Modeling and Control Approach for Dual Clutch Transmission Vehicles Starting Process Based on State-Dependent Autoregressive With Exogenous Model

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ABSTRACT To solve the difficulties of modeling the starting system of dual clutch transmission (DCT) vehicles, a state-dependent autoregressive with exogenous variables (SD-ARX) model whose functional coefficients are approximated by sets of radial basis function (RBF) networks is proposed to describe the dynamic characteristics of DCT vehicles starting process in this study. The validity of this modelling approach is verified via a real vehicle test. On this basis, a nonlinear predictive controller based on SD-ARX model is designed. In addition, the physical constraints of this system, including control variables (change rate of engine torque and clutch torque) and state variables (engine speed and clutch speed), are also taken into account during the controller design process via limiting the relevant parameters in particle swarm optimization or setting saturation demand in control program. To verify the validity and merits of the proposed control approach, many sets of simulation analysis in different driving intentions are conducted. Simulation results shown that: the proposed control approach can well control the starting process of DCT vehicles and effectively reflect the demand of driver’s intention; compared with the conventional control method, SD-ARX-MPC can improve the starting performance; the proposed approach is robust to a certain extent according to simulation results under changed starting conditions.

INDEX TERMS Starting control, radial basis function (RBF) networks, data-driven, state-dependent autoregressive with exogenous (SD-ARX) model, predictive control, nonlinear system.

I. INTRODUCTION
In recent years, dual clutch transmission (DCT) has attracted significant attention in the automotive industry, owing to its high fuel efficiency and transmission efficiency [1], [2].

Starting control is a significant and difficult challenge in DCT-related research [3], [4], as the driving intention is also a key factor to consider (in addition to the starting performance).

There have been many studies on the starting control for DCT vehicles, mainly incorporating physical-model-based optimal control [5]–[7] and intelligent control (particularly, fuzzy control) [8], [9]. For instance, in Ref. [10], a fuzzy reasoning method was proposed to obtain the clutch engagement speed using the driver’s intention, speed difference of the drive-driven plates, and engine speed as inputs. In Ref. [11], a variable structure feedback fuzzy control method was proposed to realize accurate clutch pressure control considering the jerk, driver’s intention, and starting time. In addition, in Ref. [12], the driver’s intention and the changing ratio of the friction work and friction work difference were considered, and a fuzzy control method for improving the starting performance of a heavy truck equipped with a DCT was proposed.

With respect to physical-model-based optimal starting control some references have used linear quadratic optimal methods to optimize the clutch engagement pressure or torque [13], [14], whereas others have considered the...
minimum engine speed and maximum clutch torque based on the minimum principle, and have realized optimal starting control [15], [16]. In Ref. [17], a linear quadratic regulator for designing a starting control strategy for a dry clutch was proposed, and the minimum time and change rate of the clutch force were used as the objective function. Additionally, in Ref. [18] a model predictive control (MPC) method based on dual-clutch torque distribution of DCT vehicles was proposed. Based on the minimum value principle, the authors of Ref. [3] used the starting time and shock degree as the objective function, and optimized the clutch torque under different accelerator pedal openings. In Ref. [19], linear quadratic output regulators considering disturbances and modeling errors of the observer were proposed; the simulation results showed that the impacts of modeling accuracy and disturbances could be weakened to a certain extent.

An intelligent control approach can effectively reflect the driver’s intention and decrease the impacts of modeling errors, but the control effects mainly rely on generated fuzzy rules and cannot realize dynamic optimal control during a dynamic starting process. Additionally, although a physical-model-based optimal control method may achieve global or local optimization during the clutch engagement process, the DCT starting process is a complex and nonlinear system. Additionally, as the system characteristics change with long-term aging and different driving conditions, large modeling errors can arise. Hence, it is fairly difficult and time-consuming to build a precise and efficient physical model [20], [21].

The radial basis function/autoregressive exogenous (RBF-ARX) model is a globally nonlinear and locally linear model, and can describe the nonlinear dynamic characteristics of a class of smooth nonlinear and nonstationary systems in which the working points change with time [22], [23]. The structure of the RBF-ARX model and its parameter optimization method were proposed for effectively characterizing nonlinear systems [22], [23]. Recently, an offline-identified RBF-ARX model-based predictive control has been applied in some real industrial systems; the satisfactory nonlinear modeling accuracy and significant effectiveness of the proposed MPC were verified [24]. In Ref. [25], an RBF-ARX model was used to describe the dynamic behavior of a quad-rotor, and used this model as an internal predictor in a receding horizon predictive controller to solve an attitude control issue. In Ref. [26], an RBF-ARX model was used to characterize the nonlinear relationships between the heading angle deviation, track error, and rudder angle, and combined with an MPC for the curve tracking of the route. In Refs. [22], [23] RBF-ARX modeling and an MPC approach were used to design a nonlinear NOx decomposition process for thermal power plants. Furthermore, some stability analyses regarding the offline-identified RBF-ARX model-based MPC were also provided in Ref. [23]. In Ref. [27], a time-varying locally linear RBF-ARX-MPC was used to achieve step response process control in a wide range of magnetic levitation systems. Based on this model, a predictive controller was designed for stabilizing a magnetic levitation ball at a given position, or for making it track a desired trajectory. In addition, the validity of the proposed control method was verified via comparisons with a conventional proportional–integral–derivative (PID) controller and a linear ARX-MPC. In Ref. [28], a neural network-based input–output feedback linearization slip control was proposed for anti-lock braking systems, aiming at the high uncertainty associated with the estimation of the slip. In Ref. [29], a method for generating a fuzzy model based on simulation data was proposed, and this method was applied as a surrogate model for the determination of optimal green period ratios and traffic light cycle times. Additionally, the authors of Ref [30] proposed a cost-effective approach to the design of nonlinear state-space control systems, and the method was exemplified via a real-world process represented by laboratory equipment (a pendulum-cart system). The authors noted that a limitation of their study was its dependence on a physical model, and that their future work would focus on combining the data-driven modeling approach and proposed control method. The motivation for the data-driven modeling approach to some physical systems is based on the impacts of inaccurate physical models and the uncertain parameters of the relevant systems.

