Abstract. With new data available from the SPS, at 40 and 80 GeV/A, I review the systematics of bulk hadron multiplicities, with prime focus on strangeness production. The classical concept of strangeness enhancement in central AA collisions is reviewed, in view of the statistical hadronization model which suggests to understand strangeness enhancement to arise chiefly in the transition from the canonical to the grand canonical version of that model. I.e. enhancement results from the fading away of canonical suppression. The model also captures the striking strangeness maximum observed in the vicinity of $\sqrt{s} \approx 8$ GeV. A puzzle remains in the understanding of apparent grand canonical order at the lower SPS, and at AGS energies.

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1. Hadron multiplicity and strangeness enhancement

The first SPS experiments with $^{32}$S-beams at 200 GeV/A showed an enhancement of various strange particle multiplicities, chiefly $K^+$, $\Lambda$, and $\bar{\Lambda}$, relative to pion multiplicities, in going from peripheral to central $S + (S, Ag, Au)$ collisions [1]. This observation appeared to be in-line with the pioneering analysis of Rafelski and Müller [2] who first linked strangeness enhancement to the advent of transition from the hadronic to a partonic phase. This offered lower effective $s\bar{s}$ threshold, shorter dynamical relaxation time toward flavour equilibrium, and an additional, nontrivial effect of relatively high net baryon number or baryochemical potential: the light quark Fermi energy levels move up, perhaps even to the $s$-quark mass at high $\mu_B$, and the Boltzmann penalty factor for the higher mass $s\bar{s}$ pair creation might be removed. This latter aspect was mostly ignored in the late 1980’s but receives fresh significance as we become increasingly aware of the crucial role of $\mu_B$. 
It is the purpose of this report to present a sketch of our recent progress, both in gathering far superior data and in the understanding of the statistical model that was rudimentarily anticipated in such early strangeness enhancement speculations.

![Graphs showing kaon to pion multiplicity ratios for Pb+Pb at 158 GeV/A.](image)

**Fig. 1.** Negative and positive kaon to pion multiplicity ratios for Pb+Pb at 158 GeV/A, as a function of collision centrality given by R-b/2 where R is the nuclear radius. Data for min. bias pp and for central light nucleus collisions are also given, Ref.3.

Fig. 1 shows a modern version of the typical strangeness enhancement phenomena. Negative and positive kaon to average charged pion multiplicity ratios obtained by NA49 [3] in Pb+Pb SPS collisions at 158 GeV/A (corresponding to √s=17.3 GeV) are plotted for a sequence of collision centrality conditions from peripheral to central. At the peripheral end the minimum bias p+p point [3] matches with the trend. The centrality scale employed here is unusual but it leads to interesting conclusions. The raw data bins are ordered in NA49 data by decreasing projectile spectator energy as recorded in a zero degree calorimeter. This information is converted to mean participant nucleon number, or impact parameter b, by a Glauber calculation. Neither of these scales turn out to be satisfactory [3] in merging data from central light nuclei collisions, C+C, Si+Si and S+S, with the various centralities of the Pb+Pb collisions. For example, a central S+S collision has b ≈ 2 fm and N_{part} ≈ 57 but on a b scale the K⁺/π value is about 40% lower than the b = 2 result for the much heavier PbPb system. Inversely on the N_{part} scale: N_{part}=57 corresponds to very peripheral Pb+Pb and the central S+S result is about 40%
higher than the Pb+Pb curve. A central collision of a relatively light nuclear pair thus behaves quite differently from a very peripheral heavy nuclear collision where only the dilute Woods-Saxon density tails interact! The scale of Fig. 1 is an intuitive guess \[ \frac{R-b}{2} \] where \( R \) is the radius of the colliding nuclear species. It might be connected with the energy density reached in the primordial collision volume. We see that the central light nuclear collision data now merge with the Pb+Pb centrality scale. Similar NA49 data \[ 5 \] exist for \( \Phi \) and \( \Xi^- \) (892) production. The "strangeness enhancement factor" is also given oftentimes as the production ratio of AA central/(pp min. bias times 0.5 \( N_{\text{part}} \)). In the case of Fig. 1 it would be roughly two. Multistrange hyperons \[ 7, 8 \] show factors between 4 and 15.

