Liquid Energy Reduction with Centrifugal Rotating Motion

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Abstract. This article provides equation for calculating the pressure loss for a fluid and energy for a rolling ball moving in a logarithmic spiral from the action of inertial forces. Classical formulas give incorrect results when calculating coils with a changing twist radius. The calculation of the parameters of the fluid flow in the spiral bend and in the blades of the centrifugal impeller can now be determined only by numerical methods of calculation and, as such, formulas reflecting the energy losses associated with inertial forces do not yet exist. Based on the results of experimental and theoretical work, formulas were obtained that describe the energy losses of a liquid and a ball, when they move along a logarithmic spiral. For the correct derivation of formulas for a moving fluid, it was first necessary to obtain formulas for a rolling ball and a material point. The movement in a logarithmic spiral is the basis for the movement of particles in a vortex.

1. Introduction
In the turbulent motion of a liquid, local vortices are clearly traced, constantly arising and damping throughout the entire volume of the liquid. The movement of fluid in a local vortex occurs along a circular path from the periphery to the center or vice versa. The energy losses in the fluid during such a motion are associated, first of all, with the friction of the fluid particles with each other, as well as the losses due to inertial forces. In mathematical modeling, it is not customary to separate the inertial component of energy losses separately. When using empirical-theoretical formulas for determining energy losses when a fluid moves through a straight pipe, energy losses associated with inertial processes during vortex formation are not released. In this paper, the authors propose a physical model describing the energy loss of a liquid during its vortex motion, associated with inertial forces acting on a particle or a separate elementary volume of the liquid.

2. Currentness
The currentness of the work is due to the need to isolate potential energy losses during the vortex movement of the liquid, the movement of the liquid in the spiral outlet of a centrifugal pump, as well as in coils of variable radius.

3. Scholarly importance
Calculation of the parameters of the fluid flow in the spiral bend and in the blades of the centrifugal impeller can now be determined only by numerical methods of calculation and, as such, formulas reflecting the energy losses associated with inertial forces do not yet exist. The energy losses of the liquid, as shown by calculations and the results of experimental measurements, are of the same order of
magnitude as the friction losses along the length. Therefore, it is important to highlight them and be able to calculate. Energy losses in a vortex can now also be determined only using specialized programs. The authors seek to isolate energy losses during vortex motion of a liquid.

4. Research objective
For a coil made of a round pipe of a logarithmic structure, isolate the energy losses from inertial forces associated with the transition from one radius to another.

The model is based on the assumption that the trajectory of fluid motion during vortex motion follows a logarithmic spiral, since a very large number of natural processes are described by logarithmic or exponential laws. In order to build a physical model and obtain formulas for taking into account inertial losses during vortex motion, the authors created a special coil in the form of a logarithmic spiral (figure 1).

5. Theoretical part
The theory of calculating cylindrical coils of constant turning radius was also considered [1]. Classically it is considered that if the radius of the spiral coil twist or the radius of the pipe turn is more than five nominal diameters, then there are no local resistance losses. An example would be a conventional coiled coil with a constant turning radius, i.e. cylindrical coil. Thus, if the fluid flow turns over a large radius, i.e. the turn for the flow occurs rather slowly, then there is no circulation in the flow and, accordingly, there is no loss of potential energy for this circulation (vortex formation), and there are no energy losses associated with transitions from one radius to another. In the event that the coil will have a trajectory of a logarithmic spiral with a turning radius that varies exponentially

$$ R = R_0 \cdot \exp(b \cdot \varphi), $$

where $ R $ – current turning radius;
$R_0$ – initial turning radius;

$\varphi$ – angle of rotation from $R_0$ to $R$;

$b$ – swirl coefficient or logarithmic coefficient according to the formula (2), the formulas given in [1] do not give the correct result of potential energy losses.

Figure 2a shows the actual pressure loss in the expanding coil and the pressure loss calculated according to [1] and [2] at a constant radius equal to the outer radius (0.34 m) of the spiral coil. Figure 2b shows the actual pressure loss in the spiral coil in the form of a converging coil and the pressure loss calculated according to [1] and [2] at a constant radius equal to the outer radius (0.34 m) of the spiral coil. Figure 3a shows the actual pressure loss in the spiral expanding coil and the pressure loss calculated according to [1] and [2] at a constant radius equal to the inner radius (0.16 m) of the spiral coil. Figure 3b shows the actual pressure loss in the coiled coil and the pressure loss calculated according to [1] and [2] at a constant radius equal to the inner radius (0.16 m) of the coil.

