Turbulence, instabilities and passive scalars in rotating channel flow

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Abstract. Fully developed channel flow with a passive scalar rotating about the spanwise axis is studied by direct numerical simulations. The Reynolds number based on the bulk mean velocity $Re_b$ is up to 30000, substantially higher than in previous studies, and the rotation rates cover a broad range. Turbulence on the stable channel side is less strongly damped at moderate rotation rates than in channel flow at lower $Re_b$. At high rotation rates and sufficiently high $Re_b$, intermittent strong instabilities occur on the stable side caused by rapidly growing modes resembling two-dimensional Tollmien-Schlichting waves which at some instant become unstable and break down into intense turbulence. The turbulence decays and after some time the waves form again and the process is repeated in a cyclic manner.

Rotation also strongly affects the mean passive scalar profiles and turbulent scalar fluxes. Large scalar fluctuations are observed on the border between the stable and unstable channel sides. While in non-rotating channel flow the turbulent Prandtl number of the passive scalar is about one like in other shear flows, it is much smaller in the rotating cases.

1. Introduction

Rotation has a pronounced influence on turbulent shear flows and can either damp or amplify turbulence as shown by direct numerical simulations (DNS) and linear rapid distortion theory (Brethouwer 2005). Many studies of wall-bounded flows subject to system rotation have been conducted because of their practical importance and especially plane channel flow with spanwise rotation has received considerable attention. In this case, there is no direct influence on the mean flow, i.e. laminar Poiseuille flow is unaffected by rotation. However, a key finding of the experimental study by Johnston et al (1972) was that moderate spanwise rotation amplifies and damps turbulent stresses on the so-called unstable and stable channel side, respectively. As a result, the mean velocity and other profiles become asymmetric. Those findings were later corroborated by direct numerical simulations (DNS) of Kristoffersen & Andersson (1993) at $Re_r = u_r h / \nu = 194$, where $u_r$ is the friction velocity, $h$ the channel half width and $\nu$ the kinematic viscosity. They also found that the mean velocity profile is linear in the core of the channel with a slope equal to $dU/dy \simeq 2\Omega$ implying that the absolute mean vorticity is zero.

The effect of rapid spanwise rotation of turbulent channel flow was recently investigated by Grundestam et al (2008) through DNS at $Re_r = 180$ and rotation numbers $Ro_h = 2\Omega h / U_b$ up to 2.49, where $\Omega$ is the imposed spanwise system rotation. They observed that at high rotation rates the turbulent stresses were small on both sides and the mean velocity approached the laminar Poiseuille profile. An inviscid linear stability analysis by Wallin et al (2011) showed...
that all oblique modes, i.e. modes with spanwise wave number $k_z \neq 0$, are damped if $Ro_h \geq 3.0$. If viscous effects are included the linear analysis showed all oblique modes to be stable above a critical $Ro_h$ slightly less than 3.0. The analysis was confirmed by DNS showing that above the critical $Ro_h$ a perturbed flow relaminarizes. However, modes with $k_z = 0$, the so-called Tollmien-Schlichting waves, are not suppressed by spanwise rotation. DNS of channel flow with $Ro_h$ slightly above the critical value showed that these modes can grow if the computational domain is large enough.

DNS of homogeneous shear flow (Brethouwer 2005) and spanwise rotating channel flow at $Re_\tau = 150$ (Nagano & Hattori 2003) and 194 (Liu & Lu 2007) have also shown that rotation strongly influences the rate and direction of scalar transport as well as scalar fluctuations. This is to be expected since the mixing process is tightly coupled to the turbulence.

Our objective is to study fully developed spanwise rotating channel flow with a passive scalar through DNS at higher $Re_b$ than in previous studies and for a broad range of $Ro_b$. The intention is to clarify the effect of viscosity on the stabilization of the flow by rotation and the effect of rapid rotation on scalar transport. Furthermore, we want to examine the parameter regime for which instabilities can occur. Besides, DNS data at higher $Re_b$ are valuable for turbulence modelling both in the RANS context and for subgrid-scale modelling in LES.

