Bistability in a magnetic and nonmagnetic double-quantum-well structure mediated by the magnetic phase transition

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Abstract

The hole distribution in a double quantum well (QW) structure consisting of a magnetic and a nonmagnetic semiconductor QW is investigated as a function of temperature, the energy shift between the QWs, and other relevant parameters. When the itinerant holes mediate the ferromagnetic ordering, it is shown that a bistable state can be formed through hole redistribution, resulting in a significant change in the properties of the constituting magnetic QW (i.e., the paramagnetic-ferromagnetic transition). The model calculation also indicates a large window in the system parameter space where the bistability is possible. Hence, this structure could form the basis of a stable memory element that may be scaled down to a few hole regime.

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Spintronics approaches the brink of applications in the micro- and nano-electronics. As soon as the semiconductor compounds doped with magnetic ions reveal room temperature ferromagnetism, the creation of competitive electronic devices will become the barest necessity. The study of different structures and their spin effects, which could result in spin devices at lower temperatures, is already a topical problem especially when we take into the account the continued improvement in the critical temperature $T_c$ of the paramagnetic-ferromagnetic (PM-FM) phase transition.

In this Letter, we explore the formation of a bistable state and its device application in a double quantum well (QW) semiconductor structure with respect to the distribution of itinerant holes between the constituting nonmagnetic and magnetic QWs. The calculation illustrates that the interplay between the free carriers and the magnetic ion spins is the key to achieving the bistability where the magnetic QW can switch between the PM and FM phases in a controlled manner.

The structure under investigation is illustrated in Fig. 1. Consider a magnetic QW (MQW) and a nonmagnetic QW (NQW) of widths $L_{wM}$ and $L_{wN}$, respectively, separated by a barrier. The width $L_b$ and the height of the barrier should be large enough to form non-coherent single QW states and yet not too large in order to enable hole redistribution when a gate bias is applied. The total 2D hole concentration $n_h^0$ is assumed to be a constant and the FM-PM transition in the MQW to be mediated by two-dimensional (2D) free holes. If the hole energy in the NQW is lower than that in the MQW, one can expect that the stable state will correspond to hole localization primarily in the NQW with a small leakage in the MQW, which is in a PM phase (see the schematic on the left in Fig. 1). When a proper bias is applied, holes from the NQW can be transferred to the MQW via tunneling, over-barrier injection, etc. As the hole density in the MQW surpasses a certain threshold at a given operating temperature, the layer undergoes the PM-FM transition. When the hole exchange interaction with the FM-ordered ion spins in the MQW is strong, it can reduce the total free energy below that of the initial state with the PM phase even after the bias is switched off. Then, the holes will remain confined in the MQW and the FM state be maintained (the schematic on the right in Fig. 1). When a reverse bias pulse is applied, the holes are drained out of the MQW into the NQW, and the MQW will return to the PM state. Hence, if realized, these two stable states can coexist under the same external condition and the switching between them is mediated by the electrically-controlled PM-FM
phase transition. It is expected that the structure can operate up to a temperature slightly below the saturated maximum of $T_c$, which can reach room temperature or higher in some material systems.

To analyze this problem, typically one would derive the magnetic Hamiltonian $H_m$, which describes the effect of the effective ferromagnetic inter-ion spin-spin interaction in the presence of free holes. Then the calculation of the free energy $F$ of the total system consisting of magnetic ions and free carriers, which occupy the MQW with a concentration of $n_{hM}$ and the NQW with $n_{hN}$ ($n_{hM} + n_{hN} = n_0^h$), leads to the carrier population factor $\eta = n_{hM}/n_0^h$ and $1 - \eta = n_{hN}/n_0^h$ at each QW that minimizes $F = F(\eta)$. The existence of two local minima with respect to $\eta$ demonstrates the bistability of the system under consideration.

