Convergence Rate For Low-Pass Infinite Impulse Response Digital Filter

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Abstract. The hybrid optimization technique is used to design a low-pass infinite impulse response (IIR) digital filter, to improve the coefficients of adaptive IIR digital filter, thus ensure stability. In this paper, the newly technique of hybrid particle swarm optimization (PSO) algorithm is proposed. This technique is a dynamic and static topology with a PSO algorithm, which called dynamic and static PSO (DS-PSO) algorithm. In the simulation, the low-pass IIR digital filter 8th order is designed. The fitness function problem is discussed based on values of the ripple of the passband, a ripple of the stopband, and a transition band. Thus, the proposed algorithm results are compared with previous studies results. From results comparison, results have shown the convergence rate with the DS-PSO algorithm outperformed the convergence rates using fuzzy gravitational search algorithm (FGSA), gravitational search algorithm (GSA), and differential evolution (DE) with mean values of 55.21%, 57.80%, and 70.40%, respectively.

Keywords: Adaptive filter; Fitness function; Optimization Algorithm; Particle Swarm Optimization; Signal Processing.

1. Introduction
Digital filter have gained much attention, due to those filters use in the signal processing of many applications as control systems, image processing, biomedical engineering, communication systems, and others [1]. Those filters are two types, the first type is the infinite impulse response (IIR) digital filter, and the second type is the digital finite impulse response (FIR) filter. However, digital filter is the two types of adaptive filters. Also, these digital filter have different outputs, where the IIR digital filter output depends on past output and input samples; The output of the FIR digital filter depends only on past and present input samples. But, the IIR digital filter is essentially a choice for many researchers, because it requires system parameters fewer than in the FIR digital filter [2].

The design of IIR digital filter, one of technicals used to design digital filter is the optimization algorithms [3]. Certain conditions must be met when the process of improving the design of the IIR digital filter, such as the determination of the lowest order of the filter, the stability of the filter, the minimum value of the passband and the magnitude of the stopband ripple.

The most important previous studies of IIR digital filter design, Judhisthir Dash et al. propose the design and implementation of IIR digital filter using a robust swarm and an evolutionary optimization algorithm, whereby those filter best coefficients are sought by using the modern firefly algorithm (FA)
modified version, known as the improved firefly algorithm (IFA), to provide stability in the frequency magnitude response [1].

Nikhil Agrawal et al. proposed a method for IIR digital filter design with an almost linear phase response, where is provided by fractional derivative constraints (FDC), whereby the FDC optimality is ensured by a new sorting mechanism based on greedy. Also, the FDC optimal value is achieved by minimizing total error, when using an improved swarm based on an optimization algorithm, formulated by linking a scout bee mechanism of an artificial bee colony algorithm (BCA) to the particle swarm optimization (PSO) algorithm and [2], [3]. It's designed to optimize the hybrid particle swarm.

Lijia Chen and et al. proposed a method of evolutionary for designing a digital filter with a diverse structure, this evolutionary method is the differential evolution (DE) algorithm, where used adaptive multiple elites guide composite, along with a transformation mechanism that does not need to use known circuit structures [4].

Giovanni Pepe and et al. presented an evolutionary algorithm for designing a stable IIR digital filter, arranged as a series of 2nd order sections (SOS) and used as a GSA algorithm, this process seeks ideal coefficients based on a function of fitness, possibly leading to unstable filtering coefficients [5].

Saha et al proposed the GSA algorithm for solving non-linear IIR digital filter design optimization problems [6], to improve 8th order IIR digital filter design by used GSA with wavelet mutation [7]. Next, Danilo Pelusi et al. proposed the optimal IIR digital filter design with fuzzy gravity search algorithm (FGSA) [8]. This is done with the help of the revised GSA algorithm and the design of suitable fuzzy inference systems (FIS).