Based on the above-mentioned analysis, this study proposes a novel optimal control approach based on an RBF-ARX modeling approach, to overcome the impacts of inaccurate physical models and the uncertain parameters in the starting systems of DCT vehicles [22]. A nonlinear model predictive controller based on a multivariable state-dependent autoregressive with exogenous variables (SD-ARX) model is considered. To obtain a global model of the starting process, a multivariable SD-ARX model is used to characterize the nonlinear dynamics of the starting process, and its functional coefficients comprise a set of RBF networks. After the model is identified offline using the Levenberg-Marquardt method and least squares method [22], a multistep state-space predictive model structure is proposed. Based on these models, which effectively describe the characteristics of DCT vehicles’ starting process, a predictive control strategy is designed. In each sampling interval, the SD-ARX model is used as an internal model or predictor of the MPC, and may be regarded as a locally linear ARX model. This approximated linearization ARX model’s parameters vary with time, as the working point of starting process changes with time, and its dynamic behavior is nonlinear. Additionally, the physical constraints of the starting process are also considered during the online optimization process.

The main contributions of the proposed control approach are as follows. First, a novel modeling approach is proposed for the starting process of DCT vehicles. The nonlinear and complex starting process is considered as equivalent to the SD-ARX model. Data-driven modeling based on input–output data is achieved, and a relevant real vehicle test is conducted to verify its validity. Second, with respect to control of the starting process, the data-driven modeling...
approach is combined with MPC theory, and the proposed SD-ARX-MPC is used to optimize the starting process of DCT vehicles. The relevant physical constraints are also considered. Third, simulation tests are conducted with different intentions and starting conditions, to thereby validate the entire control approach. This proposed control approach can avoid the dependence on physical modeling in the conventional optimal control methods, and can improve the starting performances of DCT vehicles.

The reminder of this study is organized as follows: Section II proposes the data-driven SD-ARX modeling principle and provide the details of the validation of this technique using a real vehicle test, and Section III proposes SD-ARX-MPC strategy based on the obtained SD-ARX model. Based on the MATLAB/Simulink software platform, Section IV presents the relevant simulation analysis and verify the merits of SD-ARX-MPC via comparison with a conventional control method, and the robustness of the proposed control approach.

II. STATE-DEPENDENT AUTOREGRESSIVE WITH EXOGENOUS VARIABLES (SD-ARX) MODELING AND VERIFICATION

A. SD-ARX MODEL FOR DCT VEHICLES STARTING PROCESS

Ref. [22] presented the nonlinear system, which could be described as follows:

\[ y(t) = f(w(t - 1)) + \xi, \quad \tilde{w}(t - 1) = [y(t - 1), \ldots, y(t - k_y), u(t - 1), \ldots, u(t - k_y), v(t - 1), \ldots, v(k - k_y)], \quad (1) \]

where \( y(t) \) represents the output of the system; \( u(t) \) represents the input of the system; \( v(t) \) represents the measurable disturbances, and \( \xi \) represents the Gaussian white noises. Assuming that \( f(\cdot) \) is continuously differentiable at any equilibrium point, it can be expanded in a Taylor series; thus, Model (1) can be described as the following state-dependent autoregressive Model (2) as follows:

\[
\begin{align*}
Y(t) &= \sum_{i=1}^{k_y} \bar{A}_i(\tilde{w}(t - 1))Y(t - i)^T \\
&\quad + \sum_{i=1}^{k} \bar{B}_i(\tilde{w}(t - 1))U(t - i)^T + \Theta(\tilde{w}(t - 1)) + \Xi(t) \\
(4)
\end{align*}
\]

In the above, \( k_y, k_\beta, k_\gamma, \dim(\tilde{w}(t - 1)) \) represents the orders of the model; \( z_{jk}, \lambda^i_f \) represents the center point and non-linear weight coefficients of the Gaussian RBF network, respectively; \( \phi_\beta, \phi_0 \) are the Gaussian nonlinear state-dependent coefficients, and vary with \( \tilde{w}(t - 1); c_\beta^i, c_f^i \) are the weight coefficients; \( \| \nabla \|_2 = \nabla^T \nabla \) represents the second norm of a vector; \( \Xi(t) \) represents the Gaussian white noise, independent of observations; and \( \tilde{w}(t - 1) \) in Model (2) represents the indexes of the model, and gradually represents the input/output data of the system.

