A Feynman-graph generator for any order of coupling constants

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abstract

A computer program has been developed which generates Feynman graphs automatically for scattering and decay processes in non-Abelian gauge theory of high-energy physics. A new acceleration method is presented for both generating and eliminating graphs. This method has been shown to work quite efficiently for any order of coupling constants in any kind of theoretical model. A utility program is also available for drawing generated graphs. These programs consist of the most basic parts of the GRACE system, which is now used to automatically calculate tree and one-loop processes.

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1 Introduction

In regard to recent projects concerning experimental high-energy physics with increasing available energy, accurate theoretical analysis has been required for increasing the number of scattering processes. Since electro-weak theory combined with QCD, known as the standard model, is considered to be a basic theory among various theoretical models, exact perturbative calculations within the framework of this model are considered to be the standard for theoretical predictions. Because of the complexity of interactions, the number of Feynman graphs for one process becomes much greater than that in QED. There appear several tens to hundreds of graphs in the tree process and several hundreds to thousands in the one-loop process. In addition, in contrast to pure QCD, one cannot ignore various mass parameters which play important roles. These situations make a theoretical calculation of the amplitude more difficult and tedious. At present, it seems that the amount of labor necessary for exact calculations has almost reached the limit of hand calculations. It is thus natural to utilize computers to carry out such work. Several groups have started independently to develop computer systems which automate the perturbative calculations in the standard model\[1, 2, 3, 4, 5\].

Automatic calculations of the tree processes and some part of the one-loop processes have already been achieved by these systems.

An automatic calculating system starts with the generation of Feynman graphs for a given physical process. Feynman-graph generation for all orders in QED is not difficult, since the problem can be reduced to a problem equivalent to tree-graph generation\[6, 7\]. For electro-weak theory, or even in the $\phi^3$ model, however, the situation is different. They include self-interactions of particles, whose vertices are symmetric under exchanging interacting particles. This symmetry of the vertices results in a complicated structure of the internal symmetry in a Feynman graph.

It is not difficult to generate a sufficient set of Feynman graphs by simply connecting the vertices. The problem, however, is to eliminate duplicated graphs. It is hard to analyze the structure of the internal symmetry of graphs so as to avoid duplicated graphs controlling the graph generation process. Usually, a newly generated graph is discarded when it is found to be topologically equivalent to one already generated. Although explicit comparisons of graphs are, of course, possible, any known algorithm consumes time, which increases as an exponential of the number of vertices. A graph comparison
is generally not considered in the category of polynomial time complexity\[8\]. There is another problem in an explicit comparison method. It requires to save all generated graphs in order to recall graphs for comparisons. In a practical problem, the list of generated graphs grows too long to keep in the main memory of a computer. So the list is recorded on a secondary memory, usually on disk space. Frequent access to a secondary memory makes program very slow. This problem is solved by an orderly algorithm\[9\]. An orderly algorithm judges whether a newly generated graph is necessary to be kept or not, looking only at the graph without recalling already generated graphs. Based on this algorithm, a Feynman-graph generator for a wide variety of models was developed by P.Nogueira\[10\].

The aim of this paper is to present a computer program for Feynman-graph generation that is sufficiently fast for practical use. For this purpose, we have developed a new method of graph generation which accelerates the orderly algorithm. We classify vertices in such a way that there is no topologically equivalent vertices in different classes. With this classification, we are able to reduce the number of graphs to which the orderly algorithm is applied. The classification method of vertices which we use is an empirical one developed by graph theorists in order to determine the equivalence of two given graphs\[11\]. We have modified this method so as to be suitable for graph generation.

In order to make the problem simpler, we assume the following conditions:

1. Vacuum-to-vacuum graphs are not considered.

2. External particles of the graphs are assumed to be topologically different.

The first condition stems from the fact that the purpose of our automatic system is to calculate physical processes, namely the scattering and decay of particles. With the second condition, we have a one-to-one correspondence between the Feynman graphs and the Feynman amplitudes. One can generalize the method to loosen these restrictions without any essential difficulty, since the orderly and classification algorithms are still applicable. However, it must make the program more complicated. This generalization is discussed in section \[5\].

In the next section we introduce our basic methods. An explanation concerning the orderly algorithm is also given.
In order to accelerate orderly algorithm, we classify vertices in section 3. A pre-selection rule of generated graph, which is necessary to make the orderly algorithm consistent with vertex classification, is presented in this section.

The method of graph generation is described in section 4. The vertex-classification algorithm has been combined in order to eliminate graph in intermediate steps.

In section 5 we introduce our implementation of algorithms and utility programs.

A summary and comments are given about the physical processes calculated by the GRACE system up to now. A generalization of our program without the two conditions mentioned above is also discussed.

Appendices A, B, C and D comprise a brief manual of the programs and file formats.

2 Basic method

A graph consists of a finite number of nodes and edges. A node is either an external particle or a vertex. An edge is either a propagator or a connection between an external particle and a vertex. Throughout this paper, \( N \) denotes a set of nodes and \( E \subset \{(u, v)|u, v \in N\} \) does a set of edges. A graph \( G = (N, E) \) is defined mathematically as a pair of these sets. The degree \( \text{deg}(v) \) of node \( v \) is defined as the number of edges attached to the node \( v \):

\[
\text{deg}(v) = |\{(u, v) \in E\}|
\]

(1)

The degree of an external particle is 1. An edge has several attributes in a Feynman graph, such as the number of multiple connections between nodes and the name of a particle of a propagator. In order to make our discussion simpler, we drop these attributes in the following description. A node is labeled by a positive integer, with which we identify a node. We regard a set of integers \( \{1, \ldots, |N|\} \) as the set of nodes \( N \).

Let us consider a graph \( G = (N, E) \) and a permutation \( p \) mapping the set of nodes \( N \) onto itself. The permutation \( p \) is generalized to act on the set of edges by defining \( p(u, v) = (pu, pv) \) for \( (u, v) \in E, u, v \in N, \) and is generalized to act on the graph \( G \) as \( pG = p(N, E) = (pN, pE) = (N, pE) \). Two graphs, \( G = (N, E) \) and \( G' = (N, E') \), are isomorphic when there exists
such a permutation $p$ that maps the graph $G$ to $G' = pG$. This condition is equivalent to

$$(u, v) \in E \iff (pu, pv) \in E'.$$  \hspace{1cm} (2)

Isomorphic graphs are different each other only in the way of labeling nodes.

A permutation $p$ is an automorphism of a graph $G$ when $pG = G$. The set of all automorphisms of a graph $G$ forms a group $\Gamma_G$ called an automorphism group.

A graph can be expressed as a matrix, called adjacency matrix, whose element $a_{ij}$ carries the number of connections between nodes $i$ and $j$. We show an example of isomorphic graphs and their adjacency matrix in Fig. 1.

\begin{figure}[h]
\centering
\begin{subfigure}{0.4\textwidth}
\centering
\begin{tabular}{cccccccc}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
\end{tabular}
\caption{(a)}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\begin{tabular}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
\end{tabular}
\caption{(b)}
\end{subfigure}
\caption{Example of adjacency matrix.}
\end{figure}
In this and the next sections we consider a simple method of graph generation. First, necessary number of nodes are generated and then the nodes are connected in all possible ways. This method creates all possible graphs. Topologically equivalent graphs may appear several times. In order to eliminate duplicate graphs, we use a systematic method, an orderly algorithm, which judges a newly generated graph being necessary or not without recalling any of already generated graphs.

