Oscillations in the inflaton potential?

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We consider a class of inflationary models with small oscillations imprinted on an otherwise smooth inflaton potential. These oscillations are manifest as oscillations in the power spectrum of primordial perturbations, which then give rise to oscillating departures from the standard cosmic microwave background power spectrum. We show that current data from the Wilkinson Microwave Anisotropy Probe constrain the amplitude of a sinusoidal variation in the inflaton potential to have an amplitude less than $3 \times 10^{-5}$. We anticipate that the smallest detectable such oscillations in Planck will be roughly an order of magnitude smaller, with slight improvements possible with a post-Planck cosmic-variance limited experiment.

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I. INTRODUCTION

Cosmic microwave background (CMB) experiments continue to be consistent with the simplest predictions of inflationary models, even as the data become increasingly precise \cite{1}. Constraints on the amplitude and spectral index of primordial density perturbations, and on the amplitude of the inflationary gravitational-wave background, can now be used to constrain the parameter space of the inflaton potential. In most of the current analyses the inflaton potential is parameterized in terms of its amplitude $V(\phi)$ and first and second derivatives, $V'(\phi)$ and $V''(\phi)$.

Given the recent advances in data quality, as well as the improvements anticipated with forthcoming experiments (e.g., the Planck satellite \cite{2}, to be launched within the next year), it is worth asking whether the data can be used to study more complicated forms of the potential. For example, Adams et al. \cite{3} showed that supergravity-inspired models may give rise to inflaton potentials with a large number of steps. Each step corresponds to a symmetry-breaking phase transition in a field coupled to the inflaton. The inflaton mass then changes suddenly when each transition occurs. These steps are responsible for unusual inflaton dynamics, often represented as a hybrid inflation model \cite{4}, and they will create oscillating features in the primordial power spectrum. Oscillations can also be directly imprinted on the inflaton potential itself, due to some trans-Planckian physics \cite{5,6,7}. Finally, some other mechanisms may create features in the primordial power spectrum \cite{8}. There may also be empirical motivations to consider more complicated potentials, as several CMB analyses suggest that the CMB power spectrum may be better fit by primordial power spectra with features than by smooth power spectra \cite{8,9,10,11,12}.

In this paper, we consider a class of inflationary models that feature periodic oscillations imprinted on a smooth inflaton potential, which then give rise to oscillations in the primordial power spectrum. We look for these oscillations in the Wilkinson Microwave Anisotropy Probe (WMAP) data, and then determine the smallest oscillation amplitude that will be probed with forthcoming experiments. Our work is somewhat similar to that in Ref. \cite{13}, which considers oscillations in the primordial power spectrum that arise from a step in the inflaton potential, but differs in that we consider wiggles in the inflaton potential itself. There is also related work in Ref. \cite{14}, which considers oscillations in the CMB power spectrum from a rapid phase transition during inflation. However, the work most closely related to ours is that in Ref. \cite{15}, which considers oscillations in the inflaton potential in natural-inflation models. While they focused on constraints from existing data, we forecast also the detectability with future measurements.

The plan of this paper is as follows. Section II introduces the model and discusses the calculation of the power spectrum. Section III presents results of our search for oscillations in the WMAP data. Section IV presents results for the primordial, matter, and CMB power spectra. Section V presents numerical results for the primordial power spectra. Section VI presents results of our search for oscillations in the WMAP data. Section VII discusses the forecasts for the detectability of oscillations in future experiments; we consider here both Planck and a post-Planck cosmic-variance limited experiment. We summarize and provide some concluding remarks in Section VII.

II. THE OSCILLATING MODEL

In order to study the effects of small oscillations in the inflaton potential, we begin with a simple base inflationary model that is consistent with current data. We consider the simplest such model, namely a quadratic inflaton potential, $V_0(\phi) = \frac{1}{2}m^2\phi^2$, with $m \simeq \sqrt{8\pi} \times 10^{-6} M_{\text{Pl}}$. This potential has a corresponding CMB amplitude $A_s = 1.2 \times 10^{-9}$ for a $\Lambda$CDM model. Once normalized with the best-fit WMAP5 data, we get $n_s = 0.96$ for a pivot scale $k_0 = 0.002$ Mpc$^{-1}$, consistent with current constraints \cite{16}.

