Majorana Neutrino Masses from Flavor Symmetries

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Abstract

In this talk we discuss the implications of the Minimal Supersymmetric Standard Model augmented by a single $U(1)$ anomalous family symmetry for neutrino masses and mixing angles. The left-handed neutrino states are provided with Majorana masses through a dimension-five operator in the absence of right handed neutrino components. Assuming symmetric lepton mass matrices, the model predicts inverse hierarchical neutrino mass spectrum, $\theta_{13} = 0$ and large mixing while at the same time it provides acceptable mass matrices for the charged fermions.

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1. Introduction

Enlarging the gauge symmetry of the Standard Model is a common solution to some of its problems. A natural candidate would be an additional $U(1)$ family symmetry that is broken at some high scale $M$, a scenario proposed some time ago for the explanation of the charged fermion mass hierarchy [1, 2, 3] as well as for the realization of the quark-lepton complementarity (QLC) [4] more recently [5, 6]. This is motivated by the experience from string model building which has shown that a natural step towards this simplification is to assume the existence of $U(1)$ symmetries which distinguish the various families.

Experimental facts [7] suggest that the Yukawa couplings related to neutrino masses are highly suppressed compared to those of quarks and charged leptons while their mixing is much larger than that of the quark sector. Therefore, exploring whether the neutrino oscillation data could be interpreted in the context of an extension of the Standard Model gauge symmetry is an interesting possibility. We will use only the minimal fermion spectrum of the Minimal Supersymmetric Standard Model (MSSM) without introducing right handed neutrinos [8]. Thus, we will provide Majorana masses for all three neutrinos from the lepton number violating operator [9], which has the form

$$(\bar{L}cH)(LH)$$

where $M$ stands for a large scale that will turn out to be of the order $10^{13-14}$ GeV. This scale is quite low to be identified with the GUT or the string scale in the context of the heterotic string theory, however, it is compatible with the effective gravity scale in theories with large extra dimensions obtained in the context of Type I string models.

In this talk, we explore the possibility that neutrino masses and mixing can be interpreted with the help of an additional anomalous $U(1)$ family symmetry which at the same time is responsible for the generation of charge fermion mass hierarchy. This symmetry could be anomalous and anomaly cancellation is assumed to happen in the context of a fundamental theory valid above the scale $M$. We show that in a generic model an additional abelian symmetry can account for atmospheric data and predicts $\theta_{13} = 0$. We also show how secondary effects possibly arising from additional singlet(s) or some alternative mechanism, as supersymmetry breaking, can under certain assumptions render the model compatible with all recent experimental data. We finally derive explicit charge assignments that reproduce the above results.

2. Description of the Model

We consider the MSSM with gauge symmetry $G_{SM} = SU(3) \times SU(2)_L \times U(1)_Y$ as an effective field theory below a scale $M$ of a fundamental theory. In the context of the $G_{SM}$ symmetry, all gauge invariant Yukawa terms relevant to quark and charged lepton masses appearing at the tree-level superpotential are

$$W = y_{ij}^u Q_i U^c_j H_2 + y_{ij}^d Q_i D^c_j H_1 + y_{ij}^e L_i E^c_j H_1.$$  (2)
Table 1: $U(1)_X$ charge assignments for MSSM fields. The $U(1)_X$ charges of the two extra singlet fields $\Phi$ and $\bar{\Phi}$, are taken to be $+1$ and $-1$ respectively.

In the case of models constructed in the framework of string theory, there are explicit examples where the MSSM fields are charged under (at least) one additional abelian anomalous ($U(1)_X$) factor that prevents terms not invariant under this symmetry from appearing in (2). Usually, the appearance of the additional $U(1)_X$ symmetry is accompanied by at least a pair of MSSM singlets ($\Phi$, $\bar{\Phi}$) with opposite $U(1)_X$-charges. $\Phi$ and $\bar{\Phi}$ can acquire vevs leading to the breaking of the extra abelian symmetry.

