Viable Palatini-f(R) cosmologies with generalized dark matter

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We study the formation of large-scale structure in universes dominated by dark matter and driven to accelerated expansion by f(R) gravity in the Palatini formalism. If the dark matter is cold, practically all of these models are ruled out because they fail to reproduce the observed matter power spectrum. We point out that if the assumption that dark matter is perfect and pressureless at all scales is relaxed, nontrivial alternatives to a cosmological constant become viable within this class of modified gravity models.

I. INTRODUCTION

The f(R) theories of gravity have been intensively explored in recent years as possible alternatives to dark energy [1]. This class of extensions of general relativity is defined via a seemingly simple generalization of the Einstein-Hilbert action by allowing nonlinear interactions of the Ricci scalar as

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R_{\mu\nu}(\Gamma)) + S_m, \]

(1)

where \( S_m \) is the matter action. When \( \Gamma \) is taken to be the metric-compatible (Christoffel) connection, the action [11] represents the so called metric f(R) models [2, 3, 4, 5, 6]. Though a realistic universe expansion history may be reconstructed from these models [7, 8], they seem to be ruled out as alternatives to dark energy because of their consequences to the Solar system physics [9, 10, 11], see however [12]. If the connection \( \Gamma \) is promoted to an independent variable, the action [11] represents the f(R) models in the so called Palatini formalism [13, 14, 15, 16, 17, 18], which instead appear to pass the Solar system tests [19]. The viability of the small-scale limit of these Palatini-f(R) models has been an issue of debate [20, 21]. A recent claim of violations of the equivalence principle [22], though partly erroneous [23], has resurfaced doubts about the physical tenability of the Palatini-f(R) gravity [22, 24]. These have been already considerably clarified by Kainulainen et al [11], and the problems which perhaps remain at tiny scales could also dissipate if one entertains these models as a macroscopic limit of a possibly more fundamental description of spacetime with the aid of truly metric-affine degrees of freedom [25].

This suggests it worthwhile to seriously reconsider the cosmology of the Palatini-f(R) models. Expansionwise, they can generate a viable sequence of radiation dominated, matter dominated and accelerating era matching with the constraints, as has been shown for various parameterizations of the function \( f(R) \), most often with some power-law forms (with one, two or three powers of \( R \)), but also with square-root, logarithmic and exponential forms for the curvature correction terms [26, 27, 28, 29, 30]. However, the inhomogeneous evolution present in any realistic universe has not been successfully reconciled with observations in these models. At the background level the additional derivative terms in the field equations due to nonlinearity in \( f(R) \) can play the role of an effective smooth dark energy. Meanwhile the spatial gradients of these extra terms cannot be neglected for cosmological perturbations [31] which then assume unusual behaviour that is at odds with the observed distribution of galaxies [32] and with the cosmic microwave background spectrum [33], even if the nonlinear part of the action is exponentially suppressed at late times [34]. In quantitative terms, parameterizing the \( f(R) \sim R^\beta \) data analysis constrains |\( \beta \)| < 10^{-5} [31, 32]. Thus, while these models are allowed by local experiments, they are extremely tightly constrained by cosmological data.

In this paper our purpose is to investigate under which conditions these cosmological constraints on the Palatini-f(R) could be loosened. Our approach is to allow generalized dark matter (GDM) with possible (isotropic or anisotropic) pressures in the matter sector. The gravity sector we assume to be given by a general nonlinear Lagrangian \( f(R) \) in the Palatini formalism. If the dark matter is cold, practically all of these models are ruled out because they fail to reproduce the observed matter power spectrum. We point out that if the assumption that dark matter is perfect and pressureless at all scales is relaxed, nontrivial alternatives to a cosmological constant become viable within this class of modified gravity models.

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the properties of dark matter, the large-scale structure in these models can be consistent with evolution equations. Appendix A contains the full linearized evolution equation.

II. COSMOLOGY WITH GENERALIZED DARK MATTER

In the ΛCDM model the CDM component consists possibly of weakly interacting massive particles or something else which should be at cosmological scales very accurately approximated as an exactly pressureless and perfect fluid. These properties are deduced from the observations of large-scale clustering properties of matter, assuming the gravitational dynamics to be governed by GR. The so called hot dark matter scenario, as an example, is excluded because of the finite pressure in a hot component inhibits matter from clustering at subhorizon scales efficiently enough to explain the observed amount of structure at those scales when the observations are interpreted within the framework of a cosmological constant model 37.

At present study we are focusing on an alternative gravity model featuring deviations from general relativity at the scales where observations require to invoke dark matter. The question we then ask is not "what gravity models are consistent with the CDM cosmology", but rather "what properties of GDM are required for cosmological viability of a given modified gravity model?".

