Gluonic Reggeons†

R. Kirschner† and L. Szymanowski†#

†Naturwissenschaftlich-Theoretisches Zentrum
und Institut für Theoretische Physik, Universität Leipzig
Augustusplatz 10, D-04109 Leipzig, Germany

# Soltan Institut for Nuclear Studies, Hoża 69, 00-681 Warsaw, Poland

Abstract: Contributions from gluon interactions, which are non-leading in high-energy semi-hard processes, are studied and represented in terms of reggeon exchanges. Unlike the leading gluonic reggeon, related to the BFKL pomeron, the non-leading reggeons are sensitive to the spin and transverse momentum distributions of scattering partons. There are several gluonic reggeons with poles in the vicinity of angular momentum $j = 0$ contributing to the perturbative Regge asymptotics of QCD. We extend the high-energy effective action including sub-leading terms which describe these reggeons and their interactions with scattering quarks and gluons in the multi-Regge approximation.

Dedicated to the memory of V. N. Gribov

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1 Non-leading asymptotics

In the semi-hard region, where the momentum scale controlling the coupling is large but much smaller than the scattering energy $\sqrt{s}$, the behaviour of scattering processes is dominated by reggeized gluon exchange. The exchange of two reggeized and interacting gluons results in the BFKL pomeron [1].

In that region the contributions from quark exchange are suppressed at least by one power of $s^{1/2}$ for each exchanged fermion. These contributions dominate, if flavour quantum numbers in the $t$-channel exclude the pomeron exchange.

There are contributions from gluon interaction, which are down by one or more powers of $s$ compared to the leading gluon exchange related to the BFKL pomeron. This is evident e.g. from the gluon elastic scattering tree amplitude. Non-vacuum quantum numbers, odd $C$ and $P$ parity, can also be transferred by gluons.

Consider the scattering with small momentum transfer of a gluon or a quark of high momentum on a source of colour fields. The leading interaction contributes to the amplitude proportional to the first power of the large momentum. It conserves helicity of the high momentum quark or gluon and it is not sensitive to the details of the colour source as its distribution in the transverse (impact parameter) plane or to its spin structure. However such details are resolved by the interactions resulting in a contribution to the amplitude suppressed by powers of the large momentum compared relative to the leading one.

Our aim is to investigate the non-leading gluon exchange with contributions to the amplitude down by one power of $s$ compared to the leading behaviour. There are measurable quantities of interest, where the non-leading gluon exchange is essential and contributes not just as a small correction. The flavour singlet spin structure function $g_1(x)$ is determined in the region of small $x$ mainly by gluon exchange, where one of the exchanged gluons is of the non-leading type. The spin structure function of the photon $F_3^\gamma(x)$ related to the helicity flip amplitude is dominated at small $x$ by exchange of two subleading gluons.

Describing the high energy behaviour it is convenient to consider the Mellin transform of the amplitude with respect to $s$. It is essentially the $t$-channel partial wave and the Mellin variable is the complex angular momentum $j$. Whereas the leading gluon exchange and the BFKL pomeron cor-
respond to Regge singularities near $j = 1$, the non-leading gluon exchanges studied here appear as reggeons with poles near $j = 0$.

In this letter we show that there are several gluonic reggeons near $j = 0$. One of them transfers positive parity like the leading gluonic reggeon. Three reggeons transfer negative parity. Moreover there are reggeons carrying colour states different of the one of the gluon. In analogy to the BFKL pomeron the colour singlet $t$-channel exchanges important for physical processes are built from two or more gluonic reggeons with pair interaction due to emission and absorption of $s$-channel gluons. Avoiding all technical details in this paper we are going to discuss the basic ideas of the approach and to explain the results.

Our tool for investigating the non-leading gluon exchanges is the high-energy effective action. It has been introduced originally as a convenient starting point for improving the leading $\ln s$ approximation for the leading gluon exchange \[2\]. The terms of this action related to leading gluon and quark exchanges are known in the approximation corresponding to the multi-Regge kinematics for $s$-channel intermediate state. The configuration of multi-Regge kinematics of a multiparticle intermediate state is the one giving the leading $\ln s$ contribution. This configuration is characterized by large sub-energies of pairs of particles (equivalently, large rapidity gaps) and restricted momentum transfers. Now we are going to include the terms related to the non-leading gluon exchange. A treatment of the leading gluon exchange going beyond the multi-Regge approximation is given in \[3\].

