1−− and 0++ heavy four-quark and molecule states in QCD

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1st introduction and a short review on the 1++ channel

A large amount of exotic hadrons which differ from the “standard” cc charmonium and bb bottomonium radial excitation states have been recently discovered in B-factories through J/ψπ+π− and τnππ processes and have stimulated different theoretical interpretations. Most of them have been assigned as four-quarks or molecule states [1]. In previous papers [2, 3], some of us have studied, using exponential QCD (spectral) sum rules (QSSR) [4–6] and the double ratio of sum rules (DRSR) [7–10], the nature of the X(3872) 1++ states found by Belle [11] and confirmed by Babar [12], CDF [13] and D0 [14]. If it is a (cq)(cq) four-quark or D – D∗ molecule state, one finds [2]:

\[ X_c = (3925 ± 127) \text{ MeV}, \quad \text{with} \quad \sqrt{t_c} = (4.15 ± 0.03) \text{ GeV} \]  

(1)

corresponding to a t−-value common solution of the exponential Laplace (LSR) and Finite Energy (FESR) sum rules. While in the b-meson channel, using mb = 4.26 GeV, one finds [2]:

\[ X_b = (10144 ± 104) \text{ MeV}, \quad \text{with} \quad \sqrt{t_e} = (10.4 ± 0.02) \text{ GeV}. \]  

(2)

By assuming that the mass of the radial excitation \( X_0' \approx \sqrt{t_c} \), one can also deduce the mass-splitting:

\[ X_c' - X_c \approx 225 \text{ MeV} \approx X_b' - X_b \approx 256 \text{ MeV}, \]  

(3)

which is much lower than the ones of ordinary \( \Upsilon \) and \( \Upsilon(1S) \) states:

\[ \psi(2S) - \psi(1S) \approx 590 \approx \Upsilon(2S) - \Upsilon(1S) \approx 560 \text{ MeV}, \]  

(4)

suggesting a different dynamics for these exotic states.

2. QCD Analysis of the 1−− and 0++ channels

In the following, we extend the previous analysis to the case of the 1−− and 0++ channels and improve some existing analysis from QCD (spectral) sum rules in the 1++ channel [15, 16]. The results will be compared with the experimental 1++ candidate states: \( Y(4260), Y(4360), Y(4660), Y(10890) \). These states have been seen by Babar [17] and Belle [18, 19] and which decay into J/ψπ+π− and τnππ around the \( \Upsilon(5S) \) mass. These states cannot be identified with standard cc charmonium and bb bottomonium radial excitations and have been assigned to be four-quark or molecule states or some threshold effects.

- Interpolating currents

We assume that the \( Y \) states are described either by the lowest dimension (without derivative terms) four-quark and molecule \( D_s D_s' \) vector currents \( J_\mu \) given by:

\[ J_\mu = \frac{e_{\mu\nu} e_{\rho\kappa}}{\sqrt{\gamma}} \left[ \langle s_Q c_Q \rangle \langle \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\kappa \rangle \langle \gamma^\gamma \gamma \gamma \rangle \right] \]

(5)

and

\[ J_\mu^{\text{mol}} = \frac{1}{\sqrt{2}} \left[ \langle s_Q c_Q \rangle \langle \gamma^\mu \gamma^\nu \rangle \langle Q \rangle + \langle \gamma^\gamma \gamma \gamma \rangle \langle \gamma^\mu \gamma^\nu \rangle \right] \]

(6)

Keywords: QCD spectral sum rules, four-quark and molecule states, heavy quarkonia.
**QCD input parameters**

The QCD parameters are given in Table 1 and we shall work with the running light quark parameters [20, 21].

