Cooperative Stochastic Multi-agent Multi-armed Bandits Robust to Adversarial Corruptions

Junyan Liu  
University California San Diego  
jul037@ucsd.edu

Shuai Li  
Shanghai Jiao Tong University  
shuaili@sjtu.edu.cn

Dapeng Li  
Nanjing University of Posts and Telecommunications  
dapengli@njupt.edu.cn

Abstract

We study the problem of stochastic bandits with adversarial corruptions in the cooperative multi-agent setting, where $V$ agents interact with a common $K$-armed bandit problem, and each pair of agents can communicate with each other to expedite the learning process. In the problem, the rewards are independently sampled from distributions across all agents and rounds, but they may be corrupted by an adversary. Our goal is to minimize both the overall regret and communication cost across all agents. We first show that an additive term of corruption is unavoidable for any algorithm in this problem. Then, we propose a new algorithm that is agnostic to the level of corruption. Our algorithm not only achieves near-optimal regret in the stochastic setting, but also obtains a regret with an additive term of corruption in the corrupted setting, while maintaining efficient communication. The algorithm is also applicable for the single-agent corruption problem, and achieves a high probability regret that removes the multiplicative dependence of $K$ on corruption level. Our result of the single-agent case resolves an open question from Gupta et al. [2019].

1 Introduction

The multi-armed bandit (MAB) problem is one of the most fundamental problems in online learning. Motivated by the emerging need for cooperative multi-agent learning in large-scale systems, researchers sought to expedite the learning process for MAB problem by exploiting the multi-agent cooperation. Examples include recommendation systems [Sankararaman et al. 2020], forage task of robotics [Sugawara et al. 2004], channel allocation in wireless networks [Kalathil et al. 2014, Liu et al. 2013]. The learning process of those applications is inherently distributed, and due to geographical dispersion, the communication between each agent often comes at a heavy price. Therefore, apart from the exploration-exploitation trade-off in the basic MAB problem, the multi-agent MAB problem needs to tackle with an additional trade-off between the regret and communication cost.

The multi-agent MAB problem is typically studied in the (i) stochastic setting [Szörényi et al. 2013, Hillel et al. 2013, Tao et al. 2019, Wang et al. 2020, Bistritz and Leshem 2018, Rosenski et al. 2016, Martinez-Rubio et al. 2019, Dubey and Pentland 2020], i.e., the rewards are always sampled from some underlying distributions, or (ii) adversarial setting [Awerbuch and Kleinberg 2005, Kanade et al. 2012, Cesa-Bianchi et al. 2019, Ito et al. 2020, Bar-On and Mansour 2019], i.e., the rewards are always corrupted/manipulated by an adversary. However, these two extremes are not appropriate for many real-world situations, e.g., click fraud [Lykouris et al. 2018], fake review-
The corruption level across all agents and rounds is measured as
\[ C = \sum_{t=1}^{T} \sum_{v=1}^{V} \left\| r_v(t) - r_v^S(t) \right\|_{\infty}. \] (1)

The adversary is allowed to be adaptive, in the sense that the corruption at round \( t \) is determined as a function of all previous arm selections and corresponding rewards. The corruption in any round is independent of the choice of arms in the same round across all agents. It is noteworthy that the corruption level \( C \) is a random variable that depends on the randomization of stochastic rewards and the choices of agents.

The goal of agents is to minimize the communication costs that measure the total bits transmitted between all agents and the pseudo-regret (regret, for short unless otherwise stated) defined as
\[ R_T = \sum_{t=1}^{T} \sum_{v=1}^{V} \left( \mu^{\ast} - \mu_{v\ast(t)} \right), \] (2)
where \( i^* \) is the optimal arm such that \( i^* = \arg \max_{i \in [K]} \mu_i \), and \( \mu_{i^*} \) is its corresponding mean. For any suboptimal arm \( i \neq i^* \), the gap of arm \( i \) is \( \Delta_i = \mu_{i^*} - \mu_i \in [0, 1] \) and \( \Delta_{\min} = \min_{i: \Delta_i > 0} \Delta_i \).

### 1.2 Related Work

#### Multi-agent MAB

The multi-agent MAB problem is extensively studied in either stochastic setting or adversarial setting.

In the stochastic setting, one line of research [Awerbuch et al. 2005, Chakraborty et al. 2017, Wang et al. 2020, Tao et al. 2019, Martinez-Rubio et al. 2019, Dubey and Pentland 2020] study the cooperative multi-agent model that allows agents to communicate with other agents. Those models can be roughly categorized into two types. On the one hand, the works of [Hillel et al. 2013, Buccapatnam et al. 2015, Chakraborty et al. 2017, Wang et al. 2020, Tao et al. 2019] allows each pair of agents to communicate in a predefined round, and the regret is improved by some \( V \)-dependent factors, e.g., \( 1/V \) or \( 1/\sqrt{V} \). This type of model is most relevant to ours without corruption, i.e., \( C = 0 \). On the other hand, the works of [Landgren et al. 2016a,b, 2021, Martinez-Rubio et al. 2019, Dubey and Pentland 2020] restricts the communication over the network, wherein agents can only communicate with their neighbors encoded by an undirected communication graph with an unknown structure. Another branch of works [Bistritz et al. 2020, Kalathil et al. 2014, Liu et al. 2013, Tibrewal et al. 2019, Rosenski et al. 2016, Bistritz and Leshem 2018] investigates a competitive setting where the reward becomes zero or the reward is split when two or more agents select the same arm simultaneously. These competitive models are considered for some real-world applications wherein the communication between agents is almost impossible, and sometimes agents need to learn from the collision rather than communicating information. This setting essentially differs from ours which leverage the balance between the regret and the communication.

The adversarial multi-agent MAB problem is first examined in [Awerbuch and Kleinberg 2005] which considers some dishonest agents who do not follow the predefined protocol and may send fake observations. Subsequently, the authors of [Cesa-Bianchi et al. 2019] study the adversarial setting for multiple agents connected by a general communication graph with potential delays, and prove an averaged regret over all agents. Then, authors of [Bar-On and Mansour 2019] show that all agents can simultaneously maintain a low individual regret under the same setting of [Cesa-Bianchi et al. 2019]. Moreover, the work of [Cesa-Bianchi et al. 2020] studies the adversarial multi-agent problem in an asynchronous setting where only some of the agents are active at each round, and agents share feedback in an undirected graph. Unlike our model, all of the above works pessimistically assume that the unknown environment is completely adversarial, and thus achieve a \( \tilde{O}(\sqrt{T}) \) (ignoring factors of delay and network properties) bound even if the environment is completely stochastic. As a result, those cooperative (coop)-Exp3 algorithms cannot obtain a better \( O(\log T) \) bound when the environment is close to being stochastic (i.e., under moderate corruptions). This paper allows the environment to adaptively transit between the stochastic one and the adversarial one, as the adversary to inject the different amount of corruption.

#### Stochastic Bandits with Corruption

Lykouris et al. [Lykouris et al. 2018] first introduce the corrupted setting and incorporate the corruption level \( C \) in the regret bound as \( O(\sum_{i \neq i^*} C/\Delta_i) \). Subsequently, the work of [Gupta et al. 2019] improves the regret bound of [Lykouris et al. 2018] and derives a \( O(KC) + \tilde{O}(\sum_{i \neq i^*} 1/\Delta_i) \) bound which enjoys an additive term of corruption \( C \). The authors of [Kapoor et al. 2019] consider a probabilistic corruption model where the corruption in the current round is independent of previous corruption. Even though authors of [Kapoor et al. 2019] obtain the \( O(C) + \tilde{O}(\sum_{i \neq i^*} 1/\Delta_i) \) bound, but the algorithm of [Kapoor et al. 2019] is not robust to the attack policy which does not follow some specific probabilistic models. The works of [Zimmert and Seldin 2021, Masoudian and Seldin 2021] adapt online mirror descent (OMD) technique with Tsallis entropy to achieve the expected regret of \( O(\mathbb{E}[C]) + O(\sum_{i \neq i^*} 1/\Delta_i) \) in the corrupted MAB problem. Note that the works of [Zimmert and Seldin 2021, Masoudian and Seldin 2021] use a different regret metric, and the additive term of \( \mathbb{E}[C] \) is necessary if we convert their results to \( \mathbb{E}[R_T] \) (see Appendix C.2). Besides, the corrupted setting is also considered for the linear bandits of [Li et al. 2019, Bogunovic et al. 2020, 2021, Lee et al. 2021]. The work of [Li et al. 2019, Bogunovic et al. 2021] incurs a \( \tilde{O}(C^2) \) or \( \tilde{O}(C/\Delta_{\min}) \) dependence of corruption for regret when the adversary exactly knows the currently chosen arm, while [Lee et al. 2021] provides a high probability regret which
has additive dependence of $O(C)$, when the adversary is unaware of the current choice. The result in [Lee et al. 2021] becomes $O(C) + \tilde{O}(K^{1.5}/\Delta_{\min})$ in the MAB setup. All of the above works consider the corruption for single-agent MAB problem, little is known about the corruption in the multi-agent MAB problem. Further, extending existing single-agent algorithms toward multi-agent ones may fail to balance the communication-regret trade-off due to some technical challenges as discussed in Subsection 3.1.

1.3 Contributions

Our main contributions are as follows:

- This paper, to the best of our knowledge, first studies the adversarial corruption problem for the multi-agent setting. For this problem, we present a lower bound $\mathbb{E}[R_T'] = \Omega(\mathbb{E}[C] - \sqrt{VT\log(VT)})$ and $\mathbb{E}[R_T] = \Omega(\mathbb{E}[C] + \mathbb{E}[R_T])$ for any algorithm with $V$ agents (Theorem 1) where $R'_T = \max_{i \in [K]} \sum_{t=1}^{T} \sum_{v=1}^{V} (r_{v,i}(t) - r_{v,i}(0)(t))$ and $R_T$ in (2). Different from previous results of Lykouris et al. [2018], Auer and Chiang [2016] that only give the lower-bound of $\mathbb{E}[R_T']$, we present the lower bounds of both $\mathbb{E}[R_T']$ and $\mathbb{E}[R_T']$ for any bandit algorithm with $V \geq 1$. Our results show that the additive term $\mathbb{E}[C]$ is unavoidable for $\mathbb{E}[R_T']$ and $\mathbb{E}[R_T']$.

- We propose a new algorithm (Algorithm 1) with $V \geq 1$ agent(s), which is agnostic to the corruption level $C$. The proposed algorithm achieves a high probability regret $R_T = O(VC) + \tilde{O}(K/\Delta_{\min})$ and expected regret $\mathbb{E}[R_T] = O(V\mathbb{E}[C]) + \tilde{O}(K/\Delta_{\min})$, while maintaining a $O(K\log T)$ communication cost (Theorem 2). The proposed algorithm is applicable for the single-agent setting, but existing robust single-agent algorithms cannot be trivially extended to the multi-agent setting (see discussion 5.1). Our algorithm achieves a high probability regret $O(C) + \tilde{O}(K/\Delta_{\min})$ for $V = 1$, which resolves an open question from Gupta et al. [2019].

- We further discuss the results evaluated by some other possible regret metrics. We show that any algorithm in the corrupted problem will suffer $\mathbb{E}[R_T'] = \Theta(\mathbb{E}[R_T'] + \mathbb{E}[C])$ and $\mathbb{E}[R_T'] = \Theta(\mathbb{E}[R_T'] + \mathbb{E}[C])$ (Theorem 3) where $\mathbb{E}[R_T'] = \max_{i,v} \mathbb{E}[\sum_{t=1}^{T} \sum_{v=1}^{V} (r_{v,i}(t) - r_{v,i}(0)(t))]$ is the (pseudo)-regret defined in the adversarial regime, which cannot be trivially converted to (pseudo)-regret $R_T$ defined in the stochastic regime, when $C \neq 0$.

2 Lower Bound

In this section, we present the lower bound for this problem. Since it is challenging to directly lower-bound $R_T'$ in the corrupted setting, we first lower-bound another regret notation and then bridge both metrics by considering their expectation. Let define the regret as $R'_T$ which measures the difference between the reward from the arm that contributes to the maximum cumulative rewards, and the rewards observed by the agents in hindsight:

$$R'_T = \max_{i \in [K]} \sum_{t=1}^{T} \sum_{v=1}^{V} (r_{v,i}(t) - r_{v,i}(0)(t)),$$

Note that $R'_T$ can be either positive or negative, whereas the pseudo-regret $R_T$ is always non-negative because it measures the difference of means (i.e., the expectation of stochastic rewards).

**Theorem 1. (Lower bound)** For any $V \geq 1$, if a bandit algorithm with $V$ agent(s) that guarantees a pseudo-regret bound as $O(\log(VT)/\Delta)$ in stochastic environments with two arms and $|\mu_1 - \mu_2| = \Delta \in (1/4, 1/2)$ for sufficiently large $T$, then, there is an corrupted instance with corruption level $C = VT^\alpha$ for $\alpha \in (0, 1)$ such that with at least a constant probability $R'_T = \Omega(C - \sqrt{VT\log(VT)})$. Then, $\mathbb{E}[R'_T] = \Omega(\mathbb{E}[C] - \sqrt{VT\log(VT)})$ and $\mathbb{E}[R_T'] = \Omega(\mathbb{E}[C] + \mathbb{E}[R_T'])$.

**Remark 1.** Theorem 1 shows that the additive term of expected corruption $\mathbb{E}[C]$ is unavoidable in $\mathbb{E}[R_T']$ for any algorithm that can achieve a near-optimal regret in the stochastic setting. The lower bound of $R'_T$ appear in some previous works for the single-agent corruption setting.
Wang et al. [2020] for multi-agent bandits. Note that the lower bound of $\mathbb{E}[R_T]$ does not imply a lower bound of $r_T$, and the lower bound of $R_T$ remains as an open question.

3 Our Algorithm

This section presents our algorithm in Algorithm 1. The algorithm proceeds in epochs indexed by $\tau$ whose length $N(\tau)$ with exponential increase depending on a global error level $\varepsilon(\tau)$ (line 7). The algorithm maintains a global active arm set $\mathcal{A}(\tau)$, and a global bad arm set $\mathcal{B}(\tau)$ such that $|K| = \mathcal{A}(\tau) \cup \mathcal{B}(\tau)$. Besides, each agent $v$ is endowed with an agent-specific arm set $\mathcal{K}_v(\tau) \subseteq [K]$, an agent-specific active arm set $\mathcal{A}_v(\tau)$, and an agent-specific bad arm set $\mathcal{B}_v(\tau)$ such that $\mathcal{K}_v(\tau) = \mathcal{A}_v(\tau) \cup \mathcal{B}_v(\tau)$ and each agent has the same size of $\mathcal{K}_v(\tau)$, i.e., $|\mathcal{K}_v(\tau)| = K$. We further have that $\cup_{v=1}^V \mathcal{A}_v(\tau) = \mathcal{A}(\tau)$ and $\cup_{v=1}^V \mathcal{B}_v(\tau) = \mathcal{B}(\tau)$. Each arm $i$ is associated with an arm-specific error level $\varepsilon_i(\tau)$. The error level $\varepsilon_i(\tau)$ controls the expected number of pulls of arm $i$ in epoch $\tau$, denoted by $N_{v,i}(\tau)$.

At the beginning of epoch $\tau$, Algorithm 1 runs ARM ALLOCATION subroutine to divide the arm space into total $V$ parts and thus each agent $v$ only needs to learn a small arm space $\mathcal{K}_v(\tau)$ (line 5). In each round, every agent $v \in [V]$ samples an arm $i \in \mathcal{K}_v(\tau)$ with the probability $p_{v,i}(\tau)$ (line 9), given by

$$p_{v,i}(\tau) = \begin{cases} 
\frac{\varepsilon_i(\tau)}{K}, & i \in \mathcal{B}_v(\tau), \\
\frac{\varepsilon_i(\tau)}{K} - \left(1 - \sum_{j \in \mathcal{B}_v(\tau)} \frac{1}{|\mathcal{A}_v(\tau)|}\right), & i \in \mathcal{A}_v(\tau).
\end{cases}$$

At the end of epoch $\tau$, the algorithm first proceeds in Step 1 in which each agent $v$ updates the estimator $\hat{\mu}_{v,i}(\tau)$ for all $i \in \mathcal{K}_v(\tau)$, and then broadcasts the arm sets i.e., $\mathcal{A}_v(\tau)$ and $\mathcal{B}_v(\tau)$ and estimation to the leader agent (line 14). The leader agent constructs a global estimator $\hat{\mu}_i(\tau)$ for each arm $i \in [K]$ by averaging estimators over all agents. Then, the leader agent proceeds in Step 2 which reactivates the seemingly-good arms from $\mathcal{B}(\tau)$ whose estimations are inconsistent with previous ones (line 17), and then deactivates seemingly-bad arms which contribute to less reward in this epoch (line 19). Finally, Step 3 updates the global error level $\varepsilon(\tau)$ and arm-specific error level $\varepsilon_i(\tau)$ for the next epoch (line 22-24) according to the results of Step 2. We then explain these three important ingredients of our algorithm, including arm allocation, robust reactivation and deactivation, and error level update in detail.