We then superimpose on this smooth potential a sim-
soidal fluctuation to give a potential of the form
\[ V(\phi) = \frac{1}{2} m^2 \phi^2 \left[ 1 + \alpha \sin \left( \frac{\phi}{\beta M_{Pl}} + \delta \right) \right], \quad (1) \]

parameterized by an amplitude of oscillations and a parameter \( \beta \) that characterizes the frequency. The amplitude \( \alpha \) is assumed to be small so that the inflaton does not get stuck in one of the local minima introduced in the potential by the oscillations; we discuss the precise constraint later. We choose the phase \( \delta = 0 \) and explain below why our results will be the same for different values.

The homogeneous dynamics are dictated by the Friedmann equation for the scale factor and the inflaton equation of motion. The Friedmann equation (in units where \( 8 \pi G = c = \hbar = 1 \)) for a universe containing a scalar field \( \phi(t) \) with potential \( V(\phi) \) is
\[ 3H^2 \equiv 3 \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad (2) \]
and the scalar-field equation of motion is
\[ \ddot{\phi} + 3H \dot{\phi} = -\frac{dV}{d\phi}. \quad (3) \]

We solve these coupled differential equations for the scale factor \( a(t) \) and scalar field \( \phi(t) \) numerically. The solution is insensitive to our choice of initial conditions for \( \phi(t) \), as the solution exhibits an attractor behaviour if the field begins high enough in the potential \( [18] \).

We then turn our attention to the perturbations. We express the power spectrum \( P_k \) of the primordial curvature perturbation with the horizon-crossing approximation \( [17][18] \).
\[ P_k = [1 - 2(2C + 1)\epsilon_H + 2C\eta_H] \left( \frac{H^2}{2\pi|\phi|} \right)^2, \quad (4) \]
where \( C = -2 + \ln 2 + b \simeq -0.73 \), with \( b \) the Euler–Mascheroni constant. The right-hand side is evaluated at \( k = aH \), and the Hamilton–Jacobi slow-roll parameters are
\[ \epsilon_H \equiv \frac{1}{2} \frac{\dot{\phi}^2}{H^2}, \quad \eta_H \equiv -\frac{\ddot{\phi}}{H \dot{\phi}}. \quad (5) \]

The next step is to relate a value of \( \phi \) to a comoving wavenumber \( k \) that crosses the horizon at that value of \( \phi \). To do so, we note that the number of e-foldings of inflation between a time \( t \) and the end of inflation is
\[ N(t) \equiv \ln \frac{a(t_{\text{end}})}{a(t)}, \quad (6) \]
and in terms of \( k \), it is
\[ N(k) \simeq 55 - \ln \frac{k}{a_0 H_0}, \quad (7) \]
where \( a_0 \) is the scale factor today (which we choose to be \( a_0 = 1 \)), and \( H_0 = (h/3000) \text{ Mpc}^{-1} \) with \( h \simeq 0.72 \). Note that the uncertainty around the 55 e-foldings of inflation at horizon crossing is about 5, and comes mainly from the uncertainty in the reheating process \( [19] \).

The distance scale \( \lambda \) that exits the horizon varies roughly as the exponential of the change \( \delta \phi \) in the inflaton \( \phi \). Hence an oscillation in \( \phi \) in the inflaton potential should give rise to oscillations in the logarithm of the wavenumber \( k \) in the primordial power spectrum, and thus in the logarithm of the CMB multipole moment \( \ell \).

We then calculate the temperature and polarization (TT, EE, and TE) spectra for the model using the \textsc{Cmb} code \[20\], using the current best-fit parameters for a \( \Lambda \text{CDM} \) model from the WMAP 5-year results \[16\]. We neglect the B-mode polarization, as it comes about either through gravitational waves or gravitational lensing and is always small compared with the E-mode polarization.

### III. Power Spectra

We begin by showing in Fig. 11 the primordial power spectrum, as well as the present matter power spectrum, for the standard smooth potential and for an oscillating potential with parameters chosen to be \( [\alpha, \beta] = [5 \times 10^{-4}, 3 \times 10^{-2}] \). The oscillations in the power spectra are nearly sinusoidal in log \( k \) with almost constant amplitude. The corresponding CMB angular power spectrum, shown in Fig. 22 reflects this behaviour. The mapping from the three-dimensional matter power spectrum to the two-dimensional CMB power spectrum slightly smooths the wiggles. In this example, the frequency \( \beta \) of the inflationary potential’s oscillations has been chosen so that one of the primordial oscillation peaks lines up with the first acoustic peak. However, this coincidence will not be generic.

