CPT and Lorentz violation as signatures for Planck-scale physics

Ralf Lehnert
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A. Postal 70-543, 04510 México D.F., Mexico
E-mail: ralf.lehnert@nucleares.unam.mx

Abstract. In recent years, the breakdown of spacetime symmetries has been identified as a promising research field in the context of Planck-scale phenomenology. For example, various theoretical approaches to the quantum-gravity problem are known to accommodate minute violations of CPT invariance. This talk covers various topics within this research area. In particular, some mechanisms for spacetime-symmetry breaking as well as the Standard-Model Extension (SME) test framework will be reviewed; the connection between CPT and Lorentz invariance in quantum field theory will be exposed; and various experimental CPT tests with emphasis on matter–antimatter comparisons will be discussed.

1. Introduction
The discrete spacetime transformations of charge conjugation (C), parity inversion (P), and time reversal (T), as well as various combinations like CP and CPT have played a key role in fundamental physics since the 20th century. For example, prior to the 1950s, parity was widely believed to be an exact symmetry of nature. This belief changed when convincing experimental evidence of parity violation in the beta decay of $^{60}$Co was found\(^1\). This discovery paved the way for a viable theoretical description of the weak force. At present, it is known that not only P, but also other discrete spacetime transformations (e.g., T and CP) are not associated with exact symmetries in nature. Experimental and theoretical studies in this field have remained an active and important research area, which may give insight into physics beyond the Standard Model.

The role of the CPT transformation is special in this context. While there are no particular theoretical reasons why C, P, T, CP, CT, and PT should be conserved, CPT invariance must hold in conventional physics\(^2\): the celebrated CPT theorem states that under mild assumptions Lorentz symmetry implies CPT invariance in a unitary quantum field theory. In other words, relativity and quantum mechanics essentially require CPT to be conserved. It follows that CPT tests are a tool for probing the foundations of physics. This fact together with the ultrahigh sensitivities attainable in CPT-violation searches have led to a recent revival of interest in this subject.

The CPT transformation provides a connection between a particle and its antiparticle. This suggests that CPT conservation is associated with a symmetry between matter and antimatter. One can prove that this is indeed the case: the magnitudes of the mass, charge, decay rate,

\(^1\) However, it had been suggested several times and in different contexts that parity might, in fact, be violated.
gyromagnetic ratio, and other intrinsic properties of a particle are identical to those of its antiparticle if CPT invariance holds. Such arguments can also be applied to systems of particles and their dynamics. For example, atoms and their corresponding anti-atoms must have the same spectra, and a particle-reaction process and its CPT conjugate must exhibit equal reaction cross sections. It follows that experimental matter–antimatter comparisons can serve as probes for the validity of CPT symmetry.

However, CPT symmetry can also be tested in other systems. The basic idea is that if CPT is violated one of the assumptions necessary to prove the CPT theorem must be relaxed. In the context of axiomatic field theory, one can prove rigorously that CPT violation implies Lorentz breakdown if quantum-mechanical probability conservation is to be maintained. We remark, however, that the converse of this fact—namely that Lorentz violation implies CPT breaking—is not true in general. In any case, it follows that in conventional quantum mechanics, CPT-violation searches are at the same time Lorentz-symmetry tests. This offers the possibility of probing CPT invariance via certain experiments designed to test Lorentz symmetry.

The present talk develops these particular ideas further. We begin by reviewing the theoretical motivations for considering CPT violation and the associated Lorentz breakdown. We then recall the Standard-Model Extension (SME)—the framework for the description of CPT- and Lorentz-violating effects at low, presently attainable energies. The final part discusses a few possibilities for testing these ideas.

2. Motivations and model for CPT and Lorentz violation

For the identification and analysis of experimental CPT- and Lorentz-violation searches, a test framework is needed. This framework must allow for small departures from exact CPT and Lorentz symmetry. The SME mentioned above is such a framework. It has been developed over the last decade, and it is constructed to be relatively general and independent of the (thus far unknown) details of the underlying physics. At the same time, the SME maintains numerous desirable features of conventional physics. We begin this section by sketching the line of reasoning that has been employed to establish the SME.

In a first step, it is necessary to decide how to implement CPT and Lorentz violation into a test model. Features associated with the requirement of coordinate independence provide one possible basis for classifying departures from Lorentz symmetry. Coordinates, which are a pure product of human thought, label spacetime points in a largely arbitrary way; they are descriptive tools and as such they lack physical reality. Model predictions must therefore remain independent of the chosen coordinates. This can be achieved by working on a spacetime manifold and representing physical quantities by geometric objects like tensors or spinors. However, this principle does not fix the type of the underlying manifold: for example, both Lorentzian and Galilean spaces would equally be consistent with coordinate independence.

