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A real option analysis for stochastic disease control and vaccine stockpile policy: An application to H1N1 in Korea

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ABSTRACT

Vaccination is mostly used for controlling the diffusion of an infectious disease. This paper attempts to bridge a gap between economic model and epidemiological model to analyze the optimal vaccination strategy when the diffusion of pandemic disease follows a stochastic process. Impulsive vaccination is considered as an effective option to control an infectious disease. A real option model under stochastic Susceptible-Infected-Susceptible (SIS) environment is developed to examine the optimal vaccination threshold when the social costs and benefits of vaccination efforts are considered. A numerical illustration is provided for the case of H1N1 in Korea to show the herd immunity level as a policy rule to suppress epidemic. Policy implications are discussed regarding the vaccine stockpile as a countermeasure to epidemic diffusion.

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1. Introduction

Vaccination is one of the most popular methods for controlling contagious diseases such as H1N1 influenza. As well known, the primary objective of vaccination policy is to minimize the social impacts of pandemic by protecting susceptible individuals from possible infection. In this regard, it is important to establish proper vaccination strategy to prevent the diffusion of a contagious disease at the outset. The decision on the scale of vaccination and the required level of vaccine stockpile is a critical issue, particularly when the disease diffusion process is so fast that immediate vaccination would be only the most viable and effective measure to prevent pandemic.

Numerous models for infectious diseases have been developed to lead to a better understanding of how vaccination programs affect the eradication of the disease, particularly since the pathbreaking works by Anderson and May (1982). However, more originally, mathematical models of infectious diseases can be traced back to 1776 when Daniel Bernoulli used a smallpox model to discuss effectiveness of mass vaccination (inoculation). His basic assertion in building mathematical model is well stated in his remark, “I simply wish that, in a matter which so closely concerns the wellbeing of the human race, no decision shall be made without all the knowledge which a little analysis and calculation can provide.” His fundamental question in regards to the smallpox problem rampant in his era is whether the general population needs to be vaccinated against smallpox when there is a fatal risk of such inoculation measures. Afterwards, many pivotal research including Kermack and McKendrick (1927) have contributed to establish a field of mathematical epidemiology and their works are comprehensively summarized in Anderson and May (1982).

For epidemiological models, however, economic considerations are largely ignored in finding control methods against an infectious disease. The conventional epidemiological approach measures the outputs of disease control, for example, by the numbers of infections/deaths and hospitalizations but without economic considerations on possible benefits and costs. Notably, economic model provides a useful assessment framework for cost–benefit of public health policy in case of disease outbreak. This paper is an attempt to incorporate such economic aspects into epidemiological model in order to reconcile the aforementioned gap between the distinct two fields.

There have been several efforts to find optimal control methods against an infectious disease in a framework of economic optimization model. One of the seminal papers in economics literature regarding the disease control is Laxminarayan and Brown (2001) who consider antibiotics as a natural resource and show that models of resource extraction may be helpful in understanding how to manage the effectiveness of antibiotics. Horan and Wolf (2004) use a two-state linear control model to examine the socially optimal management of disease in a valuable wildlife population and examine the case of bovine tuberculosis among Michigan white-tailed deer.

This paper develops an economic model coupled with an epidemiological approach of stochastic Susceptible-Infected-Susceptible (SIS) model which allows a dynamic process with interactions between the infected and the susceptible groups (Kermack and McKendrick 1927). There are a few papers providing economic models coupled with the epidemiological SIS model. Goldman and Lightwood (1996) study the effect of treatment when the disease has a pattern of susceptibility, infection, and recovery under the framework of SIS model. Similar types of SIS model are presented in Gersovitz and Hammer (2004) and Francis (2004) who analyze social planner’s problems for preventing the spread of disease and identifying optimal subsidy for...
flu vaccination. More recently, Anderson et al. (2012) use the standard SIS model to solve an optimization problem for allocating limited budgets in an effort to minimize the discounted social cost of infection. Similar to them, this paper adopts a SIS model since it allows dynamic processes with interactions between the infected and susceptible population.

