The coupling of a young stellar disc with the molecular torus in the Galactic Centre

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ABSTRACT

The Galactic Centre hosts, according to observations, a number of early-type stars. About one half of those which are orbiting the central supermassive black hole on orbits with projected radii $\gtrsim 0.03$ pc form a coherently rotating disc. Observations further reveal a massive gaseous torus and a significant population of late-type stars. In this paper, we investigate, by means of numerical $N$-body computations, the orbital evolution of the stellar disc, which we consider to be initially thin. We include the gravitational influence of both the torus and the late-type stars, as well as the self-gravity of the disc. Our results show that, for a significant set of system parameters, the evolution of the disc leads, within the lifetime of the early-type stars, to a configuration compatible with the observations. In particular, the disc naturally reaches a specific – perpendicular – orientation with respect to the torus, which is indeed the configuration observed in the Galactic Centre. We, therefore, suggest that all the early-type stars may have been born within a single gaseous disc.

Key words: methods: numerical – stars: kinematics and dynamics – Galaxy: nucleus.

1 INTRODUCTION

Over the past two decades, nearly 200 early-type stars have been revealed in the innermost parsec of our Galaxy (see Genzel, Eisenhauer & Gillessen 2010 for the most recent review; Allen, Hyland & Hillier 1990; Genzel et al. 2003; Ghez et al. 2003, 2005; Paumard et al. 2006; Bartko et al. 2009, 2010). Observations suggest that these stars are orbiting a highly concentrated mass, which is associated with the compact radio source Sgr A*. It is widely accepted that this source is powered by a supermassive black hole (SMBH). Its mass and distance from the Sun are estimated to be approximately $4 \times 10^6 M_\odot$ and 8 kpc, respectively (Ghez et al. 2003; Eisenhauer et al. 2005; Gillessen et al. 2009a,b; Yelda et al. 2010).

According to the most recent observations of Bartko et al. (2009, 2010), the majority (136) of the early-type stars observed in the Sgr A* region are located at projected distance $0.03 \lesssim r \lesssim 0.5$ pc from the SMBH. Roughly one half of these stars appear to form a coherently rotating disc-like structure, the so-called clockwise system (CCWS; first mentioned by Genzel et al. 2003). Given these two discs, a significant number of the early-type stars are still not belonging to either of the structures.

Observations have further established that all the early-type stars between 0.03 and 0.5 pc from the SMBH are either Wolf–Rayett stars or O- or early B-stars (WR/OB stars; Bartko et al. 2009, 2010). Evolutionary phases of individual stars indicate that all of them have been formed $6 \pm 2$ Myr ago within a short period of time, probably not exceeding 2 Myr (Paumard et al. 2006). The presence of such stars so close to a SMBH is rather surprising. In particular, the tidal field of the SMBH is strong enough to prevent standard star formation mechanisms. Hence, various hypotheses have been suggested to explain the origin and configuration of the WR/OB stars observed in the Sgr A* region.

In situ fragmentation of a self-gravitating gaseous disc is probably the currently most widely accepted formation scenario for the stars of the CWs (Levin & Beloborodov 2003; Paumard et al. 2006). This process was theoretically predicted to form stars in active galactic nuclei around SMBHs of masses $10^6–10^8 M_\odot$ (Collin & Zahn 1999). However, as it naturally forms stars in a single disc-like structure, it fails to explain the origin of the stars observed outside the CWs. Many authors have, therefore, been seeking a mechanism that could have scattered these outliers from the parent disc plane.

