Application of credible value at risk in predicting Indonesia’s stock market return

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Abstract. Risk is a probability of loss. In financial terms, loss can be interpreted as a possibility that an actual return on an investment will be lower than expected returns. Recent studies develop a new type of risk measures called credible value at risk (CrVaR). Credible value at risk is a model obtained by combining credibility theory and one of the most used risk measures, value at risk (VaR). Credibility theory is a model which gives a proper weight for both information and VaR is used to calculate maximum loss with the specific level of certainty and specific time frame. The combination of credibility theory and VaR is required to get a better value at risk estimation based on individual and group experiences. This paper discusses the model of credible value at risk, its parameter estimation, and focuses on the implementation of credible value at risk to predict the future rate of return from Indonesia’s stock market data.

Keywords: Credibility theory, risk, Value at Risk

1. Introduction
According to Bank Indonesia, risk is a possibility of loss because of a certain accident. There are some well-known risk measures and the most used risk measure is value at risk (VaR) because of its simplicity. VaR can be interpreted as the maximum loss occurs with a specific choice of confidence level and time frame. In practice, to predict future risk based on individual information will produce a less accurate prediction. Therefore, we need to consider other similar information, such as group information, to get a better prediction.

To combine both individual and group information, credibility theory is required to give a proper weight for each information so that the better estimation will be obtained. Bühlmann [1] introduced a modern credibility model which contain the combination of assets with several assumptions. With Bühlmann credibility theory, the proper weight (called as credibility risk factor) that is necessary for individual and group risk ranged between 0 and 1. As the credibility risk factor value closer to 1, the more credibility is given to individual risk more than the group risk. By combining Bühlmann credibility theory with VaR, the better estimation will be produced as a result of the joined information not only from an individual but another similar individual as well. This paper will focus on the implementation of credible value at risk to predict the worst loss from Indonesia’s stock market data.
2. Credible value at risk model

Before we discuss further on the implementation of credible value at risk [2] to estimate the worst loss from Indonesia’s stock market data, we will review order of statistics in short, as this is one of the main theories needed for Credible Value at Risk.

As per Hogg et al. [3], let \( X_i, i = 1, 2, ..., n \), denotes a random sample from a distribution of continuous type with pdf \( f(x) \). Suppose that \( Y_1 \) be the smallest of \( X_1, Y_2 \) the next \( X_1 \) in order of magnitude, ..., and \( Y_n \) the largest of \( X_1 \). That is, \( Y_1 < Y_2 < \cdots < Y_n \) represents \( X_1, X_2, ..., X_n \) are arranged in ascending order of magnitude. \( Y_i, i = 1, 2, ..., n \), called the \( i^{th} \) order statistic of random sample \( X_1, X_2, ..., X_n \).

We will briefly introduce the illustration, assumptions used in this model, and the parameters’ estimation used to derive credible value at risk. Suppose that a portfolio \( X \) consists of some assets \( X_{ij}, i = 1, 2, ..., m \). Each \( X_{ij} \) contains observed return from asset \( j \) from \( t^{th} \) period \( (X_j = (X_{1j}, X_{2j}, ..., X_{nj})) \). Besides, there is a parameter \( \theta_j \) that is unknown and expressed the risk parameter which characterized the risk from \( j^{th} \) asset. Now, suppose that \( \xi \) is the choice of quantile for credible value at risk estimation. Quantile is chosen to determine how accurate the outcome will be. Based on [4], the assumptions follow as

1) Given \( \theta_j = \theta_j \), observations \( X_{1j}, ..., X_{nj} \) are independent with the same distribution function,
2) \( \theta_j \) is a random variable with identical and independent uniform distribution,
3) If \( \xi_{pj} \) assumed to be unbiased, \( \xi_{pj} = E(\xi_{pj}|\theta_j) \), where \( \xi_{pj} \) denotes risk measure using Value at Risk with confidence interval \( p \) from sample \( j \) and \( \overline{\xi}_{pj} \) stands for the estimation,
4) \( Var(\overline{\xi}_{pj}|\theta_j) = \frac{1}{n}\sigma^2_{\xi_p}(\theta_j) \) and,
5) Assets’ return data have the same number of observations.

Using Buhlmann credibility model, the so-called credible value at risk model will have the similar form, which is

\[
\xi_{pj}^{cred} = Z_{pj}\overline{\xi}_{pj} + (1 - Z_{pj})\Xi_p
\]

with

\[
Z_{pj} = \frac{\Psi_p}{\sigma^2_{\xi_p} + \Psi_p}
\]

and

\[
\Xi_p = \frac{1}{m}\sum_{j=1}^{m}\overline{\xi}_{pj}
\]

where

- \( \Xi_p = E[\Xi_p(\theta_j)] \) represents the overall portfolio mean,
- \( \sigma^2_{\xi_p} = \frac{1}{n}E[\sigma^2_{\xi_p}(\theta_j)] \) represents the mean variance of the portfolio, and
- \( \Psi_p = Var[\Xi_p(\theta_j)] \) represents the variance of mean of the portfolio.