The above reasoning suggests one possibility for the implementation of Lorentz breakdown: the underlying spacetime structure is no longer Lorentzian, so that inertial frames are not connected via the usual Lorentz transformations. In other words, covariance under Lorentz transformations is replaced by covariance under some other symmetry transformation. Such deformations of Lorentz symmetry have been discussed in the literature. However, their interpretation—and in particular their CPT properties—remain unclear at present. These ideas are not directly employed in the construction of the SME.

A second possibility for the implementation of CPT and Lorentz violation maintains the conventional Lorentzian spacetime structure and employs a nontrivial vacuum instead. Such a vacuum can be described by a nondynamical tensorial background; such a background could...
lead to direction- and boost-dependent physics, for example. This situation is somewhat similar to electrodynamics in macroscopic media: covariant behavior under Lorentz transformations of the Minkowski frame (i.e., coordinate independence) is maintained, but the propagation of light need not be isotropic and can be slower than $c$. The SME incorporates this type of CPT and Lorentz violation.

Such a nontrivial vacuum with CPT and Lorentz violation can now be implemented in a model Lagrangian, which is to be interpreted as a low-energy effective field theory. The motivation for this approach is the following. Effective field theories have been exceptionally successful in particle, nuclear, and condensed-matter physics. In fact, the conventional Standard Model (SM) and General Relativity (GR) are usually viewed as effective field theories; leading-order CPT- and Lorentz-breaking corrections outside the framework of effective field theory therefore seem somewhat contrived. Moreover, this approach can naturally incorporate practically the entire basis of known physics—the SM and GR—as limiting cases. These two features (i.e., a theoretically well understood framework containing all of present-day physics) ensure broadest applicability of the SME for all currently feasible experiments.

With the above considerations, the SME Lagrangian $\mathcal{L}_{\text{SME}}$ now takes the form

$$
\mathcal{L}_{\text{SME}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{GR}} + \delta \mathcal{L}_{\text{SME}}.
$$

Here, $\mathcal{L}_{\text{SM}}$ and $\mathcal{L}_{\text{GR}}$ denote the Lagrangians of the SM and GR, respectively. It is thus evident that the SME incorporates the entire foundation of established physics, as argued above. The CPT- and Lorentz-violating corrections are contained in $\delta \mathcal{L}_{\text{SME}}$, which is formed by contracting the stipulated vacuum background tensors with SM or GR fields. For example, $\delta \mathcal{L}_{\text{SME}}$ includes the term $b_\mu \bar{\psi} \gamma^5 \gamma^\mu \psi$, where $\psi$ is, e.g., the electron field of the SM. The background vector $b_\mu$ is assumed to be caused by underlying physics. This sample term violates both CPT and Lorentz symmetry, and it is $b_\mu$ that experiments can measure or constrain. We remark that physically desirable features, such as power-counting renormalizability and $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ gauge invariance, are often imposed in the literature; this special case is then referred to as the minimal SME. Various studies have solidified the theoretical foundations of the minimal SME [6].

Thus far, the SME has been constructed by hand without reference to physics beyond the SM. In the remaining part of this section, we list a few mechanisms in underlying physics that can generate the tensorial backgrounds contained in the SME, which are responsible for violating CPT and Lorentz invariance.

**Spontaneous CPT and Lorentz breakdown in string theory.**—From a theoretical viewpoint, spontaneous symmetry violation (SSV) provides an attractive mechanism for generating CPT and Lorentz breakdown. SSV is well established in solid-state physics, and in the electroweak model it is believed to be responsible for creating the masses of elementary particles. The essence of SSV is that a symmetric zero field value does not correspond to the lowest-energy state. Instead, non-vanishing vacuum expectation values (VEVs) are energetically preferred. In string field theory, one can demonstrate that SSV can produce VEVs of vector and tensor fields, which can then be related to the CPT- and Lorentz-violating background in the SME (e.g., $b_\mu$ in the example in the previous paragraph) [7].

**Nontrivial spacetime topology.**—The basic idea behind this approach is the possibility that one of the usual three spatial dimensions is compactified. On observational grounds, the compactification radius $R$ would clearly have to be very large. In any case, the local structure of flat Minkowski space is preserved. The finite size of the compactified dimension implies periodic boundary conditions, which in turn lead to a discrete momentum spectrum, so that a Casimir-type vacuum emerges. It is then intuitively reasonable that a vacuum of this type can possess a preferred direction along the compactified dimension. Indeed, one can show that
such a situation is described effectively by certain SME terms \cite{8}; the corresponding background vectors are inversely proportional to the compactification radius $R$.

