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The solution of nonisothermal flow equations in three – dimension reservoir with wells on heterogeneous computing systems

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Abstract. The three-dimensional problem of two-phase nonisothermal fluid flow in the reservoir with the system of wells is considered. Hot water injection into the reservoir is expected at injection wells. To determine the temperature use the law of conservation of energy (first law of thermodynamics). The temperature of the fluid and of the skeleton are considered the same. Fluid viscosities are taken by temperature functions. The solution algorithm based on the decomposition method is proposed. The algorithm are implemented on a new generation of computing systems - heterogeneous supercomputers built on the basis of modern CPUs and graphics accelerators.

1. Introduction
Research of the processes of development of hydrocarbon deposits using mathematical models of multiphase fluid flow in porous media is an important task. Mathematical models of such processes are systems of coupled nonlinear unsteady partial differential equations [1, 2]. In the development of oil and gas reservoirs the fluids contained in them can reach a temperature different from the reservoir temperature. The temperature change in the reservoir can occur due to the action of thermodynamic effects during the movement of fluids in a porous medium [3]: barothermal effect, the effect of phase transformations, injection into the reservoir of various displacers with a temperature different from the initial (cold or hot water, steam), the implementation of various thermochemical oxidation processes. At the same time, the displacement coefficients, phase permeabilities, etc., change, as a result of which the temperature significantly affects both the current filtration characteristics and the final oil recovery. In this paper, the model problem of hot water injection into the reservoir was considered. To determine the temperature used the law of conservation of energy (first law of thermodynamics). The temperature of the fluid and of the skeleton was considered to be the same. The viscosity of the fluid was taken as functions of temperature. The solution algorithm based on the decomposition method is proposed. The algorithm are implemented on a new generation of computing systems - heterogeneous supercomputers built on the basis of modern CPUs and graphics accelerators.
2. Problem statement
We will consider the three-dimensional confined reservoir. The filtration flow is subject to Darcy’s law. It is required to determine the pressure field \( P \), temperature field \( T \), saturation fields \( S_\alpha \), \( \alpha = o, w \) from the system of equations

\[
\text{div}(\mathbf{q}_o + \mathbf{q}_w) = 0 ,
\]

\[
\mathbf{q}_\alpha = K_\alpha \text{grad} p \text{ in } D , \quad \alpha = o, w ,
\]

\[
\frac{\partial}{\partial t} \left( m \sum_\alpha \rho_\alpha S_\alpha U_\alpha + (1-m) \rho_s C_s T \right) + \text{div} \left( \sum_\alpha \rho_\alpha \mathbf{q}_\alpha H_\alpha \right) - \text{div} (\kappa_T \text{grad}(T)) = 0
\]

\[
\text{div}(\mathbf{q}_w) + m \tilde{c} S_w / \partial t = 0 .
\]

\[
S_w + S_0 = 1 ,
\]

Boundary conditions:

\[
p = p_1 \text{ on } \Gamma_1 ,
\]

\[
-(K_o + K_w)c p / \partial n = q_{\ell_n} \text{ on } \Gamma_2 ,
\]

\[
p |_{\partial \Gamma} = P_{\ell} , \quad l = 1,...,N ,
\]

\[
T = T_{w_l} , \quad l = 1,...,M ,
\]

\[
T = T_c \text{ on } \Gamma ,
\]

\[
S_w = S_{w_l} \text{ on } \Gamma_3 , \quad S_w = S_{w_l} , l = 1,...,M .
\]

