A review on the current applications of genetic algorithms in mean-variance portfolio optimization

Ortalama-varyans portföy optimizasyonunda genetik algoritma uygulamaları üzerine bir literatür araştırması

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Abstract
Mean-variance portfolio optimization model, introduced by Markowitz, provides a fundamental answer to the problem of portfolio management. This model seeks an efficient frontier with the best trade-offs between two conflicting objectives of maximizing return and minimizing risk. The problem of determining an efficient frontier is known to be NP-hard. Due to the complexity of the problem, genetic algorithms have been widely employed by a growing number of researchers to solve this problem. In this study, a literature review of genetic algorithms implementations on mean-variance portfolio optimization is examined from the recent published literature. Main specifications of the problems studied and the specifications of suggested genetic algorithms have been summarized.

Keywords: Portfolio management and optimization, Mean-variance model, Evolutionary algorithms, Genetic algorithm

1 Introduction: Portfolio optimization and mean-variance model

Investors’ desire is to have a non-decreasing fund even if the market is losing value. It is not always possible to achieve this by investing on only one security. In a financial market, it is rare that all securities gain or lose value at the same time.

Therefore, an investor should use a diversification strategy, such as forming a portfolio, to spread the risk among assets. The main question in portfolio management is to decide on the assets and weights for a better investment. A fundamental answer to the problem of portfolio management was given by the mean-variance model [1],[2]. Mathematical formulation of unconstrained portfolio optimization problem (UCPO) according to Markowitz’s standard mean-variance approach is given as follows where parameter $N$ represents the number of available assets, $\mu_i$ represents the expected return of asset $i$, $\sigma_{ij}$ represents the covariance between asset $i$ and asset $j$, $R^*$ represents the expected return at the desired level and variable $w_i$ represent the proportion of asset $i$.

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}$$

Subject to:

$$\sum_{i=1}^{N} w_i \mu_i = R^*$$

$$\sum_{i=1}^{N} w_i = 1$$

($4$)

Equation (1) minimizes risk of the portfolio while equation (2) ensures that expected return ($R^*$) is at the desired level. Equation (3) guarantees that proportions add to one while the proportion of an asset neither can be less than zero nor can be greater than one (Equation (4)). In practice, it is possible to calculate an optimal solution for a particular data set with this formulation. Solving this formulation by varying values of the expected return, an efficient frontier can be found as a non-decreasing curve. This frontier represents the balance of expected return corresponding to risk that must be accepted. Figure 1 demonstrates a standard efficient frontier.

![Figure 1: An example of standard efficient frontier.](image-url)

The mean-variance model has been extended throughout the decade by introducing additional real-world constraints such as the cardinality constraints that impose a predetermined
limit on the number of assets \( (K) \) to be held in the portfolio and the quantity constraints which restrict the proportion of each asset in the portfolio to satisfy lower \( (\varepsilon_i) \) and upper \( (\delta_i) \) bounds. The mixed integer nonlinear programming formulation of cardinality constrained portfolio optimization (CCPO) problem is given as follows with additional parameters: \( K \) representing the desired number of assets to be held in the portfolio, \( \varepsilon_i \) representing the minimum proportion of asset \( i \), \( \delta_i \) representing the maximum proportion of asset \( i \) and \( z_i \) representing a binary variable whether or not an asset \( i \) is held in the portfolio \[3\]:

\[
\min \lambda \left[ \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \right] - (1 - \lambda) \sum_{i=1}^{N} w_i \mu_i \tag{5}
\]

Subject to:

\[
\sum_{i=1}^{K} w_i = 1 \tag{6}
\]

\[
\sum_{i=1}^{K} z_i = K \tag{7}
\]

\[
\varepsilon_i z_i \leq w_i \leq \delta_i z_i \quad i = 1, \ldots, N \tag{8}
\]

\[
z_i \in (0,1) \quad i = 1, \ldots, N \tag{9}
\]

\[
1 \leq w_i \leq 1, \quad i = 1, \ldots, N \tag{10}
\]

\[
0 \leq \varepsilon_i \leq 1, \quad i = 1, \ldots, N \tag{11}
\]

