Impurity effect of Lambda hyperon on collective excitations of atomic nuclei

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Abstract

Taking the ground state rotational band in $^{24}$Mg as an example, we investigate the impurity effect of Λ hyperon on collective excitations of atomic nuclei in the framework of non-relativistic energy density functional theory. To this end, we take into account correlations related to the restoration of broken symmetries and fluctuations of collective variables by solving the eigenvalue problem of a five-dimensional collective Hamiltonian for quadrupole vibrational and rotational degrees of freedom. The parameters of the collective Hamiltonian are determined with constrained mean-field calculations for triaxial shapes using the SGII Skyrme force. We compare the low-spin spectrum for $^{24}$Mg with the spectrum for the same nucleus inside $^{25}_{\Lambda}$Mg. It is found that the Λ hyperon stretches the ground state band and reduces the $B(E2 : 2^+_1 \rightarrow 0^+_1)$ value by $\sim 9\%$, mainly by softening the potential energy surface towards the spherical shape, even though the shrinkage effect on the average proton radius is only $\sim 0.5\%$.

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I. INTRODUCTION

Since the first discovery of Λ-hypernuclei by observing cosmic-rays in emulsion chambers [1], hypernuclei, which are nuclei with one or more of the nucleons replaced with hyperons, have been used as a natural laboratory to study hyperon-nucleon and hyperon-hyperon interactions, properties of hadrons in nuclear environment, and in particular the impurity effect of hyperon in nuclear medium [2–4]. Due to the absence of Pauli’s principle between the nucleon and the Λ particle, a Λ hyperon can probe deeply into the interior of nuclear medium and have important influences on its properties, including softening the equation of state [5], modifying the shape and size of finite nucleus [6], changing the nuclear binding and thus the driplines of neutrons and protons [7] as well as the fission barrier heights in heavy nuclei [8].

In the past decade, many high-resolution γ-ray spectroscopy experiments using germanium detector arrays (Hyperball) have been carried out for Λ-hypernuclei [9] to understand the nature of Λ-nucleon interaction in nuclear medium and the impurity effect of a Λ on nuclear structure. In particular, the facilities built at J-PARC will provide an opportunity to perform hypernuclear γ-ray spectroscopy study with high precision by improving the quality of the secondary mesonic beam [10]. These facilities offer useful tools to study the low-lying states of hypernuclei, especially those of medium and heavy hypernuclei. To date, there are many experimental data not only on the single-Λ binding energy but also on the hypernuclear γ-ray spectroscopy that allow us to study the Λ-nucleon interaction, nuclear medium effects of baryons and impurity effects induced by a Λ hyperon in much greater detail [9].

The theoretical studies for the hypernuclear γ-ray spectroscopy are mainly performed with the cluster model [4, 11], few-body model [12, 13], and shell model [14]. The energy level scheme, M1 and E2 transition rates in low-lying states of light Λ-hypernuclei have been investigated with either a one-boson exchange potential or a parameterized spin-dependent Λ-nucleon interaction. Due to the numerical difficulty, the application of these models to medium and heavy hypernuclei is greatly limited. It is noted that, recently, the framework of few-body model has been extended to the case of five-body and used to study the energy levels of the double Λ-hypernucleus, $^{11}_{\Lambda \Lambda}$Be [15].

The framework of nuclear energy-density functionals (EDF) is nowadays one of the most
important microscopic approaches for large-scale nuclear structure calculations in medium and heavy nuclei [16] and has already been extended to study hypernuclei [17–24]. Recently, both the non-relativistic Skyrme-Hartree-Fock (SHF) theory [25–27] and the relativistic mean-field (RMF) theory [28] have been applied to study the impurity effect of Λ hyperon on the deformation of Λ-hypernuclei. The predicted energy surface is somewhat soft, in which case a large shape fluctuation effect of collective vibration might be expected. Furthermore, the static single-reference (SR) EDF is characterized by symmetry breaking (e.g., translational, rotational, particle number), and can provide only an approximate description of bulk ground-state properties. Therefore, to calculate excitation spectra and electromagnetic transition rates in individual hypernuclei, it is necessary to extend the SR EDF framework to include collective correlations related to restoration of broken symmetries and to fluctuations of collective coordinates.

