Observational constraints on successful model of quintessential Inflation

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Abstract. We study quintessential inflation using a generalized exponential potential $V(\phi) \propto \exp(-\lambda \phi^n/M_{Pl}^n)$, $n > 1$, the model admits slow-roll inflation at early times and leads to close-to-scaling behaviour in the post inflationary era with an exit to dark energy at late times. We present detailed investigations of the inflationary stage in the light of the Planck 2015 results, study post-inflationary dynamics and analytically confirm the existence of an approximately scaling solution. Additionally, assuming that standard massive neutrinos are non-minimally coupled, makes the field $\phi$ dominant once again at late times giving rise to present accelerated expansion of the Universe. We derive observational constraints on
the field and time-dependent neutrino masses. In particular, for $n = 6$ (8), the parameter $\lambda$ is constrained to be, $\log \lambda > -7.29$ (−11.7); the model produces the spectral index of the power spectrum of primordial scalar (matter density) perturbations as $n_s = 0.959 \pm 0.001 (0.961\pm0.001)$ and tiny tensor-to-scalar ratio, $r < 1.72\times 10^{-2}$ ($2.32\times 10^{-2}$) respectively. Consequently, the upper bound on possible values of the sum of neutrino masses $\Sigma m_\nu \lesssim 2.5 \text{ eV}$ significantly enhances compared to that in the standard $\Lambda$CDM model.

**Keywords:** dark energy theory, inflation, modified gravity

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1 Introduction

It is remarkable that according to the standard cosmological paradigm, both early and late-time phases of the Universe history require accelerated expansion. Inflation [1–5] naturally provides necessary initial conditions, including small perturbations, for the subsequent isotropic hot Big Bang (radiation dominated) stage at early times, whereas the qualitatively similar late-time cosmic acceleration [6–11] beautifully resolves the age crisis and fits the SNIa luminosity distance and baryon acoustic oscillations (BAO) data. Hence, the radiation dominated (RD) and matter dominated (MD) stages of the Universe evolution are sandwiched between two phases of accelerated expansion. In this perspective, it sounds quite reasonable to imagine a common origin for both the phases, in other words, a model of inflation which could also give rise to late-time acceleration.

The idea of unification of inflation and late-time acceleration (even before an observational discovery of the latter) was first briefly mentioned in [12], and then further developed in [13] where the name *quintessential inflation* was introduced, too, see also [14–21]. Later it was implemented in the framework of braneworld cosmology in [22–25] (for a review see ref. [26], see also ref. [27] on the related theme).

It is indeed quite challenging to describe both phases of acceleration using a single scalar field minimally couple to gravity, without affecting the thermal history of the universe which has been verified to a good accuracy. Let us spell out the broad requirements on the field, if it is to unify both the phenomena. In order to facilitate slow-roll, the field potential should exhibit shallow behaviour at early times, followed by a steep region for most of the universe history, turning shallow once again at late times. Since the inflaton scalar field survives till late in this picture, conventional mechanisms of creation and heating of matter after the end of inflation by the inflaton field itself, either perturbative ones [28–30] or using the non-perturbative broad parametric resonance (preheating) [31, 32], are not operative, while those based on the effect of gravitational particle creation [2, 33–36] may appear to be insufficient, too (though this depends on a value of the dimensionless model parameter $\lambda$ and requires special consideration\(^1\)). A good alternative may be provided by preheating

\(^1\)Since gravitons are created by the same effect, too, in this case the problem of gravitational wave overproduction may arise.
based upon instant particle production [37], which can successfully address the problem of relic gravitational waves generated during inflation [1, 38–40] and after it [41].

After inflation ends, the scalar field enters the kinetic regime [12, 23, 25, 42], and overshoots the background and freezes on its potential. During evolution, the background energy density becomes comparable to the field energy density, thereafter the field evolution crucially depends upon the degree of steepness of its potential. In case of a steep exponential potential [20, 22, 23, 43–45], the field would track the background being sub-dominant, in the so-called scaling regime [46]. If the potential is less steeper than the exponential function, the field energy density would move towards the background, as it happens in case of inverse power-law potentials. However, in the opposite case, the field would move away from the background, overshoot it and freeze on its potential, and this latter behavior would keep repeating.

A new phenomenon occurs if we consider the class of potentials $V(\phi) \sim \exp(-\lambda \phi^n / M_{Pl}^n)$ [47]. First, one can realize successful inflation in this model for suitable values of $\lambda$ and $n$. Second, in the post-inflationary evolution, the model exhibits an interesting behavior. Indeed, since $\Gamma = V_{\phi \phi} V / V^2_{\phi} \rightarrow 1$ for large values of $\phi$, the model can give rise to an approximately scaling solution at late stages, before which the field exhibits the aforementioned behavior. With the steep exponential potential, one could invoke high energy brane corrections [22, 23, 25, 48, 49] to facilitate slow-roll. Assuming the required late-time features in the potential, this construction could give rise to viable post-inflationary evolution.

Although the simple exponential potential does not comply with the observational constraints related to inflation, the generalized exponential gives rise to slow-roll for small values of $\phi$ [18, 20, 21, 25, 26]. In this case we have one extra parameter at hand that might allow us to satisfy all observational constraints for inflation. At the same time, the model enjoys all the benefits of the steep exponential case, as it effectively mimics it at late stages.

Because of its approximately scaling behaviour, the field $\phi$ by itself may not provide the late time transition to the present accelerated expansion of the Universe. To achieve this, some new element has to be added to the model. One possibility is to assume a non-minimal coupling of the field $\phi$ to all matter [50, 51]. In particular, a large coupling constant gives rise to a minimum of the effective potential which can trap the field producing an attractor of the dynamical system that mimics the cosmological-constant-like behavior. However, the latter is undesirable as the said regime can be reached soon after the transition from radiation to matter dominance in the early Universe. Thus, to have a sufficiently long MD stage required by observations, it is absolutely essential to leave the main part of non-relativistic matter (including cold dark matter and baryons) intact. On the other hand, a non-minimal coupling of $\phi$ to neutrinos only, that makes neutrino masses time-dependent, is well possible, and it can safely trigger a late-time transition to the present accelerated expansion of the Universe leaving the duration of the MD stage practically unchanged [52–62].

