Neutrino Oscillations and Other Key Issues in Supersymmetric
\[ \text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \]

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Abstract

We try to gain an understanding of the recent Superkamiokande data on neutrino oscillations and several other important phenomenological issues within the framework of supersymmetric \( \text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \) \((\equiv G_{422})\). By supplementing \( G_{422} \) with a \( U(1)-\mathcal{R} \) symmetry, we can provide an explanation of the magnitude \( M_G \) \((\sim 10^{16} \text{ GeV})\) of the \( G_{422} \)-symmetry breaking scale, resolve the MSSM \( \mu \) problem, and understand why proton decay has not been seen \((\tau_p \gg 10^{34} \text{ yr})\). The family dependent \( \mathcal{R} \)-symmetry also helps provide an explanation of the charged fermion mass hierarchies as well as the magnitudes of the CKM matrix elements. Several additional heavy states in the mass range \( 10^4 - 10^7 \text{ GeV} \) are predicted, and the MSSM parameter \( \tan \beta \) turns out to be of order unity. The atmospheric neutrino problem is explained through \( \nu_\mu - \nu_\tau \) mixing with \( \sin^2 2\theta_{\mu\tau} \simeq 1 \). The resolution of the solar neutrino puzzle is via the small angle MSW oscillations and necessarily requires a sterile neutrino \( \nu_s \) which, thanks to the \( \mathcal{R} \)-symmetry, has a tiny mass.

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1 Introduction

The Superkamiokande collaboration [1] may have presented the first ‘real’ experimental evidence of physics beyond the standard model through their observations of the atmospheric and solar neutrino puzzles. These are most simply resolved by invoking neutrino oscillations, thereby suggesting that one or more of the known neutrinos has a mass greater than $\sim 10^{-1} - 10^{-2}$ eV or so. This value is far in excess of $\sim 10^{-5}$ eV that arises from the ‘standard model’ dimension five operators. Clearly, physics beyond the standard model is called for, and supersymmetric unification is one good way to proceed.

Acceptable neutrino masses can be realized in several ways, but let us list just two of them. One is the well-known see-saw mechanism [2] in which we invoke the existence of a right-handed neutrino. In the second scheme [3] an $SU(2)_L$ scalar triplet naturally acquires a VEV of order $M_W^2/M$, where $M$ denotes the superheavy mass of the triplet field. The neutrinos then acquire mass through their coupling to the triplet VEV (Lepton number is not spontaneously broken in either of these schemes). Of course, it is possible to consider models in which both these mechanisms are simultaneously present.

In this paper we will focus on the $SU(4)_c \times SU(2)_L \times SU(2)_R$ ($\equiv G_{422}$) scheme [4] which automatically introduces the right-handed neutrino. This scheme has a number of important features as pointed out in ref. [5]. First, in contrast to $SU(5)$ or $SO(10)$, it can provide a relatively straightforward resolution of the MSSM $\mu$ problem. Second, once the hierarchy and $\mu$ problems are resolved it usually implies [3] an essentially stable proton. Third, it contains a neat mechanism for generating the observed baryon asymmetry via a lepton asymmetry, with the right-handed neutrinos playing an essential role. Last, but not least, the gauge symmetry $G_{422}$, again in contrast to $SU(5)$ or $SO(10)$, can arise from fairly straightforward superstring constructions [6].

The scheme presented here relies on the minimal ‘higgs’ structure which gives rise to MSSM at low energies. Remarkably, the gauge hierarchy and $\mu$ problems are resolved within the minimal ‘higgs’ framework [4]. An essential role in achieving this is played by a (family dependent) $U(1)$ $R$-symmetry. This symmetry also plays a crucial role in understanding the magnitude of the $G_{422}$ symmetry breaking scale $M_G$ ($\sim 10^{16}$ GeV), which is expressed in terms of $M_P$ (reduced Planck scale $= 2.4 \cdot 10^{18}$ GeV) and $m_{3/2}$ (= gravitino mass $\sim$ TeV). The $R$-symmetry also implies an essentially stable proton ($\tau_p \sim 10^{60}$ yr) by strongly suppressing the dimension five operators from the colored triplets as well as the nonrenormalizable Planck scale operators.

The $R$-symmetry also plays an essential role in realizing the observed charged fermion mass hierarchies as well as the magnitudes of the CKM matrix elements [4]. We are led to introduce a light sterile neutrino in order to resolve the solar neutrino puzzle via the small angle MSW solution [3], while the atmospheric neutrino puzzle is explained through

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4The resolution of the $\mu$ problem here will be different from the one given in [3].
large $\nu_\mu - \nu_e$ mixing. We arrange for the sterile neutrino to be light by exploiting the $R$-symmetry \[\text{[9, 10]}\].