In recent years several accurate and efficient methods and algorithms have been developed that perform the restoration of rotational symmetries in 3D Euler space broken by the static nuclear mean field and take into account fluctuations around the mean-field minimum [29–32]. The most effective approach to configuration mixing calculations is the generator coordinate method (GCM). Within these methods, the energy spectrum and electromagnetic transition rates of low-lying excited states in both light and heavy nuclei have been successfully reproduced. However, these approaches are currently developed only for even-even nuclei, which cannot be extended straightforwardly to study the γ-ray spectra of single-Λ hypernuclei by simply adding hyperon degree of freedom.

At present, the extension of 3D angular momentum projected GCM (3DAMP+GCM) method to single-Λ hypernuclei based on triaxial symmetry-breaking intrinsic states is still much complicated and its applications to medium-heavy and heavy nuclei would be computationally demanding. As an alternative approach to the 5D quadrupole dynamics that restores rotational symmetry and allows for fluctuations around the triaxial mean-field minima, a 5D collective Bohr Hamiltonian (5DCH) has been formulated with deformation-dependent parameters determined by microscopic selfconsistent mean-field calculations [33–36]. In this work, we will construct a 5DCH with the parameters derived from the Skyrme-Hartree-Fock calculations for the nuclear core in a single Λ-hypernucleus and calculate the corresponding low-spin excitation spectra. The impurity effect of Λ hyperon on the collective motion of an atomic nucleus will be examined by studying the modifications of collective excitation
spectrum. In this way, we will in this paper concentrate on the modification of the core nucleus due to the addition of a Λ particle, leaving the evaluation of the spectrum of the whole hypernucleus as a future work.

The paper is organized as follows. In Section II we present a brief outline of the 5DCH method and the Skyrme-Hartree-Fock approach for Λ-hypernucleus. The collective potential energy surface, parameters in collective Hamiltonian as well as the resultant collective excitation spectra for $^{24}$Mg and the same nucleus inside $^{25}_{\Lambda}$Mg are given in Section III. A brief summary and an outlook for future studies are included in Section IV.

II. THE METHOD

A. Collective Hamiltonian in five dimension

The collective Hamiltonian that describes the nuclear excitations of quadrupole vibrations, 3D rotations, and their couplings can be written in the form:

$$\hat{H} = \hat{T}_{\text{vib}} + \hat{T}_{\text{rot}} + V_{\text{coll}},$$

where $V_{\text{coll}}$ is the collective potential. The vibrational kinetic energy reads,

$$\hat{T}_{\text{vib}} = -\frac{\hbar^2}{2\sqrt{wr}} \left\{ \frac{1}{\beta^4} \left[ \frac{\partial}{\partial \beta} \sqrt{\frac{r}{w}} \beta^4 B_{\gamma \gamma} \frac{\partial}{\partial \beta} + \frac{\partial}{\partial \beta} \sqrt{\frac{r}{w}} \beta^3 B_{\beta \gamma} \frac{\partial}{\partial \beta} \right] + \frac{1}{\beta \sin 3\gamma} \left[ -\frac{\partial}{\partial \gamma} \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta \beta} \frac{\partial}{\partial \gamma} \right] + \frac{1}{\beta} \frac{\partial}{\partial \gamma} \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta \gamma} \frac{\partial}{\partial \gamma} \right\},$$

and the rotational kinetic energy,

$$\hat{T}_{\text{rot}} = \frac{1}{2} \sum_{\kappa=1}^{3} \frac{\hat{J}_{\kappa}^2}{\mathcal{I}_{\kappa}},$$

with $\hat{J}_{\kappa}$ denoting the components of the angular momentum in the body-fixed frame of a nucleus. It is noted that the mass parameters $B_{\beta \beta}$, $B_{\beta \gamma}$, $B_{\gamma \gamma}$, as well as the moments of inertia $\mathcal{I}_{\kappa}$, depend on the quadrupole deformation variables $\beta$ and $\gamma$,

$$\mathcal{I}_{\kappa} = 4B_{\kappa} \beta^2 \sin^2(\gamma - 2\kappa\pi/3), \quad \kappa = 1, 2, 3.$$

