Mobility operator resource-pooling contract design to hedge against network disruptions

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Abstract
Public transportation delays due to systematic failures have a major impact on network users. We propose designing capacity pooling contracts to facilitate horizontal cooperation among operators to mitigate those costs and improve service resilience. When two or more public transport providers agree upon sharing resources, the total transportation costs can be reduced due to added flexibility in the system. These operators may contribute capacity to be used in cost-effective routes owned by other operators. We formulate a two-stage stochastic model to determine the cost savings under different collaboration scenarios. We provide several solution methods: a deterministic equivalent problem, the L-shaped method, and sample average approximation. Coalitional stability under Shapley value, nucleolus, and τ-value are tested. The proposed model is applied to a regional multimodal network in the Randstad area of the Netherlands, for four operators, 80 origin-destination pairs, and over 1400 links where disruption data is available. Using the proposed method, we identify stable cost allocations among four operating agencies that could yield a 44% improvement in overall network performance over not having any risk pooling contract in place.

Keywords: Two stage-stochastic programming, network disruptions, cost allocation mechanisms, horizontal collaboration, multicommodity flow problem

1. Introduction

Mobility services face the uncertainty of disruptions that can cause significant costs and delays. These disruptions are caused by non-recurrent events (natural or man-made) that reduce transportation supply and can cause breakdown conditions on other parts of the network. In large multimodal urban networks, the lack of coordination can lead to significant financial risks and needs to be addressed beforehand to mitigate them. For example, Hurricane Sandy caused $400 million to the U.S. public transit system (Levin, 2012) in which 25% of the total cost was from capital expenses.

These disruption costs may be mitigated with capacity available from other parts of the system (Pender et al., 2013), or from other operators in a multimodal environment or Mobility-as-a-Service (MaaS) platform. Examples include redirecting buses from one line to serve a degraded line (e.g. bus bridging and shuttle planning: Hu et al., 2016; Jin et al., 2016, Van der Hurk et al., 2016; Zhang and Lo, 2020), or for Mobility-on-Demand fleets to provide coverage during disrupted service (e.g. Tyndall, 209; Fang et al., 2020). In this manner, risks of service degradation due to disruptions are reduced by having multiple operators pool their resources together. As multi-
modal travel becomes essential in large megacities, the benefits of resource consolidation can provide a much better service with fewer delays and disruptions. Such a multi-modal system requires operators to agree to a contract in advance, which offers a new design problem for mobility operators (Hensher, 2017).

The research question that follows is how to design such a contract between mobility operators such that they are incentivized to commit a required amount of capacity? Without pre-disruption agreements, mobility operators may choose not to share resources post-disaster or, worse still, may even leverage the tragedy to gain further profits (Hawkins, 2018). We propose a resource-sharing insurance contract mechanism designed to pool the resources of transportation operators (public transit operators, shared-use mobility providers, taxi companies, etc.) to hedge against disruptions.

The novel problem addressed in this study is distinctive in that the value of a coalition of mobility operators serving in a capacitated multicommodity flow network is determined by a two-stage stochastic program, which has not been studied before. In the first stage, the model determines the optimal amount of resources that each operator needs to contribute to a pool such that the second stage disruption scenarios allow disrupted operators to draw from that pool. These contributions are measured in capacity units (either relocating vehicles to other links or providing part of their infrastructure to accommodate multimodal trips). The model is coupled with different alternative cost allocation mechanisms to identify conditions needed to ensure stability of the contract. We propose solution algorithms using L-shaped method common to stochastic programming and sample average approximation method (SAA) to reach satisfying solutions.

Computational tests are conducted with the model and algorithms in different-sized instances. For the larger network instance, we evaluate a four-agency insurance contract for the urban multimodal network in the southern ring (Zuidvleugel) of the Randstad area in Netherlands where real disruption data is available. The multimodal case study network offers an interesting case because it includes multiple operating agencies that operate different fleet types and the results provide a characterization of their bargaining power in setting up the contract.

The rest of the paper is organized as follows. Section 2 provides an overview of the literature on two-stage stochastic network problems and game-theoretic profit allocation methods. Section 3 describes the formulation of the two-stage stochastic programming model and solution algorithms. In Section 4, we provide illustrative examples and discuss potential shortcomings of simple allocation mechanisms to further show how cost allocation mechanisms impact the design. We also provide a computational evaluation of the proposed solution methods. Section 5 demonstrates the use of our contract design model in a real case study using data from the multimodal Randstad Zuidvleugel network. Section 6 discusses the key findings and offers concluding remarks and potential future research directions.

2. Literature review

Capacity planning is critical for the robustness of transportation networks that face major disruptions (Chen et al., 1999; Cats and Jenelius, 2015). Disaster mitigation against these disruptions has been studied extensively in the context of pre-disaster relief planning. Solution strategies include retrofitting or allocating reserve capacity in anticipation of disruptions (Miandoabchi and Farahani, 2011; Wang et al., 2015). The multicommodity flow problem was first used in Haghani and Oh (1996) to model disaster relief planning as a large-scale deterministic time-space network. The two-stage stochastic programming approach has been used extensively...
in pre-disaster relief network planning (Barbarosoğlu and Arda, 2004; Liu et al., 2009; Rawls and Turnquist, 2010; Peeta et al., 2010; Noyan, 2012; Hong et al., 2015; Klibi et al., 2018; Elçi and Noyan, 2018). A more comprehensive account of two-stage stochastic problems in disaster relief network planning can be found in Grass and Fischer (2016). Common solution methods used in these problems include the L-shaped method for two-stage stochastic programming (e.g. Liu et al., 2009; Rawls and Turnquist, 2010; Miller-Hooks et al., 2012) as well as Monte Carlo-based sample average approximation of the disruption scenarios (e.g. Chen and Yang, 2004; Peeta et al., 2010; Miller-Hooks et al., 2012; Chow and Regan, 2014).

The underlying problem in those studies assumes a single centralized decision-maker. On the contrary, many systems are operated by multiple co-existing operators (Chow and Sayarshad, 2014) in which a centralized operation cannot be assumed, particularly in multimodal networks (see Rasulkhani and Chow, 2019) or MaaS platforms (see Pantelidis et al., 2019; Ma et al., 2019). There are studies looking at cascading failures in interdependent systems to measure the effects of one system on another (e.g. traffic-electric interactions: Fotouhi et al., 2017). However, there are no studies that deal with design of incentives for multiple transport operators to collaborate in anticipation of disruptions. Game-theoretic models have been used in disaster planning, although the players other than the network decision-maker involve either an attacker (Jin et al., 2015) or an “evil entity” that represents worst case disasters (Bell et al., 2008), leading to a class of network retrofit models called fortification problems under interdiction (e.g. Church and Scaparra, 2007) or network fortification games (Smith and Lim, 2008). We need cooperative game methods that deal with coalition formation as part of horizontal collaboration (see Doukidis et al., 2007) between multiple operators.