The high-energy effective action is obtained from the QCD action by separating the momentum modes of the gluon and quark fields into those relevant for scattering, for exchange and remaining "heavy modes" \[4\], \[5\]. The latter are integrated out approximately.

An important feature of the QCD, which becomes apparent in the intermediate steps, is the factorizability in the $t$-channel of the quartic terms describing effectively the high-energy $2 \to 2$ scattering. Also on the non-leading level this factorizability holds. This is essential for identifying the reggeons and the scattering vertices entering the effective action.

The structure of the fermionic terms in the QCD high-energy effective action can be read off from the ones in supersymmetric Yang-Mills theory \[4\]. The supersymmetry subgroup of the latter theory compatible with the chosen gauge allows to reconstruct fermionic terms out of gluonic ones. The scattering vertices as well as reggeons form multiplets of this subgroup. It
is interesting that the leading and some non-leading gluonic reggeons are in the same multiplet.

Some of the technical steps in our procedure can be justified only in the framework of perturbation theory, the applicability of which is restricted to the semi-hard region. Referring to this perturbative Regge region we can give the inverse derivatives appearing in the calculations and in the final result a meaning, since there the typical longitudinal momenta are not small and small transverse momenta should not give essential contribution either. Technically inverse derivatives come in by adopting the light-cone axial gauge and by integrating over the heavy modes. Thus non-local interactions are an essential feature of the high-energy effective action. With improvements the effective action concept will be useful also for a non-perturbative treatment of peripheral high-energy scattering.

2 Separation of modes

We follow the derivation of the effective action invented for the leading gluon exchange and quark exchange keeping now all terms of the non-leading (by one power of \(s\)) contributions [4]. The separation of modes is the first essential step.

It is convenient to start from the Yang-Mills action in the light-cone axial gauge \(A_- = 0\),

\[
\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)}
\]

\[
\mathcal{L}^{(2)} = -2A^{*a}(\partial_+ \partial_- - \partial \partial^*)A^a
\]

\[
\mathcal{L}^{(3)} = -\frac{g}{2}J^{a}A^a - \frac{g}{2}j^{a}A^a
\]

\[
\mathcal{L}^{(4)} = \frac{g^2}{8}J^{a}\partial_-^2J^{a} - \frac{g^2}{8}j^{a}j^{a}
\]

We use light-cone components for the longitudinal part of vectors and complex numbers for the transverse part [4]. The space-time derivatives are normalized such that \(\partial_- x_+ = \partial_+ x_- = \partial x = \partial^* x^* = 1\). The gluon field is represented by the transverse gauge potential \(A^a, A^{*a}\). It enters the interaction terms [4] in the combinations \(A^{a} = \partial^- (\partial A^a + \partial^* A^{*a})\), \(A^{*a} = i(\partial A^a - \partial^* A^{*a})\) and in the currents

\[
J^{a} = i(A^{*}T^{a} \frac{\partial}{\partial x} A), \quad j^{a} = (A^{*}T^{a} A)
\]
In the following we encounter besides of the longitudinal components $J_a$ also the transverse components $J^a, J^{a\ast}$ of the vector current (obtained by replacing $\partial_-$ by $\partial^\ast$ and $\partial$, respectively). We use the abbreviation $(AT^aB) = -i f^{abc}A^bB^c$ with $f^{abc}$ the structure constants of SU($N$).

The notations are chosen such that there is a close relation to the leading terms of the effective action [4]: The expression $A^a_t$ describes the leading gluonic reggeon and the current $J_a^\ast$ determines the leading scattering vertex. We shall see that some of the non-leading gluonic reggeons are described by the expression $A^a_\ast$ in terms of the original gluon field $A^a$. From the point of view of momentum representation $A^a_t$ and $A^a_\ast$ represent the projections of the transverse gauge potential $A^a(k)$, the first parallel to the transverse part of its momentum $k^\mu$ and the second orthogonal to $k^a$. The non-leading scattering vertices involve besides of the current $j^a$ other currents like $J^a, J^{a\ast}$. Removing the redundant field components in the light-cone gauge is convenient because now the complex field $A^a$ is directly related to the gluonic degrees of freedom. However introducing this gauge is a technical step which can be avoided since the effective action is gauge invariant.