Coefficient turn or logarithmic coefficient

![Graph of pressure loss](image)

| Graph Details | Equation |
|---------------|----------|
| a) expanding spiral | $R_F = 0.34$ m, $R_0 = 0.16$ m |
| b) tapering spiral | $R_F = 0.16$ m, $R_0 = 0.34$ m |

1 – actual measured pressure loss in the spiral coil $R_F = 0.34$ m, $R_0 = 0.16$ m and $R_F = 0.16$ m, $R_0 = 0.34$ m;

2 – pressure loss calculated according to [1] at $R = 0.34$ m; 3 – the same, according to [2]

**Figure 2.** Pressure loss in the helical coil and a coil of constant radius $R = 0.34$ m.
Figure 3. Pressure loss in the helical coil and a coil of constant radius $R = 0.16$ m.

$$b = \frac{1}{\varphi_F - \varphi_0} \cdot \ln \left( \frac{R_F}{R_0} \right),$$

where $R_F$ – final turning radius;

$\varphi_0$, $\varphi_F$ – respectively, the start and end angle of rotation for the radii of rotation $R_0$ and $R_F$.

The $b$ coefficient can be either positive or negative.

To create a mathematical model of the movement of a fluid along a logarithmic trajectory, the authors created a model of the movement of a material point. Below are the equations obtained by the authors to describe the kinematics of the ball’s motion.

Basic energy equation

$$m \cdot g \cdot \sin (\alpha) \cdot L = \frac{J_{\mu} \cdot \omega^2}{2} + \frac{J \cdot \omega^2}{2} \pm A_{in},$$

where $m$ – ball mass;

$g$ – acceleration of gravity;

$\alpha$ – the angle of inclination of the spiral to the horizon;

$L$ – distance traveled by the ball

$$L = R_0 \cdot \frac{1 + b^2}{b} \cdot [\exp(b \cdot \varphi) - 1];$$

$J_{\mu}$ – own axial moment of inertia of the ball;

$J$ – moment of inertia of a ball as a material point rotating around an axis;

1 – actual measured pressure loss in the spiral coil $R_F = 0.34$ m, $R_0 = 0.16$ m

and $R_F = 0.16$ m, $R_0 = 0.34$ m;

2 – pressure loss calculated according to [1] at $R = 0.16$ m; 3 – the same, according to [2]
\( \omega_{uu} \) – own angular velocity of the ball;
\( \omega \) – angular velocity of the ball as a material point rotating around an axis;
\( A_{in} \) – work of inertia force according to the equation
\[
A_{in} = \frac{2 \cdot m \cdot b}{\sqrt{1 + b^2}} \cdot \int v^2 \cdot d\varphi,
\]
where \( v \) – the absolute speed of the ball. the absolute speed of the ball.

The sign before the work of the inertial force depends on the type of spiral:
– for a tapering spiral \( (b < 0) \) the sign "-" is used;
– for an expanding spiral \( (b > 0) \), the "+" sign is used.

The solution to equation (3) is the absolute linear velocity function
\[
v^2 = \frac{10}{7} \cdot \frac{g \cdot \sin(\alpha) \cdot R_0 \cdot \sqrt{1 + b^2}}{(b - 2 \cdot A)} \cdot \left[ \exp(b \cdot \varphi) - \exp(2 \cdot A \cdot \varphi) \right].
\]

The angular velocity is determined by the equation
\[
\omega^2 = \frac{10}{7} \cdot \frac{g \cdot \sin(\alpha)}{(b - 2 \cdot A) \cdot R_0 \cdot \exp(2 \cdot b \cdot \varphi) \cdot \sqrt{1 + b^2}} \cdot \left[ \exp(b \cdot \varphi) - \exp(2 \cdot A \cdot \varphi) \right].
\]

The absolute linear velocity is the direction vector that describes the spiral path. This vector is displaced relative to the vector of tangential velocity by an angle.

