Exploring binding energy and separation energy dependences of HBT strength

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(Dated: January 22, 2022)

Hanbury Brown-Twiss (HBT) results of the nucleon-nucleon correlation function have been presented for the nuclear reactions with neutron-rich projectiles (Be isotopes) using an event-generator, the Isospin-Dependent Quantum Molecular Dynamics model. We explore that the relationship between the binding energy per nucleon of the projectiles and the strength of the neutron-proton HBT at small relative momentum. Moreover, we reveal the relationship between the single neutron separation energy and the strength of the halo neutron-proton HBT. Results show that neutron-proton HBT results are sensitive to binding energy or separation energy.

PACS numbers: 25.10.+s, 25.70.Mn, 21.45.+v, 27.20.+n

I. INTRODUCTION

Hanbury Brown-Twiss (HBT) technique was presented for the astrophysical measurements a few decades ago, and it reveals information about the angular diameters of distant stars 1. More recently, it has been widely used in other fields, for instance, the analogous correlations in semiconductors and in free space aiming at the fermionic statistics of electrons 2, 3, 4. It has also become an important tool in high energy region since it can be utilized to measure the evolving geometry of the interaction zone while being applied to the studies to search for a possible quark-gluon plasma and study the properties of the predicted new state of matter 5. In the past several years it has also significant theoretical development and widespread application in subatomic physics 6, 7, 8, 9, 10, 11, 12. The emission time and source size in the nuclear reaction can be extracted by the nuclear HBT technique, which is one kind of intensity interferometry. In the applications of experimental and theoretical heavy ion reactions at intermediate energy, various aspects have been investigated via the correlation functions, such as the dependences of the isospin of the emitting source 13, the impact parameter 14, the nuclear symmetrical energy 15 and the total momentum of nucleon pairs 16 and so on. The details about the EOS and the collision process can be revealed from the correlation functions well.

Some groups have applied HBT technique to study the exotic nuclear reaction recently 17, 18, 19. Studies have been performed for many years for exotic nuclei with the increasing availability of radioactive nuclear beams around the world. Among the various techniques to investigate on the exotic nuclei, the measurements of the total interaction cross section and the fragment momentum distribution of the projectile are the main methods to explore the exotic structure in the past years 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30. Some nuclei like 6He 21, 22, 23, 11Li 21, 14Be 11, 24, 25, 26 are considered as two-neutron halo ones and 11Be 22, 23. 15C 31, 32, 22O 33 are as one neutron halo, and the proton halo structure has been also proved to exist in the structure of the nuclei 8B 21, 12N 31, 23Al 32 etc. In terms of the structure of the halo, integral measurements, such as total reaction cross sections, are only sensitive to the overall size. Dissociation reactions, in which the core and/or nucleons are detected in the final state, can provide some structure information 17-the major difficulty being the relationship between the initial and final states as dictated by the disturbing effects of the reaction 18. So it is very interesting to investigate the exotic nuclei via HBT technique further. As we know, the binding energy and the nucleon(s)-separation energy are important for the structure of the nuclei. The former indicates the stable level of the nucleus and the nucleon-nucleon relationship among the nucleus, and the later is a good criterion to verify the possibility of the exotic nucleus. These two have been studied through the density calculation by RMF theory in the past years. Researches about these two factors via the nuclear collisions systematics are needed. In this Letter, we expect to explore the relationship between these two factors and the nucleon-nucleon correlation function value at very small relative momentum with help of Isospin-Dependent Quantum Molecular Dynamics (IDQMD) model which can describe the reaction dynamics on event by event basis.

II. HBT TECHNIQUE

Firstly, we would like to recall the HBT technique. As we know the wave function of relative motion of light identical particles, when emitted in close proxim-
ity in space-time, is modified by the final-state interaction (FSI) and quantum statistical symmetries (QSS), and this is the principle of the intensity interferometry, i.e. HBT. The correlation function is defined as the ratio between the measured two-particle distribution and the product of the independent single-particle distributions:

$$C(p_1, p_2) = \frac{dn^2/dp_1dp_2}{dn/dp_1dn/dp_2},$$

where $d^2n/dp_1dp_2$ represents the correlated two-particle distribution and $dn/dp_1$ and $dn/dp_2$ is the independent single-particle distributions of particle 1 and 2, respectively. Usually, the projection $C(q) = c_0 \frac{N(q)}{D(q)}$ onto the relative three-momentum $q = \frac{1}{2}(|p_1^2 - p_2^2|)$ is used as the momentum correlation function, where the measured distribution of pairs $(N(q))$ is divided by a reconstructed distribution of uncorrelated pairs $(D(q))$. $c_0$ is a normalized coefficient so that $C(q)$ tends to 1 at high relative momentum, where the effects of FSI and QSS vanish. The deviation of $C(q)$ from 1 thus reflects the information of the emission source. Other effects, arising from the form of the single-particle distributions or the experimental acceptances, are eliminated by the denominator of Eq. (1).

