The pion pole in hard exclusive vector-meson leptoproduction

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Abstract. Exploiting a set of generalized parton distributions (GPDs) derived from analyses of hard exclusive leptoproduction of $\rho^0$, $\phi$ and $\pi^0$ mesons, we investigate the $\omega$ spin density matrix elements (SDMEs) recently measured by the HERMES Collaboration. It turns out, from our study, that the pion pole is an important contribution to $\omega$ production. It will be treated as a one-particle exchange since its evaluation from the GPD $E$ considerably underestimates its contribution. As an intermediate step of our analysis, we extract the $\pi\omega$ transition form factor for photon virtualities less than 4 GeV$^2$. From our approach we achieve results for the $\omega$ SDMEs in good agreement with the HERMES data. The role of the pion pole in exclusive $\rho^0$ and $\phi$ leptoproduction is discussed too.

1 Introductory remarks

It has long been known that pion exchange plays an important role in photo- and lepton production of $\omega$ mesons [1] (see, for instance, the review [2]). Pion exchange also contributes to other reactions as, for instance, to exclusive $\omega$ production in proton-proton collisions at high energies [3]. The residue of the pion pole in $\omega$ lepton-production includes the $\gamma^*\pi\omega$ vertex function. An analysis of the pertinent processes therefore allows for an extraction of information on this vertex. The recent HERMES measurement [4] of the SDMEs for electroproduced $\omega$ mesons at fairly large values of the photon virtuality, $Q^2$, small Bjorken-$x$, $x_{\text{BJ}}$, and small invariant momentum transfer, $t$, offers a unique possibility to learn about the $\gamma^*\pi\omega$ vertex function in the space-like region which, at small $t$, can be regarded as the $\pi\omega$ transition form factor. The extracted information is complementary to that on the form factor in the time-like region derived from data [5–7] on electron-positron annihilation into $\pi^0\omega$, see [8] for a recent analysis.

A particular combination of $\omega$ SDMEs isolates the so-called unnatural-parity ($U$) contribution to the $\gamma^*p \rightarrow \omega p$ cross section which, for the kinematics of the HERMES experiment, is strongly dominated by pion exchange. The $\pi\omega$ form factor can easily be extracted from this combination of SDMEs provided the natural-parity ($N$) contribution to the cross section is known. Since this is not the case experimentally, at present, we are forced to rely on our detailed analysis of exclusive meson lepton-production at small $x_{\text{BJ}}$ within the handbag approach [9–11]. In combination with results of a GPD analysis of the electromagnetic form factors [12,13], we have extracted a set of GPDs which allows us to compute the natural-parity contribution to the $\gamma^*p \rightarrow \omega p$ cross section and, subsequently, the $\pi\omega$ transition form factor from the HERMES data on the $\omega$ SDMEs [4]. There is also a CLAS measurement of these SDMEs [14]. These data are, however, characterized by large $x_{\text{BJ}}$, small $W$ and rather large $t$. Since the set of GPDs we have determined from meson leptonproduction is optimized for small $x_{\text{BJ}}$, we cannot reliably compute from it the natural-parity contribution to the $\gamma^*p \rightarrow \omega p$ cross section for the kinematics of the CLAS experiment and therefore we are unable to determine the $\pi\omega$ transition form factor from these data.

In the next section we discuss the role of the pion pole in vector-meson lepton production. In sect. 3 we extract the $\pi\omega$ form factor from the HERMES data. Section 4 is devoted to a discussion of various partial cross sections, which can be obtained from combinations of SDMEs and to a comparison of the experimental results with the theoretical ones obtained from the handbag approach in combination with the pion-exchange contribution. This further probes the $\pi\omega$ form factor extracted in sect. 3. In sect. 5 we present our results for the SDMEs and, in sect. 6, we comment on spin asymmetries. Our summary is given in sect. 7.

2 The pion pole

Here, in this section, we are going to discuss one-pion exchange in lepton production of vector mesons ($V = \rho^0, \omega$).
The momenta, helicities and masses for the process \( \gamma^* p \rightarrow V p \) are specified in fig. 1. In analogy to \( \pi^+ \) lepton production [15] one may write the pion-exchange contribution to the helicity amplitudes of vector-meson lepton production as

\[
\mathcal{M}_{\mu'\mu;\nu'}^{\text{pole}} = i g_{\pi NN} F_{\pi NN}(t) \frac{\bar{u}(p',\nu') \gamma_\nu u(p,\nu)}{t - m_N^2} \times (V; q', \mu' | j^0(0) \cdot \epsilon_\gamma(\mu) | \pi; q_\pi = q' - q),
\]

(1)

where \( g_{\pi NN} \) is the pion-nucleon coupling constant for which we adopt the value \( 13 \pm 0.07 \) GeV as in [11]. \( F_{\pi NN} \) is a form factor that describes the \( t \)-dependence of the pion-nucleon coupling. In concord with [11] this form factor is parametrized as

\[
F_{\pi NN} = \frac{A_N^2}{4} \frac{\Lambda_N^2 - m_N^2}{\Lambda_N^2 - t},
\]

(2)

in the small \(-t\) region (\( \lesssim 0.5 \) GeV\(^2\)). For the parameter \( \Lambda_N \) we take the value \( 0.44 \pm 0.07 \) GeV as in [11].

