Estimation of drying parameters in rotary dryers using differential evolution

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Abstract. Inverse problems arise from the necessity of obtaining parameters of theoretical models to simulate the behavior of the system for different operating conditions. Several heuristics that mimic different phenomena found in nature have been proposed for the solution of this kind of problem. In this work, the Differential Evolution Technique is used for the estimation of drying parameters in realistic rotary dryers, which is formulated as an optimization problem by using experimental data. Test case results demonstrate both the feasibility and the effectiveness of the proposed methodology.

1. Introduction
Inverse problems or parameter identification problems arise from the necessity of obtaining parameters of theoretical models in such a way that the models can be used to simulate the behavior of the system for different operating conditions. The estimation procedure consists in obtaining the model parameters through the minimization of the difference between calculated and experimental values. These problems arise frequently in process engineering with applications in process control, fermentation process, and drying, among others.

Traditionally, inverse problems have been treated by two different approaches: the deterministic approach that makes use of the Variational Calculus and the non-deterministic one that it is based on the process of natural selection, i.e., in the genetics of the populations or, alternatively, in purely structural methodologies. The use of the non-deterministic approach is getting increasing attention in the last decade, mainly due to the fact that they do not use derivatives and they can be easily implemented. Besides, non-deterministic approaches are in general more robust than the deterministic ones when dealing with inverse problems.

One of the non-deterministic techniques is the so-called Differential Evolution (DE) algorithm, which is considered as a structural approach. It has shown to be a viable alternative for handling realistic problems. Its main characteristics are the following: simple conceptual base and easiness for implementation.

According to the literature, a challenging inverse problem has to do with the determination of the parameters of the drying process through a cascading rotary dryer. This equipment is commonly used

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to dry particulate material in a range of food and mineral processing industries. The experimental shape of stationary profiles of temperature and humidity for the gas phase and temperature and moisture for the solid phase in a counter-current cascading rotary dryer can be used to test a mathematical model and to find optimal values of heat and mass transfer coefficients.

In the present work, DE is used for estimating the characteristic parameters of the constitutive equations for the drying kinetics and the heat loss coefficient in a rotary dryer pilot plant for which the fertilizer granulates simple super-phosphate (SSPG) is used as wet material. The results are compared with real experimental data, indicating that the proposed approach characterizes a promising methodology for solving inverse problems related to drying.

This work is presented as follows. Section 2 presents the general aspects regarding the rotary dryer. In sections 3 and 4 the mathematical formulation of the inverse problem and the approach used are presented, respectively. In section 5 the results obtained are shown and discussed. Finally, the conclusions are outlined in section 6.

2. The Rotary Dryer

The rotary dryer consists basically of a cylindrical shell inclined at a small angle with respect to the horizontal position. To promote gas-solid contact, the dryer is equipped with lifting flights, placed parallel along the length of the shell, which lift solids and make them rain across the section of the dryer. The rotary dryer (a) and a scheme of discharge of solids by the flights inside the cylinder (b) are shown in Figure 1

![Figure 1. Schematic representation of a rotary dryer and the lifting flights.](a) (b)

The modeling of drying in this equipment needs constitutive equations, such as drying kinetics and heat loss coefficient equations, among others. These constitutive equations are expressed in terms of a set of parameters that are fitted for each type of material used. For this aim specific time-consuming experiments are necessary. The solution of the model for the simultaneous mass and heat transfer in this dryer are obtained by using a set of Two Point Boundary Value Problem for heat and mass balance that describe the variation of the humidity and temperature of air and the moisture and temperature of the wet solids along the shell. Thus, the development of a computational method for estimating the parameters of the constitutive equations of the model of heat and mass transfer to save time-consuming experimentation in laboratory is an important issue.

3. Mathematical Description of Rotary Dryer Process

By applying mass and energy balances to both phases in the discrete element of volume shown in figure 2, a set of differential equations is obtained for describing the profiles of temperature and humidity for the gas phase and temperature and moisture for the solid phase in a counter-current cascading rotary dryer.
Figure 2. Control volume for mass and energy balances.