It has been shown by Cuadra, Armitage & Alexander (2008) that two-body relaxation of the parent disc does not yield the observed large inclinations of the outliers with respect to the disc plane.
According to Kocsis & Tremaine (2010), some of them may have been brought to their positions by vector resonant relaxation between the disc and the cluster of late-type stars, which has also been reported in the Sgr A* region (Genzel et al. 2003; Schödel et al. 2007; Do et al. 2009). However, it is still unclear whether this process can explain the origin of the stars with line-of-sight angular momenta opposite to that of the stars within the CWS. In order to overcome this issue, Lückmann, Baumgardt & Kroupa (2008) have considered mutual interaction of two self-gravitating discs at large angles relative to each other. Although this mechanism indeed yields the observed configuration of the WR/OB stars, it needs rather special initial conditions. In particular, the two discs must have been formed at specific angles with respect to each other in order to stand for the CWS and CCWS. Moreover, as all the WR/OB stars seem to be coeval (Paumard et al. 2006), the two discs must have had started to form stars at the same time. Since this is not very likely, the need of such special initial conditions represents the major drawback of this scenario. Similar scenarios, such as the interaction of two gaseous streams (Hobbs & Nayakshin 2009), suffer from the same problem.

Šubr, Schovancová & Kroupa (2009) and Šubr (2010) have suggested that all the WR/OB stars in the Sgr A* region may have been born in a single gaseous disc. They argue that the stars observed outside the CWS represent the outer parts of the parent disc, that have been partially disrupted by the gravity of the circumnuclear disc (CND). The CND is a clumpy molecular torus, that is located between 1.6 and 2.0 pc from Sgr A* (Christopher et al. 2005). The total mass of this structure, which is almost perpendicular to the CWS (Paumard et al. 2006), is estimated to be of the order of $10^4 M_\odot$. Šubr et al. (2009) claim that the gravity of the CND would cause differential precession of the individual orbits in the parent stellar disc. Such a process would force the stars from the outer parts of the disc to leave the disc plane, while the inner parts of the disc would remain untouched. This core would be identified as the CWS today.

In this paper, we further investigate the hypothesis of Šubr et al. (2009). In particular, we include the self-gravity of the parent stellar disc, and follow its orbital evolution in a pre-defined external potential by means of numerical N-body computations. In addition to the SMBH and the CND, the external potential includes the gravity of the cluster of late-type stars. Even though its density profile is still unclear, its potential may be considered, in the first approximation, to be spherically symmetric and centred on the SMBH.

In the following section, we briefly introduce the problem of stellar dynamics in a perturbed Keplerian potential and describe our model. The results of our calculations are presented in Section 3 and discussed in Section 4. We conclude our work in Section 5.

2 ORBITAL EVOLUTION OF A STELLAR DISC IN AN EXTERNAL POTENTIAL

The gravitational potential in the vicinity of the CWS induced by the SMBH is, to a very high accuracy, Keplerian. Hence, it is useful to describe the individual stellar orbits by means of Keplerian orbital elements: semi-major axis $a$, eccentricity $e$, inclination $i$, longitude of the ascending node $\Omega$ and argument of pericentre $\omega$. The orbital evolution of the disc can then be investigated by following the evolution of these elements. If the only component of the external potential were Keplerian gravity of the SMBH, and if the stars in the disc were treated as test particles, the orbital elements would remain constant in time. On the other hand, with any additional potentials included, some of the elements may undergo complex secular evolution.

Šubr et al. (2009) have investigated the influence of the CND upon the orbital evolution of the disc. They have considered the disc to be surrounded by the cluster of late-type stars, and the stars in the disc to be test particles. They claim that the evolution of individual stellar orbits in the disc is dominated by two processes: (i) precession of the ascending node and (ii) periodic oscillations of eccentricity and inclination (Kozai oscillations; independently theoretically predicted by Kozai 1962 and Lidov 1962). These oscillations are diminishing with increasing mass of the cluster of late-type stars. They become fully negligible when the mass, $M_c$, of the cluster within the radius $R_{CND}$ of the CND fulfills the approximate condition $M_c \gtrsim 0.1 M_{CND}$, where $M_{CND}$ stands for the mass of the CND. In that case, the first time derivative of $\Omega$ becomes constant and can be written as

$$\frac{d\Omega}{dt} = -\frac{3}{4} \frac{\cos i}{T_K} \frac{1 + \frac{1}{2} e^2}{\sqrt{1 - e^2}}$$

where $G$ denotes the gravitational constant and $M_\star$ represents the mass of the SMBH. According to this formula, the rate of precession strongly depends upon the semimajor axis of the orbit. Hence, the outer parts of the disc are more affected by the precession than the inner parts and, therefore, the disc becomes warped or, eventually, completely disrupted.