Equation (6) and equation (10) are inherited from the original Markowitz formulation. Equation (7) guarantees that exactly \( K \) assets are held in the portfolio while equation (8) restricts the proportion of an asset to be between predetermined values of minimum and maximum limits with decision variable defined in Equation (9). Equation (11) defines variable domains. The quadratic objective function given in equation (5) seeks the best trade-offs between two conflicting objectives, maximizing return and minimizing risk. In the equation (5), a \( \lambda \) parameter is used to trace the efficient frontier by gradually increasing the value of \( \lambda \) from 0 to 1. Thus, a weighted sum of two objectives is obtained. The resulting single objective aims to construct the cardinality constrained efficient frontier that represents the best balance of expected return and the risk that must be accepted since both objectives cannot be simultaneously achieved. Transaction costs are also considered as additional real life constraints in the literature \[4\]-\[7\]. It is not convenient to find an optimal efficient frontier in practice when real life constraints are taken into account. In fact, calculating an optimal portfolio for the standard mean-variance model is known to be NP-hard \[8\] since a classical quadratic optimization problem becomes NP-hard if a single cardinality constraint is added to the formulation \[9\]. Therefore, in the literature, several computationally efficient solution approaches have been developed in order to calculate the efficient frontier. Among those approaches, genetic algorithms (GA) is one of the most preferred algorithm for solving the problem. Metaxiotis and Liagkouras \[10\] presented a review of multi-objective evolutionary algorithms applied to portfolio management problem in a broad problem perspective. In this study, however, a comprehensive review of GA applications including single and multi-objective implementations specifically in mean-variance portfolio optimization is conducted. This aim of this review is to reveal the problem specifications considered and the key strategies of GA utilized to solve mean-variance portfolio optimization problem types. Section 2 presents the genetic algorithm implementations for mean-variance portfolio optimization while Section 3 concludes the paper with a discussion of future research directions.

2 Genetic algorithms for mean-variance portfolio optimization

GA, firstly introduced by Holland \[11\], is a search method that can be modified to solve complex optimization problems. In GA, a set of iterative search procedures based on biological natural selection and genetic inheritance principals is executed. A population of solutions is updated over generations using selection, crossover and mutation strategies. Each individual that is evaluated in the population represents a potential solution to the problem in hand. Individuals form new individuals a stochastic transformation of individuals is achieved by genetic operators such as crossover and mutation. Crossover provides better solutions to be constructed from good solutions by a random, yet structured change of genetic materials. The role of mutation is to obtain lost or unexplored genetic materials, thereby preventing premature convergence and stuck in local optima. After several iterations, the algorithm converges a (near) optimal solution. Basic steps of the GA are given in Table 1.

| Step | Procedure |
|------|-----------|
| 1    | Generate initial population. |
| 2    | Evaluate fitness of each individual in the population. |
| 3    | Select the set of individuals for applying genetic operators. |
| 4    | Apply genetic operators and evaluate new fitness values. |
| 5    | Form new generation according to fitness values. |
| 6    | Go to step 3 if termination criteria are not satisfied. |
| 7    | End evolution and report the results. |

Table 1: Basic steps of GA.

Components of a typical GA are summarized below:

- Genetic representation (Encoding strategy): The solution of the problem that is formed by binary, integer or real numbers,
- Chromosome: A solution of encoding,
- Population: A set of chromosomes,
- Fitness: A function that evaluates how good a solution is,
- Genetic operators: Procedures such as crossover and mutation that provide to obtain new population from the current population,
- Control parameters: Input parameters such as population size, crossover and mutation rates.

Goldberg \[12\] pointed out search and optimization applications of GA in different areas. Efficient portfolio selection is one of the main concerns of researchers who practice in financial optimization domain. One of the most preferred solution approach for portfolio optimization is GA. Several researchers applied GA variants for solving portfolio optimization problems since 1998 \[12\]. In this study, 44 articles published in conferences and refereed journals between 1998-2016 are examined. Figure 2 shows the number of papers published in recent years for GA implementation on mean-variance portfolio optimization with respect to publication years. Among these studies published in the literature, 3 types of problems; UCPO, CCP and Portfolio
Optimization with Transaction Costs (POTC) come into prominence among other types (See Figure 3).