The plan of the paper is as follows. In section 2 we consider the symmetry breaking of $G_{422}$ as well as the origin of the GUT scale. Two important parameters make an appearance here. One is $\epsilon_R \sim 0.2$ (equal in magnitude to the Wolfenstein parameter $\lambda$), which is the ratio of the $U(1)$- $R$ symmetry breaking scale and $M_P$, while the second parameter is $\epsilon_G = M_G / M_P \sim 10^{-2}$ . In section 3 we discuss how the charged fermion mass hierarchies and the magnitudes of the CKM matrix elements can be understood by exploiting the family dependent $R$-symmetry. We are led to introduce additional ‘matter’ supermultiplets in the $(6, 1, 1)$ and $(1, 2, 2)$ representations of $G_{422}$ . It turns out that the MSSM parameter $\tan \beta$ has to be of order unity. In section 4 we discuss neutrino masses and mixings in detail and show the need for a light sterile neutrino which, thanks to the $U(1)$ - $R$- symmetry, is easily arranged. In section 5 we discuss the stability of the proton and the important role played (yet again!) by the $R$-symmetry. The conclusions are presented in section 6.

2 \quad SU(4)_c \times SU(2)_L \times SU(2)_R \text{ Symmetry} \\
\text{Breaking and Origin of } M_G

The ‘higgs’ sector \[\text{[4]}\] of the $SU(4)_c \times SU(2)_L \times SU(2)_R$ model consists of the following superfields:

\[
H \sim (4, 1, 2) , \quad \bar{H} \sim (\bar{4}, 1, 2) , \\
h \sim (1, 2, 2) , \quad D_6 \sim (6, 1, 1) ,
\]

where

\[
(4, 1, 2) = \left( \frac{\bar{u}_c^1}{d_1}, \frac{\bar{u}_c^2}{d_2}, \frac{\bar{u}_c^3}{d_3}, \nu^c \right) , \quad (\bar{4}, 1, 2) = \left( u_c^1, u_c^2, u_c^3, \bar{d}_c^1, \bar{d}_c^2, \bar{d}_c^3, \bar{e}_c \right) , \\
(1, 2, 2) = (h_u, h_d) , \quad (6, 1, 1) = (D^c, \bar{D}^c) .
\]

The $H$ and $\bar{H}$ fields provide the breaking of $G_{422}$ to $SU(3)_c \times SU(2)_L \times U(1)_Y$ after their components $\nu^c$ and $\nu^c$ develop nonzero VEVs. The states $\bar{u}_c + u_c$ and $\bar{e}_c + e_c$ from $H + \bar{H}$ are ‘goldstones’ absorbed by the appropriate gauge fields, while $\bar{d}_c$ and $d_c$ form physical massive states through mixings with the corresponding fragments from $D_6$. The superfield $h$ ‘unifies’ the pair of MSSM electroweak ‘higgs’ doublets. For obtaining

\[\text{[5]}\] We assume the existence of an unbroken $Z_2$ ‘matter’ parity which distinguishes the higgs and matter superfields.
Table 1: $\mathcal{R}$ charges of the scalar superfields and the superpotential.

| $\mathcal{R}$ | $W$ | $H$ | $\bar{H}$ | $D_6$ | $h$ | $X$ |
|---------------|-----|-----|----------|-------|-----|-----|
| $R$           | $\frac{R}{10} + \frac{1}{2} R_X$ | $\frac{R}{10} - \frac{1}{2} R_X$ | $\frac{4}{5} R - R_X$ | $\frac{2}{3} R + \frac{1}{2} R_X$ | $\frac{R}{24}$ |

an all order doublet-triplet hierarchy and the desirable pattern of symmetry breaking, we introduce an additional singlet superfield $X$ and the crucial $U(1)-\mathcal{R}$ symmetry. The latter will be important also for understanding the hierarchies of fermion masses and mixings. The $\mathcal{R}$ charges of the scalar superfields and superpotential are presented in Table (1).

The scalar superpotential

$$W = M_P^3 \left( \frac{\bar{H}H}{M_P^2} \right)^5 + M_P^3 \left( \frac{X}{M_P} \right)^{24} + D_6HH + \left( \frac{X}{M_P} \right)^2 D_6\bar{H} \bar{H}$$

(4)
is the most general allowed under the $\mathcal{R}$ symmetry. The first two terms in (4), together with the quadratic soft terms

$$V_{SSB}^m = m_{3/2}^2 \left( |H|^2 + |\bar{H}|^2 + |X|^2 \right) ,$$

(5)
(which emerge in $N = 1$ SUGRA after SUSY breaking) lead to the nonzero VEVs $\langle \nu_H \rangle \equiv \langle H \rangle$ and $\langle \nu_{\bar{H}} \rangle \equiv \langle \bar{H} \rangle$ with magnitudes:

$$\langle \bar{H} \rangle \sim \langle H \rangle \sim M_P \left( \frac{m_{3/2}}{M_P} \right)^{1/8} , \quad \langle X \rangle \sim M_P \left( \frac{m_{3/2}}{M_P} \right)^{1/22} .$$

