Dimension-seven operator contribution to the
top quark anomalous interactions

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Abstract

The contribution of dimension-seven operators to anomalous FCNC-interactions of the $t$-quarks with a photon and a gluon is considered. The phenomenological Lagrangian and Feynman rules are derived. There are evaluated the expressions for the widths FCNC-decays of the $t$-quark into light quark and two photons, two gluons, a photon and a gluon, and three quarks.
1 Introduction

At present, it is not known what type of New Physics (NP) beyond the Standard Model framework will be responsible for possible deviations from predictions of the Standard Model (SM). Multiple scenarios of SM extensions lead to different predictions in the $t$-quark sector with their own a specific set of types of interactions and parameters (coupling constants, the masses of new objects). At the same time, different scenarios often predict the same or very similar effects, leading to processes with identical final states.

To describe the various anomalous interactions of $t$-quarks it is widely used a universal approach based on the formalism of an effective field theory \[1\]. In this approach, the anomalous interactions of $t$-quarks are described by a model-independent manner through the use of an effective (phenomenological) Lagrangian \[2, 3, 4, 5\]. This Lagrangian must be gauge-invariant with respect to the SM gauge group (otherwise, the input anomalous interactions immediately would lead to contradictions with modern precision measurements) and consists of the terms with an increasing dimension, suppressed by increasingly higher degrees of the NP-scale. Such a Lagrangian of the anomalous $t$-quark interactions can be presented in the following form \[1, 3, 4\]:

$$L_{QFT} = L_{SM} + \kappa_4 \bar{\psi}_q \hat{O}^{(4)} \psi_t + \frac{\kappa_5}{\Lambda} \bar{\psi}_q \hat{O}^{(5)} \psi_t + \frac{\kappa_6}{\Lambda^2} \bar{\psi}_q \hat{O}^{(6)} \psi_t + \cdots$$  \hspace{1cm} (1)

where $L_{SM}$ is the SM Lagrangian (see, e.g. \[9\]), $\Lambda$ is new physics mass scale, $\kappa_i$ are the anomalous couplings.

The promising directions of searching for NP in the $t$-quark sector are processes with the Flavour changing neutral currents (FCNC) $t \to \gamma(g, Z) + c(u)$.

Within the SM framework such processes are highly suppressed (due to loop diagrams) \[2\]: $B(t \to q \gamma/g/Z) < O(10^{-11} \div 10^{-13})$, which makes them practically impossible to observe in the experiment. Thus, the experimental observation of the $t$-quark FCNC interactions $t$-quarks will unambiguously indicate the existence of NP beyond the SM.

The FCNC processes with the dimension-5 and 6 operators were analyzed in detail earlier and there were built expressions for all possible FCNC interactions of $t$-quarks (see, for example, \[3, 4\]).

In this article we consider the role of the dimension-seven operator contributions to anomalous FCNC-interaction of $t$-quarks with a photon and gluon ($t\gamma q$ and $t g q$):

$$L_{FCNC}^{(7)} = \frac{\kappa}{\Lambda^3} \bar{\psi}_q \hat{O}^{(7)} \psi_t, \quad q = u, c$$

In this article, the upper $q$-quark in the $(tVq)$ interaction will be denoted by the symbol $u$. Therefore, all the results will remain the same for $u$- or $c$-quarks.

2 Anomalous FCNC interaction Lagrangian

In this article, when constructing the operators $\hat{O}^{(n)}$ there are considered the interactions with only one external massless gauge boson $V$ ($V = \gamma, g$). Due to a gauge invariance, the interacting boson enters into operators in the form of the field strength tensor $F^{\mu\nu}$ (and the
dual tensor $\tilde{F}_{\mu\nu}$):

$$\mathcal{L}_{\text{FCNC}} = \frac{1}{\Lambda^3} \bar{\psi}_u \kappa_1 \tilde{W}^{(7)}_{\mu\nu} \psi_t F_{\mu\nu} + \frac{1}{\Lambda^3} \bar{\psi}_a \kappa_2 \tilde{W}^{(7)}_{\mu\nu} \psi_t \tilde{F}_{\mu\nu}$$  \hspace{1cm} (2)

where $A_{\mu}, B^a_{\mu}$ are the photon and gluon fields, $\xi_i, \zeta_i$ - are the anomalous couplings (in the general case, the complex numbers).

In the general case, the operator $\tilde{W}^{(n)}$ is constructed from the Dirac, Gell-Man and covariant derivatives $D^\mu, D^{*\mu}$ [9]:

$$\tilde{D}^\mu = D^\mu = \bar{\psi} \gamma^\mu - i e_q A^\mu - i g_s t^a G^a_{\mu}$$

$$\tilde{D}^{*\mu} = D^{*\mu} = \bar{\psi} \gamma^{\mu} + i e_q A^\mu + i g_s t^a G^a_{\mu}$$

where $e_q$ is the electric charge of the quark, $g_s$ is the constant of the strong interactions, and $t^a$ - Gell-Mann matrices. The covariant derivative $D^\mu$ acts on the spinor $\psi_t$, and $D^{*\mu}$ - to the antispinor $\bar{\psi}_t$. We note that, because of covariant derivatives, in Lagrangians there are terms, describing the interaction with one, two and three bosons. In this article we present an explicit form of expressions only for the interaction with one and two bosons.

