Electric machine with combined excitation in transport

D L Kaluzhsky1, V V Biryukov2, A D Mekhtiyev3, A D Alkina4

1 Department of Electronics and Electrical Engineering, Novosibirsk State Technical University, 20, Karl Marx Ave., Novosibirsk, 630073, Russia, E-mail:
2 Department of Electrical Engineering Complexes, Novosibirsk State Technical University, 20, Karl Marx Ave., Novosibirsk, 630073, Russia
3 Department of Technology and Communication Systems, Karaganda State Technical University, 56, Nursultan Nazarbayev Ave., Karaganda, 100027, Republic of Kazakhstan
4 Department of Technology and Communication Systems, Karaganda State Technical University, 56, Nursultan Nazarbayev Ave., Karaganda, 100027, Republic of Kazakhstan

verp307@mail.ru

Abstract. One of the factors of the electric rolling stock competitiveness of any type of transport is the amount of energy consumed by its electric drive for transporting passengers. During movement, most of the energy is consumed as a heat energy loss in electric traction machines. It is especially important to reduce energy losses in the drive of vehicles with an autonomous power plant. In order to reduce losses in the mechanical part of drives and to increase reliability of their work, it is advisable to use an individual electric drive for each wheel of the rolling stock running gear, especially rail vehicles. In the electrical part of the drive, most of the energy loss is due to the traction motor. Electric motors with the rigid electromechanical characteristic are known to be most suitable for electric traction. These include electric machines with a combined excitation system, which can significantly increase their efficiency and reduce energy consumption by regulating the magnetic field of the machines. The article discusses one of the options of this kind of machines.

1. Introduction
The global trend in the development of energy-efficient traction drives of vehicles shows that improvement is taking place in a number of areas, the main of which are as follows [1-3]:
- reducing the own tare weight of the rolling stock;
- optimizing kinematic schemes of traction drives;
- increasing the power of traction engines;
- developing and implementing new traction engines with the best technical and economic indicators;
- improving power circuits and power management systems for traction engines;
- maximum reducing energy consumption for implementing the movement, including that due to regenerative braking.

Let us consider in more detail the above conditions, taking into account the requirements for the rolling stock.
The energy of movement of any vehicle can be conditionally divided into two components: the useful energy that is spent for transportation of passengers (goods), and the technological energy needed to move the rolling stock (containers). For example, in percentage terms, the mass of the passenger load of the tram does not exceed 25…30% of its total mass. We note that we cannot expect a significant reduction in the weight of the body while maintaining its strength properties due to the use of new structural materials, since plastic elements instead of metal are used in almost all models of passenger vehicles of the latest design.

A different story is the running gears (chassis). For a number of reasons, the most widely used models are those with running gears of the trolley design. For example, a retrospective analysis shows that as the tram developed, the bogies underwent dramatic changes, as a result of which frameless structures with a lower mass are used on modern types of wagons. A striking example of this type is the tram carriage shown in Fig. 1. Its distinctive feature is the modular layout principle, in which the axles of the wheelsets are absent as such, the front and rear wheels are combined into modules of one side, and the trolley is formed by connecting the modules with each other through three tubular section elements. The undoubted advantage of this solution is increased reliability of the drive and attractiveness of the tram due to the maximum reduction of the cabin floor level not only on storage sites, but also along its entire length, which allows solving the problem of boarding passengers with disabilities.

Another not less important advantage of this solution in comparison with analogues is the utmost simplification of the process of energy recovery to the network (on-board storage), since each individual trolley drive (Figure 1) is able to solve this problem independently, operating with instantaneous data of its traction motor load nature [4,5]. Therefore, when the rolling stock moves on a separate roadbed according to the classical scheme, during recovery it becomes possible to return more than 20% of energy consumed during start-up.

Let us note that the use of an individual traction drive on each wheel worsens the anti-skid properties. To eliminate this negative factor in the future, a group drive can be used, in which one traction motor is used to drive both wheels of the module, but such solution will require a radical improvement of the overall dimensions of the existing electric machines.

