A Novel Cutting Method for Face-Hobbed Spiral Bevel Gears Based on Tool Path Modification

Lingzhi Li\textsuperscript{1}, Wenchao Guo\textsuperscript{2*}, Huawei Yang\textsuperscript{1} and Shimin Mao\textsuperscript{2}

\textsuperscript{1}State Key Laboratory of Comprehensive Technology on Automobile Vibration and Noise & Safety Control, No.1 Xinhongqi Street, Changchun, 130013, China
\textsuperscript{2}School of Mechanical Engineering, Xi’an Jiaotong University, Xianning West Road 28, Xi’an, 710049, China
*Email: wc_guo@163.com

Abstract. A novel tooth surface design method of face-hobbed spiral bevel gears is proposed, which utilizes a solid cutter without axis tilting through tool path control combining with optimized cutter blade profile to control tooth contact patterns and realize tooth lengthwise chamfering. Firstly, a gear tooth surface generating plan of tool path controlling actively is developed, the mathematical model of the tool path is built, and the equations are given in detail, the determination method of all control parameters is also described. Next, the tooth surface functions of generating gear and machined gear are established based on a mathematic model of the traditional cradle machine tool. Then, the machine setting is also given in concise equations. Finally, the tooth contact analysis is also conducted for a specific gear pair to validate the validity and the reasonability of the new tooth modification method.

1. Introduction

The face-hobbed spiral bevel gears are processed with continuous indexing method, and include three Gear systems: Klingelnberg, Oerlikon and Gleason. The former is called Cyclo-palloid system, which utilizes generating method with a two-part head cutter, and the two later methods usually utilize Format method with a solid cutter head.

For the past few years, many scholars have conducted research in face-hobbed mechanism. A head cutter with bicircular profile and optimal diameter was developed by Simon [1], and the corresponding tooth surface design method was presented, and the homologous tooth contact analysis was also discussed [2], in order to reduced transmission error amplitudes polynomial function was utilized to change the cradle radial setting and the velocity of the machine tool during the pinion tooth surface generating [3,4], a calculation method of load distribution was proposed to evaluate the tooth modification influence by loaded tooth contact analysis [5], in order to obtain the minimize tooth contact pressure and gear transmission error, the optimization methodologies were developed [6], the optimal tooth modification, the execution motion controlling on the CNC hypoid generation and the misalignment influence were also discussed [7,8]. The effect on transmission performance caused by arc cutter blade was discussed by Kawasaki [9,10]. A common continuous indexing kinematics model and four parts profile of cutter blade were established by Fan [11]. The ultimate motion concept and tooth deviation correction were also developed by Fan [12,13]. An optimal cutter blade profile was also developed to eliminate gear tooth edge contact when a large amount of misalignment was existed [14].

In the meanwhile, the design and processing software have been exploited by Klingelnberg-Oerlikon and Gleason to obtain gear processing scheme automatically, however, two questions still exist: One is...
that the Klingelnberg two-part cutter is quite complicated to design and manufacture, and its rotation speed is difficult to improve which directly influence the quality of tooth surface and working efficiency. The other is that the structure of Oerlikon machine is too complex because of its tilt and swivel mechanisms, which immensely increases the calculation amount of tooth modification, meanwhile, one more linkage axis is needed during generating method because of cutter axis tiling.

In response to this problem, A tool path control method for tooth surface modification is proposed in this paper, which uses a solid cutter without axis tilting, and the tool path control and cutter profile parameters can be acquired in accordance with the second order meshing characteristics of the gear pair.

2. Tool Path Control

Face-hobbed spiral bevel gears are machined by the principle of imaginary crown gear, and the tooth surface of the crown gear is kinematically formed by the trace of the cutter blade cutting-edge [10]. The tooth trace of crown gear is a portion of extend epicycloid in pith plane, as shown in figure 1. In common, there is a constant rotating ratio between cutter (roll circle) and crown gear (base circle) during cutting. If we add an auxiliary variable to the constant ratio, which will make the rolling motion with sliding, so does the generating gear tooth trace. According to this idea, the tool path control method for tooth surface design is proposed, in which the rotating ratio of cutter and crown gear changes periodically to realize tooth crown (longitudinal correction), and the profile crown with circular arc profile of cutter blade.

\[
\theta_C = -\frac{z_k}{z_p} (\theta_K - \theta_{K0})
\]

(1)

where, \(\theta_C\) is the rotation angle of generating gear for its tooth surface forming, \(\theta_K\) is the rotation angle of cutter, \(\theta_{K0}\) is the cutter starting position, \(z_k\) and \(z_p\) are the number of cutter blade groups and crown gear teeth.