To establish a successful mechanism, the joint benefits of collaborating among the members of the collaboration should be distributed in a stable manner (Özener and Ergun, 2008). There are several approaches in the literature to ensure stability of horizontal collaborations. Determining the allocations obtained for participating in a resource pooling contract is just as important as the cost savings estimation model itself. Depending on the value of these allocations, operators may be incentivized to participate in a risk-pooling contract or choose to abstain. Different allocation rules may result in different payoffs for cooperating operators. Myerson (1980) defines these allocation rules as follows:

Definition 1. An allocation rule is a function $X: S \rightarrow \mathbb{R}^{|F|}$ mapping each coalition structure $s \in S$, where $S$ is the set of all possible coalitions, onto a payoff allocation:

$$X(s) = (X_1(s), X_2(s), ..., X_{|F|}(s))$$

$X_f(s)$ is the payoff allocated to operator $f \in F$, under the coalitional structure $s \in S$. $V(s)$ is the characteristic function payoff.

In the equal-gains allocation scheme, we aim to provide the same satisfaction level to all operators. In the proportional method, the satisfaction of operators is proportional to their pool contribution. The total savings CS$(s)$ are the same as before. The core (Gilles, 1953) is a cost allocation concept that ensures no player in a coalition would break away. For example, any game that is convex has a non-empty core and therefore a stable allocation solution. The unique stable set of the convex game coincides with the core. Another mechanism is the Shapley value (Shapley, 1951). The Shapley value is a merit-based allocation mechanism that determines the value of each players contribution to the coalition. It expresses the core center of gravity in convex games. Other mechanisms include the $\tau$-value proposed by Tijs and Driessen (1986), and the nucleolus Schmeidler (1969). The equal satisfaction allocation, which is non-CGT, may not always belong
within the core. Only cooperative game-theoretic methods will always belong in the core (when it is non-empty) and consequently guarantee stability and fairness of a coalition. In general, transportation games are superadditive but not convex (Tuljak-Suban, 2018).

The simple cost allocation mechanisms (equal, proportional, etc.) have been used in the literature, as summarized in Kolker (2017). Overall, these types of simple allocation rules do not guarantee a fair and equitable distribution of the attained benefits of the collaboration (Cruijssen et al., 2007; D’Amours and Rönnqvist, 2010). For example, Schotanus et al. (2008) illustrate how the Equal Price method may lead to unfair allocations for larger purchasing organizations. Also, proportional allocation schemes may be easy to implement but they are not considered stable (Özener and Ergun, 2008). Cost allocations based on a core are used as side payments in Agarwal and Ergun (2008) to incentivize cargo ship operators to consolidate their capacities to transfer goods more efficiently. Özener and Ergun (2008) proposed several cost-allocation schemes related to the core that were also applied to shipping collaborations.

The Shapley value has been considered as a payoff allocation mechanism in resource pooling contracts (Reinhardt and Dada, 2005). Lozano et al. (2013) use a minimum flow problem to estimate cost savings for every sub-coalition of operators. These savings are allocated using such allocation methods as Shapley value, nucleolus and \(\tau\)-value. Kellner and Otto (2012) also use a similar approach in allocating \(CO_2\) emissions of different shipments in a road transport route.

None of the cooperative game theoretic approaches have considered multimodal urban transportation systems under a two-stage stochastic programming setting.

Our contract design approach uses a multicommodity flow problem to model costs in the transportation system for a given coalition and allocates capacities based on savings achieved from horizontal cooperation between operators (capacity pooling). It shares some commonalities with Lozano et al. (2013). In that study a core transportation model is solved multiple times to obtain the potential cost savings solution for every sub-coalition of operators. Finally, these savings are allocated using cooperative game theoretic (CGT) allocation methods such as Shapley value, nucleolus and \(\tau\)-value. A key difference between the two studies is that Lozano et al. (2013) is a purely deterministic model. We use a two-stage stochastic model that captures the stochasticity of capacities that are subject to disruptions.

### 3. Proposed methodology

#### 3.1 Problem statement

A group of operators \(F\) own and operate links \(A_f, A = \bigcup_{f \in F} A_f\), in a network \(G(N,A)\) that serve passengers corresponding to a set of origin-destination (OD) pairs \(S\). The links have capacities that are transferable, e.g. service lines with vehicle frequencies that may be reassigned to serve another line. This may also represent shared mobility services in which an effective capacity can be assigned to an OD pair (e.g. a carshare network can be modeled as a complete graph from which steady state capacities can be obtained for each OD pair). The network is subject to disruption scenarios \(\omega \in I\) where one or more links are disrupted, i.e. their capacities are dropped to zero. The model can be trivially extended to consider intermediate capacity degradations instead (see Chow and Regan, 2014).

The contract design problem for capacity sharing is to determine the amount of capacity \(b_f\) that each operator \(f \in F\) is willing to contribute to a pool. A contribution of \(b_f = 0\) implies the operator \(f \in F\) does not participate in the pool. In the event of a disruption, an impacted operator
\( f \in F \) can freely borrow capacity \( e_{a}^{f} \) to link \( a \in A_{f} \). The borrowed capacity comes from capacities \( g_{a} \) at links \( a \in A_{f}' \) operated by other operators \( f' \in F \backslash f \). The capacity sharing agreement assumes that the savings from the capacity sharing agreement is transferable to the coalition members. In that case, stability of a capacity sharing agreement is determined using one of three alternative cost-sharing mechanisms: Shapley value, nucleolus, and \( \tau \)-value. An empty set implies an unstable contract for that coalition under that mechanism.

### 3.2 Proposed two-stage stochastic program formulation

The capacity pooling problem is defined as a two-stage stochastic linear problem with fixed resources. The goal is to minimize the expected total flow costs given a disruption scenario \( \omega \in I \), which can indicate the failure of multiple links (and model correlated network disruptions: see Lo and Tung, 2003; Sumalee and Watling, 2008; Chow and Regan, 2014).

The stochastic program is divided into two stages. The first stage decision variables \( b^{T} = (b_{1}, b_{2}, \ldots, b_{|F|}) \) denote the resource contributions to the pool by each operator \( f \in F \) and are made prior to knowing the outcome of the random disruption scenario. The second stage variables are made after a random event is realized: flows \( x_{a}^{s} \) on links \( a \in A \) for OD \( s \in S \), capacity contributions \( g_{a}, a \in A_{f} \), made by operator \( f \), and excess capacity \( e_{a}', a' \in A_{f}' \), borrowed by operator \( f' \) for link \( a' \). The available capacity is bounded by the first stage decisions.

