Optimization of passive activator parameters for printing machines

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Abstract. This paper is devoted to the mathematical calculation and experimental researching of ink apparatus using in printing machines. We show the existence of structuring process in ink boxes and the formation of so-called quasi-solid in the printing ink. Our geometric model is based on the mathematical problem of covering some planar domains by the set of circles. All circles belong to one-parameter family and it is a special feature of the problem. Using this model we get the method of calculation some optimal dimensions of passive activators using for destruction the quasi-solid. Calculations are based on experimental data and it is achieved by variation of several parameters. In this paper we describe the system with only one passive activator but there exist the opportunity to optimize the system with several ones. Software realization is not given in the paper but it is based on the algorithms that are described. Experimental data were received with using real equipments and materials of printing industry. Specifically, we investigated the generating of quasi-solids in the ink boxes and we got some results for using the passive activators.

1. Introduction
Printer’s ink is a high-concentrated suspension. One of its properties is to form an inner structure having a certain mechanical stability. In order to destroy the structure it is necessary to make a certain effort which is called a minimal shift load. Both stable and periodic rotation of the ink generates a circulated current in ink box of printing machine. Shift loads between neighboring ink layers are diminished from outlying layers to the central ones. Near the centre of rotation the values of shift loads are less than the value of minimal shift load and when the rotary motion of the ink takes place this property generates the inner structure. In this case a certain part of the ink rotates without intermixing, i.e. it rotates as a solid. This part of suspension is called a quasi-solid.

Based on some results of our researches we have concluded that cubic content of the quasi-solid is about 20 % from cubic content of the ink which is in the ink box [1]. Such a property exercises a negative influence on ink-supply into the ink apparatus of printing machine and it leads to defect printed matter. In some cases the quasi-solid increases its cubic content to such an extent that an air clearance comes into being between ductor cylinder and the ink. Then the ink-supply into the ink apparatus is stopped.

Fig. 1 shows the scheme of ink rotation in the ink box. The ductor cylinder is denoted by 1, the metal plate is denoted by 2, the ink is marked by figure 3 and the quasi-solid is marked by figure 4.

In order to eliminate structuring of the ink we have proposed to use an activator. The activator is a cylinder which is put in ink parallel to the axis of the ductor cylinder. The length of the activator is a little smaller than the length of the ink box. It is necessary for free rotation of the activator. Activator’s rotation in itself is due to a certain capacity of friction, i.e. it takes place without external energy. Such an activator is called an activator of passive type (APT).

Printing machine should be provided with a set of APT. Only one activator from the set may be used while printing. Selection out of the ATP set is dependent on cubic content of the ink in the ink box. We have determined only minimal and maximal diameters of APT. Fig. 2 shows their positions in the ink box. The minimal activator is denoted by 3 and the maximal one is denoted by 4. Our aim is to compute the number of APT in the set and to find their diameters.
2. General geometric model

To solve the APT problem we used a geometric model. Our model is based on mathematical problem of covering some set in a plane by circles. There are many variants of covering some set; e.g. the variant of covering by minimal number of circles with the circle diameters being given, the variant of covering a set by a set of given number of equal circles and so on. Some recent contributions to the problem of covering are in [2 – 5].

The scheme of our geometric model is shown in Fig. 3.

Let us assume that an arbitrary planar non-convex quadrangular closed domain $S$ is given. We want to cover the domain $S$ by a set of circles keeping the following conditions.

1. All circles of the set belong to one-parameter family and they may have different diameters.
2. All borders of $S$ belong to parabolic type of plane curves. We denote these borders by $F_1$, $F_2$, $F_3$, $F_4$.
3. Two opposite curved borders, say $F_1$ and $F_2$, of the domain $S$ are the envelopes of the family.
4. All circles of the family may cover each other partly but not fully.
5. The number of circles must be a finite number. It must be a minimal number but it must be sufficient to cover the domain $S$ almost fully.
6. The first and the last circles of the family may come out from those borders which are not the envelopes of the family, i.e. the borders $F_3$ and $F_4$.

7. A sub-domain of $S$ which is not covered by the family must have a minimal area. We denote this sub-domain by $S_2$.

![Diagram](image)

**Figure 3.** The scheme of general model for the problem of covering the planar closed domain

Special cases of the problem may be solved by precise analytical methods, e.g. if $S$ is a rectangle or trapezium. In general case and because of our problem is a multi-parameter one we prefer to use an approximation, numerical methods and algorithms based on some iterative processes.

The iterative process for our case may be realized in the following steps.

1. **Step 1:** Points $A = F_1 \cap F_3$, $B = F_2 \cap F_3$, $C = F_2 \cap F_4$, $D = F_1 \cap F_4$ are determined (see Fig. 3).

