Pose Synchronization for Quadrotor Networks under Fixed General Interconnection Topology: A Passivity Approach

Tatsuya Ibuki*, Satoshi Nakano**, Mahato Endou*, and Mitsuji Sampei*

Abstract: In this paper, a distributed 3-D pose synchronization law is proposed for a group of networked quadrotors. The quadrotor network consisting of multiple quadrotors with dynamics and interconnection topology between them is first defined, and then, the definition of pose synchronization is provided. A novel pose synchronization law is next proposed, where linear approximation and passivation approaches are employed in the controller design. The former allows us to independently deal with the yaw, vertical, and horizontal motion dynamics of the quadrotor network, and the latter enables us to apply a passivity-based output synchronization technique. Convergence is next analyzed, and finally, the present pose synchronization law is demonstrated via simulation and an experiment to show its validity.

Key Words: quadrotor network, cooperative control, synchronization, passivity.

1. Introduction

Cooperative control in multi-agent systems gains huge interests in these decades [1]–[3], thanks to technological innovation and information technology such as small, accurate mobile robots/sensors, and low computational costs. The main goal of the cooperative control is to achieve desired motion coordination of a group of networked robots for better performance, robustness, and so on. In theoretical aspects, numerous researchers have proposed distributed cooperative control mechanisms including basic consensus [4], [5], synchronization [6]–[8], coverage [9], [10], and other coordination [11]–[13].

The objective of this paper is to achieve 3-D pose (position and attitude) synchronization as a kind of motion coordination, where all the 3-D poses converge to a common one (or desired formation) with a desired trajectory. Passivity of agent dynamics often plays a central role for cooperative control design since it makes convergence analysis easy to tackle via energy-based techniques. For example, passivity-based approaches are applied to synchronization problems in vector space [14] and extended to those in 3-D space [2], [15]. The authors in [3] also present passivity-based schemes for vector/3-D space. However, the authors in [2], [3], [15] assume that agents are fully-actuated (i.e. possible to control 3-D positions/orientations independently), which makes the present control schemes hard to implement. Similarly, in 3-D motion coordination research, actual dynamics of 3-D mobile vehicles is often neglected due to complexity of those models as in [16]–[18].

In this work, quadrotors are introduced as the 3-D mobile vehicles in view of mechanical simplicity and great mobility. Especially, the 3-D motion dynamics is explicitly considered in a pose synchronization problem, which makes controller design challenging. The quadrotors are rapidly spread worldwide, and numerous studies are devoted to their applications to various tasks such as building inspection at a high altitude [19], imagery collection in disaster sites [20], and parcel delivery across the sky [21]. On the other hand, the quadrotors have only four rotors as inputs for the six degrees of freedom: they are underactuated. As a result, they have to rotate their bodies for horizontal motion and cannot hover except for one typical attitude. We thus directly design individual synchronization laws for each 3-D position and yaw angle of the quadrotors, and then, the remaining roll/pitch angle analysis is provided.

We first define quadrotor networks composed of multiple quadrotors with dynamics and interconnection topology between them. The quadrotor dynamics is then linearized to completely decouple the yaw angle, vertical and horizontal motion dynamics. This makes controller design tractable, and we can apply a passivation technique and a passivity-based synchronization scheme to each dynamics. The advantage of the present approach is that in spite of dealing with the dynamical models, the control law does not require acceleration information which is hard to sense in general. This is based on the usage of the special property of the quadrotors: rotating their bodies generates thrust force in the horizontal directions. The present method is then verified through convergence analysis, simulation, and experimental demonstration.

The main contribution of this paper is to explicitly consider the quadrotor dynamics for 3-D motion coordination problems, where the special property of the quadrotor motion, explained above, is exploited for the controller design. The other contribution is to provide the convergence analysis and experimental demonstration as well as simulation verification. This paper heavily revises the conference paper [22], which includes providing the detailed analysis and improving both the simulation and experimental demonstration. Especially, compared with the simple experimental setting in [22] where only three quadrotors with unweighted and bidirectional interconnection topology are utilized, this paper increases the number of quadrotor testbeds and demonstrates general interconnection topology to validate the main result.

*Department of Systems and Control Engineering, Tokyo Institute of Technology, Tokyo 152-8550, Japan
**Department of Mechanical and Control Engineering, Tokyo Institute of Technology, Tokyo 152-8550, Japan
E-mail: ibuki@sc.e.titech.ac.jp
(Received November 3, 2017)
(Revised December 29, 2017)
2. Quadrotor Network

2.1 Quadrotor Dynamics

This work considers networked quadrotors \( V \in \{1, \ldots, n\} \) in 3-D space (Fig. 1). An inertial coordinate frame is denoted by \( \Sigma_w \) and body-fixed frames are \( \Sigma_i, i \in V \). The origin of \( \Sigma_i \) is located on the center of quadrotor \( i \)'s body. We represent the position of quadrotor \( i \) in \( \Sigma_w \) by \( p_{wi} = [x_i, y_i, z_i]^T \in \mathbb{R}^3 \) and the orientation by XYZ Euler angles: \( \xi_{wi} = \begin{bmatrix} \gamma_i \beta_i \alpha_i \end{bmatrix} \in \mathbb{R}^3 \), with \( \gamma_i, \beta_i \in (\pi/2, \pi/2), \alpha_i \in (-\pi, \pi) \). Then, the rotation matrix of \( \Sigma_i \) relative to \( \Sigma_w \) is derived as

\[
R_{wi}(\gamma_i, \beta_i, \alpha_i) = R_z(\gamma_i)R_y(\beta_i)R_x(\alpha_i) \in SO(3),
\]

where \( R_x, R_y, R_z \in SO(3) \) are respectively the rotation matrices with respect to \( x, y, \) and \( z \)-axes [23].