Unlike them, however, a stochastic nature of disease diffusion is incorporated into a SIS model in this paper. In addition, the paper focuses on the decision on the optimal level of vaccine stockpile based on an economic-epidemiological model. More specifically, the vaccine stockpile model is developed from the perspectives of a real option, since, when the vaccination is once carried out for the substantial number of susceptible group, it is largely irreversible, thereby creating option values for the vaccination particularly when the infectious disease follows a stochastic diffusion process. Real option approach enables us to capture the stochastic nature of disease diffusion, irreversible vaccination costs and irreversible damage costs from the disease outbreak in a unified framework (Dixit and Pindyck, 1994). For example, a real option model to optimally manage stochastic pest population are presented in Saphores (2000) who proves the potentiality of biological resource economics models in studying disease control under stochastic environment. Hsu and Schwartz (2008) develop a real option model to study vaccine investment problem to treat the disease and they examine the effects of various types of research incentive contracts on the vaccine investment. In contrast with those papers which do not study epidemiological aspects of disease diffusion, this paper attempts to combine explicitly the study of the disease control and vaccine stockpile.

In particular, the model is presented to be compatible with impulsive vaccination strategy which is applied to a constant fraction of susceptible population (Gakkhar and Negi, 2008). Vaccine application in response to the outbreak of H1N1 is a typical example of impulsive vaccination in which a fraction of susceptible group (e.g., agents working in hospital, school and military sector) is directed to be vaccinated. Hence, the paper develops a model for the optimal impulsive vaccination taking into account economic benefits and costs of vaccine application. It must be noted that the presented model is not suitable for the continuous vaccinations (age-structured vaccinations) such as in controlling smallpox, rubella and measles.

The model comparison is based on the herd immunities which concept will be in detail discussed in the next section. The herd immunity is a popular measure in epidemiological model when controlling a disease and to describe a form of threshold where the vaccination of a significant portion of a population provides immunity protection in society. Conventional herd immunity in epidemiological model and herd immunity based on an economic-epidemiological model are compared to show the differences of vaccine stockpile levels between them. In addition to the development of the herd immunity, the presented economic-epidemiological model allows us to introduce other interesting measure, i.e., the vaccination threshold which cannot be derived from the conventional epidemiological approach. The vaccination threshold is the point at which it is optimal to implement the impulsive vaccination while taking into account the vaccination costs and benefits. After developing an economic-epidemiological model in the next section, an empirical illustration is provided using the H1N1 case in Korea, since H1N1 influenza represents a typical example where the impulsive vaccination is concentrated in the short time windows to prevent pandemic.

The paper is structured as follows: a stochastic SIS and real option model is presented in Section 2 to analyze the optimal vaccinations strategies in terms of the herd immunities and vaccination thresholds. Section 3 provides an illustration focusing on the H1N1 case in Korea. Conclusion is presented in Section 4.

2. Model

This section presents an optimal vaccination model which is augmented with epidemiological characteristics. For doing so, more specifically, the concept of herd immunity threshold (or critical immunization value) is considered for the vaccination policy rule which is one of the most widely applied rules in epidemiology for the task of controlling contagious disease. As the herd immunity represents the smallest proportion of a population that needs to be vaccinated to prevent pandemic, it could be used as a basis for the level of vaccine stockpiles.

The presented model is based on the SIS model that is also called as a compartmental model in epidemiology literature. The SIS model is a mathematical model to analyze an infectious disease and first discussed by Kermack and McKendrick (1927). The other popular model is the SIR (Susceptible-Infected-Recovered) model which is not considered here because the SIR model is suitable for the disease control when vaccination provides (pseudo) permanent immunity effects.

For simplicity, the total population \( N(t) \) is normalized to unity as in many classical SIS models (Aadland et al., 2013; Goldman and Lightwood, 1996; Rowthorn and Toxvaerd, 2012). It implies that the population of \([0,1]\) consists of a continuum of infinitely lived agents who can be in susceptible or infected state. The normalized susceptible and infected groups are denoted, respectively, by \( S(t) \) and \( I(t) \) so that \( S(t) + I(t) = 1 \) (Kang and Castillo-Chavez, 2014). The evolution of stochastic susceptible and infected groups are specified as follows:

\[
dS = -\beta S I dt + \gamma I dt + \sigma_S dW_S,
\]

\[
dl = \beta S I dt - \gamma I dt - \mu I dt + \sigma_I dW_I,
\]

