Spatial vector autoregressive model with calendar variation and its application

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Abstract. The Vector Autoregressive (VAR) model can be used to determine the relationships among several interacting variables in time series data. While the Spatial VAR (SpVAR) model was developed to accommodate temporal dynamics and spatial dynamics simultaneously. Time series data in economics are often influenced by events such as holidays occurring based upon the lunar calendar. Hence, it happens on different dates and months each year. Such holidays are called calendar variations. The purpose of this paper is to develop a SpVAR model with the effects of calendar variations, discuss the parameter estimation method and apply the model to Inflation and Money Supply data in three cities in West Java, Indonesia. Parameters are estimated by using Full Information Maximum Likelihood. The result for the application is there is a relationship between Money Supply in Cirebon and Inflation in Bandung and Tasikmalaya. Also, there are effects of calendar variation on Inflation and Money Supply in all three cities.

1. Introduction
In studying an economic phenomenon, we are often interested in exploring the interrelationships between variables. For example is how the relationship between Inflation, Money Supply, and Interest Rates [1]. The other examples are the studies about relationship between Inflation, Interest Rate, Stock Index, and The Dollar Exchange Rate and the linkage of six stock indexes in the world [2][3]. For time-series data, the model used for this purpose is Vector Autoregressive. Vector Autoregressive of order p (VAR (p)) was developed and used to determine the interrelationships of several interrelated variables in time-series data [4][5]. Parameter estimation of VAR (p) model can be done by using the Ordinary Least Square (OLS) or Maximum Likelihood Estimator (MLE) method [6]. It can also be estimated by the Bayesian approach [7][8][9][10][11].

Some economic variables also indicate the existence of a space-time relationship. For example, inflation in seven cities in East Java Indonesia has a space-time relationship [12]. Models for Spatio-temporal analysis include Vector Autoregressive, Space-Time Autoregressive (STAR), and Generalized Space-Time Autoregressive (GSTAR) [13][14][15][16]. GSTAR has been developed by several researchers[17][18][19][20][21].
Some variables are even linked to other variables and have a space-time relationship. Case study research like this was done in manufacturing and in macroeconomics using Spatial Vector Autoregressive (SpVAR) model [22][23]. SpVAR is a model that combines temporal dynamic and spatial dynamics. In this model, the value of the \( k \)-th variable \((k=1,2,...,K) \) on the \( n \)-th location \((n=1,2,...,N) \) and at time \( t=1,2,...,T \) depends on the temporal lag, spatial lag at time \( t \), spatial lag at time \( t-\tau \) \((\tau>0) \) of the \( k \)-th variable and other variables. SpVAR is classified into two, namely SpVAR with spatial autocorrelation and without spatial autocorrelation [24]. Giacinto [25] also proposed SpVAR models with some differences with those offered by Beenstock and Felsenstein [24]. In SpVAR models, coefficients at all locations are considered equal, and parameter estimation is performed by forming a reduced form [24]. Then the reduced form is obtained by estimating each endogenous variable as a separate block. The estimation method in each block is SUR. While in another model, the coefficient at each location can be different, and parameter estimation is done by FIML (Full Information Maximum Likelihood) [25]. Also, the coefficient in this model is also restricted.

Timeseries data is often influenced by events such as holidays, government policies, natural disasters, or other events. Some holidays occur every year but at different times (dates and months). Such holidays usually occur because of the feast of a particular religion that uses the lunar calendar and affects the time-series data, especially in the economic field, because the consumption of some goods at that time increased. Some research regarding the effect of calendar variation on univariate time-series has been carried out [26][27][28][29]. Calendar Variation models are very useful. There are some usability of calendar variation models; 1) to predict sales of Muslim boy’s clothes that are affected by the effects of Ramadan, 2) to estimate the currency Netflow influenced by the Eid Al Fitr holiday, 3) to estimate the money inflow and outflow in Surabaya, Malang, Kediri, and Jember; 4) to forecast the outflow and inflow of currency at the Indonesian Central Bank or Bank Indonesia (BI) in the Sulawesi Region, 5) to forecast the money inflow and outflow in Central Java, and 6) to forecast the sales of Apple Washington and Orange Kintamani that is influenced by the Hindu holiday in Bali [30][31][32][33][34][35]. Effect of Calendar Variation on multivariate time series was discussed in the Generalized Space-Time Autoregressive Model with Exogenous Variable (GSTARX) and using Generalized Least Square (GLS) as the estimation method, so the model is called GSTARX – GLS [19].

