12C and the triple-α reaction rate

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Abstract. We briefly review the role of the triple-α reaction in astrophysics and discuss the uncertainties associated with the determination of the reaction rate. We summarize the results of three recent experimental studies of the breakup of the Hoyle state in 12C into three α particles, and we show how these results eliminate an often overlooked source of uncertainty in the determination of the triple-α reaction rate. Finally, we contemplate whether improved studies of the breakup of the Hoyle state can teach us something about the structure of the Hoyle state.

1. Introduction
The cross section of a nuclear reaction $A + B \rightarrow X + Y + \ldots$ can be enhanced by many orders of magnitude if the compound nucleus, $C = AB$, has a state with the appropriate quantum numbers at an energy corresponding to the combined energy of the reactants, $A$ and $B$; we then speak of resonance in the channel $A + B$. Resonances play a very important role in astrophysics by speeding up nuclear reactions that otherwise would proceed very slowly. Furthermore, because thermal energies in stars tend to be small compared to nuclear excitation energies, the astrophysically important resonances are usually the ones nuclear physicists consider to be “low energy”, i.e. for a nuclear state in the compound nucleus, $C$, to be astrophysically important, it must be close to the combined rest mass of $A$ and $B$, i.e. close to the decay threshold.

At the same time, it is close to threshold that we find the best examples of cluster structures in nuclei. Thus, there exists a remarkable connection between stellar phenomena and the clustering aspects of nuclear structure. The famous Hoyle state in 12C provides arguably the best example of this connection. Were it not for the tendency of nucleons to cluster into α particles, the Hoyle state would not exist, and without the Hoyle state little carbon would be produced in stars [1]. (And without carbon, we humans would not exist.)

2. Astrophysical importance
The triple-α reaction, i.e. the fusion of three α particles into a 12C nucleus, plays a role in many astrophysical processes, but its great importance in astrophysics is to be attributed to two circumstances in particular: Firstly, it is the main mechanism behind stellar synthesis of carbon, essential to life as we know it on earth, and secondly, it is the mechanism by which the first generation of stars, created from the hydrogen and helium ashes of the Big Bang, were able to bridge the $A = 5$ and $A = 8$ mass gaps, thus allowing nucleosynthesis to proceed.

Carbon is mainly created during two phases of stellar evolution: the horizontal branch (HB) phase where helium burns in the core of the star, and the asymptotic giant branch (AGB) phase where helium burns in a shell surrounding the core. During these phases the star is in hydrostatic
equilibrium and helium burns at temperatures between 0.1 GK and 2 GK (1 GK = \(10^9\) K). The triple-\(\alpha\) reaction and the \(^{12}\text{C}(\alpha, \gamma)\) reaction operate side by side, and their relative rates determine the relative abundances of carbon and oxygen created. The triple-\(\alpha\) reaction proceeds as a two-step process, strongly enhanced by two low-lying \(s\)-wave (\(\ell = 0\)) resonances,
\[
\alpha + \alpha \rightarrow ^{8}\text{Be}(gs), \quad \alpha + ^{8}\text{Be}(gs) \rightarrow ^{12}\text{C}(\text{Hoyle}),
\]
(1)
the first being due to the ground state in \(^{8}\text{Be}\) and the second being due to the Hoyle state in \(^{12}\text{C}\).

Furthermore, the triple-\(\alpha\) reaction plays an important catalytic role in various explosive stellar processes in which the peak temperature exceeds 2 GK. Examples are given in Table 1. At such elevated temperatures, higher-lying resonances begin to contribute, and the triple-\(\alpha\) reaction is no longer completely dominated by the low-lying \(s\)-wave resonances.

At temperatures somewhat below 0.1 GK, the thermal energy is too small for the triple-\(\alpha\) reaction to proceed sequentially via the \(s\)-wave resonances; instead the reaction proceeds directly by some three-body mechanism. The rate of the triple-\(\alpha\) reaction below 0.1 GK is important for the evolution of the first generation of stars. Since they were created from the hydrogen and helium ashes of the Big Bang, they lacked the carbon necessary for the CNO-I cycle to operate, which therefore had to be produced by the triple-\(\alpha\) reaction, but at the relatively low temperature sustained by the \(pp\) chain [2].

Table 1 gives an overview of the three temperature domains of the triple-\(\alpha\) reaction, including the most important astrophysical sites.

### Table 1. Temperature domains of the triple-\(\alpha\) reaction.

| \(T\) (GK) | Burning        | Reaction mechanism         | Astrophysical sites                                               |
|------------|----------------|---------------------------|-------------------------------------------------------------------|
| 0.1–2      | hydrostatic    | \(s\)-wave resonances     | – helium core burning in HB stars                                 |
|            |                |                           | – helium shell burning in AGB stars                               |
| < 0.1      | hydrostatic    | direct three-body         | – first-generation stars                                          |
| > 2        | explosive      | higher-lying resonances    | – shock fronts of core-collapse supernovae                        |
|            |                |                           | – X-ray bursts                                                    |
|            |                |                           | – helium flashes during AGB phase                                 |

