We devise the optimal form of Gaussian resource states enabling continuous variable teleportation with maximal fidelity. We show that a nonclassical optimal fidelity of $N$-user teleportation networks is necessary and sufficient for $N$-party entangled Gaussian resources, yielding an estimator of multipartite entanglement. This entanglement of teleportation is equivalent to entanglement of formation in the two-user protocol, and to localizable entanglement in the multi-user one. The continuous-variable tangle, quantifying entanglement sharing in three-mode Gaussian states, is operationally linked to the optimal fidelity of a tripartite teleportation network.

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Quantum teleportation using quadrature entanglement in continuous variable (CV) systems is in principle imperfect, due to the impossibility of achieving infinite squeezing. Nevertheless, by considering the finite quantum correlations between the quadratures in a two-mode squeezed Gaussian state, a realistic scheme for CV teleportation was proposed, and experimentally implemented to teleport coherent states with a fidelity up to $F = 0.70 \pm 0.02$. Without using entanglement, by purely classical communication, an average fidelity of $F_{\text{cl}} = 1/2$ is the best that can be achieved if the alphabet of input states includes all coherent states with even weight. The original teleportation protocol was generalized to a multi-user teleportation network requiring multipartite CV entanglement in Ref. [4]. This network has been recently demonstrated experimentally by exploiting three-mode squeezed Gaussian states, yielding a best fidelity of $F = 0.64 \pm 0.02$. The fidelity, which quantifies the success of a teleportation experiment, is defined as $F = \langle \psi^{\text{in}} | \hat{q}^{\text{out}} | \psi^{\text{in}} \rangle$, where “in” and “out” denote the input and the output state. $F$ reaches unity only for a perfect state transfer, $\hat{q}^{\text{out}} = | \psi^{\text{in}} \rangle \langle \psi^{\text{in}} |$. To accomplish teleportation with high fidelity, the sender (Alice) and the receiver (Bob) must share an entangled state (resource). The sufficient fidelity criterion states that, if teleportation is performed with $F > F_{\text{cl}}$, then the two parties exploited an entangled state. The converse is generally false, i.e. some entangled resources may yield lower-than-classical fidelities.

In this Letter we investigate the relation between the fidelity of a CV teleportation experiment and the entanglement present in the resource states. We show that the optimal fidelity, maximized over all local single-mode operations (at fixed amounts of noise and entanglement in the resource), is necessary and sufficient for the presence of bipartite (multipartite) entanglement in two-mode (multimode) Gaussian resources. Moreover, it allows for the definition of the entanglement of teleportation, an operative estimator of bipartite (multipartite) entanglement in CV systems. Remarkably, in the multi-user instance, the optimal shared entanglement is exactly the localizable entanglement, originally introduced for spin systems, which thus acquires for Gaussian states a suggestive operative meaning in terms of teleportation processes. In the CV scenario, a recent study on entanglement sharing led to the definition of the residual CV tangle, or contangle $E_{\text{r}}$, as a tripartite entanglement monotone under Gaussian LOCC for three-mode Gaussian states. This measure too is here operationally interpreted via the success of a three-party teleportation network. Besides these fundamental theoretical results, our findings are of important practical interest, as they answer the experimental need for the best preparation recipe for entangled squeezed resources, in order to implement CV teleportation with the highest fidelity.

