$B^0$ and $B^0_s$ decays into $J/\psi$ plus a scalar or vector meson

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Abstract

We extend a recent approach to describe the $B^0$ and $B^0_s$ decays into $J/\psi f_0(500)$ and $J/\psi f_0(980)$, relating it to the $B^0$ and $B^0_s$ decays into $J/\psi$ and a vector meson, $\phi$, $\rho$, $K^*$. In addition the $B^0$ and $B^0_s$ decays into $J/\psi$ and $\kappa(800)$ are evaluated and compared to the $K^*$ vector production. The rates obtained are in agreement with available experiment while predictions are made for the $J/\psi$ plus $\kappa(800)$ decay.

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I. INTRODUCTION

The $B_s^0$ decays into $J/\psi$ plus $f_0(500)$ or $f_0(980)$ is capturing the attention of both experiment and theory. A striking result observed in LHCb is that in the $B_s^0$ decay a pronounced peak for the $f_0(980)$ is observed [1] while no appreciable signal is seen for the $f_0(500)$. These results have been corroborated by Belle [2], CDF [3], D0 [4] and again LHCb [5, 6] collaborations. Conversely, in [7] the $\bar{B}^0$ into $J/\psi$ and $\pi^+\pi^-$ is investigated and a signal is seen for the $f_0(500)$ production while only a very small fraction is observed for the $f_0(980)$ production.

Estimations of the order of magnitude of rates for some of these reactions have been done using light cone QCD sum rules under the factorization assumption [8]. Also the experimental data has served as a basis of discussion on the possible nature of the scalar mesons as $q\bar{q}$ or tetraquark [9].

More recently a simple approach based on the final state interaction of mesons provided by the chiral unitary approach has been applied that allows to calculate all these rates relative to one of them [10]. The work isolates the dominant weak decay mechanism into $J/\psi$ and a $q\bar{q}$ pair. After this, the $q\bar{q}$ pair is hadronized, and meson meson pairs are produced with a certain weight. These mesons are then allowed to interact and for this, the chiral unitary approach for meson meson interaction [11–16] is used. This approach uses a full unitary scheme by means of the Bethe-Salpeter equation in coupled channels [17, 18], extracting the kernel, or potential, from the chiral Lagrangians [19, 20]. The success of this approach to deal with meson meson interaction, and with reactions in which the $f_0(500)$ or $f_0(980)$ and other resonances are produced, is remarkable (see [10] for a detailed list of reactions studied), but the closest ones are the $J/\psi \to \phi(\omega)\pi\pi$ where different signals for the $f_0(500)$, $f_0(980)$ are observed depending on the reaction [21–25]. The idea of these latter works in which the final state interaction of pairs of mesons is explicitly taken into account has been also followed in weak decays similar to those discussed above, concretely in the $B \to \pi\pi K$ decay [26, 27].

The related experimental work on vector meson production is more abundant. The $B_s^0 \to J/\psi K^{*0}$ is studied recently in [3, 28] (see the particle data book (PDG) for more experiments [29]), and the $B^0 \to J/\psi K^{*0}$ is studied in [30, 31] among others [29]. The $\rho$ production is studied as a part of the spectra of the $B^0$ decay into $J/\psi$ and $\pi^+\pi^-$ in [7, 32].

In the present paper we review shortly the work of [10] and complement it by evaluating the rates of $B^0$ and $B_s^0$ decays into $J/\psi$ and a vector, $\phi$, $\rho$, $K^*$. In addition we also evaluate the rates of $B^0$ and $\bar{B}_s^0$ decays into $J/\psi$ and $\kappa$. This allows us to compare the rates for $K^*$ and $\kappa$ production, as well as $f_0(500)$, $f_0(980)$ with $\rho$ production, and $\kappa$ production with $f_0(500)$, $f_0(980)$ production. The work exploits flavor symmetries and dynamics of meson meson interaction and factorizes the matrix elements of the weak process, which are not explicitly evaluated. These latter ones are shared by different reactions such that at the end, by using only two rates from experiment, we can produce all the mass distributions for all the different reactions possible.