where \( \beta \) represents a transmission rate (a force of infection) at which susceptible agents become infected. The incidence of infection, \( \beta S I \), means that initially susceptible individuals may become infected depending on the infective interactions between the susceptible and infected groups. The recovery rate from the disease is \( \gamma \) and the mortality rate is \( \mu \). Note that the transmission rate is associated with the number of susceptible whereas the recovered rate is associated with the number of the infected. For each \( i = S \) and \( I \), the increment of the standard Wiener process \( W_i \) is characterized by \( E(d W_i) = 0 \), \( \text{Var}(d W_i) = dt \) and the associated volatility parameter \( \sigma_i \). By means of \( S = 1 - I \), Eq. (2) is transformed into

\[
dl = l(\alpha - \beta S) dt + \sigma_I dW
\]

where \( \alpha = \beta - \gamma - \mu \), measuring the force of infection moving to the infected population. It is interesting to see that Eq. (3) is a stochastic version of logistic function which is extensively used in biological economics. For example, when there is no uncertainty with \( \sigma = 0 \), Eq. (3) is degenerated into a standard deterministic logistic function \( dI/dt = acl(1 - I/K) \) where \( K = \alpha/\sigma \). The carrying capacity denoted as \( K \) defines the maximum viable equilibrium population size that the environment can sustain (Clark, 1990). This maximum property is also confirmed by studying the solution to the logistic function,

\[
l(t) = \frac{l(0)Ke^{at}}{K + l(0)e^{at} - 1}
\]

where \( \lim_{t \to -\infty} l(t) = K \).

In epidemiological literature, there is a similarly interesting measure which is useful to examine equilibrium property of time-varying size of infected group. It is called the basic reproduction number that is defined as \( R_0 = \beta(\gamma + m) \) and it represents the average number of secondary infections being produced by the primary infection in the susceptible population. For instance, an infectious disease with the value of \( R_0 = 2 \) would produce two new secondary cases from the primary infection. It is immediate to see that the solution to the logistic function of \( dI/dt = acl(1 - I/K) \) asymptotically converges to 0 as \( t \to \infty \) if \( R_0 < 1 \) and approaches \( 1 - 1/K \) if \( R_0 > 1 \) (Robeva et al., 2008).

\footnote{Time \( t \) is suppressed for notational convenience unless it is required for clarity.}
Since $R_0$ helps us determine the prevalence scale of a disease over the course of its infectious period, we are able to identify the appropriate rate of vaccinated population in order to prevent pandemics. For this purpose, conventional epidemiological model defines the herd immunity threshold (HIT) as $H_0 = 1 - 1 / R_0$ to indicate the smallest proportion of a population that needs to be immune in order for an infectious disease to become stable or die out. For the case of smallpox which average estimates of $R_0$ is 6, the HIT of $H_0$ is 0.83 which means that 83% of susceptible population needs to be vaccinated to achieve herd immunity. At this moment, it is important to understand that the herd immunity is defined as the process when a substantial percentage of the population (or the herd) is vaccinated so as to reduce the risk of infection for the rest of the community. The information on HIT is in particular critical in establishing the prudential vaccine stockpile policy so that the mass vaccination can be applied at any necessary time to curb progressive development of the disease.

In the rest following part of this section, the HIT of H1N1 influenza is derived from an economic-epidemiology model and it is compared with the conventional HIT($H_0$) to discuss policy implications on vaccine stockpiles. In addition, one advantage of economic-epidemiological model is that it allows us to explicitly identify the vaccination threshold, which is not feasible in conventional epidemiological approach, from where it is optimal to carry out the impulsive vaccination.

Denote $V(I)$ as an option value for impulsive mass vaccination when $\sigma \neq 0$ and $I$ follows a stochastic process (Eq. (3)). When the infected is almost zero, it is absolutely less likely to exercise the vaccination option. Hence, the appropriate boundary condition is $\lim_{t \to 0} V(I) = 0$. Note that the impulsive vaccination greatly simplifies the vaccination problem as a stationary problem by making the optimal exercise threshold $I^*$ time-independent. Then, prior to the exercise of vaccination option, $V(I)$ must satisfy the following Eq. (5) from the Ito’s lemma:

$$\frac{1}{2} \sigma^2 I^2 V''(I) + \alpha d\left(1 - \frac{\beta}{\alpha}\right) V'(I) - \rho V(I) = 0$$

