e^+e^- BEAM-BEAM PARAMETER STUDY FOR A TeV-SCALE PWFA LINEAR COLLIDER

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Abstract

We perform a beam-beam parameter study for a TeV-scale PWFA (particle-driven plasma wakefield acceleration) e^+e^- linear collider using GUINEA-PIG simulations. The study shows that the total luminosity follows the 1/√σz scaling predicted by beamstrahlung theory, where σz is the rms beam length, which is advantageous for PWFA, as short beam lengths are preferred. We also derive a parameter set for a 3 TeV PWFA linear collider with main beam parameters optimised for luminosity and luminosity spread introduced by beamstrahlung.

Lastly, the study also compare the performance for scenarios with reduced positron beam charge at 3 TeV and 14 TeV with CLIC parameters.

INTRODUCTION

In the blow-out regime of PWFA (particle-driven plasma wakefield acceleration), a dense ultra-relativistic drive beam is used to excite a plasma wake, where plasma electrons are expelled from the region close to the propagation axis, leaving only positively charged ions behind to form a plasma ion bubble cavity. Inside the plasma ion cavity accelerating gradients in the multi-GV/m level can be used to accelerate a trailing main beam.

A previous parameter study on a 1.5 TeV PWFA (particle-driven plasma wakefield acceleration) accelerator derived a parameter set that can provide reasonable stability, energy spread and efficiency for electron acceleration. This study adopted the parameter set in [2], assuming that positrons can be accelerated in a similar manner, and optimised the main beam parameters at the interaction point (IP) for an e^+e^- collider with respect to luminosity and luminosity spread introduced by beam-beam effects. The preference of short beams in PWFA can be exploited to reduce the beam sizes accordingly without increasing the level of beamstrahlung, while achieving a higher luminosity.

Furthermore, this study also examined asymmetric collision scenarios with reduced numbers of positrons at 3 TeV and 14 TeV.

BEAMSTRahlung THEORY

Colliding beams in a linear collider are focused to small transverse dimensions in order to reach high luminosity. This gives rise to intense electromagnetic fields that will bend the trajectories of particles in the opposite beam, and cause the particles to emit radiation in the form of beamstrahlung, and hence lose energy. A large fraction of particles will therefore collide with a less than nominal energy, and form a luminosity spectrum.

Beamstrahlung Parameter

Beamstrahlung can be charaterised by the critical energy defined at half power spectrum [8]

$$ E_\text{c} = \hbar \omega_\text{c} = \frac{3 \hbar \gamma^3 c}{2 R}, \quad (1) $$

where $R$ is the bending radius of the particle trajectory.

It is however more convenient to use the dimensionless Lorentz invariant beamstrahlung parameter defined as [4,5]

$$ \Upsilon = \frac{\gamma^6}{m_e c^3 (p^\mu F^\nu_\mu F^\nu)_1^{1/2}}, \quad (2) $$

where $p^\mu$ is the four-momentum of the particle, and $F^\mu_\nu$ is the electromagnetic field tensor of the beam field. The beamstrahlung parameter can also be written as

$$ \Upsilon = \frac{2 \hbar \omega_\text{c}}{3 \gamma} = \gamma \frac{(E + eB)}{B_\gamma}, \quad (3) $$

where $\gamma$ is the energy of a particle before emitting radiation and $B_\gamma = m_e^2 c^2 / (e \hbar) = 4.4140 \text{GT}$ is the Schwinger critical field. $\Upsilon$ can be interpreted as a measure for the strength of the electromagnetic fields in the rest frame of the electron in units of $B_\gamma$. Since fields above $B_\gamma$ are expected to cause non-linear QED effects, $\Upsilon \ll 1$ is associated with the classical regime, while $\Upsilon \gg 1$ corresponds to the (deep) quantum regime.

$\Upsilon$ is not constant during collision. For Gaussian beams with $N$ particles, horizontal rms beam size $\sigma_x$, vertical rms beam size $\sigma_y$ and rms beam length $\sigma_z$, the average and maximum $\Upsilon$ can be approximated as

$$ \langle \Upsilon \rangle \approx \frac{5}{6} \frac{N r_e^2 \gamma}{\alpha \sigma_x (\sigma_x + \sigma_y)} \quad \Upsilon_{\text{max}} \approx \frac{12}{5} \langle \Upsilon \rangle, \quad (4) $$

where $r_e$ is the classical electron radius and $\alpha$ is the fine structure constant.