II. FORMALISM FOR SCALAR MESON PRODUCTION

Following [9] and [1–7, 32] we take the dominant weak mechanism for $\bar{B}_s^0$ and $\bar{B}_s^0$ decays as depicted in Fig. 1. The case of $B^0$ ($B_s^0$) is identical to that of $\bar{B}_s^0$ ($\bar{B}_s^0$), changing the particles by their antiparticles. In the (a) diagram, in addition to the $J/\psi$, a primary pair of $d\bar{d}$ quarks are produced from the $B^0$ decay, while a $s\bar{s}$ pair is produced in the case of
the $B_s^0$ decay, diagram (b). These two cases are those studied in [10]. In addition we can also produce a $s\bar{d}$ pair in the $\bar{B}^0$ decay and a $d\bar{s}$ pair in the $\bar{B}_s^0$ decay, diagrams (c) and (d). These two latter cases are new and we study them here. If we look for production of scalar mesons, $f_0(500)$, $f_0(980)$ and $\kappa$, one identifies them by looking at $\pi^+\pi^-$ production in the case of $f_0(500)$, $f_0(980)$ and $\pi K$ for the case of the $\kappa$. We have to produce two mesons, which means that the $q\bar{q}$ pair must hadronize. To accomplish this we follow the approach of [33] and complement the primary $q\bar{q}$ pair by another $q\bar{q}$ pair with the quantum numbers of the vacuum $u\bar{u} + d\bar{d} + s\bar{s}$ (see Fig. 2). Then we realize that the $q\bar{q}$ matrix $M$

$$M = \begin{pmatrix}
    u\bar{u} & u\bar{d} & u\bar{s} \\
    d\bar{u} & d\bar{d} & d\bar{s} \\
    s\bar{u} & s\bar{d} & s\bar{s}
\end{pmatrix}$$  \hspace{1cm} (1)

has the property

$$M \cdot M = M \times (u\bar{u} + d\bar{d} + s\bar{s}).$$  \hspace{1cm} (2)

FIG. 2: Schematic representation of the hadronization $q\bar{q} \rightarrow q\bar{q}(u\bar{u} + d\bar{d} + s\bar{s})$.

Now, in terms of mesons, neglecting the $\eta_1$ singlet that corresponds mostly to the $\eta'$, which we omit in the coupled channels because its large mass, the matrix $M$ corresponds to

$$\phi = \begin{pmatrix}
    \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta \\
    \pi^- \\
    K^-
\end{pmatrix} \begin{pmatrix}
    \pi^+ \\
    -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta \\
    K^0
\end{pmatrix} \begin{pmatrix}
    K^+ \\
    \bar{K}^0 \\
    -\frac{2}{\sqrt{6}}\eta
\end{pmatrix},$$  \hspace{1cm} (3)
where \( \eta \) is actually \( \eta_s \), but can be considered the \( \eta \) for practical purposes. Hence, in terms of two pseudoscalars we have the correspondence:

\[
\begin{align*}
dd(u \bar{u} + d \bar{d} + s \bar{s}) & \equiv (\phi \cdot \phi)_{22} = \pi^- \pi^+ + \frac{1}{2} \pi^0 \pi^0 - \frac{1}{\sqrt{3}} \pi^0 \eta + K^0 \bar{K}^0 + \frac{1}{6} \eta \eta, \\
n\bar{s} s(u \bar{u} + d \bar{d} + s \bar{s}) & \equiv (\phi \cdot \phi)_{33} = K^- K^+ + K^0 \bar{K}^0 + \frac{4}{6} \eta \eta, \quad (4) \\
ss(u \bar{u} + d \bar{d} + s \bar{s}) & \equiv (\phi \cdot \phi)_{32} = K^- \pi^+ - \frac{1}{\sqrt{2}} K^0 \pi^0 - \frac{1}{\sqrt{6}} \eta \bar{K}^0, \\
ss(u \bar{u} + d \bar{d} + s \bar{s}) & \equiv (\phi \cdot \phi)_{23} = \pi^- K^+ - \frac{1}{\sqrt{2}} K^0 \pi^0 - \frac{1}{\sqrt{6}} \eta K^0.
\end{align*}
\]