However, no model can accommodate the three aspects needed in economic modeling, as previously explained, namely, the relationship between variables, space-time relationships, and calendar variation effects at once. Therefore, the purpose of this research is to form a model that accommodates the existence of the relationship between variables, space-time relationships, and the impact of calendar variations. The model we propose is the Spatial Vector Autoregressive Model with Calendar Variations. This paper also discusses methods for estimating the parameters and application of this model.

**2. Methods**

In this section, the Vector Autoregressive model and Spatial Vector Autoregressive model are discussed. The Spatial Vector Autoregressive model is an extension of the Vector Autoregressive (VAR) Model. The VAR model is a statistical model used to analyze the relationship between several variables that affect each other. Vector Autoregressive model of order \( p \) with \( K \) variables is expressed in equation (1).

\[
y_t = A_{1y}y_{t-1} + \ldots + A_{Ky}y_{t-p} + u_t
\]

Where \( y_t = (y_{1t},...,y_{Kt})' \), \( A_i \) is \( K \) by \( K \) is the coefficient matrix of order \( i (i=1,2,...,p) \) and \( u_t \) is white noise with nonsingular covariance matrix \( \Sigma_u \) [36].

Suppose there are two variables, then model in equation (1) can be described as

\[
\begin{bmatrix}
y_{1t} \\
y_{2t}
\end{bmatrix} = \begin{bmatrix}
a_{11}^{(1)} & a_{12}^{(1)} \\
a_{21}^{(1)} & a_{22}^{(1)}
\end{bmatrix} \begin{bmatrix}
y_{1t-1} \\
y_{2t-1}
\end{bmatrix} + \ldots + \begin{bmatrix}
a_{11}^{(p)} & a_{12}^{(p)} \\
a_{21}^{(p)} & a_{22}^{(p)}
\end{bmatrix} \begin{bmatrix}
y_{1t-p} \\
y_{2t-p}
\end{bmatrix} + \begin{bmatrix}
u_{1t} \\
u_{2t}
\end{bmatrix}
\]

Or in another form, can be written in equation system (2).
\[ y_t = a_{11}^{(i)} y_{t-1} + a_{12}^{(i)} y_{2t-1} + \ldots + a_{1l}^{(i)} y_{lt-p} + a_{21}^{(i)} y_{2t-p} + u_t, \]
\[ y_{2t} = a_{21}^{(i)} y_{t-1} + a_{22}^{(i)} y_{2t-1} + \ldots + a_{2l}^{(i)} y_{lt-p} + a_{22}^{(i)} y_{2t-p} + u_{2t}. \]

(2)

The equation system **Error! Reference source not found.** states that the first variable (e.g., inflation) is a linear function of the lag of the first variable and the lag of the second variable (e.g., money supply). Besides, the second variable is also a linear function of the lag of the first and second variables.

While, the Spatial Vector Autoregressive model with spatial order 1 and temporal order \( p \) SpVAR [(1, \( p \))] is expressed by equation (3) [25].

\[ y_t = B_y y_{t-1} + \ldots + B_p y_{t-p} + \eta_t, \]

(2)

Where \( y_t = \{ y_{1t}, y_{2t}, \ldots, y_{N1t}, y_{12t}, y_{22t}, \ldots, y_{N2t}, \ldots, y_{Kt}, y_{2Kt}, \ldots, y_{NKt} \} \), Where \( N \) is the number of location and \( K \) is the number of variables. \( y_{nkr} \) is the value of the \( k \)-th variable observed at \( n \)-th location at time \( t \). This is the difference between VAR model and SpVAR model. In the VAR model, the \( k \)-th variable at time \( t \) is only observed in one location so that \( y \) only has two indexes, namely \( k \) and \( t \), which indicate variable and time. Whereas in the SpVAR model, the \( k \)-th variable at time \( t \) is observed in several locations so that the variable \( y \) has three indexes, namely \( n \), \( k \), and \( t \), which indicate location, variable, and time.

\[ \eta_t = [\eta_{11}, \eta_{12}, \ldots, \eta_{N1}, \eta_{12t}, \eta_{22t}, \ldots, \eta_{N2}, \ldots, \eta_{K1}, \eta_{2K1}, \ldots, \eta_{NK}] \],

\[ E(\eta_t) = 0, E(\eta_t \eta_t') = \Sigma, E(\eta_t \eta_i') = 0, i = 1, 2, \ldots, p \]

\( \Sigma \) is \( NK \) by \( NK \) positive definite covariance matrix where \( N \) is the number of location and \( K \) is the number of endogenous variable.

