Static and dynamic phases for magnetic vortex matter with attractive and repulsive interactions

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Abstract

Exotic vortex states with long range attraction and short range repulsion have recently been proposed to arise in certain superconducting hybrid structures such as type-I/type-II layered systems as well as multi-band superconductors. In previous work it has been shown that such systems can form clump or phase separated states, but little is known about how they behave in the presence of pinning and under an applied drive. Using large scale simulations we examine the static and dynamic properties of such vortex states interacting with random and periodic pinning. In the absence of pinning this system does not form patterns but instead undergoes complete phase separation. When pinning is present there is a transition from inhomogeneous to homogeneous vortex configurations similar to a wetting phenomenon. Under an applied drive, a dynamical dewetting process can occur from a strongly pinned homogeneous state into pattern forming states, such as moving stripes that are aligned with the direction of drive or moving labyrinth or clump phases. We show that a signature of the exotic vortex interactions observable with transport measurements is a robust double peak feature in the differential resistance curves. Our results should be valuable for determining whether such vortex interactions are occurring in these systems and also for addressing the general problem of systems with competing interactions in the presence of random and periodic pinning.

(Some figures may appear in colour only in the online journal)

1. Introduction

Vortices in superconductors interacting with random or periodic pinning provide a model system for studying the interplay between particle interactions that favor one type of ordering and substrate interactions that favor some alternative symmetry or a disordered state [1]. In such systems a rich variety of liquid [1], glassy [2], and incommensurate static phases [3, 4] arise, and under an external drive numerous dynamical phases [1, 5–7] occur that produce distinct transport measurement features [5, 8–10]. Most studies of vortices in type-II superconductors consider purely repulsive interactions; however, modified interactions for vortices consisting of short range repulsion and long range attraction have recently been proposed to arise in certain multi-band superconductors or type-I/type-II layered systems. Imaging experiments of Moshchalkov et al [11] in the multi-band superconductor MgB$_2$ revealed inhomogeneous, stripe-like vortex configurations [11–13], leading the authors to claim that the vortex interactions share properties of type-I and type-II materials, with the type-I characteristics dominating at longer length scales. To support this, they conducted molecular dynamics simulations of vortices with pairwise interactions that combined long range attraction and short range repulsion. Earlier work on type-I/type-II hybrids also showed that the system would have characteristics of both states, producing inhomogeneous vortex arrangements [14].

This initial work has been followed by much activity in the field, including studies of vortex coalescence, clump states, and complete phase separation [11, 15–28].

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One of the first questions is whether there is an effective vortex–vortex interaction with the long range attraction and short range repulsion that could be used to model this system. Recently Zhao et al adopted a two Bessel function vortex interaction form that is repulsive at short range and attractive at longer range, and showed that this interaction produces inhomogeneous vortex clusters [29]. In multi-band superconductors, non-pairwise vortex interactions can also be important; however, in many cases the non-pairwise interaction forces are small, permitting a pairwise approach to the vortex interactions, as discussed in [30]. In principle a full Ginzburg–Landau approach to simulation of the dynamics and pinning of such systems would be optimal; however, due to computational constraints this approach is generally not feasible, and even for purely repulsive vortex interactions, only limited full studies can be performed to explore the collective response of interacting vortices to pinning and driving currents.

With an effective pairwise vortex interaction potential, large scale studies of the dynamical response of the system can be performed, as has been successfully demonstrated in simulations of repulsively interacting vortex systems [7, 10]. In the work of Zhao et al [29] with pairwise interaction potentials of a two Bessel function form, it was shown that multi-band type superconducting vortices form clumps or inhomogeneous states. These configurations were obtained using a rapid quench from a uniform state, meaning that the system may not have reached a ground state. It is possible that if such vortex interactions occur in actual multi-band superconductors, then inhomogeneous vortex configurations could appear due to rapid quenching. Even if this is the case, once the vortices are driven the patterns can change, so it is desirable to understand the way in which the patterns would evolve and whether there are any signatures that would appear in experimentally measured quantities such as current–voltage curves. Although there have been imaging experiments that have been interpreted as showing the existence of long range attractions between the vortices [11, 13], it is crucial to understand the effects of pinning as well as an applied drive, since even for vortices with purely repulsive interactions, strong pinning can produce highly inhomogeneous flux distributions [31–33].