where $\rho$ is the discount rate. The complex form of Eq. (5) is generally non-tractable to be explicitly solved. However, the general solution to homogeneous Eq. (5) can be found through the function transformation as shown in Sarkar (2003) and Dias and Shackleton (2010). Let $Y(I) = V(I) / f(I)$ where $\sigma$ is a constant to satisfy a characteristic equation that is introduced shortly. Then, from $V(I) = Y(I)f(I)$, we obtain $V'(I) = Y'(I)f(I) + aY - bYf(I)$ and $V''(I) = a(a - 1)Y^2 + 2aYf(I) - Yf(I) + f''(I)$. That $\frac{1}{2} \sigma^2 I^2 V''(I)$ is substituted into Eq. (5). This process yields

$$\frac{1}{2} \sigma^2 I^2 \left[a(a-1)Y'' - 2aYf''Y'' + f''\right] + \alpha d\left(1 - \frac{\beta}{\alpha}\right) \left(Y' - \rho Y\right) = 0.$$  

Eq. (6) is rearranged into

$$\frac{1}{2} \sigma^2 I^2 \left[2aYf''Y'' + \rho Y + aYf' - f\right] = 0.$$  

Eq. (7) must hold for any value of $I$, so the terms in the brackets must equal zero. From the first bracketed terms in the LHS, we obtain the characteristic equation $a(a - 1)Y'' + 2aYf'' - f = 0$. This characteristic equation is solved with the root $\alpha = F(a, \sigma) + \sqrt{F(a, \sigma) + 2\rho / \sigma^2}$ where $F(a, \sigma) = 0.5 - \alpha \sigma^2$. The second bracketed terms in Eq. (7) is rearranged as

$$Y' + 2\left(\frac{\alpha - \alpha \beta}{\sigma^2} + a\right)Y - 2\alpha a Y'' = 0.$$  

Using the change of variables, let $X = \beta I / \sigma^2$. Then, from $Y(I) = y(x)(\sigma^2/\beta) = y(x)$, we have $y'(x) = Y'(I)(\sigma^2/\beta)$ and $y''(x) = Y''(I)(\sigma^2/\beta)^2$ that are substituted into Eq. (8) to obtain

$$\frac{x\alpha}{\sigma^2}y''(x) + \frac{\beta}{\sigma^2} \left[2a + 2a - x\right]y' - 2\alpha a \beta \sigma^2 y(x) = 0.$$  

Now, dividing Eq. (9) with $\sigma^2/\beta$, we have

$$xy' + (b - x)y - 2ay = 0$$

where $b = 2a(\alpha^2 + 2/a)$. Note that the second-order differential Eq. (10) is the so-called Kummer’s equation as presented in Abramowitz and Stegun (1972, Eq. 13.1.11). Since there are alternative solutions to Kummer’s equation, it is necessary to choose the ones that are simpler and easier to apply boundary conditions (Dias and Shackleton, 2010; Abramowitz and Stegun, 1972). This leads to the well-known solution

$$y(x) = A\Phi(2a, b; x) + B(2a; b; x)$$

where

$$\Phi(2a, b; x) = 1 + \sum_{i=1}^{n} \frac{(2a)_{i} x^i}{b^i i!},$$

and

$$U(2a; b, x) = \Gamma(1-b) \Phi(2a, b; x) + \Gamma(1-b) \frac{(b-1)}{(b-1)!} x^{b-1} \Phi(2a-b+1, 2-b, x),$$

are the confluent hypergeometric functions. $(2a, (b))$, denote Pochhammer symbols for the rising factorial and $\Gamma(z)$ is the Gamma function, $\Gamma(z) = e^{-z} \Gamma(z)$ d s. To satisfy $\lim_{t \to 0} V(I) = 0$, we limit our focus on a case when the constant term $B = 0$ in Eq. (11). By reinstating the original variables from the transformed variables (Dias and Shackleton, 2010), we finally obtain

$$V(I) = A\Phi(I)^{p}$$

where $\Phi(I) = 1 + \sum_{i=1}^{n} \frac{(2a)_{i} x^i}{b^i i!}$ and $A$ is the option constant term that will be determined using the boundary conditions that are presented below.

