Hypermultiplet dependence of the effective action in \( \mathcal{N} = 2 \) superconformal theories

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Abstract

I review the approach [1] to the one-loop low-energy effective action in the hypermultiplet sector for \( \mathcal{N} = 2 \) superconformal models. Any such a model contains an \( \mathcal{N} = 2 \) vector multiplet and some number of hypermultiplets. We found a general expression for the low-energy effective action in the form of a proper-time integral. The leading space-time dependent contributions to the effective action are derived and their bosonic component structure is analyzed. The component action contains terms with three and four space-time derivatives of component fields and has the Chern-Simons-like form.

1 Introduction

I am very glad to take part in this book devoted to celebration of the 60 birth day of remarkable scientist and my dear friend Ioseph L. Buchbinder.

Four-dimensional \( \mathcal{N} = 2 \) supersymmetric gauge theories are formulated in terms of \( \mathcal{N} = 2 \) vector multiplet coupled to a massless hypermultiplets in certain representations \( R \) of the gauge group \( G \). All such models possess only one-loop divergences [2] and can be made finite at certain restrictions on representations and field contents. In the model with \( n_\sigma \) hypermultiplets in representations \( R_\sigma \) of the gauge group \( G \) the finiteness condition has simple and universal form

\[
C(G) = \sum_\sigma n_\sigma T(R_\sigma),
\]

where \( C(G) \) is the quadratic Casimir operator for the adjoint representation and \( T(R_\sigma) \) is the quadratic Casimir operator for the representation \( R_\sigma \). A simplest solution to Eq.(1) is \( \mathcal{N} = 4 \) SYM theory where \( n_\sigma = 1 \) and all fields are taken in the adjoint representation. It is evident that there are other solutions, e.g. for the case of SU(\( N \)) group and hypermultiplets in the fundamental representation one gets \( T(R) = 1/2, C(G) = N \) and \( n_\sigma = 2N \). A number of \( \mathcal{N} = 2 \) superconformal models has been constructed in the context

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of AdS/CFT correspondence (see e.g. [3], the examples of such models and description of structure of vacuum states were discussed in details e.g. in Ref. [4]).

In this paper we study the structure of the low-energy one-loop effective action for the $\mathcal{N} = 2$ superconformal theories. The effective action of the $\mathcal{N} = 4$ SYM theory and $\mathcal{N} = 2$ superconformal models in the sector of $\mathcal{N} = 2$ vector multiplet has been studied by various methods. However a problem of hypermultiplet dependence of the effective action in the above theories was open for a long time.

The low-energy effective action containing both $\mathcal{N} = 2$ vector multiplet and hypermultiplet background fields in $\mathcal{N} = 4$ SYM theory was first constructed in Ref. [5] and studied in more details in [6]. In this paper we will consider the hypermultiplet dependence of the effective action for $\mathcal{N} = 2$ superconformal models. Such models are finite theories as well as the $\mathcal{N} = 4$ SYM theory and one can expect that hypermultiplet dependence of the effective action in $\mathcal{N} = 2$ superconformal models is analogous to one in $\mathcal{N} = 4$ SYM theory. However this is not so evident. The $\mathcal{N} = 4$ SYM theory is a special case of the $\mathcal{N} = 2$ superconformal models, however it possesses extra $\mathcal{N} = 2$ supersymmetry in comparison with generic $\mathcal{N} = 2$ models. As it was noted in [3] just this extra $\mathcal{N} = 2$ supersymmetry is the key point for finding an explicit hypermultiplet dependence of the effective action in $\mathcal{N} = 4$ SYM theory. Therefore a derivation of the effective action for $\mathcal{N} = 2$ superconformal models in the hypermultiplet sector is an independent problem.

In this paper we derive the complete $\mathcal{N} = 2$ supersymmetric one-loop effective action depending both on the background vector multiplet and hypermultiplet fields in a mixed phase where both vector multiplet and hypermultiplet have non-vanishing expectation values. The $\mathcal{N} = 2$ supersymmetric models under consideration are formulated in harmonic superspace [7]. We develop a systematic method of constructing the lower- and higher-derivative terms in the one-loop effective action given in terms of a heat kernel for certain differential operators on the harmonic superspace and calculate the heat kernel depending on $\mathcal{N} = 2$ vector multiplet and hypermultiplet background superfields. We study a component form of a leading quantum corrections for on-shell and beyond on-shell background hypermultiplets and find that they contain, among the others, the terms corresponding to the Chern-Simons-type actions. The necessity of such manifest scale invariant $P$-odd terms in effective action of $\mathcal{N} = 4$ SYM theory, involving both scalars and vectors, has been pointed out in [8]. Proposal for the higher-derivative terms in the effective action of the $\mathcal{N} = 2$ models in the harmonic superspace has been given in [9]. We show how the terms in the effective action assumed in P.C. Argyres at al. can be actually computed in supersymmetric quantum field theory.

2 The model and background field splitting

$\mathcal{N} = 2$ harmonic superspace has been introduced in [10] extending the standard $\mathcal{N} = 2$ superspace with coordinates $z^M = (x^m, \theta^i, \bar{\theta}_i) (i = 1, 2)$ by the harmonics $u^\pm_i$ parameterizing the two-dimensional sphere $S^2$: $u^+ u^- = 1$. $u^\pm = u^\pm_i$.

The main advantage of harmonic superspace is that the $\mathcal{N} = 2$ vector multiplet and hypermultiplet can be described by unconstrained superfields over the analytic subspace with the coordinates $\zeta^M \equiv (x^m_A, \theta^{+\alpha}, \bar{\theta}^{+\dot{\alpha}}, u^\pm_i)$, where the so-called analytic basis is defined by

$$x^m_A = x^m - i\theta^+ \sigma^m \bar{\theta}^- - i\theta^- \sigma^m \bar{\theta}^+, \quad \theta^ \pm = u^+ \theta^i, \quad \bar{\theta}^ \pm = u^\mp \bar{\theta}^i.$$ (2)
The \( \mathcal{N} = 2 \) vector multiplet is described by a real analytic superfield \( V^{++} = V^{++ \ell}(\zeta)T_1 \) taking values in the Lie algebra of the gauge group. A hypermultiplet, transforming in the representation \( R \) of the gauge group, is described by an analytic superfield \( q^+(\zeta) \) and its conjugate \( \tilde{q}^+(\zeta) \).

