Multiple Particle Scattering in Quantum Point Contacts

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Recent experiments performed on weakly pinched quantum point contacts, have shown a resistance that tend to decrease at low source drain voltage. We show that enhanced Coulomb interactions, prompt by the presence of the point contact, may lead to anomalously large multiple-particle scattering at finite bias voltage. These processes tend to decrease at low voltage, and thus may account for the observed reduction of the resistance. We concentrate on the case of a normal point contact, and model it by a spinfull interacting Tomonaga-Luttinger liquid, with a single impurity, connected to non interacting leads. We find that sufficiently strong Coulomb interactions enhance two-electron scattering, so as these dominate the conductance. Our calculation shows that the effective charge, probed by the shot noise of such a system, approaches a value proportional to $e^* = 2e$ at sufficiently large backscattering current. This distinctive hallmark may be tested experimentally. We discuss possible applications of this model to experiments conducted on Hall bars.

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Introduction.-In the past decades, one dimensional (1D) systems have drawn extensive experimental and theoretical work. These studies have demonstrated the profound effects of interactions in low-dimensional systems. The theoretical model commonly used to describe 1D systems is the Tomonaga-Luttinger liquid (TLL). One way to experimentally examine the behavior of the 1D system, is to introduce backscattering, e.g., due to confinement by a quantum point contact (QPC). Within the framework of the TLL model, a single impurity scatterer reduces the conductance, as the energy scale $\Theta = \max(T, eV)$, decreases. Here $T$ denotes the temperature, and $V$ is the bias voltage. The reduced conductance displays the fact that reflected particles at the QPC modulate the local electron density, and enhance the reflection of other electrons. In the case of a finite size wire, the reduction of the conductance is cutoff by the interacting length, and does not go to zero as $\Theta \to 0$. Instead, at low $\Theta$ corresponding to large distances, the interactions induced by the QPC are fairly screened, and the dependence of the conductance on $\Theta$ reflects the state of the bulk system, away from the QPC.

While some experiments support the TLL model, recent transport measurements appear to be inconsistent with its predictions. Within the interacting TLL, with non interacting leads, the conductance of a system in which the basic excitations are electrons, e.g., a normal QPC or a Hall bar at filling $\nu = 1$, is expected to be independent of $\Theta$ as $\Theta \to 0$. Similarly, for a system in the fractional Hall regime, $\nu = \frac{1}{3}$, the conductance is expected to decrease as $\Theta \to 0$. These theoretical predictions are at odds with measurements performed on long QPCs, and Hall bars at integer and fractional fillings, where the conductance is shown to increase as $\Theta \to 0$.

At low $\Theta$, the state of the system away from the point contact, where equilibration occurs, affects the conductance and determines the allowed scattering processes. For example, if at the place where equilibration occurs, the system is described as a spinfull non interacting TLL, the basic excitations are electrons. Consequently, at low energies, only electrons can be reflected or transmitted across the constriction. In the same way, at $\nu = 1/3$, only quasi-particles can hop from one edge to the other, at low energies. This symmetry argument does not forbid, however, multiple particle scattering processes, in which an integer number of particles are simultaneously scattered back.

We focus our study on a normal QPC, and model it by an interacting spinfull TLL, with non interacting leads. We will show that the presence of strong or long ranged Coulomb interactions generate two-electron scattering, which are relevant (in the RG sense), and may dominate the conductance at high bias $eV > T_L$. Here $T_L$ is the energy associated with the interacting length. The physical origin of the enhanced two-electron scattering in long QPC’s, may be deduced from extremely low densities, when the electrons at the QPC form a periodic structure called a Wigner crystal (WC). The reciprocal lattice spacing of the WC appears to be $K = 4k_F = 2n$, where $n$ is the density, and thus it induces the backscattering of two electrons, simultaneously. At intermediate densities a similar density modulation appears, which increases the scattering of two electrons relative to the scattering of one. At low bias $eV < T_L$, as interactions are screened, these processes decrease as the bias is lowered, while the single electron process remain unchanged. As a result, the conductance is increased as the bias is reduced (Fig. 1), in agreement with the experimental results.

