QUANTUM SIMULATION

Experimental reconstruction of the Berry curvature in a Floquet Bloch band

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Topological properties lie at the heart of many fascinating phenomena in solid-state systems such as quantum Hall systems or Chern insulators. The topology of the bands can be captured by the distribution of Berry curvature, which describes the geometry of the eigenstates across the Brillouin zone. Using fermionic ultracold atoms in a hexagonal optical lattice, we engineered the Berry curvature of the Bloch bands using resonant driving and show a full momentum-resolved measurement of the ensuing Berry curvature. Our results pave the way to explore intriguing phases of matter with interactions in topological band structures.

Topological properties are essential for our understanding of many fascinating systems, such as topological superconductors or topological insulators, which conduct only at their edges (1, 2). The topology of the bulk band is quantified by the Berry curvature (2), the integral of which over the whole Brillouin zone is a topological invariant called the Chern number. According to the bulk boundary correspondence principle, the Chern number determines the number of chiral conducting edge states (2). Although edge states have been directly observed in a variety of lattice systems—ranging from solid-state systems to photonic waveguides, and even coupled mechanical pendula (3–7)—the underlying Berry curvature as the central measure of topology is not easily accessible. In recent years, ultracold atoms in optical lattices have emerged as a platform with which to study topological band structures (8, 9), and these systems have seen considerable experimental and theoretical progress. Whereas in condensed-matter systems, topological properties arise thanks to external magnetic fields or intrinsic spin-orbit coupling of the material, in cold atom systems they can be engineered by periodic driving analogous to illuminated graphene (10, 11). The resulting Floquet system can have topological properties very different from those of the original system (12). The driving can, for example, be realized through lattice shaking (13–17) or Raman coupling (18–20) with high-precision control in a large parameter space. In particular, the driving can break time-reversal symmetry (14, 15, 17) and thus allows for engineering nontrivial topology (17, 19). In quantum gas experiments, topological properties have been probed via the Hall drift of accelerated wave packets (17, 19), via an interferometer in momentum space (21, 22), and via edge states (23, 24), but so far, the full underlying Berry curvature was not measured quantitatively.

We measured the Berry curvature with full momentum resolution based on a method proposed in (25, 26). We performed a full tomography of the Bloch states across the entire Brillouin zone by observing the dynamics at each momentum point after a projection onto flat bands. The topological bands were engineered through resonant dressing of the two lowest bands of an artificial boron nitride lattice and feature a rich distribution of Berry curvature. Other relevant quantities such as the Berry phase or the Chern number can easily be obtained from the Berry curvature, which is thus the central concept for the description of topology.

Our system consists of ultracold fermionic atoms in a hexagonal optical lattice (27) formed by three interfering laser beams. With an appropriate polarization (28), a variable energy offset $\Delta_{\text{AB}}$ between the A and B sites (Fig. 1A), which breaks inversion symmetry, can be engineered. With the emerging band gap $h\Delta_{\text{AB}}$, the Dirac points at K and K' become massive, and for a large offset, the bands are flat (Fig. 1B) (28). This is a key ingredient for our tomography, because the flat band acts as the reference frame in which we reconstruct the eigenstates. Then as a central experimental method, we could accelerate the lattice on circular trajectories in real space by modulating the phases of the three lattice beams, thus realizing circular shaking (13–17). When the shaking frequency is near resonant with respect to a band transition, the two bands couple and form two new dressed Floquet bands. In Fig. 1C, we show the dressed Floquet bands for different accessible driving amplitudes. Apart from the dramatic change in the dispersion relation, the topological properties of the bands are changed. This manifests itself in the creation of a new Dirac point at the $\Gamma$-point and the annihilation of a Dirac point at the K point (Fig. 1D). A threefold symmetry also becomes visible in the dispersion relation (Fig. 1E).

