Protected Areas as a Place for Biodiversity Conservation

M D Vasilyev¹, N V Vasilyeva¹, Yu I Trofimtsev¹

¹Institute of mathematics and informatics, North-Eastern Federal University after M.K. Ammosov, Belinski st., 42, Yakutsk, 677016, Russia

E-mail: 1767700@mail.ru

Abstract. The technogenic impact is least felt in specially protected areas whose task is to preserve rare species of animals and plants. Population dynamics is investigated effectively using mathematical models to explore various scenarios of interaction and development of populations, as well as assess the impact of the environment on the spatial-temporal dynamics of biological communities. The paper considers a non-linear model of a protected population, a feature of which is the presence of a bilocal area and the exchange of individuals between its parts. It is a system of nonlinear parabolic partial differential equations that describes the dependence of migration flows on the uneven distribution of individuals on a heterogeneous area with a generalized resource. The influence of the generalized resource on the distribution of the density of parts of a population is investigated using a computational experiment. A specific feature of the model is the presence of a variable cross-diffusion coefficient, which makes it possible to describe the process of excessive concentration of individuals in the most favorable places in terms of resource availability, as a result of which individuals of the population are dispersed in search of a new resource source. The simulation results can be used in the determination of volume of fishing of populations on the unprotected part of the area, the organization of feeding points in a protected part.

1. Introduction
One of the methods of biodiversity conservation in the conditions of industrial development of the regions is the creation of specially protected areas. Population dynamics is adequately describ using mathematical modeling methods [1, 2]. The models are systems of partial differential equations of parabolic type [3, 4, 5, 6, 7, 8], approximable by difference operators [9, 10, 11]. The generalization of parabolic equations system constructed at [12] is considered in this article. Variable cross-diffusion coefficients describing the effect of the protected and unprotected parts of the population on each other are introduced in the system. The goal of the research is to describe the effect of the competitive interaction of population at bilocal areal with resource.

2. Mathematical model
Let the two-dimenstional areal of the population be divided under the influence of certain conditions into two parts. In a mathematical model, without loss of generality, such an area can be represented in a rectangle. The model describing population development in a bilocal habitat defined by generalized resource has the form of a system of non-linear parabolic equations:
\[
\begin{align*}
\frac{\partial u}{\partial t} &= \nabla (\varepsilon_1(u,v)u - p_1(u)v - q_1u\nabla r) + f_1(u,v), \\
\frac{\partial v}{\partial t} &= \nabla (\varepsilon_2(u,v)v - p_2(v)u - q_2v\nabla r) + f_2(u,v).
\end{align*}
\] (1)

Here \(u(x,y,t)\), \(v(x,y,t)\) population densities, divided into two - non-secured and secured, respectively. Migrational flows are defined by diffusional coefficients \(\varepsilon_1(u,v)\), \(\varepsilon_2(u,v)\), functions of directed migration (cross-diffusion) \(p_1(u)\), \(p_2(v)\) and generalized resource function \(r(x,y)\). The parameters \(q_1\) and \(q_2\) describe the degree of influence of the value of the generalized resource on the density of parts of the population. Density growth of population parts are defined by functions \(f_1\) and \(f_2\):

\[
\begin{align*}
 f_1(u,v) &= m_1u + d_1(v - u) - c_1u^2 - a_1u - b_1, \\
 f_2(u,v) &= m_2v + d_2(u - v) - c_2v^2.
\end{align*}
\] (2)

Here \(m_1\) and \(m_2\) – population growth coefficients; \(d_1\) and \(d_2\) – coefficients of specimen exchange between secured and non-secured part of the habitat; \(c_1\), \(c_2\) – coefficients of internal competition of population. Addend \(a_1u\) is interpreted as a measure of the supervised population impact, addend \(b_1\) is a measure of unlimited industrial impact. Coefficients \(m_1\), \(m_2\) – plus-minus signs or zeroes, other coefficients are non-negative.

Selection of parameter values for the growth functions \(f_1\) and \(f_2\), which involve stable stationary states, forces occurrence of stable stationary distributions as two-dimensional structures, inside the model. This allows for the description of steady scenarios of the population development under the condition of anthropogenic influence. Case of bifurcations was also examined.

Diffusional coefficients \(\varepsilon_1(u,v)\) and \(\varepsilon_2(u,v)\) are positive, and depend on population densities:

\[\varepsilon_i(u,v) = \alpha_{i1} + \alpha_{i2}uv, i = 1,2.\]

Here \(\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}\) are presented as matrices of second derivatives with non-negative elements.

On boundaries of habitat \(\Omega = [0,l_1] \times [0,l_2]\) conditions of Dirichlet and Neumann have been set. System (1)-(2) has following initial values:

\[u(x,y,0) = u_0(x,y), \quad v(x,y,0) = v_0(x,y).\]

Arbitrary, initial distributions of population densities were set as Gaussian distributions. Of course, other functions may also serve as initial values.