An orderly algorithm compares graphs through values of a function, called coding, which maps a graph to an integer in a way that different isomorphic graphs are mapped to different values. Such a function is easily realized, for example, in regarding the elements of the adjacency matrix as a sequence of digits of an integer.

In order to pick up a representative from a set of isomorphic graphs, An orderly algorithm selects a graph $G$ when it satisfies the following condition with coding $f$:

$$f(G) = \max_{p \in S_{|N|}} f(pG),$$  \hspace{1cm} (3)

where $S_{|N|}$ is the symmetry group of the set of nodes $N$ (the set of all permutations acting on $N$). A graph $G$ is indirectly compared with graph $pG$ through their values of the coding $f$.

Combining this selection condition with graph generation process, an orderly algorithm generates graphs in the following procedure:

1. Generate all the possible graphs.
2. Apply all possible permutations to each graph.
3. Discard a graph when it has a smaller coding value than ones of permuted graphs.

This method requires to keep only two graphs, a tested graph and its permuted one, which are small enough to keep in the main memory of a computer.

When a permutation $p$ satisfies selection condition (3), $p$ is an element of automorphism group $\Gamma_G$ of $G$. One can construct $\Gamma_G$ explicitly in collecting all of such permutations. The symmetric factor $|\Gamma_G|$ necessary for calculating Feynman amplitudes is evaluated in this way.
A simple application of this algorithm requires comparisons among graphs corresponding to $O(|N|!)$ permutations for each of $O(|N|!)$ isomorphic graphs. In order to decrease the number of comparisons in the orderly algorithm, we consider to replace $S_{|N|}$ in (3) by its subgroup.

### 3 Vertex classification

We consider a classification $\{N_i\}$ ($\cup_i N_i = N$, $N_i \cap N_j = \emptyset$ for $i \neq j$) of the set of nodes $N$ such that any two nodes in different classes are topologically different. We do not require that nodes in a class are topologically equivalent. In other words, each class of nodes is a direct sum of the orbits in $\Gamma_G$ (an orbit of a node $v$ in $\Gamma_G$ is the set $\{u|\exists p \in \Gamma_G, pv = u\}$). We call such a classification a consistent classification. With this classification of nodes, we construct a group

$$S(\{N_i\}) = S_{|N_1|} \otimes S_{|N_2|} \otimes ...$$  

(4)

where $S_{|N_i|}$ is the symmetry group acting on the set $N_i$. We replace the group $S_{|N|}$ in condition (3) by this subgroup $S(\{N_i\})$. This replacement reduces the number of permutations from $|N|!$ to $|N_1|! \times |N_2|! \times ...$. Corresponding to this limitation of comparisons in the orderly algorithm, one must limit the set of generated graphs. We present the way of limiting graphs after describing our method of node classification.

We adopt a method of node classification used in graph theory[11]. First, nodes are classified into $\{N_i^{(0)}\}$ by simple topological properties. Since we consider that external particles are topologically different from each other, they are put into different classes. The i-th external particle is put to the i-th class. Other nodes are classified by their degree such that:

$$u, v \in N_i^{(0)} \iff \deg(u) = \deg(v)$$

$$u \in N_i^{(0)}, v \in N_j^{(0)}, \deg(u) < \deg(v) \Rightarrow i < j$$  

(5)

We call this initial classification $\{N_i^{(0)}\}$ a primitive classification. Primitive classifications $\{N_i^{(0)}\}$ and $\{M_i^{(0)}\}$ of isomorphic graphs $G$ and $pG$, respectively, satisfy:

$$pN_i^{(0)} = M_i^{(0)}$$  

for all $i$,  

(6)
since relabeling of nodes does not change topological properties of nodes such as \( \deg(pv) = \deg(v) \).

Starting from the primitive classification, the classes \( \{N_i^{(k)}\} \) are refined to \( \{N_i^{(k+1)}\} \) iteratively. We consider a vector \( a_v[i], i = 0, 1, \ldots \) for a node \( v \in N_j^{(k)} \). The zeroth element \( a_v[0] \) of the vector keeps its current class number, and the \( i \)-th element keeps the number of edges which connect \( v \) with nodes in the \( i \)-th class \( N_i^{(k)} \):

\[
\begin{align*}
a_v[0] &= j, \quad \text{for } v \in N_j^{(k)}, \\
a_v[i] &= |\{(u, v) \in E|u \in N_i^{(k)}\}| \quad \text{for } i > 0.
\end{align*}
\]

(7)

The nodes are classified by the values of these vectors. New classes are numbered in increasing order, which is evaluated in the lexicographical ordering “\(<\)” of vectors. This process is repeated until classes can no more be refined. Thus the nodes are classified only by their topological properties. We summarize this refining method of classification as follows:

\[
\begin{align*}
\text{refine}(&\{N_i^{(k)}\}) \\
\{ &\text{for all (class } j) \{ \\
&\quad \text{for all (} v \in N_j^{(k)} \text{) } \{ \\
&\quad \quad a_v[0] = j; \\
&\quad \quad \text{for all (class } i) \\
&\quad \quad \quad a_v[i] = |\{(u, v) \in E|u \in N_i^{(k)}\}|; \\
&\quad \}\} \\
&\quad \langle \text{construct } \{N_i^{(k+1)}\} \text{ such that} \\
&\quad \quad a_v = a_w, v \in N_i^{(k+1)}, w \in N_j^{(k+1)} \Rightarrow i = j; \\
&\quad \quad a_v < a_w, v \in N_i^{(k+1)}, w \in N_j^{(k+1)} \Rightarrow i < j; \\
&\quad \rangle; \\
&\text{return } \{N_i^{(k+1)}\};
\}
\end{align*}
\]

We call a sequence of this classification of nodes \( \{N_i^{(k)}\} (k = 0, \ldots, K) \) a refinement sequence.

We show the following properties for a refinement sequence.
Property 3.1 For any $N_i^{(k+1)}$ there exists an input class $N_j^{(k)}$ includes $N_i^{(k+1)}$:

$$N_i^{(k+1)} \subset N_j^{(k)}.$$  \hspace{1cm} (8)

This property states that $N_i^{(k+1)}$ is a refined class of $N_j^{(k)}$.

Property 3.2 Ordering of classes are kept by the refinement procedure:

$$\forall i \forall j [N_i^{(k+1)} \subset N_j^{(k)}], N_j^{(k+1)} \subset N_j^{(m)}, i < j \Rightarrow l \leq m].$$  \hspace{1cm} (9)

This property is evident from the fact that the algorithm renumbers the new classes according to the lexicographical ordering of the vector $a_v$, which keeps the old class number at the first element.

