ON TIME VARIATION OF $G$ IN MULTIDIMENSIONAL MODELS WITH TWO CURVATURES

H. Dehnen$^{1,a}$, V. D. Ivashchuk$^{2,b,c}$, S.A. Kononogov$^{3,b}$ and V.N. Melnikov$^{4,b,c}$

$^a$ Universität Konstanz, Fakultät für Physik, Fach M 568, D-78457, Konstanz
$^b$ Centre for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya St., Moscow 119361, Russia
$^c$ Institute of Gravitation and Cosmology, Peoples’ Friendship University of Russia, 6 Miklukho-Maklaya St., Moscow 117198, Russia

Expressions for $G$ are considered in a multidimensional model with an Einstein internal space and a multicomponent perfect fluid. In the case of two non-zero curvatures without matter, a mechanism for prediction of small $G$ is suggested. The result is compared with exact (1+3+6)-dimensional solutions. A two-component example with two matter sources (dust + 5-brane) and two Ricci-flat factor spaces is also considered.

1. Introduction

The idea of possible slow (cosmological) time variations of fundamental physical constants, the gravitational constant $G$ in particular, came out from Dirac’s analysis in 1937 of some relations between macro- and micro-world phenomena. His Large Numbers Hypothesis (LNH) was the origin of many further theoretical and experimental explorations of time-varying $G$. According to the LNH, $G/G$ should have approximately the Hubble rate. Although it has become clear in the recent decades that the Hubble rate is too high to be compatible with experiment, the enduring legacy of Dirac’s bold stroke is the acceptance by modern theories of non-zero values of $G/G$ as being potentially consistent with physical reality.

After Dirac’s original hypothesis, some new ideas appeared as well as generalized theories of gravity admitting variations of the effective gravitational coupling.

Different theoretical schemes lead to temporal variations of the effective gravitational constant:

1. Empirical models and theories of Dirac type, where $G$ is replaced with $G(t)$.
2. Numerous scalar-tensor theories of Jordan-Brans-Dicke type where $G$ depends on the scalar field $\sigma(t)$.
3. Gravitational theories with a conformal scalar field arising in different approaches [1,2].
4. Multidimensional unified theories arising from supergravities and superstrings and a future possible M-theory, containing are dilaton fields and effective scalar fields appearing in our 4-dimensional spacetime from extra dimensions [9]. They may also help in solving the problem of a variable cosmological constant from Planckian to present values.

A striking feature of most modern scalar-tensor and unification theories is that they do not admit a unique and universal constant values of physical constants, including the Newtonian gravitational coupling constant $G$. In this paper, we briefly set out the results of some calculations which have been carried out for various theories, and discuss various bounds that may be suggested by multidimensional theories. Although the bounds on $G$ and $G(r)$ are, in some classes of theories, rather wide on purely theoretical grounds as a result of adjustable parameters, we note that the observational data concerning other phenomena may place limits on the possible range of these adjustable parameters.

Here we restrict ourselves to the problem of $G$ (for $G(r)$ see [1,2,3,6,9]). We show that various theories predict $G/G$ of the order $10^{-12}$/yr or smaller. The significance of this fact for experimental and observational determinations of the value of or upper bound on $G$ is the following: any determination with error bounds significantly below $10^{-12}$ will be typically compatible with only a small portion of the existing theoretical models and will therefore cast serious doubt on the viability of all other models. In short, a tight bound on $G$, in conjunction with other astrophysical observations, will be a very effective “theory killer”. In other words, it may be a new test of cosmological models in addition to standard cosmological tests.

Some estimations for $G$ were made long ago in the framework of GR with a conformal scalar field [4,5] and in general scalar tensor theories using the values of the cosmological parameters ($\Omega$, $H$, etc.) [2,10]. With modern values, they predict $G/G$ at the level of $10^{-12}$/yr and smaller (see also recent estimations by A. Miyazaki [11], predicting time variations of $G$ at the level of $10^{-13}$/yr) for a Machian-type cosmological solution in the Brans-Dicke theory and Fujii’s estimations [12].

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1 e-mail: Heinz.Dehnen@uni-konstanz.de
2 e-mail: rusgs@phys.msu.ru
3 e-mail: kononogov@vniims.ru
4 e-mail: melnikov@phys.msu.ru
The most reliable experimental bounds on Ġ/G (radar ranging of spacecraft dynamics \[13, 14\] and laser lunar ranging \[18\]) give the limit of \(10^{-12}/\text{yr}^2\). There also exist some model-dependent measurements of Ġ from Big Bang nucleosynthesis at the level of a few units of \(10^{-13}/\text{yr}\).

