On size multipartite Ramsey numbers for stars

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Abstract

Burger and Vuuren defined the size multipartite Ramsey number for a pair of complete, balanced, multipartite graphs \(m_j(K_{a\times b}, K_{c\times d})\), for natural numbers \(a, b, c, d\) and \(j\), where \(a, c \geq 2\), in 2004. They have also determined the necessary and sufficient conditions for the existence of size multipartite Ramsey numbers \(m_j(K_{a\times b}, K_{c\times d})\). Syafrizal et. al. generalized this definition by removing the completeness requirement. For simple graphs \(G\) and \(H\), they defined the size multipartite Ramsey number \(m_j(G, H)\) as the smallest natural number \(t\) such that any red-blue coloring on the edges of \(K_{j\times t}\) contains a red \(G\) or a blue \(H\) as a subgraph. In this paper, we determine the necessary and sufficient conditions for the existence of multipartite Ramsey numbers \(m_j(G, H)\), where both \(G\) and \(H\) are non complete graphs. Furthermore, we determine the exact values of the size multipartite Ramsey numbers \(m_j(K_{1,m}, K_{1,n})\) for all integers \(m, n \geq 1\) and \(j = 2, 3\), where \(K_{1,m}\) is a star of order \(m + 1\). In addition, we also determine the lower bound of \(m_3(kK_{1,m}, C_3)\), where \(kK_{1,m}\) is a disjoint union of \(k\) copies of a star \(K_{1,m}\) and \(C_3\) is a cycle of order 3.

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1. Introduction

The classical Ramsey number \(r(a, c)\) is the smallest natural number \(j\) such that any red-blue coloring of the edges of \(K_j\), necessarily forces a red \(K_a\) or a blue \(K_c\) as subgraph. The size multipartite Ramsey number is one of generalizations of the classical Ramsey number. Burger and Vuuren [1] gave a definition of the size multipartite Ramsey numbers for a pair of complete, balanced, multipartite graphs, as follows. Let \(a, b, c, d\) and \(j\), be natural numbers with \(a, c \geq 2\), the
size multipartite Ramsey number \( m_j(K_{axb}, K_{cxd}) \) is the smallest natural number \( t \) such that any red-blue coloring of the edges of \( K_{jxt} \), necessarily forces a red \( K_{axb} \) or a blue \( K_{cxd} \) as subgraph. They also determined \( m_j(K_{2x2}, K_{3x1}) \), for \( j \geq 1 \) and have established the following existence of size multipartite Ramsey numbers.

**Theorem 1.1.** *(The existence of size numbers) [1]*

The size multipartite Ramsey numbers \( m_j(K_{axb}, K_{cxd}) \) exists for any \( a, c \geq 2 \) and \( b, d \geq 1 \) if and only if \( j \geq r(a, c) \).

Syafrizal et al. [10] generalized this definition by removing the completeness requirement. For simple graphs \( G \) and \( H \), they defined the size multipartite Ramsey number \( m_j(G, H) \) as the smallest natural number \( t \) such that any red-blue coloring on the edges of \( K_{jxt} \) contains a red \( G \) or a blue \( H \) as a subgraph. The size bipartite Ramsey numbers for stars versus paths \( m_2(K_{1,m}, P_n) \), for \( m, n \geq 2 \) given by Hattingh and Henning [3]. In 2007, Syafrizal et al. [11] determined the size multipartite Ramsey numbers for stars versus \( P_3 \). Then, Surahmat et al. [9] gave the size tripartite Ramsey numbers for stars versus \( P_n \), for \( 3 \leq n \leq 6 \). Furthermore, we gave the size multipartite Ramsey numbers for stars versus cycles [5] and the size tripartite Ramsey numbers for a disjoint union of \( m \) copies of a star \( K_{1,n} \) versus \( P_3 \) [6]. In 2017, Jayawardene et al. [4] and Effendi et al. [2] determined the size multipartite Ramsey numbers for stars versus paths. Then, we also gave the size multipartite Ramsey numbers for stars versus paths and cycles [7], that complete the previous results given by Syafrizal and Surahmat. Recently, we determined \( m_j(mK_{1,n}, H) \), where \( H = P_3 \) or \( K_{1,3} \) for \( j \geq 3, m, n \geq 2 \) [8].

