Magnetic Brane of Cubic Quasi-Topological Gravity in the Presence of Maxwell and Born-Infeld Electromagnetic Field

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The main purpose of the present paper is analyzing magnetic brane solutions of cubic quasi-topological gravity in the presence of a linear electromagnetic Maxwell field and a nonlinear electromagnetic Born-Infeld field. We show that, the mentioned magnetic solutions have no curvature singularity and also no horizons, but we observe that there is a conic geometry with a related deficit angle. We obtain the metric function and deficit angle and consider the behavior of them. We show that the attributes of our solution are dependent of cubic quasi-topological coefficient and the GB parameter.

I. INTRODUCTION

The existence of the extra dimensions can improve our imagination over our knowledge. In recent years, several higher-dimensional models have been introduced. Actions of quasi-topological gravity and maxwell electrodynamics have been the subject of important interest in this years. In the low energy limit, string theories give rise to effective models of gravity in higher dimensions and the effective electrodynamics action for the open string ending on D-branes can be written in a Born-Infeld form. This is because of the fact that both of them happen in the low energy limit of string theory [1]. One of the generalization of the Einstein-Hilbert action to higher dimensional space-time, and higher order gravity with second order equation of motion, is the quasi-topological action which consist of cubic and quartic quasi-topological terms of Riemann tensor [2–8]. Because of the topological origin of the quasi-topological terms, the second term of the quasi-topological action (the Gauss-Bonnet term) does not have any dynamical effect in four dimensions. In five dimensions or more, the cubic term just contributes to the equations of motion. In this paper, we are dealing with the type of the space-times generated by static sources which are horizonless and have non-trivial external solutions. These kinds of solutions have been explained by many authors in four dimensions. In Ref. [9, 10], the static solutions of Einstein gravity for the cylindrically symmetric metric in four dimensions were considered. The same static solutions for cosmic string theory were investigated in Ref. [11, 12]. These static solutions have no horizon and have a conical singularity. Hoffmann was the first one that obtained a static solution for Einstein gravity with the nonlinear Born-Infeld electromagnetic field [13]. The extension of magnetic solutions to the higher curvature gravity with the linear and nonlinear electromagnetic field has also been done [14–27].

In this paper, we want to obtain (n + 1)-dimensional magnetic solutions of quasi-topological gravity in the presence of the electromagnetic field in Sec. II. Sec. III will start with presenting the metric for the static horizonless solutions. Using this metric, we calculate the magnetic solution of cubic Quasi-Topological-Maxwell gravity in Subsec. III A and analyze the magnetic brane of cubic Quasi-Topological-Born-Infeld gravity in Subsec. III B. In these two subsections, the behavior of the metric function and the deficit angle is considered and the effect of the quasi-topological coefficient and the Gauss-Bonnet parameter is obtained. Finally, we finish our research with some concluding remarks in Sec. IV.

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II. QUASI-TOPOLOGICAL ACTION

Here, we will present the action of quasi-topological gravity up to third order in \((n+1)\) dimensions in the presence of an electromagnetic field as follows \([3,8]\):

\[
I_G = \frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} \left[-2\Lambda + \mathcal{L}_1 + \lambda \mathcal{L}_2 + \mu \mathcal{X}_3 + L(F)\right].
\]

where \(\Lambda = -n(n-1)/2l^2\) is the cosmological constant, \(\mathcal{L}_1 = R\) is just the Einstein-Hilbert Lagrangian, \(\mathcal{L}_2 = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2\) is the second order Lovelock (Gauss-Bonnet) Lagrangian, \(\mathcal{X}_3\) is the curvature-cubed Lagrangian of the quasi-topological gravity \([3,8]\):

\[
\mathcal{X}_3 = R_{abcd}R^{abcd} + \frac{1}{(2n-1)(n-3)} \left(\frac{3(3n-5)}{8} R_{abcd}R^{abcd} R - 3(n-1)R_{abcd}R^{abc}R^{de} + 3(n+1)R_{abcd}R^{ac}R^{bd}\right) + 6(n-1)R_{ab}R_{bc}R_{cd} - \frac{3(3n-1)}{2} R_{ab}R_{bc}R + \frac{3(n+1)}{8} R^3).
\]