Thus classes $N_i^{(k)}$ are divided to:

$$N_1^{(k)} = N_1^{(k+1)} \cup N_2^{(k+1)} \cup ... \cup N_{i_1}^{(k+1)}$$
$$N_2^{(k)} = N_{i_1+1}^{(k+1)} \cup N_{i_1+2}^{(k+1)} \cup ... \cup N_{i_1+i_2}^{(k+1)}$$  \hspace{1cm} (10)

Property 3.3 When nodes of two isomorphic graphs $G = (N, E^G)$ and $H = pG = (N, E^H)$, $p \in S_{|N|}$, are classified into $\{N_i^{(k)}\}$ and $\{M_i^{(k)}\}$, respectively, the following relation holds for all $k$:

$$\forall i [pN_i^{(k)} = M_i^{(k)}] \Rightarrow \forall j [pN_j^{(k+1)} = M_j^{(k+1)}]$$  \hspace{1cm} (11)

Proof. Let us take a node $v \in N_i^{(k)}$. We consider vectors $a_v^{G}$ and $a_v^{H}$ in eq. (8) which are used to construct $\{N_j^{(k+1)}\}$ and $\{M_j^{(k+1)}\}$ from $\{N_j^{(k)}\}$ and $\{M_j^{(k)}\}$, respectively. Their zero-th elements are equal $a_v^{G}[0] = a_v^{H}[0] = i$ since $pv \in M_i^{(k)}$. We obtain for $l > 0$:

$$a_v^{G}[l] = |\{(v, w) \in E^G | w \in N_i^{(k)}\}|$$
$$= |\{(pv, pw) \in pE^G | pw \in pN_i^{(k)}\}|$$
$$= |\{(pv, pw) \in E^H | pw \in M_i^{(k)}\}|$$
$$= a_v^{H}[l]$$  \hspace{1cm} (12)
Thus equation $a_v^G = a_v^H$ is hold for any node $v$. Since the numbering of refined classes are determined only by these vectors, it is easy to see

$$v \in N_j^{(k+1)} \iff pv \in M_j^{(k+1)},$$

(13)

which implies $pN_j^{(k+1)} = M_j^{(k+1)}$. □.

Since $pN_i^{(0)} = M_i^{(0)}$ is satisfied for all $i$ in eq. (3), the above property result in:

$$pN_i^{(k)} = M_i^{(k)}$$

for all $i$ and $k$. (14)

Especially, the length of refining sequences of two isomorphic graphs are equal.

Now we consider the way of limiting graphs to be applied to the orderly algorithm. We impose the following pre-selection condition for the classification $\{N_i\}$ of nodes of a generated graph:

$$\forall i \forall j \forall u \in N_i \forall v \in N_j \left[ i < j \Rightarrow u < v \right],$$

(15)

where the ordering of the nodes is evaluated in terms of the labeling numbers of the nodes. Classes satisfying this condition are expressed as:

$$N_1 = \{1, ..., |N_1|\}$$
$$N_2 = \{|N_1| + 1, ..., |N_1| + |N_2|\}$$

... (16)

We consider this condition not only for the final classification but also classifications appearing in a refinement sequence.

**Property 3.4** Classification $\{N_i^{(k)}\}$ is necessary to satisfy condition (14) for $\{N_i^{(k+1)}\}$ to satisfy the same condition.

**Proof.** Let $\{N_i^{(k+1)}\}$ satisfy the condition (15). A class $N_j^{(k+1)}$ is expressed as eq. (16):

$$N_j^{(k+1)} = \{s_j^{(k+1)} + 1, s_j^{(k+1)} + 2, ..., s_{j+1}^{(k+1)}\},$$

(17)

where $s_j^{(k+1)} = \sum_{t<j} |N_t^{(k+1)}|$. 

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Let a class $N_i^{(k)}$ be decomposed to $b_i$ classes of $\{N_i^{(k+1)}\}$. From eq. (10), we obtain,

$$N_i^{(k)} = N_{c_i+1}^{(k+1)} \cup N_{c_i+2}^{(k+1)} \cup ... \cup N_{c_{i+1}}^{(k+1)},$$  \hspace{1cm} (18)

where $c_i = \sum_{j<i} b_j$. The following relation holds:

$$s_{c_i+1}^{(k+1)} = \sum_{l<c_i+1} \left| N_l^{(k+1)} \right|$$
$$= s_{c_i+1}^{(k+1)} + \sum_{l=c_i+1}^{c_i} \left| N_l^{(k+1)} \right|$$
$$= s_{c_i+1}^{(k+1)} + \left| N_{c_i}^{(k)} \right|$$
$$= s_{c_i+1}^{(k+1)} + s_i^{(k)} - s_i^{(k-1)}.$$  \hspace{1cm} (19)

Combining with $c_1 = 0$ and $s_{c_1+1}^{(k+1)} = s_i^{(k)} = 0$, we obtain

$$s_{c_i+1}^{(k+1)} = s_i^{(k)}.$$  \hspace{1cm} (20)

Equations (18), (17) and (20) lead

$$N_i^{(k)} = \bigcup_{j=1}^{b_i} \left\{ s_{c_i+j}^{(k+1)} + 1, s_{c_i+j}^{(k+1)} + 2, ..., s_{c_i+j+1}^{(k+1)} \right\}$$
$$= \left\{ s_{c_i+1}^{(k+1)} + 1, ..., s_{c_i+1}^{(k+1)} \right\}$$
$$= \left\{ s_i^{(k)} + 1, ..., s_i^{(k)} \right\}.$$  \hspace{1cm} (21)

This expression is in the form of eq. (16). Thus $\{N_i^{(k)}\}$ satisfies condition (15). $\square$

With this property, we can eliminate a graph before reaching to the end of the refinement sequence, when an intermediate classification in a classification sequence is found not to satisfy condition (13). So we can decrease the number of refinement steps for graphs which are to be discarded.

We show that graph selection using this condition (13) leaves enough graphs.

**Property 3.5** For each graph $G$, there exists such a graph that is automorphic to $G$ and satisfies condition (13).
Proof. We consider the case when \( \{N_i\} \) does not satisfy condition (15). We define a permutation \( p \) of nodes by the following procedure:

```c
void renumber()
{
    count = 1;
    for (i = 1; i <= (The number of classes); i++){
        for all \( (v \in N_i) \) {
            (let \( pv = \) count);
            count = count + 1;
        }
    }
}
```

It is easy to see that classification \( \{pN_i\} \) is expressed as eq. (16). So the graph \( pG \) satisfies condition (15) \( \square \).

We select graphs by the following condition instead of eq. (3):

\[
f(G) = \max_{p \in S(\{N_i\})} f(pG) \quad (22)
\]

Our modified orderly algorithm becomes:

1. Generate all the possible graphs.
2. Select graphs by condition (15).
3. Apply all permutations in \( S(\{N_i\}) \) to each graph.
4. Discard a graph when it has a smaller coding value than ones of permuted graphs.

In order to show that two selection rules (15) and (22) are consistent each other, we prove the following relation for a graph \( G \) satisfying (15):

\[
\{pG | p \in S_{|N_i|}, pG \text{ satisfies condition (15)}\} = \{pG | p \in S(\{N_i\})\}. \quad (23)
\]

Property 3.6 When nodes of a graphs \( G \) are classified into \( \{N_i\} \) satisfying condition (13) and a graph \( H = pG \) is isomorphic to \( G \) with permutation \( p \), necessary and sufficient condition that a graph \( H \) satisfies condition (15) is that the permutation \( p \) is an element of the subgroup \( S(\{N_i\}) \).
Proof. Let the nodes of isomorphic graph $H = pG$ be classified into \(\{M_j\}\). From eq. (14), a class $N_i$ of $G$ is mapped by $p$ to the class $M_i = pN_i$ with the same index.

