Effect of passenger uncertainty on the inertial properties of a railway coach

Xuejun Gao¹*, Yinghui Li²

1. State Key Laboratory of Geohazard Prevention and Geoenvironment Protection, Chengdu University of Technology, Chengdu 610059, Sichuan Province, China;
2. School of Mechanics and Engineering, Southwest Jiaotong University, Chengdu 610031, Sichuan Province, China.

Abstract: The rotation transformation matrix and translation transformation matrix are derived. They are combined to study the variation of inertial properties of the loaded coach with seating and standing passengers. After that, a CRH2 (China Railway Highspeed) motor coach and Chinese adults in statistical terms are illustrated for precise modelling. It is indicated that CG (Center of Gravity) positions and moments of inertia are all close to linear varying with passenger numbers but at different slopes before and after full-load. It is also found that yaw moment of inertia and pitch moment of inertia are highly correlated. The mass has larger correlation on CG z than CG x and CG y, and larger correlation on roll moment of inertia than yaw and pitch moment of inertia. It may offer some instructions and reference for more realistic simulation of railway vehicle dynamics and measure experiments.

Key words: passenger uncertainty; inertial property; railway coach; transformation matrix; correlation coefficient

Corresponding author: Gao Xue-jun (1979- ), male, Ph.D., professor, +86-28-84078955, gaoxj3000@sina.com

1 Introduction

The advantages of large carrying capacity, high running speed, low energy consumption and low transportation cost of railway transportation make it always be a fascinating topic for railway engineers and researchers. Railway vehicle dynamics is one of the important aspect of studying dynamic performance of vehicles. For this the vehicle models are usually formulated as a multi-body system including many rigid bodies such as car body, bogie frame, axle-box and wheel-set, and still including many elastic components such as primary suspensions and secondary suspensions etc. It is usually assumed that the characteristic parameters of all rigid bodies and suspension components are determined in earlier studies [1, 2]. However, there are some uncertainties [3-5] of many parameters due to manufacturing, installation, use or other reasons. Meanwhile, the values of some parameters may be varied with time and space during the operational period.

Many researchers have noticed these uncertainties and done some works during this decade. Lu et al. [6] established parametric finite element model and used Monte-Carlo (MC) method to study reliability and parameter sensitivities of the plate thickness, service load and material constants of the bogie frame. Shen [7] did sensitivity analysis of suspension parameters of a high-speed Electric Multiple Units (EMU) based on orthogonal experiments, and then presented an improved niche genetic algorithm to optimize the dynamic performance of the EMU. Mazzola et al. [8] adopted three methods, one-at-time method, MC simulation method based on Latin Hypercube Sampling (LHS) and MC method based on Design of Experiments (DOE), to investigate the propagation of suspension uncertainties to the critical speed of the vehicle. Suarez et al. [9] took the mass of rigid bodies and the height of CG as random parameters, and did 216 dynamics simulations to perform sensitivity analysis. It was concluded that the mass and the vertical moment of inertia are most sensitive parameters to dynamic behaviour. Bigoni et al. [10] used a stochastic Cooperrider truck model with two degrees of freedom to investigate the computational performance and convergence of the advanced Uncertainty Quantification (UQ) methods. The generalized polynomial chaos in the stochastic collocation form is highlighted and compared with MC and quasi-Monte-Carlo (QMC) methods. They [11] also applied high-dimensional model representation to global sensitivity analysis (GSA) of the Cooperrider bogie running on curved track with normal distribution of suspension parameters. It was found that the steering suspension components account for most of the variance of the critical speed. Luo et al. [12] used stochastic...
dynamics method to predict dynamics performance and optimize suspension parameters of a high-speed train where the suspension and wheel-rail parameters etc. are considered as random distribution. They found that the optimized suspension parameters have a wide range of line operation adaptability compared with conventional dynamic analysis. Bideleh et al. [13] studied the effects of the bogie suspension components on the wear, safety, and ride comfort. The multiplicative dimensional reduction method is used for GSA of the bogie with symmetric/asymmetric suspensions and straight/curved track. Gao et al. [14] conduct local sensitivity analysis (LSA), GSA and regional sensitivity analysis (RSA) of a railway bogie with independent and normal distribution of left-right suspension components. They found RSA is a good choice for importance evaluation and stability control due to its good efficiency and trusty precision. They [15] also found that different front-back symmetrical suspension components usually have different sensitivity indices to the critical speed of the bogie.

