Aperiodic arrays of quantum dots: influence of external magnetic and electric fields

N E Kaputkina¹, Yu E Lozovik², R F Muntyanu¹ and Yu Kh Vekilov¹
¹State Technological University "Moscow Institute of Steel and Alloys", Moscow, Russia
²Institute of Spectroscopy, Academy of Sciences of Russia, Troitsk, Russia

E-mail: kaputkina@mail.ru

Abstract. Electronic and excitonic excitations in aperiodic sequences of quantum dots (Thue-Morse, Cantor, Fibonacci, Double-period) were studied in external electrical and magnetic field. Single-particle and two-particle tunneling probability was taken into consideration. Transmission coefficient was determined using quasi-classical approximation and range of values of confining potential and interdot distances when tunneling is essential was estimated. An external electrical and magnetic field effect on electron localization was taken into consideration, an effective steepness of confining potential in magnetic field was appeared as control parameter of the problem. Energy spectrum of aperiodic quantum dot sequences in external magnetic field was obtained. Possibility to tune the state of the system by magnetic field was studied. The increase of the external electrical field shifts the energy states of the particle in a quantum dot and contributes to particle localization. The localization of the excitations is possible at the finite values of the perturbation in the case of aperiodic sequences of quantum dots (contrary to the case of periodical sequences).

1. Introduction
We investigate the energy spectra and tunneling of electronic and excitonic excitations in aperiodic sequences of quantum dots (QDs). These objects are perspective for nanoelectronics and optoelectronics [1]. The most investigated heterostructures are based on 2 types of semiconductors: GaAs and its solid solutions \( \text{Al}_x\text{Ga}_{1-x}\text{As} \) or \( \text{In}_x\text{Ga}_{1-x}\text{As} \) with different composition \( x = 0.1 – 1 \). These materials have direct gap structure with energy gap width \( E_g \sim 1 \text{eV} \). \( m_e = 0.07m_0 \) for GaAs effective electron mass in the conduction band (\( m_0 \) is vacuum electron mass).

One-dimensional periodic structures are of interest as possible photonic crystals [see [2] and Refs therein], waveguides [3-6]. A lot of interesting works were done for quasi-periodic chains of atoms (see [7] and Refs therein). The energy spectrum of horizontal and vertical molecules of QDs were studied in detail [8-11]. Aperiodic chains of QDs have a lot of advantages. With the help of the external parameters it is possible to control such characteristics as the particle localization, tunneling probability, energy spectrum of a quasi-crystalline sequences of QDs. For example, tuning the magnetic field and the value of bias voltage across the double dot it is possible to control the effect of resonance tunneling. This process can occur in the case when the corresponding levels in neighboring QDs align and are situated within...
the bias window. The transport process is registered by sharp resonance peaks in I-V curves [12]. In the case of slow relaxation the particle is “trapped” within the single QD. We show how such parameters as the steepness of confining parabolic potential, the distance between neighboring QDs, the external transverse magnetic field influence on the localization of excitations and the resonance tunneling. The smoothness of the energy spectra of aperiodic sequences of QDs was studied using the method of level statistics. The energy spectra of Fibonacci, Thue-Morse, Cantor, Double-period sequences exhibit singular behavior. The detailed analysis of energy and wave-functions spectra will be published elsewhere.

One can tune the system state by changing controlling parameters, the steepness of confining parabolic potential, the distance between neighboring QDs, external magnetic and electrical fields. The influence of magnetic and electrical field on the localization of excitations, the energy states positions, and the resonance tunneling is analyzed in detail below.

2. Eigenstates and eigenfunctions of excitations in a single QD and the influence of magnetic field.

The particles in QDs are confined by a harmonic potential of steepness $\alpha$ . The Hamiltonian for a single dot is

$$H = -\frac{\hbar^2}{2\mu} \frac{d^2}{dx^2} + \alpha x^2$$  \hspace{1cm} (1)

where $\mu$ is the effective mass of the particle. The energy spectrum and normalized eigenfunctions of a particle in single QD in one-dimensional case are

$$E_n = \frac{n}{2\sqrt{\alpha \beta}}$$  \hspace{1cm} (2)

$$\psi_n(x) = \left( \frac{\beta}{\pi} \right)^{1/4} \frac{1}{\sqrt{2^n n!}} \exp \left\{ \beta x^2 / 2 \right\} H_n \left( \sqrt{\beta} x \right)$$  \hspace{1cm} (3)

Where $\alpha$ is the steepness of confining potential, $x$ is the distance between the center of the QD and the particle; $H_n \left( \sqrt{\beta} x \right)$ are Hermite polynomials, and $\beta = \sqrt{2\mu \alpha / \hbar}$.