Bulk strangeness enhancement in central collision is a nuclear feature, absent in pp collisions. Of course we lack a detailed picture about "centrality" in pp collisions but we could still employ e. g. the total charged particle multiplicity to select more or less "violent" collisions. Fig. 2 shows the \( K^+/\pi \) ratio of pp at 158 GeV versus charged particle multiplicity, to be essentially flat \[ 9 \]. Similar findings are made up to Tevatron \( \sqrt{s} = 1.8 \) TeV \( pp \) collisions \[ 10 \]: the \( K^+/\pi \) ratio is 50% higher here but also independent of \( N_{\text{ch}} \).

\[ \text{Fig. 2. The multiplicity ratio of positive kaons to pions in pp collisions at 158 GeV, as a function of charged particle multiplicity, Ref.9.} \]
tiality” of interactions at the microscopic level, i.e. with the number of successive collisions if one may employ a naive Glauber picture: with the size and density of the primordial interaction zone. Unfortunately this formal statement does not give us much deeper insight because if we knew how to describe a second, third etc. collision of a hadron, within fm/c space-time distance we would have probably resolved the key issue: does it dissolve into a parton cascade from which the final hadrons are reconstituted? Proton-nucleus collisions must hold a key to this question but nobody has succeeded in isolating the second, third, n’th successive collision of the projectile, as of yet [11, 12].

Fig. 3. NA49 data for hadron multiplicities in pp collisions at 158 GeV confronted with the canonical model of Becattini, Ref.15.

At the moment we thus forgo pA as an intermediate step although it certainly also features changes in the hadronic production ratios [13] and base the analysis on comparing pp to AA. Fig. 3 shows the hadronic multiplicities, from pion to cascade hyperon, obtained by NA49 for min. bias pp at √s=17.3 GeV [14]. The data are confronted with the Hagedorn statistical model in its canonical Gibbs ensemble form as employed by F. Becattini [15], leading to very good agreement (as it is well known also for other elementary collisions and energies [16]). The three parameters are T=186±7 MeV, a reaction volume of 6 fm³, and a total of about 0.5 sσ pairs. The apparent validity of a statistical weight-dominated picture of
phase-space filling is not well understood already since Hagedorn’s time. It is clear, however, that the apparent canonical "hadrochemical equilibrium" pattern can not result from "rescattering" of produced hadrons: there is none. In Hagedorn’s view [17] a creation "from above" must hold the key to the apparent maximum entropy state, i.e. the QCD process of hadronization [18]. This pattern and T-value are a fingerprint of QCD hadronization - do AA collision data at high $\sqrt{s}$ also confirm this picture (they must, of course, also result from a hadronization process)?

2. AA collisions in the Grand Canonical Model

Fig. 4 shows the grand canonical fit by Becattini to the NA49 data from central Pb+Pb at 158 GeV/A [18]. The temperature is $160 \pm 5$ MeV and $\mu_B = 240$ MeV; besides, this model employs the much discussed strangeness undersaturation factor $\gamma_s = 0.8$.

![Graph](image)

**Fig. 4.** Hadron multiplicities for central PbPb collisions at 158 GeV/A from NA49 confronted with the grand canonical statistical model, Ref.18.

Leaving the second order concern about $\gamma_s$ to the theoretical community I note here that Braun-Munzinger et al. [19] fit a set of data at the same top SPS energy without introducing a $\gamma_s$; they report $T = 170 \pm 5$ MeV, at $\mu_B = 270$ MeV, close enough. There are also studies of the new RHIC STAR data [20] at $\sqrt{s} = 130$ GeV by this model [21] and by Kaneta and Xu [22], averaging at $175 \pm 5$ MeV and
\( \mu_B = 48 \text{ MeV} \). And the new, still preliminary data of NA49 \[23\] at 80 and 40 GeV/A have resulted in Becattini fit values of \( T = 155 \text{ MeV} \), \( \mu_B = 270 \text{ MeV} \) and \( T = 150 \text{ MeV} \), \( \mu_B = 395 \text{ MeV} \), respectively. I will return shortly to a further discussion of the grand canonical approach but wish to, first of all, show an overall impression from these analyses which are confronted in Fig. 5 with the sensational new lattice QCD calculations at finite \( \mu_B \) by Fodor and Katz \[24\].