**Cosmologically varying scalars.**—A varying scalar, such as a varying coupling or a cosmological scalar field, is typically associated with the breakdown of translational symmetry. This feature occurs regardless of the mechanism responsible for the spacetime dependence. Since translations are closely intertwined with the Lorentz transformations within the Poincaré group, it is unsurprising that the translation-symmetry violation can also affect Lorentz invariance. Consider, for example, a physical system with a varying coupling $\alpha(x)$ and two scalar fields $\phi$ and $\Phi$. Suppose further that the Lagrangian for this system contains a kinetic-type term of the form $\alpha(x) \partial^{\mu} \phi \partial_{\mu} \Phi$. A suitable integration by parts then generates the term $-(\partial^{\mu} \alpha) \phi \partial_{\mu} \Phi$ while leaving unaffected the equations of motion, and thus the physics. If the variation of $\alpha(x)$ is slow (say on cosmological scales), the gradient $(\partial^{\mu} \alpha)$ can be taken approximately constant on laboratory scales. It is then apparent that the external nondynamical gradient can be identified with one of the background vectors in the SME \cite{9}.

**Non-commutative field theory.**—This popular approach to physics beyond the SM postulates that coordinates are no longer real numbers, but rather operators satisfying nontrivial commutation relations such as $[x^{\mu}, x^{\nu}] = i\theta^{\mu\nu}$. The quantity $\theta^{\mu\nu} \neq 0$ is often taken as spacetime constant and selects preferred directions in the non-commutative space. To interpret such models physically, one can transform them into ordinary field theories via the Seiberg–Witten map. The resulting field theory on conventional Minkowski space still contains the nondynamical constant $\theta^{\mu\nu}$, which acts as a background tensor, i.e., SME terms are generated \cite{10}.

**Loop quantum gravity.**—Another widely known approach to a quantum version of General Relativity is loop quantum gravity. In semiclassical calculations, various results have been derived that indicate Lorentz violation in electrodynamics and for fermions under certain reasonable assumptions \cite{11}. An effective description of these effects is contained in the SME.

3. Experimental CPT-violation searches

The SME framework discussed in the previous section can now be employed to predict experimental signatures for CPT and Lorentz violation. For example, a key concept in the context of Relativity theory is the speed of light and its constancy; the SME predicts deviations from this concept that can be searched for experimentally \cite{12}. But CPT and Lorentz symmetry also provide the basis for many other properties in numerous physical systems. Accordingly, a large number of additional CPT and Lorentz tests have been analyzed within the context of the SME \cite{13}. In this section, we review a few of these ideas with focus on those tests that not only bound Lorentz breaking but also CPT violation.

**Spectropolarimetry of cosmological sources.**—The photon sector of the minimal SME contains one type of coefficient that violates both CPT and Lorentz symmetry, the Chern–Simons-type $(k_{AF})^{\mu}$ term. For example, this term leads to birefringence in the propagation of electromagnetic waves \cite{14}, vacuum Cherenkov radiation \cite{15}, and shifts in cavity frequencies \cite{16}. These are effects that can be searched for experimentally. Birefringence studies in cosmic radiation are particularly well suited because the extremely long propagation distances translate into ultrahigh sensitivity to this type of Lorentz and CPT violation. An analysis of experimental data from cosmological sources has yielded a limit on $(k_{AF})^{\mu}$ at the level of $10^{-42}$GeV \cite{14}.

**Studies involving cold antihydrogen.**—Comparisons of the spectra of hydrogen ($H$) and antihydrogen ($\bar{H}$) are well suited for CPT and Lorentz tests. Among the various transitions that can be considered, the unmixed 1S–2S transition appears to be an excellent candidate: its projected experimental resolution is expected to be about one part in $10^{18}$, which is promising in light of potential Planck-suppressed quantum-gravity effects. On the other hand, the corresponding leading-order SME calculation establishes identical shifts for free $H$ or $\bar{H}$ in the initial and final states with respect to the conventional energy levels. From this perspective,
the 1S–2S transition is actually less suitable for the measurement of unsuppressed CPT- and Lorentz-violating signals. The largest non-trivial contribution to this transition within the SME test framework is produced by relativistic corrections, and it is multiplied by two additional powers of the fine-structure parameter $\alpha$. The expected energy shift, already at zeroth order in $\alpha$ expected to be minuscule, is therefore associated with an additional suppression factor of more than ten thousand \[17].