In order to make the vehicle dynamics model more realistic, model parameters should be carefully and accurately determined with good method and effective test equipment. However, even if we don’t talk about parameter uncertainties from manufacturing, installation or experiment. There are still some uncertainties hard to quantify in railway vehicle system, such as passenger uncertainties for passenger train, cargo uncertainties for freight train etc. These uncertainties make the study of vehicle dynamics be more complex, and thus the result is not quite convincing to some extent. Therefore, it should be clear about the effects of these uncertainties on the vehicle performance.

In this paper, effect of passenger uncertainty on the inertial properties of a railway coach is studied. The expressions of inertial tensor using the rotation transformation matrix and translation transformation matrix are derived. A CRH2 motor coach and Chinese adults in statistical terms are illustrated to discuss how passengers, including passenger numbers and the proportion of female, affect CG positions and moments of inertia of the coach. The correlation analysis is also conducted between different inertial parameters to provide comprehensive conclusions. The results provide a kind of methodology for similar problems and a validation method for some measure experiments.

2 Transformation Matrix

2.1 Rotation transformation matrix

One case, suppose that there is an inertial reference coordinate system \((OXYZ)\), a body with coordinate system \((OX'Y'Z')\) and CG position \(O'\) which shows in Fig. 1(a). Two coordinate systems have the same coordinate origins \(O(O')\). The rectangular coordinates are used here and the coordinate system \((OX'Y'Z')\) can be transformed to the \((OXYZ)\) by respectively rotating, \(\theta_1\) angle about \(X'\)-axis, \(\theta_2\) angle about \(Y'\)-axis and \(\theta_3\) angle about \(Z'\)-axis. As a case of rotating about \(Z'\)-axis shows in Fig. 1(b), the transformation matrix is
In the above equation, the denotations of $c_i = \cos(\theta_i)$, $s_i = \sin(\theta_i)$ for $i=1-3$ are used for simplicity. Similarly, the transformation matrix about $X'$-axis and $Y'$-axis are

$$\begin{bmatrix}
1 & 0 & 0 \\
0 & c_1 & -s_1 \\
0 & s_1 & c_1
\end{bmatrix}, \quad
\begin{bmatrix}
c_2 & 0 & s_2 \\
0 & 1 & 0 \\
-s_2 & 0 & c_2
\end{bmatrix}$$

Therefore, the rotation transformation matrix from the body coordinate system ($O'X'Y'Z'$) to the inertial reference system ($OXYZ$) is

$$A_{\text{rot}} = A_1 A_2 A_3 = \begin{bmatrix}
c_2 & c_3 & -c_2 s_3 & s_2 \\
c_1 s_3 + c_3 s_1 & c_2 c_3 - s_2 s_3 & c_3 s_1 & -c_2 s_1 \\
s_1 s_3 - c_1 c_3 s_2 & c_2 s_3 - c_1 s_2 s_3 & c_1 s_1 & c_2 s_1 \\
\end{bmatrix}$$

The transformation matrix $A_{\text{rot}}$ is an identity orthogonal matrix, that is, its transposed matrix is equal to its inverse matrix.

Without loss of generality, it is supposed that the moments of inertia of the body are $J_{xx}$ about $X'$-axis, $J_{yy}$ about $Y'$-axis and $J_{zz}$ about $Z'$-axis, and the products of inertia are $J_{xy}$ about $X'Y'$ plane, $J_{yz}$ about $Y'Z'$ plane and $J_{zx}$ about $Z'X'$ plane. Then the inertial tensor of the body about coordinate system ($O'X'Y'Z'$) can be expressed as a matrix as follows

$$J_c = \begin{bmatrix}
J_{xx} & J_{xy} & J_{xz} \\
J_{xy} & J_{yy} & J_{yz} \\
J_{xz} & J_{yz} & J_{zz}
\end{bmatrix}$$

If the coordinate axis $O'X'$, $O'Y'$ and $O'Z'$ are the symmetrical axis of the body, the non-diagonal elements of above tensor matrix, i.e., the products of inertia are all equal to zero $J_{xy} = J_{yz} = J_{zx} = 0$. The tensor matrix of $J_c$ is simplified as a diagonal matrix.