The current matrix element in (1) reads

\[
\langle V; q', \mu' | j^0(0) | \pi; q_\pi = q' - q \rangle = e_\alpha g^{\gamma_\pi V}(Q^2, t) e_{\lambda\rho\sigma} q^{\lambda} \epsilon_\gamma^*(\mu') q^{\rho \sigma},
\]

(3)

where \( t = (q' - q)^2 \) is the virtuality of the pion. At small \(-t\), i.e. near the pole, but large \( Q^2 \) one may ignore the \( t \)-dependence of the vertex function \( g_{\gamma^* \pi V} \) and regard it as the \( \pi V \) transition form factor

\[
g_{\pi V}(Q^2) \equiv g^{\gamma_\pi V}(Q^2, m_N^2) \equiv g_{\gamma^* \pi V}(Q^2, t).
\]

(4)

Chernyak and Zhitnitsky [16] have sketched the calculation of the \( \pi V \) transition form factor within perturbative QCD. This is a complicated task: two- and three-particle configurations have to be taken into account as well as leading- and higher-twist wave functions. In the collinear limit some of the convolutions are infrared singular which, according to [16], may be regularized by Sudakov effects and parton transverse momenta in the denominators of the hard propagators. This is precisely the method we have used in our analysis of meson lepton production [9,10] for the regularization of infrared singularities occurring in photon-meson transition amplitudes other than \( \gamma^*_L \rightarrow V_L \) (here and in the following the label \( L(T) \) denotes a longitudinally (transversely) polarized photon or meson).

An outcome of this perturbative calculation of the transition form factor is that

\[
g_{\pi V} \sim 1/Q^4
\]

at large \( Q^2 \). Another result is that the ratio of the \( \pi \rho \) and \( \pi \omega \) form factor is governed by the quark charges

\[
g_{\pi \rho} \simeq \frac{e_u + e_d}{e_u - e_d} g_{\pi \omega},
\]

(6)

leaving aside possible differences in the \( \rho \) and \( \omega \) wave functions. The quark charges \( e_a \) are given in units of the positron charge \( e_0 \). We note that the \( 1/Q^2 \) fall has already been pointed out in [17,18]. Chernyak and Zhitnitsky estimated the strength of the \( \pi \rho \) form factor to amount to

\[
g_{\pi \rho} \simeq 0.4 \text{GeV}^3/Q^4
\]

(7)

at large \( Q^2 \). Similar numerical values for this form factor have been obtained from light-cone sum rules [19,20]. In contrast to [16] only soft contributions are taken into account in the latter work.

Experimentally, the transition form factors are unknown in the space-like region except at \( Q^2 = 0 \) where they control the radiative decays of the vector mesons

\[
\Gamma(V \rightarrow \pi \gamma) = \frac{1}{24} \alpha_{\text{em}} g_{\pi V}(0)^2 m_V^5 \left[ 1 - m_\pi^2 / m_V^2 \right]^2
\]

(8)

(see, for instance, [21]). From the branching ratios of the radiative decays of the vector mesons and the total decay widths quoted in [22] one finds

\[
|g_{\pi \omega}(0)| = (2.30 \pm 0.04) \text{GeV}^{-1},
\]

\[
|g_{\pi \rho}(0)| = (0.85 \pm 0.06) \text{GeV}^{-1}.
\]

(9)

\(^1\) These values agree with those quoted in [3] if the different normalization in the latter work is considered.
Approximately, these values also obey the charge ratio (6) although the radiative decays of the vector mesons are not controlled by perturbative QCD.

The pion-pole contribution to the (light-cone) helicity amplitudes of $\gamma^* p \to V p$ can readily be worked out from (1) and (3):

$$\mathcal{M}_{++}^{\text{pole}} = -\mathcal{M}_{--}^{\text{pole}} = \frac{\varrho_{\pi V}}{t-m_{\pi}^2} \frac{m_{\pi} Q^2}{\sqrt{1-\xi^2}} \times \left[ 1 - \xi^2 \frac{4m^2 - t}{2} \right],$$

$$\mathcal{M}_{00}^{\text{pole}} = -\mathcal{M}_{00}^{\text{pole}} = \frac{\varrho_{\pi V}}{t-m_{\pi}^2} \frac{\sqrt{-t} Q^2}{2},$$

$$\mathcal{M}_{++0-}^{\text{pole}} = \mathcal{M}_{--0+}^{\text{pole}} = \frac{\varrho_{\pi V}}{t-m_{\pi}^2} \frac{\sqrt{-t} Q^2}{2},$$

$$\mathcal{M}_{+-0+}^{\text{pole}} = \mathcal{M}_{-0+0}^{\text{pole}} = \frac{\varrho_{\pi V}}{t-m_{\pi}^2} \frac{\sqrt{-t} Q^2}{2}. \tag{10}$$

Here, the form factors for the coupling of the pion to the vector meson and to the proton are combined in the quantity

$$\varrho_{\pi V} = e_0 g_{\pi V}(Q^2) g_{\pi NN} F_{\pi NN}(t). \tag{11}$$

The skewness, $\xi$, is related to $x_{ Bj}$ by

$$\xi = \frac{x_{ Bj} (1 + \frac{m_{\pi}^2}{Q^2})}{2 - x_{ Bj}} \tag{12}$$

in the photon-proton center-of-mass system specified by $p = \bar{p} - \Delta/2$ and $p' = \bar{p} + \Delta/2$ where $\bar{p} = (p + p')/2$ and $\Delta = p' - p$. As usual, $t' = t - t_0$, where

$$t_0 = -4m^2 \frac{\xi^2}{1-\xi^2} \tag{13}$$

is the minimal value of $-t$ attainable in the scattering process $\gamma^* p \to V p$. The contribution of the pion pole to the $\gamma_+^* \to V_T$ and $\gamma_-^* \to V_T$ cross sections reads

$$\frac{\mathrm{d}\sigma^{\text{pole}}}{\mathrm{d}t}(\gamma_+^* \to V_T) = \frac{1}{2\kappa(t-m_{\pi}^2)^2} \frac{-t}{Q^2} \frac{\varrho_{\pi V}}{t-m_{\pi}^2} \left[ 1 - 2\xi^2 \frac{4m^2 - t}{Q^2} \right],$$

$$\frac{\mathrm{d}\sigma^{\text{pole}}}{\mathrm{d}t}(\gamma_-^* \to V_T) = \frac{1}{\kappa(t-m_{\pi}^2)^2} \frac{tt'}{1-\xi^2} Q^2 \frac{\varrho_{\pi V}}{t-m_{\pi}^2}, \tag{14}$$

where $\kappa$ denotes the phase-space factor and $\Lambda$ the familiar triangle function

$$\kappa = 16\pi(W^2 - m_{\pi}^2) \sqrt{\Lambda(W^2, -Q^2, m_{\pi}^2)}. \tag{15}$$

