Pomeron and its ups and downs

(My personal point of view)

Eugene Levin

Department of Particle Physics, School of Physics and Astronomy, Tel Aviv University, Tel Aviv, 69978, Israel
and
Departamento de Física, Universidad Técnica Federico Santa María and Centro Científico-Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile
E-mail: leving@post.tau.ac.il, eugeny.levin@usm.cl

Abstract. In this talk we review the main ideas on the Pomeron structure that have been developed during the half of century. Doing this I believe that I pay the tribute to my life long unfailing friend Aliosha Kaidalov who, as well as me, belongs to the Pomeron generation of high energy theoretical physicists.

1. Introduction

Aliosha Kaidalov, as well as me, belongs to Pomeron generation of high energy physicists. The structure of the Pomeron was, is and, I hope, will be the topic that has inspired us, exited us and, may be, even spoilt us for half of century. Therefore, I believe that presenting a brief review of our achievements and failures I pay my tribute to the memory of Aliosha,

The talk is organized in the following way. First, I give a brief history of Pomeron as a projection on my life in physics. I hope that you would feel the pulse of hopes and disappointments in the dry language of a scientific talk. Second, I share with you the four ideas on the Pomeron structure that made me happy, After this I give very personal and short review of the Pomeron that arises from high energy phenomenology, Finally I describe my the last paper on the subject of modeling confinement in the BFKL Pomeron.

2. Fifty years with the Pomeron

2.1. 1966 - 1974 — Frontier of high energy physics

The problem of finding of high energy asymptotic behaviour of the scattering amplitude was one of the priority problem of the particle physics. At that time we believed that the future theory will be based on three principles: analyticity, crossing symmetry and the unitarity constrains in t and s channels. In the framework of this approach the high energy asymptotic behaviour was needed to specify the use of the dispersion relation: our main tool to implement the above principles. We started to do research when the main approach to high energy amplitude at high energy had been formulated and it was based on the Reggeon approach. The theory
of the Reggeon exchange had been developed but our generation solved two problems: the contribution the multi-Pomeron exchanges and the $s$-channel structure of the Pomeron exchange. Remembering that time I surprise how much we learned having so poor theoretical tool in hand. I believe that it happened because we were familiar with existing quantum field theories such as QED and other theoretical models but also because we had feedback interaction with the experimental physicists. We were searching not only how our approach describes the data but also what features in the data have general character that have to be implemented in the theoretical approach.

Basically we have very simple approach [1, 2, 3] based on the parton model, which can be summarized in two pictures of Fig. 1.

Fig. 1-a states that the total cross section for the single Pomeron exchange is equal to

$$\sigma = \sum_{n=2}^{\infty} \int_{0}^{Y} d y_{1} \int_{0}^{y_{1}} d y_{2} \ldots \int_{0}^{y_{n-1}} d y_{n} \prod_{i=2}^{n-1} d^{2} p_{i,T}$$

(1)

where $\Psi$ is the wave function of the partons (point-like particles) which have restricted transverse momentum $p_{i,T} \leq \mu$ and $\mu$ does not depend on total energy. We assume that partons are distributed uniformly in the rapidity range $(0, Y)$ and the integral over $d y_{n}$ converges.

Fig. 1-b illustrates the Gribov’s diffusion picture in the transverse plane of the partons in the parton cascade. It based on the uncertainty principle that $\Delta b p_{i,T} \sim 1$ for emission of the parton in the cascade. This figure shows that after $n$-emissions the partons are distributed in the area with radius $b_{n}^{2} = (1/\mu^2) n$ and since $n \propto Y$ we get $R^{2} \propto (1/\mu^2) Y = \alpha' p_{T} Y$.