and \(L(F)\) is an arbitrary Lagrangian of the electromagnetic field, where we use the maxwell lagrangian as a linear electromagnetic field in the subsection (III A) and the Born-Infeld lagrangian as a nonlinear electromagnetic field in the subsection (III B). Note that \(\mathcal{X}_3\) is only effective in dimensions greater than four and they become trivial in six and eight dimensions \([3,8]\).

III. STATIC MAGNETIC BRANES

In this section, we calculate the solutions of the cubic quasi-topological gravity in the presence of a linear and nonlinear electromagnetic field. We will work with the following metric \([10,18]\):

\[
ds^2 = -\rho^2/l^2 dt^2 + \frac{d\rho^2}{f(\rho)} + l^2 g(\rho) d\phi^2 + \rho^2 \sum_{i=1}^{n-1} d\theta_i^2
\]

where \(l\) is a scale factor related to the cosmological constant and \(\sum_{i=1}^{n-1} d\theta_i^2\) is the Euclidean metric. Using this metric, we want to obtain the magnetic solutions with no horizon. Therefore, instead of using Schwarzschild metric \([(g_{\rho\rho})^{-1} \propto (g_{tt})\) and \((g_{\phi\phi}) \propto \rho^{-2}\)], we use the metric like \([(g_{\rho\rho})^{-1} \propto (g_{\phi\phi})\) and \((g_{tt}) \propto -\rho^{-2}\)]. In this metric, \(f(\rho)\) and \(g(\rho)\) are arbitrary functions of \(\rho\) (the radial coordinate) and we should find the values of them. Here, \(\phi\) is the angular coordinate and it ranges in \(0 \leq \rho < 2\pi\) and it is dimensionless.

A. The Magnetic Solutions of Quasi-Topological-Maxwell Gravity

By using the metric \([8\)], we can obtain the horizonless solutions that are of our interest. First, we want to obtain the solution of quasi-topological gravity in the presence of the linear maxwell electromagnetic field. The lagrangian of maxwell electromagnetic field is

\[
L(F) = -F^2
\]

where \(F^2 = F_{\mu\nu}F^{\mu\nu}\) is the maxwell invariant, \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\) is the electromagnetic field tensor and \(A_\mu\) is the vector potential. Using the metric \([8\]) and

\[
A_\phi = h(\rho).
\]

for the vector potential, we can obtain the below action per unit volume by integrating by parts as

\[
I_G = \frac{(n-1)}{16\pi l^2} \int dt d\rho \left[\rho^n (1 + \psi + \mu_2 \psi^2 + \mu_3 \psi^3)\right] + \frac{\rho^{n-1} h' \rho^2}{N(\rho)^2 (n-1)}.
\]
where $\psi = l^2 \rho^{-2} f(\rho)$ and $g(\rho) = N(\rho)^2 f(\rho)$. The dimensionless parameters $\mu_2, \mu_3$ are defined as:

$$\mu_2 \equiv \frac{(n-2)(n-3)}{l^2} \lambda, \quad \mu_3 \equiv \frac{(n-2)(n-5)(3n^2 - 9n + 4)}{8(2n-1)l^3} \mu,$$

Varying the action $\mathcal{L}$ with respect to $\psi(\rho)$ yields

$$\left(1 + 2\mu_2 \psi + 3\mu_3 \psi^2\right) \frac{dN(\rho)}{d\rho} = 0,$$

which shows that $N(\rho)$ should be a constant. Variation with respect to $h(\rho)$ and substituting $N(\rho) = 1$ gives

$$(n - 1)h' + \rho h'' = 0,$$

So, we can calculate the vector potential as

$$h(\rho) = -\frac{q}{\rho^{n-2}},$$

where $q$ is related to the charge parameter which is an integration constant. Variation with respect to $N(\rho)$ and substituting $N(\rho) = 1$ gives