With regard to the large $Q^2$ behavior of the $\pi V$ form factor these cross sections are suppressed by $1/Q^2$ and $1/Q^4$ as compared to the asymptotically leading $\gamma_+^* \to V_L$ one which is known to fall as $1/Q^2$ at fixed $x_{ Bj}$ or $\xi$ [23]. The pion-pole contribution to the $\gamma_+^* \to V_L$ and $\gamma_-^* \to V_L$ cross sections are even stronger suppressed and therefore neglected in this work. Its contribution to the $\gamma_+^* \to V_L$ cross section is strictly zero.

A structure similar to (14) is also found for the pion-pole contribution to the longitudinal cross section of $\pi^+ \to V_L$ leptoproduction [10]

$$\frac{\mathrm{d}\sigma^{\text{pole}}}{\mathrm{d}t}(\gamma_+^* \to \pi^+) = \frac{1}{\kappa(t-m_{\pi}^2)^2} \frac{-t}{Q^2} \frac{\varrho_{\pi V}}{t-m_{\pi}^2}, \tag{16}$$

where $\varrho_{\pi V} = \sqrt{2} e_0 F_{\pi}(Q^2) g_{\pi NN} F_{\pi NN}(t)$ and $F_{\pi}(Q^2)$ being the electromagnetic form factor of the pion. Characteristic of pion exchange is the factor $-t/(t-m_{\pi}^2)^2$ in (14) and (16) with a zero at $t = 0$ and a maximum at $t = -m_{\pi}^2$, see fig. 1. This factor dominates the $t$-dependence of the relevant cross sections at small $-t$. Depending on the value of the skewness the maximum of this factor and, hence, of the cross sections, lies in or out of the physical scattering region.

A reliable extraction of the $\pi V$ transition form factor or the electromagnetic form factor of the pion from experiment necessitates data as close as possible to the position of the pion pole in order to see the characteristic $t$-dependence of pion exchange and to justify the neglect of the virtuality of the exchanged pion at the $\gamma^* \pi V$ vertex. This can only be achieved for sufficiently small skewness. The extraction of the transition form factor at large $Q^2$ therefore requires large $\gamma^* p$ c.m.s. energy, $W$, with the consequence of a small pion-pole contribution. A reasonable compromise seems to be an energy $W \approx 3$–$8$ GeV for a measurement of the form factor in the range $Q^2 = 2$–$5$ GeV$^2$. This is the case for the HERMES experiment [4] and can perhaps be realized at the upgraded JLab.

Finally, we want to remark that one may use a reggeized version of pion exchange. However, owing to the Goldstone-boson nature of the pion the corresponding Regge trajectory, fixed by the pion and the $\pi(1670)$, is rather flat. Since we are only interested in very small values of $t$ and a rather narrow range of energy both variants of pion exchange differ only slightly.

### 3 Extraction of the $\pi\omega$ transition form factor

For a reliable extraction of the $\pi\omega$ or the pion electromagnetic form factor it is important to reduce the background, i.e., contributions from other dynamical mechanisms. In the case of the electromagnetic form factor of the pion this is achieved by the familiar longitudinal/transverse separation of the cross section for $\pi^+$ production. For the case at hand, the isolation of the unnatural-parity cross section ensures the background suppression. The natural/unnatural parity separation can be accomplished with the help of a particular combination of SDMEs

$$U_1 = 1 - \gamma_{04}^{04} + 2\gamma_{04}^{01} - 2\gamma_{11}^{04} - 2\gamma_{11}^{11} \tag{17}$$

The normalization of $U_1$ is related to the transverse and longitudinal differential cross sections by $\mathrm{d}\sigma/\mathrm{d}t =$
\[ \frac{d\sigma_T}{dt} + \varepsilon d\sigma_L/dt \], where \( \varepsilon \) is the ratio of the longitudinal and transverse photon fluxes (for HERMES kinematics: \( \varepsilon \approx 0.8 \)). The helicity amplitudes occurring in (17) are their unnatural-parity parts, defined by

\[ M^{U}_{\mu'\nu',\mu\nu} = \frac{1}{2} \left[ M_{\mu'\nu',\mu\nu} - (-1)^{\alpha-\beta} M_{-\mu'\nu',-\mu\nu} \right]. \tag{18} \]

The natural-parity parts of the helicity amplitudes are analogously defined:

\[ M^{N}_{\mu'\nu',\mu\nu} = \frac{1}{2} \left[ M_{\mu'\nu',\mu\nu} + (-1)^{\alpha-\beta} M_{-\mu'\nu',-\mu\nu} \right]. \tag{19} \]

The \( N \)- and \( U \)-type amplitudes possess the property

\[ M^{-}_{\mu'\nu',\mu\nu} = (-1)^{\alpha-\beta} M^{N}_{\mu'\nu',\mu\nu}, \]

\[ M^{U}_{-\mu'\nu',-\mu\nu} = (-1)^{\alpha-\beta} M^{U}_{\mu'\nu',\mu\nu}. \tag{20} \]

The exchange of a particle of either natural or unnatural parity leads to such a behavior (see the pion-exchange amplitude (10)). In the differential cross sections there is no interference between the natural- and unnatural-parity amplitudes.

