Sliding Mode Control for Optimal Torque Transmission of Dry Dual Clutch Assembly of A Two-Speed Electric Vehicle During Launch

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Abstract. As fork-lever actuator has significant advantages in structure dimensions and manufacturing prices, and is able to respond to drivers’ demands in very quick and precise manner, it becomes more and more popular in conventional and electric vehicles (EVs) equipped with dry dual clutch transmission (DDCT). To investigate optimal dynamic behaviors of EV equipped with DDCT, displacements of little ends of diaphragm springs and change rates of such displacements are defined key state variables, and dynamic model of such kind of actuator is built and integrated into DDCT driveline dynamic model of an EV with two-speed. Then, an affine nonlinear launch dynamic model for the whole DDCT system is proposed. Further, optimal torques of electric motor and dual clutch are derived, based on Pontryagin’s minimum principle, mainly for the purpose of smooth power transmission from electric motor to dual clutch. Finally, a sliding mode control algorithm for the feedback linearization system is derived to improve tracking accuracy at the presence of strong nonlinearities and modelling uncertainties. Through careful observation of numerical simulation results from MATLAB/Simulink platform, it can be concluded that the proposed optimal control algorithm is able to improve launch quality to fine level, and sliding mode algorithm for tracking control acquires strong robustness to modeling uncertainties and nonlinearities.

1. Introduction
Significant merit in environment protection is key reason for rapid development of electric vehicles (EVs) in recent years. So far, most kind of EVs are equipped with multi-speed transmission gearboxes [1-3], mainly for eliminating low driving efficiency conditions at high driving speed in single-speed EV [4]. However, structure complexity and comparatively low power efficiency during transmission make its applications insignificant [5]. At present, proper choice of gear number for fine compromise of power transmission efficiency and manufacturing cost is two [6-8]. Further, to eliminate jerk and to prohibit power interruption during shift, as demonstrated in conventional vehicles [9-10], dry dual-clutch transmission (DDCT) is also an ideal choice for two-speed EV.

In this paper, optimal torque transmission process during launch is investigated. So far, many published researches[11-15] have thoroughly investigated optimal ways of coordinating dynamic behaviors of dual clutches and that of power source. Main research approaches focus on comprehensively optimizing dual clutch friction work and shock intensity. However, as friction work is generally approximated to weighted sum of square of angular velocity difference and that of dual clutch friction torque, such optimization results of friction works lead to large deviations from real optimal values.
Precise tracing to optimal torques of electric motor and dual clutch during launch should also be concerned. So far, dynamics of electro-hydraulic actuators have been thoroughly investigated for control research purposes [16,17]. Fork-lever actuator swayed by ball-screw driving rollers becomes popular in recent years [18,19]. It is compact in structure, and can quickly response to control demand precisely. Unfortunately, dynamic models and key structure parameters published by manufacturers and researchers are comparatively less, and some modelling techniques are complex [5].

In view of above theory deficiency in such technical development, the present research proposes a dynamic modelling technique for the fork-lever actuator, builds relation between displacement of little end of diaphragm spring and control current of motor, and transforms such relation to state space model. Then, state space model of the fork-lever actuator is integrated into the launch dynamic model of the EV’s DDCT system, and affine nonlinear dynamic model of the system during launch is proposed. And then, optimal torques of electric motor and dual clutch are derived, based on Pontryagin’s minimum principle, mainly for smoothly transmitting power from electric motor to dual clutch. Further, at the presence of strong nonlinearities and modelling uncertainties in the dynamic model, a sliding mode control algorithm based on feedback linearization control theory is derived to improve tracking accuracy to a very fine level. Finally, effectiveness of the proposed optimal control algorithm and sliding mode tracking control algorithm is verified according to numerical simulation results from MATLAB/Simulink platform.