In this case, the coupling forms dynamically as massive neutrinos become non-relativistic at late times. Such a coupling seems to be a natural device for triggering the transition from the scaling regime to the late-time acceleration.

In this paper, we explore aspects of quintessential inflation using the new generalized exponential potential 2.9 for the inflaton-quintessence scalar field $\phi$, both without and with its coupling to massive neutrinos according to the formula 2.8. In section 2 we describe the model setup, and in section 3 we present analytical and numerical details of the inflationary stage in this model. Section 4 includes the major part of our work and is devoted to investigations of observational constraints on the sum of neutrino masses. This section additionally includes
an analytical proof of the existence of a scaling solution in the model under consideration. Our results are summarized in section 5.

2 Non-minimally coupled massive neutrino matter to dark energy

We are interested in a scenario which could give rise to successful inflation at early times followed by a viable post inflationary evolution with a possibility of exit to acceleration at late stages. In order to accomplish the underlying idea, we consider the following action (we use the metric signature, (+, −, −, −)),

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{M^2_{Pl}}{2} R + \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right] + S_m + S_r + S_\nu(\phi, \Psi), \]  

(2.1)

where \( S_m, r, \nu \) correspond to the actions of matter, radiation and neutrino matter, respectively. We assume here that only massive neutrino matter is non-minimally coupled to the scalar field \( \phi \) with the Lagrangian,

\[ L_\nu = i \bar{\Psi} \gamma^\lambda \partial_\lambda \Psi - m_\nu \bar{\Psi} \Psi = i \bar{\Psi} \gamma^\lambda \partial_\lambda \Psi - m_\nu e^{\beta \phi} \bar{\Psi} \Psi. \]  

(2.2)

Varying the action (2.1) with respect to the metric and keeping in mind the Friedmann-Lemaître-Robertson-Walker (FLRW) spatially flat background, we obtain the evolution equations

\[ 3H^2 M^2_{Pl} = \frac{1}{2} \dot{\phi}^2 + V(\phi) + \rho_m + p_r + \rho_\nu, \]  

(2.3)

\[ \left( 2\ddot{H} + 3H^2 \right) M^2_{Pl} = -\frac{1}{2} \dot{\phi}^2 + V(\phi) - p_m - p_r - p_\nu, \]  

(2.4)

where \( \rho_i \) (\( p_i \)) are the energy densities (pressures) of the corresponding sectors. Additionally, varying the action (2.1) with respect to the scalar field \( \phi \), we derive its equation of motion, which reads as

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{dV_{\text{eff}}}{d\phi} = 0, \]  

(2.5)

where \( V_{\text{eff}} \) is the effective potential such that \( dV_{\text{eff}}/d\phi = dV(\phi)/d\phi + \beta(\rho_\nu - 3p_\nu)/M_{Pl} \). Consequently, the evolution equation for neutrino matter becomes

\[ \dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = \frac{\beta}{M^2_{Pl}} \dot{\phi}(\rho_\nu - 3p_\nu). \]  

(2.6)

Let us note that the evolution of the radiation and matter sectors is the standard one. For the neutrino sector, we have

\[ m_{\nu, \text{eff}}(\phi) = m_{\nu, 0} e^{\beta \phi/M_{Pl}}, \]  

(2.7)

and hence the neutrino matter behaves as radiation and non-relativistic matter during early and late times, respectively. Using the expression (2.7), the effective potential can be re-written as

\[ V_{\text{eff}}(\phi) = V(\phi) + (\rho_{\nu, 0} - 3p_{\nu, 0}) e^{\beta \phi/M_{Pl}}, \]  

(2.8)

where \( \rho_{\nu, 0} \) represents the neutrino energy density with a constant mass, \( m_{\nu, 0} \), i.e., \( \rho_\nu = \rho_{\nu, 0} e^{\beta \phi/M_{Pl}} \). Clearly, the coupling massive neutrino matter with field builds up dynamically at late stages as massive neutrinos turn non-relativistic.
As discussed in the Introduction, the unification of inflation and dark, energy requires that the field potential should be shallow, satisfying the slow-roll condition, at the inflationary stage, and steep after the end of inflation. Since the underlying field potential is typically of a runaway type that is a characteristic feature of quintessential inflation, the model belongs to the category of a non-oscillatory type. Thereby one needs an alternative mechanism of preheating. E.g. preheating may proceed through the instant particle production.

In what follows, we shall focus on the generalized exponential potential
\[
V(\phi) = V_0 e^{-\lambda \phi^n/M_{Pl}^n}, \tag{2.9}
\]
which can successfully unify inflation and dark energy without interfering with the thermal history of the universe. If \( V(\phi) \) is analytic at the origin, \( \phi = 0 \), that is typically expected from a more fundamental microscopic field-theoretical model leading to this effective potential, then \( n \) is a positive integer. Furthermore, it should be an even positive integer if we want the potential to be bounded for all real values of \( \phi \). However, all formulas in section 3 are valid for any \( n > 2 \). The case \( n = 2 \) requires special consideration, but it cannot produce a good fit to the measured value of the slope of the scalar power spectrum \( n_s - 1 \) as follows from eq. (3.18) in the next section in the limit \( n \to 2 \). Let us note that the slope of the potential is given by \( n \lambda \phi^n/M_{Pl}^n \), and thus slow roll is ensured for sufficiently small values of \( \phi \) which, however, can much exceed \( M_{Pl} \) for \( \lambda \ll 1 \). This does not lead to any problems in the UV regime since the potential is bounded for even \( n \) and can be considered as a perturbation for energies \( E \gg V_0 \), but still much less than the Planck one. As a result, the field acquires the approximate shift symmetry at these energies, so all values of \( \phi \) are possible.

