Quantum Zeno and anti-Zeno effect in atom-atom entanglement induced by non-Markovian environment

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The dynamic behavior of the entanglement for two two-level atoms coupled to a common lossy cavity is studied. We find that the speed of disentanglement is a decreasing (increasing) function of the damping rate of the cavity for on/near (far-off) resonant couplings. The quantitative explanations for these phenomena are given, and further, it is shown that they are related to the quantum Zeno and anti-Zeno effect induced by the non-Markovian environment.

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I. INTRODUCTION

Quantum entanglement plays an essential role in quantum information science [1]. However, entanglement is very fragile due to the influence of the environment, which is recognized as the major obstacle for the final application of quantum information processing. For example, a phenomenon called “entanglement sudden death” [2, 3, 4] shows that two initially entangled qubits may disentangle completely in a finite time due to spontaneous emission. So in order to keep the atomic entanglement for a long time, spontaneous emission should be suppressed. Several ways were proposed for this purpose. One way widely applied is to place the qubits in a structured environment, say, microcavity [5, 6] or in the photonic band gap of photonic crystals [7], such that the qubits are separated from the environment. Another way is to dynamically control the coupling between the system and the environment, by, e.g., quantum Zeno effect [8]. In practice, people may combine the two ways [8, 3, 10, 11, 12, 13, 14]. In fact, the two methods have something in common.

In this paper, we investigate the entanglement dynamics of two two-level atoms in a common cavity, a non-Markovian environment. We find that when the atoms are on or near resonant with the cavity, the speed of the disentanglement decreases as the quality factor of the cavity increases, and when the atoms are far-off-resonant with the cavity, the speed of the disentanglement increases as the quality factor of the cavity decreases. These phenomena can be related to quantum Zeno [10, 15, 16] and anti-Zeno effect [11, 12, 13, 14].

II. THEORETICAL MODEL

Consider two spontaneously emitting two-level atoms with a common zero-temperature bosonic reservoir, and in the Lamb-Dicke limit [17], the dipole-dipole interaction is negligible. Under the rotating wave approximation (RWA), the Hamiltonian of this composite system plus the reservoir is given by (\(h = 1\)) [8, 17, 18]:

\[
H = H_0 + H_{\text{int}},
\]

with

\[
H_0 = \omega_1 \sigma_+^{(1)} \sigma_-^{(1)} + \omega_2 \sigma_+^{(2)} \sigma_-^{(2)} + \int_{-\infty}^{\infty} d\omega k \omega b^\dagger(\omega_k)b(\omega_k),
\]

\[
H_{\text{int}} = (\alpha_1 \sigma_+^{(1)} + \alpha_2 \sigma_+^{(2)}) \int_{-\infty}^{\infty} d\omega_k g(\omega_k)b(\omega_k) + \text{h.c.}
\]

Here, \(\sigma_+^{(j)}\) and \(\omega_j\) are the inversion operators and transition frequency of the \(j\)th qubit \((j = 1, 2)\), and \(b(\omega_k)\), \(b^\dagger(\omega_k)\) are the annihilation and creation operators of the field mode of the reservoir. The mode index \(k\) contains several variables which are two orthogonal polarization indices and the propagation vector \(\vec{k}\). To measure the coupling strength of the atoms to the cavity mode determined by the atom’s relative position in the cavity, we introduce the dimensionless constant \(\alpha_j\) [8]. In the following discussion, we introduce vacuum Rabi frequency \(R = W(\alpha_1^2 + \alpha_2^2)^{1/2}\) and relative coupling strengths \(r_j = \alpha_j(\alpha_1^2 + \alpha_2^2)^{-1/2}\) \((j = 1, 2)\).

For an initial state of the form

\[
|\psi(0)\rangle = (c_{10}|e\rangle_1|g\rangle_2 + c_{20}|g\rangle_1|e\rangle_2)|0\rangle_E,
\]

since \([H, N] = 0\), where \(N = \int_{-\infty}^{\infty} d\omega_k b^\dagger(\omega_k)b(\omega_k) + \sigma_+^{(1)} \sigma_-^{(1)} + \sigma_+^{(2)} \sigma_-^{(2)}\), the time evolution of the total system is confined to the subspace spanned by the bases \(|e\rangle_1|g\rangle_2|0\rangle_E\), \(|g\rangle_1|e\rangle_2|0\rangle_E\), \(|g\rangle_1|g\rangle_2|1_k\rangle_E\):