2. Affine nonlinear dynamic modelling of a dry-type two-speed dual-clutch transmission for an electric vehicle during launch

The DDCT assembly of two-speed EV is composed of electric motor, dry dual clutch body, gearbox and fork-lever actuators, as shown in figure 1.

![Figure 1. The dry dual clutch system with fork-lever actuators driven by motor-screw-roller assemblies[5]](image)

To simplify computation procedure to a reasonable level, elasticity-damping properties of the DDCT driveline are not taken into consideration in the present research. As a result, only moments of inertia of rotational components should be considered, as mentioned in [21]. Consequently, dynamic model of DDCT launch operation can be derived and presented in equation (1) and figure 2.
Figure 2. The simplified dynamic model of the DDCT driveline of two-speed EV

\[
\begin{align*}
J_e \ddot{\omega}_e &= T_e - (T_{co} + T_{ce}) \\
J_{co} \ddot{\omega}_{co} &= T_{co} - T_{mo} \\
J_{ce} \ddot{\omega}_{ce} &= T_{ce} - T_{me} \\
(J_{oo} + J_{oe} + J_o) \dot{\omega}_o &= T_{mol1} + T_{mel} - T_m \\
(J_{out} + J_M) \dot{\omega}_T &= T_m - T_r 
\end{align*}
\] (1)

The resisting torque \( T_r \) can be calculated as follows:

\[
T_r = r_w \left( F_f + F_a + F_i \right) 
\] (2)

and tire rolling friction resistance \( F_f \), wind resistance \( F_a \) and gradient resistance \( F_i \) can be calculated as follows:

\[
F_f = (M + M_H) \cdot g \cdot f 
\] (3)

\[
F_a = C_D \cdot A \cdot v^2 \frac{1}{21.25} 
\] (4)

\[
F_i = (M + M_H) \cdot g \cdot \sin \alpha 
\] (5)

with \( k = \frac{r_w}{l_o} \), \( f = f_o \left( 1 + \frac{v^2}{19440} \right) \), where \( f \) is rolling resistance coefficient, \( f_o \) is static rolling resistance coefficient, \( g \) is acceleration of gravity, \( v \) is vehicle velocity, \( r_w \) is wheel rolling radius, \( A \) is frontal area of the vehicle, \( C_D \) is air resistance coefficient, \( \alpha \) is road gradient, \( M \) is total vehicle mass without load, \( M_H \) is mass of the load.

Dual clutch friction torque is modulated solely through displacement and its change rate of little end of diaphragm spring. Displacement change rate is in strong correlation with velocity of screw driven by motor of the fork-lever actuator. Naturally, dual clutch friction torque can be directly modulated through motor control current \( I_d \) of the fork-lever actuator. Since displacement of little end of diaphragm spring \( \delta_{pl} \) can be modulated by proper control of motor current, as indicated by Wu [20], integration of fork-lever actuator dynamics into that of DDCT driveline is inevitable to derive tracking control algorithm.
Mathematical relation between displacement of little end of diaphragm spring $\delta_{p1}$ and motor current $I_d$ is shown in equation (6)[20].

$$\ddot{\delta}_{p1} = -\frac{p^2}{4\pi^2\eta_s J_m}\left[ \frac{L\alpha - \delta_{p1} - \delta_k (\delta_{p1})}{L} + \mu_+ + \mu_0 \left( 1 - \frac{L\alpha - \delta_{p1} - \delta_k (\delta_{p1})}{L} \right) \right].$$

$$\left[ F_{p1} (\delta_{p1}) + F_k (\delta_k (\delta_{p1})) \right] \left( \frac{d^2 y_p}{d\delta_{p1}^2} + \frac{dy_p}{d\delta_{p1}} \right) \ddot{\delta}_{p1} + \frac{pp_m N \Phi_m}{2\pi J_m \cdot (dy_p/d\delta_{p1})} I_d$$

Physical meanings of $y_p, \delta_k (\delta_{p1}), F_k [\delta_k (\delta_{p1})], \alpha, F_{p1} (\delta_{p1})$ are defined in figure 3.

