Kuiper-belt Binary Formation through Exchange Reactions

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Recent observations\textsuperscript{1, 2, 3} have revealed an unexpectedly high binary fraction among the Trans-Neptunian Objects (TNOs) that populate the Kuiper Belt. The TNO binaries are strikingly different from asteroid binaries in four respects\textsuperscript{2}: their frequency is an order of magnitude larger, the mass ratio of their components is closer to unity, and their orbits are wider and highly eccentric. Two explanations have been proposed for their formation, one assuming large numbers of massive bodies\textsuperscript{4}, and one assuming large numbers of light bodies\textsuperscript{5}. We argue that both assumptions are unwarranted, and we show how TNO binaries can be produced from a modest number of intermediate-mass bodies of the type predicted by the gravitational instability theory for the formation of planetesimals\textsuperscript{6}. We start with a TNO binary population similar to the asteroid binary population, but subsequently modified by three-body exchange reactions, a process that is far more efficient in the Kuiper belt, because of the much smaller tidal perturbations by the Sun. Our mechanism can naturally account for all four characteristics that distinguish TNO binaries from main-belt asteroid binaries.
The TNO binary 1998WW31 has the following dynamical properties: mass ratio $m_2/m_1 \sim 0.7$, eccentricity $e \sim 0.8$, semimajor axis $a \sim 2 \times 10^4$ km, and inferred radii $r_1 \sim 1.1 r_2 \sim 10^2$ km, where $m_1(r_1)$ and $m_2(r_2)$ are the masses (radii) of the primary and the secondary, hence $a/r_1 > 10^2$. All this stands in stark contrast to the properties of typical main belt asteroid binaries, where $m_2/m_1 \ll 1$, $e \sim 0$, and $a/r_1 \lesssim 10$.

Asteroid binaries are thought to have been formed by collisions, in a scenario similar to the leading candidate for the formation of the Earth-Moon system: two asteroids collide, leaving a small fraction of their combined matter with a large fraction of their relative angular momentum in a disk. Some of the disk matter then quickly coagulates into a small companion. The observed characteristics, $m_2/m_1 \ll 1$, $e \sim 0$, and $a/r_1 \lesssim 10$, are all natural consequences of this scenario.

Clearly, we need a different mechanism for the formation of 1998WW31. We can look for an analogy by considering dynamical binary formation among stars in dense stellar systems, where there are three channels: 1) tidal capture, where tidal dissipation during a close encounter between two single stars leaves the system bound; 2) three-body binary formation, in a simultaneous close encounter between three single stars where one of the three stars carries the excess energy away, leaving the other two stars bound; and 3) exchange reactions, where a single star encounters a binary, and replaces one of its original members.

Channel 1 is analogous to the standard scenario for asteroid binary formation. It will indeed occur: each TNO has grown through accretion, and much of this accretion has happened through collisions with an object comparable in mass to the growing TNO itself; in some of these collisions there will be too much angular momentum to form a single stable body, and in these cases it is unavoidable to form tight circular binaries of strongly unequal mass. Such binaries, when formed as intermediary stages during the growth of a TNO, are short-lived. The cross section for disruption of a binary is related to the cross section for accretion by the ratio of the sizes of the binary orbit and the TNO primary. Therefore, the binary will be broken up by a perturbation of a third body with mass comparable to the primary well before such a body will hit the primary itself, in the vast majority of cases.

Channel 2 would require a near-simultaneous encounter of three massive objects with low enough velocities to allow an appreciable chance to leave two of the objects bound. For this to work, the random velocities of the most massive objects should be significantly lower than their Hill velocities. Under such conditions, this channel could play a role, as pointed out by Goldreich et al., who assumed that there are $\sim 10^9$ 100 km–sized object embedded in a sea of small (<1 km) objects. This assumption, however, is at odds with Goldreich and Ward’s theory for the formation of planetesimals through gravitational instability, and it is hard to see how objects in the Kuiper Belt could form from non-gravitational coagulation, because the time scales are far too long. In contrast, the gravitational instability theory predicts the size of the initial bodies to be $10 – 100$ km. Starting with these larger bodies would make channel 2 ineffective, because the velocity dispersion would be higher than the Hill velocity.

Recently, Weidenschilling proposed a variation on the idea of using interactions
between three unbound bodies in order to create a binary. He studied how a third massive body could capture the merger remnant from a collision of two massive bodies if the third body were near enough during the time of the collision. This mechanism seems unlikely to work, however, since it requires a number density of massive objects about two orders of magnitude higher than the value consistent with present observations.