The inertial tensor matrix of the body about the reference coordinate system ($OXYZ$) can be obtained by the following expression [16] through the rotation transformation matrix $A_{\text{rot}}$

$$J_i = A_{\text{rot}}^T J_c A_{\text{rot}}$$

It can be easily proved that the square matrix $J_i$ is similar with the matrix $J_c$ due to the orthogonality characteristic of the transformation matrix $A_{\text{rot}}$. That is, they have same eigenvalues and characteristic polynomial, and moreover the following expressions are always true

$$\text{rank}(J_i) = \text{rank}(J_c), \quad \det(J_i) = \det(J_c), \quad \text{tr}(J_i) = \text{tr}(J_c)$$

where $\text{rank}(\cdot)$, $\det(\cdot)$ and $\text{tr}(\cdot)$ are operators of rank, determinant value and trace of a matrix.

### 2.2 Translation transformation matrix

Another case, the definitions of two coordinate systems are same as Fig. 1 except that two coordinate systems have different coordinate origins and the corresponding axes are parallel with each other which shows in Fig. 2. Suppose that the mass of the body is $m$ and the values of CG position $O'$ are ($x$, $y$, $z$) in the inertial reference coordinate system ($OXYZ$). Then the parallel-axis theorem can be directly applied to compute the inertial tensor of
the body in the inertial reference system.

\[
J_z = J_c + mA_{\text{tra}}
\]  

(7)

In above equation, the translation transformation matrix \(A_{\text{tra}}\) is

\[
A_{\text{tra}} = \begin{bmatrix}
y^2 + z^2 & -xy & -xz \\
-xy & z^2 + x^2 & -yz \\
-xz & -yz & x^2 + y^2
\end{bmatrix}
\]  

(8)

If three mass centre axes of the body are originally not parallel to those of the inertial coordinate system, the rotation transformation matrix should firstly be used to make them parallel. After that the translation transformation can still be conducted to compute the inertial tensor of the body in the inertial reference system. It is in the form of

\[
J_z = A_{\text{rot}}J_cA_{\text{rot}}^T + mA_{\text{tra}}
\]  

(9)

3 Description of the model

3.1 The railway coach

A CRH2 motor coach of second class [18] is used as the research object to study the effect of passenger uncertainties on the inertial properties of the coach. The schematic diagram is shown in Fig. 3. It is designed and produced by Qingdao Sifang Corporation of China Railway Rolling Stock Corporation (CRRC) through importation and absorption of advanced technologies from Japanese Kawasaki Company. The length of the coach is 24500 millimetre, the width is 3380 millimetre and the height is 3500 millimetre in the outline. The bottom of the coach is 200 millimetre and the floor is 1300 millimetre both above the rail top. The inner width is 3095 millimetre, including 1480 millimetre of three seats, 1015 millimetre of two seats and 600 millimetre of aisle. There are 2605 millimetre and 1675 millimetre length regions for doors, drinking water, luggage, dustbin, etc. at two ends of the coach. The standard seating capacity is 100 seats with 20 rows and five seats in each row, and the seat region is 20220 millimetre in length size and the bottom of the seat is 1625 millimetre above the rail top.

In the analysis, it is assumed that the z-axis is upwards, the y-axis points to the regions of three seats and the x-axis is orthorhombic to the y-z plane and satisfying the right-hand screw rule. The floor plane is choose as x-y plane. Moreover, it is also assumed that the coordinate origin is located in the geometric centre in the x-y plane when the coach is in the condition of curb weight and no passenger. That is to say, half of 24500 millimetre in the length direction and 3095 millimetre in the width is the coordinate origin. The coordinate axes \(x, y, z\) and coordinate origin \(O\) are also illustrated in Fig. 3. In the context of railway vehicle dynamics [19], there are more specific terms about moments of inertia of rigid bodies. The moment of inertia around x-axis is usually called roll moment of inertia, pitch moment of inertia is around y-axis and yaw moment of inertia is around z-axis.
3.2 Inertial properties of Chinese adult

Different peoples in different countries often have different inertial properties. In the meantime, a same person in different ages still has different inertial properties. To a Chinese people, it often includes some big parts such as head and neck, upper torso, lower torso, thigh, calves, foot, upper arm, fore arm and hand in standard [20] which shows in Fig. 4. Each part has its own mass, CG position and moments of inertia. All parts are organically composed a person with general inertial properties. For simplicity, a body coordinate system ($O'X'Y'Z'$) is also established in Fig. 4 where the $O'$ is the CG position, $X'$-axis is in the forward direction, $Y'$-axis is in the left direction and $Z'$-axis is upwards. The $X'$-$Z'$ plane is the symmetrical plane of the body.