Interpretation of correlation functions measured in heavy-ion collisions requires understanding the relationship of the parameters extracted from fitting the data and the true single-particle distributions at freeze-out. This relationship can be established by using an event generator that models the collision dynamics, particle production, and then constructing a two-particle correlation function. The event-generator correlation functions are constructed from the positions and momenta represented by the single-particle distributions or the experimental acceptances, are eliminated by the denominator of Eq. (1).

III. MODEL DESCRIPTION

The Quantum Molecular Dynamics (QMD) approach is an n-body theory to describe heavy ion reactions from intermediate energy to 2 GeV/n. It includes five important parts: the initialization of the target and the projectile; the propagation in the effective potential; the collisions between the nucleons; the Pauli blocking effect and the numerical tests. A general review about the QMD model can be found in [36]. The IDQMD model is based on the QMD model affiliating the isospin factors.

As we know, the dynamics in heavy-ion collisions (HIC) at intermediate energies is mainly governed by three components, the mean field, two-body collisions, and Pauli blocking. Therefore, for an isospin-dependent reaction dynamics model it is important for these three components to include isospin degrees of freedom. What is more essential, in initialization of projectile and target nuclei, the samples of neutrons and protons in phase space should be treated separately because there exists a large difference between neutron and proton density distributions for nuclei far from the $\beta$-stability line. Particularly, for neutron-rich nucleus one should sample a stable initialized nucleus with neutron-skin structure and therefore one can directly explore the nuclear structure effects through a microscopic transport model. The IDQMD model has been improved based on the above ideas.

In the IDQMD model, the nuclear mean field can be parametrized by

$$U(\rho, \tau_z) = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^\gamma + \frac{1}{2} (1 - \tau_z) V_c + C \left( \frac{\rho_n - \rho_p}{\rho_0} \right) \tau_z + U \gamma uk$$

with $\rho_0$ the normal nuclear matter density (here, 0.16 $fm^{-3}$), $\alpha = -124$ MeV, $\beta = 70.5$ MeV and $\gamma = 2.0$, $\rho_0$, $\rho_n$, and $\rho_p$ are the total, neutron, and proton densities, respectively; The coefficients $\alpha$ and $\beta$ are parameters for nuclear equation of states (EOS). $\tau_z$ is the $z$th component of the isospin degree of freedom, which equals 1 or -1 for neutrons or protons, respectively; $V_c$ is the coulomb potential; and $U \gamma uk$ is Yukawa (surface) potential which has the following form:

$$U \gamma uk = \frac{V_u}{2m} \sum_{i \neq j} \frac{1}{r_{ij}} \exp(Lm^2) \left[ \exp(-mr_{ij}) \text{erf}(\sqrt{Lm} - r_{ij}/\sqrt{4L}) - \exp(mr_{ij}) \text{erf}(\sqrt{Lm} + r_{ij}/\sqrt{4L}) \right]$$

with $V_u = 0.0074$ GeV, $m = 1.25$ $fm^{-1}$ and $L = 2.0$ $fm^2$. The relative distance $r_{ij} = |r_i - r_j|$. In our task, the so-called soft EOS with an incompressibility of $K = 200$ MeV is used and the symmetry strength $C = 32$ MeV [36].

The NN cross section is the experimental parametrization which is isospin dependent. Recently, studies of collective flow in HIC at intermediate energies have revealed the reduction of the in-medium NN cross sections [33]. An empirical expression of the in-medium NN cross section [14] is used:

$$\sigma_{NN}^{med} = (1 + f \frac{\rho}{\rho_0}) \sigma_{NN}^{free}$$

with the factor $f \approx 0.2$ which has been found to better reproduce the flow data [13]. Here $\sigma_{NN}^{free}$ is the experimental NN cross section [35]. The neutron-proton cross section is about three times larger than the
neutron-neutron or proton-proton cross section below 300 MeV/nucleon.