When a permutation $p$ is an element of $S(\{N_i\})$, each class $N_i$ is kept invariant under $p$, that is, $N_i = pN_i = M_i$. Since $\{N_i\}$ satisfies condition (15), $\{M_i\}$ satisfies the same condition.

Inversely, we assume that the graph $H$ satisfies the condition (15). Since $|N_i| = |pN_i| = |M_i|$ holds for all $i$, it is easy to see from expression (15) that $N_i = M_i = pN_i$ is satisfied for all $i$. This implies that the permutation $p$ keeps $\{N_i\}$ invariant and that $p$ is an element of $S(\{N_i\})$.

We have first confirmed that necessary graphs are kept in step 2 of our modified algorithm. Then surviving isomorphic graphs are transformed each other by an element of subgroup $S(\{N_i\})$. They are compared through the values of coding in step 4 and duplicated graphs are properly eliminated.

In the next section we combine step 1 and 2 in order to decrease the number of generated graphs.

4 Graph generation and selection

Here we describe how to generate graphs in combination with elimination of graphs.

The first step of graph generation is to prepare nodes. A node is to be connected to several edges. We consider such an imaginary object, let us call it a leg, that is a place in a node where an edge is to be tied to. A fixed number of legs is assigned to each node at the beginning. The number of legs becomes equal to the degree of the node when connection process terminates.

The next step is to connect nodes by edges. The nodes are connected iteratively starting from one fixed external particle, which we call the root of the graph. In order to make our way of connections systematic, we define level of node $v$ as the distance of the node from the root measured by the minimum number of edges among paths connecting $v$ and the root. In this step there appear intermediate configurations of graphs, in which only a part of legs of nodes are tied with edges and some nodes may remain isolated from other nodes. We consider the level of an isolated node is infinity.

Nodes are connected in the following way. The root, with one leg, is first connected to another node. The connected node becomes the only one node
at the first level. Then this node is connected to other nodes until all of its legs are connected. These connections define the set of nodes in the second level. Then, the all legs of all nodes in the second level are connected to others. In this way, nodes are connected in increasing order of the value of level. Once a level of a node has a finite value, its value is not changed. This process proceeds in a recursive way so as to exhaust all possibilities of the connections. A skeleton description of the algorithms is given as follows:

```c
void gsconn()
{
    node ns, nt;

    ns = (find node at the lowest level with free legs);
    if(no more such nodes) {
        if(the graph is a connected graph) {
            if(the graph is accepted by the orderly algorithm)
                (a new graph is obtained);
        }
    }
    else {
        for all (node nt with free legs) {
            (connect ns to nt);
            gsconn();
            (disconnect ns from nt);
        }
    }
}
```

For efficient graph generation, it is important to eliminate unnecessary intermediate graphs in as earlier stage as possible, since a large number of graphs may be produced in exhausting all possible additional connection to the intermediate graph. We eliminate graphs with condition (15) not only in the final form of the generated graphs, but also in intermediate graphs.

One must notice that the classification of an intermediate graphs is not always consistent with that of the final form of the graphs. For example, we consider a final form of the graph of Fig.1a and its intermediate graph Fig.1b. The set of classes of nodes of Fig.1a is

\[ \{(1), (2), (3), (4, 5), (6)\} \]
Figure 2: Example of a two-loop graph and its intermediate configurations.

while that of Fig.1b is

\{(1), (2), (3), (4), (5), (6)\}.

This example shows that nodes 4 and 5 are topologically equivalent in the final form of the graph, although they are not always equivalent along the way of graph generation. When one applies condition (15) to such graphs, one loses necessary graphs. It is necessary to apply the condition (15) only to the consistent classification.

We can recover a consistent classification by considering an intermediate graph \(G[l] = (N, E[l])\) at the time when all of the legs of all of the nodes in level \(l\) are just connected. These intermediate graphs form a finite series,

\[G[0] \subset G[1] \subset \ldots \subset G[m-1] = G,\]

where \(m\) is the maximum level of the nodes in graph \(G\). Fig.1c represents an example of \(G[2]\). The set of edges \(E[l]\) is expressed as:

\[E[l] = \{(u, v) \in E | \min(\text{level}(u), \text{level}(v)) \leq l\}.\] (24)

We show the following property for the automorphism group \(\Gamma_{G[l]}\) of \(G[l]\):
Property 4.1 The following relation holds for all \( l \): 

\[
\Gamma_{G[l+1]} \subset \Gamma_{G[l]}.
\]  

Proof. If this is not satisfied, there exists a permutation \( p \in \Gamma_{G[l+1]} \) which is not an element of \( \Gamma_{G[l]} \). This means that there exists an edge \((u, v) \in E[l]\) and \((pu, pv) \notin E[l]\). On the other hand, since \( p \) is an automorphism in \( \Gamma_{G[l+1]} \) and \( \text{level}(pu) = \text{level}(u) \), we get:

\[
\min(\text{level}(pu), \text{level}(pv)) = \min(\text{level}(u), \text{level}(v)) \leq l,
\]

which implies \((pu, pv) \in E[l]\). \( \square \).

Relation (25) implies \( \Gamma_G = \Gamma_{G[l-1]} \subset \Gamma_{G[l]} \). Thus an orbit of \( \Gamma_{G[l]} \) is a direct sum of the orbits of \( \Gamma_G \). Since a class of nodes constructed for \( G[l] \) is a direct sum of orbits of the automorphism group \( \Gamma_{G[l]} \), the class is a direct sum of the orbits of \( \Gamma_G \). The classification of nodes in \( G[l] \) is therefore consistent with the classification in \( G \).

We can now eliminate irrelevant intermediate graphs with condition (15) applying to \( G[l] \). Furthermore, it is possible to use the classification of nodes in \( G[l-1] \) as an input for constructing the classification in \( G[l] \) in our class refinement algorithm. This method decrease the total number of class refinement. The final form of our algorithms is as follows:

```c
void gsconn(int lvl)
{
    node ns, nt;

    if (all legs of the nodes in the level lvl have been connected) {
        (refine classes of the nodes);
        if(! (class ordering condition (15) is satisfied))
            return;
        lvl++;
    }
    ns = (find node at the level lvl with free legs);
    if(no more connectable legs) {
        if(the graph is connected one) {
            if(the graph is accepted by the orderly algorithm)
```
(a new graph is obtained);

} else {
  for all (node nt with free legs) {
    (connect ns to nt);
    gsconn();
    (disconnect ns from nt);
  }
}

5 Implementation

We have implemented the method described above as a computer program written in C language. The program generates Feynman graphs in which particles are assigned to propagators. The particles of the propagators are determined in accordance with a table of particles and interactions defined by the users. It is possible that duplicated graphs are produced in this particle assignment process. They are eliminated again by the orderly algorithm with extended coding of graphs with particle attributes on the edges.