The diagrams of Fig. 1 share the same dynamics and are only differentiated by the different matrix element of the Cabbibo-Kobayashi-Maskawa (CKM) matrix. From the second \( qqW \) vertex in the diagrams we have \( V_{cd} \) in diagrams (a) and (d) and \( V_{cs} \) in (b) and (c). These matrix elements are related to the Cabbibo angle

\[
\begin{align*}
V_{cd} &= -\sin \theta_c = -0.22534, \\
V_{cs} &= \cos \theta_c = 0.97427. \quad (5)
\end{align*}
\]

The next step consist in allowing the pair of mesons originated in the first step to interact among themselves and their coupled channels, since this interaction is what gives rise dynamically to the low lying scalar mesons in chiral unitary theory. This is depicted diagrammatically in Fig. 3 for \( \pi^+ \pi^- \) and \( \pi^+ K^- \) or \( \pi^- K^+ \) production.

We can see that \( \pi^+ \pi^- \) is obtained in the first step in the \( \bar{B}^0 \) decay but not in \( \bar{B}^0_s \) decay. In this latter case, upon rescattering of \( KK \) we also can get \( \pi^+ \pi^- \) in the final state. Since the \( f_0(980) \) couples strongly to \( K \bar{K} \) and the \( f_0(500) \) to \( \pi \pi \), the meson meson decomposition of Eq. (4) is already hinting that the \( \bar{B}^0 \) decay will be dominated by \( f_0(500) \) production and \( \bar{B}^0_s \) decay by \( f_0(980) \) production. This is indeed what was found in [10].

The primary production and rescattering of the mesons is taken into account as follows: Let us call \( V_P \) the production vertex containing all dynamical factors common to the four reactions. The \( \pi^+ \pi^- \) or \( \pi K \) production will proceed via primary production or final state interaction as depicted in Fig. 3.

The amplitudes for \( \pi^+ \pi^- \) and \( \pi K \) production are given by

\[
\begin{align*}
t(\bar{B}^0 \rightarrow J/\psi \pi^+ \pi^-) &= V_P V_{cd} (1 + G_{\pi^+ \pi^-}^{+} t_{\pi^+ \pi^-}^{\rightarrow \pi^+ \pi^-} + \frac{1}{22} G_{\pi^0 \pi^0}^{\rightarrow \pi^+ \pi^-} + \frac{1}{2} G_{\eta \eta}^{t_{\eta \eta}^{\rightarrow \pi^+ \pi^-}} + \frac{1}{2} G_{\eta \eta}^{t_{\eta \eta}^{\rightarrow \pi^+ \pi^-}}), \\
t(\bar{B}^0_s \rightarrow J/\psi \pi^+ \pi^-) &= V_P V_{cs} (G_{\pi^+ \pi^-}^{+} t_{\pi^+ \pi^-}^{\rightarrow \pi^+ \pi^-} + G_{\pi^0 \pi^0}^{\rightarrow \pi^+ \pi^-} + \frac{41}{62} G_{\eta \eta}^{t_{\eta \eta}^{\rightarrow \pi^+ \pi^-}} + \frac{41}{62} G_{\eta \eta}^{t_{\eta \eta}^{\rightarrow \pi^+ \pi^-}}), \\
t(\bar{B}^0 \rightarrow J/\psi \pi^- K^+) &= V_P V_{cs} (1 + G_{\pi^- K^+}^{+} t_{\pi^- K^+}^{\rightarrow \pi^- K^+} - \frac{1}{\sqrt{2}} G_{\pi^0 K^0}^{t_{\pi^0 K^0}^{\rightarrow \pi^- K^+}} - \frac{1}{\sqrt{6}} G_{\eta \eta}^{t_{\eta \eta}^{\rightarrow \pi^- K^+}}), \\
t(\bar{B}^0_s \rightarrow J/\psi \pi^- K^+) &= V_P V_{cd} (1 + G_{\pi^- K^+}^{+} t_{\pi^- K^+}^{\rightarrow \pi^- K^+} - \frac{1}{\sqrt{2}} G_{\pi^0 K^0}^{t_{\pi^0 K^0}^{\rightarrow \pi^- K^+}} - \frac{1}{\sqrt{6}} G_{\eta \eta}^{t_{\eta \eta}^{\rightarrow \pi^- K^+}}), \\
\end{align*}
\]
where $G_i$ are the loop functions of two meson propagators