**Second** restriction on the form of the operators $\tilde{W}^{(n)}$ consists in the following. When constructing such operators, it is assumed that covariant derivatives act only on quarks (spinors). In this case, they must have convolution by indices with a strength tensor. This assumption restricts the type of the operators $\tilde{W}^{(n)}$. Indeed, let us consider the contribution of the dimension-seven operator:

$$\bar{\psi}_u \tilde{D}^{*\gamma^\mu D^\nu} \psi_t F_{\mu\nu}, \quad \tilde{D}^* = \tilde{D}^{\alpha\gamma^\alpha}$$

In this case, the action of the derivative $\tilde{D}^*$ (which is not related to interacting boson), can be treated, for example, as a form-factor:

$$\frac{\kappa^{(7)}}{\Lambda^3} \bar{\psi}_u \tilde{D}^{*\gamma^\mu D^\nu} \psi_t F_{\mu\nu} = \frac{\kappa^{(7)}}{\Lambda^2} \bar{\psi}_u \left( \frac{\tilde{D}^{\alpha\gamma^\alpha}}{\Lambda} \right) \gamma^\mu D^\nu \psi_t F_{\mu\nu}, \quad \frac{\kappa^{(7)}}{\Lambda^2} \bar{\psi}_u \left( \frac{\tilde{D}^{\alpha\gamma^\alpha}}{\Lambda} \right) \Rightarrow \frac{\kappa^{(7)}}{\Lambda^2} \bar{\psi}_u$$

$$\Rightarrow \frac{\kappa^{(7)}}{\Lambda^3} \bar{\psi}_u \tilde{D}^{*\gamma^\mu D^\nu} \psi_t F_{\mu\nu} \approx \frac{\kappa^{(7)}}{\Lambda^2} \bar{\psi}_u \gamma^\mu D^\nu \psi_t F_{\mu\nu}$$

The last expression is, in fact, represented by the contribution of an operator of dimension-six! Thus, taking into account these two assumptions (on the interaction with only one boson and second, described above) the phenomenological Lagrangian of FCNC interactions can be constructed only from the operators of dimensions-five, six and seven.

### 2.1 The anomalous interaction with a photon

Dimension-seven operator for anomalous $t$-quark FCNC-interaction with a photon comprises four gauge-invariant terms:

$$\tilde{W}^{(7)}_{\gamma} \mu\nu : D^\mu D^\nu, \quad D^{*\mu} D^{*\nu}, \quad D^{*\mu} D^\nu, \quad D^\nu D^{*\mu}$$
The Lagrangian of the anomalous couplings. We note that in the Lagrangian the operators are equal to each other, and the last two are related by the relation:

\[
D^\mu D^\nu = D^{*\mu} D^{*\nu} = -\frac{1}{2} \Delta^{\mu\nu}
\]

\[
D^\nu D^{*\mu} = D^{*\mu} D^\nu - \Delta^{\mu\nu}
\]

\[
\Delta^{\mu\nu} = i e_q F^{\mu\nu} + i g_s t^a G_a^{\mu\nu}
\]

Therefore, the operator \( \tilde{W}_\gamma^{(7)} \mu\nu \) comprises only two independent structures \( (D^{*\mu} D^\nu + \Delta^{\mu\nu}) \).

The Lagrangian of \( t \)-quark FCNC-interaction with a photon has the form:

\[
\mathcal{L}_{FCNC}^{(7)}(t \gamma q) = \frac{e_q}{\Lambda^3} \bar{\psi}_u \psi_t F_{\mu\nu} + \frac{e_q}{\Lambda^3} \bar{\psi}_u \psi_t \tilde{F}_{\mu\nu}
\]

\[
\tilde{W}^{\mu\nu} = \kappa_1 D^{*\mu} D^\nu - \kappa_2 \frac{\Delta^{\mu\nu}}{2}; \quad \tilde{W}^{\mu\nu}_D = \kappa_3 D^{*\mu} D^\nu - \kappa_4 \frac{\Delta^{\mu\nu}}{2}
\]

where \( \xi_i, \zeta_i \) are the anomalous couplings.

In what follows, the common numerical factors (of the type \( \pm 1, 1/2, i, ... \)) are included in the anomalous couplings. We note that in the Lagrangian the operators \( D^{*\mu} D^\nu \times \tilde{F}_{\mu\nu} \) and \( \Delta^{\mu\nu} \times \tilde{F}_{\mu\nu} \) result in the same expressions for terms describing the interaction with one or two bosons. Therefore, in this article we rely on

\[
\kappa_4 = \kappa_3
\]