![Figure 3](image-url)

(a) The kinematic analysis of roller-fork mechanism
(b) The analysis of forces acting on the couple of rollers
(c) The analysis of forces acting on the fork lever

Figure 3. Kinematic and dynamic analyses of fork-lever actuators driven by motor-screw-roller assemblies[20]

To investigate optimal launch process whilst derive tracking control algorithm based on sliding mode control theory and feedback linearization theory, affine nonlinear dynamics of the controlled system should be proposed. State variables, control inputs and outputs of the dynamic system are defined and presented as following:

$$x_1 = \omega_e, x_2 = \omega_n, x_3 = \begin{cases} \delta_{pio}, & \text{odd gear launch} \\ \delta_{plo}, & \text{even gear launch} \end{cases}, \quad x_4 = \begin{cases} \dot{\delta}_{pio}, & \text{odd gear launch} \\ \dot{\delta}_{plo}, & \text{even gear launch} \end{cases}$$

$$u_1 = T_e, u_2 = \begin{cases} I_{do}, & \text{odd gear launch} \\ I_{de}, & \text{even gear launch} \end{cases}$$

$$y_1 = \omega_e, y_2 = \begin{cases} \delta_{pio} \\ \delta_{plo} \end{cases}$$

To integrate nonlinear dynamics of fork-lever actuator into that of DDCT driveline, equations (7) and (8) are substituted into equations (1) and (6). Then, affine nonlinear dynamic equation of the whole DDCT system is derived and presented as following:
\[ \dot{x} = f(x) + \sum_{i=1}^{2} g_i(x)u_i \]  
(10)

where

\[ x = [x_1, x_2, x_3, x_4]^T \]

\[ f(x) = \left[ \begin{array}{c} -\frac{\mu \cdot F_{p,lo}(x_5)}{J_e} - M_r \cdot x_4 + B_e(x_3) \cdot A_o(x_3) + C_e(x_3) \cdot A_o(x_3) + D_e(x_3) \cdot A_o(x_3) \end{array} \right]^T, \]  
odd gear launch

\[ \left[ \begin{array}{c} -\frac{\mu \cdot F_{p,lo}(x_5)}{J_e} - M_r \cdot x_4 + B_e(x_3) \cdot A_o(x_3) + C_e(x_3) \cdot A_o(x_3) + D_e(x_3) \cdot A_o(x_3) \end{array} \right]^T, \]  
even gear launch

\[ g_1(x) = \left[ \frac{1}{J_e}, 0, 0, 0^T \right] \]

\[ g_2(x) = \left[ \begin{array}{c} 0, 0, 0, \frac{p_0\Phi_N}{\pi J_e A_o(x_3)} \end{array} \right]^T, \]  
odd gear launch

\[ \left[ \begin{array}{c} 0, 0, 0, \frac{p_0\Phi_N}{\pi J_e A_e(x_3)} \end{array} \right]^T, \]  
even gear launch

Analytical expressions of \( A_e(x_3), B_e(x_3), C_e(x_3), D_e(x_3), A_o(x_3), B_o(x_3), C_o(x_3), D_o(x_3) \) are very complex, corresponding details are omitted here.

3. Optimal process control

To seek optimal solution of smooth power transmission during launch whilst derive analytical control law for comprehensive optimization of friction work, shock intensity and electric motor acceleration, a nonlinear objective functional including friction work, shock intensity and angular acceleration of electric motor is established and presented as following:

\[ J = \int_t^T Q_1 \cdot T_{co}(\omega_e - i_o_o_e_o) + k^2Q_2 \cdot \left( \frac{T_{co} - M_r}{J_e} \right)^2 + Q_3 \cdot \left( \frac{J_e - J_o}{J_e} \right)^2 \]  
odd gear launch

\[ \int_t^T Q_1 \cdot T_{ce}(\omega_e - i_o_o_e_o) + k^2Q_2 \cdot \left( \frac{T_{ce} - M_r}{J_e} \right)^2 + Q_3 \cdot \left( \frac{J_e - J_o}{J_e} \right)^2 \]  
even gear launch