To begin, we will derive an evolution equation for the inhomogeneities in a general fluid. To characterize small perturbations about the background we can without loss of generality adopt the longitudinal Newtonian gauge 38, 39, which is defined by including the two gravitational potentials Ψ and Φ to the Robertson-Walker line-element as

\[ ds^2 = a^2(\tau)[-d\tau^2 + (1 - 2\Phi)g^{(3)}_{ij}dx^idx^j], \]

where \( g^{(3)}_{ij} \) is the spatial 3-metric in a flat universe reduces to the Dirac delta function \( g^{(3)}_{ij} = \delta_{ij} \). The components of the energy-momentum tensor for a general fluid including scalar perturbations can be written as

\[ T^0_0 = -\bar{\rho}(1 + \delta^N), \]

\[ T^0_i = -(\bar{\rho} + \bar{p})v^i_N, \]

\[ T^j_i = \bar{\rho}(w + c^2\delta^N)\delta^j_i + \left(\nabla^i
\nabla_j + \frac{1}{3}\delta^j_i\nabla^k\nabla_k\right)\Pi. \]

where an overbar means the background value, \( \delta^N \) is the overdensity and \( v^N \) the velocity potential evaluated in the Newtonian gauge. A nonzero equation of state \( w = p/\rho \) would imply that the properties of dark matter differ from CDM already in their effects to the overall expansion of the universe. Then the pressure perturbation \( \delta p \) could differ from zero as well. Our description is not completely general as we do not now allow entropic pressure, but assume \( \delta p = c^2\delta \rho \), where \( c^2 \equiv \dot{\rho}/\rho \) is the sound speed (an overdot denoting a derivative with respect to the conformal time \( \tau \)). The anisotropic stress is constructed from the scalar potential \( \Pi \) with the aid of the covariant derivatives \( \nabla_k \) of the metric \( g^{(3)}_{ij} \). The last term in Eq.(3) can appear only at the perturbative level in a Friedmann-Lemaitre universe. In the following, it will be useful to employ the gauge-invariant variable

\[ \delta = \delta^N + 3H(1 + w)v^N/k, \]

This \( \delta \) is equal to the fractional overdensity in the comoving gauge, where the velocity potential vanishes. On the other hand, it is proportional to the velocity potential in the uniform-density gauge 34.

Following the method of 31, one may then derive a generalized evolution equation for \( \delta \) in the case where that energy-momentum tensor is given by Eq.(3). Here we report only the result, which is

\[ \ddot{\delta} = D_1H\dot{\delta} + \left(D_2H^2 + D_3k^2\right)\delta + P_1H\Pi + P_2H^2\Pi, \]

where the dimensionless coefficients are given explicitly in the Appendix A, Eq.(7) determines completely the behaviour of cosmological fluctuations with freedom to choose any \( f, w, c^2 \) or \( \Pi \). Once the comoving matter fluctuation \( \delta \) is solved from this second order differential equation, the corresponding matter variables in any other gauge, as well any metric perturbation, in particular the potentials \( \Psi \) and \( \Phi \) in Eq.(2), are uniquely fixed by the solution for \( \delta \), and can be found as usually by plugging this solution into the gauge transformation or constraint equations determining the relationships between the different variables. When \( w = c^2 = \Pi = 0 \), Eq.(7) reduces to the cases considered in 31. The term of main interest to us here is the gradient term Eq.(A4), whose effects easily spoil the agreement of these models with observations. This has been extensively discussed elsewhere 31, 32, 33, 34, 35, 38 and here we will just discuss the possibilities of alleviating the effects of this scale-dependent term. Let us though note that its presence stems from the new effective matter sources peculiar to the particular class of modified gravity models. It has been recently found that such a scale-dependent term is absent in various modified gravity theories within the metric formalism 39, 40, 41, 42.

1 We assume this for simplicity since if \( w = 0 \) (which is satisfied by baryons and CDM), the entropic pressure should identically vanish. On the other hand, a small sound speed described by \( c^2 \) and negligible for the background, could be conceivable for dark matter even when \( w = 0 \) (this is indeed the standard description of baryons).
III. A SPECIFIC EXAMPLE ESCAPING THE CONSTRAINTS

One can notice that the dynamics of perturbations in a perfect fluid (with \( \Pi = 0 \)) can be scale-independent in the case that the sound speed of the fluid would obey

\[
e^2 = -\frac{1}{6} \frac{\log F}{\log a},
\]

where we have defined \( F \equiv \partial f/\partial R \). In the following we will however study an imperfect case with \( \Pi \neq 0 \).

