Atmospheric neutrino oscillations and maximal $\nu_\mu - \nu_\tau$ mixing in unified models

B. C. Allanach\textsuperscript{1}

\textsuperscript{1} DAMTP, Silver Street, Cambridge, Cambs., CB3 9EW, U. K.

Abstract

The recent data describing the evidence for neutrino oscillations by Super Kamiokande implies that two neutrinos are very strongly mixed. We ask the question: can approximate maximal neutrino mixing be natural in unified (GUT or string) models? We attempt to answer this question in as much generality as possible. Without specifying a particular model, we are able to show that a gauged family symmetry can naturally provide the required maximal mixing between $\mu$ and $\tau$ neutrinos. The second and third family neutrino mass eigenstates are almost degenerate with masses of order 0.2 eV.
1 Introduction

The recent data from the Super-Kamiokande collaboration [1, 2] provides compelling evidence for the existence of neutrino oscillations [2] and therefore also neutrino masses. We take the simple view here that to a good approximation the data are the result of only two neutrino flavours oscillating. This is true if mixing angles between any other neutrino species are small. There are strong experimental hints that this is indeed the case: the LSND results [3], and the small-angle MSW solution [4] to the solar neutrino problem. \( \nu_e - \nu_\mu \) oscillations as an explanation of the atmospheric neutrino effect are disfavoured by the CHOOZ data [5, 6, 7] and by the ratio of upward to downward events measured by Super-Kamiokande for \( \mu \)-like and \( e \)-like signatures [1]. The atmospheric anomaly could then be due to \( \nu_\mu \) oscillations involving a sterile neutrino \( \nu_s \) and/or \( \nu_\tau \) [8, 9]. In the following, we assume that \( \nu_\mu \) mixes dominantly with \( \nu_\tau \) rather than \( \nu_s \). This hypothesis could in principle be checked by examining the ratio of charged to neutral current events [10] or the up-down ratio of contained inclusive multi-ring events [11].

To interpret the atmospheric data as an oscillation between two neutrinos, we require \( \Delta m^2 \sim 5 - 50 \times 10^{-4} \) eV\(^2\) and \( \sin^2 2\theta \sim 1 \ [5] \). Naively, one might think there is a conflict between the usual predictions of unified theories and these parameters. Quark-lepton unification at some high (GUT or string) scale is a common prediction of unified theories. This would imply that the mixings of quarks and their leptons be equal to within factors of order 3 or so, which could arise from renormalisation effects. This is of course in conflict with charged fermion mass/mixing data and is so too simplistic. Non-renormalisable operators involving unified Higgs can be employed to explain further factors of around 3, leading to successful predictions of both GUT [12] and string-inspired [13] models. Even within this context, it is necessary to understand where the mass suppression of the neutrinos with respect to the charged fermions arises, because the Dirac neutrino masses are equal to the up-quark masses at the unification scale. This suppression has a natural explanation in terms of the see-saw mechanism. In the see-saw mechanism, heavy right-handed neutrinos of mass much larger than the electroweak scale are introduced which suppress the effective light neutrino masses. These right handed neutrinos are naturally present as the partner of the right handed electron in models containing the symmetry SU(2)\(_R\), such as left-right symmetric models [14], GUTs [15] and string-inspired models [17]. Their mass terms are not constrained to be the same magnitude as that of the quarks and so their large size (as well as hierarchies between them) can be well motivated [17]. In fact, their size is often related to the scale of SU(2)\(_R\) breaking (or the breaking of a group of which SU(2)\(_R\) is a subgroup).

Another problem is to generate such a large (maximal) mixing angle when the quarks are known to have small mixings, as described by the CKM matrix. There have been some specific proposals to solve this problem, for example by imposing discrete family symmetries [18, 19]. In ref. [20], a phenomenological texture is assumed for the Dirac and Majorana mass matrices of the \((e, \mu, \tau)\) neutrinos which reproduces large
mixing between $\nu_{\mu}$ and $\nu_{\tau}$. The authors use the Georgi-Jarlskog ansatz \cite{12} for down-quark and lepton unification, which naively produces a $\sin^2 2\theta_{e\mu}$ mixing angle too big to satisfy the small angle MSW explanation of the solar neutrino problem. However, if in this scheme the $\nu_{\mu}, \nu_{\tau}$ neutrinos are maximally mixed, the $\sin^2 2\theta_{e\mu}$ mixing angle is brought into line with what is required. The form of the down-quark Dirac mass matrix is different to that of the up-quark Dirac matrix and so would not seem to be a natural prediction of theories that unify up and down quarks, as is often the case. In ref. \cite{21}, a large mixing is generated by using a spontaneously broken gauged U(1)$_F$ family symmetry \cite{22}. This works very well and in refs. \cite{21, 16}, examples that generate large mixings were found. This approach will be utilised later to motivate a class of texture. The implications of this family symmetry for the neutrino mass structure was also investigated in ref. \cite{23}. Another recent specific SO(10) model \cite{15} has maximal mixing of the charged $\mu$ and $\tau$ leptons.