The two-user CV teleportation protocol would require, to achieve unit fidelity, the sharing of an ideal (unnormalizable) Einstein-Podolski-Rosen (EPR) resource state ($10$), i.e. the eigenstate of relative position and total momentum of a two-mode radiation field. An arbitrarily good approximation of the EPR state is represented by two-mode squeezed Gaussian states with squeezing parameter $r \to \infty$. In a CV system consisting of $N$ canonical bosonic modes, and described by the vector $X = \{ \hat{x}_1, \hat{p}_1, \ldots, \hat{x}_N, \hat{p}_N \}$ of the field quadrature operators, Gaussian states (such as thermal, coherent, squeezed states) are fully characterized by the first statistical moments (arbitrarily adjustable by local unitaries: we will set them to zero) and by the $2N \times 2N$ covariance matrix (CM) $\sigma$ of the second moments $\sigma_{ij} = 1/2 \{ \hat{x}_i, \hat{x}_j \}$. A two-mode squeezed state can be, in principle, produced by mixing a momentum-squeezed state and a position-squeezed state, with squeezing parameters $r_1$ and $r_2$ respectively, through a 50:50 ideal (lossless) beam splitter. In practice, due to experimental imperfections and unavoidable thermal noise the two initial squeezed states will be mixed. To perform a realistic analysis, we must then consider two thermal squeezed single-mode states, described by the following quadrature operators in Heisenberg picture

$$\hat{x}_1^{sq} = \sqrt{n_1} e^{r_1} \hat{x}_1^0, \quad \hat{p}_1^{sq} = \sqrt{n_1} e^{-r_1} \hat{p}_1^0,$$

$$\hat{x}_2^{sq} = \sqrt{n_2} e^{-r_2} \hat{x}_2^0, \quad \hat{p}_2^{sq} = \sqrt{n_2} e^{r_2} \hat{p}_2^0,$$

where the suffix “0” refers to the vacuum. The action of an ideal (phase-free) beam splitter operation on a pair of modes $i$ and $j$ is defined as $B_{i,j}(\theta) : \hat{a}_i \to \hat{a}_i \cos \theta + \hat{a}_j \sin \theta$, $\hat{a}_j \to \hat{a}_i \sin \theta - \hat{a}_j \cos \theta$, where $\hat{a}_k = (\hat{x}_k + i\hat{p}_k)/2$ is the annihilation operator of mode
When applied to the two modes of Eqs. (12), the beam splitter entangling operation \((\theta = \pi/4)\) produces a symmetric mixed state \([13]\), depending on the squeezings \(r_{1,2}\) and on the thermal noises \(n_{1,2}\). The noise can be difficult to control and reduce in the lab, but it is quantifiable. Now, keeping \(n_1\) and \(n_2\) fixed, all states produced starting with different \(r_1\) and \(r_2\), but with equal average \(\bar{r} \equiv (r_1 + r_2)/2\), are completely equivalent up to local unitary operations and possess, by definition, the same entanglement. Let us recall that a two-mode Gaussian state is entangled if and only if it violates the positivity of partial transpose (PPT) condition \(\eta \geq 1\) \([14]\). The quantity \(\eta\) is the smallest symplectic eigenvalue of the partially transposed CM, which is obtained from the CM of the Gaussian state by performing transposition (time reversal in phase space \([14]\)) in the subspace associated to either one of the modes. The CM \(\sigma\) of a generic two-mode Gaussian state can be written in the block form \(\sigma = \begin{pmatrix} \alpha & \gamma \\ \gamma^* & \beta \end{pmatrix}\), where \(\alpha\) and \(\beta\) are the CM’s of the individual modes, while the matrix \(\gamma\) describes intermodal correlations. One then has \(2\eta = \Sigma(\sigma) - \sqrt{\Sigma^2(\sigma) - 4\text{Det}\sigma}\), where \(\Sigma(\sigma) = \text{Det}\alpha + \text{Det}\beta - 2\text{Det}\gamma\) \([15]\). The parameter \(\eta\) also provides a quantitative characterization of CV entanglement, because the logarithmic negativity and, equivalently, for symmetric states (\(\text{Det}\alpha = \text{Det}\beta\)), the entanglement of formation \(E_F\), are both decreasing functions of \(\eta\). For symmetric Gaussian states the bipartite entanglement \(E_F\) reads \([12]\):

\[
E_F(\sigma) = \max \{0, f(\eta)\},
\]

with

\[
f(x) = \frac{(1+x)^2}{4x} \log \frac{(1+x)^2}{4x} - \frac{(1-x)^2}{4x} \log \frac{(1-x)^2}{4x}.
\]