The classical action of \( \mathcal{N} = 2 \) SYM theory coupled to hypermultiplets consist of two parts: the pure \( \mathcal{N} = 2 \) SYM action and the \( q \)-hypermultiplet action in the fundamental or adjoint representation of the gauge group. Written in the harmonic superspace its action reads

\[
S = \frac{1}{2g^2} \text{tr} \int d^8z \mathcal{W}^2 + \frac{1}{2} \int d\zeta(-4)q^+_a f(D^{++} + igV^{++})q^{+a} , \tag{3}
\]

where we used the doublet notation \( q^+_a = (q^{\pm}, \tilde{q}^\pm) \). By construction, the action (3) is manifestly \( \mathcal{N} = 2 \) supersymmetric. Here \( d\zeta(-4) = d^4x d^4\theta d\bar{\theta} d\bar{u} \) denotes the analytic subspace integration measure and

\[
D^{++} = D^{++} + iV^{++} , \quad D^{++} = \partial^{++} - 2i\theta^m \sigma^m \bar{\theta}^\alpha \partial_\alpha , \quad \partial^{++} \equiv u^i \frac{\partial}{\partial u^{-i}}
\]

is the analyticity-preserving covariant harmonic derivative. It can be shown that \( V^{++} \) is the single unconstrained analytic, \( D_\alpha V^{++} = 0 \), prepotential of the pure \( \mathcal{N} = 2 \) SYM theory, and all other geometrical object are determined in terms of it. So, the covariantly chiral superfield strength \( \mathcal{W} \)

\[
\mathcal{W} = -\frac{1}{4}(D^+)^2V^{--} , \quad \bar{\mathcal{W}} = -\frac{1}{4}(D^+)^2V^{--} . \tag{4}
\]

is expressed through the (nonanalytic) real superfield \( V^{--} \) satisfying the equation

\[
D^{++}V^{--} = D^{--}V^{++} + i[V^{++}, V^{--}] = 0 .
\]

This equation has a solution in form of the power series in \( V^{++} \) [11].

For further use we will write down also the superalgebra of gauge covariant derivatives with the notation \( D^\pm_{(a, \dot{a})} = D^i_{(a, \dot{a})}u^\pm_i \):

\[
\{D^+_\alpha, D^-_{\beta}\} = -2i\varepsilon_{\alpha\beta}\mathcal{W} , \quad \{D^+_{\dot{\alpha}}, D^-_{\dot{\beta}}\} = 2i\varepsilon_{\dot{\alpha}\dot{\beta}}\mathcal{W} , \tag{5}
\]

\[
\{D^+_{\dot{\alpha}}, D^-_{\dot{\beta}}\} = -\{D^+_{\dot{\alpha}}, D^-_{\dot{\beta}}\} = 2iD^+_{\dot{\alpha}\dot{\beta}} ,
\]

\[
[D^+_{\alpha}, D^\pm_{\beta\dot{\beta}}] = \varepsilon_{\alpha\beta\dot{\beta}}D^\pm_{\beta\dot{\beta}}\mathcal{W} , \quad [D^\pm_{\dot{\alpha}}, D^\pm_{\dot{\beta}\dot{\beta}}] = \varepsilon_{\dot{\alpha}\dot{\beta}}D^\pm_{\dot{\beta}\dot{\beta}}\mathcal{W} ,
\]

\[
[D^\pm_{\alpha\dot{\alpha}}, D^\pm_{\beta\dot{\beta}}] = \frac{1}{2i} \{\varepsilon_{\alpha\beta\dot{\alpha}\dot{\beta}}D^-_{\dot{\alpha}\dot{\beta}}\mathcal{W} + \varepsilon_{\dot{\alpha}\dot{\beta}\alpha\beta}D^-_{\alpha\beta}\mathcal{W} \} = \frac{1}{2i} \{\varepsilon_{\alpha\beta\dot{\alpha}\dot{\beta}}\bar{F}_{\dot{\alpha}\dot{\beta}} + \varepsilon_{\dot{\alpha}\dot{\beta}\alpha\beta}F_{\alpha\beta} \} .
\]

The operators \( D^+_{\alpha} \) and \( D^+_{\dot{\alpha}} \) strictly anticommute

\[
\{D^+_\alpha, D^+_\dot{\beta}\} = \{D^+_\dot{\alpha}, D^+_\beta\} = \{D^+_{\alpha}, D^+_{\dot{\beta}}\} = 0 . \tag{6}
\]

A full set of gauge covariant derivatives includes also the harmonic derivatives \( (D^{++}, D^{--}, D^0) \), which form the algebra \( su(2) \) and satisfy the obviously commutation relations with \( D^\pm_{\alpha} \) and \( D^\pm_{\dot{\alpha}} \).

The action (3) possesses the superconformal symmetry \( SU(2, 2|2) \) which is manifest in the harmonic superspace approach. The low energy effective action at a generic vacuum of \( \mathcal{N} = 2 \) gauge theory includes only massless \( U(1) \) vector multiplets and massless
neutral hypermultiplets, since charged vectors and charged hypermultiplets get masses by the Higgs mechanism. The moduli space of vacua for the theory under consideration is specified by the following conditions [12]:

\[
[\bar{\phi}, \phi] = 0, \quad \phi f_i = 0, \quad \bar{f}^i \bar{\phi} = 0 \quad \bar{f}^i T_I f^j = 0.
\]  

(7)

Here the \(\phi, \bar{\phi}\) are the scalar components of \(\mathcal{N} = 2\) vector multiplet and complex scalars \(f_i\) are the scalar components of the hypermultiplet.

The structure of a vacuum state is characterized by solutions to Eqs. (7). These solutions can be classified according to the phases or branches of the gauge theory under consideration. In the pure Coulomb phase \(f_i = 0, \phi \neq 0\) and unbroken gauge group is \(U(1)^{\text{rank}(G)}\). In the pure Higgs phase \(f_i \neq 0\) and the gauge symmetry is completely broken; there are no massless gauge bosons. In the mixed phases, i.e. on the direct product of the Coulomb and Higgs branches (some number of \(\phi, \bar{\phi}\) is not equal to zero and some number of \(f_i\) is not equal to zero) the gauge group is broken down to \(\tilde{G} \times K\) where \(K\) is some Abelian subgroup.

Further we impose the special restrictions on the background \(\mathcal{N} = 2\) vector multiplet and hypermultiplet. They are chosen to be aligned along a fixed direction in the moduli space vacua; in particular, their scalar fields should solve Eqs. (7):

\[
V^{++} = V^{++}(\zeta) H, \quad q^+ = q^+(\zeta) \Upsilon.
\]  

(8)

Here \(H\) is a fixed generator in the Cartan subalgebra corresponding to Abelian subgroup \(K\), and \(\Upsilon\) is a fixed vector in the \(R\)-representation space of the gauge group, where the hypermultiplet takes values, chosen so that \(H\Upsilon = 0\) and \(\Upsilon T_I \Upsilon = 0\). Eq. (8) defines a single \(U(1)\) vector multiplet and a single hypermultiplet which is neutral with respect to the \(U(1)\) gauge subgroup generated by \(H\).