We will show below that the two-electron scattering has a distinctive signature on the shot-noise of the system. As two-electron scattering processes dominate the conductance, the shot-noise reveals current fluctuations with a charge that approaches $2e$, see Fig. 2. This model can be applied to other 1D systems, e.g., a Hall bar at integer and fractional filling, and may account for the observed conductance behavior.
(spin degenerate) mode crosses the constriction, yet sufficiently far from complete depletion, such that the TLL model is applicable. We divide the constriction created by the QPC, into two distinct regions. The section near the QPC is populated by a single 1D mode, and is thus characterized by relatively low electronic density. Hence, it is modeled by an interacting TLL. The two segments at the broadening of the constriction are populated by a finite number of 1D modes, such that screening is efficient. These sections are modeled by non interacting TLLs. The varying transversal width of the constriction generates a potential modulation in the middle section, which may give rise to single electron scattering. Since we are interested in the low energy properties of the system, the precise form of the potential is not essential. Therefore, we model this scattering potential as a local impurity scatterer.

The bosonic action of the TLL with single electron scattering is given by $S_0 + S_1$, where

$$S_0 = \frac{1}{\pi} \int \frac{d\omega}{2\pi} \left[ \log \left( \frac{\omega}{K_\rho(\omega)} \right) |\phi_\rho(\omega)|^2 + |\omega| |\phi_\sigma(\omega)|^2 \right],$$

$$S_1 = -\lambda_1 \int d\tau \epsilon_F \cos \sqrt{2}\phi_\rho(0, \tau) \cos \sqrt{2}\phi_\sigma(0, \tau),$$

where $\phi_\rho$ and $\phi_\sigma$ are related to the charge and spin densities via $\rho(x) = -\sqrt{2/\pi} \partial_x \phi_\rho$, $\sigma_z(x) = -\sqrt{2/\pi} \partial_x \phi_\rho$, $\lambda_1$ is the dimensionless strength of the single-electron scattering potential, and $\epsilon_F$ is the Fermi energy (which serves as the cutoff energy), defined as the energy difference between the chemical potential and the bottom of the band, in the 1D section. It is therefore set by the local gate voltage. The interaction parameter $K_\rho(\omega)$ is

$$K_\rho(\omega) = 2 |\omega| \int \frac{dq}{2\pi} K_\rho(q) u_\rho(q)^2 q^2,$$

with $u_\rho(q) = (v_F + g_4(q)/\pi^2 - g_2(q)/\pi^2)$ and $K_\rho(q) = 2 \sqrt{\pi v_F + g_4(q)/\pi^2 - 2 g_2(q)/\pi v_F + g_4(q)/\pi^2}. \,$

Here $g_4(q)$ and $g_2(q)$ are the Fourier transforms of the Coulomb interaction between charge density of the same and opposite chirality, respectively (screening due to external metallic gates should be taken into account). The Coulomb interaction is taken to be $V(x) = e^2/(\epsilon_r \sqrt{x^2 + d^2}) e^{-\sqrt{x^2 + d^2}/\alpha}$, where $\alpha$ is the screening length, $\epsilon_r$ is the dielectric constant, and the short distance singularity is cut off by the finite width of the wire $d$. This has a Fourier transform $V(q) = 2 \pi \epsilon_r e^{2}/(\epsilon_r \sqrt{x^2 + d^2}) e^{-\sqrt{x^2 + d^2}/\alpha}$.

$$-2e^2/\epsilon_r \log [d \sqrt{q^2 + (1/\alpha)^2}].$$

Using Eq. (2), with $g_2(q) = g_4(q) = V(q)$ one obtains

$$K_\rho(\omega) \approx 1/\hat{r}_s \sqrt{2} \log \left[ (v_F \hat{r}_s) / \left( d \sqrt{\omega^2 + \omega_{SC}^2} \right) \right],$$

where $\hat{r}_s = \sqrt{4e^2/(\epsilon_r \pi v_F)}$, and $\omega_{SC} = \sqrt{v_F \hat{r}_s}/\alpha$.

