Spontaneous breaking of superconformal invariance in (2+1)D supersymmetric Chern-Simons-matter theories in the large N limit

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In this work we study the spontaneous breaking of superconformal and gauge invariances in the Abelian $\mathcal{N} = 1, 2$ three-dimensional supersymmetric Chern-Simons-matter (SCSM) theories in a large $N$ flavor limit. We compute the Kählerian effective superpotential at subleading order in $1/N$ and show that the Coleman-Weinberg mechanism is responsible for the dynamical generation of a mass scale in the $\mathcal{N} = 1$ model. This effect appears due to two-loop diagrams that are logarithmic divergent. We also show that the Coleman-Weinberg mechanism fails when we lift from the $\mathcal{N} = 1$ to the $\mathcal{N} = 2$ SCSM model.

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I. INTRODUCTION

The $AdS/CFT$ correspondence which relates a special weak (strong) coupled string theory to a strong (weak) coupled superconformal field theory \cite{Maldacena}, opened a new freeway in the direction of the understanding of strong coupled gauge field theories. Several aspects of the correspondence have been studied \cite{Krasnov, Minwalla}. In particular, the $AdS_4/CFT_3$ correspondence have attracted great attention in the literature due to its contribution for the development of the understanding of some condensed matter effects, especially the superfluidity \cite{Gubser} and the superconductivity \cite{Gross, Susskind}. Recently, Gaiotto and Yin suggested that various $\mathcal{N} = 2, 3$ three-dimensional SCSM theories are dual to open or closed string theories in $AdS4$ \cite{Gaiotto}. These SCSM model are superconformal invariants, an essential ingredient to relate them to $M2$ branes \cite{4, 5, 6}.

On the other hand, it is known that in a three-dimensional non-supersymmetric Chern-Simons-matter theory the conformal symmetry is dynamically broken \cite{Isham} by the Coleman-Weinberg mechanism \cite{ColemanWeinberg} in two loop approximation; the same is also true for the superconformal invariance of...
the Abelian, $D = (2 + 1)$, $\mathcal{N} = 1$ SCSM model \cite{13}, after two loops corrections to the effective (super) potential. For the $\mathcal{N} = 2$ model, on the other hand, this mechanism fails to induce a breakdown of this symmetry.

In this work we study the spontaneous breaking of the superconformal and gauge invariances in the three-dimensional Abelian $\mathcal{N} = 1, 2$ SCSM theories in the large $N$ flavor limit approximation. In the section \[\text{II}\] it is shown that the dynamical breaking of superconformal and gauge invariances in the $\mathcal{N} = 1$ SCSM model is compatible with $1/N$ expansion, determining that the matter self-interaction coupling constant $\lambda$ must be of the order of $g^6/N$, while no restriction to the gauge coupling $g$ has to be imposed. In the section \[\text{III}\] it is discussed that similarly to what happens in the perturbative approach \cite{13} the Coleman-Weinberg mechanism in the $1/N$ expansion is not feasible for the $\mathcal{N} = 2$ extension of SCSM model. This happens because the coupling constants are constrained by the conditions that minimize the effective superpotential. In the section \[\text{IV}\] the last comments and remarks are presented.

\[\text{II. } \mathcal{N} = 1 \text{ SUSY CHERN-SIMONS-MATTER MODEL}\]

The $\mathcal{N} = 1$ three-dimensional supersymmetric Chern-Simons-matter model (SCSM) is defined by the classical action

\[S = \int d^5z \left\{ -\frac{1}{2} \Gamma^\alpha W_\alpha - \frac{1}{2} \nabla^\alpha \Phi_a \nabla_\alpha \Phi_a + \lambda (\bar{\Phi}_a \Phi_a)^2 \right\}, \]

(1)

where $W^\alpha = (1/2)D^\beta D^\alpha \Gamma_\beta$ is the gauge superfield strength with $\Gamma_\beta$ being the gauge superfield, $\nabla^\alpha = (D^\alpha - ig^\alpha)$ is the supercovariant derivative, and $a$ is an index that assume values from 1 to $N$, where $N$ is the number of flavors of the complex superfields $\Phi$. We use the notations and conventions as in \cite{14}. When a mass term $\mu(\bar{\Phi}_a \Phi_a)$, with $\mu > 0$, is present in the matter sector, the SCSM model exhibits spontaneous breaking of gauge invariance and a consequent generation of mass for the scalar and gauge superfields at tree level \cite{13}.