\[ B_i = \begin{bmatrix} A_{11}^{(i)} & A_{12}^{(i)} & \ldots & A_{1K}^{(i)} \\ A_{21}^{(i)} & A_{22}^{(i)} & \ldots & A_{2K}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ A_{K1}^{(i)} & A_{K2}^{(i)} & \ldots & A_{KK}^{(i)} \end{bmatrix}, i = 1, 2, \ldots, p . \]

\( B_i \) is unrestricted \( NK \) by \( NK \) coefficient matrix. For example, suppose there are two variables observed in four locations, the coefficient matrix \( B_i \) is an \( 8 \times 8 \) matrix and contains 64 or \( (NK)^2 \) parameters that must be estimated. The more locations observed, the more parameters that must be estimated, and this can be a problem. To overcome this problem, then Di Giacinto [25] restrict the coefficient of SpVAR by setting \( A_{r}^{(i)} \) as

\[ A_{r}^{(i)} = \sum_{l=0}^{1} b_{r}^{(i)} W_{r}^{(i)}', \quad b_{r}^{(i)} = \text{diag}\{[\beta_{1r}^{(i)}, \ldots, \beta_{Kr}^{(i)}]\}, \quad k, r = 1, \ldots, K; h = 1, \ldots, p , \]

(3)

where \( W_{r}^{(i)} \) is \( N \times N \) spatial weight matrix of order \( l \), the element of \( W_{r}^{(i)} \) being \( w_{r}^{(i)}(i, j) \) which are known apriori and assumed to have non-negative value and positive if location \( i \) and \( j \) are neighbor of order \( l \) according to given spatial ranking and zero if \( i = j \). Spatial weights are assumed to be fixed all the time. The autoregressive coefficient is assumed to vary between locations. With the restriction on (3), the number of parameters that must be estimated decreases to as much as \( 2NK^2 \). Equation (4) is an example of the SpVAR model with two endogenous variables, three locations, and spatial order is one, and temporal order is one.
\[ y_i = B_i y_{i-1} + \eta_i. \]  

Where

\[
B_i = \begin{bmatrix}
A_{i1} & A_{i2} \\
A_{i1} & A_{i2}
\end{bmatrix}
\]

\[ A_{i1} = \beta_{i10} W_{i1} + \beta_{i11} W_{i1}^{(1)} = \beta_{1i} + \beta_{1i1} W_{i1}, \]

\[ A_{i2} = \beta_{i20} W_{i2} + \beta_{i12} W_{i2}^{(1)} = \beta_{12} + \beta_{121} W_{i2}, \]

\[ A_{i1} = \beta_{i10} W_{i1} + \beta_{i11} W_{i1}^{(1)} = \beta_{1i} + \beta_{1i1} W_{i1}, \]

\[ A_{i2} = \beta_{i20} W_{i2} + \beta_{i22} W_{i2}^{(1)} = \beta_{12} + \beta_{122} W_{i2}. \]

Furthermore, it can be described by equation (6) until equation (11).

\[ y_{1t} = \beta_{110} y_{1t-1} + \beta_{111} \left[ w_{t1}^{(1)} (1,2) y_{2t-1} + w_{t1}^{(1)} (1,3) y_{3t-1} \right] + \beta_{112} \left[ w_{t2}^{(1)} (1,2) y_{2t-1} + w_{t2}^{(1)} (1,3) y_{3t-1} \right] + \eta_{1t} \]

\[ y_{2t} = \beta_{210} y_{1t-1} + \beta_{211} \left[ w_{t1}^{(1)} (2,1) y_{1t-1} + w_{t1}^{(1)} (2,3) y_{3t-1} \right] + \beta_{212} \left[ w_{t2}^{(1)} (2,1) y_{2t-1} + w_{t2}^{(1)} (2,3) y_{3t-1} \right] + \eta_{2t} \]

\[ y_{3t} = \beta_{310} y_{1t-1} + \beta_{311} \left[ w_{t1}^{(1)} (3,1) y_{1t-1} + w_{t1}^{(1)} (3,2) y_{3t-1} \right] + \beta_{312} \left[ w_{t2}^{(1)} (3,1) y_{2t-1} + w_{t2}^{(1)} (3,2) y_{3t-1} \right] + \eta_{3t} \]

\[ y_{12t} = \beta_{120} y_{1t-1} + \beta_{121} \left[ w_{t1}^{(1)} (1,2) y_{1t-1} + w_{t2}^{(1)} (1,3) y_{3t-1} \right] + \beta_{122} \left[ w_{t2}^{(1)} (1,2) y_{2t-1} + w_{t2}^{(1)} (1,3) y_{3t-1} \right] + \eta_{12t} \]

