Dark Energy and Dark Gravity

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Observations provide increasingly strong evidence that the universe is accelerating. This revolutionary advance in cosmological observations confronts cosmological theory with a tremendous challenge, which it has so far failed to meet. Explanations of the acceleration within the framework of general relativity are plagued by difficulties. General relativistic models are nearly all based on a dark energy field with fine-tuned, unnatural properties. There is a great variety of models, but all share one feature in common – an inability to account for the gravitational properties of the vacuum energy. Speculative ideas from string theory may hold some promise, but it is fair to say that no convincing model has yet been proposed. An alternative to dark energy is that gravity itself may behave differently from general relativity on the largest scales, in such a way as to produce acceleration. The alternative approach of modified gravity provides a new angle on the problem, but also faces severe difficulties, including the problem of explaining why the vacuum energy does not gravitate. The lack of an adequate theoretical framework for the late-time acceleration of the universe represents a deep crisis for theory – but also an exciting challenge for theorists. It seems likely that an entirely new paradigm is required to resolve this crisis.

I. INTRODUCTION

The current “standard model” of cosmology is the inflationary cold dark matter model with cosmological constant, usually called LCDM, which is based on general relativity and particle physics (i.e., the Standard Model and its minimal supersymmetric extensions). This model provides an excellent fit to the wealth of high-precision observational data, on the basis of a remarkably small number of cosmological parameters \[1\]. In particular, independent data sets from CMB anisotropies, galaxy surveys and supernova luminosities, lead to a consistent set of best-fit model parameters – which represents a triumph for LCDM.

The standard model is remarkably successful, but we know that its theoretical foundation, general relativity, breaks down at high enough energies, usually taken to be at the Planck scale,

\[ E \gtrsim M_p \sim 10^{16} \text{TeV}. \] (1)

The LCDM model can only provide limited insight into the very early universe. Indeed, the crucial role played by inflation belies the fact that inflation remains an effective theory without yet a basis in fundamental theory. A quantum gravity theory will be able to probe higher energies and earlier times, and should provide a consistent basis for inflation, or an alternative that replaces inflation within the standard cosmological model (for recent work, see e.g. Refs. \[2\]).

An even bigger theoretical problem than inflation is that of the late-time acceleration in the expansion of the universe \[3\]. In terms of the fundamental energy density parameters, the data indicates that the present cosmic energy budget is given by

\[ \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2} \approx 0.75, \quad \Omega_m \equiv \frac{8\pi G \rho_m}{3H_0^2} \approx 0.25. \] (2)

The Friedman equation is

\[ H^2 = \frac{8\pi G}{3} \left( \rho_m + \rho_r \right) + \frac{\Lambda}{3} - \frac{K}{a^2} \]

\[ = H_0^2 \left[ \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda + \Omega_K (1+z)^2 \right], \] (3)

where \( z = a^{-1} - 1 \) and \( a(0) = 1 \). Together with the conservation equations it gives

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho_m + 2\rho_r - 2\Lambda \right). \] (4)

Equations (2) and (4) lead to the dramatic conclusion that the universe is currently accelerating,

\[ \ddot{a}_0 > 0. \] (5)
FIG. 1: Observational constraints in the \((\Omega_m, \Omega_\Lambda)\) plane: joint constraints (left) (from [4]); recent compilation of supernova constraints (right) (from [5]).

The data further indicates that the universe is (nearly) spatially flat,

\[ |\Omega_K| \ll 1. \]  

These results are illustrated in Fig. 1.

Within the framework of general relativity, the acceleration typically originates from a dark energy field with negative pressure,

\[ w \equiv \frac{p}{\rho} < -\frac{1}{3}, \]  

such as vacuum energy \((w = -1)\) or a dynamical scalar field ("quintessence", \(w > -1\)). So far, none of the available models has a natural or convincing explanation.

For the simplest option of vacuum energy, i.e., the LCDM model, the observed value of the cosmological constant is overwhelmingly smaller than the prediction of current particle physics:

\[ \rho_{\Lambda, \text{obs}} \sim \Lambda \approx H_0^2 M_p^2 \sim (10^{-33} \text{eV})^2 (10^{19} \text{GeV})^2 \sim (10^{-3} \text{eV})^4, \]  

\[ \rho_{\Lambda, \text{theory}} \sim M_{\text{fundamental}}^4 \geq M_{\text{susy}} \gtrsim 1 \text{ TeV}^4 \gg \rho_{\Lambda, \text{obs}}. \]  

The fundamental energy scale \(M_{\text{fundamental}}\) could be as high as the Planck scale, \(~10^{16} \text{TeV}\), but if there is supersymmetry, then a cancellation of fermion and boson contributions to the vacuum energy applies above the supersymmetry scale, \(M_{\text{susy}} (~1 \text{ TeV})\).

