Active disturbance rejection control for fractional-order PID control system research

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Abstract. The active disturbance rejection control (ADRC) extended to the fractional-order control system, fractional-order proportional–integral (PI) and proportional–integral–derivative (PID) controllers are the most commonly used controller. This paper presents a simple fractional-order control system combined with ADRC control system. By treating the fractional-order as a common disturbance and actively rejecting it, an ADRC can achieve the desired response. External disturbance, sensor noise, and parameter disturbance is also estimated using extended state observer. Simulation on ADRC fractional order model stability analyzes. The results showed that, ADRC fractional order control system object has good control performance.

1. Introduction
Active disturbance rejection control (ADRC) presents a novel perspective [1], originating by Han and the active disturbance rejection control proportional-integral-derivative (PID) [2,3] using tracking differentiator. Fractional-order system and control have been studied by many researchers in the past decade because fractional calculus can model real-world phenomena more precisely [4,5]. ADRC technology made over the years, at home and abroad has been a large number of applications. In the United States, NASA spacecraft solar power stabilizing device, aircraft jet engine is controlled by the ADRC control technology. In Japan, disturbance rejection control technology is also used in high-precision displacement control, temperature control. At home, in the field of power systems, chemical systems, precision machining, military systems have successfully applied in ADRC technology.

For fractional-order model, fractional-order controller can be naturally considered as the best controller [6,7]. However, the active disturbance rejection controller (ADRC), proposed as an alternative paradigm for control system design, offers a novel perspective [8,9]. Stability and frequency response are also researched [10].

ADRC has been applied to DSP-based power converter [11], delay system [12], motion control [13,14], hysteretic system [15], high performance turbofan engines [16], flight control [17], interconnected power system [18], decoupling, micro-electro-mechanical systems (MEMS) gyroscopes [19], noncircular turning process [20], and coordinated robust nonlinear boiler–turbine–generator control system [21].

2. Active disturbance rejection control
ADRC is not totally dependent on the mathematical model of the system, but by the size and direction of the desired trajectory and the actual trajectory error to implement. That is based on the process error to suppress or eliminate the error of the method. ADRC has a special advantage, and has been widely used. ADRC is mainly composed of three parts: tracking differentiator (TD), extended state observer
(ESO), nonlinear state error feedback (NLSEF). TD role is setting the size of the arrangement according to the transition process, extract its differential signals. The convergence of ESO for nonlinear systems with uncertainty was offered in ESO is applied to the input and output of the controlled object to estimate the state of the system and the sum of the disturbance. NLSEF control amount calculated according to the system error. Disturbance estimation and compensation, disturbance compensation is estimated on the ESO estimated total disturbance compensation to calculate the amount of control NLSEF, obtain the actual controlled variable the controlled object.

In this paper, an example of second order ADRC. Fig. 1 shows the second-order ADRC structure.

In order to protect the superiority on the ADRC structure unchanged, While addressing the ADRC parameter tuning problems, some optimization algorithms are applied to ADRC parameter tuning, including genetic algorithm and particle swarm optimization (PSO). Some researchers have proposed PSO applied ADRC flight controller design theory, but the classical PSO easily converge to local optima. To solve this problem, Liu chaotic particle swarm algorithm is proposed to solve the problem ADRC parameter tuning.

2.1. Active disturbance rejection control algorithm
Considering a second-order control system, that the set value is \( r(t) \), the controlled variable of the system is \( u(t) \), the output of the system is \( y(t) \), and the external disturbance is \( w(t) \). A typical second-order system ADRC algorithm is as follows:

\[
\begin{align*}
\text{TD:} \\
\dot{r}_1 &= r_2 \\
\dot{r}_2 &= fst(r_1, r_2, r(t), r, h) \\
\text{ESO:} \\
\dot{e}_i &= z_i(t) - y(t) \\
\dot{z}_i &= \beta_{i0} \cdot fal(e, a_i, \delta_i) \\
\dot{z}_3 &= \beta_{10} \cdot fal(e, a_3, \delta_1) + b u(t) \\
\text{NLSEF:} \\
e_r &= r_1 - z_1, e_2 = r_2 - z_2 \\
\dot{u}_0 &= k_1 \cdot fal(e_r, a_1, \delta_2) + k_2 \cdot fal(e_2, a_2, \delta_2) \\
u &= \frac{u_0 - z_1}{b_0}
\end{align*}
\]

The \( r_0, \beta_{01}, \beta_{02}, \beta_{03}, r, a, h, b_0 \) are the parameters of the controller, and \( r_0 \) is determined on the basis of the need of the transition speed and the withstanding capability of the system; the parameters
\( \beta_{01}, \beta_{02}, \beta_{03} \) are determined by the sampling step length the system used, that no matter what kind of the object is, the same \( \beta_{01}, \beta_{02}, \beta_{03} \) can be used with the same sampling step length.