Numerical solution of nonlinear differential equations with given boundary conditions can be constructed using variational or grid methods. Various methods for approximating a continuous grid function and discretization of differential operators are used, with a subsequent reduction of the solution of the boundary value problem for differential equations to the solution of a system of nonlinear equations. Onwards, the computational implementation of problem and visualization of the results has been developed in Python.

3. Results

System (1)-(2) depicts dynamics of the reserved population in bilocal habitat with generalized resource, in the case of non-zero density distributions \(u(x,y,t)\) and \(v(x,y,t)\) are formed:

\[
\begin{align*}
\alpha_{11} &= \alpha_{21} = 0.03, \quad \alpha_{12} = \alpha_{22} = 0.01, \quad q_1 = q_2 = 0.03, \\
m_1 &= 0.95, \quad m_2 = 1.2, \quad d_1 = 0.7, \quad d_2 = 0.5, \quad c_1 = 0.25, \quad c_2 = 0.65, \quad a_1 = 0.55, \quad b_1 = 0.1
\end{align*}
\] (3)
The habitat in question can possess a few localized zones abundant with resources, or one narrow resource zone that encompasses only one part of the habitat.

The functions describing cross-diffusion in both parts of the population are introduced into the system (1). Variable cross-diffusion coefficients allow you to simulate the effects of creating herds of individuals of the population. Initially, individuals of the population concentrate in the locations of the greatest resource value, reaching the maximum possible volume in this part of bilocal areal. Then, the effect of overfilling the ecological niche is triggered and the distribution of individuals begins in search of a less competitive environment. Further, with a decrease in the population density in the most favorable places in terms of the amount of the resource, the process of concentration and divergence of individuals repeats. During the breeding season, it is also possible to creating herds without the influence of a generalized resource, which then disintegrate.

Figure 1 shows initial distributions of densities \( u(x, y, t) \) and \( v(x, y, t) \) in the case of different initial values \( u_0(x, y) \) and \( v_0(x, y) \) and parameters of system (1)-(2). Arbitrary, initial distributions of population densities were set as two-dimensional Gaussian distributions. Of course, other functions may also serve as initial values. On boundaries of habitat conditions of Dirichlet and Neumann have been set.

![Figure 1](image.png)

**Figure 1.** Initial density distributions \( u \) and \( v \) under of two localized abundant zone with extremum at secured territory.

Under conditions of parameters (3) and set boundaries conditions the resource function \( r(x, y) \) - defines two prosperous zones, its influence on a migrations of both part population is investigated. Figure 2 shows the resource function on bilocal areal, which defined as a surface with extremum at secured territory.
Figure 2. Resource function $r(x, y)$.

Figure 3 shows a case of influence cross-diffusion and resource function defining two abundant zones. Calculations show, that up to $t = 3$ under positive influence of cross-diffusion parameters values a both part of population strives to completely fill the prosperous zones and have direct migration toward each other.

Figure 3. Influence of cross-diffusion and resource functions for density distributions $u$ and $v$, $t = 3$.

Figure 4 shows intermediate distributions ($t = 5$) under influence of cross-diffusion function negative values. In this way, the positive value of the cross-diffusion parameter means the directed migration of one part of the population towards the highest concentration of the other part, and vice versa, a negative sign of the cross-diffusion parameter of one part of the population means a directed migration to the smallest concentration of the other part. Also, in this case, profile of distribution is identical to the profile of resource function $r(x, y)$. 
Figure 4. Influence of cross-diffusion and resource functions for density distributions $u$ and $v$, $t = 5$.

Figure 5 shows a final distributions $u(x, y, t)$ and $v(x, y, t)$ under the conditions of resource function $r(x, y)$ in the case of variable cross-diffusion parameters. Distribution of population in habitat is influenced by an introduced resource function.

For comparison, under the same conditions (3), the case when the values of the cross-diffusion parameter are positive constants is considered (figure 6). Here, from the beginning of the migration process, both parts of the population fill the adjacent abundance zone. The final distributions $u(x, y, t)$, $v(x, y, t)$ possess more distinctive extremums.

Figure 5. Influence of cross-diffusion and resource functions for final density distributions $u$ and $v$, $t = 12$.

4. Conclusion
The assignment of cross-diffusion coefficients in the form of functions allows us to describe some features of the location of the population in the area.

Generalized resource function has a distinctive influence over final density distributions of both parts of the population. As a result, despite the competitive process in bilocal areal, population density profiles repeat the profile of the generalized resource. Generalized resource in the model of secured population has a positive effect on survival of a population part that exists under industrial anthropogenic impact.
Figure 6. Final density distributions $u$ and $v$ under influence of two localized abundant zone and of positive constant values of cross-diffusion parameters, $t = 12$.

5. References

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