In this paper, we determine the necessary and sufficient conditions for the existence of the size multipartite Ramsey numbers \( m_j(G, H) \), where both \( G \) and \( H \) are non complete graphs. Furthermore, we determine the exact values of the size multipartite Ramsey numbers \( m_j(K_{1,m}, K_{1,n}) \) for all integers \( m, n \geq 1 \) and \( j = 2, 3 \). In addition, we also determine the lower bound of \( m_3(kK_{1,m}, C_3) \).

We call some basic definitions that will be used in this paper, as follows. Let \( G \) be a finite and simple graph. Let vertex and edge sets of graph \( G \) are denoted by \( V(G) \) and \( E(G) \), respectively. Vertex colorings in which adjacent vertices are colored differently are proper vertex colorings. A graph \( G \) is \( k \)-colorable if there exists a proper vertex coloring of \( G \) from a set of \( k \) colors. A matching of a graph \( G \) is defined as a set of edges without a common vertex. A matching of maximum size in \( G \) is a maximum matching in \( G \). The maximum degree of \( G \) is denoted by \( \Delta(G) \), where \( \Delta(G) = \max \{ d(v) | v \in V(G) \} \). The minimum degree of \( G \) is denoted by \( \delta(G) \), where \( \delta(G) = \min \{ d(v) | v \in V(G) \} \). A star \( K_{1,n} \) is the graph on \( n + 1 \) vertices with one vertex of degree \( n \), called the center of this star, and \( n \) vertices of degree 1, called the leaves. A disjoint union of \( k \) copies of a star \( K_{1,m} \), a cycle of order \( n \), and a path of order \( n \) are denoted by \( kK_{1,m}, C_n, \) and \( P_n \), respectively.

2. Results

For any non complete graphs \( G \) and \( H \), we will determine the necessary and sufficient conditions for the existence of the size multipartite Ramsey numbers \( m_j(G, H) \). In order to do so,
we recall the definition of the chromatic number of a graph \(G\), denoted by \(\chi(G)\), which is the minimum positive integer \(k\) for which \(G\) is \(k\)-colorable.

**Lemma 2.1.** In every proper vertex coloring of a simple graph \(G\), the maximum number of the vertices in \(G\) with the same color is \(|V(G)| - \chi(G) + 1\).

**Proof.** Let \(c\) be a proper vertex coloring of \(G\), with \(\chi(G)\) color, that is \(c : V(G) \to \{1, 2, ..., \chi(G)\}\). Let \(C_i = \{v \in V(G) | c(v) = i\}\). Without lost generality, let \(|C_1| \leq |C_2| \leq ... \leq |C_{\chi(G)}|\). Since for \(1 \leq i \leq \chi(G) - 1\), we have \(|C_i| \geq 1\), then \(|C_{\chi(G)}| \leq |V(G)| - \chi(G) + 1\).

**Theorem 2.1.** Let \(G\) and \(H\) be two non complete graph. The multipartite Ramsey numbers \(m_j(G, H)\) are finite if and only if \(j \geq \max\{\chi(G), \chi(H)\}\).

**Proof.** Let \(m_j(G, H) = t < \infty\), that is \(K_{jxt} \to (G, H)\). If \(K_{jxt} = F_1 \oplus F_2\), then \((F_1 \not\in G \Rightarrow F_2 \subseteq H)\) or \((F_2 \not\in H \Rightarrow F_1 \supseteq G)\). This implies that \(j \geq \chi(H)\) and \(j \geq \chi(G)\). Therefore, \(j \geq \max\{\chi(H), \chi(G)\}\).