Arm allocation. The main idea of arm allocation is to divide the arm space into several parts which are allocated with agents, and then each agent only needs to (i) learn a small arm space, and (ii) broadcast a small message with $O(K/V)$ size. The details of arm allocation is presented in Algorithm 2. Each agent $v \in [V]$ is allocated with exactly $\hat{K} = \lceil K/V \rceil + 1$ arms such that $\cup_{v=1}^V \mathcal{K}_v(\tau) = [K]$, and each agent has the same number of arms. Note that we need $\mathcal{K}_v(\tau)$ to have at least one active arm $i$ such that $i \in \mathcal{A}(\tau)$, and otherwise $\sum_{i \in \mathcal{K}_v} p_{v,i}(\tau) \neq 1$. To this end, we add an additional arm $\hat{i}^*$ named empirical best arm for each $\mathcal{K}_v(\tau)$. The empirical best arm is given by

$$\hat{i}^* = \arg\max_{j \in \mathcal{A}(\tau) \cup \mathcal{H}(\tau)} \hat{\mu}_j(\tau) + 2\varepsilon_j(\tau),$$

whose corresponding estimation is denoted by $\hat{\mu}^*$. Adding arm $\hat{i}^*$ can ensure that $\sum_{i \in \mathcal{K}_v} p_{v,i}(\tau) = 1$ because the arm $\hat{i}^*$ identified in epoch $\tau$ must be active in epoch $\tau + 1$ according to the deactivation rule (regard the initialization as epoch $\tau = 0$).

The similar idea of dividing arm space appears in, for example Hillel et al. [2013], Tao et al. [2019], Wang et al. [2020] for multi-agent bandits. Note that compared with the ones in previous works, our algorithm has a unique difference. The algorithms in Hillel et al. [2013], Tao et al. [2019], Wang et al. [2020] only allocate active arms for agents, thereby the arm space shrinking in proceeding epochs, but our algorithm needs to ensure that all arms have chance to be pulled for every epoch due to the presence of the adversary. As a consequence, we do not shrink the arm space, but
Algorithm 1 Cooperative Bandit Algorithm Robust to Adversarial Corruptions

Require: Time horizon $T$, and confidence $\delta \in (0, 1)$.
1: Set $\tau = 1$, $N(\tau) = 0$, and $\epsilon(\tau) = \epsilon(\tau) = \frac{1}{\tau^2}$ for all $i$. Randomly sample $\hat{i}^*$ from $[K]$.
2: Set $A(\tau) = [K]$, $B(\tau) = 0$, and $H(\tau) = 0$ for $\tau = 1$. Set $d_i = 1$ for all $i$.
3: Select a leader agent arbitrarily or according to any predefined strategy.
4: Warm-up: Arm allocation:
5: Run Algorithm 2 to get $K$, $K_0(\tau)$, and $A_0(\tau)$, and $B_0(\tau)$.
6: for $\tau = 1, 2, 3, \ldots$ do
7: Set $N(\tau) = 3K \log((8K \log_4 T)/\delta)/\epsilon^2(\tau)$, and $T(\tau) = T(\tau - 1) + N(\tau)$.
8: for $t = T(\tau - 1) + 1 \leq t \leq T(\tau)$ do
9: Each agent $v$ pulls an arm $i_v(t)$ with probability $p_{v,i}(\tau)$ and observes reward $r_{v,i}(t)$.
10: end for
11: Step 1: Parameter estimation: (run by each agent)
12: Update estimation $\hat{\mu}_{v,i}(\tau)$ for each agent $v$ and arm $i$ in epoch $\tau$.
\[
\hat{\mu}_{v,i}(\tau) = \frac{\sum_{t \in T(\tau)} r_{i_v(t)}(\tau) 1\{i_v(t) = i\}}{N_{v,i}(\tau)},
\]
13: where $N_{v,i}(\tau) = p_{v,i}(\tau)N(\tau)$ and $T(\tau) = \{t : T(\tau - 1) + 1 \leq t \leq T(\tau)\}$.
14: Each agent $v$ broadcasts $A_0(\tau)$, $B_0(\tau)$, and $\{\hat{\mu}_{v,i}(\tau)\}_{i \in K_v(\tau)}$ to the leader agent.
15: The leader agent updates estimation $\hat{\mu}_i(\tau)$ for each arm $i$ in epoch $\tau$.
\[
\hat{\mu}_i(\tau) = \frac{1}{|\{v \in [V] : i \in K_v(\tau)\}|} \sum_{v \in [V], i \in K_v(\tau)} \hat{\mu}_{v,i}(\tau).
\]
16: Step 2: Robust reactivation and deactivation: (run by the leader agent)
17: Reactivation: Identify $H(\tau) = \left\{ i \in B(\tau) : \max_{j \in A(\tau)} \hat{\mu}_j(\tau) - \hat{\mu}_i(\tau) < 4\epsilon(d_i) \right\}$.
18: Set $\bar{\mu}_i(\tau) = \max_{j \in A(\tau) \cup H(\tau)} \{\hat{\mu}_j(\tau) + 2\epsilon_j(\tau)\}$ and find $\hat{i}^*$ such that $\bar{\mu}_i(\tau) = \hat{\mu}_{\hat{i}^*}(\tau) + 2\epsilon_{\hat{i}^*}(\tau)$.
19: Deactivation: Identify $M(\tau) = \{i \in A(\tau) \cup H(\tau) : \hat{\mu}_i(\tau) > 14\epsilon(\tau)\}$.
20: Set $A(\tau + 1) = (A(\tau) \cup H(\tau)) \setminus M(\tau)$, and $B(\tau + 1) = (B(\tau) \setminus H(\tau)) \cup M(\tau)$.
21: Step 3: Error level update: (run by the leader agent)
22: Set $\epsilon(\tau + 1) = \epsilon(\tau)/2$.
23: Set $\epsilon_i(\tau + 1) = \epsilon(\tau + 1)$ and $d_i = \tau$ for $\forall i \in A(\tau + 1)$.
24: Set $\epsilon_i(\tau + 1) = \epsilon(d_i)$ for $\forall i \in B(\tau + 1)$.
25: Leader broadcasts $\hat{i}^*$ for each agent $v$, and all agents update $\hat{i}^*$.
26: Leader broadcasts $\epsilon_i(\tau + 1)$ for $i \in K_v(\tau + 1)$, $A_v(\tau + 1)$, and $B_v(\tau + 1)$ for each agent $v$.
27: end for

Algorithm 2 ARM ALLOCATION

Input: Agent number $V$, arm number $K$, $\hat{i}^*$, $A(\tau)$, and $B(\tau)$.
1: Set $\hat{K} = [K/V] + 1$. Find a minimum $\hat{v} \in [V]$ such that $\hat{v}[K/V] \geq K$.
2: Set $K_v(\tau) = \{(v - 1)K/V + 1, \ldots, v[K/V]\}$ for $v = 1, \ldots, \hat{v} - 1$, if $\hat{v} \geq 2$. For $v = \hat{v}$, we first set $K_v(\tau) = \{(v - 1)K/V + 1, \ldots, \hat{v} - 1\}$ and then randomly sample $\hat{v}[K/V] - K$ arms from $[K] \setminus K_v(\tau)$ to merge them to obtain a new $K_v(\tau)$. For those $v = \hat{v} + 1, \ldots, V$, if exist, we randomly sample $[K/V]$ arms for them.
3: Update $K_v(\tau) = K_v(\tau) \cup \{\hat{i}^*\}$, $A_v(\tau) = A(\tau) \cap K_v(\tau)$, and $B_v(\tau) = B(\tau) \cap K_v(\tau)$.
Return: $\hat{K}$, $K_v(\tau)$, $A_v(\tau)$, and $B_v(\tau)$.

instead shrink the probability of pulling arm. This idea will be clear in the analysis of Step 3, error level update.

Robust reactivation and deactivation. This design is motivated by the fact that an adversary might trick traditional active arm elimination (AAE) methods [Even-Dar et al. [2006], Bubeck et al. [2013].
Then, we have that we now provide the main results of nearly instance-optimal regret bound and communication cost.

Theorem 2 also reveals that in the single-agent setting, i.e., \( V = 1 \), the proposed algorithm can achieve a high probability regret as \( \tilde{O}(C) + \tilde{O}(K/\Delta_{\text{min}}) \). Our expected regret bound

---

**Lykouris et al. [2018]** to eliminate the optimal arm permanently in the corrupted setting. To address this issue, we allow those deactivated (eliminated) arms to be reactivated again. Specifically, the algorithm first checks whether the estimators of some bad arms in \( B(\tau) \) suddenly become better than those of arms in \( A(\tau) \). If there exist such arms, we put them into a temporary set \( H(\tau) \). An interesting observation is that the reactivation will not negatively impact the regret performance in the stochastic setting without corruption. This is due to the following lemma that \( H(\tau) \) with high probability is always an empty set, and thus reactivation will not be triggered by the algorithm.

**Lemma 1.** In the stochastic setting, with probability at least \( 1 - \delta \), \( H(\tau) = \emptyset \) for all \( \tau \).

The algorithm does not directly reactivate those arms in \( H(\tau) \) but instead puts them together with arms in \( A(\tau) \) for the deactivation step. In the deactivation step, the algorithm compares the estimators of \( \hat{i}^* \) with each arm \( i \in A(\tau) \cup H(\tau) \). One can see that not all arms in set \( H(\tau) \) will be successfully reactivated, but the algorithm only reactivates those arms that suffice

\[
\{ i \in A(\tau) \cup H(\tau) : \hat{\mu}_i(\tau) - \mu_i(\tau) \leq 14\epsilon(\tau) \}. \tag{6}
\]

Adding arms in \( H(\tau) \) for deactivation forces the adversary to inject more corruption. Specifically, if an adversary hopes to reactivate some target arms, the deactivation step forces the adversary to inject large enough corruption to guarantee that the estimators of target arms are close to \( \mu^* \) so that they can be successfully reactivated.

**Error level update.** The high-level idea of this design is to use the error level to control the expected number of pulls of arm. The arm \( i \) with a large gap \( \Delta_i \) will be assigned with a large error level and thus will be pulled a few times in the epoch. Our algorithm maintains a global error level \( \epsilon(\tau) \) and an arm-specific \( \epsilon_i(\tau) \). For arms \( i \in A(\tau) \), we set \( \epsilon_i(\tau) = \epsilon(\tau) \), and for \( i \in A(\tau) \), set \( \epsilon_i(\tau) = \epsilon(d_i) \) where \( d_i \) is the last epoch up to the current epoch such that arm \( i \) holds (6).

The expected number of pulls of a bad arm \( i \) in a agent \( v \) is \( N_{v,i}(\tau) = p_{v,i}(\tau)N(\tau) \), which depends on \( 1/\epsilon_i^2(d_i) \). Hence, if the arm is deactivated earlier, then, it will be pulled less in expectation. This further implies that in the stochastic setting, if the arm \( i \) has a large arm gap \( \Delta_i \), it will be deactivated early so that it incurs less regret.

**Remark 2.** Algorithm 1 can naturally reduce to a single-agent one without any additional procedure. In this case, one can drop all for those parameters or random variables, e.g., \( \hat{\mu}_{v,i}(\tau) \) and \( N_{v,i}(\tau) \). Then, we have that \( A_v(\tau) = A(\tau) \), \( B_v(\tau) = B(\tau) \), and \( K_v(\tau) = K \), and thus the leader agent runs the entire algorithm.

### 4 Regret and Communication Analysis

We now provide the main results of nearly instance-optimal regret bound and communication cost of Algorithm 1.

**Theorem 2. (Upper bound)** Algorithm 1 which is agnostic to the corruption level \( C \), with probability \( 1 - \delta \), incurs \( O(K \log T) \) communication cost and regret as

\[
O \left( VC + \frac{K \log T \log((V K \log T)/\delta)}{\Delta_{\text{min}}} \right),
\]

and incurs the expected regret \( \mathbb{E}[R_T] \) as \( O(V \mathbb{E}[C] + \frac{K \log T \log(V T)}{\Delta_{\text{min}}}) \).

**Remark 3.** Although \( \tilde{O}(K/\Delta_{\text{min}}) \) in Theorem 2 is slightly weaker than \( \tilde{O}(\sum_{i \neq \hat{i}^*} 1/\Delta_i) \), the latter one is also controlled by \( \Delta_{\text{min}} \). Note that our regret can be also written in a summation form with some algorithm-dependent random variables, i.e., summing over active arms \( i \in A_v(\tau) \), and over bad arms \( i \in B_v(\tau) \), respectively. The regret of summation form can be found in Appendix F.3.

Theorem 2 shows that in the uncorrupted setting, i.e., \( C = 0 \), our regret bound recovers the near-optimal regret of [Wang et al. 2020] up to a logarithmic factor, while maintaining efficient communication with a linear dependence on \( K \) and a logarithmic dependence on \( T \). One can also see that in the uncorrupted setting, our instance-dependent bound only has a \( \log(V) \) dependence of agent number, which implies that the algorithm enjoys the speedup of learning.

Theorem 2 also reveals that in the single-agent setting, i.e., \( V = 1 \), the proposed algorithm can achieve a high probability regret as \( \tilde{O}(C) + \tilde{O}(K/\Delta_{\text{min}}) \).
Then, we hold that
\
E[R_T] \text{ matches the state-of-the-art regret bounds } \tilde{O}(E[C] + K/\Delta_{\min} + \sqrt{KE[C]/\Delta_{\min}}) \text{ e.g., Zimmert and Seldin \cite{Zimmert2021}, Masoudian and Seldin \cite{Masoudian2021} (up to a logarithmic factor). Recall that the lower bound in Theorem 1 shows that } E[C] \text{ is unavoidable for any algorithm, which corroborates that our regret bound is tight. Note that the regret bounds in Zimmert and Seldin \cite{Zimmert2021}, Masoudian and Seldin \cite{Masoudian2021} are given as } O(K/\Delta_{\min} + \sqrt{KE[C]/\Delta_{\min}}) \text{ instead of } \tilde{O}(E[C] + K/\Delta_{\min} + \sqrt{KE[C]/\Delta_{\min}}). \text{ This is because they use the regret metric defined for adversarial regime, but we use the regret } R_T \text{ following other single-agent corruption models \cite{Gupta2019, Kapoor2019, Lee2021}. We discuss these regret notions in Subsection 5.3 and Appendix G.2.}

\textbf{Proof Sketch} \text{ In this subsection, we provide a proof sketch for regret bound in Theorem 2, and show the way to resolving the open question from \cite{Gupta2019}, i.e., removing the multiplicative dependence of } K \text{ on } C. \text{ Before sketching the proof, let } N_{v,i}(\tau) \text{ be the total number of pull of arm } i \text{ by agent } v \text{ in epoch } \tau \text{ and define } C(\tau) = \max_{i \in [K]} \sum_{v=1}^{V} \sum_{t \in T(\tau)} |r_{v,i}(t) - r_{v,i}(t)| \text{ if arm } i \text{ is not allocated to agent } v, \text{ then, } |r_{v,i}(t) - r_{v,i}(t)| = 0. \text{ Then, the following lemma presents a good event } E, \text{ showing that } \mu_{v,i}(\tau) \text{ and } \tilde{N}_{v,i}(\tau) \text{ are close to their actual expectations, respectively.}

\textbf{Lemma 2. Let define event } E \text{ as}
\[ E = \left\{ \forall v, i, \tau : |\mu_{i}(\tau) - \mu_{i}| \leq \mu_{i} + 2\epsilon_{\iota}(\tau) + \frac{2C(\tau)}{N(\tau)}, \tilde{N}_{v,i}(\tau) \leq 3N_{v,i}(\tau) \right\}. \tag{7} \]

Then, we hold that \( \mathbb{P}[E] \geq 1 - \delta. \)

We rewrite the regret as
\[ R_T = \sum_{\tau} \sum_{v} \left( \sum_{i \neq j} \sum_{i \in A(\tau)} \Delta_{i} \tilde{N}_{v,i}(\tau) + \sum_{i \neq j} \sum_{i \in B(\tau)} \Delta_{i} \tilde{N}_{v,i}(\tau) \right). \]

Under event \( E, \tilde{N}_{v,i}(\tau) \Delta_{i} \) can be upper-bounded by \( 3R_{v,i}(\tau) \) where \( R_{v,i}(\tau) = N_{v,i}(\tau) \Delta_{i}. \)

The main idea is to bound \( R_{v,i}(\tau) \) by considering two cases including \( \Delta_{i} > 32\eta(d_{i}) \) and \( \Delta_{i} \leq 32\eta(d_{i}) \) where \( \eta(d_{i}) = \frac{C(s)}{2^{d_{i}}N(s)} \). On the one hand, \( \Delta_{i} > 32\eta(d_{i}) \) implies a small volume of corruption. In this case, \( R_{v,i}(\tau) \) is bounded by \( O(1/\Delta_{i}) \). On the other hand, \( \Delta_{i} \leq 32\eta(d_{i}) \) implies a large volume of corruption on arm \( i \) in agent \( v \) during epoch \( \tau \). In this case, the regret from all \( R_{v,i}(\tau) \) is bounded by \( O(VC) \).