$$\mu_3 \psi^3 + \mu_2 \psi^2 + \psi + \kappa = 0,$$

where

$$\kappa = 1 - \frac{m}{\rho^n} + \frac{q^2}{\rho^{2(n-2)}(n-1)}$$

and $m$ is an integration constant which is like the mass of the space-time. The only real solution of Eq. (10) is

$$f(\rho) = \frac{\rho^{(n-2)}}{l^2} \left(\frac{4\lambda^3}{81 \mu^3} + \frac{2\lambda}{9 \mu^2} + \frac{2\kappa}{3 \mu} - \frac{\lambda}{3 \mu}\right).$$

Figures 1 and 2 indicate that there is a curvature singularity at $\rho = \rho_0$. In Fig. 1, when we increase the coefficient value of quasi-topological gravity, $\mu$, the value of $\rho_0$ increases. And it becomes clear that the metric function, $f(\rho)$, is positive for the large value of $\rho \gg \rho_0$. Fig. 2 shows that increasing value of the GB parameter leads to decreasing $\rho_0$.

![FIG. 1: The overlay plot of $f(\rho)$ versus $\rho$ for $\mu = 1$ (solid), $\mu = 1.1$ (dotted) and $\mu = 1.2$ (dashed). Here, $l = 1$, $\lambda = 10$, $q = 0.1$ and $m = 0.01$.](image)
\( f(\rho) \) becomes zero at the radius of horizon. But, we see the space-time will never achieve \( \rho = 0 \). Suppose that \( r_+ \) is the largest real root of \( f(\rho) = 0 \). So, the function \( f(\rho) \) is negative for \( \rho < r_+ \) and positive for \( \rho > r_+ \) and therefore, we have

\[
\kappa(r_+) = 1 - \frac{m}{r_+^n} + \frac{q^2}{r_+^{2(n-2)/(n-1)}} \tag{13}
\]

For \( \rho < r_+ \), \( g_{\rho\rho} \) can not be negative. This leads to the change of signature of the metric from \((n-1)_+\) to \((n-2)_+\). Therefore, it shows that we use an incorrect extension. So, we do the below transforming:

\[
r = \sqrt{\rho^2 - r_0^2} \Rightarrow d\rho^2 = \frac{r^2}{r^2 + r_0^2} dr^2 \tag{14}
\]

By the above transforming, the metric becomes

\[
d\!s^2 = -\frac{(r^2 + r_0^2)}{l^2} dt^2 + \frac{r^2 dr^2}{(r^2 + r_0^2)f(r)} + l^2 g(r) d\phi^2 + \frac{(r^2 + r_0^2)^{n-1}}{l^2} \sum_{i=1}^{n-1} d\theta_i^2 . \tag{15}
\]

where now the functions \( h(r) \), \( \kappa \) and \( f(r) \) are

\[
h(r) = -\frac{q}{(r^2 + r_0^2)^{(2-n)/2}}, \tag{16}
\]

\[
\kappa = 1 - \frac{m}{(r^2 + r_0^2)^{n/2}} + \frac{q^2}{(r^2 + r_0^2)^{(n-2)/(n-1)}} \tag{17}
\]

\[
f(r) = \frac{(r^2 + r_0^2)^{(n-2)/2}}{l^2} \left( \frac{4\lambda^3}{81\mu^3} + \frac{2\kappa}{3\mu} - \frac{\lambda}{3\mu} \right) . \tag{18}
\]

where, \( m \) can take the following value

\[
m = \frac{18\lambda\mu^6 + 4\lambda^3\rho^6 + 54\mu^2\rho^6 + 18\mu^2q^2 + 27\lambda\mu^2\rho^6}{54\rho^2\mu^2} \tag{19}
\]