In this paper, we consider vertical body acceleration \( a_i \in \mathbb{R} \) and 3-D angular body velocity \( \omega_i = [\omega_{i, x} \omega_{i, y} \omega_{i, z}]^T \in \mathbb{R}^3 \) as control inputs of quadrotor \( i \), because many actual quadrotors are controlled by those commands. This setting is also used in some research on quadrotors as in [24]. The translational motion in \( \Sigma_w \) is written by

\[
\ddot{p}_{wi} = R_{wi}(\gamma_i, \beta_i, \alpha_i) \begin{bmatrix} 0 \\ 0 \\ a_i \end{bmatrix}, \quad (1)
\]

where \( g > 0 \) is the gravitational acceleration. On the other hand, the relation between \( \xi_{wi} \) and \( \omega_i \) is given as

\[
\dot{\xi}_{wi} = \begin{bmatrix} \cos \alpha_i \cos \beta_i & \sin \alpha_i \cos \beta_i & 0 \\ -\sin \alpha_i \cos \beta_i & \cos \alpha_i \cos \beta_i & \sin \alpha_i \\ \sin \beta_i & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_{i, x} \omega_{i, y} \omega_{i, z} \end{bmatrix} \in \mathbb{R}^3.
\]

We next derive linearized dynamics to make controllers easy to design. Applying linear approximation around \( \gamma_i, \beta_i, \omega_{i, x}, \omega_{i, z} = 0 \) to the dynamics (1), (2) yields

\[
\ddot{p}_{wi} = R_{wi}(\gamma_i, \beta_i, \alpha_i) \begin{bmatrix} 0 \\ 0 \\ a_i \end{bmatrix}, \quad \dot{\xi}_{wi} = \begin{bmatrix} \cos \beta_i & \sin \beta_i \\ -\cos \alpha_i \cos \beta_i & \cos \alpha_i \sin \beta_i & \sin \alpha_i \cos \beta_i \\ \sin \alpha_i \sin \beta_i & \cos \alpha_i \sin \beta_i & \cos \alpha_i \cos \beta_i \end{bmatrix} \begin{bmatrix} \omega_{i, x} \omega_{i, y} \omega_{i, z} \end{bmatrix}.
\]

Moreover, we introduce the input transformation

\[
u_i = \begin{bmatrix} u_{i, 1} \\ u_{i, 2} \\ u_{i, 3} \\ u_{i, 4} \end{bmatrix} = \begin{bmatrix} a_i - g \\ \omega_{i, x} \cos \alpha_i - \omega_{i, z} \sin \alpha_i \\ \omega_{i, x} \sin \alpha_i + \omega_{i, z} \cos \alpha_i \\ \omega_{i, z} \end{bmatrix} \in \mathbb{R}^4. \quad (3)
\]

Then, by another approximation around \( u_{i, 1} = 0 \), the following linearly approximated quadrotor dynamics is obtained.

\[
\ddot{p}_{wi} = \begin{bmatrix} g \beta_i \\ -g \gamma_i \\ \dot{\xi}_{wi} = \begin{bmatrix} u_{i, 2} \\ u_{i, 3} \\ u_{i, 4} \end{bmatrix}. \quad (4)
\]

Remark 1 The conditions \( \omega_{i, z}, \omega_{i, y}, \gamma_i, \beta_i \approx 0 \) for the linearized dynamics (4) hold when unrealistically large angular velocity is not generated for horizontal motion of the quadrotors. This is reasonable since the quadrotors cannot keep hovering when largely rotating their roll/pitch angles. On the other hand, \( u_{i, 1} = a_i - g \approx 0 \) is interpreted that the hovering state \( a_i = g \) is considered as a standard state.

Remark 2 The linearized dynamics (4) implies that the dynamics of the 3-D position and yaw angles is formed by integrators, which enables to employ passivation techniques as discussed later. We note here that the dynamics of \( x_i, (y_i) \) is given by triple integrators because the dynamics of \( \beta_i (\gamma_i) \) is formed by a single integrator.

2.2 Interconnection Topology between Quadrotors

Interconnection topology of the networked quadrotors is given by a fixed weighted digraph \( G \equiv (V, \mathcal{E}, \mathcal{W}) \); \( V \) represents a node set, \( E \subset V \times V \) an edge set meaning information flow between nodes, and \( \mathcal{W} \) an weight set representing importance/reliability of information [25]. Here, the weights \( w_{ij} \in \mathcal{W} \) satisfy \( w_{ij} > 0 \) for \( (j, i) \in \mathcal{E} \) and \( w_{ij} = 0 \) otherwise. These notations give a useful definition \( \mathcal{N}_j := \{ j \in V \mid (j, i) \in \mathcal{E} \} \) as the neighbor set of quadrotor \( i \), where \( j \in \mathcal{N}_i \) means that quadrotor \( i \) receives information from quadrotor \( j \).

Let us next define a condensation digraph of \( G \), represented by \( G' \) (refer to [1] for the details). Subgraphs \( G_j \subseteq G \) are defined as strongly connected components of \( G \) if \( G_j \) is strongly connected and any other subgraphs of \( G \) strictly containing \( G_j \) are not strongly connected (Fig. 2). Also, \( G_j \) is defined as \( (\emptyset, \emptyset, \emptyset) \) if node \( h \) of \( G \) does not form any strongly connected components. We then regard each \( G_j \) as node \( H_j \) of \( G' \), and \( (H_i, H_j) \) is defined as an edge of \( G' \) if \( G \) has at least one directed edge from a node of \( G_j \) to that of \( G_j \). The definition of the condensation digraph \( G' \) yields the following properties [1].

- The condensation digraph \( G' \) is acyclic.
- If the digraph \( G \) has a directed spanning tree, \( G' \) also has a directed spanning tree.
- If \( G' \) contains a directed spanning tree, then the root of the tree, denoted by \( H_1 \) in this work, is uniquely determined.

![Fig. 1 Quadrotor model.](image1)

![Fig. 2 Digraph G and corresponding condensation digraph G'.](image2)
• If $G$ has a directed spanning tree, then there exists at least one node in $G$ having the root $H_1$ as only one parent node (e.g. $H_3$ in Fig. 2).

Throughout this paper, a group of networked quadrotors with the dynamics (1) and (2) and a digraph $G$ is called quadrotor network $\Sigma$.

### 3. Goal and Output Synchronization

#### 3.1 Research Objective

The objective of this work is to propose a distributed control law for the quadrotor network $\Sigma$ to achieve pose synchronization defined as follows.