Absolute linear acceleration
\[
a = \frac{5}{7} \cdot \frac{g \cdot \sin(\alpha)}{(b - 2 \cdot A)} \cdot \left\{b - 2 \cdot A \cdot \exp[(2 \cdot A - b) \cdot \varphi]\right\}.
\]

Angular acceleration
\[
\varepsilon = \frac{5}{7} \cdot \frac{g \cdot \sin(\alpha)}{(b - 2 \cdot A) \cdot R_0 \cdot \exp(b \cdot \varphi) \cdot \sqrt{1 + b^2}} \cdot \left\{-b - 2 \cdot (A - b) \cdot \exp[(2 \cdot A - b) \cdot \varphi]\right\}.
\]

When moving along a circle, the linear velocity vector, which determines the trajectory of motion, describes a circle and is tangent to this circle. When moving in a spiral, this vector deviates from the tangent to the circle by an angle \( \gamma \). Therefore, the work of centrifugal force takes on non-zero values. It is also worth noting that the instantaneous center of rotation (imaginary center of rotation) does not coincide with the geometric center of the spiral. Below are figures 4 and 5, explaining the directions of the vectors of velocities and accelerations, as well as their deviation by an angle \( \gamma \).
Figure 4. Velocity vectors of a ball moving in a tapering spiral.

Figure 5. Vectors of acceleration of a ball moving in a tapering spiral.

To check the correctness of the presented kinematics of a ball moving in a spiral, an experiment was carried out. The bottom line was to determine the distance of the ball departure from the chute, inclined to the horizon at an angle of 30º, located at the exit from the spiral.

For the considered spiral, the velocity at its exit according to equation (6) was:
- for a tapering spiral: 1.154 m / s;
- for expanding - 1.539 m / s.

For these speeds, the estimated distance of departure from the chute at an angle of 30º to the horizon:
- for a tapered spiral: 117 mm;
- for expanding – 209 mm.

The actual value of the ball protrusion was:
- for a tapered spiral: 118 mm;
- for expanding – 181 mm.

The error was:
- for a tapering spiral: 0.8%;
- for expanding – 15.5%.

Taking into account the fact that the gutter could have deviations of plus or minus 5º, the calculation results turn out to be quite correct.

Extending rotary motion along a logarithmic spiral in the case of fluid movement in a pipe with a circular cross-section, the following should be assumed:
- linear absolute \( v \) and tangential velocities \( v_{\tau} \) - a constant value due to the continuity equation for a stationary flow;
- the basic energy equation should be written in terms of pressure in the following form
\[-dP = dP_X \pm 2 \cdot \frac{\rho \cdot b}{\sqrt{1 + b^2}} \cdot v^2 \cdot d\varphi, \tag{10}\]

where \( dP \) – the pressure drop of the liquid between the end and the beginning of the spiral;
\( dP_X \) – friction pressure loss along the length of the spiral;
\( \rho \) – fluid density;
\( v \) – is determined by equation
\[ v = v_T \cdot \sqrt{1 + b^2} = \omega \cdot R \cdot \sqrt{1 + b^2}, \tag{11}\]

where \( v_T \) – tangential velocity which is determined by the equation
\[ v_T = \frac{Q}{S}, \tag{12}\]

where \( Q \) – liquid flow rate in a spiral pipeline;
\( S \) – cross-sectional area.

If the spiral is a pipeline, then \( P_X \) it is determined by the classical Darcy-Weisbach equation.

The loss of fluid pressure from the Coriolis force, as well as from the centrifugal force of inertia, is proposed to be determined by the equation
\[ P_{in} = P_{cor} = 2 \cdot \frac{\rho \cdot v^2}{\sqrt{1 + b^2}} \cdot \ln \left| \frac{R}{R_0} \right| = 2 \cdot \frac{\rho \cdot \omega^2 \cdot R^2 \cdot \sqrt{1 + b^2}}{\sqrt{1 + b^2}} \cdot \ln \left| \frac{R}{R_0} \right| = 2 \cdot \frac{\rho \cdot \frac{Q^2}{S^2} \cdot \sqrt{1 + b^2}}{\sqrt{1 + b^2}} \cdot \ln \left| \frac{R}{R_0} \right|. \tag{13}\]

