Representation of coherent structures and turbulence spectra from a virtual SpinnerLidar for future LES wake validation

Kenneth Brown, Alan Hsieh, Thomas Herges, and David Maniaci
Sandia National Laboratories, Albuquerque, NM, USA

kbrown1@sandia.gov

Abstract. Work has begun towards model validation of wake dynamics for the large-eddy simulation (LES) code Nalu-Wind in the context of research-scale wind turbines in a neutral atmospheric boundary layer (ABL). Interest is particularly directed at the structures and spectra which are influential for wake recovery and downstream turbine loading. This initial work is to determine the feasibility of using nacelle-mounted, continuous-wave lidars to measure and validate wake physics via comparisons of full actuator line simulation results with those obtained from a virtual lidar embedded within the computational domain. Analyses are conducted on the dominant large-scale flow structures via proper orthogonal decomposition (POD) and on the various scales of wake-added turbulence through spectral comparisons. The virtual lidar adequately reproduces spatial structures and energies compared to the full simulation results. Correction of the higher-frequency turbulence spectra for volume-averaging attenuation was most successful at locations where mean gradients were not severe. The results of this work will aid the design of experiments for validation of high-fidelity wake models.

Keywords: virtual lidar, large eddy simulation, wake physics, coherent structures, proper orthogonal decomposition, turbulence spectra, model validation, wind energy

1. Introduction

1.1. Overview of Wake Physics

Wake flow physics is an important subject in the areas of siting, optimization, control, and loads analyses of wind farms, especially because of the implications of the wake flow for downstream turbines. The wake development begins in the near-wake where a rich mix of coherent flow features propagating downstream from an individual turbine include a velocity deficit on the scale of the rotor diameter, swirling motion opposite the blade rotation, an initially two-dimensional wake structure from the tower structure, helicoidally-tracking vorticity (tip, root, and shed components), a columnar hub vortex, lateral spreading due to a pressure differential with the freestream, and entrainment of mean-flow kinetic energy which promotes recovery, as well as any structures which might be recognizable from the ABL inflow. The near-wake region of organized flow spans approximately 2 - 4 diameters downstream depending on atmospheric conditions before the transition to a far-wake region which features bulk meandering at wavelengths on the order of two rotor diameters or greater as well as increasingly homogeneous flow [1-3].
1.2. Validation of Simulations
Relatively few studies have delved into higher-order validation of wake dynamics quantities such as through analyses of coherent structures and turbulence spectra. One exception is the work of Andersen et al. [1] whose POD analysis showed streamwise planar measurements in the near wake which had more gradual energy roll-off with mode number than LES simulations. Jimenez et al. [4] validated lateral and vertical coherence predicted by LES versus wind turbine measurements with good agreement between the two. Despite such insights on the validation of wake dynamics from the existing literature, there remains a need to guide the improvement, calibration and validation of numerical models using high-quality wind tunnel and field experiments [5].

Validation is underway of the wake modeling fidelity of the emerging LES code Nalu-Wind using actuator line formulation [6]. The mean wake produced by Nalu-Wind for a research-scale wind turbine was shown in [7] to compare favorably with measurements for a range ofABL stability conditions. The coherent structures and turbulence spectra of the wake predictions have not yet been validated which is the ultimate goal of the work begun here.

The present scope is to determine the feasibility of using nacelle-mounted, continuous-wave lidar for model validation with emphasis on the ability to resolve spatial and spectral aspects of the wake physics including large-scale structures as well as finer-scale wake-added turbulence. For the present work, we do not yet consider experimental results but use LES as the source of truth against which to assess the capabilities of a virtual lidar embedded in the LES domain.

Section 2 introduces the future validation case that is the basis for the design of the LES domain and sampling strategies as described in Section 3. Section 4 contains our main analysis in two-fold fashion, first benchmarking the virtual lidar results against the full simulation data in terms of spatial POD modes before inspecting the spectral cascade. Conclusions and future work are discussed in Section 5.