Here not a specific people is considered and the statistical inertial properties of Chinese adults are used. According to the sampling survey and statistical analysis, the CG $z$ position from the head point, moments of inertia $J_{xx}$, $J_{yy}$ and $J_{zz}$ for a Chinese adult male can be regressed as a linear equation with two parameters [21] as follows

$$Z = B_0 + B_1X_1 + B_2X_2$$  \hspace{1cm} (10)

where $B_0$, $B_1$ and $B_2$ are regression coefficients. They are different for the computation of CG $z$ position and moments of inertia. $X_1$ and $X_2$ are the mass and height of the adult male. Their units are separately kilogram and millimetre in the equation.
Fig. 4 Parts of Chinese people

The CG z position from the foot bottom is

\[ Z_c = X_2 - Z = -B_0 - B_1 X_1 - (B_2 - 1) X_2 \]  \hspace{1cm} (11)

For Chinese adult females, the above methods still hold effective only with different coefficients. The coefficients of CG z and moments of inertia for Chinese adults are listed in detail in Table 1.

|        | CG z | J_{xx} | J_{yy} | J_{zz} |
|--------|------|--------|--------|--------|
| male   | -32.2975 | -25397472.8 | -27319232.8 | -290702.3 |
| female | -95.1467 | -17309877.5 | -17803583.7 | -241653.3 |

It should be noted that the coefficients for the moments of inertia of \(J_{xx}\) and \(J_{yy}\) are exchanged with each other since the \(X'\)-axis and \(Y'\)-axis are switched with each other compared with the reference [20]. For general Chinese adults, the value of \(J_{xx}\) is the largest, \(J_{yy}\) is the second and \(J_{zz}\) is the lowest among three moments of inertia.

4 Numerical results and discussions

A detailed CRH2 coach is considered for numerical analysis. The CG z position of the coach in curb weight state is 1520 millimetre above the rail top – namely it is 220 millimetre in the \(z\)-axis of inertial reference frame of Fig. 3. The inertial parameters, including CG position, mass, roll moment of inertia \(J_{cx}\), pitch moment of inertia \(J_{cy}\) and yaw moment of inertia \(J_{cz}\) of the empty coach, are all invariable. In Table 2, parameters and their values for the coach and Chinese standing adults are presented.

For Chinese standing adults, their masses and heights are supposed to be independent and normally distributed around their nominal values. The nominal values and the standard deviation [22] are also given in Table 2. In the meanwhile, the rotation angles abound \(Z'\)-axis are also considered and randomly produced from -180 to 180 degree. These data for standing adults can be used to produce the masses and heights of different passengers, and then they are substituted into equation (11) for CG z position computation and equation (10) for moments of inertia computation. For the CG x and CG y position of standing passengers, they are consistent with the CG x and CG y
Table 2 Parameters and their values for the coach and Chinese standing adults

| Parameters                                      | Nominal value       | Standard deviation |
|------------------------------------------------|---------------------|--------------------|
| CG position of the coach/mm                    | (0, 0, 220)         | 0                  |
| Mass of the coach/kg                           | 34.934×10³          | 0                  |
| Roll moment of inertia of the coach $J_{cx}$/kg·m² | 112.1×10³          | 0                  |
| Pitch moment of inertia of the coach $J_{cy}$/kg·m² | 1695.4×10³         | 0                  |
| Yaw moment of inertia of the coach $J_{cz}$/kg·m² | 1599.9×10³         | 0                  |
| Mass of the adult male/kg                      | 66.2                | 4.5                |
| Height of the adult male/mm                    | 1671                | 55.0               |
| Mass of the adult female/kg                    | 57.3                | 4.5                |
| Height of the adult female/mm                  | 1558                | 45.0               |

For Chinese seating adults, the CG z position and moments of inertia are related to those of standing passengers. They can be computed by the inertial properties of each part of the body and then organized as a whole using mass center formula and the parallel-axis theorem. It is a simple but time-consuming work through some simulations of the adults in seating and standing states. To decrease the computation cost, the CG z position in seating state is reduced by the factor of 0.7176 and 0.7087 to the standing passengers separately for male and female passengers through some tests [23]. The moments of inertia are also changed by the factor of 0.5592, 0.6346 and 2.3347 separately about x-axis, y-axis and z-axis of new CG position of the male passenger. For female passenger, the values are 0.6035, 0.6838 and 2.3803. Moreover, the rotation angles around $Z'$-axis are all set to zero. For the CG x and CG y position of seating passengers, it is assumed that they are identical to the CG x and CG y of the seats of the coach.