![Fig. 5. The lattice QCD phase boundary in the plane of \( T \) vs. \( \mu_B \), Ref.24. The hadronization points captured by grand canonical analysis for SPS and RHIC energies are also shown.](image)

The latter predict the \( T, \mu \) dependence of the QCD phase transformation which in this model consists of a crossover for all \( \mu > 650 \text{ MeV} \), i.e. in the SPS-RHIC domain. Note that physics observables can change rapidly in a crossover, too: the familiar steep rise of e.g. lattice \( \epsilon/T^4 \) at \( T_c \) does not, by itself, reveal the order of the phase transformation \[25\]. Anyhow: the hadronization points from grand canonical ensemble analysis merge with the phase transformation site of lattice QCD at top SPS and RHIC energy. Quite a sensational result, but also a plausible one \[24\] if we recall that Ellis and Geiger did already point out in 1996 that hadronic phase space weight dominance appears to result from the colour-flavour-spin-momentum "coalescence" that occurs at hadronization \[27\]. Unfortunately a rigorous QCD treatment of the parton to hadron transition is still missing.

At this point the following objection is always raised: if the same basic model describes hadronic yield ratios in \( pp \), \( e^+e^- \) and in central AA collisions, Figs. 3
and 4, what is special about AA, as you will not tell us now that a QGP is also formed in pp?! Answer: on the one hand both collision systems reveal the QCD hadronization process which features, furthermore, the Hagedorn limiting hadronic temperature $T_H$. At top SPS and at RHIC energy $T$ (hadronic ensemble) $\approx T_H \approx T_c$ (QCD), this is the common feature; it should not be a chance coincidence. On the other hand hadronization appears to occur under dramatically different conditions in AA collisions, as captured in the transition from a canonical to a grand canonical description. Inspection of Fig. 3 and 4 shows that the hadronic population ratios are quite different: the falloff from pions to cascade hyperons in the former case is about four orders of magnitude whereas it reduces to three in the grand canonical situation: strangeness enhancement! In the canonical case the small reaction fireball volume is strongly constrained by local conservation of baryon number, strangeness neutrality and isospin whereas these constraints fade away in the GCE which represents a situation in which, remarkably, these conservation laws act only on the average, over a rather large volume, as captured by a collective chemical potential $\mu = \mu_B + \mu_s + \mu_I$. On top of this, $\mu_s$ is eliminated by overall strangeness neutrality, and $\mu_I$ by overall charge conservation. This leaves one global quantity $\mu_B$ essentially in charge of all the conservation task. Note that the statistical model does predict nothing, it merely captures this most remarkable feature of the data. Its observed success implies some kind of long range collective behaviour in the hadronizing source, the origin of which is yet unknown, but must be specific to central AA collisions. Strangeness enhancement is the fading away of canonical constraints, in the terminology of the statistical model [28].

This aspect has been recently studied in all detail by Cleymans, Redlich, Tounsi and collaborators [28, 29]. Fig. 6 illustrates their results concerning the transition from canonical to grand canonical behaviour with increasing number of participants, i.e. overall "source" size. It is intuitively clear that it should occur, first, in singly strange hadrons, the increase occurring with offset (but having a larger specific effect on the yields per participant) in S=2,3 hyperons.

A further, appropriate critical question: how can we understand the other aspect of Fig. 5, i.e. the steep falloff from the QCD transition domain occurring at the lower SPS energies? We even have a further GCE analysis, at top AGS energy, by Braun-Muzinger et al. [21], for central Si+Au collisions at 14.6 GeV/A, shown in Fig. 7.

The result is $T=125$ MeV, $\mu_B=540$ MeV, far below the $T$ scale of Fig. 4. The picture of a direct parton to hadron transition is intuitively inapplicable at these lower energies. Still the overall dynamical trajectory that ends in hadronic chemical (abundance) freezeout should arrive there "from above" as hydrodynamical models [30] show. How can the expanding hadronic system maintain conditions near equilibrium, or acquire them? We do not know. Clearly a primitive hadronic point-like gas would not accomplish this, due to relaxation times far exceeding the expansion time scale (volume doubling occurring every about 4 fm/c). However, the hadronic system is very dense along its trajectory, and it is thus a quantum mechanical coherent state that decays to the finally observed hadron ensemble.
Unlike a quasi-classical, albeit dense "gas" it might thus ignore the classical concept of a relaxation time. Recall the nucleus, also still a dense system: we do not invoke relaxation time in a transition within such a quantal medium, such as β-decay. And yet "Fermis Golden Rule" asserts that the transition strength depends "only" on the squared matrix element times final state phase space volume weight plus global conservation laws. And we know that the phase space factor oftentimes far overrides the matrix element, in the net decay strength. Vague hints, at present! High density hadronic matter behaviour is essentially unknown: an old and new research paradigm. At top SPS and at RHIC energy, in turn, the increasingly "explosive" nature of hadronic and partonic expansion may almost instantaneously dilute the hadronizing source toward chemical freezeout, as indicated by $T(GCE) \approx T(Hagedorn) \approx T_c(QCD)$.