\[ y_{22t} = \beta_{220} y_{2t-1} + \beta_{221} \left[ w_{t1}^{(1)} (2,1) y_{1t-1} + w_{t2}^{(1)} (2,3) y_{3t-1} \right] + \beta_{222} \left[ w_{t2}^{(1)} (2,1) y_{2t-1} + w_{t2}^{(1)} (2,3) y_{3t-1} \right] + \eta_{22t} \]

\[ y_{32t} = \beta_{320} y_{3t-1} + \beta_{321} \left[ w_{t1}^{(1)} (3,1) y_{1t-1} + w_{t2}^{(1)} (3,2) y_{2t-1} \right] + \beta_{322} \left[ w_{t2}^{(1)} (3,1) y_{1t-1} + w_{t2}^{(1)} (3,2) y_{2t-1} \right] + \eta_{32t} \]

The example shows that the k-th variable observed at n-th location at time t is a linear function of the lag of the k-th variable at the same location, a weighted average of the lag of the k-th variable at other locations, lag of other variables in the same location and weighted averages of the lag of the other variables in other locations.

The proposed Spatial Vector Autoregressive model with calendar Variation will be discussed in the next section.

3. Result and discussion

In this section, we propose a Spatial Vector Autoregressive Model with Calendar Variation. This model accommodates three aspects of space-time modeling, namely variable interrelationship, space-time relationship, and calendar variation effect. This section discusses the model, parameter estimation method, and the application of the model for Inflation and Money Supply data in the three cities of West Java, Indonesia, namely Bandung, Tasikmalaya, and Cirebon.

3.1. The proposed model

Spatial Vector Autoregressive Model with Calendar Variation is formed by adding a dummy variable to Spatial Vector Autoregressive to accommodate the Calendar Variation effect. The SpVAR (1, p) model with calendar variation K endogenous variables and N locations that we propose can be written as

\[ y_t = X_t y + B_1 y_{t-1} + \cdots + B_p y_{t-p} + \xi_t, \]

Where:

\[ y_t = y_{1t}, y_{2t}, \ldots, y_{Kt} \]

\[ X_t = \begin{bmatrix} x_{1t}, x_{2t}, & \cdots, & x_{Kt} \end{bmatrix} \]

\[ B_1, B_2, \ldots, B_p \text{ are } K \times K \text{ matrices} \]

\[ \xi_t \text{ is a noise term} \]
\[
y_i = \left[ y_{1i}, y_{2i}, \ldots, y_{N_{ii}}, y_{12i}, y_{22i}, \ldots, y_{1K_i}, y_{2K_i}, \ldots, y_{NK_i} \right], \frac{1}{2}
\]

\[y_{nkt}\] is the value of the \(k\)-th variable on the \(n\)-th location at time \(t\),

\[
\gamma = \left[ \gamma_{i1}, \gamma_{i2}, \ldots, \gamma_{iN_{ii}}, \gamma_{i12}, \gamma_{i22}, \ldots, \gamma_{i1K_i}, \gamma_{i2K_i}, \ldots, \gamma_{iNK_i} \right],
\]
is a vector of parameters representing the calendar effects.

\[
\gamma_{nkt} = \left[ \gamma_{nk1}, \gamma_{nk2}, \ldots, \gamma_{nk{mn}} \right], \ m\ is\ the\ number\ of\ the\ dummy\ variable\ for\ calendar\ variation,
\]

\[
X_i = diag\left(x_{i1}, \ldots, x_{iN_{ii}}, \ldots, x_{iK_i}, \ldots, x_{iNK_i}\right), \ represents\ calendar\ variation\ intervention.
\]

\[
\xi_i = \left[ \xi_{i1}, \xi_{i2}, \ldots, \xi_{i{nk}} \right],
\]
is error from the model of the \(k\)-th variable on the \(n\)-th location at time \(t\), and it is assumed

\[
\xi_i \sim N(\theta, \Sigma),
\]

\[
\Sigma = \begin{bmatrix}
E(\xi_{i1}\xi_{i1}') & E(\xi_{i1}\xi_{i2}') & \ldots & E(\xi_{i1}\xi_{i{nk}}') \\
E(\xi_{i2}\xi_{i1}') & E(\xi_{i2}\xi_{i2}') & \ldots & E(\xi_{i2}\xi_{i{nk}}') \\
\vdots & \vdots & \ddots & \vdots \\
E(\xi_{i{nk}}\xi_{i1}') & E(\xi_{i{nk}}\xi_{i2}') & \ldots & E(\xi_{i{nk}}\xi_{i{nk}}')
\end{bmatrix}, \quad \text{where} \quad \xi_{ik} = \left[ \xi_{ik1}, \xi_{ik2}, \ldots, \xi_{ik{nk}} \right].
\]