Let us see what effect the mechanical characteristics of the rolling stock have on the traction motors parameters. It is well known that acceleration to the maximum permissible speed and braking, in order to ensure driving comfort, should be carried out at accelerations not exceeding 1.5 m/s². At the same time, the acceleration rise rate is limited to 2m/s³, which imposes certain conditions on the operation of the power control system of the traction motor. As an example, Figure 2 shows the curves of the acceleration dependence at start-up for drives of the same power of a vehicle with a DC motor of series excitation (curve 1), an asynchronous motor and a DC motor of independent excitation (curve 2).
It can be seen from the above Figure that the series excitation DC machine is competitive in its traction and energy characteristics to other types of engines only until it reaches the automatic curve. The \( v_{\text{auto}} \) point for engines of different power of the tram lies in the range of 18...20 km/h; for freight trains it is 45...50 km/h; for passenger electric trains it is more than 70 km/h. With further acceleration, the dynamics of the rolling stock with a DC motor of series excitation worsens as the speed increases.

The presented data convincingly prove that all modern vehicles should be equipped with electric drives capable of:

1) forming a rigid mechanical characteristic that allows the rolling stock to perform high-speed maneuvers and develop the needed traction force for \( v > v_{\text{auto}} \);
2) ensuring the operation of electric vehicles with high efficiency;
3) ensuring the minimum dimensions and weight of the traction engine and control system;
4) ensuring synchronization of the operation of all traction electric motors in order to avoid the emergence of modes of skidding and slipping. As it is shown in [4], motor control should be carried out from individual energy regulators;
5) forming the conditions for energy recovery in the network.

Let us evaluate the operational properties of electric machines used in transport in order to establish the extent to which they satisfy the above requirements.

The intense development of the theory of electric machines and their control from the beginning of the 60-s led to the appearance of various options for their execution and use, including those in transport [5-13].

Asynchronous motors seem more reliable, because in them there is no such element as a collector that, according to statistics, accounts for up to 70% of electrical equipment failures. As for the overall dimensions and energy parameters of electric machines, today the situation is as follows.

DC collector motors, asynchronous machines with a phase rotor and synchronous machines with electromagnetic excitation (from windings) have comparable technical characteristics. A lower weight and smaller dimensions can be obtained by using an asynchronous squirrel-cage rotor motor in the traction drive as the field tests evidence. According to numerous literature sources, for example [14–26], a synchronous motor with permanent magnets on a rotor is able to demonstrate the best performance.

Indeed, after development of the first highly coercive magnets in the second half of the 70-s and the subsequent sharp drop in prices for them, a reliable and not requiring energy consumption source of the excitation flux appeared in electric machines. Of the heat losses on the rotor of such engine, only additional losses remained due to the tooth harmonics of the spatial magnetic field. The electrical losses in the rotor windings were completely reset to zero, without which, as it is known, an induction motor cannot function, and this is at least 30...40% of all heat losses! Increasing the efficiency at a fixed generating capacity led to decreasing the consumption of cooling air, to a 1.5...2.0-fold decrease (in comparison with AM) of the mass and dimensions of the machine. As a result, in aircraft manufacturing, space drives, medicine, robotic systems and in a number of other leading areas of
science and technology, electric machines of this type occupy a privileged position. Why then is a synchronous motor with permanent magnets extremely rarely used in transport?

