Edge Modes Waves in Superlattices in Quantum Hall Effect Regime

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Abstract

The wave propagation of edge modes in a superlattice of 2D electron Gases in quantum Hall regime is investigated. After introducing surfaces charge and current densities at the edge, the Maxwell equations are solved for waves running along the boundary. The constitutive relations expressing the edge charge and current densities in terms of the fields at the boundary are derived. One of them is similar to the London equation for superconductor currents. The dispersion relation and wave polarizations for the momenta region \( w/c < k \) are also obtained for propagation along the borderlines of the electron gases. It follows that the modes have no dispersion at any frequency. The static limit solutions complete the definition at the boundary of the formerly determined interior field configurations showing a Meissner like effect. The results underline that various of the current theoretical approaches to edge excitations could be appropriate for superlattice structures but can fail to describe standard planar samples.
This work is devoted to investigate the propagation properties of edge modes in superlattices formed by 2D electron gases under quantum Hall regime. Such structures are constituted by independent semi-planes in which the electrons are mobile only within their semi-plane. The distance between the planar gases will be assumed to be small with respect to the wavelengths under investigation. Then the system can be considered as an anisotropic medium. The typical width of the 2D electron gases is the order of \(10 - 100\) Å and the distance between them can be of the order of hundreds or thousands Å in normal situations. The physical properties of the excitations at the border of samples showing fractional or integral QHE have been the objective of study from early times after the discovery of the effect. The motivation for their study got further strength after the development of the Buttiker description. One of the points of central interest was the study of the electromagnetic properties near the edges. In spite of the extensive attention to this problem in the literature, in our view, there remain yet important unsolved questions. Among them, stays the structure of the edge field configurations in planar samples. To be specific, the dynamics of current and charges at the border furnishing the gauge invariance (or charge conservation) of the action describing the QHE (Chern-Simons action) are not yet fully understood for realistic planar samples. We think that the main difficulty in the development of such an understanding is related with the assumption about the 2 space and one time nature of the electromagnetic action. It is certain that the Chern-Simons contribution has effectively this structure due to the planar character of the currents. However, the electromagnetic action should retain its usual 3-space +1 time structure because this form accounts for the real electromagnetic field to be non planar for 2D samples. A justification of these statements was done in Refs. The effective Maxwell equations derived in that work were investigated in Refs. These equations generalized previous ones valid for the static limit. Their study furnished interesting conclusions as the existence of: Meissner effect in superlattices and surface waves in planar samples. Further they also allowed to propose a natural model for a surprising tunneling phenomena in strip samples in QHE regime. Therefore, when superlattice structures of planar QHE samples are considered the field distribution can be two dimensional, that is, invariant along the normal directions to the planes. However, the pure 2D samples seem to need for a more involved analysis mainly due to the singular behavior of strictly filamentary edge charge and current densities. The analysis of this problem will be considered elsewhere.

The objective of the present work is to investigate the propagation properties of edge mode waves in superlattice of planar samples. It should be stressed that the discussion done in Ref. was restricted by the very low momentum approximation. The interpretation for the equations to be valid for superlattices also introduce the possibility of studying the propagation of modes in any direction ranging from the parallel to the normal to the planar gases. After introducing surface charge and current densities at the edge, the Maxwell equations are solved for waves propagating within the boundary. The constitutive relations expressing the densities in terms of the field values at the boundaries are determined. Their form
resembles the London equation for the superconductor currents. In the momenta region \( w/c < k \) the dispersion and polarization vectors have been derived. The propagation along the direction parallel to the planes turns to be non dispersive at any momenta, then verifying the assumption posed in [10].

It can be useful to underline again that our discussion is closely linked with that given by Wen [10] which in common with other authors have pointed out the relevance of the Chern-Simons action for the description of the electromagnetic properties of QHE samples. However, as it was indicated in [12], the treatment in [10] turns to be appropriate only for strictly 2+1 dimensional systems. That is, it should become applicable for superlattice arrays of 2D electronic gases. One of the objectives of this work is to underline this circumstance.