For a starting system with multiple inputs and multiple outputs (MIMO) that contains measurable disturbances in this study, Model (1) can be extended into (3):

Here, \( Y(t) \) represents the output of this system, i.e., the angular speed of the engine \( \omega_c \), and the angular speed difference of the drive-driven plates \( \omega_{dc} \). \( U(t) \) represents the inputs of this system, i.e., clutch torque \( T_c \) and engine torque \( T_e \); \( V(t) \) represents the measurable disturbance, i.e., the resistance torque \( T_r \), and its value can be obtained via a designed observer [31].

Many types of functions have been used to approximate the unknown nonlinear function \( f(\cdot) \) that maximizes the similarity of the model. A general method is the SD-ARX model, which can be described as follows:

In the above, \( \tilde{w}(t - 1) \) is regarded as the state vector at time \( t \), i.e., the variable causing the working point of system to vary with time. Generally, it contains the inputs, outputs, or other variables for representing the nonlinear dynamic characteristics of the objective system. The state-dependent coefficients of this model are given by \( \Theta(\tilde{w}(t - 1)), \bar{A}_i(\tilde{w}(t - 1)), \) and \( \bar{B}_i(\tilde{w}(t - 1)) \).

The main idea of the SD-ARX Model (4) is to achieve a local linearization of the nonlinear ARX Model (3) via introducing a locally linear ARX model with state-dependent coefficients. At any working point, a locally linearized ARX model can be easily obtained by fixing the state vector \( \tilde{w}(t - 1) \) in the SD-ARX Model (4), and the model can represent the global dynamic characteristics of the system via the changeable \( \tilde{w}(t - 1) \) at each working point.

Model (4) can be rewritten in a matrix polynomial form, as follows:

\[
Y(t) = \sum_{i=1}^{k_y} \Lambda_{i,x-1}Y(t - i)^T \\
+ \sum_{i=1}^{k} \bar{B}_{i,x-1}U(t - i)^T + \Theta(w(t - 1)) + \Xi(t) \\
(5)
\]
Model (5) can be converted into a state space model by defining a state vector, as follows:

\[
X(t) = \begin{bmatrix} \ell_1^T, \ell_2^T \end{bmatrix}^T
\]

\[
\ell_1^T = \begin{bmatrix} x_{1,1}, x_{2,1}, \ldots, x_{k,1} \end{bmatrix}^T
\]

\[
x_{k,x}^T = [\omega_c(t), \alpha_{ec}(t)]^T
\]

\[
x_{k,x}^T = \sum_{i=1}^{k} \sum_{j=1}^{2} a_{ij} \phi_i(t) \phi_j(t-i) + \sum_{i=1}^{k} \sum_{j=1}^{2} b_{ij} \phi_i(t) \phi_j(t-i)
\]

\[\begin{cases}
\hat{a}_{ij} = 0, (k > \alpha_i), \hat{b}_{ij} = 0, (k > \beta_j),
\hat{a}_{ij} = \max(\alpha_i, \beta_j), k = 2, 3, \ldots, k_n, l = 1, 2
\end{cases}
\]

The state space model corresponding to model (6) can then be described as follows:

\[
\begin{align*}
X(t) &= A_{t-1}X(t-1) + B_{t-1}U(t-1) + \Phi_{t-1} + \Xi(t), \\
Y(t) &= C_{t-1}X(t).
\end{align*}
\]

The above description is subject to the following:

\[
\begin{align*}
A_{t-1} &= \begin{bmatrix}
\hat{a}_{11} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \hat{a}_{1n}
\end{bmatrix}_{n \times n} \\
B_{t-1} &= \begin{bmatrix}
\hat{b}_{11} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \hat{b}_{1n}
\end{bmatrix}_{n \times 2} \\
C_{t-1} &= \begin{bmatrix}
\chi & 0 \\
0 & \chi
\end{bmatrix}_{2 \times 2n} \\
\Phi_{t-1} &= \begin{bmatrix}
\hat{\Phi}_1(t-1) & \hat{\Phi}_2(t-1)
\end{bmatrix}^T
\end{align*}
\]

\[
\begin{align*}
\hat{a}_{ij} &= \begin{bmatrix}
\hat{a}_{ij}^1 & 0 & \cdots & 0 \\
0 & \hat{a}_{ij}^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \hat{a}_{ij}^{n-1}
\end{bmatrix}_{n \times n}, \\
\alpha_{ij} &= \begin{bmatrix}
\hat{a}_{ij}^1 & 0 & \cdots & 0 \\
\hat{a}_{ij}^2 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\hat{a}_{ij}^{n-1} & 0 & \cdots & 0
\end{bmatrix}_{n \times n}, \\
\hat{b}_{ij} &= \begin{bmatrix}
\hat{b}_{ij}^1 & \hat{b}_{ij}^2 & \cdots & \hat{b}_{ij}^n
\end{bmatrix}^T, \\
\chi &= [1, 0, \cdots, 0]^T, \\
\hat{\Phi}_1(t-1) &= [\phi_1(t-1), 0, \cdots, 0]^T, l = 1, 2
\end{align*}
\]

The state vector \(X(t)\) at the current time \(t\) can be obtained by Model (7), according to the current outputs \(Y(t)\), past input/output data, and identified MIMO SD-ARX Model (4). The state space Model (7) can be utilized to predict the status of future outputs, based on the SD-ARX Model (4).