2. Validation Case
This section introduces the physical layout and flow parameters for the experimental benchmark study which is the template for the computational domain in this study. The experimental data are derived from measurements at the Scaled Wind Farm Technology (SWiFT) facility [8] in Lubbock, Texas, USA. As shown in figure 1, the baseline instrumentation includes three research turbines (WTG) and two meteorological towers (MET). The SWiFT turbines are variable speed, collective pitch Vestas V27 machines with a hub height of 32.1 meters and a rotor diameter, $D$, of 27 meters. The boundary conditions specified in the simulations below come from time-averaged measurements over six 10-minute intervals by META in a neutral boundary layer as described in [7]. The mean tip-speed ratio, $\lambda$,
height wind speed, streamwise turbulence intensity, and power-law exponent of the vertical velocity profile were 7.3, 8.69 m/s, 10.7%, and 0.14, respectively. Experimental data used for validation come from a series of measurements using the DTU SpinnerLidar mounted on turbine WTGa1 which have previously been used for benchmarking time-averaged predictions from various models [7].

Some details about this lidar measurement configuration are useful for understanding the analysis below of its computational counterpart. The lidar scans were completed over a rosette pattern in 2 s intervals and consisted of 984 points measured at locations between 2 – 5D downstream. The lidar probe length provides filtering in the beamwise (near-streamwise) direction which offers a degree of temporal anti-aliasing filtering since flow features are predominantly convected downstream [9]. As a single lidar only provides line-of-sight information along the beam direction, the POD analysis below closely represents a one-component slice-POD analysis [10]. When interpreting the POD results, the non-simultaneity of the sampling at different locations over the 2 s scan period should be remembered.

3. Computational Setup

3.1. LES Domain

The multi-physics, massively parallel LES code Nalu-Wind [6] was used to simulate the turbulent ABL. Nalu-Wind solves the Navier-Stokes equations in the low-Mach number approximation and applies a one-equation, constant coefficient, turbulent kinetic energy (TKE) model for the subgrid scale stresses. The coupled dynamic response of the wind turbines is performed through the OpenFAST software suite developed at the National Renewable Energy Laboratory (NREL) [11]. In the present simulations, the actuator line model [12] is applied to project the non-volumetric actuator forces computed in OpenFAST into the fluid volume using a uniform Gaussian projection function.

Simulating wind turbines within a turbulent ABL is a two-step process in Nalu-Wind. First, the neutral ABL was established using a precursor simulation. The boundary layer was initialized with small perturbations near the surface to accelerate turbulence development, and the precursor was run for 20,000 seconds to establish a fully developed turbulent flow. The simulation domain was 3 km by 3 km by 1 km in the x-, y-, and z-directions, respectively, with x representing the streamwise direction, y representing the transverse direction, and z representing the vertical direction. The precursor simulation used a 10 m uniform resolution mesh.

Second, the turbine was added to the domain. The turbine, which was coupled with OpenFAST using the actuator line model, was located near the center of the domain while matching the configuration of the SWiFT site including the turbine tower but excluding the nacelle. Replacing the precursor mesh, a refined mesh was introduced with four levels of refinement around the turbines; the minimum grid spacing was 0.625 meters.

The turbine was simulated for over 3434 seconds using a time step of 0.02 seconds, and the first 60 seconds were discarded to eliminate transient effects. The time history of the hub-height freestream velocity, \( U_{\infty} \), from the simulated WTGa1 turbine is shown in figure 2. Another turbine, WTGa2, was also operating in the simulation domain though is not considered further. Potential effects of sampling through the second turbine’s rotor plane should be considered when interpreting the results. For more

Figure 2. Time history of the simulated hub-height freestream velocity, \( U_{\infty} \), from the upstream turbine (WTGa1). The time range used for the present analysis is from 60 to 3434 s.
information and details about the methodology, meshes, and specifications used in the present and other similar Nalu-Wind simulations, readers are referred to [13].

3.2. Planar Sampling

Wake planes were sampled at 0.5D, 2D, and 5D downstream of WTGa1. Spectral processing of the planar data is performed at the finest 0.02 s data rate to prevent any aliasing effects. Prior to the POD analysis, the data are averaged into 2 s blocks for computational memory reasons to give a total of 1688 frames. Such averaging should have little impact on the large-scale flow structures considered in the POD analysis and serves to minimize the effects of unwanted periodicity from the induction plane and wake of WTGa2 for the data at 5D.

Based on the data at 0.5D, 2D, and 5D, integral timescales in the wake, \( \tau_{\text{wake}} \), were computed to be 30, 14, and 7 s, respectively, from the average across the swept area of the integral of the autocorrelation up to the first zero crossing. The value of the integral timescale in the wake, rather than the conventional freestream value, is reported since the number of independent flow realizations in the wake is an important metric for the convergence of POD calculations. Furthermore, this waked value is relevant when assessing the effect of the non-simultaneous data capture over the 2 s lidar scan pattern.

