Markovian Modelling and Calibration of IBMQ Transmon Qubits

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In the design of quantum devices, it is crucial to account for the interaction between qubits and their environment to understand and improve the coherence and stability of the quantum states. This is especially prevalent in Noisy Intermediate Scale Quantum (NISQ) devices in which the qubit states quickly decay through processes of relaxation and decoherence. Ideal quantum states evolve according to Markovian dynamics, which modern devices are not always capable of maintaining. NISQ devices, such as those offered by IBM through their cloud-based open-access IBM Quantum Experience (IBM QE) are regularly calibrated to provide the users with data about the qubit dynamics which can be accounted for in the design of experiments on the quantum devices. In this paper we demonstrate a method of verifying the Markovianity of the IBMQ devices while extracting multiple calibration parameters simultaneously through a simplified process which can be modified for more complex modelling of qubit dynamics.

I. INTRODUCTION

The engineering of quantum computers is based on the ability to create, manipulate, and measure a collection of quantum states. These procedures necessarily require the presence of the quantum states in a surrounding environment. When in contact with such an environment, with many additional degrees of freedom for interaction, the theory of open quantum systems (OQS) is required to describe the dynamics of the quantum states involved [1]. This framework provides insight into the mechanisms and behaviours of interacting quantum states dissipating energy and losing coherence within quantum devices. These mechanisms of dissipation are forms of quantum noise, which is a significant hurdle in the improvement of modern quantum technologies, known as NISQ devices [2].

The open system dynamics are described by a quantum master equation which is typically an extension of simple unitary evolution of a quantum system to include dissipative effects introduced by the environment. Solving the master equation for a particular system provides the quantum channel, in the form of a dynamical map, which is a conduit for the system’s evolution based on all of the present influences [3]. In the most general form, master equations include too many considerations to be easily solved for all scenarios. However, with reasonable simplifying assumptions, such as the Born-Markov approximation which treats the system as memoryless, a Markovian master equation such as the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) form can be applied to many implementations of OQS in quantum devices [4] [5].

Ideal quantum devices fall into the Markovian regime and can be accurately described by the GKSL master equation, as the energy which they dissipate will leave the system, and not remain present to alter future dynamics through dissipated information returning to the system to influence the evolution in ways which are much harder to predict, as in non-Markovian dynamics. Despite this distinction between regimes being very influential on the performance of quantum technologies, it is not often a focal point of discussions of the capabilities of the devices.

One such example is the IBMQ set of quantum devices, openly accessible through the IBM QE platform [6], which has been the test-bed of a lot of quantum research of quantum computation and OQS models. These devices serve as a prime example of the current state of NISQ technologies, and as such have been used in support of competing claims of the compatibility of Markovian descriptions with the dynamics present in these devices under various conditions. Like all modern superconducting qubit devices, the primary decay mechanisms which bottle-neck performance are relaxation and decoherence, typically characterised by $T_1$ and $T_2$ times, respectively. These calibration benchmarks, among others, have become standard measurements of performance of quantum computers, as reflected by the amount of research published on the topic in recent years [7] [12].

In the case of this work, the quantum devices being investigated are IBM’s superconducting transmon qubit processors, which operate through the use of super-cooled superconductors making use of artificial atoms comprised of Cooper Pairs (CPs). It is the presence, or lack thereof, of these pairs in the circuits that are the basis of the two-level-system (TLS) necessary for a quantum computational basis. The transmon architecture of an anharmonic quantum oscillator introduces the non-linearity between energy levels which ensures that the quantum states can be individually addressed without accidental driving of higher energy levels [13] [14].

Although there has been much research into all of these topics separately, there is a lack of investigation into the overlap of them all, particularly in the realm of application to currently operational NISQ devices. Recent research, such as that of Pollock et al. [15] [16], and Samach

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et al. \cite{17}, has shed more light on this area and serves as the inspiration for the present work.

The focus of this research is on the characterisation of benchmarking metrics, such as $T_1$ and $T_2$ times, into the Markovian or non-Markovian regimes to verify the applicability of simplifying assumptions used in the use of NISQ devices as test-beds for quantum dynamics. The methods used here are tested on IBMQ devices due to their accessibility and role as a flagship of the NISQ era, though the results and procedures are generalisable to other quantum devices. Furthermore, we offer a new method for real-time calibration of several hardware parameters within one comprehensive experimental process, which provides quantitative insight into the performance of each device to be accounted for in future experiments. We also offer insight into the necessity of broad-scale tomography of these devices by showing situations in which a series of smaller tomography procedures can be equally effective and more efficient than the typical computationally expensive standard full tomography procedures.

This paper is structured as follows. In section II, we discuss the necessary theory behind the fields which overlap in this work. In section III, the experimental methodology is outlined. In section IV, the experimental results are discussed in greater detail along with their implications. In section V, we summarise and provide concluding remarks.

II. BACKGROUND THEORY

Transmon qubits, used in many modern superconducting quantum processors such as those of IBM, are made of electronic circuits which create Josephson junctions through connecting superconducting islands to superconducting reservoirs through an insulating barrier. This allows for a driving voltage to allow Cooper Pairs to tunnel through the junction forming a controllable artificial two-level-system wherein the number of excess CPs defines the quantum number. The transmon architecture introduces anharmonicity into the system which prevents the accidental addressing of higher energy levels and constrains the qubits to the computational basis $|0\rangle$ and $|1\rangle$.

To allow for the full potential of quantum computation to be realised, these qubits cannot only be used individually, but in conjunction with each other for a full ensemble where the exponential performance increase can be obtained. This requires the ability of the qubits to connect to other qubits in the processor to create entanglement and superposition. This is obtained through coupling junctions between the qubits which have coupling strengths defined by the hardware parameters of the circuit, similarly to the qubit frequency. The organisation of these couplings give rise to a plethora of topologies which carry with their own unique dynamics. For example, one could expect that a qubit connected to several neighbours would undergo different dynamics to a qubit with only one coupling junction.