We separate the field into modes $A = A_t + A_s + A_1$. $A_t$ are the momentum modes typical for exchanged gluons, $A_s$ are the modes typical for scattering gluons and $A_1$ are the heavy modes, which do not contribute directly to the scattering or exchange and will be integrated out.

The modes are separated according to the multi-Regge kinematics, i.e. the momentum configuration of a multi-particle $s$-channel (intermediate) state ($p_l, l = 0, 1, ..., n$) giving the dominant contribution in the leading $\ln s$ approximation. Decomposing the transferred momenta $k_l = p_A - \sum_{i=0}^{l-1} p_i$ with respect to the (almost light-like) momenta of incoming particles $p_A, p_B$

$$k^\mu = \sqrt{2s}(k_+p_B^\mu + k_-p_A^\mu) + k'^\mu,$$

the multi-Regge kinematics is characterized by the conditions

$$k_{+n} \gg \ldots \gg k_{+1}, \quad k_{-n} \ll \ldots \ll k_{-1},$$

$$k_{+l}k_{-l} \ll |\kappa_l|^2, \quad s_l = k_{+l-1}k_{-l+1} \gg |\kappa_l|^2,$$

$$\prod_{l=1}^n s_l = s \prod_{l=2}^n |\kappa_l - \kappa_{l-1}|^2.$$

Here $\kappa_l$ denotes the transverse (with respect to $p_A, p_B$) part of the momentum $k$. It is represented by a 4-vector in (3) and in the following it will be
represented by a complex number keeping the same notation. The longitudinal momenta are strongly ordered. The subenergies $s_l$ are large compared to the transferred momenta. The longitudinal contribution to the transferred momenta squared is small. In loops the main contribution from s-channel intermediate particles arises from the vicinity of the mass shell. Therefore the modes $A_t$, $A_s$, $A_1$ are characterized by the following conditions

\begin{align*}
A_t & : |k_- k_+| \ll |\kappa|^2 \\
A_s & : |k_- k_+ - |\kappa|^2| \ll |\kappa|^2 \\
A_1 & : |k_- k_+| \gg |\kappa|^2 .
\end{align*}

(4)

We introduce the mode separation into the action (1) by substituting $A$ by $A_s + A_t + A_1$. The kinetic term decomposes into three, one for each of the modes, which follows immediately from momentum conservation.

Consider now the triple term $\mathcal{L}^{(3)}$ of the action (1). Introducing the mode separation results in many terms, the most important of them for our discussion are those where two of the fields have longitudinal momenta of the same order and the third one has much larger or much smaller longitudinal momentum. We denote by $\mathcal{L}^{(3)}_1$ the terms with one of the three fields in the exchange mode $A_t$ and two in the scattering or heavy mode $A_s + A_1$. The field in the $A_t$ mode will be written as the last factor in each of those terms and decomposed into the expressions $A^a_+$ and $A^a$ appearing in the action (1), $\partial A_t = \frac{1}{2}(\partial A_{t+} - iA_{t-})$. Adopting this convention allows in the following to omit the subscripts ($t$, $s$, 1) referring to the range of modes. We use the currents $J^a$, $J^a$, $J^a$, $j^a$ introduced above to express the two field in the modes $A_s + A_1$ in some of these terms. In this way the considered part of the separated triple term $\mathcal{L}^{(3)}_1$ can be written by a straightforward rearrangement of terms as

\begin{align*}
\mathcal{L}^{(3)}_1 &= -\frac{g}{2} [J^a - \frac{1}{2} \frac{\partial}{\partial \kappa}(\partial J^a + \partial^* J^{a*})] A^a_+ \\
&\quad - i \frac{\partial^2}{\partial x} \left( \frac{1}{\partial_-}(\partial A + \partial^* A^*) T^a A \right) - i \frac{\partial^2}{\partial x} \left( \frac{1}{\partial_-}(\partial A + \partial^* A^*) T^a A^* \right) A^a_+ \\
&\quad - [j^a + \frac{i}{4} \frac{\partial}{\partial x}(\partial J^a - \partial^* J^{a*})] - \frac{1}{2} \frac{\partial}{\partial x} \left( \frac{1}{\partial_-}(\partial A + \partial^* A^*) T^a A \right) \\
&\quad + \frac{1}{2} \frac{\partial}{\partial x} \left( \frac{1}{\partial_-}(\partial A + \partial^* A^*) T^a A^* \right) A^a 
\end{align*}

(5)
It is understood that the second factor in each term \((A^a_+ \text{ or } A'^a)\) is in the \(A_t\) modes.