For the SDMEs we use the notation introduced by Schilling and Wolf [24]. In the relations of the SDMEs to the helicity amplitudes (10). In the differential cross sections there is no interference between the natural- and unnatural-parity amplitudes. This convention has already been used for the pion-pole contributions (10) for the pion-pole amplitude. In the relations of the SDMEs to the helicity amplitudes (10). In the differential cross sections there is no interference between the natural- and unnatural-parity amplitudes. This convention has already been used for the pion-pole contributions (10).

Since the unseparated cross section for \( \omega \) production has not been measured by the HERMES Collaboration we have to take care of its unnatural-parity part. Consequently, we rewrite (17) as

\[ \sum_{\nu'} \left[ |M^{U}_{\mu'\nu',++}|^2 + 2\varepsilon |M^{U}_{\mu'\nu',0+}|^2 \right] = \frac{U_1}{2 - U_1} \left\{ \sum_{\nu'} \left[ |M^{N}_{\mu'\nu',++}|^2 + \varepsilon |M^{N}_{\mu'\nu',0+}|^2 \right] + |M^{N}_{0++,++}|^2 + \frac{1}{2} |M^{N}_{0-++,+}|^2 \right\}. \tag{21} \]

The left-hand side of this equation, dominated by the pion-pole contribution (10), includes terms quadratic and, provided there is a background to these amplitudes, linear in the form factor \( g_{\pi\omega} \). For this possible background as well as for the amplitudes on the right-hand side of (21) we exploit results from our previous studies of hard exclusive meson production at small skewness within the handbag approach [9–11] in which the helicity amplitudes are given by convolutions of hard subprocess amplitudes and GPDs. The partonic subprocess has been calculated by us within the modified perturbative approach in which quark transverse momenta and Sudakov suppressions are taken into account. From this analysis we have extracted a set of GPDs for gluons and quarks which include \( H \), \( E \), and \( H \) as well as the transversity GPDs \( E_T = 2H_T + E_T \) and \( H_T \). The GPD \( E \) has been neglected. In principle, pion exchange contributes to \( \tilde{E} \) [26, 27]. We however replace this contribution by the one-particle exchange term discussed in the preceding section (see also the next section). The parametrizations of the GPDs are specified in [9, 11]. The convolutions of \( H \), \( E \) (contributing to \( \gamma^\pi_T \to V_L \) and \( \gamma^\pi_T \to V_T \) transition amplitudes) and \( E_T \) (contributing to \( \gamma^\pi_T \to V_L \) transitions) behave like natural-parity exchanges (19), those of \( H \) (feeding the \( \gamma^\pi_T \to V_T \) amplitudes) like unnatural parity (18). A special case is the convolution of \( H_T \) which fixes the helicity non-flip amplitude \( M_{0-++,+} \). Its parity counterpart, \( M_{0-++,+} \), is suppressed by \( 1/Q^2 \). Hence, the convolution of \( H_T \) does not have a definite parity [11, 28]. No parity label is therefore assigned to the amplitude \( M_{0-++,+} \) in (21). The \( \gamma^\pi_T \to V_L \) and \( \gamma^\pi_T \to V_T \) transition amplitudes are assumed to be zero except of the pion-exchange contribution to the first ones\(^2\). We stress that our handbag approach is designed for \( \zeta < 0.1 \), \( -t' < 0.7 \text{GeV}^2 \), \( W > 4 \text{ GeV} \), and \( Q^2 > 2 \text{ GeV}^2 \).

The SDME combination \( U_1 \) appearing in (21) is known from the HERMES measurement [4]. The natural-parity amplitudes in (21) as well as a background to the amplitude \( M^{U,++} \) is evaluated from the above described set of GPDs along the lines we computed \( \rho^0 \) production [9]. For this computation we apply the same wave functions for the \( \omega \) as for the \( \rho^0 \) except that the decay constants of the \( \rho^0 \) are replaced by those of the \( \omega \) (\( f_\omega = 187 \text{ MeV} \) and 149 MeV for longitudinally and transversely polarized \( \omega \)-mesons, respectively). It turns out that the contribution of \( H \) to \( M^{U,++} \) is dominantly imaginary at small \(-t\) with the consequence of a very small interference with the pion-exchange amplitudes. Therefore, the two solutions of (21) for \( g_{\pi\omega} \) differ only by the sign within errors. In other words, we are only able to extract the absolute

\(^2\) A non-zero \( \gamma^\pi_T \to V_T \) amplitude could be generated by gluon transversity [29].
value of the form factor from $U_1$ for which we take the average of the two solutions. Our results for the form factor are shown in fig. 2. We also quote a result for the form factor at $Q^2 = 1.284 \text{GeV}^2$ for which HERMES has measured the $\omega$ SDMEs too. This form factor value is to be taken with a grain of salt, since the handbag amplitudes are not probed against experiment. Our results for the form factor are apparently consistent with the $Q^2 = 0$ value. The $Q^2$-dependence of the results can be parametrized as

$$|g_{\pi\omega}(Q^2)| = \frac{2.3 \text{GeV}^{-1}}{1 + Q^2/a_1^2 + Q^4/a_2^4},$$

(22)

with $a_1 = 2.7 \text{GeV}$ and $a_2 = 1.2 \text{GeV}$. This parametrization will be used if the form factor is needed at values of $Q^2$ other than 2 or $4 \text{GeV}^2$. For comparison we have also shown in fig. 2 the perturbative and the light-cone sum rule results for the $\pi\rho$ form factor [16] multiplied by the charge ratio (6). They are substantially smaller than our results extracted from the HERMES data on the $\omega$ SDMEs. Our error assessment does not only include the experimental errors of $U_1$ but also the uncertainties of the handbag amplitudes as well as those of the pion-nucleon coupling. The errors would be considerably smaller if the $Q^2$ cross section were known from experiment with an error smaller than, say, 10%. In the absence of such data our results for the $\pi\omega$ form factor evidently depend on the model used for the natural-parity amplitudes and one may wonder whether we have merely extracted an effective parameter. Nevertheless, the consistency of the entire approach supports its interpretation as an realistic estimate of the $\pi\omega$ transition form factor.

