Application of vogel and stedinger (V–S) empirical procedure to develop storage reliability yield relationships for Kainji reservoir system

Abstract
Storage reservoirs tend to be large and complex systems requiring equally complex mathematical models to simulate their behaviour. What are lacking are simple, reasonably accurate methods which give insight into a wide range of reservoir storage system characteristics and reliability indices. Such methods would be useful for the education of water supply analysts. This study tends to apply the Vogel Stedinger (V–S) procedure to simulate and develop a mathematical model to predict reservoir storages of a within year system. Inflows, out flows and reservoir levels of Kainji reservoir were obtained between 1991 and 2014. Reservoir storages were determined and Vogel Stedinger parameter variables ($S_p$ is the $p$th quartile of the distribution of required reservoir capacity for 100% failure free operation over a specified planning period N, $Z_p$ is the standardised Normal variate at $p$%, $\mu$ and $\sigma$ are mean and standard deviation of the logarithms of the storages and $Y_1$ is the lower bound of the storage) determined and were used to develop an empirical model for predicting storages and other V-S parameters. The R² Value (0.72) of the model indicates the strength and reliability of the model. The model could be useful to reservoir managers in overcoming extreme events of flood and drought.

Keywords: reservoir capacity, mathematical model, simulate storages, and reliability

Introduction
The dynamics in hydrology which results from climate change, leads to variation in key reservoir variables like the inflow, storage and outflow are some problems that constitute reasons why, water managers face the challenge of making available adequate quantities of water for drinking, agricultural and other uses and also because of the geometrically increasing population pressure and socio-economic development, increase the needs and demands for particular water flows.1 Challenges remain widespread and reflect severe problems in the management of water resources in many parts of the world. These problems will intensify unless effective and concerted actions are taken.2 The role of water-storage reservoirs, therefore, is to impound water during periods of higher flows, thus preventing flood disasters, and then permit gradual release of water during periods of lower flows.

Storage Yield Relationships (S Y R) are the traditional tool used by water resource engineers to determine the required capacity of a storage reservoir to maintain a pre-specified reservoir release. Essentially, two schools of thought exist regarding the development of Storage Reservoir Yield (S-R-Y) relationships. In the USA, S-R-Y relationships are usually based on an interpretation of reliability which depends upon the most critical draw down period of a reservoir over its planning horizon. According to Vogel,1 basically, these methods utilize the automated equivalent of Rippl’s mass curve approach (M C A), known as the sequent peak algorithm,4 in conjuction with stochastic streamflow models to obtain the probability of no-failure reservoir operations,(p) corresponding to a specific reservoir capacity yield combination. Alternatively, in Australia and else-where, a common approach to estimating the S. R.Y relationship is to determine the steady-state probability (S S P) of failure, (q) corresponding to a specific reservoir capacity yield combination.6 Essentially, reliability gives a measure of certainty that a given yield can be met without failure. The reliability index is between 0 and 1 and can be expressed in one of the three ways: (1) annual, (2) time based (3) volumetric. Understanding the reliability and uncertainty associated with water supply yields derived from surface water reservoirs is central for planning purposes.7

The Vogel and Stedingar empirical procedure estimates the expected valued and variance of reservoir storage capacity assuming both the inflows to the reservoir and the standardised storages are lognormally distributed. Although the method is based on empiricall procedure, its application is straightforward.8,9 While behaviour (or simulation) analysis is a simple and visual procedure to estimate storage capacity and is not restricted by the characteristics of the inflows; unlike some of the analytical approaches, evaporation and operating rules that are a function of reservoir storage levels can be easily taken into account.8

Vogel and Stedinger,9 showed that for a reservoir system fed by AR (1) lognormal stream flows, the standardised storage C (capacity divided by the standard deviation) for a failure free operation (S.P.A approach) is a random variable described by a three parameter Lognormal distribution. The form of the V.S relationship is:

$$S_p = \sigma [Y_1 + \exp(\mu + Z_p \sigma_1)] \quad (1)$$

Where $S_p$ is the $p$th quartile of the distribution of required reservoir capacity for $p$% failure-free operation over a specified planning period N, $Z_p$ is the standardised Normal variate at $p$%, $\mu$ and $\sigma_1$ are mean and standard deviation of the logarithms of the storages and $Y_1$ is the lowerbound of the storage.
The Vogel and Stadinger empirical procedure estimates the expected value and variance of sequent peak algorithm (SPA) reservoir storage capacity assuming both the inflows to the reservoir and the standardized storages are lognormally distributed. Although the method is based on six equations and 22 parameters its application is straightforward. Being an empirical procedure as reported by Mc-Mahon and Vogel, it should only be applied within the range of values of m, Cs, N and q that were used to define the 22 parameters. Faith and Richard reported that understanding the reliability and uncertainty associated with water supply yields derived from surface water reservoirs is central for planning purposes. In their study they used a global dataset of monthly river discharge, to introduce a generalized model for estimating the mean and variance of water supply yield, Y, expected from a reservoir for a prespecified reliability, R, and storage capacity, S assuming a flow record of length n. The generalized storage-reliability-yield (SRY) relationships reported by them have numerous water resource applications ranging from preliminary water supply investigations, to economic and climate change impact assessments.

According to Longobardi et al., a number of studies presented in the recent past, have demonstrated that the combination of a simulation approach coupled with a performance assessment via indices evaluation is a valuable tool to measure the sensitivity of reservoirs to climate variability and prolonged droughts. An example indicates how Faith and Richard used generalized SRY relationship combined with a hydroclimatic model to determine the impact of climate change on surface reservoir water supply yields. They documented that the variability of estimates of water supply yield are invariant to characteristics of the reservoir system, including its storage capacity and reliability. Standardized metrics of the variability of water supply yields were shown to depend only on the sample size of the inflows and the statistical characteristics of the inflow series. Abdesselam and Sylvain used empirical models to compares the two main statistical approaches to calculate the effective discharge (the empirical method based on histograms of sediment supply by discharge classes and an analytical calculation based on a hydrological probability distribution and on a sediment rating curve) to a very simple proxy: the half-load discharge, i.e. the flow rate corresponding to 50% of the cumulative sediment yield. Three types of discharge subdivisions were tested. In the empirical approach, two subdivisions provided effective discharge close to the half-load discharge.

According to Thomas et al., Annual and monthly streamflows for 729 rivers from a global data set were used to assess the adequacy of five techniques to estimate the relationship between reservoir capacity, target draft (or yield) and reliability of supply. The techniques examined were extended deficit analysis (EDA), behaviour analysis, sequent peak algorithm (SPA), Vogel and Steding empirical (lognormal) method and Phien empirical (Gamma) method. In addition, a technique to adjust SPA using annual flows to account for within-year variations were assessed. Of their nine conclusions the key ones were, that, EDA is a useful procedure to estimate streamflow deficits and, hence, reservoir capacity for a given reliability of supply. Secondly, that the behaviour method is suitable to estimate storage deficits but has limitations if an annual time step is adopted. Thirdly, that in contrast to EDA and behaviour analysis which are based on time series of flows, if only annual statistics are available, the Vogel and Steding empirical method compares favourably with more detailed simulation approaches.

Materials and methods

Materials

The hydrological data employed for this study was reservoir storages.

The study area

Geographically, Kainji hydroelectric dam is located in New Bussa town now headquarter of Borgu local government area of Niger State, Nigeria. The lake created behind the dam span between latitude 9°8’ to 10°7’ and between longitude 4°5’ to 4°7’ E with reference point 9.54N and 4.38E northwest of the Federal Capital Territory (FCT, Abuja).

Hydrology of the Niger river system

The average rainfall at the headwaters of Niandan and Milo rivers at the source of the Niger at the Fouta Djallon Mountains in Guinea and its exit to the sea in Nigeria is 2200mm. The river flow regime is characterized by two distinct flood periods occurring annually namely the White and Black floods. The black flood derives its flow from the tributaries of the Niger outside Nigeria (flow lag October to May) and arrives at Kainji reservoir (Nigeria) in November and lasts until March at Jebba after attaining a peak rate of about 2,000m³/sec in February. The White flood is a consequent of flows from local tributaries especially the Sokoto-Rima and Malendo river systems. The White flood is heavily laden with silts and other suspended particles (flow lag June to September) and arrives Kainji at August in the pre-Kainji Dam River Niger having attained a peak rate of 4,000 to 6,000m³/sec in September-October in Jebba. The critical low flow period into the Kainji reservoir is March and July each year (Figure 1).

Data base

The data base for this study constitutes reservoir storages from 1991 to 2014.