In the analysis, the rate of the male or female is also researched for a more realistic simulation of the actual situation. According to the regulation [24], the overcrowding rate of twenty percent is permitted for short trips of CRH coach of second class. However, it may exceed the standard at some lines in some cases, for example, Spring Festival peak travel season. Here the maximum overcrowding rate of fifty percent is studied. Fig. 5 shows a case of positions of 150 passengers in the coach with fifty percent of female, including 100 seating passengers and 50 standing passengers. The overall number of male is 75 and the female is 75 which are respectively denoted by blue circles and red circles. The minimum distance of 150 millimetre between standing passengers is set in the computation for real representation of actual circumstances.

4.1 CG position

Firstly, the CG position of the coach with different passengers are studied. The probability of the adult female is set to $p=0.5$.

Fig. 6 shows error bar of CG x of the coach with passenger numbers. The abscissa is the passenger numbers from 10 to 150, and the ordinate is the x coordinate value of CG. For each working condition, the 50000 data points with normal distribution are computed for statistical analysis of their mean values, standard deviation, minimum and maximum values. In the diagram, the mean value is denoted by a small circle, the standard deviation is showed.
by half of the height of a narrow rectangle, and the short horizontal lines located at lower and upper positions are minimum and maximum values in this case. It is seen from the diagram that the mean of x coordinate values are all negative since the passenger zones are slightly to the back of the coach in Fig. 3. The mean value is -8.88 millimetre with 10 passengers to -75.97 millimetre with 100 passengers where the limit value is reached and all passengers are in seat. After that, with increasing numbers of standing passengers, the mean value is slowly down to -70.56 millimetre with 150 passengers. Furthermore, it is also found that the standard deviation, the range of minimum and maximum values at 100 passengers point are the minimum since all passengers are in seat and there is no empty seat. It greatly decreases the deviation. The largest standard deviation and the range are both near the point of 150 passengers where the value are 63.85 and 563.90 millimetre.

Fig. 6 Error bar of CG x of the coach with passenger numbers (p=0.5)

Fig. 7 Error bar of CG y of the coach with passenger numbers. The coordinate representations are the same as Fig. 6. The mean values of the y coordinate are always increasing and positive when the seated passengers are increased from 10 to 100 numbers because the y-axis of the inertial coordinate system points to the direction of
three seats region. After that, the standing passengers begin to increase, the mean values of the y coordinate start to decrease toward the negative direction since the centre of aisle region is partial to the two seats regions which is in the negative y coordinate direction. There is a minimum mean y coordinate in absolute values when all passenger numbers are close to 130. It can also be seen that the standard deviation, the range of minimum and maximum values of the y coordinate are all small when there is no empty seat, i.e. passenger numbers are from 100 to 150 compared with other conditions.

Fig. 8 shows error bar of CG z of the coach with passenger numbers. The ordinate axis with number of 220 millimetre is the CG z position of the empty coach in the inertial reference system. Since the CG z position of the empty coach is lower than the bottom of the seat which is 325 millimetre above the floor, in other words, the CG z position of adult passengers, in spite of male or female and seating or standing, are all higher than that of the empty coach, thus the CG z positions are all increased along. With increasing of passenger numbers, the CG z is linearly increased with different slopes. The slope is bigger when the passenger numbers exceed 100 because the standing passengers make more contribution to the CG z. This is also true for the range of minimum and maximum values of CG z since standing passengers have much bigger variation of CG z than that of seating passengers. Meanwhile, it is found that the CG z is increased 63.26 millimetre when the coach is in full load of 100 passenger numbers compared with empty coach. The values are 81.20 millimetre and 106.23 millimetre if the passenger numbers increased to 120 and 150 separately.

![Fig. 8 Error bar of CG z of the coach with passenger numbers (p=0.5)](image)

4.2 Mass and moments of inertia

Secondly, the mass and moments of inertia of the coach are discussed.

Fig. 9 shows increase rate of mass and moments of inertia of the coach versus passenger numbers. The mean value is only illustrated for each working condition. The symbols $J_x$, $J_y$ and $J_z$ are moments of inertia around x-axis, y-axis and z-axis with new CG position but all coordinate axes are still parallel to original inertial reference coordinate axes. It is seen from the diagram that the increase rate of mass is completely increased by linear way with increasing numbers of passengers. It can be regarded as part of validity and effectiveness of the method and program. In the meantime, the increase rate of moments of inertia $J_x$, $J_y$ and $J_z$ are all linearly increased with different slopes before and after the point of 100 passenger numbers. The slopes of moments of inertia $J_y$ and $J_z$ are smaller before the point but larger after the point, while moment of inertia $J_x$ is reversed.
Fig. 9 Increase rate of mass and moments of inertia of the coach versus passenger numbers (p=0.5)