(6)

For $m_{3/2} = 10^3$ GeV and $M_P = 2.4 \cdot 10^{18}$ GeV, we have:

$$\epsilon_G \equiv \frac{\langle H \rangle}{M_P} \sim \left( \frac{m_{3/2}}{M_P} \right)^{1/8} \sim 10^{-2} ,$$

$$\epsilon_{\mathcal{R}} \equiv \frac{\langle X \rangle}{M_P} \sim \left( \frac{m_{3/2}}{M_P} \right)^{1/22} \sim 0.2 .$$

(7)

We observe that the $\mathcal{R}$ symmetry (broken at scale $\langle X \rangle$) helps generate the GUT scale with magnitude $M_G \simeq \epsilon_G M_P \sim 10^{16}$ GeV.

The last two terms in (4) are responsible for the generation of masses of the colored triplet fragments. Substituting the nonzero VEVs of the scalars in these two terms the masses of the decoupled states are given by:
Note that the electroweak higgs doublets are massless to all orders in the unbroken SUSY limit. One source for obtaining the desirable value of the MSSM $\mu$ term can be the mechanism suggested in ref. [11]. It generates the $\mu$ term through the Kähler potential, and can be applied also in our scheme. Consider the coupling between the electroweak doublets and the hidden sector field $Z$:

$$\Delta K_h = \frac{Z}{M_P} h^2.$$  \label{eq:9}

For $\langle F_Z \rangle \sim m_3^2/2 M_P$, one obtains $\mu \sim m_3^2/2$.

In summary, the $U(1)_R$ symmetry leads to the desirable gauge hierarchy and is crucial for the generation of the GUT scale.

3 Horizontal $U(1)_R$ Symmetry: Charged Fermion Masses and Mixings

In the simplest approach to $G_{422}$ the ‘matter’ sector consists of the superfields $F_\alpha \sim (4, 2, 1)_\alpha$, $\bar{F}_\alpha \sim (\bar{4}, 1, 2)_\alpha$ ($\alpha$ is a family index), where $F_\alpha \supset (q, l)_\alpha$, $\bar{F}_\alpha \supset (\bar{u}^c, \bar{d}^c, \bar{e}^c)_\alpha$, and the masses of the quark-lepton families are generated through the coupling $A_{\alpha\beta} F_\alpha \bar{F}_\beta h$. If the family dependent $A_{\alpha\beta}$ couplings are taken as field independent Yukawa constants, one obtains the unacceptable asymptotic relations $\hat{Y}_u = \hat{Y}_d = \hat{Y}_e$ and no CKM mixings emerge. However, if appropriate entries of $A$ are $(\frac{HH}{M^2})$ ($M$ is some cut-off) dependent operators then, due to the many possible convolutions of $G_{422}$ group indices, the unwanted mass relations are avoided and physical mixings can emerge. The mixing angles and values of Yukawa couplings depend on the physical cut-off $M$, and taking it $M_P$ (which could be considered as a natural cut-off of the theory), one observes that all the mixing angles are much smaller than the corresponding measured values. Consequently, for obtaining reasonable values of the CKM matrix elements the cut-off $M$ should be taken to be somewhat lower \[13-17\], assuming that the corresponding operators are obtained after decoupling of some states whose masses lie between the scales $M_G$ and $M_P$. Refs. [14, 16] present the classification of corresponding operators and some needed $G_{422}$ representations of the decoupled states.

Our approach here is rather different. The cut-off parameter of all nonrenormalizable operators which we consider will be the Planck mass. For obtaining the desirable hier-

\[m_{T_1} \simeq M_G \simeq M_P \epsilon_G, \quad m_{T_2} \simeq M_G \epsilon_R^2 \simeq M_P \epsilon_G \epsilon_R^2.\]  \label{eq:8}

For an explanation of the origin of GUT scale and all order natural gauge hierarchy in $SU(3)^3$ and flipped $SU(6)$ models, see \[12\] and \[10\] respectively.
Table 2: $R$ charges of the ‘matter’ superfields under the $\mathcal{R}$ symmetry.

|   | $F_1$ | $F_2$ | $F_3$ | $\bar{F}_1$ | $\bar{F}_2$ |
|---|---|---|---|---|---|
| $\mathcal{R}$ | $\frac{3}{10}R - \frac{7}{2}R_X$ | $\frac{3}{10}R - \frac{5}{2}R_X$ | $\frac{3}{10}R - \frac{1}{2}R_X$ | $\frac{3}{10}R - 3R_X$ | $\frac{3}{10}R - R_X$ |
| $\mathcal{F}_3$ | $g_1$ | $g_2$ | $g_3$ | $f_\alpha$ |
| $\mathcal{R}$ | $\frac{3}{10}R$ | $\frac{1}{3}R - \frac{5}{2}R_X$ | $\frac{1}{5}R - \frac{1}{2}R_X$ | $\frac{1}{5}R - \frac{1}{2}R_X$ | $\frac{2}{5}R - (9 - \alpha)R_X$ |

Archies of Yukawa couplings and mixings we introduce three additional pairs of ‘matter’ superfields:

$$g_\alpha \sim (6, 1, 1)_\alpha, \quad f_\alpha \sim (1, 2, 2)_\alpha.$$ 

These representations turn out to be the most economical ones for building a phenomenologically acceptable ‘matter’ sector of our model.