The topological properties are not captured by the mere dispersion relation but by the Berry curvature, which describes the winding of the eigenstates across the Brillouin zone. Therefore, a complete tomography of the eigenstates of a Bloch band is mandatory for a measurement of the Berry curvature. The key idea behind constructing a Floquet lattice is to reconstruct the eigenvectors from dynamics after a projection onto flat bands (26). Consider the Bloch sphere (Fig. 2A), whose poles are given by $|k\cdot A\rangle$ and $|k\cdot B\rangle$, which are the Bloch states restricted to the A and B sublattices, respectively. The lower band can be written as $|k\rangle = \sin (\theta_2/2)|k\cdot A\rangle - \cos (\theta_2/2)\exp (i\theta_1)|k\cdot B\rangle$, and after a projection onto flat bands, the state oscillates around $|k\cdot B\rangle$, with the frequency $\gamma_k$.

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Fig. 1. Engineering of the topology by band dressing. (A) Three laser beams intersecting at 120° angles interfere to form a tunable honeycomb lattice (28) with a variable offset energy $h_{\text{AB}}$ between A and B sites. (B) Red and blue lines indicate the two lowest bands of the honeycomb lattice plotted along a high-symmetry path (K, M, K', G, and K). The lattice is accelerated along a circular trajectory, thus breaking time-reversal symmetry. Shown are dressed Floquet bands for different shaking cases, illustrating the dramatic change of the dispersion relations of the bare and dressed bands from (B). The lattice is accelerated along a circular trajectory, thus breaking time-reversal symmetry. (C) Circular shaking of the lattice (inset) with frequency $\nu$ resonantly ($\nu = \nu_{\text{AB}}$) couples the two bands from (B). The lattice is accelerated along a circular trajectory, thus breaking time-reversal symmetry. The arrow indicates the direction of the phase winding around the Dirac point. (D) Two-dimensional dispersion relations of the bare and dressed cases, illustrating the dramatic change of the topology. The arrow indicates the direction of the phase winding around the Dirac point. (E) Two-dimensional dispersion relations of the bare and dressed cases, illustrating the dramatic change of the topology. The arrow indicates the direction of the phase winding around the Dirac point.

Fig. 2. Dynamical measurement of the topology of the dressed bands. (A) Illustration of the experimental protocol. (i) We start in a band insulator in the lowest undressed band and (ii) adiabatically ramp up the coupling strength (shaking amplitude). Now the Bloch state at each quasimomentum is a superposition of $|k,A\rangle$ and $|k,B\rangle$, pointing in different directions on the Bloch sphere. (iii) When the dressing is switched off, the state is projected onto the flat bands and rotates on the Bloch sphere with the band difference $\nu$. After time-of-flight, this yields an oscillation of the density at each momentum. (B) Experimental momentum distributions for different hold times after the projection onto flat bands. (C) Oscillations at different quasimomenta. The solid lines are sinusoidal fits. From the amplitude and phase of the oscillation, we reconstruct the dressed state according to Eq. 1. The experimental parameters are $V_f = 15.15(10)E_r$, $\nu_{\text{AB}} = 11.65(11)$ kHz, $\nu = 11$ kHz, and a shaking amplitude of 223 nm (28). $E_r = \hbar^2/2m\lambda^2 = 4.41$ kHz is the recoil energy, where $\lambda = 1064$ nm is the lattice wavelength and $m$ is the atomic mass of $^{40}$K. The lattice depth $V_f$ is defined in (28).

Given by the energy difference of the flat bands. Then, the momentum distribution after time-of-flight is given by

\[ n(k, t) = f(k)[1 - \sin(\theta_k)\cos(\phi_k + 2\pi\nu_k t)] \]

from which both $\theta_k$ and $\phi_k$ can be easily obtained, yielding the desired tomography of the eigenstates for each quasimomentum. The method we use hence allows for a direct reconstruction of the Berry curvature according to

\[ \Omega(k) = \frac{1}{2}(\partial_k \hat{h} \times \partial_{\nu_k} \hat{h}) \cdot \hat{n} \]

