Magnetic-field switchable metal-insulator transitions in a quasi-helical conductor

Bernd Braunecker,1 Anders Ström,2 and G. I. Japaridze3,4

1Departamento de Física Teórica de la Materia Condensada, Facultad de Ciencias, Universidad Autónoma de Madrid, 28049 Madrid, Spain
2Department of Physics, University of Gothenburg, SE 412 96 Gothenburg, Sweden
3Andronikashvili Institute of Physics, Tamarashvili 6, 0177 Tbilisi, Georgia
4Ilia State University, Cholokashvili Avenue 3-5, 0162 Tbilisi, Georgia

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We study Anderson localization in disordered helical conductors that are obtained from one-dimensional conductors with spin-orbit interaction and a magnetic field, or from equivalent systems. We call such conductors “quasi-helical” because the spins of the counterpropagating modes are not perfectly antiparallel and have a small spin-wavefunction overlap that is tunable by the magnetic field. Due to the overlap, disorder backscattering is possible and allows a localization transition. A conductor can pass through two localization transitions with increasing field, one from the conventionally localized system to the quasi-helical conductor (with localization length exceeding the system length), and one at a higher field again to a localized state, due now, however, to backscattering below the magnetic field induced pseudo-gap. We investigate these transitions using a unified two-step renormalization group approach.

I. INTRODUCTION

Over 50 years ago, Anderson showed that a metal-insulator transition can arise due to localization of particles by scattering on a disorder potential. Since then, Anderson localization has evolved into an important topic of condensed matter physics, photonics, and ultracold atom gases. While the basic localization mechanism can be understood on a single-particle basis, a complete investigation needs to include further interactions, especially if they compete against localization. To study such a competition it would be advantageous to control the interactions externally, and to pass through the metal-insulator transition on demand. Here we show that this can indeed be achieved in the presently much investigated one-dimensional (1D) helical conductors through an external magnetic field.

Helical conductors are characterized by spin-filtered transport, in which opposite spins (or Kramers partners) are bound to the right (R) and left (L) moving conduction modes. In conventional 1D conductors, even weak disorder is already sufficient to turn the conductor into an insulator by disorder-induced backscattering between the Fermi points ±kF. In a helical conductor, however, backscattering is only possible together with a spin-flip, and the conductor is insensitive to normal, spin-preserving disorder scattering.

Helical conductors appear at the edges of topological insulators or in quantum wires or nanotubes with strong spin-orbit interaction (SOI). They have attracted much attention recently as they allow for spin-filtering, Cooper pair splitting, and, if in contact with a superconductor, the realization of Majorana end states. While localization cannot occur in a perfect helical conductor, many of the investigated conductors are imperfect and we call them “quasi-helical”: the spins moving in opposite directions are not perfectly antiparallel. They provide the handle to tune localization externally. Examples are semiconductor nanowires with strong SOI in the presence of a uniform magnetic field or, without SOI, of a spiral magnetic field which can also take the form of a spiral Overhauser field due to ordered nuclear spins.

The SOI shifts the spin ± bands by the wave vectors ±q0 (Fig. 1 dashed lines). A magnetic field Bz perpendicular to the ± axis has now two effects. It lifts the degeneracy at k = 0 by opening a pseudo-gap (Fig. 1 solid lines), and it breaks time-reversal symmetry. Turning the Fermi level to the center of the pseudo-gap by letting kF = q0 allows conduction only through the modes close to momenta ±2q0 with opposite spins. Through Bz, however, these spins are no longer antiparallel, disorder backscattering becomes again possible, and localization can occur.

In this paper we provide a unified approach to such localization, taking into account disorder, magnetic field, spin overlaps, and electron interactions. We formulate a two-step renormalization group (RG) approach within the Luttinger liquid (LL) framework that provides us with a transparent picture of the underlying physics. We consider the zero temperature case, valid if the temperature is smaller than any of the gap sizes.