Based on this simple picture we found out how to calculate (i) the two particle correlations (Mueller diagrams [4, 5] see Fig. 2-a), (ii) the contribution of the multi-Pomerton exchange to the correlations functions[7, 8] (see Fig. 2-b) and (iii) to the processes of multi-particle production (AGK cutting rules[6], see Fig. 3); (v) learned the origin and main properties of the large mass diffractive production (see Fig. 2-c) as well as the (vi) structure of maxima and minima in elastic scattering (see Fig. 2-d); (vii) the general properties of the multiplicity distributions (KNO scaling or $\sigma_{n}/\sigma_{mn} = F(n/<n>)$ have been explained [9] (see Fig. 2-e) and so on.

Our knowledge of Pomeron interaction was summarized in the Gribov Pomeron calculus[10] which can be formulated as the following path integral:
Figure 2. Diagrams for different processes: the two particle correlations (Fig. 2-a), two particles long rapidity range correlations (Fig. 2-b), the large mass diffractive production (Fig. 2-c), structure of maxima and minima in elastic scattering (Fig. 2-d) and KNO scaling (Fig. 2-e).

$$Z[\Phi, \Phi^+] = \int D\Phi D\Phi^+ e^S$$ with $$S = S_0 + S_I + S_E$$

$$S_0 = \int dY \Phi^+(Y, b) \left\{- \frac{d}{dY} + \Delta + \alpha_s \nabla^2 \right\} \Phi(Y, b);$$

$$S_I = G_{3P} \int dY \left\{ \Phi(Y, b) \Phi^+(Y, b) \Phi^+(Y, b) + h.c. \right\}$$

(2)

$S_E$ specifies the interaction with hadrons or nuclei. $\Phi(Y, r)$ describes the Pomeron with rapidity $Y$ and impact parameter $b$. It turns out that Eq. (2) has a simple statistical interpretation and can be re-written as the equation for probability $P_n$ to have $n$-Pomerons at rapidity $Y$. The equation takes the form[11, 12]

$$- \frac{\partial P_n(y, b)}{\partial y} + \alpha'_P \nabla^2 P_n(y, b) = G_{3P} \left\{ -n P_n(y, b) + (n-1) P_{n-1}(y, b) \right\}$$

(3)

$$+ G_{3P} \left\{ -n(n-1) P_n(y, b) + (n+1) n P_{n+1}(y, b) \right\}$$

However, the problem of Pomeron interactions has not been solved and we failed to find theoretical arguments for restricting the number for Pomeron interaction vertices as well as for finding term $S_E$ in Eq. (2).

2.2. 1974 - 1990 $\rightarrow$ Dark ages

In 1974 QCD was suggested and the theory of strong interaction got the solid theoretical basis. At first sight, both the idea of the Pomeron and its importance have no roots in QCD and
they have been well forgotten at least for 15 years. During these years I had only a couple of publications on Pomeron. However, the problem of high energy amplitude in QCD was and is one of the most difficult problems which has been tried to solve during these years. The main ingredients of all attempt to approach this problem is so called BFKL Pomeron which is the solution to the BFKL evolution equation[13]. The main features of the BFKL Pomeron looks as follows:

- Scattering amplitude behaves as $s^{\Delta_{BFKL}}$ with $\Delta_{BFKL} = 4 \ln 2 \alpha_S$;
- $\sigma$ Eq. (1) but the cross section $\sigma_{\text{parton}} (y_n, p_{n,T})$ is a constant versus $y$;
- No Gribov’s diffusion, $< b^2 > = \text{Const}$;
- Diffusion in $\ln k^2_T$;
- The BFKL Pomeron interaction can be described by BFKL Pomeron calculus with the action which is very close to Eq. (2)[14];

By now the theoretical approach based on the BFKL Pomeron, has reached a mature stage (see Ref.[15]), we learned a lot about the high energy asymptotic behaviour of the scattering amplitude for deep inelastic and other processes. However, the soft scattering has been still elusive.