With the restriction of the equation \textbf{Error! Reference source not found.}, the equation (12) can be written in equation (13).

\[
y_i = X_i \gamma + V_i \beta + \xi_i
\]  
(13)

Where:

\[
V_i = \left[ V_{1i}, V_{2i}, \ldots, V_{pi} \right],
\]

\[
\beta = \left[ \beta^{(1)}, \beta^{(2)}, \ldots, \beta^{(p)} \right]',
\]

\[
V_{1i} = diag\left(v_{11,i-1}, \ldots, v_{N_{11},i-1}, \ldots, v_{1K_{11},i-1}, \ldots, v_{NK_{11},i-1}\right).
\]

\[
v_{nk,i-1} = \left[ y_{n1,i-1}, y_{n2,i-1}^{(k1)}, \ldots, y_{n2,i-1}^{(kk)}, \ldots, y_{NK,i-1}, y_{NK,i-1}^{(kk)} \right].
\]

\[
y_{nk,i-1}^{(kj)} = \sum_{u=1}^{N} w_{i j}(n,u)y_{uk,i-1},
\]

\[
\beta^{(i)} = \left[ \beta^{(i)}_{11}, \beta^{(i)}_{21}, \ldots, \beta^{(i)}_{N1}, \beta^{(i)}_{12}, \beta^{(i)}_{22}, \ldots, \beta^{(i)}_{N2}, \ldots, \beta^{(i)}_{1K}, \beta^{(i)}_{2K}, \ldots, \beta^{(i)}_{NK} \right], i = 1, 2, \ldots, p
\]

and \(\beta^{(k)}_{nk} = \left[ \beta^{(k)}_{nk1}, \beta^{(k)}_{nk2}, \ldots, \beta^{(k)}_{nkK}, \beta^{(k)}_{nkK} \right].
\]

The equation (13) can also be written in equation (14).

\[
y_i = Z_i \theta + \xi_i,
\]  
(14)
Where

\[ Z_t = \begin{bmatrix} X_t, V_{1t}, V_{2t}, \ldots, V_{pt} \end{bmatrix} \quad \text{and} \quad \theta = \begin{bmatrix} \gamma \\ \beta^{(1)} \\ \beta^{(2)} \\ \vdots \\ \beta^{(p)} \end{bmatrix} \]

The equation (14) can be elaborated to become

\[
[ y_{1t} \\
 y_{2t} \\
 \vdots \\
 y_{N_t} \\
 y_{Kt} \\
 \vdots \\
 y_{NK_t} ] = [ X_t, V_{1t}, V_{2t}, \ldots, V_{pt} ] \begin{bmatrix} \gamma \\ \beta^{(1)} \\ \beta^{(2)} \\ \vdots \\ \beta^{(p)} \end{bmatrix} + [ \xi_{1t} \\
 \xi_{2t} \\
 \vdots \\
 \xi_{N_t} \\
 \xi_{Kt} \\
 \vdots \\
 \xi_{NK_t} ]
\]

In the next subsection, we explain the parameter estimation of SpVAR model with Calendar Variation.

3.2. Parameter estimation

Parameter estimation can be performed using the Full Information Maximum Likelihood (FIML) method. If there are \( T \) observations \((t = 1, 2, \ldots, T)\), then the model (14) can be written in equation (15).
Error is assumed to have a normal multivariate distribution or \( \xi \sim N(0, \Omega) \) where \( \Omega = I_T \otimes \Sigma \), so that \( y \sim N(Z_\theta, \Omega) \). The likelihood function can be written as

\[
L(\theta, \Sigma) = \frac{1}{(2\pi)^{\frac{NKT}{2}}} |\Omega^{-\frac{1}{2}}| \exp \left[ -\frac{1}{2} (y - Z_\theta)' \Omega^{-1} (y - Z_\theta) \right]
\]

(16)  

Natural log of equation (16) can be written as

\[
\ln L(\theta, \Sigma_\xi) = -\frac{NKT}{2} \ln(2\pi) + \frac{1}{2} \ln |(I_r \otimes \Sigma_\xi)^{-1}| - \frac{1}{2} (y - Z_\theta)' (I_r \otimes \Sigma_\xi)^{-1} (y - Z_\theta) 
\]