2. Ġ in \((4 + 3 + N)\)-dimensional cosmology with a multicomponent anisotropic fluid

We consider here a \((4 + N)\)-dimensional cosmology with an isotropic 3-space and an Einstein internal space. The Einstein equations provide a relation between Ġ/G and other cosmological parameters.

2.1. The model

Let us consider a \((4 + N)\)-dimensional theory with the gravitational action

\[
S_g = \frac{1}{2\kappa^2} \int d^{4+N}x \sqrt{-g} R, \tag{1}
\]

where \(\kappa^2\) is the fundamental gravitational constant. Then the gravitational field equations are

\[
R^M_{\dot{M}} = \kappa^2 \left( T^M_{\dot{M}} - \delta^M_{\dot{M}} \frac{T}{N+2} \right), \tag{2}
\]

where \(T^M_{\dot{M}}\) is the \((4+N)\)-dimensional energy-momentum tensor, \(\dot{T} = T^M_{\dot{M}}\) and \(M, \dot{M} = 0, \ldots, N+3\).

For the \((4+N)\)-dimensional manifold we assume the structure

\[
M^{4+N} = \mathbb{R}_t \times M^3_k \times K^N, \tag{3}
\]

where \(\mathbb{R}_t\) is the 1-dimensional time manifold, \(M^3_k\) is a 3D space of constant curvature, \(M^3_k = S^3, R^3, L^3\) for \(k = +1, 0, -1\), respectively, and \(K^N\) is an \(N\)-dimensional Einstein manifold.

The metric is taken in the form

\[
ds^2 = g_{MN} dx^M dx^N = -dt^2 + a^2(t) g^{(3)}_{ij}(x) dx^i dx^j + b^2(t) g^{(N)}_{mn}(y) dy^m dy^n, \tag{4}
\]

where \(i, j, k = 1, 2, 3; m, n, p = 4, \ldots, N+3; g^{(3)}_{ij}, g^{(N)}_{mn}\), \(a(t)\) and \(b(t)\) are the metrics and scale factors of \(M^3_k\) and \(K^N\), respectively.

For \(T^M_{\dot{M}}\) we adopt the expression for a multicomponent (anisotropic) fluid

\[
(T^M_{\dot{M}}) = \sum_{\alpha=1}^{m} \text{diag}(-\rho^\alpha(t), p^\alpha(t) \delta^i_j, p^\alpha(t) \delta^m_n). \tag{5}
\]

Under these assumptions, the Einstein equations take the form

\[
\frac{3\ddot{a}}{a} + \frac{N\ddot{b}}{b} = \frac{\kappa^2}{N+2} \sum_{\alpha=1}^{m} [-(N+1)\rho^\alpha - 3p^\alpha - Np_N^\alpha], \tag{6}
\]

\[
\frac{\ddot{a}}{a} + \frac{2a^2}{a^2} + \frac{N\dot{a} \dot{b}}{ab} + \frac{2k}{a^2} = \frac{\kappa^2}{N+2} \sum_{\alpha=1}^{m} [\rho^\alpha - 3p^\alpha + 2p_N^\alpha]. \tag{8}
\]

Here

\[
R_{mn}[g^{(N)}] = \lambda g^{(N)}_{mn}, \tag{9}
\]

\(m, n = 1, \ldots, N\), where \(\lambda\) is constant.

The 4-dimensional density is

\[
\rho^{(4)}(t) = \int_\mathbb{R} d^N y \sqrt{g^{(N)}} b^N(t) \rho^\alpha(t) = \rho^\alpha(t) b(t), \tag{10}
\]

where we have normalized the factor \(b(t)\) by putting

\[
\int_\mathbb{R} d^N y \sqrt{g^{(N)}} = 1. \tag{11}
\]

On the other hand, to get the 4D gravity equations, one should put \(8\pi\rho^{(4)}(t) = \kappa^2 \rho^{\alpha}(t)\). Consequently, the effective 4D gravitational “constant” \(G(t)\) is defined by

\[
8\pi G(t) = \kappa^2 b^{-N}(t) \tag{12}
\]

whence its time variation is expressed as

\[
\dot{G}/G = -N\ddot{b}/b. \tag{13}
\]