Goldreich et al. have proposed another mechanism, based on the dynamical friction from a sea of smaller bodies that can turn a hyperbolic encounter between two massive bodies into a bound orbit under favorable conditions. Effectively, this mechanism makes use of a superposition of three-body encounters, since each light body interacts independently with the two heavier ones, and in that sense it is another variant on channel 2. As we mentioned above, the gravitational instability theory for the formation of planetesimals would exclude the existence of such a sea of small objects, and since the alternative theory of nongravitational agglomeration does not seem to work, we will explore the consequences of dropping channel 2, which leaves us with channel 3.

Channel 3 can only operate when there are already binaries available for encounters. The only binaries that are expected to be formed frequently are the ones produced in channel 1, so we should consider channel 1 and 3 in tandem. What remains to be done is to check whether the resulting properties of the binaries produced are the ones we are looking for in the Kuiper belt, and to check whether the formation efficiency is high enough to explain the abundant presence of surviving TNO binaries.

Starting with the first task, consider a relatively massive TNO primary in a binary orbit with a much less massive secondary, embedded in a sea of smaller particles, most of which are far less massive than either of the binary components. The smallest particles, when they come close, will simply accrete on either the secondary or primary or just pass through the system. The cumulative effect may drive the binary components to collide, which is fine, since then sooner or later a large impact is likely to create again a new temporary binary of a similar type. However, if the binary encounters a particle with a mass that is comparable to the mass of the primary component, the most likely result is an exchange reaction, in which the incoming object replaces the original secondary. Figure 1 shows an example of such a reaction.

Using the impulse approximation for the replacement, together with conservation of energy and specific angular momentum, we can make a crude estimate for the semimajor axis and eccentricity of the newly formed binary in terms of the corresponding values for the original binary. Viewing the replacement as an isolated two-body encounter, we see immediately that the asymptotic escape speed of the initial secondary must be comparable to that of the incoming velocity at infinity of the third body. Since the latter is much smaller than the relative velocity of the binary components (as a necessary condition for runaway growth), the binding energy of the binary will not change much during the exchange, hence \( a_1 m_2 / a_0 \approx m_1 m/a \) where \( a \) is the new semimajor axis after the exchange. This implies \( a/a_0 \approx m/m_2 \gg 1 \): the size of the orbit of the binary is increased in proportion to the increase in mass of the object orbiting the primary. Under the impulse approximation, the interaction
Figure 1: An example of a binary–single-body exchange interaction, in the ‘(massive, light) meets massive’ category discussed in this paper. Bodies 1 and 2 have masses $m_1 = 1$ and $m_2 = 0.1$, respectively, forming a binary with an initially circular orbit. Body 3, with mass $m_3 = 1$, encounters the binary on an initially parabolic orbit. In panel (a), the whole scattering process is shown. Panel (b) shows the complex central interaction in more detail, while panels (c) and (d) show the orbits of the initial and final binary, respectively. Note that the final binary orbit is highly eccentric and much wider than the initial circular binary orbit.
happens in a space small compared to the distance $a_0$ to the primary. Neglecting factors of $\lesssim 2$ related to reduced mass, this implies that the angular momentum per unit mass of the newly bound particle with respect to the primary is the same as that of the previous companion. Conservation of specific angular momentum of the system then gives $m_2a_0(1 - e_0) \approx ma(1 - e)$ which gives $1 - e \approx m_2/m \ll 1$: the eccentricity of the new binary is almost unity, and the orbit is very elongated.

While the impulse approximation may not be a very good assumption, at least qualitatively it predicts that exchange reactions provide us with just what we wanted: binaries with small mass ratios in large highly eccentric orbits. In order to perform a more quantitative check, we have run a series of scattering experiments to obtain the relevant cross sections. Here, we assumed that the initial binary has a mass ratio of $20:1$ and semimajor axis $a_0 = 20r_1$, where $r_1$ is the radius of the primary. These values are typical for main-belt binary asteroids, with $m_2/m_1 < 0.1$, and separations $5 - 40$ times the radius of the primary. We choose parabolic relative orbits for the single body approaching the binary, with periastron distances uniformly distributed between 0 and $20a_0$. In case of resonance reactions, where the incoming body is captured, and the whole system undergoes a complicated three-body dance, we only followed the system as long as all three bodies stayed within a maximum distance of $1000a_0$ from the other two. If this condition was violated for any of the three bodies, we considered that body to escape, due to the perturbing tidal field of the Sun, which in the Kuiper belt corresponds to a Hill radius of order of $10^3a_0$.