$$G_i(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P - q)^2 - m_1^2 + i\varepsilon} \frac{1}{q^2 - m_2^2 + i\varepsilon},$$

with $m_1, m_2$ the masses of the mesons in the $i$-channel, $q$ the four-momentum of one meson, and $P$ the total four-momentum of the system, thus, $s = P^2$. The integral is performed integrating exactly the $q^0$ variable and implementing a cut off $\Lambda$ of the order on 1 GeV/c for the three momentum (in [10] it was taken as 600 MeV/c after including explicitly the $\eta\eta$ channel). The elements $t_{ij}$ are the scattering matrices for transitions of channel $i$ to $j$. According to [11] this matrix is given by

$$t = [1 - VG]^{-1}V,$$

and the $V$ matrix is taken from [11] complemented with the matrix elements of the $\eta\eta$ channels, which we have taken from [34]. Explicit forms of the potential are given in [10]. Note that we include the factor $1/2$ before the $G$ function in the case of identical particles. As discussed in [10] the unitary normalization of the identical states is used to get the $V$ and $T$ matrices by means of Eq. (8), but the good normalization must be used for the $t$-matrices in Eqs. (6).
In addition, to deal with the $\kappa$ we have to solve the Bethe-Salpeter equation of Eq. (8) with the channels $\pi^-K^+$, $\pi^0K^0$ and $\eta K^0$. These channels are numbered orderly as 1, 2 and 3. The matrix elements projecting into $S$-wave are taken from [34] and given by

$$V_{11} = -\frac{1}{4f^2}s,$$
$$V_{12} = -\frac{1}{2\sqrt{2}f^2}(3s + m_\pi^2 + m_K^2),$$
$$V_{13} = -\frac{1}{6\sqrt{6}f^2}(-\frac{9}{2}s + 9m_K^2 - \frac{1}{2}m_\pi^2 + \frac{3}{2}m_\eta^2),$$
$$V_{22} = -\frac{1}{12f^2}(3s + 3m_K^2 + 3m_\pi^2),$$
$$V_{12} = -\frac{1}{12f^2}(\frac{9}{2}s + 9m_K^2 + 3m_\eta^2 - 2m_\pi^2),$$

with $f = 93$ MeV the pion decay constant.

In Eq. (6) we made use of the fact that both the $f_0(500)$ and $f_0(980)$ appear in relative $L = 0$ meson-meson orbital angular momentum, and then $\pi\pi$ in the final state selects $I = 0$, hence, the $\pi^0\eta$ intermediate state does not contribute.

One final element of information is needed to complete the formula for $d\Gamma/dM_{inv}$, with $M_{inv}$ the $\pi^+\pi^-$ or $\pi K$ invariant mass, which is the fact that we need a $L' = 1$ orbital angular momentum for the $J/\psi$ in a $0^- \rightarrow 1^-0^+$ transition to match angular momentum conservation. In [10] we assumed $V_p = A p_{J/\psi} \cos \theta$, although the combination with spin produces a different angular dependence. In practice the explicit form does not matter since it is the same for the different reactions and we only care about ratios between them. The only thing that matters is the presence of the factor $p_{J/\psi}$. Thus we follow the formalism of [10] and write

$$\frac{d\Gamma}{dM_{inv}} = \frac{1}{(2\pi)^3} \frac{1}{4m_B^2} \sum \sum \sum |f_{B^0 \rightarrow J/\psi \pi^+\pi^-}|^2,$$

where the factor $1/3$ is coming from the integral of $\cos^2 \theta$ and $f_{B^0 \rightarrow J/\psi \pi^+\pi^-}$ is $t_{B^0 \rightarrow J/\psi \pi^+\pi^-}/(p_{J/\psi} \cos \theta)$, which depends on the $\pi^+\pi^-$ invariant mass. In Eq. (10) $p_{J/\psi}$ is the $J/\psi$ momentum in the global CM frame ($\vec{B}$ at rest) and $\vec{p}_\pi$ is the pion momentum in the $\pi^+\pi^-$ rest frame,

$$p_{J/\psi} = \frac{\lambda^{1/2}(M_B^2, M_{J/\psi}^2, M_{inv}^2)}{2M_B}, \quad \vec{p}_\pi = \frac{\lambda^{1/2}(M_{inv}^2, m_\pi^2, m_\eta^2)}{2M_B}.$$

