Movement of particles of concrete mixture aggregate subjected to vibration by cylindrical punches

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Abstract. For compaction of the concrete mixture in the manufacture of reinforced concrete products using the formless production method on long stands, as a universal and widespread method, vibration is used. The aim of the study was to establish the regularities of the process of spreading a cylindrical punch of a continuous formless machine of vibration oscillations on the aggregate particles of a viscous concrete mixture in the form of crushed stone and sand. The chaotic movement of filler particles of various sizes with an arbitrary number of degrees of freedom, due to their large number, movement, and exchange of impulses, is considered. The problems of the development of mathematical descriptions of the motion of spherical filler particles of different grain size in the vibration of a cylindrical punch with a viscous environment and determination of their average values of travel speed and distance through random motion were solved, the average values for these parameters at vibration of the punch with frequencies and amplitudes that are applied in industrial conditions were found. In the course of the work, the method of mathematical modeling was used. The result of this work is to establish the possibility, for particles of different sizes, of varying the average values of the speed of movement and the path traversed in a viscous environment, carried out by changing the characteristics of the vibration of the punch. The regularities of changes in the parameters under consideration for filler particles of various sizes depending on the frequency and amplitude of vibration oscillations of the punch surface are considered.

1. Introduction

It is known that the main properties of such a popular building material as concrete, reinforced concrete products and structures made of it in industrial conditions, are laid at the stage of forming its structure [1]. Various exposure conditions at the production stage can significantly increase these properties in order to form a dense and stable structure [2]. The effectiveness of these effects, mainly mechanical, provided by forming units, depends on the method in which the particles of aggregate and cement paste would form a uniform structure in the formed volume of the concrete product. Vibrating, as one of the most universal methods, has deservedly found wide distribution in various technological complexes. These include, in particular, the continuous production of prestressed long-length products without formwork, in which various functions are performed by aggregates relative to a stationary stand [3-5]. In Russia, the general use of forming machines of this type is over 80% [6].

The process of transmitting vibrations to a viscous concrete environment is provided by surface, deep or volumetric vibration, and when forming products of large cross sections, the effectiveness of
the latter has been proven with unchanged characteristics and the content of the binder. Therefore, a significant part of the available scientific and technical literature is devoted to the description of vibration techniques of volumetric impact. However, scientists note its insufficiency in the field of scientific description of the process of transmitting vibration to a viscous concrete environment by the working bodies of a continuous formless forming machine in the form of a vibroform with fixed punches [7]. The complexity of the description is represented by the process of movement of concrete mixture particles, which is provided by rationally selected amplitude-frequency characteristics of vibration of the vibroform, allowing to provide the necessary characteristics of the formed product. In this regard, there is a need to develop a mathematical description of the speed of movement and displacement of particles in a viscous concrete environment, depending on the design of the vibroform punch and the rheological characteristics of the mixture.

2. Materials and methods
Let us consider the punch of a cylindrical shape. When vibrating, its surface has a cylindrical axial symmetry, so it is advisable to describe the action of vibrations in the coordinate system \((r, \varphi, z)\) with the center on the axis (figure 1), taking into account the movement of the aggregate of filler particles in a volume equal to:

\[
V = \pi L \cdot l \cdot (2R_0 + l),
\]

where \(L\) – length of the punch; \(l\) – the size of the propagation zone of intense vibration in the environment; \(R_0\) – the radius of the cylindrical surface of the punch, we take \(R_0\) equal to 0.078 m in accordance with the reinforced concrete slabs manufactured by the industry of the Russian Federation.

![Figure 1](image-url)

**Figure 1.** Scheme for determining the propagation of vibration in the volume of a viscous concrete medium, excited by the cylindrical surface of the punch.

As a result of vibration, the filler particles leave the equilibrium position they occupied in the viscous medium. The resulting movement of environmental particles leads to their interaction and exchange of impulses. As a result of a large number of material particles, the exchange of impulses contributes to their chaotic movement. Consequently, the material particles behave like ideal gas particles when the viscous environment vibrates [8]. Based on the above, we can conclude that the movement of material particles in a viscous environment can be described in the framework of statistical physics.

The velocity of a material particle in a viscous environment as a result of vibration is probabilistic and can be described using the velocity distribution function.

Due to the axial symmetry, it is convenient to present the velocity distribution function in the following form:
where $B_0$ – normalization constant; $m$ – particle mass; $v_r$ – the value of the particle velocity in a plane perpendicular to the cylinder axis $z$; $E_0$ – the average value of the energy of motion of a viscous mixture as a result of vibration during the oscillation period.