**Definition 1** The quadrotor network $\Sigma$ is said to achieve pose synchronization if the following equations hold for all $i, j \in \mathcal{V}$.

\[
\lim_{t \to \infty} \|p_{\alpha i}(t) - p_{\alpha j}(t)\| = 0, \quad \lim_{t \to \infty} \|\dot{p}_{\alpha i}(t) - v_{\alpha i}(t)\| = 0, \quad (5a)
\]

\[
\lim_{t \to \infty} |\dot{\alpha}_i(t) - \alpha_j(t)| = 0, \quad \lim_{t \to \infty} |\dot{\alpha}_i(t) - \omega_{\alpha i}(t)| = 0, \quad (5b)
\]

\[
\lim_{t \to \infty} |\beta_i(t) - \beta_j(t)| = 0, \quad \lim_{t \to \infty} |\gamma_i(t) - \gamma_j(t)| = 0. \quad (5c)
\]

Here, $v_{\alpha i} = [v_{\alpha i, 2} \ v_{\alpha i, 3} \ vd_i] \in \mathbb{R}^3$ and $\omega_{\alpha i} \in \mathbb{R}$ are respectively common desired linear velocity and yaw angular one in $\Sigma_{\alpha}$ provided by smooth functions on time $t$, and $\gamma_{\alpha i}, \beta_{\alpha i} \in (-\pi/2, \pi/2)$ are common desired roll and pitch angles.

The first equations of Eqs. (5a) and (5b) mean that the 3-D positions and yaw angles of all the quadrotors converge to common values\(^1\). Then, Eq. (5c) is required for successful tracking to the common desired trajectory $v_{d i}$ (Fig. 3), where $g\beta_{\alpha i} = \dot{v}_{d i}$ and $-\gamma_{\alpha i} = \dot{v}_{d i}$ corresponding to the dynamics (4) should hold.

#### 3.2 Passivity-Based Output Synchronization

A preliminary result is first introduced based on [14]. Consider the following nonlinear system.

\[
\begin{align*}
\dot{x}_p &= f(x_p, u_p), \quad x_p \in \mathbb{R}^m, \quad u_p, y_p \in \mathbb{R}^l.
\end{align*}
\]  

(6)

Then, passivity is defined as follows [26].

**Definition 2** The system (6) is passive if there exists a continuously differentiable positive semi-definite function $S(x_p) \geq 0$, called storage function, such that

\[
\dot{S} = \frac{\partial S}{\partial x_p} f(x_p, u_p) \leq y_p^T \beta_{\alpha i}.
\]

Let us next consider the multi-agent system

\[
\begin{align*}
\dot{x}_{i,p} &= f(x_{i,p}, u_{i,p}), \quad x_{i,p} \in \mathbb{R}^m, \quad u_{i,p}, y_{i,p} \in \mathbb{R}^l, \quad i \in \mathcal{V},
\end{align*}
\]  

(7)

where each dynamics is assumed to be passive, and define output synchronization as

\[
\lim_{t \to \infty} |y_{j,p}(t) - y_{j,p}(t)| = 0 \quad \forall i, j \in \mathcal{V}. 
\]  

(8)

Then, we have the following proposition as an extended result of Theorem 1 in [14].

**Proposition 1** The following synchronization law for the system (7) achieves the output synchronization (8) if the digraph $G$ corresponding to the interconnection topology contains a directed spanning tree.

\[
u_{i,p} = K_i \sum_{j \in \mathcal{N}_i} w_{ij}(y_{j,p} - y_{i,p}), \quad K_i > 0, \quad i \in \mathcal{V}. 
\]  

(9)

**Proof.** See Appendix A.1.

Proposition 1 means that if each dynamics in a multi-agent system is passive, the outputs are synchronized by the synchronization law (9) based on summation of output errors with respect to neighbors. This plays a key role in our main result.

### 4. Passivity-Based Pose Synchronization

We propose a 3-D pose synchronization law based on the linearized dynamics (4). Since the 3-D positions and yaw angles in Eq. (4) are completely decoupled, we can consider the synchronization problems independently. Each dynamics is first passivated, and we then employ Proposition 1 to show the synchronization of each state.

#### 4.1 Yaw Angle Synchronization

The first partial objective is yaw angle synchronization defined as Eq. (5b). We obtain the yaw angle dynamics from Eq. (4) as

\[
\dot{\alpha}_i = u_{i,\alpha}, \quad i \in \mathcal{V}. 
\]  

(10)

The dynamics (10) is given by a single integrator, and thus passive from $u_{i,\alpha}$ to $\sin(\alpha_i/2)$ with respect to the storage function $S_{\alpha}(\alpha_i) := 2(1 - \cos(\alpha_i/2)) \geq 0$.

The passivity motivates us to propose the yaw angle synchronization law

\[
u_{i,\alpha} = k_{\alpha,\alpha} \sum_{j \in \mathcal{N}_i} w_{ij} \sin \frac{\alpha_j - \alpha_i}{2} + \omega_{\alpha i}, \quad k_{\alpha,\alpha} > 0, \quad i \in \mathcal{V}
\]  

(11)

which is formed by the output errors with respect to neighbors similarly to the output synchronization law (9). Then, we have the following proposition.

**Proposition 2** Under the digraph $G$ containing a directed spanning tree, the synchronization law (11) for the system (10) achieves the yaw angle synchronization (5b).

**Proof.** See Appendix A.2.

**Remark 3** Only in this subsection, the synchronization law (11) is slightly different from the original output synchronization law (9): sinusoidal functions are introduced. This is motivated by well-known Kuramoto oscillators [6]–[8] and often
used for attitude synchronization problems [2], [8], [15]. On
the other hand, the angle differences are divided by 2 in the
sinusoidal functions differently from the original Kuramoto
oscillators or our previous work [2], [15]. This guarantees the
synchronization in the global region \( \alpha_i \in (\pi/2, \pi/2) \) instead of
\( \alpha \in (-\pi/2, \pi/2) \) shown in [2], [15], which is one of our con-
tributions (notice that linearization is not applied for \( \alpha_i \) in this
work).