In the steep region away from the origin, the scalar field rolls very fast, and therefore the energy density \( \rho_\phi \) overshoots the background, but it evolves as a scaling solution at late times, as \( \Gamma \to 1 \) for large values of \( \phi \). Relativistic and non-relativistic fluids dominate the universe in this region, until neutrino-matter becomes non-relativistic that happens at late times. This then leads to the building of the non-minimal coupling of \( \phi \) with massive neutrino matter, which in turn triggers a minimum in the effective potential. The slowly rolling scalar field around the minimum can then mimic the cosmological-constant-like behavior at late times. There are very few potentials that can simultaneously pass observational constraints from inflation to late-time evolution of the universe. In figure 1 this potential is illustrated with \( \lambda = 1 \), where the solid, dashed and dot-dashed lines correspond to \( n = 9, 6 \) and 3, respectively. We observe that when \( \lambda \phi^n/M_{Pl}^n \ll 1 \) with a large \( n \), the shallow potential may give rise to inflation. As \( \lambda \phi^n/M_{Pl}^n \gg 1 \), a scaling solution exists in the steep region.

In the next section we will explicitly consider the inflationary stage in this model and confront it with the Planck 2015 results.

3 Inflation

In the inflationary era, the evolution of the Universe is driven by the scalar field, in which case the contributions from \( S_r \), \( S_m \) and \( S_\nu \) should be ignored; they become relevant after inflation only. Using the standard expressions of slow-roll parameters, namely
\[
\epsilon \equiv \frac{M_{Pl}^2}{2} \left( \frac{V_\phi}{V} \right)^2, \quad \eta \equiv \frac{M_{Pl}^2 V_{\phi \phi}}{V}, \quad \xi \equiv \frac{M_{Pl}^4 V_\phi V_{\phi \phi \phi}}{V^2}, \tag{3.1}
\]
we can derive the expressions for the scalar and tensor spectral indexes \( (n_s, n_t) \), tensor-to-scalar ratio \( (r) \) and scalar spectral index running \( (\alpha_s \equiv d n_s/d \ln k) \), as
\[
\begin{align*}
n_s - 1 &= -6 \epsilon + 2 \eta, \tag{3.2} \\
n_t &= -2 \epsilon, \tag{3.3}
\end{align*}
\]
The generalized exponential potential $V(\phi) = V_0 e^{-\lambda \phi^n/M_{pl}^n}$, with $\lambda = 1$, where $n = 9$ (solid line), 6 (dashed line) and 3 (dot-dashed line), respectively.

$$r = 16 \epsilon,$$

$$\alpha_s = 16 \eta - 24 \epsilon^2 - 2 \xi.$$  

The weakly $k$-dependent primordial spectra of scalar and tensor perturbations can be expanded in powers of $\ln(k/k_s)$ around some pivot comoving scale $k_s$:

$$\ln P_s(k) = \ln A_s + (n_s - 1) \ln \left(\frac{k}{k_s}\right) + \alpha_s \left[\ln \left(\frac{k}{k_s}\right)\right]^2,$$

$$\ln P_t(k) = \ln A_t + n_t \ln \left(\frac{k}{k_s}\right).$$

Inserting the potential (2.9) into eqs. (3.2), (3.4) and using the standard expression for the primordial scalar spectrum $P_s$ itself, too, we get

$$P_s = \frac{V^3}{12 \pi^2 M_{Pl}^4 V_0^2} = \frac{V_0 e^{-\lambda \phi^n/M_{Pl}^n}}{12 \pi^2 \lambda^2 M_{Pl}^2 \phi^{2n-2}},$$

$$n_s - 1 = -n\lambda \left(\frac{\phi}{M_{Pl}}\right)^{n-2} \left(2n - 2 + n\lambda \left(\frac{\phi}{M_{Pl}}\right)^n\right),$$

$$r = 8n^2 \lambda^2 \left(\frac{\phi}{M_{Pl}}\right)^{2n-2},$$

where, as usually, $\phi(t)$ has to be estimated at the moment $t = t_k$ of the first Hubble radius crossing of a given comoving scale $k^{-1}a(t)$ during inflation, i.e. when $k = a(t_k)H(t_k)$. When it follows the observational result $r \lesssim 3(1 - n_s)$ [63] that the range of $\phi$ corresponding to the cosmological scales at which CMB fluctuations are observed lies in the region where the potential $V(\phi)$ is concave, $V_{\phi\phi} < 0$, and where the second term in large round brackets in eq. (3.9) is significantly less than the first one: their ratio

$$\frac{n\lambda}{2n - 2} \left(\frac{\phi}{M_{Pl}}\right)^n = \frac{r}{8(1 - n_s) - r} \lesssim 0.6.$$
The latter inequality can be made about twice stronger using the more recent combined data from the BICEP2, Keck Array and Planck collaborations [64]. Note one more useful relation which does not contain the parameter $\lambda$:

$$\frac{2\phi^2}{(n-1)^2M_{\text{Pl}}^2} = \frac{r}{(1 - n_s - \frac{1}{2})^2}. \quad (3.12)$$

The usual conditions for the end of inflation are given by either $\epsilon(\phi = \phi_{\text{end}}) = 1$, or $|\eta| (\phi = \phi_{\text{end}}) = 1$ depending on which of them occurs earlier. So, if $\lambda \ll 1$, we have large-field inflation which ends at

$$\phi = \phi_{\text{end}} = M_{\text{Pl}} \left( \frac{2}{n^2\lambda^2} \right)^{\frac{1}{2n-2}} \gg M_{\text{Pl}}. \quad (3.13)$$

when both $\epsilon$ and $\eta$ approach unity and

$$V_{\text{end}} \ll V_0, \quad \ln(V_{\text{end}}/V_0) \sim -\frac{\phi_{\text{end}}}{M_{\text{Pl}}} \sim -\lambda^{-\frac{1}{n-1}}. \quad (3.14)$$

In the opposite case $\lambda \ll 1$, we get small-field inflation of the hilltop type [65] which ends when $|\eta| = 1, \phi = \phi_{\text{end}} \sim M_{\text{Pl}} \lambda^{-1/(n-2)} \ll M_{\text{Pl}}, V_{\text{end}} \approx V_0$. However, only the former case can be used for quintessence applications.

