in general Newton’s third law is not compatible with the principles of special relativity and the total force on a system of a current loop and a permanent magnet system is not zero. Still momentum is conserved if one takes the field momentum into account.

We have developed simple formulae for the total force in various cases including the case of equal radii of magnet and current loop and the case of a small current loop. This was extrapolated to the case of multiple loops and a specific example was done that the force of the system is equals its weight.

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