Our basic assumption is that the Dirac lepton/neutrino mass matrices have small mixings, comparable to that of the quarks. This is motivated by quark-lepton unification. It is then natural to propose that the observed maximal mixing is due to maximal mixing of the heavy right handed neutrinos. Because the mixing in the Majorana sector is not related by the vertical gauge symmetry to that of the quarks, it can have a different form. This then filters through the see-saw mechanism to provide maximally mixed light left-handed neutrinos. Here, we analyse a specific class of texture in detail. We check that the corrections expected from our approximations do not change the qualitative result. We then provide physical motivation for this ansatz in terms of a U(1)$_F$ gauged family symmetry \textit{a la} refs. \cite{21, 16}, and derive constraints on the quantum numbers of the right-handed $\nu_{\tau}, \nu_{\mu}$.

### 2 Majorana Mass Ansatz

Here, we postulate that mass terms involving $\nu_{e,s}$ don’t affect the atmospheric neutrino oscillations, allowing us to concentrate on $\nu_{\mu}, \nu_{\tau}$ masses only. The outcome of the see-saw mechanism is two light approximately left-handed neutrinos whose Majorana mass matrix is approximately

$$m_\nu = -m_D^T M^{-1}_R m_D,$$

where $m_D$ is the Dirac neutrino mass matrix of $\nu_{\mu}$ and $\nu_{\tau}$ (set equal to the charm-top quark mass matrix through quark-lepton unification) and $M_R$ is the Majorana mass matrix of the right-handed $\nu_{\mu}, \nu_{\tau}$. We expect that the charm-top quark mass matrix is approximately diagonal in the SU(2)$_L$ current basis. Otherwise, the measured smallness of $|V_{cb}| \sim 0.03$ would require a large cancellation with an element in the down quark-
sector (which we consider unnatural\(^1\)). Thus \(m_D\) is of the order

\[
m_D \sim \left( \begin{array}{cc} m_c & 0 \\ \xi m_t & m_t \end{array} \right),
\]

(2)

where \(\xi \sim O(|V_{cb}|/2) \ll 1\). The zero in the (1,2) position of \(m_D\) is only approximate: a non-zero element has negligible effect if it is much smaller than \(m_t\). \(m_D\) is approximately diagonal in the sense that the orthogonal diagonalising matrices \(U, V\) defined by

\[
U^T m_D V = \text{diag}(m_c, m_t)
\]

(3)

are approximately \(\frac{1}{2}\). In many models, see for example ref. [17], Eq. (2) is a prediction valid at a particular high scale rather than an order of magnitude statement. \(M_R\) is the two by two right-handed neutrino Majorana mass matrix whose eigenvalues are expected to be much greater than \(m_t\).

We now postulate that the form

\[
M_R \approx \left( \begin{array}{cc} 0 & M \\ M & 0 \end{array} \right)
\]

(4)

will provide maximal mixing between the light \(\nu_\tau, \nu_\mu\). This possibility has already been included in some of the models of ref. [21, 16]. Using Eq. (1), we determine the effective light neutrino mass matrix

\[
m_\nu \approx m' \left( \begin{array}{cc} 2\xi & 1 \\ 1 & 0 \end{array} \right),
\]

(5)

where \(m' \equiv m_c m_t / M\). We can already see the property of maximal mixing from Eq. (5) from the smallness of diagonal entries in comparison to the off-diagonal ones. We now have two approximately degenerate neutrinos, with masses

\[
m_2 = m'(1 - \xi + O(\xi^2)),
\]

\[
m_3 = m'(1 + \xi + O(\xi^2))
\]

(6)

respectively. Therefore, \(\Delta m^2_{23} \approx 4m'^2\xi + O(\xi^3)\). Using \(\Delta m^2_{23} = 3 \times 10^{-3} \text{ eV}^2\) to explain the atmospheric data and substituting \(\xi = |V_{cb}|/2\) yields \(m' \sim O(0.2) \text{ eV}\). This value of \(m'\) corresponds to \(M \sim O(10^{12}) \text{ GeV}\). The energy density of relic neutrinos is [25]

\[
\Omega_\nu h^2 = \sum_i \frac{m_{\nu_i}}{94 \text{ eV}} \times 4.3 \times 10^{-3}
\]

(7)

where \(h = H/100 \text{ Km/sec/Mpc}\) is the present Hubble parameter. So for \(h > 0.5\), one has \(\Omega_\nu < 0.017\), i.e. there would be at most a 2% component of hot dark matter if \(\Omega_{\text{total}} = 1\).