In the presence of Coulomb repulsion, a reflected electron at the QPC may cause additional electrons to be simultaneously reflected. An RG analysis shows that the two-electron scattering is the principal process of such multiple-electron scattering. Long range interaction further enhances these processes, due to the formation of a periodic density modulation. We thus introduce the two-electron scattering

$$S_2 = -\lambda_2 \sum_s \int d\tau (\pi/k_F)^2 \epsilon_F \times \left\{ \begin{array}{l} R_s^0(0, \tau) R_{1s}(0, \tau) L_s(0, \tau) L_{s}(0, \tau) + h.c. \end{array} \right\} = -\lambda_2 \epsilon_F \int d\tau \cos 2\emptyset_{\rho}(0, \tau).$$

Using perturbation theory in the Coulomb interaction, one obtains a lower bound assessment for the bare two-electron scattering

$$\lambda_2^0 = \gamma \left( \lambda_1^0 \right)^2 \left[ e^2/(\epsilon_r \hbar v_F) \right] \log (\lambda_F/d),$$

where $\gamma$ is of order 1. The factor $\log (\lambda_F/d)$ is due to the long range nature of the Coulomb interaction, and may be modified where short range interactions are considered. Eq. (4) shows that the two-electron scattering process involves a pair of spin up and spin down electrons. Therefore as the degeneracy is lifted, in the presence of a magnetic field, these scattering events will become scarce, and the conductance resumes its spin polarized non interacting value, $G = (1/2) G_0$. $G_0 = 2e^2/h$, in agreement with the experimental observations.

The scaling equations for the single-electron $\lambda_1$, and the two-electron $\lambda_2$ scattering processes are

$$d\lambda_1/d(-\log \Lambda) = \lambda_1 \left[ 1 - (1 + K_\rho(\Lambda))/2 \right],$$

$$d\lambda_2/d(-\log \Lambda) = \lambda_2 \left[ 1 - 2K_\rho(\Lambda) \right].$$

where $\Lambda$ is the running scale. When short range Coulomb interactions are considered, i.e., $\omega < \omega_{SC}$, the interaction parameter $K_\rho$ is $\Lambda$ independent. Eq. (6) demonstrates that in the presence of strong short range interactions, namely for $K_\rho < 1$, the two-electron scattering renormalizes faster than the single electron process. This tendency is enhanced in the presence of long range Coulomb interactions, i.e. $\omega > \omega_{SC}$, as can be seen by solving Eq. (6) in the proper limit.

**Conductance.** To find corrections to the conductance due to one- and two-electron scattering, we calculate the backscattering current at different energy scales. Those are the cutoff energy $\epsilon_F = v_F/\lambda_F$, the energy associated with the interacting length $L_L = u_F/L$, and the energy scale governed by the experiment $\Theta$. We solve the flow equations for two distinct regions. At high energies $T_L < \Lambda < \epsilon_F$, the relevant distances are shorter than the interacting length $L_L$, and the two scattering processes, $\lambda_1$, and $\lambda_2$ scale as Eq. (6), with $K_\rho(\Lambda)$ given by Eq. (3).

For $T_L < \Theta$, this behavior is carried down to $\Lambda < \Theta$. However, if $\Theta < T_L$ there are energy scales for which the relevant distances are longer than $L$, $\Theta < \Lambda < T_L$. At these energies the interactions are absent, $K_\rho = 1$, and
the single-electron scattering becomes marginal, while the two-electron scattering is irrelevant [see Eq. (5)].