At the tree level and energies below the symmetry breaking scale, we have free field massless dynamics of the \(\mathcal{N} = 2\) vector multiplet and the hypermultiplet aligned in a particular direction in the moduli space of vacua. Thus the low energy propagating fields are massless neutral hypermultiplets and \(U(1)\) vector which form the on shell superfields possessing the properties

\[
(D^{\pm})^2 W = (\bar{D}^{\pm})^2 \bar{W} = 0,
\]  

(9)

\[
D^{++} q^+ a = (D^{--})^2 q^+ a = D^{--} q^- a = 0, \quad q^- a = D^{--} q^+ a, \quad D^{(a,\bar{a})} q^- a = 0.
\]

The equations (9) eliminate the auxiliary fields and put the physical fields on shell.

At the quantum level, however, exchanges of virtual massive particles produce the corrections to the action of the massless fields. We quantize the \(\mathcal{N} = 2\) supergauge theory in the framework of the \(\mathcal{N} = 2\) supersymmetric background field method [13] by splitting the fields \(V^{++}, q^+ a\) into the sum of the background fields \(V^{++}, q^+ a\), parameterized according to (8), and the quantum fields \(v^{++}, Q^+ a\) and expanding the Lagrangian in a power series in quantum fields. Such a procedure allows us to find the effective action for arbitrary \(\mathcal{N} = 2\) supersymmetric gauge model in a form preserving the manifest \(\mathcal{N} = 2\) supersymmetry and classical gauge invariance in quantum theory.

In the background-quantum splitting, the classical action of the pure \(\mathcal{N} = 2\) SYM theory can be shown to be given by

\[
S_{\text{SYM}}[V^{++} + v^{++}] = S_{\text{SYM}}[V^{++}] + \frac{1}{4} \int d\zeta^{(-4)} du v^{++} (D^+)^2 W_\lambda
\]  

(10)
\[-\text{tr} \int d^4z \sum_{n=2}^{\infty} \frac{(-ig)^{n-2}}{n} \int du_1...du_n \frac{v^+_{\lambda}(z, u_1)...v^+_{\lambda}(z, u_n)}{(u_1^+ u_2^+)...(u_n^+ u_1^+)}.
\]

\(\mathcal{W}_\lambda\) and \(v^+_{\lambda}\) denote the \(\lambda\) and \(\tau\)-frame forms of \(\mathcal{W}\) and \(v^+\) respectively. The hypermultiplet action becomes

\[
S_H(q + Q) = S_H[q] + \int d\zeta (-4) du Q^+_a D^+ q^{+a} + \frac{1}{2} \int d\zeta (-4) du q^{+a} i v^+ q^{+a} \quad (11)
\]

\[
+ \frac{1}{2} \int d\zeta (-4) du \{ Q^+_a D^+ q^{+a} + Q^+_a i v^+ q^{+a} + q^{+a} i v^+ Q^{+a} + Q^+_a i v^+ Q^{+a} \}.
\]

The terms linear in \(v^+\) and \(q^+\) in \((10), (11)\) determines the equation of motion and this term should be dropped when considering the effective action.

To construct the effective action, we will follow the Faddeev-Popov Ansatz. We write the final result for the effective action \(\Gamma[V^+, q^+]\)

\[
e^{-\text{tr} \int d^4z} = e^{-\text{tr} \int d^4z} D^+ D^+ b D^+ c D^+ \varphi e^{iS_q},
\]

where \(\Box = -\frac{1}{2}(D^+)^4(D^-)^2\) and action \(S_q\) is as follows

\[
S_q = S_2[V^+, Q^+, b, c, \varphi, V^+, q^+] = S_2[V^+, Q^+, b, c, \varphi, V^+, q^+] + S_{\text{int}},
\]

\[
S_2 = -\frac{1}{2} \text{tr} \int d\zeta (-4) du v^+ \Box v^+ + \text{tr} \int d\zeta (-4) du b(D^+)^2 c
\]

\[
+ \frac{1}{2} \text{tr} \int d\zeta (-4) du v^+ \Box \varphi v^+ + \frac{1}{2} \text{tr} \int d\zeta (-4) du \{ Q^+_a D^+ Q^{+a} + Q^+_a i v^+ Q^{+a} + q^{+a} i v^+ Q^{+a} \},
\]

This equations completely determine the structure of the perturbation expansion for calculating the effective action \(\Gamma[V^+, q^+]\) of the \(N = 2\) SYM theory with hypermultiplets in a manifestly supersymmetric and gauge invariant form. The action \(S_2\) defines the propagators depending on background fields. In the framework of the background field formalism in \(N = 2\) harmonic superspace there appear three types of covariant matter and gauge field propagators. Associated with \(\Box\) is a Green’s function \(G^{(2,2)}(1|2)\) which satisfies the equation \(\Box G^{(2,2)}(1|2) = -\frac{1}{2} \delta^{(2,2)}(1|2)\), is

\[
G^{(2,2)}(1, 2) = -\frac{1}{2} \frac{1}{\Box_1 \Box_2} (D^+_1)^4 (D^+_2)^4 \left\{ 1 \delta_{12} (z_1 - z_2) (D_2^-)^2 \delta^{(-2, 2)}(u_1, u_2) \right\}.
\]

The \(Q^+\) hypermultiplet propagator associated with the action \((13)\) has the form

\[
G^{(1,1)}_b(1|2) = -\frac{1}{2} \frac{1}{\Box_1} (D^+_1)^4 (D^+_2)^4 \left\{ 1 \delta_{12} (z_1 - z_2) \frac{u_1^- u_2^-}{u_1^+ u_2^+} \right\}.
\]

It is not hard to see that this manifestly analytic expression is the solution of the equation \((D^+_1)^2 G^{(1,1)}(1|2) = \delta_3^{(3,1)}(1|2)\). For the hypermultiplet of the second type described by a chargeless real analytic superfield \(\omega(z, u)\) the equation for Green’s function is

\[
(D^+_1)^2 G^{(0,0)}(1|2) = \delta_4^{(4,0)}(1|2).
\]

The suitable expression for \(G^{(0,0)}(1|2)\) is

\[
G^{(0,0)}(1|2) = -\frac{1}{\Box_1} (D^+_1)^4 (D^+_2)^4 \left\{ 1 \delta_{12} (z_1 - z_2) \frac{u_1^- u_2^-}{u_1^+ u_2^+} \right\}.
\]

The operator \(\Box = -\frac{1}{2}(D^+)^4(D^-)^2\) transforms each covariantly analytic superfield into a covariantly analytic and, using algebra \((5)\), can be rewritten as second-order d’Alembertian-like differential operator on the space of such superfields. The coefficients of this operator depend on background superfields \(\mathcal{W}, \mathcal{W}\).
3 Structure of the one-loop effective action

Consider the loop expansion of the effective action within the background field formulation. A formal expression of the one-loop effective action $\Gamma[V^{++}, q^+]$ for the theory under consideration is written in terms of a path integral as follows (12), where the full quadratic action is defined in Eq. (13). Here $v^{++}$ is a quantum vector superfield taking values in the Lie algebra of the gauge group and $b, c$ are two real analytic Faddeev-Popov fermionic ghosts and $\varphi$ is the bosonic Nielsen-Kallosh ghost, all in the adjoint representation of the gauge group.