Assuming natural values of the Yukawa couplings $\lambda_{ij}$ in (2) (i.e., order one), and taking into account the observed low energy hierarchy of the fermion mass spectrum, we infer that only couplings associated with the third generation should remain invariant at tree-level. Mass terms for the lighter fermions are to be generated from higher order non-renormalizable superpotential couplings. Such higher order invariants are formed by adding to the non-invariant tree-level coupling an appropriate number of $U(1)_X$-charged singlet fields which compensate the excess of the $U(1)_X$-charge. In the case supersymmetric models, the magnitudes of the singlet vevs $\langle \Phi \rangle$ and $\langle \bar{\Phi} \rangle$ are related by the $D$-flatness conditions of the superpotential, while perturbative considerations require that the vevs for the singlet fields are about one order of magnitude below the effective theory scale $M$ scale, therefore lighter generations couplings will be suppressed by powers of $\lambda$, $\bar{\lambda}$ where

$$\lambda = \frac{\langle \Phi \rangle}{M}, \quad \bar{\lambda} = \frac{\langle \bar{\Phi} \rangle}{M}$$

Introducing the generic charge $U(1)_X$-charge assignments of Table 1 the charges of the entries of the corresponding mass matrices are

$$C_{ij}^u = q_i + u_j, \quad C_{ij}^d = q_i + d_j, \quad C_{ij}^e = \ell_i + e_j.$$ (4)

Restricting the analysis to the investigation of symmetric fermion mass matrices we obtain the following constraints $q_i + u_j = q_j + u_i$, $q_i + d_j = q_j + d_i$, $\ell_i + e_j = \ell_j + e_i$. Moreover, the requirement that the third generation mass couplings appear at tree-level imposes the additional constraints $q_3 + u_3 + h_2 = 0$, $q_3 + d_3 + h_1 = 0$, $\ell_3 + e_3 + h_1 = 0$. Since in our configurations the top, bottom and $\tau$-Yukawa couplings are equal at the high scale $M$, up to order one coefficients, the difference between the top mass ($m_t$) and the bottom mass ($m_b$) must arise mainly from a large Higgs vev ratio $\tan \beta = \frac{v_2}{v_1} \gg 1$.

The general form of the superpotential couplings contributing to the fermion mass matrices has been studied in [8]. Here, we will concentrate on the neutrinos. These are
massless at tree-level, however, the non-renormalizable mass term \((1)\) leads directly to a light Majorana mass matrix involving only the left handed components \(\nu_L\). Therefore, defining \(\varepsilon^k = \lambda^k\) if \(k = [k] < 0\), \(\varepsilon^k = \bar{\lambda}^k\) if \(k = [k] > 0\) and \(\varepsilon^k = 0\) if \(k \neq [k]\) (where \([k]\) stands for the integer part of \(k\)) the mass term takes the form

\[
W_{n.r.}^{(2)} = \frac{\zeta^\alpha_\nu^\beta}{M} \bar{\nu}_i^\nu L_a \sigma^\nu (\bar{L}_c^a H_2^i \varepsilon_{ij}) (H_2^k L^k_{\beta k}) \equiv \zeta^\alpha_\nu^\beta \bar{\nu}^\nu_{La} \nu_{L\beta}
\]  

with \(v_2 = \langle H_2 \rangle \approx O(m_W)\) and \(C^\nu_{ij} = 2h_2 + \ell_i + \ell_j\).

For the quarks we impose

\[
q_1 - q_3 = \frac{n}{2}, \quad q_2 - q_3 = \frac{m}{2}
\]

where \(m + n \neq 0\), \(m, n = \pm 1, \pm 2, \ldots\) (6)

Details for the quark sector can be found in \([10]\).