According to optimization technique proposed on the basis of Pontryagin’s minimum principle, analytical solutions of electric motor torque and dual clutch friction torque to the optimal launch control are derived and presented as following:

\[ T_e = \left\{ \begin{array}{ll} \frac{Q_1}{2Q_3J_e}T_{co}(t) \left[ e^{2(j(t))} - 1 \right] - \frac{\gamma_1}{2Q_3J_e}, & \text{odd gear launch} \\
\frac{Q_1}{2Q_3J_e}T_{ce}(t) \left[ e^{2(j(t))} - 1 \right] - \frac{\gamma_1}{2Q_3J_e}, & \text{even gear launch} \end{array} \right. \]  
(12)
Substitution of such optimal solutions into dynamic equation (1) yields optimal solutions to angular velocities of electric motor and dual clutch. Corresponding details are omitted here.

4. Sliding mode control for the feedback linearization system

At the presence of strong nonlinearities and modelling uncertainties in affine nonlinear dynamic equation (10), sliding mode control algorithm based on feedback linearization control theory is proposed for precise tracking of optimal angular velocities.

First of all, relative degree of the affine nonlinear dynamic system is calculated, based on differential geometry theory [22]. As it is equal to number of state variables, a fully controllable linear dynamic system topologically equivalent to equation (10) can be derived and presented as following:

\[ \dot{z} = A_z z + B v \]  

where

\[ z = z_1, z_2, z_3^T, v = v_1, v_2^T, A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \]

Then, differential homeomorphic mapping between affine nonlinear dynamic system and fully controllable linear dynamic system is derived as following:

\[ z_4 = h_1(x) = x_1, z_2 = h_2(x) = x_3, z_3 = L_f h_2(x) = x_4, \]

\[ \dot{z}_4 = \dot{x}_2 = \begin{cases} \left[ \frac{-\mu \cdot F_{p3e}(z_2)}{J_e} \left( \mu \cdot F_{p3e}(z_2) - M_r \right), \frac{B_r(z_2)}{A_r(z_2)} + \frac{C_r(z_2)}{A_r(z_2)} z_3 + \frac{D_r(z_2)}{A_r(z_2)} z_3^2 \right]^T, & \text{odd gear launch} \\ \left[ \frac{-\mu \cdot F_{p3e}(z_2)}{J_e} \left( \mu \cdot F_{p3e}(z_2) - M_r \right), \frac{B_r(z_2)}{A_r(z_2)} + \frac{C_r(z_2)}{A_r(z_2)} z_3 + \frac{D_r(z_2)}{A_r(z_2)} z_3^2 \right]^T, & \text{even gear launch} \end{cases} \]

Further, tracking control algorithm for fully controllable linear dynamic system is developed. To enhance tracking control robustness to modelling uncertainties, sliding mode control strategy is proposed here. The sliding mode surfaces are proposed in the following form:

\[ s_1 = e_1, s_2 = c_2 e_2 + e_3 \]  

in which \( c_{21} = 10^3 \). As for reaching laws, exponential forms are proposed and presented as following:

\[ \dot{s}_1 = -c_1 sgn s_1 - k_1 s_1, \dot{s}_2 = -c_2 sgn s_2 - k_2 s_2 \]  

in which \( c_1 = 10^{-2}, k_1 = 5; c_2 = 10^{-2}, k_2 = 2.5 \times 10^3 \). Consequently, sliding mode control inputs for the topologically equivalent linear system can be derived:

\[ v_1 = \dot{z}_{4r} - \dot{c}_1 sgn s_1 - k_1 s_1 \]

\[ v_2 = \dot{z}_{3r} - c_2 e_2 + c_2 sgn s_2 - k_2 s_2 \]