2.3. 1991 - present —— Renaissance

In 1991 I came to DESY for long visit and to my great surprise I noted, after staying six months, that the majority of questions that I was asked, were related to the high energy Regge phenomenology. Frankly speaking, I had forgot what was a status of this phenomenology. Fortunately, Asher Gotsman, was also visiting DESY and stayed in the next door room. We started to clear up the situation and from that time A. Gotsman, U.Maor and me worked on the Regge phenomenology for soft interaction at high energy. Below, I will discuss what kind of approach we developed. During the past two decades I have followed closely the theoretical development of the understanding of the Pomeron structure and I will share with you four ideas on this subject that made me happy.

3. Four theoretical approaches that made me happy

3.1. Scaling anomaly (breakdown of scale invariance)

As we have mentioned perturbative QCD leads to the BFKL Pomeron after summing all diagrams that have contribution:$(\alpha_S \ln s)^n$. Generally speaking we expect that non-perturbative corrections stems from the region of low transverse momenta of gluon ($\Lambda_{QCD}$). In Ref.[16] we made an observation that strong gluon fields in QCD vacuum lead to sufficiently large produced mass in the BFKL ladder (see the second diagram of Fig. 4).

Indeed, due to scaling anomaly, $\alpha_S F^{\mu\nu,a} F_{\mu\nu,a} \propto \theta_\mu^a \propto \alpha_S^0$. Therefore, in the diagrams of Fig. 4 with two produced gluon in the rank of the ladder we see that contribution to the high energy asymptotic behaviour is proportional to $\ln^n s$, Using $< \pi^+\pi^-|\theta_\mu^a|0 > = M^2$ from chiral Lagrangian. Using the sum rules for $\theta_\mu^a$ [17]

$$\Pi(0) = i \int dx \langle 0|T \{ \theta_\mu(x) \theta_\mu(0) \} |0 \rangle = -4 \langle 0|\theta_\mu^a|0 \rangle = -16 \epsilon_{\text{vac}} \neq 0 \quad (4)$$

Plugging in $< \pi^+\pi^-|\theta_\mu^a|0 > = M^2$ and subtraction perturbative contribution we obtain that

$$\Pi(0) = \int \frac{dM^2}{M^2} [\rho_0^{\text{phys}}(M^2) - \rho_0^{\text{pt}}(M^2)] = 16 |\epsilon_{\text{vac}}| \quad (5)$$
And using

\[ \rho^\text{phys} (M^2) \rightarrow \frac{M^2 > M_0^2}{\rho^\text{pt} (M^2)} \]

we obtain that

\[ M_0^2 \approx 32\pi \left\{ \left| \epsilon_{\text{vac}} \right| \right\}^{\frac{1}{2}} \]

Numerically: \( \epsilon_{\text{vac}} \approx -0.24 \text{ GeV}^4 \) and \( M_0^2 = 4 \div 6 \text{ GeV}^2 \) which gives the following intercept

\[ \Delta = \frac{1}{48} \ln \frac{M_0^2}{4m^2} = 0.082 \div 0.09 \]

For me this was the first time we obtain reasonable Pomeron intercept based on non-perturbative approach.

### 3.2. Instantons and soft Pomeron

In previous subsection we showed that the strong semi-classical field can play a crucial role in the soft Pomeron structure. Strong fields correspond to classical solution of QCD and there exists a well known such a solution: instantons (see Ref.[18] and reference therein). In our paper[19] we build the soft Pomeron from instantons using the simple multi-peripheral approach (see Fig. 5).

In Fig. 6 we plotted the soft Pomeron trajectory as a function of \( t\rho_0 \) where \( \rho_0 \) is the typical scale of the QCD instanton.

Therefore, we demonstrated again that the strong gluon field can be dominant contribution to the soft Pomeron structure.

### 3.3. AdS-CFT correspondence and the Pomeron in N=4 SYM

In 70’s Prof. Gribov, working on the Pomeron interactions in framework of the Pomeron calculus, told us, the young scientists in his department, that he needs a particle the same as photon but with vacuum quantum numbers. Indeed, his approach led to weak Pomeron coupling [21] and universality: \( \Delta \mathcal{P} = 0 \); \( g_h = \text{Universal constant} \), \( G_3 \mathcal{P} = 0 \).