with $\hat{n} = [\sin(\theta_k)\cos(\phi_k), \sin(\theta_k)\sin(\phi_k), \cos(\theta_k)]$. Our experimental sequence for this state tomography is sketched in Fig. 2. We start with a...
cloud of $5 \times 10^4$ single-component fermionic $^{40}$K atoms forming a noninteracting band insulator in the undressed lattice. Thanks to a large offset between the A and B sites, leading to a band gap of $v_{AB} = 11.65(11)$ kHz, the undressed bands are flat, so that for all quasimomenta $|k\rangle = |k, \beta\rangle$. We adiabatically ramped up the shaking amplitude to 223 nm within 5 ms at a shaking frequency of $\nu = 9$ kHz and then ramped the frequency to $\nu = 11$ kHz within 2 ms. By suddenly switching off the dressing, we projected onto the bare flat bands, so that Eq. 1 can be applied. In Fig. 2B, we show typical time-of-flight images for different hold times in the flat bands. The images feature dynamics with very large contrast. Time evolutions for different quasimomenta are shown in Fig. 2C, revealing the pure sinusoidal oscillations with clearly distinct amplitudes $\sin(\theta_k)$ and phases $\phi_k$, which are obtained by a simple fit to Eq. 1. We observed very large and long-lived oscillations after the projection, yielding relative amplitudes of up to 0.8. Additionally, with more than 2800 pixels in the first Brillouin zone, the resolution in momentum space is very high.

As the central result, we reconstructed the Berry curvature of the dressed band structure from these fits, which fully visualize the Bloch states (Fig. 3A). The amplitude map features a pronounced threefold symmetry, illustrating the breaking of equivalence between the K and K’ points. The amplitude has a maximum at the K point and is zero at the K’ and $\Gamma$ points. Even more striking is the very distinct threefold symmetry of the phase map with nearly discrete values of 0, $2\pi/3$, and $4\pi/3$. Where the amplitudes are zero at the K’ and $\Gamma$ points, the phase map correspondingly displays vortices. The phase vortices are clear signatures of Dirac points, which constitute topological defects. Furthermore, the data clearly show that we annihilated the Dirac point of the undressed hexagonal lattice at K and created a Dirac point in the dressed system at the $\Gamma$ point, changing the topology of the band. The resulting Berry curvature is localized at the new Dirac points and also shows this clear threefold symmetry. It has opposite signs at the two Dirac points, which results from the opposite chirality of the phase vortices. By inverting the chirality of the shaking, we instead annihilated the Dirac point at the K’ point and inverted the chirality of the phase windings, which also resulted in an inverted symmetry in the Berry curvature. All quantities agree well with a Floquet theory calculation (Fig. 3B), based on a tight-binding model as described in (28). In Fig. 3C, we plot the Berry curvature pixel-wise evaluated along a high-symmetry path, illustrating the very good agreement with the theory.

As mentioned above, with the fully momentum-resolved Berry curvature, we can easily obtain the further relevant quantities such as the Berry phase or Chern number. A discussion of the respective Berry phases is available in (28). The integral over the closed area of the full first Brillouin zone must be quantized to $2\pi$ times the integer Chern number $C$. From our data, we obtain $C = 0.005(6)$ and $C = -0.016(6)$ for the two different shaking lengths $|b|$ squared (28).

Fig. 3. Momentum-resolved measurement of the Berry curvature. (A) Amplitude (left column) and phase (middle column) obtained from the fits to the oscillations from Fig. 2, performed for each pixel in and around the first Brillouin zone (hexagon). From those fit results, we obtain, as our central result, the momentum-resolved Berry curvature (right column). The experiment was performed for different chiralities of the lattice shaking, destroying the Dirac point at either K (top) or K’ (bottom). (B) Theoretical results from a tight-binding Floquet calculation (28) by using the experimental parameters, yielding a very good agreement. (C) (Top left) A detailed view of a phase vortex [ (A), red square], illustrating the high momentum resolution of our method. The plots show the experimental Berry curvature (blue points) along the high-symmetry path in comparison with the theoretical calculation (red solid lines). In (B) and (C), the Berry curvature is given in units of the inverse reciprocal lattice vector length $|b|$ squared (28).
chiralities (28), which clearly confirms this quantization within the experimental errors. Our measurements demonstrate that even when the global topology has Chern number zero, the distribution of Berry curvature can be very rich. Our measurement scheme can be readily extended to characterize bands with Chern numbers different from zero (47, 49). In principle, one could start in a shallow lattice, where reaching nonzero Chern numbers is feasible, and for the deeper lattices where the bands appear as effectively degenerate)

To extend to high-spin systems (29) or to strongly interacting spin mixtures, which are expected to lead to interesting many-body phases (30–32).