Previous work has shown that a variety of periodic clump crystals and stripe structures can arise in systems with competing long range repulsive and short range attractive interactions or with two step repulsive interactions such as in certain models of charge ordering in Mott-insulators as well as some fillings for two-dimensional (2D) electron gas systems in magnetic fields [34–37]. When driven, these systems exhibit distinct features in the velocity–force curves as they transition from a disordered state at low drive to a dynamically ordered state at high drives. It is not known whether similar structures or dynamically induced disorder–order transitions would occur for systems with long range attractive and short range repulsive interactions of the type proposed to occur for vortices in multi-band superconductors. Numerical simulation of a Ginsburg–Landau description of vortices in these materials revealed an effective pairwise energy consisting of short range repulsion and long range attraction, and in molecular dynamics calculations using this potential, the system completely phase separates into a single giant vortex cluster [26]. The formation of a single vortex cluster was also observed in Ginsburg–Landau simulations in [30].

In order to address the ground state ordering of the modified vortex interaction system with and without quenched disorder, we perform large scale numerical simulations. In the absence of pinning, we find that for long simulated annealing times, pattern formation in the form of lattices of clumps or stripes does not occur and the system completely phase separates into a single giant clump, while for shorter annealing times the system can become trapped in metastable states exhibiting pattern formation. For the long annealing times, addition of pinning induces a crossover from the phase separated state to a homogeneous, pattern-free vortex state. This behavior is very similar to the wetting–dewetting transition found for liquids of mutually attractive atoms on surfaces. When the atomic attraction dominates, the atoms clump or form a single dewetted drop, while when the surface attraction dominates, the atoms spread out and wet the surface [38].

Another open question is how the transport response for vortices with modified interactions differs from that of vortices with purely repulsive interactions, and whether transport could be used as a probe of the nature of the vortex interactions. We find that in the presence of strong pinning, vortices with modified interactions that do not form patterns in the absence of a drive organize into patterns such as stripes or labyrinth structures under an applied drive. The dynamic phase transition into inhomogeneous clump or stripe states produces a robust double peak in the differential resistance. This is in contrast to the case for vortices with only repulsive interactions, where a single peak in the differential resistance occurs under an applied drive when the system undergoes a dynamical ordering into a moving smectic or anisotropic crystal [7–10]. This indicates that the formation of dynamic patterns can be deduced from transport measurements, providing a clear method for determining whether vortices with modified interactions exist in a sample without requiring imaging. The dynamic patterns also have a number of interesting characteristics, including the formation of stripes aligned with the direction of the applied force. Our results are general and can be applied to other systems of particles with competing long range attraction and short range repulsion in the presence of a substrate, such as colloidal particles or vortices in Bose–Einstein condensates [39].

2. Simulation

We simulate a two-dimensional (2D) slice containing \( N_p \) pinning sites of a three-dimensional system of \( N_v = 400 \) stiff vortex lines with periodic boundary conditions in the \( x \) and \( y \) directions. The vortex dynamics are obtained by integrating the following equation of motion: 
\[
\eta(d\mathbf{R}_i/dt) = F_i^{vv} + F_i^{vp} + F_i^p + \mathbf{F}_i^f,
\]
where \( \eta \) is the damping constant and \( \mathbf{R}_i \) is the position of vortex \( i \). We use the vortex–vortex interaction
force proposed for the type-I/type-II hybrid materials and multi-band superconductors [19],

\[ F_{j}^{V} = \sum_{j \neq i} [aK_{1}(bR_{ij}/\lambda) - K_{1}(R_{ij}/\lambda)]\hat{R}_{ij}. \]  