In the tool path controlling, \(\theta_C\) can be expressed by

\[
\theta_C = -\frac{z_k}{z_p} (\theta_K - \theta_{K0}) + c_2 (\theta_K - \theta_{KM})^2 + c_3 (\theta_K - \theta_{KP})^3 + L + c_n (\theta_K - \theta_{KP})^n
\]

(2)

where, \(c_n\) are tool path control parameters, and \(\theta_{KM}\) is the cutter rotation angle when tooth surface calculated point is cut.

Currently, gear meshing characteristic parameters are second order, that it is to say, only coefficient \(c_2\) could affect the gear meshing performance. The other coefficients are usually used to gear tooth chamfer, and the chamfering starting point is depended on \(\theta_{KP}\). In this case, gear tooth is divided into
three parts, so is the cutter edge point traces, as shown in figure 2. The equation (2) becomes a piecewise function (for left hand cutter)

\[
\begin{align*}
\theta_c &= -\frac{z_p}{z_p} (\theta_k - \theta_{KMI}) + c_2 \cdot (\theta_k - \theta_{KME})^2 + c_4 \cdot (\theta_k - \theta_{KPE})^4 \\
\theta_c &= -\frac{z_p}{z_p} (\theta_k - \theta_{KMI}) + c_2 \cdot (\theta_k - \theta_{KME})^2 \\
\theta_c &= -\frac{z_p}{z_p} (\theta_k - \theta_{KMI}) + c_2 \cdot (\theta_k - \theta_{KME})^2 + c_4 \cdot (\theta_k - \theta_{KPE})^4
\end{align*}
\]

(3)

where, \(\theta_{KMI}\) and \(\theta_{KME}\) are the cutter rotation corresponding to chamfering start in toe and heel respectively.

![Figure 2. Cutter edge point trace in tool path controlling.](image)

In reality, there are multiple cutter blade groups in a cutter head, therefore, the real rotation of the generating gear can be expressed by a piecewise function with a period of \(2 \pi / z_K\). It means that every cutter blade group is correspond to a period, which consists of four segments: inner blade cutting (A), transition from inner to outer blade (B), outer blade cutting (C), and transition from outer to inner blade (D), as shown in figure 3. P1 and P2 are the effective cutting process for inner and outer blades respectively.

![Figure 3. The period of the generating gear function.](image)

Tool path controlling method with circular cutter blade profile can be used to correct tooth surface for both pinion and wheel, which could determine the tool path controlling factors C and radius of cutter blade profile in accordance with the second order meshing characteristics at the calculated point M: the long axis length of instantaneous contact ellipse, tangential direction of the tooth contact trace, and instantaneous angular acceleration.

3. Tooth Surface of Face-Hobbed Spiral Bevel Gear

Based on the traditional cradle machine, a mathematic model for face hobbing method is successfully established and the relative coordinate positions of cutter, generating gear and machined gear are shown in figure 4. \(S_m\) is machine tool static coordinate system, \(S_k\) is cutter moving coordinate system which rotates with cutter axis \(z_k\), \(S_c\) is generating gear moving coordinate system which rotates with its own
axis $z_k$ and $S_i$ is machined gear moving coordinate which rotates with gear axis $x_i$. The subscript ‘s’ in figure 4 means that the coordinate system is static.

![Figure 4. The relative coordinate positions of cutter and machined gear in machine system.](image)

According to the motion relation of cutter and generating gear explained in Eq.(3), assuming that the coordinates of cutter profile in $S_k$ is $r_k = (x_k, y_k, z_k)$, the tooth surface equation of generating gear can be expressed by

$$r^{(w)}(x, y, z) = M_c x = M_c x_k y_k z_k = \begin{pmatrix} \cos(\theta_k - \Theta_c) x_k - \sin(\theta_k - \Theta_c) y_k + M_d \cos(mq_H - \Theta_c) \\ \sin(\theta_k - \Theta_c) x_k + \cos(\theta_k - \Theta_c) y_k + M_d \sin(mq_H - \Theta_c) \\ z_k m x_i m_n \end{pmatrix}$$

where, the “−” is used for left-hand gears, and the “+” is used for right-hand gears. $M_d$ and $m_H$ are the radial distance and angular distance of the cutter head, which will be described in next section. $x_i$ is the profile shift coefficient and $m_n$ is the mean normal module.