Finally, such derived sliding mode control inputs should be equivalently transformed into those for original affine nonlinear dynamic system, as presented in equation (10). Therefore, following nonlinear feedback control law should be utilized:

\[ u = A^{-1}(x) \left[ v - b(x) \right] \]
where
\[ u = [u_1, u_2]^T, \quad b(x) = \left[ L_1^0 h_1(x), L_2^0 h_2(x) \right]^T, \quad A(x) = \text{diag} \left[ L_{g_1}^0 h_1(x), L_{g_2}^0 L_1^0 h_2(x) \right]. \]

5. Numerical evaluation of the proposed strategy of optimal control and sliding mode control

Effectiveness of proposed strategy of optimal control and sliding mode control is numerically evaluated on the MATLAB/Simulink platform. The launch condition for simulation is set to be a typical 1\textsuperscript{st} gear launch on a 6\degree ramp road. Nominal values of clutch friction coefficients are set to be 0.3. Wear of friction plate wear is set to be 1mm. Other parameters of the fork-lever actuator dynamic system are presented in Table 1.

| Parameter | Value | Unit |
|-----------|-------|------|
| \( p_m \) | 5 | pairs |
| \( \Phi_m \) | 776 | Wb |
| N | 30 | turns |
| \( \mu_{co,\mu_{ce}} \) | 0.3, 0.3 | |
| \( R_{co,R_{ce}} \) | 0.097,0.091 | m |
| z | 2 | |
| L | 0.1 | m |
| \( \alpha_i \) | 0.17 | rad |
| J_m | 0.005 | kg*m\(^2\) |
| \( \eta_s \) | 0.95 | |
| \( \mu_{rb,\mu_{rs}} \) | 0.008,0.01 | |

The simulation results for optimal launch control investigation are presented in figure 4. By surveying numerical simulation results, it can be concluded that gradual increase in \( Q_1/Q_2 \) value yields gradual increase of shock intensity and gradual decrease of friction work, as shown in figure 4 (b) and (c). Also, synchronization time during launch is correspondingly shortened gradually to a rational value, as shown in figure 4 (a). Consequently, proper adjustment of \( Q_1/Q_2 \) value in accordance with actual road condition is crucial to achieve satisfied launch quality at any time.

(a) Angular velocities of electric motor and dual clutch
The simulation results for tracking control investigation are presented in figure 5. After careful observation of simulation results, it can be concluded that optimal angular velocities of electric motor and dual clutch, as well as displacement of little end of diaphragm spring, are precisely traced through sliding mode control for the sliding mode system, as shown in figure 5 (a) (b). Consequently, precise regulation of motor current of actuator and electric motor torque, as shown in figure 5 (c) (d), makes it able to ensure all angular velocities tracing optimal trajectories at the presence of modelling uncertainties and strong nonlinearities.

(a) Angular velocities of electric motor and dual clutch

(b) Friction work

(c) Shock intensity

Figure 4. Optimal control quality during launch
6. Concluding remarks

An affine nonlinear launch dynamic model for the whole DDCT system is proposed to investigate optimal launch dynamic behaviours, tracking control accuracy for angular velocities of electric motor and dual clutch during launch. Then, optimal control torques of electric motor and dual clutch are derived in analytical form, based on Pontrygain’s minimum principle. Further, to precisely trace derived optimal angular torques and velocities of DDCT, sliding mode control algorithm based on...
feedback linearization control theory is proposed, and accurate tracking control inputs for original affine nonlinear dynamic system are derived. Finally, effectiveness of proposed optimal tracking control algorithm is testified through numerical simulations on MATLAB/Simulink platform. The simulation results verify that the proposed optimal control algorithm is able to improve launch quality to fine level, and excellent tracking to optimal angular velocities of electric motor and dual clutch are realized at the presence of modelling uncertainties and strong nonlinearities.

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