Omitting the trivial computations and using the the momentum representation, for each terms from \( \tilde{W}^{(7)} \) and \( \tilde{W}_D^{(7)} \) we have

\[
\bar{\psi}_u D^{*\mu} D^\nu \psi_t F_{\mu\nu} \quad \rightarrow \quad \bar{u}_u \kappa_1 [\hat{w}_1 + \hat{X}_2 + \hat{U}_2 + \hat{V}_1(3)] u_t
\]

\[
\bar{\psi}_u \Delta^{\mu\nu} \psi_t \tilde{F}_{\mu\nu} \quad \rightarrow \quad \bar{u}_u \kappa_2 [2 \hat{X}_1 + \hat{U}_1 + \hat{V}_1(3)] u_t
\]

\[
\bar{\psi}_u D^{*\mu} D^\nu \psi_t \tilde{F}_{\mu\nu} \quad \rightarrow \quad \bar{u}_u \kappa_3 [2 \hat{X}_3 + \hat{U}_3 + \hat{V}_3(2)] u_t
\]

where \( \bar{u}_u \) and \( u_t \) are the spinors of the light \( u \) and \( t \)-quarks with momenta \( p_2 \) and \( p_1 \), respectively. The expressions \( \hat{V}_i(3), i = 1, 2 \) describe interactions with three bosons. The explicit form of which is not given in this article. For the rest, we have (below, each expression contains the total coefficient \( \Lambda^{-3} \)):

\[
\begin{align*}
w_1 &= e_q p_1^\mu p_2^\nu (q^\mu g^{\nu\alpha} - q^\nu g^{\mu\alpha}) A^\alpha \\
X_1 &= e_q^2 \left[ (q_1 q_2) g^{\alpha\beta} - q_2^\beta q_2^\alpha \right] A_1^\alpha A_2^\beta \\
X_2 &= e_q^2 \left[ (q_1 + q_2)^2 g^{\alpha\beta} - q_2^\beta (q_1 + q_2)^\beta - q_1^\beta (q_1 + q_2)^\alpha \right] A_1^\alpha A_2^\beta \\
X_3 &= e_q^2 e^{\mu\nu\alpha\beta} q_1^\mu q_2^\nu A_1^\alpha A_2^\beta \\
U_1 &= e_q g_s t^a \left[ (q_1 q_2) g^{\alpha\beta} - q_2^\beta \right] A^a (q_1) B_2^\beta (q_2) \\
U_2 &= e_q g_s t^a (q_1 + q_2)^\lambda (q_1^\lambda g^{\alpha\beta} - q_2^\beta g^{\alpha\lambda}) A^a (q_1) B_2^\beta (q_2) \\
U_3 &= e_q g_s t^a e^{\mu\nu\alpha\beta} q_1^\mu q_2^\nu A^a (q_1) B_2^\beta (q_2)
\end{align*}
\]

here \( q_1, q_2 \) are the momenta of the bosons. The corresponding Feynman rules are given in the Appendix.
2.2 The anomalous interaction with a gluon

Dimension-seven operator for anomalous $t$-quark FCNC-interaction with a gluon comprises three gauge-invariant terms:

$$\hat{W}^{\alpha\mu\nu} : D^{*\mu} t^a D^\nu, \quad t^a D^\mu D^\nu, \quad D^{*\mu} D^{*\nu} t^a$$

Also, as in the case of interaction with a photon, the contributions of operators $t^a D^\mu D^\nu$ and $D^{*\mu} D^{*\nu} t^a$ are equal to each other. Thus, the Lagrangian of dimension-seven, describing the interaction with the gluon, has the form:

$$\mathcal{L}_{\text{FCNC}}^{(7)}(t g q) = \frac{g_s}{\Lambda^3} \bar{\psi}_u \hat{W}^{\alpha\mu\nu} \psi_t G_{\mu\nu} + \frac{g_s}{\Lambda^3} \bar{\psi}_u \hat{W}_D^{\alpha\mu\nu} \psi_t \tilde{G}_{\mu\nu}$$

(5)

$$\hat{W}^{\alpha\mu\nu} = \lambda_1 D^{*\mu} t^a D^\nu - \lambda_2 t^a \frac{\Delta_{\mu\nu}}{2}; \quad \hat{W}_D^{\alpha\mu\nu} = \lambda_3 D^{*\mu} t^a D^\nu - \lambda_4 t^a \frac{\Delta_{\mu\nu}}{2}$$

$$\lambda_i = \xi_i^g + \zeta_i^g$$

where $\xi_i^g, \zeta_i^g$ are anomalous couplings, $\bar{\psi}_u$ and $\psi_t$ are spinors describing the light $u$ and $t$-quarks, the strength tensor $G_{\alpha\mu\nu}$ is defined above (2).

After transition into momentum space for each term in the Lagrangian (5) we get:

$$\bar{\psi}_u D^{*\mu} t^a D^\nu \psi_t G_{\mu\nu} \rightarrow \bar{u}_u \lambda_1 [w_1^u + U_0 + Y_2 + V_{3,4}] u_t$$

$$\bar{\psi}_u t^a D^\mu D^\nu \psi_t G_{\mu\nu} \rightarrow \bar{u}_u \lambda_2 [U_1 + Y_1 + V_{3,4}] u_t$$

$$\bar{\psi}_u t^a D^{*\mu} D^\nu \psi_t G_{\mu\nu} \rightarrow 2 \bar{u}_u \lambda_3 [U_3 + Y_3 + V_{3,4}] u_t$$