The analysis here presented can be considered as complementing the results of [10] by determining the structure of the electromagnetic propagation outside the infrared non dispersive region considered in that reference. Furthermore, the results for the static field configurations reproduce the ones obtained in [12] which were used to argue the existence of a Meissner like effect in superlattices [12]. Here the solution is fully determined by finding the charge and current at the border in terms of the dynamical fields and the outside field values. As it was mentioned above, the current satisfies a sort of London relation.

It should be stressed that these particular static solutions can be used to investigate the physical nature of the transitions between different plateaus at varying magnetic fields. Such a study would be one of the possible extensions of the present work.

Another worth to be addressed task can be the study along the same lines followed here of single planar samples. This task would have as a consequence a better understanding of the transient propagation at the border of realistic planar samples which has been the subject of experimental examination [20].

1 The Maxwell equations.

Let us consider the Maxwell equations for superlattices of 2D electron gases in QHE regime discussed in [12]:

\[
\square A_\mu (r, x_4) = -\frac{4\pi}{c} J_\mu (r, x_4),
\]

\[
\partial_\mu A_\mu = 0,
\]

where the current density has two contributions, the volumetric currents associated with the array of independent 2D electron gases and the surface sources at the border which should be determined in order to satisfy the Maxwell equations, boundary conditions and charge conservation. Thus, we write

\[
J_\mu (r, x_4) = J^{sp}_\mu (r, x_4) + J^{edge}_\mu (r, x_4)
\]
where the volumetric current in terms of the vector potential has the following form [12]

\[ J_\mu^v(r, x_4) = i\sigma_H I_0 \Theta(x_1) \varepsilon^{\alpha\mu\nu\sigma} n_\alpha \frac{\partial A_\mu}{\partial x_\sigma} + cI_0 \Theta(x_1) \chi e^* \]

\[ * \left[ P_{\mu\nu} u_\alpha^2 \frac{\partial^2 A_\nu}{\partial x_\alpha^2} - u_\mu P_{\alpha\beta} \frac{\partial^2 A_\nu}{\partial x_\alpha \partial x_\beta} - u_\mu P_{\alpha\beta} \frac{\partial^2 A_\nu}{\partial x_\alpha \partial x_\beta} + u_\mu u_\nu P_{\alpha\beta} \frac{\partial^2 A_\nu}{\partial x_\alpha \partial x_\beta} \right] (3) \]

In the above equation \( \sigma_H \) is the Hall conductivity, \( I_0 \) is the longitudinal density of semi-planes in the \( x_3 \) direction, \( \chi e^* \) is the planar dielectric constant which occurs to be inversely dependent of the magnetic field. The sources at the border are written as:

\[ J_\mu \text{edge} (r, x_4) = I_0 j_\mu \delta(x_1) \] (4)

and the various 4-vectors and the tensor involved in (3) are defined by

\[ j_\mu = (0, j_\text{edge}, 0, ic\rho_\text{edge}) \],

\[ u_\mu = (0, 0, 0, 1) \],

\[ n_\mu = (0, 0, 1, 0) \],

\[ P_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \],

where the auxiliary charge \( \rho_\text{edge} \) and the current \( j_\text{edge} \) densities are introduced. The also auxiliary 4-vectors \( u_\mu \) and \( n_\mu \) correspond respectively to the 4-velocity of the sample and a 4-vector which is orthogonal to the planar electron gases forming the superlattice. \( P_{\mu\nu} \) is the projector tensor on the space orthogonal to \( n_\mu \). These quantities were introduced in Ref. [12]. Such covariant entities were useful there for clarifying how the Chern-Simons topological action is involved in the description of the QHE phenomena.