#### Notation

**Parameters**
- \( I \): finite probability space of disruption scenarios
- \( K \): set of discrete scenarios representing disruption space \( I \)
- \( F \): set of operators
- \( G(N, A) \): network \( G \) of nodes \( N \) and links \( A \) which can be separated into disjoint sets \( A_{f}, f \in F \)
- \( S \): set of OD pairs
- \( \xi(\omega) \in \mathbb{Z}_{2}^{|A|} \): indicator \((0,1)\) vector of length \( |A| \) corresponding to link disruptions in scenario \( \omega \in I \)
- \( c_{a} \): travel cost on link \( a \in A \)
- \( d^{s} \): demand amount of O-D pair \( s \in S \)
- \( O(s), D(s) \): origin and destination nodes of \( s \in S \)
- \( w_{a}(\omega) \): capacity at link \( a \in A \) under scenario \( \omega \in I \)
- \( M \): big \( M \) notation
- \( E_{\xi} \): expectation across random events \( \xi \)
- \( N_{i}(+) \): set of links inbound to node \( i \)
- \( N_{i}(-) \): set of links outbound from node \( i \)
- \( p_{k} \): probability of discrete scenario \( k \in K \) occurring

**Decision variables**
\( x^s_\alpha \): flow on link \( \alpha \in A_f \) for OD pair \( s \in S \)

\( e_\alpha \): capacity allocated to link \( \alpha \in A_f \) for operator \( f \in F \)

\( g_\alpha \): capacity contributed from link \( \alpha \in A_f \) by operator \( f \in F \)

\( b_f \): capacity contributed by operator \( f \) to the pool

Following the above notation, the formulation of the two-stage stochastic programming problem is shown in Eqs. (1) – (9). Eqs. (1) – (2) comprise the first-stage problem and (3) – (9) provide the formulation of the second-stage problem.

\[
\Phi(F) = \min_b E_\xi Q(b^T, \xi(\omega))
\]

subject to

\( b \geq 0 \) \hspace{2cm} (2)

where

\[
Q(b^T, \xi(\omega)) := \min_x \sum_{s \in S} \sum_{\alpha \in A} c_\alpha x^s_\alpha
\]

Subject to

\[
\sum_{\alpha \in N_i(\ast)} x^s_\alpha - \sum_{\alpha \in N_i(\ast)} x^s_\alpha = \begin{cases} 
  d^s & \text{if } i = O(s) \\
  -d^s & \text{if } i = D(s) \\
  0 & \text{otherwise} 
\end{cases} \hspace{2cm} \forall i, s \hspace{2cm} (4)
\]

\[
\sum_{s \in S} x^s_\alpha \leq w_\alpha(\omega) + e_\alpha - g_\alpha \hspace{2cm} \forall \alpha \in A_f, f \in F \hspace{2cm} (5)
\]

\[
\sum_{\alpha \in A_f} e_\alpha \leq \sum_{f \in F \setminus \{f\}} b_f \hspace{2cm} \forall f \in F \hspace{2cm} (6)
\]

\[
\sum_{g \in A_f} g_\alpha = b_f \hspace{2cm} \forall f \in F \hspace{2cm} (7)
\]

\[
x^s_\alpha \geq 0 \hspace{2cm} \forall \alpha \in A_f, f \in F, s \in S \hspace{2cm} (8)
\]

\[
e_\alpha, g_\alpha \geq 0 \hspace{2cm} \forall \alpha \in A_f, f \in F \hspace{2cm} (9)
\]

The objective value (1) is the minimization of the total expected flow costs over a random disruption event \( \xi \). The function \( Q \) is a recourse function or expected second-stage value function. \( \Phi(F) \) represents the cost achieved through the horizontal collaboration of all network operators denoted by the set \( F \). Eq. (2) imposes non-negativity constraints on first-stage capacity commitments \( b \). The second-stage objective (3) is the minimization of total flow costs for a realization of \( \xi(\omega) \), where \( \omega \in I \). Constraint (4) expresses the flow conservation constraints for the second-stage problem. Constraint (5) denotes the link capacity \( w_\alpha(\omega) \) for a realization \( \omega \) where
\( w_a(\omega) = 0 \) if \( \xi_a(\omega) = 1 \), which can be expanded using excess capacity \( e \) or removed from the current link to lend to other links using contributed capacity \( g \). Constraint (6) provides an upper bound on resource available for each operator. Operators have no incentive to borrow from their own committed capacity by design. Constraint (7) is a balance condition that requires pool contributions to originate from operators link capacities. Finally, constraints (8) – (9) enforce non-negativity for all second-stage decision variables. The objective value without any pooling minus Eq. (1) indicates the cost savings which translate to the value of the coalition, which is used for determining the stability of the contract agreement.

The computational complexity of solving a multicommodity flow problem on a directed graph as a linear program has a complexity of \( O(n^4) \), where \( n \) is the number of nodes. When considering disruption probabilities for each of \( m \) links, the computational complexity increases to \( O(2^m n^4) \).

3.2 Solution methods

The problem grows in size very quickly. For example, a network with four independent link disruption events would have \( 2^4 = 64 \) scenarios, while 10 links would have 1024 scenarios. We investigate three solution methods for the proposed model in Eq. (1) – (9). First, we provide a brief overview of the deterministic equivalent problem (DEP) that provides an equivalent to the two-stage stochastic model in the form of a LP formulation (section 3.2.1). Second, we design an L-shaped method to solve the DEP using decomposition (section 3.2.2). The method uses sub-problems to generate feasibility and optimality constraints until it converges to the optimum. Both methods require scenario enumeration which can be very computationally costly. We investigate using sample average approximation method (SAA) based on Monte Carlo simulation (section 3.2.3) to focus on a restricted scenario set. The SAA does not require scenario enumeration and is appropriate for large network instances.

3.2.1 Deterministic Equivalent Problem (DEP)

The deterministic equivalent problem (DEP) associated with Eqs. (1) – (9) can be formulated when the outcome space \( I \) is modeled with a discrete set of scenarios \( K \) with probabilities \( p_k \) associated with each discrete scenario \( k \in K \). Given a number of finite scenarios \( k = 1, 2, \ldots, |K| \) the expected second-stage value function can be expressed as the linear weighted expectation of independent scenario outcomes:

\[
E_{\xi} Q(b^T, \xi(\omega)) = \sum_{k=1}^{K} p_k c^T x
\]  

Eq. (10.1) represents the total expectation over a finite number of scenarios \( K \). The second-stage decision variables are expanded into the scenario dimension. The complete formulation of the DEP is included in the Appendix.

3.2.2 The L-shaped method

The DEP can be solved using a decomposition method commonly known as the L-shaped method because of the block structure of \( K \) independent scenarios. This decomposition method was introduced by Van Slyke and Wets (1969) to solve linear stochastic programs and was shown to greatly reduce computational efforts required to generate a solution. An illustration of the block structure in a stochastic program is shown in Figure 1, where \( A \) is the first stage constraints, and
the second stage constraints can be divided into a scenario dependent portion \( T_k, k \in K \), and an independent portion \( W \).

![Block structure of the L-shaped method. (source: Hoppe, 2007)](image)

Figure 1. Block structure of the L-shaped method. (source: Hoppe, 2007)

We provide a summary of the notation and the L-shaped algorithm.