We will consider the $\mathcal{R}$ symmetry as ‘horizontal’ and prescribe distinct $R$ charges to fermions from different families in order to obtain a natural explanation of the hierarchies of Yukawa couplings and CKM matrix elements. The transformation properties of the various ‘matter’ superfields are presented in Table 2.

The $F_\alpha \bar{F}_\beta h$ type couplings are schematically written as

$$F_1 \left( \frac{X_{MP}}{M_P} \right)^6, \quad F_2 \left( \frac{X_{MP}}{M_P} \right)^4, \quad F_3 \left( \frac{X_{MP}}{M_P} \right)^3 \right) \cdot h,$$

and upon diagonalization yields the up quark Yukawa matrix:

$$\hat{Y}_u^D = \text{Diag} \left( \epsilon_R^6, \epsilon_R^3, 1 \right),$$

\footnote{It is worth noting that these additional states, together with $F_\alpha$, $\bar{F}_\alpha$ and singlet states $\mathcal{N}_2, \mathcal{N}_3$ and $\nu_\alpha$ which we will introduce later for accommodating the Superkamiokande data, constitute the (three) $27_\alpha$-plets of $E_6$.}
where the top quark Yukawa coupling

$$\lambda_t \sim 1 ,$$  \hspace{1cm} (13)

and

$$\lambda_u : \lambda_c : \lambda_t \sim \epsilon_R^6 : \epsilon_R^3 : 1 .$$  \hspace{1cm} (14)

The couplings involving the $g$ states read:

$$F_1 \left( \begin{array}{ccc} g_1 \left( \frac{X}{M_P} \right)^5 & g_2 \left( \frac{X}{M_P} \right)^3 & g_3 \left( \frac{X}{M_P} \right)^3 \end{array} \right) \frac{H h}{M_P} , \quad F_2 \left( \begin{array}{ccc} \frac{X}{M_P} & \frac{X}{M_P} & \frac{X}{M_P} \end{array} \right) \left( \frac{H h}{M_P} \right)^3 M_P , \quad F_3 \left( \begin{array}{ccc} \frac{X}{M_P} & \frac{X}{M_P} & \frac{X}{M_P} \end{array} \right) \left( \frac{H h}{M_P} \right)^3 M_P .$$ \hspace{1cm} (15)

$$g_1 \left( \begin{array}{ccc} \frac{X}{M_P} & \frac{X}{M_P} & \frac{X}{M_P} \end{array} \right) \left( \frac{H h}{M_P} \right)^3 M_P , \quad g_2 \left( \begin{array}{ccc} \frac{X}{M_P} & \frac{X}{M_P} & \frac{X}{M_P} \end{array} \right) \left( \frac{H h}{M_P} \right)^3 M_P , \quad g_3 \left( \begin{array}{ccc} \frac{X}{M_P} & \frac{X}{M_P} & \frac{X}{M_P} \end{array} \right) \left( \frac{H h}{M_P} \right)^3 M_P .$$ \hspace{1cm} (16)

Without loss of generality we consider a basis in which the couplings (11) and (16) are diagonal, and the matrix relevant for down-quark masses will be:

$$\hat{M}_d = \begin{pmatrix} D_c^g & d_c \\ \hat{m}'_d & \hat{Y}_D h_d \\
\hat{M}_g & \hat{M}_g F \end{pmatrix} ,$$ \hspace{1cm} (17)

where, taking into account (12) and (16),

$$\hat{m}'_d = \begin{pmatrix} \epsilon_R^5 & \epsilon_R^3 & \epsilon_R^3 \\ \epsilon_R^4 & \epsilon_R^2 & \epsilon_R^2 \\ \epsilon_R^2 & 1 & 1 \end{pmatrix} \epsilon_G h_d , \quad \hat{M}_g F = \begin{pmatrix} b_{11} \epsilon_R^5 & b_{12} \epsilon_R^3 & b_{13} \epsilon_R^2 \\ b_{21} \epsilon_R^3 & b_{22} \epsilon_R^3 & b_{23} \\ b_{31} \epsilon_R^2 & b_{32} \epsilon_R^2 & b_{33} \end{pmatrix} M_P \epsilon_G \epsilon_R ,$$ \hspace{1cm} (18)

$$\hat{M}_g = \text{Diag} \left( \epsilon_R^4 , 1 , 1 \right) M_P \epsilon_G \epsilon_R$$ \hspace{1cm} (19)

and $\hat{Y}_D$ is given in (12). For $b_{12}, b_{13} < 1/3$ the states $\bar{D}_g^c$ and $d_c$ can integrated out, and so for the down quark mass matrix we obtain:
\[ \hat{m}_d = \hat{m}'_d - Y_u^D M_g^{-1} M_g h_d \simeq q_1 \begin{pmatrix} d_1' \qquad d_2' \qquad d_3' \end{pmatrix} \begin{pmatrix} q_1 \qquad q_2 \qquad q_3 \end{pmatrix} \cam \epsilon_G h_d. \] (20)