Proper weight (credibility risk factor \( Z_{pj} \)) for individual information will be obtained by minimizing mean squared error between the estimator and its estimation. The rest of the proof can be seen in [4].

In the model, we could see that there are some parameters that we need to estimate to get the value of it. There is \( \sigma^2_{\xi_p} \), the mean variance of the portfolio for \( p^{th} \) quantile and \( j^{th} \) asset. We need to estimate this parameter using quantile confidence interval estimation [5] and order statistic [6]. Using both methods, we will obtain,
\[
\frac{\bar{\sigma}^2_{\xi_p}(\theta_j)}{4} = \frac{n(x_l - x_k)^2}{n \alpha(1-\alpha) / 2}
\]

where

- \(k = np - z_{1-\alpha/2}\sqrt{np(1-p)}\) and \(l = np + z_{1-\alpha/2}\sqrt{np(1-p)}\),
- \(x_k\) is the value of \(k^{th}\) order statistics,
- \(n\) represents the number of observations,
- \(\alpha\) represents the confidence level, and
- \(z_{1-\alpha/2}\) is the critical value given the confidence level \(\alpha\).

After obtaining the estimation of \(\bar{\sigma}^2_{\xi_p}(\theta_j)\), the other parameters’ estimation will be obtained by method of moment. With preceding assumptions, the estimation for \(\bar{\xi}_p\), \(\bar{\sigma}^2_{\xi_p}\), and \(\Psi_p\) follow as result

\[
\overline{\xi_p} = \frac{1}{m} \sum_{j=1}^{m} \xi_{pj}, \\
\overline{\sigma^2_{\xi_p}} = \frac{1}{nm} \sum_{j=1}^{m} \overline{\sigma^2_{\xi_p}(\theta_j)}, \\
\Psi_p = \frac{1}{m} - \frac{1}{m} \sum_{j=1}^{m} (\xi_{pj}^2 - \overline{\xi_p}^2) - \overline{\sigma^2_{\xi_p}}.
\]

3. Application of credible value at risk for Indonesia’s stock market data

For the implementation of credible value at risk, this paper uses the data from 10 banks that have been registered in Indonesia stock exchange. Furthermore, there are 736 observations from each bank, ranging from 9 November 2014 to 9 November 2017. These data can be downloaded online from Yahoo Finance. Figure 1 shows the rate of return plot based on actual data and table 1 gives the descriptive statistics from every bank.

**Figure 1.** Rate of Return Plot from 10 Assets (a) BCA, (b) BNI, (c) Mandiri, (d) Danamon, (e) BTN, (f) Maybank, (g) CIMB Niaga, (h) Permata, (i) BRI and (j) Sinarmas.
Figure 1 (continued). Rate of Return Plot from 10 Assets (a) BCA, (b) BNI, (c) Mandiri, (d) Danamon, (e) BTN, (f) Maybank, (g) CIMB Niaga, (h) Permata, (i) BRI and (j) Sinarmas.
From figure 1 and table 1, we can calculate the value at risk and variance from every asset and calculate the parameters’ value from credible value at risk model. Using R, we can choose specific quantile (90th, 95th, 99th) and 5 % confidence level for calculating variance of each asset. Therefore, we can obtain table 4. Using the information given in table 4, thus we can obtain the parameters’ value using the formulas in section 2 and the result shown in table 2. As we can see, the credibility risk factor $Z_{pj}$ become smaller as the chosen quantile grow bigger. This happened because the value of $\widetilde{\sigma}_p^2$, mean variance that measures the variability of the portfolio, is increasing.