Let \(j \geq \max\{\chi(G), \chi(H)\}\). We show that \(m_j(G, H)\) is finite. We construct an positive integer \(t\) such that \(K_{jxt} \to (G, H)\). Let \(p = |V(G)| - \chi(G) + 1\), \(q = |V(H)| - \chi(H) + 1\) and \(t = p + q\). Note that \(V(K_{jxt}) = V(K_{jxp}) \cup V(K_{jxq})\). Based on Lemma 2.1, \(p\) and \(q\) are the maximum number of the same colored vertices in \(G\) and \(H\), respectively, so \(K_{jxp} \supseteq G\) and \(K_{jxq} \supseteq H\). Therefore, \(K_{jxt} \to (G, H)\). Then, \(m_j(G, H) \leq t\). Since graph \(G\) and \(H\) are finite graph, so \(|V(G)|, |V(H)|, \chi(G)\) and \(\chi(H)\) are finite. So, \(m_j(G, H) \leq t < \infty\). Then, \(m_j(G, H)\) is finite.

**Theorem 2.2.** For positive integers \(m\) and \(n\), we have \(m_2(K_{1,m}, K_{1,n}) = m + n - 1\).

**Proof.** We will show that \(m_2(K_{1,m}, K_{1,n}) \geq m + n - 1\). We consider a red-blue coloring on the edges of graph \(K_{2 \times (m+n-2)} = F_R \oplus F_B\), such that \(F_R\) is a \((m-1)\)-regular graph. By Handshaking Lemma, it is possible since the sum of the degrees of the vertices of \(F_R\) is even. Then, \(F_R \not\in K_{1,m}\).

We have \(d(v) = m + n - 2 - (m - 1) = n - 1\), for any \(v \in F_B\). Hence, \(F_B \not\in K_{1,n}\).

Now, we will show that \(m_2(K_{1,m}, K_{1,n}) \leq m + n - 1\). We consider any red-blue coloring on the edges of graph \(K_{2 \times (m+n-1)} = G_R \oplus G_B\), such that \(G_R \not\in K_{1,m}\). This implies that \(\Delta(G_R) \leq m - 1\). Therefore, \(\delta(G_B) \geq m + n - 1 - (m - 1) = n\). Then, \(G_B \supseteq K_{1,n}\).

**Theorem 2.3.** For positive integers \(m\) and \(n\), we have

\[
m_3(K_{1,m}, K_{1,n}) = \begin{cases} \frac{m}{2}, & \text{for } m \equiv 2 \mod 4, n = 1, 2 \\ 2 \left\lfloor \frac{m+1}{4} \right\rfloor + 2 \left\lfloor \frac{n}{4} \right\rfloor, & \text{for } m \equiv 2 \mod 4, n \equiv 3 \mod 4 \\ 2 \left\lfloor \frac{m-1}{4} \right\rfloor + 2 \left\lfloor \frac{n}{4} \right\rfloor, & \text{for } m \equiv 4 \mod 4, n \equiv 1 \mod 4 \\ \frac{m-1}{2} + \left\lfloor \frac{n}{2} \right\rfloor, & \text{for } m \equiv 1 \mod 2, n \geq 1 \\ 2 \left\lfloor \frac{m+1}{4} \right\rfloor + 2 \left\lfloor \frac{n}{4} \right\rfloor + 1, & \text{for } m \equiv 2 \mod 4, n \neq 3 \mod 4, n \geq 4, \\ 2 \left\lfloor \frac{m-1}{4} \right\rfloor + 2 \left\lfloor \frac{n}{4} \right\rfloor + 1, & \text{for } m \equiv 4 \mod 4, n \neq 1 \mod 4. \end{cases}
\]

**Proof.** **Case 1.** \(m_3(K_{1,m}, K_{1,n}) = \frac{m}{2}\), for \(m \equiv 2 \mod 4, \text{and } n = 1, 2\).
For $n = 1$, we will use the property that $m_3(K_{1,m}, K_1) \leq m_3(K_{1,m}, K_{1,1})$. It is clear that $m_3(K_{1,m}, K_1) = \frac{m}{2}$. Therefore, $m_3(K_{1,m}, K_{1,1}) \geq \frac{m}{2}$. If $K_{3 \times \frac{m}{2}}$ contains no a blue $K_{1,1}$, then $K_{3 \times \frac{m}{2}}$ contains a red $K_{1,m}$, since $d(v) = m$, for any $v$ in $K_{3 \times \frac{m}{2}}$. Hence, $m_3(K_{1,m}, K_{1,1}) \leq \frac{m}{2}$.