The program has the following options for graph generation:

1. To pick up only one-particle irreducible graphs\[13\].

2. Not to generate graphs with a self-energy part at an external particle line.

3. Not to generate graphs with a self-energy part at an internal particle line.

4. To generate graphs with counter terms for renormalizing the theory. Counter terms are automatically generated when their interactions are of the same form as tree vertices. Other counter terms can be added by users (see appendix C).

5. To generate a skeleton graphs, in which looped one-particle irreducible subgraphs are considered as to be blobs. The generated graphs are of the tree type, but with blob vertices.
6. Not to assign particles to the internal lines (only topology).

The way to specify these options is described in appendices A and B.

We also provide a graph-drawing facility on the X-window system and on PostScript files. We show an example of output figures in Fig.3.

Figure 3: Example of drawn graphs.

The total number of graphs depends on the conditions of the graph selection. In special cases, they are analytically calculable. In the $\phi^4$ model including $\phi^3$ interaction, the method of enumerating connected graphs is developed in a graph-theoretical method [14, 15]. When there appear different kinds of particles in the model, a weighted sum by the symmetric factor can be calculated in zero-dimensional field theory [16]. These numbers of graphs are calculated and listed in ref. [10]. We have checked our program by comparing with these numbers. Since for up to 5-loop graphs the orderly algorithm is not necessarily required to generate unique graphs [12], we have also checked the number of graphs for 6-loop tadpole, which includes 90156
graphs. The number of one-particle irreducible graphs is checked using a
recursion formula for one- and two-loop graphs.

| A | B | C | D  | E  | F  |
|---|---|---|----|----|----|
|   |   |   | G  | H  | G  | H  | G  | H  |
| 1 | 1 | 1 | 1  | —  | 1  | —  | 1  | —  |
| 1 | 2 | 6 | 15 | —  | 6  | —  | 6  | —  |
| 1 | 3 | 46 | 1001 | —  | 49 | —  | 49 | —  |
| 1 | 4 | 471 | 210880 | 152.1 | 570 | 152.1 | 570 | —  |
| 1 | 5 | 5961 | 10036865 | 96948.8 | 7451 | 16.5 | 127429 | 481.9 |
| 1 | 6 | 90156 | 12 | 9 | — | 1 | — |
| 2 | 1 | 3 | 5  | —  | 3  | —  | 3  | —  |
| 2 | 2 | 29 | 240 | —  | 29 | —  | 29 | —  |
| 2 | 3 | 351 | 40302 | 25.4 | 361 | 27.0 | 361 | —  |
| 2 | 4 | 5076 | 16201383 | 14280.2 | 5435 | 11.2 | 92739 | 291.3 |
| 2 | 5 | 84749 | 12 | 1 | — | 1 | — |
| 3 | 0 | 1 | 1  | —  | 1  | —  | 1  | —  |
| 3 | 1 | 14 | 49 | —  | 14 | —  | 14 | —  |
| 3 | 2 | 217 | 7113 | 4.0 | 217 | 4.4 | 217 | —  |
| 3 | 3 | 3729 | 2486650 | 1981.2 | 3777 | 2207.8 | 3777 | 6.6 |
| 3 | 4 | 70600 | 12 | 1 | — | 1 | — |
| 4 | 0 | 4 | 7  | —  | 4  | —  | 4  | —  |
| 4 | 1 | 99 | 1056 | —  | 99 | —  | 99 | —  |
| 4 | 2 | 2214 | 344742 | 254.6 | 2214 | 281.6 | 2214 | 3.4 |
| 4 | 3 | 50051 | 217154470 | 214880.7 | 50382 | 113.5 | 72774 | 194.0 |
| 5 | 0 | 25 | 100 | —  | 25 | —  | 25 | —  |
| 5 | 1 | 947 | 38960 | 27.2 | 947 | 29.7 | 947 | 1.4 |
| 5 | 2 | 28365 | 24532920 | 22388.4 | 28365 | 56.9 | 28365 | 56.9 |
| 6 | 0 | 220 | 2750 | —  | 220 | 1.3 | 220 | —  |
| 6 | 1 | 11460 | 2214170 | 1862.0 | 11460 | 21.3 | 11460 | 21.3 |
| 7 | 0 | 2485 | 121520 | 41.1 | — | 2485 | 3.3 |

Table 1: Efficiency of acceleration in the $\phi^4$ model including $\phi^3$ interaction.

The performance of the program is shown in Table 1 in the case of connected graphs in the same model. Columns A, B and C represent the number of external particles, the number of loops and the number of graphs, respectively. The theoretical prediction of the number of graphs is calculated by using a program written in formula manipulate language REDUCE, which takes more cpu-time than our graph-generation program. Columns

---

3 The corresponding value in [12] is not correct.
D, E and F correspond to simple application of the orderly algorithm (D), application of condition (15) without (E) and with (F) elimination of intermediate graphs. Sub-columns G and H are the number of graphs applied to the orderly algorithm eq. (3) or (22) and cpu-time in seconds measured on SUN-IPX with “-a -o /dev/null” options (described in appendix A). This result shows that elimination of intermediate graphs is effective enough; it reduces cpu-time in several order of magnitudes.

6 Summary and comments

We have developed a method of Feynman graph generation, which accelerates the orderly algorithm, based on a node classification method and systematic elimination of intermediate graphs. This method is implemented as a computer program. The order of the coupling constants of a physical process is not limited. Graphs can be generated based on a user-defined model; in this paper we adopt electro-weak theory combined with QCD as an example. The program has several options of graph selection, including to generate renormalization counter terms. Although the asymptotic behavior of the execution time is proportional to the factorial of the number of nodes, the program is sufficiently fast for practical use.

This program is a part of the GRACE system, which automates computation of tree and one-loop scattering processes. With this system, the cross sections including one-loop corrections in the processes $e^+e^- \rightarrow HZ$ and $e^+e^- \rightarrow t\bar{t}$, and one-loop gluon corrections to the process $e^+e^- \rightarrow q\bar{q}\gamma$ are automatically calculated\[2\]. For the tree case, the cross sections of the scattering processes with up to five final particles are calculated as shown in table 2. The number of graphs is counted in covariant and Feynman gauges for tree and one-loop processes, respectively. In both cases, the interactions among a Higgs particle and two light fermions are neglected. Integration over phase space is calculated using the integration package BASES[17].
| Process                              | #Graph | Reference |
|-------------------------------------|--------|-----------|
| $e^+e^- \rightarrow W^+W^-\gamma$   | 18     | [18a]     |
| $\rightarrow \nu_e\bar{\nu}_eZ^0$   | 9      | [18b]     |
| $\rightarrow \nu_e\bar{\nu}_eH$    | 2      | [18b]     |
| $\rightarrow e^+e^-H$               | 2      | [18b]     |
| $\rightarrow Z^0Z^0H$               | 4      | [18b]     |
| $\rightarrow W^+W^-H$               | 11     | [18b]     |
| $\rightarrow Z^0Z^0Z^0$             | 9      | [18b]     |
| $\rightarrow W^+W^-Z^0$             | 20     | [18b]     |
| $\rightarrow t\bar{t}Z^0$           | 9      | [18b]     |
| $\rightarrow t\bar{t}H$             | 6      | [18b]     |
| $\rightarrow H\bar{H}Z^0$           | 6      | [18b]     |
| $\rightarrow \gamma\gamma\gamma$   | 9*     | —         |
| $\gamma\gamma \rightarrow e^+e^-Z^0$| 6      | —         |
| $\gamma e \rightarrow eW^+W^-$       | 18     | —         |
| $gg \rightarrow q\bar{q}\gamma$    | 18     | —         |