More than decade ago such particle have been found. It turns out that due to AdS-CFT correspondence[20] the theory that can be solved in the limit of large coupling constant has
been suggested. This theory: N=4 SYM, in the limit of small coupling has all typical property of QCD including the BFKL Pomeron. At large coupling this theory leads to the soft Pomeron exchange\[22, 23\]. It is instructive to notice that the Pomeron in this theory is not moving pole on the angular momentum, but the standing cut with all properties of the BFKL Pomeron.

**Glossary** \(\equiv\) AdS-CFT correspondence:

| N=4 SYM | QCD |
| --- | --- |
| Reggeized graviton \(\Rightarrow\) BFKL Pomeron | \(r\) (dipole size) |
| \(z\) | \(\omega_{BFKL}\) (intercept of BFKL Pomeron) |
| \(1 - 2/\sqrt{\lambda}\) | \(D_{BFKL}(\omega(\nu) = \omega_{BFKL} - D\nu^2)\) |
| \(2/\sqrt{\lambda}\) | |

**Figure 5.** Instanton contribution to the soft Pomeron

**Figure 6.** The soft Pomeron trajectory, as a function of \(t\rho^2\), where \(\rho_0\) is the typical scale of the QCD instanton, from Ref.[19].
$z$ is the fifth coordinate which has been introduced. The properties of the Pomeron in this
approach look as follows.

\[ \lambda \gg 1 \]
\[ \Delta_{IP} \rightleftharpoons \text{large (} \sim 0.3? \text{)} \]

Picture is taken from Ref.[24].

\[ \alpha'_{IP} = 0 \]

only Good-Walker mechanism for diffraction
\[ G_{3IP} = 0 \]

Picture is taken from Ref.[23]

Such Pomeron arises in top-down scenario in 10-dimensional de-Sitter space. However, even
in bottom-up scenario ( see Ref.[25] and references therein) where the string approach used in
5-dimensional space the resulting Pomeron is very similar to one described above.

In all approaches that we have discussed, is one major deficiency: they do not reproduce
correct behavior at large impact parameter ( $A \propto e^{-\mu b}$). This fact makes then interesting but
questionable. Having this in mind we decided to build the high energy phenomenology which
reproduce the main property of N=4 SYM Pomeron but has correct $b$ dependence.

4. N=4 SYM and QCD motivated phenomenology

The presentation of our attempt to develop such phenomenology is given in the lecture of
E.Gotsman [26]. Here I would like only to outline our approach to emphasize the fact that
we tried to take into account the properties of the Pomeron both in N=4 SYM and in QCD.

Building our model we assume:

- Pomeron is a Regge pole;
- $\Delta_{IP}$ is large (0.2 ÷ 0.3);
- $\alpha'_{IP} = 0$;
- Large Good- Walker component, two channel model;
Only $G_{3P}$ as in QCD;

- $G_{3P}$ is small (in QCD $G_{3P} \propto \alpha_S$ and in N=4 SYM $G_{3P} \ll 2/\sqrt{\lambda}$);

Our approach can be formulated in terms of functional integral

$$Z[\Phi, \Phi^+] = \int D\Phi D\Phi^+ e^S \quad \text{with} \quad S = S_0 + S_I + S_E$$  \hspace{1cm} (9)

Where $S_0$ describes the Pomeron without interaction. $S_I$ is responsible for interaction between Pomerons while $S_E$ is the term that describe the interaction of the Pomeron with the colliding particles. They are equal to

$$S_0 = \int dY \Phi^+(Y) \left\{ - \frac{d}{dY} + \Delta \right\} \Phi(Y);$$  \hspace{1cm} (10)

$$S_I = G_{3P} \int dY \left\{ \Phi(Y) \Phi^+(Y) \Phi^+(Y) + \text{h.c.} \right\};$$  \hspace{1cm} (11)

$$S_E = - \int dY \sum_{i=1}^{2} \left\{ \Phi(Y) g_i(b) + \Phi^+(Y) g_i(b) \right\};$$  \hspace{1cm} (12)

This model is so simple that can be solved. In Ref.[26] you can see that we achieved the good description of soft interaction from energy 20 GeV till LHC energy.