For the mixed two-mode states considered here, we have

\[
\eta = r_1 r_2 e^{-(r_1 + r_2)}.
\]

The entanglement thus depends both on the arithmetic mean of the individual squeezings, and on the geometric mean of the individual noises, which is related to the purity of the state \(\mu = (n_1 n_2)^{-1}\). The teleportation success, instead, depends separately on each of the four single-mode parameters. The fidelity (averaged over the complex plane) for teleporting an unknown single-mode coherent state can be computed by writing the quadrature operators in Heisenberg picture \([2,17]\):

\[
F = \phi^{-1/2}, \quad \phi = \left\{ \left(\hat{x}_{tel}^2 \right)^2 + 1 \right\} \cdot \left\{ \left(\hat{p}_{tel}^2 \right)^2 + 1 \right\} / 4,
\]

where \(\left\langle \hat{x}_{tel}^2 \right\rangle\) and \(\left\langle \hat{p}_{tel}^2 \right\rangle\) are the variances of the canonical operators \(\hat{x}_{tel}\) and \(\hat{p}_{tel}\) which describe the teleported mode. For the utilized states, we have \(\hat{x}_{tel} = \hat{x}^{in} - \sqrt{2n_2} e^{-r_2} \hat{x}_2^{in}, \hat{p}_{tel} = \hat{p}^{in} + \sqrt{2n_1} e^{-r_1} \hat{p}_2^{in}\), where the suffix “in” refers to the input coherent state to be teleported. Recalling that, in our units \([11]\), \(\left\langle \hat{x}^2 \right\rangle = \left\langle \hat{p}^2 \right\rangle = 1\), we can compute the fidelity from Eq. (5), obtaining \(\phi(r_{1,2}, n_{1,2}) = e^{-2(r_1 + r_2)}(e^{2r_1} + n_1)(e^{2r_2} + n_2)\). It is convenient to replace \(r_1\) and \(r_2\) by \(\bar{r}\) and \(d \equiv (r_1 - r_2)/2\):

\[
\phi(\bar{r}, d, n_{1,2}) = e^{-4\bar{r}}(e^{2(d+\bar{r})} + n_1)(e^{2(d-\bar{r})} + n_2).
\]

Maximizing the fidelity for given entanglement and noises of the Gaussian resource state (i.e. for fixed \(n_{1,2}, \bar{r}\)) simply means finding the \(d = d^{opt}\) which minimizes the quantity \(\phi\) of Eq. (6). Being \(\phi\) a convex function of \(d\), it suffices to find the zero of \(\partial \phi / \partial d\), yielding \(d^{opt} = \frac{1}{2} \log \frac{n_1}{n_2}\). For equal noises, \(d^{opt} = 0\), indicating that the best preparation of the entangled resource state needs two equally squeezed single-mode states, in agreement with the results presented in Ref. \([13]\) for pure states. For different noises, however, the optimal procedure involves two different squeezings such that \(r_1 - r_2 = 2d^{opt}\). Inserting \(d^{opt}\) in Eq. (6) we have the optimal fidelity

\[
\mathcal{F}^{opt} = 1/(1 + \eta),
\]

where \(\eta\) is exactly the lowest symplectic eigenvalue of the partial transpose, defined by Eq. (4). Eq. (7) clearly shows that the optimal teleportation fidelity depends only on the entanglement of the resource state, and vice versa. In fact, the fidelity criterion becomes necessary and sufficient for the presence of the entanglement, if \(\mathcal{F}^{opt}\) is considered: the optimal fidelity is classical for \(\eta \geq 1\) (separable state) and greater than the classical threshold for any entangled state. Moreover, \(\mathcal{F}^{opt}\) provides a quantitative measure of entanglement completely equivalent to the two-mode entanglement of formation, namely (from Eqs. (5,7)) \(E_F = \max \{0, f(1/\mathcal{F}^{opt} - 1)\}\). In the limit of infinite squeezing (\(\bar{r} \to \infty\)), \(\mathcal{F}^{opt}\) reaches 1 for any amount of finite thermal noise. On the other extreme, due to the convexity of \(\phi\), the lowest fidelity (maximal waste of entanglement) is attained at one of the boundaries \(d = \pm \bar{r}\), meaning that one of the squeezings \(r_{1,2}\) vanishes. For infinite squeezing, the worst fidelity cannot exceed \(1/\sqrt{\max\{n_1, n_2\}}\), falling below 1/2 for strong enough noise.