The formulas for the $\pi K$ invariant mass distribution are similar to Eqs. (10) and (11), but with $M_{inv}$ being the $\pi K$ invariant mass and substituting one of the $m_\pi^2$ in $\vec{p}_\pi$ by $m_K^2$.

### III. Formalism for Vector Meson Production

The diagrams of Fig. 1 without the hadronization can serve to study the production of vector mesons, which are largely $q\bar{q}$ states [35-37]. Since we were concerned up to now only about ratio of the scalars, the factor $V_p$ was taken arbitrary. In order to connect the scalar meson production with the vector production we need a factor $V_H$, associated to the hadronization. Here, instead, the spin of the particles requires $L' = 0, 2$ and with no rule
preventing \( L' = 0 \) we assume that it is preferred, hence the \( p_{J/\psi \cos \theta} \) is not present now. Then we find immediately the amplitudes associated to Fig. 1,

\[
\begin{align*}
t_{\bar{B}^0 \rightarrow J/\psi \rho^0} &= -\frac{1}{\sqrt{2}} \bar{V}_P V_{cd}, \\
t_{\bar{B}^0 \rightarrow J/\psi \omega} &= \frac{1}{\sqrt{2}} \bar{V}_P V_{cs}, \\
t_{\bar{B}^0 \rightarrow J/\psi K^{*0}} &= V'_P V_{cs}, \\
t_{\bar{B}^0 \rightarrow J/\psi K^{*0}} &= \bar{V}'_P V_{cd} \tag{12}
\end{align*}
\]

where \((-\frac{1}{\sqrt{2}})\) is the \( \rho^0 \) component in \( d\bar{d} \) and \((\frac{1}{\sqrt{2}})\) that of the \( \omega \). In order to determine \( \bar{V}'_P \) versus \( \bar{V}_P \) in the scalar production we use the well measured ratio \([5, 29]\),

\[
\frac{\Gamma_{\bar{B}^0 \rightarrow J/\psi f_0(980); f_0(980) \rightarrow \pi^+\pi^-}}{\Gamma_{\bar{B}^0 \rightarrow J/\psi}} = (13.9 \pm 0.9) \times 10^{-2}. \tag{13}
\]

The width for \( J/\psi V \) vector decay is now given by

\[
\Gamma_{V_i} = \frac{1}{8\pi} \frac{1}{m_{\bar{B}^0}} |t_{\bar{B}^0 \rightarrow J/\psi V_i}|^2 p_{J/\psi} \tag{14}
\]

Eqs. (12) allow us to determine ratios of vector production with respect to the \( \phi \)

\[
\begin{align*}
\frac{\Gamma_{\bar{B}^0 \rightarrow J/\psi \rho^0}}{\Gamma_{\bar{B}^0 \rightarrow J/\psi \phi}} &= \frac{1}{2} \frac{V_{cd}}{V_{cs}} \frac{m_{\bar{B}^0}}{p_{\rho^0}} \frac{m_{\rho^0}}{m_{\bar{B}^0}} = 0.0263, \\
\frac{\Gamma_{\bar{B}^0 \rightarrow J/\psi \omega}}{\Gamma_{\bar{B}^0 \rightarrow J/\psi \phi}} &= \frac{1}{2} \frac{V_{cd}}{V_{cs}} \frac{m_{\bar{B}^0}}{p_{\omega}} \frac{m_{\omega}}{m_{\bar{B}^0}} = 0.0263, \tag{15}
\end{align*}
\]

\[
\begin{align*}
\frac{\Gamma_{\bar{B}^0 \rightarrow J/\psi K^{*0}}}{\Gamma_{\bar{B}^0 \rightarrow J/\psi \phi}} &= \frac{m_{\bar{B}^0}^2}{m_{\bar{B}^0}^2} \frac{p_{K^{*0}}}{p_{\phi}} = 0.957, \\
\frac{\Gamma_{\bar{B}^0 \rightarrow J/\psi K^{*0}}}{\Gamma_{\bar{B}^0 \rightarrow J/\psi \phi}} &= \frac{|V_{cd}|^2}{|V_{cs}|^2} \frac{p_{K^{*0}}}{p_{\phi}} = 0.0551.
\end{align*}
\]