**B. VERIFICATION OF SD-ARX-MODELING**

To validate the effectiveness and practicability of the data-driven SD-ARX modeling for the DCT starting process, a real vehicle test is conducted. Some groups of input–output data under closed-loop control during the starting process are collected and utilized for SD-ARX modeling, and other groups of input–output data are utilized to validate the effectiveness of the obtained SD-ARX model. The relevant equipment used in the test are shown in Fig. 1. In this test, the sample frequency is 100 Hz. The speed signals can generally be obtained via the speed sensor; the engine torque can be obtained from the controller area network, after simple conversion according to the dynamic engine torque model; the clutch torque can be converted from the oil pressure signal; and the load resistance torque can be calculated by the designed torque observer [31]. One group of input–output data sets is shown in Fig. 2.

**FIGURE 1. Test equipment for real vehicle data collection. (a) Starting process of DCT vehicle. (b) Real-time display of vehicle data.**

**FIGURE 2. One set of input-output data.**

Figs. 3 and 4 show a comparison between the actual observation data and identified output generated by the
FIGURE 3. Training data set. (a) Actual output and SD-ARX model output of $\omega_e$. (b) Residual value of $\omega_e$. (c) Actual output and SD-ARX model output of $\omega_{ec}$. (d) Residual value of $\omega_{ec}$.

FIGURE 4. Testing data set. (a) Actual output and SD-ARX model output of $\omega_e$. (b) Residual value of $\omega_e$. (c) Actual output and SD-ARX model output of $\omega_{ec}$. (d) Residual value of $\omega_{ec}$. 
SD-ARX model, and the modeling errors for the training data and testing data, respectively. All the vehicle tests are conducted on a dry flat road, but the vehicle masses and slopes are slightly different. Among them, six groups of data sets are used to train the SD-ARX model, whereas another four groups of data sets are used to test the validity of the obtained data-driven model. It can be seen from Figs. 3 and 4 that the identified SD-ARX model outputs are relatively close to the observation output, regardless of the training data set or testing data set. Specifically, the errors of the training data and testing data are both within ±5 rad/s. It can be seen that the obtained SD-ARX model can well characterize the DCT vehicle starting system. Therefore, based on the data-driven model, an SD-ARX model-based predictive controller can be designed.

III. MODEL PREDICTIVE CONTROL (MPC) CONTROLLER BASED ON SD-ARX MODEL

According to Section II, one-step prediction values of state variables can be obtained; according to Section III, multi-step prediction values of state variables can be obtained. On this basis, the prediction status (including the engine angular speed and angular difference of the drive-driven plates) during the prediction time domain can be obtained. According to the principle of MPC, a rolling optimization of control variables (including clutch torque and engine torque) in the SD-ARX-MPC controller can be realized. A schematic of the control strategy for realizing starting control based on SD-ARX modeling is shown in Fig. 5. Specifically, the input-output data in each step is transferred to the SD-ARX model obtained via offline identification, which is used to predict the future system status. Based on the predictive system status and relevant objective function of the MPC, the optimized \( T_c \) and \( T_r \) are calculated via a particle swarm optimization algorithm, and the obtained \( T_c \) and \( T_r \) are used to control the starting process.

**FIGURE 5. SD-ARX-MPC control approach of DCT vehicles starting process.**

A. MULTISTEP-AHEAD PREDICTIVE MODEL FOR MPC

To design an effective multistep nonlinear model predictive controller for starting control of DCT vehicles, a multistep-ahead predictive model is built. According to the one step ahead state space Model (7), a multistep-ahead state space model can be created. To this end, the relevant vectors of the signals are defined as follows:

\[
\begin{align*}
\dot{X}(t) &= [\dot{X}(t+1)^T, \dot{X}(t+2)^T, \ldots, \dot{X}(t+N)^T]^T \\
\dot{Y}(t) &= [\dot{Y}(t+1)^T, \dot{Y}(t+2)^T, \ldots, \dot{Y}(t+N)^T]^T \\
Y_r(t) &= [Y_r(t+1)^T, Y_r(t+2)^T, \ldots, Y_r(t+N)^T]^T \\
\hat{U}(t) &= \left[ \hat{U}(t)^T, \hat{U}(t+1)^T, \ldots, \hat{U}(t+N_u-1)^T \right]^T \\
\hat{F}_f &= [\Phi_f^T, \Phi_{f+1}^T, \ldots, \Phi_{f+N-1}^T]^T \\
\end{align*}
\]

Here, \( N \) represents the predictive horizon, and \( N_u (N_u < N) \) represents the control horizon; in this study, \( N = N_u = 4 \). \( \hat{Y}(t) \) represents the identified data-driven model predictive outputs, and \( Y_r(t) \) represents the target output trajectory. Based on Model (7) at time \( t \), the \( j (j = 1, 2, \ldots, N) \) step-ahead predictive state and output variables can be given as follows:

\[
\begin{align*}
\dot{X}(t) &= \bar{A}_t X(t) + \bar{B}_t \hat{U}(t) + \bar{G}_t \Phi_f, \\
\hat{Y}(t) &= \bar{C} \hat{X}(t).
\end{align*}
\]

The state matrices \( \bar{A}_t, \bar{B}_t, \bar{G}_t \) and \( \bar{C} \) can be deduced (10)–(14), as shown at the bottom of the next page.