The coordinate transformation from the generating gear’s system to machined gear’s system can be expressed by

$$M_{c'} = M_{c'} M_{s'H} M_{MC}$$

So, the tooth surface equation of machined gear can be expressed by
\[
\begin{align*}
\mathbf{r}^{\text{in}} &= \begin{pmatrix}
x_i \\
y_i \\
z_i \\
1
\end{pmatrix} = \begin{pmatrix}
x_e \\
y_e \\
z_e \\
1
\end{pmatrix} = \\
&= \begin{pmatrix}
\cos \delta_m \cos \varphi_c x_i - \cos \delta_m \sin \varphi_c y_i + \sin \delta_m z_i + E_x \\
(\sin \varphi_i \sin \delta_m \cos \varphi_c + \cos \varphi_i \sin \varphi_c) x_i + (\sin \varphi_i \sin \delta_m \sin \varphi_c + \cos \varphi_i \cos \varphi_c) y_i \\
+ \sin \varphi_i \cos \delta_m z_i + E_x \cos \varphi_i - X_s \sin \varphi_i \cos \delta_m \\
(\cos \varphi_i \sin \delta_m \cos \varphi_c - \sin \varphi_i \sin \varphi_c) x_i + (\cos \varphi_i \sin \delta_m \sin \varphi_c - \sin \varphi_i \cos \varphi_c) y_i \\
+ \cos \varphi_i \cos \delta_m z_i - E_x \sin \varphi_i - X_s \cos \varphi_i \cos \delta_m
\end{pmatrix}
\end{align*}
\]

where, \( \varphi_i \sin \varphi_c \) is the rotation angle of machined gear and generating gear, and the latter is solved by the meshing function between the two gears. \( X_s \), \( E_x \), and \( \delta_m \) are workpiece mountings, which will be described in next section (generally, \( X_s = 0 \)).

### 4. Machine Settings

Based on geometry design of gear pair, the pitch cone apex of the generating gear coincides with the wheel for the gear pair without offset. If offset exists, the two pitch cone apexes do not coincide but be close to each other, the bigger the gear ratio the closer the pitch cone apexes. Consequently, in this method, always assuming that the pitch cone apex of the generating gear always coincides with the gear pitch cone apex and workoffset is imposed on the pinion.

Machine settings include cutter head settings and workpiece mountings. As discussed in above, cutter head does not tilt in this method, so only cutter radial distance \( M_d \) and centre roll position \( q_H \) remain to be calculated. Figure 5 shows a left-hand cutter head setting in the initial cutting position, and their calculations are expressed by

\[
\begin{align*}
M_d &= \sqrt{R_m^2 + r_0^2 - 2R_m r_0 \sin(\beta_m - \nu)} \\
q_H &= \arccos \left( \frac{M_d^2 + R_m^2 - r_0^2}{2M_d R_m} \right) + \zeta_{mp}
\end{align*}
\]

Where, \( R_m \) is mean cone distance of generating gear which is equal to the wheel (\( R_m = R_{mg} \)), \( r_0 \) is the nominal radius of cutter, \( \beta_m \) is the mean spiral angle of generating gear which is equal to the wheel (\( \beta_m = \beta_{mg} \)), \( \nu \) is the cutter lead angle, \( \zeta_{mp} \) is the offset angle in pitch plane.

**Figure 5.** Left-hand cutter position.

**Figure 6.** Workpiece mounting.

The workpiece mountings include workoffset \( E_s \), pitch apex beyond cross point \( E_x \), sliding base \( Ws \) and gear mounting angle \( \delta_M \), as shown in figure 6. Since the pitch cone of the generating gear is always a plane because of cutter head without tilting, the sliding base can be taken as zero for both pinion and gear, and the gear mounting angle is equal to the pitch angle of gear to be cut.
Based on the gear’s pitch apex coincides with the generating gear’s, its mountings are expressed by
\[
\begin{align*}
E_x &= E_y = W_x = 0 \\
\delta_m &= \delta_{0y}
\end{align*}
\]
Where, \( \delta_{0y} \) is the wheel pitch angle.

Because all the workoffset is composed on pinion, its mountings can be expressed by
\[
\begin{align*}
W_x &= 0 \\
E_y &= \frac{R_{mp} - R_w \cos \zeta_{mp}}{\cos \delta_{ml}} \\
E_x &= R_w \sin \zeta_{mp} \\
\delta_m &= \delta_{0p}
\end{align*}
\]
Where, \( R_{mp} \) is the middle pitch cone distance of pinion, and \( \delta_{0p} \) is the pinion pitch angle.

