Electroweak Effective Operators and Higgs Physics

Chien-Yi Chen\(^{(1)}\), S. Dawson\(^{(1)}\) and Cen Zhang\(^{(2)}\)

\(^{(1)}\) Department of Physics, Brookhaven National Laboratory, Upton, New York, 11973
\(^{(2)}\) Centre for Cosmology, Particle Physics and Phenomenology (CP3)
Université Catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium

(Dated: November 27, 2013)

Abstract

We derive bounds from oblique parameters on the dimension-6 operators of an effective field theory of electroweak gauge bosons and the Higgs doublet. The loop-induced contributions to the \(\Delta S\), \(\Delta T\), and \(\Delta U\) oblique parameters are sensitive to these contributions and we pay particular attention to the role of renormalization when computing loop corrections in the effective theory. Limits on the coefficients of the effective theory from loop contributions to oblique parameters yield complementary information to direct Higgs production measurements.
I. INTRODUCTION

With the discovery of a particle with the general appearance of a Higgs boson, the focus has turned to the detailed measurements of the properties of the Higgs boson candidate. This involves the measurement of as many Higgs production and decay channels as possible, along with the limits inferred from precision electroweak measurements\[1\]. Currently, all measurements are within $\sim 2 - 3\sigma$ of the predictions of the Standard Model (SM). A deviation from these predictions could be a signal of physics beyond the Standard Model.

A search for new physics in the Higgs sector can either involve examination of specific models or the use of effective field theories which respect the symmetries of the low energy physics. In this paper, we follow the later approach and assume that the Standard Model is a good approximation to physics at the weak scale and that all new physics is at a high scale. If the new physics is at a scale much higher than that probed experimentally, then an effective theory can be written in terms of an expansion in higher dimension operators,

$$L \sim L_{SM} + \Sigma_i \frac{f_i}{\Lambda^2} O_i + \ldots.$$ \hspace{1cm} (1)

The lowest dimension operators, $O_i$, which contribute to processes involving gauge bosons are dimension-6. The complete set of $SU(3) \times SU(2)_L \times U(1)_Y$ operators is quite large\[2, 3\], but the assumption of flavor conservation, CP conservation, and Standard Model (SM) physics in the lightest 2 fermion generations reduces the set of allowed operators considerably\[4\]. In this work, we further restrict the set of operators to those involving only the Higgs doublet and electroweak boson sectors\[5–9\]. The Lagrangian of Eq. (1) is valid at low energy ($\sim m_Z$) and reflects our ignorance of high scale physics.

At each order in the expansion in $1/\Lambda^n$, divergences appear in loop diagrams, which can be absorbed by renormalizing the coefficients of the operators appearing at one lower level in the loop expansion but at the same order in $1/\Lambda^n$, yielding a theory that is finite order by order in the expansion in $1/\Lambda$. The low energy effective field theory has the advantage that it can be matched to many possibilities for high scale physics. The Lagrangian of Eq. (1) can be used at energy scales much below $\Lambda$, where the observed physics approximates the SM up to small corrections. A specific model of new physics will predict the coefficients at the high scale, $f_i(\Lambda)$. The predictions must then be matched with experimental limits from low energy measurements, $f_i(\sim m_Z)$, and the renormalization group used to run the coefficients from the scale of the high energy predictions to that of the lower energy measurements\[10–14\].
New physics in the electroweak sector can be restricted by measurements of the oblique parameters, $\Delta S$, $\Delta T$, and $\Delta U$, by restrictions on the deviations of the three gauge boson vertices and Higgs branching ratios from the Standard Model predictions, along with many SM processes. These effects and the resulting limits on the coefficients, $f_i$, of the dimension $-6$ operators have been studied by many authors\cite{9, 15–23}. The new feature of our work is a careful study of the dependence of the predictions on the renormalization scheme required by the effective theory formalism\cite{5, 24, 25}. A complete analysis would include fermion operators and a careful choice of basis. Such a study is not necessary to illustrate our main point, however, which concerns the numerical effects of the renormalization scheme in the effective framework.

In Sec. II, we review the low energy effective Higgs electroweak theory and make the connection between our notation and the commonly used strongly interacting light Higgs (SILH) basis\cite{26}. Section III contains analytic results for the oblique parameters, along with a discussion of the pinch technique needed to obtain gauge invariant results. Detailed appendices contain analytic results for the contributions to $\Delta S$, $\Delta T$ and $\Delta U$, as well as the required pinch contributions, in an $R_\xi$ gauge. Numerical fits to the coefficients of the effective operators resulting from the oblique parameters and a discussion of the dependence on the renormalization scheme used to render the effective field theory finite are given in Sec. IV. As pointed out in Refs. \cite{24, 25}, our limits are considerably weaker than those obtained using only the contributions to the oblique parameters proportional to $\log(\Lambda)$ given in Ref. \cite{8}. We also compare the restrictions on the $f_i$ couplings from the oblique parameters with those obtained from the the deviations of the experimental results from Standard Model predictions of Higgs branching ratios. Finally, Sec. V contains some conclusions.

II. EFFECTIVE THEORY

The effective Lagrangian we consider contains the $SU(2)_L \times U(1)_Y$ gauge fields and the Higgs $SU(2)_L$ doublet $\Phi$. We assume that fermion interactions with the gauge bosons are those given by the Standard Model and that all new physics respects the $SU(2)_L \times U(1)_Y$ gauge invariance and conserves $C$ and $P$. Possible new physics effects in the fermion sector are not considered here.
There are 4 operators which affect the gauge boson 2–point functions at tree level \cite{5, 27},
\[
\mathcal{O}_{DW} = -\frac{g^2}{4} Tr \left( \left[ D_\mu, \sigma^a \cdot W_{\nu\rho}^a \right] \left[ D^\mu, \sigma^b \cdot W^{b,\nu\rho} \right] \right)
\]
\[
\mathcal{O}_{DB} = -\frac{g'^2}{2} (\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho})
\]
\[
\mathcal{O}_{BW} = -\frac{gg'}{4} \Phi^\dagger B_{\mu\nu} \sigma^a \cdot W^{a,\mu\nu} \Phi
\]
\[
\mathcal{O}_{\Phi,1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi),
\]
\[
\mathcal{O}_{\Phi,2} = \frac{1}{2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi).
\]
where,\(^1\)
\[
D_\mu = \partial_\mu - ig^2 B_\mu - \frac{ig'}{2} g^2 W^a_\mu
\]
\[
B^{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu
\]
\[
W^{\pm,\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \mp ig(W^3_\mu W^\nu_\pm - W^3_\nu W^\pm_\mu)
\]
\[
W^{3,\mu\nu} = \partial_\mu W^3_\nu - \partial_\nu W^3_\mu - ig(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu).
\]
\(\mathcal{O}_{BW}\) gives \(B - W^3\) mixing at tree level and contributes to \(\Delta S\), while \(\mathcal{O}_{\Phi,1}\) affects \(m_Z\), but not \(m_W\), at tree level and thus contributes to \(\Delta T\).

There are 6 bosonic operators which contribute to the oblique parameters at 1–loop,
\[
\mathcal{O}_{WWW} = -\frac{i g^3}{8} Tr \left( \sigma^a \cdot W^a_{\nu,\mu} \sigma^b \cdot W^b_{\rho,\nu} \sigma^c \cdot W^c_{\mu,\rho} \right)
\]
\[
\mathcal{O}_W = ig^2 (D_\mu \Phi)^\dagger \sigma^a \cdot W^{a,\mu\nu} (D_\nu \Phi)
\]
\[
\mathcal{O}_B = ig'(D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi)
\]
\[
\mathcal{O}_{WW} = -\frac{g^2}{4} \Phi^\dagger \sigma^a \cdot W^{a,\mu\nu} \sigma^b \cdot W^b_{\mu\nu} \Phi
\]
\[
\mathcal{O}_{BB} = -\frac{g'^2}{4} \Phi^\dagger B^{\mu\nu} B_{\mu\nu} \Phi
\]
\[
\mathcal{O}_{\Phi,2} = \frac{1}{2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi).
\]

The effects of \(\mathcal{O}_{BB}\) and \(\mathcal{O}_{WW}\) on 3-gauge boson vertices can be eliminated by wave function renormalization and coupling redefinition, leaving only contributions to effective \(VVH\) vertices.

\(^1\) The convention for \(D_\mu\) differs from that of Refs. \cite{4, 8, 9} leading to some minus signs in the literature, relative to ours, which are described in Appendix A.
In addition, there are 2 operators that are neglected in our analysis,

\[ \mathcal{O}_{\Phi,3} = \frac{1}{3}(\Phi^\dagger \Phi)^3 \]
\[ \mathcal{O}_{\Phi,4} = (D_\mu \Phi)^\dagger (D^\mu \Phi)(\Phi^\dagger \Phi). \]

(5)

\( \mathcal{O}_{\Phi,3} \) only affects the Higgs self interactions, and is irrelevant for the oblique parameters. By using the equations of motion, \( \mathcal{O}_{\Phi,4} \) can be written as a linear combination of \( \mathcal{O}_{\Phi,2} \) and dimension-6 Yukawa operators. The latter have no effect on the self energies of gauge bosons, and thus \( \mathcal{O}_{\Phi,4} \) can also be excluded from our operator basis.

Finally, the Lagrangian for Higgs physics we consider is,

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{f_{DW}}{\Lambda^2} \mathcal{O}_{DW} + \frac{f_{DB}}{\Lambda^2} \mathcal{O}_{DB} + \frac{f_{BW}}{\Lambda^2} \mathcal{O}_{BW} + \frac{f_{\Phi,1}}{\Lambda^2} \mathcal{O}_{\Phi,1} + \frac{f_{\Phi,2}}{\Lambda^2} \mathcal{O}_{\Phi,2} \]
\[ + \frac{f_{WWW}}{\Lambda^2} \mathcal{O}_{WWW} + \frac{f_{W}}{\Lambda^2} \mathcal{O}_{W} + \frac{f_{B}}{\Lambda^2} \mathcal{O}_{B} + \frac{f_{WW}}{\Lambda^2} \mathcal{O}_{WW} + \frac{f_{BB}}{\Lambda^2} \mathcal{O}_{BB}. \]

(6)

The relationship between the coefficients of Eq. 6 and those of the SILH Lagrangian are given in Appendix A for convenience.

### III. RESULTS

We are interested in the contributions to the oblique parameters from the operators of Eq. 6. The computation requires both the gauge boson 2− point functions and the pinch contributions in order to get gauge invariant results [28]. The 2− point functions are defined as,

\[ \Pi_{XY}^{\mu\nu}(q^2) = g^{\mu\nu} \Pi_{XY}(q^2) - p^\mu p^\nu B_{XY}(q^2), \]

(7)

for \( XY = WW, ZZ, \gamma\gamma \) and \( Z\gamma \). The contributions (including tadpole diagrams) to \( \Pi_{XY}(q^2) \) are given in Appendix B in Rξ gauge. The \( \Pi_{XY} \) functions are gauge dependent and ultra-
violet divergent.

The pinch contributions are defined in terms of the corrections to the \( \mathcal{F} \gamma^\mu P_L f' V^\mu \) vertices, where \( P_L = \frac{1}{2}(1-\gamma_5) \). Only left-handed contributions arise because there is always a coupling to a \( W \) boson. The vertex corrections are of the form,

\[ \Delta \Gamma^{Vff'}_\mu(q^2) = \gamma_\mu P_L \Delta \Gamma^{Vff'}_L(q^2) \]

(8)

\[ \Delta \Gamma^{Vff'}_L(q^2) \]

2 Results in the unitary gauge can be found in Ref. [28].
We normalize,
\[
\Delta \Gamma_{L}^{Vf}(q^2) = gT^f_3 \Delta \Gamma_{L}^{V}(q^2) \quad V=Z, \gamma
\]
\[
\Delta \Gamma_{L}^{Wf'}(p^2) = \frac{g}{\sqrt{2}} \Delta \Gamma_{L}^{W}(q^2) \quad V=W.
\] (9)

Expressions for the pinch contributions, $\Delta \Gamma_{L}^{V}(q^2)$, in $R_\xi$ gauge are given in Appendix C.