The number of the e-foldings counted from the end of inflation can be evaluated as

$$N = \int_{t}^{t_{\text{end}}} H dt' = -M_{\text{Pl}}^{-2} \int_{\phi}^{\phi_{\text{end}}} V(\phi') d\phi' / dV(\phi') d\phi' \quad (3.15)$$

$$= \frac{1}{n\lambda(n-2)} \left[ \left( \frac{\phi}{M_{\text{Pl}}} \right)^{2-n} - \left( \frac{2}{n^2\lambda^2} \right)^{\frac{2-n}{2}} \right]. \quad (3.16)$$

Note the specific and new feature of large-field inflation in our case: the contribution of the upper limit of integration to $N$ is not small compared to unity. Thus, exit from inflation is a rather prolonged one. However, for the values $n = 6$ and $n = 8$ producing the best fit to observational data (see the table II below) and the corresponding admissible ranges of $\lambda$, this contribution is $\lesssim 4$. Thus, it is small compared to the main contribution from the lower limit of integration, $N \approx 50 - 60$. Inverting the relation (3.15), we acquire

$$\frac{\phi}{M_{\text{Pl}}} = \left[ n(n-2)\lambda N + \left( \frac{2}{n^2\lambda^2} \right)^{\frac{2-n}{2}} \right]^{\frac{1}{n-1}}. \quad (3.17)$$

Using eqs. (3.1)–(3.5) and (3.17) with $\lambda \ll 1, n > 2$ and $N \gg 1$, one can simplify the expressions of $n_s, n_t, r$, and $\alpha_s$ as functions of $(N, n, \lambda)$. For example, the expression of scalar field at commencement of inflation is reduced to $\phi/M_{\text{Pl}} \simeq [n(n-2)\lambda N]^{-\frac{1}{n-2}}$, while eqs. (3.2)–(3.5) become

$$n_s - 1 \simeq -\frac{2(n-1)}{(n-2)N} - \frac{[n(n-2)\lambda N]^{-\frac{2}{n-2}}}{(n-2)^2N^2} \lesssim -2(n-1)/(n-2)N, \quad (3.18)$$

$$n_t \simeq -\frac{[n(n-2)\lambda N]^{-\frac{2}{n-2}}}{(n-2)^2N^2} < 0, \quad (3.19)$$
Figure 2. (a) The tensor-to-scalar ratio \( r \) as a function of the scalar spectral index \( n_s \) in the case of the generalized potential \( V(\phi) = V_0 \exp(-\lambda \phi^n / M_{\text{Pl}}^n) \), with \( N = 60 \) and \( \lambda \leq 10^{-4} \), where the contours present the 1\( \sigma \) and 2\( \sigma \) bounds in the \( \Lambda \)CDM scenario, respectively. (b) \( 1 - n_s \) (solid curves) and \( r \) (dashed curves) as functions of the model parameter \( \lambda \), where the blue, green, red and cyan lines correspond to \( n = 5, 6, 7 \) and \( 8 \), respectively.

\[
\alpha_s \simeq -2(n-1)(n-2)N^2 + 6(n-1)(n-2)\lambda N \geq -2(n-1)/(n-2)N^2 \quad (3.21)
\]
corresponding to bounds on \( n_s, r, n_t \) and \( \alpha_s \), respectively.

In figure 2 we vary the model parameter \( \lambda \) to depict the scalar spectral index \( n_s \) and tensor-to-scalar ratio \( r \) with \( V = V_0 \exp(-\lambda \phi^n / M_{\text{Pl}}^n) \) and \( N = 60 \). The contour plots in figure 2a show the Planck 2015 results for the \( \Lambda \)CDM concordance model [63] which yield \( n_s \gtrsim 0.952 \) and \( r \lesssim 0.09 \) within 2\( \sigma \) confidence level, and the curves \( r(n_s) \) for the generalized exponential potential at hand. In figure 2 the dependence of \( 1 - n_s \) and \( r \) on \( \lambda \) is depicted. It is seen that, for a sufficiently large \( \lambda \), \( n_s - 1 \) quickly approaches the \( \lambda \)-independent value given by the r.h.s. of the last inequality in eq. (3.18). As a result, a best fit value of \( n \) is mainly determined by the measured value of \( n_s - 1 \), while the upper limit on \( r \) produces the lower limit on \( \lambda \) that can be seen already from eq. (3.11). Namely, from eqs. (3.18) and (3.20) and figure 2b, we find that \( 2(n-1)/(n-2)N \lesssim 1 - n_s \lesssim 0.048 \) and \( 0 \lesssim r \lesssim 0.09 \), which lead to \( \lambda \gtrsim 10^{-7}, 10^{-9}, 10^{-11} \) and \( 10^{-14} \) with \( n = 5, 6, 7 \) and \( 8 \), respectively. It is clear from the preceding discussion that our model based upon the generalized potential (2.9) provides an accurate description of inflationary phase. Hence, in the following section we shall investigate post-inflationary evolution.

4 Post-inflationary evolution

After the end of inflation, the kinetic stage follows when the potential \( V(\phi) \) can be neglected. Thus, the effective equation of state (EoS) of the field \( \phi \) is \( p_\phi = \rho_\phi \). Then

\[
a(t) \propto t^{1/3}, \quad \phi = \phi_{\text{end}} + \sqrt{2/3}M_{\text{Pl}}\ln t/t_{\text{end}}, \quad \rho_\phi = 3M_{\text{Pl}}^2H^2 = M_{\text{Pl}}^2/3t^2. \quad (4.1)
\]
To get a transition to the RD stage (the hot Big Bang), usual matter has to be created and heated. Since the unified potential (2.9) is of a runaway type, the standard parametric resonance mechanism is not applicable in this case. However, there exist sufficiently effective alternative mechanisms for this purpose, see e.g. [37]. In the following discussion we shall study post-inflationary evolution of the field $\phi$ in the presence of another background matter with the EoS $p_b = w_b \rho_b$, $0 \leq w_b < 1$ ($w_b = 1/3$ at the hot RD stage) without invoking a concrete mechanism of its creation and thermalization.