\textbf{Case 1: } \( \Delta_{i} \leq 32\eta(d_{i}) \). This case uses \( \Delta_{i} \leq 32\eta(d_{i}) \) to bound \( R_{v,i}(\tau) \leq 32N_{v,i}(\tau)\eta(d_{i}) = O(VC). \) Although \( \text{Gupta et al.} \cite{Gupta2019} \text{ uses a similar construction of } \eta(d_{i}), \text{ their algorithm suffers a multiplicative dependence of } K. \text{ As a result, extending their algorithm for the multi-agent case might incur an additive term as } O(VC^2). \)

\text{On the contrary, our algorithm can remove the dependence of } K \text{ thanks to the differential treatment of active arm } i \in A(\tau) \text{ and bad arm } i \in B(\tau). \text{ Specifically, for } i \in A(\tau), \text{ we have a factor } 1/|A(\tau)| \text{ in } N_{v,i}(\tau), \text{ which offsets the summation } \sum_{i \neq i', i \in A(\tau)} 1. \text{ As for } i \in B(\tau), \text{ } \tilde{N}_{v,i}(\tau) \text{ has a factor } 1/K \text{ which offsets the summation } \sum_{i \neq i', i \in B(\tau)} 1.

\textbf{Case 2: } \( \Delta_{i} > 32\eta(d_{i}) \). Note that the condition \( \Delta_{i} > 32\eta(d_{i}) \) directly bounds the corruption term in (7) as
\[ \Delta_{i} > 32\eta(d_{i}) = \sum_{s=1}^{d_{i}} \frac{32C(s)}{2^{d_{i} - 4s}N(s)} \geq \frac{32C(d_{i})}{N(d_{i})}. \]

Then, we can bound \( R_{v,i}(\tau) \) for this case by making use of the concentration of estimators and some algorithm properties. Here, we provide some properties of Algorithm 1 that are used for the proof sketch. All properties can be found in Appendix \text{E}.

\textbf{Lemma 3. For any } \tau, \text{ Algorithm 1 holds that (i) if } i \in A(\tau), \text{ then, } \epsilon(\tau) = \epsilon_{i}(\tau) = \epsilon(d_{i})/2; \text{ (ii) if } i \in B(\tau), \text{ then, } \epsilon_{i}(\tau) = \epsilon(d_{i}).

As \( R_{v,i}(\tau) = N_{v,i}(\tau)\Delta_{i}, \text{ we thus turn to bound } N_{v,i}(\tau). \text{ Recall that } N_{v,i}(\tau) = p_{v,i}(\tau)N(\tau) = O(1/c_{i}^{2}(\tau)). \text{ Further, Lemma 3 connects } \epsilon(d_{i}) \text{ and } \epsilon_{i}(\tau) \text{ as } \epsilon_{i}(\tau) = \epsilon(\tau) = \epsilon(d_{i})/2 \text{ if arm } i \in A(\tau), \text{ and } \epsilon_{i}(\tau) = \epsilon_{i}(d_{i}) \text{ if arm } i \in B(\tau). \text{ Thus, we have that } N_{v,i}(\tau) = O(1/c_{i}^{2}(d_{i})). \text{ One can see that we hope to prove } \epsilon(d_{i}) \geq \Delta_{i} \text{ to obtain a bound } N_{v,i}(\tau) = O(1/\Delta_{i}^{2}). \text{ The following lemma gives the desired result.}
Lemma 4. If \( \Delta_i > 32\eta(d_i) \), then, with probability at least \( 1 - \delta \), \( \epsilon(d_i) \geq \Delta_i/32 \).

Lemma 4 makes use of robust reactivation and deactivation by considering cases including \( i^* \in A(d_i) \cup H(d_i) \) and \( i^* \in B(d_i) \setminus H(d_i) \). On the one hand, if \( i^* \in A(d_i) \cup H(d_i) \), we use the property of deactivation, i.e., the optimal arm \( i^* \) must have \( 0 \), which leads to \( \epsilon(d_i) \geq \Delta_i/32 \). For \( i^* \in B(d_i) \setminus H(d_i) \), we use the property that the optimal arm \( i^* \) is never deactivated at the beginning of epoch \( d_i \), which implies that \( 4\epsilon(d_v) \leq \max_{j \in A(d_i)} \hat{\mu}_j(d_i) - \hat{\mu}_{i^*}(d_i) \). From this, we can get that \( \epsilon(d_i) \geq \Delta_i/32 \). Combining the above analysis, we complete the proof of regret bound. The details of this proof can be found in Appendix 4.

5 Discussion

We conclude this paper with some discussions of our model and results.

5.1 Challenges of Extending Single-agent toward Multi-agent

We here present two challenges that make the adversarial corruption problem for the multi-agent MAB setup non-trivial. The first challenge is that the standard concentration inequalities for dependent random variables cannot be directly applied for the multi-agent case. Concretely, the process that all agents simultaneously observe \( V \) realized values of rewards for \( T \) rounds cannot be simulated by the process that a single agent sequentially observes a realization of reward for \( VT \) rounds, when the realizations of rewards depend on the previous history, e.g., corruptions and arm selections. Extending the standard concentration bounds for the multi-agent setting requires a non-trivial analysis [Landgren et al. (2016b, 2021)].

The other challenge is that simply extending existing single-agent robust algorithms, e.g., Gupta et al. [2019], Zimmert and Seldin [2021], Masoudian and Seldin [2021], Lee et al. [2021] may fail to balance the communication-regret trade-off. For example, the algorithms of Zimmert and Seldin [2021], [Masoudian and Seldin 2021] need agents to share observations and update estimators in each round, which yields total \( O(V KT) \) communication cost. A straightforward idea of reducing the communication cost is to adapt the epoch/phase-based algorithms Gupta et al. [2019], [Lee et al. 2021] for multi-agent setting. One may divide the epoch into \( V \) parts, where \( V \) agents simultaneously proceed in a smaller epoch. However, this does not suggest that the communication cost of epoch-based algorithms Gupta et al. [2019], Lee et al. [2021] is comparable to ours. Catoni estimator used in Lee et al. [2021] requires each agent to broadcast a loss estimator sequence whose size is equal to the epoch length. As a consequence, this yields total \( O(KT) \) communication cost. The algorithm Gupta et al. [2019] might incur \( O(KV \log T) \) communication cost that is \( V \) times larger than ours, because each agent needs to share the observations of all \( K \) arms for at most \( \log T \) epochs. Moreover, the regret of Gupta et al. [2019] has a multiplicative dependence of \( K \), and thus the corruption term in regret might be \( O(VKC) \) in the multi-agent setting.

5.2 Corrupted Setting versus Adversarial Setting

In the corrupted setting, the rewards are assumed to be initially drawn from fixed and unknown distributions, whereas in the adversarial setting, the rewards may not follow any distribution even before the adversarial attack. The corrupted setting is appropriate for those scenarios where agents are more interested in the stochastic structure, i.e., \( \mu_i \), than the actually observed reward, i.e., \( r_{v,i}(t) \) that is critical in the adversarial setting. For instance, the platform aims to recommend ads that maximize the preference of users, and thus the user’s preference (i.e., \( \mu_i \)) is what the platform cares about. As a consequence, the algorithms of the corrupted model Li et al. [2019], Bogunovic et al. [2021] are commonly evaluated by the regret defined in the stochastic regime (e.g., \( R_T \) or \( E[R_T] \)), which is different from that defined in the adversarial regime (this will be clear in the next subsection). Moreover, since the corruption problem concerns the stochastic structure, it is typically assumed that the total corruption level \( C \) is moderate, which implies that the environment in the corrupted problem is more close to being stochastic than adversarial.
5.3 Alternative Regret Notions

The bandit algorithms in adversarial setting, e.g., EXP3 is typically measured by regret as \( R_T = \max_i \mathbb{E}\left[ \sum_{t=1}^T \sum_{v=1}^V (r_{v,i}(t) - r_{v,i}(t)) \right] \). Note that \( R_T \) is also called pseudo-regret \cite{lattimore2020bandit}, but it is defined in the adversarial regime and is essentially different from pseudo-regret \( R_T \) (see (2)) defined in the stochastic regime, when \( C \neq 0 \). One can see that \( R_T \) coincides with the \( \mathbb{E}[R_T] \) in the uncorrupted setting as \( r_{v,i}(t) = r_{v,i}(t) \), but \( R_T \) cannot be trivially converted to \( \mathbb{E}[R_T] \) in the presence of an adversary. The following theorem connects notations of \( R_T, R_T, \) and \( R'_T \) (see (3)).

**Theorem 3.** For any algorithm of \( V \geq 1 \) agents, with probability at least \( 1 - 1/VT \), we have that \( R_T = O(R_T + \mathbb{E}[C] + V \log( VT )) \). Then, \( \mathbb{E}[R_T] = \Theta(\mathbb{E}[R_T] + \mathbb{E}[C]) \) and \( \mathbb{E}[R_T] = \Theta(\mathbb{E}[R'_T] + \mathbb{E}[C]) \) hold for the two-armed bandit instance.

Theorem 3 shows that as for the two-armed bandit instance, the expected regret \( \mathbb{E}[R_T] \) in stochastic regime can imply the lower bounds and upper bounds of both pseudo-regret \( R_T \) and expected regret \( \mathbb{E}[R'_T] \) in adversarial regime. The regret \( R_T \) and \( \mathbb{E}[R'_T] \) can in turn give the the lower bound and upper bound of \( \mathbb{E}[R_T] \).

5.4 Open Questions

We show a lower bound of \( \mathbb{E}[R_T] \) for our problem, but the lower bound of \( R_T \) remains as an open question. Our work leaves a gap between \( \Omega(V \mathbb{E}[C]) \) in upper bound and \( \Omega(\mathbb{E}[C]) \) in lower bound. Bridging this gap while maintaining an efficient communication is an interesting open question. Designing a multi-agent algorithm to achieve the best of three worlds, i.e., stochastic, corrupted, and adversarial settings, is another compelling question. It is also interesting to consider the multi-agent corruption problem for the constrained communication model wherein agents are connected in an unknown graph and only allowed to communicate with neighbors.

References

Arpit Agarwal, Shivani Agarwal, and Prathamesh Patil. Stochastic dueling bandits with adversarial corruption. In *ALT*, volume 132, pages 217–248, 2021.

Pragnya Alatur, Kfir Y. Levy, and Andreas Krause. Multi-player bandits: The adversarial case. *J. Mach. Learn. Res.*, 21:77:1–77:23, 2020.

Peter Auer and Chao-Kai Chiang. An algorithm with nearly optimal pseudo-regret for both stochastic and adversarial bandits. In *COLT*, volume 49, pages 116–120, 2016.

Baruch Awerbuch and Robert D. Kleinberg. Competitive collaborative learning. In *COLT*, volume 3559, pages 233–248, 2005.

Yogev Bar-On and Yishay Mansour. Individual regret in cooperative nonstochastic multi-armed bandits. In *NeurIPS*, pages 3110–3120, 2019.

Alina Beygelzimer, John Langford, Lihong Li, Lev Reyzin, and Robert E. Schapire. Contextual bandit algorithms with supervised learning guarantees. In *AISTATS*, 2011.

Ilai Bistritz and Amir Leshem. Distributed multi-player bandits - a game of thrones approach. In *NeurIPS*, pages 7222–7232, 2018.

Ilai Bistritz, Tavor Z. Baharav, Amir Leshem, and Nicholas Bambos. My fair bandit: Distributed learning of max-min fairness with multi-player bandits. In *ICML*, volume 119, pages 930–940, 2020.

Ilija Bogunovic, Andreas Krause, and Jonathan Scarlett. Corruption-tolerant gaussian process bandit optimization. In *AISTATS*, 2020.

Ilija Bogunovic, Andreas Krause, and Jonathan Scarlett. Stochastic linear bandits robust to adversarial attacks. In *AISTATS*, 2021.
Sébastien Bubeck and Aleksandrs Slivkins. The best of both worlds: Stochastic and adversarial bandits. In *COLT*, volume 23, pages 42.1–42.23, 2012.

Sébastien Bubeck, Tengyao Wang, and Nitin Viswanathan. Multiple identifications in multi-armed bandits. In *ICML*, volume 28, pages 258–265, 2013.

S. Buccapatnam, J. Tan, and L. Zhang. Information sharing in distributed stochastic bandits. In *INFOCOM*, pages 2605–2613, 2015.

Nicolò Cesa-Bianchi, Claudio Gentile, and Yishay Mansour. Delay and cooperation in nonstochastic bandits. *J. Mach. Learn. Res.*, 20(17):1–17:38, 2019.

Nicolò Cesa-Bianchi, Tommaso Cesari, and Claire Monteleoni. Cooperative online learning: Keeping your neighbors updated. In *ALT*, volume 117, pages 234–250, 2020.

Mithun Chakraborty, Kai Yee Phoebe Chua, Sanmay Das, and Brendan Juba. Coordinated versus decentralized exploration in multi-agent multi-armed bandits. In *IJCAI*, pages 164–170, 2017.

Abhimanyu Dubey and Alex ’Sandy’ Pentland. Cooperative multi-agent bandits with heavy tails. In *ICML*, volume 119, pages 2730–2739, 2020.

Eyal Even-Dar, Shie Mannor, and Yishay Mansour. PAC bounds for multi-armed bandit and markov decision processes. In *COLT*, volume 2375, pages 255–270, 2006.

Anupam Gupta, Tomer Koren, and Kunal Talwar. Better algorithms for stochastic bandits with adversarial corruptions. In *COLT*, volume 99, pages 1562–1578, 2019.

Eshcar Hillel, Zohar Shay Karnin, Tomer Koren, Ronny Lempel, and Oren Somekh. Distributed exploration in multi-armed bandits. In *NeurIPS*, pages 854–862, 2013.

Shinji Ito, Daisuke Hatano, Hanna Sumita, Kei Takemura, Takuro Fukunaga, Naonori Kakimura, and Ken-ichi Kawarabayashi. Delay and cooperation in nonstochastic linear bandits. In *NeurIPS*, 2020.

Dileep M. Kalathil, Naumaan Nayyar, and Rahul Jain. Decentralized learning for multiplayer multi-armed bandits. *IEEE Trans. Inf. Theory*, 60(4):2331–2345, 2014.

Varun Kanade, Zhenming Liu, and Bozidar Radunovic. Distributed non-stochastic experts. In *NIPS*, pages 260–268, 2012.

Sayash Kapoor, Kumar Kshitij Patela, and Purushottam Kar. Corruption-tolerant bandit learning. *Mach. Learn.*, 108(4):687–715, 2019.

Peter Landgren, Vaibhav Srivastava, and Naomi Ehrich Leonard. Distributed cooperative decision-making in multiarmed bandits: Frequentist and bayesian algorithms. In *CDC*, pages 167–172. IEEE, 2016a.

Peter Landgren, Vaibhav Srivastava, and Naomi Ehrich Leonard. On distributed cooperative decision-making in multiarmed bandits. In *ECC*, 2016b.

Peter Landgren, Vaibhav Srivastava, and Naomi Ehrich Leonard. Distributed cooperative decision making in multi-agent multi-armed bandits. *Autom.*, 125:109445, 2021.

Tor Lattimore and Csaba Szepesvári. *Bandit algorithms*. Cambridge University Press, 2020.

Chung-Wei Lee, Haipeng Luo, Chen-Yu Wei, Mengxiao Zhang, and Xiaojin Zhang. Achieving Near Instance-Optimality and Minimax-Optimality in Stochastic and Adversarial Linear Bandits Simultaneously. In *arXiv:2102.05858*, 2021.

