Paradox of integration—Cellular automata approach

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We discuss the self-deprecating strategy introduced by Peter Blau as one of stages of the process of social integration. Recently we have introduced a two-dimensional space of status, real and surface one \((A,B)\), and we have demonstrated that with this setup, the self-deprecating strategy efficiently prevents the rejection [Malarz and Kułakowski [1], arXiv:1903.04291]. There, the process of reducing the conflict was described by master equations, i.e. a set of differential equations describing evolution of density \(v(A,B)\) of actors of status \((A,B)\). Here we reformulate the problem in terms of probabilistic asynchronous cellular automata. The obtained results for number \(n(A,B)\) of actors of status \((A,B)\) are qualitatively the same as in the previous approach. Namely, an enhancement of the surface status compensates a deficiency of the real one. Yet, the results depend on the order of application of the automaton rules; the above results are obtained for the backward typewriter order.

Keywords: Self-deprecating strategy; Cellular automata

I. INTRODUCTION

According to Peter Blau, social integration of a group of adults is divided in two stages [2]. In first stage, actors demonstrate their strongest points, to achieve social status as high as possible. This stage can be painful for some, who have no virtues to present; they respond with fear and hostility. As Blau puts it: “The more successful A is in impressing B and earning BaÁ AZs high regard, the more displeasure he causes to C, whose relative standing in the eyes of B has suffered. All group members simultaneously play the role of A, B, and C in this schema, which greatly complicates the competitive process” [2, p. 44]. This leads to a paradoxical phenomenon: persons most skillful, intelligent and physically attractive meet with rejection and hostility. To neutralise this, in the second stage intelligent persons know and apply a clever strategy: they seemingly reduce their advantage, demonstrating their weak points in less important aspects of status. To cite Blau again: “Having first impressed us with his Harvard accent and Beacon Hill friends, he may later tell a story that reveals his immigrant background” [2, p. 48]. It is worth to mention that this strategy is not efficient if applied by an actor too weak or too strong. In first case, it is read as a fake suggestion of non-existent strong points [2, pp. 48-49], in the second—as an arrogant demonstration of lack of understanding of difficulties of the others’ life [3]. As a fairly complex social process, the social integration is a promising playground for an interdisciplinary research, between social sciences and computational modelling.

The self-deprecating strategy (SDS) depicted by Blau has been the subject of a series of recent papers [1, 4, 5]. Yet, it is only in the last [1] where the model results successfully reproduced the efficiency of SDS as a tool to reduce the fear-driven rejection. This was achieved by an introduction of two-dimensional space of status, with real and surface dimensions. Along these two axes, two processes were competing; fear-induced rejection along the real axis, and preventive praising, which drives the status of an opponent along the surface axis. The formalism applied was a set of differential equations, with both probability distribution of agents in the space of status and the related cumulative distribution involved as variables. In this sense, the description [1] was non local; the time evolution of the status of actors depend not only on their direct neighbours, but also on those fairly distant on the status plane.

The aim of this text is to check, how the results depend on this particular assumption on the locality. To achieve this purpose, the problem is reformulated in the frames of probabilistic asynchronous cellular automata, the formalism local by definition [6–8]. Here, the time evolution of the positions of actors in the status plane depends only on their direct neighbours in this plane. Besides this, we keep the time distribution of actors as the variable, as was done in [1].

We have found that the results reveal a direct dependence on the order of applying the automaton rules. As a full exploration of this dependence needs a thorough study, here we report only the results obtained for the backward typewriter order. The reason is that for this particular order, the results appear to be qualitatively the same, as those obtained with the differential equations [1]. This unexpected similarity induced us to present a separate report, which is the content of this text. Yet, the research should be taken as a pilot study.

In the next section, the automaton is described in detail. Third section is devoted to our numerical results, presented in form of computer animations. The same way of presentation was used in [1]. A short discussion is closing the text.

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FIG. 1. The spatial–temporal evolution of the density of actors status \( n(A, B) \) after \( t = 300 \) time steps and various values of \( \alpha \).

The rule that actors with different artificial status B do not interact assures, that the action to increase the status B of a neighbour neutralises his rejection and allows to preserve own status A. This rule is not possible if we have only one-dimensional status, as was assumed earlier [4, 5].

II. MODEL

Two processes are competing in the time evolution. First is the fear-driven rejection, which takes place by a lowering of status \( A \) (real status) of an actor by his direct neighbour with lower status \( A \) and the same status \( B \) (surface status). This process takes place with probability \( \alpha \). The second process is an enhancement of status \( B \) (surface status) of an actor by his direct neighbour with higher status \( A \) and the same status \( B \). This process takes place with probability \( 1 - \alpha \). Actors with different

| \( \alpha \) | URL                        |
|--------|---------------------------|
| 0.00   | www.zis.agh.edu.pl/files/asynchr000.gif |
| 0.25   | www.zis.agh.edu.pl/files/asynchr025.gif |
| 0.50   | www.zis.agh.edu.pl/files/asynchr050.gif |
| 0.75   | www.zis.agh.edu.pl/files/asynchr075.gif |
| 1.00   | www.zis.agh.edu.pl/files/asynchr100.gif |
status \(B\) do not interact.