Two additional quantities that appear in the expression for the vibrational energy, that is, $r = B_1B_2B_3$, and $w = B_{\beta \beta}B_{\gamma \gamma} - B_{\beta \gamma}^2$, determine the volume element in the collective space.
The corresponding eigenvalue problem is solved by expansion of eigenfunctions in terms of
a complete set of basis functions that depend on the deformation variables $\beta$ and $\gamma$, and the
Euler angles $\phi$, $\theta$ and $\psi$ [37].

The dynamics of the collective Hamiltonian is governed by seven collective quantities,
that is, the collective potential $V_{\text{coll}}$, three mass parameters $B_{\beta\beta}$, $B_{\beta\gamma}$, and $B_{\gamma\gamma}$, and three
moments of inertia $I_i$. These quantities are functions of the intrinsic deformations $\beta$ and $\gamma$
and will be determined by Skyrme-Hartree-Fock calculations with constraints on the mass
quadrupole moments.

B. Skyrme-Hartree-Fock approach for $\Lambda$-hypernucleus

In Ref. [27], the computer code ev8 [38] of SHF+BCS approach has already been extended
for the study of $\Lambda$ hypernuclei. Therefore, in the following, we start from this approach to
calculate the seven collective quantities in the 5DCH, as shown in Eq. (1).

In the SHF+BCS approach for $\Lambda$ hypernucleus, the total energy $E$ can be written as the
integration of three terms,

$$E = \int d^3r \left[ \mathcal{E}_N(r) + T_\Lambda(r) + \mathcal{E}_{N\Lambda}(r) \right],$$

where $\mathcal{E}_N(r)$ is the standard nuclear part of energy functional, including both $ph$-channel of
the Skyrme force and $pp$-channel of the $\delta$-force, as well as the kinetic energy density for the
nucleons [38, 39]. $T_\Lambda(r) = \frac{\hbar^2}{2m_\Lambda} \tau_\Lambda$ is the kinetic energy density of $\Lambda$ hyperon.
$\mathcal{E}_{N\Lambda}(r)$ is the interaction energy density between the $\Lambda$ and nucleons given in terms of the $\Lambda$ and nucleon
densities [40],

$$\mathcal{E}_{N\Lambda} = t_0^\Lambda (1 + \frac{1}{2} t_0^\Lambda) \rho_\Lambda \rho_N + \frac{1}{4} (t_1^\Lambda + t_2^\Lambda) (\tau_\Lambda \rho_N + \tau_N \rho_\Lambda)$$
$$+ \frac{1}{8} (3t_1^\Lambda - t_2^\Lambda) (\nabla \rho_N \cdot \nabla \rho_\Lambda) + \frac{1}{4} t_3^\Lambda \rho_\Lambda \rho_N^2 + 2 \rho_a \rho_p$$
$$+ \frac{1}{2} W_0^\Lambda (\nabla \rho_N \cdot \mathbf{J}_\Lambda + \nabla \rho_\Lambda \cdot \mathbf{J}_N) \tau_N \rho_\Lambda. \quad (6)$$

Here, $\rho_\Lambda, \tau_\Lambda$ and $\mathbf{J}_\Lambda$ are respectively the particle density, the kinetic energy density, and the
spin density of the $\Lambda$ hyperon. These quantities are given in terms of the single-particle
wave-function of $\Lambda$ and occupation probabilities [39]. $t_0^\Lambda, t_1^\Lambda, t_2^\Lambda, t_3^\Lambda$, and $W_0^\Lambda$ are the Skyrme
parameters for the $\Lambda N$ interaction.
The pairing correlation between the nucleons is taken into account in the BCS approximation. The density-dependent $\delta$-force is adopted in the $pp$ channel,

$$V(r_1, r_2) = -g \frac{1 - \hat{P}^\sigma}{2} \left[ 1 - \frac{\rho(r_1)}{\rho_0} \right] \delta(r_1 - r_2), \quad (7)$$

where $\hat{P}^\sigma$ is the spin-exchange operator, and $\rho_0 = 0.16 \text{ fm}^{-3}$.