We call \( m \) as \( m_0 \) when \( r = 0 \). One can calculate the derivative of \( f(r) \) that it becomes as

\[
f'(r) = 0 \tag{20}
\]
The function $f(r)$ given in Eq. (18) in the whole space-time is positive and becomes zero at $r = 0$. And in the range $0 \leq r < \infty$, the Kretschmann scalar does not diverge. Thus, the space-time has a conical singularity at $r = 0$, since

$$\lim_{r \to 0} \left( \frac{1}{r} \sqrt{\frac{g_{\phi\phi}}{g_{rr}}} \right) \neq 1 \quad (21)$$

When the radius $r$ tends to zero, the limit of the ratio “circumference/radius” is not $2\pi$, so there is a conical singularity at $r = 0$. We can remove the conical singularity if we identify the coordinate $\phi$ with the period

$$\text{period}_\phi = 2\pi \lim_{r \to 0} \left( \frac{1}{r} \sqrt{g_{\phi\phi} g_{rr}} \right) = 2\pi (1 - 4\tau) \quad (22)$$

That $\tau$ is

$$\tau = \frac{1}{4} \left( 1 - \frac{2}{lr^2 f''} \right) \quad (23)$$

In above equation $f''_0$ is the value of the second derivative of $f(r)$ at $r = 0$, that we calculated it as

$$f'' = \frac{2}{l^2} \left\{ \frac{2\lambda}{9\mu^2} + \frac{4\lambda^3}{81\mu^4} + \frac{2}{3\mu} \left( 1 - \frac{1}{54r_+^6\mu^2} (18\lambda \mu r_+^6 + 4\lambda^3 r_+^6 + 54\mu^2 r_+^6 + 18\mu^2 q^2 + 27\lambda \mu^2 r_+^6) + \frac{q^2}{r_+^6} \right) \right\} \quad (24)$$

From the above analysis, we can conclude that near the origin, $r = 0$, the metric (15) may be written as

$$ds^2 = \frac{r^2}{l^2} \left( -dt^2 + dX_1^2 + dX_2^2 \right) + \frac{dr^2}{r^2 f} + l^2 r^2 f'' \, d\phi^2 \quad (25)$$

![FIG. 3: The overlay plot of $\delta$ versus $r_0$ for $\mu = 1$ (solid), $\mu = 1.1$ (dotted) and $\mu = 1.2$ (dashed). Here, $l = 1$, $\lambda = 10$ and $q = 0.1$.](image)

We describe a space-time by this metric which is locally flat but has a conical singularity at $r = 0$ with a deficit angle, $\delta = 8\pi \tau$ [24]. By this transformation, we remove the imaginary parts of the metric. Then, we check the effects of different parameters of quasi-topological action on the deficit angle of the space-time. For this purpose, we plot $\delta$ versus the parameter $r_0$. This is shown in Figures [3] and [4] which find that the deficit angle $\delta$ is an increasing function of $r_0$. In addition, deficit angle diagrams (Fig. [3], [4]) show that there is a minimum for $r_0$ in which for $r_0 < r_{0\text{min}}$, the calculated values for deficit angle are not real. In Fig. [3], one can find $r_{0\text{min}}$ increases as the $\mu$ parameter of the quasi-topological action increases, whereas in Fig. [4], for increasing the $\lambda$ parameter of Gauss-Bonnet action, $r_{0\text{min}}$ decreases.
B. The Magnetic Solution of Quasi-Topological-Born-Infeld Gravity

Here we use the metric (3) and the lagrangian of the electromagnetic Born-Infeld field as

\[ L(F) = 4\beta^2 \left( 1 - \sqrt{1 + \frac{F^2}{2\beta^2}} \right). \]  

where \( F = F_{\mu\nu}F^{\mu\nu} \), \( F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \) is the electromagnetic field tensor and \( A_{\mu} \) is the vector potential. One may note that in the limit \( \beta \rightarrow \infty \) reduces to the standard Maxwell form \( L(F) = -F^2 \). By substituting the metric (3) and the value of \( L(F) \) as the lagrangian of the electromagnetic Born-Infeld field in the action (1), we have

\[ I_G = \frac{(n-1)}{16\pi l^2} \int dt \int d\rho \left[ \rho^n(1 + \psi + \tilde{\mu}_2\psi^2 + \tilde{\mu}_3\psi^3) \right]' + \frac{4l^2\beta^2\rho^{(n-1)}(1 - \sqrt{1 - \frac{h'^2}{\beta^2}})}{(n-1)}. \]  

(27)

where \( \psi \) and the dimensionless parameters \( \tilde{\mu}_2 \) and \( \tilde{\mu}_3 \) are defined like the previous section.