3. Energy dependence of strangeness yields

From combination of AGS, SPS and RHIC hadron multiplicities we can construct the energy dependence of various strange particle yields relative to the pion yield which carries the main fraction of light quark production. As an example Fig. 8
shows the total Λ and cascade hyperon yield ratio relative to π⁺, as a function of √s. A distinct maximum is visible in the Λ/π⁺ yield ratio. Similar maxima occur e. g. in the K⁺/π⁺ ratio, at √s ≈ 6−8 GeV.

Recent work with the grand canonical hadronization model, by Braun-Munzinger, Cleymans, Oeschler, Redlich and Stachel has shown that such, at first sight remarkable, nonmonotoneous behaviour is, again, semiquantitatively captured. They interpolate among the various GCE fits at increasing √s, to obtain a continuous hadronic freezeout trajectory in the T, µB plane. The result is shown in Fig. 8 to reproduce the overall features of the data. Furthermore they showed that these separate strange to nonstrange Λ or K⁺ to pion yield dependences on √s are the consequence of a more general maximum in the "Wroblewski-ratio", λ ≡ 2(s+γ)/(u+π+d+d) at similar √s. This finding is illustrated in Fig. 9. The Wroblewski λ dependence on T and µB is shown in the plane of T, µB in a set of curves each corresponding to a fixed value of λ, from 0.3 to 0.8, as obtained from the GCE model. The (dashed) hadronic freeze-out curve intersects these lines, steeply at first, from high µB downwards ("strangeness enhancement") to µB ≈ 450 MeV where √s=6-8 GeV is implied. It peaks there at λ =0.65 indicating a maximum global strangeness to nonstrangeness ratio, the reflection of which we saw in Fig. 8. Then λ falls back to 0.4 toward top SPS and RHIC energies.

Cleymans has shown that this general evolution of the strange to nonstrange hadronic population is the consequence, at the level of the statistical model, of qualitatively different trends concerning the basic parameters. While µB drops to zero continuously with increasing √s the apparent hadronization temperature turns into saturation (after a similar steeply ascending passage initially) toward T = 170±10 MeV, above about √s= 6-8 GeV where it has already reached T = 140 − 150 MeV. This picture can actually be even recast in the terminology of
microscopic collisions: at lower $\sqrt{s}$ strangeness production is "encouraged" by associated production channels owing to the prevailing high net baryon density, then to turn over to $s\pi$ production from free fireball energy \[34\]. This picture would, alone by itself, indicate merely a saturation of the relative strangeness yield. An additional feature sets in with the advent of limiting hadron temperature which can not be understood from a continuous evolution of hadronic collision energy alone. It signifies the advent of partonic phase dynamics. Actually, "no reasonable person would doubt that toward $\sqrt{s} = 200$ GeV the most simple picture arises from the interaction of quarks and gluons", to quote Lerry McLerran \[35\].

Thus, in concluding this report, let me recapitulate that the Hagedorn statistical model does not predict anything except for the existence of a limiting temperature. It reacts to the data in its particular language, and from this reflection we infer that hadron multiplicities obey a grand canonical order which indicates that an extended, collectively interacting "fireball" of strongly interacting matter is formed in AA collisions, that merges closely with the QCD phase transformation boundary toward top SPS, and RHIC energy, thus providing for an estimate of the QCD critical temperature. At lower $\sqrt{s}$ the overall behaviour of the hadronic multiplicities may reflect the advent of this phase boundary, but at present we lack an appropriate understanding of dense hadronic matter dynamics to fully comprehend the apparent validity of the grand canonical model, also in this domain.
Fig. 9. Lines of constant Wroblewski $\lambda$ parameter in the $T, \mu_B$ plane in the GCE model, intersected by the hydrostatic chemical freezeout curve (dashed), from Ref.32.

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