For the leptons we define the parameters \(2n' = l_1 - l_3\) and \(2m' = l_2 - l_3\), where \(m', n'\) are integers and the associated \(U(1)_X\)–charge matrix takes the form

\[
C_e = \left( \begin{array} {cccc}
\frac{n'}{2} & \frac{m'+n'}{2} & \frac{n'}{2} \\
\frac{m'}{2} & \frac{n'}{2} & 0 \\
\frac{m'}{2} & \frac{n'}{2} & 0 \\
\end{array} \right)
\]  

(7)

The zero charge in the position 33 of the above charge-matrices is due to the fact that we demand the appearance of the corresponding Yukawa couplings at the tree-level superpotential. For the remaining entries, a proper power of the appropriate expansion parameter is needed.

We can re-express the generic fermion charges of Table 1 in terms of the new parameters which we choose to be \(m, n, m', n'\) that appear in the quark and charged lepton matrices and \(q_3, \ell_3, h_2, h_1\). The resulting assignments are presented in Table 2.

The \(U(1)_X\)–charge entries for the light Majorana neutrino mass matrix take the form

\[
C_\nu = \left( \begin{array} {ccc}
\frac{n'}{2} + A & \frac{m'+n'}{2} + A & \frac{n'}{2} + A \\
\frac{m'}{2} + A & \frac{n'}{2} + A & A \\
\frac{m'}{2} + A & A & A \\
\end{array} \right)
\]  

(8)

Table 2: Fermion \(U(1)_X\) charge assignments after introducing the integer parameters \(m, n\) and \(m', n'\) that appear in the quark and charge lepton matrices respectively.
where we have introduced the new parameter $A = 2(l_3 + h_2)$. We observe that the neutrino $U(1)_\chi$-charge entries differ from the corresponding charged leptonic entries by the constant $A$

$$C_{ij}^\nu = C_{ij}^e + A$$

3. Neutrino Masses and Mixing

In this section we search for explicit $U(1)_\chi$ charge assignments for MSSM particles that provide phenomenologically acceptable mass textures for all MSSM fermions and in particular for neutrinos. The basic structure of the mass matrices and mixing angles which meet the phenomenological requirements can be obtained without referring to a set of particular $U(1)_\chi$-charges. Explicit examples with sets of charges for all fermion and Higgs fields will be given in the end of this section. Before we present viable cases, we should note that our procedure exhibits here the basic structure of the mass matrices and mixing. The most striking feature, is that the extension of the $G_{SM}$ symmetry to include an $U(1)_\chi$ anomalous factor can reproduce the correct hierarchy of all fermion fields while at the same time the recent neutrino oscillation data are interpreted to a good approximation by a lepton mixing matrix involving two mixing angles, one originating from the charged leptonic matrix matrix and the second by the light Majorana mass matrix. However, at this level of analysis the value of the non-vanishing coefficients of the Yukawa superpotential terms are unknown, since their calculation requires a detailed knowledge of the fundamental theory above the scale $M$ (possibly string theory). Hence, in the present analysis, we restrict ourselves in the description of the general characteristics of the theory, which are nevertheless very interesting.

We first note that in our framework the leptonic matrices depend on $m', n'$. We can fix the parameters $m, n$, so that a correct hierarchical quark mass spectrum is obtained [8]. The lepton sector can be then worked out independently, choosing appropriate values for the two additional parameters $m'$ and $n'$.

In order to obtain a viable set of lepton mass matrices and mixing, a systematic search shows that the charge parameters $m', n'$ should be $n' = \text{odd}$, $m' = \text{even}$. Under this choice the charged lepton mass matrix takes the form

$$M_e = m_0^e \begin{pmatrix} \delta \varepsilon^{m'} & 0 & 0 \\ 0 & \varepsilon^{m'} & \alpha \varepsilon^{\frac{m'}{2}} \\ 0 & \alpha \varepsilon^{\frac{m'}{2}} & 1 \end{pmatrix}$$

where we have explicitly introduced two (out of three) order-one parameters $\alpha$ and $\delta$ that account for the Yukawa couplings and renormalization effects.