In formula (1) and further on, we assume all angles to be measured in the frame where the CND lies in the $xy$-plane. Consequently, inclination of the orbit $i$ is the angle between the symmetry axis of the CND and angular momentum of the star.

With the gravity of the stars in the disc included, the orbital elements of individual orbits undergo random variations due to two-body relaxation. As a result, the mean value of eccentricity and inclination of the orbits increases. These changes are, by themselves, too small to have a significant impact on the overall shape of the disc (Cuadra et al. 2008). However, according to formula (1), they accelerate the differential precession of the orbits. Let us, therefore, further investigate the differential precession of the orbits within a self-gravitating disc. For this purpose, we introduce the model of the Galactic Centre in the following way.

(i) The SMBH of mass $M_\star = 4 \times 10^6 M_\odot$ is considered to be a source of Keplerian potential.

(ii) The CND is modelled as a single massive particle of mass $M_{CND}$ orbiting the SMBH on a circular orbit of radius $R_{CND} = 1.8$ pc (see Section 4.2 for the discussion of this approximation).

(iii) The cluster of late-type stars is represented by a smooth power-law density profile, $\rho(r) \propto r^{-\alpha}$, and mass $M_c$ within the radius $R_{CND}$.

(iv) The early-type stars in the disc are treated as $N$ gravitating particles, $m \in [m_{min}, m_{max}]$, distributed according to a power-law mass function $dN \propto m^{-\beta} dm$.

The stellar orbits in the disc are constructed to be initially geometrically circular. However, due to the additional spherically symmetric component of the gravitational potential (cluster of late-type stars), the osculating eccentricities do not truthfully describe real curvature of the orbits in space. For example, the initial osculating eccentricity of the outermost orbits in the disc is $\approx 0.1$ for $M_c = 0.1$ and $\beta = 7/4$. Initial radii of the orbits are, in accord with the
observations (Paumard et al. 2006; Bartko et al. 2009; Bartko et al. 2010), generated randomly between 0.04 and 0.4 pc. Their distribution obeys dV ∝ a3/2 da. The disc is initially thin with half-opening angle Δθ ≲ 5°. The initial inclination of the disc plane with respect to the CND, which is defined by the mean angular momentum of the stars in the disc, is denoted iCWS.

We follow the evolution of the latter system numerically by means of the N-body integration code GRADYN (Aarseth 2003). The gravitational potentials of both the SMBH and the cluster of late-type stars have been incorporated into the original code as additional external potentials.

3 RESULTS

The evolution of the stellar disc depends upon the shape of the gravitational potential, which is determined by the parameters M_{CND}, M_{c}, β, N, m_{min}, m_{max}, α, Δθ and Δθ_{CWS}. Hence, in order to investigate the evolution properly, it is necessary to cover all the reasonable values of these parameters. On the other hand, in order to demonstrate the results of our calculations, it is useful to define a ‘canonical’ model with the parameters set to the values listed in Table 1.

The observations suggest that all the WR/OB stars have mass m ≳ 12M⊙ (Paumard et al. 2006; Bartko et al. 2009, 2010). However, since it is likely that a number of undetected less massive early-type stars exist in the Sgr A* region, we consider m ∈ [4M⊙, 120M⊙] in the ‘canonical’ model. Nevertheless, for a more convenient comparison with the currently available observational data, we display properties of only a subset of the stars with mass m ≥ 12M⊙ in figures.