Beamstrahlung and Luminosity

In the quantum regime with $\Upsilon \gg 1$, the average number of emitted photons per electron during the collision for a Gaussian beam can be approximated as [5]

$$ n_\gamma \approx 2.54 \frac{\sigma^2 \sigma_z}{r_e \gamma} \langle \Upsilon \rangle^{2/3} = 2.25 \left( \frac{\sigma^2 \sqrt{r_e \sigma_z} N}{\sqrt[N]{(\sigma_x + \sigma_y)}} \right)^{2/3}. \quad (5) $$
The total luminosity for a linear collider is given by

$$L = \frac{N^2}{4\pi \sigma_x \sigma_y} n_b f_t = \frac{N}{4\pi \sigma_x \sigma_y} f_t P_b/\mathcal{E}_b,$$  \hspace{1cm} (6)

where $n_b$ is the number of beams per pulse, $f_t$ is the repetition rate of pulses, $P_b = n_b f_t N \mathcal{E}_b$ is the beam power per beam, $\mathcal{E}_b$ is the beam energy and $H_D$ is a correction factor usually in the range 1.5 - 2 that takes into account the combined effect of the hourglass effect and disruption enhancement due to the attractive force that the two colliding bunches exert on each other. Since $L \propto 1/(\sigma_x \sigma_y)$ and $n_y \propto 1/((\sigma_x + \sigma_y)^{2/3})$, choosing a flat beam with $\sigma_x \gg \sigma_y$ can limit $n_y$ without sacrificing luminosity. This gives the following relation on $\sigma_x$ and $n_y$:

$$\sigma_x = 3.38 \frac{\alpha^2 N}{n_y^{3/2}} \frac{r_c \sigma_y}{\gamma}.$$  \hspace{1cm} (7)

Inserting this into the equation for the total luminosity, we obtain

$$L = \frac{0.30 H_D}{4\pi \alpha^2} \sqrt{\frac{\gamma}{r_c \sigma_y \sigma_x}} \frac{1}{\mathcal{E}_b} \eta P_{AC},$$  \hspace{1cm} (8)

where $\eta$ is the total (wall-plug to beam) conversion efficiency, $P_{AC}$ the wall-plug power for beam acceleration and $\mathcal{E}_b$ is the beam energy.

Equation (5) shows that for $T \gg 1$, a shorter beam can suppress beamstrahlung. This implies that $\sigma_x$ can be reduced accordingly for a flat beam, as described by equation (7), without increasing $n_y$. Consequently, the luminosity can be increased for shorter beams, as outlined by equation (8). This is particularly advantageous for PWFA, since short beams are preferred in PWFA due to the high plasma frequency. E.g. for a plasma with density $n_0 = 10^{16}$ cm$^{-3}$, the plasma wavelength is $\lambda_p = 334$ μm. For comparison, $\lambda_{RF} = 2.51$ cm in CLIC.

**BEAM-BEAM PARAMETER SCAN**

We performed beam-beam simulations using GUINEA-PIG [1], where we optimised collisions of $e^+e^-$ beams with respect to luminosity spread by performing parameter scans over $\beta_x$, $\beta_y$ and $\sigma_z$.

**Equal $e^+e^-$ Beam Charges**

In this study, we assumed that the number of particles in both the $e^+$ and the $e^-$ beams are the same, and that $\beta_y$ can be made arbitrarily small regardless of technical constraints. Furthermore, we define the peak luminosity $L_{0.01}$ as the part of the luminosity corresponding to the quark of mass energy $\sqrt{s} > 0.99 \sqrt{m_0}$, where $\sqrt{m_0}$ is the nominal centre of mass collision energy. The acceptable level of luminosity spread is chosen to be $L_{0.01}/L \approx 1/3$, where $L$ is the total luminosity.

Both beams have $N = 5 \cdot 10^9$ particles and were collided at $\sqrt{s} = 3$ TeV. For each pair of $\beta_y$ and $\sigma_z$, we kept only the results given by an optimal $\beta_x$ that corresponds to $L_{0.01}/L \approx 1/3$. The corresponding results for $L$ and $L_{0.01}$ are shown in fig. 1 and 2 respectively. The unit $b x^{-1}$ denotes “per beam crossing”.