3.3. Lidar Sampling

To mimic the real-life lidar measurements, a virtual lidar is embedded in the computational domain [see also 14]. Along with the planar sampling described above, the flowfield is sampled along a radial vector emanating from the mounted lidar position that scans through the 984-point rosette pattern depicted in figure 1. The scanning vector extends 200 m and acquires the line-of-sight velocity at discrete points matching the resolution of the finest grid scale (0.625 m). To simplify the data extraction from the simulation, all points along the sampling vector at all 984 positions within the scan pattern are acquired at each 0.02 s time step. Since the DTU SpinnerLidar acquires 984 points at a rate faster than 0.02 s, the values along the vectors are linearly interpolated in time to match the lidar sample schedule from the beginning position to the end, smoothing out the scanning effect of the lidar over the 2 s scan.

Next, a truncated window probe volume weighting is applied along each vector to obtain results corresponding to the desired focus distances. The truncated window weighting is similar to but more physical than the Lorentzian weighting function [15]. The laser wavelength and effective telescope radius are 1.55 \( \mu m \) and 24 mm, respectively, for the SpinnerLidar. The probe length, as described by the full-width half-maximum value, is roughly 7 m at a focal range of 53 m and 26 m at a range of 103 m [16]. The ranges used below span between 2D and 5D which represent the approximate limits of the SpinnerLidar for the SWiFT turbines.

For the work presented below, the lidar line-of-sight velocity was projection-corrected to the streamwise direction [17], and the camber of the lidar arc was neglected. These approximations should be borne in mind when interpreting the results.

4. Results

For information on the time-averaged velocity profiles and reference inflow conditions for the case considered below, readers are directed to [7]. We here focus our attention on the study of wake dynamics which will be examined through the lenses of POD and spectral analyses in the following two subsections. Note that all analyses are performed in the fixed frame of reference.

4.1. Flow Structure

To capture large-scale coherent flow structures, POD works to systematically filter out all but the most relevant energy-containing features of the flow. This is accomplished by searching for the set of orthogonal basis functions, or eigenmodes, which most optimally capture the variance – or in the case of fluid flow, the turbulent kinetic energy (TKE) – of a set of observations. The eigenvalue, \( \lambda_j \), corresponding to the \( j^{\text{th}} \) mode is the mean TKE of the mode. Here, we will consider the streamwise component of TKE as a surrogate for the full value. A common formulation of POD as used below is a
space-only form of POD, the theoretical details of which are clearly conveyed in [18] and omitted here for brevity.

For the planar data, the sample points are uniformly distributed in space, so no special consideration for the spatial weights must be made in the POD calculation other than to set the weighting matrix to an identity one [19]. For the lidar data, the sample points are initially non-uniform, so they are first interpolated to the same grid as the planar data using triangulation-based interpolation. Delaying the interpolation until after the POD calculation (and erroneously applying the uniform weighting) was found to qualitatively alter the higher-order mode shapes starting by the fourth mode for the $5D$ location.

The number of snapshots used in the analysis is determined similarly as in [20]. To assess convergence of the POD modes, the total simulation length was held constant while the interval between frames was incrementally decreased as shown in figure 3 for both the planar and virtual lidar data. For a POD calculation with $N$ frames, the normalized $\text{TKE}$ represented in the modes is denoted by $\eta^N$ as defined in figure 3. All cases are converged after $N = 844$ frames, where convergence is achieved when $E(N) < 0.005$ (see figure 3 for notation). In addition to the convergence of mode energies, the first 9 modes are qualitatively similar in structure between the $N = 844$ and $N = 1688$ cases. The convergence of mode energy and structure is thus achieved by $N = 844$, and the analysis below is conservatively performed at the full $N = 1688$.

Note that the total simulation time corresponds to $112 - 491 \tau_{\text{wake}}$ depending on the streamwise location, so the results may not be statistically converged even though the mode energies have converged for the given simulation time [see ref. 21 for context]. It is worth noting, however, that any incomplete convergence of the snapshots is manifested in both the planar data and the lidar data, so this potential nonconvergence issue does not preclude a useful analysis in this article.

Normalized mode shapes are plotted in figure 4. The first mode at $0.5D$ is unique from all other modes in the decomposition as it is relatively broad (i.e. – no zero crossings outside of the small tower wake region). The relatively large and slowly-varying magnitude of the expansion coefficients (not shown) for this mode are similar to the time series of velocity in figure 2 and confirm that this mode represents the low frequency change in effective freestream velocity over the entire rotor plane due to large-scale ABL structures. This broad mode shape is not present at the $2D$ and $5D$ locations which suggests that smaller structures become relatively more energetic as the wake propagates downstream.