Kennedy and Eberhart [9] PSO algorithm introduced. Often, PSO algorithm is used for improvement problems since PSO algorithm has relative speed and simplicity butter than other algorithms. Also, application of various PSO algorithm variants to IIR digital filter design has been reported in the literature, e.g.: Krusienksi et al., the PSO algorithm for Adaptive IIR Filter Structures [10]. The PSO algorithm is used to estimate the coefficients of a IIR digital filter and to find a suitable structure for this filter [11]. Hartmann et al, determination of IIR filter coefficients using a PSO algorithm with its application to reconstruct lost cardiovascular signals [12]. Jiang et al. proposed the hybrid algorithm combining PSO algorithm and GSA algorithm to find the optimal IIR system coefficients set [13]. P. Upadhyay et al., a new design method based on firefly algorithm for IIR system identification problem [14]. Lagos-Eulo-gio, a new design method for adaptive IIR system identification using hybrid cellular PSO algorithm and DE algorithm [15]. Shafaati et al. improving the IIR digital filter by an improved chaotic harmonic search algorithm (CSH) [16]. Also, A. T. Alsahlanee designed the IIR digital filter with the PSO algorithm [17].

In most cases, the hybrid algorithm performs better than the individual algorithm. For PSO algorithm is the same like many optimization algorithms, PSO algorithm may also converge early in local optima. But using multiple neighborhoods reduces the convergence problem in the local optima of the PSO algorithm to a certain extent. Therefore, in this paper, the PSO algorithm novel version is used. This version is as multiple topologies with the PSO algorithm, where these multiple topologies are dynamic and static topologies, that designates multiple neighborhoods for each particle. This novel version called the name dynamic and static with the PSO (DS-PSO) algorithm, where the DS-PSO algorithm encourages exploration and exploits existing knowledge about the space of search [18].

This paper aims to diversify the adaptive IIR digital filter coefficients to ensure IIR digital filter stability. Therefore, the problem with a fitness function for the errors dependent on the values of pass-band ripple, stopband ripple, and transition band is discussed. The aim is achieved by designing low-pass IIR digital filter 8th order. Also, the simulation model is provided to demonstrate the effectiveness of proposed method, and to compare it with simulation models of FGSA, GSA, and DE algorithms in [8].
This paper is organized as follows. In the next section, coefficients of adaptive IIR digital filter are described. In section 3, the fitness function for the errors is explained. In section 4, the DS-PSO algorithm is presented. In section 5, a simulation and results are described. Finally, in section 6, conclusions are drawn.

2. Coefficients of adaptive IIR digital filter

The problem of defining an IIR system involves the diversification of adaptive IIR digital filter coefficients. This is by using an algorithm to match unknown IIR digital filter coefficients when applying similar income signals to an unknown system and an adaptive filter.

Through the adaptive IIR system in [19], [20], a block diagram for an adaptive IIR system was designed using the DS-PSO algorithm of this paper as shown in Figure 1.

![Figure 1. Block diagram of an adaptive IIR filter with DS-PSO algorithm.](image)

where \(x(k)\) is the input signal to both systems, \(y(k)\) and \(y_o(k)\) are an output of an unknown IIR system and an adaptive IIR system, respectively. As for, \(v(k)\) is a noise signal added to \(y(k)\), \(d(k)\) is output signal of an unknown system with noise, \(e(k)\) is a deviation between outputs of both systems, and \(r(k)\) is the rate change in adaptive IIR filter coefficients introduced by an optimization algorithm to minimizes \(e(k)\).

The adaptive IIR system depicts an input-output relationship by equation (1) [19].

\[
y(k) + \sum_{i=1}^{n} a_i y(k - i) = \sum_{i=0}^{m} b_i x(k - i) \quad (1)
\]

where \(x(k)\) the input of a filter, while \(y(k)\) is the output. The filter order is determined by \(n\) with \(n \geq m\). It is assuming that \(a_0 = 1\) [6], then filter coefficients \((a_i)\) and \((b_i)\) to be real-valued, coefficients for filter taps and control of a frequency response of filter, and coefficients of numerator and denominator, respectively for a design. The non-negative integers \(n\) and \(m\) can be chosen arbitrarily and need not be equal. The IIR digital filter transfer function is expressed as (2):

\[
H(z) = \frac{\sum_{i=0}^{m} b_i z^{-i}}{1 + \sum_{i=1}^{m} a_i z^{-i}} \quad (2)
\]
where \( z = e^{j\omega} \), \( m \) is the numerator polynomial degree in \((z^{-1})\), \( m \) will call the numerator degree, and \( n \) will call the denominator degree. To facilitate arithmetic operations, the value of the coefficient \( a_0 \) is taken as 1 [19].

