SEARCH FOR FLAVOURED MULTIQUARKS IN A SIMPLE BAG MODEL

S. Zouzou

Institut de Physique, Université de Constantine, Algérie

J.-M. Richard

Institut des Sciences Nucléaires, Université Joseph Fourier - CNRS - IN2P3

53, avenue des Martyrs, Grenoble, France

(March 26, 2022)

Abstract

We use a bag model to study flavoured mesonic (Qqq̄) and baryonic (QQqq̄) states, where one heavy quark Q is associated with light quarks or antiquarks, and search for possible stable multiquarks. No bound state is found. However some states lie not too high above their dissociation threshold, suggesting the possibility of resonances, or perhaps bound states in improved models.

12.40. Aa, 14.20.Kp, 14.40.Jz
I. INTRODUCTION

The search for multiquark states is no longer as fashionable as it used to be at the end of the 70’s [1,2], but remains a very important issue of the physics of confinement. A renewed interest is however observed in the physics of hadrons with heavy flavour, and there is a rich literature on heavy-quark symmetry, heavy-quark effective theory, etc. [3,4]. The structure of flavoured mesons ($Qar{Q}$) and baryons ($Qqq$) and ($QQq$) seems better understood now, and reliable extrapolations towards exotic configurations like the “Tetraquark” ($Qqar{q}ar{q}$) and the “Pentaquark” ($Qqqqq$) sectors will become feasible. (Here and throughout this paper, $Q$ denotes a charmed or bottom quark, $q$ a light quark, which is either the strange $s$, or an ordinary $q_0 = u$ or $d$.) It is useful to summarize what can be learned on these flavoured exotics from simple models before considering more sophisticated approaches.

Many papers have been devoted to multiquark spectroscopy in simple potential models or in bag models of various types. It is impossible to list all relevant references. Let us mention, for instance: in the “Hexaquark” or dibaryon sector, the pioneering work by Jaffe on the H particle ($ssuudd$) [5], and subsequent studies by the Nijmegen group [3,4]; in the 5-quark sector, again the work of the Nijmegen group [3,4] or, more recently by Lipkin and the Grenoble group on the flavoured pentaquark [8]; in the 4-quark sector, the numerous speculations about scalar mesons [9] or the observation that flavour-independence inevitably leads to stable tetraquark states ($QQqar{q}$) if the mass ratio $m(Q)/m(q)$ is large enough [10].

The various tentative multiquarks experience different aspects of quark forces. A deeply bound state would test how the Coulomb-plus-linear potential of quarkonium applies when more than two or three constituents are lying close together. On the other hand, the loosely bound meson–meson molecules, predicted by several authors [11,12], would be more sensitive to the long-range part, or say, the Van-der-Waals or Yukawa regime of the interaction.

The analogy with atomic physics might provide some guidance, to guess which flavour configurations are the most favourable for stable multiquarks. The reason is that in both cases, we have a central potential that does not depend on the mass of the constituents,
a property called universality, or flavour independence. For instance, the H⁻ ion \( (pe^-e^-) \) with \( m(p) \gg m(e) \) has a slightly larger relative energy (as compared to its threshold) than the positronium ion \( (e^+e^-e^-) \) \[16\]; in the 4-charge sector, the so-called positronium hybride \( (pe^+e^-e^-) \) is stable against dissociation into \( (pe^-) + (e^+e^-) \), by a small margin \[17\]: these observations are felt as an encouragement to study singly-flavoured multiquarks in hadron spectroscopy.

Of course, there are dramatic differences between colour forces and Coulomb forces. In particular, the spin-dependent corrections play a more important role in hadrons than in atoms. This led to speculations on states bound essentially by the coherent effect of hyperfine forces (also called chromomagnetic forces). A systematic study of chromomagnetism for multiquarks was carried out by de Swart et al. \[18, 19\], and further developed by several authors \[18, 20\]. We shall use their techniques and results along this work.

To summarize, collective binding of multiquark configurations might result either from peculiar asymmetries of the constituent masses, as in molecular physics, or from a favourable arrangement of the spins. When building a model, one should combine the effects of electric confinement and magnetic forces. In the potential model approach, the former is described by a flavour-independent central potential, and the latter by the spin–spin potential. In the bag picture we shall adopt, we have quarks moving inside a cavity, and chromoelectric as well as chromomagnetic interactions between them.

Our paper is organized as follows. In Sec. II, we briefly present the bag model we shall use. In Sec. III, we apply this model to tetraquarks with a single heavy flavour, \( (Qq\overline{q}q) \), while the pentaquark case \( (\overline{Q}qqqq) \) is presented in Sec. IV. The discussion in Sec. V is guided by the comparison with potential models. We conclude that our model does not predict the existence of stable flavoured multiquarks, but can select the most favourable configurations, to be studied in a more elaborated theoretical framework.
II. MODEL

The bag model is well known, and hardly needs to be explained in detail. We shall restrict ourselves to a quick review, with however all the necessary equations, to make this paper self contained. For a review on the bag model, see, e.g. Refs. [21–23].

A. The MIT bag

The original fit by the MIT group [24] of groundstate mesons and baryons with \(u, d\) and \(s\) quarks corresponds to the static cavity approximation of the bag model [25]. The quark wave functions are given by the free Dirac equation inside a sphere of radius \(R\). The linear boundary conditions fix the wave number \(x/R\) of each quark species as the lowest solution of

\[
x = \left( 1 - mR - \sqrt{x^2 + m^2 R^2} \right) \tan x.
\]

The corresponding energy is \(\omega = (x^2 + m^2 R^2)^{1/2}/R\). For a given radius, the hadron energy,

\[
E(R) = \frac{4}{3} \pi BR^3 + \sum_i \omega_i - \frac{Z_0}{R} + \delta E_e + \delta E_m,
\]

combines the volume energy, the kinetic energy, the zero-point energy (which presumably includes many other corrections in an effective way), and (chromo-)electric and magnetic corrections. The electric term is

\[
\delta E_e = \frac{\alpha_e}{2R} \sum_{i,j} \tilde{\lambda}_i \cdot \tilde{\lambda}_j f_{i,j}.
\]