In the vector sector of the $\mathcal{N} = 2$ SYM theory where the matter hypermultiplet are integrated out, the one-loop effective action $\Gamma[V^{++}]$ reads

$$\Gamma[V^{++}] = \frac{i}{2} \text{Tr}_{(2,2)} \ln \square - \frac{i}{2} \text{Tr}_{(4,0)} \ln \square - \frac{i}{2} \text{Tr}_{ad} \ln (D^{++})^2 + i \text{Tr}_q \ln D^{++} + \frac{i}{2} \text{Tr}_{R_q} \ln (D^{++})^2.$$  

(17)

Currently, the holomorphic and non-holomorphic parts of the low-energy effective action $\mathcal{N} = 2, 4$ SYM theory on the Coulomb branch, including Heisenberg-Euler type action in the presence of a covariantly constant vector multiplet, are completely known. The general structure of the low-energy effective action in $\mathcal{N} = 2, 4$ superconformal theories is [14]:

$$\Gamma = S_{cl} + c \int d^2 z \ln \mathcal{W} \ln \mathcal{W} + \int d^2 z \ln \mathcal{W} \Lambda \left( \frac{D^4 \ln \mathcal{W}}{\mathcal{W}^2} \right) + c.c. + \int d^2 z \Upsilon \left( \frac{\tilde{D}^4 \ln \mathcal{W}}{\mathcal{W}^2}, \frac{D^4 \ln \mathcal{W}}{\mathcal{W}^2} \right),$$

where $\Lambda$ and $\Upsilon$ are holomorphic and real analytic function of the (anti)chiral superconformal invariants. The $c$-term is known to generate four-derivative quantum corrections at the component level which include an famous $F^4$ term.

The hypermultiplet dependent part of the effective action in $\mathcal{N} = 4$ SYM theory in leading order is also known [15]. For further analysis of the effective action it is convenient to diagonalize the action of quantum fields $S^{(2)}$ using a special shift of hypermultiplet variables in the path integral

$$Q^{+a} = \xi^{+a} + i \int d\zeta_2^{(-4)} q^{+b}(2)v^{++}(2)G_b^{(1,1)}(1|2),$$  

(18)

$$Q^{-a}_a = \xi^{-a}_a - i \int d\zeta_2^{(-4)} G^{a(1,1)}_a(1|2)v^{++}(2)q^+_b(2),$$

where $\xi^{+a}, \xi^{-a}_a$ are the new independent variables in the path integral. It is evident that the Jacobian of the replacement (18) is equal to unity. Here $G^{a(1,1)}_b(1|2)$ is the background-dependent propagator [15] for the superfields $Q^{+a}, Q^{+b}_a$. In terms of the new set of quantum fields we obtain for the following hypermultiplet dependent part of the quadratic action

$$S_H^{(2)} = -\frac{1}{2} \int d\zeta^{(-4)} \xi^{a+} D^{++} \xi^{a+}_a - \frac{1}{2} \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} q^{+a}(1)v^{++}(1)G^{b(1,1)}_a(1|2)v^{++}(2)q^+_b(2).$$  

(19)

Then the vector multiplet dependent part of the quadratic action gets the following non-local extension

$$S_v^{(2)} = -\frac{1}{2} \text{tr} \int d\zeta_1^{(-4)} v^{++}_1 + \int d\zeta_2^{(-4)} \left( \square \delta^{(2,2)}_A(1|2) + q^{+a}(1)G^{b(1,1)}_a(1|2)q^+_b(2) \right) v^{++}_2.$$  

(20)
Expression (20), written as an analytical nonlocal superfunctional, will be a starting point for our calculations of the one-loop effective action in the hypermultiplet sector. Our aim in the current and later sections is to find the leading low-energy contribution to the effective action for the slowly varying hypermultiplet when all derivatives of the background hypermultiplet can be neglected. We will show that for such a case the non-local interaction is localized.

Using the relation \( v_2^{++} = \int d\zeta_3^{-4} \delta_A^{(22)}(2|3)v_3^{++} \) one can rewrite expression for \( S_v^{(2)} \) in the form

\[
S_v^{(2)} = -\frac{1}{2} \text{tr} \int d\zeta_1^{-4} v_1^{++} \int d\zeta_2^{-4} (\Box \delta_A^{(22)}(1|2)) + \int d\zeta_3^{-4} q^a(1) G^{(1,1)}(1|3) q^{+}(3) \delta_A^{(22)}(3|2) v_2^{++}.
\]

Then we use the explicit form of the Green function \( (15) \) and the relation allowing us to obtain the on-shell harmonic dependence of hypermultiplet.

Expression (20), written as an analytical nonlocal superfunctional, will be a starting point for our calculations of the one-loop effective action in the hypermultiplet sector.

As a result, is it sufficient to consider only the third term in the braces. Since we keep only contributions without derivatives, the above terms can be neglected. As a result, it is sufficient to consider only the third term in the braces.

Now we apply the relation \( f d\zeta_3^{-4} (D_3^+)^4 = f d^{12}z \), allowing to integrate over \( z_3 \), and obtain

\[
-\int du_3 q^a(1) \frac{(u_3^+ u_3^+)^2}{(u_3^+ u_3^+)} q^+(u_3, z_1) \delta_A^{(22)}(u_3, z_1|2).
\]

Then one uses the on-shell harmonic dependence of hypermultiplet \( q^a(3) = u_3^+ q^a \) and take the coincident limit \( u_1 = u_3 \) (conditioned by \( \delta_A^{(22)}(u_3, z_1|2) \)). After that we get

\[
\int du_3 \frac{u_3^+}{u_3^+} = -u_1^-. \quad \text{As a result, the term under consideration has the form}
\]

\[
q^+(1) q^-(1) \delta_A^{(22)}(1|2),
\]

where the expression \( q^+(1) q^-(1) = q^a q_a \) is treated further as the slowly varying superfield and all its derivatives are neglected. Namely such an expression was obtained in \( \text{[6]} \) by summation of harmonic supergraphs.
Thus, the second term in (21) becomes local in the leading low-energy approximation. As a result, the operator in action $S_v^{(2)}$ determining the effective background covariant propagator of the quantum vector multiplet superfield $v_I^{++}$ takes the form

$$\left(\Box_{IJ} + q^{+a}(z_1, u_1)\{T_I, T_J\}q_a^-(z_1, u_1)\right)\delta_A^{(2)}(1|2),$$

(25)

where

$$\Box_{IJ} = \text{tr}(T_I \Box T_J) + \frac{i}{2} T_I[D^{+a}\mathcal{W}, T_J]D_a^- + \frac{i}{2} T_I[D_a^+\mathcal{W}, T_J]D_a^- + T_I[\mathcal{W}, [\mathcal{W}, T_J]].$$

Here $\Box = \frac{1}{2}D^{\alpha\dot{\alpha}}D_{\alpha\dot{\alpha}}$ is the covariant d’Alembertian.