Initial conditions:

\[
T = T^0 \text{ in } D ,
\]

\[
S_w = S^0_w \text{ in } D ,
\]

where \( D \) is the reservoir(solution domain), \( m \) is the porosity, \( \mathbf{q}_\alpha \) is the velocity vector of the phases, \( K_\alpha \) is the conductiviy of the phases, \( \rho_\alpha \) is the densities of the phases, \( U_\alpha = C_\alpha T \) is the internal energies of the phases, \( C_\alpha \) is the phases heat capacity, \( \rho_s \) is the density of the skeleton, \( C_s \) is the heat capacity of the reservoir, \( H_\alpha = U_\alpha + p / \rho_\alpha \) is the enthalpies of the phases, \( \kappa_T \) is the thermal conductivity of the reservoir, \( N \) is the number of the injector wells, \( M \) is the number of the producer wells. At the initial time, the pressure, temperature and saturations in the reservoir are considered known. On the outer surface of the reservoir the boundary conditions of the 1st(\( \Gamma_1 \)) or 2nd(\( \Gamma_2 \)) kind are set. Usually this is the initial reservoir pressure, the condition of non-flow and saturations of the reservoir with phases on the external surface through which the fluids enter. The solution domain \( D \) is represented by a multi-connected area, the inner surfaces of which are determined by the surfaces of the wells in the intervals of reservoir opening. The problem is solved numerically on the grid, thickening to wells.

3. Solution decomposition method
At each time step, systems of nonlinear equations (1)-(13) for determining the pressure field \( P \), temperature field \( T \) and a system for determining the saturation fields \( S_\alpha \) are jointly solved. To solve
the saturation equations a new method of domain decomposition is proposed based on a combination of elements of explicit and implicit schemes. Decomposition of explicit schemes is not difficult, but the size of the cells requires a small step in time which leads to large computational costs. Decomposition of implicit schemes requires the use of a predictor-corrector procedure. In the proposed method, at each time step, the saturation grid equations for the thickening regions are solved independently using an implicit scheme. The coordination of the solutions obtained with the solution on the coarse grid is achieved through a combination of elements of the explicit and implicit schemes in the determination of the saturation of the cells surrounding the gathering sites, without using the predictor-corrector procedure.

To solve the problem of two-phase fluid filtration in [4] the decomposition method based on a combination of elements of explicit and implicit schemes is proposed. The proposed method consists of the following stages.

a) The total flow rate of the liquid leaving the grid cells surrounding the well zones per unit time is calculated

\[ Q_{oi}^{n+1} = \left( p_i^{n+1} - p_j^{n+1} \right) / R_{i,j}^{n} \]

where \[ R_{i,j} = A_{i,j} / \left( K_{wi} + K_{ao} \right) + A_{j,i} / \left( K_{wj} + K_{ao} \right) \], \[ A_{i,j} \] is the coefficients that take into account the resistance of the fluid flow between \( i \)-th and \( j \)-th cells, \( p_i \) is the pressure in \( i \)-th cell, \( p_j \) is the pressure in \( j \)-th cell. For parallelepipeds \[ A_{i,j} = L_{i,j} / D_{i,j} \], where \( D_{i,j} \) is total area of the boundary surface \( i \)-th and \( j \)-th cells, \( L_{i,j} \) is the distance from the node value of the \( i \)-th cell to the common boundary surface. For curvilinear elements, the coefficients are calculated numerically taking into account the flow movement along the normal to the opening interval. Resistance to fluid flow in flow through the faces of the outer and inner surfaces was taken into account only at the expense of the boundary cells.

b) For the total flow coming out of the grid cells surrounding the well zones, the phase flows are calculated using an explicit scheme.

- \[ Q_{w,j}^{n+1,ex} = (K_w/(K_w + K_o))^{op} Q_{j}^{n+1} \]
- where \( (K_w/(K_w + K_o))^{op} = \left( (K_w/(K_w + K_o))^{jt} \right) \)

\[ p_i \geq p_j \]

\[ p_i < p_j \]

c) For each well area, the saturation is calculated independently using an implicit scheme from the system of equations

\[ m_i V_i (S_{wi}^{n+1} - S_{wi}^{n}) / \Delta t = \sum Q_{w,j}^{n+1} \]

where the sum is taken from the \( j \)-th cells surrounding the \( i \)-th cell, \[ Q_{w,j}^{n+1} = Q_{w,j}^{n+1,ex} \] for \( j \)-th grid cells, from which the flow enters the near well zone. In other cases \[ Q_{w,j}^{n+1} = Q_{w,j}^{n+1,lm} = (K_w/(K_w + K_o))^{ip} Q_{j}^{n+1} \]. Phase flow \[ Q_{w,j}^{n+1,ex} \] are boundary conditions at the solution of system (15).