Yingkai Li, Edmund Y. Lou, and Liren Shan. Stochastic linear optimization with adversarial corruption. In *arXiv:1909.02109*, 2019.

Haoyang Liu, Keqin Liu, and Qing Zhao. Learning in a changing world: Restless multiarmed bandit with unknown dynamics. *IEEE Trans. Inf. Theory*, 59(3):1902–1916, 2013.
Thodoris Lykouris, Vahab S. Mirrokni, and Renato Paes Leme. Stochastic bandits robust to adversarial corruptions. In *STOC*, 2018.

David Martínez-Rubio, Varun Kanade, and Patrick Rebeschini. Decentralized cooperative stochastic bandits. In *NeurIPS*, pages 4531–4542, 2019.

Saeed Masoudian and Yevgeny Seldin. Improved analysis of robustness of the tsallis-inf algorithm to adversarial corruptions in stochastic multiarmed bandits. In *arXiv:2103.12487*, 2021.

Jonathan Rosenski, Ohad Shamir, and Liran Szlak. Multi-player bandits - a musical chairs approach. In *ICML*, volume 48, pages 155–163, 2016.

Abishek Sankararaman, Ayalvadi Ganesh, and Sanjay Shakkottai. Social learning in multi agent multi armed bandits. In *SIGMETRICS*, pages 29–30, 2020.

Chengshuai Shi, Wei Xiong, Cong Shen, and Jing Yang. Decentralized multi-player multi-armed bandits with no collision information. In *AISTATS*, volume 108, pages 1519–1528, 2020.

Ken Sugawara, Toshiya Kazama, and Toshinori Watanabe. Foraging behavior of interacting robots with virtual pheromone. In *IROS*, pages 3074–3079, 2004.

Balázs Szörényi, Róbert Busa-Fekete, István Hegedüüs, Róbert Ormándi, Márk Jelasity, and Balázs Kégl. Gossip-based distributed stochastic bandit algorithms. In *ICML*, volume 28, pages 19–27, 2013.

Chao Tao, Qin Zhang, and Yuan Zhou. Collaborative learning with limited interaction: Tight bounds for distributed exploration in multi-armed bandits. In *FOCS*, pages 126–146, 2019.

Harshvardhan Tibrewal, Sravan Patchala, Manjesh Kumar Hanawal, and Sumit Jagdish Darak. Distributed learning and optimal assignment in multiplayer heterogeneous networks. In *INFOCOM*, pages 1693–1701, 2019.

Yuanhao Wang, Jiachen Hu, Xiaoyu Chen, and Liwei Wang. Distributed bandit learning: Near-optimal regret with efficient communication. In *ICLR*, 2020.

Pan Zhou, Jie Xu, Wei Wang, Yuchong Hu, Dapeng Oliver Wu, and Shouling Ji. Toward optimal adaptive online shortest path routing with acceleration under jamming attack. *IEEE/ACM Trans. Netw.*, 27(5):1815–1829, 2019.

Julian Zimmert and Yevgeny Seldin. Tsallis-inf: An optimal algorithm for stochastic and adversarial bandits. *J. Mach. Learn. Res.*, 22:28:1–28:49, 2021.
A Concentration inequalities

Lemma 5. (Hoeffding-Azuma’s inequality for martingales, Bubeck and Slivkins [2012], Theorem 4.2). Let \( X_0, X_1, \ldots, X_n \) be a martingale difference sequence with zero mean and \( |X_i - X_{i-1}| \leq m_i \) almost surely for all \( i \geq 1 \). Then, we have for all \( \delta > 0 \),

\[
\mathbb{P} \left( \left| \sum_{i=1}^{n} X_i \right| > \frac{\log(2/\delta)}{2} \sum_{i=1}^{n} m_i \right) \leq \delta.
\]

Lemma 6. (Freedman’s concentration inequality, Beygelzimer et al. [2011], Theorem 1). Let \( X_1, \ldots, X_n \) be a martingale difference sequence with zero mean and \( |X_i| \leq M \) almost surely for all \( i \). Let \( V = \sum_{i=1}^{n} \mathbb{E} \left[ X_i^2 | X_1, \ldots, X_{i-1} \right] \) be the cumulative variance of the martingale. Then, we have for all \( \delta > 0 \),

\[
\mathbb{P} \left( \sum_{i=1}^{n} X_i > \frac{V}{M} + M \log(1/\delta) \right) \leq \delta.
\]

B Notations

For ease of reading, we list here key notations that will be used in this Appendix.

- \( T, V, K \) : Time horizon, agent number, and arm number.
- \( i^*, \hat{i}^* \) : Best arm and empirical best arm. See \( \text{[3]} \) for the detail of \( \hat{i}^* \).
- \( N_{v,i}(\tau) \) : The real number of times that arm \( i \) is pulled by agent \( v \) in epoch \( \tau \).
- \( \hat{N}_{v,i}(\tau) \) : The expected number of times that arm \( i \) is pulled by agent \( v \) in epoch \( \tau \).
- \( p_{v,i}(\tau) \) : The probability that arm \( i \) is pulled by agent \( v \) in epoch \( \tau \).
- \( T(\tau) \) : The total number of rounds up to the end of epoch \( \tau \), and \( T(\tau = 0) = 0 \).
- \( \hat{T}(\tau) \) : The set of rounds in epoch \( \tau \), and \( \hat{T}(\tau) = \{ t : T(\tau - 1) + 1 \leq t \leq T(\tau) \} \).
- \( y_{v,i}(t) \) : An independent draw from a Bernoulli distribution with mean \( p_{v,i}(\tau) \) for \( t \in \hat{T}(\tau) \).

\( y_{v,i}(t) = 1 \) if \( i_v(t) = i \), and \( y_{v,i}(t) = 0 \), otherwise.

- \( \hat{\mu}_{v,i}^S(\tau) \) : \( \hat{\mu}_{v,i}^S(\tau) = \sum_{t \in \hat{T}(\tau)} r_{v,i}(t) \mathbb{1}\{i_v(t) = i\} / \hat{N}_{v,i}(\tau) \) is the estimation of arm \( i \) computed by stochastic rewards from agent \( v \) in epoch \( \tau \).

- \( C_{v,i}(t) \) : \( C_{v,i}(t) = |r_{v,i}(t) - r_{v,i}^\ast(t)| \) is the corruption on arm \( i \) at round \( t \).

- \( C_{v,i}(\tau) \) : \( C_{v,i}(\tau) = \sum_{t \in \hat{T}(\tau)} C_{v,i}(t) \) is the corruption added on arm \( i \) in agent \( v \) in epoch \( \tau \).

- \( C(\tau) \) : \( C(\tau) = \max_{i \in [K]} \sum_{v=1}^{V} C_{v,i}(\tau) \) is the corruption level in epoch \( \tau \).

- \( \hat{C}_{v,i}(t) \) : \( \hat{C}_{v,i}(t) = C_{v,i}(t) y_{v,i}(t) \) is the actual corruption added on arm \( i \) in agent \( v \) at round \( t \).

- \( \hat{C}_{v,i}(\tau) \) : \( \hat{C}_{v,i}(\tau) = \sum_{t \in \hat{T}(\tau)} \hat{C}_{v,i}(t) \) is the actual corruption added on arm \( i \) in agent \( v \) in epoch \( \tau \).

- \( \mathcal{F}_t \) : The smallest \( \sigma \)-algebra containing all information up to round \( t \).

Note that in the above definitions, if agent \( v \) is not allocated with arm \( i \), then, let associated random variables be zero. For example, we set \( \hat{C}_{v,i}(\tau) = 0 \) and \( C_{v,i}(\tau) = 0 \), if agent \( v \) does not have arm \( i \).

C Lower Bound: proof of Theorem 1

To lower-bound the pseudo-regret \( R_T \), we start our proof from another regret notation \( R'_T \), defined as follows

\[
R'_T = \max_{i \in [K]} \sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,i}(t) - r_{v,i}(t) \right).
\]

The proof is divided into three parts by lower-bounding (i) \( R'_T \), (ii) \( \mathbb{E}[R'_T] \), and (iii) \( \mathbb{E}[R_T] \) where \( R_T \) is defined in \( \text{[2]} \). In the following, we use \( \mathbb{P}_{\text{sto}[\cdot]} \) and \( \mathbb{E}_{\text{sto}[\cdot]} \) to denote the probability and expectation in stochastic setting, respectively. Similarly, the probability and expectation in corrupted setting is denoted by \( \mathbb{P}_{\text{cor}[\cdot]} \) and \( \mathbb{E}_{\text{cor}[\cdot]} \), respectively.
C.1 Preliminaries: two-armed setting and adversary

**Setting:** consider a two-armed bandit instance where $V$ agents interact with arm 1 and arm 2. The arm 1 is with Bernoulli reward $\mu_1 = 1/2 - \Delta$ and the arm 2 is with a constant reward as $r_2 = \mu_2 = 1/2$. We divide the rounds into some intervals of increasing length $3^\ell T^\alpha$ for $\ell = 1, 2, \ldots, L$. For simplicity, we assume $3^\ell T^\alpha$ is an integer (If $3^\ell T^\alpha$ is not an integer, the length can be modified as $3^\ell [T^\alpha]$). Note that interval $L$ might be incomplete as the maximum round is $T$. Therefore, $L \geq \frac{1}{\log 3} \log T$.

Let $\tilde{N}_1(\ell^*)$ be the number of times that arm 1 is pulled in the interval $\ell^*$ across all agents. For any algorithm with pseudo-regret $O(\log(T)/\Delta)$, there is an interval $\ell^* < L$ such that $\mathbb{E}_{sto} \left[ \tilde{N}_1(\ell^*) \right] \leq Y$ where for the constant $B_0 > 0$, $Y$ is given by

$$Y = \frac{B_0}{(1 - \alpha) \Delta^2}.$$

**Adversary:** We create an adversary who corrupts the Bernoulli distribution of arm 1 by setting $\mu_1 = 1/2 + \Delta$, and does nothing for arm 2. Before interval $\ell^*$, the adversary does not inject any corruption, but starts to inject the corruption in interval $\ell^*$ and beyond. Such a corruption strategy is applied for all agents. Let $t_{\ell^*}$ be the round at the end of interval $\ell^*$. Define events $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_3$ as

$$\mathcal{E}_1 = \left\{ \tilde{N}_1(\ell^*) \leq 4Y \right\},$$

$$\mathcal{E}_2 = \left\{ \sum_{t=1}^{T} \sum_{v=1}^{V} r_{i_v(t)}(t) < \frac{1}{2} V T + (T - t_{\ell^*}) V \Delta + 4YV \Delta + \sqrt{2VT \log(VT)} \right\},$$

$$\mathcal{E}_3 = \left\{ \sum_{t=1}^{T} \sum_{v=1}^{V} r_{v,1}(t) \geq \frac{1}{2} V T + 3V T^\alpha + \Delta V (T - t_{\ell^*}) - \sqrt{2VT \log(VT)} \right\},$$

where $\mathcal{E}_1^c$, $\mathcal{E}_2^c$, and $\mathcal{E}_3^c$ are the complementary events, respectively.

C.2 Lower-bounding $R^\ell T$

The proof of $R^\ell_T$ adapts some basic techniques from the single-agent bandit problem, e.g., [Auer and Chiang 2016]. The proof is divided into the following steps.

**Step 1:** analyze $\mathcal{E}_1$. By Lemma 12 in [Auer and Chiang 2016], we have

$$\mathbb{P}_{cor} (\mathcal{E}_1) \geq \frac{1}{16} \exp \left( -64\Delta^2 Y \right).$$

For simplicity, we dub $p_1 = \frac{1}{16} \exp \left( -64\Delta^2 Y \right)$.

**Step 2:** analyze $\mathcal{E}_2$. From the construction of the corruption, the following holds under event $\mathcal{E}_1$.

$$\sum_{t=1}^{T} \sum_{v=1}^{V} \mathbb{E}_{cor} [r_{i_v(t)}(t) | \mathcal{F}_{t-1}] \leq \frac{1}{2} \left( V t_{\ell^*} - \tilde{N}_1(\ell^*) \right) + \tilde{N}_1(\ell^*) \left( \frac{1}{2} + \Delta \right) + V (T - t_{\ell^*}) \left( \frac{1}{2} + \Delta \right)$$

$$\leq \frac{1}{2} V T + (T - t_{\ell^*}) V \Delta + 4YV \Delta. \quad (9)$$

Now, we construct a martingale difference sequence $\{ D_i(t) \}_{t=0}^\infty$ where $D_i(t) = \sum_{v=1}^{V} (r_{i_v(t)}(t) - \mathbb{E}[r_{i_v(t)}(t) | \mathcal{F}_{t-1}])$. By Hoeffding-Azuma’s inequality and union bound with $\mathcal{E}_1$, we have

$$\mathbb{P}_{cor} (\mathcal{E}_2^c) \leq 1 - \frac{1}{16} \exp \left( -64\Delta^2 Y \right) + \frac{1}{(VT)^2}.$$
**Step 3: analyze** $E_3$. Define $L(\ell^*)$ as the total number of rounds in interval $\ell^*$ and then we get for arm 1 that

$$
\sum_{t=1}^{T} \sum_{v=1}^{V} \mathbb{E}_{\text{cor}}[r_{v,1}(t)] = \frac{1}{2}VT + \Delta V \left(2L(\ell^*) - t_{\ell^*}\right) + \Delta V \left(T - t_{\ell^*}\right)
$$

$$
\geq \frac{1}{2}VT + 3\Delta VT^\alpha + \Delta V \left(T - t_{\ell^*}\right),
$$

where $2L(\ell^*) - t_{\ell^*}$ is bounded by

$$
2L(\ell^*) - t_{\ell^*} = L(\ell^*) - \sum_{\ell=1}^{\ell^* - 1} L(\ell) \geq 3T^\alpha.
$$

By Hoeffding-Azuma’s inequality, we have for arm 1 that

$$
P_{\text{cor}}(E_3) \leq \frac{1}{(VT)^2}.
$$

**Step 4: arm 1 contributes to more total rewards than arm 2.** Since the adversary uses the same corruption strategy across all agents, we only focus our analysis of Step 1 on a single agent $v$. According to our construction, the total rewards of arm 2 for agent $v$ are $\mathbb{E}_{\text{cor}}[\sum_{t} r_{v,2}(t)] = \frac{1}{2}T$, and the total rewards of arm 1 for agent $v$ are presented in (10). Let $\Delta_v(t) = \sum_{t} r_{v,2}(t) - \sum_{t} r_{v,1}(t)$. As all rewards are independent, we use Hoeffding inequality to get

$$
P_{\text{cor}} \left[ \sum_{t=1}^{T} r_{v,2}(t) > \sum_{t=1}^{T} r_{v,1}(t) \right] \leq P_{\text{cor}} \left[ \sum_{t=1}^{T} \Delta_v(t) - \mathbb{E}_{\text{cor}}[\Delta_v(t)] > 3\Delta VT^\alpha + \Delta V \left(T - t_{\ell^*}\right) \right]
$$

$$
\leq \exp \left( -\frac{(3VT^\alpha + V \left(T - t_{\ell^*}\right))^2}{2T} \right) = p_2.
$$

Thus, given any agent $v$, with probability at least $1 - p_2$, the total rewards of arm 2 is less than the total rewards of arm 1 over time horizon $T$. Then, by a union bound over all $V$ agents, $R_T'$ with probability at least $1 - Vp_2$, is

$$
R_T' = \max_{i \in [K]} \sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,i}(t) - r_{v,2}(t) \right) = \sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,1}(t) - r_{v,2}(t) \right).
$$

**Step 5: put together.** Take a union bound, and then use the fact that $C = VT^\alpha$, (9), (10), and (11) to get the probability at least $p_1 = \frac{2}{(VT)^2} - Vp_2$ (Note that for a sufficiently large $T$, one can have that $p_1 > Vp_2$),

$$
\sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,1}(t) - r_{v,2}(t) \right) \geq \Delta C - 4Y\Delta - 2\sqrt{2VT \log(VT)}.
$$

where $4\Delta Y = \frac{4\Delta}{1-\alpha}$, $\leq \sqrt{2VT \log(VT)}$ with a large $T$. Combining (12), we have the following with probability at least $p_1 = \frac{2}{(VT)^2} - Vp_2$

$$
R_T' = \sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,1}(t) - r_{v,2}(t) \right) \geq \Delta C - 3\sqrt{2VT \log(VT)}.
$$

**C.3 Lower-bounding $E[R_T']$**

**Modification of adversary.** To lower bound $E[R_T']$, we slightly modify the corruption strategy. Specifically, if there is a round $t \leq t_{\ell^*}$ such that the number of pulls of arm 1 in interval $\ell^*$ exceeds $4Y$, then for all remaining rounds, the adversary does not corrupt any more.
After modifying the corruption strategy, one can observe that under $\tilde{N}_1(\ell^*) \leq 4Y$, the adversary will always inject corruption in interval $\ell^*$ and beyond. Therefore, from the construction of interval, we have that under $\tilde{N}_1(\ell^*) \leq 4Y$, with probability at least $1 - Vp_2$, $R_T^r = \sum_{t=1}^T \sum_{v=1}^V (r_v,1(t) - r_{i,v}(t))$. Thus, the analysis in Appendix C.2 is also applicable for regret analysis after modifying the adversary, under $\tilde{N}_1(\ell^*) \leq 4Y$. Under $\tilde{N}_1(\ell^*) > 4Y$, it is known that there should exist a round such that the adversary stops injecting corruption. In this case, with a high probability, $R_T^r = \sum_{t=1}^T \sum_{v=1}^V (r_v,2(t) - r_{i,v}(t))$ as the adversary stops injecting corruption in interval $\ell^*$, and arm 2 will yield more rewards than that of arm 1.