Gauge invariant 2−point functions can be constructed by forming the combinations,
\[
\Pi_{WW}(q^2) = \Pi_{WW}(q^2) + 2(q^2 - m_W^2) \Delta \Gamma_{L}^{W}(q^2)
\]
\[
\Pi_{ZZ}(q^2) = \Pi_{ZZ}(q^2) + 2c(q^2 - m_Z^2) \Delta \Gamma_{L}^{Z}(q^2)
\]
\[
\Pi_{\gamma Z}(q^2) = \Pi_{\gamma Z}(q^2) + s q^2 \Delta \Gamma_{L}^{Z}(q^2) + c(q^2 - m_Z^2) \Delta \Gamma_{L}^{\gamma}(q^2)
\]
\[
\Pi_{\gamma \gamma}(p^2) = \Pi_{\gamma \gamma}(p^2) + 2s q^2 \Delta \Gamma_{L}^{\gamma}(q^2),
\] (10)

where $c \equiv \cos \theta_W$ and $s \equiv \sin \theta_W$. We have explicitly verified the cancellation of the $\xi$ gauge parameters.

This allows the construction of gauge invariant oblique parameters\[28,\]
\[
\alpha \Delta S = \left(\frac{4s^2c^2}{m_Z^2}\right) \left\{ \Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0) - \Pi_{\gamma \gamma}(m_Z^2) \right. \\
- \left. \frac{c^2 - s^2}{cs} \left( \Pi_{\gamma Z}(m_Z^2) \right) \right\}
\]
\[
\alpha \Delta T = \left(\frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}\right)
\]
\[
\alpha \Delta U = 4s^2 \left\{ \frac{\Pi_{WW}(m_W^2)}{m_W^2} - \frac{\Pi_{WW}(0)}{m_W^2} - c^2 \left( \frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2} \right) \right. \\
- \left. 2sc \left( \frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} \right) - s^2 \frac{\Pi_{\gamma \gamma}(m_Z^2)}{m_Z^2} \right\}.
\] (11)

The oblique parameters effectively capture the new physics consequences on low energy observables with the assumption that all new physics is at a scale much larger than $M_Z$ and that the new physics contributes only to the 2-point functions. The non-pinch terms which contribute to the vertex function are hence not included here, but generate additional contributions from the effective Lagrangian which is not captured in the STU formalism.

---

3 Gauge invariant expressions for the 2−point functions are found in Refs.\[24, 25\]. Their construction differs from ours by finite, gauge invariant terms, as explained in Appendix C.
We parameterize the oblique parameters as,

\[
\Delta S = C_S \frac{1}{\epsilon} \left( \frac{4\pi \mu^2}{m_Z^2} \right)^\epsilon \Gamma(1 + \epsilon) + R_S
\]

\[
\Delta T = C_T \frac{1}{\epsilon} \left( \frac{4\pi \mu^2}{m_Z^2} \right)^\epsilon \Gamma(1 + \epsilon) + R_T
\]

\[
\Delta U = C_U \frac{1}{\epsilon} \left( \frac{4\pi \mu^2}{m_Z^2} \right)^\epsilon \Gamma(1 + \epsilon) + R_U ,
\]  

(12)

where

\[
C_S = \frac{m_H^2}{8\pi} \left\{ \frac{f_B + f_W}{\Lambda^2} \right\} + \frac{m_Z^2}{24\pi \Lambda^2} \left\{ f_B(20c^2 + 7) - 3f_W 
+ 24(s^2f_{BB} + c^2f_{WW}) + 36c^2g^2f_{WW} + \frac{8e^2}{g^2}f_{\Phi,2} \right\}
\]

\[
C_T = \frac{1}{16\pi c^2} \left\{ 9m_W^2 \left( \frac{f_B + f_W}{\Lambda^2} \right) + m_H^2 \frac{f_B}{\Lambda^2} - 12m_W^2 \frac{f_{\Phi,2}}{g^2 \Lambda^2} \right\}
\]

\[
C_U = \frac{m_Z^2}{6\pi \Lambda^2} s^2 f_W .
\]  

(13)

The logarithmic contributions to the oblique parameters coefficients have been obtained in Ref. [8] and are obtained by the replacement in Eq. [12]

\[
\frac{1}{\epsilon} \left( \frac{4\pi \mu^2}{m_Z^2} \right)^\epsilon \Gamma(1 + \epsilon) \rightarrow \ln \left( \frac{\Lambda^2}{m_Z^2} \right).
\]  

(14)

All other terms are dropped, which gives the leading logarithmic result,

\[
\Delta S^{LL} \rightarrow C_S \log \left( \frac{\Lambda^2}{m_Z^2} \right)
\]

\[
\Delta T^{LL} \rightarrow C_T \log \left( \frac{\Lambda^2}{m_Z^2} \right)
\]

\[
\Delta U^{LL} \rightarrow C_U \log \left( \frac{\Lambda^2}{m_Z^2} \right).
\]  

(15)

We take as inputs,

\[
m_H = 126 \text{ GeV}, \quad m_Z = 91.1875 \text{ GeV}, \quad m_W = 80.399 \text{ GeV}
\]

\[
G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \quad m_t = 173 \text{ GeV}.
\]  

(16)
All other inputs are obtained using the tree level relationships of the SM. Eq. 15 becomes,

\[ \Delta S^{LL} = 0.015 f_B + 0.0014 f_W + 0.0028 f_{BB} + 0.01 f_{WW} + 0.006 f_{WWW} + 0.0016 f_{Φ,2} \]

\[ \times \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \left[ \frac{\log(\Lambda^2/m_Z^2)}{\log(1 \text{ TeV}^2/m_Z^2)} \right] \]

\[ \Delta T^{LL} = 0.013 f_B + 0.007 f_W - 0.0047 f_{Φ,2} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \left[ \frac{\log(\Lambda^2/m_Z^2)}{\log(1 \text{ TeV}^2/m_Z^2)} \right] \]

\[ \Delta U^{LL} = 0.0005 f_W \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \left[ \frac{\log(\Lambda^2/m_Z^2)}{\log(1 \text{ TeV}^2/m_Z^2)} \right] \]

(17)

Numerical fits to the oblique parameters using the logarithmic contributions of Eq. 17 have been given in Refs. 15, 30. The terms not associated with the divergences are,

\[ R_S = -\frac{4\pi v^2}{\Lambda^2} f_{BW} + R_{S1} + R_{S2} \log(c) + R_{S3} \log \left( \frac{m_H}{m_Z} \right) \]

\[ R_T = -\frac{v^2}{2\alpha \Lambda^2} f_{Φ,1} + R_{T1} + R_{T2} \log(c) + R_{T3} \log \left( \frac{m_H}{m_Z} \right) \]

\[ R_U = \frac{g^2 s^2 8\pi v^2}{c^2 \Lambda^2} f_{DW} + R_{U1} + R_{U2} \log(c) + R_{U3} \log \left( \frac{m_H}{m_Z} \right) \].

(18)

Analytic results for \( R_S, R_T, \) and \( R_U \) are given in Appendix D. Numerically we find,

\[ R_S = \left\{ -0.76 f_{BW} \right. + 10^{-3} \left. (1.48 f_B - 1.4 f_W - 0.2 f_{BB} - 0.71 f_{WW} \right. \]

\[ + 0.66 f_{WWW} + 1.96 f_{Φ,2} \left) \right. \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \]

\[ R_T = \left\{ -4.0 f_{Φ,1} \right. - 10^{-3} \left. (0.13 f_B + 0.12 f_W - 3.97 f_{Φ,2} \right) \right. \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \]

\[ R_U = \left\{ 0.20 f_{DW} \right. + 10^{-3} \left. (-0.02 f_B + 2.06 f_W + 0.14 f_{WW} \right. \]

\[ + 2.1 f_{WWW} - 0.25 f_{Φ,2} \left) \right. \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \].

(19)

The effective field theory is defined at the weak scale, \( \mu \sim m_Z \), and encapsulates the effects of potential new physics which may occur at high scales. The divergences which arise at 1-loop, Eq. 12 can be eliminated by renormalizing the coefficients which enter the tree level results. The theory is thus rendered finite order by order in the expansion in powers.

---

4 Since \( f_{BW}, f_{Φ,1}, \) and \( f_{DW} \) contribute at tree level, we do not consider the much smaller contributions of these operators at 1-loop to the oblique parameters.
of $1/\Lambda^2$ \cite{5, 7, 24, 25, 31}. Using $\overline{MS}$ renormalization, the renormalized coefficients relevant for a study of the oblique parameters are,

$$f_{BW}(\mu) = f_{BW} - \frac{1}{\epsilon} (4\pi)^\epsilon \Gamma(1 + \epsilon) C_S$$

$$f_{DW}(\mu) = f_{DW} - \frac{1}{\epsilon} (4\pi)^\epsilon \Gamma(1 + \epsilon) C_U$$

$$f_{\Phi,1}(\mu) = f_{\Phi,1} - \frac{1}{\epsilon} (4\pi)^\epsilon \Gamma(1 + \epsilon) C_T.$$  \hspace{1cm} (20)

This renormalization prescription is equivalent to that of \cite{5}. The large logarithms of Ref. \cite{8} have been eliminated by the renormalization of the tree level couplings and the only remaining contributions to the oblique corrections are the finite contributions. Our final result, taking $\mu = m_Z$, as appropriate for the low energy effective Lagrangian, is,

$$\Delta S = R_S$$

$$\Delta T = R_T$$

$$\Delta U = R_U.$$ \hspace{1cm} \hspace{1cm} (21)

Note that this result is quite different from that of Ref. \cite{8}, since the $\log(\Lambda)$ terms have all been cancelled by the renormalization of the tree level coefficients. This is in agreement with the leading result of Ref. \cite{10} for $\Delta S$ which was obtained by scaling the coefficients in a BSM model from $\Lambda$ to $m_Z$.

IV. PHENOMENOLOGY

A. Results from Fits to Oblique Parameters

We do a $\chi^2$ fit to the oblique parameters, using the results of the GFITTER group\cite{1},

$$\Delta S = 0.03 \pm 0.10$$

$$\Delta T = 0.05 \pm 0.12$$

$$\Delta U = 0.03 \pm 0.10$$ \hspace{1cm} (22)

with the correlation matrix,

$$\rho = \begin{pmatrix} 1.0 & 0.891 & -0.540 \\ 0.891 & 1.0 & -0.803 \\ -0.540 & -0.803 & 1.0 \end{pmatrix}.$$ \hspace{1cm} (23)
FIG. 1: Limits from the oblique parameters on \( f_{\Phi,1} \) and \( f_{BW} \) for \( \Lambda = 1 \, \text{TeV} \). These operators contribute at tree level and are significantly restricted. The curves (from outer to inner) are 99, 95, and 68 % confidence level.