Vogel Steding empirical (V–S) empirical procedure

The V-S procedure was employed to develop S-R-Y relationships, it is based on the premise that a reservoir system fed by AR (1) lognormal inflows, the standardized storage capacity C for a failure free operation is a random variable. The general V-S relationship was employed:

$$S_p = \sigma_s \vartheta + \exp(\mu_s + Z_{p\%} \sigma_s)$$

Where:

$$u_i = \ln \left( \frac{\mu_i - \vartheta}{\sigma_i} \right) \left( 1 + \frac{\sigma_i^2}{(\mu_i - \vartheta)^2} \right) - 0.5$$

$$\sigma_i^2 = - \ln \left( \frac{\mu_i - \vartheta}{\sigma_i} \right) \left( 1 + \frac{\sigma_i^2}{(\mu_i - \vartheta)^2} \right)$$

Where $S_p$ is the $p^{th}$ quantile of the distribution of required reservoir capacity for 100% failure-free operation over a specified planning period N, $\vartheta$ is the standardised Normal variate at $p\%$, $\sigma$ is the standard deviation of annual streamflows, $\mu$ and $\sigma$ are mean and standard deviation of the logarithms of the storages, $\vartheta$ is the lower bound of the storage (Table 1).
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Figure 1 Location of Kainji Hydroelectric Dam.
Source: Salami (2013).

Table 1 Reservoir working table for Kainji Dam

| Month | Inflow (m$^3$/sec) | Antecedent reservoir storage (Mm$^3$) | Antecedent reservoir level(m) | Antecedent reservoir release (Mm$^3$) |
|-------|-------------------|--------------------------------------|-------------------------------|---------------------------------------|
| Apr   | 131.95            | 12634.34                             | 137.13                        | 1036.625                              |
| May   | 81.46             | 12587.79                             | 134.94                        | 925.5417                              |
| Jun   | 122.26            | 12560.91                             | 132.85                        | 786.3333                              |
| Jul   | 655.99            | 12556.17                             | 132.07                        | 626.375                               |
| Aug   | 1862.46           | 12696.2                              | 133.21                        | 766.25                                |
| Sep   | 2743.94           | 13141.93                             | 136.88                        | 1246.917                              |
| Oct   | 2030.27           | 13803.16                             | 140.26                        | 1443.167                              |
| Nov   | 1548.73           | 14271.61                             | 140.64                        | 1112.417                              |
| Dec   | 1708.73           | 14602.58                             | 140.75                        | 1138.583                              |
| Jan   | 1617.77           | 12000                                | 141.21                        | 1186.5                                |
| Feb   | 1334.49           | 12331.41                             | 140.37                        | 1177.042                              |
| Mar   | 642.57            | 12570.94                             | 139.71                        | 1064.542                              |

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**Vogel Stedinger (V–S) relationships**

The result of the Vogel Stedinger relationship is as presented below:

The V-S procedure was employed to develop S-R-Y relationships, it is based on the premise that a reservoir system fed by AR(1) lognormal inflows, the standardized storage capacity $C$ for a failure free operation is a random variable. The developed S-R-Y model was used to simulate the reservoir storages. Figure 2 depicts the observed and simulated storages. The developed models’ extent of correlation was also tested with a value of 0.72 imperatively showing strong correlation between the observed storages and the simulated storages. It also shows that the model is strong (Table 2).

![Figure 2 Developed S-R-Y Relationships (Vogel Stedinger V-S).](image)

**Table 2 Developed models for S-R-Y relationships**

| Model type | Developed model | $R^2$ |
|------------|-----------------|-------|
| V-S        | $S_p = 0.998[(174.482) + \exp(0.4142 + Z_p 0.843)]$ | 0.72  |

V-S, Vogel–Stedinger; $S_p$, Required storage capacity; $Z_p$, standardize normal variate.

**Conclusion**

The flowing conclusions are drawn from the study;

a. The Vogel and Stadinger empirical procedure estimates the expected value and variance of (SPA) reservoir storage capacity assuming both the inflows to the reservoir and the standardized storages are lognormally distributed.

b. The V-S procedure was employed to develop S-R-Y relationships.

c. The developed S-R-Y model was used to simulate the reservoir storages.

d. The extent of correlation between the observed storages and the simulated storages indicate the strength of the model.

e. The developed model can be used by the reservoir managers to predict future storages of the reservoir consequently helping in preparing for extreme events of flooding and drought.

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**Conflicts of interest**

The authors declare that there are no conflicts of interest.

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