Table 1 shows the cross sections for those processes in which the initial binary membership is altered. Channels (a), (c) and (e) result in binaries with two massive components, while channels (b) and (d) produce binaries with large mass ratios. There are four remaining channels that do not produce any binary: a triple merger $1+2+3$, and three channels in which two of the three bodies merge while the third body escapes. The cross sections of these four cases are summed together under (f). Note that about 80% of these interactions result in binaries with two massive components. These cross sections were calculated using a scattering code that incorporated an effective tidal cut-off, and the results were checked independently through a comparison with the starlab three-body scattering package.

Figure 2 shows the normalized differential cross sections for the semimajor axis $a$ and eccentricity $e$, for forming a final binary with two massive components. The distribution for the semi-major axis is strongly peaked at $a = 20$, in good agreement with the simple argument presented above. Similarly, the eccentricity peaks at 0.95, as expected. For the physical values of the semi-major axis, we have assumed a radius $r_1 = 75$km, the estimated size of the primary of 1998WW31, based on the mass deduced from the binary motion and an assumed mean density of $1g/cm^3$. Figure 3 shows the distribution of the final binary in the $a,e$ plane. The orbital elements of 1998WW31 are consistent with the binary having formed through the processes modeled here.

We are now in a position to confront our second task: to check whether the exchange channel is efficient enough to produce the observed binaries. A straightforward approach would be to derive semi-analytic estimates or to perform numerical simulations within the context of a model for the protoplanetary disk. Such an approach
Figure 2: Normalized differential cross sections for the formation of a ‘massive-massive’ binary, under the conditions specified in the text (channels a, c and e in table 1), with respect to the semi-major axis $a$ (top panel), and eccentricity $e$ (bottom panel) of the final binary. The initially circular binary has $a = 1$ in the dimensionless units used for $d\sigma/da$, while the physical units are given for reference at the top of the figure. The filled points are the total values for the differential cross sections, while the open circles are the contributions from the merger channels (c and e in table 1). Note the double-peaked structure in the top panel: the sharp peak toward $a \sim 20$ arises from non-resonant exchanges, where the final binary has an energy comparable to that of the initial binary; the broad peak around $a \sim 10$ arises from resonant exchanges, where the memory of the initial binary is wiped out, leading on average to more strongly hyperbolic escape in which a harder binary is formed.
Figure 3: Orbital properties of ‘massive-massive’ binaries formed in our scattering experiments: $a$ and $e$ have the same meaning and units as in fig. 2. Contributions from exchange reactions, channel (a) in table 1, are limited by energy conservation to $a \lesssim 20$, and give rise to the horizontal rim in the middle of the figure. Contributions involving mergers, channels (c) and (e) in table 1, can lead to $a$ values all the way to the Hill radius $a \approx 10^3$, but are limited by angular momentum conservation to increasingly high $e$ for increasing $a$. The star symbol shows the observed orbit for 1998WW31. Boxes around the star indicate the observational 1- and 2-$\sigma$ error bars.
Table 1: Cross sections $\sigma$ for various configuration-changing channels in binary–single-body scattering. The gravitational focusing factor $v^2$ is scaled out in order to obtain finite values in the parabolic limit, where $v$ is the initial relative velocity between binary and single body at infinity. We use units in which $G = m_1 = m_3 = a = 1$, where $G$ is the gravitational constant, $m_1$ and $m_3$ are the masses of the heaviest body in the binary and the single body, respectively, and $a$ is the initial semi-major axis of the binary. The mass of the lighter body in the binary is $m_2 = 0.05$. The radii are $r_1 = r_3 = 0.05$ and $r_2 = r_1(m_2/m_1)^{1/3} \approx 0.01842$. The scattering processes are coded as follows: $(x, y)$ indicates a binary in the final state with components $x$ and $y$, while $p + q$ indicates the product of a merger between bodies $p$ and $q$. A single body $z$ in the final state is indicated by $(, z)$. The physical meaning of the six channels is as follows: (a) an exchange reaction resulting in a massive–massive binary; (b) an exchange reaction resulting in a massive–light binary; (c) a merger resulting in a massive–massive binary; (d) a merger resulting in a twice-as-massive–light binary; (e) a merger resulting in a massive–massive binary; (f) no binary is left, after three-body merging or two-body merging followed by escape.

| channel: | (a) | (b) | (c) | (d) | (e) | (f) |
|----------|-----|-----|-----|-----|-----|-----|
| process: | (1,3),2 | (2,3),1 | (1+2,3) | (1+3,2) | (2+3,1) | no binary |
| $\sigma v^2$: | 12.1 | 1.3 | 0.9 | 1.3 | 0.9 | 1.2 |
an accretion disk that is likely to form a small companion. In two of the three cases in which no binary is produced, we have to wait until another major collision occurs. Let us introduce this typical waiting time as $T$. What will happen, and how fast will it happen, in the remaining one out of three cases in which we form a binary?

