The vortex core in chiral $p$-wave superconductors exhibits various properties owing to the interplay between the vorticity and chirality inside the vortex core. In the chiral $p$-wave superconductors, the site-selective nuclear spin-lattice relaxation rate $T_1^{-1}$ is theoretically studied inside the vortex core within the framework of the quasiclassical theory of superconductivity. $T_1^{-1}$ at the vortex center depends on the sense of the chirality relative to the sense of the magnetic field. The effect of a tilt of the magnetic field upon $T_1^{-1}$ is investigated. The effect of the anisotropy in the superconducting gap and the Fermi surface is then investigated. The result is expected to be experimentally observed as a sign of the chiral pairing state in a superconducting material $\text{Sr}_2\text{RuO}_4$.

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1. INTRODUCTION

Site-selective nuclear magnetic resonance (NMR) method is a powerful tool for investigating the electronic structure inside vortex cores in the mixed state of type-II superconductors. We have theoretically studied the site-selective nuclear spin-lattice relaxation rate $T_1^{-1}$ inside a vortex core in the case of an isotropic chiral $p$-wave superconductivity $d = \tilde{z}(k_x \pm i k_y)$. We found that $T_1^{-1}$ was suppressed and almost vanished in the $k_x - i k_y$ state owing to the interplay between the vorticity and chirality inside the vortex core (here, the magnetic field was applied in positive direction of the $z$ axis). In this paper, we investigate the effect of a tilt of the magnetic fields
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on $T_1^{-1}$. We then investigate the effect of the anisotropy in the superconducting gap and the Fermi surface. Our result is expected to be experimentally observed as a sign of the chiral pairing state in a superconducting material Sr$_2$RuO$_4$.\[2]

2. QUASICLASSICAL THEORY

To investigate $T_1^{-1}$, we utilize the quasiclassical theory of superconductivity. We consider the quasiclassical Green function

$$\hat{g}(i\omega_n, \mathbf{r}', \mathbf{k}) = -i\pi \begin{pmatrix} g & if \\ -if & -g \end{pmatrix},$$

which is the solution of the Eilenberger equation,$^8$

$$i\mathbf{v}_F(\mathbf{k}) \cdot \nabla \hat{g} + [i\omega_n \hat{\tau}_z - \hat{\Delta}, \hat{g}] = 0,$$

where the superconducting order parameter is $\hat{\Delta}(\mathbf{r}', \mathbf{k}) = [(\hat{\tau}_x + i\hat{\tau}_y)\Delta(\mathbf{r}', \mathbf{k}) - (\hat{\tau}_x - i\hat{\tau}_y)\Delta^*(\mathbf{r}', \mathbf{k})]/2$ and $\hat{\tau}_i$ the Pauli matrices. $\mathbf{v}_F(\mathbf{k})$ is the Fermi velocity. The vector $\mathbf{r}' = (x', y', z') = (r \cos \phi, r \sin \phi, z')$ is the center of mass coordinate, where the magnetic field is applied along the $z'$ axis. The unit vector $\mathbf{\bar{k}} = (\bar{k}_x, \bar{k}_y, \bar{k}_z) = (k_\perp \cos \theta, k_\perp \sin \theta, k_z)/\sqrt{k_\perp^2 + k_z^2}$ represents the wave number of relative motion of the Cooper pairs in the crystallographic coordinate frame.

From the spin-spin correlation function,$^2$ we obtain the expression for $T_1^{-1}$ in terms of $\hat{g}$,

$$\frac{T_1^{-1}(T)}{T_1^{-1}(T_c)} = \frac{1}{4T_c} \int_{-\infty}^{\infty} \frac{d\omega}{\cosh^2(\omega/2T)} W(\omega, -\omega),$$

where the spectral function $\hat{a}(\omega, \mathbf{r}', \mathbf{\bar{k}}) = (a_{ij})$ is given as

$$\hat{a}(\omega, \mathbf{r}', \mathbf{\bar{k}}) = -i\frac{2\pi}{\tau_3} \hat{\tau}_3[\hat{g}(i\omega_n \rightarrow \omega - i\eta, \mathbf{r}', \mathbf{\bar{k}}) - \hat{g}(i\omega_n \rightarrow \omega + i\eta, \mathbf{r}', \mathbf{\bar{k}})],$$

the symbol $\langle \cdots \rangle$ represents the average over the Fermi surface, and $\eta$ is a small positive constant roughly representing the impurity effect. $T$ is the temperature and $T_c$ the superconducting transition temperature.

3. EFFECT OF TILT OF MAGNETIC FIELD

We define $\gamma$ as the angle between the magnetic field ($\parallel z'$ axis) and the $z$ crystallographic axis ($\parallel \bar{k}_z$ axis). The isotropic cylindrical Fermi surface
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![Graph](image)

Fig. 1. $T_{C}^{-1}$ vs $T$ at the vortex center. The magnetic field is tilted from the positive direction of the $z$ crystallographic axis. The angle $\gamma$ between the magnetic field and the $z$ crystallographic axis is set to $\gamma = 85^\circ$ (solid lines) and $\gamma = 0^\circ$ (dashed lines). The parameter $\eta = 0.2\Delta_{0}$ ($\Delta_{0}$ is the gap amplitude at $T = 0$).

is assumed and the Fermi velocity is $v_{F}(\bar{k}) = (v_{F}\cos\theta, v_{F}\sin\theta, 0)$ in the crystallographic coordinate frame. We assume that the layer perpendicular to the $z$ crystallographic axis is isotropic. In this section, we consider the isotropic chiral $p$-wave pairing $d = \bar{z}(\bar{k}_{x} \pm i\bar{k}_{y}) = \bar{z}\exp(\pm i\theta)$. We assume that this pairing is unchanged by a tilt of the magnetic field.