The parameters \( f_{BW} \) and \( f_{\Phi,1} \) contribute to \( \Delta S \) and \( \Delta T \) at tree level. We show the limits in Fig. 1.\(^5\) These coefficients are highly restricted by the electroweak data and we ignore them in the remaining fits, where we obtain limits pairwise on various coefficients. We do not perform a global fit to the coefficients, since our point is simply to illustrate the numerical effects of the renormalization. Taking the 95% confidence level limits, and assuming \( f_{BW}(m_Z) \) and \( f_{\Phi,1}(m_Z) \) are \( \mathcal{O}(1) \), the fit implies \( \Lambda > 1.8 \, \text{TeV} \). Even though they are numerically constrained, \( f_{BW} \) and \( f_{\Phi,1} \) cannot be set to 0 at the beginning, since they play a critical role in the renormalization, as seen in Eq. 20\(^{24, 25}\).

In Fig. 2 we show the allowed region for \( \Lambda = 1 \, \text{TeV} \) in the \( f_{BB}(m_Z) \) and \( f_{WW}(m_Z) \) plane, setting all other coefficients to zero. The left-hand side shows the result using the leading logarithmic result of Eq. 19. This figure is in agreement with Fig. 6 of Ref. 16\(^\text{1} \). After renormalizing the coefficients of the effective theory, as in Eq. 21, the result is shown on the right-hand side of Fig. 2. In Fig. 3 we show the allowed region for \( \Lambda = 1 \, \text{TeV} \) in the \( f_{W}(m_Z) \) and \( f_{WW}(m_Z) \) plane, setting all other coefficients to zero. Again, the left-hand side shows the result using the leading logarithmic result of Eq. 19, while the right-hand side shows the result after renormalization of the couplings. As emphasized in Refs. 24, 25\(^\text{1} \), the

\(^5\) Ref. 32 has done a similar analysis and our fit to \( f_{BW} \) and \( f_{\Phi,1} \) agrees with theirs.
FIG. 2: (a) Limits from the oblique parameters on $f_{BB}$ and $f_{WW}$ for $\Lambda = 1 \text{ TeV}$, using the leading logarithmic results of Eq. 17. The curves (from outer to inner) are 99, 95, and 68 % confidence level. (b) Same as (a) except using the renormalized values of the coefficients, $f_{BB}(m_Z)$ and $f_{WW}(m_Z)$, Eqs. 19 and 21.

limits are considerably weakened once the renormalization procedure, Eq. 20 is applied. In fact, taking $f_i \sim 1$, we see that no useful limits can be inferred.

B. Implications for Higgs Decays

In this section, we demonstrate the complementarity of limits from oblique parameters to those obtained from measurements of Higgs branching ratios.
FIG. 3: (a) Limits from the oblique parameters on $f_W$ and $f_{WW}$ for $\Lambda = 1 \, TeV$, using the leading logarithmic results of Eq. [17]. The curves (from outer to inner) are 99, 95, and 68 % confidence level. (b) Same as (a) except using the renormalized values of the coefficients, $f_W(m_Z)$ and $f_{WW}(m_Z)$, Eqs. [19] and [21].

In the effective theory, the decay $H \rightarrow W^+W^-$ is modified\[6, 9, 12\],

$$\mu_{WW} \equiv \frac{\Gamma(H \rightarrow W^+W^-)}{\Gamma(H \rightarrow W^+W^-)}|_{SM}$$

$$\begin{align*}
&= \frac{1}{4 - 4x_W + 3x^2_W} \left[ 2 \left( x_W + 2f_W(m_Z)\frac{m^2_W}{\Lambda^2} + 2f_{WW}(m_Z)\frac{m^2_W}{\Lambda^2}(2 - x_W) \right)^2 \\
&\quad + \left( 2 - x_W + 2f_W(m_Z)\frac{m^2_W}{\Lambda^2} + 2f_{WW}(m_Z)\frac{m^2_W}{\Lambda^2}x_W \right)^2 \right] - \frac{2}{g^2 \Lambda^2}(f_{\Phi,1} + 2f_{\Phi,2}) \\
&\sim 1 + \left[ .0086f_{WW}(m_Z) + .017f_W(m_Z) \\
&\quad -.03f_{\Phi,1}(m_Z) - .06f_{\Phi,2}(m_Z) \right] \left( \frac{1 \, TeV}{\Lambda} \right)^2 + \mathcal{O} \left( \frac{1}{\Lambda^4} \right),
\end{align*}$$

(24)

where $x_W = 4m_W^2/m_H^2$ and we have made explicit the dependence of the coefficients on the scale. The $f_{\Phi,1}$ and $f_{\Phi,2}$ contributions come from the Higgs wave function renormalization. Since we have assumed that there are no non-SM corrections to the fermion-Higgs couplings, we can use the measurements of $H \rightarrow W^+W^-$ from gluon fusion to limit $f_{WW}$ and $f_W$ in

\[6\] Note our differing convention for the sign of $f_W$ from these references.
Eq. 24 [33, 34]:

\[
\mu_{WW} = 0.68 \pm 0.20 \quad (CMS)
\]

\[
\mu_{WW} = 0.99 \pm 0.30 \quad (ATLAS).
\] (25)

In Fig. 4 we show the allowed region in \( f_{WW}(m_Z) \) versus \( f_W(m_Z) \) from \( H \to W^+W^- \). In this case, the limits from Higgs decay and from the oblique parameters are similar. Note that the scale of Fig. 4, \( f_i/\Lambda^2 \approx 200 \), makes these limits meaningless.

Only \( f_{BB}, f_{WW}, \) and \( f_{BW}(m_Z) \) contribute to \( H \to \gamma\gamma \). Using the well known SM results [35] we find,

\[
\mu_{\gamma\gamma} \equiv \frac{\Gamma(H \to \gamma\gamma)}{\Gamma(H \to \gamma\gamma)_{SM}}
= \left\{ 1 + \left( \frac{I_{\text{real}}}{I_{\text{real}} + I_{\text{imag}}} \right) \frac{8\pi^2 v^2}{\Lambda^2} \left[ f_{BB}(m_Z) + f_{WW}(m_Z) - f_{BW}(m_Z) \right] \right\}^2
\sim 1 + 1.47 \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 \left[ f_{BB}(m_Z) + f_{WW}(m_Z) - f_{BW}(m_Z) \right] + O \left( \frac{1}{\Lambda^4} \right)
\] (26)

where

\[
I_{\text{real}} = \sum_f N_C Q_f^2 F_{1/2}^{\text{real}}(x_f) + F_1^{\text{real}}(x_W)
\]

\[
I_{\text{imag}} = \sum_f N_C Q_f^2 F_{1/2}^{\text{imag}}(x_f) + F_1^{\text{imag}}(x_W),
\] (27)
FIG. 5: Comparison of limits from oblique parameters (blue bands) and 95% confidence level limits from the measurement of gluon fusion production and the subsequent $H \rightarrow \gamma\gamma$ decay (red vertical line).

$x_f = 4m_f^2/m_H^2$ and expressions for $F_{1/2}$ and $F_1$ are found in Ref. [35]. We can do a simple fit to the ATLAS and CMS results for $H \rightarrow \gamma\gamma$, [33, 34]

\[
\mu_{\gamma\gamma} = 0.77 \pm 0.27 \quad (CMS)
\]
\[
\mu_{\gamma\gamma} = 1.55 \pm 0.31 \quad (ATLAS) .
\]

(28)

Fig. 5 shows the 95% confidence level limits from the gluon fusion of the Higgs boson, with the subsequent decay to $\gamma\gamma$, and contrasts the limit from the oblique parameters, (setting $f_{BW}(m_Z) = 0$). The error band on the $H \rightarrow \gamma\gamma$ limits is not apparent on this scale, and again it is clear that the constraints from the oblique parameters cannot compete with those from Higgs decay even though they have a different shape in the $f_{WW}$ and $f_{BB}$ planes.

In a similar fashion, we can find the contribution to $H \rightarrow Z\gamma$,

\[
\mu_{Z\gamma} \equiv \frac{\Gamma(H \rightarrow Z\gamma)}{\Gamma(H \rightarrow Z\gamma)|_{SM}}
\]

\[
= 1 + \frac{2A_{real}}{A_{real}^2 + A_{imag}^2} \frac{2\pi sc m_Z^2}{\alpha \Lambda^2} g_1 + O\left(\frac{1}{\Lambda^4}\right)
\]

(29)

where

\[
g_1 = f_B(m_Z) - f_W(m_Z) + 4s^2 f_{BB}(m_Z) - 4c^2 f_{WW}(m_Z) + 2(c^2 - s^2) f_{BW}(m_Z),
\]

(30)
FIG. 6: Limits from oblique parameters which influence the decay $H \rightarrow Z\gamma$. The curves (from outer to inner) are 99, 95, and 68 % confidence level.

and $A_{\text{real}}$ and $A_{\text{imag}}$ are the real and imaginary contributions of the sum of the fermion and $W$ loops to the SM $H \rightarrow Z\gamma$ decay and can be found in Ref. [35]. In Fig. 6 we show the limits on combinations of $f_i(m_Z)$ which influence the decay $H \rightarrow Z\gamma$. Neglecting all parameters except $f_W(m_Z)$ and $f_B(m_Z)$, Fig. 6 corresponds to

$$-80 < \left[ f_B(m_Z) - f_W(m_Z) \right] \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 < 35,$$

which is not strong enough to give a limit on $\mu_{Z\gamma}$.

V. CONCLUSION

We have re-examined the prescription of Ref. [8] for obtaining limits on the couplings of an effective low energy theory of Higgs-gauge boson interactions by approximating $\Delta S$, $\Delta T$ and $\Delta U$ by the leading logarithmic contributions. After renormalizing the coefficients of the operators which affect tree level results, however, the remaining contributions to the oblique parameters have no logarithmic enhancement and the leading logarithmic approximation is inaccurate. We give both analytic and numerical results for the oblique parameters. Fits to the couplings of the effective theory using our prescription for $\Delta S$, $\Delta T$ and $\Delta U$ show that no meaningful limit on the couplings which contribute at one-loop can be obtained from the
oblique parameters. This is in contrast to the couplings which contribute at tree level which are tightly constrained. This is in agreement with the results of Ref. \[24, 25\]. Limits on the couplings, \( f_i \) can, however, be extracted from Higgs decays and measurements of 3-gauge boson vertices and a complete fit was given in Ref. \[30\].

**Acknowledgement**

The work of C.-Y. Chen and S. Dawson was supported by the U.S. Department of Energy under DE-AC02-98CH10886. C. Zhang was supported by IISN Fundamental Interactions convention 4.4517.08. We thank S. Willenbrock for helpful comments on the manuscript and M.C. Gonzalez-Garcia for pointing out the need to include \( O_{\phi, 2} \) in our analysis. C. Zhang thanks H. Mebane, N. Greiner, and S. Willenbrock for their previous collaboration.
Appendix A: Relations between HISZ and SILH Basis

In this Appendix we present a prescription for converting between our effective Lagrangian and others which are frequently used in the literature. We begin by comparing our results with those of Ref. [5] which we label as HISZ. Ref. [5] uses as their convention, $D_H^{\text{HISZ}} \equiv \partial_\mu + ig\frac{1}{2}B_\mu + ig\frac{\sigma^a}{2}W^a_\mu$, which is opposite from ours. To convert our final results to those of HISZ, the substitutions, $f_{WW} \rightarrow -f_{WW}^{\text{HISZ}}, f_W \rightarrow -f_W^{\text{HISZ}}$ and $f_B \rightarrow -f_B^{\text{HISZ}}$ must be made in Eq. 12.