The energy of the transmon qubits manifests in the form of qubit frequency, which is a parameter which can be directly controlled and maintained to a high degree of accuracy which assists in the individual addressing of the qubits. This frequency is typically on the order of $\sim 5\ \text{GHz}$. Additionally, to allow for the use of superconducting phenomena, the circuits need to be below the critical temperature at which the materials use become superconducting. In the case of most transmon devices the processors are kept at $\sim 10\ \text{mK}$, although this number is not stringently controlled due to the ambient fluctuations in temperature stemming from various sources.

These temperature fluctuations are one of many sources of noise in the system which deteriorates the ability of the quantum states to carry information. The property of a quantum state to maintain its information without dissipating and reaching equilibrium with its environment is known as coherence. It is this coherence which allows for quantum computation as without it no information can be manipulated through a computation, and due to the natural fragility of quantum states which are consistently undergoing interactions with the environment, this window of coherence in which computations can be performed is very small. This characteristic time is the coherence time of a qubit, and in the case of NISQ transmon devices is on the scale of $\sim 100\ \mu s$.

There are two dominant forms of a coherent quantum state decaying in a quantum computer, namely the processes of relaxation and decoherence, which are typical performance benchmarks inherited from older realisations of quantum experiments, such as NMR. These processes effectively describe the evolution of an excited quantum state decaying to the ground state through simple spontaneous emission (relaxation), and the transverse coupling to environmental noise to reach equilibrium with the rest of the system (decoherence). These processes have been depicted in a Bloch sphere representation in fig. 1.

The dynamics of these qubits is typically described only by the system Hamiltonian, which in the case of IBMQ transmon qubits has the form of a Duffing oscillator. For example, for a single qubit device, the Hamiltonian in terms of qubit frequency, $\omega_{q,i}$, and anharmonicity, $\Delta_i$, is

$$\mathcal{H} = \frac{\omega_q}{2}(1 - \sigma^z) + \frac{\Delta_i}{2}(O^2 - O) + \Omega_dD(t)\sigma^x,$$

where $O_i = b_i^\dagger b_i$, $b_i^\dagger = \sigma^+$, $b_i = \sigma^-$, $b_i^\dagger + b_i = \sigma^x$ are the operator transformations used and $\Omega_d$, $D(t)$ being qubit-drive parameters. The $\sigma$-operators throughout this work refer to the Pauli matrices. Similarly, but with more detailed structure with included inter-qubit coupling, $J$, the 2-qubit Hamiltonian is
The Hamiltonians in (1) and (2) are essentially identical to the simplified versions,
\[ \mathcal{H} = \frac{\omega_0}{2} \sigma^z, \]

\[ \mathcal{H} = \frac{\omega_0}{2} \sigma^z + \omega_1 \sigma^z + J_{0,1} (\sigma_0^+ \sigma_1^- + \sigma_0^- \sigma_1^+), \]

for shallow quantum circuits where the anharmonicity and driving parameters do not have the opportunity to make any significant difference. These Hamiltonians have a clear assumption concerning the axes involved in the definitions of the parameters, in that they are rigid along each axis involved, meaning that the qubit frequency is assumed to only have a component along the z-axis, and the jump operators of the coupling are assumed to flip the qubit Bloch-orientations perfectly and only along one direct axis of coupling.

A more generalised form of these Hamiltonians needs to be investigated to uncover any underlying deviations to the initial assumptions. For the single-qubit case, this generalised Hamiltonian has the following form,
\[ \mathcal{H} = \frac{1}{2} (\vec{\omega} \cdot \vec{\sigma}), \] where \( \vec{\omega} = (\omega_x, \omega_y, \omega_z) \) \& \( \vec{\sigma} = (\sigma_x^+, \sigma_y^+, \sigma_z^+) \).

Similarly, for the two-qubit case, there are terms for each qubit, denoted by subscripts, and can be expressed as
\[ \mathcal{H} = \frac{1}{2} (\vec{\omega}_0 \cdot \vec{\sigma}_0) + \frac{1}{2} (\vec{\omega}_1 \cdot \vec{\sigma}_1) + \vec{\sigma}_0 \vec{J} \vec{\sigma}_1, \]

where the new coupling matrix term is defined as
\[ \vec{\sigma}_0 \vec{J} \vec{\sigma}_1 = (\sigma_0^x, \sigma_0^y, \sigma_0^z) \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix} \begin{pmatrix} \sigma_1^x \\ \sigma_1^y \\ \sigma_1^z \end{pmatrix}. \]

This is a far more inclusive description of the qubit Hamiltonian as it will give insight into the behaviour of the qubits which is not limited to the ideal axes, but rather includes external influences which can give insight into possible noise sources which are not accounted for. The Hamiltonian is worth this much focus and investigation as it plays a crucial role in the modelling of the qubit dynamics for this investigation.

