Mechanical Analysis of the S&P 500 Index Time Series

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Abstract. Daily values of the stock market index S&P 500 are considered as a coordinate of an
oscillating particle in discrete time. These driven nonlinear oscillations are occasionally
damped, amplified and unstable. Large changes of the coordinate are usually preceded by
exceptional values of the acting force parameters. Therefore, mechanical analysis enables an
estimation of the risk of stock market crash. In order to better understand the problem of
predictability, we have considered some artificial time series with combination of chaos and
stochasticity, generated by the appropriate differential equation of motion. We indeed can
roughly predict the second half of the time series, with low enough level of stochasticity, if we
know the first half of the series.

1. Introduction
With market liberalization, freer capital flows and stock market security, international financial
markets are becoming increasingly more closely connected and increasingly interdependent. In such
relationship, the stock exchange puts the most efficient market institution for the rapid purchase and
sale of various types of stock exchange material. Stock exchanges are the most sensitive financial
institutions, because any positive or negative change in them directly affects the decisions of investors,
so instability or large oscillations in stock markets can create panic among investors. The most
important resource for trading on stock exchanges is information. Throughout history, many concepts
have emerged that have attempted to predict the decline of the stock market, relationship between
variables that determine trading and stock market turnover. There was one traditional approach in
economics without adaptation, innovation or structural change and with presumed equilibrium. In
1987 a conference The Economy as an Evolving Complex System is organized by Nobel laureate in
economy Kenneth Arrow and Nobel laureate in physics Philip Anderson. This started a new approach
to economy as a complex system that is evolving [1]. Various methods for stock market crash
indicator have been proposed. Quax et al. propose that analysis of financial time series could provide a
signal indicating upcoming Lehman Brothers bankruptcy that started global financial crisis [2].
Saracco et al. analysed global trading network and find that it was possible to have indication of global
financial crisis [3]. Bouchard et al. claim that classical economics lacks methods for predicting and
avoiding crises and that it needs a scientific revolution [4]. Stavroglo et al. propose a method based
on symbolic dynamics, which probes beneath the surface of abstract causality and unveils the nature of
causal interactions [5].

The term "econophysics" was first used in the mid-1990s. It was imagined that physicists work on
economic problems by testing conceptual new approaches borrowed from the physical sciences. It is
no secret that many physicists have long been interested in the study of economics like Jan Tinbergen,
who won the first Nobel Memorial Prize in Economics Sciences in 1969 for having developed and
applied dynamic models for the analysis of economic processes. Econophysics was started in the subfield of statistical mechanics. Basic tools of econophysics are statistical methods taken from statistical physics, but there are also models taken from mechanics, dynamics, chaos theory, and other disciplines in addition to physics that can be used in economics. It could be done because we can observe financial market as a complex system where risk tolerance of individuals is crucial for system resilience. The market crash is usually preceded by a bubble, an anomalous increase in asset prices regarding to the economy [6]. Prediction of complex time series is not unrealistic [7]. The dynamics of beliefs play a central role in macroeconomics and finance [8].

2. Method

In the model presented in this paper, we present the analysis of the stock exchange index through mechanical analysis.

We assume that \( x_n \) \((n = 1, 2, 3, \ldots)\) are the daily values of the stock exchange index S&P 500. First we look at the coordinate of a particle of unit mass in discrete time and we define the velocity and force acting on the particle in time \( n + j \).

Velocity is given by

\[
v_{n+j} = x_{n+j} - x_{n+j-1}, n = 2, 3, 4, \ldots, 12; j = 0, 1, 2, \ldots
\]

and force is given by

\[
v_{n+j} - v_{n+j-1} = a(x_{n+j} - S) + a_2(x_{n+j} - S)^2 + a_3(x_{n+j} - S)^3 + bv_{n+j}
+ w + \sum_{i=2}^{6} c_i \cos(6.28(n+j)/i)
\]

where we assumed that \( S \) is given by

\[
S = \frac{1}{12} \sum_{n=1}^{12} x_{n+j}; j = 0, 1, 2, \ldots
\]

The first term on the right side of relation (2) represents the elastic force, if \( a < 0 \). In the case of \( a > 0 \), the equilibrium for \( x_{n+j} = S \) is unstable. If \( b < 0 \), the fourth term is damping force, and for \( b > 0 \) this is amplifying force.