The HISZ (Hagiwara-Ishihara-Szalapski-Zeppenfeld) operator basis [5] has 11 operators involving Higgs and EW gauge fields:

$$O_{DW}^{HISZ}, O_{DB}^{HISZ}, O_{BW}^{HISZ}, O_{\Phi,1}^{HISZ}, O_{\Phi,2}^{HISZ}, O_{\Phi,3}^{HISZ}, O_{WW}^{HISZ}, O_{WW}^{HISZ}, O_{BB}^{HISZ}, O_{W}^{HISZ}, O_{B}^{HISZ}. $$

The SILH (Strongly-Interacting Light Higgs) operator basis [26] has the same number of operators (involving Higgs and EW gauge fields):

$$O_H^{SILH}, O_T^{SILH}, O_6^{SILH}, O_W^{SILH}, O_B^{SILH}, O_{2W}^{SILH}, O_{2B}^{SILH}, O_{BB}^{SILH}, O_{HW}^{SILH}, O_{HB}^{SILH}, O_{3W}^{SILH}. $$

where we use the definition in [13]. The operators are the same, but the normalization factors are more convenient.

The connection between operators are:
From SILH to our convention:

\[ \mathcal{O}^{\text{SILH}}_H = \mathcal{O}_{\Phi,2} \]
\[ \mathcal{O}^{\text{SILH}}_T = \mathcal{O}_{\Phi,2} - 2\mathcal{O}_{\Phi,1} \]
\[ \mathcal{O}^{\text{SILH}}_6 = 3\lambda\mathcal{O}_{\Phi,3} \]
\[ \mathcal{O}^{\text{SILH}}_W = 2\mathcal{O}_W - \mathcal{O}_{WW} - \mathcal{O}_{BW} \]
\[ \mathcal{O}^{\text{SILH}}_B = 2\mathcal{O}_B - \mathcal{O}_{BB} - \mathcal{O}_{BW} \]
\[ \mathcal{O}^{\text{SILH}}_{2W} = \frac{1}{2g^2}\mathcal{O}_{DW} - \frac{2}{g^2}\mathcal{O}_{WWW} \]
\[ \mathcal{O}^{\text{SILH}}_{2B} = \frac{1}{2g'^2}\mathcal{O}_{DB} \]
\[ \mathcal{O}^{\text{SILH}}_{BB} = -4\mathcal{O}_{BB} \]
\[ \mathcal{O}^{\text{SILH}}_{HW} = 2\mathcal{O}_W \]
\[ \mathcal{O}^{\text{SILH}}_{HB} = 2\mathcal{O}_B \]
\[ \mathcal{O}^{\text{SILH}}_{3W} = \frac{2}{3g^2}\mathcal{O}_{WWW} \]

\[ \text{(32)} \]

From our convention to SILH:

\[ \mathcal{O}_{DW} = 2g^2\mathcal{O}^{\text{SILH}}_{2W} + 6g^2\mathcal{O}^{\text{SILH}}_{3W} \]
\[ \mathcal{O}_{DB} = 2g'^2\mathcal{O}^{\text{SILH}}_{2B} \]
\[ \mathcal{O}_{BW} = -\mathcal{O}^{\text{SILH}}_B + \frac{1}{4}\mathcal{O}_{BB} + \mathcal{O}_{HB}^{\text{SILH}} \]
\[ \mathcal{O}_{\Phi,1} = \frac{1}{2}\mathcal{O}^{\text{SILH}}_H - \frac{1}{2}\mathcal{O}^{\text{SILH}}_T \]
\[ \mathcal{O}_{\Phi,2} = \mathcal{O}^{\text{SILH}}_H \]
\[ \mathcal{O}_{\Phi,3} = \frac{1}{3\lambda}\mathcal{O}^{\text{SILH}}_6 \]
\[ \mathcal{O}_{WWW} = \frac{3g^2}{2}\mathcal{O}^{\text{SILH}}_{3W} \]
\[ \mathcal{O}_{WW} = \mathcal{O}^{\text{SILH}}_B - \mathcal{O}^{\text{SILH}}_W - \mathcal{O}^{\text{SILH}}_{HB} + \mathcal{O}^{\text{SILH}}_{HW} - \frac{1}{4}\mathcal{O}^{\text{SILH}}_{BB} \]
\[ \mathcal{O}_{BB} = -\frac{1}{4}\mathcal{O}^{\text{SILH}}_{BB} \]
\[ \mathcal{O}_W = \frac{1}{2}\mathcal{O}^{\text{SILH}}_{HW} \]
\[ \mathcal{O}_B = \frac{1}{2}\mathcal{O}^{\text{SILH}}_{HB} \]

\[ \text{(33)} \]

The connections between coefficients are:
• From SILH to our convention:

\[
\begin{align*}
  c_{SILH}^H &= \frac{1}{2} f_{\Phi,1} + f_{\Phi,2} \\
  c_{SILH}^T &= -\frac{1}{2} f_{\Phi,1} \\
  c_{SILH}^6 &= \frac{1}{3\lambda} f_{\Phi,3} \\
  c_{SILH}^W &= -f_{WW} \\
  c_{SILH}^B &= -f_{BW} + f_{WW} \\
  c_{SILH}^{2W} &= 2g^2 f_{DW} \\
  c_{SILH}^{2B} &= 2g^2 f_{DB} \\
  c_{SILH}^{BB} &= -\frac{1}{4} f_{BB} + \frac{1}{4} f_{BW} - \frac{1}{4} f_{WW} \\
  c_{SILH}^{HW} &= \frac{1}{2} f_{W} + f_{WW} \\
  c_{SILH}^{HB} &= \frac{1}{2} f_{B} + f_{BW} - f_{WW} \\
  c_{SILH}^{3W} &= \frac{3}{2} g^2 f_{WWW} + 6g^2 f_{DW}
\end{align*}
\]

(34)

• From our convention to SILH:

\[
\begin{align*}
  f_{DW} &= \frac{1}{2g^2} c_{SILH}^{2W} \\
  f_{DB} &= \frac{1}{2g^2} c_{SILH}^{2B} \\
  f_{BW} &= -c_{SILH}^B - c_{SILH}^W \\
  f_{\Phi,1} &= -2c_{SILH}^T \\
  f_{\Phi,2} &= c_{SILH}^H + c_{SILH}^T \\
  f_{\Phi,3} &= 3\lambda c_{SILH}^6 \\
  f_{WWW} &= \frac{2}{3g^2} c_{SILH}^{3W} - \frac{2}{g^2} c_{SILH}^{2W} \\
  f_{WW} &= -c_{SILH}^W \\
  f_{BB} &= -c_{SILH}^B - 4c_{SILH}^{BB} \\
  f_{W} &= 2c_{SILH}^W + 2c_{SILH}^{HW} \\
  f_{B} &= 2c_{SILH}^B + 2c_{SILH}^{HB}
\end{align*}
\]

(35)
Appendix B: Self Energies in $R_\xi$ Gauge

In this appendix we present the detailed formulas of the self-energies that contribute to $\Pi_{XY}$ defined in Eq. 17 in $R_\xi$ gauge. We use different gauge parameters $\xi_W$ and $\xi_Z$ for the W and Z bosons, respectively. $\Pi_{XY}$ can be written as

$$\Pi_{XY}(q^2) = \Sigma_if_i\left\{\Pi_{XY,FG}^i(q^2) + \Pi_{XY,\xi}^i(q^2)\right\},$$  

where $XY$ represents $\gamma\gamma$, $Z\gamma$, $ZZ$ and $WW$ and $\Pi^i$ is the part of the 2-point function which is proportional to $f_i$. $\Pi_{XY,FG}$ is independent of $\xi$ and contains results in Feynman gauge and the second term collects terms that vanish when $\xi = 1$. In the following, we express our results in terms of scalar integral functions $A_0$ and $B_0$ [36]. Only non-zero contributions are listed here. We separate the contributions proportion to each $f_i$.

$O_B$:

$$\Pi_{\gamma\gamma,FG} = -\frac{f_B}{\Lambda^2 \frac{1}{72\pi}} q^2 \alpha \left\{ 3(20m_W^2 + q^2)B_0(q^2, m_W^2, m_W^2) - 6A_0(m_W^2) + 2(q^2 - 6m_W^2) \right\}$$  

$$\Pi_{\gamma\gamma,\xi} = \frac{f_B}{\Lambda^2 \frac{\alpha}{48\pi m_W^2}} \left\{ -2q^2 (-2q^2 m_W^2 (\xi_W - 2) + m_W^4 (\xi_W^2 + 4\xi_W - 5) + q^4) B_0 (q^2, m_W^2, m_W^2, m_W^2) + (q^6 - 4q^4 m_W^2 \xi_W) B_0 (q^2, m_W^2, m_W^2, m_W^2) + (8q^4 m_W^2 + q^6) B_0 (q^2, m_W^2, m_W^2) - 2q^2 m_W^2 (\xi_W + 5) A_0 (m_W^2) + 2q^2 m_W^2 (\xi_W + 5) A_0 (m_W^2) \right\}$$

$$\Pi_{\gamma Z,FG} = \frac{f_B}{\Lambda^2 \frac{\alpha}{288\pi cs}} \left\{ 3 \left( 4m_Z^2 (m_H^2 + q^2) - 2q^2 m_H^2 + m_H^4 - 5m_Z^4 + q^4 \right) B_0 (q^2, m_H^2, m_Z^2) + 3q^2 (16 (5s^2 - 2) m_W^2 + q^2 (4s^2 - 1)) B_0 (q^2, m_W^2, m_W^2) + 3A_0 (m_Z^2) \right\}$$

$$\Pi_{\gamma Z,\xi} = \frac{f_B}{\Lambda^2 \frac{\alpha}{96\pi csm_W^2}} \left\{ \left( m_W^2 + q^2 (2s^2 - 1) \right) (4q^2 m_W^2, \xi_W - q^4) B_0 (q^2, m_W^2, m_W^2, m_W^2) + 2 \left( m_W^2 + q^2 (2s^2 - 1) \right) (m_W^2, q^2, m_W^2, m_W^2) - \left( 8q^2 m_W^2 + q^4 \right) \left( m_W^2 + q^2 (2s^2 - 1) \right) B_0 (q^2, m_W^2, m_W^2) \right\}$$

$$\Pi_{ZZ,FG} = \frac{f_B}{\Lambda^2 \frac{\alpha}{144\pi c^2 q^2}} \left\{ \right\}$$
\[ (-3m_H^1 (m_Z^2 + q^2) + 6m_H^2 (-4q^2 m_Z^2 + m_Z^4 + q^4) - 3 (m_Z^4 - q^4) (m_Z^2 - q^2)) B_0 (q^2, m_H^2, m_Z^2) + 3q^4 (q^2 (1 - 2s^2) - 8 (5s^2 - 4) m_W^2) B_0 (q^2, m_W^2, m_Z^2) + 3A_0 (m_H^2) (m_Z^2 + q^2) (m_Z^2 - m_Z^2 + q^2) + 3 A_0 (m_Z^2) (-m_Z^2 (m_Z^2 + q^2) - 10q^2 m_Z^2 + m_Z^4 + q^4) + 6q^4 (2s^2 - 1) A_0 (m_Z^2) + 2q^2 (3m_H^2 (m_Z^2 + q^2) - 2q^2 ((3 - 6s^2) m_W^2 + q^2 s^2) + 2q^2 m_Z^2 + 3 m_Z^4) \} \]

\[ \Pi_{ZZ,\xi} = \frac{f_B}{\Lambda^2 48\pi q^2} \left\{ (m_Z^2 - q^2) (q^4 - 4q^2 m_W^2 \xi_W) B_0 (q^2, m_W^2 \xi_W, m_W^2 \xi_W) - 2 (m_Z^2 - q^2) (-2q^2 m_W^2 (\xi_W - 2) + m_W^4 (\xi_W + 4\xi_W - 5) + q^4) B_0 (q^2, m_W^2, m_W^2 \xi_W) + (8q^2 m_W^2 + q^4) (m_Z^2 - q^2) B_0 (q^2, m_W^2, m_W^2) + 2m_W^2 (\xi_W + 5) A_0 (m_W^2) (q^2 - m_Z^2) + 2m_W^2 \{ \xi_W + 5 \} (m_Z^2 - q^2) A_0 (m_W^2 \xi_W) \right\} \] (41)

