Quantum metallic state in the titanium sesquioxide heterointerface superconductor

Guanqun Zhang,1,* Yixin Liu,2,3,* Lijie Wang,1 Huanyi Xue,1 Zhongfeng Ning,1 Chunlei Gao,1,4 Zhenghua An,1,4 Gang Mu,2,3,4 Yan Chen,1 and Wei Li1,‡
1 State Key Laboratory of Surface Physics and Department of Physics, Fudan University, Shanghai 200433, China
2 State Key Laboratory of Functional Materials for Informatics, Shanghai Institute of Microsystem and Information Technology, and Center for Excellence in Superconducting Electronics, Chinese Academy of Sciences, Shanghai 200050, China
3 University of Chinese Academy of Sciences, Beijing 100049, China
4 Institute for Nanoelectronic Devices and Quantum Computing, Fudan University, Shanghai 200433, China

(Dated: November 9, 2022)

The emergence of the quantum metallic state marked by a saturating finite electrical resistance in the zero-temperature limit in a variety of two-dimensional superconductors injects a new momentum to the realm of unconventional superconductivity. Despite much research efforts over last few decades, there is not yet a general consensus on the nature of this unexpected quantum metal. Here, we report the unique quantum metallic state within the hallmark of Bose-metal characterized by the saturated resistance and simultaneously vanished Hall resistance in the titanium sesquioxide heterointerface superconductor Ti$_2$O$_3$/GaN. Strikingly, the quantum bosonic metallic state proximate to the two-dimensional superconductivity-metal transition tuned by magnetic fields persists in the normal phase, suggesting that the existence of composite bosons formed by electron Cooper pairs survives even in the normal phase. Our work marks the observation of the preformed electron Cooper pairs in heterointerface superconductor and sheds new light on understanding the underlying pairing mechanism of unconventional superconductivity.

Introduction.—The discovery of heterointerface superconductivity has opened up intriguing venues in condensed matter physics and materials science, which stimulates enormous research attention to unveil the underlying rich physical properties and provide an exciting opportunity to the development of novel mesoscopic superconducting circuits [1–4]. A variety of emergent appealing quantum behaviors has been observed in the SrTiO$_3$ and KTaO$_3$ heterointerface superconductors [5, 6], including the strong Rashba-like spin-orbit coupling [7–10] and the interfacial ferromagnetism [11,12], as well as the gate-tunable superconductivity [13,14]. Remarkably, the superconducting to insulating quantum phase transition can be continuously controllable through an application of gate voltage without introducing unintentional disorders. In the intermediate regime between the superconducting and insulating phases, it is interesting to point out that the quantum metallic state characterized by saturating a finite electrical resistance is stabilized in the ultralow temperature limit [15,16]. Qualitatively similar behavior has also been reported in disordered thin films [20–27], exfoliated two-dimensional superconductors under applied magnetic fields [28,30] and artificially patterned superconducting islands [31,32], the existence and origin of such an unexpected quantum metallic state, however, remain under intense scrutiny [33,35].

Very recently, an impressive quantum metallic-like state has also been experimentally observed at a finite high temperature at the superconducting heterointerface between the Mott insulating Ti$_2$O$_3$ and the polar semiconducting GaN via state-of-the-art heterostructure engineering [36]. Specifically, at the verge of interfacial superconductivity, the electrical resistance in the normal phase behaves a wide range of temperature-independent plateau as a manifestation of the quantum metallic state. Additionally, the transverse Hall resistance is simultaneously found to be vanished in this plateau regime, unveiling the existence of electron-hole symmetry inherent to the quantum metallic state within the hallmark of the Bose-metal scenario [37,38]. These results are in stark contrast to the basic picture of the emergence of the quantum metallic phase in the ultralow temperature limit [33,39]. Thus, it is important to clarify experimentally the connection between this quantum bosonic metallic state observed in the normal phase and that proximate to a superconducting to insulating phase transition tuned by fields at the ultralow temperatures.

