On the wave-particle duality within the framework of modeling single-photon interference

A P Davydov\textsuperscript{1,3} and T P Zlydneva\textsuperscript{2}

\textsuperscript{1}Department of Applied and Theoretical Physics, Nosov Magnitogorsk State Technical University, 38 Lenin prospect, Magnitogorsk 455000, Russia
\textsuperscript{2}Department of Applied Mathematics and Informatics, Nosov Magnitogorsk State Technical University, 38 Lenin prospect, Magnitogorsk 455000, Russia

\textsuperscript{3}E-mail: ap-dav@yandex.ru

Abstract. The general results of constructing and modeling the photon wave function in the coordinate representation in the framework of quantum mechanics are presented in order to explain single-photon interference phenomena, in particular Young's experiment. The relevance of this function is emphasized in connection with modern single-photon and two-photon experiments. The necessity of replacing the explanation of obtaining coherent light sources using the concept of a train of supposedly real electromagnetic waves emitted by one atom, by the conditional emission by this atom of the photon wave function in the coordinate representation, which then experiences diffraction and interference, is substantiated. A single-photon wave function is simulated based on the consideration of the electric dipole radiation of an atom in classical electrodynamics, and a conclusion is drawn on the possibility to explain Young's experiment. It is argued that although the photon wave function (as well as of particles having mass) is not a physical object, the application of this function significantly weakens the problem of wave-particle duality of light. It is assumed the "nature" of this function reflects the properties of a physical vacuum, but new experiments are required to clarify it.

1. Introduction

It is customary to explain most electromagnetic phenomena within classical electrodynamics, which is simpler and more intuitive than quantum theory, since both approaches give similar results in many cases. For example, it is sometimes believed that both theories give the same formula for the Lorentzian shape of the spectral line of atomic radiation [1], despite the fact that they have differences. From our point of view, the identical results of both theories stem from the quantum nature of the Maxwell equations established by Majorana [2], since from these equations it is possible to derive an equation of the Schrödinger type, obtain its solution as the photon wave function in the coordinate representation, and apply this function to the description of single-photon states [3–6].

The relevance of this function is currently growing again, due to the use of single and two-photon states in quantum teleportation, quantum cryptography and promising ideas for implementing quantum computing. In these experiments, the transmission of one photon (or two connected, "entangled") at certain distances is obviously associated with the propagation in space of the wave function of this photon (or photons) in the coordinate representation.

Also, the wave function of the photon in the coordinate representation becomes demanded from a number of other considerations. For instance, in experiments with single-photon interference, splitters...
are widely used, dividing the photon propagation into two possible channels. Such a separation cannot be interpreted in any way as a division of a train of real electromagnetic waves, since the photon energy is not divided in half, and the whole photon is "sent" either along one path or the other, nevertheless showing wave properties under the corresponding experimental conditions. Using the mathematical apparatus of probability amplitudes, quantum theory correctly explains the results of all single-photon interference experiments, often simultaneously appealing, from a metaphysical point of view, to the concept of the wave function of a “propagating” photon. However, it, in the coordinate representation, is not given and is not used in these cases, since the recipe for its exact construction is little known, despite its presence in the literature (see [3–6] and Refs. there).

A similar situation arises when explaining the production of coherent radiation from non-laser sources (by both methods: dividing the amplitude and front of the "wave"). In this case, it is necessary to replace the often-held idea of the separation of a train of supposedly real electromagnetic waves emitted by one atom and occupying an allegedly certain volume in space with energy and momentum continuously distributed in it, by the concept of separation of the photon wave function in the coordinate representation "radiated" by this atom [6].

Although in the literature the question of constructing the photon wave function in the coordinate representation has been sufficiently developed, its practical construction (of the wave packet) requires finding the coefficients $b(k, \lambda)$ (see formula (2) below) for each specific application of this function.

The aim of the article is to demonstrate how these coefficients can be found for the case of electric dipole radiation, which was originally specified in the framework of classical electrodynamics and apply this function to the explanation of single-photon interference in Young’s experiment.