Turning to the neutrino sector the Majorana neutrino mass matrix takes the form

$$M_\nu^0 = m_0^\nu \begin{pmatrix} 0 & -\varepsilon^{\frac{m' + n'}{2}} + A & \zeta \varepsilon^{\frac{n'}{2}} + A \\ -\varepsilon^{\frac{m' + n'}{2}} + A & 0 & 0 \\ \zeta \varepsilon^{\frac{n'}{2}} + A & 0 & 0 \end{pmatrix}$$
Table 3: Examples of $U(1)_X$ charges which lead to the neutrino mass matrix structure discussed in the text.

where $\zeta$ stands for an order one coefficient. This mass matrix can be diagonalised by a unitary matrix $V_\nu(\omega)$, where $\tan \omega = \zeta \varepsilon^{-m'/2}$, and can lead to bimaximal mixing in the case that the two mass matrix elements are equal.

The leptonic mixing matrix $U^0_l = V_\nu(\phi) V_\nu(\omega)$ is given by

$$U^0_l = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{\cos(\phi + \omega)}{\sqrt{2}} & \frac{\cos(\phi + \omega)}{\sqrt{2}} & \sin(\phi + \omega) \\ -\frac{\sin(\phi + \omega)}{\sqrt{2}} & \frac{\sin(\phi + \omega)}{\sqrt{2}} & \cos(\phi + \omega) \end{pmatrix}$$

(12)

where $\tan(2\phi) = 2a \varepsilon^{m'/2}/(1 - \varepsilon^{m'})$ while for the mass square differences

$$\Delta m_{\text{atm}}^2 = \Delta m_{23}^2 = (m^\nu_0)^2 \varepsilon^{2A+m'+n'}(1 + \zeta^2\varepsilon^{-m'})$$

$\Delta m_\odot^2 = \Delta m_{12}^2 = 0$

(13)

The above results exhibit a number of interesting properties of the model, that are worth mentioning at this point. We first observe that the model predicts an inverted neutrino mass hierarchy. We further point out that the $U(1)_X$ symmetry implies large mixing effects in the neutrino mass matrix, in contrast to the situation of the charged fermion sector where the mixing is small. Moreover, at this level of approximation, a zero-entry for the element $U_{13}$ is predicted in the mixing matrix. The rest of the elements are determined by two angles, $\phi$ arising from the charged lepton mass matrix diagonalisation and $\omega$ arising from the neutrino mass matrix.

Working out specific cases we aim to find explicit sets of $U(1)_X$ charges which interpret the neutrino data in the context of the above scenario. For the specific solutions we have set $m'$ even and $n'$ odd. Then, from the formulae of Table 2, we find that the leptons have fractional $U(1)_X$-charges of the form $\frac{2k+1}{4}$, with $k$ integer.

Choosing for example, the values $m = 4$, $n = 8$, $m' = 2$, $n' = 7$, $h_1 = 2$, $h_2 = 0$, $A = -\frac{5}{2}$, we obtain the charge assignments of solution A of Table 3 and the following fermion mass...
matrices for the quarks

\[ M_{u,d} \sim m_{0}^{u,d} \begin{pmatrix} \varepsilon^8 & \varepsilon^6 & \varepsilon^4 \\ \varepsilon^6 & \varepsilon^4 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1 \end{pmatrix}. \]  

(14)

(which is the texture discussed in [10]), the charged leptons

\[ M_{e} \sim m_{0}^{e} \begin{pmatrix} \delta \varepsilon^7 & 0 & 0 \\ 0 & \varepsilon^2 & a \varepsilon \\ 0 & a \varepsilon & 1 \end{pmatrix}. \]  

(15)

and the neutrinos

\[ M_{\nu}^{0} \sim m_{0}^{\nu} \begin{pmatrix} 0 & -\varepsilon^2 & \zeta \varepsilon \\ -\varepsilon^2 & 0 & 0 \\ \zeta \varepsilon & 0 & 0 \end{pmatrix}. \]  

(16)