For notational simplicity, we dub $p_3 = p_1 - \frac{2}{(Vt)^2} - Vp_2$. In the following, all expectations are taken for the corrupted setting, and hence we use $E[\cdot]$ to avoid clutter. Then, we write $E[R_T^r]$ as

$$E[R_T^r] = E[R_T^r | \tilde{N}_1(\ell^*) \leq 4Y]P(\tilde{N}_1(\ell^*) \leq 4Y) + E[R_T^r | \tilde{N}_1(\ell^*) > 4Y]P(\tilde{N}_1(\ell^*) > 4Y),$$

and then use the result of [13] to bound the first term as

$$E[R_T^r | \tilde{N}_1(\ell^*) \leq 4Y] \geq \left( \Delta E[C] - 3\sqrt{2VT\log(VT)} \right) P(\tilde{N}_1(\ell^*) \leq 4Y, R_T^r \geq \Delta C - 3\sqrt{2VT\log(VT)}) - VT \cdot P(\tilde{N}_1(\ell^*) \leq 4Y, R_T^r < \Delta C - 3\sqrt{2VT\log(VT)}) - \frac{2}{VT},$$

where the last inequality is due to the following steps

- First, the following holds

$$P(\tilde{N}_1(\ell^*) \leq 4Y, R_T^r < \Delta C - 3\sqrt{2VT\log(VT)}) = P(R_T^r < \Delta C - 3\sqrt{2VT\log(VT)} | \tilde{N}_1(\ell^*) \leq 4Y) P(\tilde{N}_1(\ell^*) \leq 4Y).$$

- Second, from the analysis in Appendix C.2 we have known that $P(\mathcal{E}_2 | \tilde{N}_1(\ell^*) \leq 4Y) \leq 1/(VT)^2$ and $P(\mathcal{E}_3 | \tilde{N}_1(\ell^*) \leq 4Y) \leq 1/(VT)^2$.

- Finally, by a union bound, the following holds.

$$P\left( \sum_{t=1}^T \sum_{v=1}^V (r_v,1(t) - r_{i,v}(t)) < \Delta C - 3\sqrt{2VT\log(VT)} | \tilde{N}_1(\ell^*) \leq 4Y \right) \leq \frac{2}{(VT)^2}.$$

By a similar method, we have that

$$E[R_T^r | \tilde{N}_1(\ell^*) > 4Y] \geq \left( \Delta E[C] - 3\sqrt{VT\log(VT)} \right) P(\tilde{N}_1(\ell^*) > 4Y) - \frac{3}{VT},$$

where the last inequality bounds $\sum_{t=1}^T \sum_{v=1}^V r_{v,1}(t) - r_{i,v}(t) \geq -VT$, and the probability $P(R_T^r = \sum_{t=1}^T \sum_{v=1}^V (r_v,2(t) - r_{i,v}(t)) | \tilde{N}_1(\ell^*) > 4Y)$ is at most $1/(VT)^2$ since the corruption injected by the adversary is at most $4Y$ and the environment is close to being stochastic.

Combing the above, we get for some constant $0 < B_1 < p_3$.

$$E[R_T^r] \geq B_1 \left( E[C] - \sqrt{VT\log(VT)} \right). \tag{14}$$
C.4 Lower-bounding $E[R_T]$

In the two-armed setting, we have that $E[R_T] = E[\sum_{t=1}^T \sum_{v=1}^V (r_{v,1}^S(t) - r_{v,2}^S(t))]$. Thus, to lower-bound $E[R_T]$, we need to connect $\sum_{t=1}^T \sum_{v=1}^V (r_{v,1}^S(t) - r_{v,2}^S(t))$ and $R'_T$. From the analysis from Appendix C.3, we know that with probability at least $1 - Vp_2$, $R'_T = \sum_{t=1}^T \sum_{v=1}^V (r_{v,1}(t) - r_{i_v(t)}(t))$ under $\tilde{N}_1(\ell^*) \leq 4V$, whereas with high probability, $R'_T = \sum_{t=1}^T \sum_{v=1}^V (r_{v,2}(t) - r_{i_v(t)}(t))$ under $\tilde{N}_1(\ell^*) > 4Y$. We first decompose $\sum_{t=1}^T \sum_{v=1}^V (r_{v,1}(t) - r_{i_v(t)}(t))$ as

$$\sum_{t=1}^T \sum_{v=1}^V (r_{v,1}(t) - r_{i_v(t)}(t))$$

$$= \sum_{t=1}^T \sum_{v=1}^V (r_{v,1}^S(t) - r_{v,2}^S(t)) + \sum_{t=1}^T \sum_{v=1}^V (r_{i_v(t)}^S(t) - r_{v,2}^S(t)) + \sum_{t=1}^T \sum_{v=1}^V (r_{v,1}(t) - r_{i_v(t)}^S(t))$$

$$= \sum_{t=1}^T \sum_{v=1}^V (r_{v,1}^S(t) - r_{i_v(t)}^S(t)) + \sum_{t=1}^T \sum_{v=1}^V (r_{i_v(t)}^S(t) - r_{v,2}^S(t)) + C$$

Then, we decompose $\sum_{t=1}^T \sum_{v=1}^V (r_{v,2}(t) - r_{i_v(t)}(t))$ as

$$\sum_{t=1}^T \sum_{v=1}^V (r_{v,2}(t) - r_{i_v(t)}(t))$$

$$= \sum_{t=1}^T \sum_{v=1}^V (r_{v,2}^S(t) - r_{v,2}^S(t)) + \sum_{t=1}^T \sum_{v=1}^V (r_{i_v(t)}^S(t) - r_{v,2}^S(t)) + \sum_{t=1}^T \sum_{v=1}^V (r_{v,2}(t) - r_{v,2}^S(t))$$

$$= \sum_{t=1}^T \sum_{v=1}^V (r_{v,2}^S(t) - r_{v,2}^S(t)) + \sum_{t=1}^T \sum_{v=1}^V (r_{i_v(t)}^S(t) - r_{v,2}^S(t)) + C.$$

Take expectation over $R'_T$ under $\tilde{N}_1(\ell^*) \leq 4Y$ to have

$$E[R'_T|\tilde{N}_1(\ell^*) \leq 4Y]$$

$$\leq E \left[ \sum_{t=1}^T \sum_{v=1}^V (r_{v,2}^S(t) - r_{v,2}^S(t)) + C \right| \tilde{N}_1(\ell^*) \leq 4Y \right] - \Delta VT$$

$$+ E \left[ \sum_{t=1}^T \sum_{v=1}^V (r_{i_v(t)}^S(t) - r_{v,2}^S(t)) \right| \tilde{N}_1(\ell^*) \leq 4Y \right] + 1,$$

where the inequality is due to the following reasons. First, the generation of $r_{v,1}^S(t) - r_{v,2}^S(t)$ for each round $t$ is independent of the history. Then, the summation over $E[r_{v,1}^S(t) - r_{v,2}^S(t)]$ can be bounded by $-\Delta VT$. Second, for a sufficiently large $T$, one can simply bound $Vp_2 \leq 1/VT$, so that $R'_T = \sum_{t=1}^T \sum_{v=1}^V (r_{v,1}(t) - r_{i_v(t)}(t))$ with probability at least $1 - 1/VT$, and the expected regret with the remaining probability of $R'_T = \sum_{t=1}^T \sum_{v=1}^V (r_{v,2}(t) - r_{i_v(t)}(t))$ can be trivially bounded by 1.

Again, take expectation over $R'_T$ under $\tilde{N}_1(\ell^*) > 4Y$ to have

$$E[R'_T|\tilde{N}_1(\ell^*) > 4Y]$$

$$\leq E \left[ \sum_{t=1}^T \sum_{v=1}^V (r_{v,2}^S(t) - r_{v,2}^S(t)) \right| \tilde{N}_1(\ell^*) > 4Y \right]$$

$$+ E \left[ \sum_{t=1}^T \sum_{v=1}^V (r_{i_v(t)}^S(t) - r_{v,2}^S(t)) \right| \tilde{N}_1(\ell^*) > 4Y \right] + 1.$$
where under this event, the probability of \( R_T^r = \sum_{t=1}^T \sum_{v=1}^V (r_{v,1}(t) - r_{i_v(t)}(t)) \) is at most \( 1/VT \) for a sufficiently large \( T \) and thus the expected regret can be also trivially bounded by 1.

Then, we bound (15) and (16), respectively.

**Under \( \tilde{N}_1(\ell^*) > 4Y \).**

In this case, the adversary stops injecting the corruption at a certain round \( t \) such that \( t \leq t_{\ell^*} \). According to the corruption strategy, it is known that \( \mathbb{E}[r_{i_v(t)}^s(t) - r_{i_v(t)}(t)] = -2\Delta \) for \( i_v(t) = 1 \), and \( r_{i_v(t)}^s(t) - r_{i_v(t)}(t) = 0 \) for \( i_v(t) = 2 \). Under \( \tilde{N}_1(\ell^*) > 4Y \), the corruption in interval \( \ell^* \) ends upon the number of pulls of arm 1 exceeds 4Y. From the analysis of Step 1 in Appendix C.2, we know that the algorithm will not detect the corruption on arm 1 in interval \( \ell^* \), and thus the algorithm behaves as the environment was almost stochastic. Such a corruption is \( O(1/\Delta) \), which implies that the expected corruption level is \( \mathbb{E}[C|\tilde{N}_1(\ell^*) > 4Y] = B_2/\Delta \) for a suitable constant \( B_2 > 0 \).

\[
\mathbb{E} \left[ \sum_{t=1}^T \sum_{v=1}^V (r_{i_v(t)}^s(t) - r_{i_v(t)}(t)) \right| \tilde{N}_1(\ell^*) > 4Y] = \mathbb{E} \left[ \sum_{t=1}^T \sum_{v=1}^V (r_{i_v,t}^s(t) - r_{i_v,t}(t)) \right| 1\{i_v(t) = 1\} \tilde{N}_1(\ell^*) > 4Y] = -B_2/\Delta \tag{17}
\]

We use (16), (17), and \( \mathbb{E}[C|\tilde{N}_1(\ell^*) > 4Y] = B_2/\Delta \) to get

\[
\mathbb{E} \left[ \sum_{t=1}^T \sum_{v=1}^V (r_{i_v,t}^s(t) - r_{i_v,t}(t)) \right| \tilde{N}_1(\ell^*) > 4Y] \geq \mathbb{E}[R_T^r|\tilde{N}_1(\ell^*) > 4Y] + \frac{B_2 \log(VT)}{\Delta} - 1 \tag{18}
\]

\[
\geq \mathbb{E}[R_T^r|\tilde{N}_1(\ell^*) > 4Y] + \mathbb{E}[C|\tilde{N}_1(\ell^*) > 4Y] - 1.
\]

**Under \( \tilde{N}_1(\ell^*) \leq 4Y \).**

Next, we connect \( \mathbb{E}[\sum_{t=1}^T \sum_{v=1}^V (r_{i_v,t}^s(t) - r_{i_v,t}(t)) + C|\tilde{N}_1(\ell^*) \leq 4Y] \) and \( \mathbb{E}[R_T^r|\tilde{N}_1(\ell^*) \leq 4Y] \) by considering the following two cases.

**Case 1**: \( \ell^* = 1 \). This case implies that the adversary injects the corruption from the beginning to the end. In this case, the reward of arm 1 is always sampled from a fixed Bernoulli distribution with mean \( 1/2 + \Delta \), while arm 2 is with a constant reward \( 1/2 \). Such a setting can be simulated as a “mirror stochastic” setting (compared with \( \mu_1 = 1/2 - \Delta \) and \( \mu_2 = 1/2 \)). In this setting, the suboptimal arm (in this mirror stochastic setting) is arm 2. Thus, for any algorithm that enjoys a \( O(\log VT/\Delta) \) pseudo-regret, the expected number of pull of the suboptimal arm is \( B_3 \log(FT)/\Delta^2 \) for a suitable constant \( B_3 > 0 \).

Define \( \tilde{N}_2 \) as the number of pulls of arm 2 over all rounds and agents. Then, we have

\[
\mathbb{E} \left[ \sum_{t=1}^T \sum_{v=1}^V (r_{i_v,t}^s(t) - r_{i_v,t}(t)) \right| \tilde{N}_1(\ell^*) \leq 4Y] = \mathbb{E} \left[ \sum_{t=1}^T \sum_{v=1}^V (r_{i_v,t}^s(t) - r_{i_v,t}(t)) I\{i_v(t) = 1\} \right| \tilde{N}_1(\ell^*) \leq 4Y] \tag{19}
\]

\[
= -2\Delta \left( VT - \mathbb{E}[\tilde{N}_2|\tilde{N}_1(\ell^*) \leq 4Y] \right)
\]

\[
= -2\Delta VT + \frac{B_3 \log(FT)}{\Delta}
\]

\[
\leq -\frac{3}{2} \Delta VT,
\]

18
where the last inequality follows that $B_3 \log(VT)/\Delta \leq \Delta VT/2$ for a sufficiently large $T$.

Under $\tilde{N}_1(\ell^*) \leq 4Y$, the corruption takes place in all rounds, the expected corruption level is $\mathbb{E}[C|\tilde{N}_1(\ell^*) \leq 4Y] \leq 2\Delta VT$. We use (15), (19), and $\mathbb{E}[C|\tilde{N}_1(\ell^*) \leq 4Y] \leq 2\Delta VT$ to get

$$\mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,2}(t) - r_{i_v(t)}^{S}(t) \right) \left| \tilde{N}_1(\ell^*) \leq 4Y \right. \right] \geq \mathbb{E}[R_T'|\tilde{N}_1(\ell^*) \leq 4Y] + \frac{5}{2}\Delta VT - \mathbb{E}[C|\tilde{N}_1(\ell^*) \leq 4Y] - 1 \quad (20)$$

$$\geq \mathbb{E}[R_T'|\tilde{N}_1(\ell^*) \leq 4Y] + \frac{1}{4}\mathbb{E}[C|\tilde{N}_1(\ell^*) \leq 4Y] - 1.$$

**Case 2:** $\ell^* \geq 2$. According to the corruption strategy (if corruption occurs, then, the adversary always drags down the reward of arm $1$), $\sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,2}(t) - r_{i_v(t)}^{S}(t) \right) \leq 0$. Under $\tilde{N}_1(\ell^*) \leq 4Y$, (15) is upper-bounded as

$$\mathbb{E}[R_T'|\tilde{N}_1(\ell^*) \leq 4Y] \leq \mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,2}(t) - r_{i_v(t)}^{S}(t) \right) \left| \tilde{N}_1(\ell^*) \leq 4Y \right. \right] - \Delta VT + \mathbb{E}[C|\tilde{N}_1(\ell^*) \leq 4Y]. \quad (21)$$

As $\ell^* \geq 2$, under $\tilde{N}_1(\ell^*) \leq 4Y$, the corruption is upper-bounded as

$$C \leq 2\Delta V \left( T - \sum_{\ell=1}^{\ell^*-1} L_1 \right) = 2\Delta V \left( T - T^\alpha \frac{3\ell^* - 1}{2} \right) \leq 2\Delta (VT - 4C),$$

which immediately leads to

$$\Delta VT \geq \left( \frac{1}{2} + 4\Delta \right) \mathbb{E}[C|\tilde{N}_1(\ell^*) \leq 4Y]. \quad (22)$$

Using (21), (22) and (20), we lower bound $\mathbb{E}[R_T]$ as

$$\mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,2}(t) - r_{i_v(t)}^{S}(t) \right) \left| \tilde{N}_1(\ell^*) \leq 4Y \right. \right] \geq \mathbb{E}[R_T'|\tilde{N}_1(\ell^*) \leq 4Y] + \left(4\Delta - \frac{1}{2}\right) \mathbb{E}[C|\tilde{N}_1(\ell^*) \leq 4Y]. \quad (23)$$

**Put two cases together.** Since $\Delta \in (1/4, 1/2) \cup (4\Delta - \frac{1}{2})$ is positive for $\Delta \in (1/4, 1/2)$. Combing the above analysis, we have that

$$\mathbb{E}[R_T] = \mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,2}(t) - r_{i_v(t)}^{S}(t) \right) \left| \tilde{N}_1(\ell^*) \leq 4Y \right. \right] \mathbb{P} \left( \tilde{N}_1(\ell^*) \leq 4Y \right)
+ \mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} \left( r_{v,2}(t) - r_{i_v(t)}^{S}(t) \right) \left| \tilde{N}_1(\ell^*) > 4Y \right. \right] \mathbb{P} \left( \tilde{N}_1(\ell^*) > 4Y \right)
= \Omega \left( \mathbb{E}[R_T'] + \mathbb{E}[C] \right),$$

where $\mathbb{P}(\tilde{N}_1(\ell^*) \leq 4Y) = p_1$ (see Appendix C.2), and thus we get the desired result.