(17)  

To obtain \( \theta \) that maximize the likelihood, then equation (17) is differentiated with respect to \( \theta \) and is equated to zero. Equation (17) can be elaborated as

\[
\ln L(\theta, \Sigma_\xi) = -\frac{NKT}{2} \ln(2\pi) + \frac{T}{2} \ln |\Sigma_\xi| - \frac{1}{2} (y - Z_\theta)' (I_r \otimes \Sigma_\xi)^{-1} (y - Z_\theta) + \theta' Z (I_r \otimes \Sigma_\xi^{-1}) y
\]

(18)  

Equation (18) is differentiated with respect to \( \theta \) to get

\[
\frac{\partial \ln L(\theta, \Sigma_\xi)}{\partial \theta} = -\frac{1}{2} [2Z'(I_r \otimes \Sigma_\xi^{-1}) Z \theta - 2Z'(I_r \otimes \Sigma_\xi^{-1}) y]
\]

(19)  

\( \theta \) is estimated by \( \hat{\theta} \) and equation (19) is equated to zero, to get

\[
-\frac{1}{2} [2Z'(I_r \otimes \Sigma_\xi^{-1}) Z \hat{\theta} - 2Z'(I_r \otimes \Sigma_\xi^{-1}) y] = 0
\]

(20)  

\[
\hat{\theta} = (Z'(I_r \otimes \Sigma_\xi^{-1}) Z)^{-1} Z'(I_r \otimes \Sigma_\xi^{-1}) y
\]

(21)  

If \( \Sigma_\xi \) is not known, then \( \Sigma_\xi \) is estimated using OLS (\( \Sigma_{\text{OLS}} \)) first and then used \( \Sigma_{\text{OLS}} \) to get \( \hat{\theta} \) in equation (21). Furthermore, to obtain the estimator of \( \Sigma_\xi \), equation

Error! Reference source not found. is differentiated with respect to \( \Sigma_\xi^{-1} \) and is equated to zero.

\[
\ln L(\theta, \Sigma) = -\frac{NKT}{2} \ln(2\pi) + \frac{T}{2} \ln |\Sigma_{\xi|^{-1}}| - \frac{1}{2} (y - Z_\theta)' (I_r \otimes \Sigma_{\xi|^{-1}})(y - Z_\theta)
\]

(22)  

\[
= -\frac{NKT}{2} \ln(2\pi) + \frac{T}{2} \ln |\Sigma_{\xi|^{-1}}| - \frac{1}{2} \xi'(I_r \otimes \Sigma_{\xi|^{-1}}) \xi
\]
\[
\frac{\partial \ln L(0, \Sigma)}{\partial \Sigma^{-1}} = \frac{1}{2} \Sigma' - \frac{1}{2} \sum_{i=1}^{T} \xi_i \xi'_i \tag{22}
\]

\(\Sigma'_i\) is estimated by \(\hat{\Sigma}'_i\) and equation (22) is equated to zero.

\[
\frac{1}{2} \hat{\Sigma}'_i - \frac{1}{2} \sum_{i=1}^{T} \xi_i \xi'_i = 0
\]

\[
\hat{\Sigma}'_i = \frac{1}{T} \sum_{i=1}^{T} \xi_i \xi'_i.
\]

\(\hat{\Sigma}'_i\) is a symmetric matrix, so that

\[
\hat{\Sigma}'_i = \frac{1}{T} \sum_{i=1}^{T} \xi_i \xi'_i = \frac{1}{T} \sum_{i=1}^{T} (y_i - Z_i \theta)(y_i - Z_i \theta)'.
\]

\(\hat{\Sigma}_i\) in equation (23) can be used to get \(\hat{\theta}\) in equation (21). This procedure is done iteratively until \(\hat{\theta}\) and \(\hat{\Sigma}_i\) are convergence.