The order parameter around a vortex is expressed as $\Delta(r', \bar{k}) = |\Delta(r)|\exp(i\phi)\exp(\pm i\theta)$. On the basis of an analysis of the so-called zero-core vortex model, the matrix elements of $\hat{g}$ at the vortex center $r = 0$ are approximately obtained as

$$g = \sqrt{\omega_{n}^{2} + |\Delta|^{2}\omega_{n}^{-1}}, \quad f = -\tilde{\Delta}\omega_{n}^{-1}, \quad f^\dagger = \tilde{\Delta}^{*}\omega_{n}^{-1},$$

where $\tilde{\Delta} = |\Delta(r \to \infty)|\exp(i\phi)\exp(\pm i\theta)$ and $(\cos\phi, \sin\phi) \parallel (v_{Fx}'', v_{Fy}')$ in the plane perpendicular to the $z'$ axis, i.e., to the magnetic field. Without loss of generality, we tilt the magnetic field in the $z$-$y$ plane. In the present case, it follows that $\cos\phi = \cos\theta$ and $\sin\phi = \sin\theta\cos\gamma$.

Inserting these matrix elements of $\hat{g}$ into Eq. (5) and taking the Fermi-surface average $\langle \cdots \rangle = \int \cdots \sin\theta/2\pi$ in Eq. (4), we numerically calculate $T_{C}^{-1}(T)$ at the vortex center. We show the result for $\gamma = 85^\circ$ in Fig. 1. Increasing the tilt angle $\gamma$, we observe in our numerical results that $T_{C}^{-1}(T)$ are almost unchanged by $\gamma$ up to $\gamma \sim 50^\circ$. Above $\gamma \sim 50^\circ$ ($\gamma < 90^\circ$), each $T_{C}^{-1}(T)$ in the $\bar{k}_{x} \pm i\bar{k}_{y}$ states gradually deviates from $T_{C}^{-1}(T)$ of $\gamma = 0^\circ$ and the difference in $T_{C}^{-1}(T)$ between the two chiral states gradually becomes small. At $\gamma = 90^\circ$, $T_{C}^{-1}(T)$ of the two chiral states coincide each other. As seen in Fig. 1, however, even at a large tilt angle, the difference in $T_{C}^{-1}(T)$
Fig. 2. $T^{-1}_1$ vs $T$ at the vortex center. The magnetic field is applied in positive direction of the $z$ crystallographic axis. The parameter $\eta = 0.2\Delta_0$ ($\Delta_0$ is defined such that the pair potential is $\Delta(k) = \Delta_0(\sin k_x \pm i \sin k_y)$ at $T = 0$).

between the $\vec{k}_x \pm i \vec{k}_y$ states is still noticeable. At $\gamma = 85^\circ$ (the solid lines in Fig. 2), $T^{-1}_1(T)$ in the $\vec{k}_x + i \vec{k}_y$ state is two times larger at $T \sim 0.4T_c$ than that in the $\vec{k}_x - i \vec{k}_y$ state.

4. EFFECT OF ANISOTROPY

Within the present framework based on the quasiclassical theory, we calculate $T^{-1}_1(T)$ using an anisotropic gap $d = \vec{\Delta}(\sin k_x \pm i \sin k_y)$ and an anisotropic dispersion relation $\varepsilon_k = -2t(\cos k_x + \cos k_y) - 2t'(\cos(k_x + k_y) + \cos(k_x - k_y))$, ($t' = 0.47t$ and the chemical potential $\mu = 1.2t$). In this section, the magnetic field is applied in positive direction of the $z$ crystallographic axis, namely $\gamma = 0^\circ$. This physical situation is the same as that of a calculation of $T^{-1}_1(T)$ by Takigawa et al.12

We show our result for $T^{-1}_1(T)$ in Fig. 2. We find that, even in this case of the anisotropic gap and the anisotropic Ferm surface, the difference in $T^{-1}_1(T)$ between the $\sin k_x \pm i \sin k_y$ states is noticeable and $T^{-1}_1(T)$ in the $\sin k_x - i \sin k_y$ state is suppressed in wide $T$ region in comparison with that in the $\sin k_x + i \sin k_y$ state. This result is in contrast to a corresponding theoretical result for $T^{-1}_1(T)$ by Takigawa et al. In their result, there is not such suppression of $T^{-1}_1(T)$ as seen in our Fig. 2. While the same anisotropy is taken into account in both calculations, their result is different from ours. A reasonable origin of this difference may be as follows. The calculation of $T^{-1}_1(T)$ by Takigawa et al. is in the quantum limit ($k_F \xi \sim 1$) where the energy spectrum inside the vortex core is quantized and it dominantly
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determines $T_1^{-1}(T)$ as pointed out by them while we base our calculation on the quasiclassical theory relevant in the opposite limit $k_F\xi \gg 1$ where the vortex core spectrum is continuous. Now, in the case of the material Sr$_2$RuO$_4$ the coherence length is not so small ($\xi \sim 660 \AA$), namely $k_F\xi \gg 1$, and therefore our result based on the quasiclassical theory is relevant to this material.

5. SUMMARY

Within the framework of the quasiclassical theory of superconductivity, we numerically calculated the site-selective nuclear spin-lattice relaxation rate $T_1^{-1}$ at the vortex center in the chiral $p$-wave superconductors. The case of the tilted magnetic field and the case of the anisotropic gap and Fermi surface were investigated. Our result (Fig. 1) can be experimentally observed as a sign of the chiral pairing state in Sr$_2$RuO$_4$ by applying the magnetic field tilted from the $z$ crystallographic axis by $\sim 85^\circ$.

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