2. **Step 2:** The domain $S$ is inscribed into rectangle $S_1$ which is called a rectangle of coordinate closure.

3. **Step 3:** A regular set $P$ of points $(x, y)_{i, j}$ having a sufficiently small intervals is generated within the rectangular domain $S_1$. Here $x_A \leq x \leq x_D$, $y_B \leq y \leq y_D$.

4. **Step 4:** Let $M$ denote the matrix which has a number of columns and a number of rows equal to the numbers of points $(x, 0)_{i, 0}$ and $(0, y)_{0, j}$ correspondently. Thus, $M$ is an analytical image of the set $P$.

5. **Step 5:** The family of the circles (i.e. their centers and diameters) is determined by one-parameter parabolic functions.

6. **Step 6:** A minimal number of the circles which are sufficient to cover the domain $S$ is computed. We denote this number by $k$.

7. **Step 7:** A primary parameterization of $k$ circles and a step of iteration process are given.

8. **Step 8:** Let $S_2$ denote the domain which is covered by $k$ circles and which is located inside the domain $S$. We examine every point $(x, y)_{i, j}$ of the set $P$ and if it is inside the domain $S_2$ we assign the value 1 to corresponding element of matrix $M$. If it is not inside the domain $S_2$ we assign the value 0 to the element of $M$.

9. **Step 9:** A sum of all nonzero element of $M$ is computed.

10. **Step 10:** We vary all $k$ parameters step by step and every time we return to Step 7. We stop the algorithm when any variation of parameters does not increase the number of nonzero elements in the matrix $M$. Final values of parameters give us a solution of the APT problem. This is a simplest algorithm for APT problem.

### 3. Special geometric model
The special geometric model adapted to the solution of the APT problem is shown in Fig. 4. Our goal is to find the parameters of only one covering circle, i.e. \( k = 1 \). The center and radius of this circle are denoted by \( O_k \) and \( R_k \) correspondently.

![Figure 4. The scheme for calculation the parameters of the covering circle](image)

We choose the coordinate system so that \( O \) is at the center of the ductor cylinder and the axis denoted by \( x \) is parallel to the line of metal plate which is denoted by \( L \). Let \( x_{\text{min}}, y_{\text{min}}, R_{\text{min}} \) are the parameters of the minimal circle and let \( x_{\text{max}}, y_{\text{max}}, R_{\text{max}} \) are the parameters of the maximal circle of the family. All these parameters are given.

The domain which is covered by the circle \((O_k, R_k)\) is located between the circles having \( R_{\text{min}} \) and \( R_{\text{max}} \) and between the circle \((O, R)\) and the line \( L \).

Then the circle of ductor cylinder is expressed by the equation

\[
x^2 + y^2 - R^2 = 0.
\]

Line \( L \) has the equation

\[
y = -a.
\]

The equation of parabolic curve as a locus of all centers of all APT is

\[
y = \frac{x^2 - (a + R)^2}{2(a + R)}.
\]

The set of APT circles is expressed by

\[
R_k = a + y.
\]

Here, \( x \) is the parameter of the family and \( x \geq (R^2 - a^2)^{1/2} \).

Hence, we have

\[
\begin{align*}
x_A &= R \cdot x_{\text{min}} / (x_{\text{min}}^2 + y_{\text{min}}^2)^{1/2}, \quad y_A = x_A \cdot y_{\text{min}} / x_{\text{min}} \\
x_B &= x_{\text{min}}, \quad y_B = -a.
\end{align*}
\]
\[ x_C = R \cdot x_{\text{max}} / (x_{\text{max}}^2 + y_{\text{max}}^2)^{1/2}, \quad y_C = x_C \cdot y_{\text{max}} / x_{\text{max}}, \]
\[ x_D = x_{\text{max}}, \quad y_D = -a. \]

The parameters \( O_k \) and \( R_k \) are computed by numerical method. Computational algorithm may be as follows.

Step 1: The matrix \( M = (M_{i,j}) \), \( M_{i,j} = 0 \), \( 1 \leq i, j \leq 100 \), is generated.