Although conceptually correct, this approach faces the difficulty of specifying the mechanisms responsible for FM ordering in the MQW (for details on various mechanisms, refer to those cited in Ref. 4). To circumvent this problem, which is beyond the scope of the present study, we develop a semi-phenomenological approach that utilizes the data extracted from routine experimental measurements of magnetism. Namely, we assume that the magnetic part of the free energy $F_M$ can be expanded with the magnetization $M$ (order parameter) according to Landau theory. This expansion approximates satisfactorily the veritable dependence in the whole temperature interval we are interested in. Assuming the easy magnetization axis is directed along the growth axis of the sample and a magnetic field $\vec{B}$ parallel to $\vec{M}$, the $F_M$ expansion in the most general form reads

$$F_M = -a(T_c - T)M^2 + bM^4 - \frac{1}{2}MB.$$  

It can be shown that the parameters $a$ and $b$ of Landau expansion in Eq. (1) can be expressed in terms of fundamental properties of the magnet: The Curie-Weiss law for the magnetic susceptibility $\chi = C_0/(T - T_c)$ at $T > T_c$ defines $a = 1/4C_0$, while the spontaneous magnetization $M_s = [a(T_c - T)/2b]^{1/2} = M_0\sqrt{1 - T/T_c}$ for $T < T_c$ (that minimizes $F_M$ at $B = 0$) provides $b = aT_c/2M_0^2$. By these relations, all parameters in $F_M$ in Eq. (1) are determined. $T_c$ can be found separately as a function of the magnetic layer carrier population.

Since we are looking for the free hole distribution over the magnetic and nonmagnetic QWs at $B = 0$, the magnetization $M$ in Eq. (1) takes the equilibrium value. Hence,

$$F_M = -\frac{a^2(T_c - T)^2}{4b}, \quad T < T_c; \quad F_M = 0, \quad T > T_c.$$  

(2)
It is important to note that Eq. (2) includes the dependence on the free hole concentration in the MQW via the critical temperature \( T_c = T_c(\eta) \).

The total free energy of the system can be obtained if Eq. (2) is supplemented with the free energy of the hole gas:

\[
F_h = F_{2D}(\eta) + U(\eta) + C(\eta). \tag{3}
\]

The first term in Eq. (3) accounts for the kinetic energy of the hole gas in both QWs, \( U(\eta) \) is the hole potential that is different for the MQW and the NQW, and \( C(\eta) \) is the energy of the Coulomb interaction between the QWs. Assuming the parabolic dispersion law with an effective mass \( m \) for 2D holes and that only the lowest sub-bands of the QWs can be populated by holes with \( L_{wM} = L_{wN} \), one can find the kinetic energy for the hole gas as

\[
F_{2D}(\eta) = k_B T \left\{ \eta f_1 \left( \frac{\varepsilon_0^F}{k_B T} \eta \right) + (1 - \eta) f_1 \left( \frac{\varepsilon_0^F}{k_B T} (1 - \eta) \right) \right\} n_h^0, \tag{4}
\]

where \( k_B \) is the Boltzmann constant, \( \varepsilon_0^F = \pi \hbar^2 n_h^0 / m \) is the Fermi energy of 2D holes with concentration \( n_h^0 \), and

\[
f_1(x) = \ln (e^x - 1) + \frac{1}{x} Li_2 (1 - e^x) \tag{5}
\]

with a polylogarithmic function \( Li_2 (x) = \int_0^x dt \ln (1 - t) / t \). At low temperature \( T \ll \varepsilon_0^F / k_B \), Eq. (4) describes the sum of degenerate carrier energies in both QWs. The low temperature assumption is commonly used in the works on the FM ordering in MQWs. For the present purpose, however, we need to account for the arbitrary relation between \( \varepsilon_0^F \) and \( k_B T \) according to Eq. (4). The contribution of the energy shift \( \Delta U \) between the MQW and the NQW (Fig. 1) is accounted for in the term

\[
U(\eta) = \Delta U \eta n_h^0, \tag{6}
\]

while the Coulomb energy in the strong confinement limit takes the form

\[
C(\eta) = \frac{2\pi e^2}{\epsilon} L_{bn_h^0}^2 \left( \eta - \frac{1}{2} \right)^2, \tag{7}
\]

where \( e \) is the electron charge and \( \epsilon \) the dielectric constant.