It is possible to show that when the external transverse magnetic field is applied the eigenenergies and normalized eigenfunctions have the similar form as in the previous case but $\alpha$ is replaced on the effective steepness of the confinement potential $\lambda = (\mu/2)B/\mu^2 + \alpha$, where $B$ is the external magnetic field.

The wave functions of the Fibonacci generations were obtained by the method of Komoto, Kadanoff, and Tang [14]. Their critical behavior was proved by the analysis of the $2p$-norm[13].

The solution of the secular equation gives the first order corrections to the energy by perturbation theory:

$$E_{1/2} = 0.5(H_{11} + H_{22}) \pm 0.5 \left\{ H_{11} - H_{22} \right\}^2 + 4H_{12}^2$$  \hspace{1cm} (5)

where the matrix elements are:
\[
H_{21} = \int \phi_n^{(0)}(x) H \phi_n^{(0)}(x_1) dx_1 dx_2, \quad H_{22} = \int \phi_n^{(0)}(x) H \phi_n^{(0)}(x_2) dx_1 dx_2,
\]
where \(\phi_n^{(0)}\) are the normalized eigenfunctions of the particle in the 1st and 2nd QD respectively.

The energy of the particle interaction \(H' = \frac{1}{\phi_0 \epsilon_0} \frac{\alpha^2}{\phi} + x_2 - x_1\) is considered as the perturbation. The calculation in the case of excitons is obtained by changing the effective mass from \(\mu = 0.07 m_0\) to \(M = 0.57 m_0\) (only heavy holes are considered, the existence of the light holes is neglected).

Now, it is possible to estimate the distance between the centers of QDs when the first order corrections to the energy of the particle become inessential. It is 180Å when \(\alpha = 10^3 J/m^3\) for the electron. Thus, we can use the pair interaction to obtain the energy spectrum of an aperiodic sequence of QDs. In contrast to aperiodic sequences of atoms, for aperiodic sequences of QDs rather small magnetic fields are essential.

3. The tunneling probability and the influence of magnetic field.

The expression for the single-particle and two-particle tunneling probability was obtained using quasi-classical approximation:

\[
\frac{1}{\hbar} \int_{a}^{b} |p(x)| dx, \quad E = \int_{a}^{b} |U(x)| dx
\]

is the particle pulse. \(E\) is the particle energy. \(U(x)\) is the potential energy. \(a, b\) are the turning points.

The energy level splitting versus \(\alpha\) and \(d\) dependence is given by formula:

\[
E_2 - E_1 = \frac{\hbar}{m} \exp \left( -\frac{1}{\hbar} \int_{a}^{b} |p(x)| dx \right), \quad p(x) = \sqrt{2m(E - U(x))}; \quad a \text{ and } b \text{ are the turning points.}
\]

![Figure 1](image)

Figure 1. The electron and exciton tunneling probability without magnetic field and in magnetic field versus the distance between the centers of the QD pair.

The external magnetic field increasing brings to particle probability peaks close in, thus particle is localized within some QD and B acts like effective steepness potential (see Figure 1).

The expression for the tunneling probability by the perturbation theory is used to show the resonance tunneling of the excitations between the pair of QDs:

\[
W_{fi} = \frac{2\pi}{\hbar} \int_{\delta \epsilon_f - E_i} ^{\delta \epsilon_f} \Phi_f \Phi_i \| d\nu_f \, ,
\]
where \( V_{ji} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \Psi_i^{(0)} | H | \Psi_j^{(0)} \rangle dx_1 dx_2 \) is the matrix element of interaction, and \( dV_j \) is the density of states.

The particle tunneling is possible only in the case when the quantum states in the neighboring QDs are aligned. The finite perturbation destroys the alignment of quantum states of QDs and localizes the excitations (see [13]).

4. The influence of the external electrical field on the energy spectrum of aperiodic sequences of QDs

Let QD is located in the GaAs wire and confined by the Al\(_x\)Ga\(_{1-x}\)As layers in two directions (the energy gap of last layer is larger and increases with \( x \)). Particles are confined by the parabolic potential in the third direction of the GaAs wire (produced, e.g., by lithography). Only single electron in each QD is considered and spin effects are not taken into account. The parabolic potential is obtained by the gate voltages.