The slopes of the increase rate of mass and moments of inertia $J_x$, $J_y$ and $J_z$ are listed in Table 3 where the slopes of CG coordinates are also given. The point of 100 passengers is regarded as a break point of the slope concluded from Fig. 6 to Fig. 9.

| Parameter                  | Before 100 passengers | After 100 passengers |
|----------------------------|-----------------------|----------------------|
| CG $x$                     | $-7.4151 \times 10^{-1}$ | $1.0364 \times 10^{-1}$ |
| CG $y$                     | $1.0136 \times 10^{-1}$ | $-3.3913 \times 10^{-1}$ |
| CG $z$                     | $6.2072 \times 10^{-1}$ | $8.5923 \times 10^{-1}$ |
| Increase rate of mass      | $1.7673 \times 10^{-3}$ | $1.7673 \times 10^{-3}$ |
| Increase rate of moment of inertia $J_x$ | $6.5163 \times 10^{-4}$ | $3.2350 \times 10^{-4}$ |
| Increase rate of moment of inertia $J_y$ | $1.2546 \times 10^{-3}$ | $1.8401 \times 10^{-3}$ |
| Increase rate of moment of inertia $J_z$ | $1.3576 \times 10^{-3}$ | $1.9343 \times 10^{-3}$ |

Fig. 10 Increase rate of mass and moments of inertia of the full load coach with probability of the adult female
In above analysis, the probability of the adult female are all set to $p=0.5$. Here the effect of probability is further researched. Fig. 10 shows the increase rate of mass and moments of inertia of the full load coach with probability of the adult female. The linear decreasing tendency is obvious for the four parameters with increasing of the female probability. That is to say, the higher of the proportion of female is, the more of the female numbers are when the total number is constant. It decreases total mass and thus decrease the moments of inertia to some extent.

### 4.3 Correlation analysis

Finally, the correlation analysis is conducted and the probability of the adult female is set to $p=0.5$ for general condition. Pearson’s method [25] is used to compute linear correlation coefficient since the data of the considered parameters is near normal and independent distribution with one peak through data validation. It computes the correlation coefficient $\rho_{X,Y}$ of the vector $X$ and $Y$ by the equation

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y}$$

(12)

where $\text{cov}(\cdot, \cdot)$ denotes the covariance of the two vectors and $\sigma$ is the standard deviation.

Fig. 11 shows correlation coefficient map of the full-load coach. The parameters are the mass, CG $x$, CG $y$, CG $z$ and moments of inertia $J_x$, $J_y$, and $J_z$. Numbers in squares are the corresponding correlation coefficients between horizontal and vertical parameters. It is found that the maximum correlation coefficient for different parameters is 0.9998 between moments of inertia $J_y$ and $J_z$, which means they are highly correlated. The strong correlation exists between the mass and CG $z$ whose value is 0.7303. There is slight correlation between the parameter groups (CG $x$, $J_z$), (CG $x$, $J_y$) and (CG $y$, $J_x$). The rest of parameter groups have negligible correlation since the correlation values are less than 0.1. It should be noted that the correlation relation between two parameters is changed with different passenger numbers.

|               | Mass | CG $x$ | CG $y$ | CG $z$ | $J_x$ | $J_y$ | $J_z$ |
|---------------|------|--------|--------|--------|-------|-------|-------|
| Mass          | 1    | -0.005652 | 0.004414 | 0.7303 | 0.09057 | 0.05083 | 0.05203 |
| CG $x$        | 1    | -0.005143 | -0.00499 | -0.002096 | -0.1651 | -0.165 |
| CG $y$        | 1    | 0.001551 | -0.1022 | -0.0008772 | -0.002989 |
| CG $z$        | 1    | 0.08468 | 0.04158 | 0.04206 |
| $J_x$         | 1    | 0.006055 | 0.02664 |
| $J_y$         | 1    | 0.9998 |
| $J_z$         | 1    |   |

Fig. 11 Correlation coefficient map of the full-load coach ($p=0.5$)

In railway vehicle engineering, the correlation effects of mass on other parameters are desirable to some extent since the mass is relatively convenient to measure. In Fig. 12, the correlation coefficients of CG with the mass at different passenger numbers is given. It can be seen from the diagram that passenger numbers have great effect on the correlation coefficient of CG $z$ and the positive correlation are all along. With increasing numbers of passengers, the correlation coefficient becomes smaller and smaller, particular after 100 passenger numbers where it undergoes a sudden drop from 0.7303 to 0.2050. After that, the value is slowly decreased to 0.1100 with 150 passengers. However, there are both positive and negative correlation coefficients between the mass and CG $y$ or CG $z$. The correlation coefficients are all lower than 0.01 within the considered passenger numbers range.
Fig. 12 Correlation coefficients of CG with the mass at different passenger numbers (p=0.5)