The system matrices \( A_t, B_t, G_t \) and \( C \) in model (9) can be calculated by (10-14). Knowledge of the working point state prediction \( \hat{v}(t+j|t) (j = 1, 2, \ldots, N - 1) \) is required. Generally, the value of \( \hat{v}(t+j|t) \) can be simplified by single-point linearization [32].

B. NONLINEAR MPC STRATEGY FOR STARTING PROCESS OF DCT VEHICLES

According to the multiple step-ahead state space Model (9), the prediction of outputs can be also described as follows:

\[
\begin{align*}
\hat{Y}(t) &= G_t \hat{U}(t) + Y_0(t) \\
G_t &= \bar{C} \bar{B}_t \\
Y_0(t) &= \bar{C} \bar{A} \bar{x}(t) + \bar{C} \bar{G} \Phi_f \\
\end{align*}
\]

The control move sequence \( \Delta \hat{U}(t) \) can be defined as follows:

\[
\begin{align*}
\Delta \hat{U}(t) &= [\Delta u(t)^T, \Delta u(t+1)^T, \ldots, \Delta u(t+N_u-1)^T]^T \\
\Delta u(t) &= u(t) - u(t-1)
\end{align*}
\]

Excessive variation of the clutch torque will affect the smooth operation of the powertrain system [5]. To achieve a fast and smooth starting process and reflect the driver’s intention, the objective function can be defined as follows:

\[
\begin{align*}
\min J &= \left\| \hat{Y}(t) - Y_r(t) \right\|_Q^2 + \left\| U(t) \right\|_R_1^2 + \left\| \Delta U(t) \right\|_R_2^2 \\
\text{s.t.} \, \hat{Y}_{\min} \leq \hat{Y}(t) \leq \hat{Y}_{\max}, \, U_{\min} \leq U(t) \leq U_{\max}, \\
\Delta U_{\min} \leq \Delta U(t) \leq \Delta U_{\max}
\end{align*}
\]
such matrices. Among them, \( Q \) represents a positive definite diagonal weighting matrix; control moves and control levels can be penalized via such matrices. Among them, \( Q \) represents the punishment coefficient matrix for the state variables following the target trajectory. Here, the state variables are the engine angular speed and the angular difference of the drive-driven plates. The value of \( Q \) is adjusted to reflect the demands of the driver's intention and those of friction work to some extent; a larger value of \( Q \) can decrease the generation of friction work. The value of \( R_2 \) affects the change rates of the clutch and engine torque, and a larger value of \( R_2 \) helps to achieve a smoother starting process. A quadratic form of this optimization problem that is equivalent to (17) can be transformed into the following form, as in (18), via substituting (15) into (17).

In the above, \( Y_r(r) \) represents the target trajectory. In this study, \( \omega_{ref} \) is the preset engine speed, and its value is
determined by the accelerator pedal opening [19]; \( \omega_{ec}(t + i) = \beta \omega_{ec}(t), \ i = 1, 2, \ldots, N_e, \ \beta < 1 \), indicating that the objective of this control strategy is to drive the speed difference of the drive-driven plates to zero; its value is set to 0.9 in this study.

The corresponding physical constraints mainly include \( T_c \in [T_{c_{\min}}, T_{c_{\max}}], \ T_e \in [T_{c_{\min}}, T_{c_{\max}}], \ T_e \in [T_{e_{\min}}, T_{e_{\max}}] \), \( \omega_e \in [\omega_{e_{\min}}, \omega_{e_{\max}}], \ \omega_e \geq 0 \). In particular, \( T_{c_{\max}} = 210N \cdot m \) denotes the maximum value of the engine torque; \( T_{c_{\min}} = 0 \) denotes the minimum value of the engine torque; \( T_{e_{\max}} = 280N \cdot m \) denotes the maximum value of the clutch torque; \( T_{e_{\min}} = 0 \) denotes the minimum value of the clutch torque; \( T_{e_{\max}} = 750N \cdot m/s \) denotes the maximum increase value per second of the clutch torque; \( T_{e_{\min}} = -750N \cdot m/s \) denotes the maximum decrease value per second of the clutch torque; \( \omega_{e_{\max}} = 628rad/s \) is the maximum value of the engine speed in normal work conditions; and \( \omega_{e_{\min}} = 83rad/s \) is the minimum value of the engine speed in normal work conditions.

\[
\begin{align*}
\hat{U}(t) &= U_0(t - 1) + F \Delta \hat{U}(t), \\
U_0(t - 1) &= [U(t - 1)^T, U(t - 1)^T, \ldots, U(t - 1)^T]^T.
\end{align*}
\]

(19)

The constant matrix \( F \) relates to the order of inputs and the form of \( U_0(t-1) \), and \( F \) can be described as follows:

\[
F = \begin{bmatrix}
I & 0 & 0 & 0 \\
I & I & 0 & 0 \\
\vdots & \vdots & \ddots & 0 \\
I & I & \ldots & I
\end{bmatrix}.
\]

(20)

where \( I \) represent the identity matrix that has the same order as the control variable.