Another transition that can be employed for CPT- and Lorentz-violation searches is the spin-mixed 1S–2S transition. When $\text{H} \text{or } \text{H}^\overline{\text{b}}$ is confined with magnetic fields—such as in a Ioffe–Pritchard trap—the 1S and the 2S levels are each split due to the usual the Zeeman effect. In the framework of the SME, one can demonstrate that in this case the 1S–2S transition between the spin-mixed states is indeed shifted by CPT and Lorentz breaking at leading order. A disadvantage from a practical viewpoint is the $\vec{B}$-field dependence of this transition, so that the experimental sensitivity is limited by the size of the inhomogeneity in the trapping magnetic field. The development of novel experimental techniques might circumvent this issue, and resolutions close to the natural linewidth might then be achievable \[17].

A third transition suitable for CPT- and Lorentz-violation searches is the hyperfine Zeeman transitions within the 1S state.\[3 Even in the limit of a vanishing magnetic field, the SME predicts leading-order effects for two of the transitions between the Zeeman-split states. We mention that this result may also be practical from an experimental point of view because various other transitions of this type (e.g., the conventional Hydrogen-maser line) can be well resolved in measurements \[17, 18].

Tests in Penning traps.—The SME predicts not only that atomic energy levels can be affected by the presence of CPT and Lorentz violation, but also, e.g., proton and antiproton levels in Penning traps. A perturbative calculation establishes that only one SME coefficient (a $b^\mu$-type background vector mentioned in the previous section) contributes to the transition-frequency difference between the proton and its antiparticle at leading order. More specifically, the anomaly frequencies are shifted in opposite directions for protons and antiprotons. This effect can be employed to extract a clean experimental bound on the proton’s $b^\mu$ \[19].

Neutral-meson interferometry. A widely known standard CPT test involves the comparison of the K-meson’s mass to that of its antimeson: even very small mass differences would be measurable in Kaon-oscillation experiments. Although the SME contains only one mass parameter for a given quark species and the corresponding antiquark species, these particles are nevertheless affected differently by the CPT- and Lorentz-violating background in the SME. This allows the dispersion relations for mesons and antimesons to differ, so that mesons and antimesons can have distinct energies. It is this difference in energy that ultimately affects interferometric experiments and is therefore potentially observable in such systems \[20, 21]. Note that not only the K-meson but also other neutral mesons can be studied. Note also that besides CPT violation, Lorentz breaking is involved as well, so that boost- and rotation-dependent signals can be searched for.\[4

Acknowledgments
The author wishes to thank the organizers for arranging this stimulating meeting and for subsidizing my attendance. This work was also supported in part by the European Commission under Grant No. MOIF-CT-2005-008687 and by CONACyT under Grant No. 55310.

References
[1] C.S. Wu, E. Ambler, R.W. Hayward, D.D. Hoppes, and R.P. Hudson, Phys. Rev. 105, 1413 (1957).
[2] See, e.g., R.F. Streater and A.S. Wightman, *PCT, spin statistics and all that*, Benjamin/Cummings (1964).

3 See also E. Widmann’s contribution to these proceedings.
4 See also A. Di Domenico’s contribution to these proceedings.
[3] O.W. Greenberg, Phys. Rev. Lett. 89, 231602 (2002) [arXiv:hep-ph/0201258].

[4] D. Colladay and V.A. Kostelecký, Phys. Rev. D 55, 6760 (1997) [arXiv:hep-ph/9703464]; Phys. Rev. D 58, 116002 (1998) [arXiv:hep-ph/9809521]. V.A. Kostelecký and R. Lehnert, Phys. Rev. D 63, 065008 (2001) [arXiv:hep-th/0012060]; V.A. Kostelecký, Phys. Rev. D 69, 105009 (2004) [arXiv:hep-th/0312310]; R. Bluhm and V.A. Kostelecký, Phys. Rev. D 71, 065008 (2005) [arXiv:hep-th/0412320].

[5] See, e.g., R. Lehnert, Phys. Rev. D 68, 085003 (2003) [arXiv:gr-qc/0304013].