Due to this second kind of matter, the stage eq. (4.1) ends at some moment $t = t_b$. After that the Universe expands as $a(t) \propto t^q$, $q = \frac{2}{3(1 + w_b)}$. Let us first argue on heuristic grounds that there exists an approximately scaling late-time solution for $\phi$ in the case of the potential (2.9). By defining the dimensionless variables

$$x = \frac{\dot{\phi}}{\sqrt{6} H M_{\text{Pl}}}, \quad y = \sqrt{\frac{V}{3 H M_{\text{Pl}}}}, \quad \lambda = -M_{\text{Pl}} V_{\phi} \sqrt{\frac{V}{3 H M_{\text{Pl}}}}, \quad \Gamma = \frac{V V_{\phi \phi}}{V_{\phi}^2},$$

(4.2)

we can transform the equations of motion into the autonomous form as

$$\frac{dx}{dN} = f(x, y), \quad \frac{dy}{dN} = g(x, y),$$

(4.3)

$$\frac{d\lambda}{dN} = -\sqrt{6} \lambda^2 x (\Gamma - 1),$$

(4.4)

where $N = \ln a$ and $f$ and $g$ are functions of $x$ and $y$ whose explicit forms are not required for the present discussion. The third equation becomes redundant in the case of exponential potential. We mention that the potential (2.9) is steeper than the standard exponential function in the post inflationary era (slope $\sim \phi^{n-1}$, $n > 1$). Moreover, the function $\Gamma$ exhibits a very interesting feature, namely

$$\Gamma = 1 - \frac{(n-1)}{n} M_{\phi}^n \frac{\phi^n}{\phi^n}, \quad n > 1,$$

(4.5)

thereby $\Gamma \to 1$ for large values of the field, and thus the scaling solution would emerge as an attractor at late times. [47]. It is therefore clear that the dynamical system under consideration mimics scalar field with exponential potential at late stages.

### 4.1 Approximately scaling solution and exit to late-time acceleration

In this subsection we shall explicitly demonstrate the existence of the scaling solution. We shall focus on the scaling behavior in the RD and MD stages, followed by the dark energy epoch. In the RD and MD epochs the neutrino masses are negligible and neutrino matter behaves as radiation, hence $V_{\text{eff}}(\phi) = V(\phi)$, and thus (2.5) results in the field equation of the minimally coupled quintessence case, namely

$$\ddot{\phi} + 3H \dot{\phi} = -\frac{dV}{d\phi} = \frac{n \lambda \phi^{n-1}}{M_{\text{Pl}}^n} \exp \left( -\frac{\lambda \phi^n}{M_{\text{Pl}}^n} \right).$$

(4.6)

Using eqs. (2.3) and (2.4) and keeping in mind the definition

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{1}{2} \dot{\phi}^2 - V(\phi) = \frac{1}{2} \dot{\phi}^2 + V(\phi),$$

(4.7)
we obtain the expression

\[ \frac{dV}{d\phi} = \frac{1 - w_\phi}{1 + w_\phi}, \]

(4.8)

where \( w_\phi \) has been assumed to be constant in view of the scaling solution we are interested in. For the background dominated fluid with equation of state \( w_b \), the Hubble parameter has the form \( H = 2/3(1 + w_b) \cdot t^{-1} \). Substituting eq. (4.8) into (4.6), one readily obtains

\[ \ddot{\phi} + \frac{3H(1 + w_\phi)}{2} \dot{\phi} = 0, \]

(4.9)

such that for \( w_\phi = const \), \( \dot{\phi} \sim t^p \) is a solution of eq. (4.9), where \( p \) is a real number. However, this solution does not satisfy the field equation, since the time exponents of different terms in the equation do not match with each other which is not surprising as \( w_\phi \) is not constant in general. Instead, the asymptotic late-time solution of eqs. (4.6) for the potential (2.9) has the form of the following series:

\[ \lambda \left( \frac{\phi}{M_{Pl}} \right)^n = C_1 \ln(H_1 t) + C_2 \ln \ln(H_1 t) + \ldots, \]

(4.10)

where \( C_1, C_2 \) and \( H_1 \) are constant parameters to be determined. Here, we assume \( \lambda \ll 1 \), so that the scalar field undergoes the scaling regime with \( \phi \gg M_{Pl} \) and \( \ln(H_1 t) \gg 1 \) (note that \( H_1 \) is not of the order of \( t_b \) as is seen from the expression eq. (4.12) below).

After substituting the Ansatz (4.10) into eq. (4.6), the latter equation becomes

\[ \frac{(3q - 1)C_1^{\frac{1}{n}} \lambda^{-\frac{1}{n}} M_{Pl} (\ln(H_1 t))^{\frac{1-n}{n}}}{nt^2} = nC_1^{\frac{n-1}{n}} \lambda^{\frac{1}{n}} V_0 (\ln(H_1 t))^{\frac{n-1}{n} - C_2} M_{Pl} (H_1 t)^{C_1} \]

(4.11)

in the leading approximation. Comparing the powers and coefficients in the l.h.s. and r.h.s. of eq. (4.11), we find that

\[ C_1 = 2, \quad C_2 = 2 - \frac{2}{n}, \quad H_1^2 = \frac{2^{n-2} n^2 \lambda^{\frac{2}{n}} V_0}{(3q - 1) M_{Pl}^2}. \]

(4.12)

In addition, it is easy to check that

\[ \frac{\ddot{\phi}}{\dot{\phi}} \approx -\frac{1}{t} - \frac{\phi - 1}{nt \ln(H_1 t)} \frac{\lambda^{\phi_n/M_{Pl}^n} \gg 1}{\ln(H_1 t)} \approx \frac{1}{t}, \]

(4.13)

which clearly satisfies eq. (4.9) provided that we make the following identification:

\[ w_\phi = w_b. \]

(4.14)

In fact, the equation of state parameter for the field in the asymptotic regime (4.10) has the following form,

\[ w_\phi = w_b + O \left( \frac{\ln \ln(H_1 t)}{\ln(H_1 t)} \right), \]

(4.15)
which shows that the system gradually settles to the approximately scaling regime. As a result, the relative field energy density ratio at this regime can be solved as

$$\Omega_\phi = \frac{\rho_\phi}{3M_{Pl}^2H^2} \approx \frac{V(\phi)t^2}{q(3q-1)M_{Pl}^2} = \left(2n^2\lambda^2 \left(\ln(H_1t)\right)^{2-\frac{3}{q}}\right)^{-1} = \frac{2}{qn^2\lambda^2} \left(\frac{M_{Pl}}{\phi}\right)^{2n-2}. \quad (4.16)$$