As a result, we find that varying Bz can cause localization transitions of two kinds. A localized system at Bz = 0 can make a transition to a quasi-helical conductor at a critical field Bz = lying (for the example of InAs nanowires) in the range up to ~ 1 T. This transition is quite abrupt and appears when the Bz generated pseudo-gap overcomes the disorder gap, and the quasi-helical state is lower in energy. The transition is marked by a rapid increase of the localization length 0 , and the system is conducting if 0 is greater than the system length L. Through the equal-spin overlap, 0 becomes strongly Bz dependent and decreases with increasing field. At a larger critical field 2Bz > ~ 1 T, when 0 < L, backscattering between ±2q0 causes again localization, resulting in a 4q0 modulated density wave co-
existing with the transverse spin polarization generated by $B_z$. This coexistence of density wave and uniform polarization distinguishes this phase from conventional localization, and we shall call it “sub-gap localization.” For very strong disorder, the helical phase is suppressed ($B_\text{ph}^* = B_\text{ph}^*$), and a transition takes place directly between the two localized phases. We show concrete results for InAs nanowires in Sec. VI below. The underlying physics, however, is general and applies to many materials, except for the edge states of topological insulators due to their different band structure. For the latter, quasi-helicity can still be obtained, but has different interesting consequences.\cite{13}

The plan of the remainder of the paper is as follows. In Sec. II we introduce the model of the quasi-helical conductor. The approach for its solution in the presence of disorder is discussed in Sec. III. In Sec. IV we present the necessary background of the bosonization framework, and in Sec. V we discuss the technical details of the two-step RG approach. Section VI contains the discussion of the results for the example of InAs nanowires and the conclusions.

II. MODEL

We consider a generic interacting 1D quantum wire of length $L$ with SOI and an external magnetic field, described by the Hamiltonian

$$
H = \sum_{s,s'} \int dx \, \psi_{s'}^\dagger(x) \left[ \left( \frac{\hat{p}^2}{2m} - \mu \right) \delta_{s,s'} + \alpha_R \sigma_s^z \hat{p} 
+ \Delta_Z \sigma_{s,s'}^z \right] \psi_{s'}(x) + U_{\text{dis}}.
$$

(1)

Here $\psi_s(x)$ are the electron operators at position $x$ for spin $s = \uparrow, \downarrow = +, -$, $p = -i \hbar \partial_x$ is the momentum operator, $\mu$ the chemical potential, $m$ the band mass, $\alpha_R$ the Rashba SOI strength, and $\Delta_Z = \mu_B g_B B_z/2$ the Zeeman interaction strength, with Bohr magneton $\mu_B$ and Landé $g$-factor $g$. $U_{\text{dis}}$ is a general electron-electron interaction and $U_{\text{dis}}$ the disorder potential. $\sigma^z$ are the spin Pauli matrices, with the spin axes chosen such that $\alpha_R$ couples to $\sigma^x$ and $B_z$ to $\sigma^y$. Equation (1) describes a quantum wire with a single transverse subband, and we exclude the influence of higher subbands.\cite{14,15} Realizations of Eq. (1) are found in GaAs, InAs,\cite{11,16,17} Ge/Si,\cite{18,19} or InSb nanowires.\cite{20}

Without $U_{\text{dis}}$ and $U_{\text{dis}}$, Eq. (1) leads to the bands shown in Fig. 1. The SOI shifts the $\uparrow, \downarrow$ dispersions by $\pm q_0 = \pm m \alpha_R/\hbar$. The $B_z$ field opens a pseudo-gap at momentum $k = 0$ by spin-flip scattering. We denote the lower (upper) resulting band by $a$ ($b$). Diagonalizing $H$ leads to the dispersions $E_{a,b}(k) = \hbar^2 k^2/2m \mp \sqrt{\Delta_Z^2 + \hbar^2 (\alpha_R^2 k^2)}$, and the wave functions $|a,k\rangle = u_{-k} |\uparrow\rangle - u_k |\downarrow\rangle$, $|b,k\rangle = u_k |\uparrow\rangle + u_{-k} |\downarrow\rangle$, with $u_k = 1 + \hbar \alpha_R k/\sqrt{\Delta_Z^2 + \hbar^2 (\alpha_R^2 k^2)}$\cite{14} and $u_{-k} = 1 - \hbar \alpha_R k/\sqrt{\Delta_Z^2 + \hbar^2 (\alpha_R^2 k^2)}$. Overlaps of the form $\langle a/b,k | a/b,k' \rangle$ weight the disorder backscattering amplitudes at a given energy. Figure 2 shows the two overlaps which will be relevant for the localization transition.