$$\bar{\psi}_u t^a D^\mu D^{*\nu} \psi_t G_{\mu\nu} \rightarrow -2 \bar{u}_u \lambda_4 [U_3 + Y_3 + V_{3,4}] u_t$$

where $\bar{u}_u$ and $u_t$ are spinors, describing a light $u$ and $t$-quarks with momenta $p_2$ and $p_1$, respectively. Values of $V_{3,4}$ describe interactions with 3 and 4 bosons. The explicit form of which is not given in this article. For the rest, we have (below), each expression contains the total coefficient $\Lambda^{-3}$:

$$w_1^u = g_s t^a p_1^\mu p_2^\nu (q^\mu g^{\nu\alpha} - q^\nu g^{\mu\alpha}) B_{a\alpha}$$

$$U_0 = e_q g_s t^a \left[ ((q_1 + q_2) q_2) g^{\alpha\beta} - q_2^\alpha (q_1 + q_2)^\beta \right] A_{\alpha} B_{\beta}^{a\alpha 2}$$

$$U_1 = e_q g_s t^a \left[ (q_1 q_2) g^{\alpha\beta} - q_2^\alpha q_1^\beta \right] A^\alpha (q_1) B_{\beta}^{a\alpha} (q_2)$$

$$U_3 = e_q g_s t^a \varepsilon^{\mu\nu\alpha\beta} q_1^\mu q_2^\nu A_{\alpha} B_{\beta}^{a\alpha 2}$$

$$Y_2 = g_s \left\{ \begin{array}{l}
\quad (p_1 q_2) - (p_2 q_1) g^{\alpha\beta} - p_2^\mu p_1^\nu - p_1^\mu p_2^\nu - q_2^\mu q_1^\nu + p_2^\mu q_1^\nu \nonumber \\
+ \quad t^b t^a \left[ ((p_1 q_1) - (p_2 q_2)) g^{\alpha\beta} - p_2^\mu p_1^\nu - p_1^\mu p_2^\nu + q_2^\mu q_1^\nu - p_2^\mu q_1^\nu \right] \right\} B_{\alpha}^a B_{\beta}^b$$

$$Y_1 = g_s \left( \frac{1}{3} \delta^{\mu\nu} + d^{\mu\nu} \varepsilon^\lambda \right) [(q_1 q_2) g^{\alpha\beta} - q_2^\alpha q_1^\beta] B_{\alpha}^{a\mu} B_{\beta}^{b\nu}$$

$$Y_3 = g_s \left( \frac{1}{3} \delta^{\mu\nu} + d^{\mu\nu} \varepsilon^\lambda \right) q_1^\mu q_2^\nu + 2 t^k f^{kab} p_1^a p_1^\nu \varepsilon^{\mu\nu\alpha\beta} B_{\alpha}^{a\mu} B_{\beta}^{b\nu}$$

$$Y_4 = g_s \left( \frac{1}{3} \delta^{\mu\nu} + d^{\mu\nu} \varepsilon^\lambda \right) q_1^\mu q_2^\nu B_{\alpha}^{a\mu} B_{\beta}^{b\nu}$$

(6)

Here $q_1, q_2$ are the momenta of the bosons. The corresponding Feynman rules are given in the Appendix.

3 $t$-quark decay widths

We note that the amplitudes (T) containing the anomalous interaction vertex with one real boson (photon or gluon), are always equal to zero. Indeed, taking into account the law of
conservation of momentum and choosing a calibration, which ensures the Lorentz condition 
\((qV) = 0\), for such vertices we obtain:

\[
T \propto w_1 = p_1^\mu p_2^\nu (q^\mu g^\nu - q^\nu g^\mu) V^\alpha; \quad V^\alpha = A_\alpha^\alpha, \quad p_1 = p_2 + q, \quad (qV) = 0, \quad q^2 = 0
\]

therefore, in contrast to operators of dimension-5 and 6, for the considered interaction due to dimension-seven operators in the lowest order perturbation theory the \(t\)-quark can decay into the following three-body channels:

\[
t \rightarrow u \gamma \gamma, \quad t \rightarrow u \gamma g, \quad t \rightarrow u g g
\]

The diagrams describing these processes are presented below.

Figure 1: The diagrams describing the \(t\)-quark decays. Here \(p_1\) and \(p_2\) are the momenta of \(t\) and \(u\)-quarks, respectively, \(q_1\) and \(q_2\) are the momenta of the gauge bosons or \(\bar{q}q\) pair.