At \( \Theta < T_L \), the backscattering current for both short and long range interactions is

\[
\langle I_B(t) \rangle = G_1 V + G_2 V \left[ (\pi T / T_L)^2 + (e V / T_L)^2 \right],
\]

where \( G_1 \propto G_0 \lambda_1^2 \) and \( G_2 \propto G_0 \lambda_2^2 \), and \( \lambda_1 \) and \( \lambda_2 \) are the renormalized parameters for one- and two-electron scattering, respectively. For short range interaction \( \lambda_1 \) and \( \lambda_2 \) are found using Eqs. (6) and (9), with \( \omega < \omega_{SC} \approx \epsilon_F \)

\[
\lambda_1 = \lambda_1^0 (T_L / \epsilon_F)^{2 K_\rho - 1}; \lambda_2 = \lambda_2^0 (T_L / \epsilon_F)^{2 K_\rho - 1}.
\]

For long range interactions \( \lambda_1 \) and \( \lambda_2 \) are found using Eqs. (6) and (6), with \( \omega > \omega_{SC} \approx T_L \).

\[
\lambda_1 = \lambda_1^0 (T_L / \epsilon_F)^{2 K_\rho - 1}; \lambda_2 = \lambda_2^0 (T_L / \epsilon_F)^{2 K_\rho - 1}.
\]

Notice that Eqs. (5) and (6) correspond to the two extreme limits \( \omega_{SC} \approx \epsilon_F \) and \( \omega_{SC} \approx T_L \), respectively. The intermediate case \( T_L < \omega_{SC} < \epsilon_F \) (not considered here), interpolates between the two.

When short range interactions are considered, one can obtain an analytic expression for both \( \Theta > T_L \) and \( \Theta < T_L \). Following Ref. 18, the backscattering current can be shown to have two independent contributions \( \langle I_B(t) \rangle = \langle I_{B_1}(t) \rangle + \langle I_{B_2}(t) \rangle \), where (with \( \beta = 1 / k_B T \))

\[
\langle I_{B_1}(t) \rangle = G_0 V \left( \frac{\lambda_1^0}{4} \right)^2 f_{K_\rho,1} \left( \frac{1}{\beta \epsilon_F} \epsilon V / 2h \right)
\]

\[
\langle I_{B_2}(t) \rangle = G_0 V \left( \frac{\lambda_2^0}{4} \right)^2 f_{K_\rho,2} \left( \frac{1}{\beta \epsilon_F} \epsilon V / 2h \right),
\]

\[
f_{\nu}(x, y) = \frac{2^{\nu-1} \pi^{\nu-1} x^{\nu-1}}{y^\nu \Gamma(\nu)} \sinh(y) \left[ \frac{\nu}{2} + \frac{i y}{\pi} \right] \right|^2 .
\]

In Fig. 1 we present the conductance \( G = I / V \) in the presence of both one- and two-electron scattering, and short range Coulomb interactions as a function of bias voltage, for different cutoff energies. The conductance is shown to increase as the bias voltage decreases. The kinks in Fig. 1 are due to the abrupt change in the parameter \( K_\rho \), taken to be \( K_\rho < 1 \) in the vicinity of the QPC, and \( K_\rho = 1 \) at the broadening of the constriction. In an experimental realization, the transition between the two is smooth, and the kinks will be softened. A study of Eq. (9) shows that for \( T \ll \epsilon V \) and \( K_\rho < 1 / 4 \), the contribution of the two-electron scattering to the differential conductance may change sign. As a result the differential conductance at \( eV > T_L \) may exceed the universal value \( 2 e^2 / h \), and the curvature at the high bias regime changes sign (not shown). From Eq. (7) it follows that at \( \Theta < T_L \), the backscattering current is dominated by single-electron scattering, \( \langle I_B(t) \rangle \approx \langle I_{B_1}(t) \rangle \). Conversely at \( \Theta \lesssim T_L \), for an interacting system that exhibits an enhanced two-electron scattering, the current is given by \( \langle I_B(t) \rangle \approx \langle I_{B_2}(t) \rangle \). At \( \epsilon V \), the crossover voltage

\[
V^* = \sqrt{G_1 G_2 / \pi^2 T_L^2}.
\]

Noise.-The occurrence of multi-particle scattering is accompanied by a distinctive signature on the shot noise of the system. Shot noise measures the size of charge-transfer events. As multi-particle scattering becomes frequent, the events that determine the shot noise consist of a charge which is larger than the single particle charge.