III. APPROACH

To study a disordered interacting conductor in 1D, it is generally convenient to bosonize Eq. (1) and use an RG approach. For the present model, however, this approach exhibits some peculiarities. First, the overlap integrals depend on $k$ and the problem is no longer scale invariant, a central assumption for RG. Second, the low-energy physics below the scale set by $\Delta_Z$ is described mostly by the band $a$. Hence, the RG procedure of reducing the en-
energy scale can lead to a problematic crossing the bottom of the b band, which also invalidates the bosonization formulation that neglects the band curvature. While the exact treatment of every aspect would require a separate investigation, the following two observations allow us to still implement a global RG scheme capturing the relevant physics in a transparent way.

First, Fig. 2 shows that the spin overlaps above \( \Delta_Z \) approach their asymptotics only slowly with increasing energy \( E \) (as \( |\langle s | R | a, L \rangle|^2 \sim \Delta_Z^2 / 2 m a^2 E \) and \( |\langle s | R | b, L \rangle|^2 \sim 1 - \Delta_Z^2 / 4 E^2 \)). Considering constant overlaps evaluated, e.g., at half of the high energy cutoff scale yields a valid, rather conservative estimate of the true influence of the overlaps. Second, the interactions also renormalize \( \Delta_Z \) and it grows quickly under the RG flow. Hence the reduced RG bandwidth \( \Delta_Z \) for the modes originating from the true band bottom. While the description of the proper overlap integrals separately, the standard lines of a spinless LL, and localization backscattering (the second RG step) follows from LL theory for the modes originating from the pseudogap. The corresponding Hamiltonian is obtained \( H_{\text{bos}} \) by suppressing all \( b \)-related fields, which leads to 
\[
H_{\text{bos}} = (u/\sqrt{2}) \int dx [K^{-1}_a(\partial_z \varphi_a)^2 + K_\sigma(\partial_z \varphi_\sigma)^2],
\]
with \( u^2 = \langle \frac{1}{4} [u^2_R + u^2_L + u^2_\sigma (K_\sigma K_- + K^{-1}_\sigma K^-_\sigma)] \rangle \) and \( K^2_\sigma = K_\sigma K^{-1}_\sigma (u_\sigma K_\sigma + u_\sigma K^{-1}_\sigma) (u_\sigma K_\sigma + u_\sigma K^{-1}_\sigma) \). We have neglected here also a marginal coupling between \( a \) and \( b \) fields because it affects mostly the fermionic responses but not the bosonic theory.

