Realization of matched filter on spin-wave delay line for amplitude modulated signals

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Abstract. Matched filter based on active ring resonator with YIG delay line was estimated in this work. Complex transmission characteristic including microstrip meander antennae frequency response was described. Assuming of influence of meander antennae provided possibility of matched filtration estimation. Deformation of input square pulse was considered for clear signal processing and signal against noise. Distribution of pulse deformation with frequency shift from central frequency was calculated for both cases. Signal to noise ratio was achieved in -20 dB.

1. Introduction
Ferrite phase shifters or delay lines are nowadays commonly used in microwave and radar technology [1]. High values of delay time and inherent dispersion typical for the ferrite waveguide structures provide wide functionality for microwave applications [2]. Closed loop circuits consisted of delay lines with high values of delay time and amplifiers are widely used for low-noise oscillators design [3]. From the point of view of resonant nature, below the self-oscillation threshold (open-loop gain less unity) these circuits are regarded as linear filters that have narrow comb shape of the transfer function [4]. Each frequency band has a high quality factor (more than thousand for the open-loop gain approached to unity) and linear (or quasi-linear) phase behavior. Narrow bands with high quality factor are similar to real spectral part of wide class of signals, which consists manipulations on amplitude in time domain. It could be a simple amplitude modulation (manipulation) or manipulation on parts of the complex signal. The combination of these features is attractive for narrow comb matched filter.

This work is devoted to investigation of matched filter based on the active ring resonator (ARR) with spin-wave delay line. The circuit consists of following elements: a spin wave delay line based on a single crystal yttrium-iron garnet (YIG) film on a gadolinium gallium garnet (GGG) substrate and two short-circuited microstrip antennae; a wideband amplifier and a variable attenuator connected in series to the delay line; two directional couplers serving as the loop input and output. Signal after the directional coupler is fed back to the input of the spin wave delay line. The resonant frequencies are defined by phase shift of the waves circulating in the loop. In the case the delay time of all electrical interconnections negligibly small the phase shift is determined by dispersion and geometry of spin wave delay line. Since the loop support waves circulating until complete decay, the resonator is fundamentally multi-mode, with the mode spacing determined by the delay time. Due to strong dispersion of the spin wave the mode spacing is also strongly depends on frequency. Transmission characteristic of the ARR with spin wave delay line consists of series of passbands that coincide with resonant frequencies. The
form of the transmission coefficient is defined by a damping decrement of the spin waves in the delay line and transfer function of the spin wave antennae. As result, design of delay line can be configured for necessary signal. For periodic AM train main parameters are frequency of modulation (time of period) and duty cycle. Period of the signal determinates FSR between passbands and their direct loss, in other words, filtration is matched when duration of signal period is equal to group delay time of ARR at the carrier frequency. Full-band range can be chosen by value of duty cycle. Duty cycle defines envelope of amplitude spectral density of the incoming signal. When the duty cycle more than 50%, the major part of spectral power is concentrated around carrier frequency, otherwise, the power has wider distribution in frequency range. This fact defines criterion of full-band range. For common YIG films this is frequency response of the simple delay line.

2. Theory of transfer function

A complex transmission coefficient of the matched filter can be written in the following form

$$H(f) = H_{in}(f)H_{out}(f)\sqrt{H_p(f)} \exp[i\phi(f)], \quad (1)$$

where $H_{in}(f)$ and $H_{out}(f)$ is the transmission coefficients of input and output spin-wave antennae, $H_p(f)$ is the power transmission coefficient and $\phi(f)$ is the phase-frequency characteristic of the active ring resonator [5] and have following forms:

$$H_p(f) = \frac{1}{2} \frac{e^{g - \alpha(f)d}}{\cosh(g - \alpha(f)d) - \cos(k(f)d)}, \quad (2)$$

$$\phi(f) = \arctan\left[\frac{\sin\left(k(f)d\right)}{\exp(g - \alpha(f)d) - \cos\left(k(f)d\right)}\right] \pm R\pi, \quad (3)$$

where $R = 0,1,2,...$, $g$ is gain in circuit, $d$ - distance between input and output antennae, $\alpha$ - damping decrement and $k(f)$ is wave number.