It is this complex set of dynamics which reinforces the necessity of the theory of open quantum systems to model and predict the evolution of a quantum device performing a calculation or execution of a quantum algorithm. In the framework of OQS, these dynamics are typically encapsulated in a dynamical map, \( \Lambda(t) \), which is a collection of completely positive trace-preserving (CPTP) maps, that constructs a quantum channel describing the path of evolution of a collection of interacting quantum states represented by a density matrix, \( \rho \). This evolution of the collection of quantum states is easily expressed in terms of a dynamical map as
\[ \rho(t) = \Lambda(t) \rho(0). \]

In most cases of describing quantum devices, the dynamical map will satisfy a time-local master equation,
\[ \frac{d}{dt} \Lambda(t) \Lambda(t) \Rightarrow \frac{d}{dt} \rho(t) = \mathcal{L}(t) \rho(t). \]

This introduces the Lindbladian generator, \( \mathcal{L}(t) \) which has the form of
Fig. 1: A Bloch sphere representation of the noise processes of relaxation (a), pure dephasing (b), and decoherence (c), respectively. Relaxation is the process of an excited state, \(|1\rangle\), decaying to the ground state, \(|0\rangle\). Pure dephasing is the fluctuations along the \(x-y\) axis from the initial \(|+\rangle\) state in this example. Decoherence is the combination of starting in the ground state, moving to the transverse axis, dephasing, and going back to the ground state.

\[
\mathcal{L}(t)\rho = -i [\mathcal{H}(t), \rho] + \sum_{\alpha} \gamma_{\alpha} \left( A^\dagger A_{\alpha} \rho - \frac{1}{2} \{ A^\dagger A_{\alpha}, \rho \} \right),
\]

where the first term describes the unitary evolution of the system in natural units, the coefficients \(\gamma_{\alpha}\) represent decay rates of the system, and \(A^\dagger A_{\alpha}\) represents noise (or “jump”) operators. The form of \((10)\) is the GKSL generator corresponding to the master equation which describes Markovian dynamics, which neglects memory effects of the system and assumes that any information dissipated from the system does not return to influence future dynamics. There are many descriptions of non-Markovian dynamics due to its much broader scope, however in this work the simple definition we will assume is that non-Markovian dynamics are any that are not accurately described by the GKSL master equation.

This Markovian framework allows for the extraction and verification of more claimed qubit parameters from the calibration metrics, such as the relaxation and decoherence times, as well as an extraction of information not included in the calibration, such as the qubit temperatures. These capabilities are all included in the GKSL master equation, which for this application is expressed, for a single qubit, as

\[
\frac{d}{dt} \rho = -i [\mathcal{H}, \rho] + \gamma \left( \langle n \rangle + 1 \right) \left( \sigma^- \rho \sigma^+ - \frac{1}{2} \{ \sigma^+ \sigma^-, \rho \} \right) + \gamma \left( \langle n \rangle \right) \left( \sigma^+ \rho \sigma^- - \frac{1}{2} \{ \sigma^- \sigma^+, \rho \} \right),
\]

where \(\gamma\) is the emission coefficient, and acts as an inverse of the relaxation and decoherence times, and \(\langle n \rangle\) represents the average number of photons emitted as the density matrix evolves, which is represented by

\[
\langle n \rangle = \frac{1}{e^{\hbar \omega / k_B T} - 1}.
\]

This encapsulates the ideal Markovian dynamics of the system, even at absolute zero, and allows for the extraction of decay times through the emission coefficient and temperature through the photon number. For the \(N\)-qubit case, the expression is extended to include the density matrix evolution for each qubit, expressed as

\[
\frac{d}{dt} \rho = -i [\mathcal{H}, \rho] + \sum_{i=0}^{N-1} \left[ \gamma_i \left( \langle n_i \rangle + 1 \right) \left( \sigma_i^- \rho \sigma_i^+ - \frac{1}{2} \{ \sigma_i^+ \sigma_i^-, \rho \} \right) + \gamma_i \langle n_i \rangle \left( \sigma_i^+ \rho \sigma_i^- - \frac{1}{2} \{ \sigma_i^- \sigma_i^+, \rho \} \right) \right].
\]
the $|1\rangle^n$ state. After this excitation, the qubits are left to decay for a variable time, here set to be a period of 296 $\mu$s, after which the states are measured in the computational basis and the state distribution calculated.

For the $T_2$ procedure, the Hahn echo sequence [18] is used, depicted in (QC2), which consists of a set of $n$ qubits being initialised in the $|0\rangle^n$ state, after which a set of $\pi/2$ rotations is applied, around the $x$- or $y$-axis. Succeeding this is a delay period of variable time followed by a $\pi$ rotation around the same axis as the first rotation, and then another delay time of the same period, and a final $\pi/2$ rotation in the same direction to return the state to $|0\rangle^n$ where it can be measured.

If the coupling is strong enough and not shielded in some way, then the decay of the excited qubit will directly influence the stationarity of the other. By performing the experiment with the exclusion of excitation operators entirely, shown in (QC6), the qubits are left in the ground state to idle for the delay period to investigate if there are any external sources which unintentionally excite the subsystem of qubits.

To provide further insight into the relaxation and decoherence mechanisms of the qubits, the procedures described before can be combined into new composite systems to show the dynamics between various decay mechanisms, and how the decay of one qubit in the system might influence its neighbours which are ideally excluded from the subsystem. To achieve these, the 2-qubit ensemble is modified into new quantum circuits, the first of which has one qubit undergoing a standard $T_1^*$ experiment, while its neighbour undergoes a $T_1$ experiment, having the first $H$ gate be replaced by an $X$ gate and the
second $H$ replaced by an identity gate, $I$, illustrated in (QC7).

\[ |0\rangle \xrightarrow{H} \xrightarrow{\text{Delay}(\Delta t)} H \xrightarrow{\text{Delay}(\Delta t)} |0\rangle \quad (\text{QC7}) \]

Similarly, the other modified circuit has the first qubit undergo a $T_2$ sequence, while the neighbour qubit simply stays in its idle state from the $|0\rangle$ initial state, as shown in (QC8).

\[ |0\rangle \xrightarrow{H} \xrightarrow{\text{Delay}(\Delta t)} H \xrightarrow{\text{Delay}(\Delta t)} |0\rangle \quad (\text{QC8}) \]

All of these circuits return a set of average state distributions for each time step and show how the states evolve through these different scenarios. To elucidate what should happen from the theoretical standpoint, it is necessary to return to the master equation (13). This equation is very complicated to solve analytically, so numerical methods must be used to obtain useful information, the details of which will be discussed in the next section. A solution for this equation can be found for a set of parameters, $\vec{x}$, which is dependent on the form of the master equation and the Hamiltonian used. For example, the single-qubit master equation solution with the simple Hamiltonian (3) is a function of 4 parameters, $\vec{x} = (t, \omega, \gamma, T)$, being time, qubit frequency, emission rate, and temperature. However for the general Hamiltonian (5), the solution is a function of 6 parameters, $\vec{x} = (t, \omega_x, \omega_y, \omega_z, \gamma, T)$.