Upon the exercise of impulsive vaccination, the instantaneous net benefit is realized that is the vaccination benefit less the variable vaccination cost:

$$\omega_\nu (1-I) - c\nu^{2}(1-I)N,$$

given the vaccine effective rate $\nu$, unit benefit $\nu$, vaccination rate $\nu$, and the unit cost $c$ of the vaccination. The first and second terms in Eq. (13) represent the vaccination benefit less variable vaccination costs, respectively. By the definition of impulsive vaccination which is intensively applied during the short time period, the social net benefit is instantly realized immediately after the vaccination. By inserting the optimal vaccination rate, $\nu^* = \omega / 2c$, from the first order condition of net benefit maximization, the instantaneous payoff (Eq. (13)) is changed to $(1-I)\nu^2 g^2 2N / 4c$. Then, the impulsive vaccination is carried out immediately at the optimal threshold $I^*$ that is identified by the following boundary conditions:

$$A\Phi(1)^{p} = (1-I)\nu^2 g^2 2N / 4c,$$

$$aA\Phi(1)^{p-1} + A\Phi^{'}(I) = -4\nu^2 g^2 N / 4c.$$

}
Eq. (14) represents the so-called value matching condition, implying the equivalence of vaccination option value and the net payoff of vaccination. When there exists the waiting value measured by $Adv(t)I^2 - (1 - I)w^2g^2N/4c$ in Eq. (14), it is better to postpone the impulsive vaccination. Eq. (15) is the smooth pasting condition which is intuitively about the rule of marginal benefit and cost. Solving Eqs. (14) and (15), we get $a(1 - I) + \Phi'(I)(I - I^2) / \Phi(I) + I = 0$ that is used to numerically identify $I^*$ due to the non-availability of the closed form solution. Then, the option constant term $A$ is given by

$$A = \frac{(I - a - I^*)^2 Nw^2g^2}{4c\Phi(I)}$$

which fully characterizes the option value $V(I) = Adv(I)I^2$.

Fig. 1. The infection rates for H1N1 in Korea (Aug. 2009–Jan. 2010).

Fig. 2. Contour map for $\alpha$ and $t$. 

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intrinsinc growth rate, \( \alpha \), and the carrying capacity, \( K \), explained in the previous section. From this, \( \beta \) is recovered by setting \( \beta = \alpha/K \) A non-linear least square method is adopted to estimate parameters in Eq. (4) after applying initial values that are approximately verified in an inflection point in Fig. 1. The contour map illustrated in Fig. 2 confirms an existence of minimum value for the least square errors at around \( \alpha = 0.4 \) and its corresponding \( t \). Using them as initial values, the estimation of non-linear least square provides \( \alpha = 0.496 \) and \( \beta = 0.4799 \). The estimate of volatility parameter is \( \sigma = 0.0683 \) that is calculated from the log-normal distribution of residual values of the least squares. The fitted curve of logistic function is depicted in Fig. 1 to be compared with the actual values. The reported mortality rate in Korea is \( m = 0.005 \) (KCDC, 2010). The recovery rate is assumed to be \( \gamma = 0.3149 \) based on \( R_0 = \beta/(\gamma + m) \) so that it could be consistent with the mid-point of \( R_0 \in [1.2, 2.0] \) as reported in Suh et al. (2010). The vaccine effectiveness rate is assumed to be \( \omega = 0.6 \) following Suh et al. (2010) and Song et al. (2011). The unit benefit from the vaccination is calculated by relying on KCDC (2010) that provides the costs of severe inpatients cares for adults and children. ³ By taking the weighted average for inpatient costs, we finally obtain \( g = 1.9 \) and \( c = 1.2 \). The annual discount rate is assumed to be 0.1, which means the weekly discount rate \( \rho = 0.0019 \).