Thus, on the one hand, we abandon the misconception that an atom emits a classical electromagnetic field when explaining interference, etc. On the other hand, we use classical radiation formulas to construct the photon wave function. Therefore, at this stage, we simulate the photon wave function, and conclusions about its adequacy can only be made after it successfully describes the real experiments. The construction of this function has the character of modeling also for the reason that we select polarization vectors in a certain way, which also determine its structure.

In our opinion, the description with its help of the diffraction and interference phenomena significantly weakens the problem of wave-particle duality of light and particles. Indeed, if for particles having mass, their wave properties are associated with the possibility of applying a wave function to them in the coordinate representation, then for a photon the wave function in the coordinate representation is practically not used, being prohibited in most books on quantum mechanics, starting with [7]. Obviously, using it to explain the interference of light equalizes photons and particles in the rights to the “wave part” of wave-particle duality.

2. The photon wave function in the coordinate representation

The wave function of a photon is often spoken of as a supposedly “physical object”, using concepts such as radiation and propagation of a wave packet, diffraction of it by obstacles, etc. However, the wave function itself is not a physical object: only its square modulus can be measured in an experiment (more precisely - probability density), not to mention the fact that it does not have such attributes of objective existence as mass, electric charge, or other “internal” characteristics. Therefore, the question arises of what kind of physics “stands” behind the wave function, its “radiation”, “propagation”, “collapse”, etc. Clarification of this physics should follow the path to a fundamentally new theory affecting the physical vacuum. This requires fundamentally new experiments, the essence of which remains to be seen. In our opinion, a photon is a quasiparticle of a physical vacuum, the nature of which should be detected at Planck distances. In a first approximation, a photon can be considered as a quasiparticle similar to a magnon in a solid - as a consequence of the propagation in space of some excited quantum state of a physical vacuum [8].

For a long time it was considered [9–13] that the wave function of a photon in the configurational representation could not be constructed. A definite result, delimiting the terminology and the field of
application of the photon wave function, was summarized in [14]. The introduction of this function into science was facilitated by the creation of sources [15, 16] and detectors [17–18] of single photons, as well as experiments with their participation (the first experiment was carried out in [19]).

For the first time, the “localization” of a photon, determined by the probability density, was considered in [20, 21]. Further substantiation of the construction of the coordinate single-particle wave function of the photon was carried out in [22–27] and others.

Vectors describing circularly polarized plane waves, which are eigenfunctions of quantum mechanical operators of momentum, energy and helicity of a photon, are found in [3, 4], namely

\[ \Psi^{(\pm)}(r, t) = \frac{(Oe)e^{\pm i(kr + \beta ct)}}{(2\pi)^{3/2}} \delta_0^{(k)}, \quad \eta^{(\pm)}_{k, \mp}(r, t) = \frac{(Oe)e^{\pm i(kr + \beta ct)}}{(2\pi)^{3/2}} \delta_0^{(k)}, \]

which are also solutions of Maxwell’s equations in the Majorana form [2] and are used to construct the wave function of the photon – the wave packet

\[ \Psi^{(\pm)}(r, t) = \int \frac{b(k, \pm 1)}{(2\pi)^{3/2}} \left( 1 \right) e^{i(kr + \beta ct)} d^3k + \int \left[ \frac{b(-k, \mp 1)}{(2\pi)^{3/2}} \right] \left( 0 \right) e^{i(kr + \beta ct)} d^3k, \]

where \((Oe)\) is the unit of measurement (oersted) of the quantities \(\xi, \eta\); the signs “+” in the upper indices correspond to a photon with positive and hypothetical negative energy, and “\(+\)” or “\(-\)” in the lower indices and in \(b(k, \pm 1)\) correspond to helicity \(\lambda\). Vectors \(e_\lambda(k) = [e_\alpha(k) + i\lambda e_{\lambda H}(k)]/\sqrt{2}\), where \(e_\alpha, e_{\lambda H}\) are real unit mutually orthogonal vectors forming the right-handed triad with \(n = k/k\):

\[ |e_\alpha| = |e_{\lambda H}| = 1; \quad (e_\alpha, n) = (e_{\lambda H}, n) = (e_\alpha e_{\lambda H}) = 0; \quad e_{\lambda H} = [n \times e_\alpha]; \quad n = i\lambda \left[ e_\alpha \times e_\alpha^* \right]. \]