### 3.3. Application to West Java inflation and money supply data

To apply SpVAR model with calendar variation, we use the simplest model of SpVAR (1,1) model with calendar variation for Inflation and Money Supply data in three cities in West Java, Indonesia, namely Bandung, Tasikmalaya, and Cirebon. The data is monthly data from January 2003 until December 2013. We use three models for this data, namely the SpVAR (1,1) model, SpVAR model (1,1) with the scheme 1 calendar variation, and the SpVAR (1,1) model with the scheme 2 calendar variation. In the SpVAR (1,1) with the scheme 1 calendar variation we use two dummy variables for the calendar variations. They’re presented in equation (24).

\[
\begin{align*}
x_{1,t} &= \begin{cases} 
1, & \text{if } t \text{ is the month of Eid al Fitr} \\
0, & \text{if } t \text{ is not the month of Eid al Fitr},
\end{cases} \\
x_{2,t} &= \begin{cases} 
1, & \text{if } t \text{ is a month before Eid al Fitr} \\
0, & \text{if } t \text{ is not a month before Eid al Fitr},
\end{cases}
\tag{24}
\end{align*}
\]

In the SpVAR (1,1) model with the scheme 2 calendar variations, we used eight dummy variables for calendar variations that are presented in equation (25).

\[
\begin{align*}
m_{0,i,t} &= \begin{cases} 
1, & \text{if } t \text{ is a month before Eid al Fitr with Eid al Fitr in week } i \\
0, & \text{for other month}
\end{cases} \\
m_{i,t} &= \begin{cases} 
1, & \text{if } t \text{ is Eid al Fitr month with Eid al Fitr in week } i \\
0, & \text{for other month}
\end{cases}
\tag{25}
\end{align*}
\]

for \(i = 1, 2, 3, 4\)

Next, we select the best model of all three models based on the smallest MSE values.

Prior to modeling, a stationary test was performed using Levin, Lin and Chu [37] test. The result of this test for inflation data is stationary at the level, whereas the test result for Money Supply is stationary on the difference data.

Furthermore, parameter estimation and MSE comparison were performed to select the best model of the three models tested. Table 1 shows the MSE of the three models.
Table 1. MSE of the Models.

| Variable                    | MSE SpVAR(1,1) | MSE SpVAR(1,1) with Calendar Variation Scheme 1 | MSE SpVAR(1,1) with Calendar Variation Scheme 2 |
|-----------------------------|----------------|-------------------------------------------------|-------------------------------------------------|
| Inflation in Bandung        | 0.767723       | 0.682294                                        | 0.533444                                        |
| Inflation in Tasikmalaya    | 1.106292       | 0.984047                                        | 0.772757                                        |
| Inflation in Cirebon        | 1.119582       | 0.931415                                        | 0.66926                                        |
| Money Supply in Bandung     | 0.951292       | 0.776265                                        | 0.713687                                        |
| Money Supply in Tasikmalaya | 0.017639       | 0.014197                                        | 0.01367                                        |
| Money Supply in Cirebon     | 0.053777       | 0.041452                                        | 0.037567                                        |

Table 1 shows that the calendar effect component can reduce MSE. It happens for both schemes, but scheme 2 reduce MSE more than scheme 1. The calendar effect component scheme 2 can reduce MSE by 22.50% to 40.22%. While the calendar effect component scheme 1 can only reduce MSE by 11.05% to 22.92%. So, we select the SpVAR (1,1) model with the calendar variation scheme 2 as the best model because it has the smallest MSE value.

1. SpVAR(1,1) Model with Calendar Variation for Inflation in Bandung is
   \[
   \hat{y}_{1t} = 0.2708 + 4.0612m_{4t} + 0.6979m_{3t} + 0.3669m_{3t} + 0.0261m_{4t} \\
   + 0.2360m_{4t} + 0.0658m_{4t} + 0.2286m_{4t} + 0.3608m_{4t} + 0.2324y_{1t-1}^{*} \\
   + 0.0749y_{12,t-1}^{*} + 0.0750[0.5y_{21,t-1}^{*} + 0.5y_{31,t-1}^{*}] - 0.1410[0.5y_{22,t-1}^{*} + 0.5y_{32,t-1}^{*}]
   \]

2. SpVAR(1,1) Model with Calendar Variation for Inflation in Tasikmalaya is
   \[
   \hat{y}_{2t} = 0.4276 + 4.8401m_{4t} + 0.8164m_{3t} + 0.1131m_{3t} + 0.1552m_{4t} \\
   - 0.3494m_{4t} + 0.0669m_{4t} + 0.3842m_{4t} + 0.4623m_{4t} + 0.0055y_{21,t-1}^{*} \\
   - 0.5740y_{22,t-1}^{*} + 0.1144[0.5y_{11,t-1}^{*} + 0.5y_{31,t-1}^{*}] + 0.1626[0.5y_{12,t-1}^{*} + 0.5y_{32,t-1}^{*}]
   \]