Now one can analyze the total free energy \( F = F_M + F_h \) with respect to possible FM phase transitions in the MQW. Considering that the realistic dependence \( T_c = T_c(n_{hM}) \) should be taken from the experiments, we specify our analysis by a model that can be applied to
a typical diluted magnetic semiconductor. Assuming that the total magnetization of the
MQW stems mainly from the magnetic ions and \( n_h^0 \ll n_m L_{wM} \) (\( n_m \) is the 3D magnetic ion
concentration in the MQW), one can easily find the parameters \( a = \frac{3}{\varepsilon_F^0 n/k_B T_c^0} \)
and \( b = 3k_B T_c^0/[8S^3(S + 1)g^4 \mu_B^4 n_m^3] \), where \( g \) is the magnetic ion \( g \)-factor with spin \( S \) and \( \mu_B \) denotes the Bohr magneton.

In order to describe the dependence of \( T_c \) on \( \eta \), we propose an approximation
\[
T_c = T_c^0 \left( 1 - e^{-\alpha \varepsilon_F^0 \eta/k_B T_c^0} \right),
\]
where \( T_c^0 \) is the asymptotic (at a high enough \( n_h M \)) value of the critical temperature and \( \alpha \) is the fitting parameter that adjust the dependence [Eq. (8)] to the experiments. In
the following, we assume \( \alpha = 1 \) since it describes the experimental results satisfactorily.\(^6,7\)

Combining this approximation with Eqs. (2), (4), (6), and (7), we find the trial function in
the form of the free energy per hole normalized by the energy unit \( k_B T_c^0 \)
\[
F = -\frac{3}{8S + 1} \nu \frac{\varepsilon_F^0}{t_c(\eta)} [t_c(\eta) - t]^2 \theta(t_c(\eta) - t)
+t \left\{ \eta f_1 \left( \frac{r \varepsilon_F^0 \eta}{t_c(\eta)} \right) + (1 - \eta) f_1 \left( \frac{r \varepsilon_F^0 (1 - \eta)}{t_c(\eta)} \right) \right\}
+u \eta + w(2 \eta - 1)^2 + F_{ex},
\]
where \( \nu = n_m L_w / n_h^0 \), \( t = T/T_c^0 \), \( t_c(\eta) = T_c/T_c^0 \), \( u = \Delta U/k_B T_c^0 \), \( w = \pi e^2 L_b n_h^0 / 2k_B T_c^0 \), \( r = \varepsilon_F^0 / k_B T_c^0 \), \( t_c(\eta) = 1 - \exp(-r \eta) \), and \( \theta(x) \) is the Heaviside step function. For completeness,
Eq. (9) also includes the exchange energy of free carriers \( F_{ex} \), which can influence the effect of
bistability.\(^6,7\) We performed the calculation of \( F_{ex} \) in a Hartree-Fock approximation following
Ref. [8], where the exchange and the correlation potentials of a 2D gas were found at finite
temperatures. The final result of our calculation has a proper analytical approximation in
terms of the dimensionless parameter \( r \)
\[
F_{ex} \approx -\frac{2^{5/2} e^2 \sqrt{n_h^0}}{3 \sqrt{\pi} e k_B T_c^0} \times
\left\{ \eta^{3/2} \phi \left( \frac{r \varepsilon_F^0 \eta}{t_c(\eta)} \right) + (1 - \eta)^{3/2} \phi \left( \frac{r \varepsilon_F^0 (1 - \eta)}{t_c(\eta)} \right) \right\},
\]
where \( \phi(x) = 1 - e^{-x^2/2 \lambda^2} + x^b / (2^{b+1} x^b) \), \( b = 0.3 + 0.2 \theta(x - 1) \), \( c = 2.2 - 0.2 \theta(x - 1) \).
Note that Eq. (10) takes a form similar to the free energy expression used for analyzing the
spin/charge separation of magnetic semiconductors near a PM-FM phase transition.\(^9\)
For a numerical evaluation, let us assume the following "typical" values for the parameters of the double QW structure: \( m = 0.3m_0 \) (\( m_0 \) is the free electron mass), \( \epsilon = 12.9 \), \( S = 5/2 \), \( L_{wM} = L_{wN} = 10 \) nm, \( L_b = 5 \) nm, \( n^0_h = 10^{12} \) cm\(^{-2} \), \( n_m = 1.3 \times 10^{21} \) cm\(^{-3} \), and \( T_c^0 = 100 \) K.