Figure 2: Number of papers published between 1998-2016 for GA implementation on mean-variance portfolio optimization.

Figure 3: Problems types studied in mean-variance portfolio optimization.

Studies in the literature can be classified in many ways. In this review, two classification schemes for studies that apply GA to mean-variance portfolio optimization are used. First classification is formed according to the problem specifications such as data type, compared methods, problem type and coded programming language. Table 2 provides an up-to-date list of problem specifications. As summarized in Table 2, experimental settings are generally carried out on either real world applications or hypothetical data sets for benchmarking purposes. Methods reported in the literature are generally compared against other metaheuristics either taken from the literature or coded by the authors themselves. Literature analysis show that most of the problems types considered so far consist of UCPO, CCPO and POTC. Most of recent studies focused on CCPO and POTC while UCPO provided a basis for other types of problems.

The second classification scheme is formed according to applied algorithm specifications such as the generation methodology of initial population, size of the population, chromosome representation, crossover type and rate, mutation type and rate, type of selection mechanism, survival type, feasibility construction and termination criteria. Table 3 provides an up-to-date list of algorithm specifications. As summarized in Table 3, single objective GA are widely applied while some multi-objective GA are also suggested for mean-variance portfolio optimization. A two-stage GA is employed in [14] that firstly identifies good quality assets in terms of asset ranking and then optimizes investment allocation in the selected good quality assets. Some hybrid strategies are also suggested as in [15] that utilize quadratic programming approach with GA, in [16] that combines GA with simulated annealing approach and in [17] that utilizes a position displacement strategy of the particle swarm optimization methodology with GA.

GA implementations in the literature shows that initial population is widely preferred to be randomly generated while just a few studies [15],[18] employed heuristic approaches for the construction of initial population. In the studies examined, population size (PS) parameter is set to be 20 as minimum and 2000 as maximum. However, most of the researchers zoomed in the range of 100 and 300 for PS parameter. Binary and real valued chromosome representation are observed to be popular although some other representation strategies such as integer based, tree based are also utilized. Several studies differentiate from each other with the use of crossover operators such as uniform, BLX-Alpha, one point, n-point and mutation operators such as swap, one-point, Gaussian, guided, bit-flipping strategies. Tournament selection is mostly utilized while roulette wheel selection is also used. Elitism and ranking strategies are generally employed for survival of population. Feasibility of chromosomes are ensured by repair or penalty functions. It is observed that iteration number is used as the termination criterion in all of the studies examined.

Although several authors used the data as downloaded from the mentioned OR-Library to test their proposed algorithm, unfortunately, there is a limited number of papers that provide a performance comparison against other published papers in the literature.

In terms of evaluation approaches, there are two types of methodologies in the literature, namely weighted sum and pareto based approaches. As for weighted sum approach, studies make use of equation (5) given in Section 1 by combining two conflicting objectives: risk minimization and return maximization. On the other hand, pareto based methodology, especially used in multi-objective evolutionary algorithms, considers two objectives separately by systematically removing dominated solutions from the heuristic frontier during the search in solution space. Table 3 summarizes the classification.

3 Introduction
Portfolio optimization is a significant problem that intrigues investors and challenges researchers. As GA was established to be a popular technique in the optimization field, the application of GA to optimization problems related to portfolio selection has expanded since 2000 as the problem is known to be NP-Hard. Two different classifications are introduced. Firstly, the main specifications of the problems were summarized, and then implemented GAs with chromosome representations, genetic operators and the fitness functions used for performance evaluation were discussed. 44 articles were examined and grouped in chronological order.

Although, there are several implementations of GA for mean-variance portfolio optimization problem, unfortunately, the improvements and enhancements made to the algorithms’ main framework is not evidently noticeable since there is a limited number of papers that provide a benchmark based comparison against other published studies in the literature.

Therefore, future studies should definitely consider such a comparison that may lead the way towards a better algorithmic design and related software implementations.