Up to this point, we have obtained all parameter’s value from credible value at risk model. Hence, to calculate the worst loss estimation with the chosen quantile, we need to substitute those value to the model. The result of Credible Value at Risk calculation for each asset with 99th quantile chosen is given in table 3. Higher chosen quantile provides more accurate result compared to lower quantile.

| Table 1. Descriptive statistics from each asset |
|-----------------------------------------------|
| BCA  | Sinarmas | BRI  | Danamon | Maybank |
| Minimum | -0.05589354 | -0.1449893 | -0.07175926 | -0.1089109 | -0.07303371 |
| Median  | 0 | 0 | 0 | 0 | 0 |
| Mean    | 0.0007687703 | 0.001946466 | 0.0007541 | 0.00082918 | 0.0005362 |
| Maximum | 0.07070707 | 0.25 | 0.078947 | 0.1906158 | 0.3434343 |
| Variance | 0.000164648 | 0.0007838693 | 0.0003332 | 0.000739883 | 0.000793997 |
| Standard deviation | 0.01283152 | 0.02799767 | 0.018253 | 0.0272008 | 0.02817795 |

| Table 2. Credible value at risk parameter’s value |
|-----------------------------------------------|
| $\xi_p$ | $\sigma_p^{-2}$ | $\Phi_p$ | $Z_{pj}$ |
| 90% | -0.021345709 | 0.00000022936 | 0.0000167717 | 0.87969605 |
| 95% | -0.03147612 | 0.0000060277 | 0.0000280229 | 0.822978408 |
| 99% | -0.054780699 | 0.0000646410 | 0.0000755832 | 0.539016673 |
Table 3. Credible value at risk with 99th quantile

|         | BCA       | Sinarmas  | BRI       | Danamon   | Maybank   |
|---------|-----------|-----------|-----------|-----------|-----------|
| VaR     | -0.03606  | -0.0735   | -0.0473   | -0.0631   | -0.0584   |
| \( \xi_p \) | -0.054780699 |          |           |           |           |
| CrVaR   | -0.05073974 | -0.06486398 | -0.05073974 | -0.05926156 | -0.05672744 |

|         | CIMB Niaga | Permata   | BTN       | BNI       | Mandiri   |
|---------|------------|-----------|-----------|-----------|-----------|
| VaR     | -0.059     | -0.0683   | -0.0531   | -0.0424   | -0.0467   |
| \( \xi_p \) | -0.054780699 |          |           |           |           |
| CrVaR   | -0.057062449 | -0.06206928 | -0.05388449 | -0.04808217 | -0.05042635 |

Table 4. Value at risk and variance from each asset

|         | p         | BCA       | Sinarmas  | BRI       | Danamon   | Maybank   |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|
| Value at risk | 90 %     | -0.01247  | -0.019    | -0.021    | -0.0297   | -0.0213   |
|          | 95 %     | -0.02049  | -0.0395   | -0.0295   | -0.0408   | -0.0316   |
|          | 99 %     | -0.03606  | -0.0735   | -0.0473   | -0.0631   | -0.0584   |
| Variance | 90 %     | 0.00043   | 0.00291   | 0.00149   | 0.00194   | 0.00253   |
|          | 95 %     | 0.00181   | 0.01294   | 0.00347   | 0.00296   | 0.00419   |
|          | 99 %     | 0.01218   | 0.04694   | 0.04829   | 0.04887   | 0.02202   |

|         | p         | CIMB      | Permata   | BTN       | BNI       | Mandiri   |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|
| Value at risk | 90 %     | -0.0242   | -0.0239   | -0.0211   | -0.0208   | -0.02     |
|          | 95 %     | -0.0332   | -0.0333   | -0.0287   | -0.0283   | -0.0292   |
|          | 99 %     | -0.059    | -0.0683   | -0.0531   | -0.0424   | -0.0467   |
| Variance | 90 %     | 0.00179   | 0.00168   | 0.00135   | 0.00129   | 0.0014854 |
|          | 95 %     | 0.00308   | 0.01024   | 0.0014    | 0.00246   | 0.0018079 |
|          | 99 %     | 0.1612    | 0.06245   | 0.03781   | 0.03556   | 0.0121838 |

Risk measurement result using credible value at risk can be found in table 4 below. The interpretation of this table is, for example, maximum loss with a 99th quantile based on 736 past observations for BCA is about -5.073974 %, while using the original value at risk calculation, the maximum loss is -3.606 %. This percentage can be interpreted by the investor as the worst loss that will occur in the next one day using credible value at risk calculation -5.073974 % and investors could take this number as a precaution. Therefore, they can prepare more money to overcome the loss occurred.

4. Conclusion
Following the result in previous section, we can see that the credible value at risk estimation will produce a better, more conservative risk measurement and simultaneously not be much differ from the basic value at risk calculation. However, not every asset will have a better risk estimation by using credible value at risk. Out of 10 companies chosen as samples, only 50 % of it produce better risk estimation using credible value at risk rather than the original value at risk method. Theoretically, combination of
individual and group risk would yield a better estimation of risk. Besides, there are still many other aspects should be considered, such as outliers and the size of one firm compared to other. Hence, credible value at risk calculation can be implemented better for the assets that are withdrawn from a random data without some significant outliers.

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