**Algorithm Notation**

**Parameters**
- \( l, z, m \): feasibility cut, optimality cut and algorithm iteration step
- \( D, d \): Feasibility cut matrix coefficient and RHS bound vector
- \( R, r \): Optimality cut matrix coefficient and RHS bound vector
- \( c^T \): Second-stage cost vector
- \( p_k \): Probability of scenario \( k \)
- \( W \): scenario independent portion of second-stage problem coefficient matrix
- \( T_k \): scenario dependent portion of second-stage problem coefficient matrix
- \( h_k \): RHS bound vector of second-stage problem

**Decision variables**
- \( \theta \): Approximation of the second-stage objective function \( E_\xi Q(b^T, \xi(\omega)) \)
- \( b \): First-stage decision variable vector (capacity contributions)
- \( v^+, v^- \): Feasibility cut sub-problem decision variables
- \( x \): Second-stage decision variables
- \( \sigma^m \): Lagrange multipliers obtained from sub-problem \( J_k \) at iteration \( m \)
- \( \pi^m_k \): Lagrange multipliers obtained from sub-problem \( \hat{J}_k \) at iteration \( m \)

The method, shown in **Algorithm 1**, exploits the block structure of the DEP problem of the previous section 3.2.1 to solve the problem in an iterative manner. The master problem includes all first-stage decision variables and constraints (step 4). The second-stage objective \( E_\xi Q(b^T, \xi(\omega)) \) is substituted by the decision variable \( \theta \). The algorithm adds optimality cuts (steps
Algorithm 1. L-shaped method (source: Van Slyke and Wets (1969))

1. Set $l, z, m, d, r = 0$ and $D, R = 0_{[b]}$
2. $m = m + 1$
3. If $l = 0$: set $\theta = 0$ and $\theta^m = -\infty$
4. Solve \{min $J := \theta |Db \geq d, Rb + \theta \geq r, b \geq 0, \theta \in R$\} %Optimal solution is $(b^m, \theta^m)$
5. For $k = 1, \ldots, K$
6. Solve \{min $\tilde{f}_k := \sum_{i=1}^{[h_k]} v_i^+ + v_i^- \ |Wx + v^+ - v^- = h_k - T_k b^m, x, v^+, v^- \geq 0$\}
7. If $\tilde{f}_k > 0$ :
8. Add vector row $(\sigma^m)^T T_k$ to matrix $D$ and scalar $(\sigma^m)^T h_k$ to vector $d$
9. Set: $r = r + 1$ and go to step 2
10. For $k = 1, \ldots, K$
11. Solve \{min $\tilde{f}_k := c^T x \ |Wx = h_k - T_k b^m, x \geq 0$\}
12. Add vector row $\sum_{k=1}^K p_k (\pi_k^m)^T T_k$ to matrix $R$ and scalar $\sum_{k=1}^K p_k (\pi_k^m)^T h_k$ to vector $r$
13. Set $z = z + 1$ and $J^m = \sum_{k=1}^K p_k (\pi_k^m)^T h_k - \sum_{k=1}^K p_k (\pi_k^m)^T T_k b^m$
14. If $\theta^m \geq J^m$: terminate else go to step 2

Matrix $W$ corresponds to second-stage variable coefficients and $T_k$ to first-stage variable coefficients of Eqs. (4) – (9).

3.2.3 Sample average approximation method (SAA)

To further enhance the efficiency of the method, we consider Monte Carlo simulation to obtain a sample $L$ of the scenarios $K$, known as sample average approximation method (Shapiro and Phipott, 2007). This can be run in combination with L-shaped method (e.g. Miller-Hooks et al., 2012) or with DEP. This method works best for cases where the total number of scenarios is very large or even infinite. We can generate a sample: $\tilde{\xi}(1), \ldots, \tilde{\xi}(L)$ of $L$ replications where $j = 1, \ldots, L$ from the random vector $\xi$. The sample mean is shown in Eq. (11).

$$\min_b \left\{ \tilde{Q}_L(b) := \frac{1}{L} \sum_{j=1}^L Q(b, \xi(j)) \right\}$$

3.3 Cost allocation mechanisms used

After determining the commitments of the operators, the stability of the coalition is determined using one of several alternative cost allocation mechanisms reviewed in Section 2. The equal-gains allocation mechanism requires computing the total savings. The total savings are calculated as: $CS(V) = \Phi(\emptyset) - \Phi(V)$, where $\Phi(V)$ is the objective value of (1) that is accompanied with
additional constraints as shown in Algorithm 2. \(\Phi(\emptyset)\) reflects stand-alone operations where operators are not sharing resources. Then, the allocated payoff for each operator is the same and given by Eq. (12).

\[
X_f^{eq} = \frac{CS(V)}{|F|}
\]  

(12)

In the proportional cost allocation mechanism, the satisfaction is given by the proportion contributed by each operator, i.e. Eq. (13).

\[
X_f^{pr} = \frac{b_f}{\sum_{f \in F} b_f} CS(V)
\]  

(13)

As for the Shapley value, a value function needs to be computed. The values of each coalition subset in the contract design problem are computed using Algorithm 2 adapted from Shapley (1951) to our model. In step 2, we compute all sub-coalitions \(V\) where the number of sub-coalitions is \(|F|!\). In steps (4) – (6) we iteratively solve the cost-savings resource pooling model of Eq. (1) – (9) \(|F|!\) times to compute the potential savings for sub-coalition \(V \in V'\). The constraint at step 5 along with constraints (10) and (11) block players outside the coalition \(V\) from using pooled resources.

Algorithm 2. Calculate sub-coalition payoffs

1. Begin
2. Compute the set of all possible sub-coalitions \(V'\)
3. For sub-coalition \(V \in V'\): do:
   4. For any operator \(f \not\in V\) do:
   5. Add constraint to the problem: \(b_f, e_a, g_a = 0 \ \forall \ a \in A_f\)
   6. \(\Phi(V) \leftarrow\) solve problem (5.1) - (5.9)
7. Compute subsidies under allocation rule \(X(V)\)
8. End.

*\(|V'| = |F|!, \ 0 \leq |V| \leq |F|*

Having computed the values, Eq. (14) is used to obtain the Shapley value.

\[
Sh_f = \sum_{V \subset F} \left(\frac{(V - 1)! (F - V)!}{V!}\right) \left[\Phi(V) - \Phi(V - f)\right]
\]

(14)

This Shapley formula asserts that each player's total gain from a coalition structure is a weighted average of his contributions to all players in smaller coalition structures.

The nucleolus also lies in the core if that exists (Schmeidler, 1969). The nucleolus is calculated by finding the imputations that minimize the maximum dissatisfaction. The excess of Eq. (15) represents the dissatisfaction for the coalition \(V\):
\[ e(X, V) = \Phi(V) - \sum_{j \in V} X_j \] (15)

Also, the core of a game is expressed as a set of imputations: \( \sum_{i \in S} X_i \geq \Phi(V) \). The \( \tau \)-value is expressed as the unique efficient payoff on interval \([m(V), M(V)]\) (Tijs, 2003). \( M(V) \) is the marginal contribution allocation vector where each element \( M_i(V) \) expresses the marginal contribution of player \( i \) to the grand coalition:

\[ M_i(V) = \Phi(F) - \Phi(F/\{i\}) \] (16)

This vector is also called the set of Utopia payoffs. The minimum rights vector \( m(V) \) is:

\[ m_i(F) = \max_V (\Phi(V) - \sum_{j \in V \setminus \{i\}} M_j(V)) \] (17)

The concept of the minimum rights vector is quite similar to the nucleolus but instead of minimizing dissatisfaction, \( m_i(V) \) represents the least-amount that player \( i \) can ask in the grand coalition.