The vector \(e_\alpha\) does not change when changing the direction of the vector \(n: e_\alpha(n) = e_\alpha(-n)\), and from (3) the other equalities follow:

\[ (e_\alpha^* e_\beta) = \delta_{\alpha\beta}; \quad e_\alpha^* e_\beta = \delta_{\alpha\beta}; \quad e_\beta(n) = e_\beta(-n); \quad [e_\alpha(k)]^* = e_{-\alpha}(k) = e_\alpha(-k). \]

The function (2) arises directly as a solution of the Schrödinger equation justified in [3, 4, 6]. It is normalized to the unit probability and satisfies the continuity equation for the probability density \(\rho^{(\pm)}(r, t)\) of photon detection in the vicinity of the point \(r\) at the moment \(t\). For a free photon or one experiencing diffraction, the coefficients \(b(k, \pm 1)\) must be determined by the initial and boundary conditions. However, many patterns can be studied by modeling its wave function. In any case, these coefficients must satisfy the normalization condition

\[ \left\langle \Psi^{(\pm)} \right| \left| \Psi^{(\pm)} \right\rangle = \int \rho^{(\pm)}(r, t) d^3r = \int d^3k \rho_p^{(\pm)}(k) = \int d^3k \left\{ \left| b(k, \pm 1) \right|^2 + \left| b(-k, \mp 1) \right|^2 \right\} = 1. \]

3. Modeling the femtosecond laser radiation

In [3, 4, 28–32], a wave packet with a Gaussian distribution over the photon momenta was simulated, for which the coefficients were chosen in the form

\[ b(k, \pm 1) = \left| b(-k, \mp 1) \right| = \sqrt{\alpha_1^2 \alpha_2 \alpha_3} \exp \left[ -\frac{1}{2} \left( \alpha_1^2 k_x^2 + \alpha_2^2 k_y^2 + \alpha_3^2 (k_z \mp k_0)^2 \right) - ikr_0 \right]. \]
\[ e_t (\mathbf{k}) = \begin{pmatrix} e_{1x} \\ e_{1y} \\ e_{1z} \end{pmatrix} = \begin{pmatrix} 1 - (1 - \cos \theta) \cos^2 \varphi \\ - (1 - \cos \theta) \sin \varphi \cos \varphi \\ - \sin \theta \cos \varphi \end{pmatrix} \]
\[ e_{H} (\mathbf{k}) = \begin{pmatrix} e_{Hx} \\ e_{Hy} \\ e_{Hz} \end{pmatrix} = \begin{pmatrix} - (1 - \cos \theta) \sin \varphi \cos \varphi \\ \cos \theta (1 - \cos \theta) \cos^2 \varphi \\ - \sin \theta \sin \varphi \end{pmatrix} \]  
\[ \begin{pmatrix} 1 - (1 + \cos \theta) \cos^2 \varphi \\ - (1 + \cos \theta) \sin \varphi \cos \varphi \\ \sin \theta \cos \varphi \end{pmatrix} \]

\[ \begin{pmatrix} (1 + \cos \theta) \sin \varphi \cos \varphi \\ \cos \theta (1 + \cos \theta) \cos^2 \varphi \\ - \sin \theta \sin \varphi \end{pmatrix} \]  

(7)

where \( \theta \) and \( \varphi \) determine the vector \( \mathbf{k} \) in the spherical coordinate system, and at \( \pi/2 < \theta \leq \pi \) as

\[ e_t (\mathbf{k}) = \begin{pmatrix} e_{1x} \\ e_{1y} \\ e_{1z} \end{pmatrix} = \begin{pmatrix} 1 - (1 + \cos \theta) \cos^2 \varphi \\ - (1 + \cos \theta) \sin \varphi \cos \varphi \\ \sin \theta \cos \varphi \end{pmatrix} \]
\[ e_{H} (\mathbf{k}) = \begin{pmatrix} e_{Hx} \\ e_{Hy} \\ e_{Hz} \end{pmatrix} = \begin{pmatrix} (1 + \cos \theta) \sin \varphi \cos \varphi \\ \cos \theta (1 + \cos \theta) \cos^2 \varphi \\ - \sin \theta \sin \varphi \end{pmatrix} \]  