**D Proof of Lemma 2**

**Lemma 7.** For any fixed $v, i, \tau$ the following holds

$$\mathbb{P} \left( \tilde{N}_{v,i}(\tau) \leq 3N_{v,i}(\tau) \right) \geq 1 - \frac{\delta}{2V K \log_4 T}. \quad (24)$$
We complete the proof as in Appendix B. Then, \( \{D_{v,i}(t)\}_{t=0}^\infty \) is a martingale difference sequence with respect to the filtration \( \{\mathcal{F}_t\}_{t=0}^\infty \). Then, \( \mathbb{E}[D_{v,i}(t)|\mathcal{F}_{t-1}] = 0 \), and then the martingale variance is equal to \( \mathbb{E}[D_{v,i}^2(t)|\mathcal{F}_{t-1}] \), which is bounded as

\[
V = \sum_{t \in \mathcal{T}(\tau)} \mathbb{E}[D_{v,i}^2(t)|\mathcal{F}_{t-1}] \leq \sum_{t \in \mathcal{T}(\tau)} \text{Var}(y_{v,i}(t)) \leq p_{v,i}(\tau)N(\tau).
\]

As \( |D_{v,i}(t)| \leq 1 \), applying Freedman’s inequality (see Lemma 5),

\[
\sum_{t \in \mathcal{T}(\tau)} D_{v,i}(t) \leq V + \log(1/\delta') = p_{v,i}(\tau)N(\tau) + \log(1/\delta').
\]

Then, by choosing \( \delta' = \delta/(2VK \log_4 T) \), we get

\[
\dot{N}_{v,i}(\tau) = \sum_{t \in \mathcal{T}(\tau)} (D_{v,i}(t) + p_{v,i}(\tau)) \leq 2p_{v,i}(\tau)N(\tau) + \log((2VK \log_4 T)/\delta) \leq 3p_{v,i}(\tau)N(\tau),
\]

where the last inequality holds as \( p_{v,i}(\tau)N(\tau) > \log((8VK \log_4 T)/\delta) > \log((2VK \log_4 T)/\delta) \).

We complete the proof as \( p_{v,i}(\tau)N(\tau) = N_{v,i}(\tau) \).

\[\square\]

**Lemma 8.** For any fixed \( v, i, \tau \), and any corruption level \( C \), the following holds

\[
\mathbb{P} \left( \dot{C}_{v,i}(\tau) \leq 2p_{v,i}(\tau)C(\tau) + \log((8VK \log_4 T)/\delta) \right) \geq 1 - \frac{\delta}{8VK \log_4 T}.
\]

**Proof.** We define \( D_{v,i}(t) = C_{v,i}(t)(y_{v,i}(t) - p_{v,i}(\tau)) \) where the definition of \( y_{v,i}(t) \) is given in Appendix B. Then, \( \{D_{v,i}(t)\}_{t=0}^\infty \) is a martingale difference sequence with respect to the filtration \( \{\mathcal{F}_t\}_{t=0}^\infty \). We have that \( \mathbb{E}[D_{v,i}(t)|\mathcal{F}_{t-1}] = 0 \) and \( C_{v,i}(t) \) is deterministic given \( \mathcal{F}_{t-1} \), i.e., \( \mathbb{E}[C_{v,i}(t)|\mathcal{F}_{t-1}] = C_{v,i}(t) \) (as the corruption at round \( t \) is a function of the history information).

Then, the martingale variance \( V \) is equal to \( \mathbb{E}[D_{v,i}^2(t)|\mathcal{F}_{t-1}] \), which is bounded as:

\[
V = \sum_{t \in \mathcal{T}(\tau)} \mathbb{E}[D_{v,i}^2(t)|\mathcal{F}_{t-1}] \leq \sum_{t \in \mathcal{T}(\tau)} C_{v,i}(t) \text{Var}(y_{v,i}(t)) \leq p_{v,i}(\tau)C_{v,i}(\tau) \leq p_{v,i}(\tau)C(\tau).
\]

As \( |D_{v,i}(t)| \leq 1 \), applying Freedman’s inequality (see Lemma 6),

\[
\sum_{t \in \mathcal{T}(\tau)} D_{v,i}(t) \leq V + \log(1/\delta') \leq p_{v,i}(\tau)C(\tau) + \log(1/\delta').
\]

Then, by choosing \( \delta' = \delta/(8VK \log_4 T) \), we get with probability at least \( 1 - \delta/(8VK \log_4 T) \),

\[
\dot{C}_{v,i}(\tau) = \sum_{t \in \mathcal{T}(\tau)} (D_{v,i}(t) + C_{v,i}(t)p_{v,i}(\tau)) \leq 2p_{v,i}(\tau)C(\tau) + \log((8VK \log_4 T)/\delta),
\]

which concludes the proof.

\[\square\]

**Lemma 9.** For any fixed \( v, i, \tau \), the following holds

\[
\mathbb{P} \left( |\dot{\mu}_{v,i}(\tau) - \mu_i| > \frac{20}{9} \epsilon_i(\tau) \right) \leq \frac{\delta}{4VK \log_4 T}.
\]

**Proof.** We define \( D_{v,i}(t) = \epsilon_{v,i}(t)y_{v,i}(t) - \mu_i p_{v,i}(\tau) \) where the definition of \( y_{v,i}(t) \) is given in Appendix B. Then, \( \{D_{v,i}(t)\}_{t=0}^\infty \) is a martingale difference sequence with respect to the filtration \( \{\mathcal{F}_t\}_{t=0}^\infty \). By Hoeffding-Azuma’s inequality (see Lemma 5), with probability at most \( \delta' \),

\[
\left| \sum_{t \in \mathcal{T}(\tau)} D_{v,i}(t) \right| \leq \sqrt{\frac{\log(1/\delta')}{2} \sum_{t \in \mathcal{T}(\tau)} |D_{v,i}(t) - D_{v,i}(t-1)|}.
\]
The left term in the above can be bounded as
\[
\sqrt{\frac{\log(2/\delta')}{2} \sum_{t \in T(\tau)} |D_{v,i}(t) - D_{v,i}(t-1)|} \leq \sqrt{\frac{\log(2/\delta')}{2} \sum_{t \in T(\tau)} \max\{y_{v,i}(t-1), y_{v,i}(t)\}}
\]
\[
\leq \sqrt{\frac{\log(2/\delta')}{2} \sum_{t \in T(\tau)} (y_{v,i}(t-1) + y_{v,i}(t))}
\]
\[
\leq \log(2/\delta')N_{v,i}(\tau)
\]
\[
\leq 3\log(2/\delta')N_{v,i}(\tau),
\]
where the second inequality is due to the fact that \(\max\{\alpha, \beta\} \leq \alpha + \beta\) holds for any \(\alpha, \beta \geq 0\), and the last inequality uses Lemma [7]. According to the definition of \(D_{v,i}(t)\), we get
\[
\frac{\sum_{t \in T(\tau)} D_{v,i}(t)}{N_{v,i}(\tau)} = \frac{\sum_{t \in T(\tau)} (r_{v,i}^S(t)y_{v,i}(t) - \mu_i p_{v,i}(\tau))}{N_{v,i}(\tau)} = \mu_{v,i}(\tau) - \mu_i.
\]
By choosing \(\delta' = \delta/4VK\log_4 T\), we have
\[
\mathbb{P}\left(|\hat{\mu}_{v,i}^S(\tau) - \mu_i| > \sqrt{\frac{3\log(8VK\log_4 T/\delta)}{N_{v,i}(\tau)}}\right) \leq \frac{\delta}{4VK\log_4 T}.
\]
For \(i \in A_v(\tau)\), we have
\[
\sqrt{\frac{3\log(8VK\log_4 T/\delta)}{N_{v,i}(\tau)}} = \sqrt{\frac{|A_v(\tau)|\epsilon^2(\tau)}{K(1 - \sum_{j \in B_v(\tau)} p_{v,j}(\tau))}} \leq \frac{2\sqrt{3}}{3} \epsilon(\tau) < \frac{14}{9} \epsilon_i(\tau),
\]
where \(\sum_{j \in B_v(\tau)} p_{v,j}(\tau)\) is bounded by
\[
\sum_{j \in B_v(\tau)} p_{v,j}(\tau) \leq \sum_{j \in B_v(\tau)} \frac{\epsilon^2(\tau)}{\epsilon_j^2(\tau)K} \leq \frac{1}{4}.
\]
(26)
For \(i \in B_v(\tau)\), \(\sqrt{3\log(8VK\log_4 T/\delta)/N_{v,i}(\tau)} = \epsilon_i(\tau) < \frac{14}{9} \epsilon_i(\tau)\) also holds. As a result, we complete the proof.

Lemma 10. For any fixed \(v, i, \tau\), the following holds
\[
\mathbb{P}\left(|\hat{\mu}_{v,i}(\tau) - \mu_i| > \mu_i + 2\epsilon_i(\tau) + \frac{2C(\tau)}{N(\tau)}\right) \leq \frac{\delta}{2VK\log_4 T}.
\]
(27)

Proof. We have
\[
\hat{\mu}_{v,i}(\tau) \leq \frac{\sum_{t \in T(\tau)} (r_{v,i}^S(t) + \hat{C}_{v,i}(t))}{N_{v,i}(\tau)} \leq \frac{\sum_{t \in T(\tau)} (r_{v,i}^S(t) + \hat{C}_{v,i}(t))}{N_{v,i}(\tau)}.
\]
By Lemma [8] and the fact \(p_{v,i}(\tau)N(\tau) = N_{v,i}(\tau)\), with probability at least \(1 - \delta/(8VK\log_4 T)\), we have for any \(i\) that
\[
\frac{\sum_{t \in T(\tau)} \hat{C}_{v,i}(t)}{N_{v,i}(\tau)} \leq \frac{2p_{v,i}(\tau)C_{v,i}(\tau) + \log((8VK\log_4 T)/\delta)}{N_{v,i}(\tau)} \leq \frac{2C(\tau)}{N(\tau)} + \frac{\log((8VK\log_4 T)/\delta)}{N_{v,i}(\tau)}.
\]
Then, for \(i \in A_v(\tau)\), we have that \(\epsilon(\tau) = \epsilon_i(\tau)\), which further gives
\[
\frac{\log((8VK\log_4 T)/\delta)}{N_{v,i}(\tau)} = \frac{|A_v(\tau)|\epsilon^2(\tau)}{3K(1 - \sum_{j \in B_v(\tau)} p_{v,j}(\tau))} \leq \frac{4}{9} \epsilon(\tau) = \frac{4}{9} \epsilon_i(\tau),
\]

21
where in the last inequality, we bound $|A_v(\tau)| / \tilde{K} \leq 1$, use (26) to bound \( 1 - \sum_{j \in B_v(\tau)} p_{v,j}(\tau) \), and bound $\epsilon^2(\tau) < \epsilon(\tau)$. For $i \in B_v(\tau)$,
\[
\frac{\log((8VK \log_4 T)/\delta)}{N_{v,i}(\tau)} = \frac{\epsilon^2(\tau)}{3} < \frac{\epsilon_i(\tau)}{3} < \frac{4}{9} \epsilon_i(\tau).\]

Then, using a union bound on similar proof of $\sum_{i \in \mathcal{T}(\tau)} -\tilde{C}_{v,i}(t) / N_{v,i}(\tau)$, the following holds
\[
\mathbb{P}\left( \left| \sum_{i \in \mathcal{T}(\tau)} \tilde{C}_{v,i}(t) \right| / N_{v,i}(\tau) > \frac{2C(\tau)}{N(\tau)} + \frac{4}{9} \epsilon_i(\tau) \right) \leq \frac{\delta}{4V K \log_4 T}. \tag{28}\]

By a union bound over (25) and (28), we get $|\hat{\mu}_{v,i}(\tau) - \mu_i| \leq \mu_i + 2\epsilon_i(\tau) + 2C(\tau)/N(\tau)$.

\[\square\]

Lemma 2 (restated). Let define event $\mathcal{E}$ as
\[
\mathcal{E} = \left\{ \forall v, i, \tau : |\hat{\mu}_i(\tau) - \mu_i| \leq \mu_i + 2\epsilon_i(\tau) + \frac{2C(\tau)}{N(\tau)}, \tilde{N}_{v,i}(\tau) \leq 3N_{v,i}(\tau) \right\}.
\]

Then, we hold that $\mathbb{P}[\mathcal{E}] \geq 1 - \delta$.

Proof. We apply union bound over (27) and (24) such that
\[
\mathbb{P}\left( |\hat{\mu}_{v,i}(\tau) - \mu_i| > \mu_i + 2\epsilon_i(\tau) + \frac{2C(\tau)}{N(\tau)} \lor \tilde{N}_{v,i}(\tau) > 3N_{v,i}(\tau) \right) \leq \frac{\delta}{V K \log_4 T}.
\]

Again, by a union bound, we hold $|\hat{\mu}_{v,i}(\tau) - \mu_i| > \mu_i + 2\epsilon_i(\tau) + \frac{2C(\tau)}{N(\tau)}$ for all $v, i, \tau$ (at most $\log_4 T$ epochs). Based on the definition of $\hat{\mu}_i(\tau)$, for all $v, i, \tau$, we have that
\[
\hat{\mu}_i(\tau) - \mu_i = \frac{1}{|\{v \in [V] : i \in \mathcal{K}_v(\tau)\}|} \sum_{v \in [V] : i \in \mathcal{K}_v(\tau)} (\hat{\mu}_{v,i}(\tau) - \mu_i) \leq \mu_i + 2\epsilon_i(\tau) + \frac{C(\tau)}{N(\tau)}.
\]

Combing the above analysis, we get the desired result.

\[\square\]

E Algorithm Properties: proof of Lemma 1 and Lemma 3

As the proof of Lemma 1 will use the results of Lemma 3, we first prove Lemma 3. Here, we provide all useful properties of our algorithm, and these properties will be used for the following proof.

Lemma 3 (restated). The following holds.

(i) If $i \in A(\tau)$, then $\epsilon(\tau) = \epsilon(d_i)/2$;

(ii) $7\epsilon(d_i) \geq \epsilon_i(d_i)$.

(iii) If $i \in B(\tau)$, then $\epsilon_i(\tau) = \epsilon(d_i)$.

(iv) If $i \in B(\tau)$, then $\epsilon(d_i) = \epsilon_i(\tau) \geq 2\epsilon(\tau)$;

(v) $\epsilon(\tau) \leq \epsilon_i(\tau)$ for all arms $i \in [K]$;

Proof. Proof of (i): In our algorithm, $i \in A(\tau)$ implies that arm $i$ must hold (6) at epoch $\tau - 1$ and therefore the algorithm sets $d_i = \tau - 1$ for epoch $\tau$. As $\epsilon(\tau) = \epsilon(\tau - 1)/2$, the proof is evident.