By taking as input the branching ratio of \( \bar{B}_s^0 \rightarrow J/\psi \phi \)

\[
BR(\bar{B}_s^0 \rightarrow J/\psi \phi) = (10.0^{+3.2}_{-1.8}) \times 10^{-4}, \tag{16}
\]

we obtain the other four branching ratios

\[
\begin{align*}
BR(\bar{B}_s^0 \rightarrow J/\psi \rho^0) &= (2.63^{+0.84}_{-0.47}) \times 10^{-5}, \\
BR(\bar{B}_s^0 \rightarrow J/\psi \omega) &= (2.63^{+0.84}_{-0.47}) \times 10^{-5}, \\
BR(\bar{B}_s^0 \rightarrow J/\psi K^{*0}) &= (9.57^{+3.1}_{-1.7}) \times 10^{-4}, \\
BR(\bar{B}_s^0 \rightarrow J/\psi K^{*0}) &= (5.51^{+1.7}_{-1.0}) \times 10^{-5}. \tag{17}
\end{align*}
\]

The experimental numbers are \([29]\)

\[
\begin{align*}
BR(\bar{B}_s^0 \rightarrow J/\psi \rho^0) &= (2.58 \pm 0.21) \times 10^{-5}, \\
BR(\bar{B}_s^0 \rightarrow J/\psi \omega) &= (2.3 \pm 0.6) \times 10^{-5}, \\
BR(\bar{B}_s^0 \rightarrow J/\psi K^{*0}) &= (1.34 \pm 0.06) \times 10^{-3}, \\
BR(\bar{B}_s^0 \rightarrow J/\psi K^{*0}) &= (4.4 \pm 0.9) \times 10^{-5}. \tag{18}
\end{align*}
\]
We can see that the agreement is good within errors, taking into account that the only theoretical errors in Eq. (17) are from the experimental branching ratio of Eq. (16). In the case of \( BR(\bar{B}^0 \to J/\psi K^{*0}) \) the agreement is border line because of the small experimental errors. Admitting only 5% extra error from the theory, the agreement is quite good. Note also that the experimental \( BR \) for \( \bar{B}^0 \to J/\psi K^{*0} \) of the PDG has abnormally small errors, the most recent measurement from BABAR gives \( (1.33^{+0.22}_{-0.21}) \times 10^{-3} \) [38].

The next step is to compare the \( \rho \) production with \( \rho \to \pi^+\pi^- \) decay with \( \bar{B}^0 \to J/\psi f_0; f_0 \to \pi^+\pi^- \) (\( f_0(\equiv f_0(500), f_0(980) \)). In an experiment that looks for \( \bar{B}^0 \to J/\psi \pi^+\pi^- \), all these contributions will appear together and only a partial wave analysis will disentangle the different contributions. This is done in [7, 32]. There (see Fig. 13 of [32]) one observes a peak of the \( \rho \) and a \( f_0(500) \) distribution, with a peak of the \( \rho^0 \) distribution about a factor 6 larger than that of the \( f_0(500) \). The \( f_0(980) \) signal is very small and not shown in the figure.