Table 2a Tree processes with the final 3-body

| Process                              | #Graph | Reference |
|-------------------------------------|--------|-----------|
| $e^+e^- \rightarrow \nu_e\bar{\nu}_eW^+W^-$ | 60     | [18c]     |
| $\rightarrow \nu_{\mu}\bar{\nu}_{\mu}(\tau)W^+W^-$ | 36     | [18c]     |
| $\rightarrow e^+e^-W^+W^-$            | 114    | [18c]     |
| $\rightarrow \nu_e\bar{\nu}_eZ^0Z^0$   | 57     | [18c]     |
| $\rightarrow e^+\nu_eW^-Z^0$           | 88     | [18c]     |
| $\rightarrow e^+e^-Z^0Z^0$             | 86     | [18c]     |
| $\rightarrow e^+e^-\gamma\gamma$      | 80     | [18c]     |
| $\rightarrow \mu^+(\tau^+)\mu^-(\tau^-)\gamma\gamma$ | 40     | [18c]     |
| $\rightarrow \nu_e\bar{\nu}_ebb$      | 21     | [18c]     |
| $\rightarrow \nu_{\mu}\bar{\nu}_{\mu}bb$ | 11     | [18c]     |
| $\rightarrow e^+\nu_etb$               | 21     | [18c]     |
| $\rightarrow W^+W^-\gamma\gamma$      | 138    | —         |
| $\rightarrow e^-\nu_eud$               | 24     | [18c]     |
| $\rightarrow ud\bar{u}d$               | 69     | [18c]     |
| $\rightarrow q\bar{q}g\gamma$         | 21     | —         |
| $\rightarrow e^+e^-bb$                 | 50     | —         |
| $\gamma e \rightarrow \nu_eW^-HH$      | 40     | —         |

Table 2b Tree processes with the final 4-body
Table 2c Tree processes with the final 5-body

| Process                        | #Graph | Reference |
|-------------------------------|--------|-----------|
| $e^+ e^- \rightarrow \mu^+ \nu_\mu lb\gamma$ | 71     | —         |
| $\rightarrow e^- \bar{\nu}_e ud\gamma$ | 142    | [18h]     |

Table 2d One-loop processes with the final 2-body

| Process                        | #Graph | Reference |
|-------------------------------|--------|-----------|
| $e^+ e^- \rightarrow HZ$      | 89     | [2]       |
| $\rightarrow tt$             | 50     | [2]       |

Table 2e One-loop processes with the final 3-body

Gluon correction with only final radiations.

Table 2: Scattering processes calculated by the GRACE system.
In the following we discuss how to loosen the restrictions for graph generation assumed in the introduction. The second restriction regarding identical external particles can be removed by changing the primitive classification so as to put identical particles in the same class. At the same time, the root node should be replaced by a class of nodes.

The first restriction of excluding vacuum-to-vacuum graphs is complicated to loosen. It is possible that the primitive classification contains only one class, such as in vacuum-to-vacuum graphs in the $\phi^3$ model. In this case, node classification and acceleration methods do not work anymore. However, one can artificially fix one node as the root of the graph.

We consider a set of graphs $A$ generated by our method with fixing node 1 among $n = |N|$ nodes and selected by condition (15). Let us also consider the true set of vacuum-to-vacuum graphs $T$. For any graph $G \in T$, one can construct a set of isomorphic graphs $I_G = \{pG \mid p \in S_{n-1}\}$, where $S_{n-1}$ is the symmetric group acting on the set of the nodes $\{2, \cdots, n\}$. We consider to select a representative graph from $I_G$ by applying condition (15) after classifying the nodes with fixing node 1. The selected graph belongs also to $A$, since $A$ is constructed by the same selection rule applied to all possible graphs. The set of graphs $A$ thus contains at least an isomorphic graph to any graph of $T$, though some graphs in $A$ are duplicated.

The duplication of graphs is to be tested by the original orderly algorithm (3). Since we have already eliminated some of the duplicated graphs by condition (15), we must limit the set of graphs $\{pG \mid G \in A, p \in P\}$ by the same condition before comparing the values of the coding. The orderly algorithm is then changed to the following form:

```python
compare(G)
{    accept = True;
    for all \((pG, p \in S_{|N|})\) \{
        H = pG;
        (classify node of \(H\) with fixing node 1);
        if(condition (15) is satisfied) \{
            if\(\text{coding}(G) < \text{coding}(H)\) \{
                accept = False;
                break;
            \}
        \}
    }
}
```
if(accept)
    (accept graph $G$);
}

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26
A Installation and execution

The source code is available by anonymous ftp from ftp.kek.jp in the directory kek/minami/grc. It has two programs: one generates Feynman graphs; the other draws generated graphs, which work on UNIX with the X-Window system.

The graph-generation program grc reads two files, a process file and a model file. It then creates an output file named “out.grf”, which keeps the information about the generated graphs. The format of the input files is described in appendices B and C.

The graph-drawing program grcdraw reads the model and the output file of the generation program, and then draws the graphs on the display with the X-Window system. This program is not so intelligent as to display graphic objects beautifully; however, it is still useful for checking the generated graphs. This program has several subcommands.

The procedure of installation is:

1. Uncompress and expand the source code file.
   
   In a new directory, run the following command:

   \[
   \text{zcat grc.tar.Z} | \text{tar xvf -}
   \]

2. Editing Makefile.

   The file Makefile in the sub-directory src controls the way of compiling source code. The path-names of include files and libraries of Xlib should be changed in accordance with the system configuration. If Athena widgets (included in the standard distribution of X-Window system) is installed in your system, use the line in the Makefile:

   \[
   \text{XCOPT} = -\text{DTOOLKIT} -\text{DX11}
   \]

   If not, use

   \[
   \text{XCOPT} = -\text{DX11}
   \]

3. Compilation.

   In the sub-directory src, run make command.
4. Testing the Feynman-graph generator.
   In the sub-directory sample, run the following command

   ../src/grc

   The program generates 4 tree graphs for the $e^+ e^- \rightarrow W^+ W^-$ process,
   316 one-loop graphs of the same process and 28 tree graphs for $e^+ e^- \rightarrow W^+ W^- \gamma$.

5. Testing the Feynman-graph drawer.
   In the same sub-directory sample, run the following command

   ../src/grcdraw -h
   ..src/grcdraw

   Try to type “f”, “b”, “n”, “p”, “g” “q”.
   If you have installed with Athena widgets, buttons will appear on the display.

**Feynman-graph generator**

The command syntax of grc command is

```
grc [options] [process-file-name]
```

The *process-file-name* is an input file specifying the physical process and
options, the format of which is described in appendix [3]. The default process
file name is “in.prc”.

The command line options of the grc command are:

- `-a`
  Skip particle assignment.

- `-c`
  Do not generate an output file, but only count the number of graphs.

- `-o output-file-name`
  Specify the output file name. The default output file is “out.grf”.

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• -h
  Print help message.