We studied a sequence model based on the pairs of QDs and the distances between their centers as the structural elements of the sequences. The parameters of each QD were identical. Two distances \( d_A=400 \, \text{Å}, \, d_B=200 \, \text{Å} \) were used to construct such sequences as Fibonacci, Thue-Morse, Double-period, Cantor, and \( d = 300 \, \text{Å} \) was used for the periodic sequence construction. For simplicity we suppose that the electrons in each dot occupy the same level \( n=3 \). The number of words in each alphabet equals to 20.

The substitution rules for different types of sequences are following:

- Fibonacci sequence: \( \sigma_f \mid A \to AB, B \to A \);
- Thue-Morse sequence: \( \sigma_{TM} \mid A \to AB, B \to BA \);
- Period-doubling sequence: \( \sigma_{PD} \mid A \to AB, B \to AA \);
- Cantor sequence: \( \sigma_c \mid A \to ABA, B \to BBB \).

One-dimensional sequences of QDs are arranged between two leads. At the moment when an electron leaves it’s state at the last QD and tunnels into the right lead the resonant tunneling is possible under the influence of the external electrical field when the energy states in the neighboring dots are aligned. The increase of the steepness of the parabolic potential and the external magnetic field contributes to particle localization [13]. The tunneling probability decreases with growth of magnetic field. The energy spectra of the aperiodic sequences are obtained using the approximation of pair-correlation interaction. Coulomb interaction energy and the electrical field acted on charge transfer were considered as the perturbations in the case when their values are much less than the corresponding interlevel distance. The energy spectra were obtained and plotted versus the external electrical field. 

\[
E_{n,k} = \hbar \sqrt{\alpha/\mu(n+1/2)} - q^2 F^2_k /(2\alpha) \]  

is the energy of the electron at the \( n \)-th state in the QD.

\( 1 \leq n \leq N \) is a generation index; \( N \) is the number of words in the alphabet; index \( k \) denotes the number of the applied electrical field values; \( \alpha \) is the steepness of confining potential, and \( \mu \) is the electron’s effective mass in GaAs; \( F \) is the external electrical field.

It is important to estimate the values of electrical field and the distances when one can consider the external electrical field as a perturbation. The typical value of electron’s energy in the QD is about several meV ( \( \alpha \sim 10^{-8} \, J/m^2 \)), the typical value of the first-order correction to the energy \( E \) is \((10^{-4} \div 10^{-5}) \cdot E \). So the values of the electrical field must not exceed \( 10^7 \, V/m \).

According to our calculations the Coulomb energy of interaction can be considered as a perturbation at the distances of order \( \sim 550 \, \text{Å} \) for Fibonacci, Thue-Morse, Double-period and Cantor sequences.

Matrix elements required to determine the first-order corrections to the energy are:

\[
H_{i,f} = (4\pi\varepsilon_0)^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\langle \Psi_i(\rho_i) \right| \rho_i \left( d + \rho_i - \rho_f \right) \right| \Psi_f(\rho_f) \rangle d\rho_i d\rho_f \quad \text{8) }
\]
\[ V_{i,f} = \int_{-\infty}^{\infty} \Psi_i(\rho_i) F(d + \rho_i - \rho_f) \Psi_f(\rho_f) d\rho_i d\rho_f, \quad (9) \]

where \( d \) is the distance between the centers of QDs. \( \rho_i \) is the distance between the \( i \)-th QD and the position of the particle.

![Fibonacci sequence energy spectrum](image1)

![Cantor sequence energy spectrum](image2)

![Periodic sequence energy spectrum](image3)

![Double-period sequence energy spectrum](image4)

![Thue-Morse sequence energy spectrum](image5)

**Figure 2** The energy spectrum of Fibonacci, Thue-Morse, Cantor, Double-period, Periodic sequences versus the external electrical field \((0,2,4,6,8) \cdot 10^3 \text{ V/m.}\) The distances \( dA = 400 \text{ Å}, dB = 200 \text{ Å} \) were used for Fibonacci, Thue-Morse, Double-period, Cantor sequences, and \( d = 300 \text{ Å} \) was used for the periodic sequence construction. The electrons in each dot occupy the same level \( n = 3 \). The number of words in each alphabet equals to 20.