\[
|\psi(t)\rangle = c_1(t) e^{-i\omega_1 t}|e\rangle_1|g\rangle_2|0\rangle_E + c_2(t) e^{-i\omega_2 t}|g\rangle_1|e\rangle_2|0\rangle_E
+ \int_{-\infty}^{\infty} d\omega_k c_{1k}(t) e^{-i\omega_k t}|g\rangle_1|g\rangle_2|1_k\rangle_E,
\]

where \(|1_k\rangle_E\) is the state of the reservoir with only one exciton in the \(k\)th mode. Here, we consider the case in which the two
The entanglement of the two atoms can be evaluated by solving Schrödinger’s equation and eliminating the coefficients $c_{\alpha_k}(t)$, one has

$$
\dot{c}_1(t) = -\int_0^t dt f(t-t_1) \alpha_1[c_1(t_1) + \alpha_2c_2(t_1)],
$$

(3a)

$$
\dot{c}_2(t) = -\int_0^t dt f(t-t_1) \alpha_2[c_1(t_1) + \alpha_2c_2(t_1)],
$$

(3b)

where the correlation function takes the form:

$$
f(t-t_1) = \int_{-\infty}^{\infty} d\omega J(\omega) e^{-i(\omega_0-\omega)t-t_1}.
$$

As discussed in [8], there is a subradiant state, $|\psi_-\rangle = r_2|e\rangle_1|g\rangle_2 - r_1|g\rangle_1|e\rangle_2$, which does not decay in time, and the only relevant time evolution is the superradiant state $|\psi_+\rangle = r_1|e\rangle_1|g\rangle_2 + r_2|g\rangle_1|e\rangle_2$.

For the case of two atoms interacting with a cavity field in presence of cavity losses, the spectral density function takes the form

$$
J(\omega_k) = W^2 \lambda/\pi[(\omega_k - \omega_c)^2 + \lambda^2],
$$

(4)

where $W$ is the transition strength, $\omega_c$ is the center of the spectrum, and $2\lambda$ is the full width at half maximum (FWHM) of the spectral function. By employing Fourier transform and residue theorem, we get the explicit form $f(t-t_1) = W^2 e^{-i(\omega_0-\omega)t-t_1}$, where the quantity $1/\lambda$ is the reservoir correlation time.

Using Laplace transform, we get the solutions of Eqs. (3a) and (3b):

$$
c_1(t) = r_2\beta_- + r_1\beta_+ \mathcal{E}(t),
$$

(5a)

$$
c_2(t) = -r_1\beta_- + r_2\beta_+ \mathcal{E}(t),
$$

(5b)

where $\beta_\pm = \langle \psi_\pm | \mathcal{E}(t) \rangle = (s_+ + \lambda + i\delta)e^{s_+ t}/(s_+ - s_-) - (s_- + \lambda + i\delta)e^{-s_- t}/(s_+ - s_-)$, and $s_\pm$ are the roots of the equation for $s$: $s^2 + (\lambda + i\delta)s + R^2 = 0$, where $\delta = \omega_c - \omega_0$ is the detuning. In the $|e\rangle_1|e\rangle_2, |e\rangle_1|g\rangle_2, |g\rangle_1|e\rangle_2, |g\rangle_1|g\rangle_2$ basis, the reduced density matrix of the two atoms is given by:

$$
\rho_a(t) = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & |c_1(t)|^2 & c_1(t)c_2^*(t) & 0 \\
0 & c_1(t)c_2^*(t) & |c_2(t)|^2 & 0 \\
0 & 0 & 0 & 1 - |c_1(t)|^2 - |c_2(t)|^2
\end{pmatrix}.
$$

(6)

The entanglement of the two atoms can be evaluated by concurrence $C(t)$ [19]. For $\rho_a$ [Eq. (6)], its concurrence can be derived from [19], as

$$
C(t) = 2 |c_1(t)c_2^*(t)| = 2 |c_1(t)||c_2(t)|.
$$

(7)

III. NUMERICAL RESULTS

We focus on the concurrence as a function of time $t$ in weak-coupling regime, $W < \lambda/2$. In this regime, the concurrence of the two atoms undergoes nearly irreversible exponential decay. Similar behaviors mentioned below take place for strong-coupling regime in the time scale of Rabi oscillation.

We compare the entanglement dynamics of the two atoms initially in the maximal entanglement states $|\psi_+\rangle = \sqrt{2}(|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2)$ for three different values of full width at half maximum (FWHM) of the spectral function, namely, $2\lambda = 10, 16, 20$.