We are dealing with a classically superconformal model, and our aim in this work is to look for the possibility of dynamical breaking of the superconformal and gauge invariances in the $1/N$ expansion. To do this, let us redefine our coupling constants, $\lambda \to \frac{\lambda}{N}, g \to \frac{g}{\sqrt{N}}$, and shift the $N$-th component of the set of superfields $\Phi_a$ (bar $\Phi_a$) by the classical background superfield $\sigma_{cl} = \sigma_1 - \theta^2 \sigma_2$ as follows

\[\bar{\Phi}_N = \frac{1}{\sqrt{2}} \left( \Sigma + \sqrt{N} \sigma_{cl} - i\Pi \right), \]

\[\Phi_N = \frac{1}{\sqrt{2}} \left( \Sigma + \sqrt{N} \sigma_{cl} + i\Pi \right), \]

(2)
with the vacuum expectation values (VEV) of the quantum superfields, i.e., \( \langle \Sigma \rangle = \langle \Pi \rangle = \langle \Phi_j \rangle = 0 \) vanishing at any order of \( 1/N \). The index \( j \) runs over: \( j = 1, 2, \cdots (N - 1) \). To investigate the possibility of spontaneous breaking of gauge/superconformal symmetry is enough to obtain the Kählerian superpotential \[13, 16\], i.e., to consider the the contributions to the superpotential, where supersymmetric derivatives \( (D^\alpha, D^2) \) acts only on the background superfield \( \sigma_{cl} \).

The action written in terms of the real quantum superfields \( \Sigma \) and \( \Pi \) and the \( (N - 1) \) complex superfields \( \Phi_j \) with vanishing VEVs is given by

\[
S = \int d^5z \left\{ -\frac{1}{2} \Gamma^a W_a - \frac{g^2 \sigma_2}{2} \Gamma^2 + \frac{g}{2} \left( \sigma_{cl} D^\alpha \Pi \Gamma_\alpha + \Pi \Pi^\alpha D^\alpha \sigma_{cl} \right) + \bar{\Phi}_j (D^2 + \lambda \sigma_{cl}^2) \Phi_j + \frac{1}{2} \Sigma (D^2 + 3 \lambda \sigma_{cl}^2) \Sigma \\
+ \frac{1}{2} \Pi (D^2 + \lambda \sigma_{cl}^2) \Pi + i \frac{g}{2 \sqrt{N}} \left( D^\alpha \bar{\Phi}_j \Gamma_\alpha \Phi_j + \bar{\Phi}_j \Gamma_\alpha D^\alpha \Phi_j \right) + \frac{g}{2 \sqrt{N}} \left( D^\alpha \Pi \Gamma_\alpha \Sigma + \Pi \Pi^\alpha D^\alpha \Sigma \right) \\
- \frac{g^2}{2N} \left( 2 \bar{\Phi}_j \Phi_j + \Sigma^2 + \Pi^2 \right) \Gamma^2 + \frac{\lambda}{N} \left( \bar{\Phi}_j \Phi_j \right)^2 + \frac{\lambda}{4N} \left( \Sigma^2 + \Pi^2 \right)^2 + \frac{\lambda}{N} (\Sigma^2 + \Pi^2) \bar{\Phi}_j \Phi_j \\
+ \frac{1}{N} \sigma_{cl} \Sigma \left( 2 \bar{\Phi}_j + \Sigma^2 + \Pi^2 - \frac{\lambda}{N} \Gamma^2 \right) + \sqrt{\frac{\lambda}{N}} \left( \lambda \sigma_{cl}^2 + D^2 \sigma_{cl} \right) \Sigma + N \sigma_{cl} D^2 \sigma_{cl} + \frac{\lambda}{4} \sigma_{cl}^4 \\
- \frac{1}{4\alpha} \left( D^\alpha \Gamma_\alpha + \alpha g \sigma_{cl} \Pi \right)^2 + \hat{c} D^2 \hat{c} + \alpha \frac{g^2 \sigma_{cl}^2}{2} \hat{c} \hat{c} + \frac{\alpha}{2 \sqrt{N}} g \sigma_{cl} \Sigma \Sigma + \mathcal{L}_{ct} \right\},
\]  