The second and third modes (first and third for the $5D$ case) exhibit dipole character about the vertical and horizontal symmetry planes, respectively. The outer shape of the modes traces the tips of the blades at the $0.5D$ location while the locations further downstream show more expansive structures. Despite some blurring of structures possibly due to incomplete convergence, it can be understood that mode 3 is the orthogonal pair of mode 2 at the $0.5D$ and $2D$ locations. Orthogonal mode pairing is not uncommon in POD analyses [22] and occurs when two modes represent the same
Figure 4. Cross-stream slices of the mean flow and POD modes at the indicated streamwise locations. Each contour is scaled on its maximum absolute value (red = 1 and blue = -1). The rotor area is indicated by the white outline, and the rosette scan pattern is overlaid in gray for the lidar cases. Values above the mode shapes are the percent of explained variation in streamwise TKE per mode. Note that the unusual contour of the mean flow data at 5D is due to the influence of WTGa2’s rotor (without nacelle).
structure but are out of phase by $\pi/2$. The $2-3$ mode pairing is present in both the planar and lidar results and represents the bulk motion, or meandering, of the wake as it responds to the stream-normal forcing of the ABL inflow. As will be shown further below in figure 5, the second mode contains significantly more energy than the third mode since the motion of the latter is damped by the presence of the ground.

At least at the $2D$ and $5D$ locations, the fourth through sixth modes also have defined structures with modes 4 and 5 being orthogonal pairs of an elongated structure and mode 6 being a quadrupole, as shown. These modes and others at higher order describe the fluctuating distortions of the wake deficit. The dipole and quadrupole modes, which are visible in both planar and lidar data at each streamwise location shown, are similar to those of the slice-POD in Sorensen et al. [2]. The different mode shapes have varying significance for predicting wake recovery while all the mode types are important from a validation standpoint since they describe the large-scale structures that will impact downstream turbines.

At the nearer lidar measurement position of $2D$, the lidar successfully captures the qualitative character of the planar mode shapes for all modes save potentially the first. At the farther wake position of $5D$, discrepancies with the planar data for the first two modes are apparent which could be a result of sampling at the rotor plane of the downstream turbine.

Figure 5 plots the fraction of energy in the flow decomposition, $\lambda_j/\sum \lambda$, contained in the six modes shown in figure 4 versus their streamwise position for both the virtual lidar and planar dataset. For the first mode at the $0.5D$ location, the modulation of effective freestream velocity due to the large-scale ABL structures contributes more than three times more energy than any other single mode due to the relatively high 10.7% turbulence intensity in the inflow. The relative contribution of this mode is reduced moving to $2D$, plausibly because mean-flow kinetic energy is converted to TKE through the production term in the Reynolds stress transport equation. This production apparently causes growth of $\lambda_j/\sum \lambda$ for each of the other five modes between the $0.5D$ and $2D$ locations. Moving from $2D$ to $5D$, the total $\lambda_j/\sum \lambda$ of modes 2 - 6 continues to increase by 0.042 and 0.145 for the planar and lidar cases, respectively. The above findings should be taken with caution since it is noted that the relative area of the wake region in the POD analysis window increases with increasing focal range as shown by comparing the mean flow contours for the $0.5D$ and $2D$ locations in figure 4 (the effect at $5D$ is countered by the induction of WTGa2). The coarsening resolution of the lidar at larger focal ranges also adds uncertainty to the values of $\lambda_j/\sum \lambda$.

![Figure 5](image-url)
While the virtual lidar shows agreement with planar results at the 2D location, the results diverge at the 5D location for higher modes. Modes 4 - 6 for the lidar all show an unexpected drop in $\lambda_i/\sum \lambda$ leading into the 5D location. This drop is hypothesized to be primarily a result of the beamwise averaging through the probe volume which causes flow in the wake of the downstream turbine to be averaged into the lidar results at the 5D location. The roughly quadratic growth of the lidar probe volume with measurement range may also be a contributing factor.

4.2. Flow Scales

The spectral behavior of a flow is studied through the power spectral density of well-resolved time series at representative locations in the flow. Such is the case in figure 6 which is given at $x = 2D$ for both the planar and virtual lidar data. The points for comparison are at the tip of the rotor where the wake-added turbulence is prominent. The data show conformance to the -1 scaling of the production subrange and the -5/3 scaling of the inertial subrange. We note that the frequency range of the lidar is limited due to the 0.5 Hz sampling ceiling of the SpinnerLidar which prevents the capture of the first blade passage Strouhal number, $St = fD/\omega$, at around 9.6.