Taking into account the average value of the square of the sine for a time equal to the period $<\sin^2(\omega_0 t + \phi_0)>$, the average value of the motion energy of a viscous mixture takes the form:

$$E_0 = \frac{1}{2} \rho_v \sqrt{V \cdot A_0^2 \cdot \omega_0^2} \cdot \psi(r),$$

here $\rho_v$ – mixture density; $A_0$ and $\omega_0$ – amplitude and frequency of cylinder surface vibrations;

$$\psi(r) = \sqrt{\frac{R_v}{r}} \exp\left(-\frac{r-R_v}{l}\right)$$

To find the maximum value of the velocity of a material particle in a viscous environment under the influence of vibration, the first derivative of (2) must be equal to zero:

$$B_0 \cdot \exp\left(-\frac{mv_r^2}{2 \cdot E_0}\right) \left(1 - \frac{v_r^2}{E_0}\right) = 0.$$  
(5)

Expression (5) turns to zero if:

$$v_r = v_{max} = \sqrt{\frac{E_0}{m}}.$$  
(7)

Substituting (3) in (7) leads to the following result:

$$v_{max} = \sqrt{\frac{\rho_v \cdot V}{2 \cdot m}} \cdot A_0 \cdot \omega_0 \cdot \psi(r).$$  
(8)

Based on (2) the probability of different values of the velocities of the material particles in a viscous environment under the action of vibration can be written as:

$$d\omega = f(v_r) \cdot dv_r.$$  
(9)

The normalizing constant $B_0$ can be found based on the normalization condition:

$$\frac{v_{max}}{0} d\omega = 1.$$  
(10)

Applying (10) to (9) with (2) in mind, we get the following expression:

$$B_0 \int_0^{v_{max}} v_r \ exp\left(-\frac{mv_r^2}{2 \cdot E_0}\right) \cdot dv_r = 1.$$  
(11)

We transform:

$$I_1 = \int_0^{v_{max}} v_r \ exp\left(-\frac{mv_r^2}{2 \cdot E_0}\right) \cdot dv_r = \frac{1}{\frac{m}{2 \cdot E_0}} = \frac{2 \cdot E_0}{m} \int_0^{v_{max}} \exp\left(-\frac{m}{2 \cdot E_0} \cdot \xi\right) \cdot d\xi = \frac{E_0}{m} \int_0^{v_{max}} e^{-\frac{m}{2 \cdot E_0} \cdot \xi} \cdot d\xi = \frac{E_0}{m} \int_0^{v_{max}} e^{-\frac{m}{2 \cdot E_0} \cdot \xi} \cdot d(\xi^2).$$  
(12)

where $\xi_{max}$ – coefficient of penetration of vibrations to depth $l$;

$$\xi_{max} = \frac{m}{2 \cdot E_0}.$$  
(13)

Substituting (7) in (13) leads to the following result:
\[ \xi_{\text{max}} = \frac{\sqrt{2}}{2}. \]  

(14)

Calculating the integral in (12), taking into account (14), allows obtaining the following relation:

\[ I_1 = \frac{E_0}{m} \left( 1 - e^{-\frac{1}{2}} \right). \]  

(15)

Based on (11) and (15), we find the value of the constant \( B_0 \):

\[ B_0 = \frac{1}{I_1} = \frac{m}{E_0} \cdot \frac{1}{1 - e^{-\frac{1}{2}}}. \]  

(16)

Substituting (16) in (2) allows getting the final form of the speed distribution function:

\[ f(v_r) = \frac{m}{E_0} \cdot \frac{v_r \cdot \exp \left( \frac{-mv_r^2}{2E_0} \right)}{1 - e^{-\frac{1}{2}}} . \]  

(18)

Based on the velocity distribution function (18), it is possible to find the average value of the velocity of a material particle in a viscous environment subject to vibration relative to the initial equilibrium position:

\[ v_{cp} = \frac{1}{v_r} \int_0^\infty f(v_r) dv_r . \]  

(19)

Substituting (18) in (19) gives the ratio:

\[ v_{cp} = \frac{m}{E_0} \cdot \frac{1}{1 - e^{-\frac{1}{2}}} \cdot \frac{v_r \cdot \exp \left( \frac{-mv_r^2}{2E_0} \right)}{1 - e^{-\frac{1}{2}}} . \]  

(21)