Here \( K_{1}(r) \) is the modified Bessel function, \( R_{ij} = |\mathbf{R}_{i} - \mathbf{R}_{j}| \), \( \hat{R}_{ij} = (\mathbf{R}_{i} - \mathbf{R}_{j})/R_{ij} \), and \( \lambda \) and \( \lambda/b \) are the penetration depths in the two bands. A double Bessel function interaction potential was previously proposed for low \( \kappa \) depths in the two bands. A double Bessel function interaction potential was previously proposed for low \( \kappa \) superconductors at low fields [40], but non-monotonicity of the vortex interaction requires the existence of multiple bands and does not occur for single bands or single-component Ginzburg–Landau models [14]. The system size is \( L \times L \) with \( L = 64 \). In previous pin-free simulations of this model, the constants \( a \) and \( b \) were varied to change the relative strength of the attractive term and modify the distance \( r_{c} \) at which the interaction force changes sign [29]. Long range attraction and short range repulsion are obtained when \( a > b \) and \( b > 1.0 \). The coefficient \( a = K_{1}(r_{c})/K_{1}(br_{c}) \), and we set \( r_{c} = 2.1\lambda \) and \( b = 1.1 \). Since the interaction falls off rapidly at large distances we cut off the potential for \( R_{ij} > 8\lambda \). We model the pinning sites as attractive parabolic traps with maximum force \( F_{p} \) and radius \( R_{p} = 0.3\lambda \), as previously used in simulations of repulsive vortices [4, 7]. Periodic pinning arrays can be fabricated using lithographic techniques [3]. Here, we consider both a square pinning array and randomly distributed pinning. The ratio \( N_{p}/N_{v} \) of the number of pinning sites to the number of vortices is reported in terms of \( B_{\phi}/B \), where \( B_{\phi} \) is the field at which there is one vortex per pinning site and \( B \) is the vortex density, held fixed throughout this work. The thermal force term \( F_{i}^{T} \) has the following properties: \( \langle F_{i}^{T}(t) \rangle = 0 \) and \( \langle F_{j}^{T}(t)F_{j}^{T}(t') \rangle = 2nk_{B}T\delta(t-t') \). After performing simulated annealing we apply an external drive \( F_{\delta} = F_{0}\hat{x} \) and measure the resulting vortex velocity \( \langle v_{i} \rangle = N_{v}^{-1}\sum_{i} (d\mathbf{R}_{i}/dt) \cdot \hat{x} \), which would correspond to an experimentally measured current–voltage curve.

We note that in real superconducting systems there may be an additional demagnetization effect arising from the bending of the field lines outside of the sample, which could induce an effective repulsion at long range. Even in systems where this occurs, the longer range attraction and shorter range repulsion will still lead to larger clumps in clean systems. Additionally, for thicker samples and higher fields this effect would be strongly reduced. It is important to understand what types of patterns can occur without this additional complication in order to determine whether the proposed vortex interactions with long range attraction and short range repulsion can describe the dynamical response of the experimental samples.

### 3. Annealing times and pinning effects

We first study the model without pinning and conduct simulated annealing studies by starting from a high temperature molten state and slowly cooling to \( T = 0 \) during a time \( \tau_{s} \). We find that for instantaneous annealing (small \( \tau_{s} \)) the vortices do not form a triangular lattice but instead fall into a disordered assembly of clumps, labyrinths, or voids. A similar set of patterns was observed for this same model in [29], where we presume that an instantaneous quenching of the system was performed. The structures we obtain coerce as the annealing time is increased, and for long annealing times \( \tau_{a} \geq 1 \times 10^{6} \) simulation time steps, the system completely phase separates into a single clump, rather than forming ordered or partially disordered arrangements of stripes or clumps of the type found for systems with long range repulsion and short range attraction [35, 36]. Such complete phase separation was observed in molecular dynamics simulations of this same modified vortex interaction model [26]. These results suggest that the stripes and inhomogeneous patterns imaged in MgB\(_{2} \) [11, 13] would not be produced purely by the proposed vortex interactions in equation (1), assuming that only very weak pinning is present in the sample, although demagnetization effects and the possible trapping of the system in a metastable nonequilibrium state could play a role in the experiments. Even type-I superconductors do not always show complete phase separation. It is also possible that equation (1) does describe the vortex interactions present in this material but that pinning arrests the phase separation or causes the ground states to break apart. Even if it is true that the pinning could stabilize pattern forming structures, it is also possible for inhomogeneous pinning to produce similar structures for vortices with purely repulsive interactions, so the structures imaged in [11, 13] do not provide conclusive evidence of the nature of the vortex interaction potential. Another possibility we explore later is that the vortices could have been in motion leading to dynamic pattern formation.