\[ \Pi_{WW,FG} = \frac{f_B}{\Lambda^2 144\pi q^2} \left\{ 12 (m_W^3 - q^2 m_W^2)^2 B_0 (q^2, 0, m_W^2) - 3 (-5q^4 m_Z^2 - m_W^4 (7m_Z^2 + 8q^2) + 4q^2 m_Z^4) + 2m_W^2 (-14q^2 m_Z^2 + m_Z^4 + 2q^4) + 4 m_W^6 + m_W^6) B_0 (q^2, m_W^2, m_W^2) - 3m_Z^2 A_0 (m_W^2) (3m_W^2 + m_Z^2 + 5q^2) - 3A_0 (m_Z^2) (-m_W^2 (3m_Z^2 + 4q^2) + 5q^2 m_Z^2 + 4m_W^4 - m_Z^4) + 2q^2 m_Z^2 (-3m_W^2 + 3 m_Z^2 - q^2) \right\} \] (42)

\[ \Pi_{WW,\xi} = \frac{f_B}{\Lambda^2 48\pi q^2} \left\{ (m_W^2 - q^2)^2 (m_W^2 + 5q^2) B_0 (q^2, 0, m_W^2) + (q^2 - m_W^2) (4q^2 m_W^2 \xi_W + m_W^4 \xi_W^2 - 5q^4) B_0 (q^2, 0, m_W^2 \xi_W) + (m_W^2 - q^2) (4q^2 m_W^2 \xi_W + m_W^4 \xi_W^2 + 4q^4 - 2m_W^2 \xi_W) + m_W^2 \xi_W^2 + m_Z^2 - 5q^4) \right\} \]

\[ \times B_0 (q^2, m_W^2, m_W^2 \xi_W) + (q^2 - m_W^2) (-2 m_W^2 (m_Z^2 - 2q^2) + 4q^2 m_Z^2 + m_W^4 + m_Z^4 - 5q^4) \]

\[ \times B_0 (q^2, m_W^2, m_W^2) + m_W^2 (\xi_W - 1) A_0 (m_Z^2) (m_W^2 - q^2) + m_W^2 \{ m_W^2 - q^2 \} A_0 (m_W^2 \xi_W) + m_W^2 A_0 (m_W^2) (q^2 - m_W^2) \} \] (43)

\[ \mathcal{O}_W: \]

\[ \Pi_{\gamma\gamma,FG} = -\frac{f_W}{\Lambda^2 \sqrt{72\pi}} q^2 \alpha \left\{ 3(20m_W^2 + q^2) B_0 (q^2, m_W^2, m_W^2) - 6A_0 (m_W^2) + 2(q^2 - 6m_W^2) \right\} \] (45)

\[ \Pi_{\gamma\gamma,\xi} = \frac{f_W}{\Lambda^2 48\pi m_W^2} \left\{ -2q^2 (-2q^2 m_W^2 (\xi_W - 2) + m_W^4 (\xi_W^2 + 4\xi_W - 5) + q^4) B_0 (q^2, m_W^2, m_W^2 \xi_W) \right\} \]
\[ \Pi_{\gamma,FG} = \frac{f_w}{\Lambda^2} \sum_{\gamma Z,F G} \{ -3 \left( 4 m_{\gamma}^2 (m_h^2 + q^2) - 2 q^2 m_{t}^2 + m_{\gamma}^2 - 5 m_{\gamma}^2 + q^4 \right) B_0 (q, m_{\gamma}^2, m_{t}^2) \
-2 q^2 m_{\gamma}^2 (\xi_w + 5) A_0 (m_{\gamma}^2) + 2 q^2 m_{w}^2 (\xi_w + 5) A_0 (m_{w}^2 \xi_w) \} \quad (46) \]

\[ \Pi_{\gamma,\xi} = \frac{f_w}{\Lambda^2} \sum_{\gamma Z,\xi} \{ q^2 (2 q^2 - m_{\gamma}^2) (q^2 - 4 m_{\gamma}^2 \xi_w) B_0 (q, m_{\gamma}^2 \xi_w, m_{\gamma}^2 \xi_w) \\n+ (4 m_{\gamma}^2 (\xi_w - 1) (m_{\gamma}^2 (\xi_w + 2) - q^2 (\xi_w + 5)) + 4 q^2 m_{\gamma}^2 (m_{\gamma}^2 (\xi_w + 4) + 2 q^2 (\xi_w - 2)) - 4 (2 q^4 m_{\gamma}^2 + q^6) B_0 (q^2, m_{\gamma}^2, m_{\gamma}^2 \xi_w) \\n+ (9 q^4 m_{\gamma}^2 - 8 m_{\gamma}^2 (3 q^2 m_{\gamma}^2 - 2 q^4) + 2 q^6) B_0 (q^2, m_{w}^2, m_{w}^2) \\n+A_0 (m_{\gamma}^2) (m_{\gamma}^2 (4 m_{\gamma}^2 (\xi_w + 2) - 4 q^2 (\xi_w + 5)) - 10 q^2 m_{\gamma}^2) \\n+A_0 (m_{\gamma}^2 \xi_w) (m_{\gamma}^2 (4 q^2 (\xi_w + 5) - 4 m_{\gamma}^2 (\xi_w + 2)) + 10 q^2 m_{\gamma}^2) \\n-4 q^2 m_{\gamma}^2 \xi_w (\xi_w - 1) \} \quad (47) \]

\[ \Pi_{zz,FG} = \Pi_{zz,\xi} = \frac{f_w}{\Lambda^2} \sum_{\gamma Z,FG} \{ (m_{\gamma}^2 - q^2) (q^6 - 4 q^4 m_{w}^2 \xi_w) B_0 (q^2, m_{w}^2 \xi_w, m_{w}^2 \xi_w) \\n-2 (m_{\gamma}^2 - q^2) (5 q^4 m_{\gamma}^2 - 2 q^2 m_{w}^2 (2 m_{\gamma}^2 (\xi_w + 1) + q^2 (\xi_w - 2)) + m_{w}^2 (\xi_w - 1) (m_{\gamma}^2 (\xi_w - 1) + q^2 (\xi_w + 5)) + 4 q^2 (\xi_w + 5)) B_0 (q^2, m_{w}^2, m_{w}^2 \xi_w) \\n+ (m_{\gamma}^2 - q^2) (10 q^4 m_{\gamma}^2 + 8 q^2 m_{w}^2 (q^2 - 2 m_{\gamma}^2) + q^6) B_0 (q^2, m_{w}^2, m_{w}^2) \\n-2 A_0 (m_{w}^2) (m_{\gamma}^2 - q^2) (m_{w}^2 (m_{\gamma}^2 (1 - \xi_w) + q^2 (\xi_w + 5)) + 5 q^2 m_{\gamma}^2) \\n-2 A_0 (m_{w}^2) (m_{\gamma}^2 - q^2) (m_{w}^2 (m_{\gamma}^2 (1 - \xi_w) + q^2 (\xi_w + 5)) + 5 q^2 m_{\gamma}^2) \} \quad (48) \]
\[ \Pi_{WW,FG} = - \frac{f_w}{\Lambda^2} \frac{\alpha}{144 \pi q^2 s^2} \{ (m^2_W - q^2)^2 (2 m^2_W (5 q^2 - m^2_Z \xi_Z) + (q^2 - m^2_Z \xi_Z)^2 + m^4_W) B_0 (q^2, m^2_W, m^2_Z \xi_Z) \\
- q^2 (q^2 - m^2_W) (-2 q^2 (m^2_W \xi_W + m^2_Z \xi_Z) + (m^2_W \xi_W - m^2_Z \xi_Z)^2 + q^4) \\
\times B_0 (q^2, m^2_W, m^2_Z \xi_Z) \\
- (m^2_W - q^2) (m^2_W - m^2_Z) (4 q^2 m^2_W \xi_W + m^4_W \xi_Z - 5 q^4) B_0 (q^2, 0, m^2_W) \\
+ (m^2_W - q^2) (4 q^2 m^2_W + m^4_W - 5 q^4) (m^2_W - m^2_Z) B_0 (q^2, 0, m^2_W) \\
+ (m^2_W - q^2) (-4 q^4 m^2_Z - m^4_W \xi_W (2 m^2_Z + q^2 (\xi_W - 4)) + 5 q^2 m^4_Z \\
+m^2_W (-4 q^2 m^2_Z (\xi_W - 1) + m^4_Z + q^4 (2 \xi_W - 5)) + m^6_W \xi_Z - q^6) B_0 (q^2, m^2_Z, m^2_W \xi_Z) \\
- (q^2 - m^2_W) (4 q^4 m^2_Z + m^4_W (4 m^2_Z - 13 q^2) - 5 q^2 m^4_Z - 2 m^2_W (-q^2 m^2_Z + m^2_Z - 7 q^4) \\
- 2 m^6_W + q^6) B_0 (q^2, m^2_W, m^2_Z) \\
+ m^2_W A_0 (m^2_Z) (m^2_W - q^2) (m^2_W (\xi_W - 2) + m^2_Z + q^2 (- (\xi_W + 10))) \\
+ m^2_W (m^2_W - q^2) A_0 (m^2_W \xi_Z) (m^2_W - m^2_Z \xi_Z + q^2 (\xi_W + 10)) \\
- m^2_Z (q^2 - m^2_W) A_0 (m^2_W \xi_Z) (m^2_W (1 - \xi_W) + q^2 (\xi_Z + 10)) \\
+ m^2_A_0 (m^2_W) (q^2 - m^2_W) (m^2_W (1 - \xi_Z) + q^2 (\xi_Z + 10)) \\
+ 2 q^2 m^2_W m^2_Z (q^2 - m^2_W) (\xi_W + \xi_Z - 2) \} \] 