For $n = 1$ (the standard exponential potential), this expression reduces to the well known result $\Omega_\phi = \frac{2}{q\lambda^2} = \text{const}.$

It follows from eq. (4.16) that the approximately scaling regime begins at the moment $t = t_s$ when $\ln(H_1t) \sim \lambda^{-\frac{1}{q}} \gg 1$ and $\phi \sim M_{Pl}\lambda^{-\frac{1}{q}} \sim \phi_{end},$ where $\phi_{end}$ was defined in eq. (3.13). However, in fact $t_s \gg t_b,$ so it cannot begin immediately after the moment when $\rho_\phi = \rho_b.$ The reason for this is that the regime eq. (4.16) requires $\frac{\dot{\rho}}{2V(\phi)} = (1 + w_b)/(1 - w_b) = 1/(3q - 1) \sim 1,$ while $V(\phi) \ll \lambda^2$ during the kinetic stage eq. (4.1), and at $t = t_b$ in particular. Thus, another, a rather short intermediate stage should occur between the kinetic and scaling regimes until $\rho_b \propto t^{-2}$ falls down to the value of $V(\phi).$ At this stage

$$a(t) \propto t^q, \; q > \frac{1}{3}, \; \dot{\phi} \propto a^{-3} \propto t^{-3q}, \; \phi \approx \text{const}, \; V(\phi) \approx V_1 = \text{const}. \quad (4.17)$$

Thus,

$$t_s \sim \frac{M_{Pl}}{\sqrt{V_1}} \sim t_b \left(\frac{\rho_b(t_b)}{V_1}\right) \gg t_b. \quad (4.18)$$

The fact that scaling at the stage (4.10), (4.16) is only an approximate one in our model is crucial since it leads to $\Omega_\phi \ll 1$ at the RD and MD stages (apart from a short period around $t = t_s$ at the beginning of scaling). This provides a possibility to satisfy observational upper limits on the amount of early dark energy (quintessence in our case) at the Big Bang Nucleosynthesis (BBN) period in the early Universe ($q = 1/2$) and at the recombination moment at the MD stage (the redshift $z \approx 1100$ when $q \approx 2/3$). From BBN, we have the constraint $\Omega_\phi < 0.045$ [66]. The most recent result on the primordial $^4$He abundance [67] leads to the effective neutrino number $\Delta N_{eff}$ to be $0.53 \pm 0.50$ at 99% confidence level, corresponding to $\Omega_\phi < 0.11$ with $N_{eff} = 3.046$. The upper limit from recombination is an order of magnitude stronger: $\Omega_\phi < 0.0036$ in ref. [68]. Consequently, from eq. (4.16) we find that

$$\ln(\phi/M_{Pl})_{BBN}^{n-1} > 6.03 \quad \text{and} \quad \ln(\phi/M_{Pl})_{rec}^{n-1} > 28.9. \quad (4.19)$$

Subsequently, by combining with the numerical values of $\lambda(\phi/M_{Pl})_{BBN}^{n-1} \simeq 170$ and $\lambda(\phi/M_{Pl})_{rec}^{n-1} \simeq 220$ with $V_0/H_0^2M_{Pl}^2 = 10^{100},$ the second condition in eq. (4.19) is reduced to

$$n\lambda^{\frac{1}{n}} > 28.9 \times (220)^{\frac{1}{n-1}}. \quad (4.20)$$

Clearly, eq. (4.20) is always valid when $n \gg 1,$ while the allowed parameter space for small numerical values of $n$ is shown in figure 3.

Now let us consider transition from the MD stage to the stage of dark energy dominance which occurs at the present epoch. In the absence of non-minimal coupling of neutrinos to quintessence ($\beta = 0$ in eq. (2.2)), the approximately scaling MD stage would run forever. A non-zero $\beta$ changes the effective potential by adding a $\phi$-dependent term to it according to eq. (2.8). As follows from the relation (4.10), the scalar field grows in time during the
Figure 3. The gray region shows the allowed parameter space from the recombination constraint.

RD and MD epochs, and the growth of $\phi$ results in the increase of neutrino masses. With a large enough $\phi$, neutrino-matter turns into non-relativistic, and the neutrino contribution in the effective potential (2.8) is no longer negligible. Hence, the coupling to the field builds up, giving rise to the minimum in the effective potential; the original potential is of a steep run-away type. As the scalar approaches the minimum of the effective potential, it oscillates around it and finally settles down in the minimum at $\phi_m$, and the universe in the scenario under consideration enters the dark-energy dominated stage. In this case, the numerical value of the scalar at present time, namely $\phi_0$, is approximately equal to $\phi_m$, i.e.

$$\phi_0 \simeq \phi_m = M_{\text{Pl}} \left[ \ln \left( \frac{V_0}{\rho_{\phi,0}} \right) \right]^{\frac{1}{n}},$$

(4.21)

derived by assuming $\rho_{\phi,0} = \dot{\phi}^2/2 + V(\phi) \simeq V(\phi)$, where $\rho_{\phi,0}$ is the energy density of the scalar at present time. Combining (4.21) with

$$\left. \frac{dV_{\text{eff}}}{d\phi} \right|_{\phi=\phi_m} = 0,$$

(4.22)

we obtain

$$\beta = \frac{\lambda n \phi_0^{n-1} \rho_{\phi,0}}{M_{\text{Pl}}^{n-1} \rho_{\nu,0}}.$$

(4.23)