The Fibonacci Thue-Morse, Double-period and Cantor sequences are based on the two-letter alphabet \((A, B)\), so the first-order corrections to the energy are:

\[ E_{Q,n,k}^{(1)} = N_{A_k} \cdot E_{Ak}^{(1)} + N_{B_k} \cdot E_{Bk}^{(1)} + N_{AB_k} \cdot E_{ABk}^{(1)} + N_{AA_k} \cdot E_{AAk}^{(1)} + N_{BB_k} \cdot E_{BBk}^{(1)} , \quad (10) \]

where \( N_{A_k}, N_{B_k} \) are the numbers of letters \( A \) and \( B \) in the \( n \)-th word; \( N_{AB_k}, N_{AA_k}, N_{BB_k} \) are the numbers of neighboring pairs of letters, \( E_{Ak}^{(1)}, E_{Bk}^{(1)}, E_{ABk}^{(1)}, E_{AAk}^{(1)}, E_{BBk}^{(1)} \) are the first-order
corrections to the energy of the word $A$, $B$, and pairs of words $AB$, $AA$, $BB$ respectively, calculated under the influence of the $k$-th value of electrical field.

Energy spectrum of an aperiodic sequence:

$$E_{Q_{n,k}} = N_{A_{k}} \cdot E_{A_{n,k}} + N_{B_{k}} \cdot E_{B_{n,k}} + E^{(1)}_{Q_{n,k}}$$  \hspace{1cm} (11)

Energy spectrum of the periodic sequence:

$$E_{P_{n,k}} = k \cdot E_{n,k} + (k-1) \cdot E^{(1)}_{P_{k}}$$  \hspace{1cm} (12)

where $E^{(1)}_{P_{k}}$ denotes the first-order corrections to the energy of the identical pairs of QDs.

If one compares the dependence of the aperiodic sequences with the dependence of the periodic sequence (see Figure 2) then one can notice that a much less value of the electrical field for the periodic sequence is required to shift the energy of the $10^{th}$ word to the previous value for example. This supports our idea that the transport blocking occurs at the infinitively small values of perturbation in the case when periodic sequences are considered, and finite values of perturbations are needed to block the transport in the case of aperiodic sequences.

5. Conclusions

The energy spectra of aperiodic sequences Thue-Morse, Cantor, Fibonacci, Double-period were obtained in the external electrical and magnetic field. The increase of the external electrical field shifts the energy states of the particle in a QD and contributes to particle localization. The tunneling of the excitations is possible only in the case when the quantum states of neighboring QDs are aligned. The transport blockage of the excitations is possible at the finite values of the perturbation in the case of aperiodic sequences of QDs while relatively small values of perturbations (external fields etc.) are required to block the transport in the case of periodical sequences. External magnetic field influences also on the wave functions overlapping and tunneling probability. It is possible to tune the state of the system by magnetic field.

Acknowledgments This work was supported by the Russian Foundation for Basic Research (grants 08-02-01461-0, 09-02-001447-a, 08-02-00685-a) and the Russian Government Program “Development of High School Potential” (grant 2.1.1.1552).

References

[1] Kulbachinskii V A 2003 *Semiconductor physics and technics*, 2003, vol 37, issue 1
[2] Busch K, von Freymann G, Linden S, Mingaleev S F, Tkeshelashvili L, Wegener M 2007 *Physics reports* vol 444 pp. 101-202
[3] Jones A L 1965 *J. Opt. Soc. Am.* vol 55 p 261
[4] Somekh S, Garmire E, Yariv A, Garvin H L, Hunsberger R G 1973 *Appl. Phys. Lett.* vol 22 p 46
[5] Stegeman G I, Segev M 1999 *Science*, vol 286 p 1518
[6] Christodoulidis D M, Lederer F, Silverberg I 2003 *Nature* vol 424 p 817
[7] Albuquerque E L, Cottam M G 2003 *Physics reports* vol 376 pp. 225-337
[8] Kaputkina N E, Lozovik Yu E 1998 *Physics of the Solid State* vol 40 pp 1929-1934
[9] Kaputkina N E, Lozovik Yu E 1998 *Physics of the Solid State* vol 40 pp 1594-1599
[10] Lozovik Yu E, Kaputkina N E 1998 *Physica Scripta* vol 57 p 542
[11] Kaputkina N E, Lozovik Yu E 2006 *J. Phys.: Condens. Matter*, vol 18 pp S2169–S2174
[12] Van der Wiel V G, De Franceschi S, Elzerman J M, Fujisawa T, Tarucha S, Kouwenhoven L P 2003 *Reviews of modern physics* vol 75 pp 1-21
[13] Kaputkina N E, Lozovik Yu E, Muntyanu R F, Vekilov Yu Kh 2008 *Philosophical Magazine*, vol 88 issue 13 pp 2253-2259
[14] Komoto M, Kadanoff L P, and Tang C 1983 *Phys. Rev. Lett.* vol 50 p 1870