It vanishes if a single quark species is involved, since the colour operators \(\tilde{\lambda}_i \cdot \tilde{\lambda}_j\) sum up to zero for a colour singlet. For \(K, K^*, \Lambda, \Sigma, \Sigma^*, \Xi,\) or \(\Xi^*\), the value of

\[
\delta E_e = \frac{8\alpha_e}{3R} (f_{s,s} + f_{0,0} - 2f_{0,s})
\]

turns out to be very small. The strength \(f_{i,j}\) is calculated out of the quark densities.
\[ f_{i,j} = \int_0^1 \rho_i \rho_j \frac{du}{u^2} \]  
\[ \rho = \frac{\omega(x' - \sin^2 x'/x') - m(\cos x' \sin x' - \sin^2 x'/x')} {\omega(x - \sin^2 x/x) - m(\cos x \sin x - \sin^2 x/x)}, \]  

where \( x' = ux \). The magnetic term reads

\[ \delta E_m = -\frac{3\alpha_c}{R} \sum_{i<j} \bar{\sigma}_i \cdot \bar{\sigma}_j \bar{\lambda}_i \cdot \bar{\lambda}_j g_{i,j}, \]

with the strength \( g_{i,j} \) expressed in terms of (reduced) magnetic moments and densities

\[ g_{i,j} = \tilde{\mu}_i \tilde{\mu}_j + 2 \int_0^1 \mu_i \mu_j du, \]

\[ \tilde{\mu} = \frac{4\omega R + 2mR - 3}{12\omega R(\omega R - 1) + 6mR}, \]

\[ \bar{\mu} = \frac{(\omega R - mR)x} {[2\omega R(\omega R - 1) + mR] \sin^2 x} \left( \frac{1 - 3 \sin(2x')/4x' + \cos(2x')/2}{3x'} \right), \]

where, again, \( x' = ux \). As a result of the non-linear boundary conditions, the bag energy (4) is minimized with respect to \( R \). The results given in Table III of Ref. [24] correspond to the set of parameters and masses

\[ B^{1/4} = 0.145 \text{ GeV}, \quad Z_0 = 1.81, \quad \alpha_c = 0.55, \]  
\[ m_0 = 0, \quad m_s = 0.279 \text{ GeV}, \]

**B. Bag with one heavy quark**

Numerous improved or modified versions of the MIT bag model have been developed, with a variety of purposes. One problem was to adapt the model to include heavy quarks. In the case of quarkonium states \((QQ)\), or triple-flavour baryons \((QQQ)\), an “adiabatic” bag model was proposed \([26, 30]\): for fixed interquark separations, the shape and size of the bag is adjusted to minimize the energy of the gluon field, and this minimum is read as the interquark potential, and inserted into the Schrödinger equation.

The case of hadrons with one heavy quark was treated by Izatt et al. \([31]\). They first introduce a running coupling constant, which depends on the bag radius (we keep here the MIT notation \( \alpha_c = \alpha_s/4 \))
\[ \alpha_c = \frac{\pi}{18 \ln (1 + 1/(\Lambda R))}. \]  

Another change is that the bag radius \( R \) is adjusted in the approximation where only the volume, kinetic (and mass), and zero-point energies are included. The electric and magnetic terms are then computed with this radius already fixed.

The model holds for ordinary hadrons, but in this case a centre-of-mass correction is applied to the total energy \( E \), leading to the hadron mass

\[ E' = \left[ E^2 - \sum_i \bar{x}^2_i/R^2 \right]^{1/2}. \]  

For hadrons with charm or beauty, it is assumed that the heavy quark stays at the centre of the bag. No centre-of-mass correction is applied. The electric and magnetic terms involving the heavy quark are modified as follows (\( M \) denotes the mass of the heavy quark, \( m \) that of a light quark)

\[ f_{M,M} = -1 \]  
\[ f_{M,m} = N_x \omega R \left[ \frac{\sin^2 x}{2x^2} + \frac{\sin x \cos x}{x} - \frac{3}{2} + \int_0^{2x} \frac{1 - \cos u}{u} du \right] - N m R \left[ \frac{\sin^2 x}{2x} - \frac{x}{2} \right] \]  
\[ N^{-1} = \omega R (x - \sin^2 x/x) - m R \sin x (\cos x - \sin x/x) \]  
\[ \tilde{\mu}(M) = \frac{1}{(2MR)} \]  
\[ g_{M,m} = \tilde{\mu}(M) \tilde{\mu}(m) \left[ 1 + \frac{2x^3/3 - x + \sin(2x)/2}{x - 3 \sin(2x)/4 + x \cos(2x)/2} \right] \]  

We shall use the model of Ref. [31] with a slightly modified set of parameters

\[ B^{1/4} = 0.1383 \text{ GeV}, \quad \Lambda = 0.400 \text{ GeV}, \quad Z_0 = 0.574, \]  
\[ \bar{x}_0 = x_0 \ (= 2.042..), \quad \bar{x}_s = 2.3, \]  
\[ m_0 = 0 \quad m_s = 0.273 \text{ GeV}, \quad m_c = 2.004 \text{ GeV}, \quad m_b = 5.360 \text{ GeV}. \]  

A reasonable description of the spectrum of ordinary, charmed or beautiful hadrons can be achieved. This is illustrated in Table I. We are not too worried anyhow about obtaining a very good fit, since we shall use our computed masses instead of the experimental ones, when estimating the thresholds of multiquark candidates. One can check in Table I that the
model does not overestimate the strength of magnetic forces, which are a potential source of collective binding.

This simple model explicitly incorporates the property of “heavy quark symmetry”, since the bag radius and the light-quark wave function are exactly the same for \( (c\bar{q}) \) and \( (b\bar{q}) \), or \( (cqq) \) and \( (bqq) \). We do expect some recoil corrections in the case of charm, but we hope they are somehow incorporated in fixing the values of the parameters.