Gas humidity:

\[
\frac{dW}{dz} = -\frac{R_w H}{G_f}
\]  

(1)

Solid moisture content:

\[
\frac{dM}{dz} = -\frac{R_w H}{G_S}
\]  

(2)

Gas temperature:

\[
\frac{dT_f}{dz} = \frac{\left[UaV(T_f - T_S) + R_w H (\lambda + Cp_f T_f) + U_p \pi DL(T_f - T_{amb})\right]}{G_f (Cp_f + WCp_v)}
\]  

(3)

Solid temperature:

\[
\frac{dT_S}{dz} = \frac{\left[UaV(T_f - T_S) + R_w H Cp_f T_S - R_w H [\lambda + Cp_v (T_f - T_S)]\right]}{G_S (Cp_S + MCp_l)}
\]  

(4)

Boundary Conditions: \( T_f(1) = T_{f0}; T_S(0) = T_{S0}; W(1) = W_0; M(0) = M_0. \)

The local drying rate is expressed by equation (5):

\[
-R_w = \frac{(MR - 1)(M_0 - M_{eq})}{t}
\]  

(5)

In this work, the dimensionless moisture (\( MR \)) was evaluated by the Page’s equation [1], according to equation (6).
\[ MR = \exp \left( -C_1 \exp \left( \frac{-C_2}{T_f} \right) r_f \right) \]

where \( t \) is available through relation between the position in the dryer (\( z \)) and the solid flowing velocity (\( v_s \)):

\[ t = \frac{z}{v_s} = \left( \frac{TR}{L} \right) z \]

The sorption isotherm is given by equation (8) and was calculated by using the modified Halsey equation [2] as obtained in laboratory specifically for the material used in this work.

\[ M_{eq} = \left( \frac{-\exp(-0.0445 T_s + 2.0795)}{\ln(UR)} \right)^{1.4349} \]

The volumetric heat transfer coefficient is described by equation (9) and the heat loss coefficient by equation (10).

\[ Ua = 3.535(G_f)^{0.289}(G_s)^{0.541} \]

\[ U_p = k_p(G_f)^{m_p} \]

where \( G_f \) and \( G_s \) are given by:

\[ G_f = \frac{1.510^{-3} AP MM_{ar}}{R(T_f + 273.15)(1+W_0)} \]

\[ G_s = \frac{G_{SL}}{1+M_0} \]

The holdup of solid dry in the dryer is:

\[ H = \frac{G_s TR z}{1+M_0} \]

The latent heat is given by Equation (14) [3]:

\[ \lambda = 2492.71 - 2.144 T_s - 0.001577 T_s^2 - 7.3353 \times 10^{-6} T_s^3 \]

The heat loss is given by a correlation given in Douglas et al. [4]:

\[ Q_p = U_p \pi DL(T_a - T_{amb}) \]

More details about the hypotheses used and the development of the model can be found in [5].
4. Mathematical Formulation of the Inverse Problem

The inverse problem consists in the determination of PAGE equation parameters (C_1, C_2 and C_3) and the heat loss coefficient (k_P and m_P) that minimize the difference between the experimental and calculated values:

\[
f = \frac{1}{M_{\text{max}}^2} \sum_{i=1}^{n=5} (M_{\text{sim}}^i - M_{\text{exp}}^i)^2 + \frac{1}{T_{s_{\text{max}}}^2} \sum_{i=1}^{n=5} (T_{s_{\text{sim}}}^i - T_{s_{\text{exp}}}^i)^2 + \frac{1}{T_{f_{\text{max}}}^2} \sum_{i=1}^{n=5} (T_{f_{\text{sim}}}^i - T_{f_{\text{exp}}}^i)^2
\]

subjected to the constraint equations (1) to (15) and by the side constraints: -10 \leq C_1, C_2, C_3, k_P, m_P \leq 500.

The variables \(\Omega_{\text{sim}}\) and \(\Omega_{\text{exp}}\) are respectively the simulated and experimental values (\(\Omega = [M W T_s T_f]\)), \(M_{\text{max}}, T_{s_{\text{max}}}, T_{f_{\text{max}}}\) are the maximum experimental values observed, and \(n\) represents the number of points (experimental points) used in the estimation procedure.

It should be emphasized that due to the difficulty of accurately measuring the gas humidity (W) in the dryer the experimental data of this variable were not used.

For the solution of the inverse problem described above, the authors have chosen the Differential Evolution (DE) [6] algorithm to minimize the objective function.

5. Solution of the Inverse Problem using Differential Evolution

Differential Evolution [6] is an improved version of the Goldberg’s Genetic Algorithm (GA) [7] for faster optimization. Unlike simple GA that uses binary coding for representing problem parameters, DE is a simple yet powerful population based, direct search algorithm for globally optimizing functions with real valued parameters and that differs from other evolutionary algorithms in the mutation and recombination phase. According to Storn and Price (1995), DE has as main advantages the simple structure, easiness of coding, speed and robustness. However, main difficulty with the technique appears to lie in the slowing down of convergence as the region of global minimum and stagnation.