In the case of scattering with a large momentum component \(k_\parallel\) the term with \(J^a\) contributes to order \(\mathcal{O}(k_\parallel)\), the terms with \(J^a, J'^a\) and \(j^a\) to \(\mathcal{O}(k^2)\) and the other terms to \(\mathcal{O}(k^{-1})\). In describing scattering with large \(k_+\) the ordering goes in the reverse direction.

### 3 Quasi-elastic scattering

In order to identify the perturbative reggeons we study the contributions to the \(2 \to 2\) high energy gluon scattering at the tree level. Introducing the mode separation into the quartic term \(L^{(4)}\) of the action \((\mathbb{1})\) we see that the term with all four fields in the scattering modes \(A_s\) contributes to the elastic scattering. We write this contribution where two fields have momenta with large \(k_\parallel\) and two with large \(k_+\) and both close to the mass-shell,

\[
L^{(4)}_{\text{scatt}} = \frac{g^2}{4} J^a \frac{1}{\partial_\parallel} J^a - \frac{g^2}{2} J^a j^a - \frac{g^2}{4} j^a_D j^a_D .
\]  

This result is obtained directly from \(L^{(4)}\) \((\mathbb{1})\) by selecting the relevant terms in the mode decomposition and expressing the two fields with large \(k_\parallel\) in terms of a current appearing at the first factor and the two with large \(k_+\) in terms of the second current in each term. Adopting the convention that the first factor carries the modes with large \(k_\parallel\) we suppress any subscript referring to the modes. This expression applies also in the more general case if the second current in each term is built from fields with all modes \(A_t + A_s + A_1\) with large \(k_+\). The currents \(j^a_D\) are not in the adjoint representation but in the reducible representation (index \(r\)) appearing as the symmetric part in the decomposition of the product of two adjoint representations, (indices \(a,b,c,d,e\))

\[
j^a_D = (A^r D^r A) ,
(T^a)^r_{ab} (T^c)^c_{cd} + (T^a)^r_{ac} (T^c)^c_{bd} = (D^r)^{ab} (D^r)^{cd} .
\]

The symmetric representation appears because we represent interaction terms without a gluon in the \(t\)-channel in form of \(t\)-channel exchange.
We write down all quartic terms contributing to the scattering of a gluon with large \( k_- \) with the two other fields in all modes (see Fig. 1)

\[
\mathcal{L}^{(4)}_{\text{tot}} = \mathcal{L}_{\text{scatt}}^{(4)} + \langle \mathcal{L}^{(3)}_1^{\text{tot}} \rangle_{A_t} + \langle \mathcal{L}^{(3)}_1^{\text{tot}} \rangle_{A_1} .
\]  

(8)

The second term is obtained by substituting the product of the two fields in the exchange modes \( A_t \) by their propagator. The analogous contraction is done in the last term with \( A_1 \) modes, where \( \mathcal{L}^{(3)}_1 \) is restricted to the first order in \( A_1 \) [4]. This is the simplest but essential contribution from the integration over "heavy" modes. In our approximation we have to sum only the contributions from one-particle intermediate states out of the modes \( A_1 \). Technically this can be done by eliminating \( A_1 \) using the equations of motion linearized with respect to \( A_1 \).

We avoid to write here the resulting explicit form of \( \mathcal{L}^{(4)}_{\text{tot}} \) (8). We disregard terms \( \mathcal{O}(k_-^{-1}) \) and use the condition that two fields carrying momenta with large \( k_- \) are close to mass-shell. Those two fields we write as the first factor in each term. It turns out that this factor can be expressed in terms of one of the currents \( J^a, J^a, J^{*a}, j^a, j^r_D \) in each term. This observation is a result of an involved calculation. It is an important step because it is the first signal of the expected factorizability which we are going to discuss now.