The size of the parameter vector quickly grows for multi-qubit states, as in the example of a 2-qubit subsystem, the solution will require 8 parameters for the simple Hamiltonian (4), and 20 parameters for the generalised Hamiltonian (6). Nonetheless, these equations can be numerically solved as a function of these parameters and initial states from the qubits at the start of the delay period, to return a time-series of the evolution of the density matrix. The solution of the master equation can be expressed as the integral of

\[
\frac{d}{dt} \rho(t) = \mathcal{L}(t) \rho(t) \quad \rho_0 = \rho(t = 0) = |1\rangle \langle 1|, \quad (16)
\]

in the case of the single-qubit $T_1$ sequence, where the system starts in the state $|0\rangle$ and is excited to $|1\rangle$. This evolution through the delay periods in the quantum circuits can be combined with the quantum gates as operators in the construction of the quantum channels to describe the entire evolution of the system. For example, in the single-qubit $T_1$ experiment, the delay period will be described by a master equation solution $\rho_d(t)$, and the excitation gates are Pauli $X$-matrices, $\sigma^x$, so the quantum channel is described by

\[
\mathcal{E}(\rho) = \sigma^x \rho_d(t) \sigma^x. \quad (17)
\]

The full solution of the master equation for a set of parameters which is passed through the Kraus form of the quantum channel is then a set of values which are comparable to the experimental results which are obtained through the respective quantum circuit. Through the use of the parameters provided by the periodic device calibration, the master equation solution can be compared directly to the experimental results to verify the accuracy of the calibration data. This verification process allows for all of the hardware parameters to be verified in conjunction with one another, rather than the independent experiments which were used to extract those values initially.

Furthermore, this method provides the capability to improve upon the parameter extraction in the case that the claimed parameters do not match the experimental observations. The parameters used in the master equation solution can be iteratively varied to provide different results, until a parameter set is found which accurately matches the experimental data. This means that the parameter set can be optimised for the smallest difference between the numerical and experimental results.

This can be achieved very easily through conventional optimisation methods which have shown extreme success in achieving this form of outcome. For example, this method is used extensively in the field of machine learning, where the learning model needs to have its parameters varied to match the dataset which it is learning, thereby increasing its predictive accuracy. The optimisation method used to achieve this, and which is used in this work, is gradient descent. This is an iterative algorithm to find the minimum point of a differentiable function, through finding the steepest path towards this minimum through calculating the negative gradient of the function which finds the direction of the fastest change in the function, for a fixed step-size $\alpha$,

\[
f_{n+1} = f_n - \alpha \nabla f_n. \quad (18)
\]

In this form, the method for optimising this function is very inefficient and prone to error due to the solution landscape created by the many parameters. In this work, the more sophisticated and accurate gradient-descent method of the Adaptive Moment Estimation (Adam) optimiser is used. The details of this algorithm are
III. EXPERIMENTAL PROCEDURE

Communication with the quantum devices is done through the Qiskit SDK [20], which allows for the extraction of backend configuration information, as well as the construction of quantum circuits which are converted to basis gate circuits to perform the experiments. Once these circuits are run, the data can be extracted and used as desired. Qiskit also offers many built-in functionality such as readout error mitigation and state tomography procedures.

Qiskit allows for a selection of several backend devices to be worked with, and for each offers a set of calibration data as well as device properties such as the qubit topology. This information was used in this work as the parameters of the numerical solutions to verify the accuracy of the calibrations performed on the backend side. This information also provided scales of extra errors, such as readout and gate errors, which could be accounted for in analysing the data.

This work made extensive use of the ibmq_armonk v2.4.23 device for single-qubit experiments, as well as the ibmq_santiago v1.3.40 and ibmq_manila v1.0.19 devices for multi-qubit circuits as these devices have 5 qubits each. These devices are examples of IBM’s Canary and Falcon architectures, respectively. The topology of these devices is a simple linear structure, as in Figure 2, and allows for the investigation of coupling effects between neighbours.

Fig. 2: Qubit topology of 5-qubit ibmq_santiago v1.3.40 and ibmq_manila v1.0.19 devices. The colours of the qubits represent the frequency, with darker meaning a lower frequency and lighter meaning a higher value.

Once the backend was selected the circuits could be constructed. The circuits introduced in the last section needed to be modified slightly to obtain useful data. Due to the nature of quantum measurement, the state distribution could not be continuously measured, otherwise the Quantum Zeno Effect [21] would alter the data completely by not allowing the system to undergo the desired decay. This means that running the circuits as presented, for example, would return only one time-slice of the data and the bit-string probability distribution at that snapshot in time. To avoid this, a series of circuits needed to be constructed and run with the delay time being varied to allow for each time step to be a new point in the dataset. For this experiment, the delay time was varied through a period of 0 s to 296 µs in steps of 4 µs for a total of 75 quantum circuits corresponding to as many data points.

In the execution of these experiments, there are unavoidable errors in the process which detract from the fidelity of the desired experimental results. These errors are primarily SPAM errors of readout and gate execution, which are not important in the present investigation but plague the data nonetheless. These errors must be accounted for to obtain meaningful data. The gate errors are avoided by using time-scales small enough for the gate execution times to be insignificant, and the actual errors are combated by statistical rigour. As mentioned in previous sections, the probabilistic nature of quantum system demands that experiments and measurements be made multiple times to create an ensemble with a reliable probability distribution from which average values can be extracted. In this experiment each experiment was run for 8 192 iterations, or “shots” in Qiskit, which ensures that the variance between experiments is suppressed enough to avoid the influence of gate errors.