The force \( w \) is constant in a short time interval \((3 \leq n \leq 12)\). The sixth term is the driving force (circular frequencies of 6.28/2 to 6.28/6). By including \( n = 3, 4, \ldots, 12 \) in (2), we get a system of ten linear equations. By solving this system, we get the parameters: \( a, a_2, a_3, b, w, c_2, c_3, c_4, c_5, c_6 \) (for a certain time shift value \( j \)).

The force parameters have significantly unpredictable change when switching from one short time interval to the next one. Here we will show that it is possible, knowing the values of the stock index for six months, predict major changes in the next six months. Approximate predictability is the result of a combination of chaos and stochasticity [9].

Let us see for example daily values of index S&P 500 for 2019. If we use \( j = 84 \), we calculate \( x_{85}, x_{86}, \ldots, x_{96} \) \((S=2872)\). Then, system of equations (2) gives us parameters: \( a = 3.8, a_2 = 0.2, a_3 = 0.003, b = 1.7, w = -11.3, c_2 = -8.4, c_3 = -17.5, c_4 = 26, c_5 = -12, c_6 = 21 \).

If \( j = 85 \), we calculate with \( x_{86}, x_{87}, \ldots, x_{97} \) \((S=2865)\), and then we have: \( a = -4.4, a_2 = 0.7, a_3 = 0.02, b = -5, w = -70.4, c_2 = -0.8, c_3 = -99, c_4 = -207, c_5 = -212, c_6 = 75 \). The force parameters change unpredictably when switching from one short time interval to another.

3. Index S&P 500 from year 2003 to year 2019

A mean value of \( a \) less than -1 and mean value of \(|c_5|\) less than 50 together announce an increase of \( S \) no more than 200 (Table 1). If the mean value of \( a \) is greater than 2 and the mean value of \( w \) is less than -25, a drop of more than 400 occurs. The mean value of the parameter \( a \) greater than 2 and the mean value \(|c_5|\) greater than 100 together announce a decrease in \( S \) by more than 400. Mean value of \( a \) less than -4 and mean value of \(|c_5|\) greater than 100 together announce an increase of \( S \) more than 150.

If the mean value \( a \) is negative and the mean value \( w \) is positive, there is no large drop of \( S \) (Table 1).
The growth rate of real estate prices in the United States, which was the highest in 2005, fell sharply in 2006. In the transition from 2005 to 2006 year there was a sharp jump in all force parameters (Table 2).

**Table 2.** Average values of the force parameters in the first half of year. Driving force and $<\theta>$ increase in years 2004-2006, announcing large changes of $S$.