The combination $U_1$ for $\omega$ production has also been measured by the CLAS Collaboration [14]. Its values are about as large as the HERMES data. However, since for the CLAS data $W$ lies in the range 1.8–2.5 GeV while the range of $Q^2$ is similar to that of the HERMES data, $-t$ is larger than $0.5 \text{GeV}^2$ in this experiment. In this kinematical situation neither the parametrization of the pion exchange (see (2), (4)) nor the natural-parity handbag amplitudes are reliably known which prevents an estimate of the $\pi\omega$ form factor. We will return to this issue in the next section.

4 Partial cross sections

With the $\pi\omega$ form factor at disposal we are in the position to evaluate various partial cross section and SDMEs and to compare the results with the HERMES data [4]. The partial cross sections we are using, are defined by

$$\frac{d\sigma(\gamma^* \rightarrow V_j)}{dt} = \frac{1}{k_{\mu_i^\prime \nu_i}} \sum_{\mu_i^\prime \nu_i} |M_{\mu_i^\prime \nu_i^\prime \mu_i}|^2,$$

(23)

where $i$ and $j$ being either $L$ or $T$ and $\mu_{L(T)} = 0(\pm 1)$. The statistical factor is $s_L = 1$, $s_T = 2$. Neglecting $\gamma^*_T \rightarrow V_L$ transitions and the amplitude $M_{0-,-1-}$, we can decompose the full differential cross section for the process $\gamma^* p \rightarrow V p$ into

$$\frac{d\sigma}{dt} = \frac{d\sigma^N}{dt}(\gamma^*_T \rightarrow V_T) + \frac{d\sigma^U}{dt}(\gamma^*_T \rightarrow V_T) + \frac{d\sigma}{dt}(\gamma^*_T \rightarrow V_L) + \epsilon \frac{d\sigma^N}{dt}(\gamma^*_L \rightarrow V_L) + \epsilon \frac{d\sigma^U}{dt}(\gamma^*_L \rightarrow V_T).$$

(24)

To begin with, we show the $Q^2$- and $t$-dependence of the quantity $U_1$ in fig. 3. It measures the cross section ratio (cf. (17))

$$U_1 = 2 \frac{d\sigma^U(\gamma^*_T \rightarrow V_T) + \epsilon d\sigma^U(\gamma^*_L \rightarrow V_T)}{d\sigma}.$$

(25)
The $Q^2$-dependence, now evaluated from the interpolation (22), is of course perfectly reproduced since we extracted the $\pi\omega$ form factor from this quantity. The $t$-dependence of our results for $\omega$ production is a bit too mild as compared to the HERMES data. We stress that the $t$-dependent HERMES data are an average of all data for $Q^2 > 1\text{GeV}^2$ [4]. Since the trends of our results towards low $Q^2$ ($< 2\text{GeV}^2$) are in general in reasonable agreement with experiment we do not think that the inclusion of the low $Q^2$ data in the averages is the source of the rather flat $t$-dependence of our results. The reasonable value of the $\pi\omega$ form factor at $Q^2 = 1.284\text{GeV}^2$ (see fig. 2) supports this supposition.

In order to demonstrate the importance of the pion pole we also show results in fig. 3 for which the pion pole is disregarded. In this case $U_1$ is dramatically reduced since the unnatural-parity amplitudes are only fed by the GPD $\tilde{H}$. The $t$-dependence is much flatter without the pion-pole contribution, i.e. the differential cross sections appearing in (25) have similar $t$-dependencies except of the pion-pole contribution. Predictions for $U_1$ at $W = 3.5$ and 8 GeV are also displayed in fig. 3. A strong energy dependence of the pion pole contribution is to be noticed. It is very large also displayed in fig. 3. A strong energy dependence of the rather flat $\tilde{G}_{\rho NN}$ form factor at $Q^2 = 3\text{GeV}^2$ but small at 8 GeV. In the latter case the results are close to those without the pion pole. Note the maximum of $U_1$ at $t' \simeq -0.05\text{GeV}^2$ and $W = 8\text{GeV}$ which is a consequence of the factor $-t/(t-m^2_\pi)^2$ discussed in sect. 2, see fig. 1.

It is well known that the pion pole contributes to the GPD $\tilde{E}$ [26,27]:

$$\tilde{E}_{\text{pole}}^{u} = -\tilde{E}_{\text{pole}}^{d}$$

$$= -\Theta(|x| \leq \xi) \frac{m \Gamma_\pi g_{\pi NN} F_{\pi NN}(t)}{2\xi \tilde{E}_{\omega}^{\rho^0}} \phi_{\pi} \left( \frac{x+\xi}{2\xi} \right),$$

where $\Gamma_\pi$ is the decay constant and $\phi_{\pi}$ the distribution amplitude of the pion. Evidently, the charge factor between $\omega$ and $\rho^0$ production quoted in (6) immediately follows from (26) since the combinations $e_u \tilde{E}_u^u + e_d \tilde{E}_d^d$ contribute to $\omega$ and $\rho^0$ production, respectively. It is well known that an evaluation of the pion-pole contribution to $\pi^+\omega$ leptoproduction from $\tilde{E}_{\text{pole}}$ underestimates it by far since this way only the one-gluon-exchange contribution to the electromagnetic form factor of the pion in (16) is taken into account which is known to amount to only 30–50% of its experimental value [30]. To probe the contribution from $\tilde{E}_{\text{pole}}$ in $\omega$ leptoproduction we evaluate it from the handbag graphs along the same lines as we did for $\tilde{H}$, see [9]. Since $\tilde{E}_{\text{pole}}$ contributes to the $\gamma^*_\tau \rightarrow \omega_\tau$ transition amplitude while quark and antiquark forming the meson, possess opposite helicities one unit of orbital angular momentum is required in the $\omega$ wave function, which is represented by a factor $k_\perp$ [9, 31]. A result similar to that for $\pi^+\omega$ production is obtained—the pion pole contribution to $\omega$ production evaluated through (26), is about a factor of 3 smaller than the one-particle exchange contribution, see sect. 2. This also holds true for $\rho^0\omega$ production which motivated us to neglect the pion-pole contribution in our previous analysis of this process [9].