Storn and Price [6] gave the basic principles of DE for a single strategy. Later on, they suggested ten different strategies for DE [8]. The crucial idea behind DE is a scheme for generating trial parameter vectors. Basically, DE adds the weighted difference between two population vectors to a third vector. The key parameters of control in DE are: \(N\) the population size, \(CR\) the crossover constant, and \(F\) the perturbation rate or scaling factor. Price and Storn [8] have given some simple rules for choosing key parameters of DE for any given application. Normally, \(N\) should be about 5 to 10 times the dimension (number of parameters in a vector) of the problem. As for \(F\), it lies in the range 0.4 to 1.0. Initially \(F = 0.5\) can be tried, then \(F\) and/or \(N\) is increased if the population converges prematurely. A generic DE algorithm is presented below.

### Differential Evolution

Initialize and evaluate population \(Pop\) while (not done) {
  for \((i = 0; i < N; i++)\) {
    \textbf{Create candidate} \(Cand[i]\)
    Evaluate \(Cand[i]\)
    if (\(Cand[i]\) is better than \(Pop[i]\))
      \(Pop[i] = Cand[i]\)
    else
      \(Pop[i] = Pop[i]\)
  }
}

### Create candidate \(Cand[i]\)

Randomly select parents \(Pop[i_1], Pop[i_2]\), and \(Pop[i_3]\) where \(i_1, i_2, \text{ and } i_3\) are different.
Create initial candidate \(Cand[i] = Pop[i_1] + F \times (Pop[i_2] - Pop[i_3])\).
Create final candidate \(Cand[i]\) by crossing over the genes of \(Pop[i]\) and \(Cand[i]\) as follows:
for \((j = 0; j < N; j++)\) {
  if \((r < CR)\)
    \(Cand[i][j] = Cand[i][j]\)
  else
    \(Cand[i][j] = Pop[i][j]\)
}
where \( N \) is the population size, \( Pop \) is the population of the current generation, \( Pop' \) is the population to be formed for the next generation, \( Cand[i] \) is the candidate solution with population index \( i \), \( Cand[i][j] \) is the \( j \)-th entry in the solution vector of \( Cand[i] \), \( r \) is a random number between 0 and 1, \( CR \) is the probability of crossover, and \( F \) is the scaling factor.

DE has been successfully applied in various fields. Some of the successful applications of DE include: digital filter design [9], batch fermentation process [10], synthesis and optimization of heat integrated distillation system [11], optimization of robotic systems [12], solution of multi-objective optimal control problems with index fluctuation applied to fermentation process [13], multi-objective optimization of mechanical structures [14], apparent thermal diffusivity estimation of the banana drying [16], and other applications [15].

6. Results and Discussion

In order to evaluate the performance of the DE algorithm we have considered the three test cases listed in table 1.

| Case  | \( M(z=0)=0.1124 \) | \( W(z=1)=0.0057 \) | \( T_s(z=0)=32.3 \, ^{\circ}C \) | \( T_f(z=1)=94.8 \, ^{\circ}C \) | \( UR = 0.1721 \) |
|-------|--------------------|------------------|------------------|------------------|----------------|
| Case 1| \( M(z=0)=0.1384 \) | \( W(z=1)=0.0048 \) | \( T_s(z=0)=25.0 \, ^{\circ}C \) | \( T_f(z=1)=74.6 \, ^{\circ}C \) | \( UR = 0.1902 \) |
| Constant conditions | \( CP_v=1.02577 \, \text{kJ/(kg}^{\circ}\text{C)} \) | \( CP_p=1 \, \text{kJ/(kg}^{\circ}\text{C)} \) | \( CP_f=4.1868 \, \text{kJ/(kg}^{\circ}\text{C)} \) | \( CP_r=1.1723 \, \text{kJ/(kg}^{\circ}\text{C)} \) | \( A=\pi r^2 \) m\(^2\) | \( r=0.15 \, m \) | \( MM_{ar}=28.9 \, \text{g/mol} \) | \( R=8.2x10^{-5} \, \text{atm m}^3/(\text{mol K}) \) | \( P=0.91 \, \text{atm} \) | \( L=1.40 \, m \) | \( V=LA \) m\(^3\) | \( D=2r \, m \) | \( T_{amb}=35^{\circ}C \) | \( \alpha=3^{\circ} \) | \( Y_{aq}=0.209 \) | \( t_{aq}=0.209 \) | \( TR=327 \, s \) (\( 0 \leq t \leq TR \)) |

The parameters used in DE algorithm are the following: 15 individuals in the population, 250 generations, perturbation rate and crossover probability both equal to 0.8, and DE/rand/1/bin strategy. In the solution of the direct problem 10 collocation points was used. The simulations were performed in a microcomputer PENTIUM IV with 3.2 GHz and 2 GB of RAM. Table 2 presents the results obtained for the two cases by considering 20 executions of the algorithm for obtaining the average values.