| year | $<a_2>$ | $<a_3>$ | $<b>$ | $<|c_2|>$ | $<|c_3|>$ | $<|c_4|>$ | $<|c_6|>$ |
|------|--------|--------|-------|----------|----------|----------|----------|
| 2003 | 0.16   | -0.01  | 1.34  | 17.79    | 34.12    | 44.28    | 44.06    |
| 2004 | 0.05   | -0.04  | 1.45  | 9.23     | 13.66    | 29.21    | 26.99    |
| 2005 | 0.31   | -0.005 | 1.46  | 11.00    | 37.79    | 44.91    | 43.15    |
| 2006 | 2.98   | -0.64  | 3.50  | 60.39    | 131.12   | 73.04    | 132.13   |
| 2007 | 1.06   | -0.07  | 0.55  | 37.30    | 101.20   | 107.06   | 118.44   |
| 2008 | 0.29   | -0.03  | -0.75 | 90.76    | 170.95   | 201.64   | 285.38   |
| 2009 | -0.01  | -0.02  | 3.29  | 221.33   | 426.69   | 125.80   | 545.15   |
| 2010 | 0.07   | -0.01  | 1.36  | 20.51    | 37.40    | 36.72    | 46.94    |
| 2011 | 0.13   | 0.001  | 2.01  | 12.68    | 24.21    | 35.90    | 42.50    |
| 2012 | 0.07   | 0.004  | 0.82  | 10.94    | 16.78    | 20.68    | 37.92    |
| 2013 | -0.21  | -0.05  | -1.26 | 10.60    | 66.35    | 44.86    | 114.47   |
| 2014 | 0.09   | -0.005 | 1.01  | 11.24    | 34.03    | 31.75    | 43.14    |
| 2015 | -0.59  | 0.21   | -6.57 | 38.85    | 132.03   | 91.09    | 144.52   |
| 2016 | -0.05  | 0.001  | 0.41  | 22.67    | 137.75   | 148.22   | 141.51   |
| 2017 | 0.19   | -0.001 | 1.04  | 8.44     | 20.79    | 21.75    | 27.51    |
| 2018 | 0.03   | 0.0006 | 2.70  | 31.81    | 86.98    | 73.21    | 144.40   |
| 2019 | 0.08   | 0.002  | 1.12  | 22.65    | 23.38    | 49.44    | 82.83    |
Time dependence of $S$, $a$, $c_5$ and $w$, for particular years, is presented in figures 1-5. High maximum (minimum) of the force parameter $w$ announces growing (decline) of $S$.

**Figure 1.** Year 2008; $S$-1100 (red), $a/4.3$ (blue), $c_5/60$ (green), $w/42$ (gold).

**Figure 2.** Year 2004; $S$-1080 (red), $a/15$ (blue), $c_5/200$ (green), $w/90$ (gold).

**Figure 3.** Year 2009; $S$-830 (red), $a/3$ (blue), $c_5/30$ (green), $w/70$ (gold).

**Figure 4.** Year 2016; $S$-2000 (red), $a/4$ (blue), $c_5/50$ (green), $w/30$ (gold).
Decline of the stock market index during 2008 year (Figure 1) is preceded by high instability (blue), large driving force (green) and deep minimum of the force parameter \( w \) (gold).

Using data from the second half of 2019, we cannot predict a decline in the middle of 2020, which was caused by an epidemic. Among investors there was no fear of an epidemic in the second half of 2019. The measures introduced due to the epidemic were such that an economic downturn could have been predicted without any science. Stock market crash is the rule unexpected and enigmatic. Method proposed in this paper would have practical importance very often, when extremely great calamity is highly improbable.

4. Artificial time series

In classical mechanics, most important law is Newton’s second law of motion

\[
m \frac{d^2 z}{dt^2} = F(z, v, t).
\]  
(4)

If the force \( F(z, v, t) \) is a linear function of \( z \) and \( v \), with known initial conditions we have complete predictability (we calculate \( z(t) \) and \( v(t) \)). If \( F(z, v, t) \) is a nonlinear function in \( z \) and \( v \), sensitivity to initial conditions and force parameters is possible (chaos). A very small change in the initial condition or force parameter significantly affects value of \( z \) in large time interval. In that case, we lose long-term predictability but short-term predictability remains. If \( F(z, v, t) \) is fluctuating function of time, it is possible that we have complete stochasticity. Then we lose predictability. We consider here a special case in which the parameters of the force \( F(z, v, t) \) have the form of the sum of a deterministic and a fluctuating term (a combination of chaos and stochasticity). In this particular case we can, if the size of the deterministic term is sufficient large, knowing the first half of the time series \( z(n\Delta t) \), roughly predict the second half of the time series.