The expression for the symmetrized backscattering current correlator is found to be

\[
\langle \delta I_B^2 \rangle_{\omega = 0} = \frac{2 e |\langle I_{B_1} \rangle|}{\tanh(\beta \epsilon V / 2h)} + \frac{2 e |\langle I_{B_2} \rangle|}{\tanh(\beta \epsilon V / 2h)}. \tag{11}
\]

The noise behavior, depicted in Fig. 2, is affected by the competition between the single and two-electron scattering events. At small \( V \ll V^* \) corresponding to small backscattering current, two-electron scattering are scarce. Thus, the charge defined as the ratio \( S / (2 |\langle I_B \rangle|) \) is \( e^* = e \). At \( V > V^* \), corresponding to large backscattering current, two-electron scattering dominant the conductance, and the charge approaches \( e^* \approx 2 e \).

While other existing models, e.g., the Kondo model, can account for increasing conductance at small SD bias, the doubling of the charge is a unique feature of the model presented here. Moreover, the two models have different ranges of validity. The use of the Kondo model is justified at very low densities, where a single electron is believed to be localized at the point contact. Conversely the model presented here is valid at densities high enough to consider the TLL model. Thus, noise measurements performed at different ranges of parameters, can be used to distinguish the two behaviors.

Conclusion.- We have studied the spinfull TLL model with one- and two-electron scattering. Under proper conditions, the two-electron scattering processes dominate the conductance of the system. We predict that at this limit, the shot noise will approach \( e^* = 2 e \), and suggest noise measurements as a way to substantiate the validity.
FIG. 2: $S(\omega = 0) = \langle \delta I^2 \rangle_{\omega=0}$, as a function of $I_B$ (solid line), calculated using Ref. 24 with $\langle \delta I^2 \rangle_{\omega=0}$ given by Eq. 24 at $eV < T_L$. Auxiliary plots represent the noise calculated with $\langle \delta I^2 \rangle_{\omega=0} = \frac{2 e^2 \epsilon}{\cosh(eV/2k_B)}$, where $\epsilon = e$ (dotted line), and $\epsilon = 2e$ (dashed line). $I^*$ is given by Eq. 4 with $V = V^*$. We have used the parameters of Fig. 3 with $\epsilon_F = 7K$. Inset: Temperature dependence of the total noise, at $T$ from 80mK to 980mK in 100mK steps (thermal noise was subtracted).

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of our model.

The interacting TLL with a single impurity scatterer, describes a large class of systems at the low energy limit. An example of which is the Hall bar contracted by means of a QPC. Within the TLL model, the backscattering of electrons at $\nu = 1$, and of quasi particles at $\nu = \frac{1}{2}$ cannot account for the measured enhancement in the conductance at low bias voltage. Conversely, multiple particle backscattering prompt by the reduced density near the QPC, become scarce as the bias is lowered, leading to an enhanced conductance, concurring with the transport measurements. In a system dominated by n-particle scattering at high energies, we expect that the shot noise, at large backscattering current, will approach $e^* = ne$. For example, at $\nu = 1/3$, an enhanced three quasi-particle scattering, will result in an effective charge, that approaches $e^* = 3ne = e$, at large backscattering current. The challenging task remains to find a microscopic model that predicts such an enhanced multiple particle scattering in the quantum Hall system.

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The relation between the zero frequency spectral density of the noise of the transmitted current, \( \langle \delta I_T^2 \rangle_{\omega=0} \), and of the backscattering one, \( \langle \delta I_B^2 \rangle_{\omega=0} \), is given by \( S(\omega = 0) = \langle \delta I_T^2 \rangle_{\omega=0} = 4T G_0 - 8T \partial V \langle I_B \rangle + \langle \delta I_B^2 \rangle_{\omega=0} \).

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