### 4.2 Vertical Position Synchronization

The second partial objective is vertical position synchroniza-
tion defined as a part of Eq. (5a):

\[
\begin{align*}
\lim_{t \to \infty} |z_i(t) - z_j(t)| &= 0 \\
\lim_{t \to \infty} |\dot{z}_i(t) - \dot{z}_j(t)| &= 0 \quad \forall i, j \in \mathcal{V}.
\end{align*}
\]

We obtain the vertical position dynamics from Eq. (4) as

\[
\ddot{z}_i = v_{iz}, \quad \dot{v}_{iz} = u_{i1}, \quad i \in \mathcal{V},
\]

where \( v_{iz} \in \mathbb{R} \) is newly introduced as an internal state. Notice
here that the dynamics (13) given by double integrators is not
passive from \( u_{i1} \) to \( z_i \). We thus passivate the system (13).

Let us first apply the state transformation \( \tilde{z}_i(t) := z_i(t) - \int_0^t v_{iz}(\tau) d\tau \in \mathbb{R} \) to Eq. (13), which
provides

\[
\begin{align*}
\ddot{\tilde{z}}_i &= v_{i\tilde{z}} - v_{iz} \quad \forall i \in \mathcal{V}.
\end{align*}
\]

We next employ the input transformation

\[
u_{i1} := \tilde{v}_{i\tilde{z}} + u_{i1} - k_{i1} e_{i\tilde{z}} - k_{i2} u_{i2}, i \in \mathcal{V},
\]

where \( u_{i2} \in \mathbb{R} \) is a new input, \( e_{i\tilde{z}} := \dot{\tilde{z}}_i - v_{i\tilde{z}} \in \mathbb{R} \), and \( k_{i1}, k_{i2} > 0 \).

Then, the following lemma holds.

**Lemma 1** If \( k_{i1} > k_{i2} \) holds, the system (14), (15) is passive
from \( u_{i2} \) to \( \tilde{z}_i \) with respect to the storage function

\[
S_{i\tilde{z}} := \frac{1}{2(k_{i1} - k_{i2})} |e_{i\tilde{z}}|^2 + \frac{1}{k_{i1}} \left( \frac{1}{2} \right) |\dot{z}_i|^2 
\]

**Proof.** Direct calculation yields \( S_{i\tilde{z}} \geq 0 \).

An image of the passivation technique is shown in Fig. 4.

Lemma 1 motivates us to propose the vertical position synchro-
nization law

\[
u_{i2} = k_{i2} \sum_{j \in \mathcal{N}_i} w_{ij}(\tilde{z}_j - \tilde{z}_i), k_{i2} > 0, \quad i \in \mathcal{V},
\]

and we have the following proposition.

**Proposition 3** Under the digraph \( G \) containing a directed spanning
tree and \( k_{i1} > k_{i2} \) \( \forall i \in \mathcal{V} \), the synchronization law (15) and
(16) for the system (13) achieves the vertical position synchro-
nization (12).

**Proof.** See Appendix A.3.

Let us now explain the structure of the present vertical po-
sition synchronization law (15) and (16). Eqs. (15) and (16)
yield

\[
u_{i1} = \tilde{v}_{i\tilde{z}} + k_{i1}(v_{i\tilde{z}} - \tilde{z}_i) + k_{i2} \sum_{j \in \mathcal{N}_i} w_{ij}(\tilde{z}_j - \tilde{z}_i)
\]

\[
+ k_{i3}(k_{i1} - k_{i2}) \sum_{j \in \mathcal{N}_i} w_{ij}(\tilde{z}_j - \tilde{z}_i), \quad i \in \mathcal{V}.
\]

We see from this form that the first two terms are tracking in-
puts to the desired trajectory \( v_{i\tilde{z}} \), and the remaining terms mean
vertical motion synchronization inputs. Namely, the present
passivity-based synchronization law has intuitive structure.

### 4.3 Horizontal Position Synchronization

The final objective is horizontal position synchronization de-
defined as the remaining part of Eq. (5):

\[
\begin{align*}
\lim_{t \to \infty} |x_i(t) - x_j(t)| &= 0 \\
\lim_{t \to \infty} |\dot{x}_i(t) - \dot{x}_j(t)| &= 0 \\
\lim_{t \to \infty} |y_i(t) - y_j(t)| &= 0 \\
\lim_{t \to \infty} |\beta_i(t) - \beta_j(t)| &= 0.
\end{align*}
\]

This paper discusses only the x-axis motion because the same
argument holds for the y-axis one. We obtain the x-axis motion
dynamics from Eq. (4) as

\[
\begin{align*}
\dot{x}_i &= v_{ix}, \quad v_{ix} = g\beta_i, \quad \beta_i = u_{i3}, \quad i \in \mathcal{V}.
\end{align*}
\]

Here, an internal state \( v_{ix} \in \mathbb{R} \) is newly introduced. We note
that the dynamics (17) given by triple integrators is not passive
from \( u_{i3} \) to \( x_i \). The system (17) is thus passivated similarly to
the vertical motion case.

We first apply the state transformation \( \tilde{x}_i(t) := x_i(t) - \int_0^t v_{ix}(\tau) d\tau \in \mathbb{R} \) to Eq. (17), which provides

\[
\begin{align*}
\dot{\tilde{x}}_i &= v_{i\tilde{x}} - v_{ix} \\
\dot{v}_{i\tilde{x}} &= g\beta_i, \quad \beta_i = u_{i3}, \quad i \in \mathcal{V}.
\end{align*}
\]

The following input transformation is next employed.

\[
u_{i3} := \beta_i - k_{i5} e_{i\beta} + k_{i6} e_{i\beta}, \quad i \in \mathcal{V}.
\]

Here, \( e_{i\beta} := \dot{\beta}_i / g - u_{i3} \in \mathbb{R} \) (\( u_{i3} \in \mathbb{R} \) is a new input), \( \beta_i := \beta_i + u_{i3}, \quad k_{i5}, k_{i6} > 0, e_{i\beta} := \dot{\beta}_i / g \in \mathbb{R} \),
and \( k_{i5}, k_{i6} > 0 \). Then, the following lemma holds.