In all calculations we set: \(m\) is the mass of the \(t\)-quark, the masses of light quarks are assumed to be zero. We use axial gauge \([8]\):

\[
\sum_{pol} V^\mu V^\nu = \rho^{\mu \nu}(q) = -g^{\mu \nu} + \frac{q^\mu n^\nu + n^\mu q^\nu}{(qn)} - \frac{n^2 q^\mu q^\nu}{(qn)^2}; \quad \rho^{\mu \nu} n_\nu = 0
\]

where \(q\) is the gauge boson momentum, \(n\) is gauge fixing 4-vector. In what follows we take it a sum of \(q_1\) and \(q_2\). Then, we get:

\[
n = q_1 + q_2 \rightarrow \rho^{\mu \nu}(q_1) = \rho^{\mu \nu}(q_2) = \rho^{\mu \nu} = -g^{\mu \nu} + \frac{q_1^\mu q_2^\nu + q_2^\mu q_1^\nu}{(q_1 q_2)}\]

\[
\rho^{\mu \nu} \rho^{\alpha \beta} = \rho^{\alpha \beta}; \quad \rho^{\mu \nu} g_{\mu \nu} = 2
\]

As is well known, the \(1 \rightarrow 3\) decay is described by two independent invariants. In this article, the following variables are selected:

\[
q^2 = (q_1 + q_2)^2; \quad x = \frac{q^2}{m^2}, \quad y = \frac{(p_2 + q_2)^2}{m^2}, \quad x + y \leq 1, \quad 0 \leq \{x, y\} \leq 1
\]

Then, we get the following form for the width for each decay channels:

\[
d\Gamma(t \rightarrow uab) = \frac{m}{96\pi} |T(t \rightarrow uab)|^2 \, dx \, dy
\]
where $|T|^2$ is the square of the amplitude (with averaging on the spin and color of the initial $t$-quark is included in expression (10)), $a$ and $b$ are the corresponding final states (photons, gluons, and quarks).

$t \to u\gamma\gamma$ decay

The amplitude of the decay of the $t$-quark into two photons is described by a single Feynman diagram (as a result, the amplitude squared equals:

$$T(t \to u\gamma\gamma) = \frac{e_q^2}{\Lambda^3} \bar{u}(p_2)(T_1 + T_2 + T_3)u(p_1)$$

$$T_1 = \kappa_1 \left[ (q_1 + q_2)^2 g^{\alpha\beta} - q_1^2 (q_1 + q_2)^\beta - (q_1 + q_2)^\alpha q_1^\beta \right] A_1^\alpha A_2^\beta$$

$$T_2 = 2\kappa_2 \left[ (q_1 q_2) g^{\alpha\beta} - q_1^2 q_2^\beta \right] A_1^\alpha A_2^\beta$$

$$T_3 = 2\kappa_3 \varepsilon^{\mu\nu\alpha\beta} q_1^\mu q_2^\nu A_1^\alpha A_2^\beta, \quad \kappa_i = \xi_i^7 + \zeta_i^7 \gamma^5$$

where $q_1$ and $q_2$ are the momenta of the photons. Using the axial gauge (8) we get:

$$q_1^2 = q_2^2 = 0; \quad (q_1 A_1) = (q_1 A_2) = (q_2 A_1) = (q_2 A_2) = 0$$

$$T_1 + T_2 = 2(\kappa_1 + \kappa_2) B_0; \quad T_3 = 2\kappa_3 B_1$$

$$B_0 = (q_1 q_2)(A_1 A_2) = (q^2/2)(A_1 A_2); \quad B_1 = \varepsilon^{\mu\nu\alpha\beta} q_1^\mu q_2^\nu A_1^\alpha$$

$$|B_0|^2 = q_1^4 A_1^\mu A_2^\mu A_1^\alpha A_2^\alpha = \frac{q_1^4}{4} \tilde{\rho}_0 \tilde{\rho}_0 = \frac{1}{2} q_1^4$$

$$|B_1|^2 = \varepsilon^{\mu\nu\alpha\beta} \varepsilon^{\mu'\nu'\alpha'\beta'} q_1^\mu q_2^\nu q_1^{\mu'} q_2^{\nu'} A_1^\alpha A_2^\beta A_1^{\alpha'} A_2^{\beta'} = 2(q_1 q_2)^2 = \frac{1}{2} q_1^4$$

$$|\bar{u}(p_2)\kappa_i u(p_1)|^2 = \text{Tr}(\hat{p}_1 + m)(\xi^* - \zeta^* \gamma^5)\hat{p}_2(\xi + \zeta \gamma^5) = 2(|\xi|^2 + |\zeta|^2)(m^2 - q^2)$$

As a result, the amplitude squared equals:

$$|T|^2 = 4 \left( \frac{3}{2} \right) \frac{e_q^4}{\Lambda^6} \mathcal{K}_{\gamma\gamma}(m^2 - q^2)q^4$$

$$\mathcal{K}_{\gamma\gamma} = |\xi_1^2 + \xi_2^2|^2 + |\xi_3^2|^2 + |\zeta_1^2 + \zeta_2^2|^2 + |\zeta_3^2|^2$$

where 3 is color coefficient, and 2 in the denominator takes into account the identity of the final photons. Below we present the expressions for the $t$-quark decay width:

$$d\Gamma(t \to u\gamma\gamma)/dx dy = e_q^4 \alpha_e^2 \mu_\Lambda^6 \left( m/16\pi \right) \mathcal{K}_{\gamma\gamma} x^2 (1 - x) \left\{ \mu_\Lambda^6 \left( m/480\pi \right) \mathcal{K}_{\gamma\gamma} \right\} \quad (11)$$

where $\alpha_e$ is the fine structure constant and we use the notation

$$\mu_\Lambda^6 = \left( \frac{m}{\Lambda} \right)^6$$

$t \to u\gamma g$ decay

The decay of the $t$-quark into $u$-quark, the photon and the gluon occurs due to FCNC interactions induced by a photon and a gluon. The amplitude is described by a single Feynman diagram (as a result, the amplitude squared equals:

$$T(t \to q\gamma g) = \frac{e_q g_s}{\Lambda^3} \bar{u}(p_2)(T_1^{\gamma g} + T_2^{\gamma g} + T_3^{\gamma g})u(p_1)$$