In this Letter, we use a magnetic field to tune the superconducting behaviors at the heterointerface of Ti$_2$O$_3$/GaN through the superconductivity-metal transition. Firstly, in the absence of magnetic field, the two-dimensional superconductivity and the emergent quantum bosonic metal in the normal phase in the Ti$_2$O$_3$/GaN are briefly revisited. Secondly, the field driving superconductivity into the quantum bosonic metallic state is identified by the indications of the saturated resistance and simultaneously vanished Hall resistance at the ultralow temperatures. Thirdly, this quantum bosonic metallic state located in the normal phase is uniquely linked to that proximate to the two-dimensional superconductivity-metal transition tuned by fields, pointing to a common origin of Bose-metal. These intriguing results suggest that the existence of composite bosons formed by electron Cooper pairs as the pre-
formed superconducting electron Cooper pairs persists in the normal phase, which is essential for understanding the underlying pairing mechanism of unconventional superconductivity [40].

Experimental results.—The antiferromagnetic Mott insulating Ti$_2$O$_3$ thin film with a narrow-band gap of 0.1 eV [41, 42] is epitaxially grown by pulsed laser deposition on top of a (0001)-oriented GaN substrate with an N-polar terminated face. The detailed sample growth and characterization are described in the supplementary materials (SM) [43] and in previous study [36]. The thickness of the Ti$_2$O$_3$ thin film is about 90 nm, as determined by an atomic force microscope. Before proceeding with the electrical transport measurements, the active area of the Ti$_2$O$_3$ thin film device is patterned into a Hall bar configuration (see Fig. S1 in SM [43]). All transport measurements in this study are performed on the same device. To exclude an inevitable misalignment effect of the transverse contact pads in the Hall bar device, mixing between longitudinal electrical resistance $R_s$ and transverse Hall resistance $R_{xy}$ have been minimized in the data as presented by reporting the field symmetrized longitudinal electrical resistances and anti-symmetrized transverse Hall resistances, respectively.

Figure 1(a) shows the $R_s(T)$ as a function of temperature ranging from 1.5 to 300 K at zero magnetic field at the heterointerface of Ti$_2$O$_3$/GaN. In the low temperature regime, we observe the conspicuous signal of superconducting transition to zero-resistance state measured to the limit of our instrument resolution. The critical temperature is $T_c = 3.98$ K, as defined by where the resistance is at the midpoint of the normal phase value at 4.5 K. To further clarify the intrinsic nature of the interfacial superconductivity at the heterointerface between the Mott insulator Ti$_2$O$_3$ and the polar semiconductor GaN, we have characterized sample with a series of measurements. Firstly, the out-of-plane polar angular dependence of the upper critical field, $\mu_0 H_{c2}^{\theta}$, defined as the magnetic field at the midpoint of the electrical resistance transition ($\theta$ represents the angle between a magnetic field and the perpendicular direction to the surface of Ti$_2$O$_3$/GaN), is shown in Fig. 1(b). When the magnetic field is parallel to the film ($\theta = 90^\circ$), $\mu_0 H_{c2}^{90^\circ}$ is apparently much higher than that in the perpendicular field, $\mu_0 H_{c2}^{0^\circ}$. This strong anisotropy in the observed upper critical fields points to the two-dimensional superconducting feature in the Ti$_2$O$_3$/GaN (also see Fig. S2 in SM [43]). Furthermore, at around $\theta = 90^\circ$, a cusp-like peak is clearly resolved [Fig. 1(b), inset] and is qualitatively distinct from the three-dimensional anisotropic Ginzburg-Landau model but is well described by the two-dimensional Tinkham model [44], as frequently observed in interfacial superconductivity [30, 45], surface of doped SrTiO$_3$ [46, 47] and layered transition metal dichalcogenides [48, 49]. Secondly, the current-voltage (I-V) curves at various temperatures close to $T_c$ in the log-log scale are shown in Fig. 1(c). A power-law dependence of $V \propto I^\alpha$ behavior is clearly resolved and the extracted exponent $\alpha$ increases monotonically with decreasing temperature [Fig. 1(c), inset]. At the Berezinskii-Kosterlitz-Thouless (BKT) transition temperature $T_{BKT}$, which defines the dissociation of vortex-antivortex pairs in two-dimensional superconductors obeying the universal scaling relation $V \sim I^3$ [50, 51], we thus determine $T_{BKT} = 3.3$ K from where $\alpha = 3$ interpolates, consistent with the $T_c$ as defined in Fig. 1(a).