For $m = n = 2$, it is clear that $m_3(K_{1,m}, K_{1,n}) \geq \frac{m}{2}$. For $m \equiv 6 \mod 4$ and $n = 2$, we consider a red-blue coloring on the edges of graph $K_{3 \times (\frac{m}{2} - 1)}$, such that $K_{3 \times (\frac{m}{2} - 1)}$ contains a maximum blue matching graph. Since $\frac{m}{2} - 1$ is even, the blue graph is a $1-$regular graph. This implies that graph $K_{3 \times (\frac{m}{2} - 1)}$ contains red $(m - 3)$-regular graph. So $K_{3 \times (\frac{m}{2} - 1)}$ contains no a red $K_{1,m}$. Then, $m_3(K_{1,m}, K_{1,2}) \geq \frac{m}{2}$. Furthermore, we consider any red-blue coloring on the edges of graph $K_{3 \times \frac{m}{2}}$, such that graph $K_{3 \times \frac{m}{2}}$ contains no a blue $K_{1,2}$. This implies that the maximum degree of blue graph is 1. Since $\frac{m}{2}$ is odd, then there is at least one vertex $v$, where $d(v) = 0$ in blue graph and $d(v) = m$ in red graph. Then, $K_{3 \times \frac{m}{2}}$ contains a red $K_{1,m}$. Therefore, $m_3(K_{1,m}, K_{1,2}) \leq \frac{m}{2}$.

**Case 2.** For $(m \equiv 2 \mod 4$ and $n \equiv 3 \mod 4$), let $t = 2[\frac{m+1}{4}] + 2[\frac{n}{4}]$ and for $(m \equiv 4 \mod 4$ and $n \equiv 1 \mod 4$), let $t = 2[\frac{m+1}{4}] + 2[\frac{n}{4}]$.

We consider a red-blue coloring on the edges of graph $K_{3 \times (t-1)} = F_R \oplus F_B$, such that $d(v_1) = m - 2$, for a vertex $v_1 \in V(F_R)$ and $d(v) = m - 1$, for any $v \in V(F_R) - \{v_1\}$. By **Handshaking Lemma**, it is possible since the sum of the degrees of the vertices of $F_R$ is even. Then, $F_R \not\supseteq K_{1,m}$. We distinguish the following two cases, to show that $m_3(K_{1,m}, K_{1,n}) \geq t$.

**Case a.** For $m \equiv 2 \mod 4$ and $n \equiv 3 \mod 4$.

We have $d(v_1) = 2t - m = 4[\frac{m+1}{4}] + 4[\frac{n}{4}] - m = m - 2 + n + 1 - m = n - 1$, for $v_1 \in V(F_B)$ and $d(v) = 2t - m - 1 = 4[\frac{m+1}{4}] + 4[\frac{n}{4}] - m - 1 = m - 2 + n + 1 - m - 1 = n - 2$, for any $v \in V(F_B) - \{v_1\}$. Then, $F_B \not\supseteq K_{1,n}$.

**Case b.** For $m \equiv 4 \mod 4$ and $n \equiv 1 \mod 4$.

We have $d(v_1) = 2t - m = 4[\frac{m+1}{4}] + 4[\frac{n}{4}] - m = m - 4 + n + 3 - m = n - 1$, for $v_1 \in V(F_B)$ and $d(v) = 2t - m - 1 = 4[\frac{m+1}{4}] + 4[\frac{n}{4}] - m - 1 = m - 4 + n + 3 - m - 1 = n - 2$, for any $v \in V(F_B) - \{v_1\}$. Then, $F_B \not\supseteq K_{1,n}$.