In starting, let us search for traveling waves of the following form

\[ A_\mu = F_\mu e^{ik_1x_1 + ik_2x_2 + wx_4/c}, \] (8)

\[ j_\text{edge} = i_0 e^{ik_2x_2 + wx_4/c}, \] (9)

\[ \rho_\text{edge} = \rho_0 e^{ik_2x_2 + wx_4/c} \] (10)

which already assumes the harmonic character in the direction of the \( x_2 \) axis. The \( x_3 \) axes is orthogonal to the electron gases.
Here we will consider the situation in which only the Hall conductivity response of the medium is retained. This assumption corresponds to a limit of high magnetic fields when $\chi_e = 0$ because the effective dielectric constant is inversely proportional to the magnetic field. Accordingly, the Maxwell equations for internal region are reduced to:

$$\left( k^2 \delta_{\mu \nu} - \frac{4\pi}{c} \sigma_H I_0 e^{\alpha_{\mu \sigma \nu} n_\alpha k_\sigma} \right) F_\nu = 0 \quad (11)$$

where the 4-vector $k$ entering in this relation is defined by

$$k_\sigma = \begin{pmatrix} k_1 \\ k_2 \\ 0 \\ \frac{w}{c} \end{pmatrix} \quad (12)$$

At the exterior the same expression will be valid after fixing $\sigma_H = 0$.

Equations (11) have two types of volumetric waves satisfying the dispersion relations $k^2 = 0$ and $k^2 + \sigma^2 = 0$ where

$$\sigma = \frac{4\pi}{c} \sigma_H I_0.$$

The polarization vectors of both modes can be written in the common form

$$F_\mu = \begin{pmatrix} \frac{(k_2 k_3 - k_2 \sigma)}{k_3^2 + \sigma^2} F_4 \\ \frac{(k_2 k_3 + k_2 \sigma)}{k_3^2 + \sigma^2} F_4 \\ F_3 \\ F_4 \end{pmatrix}. \quad (13)$$

where $F_3$ is independent of $F_4$. Now we will consider the two spatial regions of the problem: the free space at $x_1 < 0$ and the superlattice bulk $x_1 > 0$.

### 1.1 Region $x_1 < 0$

In this zone the waves always satisfy $k^2 = 0$ and then the vector potential takes the explicit form

$$A_{\mu}^{ext} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} e^{\lambda x_1 + i k_2 x + \frac{w x_4}{c}} \quad (14)$$

where the real constant

$$\lambda = \sqrt{k_2^2 - \frac{w^2}{c^2}} \quad (15)$$

defines a decaying character of the fields at $x_1 \to -\infty$.

After also imposing the Lorentz gauge condition $\partial_\mu A_\mu = 0$, the following relation arises:
\[ \lambda A_1 + ik_2 A_2 + \frac{w}{c} A_4 = 0. \] (16)

### 1.2 Region \( x_1 > 0 \)

In this zone two possible waves exist. Those obeying \( k^2 = 0 \) and having a vector potential in the following form:

\[ A^{(2)}_{\mu} = \begin{pmatrix} \frac{q_2}{k_4} \\ \frac{k_2}{k_4} \\ \frac{k_4}{k_4} \\ A_3/C_1 \\ 1 \end{pmatrix} C_1 e^{-\lambda x_1 + i k_2 x + w x_4/c}, q_2 = i \lambda, \] (17)

and modes satisfying \( k^2 + \sigma^2 = 0 \) and having polarizations

\[ A^{(1)}_{\mu} = \begin{pmatrix} \frac{(k_2 q_1 - k_2 \sigma)}{k_1^2 + \sigma^2} \\ \frac{(k_2 q_1 + \sigma q_1)}{k_1^2 + \sigma^2} \\ 0 \\ 1 \end{pmatrix} C_2 e^{-\lambda_1 x_1 + i k_2 x + w x_4/c} \] (18)

where

\[ \lambda_1 = \frac{q_1}{\lambda} = \sqrt{k_2^2 - \frac{w^2}{c^2} + \sigma^2} \] (19)

which again defines a decaying behavior now at \( x_1 \to \infty \).