d) The saturation is calculated for the grid cells surrounding the borehole zones

\[ S_{wi}^{n+1} = S_{wi}^{n} + (\Delta t / m_i V_i) \sum Q_{w,j}^{n+1} \]

where the values \[ Q_{w,j}^{n+1} = Q_{w,j}^{n+1,lm} \] are taken from the solution of the system of equations (15). In other cases \[ Q_{w,j}^{n+1} = Q_{w,j}^{n+1,ex} \].


4. Numerical results

The proposed algorithm was tested in solving a model three-dimensional problem of two-phase filtration of liquids with a different number of vertical producing and injection wells. The reservoir consisted of 10 layers (≈1 km×1 km×0.018 km) with the layers thicknesses \(d_1 = 1\) m, \(d_2 = 1\) m, \(d_3 = 3\) m, \(d_4 = 1\) m, \(d_5 = 1\) m, \(d_6 = 2\) m, \(d_7 = 1\) m, \(d_8 = 2\) m, \(d_9 = 5\) m and absolute permeability's \(k_1 = 10^2\) d, \(k_2 = 10^2\) d, \(k_3 = 25×10^3\) d, \(k_4 = 10^2\) d, \(k_5 = 10^3\) d, \(k_6 = 10^2\) d, \(k_7 = 5×10^2\) d, \(k_8 = 10^2\) d, \(k_9 = 10^3\) d, \(k_{10} = 15×10^3\) d, respectively. On the top of the reservoir was considered impermeable, on the side surfaces and the bottom of the reservoir was set pressure \(p_T = 125\) atm, on the wells \(P_k = 30\) atm, on the side surface of the water saturation \(S_w = 0\), on the sole \(S_w = 1\). Initial saturation \(S_w = 0\). The viscosity of the oil at a given temperature can be estimated by knowing the viscosity at two other temperatures \(\mu_0 = \mu_T \exp[(T_2 - T_1)/(T_2 - T_1)\ln(\mu_2/\mu_1)]\). The water viscosity is given by the Bingham formula \(\mu_w = (0.021482(T - 8.435) + (8078.4 + (T - 8.435)^2)^{0.5})^{-1}\).

Oil density \(\rho_o = 0.882\)g/cm\(^3\), water density \(\rho_w = 1\)g/cm\(^3\). Specific heat capacity \(C_w = 4.182\)kJ/(kg·K), \(C_o = 1.8\)kJ/(kg·K). The specific heat of the skeleton is \(C_z = 0.92\)kJ/(kg·K). Thermal conductivity of the reservoir \(\kappa_T = 1.7W/(m·K)\), initial temperature \(T^0 = 300K\). The relative permeability's were taken as linear functions of saturations. Each autopsy interval was simulated in a circular cylinder with base radius \(r = 0.1\) m and is closed at the top and bottom spherical surfaces of radius \(r = 0.1\) m. Thus, for each point on the surface of the opening intervals of the normal vector uniquely determined.