We now extend our analysis to a quantum teleportation-network protocol, involving \(N\) users who share a genuine \(N\)-partite entangled Gaussian resource, completely symmetric under permutations of the modes \([6]\). Two parties are randomly chosen as sender (Alice) and receiver (Bob), but this time, in order to accomplish teleportation of an unknown coherent state, Bob needs the results of \(N - 2\) momentum detections performed by the other cooperating parties. A nonclassical teleportation fidelity (i.e. \(F > F^d = 1/2\)) between any pair of parties is sufficient for the presence of genuine \(N\)-partite entanglement in the shared resource, while in general the converse is false (see e.g. Fig.1 of Ref. \([6]\)). Our aim is to determine the optimal multi-user teleportation fidelity, and to extract from it a quantitative information on the multipartite entanglement in the shared resources. We begin by considering a mixed momentum-squeezed state described by \(r_{1,2}\) as in Eq. (1), and \(N - 1\) position-squeezed states of the form \([6]\):

\[
\hat{N}_{1\ldots N} \equiv B_{N-1,N}(\pi/4)B_{N-2,N-1}(\cos^{-1}1/\sqrt{3}) \ldots B_{1,2}(\cos^{-1}1/\sqrt{N}).
\]

The resulting state is a completely symmetric mixed Gaussian state of a \(N\)-mode CV system, parametrized by \(n_{1,2}, \bar{r}\) and \(d\). Once again, all states with equal \(\{n_{1,2}, \bar{r}\}\) belong to the same iso-entangled class of equivalence. For \(\bar{r} \to \infty\) and for \(n_{1,2} = 1\) (pure states), these states reproduce the (unnormalizable) CV Greenberger-
Horne-Zeilinger (GHZ) \(^{19}\) state \(\int dx|x, x, \ldots, x\rangle\), an eigenstate with total momentum zero and all relative positions \(x_i - x_j = 0\) (\(i, j = 1, \ldots, N\)). Choosing randomly two modes, denoted by the indices \(k\) and \(l\), to be respectively the sender and the receiver, the teleported mode is described by the following quadrature operators \((\text{see Refs. } 6, 17\text{ for further details})\): \(\hat{x}_{tel} = \hat{x}_n - \hat{x}_{rel}\), \(\hat{p}_{tel} = \hat{p}_n + \hat{p}_{tot}\), with \(\hat{x}_{rel} = \hat{x}_k - \hat{x}_l\) and \(\hat{p}_{tot} = \hat{p}_k + \hat{p}_l + gN\sum_{j\neq k,l}\hat{p}_j\), where \(g\) is an experimentally adjustable gain. To compute the teleportation fidelity from Eq. (5), we need the variances of \(\hat{x}_{rel}\) and \(\hat{p}_{tot}\). From the action of the \(N\)-splitter, we have

\[
\langle (\hat{x}_{tel})^2 \rangle = 2n_2e^{-2(\eta_d - \eta)} , \quad \langle (\hat{p}_{tot})^2 \rangle = \left\{ 2 + (N - 2)gN \right\}^2 n_1 e^{-2(\eta_d + \eta)} + 2 \left[ gn_1 - 1 \right]^2 (N - 2)n_2 e^{2(\eta_d - \eta)} \right\} / 4 .
\]