**Lemma 2** If \( (k_{i3} - k_{i4})k_{i5} - k_{i6} > 0 \) holds, the system (18), (19) is passive from \( u_{i3} \) to \( \tilde{x}_i / g \) with respect to the storage function

\[
S_{i\tilde{x}} := \frac{1}{2k_{i4}((k_{i3} - k_{i4})k_{i5} - k_{i6})} \left( e_{i\beta}^2 + \frac{e_{i\beta}^2}{k_{i6}} \right) \geq 0,
\]

**Proof.** Direct calculation yields \( S_{i\tilde{x}} = -e_{i\beta}^2 / k_{i4} - e_{i\beta}^2 / (k_{i4}k_{i6}) + u_{i3}^2 / g \).

![Image 4](image-url) Image of passivation with \( v_{i\tilde{z}} \equiv 0 \).
The passivation structure is similar to that in Fig. 4. Lemma 2 motivates us to propose the synchronization law
\[ u_{i,x} = \frac{k_{i,x}}{g} \sum_{j \in N_i} w_{ij}(\dot{x}_j - \dot{x}_i), \quad k_{i,x} > 0, \quad i \in \mathcal{V}, \quad (20) \]
and we have the following proposition.

Proposition 4 Under the digraph \( G \) containing a directed spanning tree and \( (k_{i,3} - k_{i,4})k_{i,5} - k_{i,6} > 0 \) \( \forall i \in \mathcal{V} \), the synchronization law (19), (20) for the system (17) achieves the horizontal position synchronization associated with the \( x \)-axis motion.

Proof. See Appendix A.4.

Let us now explain the structure of the present horizontal position synchronization law (19) and (20). Eqs. (19) and (20) yield
\[ u_{i,3} = \dot{\beta}_d + (k_{i,3} + k_{i,5})(\dot{\beta}_d - \dot{\beta}_i) + \frac{k_{i,3}k_{i,5} - k_{i,6}}{g}(v_{d,3} - \dot{x}_i) + k_{i,3} \sum_{j \in N_i} w_{ij}(\dot{\beta}_j - \dot{\beta}_i) + \frac{k_{i,3}(k_{i,3} - k_{i,4} + k_{i,5})}{g} \sum_{j \in N_i} w_{ij}(\dot{x}_j - \dot{x}_i) + k_{i,3}(k_{i,3} - k_{i,4})(k_{i,5} - k_{i,6}) \sum_{j \in N_i} w_{ij}(x_j - x_i), \quad i \in \mathcal{V}. \]

We see from this structure that the first three terms are tracking inputs to the desired trajectories \( v_{d,3}, \dot{\beta}_d \), and the remaining terms mean motion synchronization inputs in the \( x \)-axis direction. Moreover, a positive feature of the present law is to replace the acceleration state \( \ddot{x}_i \) by the pitch angle \( \dot{\beta}_i \) based on the special property of the quadrotor (linearized) dynamics, i.e., \( \ddot{x}_i = g\ddot{\beta}_i \), which means that rotating the bodies generates thrust force in the horizontal directions. This replacement helps implementation in the sense of sensing accuracy.

4.4 Pose Synchronization for Quadrotor Network

We finally summarize Propositions 2–4 as follows.

Theorem 1 Under the digraph \( G \) containing a directed spanning tree, \( k_{i,1} > k_{i,2}, (k_{i,3} - k_{i,4})k_{i,5} - k_{i,6} > 0 \) \( \forall i \in \mathcal{V} \), and the corresponding gain conditions for the \( y \)-axis motion, the quadrotor network \( \Sigma \) with the present synchronization law (3), (11), (15), (16), (19), (20), and the corresponding law for the \( y \)-axis motion locally achieves the pose synchronization (5).

Proof. This argument is proved by considering Propositions 2–4 and that of the \( y \)-axis motion simultaneously.

This result provides only local stability analysis, and convergence analysis in wider regions is one of our future directions.

5. Verification

5.1 Simulation

The effectiveness of the present pose synchronization law is shown via simulation with 20 quadrotors represented by the original dynamics (1) and (2). The interconnection topology depicted in Fig. 2 is utilized in this verification. We randomly set the initial states with \( x_i(0), y_i(0), z_i(0) \in [-3,3] m, \dot{x}_i(0), \dot{y}_i(0), \dot{z}_i(0) \in [-1.5,1.5] m/s, \gamma_i(0), \beta_i(0) \in [-0.1,0.1] \text{rad}, \alpha_i(0) \in [-1,1] \text{rad}, \text{and } \dot{\gamma}_i(0) = \dot{\beta}_i(0) = \dot{\alpha}_i(0) = 0 \) for all \( i \in \mathcal{V} \).

The controller gains are set as \( k_{1,1,i} = 1.0, k_{1,4,i} = 4.0, k_{2,2,i} = 1.0, k_{i,3} = 2.0, k_{i,4} = 0.5, k_{i,5} = 5.0, k_{i,6} = 0.2 \) \( \forall i \in \mathcal{V} \) which satisfy the conditions in Theorem 1. We finally set the common desired velocity as \( v_y = [0.2 - 0.2 \sin \pi/5, 0] \) m/s and \( \omega_d = 0.05 \) rad/s.

The simulation results illustrated in Fig. 5 show that the present pose synchronization law successfully achieves the 3-D pose synchronization in the sense of Eq. (5).

5.2 Experiment

We next demonstrate the present method via an experiment. The experimental environment is shown in Fig. 6 (a). As illustrated in Fig. 6 (b), 4 quadrotors with interconnection topology containing a directed spanning tree are employed, and position biases are introduced in the horizontal position synchronization law to avoid collision. In the current system, thrust force of each rotor cannot be measured, and thus local feedback controllers to achieve desired acceleration inputs \( a_i \) with high accuracy cannot be integrated. Therefore, we demonstrate only the yaw angle and horizontal position synchronization here. The controller gains are set as \( k_{1,1,x,y,z} = 2, k_{2,2,x,y,z} = 0.5, k_{3,3,x,y,z} = 1, k_{4,4,x,y,z} = 0.5, k_{5,3} = 3.0, k_{5,4} = 0.1, k_{5,5} = 9.8, k_{5,6} = 0.3 \) \( \forall i \in \mathcal{V} \). The common desired velocity is set as \( v_{d,x} = 0 \) m/s, \( v_{d,y} = 0 \) m/s and \( \omega_d = 0.05 \pi \cos(0.25 \pi t) \) rad/s due to the limited workspace. The total sampling time is about 20 ms.