Varying the action (27) with respect to \( \psi(\rho) \) yields

\[ (1 + 2\tilde{\mu}_2\psi + 3\tilde{\mu}_3\psi^2) \frac{dN(\rho)}{d\rho} = 0, \]  

(28)

which shows that \( N(\rho) \) should be a constant. We obtain the below equation by Variation of the action (27) with respect to \( h(\rho) \) and using \( N(\rho) = 1 \) as

\[ 3h'(\beta^2 - h'^2) + rh''\beta^2 = 0, \]  

(29)

Now, we can show that the vector potential can be written as

\[ h(\rho) = -\sqrt{\frac{(n-1)}{2n-4}} \frac{q}{\rho^{n-2}} \Gamma(\eta), \]  

(30)

where \( q \) is is related to the charge parameter and it is an integration constant and \( \eta \) is

\[ \eta = \frac{(n-1)(n-2)q^2}{2\beta^2\rho^{2n-2}}. \]  

(31)

In Eq. (30), \( \Gamma \) is the hypergeometric function that we show the form of it here, 

\[ _2F_1 \left( \begin{array}{c} \frac{1}{2} \end{array} \right| \begin{array}{c} n-2 \n-2 \end{array}, \begin{array}{c} 3n-4 \n-2 \end{array}, -z \right) = \Gamma(z). \]  

(32)
The hypergeometric function $\Gamma(\eta) \to 1$ as $\eta \to 0$ ($\beta \to \infty$) and therefore $h(\rho)$ of Eq. (30) reduces to the gauge potential of Maxwell field. Variation with respect to $N(\rho)$ and substituting $N(\rho) = 1$ gives

$$\hat{\mu}_3 \psi^3 + \hat{\mu}_2 \psi^2 + \psi + \kappa = 0,$$

where

$$\kappa = \hat{\mu}_0 - \frac{m}{\rho^n} + \frac{4l^2 \beta^2}{n(n-1)} [1 - \sqrt{1 + \eta - \frac{n}{n-2} F(\eta)}]$$

and $m$ is an integration constant which is related to the mass of the space-time.

$$m = \{(2n-2)(n-1)\lambda \mu \rho^{n-2} AB + (2n-4)\lambda^3 \rho^{n-2} AB + (2n-2)(n-1)^2 \mu^2 \rho^{n-2} AB$$

$$- (2n-2)(n-1)\mu^2 (l^2 q^2) \hat{\beta} + (2n-2)(n-1)\mu^2 (l^2 q^2) \hat{\beta} A$$

$$+ (2n-2)(n-1)\mu^2 (l^2 q^2) \hat{\beta} ((-\frac{q^2}{l^2 \rho^{2n-2} \beta^2 - q^2}) \hat{\beta} A_1 \left[\frac{1}{2}, \frac{n-2}{2}, \frac{3n-4}{2n-2}, A^2 \right]$$

$$+ (n-1)^3 \lambda \mu^2 \rho^{n-2}(AB))/\{(2n-2)(n-1)^2 \mu^2(AB)\}$$

where

$$A = \sqrt{\frac{l^2 \rho^{2n-2} \beta^2}{l^2 \rho^{2n-2} \beta^2 - q^2}}$$

$$B = \left(\frac{q^2}{l^2 \rho^{2n-2} \beta^2 - q^2}\right)^{\frac{1}{2}}$$

After that, we can calculate the $f(\rho)$ function that leads the same function as the $f(\rho)$ in the previous section (Eq. 12), but the value of $\kappa$ is different. We can see the behavior of $f(\rho)$ function versus $\rho$ in Fig. 5 and 6 that there is a curvature singularity at $\rho = \rho_0$. We can find the same results as obtained in the previous section for Quasi-Topological-Maxwell gravity. Here, in the presence of a nonlinear electromagnetic Born-Infeld field, we can see that the effect of $\beta$ is negligible.