where the last line of above equation is the \( R_\xi \) gauge-fixing term and the corresponding Faddeev-Popov terms, plus counterterms of renormalization represented by \( \mathcal{L}_{ct} \). The term \(-\frac{g \sigma_{cl}}{2} D^2 \Pi \Gamma_\alpha \) is responsible for the mixing between the scalar superfield \( \Pi \) and the gauge superfield \( \Gamma^\alpha \).

The introduction of an \( R_\xi \) gauge-fixing eliminate this mixing, in the approximation considered.

From the action above, Eq.(3), we can compute the free propagators, Figure(II) of the model as

\[
\langle T \Phi_i(k, \theta) \bar{\Phi}_j(-k, \theta') \rangle = -i \delta_{ij} \frac{D^2 - M_0}{k^2 + M_0^2} \delta(2)(\theta - \theta'),
\]

\[
\langle T \Sigma(k, \theta) \Sigma(-k, \theta') \rangle = -i \frac{D^2 - M_1^2}{k^2 + M_1^2} \delta(2)(\theta - \theta'),
\]

\[
\langle T \Pi(k, \theta) \Pi(-k, \theta') \rangle = -i \frac{D^2 - M_2^2}{k^2 + M_2^2} \delta(2)(\theta - \theta'),
\]

\[
\langle T \Gamma_\alpha(k, \theta) \Gamma_\beta(-k, \theta') \rangle = -i \frac{\left( D^2 - M_A \right) D^2 D_\alpha D^\beta}{k^2 (k^2 + M_A^2)} \delta(2)(\theta - \theta'),
\]

\[
\langle T c(k, \theta) \bar{c}(-k, \theta') \rangle = -i \frac{D^2 + \alpha M_A \bar{c}}{k^2 + \alpha^2 M_A^2} \delta(2)(\theta - \theta'),
\]

where

\[
M_0 = \lambda \sigma_{cl}^2, \quad M_1 = 3 \lambda \sigma_{cl}^2, \quad M_A = \frac{g^2 \sigma_{cl}^2}{2}, \quad M_2 = \lambda \sigma_{cl}^2 - \frac{\alpha}{2} M_A.
\]
It is important to notice that these propagators are obtained as an approximation, where we are neglecting any superderivative acting on background superfield $\sigma_{cl}$. This approximation is the enough to obtain the three-dimensional Kählerian effective superpotential, as described in [17]. It does not allow us to evaluate the higher order quantum corrections of the auxiliary field $\sigma_2$. One way to do this, is to approach the effective superpotential by using the component formalism, as was done in the Wess-Zumino model in [18]. Even though our aim is to study the SCSM model in the large $N$ limit, one more approximation will be considered: we will restrict to small values of the coupling $\lambda$, a choice to be justified later, when we will show that $\lambda$ must be of the order of $g^6/N$.