The optimal fidelity can be found in two straightforward steps: 1) minimizing \(\langle (\hat{p}_{tot})^2 \rangle\) with respect to \(g\) (i.e. finding the optimal gain \(g^\text{opt}\)); 2) minimizing the resulting \(\phi\) with respect to \(d\) (i.e. finding the optimal \(d^\text{opt}_N\)). The results are

\[
g^\text{opt}_N = 1 - N / \left[ (N - 2) + 2e^{4\eta_2}n_2/n_1 \right] ,
\]

\[
d^\text{opt}_N = \bar{r} + \log \left\{ N / \left[ (N - 2) + 2e^{4\eta_2}n_2/n_1 \right] \right\} / 4 .
\]

Inserting Eqs. 8, 10 in Eq. 5, we find the optimal teleportation-network fidelity, which can be put in the following general form for \(N\) modes

\[
F_N^\text{opt} = \frac{1}{1 + \eta_N} , \quad \eta_N = \sqrt{\frac{Nn_1n_2}{2e^{4\eta} + (N - 2)n_1/n_2}} .
\]

For \(N = 2\), \(\eta_2 = \eta\) from Eq. 4, showing that the general multipartite protocol comprises the standard bipartite case. By comparison with Eq. 4, we observe that, for any \(N > 2\), the quantity \(\eta_N\) plays the role of a generalized symplectic eigenvalue, whose physical meaning will be clear soon. Before that, it is worth commenting on the form of the optimal resources, focusing for simplicity on the pure-state setting \((n_{1,2} = 1)\). The optimal form of the shared \(N\)-mode symmetric Gaussian states, for \(N > 2\), is neither unbiased in the \(x_i\) and \(p_i\) quadratures (like the states discussed in Ref. 18 for three modes), nor constructed by \(N\) equal squeezers \((r_1 = r_2 = \bar{r})\). This latter case, which has been implemented experimentally for \(N = 3\) \(^{17}\), is clearly not optimal, yielding fidelities lower than 1/2 for \(N \geq 30\) and \(\bar{r}\) falling in a certain interval \(^{3}\). The explanation of this paradoxical behaviour, provided by the authors of Ref. 3, is that their teleportation scheme might not be optimal. Our analysis shows instead that the problem does not lie in the protocol, but rather in the employed states. If the shared \(N\)-mode resources are prepared (or locally transformed) in the optimal form of Eq. 10, the teleportation fidelity is guaranteed to be nonclassical (see Fig 1) as soon as \(\bar{r} > 0\) for any \(N\), in which case the considered class of pure states is genuinely multipartite entangled \(^{17,20}\). Therefore a nonclassical optimal fidelity is necessary and sufficient for the presence of multipartite entanglement in any multimode symmetric Gaussian state, shared as a resource for CV teleportation. On the opposite side, the worst preparation scheme of the multimode resource states, even retaining the optimal protocol \((g = g_N^\text{opt})\), is obtained setting \(r_1 = 0\) if \(n_1 > 2n_2e^{2\bar{r}} / (N e^{2\bar{r}} + 2 - N)\), and \(r_2 = 0\) otherwise. For equal noises \((n_1 = n_2)\), the case \(r_1 = 0\) is always the worst one, with asymptotic fidelities (in the limit \(\bar{r} \to \infty\)) equal to \(1 / \sqrt{1 + Nn_{1,2}/2}\), so rapidly dropping with \(N\) at given noise.