Moment of inertia:
\[-M_{in} = 2 \cdot \rho \cdot \omega^2 \cdot R \cdot \sqrt{1 + b^2} \cdot \frac{R \cdot S}{\sin(\gamma)} \cdot \ln \left| \frac{R}{R_0} \right| \cdot b \cdot R = 2 \cdot F_{in} \cdot b \cdot R; \tag{14}\]
\[-M_{cor} = 2 \cdot \rho \cdot \omega \cdot v_n \cdot \frac{R \cdot S}{\sin(\gamma)} \cdot \ln \left| \frac{R}{R_0} \right| \cdot R \cdot \sqrt{1 + b^2} = F_{cor} \cdot R \cdot \sqrt{1 + b^2}. \tag{15}\]

The work of the forces of inertia
\[ A_{in} = A_{cor} = 2 \cdot \rho \cdot v_T^2 \cdot \left( 1 + b^2 \right) \cdot \frac{R_0 \cdot S}{b} \cdot \left[ \ln \left| \frac{R}{R_0} \right| - 1 \right] \cdot \exp(b \cdot \varphi) + 1 \]
\[ = 2 \cdot \rho \cdot v_T^2 \cdot \frac{R_0 \cdot S}{b} \cdot \left[ \ln \left| \frac{R}{R_0} \right| - 1 \right] \cdot \exp(b \cdot \varphi) + 1. \tag{16}\]

6. Experimental findings
To check the presented formulas, a series of experiments was carried out on a coil in the form of a logarithmic spiral [4]. The coil was located both on the expansion and on the narrowing of the spiral (figures 1b). The results of measurements of pressure losses in a spiral coil with an expanding and contracting logarithmic spiral and calculations of pressure losses from inertial forces are presented in figure 6.
a) expanding spiral

\[ R_F = 0.34 \text{ m}, \quad R_0 = 0.16 \text{ m} \]

b) tapering spiral

\[ R_F = 0.16 \text{ m}, \quad R_0 = 0.34 \text{ m} \]

1 – approximation of the total pressure loss in the spiral coil;

2 – actual, total pressure loss;

3 – actual pressure loss from inertial forces in the spiral coil;

4 – approximation of pressure losses from inertial forces;

5 – calculated (according to the presented method) pressure loss from inertial forces

Figure 6. Pressure loss in the helical coil in the form of a logarithmic spiral.

The deviation of the theoretical curve 5 from the approximation curve 4 is explained by the presence of a transverse circulation motion of the liquid, which is the greater, the greater the flow rate of the liquid. Investigation of the transverse circulating motion of a liquid, and obtaining theoretical formulas for it, is a topic for further work.

7. Conclusion

The formulas obtained correctly (with an accuracy of 2%) describe the energy losses when a fluid moves through a pipe in the form of a logarithmic coil. Separately, the energy losses associated with the Coriolis force (for expanding spirals) and with the centrifugal force of inertia (for narrowing spirals) were highlighted. Moreover, these two types of forces exist simultaneously for both types of spirals. The moments of these forces are equal to each other, but play a different role. For example, for an expanding spiral, it is the Coriolis force that takes away the useful power of the energy source, reducing the angular velocity of the fluid, and the centrifugal force of inertia uses this energy and performs work to move the fluid from a smaller radius to a larger one. In the case of a tapering spiral, the inertial force takes away the useful power of the source, and the Coriolis force increases the angular velocity of the fluid. Nothing of the kind occurs in spiral coils with a constant twist radius - there the inertial forces do not perform work, since the so-called displacement angle \( \gamma \) is absent. Nature uses a vortex as a tool for transferring momentum. The illustrations [5] clearly show the spiral structure of the vortex, which is close to the logarithmic spiral. The movement of a liquid along a spiral pipe can become a prototype of the model of the movement of a particle in a vortex and energy losses during vortex formation associated with inertial forces. In the spiral branch of a centrifugal main pump, the trajectory of motion of liquid particles, also with certain assumptions, can be considered as an expanding spiral. Therefore, the model
for calculating energy losses from inertial forces when a fluid moves along a logarithmic spiral can also be used to improve or modernize the spiral branch of a centrifugal pump.

8. References

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