Now, we consider any red-blue coloring on the edges of graph $K_{3 \times t} = G_R \oplus G_B$, such that $G_R \not\supseteq K_{1,m}$. This implies that $\Delta(G_R) \leq m - 1$. We distinguish the following two cases, to show that $m_3(K_{1,m}, K_{1,n}) \leq t$.

**Case a.** For $m \equiv 2 \mod 4$ and $n \equiv 3 \mod 4$.

$\delta(G_B) \geq 2t - (m - 1) = 2t - m + 1 = m - 1 + 2[\frac{n}{2}] - m + 1 = n + 1$, since $n$ is odd. Then, $G_B \supseteq K_{1,n}$.

**Case b.** For $m \equiv 4 \mod 4$ and $n \equiv 1 \mod 4$.

$\delta(G_B) \geq 2t - (m - 1) = 2t - m + 1 = 4[\frac{m-1}{4}] + 4[\frac{n}{4}] - m + 1 = m - 4 + n + 3 - m = n + 1$. Therefore, $G_B \supseteq K_{1,n}$.

**Case 3.** For $m \equiv 1 \mod 2$ and $n \geq 1$, let $t = \frac{m+1}{2} + \lceil \frac{n}{2} \rceil$, for $m \equiv 2 \mod 4$ and $n \not\equiv 3 \mod 4$, let $t = 2[\frac{m+1}{4}] + 2[\frac{n}{4}] + 1$, and for $m \equiv 4 \mod 4$ and $n \not\equiv 1 \mod 4$, let $t = 2[\frac{m-1}{4}] + 2[\frac{n}{4}] + 1$.

We consider a red-blue coloring on the edges of graph $K_{3 \times (t-1)} = F_R \oplus F_B$, such that $F_R$ is a $(m - 1)$-regular graph. By **Handshaking Lemma**, it is possible since the sum of the degrees of the vertices of $F_R$ is even. Then, $F_R \not\supseteq K_{1,m}$. We have $d(v) = 2(t - 1) - (m - 1)$. We distinguish the following three cases, to show that $m_3(K_{1,m}, K_{1,n}) \geq t$. 

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Case a. For \( m \equiv 1 \mod 2 \) dan \( n \geq 1 \).
\[
d(v) = 2t - m - 1 = m - 1 + 2 \left\lceil \frac{n}{2} \right\rceil - m - 1 = 2 \left\lceil \frac{n}{2} \right\rceil - 2 < n,
\]
for any \( v \) in \( F_B \). Then, \( F_B \not\supseteq K_{1,n} \).

Case b. For \( m \equiv 2 \mod 4 \) and \( n \not\equiv 3 \mod 4 \).
\[
d(v) = 2t - m - 1 = 4 \left\lceil \frac{m+1}{4} \right\rceil + 4 \left\lceil \frac{n}{4} \right\rceil + 2 - m - 1 = m - 2 + 4 \left\lceil \frac{n}{4} \right\rceil - m + 1 = 4 \left\lceil \frac{n}{4} \right\rceil - 1 \leq n - 1,
\]
for any \( v \) in \( F_B \). Then, \( F_B \not\supseteq K_{1,n} \).

Case c. For \( m \equiv 4 \mod 4 \) and \( n \not\equiv 1 \mod 4 \).
\[
d(v) = 2t - m - 1 = 4 \left\lfloor \frac{m-1}{4} \right\rfloor + 4 \left\lceil \frac{n}{4} \right\rceil + 2 - m - 1 = m - 4 + 4 \left\lceil \frac{n}{4} \right\rceil - m + 1 = 4 \left\lceil \frac{n}{4} \right\rceil - 3 < n,
\]
for any \( v \) in \( F_B \). Then, \( F_B \not\supseteq K_{1,n} \).