IV. SIMULATION RESULTS AND ANALYSIS

The communication protocol of the transmission control unit (TCU) equipped in a real vehicle is confidential information. A real vehicle test cannot be conducted at this moment without the help of Changan Automobile Co. Ltd. Hence, an appropriately simplified virtual vehicle is designed in the MATLAB/Simulink platform for verifying the validity of the SD-ARX-MPC. Specifically, two groups of open-loop input data are designed under different starting conditions, and the generated input–output data set is divided into training data and testing data, to thereby train and obtain an effective data-driven SD-ARX model. Based on the obtained SD-ARX model in the simulation environment, the SD-ARX-MPC controller can be verified, based on the simulation vehicle. The simulation vehicle consists of an engine, five-speed wet dual-clutch transmission, wheel model, and drive shafts. The simplified structure is shown in Fig. 6, and the relevant parameters are listed in Table 1.

A. DESCRIPTION OF DRIVELINE SYSTEM

To describe the starting system of DCT vehicles in the MATLAB/Simulink software platform, some assumptions are presented: 1) this system consists of elastic links that lack inertia and inelastic inertial links, and all relevant components exist in the form of a lumped mass; 2) the elasticity of the bearing and bearing seat, the meshing elasticity of the gears, and the transverse vibration of the shaft can be neglected; 3) the heat decay of the clutch and the impact of torsional vibration in the driveline system can be neglected; and 4) the engagement process of the synchronizer and relevant system gaps can be neglected.

FIGURE 6. Structural diagram of DCT driveline system.

In the above, \( T_{c1} \) and \( T_{c2} \) represent the transmitted torques of clutches 1 and 2, respectively; \( I_3 \) is the equivalent rotating inertia of the clutch 1 shock absorber (passive part), input shaft 1, and relevant odd gear; \( \omega_3 \) is the angular speed of input shaft 1; \( I_4 \) is the equivalent rotating inertia of the clutch 2 shock absorber (passive part), input shaft 2, and relevant even gear; \( \omega_4 \) is the angular speed of input shaft 2; \( I_5 \) is the equivalent rotating inertia of the intermediate shaft 1, relevant gear, and main reducer 1 driving part; \( \omega_5 \) is the angular speed of the intermediate shaft; \( I_6 \) is the equivalent rotating inertia of the passive part of the main reducer, differential, half shaft, and wheel; \( \omega_6 \) is the angular speed of the half shaft; \( I_7 \) is the equivalent rotating inertia of the entire vehicle to the output shaft; \( \omega_7 \) is the angular speed of the vehicle wheel; and \( k_1 \) and \( k_2 \) represent the corresponding torsional stiffness of the shock absorbers of clutches 1 and 2, respectively.

1) ENGINE MODEL

A steady state torque model of the engine can be seen in Fig. 7. Generally, the engine runs in an unsteady state. In this study, a correction coefficient is considered to modify the engine torque in the steady state condition, as shown in (21), and the modified variable is defined as the dynamic output torque of the engine.

\[
T_e = T_e' - \lambda \dot{\omega}_e
\]

(21)

Here, \( T_e' \) is the engine torque in steady state conditions, and \( \lambda \) is the decreased coefficient of the engine torque in unsteady working conditions (0.03 in this study).

2) MODEL OF STARTING DYNAMICS OF 5-SPEED WET DCT

The starting process of DCT vehicles consists of two phases: a frictional sliding stage and synchronization stage, which are presented as follows.

(1) Frictional Sliding Stage

In this study, the first gear clutch is used in the starting process. The DCT vehicle dynamics can be described as a
Y. Yang et al.: Modeling and Control Approach for DCT Vehicles Starting Process Based on SD-ARX Model

| Parameters                           | Values |
|--------------------------------------|--------|
| vehicle mass                        | $m_c = 1600$ (kg) |
| tire rolling resistance coefficient  | $f_r = 0.018$ |
| wheel radius                         | $r_w = 0.289$ (m) |
| rotating inertia mass                | $I_e = 1.2$ |
| coefficient of air resistance        | $C_a = 0.32$ |
| effective frontal area               | $A = 2.04$ (m$^2$) |
| sliding adhesion coefficient of road surface | $\mu = 0.75$ |
| transmission efficiency              | $\eta = 0.93$ |
| engine inertia                       | $I_e = 0.25$ (kg$^2$m$^{-2}$) |
| inertia of clutch 1                  | $I_{i1} = 0.0056$ |
| inertia of clutch 2                  | $I_{i2} = 0.0056$ |
| inertia of output shaft              | $I_o = 124$ (kg$^2$m$^{-2}$) |
| ratio of gear 1                      | $i_1 = 3.2562$ |
| ratio of gear 2                      | $i_2 = 1.7064$ |
| ratio of gear 3                      | $i_3 = 1.2301$ |
| ratio of gear 4                      | $i_4 = 0.8988$ |
| ratio of gear 5                      | $i_5 = 0.6894$ |
| ratio of final drive gear            | $i_g = 4.4705$ |
| torsional stiffness of shock absorber of clutch 1 | $k_{c1} = 4000$ |
| torsional stiffness of shock absorber of clutch 2 | $k_{c2} = 4000$ |
| structural damping coefficient of shock absorber of clutch 1 | $c_{c1} = 13.6$ |
| structural damping coefficient of shock absorber of clutch 2 | $c_{c2} = 13.6$ |
| equivalent torsional stiffness of vehicle half shaft with tire | $k_{eq} = 16300$ |
| rotational viscous damping coefficient of vehicle half shaft with tire | $c_{eq} = 311$ |