3. SpVAR(1,1) Model with Calendar Variation for Inflation in Cirebon is
   \[
   \hat{y}_{3t} = 0.2588 + 5.233m_{4t} + 1.043m_{4t} + 1.1550m_{3t} + 0.1834m_{4t} - 0.6675m_{4t} \\
   + 0.4051m_{4t} + 0.3550m_{4t} + 1.0589m_{4t} + 0.0708y_{31,t-1}^{*} - 0.1402y_{32,t-1}^{*} \\
   + 0.1651[0.5y_{11,t-1}^{*} + 0.5y_{31,t-1}^{*}] + 0.1931[0.5y_{12,t-1}^{*} + 0.5y_{32,t-1}^{*}]
   \]

4. SpVAR(1,1) Model with Calendar Variation for Money Supply in Bandung is
   \[
   \hat{y}_{12t} = -0.0377 + 1.9366m_{3t} + 1.2974m_{4t} + 0.600m_{3t} + 0.5100m_{4t} \\
   - 0.2557m_{4t} + 0.5860m_{4t} + 2.3824m_{4t} + 2.0565m_{4t} - 0.1014y_{11,t-1}^{*} \\
   - 0.5388y_{12,t-1}^{*} - 0.1028[0.5y_{21,t-1}^{*} + 0.5y_{31,t-1}^{*}] + 0.1366[0.5y_{22,t-1}^{*} + 0.5y_{32,t-1}^{*}]
   \]

5. SpVAR(1,1) Model with Calendar Variation for Money Supply in Tasikmalaya is
   \[
   \hat{y}_{23t} = -0.0085 + 0.2986m_{4t} + 0.1493m_{4t} + 0.0848m_{4t} + 0.1361m_{4t} \\
   - 0.0940m_{4t} + 0.1133m_{4t} + 0.2896m_{4t} + 0.1996m_{4t} - 0.0046y_{21,t-1}^{*} \\
   - 0.3718y_{22,t-1}^{*} - 0.0158[0.5y_{11,t-1}^{*} + 0.5y_{31,t-1}^{*}] - 0.0190[0.5y_{12,t-1}^{*} + 0.5y_{32,t-1}^{*}]
   \]
6. SpVAR(1,1) Model with Calendar Variation for Money Supply in Cirebon is
\[
\hat{y}_{3n_t} = -0.0100 + 0.5985m_{b1t} + 0.3301m_{b2t} - 0.0118m_{b3t} + 0.1708m_{b4t} - 0.1805m_{b6t}
+0.2275m_{b7t} + 0.5315m_{b8t} + 0.4370m_{b9t} + 0.0080y_{31,t-1} - 0.4893y_{12,t-1},
\]
(31)

\[
-0.0584\left[0.5y_{11,t-1} + 0.5y_{21,t-1}\right] - 0.0008\left[0.5y_{12,t-1} + 0.5y_{22,t-1}\right]
\]

Where \(\hat{y}_{ntk}\) is the predicted value of the \(k\)-th variable at \(n\)-th location at time \(t\). In this paper, \(n = 1\) is Bandung, \(n = 2\) is Tasikmalaya and \(n = 3\) is Cirebon, while \(k = 1\) is inflation and \(k = 2\) is Money Supply.

From equation (31), it can be seen that there is a relationship between Money Supply in Cirebon with Inflation in Bandung and Tasikmalaya and Money Supply in Cirebon a month before. While Inflation in Bandung, Money Supply in Bandung, and Money Supply in Tasikmalaya (equation (26), (29) and (30)) relate to the variable itself one month before.

Meanwhile, equations (26) to equation (31) show that the significant dummy variable of calendar variations in the six equations varies. However, there are two significant dummy variables in the six equations, namely \(m_{b1t}\) and \(m_{b2t}\). The coefficients of these two variables are all positive, meaning that if Eid al Fitr occurs in the first or second week, then Inflation and Money Supply in three cities in Western Java in the month of Ramadhan (one month before Eid al Fitr) will increase.

4. Conclusion
The SpVAR model with calendar variation is an extension of the SpVAR model, which is a model that can be used to find out the relationship between several variables at the same location, but the time is different, also the relationship between several variables at different location and time. The SpVAR model with calendar variation accommodates holiday effects that occur based on the lunar calendar, which is a holiday occurring every year but at different times (dates and months). The parameter estimation of SpVAR model with calendar variation can be done using FIML method. From the results of the application, there is a relationship between Money Supply in Cirebon and Inflation in Bandung and Tasikmalaya. It shows there is a relationship between variables, and there is a space-time relationship as well. Besides, there are effects of calendar variation on Inflation and Money Supply in all three cities.

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