The $1/N$ expansion is characterized by a mixing of loop contributions at the same level in the $1/N$ approximation. The leading order in $1/N$ expansion is given by the tree level contribution,

$$
\Gamma_{\text{tree}} = \int d^5 z N \frac{\lambda}{4} \sigma_{cl}^4, \tag{6}
$$

plus the one-loop contribution that come from the trace of the superdeterminants of the complex superfields, plus a two-loop contribution that comes from the diagram Figure2(a). The traces of superdeterminants are given by:

$$
\Gamma_{1\text{loop}} = \frac{i}{2} (N - 1) \text{Tr} \ln[D^2 + M_0] + \frac{i}{2} \text{Tr} \ln[D^2 + M_1] \\
+ \frac{i}{2} \text{Tr} \ln[D^2 + M_2] + \frac{i}{2} \text{Tr} \ln[D^2 + \alpha M_A] \\
+ \frac{i}{2} \text{Tr} \ln \left[ - \frac{i}{2} \left( 1 - \frac{1}{\alpha} \right) \partial^\alpha + \frac{C^{\beta \alpha}}{2} \left( 1 + \frac{1}{\alpha} \right) D^2 + C^{\beta \alpha} M_A \right]. \tag{7}
$$

Proceeding as described in [17], this one-loop contribution to the effective action can be written:

$$
\Gamma_{1\text{loop}} = \frac{1}{16\pi} \int d^5 z \left\{ (N - 1) \left[ \lambda \sigma_{cl}^2 \right]^2 + \left[ 3 \lambda \sigma_{cl}^2 \right]^2 + \left| \lambda \sigma_{cl}^2 - \alpha g^2 \sigma_{cl}^4 \right|^2 \\
+ \left[ g^2 \sigma_{cl}^2 \right]^2 + \left[ \alpha g^2 \sigma_{cl}^2 \right]^2 \right\}. \tag{8}
$$

The two-loop contributions, drawn in Figure2, are given by

$$
\Gamma_{2\text{loop}} = \int d^5 z \left\{ (N + 2) \frac{\lambda^3}{16\pi} + \frac{\lambda}{16\pi} |\lambda| + \frac{\alpha}{2} g^2 - \frac{1}{64\pi^2} g^4 |\lambda| (1 + \alpha|\alpha|) \\
+ \frac{g^2}{64\pi^2} \left[ C_2(\epsilon, \lambda, g) + \left( 2 \lambda^2 (1 + \alpha) + \frac{g^4}{16} (3 - \alpha^2) - \alpha^2 \lambda g^2 \right) \ln \left( \frac{\sigma_{cl}^2}{\mu} \right) \right] \\
- \frac{\lambda^3}{2\pi^2} \left[ C_1(\epsilon, \lambda) + \ln \left( \frac{\sigma_{cl}^2}{\mu} \right) \right] \right\} \sigma_{cl}^4, \tag{9}
$$
where
\[ C_1(\epsilon, \lambda) = -\frac{1}{2} \left[ \frac{1}{\epsilon} - \gamma + 1 - \ln \left( \frac{25\lambda^2}{4\pi} \right) \right], \]
\[ C_2(\epsilon, \lambda, g) = \frac{1}{8} \left\{ 6|\lambda|g^2(1 + \alpha|\alpha|) - 2\lambda^2(3 - \alpha) + 2 \left( 8\lambda^2 + \frac{3}{4}g^4 \right) \ln \left( \frac{g^2 + 4|\lambda|}{2} \right) \right. \]
\[ + \left. \frac{1}{\epsilon - \gamma + \ln 4\pi + 1} \left[ 8\lambda^2(1 + \alpha) + \frac{g^4}{4}(3 - \alpha^3) - 4\alpha^2\lambda g^2 \right] \right\}. \]  
(10)
The integrals were evaluated using the regularization by dimensional reduction [19]. In three dimensions this regularization scheme avoids any divergence at one-loop level, and so, no mass renormalization is necessary.