Notably, at the verge of superconducting phase in the Ti$_2$O$_3$/GaN, we observe a pronounced temperature-independent $R_s$ in the normal phase that saturates a finite resistance with a wide temperature range of 10 K [Fig. 1(a) and Fig. S3 in SM [43]]. In addition, the Hall resistance $R_{xy}$ is also measured on the same footing and is strikingly found to be vanished in this resistance plateau regime within the measurement resolution [see

![Graphs and Data](image-url)
emerges, whereas the $R_{xy}$ does not seem to change from zero, which is reminiscent of the quantum metallic state at low temperatures. Such behavior with both zero Hall resistance and finite longitudinal resistance has also been observed previously in disordered two-dimensional superconductor [37, 35]. The observation that the $R_{xy}$ remains negligible at a finite magnetic field suggests that such electron-hole symmetry survives. This apparent electron-hole symmetry behavior heralds the appearance of what has been termed the "elusive" Bose-metal in the literature [35, 52]. Interestingly, we have applied fields as large as 12 T well above the superconducting critical field $\mu_0 H_C \leq 0.2$ T, zero-resistance superconducting phase is still reached down to $T = 1.5$ K (see Fig. 2(a)], the lowest temperature of this measurement limited. Additionally, the $R_{xy}$ is found to be zero in this superconducting phase regime (see Fig. 2[b]) as expected intuitively because of the electron-hole symmetry of the Bogoliubov quasiparticles that make up the superconducting electron Cooper pairs [37]. When the field is further increased, a metallic state with a finite $R_s$ emerges.

FIG. 2. Two-dimensional superconductivity-quantum bosonic metal transition driven by magnetic fields in Ti$_2$O$_3$/GaN. (a) Longitudinal electrical resistance $R_s$ and (b) transverse Hall resistance $R_{xy}$ as a function of temperature with the application of various strengths of out-of-plane magnetic field.

Next, we proceed to gain further insight into the unique quantum bosonic metallic state in the Ti$_2$O$_3$/GaN, the perpendicular magnetic field is applied to investigate the two-dimensional superconductivity-metal transition. In Fig. 2, we show the temperature-dependent $R_s$ and the corresponding Hall resistance $R_{xy}$ with the application of various strengths of field. Upon increasing the fields, the superconducting phase is progressively suppressed. If the applied field is $\mu_0 H \leq 0.2$ T, zero-resistance superconducting phase is still reached down to $T = 1.5$ K [see Fig. 2(a)], the lowest temperature of this measurement limited. Additionally, the $R_{xy}$ is found to be zero in this superconducting phase regime [see Fig. 2(b)] as expected intuitively because of the electron-hole symmetry of the Bogoliubov quasiparticles that make up the superconducting electron Cooper pairs [37]. When the field is further increased, a metallic state with a finite $R_s$ emerges.

FIG. 3. Quantum bosonic metallic state in Ti$_2$O$_3$/GaN tuned by fields at the ultralow temperatures. (a) Arrhenius plot of the longitudinal electrical resistance $R_s$ for various magnetic fields perpendicular to the surface of Ti$_2$O$_3$/GaN. In the ultralow temperature regime, the resistance saturates a finite value, indicative of the emergent quantum metallic state. (b) Extracted the field dependence of the saturated resistance at 53.5 mK in (a). The data scale to the power-law of Bose-metal $R_s \sim (H - H_0)^v$ with $H_0 = 0.5$ T and $v = 0.82$, suggestive of the Bose-metal scenario of the quantum metallic state.
FIG. 4. Experimental phase diagram of $\mu_0 H$-$T$ for Ti$_2$O$_3$/GaN, including the insulator, quantum bosonic metallic state, and two-dimensional superconductivity (2D SC). Here, it should be noted that the unique quantum bosonic metallic state is characterized by the saturated resistance and simultaneously vanished Hall resistance.