From (20), the down quark Yukawa couplings are:

\[ \lambda_b \sim \epsilon_G (\sim 10^{-2}) \],
\[ \lambda_d : \lambda_s : \lambda_b \sim \epsilon_5^R : \epsilon_2^R : 1. \] (21)

These values lead us to conclude that the MSSM parameter \( \tan \beta \) (\( \equiv \langle h_u \rangle / \langle h_d \rangle \)) is of order unity. For the CKM matrix elements we obtain:

\[ V_{us} \sim \epsilon_R, \quad V_{ub} \sim \epsilon_3^{R}, \quad V_{cb} \sim \epsilon_2^{R}, \] (22)

which are in good agreement with the observations!

Turning now to the lepton sector, the fields \( f_\alpha \) are crucial for obtaining values of the charged lepton Yukawa couplings that are consistent with the \( \tan \beta \sim \text{unity} \) regime. The couplings containing these fields are:

\[
\begin{align*}
F_1 \left( \begin{array}{ccc} f_1 \frac{X}{M_P}^{12} & f_2 \frac{X}{M_P}^{11} & f_3 \frac{X}{M_P}^{10} \\
F_2 & \begin{array}{ccc} f_1 \frac{X}{M_P}^{11} & f_2 \frac{X}{M_P}^{10} & f_3 \frac{X}{M_P}^{9} \\
F_3 & \begin{array}{ccc} f_1 \frac{X}{M_P}^{9} & f_2 \frac{X}{M_P}^{8} & f_3 \frac{X}{M_P}^{7} \\
\end{array} & \bar{H} \bar{H} = 0 \end{array} & \bar{H} \bar{H}^T = \frac{X}{M_P} \\
\end{array} \right),
\end{align*}
\] (23)

and, in the basis where (24) is taken diagonal, we will have:

\[ \hat{M}_e = \epsilon_c \left( \begin{array}{cc} l_f & l \\
\bar{e}_f & M_f \end{array} \right) \hat{M}_{ff}, \] (25)

with
\[
\hat{M}_{fF} = \begin{pmatrix}
c_{11} \epsilon_{R}^5 & c_{12} \epsilon_{R}^4 & c_{13} \epsilon_{R}^2 \\
c_{21} \epsilon_{R}^4 & c_{22} \epsilon_{R}^2 & c_{23} \epsilon_{R} \\
c_{31} \epsilon_{R}^2 & c_{32} \epsilon_{R} & c_{33}
\end{pmatrix}
M_P \epsilon_{G}^3 \epsilon_{R}^7 , \quad \hat{M}_f = \text{Diag} (\epsilon_{R}^4, \epsilon_{R}^2, 1) M_P \epsilon_{G}^2 \epsilon_{R}^{12}
\]
(26)

For \(c_{13}, c_{23} < 1/5\), the states \(l - \bar{e}_f\) can be integrated out and for the charged lepton matrix one obtains:
\[
\hat{m}_e = Y_u^D \hat{M}_{fF}^{-1} \hat{M}_f h_d .
\]
(27)

More explicitly,
\[
\hat{m}_e = e_c \begin{pmatrix}
ob_{f_1}' & l_{f_1}' & l_{f_2}' \\
e_{f_3}' & e_{f_3}' & e_{f_3}' \\
e_{f_2}' & e_{f_2}' & e_{f_2}'
\end{pmatrix}
\begin{pmatrix}
\epsilon_{R}^5 \\
\epsilon_{R}^4 \\
\epsilon_{R}^3
\end{pmatrix}
\begin{pmatrix}
\epsilon_{G}^6 \\
\epsilon_{G}^4 \\
\epsilon_{G}^2
\end{pmatrix}
\begin{pmatrix}
\epsilon_{G}^7 \\
\epsilon_{G}^5 \\
\epsilon_{G}^3
\end{pmatrix}
\]
(28)

Diagonalizing (28) we find the charged lepton Yukawa couplings to be:
\[
\lambda_\tau \sim \frac{\epsilon_{G}^5}{\epsilon_{G}} \sim 10^{-2} (\sim \lambda_b) ,
\]
\[
\lambda_e : \lambda_\mu : \lambda_\tau \sim \epsilon_{R}^5 : \epsilon_{R}^2 : 1 .
\]
(29)

Thus, with the help of \(g + f\) states and imposing a horizontal \(\mathcal{R}\) symmetry, we can indeed realize the desirable hierarchies of Yukawa couplings and magnitudes of CKM mixing angles, as given by (13), (14), (21), (29) and (22) respectively.