### 3. Determination of the reaction rate

In the intermediate-temperature domain (0.1–2 GK) the rate of the triple-\(\alpha\) reaction is fully determined by the properties of the Hoyle state, which can be measured in laboratory experiments. Precisely for this reason the rate is known relatively well, even though it cannot be directly measured. Currently, the rate is known with 10% precision, but efforts are underway to reduce this to 5% [3]. The determination of the reaction rate is far more complicated in the low- and high-temperature domains. In the high-temperature domain (> 2 GK) the rate depends on the properties of not one, but several states in \(^{12}\text{C}\), which are not well established. This is especially true for the possible \(2^+\) state around 9.6 MeV [4, 5, 6, 7, 8]; see also the contributions of M. Itoh and M. Gai. In the low-temperature domain (< 0.1 GK) sophisticated three-body models are necessary to calculate the rate [9].
When the triple-\(\alpha\) reaction proceeds through the two low-lying \(s\)-wave resonances, the reaction rate is given by,

\[
R \propto \frac{\Gamma_{3\alpha} \Gamma_{\text{rad}}}{\Gamma} T^{3/2} \exp \left( -\frac{Q}{kT} \right),
\]

(2)

where \(Q\) is the energy of the Hoyle state relative to the \(\alpha + \alpha + \alpha\) threshold, \(\Gamma_{\alpha_0}\) is the partial \(\alpha\)-decay width to the ground state of \(^8\)Be, \(\Gamma_{\text{rad}}\) is the radiative width and \(\Gamma = \Gamma_{\alpha} + \Gamma_{\text{rad}}\) is the total width, \(\Gamma_{\alpha}\) being the total \(\alpha\)-decay width. The current evaluation of the reaction rate assumes that \(\Gamma_{3\alpha} = \Gamma_{\alpha_0}\), i.e. the breakup of the Hoyle state into three \(\alpha\) particles proceeds exclusively as a sequential two-step process via the ground state of \(^8\)Be. Further using that \(\Gamma_{\text{rad}}/\Gamma \ll 1\), we have

\[
\frac{\Gamma_{\alpha_0} \Gamma_{\text{rad}}}{\Gamma} = \frac{\Gamma_{\alpha} \Gamma_{\text{rad}}}{\Gamma_{\alpha} + \Gamma_{\text{rad}}} \approx \Gamma_{\text{rad}},
\]

(3)

showing that the rate depends on two quantities only, \(Q\) and \(\Gamma_{\text{rad}}\). The latter is determined from the measurement of three quantities,

\[
\Gamma_{\text{rad}} = \Gamma_\gamma + \Gamma_\pi = \frac{\Gamma_\gamma + \Gamma_\pi}{\Gamma} \Gamma_\pi.
\]

(4)

The current error budget is,

\[
Q = 379.38 \pm 0.20 \text{ keV} \quad (1.2\%) \quad [10]
\]

\[
(\Gamma_\gamma + \Gamma_\pi)/\Gamma = (4.12 \pm 0.11) \times 10^{-4} \quad (2.7\%) \quad [11]
\]

\[
\Gamma_\pi/\Gamma = (6.8 \pm 0.7) \times 10^{-6} \quad (10\%) \quad [12]
\]

\[
\Gamma_\pi = 62.3 \pm 2.0 \mu\text{eV} \quad (3.2\%) \quad [3]
\]

with experiments underway to reduce the uncertainty on \(\Gamma_\pi/\Gamma\) to 5%.

4. An often overlooked assumption

Let \(\Gamma_{3\alpha} = \Gamma_{\alpha} - \Gamma_{\alpha_0}\) denote the combined width of all the direct \(\alpha\)-decay branches of the Hoyle state bypassing the ground state of \(^8\)Be. As we have seen, the assumption that \(\Gamma_{3\alpha} = 0\) is of critical importance to the determination of the triple-\(\alpha\) reaction rate. Nevertheless, the assumption had received relatively little scrutiny until recently. In 1994 an upper limit of \(\Gamma_{3\alpha}/\Gamma_{\alpha} < 0.04\) was obtained by Freer et al. [13]. However, in 2011 Raduta et al. identified two direct \(\alpha\)-decay branches with a combined branching ratio of \(\Gamma_{3\alpha}/\Gamma_{\alpha} = 0.17(5)\) [14]. If correct, this would imply \(\Gamma_{\alpha_0}/\Gamma = 0.83(5)\) and, as seen from Eq. (2), a corresponding reduction in the reaction rate in the temperature range 0.1–2 GK. The reduction is larger than the current estimate of the uncertainty on the rate and would have consequences for a number of astrophysical processes in which the triple-\(\alpha\) reaction plays a role [15, 16, 17, 18].

The consequences would be far more dramatic in the low-temperature domain, where, due to the different energy dependences of \(\Gamma_{\alpha_0}\) and \(\Gamma_{3\alpha}\) at very low energy [19], the findings of Raduta et al. [14] would imply an increase of several orders of magnitude in the reaction rate.