It follows that IIR digital filter frequency response will be as in equation (3):

\[
H(\omega) = \frac{\sum_{i=0}^{m} b_i e^{-j\omega}}{1 + \sum_{i=1}^{n} a_i e^{-j\omega}}
\]

where \( \omega \in [0, \pi] \) is a digital frequency [8].

The desired magnitude frequency response defined on a dense grid of frequency interval is as in equation (4).

\[
D_0(z) = \begin{cases} 
1, & \omega \in pb \\
\frac{-1}{sp - pb} (\omega - \omega_x) + 1, & \omega \in tb \\
0, & \omega \in sb
\end{cases}
\]

where \( sp \) is the edge frequency of stopband, \( pb \) is the edge frequency of passband, \( \omega_x \) is replaced by the desired filter design and \( tb \) is the transition band [21].

3. Fitness function for the errors

IIR digital filter optimization depends on transfer function coefficients \( b_0, \ldots, b_m \) and \( a_0, \ldots a_n \) in equation (2). Due to this, it is important to ensure that these coefficients are controlled out properly. But the problem here is a function of errors. This function takes a mean squared error (MSE) between a frequency response of an ideal and actual filter. An ideal filter has a magnitude of 1 on passband and a magnitude of 0 on stopband. So, fitness function for errors is squared difference between magnitudes of this filter and filter designed using proposed DS-PSO algorithm. Those magnitudes are sum based on the required frequency band, where are divided into the input samples total number, which the frequency response is assessed for them, MSE is as equation (5)[22].

\[
J_1(\omega) = \frac{1}{N} \sum_{k=1}^{N} (\text{Ideal}(\kappa) - \text{Actual}(\kappa))^2
\]

where \( N \) is the frequency points number used to calculate the error fitness function. As \( \text{Ideal}(k) \) and \( \text{Actual}(k) \) are the ideal and the actual filter magnitude response, respectively. A difference between them is an error between an actual and ideal filter responses. The actual filter response is calculated by equation (1). The quality of the IIR digital filter depends on the ripple of the passband, the ripple of the stopband, and the band of transition. So, it used a fitness function that takes into account these three parameters. To ensure such limitations it uses the traceability function of errors in the equation (6) [8].

\[
J_2(\omega) = \sum_{i=1}^{N} (|H(\omega_i)| - 1 - \delta_p)^2 + \sum_{i=1}^{N} (|H(\omega_i)| - 1 - \delta_s)^2
\]

where \( \delta_p \) is passband (\( pb \)) ripple with \( \omega \in pb \), and \( \delta_s \) is stopband (\( sp \)) ripple with \( \omega \in sp \) and \( |H(\omega)| \) is the absolute value of \( H(\omega) \) in equation (4), where the absolute value of \( H(\omega) \) is calculated for each with equation (11) and similarly for each \( l = 1, \ldots, N \) with equation (12).

\[
H(\omega_l) = |H(\omega)|, \text{ with } \omega = \omega_l
\]
where values of $\omega_l$ are the spaced frequency points between 0 and the passband normalized edge frequency $p_h$, and values of $\omega_s$ are spaced frequency points between 0 and the stopband normalized edge frequency $s_b$.

Hence, there is a need for the minimization of error in the fitness function of errors in equation (6); To achieve optimal design and high stability of IIR digital filter, these two orders are important for digital filter design [23].