Due to the stochastic nature of the studied system, the results should be averaged over a number of realizations with identical values of the model parameters in order to distinguish general trends from random fluctuations. For this purpose, we first considered 120 realizations of the ‘canonical’ model. It has, however, turned out that the results become statistically relevant already for 12 realizations. Hence, we consider only 12 realizations of all the other models discussed in this paper in order to shorten the necessary computational time (∼5h on 3-GHz CPU per run; ∼2000 runs in total).

Since we attempt to explain the configuration of a specific observed system, every single realization represents a possible course of its evolution. We thus show the standard deviation for some of the key quantities for a more thorough description of the set of possible evolutions.

The results for the canonical model are shown in Fig. 1. The left panel demonstrates the differential precession of the orbits in the disc. It shows the evolution of the mean value of Ω within different groups of stars, which are determined by their initial distance from the centre. It turns out that the precession of the ascending node affects more strongly the orbits in the outer parts of the disc (dotted and dot-dashed lines) than those in the inner parts (dashed and solid lines). This result is in accord with formula (1) and proves the gradual deformation of the disc.

Our results further show that the precession of the ascending node in the outer parts of the disc is globally accelerated. We attribute this effect, which becomes significant on longer time-scales, to the evolution of inclination due to two-body relaxation of the disc. Such an acceleration was not found by Subr et al. (2009) as they had neglected the gravity of the stars in the disc.

The sudden drop of ⟨Ω⟩ on the dotted line in the top-left panel of Fig. 1 is a residue of Kozai oscillations. Since the cluster of late-type stars is, in our canonical model, not massive enough to suppress the oscillations of e and i entirely, the first time derivative of Ω also varies on the time-scale of T_k.

The evolution of the mean inclination ⟨i⟩ with respect to the CND within different parts of the disc is shown in the top-right panel of Fig. 1. We can see that the inclination of the outer parts of the disc is decreasing (dotted and dot-dashed lines), while it grows and saturates at ≈90° in the inner parts (dashed and solid lines). We further find that the evolution of both Ω and i is similar for all the orbits in the inner parts of the disc. Hence, the core of the disc remains rather undisturbed and coherently changes its orientation towards perpendicular with respect to the CND. This effect can be seen in the top panel of Fig. 2, which shows the directions of angular momenta of the individual stars in the disc after 6Myr of orbital evolution for one of the realizations of the canonical model (the initial state is denoted by an empty circle). Our results prove that the compact group at inclination ≈90° is formed by the stars from the inner parts of the disc, while the remaining scattered stars represent the entirely dismembered outer parts. Hence, we see that the dynamical evolution of the initially thin stellar disc leads to a configuration similar to that observed in the Sgr A* region (see Paumard et al. 2006; Bartko et al. 2009, 2010). In particular, the core of the disc can be identified with the CWS observed today and, at the same time, the stars from the dissolved outer parts can stand for the WR/OB stars found outside the CWS.

In order to compare our results with the observations more thoroughly, we further define CWS within our model in the following iterative way. As the zeroth step, the CWS is considered to be formed by a fixed number of the innermost stars from the initial disc. In the next step, we exclude from the CWS all the stars whose angular momenta deviate from the mean angular momentum of the CWS by more than 20°. On the other hand, the stars initially from outside the CWS, which do not fulfil the latter condition, are included into the CWS. Then, we recalculate the mean angular momentum of the CWS and repeat the whole procedure iteratively until there are no changes of the CWS in between two subsequent steps. Observations suggest (Paumard et al. 2006; Bartko et al. 2009, 2010) that roughly one half of the WR/OB stars are members of the observed CWS. We thus follow within our calculations the relative number of stars, n_{CWS}/N, which belong to the CWS. As can be seen in the middle-left panel of Fig. 1, this number reaches, within our model, the value of ≈0.5 at t = 6 Myr.

As computed by Paumard et al. (2006), the normal vectors of the observed CWS and the CND fulfil n_{CWS} · n_{CND} = 0.01, which corresponds to the mutual angle of 89.4°. In order to confront this feature, we investigate the evolution of the inclination i_{CWS} of the CWS with respect to the CND. Our computations show that i_{CWS} ≈ 90° at t = 6 Myr (see the middle-right panel of Fig. 1), which is in a remarkable agreement with the observational data.