2. Numerical methodology

The incompressible Navier-Stokes equations and the advection-diffusion equation for a passive scalar are solved with pseudo-spectral code with Fourier expansions in the homogeneous streamwise $x$- and spanwise $z$-direction, and Chebyshev polynomials in the wall-normal $y$-direction. Constant scalar values of 0 and 1 are imposed at the two walls and the Prandtl number is 0.71. The computational domain size is $8\pi h \times 3\pi h$ in the $x$- and $z$-direction respectively in all cases and the spatial resolution is similar as in previous channel flow DNS. In all runs the flow rate was kept constant. Many DNS have been performed with $Re_b$ from 3000 up to 30000 and a wide range of $Ro_b$. The parameters of the DNS at the highest $Re_b$ are listed in table 1.

Table 1. DNS parameters: $N_x$, $N_y$ and $N_z$ are the number of modes in the streamwise, wall-normal and spanwise direction, respectively. $Re^u_\tau$ and $Re^s_\tau$ are based on the friction velocity on the unstable, $u^u_\tau$, and stable side, $u^s_\tau$, of the channel, respectively. $Re_\tau$ is based on $u_\tau = [(u^u_\tau)^2/2 + (u^s_\tau)^2/2]^{1/2}$.

| $Ro_b$ | $Re_b$ | $Re_\tau$ | $Re^u_\tau$ | $Re^s_\tau$ | $N_x \times N_y \times N_z$ |
|-------|--------|-----------|-------------|-------------|------------------|
| 0     | 20000  | 1000      | 1000        | 1000        | 2560 \times 385 \times 1920 |
| 0.15  | 20000  | 975       | 1108        | 821         | 2304 \times 385 \times 1728 |
| 0.45  | 20000  | 800       | 963         | 595         | 2048 \times 361 \times 1536 |
| 0.9   | 20000  | 550       | 679         | 379         | 1536 \times 257 \times 1152 |
| 1.2   | 20000  | 426       | 503         | 331         | 1152 \times 217 \times 864  |
| 1.5   | 30000  | 414       | 461         | 361         | 1024 \times 193 \times 768  |
| 2.1   | 30000  | 342       | 350         | 333         | 864 \times 193 \times 640   |
| 2.4   | 30000  | 337       | 355         | 319         | 640 \times 193 \times 512   |

3. Results

In this section, we report some results of the DNS. The statistics are not in all cases completely converged, nevertheless they show some clear trends.
3.1. Flow statistics

Figure 1 shows the mean streamwise velocity profiles scaled with $U_b$ at $Re_b = 20000$ and 30000 and various $Ro_b$. At $Ro_b = 0.9$ and 1.5 the profiles have an asymmetric shape due to the rotation. We also see an extended region in the core of the channel where the velocity profile is linear with $dU/dy \approx 2\Omega$, meaning that the sum of the imposed and mean flow vorticity is zero (Kristoffersen & Andersson 1993). At $Ro_b = 2.4$ the mean velocity approaches the parabolic profile of laminar Poiseuille flow. This has also been observed in the DNS by Grundestam et al (2008) at rapid rotation rates and indicates that the Reynolds stresses are strongly damped by rotation.

Figure 2 shows the effect of rotation on the root-mean-square (rms) of the streamwise and wall-normal velocity fluctuations scaled with $u_\tau$. Here, $u_\tau = [(u_x^2) / 2 + (u_z^2) / 2]^{1/2}$ with $u_x$ and $u_z$ the mean friction velocities on the unstable and stable channel side, respectively. We can observe the amplification of the turbulence by rotation on the unstable side for $Ro_b = 0.45$ and 1.5; in particular the wall-normal fluctuations are much more intense. However, at $Ro_b = 2.4$ the streamwise velocity fluctuations are much smaller than in nonrotating channel flow. Thus,
very rapid rotation also damps turbulence on the ‘unstable’ side. The turbulence on the stable side is in all cases damped by rotation. Especially the wall-normal fluctuations are very small for \( \text{Ro}_b \geq 1.5 \) indicating laminarization of the flow.