We note an appreciable freedom in fixing the values of the parameters, when comparing the MIT values [8] with our set of parameters [12]. The bag model does not constrain \( \alpha_c, B \), and \( Z_0 \) too much when one tries to reproduce the observed masses, and introducing explicit centre-of-mass corrections implies some new tuning. A similar discrepancy is observed when comparing [12] to the parameters used in Ref. [28] for charmonium: \( B^{1/4} = 0.235 \text{ GeV} \), and \( m_c = 1.35 \text{ GeV} \).

### III. TETRAQUARK

The model of Sec. II B can be applied to \( (Qq\bar{q}\bar{q}) \) configurations. We look at the \( J^P = 0^+ \) groundstate. The main changes, as compared to ordinary mesons and baryons, concern the electric and magnetic terms, which are now operators in the space of the possible spin–colour wave functions. One has to diagonalize the cumulated contribution of these electric and magnetic corrections. Details are provided in Appendix.

An example is examined in Table II. The various contributions to the energies are displayed for a \( (cs\bar{u}\bar{u}) \) state, and for the hadrons constituting its threshold. The bag radius is larger for \( (cs\bar{u}\bar{u}) \) than for \( (c\bar{u}) \) and \( (s\bar{u}) \) mesons, and this pushes \( (cs\bar{u}\bar{u}) \) above its dissociation threshold.

The results for all flavour configurations are shown in Table III. The best candidate seems \( (Qs\bar{u}\bar{d}) \) with isospin \( \bar{I} = 0 \). It benefits from the electric interaction between the heaviest quarks, \( Q \) and \( s \), and from the magnetic attraction between the lightest, \( u \) and \( d \), whereas its threshold \( (Q\bar{q}_0) + (s\bar{q}_0) \) combines the constituents in a less favourable way.
However $(Qsüd)$ is not bound in our bag model.

**IV. PENTAQUARK**

The calculation of $(\overline{Q}qqqq)$ can be done with the same bag model. Details on the electric and magnetic terms, and on the spin-colour wave functions are given in Appendix.

From previous studies [8], we know that the most favourable configuration are those where the quarks $(qqqq)$ are in a triplet state of the flavour group $SU(3)_F$, and essentially in a state of spin and parity $j^p = 0^+$. We thus restrict ourselves to study the flavour configurations $(\overline{Q}udss)$ and $(\overline{Q}sudd)$ (and its charge symmetric with $u \leftrightarrow d$) with total spin and parity $J^P = (1/2)^-$. The results are displayed in Table IV. We essentially agree with Ref. [32], and our conclusions are identical: the pentaquark is bound in the limit where $m(Q) = \infty$ and $m(s) = m(q_0)$, but the stability does not survive the heavy quark mass being finite and in the first place flavour $SU(3)_F$ symmetry being broken.

**V. COMMENTS AND CONCLUSIONS**

The multiquark masses listed in Tables III–IV result from the whole model. It seems useful to analyze the role of the various contributions.

Let us first consider the crudest version of the bag model, with two terms in the energy, corresponding to the volume and quark contributions

$$E = bR^3 + aR^{-1}, \quad (13)$$

where $b = 4\pi B/3$. If $a$ is independent of the radius $R$, the minimization with respect to $R$ leads to a kind of virial theorem, where the bag energy is four times larger than the volume term, and exhibits a very simple behaviour

$$E_{bag} = \min_R (E) \propto a^{3/4}. \quad (14)$$
Thus, in a model with \( n \) massless quarks in a cavity and without correction for zero-point energy, \( a = nx_0 \) grows as \( n \), and the hadron mass behaves as \( M(n) \propto n^{3/4} \). This would mean \( M(4) < 2M(2) \), \( M(6) < 2M(3) \), etc., i.e. stability for many multiquarks.

In actual models, \( a \) depends slightly on \( R \) for quarks of finite mass, and the term \( a/R \) incorporates the electric and magnetic corrections, as well as the zero-point energy. The \( n \) dependence is thus more involved.

Already, when going from \( n = 2 \) (mesons) to \( n = 3 \) (baryons), the MIT model or its modified versions do not behave as \( M \propto n^{3/4} \), which would give

\[
\frac{M(qqq)}{3} < \frac{M(q\bar{q})}{2},
\]

in contradiction with simple potential models \[33\], and with experimental data \[34\] (compare for instance the spin-averaged mass of \( \pi \) and \( \rho \) with that of \( N \) and \( \Delta \), or \( \Phi(1020) \) with \( \Omega(1672) \)). In other words, fitting of mesons and baryons simultaneously requires an empirical \( Z_0/R \) correction \[24\], and this prevents a proliferation of multiquarks in the bag model.

An illustration is provided in Table V, where the tetraquark calculation is repeated with the zero-point energy constant \( Z_0 \) set to zero. We no longer fit the hadron masses, but this is not too important, since stability is estimated using the calculated masses for the ordinary hadrons. We observe in Table V that the tetraquark mass becomes closer to the corresponding threshold, and sometimes even smaller than the threshold.

Another crucial ingredient for binding multiquarks is the strength of 2-body correlations at short distances. In investigatory scans of the spin-flavour space, one sometimes considers simplified Hamiltonians of the type

\[
H = K \sum_{i<j} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\lambda}_i \cdot \vec{\lambda}_j a_{i,j},
\]

and looks at the spectrum obtained with some crude ansatz for \( a_{i,j} \). For instance \( a_{i,j} \) can be taken as independent of the flavour of the quarks \( i \) and \( j \), and of the particular hadron one considers; or one can assume \( a_{i,j} = a'/m_im_j \), and take \( a' \) as being constant and universal; etc.
Specific dynamical calculations of the correlation factors $a_{i,j}$ have been performed, or are implicit in some of the previous works on multiquarks. Most authors used non-relativistic potential models, with a simple prescription to extrapolate the quark–antiquark potential of mesons toward the multiquark sector, for instance

$$V = -\frac{3}{16} \sum_{i<j} \tilde{\lambda}_i \cdot \tilde{\lambda}_j V_{Q\bar{Q}}(r_{ij}),$$

in naive analogy with atomic physics.