5. Numerical Example
The calculation example for a hypoid gear pair is carried out in this section. The gear geometry: shaft angle 90°, offset 25 mm, the number of teeth 8/39, the mean spiral angle, face width and outer pitch diameter of wheel are 34.63°, 40 mm and 265 mm respectively. The cutter parameters: nominal cutter radius 106.5 mm, cutter starts 5.

First of all, the calculated reference point M is selected in the center of the wheel tooth, the pinion tooth reference point \( M_P \) can be calculated according to the theoretical gear ratio \( \left( \frac{z_K}{z_p} \right) \). And then, the tool path control parameter \( c_2 \) and curvature radius of cutter blade profile \( \rho \) can be determined in accordance with the angular acceleration and the length of contact ellipse \( l \) and instantaneous angular acceleration \( \Omega \), the 4\(^{th}\) coefficient \( c_4 \) is used to gear tooth chamfer, which is determined by the length of chamfering.

Usually, the drive side of pinion tooth is concave, and the coast side is convex. In this example, wheel cutter blade is standard straight line, while the pinion is circular arc. The given meshing characteristic and chamfering parameters
\[
\begin{align*}
l_{drive} &= l_{coast} = L_{drive} = L_{coast} = 10 \text{mm} \\
ppte &= 60 \mu \text{rad}
\end{align*}
\]
Where, \( ppte \) is the peak value of transmission error, \( L_{drive} \) and \( L_{coast} \) are the length of chamfering, which is the distance from the start point of tooth chamfering to the toe/heel, for drive and coast side respectively. The calculated results of tooth modification coefficients are \( c_2 = 0.006545/-0.006362, \ c_4 = 0.5249/-1.3155, \ \rho = 900/700 \), where the former values are for inner-blade, and the latter is for outer blade. The rotation angle increments of the generating gear in one tooth control period, which is correspond to a cutter blade group, are represented in figure 7.

![Figure 7](image)

**Figure 7.** The angle increments of generating gear in a tool path control period.

According to the section above, the tool path control functions and cutter parameters are also determined, and the period of the generating gear rotation function is \( \pi/z_K \).
Based on the face-hobbing mathematical module in section 3, both pinion and wheel tooth surfaces can be established. The error surface (ease-off) of pinion tooth surface is shown in figure 8. Figure a) is original tooth surface ease-off without any corrections, and there are natural crown and twist left in the tooth surface, therefore, tooth modification must be used. Figure b), c) and d) are the tooth ease-off which only one modification factor is changed, which shows that the curvature radius of cutter blade profile only modifies tooth profile like the involute cylindrical gear, tool path control parameters could alter the longitudinal tooth crown and achieve gear tooth chamfer directly. Figure e) and f) are comprehensive tooth modification results, which indicates that Tool Path Controlling could generate the desired tooth surface crowning even tooth chamfering.

![Figure 8. Pinion tooth surface ease-off with tool path control.](image)

In order to estimate the meshing characteristics of the modified tooth surfaces, tooth contact analysis (TCA) is used to the e) and f) presented in figure 8, and the corresponding results are shown in figure 9 and figure 10. They indicate that all tooth contact patterns are in the center of the tooth surface, the peak values of transmission error (PPTE) are 58.03μrad (drive) and 61.36μrad (coast), gear tooth chamfer doesn’t affect the PPTE and changes the tooth contact pattern scarcely. Compared with the given PPTE, the drive side is less 1.97μrad and the coast side is beyond 1.36μrad, but they can be acceptable in gear design process.

![Figure 9. The 2nd comprehensive modification TCA.](image)
The TCA data of five tooth surface points close to the reference point are listed in Table 1. It shows that the average lengths of instantaneous contact ellipse are 9.778 mm (drive) and 12.396 mm (coast). The direction angles of tooth contact-point trace $\theta_i$ can also be calculated and be listed in Table with underline. Comparing the TCA results of two sides, it also implies that the less the direction angle $\theta_i$ is, the more the contact ratio is and the more the load capacity is.