Fig. 13 Correlation coefficients of moments of inertia with the mass at different passenger numbers (p=0.5)

Fig. 13 shows the correlation coefficients of moments of inertia with the mass at different passenger numbers. The correlation effect of the mass on moment of inertia $J_x$ is a bit larger than $J_y$ and $J_z$ whose coefficients are close to the same in the whole range. When the coach is full-loaded with 100 passengers, the maximum and positive correlation coefficients exist as 0.0906, 0.0508 and 0.0520 for $J_x$, $J_y$ and $J_z$ respectively.

5 Conclusions

In the paper, effect of passenger uncertainties, including different passenger numbers and different probabilities of adult female, on the inertial properties such as CG position and moments of inertia of a railway coach is investigated in detail.

The rotation transformation matrix of a body rotating with its CG at any angle is derived, and furthermore the translation transformation matrix of the body parallel moving some distance from CG to the inertial reference system is also given. They are combined to study different passengers, including seating passengers at any position and
standing passengers at any position and facing toward any angle, on the moments of inertia of the loaded coach. After that, a CRH2 motor coach of second class with 100 standard seating capacity is considered as the research object. The outline, distribution of functional regions and size of the coach are briefly illustrated. In the meanwhile, Chinese adults are considered as passengers, and the CG z position and moments of inertia are regressed as linear expressions with their masses and heights according to relevant specification.

It is indicated in the results that inertial parameters are nearly varied in linear way with different slopes before and after 100 passenger numbers. The mean values of CG x are slightly to the back of the coach, while CG z with passengers are higher than the empty coach within the considered passenger numbers range. For the mean values of CG y, they are deviated to three seats region when the passenger numbers are smaller than 130. Furthermore, it is also found that the increase rate of moments of inertia of \( J_y \) and \( J_z \) are larger than \( J_x \) with increasing of passengers numbers. There are different ranges of minimum and maximum values for different inertial parameters with different passenger numbers. The mass and inertia moments are all decreased with increasing proportion of female.

It is also concluded through correlation analysis that moments of inertia of \( J_y \) and \( J_z \) are highly correlated, the mass and CG z are strong correlated, and the correlation coefficients of other parameter groups are lower in full-loaded state of the coach. The correlation coefficients for different parameter groups are varied at different passenger numbers. There is strong correlation between the mass and CG z when the passenger numbers are lower than 100. The correlation coefficients between the mass and other inertial parameters are all lower than 0.1. That is, they have slight or negligible correlation relation.

The conclusions can be directly used to investigate the effect of passenger uncertainties on vehicle system dynamics. It can also be used to study cargo uncertainties, including the mass and the position of the cargo, on vehicle performance of the freight train. Moreover, it provides a kind of idea for similar problems and validation method for some measure experiments.

6 Declaration

Availability of data and materials
The datasets supporting the conclusions of this article are included within the article.

Competing interests
The authors declare no competing financial interests.

Funding
Supported by National Natural Science Foundation of China (Grant Nos. 11972095, 11872319 and 11472064).

Authors’ contributions
The author’s contributions are as follows: Xuejun Gao was in charge of the whole research, and wrote the manuscript; Yinghui Li proposed the research method. Both authors read and approved the final manuscript.

Authors’ Information
Xue-Jun Gao, born in 1979, is currently a professor at Chengdu University of Technology, China. He received his PhD degree from Southwest Jiaotong University, China, in 2010. His research interests include railway vehicle system dynamics, structural stability and reliability. Ying-Hui Li, born in 1964, is currently a professor at Southwest Jiaotong University, China. He received his PhD degree from Chongqing University, China, in 1999. His research interests include dynamics and control, vibration characteristics of viscoelastic materials.

Author Details
1State Key Laboratory of Geohazard Prevention and Geoenvironment Protection, Chengdu University of Technology, Chengdu 610059, China. 2School of Mechanics and Engineering, Southwest Jiaotong University, Chengdu 610031, China.
Acknowledgements
The authors sincerely thanks to Professor Hans True of Technical University of Denmark for his critical discussion during the research.