5. New constraints on \(\Gamma_{3\alpha}\)

In Ref. [20] we reported on a new measurement of the breakup of the Hoyle state into three \(\alpha\) particles. We used the \(^{11}\)B(\(^3\)He,\(d\)) reaction at 8.5 MeV to populate the Hoyle state, and we used a compact detection system consisting of four segmented \(\Delta E-E\) telescopes to measure the momenta of the deuteron and the three \(\alpha\) particles, see Ref. [21] for details. The complete kinematics information allowed us to obtain very clean data and allowed us to improve the resolution using a mathematical procedure known as kinematic fitting. We used the Dalitz-plot technique to analyse the momentum distribution of the three \(\alpha\) particles. The distribution of the
experimental data was fully consistent with the distribution expected for sequential decay via the ground state of $^8$Be (SD). We were able to put a stringent limit on a hypothetical phase-space distributed component (DDΦ) and even more stringent limits on the two components identified by Raduta et al., namely, three α particles of equal energy (DDE) and one α particle at rest and the other two with equal energy (DDL). The results are summarized in Table 2 along with the results of another recent experiment by Manfredi et al. [22].

Table 2. Experimentally determined magnitudes of various Dalitz-plot components. SD denotes sequential decay via the ground state of $^8$Be. DDE, DDL and DDΦ are associated with direct decay branches, see text for explanation. Upper limits are given at 95% confidence level. The uncertainties quoted by Raduta et al. are presumably 1σ uncertainties, i.e. 68% confidence level.

| Dalitz-plot component | Freer et al. [13] | Raduta et al. [14] | Manfredi et al. [22] | Kirsebom et al. [20] |
|-----------------------|-------------------|--------------------|----------------------|----------------------|
| SD                    | 1                 | 0.830(50)          | 1                    | 1                    |
| DDE                   | ...               | 0.075(40)          | < 0.3 × 10^{-2}      | < 0.9 × 10^{-3} |
| DDL                   | ...               | 0.095(40)          | ...                 | < 0.9 × 10^{-3} |
| DDΦ                   | < 0.03*           | ...               | < 3 × 10^{-2}        | < 5 × 10^{-3} |

*Given as 0.04 at 99.5% confidence level in Ref. [13].

6. Link to structure
The structure of the Hoyle state is a subject of long-standing interest, which has received much theoretical attention recently (lattice effective field theory [23, 24], no-core shell model [25], fermionic molecular dynamics [26] and α-cluster models [27, 28, 29]).

Raduta et al. make the premise that observation of a certain distribution of α-particle energies provides evidence for corresponding structural features of the Hoyle state. Specifically, decay with three α particles of equal energy (DDE) is linked to α-condensate structure [30, 31] and decay with one α particle at rest and the other two with equal energy (DDL) is linked to the long-discussed linear-chain structure [32]. Raduta et al. thus conclude that the observation of a DDE component constitutes direct evidence for α-condensate structure of the Hoyle state. While we dispute the suggested direct link between energy distribution and structure, because the former must be strongly influenced by the tunneling through the Coulomb barrier, it seems natural to expect some connection between energy distribution and structure, and therefore a precise measurement of the energy distribution, viz. the Dalitz-plot distribution, should provide a sensitive test of structure models if the latter is combined with a sophisticated decay model such as that described in Refs. [27, 28].

7. A comment on semantics
In the scientific literature it is commonplace to refer to the breakup mechanism as if it were an observable ammendable to measurement. This is an unprecise use of terms, which can lead to confusion in discussions between theorists and experimentalists. The observable is not the breakup mechanism, but the momentum distribution of the α particles, viz. the Dalitz-plot distribution. The task of the experimentalist is to measure this distribution; that of the theorist is to calculate it. Any interpretation in terms of a breakup mechanism is only meaningful in the context of the model adopted by the theorist. That said, the interpretation of the experimental
data presently available is straightforward and unambiguous: the breakup proceeds sequentially via the ground state of $^8$Be in at least 99.5 of every 100 occurrences.

8. Summary
The triple-$\alpha$ reaction plays a very important role in astrophysics. Under the conditions prevailing during helium burning in HB and AGB stars, the rate of the triple-$\alpha$ reaction is fully determined by the properties of the Hoyle state in $^{12}$C. A central assumption in the determination of the rate is that the breakup of the Hoyle state into three $\alpha$ particles always proceeds sequentially via the ground state of $^8$Be. Recently, Raduta et al. have reported experimental results that challenge this view: They claim observation of two direct $\alpha$-decay branches bypassing the ground state of $^8$Be with a combined branching ratio of 17(5)$\%$ [14]. This would imply a corresponding reduction in the reaction rate with significant astrophysical consequences. The results of Raduta et al. are, however, not verified in two more recent studies by Manfredi et al. [22] and Kirsebom et al. [20]. On the contrary, neither finds any evidence for the proposed direct branches. The most stringent upper limit is obtained by Kirsebom et al.: $5 \times 10^{-3}$ at 95$\%$ confidence level.

While the currently available experimental data is consistent with a purely sequential decay, future experiments with higher sensitivity may potentially uncover other, very rare, decay branches. Besides having significant implications for the triple-$\alpha$ reaction rate at low temperatures, such a discovery would also have implications for our understanding of the structure of the Hoyle state.

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