2.2. Discrete Algorithms
1. Tracking-differentiator (TD)

\[
\begin{align*}
&v_1(k+1) = v_1(k) + hv_2(k) \\
v_2(k+1) = v_2(k) + h\text{fst}(v_1(k) - v_0(k), v_2(k), r, h_0)
\end{align*}
\]

By Choosing the parameter \( r \) properly, the expectable transition process \( v_1 \) of the object can be arranged, and the differential signal \( v_2 \) can be given.

2. Extended state observer (ESO)

\[
\begin{align*}
&\varepsilon_i = z_i(k) - y(k) \\
z_i(k+1) = z_i(k) + h(z_2(k) - \beta_{01} e_i) \\
z_2(k+1) = z_2(k) + h(z_1(k) - \beta_{02} \text{fal}(e_1, a_1, \delta) + b_0 u(k)) \\
z_3(k+1) = z_3(k) - h \beta_{03} \text{fal}(e_1, a_1, \delta)
\end{align*}
\]

Where

\[
\text{fal}(e, a, \delta) = \begin{cases} 
|e|^\epsilon \sin(e), & \text{if } |e| > \delta \\
\frac{e}{e^{1-\epsilon}}, & \text{if } |e| \leq \delta 
\end{cases}, \quad \delta > 0
\]

By selecting the position \( \{a_1, a_2, \delta, \beta_{01}, \beta_{02}, \beta_{03}\} \), \( z_i, z_2 \)
apropriately, the controlled variable \( y \) and the differential of \( y \) can be estimated well, and the disturbance can be estimated using \( z_3 \).

3. Forming control amount

\[
\begin{align*}
&e_1 = v_1(k) - z_1(k) \\
&e_2 = v_2(k) - z_2(k) \\
&u_0 = \beta_1 \text{fal}(e_1, a_1, \delta_0) + \beta_2 \text{fal}(e_2, a_2, \delta_0) \\
&u(k) = u_0(k) - \frac{z_3(k)}{b_0}
\end{align*}
\]

where \( e_1, e_2 \) are the error and its differential between the arranged transition process \( v_1 \) and the estimate of the system \( z_i \), choose the appropriate \( \{\beta_1, \beta_2, a_1, a_2, \delta_0, b_0\} \) to construct the input component \( u_0 \). This is the realization of a nonlinear system the configuration, however the actual amount of control is \( u \), Where \( z_3(k) / b_0 \) is the object model and the compensation effect of the external disturbance component. The controller algorithm only need the data input and output objects \( u(k), y(k) \).

3. Fractional-order systems

Fractional calculus put study the fractional calculus operator as the basic starting point, development to the present form several different forms of definitions. The Grünwald–Letnikov’s (GL) definition is the most popular definition of fractional-order derivatives for fractional-order control and its application. Fractional PID controller made by professor I. Podlubny. The general format is abbreviated as PI\(^{\lambda}\)D\(^{\mu}\).
3.1. Fractional Calculus

\[
\frac{d^a}{dt^a} = \begin{cases} 
\frac{d^a}{dt^a}, & R(a) > 0, \\
1, & R(a) = 0, \\
\int_0^t (d \tau)^{(-a)}, & R(a) < 0,
\end{cases}
\]  

(7)

The most commonly used definition of fractional calculus is Riemann-Liouville (RL) definition and Grünwald–Letnikov (GL) definition.

\[
a D_t^a f(t) = \frac{1}{\Gamma(m-a)} \left( \frac{d}{dt} \right)^m \int_0^t \frac{f(\tau)}{(t-\tau)^{(m-a)}} d\tau
\]  

(8)

\[(m-1 < a < m), \Gamma(\cdot) \] is famous for Euler-Gamma function. GL is defined as:

\[
a D_t^a f(t) = \lim_{h \to 0} \frac{1}{\Gamma(a)} \sum_{k=0}^{\infty} \frac{(t-kh)^{(a)}}{\Gamma(k+1)} f(t-kh)
\]  

(9)

when the introduction of fractional operator \( aD_t^a \), description fractional order systems are commonly used mathematical tools Laplace transform. At time \( t = 0 \), join signal \( x(t) \) n-order Laplace transform.

\[
L\{D^a x(t)\} = s^aX(s)
\]  

(10)

For fractional calculus equations, if the time \( t =0 \) has input and output signals \( u(t) \) and \( y(t) \), its transfer function is

\[
G(s) = \frac{a_1s^{\alpha_1} + a_2s^{\alpha_2} + \cdots + a_ms^{\alpha_m}}{b_1s^{\beta_1} + b_2s^{\beta_2} + \cdots + b_ms^{\beta_m}}
\]  

(11)

where \((a_m, b_m) \in \mathbb{R}^2, (a_m, b_m) \in \mathbb{R}^2, \forall m \in N\) is among the equations.