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where

\[ T_1^{\gamma g} = \kappa_1 t^a \left[ ((q_1 + q_2)q_1) g^{\alpha \beta} - (q_1 + q_2)\delta_{1} \right] A_1^\alpha B_2^\beta \]

\[ + \lambda_1 t^a \left[ ((q_1 + q_2)q_2) g^{\alpha \beta} - q_2^\alpha q_1^\beta \right] A_1^\alpha B_2^\beta \]

\[ T_2^{\gamma g} = (\kappa_2 + \lambda_2) \left[ (q_1 q_2) g^{\alpha \beta} - q_2^\alpha q_1^\beta \right] B_1^\alpha B_2^\beta \]

\[ T_3^{\gamma g} = (\kappa_3 + \lambda_3 + \lambda_4) \epsilon^{\mu \nu \alpha \beta} \delta_{1} q_2^\mu A_1^\alpha B_2^\beta, \quad \kappa_\alpha = \xi_\alpha^7 + \xi_\alpha^7 \gamma^5, \lambda_\alpha = \xi_\alpha^9 + \xi_\alpha^9 \gamma^5 \]

Expressions for the decay widths are equal to the corresponding expressions from \([11]\) with the replacement

\[ \{ \epsilon_1^\alpha \epsilon_\alpha K_{\gamma \gamma} \} \rightarrow \frac{2}{3} \{ \epsilon_1^\alpha \epsilon_\alpha K_{\gamma \gamma} \} \]

where \(\alpha_\alpha\) is the QCD coupling. Here we have

\[ K_{\gamma \gamma} = |\xi_\gamma^7 + \xi_\gamma^7 + \xi_\gamma^7| + \xi_\gamma^7 + \xi_\gamma^7 + \xi_\gamma^7| + \xi_\gamma^7 + \xi_\gamma^7 + \xi_\gamma^7 + \xi_\gamma^7 + \xi_\gamma^7 + \xi_\gamma^7| \]

\[ t \rightarrow u g g \text{ decay} \]

The amplitude of the \(t\)-quark decay into \(u\)-quark and two gluons is described by two Feynman diagrams \((a_1\) and \(a_2\) in Fig. 1) and equals:

\[ T(t \rightarrow u g g) = \frac{g_s^2}{\Lambda^2} \bar{u}(p_2)(T_0^{gg} + T_1^{gg} + T_2^{gg} + T_3^{gg} + T_4^{gg}) u(p_1) \]

where

\[ T_0^{gg} = \lambda_1 t^a p^\mu k^\nu (q^\alpha g^{\mu \alpha} - q^\nu g^{\nu \alpha}) \frac{\rho^{\alpha \beta}(q)}{q^2} F_{abc} B_1^\alpha B_2^\beta \]

\[ T_1^{gg} = \lambda_1 t^a t^b \left[ ((p_1 q_2) - (p_2 q_1)) g^{\alpha \beta} - p_2^\alpha p_1^\beta + p_1^\alpha p_2^\beta - q_2^\alpha q_1^\beta \right] B_{1a}^\alpha B_{2b}^\beta \]

\[ + \lambda_1 t^a t^b \left[ ((p_2 q_1) - (p_1 q_2)) g^{\alpha \beta} + p_2^\alpha p_1^\beta - p_1^\alpha p_2^\beta + q_2^\alpha q_1^\beta \right] B_{1a}^\alpha B_{2b}^\beta \]

\[ T_2^{gg} = \lambda_2 \left( \frac{1}{3} \delta^{ab} + t^b k\right) \left[ (q_1 q_2) g^{\alpha \beta} - \delta_{1} \right] B_{1a}^\alpha B_{2b}^\beta \]

\[ T_3^{gg} = \lambda_3 \left[ \left( \frac{1}{3} \delta^{ab} + t^b k\right) \delta_{1} q_2^\mu \right] \epsilon^{\mu \nu \alpha \beta} B_{1a}^\alpha B_{2b}^\beta \]

\[ T_4^{gg} = \lambda_4 \left( \frac{1}{3} \delta^{ab} + t^b k\right) \delta_{1} q_2^\mu \epsilon^{\mu \nu \alpha \beta} B_{1a}^\alpha B_{2b}^\beta, \quad \lambda_\alpha = \xi_\alpha^9 + \xi_\alpha^9 \gamma^5 \]

here \(q_1, q_2\) are the photon and gluon momenta.

\[ F_{abc} = i g_s f^{abc} \left[ (-q_1 - q)^\delta g^{\alpha \beta} + (-q_2 + q_1)^\alpha g^{\delta \beta} + (q + q_2)^\beta g^{\alpha \delta} \right] ; \quad \rho^{\alpha \beta}(q) = -g^{\alpha \beta} + \frac{q^\alpha q^\beta}{q^2} \]