![Graph of f(\rho) versus \rho for different values of \mu](image.png)

FIG. 5: The overlay plot of $f(\rho)$ versus $\rho$ for $\mu = 1$ (solid), $\mu = 1.1$ (dotted) and $\mu = 1.2$ (dashed). Here, $l = 1$, $\lambda = 6$, $q = 0.3$, $\beta = 0.01$ and $m = 0.01$.

Again, here we look for curvature singularities. By using the transforming (14) and the metric (15) the functions $\eta$, $h(\rho)$ and $\kappa$ becomes:

$$\eta = \frac{(n-1)(n-2)q^2}{2 \beta^2 (\rho^2 + \gamma_0^2)^{2n-2/2}}.$$
FIG. 6: The overlay plot of $f(\rho)$ versus $\rho$ for $\lambda = 6$ (solid), $\lambda = 9$ (dotted) and $\lambda = 12$ (dashed). Here, $l = 1$, $\mu = 1.1$, $q = 0.1$, $\beta = 0.01$ and $m = 0.01$.

$$h(r) = -\sqrt{\frac{(n-1)}{2n-4}}\frac{q}{(r^2 + r^2_0)^{(n-2)/2}}\Gamma(\eta),$$

(39)

$$\kappa = \hat{\mu}_0 - \frac{m}{(r^2 + r^2_0)^{n/2}} + \frac{4l^2\beta^2}{n(n-1)}[1 - \sqrt{1 + \eta - \frac{\eta}{n-2}F(\eta)}],$$

(40)

and $f(r)$ is the same as Eq. (18) but the value of $\kappa$ is the above value. According the Eq. (20) upto Eq. (23) and calculating the second derivative of $f(r)$, we plot $\delta$ versus the parameter $r_0$ that we show this in Fig. (7) and (8). These plots show that by increasing $r_0$, the value of the deficit angle increases. Here, We can see the same conclusions as obtained in the previous section, too. The deficit angle plots (Fig. (7) and (8)) show that there is a minimum for $r_0$ and we can find that $r_{\text{0min}}$ increases as the $\mu$ parameter increases, whereas for increasing the $\lambda$ parameter, $r_{\text{0min}}$ decreases, but the $\beta$ is negligible.

FIG. 7: The overlay plot of $\delta$ versus $r_0$ for $\mu = 1$ (solid), $\mu = 1.1$ (dotted) and $\mu = 1.2$ (dashed). Here, $l = 1$, $\lambda = 6$, $q = 0.3$ and $\beta = 0.01$.

IV. CONCLUDING REMARKS

In this paper, we constructed magnetic solutions of the cubic quasi-topological gravity in the presence of a linear Maxwell field and a nonlinear Born-Infeld field. These solutions have no horizon and calculations of geometric
quantities showed the solutions do not have curvature singularity. By using a suitable radial transformation, we omitted change of signature and found a conic singularity at $\rho = 0$. Next, we investigated the effects of different parameters on deficit angle and behavior of $f(\rho)$ function. In two sections, we considered the effects of $\lambda$ and $\mu$ parameters on metric function and deficit angle in cubic quasi-topological action in the presence of the linear and nonlinear electromagnetic field. We found that the place of the root of the metric function was a increasing function of the cubic quasi-topological parameter and was a decreasing function of the GB parameter. We obtained that the $\beta$ parameter of Born-Infeld field do not have significant effect on the metric function and deficit angle. In the presence of Maxwell field and Born-Infeld field, we saw that the metric function and deficit angle have the same behavior. Therefore, we found that the parameters that modified the behavior of the metric function and the deficit angle graphs were cubic quasi-topological and the GB parameters.

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