4. Model verification tests

We demonstrate model feasibility and performance using an illustrative example. We investigate the conditions under which operators will participate in a contract and discuss some key CGT concepts that provide meaningful insights. Finally, we report the results of several cost-allocation methods and discuss some interesting findings.

4.1. Illustrative example

Consider the following network instance. Table 1 reports the O-D demand patterns for the illustrative example shown in Figure 2 and Table 2 lists out network parameters.
Let us now consider disruptive events that can potentially disrupt service on links 12 and 23. Figure 2 shows the probabilities of disruption assumed on the links with an 80% chance of failing completely, i.e. full link closure. These risk profiles may correspond to link-specific disruption events such as signal failure, demonstrations, traffic accidents or a tree falling. These independent probabilities are then used to calculate the scenario tree that enumerates all possible outcomes as described in Birge and Louveaux (2011).

![Figure 2. Illustrative instance.](image)

| O-D | Demand |
|-----|--------|
| (1,2) | 40     |
| (2,3) | 60     |
| (1,4) | 3      |
| (2,4) | 10     |

| From node | To node | Travel cost | Capacity $[f_1, f_2, f_3]$ | Failure prob $[f_1, f_2, f_3]$ |
|-----------|---------|-------------|-----------------------------|---------------------------------|
| 1         | 2       | 2           | [4,0,0]                     | [0.8,0,0]                       |
| 2         | 1       | 2           | [0,0,50]                    | [0,0,0]                         |
| 1         | 3       | 7           | [0,15,80]                   | [0,0,0]                         |
| 3         | 1       | 7           | [0,0,0]                     | [0,0,0]                         |
| 1         | 4       | 10          | [0,0,10]                    | [0,0,0]                         |
| 4         | 1       | 10          | [0,0,0]                     | [0,0,0]                         |
| 2         | 3       | 3           | [0,4,0]                     | [0.8,0,0]                       |
| 3         | 2       | 3           | [0,15,30]                   | [0,0,0]                         |
| 2         | 4       | 4           | [0,0,30]                    | [0,0,0]                         |
| 4         | 2       | 4           | [0,0,0]                     | [0,0,0]                         |
| 3         | 4       | 3           | [0,0,0]                     | [0,0,0]                         |
| 4         | 3       | 3           | [0,0,20]                    | [0,0,0]                         |
4.1.1. L-shaped method illustration
We illustrate the single-cut L-shaped solution method in Algorithm 1. At iteration 1, set $\theta^1 = -\infty$. No feasibility cuts need to be generated since $\bar{J}(y, v^+, v^-) = 0$ for all $1 \leq k \leq K$. At step 3, $J^1 = 919.6 \geq \theta^1$, so add the following optimality cut to the master problem:

\[-2b(1) + 8 b(2) + 8 b(3) + \theta \geq 919.6\]

At iteration 2: $\bar{J}(y, v^+, v^-) = 4.0$ for scenario $k = 2$. Add feasibility cut to the master problem:

\[b(1) - b(2) + b(3) \geq -34.0\]

The algorithm generates a total of 13 optimality cuts and 8 feasibility cuts before it reaches the optimal objective value, $E_\xi Q(b^T, \xi(\omega)) = 318$. Table 3 illustrates the results of the resource pooling model obtained solving the L-shaped method. The results were verified by solving the deterministic equivalent program (DEP).

| Flow Variables X(link, O-D pair, operator) | Scenario #1 | Scenario #2 | Scenario #3 | Scenario #4 | Pooled capacity $(b(1), b(2), b(3))$ | Objective value |
|------------------------------------------|-------------|-------------|-------------|-------------|---------------------------------|-----------------|
| X[(1, 2), (1, 2), 1]                     | 40          | 40          | 40          | 40          |                                 |                 |
| X[(1, 2), (1, 4), 1]                     | 3           | 3           | 3           | 3           |                                 |                 |
| X[(2, 3), (2, 3), 2]                     | 60          | 60          | 60          | 60          | $(0,30,73)$                      | 318             |
| X[(2, 4), (1, 4), 3]                     | 3           | 3           | 3           | 3           |                                 |                 |
| X[(2, 4), (2, 4), 3]                     | 10          | 10          | 10          | 10          |                                 |                 |

The results in Table 3 suggest the capacity contributions of $(0,30,73)$ are needed to maintain optimal flows. Operator $f_3$ contributes the largest amount of capacity to the pool while $f_1$ does not contribute anything. Even if $f_1$ does not contribute any resources, the operator improves the value of the objective by being able to use the pooled resources and thus provide more capacity to its’ users.

4.1.2. Operator savings allocations
We now employ Algorithm 2 to calculate the cost-savings for each coalitional structure in our illustrative example. The results are given in Table 4. We use the measure of synergy to evaluate the importance of a coalition. The synergy of a coalition $V$ is given by the ratio of savings divided by the total costs shown in Eq. (18).

\[Synergy(V) = \frac{CS(V)}{\Phi(V)}\] (18)

The measure of synergy is very important in understanding the effectiveness of each coalition. It can be a very useful tool for government agencies when deciding which transportation providers to invite to take part in the mobility contracts.
Table 4. Coalition results

| Coalition V | 𝑉(𝑠) | 𝐶𝑆(𝑉) | Synergy(V) |
|-------------|------|-------|------------|
| {∅}        | 919.6| 0.0   | 0.00       |
| {1}        | 919.6| 0.0   | 0.00       |
| {2}        | 919.6| 0.0   | 0.00       |
| {3}        | 919.6| 0.0   | 0.00       |
| {12}       | 679.6| 240.0 | 0.35       |
| {13}       | 543.6| 376.0 | 0.69       |
| {23}       | 621.2| 298.4 | 0.48       |
| {123}      | 318.0| 601.6 | 1.89       |

Table 5 summarizes the results obtained under different allocation rules for the coalition of three operators {123}. While there is no ubiquitous “appropriate” allocation method, we consider the Shapley value to be an adequate indicator for resource pooling problems. Different allocation rules will distribute subsidy amounts differently to operators conforming to the criteria that are set by each allocation rule 𝑋(𝑉). Computations were performed in TUGlab (Calvo and Rodriguez, 2006) and MatTU Games (Meinhardt, 2020).

Table 5. Comparison of different allocation mechanisms

| Allocation rule Operator | Equal Satisfaction | Proportional contribution | Shapley value | Core center | Nucleolus | 𝜏 − value | Utopia |
|--------------------------|--------------------|---------------------------|---------------|-------------|-----------|-----------|--------|
| 1                        | 200.53             | 0.00                      | 203.73        | 203.15      | 206.93    | 199.36    | 303.20 |
| 2                        | 200.53             | 175.22                    | 164.93        | 136.90      | 129.33    | 144.48    | 225.60 |
| 3                        | 200.53             | 426.38                    | 232.93        | 261.55      | 265.33    | 257.76    | 361.60 |

*subsidy amounts are given in terms of travel cost savings

The game presented in this section is non-convex (supermodularity condition is violated) but superadditive and thus all CGT methods listed in Table 5 are core allocations that maintain stability of the capacity sharing agreement. For example, if the operators agree to a cost allocation of the benefits based on the amount of resources that each operator contributes (proportional contribution), then the allocation vector does not belong in the core. This method is especially unreliable since cost-savings model solutions may not be unique.