The readout errors, however, are present throughout the device and cannot be avoided through collecting more data for each experiment, but rather needs to be calibrated for within the experimental run of executing all of the circuits. The measurement calibration functionality built into Qiskit follows a procedure of preparing all of the qubits in the system, or a defined subsystem, in a certain state, which in this case is similar to tomographic methods in that it produces all possible states, and measuring immediately afterwards. This produces a state distribution demonstrating the accuracy of the readout, which can be compared to the ideal case where the outcome from such an experiment is known analytically. These processes were applied and accounted for in each iteration of the experiments to minimise the influence of SPAM errors throughout the collection of results.

As the numerical procedure of calculating the master equation solution and optimisation algorithm is heavily based on calculus, the JAX SDK [22] was used as it provides substantial increases in numerical performance particularly through its auto-differentiation and Just-In-Time (JIT) Compiler features which assist in calculus based and iterative calculations.
Fig. 3: \(T_1\) relaxation density matrix evolution. Experimental data represented by dots and numerical data for claimed hardware parameters represented by solid lines. Results shown for 1-qubit (a), 2-qubit (b), and 3-qubit (c) cases.

The first numerical procedure is that of solving the master equation (16), which is performed through the built-in JAX function “ODEint” designed to integrate ordinary differential equations (ODEs). This method takes in an initial state density matrix which describes the \(|0\rangle^n\) state and all of the operations performed on it before the delay period begins which is the time evolution that the master equation describes. The method returns a \(75 \times 2^n \times 2^n\) array containing the 75 snapshots of the density matrix, of which the diagonal elements represent the observed states which can be measured experimentally. The parameters for the master equation solution were taken directly from the hardware configuration data from Qiskit for the most accurate comparison.

From these results, it can be seen that the claimed hardware parameters do not typically produce the same qubit dynamics as what is experimentally observed, although they are quite similar hinting at the model being correct but the parameters being inaccurate. This demonstration of inaccurate configuration claims further reinforces the need for a method to find the correct values which match the observed behaviours.

To apply the optimisation algorithm to find the correct parameters, the Adam optimiser was used based on the Jacobian vector of parameters and a least-squares function created to find the difference between each data point from the experimental dataset and the numerical calculations. The optimiser was run typically with an initial step-size being one of \(\alpha = (0.01, 0.1, 1)\), depending on the initial conditions of how different the functions were from each other. Similarly, the algorithm was run for a total number of iterations ranging from 200 to 500 depending on the case. The initial parameter set for the optimisation was usually the claimed hardware parameters, however in the cases where the initial datasets were too different the Adam optimiser could not obtain the optimal result in one application, and so was run multiple times in some cases, using each previous attempt’s optimal parameters as the initial set.

In the application of the optimisation algorithm, there was a notable discrepancy in its effectiveness depending on the quantum channel it was applied to. For example, across all of the \(T_1\) sequences, irrespective of the system size, the optimisation always found excellent fits, with
least-squares errors being minimised to values on the order of $\sim 10^{-3}$. This indicates as good of a fit as could be desired with the only differences between the numerical and experimental results being random fluctuations due to the nature of the experimental procedure measuring 75 sequential sub-experiments rather than a single sequence. A demonstration of these optimised results is depicted in Figure 4.

In order to further investigate the behaviour of the $T_1$ sequence, particularly in terms of the coupling between multiple neighbouring qubits, the modifications of the sequence using different initial states, as in the circuits QC4 & QC5, the same process of circuit compilation, execution, and fitting was followed. The results obtained by these verified the stability and isolated nature of this simple relaxation sequence, as can be seen in Figure 4. The data show an expected behaviour of the inter-qubit coupling not influencing the Markovianity of the system evolution, which is seen in the ground-state qubits staying in the ground state through the full period, while the excited state population follows a simple exponential path to equilibrium.

Although the $T_1$ sequences for all of the qubit sizes and initial states proved to be very stable and reliable, the rest of the experiments provided more intricate results. For a first example, the $T_2$ sequence for a single qubit proved to be a significantly more difficult numerical procedure, taking far longer to calculate the results despite the simple appearance of exponential decay, as seen in Figure 6a. Despite the difficulty in extracting numerical solutions and finding optimal parameters, the procedure still found great success in achieving these goals. The optimal parameters provided a very accurate fit, similarly to the $T_1$ outcome, having a least-square error on the level of $\sim 10^{-2}$ consistently.

In the 2-qubit case of the $T_2$ sequence, the results obtained show a more intricate behaviour, particularly in the combined forms of exponential decay and oscillatory forms, which offer more room for error to be identified in noisy systems. Despite this, the numerical results closely match the experimental outcome and display only a slight deviation from the claimed parameters which makes for a relatively simple optimisation procedure to offset the computational demand of the optimisation itself. This particular part of the project highlighted a slight unforeseen error in the execution of the quantum circuits, as the time-scale and component time steps exhibit a constant scaling factor of $1.3 \times$, meaning that rather than the ideal $592 \mu s$ period the procedure amounted to $\approx 769.6 \mu s$ overall. Each time step had this scaling phenomenon and it occurred consistently throughout every attempt at the experiment, which leads to the suspected phenomenon of an error in the compilation of the circuit when converting the gate-based formalism to pulse sequences and ensuring compatible scheduling. With this being the source of the behaviour, there is no significant impact to the results obtained as the numerical results simply need to be scaled to match this time scaling, as is confirmed by the accuracy of the results obtained by the optimisation sequence, seen in Figure 6b.