(52)
\[ O_{BB}: \]
\[
\Pi_{\gamma\gamma,FG} = -\frac{f_{BB}}{\Lambda^2} \frac{\alpha}{4\pi m_H^2} q^2 \left\{ m_H^2 A_0 \left( m_H^2 \right) + 6m_W^2 A_0 \left( m_W^2 \right) + 3m_Z^2 A_0 \left( m_Z^2 \right) \right\} -2 \left( 2m_W^4 + m_Z^4 \right) \}
\]
\[
\Pi_{\gamma Z,FG} = \frac{f_{BB}}{\Lambda^2} \frac{\alpha}{4\pi c sm_H^2} \left( e^2 - 1 \right) \left\{ m_H^2 m_Z^2 \left( -m_H^2 + m_Z^2 + q^2 \right) B_0 \left( q^2, m_H^2, m_Z^2 \right) -m_Z^2 A_0 \left( m_Z^2 \right) \left( m_H^2 + 3q^2 \right) + m_H^2 A_0 \left( m_H^2 \right) \left( m_Z^2 - q^2 \right) -6q^2 m_W^2 A_0 \left( m_W^2 \right) +2q^2 \left( 2m_W^4 + m_Z^4 \right) \right\}
\]
\[
\Pi_{ZZ,FG} = \frac{f_{BB}}{\Lambda^2} \frac{\alpha}{4\pi c^2 m_H^2} \left( 1 - e^2 \right) \left\{ 2m_H^2 m_Z^2 \left( -m_H^2 + m_Z^2 + q^2 \right) B_0 \left( q^2, m_H^2, m_Z^2 \right) +m_H^2 A_0 \left( m_H^2 \right) \left( 2m_Z^2 - q^2 \right) -m_Z^2 A_0 \left( m_Z^2 \right) \left( 2m_H^2 + 3 \left( m_Z^2 + q^2 \right) \right) -6q^2 m_W^4 A_0 \left( m_W^2 \right) +2 \left( 2q^2 m_W^4 + q^2 m_Z^4 + m_Z^6 \right) \right\}
\]
\[
\Pi_{WW,FG} = -\frac{f_{BB}}{\Lambda^2} \frac{\alpha}{4\pi c^2 m_H^2} \left( e^2 - 1 \right) \left\{ 2m_Z^2 \left( 2m_Z^2 - q^2 \right) -3 A_0 \left( m_Z^2 \right) \right\}
\]

\[ O_{WW}: \]
\[
\Pi_{\gamma\gamma,FG} = -\frac{f_{WW}}{\Lambda^2} \frac{\alpha}{4\pi m_H^4} q^2 \left\{ m_H^2 A_0 \left( m_H^2 \right) + 6m_W^4 A_0 \left( m_W^2 \right) +3m_Z^2 A_0 \left( m_Z^2 \right) \right\} -2 \left( 2m_W^4 + m_Z^4 \right) \}
\]
\[
\Pi_{\gamma Z,FG} = \frac{f_{WW}}{\Lambda^2} \frac{\alpha}{4\pi s m_H^4} \left( e^2 - 1 \right) \left\{ m_H^2 m_Z^2 \left( -m_H^2 + m_Z^2 + q^2 \right) B_0 \left( q^2, m_H^2, m_Z^2 \right) -m_Z^2 A_0 \left( m_Z^2 \right) \left( m_H^2 + 3q^2 \right) + m_H^2 A_0 \left( m_H^2 \right) \left( m_Z^2 - q^2 \right) -6q^2 m_W^4 A_0 \left( m_W^2 \right) +2q^2 \left( 2m_W^4 + m_Z^4 \right) \right\}
\]
\[
\Pi_{ZZ,FG} = \frac{f_{WW}}{\Lambda^2} \frac{\alpha}{4\pi s m_H^2} \left( 1 - e^2 \right) \left\{ 2m_H^2 m_Z^2 \left( -m_H^2 + m_Z^2 + q^2 \right) B_0 \left( q^2, m_H^2, m_Z^2 \right) +m_H^2 A_0 \left( m_H^2 \right) \left( 2m_Z^2 - q^2 \right) -m_Z^2 A_0 \left( m_Z^2 \right) \left( 2m_H^2 + 3 \left( m_Z^2 + q^2 \right) \right) -6A_0 \left( m_W^2 \right) \left( q^2 m_W^4 + m_Z^4 \right) +2 \left( 2q^2 m_W^4 + q^2 m_Z^4 + m_Z^6 \right) \right\}
\]
\[
\Pi_{WW,FG} = -\frac{f_{WW}}{\Lambda^2} \frac{\alpha}{4\pi s^2 m_H^4} \left\{ -2m_Z^2 m_W^2 \left( -m_H^2 + m_W^2 + q^2 \right) B_0 \left( q^2, m_H^2, m_W^2 \right) +m_H^2 A_0 \left( m_H^2 \right) \left( q^2 - 2m_W^2 \right) +2m_W^2 A_0 \left( m_W^2 \right) \left( m_H^2 + 3 \left( m_W^2 + q^2 \right) \right) +3A_0 \left( m_Z^2 \right) \left( q^2 m_Z^2 + m_W^2 \right) -2 \left( m_W^2 + 2q^2 \right) +q^2 m_Z^4 + m_W^4 \right\}
\]

\[ O_{WWW}: \]
\[
\Pi_{\gamma\gamma,FG} = f_{WWW} \frac{q^2 s^2 g_W^4}{\Lambda^2} \left\{ q^2 - 4m_W^2 \right\} B_0 \left( q^2, m_W^2, m_W^2 \right) -6A_0 \left( m_W^2 \right) +6 m_W^2 - q^2 \}
\]

(53) (54) (55) (56) (57) (58) (59) (60) (61) (62)
\[ \Pi_{\gamma Z, FG} = -\frac{f_{WW} c q^2 s g^4_w}{\Lambda^2} \frac{1}{16\pi^2} \left\{ (12m^2_W - 3q^2) \, B_0 \left( q^2, m^2_W, m^2_W \right) + 6A_0 \left( m^2_W \right) - 6 \, m^2_W + q^2 \right\} \]  
\[ \Pi_{ZZ, FG} = \frac{f_{WW} c q^2 s g^4_w}{\Lambda^2} \frac{1}{16\pi^2} \left\{ (12m^2_W - 3q^2) \, B_0 \left( q^2, m^2_W, m^2_W \right) + 6A_0 \left( m^2_W \right) - 6 \, m^2_W + q^2 \right\} \]  
\[ \Pi_{WW, FG} = -\frac{f_{WW} g^4_w}{\Lambda^2} \frac{1}{16\pi^2} \left\{ -3s^2 \left( m^2_W - q^2 \right)^2 \, B_0 \left( q^2, 0, m^2_W \right) + 3 \left( -m^2_W \left( m^2_Z + 2q^2 \right) \left( s^2 - 2 \right) \right) \right. \\
+ \left( s^2 + 1 \right) m^4_W + q^4 \left( s^2 - 1 \right) \, B_0 \left( q^2, m^2_W, m^2_Z \right) \\
\left. + 3A_0 \left( m^2_Z \right) \left( s^2 m^2_W - q^2 \left( s^2 - 1 \right) \right) + 3 q^2 A_0 \left( m^2_W \right) - 6q^2 m^2_W + q^4 \right\} \]  
\[ \mathcal{O}_{\Phi, 2}:^7 \]
\[ \Pi_{ZZ} = \frac{f_{\Phi, 2} \, m^2_Z}{\Lambda^2} \frac{1}{144\pi^2 q^2} \left[ 3 \left( m^4_H - 2m^2_H(m^2_Z + q^2) \right) + m^4_Z + 10m^2_Z q^2 + q^4 \right] \, B_0 \left( q^2, m^2_H, m^2_Z \right) \\
- 3(m^2_H - m^2_Z - 2q^2) A_0(m^2_H) + 3(m^2_H - m^2_Z - q^2) A_0(m^2_Z) - 2q^2(3m^2_H + 3m^2_Z - q^2) \]  
\[ \Pi_{WW} = \frac{f_{\Phi, 2} \, m^2_W}{\Lambda^2} \frac{1}{144\pi^2 q^2} \left[ 3 \left( m^4_H - 2m^2_H(m^2_W + q^2) \right) + m^4_W + 10m^2_W q^2 + q^4 \right] \, B_0 \left( q^2, m^2_H, m^2_W \right) \\
- 3(m^2_H - m^2_W - 2q^2) A_0(m^2_H) + 3(m^2_H - m^2_W - q^2) A_0(m^2_W) - 2q^2(3m^2_H + 3m^2_W - q^2) \]  

Appendix C: Pinch Terms in $R_\xi$ Gauge

Analytic results for $\Gamma^V_L$ as defined in Eq. [9] are given here in $R_\xi$ gauge and contain both a pinch contribution and a non-pinch contribution,

\[ \Gamma^V_L = \Gamma^V_L(pinch) + \Gamma^V_L(non - pinch). \]  

The "pinch" contribution is defined as in Refs. [37, 38], to be the contribution of the 3-point interaction which exactly cancels the internal fermion propagator. We note that $\Gamma^V_L(non - pinch)$ is gauge invariant. Ref. [24, 25] included this non-pinch contribution in their definition of $\Pi_{XY}$. From here on, we use $\Gamma^V_L$ to denote the pinch contribution only,

\[ \Pi_{ZZ} \quad \Pi_{WW} \quad O_{\Phi, 2} \]

\[ \mathcal{O}_{\Phi, 2}:^7 \]  

The contributions to $\Pi_{ZZ}$ and $\Pi_{WW}$ from $O_{\Phi, 2}$ are gauge independent without the addition of tadpole diagrams. Since the tadpole diagrams cancel in the calculation of the oblique parameters, we do not include them for the $O_{\Phi, 2}$ 2-point functions. The tadpole contributions are included in the 2-point functions listed above for all other operators.
as is used in the definition of the oblique parameters in Ref. [28]. We use different gauge parameters $\xi_W$ and $\xi_Z$ for the W and Z bosons, respectively, and they cancel separately in our final result. $\Delta \Gamma^V_L$ can be written as

$$
\Delta \Gamma^V_L = \Delta \Gamma^V_{L,FG} + \Delta \Gamma^V_{L,\xi},
$$

where $V$ represents $\gamma$, $Z$ and $W$. The first term $\Pi_{XY,FG}$ contains results in Feynman gauge and is independent of $\xi$. The second term collects terms that vanish when $\xi = 1$. In the following, we express our results in terms of scalar integral functions $A_0$ and $B_0$. Only non-zero pinch contributions are listed here, and we separate the contributions in terms proportional to each of the individual $f_i$.

$O_B$:  

$$
\Delta \Gamma^\gamma_{L,\xi} = \frac{f_B}{\Lambda^2 96\pi s m_W^2} \left\{ q^2 (8 m_W^2 + q^2) \left( -B_0 \left( q^2, m_W^2, m_W^2 \right) + (4 q^2 m_W^2 \xi_W - q^4) B_0 \left( q^2, m_W^2 \xi_W, m_W^2 \xi_W \right) \right) + 2 \left( -2 q^2 m_W^2 \left( \xi_W - 2 \right) + m_W^4 \left( \xi_W^2 + 4 \xi_W - 5 \right) + q^4 \right) B_0 \left( q^2, m_W^2, m_W^2 \xi_W \right) + 2 m_W^2 \left( \xi_W + 5 \right) A_0 \left( m_W^2 \right) - 2 m_W^2 \left( \xi_W + 5 \right) A_0 \left( m_W^2 \xi_W \right) \right\}
$$

$$
\Delta \Gamma^Z_{L,\xi} = \frac{f_B}{\Lambda^2 96\pi c m_W^2} \left\{ (q^4 - 4 q^2 m_W^2 \xi_W) B_0 \left( q^2, m_W^2 \xi_W, m_W^2 \xi_W \right) + \left( 4 q^2 m_W^2 \xi_W - 2 m_W^4 \left( \xi_W^2 + 4 \xi_W - 5 \right) - 2 q^4 \right) B_0 \left( q^2, m_W^2, m_W^2 \xi_W \right) + \left( 8 q^2 m_W^2 + q^4 \right) B_0 \left( q^2, m_W^2, m_W^2 \right) - 2 m_W^2 \left( \xi_W + 5 \right) A_0 \left( m_W^2 \right) \right\}
$$

$$
\Delta \Gamma^W_{L,FG} = \frac{-f_B}{\Lambda^2 8 \pi} c^2 m_Z^2 \left( B_0 \left( q^2, 0, m_W^2 \right) - B_0 \left( q^2, m_W^2, m_W^2 \right) \right) \tag{72}
$$

$$
\Delta \Gamma^W_{L,\xi} = \frac{f_B}{\Lambda^2 96\pi q^2} \left\{ \left( -4 q^2 m_W^2 \xi_W - m_W^4 \xi_W^2 + 5 q^4 \right) B_0 \left( q^2, 0, m_W^2 \xi_W \right) + \left( 4 q^2 m_W^2 \xi_W + m_Z^2 \left( 4 q^2 - 2 m_W^2 \xi_W \right) + m_W^4 \xi_W^2 + m_Z^2 - 5 q^4 \right) B_0 \left( q^2, m_Z^2, m_W^2 \xi_W \right) + \left( -4 q^2 m_W^2 - 4 q^2 m_Z^2 + 2 m_W^2 m_Z^2 - m_W^4 - m_Z^4 + 5 q^4 \right) B_0 \left( q^2, m_Z^2, m_Z^2 \right) + \left( 4 q^2 m_W^2 + m_W^4 - 5 q^4 \right) B_0 \left( q^2, 0, m_W^2 \right) + m_W^2 A_0 \left( m_W^2 \xi_W \right) - m_Z^2 A_0 \left( m_W^2 \right) \right\}
$$