(8)

In [3, 4, 28–32] it was stated that at \( t = 0 \) the case \( \alpha_1 = \alpha_2 = \alpha_3 = \alpha \) gives an almost spherically symmetric spatial shape of the wave packet. On the axis \( z \), which is the axis of symmetry of the wave packet, the probability density of detecting a photon in the vanguard of the packet moves almost exactly with the speed \( c \) of light in vacuum. The farther from this axis, the lower the speed of movement of the corresponding probability density. Thus, the initial "spherical" shape of the packet is transformed into a certain "cone-shaped" one, "resembling" the Vavilov-Cherenkov radiation. The spreading speed of the packet (2) is greater, the smaller its initial "radius", in agreement with the general provisions of quantum mechanics. These conclusions are in agreement [32] with the experimentally established [33] "reduction" of the "group" speed of photons compared with the phase velocity of light in vacuum, which reveals the adequacy of model (6)–(8).

4. Photon wave function corresponding to electric dipole radiation of atom

If, however, the single-photon state is determined so that it corresponds to the state of the electromagnetic field which is initially set using classical intensities \( \mathbf{E} \) and \( \mathbf{H} \), then, composing the vector \( \xi = \mathbf{E} + i \mathbf{H} \), we can find [3, 4] the coefficients defining the photon wave function (2):

\[ b(\mathbf{k}, \lambda) \equiv \frac{(\text{Oe})}{\sqrt{8\pi \hbar c}} B(\mathbf{k}, \lambda) = \frac{1}{(\text{Oe})\sqrt{8\pi \hbar c}} \int d^3r \left[ \xi^{(z)}(r, t) \right]^+ \xi(r, t). \]  

(9)

Let an electric dipole is oriented along \( z \) axis and performs harmonic oscillations with frequency \( \omega_0 = k_0 c \), emitting into the wave zone an electromagnetic field, whose intensities are known to be equal

\[ E_r = E_\varphi = 0, \quad E_\theta = A \sin \theta_r \cos (\omega_0 t - k_0 r) / r, \quad H_r = E_\theta = 0, \quad H_\varphi = A \sin \theta_r \cos (\omega_0 t - k_0 r) / r. \]  

(10)

Passing in (10) from the spherical components of the field to the Cartesian, we write

\[ \xi = \begin{pmatrix} \xi_x \\ \xi_y \\ \xi_z \end{pmatrix} = A \begin{pmatrix} \sin \theta_r \cos (\omega_0 t - k_0 r) \\ \cos \theta_r \cos \varphi_r - i \sin \varphi_r \\ \cos \theta_r \sin \varphi_r + i \cos \varphi_r \end{pmatrix}. \]  

(11)

Although in classical electrodynamics the coefficient \( A \) in (10) is expressed in terms of the dipole parameters, it is advisable to find it, for example, based on the time of radiation of the atom, equating all the radiated energy to the value \( h \nu_0 \). In this article we leave the coefficient \( A \) free.

Substitution \( \xi^{(z)}(r, t) \) from (1), as well as (7), (8), (11) into (9), gives three divergent integrals over the variable \( r \). This is due to the approximate nature of formulas (10) and the infinite flow of monochromatic radiation. To eliminate the divergences, we can introduce the cutoff factor \( \exp(-\varepsilon r) \) for the integrals and, after taking them, tend \( \varepsilon \) to zero. Then only one integral remains, giving the...
main contribution proportional to the Dirac function \( \delta(k - k_0) \). As a result, the wave function (2) takes the form:

\[
\Psi^{(+)}(r, t) = \frac{A e^{-i\omega t}}{4a} \sqrt{\frac{k_0}{\pi \hbar c}} \left[ \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \left( \begin{array}{c} iyI_1 - xI_2 \\ -ixI_1 - yI_2 \end{array} \right) + \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \left( \begin{array}{c} iyI_1 + xI_2 \\ -ixI_1 + yI_2 \end{array} \right) \right],
\]