5. BFKL Pomeron: modeling confinement

5.1. Setting the problem

In this section we discuss briefly the results of our paper (see Ref.[27]) in which we address the question of large impact parameter dependence of the scattering amplitude. I hope, that I have discussed how important this question which has not been solved in all theoretical approaches to Pomeron, including BFKL Pomeron in perturbative QCD.

During the past decade numerous attempts (see our paper [27] for references) have been made to solve this problem without decisive result. However we learned several lessons from these tries: (i) the confinement of quarks and gluon have to be included in the BFKL kernel (to include in the initial conditions is not enough); (ii) suppressing large sizes of the produced dipoles in the decay of one dipole to two dipoles we reproduce correct $b$-dependence; and (iii) since at large $b$ the amplitude is small we do not need to take into account the non-linear corrections.

Therefore, corrections from confinement have to be included in the kernel of the BFKL equation:

$$\frac{\partial N(x_{10}, b; Y)}{\partial Y} = \alpha_S \int d^2 x_{12} K(x_{12}, x_{20}|x_{10})$$

$$\times \left\{ 2 N \left( x_{12}, \tilde{b} - \frac{1}{2} \tilde{x}_{02}; Y \right) - N(x_{10}, b; Y) \right\};$$  \hspace{1cm} (13)

where kernel $K$ describe the decay of one dipole to two dipoles ($x_{10} \to x_{12} + x_{02}$). We solve this equation with the modified BFKL kernel:

$$K(x_{12}, x_{20}|x_{10}) = \frac{x_{10}^2}{x_{12}^2 x_{02}^2} e^{-B(x_{12}^2 + x_{02}^2)}$$  \hspace{1cm} (14)

The motivation for this behaviour of the wave function of a dipole in the confinement region stems from the Gaussian-like form of the wave functions of mesons in holographic AdS/QCD.
approach as well as in the phenomenology of the gluon emission at long distances. However, we will argue in conclusions that the main results of this paper do not depend on the particular form of Eq. (14).

5.2. Summary of the results

Since the lack of the room in this short version of the talk we start with the discussion of the main results of the paper.

(i) The scattering amplitude \( N(x_{10}, b, Y) \) \( \approx 2 b^2 \) \( \propto e^{4 B b^2} \) (expected);

(ii) \( N(x_{10}, b, Y) \) \( \approx \) \( e^{\omega_0 Y} \) with \( \omega_0 = \omega_{\text{BFKL}} \) (unexpected!!!);

(iii) \( \langle |b^2| \rangle = \text{Constant} (Y) \) (expected);

(iv) Saturation scale \( Q_s^2 \propto e^{\lambda Y} \) \( \lambda = \lambda_{\text{BFKL}} \) (expected);

(v) The modified BFKL Pomeron looks similar to the Pomeron in N=4 SYM and in high energy phenomenology[26]: \( \Delta_{IP} \approx 0.3 \); \( \alpha'_{IP} = 0 \) (unexpected).

We postpone the discussion of the point 2 to the next section and briefly outline the results which were expected.

Large \( b \) behaviour of the amplitude. The large \( b \) behaviour of the amplitude, we can derive directly from Eq. (13). Indeed, one can see that the main contribution at large \( b \) stems from the region where \( |\vec{b} - \vec{x}_{12}| \leq x_{10} \). At such \( x_{12} \) the equation takes the form

\[
\frac{\partial N(x_{10} \approx 2b, b; Y)}{\partial Y} = 4\alpha S \int \frac{d^2x_{12}}{4b^2} e^{-4 B b^2 - B x^2_{12}} 2 N(x_{12}, 0; Y) \propto e^{-4 B b^2}
\]