\(^1\)However, this possibility could be motivated by a suitable weakly broken discrete symmetry [24] but we do not consider this case here.
We may now ask how far from maximal mixing the neutrinos are. \( m_\nu \) is diagonalised by the 2 by 2 orthogonal rotation \( U_\nu^T m_\nu U_\nu \), where
\[
U_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 + \xi/2 & -1 + \xi/2 \\ 1 - \xi/2 & 1 + \xi/2 \end{pmatrix} + O(\xi^2). \tag{8}
\]
The full lepton mixing comes from \( U_\nu^T U_L \), where \( U_L \) is the analogous mixing of the \( \mu \) and \( \tau \) leptons, which is supposed to also be small as a requirement of quark-lepton unification. The off-diagonal elements of \( U_L \) are of order \( \xi \) and so we parameterise \( U_L \) by
\[
U_L = \frac{1}{\sqrt{1 - a\xi}} \begin{pmatrix} 1 & a\xi \\ -a\xi & 1 \end{pmatrix} \tag{9}
\]
where \( a \sim O(1) \). This predicts that
\[
\sin^2 2\theta_{\mu\tau} = 1 + \frac{1}{1/\sqrt{2} - 1/2 + 2a - a^2}\xi^2 + O(\xi^3), \tag{10}
\]
thus the mixing is maximal up to effects of order \( |V_{cb}|^2 \).

We now consider corrections to texture zeroes in \( M_R \). If we perturb the zeroes:
\[
M_R = M \begin{pmatrix} c & 1 \\ 1 & b \end{pmatrix}, \tag{11}
\]
we find that
\[
m_\nu \approx \frac{1}{1 - cb} m' \begin{pmatrix} 2\xi - b\frac{m_\nu}{m_t} - c\xi^2m_t/m_c & 1 - c\frac{m_\nu}{m_c}\xi \\ 1 - c\frac{m_\nu}{m_c}\xi & -c\frac{m_\nu}{m_c} \end{pmatrix} \tag{12}
\]
\( \xi m_t/m_c \sim O(1) \), so we see that maximal mixing is spoiled by the (2,2) element of Eq.(12) unless \( c < O(\xi) \). In this case, we obtain the same form of \( m_\nu \) as Eq.(5) for \( b < O(1) \). One should also check that the form of \( m_\nu \) in Eq.(5) is not spoiled by renormalisation (i.e. that large positive corrections don’t appear on the diagonal). It is necessary to set the model valid at scales below \( SU(2)_R \) breaking to make this check, so we merely examine a few common examples here. If the unified theory breaks straight to The Standard Model, radiative corrections are of the form\[26\]
\[
16\pi^2 \frac{d m_\nu}{d \ln \mu} = \left( \frac{1}{2} \lambda_2 - 3g_2^2 + \text{Tr}(6Y^\dagger_U Y_U + 6Y^\dagger_D Y_D + 2Y^\dagger_L Y_L) \right) m_\nu +
\]
\[
\frac{1}{2}(m_\nu Y^\dagger_L Y_L) + \frac{1}{2}(Y_L Y^\dagger_L m_\nu), \tag{13}
\]
where \( Y_{U,L,D} \) are the 3 by 3 up-quark, charged lepton and down quark Yukawa matrices respectively, \( \lambda_2 \) is the Higgs self-coupling and \( \mu \) is the \( \overline{MS} \) renormalisation scale. The dominant term on the RHS of Eq.(13) is the one proportional to \( m_\nu \) because the top-Yukawa coupling, \( (Y_U)_{33} \) is larger than all other couplings. Thus we see that the dominant correction to an entry of \( m_\nu \) is of the same order (and form), implying that the texture is not spoiled by large corrections to diagonal elements from the dominant
corrections. The 2 Higgs doublet model and MSSM have a similar form \cite{13} (but with different coefficients) to Eq.\ref{13}. Thus in these models, the above reasoning still applies if $\tan\beta$ is not extremely high. If $\tan\beta$ is high, say around 50 (as one would expect if up and down quarks are unified), one must examine the last two terms of Eq.\ref{13} because $h_\tau \equiv (Y_L)_{33} \sim O(1)$. $Y_L$ should have a similar form to $Y_U$, which we parameterise as:

$$Y_L = h_\tau \begin{pmatrix} d\xi & 0 \\ e\xi & 1 \end{pmatrix}$$

(14)

where $d, e \sim O(1)$. Then the last two terms in $16\pi^2 dm_\nu/d\ln\mu$ are of order

$$m'h_\tau^2 \left( \frac{d^2\xi^2}{2} \cdot \frac{1}{2} \cdot \frac{d\xi^2}{de} \right),$$

(15)

so the off-diagonal elements could possibly reduce the degeneracy of the two neutrinos, but this would have to be checked in detail for any specific model.

3 Family gauge symmetry

In the above section we showed how the existence of texture zeroes in the right-handed neutrino Majorana mass matrix and quark-lepton unification predicted approximately maximal mixing of $\nu_\mu, \nu_\tau$ provided $M \sim 10^{12}$ GeV. To describe the origin of these features, we now appeal to the existence of a $U(1)_F$ family gauge symmetry, which has been shown to lead to the correct hierarchies in the masses of charged fermions \cite{23} and which are common in string theories. The family symmetry is broken at a high scale by Standard Model singlet Higgs vacuum expectation values (vevs) $\langle \theta \rangle \sim \langle \bar{\theta} \rangle$, where $\theta, \bar{\theta}$ have family charges -1, +1 respectively. Hierarchies in the effective mass terms are realised by $U(1)_F$ invariant non-renormalisable operators $\bar{f}_L f_R H \epsilon^n$, where $f_L, f_R$ are generic left and right handed Standard Model fermions and $H$ is the relevant Higgs field. $\epsilon \equiv \langle \theta \rangle / M'$ is a parameter of order 0.1, suppressed by the heavy mass scale $M'$ which could be the string scale \cite{17}, or some other mass scale of heavy particles in the field theory \cite{27}. $n$ is a non-negative integer calculated from the powers of $\theta$ or $\bar{\theta}$ that are required to make the operator invariant under $U(1)_F$. The fermions are assigned family dependent charges such that the observed charged fermion mass/mixing hierarchies are well reproduced. This assignment can lead to model dependence: for example, one has to specify the number of electroweak Higgs doublets and their charges. There are many different assignments for the charged fermions which can reproduce the correct hierarchies and we assume here that this is the case. Thus we do not consider the $U(1)_F$ quantum numbers of the charged fermions, just $q_\mu, q_\tau$ (those of the right-handed $\nu_\mu, \nu_\tau$ particles respectively) suffice. Because we don’t consider these other quantum numbers, we also don’t consider the possible $U(1)_F$ anomalies. We derive constraints on $q_\mu, q_\tau$ by the requirement that they reproduce the texture zeroes of Eq.\ref{4}.
Thus we have the right handed neutrino masses of order
\[ M_R \sim M'' \left( \frac{\epsilon^{2|q_\mu|}}{\epsilon^{|q_\tau + q_\mu|}} \frac{\epsilon^{3|q_\mu + q_\tau|}}{\epsilon^2|q_\tau|} \right), \] (16)
where \( M'' \) is another mass that sets the scale of the right-handed neutrino masses (such as the GUT, string, or \( SU(2)_R \) breaking scale). Then, the requirement of approximate texture zeroes along the diagonal leads to the simultaneous constraints
\[ |q_\mu| > |q_\mu + q_\tau|/2, \quad |q_\tau| > |q_\mu + q_\tau|/2. \] (17)
These inequalities are incompatible if \( q_\mu \) and \( q_\tau \) have the same sign, and so the signs must be opposite. First, we examine the case where \( q_\mu < 0 \) and \( q_\tau > 0 \). There are two relevant cases:

\[ |q_\mu| \leq q_\tau : \quad M_R = M'' \epsilon^{q_\tau - |q_\mu|} \left( \frac{\epsilon^{3|q_\mu| - q_\tau}}{\epsilon^{q_\tau + |q_\mu|}} \right), \]
\[ |q_\mu| > q_\tau : \quad M_R = M'' \epsilon^{|q_\mu| - q_\tau} \left( \frac{\epsilon^{q_\tau + |q_\mu|}}{\epsilon^{3|q_\mu| - q_\tau}} \right). \] (18)
Requiring that the diagonal entries be smaller than the off-diagonal ones now leads to the constraint
\[ \frac{q_\tau}{3} < |q_\mu| < 3q_\tau. \] (19)
For the other case \( q_\mu > 0 \) and \( q_\tau < 0 \), a similar analysis shows that \( \mu \leftrightarrow \tau \) in Eq. (18). Whereas \( b < O(1) \) is already satisfied, to reproduce \( c < O(\xi) \) in Eq. (12), the (1,1) element must be at most of order \( \epsilon^3 \) in Eq. (13). This further restricts possible charge assignments, the result of which is shown in Fig. (1). So far, we have considered integers only for \( q_\mu, q_\tau \). If they are rational numbers, a discrete symmetry can force the zeroes to be exact. For example, if \( q_\mu = 1/3, q_\tau = -1/3 \) then U(1)$_F$ breaks to a Z$_3$ gauge symmetry and there are exact texture zeroes in the (1,1) and (2,2) entries.

As an explicit viable example of corrected texture zeroes, we choose \( q_\mu = -2, q_\tau = 2 \):
\[ M_R \sim M'' \left( \frac{\epsilon^4}{\epsilon} \frac{1}{\epsilon^4} \right) \] (20)
from which we see that the desired structure is accurately reproduced. \( M'' = 10^{12} \) GeV gives the correct \( \Delta m^2 \) for this charge assignment, and so could be the direct breaking scale of \( SU(2)_R \), generated for example in the several-stage intermediate breaking of a GUT. In general \( M'' \) has an order of magnitude upper bound of the scale of \( SU(2)_R \) breaking (or the breaking of a symmetry which contains \( SU(2)_R \)). If \( M'' \) has a non-renormalisable origin it can be orders of magnitude lower than the \( SU(2)_R \) scale. This can be the case if the tree-level mass is forbidden by the vertical gauge symmetries, as in ref. [17] for example. An example of the approximate form in Eq. (1) was found by appealing to U(1)$_F$ gauge symmetry in ref. [16], where the authors concentrate on the Giudice ansatz for Dirac masses.
Figure 1: Possible integer charges of right handed neutrinos leading to maximal mixing. A cross shows a charge assignment that leads to maximal mixing of $\nu_\mu$, $\nu_\tau$. We have only considered $|q_\mu|, |q_\tau| \leq 4$.

With $\epsilon \sim 0.1$,\

$$|q_\tau + q_\mu| \sim \log_{10} \frac{M''}{M}$$

would predict the correct order of magnitude of $M$, given $M''$. Using $M \sim 10^{12}$ GeV to explain the maximal mixing and $M'' = M_{GUT} = 10^{16}$ GeV, we observe that $|q_\tau + q_\mu| = 4$ predicts the correct order of magnitude for $M$ for this special case.

4 Conclusions

We have shown that the maximal mixing of $\nu_\mu$, $\nu_\tau$ atmospheric neutrino anomaly can be naturally obtained in unified theories. Our basic assumption is that Dirac mass mixing is small, motivated by quark-lepton unification present in unified theories. The maximal mixing occurs within the right handed neutrino Majorana mass matrix. The mechanism that motivates this ansatz is already familiar: a $U(1)_F$ family gauge symmetry spontaneously broken at a high scale. The parameters of the oscillation measured by Super-Kamiokande ($\Delta m^2$ and large mixing angle) can be set successfully by a choice of family dependent quantum numbers.

The model is general in the sense that by only considering the two relevant neutrinos, and not taking renormalisation effects into account, it is not sensitive to the particular form of extended symmetry one wants to examine. Other neutrino anomalies can be explained by oscillations without conflict with this scheme if their mixings are weak. For example, to motivate the small-angle MSW solution to the solar neutrino
problem, one might include $\nu_e$ oscillations with small angles. In particular, choosing the $U(1)_F$ charge of the right-handed electron neutrino, $q_e = p$ (where $p$ is an integer) and $q_\mu = 1/3, q_\tau = -1/3$ would yield the exact form

$$M_R = M'' \begin{pmatrix} \lambda e^{2|p|} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