V. DISORDER AND RENORMALIZATION GROUP APPROACH

Disorder can be expressed by a random potential \( U_{\text{dis}} \) with Gaussian distribution that scatters between the bands \( i, j = (a/b, L/R) \). The scattering amplitude is described by the dimensionless disorder strength \( \tilde{\mu} \), proportional to the square of the strength of each individual scattering potential, which in turn is proportional to \( (|\langle j | j \rangle|^2) \), hence \( \tilde{D}_{ij} = |\langle i | j \rangle|^{2} \tilde{D} \). Following the standard replica disorder averaging approach we obtain the scaling equations (including also the amplitude \( \tilde{\eta} \) expressing bulk backscattering, see Ref. [3])
\[
\begin{align*}
\partial_l \tilde{D}_{aa} &= (3 - K_\sigma - K^{-1}_\sigma) \tilde{D}_{aa}, \\
\partial_l \tilde{D}_{ab} &= (3 - K_\sigma - y) \tilde{D}_{ab}, \\
\partial_l \tilde{D}_{bb} &= (3 - K_\sigma - K^{-1}_\sigma) \tilde{D}_{bb}, \\
\partial_l \tilde{\eta}_s &= \tilde{\eta}_s K^{-1}_\rho (2 \tilde{D}_{ab} + \tilde{D}_{aa}) / 4 \tilde{\eta}_s, \\
\partial_l \tilde{\eta}_\sigma &= \tilde{\eta}_\sigma K^{-1}_\rho (2 \tilde{D}_{ab} + \tilde{D}_{aa}) / 2 \tilde{\eta}_\sigma, \\
\partial_l \tilde{\eta}_\rho &= \tilde{\eta}_\rho K^{-1}_\rho (2 \tilde{D}_{ab} + \tilde{D}_{aa}) / 4 \tilde{\eta}_\rho, \\
\partial_l \delta(l) &= \delta(l) (2 - (K_\rho + K^{-1}_\rho) / 2) \delta(l).
\end{align*}
\]
with \( l \) the running RG scale, \( \tilde{D}_{aa} = \tilde{D}_{(a,R),(a,L)} \), and \( \tilde{D}_{ab} = \tilde{D}_{(a,R),(b,L)} \). We have neglected scattering between \( (b,R) \leftrightarrow (b,L), (a,R) \leftrightarrow (b,R), (a,L) \leftrightarrow (b,L) \) after verifying that it has no effect. We have also defined \( \delta(l) = \Delta_Z(l) / E(l) \) with \( E(l) = hv/\kappa l \) the running effective bandwidth, for \( \kappa(l) = \kappa_0 l^3 \) and Fermi velocity.
The localization length $\xi_{loc}$ of an InAs nanowire as a function of $B_x$ for $\tilde{D} = 0.01$ and $L = 5 \, \mu\text{m}$. Close to $B_z = 0$ the system is localized due to conventional backscattering. At $B_x \sim B_x^* \approx 0.9 \, T$ the system crosses over to a quasi-helical conductor and at $B_x \sim B_x^{**} \approx 2 \, T$ to a localized phase due to sub-gap backscattering. The transition at $B_x^*$ results from the competition of the renormalization of $\Delta_Z$ and $\tilde{D}_{ab}$ and is thus expected to be quite abrupt and independent of $\tilde{L}$. The transition at $B_x^{**}$ is a strongly $\tilde{L}$ dependent cross-over.

The latter equations express the competition between disorder backscattering and the delocalizing effect by repulsive interactions and by $B_x$ induced spin-flip scattering. Localization occurs when the latter is not strong enough such that $\tilde{D}_{ab} \sim 1 \, (\tilde{D}_{aa} \text{ remains small above the gap})$ before we reach $E = \Delta_Z^* (\delta(l) \sim 1)$ or $\kappa(l) > \tilde{L}$. Otherwise we switch to the second step, described by $H_{loc}^{\text{ab}}$, with the parameters $K_a$ and $u_a$ obtained from the resulting $K_{\rho,\sigma}, u_{\rho,\sigma}$ of the first step. As argued above, the spin overlap weighting $\tilde{D}_{aa}$ can now (discontinuously) be replaced by $\langle a, L | a, R \rangle$ evaluated at $E = \mu$. The RG equations are

$$\partial_l K_a = -K_a^2 \tilde{D}_{aa}/2,$$
$$\partial_l \tilde{D}_{aa} = (3 - 2K_a) \tilde{D}_{aa},$$
$$\partial_l u_a = -u_a K_a \tilde{D}_{aa}/2, \quad (12)$$

describing, for effectively spinless fermions, the competition between localization and delocalizing superconducting fluctuations. The latter overrule disorder localization above the critical attractive interaction strength $K_a > 3/2$. However, $K_a < 1$ for repulsive interactions, disorder scattering is relevant, and localization occurs if $\tilde{D}_{aa} \sim 1$ is reached while $\kappa(l) < \tilde{L}$; otherwise the (finite) system is a helical conductor.