From the epidemiological model where \( R_0 = 1.5 \), the corresponding HIt, \( H_0 \), is 0.3334, implying that the H1N1 vaccine needs to be sufficiently stockpiled to vaccinate at least 33.34% of the susceptible population. For the economic-epidemiological model, numerical analysis is adopted to identify \( t' \) which satisfies \( \alpha(1-t) + \beta/(\gamma + m)(1-t') = 0 \). Based on the benchmark set of parameter values \( (\beta = 0.4799, \sigma = 0.0683, \gamma = 0.3149, \omega = 0.6, m = 0.005, \rho = 0.1) \), we obtain \( t' = 0.0347 \), indicating that it is optimal to commence an compulsory vaccination when the infected population reaches 3.47%. The value of \( t' \) is used to calculate the economic-epidemiological HIT, \( H_0 \). The resulting value of \( H_0 \) is 0.4179 which is approximately 23% higher than the conventional \( H_0 \). Notably, the economic-epidemiological model calls for more protective measure, approximately 42% of HIT, which is substantially higher than 33% of HIT in conventional epidemiology model approach.⁶

| % change from the benchmark² | 10%  | 20%  | 30%  |
|-----------------------------|-----|-----|-----|
| Vaccine benefit (g)         | 43.25 (28%) | 44.71 (34%) | 46.26 (38%) |
| Vaccine cost (c)            | 39.00 (17%) | 36.84 (10%) | 34.78 (4.3%) |
| Recovery rate (γ)           | 36.41 (36%) | 31.52 (56%) | 28.40 (105%) |
| Mortality rate (m)          | 41.74 (25%) | 41.70 (26%) | 41.65 (26%) |
| Discount rate (µ)           | 41.46 (24%) | 31.44 (−6%) | 28.37 (−15%) |

(²) indicates the percentage difference between \( H_0 \) and \( H_0 \)³ (g, c, γ, m, ρ) = (1.9, 1.2, 0.3149, 0.005, 0.1)

As mentioned above, the model building process could be simplified due to the property of impulsive vaccination strategy which realizes the net benefit instantaneously at the moment of vaccination. Hence, our vaccination policy is solved using relatively a simple version of optimal-stopping problem which requires only the smooth-pasting condition. However, if vaccination is carried out, for example, in a continuous framework of age-structured vaccination, the problem needs to be developed so that it may require a super-contact condition (Dumas, 1990).

Despite of simplicity based on the optimal-stopping problem, closed analytical solution is not available because of the nonlinearity of the confluent hypergeometric function \( \psi(t) \). Hence, in the next section, numerical analysis is employed to identify \( t' \). Then, the optimal herd immunity threshold in economic-epidemiological model is calculated using the formula \( H_0 = \psi(1 - t') \) to be compared with the traditional herd immunity, \( H_0 = 1 - 1/R_0 \) defined in epidemiological model. The information regarding \( H_0 \) provides an important insight on the level of vaccine stockpiles so that the vaccination could be efficiently implemented without time lapse in the middle of progressive development of disease.

3. Empirical application to H1N1 in Korea

The case of contagious H1N1 (influenza A) in Korea is considered for an empirical application of the model. In contrast with a zero-incidence of 2001 SARS in Korea, the outbreak of 2009 H1N1 was a poigniant experience because of its rapid diffusion and numerous deaths that could be otherwise prevented if there were sufficiently stockpiled vaccines from the very beginning. The exponential growth of the reported infection cases has raised issues on the scale of impulsive vaccination and the level of vaccine stockpile as a precautionary measure. For this study, the data on the reported incidents of H1N1 infection was obtained from the Korea CDC (Center for Disease Control). Real time reverse transcription polymerase chain reaction (RT-PCR) was used to diagnose 2009 H1N1 influenza virus infection for the subjects of visiting outpatients. The death number per week was 1 up to 5 at the initial outbreak stage but was rapidly increased up to 20 cases later. According to Kim et al. (2013b), the socioeconomic cost of the 2009 pandemic H1N1 in Korea is US $1.09 billion (0.14% of the national GDP). This includes the direct medical costs (29.6% of total costs), direct non-medical costs (9.7% of total costs) and indirect costs (60.8% of total costs).