| Parameters | Values | Ref. |
|------------|--------|------|
| \(\Lambda_{(\tau = 4)}\) | \((324 \pm 15)\) MeV | [22–24] |
| \(\Lambda_{(\tau = 5)}\) | \((194 \pm 10)\) MeV | [22–24] |
| \(\hat{m}_c\) | \((0.114 \pm 0.021)\) GeV | [5, 24] |
| \(m_c\) | \((1.26 \sim 1.47)\) GeV | [5, 24–27] |
| \(m_b\) | \((4.17 \sim 4.70)\) GeV | [5, 24–26] |
| \(\hat{\rho}_t\) | \((263 \pm 7)\) MeV | [5] |
| \(\kappa \equiv (\Sigma t)/(3\mu)\) | \((0.74 \pm 0.06)\) | [9] |
| \(M_0^2\) | \((0.8 \pm 0.2)\) GeV² | [28–30] |
| \((\alpha_s G^2)\) | \((7 \pm 2) \times 10^{-2}\) GeV⁴ | [22, 26, 31–37] |
| \((g' G'¹)\) | \((8.3 \pm 1.0)\) GeV² \times \((\alpha_s G^2)\) | [26] |
| \(\rho \equiv (\bar{q} q)/(|\bar{q} q|^2)\) | \((2 \pm 1)\) | [22, 28, 31] |

### 3. 1⁻⁺ four-quark state mass \(Y_{Qq}\) from QSSR

In the following, we shall estimate the mass of the 1⁻⁺ four-quark state \((\bar{Q}q)(\bar{Q}q)\), hereafter denoted by \(Y_{Qq}\). In so doing, we shall use the ratios of the Laplace (exponential) sum rule and of FESR:

\[
R_{Qcd}^{LSR}(\tau) \approx M_{Y_{qd}}^{LSR} \approx R_{Qcd}^{FESR} .
\]

- **The \(Y_{cd}\) mass from LSR and FESR for the case \(b=0\)**

Using the QCD inputs in Table 1, we show the \(\tau\)-behaviour of \(M_{Y_{cd}}^{LSR}\) from \(R_{Qcd}^{LSR}\) in Fig. 1a. One can notice from Fig. 1a that the \(\tau\)-stability is obtained from \(\sqrt{t_c} \geq 5.1\) GeV, while the \(t_c\)-stability is reached for \(\sqrt{t_c} \geq 7\) GeV. The most conservative prediction from the LSR is obtained in this range of \(t_c\)-values for \(m_c = 1.26\) GeV and gives in units of GeV:

\[
4.79 \leq M_{Y_{cd}} \leq 5.73 \quad \text{for} \quad 5.02 \leq \sqrt{t_c} \leq 7 \quad \text{and} \quad m_c = 1.26 ,
\]

\[
5.29 \leq M_{Y_{cd}} \leq 6.11 \quad \text{for} \quad 5.5 \leq \sqrt{t_c} \leq 7 \quad \text{and} \quad m_c = 1.47 .
\]

We compare in Fig. 1b, the \(t_c\)-behaviour of the LSR results obtained at the \(\tau\)-stability points with the ones from \(R_{Qcd}^{FESR}\) for \(m_c = 1.23\) GeV (running) and 1.47 GeV (on-shell). One can deduce the common solution in units of GeV:

\[
M_{Y_{cd}} = 4.814 \quad \text{for} \quad \sqrt{t_c} = 5.04(5) \quad \text{and} \quad m_c = 1.26 ,
\]

\[
= 5.409 \quad \text{for} \quad \sqrt{t_c} = 5.6 \quad \text{and} \quad m_c = 1.47 .
\]

We observe that the on-shell \(c\)-quark mass value tends to overestimate \(M_{Y_{cd}}\) [3, 26]. The same feature happens for the evaluation of the \(X(1⁺)\) four-quark state mass \([2]\). Then, we are tempted to take as a final result in this paper the prediction obtained by using the running mass \(\tilde{m}_c(m_c) = 1262(17)\) MeV. Considering the uncertainties from the Table (1), we deduce

\[
M_{Y_{cd}} = 4814(57)\text{ MeV}.
\]

Using the fact that the 1st FESR moment gives a correlation between the mass of the lowest ground state and the onset of continuum threshold \(t_c\), we shall approximately identify its value with the one of the radial excitation. Assuming that, one can deduce the mass-splitting: \(M_{Y_{cd}} - M_{Y_{cd}} \approx 226\) MeV, which is similar to the one obtained for the \(X(1⁺⁺)\) four-quark state \([2]\).