Charged lepton masses can be fit within a range of the mass matrix parameters in (14). For example, choosing \( \varepsilon \sim 0.28, \alpha \sim -1.3 \) and \( \delta \sim 2 \), the correct mass spectrum is obtained. Atmospheric neutrino oscillation mass-squared difference is then reproduced for \( M \sim 5 \times 10^{13} \text{GeV} \) modulo order one coefficients. This scale is quite low to be identified with the string scale in heterotic constructions, it is however compatible with type I superstring models where the string scale is tight to the Planck scale. Other configurations of additional \( U(1)_X \)-charges are also possible since the mass matrices under consideration do not depend on the parameters \( q_3, \ell_3 \). For example choosing solution B of Table 4 we obtain the same mass matrices as in solution A considered above.

As already noted however, at this level of analysis, the neutrino mass splitting between the first and second generation does not appear because the two eigenstates are degenerate. Moreover, the solar neutrino mixing angle is maximal, a situation disfavored by recent data. This discrepancy can be lifted however, if additional non-zero entries are generated by hierarchically smaller effects. We find it interesting that two additional entries, for example 11 and 23, smaller than the entries 12 and 13 already present at this level, would be sufficient to bring the final form of the neutrino matrix to an acceptable two-zero texture mass matrix [12], that provides the necessary mass splitting and interpret accurately the experimental data. To show that this is indeed the case, let us assume that, after the inclusion of these effects and in the basis where the charged lepton mass matrix is diagonal, the neutrino mass matrix takes the form

\[ M_{\nu} = m_{0}^{\nu} \begin{pmatrix} 2x & -\cos \bar{\omega} & \sin \bar{\omega} \\ -\cos \bar{\omega} & 0 & 2y \\ \sin \bar{\omega} & 2y & 0 \end{pmatrix}. \]  

(17)

where \( \bar{\omega} = \omega + \phi \).

The neutrino mass-squared differences have a ratio which depends on a different \( x, y \) linear combination, \( (x - y \sin(2\bar{\omega})) \),

\[ \frac{\Delta m_{12}^2}{\Delta m_{23}^2} = \frac{4(x - y \sin(2\bar{\omega}))}{1 - 2(x - y \sin(2\bar{\omega}))}. \]  

(18)
while the mixing angles are analogously corrected

\[
\tan \theta_{23} \approx \tan \bar{\omega} , \quad \tan \theta_{13} \approx 2y \cos(2\bar{\omega}) , \quad \tan \theta_{12} \approx 1 - (x + y \sin(2\bar{\omega})) \quad (19)
\]

Using the experimental data we find that experimentally acceptable \(\tan \theta_{12}\) values can be satisfied for \(x \approx [0.10 \text{ -- } 0.24]\) and \(y \approx [0.10 \text{ -- } 0.22]\), assuming \(\bar{\omega}\) to be maximal. We remark that these values in a wide portion of the acceptable range, are sufficiently smaller that the order one 12- and 13-neutrino mass matrix entries and thus our approximation is consistent.

4. Conclusions

In this talk, we have presented a simple extension of the Minimal Supersymmetric Standard model by an anomalous \(U(1)_X\) symmetry broken at some high scale \(M\) and attempted to interpret the recent neutrino experimental data using just the left-handed neutrino components. Assuming symmetric mass matrices and that the third generation of up, down quarks and charged fermions acquire masses at tree-level, we derive the general charge assignments for MSSM fermions and examine their implications for the Majorana neutrino mass matrix resulting from the dimension 5 operator \((LH)^2/M\). We find that the model leads naturally to inverted mass hierarchy for neutrinos, \(\theta_{13} = 0\) and maximal atmospheric mixing for \(M \sim 10^{13-14}GeV\). At this level the absolute masses of the lightest eigenstates are equal and solar neutrino mixing turns out to be also maximal. We show that higher appropriate order corrections lift the mass degeneracy and the solar neutrino data can be accurately described. We derive explicit fermion \(U(1)_X\) charge assignments that realize the above scenario.

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