From (18) and (26) one finds that the new triplet and doublet ‘matter’ states lie below the GUT scale:
\[
m_{t_1} \simeq M_P \epsilon_{G}^5 \epsilon_{R}^6 , \quad m_{t_2} \simeq M_P \epsilon_{G}^5 \epsilon_{R}^2 , \quad m_{t_3} \simeq M_P \epsilon_{G}^5 \epsilon_{R} , \\
m_{d_1} \simeq M_P \epsilon_{G}^3 \epsilon_{R}^{12} , \quad m_{d_2} \simeq M_P \epsilon_{G}^3 \epsilon_{R}^{10} , \quad m_{d_3} \simeq M_P \epsilon_{G}^3 \epsilon_{R}^7 .
\]
(30)

These masses vary from around \(10^4 - 10^7\) GeV. Also, from (8) the mass of one triplet pair from the ‘higgs’ sector is below \(M_G\). The ‘observed’ unification at \(M_G\) [18] of the three gauge coupling constants in the one loop approximation will still hold if the masses of these additional states satisfy the condition:
\[
\frac{m_{T_2}}{M_G} \simeq \frac{m_{d_1} m_{d_2} m_{d_3}}{m_{t_1} m_{t_2} m_{t_3}} .
\]
(31)

Substituting the masses of the corresponding triplet and doublet fragments from (8) and (30), we can see that (31) is indeed satisfied.
4 Neutrino Oscillations

From the couplings (11), (23) and (24) the matrix relevant for neutrino ‘Dirac’ masses turns out to be:

\[
\hat{M}'_{\nu} = \nu^c \begin{pmatrix}
\nu_f & \nu \\
M_f & \hat{Y}_D h_u \\
\end{pmatrix}.
\] (32)

Integrating out the $\bar{\nu}_f - \nu$ states yields for the ‘light’ sector:

\[
\hat{m}'_{D} = Y_D^T \hat{M}_f^{-1} \hat{M}_f h_u.
\] (33)

The matrix (33) coincides with and has the same orientation in family space as the charged lepton mass matrix (27), and by proper unitary transformations, can be diagonalized and expressed as:

\[
\hat{m}'_{D} = \text{Diag} (\lambda_e, \lambda_\mu, \lambda_\tau) h_u.
\] (34)

These ‘Dirac’ masses will be ‘suppressed’ by the see-saw mechanism [2]. The $\mathcal{R}$ symmetry allowed couplings:

\[
\begin{pmatrix}
\bar{F}_1 \\
\bar{F}_2 \\
\bar{F}_3
\end{pmatrix} \begin{pmatrix}
\frac{X}{M_P}^5 & \frac{X}{M_P}^3 & \frac{X}{M_P}^2 \\
\frac{X}{M_P}^3 & \frac{X}{M_P} & 1 \\
\frac{X}{M_P}^2 & 1 & 0
\end{pmatrix} \begin{pmatrix}
\bar{H} \\
\bar{H} \\
\bar{H}
\end{pmatrix} M_P \epsilon_G^4.
\] (35)

generate the ‘Majorana’ matrix for $\nu^c$ states which, in the basis in which $\hat{m}'_{D}$ is diagonal (see (34)), takes the form:

\[
\hat{M}'_R = \begin{pmatrix}
\epsilon_R^5 & \epsilon_R^3 & \epsilon_R^2 \\
\epsilon_R^3 & \epsilon_R & 1 \\
\epsilon_R^2 & 1 & \epsilon_R
\end{pmatrix} M_P \epsilon_G^4.
\] (36)

Finally, for the neutrino masses we have:

\[
\hat{m}'_{\nu} = \hat{m}'_{D} (\hat{M}'_R)^{-1} \hat{m}'_{D},
\] (37)

from which, taking into account (29),

\[
m'_{\nu_\tau} \sim \frac{\epsilon_R \lambda^2}{M_P \epsilon_G^4} h_u^2, \quad m'_{\nu_\mu} \sim \epsilon_R^2 m'_{\nu_\tau}, \quad m'_{\nu_e} \sim \epsilon_R^4 m'_{\nu_e}.
\] (38)
Table 3: $R$ charges of the singlet states $N_2, N_3$.

| $R$ | $N_2$ | $N_3$ |
|-----|-------|-------|
| $\frac{1}{5}R - \frac{13}{2}R_X$ | $\frac{1}{5}R + \frac{13}{2}R_X$ |