- **The \(Y_{cd}\) mass from LSR and FESR for the case \(b = 0\)**

We extend the previous analysis to the \(b\)-quark sector, in order to estimate the mass of \(Y_{cd}\) four-quark state. Considering, like in the case of charm, as a final estimate the one from the running \(b\)-quark mass \(\tilde{m}_b(m_c) = 4177(11)\) MeV [26], we deduce:

\[
M_{Y_{cd}} = 11256(49)\text{ MeV}.
\]

From the previous result, one can deduce the value of the mass-splitting between the 1st radial excitation and the lowest mass ground state: \(M_{Y_{cd}} - M_{Y_{cd}} \approx M_{Y_{cd}} - M_{Y_{cd}} \approx 250\) MeV, which are (almost) heavy-flavour independent and also smaller than the one of the bottomium splitting (~ 560 MeV).

In the following, we shall let the current mixing parameter \(b\), as defined in Eq. (5), free and study its effect on the results obtained in Eqs. (9) and (10). In so doing, we fix the values of \(\tau\) around the \(\tau\)-stability point and \(t_c\) around the intersection point of the LSR and FESR. The results of the analysis are shown in Fig. 2. We notice that the results are optimal at the value \(b = 0\).

For completing the analysis of the effect of \(b\), we also study the decay constant \(f_{Y_{cd}}\) defined as: \(0 f_{Y_{cd}} | Y_{Qq} \rangle = \bar{Y}_{Qq} M_{Y_{cd}}^{1/2}\). Doing the analysis, giving \(M_{Y_{cd}}\) and the corresponding \(t_c\) obtained above, one can deduce the optimal values at \(b = 0\):

\[
f_{Y_{cd}} \approx 0.08\text{ MeV} \quad \text{and} \quad f_{Y_{cd}} \approx 0.03\text{ MeV},
\]

which are much smaller than \(f_\tau \approx 132\) MeV, \(f_\tau \approx 215\) MeV and \(f_D \approx f_B = 203\) MeV [38]. One can also note that the decay constant decreases like \(1/M_Q\) which can be tested in HQET or/and lattice QCD.

- **SU(3) breaking for \(M_{Y_{cd}}\) from DRSR**

We study the ratio \(M_{Y_{cd}}/M_{Y_{cd}}\) using DRSR:

\[
\frac{M_{Y_{cd}}}{M_{Y_{cd}}} \equiv \sqrt{\frac{R_{LSR}^{Q\bar{Q}}}{R_{LSR}^{Q\bar{Q}}}} \quad \text{where} \quad Q \equiv c, b.
\]
Combining LSR and FESR, we consider the mass of the \(SU(3)\) breaking ratios, from which, we can deduce:

\[
\begin{align*}
\tau^\phi_{\eta s} & \equiv M_{1/\phi S_2}/M_{1/\phi S_1} = 1.022(0.2)_{m_b}(5)_{m_s}(2)_{c} , \\
\tau^\phi_{\eta s} & \equiv M_{1/\phi S_2}/M_{1/\phi S_1} = 1.011(1)_{m_b}(2)_{m_s}(0.2)_{c} ,
\end{align*}
\]

where \(S_3 \equiv \eta s\) is a scalar meson. Then, we obtain in MeV:

\[
M_{1/\phi S_1} = 5112(41) , \quad M_{1/\phi S_2} = 10125(40) ,
\]

corresponding to the \(SU(3)\) mass-splittings: \(\Delta M^\phi_{\eta s} \approx \Delta M^\phi_{\eta s} \approx 110\) MeV. Doing the same exercise for the octet current, we deduce the results in Table 2 where the molecule associated to the octet current is 100 (resp. 250) MeV above the one of the singlet current for \(J/\psi\) (resp \(\Upsilon\)) contrary to the \(1^-\) case discussed in [3]. The ratio of \(SU(3)\) breakings are respectively 1.022(5) and 1.010(2) in the \(c\) and \(b\) channels which are comparable with the ones in Eq.(20).

