Research on stability monitoring and early warning methods for cantilevered rocks

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Abstract. It is difficult to monitor the change of rock stability effectively and warn of the damage of rocks by using external deformation indicators. The article analyzed the change of the cantilevered rock's natural frequency during its stability decline and proved that the rock's natural frequency can reflect the rock's stability. The article analyzed the motion characteristics of cantilevered rocks and gave the calculation method of the natural frequency of cantilevered rocks. Using the formula, the natural frequency value within each stage of the rock can be calculated, and the critical natural frequency value before rock damage can be determined. Based on the theoretical equations' calculation results, the monitoring of dynamic changes in cantilevered rocks' stability and early warning of rock collapse can be accomplished using vibration sensors.

1. Introduction
Rock collapse is a common geological disaster. The speed of rock collapse is fast, and the collapse precursors are not obvious. It is difficult to determine the change in rock stability and predict the destabilization of rocks using external deformation monitoring methods[1-3]. However, the vibration characteristics of rocks have been found to correlate well with rocks' stability [4]. The energy of the high frequency vibration signal within the unstable slope will increase in the weeks before the collapse [5], and the highest frequency of rock vibration will decrease in the ten hours before the rock failure [6]. Compared to stable slopes, the vibration amplitude [7] and the spectral ratio [8] of unstable rocks under ambient vibration will increase significantly. Further studies have shown that the amplification effect of dangerous rock's spectral ratio is directional [9]. The analysis of vibration amplitude and vibration frequency spectrum ratio of slope vibration can identify the dangerous rock masses [10] and failure extent of slopes [11].

The analysis of unstable rock's vibration data under ambient vibrations shows that unstable rock's frequency curves have distinct energy peaks [12]. The frequency corresponding to the energy peaks is rock's resonant frequency [13]. Recording the rock's resonant frequency changes under ambient vibrations can evaluate rock's stability [14]. As the rock's stability decreases, the rock's resonant frequency will decrease [15]. The resonant frequency of unstable rocks will increase after reinforcement [16]. The same rock's different movement modes can have different resonant frequencies [17].

Researchers have become a consensus that the rock's resonant frequency changes as its stability changed [18-19]. However, the present theory cannot calculate the value of critical resonant frequency before rock destabilization. Without determining the value of critical resonant frequency at the time of rock instability, it is impossible to warn the rock's instability by monitoring rock's vibration information. This paper will analyze the relationship between the resonant frequency of cantilevered rocks and their structural characteristics to establish a theoretical calculation method for cantilevered rock's natural frequency. On this basis, promote the quantitative evaluation technology for rock's stability change and the early warning of rock collapse.

2. Methods
Rock's vibration response is mainly determined by its natural frequency, and the resonant frequency value of rock under ambient vibrations' influence is the same as its natural frequency value. Rock's natural frequency is determined by its mass distribution and material properties [8]. Cantilevered rock is one type of typical unstable rocks. Under gravity's action, the end of cantilevered rock connected to the slope will be subjected to bending stress. If the bending stress exceeds the rock's tensile strength, it will fail [20-21].

If a crack exists at the cantilevered end of the rock (Figure 1a), the bending stress at the tip of the crack grows as the depth of the crack \( h \) increases. When the external load on the rock does not change, the only factor that controls the rock's stability is its crack's depth \( h \).

![Figure 1](image.png)

**Figure 1.** (a) schematic diagram of cantilevered rock, (B) simplified model of cantilevered rock.

Considering the cantilevered rock's movement when it vibrates on the vertical plane as it rotation around the joint plane, we can replace the joint plane's restraint of the rock's vibration with a spring \( K \) (Figure 1b). The ability of the joint plane to restrain the rock vibration can be expressed in terms of the spring stiffness \( K_\theta \).