The HF equations for the nucleons and $\Lambda$ are obtained by varying the HF energy (5) with respect to the corresponding single-particle wave functions and are solved by discretizing individual single-particle wave functions on a three-dimensional Cartesian mesh. More details can be found in Ref. [38].

The method of quadratic constraints on the quadrupole moments of the nuclear density is used to find nuclear intrinsic wave functions (including the quasiparticle energies $E_i$, occupation probabilities $v_i$, and single-nucleon wave functions $\psi_i$) corresponding to the desired quadrupole deformations [38, 41]. With these wave functions, one can calculate the moments of inertia $I_\kappa$ in Eq. (4) using the Inglis-Belyaev formula [42, 43]

$$I_\kappa = \sum_{i,j} \frac{(u_i v_j - v_i u_j)^2}{E_i + E_j} |\langle i| \hat{J}_\kappa |j \rangle|^2, \quad (8)$$

where $\kappa = 1, 2, 3$ denotes the axis of rotation, and the summation of $i, j$ runs over the proton and neutron quasiparticle states.

The mass parameters $B_{\mu\nu}(\beta, \gamma)$ are also calculated in the cranking approximation [44]

$$B_{\mu\nu}(\beta, \gamma) = \frac{\hbar^2}{2} \left[ M_{(1)}^{-1} M_{(3)} M_{(1)}^{-1} \right]_{\mu\nu}, \quad (9)$$

with

$$M_{(n),\mu\nu}(\beta, \gamma) = \sum_{i,j} \frac{|i \langle \hat{Q}_{2\mu} | j \rangle |^2}{(E_i + E_j)^n} (u_i v_j + v_i u_j)^2. \quad (10)$$

The mass parameters $B_{\mu\nu}$ in Eq.(9) can be converted into the forms of $B_{\beta\beta}, B_{\beta\gamma}, B_{\gamma\gamma}$ with the following relations [45],

$$B_{\beta\beta} = B_{00} a_{00} \cos^2 \gamma + 2 B_{02} a_{02} \cos \gamma \sin \gamma + B_{22} a_{22} \sin^2 \gamma, \quad (11a)$$

$$B_{\beta\gamma} = (B_{22} a_{22} - B_{00} a_{00}) \cos \gamma \sin \gamma + B_{02} a_{02} (\cos^2 \gamma - \sin^2 \gamma), \quad (11b)$$

$$B_{\gamma\gamma} = B_{22} a_{22} \cos^2 \gamma - 2 B_{02} a_{02} \cos \gamma \sin \gamma + B_{00} a_{00} \sin^2 \gamma, \quad (11c)$$

where the coefficients $a_{00}, a_{02}, a_{22}$ are as follows,

$$a_{00} = \frac{9 r_0^4 A^{10/3}}{16 \pi^2}, \quad a_{02} = a_{00}/\sqrt{2}, \quad a_{22} = a_{00}/2, \quad (12)$$
FIG. 1: The collective potential $V_{\text{coll}}$, the rms radius of protons, the mass parameters $B_{\beta\beta}$, $B_{\gamma\gamma}$, and the moment of inertia along the 1-axis $I_1$ as functions of quadrupole deformation $\beta$ for $^{24}\text{Mg}$ and the nuclear core of $^{25}\Lambda\text{Mg}$ from the Skyrme-Hartree-Fock$+$BCS calculations using the SGII force \cite{46}. The $\Lambda N$ interaction energy $E_{\Lambda N}$ has been included ($w$) or excluded ($w/o$) in the collective potential $V_{\text{coll}}$ for the nuclear core of $^{25}\Lambda\text{Mg}$.

with $r_0 = 1.2$.