### IV. A Search in WMAP Data

Given the CMB predictions discussed above, it is straightforward to search the existing CMB data from WMAP for these oscillations, and we have carried out a Markov Chain Monte Carlo (MCMC) analysis to do so. Fig. 33 presents the results of this analysis, assuming that all other cosmological parameters are known. For each point, we run four Markov chains. The results provide an indication of the best-fit amplitude \( \alpha \) as a function of the assumed frequency \( \beta \). These amplitudes are comparable to their standard errors, and so we conclude that there is no evidence for oscillations in the WMAP data. Instead, we infer an upper limit \( \alpha \lesssim 3 \times 10^{-5} \) from WMAP. We note that a corresponding MCMC analysis has already been performed in Ref. \[13\] for the WMAP3 data for a similar potential. They find a precision on the amplitude of oscillations of the order of \( 10^{-5} \) for WMAP3, consistent with our results.
FIG. 1: Primordial power spectrum (top) and matter power spectrum (bottom) of the smooth inflaton potential (solid) and oscillating potential (dashed). The oscillating-potential parameters are $[\alpha, \beta]=\[5 \times 10^{-4}, 3 \times 10^{-2}\]$. The amplitude is chosen to be large to clearly show the effect of the oscillations.

FIG. 2: The CMB power spectrum corresponding to the models shown in Fig.1. The WMAP5 data are superimposed. The error bars include both the cosmic variance and instrumental noise.

FIG. 3: The best-fit amplitude $\alpha$ and its standard error, considering WMAP5 data, for our range of frequencies $\beta$.

V. DETECTABILITY

We now estimate the smallest oscillation amplitude $\alpha$ that will be detectable with future experiments. To do so, we first consider for simplicity the temperature power spectrum only. We suppose that each multipole moment $\ell$ can be measured with a standard error

$$\sigma_{\ell} = \sqrt{\frac{2}{(2\ell+1)f_{\text{sky}}}} (C_{\ell} + N_{\ell}),$$

where

$$N_{\ell} \equiv \left(\overline{\Delta T^2}\right) \exp \left(\frac{l^2\theta_b^2}{8 \ln 2}\right)$$

is the contribution from the detector noise, $\overline{\Delta T}$ is the detector noise per angular-resolution element, and $\theta_b$ is the beam width. We then estimate the error on $\alpha$ by

$$\left(\frac{1}{\sigma_\alpha}\right)^2 = \sum_{l} \left(\frac{\partial C_{\ell}}{\partial \alpha}\right)^2 \frac{1}{\sigma^2_{\ell}(\alpha)},$$

and we choose an amplitude $\alpha = 5 \times 10^{-5}$ to compute the $C_{\ell}$ derivatives. This estimate assumes that all other cosmological parameters are known, and as such, provides an optimistic estimate. However, the true value, obtained by marginalizing over all other cosmological parameters, will probably not be too much worse, as there are no strong degeneracies between these oscillations and any other cosmological parameters. The acoustic oscillations oscillate in $\ell$, while these oscillate in $\log \ell$, and so they should not be strongly degenerate.

In order to improve our results, we then include the polarization and temperature-polarization power spectra as well. In that case, the generalization of the expression for the smallest detectable oscillation takes the form

$$\left(\frac{1}{\sigma_\alpha}\right)^2 = \sum_l \sum_{A,A'} \frac{\partial C_{\ell}^A}{\partial \alpha} \frac{\partial C_{\ell}^{A'}}{\partial \alpha} \left[\Psi^{-1}\right]_{AA'}^{A'A} \frac{\partial C_{\ell}^{A'}}{\partial \alpha},$$

where $\Psi^{-1}$ is the inverse matrix of the Fisher matrix.
The smallest detectable amplitude $\sigma_\alpha$ as a function of $\beta$, for WMAP, Planck and a cosmic-variance limited experiment. The upper part of each pair includes the temperature data only, and the lower one the polarization data as well. Note that the temperature and temperature–polarization curves for WMAP and Planck are effectively degenerate and thus appear to be one curve.