The effect of rotation on the normalized production of turbulent kinetic energy \( \mathcal{P}^+ \) at \( \text{Re}_b = 10000 \) is shown in figure 3. Here we see a very clear amplification of \( \mathcal{P}^+ \) on the unstable side and damping of \( \mathcal{P}^+ \) on the stable side when \( \text{Ro}_b \) increases. The situation on the unstable side can be analysed by noting that for \( 1 - y/h \ll 1 \) we have

\[
\frac{dU^+}{dy^+} - \left( \frac{\langle uv \rangle}{u^+_r} \right) = \left( \frac{u^+_r}{u^+_r} \right)^2 \Rightarrow \mathcal{P}^+ \approx \left( \frac{u^+_r}{u^+_r} \right)^2 \frac{dU^+}{dy^+} - \left( \frac{dU^+}{dy^+} \right)^2,
\]

which means that \( \mathcal{P}^+_{\text{max}} \approx (u^+_r)^4/4 \) for a position where \( dU^+/dy^+ = (u^+_r/u^+_r)^2/2 \). This means that the maximum normalised production goes form 0.25 for the nonrotating case to about 0.56 for \( \text{Ro}_b = 0.45 \) and 0.9 (where \( u^+_r/u^+_r \approx 1.22 \)) and should decrease thereafter. This behaviour is confirmed by figure 3a. The near-wall peak of \( \mathcal{P}^+ \) on the stable side completely disappears for \( \text{Ro}_b \geq 0.9 \). Instead, there is a region where \( \mathcal{P}^+ \) is negative implying an energy transfer from the turbulence to the mean flow, which is an unusual phenomenon.

Since the turbulence is more intense, the friction velocity \( u^+_f \) on the unstable side will be higher than the friction velocity \( u^+_f \) on the unstable side in rotating channel flow. This is shown in figure 4, plotting \( \text{Re}_r \) based on \( u^+_f \) and \( u^+_f \) respectively for the different \( \text{Re}_b \) and \( \text{Ro}_b \). The differences in \( \text{Re}_r \) on the unstable and stable side are significant for \( \text{Ro}_b = 0.45 \) and 0.9, but becomes less for increasing \( \text{Ro}_b \). At very high \( \text{Ro}_b \), \( \text{Re}_r \) on both sides approaches the value for a laminar Poiseuille flow at the corresponding \( \text{Re}_b \) indicating that the flow almost relaminarizes. These results agree with the findings of Grundestam et al (2008), but our DNS extends these conclusions to a wider range of \( \text{Re}_b \).

The influence of \( \text{Re}_b \) on the flow can be examined in more detail in figure 5 showing the mean velocity profiles and the rms values of the wall-normal velocity fluctuations \( v^+ \) at \( \text{Ro}_b = 0.45 \) and 1.2 and \( \text{Re}_b = 5000, 10000 \) and 20000. The mean velocity profiles at the same \( \text{Ro}_b \) have all a very similar linear part in the core region of the channel where the absolute mean vorticity is about zero, meaning that this phenomenon is robust. The influence of \( \text{Re}_b \) can only be seen in the mean velocity gradient near the wall and in the position where the mean velocity starts to deviate from the linear profile. The intensity of \( v^+ \) on the unstable is very similar at \( \text{Ro}_b = 0.45 \) for the three different \( \text{Re}_b \), but on the stable side it reduces considerably when \( \text{Re}_b \) decreases. In contrast, at \( \text{Ro}_b = 1.2 \), \( v^+ \) is very similar on the stable side in the three DNS. The effect of \( \text{Re}_b \) on the turbulence thus differs for different \( \text{Ro}_b \).
Figure 4. $Re_\tau$ on the unstable and stable channel side at different $Ro_b$ for $Re_b = 20000$ (red symbols and lines), 30000 (blue symbols and lines), 10000 (green symbols and lines), 5000 (brown symbols and lines) and 3000 (magenta symbols and lines). The horizontal dashed lines on the right side of the figure show the $Re_\tau$ of a laminar Poiseuille flow for the corresponding $Re_b$. The $Re_\tau$ on the unstable side is always equal or higher than the one on the stable side.

Figure 5. Mean velocity profiles at $Ro_b = 0.45$ (a) and $Ro_b = 1.2$ (b) and $v^+$-profiles at $Ro_b = 0.45$ (c) and $Ro_b = 1.2$ (d) at various $Re_b$.

The small $v^+$ values on the stable side at $Ro_b = 0.45$ indicate relaminarization of the flow. This relaminarization can indeed be seen in snapshots of the streamwise velocity at $Re_b = 10000$.
in a plane parallel and close to the wall. The flow is fully turbulent on the unstable side, but on

the stable side it is mostly laminar-like with a few turbulent spots. However, in this case slow oscillations in the wall shear stress on the stable side can be observed since the turbulent spots appear to grow and shrink. At higher $Re_b$ the turbulent fraction is larger and the spots merge to form bands while at lower $Re_b$ only very localised disturbances can be seen (not shown here), which agrees with the higher and lower values $v^+$, respectively.