The collective potential $V_{\text{coll}}$ in the collective Hamiltonian is obtained by subtracting the zero-point-energy (ZPE) from the total mean-field energy \cite{35},

$$V_{\text{coll}}(\beta, \gamma) = E_{\text{tot}}(\beta, \gamma) - \Delta V_{\text{vib}}(\beta, \gamma) - \Delta V_{\text{rot}}(\beta, \gamma),$$

(13)

where $E_{\text{tot}}$ is the total energy for the nuclear core in $\Lambda$ hypernucleus. We will investigate
two options, that is, those with (w) or without (w/o) the interaction part of energy $E_{NA}$ between the $\Lambda$ and nucleons,

$$E_{tot} = \begin{cases} 
\int d^3r E_N(r), & w/o \\
\int d^3r [E_N(r) + E_{NA}(r)], & w 
\end{cases} \quad (14)$$

In the collective potential $V_{coll}$ of Eq.(13), the vibrational ZPE, $\Delta V_{vib}$ is given by,

$$\Delta V_{vib}(\beta, \gamma) = \frac{1}{4} \text{Tr}[\mathcal{M}^{-1}(3)\mathcal{M}(2)], \quad (15)$$

where $\mathcal{M}_{(n),\mu\nu}(\beta, \gamma)$ is determined by Eq.(10) with the mass quadrupole operators $(\mu, \nu = 0, 2)$ defined as,

$$\hat{Q}_{20} = 2z^2 - x^2 - y^2 \quad \text{and} \quad \hat{Q}_{22} = x^2 - y^2. \quad (16)$$

The rotational part of ZPE is a summation of three terms,

$$\Delta V_{rot}(\beta, \gamma) = \sum_{\mu=-2,-1,1} \Delta V_{\mu\mu}(\beta, \gamma), \quad (17)$$

with

$$\Delta V_{\mu\nu}(\beta, \gamma) = \frac{1}{4} \frac{\mathcal{M}_{(2),\mu\nu}(\beta, \gamma)}{\mathcal{M}_{(3),\mu\nu}(\beta, \gamma)}. \quad (18)$$

where $\mathcal{M}_{(n),\mu\nu}(\beta, \gamma)$ is determined by Eq.(10) with the intrinsic components of quadrupole operator defined as,

$$\hat{Q}_{2\mu} = \begin{cases} 
-2iyz, & \mu = 1 \\
-2xz, & \mu = -1 \\
2ixy, & \mu = -2 
\end{cases} \quad (19)$$

III. RESULTS AND DISCUSSION

Following Ref. [27], in the $ph$-channel, we adopt the SGII parameterized Skyrme force [46] for the NN interaction, and the No.1 set in Ref. [47] for the $\Lambda N$ interaction. In the $pp$-channel for nucleons, we follow Ref. [48] to use $g = 1000$ MeV fm$^3$ for both protons and neutrons. A smooth pairing energy cutoff of 5 MeV around the Fermi level is used. In the mean-field calculations, the mass quadrupole moments are constrained to the mesh-points in $\beta$-$\gamma$ plane with $\beta = 0, 0.05, 0.10, \ldots, 1.20$ and $\gamma = 0^\circ, 6^\circ, 12^\circ, \ldots, 60^\circ$. The $\Lambda$ particle occupies the lowest single-particle state throughout the constraint calculations. In the following,
we take $^{24}\text{Mg}$ as an example, and study the impurity effect of $\Lambda$ hyperon by examining the changes of collective parameters and the resultant collective excitation spectrum and related observables.