4. DS-PSO algorithm

The dynamic and static topologies with the particle swarm optimization algorithm (DS-PSO) is very similar to the standard PSO algorithm. The DS-PSO algorithm integrates dynamic and hybrid variations portions of the standard PSO algorithm. The main difference is that DS-PSO algorithm assigns multiple topologies as dynamic and static neighborhoods to each particle. However, the dynamic topologies added are aimed at encouraging the exploration of search space and avoiding early convergence. The dynamic topologies are randomly restructured during the implementation of the algorithm. For static neighborhoods, it retains the exploitative properties of the standard PSO algorithm that are not found in some dynamic versions of the algorithm. In the DS-PSO algorithm, the particles are influenced by the best neighborhood ($N_{best}$) of both topologies, not just topology with the best fitness. Each particle of swarm updates its velocity ($V$) by equation (9), in order to update its position ($P$) by equation (10).

$$V_k^{(i+1)} = w[V_k^{i} + C_1 R_1 (P_{k-best}^{i} - P_k^{i}) + C_2 R_2 (D_{k-best}^{i} - P_k^{i}) + C_3 R_3 (S_{k-best}^{i} - P_k^{i})]$$  \hspace{1cm} (9)

$$P_k^{(i+1)} = P_k^{i} + V_k^{(i+1)}$$  \hspace{1cm} (10)

Here $C_1$, $C_2$, and $C_3$ are acceleration coefficients all set to $4.1/3$ or 0.7298438, $w$ is inertia factor. As for, $R_1$, $R_2$ and $R_3$ are random numbers between [0,1] to encourage exploration for each position component of equation (9). Each component of $v_k$ is kept within a band [$V_{min}$, $V_{max}$], where $V_{min}$ and $V_{max}$ represent minimum values and maximum values for search space, in order to keep particles within the space of search.

It is clear that numerical variables of equation (9) are incorporated as acceleration components for dynamic and static neighborhoods best. It indicates that the particle $i$ is biased towards both dynamic neighborhoods best ($D_{best}$) and static neighborhoods best ($S_{best}$). This is instead of being influenced by a single $N_{best}$. So, it is the current best solutions dynamic and static topologies of the particle $i$, respectively. Also, every particle is attracted to the best dynamic, best static, and best personal, due to found three acceleration coefficients. Also, in the $i$th iteration ($I$), the velocity of the $k$th particle is updated taking into account its velocity of the present, best-acquired individual/personal position ($P_{k-best}$) best-acquired swarm position, $N_{best}$ position described in equation (10) [18]. Also In this section, the DS-PSO pseudo-code has been formatted.

**Algorithm: DS-PSO**

1. **Inputs:**
   1. $S$, the swarm size
   2. $f$, function to be optimized
   3. prob, the probability for dynamic neighborhoods restructuring
   4. max-Iterations, Iterations maximum number

2. **Outputs:**
7. \( P_k^l \), The position of the minimum value of the function is found
8. \( f(P_k^l) \), function value at this position
9. for \( i = 1; \ldots; n \) do
10. set particle \( i \) by used the random position and velocity.
11. while iterations number \(<\ max \) - Iterations do
12. for \( i = 1; \ldots; n \) do
13. \( P_k^{l,best} \) = the position of the best solution particle \( i \) is found so far
14. \( D_k^{l,best} \) = the position of the best solutions found by the particle dynamic neighborhoods so far
15. \( S_k^{l,best} \) = the position of the best solutions found by the particle static neighborhoods so far
16. \( V_k^{(l+1)} \) = particle \( i \) velocity updated from equation (9)
17. \( P_k^{(l+1)} \) = particle \( i \) position updated from equation (10)
18. Calculate \( f(P_k^{(l+1)}) \) and update \( P_k^{l,best}, D_k^{l,best}, S_k^{l,best}, \) and \( P_k^l \)
19. if the double of random \(<\ probD \) then
20. The dynamic neighborhoods restructuring
21. return \( P_k^l \) and \( f(P_k^l) \)

5. Simulation and Results

In the simulation, the MatLab 2019a program used as a work environment. The function ellip() used to design a low-pass IIR digital filter. Then call results of function within the parameters of DS-PSO algorithm used. The best values of transfer function coefficients for the IIR digital filter are also obtained in equation (2) when implementing the proposed algorithm. Then, these coefficients are used to find values in equation (3) and equation (4). Then, the value \( I_2(\omega) \) is calculated in equation (6).