In figure 1 we show the vortex and pin positions from long annealing time simulations of samples with periodic and random pinning at \( B_{\phi}/B = 0.4225 \). At \( F_{p} = 0.3 \) for square pinning, figure 1(a) shows complete phase separation of the vortices into a single clump, with all pins empty except those directly under the clump. In figure 1(b) at \( F_{p} = 0.9 \), each pin captures one vortex producing a homogeneous flux background, while the remaining vortices form a single clump. At \( F_{p} = 1.5 \) in figure 1(c), each pin captures multiple vortices, and there are patches of unpinned interstitial vortices. For larger values of \( F_{p} \), all the vortices are trapped by pins. Similar behavior occurs for random pinning, as shown in figures 1(d)–(f). The single clump state for weak pinning in figure 1(d) is followed at higher pinning strength by a phase with a coexisting clump and uniform flux background, shown in figure 1(e) for \( F_{p} = 0.9 \). The clump decreases in size as \( F_{p} \) increases, and eventually all the vortices become pinned as shown in figure 1(f) for \( F_{p} = 1.5 \). We have investigated a range of values of \( F_{p} \) and \( B_{\phi}/B \) as well as other system parameters, and in general always find the same generic phases. We find no regimes where multiple clump, stripe, or labyrinth phases are stabilized, nor are there any phases that resemble the structures observed for systems with long range repulsive and short range attractive interactions [34–36]. In analogy to the spreading of liquid drops on surfaces [38], we term the states illustrated in figure 1 dewetted (D), where all the vortices form a single clump; partially wetted (PW), where
Figure 1. The vortex positions (filled circles) and pinning sites (open circles) after annealing in the presence of periodic (a)–(c) or random (d)–(f) pinning for $B_{\phi}/B = 0.4225$ and $\tau_a = 1 \times 10^6$ simulation time steps. (a) Dewetted (D) state at $F_p = 0.3$ where most pinning sites are unoccupied. (b) Partially wetted (PW) state at $F_p = 0.9$. Each pin captures at least one vortex and the remaining vortices form a single clump. (c) Wetted (W) state at $F_p = 1.5$ where most vortices are trapped by pins. (d) D state at $F_p = 0.3$. (e) PW state at $F_p = 0.9$. (f) W state at $F_p = 1.5$. All the pins are occupied but there is still a vortex clump; and wetted (W), where the clump is absent and the majority of the vortices are trapped by pins.

Figure 2 shows the locations of the D, PW, and W regimes in the $F_p$ versus $B_{\phi}/B$ phase diagram obtained for both periodic and random pinning samples where we vary $B_{\phi}$ by changing the pinning density. To identify the locations of the phases we measure $M_n$, the fraction of pins containing at least $n$ vortices, as shown in the insets of figure 2. Both $M_1$ and $M_2$ are low in the D phase; $M_1$ is high and $M_2$ is low in the PW phase; and $M_1$ and $M_2$ are both high in the W state. For clarity, $M_n$ with $n > 2$ is not shown. We find that the system enters the W state at high $F_p$ and $B_{\phi}/B$, while for weak $F_p$ or low $B_{\phi}/B$, the D state dominates.

4. Driven phases

We now examine the driven dynamics of the vortices with modified interactions and show both that they exhibit a rich variety of dynamic phases as well as that transport measurements may be the most promising method for determining whether vortices with modified interactions are present. Studies of vortices with strictly repulsive interactions driven over random pinning find that a disordered pinned state can undergo a dynamical ordering transition in the moving state that lead to the formation of smectic type ordering [6–10]. The onset of the dynamic ordering in systems with repulsive vortex interactions is associated with
Figure 3. The vortex positions (filled circles) and pinning sites (open circles) in snapshots of the moving state for a sample with (a)–(f) random pinning and (g)–(i) periodic pinning at $B/\phi/B = 0.64$ under a drive $F_d = F_d x$ showing the formation of heterogeneous states at high drives. (a) $F_p = 0.3, F_d = 0.05$ (i). (b) $F_p = 0.3, F_d = 0.3$ (ii). (c) $F_p = 0.7, F_d = 0.25$ (iii). (d) $F_p = 0.7, F_d = 0.8$ (iv). (e) $F_p = 1.3, F_d = 2.5$ (v). (f) $F_p = 1.3, F_d = 3.8$ (vi). (g) $F_p = 0.7, F_d = 1.6$ (vii). (h) $F_p = 1.3, F_d = 0.96$ (viii). (i) $F_p = 1.3, F_d = 3.0$ (ix). The labels i–vi (vii–ix) correspond to the drives marked in figures 4(a)–(c) ((e) and (f)).