Figure 1 displays the collective potential $V_{\text{coll}}$, the rms radius of protons, the mass parameters $B_{\beta\beta}$, $B_{\gamma\gamma}$, and the moment of inertia along the 1-axis $I_1$ as functions of quadrupole deformation $\beta$ for $^{24}\text{Mg}$ and the nuclear core of $^{25}_{\Lambda}\text{Mg}$. The $\Lambda N$ interaction energy $E_{\text{NA}}$ in Eq.(5) has been included (w) or excluded (w/o) in the collective potential $V_{\text{coll}}$ for the nuclear core of $^{25}_{\Lambda}\text{Mg}$ [cf. Eq.(13)]. It is found that the $\Lambda$ hyperon has negligible influences on the moments of inertia and mass parameters of the nuclear core. However, it can lower down the barrier in the neighborhood of spherical shape and make the energy curve stiffer at large deformed region. In other words, the $\Lambda$ will reduce the collectivity of $^{24}\text{Mg}$, where the $\Lambda N$ interaction energy plays a major role.

In Fig. 2, we plot the probability distribution $\rho_{I\alpha}(\beta, \gamma)$ in $\beta$-$\gamma$ plane for the $0^+_1$ state in $^{24}\text{Mg}$ and the nuclear core of $^{25}_{\Lambda}\text{Mg}$, where the $\rho_{I\alpha}$ is defined as [49],

$$\rho_{I\alpha}(\beta, \gamma) = \sum_K |\Psi^{I}_{\alpha,K}(\beta, \gamma)|^2 \beta^3 |\sin 3\gamma|,$$

which follows the normalization condition,

$$\int_0^\infty \beta d\beta \int_0^{2\pi} d\gamma \rho_{I\alpha}(\beta, \gamma) = 1.$$
FIG. 3: The difference in the probability distribution $\rho_{I \alpha}(\beta, \gamma)$ of $0^+_1$ states between $^{25}_\Lambda$Mg (core, w/o) and $^{24}$Mg (left panel) as well as between $^{25}_\Lambda$Mg (core, w) and $^{24}$Mg (right panel).

FIG. 4: The rms proton radius for $^{24}$Mg and the nuclear core of $^{25}_\Lambda$Mg.

Here, $\Psi^{I}_{\alpha,K}(\beta, \gamma)$ is the collective wave function that corresponds to the solution of 5DCH in Eq.(1) and $\alpha = 1, 2, \cdots$, labels collective eigenstates for a given angular momentum $I$. It is shown in Fig. 2 that the $\Lambda$ shifts slightly the probability distribution of the $0^+_1$ state to the smaller deformation region. This effect can be seen more clearly from the changes in $\rho_{I \alpha}(\beta, \gamma)$ for the $0^+_1$ state after the introducing of $\Lambda$ hyperon, as shown in Fig. 3, where the differences in the probability distribution $\rho_{0,1}(\beta, \gamma)$ for the nuclear core of $^{25}_\Lambda$Mg and $^{24}$Mg are plotted. Quantitatively, the average values of $\beta(\gamma)$ are $0.54(20.0^\circ)$ for $^{24}$Mg and these values become $0.53(20.7^\circ)$ for $^{25}$Mg (core, w/o), and $0.52(20.8^\circ)$ for $^{25}$Mg (core, w).

Moreover, it is also shown in Fig. 1 that the rms radius of protons is reduced by the $\Lambda$, in particular in the neighborhood of spherical shape. However, this shrinkage effect on the proton radius of $^{24}$Mg is only $\sim 0.5\%$, as illustrated in Fig. 4, where the rms proton radius in $\beta$-$\gamma$ plane for both the $^{24}$Mg and the nuclear core of $^{25}_\Lambda$Mg from the 5DCH calculations.
FIG. 5: The low-spin spectra of the ground state band for the $^{24}$Mg (b) and the nuclear core of $^{25}$Mg (c, d) obtained by the five-dimensional collective Hamiltonian (5DCH) with the parameters determined by the Skyrme-Hartree-Fock+BCS calculations using the SGII force [46]. The $B(E2)$ values are in units of $e^2$ fm$^4$. The spectrum of $^{24}$Mg is compared with the corresponding experimental data (a), taken from Ref. [50].

are plotted.