Furthermore, it would be very helpful to analyze and compare other heuristics, exact solution approaches as well as metaheuristics applied to solve this problem in a future research study. A comparison on the performance of different approaches would help researchers to move forward in the search of discovering better methodologies for solving portfolio optimization problems.
Table 2: Mean-variance portfolio optimization problem specifications.

| Year | Researcher(s) | Performance specifications / Experimental Settings | Benchmark | Methods compared | Problem type |
|------|---------------|--------------------------------------------------|------------|-----------------|--------------|
| 1998 | Shoaib and Foster [13] | Five stocks from various markets | - | - | UCPO |
| 2000 | T J Chang et al. [3] | Hang Seng, DAX100, FTSE100, S&P100, Nikkei | OR-Library | SA, TS | CCP0 |
| 2000 | Xia et al. [4] | Six stocks | - | - | PCTC |
| 2006 | Lai et al. [14] | One-hundred stocks from Shanghai market | - | - | UCPO |
| 2006 | ChiangLin [19] | Forty-two stocks from Taiwan market | - | - | CCP0 |
| 2006 | Moral-Escudero et al. [15] | Hang Seng, DAX100, FTSE100, S&P100, Nikkei | OR-Library | Exact | CCP0 |
| 2008 | W Chen et al. [5] | Fifteen stocks from China market | - | - | PCTC |
| 2008 | Lin and Liu [6] | Taiwanese mutual fund data from the year 1997 to 2000 | - | - | UCPO |
| 2009 | Aranha and Rha [20] | Thirty stocks in FTSE from 24 April 2001 to 29 December 2006 | PSO, DE | - | UCPO |
| 2009 | Bai and Michel [22] | BSE200 and Nikkei225 | - | - | CCP0 |
| 2009 | Dong et al. [16] | Twenty stocks from Shanghai market | - | - | UCPO |
| 2009 | Shaikh and Abbas [26] | Twenty-three stocks from KSE30 | - | - | UCPO |
| 2009 | Soleimani et al. [7] | 580 and 2000 stocks randomly generated by MATLAB | - | - | POTC |
| 2010 | Anagnostopoulos and Mamanis [27] | FTSE100 | OR-Library | NSGA-2, PES, SPEA2 | CCP0 |
| 2010 | Ruiz-Torrubiano and Suarez [28] | Hang Seng, DAX100, FTSE100, S&P100, Nikkei | OR-Library | SA | CCP0 |
| 2011 | Anagnostopoulos and Mamanis [29] | Hang Seng, DAX100, FTSE100, S&P100, Nikkei | OR-Library | SDEA | |
| 2011 | Anagnostopoulos and Mamanis [30] | DAX100 | OR-Library | - | CCP0 |
| 2011 | Fu et al. [31] | Four stocks from Hong Kong market | - | - | UCPO |
| 2011 | Y Chen et al. [32] | Five-hundred stock from Tokyo market | - | - | UCPO |
| 2011 | Kremml et al. [33] | Software projects | SPEA2, NSGA-2 | - | UCPO |
| 2011 | Woodsidge-Uriahtli et al. [34] | Hang Seng, DAX100, FTSE100, S&P100, Nikkei | OR-Library | TS, SA | CCP0 |
| 2012 | Sadjadi et al. [35] | Hang Seng, DAX100, FTSE100, S&P100, Nikkei | OR-Library | - | CCP0 |
| 2013 | Li and Wang [18] | Six stocks from Chinese market | - | - | UCPO |
| 2013 | Li and Yang [36] | Eight stocks from china market | - | - | UCPO |
| 2014 | Akcoca-Trab et al. [37] | Randomly selected five stocks from Ghana market | - | - | UCPO |
| 2014 | Joglerov et al. [38] | Seventy-two stocks from MSN Money | - | - | UCPO |
| 2014 | Liagkouras and Metaxiotis [39] | Hang Seng, DAX100, FTSE100, S&P100, Nikkei | OR-Library | MOEA-PLM | CCP0 |
| 2014 | Lwini et al. [40] | Hang Seng, DAX100, FTSE100, S&P100, Nikkei, S&P500, BSE200 and Nikkei225 | OR-Library | NSGA-2, SPEA2, Russel2000 | |
| 2015 | Adebiyi Ayodele and Ayo Charles [41] | Hang Seng, DAX100 | OR-Library | SA, TS, PSO | CCP0 |
| 2016 | Hadi et al. [42] | Forty-five stocks from Egypt market | - | - | CCP0 |
| 2016 | Mashayekhi and Omran [43] | Fifty-two stocks from Iran market | - | - | CCP0 |