The answer to this question is very simple: alongside with a host of advantages, this type of machine has one significant drawback, namely, it does not allow regulating the flow of excitation, which causes many problems. So, for example, in the aircraft power supply system, a synchronous generator (SG) operates with a double change in speed. And in order for the voltage in the on-board network to be unchanged, an adjustable voltage divider must be mounted at its input, the dimensions and mass of which are an order of magnitude greater than that of an electric machine. In addition, the SG is underutilized, since it is designed based on the minimum speed of the turbine, while this mode of operation takes a short time, and during the flight itself it is necessary to take measures to reduce excess voltage. For this reason, the energy supply system of helicopters of the Russian Federation is still based on the use of DC machines as a SG with all the known disadvantages: quick wear of brushes and collectors, icing of moving contacts in winter, low efficiency, etc. Similar problems can be fixed in wind energy, where the range of variation of the rotational speed of the drive shaft exceeds its triple minimum value.

In electric transport, despite all its attractiveness, synchronous machines are also not used. The main reason is that the permanent magnet motor on the rotor at a high power frequency "selects" the entire voltage margin in the converter $U_0$.

2. Electromagnetic Processes in Synchronous Round-Rotor Machine

Let us consider a mathematical model of a synchronous round-rotor machine:

$$U_1 = E_1 + (r_1 + jx_1) \cdot I_1,$$  \hspace{1cm} (1)

$$E_1 = -j\sqrt{2\pi}f_1 w_1 k_{v1} \Phi_m,$$  \hspace{1cm} (2)

$$M = \frac{m_j p}{\sqrt{2}} \cdot w_1 k_{v1} \cdot \Phi_m I_1 \cos \gamma = 30P_{em} / (\pi n),$$  \hspace{1cm} (3)

$$U_1 \approx (0 + 1, 0)U_{1\text{max}}, \quad U_{1\text{max}} \approx U_0 / \sqrt{6},$$  \hspace{1cm} (4)

$$P_{em} = M \cdot 2\pi f_1 / p = mE_1 I_1 \cos \gamma = x_1^{-1} \cdot mU_1 E_1 \sin \theta,$$  \hspace{1cm} (5)

$$n = 60 \cdot f_1 / p \quad (1 \text{ min}), \quad \nu = 0.06\pi D_k n \cdot j_{fp}^{-1} \text{ (km / h)}. \hspace{1cm} (6)$$

Here: $r_1$ and $x_1$ are the active and inductive resistance of the armature winding phase; $w_1$ and $k_{v1}$ are the turning number in the armature winding phase and the phase factor, respectively. $I_1$ is the phase current; $\gamma$ is the phase shift between current $I_1$ and EMF $E_1$. $M$ is the torque on the motor shaft; $P_{em}$ is electromagnetic (useful) power; $\theta$ is the load angle (difference of the initial phases between voltage $U_1$ and EMF). $D_k$ is the diameter of the wheel (of electric locomotive, tran, etc.); $j_{fp}$ is the differential ratio of the gear; $p$ is the number of pole pairs of the machine.

If the excitation flux in a synchronous motor is caused by the presence of permanent magnets on the rotor, then with the increase in the speed $v$, the EMF of the phase $E_1$ also increases linearly.

It is known that there are two main ways to control a synchronous machine in motor operation. In the first one [11], using the rotor position sensor, currents are formed that have exactly the same initial phases as for the same EMF (Figure 3, a). Neglecting the voltage drop across the active resistance, we can say that in this case, in accordance with (3), $\gamma = 0$, the voltage $U_1$ is automatically generated as the sum of two vectors $E$ and $jx_1 I$. Until the speed of the electric vehicle exceeds a certain critical value (usually $v_{cr} = 45...55 \text{ km/h}$ when driving in horizontal sections of the track), the electromagnetic power $P_{em}$ at some constant current increases linearly in the function of $v$. At the same time, the moment on the shaft and, consequently, the acceleration remain unchanged.
It should be noted that in the considered speed range this control method will be optimal, since with a constant current value (and therefore, heat loss) and EMF, you can obtain the greatest moment or electromagnetic power.