Let us calculate the integral:

\[ I_2 = \int_0^{v_{Er}} v_r^2 \cdot \exp \left( \frac{-mv_r^2}{2E_0} \right) \cdot dv_r = \left[ \xi = v_r \cdot \left( \frac{m}{2E_0} \right)^{\frac{1}{2}} \right] = \left( \frac{2 \cdot E_0}{m} \right)^{\frac{1}{2}} \left[ -\frac{1}{2} e^{-\frac{1}{2}} \right] \left( \frac{1}{2} e^{-\frac{1}{2}} \right) = \left( \frac{2 \cdot E_0}{m} \right)^{\frac{1}{2}} \left[ \frac{1}{2} \right] = \left( \frac{2 \cdot E_0}{m} \right)^{\frac{1}{2}} \left[ \frac{1}{2} \right]. \]  

(22)

here \( \text{erf}(\xi) \) – function that defines the value of the probability integral [9].

Substituting (14) in (22) leads to the following relation:

\[ I_2 = \left( \frac{2 \cdot E_0}{m} \right)^{\frac{1}{2}} \left[ \frac{1}{4} \sqrt{\pi} \cdot \text{erf} \left( \frac{\sqrt{2}}{2} \right) - \frac{1}{2} e^{-\frac{1}{2}} \right] . \]  

(23)

Based on (21) – (23), we can get the following result:

\[ v_{cp} \approx 0.633 \cdot \sqrt{\frac{E_0}{m}} . \]  

(24)

If we consider the relation between the mass of a spherical material particle and its diameter, then substituting (3) with (1) in (24) allows obtaining the following relation:

\[ v_{cp} = 0.633 \cdot \sqrt{\frac{3 \cdot \rho \cdot \omega \cdot L \cdot l \cdot (2 \cdot R_0 + 1)}{2 \cdot \rho \cdot d^3} \cdot A_0 \cdot \omega_0 \cdot \psi (r)} . \]  

(25)

where \( \rho \) – particle mass; \( d \) – particle diameter.
Using the expression (25), we can find the average value of the displacement of a material particle over a time equal to the oscillation period:

\[ r_{cp} = v_{cp} \cdot \tau, \]

where \( \tau \) – the period of fluctuations of the viscous environment.

Based on (25), the formula for the average value of the particle displacement (26) is given as:

\[ r_{cp} = 2\pi \cdot 0.633 \cdot \sqrt{\frac{3 \cdot \rho_\infty \cdot L \cdot L^2 \cdot (2 - R_c + L) \cdot A_0 \cdot \psi(r)}{2 \cdot \rho \cdot d^3}}, \]

3. Results

Using the obtained expressions for constructing graphical dependencies of changes in the average values of the velocities of movement and displacement of particles subject to vibration, we set the frequency and amplitude of vibrations as variable parameters. For implementing the vibrations of the prototype we will take vibroform machine of the continuous cold forming, consisting of two parts [10].

The density of the concrete mixture \( \rho_\infty \), with which the punch directly interacts during the operation of the vibroform, according to the manufactured factory products, is assumed to be equal to 2400 kg/m\(^3\).

When selecting aggregate fractions, based on their largest and smallest sizes found in the formed product, the particle size for crushed stone is assumed to be 0.01 m in diameter, and for sand – 0.00125 m in diameter.

![Figure 2](image)

**Figure 2.** Dependences of the average values of the speed of movement of particles for a vibrating mold: a – for particles with a size of 0.01 m; b – for sand particles with a diameter of 0.00125 m.

The graphs of the functions \( v_{cp} \) from \( \omega_0 \) at \( A_0 = 0.0005...0.0007 \) m (figure 2) and \( A_0 = 0.0003...0.0004 \) m (figure 3) corresponding to the parts of the vibroform are monotonously increasing. At the same time, with a decrease in the diameter of the filler from 0.01 m to 0.00125 m, the \( v_{cp} \) increases over the entire frequency range \( \omega_0 \) of the first and second parts of the vibroform. For example, when \( \omega_0 \) is equal to 55 Hz and \( A_0 = 0.0007 \) m, the average speed of movement of a crushed stone particle with a diameter of 0.01 m corresponds to a value of 0.393 m/s (figure 2, a), and for a sand particle with a diameter of 0.00125 m (figure 2, b) increases to a value of 8.82 m/s, i.e. by 22.44 times. Similarly, the increase of the velocity of the particle depending on its diameter can be observed when the frequency and amplitude of oscillation of the punch surface of the second part of the vibrating mold is 75 Hz and 0.0004 m, respectively, where with decreasing particle diameter from
0.01 to 0.00125 m increase in $v_{cp}$ occurs from 0.283 to 6.34 m/s (figure 3, a and b).