Table 3: Genetic algorithm specifications.

| Year | Researcher(s) | Evolutionary algorithm specifications |
|------|---------------|--------------------------------------|
| 1998 | Shoaib and Foster [13] | GA Weighted sum PS=100 Binary and real value 2-Point & Rc=60% Rm=0.1% Roulette wheel - Penalty Iteration number |
| 2000 | T J Chang et al. [3] | GA Pareto based Random & PS=100 Integer and real value Uniform & Rc=100% One-point mutation & Rm=10% Roulette wheel - Repair Iteration number |
| 2000 | Xia et al. [4] | GA Pareto based Random & PS=30 Integer and real value Uniform & Rc=30% Roulette wheel - Repair Iteration number |
| 2006 | Lai et al. [14] | 2 stage GA Weighted sum Random & PS=100 Integer and real value One point crossover & Rc=50% Rm=0.5% Roulette wheel - Repair Iteration number |
| 2006 | ChiangLin [19] | GA Pareto based Random & PS=100 Integer and real value One point crossover & Rc=100% Rm=3% Roulette wheel - Iteration number |
| 2006 | Moral-Escudero et al. [15] | Hybrid GA and quadratic programming Weighted sum Heuristic & PS=100 Binary Uniform & Rc=100% Swap mutation & Rm=1% Tourname nt steady-state Repair and Penalty - |
| 2008 | W Chen et al. [5] | GA Pareto based Random & PS=30 Real value One point crossover & Rc=variable Roulette wheel - Repair Iteration number |
| 2008 | Lin and Liu [6] | GA Weighted sum Random & PS=n Integer and real value One point crossover & Rc=100% Uniform & Rc=variable Roulette wheel - Replaceme nt Penalty Iteration number |
| Year | Researcher(s) | Method | Evaluation approach | Initial population & size | Chromosome type & representation | Crossover type & rate (%) | Mutation type & rate (Rm) | Selection type | Survival type | Possibility (Repair or Penalty) | Termination criteria |
|------|---------------|--------|---------------------|---------------------------|-------------------------------|--------------------------|--------------------------|----------------|--------------|-----------------------------|-------------------|
| 2009 | Arana and Iba [20] | Tree based GA | Weighted sum | P=200 Tree based | Best-Worst Sub-tree | Swap mutation & Rm=3% | Tournament nt | Ranking | - | - | Iteration number |
| 2009 | Branke et al. [21] | envelop e-based MOEA | Pareto based Random & PS=30 | Binary and real value | Uniform & Swap mutation | Tournament nt | Ranking | - | - | Iteration number |
| 2009 | T-J Chang et al. [22] | MOEA | GA Pareto based Random & PS=100 | Binary and real value | Uniform & Rc=100% | Tournament nt | steady-state | Repair | Iteration number |
| 2009 | Li and Guo [23] | GA | Weighted sum Random & PS=100 | Binary and real value | BLX-Alpha | Bit-flipping mutation | Real values uniform | Tournament nt | - | - | Iteration number |
| 2009 | Loukeris et al. [24] | GA | Pareto based Random & PS=100 | Binary and real value | Arithmetic & Rc=80% | Non-uniform | Tournament nt | - | Penalty | Iteration number |
| 2009 | Pai and Michel [25] | GA | Pareto based Random & PS=100 | Binary and real value | Uniform & Rc=90% | Gaussian & Rm=100% | Tournament nt | Elitism | Repair | Iteration number |
| 2009 | Rong et al. [16] | Hybrid (GA and SA) | Pareto based Random & PS=20 | Binary and real value | RAR & Rc=100% | Bit-flipping mutation & Rm=1% | Tournament nt | Ranking | Repair | Iteration number |
| 2009 | Shaikh and Abbas [26] | GA | Weighted sum Random & PS=20 | - | - | - | Tournament nt | - | - | Iteration number |
| 2009 | Soleimani et al. [7] | NSGA-II, PESA, SPEA | GA Weighted sum Random & PS=100 | - | Random separate & Rc=100% | Rm=50% | Randomly | Ranking | - | - | Iteration number |
| 2010 | Anagnostopoulos and Mamanis [27] | GA | Pareto based Random & PS=200-300 | Binary and real value | Uniform & Rc=90% | Gaussian & Rm=100% | Tournament nt | Elitism | Repair | Iteration number |
| 2010 | Ruiz-Torresbiano and Suarez [28] | MOEA | Pareto based Random & PS=250 Heuristi c & PS=500 | Integer and real value | Uniform | Swap mutation & Rm=1% | Tournament nt | Elitism | Repair | Iteration number |
| 2011 | Anagnostopoulos and Mamanis [29] | MOEA | Pareto based Random & PS=500 | Integer and real value | Uniform | Swap mutation & Rm=1% | Tournament nt | Elitism | Repair | Iteration number |
| 2011 | Fu et al. [31] | GA | Pareto based Random & PS=100 | - | - | - | Tournament nt | - | - | - | |
| 2011 | Y Chen et al. [32] | GAs | Weighted sum Random & PS=100 | nodes and edges | Node Swap & Rc=20% | Guided & Rm=3% | Tournament nt | Elitism | - | Iteration number |
| 2011 | Kremmel et al. [33] | MOEA | Weighted sum Random & PS=500 | Binary | Uniform & Rc=70% | Bit-flipping mutation & Rm=3% | Tournament nt | Repair | - | Iteration number |
| 2011 | Woodside-Oriakhii et al. [34] | GA | Pareto based Random & PS=100 | Integer and real value | Uniform & Rc=100% | Swap mutation & Rm=100% | Tournament nt | Elitism | Repair | Iteration number |
| 2012 | Sadjadi et al. [35] | GA | Weighted sum Random & PS=10 Integer | One point crossover & Rc=80% | Rm=20% | Roulette wheel and Uniform | Ranking | Repair | - | - | |
| 2012 | Lu and Wang [16] | GA | Pareto based Integer & PS=60 | Integer and real value | One point crossover & Rc=80% | Rm=77.8% | - | Penalty | Iteration number |
| 2013 | Yi and Yang [36] | FGA | Weighted sum Random & PS=100 | Real value | Heuristic & Rm=3% | Roulette wheel | Elitism | - | Iteration number |
| 2013 | Ackora-Prah et al. [37] | GA | Pareto based Random & PS=50 | Real value | Uniform & Rm=20% | Roulette wheel | Elitism | - | Iteration number |
| 2014 | Joglekar [38] | MOEA | Weighted sum Random & PS=100 | Binary and real value | Simulated Binary | Guided | Tournament nt | Repair | - | - | |
| 2014 | Liwol et al. [40] | MOEA | Pareto based Random & PS=vari able | Binary and real value | Rr=90% | Polynomial | Tournament nt | Elitism | Repair | Iteration number |
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Table 3: Cont.