Every site of rectangle lattice \(G = \{(A, B) : 1 \leq A \leq L_A, 1 \leq B \leq L_B\}\) represents the number \(n(A, B)\) of actors with real status \(A\) and surface status \(B\). Initially, all sites for \(B \leq 15\) are occupied by five actors. Every time step \(t\), every pair of actors at position \(\{(A, B) \cup (A+1, B)\}\) for which \(n(A, B; t)n(A+1, B; t) > 0\) apply either SDS (with probability of \(1 - \alpha\)):

\[
n(A, B + 1; t + 1) = n(A, B + 1; t) + 1, \tag{1a}
\]

\[
n(A, B; t + 1) = n(A, B; t) - 1 \tag{1b}
\]

or the fear-driven rejection process (with probability \(\alpha\)):

\[
n(A + 1, B; t + 1) = n(A + 1, B; t) - 1, \tag{2a}
\]

\[
n(A, B; t + 1) = n(A, B; t) + 1. \tag{2b}
\]

The rules (1) and (2) are applied asynchronously to all sites, i.e. in the backward typewriter order. The rules implementation in Fortran95 [9] is available in Listing 1.

III. RESULTS

In Tab. I the links to videos showing spatial-time system evolution are presented.

The final states of system evolution for various values of probabilities \(\alpha\) are presented in Figs. 1(b)–(f). In Fig. 1(a) the common initial state of the system is presented. For \(\alpha = 0\) (Fig. 1(b)) all actors apply SDS, which yields generation of plenty actors with high surface status \(B\). On contrary, assuming \(\alpha = 1\) pushes system to the final state presented in Fig. 1(f) with all actors with minimal real status \(A = 1\) and without changing initial surface status \(B\).

In Figs. 1(b)–(f) we see that the final outcome depends on the probability \(\alpha\); indeed, the asymptotic (stationary) state depends only on one parameter \(\alpha/(1 - \alpha)\). For \(\alpha\) close to one, SDS is not active, and the rejection reduces the real status \(A\) of all actors to the minimal value. In the opposite limit \(\alpha\) close to zero, a straight line appears in the plane \((A, B)\), with a slope \(\pi/4\) to both axes. This result is the same as the one obtained in Ref. [1]. As we argued there, the angle has no real mining, because it is the consequence of the assumed scales of both statuses, both arbitrary and unverifiable.

IV. DISCUSSION

The obtained results for number \(n(A, B)\) of actors of status \((A, B)\) are qualitatively the same as in the previous approach [1]. Namely, an enhancement of the surface status \(B\) compensates a deficiency of the real one \(A\).

Examples of social applications of SDS have been listed already in Ref. [1]. The main result here is just this similarity of results obtained with different formalisms. It appears that the locality is not crucial. This is an indication, that the formalism in its present form is unable to reproduce the details mentioned in the Introduction, on the inefficiency of SDS applied by an actor too weak or too strong.

On the other hand, the dependence of the results on the order of applying the rules—known as non-abelian character of an automaton—reveals a subtle character of the simulated process. Perhaps the order of actions is important also in the social reality. One could imagine, that a quick application of SDS by a smart actor blocks the fear-driven rejection, simultaneously putting the subject actor in dependence of surface praising. How and to which extent this symbiotic relation can be transformed into a parasitic one? [10, 11] This problem remains out of scope of our simple study.

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Listing 1. Fortran95 code allowing for reproduction of videos linked in Tab. I with final states presented in Fig. 1

```fortran
!! Paradox of integration — Cellular automata approach

!! module settings
!!
implicit none

integer, parameter :: Lx=40, Ly=60, T=3e+2
real*8, parameter :: alpha=0.00d0, beta=1.0d0-alpha

end module settings

!! # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # #

program Blau_CA

use settings

implicit none

integer ii, x, y

integer, dimension (Lx,Ly) :: n

n=0

print '(A3,2A10,2A10) ', '###' , 'Lx' , 'Ly' , 'alpha ' , 'beta '
print '(A3,2I10 ,2F10.6) ', '###' ,Lx,Ly, alpha , beta

do x=1,Lx
! initial state
  do y=1,Ly/4
    n(x,y)=5
  enddo
enddo

ii=0
print *, '#/uni2423 ', ii , sum(n)
do x=1,Lx
  do y=1,Ly
    print *,x,y,n(x,y)
  enddo
enddo

do ii =1,T
  if(rand() .lt. alpha) then
    do y=1,Ly-1
      do x=Lx-1,1,-1
        if(n(x,y) .gt. 0 .and. n(x+1,y) .gt. 0) then
          n(x+1,y)=n(x+1,y)-1
          n(x,y)=n(x,y)+1
        endif
      enddo
    enddo
  else
    do x=1,Lx-1
      do y=1,Ly-1
        if(n(x,y) .gt. 0 .and. n(x+1,y) .gt. 0) then
          n(x+1,y)=n(x+1,y)+1
          n(x,y)=n(x,y)-1
        endif
      enddo
    enddo
  endif
  endif =1,T
endif

if(mod(ii ,1) .eq. 0) then
  print *, '#/uni2423 ', ii , sum(n)
do x=1,Lx
  do y=1,Ly
    print *,x,y,n(x,y)
  enddo
enddo
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