Another way to separate the natural- and unnatural-parity cross section is the combination $3$

$$P = \frac{2r^1_{-1}}{1 - r^0_{00} - 2r^0_{11}} = \frac{d\sigma^N(\gamma^*_\tau \rightarrow V_\tau) - d\sigma^U(\gamma^*_\tau \rightarrow V_\tau)}{d\sigma^N(\gamma^*_\tau \rightarrow V_\tau) + d\sigma^U(\gamma^*_\tau \rightarrow V_\tau)}. \quad (27)$$

Our results for $P$ in $\omega$ production are shown in fig. 4 and are compared to the HERMES data. Good agreement with experiment is to be noticed as well as the prominent role of the pion pole. With the pole the unnatural-parity $\gamma^*_\tau \rightarrow V_\tau$ cross section is about three times larger than the corresponding natural one at $W = 4.8\text{GeV}$, without it the ratio of the two cross sections only amounts to about 1/3. In the latter case the unnatural-parity cross section is solely fed by the $\tilde{H}$ contribution. The $N/U$ ratio strongly depends on the energy.

3 In [4] a slightly different definition of $P$ is used.
Fig. 5. \( R \) versus \( Q^2 \) (left) and versus \( t' \) (right). For other notations it is referred to fig. 3. The short-dashed line represents the cross section ratio for longitudinal and transverse photons.

Fig. 6. Left: The integrated cross sections (\( -t' \leq 0.5 \text{GeV}^2 \)) for longitudinal (\( \sigma_L \)) and transverse (\( \sigma_T \)) photons as well as the natural-parity (\( \sigma_N \)), unnatural-parity (\( \sigma_U \)) and full (\( \sigma \)) one for \( \omega \) production versus \( Q^2 \) at \( W = 4.8 \text{ GeV} \). Right: The integrated cross section for \( \omega \) production versus \( W \) at \( Q^2 = 3.3–3.5 \). The solid (dashed) line represents our results with (without) the pion pole. Data are taken from [14,34].

A frequently considered combination is

\[
R = \frac{1}{\varepsilon} \frac{r_{00}^{04}}{1 - r_{00}^{04}} = \frac{d\sigma(\gamma^*_L \rightarrow V_L)}{d\sigma(\gamma^*_T \rightarrow V_T)} + \frac{1}{3}d\sigma(\gamma^*_T \rightarrow V_L) + \varepsilon d\sigma(\gamma^*_L \rightarrow V_T),
\]

which is the ratio of the cross sections for longitudinally and transversally polarized vector mesons. It is often identified with the cross section ratio for longitudinally and transversally polarized photons: \( d\sigma_L/d\sigma_T \). This is however only true if the transitions \( \gamma^*_L \rightarrow V_T \) and \( \gamma^*_T \rightarrow V_L \) are strongly suppressed. As can be seen from fig. 5 this is not the case for \( \omega \) production: \( d\sigma_L/d\sigma_T \) is about 30% larger than \( R \) at \( t' = -0.08 \text{GeV}^2 \) because of the rather large cross section \( d\sigma(\gamma^*_L \rightarrow \omega_T) \). The cross section \( d\sigma(\gamma^*_T \rightarrow \omega_L) \), fed by the transversity GPDs, plays only a minor role. Due to the pion-pole contribution which strongly enhances the \( \gamma^*_T \rightarrow \omega_T \) cross section \( R \) is much smaller than 1. This is in sharp contrast to \( \rho^0 \) production where \( R \) is somewhat larger than 1 at \( Q^2 \approx 3 \text{ GeV}^2 \) and almost independent of energy [32,33].

In fig. 6 we display various integrated cross sections for \( \omega \) production. As expected from the discussions of \( U_1 \), \( P \) and \( R \) we find \( \sigma_T > \sigma_L \) and \( \sigma_U > \sigma_N \) (the small amplitude \( M_{0++} \) is assigned to \( \sigma_N \)). Neglect of the pion pole reverses the inequalities. All these features lead us to the conclusion that, at least for HERMES kinematics, \( \omega \) production is very different from the asymptotic result of proceeding dominantly through longitudinally polarized photons [35]. Our predictions for the \( \omega \) cross section versus \( W \) at fixed \( Q^2 = 3.3–3.5 \text{ GeV}^2 \) shown in fig. 6 reveal an interesting behavior: there is a maxi-
Now—is caused by the pion pole. At large $W$ than about 8 GeV its contribution is very small and the contribution from $\bar{\omega}$ is tiny, much smaller than the natural-parity one. Since we neglect in our parametrization of the GPDs, may generate this effect. However, since the sharp drop of the cross section between 2 and 4 GeV is also seen in $\rho^+$ production [41] but not in the $\phi$ cross section [42] this explanation must be taken with care. In any case below about 3 GeV there seems to be an additional strong dynamical mechanism beyond our handbag approach and the pion pole.

