Mathematical reasoning ability in relations and function using the problem solving approach

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Abstract. Mathematical reasoning ability is an important ability in mathematics learning because by using reasoning it can interpret mathematics. This study aims to examine the achievement and improvement of junior high school students' mathematical reasoning abilities. The method used in this study was quasi experimental with a pretest and posttest design. The independent variable of research uses a problem solving approach, while the dependent variable is mathematical reasoning ability. The sample used by the Junior High School students, SMPN 2 Cilebar in Karawang Regency was taken using a purposive technique, namely the experimental group and the control group. The research instrument was in the form of a five-question test of mathematical reasoning skills. Data processing using Microsoft Excel and Minitab software 16. The first result is the achievement of junior high school students' mathematical reasoning abilities whose learning with problem solving approach is better than students who get conventional learning. The second result is an increase in mathematical reasoning abilities of junior high school students who use a problem solving approach better than students who use conventional learning. Increased mathematical reasoning ability is classified as moderate.

1. Introduction
Mathematics as a lesson, must be given to all students to supply students with the ability to think logically, analytically, systematically, critically, and creatively, and the ability to work together [1]. According to May Lwin, logical-mathematical intelligence is the ability to handle numbers and calculations, patterns and logical and scientific thinking [2]. Mathematical learning is closely related to reasoning. Mathematical material and mathematical reasoning are two things that cannot be separated because mathematical material can be understood by reasoning while reasoning is understood by learning mathematics [1, 3]. Mathematical reasoning includes logical abilities, and systematic thinking [4]. Mathematics does not only memorize formulas and apply them but must be able to think logically [5, 6]. Reasoning ability is needed when solving problems faced so that it will affect logically decision making.

Reasoning is the main and continuous tool used when trying to understand mathematics or to solve problems in mathematics [6]. If reasoning abilities are not developed in students then for students, mathematics will only be material that follows a series of procedures and mimics examples without knowing the true meaning [7]. Students can only count numbers without being able to apply them to everyday life either changing the problems that exist around the environment into mathematical models or vice versa. This shows that the ability of students still at the level of understanding cannot interpret it. This is in accordance with the results of TIMSS and PISA assessment that students' mathematical reasoning abilities in Indonesia are still in the low category [8,
9]. Therefore students must be accustomed to reasoning so that they can integrate information obtained from other things.

Through mathematical reasoning, students can propose allegations and then compile evidence and manipulate mathematical problems and draw conclusions correctly and precisely [10]. Sumarmo defines reasoning as the process of obtaining logical conclusions based on relevant data and sources [11]. Mathematical reasoning is divided into two types, namely inductive reasoning and deductive reasoning [12]. Inductive reasoning is general or specific conclusions based on observed data. The value of truth in inductive reasoning can be true or false. Whereas deductive reasoning is a statement that draws conclusions based on general matters, which have been proven beforehand.

Haylock & Thangata states that problem solving is a situation where students use mathematical knowledge and reasoning to solve problems [13]. This shows that problem solving is also important to improve mathematical reasoning skills because in solving mathematical problems students really need knowledge and mathematical reasoning skills so they can solve these problems. Jacobsen, Eggen, and Kauchak define problem solving approaches as an approach that requires teachers to guide students in learning to solve problems through hands-on learning experiences [14]. The point is that students interact directly with the problems given by the teacher. When interacting with the problem, students are expected to be able to find mathematical concepts. Because basically the main purpose of the problem solving approach is to introduce and discover the mathematical concepts of the problems given.

The problem solving approach allows students to get their own understanding by solving a problem so students can learn from their own experiences. This is in line with the results of Mahmood's research. Which stated that the Problem Solving approach can make students able to solve everyday life problems [15]. Kirkley argues that there are five stages in problem solving, namely: 1) Identifying problems; 2) Defining the problem by collecting relevant information; 3) Explore alternative solutions, express opinions, and check back from a different perspective; 4) Carry out a settlement strategy; 5) Check and re-evaluate [6]. This is in line with the problem solving strategy according to Polya's heuristics [7]. The steps in learning with a problem solving approach include problem orientation, problem solving, class presentation, and completion of problem training.

![Figure 1. Model of Process Problem Solving Approach](image)

Based on Figure 1, students will first identify the problem by giving questions such as, "What is happening to the problem?" Or "What should be done?". That question will lead students to think more deeply so students will ask, "why did this happen?" [18]. This is part of the search for a solution to the problem. Estimates of answers obtained by students are analyzed again by sorting the priority of the problem and finding the right solution to the problem. The solutions that have been obtained are implemented to solve the overall problem.

2. Methods
The method used in this study is quasi experimental research. The treatment given in this study is a problem solving approach, while the variable that is measured is mathematical reasoning ability. The study population was students at SMPN 2 Cilebar in Karawang Regency. Samples were selected from
several classes using purposive techniques to obtain two sample groups. The first group is the experimental group that is given treatment using a problem solving approach. The second group is the control group given conventional learning. Each group numbered around 35-40 students.

This study used a pretest and posttest research design [19]. The tests given to the two groups will be the same. The test was conducted twice, namely at the beginning (pretest) and the end (posttest) of the study. Both groups will be given the same test questions, namely five questions about mathematical reasoning. This test aims to find out the mathematics learning outcomes and student learning completeness towards the material that has been taught that is related to reasoning abilities.