For A = TT, EE, TE where $[\Psi^{-1}]_{AA'}$ are elements of the inverse of $\Psi$, the covariance matrix; its elements are given in Ref. [22].

In Figs. 4 and 5, we plot the smallest detectable amplitude $\sigma_\alpha$, from these analyses, as a function of $\beta$, for WMAP (in Fig. 4 only), Planck, and a hypothetical cosmic-variance limited experiment. For WMAP, we simply consider the measured uncertainties from the 5-year results [16]. For Planck, the forecast is done considering the three most sensitive temperature channels, of specifications similar to the HFI channels of frequency 100 GHz, 143 GHz, and 217 GHz [2]. The intensity sensitivities (detector noise) of these channels are taken as 6.8 $\mu$K, 6.0 $\mu$K, and 13.1 $\mu$K respectively, corresponding to the values quoted for two complete sky surveys. These are average sensitivities per pixel, where a pixel is a square whose side is the FWHM extent of the beam. The FWHMs (beam widths) of these channels are given as 9.5 arcmin, 7.1 arcmin, and 5.0 arcmin, respectively.

The composite noise spectrum for the three temperature channels is obtained by inverse-variance weighting the noise of individual channels [23, 24]. For polarization we take only one channel, the 143 GHz channel, of sensitivity 11.5 $\mu$K, and FWHM 7.1 arcmin. Finally, for the cosmic-variance limited experiment, the fractional sky covered is taken to be 0.8 for all $\ell$, and we use simulated data out to an $\ell_{\text{max}}$ of 2000, as for Planck.

The statistical analysis presented in this Section is intended to forecast the uncertainty on parameters measured by future experiments. It is nevertheless reassuring that this forecast, when applied to WMAP, recovers roughly the limits obtained from the detailed MCMC analysis. Fig. 4 illustrates the advantage of Planck over WMAP for an oscillation search. While the smallest detectable amplitude is $O(10^{-5})$ for WMAP, it is $O(10^{-6})$ for Planck. Looking at Planck more closely, in Fig. 5 we find a smooth variation of $\sigma_\alpha$ except at some frequencies, such as $\beta = 0.03$ and $\beta = 0.04$ for instance. These deviations arise from correlations between the acoustic peaks and the primordial peaks, coming from the inflaton potential, in the CMB power spectrum. When varying the frequency, the bumps thus created are aligned differently with the acoustic oscillations. For example, the choice of $\beta = 0.03$ considered in Fig. 2 is particular as many bumps are well aligned with the acoustic peaks. This effect is however much less important at high frequency, as our results suggest.

The temperature-only and temperature–polarization curves coincide for both WMAP and Planck. This follows because the polarization amplitude is much smaller and neither WMAP nor Planck will measure polarization to the cosmic-variance limit. The contribution to the total signal-to-noise from polarization in these experiments is thus small. The degeneracy is broken in the cosmic-variance limited experiment. In this case, the precisions with which the temperature and polarization power spectra can be measured are roughly the same, resulting in roughly a $\sqrt{2}$ improvement to the oscillation sensitivity.

Figs. 4 and 5 indicate that a higher frequency of oscillations (i.e. a smaller $\beta$) allows a smaller detectable oscillation amplitude. This can be understood by considering Eq. (10), whose result is mainly governed by the difference between the chaotic and the oscillating curve, in terms of amplitudes. For a given amplitude of oscillations in the CMB power spectrum, but different frequencies, the difference between these two curves is almost the same (it would be exactly the same if the slope of the chaotic spectrum was constant). However, in our analysis, we are considering a given amplitude $\alpha$ in the inflaton potential itself. When the frequency of oscillations in the inflaton potential increases, so does that in the CMB power spectrum, but the corresponding amplitude in the CMB spectrum also increases (rather than staying constant). As such, for a given $\alpha$, the difference between the chaotic and oscillating curve increases when $\beta$ decreases, and hence $\sigma_\alpha$ decreases.
We can now also justify our neglect of the phase δ introduced in Section II. Its effect would simply be to shift the deviations on the smooth curves. In Section II, we assumed that α was sufficiently small that inflation was not interrupted, and we can now justify that assumption. The constraint on α from WMAP is already α ≲ 3 × 10⁻⁵.

In the case of the most critical values considered in our analysis, being [α, β] = [5 × 10⁻⁵, 5 × 10⁻³], we get |
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