![Figure 1: Contour plot of total luminosity $L$ vs. beam length $\sigma_x$ and vertical beta function $\beta_y$, where the horizontal $\beta_x$ for each pair of $\sigma_x$ and $\beta_y$ has been chosen such that $L_{0.01}/L \approx 1/3$.](image1)

![Figure 2: Contour plot of peak luminosity $L_{0.01}$ vs. beam length $\sigma_x$ and vertical beta function $\beta_y$, where the horizontal beta function $\beta_x$ for each pair of $\sigma_x$ and $\beta_y$ has been chosen such that $L_{0.01}/L \approx 1/3$.](image2)

Assuming $\sigma_x$ can be made sufficiently small despite technical constraints to keep $n_y$ constant as $\sigma_z$ is reduced, and that $\sigma_y$ is kept constant, eq. (8) gives the scaling $L \propto 1/\sqrt{\sigma_z}$. The luminosity is plotted against $\sigma_z$ for a selection of $\beta_y$ along with the corresponding $L \propto 1/\sqrt{\sigma_z}$ fits in fig. 3.

The $1/\sqrt{\sigma_z}$-scaling agree very well with simulation results, especially for larger values of $\beta_y$. The disagreement at small $\beta_y$ may be due to the hourglass effect, which imposes $\beta_y \geq \sigma_z$. When $\beta_y < \sigma_z$, a small beam size is only maintained over a small length, which reduces luminosity. Thus, using our range of $\sigma_z$-values, the luminosity appears to decrease faster than the $1/\sqrt{\sigma_z}$-scaling.
In a previous parameter study \cite{2} on a 1.5 TeV PWFA linear accelerator, we found a parameter set for the main beam with acceptable stability, energy spread and efficiency. This parameter set involves an electron beam with \( N = 5 \times 10^9 \) electrons and a rms beam length of \( \sigma_z = 5 \) µm. The corresponding optimised results for a \( N = 5 \times 10^9 \sigma_z = 5 \) µm beam from this study are listed in table \ref{table1} together with relevant results from \cite{3}.

The normalised amplification factor \( \Lambda / \Lambda_0 \) \cite{3} is a measure for stability used to quantify the amplification of the transverse jitter of the main beam. It is listed along with the relative rms energy spread \( \sigma_E / \langle E \rangle \) and drive beam to main beam efficiency \( \eta \) in table \ref{table1}.

In deriving this parameter set, we did not consider technological constraints on the vertical beta function. The vertical beta function \( \beta_y = 0.068 \) mm from the 3 TeV CLIC parameter set \cite{7} represents what is currently achievable, which is about an order of magnitude larger than our proposed value.

**Reduced Positron Beam Charge**

In the blow-out regime of PWFA, an electron beam will be focused by the positive ion background, while a positron beam will be defocused. Different approaches such as hollow channel plasma \cite{8} and the quasi-linear \cite{9} regime have been studied, but positron acceleration remain one of the main challenges in PWFA, as there are currently no self-consistent scheme for positron acceleration in plasma that can simultaneously provide high efficiency, low preserved emittance and mitigation of transverse instabilities.

Here we examine the effects of asymmetric \( e^+ e^- \) collisions on luminosity at two energy levels, where the number of particles \( N_{e^+} \) in the \( e^+ \) beam is only a fraction of the number of particles \( N_{e^-} \) in the \( e^- \) beam. Other beam parameters such as \( \sigma_x, \beta_x, \) and \( \epsilon_{Nx,y} \) are identical for the \( e^+ e^- \) beams, and can be found in table \ref{table1}. The results for different scenarios are summarised in table \ref{table2} together with CLIC parameters \cite{10}.

As a result of the reduced \( N_{e^+} \), the total luminosity is reduced by approximately the same factor compared to cases where \( N_{e^+} = N_{e^-} \). However, by reducing \( N_{e^-} \), the beamstrahlung from the electron beam is also reduced, which results in a narrower luminosity spectrum. Alternatively, this also allows the horizontal beam size to be further reduced without increasing \( n_y \).

Even in the \( N_{e^+} = 0.1 N_{e^-} \) scenario, a PWFA linear collider using the parameter set in table \ref{table1} can still provide a comparable luminosity level per beam crossing compared to CLIC\cite{10}. Furthermore, the \( N_{e^+} = 0.1 N_{e^-} \) scenario shown in table \ref{table2} has a luminosity spread that is significantly better than our defined tolerance of \( \mathcal{L} / \mathcal{L}_{0.01} = 1 / 3 \), which indicates that the horizontal beam size can likely be reduced even further to increase the total luminosity.

The muon collider submission to the European Particle Physics Strategy \cite{11} showed that a 14 TeV muon collider can provide a similar effective discovery potential as the 100 TeV FCC. For comparison, the same parameters for 14 TeV collision energy are shown in table \ref{table3}.