In order to compare the theoretical results with these experimental distributions we convert the rates obtained in Eqs. (17) into \( \pi^+\pi^- \) distributions for the case of the \( \bar{B}^0 \to J/\psi \rho^0 \) decay and \( K^-\pi^+ \) for the case of the \( \bar{B}^0 \to J/\psi K^{*0} \) decay. For this purpose we multiply the decay width of the \( \bar{B}^0 \) by the spectral function of

\[
\frac{d\Gamma_{\bar{B}^0 \to J/\psi \rho^0}}{dM_{\text{inv}}(\pi^+\pi^-)} = -\frac{1}{\pi} 2M_{\rho} \text{Im} \frac{1}{M_{\text{inv}}^2 - M_{\rho}^2 + i M_{\rho} \Gamma_{\rho}(M_{\text{inv}})} \Gamma_{\bar{B}^0 \to J/\psi \rho^0}, \tag{19}
\]

where

\[
\Gamma_{\rho}(M_{\text{inv}}) = \Gamma_{\rho} \left( \frac{p_{\pi}^{\text{off}}}{p_{\pi}^{\text{on}}} \right)^3, \\
p_{\pi}^{\text{off}} = \frac{\lambda^{1/2}(M_{\text{inv}}^2, m_{\pi}^2, m_{\rho}^2)}{2M_{\text{inv}}} \theta(M_{\text{inv}} - 2m_{\pi}), \\
p_{\pi}^{\text{on}} = \frac{\lambda^{1/2}(M_{\rho}^2, m_{\pi}^2, m_{\rho}^2)}{2M_{\rho}}. \tag{20}
\]

For the case of the \( \bar{B}^0 \to J/\psi K^{*0} \) (\( K^{*0} \to \pi^+ K^- \)) we have

\[
\frac{d\Gamma_{\bar{B}^0 \to J/\psi K^{*0}; K^{*0} \to \pi^+ K^-}}{dM_{\text{inv}}(\pi^+K^-)} = -\frac{2}{\pi} \frac{2M_{K^*}}{3} \text{Im} \frac{1}{M_{\text{inv}}^2 - M_{K^*}^2 + i M_{K^*} \Gamma_{K^*}(M_{\text{inv}})} \Gamma_{\bar{B}^0 \to J/\psi K^{*0}}, \tag{21}
\]

with

\[
\Gamma_{K^*}(M_{\text{inv}}) = \Gamma_{K^*} \left( \frac{p_{\pi}^{\text{off}}}{p_{\pi}^{\text{on}}} \right)^3, \\
p_{\pi}^{\text{off}} = \frac{\lambda^{1/2}(M_{\text{inv}}^2, m_{\pi}^2, m_{K^*}^2)}{2M_{\text{inv}}} \theta(M_{\text{inv}} - m_{\pi} - m_{K}), \\
p_{\pi}^{\text{on}} = \frac{\lambda^{1/2}(M_{K^*}^2, m_{\pi}^2, m_{K^*}^2)}{2M_{K^*}}. \tag{22}
\]

and similarly for \( \bar{B}^0_s \to J/\psi K^{*0}; K^{*0} \to \pi^- K^+ \).

In Eqs. (19) and (21) we have taken into account that \( \rho^0 \) decays only in \( \pi^+\pi^- \), while \( K^{*0} \) decays into \( \pi^+ K^-, \pi^0 K^0 \) with weights \( \frac{2}{3} \) and \( \frac{1}{3} \) respectively.
IV. RESULTS

In Fig. 4 we show our results for $\bar{B}_s^0 \to J/\psi \pi^+ \pi^-$ decay. $\tilde{V}_P$ has been taken equal to 1 in this arbitrary normalization. The factor $\tilde{V}_P'$ of Eqs. (12) has then been adjusted to get the ratio of Eq. (13). One can see a clear signal for $\bar{B}_s^0 \to J/\psi f_0(980), f_0(980) \to \pi^+ \pi^-$. It is also clear that there is no appreciable signal for $f_0(500)$ production as observed in the experiment [6]. This is a clean case since the $q\bar{q}$ produced was $s\bar{s}$ which has $I = 0$ and there is no $\rho^0$ production.

In Fig. 5 we show our predictions for $f_0(500), f_0(980)$ and $\rho^0$ production in $\bar{B}^0 \to J/\psi \pi^+ \pi^-$, with the same normalization as in Fig. 4.

The relative strengths and the shapes of the $f_0(500)$ and $\rho$ distributions are remarkably similar to those found in the partial wave analysis of [32]. However, our $f_0(500)$ has a somewhat different shape since in the analysis of [32], like in many experimental papers, a Breit-Wigner shape for the $f_0(500)$ is assumed which is different to what the $\pi\pi$ scattering and the other production reactions demand [39, 40].