The equation of movement of the rock as it rotates around the fulcrum is

\[
ML^2 \ddot{\theta} + 4K_\theta \dot{\theta} + 2MgL \sin \theta = 0 \quad \text{(*) MERGEFORMAT (1)}
\]

where \( M \) is the mass of the rock, \( L \) is the length of the rock, and \( \theta \) is the vertical rotation angle of the rock. From equation (1), the equation for the natural frequency \( w \) of the rock can be deduced as

\[
w = \sqrt{\frac{2MgL + 4K_\theta}{ML^2}} \quad \text{(*) MERGEFORMAT (2)}
\]

The mass and length of the rock in Eq. (2) can be measured by field investigation, where the only unknown quantity is the spring stiffness \( K_\theta \). For the rock shown in Fig. 1, the slope's ability to restrain the rock will weaken as the crack depth increases. Thus the spring stiffness is controlled by the crack depth.

Two methods are used to analyze the effect of cracks on structural stiffness, the fracture mechanics method [22] and the equivalent stiffness method [23]. The equivalent stiffness method uses a one-dimensional function to represent the structural's stiffness due to the crack. Following the equivalent stiffness method, the cross-sectional bending stiffness \( EI \) of the rock when cracks are existent is calculated as [24-25].

\[
EI(x) = \frac{EI_0}{1 + \delta \exp\left(\frac{4(x-x_i)}{3H}\right)} \quad \text{(*) MERGEFORMAT (3)}
\]

where \( EI_0 \) is the cross-sectional bending stiffness of the rock in the absence of a crack, \( EI(x) \) is the cross-sectional bending stiffness of the rock at coordinate \( x \) in the existence of a crack, \( x_i \) is the location where the crack is located, \( H \) is the height of the rock, and \( \delta \) is the calculated coefficient.
\[
\delta = \frac{h^3}{(H-h)^3}
\]

From equations (3) and (4), it can be deduced that when cracks exist at the fixed end of the rock, the cross-sectional bending stiffness of the structural plane is

\[
K_\theta = EI(x) = \frac{EBH^3(1-\nu)^2}{12}
\]

where \(\nu = h/H\) represents the proportion of the crack cut and \(E\) is the Young's modulus of the rock. The rock's natural frequency can be calculated by bringing equation (5) into equation (2). It can be seen from equations that rock's natural frequency is mainly influenced by its mass, properties and material properties, and all these parameters can be obtained through field investigation or testing. Combining equation (5) and equation (2), we can draw a simple conclusion: rock's natural frequency decreases with the increase of crack's depth. For cantilevered rocks (Figure 1a), the increase in crack's depth reduces the rock's stability. The decrease in the rock's stability is accompanied by a decrease in its natural frequency, which is consistent with the results of field monitoring.

3. Numerical Modal Analysis

To analyze the accuracy of the method mentioned above for calculating the natural frequency of cantilevered rock, a numerical model of the cantilevered rock was developed, and its vibration response was analyzed. In the numerical model, one end of the cantilevered rock was connected to the erect slope, and the width of the rock was 2 meters, and the height was 3 meters. The rock's natural frequency values were calculated for the lengths of 5 meters, 4 meters and 3 meters, respectively. The length and height of the side slope were 100 meters, and the width was 60 meters. The slope's connection plane to the rock was free, and all other sides' displacements were constrained.

The material properties of the rock and the slope were the same, its density \(\rho = 2704kg/m^3\), tensile strength \(\delta_t = 1.10E6 Pa\), Young's modulus \(E = 2.50E10 Pa\). The vertical impact force was applied to the top of the rock, and rock's response acceleration in the Z-direction was extracted at a frequency of 1000 Hz. For each length of rock, its connect length to the slope was set to 2.75 meters, 2.50 meters, 2.25 meters, 2.00 meters, 1.75 meters and 1.50 meters, and the calculation was repeated six times.

**Figure 2.** Schematic diagram of the numerical simulation model. Node1 is the action point of impact force; Node2 is the extraction point of rock's vibration acceleration.