Thus, using the $\mathcal{N} = 2$ harmonic superspace formulation of the $\mathcal{N} = 2$ SYM theory with hypermultiplets and techniques of the non-local shift we obtained that the whole dependence on the background hypermultiplet is concentrated in the quantum vector multiplet sector with the modified quadratic action. Therefore the one-loop effective action is given by the expression

$$\Gamma^{(1)}[V^{++}, q^+] = \Gamma^{(1)}_v[V^{++}, q^+] + \tilde{\Gamma}^{(1)}[V^{++}],$$

(27)

where the first term in (27) is originated from quantum vector multiplet $v_I^{++}$

$$\Gamma^{(1)}_v[V^{++}, q^+] = \frac{i}{2} \text{Tr} \ln(\Box_{IJ} + q^{+a}\{T_I, T_J\}q_a^-).$$

(28)

Second term in (27) is the contribution of ghosts and quantum hypermultiplet $\xi_a^{+}$ and does not depend on the background hypermultiplet.

As a result, the background hypermultiplet dependence of one-loop effective action is included into the operator (26), acting on $v_I^{++}$ and containing the mass matrix of the vector multiplet

$$(\mathcal{M}_v^2)_{IJ} = \text{tr} \left( [T_I, \mathcal{W}][\mathcal{W}, T_J] + (I \leftrightarrow J) \right) + q^{+a}\{T_I, T_J\}q_a^-,$$

(29)

if $q^+$ is in the fundamental representation, and

$$(\mathcal{M}_v^2)_{IJ} = \text{tr} \left( [T_I, \mathcal{W}][\mathcal{W}, T_J] + [q^{+a}, T_I][T_J, q_a^-] \right) + (I \leftrightarrow J),$$

(30)

if $q^+$ in an arbitrary matrix representation.

In the above discussion, the gauge group structure of the superfields $\mathcal{W}, q_a^+$ has been completely arbitrary. Henceforth, the background superfields will be chosen to be aligned along a fixed direction in the moduli space of vacua in such a way that their scalar fields should solve Eqs. (7). Then the hypermultiplet dependent effective action in the case under consideration takes the universal form

$$\Gamma^{(1)}_v[V^{++}, q^+] =$$

$$\frac{i}{2} n(\Upsilon) \times \text{Tr} \ln \left( \Box + \frac{i}{2} \alpha(H)(D^{+a}\mathcal{W}D_a^- + \bar{D}^{+\dot{a}}\bar{\mathcal{W}}\bar{D}^{-\dot{a}}) + \alpha^2(H)\mathcal{W}\bar{\mathcal{W}} + r(\Upsilon)q^{+a}q_a^- \right).$$

(31)

As the examples we list the values of $\alpha(H), r(\Upsilon)$ and $n(\Upsilon)$ for models considered in [4].

(i) $\mathcal{N} = 4$ SYM theory with gauge groups SU($N$), Sp(2$N$) and SO($N$). Here the hypermultiplet sector is composed of a single hypermultiplet in the adjoint representation
of the gauge group. The background was chosen such that the gauge groups are broken down as follows $SU(N) \rightarrow SU(N-1) \times U(1)$, $Sp(2N) \rightarrow Sp(2(N-2)) \times U(1)$, $SO(N) \rightarrow SO(N-2) \times U(1)$. All background fields aligned along element $H = U(1)$ of the Cartan subalgebra (with $\Upsilon = H$). The mass matrix becomes
\begin{equation}
(M^2_{\nu})_{IJ} = (\bar{W}W + q^+ a q^-)(\alpha(H))^2 \delta_{I,J}
\end{equation}
and traces in Eq. (27) produce the coefficient $n(\Upsilon)$ which is equal to the number of roots with $\alpha(H) \neq 0$, i.e. to the number of broken generators $n(\Upsilon) = \begin{cases} 2(N-1) & \text{for } SU(N), \\ 4N-2 & \text{for } Sp(2N), \\ 4N-1 & \text{for } SO(2N). \end{cases}$

The form of the mass matrix shows that in this case $r(\Upsilon) = \alpha(H).$

(ii) The model introduced in [17]. The gauge group is $USp(2N) = Sp(2N, C) \cap U(2N)$. The model contains four hypermultiplets $q^+_F$ in the fundamental and one hypermultiplet $q^+_A$ in the antisymmetric traceless representation $USp(2N)$. The background fields $\mathcal{W}, q^+_F, q^+_A$ are chosen to solve Eqs. (7) with the unbroken maximal gauge subgroup $USp(2(N-2)) \times U(1)$:
\begin{align*}
\mathcal{W} &= \frac{\mathcal{W}}{\sqrt{2}} \text{ diag}(1, 0, ..., 0, -1, 0, ..., 0), \quad q^+_F = 0, \\
(q^+_A)^{\alpha \beta} &= \frac{q^+_A}{\sqrt{2N(N-1)}} \text{ diag}(N-1, -1, ..., -1, N-1, -1, ..., -1).
\end{align*}
The mass matrix $(\mathcal{M}^2_{\nu})_{IJ}$ has been calculated in [11] and it has $n(\Upsilon) = 4(N-1)$ eigenvectors with the eigenvalue
\begin{equation}
\mathcal{M}^2_{\nu} = \bar{\mathcal{W}}\mathcal{W} + \frac{N}{N-1} q^i q_i.
\end{equation}

(iii) The $\mathcal{N} = 2$ superconformal model which is the simplest quiver gauge theory [18]. Gauge group is $SU(N)_L \times SU(N)_R$. The model contains two hypermultiplets $q^+$, $\tilde{q}^+$ in the bifundamental representations $(N, \bar{N})$ and $(\bar{N}, N)$ of the gauge group. In [4] a solutions of (7) with non-vanishing hypermultiplet components that specifies the flat directions in massless $\mathcal{N} = 2$ SYM theories has been constructed. The moduli space of vacua for this model includes the following field configuration
\begin{align*}
\mathcal{W}_L &= \mathcal{W}_R = \frac{\mathcal{W}}{N \sqrt{2(N-1)}} \text{ diag}(N-1, -1, ..., -1), \\
q^+ &= \tilde{q}^+ = \frac{q^+_A}{\sqrt{2}} \text{ diag}(1, 0, ..., 0),
\end{align*}
which preserves an unbroken gauge group $SU(N-1) \times SU(N-1)$ together with the diagonal $U(1)$ subgroup in $SU(N)_L \times SU(N)_R$ associated with the chosen $\mathcal{W}$. In such a background the mass matrix has eigenvalue
\begin{equation}
\mathcal{M}^2_{\nu} = \frac{1}{N-1} \bar{\mathcal{W}}\mathcal{W} + \frac{1}{N} q^+ a q^-.
\end{equation}
and the corresponding $n(\Upsilon) = 4(N-1)$.