In figure 4, we numerically solve the evolution equations to demonstrate the evolution history. Since the cosmic evolution after the post-inflationary epoch is insensitive to $V_0$, in our numerical calculations we choose $V_0/\rho_{m,0} = 10^{110}$, corresponding to the inflationary scale. In figure 4a, the energy density $\rho_\phi$ exhibits a tracker scaling behavior in RD and MD epochs, and the dark energy dominated stage occurs after the neutrinos become massive. Figure 4b shows the evolutions of $w_\phi$ and $w_\nu$. The EoS of the $\phi$-field behaves as radiation (matter) in the RD (MD) epoch, which is consistent with the result of relation (4.14). After the neutrino mass becomes important, $\phi$ reaches its minimum $\phi_m$ and oscillates around it at $z \lesssim 3$. This plot also demonstrates that the neutrino mass is negligible ($w_\nu = 1/3$) in early times ($z \gtrsim 10$); oscillates when $\phi$ evolves around $\phi_m$ in the intermediate region ($0 \lesssim z \lesssim 3$), and finally acquires its present value when $w_\nu = 0$ at late-times, in which dark energy is dominant.
Figure 4. (a) Energy densities of $\rho_r$ (gray-dashed), $\rho_m$ (black-solid) and $\rho_\phi$ with $\lambda = 10^{-8}$ (blue-solid) and $10^{-6}$ (red-dashed), normalized by the matter energy density $\rho_m^{(0)} \equiv \rho_m|_{z=0}$, as functions of $N \equiv \ln a$, with the potential $V(\phi) = V_0 \exp (-\lambda (\phi^n/M_{Pl}^n))$ plotted for $\lambda = 10^{-8}$ (blue-solid) and $10^{-6}$ (red-dashed) and 20 (red-dotted), with $\Omega_c h^2 = 0.118$. (b) The equation-of-state parameters $w_\phi$ and $w_\nu$ as functions of $N$, where we have used $\Sigma m_\nu = 0.45 \text{eV}$, $\Omega_m h^2 = 0.118$ and $\rho_r^{(0)}/\rho_m^{(0)} = 2.6 \times 10^{-4}$ as boundary conditions.

Finally, we would like to mention that the scalar $\phi$ within a large-scale neutrino lump deviates significantly from that at the background level, leading the neutrino masses to be negligible inside the neutrino lump, and the perturbation becomes non-linear [62, 69]. In this work we avoid such a non-linear region and we assume that the neutrino mass $m_\nu$ and the scalar $\phi$ are both homogeneous, and thus the perturbation of neutrinos in the non-minimally-coupled scenario behaves in the same way as that in $\Lambda$CDM cosmology, namely

$$
\delta_\nu = 3H \left( w_\nu - \frac{\delta p_\nu}{\delta \rho_\nu} \right) \delta_\nu - \left( 1 + w_\nu \right) \left( \theta - \frac{\bar{h}}{2} \right) .
$$

In summary, from the above discussions we deduce that the non-minimally coupled neutrino-matter scenario, with the generalized exponential potential, perfectly describes the universe evolution history. In the next subsection, we will confront this scenario with cosmological observational data.

4.2 Observational constraints

We use the CosmoMC program [70, 71] in order to extract observational constraints on the quintessential inflation scenario. In our analysis, we include the data of the cosmic microwave background (CMB) from Planck [10, 11], baryon acoustic oscillation (BAO) from Baryon Oscillation Spectroscopic Survey (BOSS) [72, 73], and Type-Ia supernova (SNIa) from Supernova Legacy Survey (SNLS) [74]. The details of the fitting procedure can be found in refs. [70, 71]. The prior for the parameters is listed in table 1.

To prevent non-analytical behaviour at $\phi = 0$ and to have a maximum of $V(\phi)$ at this point, we focus on even values of $n$. It appears that the values $n = 6$ and $n = 8$ produce the best fits to observational data. In figure 5, we depict the 2D likelihood contours for $\Omega_b h^2$, $\Omega_c h^2$, $\sum m_\nu$ and $\sigma_8$ with $n = 6$ (orange) and $n = 8$ (blue), where the contour lines represent 68% and 95% confidence levels, respectively. The scenario at hand is in agreement with observations, in which the $\chi^2$ values for $n = 6$ and 8 are equal and even less than that...
| Parameter               | Prior                                |
|-------------------------|--------------------------------------|
| Neutrino mass sum       | $0.1 \leq \Sigma m_{\nu} \leq 4.0 \text{ eV}$ |
| Model parameter $\lambda$ ($n = 6$) | $-10 \leq \log \lambda \leq -5$    |
| Model parameter $\lambda$ ($n = 8$) | $-14 \leq \log \lambda \leq -7$    |
| Baryon density          | $0.5 \leq 100\Omega_b h^2 \leq 10$  |
| CDM density             | $10^{-3} \leq \Omega_c h^2 \leq 0.99$|

Table 1. Priors for cosmological parameters with $V(\phi) = V_0 e^{-\lambda(\phi/M_{Pl})^n}$.

Figure 5. One and two-dimensional distributions of $\Omega_b h^2$, $\Omega_c h^2$, $\Sigma m_{\nu}$ and $\sigma_8$, for $n = 6$ (orange) and $n = 8$ (blue), where the contour lines represent the 68% and 95% confidence levels, respectively.

of the $\Lambda$CDM model. The cosmological quantities, such as the present dark-matter and dark-energy densities, have similar ranges as those in $\Lambda$CDM cosmology [10, 11]. However, the neutrino mass sum in the model, in both $n = 6$ and 8 cases, is enhanced to 1 eV, and the allowed window is also significantly relaxed, such that $\Sigma m_{\nu} \lesssim 2.5 \text{ eV}$.