| Points’ order | Drive side | Coast side |
|---------------|------------|------------|
|               | $L$        | $V$        | $l$ (mm) | $\theta_i$(deg.) | $L$ | $V$ | $l$ (mm) | $\theta_i$(deg.) |
| -2            | 117.725    | -2.662     | 9.944 | -- | 118.357 | -3.410 | 12.522 | -- |
| -1            | 116.987    | -2.758     | 9.934 | 7.381 | 117.794 | -3.259 | 12.408 | 164.913(15.083) |
| 0             | 116.251    | -2.853     | 9.687 | 7.362 | 117.230 | -3.108 | 12.297 | 165.075(14.925) |
| 1             | 115.517    | -2.947     | 9.669 | 7.305 | 116.669 | -2.958 | 12.436 | 165.040(14.960) |
| 2             | 114.783    | -3.041     | 9.651 | 7.290 | 116.106 | -2.811 | 12.318 | 165.257(14.743) |

In practice, the contact patterns are also expected to move toward toe or heel sometimes. This method is especially suitable for these cases without changing the machine settings even the cutter data. Figure 11 illustrates that the contact pattern is towards toe for driving side and towards to heel for coasting side. Compared with Figure 10, only the reference points are changed.

As shown in Figure 11, the PPTE becomes 64.03μrad for drive side and 67.97μrad for coast side with the contact patterns shift. The error is just increased by 6 and 6.63μrad respectively. It means that this method can utilize the same cutter and machine settings to cut gears with different positions of tooth contact pattern. In the other words, it increases the applicable range of the cutter head to some extent, which is immensely beneficial to manufacturers.

6. Conclusion
This paper mainly discusses a novel tooth surface modification and gear cutting method with tool path control for the continuous face hobbing gear system. Based on the given meshing characteristics and the position of reference point on the wheel tooth, the parameters of tool path control and cutter will be calculated directly. Where, the position of the reference point and the second-order contact characteristics can be arbitrarily selected. In this case, the tool path control method can be taken as a
tooth surface forward design method. However, the premise must be guaranteed: no more than one cutter tooth would be used at any time.

Acknowledgments
Authors are thankful for the State Key Laboratory of Comprehensive Technology on Automobile Vibration and Noise & Safety Control Open Fund (FAWSKL2020KFJJB1) of China for its financially supporting.

References
[1] Simon V 2011 Generation and Tooth Contact Analysis of Face-Hobbled Spiral Bevel Gears. *Chinese Journal of Aeronautics* 25 A9-288
[2] Simon V V 2010 Advanced manufacture of spiral bevel gears on CNC hypoid generating machine. *Journal of Mechanical Design* 132(3): 031001
[3] Simon V V 2011 Generation of hypoid gears on CNC hypoid generator *Journal of Mechanical Design* 133(12): 121003
[4] Simon V V, Influence of Tooth Modifications on Load Distribution in Face-Hobbled Spiral Bevel Gears. In *ASME 2011 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* pp.135-147
[5] Simon V V 2014 Optimal Machine-Tool Settings for the Manufacture of Face-Hobbled Spiral Bevel Gears. *Journal of Mechanical Design* 136(8): 081004
[6] Simon V V 2013 Design of face-hobbled spiral bevel gears with reduced maximum tooth contact pressure and transmission errors. *Chinese Journal of Aeronautics* 26(3): 777-790
[7] Simon V V, Minimization of the Influence of Misalignments on EHD Lubrication in Face-Hobbled Spiral Bevel Gears. In *ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, pp. V005T11A037-V005T11A037
[8] Simon V V 2014 Optimal Tooth Modifications in Face-Hobbled Spiral Bevel Gears to Reduce the Influence of Misalignments on Electrohydrodynamic Lubrication. *Journal of Mechanical Design* 136(7): 071007
[9] KAWASAKI K 2005 Effect of Cutting Edge Profile on Meshing and Contact of Spiral Bevel Gears in Cyclo-Palloid System. *Mechanics Based Design of Structures & Machines* 33(3-4)
[10] KAWASAKI K, TSUJI I, Analytical and Experimental Tooth Contact Pattern of Large-Sized Spiral Bevel Gears in Cyclo-Palloid System. In *ASME 2009 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* pp. 139-47
[11] FAN Q 2006 Kinematical simulation of face hobbing indexing and tooth surface generation of spiral bevel and hypoid gears *Gear technology* pp.30-38
[12] Fan Q 2007 Enhanced algorithms of contact simulation for hypoid gear drives produced by face-milling and face-hobbing processes" *Journal of Mechanical Design* 129(1): 31-37
[13] Fan Q 2010 Tooth surface error correction for face-hobbled hypoid gears. *Journal of Mechanical Design* 132(1): 011004
[14] Guo W C, Mao S M, Yang Y and Kuang Y H 2016 Optimization of cutter blade profile for face-hobbled spiral bevel gears, *International Journal of Advanced Manufacturing Technology* 85(14):209–216