With increasing speed $v > v_{cr}$, the EMF will increase linearly ($E_{1}'' > E_{1}'$), and the phase voltage will reach its limit value $U_{1\text{max}}$. In accordance with the vector diagram (Figure 3, b), the voltage across the inductive resistance $x_{1}I_{1}$ will start decreasing, and upon reaching the equality $E_{1} = U_{1}$ the current strength, torque and acceleration will become equal to zero.

\[ x_{1}I_{1} = \sqrt{U_{1}^2 + E_{1}^2 - 2U_{1}E_{1}\cos \theta}. \]  

Figure 5 in the form of solid lines shows the curves built according to the calculated data.

As it is seen from the graphs, at the speed $v > v_{cr}$ it is possible to maintain the moment (and therefore, acceleration) unchanged and to keep the current value close to the nominal $I_{1} \approx (1,0 \pm 0,1)I_{1\text{nom}}$, only in the speed range of $0 < v < 1.35v_{cr}$. It is obvious that, using the parametric method of controlling a synchronous machine with excitation from permanent magnets, the only thing that can be achieved is to reach a speed close to $v \approx v_{cr}$ when decreasing the torque to half the value. If it is necessary to
develop a higher traction force, or at least to ensure the vehicle operability at $v > 1.5v_{cr}$, then it is necessary to generate currents in the windings that exceed the nominal values 1.3...1.5 times. And this means that in continuous operation, the power of the power source must be increased by at least 30%, and the heat loss in the engine will increase 1.7...2.0 times.

Electromechanical characteristics of a synchronous and adjustable asynchronous drive are given in Figure 6.

![Figure 6](image)

**Figure 6.** Comparative mechanical characteristics of the machine

Comparing them, we can draw the following conclusions:

1. In the speed range of $0 < v < v_{cr}$, the moment of a synchronous motor does not change, while in the asynchronous motor it decreases approximately by 25%.

2. At the rotation frequency of $n = 1,35n_{cr}$, synchronous and asynchronous drives indicators become close, and at $n > 1,35n_{cr}$, a synchronous drive becomes worse in all respects.

3. Despite a lot of advantages of a synchronous motor with permanent magnets on the rotor, demonstrated by it at low speeds, its use in modern vehicles is unlikely. This decision is also supported by the fact that the cost of a synchronous machine is greater than that of an asynchronous one.

Theoretically, a correction of the situation would become possible if an electric motor with permanent magnets on the rotor had an additional device providing partial control of the magnetic flux. Indeed, at a fixed speed (Figure 7), to develop the required power, it is enough to keep the value $E_1 \sin \theta$ unchanged.

It is obvious that the value of the armature current under this condition will be minimal when the vector $jxI_1$ is perpendicular to the vector $U_1$. In this case, the power factor $\cos \varphi = 1.0$ and the whole energy of the source, except thermal, will be converted into mechanical energy of the engine.

Below, Figure 8 shows the curves reflecting the calculated patterns of changing the variable component of the main magnetic flux of a synchronous motor, which has a hypothetical possibility of controlling the EMF to maintain the maximum value of the power factor, and in Figure 5 dotted lines show the mechanical characteristics of such electric machine.
Figures 5 and 8 show that in the speed range of $1,06v_{cr}<v<2,0v_{cr}$ while maintaining the same value of $I_1$, a synchronous motor with combined excitation will experience decreasing the torque (and therefore, acceleration) by about 45%. In this case, the magnetic flux must reach the maximum value $\Phi=1,2\Phi_{n0}$ at the point $v\approx1,064v_{cr}$, and then, gradually decreasing, it should approach the value of $0,8\Phi_{n0}$ at $v=2v_{cr}$. Comparing the values of accelerations that can provide different types of electric drives of the same power (Figures 5 and 6), we can conclude: at the maximum speed, an asynchronous motor will develop a moment almost two times lower than that of a synchronous motor with controlled excitation.