(22)

then the angle of the MSW-type oscillations would be naturally small, as they originate from the small Dirac mixing. One conclusion of ref. [20] will hold here also if we imposed the Georgi-Jarlskog texture for down and charged lepton mass matrices, i.e. that the small $\sin^2 2\theta_{e\mu}$ mixing required becomes compatible with the size of $|V_{us}|$ because of suppression due to $\nu_\tau, \nu_\mu$ maximal mixing. However, the unknown dimensionless coupling $\lambda$ would have to be tuned to make $\nu_e$ degenerate with $\nu_{\mu,\tau}$ in order to provide the low $\Delta m^2$ values required by solar neutrino oscillations. Another possibility would be to have an approximately massless $\nu_e$, providing a candidate $\Delta m^2_{12}$ in the correct ballpark for the reported LSND oscillations. Then perhaps a sterile neutrino could allow for solar neutrino oscillations by mixing with the light $\nu_1$. If there are other large mixings of the light neutrinos (as could be implied by the vacuum oscillation or large angle MSW solutions to the solar neutrino problem), the approximation of only considering two neutrinos is incorrect. Much work [28] has recently been focussed upon this possibility.

An obvious extension of the present scheme is to include $\nu_e$ and possibly $\nu_s$ oscillations to explain the other data. An explicit treatment of the unified quark and lepton $U(1)_F$ family charges would then be important to see whether the solar and/or LSND data can be reproduced. The neutrinos responsible for the atmospheric neutrino anomaly are predicted to be approximately degenerate, with masses of the order of 0.2 eV.

**Acknowledgements**

I would like to thank W. Buchmuller, H. Dreiner, C. K. Jung, S. F. King, S. Sarkar and W. Scott for discussions about this work. This work was supported by PPARC and partly carried out at Ringberg Euroconference ‘New Trends in Neutrino Physics 1998’.

**References**

[1] Y. Fukuda *et al.*, Super-Kamiokande collaboration, Phys. Lett. B436 (1998) 33; *ibid.* Phys. Rev. Lett. 81 (1998) 1158; *ibid.* Phys. Lett. B433 (1998) 9.

[2] Y. Kajita, talk at ‘Neutrinos 98’.
[3] C. Athanassopoulos et al., Phys. Rev. Lett. 81 (1998) 1774.
[4] P. Krastev and A. Y. Smirnov, Phys. Lett. B338 282 (1994).
[5] M. C. Gonzalez-Garcia et al., Phys. Rev. D58 (1998) 033004.
[6] M. Apollonio et al., CHOOZ collaboration, Phys. Lett. B420 397 (1998).
[7] C. Giunti, [hep-ph/9802201].
[8] P. Lipari and M. Lusignoli, Phys. Rev. D58 (1998) 073005.
[9] R. Foot, R. R. Volkas, and O. Yasuda, Phys. Rev. D58 (1998) 013006.
[10] F. Vissani and A. Y. Smirnov, Phys. Lett. B432 (1998) 376.
[11] L. J. Hall and H. Murayama, Phys. Lett. B436 (1998) 323.
[12] H. Georgi and C. Jarlskog, Phys. Lett. B86, 297 (1979).
[13] B. C. Allanach, S. F. King, G. K. Leontaris, and S. Lola, Phys. Rev. D56 2632 (1997).
[14] B. C. Allanach and S. F. King, Nucl. Phys. B459 75 (1996).
[15] C. H. Albright, K. S. Babu, and S. M. Barr, Phys. Rev. Lett. 81 (1998) 1167.
[16] G. K. Leontaris, S. Lola, C. Scheich, and J. Vergados, Phys. Rev. D53 6381 (1996).
[17] B. C. Allanach, G. K. Leontaris, and S. T. Petcov, [hep-ph/9712446].
[18] Zhi-zhong Xing, [hep-ph/9804433].
[19] M. Drees, S. Pakvasa, X. Tata, and T. ter Veldhuis, Phys. Rev. D57 5335 (1998).
[20] M. Bando, T. Kugo, and K. Yoshioka, Phys. Rev. Lett. 80 3004 (1998).
[21] P. Binetruy, S. Lavignac, S. Petcov, and P. Ramond, Nucl. Phys. B496 3 (1997).
[22] L. Ibanez and G. G. Ross, Phys. Lett. B332 100 (1994).
[23] H. Dreiner et al., Nucl. Phys. B436 461 (1995).
[24] H. Fritschr and D. Holtmannspottor, Phys. Lett. B338 290 (1994); M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Rev. D57 4429 (1998).
[25] S. Sarkar, [hep-ph/9710273].
[26] P. H. Chankowski and Z. Pluciennik, Phys. Lett. B316 312 (1993).
[27] G. G. Ross, Phys. Lett. B364 216 (1995).
[28] V. Barger, S. Pakvasa, T. J. Weiler and K. Whisnant, Phys. Lett. B437 (1998) 107.