Figure 5 displays the low-spin spectra of ground state band for the $^{24}$Mg and the nuclear core of $^{25}$Mg. It is noted that the Λ stretches the spectra of ground state band. Comparing with columns (b) and (d), one finds that the Λ increases the excitation energy of $2^+_1$ state by $\sim 7\%$. Moreover, it reduces the E2 transition strength $B(E2 : 2^+_1 \rightarrow 0^+_1)$ by $\sim 9\%$, which is a little smaller than the values, 19(4)% or 16(6)% in $^6$Li [6].

IV. SUMMARY AND OUTLOOK

The impurity effect of Λ hyperon in $^{24}$Mg has been quantitatively studied in the framework of non-relativistic energy density functional theory that has been extended to include correlations related to the restoration of rotational symmetries and fluctuations of collective variables by solving the eigenvalue problem of a 5DCH for quadrupole vibrational and rotational degrees of freedom, with parameters determined by constrained self-consistent
nonrelativistic mean-field calculations for triaxial shapes using the SGII Skyrme force. The low-spin spectra for $^{24}\text{Mg}$ in both free space and with the additional $\Lambda$ have been calculated. It has been found that the $\Lambda$ hyperon shifts the collective wave function of ground state to a smaller deformation region by softening the nuclear collective potential surface in the neighborhood of spherical shape. As the consequence of this effect, the spectra of ground state band becomes stretched and the excitation energy of $2^+_1$ state is increased by $\sim 7\%$. Moreover, the $B(E2 : 2^+_1 \rightarrow 0^+_1)$ value is reduced by $\sim 9\%$. However, the shrinkage effect on the average proton radius is found to be only $\sim 0.5\%$.

As pointed out in Refs. [26–28], the influence of the addition of $\Lambda$ particle might be stronger in the relativistic mean-field approach. Therefore, it would be very interesting to extend this work to the relativistic case. In addition, to calculate directly the $\gamma$-spectra of single-$\Lambda$ hypernucleus, one has to extend the current EDF based 3DAMP+GCM or 5DCH models for the odd-mass or odd-odd nucleus. Working along this direction is in progress.

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Appendix A: Calculations of moments of inertia with the EV8 code

In the ev8 code [38], the single-particle (s.p.) wave-function of $k$-state $\Phi_k(\mathbf{r})$, discretized on a three-dimensional Cartesian mesh, is written in the 4-component form,

$$
\Phi_k = \begin{pmatrix}
\psi_{k}^{(1)} + i\psi_{k}^{(2)} \\
\psi_{k}^{(3)} + i\psi_{k}^{(4)}
\end{pmatrix}
$$

(A1)
TABLE I: Parities of four components $\Psi_k^{(\alpha)} (\alpha = 1, 2, 3, 4)$ in single-particle wave function $\Phi_k$ of $k$-state with respect to the planes $x = 0, y = 0, z = 0$. The parity of $k$-state is denoted as $p_k$.

| $\Psi_k^{(1)}$ | $x$ | $y$ | $z$ | $p_k$ |
|---------------|-----|-----|-----|------|
| $\Psi_k^{(2)}$ | $-$ | $-$ |      | $p_k$ |
| $\Psi_k^{(3)}$ | $-$ | $+$ |      | $-p_k$ |
| $\Psi_k^{(4)}$ | $+$ | $-$ |      | $-p_k$ |

where $\Psi_k^{(\alpha)} (\alpha = 1, 2, 3, 4)$ are real functions corresponding to the real and imaginary, spin-up and spin-down parts of $\Phi_k$. The time-reversed state of $\Phi_k$ are determined by

$$\Phi_k \equiv \hat{T}\Phi_k = -\begin{pmatrix} \Psi_k^{(3)} - i\Psi_k^{(4)} \\ -\Psi_k^{(1)} + i\Psi_k^{(2)} \end{pmatrix}. \quad (A2)$$