3.2. Fractional order PI\(^\lambda D\mu\) controller

Fractional PI\(^\lambda D\mu\) controller transfer function:

\[
G_c(s) = \frac{U(s)}{E(s)} = K_P + K_I s^{-\lambda} + K_D s^\mu
\]  

(12)

Where \( \mu, \lambda \) is the order of the controller, \((\mu > 0, \lambda > 0)\), KP, KI, KD are the control parameters controller. In this integral term \( s^\lambda \), that is the logarithm of the relative frequency, and its slope is \(-20\lambda \text{dB/dec}\), rather than \(-20 \text{dB/dec}\). In the time domain, the control signal \( u(t) \) can be represented as,

\[
K_p e(t) + K_I D^{\lambda} e(t) + K_D D^\mu e(t)
\]  

(13)

Fractional PI\(^\lambda D\mu\) controller is the general form of the PID controller. The traditional integer order PID controller is just a special case of fractional order PID controller. Since \( \lambda \) and \( \mu \) can be any real number, fractional PI\(^\lambda D\mu\) controller over the integer order PID controller is more flexible, by choosing different values of \( \lambda \) and \( \mu \), so that fractional order control system has better dynamic characteristics and steady state characteristics.

The active disturbance rejection control (ADRC) combined with fractional-order PID (FOPID), design ADRC fractional order PID (ADRCFOPID) controller, structure as shown Fig.2
Utilize fractional-order PID controller robustness to improve ADRC nonlinear feedback control laws, and joined the fractional integral effect. That reduces the integer order integral closed-loop becomes dull, easy to produce oscillations, and the negative impact on the amount of saturation control windup caused. At the same time, ESO also eliminates the “sum of the interference” estimation error, making the steady-state error is zero. Although ADRC has strong robustness, but its multi-parameter tuning complex. Combined with fractional order PID controller, only need to design ESO parameters, and then through the optimization of the parameters fractional order PID controller tuning parameters to achieve the desired control effect.

Lemma 1. Let us assume that the following model is open-loop stable:

\[ y(s) = H(s)u(s) \]  

\[ (s) (s)u(s) y_H = \]  

\[ s, y, \text{and} u \text{represent the differential operator, the output, and the control input. Then, the following ADRC scheme can stabilize the closed-loop system:} \]

\[
\begin{align*}
\dot{z}_1 &= z_2 - \beta_1(z_1 - y) + b_x u \\
\dot{z}_2 &= -\beta_2(z_1 - y)u = \frac{1}{b_x}(-z_2 + p_1(v - z_1))
\end{align*}
\]

\[ (15) \]

Theorem 1. Assume that the following rational-order linear-time-invariant (LTI) model is open-loop stable:

\[ y(s) = \sum_{i=0}^{r_1} a_{i,j} s^i u(s) \]

\[ \sum_{j=0}^{r_2} g_{1,j} s^j \]

\[ 0 = a_{i,0} < a_{i,1} < \ldots < a_{i,r_i} \quad (i = 1, 2) \]

\[ a_{i,1} \geq a_{i,2} \quad \text{and the real numbers} \quad g_{1,j} \quad (j = 0, 1, \ldots, r_j) \]

\[ g_{2,j} > 0 (j = 0, 1, \ldots, r_2), \quad g_{2,0} = 1 \]. Then, Eq. (16) can be stabilized by the ADRC scheme [Eq. (15)].

Proof. As rational numbers can be expressed in fractional form, q can be set equal to the lowest common multiple of the denominators in all the powers that exist in the model transfer function. This condition means that Eq. (16) can be expressed in the form of Eq. (14). Thus, the proof is completed according to Lemma 1.

4. Simulation result

To control this system, Ref. [16,17] proposed turning methods, respectively, which led to the FOPID controllers Fig. 3 shows the step response and control signal of ADRC and FOPID. For fair comparison, external disturbance and sensor noise are not considered.
Fig. 3. Comparison of ADRC and FOPID on heating–furnace model.

| Model | ADRC parameters | FOPID |
|-------|-----------------|-------|
|       | $b_2$ | $c_{O_2}$ | $c_{O_3}$ | $C(s)$ |
| Model I | 1 | 1000 | 60 | $20.5(s^{1.2} + 1)$ |
| Model I | 2 | 500 | 30 | $1.72 + 41.524s^{-0.928} + 1.59s^{0.824}$ |
| Model I | 2.5 | 500 | 10 | $6.3092(1 + 0.9435s^{1.085})$ |

Modle I
Adopting the same $\omega_c$, all step responses are clearly and interestingly the same as the desired response. Although the measurement of output $y$ is affected by sensor noise, ESO estimates $y$ and $f$ accurately. The external disturbance $w$ is also estimated and cancelled. Based on ADRC, the proposed fractional-order dynamics rejection scheme is demonstrated to be effective in fractional-order systems. Fig. 4 shows the step response of ADRC and FOPID. For fair comparison, external disturbance and sensor noise are not considered. It is clearly that ADRC is effective in fractional-order system control as FOPID.

5. Conclusion
Generally, fractional-order system is controlled by fractional-order controller. However, in this paper, a novel approach based on ADRC has been successfully applied on fractional-order systems, where fractional-order dynamics are treated as a common disturbance and actively rejected. Meanwhile, the external disturbance, sensor noise, and parameter disturbance are estimated and rejected. The stability of ADRC has been proven of ADRC. Using simple tuning parameters, ADRC can control easily fractional-order systems. Thus, ADRC is also likely appropriate in other types of fractional-order system controls.

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