| Year | Researcher(s) | Method | Evaluation approach | Initial population & size | Chromosome representation | Crossover type & rate (%) | Mutation type & rate (%) | Selection type | Survival type | Possibility (Repair or Penalty) | Termination criteria |
|------|---------------|--------|---------------------|---------------------------|--------------------------|--------------------------|--------------------------|----------------|----------------|-------------------------------|---------------------|
| 2015 | Adebisy Ayodele and Ayo Charles [41] | GDE | Weighted sum | - | - | - | - | - | - | - | - |
| 2016 | Hadi et al. [42] | GA | Pareto based | Random | Real value | - | - | - | - | - | Iteration number |
| 2016 | Mashayekhi and Omrani [43] | NSGA-II | Pareto based | Random & Binary and real value | Rec=80% Gaussian & Rm=10% | Tournament | - | Repair | - | Iteration number |

a: Number of stocks, MOEA: Multi objective evolutionary algorithm, SA: Simulated annealing, NSGA-K: Non-dominated sorting genetic algorithm, PESA: Pareto envelope-based selection algorithm, SPEA2: Strength pareto evolutionary algorithm 2, GRA: Genetic relation algorithm, PSO: Particle swarm optimization, FGA: Fuzzy genetic algorithm, GDE: Generalized differential evolution.
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