VI. RESULTS AND DISCUSSION

To give a definite example, we focus on InAs nanowires using $\kappa_R = 4 \times 10^{-11} \, eV \, m$, $\gamma = -9$, $m = 0.040 m_e$ (with electron mass $m_e$), $v_F = 2 \times 10^5 \, m/s$, $u_p = v_F/K_p$ and $u_{\rho} = v_F/K_{\rho}$, with $K_p = 0.5$, $K_{\rho} = 1$, $y = 0.1 |\langle aL | bR \rangle|^2$, and short length cutoff $\kappa = 15 \, \mu\text{m}$ (which is longer than the lattice constant $a_0 = 6.06 \, \text{Å}$, and expresses a smaller effective bandwidth). The localization length is $\xi_{loc} = \min_{ij} \kappa(l^*)/\tilde{D}_{ij}(l^*)$ with $l = l^*$ the scale at which the RG flow stops (in the first or the second step). If a $\tilde{D}_{ij}(l^*) = 1$ is reached before $\kappa(l) > \tilde{L}$, the system is localized, $\xi_{loc} < \tilde{L}$. We consider disorder strengths about $\tilde{D} \sim 0.01$, leading at $B_z = 0$ to $\xi_{loc} \sim 0.3 \, \mu\text{m}$. For a sample length of, e.g., $\tilde{L} = 5 \, \mu\text{m}$, the system is well localized. At small fields, the $\tilde{D}_{ab}$ and $B_x$ scattering processes compete, and if $B_x$ passes a critical value $B_x^* < 1 \, T$, $\Delta_Z^*(l)$ overrules the disorder backscattering and the system becomes a quasi-helical conductor, where $\xi_{loc} > \tilde{L}$ is now determined by the sub-gap disorder strength $\tilde{D}_{aa}$. At $B_x = B_x^{**} \sim 2 \, T$, $|\langle a, R | a, L \rangle|$ becomes large enough such that $\xi_{loc} < \tilde{L}$ and the system crosses over into the sub-gap localized phase. In Fig. 3 we plot this crossover behavior for $\tilde{D} = 0.01$. Tracing similar curves for various disorder strengths $\tilde{D}$ leads to the phase diagram shown in Fig. 4. For strong disorder, the quasi-helical conduction phase is absent, and the crossover takes place directly between the two localized phases.

To conclude, we note that the disordered quasi-helical system shows at $B_x^*$ a phase transition from the conventional Anderson localized phase in which $\xi_{loc}$ only weakly increases with $B_x$, to a phase in which $\xi_{loc}$ jumps to a very large value and then decreases with increasing $B_x$. For fields $B_x^* < B_x < B_x^{**}$, $\xi_{loc}$ exceeds $\tilde{L}$ and forms a quasi-helical conductor before crossing over into another localized phase. While the former localized phase is characterized by the conventional $2k_F = 2g_0$ modulated charge density wave, the latter sub-gap localized phase combines a $4g_0$ density wave with a uniform electron polarization as the order parameter. The existence of a quasi-helical conduction state between the two insulating phases as a function of $B_x$ is a peculiar behavior that could be used to test if a conductor is helical. Remarkably the conducting phase extends into the regime of quite strong disorder, which indicates that a quasi-helical conductor does not necessarily require ultraclean
samples. The boundaries of this phase in the phase diagram are controlled predominantly by $\Delta Z$, and so the $g$-factor of the material has the largest influence on the phase diagram. For instance, a similar phase diagram as Fig. 4 for InSb with $g \approx 50$ spans over $B_x = 0 - 0.1 \text{T}$ and $D = 0 - 0.002$.

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