The experimental results depicted in Figs. 6 (c)–6 (i) show that the pose synchronization is almost achieved. On the other hand, in spite of setting \( v_{d,z} = 0 \), the quadrotors slightly move in the \( y \)-axis direction probably due to the calibration errors of.
the motion capture cameras. One of our future directions is thus to overcome the technical issues by tuning local controllers and equipping sensors to measure thrust force.

6. Conclusion

This paper has studied a 3-D pose synchronization problem for a group of networked quadrotors. The quadrotor network composed of multiple quadrotors with dynamics and interconnection topology have been first defined. We have next linearized the quadrotor dynamics to obtain completely decoupled systems for yaw angle, vertical and horizontal motion, which makes controller design easy. Then, after passivating each system, we have proposed a passivity-based output synchroniza-

the motion capture cameras. One of our future directions is thus to overcome the technical issues by tuning local controllers and equipping sensors to measure thrust force.

6. Conclusion

This paper has studied a 3-D pose synchronization problem for a group of networked quadrotors. The quadrotor network composed of multiple quadrotors with dynamics and interconnection topology have been first defined. We have next linearized the quadrotor dynamics to obtain completely decoupled systems for yaw angle, vertical and horizontal motion, which makes controller design easy. Then, after passivating each system, we have proposed a passivity-based output synchroniza-

the motion capture cameras. One of our future directions is thus to overcome the technical issues by tuning local controllers and equipping sensors to measure thrust force.

6. Conclusion

This paper has studied a 3-D pose synchronization problem for a group of networked quadrotors. The quadrotor network composed of multiple quadrotors with dynamics and interconnection topology have been first defined. We have next linearized the quadrotor dynamics to obtain completely decoupled systems for yaw angle, vertical and horizontal motion, which makes controller design easy. Then, after passivating each system, we have proposed a passivity-based output synchroniza-

the motion capture cameras. One of our future directions is thus to overcome the technical issues by tuning local controllers and equipping sensors to measure thrust force.

6. Conclusion

This paper has studied a 3-D pose synchronization problem for a group of networked quadrotors. The quadrotor network composed of multiple quadrotors with dynamics and interconnection topology have been first defined. We have next linearized the quadrotor dynamics to obtain completely decoupled systems for yaw angle, vertical and horizontal motion, which makes controller design easy. Then, after passivating each system, we have proposed a passivity-based output synchroniza-

the motion capture cameras. One of our future directions is thus to overcome the technical issues by tuning local controllers and equipping sensors to measure thrust force.

6. Conclusion

This paper has studied a 3-D pose synchronization problem for a group of networked quadrotors. The quadrotor network composed of multiple quadrotors with dynamics and interconnection topology have been first defined. We have next linearized the quadrotor dynamics to obtain completely decoupled systems for yaw angle, vertical and horizontal motion, which makes controller design easy. Then, after passivating each system, we have proposed a passivity-based output synchroniza-
Appendix A  Proofs

A.1 Proof of Proposition 1

Proof. Necessity: This is shown by the contraposition; if the digraph $G$ does not contain a directed spanning tree, some outputs are not synchronized. It is obvious because in this situation, there exist at least two groups which do not exchange the output information each other.

Sufficiency: This is proved by using the condensation digraph $G'$. Since $G'$ is acyclic, we can apply an induction approach from the root of the directed spanning tree to all the leaves. In the subsequent discussion, we often refer to $G'$ in Fig. 2 to help understanding.

We first consider the root group $H_1$ and denote its node set by $\mathcal{V}_1$. Then, since this group forms a strongly connected component and does not receive any information from other groups, we can show the achievement of the output synchronization as follows. Define the Lyapunov function candidate $U_1 \geq 0$ as

$$U_1 := \sum_{i \in \mathcal{V}_1} \eta_i S_i,$$

where $S_i \geq 0$, $i \in \mathcal{V}_1$ are the storage functions of each agent's passivity, and $\eta_i > 0$, $i \in \mathcal{V}_1$ are the positive scalars satisfying the property

$$[\eta_1 \eta_2 \cdots \eta_{|\mathcal{V}_1|}]L_{\eta_1} = 0 \quad (A.1)$$

for the weighted graph Laplacian $L_{\eta_1} \in \mathbb{R}^{|\mathcal{V}_1| \times |\mathcal{V}_1|}$ corresponding to $H_1$ [2] ($| \cdot |$ for a set means its cardinality). Then, the passivity and the output synchronization law (9) yield

$$\dot{U}_1 \leq \sum_{i \in \mathcal{V}_1} \eta_i \eta_i T \Psi_{i,p} y_{i,p}$$

$$= \sum_{i \in \mathcal{V}_1} \sum_{j \in \mathcal{N}_i} \eta_i w_{ij} (y_{j,p} - y_{i,p})$$

$$= \sum_{i \in \mathcal{V}_1} \sum_{j \in \mathcal{N}_i} \frac{\eta_i w_{ij}}{2} \left(\|y_{j,p}\|^2 + \|y_{j,p}\|^2 - \|y_{i,p} - y_{j,p}\|^2\right)$$

$$= -\frac{1}{2} \sum_{i \in \mathcal{V}_1} \sum_{j \in \mathcal{N}_i} \eta_i w_{ij} \|y_{i,p} - y_{j,p}\|^2. \quad (A.2)$$

Here, the last equality holds from the graph Laplacian property (A.1). Therefore, from Eq. (A.2) and the strong connectivity, LaSalle’s invariance principle [26] concludes that the output synchronization is achieved in the group $H_1$. Namely, all the agents in $H_1$ eventually have a common output.