Let us consider the change in the $v_{cp}$ for given particle diameters and frequency $\omega_0$. With a change in the amplitude $A_0$ of the first part of the vibroform (figure 2) in the range of 35...55 Hz, the average speed of movement of the crushed stone particle increases linearly from 0.200 to 0.280 m/s, i.e. by 40%. A larger increase in this parameter is observed for the diameter of sand particles equal to 0.00125 m. Thus, at $\omega_0 = 35$ Hz and $A_0 = 0.0005...0.0007$ m, the $v_{cp}$ changes from 4.42 to 6.27 m/s, which corresponds to an increase of 41.85%.

In the section of the second part of the vibroform (figure 3), this phenomenon is observed, but with a slightly smaller percentage discrepancy, with a variable amplitude of vibrations of the punch surface in the range $A_0 = 0.0003...0.0004$ m and a given value $\omega_0 = 55$ Hz. Thus, for crushed stone with a diameter of 0.01 m, the $v_{cp}$ increases from 0.169 to 0.235 m/s (an increase of 39.05%), and for sand particles with a diameter of 0.00125 m – from 3.78 to 5.03 m/s (an increase of 33.06%). Hence, the average speed of movement of particles of a viscous environmental material is significantly affected by the amplitude-frequency mode of vibrations of the surface of the punch, which can be adjusted based on their size by changing the technological mode of the unit that excites these vibrations.

The graphical dependences of the change in the average values of the displacement of the particles of the $r_{cp}$ material from the frequency $\omega_0$ in the considered range of amplitudes of the surface of the punch (figures 4 and 5) are nonlinear decreasing in nature. It follows from the dependencies that when $\omega_0$ increasing in the considered range, the average value of the displacement of crushed stone particles in the viscous environment of the first part of the vibroform occurs by a greater 1.79 times, compared with the second for similar particles. So, a reduction in the average value of the displacement of the particles of crushed stone in the first part of the vibrating mold reaches 0.0450 m (at a frequency of 55 Hz) in comparison with the average displacement of the particles of crushed stone equal to 0.0504 m (at a frequency of 35 Hz), i.e. a decrease of 12%, and in the second part of the vibrating mold – with an average value of the displacement of the particles is equal to 0.0238 m (at a frequency of 75 Hz) in comparison with the average displacement is equal to 0.0254 m (at a frequency of 55 Hz), the decrease is 6.7%. Environment of the first part of the vibroform, in comparison with the second one, is much higher and is 2.92 times. This is evidenced by a decrease in the average values of particle displacement in the first part of the vibroform from 1.128 to 1.007 m (by 12%) and in the second from 0.556 to 0.534 m (by 4.1%), respectively. Also, based on the above, we can conclude that with a
decrease in the diameter of the spherical filler particles, the differences in the average values of their displacement increase significantly.

Figure 4. Dependences of the average values of the displacement of particles subject to vibration in the first part of the vibroform: a – for crushed stone particles with a diameter of 0.01 m; b – for sand particles with a diameter of 0.00125 m.

Figure 5. Dependences of the average values of the displacement of particles subject to vibration in the second part of the vibration form: a – for crushed stone particles with a diameter of 0.01 m; b – for sand particles with a diameter of 0.00125 m.

4. Summary
The need to develop a mathematical description of the speed of movement and displacement of particles in a viscous concrete environment, depending on the structural and technological parameters of the vibroform and the characteristics of the mixture, was substantiated. Analytical expressions are obtained that allow describing the movement of spherical filler particles in a viscous environment subjected to a cylindrical punch and determining their average values of the speed of movement $\nu_{cp}$ and the displacement $r_{cp}$. Using the obtained relations, graphical dependences of changes in the studied parameters are constructed for a composite vibroform with punches of radius $R_0 = 0.078$ m, oscillating in its first part with a frequency of $\omega_0 = 35...55$ Hz, an amplitude of $A_0 = 0.0003...0.0004$ m, and in its second part – with $\omega_0 = 55...75$ Hz and $A_0 = 0.0005...0.0007$ m. It is possible to vary the average
values of the velocity of movement and displacement of particles, taking into account their size, by changing the amplitude-frequency characteristic of the punch vibrations. The regularities of changes in the parameters under consideration for filler particles of various sizes depending on the frequency and amplitude of vibration oscillations of the punch surface are considered.

5. References

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