Step 2: The rectangular point set \( P = \{(x, y)_{i,j}\} \), \( x_A \leq x \leq x_D \), \( y_B \leq y \leq y_C \), with the steps
\[ t_x = (x_D - x_A) / p, \quad t_y = (y_C - y_B) / p, \quad p = 50; 100; 1,000; \ldots \]

is generated. Then
\[ x_i = x_A + t_x \cdot i, \quad y_j = y_B + t_y \cdot j. \]

Step 3: Let a general parameter for calculation is denoted by \( t \). The parameter \( t \) is varied, \( 0 \leq t \leq p \), and the parameters of covering circle is computed by the equations
\[ x_k = x_B + t \cdot (x_D - x_A) / p, \quad y_k = [x_k^2 - (a + R)^2] / 2(a + R), \]
\[ R_k = y_k + a. \]

Step 4: Each point \( (x, y)_{i,j} \) of the set \( P \) is analyzed by the following system of inequalities
\[ x_i^2 + y_j^2 - R^2 \geq 0, \]
\[ (x_i - x_{\text{min}})^2 + (y_j - y_{\text{min}})^2 - R_{\text{min}}^2 \geq 0, \]
\[ (x_i - x_{\text{max}})^2 + (y_j - y_{\text{max}})^2 - R_{\text{max}}^2 \geq 0, \]
\[ (x_i - x_k)^2 + (y_j - y_k)^2 - R_k^2 \leq 0. \]

Step 5: If coordinates of the point \( (x, y)_{i,j} \) are the solution of the system we assign \( M_{i,j} = 1 \). Otherwise we have \( M_{i,j} = 0 \).

Step 6: Let the number of nonzero elements of the matrix \( M \) is denoted by \( K_t \). The value of \( K_t \) is calculated.

Step 7: We take \( t = t + 1 \) and return to the step 3. And so on.

Step 8: We stop the algorithm when \( K_{t+1} < K_t \). The value of parameter \( t \) determines the parameters of the circle \( (O_k, R_k) \).

4. Experiments and results

In printing machines all ink boxes are filled with the ink proceeding from the number of products which is necessary to manufactured. It means that ink box may be filled from five up to one hundred per cent. We took this circumstance into consideration. Also we accepted the fact that in printing machines the metal plank of the ink box is located as a tangent to the ductor cylinder, i.e. \( a = R \) in the aforesaid equations (see Fig. 1 and Fig. 2).

4.1. Materials and apparatus

Extreme diameters of APT were determined by means of experiments with using the following materials and equipments.

The polypropylene tubes having their diameters equal to 13, 15, 20, 25, 30, 40 mm were applied as passive activators. Also we applied plastic spindles having their diameters from 1 up to 30 mm.

We used the following inks:
- Sicolor orange ink by “Druckfarben”, Germany;
- Quickson blue ink by VAN SON, Holland;
- PANTONE Red 032 ink by “Sun Chemical”, USA.

All experiments were fulfilled with the following machines:
- Adast Gratopress GPE (Czechia) with the diameter of the ductor cylinder is equal to 70 mm;
- Heidelberg GTO 52 (Germany) with the diameter of the ductor cylinder is equal to 60 mm;
- O. M. S. O. DM 151/6 (Holland) with the diameter of the ductor cylinder is equal to 52 mm;
- Gronhi YK 1800 (China) with the diameter of the ductor cylinder is equal to 42 mm.

4.2. Procedure
In order to determine minimal diameter of APT we analyzed all forces having an effect on activator and we ruled out the possibility of its self-braking. We varied the diameter of activators from 1 up to 40 mm. The diameter of ductor cylinders was varied from 30 up to 70 mm. Maximal diameter of activators was determined by force calculation. We took into account the forces having an effect on ductor cylinder too. Rotary moment having an effect on the activator is depended on viscosity of the ink. All measurements were carried out with settled conditions of temperature and rotation.

4.3. Results
We got the following results.
1. The rotary moment is increased with increasing of the diameter of activator and also it is increased with increasing of ink viscosity. These connections are directly proportional ones.
2. The activator having the diameter more then 40 mm stops its rotation.
3. The activator having the diameter more then 30 mm rotates with interruptions and sliding.
4. The ratio of minimal diameter of activator to the diameter of ductor cylinder is as 1 to 5 for all printing machines.
5. The ratio of maximal diameter of activator to the diameter of ductor cylinder is as 1 to 2 for all printing machines.
6. The ratio of average diameter of activator to the diameter of ductor cylinder is as 1 to 2.5 for all printing machines.

5. Conclusions
Geometric model which is based on the covering problem the planar domain by the set of circles is described. The model may be applied for calculation some optimal parameters of ink apparatus for printing machines.
The model is a set of one-dimensional family of circles having two envelopes as a border of planar domain. Two other borders of the family are parabolic curves. Calculation is based on iterative algorithm. Effectiveness of the model is confirmed by applications to real printing processes is printing industry.
The basic idea of this paper can also be applied to other types of printing technologies. So the method and algorithm which we have worked out in detail is not at all confined to ink apparatus. We do hope that our considerations will open a wide range of other applications.

6. References
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