Through $\omega$-$\phi$ mixing the pion pole also appears in electroproduction of $\phi$ mesons. The $\pi\phi$ transition from factor is related to the $\pi\omega$ one by

$$g_{\pi\phi}(Q^2) \approx \sin(\Phi_V)g_{\pi\omega}(Q^2).$$

The vector-meson mixing angle, $\Phi_V$, in the quark-flavor basis is very small, about 3 degrees, as obtained from the ratio of the $\phi \rightarrow \pi^+\pi^-$ and $\omega \rightarrow \pi^0\pi^0$ decay widths [43]. Hence, the neglect of the pion pole in $\phi$ production is beyond doubt.

5 SDMEs

In this section we compare our results for the $\omega$ SDMEs with the HERMES data [4]. Since we neglect the $\gamma_T \rightarrow V_{-T}$ transitions some of the SDMEs fall together, e.g., $r_{1-1}^1 = -\text{Im}r_{1-1}^1$; others are approximately equal. In each such case we combine the SDMEs in one plot. A number of SDMEs are very small or even zero. If this agrees with experiment we do not display these SDMEs, e.g., $r_{00}^1$ or $\text{Im}r_{10}^3$. The remaining SDMEs are shown versus $Q^2$ in fig. 8 and versus $t'$ in fig. 9. In general we observe fair agreement between our results and the HERMES data. The importance of the pion pole is clearly visible, some of the SDMEs drastically change their values if the pion pole

![Fig. 7. $U_1(\rho^0)$ and $P(\rho^0)$ versus $Q^2$ for $\rho^0$ production at $W = 4.8$ GeV and $t' = -0.13$ GeV. The data are taken from [32]. The solid (dashed) lines represent our results from the handbag approach with (without) the pion pole.](image-url)
is neglected, for instance \( r_{1-1} \). We stress that the results on the SDMEs shown in figs. 8 and 9 are evaluated from the \( \pi \omega \) transition form factor (22), assuming a positive sign for it. Choosing it to be negative leads to results which agree with the other ones within the experimental errors. Since

\[
\rho_{04} = \frac{d\sigma(\gamma_T \to V_L) + \varepsilon d\sigma^N(\gamma_L \to V_L)}{d\sigma},
\]

this SDME is sufficiently well probed by the combinations of SDMEs discussed in sect. 4, thus we do not display it here. The equality of \( r_{1-1} \) and \(-\text{Im} r_{1-1}^{N} \)

\[
r_{1-1} = -\text{Im} r_{1-1}^{N} = \frac{d\sigma^N(\gamma_T \to V_L) - d\sigma^U(\gamma_T \to V_L)}{2d\sigma},
\]

is in fair agreement with experiment as can be seen from the first rows of figs. 8 and 9. The (class B) SDMEs \( \text{Re} r_{10}^{N} \) and \( \text{Im} r_{10}^{N} \) fall practically together, since they are dominated by the real part of

\[
\sum_{\nu'} \mathcal{M}_{\nu'+,++}^{N} \mathcal{M}_{0\nu',++}^{N*}.
\]

The imaginary part of this interference term controls \( \text{Im} r_{10}^{N} \approx \text{Re} r_{10}^{N} \). It is very small in our approach in agreement with experiment within admittedly large errors. A little difference between \( \text{Re} r_{10}^{N} \) and \( \text{Im} r_{10}^{N} \) is caused by the term \( \text{Re}[\mathcal{M}_{\nu'+-0,++}^{N} \mathcal{M}_{0-++}^{N*}] \) appearing in \( \text{Im} r_{10}^{N} \). This term is proportional to the \( \pi \omega \) transition form factor and therefore depends on its sign. Given the experimental errors this has however no noticeable consequences, since this term is so small. Thus, SDMEs of class B depend on the pion pole only through the normalization \( d\sigma/dt \). The SDME \( r_{11}^{01-1} \) measures the pion-exchange cross section.
Fig. 9. Various SDMEs versus $t'$ at $W = 4.8$ GeV and $Q^2 = 2.42$ GeV$^2$. Data are taken from [4]. For other notations the reader is referred to fig. 3.

$$d\sigma^{\text{pole}}(\gamma_L \to V_T) \text{ (see (14))}$$

$$r_{10}^{04} = \frac{\varepsilon}{2} \frac{d\sigma^{U}(\gamma_L^* \to V_T)}{d\sigma}.$$  \hspace{1cm} (33)

Also for this small SDME which asymptotically decreases as $Q^{-4}$, we find reasonable, although not perfect agreement with the data.

The (class C) SDMEs Re$r_{10}^{04}$, Re$r_{10}^{10} = -\text{Im}r_{10}^{2}$ and $r_{00}^{5}$, shown in the second rows of figs. 8 and 9, are sensitive to the transversity GPDs $H_T$ and $E_T$ feeding the $\gamma_T^* \to V_L$ amplitudes [29]. For these SDMEs the pion pole mainly affects the normalization $d\sigma/dt$. The SDME $r_{00}^{5}$ is dominated by Re$M_{0+0+}M_{0+}^*$, the other three by Re$M_{0+0+}^*M_{0+}^{0+}$, Re$M_{0+0+}^*M_{0+}^{0+}$ and Re$M_{0+0+}^*M_{0+}^{0+}$. All these SDMEs are rather small and in reasonable agreement with the HERMES data which are subject to rather large errors. An exception is the $t$-dependence of $r_{00}^{5}$. The good agreement of this SDME for $\rho^0$ production with our results [29] makes it difficult to improve the results for $r_{00}^{5}(\omega)$.