Consent for publication
Not applicable

Ethics approval and consent to participate
Not applicable

References
[1] Garg V K, Dukkipati R V. Dynamics of Railway Vehicle Systems. New York: Academic Press, 1984.
[2] Zhai W M. Vehicle-Track Coupling Dynamics. 2nd ed., Beijing: China Railway Publishing House, 2002.
[3] Wojtkiewica S F, Eldred M S, Field R V, Urbina A, Red-Horse J R. Uncertainty quantification in large computational engineering models. Proceeding of the 42rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, 2001, AIAA-2001-1455.
[4] Sandu A, Sandu C, Ahmadian M. Modeling multibody systems with uncertainties. Part I: theoretical and computational aspects. Multibody System Dynamics, 2006, 15(4): 369-391.
[5] Batou A, Soize C. Random dynamical response of a multibody system with uncertain rigid bodies. Computational Methods in Stochastic Dynamics, 2013, 26: 1-14.
[6] Lu Y H, Zeng J, Wu P B, Yang F, Guan Q H. Reliability and parametric sensitivity analysis of railway vehicle bogie frame based on Monte-Carlo numerical simulation. High Performance Computing and Applications. Lecture Notes in Computer Science: Springer, Berlin, Heidelberg, 2010, 280-287.
[7] Shen W L. Sensitivity analysis of high-speed EMU parameters and parameter optimization. Southwest Jiaotong University, Master thesis, 2010.
[8] Mazzola L, Bruni S. Effect of suspension parameter uncertainty on the dynamic behaviour of railway vehicles. Applied Mechanics and Materials, 2012, 104: 177-185.
[9] Suarez B, Felez J, Maroto J, Rodriguez P. Sensitivity analysis to assess the influence of the inertial properties of railway vehicle bodies on the vehicle's dynamic behaviour. Vehicle System Dynamics, 2013, 51(2): 251-279.
[10] Bigoni D, Engsig-Karup A P, True H. Modern uncertainty quantification methods in railroad vehicle dynamics. Proceeding of the ASME 2013 Rail Transportation Division Fall Technical Conference (RTDF2013). Altoona, Pennsylvania, USA, October 15-17, 2013, 1-10.
[11] Bigoni D, Engsig-Karup A P, True H. Global sensitivity analysis of railway vehicle dynamics in curved tracks. Proceeding of the ASME 2014 12th Biennial Conference on Engineering Systems Design and Analysis (ESDA2014). Copenhagen, Denmark, June 25-27, 2014, 1-10.
[12] Luo R, Li R, Hu J B, et al. Dynamics analysis of high-speed train with stochastic parameters. Journal of Mechanical Engineering, 2015, 51(24): 90-96.
[13] Bideleh S M M, Berbyuk V. Global sensitivity analysis of bogie dynamics with respect to suspension components. Multibody System Dynamics, 2016, 37: 145-174.
[14] Gao X J, Hans T, Li Y H. Sensitivity analysis of the critical speed in a railway bogie system with uncertain parameters. Vehicle Systme Dynamics, 2019 (https://doi.org/10.1080/00423114.2019.1674345).
[15] Gao X J, Li Y H. Uncertainty analysis of the critical speed of a railway bogie. Journal of Dynamics and Control, 2020, 18(3): 56-61.
[16] Li Y Z, Wang Y G. Coordinate transformation of rotating inertial tensor of robot. Robot, 1992, 14(2): 31-35.
[17] Huang H W. Unified treatment of the theorem of parallel axes for both moments and products of inertia. Journal of Natural Science of HeiLongJiang University, 2003, 20(1): 82-85.

[18] Wang B M. Overall Composition and Bogie of High Speed Trains(2nd. edition). Chengdu: Southwest Jiaotong University Press, 2014.

[19] Wang F T. Vehicle Dynamics[M]. Beijing: China Railway Publishing House, 1981.

[20] PRC National Standard, GB/T 17245-2004 Inertial parameters of adult human body. 2004.

[21] Zheng X A. Progress in Biomechanics of Sports. Beijing: National Defense Industry Press, 1998.

[22] Report on nutrition and chronic diseases of Chinese residents. Chinese Center for Disease Control and Prevention, Beijing, 2015.

[23] Liu J M. Establishment of national standard about inertial parameters of Chinese adults. Doctor thesis, Beijing Sport University, 2004.

[24] Ministry of Railways of the PRC, Regulations of passenger transport by CRH trains (interim). Beijing, 2007.

[25] Best D J, Roberts D E. Algorithm AS 89: the upper tail probabilities of Spearman's Rho. Applied Statistics, 1975, 24: 377-379.