field dependence of the saturated resistance at the ultralow temperature of 53.5 mK in Fig. 3(a), we find that the resistance follows the power-law scaling as a function of field, $R_s \sim (H - H_{c0})^{2\nu}$ (here $H_{c0}$ is the critical field of the two-dimensional superconductivity-quantum metal transition and $\nu$ is the exponent of the superfluid correlation length) as shown in Fig. 3(b), and yields a critical exponent of $\nu = 0.82$ and a critical field of $H_{c0} = 0.5$ T, consistent with the Bose-metal scenario of the quantum metallic state [29, 30]. These independent and complementary results provide the strong compelling evidence for the intrinsic quantum bosonic metallic state in the proximity to the superconductivity-metal transition tuned by fields, uniquely connecting with the one located in the normal phase. We summarize these intriguing quantum states observed in the Ti$_2$O$_3$/GaN, including the two-dimensional superconductivity and the unique quantum bosonic metallic state, in the phase diagram of $\mu_0 H$-$T$ shown in Fig. 4.

Discussions.—Having experimentally established the unique quantum bosonic metallic state in the superconducting heterointerface of Ti$_2$O$_3$/GaN shown in the phase diagram of Fig. 4, we now would like to address a theoretical discussion, which should give us a hint to understand the underlying pairing mechanism of unconventional superconductivity [40, 53]. Theoretically, superconductivity possesses the composite bosons formed by electron Cooper pairs and their macroscopic long-range phase coherence. Either breaking the electron Cooper pairs or disrupting the long-range phase coherence is expected to be detrimental to superconductivity. When the system is two-dimensional, the long-range phase coherence of electron Cooper pairs dominates the onset of global superconductivity [54]. The destruction of long-range phase coherence induces a two-dimensional superconductivity-metal transition tuned by fields at the ultralow temperatures. Since electrons usually do not form a conventional metallic state in two dimensions prohibited by the Anderson localization [55], this ultralow temperature transition is theoretically attributed to the dephasing of superconducting electron Cooper pairs based on the bosonic model [53, 55], in which the electron Cooper pairs persist in this anomalous/quantum metallic state but long-range phase coherence is lost. As a result, the electron Cooper pairs move diffusively, leading to the metallic-like behavior in the $T \to 0$ K limit. In our experimental data shown in Fig. 2 and Fig. S5 in SM [43], the observation of vanishing Hall resistance inherent to the electron-hole symmetry is a key ingredient that supports the existence of superconducting electron Cooper pairs. Notably, these preformed electron Cooper pairs without condensation not only are very much alive when the quantum bosonic metallic state is emerged proximate to a superconducting to metallic phase transition tuned by fields at the ultralow temperatures, but also continue to exist in the normal phase (see the phase diagram of Fig. 4). These experimental results thus unambiguously provide clear evidence for the existence of the preformed electron Cooper pairs in the normal phase of two-dimensional superconductor Ti$_2$O$_3$/GaN. Besides, it is worthy pointing out that the signatures of preformed electron Cooper pairs [54] at the verge of superconductivity have also been reported in unconventional [57–59], interfacial [60], and disordered thin film superconductors [61, 62]. Therefore, we mark the ubiquitous preformed electron Cooper pairs as a new paradigm for two-dimensional superconductors, which are precursor to the emergence of unconventional superconductivity.

In summary, we have experimentally observed the unique quantum bosonic metallic state characterized by the saturated resistance and simultaneously vanished Hall resistance at the superconducting heterointerface of Ti$_2$O$_3$/GaN, in which the intriguing quantum bosonic metallic state is continuously evolved from the two-dimensional superconductivity, and even persists in the normal phase, by tuning the magnetic field and/or the temperature that gradually suppresses the long-range phase coherence of electron Cooper pairs. This finding not only provides a clear evidence for the existence of the preformed electron Cooper pairs in the normal phase of two-dimensional superconducting heterointerface, but also brings a step close to understanding the underlying physics behind preformed electron Cooper pairs that is relevant to the pairing mechanism of unconventional superconductivity.

Acknowledgments.—This work is supported by the Na-
[1] Y. Saito, T. Nojima, and Y. Iwasa, *Highly crystalline 2D superconductors*, Nat. Rev. Mater. **2**, 16004 (2017).