$O_W$:  

$$
\Delta \Gamma^\gamma_{L,\xi} = \frac{f_W}{\Lambda^2 96\pi s m_W^2} \left\{ q^2 (8 m_W^2 + q^2) \left( -B_0 \left( q^2, m_W^2, m_W^2 \right) \right) \right\}
$$
\[ \Delta \Gamma^Z_{L,FG} = -\frac{f_w}{A^2} \frac{\alpha}{4\pi s^2} c m_Z^2 B_0 (q^2, m_W^2, m_W^2) \]  
\[ \Delta \Gamma^Z_{L,\xi} = \frac{f_w}{A^2} \frac{\alpha}{96\pi q^2 m_W (c m_W - m_Z)} \left\{ (q^6 - 4q^4 m_W^2 \xi_w) B_0 (q^2, m_W^2, m_W^2 \xi_w) + \right. \]  
\[ + (2 - 2 m_W^2 (\xi_w - 2) + m_W^4 (\xi_w^2 + 4 \xi_w - 5) + q^4) B_0 (q^2, m_W^2, m_W^2 \xi_w) \]  
\[ + 2 m_W^2 (\xi_w + 5) A_0 (m_W^2) - 2 m_W^2 (\xi_w + 5) A_0 (m_W^2 \xi_w) \right\} \]  
\[ (74) \]  
\[ \Delta \Gamma^W_{L,FG} = \frac{f_w}{A^2} \frac{\alpha}{8\pi s^2} c m_Z^2 \{ (s^2 - 2) B_0 (q^2, m_W^2, m_Z^2) - s^2 B_0 (q^2, 0, m_W^2) \} \]  
\[ \Delta \Gamma^W_{L,\xi} = \frac{f_w}{A^2} \frac{\alpha}{96\pi q^2 s^2 m_Z^2} \{ \]  
\[ (m_Z^2 - q^2) (2 m_W^2 (5 q^2 - m_Z^2 \xi_z) + q^2 - m_Z^2 \xi_z)^2 + m_W^4) B_0 (q^2, m_W^2, m_Z^2 \xi_z) \]  
\[ + q^2 (-2 m_W^2 (\xi_w + m_Z^2 \xi_z) + (m_W^2 \xi_w - m_Z^2 \xi_z)^2 + q^4) B_0 (q^2, m_W^2, m_Z^2 \xi_z) \]  
\[ - (m_Z^2 - m_W^2) (4 q^2 m_W^2 \xi_w + m_W^4 \xi_w^2 - 5 q^4) B_0 (q^2, 0, m_W^2 \xi_w) \]  
\[ + (4 q^2 m_W^2 + m_W^4 - 5 q^4) (m_Z^2 - m_W^2) B_0 (q^2, 0, m_W^2) \]  
\[ + (-4 q^4 m_Z^2 - m_W^4 \xi_w (2 m_Z^2 + q^2 (\xi_w - 4)) + 5 q^2 m_z^2 \]  
\[ + m_Z^2 (-4 q^2 m_Z^2 (\xi_w - 1) + m_Z^4 + q^4 (2 \xi_w - 5)) + m_Z^6 \xi_w^2 - q^6) B_0 (q^2, m_Z^2, m_Z^2 \xi_w) \]  
\[ + (4 q^4 m_Z^2 + m_Z^4 (4 m_Z^2 - 13 q^2) - 5 q^2 m_Z^4 - 2 m_Z^2 (-q^2 m_Z^2 + m_Z^4 - 7 q^4) - 2 m_Z^6 + q^6) \]  
\[ \times B_0 (q^2, m_Z^2, m_Z^2) + m_Z^2 A_0 (m_Z^2) (m_Z^2 (\xi_w - 2) + m_z^2 + q^2 (- (\xi_w + 10)) \]  
\[ + m_Z^2 A_0 (m_Z^2) (m_Z^2 (\xi_w - 1) - q^2 (\xi_w + 10)) \]  
\[ + m_Z^2 A_0 (m_Z^2 (\xi_w + \xi_w + q^2 (\xi_w + 10)) - 2 q^2 m_Z^2 m_Z^2 (\xi_w + \xi_w + 2) \} \]  
\[ (76) \]  
\[ O_{www}: \]  
\[ \Delta \Gamma^\gamma_{L,FG} = -\frac{f_{WWW}}{A^2} \frac{3\alpha}{8\pi s} q^2 g_w^2 B_0 (q^2, m_W^2, m_W^2) \]  
\[ (79) \]
\[ \Delta \Gamma_{L,FG}^Z = -\frac{f_{WWW}}{\Lambda^2} \frac{3\alpha}{8\pi s^2 c} q^2 g^2_B_0 (q^2, m_W^2, m_Z^2) \]  
\[ \Delta \Gamma_{L,FG}^W = \frac{f_{WWW}}{\Lambda^2} \frac{-3\alpha}{8\pi s^2 q^2 g^2_w (c^2 B_0 (q^2, m_W^2, m_Z^2)) + s^2 B_0 (q^2, 0, m_W^2))} \]  

Appendix D: Analytic Results for Oblique Parameters

The finite contributions to the oblique parameters are,

\[ R_{S1} = \frac{m_Z^2}{24\pi \Lambda^2} \left\{ \frac{29}{3} (5c^2 + 1) f_B - c^2 (64c^2 + 15) f_w + 72 (s^2 f_{BB} + c^2 f_{WW}) + 72c^2 f_{WWW} g^2 
+ 3 \left( \frac{m_H}{m_Z} \right)^2 (c^2 f_B + s^2 f_w - 8c^2 f_{WW} - 8s^2 f_{BB}) + \left( \frac{m_H}{m_Z} \right)^4 [(2c^2 - 3) f_B + (2s^2 - 3) f_w] 
+ [ -48 (s^2 f_{BB} + c^2 f_{WW}) + 2 \left( \frac{m_H}{m_Z} \right)^2 [(2s^2 + 1) f_B + (2c^2 + 1) f_w + 12 (s^2 f_{BB} + c^2 f_{WW})] 
+ \left( \frac{m_H}{m_Z} \right)^4 [(2s^2 + 1) f_B + (2c^2 + 1) f_w] \right. \left( \frac{m_H}{m_Z} \right) \sqrt{4 - \left( \frac{m_H}{m_Z} \right)^2} \cos^{-1} \left( \frac{m_H}{2m_Z} \right) 
-2 \left[ (32c^2 + 1) f_B + 36c^2 f_{WWW} g^2 - (32c^4 + 8c^2 - 1) f_w \right] \sqrt{4c^2 - 1} \sin^{-1} \left( \frac{1}{2c} \right) \right\} 
+ f_{\phi,2} \frac{m_W^2 s^2}{\Lambda^2} 72\pi^2 \alpha \left[ 79 - 27 \left( \frac{m_H}{m_Z} \right)^2 + 6 \left( \frac{m_H}{m_Z} \right)^4 -6 \left( 12 - 4 \left( \frac{m_H}{m_Z} \right)^2 + \left( \frac{m_H}{m_Z} \right)^4 \right) \left( \frac{m_H}{m_Z} \right) \sqrt{4 - \left( \frac{m_H}{m_Z} \right)^2} \cos^{-1} \left( \frac{m_H}{2m_Z} \right) \right] \]  

\[ R_{S2} = \frac{m_Z^2}{12\pi \Lambda^2} \left\{ (1 - 3c^2) f_B - (1 - 10c^2) f_w - 36c^2 g^2 f_{WWW} \right\} \]

\[ R_{S3} = \frac{m_H^4}{24\pi \Lambda^2} \left\{ \left( f_B + f_w \right) \left( \frac{m_H}{m_Z} \right)^4 \right. \left( m_Z^2 - 12 + 2 (s^2 f_B + c^2 f_w) \right) \left( \frac{m_H}{m_Z} \right)^4 + 24 (s^2 f_{BB} + c^2 f_{WW}) \left( \frac{m_H^4}{m_Z^4} - 12 - 18 \frac{m_H^2}{m_Z^2} - m_Z^2 \right) \right\} 
- f_{\phi,2} \frac{m_W^2 s^2 c^2}{\Lambda^2} \frac{1}{12\pi^2 \alpha \left( m_H^2 - m_Z^2 \right)} \frac{m_H^2}{m_Z^2} \]  

\[ R_{T1} = \frac{m_Z^2}{32\pi \Lambda^2} \left\{ 15f_B + \left( \frac{3 - 23c^2}{s^2} \right) f_w \right\} + \frac{5m_H^2}{32\pi c^2 \Lambda^2} f_B - 5m_Z^2 g^2 f_{\phi,2} \]

\[ R_{T2} = \frac{m_W^2}{8\pi s^2 \Lambda^2} \left\{ 5f_B - 3 \left( \frac{m_W^2}{m_H^2 - m_W^2} + \frac{2 (c^2 + 4)}{3s^2} \right) f_w \right\} + \frac{3m_W^4}{8\pi^2 \alpha \Lambda^2 (m_H^2 - m_W^2)} f_{\phi,2} \]

\[ R_{T3} = -\frac{3}{8\pi^2 \Lambda^2 (m_H^2 - m_Z^2)} \left\{ f_B + \left( \frac{m_W^2}{m_H^2 - m_Z^2} f_w \right) \right\} + \frac{3m_H^4}{8\pi^2 \alpha \Lambda^2 (m_H^2 - m_Z^2) (m_H^2 - m_W^2)} f_{\phi,2} \]

\[ R_{U1} = \frac{m_Z^2}{24\pi \Lambda^2 c^4} \left\{ 48c^6 f_{WWW} - \left( \frac{m_H}{m_Z} \right)^4 \right. \left( 2s^2 c^4 f_B + 2 (c^4 + e^6) f_w + 24c^6 f_{WWW} \right) \]
\[- \left( \frac{m_H}{m_Z} \right)^4 \left( s^2 c^4 f_B + (c^4 + c^6) f_W \right) \left( \frac{m_H}{m_Z} \right) \sqrt{4 - \left( \frac{m_H}{m_Z} \right)^2} \cos^{-1} \left( \frac{m_H}{2m_Z} \right) \]

\[+ 4 \left[ -24 c^4 f_{WW} + 2 \left( \frac{m_H}{m_Z} \right)^2 c^2 \left( f_W + 6 f_{WW} \right) + \left( \frac{m_H}{m_Z} \right)^4 f_W \right] \]

\[\times \left( \frac{m_H}{m_Z} \right) \sqrt{4c^2 - \left( \frac{m_H}{m_Z} \right)^2} \cos^{-1} \left( \frac{m_H}{2m_W} \right) \]

\[- 2 \left[ (80 c^8 + 116 c^6 + 90 c^4 + 22 c^2 - 11) f_W + 216 f_{WW} g^2 \right] \sqrt{4c^2 - 1} \sin^{-1} \left( \frac{1}{2c} \right) \]

\[+ 2 \left[ (40 c^4 + 6 c^2 + 1) \frac{s^2}{c^2} f_B + 36 c^4 \left( 4 c^2 - 1 \right) f_{WW} g^2 \right] \sqrt{4c^2 - 1} \cos^{-1} \left( \frac{1}{2c} \right) \]