Without correction, the lidar spectra are attenuated at higher $St$ due to the spatial averaging of finer turbulence structures within the probe volume. A correction is applied which is based on the Fourier transform of the Lorentzian weighting function [23]. Note that the Lorentzian weighting function is an approximation of the truncated windowing function used for the virtual lidar sampling. For the two positions located higher in the swept area (blue and green), the corrected lidar spectra fall near their respective planar spectra except for a deviation near high frequency for the lidar data at hub-height. The authors speculate that this deviation could be tied to the bulk meandering of the wake, and residual aliasing effects are also possible in the lidar data despite the low-pass filtering effect of the volume averaging. For the lowest scan position (teal), the corrected lidar results are clearly too low which will be discussed below.

Figure 7 offers a perspective on the suitability of the corrected lidar results for capturing flow scales relevant to downstream turbine loading. Here are plotted the pre-multiplied power spectra as well as are tabulated corresponding resolved-scale streamwise turbulence intensities, $T_i = \langle (U')^2 \rangle^{0.5}/\langle U \rangle$, where $U$ is the local streamwise velocity and the prime notation denotes the fluctuating component. For reference, spectra from the freestream flow outside the waked region are also plotted. The figure emphasizes the wide range of scales over which the rotor injects $\lambda_1/\lambda_2$ into the spectrum due to instability of the mixing layer. In terms of $St$, the range of scales of injected energy spans the range of computed values, 0.04 - 0.8, aligning with those reported by [24] between 0.05 - 5. The location of maximum turbulence enhancement is roughly $St \approx 0.15$ as compared to 0.35 for [24]. Considering the distribution of energy within the spectra, the planar data are in close agreement within the inertial subrange and show wider variation at larger scales, presumably because of the spatially-dependent shear instabilities.

From figure 7, the corrected lidar data indicates the presence of the wake-added turbulence, though the current lidar correction is evidently too weak. For the lowest scan position (teal), the corrected lidar levels are even partially below the freestream levels at the same height. The authors speculate that the strong mean gradients through the beam path are responsible for this discrepancy, especially since the beamwise averaging volume at the lowest scan position includes the tower shadow and near-ground regions. In fact, for all the lidar positions, the volume averaging combined with the coarse resolution of the scan pattern smooths over the sharp gradients at the shear layer, a process for which the correction model cannot account. The gradients at the edge of the wake can be compared between the planar and lidar results from the mean flow contours of figure 4.

The discrepancy of the lidar data from the lowest scan position (teal) with the planar results is also apparent in the integrated quantity $T_i U_i$ where the lidar value is 0.103, or 42%, low. The hub-height (green) and tip-top (blue) scan positions have errors of 0.067 (24%) and 0.075 (25%), respectively. Note that sub-grid scale turbulence is not included in any of the turbulence estimates.
5. Conclusions and Future Work

Evaluation has begun of the feasibility of using nacelle-mounted, continuous-wave lidar to resolve coherent structures and turbulence spectra towards the goal of high-fidelity model validation. The LES code Nalu-Wind was employed in actuator line formulation to produce high resolution wake flows with an embedded virtual lidar. The parameters of the simulation were derived from experimental results in
a neutral ABL at the candidate site for validation measurements. The analysis of the virtual lidar’s spatial resolution was facilitated with POD which revealed that the lidar can adequately reproduce large-scale mode structures and energies compared to the full simulation results including dipole- and quadrapole-type modes. Initial attempts at correction of the higher-frequency turbulence spectral content for volume-averaging attenuation were partially successful. The results of this work aid the design of experiments for validation of high-fidelity wake models. Specifically, the need to adequately resolve sharp gradients may limit the possible configurations and/or placements of the lidar due to the smoothing which stems from probe-volume averaging. Furthermore, performing the above analyses in the meandering frame of reference would be helpful to filter out the bulk effect of the ABL forcing in the stream-normal directions. Analysis of the modes from a dynamic perspective may also yield information on which structures are most important to resolve from a wake recovery standpoint.

Acknowledgements

The authors gratefully acknowledge Torben Mikkelsen and the team at the Technical University of Denmark (DTU) for their collaboration surrounding the SpinnerLidar. The first author thanks Nicholas Hamilton of NREL for his helpful comments on the manuscript.