Proof of (ii): For this proof, let consider two cases. In the first case, the arm $i$ does not hold (6) for all epochs. We note that such a case occurs if and only if arm $i$ is deactivated in epoch $\tau = 2$. Then, this case implies that $d_i = 1$ (recall that we initialize $d_i = 1$). Hence, we hold $\epsilon(d_i) = \epsilon_i(d_i)$ as we initialize $\epsilon(\tau - 1) = \epsilon_i(\tau)$.
If arm $i$ is not deactivated in epoch $\tau = 2$, there must exist an epoch such that arm $i$ holds (6). This is due to the reason that if arm $i$ is active in epoch $\tau = 2$, then, it must suffice (6) in epoch $\tau = 1$. Thus, we have, at worst, $d_i = 1$. In this case, we have
\[
14\epsilon(d_i) \geq \max_{j \in A(\tau) \cup H(\tau)} \left\{ \hat{\mu}_j(\tau) + 2\epsilon_j(\tau) \right\} - \hat{\mu}_i(d_i) \\
\geq \hat{\mu}_i(d_i) + 2\epsilon_i(d_i) - \hat{\mu}_i(d_i) \\
\geq 2\epsilon_i(d_i),
\]
which immediately leads to $7\epsilon(d_i) \geq \epsilon_i(d_i)$. Combining two cases, we get the desired result.

Proof of (iii): This holds due to the construction of our algorithm (See Algorithm 1, line 21).

Proof of (iv): In our algorithm, $B(\tau) = \emptyset$ for $\tau = 1$. As a result, $i \in B(\tau)$ only occurs for $\tau \geq 2$. For $i \in B(\tau)$, arm $i$ does not hold (6) at epoch $\tau - 1$ and therefore the maximum possible value of $d_i$ is $\max\{1, \tau - 2\}$. If the maximum possible value of $d_i$ is 1, then, for $i \in B(\tau)$, $\epsilon(d_i = 1) = \epsilon_i(\tau) \geq 2\epsilon(\tau)$. If the maximum possible value of $d_i$ is $\tau - 2$, then, we hold $\epsilon(d_i) \geq \epsilon(\tau - 2) = 4\epsilon(\tau)$. From (iii), we have $\epsilon_i(\tau) = \epsilon(d_i) \geq 4\epsilon(\tau)$, which completes the proof.

Proof of (v): For this proof, we consider two cases. If $i \in A(\tau)$, this naturally holds $\epsilon_i(\tau) = \epsilon(\tau)$ due to the construction of our algorithm. Then, if $i \in B(\tau)$, we hold $\epsilon_i(\tau) \geq 2\epsilon(\tau) > \epsilon(\tau)$ due to (iv), which concludes the proof.

Lemma 1 (restated). In the stochastic setting, with probability at least $1 - \delta$, the following holds.

(i) The optimal arm $i^*$ is active for all $\tau$;
(ii) If $i \in A(\tau)$ and $i \in B(\tau + 1)$, then, $\Delta_i > 8\epsilon(d_i)$;
(iii) $H(\tau) = \emptyset$ for all $\tau$.

Proof. We prove this lemma by iteratively arguing (i), (ii), and (iii) for each epoch. Recall that the algorithm sets $H(\tau) = \emptyset$, $B(\tau) = \emptyset$, and $A(\tau) = [K]$ for $\tau = 1$. Then, for any $i \neq i^*$ and $\tau = 1$, the following holds
\[
\hat{\mu}_i(\tau) - \hat{\mu}_i(\tau) = \max_{j \in A(\tau) \cup H(\tau)} \{ \hat{\mu}_j(\tau) + 2\epsilon_j(\tau) \} - \hat{\mu}_i(\tau) \\
= \max_{j \in A(\tau)} \{ \hat{\mu}_j(\tau) + 2\epsilon_j(\tau) \} - \hat{\mu}_i(\tau) \\
\leq \max_{j \in A(\tau)} \{ \mu_j + 4\epsilon_j(\tau) \} - (\mu_{i^*} - 2\epsilon(\tau)) \\
= \max_{j \in A(\tau)} \{ \mu_j \} - \mu_{i^*} + 6\epsilon(\tau) \\
\leq 6\epsilon(\tau).
\] (29)

The first inequality of (29) comes from Lemma 2 with $C(\tau) = 0$. The last inequality of (29) follows $\max_{j \in A(\tau)} \{ \mu_j \} \leq \max_{j \in [K]} \{ \mu_j \} = \mu_{i^*}$. It is evident that (29) contradicts to the deactivated condition, and it holds for all $H(\tau) = \emptyset$.

We then show (iii) holds in the stochastic setting. Suppose that $\tau + 1$ is the first epoch in which there exists an arm $i \in [K]$ such that $i \in B(\tau + 1)$. Such a case implies that arm $i$ contradicts to (6) at epoch $\tau$, and thus we hold $i \in A(\tau)$, $i \in B(\tau + 1)$. Since $H(\tau) = \emptyset$ is empty before epoch $\tau + 1$, we have $i^* \in A(\tau)$ and $i^* \in A(\tau + 1)$. Then, for epoch $\tau$ and the deactivated arm $i$, the following holds
\[
\hat{\mu}^*(\tau) - \hat{\mu}_i(\tau) - 14\epsilon(\tau) \\
\geq \max_{j \in A(\tau)} \{ \hat{\mu}_j(\tau) + 2\epsilon_j(\tau) \} - \hat{\mu}_i(\tau) - 14\epsilon(\tau) \\
\geq \max_{j \in A(\tau)} \{ \mu_j \} - (\mu_i + 2\epsilon(\tau)) - 14\epsilon(\tau) \\
= \max_{j \in A(\tau)} \{ \mu_j \} - \mu_i - 16\epsilon(\tau) \\
\geq \Delta_i - 16\epsilon(\tau),
\] (30)
We define $\epsilon(\tau) = \epsilon_i(\tau)$ for $i \in \mathcal{A}(\tau)$. Recall that when arm $i$ is deactivated at $\tau$, it should satisfy $\hat{\mu}^*(\tau) - \hat{\mu}_i(\tau) - 14\epsilon(\tau) > 0$, which implies $\Delta_i > 16\epsilon(\tau)$. As $d_i = \tau - 1$ is the last epoch that arm $i$ satisfies (6), we hold $\epsilon(d_i) = \epsilon(\tau - 1) = 2\epsilon(\tau)$. Hence, we have $\Delta_i > 8\epsilon(d_i)$.

Finally, we show that in the stochastic setting, $\mathcal{H}(\tau)$ is an empty set for all epochs (i.e., the deactivated arms in $\mathcal{B}(\tau + 1)$ will not be reactivated). Again, due to the fact that $i^* \in \mathcal{A}(\tau + 1)$, the following holds for any arm $i \in \mathcal{B}(\tau + 1)$.

$$
\max_{j \in \mathcal{A}(\tau + 1)} \hat{\mu}_j(\tau + 1) - \hat{\mu}_i(\tau + 1) \\
\geq \max_{j \in \mathcal{A}(\tau + 1)} \{ \mu_j \} - 2\epsilon(\tau + 1) - (\mu_i + 2\epsilon(\tau + 1)) \\
= \Delta_i - 2\epsilon(\tau + 1) - 2\epsilon(d_i) \\
\geq 8\epsilon(d_i) - 2\epsilon(\tau + 1) - 2\epsilon(d_i) \\
\geq 4\epsilon(d_i).
$$

(31)

The inequality (a) is due to $\epsilon_i(\tau + 1) = \epsilon(d_i)$ for $i \in \mathcal{B}(\tau + 1)$ (see Lemma 3 (iii)). The inequality (b) follows the fact that $\Delta_i > 8\epsilon(d_i)$. As a consequence, (31) contradicts to the condition of reactivation, which further implies that $\mathcal{H}(\tau + 1) = \emptyset$. Subsequently, $\mathcal{A}(\tau + 1) \cup \mathcal{H}(\tau + 1) = \mathcal{A}(\tau + 1)$ holds, and this, again, leads to the argument that the arm $i^*$ will not be deactivated in epoch $\tau + 2$. By iteratively repeating the analysis of (29), (30) and (31) over all epochs, we get the desired results.

## F Regret and Communication: proof of Theorem 2

### F.1 Communication analysis

Recall that the communication occurs in line 14, line 26, and line 25. Each of those messages, including $A_v(\tau)$, $B_v(\tau)$, $\{ \hat{\mu}_{v,i}(\tau) \}_{i \in \mathcal{K}_v(\tau)}$, and $\{ \epsilon_i(\tau) \}_{i \in \mathcal{K}_v(\tau)}$ has the size at most $\tilde{K} = \lceil K/V \rceil + 1$.

Since we have that $\tilde{K} = \lceil K/V \rceil + 1 < K/V + 2 \leq 3K/V$ for $V \leq K$, the communication cost in an epoch across all agents is $O(K)$ bits. As the algorithm proceeds in at most $O(\log T)$ epochs and the communication occurs for each epoch, the total communication cost is $O(K \log T)$ bits.

### F.2 Technical lemmas for regret bound

We define $R_{v,i}(\tau) = N_{v,i}(\tau)\Delta_i$, and $\eta(d_i)$ as

$$
\eta(d_i) = \sum_{s = 1}^{d_i} \frac{C(s)}{2^{4d_i - 4s}N(s)}.
$$

**Lemma 11.** If $\epsilon(d_i) \geq \Delta_i/\alpha$ for some constant $\alpha > 1$, then, the following holds,

$$
R_{v,i}(\tau) \leq \begin{cases} 
\frac{3\alpha^2 \log((8K \log_4 T)/\delta)}{\Delta_i}, & i \in \mathcal{B}_v(\tau), \\
\frac{12\alpha^2 \tilde{K} \log((8K \log_4 T)/\delta)}{|\mathcal{A}_v(\tau)| \Delta_i}, & i \in \mathcal{A}_v(\tau).
\end{cases}
$$

**Proof.** Note that based on the construction of our algorithm, $i \in \mathcal{A}_v(\tau)$ must imply that $i \in \mathcal{A}(\tau)$. This argument is also applicable for $i \in \mathcal{B}_v(\tau)$. For $i \in \mathcal{A}(\tau)$, we use (i) of Lemma 3 to get $\epsilon(\tau) = \epsilon(d_i)/2$. Thus, in the case of $i \in \mathcal{A}(\tau)$, $R_{v,i}(\tau)$ is upper-bounded as

$$
R_{v,i}(\tau) = N_{v,i}(\tau)\Delta_i \leq \frac{3\tilde{K} \log((8K \log_4 T)/\delta)}{|\mathcal{A}_v(\tau)| \epsilon^2(\tau)} \Delta_i \leq \frac{12\alpha^2 \tilde{K} \log((8K \log_4 T)/\delta)}{|\mathcal{A}_v(\tau)| \Delta_i}.
$$

(32)

For $i \in \mathcal{B}_v(\tau)$ (thereby $i \in \mathcal{B}(\tau)$), we use (iii) of Lemma 3 to get $\epsilon_i(\tau) = \epsilon(d_i)$, and then

$$
R_{v,i}(\tau) = N_{v,i}(\tau)\Delta_i \leq \frac{3\log((8K \log_4 T)/\delta)}{\epsilon_i(\tau)} \Delta_i \leq \frac{3\alpha^2 \log((8K \log_4 T)/\delta)}{\Delta_i}.
$$

(33)
Lemma 4 (restated). If $\Delta_i > 32\eta(d_i)$, then, with probability at least $1 - \delta$, $\epsilon(d_i) \geq \Delta_i/32$.

Proof. Since the following analysis is based on the fixed arm $i$, agent $v$ and epoch $\tau$, $d_i$ is the last epoch up to epoch $\tau$ such that arm $i$ holds (6) unless an exception that for $d_i = 1$. Recall that the algorithm initializes $d_i = 1$ and $i \in A(\tau = 1)$ for all arms; then it is entirely possible that arm $i$ may not hold (6) for $d_i = \tau = 1$. Here, we first show that when $d_i = 1$, the desired result $\epsilon(d_i) = 1/14 \geq \Delta_i/32$ trivially holds because $\Delta_i \leq 1$ implies that $\Delta_i/32 \leq 1/32 < 1/14$. Then, one can bound $\epsilon(d_i)$ by considering the following three cases for $d_i \geq 2$, and thus arm $i$ must hold (6) in epoch $d_i$.

**Case 1:** $i^* \in A(d_i) \cup H(d_i)$. If $d_i$ is the last epoch up to epoch $\tau$ such that arm $i$ holds (6), we have (see line 19 of Algorithm 1), the following holds

\[
14\epsilon(d_i) \geq \max_{j \in A(\tau) \cup H(\tau)} \{\hat{\mu}_j(d_i) + 2\epsilon(d_i)\} - \hat{\mu}_i(d_i),
\]

\[
\geq \hat{\mu}_i^*(d_i) + 2\epsilon_i^*(d_i) - \hat{\mu}_i(d_i),
\]

\[
\geq \Delta_i - \frac{2C(d_i)}{N(d_i)} - 2\epsilon_i(d_i) - \frac{2C(d_i)}{N(d_i)}
\]

\[
\geq \Delta_i - \frac{4C(d_i)}{N(d_i)} - 14\epsilon(d_i)
\]

\[
\geq \Delta_i - \frac{\Delta_i}{8} - 14\epsilon(d_i),
\]

where:

- The inequality (a) is due to for $i^* \in A(d_i) \cup H(d_i)$, we have that

\[
\max_{j \in A(\tau) \cup H(\tau)} \{\hat{\mu}_j(\tau) + 2\epsilon_j(\tau)\} \geq \hat{\mu}_i^*(d_i) + 2\epsilon_i^*(d_i).
\]

- The inequality (b) follows Lemma 2.

- The inequality (c) comes from (6) of Lemma 3.

- The inequality (d) holds because

\[
\Delta_i > 32\eta(d_i) = \sum_{s=1}^{d_i} \frac{32C(s)}{2^{4d_i-4s}N(s)} \geq \frac{32C(d_i)}{N(d_i)}.
\]

From (34), we have $\epsilon(d_i) \geq \Delta_i/32$.

**Case 2:** $8\epsilon_i^*(d_i) \leq \Delta_i$ and $i^* \in B(d_i) \setminus H(d_i)$. For $i^* \in B(d_i) \setminus H(d_i)$, we have $\max_{j \in A(d_i)} \hat{\mu}_j(d_i) \geq 4\epsilon_i^*(d_i) + \hat{\mu}_i^*(d_i)$ as $i^*$ contradicts the reactivation condition (see line 17 of Algorithm 1). Further, we have

\[
\max_{j \in A(d_i) \cup H(d_i)} \{\hat{\mu}_j(d_i) + 2\epsilon_j(d_i)\} > \max_{j \in A(d_i)} \hat{\mu}_j(d_i) \geq 4\epsilon_i^*(d_i) + \hat{\mu}_i^*(d_i) = 4\epsilon_i^*(d_i) + \hat{\mu}_i^*(d_i).
\]

Note that the last inequality in (36) comes from (6) of Lemma 3 (as $i^* \in B(d_i) \setminus H(d_i)$). Again, as arm $i$ holds (6), we have

\[
14\epsilon(d_i) \geq \max_{j \in A(d_i) \cup H(d_i)} \{\hat{\mu}_j(d_i) + 2\epsilon_j(d_i)\} - \hat{\mu}_i(d_i)
\]

\[
> \hat{\mu}_i^*(d_i) + 4\epsilon_i^*(d_i) - \hat{\mu}_i(d_i)
\]

\[
\geq \mu_i + 2\epsilon_i^*(d_i) - \frac{2C(d_i)}{N(d_i)} - \mu_i - 14\epsilon(d_i) - \frac{2C(d_i)}{N(d_i)}
\]

\[
\geq \Delta_i - 14\epsilon(d_i) - \frac{\Delta_i}{8},
\]

\[
\Delta_i \geq 14\epsilon(d_i) + \frac{\Delta_i}{8},
\]

\[
\Delta_i \geq 32\eta(d_i) = \sum_{s=1}^{d_i} \frac{32C(s)}{2^{4d_i-4s}N(s)} \geq \frac{32C(d_i)}{N(d_i)}.
\]
where the second inequality uses \( 35 \); the third inequality follows Lemma 2 and (ii) of Lemma 3; the last inequality uses \( \epsilon_i^- (d_i) \geq 0 \) and (35). Then, we get \( \epsilon (d_i) \geq \Delta_i / 32 \).