The Pauli blocking effect taken in IDQMD model is treated separately according to the neutron and the proton: Whenever a collision has occurred, in the phase space we assume that each nucleon occupies a six-dimensional sphere with a volume of $\hbar^3/2$ (considering the spin degree of freedom), and then calculate the phase volume, $V$, of the scattered nucleons being occupied by the rest nucleons with the same isospin as that of the scattered ones. We then compare $2V/\hbar^3$ with a random number and decide whether the collision is blocked or not.

For the initialization of the nucleons of the target and projectile, the IDQMD model distinguishes the proton and neutron from each other. The neutron and the proton density distribution are determined from the Skyrme-Hartree-Fock (SHF) method with parameter set $SKM^*$ which can give reasonable density distribution for stable and neutron-rich nuclei [40]. One can obtain the radial positions of nucleons in the initial nuclei in terms of the Monte-Carlo method. In the model, the radial density can be written as:

$$\rho(r) = \sum_i \frac{1}{(2\pi L)^{3/2}} \exp\left(-\frac{r^2 + r_i^2}{2L}\right) \frac{L}{2rr_i} \times \left[\exp\left(\frac{rr_i}{L}\right) - \exp\left(-\frac{rr_i}{L}\right)\right]. \quad (5)$$

And the momentum distribution of nucleons is generated by means of the local Fermi gas approximation. The local Fermi momentum is given by:

$$P^i_p(\vec{r}) = \hbar(3\pi^2 \rho_i(\vec{r}))^{1/3}, (i = n, p). \quad (6)$$

The stability of the propagation of the initialized nuclei has been checked in details and can last at least 200 fm/c according to the evolutions of the average binding energies and the root mean square radii of the initialized nuclei.

### IV. RESULTS AND DISCUSSIONS

Among the exotic nuclei, $^{14}$Be nucleus, with four protons and ten neutrons has received the most attentions, both theoretically and experimentally, due to its rather unique structure [19, 24, 25, 26]. To test our approach, firstly we have analyzed the reaction of $^{14}$Be fragments into $^{12}$Be + 2n at 35 MeV/n and the target is $^{12}$C [10]. The solid line is the calculated two-halo neutrons correlation function.

The momentum distribution of $^{14}$Be extending very large which is a major reason for the abnormal larger total reaction cross section and the very narrow momentum distribution of the fragment $^{12}$Be [22]. In our work, the nucleons are defined as emitted if they do not belong to any clusters ($A \geq 2$) which are recognized by a simple coalescence model: ie. nucleons are considered to be part of a cluster if in the end at least one other nucleon is closer than $r_{\text{min}} \leq 3.5$ fm in coordinate space and $p_{\text{min}} \leq 300$ MeV/c in momentum space in the final state [21, 11]. This definition of nucleon emission is crucial for the good reproduction of the experimental data in Fig. 1. The $^{14}$Be is therefore a good test case for the HBT method.

Based upon the achievement of halo neutron - halo neutron correlation function of HBT results in IDQMD with the data, we further explore the proton-neutron correlation function as a function of binding energy or separation energy. This directly leads to the neutron density distribution of $^{14}$Be extending very large which is a major reason for the abnormal larger total reaction cross section and the very narrow momentum distribution of the fragment $^{12}$Be [22]. The target is $^{12}$C and the projectile is Be isotope. Only those events in which the neutron and the proton are emitted in the same event are accepted. The calculated results are shown in Fig. 2. The figure shows the proton-neutron correlation function for different Be isotopes and the insert of Fig. 2 shows the relationship between the strength of proton-neutron correlation function $C_{PN}$ at 5 MeV/c and the binding energy per nucleon of the projectile $E_{\text{binding}}$. The solid line of the insert is just a linear fit to guide the eyes.