[2] J. Mannhart and D. G. Schlom, *Oxide interfaces: An opportunity for electronics*, Science **327**, 1607 (2010).

[3] P. Zubko, S. Gariglio, M. Gabay, P. Ghosez, and J.-M. Triscone, *Interface physics in complex oxide heterostructures*, Annu. Rev. Condens. Matter Phys. **2**, 141 (2011).

[4] H. Y. Hwang, Y. Iwasa, M. Kawasaki, B. Keimer, N. Nagaosa, and Y. Tokura, *Emergent phenomena at oxide interfaces*, Nat. Mater. **11**, 103 (2012).

[5] N. Reyren, S. Thiel, A. D. Caviglia, L. F. Kourkoutis, G. Hammerl, C. Richter, C. W. Schneider, T. Kopf, A.-S. Rüetschi, D. Jaccard, M. Gabay, D. A. Muller, J.-M. Triscone, and J. Mannhart, *Superconducting interfaces between insulating oxides*, Science **317**, 1196 (2007).

[6] C. Liu, Y. Yan, D. Jin, Y. Ma, H.-W. Hsiao, Y. Lin, T. M Bretz-Sullivan, X. Zhou, J. Pearson, B. Fisher, J. S. Jiang, W. Han, J.-M. Zuo, J. Wen, D. D. Fong, J. Sun, H. Zhou, and A. Bhattacharya, *Two-dimensional superconductivity and anisotropic transport at KTaO$_3$ (111) interfaces*, Science **371**, 716 (2021).

[7] M. B. Shalom, M. Sachs, D. Rakhmilevitch, A. Palevski, and Y. Dagan, *Tuning spin-orbit coupling and superconductivity at the SrTiO$_3$/LaAlO$_3$ interface: A magneto-transport study*, Phys. Rev. Lett. **104**, 126802 (2010).

[8] A. D. Caviglia, M. Gabay, S. Gariglio, N. Reyren, C. Cancellieri, and J.-M. Triscone, *Tunable Rashba spin-orbit interaction at oxide interfaces*, Phys. Rev. Lett. **104**, 126803 (2010).

[9] F. Y. Bruno, S. M. Walker, S. Riccò, A. de la Torre, Z. Wang, A. Tamai, T. K. Kim, M. Hoesch, M. S. Bahrany, and F. Baumberger, *Band structure and spin-orbital texture of the (111)-KTaO$_3$ 2D electron gas*, Adv. Electron. Mater. **5**, 1800860 (2019).

[10] K. Rubi, S. Zeng, F. Bangma, M. Goiran, A. Ariando, W. Escoffier, and U. Zeitler, *Electronic subbands in the α-LaAlO$_3$/KTaO$_3$ interface revealed by quantum oscillations in high magnetic fields*, Phys. Rev. Research **3**, 033234 (2021).

[11] A. Brinkman, M. Huijben, M. van Zalk, J. Huijben, U. Zeitler, J. C. Maan, W. G. van der Wiel, G. Rijnders, D. H. A. Blank, and H. Hilgenkamp, *Magnetic effects at the interface between non-magnetic oxides*, Nat. Mater. **6**, 493 (2007).

[12] A. Ariando, X. Wang, G. Baskaran, Z. Q. Liu, J. Huijben, J. B. Yi, A. Annadi, A. Roy Barman, A. Rusingyi, S. Dhar, Y. P. Feng, J. Ding, H. Hilgenkamp, and T. Venkatesan, *Electronic phase separation at the LaAlO$_3$/SrTiO$_3$ interface*, Nat. Commun. **2**, 1192 (2011).

[13] J.-S. Lee, Y. W. Xie, H. K. Sato, C. Bell, Y. Hikita, H. Y. Hwang, and C.-C. Kao, *Titanium $d_{xy}$ ferromagnetism at the LaAlO$_3$/SrTiO$_3$ interface*, Nat. Mater. **12**, 703 (2013).

[14] P. Krantz, A. Tyner, P. Goswami, and V. Chandrasekhar, *Emergent magnetism and intrinsic anomalous Hall effect in KTaO$_3$ two-dimensional electron gases*, arXiv:2209.10534 (2022).