In the gravitational focusing regime, the cross section for significant three body interactions to occur is set by the size of the orbit $a_0$, while the cross section for direct collisions is set by the size $r$ of the TNO primary. Therefore, our newly-formed binary is likely to undergo an exchange reaction on a time scale $(r/a_0)T$, significantly smaller than $T$. As a result, $a$ will increase significantly, as we have seen above. Strong three-body interactions will subsequently occur on an even much shorter time scale $(r/a)T \ll T$. After the first exchange reaction, a very wide roughly equal mass binary exposed to encounters with a third body of comparable mass will on average shrink the orbit and randomize the eccentricity$^{13}$. As a result, the semimajor axis will shrink systematically, while at any given time the eccentricity will still be considerable, since the ‘thermal’ distribution $f(e) = 2e$ of eccentricities that follows phase space volume is tilted towards high eccentricities.

When the orbit becomes small enough, subsequent three-body encounters are more and more likely to lead to a physical collision between two or three of the TNOs involved. If all three collide, we are back where we started, and the resulting system may be a single body (with an assumed chance of $2/3$) or a strongly unequal-mass binary (chance $1/3$). If two of the bodies collide, the third one may remain in orbit, with a thermal ($e.g.$ high) eccentricity expectation, or it may escape. In the latter case, we again are back where we started. In the former case, we still have an equal-mass and likely highly eccentric binary. Even its separation will still be high, for the following reason. Long before the semimajor axis becomes comparable to the size of the bodies, a typical three-body resonance reaction is likely to lead to a collision between the three bodies, since each pass through the resonant period gives a renewed chance for a collision. The chance for collisions in resonant encounters becomes significant$^{20}$ when $r/a \sim 0.03$. For simplicity, let us assume that an exchange reaction turns a ‘giant impact’ binary into a binary with a semi-major axis of $a \sim 300r$. Each subsequent strong encounter will on average decrease $a$ by a factor$^{21} \sim 1.2$. We thus need to wait for a dozen such encounters to occur before reaching $a \sim 30r$ and facing a significant chance for a collision. The time scale for each encounter to occur is $\sim (r/a)T$. The waiting time for the last encounter in this series to occur, under these simplifying assumptions, is $(1/30)T$, while each previous waiting time was less by a factor $1.2$. Summing this series, we get a total waiting time of $(T/30)(1 + (1/1.2) + (1/1.2)^2 + \ldots) \approx (T/30)/(1 - (1/1.2)) = 0.2T$, as an estimate for the duration for those dozen encounters to happen.

Under these assumptions, in $1/3$ of the cases, we wind up with an equal-mass TNO binary with the observed properties for a period $\sim 0.2T$, compared to a $2/3$ chance to wind up with a single TNO for a period $\sim T$. This allows us to derive the rate equation for the formation and destruction of the binaries. If we denote by $N_S$ and $N_B$ the number of single bodies and the number of binaries, respectively, we have

$$\frac{dN_B}{dt} = \frac{1}{3} N_S - \frac{1}{0.2} \frac{2}{3} N_B$$
\[
\frac{dN_S}{dt} = -\frac{dN_B}{dt}
\]

If we measure time in unit of \(T\). So for the stationary state we have \(dN_B/dt = dN_S/dt = 0\), and \(N_B = 0.2N_S/2 = 0.1N_S\). Therefore, the binary fraction is \(\sim 10\%\). When accretion in the Kuiper belt region diminished, the number of single and binary objects was frozen, with a ratio similar to this steady-state value.

Clearly, this predicted binary fraction of 10% is dependent on the assumption we made for the size of a newly formed equal-mass binary, and our other estimates have also been rather crude. It is clear, however, that we expect to find any TNO as part of a very wide eccentric equal-mass binary during at least a few percent of its history. This implies that among all TNOs, after cessation of the accretion stage several percent or more were accidentally left in such a binary phase. The fact that more than 1% of the known TNOs are found to be in wide roughly equal-mass binaries is thus a natural consequence of any accretion model independent of the assumed parameters for the density and velocity dispersion of the protoplanetary disk or the duration of the accretion phase.

We conclude that we have found a robust and in fact unavoidable way to produce the type of TNO binaries that have been found. As a corollary, we predict that future discoveries of TNO binaries will similarly show roughly equal masses, large separations, and high eccentricities.

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