For $\lambda \sim 10^{-2}$, $h_u = 174$ GeV we find

$$m_{\nu_e}' \simeq 2.5 \cdot 10^{-2} \text{ eV} , \quad m_{\nu_\mu}' \simeq 10^{-3} \text{ eV} , \quad m_{\nu_\tau}' \simeq 4 \cdot 10^{-5} \text{ eV} .$$  

(39)

These values of neutrino masses and the corresponding mixing angles ($\theta_{12}' \sim \theta_{13}' \sim \epsilon_{R}, \theta_{13}'' \sim \epsilon_{R}^2$) are clearly inconsistent with the atmospheric and solar neutrino data [1]. We have to suitably modify our scheme and the mechanism that we propose is relatively simple, although its realization requires additional singlet states $N_{2,3}$ and $\nu_s$ (see footnote 7). Transformation properties of $N_{2,3}$ superfields under $R$ given in Table (3).

We begin with the atmospheric neutrino puzzle. Noting that the physical 'light' left-handed neutrinos $\nu_i$ reside mainly in the $f_i$ (see (32)), the relevant terms for the $\nu_\mu - \nu_\tau$ system are:

$$\hat{m}_{DP} = \left( \frac{X_{MP}}{M_P} \right)^{13} \hat{N}_2 \hat{N}_3 \left( \frac{X_{MP}}{M_P} \right)^{12} h , \quad \hat{M}_{N_{2,3}} = \left( \frac{X_{MP}}{M_P} \right)^{13} \hat{N}_2 \hat{N}_3 \left( \frac{X_{MP}}{M_P} \right)^{13} \hat{N}_2 \hat{N}_3 ,$$

(40)

from which we get:

$$\hat{m}_{DP} = \left( \begin{array}{cc} \epsilon_{R}^{13} & 1 \\ \epsilon_{R}^{12} & 0 \end{array} \right) h , \quad \hat{M}_{N_{2,3}} = \left( \begin{array}{cc} \epsilon_{R}^{13} & 1 \\ \epsilon_{R}^{12} & 0 \end{array} \right) M_P \epsilon_G^6 ,$$

(41)

Thus, the neutrino mass matrix involving the second and third generations has the quasi degenerate form, which provides the large mixing as well as the needed mass squared difference:

$$m_{\nu_2} \simeq m_{\nu_3} \equiv m \simeq \frac{\epsilon_{R}^{12} h_u^2}{M_P \epsilon_G^6} \simeq 5 \cdot 10^{-2} \text{ eV} ,$$

(42)
\[ \Delta m_{23}^2 = 2m^2\epsilon_R \simeq 10^{-3} \text{eV}^2, \]
\[ \sin^2 2\theta_{\mu\tau} \simeq 1. \]  
(43)

For obtaining this picture the form of \( \nu_\mu - \nu_\tau \) mass matrix in (42) is crucial, and we must ensure the absence of terms which can spoil the large mixing. The contribution from the elements of \( \hat{m}'_\nu \) (see (38)-(39)) are negligible, and also inclusion of the first generation does not change the picture.

The solar neutrino puzzle in our scheme can be explained through the small angle MSW oscillations. For this we have to invoke a new sterile state \( \nu_s \) with \( R \) charge equal to \( R_{\nu_s} = -19R_X/2 \). The relevant superpotential couplings are:

\[ W_{\nu e s} = \left( \frac{HH}{M_P^2} \right) \left( \frac{X}{M_P} \right)^{17} \nu_s f_1 h + M_P \left( \frac{X}{M_P} \right)^{43} \nu_s^2, \]  
(44)

which give

\[ \hat{m}_{\nu e \nu_s} = \nu_e \nu_s \begin{pmatrix} \nu_e & \nu_s \\ 0 & m_{\nu e \nu_s} \\ m_{\nu e \nu_s} & m_{\nu_s} \end{pmatrix}, \]  
(45)

with the corresponding entries in the range:

\[ m_{\nu e \nu_s} \sim \epsilon_G \epsilon_R^{17} h_a \sim \left( 2.3 \cdot 10^{-5} - 1.2 \cdot 10^{-4} \right) \text{eV}, \]
\[ m_{\nu_s} \sim M_P \epsilon_R^{43} \sim \left( 2 \cdot 10^{-3} - 1.3 \cdot 10^{-1} \right) \text{eV}, \]  
(46)

which for \( m_{\nu_s} \simeq 10^{-3} \text{eV}, m_{\nu e \nu_s} \simeq 5 \cdot 10^{-5} \text{eV} \) gives the oscillation parameters:

\[ \Delta m_{\nu e \nu_s}^2 \simeq m_{\nu_s}^2 \simeq 10^{-6} \text{eV}^2 \]
\[ \sin^2 2\theta_{\nu_e \nu_s} \simeq 10^{-2}, \]  
(47)

which correspond to the small angle MSW oscillations of \( \nu_e \) into the sterile state \( \nu_s \). We repeat that in our model the small masses \( m_{\nu e \nu_s} \) and \( m_{\nu_s} \) (see (43), (46)) are guaranteed by the \( R \) symmetry.