(2) Synchronization Stage
When the drive-driven plates synchronize, i.e., the starting process enters a stable operation phase, and (22) can be changed into the following form:

$$
\begin{align*}
T_e - T_{i1} & = (I_e + I_1)\dot{\omega}_e \\
T_{i1}i_1& - T_{load} = i_{d1}i_1\dot{\omega}_1 + (I_o + \frac{I_1^2}{I_{eq}})\dot{\omega}_6 \\
T_{i1} - I_1\dot{\omega}_1 & = k_1(\theta_1 - \theta_2) + c_1(\omega_1 - \omega_3) \\
I_1\dot{\omega}_6 + T_{load} & = k_o(\theta_6 - \theta_1) + c_o(\omega_6 - \omega_v)
\end{align*}
$$

In the above, $\theta_1$ and $\theta_2$ represent the angular displacements (rad) corresponding to $\omega_1$ and $\omega_3$, respectively; $\theta_6$ and $\theta_1$ are the angular displacements (rad) corresponding to $\omega_6$ and $\omega_v$, respectively; and $T_e$ represents the running resistance distance equivalent to the transmission output shaft.

3) RESISTANCE TORQUE
The starting resistance consists of the gravity, rolling resistance, acceleration resistance, and wind resistance, as shown below.

$$
T_e = (mg \sin \alpha + fmg \cos \alpha + \frac{1}{2} C_D A \rho v^2 + \delta ma) r
$$

4) EVALUATION INDEXES
The jerk and friction work are the main evaluation indexes for the starting process, and are defined below.

(1) Jerk
The value of jerk directly affects the driver’s comfort. Jerk is defined as the second derivative of the velocity ($v$), as shown in (25). Generally, the absolute value of the jerk should be less than 10 m/s$^3$.

$$
\frac{d}{dt}\frac{dv}{dt} = j
$$

(2) Friction Work
The value of friction work has a direct impact on the service life of the clutch and driving performance of the entire vehicle, and can be defined as follows:

$$
W = \int_{t_0}^{t_1} T_e \omega_e dt + \int_{t_1}^{t_2} T_e [\omega_e - \omega_v] dt
$$

In the above, $t_0$ represents the exact moment of the frictional faces of the clutch starting to contact; $t_1$ represents the moment of the DCT vehicle starting to move, and $t_2$ represents the exact moment of the drive-driven plates being synchronized.

B. SD-ARX MODELING IN SIMULATION ENVIRONMENT
To obtain a valid data-driven SD-ARX model, a system identification is initially conducted, based on simulation data. Then, the obtained model is verified.

Similar to Section II (B), the purpose of this part is to train and validate the SD-ARX model in a simulation environment. The sample frequency is set as 100 Hz. In a real vehicle test, the data set are collected under closed-loop control. While in a simulation environment, a group of open-loop inputs is designed for obtaining adequate data set samples, as shown in Fig. 8(a). This generates a significant amount of the required input–output data, and these data are utilized in the SD-ARX modeling as training data.

The identification effects are shown in Fig. 8 (b), (c), (d), and (e), in which the identification errors of $\omega_e$, are less than ±0.01 (rad/s), and the identification errors of $\omega_{eq}$ are less than ±1 (rad/s). To validate the effectiveness of the obtained
SD-ARX model, another set of input data is designed under a different starting condition, as shown in Fig. 9 (a). The verification effects according to this part of data set are shown in Fig. 9 (b), (c), (d), and (e). The identification errors of $\omega_e$ are among ±0.1 (rad/s), and the identification errors of $\omega_{ec}$ are among ±8 (rad/s), which validates the efficacy of the obtained SD-ARX model. As can be seen from Figs. 8 and 9, the nonlinear model based on SD-ARX can well reflect the dynamic characteristics of the simulation model, which lays the foundation for the verification of the SD-ARX-MPC in a simulation environment.

C. SIMULATION RESULTS AND ANALYSIS

To verify the proposed control approach based on the SD-ARX model, three steps of simulation analysis are conducted: 1) a comparative analysis with three types of starting intentions, to validate the adaptability of SD-ARX-MPC to different intentions; 2) a comparative analysis between the
FIGURE 9. Testing data. (a) Input data $T_c$ (constant throttle pedal is 10%, $m_v = 1700$ kg, slope $\alpha = 15\%$). (b) Identification effects of $\omega_e$. (c) Identification errors of $\omega_e$. (d) Identification effects of $\omega_{ec}$. (e) Identification errors of $\omega_{ec}$.

1) CONTROL EFFECTS OF SD-ARX-MPC EMPLOYING DIFFERENT DRIVING INTENTIONS

The accelerator pedal openings for three types of driver starting intentions are simplified in Fig. 10 (slow starting, normal starting, and urgent starting), and the simulation results of the SD-ARX-MPC are shown in Figs. 11 and 12.
Here, the simulation is performed for a flat road, i.e., slope \( \alpha = 0 \); and the whole vehicle load \( m_v = 1600 \) kg.