In Fig. 6 we show the results for the Cabbibo allowed $\bar{B}^0 \to J/\psi \pi^+ K^-$, superposing the contribution of the $\kappa$ and $\bar{K}^{*0}$ contributions and in Fig. 7 the results for the Cabbibo suppressed $\bar{B}_s^0 \to J/\psi \pi^- K^+$, with the contributions of $\kappa$ and $K^{*0}$.

The narrowness of the $K^*$ relative to the $\rho$, makes the wide signal of the scalar $\kappa$ to show clearly in regions where the $K^{*0}$ strength is already suppressed. While no explicit mention of the $\kappa$ resonance is done in these $B$ decays, in some analyses a background is taken that resembles very much the $\kappa$ contribution that we have in Fig. 6 [41]. The $\kappa(800)$ appears naturally in chiral unitary theory of $\pi K$ and coupled channel scattering as a broad resonance around 800 MeV, similar to the $f_0(500)$ but with strangeness [12]. In $D$ decays, concretely in the $D^+ \to K^- \pi^+ \pi^+$ decay, it is studied with attention and the links to chiral dynamics are stressed [42, 43]. With the tools of partial wave analysis developed in [32] it would be interesting to give attention to this $S$-wave resonance in future analysis.
FIG. 5: $\pi^+\pi^-$ invariant mass distributions for the $\bar{B}^0 \to J/\psi\pi^+\pi^-$ ($S$-wave) (solid line) and $\bar{B}^0 \to J/\psi\rho$, $\rho \to \pi^+\pi^-$ ($P$-wave) decays, with arbitrary normalization.

V. CONCLUSION

In this paper we have addressed the problem of the $\bar{B}^0$ and $\bar{B}^0_s$ decays into $J/\psi f_0(980)$, $J/\psi f_0(500)$ and $J/\psi \kappa(800)$. In addition we have also studied the decay of these $B$ states into $J/\psi$ and a vector meson, $\rho$, $\omega$, $\phi$, $K^{*0}$, $\bar{K}^{*0}$. We isolate the dominant mechanism for the weak decay of the $B$ meson, going to $J/\psi$ and a $q\bar{q}$ pair. This mechanism already allows us to relate the different vector decays, $J/\psi$ and $\rho$, $\omega$, $\phi$, $K^{*0}$, $\bar{K}^{*0}$, with a good agreement with experiment for the four predictions that we can make. The production of the scalar
FIG. 7: $\pi^-K^+$ invariant mass distributions for the $\bar{B}_s^0 \to J/\psi K^{*0}$, $K^{*0} \to \pi^-K^+$ (solid line) and $\bar{B}_s^0 \to J/\psi\kappa$, $\kappa \to \pi^-K^+$ (dashed line), with arbitrary normalization.

mesons is more subtle since it requires the hadronization of the $q\bar{q}$ pair into a pair of mesons. We implement this step and after that the pair of mesons are allowed to interact with their coupled channels, and this interaction generates the low lying scalar resonances, $f_0(500)$, $f_0(980)$, and $\kappa(800)$. By using the experimental rate of $J/\psi f_0(980)$ production from the $\bar{B}_s^0$ decay versus the one to $J/\psi \phi$, we can convert all the ratios of rates obtained into absolute numbers. We compare the results with experiment and find good agreement with experiment for the different observables. In particular, the ratio of $\rho$ production to $f_0(500)$ production in $\bar{B}^0$ decay is in fair agreement with the results of a recent partial wave analysis of data. We also make predictions for $J/\psi \kappa$ production versus $J/\psi K^{0s}$ and $J/\psi K^{0s}$ that can be tested in further partial wave analysis of these decays.

In the discussion about the nature of hadrons, in which the vector mesons stand as largely $q\bar{q}$ states while the low lying scalar mesons are rather dynamically generated states from the meson meson interaction, we have shown that the $B$ decays investigated here greatly support this picture. We studied together the two decay modes into $J/\psi$ scalar and $J/\psi$ vector from this perspective and we obtained a remarkable agreement with experimental results which range in several orders of magnitude.

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