Consider the contribution of \( \mathcal{L}^{(4)}_{\text{tot}} \) to high-energy scattering where the two other fields are in the scattering modes \( A_s \) too, but with large \( k_+ \). We obtain from the explicit form of (8) with factorized currents by restricting the two fields carrying large \( k_+ \) also to the modes \( A_s \)

\[
\mathcal{L}^{(4)}_{\text{tot},\text{scatt}} = \frac{g^2}{8} J^a - \frac{1}{\partial^2 a} \left( \frac{g^2}{16} \left( \partial_+ J^a - \frac{1}{\partial^2 a} \right) \right) (\partial_- J^a) \\
+ g^2 J_s j_{sR} - \frac{g^2}{2} J^a J^a - \frac{g^2}{4} J^r_D J^r_D + \mathcal{O}(s^{-1}) .
\]  

(9)

The first current in each term carries the scattering modes with large \( k_- \) and the second current the scattering modes with large \( k_+ \). The current with subscript \( R \) is obtained from the expression of the corresponding current without this subscript by substituting \( A^a \to A^a_R = -\frac{\partial}{\partial a} A^{a*} \). This corresponds to the gauge transformation leading from the gauge \( A^a_- = 0 \) to the gauge \( A^a_+ = 0 \).

It is remarkable and essential for the approach that, up to the non-leading terms, the two fields carrying large \( k_+ \) can be written in terms of currents too.
The currents of the scattering modes with large $k_+$ carrying the subscript $R$ arise in our calculation as the sum of many terms transformed by applying the equations of motion for the scattering particles, i.e. the mass-shell condition.

As the further technical remark it is useful to notice that the analogous result to Eq. (9) without the substitution $A^a \rightarrow A^a_R$ in the currents with label $R$ is obtained from the first two terms of (8) only if $\mathcal{L}^{(3)}_1$ is truncated by omitting those terms which are $O(k^{-1})$ in the large $k_-$ scattering regime.

The other contributions, in particular the ones from the heavy mode integration, result merely in the gauge transformation, i.e. in the replacement $A^a \rightarrow A^a_R$.

The result (9) has the form expected from parity symmetry: Formulating the scattering of gluon with large $k_-$ in the gauge $A^a_\pm = 0$ or one with large $k_+$ in the gauge $A^a_+ = 0$ should be the same up to exchanging $+ \leftrightarrow -$ accompanied by the corresponding gauge transformation. The result exhibits the important feature of $t$-channel factorizability which is essential for the effective action.

Each of the current times current terms in (9) can be viewed as the result of an elementary $t$-channel exchange (pre-reggeon). Here we can read off which reggeons arise. The interaction of the pre-reggeons and in particular their reggeization has to be derived from the further analysis (Sec. 5).

The first term in Eq.(9) represents the leading $O(s^1)$ contribution to the quasi-elastic scattering. Its first factor arises directly from the first term in $\mathcal{L}^{(3)}_1$. The terms of interest are the following which contribute as $O(s^0)$ to the amplitude. The second term in Eq.(9) is closely related to the leading term, both describe positive parity exchange. Its first factor arises from the second term in $\mathcal{L}^{(3)}_1$ by applying the mass-shell condition. In the pure gluodynamics considered now there are two terms describing colour octet odd parity exchange (the fourth and fifth term in Eq.(9)), where

$$j^a_s = j^a + \frac{i}{4} \partial\partial^* (\partial J^a - \partial^* J^a) .$$

$j_s$ appears in $\mathcal{L}^{(3)}_1$ as the leading terms involving $\mathcal{A}'$ exchange. The last term written explicity in Eq.(9) corresponds to the symmetric gauge group representation in the $t$-channel (compare (7)).

Introducing 5 pairs of fields for the pre-reggeon exchanges corresponding to each term in Eq.(9) the quasi-elastic high-energy scattering can be
described by the following effective action

\[
\mathcal{L}_{\text{eff,scatt}} = \mathcal{L}_{\text{kin.}} + \mathcal{L}_{s-} + \mathcal{L}_{s+}
\]

\[
\mathcal{L}_{\text{kin.}} = -2A_{s}^{a}(\partial_{-}\partial_{+} - \partial\partial^{*})A_{s}^{a} - 2A_{s}^{a}\partial\partial^{*}A_{s}^{a} + A_{s}^{a}\partial A_{s}^{a}
\]

\[
-\mathcal{A}_{s}^{a}A_{s}^{a} - 2\mathcal{A}_{s}^{a}\mathcal{A}_{s}^{a} + B^{r(+)B^{r(-)}}
\]

\[
\mathcal{L}_{s-} = -\frac{g}{2}j_{a}A_{+}^{a} - \frac{g}{4}\left(\frac{\partial_{+}}{\partial\partial^{*}}J_{a}^{a}\right)A_{s+}^{a} - gJ_{s}A_{s}^{a} - gJ_{s}A_{s}^{a} - \frac{g}{2}J_{a}B^{r(+)B^{r(-)}} .
\]

\[
\mathcal{L}_{s+} \text{ is obtained from } \mathcal{L}_{s-} \text{ by replacing the labels } + \leftrightarrow - \text{ and the currents by their partners with label } R. \text{ The result for the effective action for quasi-elastic scattering is checked by observing that each term in (9) is reproduced from (11) by integrating out the corresponding pre-reggeon.}
\]