The SW wave-number $k_{sw}(f)$ is calculated using an appropriate dispersion equation. For the surface spin waves used in this work the wave-number is calculated as [6]

$$k_{sw}(f) = -\ln\left(1 - 4\left(f^2 - f_H\left(f_H + f_M\right)\right) / f_M^2\right) / 2L, \quad (4)$$

where $L$ is a thickness of the YIG film, $f_H = |g|\mu_0H_0$, $f_M = |g|\mu_0M_0$, $|g| = 1.76 \cdot 10^{11}$ rad·s$^{-1}$·T$^{-1}$ is a gyromagnetic ratio for an electron spin, $\mu_0 = 4\pi \cdot 10^{-7}$ H·m$^{-1}$ is a vacuum permeability, $H_0$ is an external magnetic field, and $M_0$ is a saturation magnetization. The spatial damping decrement for the surface spin waves is calculated as

$$\alpha_{sw}(f) = 2\pi |g| f \Delta H \cdot L^{-1}\left(\left(f_H + f_M / 2\right)^2 - f\right)^{-1}, \quad (5)$$

where $\Delta H$ is a half of the ferromagnetic resonance line-width.

Transmission coefficients of input/output spin wave antennae forms frequency response of the delay line and defined by radiation resistance. It could be found from radiation impedance $Z_{sw}$:

$$Z_{sw} = \frac{1}{2f^2} \int_{s} j^* EdS \quad (6)$$
where $I$ is current in spin-wave antenna, $j$ – current density, $E$ – intensity of spin-wave’s electric field and $S$ – surface area of antenna. Current distribution is obtained from telegraph equations, when length of spin-wave comparable with width of microstrip line. Radiation resistance can be separated onto positive (+) and negative (-) parts respectively to delay line accordingly.

$$R_{res} = R' + R'',$$

$$R''(f) = \frac{f_{ms}H_0}{4\pi} \sum \frac{f_n - f_{ms} \pm f}{2\pi|V_g(f)|} Y(f) |j_{sk}(k_s)|^2$$

(7)

Here $n$ is number of spin-wave mode, $V_g$ – group velocity of spin-wave, $Y(f)$ – overlap integral, $j_{sk}$ – Fourier transform of the antenna’s current. In this work, meander type antennae were used for numerical calculation. Fourier transform for double elements antennae described in terms of Bessel’s functions:

$$|j_{sk}(f)|^2 = \sum J_{2i}^2 \left(\frac{k(f)W_i}{2}\right) + 2\sum_{i\neq j} J_{2i}^2 \left(\frac{k(f)W_i}{2}\right) J_{2j}^2 \left(\frac{k(f)W_j}{2}\right) \cos\left(k(s_i - s_j)\right),$$

(8)

where $W_i$ is width of $i$-th antenna element, $s_i$ – distance from origin of the antenna coordinate system. Radiation reactance is negligibly small, in case of matching inter input and output impedances of ferrite film and antennae. Transmission characteristic of the input antenna is relation between full power in microstrip ($P_0$) and power that radiates into ferrite film ($P'$). In accordance with coefficient of reflection, power response of input antenna is

$$H_{in} = \frac{P'}{P_0} = \left(1 - |I|^2\right) \frac{R'}{R' + R'' + R_0}. $$

(9)

where $R_0$ is input resistance of microstrip. With symmetrical delay line (identical input and output antennae) complex transmission coefficient can be simplified to
d

$$\hat{H}(f) = H_{in}(f)^2 \sqrt{H_p(f)} \exp[i\phi(f)]$$

(10)

3. Numerical investigation

For investigation of frequency response of ARR and matched filtration were assumed next parameters: thickness of ferrite film $L = 15 \mu m$; distance between antennae under ferrite film $d = 6 \ mm$; meander-type microstrip lines had a width $W$ of $50 \mu m$ and interspace of $300 \mu m$; saturation magnetization $M_0 = 1660$; the film is magnetized to saturation by bias magnetic field of $1160 \ Oe$ directed parallel to the plane of the film and perpendicular to the direction of the wave propagation. This setup provides full-band more than $100 \ MHz$ and FSR around $10 \ MHz$ affected by dispersion. Sequence of square pulses with $94 \ ns$ period duration and $25\%$ duty cycle was chosen as input influence.