The effective action at subleading order is obtained by adding Eqs. (6), (8) and (9) and can be cast as
\[ \Gamma = \int d^5 z \left\{ N\lambda^4 + (N + 8)\frac{\lambda^2}{16\pi} + \frac{1}{16\pi}|\lambda - \alpha\frac{g^2}{4}|^2 + (1 + 4\alpha^2)\frac{g^4}{256\pi} \right. \]
\[ + (N + 2)\frac{\lambda^2}{16\pi^2} + \frac{\lambda}{16\pi^2}|\lambda + \frac{\alpha}{2}g^2| - \frac{1}{64\pi^2}g^4|\lambda| (1 + \alpha|\alpha|) \]
\[ + \frac{g^2}{64\pi^2} \left[ C_2(\epsilon, \lambda, g) + \left( 2\lambda^2(1 + \alpha) + \frac{g^4}{16}(3 - \alpha^2) - \alpha^2\lambda g^2 \right) \ln \left( \frac{\sigma_{cl}^2}{\mu} \right) \right] \]
\[ - \frac{\lambda^3}{2\pi^2} \left[ C_1(\epsilon, \lambda) + \ln \left( \frac{\sigma_{cl}^2}{\mu} \right) \right] + B\sigma_{cl}^4 \sigma_{cl}^4 \]
\[ = - \int d^5 z \ K_{eff}, \]  
(11)
where \( K_{eff} \) is the Kählerian effective superpotential; \( B \) is a convenient counterterm to renormalize the model. It is well known that the effective (super) potential is a gauge-dependent quantity [20].

Following the renormalization procedure as described in [12], and observing that divergences larger than logarithmic does not show up, which constrains the mass counterterm to be trivial, the only necessary condition to renormalize the \( N = 1 \) SCSM model can be cast as
\[ \frac{\partial^4 K_{eff}}{\partial \sigma_{cl}^4} \bigg|_{\sigma_{cl}=v} = -4! \frac{N\lambda}{4}, \]  
(12)
where \( v \) is a mass scale independent of the Grassmanian coordinate \( \theta \). This feature means that we are evaluating the derivatives on \( K_{eff} \) at \( \sigma_{cl} = \sigma_{cl} = v \).

We determine \( B \) by solving the Eq. (12). Substituting the result in Eq. (11) we obtain the following expression for the Kählerian effective superpotential
\[ K_{eff} = -N\frac{\lambda}{4}\sigma_{cl}^4 + \frac{e}{1024\pi^2}\sigma_{cl}^4 \left[ -\frac{25}{6} + \ln \frac{\sigma_{cl}^2}{v^2} \right], \]  
(13)
\[ e = (\alpha^2 - 3)g^6 + 16\alpha^2g^4\lambda - 32(\alpha + 1)g^2\lambda^2 + 512\lambda^3. \] (14)

The renormalization of \( K_{\text{eff}} \) requires the introduction of the mass scale, \( v \), at sub-leading order in \( 1/N \), dynamically breaking the superconformal invariance of the model.

To analyze the possibility of a dynamical breaking of the gauge symmetry we have to determine if the superfield \( \sigma_d \) acquires a non-vanishing vacuum expectation value (VEV). For this we must determine the conditions for the minimum of the effective scalar potential \( V_{\text{eff}} = \int d^2\theta K_{\text{eff}} \). So, after integrating over the Grassmannian coordinates, \( V_{\text{eff}} \) can be cast as

\[ V_{\text{eff}} = -N\lambda\sigma_2\sigma_1^3 + \frac{e}{512\pi^2}\sigma_2\sigma_1^3 \left[ -\frac{22}{3} + \ln \frac{\sigma_1}{v} \right]. \] (15)

The conditions that minimize \( V_{\text{eff}} \) are

\[ \frac{\partial V_{\text{eff}}}{\partial \sigma_1} = 3\sigma_2\sigma_1^2 \left[ -N\lambda + \frac{e}{512\pi^2} \left( -\frac{19}{3} + \ln \frac{\sigma_1}{v} \right) \right] = 0, \] (16)

\[ \frac{\partial V_{\text{eff}}}{\partial \sigma_2} = \sigma_1^3 \left[ -N\lambda + \frac{e}{512\pi^2} \left( -\frac{22}{3} + \ln \frac{\sigma_1}{v} \right) \right] = 0. \] (17)