Finally, we investigate the eccentricity distribution n_e within the CWS and in the whole young stellar system after 6 Myr of orbital evolution. The corresponding histograms in the bottom-left panel of Fig. 1 show that in both cases a substantial fraction of the orbits have, in accord with the observations, moderate eccentricities. The
Figure 1. Results for the canonical model (see Table 1 for the corresponding parameters). Only properties of the stars with $m \geq 12 \, \text{M}_\odot$ are displayed. The dotted lines in the middle panels denote standard deviation for the set of 120 included realizations. Top: evolution of the mean value of $\Omega$ (left-hand panel) and $i$ (right-hand panel) within different parts of the disc for one of the realizations of this model. The solid line describes the group of the innermost stars, followed by the dashed, dot-dashed and dotted line, which correspond to successive outer groups. Middle-left panel: number of stars within the CWS (i.e. with angular momentum deviating from the mean angular momentum of the CWS by less than 20°). Middle-right panel: inclination of the CWS with respect to the CND. Bottom-left panel: eccentricity distribution for the stars after 6 Myr of orbital evolution. The empty boxes denote distribution for all the stars in the young stellar system while the grey ones represent only stars within the CWS. Bottom-right panel: mean eccentricity of all the stars in the young stellar system (solid line) and within the CWS (dashed line).
mean eccentricity of the stars within the CWS (see the dotted line in the bottom-right panel of Fig. 1) is then \( \approx 0.25 \), which is somewhat lower than the value \( 0.36 \pm 0.06 \) recently reported by Bartko et al. (2009). However, at the current level of accuracy, the observations do not provide sufficient information for a reliable determination of the orbital eccentricity for a significant number of the WR/OB stars. Hence, the eccentricity criterion should be considered only as supplemental.

4 DISCUSSION

As we have shown in the previous section, all the WR/OB stars may have been formed within a single gaseous disc and, subsequently, brought to their present location by the combined effects of two-body relaxation and differential precession. In the following, we establish a set of parameters for which the evolution of the young stellar system leads to a configuration compatible with the current observational data. For this purpose, we follow the evolution of the system for various values of the model parameters. Within the results, we then concentrate on \( N_{\text{CWS}}/N \), \( i_{\text{CWS}} \), \( c_{\text{CWS}} \), and \( \epsilon \) and confront their values at \( t = 6 \) Myr with the observations.

4.1 System parameters compatible with the observations

To begin with, we follow the evolution of the young stellar system for various values of \( M_{\text{CND}} \) and \( M_{\epsilon} \) with the other parameters fixed to their canonical values (see Table 1). The results indicate that the strongest constraints on the possible values of \( M_{\text{CND}} \) and \( M_{\epsilon} \) come from \( N_{\text{CWS}}/N \) and \( i_{\text{CWS}} \). Their values for \( M_{\epsilon} \in [0.01 M_{\star}, 3 M_{\star}] \) are depicted by the solid lines in the top panels of Fig. 3, whereas \( M_{\text{CND}} \) remains constant along each of the lines and is set to either \( 0.1 M_{\star} \) or \( 0.3 M_{\star} \) or \( 0.6 M_{\star} \). According to these results, the evolution of the young stellar disc leads to values of \( N_{\text{CWS}}/N \) which accommodate the observational constraints, if \( 0.1 M_{\star} < M_{\text{CND}} < 0.3 M_{\star} \) and \( 0.01 M_{\star} < M_{\epsilon} < 2 M_{\star} \). However, the upper limit for the mass of the cluster of late-type stars has to be reduced to \( M_{\star} \), since larger values do not lead to the observed \( i_{\text{CWS}} \approx 90^\circ \). Both observational criteria are, therefore, fulfilled if \( 0.1 M_{\star} < M_{\text{CND}} < 0.3 M_{\star} \) and \( 0.01 M_{\star} < M_{\epsilon} < 2 M_{\star} \). Moreover, the results of our computations with \( M_{\epsilon} = 0 \) show that even in this case the evolution of the young stellar system leads to a configuration which matches the observational data (due to logarithmic scale in Fig. 3, the corresponding values are not displayed). Hence, we find the final intervals \( 0.1 M_{\star} < M_{\text{CND}} < 0.3 M_{\star} \) and \( 0 < M_{\epsilon} < 2 M_{\star} \). If we substitute \( M_{\star} = 4 \times 10^5 M_{\odot} \), the intervals transform to \( 4 \times 10^5 M_{\odot} < M_{\text{CND}} < 1.2 \times 10^6 M_{\odot} \) and \( 0 < M_{\epsilon} < 4 \times 10^5 M_{\odot} \).