The control parameters for the design of low-pass IIR digital filter are setting as in table 1, where LP is Low-pass, and \( (\delta_p) \) is an order of filter. As for \( \delta_p, \delta_s, Pb, \) and \( Sb \) be as in equation (4) and equation (6).

### Table 1. The design specifications of IIR filters.

| Type Filter | \( \delta_p \) | \( \delta_s \) | \( Pb \) | \( Sb \) | \( Of \) |
|-------------|----------------|----------------|--------|--------|-------|
| LP          | 0.01           | 0.001          | 0.45   | 0.50   | 8     |

Also, a DS-PSO algorithm system analyzed. Then, control parameters are set, where values governing local and global search coefficients \( C_1, C_2, \) and \( C_3 \) are given in equation (9). As for, population cycle and size is empirically based on table 2. This to obtain appropriate values of controller parameters for different particles, where in table 2, \( V_{\text{max}} \) is the maximum value of search space, \( V_{\text{min}} \) is the minimum value of search space, \( I_{\text{max}} \) is the maximum iteration and \( Ps \) is population size (swarm size).

### Table 2. Control parameters for the algorithm.

| \( C_1 \) | \( C_2 \) | \( C_3 \) | \( V_{\text{max}} \) | \( V_{\text{min}} \) | \( I_{\text{max}} \) | \( Ps \) |
|----------|----------|----------|----------------|-------------|-------------|-------|
| 4.1/3    | 4.1/3    | 4.1/3    | 2             | -2          | 500         | 50    |

In this simulation, the DS-PSO algorithm is implemented in 30 experiments, in order to obtain the best solutions. The frequency range \([0 \text{ to } \pi]\) divided into \( N = 512 \) isometric distance sample points. Whereas in a study by Danilo Pelusi and others [8], the frequency range from \([0 \text{ to } \pi]\) are divided into \( N = 256 \) sample points at equal distance.
Generally, an optimal IIR digital filter has a small passband ripple and small stopband ripple, and the narrow transition band. In addition, the minimum of value of fitness function for errors and small value of convergence rate. In reality, the result is more robust if the data have the least standard deviation; Because of the strength, it depends on the values of the standard deviation.

Then, the results of DS-PSO algorithm are compared with the results found in ref [8], which are by using the FGSA, GSA, and DE algorithms for the design of LP IIR digital filter 8th order.

The results explained and analyzed in the following paragraphs 5.1, 5.2, 5.3, and 5.4 of in this section.

### 5.1. Coefficients of quality for LP IIR filter

Table 3 is shown the value of coefficients of quality for design 8th order of LP IIR digital filter by using a DS-PSO algorithm compare with the results of FGSA, GSA, and DE algorithms in [6], where \( PBRipple \) is a passband ripple, \( SB\text{Ripple} \) is a stopband ripple, \( tb \) is a transition band. As for, max is maximum and min is minimum. The results in table 3 indicate that the maximum and minimum values of results for passband ripple, stopband ripple, and transition band by using a DS-PSO algorithm are less than values by FGSA, GSA, and DE algorithms.

| Algorithm | \( PBRipple \) max | \( PBRipple \) min | \( SB\text{Ripple} \) max | \( SB\text{Ripple} \) min | \( tb \) max | \( tb \) min |
|-----------|---------------------|-------------------|---------------------|-------------------|----------------|----------------|
| DE        | 0.209371            | 0.092159          | 0.092159            | 0.092159          | 0.430534       | 0.188224       |
| GSA       | 0.067816            | 0.066199          | 0.016867            | 0.016857          | 0.120964       | 0.029634       |
| FGSA      | 0.065990            | 0.061591          | 0.022298            | 0.017547          | 0.101172       | 0.027451       |
| DS-PSO    | 0.010830            | 0.002330          | 0.010125            | 0.001393          | 0.059340       | 0.006023       |