A similar pattern is present in the 3-qubit case of the $T_2$ sequence, with a similar style to the 2-qubit case in having damped oscillations as well as gentle exponential curves to the asymptotic equilibrium state. The attempts of this experiment did suffer the general difficulty in extracting high fidelity results from 3-qubit experiments, however in the cases where the data were well behaved and reliable the results fitted the expected dynamics from which optimised parameters could be extracted. The nature of 3-qubit sequences being less reliable in general further emphasises the versatility and usefulness of the single qubit dynamics which can be used to predict larger subsystem dynamics as well, as will be seen later.

The results of the $T_2^*$ sequences proved to be the least reliable results of the ensemble, exhibiting a lot of stochastic errors throughout the experiments which could not be described by the numerical modelling. This is not to say that the results are necessarily non-Markovian, but rather that the sequences themselves were subject to errors and deviations between each time step, as the experimental procedure could not observe the evolution of one state from start to finish, but rather comprised of 75 experiments, each providing a snapshot of the time step of interest. This procedure relies heavily on the assumption that each replication of the experiment undergoes the same behaviour, which holds well for all of the previously mentioned sequences, however not here. This can be seen in Figure 6c where the optimal numerical modelling of the procedure with the best parameters still only roughly mimics the experimental results, with the fit becoming worse for later time steps where more random errors are prevalent, creating unpredictable jumps in the curves. In some evaluations of the experiment the results were well behaved as expected, as in the figures displayed here, while in others the results appeared to be scaled by some sinusoidal function of a constant angle scaled by time. This leads to the speculation that in some part of the calibration or measurement of the quantum devices, there is some slight offset which is not accounted for in the error mitigation schemes.

This erratic behaviour is seen throughout the modifications to the $T_2^*$ sequence, but most clearly in the single-qubit case where the experiments were run on a device with only one qubit, so any external factors could be explicitly seen. The influence of this suspected SPAM error seems to manifest in providing some oscillatory factor while also exaggerating the peaks and troughs of existing oscillations, which can be seen, albeit in a less evident form, throughout the 2-qubit examples. In the case of a 3-qubit version of this sequence, the results are far less reliable, as the fluctuations in the trajectories of each of the 8 states quickly overwhelm any expected behaviour and blend into being all within the same range. This poses a problem as this equilibrium state is reached too rapidly for there to be any clear indication of which results are fluctuations and which are following the correct
Fig. 4: Optimised $T_1$ relaxation density matrix evolution. Experimental data represented by dots and numerical data for optimised parameters represented by solid lines. Results shown for 1-qubit (a), 2-qubit (b), and 3-qubit (c) cases.

| Claimed   | Experimental |
|------------|--------------|
| $\omega$ (GHz) | $T_1$ (µs) | $T_2$ (µs) | $T$ (mK) |
| $q_0$ | 31.42 | 100.24 | 31.42 | 101.23 | 47.96 |
| $q_1$ | 30.47 | 106.95 | 30.47 | 108.31 | 54.20 |
| $q_2$ | 30.05 | 101.45 | 30.05 | 105.92 | 50.30 |

TABLE I: Claimed and experimental results for the single-qubit experiments to measure the qubit frequency, $\omega$, relaxation time, $T_1$, and qubit temperature, $T$. 

path, especially considering that the readouts of the values over a set of 8192 iterations of the sequence means that the fluctuations are still on the scale of the variance between iterations.

This is fortunately not the case for other sequences on the 3-qubit scale, but still limits the focus of the results to the 1- and 2-qubit cases. Furthermore, in the case of the $T_2^*$ sequences and modifications thereof, there were not any claimed parameters of the $T_2^*$ time for the qubits accessible through calibration data, as only $T_1$ and $T_2$ times were included there, which typically matched fairly well to the experimental results but still warranted the optimisation process. Thus, in the optimisation of the $T_2^*$ numerical results, the $T_2$ parameters were used as initial conditions, but lead to unexpected optimal parameters which do not reflect the standard hardware parameters but are rather slightly warped to match the sequences where gates such as the Hadamard are involved. For further elucidation of this, the next section will go into a thorough analysis and discussion of the obtained results.

IV. ANALYSIS AND DISCUSSION

Apart from the $T_2^*$ sequences, which provided lower fidelity data, all of the experimental procedures proved to attain successful and reliable data which in turn provided valuable insight. The data allowed for very effi-
Fig. 5: Optimised $T_1$ relaxation density matrix evolution for varied initial conditions. Experimental data represented by dots and numerical data for optimised parameters represented by solid lines. Results shown for initial states $|10\rangle$ (a), $|01\rangle$ (b), and $|11\rangle$ (c).

icient optimisation of hardware parameters, allowing for the probing of these values without direct access to their measurements from a relatively simple theoretical model. This section will focus on the detailed analysis of these results and the numerical accuracy of all the data obtained.

In the case of single qubit experiments, all of the sequences proved to be highly successful and reliable to extract information, mostly due to the simple forms of the qubit dynamics. These results didn’t have any surprising features, as the experimental results had high fidelity and inter-iteration consistency, while the numerical results were based on a simple form of the GKSL Master Equation which made the computation very quick and easy, especially in the optimisation process which thereby allowed for more iterations to be run and higher accuracy to be achieved. The set of hardware parameters which were extracted was the smallest of all the subsystems, due to the simple Hamiltonian form, which did provide useful insight despite the simplicity. The Hamiltonian parameters extracted, being the qubit frequencies along each Bloch sphere axis ($\omega_x, \omega_y, \omega_z$), showed that the simplifying assumption given by the backend providers of there only being a $z$-component is not entirely accurate. This was shown by there being small, but significant, contributions to the qubit frequency along the $x$- and $y$-axes, with the vector norm of these being roughly equivalent to the claimed hardware parameter, within $\sim 1$ GHz. This would ordinarily not pose any issue for the function of the devices, so the backend claim of the simplified model is effectively correct in most use cases, however this general form should not be ignored as it can prove to be a significant factor in the scalability of larger devices with potential resonance with neighbouring qubits.