The intervals for allowed \( M_{\text{CND}} \) and \( M_{\epsilon} \) are not affected if we evaluate the eccentricity criterion. As can be seen in the bottom panels of Fig. 3 (solid lines), all the considered values of \( M_{\text{CND}} \) and \( M_{\epsilon} \) lead to similar values of both \( (i_{\text{CWS}}) \) and \( (\epsilon) \), which satisfy the observational constraints. Nevertheless, our results show that the orbital eccentricities in the young stellar system reach slightly higher values for lower \( M_{\star} \). We attribute this effect to Kozai oscillations, which are less suppressed by the cluster of late-type stars. Furthermore, our results suggest that also for \( M_{\epsilon} \approx M_{\star} \), all the orbits in the young stellar system gain somewhat larger eccentricities, regardless the mass of the CND. Around the same value, \( i_{\text{CWS}} \) appears to be more sensitive upon the variations of \( M_{\star} \), and \( N_{\text{CWS}}/N \) reaches its minima (see the solid lines in the top panels of Fig. 3). Hence, it seems that all of these effects are somehow connected with a stronger influence of the CND on the dynamical evolution of the young stellar disc. However, at this point, we cannot provide any explanation of this effect.

In order to investigate whether the suggested intervals for \( M_{\text{CND}} \) and \( M_{\epsilon} \) depend upon the density profile of the cluster of late-type stars, we model the evolution of the young stellar system also for \( \beta = 1/2 \) and \( M_{\star} \in [0.01 M_{\star}, 3 M_{\star}] \). The other parameters remain at their canonical values (see Table 1). The dotted line in Fig. 3 proves that \( N_{\text{CWS}}/N \), as well as \( i_{\text{CWS}} \) and both \( (i_{\text{CWS}}) \) and \( (\epsilon) \), reach the same values as in the case with \( \beta = 7/4 \), except for the neighbourhood of the point \( M_{\star} \approx M_{\epsilon} \). The absence of the resonant effects observed in this case can be interpreted as a consequence of the different mass of the cluster of late-type stars enclosed within the young stellar disc, due to the different value of \( \beta \). However, since the value \( M_{\epsilon} \approx M_{\star} \) represents only the approximate upper boundary of the suggested interval for \( M_{\star} \), the latter effect is, for the purpose of this study, rather insignificant.

Similarly, in order to test the dependence of the intervals upon the mass function of the young stellar disc itself, we perform a set of computations with the disc treated as a group of 136 single mass stars each with mass \( m = 50 M_{\odot} \). The other parameters are set to their canonical values. In this case, the total mass of the disc is the same as in all the models, which we have presented so far (\( \approx 6.8 \times 10^5 M_{\odot} \)). As demonstrated by the dashed line in Fig. 3, none of the
results depends upon the mass function of the disc if its total mass is preserved.

