ELLIPTIC FLOW IN A FINAL STATE INTERACTION MODEL

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We propose a final state interaction model to describe the fixed $p_T$ suppression of the yield of particles at all values of $p_T$. We make an extension of the model to the motion in the transverse plane which introduces a dependence of the suppression on the azimuthal angle $\theta_R$.

We obtain values of the elliptic flow $v_2$ close to the experimental ones for all values of $p_T$.

1 Introduction

The RHIC data show azimuthal anisotropy in the production of particles in heavy ion collisions. This asymmetry is currently named elliptic flow $v_2$. Two main interpretations has been applied in order to explain the data. In the framework of the hydrodynamical models, it is considered as a signal of early thermalization (Quark Gluon Plasma formation): particles tend to go in the direction of the strongest pressure gradients, hence preferably in the collision plan. On the other hand, at larger transverse momenta, measurements of azimuthal anisotropy are also relevant to the observation of jet quenching – final state interactions. In the second scenario, due to the asymmetry of the overlap region of the two nuclei, the average amount of matter traversed by a parton depends on its azimuthal direction with respect to the reaction plane, which leads to an azimuthal anisotropy of the emitted jets. Our mechanism consists on a final state interactions approach applied to the whole $p_T$ region. In this approach, we can reproduced the large $p_T$ suppression and the elliptic flow data. In this way, the $v_2$ in our approach becomes medium dependent, while in other approaches the $v_2$ corresponds to the gradient of the medium.

2 The final state interaction model

The interaction of a particle or a parton with the medium is described by the gain and loss differential equations which govern final state interactions:

$$\tau \frac{d\rho_i}{d\tau} = \sum_{k,\ell} \sigma_{k\ell} \rho_k \rho_\ell - \sum_k \sigma_{ik} \rho_i \rho_k ,$$

where $\rho_i \equiv dN^{AA\rightarrow i}(b)/dyd^2s$ are transverse densities and $\sigma_{ij}$ are the final state interaction cross-sections. The first term of Eq. 1 describes the gain in type $i$ particle yield resulting from the interaction of $k$ and $\ell$. The second one corresponds to the loss of type $i$ particles resulting from its interaction with particle $k$. Consider a $\pi^0$ produced at fixed $p_T$ interacting with the hot medium. In the interaction, with cross-section $\sigma$, the $\pi^0$ suffers a decrease in its transverse
momentum with a $p_T$-shift $\delta p_T$. This produces a loss in the $\pi^0$ yield in a given $p_T$ bin. There is also a gain resulting from $\pi^0$'s produced at $p_T + \delta p_T$ and that are shifted to smaller values of $p_T$. The gain and loss differential equation for pions is then

$$\tau \frac{d\rho_{\pi^0}}{dp_T} = -\sigma \rho_{\text{medium}} \rho_{\pi^0}(b, s, y, p_T) + \sigma \rho_{\text{medium}} \rho_{\pi^0}(b, s, y, p_T + \delta p_T). \quad (2)$$

Due to the steep fall-off of the $p_T$ spectrum the loss is larger than the gain, resulting in a net suppression of the $\pi^0$ yield at a given $p_T$. Our equations have to be integrated between initial time $\tau_0$ and freeze-out time $\tau_f$. The solution depends only on the ratio $\tau_f/\tau_0$. We use the inverse proportionality between proper time and densities, $\tau_f/\tau_0 = \rho(b, s, y)/\rho_{pp}(y)$, where $\rho_{pp}(y)$ corresponds to the density per unit rapidity for $pp$ collisions at $\sqrt{s} = 200$ GeV= 2.24 fm$^{-2}$ and $\rho(b, s, y)$ is the density produced in the primary collisions. Our densities can be either hadrons or partons. In fact, at early times, densities are very high and hadrons not yet formed, so our equations describe final state interactions at a partonic level. At later times we have interactions of full fledged hadrons, and, thus, $\sigma$ represents an effective cross-section averaged over the interaction time.

Integrating eq. (2) from $\tau_0$ to $\tau_f$ and taking $\rho(b, y, p_T)_{\pi^0} = dN_{\pi^0}/dbdydp_T$, we obtain the suppression factor $S_{\pi^0}(b, y, p_T)$ of the yield of $\pi^0$'s at given $p_T$ and at each impact parameter, due to its interaction with the dense medium:

$$S_{\pi^0}(y, p_T, b) = \exp \left\{-\sigma \rho(b, y) \left[1 - \frac{N_{\pi^0}(p_T + \delta p_T)}{N_{\pi^0}(p_T)}(b) \right] \frac{\ln \left(\frac{\rho(b, y)}{\rho_{pp}(y)}\right)}{\rho_{pp}(y)} \right\} \quad (3)$$

When $\delta p_T$ tends to $\infty$, the gain term vanishes, and the survival probability has the same expression as in the case of $J/\psi$ suppression without $c\bar{c}$ recombination. If $\delta p_T$ is equal to 0, the loss and gain terms are identical and the survival probability becomes one.