(12)

where

\[
I_1 = \frac{2}{\sqrt{a k_0}} \sum_{n=0}^{\infty} \frac{(-2)^n (k_0 z)^{2n} \Gamma(n+1/2)}{(ak_0)^n (2n)!} J_{n+3/2}(ak_0),
\]

(13)

\[
I_2 = \frac{i}{\sqrt{a k_0}} \sum_{n=0}^{\infty} \frac{(-2)^n (k_0 z)^{2n+1} \Gamma(n+1/2)}{(ak_0)^{n+1} (2n+1)!} J_{n+5/2}(ak_0),
\]

(14)

\[
I_3 = 4 \sum_{n=0}^{\infty} \frac{(-1)^n (k_0 z)^{2n+1} F_2(2, 1, n+5/2; - (ak_0)^2/4)}{(2n+1)(2n+3)(2n)!},
\]

(15)

where \( F_2 \) is the generalized hypergeometric function. In the case \( ak_0 \gg 1 \) and \( |z| \ll a \), in particular applicable to explaining Young's experiment, where \( a = \sqrt{x^2 + y^2} \), series (13)–(15) converge quickly.

Taking into account the terms only with \( n = 0 \), assuming \( z/a \approx z/r = \cos \theta_r \), from (12) we get:

\[
\Psi^{(+)}(r, t) = \frac{A e^{-i\omega t}}{4a \sqrt{\pi \hbar c}} \left[ \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \left( \begin{array}{c} -i \cos (ak_0) \\ \sin (ak_0) \cos \theta_r \end{array} \right) + \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \left( \begin{array}{c} -i \cos (ak_0) \\ -\sin (ak_0) \cos \theta_r \end{array} \right) \right].
\]

(16)

Using the same method as in [6, 31], it can be shown that this function describes a single-photon interference picture of Young's experiment.

5. Conclusion

The results of our modeling of photon wave packet allow to illustrate the possibility of a single-photon approach to the description of electromagnetic phenomena. In particular, it appears that those aspects of interference and diffraction such as the interference pattern of Young's double-slit experiment, which were described in the language of classical electrodynamics, obviously can be described in the language of quantum mechanics without the involvement of the apparatus of second quantization of the electromagnetic field. This significantly expands the scope of “ordinary” quantum mechanics and considerably reduces the problem of wave-particle duality in the present level of our knowledge.

References

[1] Heitler V 1940 *Quantum Theory of Radiation* (Moscow, Leningrad: State Publishing House of Techno-Theoretical Literature) p 129
[2] Mignani R, Recami E and Baido M 1974 About a Dirac-like equation for the photon, according to Ettore Majorana // Lett. N. Cim. 11 No 12 pp 568-72
[3] Davydov A P 2015 Quantum mechanics of photon: wave function in coordinate representation *Elektromagnitnye volny i elektronnye sistemy* 20 No 5 pp 43-61
[4] Davydov A P 2015 *The photon wave function in the coordinate representation* (Magnitogorsk: Publishing House of Nosov MSTU)
[5] Davydov A P 2015 Modeling the propagation in three-dimensional space of a photon wave packet *Actual problems of modern science, technology and education: mater. 73rd international scientific and technical conf.* (Magnitogorsk: Publishing House of Nosov MSTU vol 3) pp 133-7
[6] Davydov A P and Zlydneva T P 2018 The interference of electromagnetic waves from the point
of the photon wave function in the coordinate representation *Ehlektromagnitnye volny i ehlektronnye sistemy* 23 No 8 pp 27-38

[7] Landau L and Peierls R 1930 Quantenelectrodynamik im Konfigurationsraum Zeit. F. Phys. 62 pp 188-98

[8] Davydov A P 2001 Extreme maximons, the structure of fundamental particles, QED, GR and RTG of A. A. Logunov *Ehlektromagnitnye volny i ehlektronnye sistemy* 6 5 4-13