An interesting result of such calculations is that the short-range 2-body correlation is generally smaller in a multiquark state than in ordinary hadrons. This tends to weaken or to kill the binding of multiquark predicted by simple chromomagnetic models like (16).

In our bag model, we observe a similar effect: the bag radius $R$ is larger for multiquarks than for ordinary hadrons, and this lowers the strength of chromomagnetic terms.

To summarize, we confirm the conclusions of previous studies of the flavoured pentaquark: the stability predicted on the basis of simple chromomagnetic calculations disappears due to the lack of short-range correlations between light quarks. However, the configuration ($buuds$) is found not too far from being bound, and deserves to be further studied. Similarly, none of the possible tetraquark configurations ($Qq\bar{q}\bar{q}$) is found stable in our model, but those where the antiquarks have strangeness $S = 0$ and isospin $I = 0$ seem the best candidates in the event that improving the model would provides some additional attraction.

ACKNOWLEDGMENTS

We would like to thank our colleagues of Grenoble for several fruitful discussions and for the hospitality extended to S.R.Z. during a visit where this work was completed, and R. Barrett for useful comments on the manuscript. J.M.R. benefitted from the warm hospitality of the ECT* at Trento.
APPENDIX A:

We now provide some details about the spin–colour wave functions, and the electric and magnetic terms, for tetraquark and pentaquark states. The simplest case is \((Qs\bar{s}\bar{s})\) or \((Qq\bar{q}_0\bar{q}_0)\), with the antidiquark in an isospin \(\bar{I} = 1\) state. We can use the spin–colour basis

\[
|1\rangle = |(\bar{3}, 3), (1, 1)\rangle, \quad |2\rangle = |(6, 6), (0, 0)\rangle,
\]

to combine the diquark, of colour \(\bar{3}\) or 6 and spin 0 or 1, with the antidiquark, whose spin and colour are correlated by the Pauli principle. The electric term is scalar, with value

\[
\delta E_e = -\frac{8\alpha_c}{3R} \left[ 2f_{M,m} - f_{m,m} - f_{M,M} \right],
\]

and one has to diagonalize the colour–spin operator arising in the magnetic term

\[
\delta E_m = -\frac{3\alpha_c}{2R} (g_{m,m} + g_{M,m}) \begin{pmatrix} 16/3 & 8\sqrt{6} \\ 8\sqrt{6} & -8 \end{pmatrix}
\]

The eigenvalue of interest in the above matrix is \((-8 + 8\sqrt{241})/6 \simeq 19.4\).

Another simple case is \((Qq_0\bar{q}_0\bar{q}_0)\) with isospin \(\bar{I} = 0\) for the antiquarks. The basis is now

\[
|3\rangle = |(\bar{3}, 3), (0, 0)\rangle, \quad |4\rangle = |(6, 6), (1, 1)\rangle.
\]

The electric term is unchanged. The magnetic term is similar to (A3), besides the matrix which is now

\[
\begin{pmatrix} 16 & 8\sqrt{6} \\ 8\sqrt{6} & 88/3 \end{pmatrix}
\]

and whose largest eigenvalue is \((136 + 8\sqrt{241})/6 \simeq 43.4\).

For \((Qq_0\bar{s})\), or \((Qs\bar{q}_0\bar{q}_0)\) with \(\bar{I} = 1\), the basis (A1) is still appropriate, and the electric term diagonal. The matrix elements of interest are

\[
(\delta E_e)_{1,1} = \frac{8\alpha_c}{3R} (f_{1,1} + f_{2,2} + f_{3,3} - f_{1,2} - f_{1,3} - f_{2,3}), \\
(\delta E_e)_{2,2} = \frac{4\alpha_c}{3R} (2f_{1,1} + 2f_{2,2} + 5f_{3,3} + f_{1,2} - 5f_{1,3} - 5f_{2,3}),
\]
where 1 stands for the heavy quark, 2 for $q_0$ (for $s$), and 3 for $\bar{s}$ (for $\bar{q}_0$), in the case of $(Qq_0\bar{s}\bar{s})$ (respectively $(Qs\bar{q}_0\bar{q}_0)$ with $\bar{I} = 1$).

For $(Qs\bar{q}_0\bar{q}_0)$ with $\bar{I} = 0$, we use the basis (A4) and the electric and magnetic terms of interest are

\[
\begin{align*}
(\delta E_m)_{1,1} &= -\frac{8\alpha_c}{R} (2g_{1,3} - g_{1,2} + 2g_{2,3} - g_{3,3}), \\
(\delta E_m)_{2,2} &= \frac{12\alpha_c}{R} (g_{1,2} + g_{3,3}), \\
(\delta E_m)_{1,2} &= -\frac{12\sqrt{6}\alpha_c}{R} (g_{1,3} + g_{2,3}),
\end{align*}
\]

where we use the labeling 1 = $Q$, 2 = $s$, and 3 = $q_0$.

For $(Qq_0\bar{s}\bar{s})$ (the labeling corresponds to this order, and is such that $m_2 = m_4$), the whole basis (A1,A4) contributes, and the non-vanishing matrix elements are

\[
\begin{align*}
(\delta E_e)_{1,1} &= (\delta E_e)_{3,3} = \frac{4\alpha_c}{3R} (3f_{1,1} + 3f_{2,2} + 3f_{3,3} - f_{1,2} - 3f_{1,3} - 3f_{2,3} + f_{3,3}), \\
(\delta E_e)_{1,4} &= (\delta E_e)_{2,3} = \frac{2\sqrt{2}\alpha_c}{R} (f_{1,2} - f_{1,3} - f_{2,2} + f_{2,3}), \\
(\delta E_e)_{2,2} &= (\delta E_e)_{4,4} = \frac{2\alpha_c}{3R} (4f_{1,1} - 3f_{1,2} - 5f_{1,3} + 3f_{2,2} - 3f_{2,3} + 4f_{3,3})
\end{align*}
\]