3 Numerical results

In order to perform numerical calculations, we need the $p_T$ distribution of the $\pi^0$'s. We use the following parametrization for the ratio $R_{AA}^0(b, p_T)$ in the absence of final state interactions:

$$R_{AA}^0(b, p_T) = R_{AA}^0(b, p_T = 0) \left(\frac{p_T + p_{0AA}(b)}{p_T + p_{0pp}}\right)^{-n} / \left(\frac{p_{0AA}(b)}{p_{0pp}}\right)^{-n} \quad (4)$$

where $p_0(b) = (n - 3)/2 < p_T >, n = 9.99$ and $< p_T >$ is the experimental value of $< p_T >$ at each $b$. We have also tried different parametrization\[2\] for $R_{AA}^0(b, p_T)$. Our final result depends little on the form of $R_{AA}^0$ taking a $p_T$-shift $\delta p_T$ of the form $p_T^{3/2}/C$. To the value $R_{AA}^0$ we apply the correction due to the suppression factor $S_{\pi^0}$, $R_{AA}(b, p_T) = R_{AA}^0(b, p_T) S_{\pi^0}(b, p_T)$. Our results are shown in Fig. 1. The dashed lines correpsonds to the result obtained using eq. (1) for $R_{AuAu}$ and a $p_T$-shift given by $\delta p_T = p_T^{3/2}/(20 \text{ GeV}^{1/2})$. The continuous lines are obtain with $R_{AA}(b, p_T \geq 5 \text{ GeV}) = 1$ and the $p_T$-shift given by $\delta p_T = p_T^{3/2}/(20 \text{ GeV}^{1/2})$ for $p_T < 2.9$ GeV and $\delta p_T = p_T^{0.8}/(9.5 \text{ GeV}^{1/2})$ for $p_T \geq 2.9$ GeV.
4 Elliptic flow

Our final state interaction model takes into account the longitudinal expansion with no consideration for the motion in the transverse plane. Elliptic flow, on the contrary, results from an asymmetry in the azimuthal angle, so the motion in the transverse plane plays a fundamental role. We propose a simple extension of the model, taking into account the different path length of the particles or partons in the transverse plane for each value of its azimuthal angle $\theta_R$ – measured with respect to the reaction plane. At $y^* \sim 0$, the path length $R_{\theta_R}$, measured from the center of the interaction region (overlap of the colliding nuclei) is given by

$$R_{\theta_R}(b) = R_A \frac{\sin(\theta_R - \alpha)}{\sin \theta_R},$$

where $R_A = 1.05 \, A^{1/3} \, \text{fm}$ is the nuclear radius and $\sin \alpha = b \sin \theta_R / 2R_A$. Our ansatz consists in the following replacement in eq. (3)

$$\rho(b, y) \rightarrow \rho(b, y) \frac{R_{\theta_R}}{<R_{\theta_R}>}.$$  

This is motivated by the fact that the duration of the interaction, as well as the density of the medium traversed by the particles, are expected to be proportional to the path length $R_{\theta_R}$ inside the overlap region of the colliding nuclei. Due to the division by $<R_{\theta_R}>$ in eq. (6), the results of section 3 are unchanged. With the replacement (6), the survival probability becomes $\theta_R$-dependent, and the elliptic flow can be obtained as

$$v_2(p_T, b) = \frac{\int_0^{90^\circ} \! d\theta_R \, \cos 2\theta_R \, S^{\theta_R}_{\pi^0}(p_T, b)}{\int_0^{90^\circ} \! d\theta_R \, S^{\theta_R}_{\pi^0}(p_T, b)},$$

Clearly, when the particle or parton moves along the reaction plane $\theta_R = 0^\circ$ its path length will have its minimal value and the survival probability its maximal one. On the contrary, for $\theta_R = 90^\circ$ the path length will be maximal and the survival probability minimal.

Using the same value $\sigma = 1.3 \, \text{mb}$ of the final state interaction cross-section and the same $p_T$-shift introduced in section 3 in order to describe the experimental values of $R_{AA}(b, p_T)$, we obtain the values of $v_2$ versus $p_T$ and centrality shown in Fig. 2.
Figure 2. Left: $v_2$ vs. $p_T$ at centrality 13 %-26 %. Right: $v_2$ vs. the number of participants for different values of $p_T$: 0.4 (lower), 0.75 (middle) and 1.35 GeV (top). Data are from PHENIX (black), STAR (open).

We find a good agreement with data for the $p_T$ and the centrality dependence. For the mass dependence, in Fig. 3, our $v_2$ of mesons falls below that of baryons for $p_T > 2$ GeV, in agreement with data, while the hydrodynamical model predicts the same mass ordering for $v_2$ at all $p_T$.

Figure 3. Left: Values of $R_{AuAu}(b, \theta_R)$ as a function of the azimuthal angle $\theta_R$ for $p_T \geq 4$ GeV in various centrality bins. Right: $v_2$ vs. $p_T$ of different particles for nb collisions. Data are from PHENIX.

We have proposed a final state interaction model which takes into account the different path length of a particle in the transverse plane for each value of its azimuthal angle. In our approach, the mechanism responsible for the large $p_T$ suppression gives a contribution to $v_2$. Elliptic flows very close to the experimental ones are obtained in the whole $p_T$ region. Although this contribution to $v_2$ results from an asymmetry in the azimuthal angle, it can be qualified as non-flow: the mechanism from which it arises (fixed $p_T$ suppression) is maximal at zero impact parameter and thermalization is not needed. We do not claim that our mechanism gives the only contribution to the elliptic flow. In our opinion, our knowledge of the dynamics of the nuclear interaction is not sufficient to disentangle all the interpretations regarding the $v_2$.

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