3.2. Scalar statistics

Now we will turn to the influence of rotation on passive scalar transport. Figure 7 shows the mean scalar profiles and rms of the scalar fluctuations at $Re_b = 20000$ and $30000$ and various $Ro_b$. When there is rotation, the mean scalar gradient is much smaller on the unstable side and larger on the stable side of the channel, indicating that the scalar transfer on the stable side becomes less efficient. Besides a near-wall peak on the unstable side, the scalar fluctuation profiles at $Ro_b = 0.45$ and $0.9$ show high intensities at the position where the mean scalar gradient becomes much steeper. This is likely the results of a combination of a large mean scalar gradient and quite intense turbulence on the border between the stable and unstable side of the channel. At $Ro_b = 2.1$ the scalar fluctuations are very intense in a large region. This is probably related to the large-scale instabilities that occur in this case as we will show later.

The influence of rotation on the streamwise and wall-normal turbulent scalar fluxes is shown in figure 8. In the nonrotating channel, the streamwise flux is larger than the wall-normal flux in a large part of the channel like in other turbulent shear flows. Rotation reduces the streamwise

Figure 6. Snapshots of the streamwise velocity fluctuations in the $x - z$ plane on the unstable and stable side at $Re_b = 10000$ and $Ro_b = 0.45$.

Figure 7. (a) Mean scalar profiles and (b) rms of the scalar fluctuations scaled with scalar wall-units $Q_w/u_T$ at $Re_b = 20000$ and $30000$ and various $Ro_b$. Here, $Q_w$ is the mean scalar flux at the wall.
turbulent scalar flux, so the flux is much more aligned with the mean scalar gradient. We can also see that rotation strongly reduces the turbulent scalar flux on the stable side and at $Ro_b = 2.1$ the scalar transport is almost entirely molecular.

A much used assumption is that the turbulent Prandtl number $Pr_T$, i.e. the ratio of the turbulent viscosity and scalar diffusivity, is about one. In figure 9 we show $Pr_T$ obtained from the DNS at various $Ro_b$. In the nonrotating case $Pr_T$ is indeed close to one. However, $Pr_T$ declines significantly when $Ro_b$ increases on the unstable side, except very near the wall. When there is rotation, models for turbulent scalar transport therefore need to be modified.

3.3. Cyclic instabilities

In some of the DNS we observed intense recurring instabilities. For example, figure 10 shows the time series of the plane averaged wall shear stresses on both walls and the rms of the streamwise velocity fluctuations integrated over the whole computational domain at $Re_b = 30000$ and $Ro_b = 1.5$ and 2.4. In the time series at $Ro_b = 1.5$ (figure 10.a) we see recurring sudden increases of the wall shear stress on the stable side and a simultaneous large increase of the streamwise velocity fluctuations. On the other hand, the wall shear stress on the unstable side is approximately constant. In this case, the instabilities appear to be confined to the stable side.

Figure 8. (a) Mean streamwise and (b) wall-normal turbulent scalar fluxes at $Re_b = 20000$ and 30000 and various $Ro_b$ in inner scales.

Figure 9. Turbulent Prandtl number $Pr_T$ on the unstable side at $Re_b = 20000$ and 30000 and various $Ro_b$. 

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Figure 10. Time series of the wall averaged $\tau_w$ on the unstable and stable side and the volume integrated rms of the streamwise velocity fluctuations at (a) $Ro_b = 1.5$ and (b) $Ro_b = 2.4$ and $Re_b = 30000$.

The time series at $Ro_b = 2.4$ (figure 10.b) shows even stronger cyclic instabilities, but in this case the instabilities can also be seen in the time series of the wall shear stress on the unstable side; in fact the instabilities appear stronger on the unstable side than the stable side. Note that period of the instabilities in quite constant and of the order of $10^3 h/U_b$ time scales, which is much longer than any of the turbulent time scales. Similar periodic-like instabilities were also seen in the DNS at $Re_b = 20000$ and $Ro_b = 0.9$ and 1.2, and $Re_b = 30000$ and $Ro_b = 2.1$. At $Re_b = 5000$ and $Re_b = 10000$ we could also observe similar instabilities but only at very rapid rotation rates. In the DNS at $Re_b = 3000$ no large-scale instabilities were observed.