[15] A. D. Caviglia, S. Gariglio, N. Reyren, D. Jaccard, T. Schneider, M. Gabay, S. Thiel, G. Hammerl, J. Mannhart, and J.-M. Triscone, *Electric field control of the LaAlO$_3$/SrTiO$_3$ interface ground state*, Nature **456**, 624 (2008).

[16] J. Biscaras, N. Bergeal, S. Hurand, C. Grossetête, A. Rastogi, R. C. Budhani, D. LeBoeuf, C. Proust, and J. Lesueur, *Two-dimensional superconducting phase in LaTiO$_3$/SrTiO$_3$ heterostructures induced by high-mobility carrier doping*, Phys. Rev. Lett. **108**, 247004 (2012).

[17] J. Biscaras, N. Bergeal, S. Hurand, C. Feuillet-Palma, A. Rastogi, R. C. Budhani, M. Grilli, S. Caprara, and J. Lesueur, *Multiple quantum criticality in a two-dimensional superconductor*, Nat. Mater. **12**, 542 (2013).

[18] Z. Chen, A. G. Swartz, H. Yoon, H. Inoue, T. A. Merz, D. Lu, Y. Xie, H. Yuan, Y. Hikita, S. Raghu, and H. Y. Hwang, *Carrier density and disorder tuned superconductor-metal transition in a two-dimensional electron system*, Nat. Commun. **9**, 4008 (2018).

[19] Z. Chen, Y. Liu, H. Zhang, Z. Liu, H. Tian, Y. Sun, M. Zhang, Y. Zhou, J. Sun, and Y. Xie, *Electric field control of superconductivity at the LaAlO$_3$/KTaO$_3$ (111) interface*, Science **372**, 721 (2021).

[20] J. Biscaras, A. F. Hebard, and R. R. Ruel, *Low-temperature insulating phases of uniformly disordered two-dimensional superconductors*, Phys. Rev. Lett. **69**, 1604 (1992).

[21] A. Yazdani and A. Kapitulnik, *Superconducting-insulating transition in two-dimensional α-MoGe Thin Films*, Phys. Rev. Lett. **74**, 3037 (1995).

[22] D. Ephron, A. Yazdani, A. Kapitulnik, and M. R. Beasley, *Observation of quantum dissipation in the vortex state of a highly disordered superconducting thin film*, Phys. Rev. Lett. **76**, 1529 (1996).

[23] N. Mason and A. Kapitulnik, *Dissipation effects on the superconductor-insulator transition in 2D superconductors*, Phys. Rev. Lett. **82**, 5341 (1999).

[24] J. A. Chervenak and J. M. Valles, Jr., *Absence of a zero-temperature vortex solid phase in strongly disordered superconducting Bi films*, Phys. Rev. B **61**, R245(R) (2000).

[25] C. Christiansen, L. M. Hernandez, and A. M. Goldman, *Evidence of collective charge behavior in the insulating state of ultrathin films of superconducting metals*, Phys. Rev. Lett. **88**, 037004 (2002).

[26] Y.-H. Lin, J. Nelson, and A. M. Goldman, *Suppression of the Berezinskii-Kosterlitz-Thouless transition in 2D superconductors by macroscopic quantum tunneling*, Phys. Rev. Lett. **109**, 017002 (2012).

[27] W. Liu, L. Pan, J. Wen, M. Kim, G. Sambandamurthy, and N. P. Armitage, *Microwave spectroscopy evidence of superconducting pairing in the magnetic-field-induced metallic state of InO$_2$ films at zero temperature*, Phys. Rev. Lett. **111**, 067003 (2013).

[28] Y. Saito, Y. Kasahara, J. Ye, Y. Iwasa, and T. Nojima, *Metallic ground state in an ion-gated two-dimensional superconductor*, Science **350**, 409 (2015).