5. \(0^{++}\) four-quark and molecule masses from QSSR

\[Y_{0q}^0\] mass and decay constant from LSR and FESR

We do the analysis of the \(Y_{0q}^0\) and \(Y_{1q}^0\) masses using LSR and FESR match. We work with the current mixing parameter \(b = 0\) from which we deduce in MeV, for the running quark masses, and respectively for \(\sqrt{t_c} = 6.5\) and 13.0 GeV:

\[
M_{Y_{0q}^0} = 6125(51)\text{ MeV} , \quad M_{Y_{0q}^0} = 12542(43)\text{ MeV} .
\]

One can notice that the splittings between the lowest ground state and the 1st radial excitation approximately given by \(\sqrt{t_c}\) is: \(M_{Y_{0q}^0} - M_{Y_{1q}^0} \approx 375\) MeV , \(M_{Y_{1q}^0} - M_{Y_{2q}^0} \approx 464\) MeV , which is larger than the ones of the \(1^-\) states, comparable with the ones of the \(J/\psi\) and \(\Upsilon\) and are (almost) heavy-flavour independent. For completeness, we calculate the sum rule for the decay constants from which we deduce:

\[
\begin{align*}
f_{Y_{0q}^0} & \approx 0.12\text{ MeV} \quad \text{and} \quad f_{Y_{1q}^0} \approx 0.03\text{ MeV} ,
\end{align*}
\]

which are comparable with the ones of the spin 1 case.

\[SU(3)\] breaking for \(Y_{0q}^0\) from DRSR

Analyzing the \(\tau\) and \(t\) behaviours of the \(SU(3)\), we deduce:

\[
\begin{align*}
\nu^\phi_{\eta s} & \equiv M_{1/\phi S_2}/M_{1/\phi S_1} = 1.011(1)_{m_b}(1.7)_{m_s}(0.7)_{c} , \\
\nu^\phi_{\eta s} & \equiv M_{1/\phi S_2}/M_{1/\phi S_1} = 1.004(1)_{m_b}(1.7)_{m_s}(0.3)_{c} ,
\end{align*}
\]

leading to the masses in MeV and theirs respective \(SU(3)\) mass-splittings:

\[
\begin{align*}
M_{Y_{0q}^0} & \approx 6192(59) , \quad M_{Y_{1q}^0} = 12592(50) , \\
\Delta M^\phi_{\eta s} & \approx 67 \approx \Delta M^\phi_{\eta s} = 50\text{ MeV} .
\end{align*}
\]
\[ M_{D_s D_s} = 5955(48) \text{ MeV}, \quad M_{B_s B_s} = 11750(40) \text{ MeV}. \]

The splittings (in MeV) between the lowest ground state and the 1\(^{st}\) radial excitation, approximately given by \( \sqrt{\Delta M} \), is:
\[
M_{D_s D_s} - M_{D_s D_s} \approx 290, \quad M_{B_s B_s} - M_{B_s B_s} \approx 270, \tag{27}
\]
which, like in the case of the 1\(^{\pm}\) states are smaller than the ones of the \( J/\psi \) and \( \Upsilon \) and almost heavy-flavour independent.

### 6. Summary and conclusions

- The three \( Y_3(4260, 4360, 4660) \) 1\(^{--}\) experimental candidates are too low for being pure four-quark or/molecule \( DD' \) and \( J/\psi S_2 \) states but can result from their mixings. The \( Y_3(10890) \) is lower than the predicted values of the four-quark and \( BB' \) molecule masses but heavier than the predicted \( T S_2 \) and \( T S_3 \) molecule states. Our results may indicate that some other exotic structure of these states are not excluded.

- For the 1\(^{--}\), there is a regularity of about (250-300) MeV for the value of the mass-splittings between the lowest ground state and the 1\(^{st}\) radial excitation roughly approximated by the value of the continuum threshold \( \sqrt{\Delta M} \) at which the LSR and FESR match. These mass-splittings are (almost) flavour-independent and are much smaller than the ones of 500 MeV of ordinary charmonium and bottomium states.

- There is also a regularity of about 50–90 MeV for the \( S(13) \) mass-splittings of the different states which are also (almost) flavour-independent.

- The spin 0 states are much more heavier (\( \geq 400 \) MeV) than the spin 1 states, like in the case of hybrid states [5].

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