Figure 1a represents calculated transmission characteristic of the matched filter based on ferrite delay line in ARR and power spectrum of square pulse train. Narrow bands of the active ring resonator with high quality factor are similar to real spectral part of width class of signals which consist manipulations on amplitude in time domain. Each single passband of spin-wave filter can overlay a separated harmonica of amplitude spectral density of periodic signal. Therefore, all other spectral part shall be isolated from useful part of spectrum that should provide higher signal to noise ratio. Utilization of meander type antennae allows for obtain symmetric passband of the delay line and filter at all. Dispersion of surface spin-waves produces non-equidistant FSR between resonances, that leads to asymmetric filtration of side lobes of the power spectrum. Left side of spectrum has less suppression than right side. In this fact, duration time of square pulse at $90\%$ level increases and decreases at $10\%$ level. Note that the duration of pulse at $50\%$ level almost not changes (fig. 1b).
Figure 1. a) Transmission characteristic of the spin-wave matched filter (dash) and arrangement of square pulse train spectrum (solid); b) Input (dash) and output (solid) signals.

Consider high-Q feature of ARR in case of matching filtration. Figure 2 represents ARR response distribution on square pulse with frequency shift near central resonant passband peak. Central position on surface corresponds to match condition between central resonant passband and carrier frequency of square pulse and centre of pulse in time domain. Duration of pulse at 50% level distribution on frequency shift shown separately on fig. 3. As can be seen, suppression of higher frequencies occurs by spin-wave dispersion together with increase of insertion loss. It appears in decrease of pulse duration. Additionally, there is strong deformation of top of the pulse when carrier frequency shifts to edge of 3dB passband level. Symmetrical behavior corresponds to phase response in bandwidth of stand-alone resonance. These results are in agreement with experimental investigation of optimum filtration in spin-wave ARR [7].

Figure 2. Distribution of the isolated pulse duration on frequency shift near resonant passband peak
Investigation of filtration makes sense in occurrence of noise mixed with useful signal. Therefore, white Gaussian noise was mixed with square pulses to estimate applicability of spin-wave matched filter. Power level of noise was assumed at 20 dB per sample of signal. So, input signal to noise ratio was -20 dB. Distribution of duration of the output signal mixed with noise on frequency shift is shown on fig. 4. Influence of spin wave dispersion on noise component is clearly visible on sides of the output pulse in the absence of frequency shift. Besides, boundary noise has almost constant amplitude relative to pulse. Contribution of high-Q resonant passband in matched filtration against white noise appears in decrease of output signal to noise ratio with shifting from matched condition.

Figure 3. Distribution of the isolated pulse duration at 50% level on frequency shift near resonant passband peak.

Input, output signals and square pulse pattern are presented on fig. 5. Output signal corresponds to matched condition of filtration. Processing of input signal with low S/N ratio provides deformation of pulse duration in 18% at 10%-level, 9% at 50%-level and 42% at 90%-level, in comparison with square pulse pattern.

Figure 4. Distribution of the isolated pulse duration mixed with white Gaussian noise on frequency shift near resonant passband peak.
In conclusion, matched filter based on active ring resonator with YIG delay line was estimated in this work. Complex transmission characteristic including microstrip meander antennae frequency response was described. Assuming of influence of meander antennae provided possibility of matched filtration estimation. Deformation of input square pulse was considered for clear signal processing and signal against noise. Signal to noise ratio was achieved in -20 dB.

**References**

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