2.2. Cosmological parameters

Some inferences concerning the observational cosmological parameters can be extracted directly from the equations without solving them \[17\]. Indeed, let us define the Hubble parameter \(H\), the density parameters \(\Omega^\alpha\) and the “deceleration” parameter \(q\) referring to a fixed instant \(t_0\) in the usual way:

\[
H = \frac{\dot{a}}{a}, \quad \Omega^\alpha = \frac{8\pi G \rho^{(4)}(t)}{3H^2} = \frac{\kappa^2 \rho^\alpha(t)}{3H^2}, \quad q = -\frac{\ddot{a}}{a}. \tag{14}
\]

Besides, instead of Ġ, let us introduce the dimensionless parameter

\[
g = \dot{G}/GH = -Na\ddot{b}/ab. \tag{15}
\]

The present observational upper bound on \(g\) is

\[
g < 0.1 \tag{16}
\]

if we take in accord with \[13, 18\]

\[
\dot{G}/G < 0.6 \times 10^{-11}/\text{yr}^{-1} \tag{17}
\]

and \(H = (0.7 \pm 0.1) \times 10^{-11}/\text{yr} \approx 70 \pm 10 \text{ km/(s. Mpc)}\).
3. A vacuum model with two Einstein spaces

Here we consider the vacuum case when $T_{P}^{M} = 0$. Let us suppose that $t_0$ is an extremum point of the function $b(t)$, i.e.,

$$\dot{b}(t_0) = 0.$$  

(18)

At this point we get $\dot{G}(t_0) = 0$. From Eqs. (14), (17), (28) we obtain that for $t = t_0$

$$\frac{3\ddot{a}}{a} + \frac{N\ddot{b}}{b} = 0,$$

(19)

$$\ddot{a} = -\frac{2a^2}{a^2} - \frac{2k}{a^2} = 0,$$

(20)

$$\ddot{b} = \frac{\lambda}{b}.$$  

(21)

Suppose that “we live” near the point $t_0$, then, according to modern observations on the acceleration of the Universe expansion (23, 24) we should put

$$\dot{a}(t_0) > 0$$

(22)

$$\dot{a}(t_0) > 0.$$  

(23)

This implies

$$k < 0$$

(24)

due to (20) and

$$\dot{b}(t_0) < 0, \quad \lambda > 0$$

(25)

due to (19) and (21). Thus our 3-dimensional space should have a negative curvature while the internal $N$-dimensional space should have a positive curvature.

From (19), (20) we obtain, using the definitions of the cosmological parameters,

$$\frac{|2k|}{H_0^2a_0^2} = 2 + |q_0|,$$

(26)

$$\frac{d_2|\lambda|}{H_0^2b_0^2} = 3|q_0|.$$  

(27)

Here $a_0 = a(t_0)$ and $b_0 = b(t_0)$. Since by assumption “we live” near the point $t_0$, we get

$$\frac{\dot{b}}{b} \approx \frac{\dot{b}_0(t - t_0)}{b_0},$$

(28)

and due to (18) and (19) we find

$$\frac{\dot{G}}{G} = -N\frac{\dot{b}}{b} \approx -N\frac{\dot{b}_0}{b_0}(t - t_0) = 3\frac{\ddot{a}_0(t - t_0)}{a_0}.$$  

(29)

The subscript “0” refers to $t_0$. Using the definitions of the cosmological parameters (14), we obtain in our approximation

$$\frac{\dot{G}}{G} \approx -3q_0H_0^2(t - t_0).$$

(30)

Recall that $q_0 < 0$, hence $\dot{G}/G > 0$ for $t > t_0$ and $\dot{G}/G < 0$ for $t < t_0$.

We also note that, in our approximation, $\dot{G}/G$ is independent of the internal space dimension $N = \dim K$.

3.1. Exact $1 + 3 + 6$ solution

Now we consider an exact solution from Ref. [25] defined on the manifold

$$M = \mathbb{R}_+ \times M^{(3)} \times M^{(6)},$$

(31)

with the metric

$$ds^2 = (f_1 f_2)^{-1/2} \left[-2f_1^{-2}(d\tau)^2 + |\lambda_i|g^{(3)}_{ij}(x)dx^i dx^j + f_2|\lambda_6|g^{(6)}(y)dy^m dy^n\right],$$

(32)

where $(M^{(3)}, g^{(3)})$ and $(M^{(6)}, g^{(6)})$ are Einstein spaces:

$$Ric[g^{(i)}] = \lambda_i g^{(i)},$$

(33)