The data processing used is testing the hypothesis regarding the difference in the two average pretest scores then posttest and the last normalized gain (N-Gain) between the experimental group and the control group. The amount of competence improvement that occurred before and after learning was calculated by the N-Gain formula from Hake, namely [20, 21]:

\[
g = \frac{\text{posttest score} - \text{pretest score}}{\text{maximal score} - \text{pretest score}}
\]

Criteria for assessing N-Gain scores are explained in the following table [21]:

| Score    | Category |
|----------|----------|
| \(g > 7\) | High     |
| \(0.3 < g \leq 0.7\) | Moderate |
| \(g \leq 0.3\) | Low      |

3. Result
The study was conducted for 10 meetings in semester 1 of class VIII with the subject of Relations and Functions the data that had been obtained was analyzed in such a way that the results of research on reasoning abilities were as follows.

| Measured Aspects                  | Problem Solving(N = 25) | Conventional(N = 26) |
|-----------------------------------|--------------------------|-----------------------|
|                                   | Pretest | Posttest | Gain | Pretest | Posttest | Gain     |
| Mathematical Reasoning Ability   | \(\bar{x}\) | 7,10     | 14,64 | 0,45    | 6,92     | 13,06    | 0,35     |
| (SMI = 24)                        |         |          |       |         |          |          |          |
| s                                 | 1,65    | 2,89     | 0,13  | 2,28    | 2,80     | 0,14     |

In Table 2, the average pretest of mathematical reasoning ability with problem solving learning is higher than conventional learning (7,10 > 6,92). Both groups experienced an average increase in posttest results. The problem solving learning group is 14,64 while the conventional learning group also increases to 13,06. But the average group still uses greater problem solving learning. Based on the results of the calculation of the standard deviation of the values of the two groups, it is seen that the standard deviation of the pretest value of the problem solving learning group is lower than the conventional learning group. This shows that the initial mathematical reasoning ability in problem solving learning groups is more diffuse. While the standard deviation in the posttest of the two groups is almost the same, which is around 2,80. This shows that the spread of the value of mathematical reasoning abilities in the two groups is almost the same.

The gain data analysis is done to find out the improvement of students' mathematical reasoning ability significantly.
Table 3. Test Results for Mathematical Reasoning Ability Tests

| Learning          | N  | \( \bar{x} \) | P-Value | Interpretation                  | Mann-Whitney test |
|-------------------|----|---------------|---------|---------------------------------|------------------|
| Problem Solving   | 25 | 0.45          | <0.010  | Not normal distribution         | 0.0131            |
| Conventional      | 26 | 0.35          | 0.127   | Normal distribution             | \( H_0 \) rejected |

Table 3 shows that the results of the test for normality from the value of problem solving learning gain are \( p < 0.010 \) and \( p \)-value <0.05 so that the data is not normally distributed. Whereas for the conventional learning gain value \( p = 0.127 \) so that the \( p \)-value > 0.05, the data for conventional learning groups are normally distributed.

Based on the Mann-Whitney test results for the gain value mathematical reasoning ability was obtained that the value of \( p = 0.0131 \) which means that the \( p \)-value <0.05 so that \( H_0 \) is rejected. So it can be concluded that the improvement of students’ mathematical reasoning ability using problem solving learning is better than conventional learning. The gain value is included in the low category because of 0.1 [21, 22].

4. Discussion

Research begins by dividing the population into two groups with learning groups using problem solving approaches and the other with conventional. Students who usually only receive material provided by teachers must adapt to problem solving approaches. Because in problem solving learning students are required to find their own problems and solve them.

At the beginning of learning using a problem solving approach, students are not familiar, so learning is still not optimal. At the next meeting, the teacher tries to motivate students to be able to find problems and find solutions to the problems given. The teacher gives an illustration and students are asked to identify any problems contained in the illustration. After the problem is found, start looking for the right solution to solve the problem. In conducting this activity, students are divided into small groups to discuss.

In problem solving learning, students will rely on intuition and capability to solve the problem given [18]. At the time of learning students are divided into several groups of 4-5 heterogeneous students. In groups students will be easier to solve problems given because students will interact with their group members. When identifying problems, students register what problems are faced and estimate the solution. The solutions obtained will be implemented in the problem and the most appropriate solution chosen. The steps of solving the problem solving approach train the thinking power of students to reason and think more critically and creatively.
Figure 2 shows that the posttest results state that students' mathematical reasoning abilities using problem solving learning are higher than conventional ones. This is also reinforced by the N-gain results (see Table 2). The problem solving approach can train students' mathematical reasoning skills because when discussions students look for problems themselves pay attention to these problems then find the most appropriate solution to solve them. From several choices of solutions obtained students must choose the best solution so that the problem can be resolved properly. The process will shape the character of students who are not unyielding in finding solutions to problems. With the many input opinions, students can make the best decisions / solutions to take. Students' ability to make decisions becomes increasingly sharp and deep. Increasingly honed the students' mathematical reasoning abilities, the achievement of other mathematical abilities will also develop [23].

5. Conclusion
Based on the explanation of the analysis of the results of the study, it was said that the mathematical panalanaran abilities of the groups learning using problem solving approaches were better than those in conventional learning. This can happen because it is caused by the use of a problem solving approach that emphasizes the search for problems and how to solve them. This is in line with the results of Perveen's research which states that problem solving approaches can improve student learning outcomes especially in middle school students [24]. So it can be concluded that the problem solving approach has a good effect on improving students' mathematical reasoning abilities

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