In the case of the 2-qubit subsystems, the data showed a similar pattern, in that the simple $T_1$ sequence provided the most accurate results which gave rise to insightful optimised parameters. These parameters showed once more that the simple effective Hamiltonian claimed by the backend provider does not show the full picture, but rather each qubit has contributions to the frequency from the transverse Bloch sphere axes, and a similar pattern is observed in the qubit coupling parameters which are not the simplified scalar values claimed to act only upon the
Experimental results for the 2-qubit experiments to measure the qubit frequency, $\omega$, relaxation time, $T_1$, inter-qubit coupling, $J$, and qubit temperature, $T$.

| $q_{01.0}$ | $31.42$ | $100.24$ | $8.31 \times 10^{-3}$ | $31.42$ | $96.49$ | $5.87 \times 10^{-3}$ | $6.62$ |
|-----------|--------|---------|----------------------|--------|---------|----------------------|-------|
| $q_{01.1}$ | $30.47$ | $106.95$ |                       | $30.47$ | $109.62$ |                       | $65.55$ |
| $q_{12.1}$ | $30.47$ | $106.95$ | $7.42 \times 10^{-3}$ | $30.47$ | $109.26$ | $5.25 \times 10^{-3}$ | $73.63$ |
| $q_{12.2}$ | $30.05$ | $101.45$ |                       | $30.05$ | $108.31$ | $6.42$                |        |

TABLE II: Claimed and experimental results for the 2-qubit experiments to measure the qubit frequency, $\omega$, relaxation time, $T_1$, inter-qubit coupling, $J$, and qubit temperature, $T$. 

$x$- and $y$- axes, but rather a $3 \times 3$ matrix of coupling along all axes. This discrepancy does not pose any significant influence to the relaxation and decoherence dynamics investigated here, but once more should be accounted for in considering the scalability of the quantum devices to avoid unwanted noise from resonances. Significantly, in terms of the temperatures of the devices, which did not have claimed calibration values but rather a general order of magnitude claim, the average photon number of the Markovian dynamics demonstrated that these claims are accurate to within ±10 mK, proving that this method is a viable approach to calibrating the device and individual qubit temperatures. It should be noted, however, that this temperature measure does have the disadvantage of being an inferred value from the photon emission contribution to state decay, and as such does carry inherent inaccuracy compared to a direct measurement.

The $T_2$ sequence for 2-qubit subsystems demonstrated interesting and well behaved dynamics, including forms of a damped oscillator as well as exponential evolution to equilibrium states, which allowed for a direct demonstration of the Markovianity assumption with a more complicated architecture. This sequence yielded similar results to the $T_1$ sequences, in terms of the hardware parameters which could be extracted, as they were consistent with expectations as well as the claimed parameters and optimised values from the $T_1$ sequences. The $T_2$ sequences proved to be slightly less reliable due to some off-resonance SPAM error, as discussed before, which would quickly deteriorate the results in most cases and enforcing the requirement of multiple attempts at the experiment to obtain accurate results. When these more accurate results were obtained, however, the optimisation process could work with great proficiency and give accurate fits to the experimental data. The hardware parameters which this optimisation led to were not as would be expected by the trends set by previous sequences. Rather, all of the hardware parameters underwent a significant scaling phenomenon, shifting them all away from claimed parameters given by the backend providers. For example, the qubit frequencies underwent a dramatic shift by up to a factor of 2x, which is characteristic of the dephasing of the system, while also having average photon number contributions to the decay rates reflecting qubit temperatures on the scale of ~1 K rather than the expected ~15 mK. Additionally, the actual decoherence time in the form of the decay rate $\gamma$ was much larger than the claimed parameters, which does not necessarily indicate a successful increase in qubit coherence, but is likely rather a relic of the sequence possessing a more complicated form which would require more careful analysis to extract hardware information.

To assist in the associated difficulties of the $T_2$ sequence, the modified sequences of (QC7) and (QC8), representing one qubit undergoing the $T_2$ sequence with its neighbour undergoing the $T_1$ and idle sequences, respectively. These modifications allowed for the damping of the resonant noise of 2 qubits being susceptible to SPAM errors, and provided a more granular look at the qubit dynamics. In the case of (QC8), this problem of resonant external noise remained persistent, though showing a scaled intensity of the errors, reinforcing the suspicion that the $H$ gates in particular are more difficult to implement effectively. In the case of (QC7), however, a new and interesting form of the qubit dynamics was seen, in Figure 6d, which coupled the simple exponential decay of the $T_1$ sequence with the damped oscillator form of the $T_2$ sequence, which allowed for proficient optimisation and extraction of hardware parameters which were consistent with the results from the separate sequences showing that the mixing of these experiments does not produce any phenomena greater than the combination of its parts.

As another example of subsystems combining in a linear and separable manner, the $T_1$ sequence is worth returning to for a focussed analysis on the predictions made by different subsystems of overlapping qubits. For this case, the single qubit results for individual neighbouring qubits, which provided very accurate and reliable behaviours, can be compared to the results obtained by 2-qubit subsystems comprised of those individual neighbours, as well as separate 2-qubit subsystems which overlap. This analysis goes one step larger to the 3-qubit system as well for an additional comparison point of the predicted hardware parameters and behaviours. For an example of the specifics of this kind of procedure, the first 3 qubits (indexed as 0, 1, and 2) of a 5-qubit system were analysed and optimised to extract parameters which agreed accurately with the claimed values, within a small margin of error which justifies the use of optimisation. Then two sets of 2-qubit subsystems, focussed on qubits 0 & 1 and 1 & 2 respectively, went through the optimisation process to produce values comparable with the single qubit case as well as the doubly predicted qubit 1 parameters. These 3 qubits comprised the final overall system to investigate the scalability of this method.
The results which this procedure produced, as seen in Tables I, II, and III were very promising in that the single qubit results demonstrated excellent accuracy to the experimental data, while having these results replicated by the 2-qubit subsystems which were internally consistent, and finally the 3-qubit system to confirm this accurate extraction. These results not only reflect the findings of the $T_1$ sequences discussed previously, about the generalised form of the Hamiltonian and more accurate decay parameters, but also confirm the viability of this method to predict larger systems of qubits. This means that multiple 2-qubit systems can be run and analyzed, similarly to tomography procedures, and then be used to extract parameters and act as a form of calibration, but then also be stitched together through tensor products to obtain a tomography of a larger system which would ordinarily grow exponentially in difficulty.