As the result, the hypermultiplet dependent effective action is given by the expression (31). In the next section we will consider the evaluation of this expression.
4 Calculation of the one-loop effective action

The expression (31) is a basis for an analysis of the hypermultiplet dependence of the effective action. In the framework of the Fock - Schwinger proper-time representation, the effective action (31) is written as follows

$$\Gamma^{(1)}[V^{++}, q^+] = \frac{i}{2} n(\Upsilon) \int d\zeta^{(-4)} du \int_{0}^{\infty} \frac{ds}{s} e^{-s(\alpha(H)(D^{+}\bar{W}D^{-} + D^{+}\bar{W}D^{-}) + \mathcal{M}^{2})} \times$$

$$\times (D^{+})^{4} \left( \delta_{12}^{4}(z - z') \delta^{(-2,2)}(u, u') \right) |_{z = z', u = u'} = \int_{0}^{\infty} \frac{ds}{s} \text{Tr} K(s),$$

where \( \mathcal{M}^{2} = \alpha^{2}(H)\bar{W}W + r(\Upsilon)q^{+}q_{-} \). Here \( K(s) \) is a superfield heat kernel, the operation Tr means the functional trace in the analytic subspace of the harmonic superspace \( \text{Tr} K(s) = tr \int d\zeta^{(-4)} K(\zeta, \zeta|s) \), where tr denotes the trace over the discrete indices. Representation of the effective action (31) allows us to develop a straightforward evaluation of the effective action in a form of covariant spinor derivatives expansion in the superfield Abelian strengths \( W, \bar{W} \). The leading low-energy terms in this expansion correspond to the constant space-time background \( D_{\alpha}D_{\beta}W = \text{const}, \ D_{\alpha}D_{\beta}\bar{W} = \text{const} \) and on-shell background hypermultiplet. However, it does not mean that we miss all space-time derivatives in the component effective Lagrangian. Grassmann measure in the integral over harmonic superspace \( d^{4}\theta^{+}d^{4}\theta^{-} \) generates four space-time derivatives in component expansion of the superfield Lagrangian. Therefore the above assumption is sufficient to obtain a component effective Lagrangian including four space-time derivatives of the scalar components of the hypermultiplet.

Calculation of the effective action (35) is based on evaluating the superfield heat kernel \( K(s) \) and lead to a final result for the hypermultiplet dependent low-energy one-loop effective action of the Heisenberg-Euler type. We remind that the whole background hypermultiplet is concentrated in \( \mathcal{M}^{2} \). The explicit form of it is:

$$\Gamma^{(1)}[V^{++}, q^+] = \frac{1}{(4\pi)^{2}} n(\Upsilon) \int d\zeta^{(-4)} du \int_{0}^{\infty} \frac{ds}{s} e^{-s(\alpha^{2}(H)\bar{W}W + r(\Upsilon)q^{+}q_{-})} \times$$

$$\times \frac{\alpha^{4}(H)}{16} (D^{+}\bar{W})^{2}(\bar{D}^{+}W)^{2} \frac{s^{2}(N^{2} - N^{2})}{\cosh(s\bar{N}) - \cosh(sN)} \frac{\cosh(s\bar{N}) - 1}{N^{2}} \frac{\cosh(sN) - 1}{N^{2}}.$$ 

Here \( N \) is given by \( N = \sqrt{-\frac{i}{2} D^{4}\bar{W}W} \). It can be expressed in terms of the two invariants of the Abelian vector field \( F = \frac{i}{4} F^{mn} F_{mn} \) and \( G = \frac{1}{4} F^{mn} F_{mn} \) as \( N = \sqrt{2(F + iG)} \). It is easy to see that the integrand in (36) can be expanded in power series in the quantities \( s^{2}N^{2}, s^{2}\bar{N}^{2} \). After change of proper time \( s \) to \( s'W \bar{W} \) we get the expansion in power of \( s^{2} \) and its conjugate. Since the integrand of (36) is already \( (D^{+}\bar{W})^{2}(\bar{D}^{+}W)^{2} \) we can change in each term of expansion the quantities \( N^{2}, \bar{N}^{2} \) by superconformal invariants \( \Psi^{2} \) and \( \bar{\Psi}^{2} \) expressing these quantities from \( \Psi^{2} = \frac{1}{W^{2}} D^{4}\ln W = \frac{1}{2W^{2}}(\frac{\mathcal{M}^{2}N_{0}^{2}}{W^{2}} + O(D^{+}W)) \) and its conjugate. After that, one can show that each term of the expansion can be rewritten as an integral over the full \( N = 2 \) superspace.

It is interesting and instructive to evaluate the leading part of the effective action (36) that exactly coincides, up to group factor \( \Upsilon \) with the earlier results [5], [6], [15]:

$$\Gamma^{(1)}_{\text{lead}} = \frac{1}{(4\pi)^{2}} n(\Upsilon) \int d^{12}z \left( \ln W \ln \bar{W} + \text{Li}_{2}(X) + \ln(1 - X) - \frac{1}{X} \ln(1 - X) \right).$$

(37)
Here $\text{Li}_2(X)$ is the Euler’s dilogarithm function. Next-to-leading corrections to (37) can also be calculated. The remarkable feature of the low-energy effective action (37) is the appearance of the factor $r(\Upsilon)/\alpha(H)$ in argument $X$. This factor is conditioned by the vacuum structure of the model under consideration and depends on the specific features of the symmetry breaking.