In the last rows of figs. 8 and 9 the (class D) SDMEs $r_{11}^{5} = r_{10}^{7} = \text{Im}r_{10}^{7}$ and $r_{11}^{8} = r_{10}^{8} = \text{Im}r_{10}^{8}$ are displayed which measure the real and imaginary part of

$$\sum_{\nu'} M_{\nu',++}^U M_{\nu',0+}^U ,$$  \hspace{1cm} (34)

respectively. Up to a small contribution from $\tilde{H}$ to $M_{+++,++}$ these SDMEs probe the pion-exchange amplitudes (10). Obviously, these SDMEs are zero if the pion-pole contribution is neglected. The imaginary part of (34) is just the interference term of the $\tilde{H}$ and the pion-pole contributions. It is non-zero although very small. Moreover, it is $\sim g_{\pi \omega}$ and, hence, changes sign together with the form factor. Inspection of figs. 8 and 9 reveals that the HERMES data [4] do not fix the sign of the transition form factor.
at these energies look similar to those at the COMPASS experiment, respectively. The results at 3.5 GeV are further away from those obtained are closer. Under neglect of the pion-pole contribution, at 8 GeV they see [29,44].

We refrain from showing predictions for the ω SDMEs at W = 3.5 and 8 GeV typical for the upgraded JLab and the COMPASS experiment, respectively. The results at these energies look similar to those at W = 4.8 GeV. At 3.5 GeV the results are further away from those obtained under neglect of the pion-pole contribution, at 8 GeV they are closer.

6 Spin asymmetries

In [29] we have investigated various spin asymmetries and it is now obligatory to check whether the results presented in [29] will be substantially changed by the inclusion of the pion pole or not. In this connection we can also examine whether there are asymmetries which are sensitive to the sign of the πV transition form factor. Expressing the asymmetries for longitudinal and transverse beam and target polarizations, A_{UT}, A_{LT}, A_{LU}, A_{UL}, A_{LL}, in terms of helicity amplitudes [29,44], we find two potentially large interference terms with the pion-pole contribution

\[ M^{N*}(\gamma_{T}^\ast \rightarrow V_T)M^{U}(\gamma_{L,T}^\ast \rightarrow V_T) \] (35)

and

\[ M^{U*}(\gamma_{T}^\ast \rightarrow V_T)M^{U}(\gamma_{L,T}^\ast \rightarrow V_T). \] (36)

The imaginary part of the latter interference term reduces to that of the contributions from \( \frac{3}{2} \) and the pion pole. This term as well as the one given in (35) change sign with the transition form factor and mainly affect \( A_{UT} \) and \( A_{UL} \). The pion pole affects all spin asymmetries through the normalization, the unseparated cross section. This effect is however substantial only for ω production at energies less than about 6 GeV. Note that the term

\[ \text{Re} \sum_{\nu, \nu'} M_{\nu+\nu',++}^{U*} M_{\nu+\nu',0+}^{U} \] (37)

contributing to the \( \cos \phi_{\ast} \) modulation of \( A_{LT} \), is zero.

Two examples of our predictions for asymmetries in ω leptoproduction are shown in fig. 10. The effects of the pion pole are particularly large for these asymmetries and the sign of the πγ form factor matters.

For \( \rho^0 \) production only little effects are generated by the pion pole. The agreement of our previous results with the experimental data on \( A_{UT} \) and \( A_{LT} \) [45–47] remains true. For ω production at \( W \approx 8 \) GeV the pion pole still affects somewhat the asymmetries, in particular the \( \sin(\phi - \phi_{\ast}) \) and \( \sin \phi_{\ast} \) modulations of the transverse target asymmetry \( A_{UT} \), which are even sensitive to the sign of the \( \pi\omega \) transition form factor (see fig. 10).

7 Summary

In the present work we have analyzed the data on the SDMEs of the omega meson measured by the HERMES Collaboration [4] recently. In this analysis we have made use of the handbag approach and exploited a set of GPDs extracted by us from data on leptoproduction of \( \rho^0, \phi \) and \( \pi^+ \) mesons [9–11]. In addition we have allowed for the pion pole which, as it turns out, plays a very important role in ω production. The coupling of the exchanged pion to the proton is known from other sources (see, for instance, [10,11]) while that to the virtual photon and the ω meson, i.e. the \( \pi\omega \) transition form factor, is fixed from the ω SDMEs. With the exception of this form factor there is no free parameter in our analysis. We have obtained reasonable values for this form factor and in general a fair description of the HERMES data on ω production

\( \phi_{\ast} \) is the orientation of the target spin vector with respect to the lepton plane and \( \phi \) specifies the azimuthal angle between the lepton and the hadron plane.
for $Q^2 \gtrsim 2 \text{GeV}^2$. For $Q^2$ less than $2 \text{GeV}^2$ the GPDs and the handbag approach are not probed against experiment. There are various approximations made in the handbag approach which become inaccurate at low $Q^2$, e.g., neglect of contributions of order $t/Q^2$, target mass corrections or higher-order perturbative corrections. As shown in [48] the NLO corrections to the handbag amplitudes become large at low skewness and low $Q^2$ in the collinear limit. In this situation a resummation is required which seems to reduce the perturbative corrections drastically [49,50].

Implications of these theoretical findings for the modified perturbative approach we are using in the calculation of the partonic subprocesses, in which quark transverse momenta are retained and Sudakov effects in next-to-leading log approximations are taken into account, are unclear. Nevertheless, ignoring these problems and working out the SDMEs for $Q^2 < 2 \text{GeV}^2$, we find agreement with the HERMES data on the same level of quality as for larger $Q^2$. We stress that the SDMEs for the $\omega$ meson do not fix the sign of the $\pi\omega$ transition form factor, some of the spin asymmetries for $\omega$ lepton production are however sensitive to this sign.

We have also commented on the role of the pion pole in $\rho$ and $\phi$ lepton production. In the latter case, the pion pole only contributes through $\omega - \phi$ mixing and leads to tiny effects which are negligible in practice. For $\rho$ production the pion pole contribution is small because of (6). It enhances the cross section by about 2% and is visible only in some of the SDMEs or combinations of SDMEs like $U_1$ and $P$.

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