\( \langle |b^2| \rangle \) dependence versus \( Y \). The general origin of the energy dependence of \( \langle |b^2| \rangle \) was found by Gribov[3] (Gribov’s diffusion) and it stems from the uncertainty principle, that at each emission the shift in impact parameter \( \Delta b \) is proportional to \( 1/p_T \) where \( p_T \) is the average transverse momentum of emitted parton (gluon). In the parton model \( p_T \) is a dimensional scale of the model that does not depend on energy (rapidity) of the parton. In this case after \( n \) emission the average shift in \( b \) is equal to \( \langle |b^2| \rangle \propto \Delta b^2 \). Since \( n \propto \ln s \) (\( s \) is the energy) we have the well known result that the interaction radius \( R^2 \propto \ln s \). However, in QCD \( p_T \) increases with \( Y \) (energy) and after first several emission it becomes so large that we can neglect that shift in \( b \). Our numerical calculation shows that, indeed, \( \langle |b^2| \rangle \) is constant at large \( Y \) (see Fig. 7).

5.3. The intercept of the modified BFKL Pomeron

In this section we wish to discuss the main result of our approach: the intercept of the modified BFKL Pomeron turns out to be the same as the BFKL Pomeron. Going to \( \omega \)-representation \( (N(x_{01}; Y) = \int \frac{d\omega}{2\pi} e^{\omega Y} N_\omega(x_{01}) ) \) one can see that the modified BFKL equation takes the form

\[
\omega N_\omega(x_{01}) = -\alpha_S \mathcal{H} N_\omega(x_{01})
\]

or

\[
E N_\omega(x_{01}) = \mathcal{H} N_\omega(x_{01})
\]
For finding the eigenfunctions and eigenvalues of Eq. (17) we need to specify the boundary conditions. At short distances Eq. (14) has the same form as the BFKL kernel and, therefore, we have

\[ N_\omega (x_{01}) \xrightarrow{x_{01} \ll 1/B} N_B^{BFKL} (x_{01}) = \left( \frac{1}{x_{01}^2} \right)^{\frac{1}{2} + i\nu} \]  \hspace{1cm} (18)

Note that the BFKL spectrum is

\[ E(\nu) = 2\psi(1) - \psi\left(-\frac{1}{2} + i\nu\right) - \psi\left(\frac{1}{2} - i\nu\right) \]  \hspace{1cm} (19)

For long distances the confinement modification of the kernel does not allow for dipoles to have large sizes leading to the boundary condition

\[ N_\omega (x_{01}) \xrightarrow{x_{01} \geq 1/B} \text{Constant} \]  \hspace{1cm} (20)

**Theory estimates: variational method.** It is well known that the energy of the ground state is less or equal to

\[ E_{\text{ground}} \equiv -\omega_0 \leq F[\{N\}] \]  \hspace{1cm} (21)

where \( F[\{N\}] \) is the following functional

\[ F[\{N\}] = \frac{\langle N^* (x_{01}) | \mathcal{H} | N (x_{01}) \rangle}{\langle N^* (x_{01}) | N (x_{01}) \rangle} \]

Choosing the set of the BFKL functions of Eq. (18) which is the complete and normalized set of functions, we find that \( F_{\text{min}}[\{N\}] = \omega_{BFKL} = 4 \ln 2 \bar{\alpha}_S \) (see Eq. (19) at \( \nu = 0 \) (see Fig. 8)

Therefore, the intercept of the modified BFKL equation could be only larger or equal to the intercept of the BFKL one.

In our paper [27] we developed two theoretical approaches: *semi-classical approach* and *diffusion approximation* which show that the modified BFKL equation does not have an intercept larger than the BFKL one. However, due to lack of room we cannot discuss these approaches here but they have been considered in our paper [27].

**Numerical calculations.**

Solving numerically we face two problems:
Figure 8. Comparison of $\omega(\gamma) = \frac{1}{2} + i\nu = F_{\text{min}}[\{N\}]$ with $\omega_{\text{BFKL}}$ given by Eq. (19).