One can notice that the behavior of the correlation functions between the proton and the neutron as a function of neutron number and binding energy in Fig. 2.
Generally the strength of HBT at very small relative momentum shows a clear dependence on the neutron number. The tendency of the $C_{PN}$ rises with the increasing $E_{\text{binding}}$. As we know, among the projectiles we studied, the number of the protons is 4 and that of the neutron are gradually increasing, and this will reflect the stability of the nuclei. With the increasing of the mass number, the mean relationship between the nucleons will change weaker. Since the strength $C_{PN}$ of correlation function symbolizes the mean relationship between the emitted proton and neutron and the binding energy per nucleon associated with the tightness between the nucleon, the tendency shown in the insert of Fig. 2 reflects that the $C_{PN}$ can reveal the compactness of the nuclei.

On the other hand, we also study the correlation functions between the proton and the most outside neutron. Here the most outside neutron is defined as the one which is the most far away from the spatial center at the FSI and it is also called as a halo neutron for simplification, even though it is not strict in physics sense. To obtain reasonable HBT results for halo neutron and proton, only those events in which the halo neutron and proton are emitted in the same event are accepted to investigate such a correlation function. The similar correlation function was obtained as $C_{PN}$ and the relationship between the strength of proton-halo neutron correlation function $C_{PH}$ at 5 MeV/c and the single-neutron separation energy of the projectile $E_{\text{sep}}$ was extracted. The symbols of Fig. 3 show the calculated result and the solid line is just a linear fit to guide the eyes. It looks that, with the increasing of the $E_{\text{sep}}$, $C_{PH}$ at 5 MeV/c rises gradually. Since the strength $C_{PH}$ of correlation function between neutron and proton increases with the decreasing of the source size, the above $E_{\text{sep}}$ dependence of $C_{PH}$ reflects that the emission source size of the neutron and proton shrinks with the separation energy of the single neutron, which is consistent with the extent of binding of single neutron via $E_{\text{sep}}$.

One should keep in mind that the $E_{\text{sep}}$ is thought as one of the important information for halo structure, for Be isotope, the nucleus $^{11}\text{Be}$ is considered to exist the halo configuration according to its extremely small one-neutron separation energy. Compared with other nuclei in Be isotopes, the $C_{PH}$ at small relative momentum of it is smaller, i.e. there exists a larger spatial diffusion. That indicates that HBT of proton and halo neutron can show the different configuration of the nuclei Be isotopes well. These observations might illustrate that the $C_{PH}$ will symbolize the relationship between the most outside neutron and the emitted proton, which is related to the extension level of the mass density distribution directly. Unfortunately, in the practical point of view, this correlation function $C_{PH}$ is almost impossible to be measured. However, it still bear some information in theoretical point of view.

V. SUMMARY

In summary, the intensity interferometry (HBT) technique has been applied to investigate its sensitivity to
the binding energy and separation energy of neutron-rich nuclei from the break-up of nuclei by convoluting the phase-space distribution generated with the IDQMD model. Firstly we gave a well-fitted halo neutron - halo neutron correlation function from the break-up of $^{14}\text{Be}$ on $C$ target. Based upon this achievement of the good fit, we explore the dependence of the proton-neutron correlation function ($C_{PN}$) at small relative momentum with the binding energy ($E_{\text{binding}}$) for Be isotopes. It was found that the correlation strength of $C_{PN}$ at small relative momentum rises with the the binding energy. This changeable tendency of $C_{PN}$ with $E_{\text{binding}}$ is here reported for the first time and it might be a potential good way to study the structure of the nuclei. Moreover, the proton-halo neutron correlation function ($C_{PH}$) is also constructed from the break-up reactions. There exists the similar relationship between the $C_{PH}$ at small relative momentum and the separation energy ($E_{\text{sep}}$) of Be isotopes as the relationship of $C_{PN}$ to $E_{\text{binding}}$. From theoretical point of view, $C_{PH}$ at small relative momentum is sensitive to $E_{\text{sep}}$ and this can be attributed to the spatial extension level of the neutron which is most far away the center of the nucleus. Of course, we recognize that it is very difficult to measure $C_{PH}$ in a practical point. However, theoretical behavior is also interesting.

Acknowledgments

This work was supported in part by the Major State Basic Research Development Program under Contract No G200077400, the Chinese Academy of Sciences Grant for the Distinguished Young Scholars of National Natural Science Foundation of China under Grant No 19725521, the National Natural Science Foundation of China under Grant No 10135030 and the Shanghai Phosphor Program Under Contract Number 03 QA 14066. Y. G. Ma would like to appreciate Dr. Scott Pratt for providing CRAB code which is used to construct the momentum correlation function from phase space data.
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