Our calculations further show that the evolution of the young stellar disc is not affected significantly if its total mass is changed within the range \(10^{-3} - 10^6\) \(M_ \odot\). On this account, all the results presented in the previous section would remain entirely unaffected even if we did not include the undetected stars with mass \(m \in [4 M_ \odot, 12 M_ \odot]\) as their overall mass represents only a small fraction of the total mass of the whole disc.

On the other hand, decreasing the total mass of the disc by reducing the mass of the individual stars \(m\) inhibits the combined effects of two-body relaxation and differential precession. Namely, we do not observe the evolution of the CWS if \(m\) becomes negligibly small, i.e. if the stars in the disc can be considered as test particles. This effect is demonstrated by the bottom panel of Fig. 2, where we set \(m = 0.004 M_ \odot\). We can see that except for the few outermost stars, which are still slightly affected by Kozai oscillations caused by the CND, the inclination of the orbits remains constant throughout the disc. Hence, our results are in accord with the findings of Šubr et al. (2009).

We have determined the intervals for \(M_{CND}\) and \(M_c\) under assumption \(\theta_{CWS} = 70^\circ\). Our results indicate that if \(\theta_{CWS} \gtrsim 60^\circ\) and both the \(M_{CND}\) and \(M_c\) fall into the determined intervals, the evolution of the young stellar system leads within 6 Myr to a configuration that agrees with the current observations. With lower values of \(\theta_{CWS}\) considered, the CWS is entirely destroyed by the differential precession before it can reach the orientation perpendicular to the CND. On the other hand, considering \(\theta_{CWS}\) closer to 90° may increase the allowed intervals for \(M_{CND}\) and \(M_c\). However, in order to fully understand the impact of different values of \(\theta_{CWS}\) on the suggested intervals, a more detailed study would be required. We will focus on this issue in our future work.

In this paper, we consider the young stellar disc to be initially thin. Our calculations show that the course of its evolution does not depend upon the value of its initial half-opening angle if \(\Delta_0 \lesssim 5^\circ\).

### 4.2 Structure of the CND

So far, we have modelled the CND as a single massive particle on a circular orbit around the SMBH. This approximation has been used instead of analytical descriptions, e.g. infinitesimally thin ring, for numerical reasons. Namely, due to its simplicity, single-particle approach minimizes the necessary computational time, and the corresponding perturbing particle can be implemented into the original NBODY6 code in a trivial way.

The single-particle approximation follows from the standard averaging technique, which is commonly applied to many problems of celestial mechanics (see e.g. Morbidelli 2002). As a consequence,
the single-particle approximation is equivalent to the model with the CND treated as an infinitesimal thin ring if the assumptions of the averaging technique are satisfied. For the studied young stellar system, these assumptions can be written as the following two conditions for the orbital period $P_p$ of the massive CND particle: (i) $P_p$ must be significantly longer than the orbital periods $P_d$ of the early-type stars in the disc, and (ii) $P_p$ must be significantly shorter than the characteristic period $P_c$ of the studied phenomena. Since $P_c \sim 10^5$, $P_p \sim 10^5$ and $P_d \sim 10^3-10^4$ yr, both conditions are fulfilled and, therefore, the use of single-particle approximation is, in our case, well justified.

The real CND is, rather than a ring-like structure, a gaseous torus, which consists of several somewhat autonomous clumps. In order to test whether the evolution of the young stellar disc can be affected by the clumpiness of the CND, we further consider the CND to be a group of $N_{\text{CND}}$ equal-mass particles. It turns out that the CND constructed in this way is unstable with respect to its own gravity. Consequently, some of the particles successively migrate towards the SMBH. These particles can eventually be identified with several gaseous streams, which are indeed observed within the radius of the CND (for one of the most recent studies, see Zhao et al. 2010). The results further show that all the combined effects of two-body relaxation and differential precession remain present (see Fig. 4 for the case with $N_{\text{CND}} = 20$). Moreover, the infalling CND particles pose a stronger perturbation for the young stellar disc. As a result, the orbits in the disc gain higher eccentricities compared to models with the CND treated as a single massive particle on a stable orbit (see the bottom panel in Fig. 4 and the bottom-left panel in Fig. 1). Similarly, the gradual deformation of the young stellar disc, as well as its eventual destruction, are also accelerated.