(i) The kernel is not Fredholm type

$$\int d^2x_{01}d^2x_{12} K (x_{12}, x_{02}|x_{01}) \to \infty$$

(ii) The kernel is singular at $x_{12} \to x_{01}$

We use the following checks of our numerical procedure:

(i) Independence on the choice of $x_{\text{min}}$ and $x_{\text{max}}$;

(ii) Numerical solution to the BFKL equation coincide with the analytic one;

(iii) Independence on value of the regulator $R$:

$$\int d^2x_{13}K^B_R (x_{12}, x_{02}|x_{10}) N (x_{12}; Y) \equiv$$

$$\int d^2x_{12} e^{-B(x_{12}^2 + x_{02}^2)} \left\{ 2 N (x_{12}; Y) - 2 \frac{x_{10}^2}{x_{12}^2 + x_{02}^2 + 2R^2} N (x_{10}; Y) \right\}$$

(iv) Independence on value of $B$;

The result is plotted in Fig. 9.

Figure 9. $d\ln N/dY$ versus $Y$ for the BFKL ($B=0$) and modified BFKL ($B=1$) equations.
5.4. Conclusions

We found out that the modified BFKL Pomeron has the same intercept $\Delta$ as the BFKL Pomeron ($\Delta = \omega_{\text{BFKL}} = 4 \ln 2 \alpha_s$) and $\alpha'_p = 0$. Therefore, the BFKL Pomeron with the modified kernel reproduces the main features of the soft Pomeron that has been found both from N=4 SYM theor and from the high energy Reggeon phenomenology.

Actually, we were surprised that the model for confinement changed so little in the BFKL Pomeron and on qualitative level, the Pomeron that emerges from the modified BFKL equation, looks quite the same at the BFKL Pomeron, both in parameters and in character of the energy behaviour. It seems that the only difference between the BFKL Pomeron and the modified BFKL Pomeron is that the second has a correct large impact parameter behaviour.

We believe that this statement does not depend on the particular form of Eq. (14). As we have mentioned the spectrum of the modified BFKL equation is determined by Eq. (17) with two boundary conditions of Eq. (18) at short distances and Eq. (20) at long distances. At first sight the condition at long distances will restrict the values of $\nu$ in the comparison with the BFKL equation. However, it is not the case. Indeed, the eigenvalues of the BFKL equation is degenerate having two eigenfunctions with positive and negative $\nu$. One can see that we can find the sum of these two eigenfunction $N(\nu) = (1/x_{12}^2)^{\frac{1}{2}} \sin (\nu \ln (1/(Bx_{12}^2))) + \phi_0)$ which is equal to constant $(\sin \phi_0)$ at $x_{12} = 1/B$. Therefore, at any $\nu$ we can satisfy the condition of Eq. (20). On the other hand for $\nu = i\kappa$ ($\kappa > 0$) we have two eigenfunctions: $(x_{12}^2)^{-\frac{1}{2}+\kappa}$ and $(x_{12}^2)^{-\frac{1}{2}-\kappa}$. The normalization condition selects out the only eigenfunction $(x_{12}^2)^{-\frac{1}{2}+\kappa}$ which has no divergency at $x_{12} \to 0$. Using this function we cannot satisfy the condition at $x_{12} \to \infty$ and, therefore, we have no solution of Eq. (17) for $\nu = i\kappa$. Hence, we expect that the spectrum for the modified Hamiltonian will be the same as the BFKL spectrum.

The independence of the spectrum of the BFKL Pomeron on the models for the confinement gives us a hope that the unknown confinement will change only slightly the equations of the CGC/saturation approach and these changes will not depend on the particular way of taking into account the long distances physics. In simple words, this paper gives a hope that the CGC/saturation approach will be still a theory in spite of needed model modifications due to confinement.

6. Resume

I wanted to demonstrate how great the progress which has been made in understanding of the Pomeron structure. However, the honest answer to the question: do we understand what is Pomeron, is no. The fact that the soft Pomeron appeared in AdS-CFT approach gives us some hope that the new generation of high energy physicists will find the answer.

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