Using the Fast Fourier Transform method, the rock's vibration acceleration curve was converted into its frequency curve (Figure 3). The frequency corresponding to the energy peak in the rock's frequency curve is the natural frequency of the rock in the vertical plane, and the frequency curve's energy peak
was effect by the depth of the crack. The rock's peak frequency decrease as the crack's depth increase for all three lengths of rocks.

Figure 3. Frequency curves of the rocks. (a) The length of the rock is 3 meters; (b) the length of the
rock is 4 meters; (c) the length of the rock is 5 meters.

4. Discussion

4.1. Correction of the formula

Use equation (2) to calculate three rock's natural frequencies in the numerical model (Table 1). In the table, the simulated value of natural frequency is \( w_1 \) and the calculated value is \( w \). The rock's natural frequency simulated values were used as the standard to compare the differences between the calculated and simulated values of the rock's natural frequency to analyze the accuracy of the formula given in equation (2).

| Crack's cutting ratio \( a \) | \( L = 5m \) | \( L = 4m \) | \( L = 3m \) |
|------------------------------|-------------|-------------|-------------|
| \( w_1 \) (Hz) | \( w \) (Hz) | \( w_1 \) (Hz) | \( w \) (Hz) | \( w_1 \) (Hz) | \( w \) (Hz) |
| 0.083 | 31.19 | 65.80 | 39.45 | 91.96 | 45.50 | 141.58 |
| 0.167 | 28.84 | 57.03 | 37.33 | 79.71 | 44.67 | 122.71 |
| 0.250 | 26.18 | 48.70 | 34.67 | 68.05 | 43.25 | 104.78 |
| 0.333 | 23.33 | 40.81 | 31.19 | 57.03 | 41.69 | 87.81 |
| 0.417 | 20.51 | 33.40 | 27.54 | 46.68 | 38.02 | 71.87 |
| 0.500 | 17.70 | 26.51 | 23.66 | 37.05 | 32.96 | 57.03 |

There is a significant error between the simulated and calculated results of the natural frequency of the rock. By expressing the ratio of the two frequency values as \( R \), there is also a linear relationship between the frequency ratio \( R \) and the crack cutting ratio \( a \):

\[
R = \frac{w(a)}{w_1(a)} = m + na
\]  

where \( m \) and \( n \) are the calculated parameters of the linear function.

![Figure 4. The ratio of rock's natural frequency](image)

When the crack cuts the rock completely, the theoretical and actual values of the rock's natural frequency are the same, both being 0. So when \( a = 1 \), \( R(1) = m + n = 1 \). From this equation (6) can be written as

\[
R = 1 - n + na
\]  

\* MERGEFORMAT (7)
The variable in equation (7) is the crack's cutting ratio \( \alpha \), and the unknown quantity is \( n \). Once we identify the values of the frequency ratio \( R \) and the crack's cutting ratio \( \alpha \) of the rock, we can bring them into equation (7) to calculate the value of the unknown quantity \( n \).

Incorporating the value of \( n \) into equation (6), the corrected formula for calculation the rock's natural frequency \( w_2 \) is

\[
w_2 = w \cdot (1 - n + na) \tag{8}\]

The formula for calculating \( w \) is shown in equation (2).

The analysis was performed with a crack's cutting ratio of 0.25 to verify the corrected formula's validity. The value of parameter \( n \) was calculated by bringing the ratio of rock's natural frequency and the crack cutting ratio into equation (7). For different rock lengths, the values of \( n \) are: 0.616 (\( L=5m \)), 0.654 (\( L=4m \)), and 0.783 (\( L=3m \)). Bringing \( n \) into equation (8), rock's natural frequencies were calculated with the modified formula for crack's cutting ratios of 0.33, 0.42 and 0.5 (Table 2).