We then use a specific expression for the shear stress potential of the cosmic fluid:

\[
\Pi = \frac{1}{1 - 3K/k^2} \left( \frac{\dot{F}}{4HF} + \frac{3}{2}e^2 \right) \delta.
\]

We have kept the prefactor \( (1 - 3K/k^2)^{-1} \) here for the sake of generality, though it reduces to unity for scales smaller than the curvature radius of the universe, and in a flat universe it equals identically 1. The first contribution in the parenthesis to \( \Pi \) is there in order to eliminate the effective matter sources due to modified gravity, and the second in order to cancel the sources due to possible isotropic pressure in any kind of warm dark matter. What one then finds with this input is that Eq. (7) can be written in a simpler form

\[
\dot{\delta} + \dot{D}_1 H \delta + \dot{D}_2 H^2 \delta = 0.
\]

This verifies our claim that the effect of gradient can be cancelled by inherent properties of dark matter.

We can even consider the case when this happens while our dark matter fluid at the background level is completely pressureless as usual\(^2\). Setting \( w = 0 \) our result will further simplify to Eq. (A7) in Appendix A. Thus we have shown that it is possible to avoid the tight constraints from structure formation, while keeping the background expansion exactly the same as in the CDM scenario. In fact, as the evolution in Eqs. (10, A7) is scale-invariant, the shape of the matter power spectrum is exactly the same for any choice of \( w, e^2 \), and in particular for any function \( f(R) \). Therefore, if the normalization is left arbitrary, no constraints at all arise from comparison with the shape of the observed matter power spectrum. However, the linear growth rate would be a potentially useful test for these models.

To illustrate this, we will study the toy model \( f \sim R^n \). This model allows an analytical treatment and its predictions have been compared with the data on the late time cosmological expansion\([26, 28, 35, 45]\). In fact, the background is simply described by an effective equation of state \( w_{\text{eff}} = -1 + (1+w)/n \) when \( K = 0 \), so that then

\[
H^2 = \frac{4n^2}{(3(1+w) - 2n)^2} \frac{1}{\tau^2},
\]

\[
\dot{H} = \left(1 - \frac{3(1+w)}{2n}\right) H^2,
\]

\[
\ddot{H} = \frac{1}{2} \left(2 - \frac{3(1+w)}{n}\right)^2 H^3.
\]

It is also easy to see also that

\[
\ddot{F} = \frac{3(1+w)(1-n)}{n} H F,
\]

\[
\ddot{F} = 3 \left(\frac{1}{3} - w + e^2 + (1+w) \left(\frac{1}{2n} - 1\right)\right) HF. \tag{15}
\]

Plugging then the expressions (11) and (14) in Eq. (10) one finds that, keeping \( w \) constant for simplicity,

\[
\delta = \frac{3(1+w) - n(1+9w)}{2n-3(1+w)} \frac{\dot{\delta}}{\tau} + \frac{6(n-2)(1+w)}{2n-3(1+w)} \frac{\delta}{\tau^2}.
\]

This admits two power-law solutions, the other one corresponding to a decaying and the other to a growing mode. Setting further \( w = 0 \) we find that the latter is characterized by the growth rate

\[
r \equiv \frac{d \log \delta}{d \log a} = \frac{n - |12 - 7n|}{4n} \tag{16}
\]

There is an observational estimate \( r \geq 1/\alpha = 0.15 \) from the 2dFGRS data \([47]\), implying that \( r = 0.51 \pm 0.11 \). For the \( f(R) \sim R^n \) model considered here, an analysis combining the supernova and baryon oscillation scale constraints shows that the background data prefers the values of the exponent \( n = 2.6 \pm 0.3 \) \([28]\). This corresponds to about \( 0.2 < r < 0.47 \), thus agreeing with the 2dFGRS constraints. Note that the linear evolution for this same model, but without making the assumption \( f(R) \sim R^n \), is in gross disagreement with the data \([31]\). One may also quantify the linear behaviour utilizing the growth index \( \gamma \) defined via \( g(a) \equiv \delta/a = e^{\int_0^a d \log a (\Omega_m(a)^{\gamma-1})} \)

\[
g(a) = e^{\int_0^a d \log a (\Omega_m(a)^{\gamma-1})},
\]

This single-parameter minimalist characterization of modified gravity effects turns out to depend both on the matter density and about the exponent \( n \), whereas the result for the growth rate \( r \) was independent of the amount of matter. We find a simple relation, \( \gamma = \log r/\log \Omega_m \). As the background data constrains quite tightly \( \Omega_m \approx 0.3 \) \([28]\), we have \( 0.62 < \gamma < 1.34 \). There are other useful probes of modified gravity effects \([49]\), but their detailed analysis is left for further studies.