Figure 11 shows the rms of the velocity fluctuations at $Re_b = 20000$ and $Ro_b = 1.2$ during the calm period in between the instabilities and at the moment of the bursting events. We see that in the calm periods the turbulence on the stable side is very weak, while during the burst events the velocity fluctuations are much stronger. On the other hand, the turbulence on the unstable side is virtually unaffected by the bursts. The instabilities in this case are thus confined to the stable side like in the DNS at $Re_b = 30000$ and $Ro_b = 1.5$ (figure 10.a), while at higher $Ro_b$ when the mean velocity profile approaches the laminar Poiseuille profile the instabilities appear to span the whole channel (figure 10.b).
It was pointed out by Wallin et al (2011) that rotation damps the oblique modes at high rotation rates, whereas the two-dimensional plane modes, the so-called Tollmien-Schlichting (TS) waves, are unaffected by rotation. They performed a DNS at $Re_\tau = 180$ and imposed a $Ro_b$ very close to the critical $Ro_b$ giving a complete suppression of all oblique modes. Usually, turbulence inhibits the growth of TS waves. However, in their DNS the turbulence was so weak that the TS waves could grow and reach very high amplitudes. The plane waves then became unstable and broke down into intense turbulence which decayed due to the imposed rotation. After some time the turbulence was weak again and TS waves started growing and the whole process repeated itself in a periodic-like manner. We observed a similar phenomenon in our DNS at high $Ro_b$. For example, figure 12 shows snapshots of the streamwise velocity in a plane close to the wall on the unstable and stable side at $Re_b = 20000$ and $Ro_b = 1.2$ before a bursting event occurs. The flow on the unstable side is obviously fully turbulent, but on the stable side we see the emergence of clear plane waves resembling TS waves with some smaller scale variations due to the remaining turbulence. Those plane waves grow and after some time become unstable and Λ-shaped vortices develop as visualized in figure 13. Then the waves break down leading to intense turbulence which then decays. After some time this process repeats itself as is evidenced by the time series.

At $Re_b = 5000$ this process is only observed very close to the critical $Ro_b$. However, when $Re_b$ is increased till $Re_b = 20000$ we can observe those instabilities at much lower $Ro_b$ far from the critical value when the turbulence on the stable side is still significant. When $Ro_b$ is not too high the instability is confined to the stable side. However, when $Ro_b$ is closer to the critical value the instability seems to span the whole channel as was indicated by figure figure 10.b. This is confirmed by a snapshot of the streamwise velocity for this run, see figure 14. In this case we can clearly see the plane wave on both sides of the channel.

Figure 12. Snapshots of the streamwise velocity fluctuations in an $x-z$ plane on the unstable (left) and stable side (right) at $Re_b = 20000$ and $Ro_b = 1.2$.

Figure 13. $\lambda_2$ visualization of an instability on the stable side at $Re_b = 20000$ and $Ro_b = 1.2$. 
Figure 14. Snapshots of the streamwise velocity fluctuations in the $x - z$ plane on the unstable (left) and stable side (right) at $Re_b = 30000$ and $Ro_b = 2.4$.

4. Conclusions

The DNS show that an imposed spanwise system rotation has a large effect on the transport of a passive scalar in a fully developed turbulent channel flow. Mean scalar profiles, scalar fluctuations and the streamwise and wall-normal turbulent scalar fluxes change significantly under the influence of rotation. The turbulent Prandtl number of the passive scalar is close to one when rotation is absent, but it is much smaller than one when rotation is present.

Since we performed DNS at various $Re_b$, we could examine the influence of the viscosity on the flow field. This appeared to be large. For example, the DNS at higher $Re_b$ revealed a cyclic instability leading to high variations in the wall shear stresses and turbulent intensities for a quite broad range of $Ro_b$. These instabilities appeared to be a result of plane waves which could grow since the turbulence was weakened under the influence of rotation. However, at lower $Re_b$ those instabilities were absent or they only occurred when $Ro_b$ was close to the critical value for which the flow completely relaminarizes.

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