Each pre-reggeon is represented by a pair of fields. The distinction by labels +, - is related to the fact that the reggeon exchange is oriented in rapidity. The fields with label + couple to scattering particles with large \(k_{-}\) only and vice versa.

4 Including fermions

In the kinematics of high-energy scattering it is natural to decompose Dirac fields \(\psi\) describing quarks into light-cone parts, \(\psi = \psi_{-} + \psi_{+}, \gamma_{-}\psi_{+} = \gamma_{+}\psi_{-} = 0\). Adopting the gauge \(A_{-} = 0\) we eliminate \(\psi_{+}\) and decompose \(\psi_{-}\) with respect to a spinor basis \(u_{ij}, i, j = \pm 4\),

\[
\psi_{-} = fu_{-} + f\bar{u}_{+}, \quad \gamma u_{-} = \gamma^{*}u_{-} = 0
\]

with two complex component field \(f\) and \(\bar{f}\). These two fields describe the two chiralities of a quark.

To describe the essential features of the fermionic terms in the high-energy effective action it is enough to look at the case of supersymmetric Yang-Mills theory, i.e. restricting to one fermion chirality \(f\) and changing to the adjoint gauge group representation.

There is a supersymmetry subgroup (with generators \(\delta_{i}, \delta_{r}\)) leaving \(A_{-}\) unchanged and acting on \(A\) and \(f\) as

\[
\delta_{i}A = f, \quad \delta_{i}A^{*} = f^{*}, \quad \delta_{i}f = 2i\partial_{-}A, \quad \delta_{i}f^{*} = 2i\partial_{-}A^{*}
\]

\[
\delta_{r}A = f, \quad \delta_{r}A^{*} = -f^{*}, \quad \delta_{r}f = -2i\partial_{-}A, \quad \delta_{r}f^{*} = 2i\partial_{-}A^{*} .
\]
The transformations $\delta_i$ and $\delta_r$ are ones considered in [4] for the pure imaginary and real transformation parameter, respectively. The subsequent application of two transformations of the same type results in the action of the derivative $\partial_-$. This subgroup can be used to reconstruct the fermionic terms in the light-cone action (1) or in the separated triple terms (5) from the gluonic ones. In particular we have used the supersymmetry (13) to obtain the fermionic contributions to the currents which we are going to explain now. 

Here we shall not discuss the terms in the effective action related to fermion exchange considered earlier [4] and rather concentrate on the fermionic contributions to the currents appearing in the scattering with gluon exchange.

By studying the action of the supersymmetry transformations (13) to the known gluonic parts of the currents we obtain the fermionic parts from the condition that applying the supersymmetry transformation twice must result in the action by the derivative $\partial_-$. The fermionic contributions to the currents ($J \rightarrow J + J_F$, where $J$ is any of the above gluonic currents) are given by

$$J^a_{-F} = (f^*T^a f), \quad J^a_{ F} = -\frac{1}{2}(\frac{1}{\partial_-} f^* T^a \leftrightarrow \partial f) . \quad (14)$$

It turns out that with fermions included the current $j^a$ splits up into two $j^a$ and $\mathcal{K}^a$, both with the gluonic part $(A^*T^a A)$ and with the fermionic parts

$$j^a_F = 0 , \quad \mathcal{K}^a_F = -\frac{i}{4}(f^* T^a \leftrightarrow \partial^{-1} f) . \quad (15)$$

After inclusion of fermions the triple terms with gluonic exchange given by Eq. (5) still have the same form (5) with the fermionic parts (14), (15) included in the currents and with the replacement

$$j^a \rightarrow \frac{1}{2}(j^a + \mathcal{K}^a) . \quad (16)$$

This splitting of the current $j^a$ is the reflection of the fact that $j^a$ and $\mathcal{K}^a$ belong to the different susy multiplets of currents (see below).