(a) x
(b) x
(c) x
(d) x
(e) x
(f) x
(g) x
(h) x
(i) x

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a single peak in experimentally measured $dV/dI$ curves $[8, 9]$ and simulated $d\langle V_x \rangle/dF_d$ curves $[7, 9, 10]$. For systems with modified interaction vortices, strongly pinned samples form the uniformly pinned states illustrated in figures 1(c) and (f). A dynamical structural transition occurs under an applied drive due to the reduced effectiveness of the pinning in the moving state, which allows the attractive part of the vortex interactions to draw the vortices into a highly heterogeneous configuration.

In figures 3(a)–(f) we show snapshots of the vortex positions in the driven state for random pinning samples with three different pinning strengths at $B/\phi/B = 0.64$. In the D state, two types of dynamics occur. When $F_p$ is very weak, the clump depins and remains intact; however, for higher $F_p$, above depinning the clump becomes mobile and sheds a trail of pinned vortices, as shown in figure 3(a) for $F_p = 0.3$ and $F_d = 0.05$. The trailing edge of the clump becomes rarefied, leading to a decrease in the effective attraction between vortices at the back of the clump that allows them to be torn away from the clump by the pins. As $F_d$ increases, the trapped vortices depin and a portion can rejoin the clump, which retains its shape for higher drives. The reassembled clump for the $F_p = 0.3$ sample is shown at $F_d = 0.8$ in figure 3(b). In figure 4(a) we plot $d\langle V_x \rangle/dF_d$ and $\sigma_v$ versus $F_d$ for the random pinning sample in figures 3(a) and (b). Here $\sigma_v$ is the standard deviation of the vortex density calculated using a 2D grid with elements equal in length to the average pinning lattice constant $a$. Homogeneous vortex configurations give small values of $\sigma_v$, while $\sigma_v$ is larger for heterogeneous configurations. The pinned clump state has a large value of $\sigma_v$. As $F_d$ is increased, $\sigma_v$ initially drops when the moving clump sheds vortices, as in figure 3(a), and then increases again when the clump reforms for higher drives, as in figure 3(b). There is a weak two peak structure in $d\langle V_x \rangle/dF_d$ reflecting the two step depinning transition, with the second peak corresponding to the drive at which all of the vortices depin. Figures 3(c) and (d) shows a sample with $F_p = 0.7$, which begins in a PW
state. Above depinning, shown in figure 3(c) for \( F_d = 0.25 \), the clump disintegrates and the vortices flow in a meandering path, while at higher drives the vortices reorder into a much more heterogeneous state containing a clump and patches of filaments, as shown in figure 3(d) at \( F_d = 3.0 \). Figure 4(b) shows that \( \sigma_v \) is high in the pinned heterogeneous PW state for this sample, passes through a local minimum near the first peak in \( d\langle V_x \rangle/dF_d \) when the clump breaks apart, and then increases again to a value higher than that at \( F_d = 0 \) in the vicinity of the second peak in \( d\langle V_x \rangle/dF_d \) where the vortices form a moving clump and filament state. In figures 3(e) and (f) we illustrate the moving states at \( F_d = 2.5 \) and 3.8 for a sample with \( F_p = 1.3 \) that has a W pinned state. The vortices form a moving stripe state near \( F_d = 2.75 \) with the stripe aligned in the direction of the applied drive, and a portion of the stripes collapse back into clumps as \( F_d \) increases; however, even for high \( F_d \), some stripes remain present. Figure 4(c) shows that in this case the \( d\langle V_x \rangle/dF_d \) curve has a pronounced double peak and that \( \sigma_v \) increases with increasing \( F_d \) as the vortex configuration becomes more homogeneous. In general, for higher \( F_p \) we find more stripe-like driven states, and the double peak feature in \( d\langle V_x \rangle/dF_d \) becomes even more prominent.