![Figure 8. Calculated laws of changing the variable component of the main magnetic flux of a synchronous motor](image)

From the above it follows that the widespread use of contactless synchronous machines in transport is hampered by one drawback: inability to change the main magnetic flux, even in small limits, near a certain average value developed by permanent magnets. Indeed, the electromagnetic excitation system, which develops only 20...25% of the main magnetic flux $\Phi_m$, will practically not worsen the overall dimensions of the engine, and the power loss in such an excitation system will be 20...25 times lower than that in the classical case when the field is only formed due to the field winding.

The study of the information-patent literature shows that the prototype for solving the problem can be a synchronous machine with combined excitation [10] presented in Figure 9.

Here, in one case, in fact two motors are located: on the right side – a synchronous one with permanent magnets on the rotor, on the left side – an asynchronous one with excitation from the ring winding located between the two stator packs. The rotor of the induction machine is also two-pack with open slots, the number of which is equal to the number of pole pairs. The geometry of the stator sheet in the synchronous and in the induction motor is the same, so the armature winding for the machines is common. This so-called electromagnetic combination of the first kind makes it possible to halve the length of the frontal parts of the armature winding and thereby significantly reduce heat loss.
When the ring winding is connected to an adjustable constant current source, a magnetic field of the same polarity is formed in the air gap under each stator pack of the induction machine. If you move along the stator bore, you can see that in the areas where the rotor slots are located, the induction is greater, and in the areas where the rotor slots are absent it is lower. Thus, neglecting higher harmonics, we can assume that the field in the gap of the induction motor changes according to the law:

\[ B(\delta, \alpha) = B_0 + \beta \cdot B_0 \cos(\delta - \alpha). \]  

(8)

Here \( \delta \) is the angle of the rotor turn; \( \beta \approx 0.3 \ldots 0.6 \) is the pulsation factor of the air gap magnetic conductivity; \( B_0 \) is the average value of magnetic induction.

The variable component of induction in (8), determined by the dimensionless coefficient \( \beta \), as well as the field developed by the magnets, is induced in the winding of the EMF armature. Since developing \( B_0 \) induction close to that developed by magnets is problematic, it is believed that an induction motor of this design in specific moment (power) is worse than a synchronous motor with permanent magnets about 2.5...3 times. Therefore, the total length of the two stator packs of the inductor machine should not be five times smaller than the synchronous stator pack, but only 1.7 to 2 times.

3. Experimental Investigation

An electric machine with combined excitation STG-9V, developed at NSTU to work as a starter-generator of a helicopter can serve as the confirmation of the correctness of the above studies. Table 1 shows the comparison of the technical characteristics of the Skin of Aerospace (USA) and STG9M (RF) machines.

| Generator type | Weight, kg | Body diameter, mm | Length, mm | Efficiency, % |
|----------------|------------|-------------------|------------|---------------|
| Skin of Aerospace | 13.65 | 140 | 270 | 79 |
| STG9M | 21.0 | 164 | 308 | 79.7 |
| STG-9V | 14.2 | 161 | 224.5 | 95.8 |

1STG Skarla Aerospace and STG9M are cooled with compressed air supplied by the external compressor
2STG-9V design where self-ventilation is used.

We note that of all the examined machines, only STG-9V meets modern requirements for the electrical equipment of aircraft, namely:

1) it has the efficiency above 95%;
2) it uses self-ventilation for cooling, rather than compressed air supplied from the compressor;
3) it does not have movable contacts (collector, slip rings, brushes);
4) it has the minimum weight and size indicators comparable or lower than those of the best world and domestic analogues.

To study the possibility of using a synchronous machine with combined excitation in electric transport, a preliminary calculation of the engine with electromagnetic combination of the second kind was carried out, in which not only electric but also magnetic circuits were combined.

The calculations show that the expected advantages of the considered option in comparison with an asynchronous drive are expressed in:
- reducing the overall dimensions by approximately 25 ... 30%;
- reducing heat loss by 35 ... 40% (mainly due to losses in the rotor);
- increasing the efficiency and power factor \( \cos \phi \);
- possibility of developing a starting torque 2 ... 3 greater than that of the AM;
- reducing the cooling air consumption by 20 ... 30%.