Also try with the help option:

  ../src/grc -h

Feynman-graph drawer

The command syntax of the grcdraw command is

  grcdraw [options] [graph-file-name]

The graph-file-name is an output file generated by the grc command, the format of which is described in appendix D. The default process file name is “out.grf”.

The command line options of the grcdraw command are:

• -w number-of-graphs
  If this option is specified (number-of-graphs)^2 graphs appear on the display.

• -h
  Print help message.

Also try with the help option:

  ../src/grcdraw -h

This program has some subcommands for selecting graphs drawn on the display. They are as follows:

- q: quit.
- n: display next process
- p: display previous process
- f: display forward graphs
- b: display backward graphs
- g: scale up the size of graphs
- s: scale down the size of graphs
- l: display particle names of internal lines (on/off)
- t: display particle names and graph number (on/off)
- <number>j: jump to the specified graph
- o: output the displayed graphs to PostScript file
When the program is installed with Athena widgets, such commands are also displayed as buttons.

B Process file

The graph-generation program works in accordance with the process file. The following example is a file specifying the one-loop and tree process of $e^+e^- \rightarrow W^+W^- \gamma$.

```
Model="all.mdl";

Process;
  ELWK=\{2, 4\};
  Initial=\{electron, positron\};
  Final =\{W-plus, W-minus\};
  Expand=Yes;
  OPI=No;
Pend;

Process;
  ELWK=3;
  Initial=\{electron, positron\};
  Final =\{W-plus, W-minus, photon\};
  Expand=Yes;
  OPI=No;
Pend;
```

The lines beginning with "\%" are ignored as comment lines.

The first non-comment line specifies the file which describes the model used for graph generation, which is described in appendix C.

Then follow descriptions of the processes. A block describing a process begins with the the line of "Process;" and ends with the line of "Pend;". In this block, descriptions are given for the order of the coupling constants, initial particles, final particles and some options.

The name of the coupling constant ("ELWK" in the above example) is defined in the model file. When the value of the coupling constant is given as a list of numbers, the program generates graphs for each value of the coupling constants with the same external particles.

The initial and final external particles are given as lists of particle names, which are defined in the model file.
The available options are:

- **OPI = Yes | No**
  Generate one-particle irreducible graphs or not.

- **Expand = Yes | No**
  Expand the looped part or generate looped parts as blobs.

- **Tadpole = Yes | No**
  Generate tadpoles or not.

- **extself = Yes | No**
  Generate the self-energy part at an external particle or not.

- **selfe = Yes | No**
  Generate whether with the self-energy part or not at an internal particle line.

- **countert = Yes | No**
  Generate renormalization counter terms or not.
C Model file

Here, we describe the format of the model file. Our definitions of particles and vertices include not only the necessary information for graph generation, but also for amplitude generation. We show an example of this file:

```
Order={ELWK, QCD};
%---------------------------------------
% gauge bosons
%---------------------------------------
Particle=W-plus["W+"]; Antiparticle=W-minus["W-"];
   PType=Vector; Charge=1; Color=1; Mass=AMW; Width=AGW;
   MValue="80.22D0"; WValue="2.12D0"; PCode=2;
Pend;
%
...
%---------------------------------------
% scalars
%---------------------------------------
Particle=Higgs["H"]; Antiparticle=Particle;
   PType=Scalar; Charge=0; Color=1; Mass=AMH; Width=AGH;
   MValue="150.0D0"; WValue="0.0D0"; PCode=31;
Pend;
%
...
%---------------------------------------
% leptons
%---------------------------------------
Particle=nu-e["nue"]; Antiparticle=nu-e-bar["~nue"];  
   PType=Fermion; Charge=0; Color=1; Mass=AMNE; Width=0;
   MValue="0.0D0"; PCode=51; Massless;
Pend;
...
Particle=electron["e-"]; Antiparticle=positron["e+"];
   PType=Fermion; Charge=-1; Color=1; Mass=AMEL; Width=0;
   MValue="0.511D-3"; PCode=55;
Pend;
...
```
Particle=u; Antiparticle=u-bar["\"u\""; 
ParticleType=Fermion; Charge=2/3; Color=3; Mass=AMUQ; Width=0; 
MValue="100.0D-3"; PCode=61; 
Pend;

%---------------------------------------
% VVV
%---------------------------------------

% FFV (FFW : without quark mixing)
%---------------------------------------
Vertex={positron, nu-e, W-minus}; ELWK=1; FName=WNE; 
FValue={R, " GWFL"}; FType="V-A"; Vend;
Vertex={anti-muon, nu-mu, W-minus}; ELWK=1; FName=WNM; 
FValue={R, " GWFL"}; FType="V-A"; Vend;
Vertex={anti-tau, nu-tau, W-minus}; ELWK=1; FName=WNT; 
FValue={R, " GWFL"}; FType="V-A"; Vend;
Vertex={nu-e-bar, electron, W-plus }; ELWK=1; FName=WEL; 
FValue={R, " GWFL"}; FType="V-A"; Vend;
Vertex={nu-mu-bar, muon, W-plus }; ELWK=1; FName=WMU; 
FValue={R, " GWFL"}; FType="V-A"; Vend;
Vertex={nu-tau-bar,tau, W-plus }; ELWK=1; FName=WTA;
Lines beginning with “%” are comment lines.

The file comprises of three parts: definitions of the names of the coupling constants, definitions of the particles and then definitions of the vertices and counter-terms, arranged in this order. A definition is a sequence of “keyword=value;” or “keyword;”. The value part may have an optional part, which is enclosed by brackets “[” and “]”.

The first keyword is Order, which define the names of the coupling constants. When multiple coupling constants appear in the model, a list of these names is specified enclosed by braces “{” and “}”. For example,
Order={QED, QCD};

is for a model including QED and QCD coupling constants.

C.1 Definition of particles

The definition of a particle begins with the keyword \texttt{Particle} and ends with the keyword \texttt{Pend}.

The value of the keyword \texttt{Particle} is the name of the particle. The name, beginning with an alphabet, should be unique among the particles defined in this file, since this name is used to identify the particle. An option to the name is a shorthand name of the particle, which is used to show the particle in the graphic output.

In the same way, the name of an anti-particle is given as the value of the keyword \texttt{Antiparticle}. When the anti-particle coincides to the particle, this part should be defined as

\texttt{Antiparticle=Particle}.

The type of particle is given by the keyword \texttt{PType}. Its value is either \texttt{Scalar}, \texttt{Vector}, \texttt{Majorana}, \texttt{Fermion} or \texttt{Ghost}.

The keyword \texttt{Charge} specifies the electric charge. The value is given as a signed integer or a rational number in the unit of the positron charge $e$.

The dimension of color representation of the particle is specified by the \texttt{Color} keyword as an integer.

The definition of the mass parameter of the particle comprises three items: the Fortran variable name of the mass, the default numerical value and a flag specifying massive or massless. Even if the particle is massless, a fictitious mass can be introduced in some part of the calculation in order to avoid an infrared divergence or mass singularity. We require a definition of a particle being massless or massive, and the Fortran variable name for the particle mass with a default value not only for a massive particle, but also a massless one.

The keyword \texttt{Massless} or \texttt{Massive} (without a value) specifies that the particle is either massless or massive, respectively. If nothing is specified, the particle is considered to be massive.