The dynamics for samples with periodic pinning show the same overall trends as the random pinning samples. In the D regime, the moving clump sheds a trail of vortices above depinning that peel away from the clump as it moves, similar to the effect found for random pinning. As \( F_d \) increases, the number of vortices captured by each pin along the trail drops until all the vortices depin and the clump reforms. The corresponding \( d\langle V_x \rangle/dF_d \), illustrated for \( F_p = 0.3 \) in figure 4(d), has a double peak feature correlated with jumps in \( \sigma_v \) to higher values as the system becomes more heterogeneous at higher \( F_d \). For the PW state, above depinning the clumps break apart and shed both pinned and interstitial vortices. As \( F_d \) increases, the clumps begin to reform until all the vortices depin and form a coexisting clump and moving labyrinth state with vortices flowing along the pinning rows, as illustrated in figure 3(g) for a sample with \( F_p = 0.7 \) at \( F_d = 1.6 \). For \( F_d > 2.75 \), the labyrinth phase gradually collapses to another clump phase. Figure 4(e) shows that in the \( d\langle V_x \rangle/dF_d \) and \( \sigma_v \) curves for the \( F_p = 0.7 \) sample, the transition to a more heterogeneous state at higher \( F_d \) appears as an increase in \( \sigma_v \). In the W state the initial depinning occurs via localized incommensurations flowing along the pinning rows in one-dimensional channels while the other vortices remain pinned, as shown in figure 3(h) for \( F_p = 1.3 \) and \( F_D = 0.96 \). For higher drives the vortices remain confined along the pinning rows above the second depinning transition and move in one-dimensional channels. As \( F_d \) increases, the
moving rows transition into the moving labyrinth state shown in figure 3(i) at $F_d = 3.0$. Figure 4(f) shows two pronounced peaks in $d(V_i)/dF_d$ for this sample, while $\sigma_e$ increases at higher drives.

For systems with purely repulsive vortex interactions there is generally at most only weak hysteresis in the velocity–force curves. For the system with long range attractive interactions, we find hysteresis for the strong pinning cases with both random and periodic pinning. The double peak in $d(V_i)/dF_d$ is a very robust feature in these systems for strong pinning and persists for larger system sizes and various filling ratios $B_d/B$. There is generally a single peak in the elastic depinning regime where the clump depins as a unit.

These results show that transport measures may be the most useful approach for determining whether vortices with modified interactions are present in multi-band or hybrid superconductors, since such vortices produce robust double peak features in the $I–V$ curves. Complementary imaging experiments should show increasing heterogeneity of the vortex structure at higher drives. We note that the dynamical driven states found here are much more inhomogeneous than the states found for systems with long range repulsive and short range attractive interactions [35].

5. Summary

In summary, we have studied vortex matter with long range attraction and short range repulsion of the type proposed to occur in type-I/type-II hybrid structures and multi-band superconductors. In the absence of pinning, this system does not form patterns or periodic arrays of stripes or bubbles, but instead completely phase separates. This behavior depends on the protocol used to prepare the states: for long annealing times, complete phase separation occurs, while for fast annealing or quenches the system forms clump type states. In the presence of random or periodic substrates we observe what we term a vortex wetting transition as a function of substrate strength or pinning density, with a transition from a heterogeneous to a homogeneous vortex state for increasing substrate strength. For long annealing times in the absence of a drive, we find that the pinning does not produce stripe-like states or multiple clumps.

Under an applied drive we find a rich variety of nonequilibrium phases, including dynamic phases that are distinct from those found for vortices with purely repulsive interactions. When driven, the homogeneous pinned state reorganizes into moving stripe or labyrinth phases, with the stripes aligned in the direction of the applied drive. In general, we find that systems of vortices with modified interactions become more heterogeneous under an applied drive, while systems of vortices with purely repulsive interactions become more homogeneous when driven. The modified vortex interaction system produces robust double peak features in the differential resistivity. In contrast, for systems with purely repulsive vortex interactions, only a single peak in the differential resistivity occurs. Thus, our results show that transport measurements can be used to test for the existence of modified vortex interactions and the different stages of dynamic ordering. Our results are also relevant to the broader class of systems with long range attraction and short range repulsion on periodic and disordered substrates, which can be realized in a number of soft matter systems.

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