[9] Berestetskii V B, Lifshitz E M and Pitaevskii L P 1982 *Quantum Electrodynamics* (New York: Pergamon Press Ltd, 2nd ed)

[10] Kramers H A 1958 *Quantum Mechanic* (Amsterdam: North-Holland)

[11] Newton T D and Wigner E P Localized states for elementary particles *Reviews of Modern Physics* 21 400-6

[12] Akhiezer A I and Berestetskii V B 1981 *Quantum electrodynamics* (Moscow: Nauka)

[13] Levich V G, Vdovin Y A and Myamlin V A 1971 *Course of theoretical physics* vol II (Moscow: Nauka)

[14] Klyshko D N 1994 Quantum optics: quantum, classical and metaphysical aspects *UFN* 164 11 1187-1215

[15] Gisin N, Ribordy G, Tittel W and Zbinden H. 2002 Quantum cryptography *Rev. Mod. Phys.* 74 1 145-90

[16] Fulconis J et al 2006 Photonic crystal fibre source of photon pairs for quantum information processing *ArXiv: preprint quant-ph/0611232*

[17] Waks E et al 2003 High efficiency photon number detection for quantum information processing *ArXiv: quant-ph/0308054*

[18] Procházka I et al 2004 Recent achievements in single photon detectors and their applications *J. Mod. Opt.* 51 9-10 1289-313

[19] Clauser J 1974 Experimental distinction between the quantum and classical field theoretic predictions for the photo-electric effect *Phys. Rev. D* 835

[20] Mandel M and Wolf E 1995 *Optical coherence and quantum optics* (Cambridge University Press)

[21] Sipe J 1995 Photon wave functions *Physical Review A* 52 1875

[22] Bialynicki-Birula I 1996 Photon wave function *Progress in Optics* vol XXXVI, ed E. Wolf (North-Holland, Amsterdam: Elsevier) pp 248-294

[23] Bialynicki-Birula I 1996 The Photon Wave Function *Coherence and Quantum Optics VII*, ed J H Eberly et al (New York: Plenum Press) pp 313-23

[24] Hawton M 1999 Photon wave functions in a localized coordinate space basis *Phys. Rev. A* 59 pp 3223-7

[25] Smith B J and Raymer M G 2007 Photon wave functions, wave-packet quantization of light, and coherence theory *New J. Phys.* 9 414-48

[26] Cugnon J 2011 The photon wave function *Open Journal of Microphysics* 1 3 41-52

[27] Saari P 20012 Photon localization revisited *Quantum Optics and Laser Experiments* ed S Lyagushyn (Croatia: InTech) pp 49-66

[28] Davydov A and Zlydneva T 2016 Modeling of short-pulse laser radiation in terms of photon wave function in coordinate representation *Paper book of the International Forum IEET-2015* (Izhevsk: Publish House of Kalashnikov ISTU) pp 51-63

[29] Davydov A P and Zlydneva T P 2018 Space-Time probability density of detection of a photon in laser beam of the femtosecond range *2018 14th International scientific-technical conf. APEIE – 44894 proceedings* vol 1, Part 4 (Novosibirsk) pp 58-69

[30] Davydov A P and Zlydneva T P 2016 The Young’s interference experiment in the light of the single-photon modeling of the laser radiation *Proc. ITSMSS-2016* pp 208-15

[31] Davydov A P and Zlydneva T P 2017 The Modeling of the Young's interference experiment in terms of single-photon wave function in the coordinate representation *Proc. IV ITSMSSM-2017* pp 257-65
[32] Davydov A P and Zlydneva T P 2016 On the reduction of free photons speed in modeling of their propagation in space by the wave function in coordinate representation 2016 13th International scientific-techn. conf. on actual problems of electronic instrument engineering (APEIE) – 39281 proceedings vol 1 (Novosibirsk) pp 233-40

[33] Giovannini D et al 2015 Spatially structured photons that travel in free space slower than the speed of light Science 347 (6224) 857-60