Let us next consider each strongly connected group having the root $H_1$ as only one parent node ($H_3$ in Fig. 2). Since the digraph $G$ contains a directed spanning tree, some agents in this group have neighbors in $H_1$. We now pick up one agent, denoted by $l$, from those neighbors in $H_1$. Then, the error dynamics between agent $l$ and agent $i$ in the current group $H_l$ is given by

$$\dot{y}_{i,p} - \dot{y}_{l,p} = \sum_{j \in \mathcal{N}_l} K_l w_{lj}(y_{j,p} - y_{l,p}) - \sum_{j \in \mathcal{N}_l} K_l w_{lj}(y_{j,p} - y_{l,p})$$

$$= \sum_{j \in \mathcal{N}_l} K_l w_{lj}(y_{j,p} - y_{l,p}) - \sum_{j \in \mathcal{N}_l} K_l w_{lj}(y_{j,p} - y_{l,p})$$

$$- \sum_{j \in \mathcal{N}_l} K_l w_{lj}(y_{j,p} - y_{l,p}) - \sum_{j \in \mathcal{N}_l \setminus \mathcal{N}_l^c} K_l w_{lj}(y_{j,p} - y_{l,p})$$

$$- \sum_{j \in \mathcal{N}_l \setminus \mathcal{N}_l^c} K_l w_{lj}(y_{j,p} - y_{l,p}). \quad (A.3)$$

where $N_l^c \subset N_l$ means the neighbors of agent $l$ within the root group $H_1$. We thus obtain the following linear system by stacking $y_{ij,p} - y_{lj,p}$ for all $l$ in the current group $H_l$, denoted by $\bar{y}_{ij,p}$.

$$\dot{\bar{y}}_{ij,p} = A \bar{y}_{ij,p} + \mu. \quad (A.4)$$

Here, $A$ is the constant matrix formed by $K_l$, $w_{lj}$ of the second, forth and fifth terms of Eq. (A.3). On the other hand, $\mu$ consists of the first and third terms, and vanishes when the root group $H_1$ achieves output synchronization.

Let us now consider the situation when $\mu = 0$ holds (i.e. synchronization within $H_1$). Then, Eq. (A.4) shows the closed loop dynamics of leader-following type output synchronization for the current group $H_l$, where $y_{ij,p}$ can be regarded as the leader’s output because it is synchronized in $H_1$. We note that this system is given by a linear time-invariant system. Namely, synchronization directly means exponential stability of its origin, which is proved by the similar analysis to the above analysis for $H_1$ (refer to [15] for details). Therefore, by regarding $\mu$ as a vanishing perturbation of the system (A.4), we can employ perturbation theory [26] to show the asymptotic stability of the origin of the perturbed system (A.4). This means that the group $H_1$ achieves the output synchronization, which results in a common output within $H_1$ and $H_3$.

Because the condensation digraph $G'$ is acyclic, each strongly connected group gradually achieves the output synchronization from the root group. Notice here that even for
the group having multiple parent groups (e.g., $H_2$ in Fig. 2 has the parent groups $H_1$, $H_3$), those groups first achieve synchronization and thus can be regarded as one leader. Therefore, by using induction, the output synchronization is achieved in all the groups.

A.2 Proof of Proposition 2

Proof. We define new yaw angles $\bar{\alpha}_i \in (-\pi, \pi]$, $i \in V$ compensating for the time variation by the common desired angular velocity $\omega_d$ as $\bar{\alpha}_i(t) := \alpha_i(t) - \int_0^t \omega_d(\tau) d\tau \in \mathbb{R}$. Then, as a preliminary, we first show that for all $\alpha_i(0) = \bar{\alpha}_i(0) \in (-\pi, \pi]$, $i \in V, \bar{\alpha}_i(t)$ does not cross $\pm \pi$ for all $t \geq 0$. Consider the quadrotor $\rho \in \mathcal{V}$ whose storage function associated with $\bar{\alpha}_\rho$ has the largest value among the group:

$$\rho(t) := \max_{i \in V} S_\alpha(\bar{\alpha}_i(t)) = \max_{i \in V} \left(1 - \cos \frac{\bar{\alpha}_i(t)}{2}\right).$$

Then, when $\bar{\alpha}_\rho \in (-\pi, \pi]$ holds, the passivity property yields

$$S_\alpha(\bar{\alpha}_\rho) = \left(\alpha_{\rho,t} - \omega_d\right) \sin \frac{\bar{\alpha}_\rho}{2} = \sin \frac{\bar{\alpha}_\rho}{2} \sum_{j \in N_\rho} k_{\rho,\omega} w_{\rho,j} \sin \frac{\bar{\alpha}_j - \bar{\alpha}_\rho}{2} = \sum_{j \in N_\rho} k_{\rho,\omega} w_{\rho,j} \left(\cos \frac{\bar{\alpha}_\rho}{2} - \cos \frac{\bar{\alpha}_j - \bar{\alpha}_\rho}{2}\right) \leq -\sum_{j \in N_\rho} k_{\rho,\omega} w_{\rho,j} \cos \frac{\bar{\alpha}_\rho}{2} \left(1 - \cos \frac{\bar{\alpha}_j - \bar{\alpha}_\rho}{2}\right) \leq 0.

Here, we use the properties that $\alpha_j - \alpha_\rho = \bar{\alpha}_j - \bar{\alpha}_\rho$ and $\cos(\bar{\alpha}_\rho/2) \leq \cos(\bar{\alpha}_j/2)$ $\forall j \in V$ from the definitions. The maximum storage function is thus non-increasing. Namely,

$$S_\alpha(\bar{\alpha}_i(t)) \leq \max_{i \in V} S_\alpha(\bar{\alpha}_\rho(t)) = S_\alpha(\bar{\alpha}_\rho(0))$$

holds for all $i \in V, t \geq 0$, which proves the argument.

We now prove the yaw angle synchronization, but consider only the root group $H_1$ here because the same argument holds for all the other groups as in Appendix A.1. Namely, we show that the root group $H_1$ with the node set $\mathcal{V}_1$ and strongly connected interconnection topology achieves the synchronization.