$i = 3, 6$. We use the notations $\lambda_3 = 2k$ and $\lambda_6 = \lambda$. In (34)

$$f_1 = |\tau^2 + \varepsilon_3|,$$

(34)

$$f_2 = -3\varepsilon_6(\tau^2 + \varepsilon_3)|1 + \tau[h(\tau, \varepsilon_3) + C_1]| + \varepsilon_3 \varepsilon_6 > 0,$$

(35)

where $C_1 = \text{const}$, $\varepsilon_i = \text{sign}(\lambda_i)$, $i = 3, 6$, and

$$h(\tau, \varepsilon_3) = \frac{1}{2} \ln \left|\frac{\tau - 1}{\tau + 1}\right|, \quad \varepsilon_3 = -1,$$

(36)

$$h(\tau, \varepsilon_3) = \arctan(\tau), \quad \varepsilon_3 = 1.$$  

(37)

As was mentioned above, we should restrict our consideration to the case when “our” 3-dimensional space has a negative curvature while the 6-dimensional “internal” space has a positive curvature, i.e.,

$$\varepsilon_3 = -1, \quad \varepsilon_6 = 1.$$  

(38)

The analysis carried out in (25) shows that the scale factor of “our” 3-space

$$a_3 = a = (f_1 f_2)^{-1/4}|\lambda_i|^{1/2}$$

(39)

has a minimum at some point $\tau_\ast$ when the branch of the solution with $\tau \in (\tau_-, \tau_\ast)$ is considered. Here $\tau_-$ and $\tau_\ast$ are roots of the equation $f_2(\tau) = 0$ belonging to the interval $(0, 1)$. In this case, the scale factor of “our” space $a_3(\tau)$ monotonically decreases in the interval $(\tau_-, \tau_\ast)$ and monotonically increases in the interval $(\tau_\ast, \tau_+)$.

The scale factor of the “internal” 6-space

$$a_6 = b = (f_1 f_2)^{-1/4}f_2^{1/2}|\lambda_6|^{1/2}$$

(40)

has a maximum at some point $\tau_0$. It monotonically increases in the interval $(\tau_-, \tau_0)$ and monotonically decreases in the interval $(\tau_0, \tau_+)$.  

Remark. For other branches of the solution with either $\tau \in (\tau_-, \tau_+)$ or $\tau \in (-\infty, \tau_\ast) \cup (|\tau_\ast|, |\tau_+| > 1)$ we get a monotonic behaviour of both scale factors $a_3(\tau)$ and $a_6(\tau)$.

Now consider our solution in synchronous time:

$$ds^2 = -dt^2 + a_3^2(t)g^{(3)}_{ij}(x)dx^i dx^j + a_6^2(t)g^{(6)}_{ij}(y)dy^m dy^n,$$

(41)
where
\[ t_s = \sqrt{2} \int_{\tau_-}^\tau d\tau' (f_1 f_2)^{-1/4} f_1^{-1}. \] (42)

The function \( t_s(\tau) \) is monotonically increasing from \( t_s(\tau_-) = 0 \) to \( T = t_s(\tau_+) \).

The 3-space scale factor has a minimum at the point \( t_0 = t(\tau_+) \). The function \( a_3(t) \) monotonically decreases from infinity to finite value in the interval \((0, t_0)\) and monotonically increases to infinity in the interval \((t_0, T)\).

The 6-space scale factor has a maximum at the point \( t_s = t(\tau_+) \). The function \( a_6(t) \) monotonically increases from zero to a finite value in the interval \((0, t_s)\) and monotonically decreases to zero in the interval \((t_s, T)\).

Only in case \( C_1 > 0 \) we get \( t_0 < 0 \) and hence in the “epoch” near \( t_0 \) we get an accelerating expansion of “our” 3-space.

4. A model with two Ricci-flat spaces and a two-component fluid

Here we consider another example when two factor spaces are Ricci-flat. In this case, excluding \( b \) from (6) and (8), we get

\[ \frac{N - 1}{3N} g^2 - g + q - \sum_{\alpha = 1}^m A^\alpha \Omega^\alpha = 0 \] (43)

with

\[ A^\alpha = \frac{1}{N + 2} [2N + 1 + 3(1 - N)\nu_3^\alpha + 3N\nu_N^\alpha], \] (44)

where

\[ \nu_3^\alpha = p_3^\alpha / \rho^\alpha, \quad \nu_N^\alpha = p_N^\alpha / \rho^\alpha, \quad \rho^\alpha > 0. \] (45)

When \( g \) is small, we get from (43)

\[ g \approx - \sum_{\alpha = 1}^m A^\alpha \Omega^\alpha. \] (46)

Note that (43) for \( N = 6, m = 1, \nu_3^1 = \nu_N^1 = 0 \) (so that \( A^1 = 13/8 \)) coincides with the corresponding Wu and Wang’s relation [15] obtained for large times in case \( k = -1 \) (see also [16]).