- significant reduction of the overall dimensions of the capacitor units of the converters due to decrease of the reactive component of the motor current.

Table 2 shows, as an example, the parameters of an asynchronous traction motor for a freight electric locomotive developed by the FSUE VNIKTI, and preliminary calculation data of a synchronous motor with combined excitation (SM–1200).

| Parameters                        | AM – 1200 | SM – 1200 |
|-----------------------------------|-----------|-----------|
| Power on the shaft, kW            | 1200      | 1200      |
| Line voltage, V                   | 2300      | 2300      |
| Current intensity, A              | 338       | 338       |
| Current frequency, Hz             | 55        | 54        |
| Rotor speed, rpm                  | 1080      | 1080      |
| Torque, Nm                        | 10600     | 10600     |
| Slip, %                           | 1.8%      | 0         |
| Total losses, kW                  | 63        | 26        |
| Efficiency, %                     | 0.95      | 0.979     |
| Power factor, p.u.                | 0.94      | 0.96...1.0|
| Starting torque, Nm               | 17870     | 19500     |
| Current on start-up, A            | 447       | 622       |
| Maximum rotor speed               | 1885      | 2200      |
| Ventilation system                | independent | independent |
| Cooling air flow, m³/s            | 1.12      | 0.9       |
| Outer diameter of the stator core, m | 0.74      | 0.64      |
| Bore diameter, m                  | 0.51      | 0.48      |
| Core length, m                    | 0.46      | 2×0.2     |
| One-sided air gap, m              | 0.002     | 0.002     |
| The outer diameter of the rotor, m | 0.506     | 0.476     |
| The inner diameter of the rotor core, m | 0.205      | 0.205     |
| Outer diameter, maximum, m        | 0.846     | 0.760     |
| Engine weight, kg                 | 2600      | ≤ 2000    |