![Figure 2. Graphic analysis of quality coefficients.](image-url)
Figure 2 shown a graphical analysis for the data of table 3, where PBRipple-max is a maximum passband ripple, PBRipple-min is minimum passband ripple, SB_Ripple-max is maximum stopband ripple, SB_Ripple-min is minimum stopband ripple, tb-max is maximum transition band, and tb-min is minimum transition band. See to Figure 2, that the indicators (PBRipple-max, PBRipple-min, SB_Ripple-max, SB_Ripple-min, tb-max, and tb-min) with DS-PSO algorithm have the smallest proportion, while that the indicators have the highest proportion with FGSA, GSA, and DE algorithms.

5.2. Convergence profile results

The convergence profile results are a maximum and minimum optimal value of the fitness function. Table 4 is shown the optimal values of fitness function for errors of design 8th order of LB IIR digital filter by using a DS-PSO algorithm compare with the results of FGSA, GSA, and DE algorithms in [8], where O_vmax is maximum the fitness function optimal value for LB IIR digital filter, and O_vmin is minimum the optimal value of fitness function for LB IIR digital filter.

The results of O_vmax and O_vmin in table 4 indicate that DS-PSO algorithm has about the standard deviation low values at order (10^{-3}) less than the values of FGSA, GSA, and DE algorithms, where results of FGSA algorithm is better than GSA and DE algorithms, while the result of O_vmax with DS-PSO algorithm is 31.41% better than O_vmax with FGSA, and O_vmin with DS-PSO algorithm is 67.31% better than O_vmin with FGSA algorithm.

Table 4. Convergence profile results.

| Algorithm | O_vmax   | O_vmin   |
|-----------|----------|----------|
| DE        | 0.016140 | 0.004195 |
| GSA       | 0.006185 | 0.004130 |
| FGSA      | 0.004364 | 0.004154 |
| DS-PSO    | 0.002993 | 0.001359 |

Figure 3. Convergence profile results.
Figure 3 shown a graphical analysis for the data of convergence profile results in table 4. See Figure 3, that indicators of $O_{V_{\text{max}}}$ and $O_{V_{\text{min}}}$ with the proposed DS-PSO algorithm have the smallest proportion, while the indicators have the highest proportion with FGSA, GSA, and DE algorithms.

5.3. Convergence rates

The calculation time of the search algorithm is a measure of the velocity at which the procedure converges. Also, calculation time is not good for assessing convergence velocity, it depends on designer skills, hardware performance, and programming language. One good method to assess the velocity of convergence is to consider the number of objective function evaluations up to the minimum value of the function [8], [21]. Therefore, the ratio between the number of fitness function calculations ($N_f$) and the number of evaluations ($N_v$) is a ratio that determines the rate of convergence, where convergence rate indicated by ($C_R$) in equation (11):

$$C_R = \frac{N_f}{N_v}$$  \hspace{1cm} (11)

Table 5 shown convergence rates of LP IIR digital filter 8th order, when is design this filter by the DS-PSO algorithm, and compare with the results of FGSA, GSA, and DE algorithms, where $C_R$ is a convergence rate for LB IIR digital filter.

The results in table 5 indicate that the value of $C_R$ with FGSA algorithm is less than values $C_R$ with GSA and DE algorithms, while $C_R$ with DS-PSO algorithm is 55.21% better than $C_R$ with FGSA. So, the value of $C_R$ by using a DS-PSO algorithm is better than the values of $C_R$ by using FGSA, GSA, and DE algorithms.