The high-energy scattering terms (9) change also by including the fermionic parts in the currents and by replacing the 4-th term on the r.h.s. of (9)

$$j^a j^a_R \rightarrow \frac{1}{2} j^a j^a_R + \frac{1}{2} \mathcal{K}^a \mathcal{K}^a_R . \quad (17)$$
According to (16) the combination of currents $j_s$ is now

$$j_s^a = \frac{1}{2} j^a + \frac{1}{2} K^a + \frac{i}{4} \frac{1}{\partial \partial^*} (\partial J^a - \partial^* J^{a*})$$  \hspace{1cm} (18)$$

This modified form of $L_{\text{eff, scatt}}$ replacing (9) in the case of fermions included is the result of an extensive calculations analogous to the one leading from Eq.(8) to Eq.(9) above.

The currents with label $R$ are expressed analogously to the ones without this label where the fields are replaced by $A_R^a, f_R^a$:

$$A_R^a = - \frac{\partial^*}{\partial} A^{a*}, \quad f_R^a = \frac{\partial^*}{\partial} f^{a*}.$$  \hspace{1cm} (19)$$

It is remarkable that the exchange fields $A^a_\pm$ and $A^a_\mp$ together with two combinations of the exchange modes $f^a_\pm$ form a quartet under (13). Correspondingly, the currents $J^a_\pm$ and $j^a_\mp$ are the even members of a supersymmetry quartet. $J^{a*}$ is part of a doublet and $K^a$ is a member of a quartet again. The other even member of the latter multiplet is $k^a = (\frac{1}{2} f^* T^a \frac{1}{2} f)$ and it is related to the subasymptotics $O(s^{-1})$. In this way we understand that there are four pre-reggeons in the gauge group states of the gluon on the non-leading level $O(s^0)$ because there are four different susy multiplets where corresponding currents $(j^a, J^a, J^{a*}, K^a)$ of this level enter.

Thus we have obtained that with fermions there are three terms contributing to the high-energy scattering in the colour octet odd parity channel and we find three reggeons near $j = 0$ in this channel related to the exchange of $A'$.

A similar effect of removing degeneracy as in (16), (17) leading to one additional reggeon compared to gluodynamics arises in the case of the other gauge group representations $j^D_\pm$.

5 Reggeon interaction

Besides of the effective vertices of scattering $L_{s \pm}$ (11) the high-energy effective action involves vertices $L_p$ of emission of particles (described by the scattering modes $A_s$) from the pre-reggeons (related to the exchange modes $A_t$). These imply reggeization and reggeon interactions. The final form of the effective
action is written as the sum of terms
\[ L_{\text{eff}} = L_{\text{kin}} + L_{s+} + L_{s-} + L_p \]  

(20)

where \( L_{\text{kin}} \), \( L_{s-} \) are given by Eq. (11).

A way to obtain \( L_p \) in the multi-Regge approximation is to consider the inelastic \( 2 \to 3 \) scattering where the final state particles are separated by large rapidity gaps. We construct the terms of order 5 in the fields which give an effective description of this inelastic scattering \( 2 \to 3 \) on the tree level (Fig. 2)

\[
L^{(5)} = <L^{(4)}_{\text{tot}} L^{(3)}_1 >_{A_t} + <L^{(4)}_{\text{tot}} L^{(3)}_1 >_{A_t} + <L^{(4)}_{\text{scatt}} >_{A_t} + <L^{(4)}_{\text{scatt}} >_{A_t} .
\]

(21)

Substituting \( L^{(4)}_{\text{tot}} \) \( ^8 \) and \( L^{(3)}_{\text{scatt}} \) \( ^3 \) into the formula (21) results in a lengthy expression which we suppress here. The technical remark presented in Sec. 3 and referring to Eq. (9) applies also here: the second term contributes mainly to replacing \( A^a \to A^a R \) in the currents of large \( k_+ \) particles. The structure of the result can be read off from the other 3 terms only.

The production vertices \( L_p \) are read off from the result by representing it as

\[
L^{(5)} = <L_{s-} L_p L_{s+} >_{A_t}
\]

(22)

and factorizing the scattering vertices \( L_{s-} \), \( L_{s+} \) in view of the kinetic terms in the effective action of quasi elastic scattering (11), (compare Fig. 2). By extensive calculation we have checked that the above factorization property (22) holds and have extracted the vertices \( L_p \).