Now we discuss some terms in the component Lagrangian corresponding to the effective action (37). Component structure of the effective action (37) has been studied [5] in the context of $\mathcal{N} = 4$ SYM theory in bosonic sector for completely constant background fields $F_{mn}, \phi, \bar{\phi}, f^i, \bar{f}_i$. However, it was pointed out above that the superfield effective action (37) allows us to find the terms in the effective action up to fourth order in space-time derivatives of component fields. Now our aim is to find such terms in the hypermultiplet scalar component sector. To do that we omit all components of the background superfields besides the scalars $\phi, \bar{\phi}$ in the $\mathcal{N} = 2$ vector multiplet and scalars $f, \bar{f}$ in the hypermultiplet and integrate over $d^4\theta + d^4\bar{\theta} = (D^-)^4(D^+)^4$. To get the leading space-time derivatives of the hypermultiplet scalar components we should put exactly two spinor derivatives on each hypermultiplet superfield. It yields, after some transformations, to the following term with four space-time derivatives on $q^\pm$ in component expansion of effective action:

$$
\Gamma^{(1)}_{\text{lead}} = \int d^4x d\phi d\bar{\phi} n(\Upsilon) \frac{1}{(4\pi)^2} \sum_{k=2}^{\infty} \frac{1}{16 k(k + 1)} \frac{X^{k-2}}{(W\bar{W})^2} X^{k-2} \{ -\bar{D}^{+\dot{\alpha}} D^{+\alpha} q_b^i \bar{D}_{\dot{\alpha}} D_{\alpha} q^{+(b \bar{D}^{-\dot{\beta}} D^{-\beta} q^{+a})} D^{+\alpha} q^{+a} - \frac{1}{2} \bar{D}^{+\dot{\alpha}} D^{+\alpha} q_b^i \bar{D}_{\dot{\alpha}} D_{\alpha} q^{+(b \bar{D}^{-\dot{\beta}} D^{-\beta} q^{+a})} D^{+\alpha} q^{+a} + \frac{1}{2} \bar{D}^{-\dot{\beta}} D^{+\alpha} q_b^i \bar{D}^{+\dot{\alpha}} D^{+\alpha} q^{+b \bar{D}^{-\dot{\beta}} D^{-\beta} q^{+a} \bar{D}_{\dot{\alpha}} D_{\alpha} q^{+a} + \frac{1}{2} \bar{D}^{-\dot{\beta}} D^{+\alpha} q_b^i \bar{D}^{+\dot{\alpha}} D^{+\alpha} q^{+b \bar{D}^{-\dot{\beta}} D^{-\beta} q^{+a} \bar{D}_{\dot{\alpha}} D_{\alpha} q^{+a} - \bar{D}^{-\dot{\alpha}} D_{\alpha} q^{+a} \bar{D}^{+\dot{\beta}} D^{-\beta} q^{+b \bar{D}^{-\dot{\alpha}} D^{+\alpha} q^{+a} \bar{D}_{\dot{\alpha}} D_{\alpha} q^{+a}} \} |_{\theta = 0}.
$$

The straightforward calculation of the components in this expression shows that among the many terms with four derivatives there is an interesting term of the special type. As the first term in expansion over variable $X_0 = \frac{r(\Upsilon)\bar{f}_i}{\alpha^2 \phi \bar{\phi}}$ we have

$$
\Gamma^{(1)}_{\text{lead}} = -\frac{1}{48\pi^2} n(\Upsilon) \left( \frac{r(\Upsilon)}{\alpha(H)} \right)^2 \int d^4x \left[ \frac{1}{(\phi \bar{\phi})^2} \varepsilon^{\mu\nu\lambda\rho} (\partial_\mu \bar{f}_i \partial_\nu f_i \partial_\lambda \bar{f}_j \partial_\rho f_j - \partial_\mu \bar{f}_i \partial_\nu \bar{f}_j \partial_\lambda f_j \partial_\rho f_j) \right]
$$

The expression (38) has a form of the Chern-Simons-like action for the multicomponent complex scalar filed. The terms of such form in the effective action were discussed in Refs. [8], [9] in context of $\mathcal{N} = 4,2$ SYM models and in Refs. [19] for $d = 6, \mathcal{N} = (2,0)$ superconformal models respectively. Here the expression (38) is obtained as a result of straightforward calculation in the supersymmetric quantum field theory.

5. Hypermultiplet dependent contribution to the effective action beyond the on-shell condition

In the above consideration a crucial point was the condition that the hypermultiplet $q^+$ satisfies the one-shell conditions (9) and the constraint $q^+ = D^{++}q^-$. Here we relax the
on-shell conditions and study some of possible subleading contributions with the minimal number of space-time derivatives in the component effective action.

Figure 1: One-loop supergraph

We consider a supergraph given in Fig.1 with two external hypermultiplet legs and with all propagators depending on the background $\mathcal{N} = 2$ vector multiplet. Here the wavy line stands for the $\mathcal{N} = 2$ gauge superfield propagator and the solid external and internal lines stand for the background hypermultiplet superfields and quantum hypermultiplet propagator respectively. For simplicity we suppose that the background field is Abelian and omit all group factors. The corresponding contribution to effective action looks like

$$i\Gamma_2 = \int d\zeta_1(-4) d\zeta_2(-4) du_1 du_2 \left( \frac{(D^+_1)^4(D^+_2)^4}{(u^+_1 u^+_2)^3} \frac{1}{\Box_1} \delta^{12}(1|2) \right) \times$$

$$\times \left( \frac{(D^+_2)^4(D^+_1)^4}{\Box_2 \Box_1} \delta^{12}(2|1)(D^-_1)^2 \delta^{(-2,2)}(u_2, u_1) \right) q^+(z_1, u_1) q^+(z_2, u_2).$$

As usually, we extract the factor $(D^+)^4$ from the vector multiplet propagator for reconstructing the full $\mathcal{N} = 2$ measure. Then we shrink a loop into a point by transferring the $\Box$ and $(D^+)^4$ from first $\delta$-function to another one and kill one integration. At this procedure the operator $\Box$ does not act on $q^+$ because we are interesting in the minimal number of space-time derivatives in the component form of the effective action. As a result, one obtains

$$i\Gamma_2 = \int d\zeta_1(-4) du_1 du_2 \left( \frac{(D^+_1)^4(D^+_2)^4}{(u^+_1 u^+_2)^3} \delta^{12}(z - z') \right) \times$$

$$\times \left( (D^-_1)^2 \delta^{(-2,2)}(u_2, u_1) \right) q^+(z_1, u_1) q^+(z_1, u_2).$$

Further we use twice the relation (22) allowing us to express the $(D^+_1)^4(D^+_2)^4$ as a polynomial in powers of $(u^+_1 u^+_2)$. Then after multiplying the $(D^+_1)^4(D^+_2)^4(D^+_1)^4$ with the distribution $1/(u^+_1 u^+_2)^3$ we obtain a polynomial in $(u^+_1 u^+_2)$ containing the powers of this quantity from 5-th to 1-st. The first order is just a contribution of the type which we considered in the previous section, because one derivation $(D^{--})^2$ is used for transformation $(u^+_1 u^+_2)$ into $(u^+_1 u^-_2)\big|_{u_1 = u_2} = 1$ in the coincident limit. Another $D^{--}$ transforms $q^+$ into $q^-$. All that has been already done in Section 4.