We generate artificial time series using the differential equation of motion

\[
\frac{d^2 z}{dt^2} = az + a_2 z^2 + a_3 z^3 + b v + w + \sum_{i=2}^{6} c_i \cos \frac{6.28 t}{i}.
\]  
(5)

Force parameters \( a, a_2, a_3, b, w, c_2, c_3, c_4, c_5 \) and \( c_6 \) contain fluctuating function \( f(t) \), whose values are between -1 and 1.

Dimensions of the parameters are:

\[
[a] = \left[ \frac{F}{ml} \right]; \quad [a_2] = \left[ \frac{F}{ml^2} \right]; \quad [a_3] = \left[ \frac{F}{ml^3} \right]; \quad [b] = \left[ \frac{F}{m} \right]; \quad [w] = |c_i| = \left[ \frac{F}{m} \right].
\]  
(6)
Only parameters of the same dimension are compared. With the help of artificial series, we understand the subsequent effect of a large change in the force parameter in a short time interval. We solve differential equation of motion (5) with the help of artificial series, we understand the subsequent effect of a large change in the force parameter in a short time interval. We solve differential equation of motion (5) with

$$z(0) = 1.5, \quad v(0) = -0.11, \quad b = 0.0001 + Qf(t), \quad a_2 = -0.003 + Qf(t), \quad a_3 = -0.001 + Qf(t), \quad c_2 = -0.3 + Qf(t), \quad c_3 = 0.4 + Qf(t), \quad c_4 = -0.3 + Qf(t), \quad c_6 = 0.2 + Qf(t), \quad Q = 0.05 \text{ (green), } 0.08 \text{ (blue), } 0.17 \text{ (red) } (\text{figure 6})$$

and

$$z(0) = 0.2, \quad v(0) = 0.5, \quad b = -0.05 + Qf(t), \quad a_2 = -0.05 + Qf(t), \quad a_3 = -0.001 + Qf(t), \quad c_2 = 0.3 + Qf(t), \quad c_3 = -0.2 + Qf(t), \quad c_4 = 0.4 + Qf(t), \quad c_6 = -0.2 + Qf(t) \text{ (figure 7).}$$

High maxima and deep minima of the force parameters at $t=2.5$ cause large future changes of $S$ (case (b) in figures 6 and 7).

Approximate predictability is result of a combination of chaos and stochasticity [9].

**Figure 6.** (a) $w = 0.052e^{-17(t-2.5)^2} + Qf(t)$, $a = -0.001e^{-17(t-2.5)^2} + Qf(t)$, $c_5 = -0.005e^{-17(t-2.5)^2} + Qf(t)$; (b) $w = 5.2e^{-17(t-2.5)^2} + Qf(t)$, $a = -0.1e^{-17(t-2.5)^2} + Qf(t)$, $c_5 = -0.5e^{-17(t-2.5)^2} + Qf(t)$

**Figure 7.** (a) $w = -0.027e^{-17(t-2.5)^2} + Qf(t)$, $a = 0.001e^{-17(t-2.5)^2} + Qf(t)$, $c_5 = -0.005e^{-17(t-2.5)^2} + Qf(t)$; (b) $w = -2.7e^{-17(t-2.5)^2} + Qf(t)$, $a = 0.1e^{-17(t-2.5)^2} + Qf(t)$, $c_5 = -0.5e^{-17(t-2.5)^2} + Qf(t)$.  

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5. Conclusion

The term "econophysics" was first used by American researchers R N Mantegna and G Yu Stanley in the mid-1990s, but the connection between physics and economics goes much further into the past. It is known that many ancient physicists were interested in the application of the laws of physics in economic trends, but because of the rigid norms that were in the field of economics, they could not gain enough importance. It is not well known that Galileo still wrote about economic topics, but his significance for economics was in his methodology and not in his views on economics. Knowledge of the basic laws of physics is very important for the study of economic flows and the stock market, because it seems possible to apply laws to situations in economics. It is great practical importance possibility to understand and roughly predict, non-mechanical complex systems, in the mechanical picture. Predictability is the result of combinations of chaos and stochasticity. This approach could work with many other complex systems, such as brain.

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