Define the Lyapunov function candidate $U_{a1} \geq 0$ as

$$U_{a1} := \sum_{i \in V_1} \eta_i S_\alpha(\bar{\alpha}_i).$$

Then, the passivity and output synchronization law (11) yield

$$\dot{U}_{a1} \geq \sum_{i \in V_1} \sum_{j \in N_i} \eta_{ij} w_{ij} \left(\cos \frac{\bar{\alpha}_i}{2} - \cos \frac{\bar{\alpha}_j}{2} - \cos \frac{\bar{\alpha}_i}{2} (1 - \cos \frac{\bar{\alpha}_j - \bar{\alpha}_i}{2})\right) = -\sum_{i \in V_1} \sum_{j \in N_i} \eta_{ij} w_{ij} \cos \frac{\bar{\alpha}_i}{2} (1 - \cos \frac{\bar{\alpha}_j - \bar{\alpha}_i}{2}). \quad (A.5)$$

Here, the second equality holds from the weighted graph Laplacian property (A.1).

Since the above preliminary guarantees $\cos(\bar{\alpha}_i/2) \geq 0$ for all $t \geq 0$, Eq. (A.5) becomes negative semi-definite. LaSalle’s invariance principle with the strong connectivity thus concludes $|\bar{\alpha}_i - \bar{\alpha}_j| = |\alpha_i - \alpha_j| \to 0$ $\forall i, j \in \mathcal{V}_1$. This means $u_{i,a} \to \omega_d$ resulting in $|\bar{\alpha}_i - \omega_d| \to 0$. Therefore, the group $H_1$ achieves the yaw angle synchronization (5b).

A.3 Proof of Proposition 3

Proof. We consider only the root group $H_1$ here because the same argument holds for all the other groups as in Appendix A.1. Define the Lyapunov function candidate $U_{z1} \geq 0$ as

$$U_{z1} := \sum_{i \in V_1} \eta_i k_{i,z} S_{i,z}.$$

Then, from the same calculation as in Eq. (A.2), we obtain

$$\dot{U}_{z1} = -\sum_{i \in V_1} \sum_{j \in N_i} \eta_i k_{i,z} k_{j,z} s_{i,z} s_{j,z} \leq 0.$$

Therefore, LaSalle’s invariance principle shows $|\bar{z}_i - \bar{z}_j| = |z_i - z_j| \to 0$ and $\bar{\omega}_z \to 0$ for all $i, j \in \mathcal{V}_1$, where the former implies $u_{i,z} \to 0$. Moreover, when $e_{i,z} = 0$ and $u_{i,z}$ are satisfied, $\bar{\omega}_z = v_{d,z}$ holds. The invariance principle thus concludes the vertical position synchronization (12).

A.4 Proof of Proposition 4

Proof. We consider only the root group $H_1$ here because the same argument holds for all the other groups as in Appendix A.1. Define the Lyapunov function candidate $U_{s1} \geq 0$ as

$$U_{s1} := \sum_{i \in V_1} \eta_i S_{i,s}.$$

Then, from the same calculation as in Eq. (A.2), we obtain

$$\dot{U}_{s1} = -\sum_{i \in V_1} \sum_{j \in N_i} \left(k_{i,s}^2 e_{i,s}^2 + k_{i,s} k_{j,s} e_{i,s} e_{j,s}\right) - \frac{1}{2 \gamma^2} \sum_{i \in V_1} \sum_{j \in N_i} \eta_i w_{ij} (\bar{\omega}_i - \bar{\omega}_j)^2 \leq 0.$$

Therefore, LaSalle’s invariance principle shows $|\bar{x}_i - \bar{x}_j| = |x_i - x_j| \to 0$ and $\bar{e}_{i,z} \to 0$ and $e_{i,\beta} \to 0$ for all $i, j \in \mathcal{V}_1$, where the first implies $u_{i,x} \to 0$. Moreover, when $e_{i,x} = 0$, $e_{i,\beta} = 0$ and $u_{i,z} = 0$ are satisfied, $\bar{x}_i = v_{d,z}$ and $\beta_i = \beta_d$ hold. The invariance principle thus concludes the horizontal position synchronization associated with the $x$-axis motion.
Tatsuya Iuki (Member)

He received his B.Eng., M.Eng. and Ph.D.Eng. degrees from Tokyo Institute of Technology, Japan, in 2008, 2010 and 2013, respectively. He is currently an Assistant Professor in the Department of Systems and Control Engineering, Tokyo Institute of Technology. He was a research fellow of the Japan Society for the Promotion of Science from 2012 to 2013. His research interests include cooperative control of robotic networks, learning based cooperative control for drone networks, vision-based estimation and control, and nonlinear control. He is a member of IEEE and so on.

Satoshi Nakano (Student Member)

He received his B.Eng. degree from Nagoya Institute of Technology and his M.Eng. degree from Tokyo Institute of Technology, Japan, in 2013 and 2015, respectively. He is currently a Ph.D. student in the Department of Mechanical and Control Engineering, Tokyo Institute of Technology. His research interests include cooperative control, nonlinear control, and vision-based estimation and control.

Mahato Endou

He received his B.Eng. degree from Tokyo Institute of Technology, Japan, in 2016. He is currently a master course student in the Department of Systems and Control Engineering, Tokyo Institute of Technology. His research interests include cooperative control of quadrotor networks and nonlinear control.

Mitsuji Sampei (Member, Fellow)

He received his B.Eng., M.Eng. and Dr.Eng. degrees in control engineering from Tokyo Institute of Technology, Japan in 1983, 1985, and 1987, respectively. From 1987 to 1991, he was a Research Associate in the Department of Electrical and Electronics Engineering, Chiba University, Japan. From 1991 to 1993 he was an Associate Professor in the same department. From 1993 to 2000 he was an Associate Professor in the Department of Mechanical and Environmental Informatics, Tokyo Institute of Technology. Since 2000 he has been with the Department of Mechanical (Systems) and Control Engineering, Tokyo Institute of Technology, as a Professor. His research interests include nonlinear control theory and its applications for nonlinear systems, control theory of nonholonomic systems, and linear control theory. He is a member of IEEE, ISCIE and so on.