Now, we consider any red-blue coloring on the edges of graph \( K_{3 \times t} = G_R \oplus G_B \), such that \( G_R \not\supseteq K_{1,m} \). This implies that \( \Delta(G_R) \leq m - 1 \). We distinguish the following three cases, to show that \( m_3(K_{1,m}, K_{1,n}) \leq t \).

Case a. For \( m \equiv 1 \mod 2 \) dan \( n \geq 1 \).
\[
\delta(G_B) \geq 2t - (m - 1) = 2t - m + 1 = m - 1 + 2 \left\lceil \frac{n}{2} \right\rceil - m + 1 = 2 \left\lceil \frac{n}{2} \right\rceil \geq n.
\]
Then, \( G_B \supseteq K_{1,n} \).

![Figure 1. A coloring for \( m_3(K_{1,3}, K_{1,6}) = 4 \).](image)

For \( m \) and \( n \) are both even, suppose that \( d(v) = m - 1 \), for any \( v \) in \( G_R \). Then, the sum of the degrees of the vertices of \( G_R \) is odd. By Handshaking Lemma, it is a contradiction. Then, there is at least one vertex \( v_1 \) in \( G_R \) such that \( d(v_1) = m - 2 \). We consider \( v_1 \) in \( G_B \) for the following two cases.

Case b. For \( m \equiv 2 \mod 4 \) and \( n \not\equiv 3 \mod 4 \).
\[
d(v_1) = 2t - m + 2 = 4 \left\lceil \frac{m+1}{4} \right\rceil + 4 \left\lceil \frac{n}{4} \right\rceil + 2 - m + 2 = m - 2 + 4 \left\lceil \frac{n}{4} \right\rceil - m + 4 = 4 \left\lceil \frac{n}{4} \right\rceil + 2 \geq n.
\]

Case c. For \( m \equiv 4 \mod 4 \) and \( n \not\equiv 1 \mod 4 \).
\[
d(v_1) = 2t - m + 2 = 4 \left\lfloor \frac{m-1}{4} \right\rfloor + 4 \left\lceil \frac{n}{4} \right\rceil + 2 - m + 2 = m - 4 + 4 \left\lceil \frac{n}{4} \right\rceil - m + 4 = 4 \left\lceil \frac{n}{4} \right\rceil \geq n.
\]
Therefore, there is a star \( K_{1,n} \) in \( G_B \), where \( v_1 \) as the center.
Theorem 2.4. For positive integers $m$ and $n$, we have

$$m_3(mK_{1,n}, C_3) \geq n \left\lceil \frac{m}{2} \right\rceil + \left\lfloor \frac{m}{2} \right\rfloor.$$

Proof. Let $t = n \left\lceil \frac{m}{2} \right\rceil + \left\lfloor \frac{m}{2} \right\rfloor$. We will show that $m_3(mK_{1,n}, C_3) \geq t$. Let $A, B$ and $C$ be three partite sets in graph $K_{3 \times (t-1)}$. We consider a red-blue coloring on the edges of graph $K_{3 \times (t-1)} = F_R \oplus F_B$ such that $F_B = K_{t-1,2(t-1)}$, where the first partite set is $A$ and the second partite set is $B \cup C$. This implies that $F_R = K_{2 \times (t-1)}$, where the partite sets are $B$ and $C$. If $m$ is even, then $|V(F_R)| = 2(t-1) = 2(n \left\lceil \frac{m}{2} \right\rceil + \left\lfloor \frac{m}{2} \right\rfloor - 1) = m(n+1) - 2 < |V(mK_{1,n})|$. Therefore, $F_R \not\subseteq mK_{1,n}$. If $m = 1$, then $F_R = K_{2 \times (n-1)}$. It is clear that $F_R \not\subseteq K_{1,n}$. If $m \geq 3$ and $m$ is odd, then $|B| = |C| = \frac{n(m+1)}{2} + \frac{m-3}{2} = \frac{m-1}{2}(n+1) + \frac{n-1}{2}$. Hence, $F_R$ only contains $(m-1)K_{1,n}$. Then, $m_3(mK_{1,n}, C_3) \geq t$. \qed

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