Then we get:

\[ |T(t \rightarrow u g g)|^2 = \frac{4 g_s^4}{3 \Lambda^2} (m^2 - q^2) \left[ 7 \chi_1 q^4 + 3 \chi_2 (m^2 - q^2)^2 \right] \]

\[ \chi_1 = |\xi_\gamma^7 - \xi_\gamma^7|^2 + |\xi_\gamma^7 + \xi_\gamma^7|^2 + |\xi_\gamma^7 - \xi_\gamma^7|^2 + |\xi_\gamma^7 + \xi_\gamma^7|^2 \]

\[ \chi_2 = |\xi_\gamma^7|^2 + |\xi_\gamma^7|^2 \]
We define the probability of the two-body (two-jet) decays as follows:

\[
d\Gamma(t \rightarrow u\bar{q}g)/dx
dy = \alpha_s^2 \mu_A^6 \left( m/72\pi \right) \left[ 7\chi_1 x^2 (1 - x) + 3\chi_2 (1 - x)^3 \right] \\
\Gamma(t \rightarrow u\bar{q}g) = \alpha_s^2 \mu_A^6 \left( m/2160\pi \right) \left[ 7\chi_1 + 18\chi_2 \right]
\]

(t \rightarrow u\bar{q}q) \text{ and } (t \rightarrow u\bar{uu}) \text{ decays}

The expressions for the decay widths of the two channels:

\[
t \rightarrow u\bar{q}q, \; q \neq u; \; t \rightarrow u\bar{uu}
\]

The first decay is described by a single diagram \( a_3 \), and the second decay - by two diagrams \( a_3 \) and \( a_4 \) (see Fig. 1). The corresponding amplitudes are:

\[
T(t \rightarrow u\bar{q}q)_{q \neq u} = \left( g_s^2 / \Lambda^3 \right) W_1; \; T(t \rightarrow u\bar{uu}) = \left( g_s^2 / \Lambda^3 \right) (W_1 - W_2)
\]

\[
W_1 = \left\{ \tilde{u}(p_2) \lambda_1 t^a u(p_1) \right\} p_1^\mu p_2^\nu \left[ g^\mu g^\nu - q^\nu g^\mu - q^\mu g^\nu \right] \frac{-\rho^{a\alpha}(q)}{q^2} \left\{ \tilde{u}(q_1) t^{\alpha} g^{\gamma \nu} v(q_2) \right\}
\]

\[
W_2 = \left\{ \tilde{u}(q_1) \lambda_1 t^a u(p_1) \right\} p_1^\nu p_2^\alpha \left[ r^\nu g^{\alpha \gamma} - r^\alpha g^{\nu \gamma} \right] \frac{-\rho^{a\alpha}(r)}{r^2} \left\{ \tilde{u}(p_2) t^{\alpha} g^{\gamma \nu} v(q_2) \right\}
\]

\[
q = q_1 + q_2; \; r = p_2 + q_2
\]

\[
\rho^{a\alpha}(q) = -g^{a\alpha} + \frac{q^\alpha q^\alpha}{q^2}; \; \rho^{a\alpha}(r) = -g^{a\alpha} + \frac{q^\alpha r^\alpha + r^\alpha q^\alpha}{(qr)} - \frac{q^2 r^\alpha r^\alpha}{(qr)^2}
\]

The expressions for the decay widths of the t-quark are equal to:

\[
d\Gamma(t \rightarrow u\bar{q}q)/dx\;dy = \alpha_s^2 \mu_A^6 \left( m/12\pi \right) \left( |\xi|^2 + |\zeta|^2 \right) (1 - x - y)(1 - x) y \\
\Gamma(t \rightarrow u\bar{q}q) = \alpha_s^2 \mu_A^6 \left( m/360\pi \right) \left( |\xi|^2 + |\zeta|^2 \right)
\]

\[
d\Gamma(t \rightarrow u\bar{uu})\;dx\;dy = \alpha_s^2 \mu_A^6 \left( m/24\pi \right) \left( |\xi|^2 + |\zeta|^2 \right) (1 - x - y)(x + y - \frac{7}{3}xy) \\
\Gamma(t \rightarrow u\bar{uu}) = \alpha_s^2 \mu_A^6 \left( 23m/8640\pi \right) \left( |\xi|^2 + |\zeta|^2 \right)
\]

Estimation of the probability the t-quark “two-body” decays

We note that during hadronization the pair of quarks (or quark and gluon) with a small invariant mass can form a hadronic jet \( j \). In this case, the decays of the t-quark in the experiment can lead to the observed two-body final states:

\[
t \rightarrow u\gamma \gamma \rightarrow j(u\gamma) + \gamma; \; u\gamma g \rightarrow j(uq) + \gamma; \; u\gamma g \rightarrow j(u\gamma) + j(g); \; ugg \rightarrow j(uq) + j(g), ...
\]

To estimate the probability of these two-body decays we require that the invariant mass of a pair of final particles from the decays \( \bar{q}q \) should be less than 40 GeV (naturally, the realistic estimates can be obtained after a detailed modeling of processes):

\[
m_{\min} \leq 40 \text{ GeV} \rightarrow \delta = \left( \frac{m_{\min}}{m} \right)^2 \simeq 0.05
\]