| Rock length \( L \) (m) | Crack's cutting ratio \( \alpha \) | \( w_1 \) (Hz) | \( w_2 \) (Hz) |
|-------------------------|----------------------------------|----------------|----------------|
| 5                       | 0.33                             | 23.33          | 24.04          |
| 4                       | 0.33                             | 31.19          | 32.17          |
| 3                       | 0.33                             | 41.69          | 41.98          |
| 5                       | 0.42                             | 20.51          | 21.39          |
| 4                       | 0.42                             | 27.54          | 28.87          |
| 3                       | 0.42                             | 38.02          | 39.05          |
| 5                       | 0.50                             | 17.70          | 18.34          |
| 4                       | 0.50                             | 23.66          | 24.93          |
| 3                       | 0.50                             | 32.96          | 34.71          |

Comparing the results of the calculation of the natural frequency of the rock \( w_2 \) with the simulated value of the natural frequency of the rock \( w_1 \), it can be seen that the natural frequency of the rock calculated by the modified formula is closer to its actual value. Based on the value of the parameter \( n \) calculated for a crack cut ratio of 0.25, the value of the corrected rock natural frequency calculation \( w_2 \) can obtained.

4.2. Monitoring of dynamic changes in rock stability

The stress \( \sigma \) at a crack in a cantilevered rock under gravity is

\[
\sigma = \frac{3mgL}{BH^2(1-\alpha)^2} \tag{9}
\]

When the stress \( \sigma \) of the rock reaches the tensile strength \( \sigma_t \) of the rock, rock will fall due to the expansion of its crack. Therefore, the stability coefficient \( F_s \) of cantilevered rock is calculated by the formula as

\[
F_s = \frac{\sigma}{\sigma_t} = \frac{3mgL}{\delta B H^2(1-\alpha)^2} \tag{10}
\]

Equation (10) demonstrates the relationship between rock stability and its crack's cutting ratio \( \alpha \). In general, we can identify the values of the calculated parameters in Eq. (10) and determine the rock's stability from a single field investigation. After measuring the crack's cutting ratio \( \alpha_0 \) of the rock, we can calculate the value of the parameter \( n \) using equation (7) as long as we continue to measure the natural vibration frequency \( w_0 \) of the rock. Further, we can establish the natural frequency calculation equation (8) of the rock.

The rock's natural frequency is a monotonically varying function for its crack's cutting ratio. Therefore, once the rock's natural frequency is monitored, the numerical solution of the crack's cutting ratio of the rock can be derived from the real-time monitoring data, and then the stability coefficient \( F_s \) of the rock can be solved. After determining the cantilevered rock's natural frequency, the monitoring of the
dynamic change of the cantilevered rock's stability coefficient can be realized by using the field vibration sensor.

![Flow chart of rock stability monitoring method based on its natural frequency](image)

**Figure 5.** Flow chart of rock stability monitoring method based on its natural frequency

5. Conclusions
The vibration monitoring results of unstable rocks show a good correlation between the resonant frequencies of rocks and their stability. Monitoring the vibration frequency at different locations of the slope can identify unstable rocks, but the quantitative relationship between rock stability and vibration frequency is still unclear. The paper analyzed the vibration of cantilevered rock, simplified the restraint of slope on rock vibration to a spring, established a simplified model of cantilevered rock vibration, and derived a formula for calculating cantilevered rock's natural frequency. The formula shows that with the increase of the crack's depth, the rock's natural frequency will decrease, which is consistent with the field monitoring results.

The article took three different sizes of cantilevered rocks as the research object, compared the differences between the numerical analysis results and the theoretical calculation results of the rock's natural frequency, and gave the calculation formula's correction method based on the field investigation or monitoring data. The calculation results of the rock's natural vibration frequency's corrected equation were closer to the numerical analysis results. Based on the theoretical model of the formula and its correction result, the article proposed a method to monitor the dynamic changes of cantilevered rock's stability. The method requires the determination of the equation of the rock's natural frequency and the critical conditions of the rock failure based on the field investigation. The crack's cutting ratio of the rock can be calculated rely on the monitor result of its natural frequency using vibration sensors, and the stability of the rock is judged by comparing the monitored values with the critical conditions of rock failure.

Acknowledgments
This study was supported by the National Key Research and Development Program of China (No. 2019YFC1509602).

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