| Case |  \( C_1 \) |  \( C_2 \) |  \( C_3 \) |  \( k_P \) |  \( m_P \) |  \( f \) |
|------|------------|------------|------------|-----------|------------|--------|
| 1    | 86.841     | 361.961    | -0.673     | 29.995    | 2.835      | 0.017  |
| Best | 98.207     | 368.033    | -0.697     | 45.735    | 3.009      | 0.017  |
| 2    | 45.729     | 387.017    | 0.159      | 37.545    | 3.103      | 0.0079 |
| Best | 44.924     | 386.232    | 0.161      | 42.556    | 3.155      | 0.0079 |

It is worth mentioning that 3765 objective function evaluations were performed and the calculation time for each set of 20 optimization runs (for averaging) was approximately 14.2 minutes. Figure 3 shows the experimental (exp) and simulated (sim) profiles by considering the best results obtained for each case.

In these figures it is possible to observe a good correspondence between the experimental data and the simulated ones for the case in which the optimal parameters were used.

Conclusions

In the present work, the DE approach yielded good estimates for the parameters of Page’s equation and the heat loss coefficient by using experimental data from a realistic rotary dryer. The results presented are very encouraging, and the approach developed for the solution of the inverse problem deserves further investigation regarding its application to more complex systems.
Figure 3. Experimental data versus simulated results.

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Nomenclature

- $A$: cross-sectional area of the dryer, $m^2$
- $C_{d}$: candidate
- $C_{p}^{f}$: specific heat of the dry air, $kJ \cdot kg^{-1} \cdot ^{\circ}C^{-1}$
- $C_{1}$: PAGE equation parameter, dimensionless [-]
- $C_{2}$: PAGE equation parameter, dimension of time $s^{-1}$
- $C_{3}$: PAGE equation parameter, dimensionless [-]
- $C_{p}$: specific heat of the liquid water, $kJ \cdot kg^{-1} \cdot ^{\circ}C^{-1}$
- $C_{p}^{s}$: specific heat of the dry solid, $kJ \cdot kg^{-1} \cdot ^{\circ}C^{-1}$
- $C_{p_{v}}$: specific heat of water vapor, $kJ \cdot kg^{-1} \cdot ^{\circ}C^{-1}$
- $D$: dryer diameter, $m$
- $F$: perturbation Rate
- $G_{f}$: mass flow rate of air, $kg \cdot dry \ air^{-1}$
- $G_{sw}$: mass flow rate of wet solid, $kg \cdot dry \ solid^{-1}$
- $H$: solids holdup on the drier, $kg \cdot dry \ solid^{-1}$
- $L$: dryer length, $m$
- $M$: moisture content of the product, $kg \cdot water \ kg^{-1} \cdot dry \ solid^{-1}$
- $MM_{ar}$: molecular mass of the air, $kg \cdot kmol^{-1}$
- $MR$: dimensionless of moisture content of $eq$
- $N_{R}$: dryer rotational speed, $rev \ min^{-1}$
- $P$: atmospheric local pressure, atm
- $P_{0}$: initial condition
- $Q_{p}$: heat loss through dryer shell in a control volume, $kJ \cdot s^{-1}$
- $R$: constant of ideal gas, $atm \cdot m^{3} \cdot mol \cdot ^{\circ}C^{-1}$
- $R_{w}$: drying rate, $kg \cdot water \cdot kg^{-1} \cdot dry \ solid^{-1} \cdot s^{-1}$
- $t$: time, $s$
- $T_{f}$: drying air temperature, $^{\circ}C$
- $T_{amb}$: temperature of ambient air, $^{\circ}C$
- $T_{s}$: product temperature, $^{\circ}C$
- $T_{R}$: residence time of the solids on the dryer, $s$
- $U_{a}$: volumetric heat transfer coefficient between air and product, $kJ \cdot m^{-3} \cdot s^{-1} \cdot ^{\circ}C^{-1}$
- $U_{p}$: heat losses coefficient, $kJ \cdot m^{-2} \cdot s^{-1} \cdot ^{\circ}C^{-1}$
- $V$: relative humidity of air, decimal
- $V_{r}$: dryer volume, $m^{3}$
- $W$: absolute humidity of air, $kg \cdot water \ kg^{-1} \cdot dry \ air^{-1}$
- $z$: dimensionless of length, [-]
- $\lambda$: vaporization latent heat of water, $kJ \cdot kg^{-1}$
- $\lambda$: molecular mass of the air, $kg \cdot kmol^{-1}$
- $\lambda$: dimensionless of moisture content of $eq$
- $\lambda$: equilibrium
- $\lambda$: initial condition