This method is, however, fairly unreliable as a self-contained single-run tomography procedure which is mostly the fault of the backend rather than the method. The backend is hindered by the amount of traffic it faces with it being a cloud-based open-access service which occasionally leads to queue times lasting longer than several calibrations which occur hourly. This leads to some experiments being run on different calibration profiles and after different reset periods which affect the hardware parameters dictating the qubit dynamics which finally leads to inconsistencies in the extracted optimal data which destroys the integrity of the tomography. Additionally, as the backend is open access, the attainable information is fairly limited, for example with the device temperatures which would be able to be monitored more efficiently with direct access to the devices. Although this method is meant to be useful especially in such cases to extract information which is not being actively monitored, there is a significant distance to cross in the feedback from the devices which are ultimately controlled by the backend compilation which provides a larger margin for error between the gate composition and the execution of the quantum circuits.

Despite the difficulties faced in this setting of the experimental process, the data show very promising results for the viability of this as a form of tomography to probe the dynamics of larger systems of connected qubits in NISQ devices. For isolated devices with direct control within a lab this method could provide significant insight into unaccounted for noise sources deteriorating the qubit coherence, and for open access devices this method can serve as an additional calibration method to obtain the most current hardware parameters for higher precision in the simulation of experiments such as quantum chemistry or open quantum systems.

| Claimed | Experimental |
|---------|--------------|
| $\omega$ (GHz) | $T_1$ (µs) | $J$ | $\omega$ (GHz) | $T_1$ (µs) | $J$ | $T$ (mK) |
| $q_0$ | 31.42 | 100.24 | $8.31 \times 10^{-3}$ | 31.42 | 98.16 | $8.31 \times 10^{-3}$ | 50.59 |
| $q_1$ | 30.47 | 106.95 | – | 30.47 | 117.77 | – | 67.92 |
| $q_2$ | 30.05 | 101.45 | $7.42 \times 10^{-3}$ | 30.05 | 108.08 | $7.42 \times 10^{-3}$ | 6.37 |

TABLE III: Claimed and experimental results for the 3-qubit experiments to measure the qubit frequency, $\omega$, relaxation time, $T_1$, inter-qubit coupling, $J$, and qubit temperature, $T$.

V. CONCLUSION

In this work the fundamentals of several fields of research, such as quantum computing, superconducting qubit hardware, and open quantum systems, were introduced and discussed to lay a foundation for the understanding of the topics and their relevance to each other. This relevance was leveraged into a discussion of the overlap of these topics in the realm of quantum noise and error mitigation, which is covered primarily by current research. The most recent and relevant research in this field was discussed and used to create a map of the current state of investigation in this topic, which showed a gap in research of a simple method to probe hardware parameters of a NISQ open-access cloud-based quantum computing service such as that offered by IBM.

The filling of this research gap was the motive of this project, which provided a simple method of using basic experimental procedures typically used to measure the relaxation and decoherence times, $T_1$ and $T_2$, of quantum states, and numerically replicated through a Markovian quantum master equation to measure how well the claimed hardware parameters fit the experimental observations. This process was expanded to use optimisation techniques to improve the congruence of numerical results and experimental data to find more accurate values of the hardware parameters which dictate the dynamics of the system.

This gave rise to interesting phenomena, such as more generalise Hamiltonians containing broader descriptions of qubit frequency and coupling, as well as proving the proficiency of this method to extract all of the hardware parameters without needing to revert to various specialised experiments which measure the parameters individually. The consistency of this method was verified through using various sizes of qubit subsystems which each produced values for hardware parameters for individual qubits, which were congruent to a reliable degree of accuracy that the process could be considered successful.

There are several hindrances which detract from the accuracy of this method, such as the queue system of...
Fig. 6: Optimised results for additional experimental circuits, including $T_2$, $T_2^*$, and combined procedures. Experimental data represented by dots and numerical data for optimised parameters represented by solid lines. Results shown for single qubit $T_2$ sequence (a), 2-qubit $T_2$ sequence (b), single qubit $T_2^*$ sequence (c), and 2-qubit combination of $T_2^*$ and $T_1$ sequences (d).

the quantum devices from their open access platform as well as the lack of direct control over the compilation of the quantum circuits to executable qubit control. The method is also susceptible to external influences such as stochastic noise and hardware impurities, which are a fault of NISQ devices in general, which also detracts from the reliability in scenarios which require high accuracy. Despite these shortcomings, this method serves as a proof of concept of subsystem Markovian tomography which can be stitched together to provide a mapping of qubit dynamics in larger systems without the exponential growth in difficulty associated with full quantum tomography.

Many more improvements can be made to this method, such as assisting the performance with machine learning methods, and performing more accurate calibration and tomography to mitigate any external errors which are unavoidable in open access NISQ devices, or finding mechanisms to improve qubit coherence times. This work also opens a new path for further investigation into more generalised models which can uncover finer details which are typically suppressed in approximation, such as qubit frequency vectors and coupling tensors, but could prove crucial in further quantum engineering in the path to fault-tolerance.

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