In table. 1 the time of solving problems with different number of wells by methods without decomposition and with decomposition is given. The results of the solution showed the superiority in time of the speed of the solution of decomposition methods in comparison with the methods without decomposition.

| The number of | Number of grid nodes | Solution time without decomposition (min) | Solution time with decomposition (min) |
|---------------|----------------------|------------------------------------------|----------------------------------------|
| wells \((M+N)\) |                      |                                          |                                        |
| 1             | 11744                | 7                                        | 6                                      |
| 50            | 365524               | 301                                      | 147                                    |
| 100           | 726721               | 722                                      | 523                                    |

The proposed algorithm was tested on a cluster consisting of 4-core computing nodes with Intel Core i7 2600 processors and equipped with NVIDIA GTX 560 Ti graphics accelerators when solving a model three-dimensional problem of two-phase fluid filtration with vertical production and injection wells. Each near-well area contained about 25000 nodes. The total number of nodes for the gathering of 200 wells was \(5 \times 10^6\). Systems of linear equations for the determination of the pressure field were solved by conjugate gradients [5] with polynomial preconditioning. Systems of nonlinear equations for determining the saturation field of the near-well zone were solved by the implicit Newton method [6].

The algorithms are parallelized using MPI processes, OpenMP and CUDA technologies. We used C++ language and Visual Studio 2008 application development environment. When solving the problem, each MPI process is allocated an equal number of thickening portions of the grid. To solve problems in the thickening areas corresponding to a single MPI process, streams are generated using OpenMP technology, which distribute these tasks to the processor cores and graphics devices. In this case, the tasks are distributed dynamically, that is, as they are solved. The problem was solved on two computing nodes with a total number of 8 cores(4 cores per node) and 2 GPU accelerators. The tasks were started in the mode of one MPI process per node, each MPI process corresponds to one GPU device. The following options for running tasks were considered:
1. On all available cores using a single MPI process and without using GPU accelerators.
2. On two cores using two MPI processes and two GPU devices. In this case, each MPI process had one GPU device and one core.
3. On all available cores using MPI processes and GPU devices. In this case, all available cluster resources (2 GPUs and 8 cores) were used.

Table 2-4 results of the decision are given. The comparison was made with the solution of the problem on one core without using GPU devices. In table 2 the acceleration of time of the solution of a problem is shown, at use of kernels of only one computing node.

| Table2. Variant 1 |
|-------------------|
| Number of cores   | Acceleration |
| 1                 | 1            |
| 2                 | 1.7          |
| 3                 | 2.1          |
| 4                 | 2.9          |

The efficiency of the use of cores reaches 72%.

| Table3. Variant 2 |
|-------------------|
| The number of GPU devices | Acceleration |
| 1                     | 17.3         |
| 2                     | 30.1         |

The efficiency of GPU devices is calculated from table 4. and reaches 80%

| Table4. Variant 3 |
|-------------------|
| The number of GPU devices | Acceleration |
| 1                     | 20.2         |
| 2                     | 36.3         |

5. Conclusion
The problem of two-phase nonisothermal fluid filtration on a grid with thickening areas by decomposition method is solved. The decomposition of the grid system of saturation equations is based on the independent solution of the equations on the thickening areas by implicit schemes and on the coordination of these solutions with the solution on the coarse grid using the elements of the explicit and implicit schemes. On the basis of the proposed method of solution, algorithms for solving the three-dimensional problem of two-phase fluid filtration on heterogeneous computing systems are constructed. The high efficiency of using heterogeneous computing systems based on graphic processors is shown.

References
[1] Chen Z, Huan G and Ma Y 2006 Computational methods for multiphase flows in porous media *SIAM* 549
[2] Aziz K and Settary 1986 Petroleum reservoir simulation *Applied science publishers* ltd, London
[3] Basniev K S, Vlasov A M, Kochina I M and Maksimov V M Podzemnaya gidravlika (M.: Nedra) 303

[4] Mazurov P A and Tsepaev A V 2006 Algoritmy dlya rasparallelivaniya resheniya zadach dvukhvaznuy fil'tratsii zhidkosti na setkakh so sgushchayushchimisya uchastkami Vychislitel'nye metody i programmirovaniye 7 (2) 115–123

[5] Larabi A and De Smedt F 1994 Solving three-dimensional hexahedral finite element groundwater models by preconditioned conjugate gradient methods Water Resour. Res 30 (2) 509–521

[6] Samarskiy A A and Gulin A V 1989 Chislennye metody (M.: Nauka) 432