[6] See, e.g., V.A. Kostelecký and A.G.M. Pickering, Phys. Rev. D 65, 056006 (2002) [arXiv:hep-th/0111123]; Phys. Rev. Lett. 91, 031801 (2003) [arXiv:hep-th/0212382]; R. Lehnert, J. Math. Phys. 45, 3399 (2004) [arXiv:hep-th/0401084]; Phys. Rev. D 74, 125001 (2006) [arXiv:hep-th/0609162]; B. Altschul and D. Colladay, Phys. Rev. D 71, 125015 (2005) [arXiv:hep-th/0411212]; B. Altschul, J. Phys. A 39, 13757 (2006) [arXiv:hep-th/0602235]; A.J. Hariton and R. Lehnert, Phys. Lett. A 367, 11 (2007) [arXiv:hep-th/0612167]; C.M. Reyes, L.F. Urrutia, and J.D. Vergara, Phys. Rev. D 78, 125011 (2008) [arXiv:0810.5379 [hep-ph]].

[7] See, e.g., V.A. Kostelecký and S. Samuel, Phys. Rev. D 39, 683 (1989); V.A. Kostelecký and R. Potting, Nucl. Phys. B 359, 545 (1991); Phys. Rev. D 63, 045007 (2001) [arXiv:hep-th/0008252].

[8] See, e.g., F.R. Klinkhamer, Nucl. Phys. B 578, 277 (2000) [arXiv:hep-th/9912169]; F.R. Klinkhamer and C. Rupp, Phys. Rev. D 70, 045020 (2004) [arXiv:hep-th/0312032].

[9] V.A. Kostelecký, R. Lehnert, and M.J. Perry, Phys. Rev. D 68, 123511 (2003) [arXiv:astro-ph/0212003]; O. Bertolami, R. Lehnert, R. Potting, and A. Ribeiro, Phys. Rev. D 69, 083513 (2004) [arXiv:astro-ph/0310344].

[10] See, e.g., S.M. Carroll, J.A. Harvey, V.A. Kostelecký, C.D. Lane, and T. Okamoto, Phys. Rev. Lett. 87 (2001) 141601 [arXiv:hep-th/0105082].

[11] J. Alfaro, H.A. Morales-Técotl, and L.F. Urrutia, Phys. Rev. D 65, 103509 (2002) [arXiv:hep-th/0108061]; J. Alfaro, H.A. Morales-Técotl, and L.F. Urrutia, Phys. Rev. Lett. 84, 2318 (2000) [arXiv:gr-qc/9909079]; J. Alfaro, M. Reyes, H.A. Morales-Técotl, and L.F. Urrutia, Phys. Rev. D 70, 084002 (2004) [arXiv:gr-qc/0404113].

[12] F.R. Klinkhamer and M. Schreck, Phys. Rev. D 78, 085026 (2008) [arXiv:0709.3217 [hep-ph]]; M.A. Hohensee, R. Lehnert, D.F. Phillips, and R.L. Walsworth, Phys. Rev. Lett. 102, 170402 (2009) [arXiv:0904.2031 [hep-ph]]; [arXiv:0809.3412 [hep-ph]].

[13] V.A. Kostelecký and N. Russell, [arXiv:0801.0287 [hep-ph]].

[14] S.M. Carroll, G.B. Field, and R. Jackiw, Phys. Rev. D 41, 1231 (1990).

[15] R. Lehnert and R. Potting, Phys. Rev. Lett. 93, 110402 (2004) [arXiv:hep-ph/0406128]; Phys. Rev. D 70, 125010 (2004) [arXiv:hep-ph/0408285].

[16] M. Mewes, Phys. Rev. D 78, 096008 (2008) [arXiv:0809.4801 [hep-ph]].

[17] R. Bluhm, V.A. Kostelecký, and N. Russell, Phys. Rev. Lett. 82, 2254 (1999) [arXiv:hep-ph/9810269].

[18] B. Juhasz et al., AIP Conf. Proc. 796, 243 (2005).

[19] R. Bluhm, V.A. Kostelecký, and N. Russell, Phys. Rev. Lett. 79, 1432 (1997) [arXiv:hep-ph/9707364]; Phys. Rev. D 57, 3932 (1998) [arXiv:hep-ph/9809543].

[20] V.A. Kostelecký and R. Potting, Phys. Rev. D 51, 3923 (1995) [arXiv:hep-ph/9501341]; V.A. Kostelecký, Phys. Rev. D 61, 016002 (2000) [arXiv:hep-ph/9909554]; Phys. Rev. D 64, 076001 (2001) [arXiv:hep-ph/0104120].

[21] F. Ambrosino et al. [KLOE Collaboration], JHEP 0612, 011 (2006) [arXiv:hep-ex/0610034].