Considering all generations together and taking into account the relevant couplings, we have checked that after decoupling of the heavy states, the results obtained for the atmospheric and solar neutrino oscillation parameters are unchanged.

\[ \text{The } \nu_\mu - \nu_\tau \text{ mass matrix of our model closely resembles the one given in ref. } [19]. \]
5 Proton Decay

For studying nucleon decay in our model we begin with the color triplet mass matrix which arises from the ‘higgs’ sector. From (4) we have:

\[
\hat{M}_T = \begin{pmatrix}
d^c_H & D^c_D \\
m_{3/2} & M_{P\epsilon_G}^2 \\
0 & 0
\end{pmatrix}.
\]  

(48)

Baryon and lepton number violating $d = 5$ operators, obtained after integrating out these triplet states, will be proportional to the elements of the matrix $\hat{M}_T^{-1}$ given by:

\[
\begin{align*}
(\hat{M}_T^{-1})_{11} &\sim 0, & (\hat{M}_T^{-1})_{22} &\sim \frac{m_{3/2}^2}{M_{P\epsilon_G}^2}, \\
(\hat{M}_T^{-1})_{12} &\sim \frac{1}{M_{P\epsilon_G}^2}, & (\hat{M}_T^{-1})_{21} &\sim \frac{1}{M_{P\epsilon_G}}.
\end{align*}
\]

(49)

From (48) and (49) we see that nucleon decay can occur if the superfields $D_6$ and $H, \bar{H}$ simultaneously have couplings with the relevant matter superfields. It is easy to verify that the couplings

\[
\begin{align*}
FFD_6, & \quad FfD_6H, \\
\bar{F}\bar{F}D_6, & \quad \bar{F}gD_6H,
\end{align*}
\]

(50)

involving the $D_6$ field are forbidden by $R$ symmetry. This fact ensures that dimension five colored triplet induced nucleon decay is absent in our model.

Similarly, non-renormalizable operators suppressed by the Planck scale such as:

\[
\begin{align*}
\mathcal{O}_L^{(1)} &= \frac{1}{M_{P}} FFFF, & \mathcal{O}_L^{(2)} &= \frac{1}{M_{P}^2} FFFFH, \\
\mathcal{O}_R^{(1)} &= \frac{1}{M_{P}} \bar{F}\bar{F}\bar{F}, & \mathcal{O}_R^{(2)} &= \frac{1}{M_{P}^2} \bar{F}\bar{F}\bar{F}gH,
\end{align*}
\]

(51)

(which can lead to $d = 5$ operators $qqql$ and $u^c u^c d^c e^c$ respectively), are also eliminated by the $R$ symmetry.

Note that it is possible that the zero entry in (48) is replaced by a term of order $m_{3/2}$ due to contributions from the Kähler potential and the hidden sector (which can also give rise to couplings of the type in (50), (51)). These effects would induce proton decay with lifetime $\tau_p \sim 10^{60} \text{ yr}$.

We have also checked that dimension five operators involving the sterile neutrino superfield $\nu_s$, such as $u^c \bar{d}^c \bar{d}^c \nu_s$, are also strongly suppressed. They also imply a proton lifetime $\sim 10^{60} \text{ yr}$. 

12
We therefore conclude that the proton is essentially stable in the $SU(4)_c \times SU(2)_L \times SU(2)_R$ scheme discussed here. Its lifetime is estimated to be $\tau_p \sim 10^{60}$ yr.

6 Conclusions

In this paper we have attempted a unified treatment of several important phenomenological problems within the framework of supersymmetric $SU(4)_c \times SU(2)_L \times SU(2)_R$ ($\equiv G_{422}$). It is quite remarkable that by supplementing $G_{422}$ with a single family dependent $U(1)-\mathcal{R}$ symmetry and $Z_2$ matter parity, one can obtain an understanding of a wide ranging set of phenomena. For instance, one can explain the origin of the symmetry scale $M_G$ of $G_{422}$, understand why the MSSM $\mu$ term is of order a TeV or so rather than $M_P$, provide an estimate of the MSSM parameter $\tan \beta$ (order unity), understand why proton decay has not been (and will not be!) seen, and gain an understanding of fermion mass hierarchies and the magnitude of the CKM matrix elements. The model predicts the existence of new ‘heavy’ (mass $\sim 10^4 - 10^7$ GeV) particles. Lastly, and perhaps most significantly, the model can also accommodate the recent Superkamiokande data. The small angle $\nu_e - \nu_s$ MSW oscillations resolve the solar neutrino puzzle, while $\nu_\mu - \nu_\tau$ oscillations (with $\sin^2 2\theta_{\mu\tau} \simeq 1$) are responsible for the atmospheric neutrino anomaly. The sterile neutrino $\nu_s$ is kept light by the $\mathcal{R}$ symmetry.

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