Influenza like H1N1 is pertinent to be analyzed under SIS framework since it may not confer long immunity. For example, three reinfection cases of the H1N1 2009 were reported in Korea during the time period covered for this study.² Fig. 1 shows the trend of infection rates during August 2009 and January 2010 by normalizing the infected population with the total population.¹ It is clear to see that the diffusion of infection follows a logistic shape as theoretically presupposed in the previous section.⁴ The estimation of logistic function yields the values of an

² Three subjects who have recovered from the contagious disease in September were again diagnosed with positive for the flu (http://synapse.koreamed.org). Similar reinfection cases were reported in Chile and Thailand (http://www.ncbi.nlm.nih.gov).
³ As of 2009, the population in South Korea is 49 million.
⁴ Similar types of logistic shape in disease diffusion process are not uncommon in other infectious disease such as SARS and Mers (Hsieh et al., 2004).
effectiveness $w$ and the transmission rate $\beta$, although the level of $H^*_0$ may be gradually lowered at substantially high transmission rates. The last part of result comes from that, among other things being equal, the higher transmission rate means the decrease of vaccines benefits. This result would come different when the social cost of pandemic grows exponentially as the number of infected increases.

Lastly, Fig. 5 provides a sensitivity analysis of $H^*_0$ with respect to the recovery rate $\gamma$ and the volatility rate $\sigma$. $H^*_0$ becomes less sensitive to $\gamma$ as the volatility rate increases, which implies an existence of robust herd immunity level when uncertainty of disease diffusion is sufficiently large. One thing to note from the above analysis is that the effects of vaccination cost and uncertainty are negatively related to the level of optimal HIT which implications are intuitively consistent with real option approach.

To summarize the above results, $H^*_0$ of economic-epidemiological model is more likely higher than $H_0$ of conventional epidemiological model as long as the discount rate is within the range of conceivable levels (for example, no larger than 10% for the case of the 2009 H1N1 in Korea). This relative ordering of $H_0$ and $H^*_0$ may hold in a robust manner, since the assumption of non-conceivably high interest rate is not particularly pertinent when considering adverse long-run effects of social costs of disease. The results presented here warrant cautious interpretations, mainly because some parameter values are speculated due to the limited information on the diffusion of H1N1 in Korea. However, the numerical illustrations and comparative statics are valid to highlight distinct differences between two models, epidemiological and economic-epidemiological models.
4. Conclusion

There are growing interests in the economics of disease control in recent years. Large body of economic literature on the vaccination policy analyzes the optimality of vaccination management in terms of benefits and costs of controlling an infectious disease. Various modeling can be applied to decisions concerning vaccination management problems, ranging from sophisticated models that consider spatial, dynamic, stochastic, and other aspects of economic and biological systems, to relatively simple decision rules using minimal information.

This paper analyzed a policy rule for impulsive vaccination when the diffusion of an infectious disease follows a stochastic process. A real option model was developed to be coupled with an epidemiological SIS model. Since the vaccination is largely irreversible, and an infectious disease follows a stochastic diffusion process, the decision making on the impulsive vaccination could be analyzed in a real option framework (Dixit and Pindyck, 1994). The differences between the economic-epidemiological model and epidemiological model were examined under the framework of the so-called herd immunity to protect massive susceptible populations from the infectious disease. A numerical illustration of H1N1 in Korea was presented to prove an applicability of the developed economic-epidemiological model. As reflected in an adage, an ounce of prevention is worth a pound of cure, the resulting policy rule suggested a more aggressive and precautionary strategy on the scale of vaccination and vaccine stockpile compared to the traditional epidemiological model.

It is critically important to build appropriate database to identify model parameters and to enhance model credibility. This job is crucial not only for improving current understanding on infectious disease modeling, but also for establishing estatue initial protective measures from the beginning of disease outbreak. One notable effort to integrate monitoring system for tracing an infectious disease is Kim et al. (2013a) which model pathological interactions between agents. However, up to this point of time, no micro-level data on pathologies of individual infections is available to identify exact diffusion process for 2009 H1N1 in Korea. Hence, a heuristic approach was adopted in the paper for the numerical illustration that leaves a caveat for interpretations.

The model presented in this paper can be suitably modified to analyze different epidemic or infectious diseases other than H1N1. Notable candidates of target disease modeling include H9N1 and MERS. As an extension of the model, SIR structure that stratifies the population into susceptible, infected and recovered states would be similarly adapted to analyze disease control problem. SIR type of model is particularly relevant when there exist long-lasting immune effects after recovery from the infection or vaccination as in measles and rubella. Another promising approach for further development is to consider network effects associated with the diffusion of a contagious disease among agents (Barrat et al., 2008) which is not unfortunately feasible for the study in this paper due to the data limitation on Korea H1N1.

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