We can see that \( \sigma_2 = 0 \) gives a vanishing \( V_{\text{eff}} \) (supersymmetric vacuum) in the minimum only if Eqs. (16) and (17) are both satisfied. The Eq. (16) is readily satisfied for \( \sigma_2 = 0 \), and the condition Eq. (17) possesses two solutions:

\[ \sigma_1 = 0, \] (18)

\[ \sigma_1 = v \exp \left\{ \frac{11}{6} + \frac{128N\pi^2\lambda}{e} \right\}. \] (19)

The first one is the trivial solution, and the complex scalar matter superfield \( \Phi_N \) does not acquire a non-vanishing VEV. This solution represents a gauge invariant phase. The other solution, Eq. (19), represents a non-vanishing VEV for the superfield \( \Phi_N \), generating masses for the gauge superfield \( \Gamma \), the scalar complex superfield \( \Phi_j \) and for the real scalar superfield \( \Sigma \).

To be consistent with the approximation we used, the minimum of effective potential must lay around \( \sigma_d \sim v \), constraining the exponential function to be approximately 1. Therefore, the coupling \( \lambda \) should satisfy

\[ \lambda = -\frac{11}{48\pi^2 N} \left[ \frac{(\alpha^2 - 3)}{16}g^6 + \alpha^2g^4\lambda - 2(\alpha + 1)g^2\lambda^2 + 32\lambda^3 \right]. \] (20)

We can see that in first order the coupling \( \lambda \) is very small, of order \( 1/48N \). This result justifies our choice of studying the model in the \( 1/N \) approximation and truncating the expansion in
powers of $\lambda$. Thus, the dynamical breaking of gauge and superconformal invariances in the $\mathcal{N}=1$ SCSM model is compatible with $1/N$ expansion presented here. The compatibility between $1/N$ expansion of $\mathcal{N}=1$ SCSM model and the Coleman-Weinberg mechanism is not a big surprise, once this effect was shown to be possible in a perturbative approach in the supersymmetric \cite{13} and non-supersymmetric \cite{11} variations of the model, where we have the freedom to play with the two independent gauge and self-interaction coupling constants, as in the original work of Coleman and Weinberg. But here we have a crucial difference. Beyond self-interaction and gauge couplings we have the parameter $N$, doing that no restriction on the order of gauge coupling $g$ be necessary.

III. $\mathcal{N}=2$ SUSY CHERN-SIMONS-MATTER MODEL

One case of interest is the extension of the number of supersymmetries of the SCSM model to $\mathcal{N}=2$ \cite{21 23}. This step is given just identifying the coupling constants $\lambda = \frac{g^2}{4}$ to eliminate fermion-number violating terms in the action written in terms of component fields, as discussed in \cite{24 25}. Performing this identification and a similar renormalization procedure through a condition like the Eq.\eqref{eq:12}, the expression of the effective Kählerian superpotential can be cast as

$$K_{\text{eff}} = -N \frac{g^2}{16} \sigma_1^2 + c(\alpha) \frac{g^6}{1024\pi^2} \sigma_1^2 \sigma_1^2 \left[ -\frac{25}{6} + \ln \frac{\sigma_1^2}{v^2} \right], \quad (21)$$

where $c(\alpha) = [3 + \alpha(5\alpha - 2)]$ is non-null for any real $\alpha$. So, for the $\mathcal{N}=2$ SCSM model, the scalar effective potential $V_{\text{eff}}$ is given by

$$V_{\text{eff}2} = -N \frac{g^2}{4} \sigma_2^2 \sigma_1^2 + c(\alpha) \frac{g^6}{512\pi^2} \sigma_2^2 \sigma_1^2 \left[ -\frac{22}{3} + \ln \frac{\sigma_1^2}{v^2} \right]. \quad (22)$$