This article was prepared by Sandia National Laboratories (SNL) Albuquerque, NM, 87185, U.S.A. SNL is a multi-mission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy’s National Nuclear Security Administration under contract DE-NA-0003525.

References

1. Andersen, S.J., et al. Comparison between PIV measurements and computations of the near-wake of an actuator disc. in Journal of Physics: Conference Series. 2014. IOP Publishing. 524 012173.
2. Sorensen, J.N., et al., Simulation of wind turbine wakes using the actuator line technique. Philos Trans A Math Phys Eng Sci, 2015. 373(2035).
3. Larsen, T.J., et al., Validation of the dynamic wake meander model for loads and power production in the Egmond aan Zee wind farm. Wind Energy, 2013. 16(4): p. 605-624.
4. Jimenez, A., et al., Large-eddy simulation of spectral coherence in a wind turbine wake. Environmental Research Letters, 2008. 3(1): p. 015004.
5. Porté-Agel, F., M. Bastankhah, and S. Shamsoddin, Wind-turbine and wind-farm flows: a review. Boundary-Layer Meteorology, 2020. 174(1): p. 1-59.
6. Domino, S., Sierra low mach module: Nalu theory manual 1.0. SAND2015-3107W, Sandia National Laboratories Unclassified Unlimited Release (UUR), 2015.
7. Dohbrw, P., et al., Multimodel validation of single wakes in neutral and stratified atmospheric conditions. Wind Energy, 2020.
8. Berg, J., et al. Scaled wind farm technology facility overview. in 32nd ASME Wind Energy Symposium. 2014. 1088.
9. Citriniti, J.H. and W.K. George, Reconstruction of the global velocity field in the axisymmetric mixing layer utilizing the proper orthogonal decomposition. Journal of Fluid Mechanics, 2000. 418: p. 137-166.
10. Glauser, M.N. and W.K. George, Application of multipoint measurements for flow characterization. Experimental Thermal and Fluid Science, 1992. 5(5): p. 617-632.
11. NWTC Information Portal (OpenFAST). 14-June-2016; Available from: https://nwtc.nrel.gov/OpenFAST.
12. Churchfield, M.J., et al. An advanced actuator line method for wind energy applications and beyond. in 35th Wind Energy Symposium. 2017. 1998.
13. Hsieh, A., et al. Multilevel Uncertainty Quantification Using CFD and OpenFAST Simulations of the SWiFT Facility. in AIAA Scitech 2020 Forum. 2020. 1949.
14. Gasch, P., et al., An LES-based airborne Doppler lidar simulator for investigation of wind profiling in inhomogeneous flow conditions. Atmospheric Measurement Techniques, In press.
15. Horváth, Z.L. and Z. Bor, Focusing of truncated Gaussian beams. Optics communications, 2003. 222(1-6): p. 51-68.
16. Mikkelsen, T., et al., A spinner-integrated wind lidar for enhanced wind turbine control. Wind Energy, 2013. 16(4): p. 625-643.
17. Kelley, C.L., et al. Wind turbine aerodynamic measurements using a scanning lidar. in Journal of Physics: Conference Series. 2018. IOP Publishing. 1037 052014.
18. Towne, A., O.T. Schmidt, and T. Colonius, Spectral proper orthogonal decomposition and its relationship to dynamic mode decomposition and resolvent analysis. Journal of Fluid Mechanics, 2018. 847: p. 821-867.
19. Schmidt, O.T. and T. Colonius, Guide to spectral proper orthogonal decomposition. AIAA Journal, 2020. 58(3): p. 1023-1033.
20. Newman, A.J., D.A. Drew, and L. Castillo, Pseudo spectral analysis of the energy entrainment in a scaled down wind farm. Renewable energy, 2014. 70: p. 129-141.
21. VerHulst, C. and C. Meneveau, Large eddy simulation study of the kinetic energy entrainment by energetic turbulent flow structures in large wind farms. Physics of Fluids, 2014. 26(2): p. 025113.
22. Hamilton, N., et al., A generalized framework for reduced-order modeling of a wind turbine wake. Wind Energy, 2018. 21(6): p. 373-390.
23. Angelou, N., et al., Direct measurement of the spectral transfer function of a laser based anemometer. Review of scientific instruments, 2012. 83(3): p. 033111.
24. Bastine, D., et al. Characterizing wake turbulence with staring lidar measurements. in Journal of Physics: Conference Series. 2015. IOP Publishing. 625 012006.