Figure 3 depicts the core of the game. Every payoff vector that lies in the core (dark polygon) is considered a fair and stable solution. The set of core points that also lie in the left-hand side of the large white triangle represent allocations where operator 2 receives no payoff but nevertheless are considered stable. For example, 𝑏^𝑇 = (0,0,207) is also an optimal solution to the problem (1)-(9) but the proportional contribution allocation is significantly different: (0,0,601.6) than Table 5.
4.2 Performance evaluation

We compare the performance of all three solution methods: L-Shaped method and DEP with enumerated scenarios, and SAA-DEP (the sample sizes are noted in Table 7). The computational results are obtained using Gurobi 9.0 in Python 3.7.3 on a 13” MacBook Air laptop i-7 laptop with 8GB RAM 1600MHZ-DDR3 and OS Catalina 10.15.4. To compare the performance of these solution methods we generate 6 random grid network instances. Table 6 provides a summary of parameters used to create these test instances. The instances can be found in Pantelidis (2020).

| Parameter          | Value                      |
|--------------------|-----------------------------|
| # of links         | $2(N - \sqrt{N})$           |
| $w_{ij}$           | Random INT[1, $N/5 + 1$]     |
| $|F|$                | $\sqrt{N}$                  |
| $c_{ij}$           | Random INT[0,100]           |
| # of scenarios     | $2 \sqrt{\pi + 4}$         |
| # of O-D pairs     | $\sqrt{N} + 4$              |
| $d^s$              | Random INT[0,100]           |
| $p_{ij}$           | Random [0.6,1]              |
Link operators and vulnerable links are assigned randomly, and their respective values are drawn from formulas listed in Table 6. For each O-D pair an alternative path is generated to ensure that flow constraints are met. The results are summarized in Table 7.

Table 7. Algorithm performance results

| Runtime  | L-shaped runtime | SAA runtime | SAA Opt. Gap | Sample size | Nodes | O-D pairs | Scenarios |
|----------|------------------|-------------|--------------|-------------|-------|-----------|-----------|
| 7 sec    | 44 sec           | 2 sec       | 0.03 %       | 100         | 16    | 8         | 256       |
| 19 sec   | 1.9 mins         | 2 sec       | 0.03 %       | 100         | 16    | 9         | 512       |
| 2.6 mins | 20.9 mins        | 27 sec      | 0.05 %       | 250         | 36    | 10        | 1024      |
| 12.3 mins| 47.4 mins        | 46 sec      | 0.02 %       | 250         | 36    | 11        | 2048      |
| 2.3 hours| 9.4 hours        | 2.4 mins    | 0.03 %       | 500         | 64    | 12        | 4096      |
| -*       | 13.7 hours       | 2 mins      | 0.05 %       | 500         | 64    | 13        | 8192      |

* Solver out of memory

The single-cut L-shaped algorithm becomes more efficient than the DEP once the number of scenarios grows sufficiently large. More scenarios cause a linear increase in the L-shaped runtime but an exponential increase in the DEP runtime. However, we should also mention that Python has difficulties handling a large number of iterations which justifies the large threshold (for 8192 or more scenarios) where the L-shaped algorithm becomes more efficient than the DEP. Overall, Gurobi solution times were very similar using different solving methods (simplex, dual simplex and Barrier method). The technological advances found in modern solvers (sparse matrices, multi-threaded computing and pre-solve methods) have improved the solution times of large linear programs. The SAA proves to be adequate for such problems both in solution quality (less than 0.1% optimality gap) and runtime. Based on the results reported in Table 7, the SAA can be solved quite efficiently alongside DEP when the sample size is sufficiently small. For very large sample sizes, the SAA can be solved instead using the L-shaped method.

5. Randstad network case study

We apply our modelling approach to design a contract between the operators of the public transport network of part of the Randstad Zuidvleugel region in the Netherlands (Figure 4). The Randstad Zuidvleugel is the southern ring of the Randstad and was used as a case study in Cats et al. (2016) to identify critical links and study disruption effects. The high demand intensity and the large number of multimodal routes imply that disruption costs will be significant for both travelers and operators. Yearly passenger disruption costs resulting from disruptions on one single light rail link in the case study network can exceed €900,000 (Cats et al., 2016). For this reason, we identify a need for operators to hedge the disruption risks and insure the most critical routes. For the Randstad network, a large dataset is available for the period between January 2011-August 2013.
5.1. Network operators

During the analysis period, the case study network consists of six different types of services that are ultimately owned by four transportation entities: Dutch Railways (NS), HTM, RET and Connexxion (now EBS). Table 8 presents a summary of transportation services provided in the case study network and Figure 5 presents a GIS figure of the 4 operators’ networks generated from GTFS data.

![Figure 4. The Randstad network. Source: (Cats et al., 2016)](image-url)

| Parameters                     | Operators | Dutch Railways | HTM | HTM Buzz | RET          | Connexxion |
|-------------------------------|-----------|----------------|-----|----------|--------------|------------|
| Capacity (1000 × passengers/hour) | NS-I  | 4.75           | 107.25 | 32.76 | 6.48          |            |
|                               | NS-L  | 2.50           | 23.2  |         |              |            |
| # of Failures (7-9AM) [min, average, max] | HTM | 0.0023, 0.023, 1 | 0.0392, 0.128, 3 | 0.0025, 0.025, 1 | 0.0011, 0.1 |
| Average link frequency (trips per hour) | HTM Buzz | 0.8            | 9.43   | 8.15     | 3.66          | 18.0       |
| Demand (1000 × passengers/hour) | RET | 5.5            | 193.3  | 8.0      | 4.5           |            |
| Total network length (km)      | Connexxion | 131.76        | 298.82 | 72.44 | 32 |
Figure 5. GIS map of the four operators for the case study from GTFS data.

The network shown in Figure 5 includes two train services (NS-I, NS-L) that belong to the national railway company NS (Dutch Railways), the Hague-based HTM urban rail (tram and light rail) service that also owns an urban bus fleet that until 2019 operated under the HTM-Buzz flag, the Rotterdam-based RET that runs the metro/light rail service between Hague and Rotterdam and a fleet of buses. Finally, the international company Connexxion owned by Transdev owns a bus fleet that provides service between Delft and Zoetermeer. It is now EBS.

Some of these transportation providers can be considered part of larger transportation agencies in terms of strategic planning and decision making. The intercity train service (NS-I) is operating between Leiden Central, The Hague, Rotterdam and Gouda. The local train service (NS-L) includes all local train services between the same stations. All train services are cordoned at Leiden Central / Gouda / Rotterdam Central. The NS (Dutch Railways) is responsible for operating both the intercity and local services in the case study area and are illustrated in Figure 4 (dashed and solid black lines).

5.2. Network characteristics

Within this area there is a high-density public transport network, consisting of train, metro, light rail, tram, regional and urban bus services that serve more than 400,000 commuters daily. An origin-destination matrix was approximated by a combination of empirical data (passenger counts,
smart card records), transport demand model estimations and for certain routes for which no information was available estimated based on the available capacity and assumed occupancy rates.