\[+ \left[ \frac{2}{3} c^2 \left( 240 c^6 - 121 c^4 - 62 c^2 + 33 \right) + 11 \pi \sqrt{4c^2 - 1} \left( 8 c^4 + 2 c^2 - 1 \right) \right] f_W \]

\[- (84 c^6 - 95 c^4 + 9 c^2 + 2) f_B + 96 s^2 c^4 f_{WW} g^2 \]

\[+ 3 \left( \frac{m_H}{m_Z} \right)^2 s^2 c^4 \left( f_B - f_W - 16 f_{WW} \right) + 2 \left( \frac{m_H}{m_Z} \right)^4 s^2 c^2 \left[ c^2 f_B - (c^2 + 2) f_W \right] \}

\[+ \frac{f_{\phi,2} m_Z^2 s^2}{\Lambda^2 24 \pi^2 \alpha c^4} \left( \frac{m_H}{m_Z} \right) \left[ -9 c^4 s^2 \left( \frac{m_H}{m_Z} \right) + 2 c^2 s^2 \left( 1 + c^2 \right) \left( \frac{m_H}{m_Z} \right)^3 \right] \]

\[+ 2 c^6 \left[ \left( \frac{m_H}{m_Z} \right)^4 - 4 \left( \frac{m_H}{m_Z} \right)^2 + 12 \right] \sqrt{4 - \left( \frac{m_H}{m_Z} \right)^2} \cos^{-1} \left( \frac{m_H}{2m_Z} \right) \]

\[- 2 \left[ 12 c^4 - 4 c^2 \left( \frac{m_H}{m_Z} \right)^2 + \left( \frac{m_H}{m_Z} \right)^4 \right] \sqrt{4c^2 - \left( \frac{m_H}{m_Z} \right)^2} \cos^{-1} \left( \frac{m_H}{2m_W} \right) \]

\[
R_{U2} = \frac{1}{12 \pi^2 \Lambda^2 m_W^2} \left\{ -24 c^2 f_{WW} m_w^2 \left( 2 m_W^4 - 4 m_H^2 m_w^2 + m_H^4 \right) + 36 \left( 1 - 6 c^2 \right) f_{WW} g^2 m_Z^2 m_W^4 \right. \]

\[+ (34 c^8 - 118 c^6 + 58 c^4 - 3 c^2 - 1) f_B m_Z^6 \]

\[+ \frac{c^2 f_W}{s^2 \left( m_W^2 - m_H^2 \right)} \left( 2 m_H^2 s^2 \left( -3 m_w^6 - 6 m_H^2 m_W^4 - m_H^4 m_W^2 + m_H^6 \right) \right) \]

\[+ \left( 18 c^8 - 146 c^6 + 24 c^4 + 55 c^2 - 11 \right) m_W^6 \left( m_H^2 - m_W^2 \right) \}

\[+ \frac{m_Z^2 s^2}{12 \pi^2 \alpha \Lambda^2 c^4 \left( m_H^2 - m_W^2 \right)} \left[ 2 c^8 - 38 c^6 \left( \frac{m_H}{m_Z} \right)^2 + 24 c^4 \left( \frac{m_H}{m_Z} \right)^4 - 7 c^2 \left( \frac{m_H}{m_Z} \right)^6 \right] \]

\[+ \left( \frac{m_H}{m_Z} \right)^8 \right\} f_{\phi,2} \]

\[
R_{U3} = \frac{m_Z^2 s^2}{12 \pi \Lambda^2} \left\{ f_{WW} \frac{24 \left( (c^2 + 1) m_H^2 - 4 m_W^2 \right)}{m_W^2} + f_W \left[ \frac{18 m_Z^2 m_W^2}{(m_H^2 - m_Z^2)(m_H^2 - m_W^2)} \right] \right. \]
\begin{align}
\left. + \left( \frac{c^4 + 2c^2 + 2}{m_W^4} m_H^4 - 6 \right) + f_B \frac{m_H^6 - m_W^2 m_Z^2 - 6m_H^2 m_Z^4 - 12m_Z^6}{m_Z^6 - m_H^2 m_Z^4} \right) \\
+ \frac{m_Z^8}{12\pi^2 \alpha^2 \Lambda^2 c^4 (m_H^2 - m_W^2)} \left( \frac{m_H}{m_Z} \right)^2 \left[ -(c^4 + c^2 + 1) \left( \frac{m_H}{m_Z} \right)^8 \\
+ (c^6 + 8c^4 + 8c^2 + 1) \left( \frac{m_H}{m_Z} \right)^6 - c^2 (7c^4 + 31c^2 + 7) \left( \frac{m_H}{m_Z} \right)^4 \\
+ 24c^4 (c^2 + 1) \left( \frac{m_H}{m_Z} \right)^2 - 36c^6 \right] f_{\Phi,2} \tag{83} \end{align}
[1] M. Baak, M. Goebel, J. Haller, A. Hoecker, D. Kennedy, et al. The Electroweak Fit of the Standard Model after the Discovery of a New Boson at the LHC. *Eur.Phys.J.*, C72:2205, 2012.

[2] W. Buchmuller and D. Wyler. Effective Lagrangian Analysis of New Interactions and Flavor Conservation. *Nucl.Phys.*, B268:621, 1986.

[3] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek. Dimension-Six Terms in the Standard Model Lagrangian. *JHEP*, 1010:085, 2010.

[4] Zhenyu Han and Witold Skiba. Effective theory analysis of precision electroweak data. *Phys.Rev.*, D71:075009, 2005.

[5] Kaoru Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld. Low-energy effects of new interactions in the electroweak boson sector. *Phys.Rev.*, D48:2182–2203, 1993.

[6] Kaoru Hagiwara, R. Szalapski, and D. Zeppenfeld. Anomalous Higgs boson production and decay. *Phys.Lett.*, B318:155–162, 1993.

[7] J. Bagger, S. Dawson, and G. Valencia. Effective field theory calculation of p p to VVX. *Nucl.Phys.*, B399:364–394, 1993.

[8] S. Alam, S. Dawson, and R. Szalapski. Low-energy constraints on new physics revisited. *Phys.Rev.*, D57:1577–1590, 1998.

[9] M.C. Gonzalez-Garcia. Anomalous Higgs couplings. *Int.J.Mod.Phys.*, A14:3121–3156, 1999.

[10] Christophe Grojean, Elizabeth E. Jenkins, Aneesh V. Manohar, and Michael Trott. Renormalization Group Scaling of Higgs Operators and h to gamma gamma). *JHEP*, 1304:016, 2013.

[11] Elizabeth E. Jenkins, Aneesh V. Manohar, and Michael Trott. Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence. 2013.

[12] Elizabeth E. Jenkins, Aneesh V. Manohar, and Michael Trott. Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and lambda Dependence. *JHEP*, 1310:087, 2013.

[13] J. Elias-Miro, J.R. Espinosa, E. Masso, and A. Pomarol. Higgs windows to new physics through d = 6 operators: Constraints and one-loop anomalous dimensions. 2013.

[14] J. Elias-Miro, J.R. Espinosa, E. Masso, and A. Pomarol. Renormalization of dimension-six
operators relevant for the Higgs decays $h \rightarrow \gamma\gamma, \gamma Z$. *JHEP*, 1308:033, 2013.

[15] Tyler Corbett, O.J.P. Eboli, J. Gonzalez-Fraile, and M.C. Gonzalez-Garcia. Constraining anomalous Higgs interactions. *Phys.Rev.*, D86:075013, 2012.

[16] Eduard Masso and Veronica Sanz. Limits on Anomalous Couplings of the Higgs to Electroweak Gauge Bosons from LEP and LHC. *Phys.Rev.*, D87(3):033001, 2013.

[17] Roberto Contino, Margherita Ghezzi, Christophe Grojean, Margarete Muhlleitner, and Michael Spira. Effective Lagrangian for a light Higgs-like scalar. *JHEP*, 1307:035, 2013.

[18] Beranger Dumont, Sylvain Fichet, and Gero von Gersdorff. A Bayesian view of the Higgs sector with higher dimensional operators. *JHEP*, 1307:065, 2013.

[19] Adam Alloul, Benjamin Fuks, and Veronica Sanz. Phenomenology of the Higgs Effective Lagrangian via FeynRules. 2013.

[20] Antonio Pich, Ignasi Rosell, and Juan Jose Sanz-Cillero. Oblique S and T Constraints on Electroweak Strongly-Coupled Models with a Light Higgs. 2013.

[21] M. Baak, A. Blondel, A. Bodek, R. Caputo, T. Corbett, et al. Study of Electroweak Interactions at the Energy Frontier. 2013.

[22] Shankha Banerjee, Satyanarayan Mukhopadhyay, and Biswarup Mukhopadhyaya. Higher dimensional operators and LHC Higgs data : the role of modified kinematics. 2013.

[23] We-Fu Chang, Wei-Ping Pan, and Fanrong Xu. An effective gauge-Higgs operators analysis of new physics associated with the Higgs. *Phys.Rev.*, D88:033004, 2013.

[24] Harrison Mebane, Nicolas Greiner, Cen Zhang, and Scott Willenbrock. Constraints on Electroweak Effective Operators at One Loop. *Phys.Rev.*, D88:015028, 2013.

[25] Harrison Mebane, Nicolas Greiner, Cen Zhang, and Scott Willenbrock. Effective Field Theory of Precision Electroweak Physics at One Loop. *Phys.Lett.*, B724:259–263, 2013.

[26] G.F. Giudice, C. Grojean, A. Pomarol, and R. Rattazzi. The Strongly-Interacting Light Higgs. *JHEP*, 0706:045, 2007.

[27] Anthony C. Longhitano. Low-Energy Impact of a Heavy Higgs Boson Sector. *Nucl.Phys.*, B188:118, 1981.

[28] G. Degrassi, Bernd A. Kniehl, and A. Sirlin. Gauge invariant formulation of the S, T, and U parameters. *Phys.Rev.*, D48:3963–3966, 1993.

[29] S. Dawson and G. Valencia. Bounds on anomalous gauge boson couplings from partial Z widths at LEP. *Nucl.Phys.*, B439:3–22, 1995.
[30] Tyler Corbett, O.J.P. Eboli, J. Gonzalez-Fraile, and M.C. Gonzalez-Garcia. Robust Determination of the Higgs Couplings: Power to the Data. Phys.Rev., D87:015022, 2013.

[31] Martin B Einhorn and Jose Wudka. Higgs-Boson Couplings Beyond the Standard Model. 2013.

[32] Marco Ciuchini, Enrico Franco, Satoshi Mishima, and Luca Silvestrini. Electroweak Precision Observables, New Physics and the Nature of a 126 GeV Higgs Boson. JHEP, 1308:106, 2013.

[33] Georges Aad et al. Measurements of Higgs boson production and couplings in diboson final states with the ATLAS detector at the LHC. Phys.Lett., B726:88–119, 2013.

[34] Serguei Chatrchyan et al. CMS-PAS-HIG-13-005.

[35] John F. Gunion, Howard E. Haber, Gordon L. Kane, and Sally Dawson. The Higgs Hunter’s Guide. Front.Phys., 80:1–448, 2000.

[36] G. Passarino and M.J.G. Veltman. One Loop Corrections for e+ e- Annihilation Into mu+ mu- in the Weinberg Model. Nucl.Phys., B160:151, 1979.

[37] Daniele Binosi and Joannis Papavassiliou. Pinch Technique: Theory and Applications. Phys.Rept., 479:1–152, 2009.

[38] S. Dawson and C.B. Jackson. Chiral-logarithmic Corrections to the S and T Parameters in Higgsless Models. Phys.Rev., D76:015014, 2007.