Just as for $\mathcal{N}=1$ case, the conditions that minimize $V_{\text{eff}2}$ are

$$\frac{\partial V_{\text{eff}2}}{\partial \sigma_1} = 3g^2 \sigma_2^2 \sigma_1 \left[ -N \frac{4}{4} + c(\alpha) \frac{g^4}{512\pi^2} \left( -\frac{19}{3} + \ln \frac{\sigma_1^2}{v^2} \right) \right] = 0, \quad (23)$$

$$\frac{\partial V_{\text{eff}2}}{\partial \sigma_2} = g^2 \sigma_1^2 \left[ -N \frac{4}{4} + c(\alpha) \frac{g^4}{512\pi^2} \left( -\frac{22}{3} + \ln \frac{\sigma_1^2}{v^2} \right) \right] = 0. \quad (24)$$

Again $\sigma_2 = 0$ gives a vanishing $V_{\text{eff}2}$ in the minimum only if Eqs. \eqref{eq:23} and \eqref{eq:21} are satisfied. Once $\sigma_2 = 0$ is the supersymmetric solution, we just have to compute the solution of Eq.\eqref{eq:21}, that are given by:

$$\sigma_1 = 0, \quad (25)$$

$$\sigma_1 = v \exp \left\{ \frac{11}{6} + \frac{32N\pi^2}{c(\alpha)g^4} \right\}. \quad (26)$$
Of course, $\sigma_1 = 0$ is the gauge symmetric solution just like $N = 1$ case. For the second solution, if the minimum of effective superpotential lies around $\sigma_{cl} \sim v$, the coupling $g$ should satisfy

$$\frac{g^4}{N} = -\frac{192\pi^2}{11c(\alpha)} \approx -\frac{172}{c(\alpha)},$$

(27)

This fact determines $g$ to be of the order of $\left(\frac{N}{c(\alpha)}\right)^{1/4}$. If we observe that in the classical action every time that the coupling constant $g$ appears it is accompanied of a factor $1/\sqrt{N}$, we can see that we have an effective coupling of the order of $1/N^{1/4}$. But, the trilinear terms proportional to $\lambda/\sqrt{N}$, when we lift from $N = 1$ to $N = 2$, will be of order of $\lambda/\sqrt{N} \rightarrow -g^2/2\sqrt{N} \approx -1/2$. Therefore, our $1/N$ expansion loses its sense. This situation is similar to what happens in the perturbative (loop) expansion, where the Coleman-Weinberg mechanism for the $N = 2$ SCSM model is not compatible with perturbation theory \cite{13}. This result is in agreement with previous works \cite{7, 26, 27}, where several aspects of $N = 2, 3$ SCSM models were studied. Moreover, the above condition constrains $g^2$ to be imaginary, compromising the unitarity of the theory.

**IV. CONCLUDING REMARKS**

Summarizing, in this Letter we studied the spontaneous breaking of the superconformal and gauge invariances in the three-dimensional Abelian $N = 1, 2$ SCSM theories in the large $N$ limit approximation. It is shown that the dynamical breaking of superconformal and gauge invariances in the $N = 1$ SCSM model is compatible with $1/N$ expansion, if the matter self-interaction coupling constant $\lambda$ is of the order of $g^6/N$, while no restriction to the order of gauge coupling $g$ has to be imposed. In the $N = 2$ extension of SCSM model it is observed that as in the perturbative approach, the Coleman-Weinberg mechanism is not possible in the $1/N$ expansion, due to the constraint between the coupling constants. It is expected that non-Abelian extensions of the SCSM model share the same properties discussed here, once that the presence of logarithmic divergent Feynman diagrams of two-loop contributions that appear at subleading order in the $1/N$ expansion will also be present in such extensions.

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Figure 1: Propagators.

Figure 2: Diagram (a) contributes to leading and subleading orders, while the other diagrams are of subleading order in the large $N$ expansion.