If \( k = 0 \), then in addition to (43), one can obtain a separate relation between \( g \) and \( \Omega^\alpha \), namely,

\[ \frac{N - 1}{6N} g^2 - g + 1 - \sum_{\alpha = 1}^m \Omega^\alpha = 0 \] (47)

(this follows from the Einstein equation \( R_0^0 - \frac{1}{2} R = \kappa^2 T_0^0 \), which is certainly a linear combination of (6)–(8).

4.1. A two-component example: dust + \((N - 1)\)-brane

Let us consider a two component case: \( m = 2 \) [22]. Let the first component (called “matter”) be dust, i.e.

\[ \nu_3^1 = \nu_N^1 = 0, \] (48)

while the second one (called “quintessence”) be an \((N - 1)\)-brane, i.e.,

\[ \nu_3^2 = 1, \quad \nu_N^2 = -1. \] (49)

We remind the reader that, as was mentioned in [19], a multidimensional cosmological model on product manifold \( \mathbb{R} \times M_1 \times \ldots \times M_n \) with fields of forms (for a review see [21]) may be described in terms of a multicomponent “perfect” fluid [20] with the following equations of state for an \( \alpha \)-s component: \( p_i^\alpha = -\rho^\alpha \) if the \( p \)-brane world volume contains \( M_i \) and \( p_i^\alpha = \rho^\alpha \) otherwise. Thus the field of form matter leads either to a \( \Lambda \)-term or to stiff matter equations of state in the internal spaces.

In this case we get from (46) for small \( g \)

\[ g \approx q - \frac{2N + 1}{N + 2} \Omega^1 + \frac{N - 1}{N + 2} \Omega^2, \] (50)

and for \( k = 0 \) and small \( g \) we obtain from (11)

\[ 1 - g \approx \Omega^1 + \Omega^2. \] (51)

Now we illustrate the formulas by the following example when \( N = 6 \) \((K^6 \) may be a Calabi-Yau manifold) and

\[ -q = \Omega^1 = \Omega^2 = 0.5. \] (52)

We get from (50)

\[ g \approx - \frac{1}{16} \approx -0.06 \] (53)
in agreement with (16). In this case the second fluid component corresponds to a magnetic (Euclidean) \( NS5 \)-brane (in \( D = 10 \) type I, Het or II A string models). We here consider for simplicity the case of a constant dilaton field.

This example tells us that, for a small enough temporal variation of \( G \), we may find estimates for \( \tilde{G} \) without considering of exact solutions. But we should select the solutions that give us an accelerated expansion of our world. We may use, for instance, the mechanism suggested above (see Sec. 3), but, instead of two curvatures, we should consider two fluid components. This may be a subject of a separate study.

5. Conclusions

In this paper we have considered a multidimensional cosmological model with an \( m \)-component anisotropic (“perfect”) fluid. The multidimensional Hilbert-Einstein equations led to relations between \( \tilde{G} \) and cosmological parameters.

In the case of two non-zero curvatures without matter, we have suggested a mechanism for predicting small
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$\dot{G}$. We conjectured that we “live” near the point $t_0$ where the time variation of $G$ is zero. When the 3-space has a negative curvature and the internal space has a positive curvature, we get, in the vicinity of $t_0$, an accelerating expansion of “our” 3-dimensional space and a small value of $\dot{G}/G$ (see $\mathbf{80}$). We have shown that this result is compatible with the exact 1 + 3 + 6 solution from $\mathbf{25}$. Recall that there only three exact solutions are known for a vacuum cosmological model with a product of two Einstein spaces, see $\mathbf{25}$. Recently, a certain interest in models of this type has appeared in the context of the acceleration problem, see $\mathbf{26}$ and references therein, but without any usage of exact solutions.

We have presented another example where two factor spaces are Ricci-flat and, for a wo-component example (dust + 5-brane) we have obtained a small enough variation of $G$. This example may be used in a future work for a generalization of the mechanism suggested for the vacuum case to models with matter sources.

Acknowledgement

The work of V.D.I. and V.N.M. was supported in part by DFG grant 436RUS113/807/0-1 and by the Russian Foundation for Basic Researches grant No. 05-02-17478. V.N.M. thanks the colleagues from the Physical Department of the University of Konstanz for hospitality during his visit in October-November 2005.

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