4. Conclusion
Already at the preliminary design stage, it was revealed that only by reducing heat losses in engines it will be possible to obtain annual energy savings of about 10 million rubles per freight electric locomotive.
References
[1] Efremov I S and Gushcho-Malkov B P 1970 Teoriya i raschet mekanicheskogo oborudovaniya podvihznogo sostava gorodskogo elektricheskogo transporta [Theory and calculation of mechanical equipment of rolling stock of urban electric transport]. (Moscow: Publishing house for architecture and construction) (in Russian).
[2] Gavrilov Y I and Mnatsakanov V A 1986 Vagony metropolitena s impulsnymi preobrazovatelyami [Subway cars with pulse converters]. (Moscow: Transport). (in Russian)
[3] Biryukov V V 2017 Konstruktsiya i raschet mekanicheskogo oborudovaniya elektropodvizhnogo sostava [Design and calculation of mechanical equipment of electric rolling stock]. (Novosibirsk: NSTU Press). (in Russian)
[4] Biryukov V V 2014 Energeticheskie aspekty funktsionirovaniya transportnykh sistem [Energy aspects of the functioning of transport systems] (Novosibirsk: NSTU Press). (in Russian)
[5] Slezhanskiy O S, Datskovskiy L K, Kuznetsov I S et al. 1983 Systems of subordinate regulation of AC electric drives with valve converters (Moscow: Energoatomizdat).
[6] Elektricheskaya mashina (Electric machine) 1989 Patent Yaponii № 61–14743. Zayavitel Matsusita denki sange
[7] Zhulovyan V V, Geraskina N M, Kaluzhskiy D L et al. 1989 Sinkhronnyy elektroprivodatel (Synchronous electric motor) 1989 Patent Yaponii № 1481875 (URSS). Publ. v BI 19 (in Russian)
[8] Kaluzhskiy D L, Pastukhov V V, Prudov N M 2012 Sinkhronnyy induktornyj dvigatel s vozbuzydeniyem ot postoyannykh magnetov (Synchronous induction motor with permanent magnet excitation) Utility model patent № 125414 05.06.12
[9] Staton D, Popescu M, Hawkins D, Wu L J and Zhu Z Q 2012 Analytical Modeling and Analysis of Open-Circuit Magnet Loss in Surface-Mounted Permanent-Magnet Machines. IEEE Transactions on Magnetics 48(3) 1234-1247.
[10] Lubin T, Mezani S, and Rezzoug A. 2012 Two-Dimensional Analytical Calculation of Magnetic Field and Electromagnetic Torque for Surface-Inset Permanent-Magnet Motors IEEE Transactions on Magnetics 48(6) 2080-2091.
[11] Grabovetskiy G V, Kuklin O G and Kharitonov S A 2009 Neposredstvennye preobrazovatelyi chastoty s estestvennoy kommutatsiyey diya elektromekhanicheskikh sistem [Direct transformers of frequencies with natural communication for electromechanical systems] (Novosibirsk: NSTU Press). (in Russian)
[12] Levin A V, Konyakhin S F, Yukhnin M M, Kharitonov S A, Korobkov D V and Makarov D V 2013 Raschet elektricheskikh parametrov sistemy generirovaniya elektroenergii nestabilnoby chastoty i stabilnogo napryazheniya [Calculating electrical parameters of the systems for generating unsteady frequency and steady voltage] Aviation industry 1 1-7 (in Russian)
[13] Bertinov A I (Ed.) 1982 Spetsialnye elektricheskiye mashiny [Special electric machines] (Moscow: Energoizdat).
[14] Nasar S A 1964 Electromagnetic theory of electrical machines IEE Proc. vol. 111, iss. 6 pp. 1123-1131
[15] J Greig J and Freeman E M 1967 Traveling wave problem in electrical machine IEE Proc. 114 (11) 1681-1683.
[16] Kazansky V M and A. I. Inkin 1969 An electromagnetic model and elements of the induction machine’s theory Induction Micro-Machines. Proc. Int. University Conf. on Induction Machines (Kaunas) pp 217-229. (in Russian)
[17] Isagusov F, Sultangazyev R, Kulikov V 2016 Diagram of equilibrium phase composition of Fe-C-Si-B system Metalurgija 55(3) 305-308. (in Russian)
[18] Freeman E M 1974 Equivalent circuits from electromagnetic theory on low-frequency induction devices IEE Proc 121 (10) 1117-1121
[19] Freeman E M and Bland T G 1976 Equivalent circuit of concentric cylindrical conductors in an axial alternating magnetic field IEE Proc 123 (2) 149-152
[20] Kazansky V M et al. 1972 The analytical study of electromagnetic field in an active volume of the n-phase asynchronous machine with a non-slot stator Proc. Conf. Induction electric motors with an allocated active stator layer (Novosibirsk: NETI Press) iss. 2, pp 41-57 (in Russian)

[21] Inkin A I and Litvinov B V 1977 The synthesis of cascade equivalent circuits of induction electric machines on the basis of standard E-H-four-terminal networks Electrotechnics 1 29-33 (in Russian)

[22] Inkin A I 2002 Electromagnetic fields and parameters of electric machines: tutorial (Novosibirsk: UKEA) p 464 (in Russian)

[23] Sarapulov F N et al. 2005 Mathemetic models of linear induction machines on the basis of equivalent circuits 2nd edition (Yekaterinburg: UGTU-UPI Press) p 431 (in Russian)

[24] Inkin A I 1997 The mathematical formulation of magnetic field in the volumes of salient-pole, The electric machines Electricity 2 30-35 (in Russian)

[25] Inkin A I 1979 Analytical solution of magnetic fields equations in discrete structures of salient-pole electric machines Electricity 8 18-21 (in Russian)

[26] Inkin A I and Blanc A V 2008 The approximate analytical calculation of exciting field of electric machines on the base of the piecewise continuous eigenfunction Electricity 6 52-56. (in Russian)