The meaning of \(\eta_N\), crucial for the quantification of the multipartite entanglement, stems from the following argument. The teleportation network 6 is realized in two steps: first, the \(N - 2\) cooperating parties perform local measurements on their modes, then Alice and Bob exploit their resulting highly entangled two-mode state to accomplish teleportation. Stopping at the first stage, the protocol describes a concentration, or localization of the original \(N\)-partite entanglement, into a bipartite two-mode entanglement 6, 17. Here, the LE is the maximal entanglement concentrable onto two modes, by unitary operations and nonunitary momentum detections performed locally on the other \(N - 2\) modes. The two-mode entanglement of the resulting state (described by a CM \(\sigma_{\text{loc}}\)) is quantified in terms of the symplectic eigenvalue \(\eta_{\text{loc}}\) of its partial transpose. Due to the symmetry of the original state and of the protocol (the gain is the same for every mode), the localized two-mode state is symmetric too. It has been proven \(^{13}\) that, for two-mode symmetric Gaussian states, the symplectic eigenvalue \(\eta\) is related to the EPR correlations by the expression \(4\eta = (\langle \hat{x}_1 - \hat{x}_2 \rangle^2 + (\hat{p}_1 + \hat{p}_2)^2 \rangle\). For the state \(\sigma_{\text{loc}}\), this means \(4\eta_{\text{loc}} = (\langle \hat{x}_{\text{rel}} \rangle^2 + (\hat{p}_{\text{tot}})^2 \rangle\), where the variances have been computed in Eq. 8. Minimizing \(\eta_{\text{loc}}\) with respect to \(d\) means finding the optimal set of local unitary operations (unafffecting multipartite entangle-
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metric Gaussian states, in terms of the so-called quantify multipartite entanglement in CV systems: the maximal fidelity achievable in the highest entanglement on a pair of modes. From Eq. (8), the optimizations are readily solved and yield the same optimal \( g_N^{\text{opt}} \) and \( d_N^{\text{opt}} \) of Eqs. (9,10). The resulting two-mode state contains a localized entanglement exactly quantified by the quantity \( \eta_N^{\text{loc}} = \eta_N \). It is now clear that \( \eta_N \) of Eq. (11) is a proper symplectic eigenvalue, being the smallest one of the partial transpose of the optimal two-mode state that can be extracted from a \( N \)-party entangled resource by local measurements on the remaining modes. Eq. (11) thus provides a bright connection between two operative aspects of multipartite entanglement in CV systems: the maximal fidelity achievable in a multi-user teleportation network [22], and the LE [8].

This results yield quite naturally a direct operative way to quantify multipartite entanglement in \( N \)-mode (mixed) symmetric Gaussian states, in terms of the so-called Entanglement of Teleportation, defined as the normalized optimal fidelity

\[
E_T = \max \left\{ 0, \frac{F_N^{\text{opt}} - F_N^{\text{cl}}}{1 - F_N^{\text{cl}}} \right\} = \max \left\{ 0, \frac{1 - \eta_N}{1 + \eta_N} \right\},
\]

and thus ranging from 0 (separable states) to 1 (CV GHZ state). A homonym but different concept has also been introduced for discrete variables [21]. The localizable entanglement of formation \( E_F^{\text{loc}} \) of \( N \)-mode symmetric Gaussian states is a monotonically increasing function of \( E_T \), namely:

\[
E_F^{\text{loc}} = f[(1 - E_T)/(1 + E_T)],
\]

with \( f(x) \) defined after Eq. (5). For \( N = 2 \) the state is already localized and \( E_F^{\text{loc}} = E_F \).

Remarkably for three-mode pure (symmetric) Gaussian states, the residual contangle \( E_T \), a tripartite entanglement monotone under Gaussian LOCC that quantifies CV entanglement sharing [3], is also a monotonically increasing function of \( E_T \), thus providing another equivalent quantitative characterization of genuine tripartite CV entanglement. In formula:

\[
E_T = \log^2 2 \sqrt{E_T - (E_T - 1) \sqrt{E_T^2 + 1}} - \frac{1}{2} \log^2 \frac{E_T^2 + 1}{E_T^2 (E_T + 1) + 1}.
\]

This finding suggests an experimental test, in terms of optimal fidelities in teleportation networks [7], to verify the promiscuous sharing of tripartite CV entanglement in pure symmetric three-mode Gaussian states, discovered in Ref. [9].

Whether an expression of the form Eq. (12) connecting \( E_T \) to the symplectic eigenvalue \( \eta_N \) remains true for generalized teleportation protocols [22] and for nonsymmetric entangled resources, is currently an open question. However, nonsymmetric Gaussian states are never optimal candidates for communication protocols, as their maximum achievable entanglement decreases with increasing asymmetry [15], and therefore they are automatically ruled out by the present analysis.

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