Figure 2 displays \( F(\eta) \) calculated at three different values of the energy shift \( \Delta U \) between the minima of PM QW and NQW \((u = \Delta U/k_B T_c^0 = 29, 5, -9)\). Clearly, curves 1 and 3 \((u = 29, -9)\) support only one stable state at \( \eta = 0 \) or 1. This means that when the MQW state lies either too high (curve 1) or too low (curve 3) compared to that of the NQW, the holes strongly prefer to be confined in one of the QWs. Even when they are transferred to the other QW through an external bias, the holes will return to the preferred state once the bias is turned off. However, one can realize a structure that has free energy minima at or near both \( \eta = 0 \) (with the MQW in the PM phase) and \( \eta = 1 \) (the MQW in the FM phase) if \( \Delta U \) is properly selected (curve 2). These two states can be stable with respect to small fluctuations under the same external conditions. We also found that the relative effect of \( F_{ex} \) is small compared to the Coulomb energy of the inter-QW carrier interaction \( w(2\eta - 1)^2 \).

This bistability can be achieved in a relatively wide range of \( \Delta U \) and \( T \) as shown in Fig. 3. As expected, the condition for \( \Delta U \) becomes less stringent with a decreasing \( T \). This is because the structure can now operate with a lower \( T_C \), which in turn requires a smaller hole density in the MQW for the PM-FM transition. The highest operating temperature will be somewhat lower than \( T_c^0 \) and is a function of the maximum possible \( n_{hM} \).

It should be emphasized again that the structure is analyzed with respect to the free hole distribution \( \eta \) that minimizes the system free energy. Hence, the height of the local maximum in the free energy (e.g., near \( \eta = 0.8 \) for curve 2 in Fig. 2) does not constitute the energy barrier separating the NQW and the MQW. In fact, details of the barrier layer shown in Fig. 1 is not considered in the present study. Rather, the stability of local minima (thus, the lifetime) depends explicitly on the magnitude of the fluctuation in \( \eta \) that can cause unwanted switching. Since the required value is large according to our calculation (e.g., \( \Delta \eta \gtrsim 0.2 \)), the system is expected to be robust against most fluctuations including thermal transitions. Hence, the lifetime at each local minima may be as long as that of conventional magnetic memory cells. A detailed analysis is necessary for a quantitative estimate.

In summary, we demonstrate bistability formation in the structure consisting of magnetic and nonmagnetic QWs. The bistability is mediated by the PM-FM transition in the MQW. In contrast to the case of coupled NQWs, our structure reveals a bistability as soon as the
conditions for the ferromagnetism in the MQW are satisfied. As a result, a room temperature operation may be achieved once a proper magnetic material is developed. Although the investigation is done for a 2D structure, it is expected that a similar principle can also be applied to the 0D system. As the size of the gate electrode shrinks, the MQW can form a gated quantum dot, which can undergo the PM-FM (or super-PM) transition by controlling population/depopulation of holes. Furthermore, one can envision a magnetic nanocrystal embedded in a nonmagnetic barrier in place of the MQW. Hence, this structure can form the basis of a stable memory element that may be scaled down to a few hole regime with very low power consumption.

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Figure captions

Fig. 1. Schematic energy diagram (valence band) of the structure in the two coexisting stable states. The mutual alignment of the hole spins (large arrows) and localized spins (small arrows) reduces the energy in the FM QW by $E_{\text{exch}} \simeq 2F_M/n^0_h$ due to their exchange interaction. The switching between these states (arch arrows) is achieved by applying an appropriate bias pulse on the gate electrode.

Fig. 2. Free energy trial function $F(\eta)$ at different values of $u (= \Delta U/k_BT_c)$. Three different scenarios are shown: curve 1 ($u = 29$) - a monostable case with holes occupying the NQW ($\eta = 0$), while the MQW is in a PM phase; curve 2 ($u = 5$) - a bistable case where the PM and FM phases coexist; curve 3 ($u = -9$) - a monostable case with holes populating the MQW ($\eta = 1$) in the FM phase.

Fig. 3. Phase diagram of the parameter space indicating the potential bistability region. A free hole density $n^0_h$ of $10^{12}$ cm$^{-2}$ is assumed.
Fig. 1: Semenov et al.
Fig. 2: Semenov et al.
Fig. 3: Semenov et al.