The Fortran name is defined by the keyword \texttt{Mass} and its default value by \texttt{MValue}. The value of keyword \texttt{MValue} is defined as a character string which is used in the Fortran code.
The Fortran name of the width and its default numerical value is given by the keywords \texttt{Width} and \texttt{WValue}, respectively.

In the above definition, we do not assume any special name of particles; one can define the particle name freely. However, amplitude-generation program is necessary to know whether a special particle appears in a Feynman graph or not. For example, the \textsc{Chanel} library offers a calculation of the amplitudes in a general covariant gauge. In the unitary gauge, $\chi$-scalars disappear from the calculation, and one must drop any Feynman graphs including them. In order to detect such kinds of particles, particularly in the amplitude generating-program, we add another keyword \texttt{PCode} with an integer particle code. The values of \texttt{PCode} are not used in the Feynman-graph generator in the current version.

\subsection*{C.2 Definition of vertices}

A definition of a particle begins with the keyword \texttt{Vertex} and ends with the keyword \texttt{Vend}.

The keyword \texttt{Vertex} defines the interacting particles in the list of their name. The direction of a particle is defined as incoming to the vertex. For example,

\begin{verbatim}
Vertex={nu-e-bar, electron, W-plus};
\end{verbatim}

defines a vertex which $e^-$ and $W^+$ are coming in and $\nu_e$ is going out.

The order of the coupling constants is given by keywords defined in the value of the keyword \texttt{Order} such as:

\begin{verbatim}
ELWK=1;
\end{verbatim}

The total order of coupling constant of tree 3- and 4-point vertices should be 1 and 2, respectively.

A program generating amplitudes requires additional information. The name of the coupling constant used in the Fortran code of the vertex is given by the \texttt{FName} keyword, whose data type and default value are given by the \texttt{FValue} keyword. The number of parameters in a coupling constants of a vertex depends on the interaction. In order to decide its number and how particles interact in a vertex of two fermions and a boson, the type of the interaction is specified by the keyword \texttt{FType}. The value of \texttt{FType} is
either "V", "A", "S", "P", "V-A", "V+A", "S-P", "S+P" or "NON". When this keyword is omitted, the fermion-boson vertex is assumed to FType="NON". The format of specification by the FValue keyword is

\[
\text{FValue}\{\langle\text{flag}\rangle, \langle\text{param1}\rangle}\}
\]

or

\[
\text{FValue}\{\langle\text{flag}\rangle, \langle\text{param1}\rangle, \langle\text{param2}\rangle}\}
\]

in accordance with the number of coupling constants. The parts \(\langle\text{param1}\rangle\) and \(\langle\text{param2}\rangle\) are embedded into the Fortran code by the Fortran-code generator. The part \(\langle\text{flag}\rangle\) is either R, I or C, corresponding to the parameter being real, pure imaginary or complex number, respectively. These keywords are not used in the graph-generation program.

One can define a counter term of renormalization as a vertex with higher order coupling constants.
D Output file

We show an example of the output file of the graph generator for the input file shown in appendix B.

```
Model="all.mdl";
Process=1;
External=4;
  0= initial electron;
  1= initial positron;
  2= final w-plus;
  3= final w-minus;
Eend;
elwk=4;Loop=1;
OPI=No;Expand=Yes;
%---------------------------------------
Graph=1;
Sfactor=1;
Vertex=4;
  0={ 1[positron]};
  1={ 2[electron]};
  2={ 3[w-plus]};
  3={ 4[w-minus]};
4[order={1,0}]={ 1[electron], 2[positron], 5[photon]};
5[order={1,0}]={ 5[photon], 6[w-minus], 7[w-plus]};
6[order={1,0}]={ 3[w-minus], 6[w-plus], 8[z]};
7[order={1,0}]={ 4[w-plus], 7[w-minus], 8[z]};
Vend;
Gend;
%---------------------------------------
Graph=2;
%
...

%---------------------------------------
Graph=316;
Sfactor=1;
Vertex=3;
  0={ 1[positron]};
  1={ 2[electron]};
  2={ 3[w-plus]};
  3={ 4[w-minus]};
```
Lines beginning with "%" are ignored as comments.
The output file is a sequence of statements. Most statements are written in the format

"keyword=value" or "keyword".
separated by colons “;”. The keyword part is an identifier (name composed of alphabet and digits). The value part is either an identifier or a list of identifiers separated by “,” and enclosed by braces “{“ and “}”. An identifier may have options enclosed by brackets “[“ and “]”. Between two brackets, a statement or a list of statements is placed.

The first non-comment line specifies the file name of the model definition which is used in graph generation.

The output file includes several processes in accordance with the input file. In this example, the output file includes three processes. Each of them begins with the line

\[
\text{Process}=\langle \text{process number} \rangle; \\
\]

and ends with the line

\[
\text{Pend}=\langle \text{the number of graph in the process} \rangle; \\
\]

The description of a process comprises descriptions of external particles, options for graph generation copied from the input file and a sequence of descriptions of the generated graphs.

The description of external particles begins with the statement

\[
\text{External}=\langle \text{the number of external particles} \rangle; \\
\]

and ends with the statement

\[
\text{Eend}; \\
\]

Between them, statements for each external particle are written in the format

\[
\langle \text{external particle number} \rangle= \text{initial} \langle \text{particle name} \rangle; \\
\]

or

\[
\langle \text{external particle number} \rangle= \text{final} \langle \text{particle name} \rangle; \\
\]

The \langle external particle number \rangle is an integer number beginning from 0.

The description of a generated graph begins with the statement

\[
\text{Graph}=\langle \text{graph number} \rangle; \\
\]

and ends with the statement
Gend;

After the Graph statement, it follows the global factor of the graph,

\[ S_{\text{factor}} = \text{inverse of global factor}; \]

The value of \( S_{\text{factor}} \) is the inverse of the symmetric factor of the graph with a sign corresponding to the permutation of the external fermions and fermion loops.

The structure of a graph is expressed by connections between the nodes. They are placed between two statements:

\[ \text{Vertex} = \text{<the number of generated vertices>}; \]

and

\[ \text{Vend}; \]

Corresponding to each node, information about the connections from the node to others is written as

\[ \text{<node number>} = \{ \text{<internal line number> [particle name]}, \ldots \}; \]

The \text{<node number>} is sequentially numbered beginning from 0. A node number corresponding to one defined as \text{<node number>} expresses the same external particle. The \text{<internal line number>} is numbered sequentially beginning from 1. Two nodes with common \text{<internal line number>} are connected by the internal line. The \text{<particle name>} is the name of a particle assigned to the internal line defined as incoming to the node. When multiple coupling constants are defined in the model, the following option is added to the \text{<node number>}

\[ \text{order} = \{ \text{order-1, order-2, ..., order-n}, \ldots \}; \]

which specifies the orders of coupling constants of the vertex, corresponding to the Order statement in the model file. Moreover, when the vertex is a blob or a counter term, the following option is also added:

\[ \text{loop} = \text{<the number of loops in the vertex>}; \]

The file ends with the End statement:

\[ \text{End} = \text{<the number of processes>}; \]
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http://arxiv.org/ps/hep-th/9408107v2
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