Passenger demand and service supply correspond to the 2-hour AM peak period (7-9 a.m.). Link capacity is defined as the product of the crush capacity per public transport vehicle and the number of public transit trips within the AM period. We assume that links are unidirectional and travel costs are calculated according to average running time of link services.

We compute coalition values, pool contributions and subsidies for the four operators present in the case study network. For this experiment we use the 80 highest-volume O-D pairs with a total demand amounting to 223,800 passengers/hour for the entire region. Some of these pairs have unique transit network paths to their destination which may be disrupted leading to infeasible solutions due to violating constraint (5). To account for that, we assume that these travelers can also use alternative modes of transportation such as carpooling, bike, taxi, etc. These alternatives are presented as additional un-capacitated direct-connection links for each network O-D pair. The cost of these links will be calculated as:

$$\text{alt. mode cost} = 10 \times (\text{cost of shortest path})$$

Where the cost of shortest path is the minimum travel path of each pair in the Randstad transit network. Table 8 presents the basic parameters of the network that are used to run the computation experiments. We conduct 10000 randomized Bernoulli trials to estimate the average, minimum and maximum number of disruptions that may occur during the morning peak hours (7-9AM) by using the exposure time over the total operation time ratio. We can see that HTM is more vulnerable to disruptions which can be attributed to higher service frequencies. The demand in Table 8 represents the commuters’ willingness to use public transit services by identifying the least-cost route of each O-D pair. Total network capacity is 176.94 (1000 \times \frac{\text{passengers}}{\text{hour}}).

5.3. Disruption data

The network disruption data reflect the disruption exposure by calculating the expected time that a link is exposed to disruptions per time period (frequency \times duration) compared to the total time public transport services are provided on this same link. Based on this premise, failure probabilities are derived from the ratio: (exposure time / total operating hours). The network disruption data were obtained from Cats et al. (2016). The disruption log information was obtained from different service providers via an API. This data contained information about disruptions for a period of 2.5 years for the train network (NS-I, NS-L), and for a period up to 3 months for the tram and bus networks. Note that disruption data is for databases dating back to 2013. Since the network includes more than 1400 links, the number of scenarios is over $2^{1400}$ and hence network disruption effects can only be captured by Algorithm 2.

5.4. Results and analysis

In this section we present a summary of the computational results using SAA. The algorithm was coded in Python 3.7.3 using Gurobi 9.0. The capacity sharing model needs to be run for every sub-coalition, resulting in 4! iterations to obtain the total transportation costs for each. We limit the sample size of the problem to $N = 200$ (6GB memory was required to solve each optimization run). To increase the robustness of our estimation, we ran the SAA five times for each sub-coalition (5 \times 4! optimization runs in total) to obtain the average to compute the cost savings of each coalition. The procedure runtime for all runs exceeded 20 hours.
5.4.1. Coalition savings results for the baseline scenario

Table 9 provides a summary of the results. Since each iteration is solved multiple times (5 times), we report the average and standard deviation of the objective value. Cost-savings and synergy measures are computed using average values of $V(s)$.

| Coalition s | $V(s)_{avg, std \times 10^{-4}}$ | $CS(s)$ | Synergy(s) |
|------------|----------------------------------|--------|------------|
| {}         | [3.25, 21]                       | 0.0    | 0.00       |
| {1}        | [3.25, 21]                       | 0.0    | 0.00       |
| {2}        | [3.25, 21]                       | 0.0    | 0.00       |
| {3}        | [3.25, 21]                       | 0.0    | 0.00       |
| {4}        | [3.25, 21]                       | 0.0    | 0.00       |
| {12}       | [2.77, 12]                       | 0.479  | 0.173      |
| {13}       | [2.77, 7.2]                      | 0.480  | 0.173      |
| {14}       | [2.88, 7.1]                      | 0.373  | 0.129      |
| {23}       | [2.42, 8.4]                      | 0.830  | 0.343      |
| {24}       | [2.55, 17]                       | 0.702  | 0.275      |
| {34}       | [3.12, 1.6]                      | 0.129  | 0.041      |
| {123}      | [1.96, 7.4]                      | 1.294  | 0.660      |
| {124}      | [2.16, 2.9]                      | 1.096  | 0.507      |
| {134}      | [2.64, 1.8]                      | 0.608  | 0.230      |
| {234}      | [2.26, 4.3]                      | 0.993  | 0.439      |
| {1234}     | [1.81, 4.9]                      | 1.436  | 0.793      |

As shown in Table 9, a capacity pooling contract can improve network performance by almost 44% ($CS = 1.436$) which is remarkable. The measure of synergy is indicative of the efficiency of a coalition. For example, the coalition {123} has a synergy measure of 0.660 which is very close to that of the grand coalition {1234} that has a synergy measure 0.793. This measure can be a very useful tool for decision-making and can assist government agencies in identifying critical operators that can benefit horizontal cooperation schemes significantly.

After identifying these cost-savings coalitions, we report several cost-allocation methods that have desirable properties in terms of fairness and stability in Table 10. These values are reported in million passenger-minutes, which can then be monetarized by choosing an appropriate value of time parameter.

| Operator          | Allocation rule | Shapley value | Nucleus | $\tau - value$ | Utopia |
|-------------------|-----------------|---------------|---------|----------------|--------|
| Dutch Railways    |                 | 0.333         | 0.372   | 0.370          | 0.443  |
| HTM               |                 | 0.575         | 0.740   | 0.736          | 0.828  |
| RET               |                 | 0.317         | 0.252   | 0.237          | 0.341  |
| Connexxion        |                 | 0.212         | 0.072   | 0.094          | 0.142  |
Every allocation rule identifies HTM as the most critical operator in the network, followed by RET. As shown in Figure 5, HTM offers a highly versatile network with many redundant paths available. RET forms the main connection between Rotterdam and Den Haag. Those could be potential reasons for their bargaining power.

The game is non-convex since supermodularity is violated (e.g. $V(123) - V(12) > V(1234) - V(124)$). The game core is illustrated in Figure 6 along with its four edges. Every subsidy allocation combination that lies inside the core represents a stable outcome. The core provides unique insights in determining the strength of each sub-coalition. The 4-dimensional core can be further broken down in 3-demensional surfaces. In that case the coalition {124} which corresponds to a resource pooling contract between Dutch Railways, HTM and Connexion is the most effective and stable agreement. This sort of analysis can provide important insights into setting strategic goals and designing insurance contracts. It also shows which players (or operators) can find a common ground more easily than others.

Figure 6. Core results for the Randstad operators (baseline).

5.4.2. **Coalition savings results with capacity reduction**

In this section, we assume a second scenario where HTM operates on capacities degraded by 80% of the original in Table 8. The network parameters are given in Table 11, where the remaining 20% capacities are highlighted in bold.