\(^2\) Though our present approach is phenomenological and not aimed at explaining the origin of dark matter stress \( \Pi \), we might note that dark matter satisfying effectively \( w = 0 \) but \( \Pi \neq 0 \) has indeed been considered \([41]\). (See also \([44]\) for a recent discussion of how to consider anisotropically stressed cosmological fluids and \([30]\) for an extensive review of structure formation with generalized dark matter).
IV. CONCLUSIONS

Inspired by the theoretical developments [22, 23], the success of the Palatini-\(f(R)\) gravities within the Solar system [19] and the recent resolutions of their physical implications [11], we reconsidered the cosmological viability of these models in view of their predictions for the large-scale structure.

Previous investigations on the subject had established that the constraints from matter power spectrum allow only models which are practically indistinguishable (by e.g. their background expansion) from the \(\Lambda CD M\) model [31, 32, 33, 34]. In the present paper we examined the robustness of those conclusions to the variations of the dark matter scenario. We pointed out that the extremely tight constraints from structure formation are valid only for the \(f(R)\)CDM models, i.e. when the universe energy density is assumed to be dominated by pressureless and perfect matter. The instabilities occurring then due to modified gravity effects, may be less severe or even absent in \(f(R)\)GDM scenarios, i.e. when dark matter is allowed to have inherent stresses. The cosmological bounds on these \(f(R)\) models could thereby be drastically loosened.

This demonstrates that a revision of the nature dark energy could also change our view of dark matter.

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APPENDIX A: EVOLUTION EQUATION

The equation (7) was written as

\[
\ddot{\delta} = D_1 H \dot{\delta} + \left( D_2 H^2 + D_k k^2 \right) \delta + P_1 H \ddot{\Pi} + P_2 H^2 \Pi. \tag{A1}
\]

Defining \(F \equiv \partial f/\partial R\), we can then write the dimensionless coefficients as the following:

\[
D_1 = \frac{-2FH (FH^2 (1 + 3c^2 - 6w) + \dot{F}) + 2\dot{FH} + \dot{\dot{F}} F \left( 2\dot{H} - H^2 (1 + 3c^2 - 6w) \right)}{FH (F + 2FH)}, \tag{A2}
\]

\[
FH^2 (F + 2FH) D_2 = 2FH \left[ FH \left( 12H^3(w - c^2) + \dot{H} H (3c^2 + 3w - 2) + \dot{\dot{H}} \right) + \dot{\dot{F}} \left( H - H^2 (1 - 3w) \right) \right] \tag{A3}
\]

\[
+ \dot{\dot{F}} F \left( -2\dot{H}^2 + H \ddot{H} + 3H^2 (H - 4H^2)(c^2 - w) \right) - 2\dot{F}^2 H \left( H - H^2 (1 - 3w) \right)
\]

\[
D_k = \frac{-\left( \dot{\dot{F}} + 6c^2 FH \right)}{3 \left( F + 2FH \right)}, \tag{A4}
\]

\[
P_1 = 2 \left( \frac{3K}{k^2} - 1 \right), \tag{A5}
\]

and

\[
P_2 = \frac{2 \left( k^2 - 3K \right) \left[ 6F^2 + 9\dot{F} FH (c^2 - w) - 2F \left( 3\dot{\dot{F}} + F (6\dot{H} + 9(c^2 - w)H^2 - k^2) \right) \right]}{3FH (F + 2FH) k^2}. \tag{A6}
\]

In the case that Eq.(9) holds and \(w = 0\), the evolution equation reduces to

\[
\ddot{\delta} = \frac{1}{2F^2 H^2 \left( F + 2FH \right)} \left\{ FH \left[ -4FH (FH^2 + \dot{F}) + 3\dot{F}^2 H + 4\dot{\dot{F}} F \left( \dot{H} - H^2 \right) \right] \ddot{\delta} \right\} \tag{A7}
\]

\[
+ \left[ 2F^2 H \left( 2FH (\dot{H} - 2\ddot{H}) + \dot{\dot{F}} (2\ddot{H} - 3H^2) \right) + 3\dot{F}^2 H^2 - 3\ddot{F} FH (\dot{H} - 2H^2) \right] \dot{\dot{\delta}}
\]

\[
+ \dot{F} F \left( -4\dot{H}^2 F + 2\dot{H} FH - (3\dot{F} + 2\dot{H} F) H^2 \right) \delta \right\}.
\]