and

\[
\begin{align*}
(\delta E_m)_{1,1} &= -\frac{8\alpha_c}{R} (g_{1,3} + g_{2,2}), \\
(\delta E_m)_{1,2} &= -\frac{6\sqrt{6}\alpha_c}{R} (g_{1,2} + g_{1,3} + g_{2,2} + g_{2,3}), \\
(\delta E_m)_{1,3} &= \frac{4\sqrt{3}\alpha_c}{R} (g_{1,2} - g_{1,3} - g_{2,2} + g_{2,3})
\end{align*}
\]
\begin{align}
(\delta E_m)_{1,4} &= \frac{12\sqrt{2}\alpha_c}{R} (g_{1,1} - g_{1,3} - g_{2,2} + g_{2,3}) \\
(\delta E_m)_{2,2} &= \frac{12\alpha_c}{R} (g_{1,1} + g_{2,3}) \\
(\delta E_m)_{2,4} &= \frac{10\sqrt{3}\alpha_c}{R} (g_{1,1} - g_{1,3} - g_{2,2} + g_{2,3}) \\
(\delta E_m)_{3,3} &= -\frac{24\alpha_c}{R} (g_{1,1} + g_{2,3}) \\
(\delta E_m)_{3,4} &= -\frac{6\sqrt{6}\alpha_c}{R} (g_{1,1} + g_{1,3} + g_{2,2} + g_{2,3}) \\
(\delta E_m)_{4,4} &= -\frac{4\alpha_c}{R} (6g_{1,1} + 5g_{1,3} + 5g_{2,2} + 6g_{2,3})
\end{align}

For the Pentaquark $Qqqqq$, the quark pairs $(2,3)$ and $(4,5)$ are either in colour 3 or 6, provided the four quarks form a colour 3 state, which neutralizes the colour of the heavy antiquark. The spins of these pairs are either 0 or 1, the spin of $(qqqq)$ being mostly $j = 0$, with a small $j = 1$ admixture, so that the spin of the pentaquark is $J = 1/2$. We consider the configurations $Qssud$ and $Quuds$ (and $Qddus$), its isospin partner) for which the quarks 2 and 3 are identical and thus have their colours and spins correlated. A possible basis is

\begin{align}
|1\rangle &= |(3,\bar{3};(1,1)0)\rangle \\
|2\rangle &= |(3,\bar{3};(1,1)0)\rangle \\
|3\rangle &= |(6,\bar{3};(0,0)0)\rangle \\
|4\rangle &= |(3,\bar{3};(1,1)1)\rangle \\
|5\rangle &= |(3,\bar{3};(1,1)1)\rangle \\
|6\rangle &= |(6,\bar{3};(0,1)1)\rangle \\
|7\rangle &= |(\bar{3},\bar{3};(1,0)1)\rangle \\
|8\rangle &= |(3,\bar{3};(1,0)1)\rangle
\end{align}

For $Qssud$, we use the labeling $1 = Q$, $2 = s$, and $3 = q_0$ to list the matrix elements

\begin{align}
(\delta E_e)_{1,1} &= \frac{8\alpha_c}{3R} (f_{1,1} + f_{2,2} + f_{3,3} - f_{1,2} - f_{1,3} - f_{2,3}) \\
(\delta E_e)_{2,2} &= \frac{4\alpha_c}{3R} (2f_{1,1} + 2f_{2,2} + 5f_{3,3} + f_{1,2} - 5f_{1,3} - 5f_{2,3}) \\
(\delta E_e)_{3,3} &= \frac{4\alpha_c}{3R} (2f_{1,1} + 5f_{2,2} + 2f_{3,3} - 5f_{1,2} + f_{1,3} - 5f_{2,3}) \\
(\delta E_e)_{4,4} &= (\delta E_e)_{7,7} = (\delta E_e)_{1,1} \\
(\delta E_e)_{5,5} &= (\delta E_e)_{8,8} = (\delta E_e)_{2,2} \\
(\delta E_e)_{6,6} &= (\delta E_e)_{3,3}
\end{align}

and
\[(\delta E_m)_{1,1} = \frac{8\alpha_c}{R} (g_{2,2} + g_{3,3} - 2g_{2,3})\]
\[(\delta E_m)_{1,4} = -\frac{8\sqrt{2}\alpha_c}{R} (g_{1,2} - g_{1,3})\]
\[(\delta E_m)_{1,6} = \frac{12\sqrt{2}\alpha_c}{R} g_{1,2}\]
\[(\delta E_m)_{1,8} = -\frac{12\sqrt{2}\alpha_c}{R} g_{1,3}\]
\[(\delta E_m)_{2,2} = \frac{4\alpha_c}{R} (2g_{2,2} - g_{3,3} - 10g_{2,3})\]
\[(\delta E_m)_{2,3} = -\frac{12\sqrt{3}\alpha_c}{R} g_{2,3}\]
\[(\delta E_m)_{2,5} = \frac{4\sqrt{2}\alpha_c}{R} (g_{1,2} + 5g_{1,3})\]
\[(\delta E_m)_{2,7} = -\frac{12\sqrt{2}\alpha_c}{R} g_{1,3}\]
\[(\delta E_m)_{3,3} = \frac{12\alpha_c}{R} (g_{2,2} - 2g_{3,3})\]
\[(\delta E_m)_{3,7} = -\frac{12\sqrt{6}\alpha_c}{R} g_{1,2}\]
\[(\delta E_m)_{4,4} = \frac{8\alpha_c}{R} (g_{2,2} + g_{3,3} - g_{1,2} - g_{1,3} - g_{2,3})\]
\[(\delta E_m)_{4,6} = \frac{24\alpha_c}{R} (g_{1,2} + g_{2,3})\]
\[(\delta E_m)_{4,8} = \frac{24\alpha_c}{R} (g_{1,3} + g_{2,3})\]
\[(\delta E_m)_{5,5} = \frac{4\alpha_c}{R} (2g_{2,2} - g_{3,3} + g_{1,2} - 5g_{1,3} - 5g_{2,3})\]
\[(\delta E_m)_{5,7} = \frac{24\alpha_c}{R} (g_{1,3} + g_{2,3})\]
\[(\delta E_m)_{6,6} = \frac{4\alpha_c}{R} (3g_{2,2} + 2g_{3,3} + 2g_{1,3})\]
\[(\delta E_m)_{6,8} = \frac{12\alpha_c}{R} g_{2,3}\]
\[(\delta E_m)_{7,7} = \frac{8\alpha_c}{R} (g_{2,2} - 3g_{3,3} - 2g_{1,2})\]
\[(\delta E_m)_{8,8} = \frac{4\alpha_c}{R} (2g_{2,2} + 3g_{3,3} + 2g_{1,2})\).