Here we consider the new contribution to the effective action containing term $(u^+_1 u^+_2)^2$ in the above polynomial:
The Eq. (43) allows us to write the leading contribution to $\Gamma_2$ as
\[
i\Gamma_2 = i \int d\zeta^{(-i)} du(D^+)^4 \frac{1}{\Gamma_2(1)} \left( \frac{\Delta^{--}}{\Gamma_2(2)} - \frac{\Delta^{-+}}{\Gamma_2(2)} \right) \delta^{12}(z - z')|_{z = z'} q^+(z, u) q^+(z, u), \tag{42}
\]
where $\Delta^{--}$ is defined in (23).

Let us consider each of the two underlined contributions separately. We use the representation
\[
\frac{1}{\Delta^{--}} \delta^{12}(z - z') = \int ds s e^{\sqrt{\Delta^{--}} \delta^{12}(z - z')}|, \tag{43}
\]
where $|$ means the coincident limit $z = z'$. Then we can apply a derivative expansion of the heat kernel. The goal is to collect the maximum possible number of factors of $D^+, D^-$ acting on $(\theta^+ - \bar{\theta}^+)(\theta^- - \bar{\theta}^-)$ and having the minimum order in $s$ in the integral over $s$. Higher orders in $s$ generate the higher spinor derivatives in the effective action. We take terms $\frac{i}{2} \bar{W}_\alpha(D^-)^2 + c.c.$ from $\Delta^{--}$ and expand the exponential so as to find $(D^-)^4$. The Eq. (43) allows us to write the leading contribution to $\Gamma_2(1)$ as follows
\[
\Gamma_2(1) = -i \int d^2 z d u \int_{0}^{\infty} ds s \int \left( \frac{d^4 p}{(2\pi)^4} e^{-s p^2} e^{s(\bar{W} \bar{W} - c)} \right) \frac{s^2}{32} \bar{W}(D^+ \alpha \bar{W} D^+ \alpha \bar{W}) \times \frac{1}{\sqrt{\Delta^{--}}} \delta^{12}(z - z') q^+ q^+ + c.c. \tag{44}
\]
After trivial integration over $p$ and $s$ this contribution has the form
\[
\Gamma_2(1) = \frac{i}{32\pi^2} \int d^2 z d u \frac{D^+ \alpha \bar{W} D^+ \alpha \bar{W}}{\sqrt{\bar{W}}} q^+(z, u) q^+(z, u)(D^-)^4 \delta^8(\theta - \bar{\theta})| + c.c. \tag{45}
\]
Now we fulfill the same manipulations with the second underlined contribution $\Gamma_2(2)$ keeping the same order in $s$ and $D^-, \bar{D}^-$ as in the expression (45). After that we see that the leading term of the form (45) is absent in $\Gamma_2(2)$. Then it is not difficult to show that the contribution (45) is rewritten as follows [we use $\int d^2 \bar{\theta}^- = \bar{D}^+ 2$]
\[
- \frac{i}{32\pi^2} \int d^4 x d^4 \bar{\theta}^+ d^2 \theta^- d u (\bar{D}^+)^2 (D^+)^2 \frac{1}{\bar{W} \bar{W}} q^+(z, u) q^+(z, u)(D^-)^4 \delta^8(\theta - \bar{\theta})| \tag{46}
\]
The non-zero result arises when all $D^+$ - factors act only on the spinor delta-function. Thus, the contribution under consideration is written as an integral over the measure $d^4 x d^4 \bar{\theta}^+ d^2 \theta^- d u (\bar{D}^+)^2 (D^+)^2 \frac{1}{\bar{W} \bar{W}} q^+(z, u) q^+(z, u)(D^-)^4 \delta^8(\theta - \bar{\theta})|$

Therefore, the hypermultiplet dependent effective action contains the term
\[
\Gamma_2 = - \frac{i}{32\pi^2} \int d^4 x d^4 \bar{\theta}^+ d^2 \theta^- \frac{1}{\bar{W} \bar{W}} \ln(\bar{W}) q^+ q^+|_{\bar{\theta}^- = 0} \tag{46}
\]
\[
- \frac{i}{32\pi^2} \int d^4 x d^4 \bar{\theta}^+ d^2 \theta^- \frac{1}{\bar{W} \bar{W}} \ln(\bar{W}) q^+ q^+|_{\theta^- = 0} .
\]
Presence of such a term in the effective action for $\mathcal{N} = 2$ supersymmetric models in subleading order was proposed in [2]. Here we have shown how this term can be derived in the supersymmetric quantum field theory.
It is interesting and instructive to find a component form of such a non-standard superfield action (46). Here we consider only a purely bosonic sector of (46). After integration over anticommuting variables, which can be equivalently replaced by supercovariant derivatives evaluated at $\theta = 0$, we obtain a Chern-Simons-like contribution to the effective action containing three space-time derivatives

$$\Gamma_2 = -\frac{1}{2\pi^2} \int d^4x \frac{1}{\phi^2} \varepsilon^{mnab} \partial_m \phi \partial_n \phi F_{ab}. \quad (47)$$

This expression is the simplest contribution to the hypermultiplet dependent effective action beyond the on-shell conditions (9) for the background hypermultiplet. Of course, there exist other, more complicated contributions including the hypermultiplet derivatives, they also can be calculated by the same method which led to (46). Here we only demonstrated a procedure which allows us to derive the contributions to the effective action in the form of integral over $3/4$ - part of the full $\mathcal{N} = 2$ harmonic superspace.

6 Summary

We have studied the one-loop low-energy effective action in $\mathcal{N} = 2$ superconformal models. The models are formulated in harmonic superspace and their field content correspond to the finiteness condition (11). Effective action depends on the background Abelian $\mathcal{N} = 2$ vector multiplet superfield and background hypermultiplet superfields satisfying the special restrictions (7), (8) which define the vacuum structure of the models. The effective action is calculated on the base of the $\mathcal{N} = 2$ background field method for the background hypermultiplet on-shell (9) and beyond the on-shell conditions. For an on-shell hypermultiplet we found the universal expression for the effective active action. For hypermultiplet beyond on-shell, we calculated the special manifestly $\mathcal{N} = 2$ supersymmetric subleading contribution which is written as an integral over $3/4$ of the full $\mathcal{N} = 2$ harmonic superspace. We believe that such contributions deserves a special study.

Acknowledgments

N.G.P is grateful to I.L. Buchbinder for collaboration and S. Kuzenko and I. McArthur for helpful discussions and correspondence. The work was supported in part by RFBR grants, project No 06-02-16346, No 08-02-00334-a, grant for LRSS, project No 2553.2008.2 and INTAS grant, project No 05-7928.

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