Table 12 provides a summary of the results. The savings of the grand coalition are 40% of the total flow costs, compared to 44% in the base case. The capacity reduction for HTM accounts for 59% of total network capacity reduction (from 176.94 to 72.58) but total costs are reduced only by 1.5% (from 3.25 to 3.30 million passenger-minutes) implying that HTM does not contribute a great amount of resources to other operators. This observation conflicts with the fact that the company should receive the greatest amount of cost allocation to ensure a stable outcome.
Table 11. Randstad network parameters (capacity degradation)

| Parameters                   | Operators       | Dutch Railways | HTM   | HTMBuzz | RET   | Connexion |
|------------------------------|-----------------|----------------|-------|---------|-------|-----------|
|                              | NS-I | NS-L | HTM   | HTMBuzz |       |           |
| Capacity (1000 x passengers / hour) | 4.75 | 2.50 | 21.45 | 4.64    | 32.76 | 6.48      |
| # of Failures (7-9AM) [min, average, max] | [0.0.023,1] | [0.0.027,2] | [0.0.392,3] | [0.0.128,2] | [0.0.025,1] | [0.0.011,1] |
| Average link frequency (trips per hour) | 0.8 | 0.23 | 1.88  | 1.63    | 3.66  | 18.0      |
| Demand (1000 x passengers / hour) | 5.5 | 7.1  | 193.3 | 12.5    | 8.0   | 4.5       |
| Total network length (km) | 131.76 | 223.04 | 298.82 | 196.43  | 72.44 | 32        |

Table 12. Savings (in million passenger-minutes) and Synergy (capacity degradation).

| Coalition s | V(s) [avg, std x 10^-1] | CS(s) | Synergy(s) |
|-------------|--------------------------|-------|------------|
| {0}         | [3.30,7.6]               | 0.0   | 0.0        |
| {1}         | [3.30,7.6]               | 0.0   | 0.0        |
| {2}         | [3.30,7.6]               | 0.0   | 0.0        |
| {3}         | [3.30,7.6]               | 0.0   | 0.0        |
| {4}         | [3.30,7.6]               | 0.0   | 0.0        |
| {12}        | [2.83,8.8]               | 0.465 | 0.164      |
| {13}        | [2.82,7]                 | 0.478 | 0.169      |
| {14}        | [2.93,10]                | 0.372 | 0.127      |
| {23}        | [2.59,120]               | 0.705 | 0.276      |
| {24}        | [2.95,9.8]               | 0.349 | 0.118      |
| {34}        | [3.17,9.8]               | 0.128 | 0.040      |
| {123}       | [2.14,6.1]               | 1.154 | 0.537      |
| {124}       | [2.61,5.3]               | 0.686 | 0.262      |
| {134}       | [2.69,0.2]               | 0.607 | 0.225      |
| {234}       | [2.43,2.1]               | 0.871 | 0.358      |
| {1234}      | [1.99,5.1]               | 1.304 | 0.653      |

The reasoning behind a large cost allocation for HTM is due to its large demand. Since many travelers’ desire to use the HTM network for their itinerary, receiving capacity support from an insurance pool will improve passenger travel times.

From Table 13 we can see that the amount of cost allocation that HTM should be getting is significantly lower than before. This means that they would agree to a much lower portion of the savings due to reduced capacity. For example, based on the Nucleolus allocation vector we can see that the subsidy amount is reduced by 35% (from 0.740 to 0.483 million passenger-minutes).
### Table 13. Cost allocations under reduced capacity setting (in million passenger-minutes)

| Operator     | Allocation rule | Shapley value | Nucleolus | \(\tau - value\) | Utopia |
|--------------|----------------|---------------|-----------|------------------|--------|
| Dutch Railways |                | 0.323         | 0.362     | 0.345            | 0.433  |
| HTM          |                | 0.445         | 0.483     | 0.490            | 0.697  |
| RET          |                | 0.384         | 0.378     | 0.381            | 0.618  |
| Connexxion   |                | 0.151         | 0.079     | 0.087            | 0.149  |

The core also confirms that HTM has reduced bargaining power. For example, coalition \{124\} shown in Figure 7 is barely stable but also leads to increased bargaining power for other agencies (e.g. RET). Due to the reduced capacity, the second highest recipient of the cost allocation is shifted from Dutch Railways in the base setting (Table 10) to RET. This suggests that the reduced capacity from HTM is covered by RET which lends it more negotiating power in setting up the agreement. In general, the capacity degradation of HTM impacts more the stability of allocations and less the total savings achieved through resource pooling.

![Figure 7](image-url)

**Figure 7.** Core results for the Randstad operators under reduced HTM capacity.

### 6. Conclusion

With the emergence of a plethora of transportation services and the deregulation of traditional transport services, risk-pooling contracts will become vital in urban transit operations. New government policies should include subsidies as incentives for operators to share resources. In this study, we have shown that the benefits of such contracts can improve overall network performance by 44% and mitigate disruptions. A two-stage stochastic programming model was proposed in order to estimate transportation costs and the efficiency of three solution methods was evaluated. The performance results showed that the well-known L-Shaped method is suitable only for a large number of scenarios. Based on this observation, sample average approximation (SAA) was solved
with DEP in the numerical experiments since the chosen sample size was relatively small. In our model application, the SAA demonstrated sufficient cost-estimation robustness for the Randstad public transport network.

In this study, we introduce the measure of synergy and the concept of the core as tools to evaluate the value and stability of potential contracts among operators. The subsidy allocation policies proposed are based in cooperative game theory and have desirable properties such as fairness and stability for games with non-empty core.

There are several directions for future research. The current resource pooling formulation does not integrate subsidies into the pool contribution decisions. However, we are also looking into a closed-loop side-payment mechanism that addresses stability as an equilibrium concept. In this case no external subsidies will be needed to ensure contract stability. Instead, a side-payment mechanism between operators will be introduced to maintain the social optimum solution. There are also many other fields that this work can be applied to beyond urban transportation: freight, airlines, other two-sided markets, and other network flow games where resource pooling is critical to face demand/supply uncertainty.

APPENDIX: Deterministic Equivalent Problem (DEP) formulation

\[
\min_b \sum_{s \in S} \sum_{k \in K} \sum_{a \in A} p_k c_{a} x_{a}^{sk}
\]

Subject to

\[
\sum_{a \in N_i(+)} x_{a}^{sk} - \sum_{a \in N_i(-)} x_{a}^{sk} = \begin{cases} 
    d^s & \text{if } i = O(s) \\
    -d^s & \text{if } i = D(s) \\
    0 & \text{otherwise} 
\end{cases} \quad \forall i, s, k \in K \quad (A2)
\]

\[
\sum_{s \in S} x_{a}^{sk} \leq w_{a}^{k} + e_{a}^{k} - g_{a}^{k} \quad \forall a \in A_f, f \in F, k \in K \quad (A3)
\]

\[
\sum_{a \in A_f} e_{a}^{k} \leq \sum_{f \in F \setminus \{f\}} b_f, \quad \forall f \in F, k \in K \quad (A4)
\]

\[
\sum_{a \in A} g_{a}^{k} = b_f \quad \forall f \in F, k \in K \quad (A5)
\]

\[
b_f \geq 0 \quad \forall f \quad (A6)
\]

\[
x_{a}^{sk} \geq 0 \quad \forall a \in A_f, f \in F, s \in S, k \in K \quad (A7)
\]

\[
e_{a}^{k}, g_{a}^{k} \geq 0 \quad \forall a \in A_f, f \in F, k \in K \quad (A8)
\]
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