For \((Quuds)\), we use the labeling \(1 = Q, 2 = q_0, \) and \(3 = s\) to list the matrix elements

\[(\delta E_e)_{1,1} = \frac{4\alpha_c}{3R} (2f_{1,1} + 3f_{2,2} + 2f_{3,3} - 3f_{1,2} - f_{1,3} - 3f_{2,3})\]
\[(\delta E_e)_{1,2} = \frac{2\sqrt{2}\alpha_c}{3R} (-f_{2,2} + f_{1,2} - f_{1,3} + f_{2,3})\]
\[(\delta E_e)_{2,2} = \frac{2\alpha_c}{3R} (4f_{1,1} + 3f_{2,2} + 4f_{3,3} - 3f_{1,2} - 5f_{1,3} - 3f_{2,3})\]

\[(\delta E_e)_{3,3} = \frac{2\alpha_c}{3R} (4f_{1,1} + 9f_{2,2} + 4f_{3,3} - 9f_{1,2} + f_{1,3} - 9f_{2,3})\]

\[(\delta E_e)_{4,4} = (\delta E_e)_{7,7} = (\delta E_e)_{1,1}\]

\[(\delta E_e)_{5,5} = (\delta E_e)_{8,8} = (\delta E_e)_{2,2}\]

\[(\delta E_e)_{6,6} = (\delta E_e)_{3,3}\]

\[(\delta E_e)_{4,5} = (\delta E_e)_{7,8} = (\delta E_e)_{1,2}\]

and

\[(\delta E_m)_{1,2} = -\frac{12\sqrt{2}\alpha_c}{R} (g_{2,2} - g_{2,3})\]

\[(\delta E_m)_{1,3} = \frac{6\sqrt{6}\alpha_c}{R} (g_{2,2} - g_{2,3})\]

\[(\delta E_m)_{1,4} = -\frac{4\sqrt{2}\alpha_c}{R} (g_{1,2} - g_{1,3})\]

\[(\delta E_m)_{1,5} = -\frac{12\alpha_c}{R} (g_{1,2} - g_{1,3})\]

\[(\delta E_m)_{1,6} = \frac{12\sqrt{2}\alpha_c}{R} g_{1,2}\]

\[(\delta E_m)_{1,7} = \frac{4\alpha_c}{R} (g_{1,2} - g_{1,3})\]

\[(\delta E_m)_{1,8} = -\frac{6\sqrt{2}\alpha_c}{R} (g_{1,2} + g_{1,3})\]

\[(\delta E_m)_{2,2} = -\frac{12\alpha_c}{R} (g_{2,2} + 2g_{2,3})\]

\[(\delta E_m)_{2,3} = -\frac{6\sqrt{3}\alpha_c}{R} (g_{2,2} + g_{2,3})\]

\[(\delta E_m)_{2,4} = -\frac{12\alpha_c}{R} (g_{1,2} - g_{1,3})\]

\[(\delta E_m)_{2,5} = \frac{2\sqrt{2}\alpha_c}{R} (7g_{1,2} + 5g_{1,3})\]

\[(\delta E_m)_{2,7} = -\frac{6\sqrt{2}\alpha_c}{R} (g_{1,2} + g_{1,3})\]

\[(\delta E_m)_{2,8} = \frac{10\alpha_c}{R} (g_{1,2} - g_{1,3})\]

\[(\delta E_m)_{3,3} = \frac{12\alpha_c}{R} (g_{2,2} - 2g_{3,3})\]

\[(\delta E_m)_{3,6} = \frac{2\sqrt{3}\alpha_c}{R} (g_{1,2} - g_{1,3})\]
\( (\delta E_m)_{3,7} = -\frac{12\sqrt{6}\alpha_c}{R} g_{1,2} \) \hfill (A14)

\( (\delta E_m)_{4,4} = \frac{4\alpha_c}{R} (g_{2,2} - 3g_{1,2} - g_{1,3} + g_{2,3}) \)

\( (\delta E_m)_{4,5} = -\frac{6\sqrt{2}\alpha_c}{R} (g_{2,2} - g_{1,2} + g_{1,3} - g_{2,3}) \)

\( (\delta E_m)_{4,6} = \frac{12\alpha_c}{R} (g_{2,2} + 2g_{1,2} + g_{2,3}) \)

\( (\delta E_m)_{4,7} = \frac{4\sqrt{2}\alpha_c}{R} (g_{2,2} - g_{1,2} + g_{1,3} - g_{2,3}) \)

\( (\delta E_m)_{4,8} = \frac{12\alpha_c}{R} (g_{2,2} + g_{1,2} + g_{1,3} + g_{2,3}) \)

\( (\delta E_m)_{5,5} = -\frac{2\alpha_c}{R} (g_{2,2} + 3g_{1,2} + 5g_{1,3} + 7g_{2,3}) \)

\( (\delta E_m)_{5,6} = -\frac{6\sqrt{2}\alpha_c}{R} (g_{2,2} - g_{2,3}) \)

\( (\delta E_m)_{5,7} = \frac{12\alpha_c}{R} (g_{2,2} + g_{1,2} + g_{1,3} + g_{2,3}) \)

\( (\delta E_m)_{5,8} = \frac{10\sqrt{2}\alpha_c}{R} (g_{2,2} - g_{1,2} + g_{1,3} - g_{2,3}) \)

\( (\delta E_m)_{6,6} = \frac{4\alpha_c}{R} (3g_{2,2} + g_{1,2} + g_{1,3} + 2g_{2,3}) \)