**Case 3:** \( 8 \epsilon_i^- (d_i) > \Delta_i \) and \( i^* \in B(d_i) \setminus \mathcal{H}(d_i) \). As \( i^* \in B(d_i) \setminus \mathcal{H}(d_i) \), \( i^* \) contradicts the reactivation condition, which implies that

\[
4 \epsilon (d_i^*) \leq \max_{j \in A(d_i)} \mu_j (d_i) - \mu_{i^*} (d_i)
\]

\[
\leq \max_{j \in A(d_i)} \{ \mu_j + 2 \epsilon_j (d_i) \} + \frac{2 C(d_i)}{N(d_i)} - \mu_{i^*} + 2 \epsilon_{i^*} (d_i) + \frac{2 C(d_i)}{N(d_i)}
\]

\[
\leq 4 C(d_i) + 2 \epsilon (d_i) + 2 \epsilon_{i^*} (d_i)
\]

\[
\leq 3 \epsilon_{i^*} (d_i)
\]

\[
\leq \frac{\Delta_i}{8} + 3 \epsilon_{i^*} (d_i),
\]

- The inequality (a) follows Lemma 2.
- The inequality (b) is due to \( \max_{j \in A(d_i)} \mu_j \leq \mu_{i^*} \).
- The inequality (c) follows (iv) of Lemma 3 that \( \epsilon (d_i) \leq \epsilon_{i^*} (d_i) / 2 \) for \( i^* \in B(d_i) \setminus \mathcal{H}(d_i) \).
- The inequality (d) holds because of (35).

Note that as \( i^* \in B(d_i) \), we have \( \epsilon (d_i^*) = \epsilon_{i^*} (d_i) \) (Recall (ii) of Lemma 3). Thus, we get \( 8 \epsilon_{i^*} (d_i) \leq \Delta_i \), which contradicts to \( 8 \epsilon_{i^*} (d_i) > \Delta_i \).

\[
\square
\]

**F.3 Proof of regret bound**

By Lemma 2, the regret, with probability at least \( 1 - \delta \), is upper-bounded as

\[
R_T = \sum_{\tau = 1}^{L} \sum_{v=1}^{V} \sum_{i \neq i^*, i \in K_v (\tau)} \bar{N}_{v,i} (\tau) \Delta_i \leq \sum_{\tau = 1}^{L} \sum_{v=1}^{V} \sum_{i \in K_v (\tau)} 3 N_{v,i} (\tau) \Delta_i = 3 R (T, A) + 3 R (T, B)
\]

where

\[
R (T, A) = \sum_{\tau = 1}^{L} \sum_{i \in A_v (\tau), i \neq i^*} R_{v,i} (\tau), \text{ and } R (T, B) = \sum_{\tau = 1}^{L} \sum_{i \in B_v (\tau), i \neq i^*} R_{v,i} (\tau).
\]

Note that we slightly abuse the notation of \( L \), and \( L \) here is the maximum epoch that the algorithm proceeds up to \( \bar{T} \).

**Bounding \( R (T, A) \)**

**Case 1:** \( \Delta_i \leq 32 \eta (d_i) \). We use Lemma 3 (ii) to get \( \epsilon (\tau) = 2 \epsilon (\tau) \) and then \( R_{v,i} (\tau) \) in this case is upper-bounded as,

\[
R_{v,i} (\tau) \leq \frac{96 K \log ([8 K \log_4 T] / \delta)}{|A_v (\tau)| 2^{\epsilon (\tau)}} \eta (d_i)
\]

\[
= \frac{384 K \log ([8 K \log_4 T] / \delta)}{|A_v (\tau)| 2^{\epsilon (d_i)}} \sum_{s=1}^{d_i} \frac{C(s)}{2^{4 d_i - 4 s}} \left( \frac{3 K \log ([8 K \log_4 T] / \delta)}{2^{d_i}} \right)
\]

\[
= \frac{128}{|A_v (\tau)|} \sum_{s=1}^{d_i} C(s) 2^{2 d_i - 2 s}.
\]

\[26\]
Then, the regret under such a case is upper-bounded as,

\[
R(T, A) \leq 128 \sum_{\tau=1}^{L} \sum_{v=1}^{V} \sum_{i \in A_v(\tau), i \neq i^*} \frac{1}{|A_v(\tau)|} \sum_{s=1}^{d_i} \frac{C(s)2^{2d_i-2s}}{2^{4d_i-4s}}
\]

\[
\leq 128 \sum_{s=1}^{L} \sum_{v=1}^{V} C(s) \sum_{i \in A_v(s), i \neq i^*} \frac{1}{|A_v(s)|} \sum_{s=1}^{d_i} \frac{2^{2d_i-2s}}{2^{4d_i-4s}}
\]

\[
\leq 128 \sum_{s=1}^{L} \sum_{v=1}^{V} C(s) \sum_{i \in A_v(s), i \neq i^*} \frac{1}{|A_v(s)|} \sum_{q=1}^{\infty} 4^{-q}
\]

\[
\leq 128 \sum_{s=1}^{L} \sum_{v=1}^{V} C(s) \leq 128VC.
\]

**Case 2:** $\Delta_i > 32\eta(d_i)$. As $V \leq K$, we have that $\bar{K} = [K/V] + 1 < K/V + 2 \leq 3K/V$. Using Lemma 11 and Lemma 4, we upper-bound $R(T, A)$ as

\[
R(T, A) \leq 128 \sum_{\tau=1}^{L} \sum_{v=1}^{V} \sum_{i \in A_v(\tau), i \neq i^*} \frac{12 \cdot 32^2 \bar{K} \log((8K \log_4 T)/\delta)}{|A_v(\tau)| \Delta_i}.
\]

**Bounding $R(T, B$)**

**Case 1:** $\Delta_i \leq 32\eta(d_i)$. For $i \in B(\tau)$, we use Lemma 13 to get $c(d_i) = c_i(\tau)$. Then, $R_{v,i}(\tau)$ is upper-bounded as,

\[
R_{v,i}(\tau) \leq \frac{96 \log((8K \log_4 T)/\delta)}{c_i^2(\tau)} \eta(d_i) \leq \frac{32 \cdot d_i}{K} \sum_{s=1}^{d_i} \frac{C(s)2^{2d_i-2s}}{2^{4d_i-4s}}.
\]

Then, $R(T, B)$ in this case is upper-bounded as,

\[
R(T, B) \leq \frac{32}{K} \sum_{\tau=1}^{L} \sum_{v=1}^{V} \sum_{i \in B_v(\tau), i \neq i^*} \frac{d_i C(s)2^{2\tau-2s}}{2^{4\tau-4s}} \leq 32 \sum_{s=1}^{L} \sum_{v=1}^{V} \sum_{i \in B_v(s), i \neq i^*} C(s) \sum_{q=1}^{\infty} 4^{-q} \leq 32VC.
\]

**Case 2:** $\Delta_i > 32\eta(d_i)$. By Lemma 11 and Lemma 4, $R(T, B)$ is upper-bounded as

\[
R(T, B) \leq \sum_{\tau=1}^{L} \sum_{v=1}^{V} \sum_{i \in B_v(\tau), i \neq i^*} \frac{3 \cdot 32^2 \log((8K \log_4 T)/\delta)}{\Delta_i}.
\]

Combing the $R(T, A)$ and $R(T, B)$, we get the regret $R_T$ as

\[
O\left(VC + \log((V \log T)/\delta) \sum_{\tau=1}^{L} \left( \sum_{v=1}^{V} \frac{\bar{K}}{|A_v(\tau)|} \sum_{i \in A_v(\tau), i \neq i^*} \frac{1}{\Delta_i} + \sum_{v=1}^{V} \sum_{i \in B_v(\tau), i \neq i^*} \frac{1}{\Delta_i} \right) \right).
\]

**F.4 Concise regret bound**

We here give a concise regret bound by lower-bounding $\Delta_i \geq \Delta_{\min}$. For $\Delta_i \leq 32\eta(d_i)$, both $R(T, A)$ and $R(T, B)$ are bounded by $O(VC)$. As a consequence, we only need to refine $R(T, A)$ and $R(T, B)$ for the case $\Delta_i > 32\eta(d_i)$.

Let first refine $R(T, A)$ for $\Delta_i > 32\eta(d_i)$. As $V \leq K$, we have $\bar{K} = [K/V] + 1 < K/V + 2 \leq 3K/V$, we upper-bound $R(T, A)$ as

\[
R(T, A) \leq \sum_{\tau=1}^{L} \sum_{v=1}^{V} \sum_{i \in A_v(\tau), i \neq i^*} \frac{36 \cdot 32^2 K \log((8K \log_4 T)/\delta)}{|A_v(\tau)| \Delta_i}.
\]
Similarly, by $\Delta_i \geq \Delta_{\min}$, $R(T, \mathcal{A})$ is upper-bounded as

$$R(T, \mathcal{A}) \leq \frac{24 \cdot 32^2 K \log_4 T \log((8K \log_4 T)/\delta)}{\Delta_{\min}}.$$ 

Then, we refine $R(T, \mathcal{B})$ as

$$R(T, \mathcal{B}) \leq \sum_{\tau=1}^L \sum_{i=1}^V \sum_{v \in \mathcal{B}_v(\tau), i \neq i^*} \frac{3 \cdot 32^2 \log((8K \log_4 T)/\delta)}{\Delta_{\min}}$$

$$\leq \sum_{\tau=1}^L \sum_{i=1}^V \frac{3 \cdot 32^2 \tilde{K} \log((8K \log_4 T)/\delta)}{\Delta_{\min}}$$

$$= \frac{9 \cdot 32^2 K \log_4 T \log((8K \log_4 T)/\delta)}{\Delta_{\min}},$$

where the second inequality is due to $\sum_{i \in \mathcal{B}_v(\tau), i \neq i^*} 1 < |\mathcal{K}_v| = \tilde{K}$, and the last inequality, again, uses the fact that $\tilde{K} = [K/V] + 1 < K/V + 2 \leq 3K/V$ for $V \leq K$.

### F.5 Expected regret

For the proof of expected regret, we set $\delta = 1/T$, and then, one can trivially bound the regret by $O(T)$ with the failure probability $1/T$. Thus, this part yields the expected regret as $O(1)$, and the expected regret from the remaining probability at least $1 - 1/T$ can be bounded by using the result of $R_T$. Thus, the expected regret is bounded by $O(V \mathbb{E}[C] + \frac{K \log(T \log(VT))}{\Delta_{\min}})$.

### G Discussion

#### G.1 Proof of Theorem 3

**Theorem 3 (rephrased).** For any algorithm of $V \geq 1$ agents, with probability at least $1 - 1/V T$, we have that $R_T = O(\mathbb{E}[R_T] + \mathbb{E}[C] + V \log(V T))$. Then, $\mathbb{E}[R_T] = \Theta(\mathbb{E}[R_T] + \mathbb{E}[C])$ and $\mathbb{E}[R_T] = \Theta(\mathbb{E}[R'_T] + \mathbb{E}[C])$ hold for the two-armed bandit instance.

**Proof.** We first show $R_T = O(\mathbb{E}[R_T] + \mathbb{E}[C] + V \log(V T))$. Recall the definition of $R_T$, and it can be rewritten as

$$R_T = \sum_{t=1}^T \sum_{v=1}^V (\mu_{v^*} - \mu_{i_v(t)}) = \sum_{t=1}^T \sum_{v=1}^V \Delta_i \mathbb{I} \{i_v(t) = i\}.$$

By constructing a martingale $R_T - \mathbb{E}[R_T]$, $D_v(t) = \{\Delta_i [\mathbb{I} \{i_v(t) = i\} - \mathbb{P} \{i_v(t) = i\}]\}_{t=0}^T$ is the martingale difference sequence. Applying Freedman’s inequality, the following holds with probability at least $1 - 1/V T$,

$$R_T - \mathbb{E}[R_T] \leq \sum_{v=1}^V \left( \log(V T) + \sum_{t=1}^T \mathbb{E} \left[ D_v(t)^2 \mid \mathcal{F}_{t-1} \right] \right)$$

$$\leq V \log(V T) + \sum_{v=1}^V \sum_{t=1}^T \Delta_i \mathbb{V} \{\mathbb{I} \{i_v(t) = i\}\}$$

$$\leq V \log(V T) + \sum_{v=1}^V \sum_{t=1}^T \Delta_i \mathbb{P} \{i_v(t) = i\}$$

$$\leq V \log(V T) + \mathbb{E}[R_T],$$

After some simple algebra, the following holds

$$R_T \leq V \log(V T) + 2\mathbb{E}[R_T]. \quad (38)$$
Recall $\mathcal{R}_T$ in [Zimmert and Seldin 2021] is given as follows. (here we write it in a generic $V$-agent form, and if we set $V = 1$, $\mathcal{R}_T$ is the same the one in Zimmert and Seldin [2021].)

$$\mathcal{R}_T = \max_i \mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} (r_{v,i}(t) - r_{i^*(t)}(t)) \right].$$

The regret $\mathcal{R}_T$ is further lower-bounded as

$$\mathcal{R}_T = \max_i \mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} (r_{v,i}(t) - r_{i^*(t)}(t)) \right] \geq \mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} (r_{v,i}(t) - r_{i^*(t)}(t)) \right] \geq \mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} (r_{v,i}(t) - r_{i^*(t)}(t)) \right] + \mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} (r_{v,i}(t) - r_{i^*(t)}(t)) \right] + \mathbb{E} \left[ \sum_{t=1}^{T} \sum_{v=1}^{V} (r_{v,i}(t) - r_{i^*(t)}(t)) \right] - 2\mathbb{E}[C]$$

which immediately leads to $\mathbb{E}[R_T] \leq \mathcal{R}_T + 2\mathbb{E}[C]$. Combing the above results and (38), then,

$$R_T \leq 2(\mathcal{R}_T + 2\mathbb{E}[C]) + V \log(VT).$$

Recall Theorem[1] that $\mathbb{E}[R_T] = \Theta(\mathbb{E}[C] + \mathbb{E}[R_T^*])$ in the two-armed instance. Since $\mathcal{R}_T \leq \mathbb{E}[R_T^*]$ and $\mathcal{R}_T = \mathbb{E}[R_T^*]$ iff adversary is oblivious, i.e., the adversary fixes his/her strategy for all rounds, and the strategy is independent of agents’ choices [Lattimore and Szepesvári 2021], we have that $\mathbb{E}[R_T] = \Omega(\mathbb{E}[C] + \mathcal{R}_T)$. Thus, from (39) and $\mathbb{E}[R_T] = \Omega(\mathbb{E}[C] + \mathcal{R}_T)$, $\mathbb{E}[R_T] = \Theta(\mathbb{E}[C] + \mathcal{R}_T)$ holds.

From (39) and the fact $\mathcal{R}_T \leq \mathbb{E}[R_T^*]$, it can be seen have that $\mathbb{E}[R_T] \leq \mathbb{E}[R_T^*] + 2\mathbb{E}[C]$. Again, using the result $\mathbb{E}[R_T] = \Theta(\mathbb{E}[C] + \mathbb{E}[R_T^*])$ in Theorem[1] we conclude that $\mathbb{E}[R_T] = \Theta(\mathbb{E}[C] + \mathbb{E}[R_T^*])$.

\[\Box\]

G.2 Discussion of regret in Zimmert and Seldin [2021] and Masoudian and Seldin [2021]

In [Zimmert and Seldin 2021] and [Masoudian and Seldin 2021], authors give the upper bound of $\mathbb{E}[R_T^*]$. We here convert $\mathbb{E}[R_T^*]$ to $\mathbb{E}[R_T]$. From Corollary 8 in Zimmert and Seldin [2021] and (39) in Theorem[3] $\mathbb{E}[R_T]$ with $V = 1$ is bounded by

$$\mathbb{E}[R_T] \leq 2 \left( \sum_{i \neq i^*} \frac{\log T}{\Delta_i} + \sqrt{\sum_{i \neq i^*} \frac{\log T}{\Delta_i} \mathbb{E}[C] + 2\mathbb{E}[C]} \right).$$

Note that the expectation over $C$ cannot be dropped, as the corruption $C$ is a random variable that depends on the randomization of stochastic rewards and the choices of agents. By a similar method, the $\mathbb{E}[R_T]$ of Masoudian and Seldin [2021] is:

$$\mathbb{E}[R_T] \leq \left( \sum_{i \neq i^*} \frac{1}{\Delta_i} \log \left( \frac{KT}{\sum_{i \neq i^*} \Delta_i} \right) + \sqrt{\sum_{i \neq i^*} \frac{1}{\Delta_i} \log \left( \frac{KT}{\sum_{i \neq i^*} \Delta_i} \right)} \mathbb{E}[C] \right) \cdot \mathbb{E}[C] + 2\mathbb{E}[C].$$

As mentioned in Masoudian and Seldin [2021], for $C = \Theta(\frac{KT}{\sum_{i \neq i^*} \Delta_i})$, adversarial pseudo-regret $R_T$ is improved by a multiplicative factor of $\sqrt{\frac{\log T}{\log \log T}}$. However, this implication does not hold for
$E[R_T]$ due to the additive term of corruption is unavoidable (see lower bound in Theorem[1]). When $C = \Theta\left(\frac{TK}{\sum_{i\neq i^*} \frac{\Delta_i}{\Delta_{i^*}}}\right)$, the expected regret grows linearly.