We define the probability of the two-body (two-jet) decays as follows:

\[
\beta[t \rightarrow jj] = \Gamma(t \rightarrow jj)/\Gamma(t \rightarrow uab)
\]

(15)
where $a$ and $b$ are two photons, gluons or light quarks. Then, for decays (7) we get:

\[
\begin{align*}
&t \to u\gamma\gamma : \beta[t \to j(u\gamma) + \gamma] = (5/2)\delta \quad \simeq 0.13 \\
&t \to u\gamma g : \beta[t \to j(u\gamma) + j] = (5/4)\delta + (5/2)\delta^3(4 - 3\delta) \quad \simeq 0.07 \\
&t \to ugg : \beta[t \to j + j] = (5/4)\delta + (5/4)(1 - (1 - \delta)^2) \quad \simeq 0.3 \\
&t \to u\bar{q}q : \beta[t \to j + j] = 5\delta(1 - \delta) \quad \simeq 0.24 \\
&t \to u\bar{u}u : \beta[t \to j + j] = (20/23)\delta(6 - 7\delta + 2\delta^2) \quad \simeq 0.25
\end{align*}
\]

(16)

Thus, it follows from the estimates (16) that approximately 25% the case of decay of $t$-quarks (7) due to considered FCNC interaction can lead to observable two-body final states.

4 Conclusion

The contribution of dimension-seven operators to anomalous FCNC interactions of the $t$-quarks with a photon and a gluon is considered. A phenomenological Lagrangian of such an interaction and the corresponding Feynman rules are derived. There are evaluated the expressions for the widths of the FCNC decays of the $t$-quark to light $u$ or $c$ quarks and $\gamma\gamma$, $\gamma g$, $\bar{q}q$. It is shown that in a notable number of cases such decays of the $t$-quark due to dimension-seven operators can lead to observable two-body (a jet and a photon or two hadron jets) to final states.

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Appendix

Here we present the Feynman rules for the anomalous FCNC interactions of $t$-quarks due to dimension-seven operators. All vertices contain the common factor $\Lambda^{-3}$ and the notation for anomalous couplings is used:

$$\kappa_i = \xi^\gamma + \zeta^\gamma \gamma^5; \quad \lambda_i = \xi^g + \zeta^g \gamma^5$$

The interaction with a photon

$$e_a \kappa_1 \bar{p}_1^\alpha p_2^\beta [q^\mu g^{\alpha \nu} - q^\nu g^{\alpha \mu}] A^\alpha$$

$$e_a \kappa_2 \bar{p}_1 \left[ (q_1 + q_2)^2 g^{\alpha \beta} - q_2^\alpha (q_1 + q_2) - (q_1 + q_2)^\gamma q_1^\gamma \right] A_1^\alpha A_2^\beta$$

$$2e_a \kappa_3 \epsilon^{\mu \nu \alpha \beta} q_1^\alpha q_2^\beta A_1^\alpha A_2^\beta$$

The interaction with a gluon

$$g_s \lambda_1 \bar{p}_1^\mu p_2^\nu [q^\mu g^{\nu \alpha} - q^\nu g^{\mu \alpha}] B_1^\alpha$$

$$g_s W_{9g} B_{1a} B_{2b}; \quad W_{9g} =$$

$$\lambda_1 t^{ab} \left[ ((p_1 q_2) - (p_2 q_1)) g^{\alpha \beta} - p_2^\alpha p_1^\beta + p_1^\alpha p_2^\beta - q_2^\alpha p_1^\beta + p_2^\gamma q_1^\gamma \right]$$

$$\lambda_1 t^{ab} \left[ ((p_1 q_1) - (p_2 q_2)) g^{\alpha \beta} - p_2^\alpha p_1^\beta + p_1^\alpha p_2^\beta - q_2^\gamma p_1^\beta + p_2^\gamma q_1^\gamma \right]$$

$$\lambda_2 \left( \delta^{ab} / 3 + t_k d_{kab} \right) \left[ (q_1 q_2) g^{\alpha \beta} - q_2^\alpha q_1^\beta \right]$$

$$\lambda_3 \left( \delta^{ab} / 3 + t_k d_{kab} \right) \left[ (q_1 q_2) g^{\alpha \beta} + 2i t_k f_{kab} p_2^\alpha p_1^\beta \right] \epsilon^{\mu \nu \alpha \beta} q_1^\alpha q_2^\beta$$

$$e_a \kappa_1 \bar{p}_1 \left[ ((q_1 + q_2) q_2^\gamma q_1^\gamma - q_2^\gamma (q_1 + q_2)^\gamma \right] A_1^\alpha B_{2b}^\beta$$

$$e_a \kappa_2 \bar{p}_1 \left[ (q_2 q_1) g^{\alpha \beta} - q_2^\alpha q_1^\beta \right] A_1^\alpha B_{2b}^\beta$$

$$e_a \kappa_3 \epsilon^{\mu \nu \alpha \beta} q_1^\alpha q_2^\beta A_1^\alpha B_{2b}^\beta$$