Table 5. Convergence rates for low-pass IIR filters.

| Algorithm | DE  | GSA | FGSA | DS-PSO |
|-----------|-----|-----|------|--------|
| $C_R$     | 0.6900 | 0.4840 | 0.4560 | 0.2042 |

Figure 4. Convergence rates for low-pass IIR filters.
Figure 4 shown a graphical analysis for data of convergence rates in table 5. See in Figure 4, that indicators of $C_R$ with DS-PSO algorithm have proportion smallest, while indicators have the highest ratio with FGSA, GSA, and DE algorithms.

5.4. **Stability of low-pass IIR filters**

Pole-Zero plots are analyzing the stability of LB IIR digital filter.

**Figure 5.** Pole-Zero scheme for LP IIR filter with DS-PSO algorithm.

Figure 5 shown Pole-Zero plots for the design of LP IIR digital filter 8th order with a DS-PSO algorithm, where the circle marks are zeros, while the cross marks are poles. Pole-Zero plots show poles within the unit circle.

Also, there is data of great interest obtained by simulating a LB IIR digital filter design with an 8th order. This data is a discrete-time IIR digital filter (real) which is shown in table 6.

**Table 6.** Discrete-Time IIR Filter (real).

| Filter Structure                  | Direct-Form II Transposed |
|-----------------------------------|---------------------------|
| Length of Numerator               | 9                         |
| Length of Denominator             | 9                         |
| Stable                            | Yes                       |
| Linear phase                      | No                        |
| Cost of Implementation            | 18                        |
| Multipliers Number                | 16                        |
| Adders Number                     | 8                         |
| Multiplications per Input Sample  | 18                        |
| Additions per Input Sample        | 16                        |

Finally, results from data in tables 3, 4, 5 and 6, graphical analysis of $m$ in Figures 2, 3 and 4 respectively, and the stability analysis in Figure 5 indicates that the proposed algorithm in this paper
produces optimal results, and better than the results of FGSA, GSA, and DE algorithms for designing LB IIR digital filter 8th order.

There are a bunch of facts that got when designing the LP IIR digital filter 8th order using the DS-PSO algorithm in simulations of this paper. Those facts as following: First, the DS-PSO algorithm is able to minimize fitness function for errors, where the best values for error fitness function are obtained; Second, DS-PSO algorithm achieves the solution best of convergence rate; Third, optimal stability. Those facts assure the very good robustness of proposed DS-PSO algorithm. Also, these facts make the DS-PSO algorithm better than FGSA, GSA, and DE algorithms for optimized design of LB IIR digital filter.

Hence, in this paper, the method proposed is an optimal method for improving the convergence ratio of LB IIR digital filter.

6. Conclusions
Optimize the convergence ratio is important when designing digital filter to obtain a digital filter with optimum stability. The proposed optimization algorithm capable of improving the design of the IIR digital filter has been described. This algorithm is a combination of dynamic and static topologies with particle swarm optimization (DS-PSO), which can optimize the error fitness function of diversification coefficients of an adaptive IIR digital filter. The paper aims to improve the fitness function of error that depends on the filter quality parameters, where the filter quality depends on passband ripple, stopband ripple, and transition band; This function of fitness takes the mean square error (MSE) between the ideal and the actual filter frequency response when the filter design by different methods. This is illustrated by designing the LP IIR digital filter 8th order in the simulations of this paper. Thus, the results showed that the filter designed with the proposed algorithm in this paper has best solution and best velocity of convergence. Therefore, the convergence ratio is improved for the low-pass IIR digital filter. The values passband ripple, stopband ripple, and transition band with DS-PSO algorithm are found to be better than the optimization methods used in previous studies. Also, the convergence rate values with proposed DS-PSO algorithm were better than those using the FGSA, GSA and DE algorithms with averages of 55.21%, 57.80% and 70.40%, respectively. The results indicate that DS-PSO algorithm is the best algorithm for designing low-pass IIR digital filter in terms of error optimization, goodness, robustness, high convergence rate, and optimal stability. Finally in this paper, a low-pass digital IIR filter is a static filter with an ideal convergence ratio. Therefore, the proposed algorithm can be the optimal method to improve the convergence ratio when designing a low-pass IIR digital filter.

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