Therefore, the components in $\Phi_k$ are connected with the components in $\Phi_k$ by the following relations,

$$\Psi_k^{(1)} = -\Psi_k^{(3)}, \quad \Psi_k^{(2)} = \Psi_k^{(4)}, \quad \Psi_k^{(3)} = \Psi_k^{(1)}, \quad \Psi_k^{(4)} = -\Psi_k^{(2)}. \quad (A3)$$

Together with the parities of the four components $\Psi_k^{(\alpha)}$ in single-particle wave function $\Phi_k$ of $k$-state with respect to the planes $x = 0, y = 0, z = 0$, as shown in Table I, the moments of inertia $I_{1,2,3}$ in Eq.(8) can be simplified as

$$I_{1,2} = 2 \sum_{i,j>0} \frac{(u_i v_j - v_i u_j)^2}{E_i + E_j} \left| \langle \bar{i} | \hat{J}_{1,2} | j \rangle \right|^2, \quad (A4a)$$
$$I_3 = 2 \sum_{i,j>0} \frac{(u_i v_j - v_i u_j)^2}{E_i + E_j} \left| \langle i | \hat{J}_3 | j \rangle \right|^2, \quad (A4b)$$

where the s.p. states $i, j$ have the same parities ($p_i = p_j$) and the non-zero matrix elements
are determined by,

\[ \langle \bar{i}|\hat{J}_1|j \rangle = \int \int \int_{-\infty}^{+\infty} dx dy dz \left[ -\Psi_i^{(3)} \left( \frac{1}{2} \Psi_j^{(3)} + i\hat{L}_x \Psi_j^{(2)} \right) + \Psi_i^{(4)} \left( \frac{1}{2} \Psi_j^{(4)} - i\hat{L}_x \Psi_j^{(1)} \right) 
+ \Psi_i^{(1)} \left( \frac{1}{2} \Psi_j^{(1)} + i\hat{L}_x \Psi_j^{(4)} \right) - \Psi_i^{(2)} \left( \frac{1}{2} \Psi_j^{(2)} - i\hat{L}_x \Psi_j^{(3)} \right) \right], \quad (A5a) \]

\[ \langle \bar{i}|\hat{J}_2|j \rangle = \int \int \int_{-\infty}^{+\infty} dx dy dz \left[ -\Psi_i^{(3)} \left( -\frac{1}{2} \Psi_j^{(3)} - i\hat{L}_y \Psi_j^{(1)} \right) + \Psi_i^{(4)} \left( -\frac{1}{2} \Psi_j^{(4)} - i\hat{L}_y \Psi_j^{(2)} \right) 
+ \Psi_i^{(1)} \left( \frac{1}{2} \Psi_j^{(1)} - i\hat{L}_y \Psi_j^{(3)} \right) - \Psi_i^{(2)} \left( \frac{1}{2} \Psi_j^{(2)} - i\hat{L}_y \Psi_j^{(4)} \right) \right], \quad (A5b) \]

\[ \langle i|\hat{J}_3|j \rangle = \int \int \int_{-\infty}^{+\infty} dx dy dz \left[ +\Psi_i^{(1)} \left( \frac{1}{2} \Psi_j^{(1)} + i\hat{L}_z \Psi_j^{(2)} \right) + \Psi_i^{(2)} \left( \frac{1}{2} \Psi_j^{(2)} - i\hat{L}_z \Psi_j^{(1)} \right) 
+ \Psi_i^{(3)} \left( -\frac{1}{2} \Psi_j^{(3)} + i\hat{L}_z \Psi_j^{(4)} \right) + \Psi_i^{(4)} \left( -\frac{1}{2} \Psi_j^{(4)} - i\hat{L}_z \Psi_j^{(3)} \right) \right]. \quad (A5c) \]

In the above equations, \( \hat{L}_\kappa (\kappa = x, y, z) \) denote the components of orbital angular momentum operator.

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