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QUANTUM SIMULATION

Bloch state tomography using Wilson lines
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Topology and geometry are essential to our understanding of modern physics, underlying many foundational concepts from high-energy theories, quantum information, and condensed-matter physics. In condensed-matter systems, a wide range of phenomena stem from the geometric features of band eigenstates, which is encoded in the matrix-valued Wilson line for general multiband systems. Using an ultracold gas of rubidium atoms loaded in a honeycomb optical lattice, we realize strong-force dynamics in Bloch bands that are described by Wilson lines and observe an evolution in the bond populations that directly reveals the band geometry. Our technique enables a full determination of band eigenstates, Berry curvature, and topological invariants, including single- and multiband Chern and Z2 numbers.

Geometric concepts play an increasingly important role in elucidating the behavior of condensed-matter systems. In band structures without degeneracies, the geometric phase acquired by a quantum state during adiabatic evolution elegantly describes a spectrum of phenomena (1). This geometric phase—known as the Berry phase—is used to formulate the Chern number (2), which is the topological invariant characterizing the integer quantum Hall effect (3). However, condensed-matter properties that are determined by multiple bands with degeneracies, such as in topological insulators (4, 5) and graphene (6), can seldom be understood with standard Berry phases. Recent work has shown that such systems can indeed be described using Wilson lines (7–10).

Wilson lines encode the geometry of degenerate states (11), providing indispensable information for the ongoing effort to identify the topological structure of bands. For example, the eigenvalues of Wilson-Zak loops (i.e., Wilson lines closed by a reciprocal lattice vector) can be used to formulate the Z2 invariant of topological insulators (7) and identify topological orders protected by lattice symmetries (8, 9). Although experiments have accessed the geometry of isolated bands through various methods, including transport measurements (3, 12, 13), interferometry (14, 15), and angle-resolved photoemission spectroscopy (16, 17), Wilson lines have thus far remained a theoretical construct (7–10).

Using ultracold atoms in a graphene-like honeycomb lattice, we demonstrate that Wilson lines can be accessed and used as versatile probes of band structure geometry. Whereas the Berry phase merely multiplets a state by a phase factor, the Wilson line is a matrix-valued operator that can mix state populations (11) (Fig. 1A). We measure the Wilson line by detecting changes in the bond populations (17) under the influence of an external force, which transports atoms through reciprocal space (18). In the presence of a force F, atoms with initial quasimomentum q(0) evolve to quasimomentum q(t) = q(0) + Ft/ℏ after a time t. If the force is sufficiently weak and the bands are nondegenerate, the system will undergo adiabatic Bloch oscillations and remain in the lowest band (18). In this case, the quantum state merely acquires a phase factor composed of the geometric Berry phase and a dynamical phase. At stronger forces, however, transitions to other bands occur, and the state evolves into a superposition over several bands.

When the force is infinite with respect to a chosen set of bands, the effect of the dispersion vanishes, and the bands appear as effectively degenerate (Fig. 1B). The system then evolves according to the formalism of Wilczek and Zee for adiabatic motion in a degenerate system (17). The unitary time-evolution operator describing the dynamics is the Wilson line matrix (19)

\[ W_{q(0)\to q(t)} = \mathcal{P} \exp[i \int dq \hat{A}_q] \]  

where the path-ordered (7) integral runs over the path C in reciprocal space from q(0) to q(t) and \( \hat{A}_q \) is the Wilczek-Zee connection, which encodes the local geometric properties of the state space. In a lattice system with Bloch states \( |\Phi_n^\tau\rangle = \delta^\tau n |\nu_n^\tau\rangle \) in the nth band at quasimomentum \( q \), where \( \tau \) is the position operator, the elements of the Wilczek-Zee connection are determined by the cell-periodic part \( |\nu_n^\tau\rangle \) of \( \mathbf{A}_n = i [\mathbf{A}_n, W_{q}] \). The diagonal elements (\( n = n' \)) are the Berry connections of the individual Bloch bands, which yield the Berry phase when integrated along a...