There is a number of terms in \( L_p \) differing by the kind of pre-reggeons in the two \( t \)-channels. Here we present as an example the part referring to the odd parity reggeons \( (A'_1, A'_2) \) in both channels (without fermions)

\[
L_p |_{A'_1, A'_2} = \frac{i g}{4} \left\{ 2(\frac{1}{\partial} A_s^{(-)} T^a \partial A_s^{(+)} ) + 2(\partial^* A_s^{(-)} T^a \frac{1}{\partial^*} A_s^{(+)} ) + (\frac{\partial^*}{\partial^*} A_s^{(-)} T^a \partial A_s^{(+)} ) + (A_s^{(-)} T^a A_s^{(+)} ) + 4(A_s^{(-)} T^a A_s^{(+)}) + 2(\frac{1}{\partial} A_s^{(-)} T^a \frac{1}{\partial^*} A_s^{(+)} ) + 2(A_s^{(-)} T^a A_s^{(+)}) + 2(\partial^* A_s^{(-)} T^a \frac{1}{\partial^*} A_s^{(+)} ) + 2(A_s^{(-)} T^a A_s^{(+)}) \right\} \frac{1}{\partial} A_s^{(+)} + \text{c.c.}
\]

(23)
The details about the effective production vertices related to the non-leading $\mathcal{O}(s^0)$ gluonic reggeons will be given elsewhere.

6 Discussion

At high energy, in the perturbative Regge region, gluon interactions contribute not only to the leading asymptotics (Regge singularity near $j = 1$, resulting in particular into BFKL pomeron) but also to subleading terms in the asymptotic expansion.

By the leading scattering vertex $J^a_-$ only the longitudinal momenta of the scattering partons can be probed. It does not feel the transverse momentum and the spin structure. The currents $(J^a, J^{a*}, j^a, j_s^a, K^a, j_D^a)$ which determine the coupling of the non-leading reggeons to the scattering gluons or quarks are sensitive to the transverse momenta and the helicity of the partons.

Extending the high-energy effective action in the multi-Regge approximation to the sub-leading terms $\mathcal{O}(s^0)$ we have identified the reggeons arising near angular momentum $j = 0$. There is a partner of the leading gluon exchange with the same parity property. In the odd parity channel we find two reggeons in the pure Yang-Mills case carrying the adjoint colour group representation. With fermions included they split off into three. The occurence of more than one reggeon in one channel is similar to the case encountered in the reggeization of a scalar interacting with gauge field [6].

Presently we are missing a good physical understanding of the role of the different reggeons, in particular in the parity-odd channel. They are distinguished by their couplings to the scattering partons. Therefore the task which should give the answer to this question is to look for physical processes where the differences encoded in the currents $(J^a, J^{a*}, j^a, j_s^a,...)$ show up. Also it would be desirable to look for physical effects of the reggeons in the symmetric colour representations. Among them there is a colour singlet reggeon, which may be of particular interest, because a single reggeon exchange of this type can contribute to physical amplitudes. The other (non-singlet) reggeons appear only in the exchanges of two or more reggeons building a colour singlet. The reggeons interact by the exchange of $s$-channel gluons (production vertices $L_p$).

Our results allow to derive two-reggeon interaction kernels in analogy to the one appearing in the BFKL equation. The analogon of the BFKL
equation for the exchange of one leading and one non-leading reggeon results in Regge singularities near \( j = 0 \) of physical significance. They may be related, for example, to the small \( x \) behaviour of (combinations of) structure functions. An interesting task will be to find out whether the remarkable symmetries of the BFKL kernel apply also to the non-leading interactions.

With this analysis we did a step towards a field theoretic understanding of non-leading Regge singularities. The methods and the validity of the results are restricted to the perturbative region.

The high-energy effective action can be considered as a new approach to the classical question of reggeization in field theory. It provides an alternative to the approach based on amplitude analysis proposed by Gell-Mann et al \[7\] and developed in \[8\]. Comparing we see that the \( t \)-channel factorizability of the amplitudes is a crucial point in both approaches.

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Figure 1: The graphical illustration of Eq. (8) for the complete quartic terms. Different line forms represent different modes: full line - scattered modes, dotted line - exchange modes, bold line - heavy modes, dashed line - the sum of all modes.
Figure 2: The graphical illustration of the terms describing inelastic scattering Eq. (21) and of the factorization leading to the production vertices Eq. (22).