As discussed in section 3, the model parameter $\lambda$ is bounded from below. However, this bound depends on the other parameter $n$ and it is smaller than $10^{-10}$ when $n \gtrsim 7$. In figure 6, we present the 1D marginalized probability plot for $\lambda$ with $n = 6$ and 8. As we can see, the cosmological data prefer a larger value of $\lambda$, corresponding to $n_s \rightarrow 0.96$ and $r \rightarrow 0$

Note for completeness that if we omit the assumption of $n$ being even and, purely phenomenologically, permit it to be any real number, we get $n = 6.74_{-0.59}^{+1.10}$ (68% confidence limits).
Figure 6. Marginalized probabilities for the potential parameter \( \lambda \) with (a) \( n = 6 \) and (b) \( n = 8 \), respectively.

| Parameter                  | Quintessential Inflation (n=6) | Quintessential Inflation (n=8) | \( \Lambda \mathrm{CDM} \) |
|----------------------------|---------------------------------|--------------------------------|-----------------------------|
| Spectral index             | \( n_s = 0.960 \pm 0.001 \)      | \( n_s = 0.961 \pm 0.001 \)    | \( n_s = 0.970 \pm 0.009 \)  |
| Tensor-to-scalar ratio     | \( r < 1.72 \times 10^{-2} \)   | \( r < 2.32 \times 10^{-2} \)  | \( r < 0.125 \)             |
| Baryon density             | \( 100 \Omega_b h^2 = 2.21^{+0.01}_{-0.04} \) | \( 100 \Omega_b h^2 = 2.21^{+0.04}_{-0.03} \) | \( 100 \Omega_b h^2 = 2.23 \pm 0.04 \) |
| CDM density                | \( \Omega_c h^2 = 0.120 \pm 0.002 \) | \( \Omega_c h^2 = 0.119^{+0.002}_{-0.001} \) | \( \Omega_c h^2 = 0.118^{+0.002}_{-0.003} \) |
| Neutrino mass              | \( \sum m_\nu = 0.96^{+1.25}_{-0.66} \) eV | \( \sum m_\nu = 1.13^{+1.41}_{-1.03} \) eV | \( \sum m_\nu < 0.24 \) eV |
| Model parameter \( \lambda \) | \( \log \lambda > -7.29 \)        | \( \log \lambda > -11.7 \)     | -                           |
| \( \sigma_s \)             | \( \sigma_s = 0.770^{+0.043}_{-0.060} \) | \( \sigma_s = 0.763^{+0.057}_{-0.062} \) | \( \sigma_s = 0.805 \pm 0.027 \) |
| \( \Delta \chi^2 \equiv \chi^2 - \chi^2_{\Lambda \mathrm{CDM}} \) | \( \Delta \chi^2 = +0.2 \)       | \( \Delta \chi^2 = -2.8 \)     | -                           |

Table 2. List of allowed regions with 95% C.L., in the case of \( V(\phi) = V_0 e^{-\lambda(\phi/M_{\mathrm{Pl}})^n} \).

in both cases. Finally, it is worth to mention that the coupling \( \beta \) is tuned to control the values of \( \phi_0 \) and \( \Omega_\phi \) in the numerical analysis (for instance, from eqs. (4.21) and (4.23), we have \( (\phi_0, \beta) \simeq (54, 1800) \) with \( (\lambda, n) = (10^{-8}, 6) \)).

Our detailed results for the non-minimally-coupled scenario are summarized in table 2.

5 Conclusions

In this work we have investigated a model of quintessential inflation based upon the generalization exponential potential \( V = V_0 \exp (-\lambda \phi^n/M_{\mathrm{Pl}}^n) \), \( n > 1 \). This simple function has remarkable properties, namely its slope behaves as \( \phi^{n-1} \), and thus it can facilitate slow roll for \( \lambda \phi^{n-1}/M_{\mathrm{Pl}}^{n-1} \ll 1 \) (if \( \lambda \ll 1 \)). For large values of the field, \( \lambda \phi^{n-1}/M_{\mathrm{Pl}}^{n-1} \gg 1 \), the parameter \( \Gamma = V_\phi V/V_\phi^2 \to 1 \), and as a consequence the system effectively mimics scaling behavior at late stages. There is a comfortable parameter space in \( (n, \lambda) \) that allows us to satisfy the observational constraints related to inflation. In figure 2 we have displayed the 2\( \sigma \) contours based upon Planck data and the predictions of the scenario at hand, which show that the model performs as desired for \( n \geq 5 \) and \( \lambda \leq 10^{-4} \). After the end of inflation, the field enters into the steep regime of the potential. It was indicated earlier that the generalized potential gives rise to an approximately scaling behavior despite being steeper than the standard exponential one. We have analytically shown here the existence of approximately scaling solutions in the followed-up radiation and matter dominated epochs. At for the late time evolution, we need a mechanism of exit from the scaling regime to dark energy. To this effect,
the massive neutrino matter seems to be a convenient device to achieve the set goal. The non-minimally coupled massive neutrino-matter contributes to the effective potential of the field $\phi$ at late stages and induces a minimum in it, where the scalar field is trapped giving rise to dark energy dominated epoch.

The non-minimal coupling crucially affects the neutrino masses. Neutrinos behave as a massless fluid in most of the cosmic history, and the masses manifest at late-times when neutrinos turn non-relativistic. As a result, the constraint on the neutrino masses from early universe observations is relaxed. By making use of the CosmoMC package, we have shown that the allowed $\Sigma m_\nu$ at the present time is enhanced to around 1 eV that is significantly larger than in the standard $\Lambda$CDM model. We conclude that our model is in excellent agreement with observations and presents a successful scheme of the unification of primordial dark energy driving inflation in the very early Universe and present dark energy producing accelerated expansion of the present Universe. We should mention that the generic feature of quintessential inflation is related to the presence of the kinetic regime after inflation, which gives rise to blue spectrum of relic gravity waves at very small scales (see ref. [75] for details).

As we pointed out, there is a significant enhancement of the allowed $\Sigma m_\nu$ in our model, which could provide room for a sterile neutrino. In this case, the sterile neutrino with mass around 1 eV, as suggested by anomalies in neutrino oscillation experiments, is possible. If such a possibility is experimentally confirmed, it might signal the demise of $\Lambda$CDM, since it is well known that incorporating the neutrinos masses leads to suppression of matter power spectrum in the framework of $\Lambda$CDM cosmology or slowly rolling quintessence. Interestingly, the generic schemes of large-scale modifications of gravity give rise to the opposite effect. Indeed, in these schemes, in an approximation valid in the domain of interest, the modification is captured replacing $G$ by $G_{\text{eff}}$, which gives enhancement in the matter power spectrum. Hence, this feature could also allow us to accommodate higher mass neutrinos and salvage sterile neutrino cosmology [76, 77]. It would be interesting to address the issue in the framework of an effective theory including the most general scalar field action à la Hordeski which leads to second order equations for $\phi$. Such an investigation is left for a future project.

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