\( (\delta E_m)_{6,7} = -\frac{6\sqrt{2}\alpha_c}{R} (g_{2,2} - g_{2,3}) \)

\( (\delta E_m)_{6,8} = \frac{6\alpha_c}{R} (g_{2,2} + g_{2,3}) \)

\( (\delta E_m)_{7,7} = \frac{8\alpha_c}{R} (g_{2,2} - 2g_{1,2} - 3g_{2,3}) \)

\( (\delta E_m)_{8,8} = \frac{4\alpha_c}{R} (2g_{2,2} + 2g_{1,2} + 3g_{2,3}) \).
TABLE I. Some ordinary or charmed hadrons in the simple bag model of Sec. II B.

| State         | Mass | Exp. |
|---------------|------|------|
| $\pi(q_0\bar{q}_0)$ | 0.11 | 0.14 |
| $\rho(q_0\bar{q}_0)$ | 0.77 | 0.77 |
| $K(q_0\bar{s})$      | 0.51 | 0.50 |
| $K^*(q_0\bar{s})$    | 0.92 | 0.89 |
| $\Phi(ss)$           | 1.07 | 1.02 |
| $N(q_0q_0q_0)$       | 0.95 | 0.94 |
| $\Delta(q_0q_0q_0)$  | 1.23 | 1.23 |
| $\Lambda(sqq_0)$     | 1.12 | 1.11 |
| $\Sigma(sqq_0)$      | 1.16 | 1.19 |
| $\Sigma^*(sqq_0)$    | 1.34 | 1.38 |
| $\Xi(ssq_0)$         | 1.31 | 1.32 |
| $\Xi^*(ssq_0)$       | 1.49 | 1.53 |
| $\Omega^-(sss)$      | 1.68 | 1.67 |
| $D(c\bar{q}_0)$      | 1.85 | 1.87 |
| $D^*(c\bar{q}_0)$    | 2.03 | 2.01 |
| $D_s(c\bar{s})$      | 1.95 | 1.97 |
| $D_s^*(c\bar{s})$    | 2.11 | 2.11 |
| $\Lambda_c(cq_0q_0)$ | 2.30 | 2.28 |
| $\Sigma_c(cq_0q_0)$  | 2.40 | 2.45 |
| $\Xi_c(csq_0)$       | 2.46 | 2.47 |
TABLE II. Various contributions to the energy of \((cs\bar{u})\), \((c\bar{u})\), and \((s\bar{u})\): volume energy, quark mass and kinetic energy, zero-point energy, electric and magnetic terms, and centre-of-mass corrections, respectively, in GeV. Also shown are the bag radius, in GeV\(^{-1}\), and the coupling \(\alpha_c\).

| State | radius | \(\alpha_c\) | volume | quark | zero-point | el.+mag. | c.o.m. | mass |
|-------|--------|--------------|--------|-------|------------|----------|--------|------|
| \(s\bar{u}\) | 4.15   | 0.37         | 0.11   | 1.13  | −0.14      | −0.21    | −0.39  | 0.51 |
| \(c\bar{u}\) | 4.23   | 0.38         | 0.12   | 0.48  | −0.14      | −0.61    |        | 1.85 |
| \(cs\bar{u}\) | 5.85   | 0.49         | 0.31   | 1.21  | −0.10      | −0.63    |        | 2.79 |

TABLE III. Four-quark configurations with charm. The mass (in GeV) is compared to the lowest dissociation threshold.

| Content | Mass | State | Mass | Content | Mass |
|---------|------|-------|------|---------|------|
| \(cs\bar{s}\bar{s}\) | 3.07 | \((c\bar{s}) + (s\bar{s})\) | 2.69 | 
| \(cq_0(\bar{q}_0\bar{q}_0)_{I=1}\) | 2.61 | \((c\bar{q}_0) + (q_0\bar{q}_0)\) | 1.96 | 
| \(cq_0\bar{s}\bar{s}\) | 2.90 | \((c\bar{s}) + (q_0\bar{s})\) | 2.46 | 
| \(cs(\bar{q}_0\bar{q}_0)_{I=1}\) | 2.79 | \((c\bar{q}_0) + (s\bar{q}_0)\) | 2.36 | 
| \(cq_0(\bar{q}_0\bar{q}_0)_{I=0}\) | 2.36 | \((c\bar{q}_0) + (q_0\bar{q}_0)\) | 1.96 | 
| \(cs(\bar{q}_0\bar{q}_0)_{I=0}\) | 2.59 | \((c\bar{q}_0) + (s\bar{q}_0)\) | 2.36 | 
| \(cq_0(s\bar{q}_0)\) | 2.51 | \((c\bar{s}) + (q_0\bar{q}_0)\) | 2.06 | 
| \(cs(\bar{q}_0\bar{s})\) | 2.73 | \((c\bar{s}) + (s\bar{q}_0)\) | 2.46 |
TABLE IV. Five-quark configurations with charm or beauty. The mass (in GeV) is compared to the lowest dissociation threshold.

| State  | Mass | Content   | Threshold | Mass |
|--------|------|-----------|-----------|------|
| \( \bar{c}s\bar{s}u\bar{d} \) | 3.16 | \((\bar{c}s) + (sud)\) | 3.07     |
| \( \bar{c}u\bar{u}d\bar{s} \) | 2.97 | \((\bar{c}s) + (uud)\) | 2.89     |
| \( \bar{b}s\bar{s}u\bar{d} \) | 6.58 | \((\bar{b}s) + (sud)\) | 6.50     |
| \( \bar{b}u\bar{u}d\bar{s} \) | 6.39 | \((\bar{b}s) + (uud)\) | 6.32     |

TABLE V. Four-quark configurations with charm, and the corresponding thresholds, in a model where the zero-point energy parameter \( Z_0 \) is set to zero.