At 14 TeV, a common luminosity goal for linear colliders is \( 40 \cdot 10^{34} \) cm\(^{-2}\) s\(^{-1}\), which can be achieved in the \( N_{e^+} = N_{e^-} \) and \( N_{e^+} = 0.1 N_{e^-} \) scenarios with total beam powers of 20 MW and 281 MW, respectively. For comparison, the

\footnotesize
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
**Parameter** & **Symbol [unit]** & **Value** \\
\hline
Plasma density & \( n_0 \) [10\(^{16}\) cm\(^{-3}\)] & 2.0 \\
Particle number & \( N \) [10\(^9\)] & 5 \\
Rms beam length & \( \sigma_z \) [µm] & 5 \\
Horizontal beta function & \( \beta_x \) [mm] & 5 \\
Vertical beta function & \( \beta_y \) [µm] & 5 \\
Normalised horizontal emittance & \( \gamma x \) [mm mrad] & 0.887 \\
Normalised vertical emittance & \( \gamma y \) [mm mrad] & 0.02 \\
Relative rms energy spread & \( \sigma_E / \langle E \rangle \) [%] & 1.1 \\
Normalised amplification factor & \( \Lambda / \Lambda_0 \) & 6 \\
Drive beam to main beam efficiency & \( \eta \) [%] & 37.5 \\
Beam power/beam & \( P_b / (f_b m_b) \) [kWs] & 1.2 \\
Beamstrahlung photons/e\(^-\) & \( n_y \) & 2.3 \\
Total luminosity/beam crossing & \( \mathcal{L} \) [10\(^{34}\) m\(^{-2}\) bx\(^{-1}\)] & 4.3 \\
Peak 1% luminosity/beam crossing & \( \mathcal{L}_{0.01} \) [10\(^{35}\) m\(^{-2}\) bx\(^{-1}\)] & 1.4 \\
\hline
\end{tabular}
\caption{Main Parameters for a 3 TeV PWFA Linear \( e^+ e^- \) Collider}
\end{table}

\footnotesize
\(^1 n_0 = 312, f_b = 50 \) Hz for CLIC.
which can be seen in the large luminosity spread. Further-
what is achievable today. Thus, study in how to achieve
Note that here the calculations in GUINEA-PIG were done with
2 CLIC parameter set
the two energy levels.
more, we also made the optimistic assumption that the same
parameter sets in table 3 have been optimised for 14 TeV,
to achieve this luminosity. However, note that none of the
plasma, and the possibility of achieving a higher luminosit y
such a small vertical beta function is required.
thus give rise to a large luminosity spread.
The proposed parameter set furthermore has a vertical beta function that is an order of magnitude smaller than what is achievable today. Thus, study in how to achieve such a small vertical beta function is required.
Due to the challenges of accelerating positron beams in a plasma, and the possibility of achieving a higher luminosity

CLIC parameter set\(^2\) requires a total beam power of 90 MW to achieve this luminosity. However, note that none of the parameter sets in table 3 have been optimised for 14 TeV, which can be seen in the large luminosity spread. Furthermore, we also made the optimistic assumption that the same emittance levels and beta functions can be maintained for the two energy levels.

### CONCLUSION

The suppression of beamstrahlung with decreasing beam length (given that the horizontal beam size can be scaled appropriately to limit beamstrahlung) is beneficial for PWFA, as short beams are preferred. The derived parameter set for a 3 TeV e\(^+\)e\(^-\) PWFA linear collider shows a promising level of luminosity, while maintaining a reasonable luminosity spread. Even for a scenario where the positron beam only contains 10% of the particles in the electron beam, the derived parameter set can still provide luminosity per beam crossing comparable to that of CLIC at 3 TeV.

At 14 TeV, the derived parameter set is able to achieve the luminosity goal of 40 \(\cdot\) 10\(^{14}\) cm\(^{-2}\) s\(^{-1}\) with a significantly lower beam power than the CLIC parameter set. These parameters are however not optimised for this energy level, and thus give rise to a large luminosity spread.

The proposed parameter set furthermore has a vertical beta function that is an order of magnitude smaller than what is achievable today. Thus, study in how to achieve such a small vertical beta function is required.

Due to the challenges of accelerating positron beams in a plasma, and the possibility of achieving a higher luminosity

\(^2\) Note that here the calculations in GUINEA-PIG were done with \(\beta_x,\gamma\) that do not take non-linear effects into account and thus differ from the values given in [1]. Here we chose \(\beta_x = 9.0\) mm and \(\beta_y = 0.147\) mm, which are matched to the spot sizes and emittances given in [1] at 3 TeV.

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