As shown in Fig. 11(a), in the beginning of the starting process, the engine torque is larger than the clutch torque; thus, the engine speed can increase to its target speed. When its target speed is nearly reached, the engine torque is almost equivalent to the clutch torque, which can keep the engine running at a constant speed when possible, as shown in Fig. 11(b). Meanwhile, the change rate of the clutch torque in urgent starting is much larger than those in the other two types of starting processes, which aids in completing the rapid starting process. In contrast, the change rate of the clutch torque in slow starting is comparatively less than those in the other types of starting processes, so as to obtain a stable starting process with low shock. Thus, they completely meet the requirements of the driver.

The evaluation indexes for different starting intentions can be seen in Fig. 12. Fig. 12(a) shows that the maximum value of the absolute values of the jerk in different intentions are all less than 10 m/s\(^3\), indicating that the standard requirements can be well satisfied. The maximum absolute value of the jerk in slow starting is smaller than those in the other two types of starting processes, to thereby match the driver’s intention for a smooth starting. The maximum absolute value of the jerk in urgent starting is larger than those in the other two types of starting processes, which can satisfy the driving intention for a rapid start, (although it reduces the ride performance). As shown in Fig. 12(b), the friction work in urgent starting is larger than those in other types of starting processes. The main reason is that the decrease of the clutch engagement time is comparatively finite whereas the engine speed is greatly increased during the urgent starting process, resulting in a large speed difference for the drive-driven plates. According to (26), the generated friction work should also become larger. Hence, the friction work in slow starting is the smallest among the three types of starting processes. The specific values of the relevant evaluation indexes are listed in Table 2.

2) COMPARISON WITH THE CONVENTIONAL CONTROL APPROACH

To verify the benefits of the proposed control strategy, a comparative analysis is conducted between the SD-ARX-MPC and a conventional control method commonly applied in the control of wet DCT vehicle starting (i.e., a constant engine speed based on a PID controller). The simulation results are shown in Fig. 13, and the specific values of the evaluation indexes are listed in Table 2. Compared with those of the PID controller, the jerk and friction work using the SD-ARX-MPC in slow starting are improved by 0.2 m/s\(^3\) and 430 J, respectively; in normal starting, by 1.4 m/s\(^3\) and 105 J, respectively; and in urgent starting, by 0.8 m/s\(^3\) and 300 J, respectively. Hence, as compared with the traditional control method, the designed control approach can effectively improve the starting performance.

3) VERIFICATION OF THE ROBUSTNESS OF SD-ARX-MPC

To verify the robustness of the SD-ARX-MPC control strategy, and taking the normal starting intention as an example, the vehicle mass and slope are changed, to demonstrate the starting processes under different starting conditions. Among them, the vehicle mass is changed to 1900 kg, and the slope is changed to 20%. As shown in Fig. 14,
Y. Yang et al.: Modeling and Control Approach for DCT Vehicles Starting Process Based on SD-ARX Model

FIGURE 12. Evaluation indexes in different intentions. (a) Jerk. (b) Friction work.

TABLE 2. Comparison of evaluation indexes between SD-ARX-MPC and conventional control method.

| Driver's Intention | Evaluation Indexes | PID | SD-ARX-MPC | Improvement Value |
|--------------------|--------------------|-----|------------|------------------|
| slow               | jerk (m/s³)        | 3.1 | 2.9        | -0.2             |
|                    | friction work (J)  | 3230| 2800       | -430             |
| normal             | jerk (m/s³)        | 7.3 | 5.9        | -1.4             |
|                    | friction work (J)  | 6525| 6420       | -105             |
| urgent             | jerk (m/s³)        | 10.3| 9.5        | -0.8             |
|                    | friction work (J)  | 11300| 11000    | -300             |

Note: + represents worse performance, and - represents better performance.

the starting process can be well achieved in these conditions, and the generated jerk and friction work are 7.2 m/s³ and 9100 J, respectively. Evidently, when changing the starting conditions, the SD-ARX-MPC can still achieve optimal control of the starting process, indicating that the proposed control method is robust to a certain extent.

FIGURE 13. Simulation results under PID controller. (a) Engine and clutch torque. (b) Rotating speed of drive-driven plates. (c) Jerk. (d) Friction work.
V. CONCLUSION

This paper proposes a novel control approach, SD-ARX-MPC, and uses this approach to control DCT vehicle starting processes. To achieve this, an SD-ARX model of the DCT starting process is obtained based on a data-driven modeling approach, and a real vehicle test is conducted to validate the effectiveness of this technique. To verify the validity of the SD-ARX-MPC, simulation analyses are conducted, and the results show that: 1) the SD-ARX-based modeling method can determine the dynamic characteristics of the DCT starting process, providing the basis for validating the proposed control approach using a real vehicle; 2) compared with the control effects of a conventional control method, the proposed control approach can better control the starting processes of DCT vehicles; and 3) the SD-ARX-MPC has a certain robustness.

The proposed control approach has some flaws, especially in regards to a large amount of calculations, and subsequent work will focus on appropriate simplifications and actual vehicle verification of the proposed control strategy. In the future, with developments in the processing speeds of computing chips and appropriate simplification of this control strategy, this approach can be applied to improve the starting performances of DCT vehicles.

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