| State               | Mass | Content   | Threshold | Mass |
|---------------------|------|-----------|-----------|------|
| \( cs\bar{s}\bar{s} \) | 3.17 | \((c\bar{s}) + (s\bar{s})\) | 3.04     |
| \( c\bar{q}_0(\bar{q}_0\bar{q}_0)_{I=1} \) | 2.71 | \((c\bar{q}_0) + (q_0\bar{q}_0)\) | 2.49     |
| \( c\bar{q}_0\bar{s}\bar{s} \) | 3.00 | \((c\bar{s}) + (q_0\bar{s})\) | 2.83     |
| \( cs(\bar{q}_0\bar{q}_0)_{I=1} \) | 2.89 | \((c\bar{q}_0) + (s\bar{q}_0)\) | 2.74     |
| \( c\bar{q}_0(\bar{q}_0\bar{q}_0)_{I=0} \) | 2.46 | \((c\bar{q}_0) + (q_0\bar{q}_0)\) | 2.49     |
| \( cs(\bar{q}_0\bar{q}_0)_{I=0} \) | 2.69 | \((c\bar{q}_0) + (s\bar{q}_0)\) | 2.74     |
| \( c\bar{q}_0(\bar{s}\bar{q}_0) \) | 2.62 | \((c\bar{s}) + (q_0\bar{q}_0)\) | 2.58     |
| \( cs(\bar{q}_0\bar{s}) \) | 2.83 | \((c\bar{s}) + (s\bar{q}_0)\) | 2.83     |
REFERENCES

[1] Chan Hong-Mo, in Proc. IV European Antiproton Symposium, Barr, France, 1978, ed. A. Fridman (CNRS, Paris, 1979).

[2] L. Montanet, C.G. Rossi and G. Veneziano, Phys. Rep. 63, 149 (1980).

[3] M.B. Wise, Proc. CCAST Symposium on Particle Physics at the Fermi Scale, to be published.

[4] B. Grinstein, Ann. Rev. Nucl. Part. Sci. 42, 10 (1992).

[5] R.L. Jaffe, Phys. Rev. Lett. 38, 195 (1977).

[6] P.J.G. Mulders, A.T.M. Aerts and J.J. de Swart, Phys. Rev. Lett. 40, 1543 (1978).

[7] A.T.M. Aerts, P.J.G. Mulders and J.J. de Swart, Phys. Rev. D17, 260 (1978); D21, 1370 (1980).

[8] C. Gignoux, B. Silvestre-Brac and J.-M. Richard, Phys. Lett. 193B, 323 (1987); H.J. Lipkin, Phys. Lett. 195B, 484 (1987); G. Karl and P. Zencskykowski, Phys. Rev. D36, 3520 (1987); D.O. Riska and N.N. Scoccola, Phys. Lett. B299, 338 (1993).

[9] R.L. Jaffe, Phys. Rev. D15, 267 (1976); 281 (1976).

[10] J.-P. Ader, J.-M. Richard and P. Taxil, Phys. Rev. D25, 195 (1982); S. Zouzou et al., Z. Phys. C30, 457 (1986); L. Heller and J.A. Tjon, Phys. Rev. D32, 755 (1985); D35 969 (1987); J. Carlson, L. Heller and J.A. Tjon, Phys. Rev. D37, 744 (1988); H.J. Lipkin, Phys. Lett. B172, 242 (1986).

[11] K. Dooley, E.S. Swanson and T. Barnes, Phys. Lett. B275, 478 (1992).

[12] T. Barnes, Proc. Int. School on Physics with Low-Energy Antiprotons, Erice (1987) ed. F. Close et al., (Plenum, N.Y.).
[13] N.A. Törnqvist, Phys. Rev. Lett. 67, 556 (1991).

[14] J. Weinstein and N. Isgur, Phys. Rev. D25, 588 (1983); D41, 2236 (1990).

[15] A.V. Manohar and M.B. Wise, CERN-TH 6744/92.

[16] E.A.G. Armour and W. Byers Brown, Accounts of Chemical Research, 26 (1993) 168.

[17] D.M. Schrader, F.M. Jacobsen, N.P. Franden, and U. Mikkelsen, Phys. Rev. Lett. 69, 57 (1992); 69, 2880 (1992) (E); and references herein.

[18] D.B. Lichtenberg and R. Roncaglia, Proc. 2nd Int. Workshop on Diquarks, Turin, 1992, eds. M. Anselmino and E. Predazzi.

[19] C. Semay and B. Silvestre-Brac, Z. Phys. C57, 275 (1993).

[20] H. Høgaasen, in From Collective states to Quarks in Nuclei, Proc. Bologna Workshop, 1980, ed. H. Arenhövel and A.M. Saruis (Springer-Verlag, Berlin, 1981).

[21] R.L. Jaffe, in Point like Structures Inside and Outside Hadrons, ed. N. Zichichi (Plenum Press, N.Y., 1982).

[22] C.E. DeTar and J.F. Donoghue, Ann. Rev. Nucl. Part. Sci. 33, 735 (1983).

[23] A.W. Thomas, in Advances in Nuclear Physics, vol. 13, ed. J. Negele and E. Vogt (Plenum Press, N.Y. 1984).

[24] T. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D12, 2060 (1975).

[25] A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn and V.F. Weisskopf, Phys. Rev. D9, 3471 (1974).

[26] P. Hasenfratz and J. Kuti, Phys. Rep. 40C, 75 (1978).

[27] W.C. Haxton and L. Heller, Phys. Rev. D22, 1198 (1980).

[28] P. Hasenfratz, R.R. Horgan, J. Kuti and J.M. Richard, Phys. Lett. 95B, 299 (1980).
[29] P. Hasenfratz, R.R. Horgan, J. Kuti and J.M. Richard, Phys. Lett. 94B, 401 (1980).

[30] J. Baacke, Y. Igarashi and G. Kasperidus, Z. Phys. C13, 131 (1982).

[31] D. Izatt, C. DeTar and M. Stephenson, Nucl. Phys. B199, 269 (1982).

[32] S. Fleck, C. Gignoux, J.M. Richard and B. Silvestre-Brac, Phys. Lett. B220, 616 (1989).

[33] See, for instance, J.-M. Richard, Phys. Rep. 212, 1 (1992) and references therein.

[34] Particle Data Group, Review of Particle Properties, Phys. Rev. D, June 1992.