Systematically using the boundary conditions now at \( x_1 \to \infty \), the following relations arise

\[ \lambda A_1 + \lambda C_1 + \lambda_1 C_2 = 4\pi i I_0 \rho_0, \] (20)

\[ \lambda A_2 + \frac{k_2}{k_4} C_1 \lambda + \frac{(k_2 k_4 + \sigma q_1)}{k_4^2 + \sigma^2} C_2 \lambda_1 = \frac{4\pi}{c} I_0 i_0, \] (21)

\[ A_1 = \frac{q_2}{k_4} C_1 + \frac{(k_4 q_1 - k_2 \sigma)}{k_4^2 + \sigma^2} C_2, \] (22)

\[ A_2 = \frac{k_2}{k_4} C_1 + \frac{(k_4 k_2 + q_1 \sigma)}{k_4^2 + \sigma^2} C_2, \] (23)

\[ A_4 = C_1 + C_2, \] (24)

\[ A_3 = 0. \] (25)

Further, after imposing the charge conservation at the boundary, which relates the edge charge or current density and the Hall volumetric currents, it follows the additional condition:
Let us consider first the static situation corresponding to $k_2 = 0, k_4 = 0$. Then, from (20)-(26) it is possible to find the following simple relations:

$$A_4 = C_2 = \frac{i c \rho_0}{\sigma_H} = -i \frac{i_0}{\sigma_H} = -i A_2$$

which closely resemble the London equations for superconductivity where a current is directed along the direction of the spatial vector potential. There exist also a connection between $\rho_0$ and $i_0$ which turns to be very simple in this case:

$$i_0 = -c \rho_0$$

The spatial dependence of the fields in this static regime is simply an exponential decay which expresses the occurrence of a Meissner like effect in the superlattice. Such a phenomenon is absent in planar samples as it was discussed in [12].

Let us return now to the general discussion. It is possible to obtain the general dispersion relation for propagating modes as the condition for the existence of nontrivial solutions of the system of linear equations (16, 20-26). The dispersion relation coming from the vanishing of the relevant determinant takes the form:

$$w - \sigma_H I_0 \left( k_2 + \frac{i w}{c} B \right) = \frac{4\pi}{\left( \lambda + \lambda_1 - 2 \lambda \frac{\eta_2}{\eta_1} \right)} + ik_2 c \left( \lambda + \lambda_1 - 2 \lambda \frac{\eta_2}{\eta_1} \right)$$

in which the new quantities appearing are defined by

$$\frac{\eta_2}{\eta_1} = \frac{\lambda A + i k_2 B + w/c}{i \frac{\lambda^2}{k_4} + i \frac{k_2^2}{k_4} + \frac{w}{c}},$$

$$A = \frac{k_4 q_1 - k_2 \sigma}{k_4^2 + \sigma^2},$$

$$B = \frac{k_2 k_4 + q_1 \sigma}{k_4^2 + \sigma^2}.$$  

In spite of its complicated appearance, the dispersion relation corresponding to the propagation parallel to the electron planes, have the simple non dispersive solution

$$w = c k_2$$

as is it predicted in [10]. Then, the present results indicate that the propagation is non dispersive even at high values of the momentum. This solution for the
dispersion relation is common for two modes. One of them is a wave with the electric fields polarized in the direction perpendicular to the electron gas planes. This orientation of the electric field produces that the wave does not interact with the superlattice because the field can not move the electron out of the planes. Also, the magnetic field produces a Lorenz force which is also perpendicular to the same planes and also does not polarize the charges.

The other mode is simple the wave propagating variant of the static edge modes associated to the quantum Hall effect (or Meissner like) currents circulating though the borders. The waves decay exponentially at the interior of the superlattice and are completely undamped at free space. The propagation properties for general directions of the momenta after the inclusion of the dielectric properties of the background medium will be discussed in more detail elsewhere.

2 Conclusions

The propagation properties for edge modes in a superlattices of 2D electron gases in QHE regime is determined. The field dependence of the edge charge and current densities which implement charge conservation also follows[7]. The solutions for the static case at zero frequency are extending the ones previously obtained in Ref. [12] to be also valid at the boundary and external region. Thus, this work fully support the global (in space) validity of the Meissner effect solutions derived in [12]. As a by-product the edge and current densities at the boundary are also determined. The constant slope of dispersion relation at any momenta indicates that the waves have are non dispersive.

The results can be generalized for the inclusion of the dielectric properties of the background media. This would be necessary in order to compare with possible future experiments. A detailed study of the propagation along the directions non parallel to the planes will be considered elsewhere. From the technical point of view the solution of the here considered task will be also helpful for the investigation of the more relevant case of planar samples which are the ones employed in most of the experiments.

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