As mentioned in [20], it is hard to exactly control the position of the atom in the optical cavity. But through numerical stimulation, we find that different values of $r_i$ show qualitatively similar behaviors. So, for simplicity, we focus on the case of equal coupling parameters, i.e., $r_1 = r_2 = \sqrt{2}/2$.

As in FIG. (1a), the atoms are on resonance with the center of the spectrum, $\delta = 0$. The concurrence decreases monotonically down to zero at the beginning. An interesting phenomenon is that the speed of disentanglement decreases as $\lambda$ increases. Similar behavior happens when the atoms are near resonance with the center of the spectrum, $\delta \ll \lambda$. In fact, this phenomenon is related to the environment induced quantum Zeno effect [10, 15, 16]. However, when the atoms are far off-resonant with the center of the spectrum, $\delta \gg \lambda$, for example, as shown in FIG. (1b), where we choose $\delta = 20$, the speed of disentanglement increases as $\lambda$ increases. This latter phenomenon is related to the anti-Zeno effect [11, 12, 13, 14].

In fact, as mentioned in [21], if the coupling strengths of the two atoms to the field are different and the dipole-dipole interaction is not negligible, there are no asymptotic entanglement, which means that even $|\psi_-\rangle$ will disentangle completely. In these cases, our numerical stimulation shows that similar
phenomena mentioned above happen.

We can give an intuitive explanation for these phenomena. As we can see in FIG. 2 the center part of the spectrum decreases monotonically as \( \lambda \) increases, while the parts which are far from the center increases as \( \lambda \) increases. We can prove that the short-time behavior of the disentanglement is determined by the modes of the spectrum which are on resonance with the atoms: the speed of the disentanglement decreases (increases) as the density of these modes decreases (increases).

Similar to \([10, 13]\), we define \( P(t) \equiv |\psi(t)|^2 \equiv e^{-Rt} \), where \( R \) is the effective decay rate. From Eqs. (5a), (5b) and (7), we can see that \( P(t) \) and \( R \) can describe the concurrence to some extent. For short-time behavior, in the first-order approximation, one yields \([13]\):

\[
R = 2\pi \int_{-\infty}^{\infty} d\omega J(\omega) F(\omega), \tag{8a}
\]

\[
F(\omega) = \frac{1}{2\pi} \sin^2 \left( \frac{\omega - \omega_c}{2} \right). \tag{8b}
\]

Since \( F(\omega) \) is a sharply steep function of \( \omega \) around \( \omega_0 \), if the width of \( F(\omega) \) is much smaller than that of \( J(\omega) \), \( R \) is mainly determined by \( J(\omega) \). In this paper, the form of \( J(\omega) \) is given by Eq. (4). Then a detailed analysis shows that if \( \lambda \) is larger than \( \omega_0 - \omega_c \), \( J(\omega) \) is a monotonic decreasing function with respect to \( \omega \), otherwise, it is a monotonic increasing function with respect to \( \lambda \). Since \( R \propto J(\omega) \), these results also go to \( R \).

IV. ENVIRONMENT INDUCED QUANTUM ZENO EFFECT AND ANTI-ZENO EFFECT

In order to relate these phenomena to environment induced quantum Zeno and anti-Zeno effect, we make a transform on the original Hamiltonian [Eq. (10)] as in \([21, 22]\):

\[
a = (\lambda/\pi)^{1/2} \int_{-\infty}^{\infty} d\omega_k \frac{b(\omega_k)}{\omega_k - \omega_c - i\lambda}. \tag{9}
\]
as the strength of damping increases. In terms of quantum measurement theory, this means that the observation made by the environment suppress the disentanglement, which is just quantum Zeno effect. When $\omega_0 - \omega_c$ is much larger than $\lambda$, that means the frequency of the observation is very small, and in the language in [12, 14], we can say that the observations are made in the anti-Zeno regime, so the observations accelerate the disentanglement. A similar discussion about the Zeno and anti-Zeno effect for nonresonant systems is given in [14], where the observations are not made by the environment.

In conclusion, we extend the study of the entanglement dynamics of two atoms in a common cavity. We find that the speed of the disentanglement of the two atoms is a decreasing (increasing) function of the damping rate of the cavity when the atoms are on/near resonance (far off resonance) with the center of the cavity modes. We give a quantitative explanation for these phenomena, and relate them to quantum Zeno and anti-Zeno effect induced by the environment. These results are helpful for understanding the related experimental phenomena and for the practical engineering of entanglement in the future.

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