[29] A. W. Tsien, B. Hunt, Y. D. Kim, Z. J. Yuan, S. Jia, R. J. Cava, J. Hone, P. Kim, C. R. Dean, and A. N. Pasupa-
th, Nature of the quantum metal in a two-dimensional crystalline superconductor, Nat. Phys. 12, 208 (2016).
[30] T. Wang, A. Yu, H. Zhang, Y. Liu, W. Li, W. Peng, Z. Di, D. Jiang, and G. Mu, Enhancement of the superconductivity and quantum metallic state in the thin film of superconducting Kagome metal \( KV_3Sb_5 \), arXiv:2105.07732 (2021).
[31] S. Eley, S. Gopalakrishnan, P. M. Goldbart, and N. C. Yang, Y. Liu, Y. Wang, L. Feng, Q. He, J. Sun, Y. Tang, C. Wu, J. Xiong, W. Zhang, X. Lin, H. Yao, H. Liu, G. Fernandes, J. Xu, J. M. Valles Jr., J. Wang, and Y. Li, Intermediate bosonic metallic state in the superconductor-insulator transition, Science 366, 1505 (2019).
[32] A. Kapitulnik, S. A. Kivelson, and B. Spivak, Anomalous metals: Failed superconductors, Rev. Mod. Phys. 91, 011002 (2019).
[33] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Boson localization and the superfluid-insulator transition, Phys. Rev. B 40, 546 (1989).
[34] B. Sacpé, T. Dubouchet, C. Chapelier, M. Sanquer, M. G. Cheng, M. Tomczyk, S. Lu, J. P. Veazey, M. Huang, P. A. Lee, N. Nagaosa, and X.-G. Wen, Doping a Mott transition phenomenon, Nature 462, 487 (2009).
[35] D. Jiang, T. Yuan, Y. Wu, X. Wei, G. Mu, Z. An, and W. Li, Strong in-plane magnetic field induced reemergent superconductivity in the van der Waals heterointerface of \( NbSe_2 \) and \( CrCl_3 \), ACS Appl. Mater. Interfaces 12, 49252 (2020).
[36] A. M. Goldman and N. Markovic, Superconductor-insulator transitions in the two-dimensional limit, Phys. Today 51, 39 (1998).
[37] P. W. Anderson, Absence of diffusion in certain random lattices, Phys. Rev. 109, 1492 (1958).
[38] I. Božovic and J. Levy, Pre-formed Cooper pairs in copper oxides and \( LaAlO_3-SrTiO_3 \) heterostructures, Nat. Phys. 16, 712 (2020).
[39] H.-B. Yang, J. D. Rameau, P. D. Johnson, T. Valla, A. Tsvelik, and G. D. Gu, Emergence of preformed Cooper pairs from the doped Mott insulating state in \( Bi_2Sr_2CaCu_2O_{8+s} \), Nature 456, 77 (2008).
[40] L. Li, Y. Wang, S. Komiyas, S. Ono, Y. Ando, G. D. Gu, and N. P. Ong, Diamagnetism and Cooper pairing above \( T_c \) in cuprates, Phys. Rev. B 81, 054510 (2010).
[41] B. L. Kang, M. Z. Shi, S. J. Li, H. H. Wang, Q. Zhang, D. Zhao, J. Li, D. W. Song, L. X. Zheng, L. P. Nie, T. Wu, and X. H. Chen, Preformed Cooper pairs in layered \( FeSe-based superconductors, Phys. Rev. Lett. 125, 097003 (2020). \)
[42] G. Cheng, M. Tomczyk, S. Lu, J. P. Veazey, M. Huang, P. Irvin, S. Ryu, H. Lee, C.-B. Eom, C. S. Hellberg, and J. Levy, Electron pairing without superconductivity, Nature 521, 196 (2015).
[43] B. Sacpé, T. Dubouchet, C. Chapelier, M. Sanquer, M. Ovadia, D. Shahar, M. Feigelman, and L. Ioffe, Localization of preformed Cooper pairs in disordered superconductors, Nat. Phys. 7, 230 (2011).
[44] K. M. Bastians, D. Chatzopoulos, J.-F. Ge, D. Cho, W. O. Tromp, J. M. van Ruitenbeek, M. H. Fischer, P. J. de Visser, D. J. Thouen, E. F. C. Driessen, T. M. Klapwijk, and M. P. Allan, Direct evidence for Cooper pairing without a spectral gap in a disordered superconductor above \( T_c \), Science 374, 608 (2021).