Hence, it appears that the models with the CND treated as a single massive particle somewhat underestimate its influence upon the dynamical evolution of the young stellar disc. On the other hand, the perturbative influence of the infalling parts of the gaseous CND would probably not be as strong as the impact of infalling point-like particles in the latter model. A more precise approach to the gas dynamics would thus be required in order to obtain a more accurate description of the CND.

5 CONCLUSIONS

We have modelled the orbital evolution of a thin self-gravitating stellar disc in a pre-defined external potential by means of $N$-body computations. In accord with the observations of the Galactic Centre, we have considered the potential to include the gravity of the SMBH, the CND and the cluster of late-type stars. The results show that for a significant set of system parameters, the evolution of the disc leads to a configuration similar to that observed in the Sgr A* region within the lifetime of the WR/OB stars. In particular, while the outer parts of the disc are entirely dismembered due to differential precession of the orbits caused by the CND, the inner parts remain undisturbed forming the CWS. Due to the influence of the CND, the CWS tends to change its orientation towards perpendicular with respect to the CND for a variety of initial configurations.

Indeed, according to the observations, the CWS and the CND are in such a specific mutual orientation. It may thus be plausible that all the WR/OB stars observed in the Sgr A* region may have been born within a single gaseous disc and, subsequently, brought to their present location by the combined effects of two-body relaxation and differential precession.

We have further determined the possible values of the mass of the CND, $M_{\text{CND}}$, which lead to a configuration compatible with the observational data. Based on several observational criteria, we have found the approximate interval: $0.1 M_\odot \lesssim M_{\text{CND}} \lesssim 0.3 M_\odot$. Although this interval represents a very strict constraint on the mass of the CND, it is in accord with the most recent observational estimate $M_{\text{CND}} \approx 10^9 M_\odot$ found by Christopher et al. (2005).
Moreover, in agreement with the conclusions of Cuadra et al. (2008),
the suggested interval for $M_{\text{CND}}$ proves that unperturbed two-body
relaxation of the young stellar disc does not lead to the observed
configuration of the WR/OB stars.

Analogously, we have found the interval for allowed values of the
mass, $M_c$, of the cluster of late-type stars enclosed within the
radius of the CND: $0 \leq M_c \leq M_\ast$. Unlike the CND, this interval
is rather wide and contains, from the observational point of view,
virtually all reasonable values of $M_c$. We have confirmed that none
of the suggested intervals depends neither upon the shape of the
stellar mass function in the young stellar disc nor the density profile
of the cluster of late-type stars.

Let us note, however, that a less symmetric, e.g. flattened, cluster of
late-type stars could have an impact on the evolution of the young
stellar disc similarly to the influence of the CND. Nevertheless,
since the current observational data do not provide any indication
of such an asymmetry, we have not considered this possibility in
our analysis. Moreover, due to the observed perpendicular mutual
orientation of the CWS and the CND, it is likely that the CND
indeed plays a crucial role in the evolution of the young stellar disc.

Our results further indicate that the angular momenta of the ma-
Jrity of the WR/OB stars, which are scattered off the CWS plane,
are expected to point to the same hemisphere with respect to the
CND. This feature, together with the mass of the CND, as well as
the mutual orientation of the CND and the CWS, represent strongly
constrained output parameters of the scenario investigated in this
paper. It would, therefore, be beneficial to concentrate observational
efforts on the corresponding quantities and improve their accuracy
in order to test the suggested hypothesis.

Finally, let us emphasize that the combined effects of two-body
relaxation and differential precession are efficient enough to trans-
form the young stellar system into a configuration which matches the
observations in a period of time shorter than the estimated life-
time of the WR/OB stars. On this account, the presence of the CND
in the system is not necessarily required during the whole evolution
of the young stellar disc.

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