In addition, the \(\Lambda\) value needs to be strongly fine-tuned to be of the same order of magnitude today as the current matter density, i.e.,

\[ \rho_\Lambda \sim \rho_m(0) \Rightarrow \Omega_\Lambda \sim \Omega_M, \]  

otherwise galaxies and then life could not emerge in the universe. The question is how this “coincidence” arises at late times, given that

\[ \rho_\Lambda = \frac{\Lambda}{8\pi G} = \text{constant}, \quad \text{while} \quad \rho_m \propto (1 + z)^3. \]  

No convincing or natural explanation has yet been proposed.
Alternatively, it is possible that there is no dark energy, but instead there is a low-energy/large-scale, i.e., “infrared”, modification to general relativity that accounts for the late-time acceleration. Schematically, we are modifying the geometric side of the field equations,

\[ G_{\mu\nu} + G_{\mu\nu}^{\text{dark}} = 8\pi G T_{\mu\nu}, \]

rather than the matter side,

\[ G_{\mu\nu} = 8\pi G (T_{\mu\nu} + T_{\mu\nu}^{\text{dark}}), \]

as in the general relativity approach. Modified gravity represents an intriguing possibility for resolving the theoretical crisis posed by late-time acceleration. However, it turns out to be extremely difficult to modify general relativity at low energies in cosmology, without violating the low-energy solar system constraints, or without introducing ghosts and other instabilities into the theory. Up to now, there is no convincing alternative to the general relativity dark energy models – which themselves are not convincing.

II. GENERAL RELATIVISTIC APPROACHES TO THE PROBLEM

The “standard” general relativistic approach is based on the cosmological constant as vacuum energy:

\[ G_{\mu\nu} = 8\pi G \left[ T_{\mu\nu} + T_{\mu\nu}^{\text{vac}} \right], \quad T_{\mu\nu}^{\text{vac}} = -\frac{\Lambda}{8\pi G} g_{\mu\nu}, \]

where the vacuum energy-momentum tensor is Lorentz invariant. This approach faces the problem of accounting for the incredibly small and highly fine-tuned value of the vacuum energy, as summarised in Eqs. (8)–(10).

String theory provides a tantalising possibility in the form of the “landscape” of vacua [6]. There appears to be a vast number of vacua admitted by string theory, with a broad range of energies above and below zero. The idea is that our observable region of the universe corresponds to a particular small positive vacuum energy, whereas other regions with greatly different vacuum energies will look entirely different. This multitude of regions forms in some sense a “multiverse”. This is an interesting idea, but it is highly speculative, and it is not clear how much of it will survive the further development of string theory and cosmology.

An alternative approach to LCDM is the classical interpretation of \( \Lambda \) (see, e.g., Ref. [7]). In this approach, \( \Lambda \) is treated as a classical geometric constant, on a par with Newton’s constant \( G \). Thus the field equations are interpreted in the geometrical way,

\[ G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \]

In this approach, the small and fine-tuned value of \( \Lambda \) is no more of a mystery than the host of other fine-tunings in the constants of nature. For example, more than a 2% change in the strength of the strong interaction means that no atoms beyond hydrogen can form, so that stars and galaxies would not emerge. However, this classical approach to \( \Lambda \) does not evade the vacuum energy problem – it simply shifts that problem to “why doesn’t the vacuum gravitate?”, i.e., why is

\[ \rho_{\text{vac}} = 0. \]

The idea is that particle physics and quantum gravity will somehow discover a cancellation or symmetry mechanism to explain why the vacuum does not gravitate. This would be a simpler solution than that indicated by the string landscape approach, and would evade the disturbing anthropic aspects of that approach. Of course, this is not a question of choice, but awaits the further development of particle physics and quantum gravity.

Within general relativity, various alternatives to LCDM have been investigated.

- **Dynamical dark energy: quintessence** [3].

Here we replace the constant \( \Lambda/8\pi G \) with the energy density of a scalar field \( \varphi \) (with standard Lagrangian), where

\[ \rho_{\varphi} = \frac{1}{2} \dot{\varphi}^2 + V(\varphi), \quad p_{\varphi} = \frac{1}{2} \dot{\varphi}^2 - V(\varphi), \]

\[ \ddot{\varphi} + 3H \dot{\varphi} + V'(\varphi) = 0. \]
The field rolls down its potential, so that the dark energy density varies through the history of the universe. “Tracker” potentials have been found for which the field energy density follows that of the dominant matter component. This offers the possibility of solving or alleviating the coincidence problem. Although these models are insensitive to initial conditions, they do require strong fine-tuning of the parameters to secure recent dominance of the field, and hence do not evade the coincidence/fine-tuning problem. More generally, the quintessence potential, somewhat like the inflaton potential, remains arbitrary, until and unless fundamental physics selects a potential. There is currently not a natural choice of potential.

In conclusion, there is no compelling reason as yet to choose quintessence above the $\Lambda$ model of dark energy.

- **Dynamical dark energy: more general models** [3].

  One example is quintessence that is coupled to cold dark matter, so that the energy conservation equations become

  \[ \dot{\phi} [\ddot{\phi} + 3H\dot{\phi} + V'(\phi)] = J, \]

  \[ \dot{\rho}_m + 3H\rho_m = -J, \]

  where $J$ is the energy exchange.

  Another case is scalar fields with non-standard kinetic term in the Lagrangian, for example,

  \[ \mathcal{L} = -F(X) - V(\phi) \text{ where } X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi. \]

  The standard Lagrangian has $F(X) = X$. Some of the non-standard $F$ models may be ruled out on theoretical grounds. An example is provided by “phantom” fields, with negative kinetic energy density, $F(X) = -X$. They have $\omega < -1$, so that their energy density grows with expansion.

  Another example is “k-essence” fields which produce late-time acceleration. The sound speed of the field fluctuations for the case of Eq. (21) is

  \[ c_s^2 = \frac{F_X}{F_{,X} + 2XF_{,XX}}. \]

  For a standard Lagrangian, $c_s^2 = 1$. But for the class of $F$ that produce accelerating k-essence models, it turns out that $c_s^2 > 1$, so that these models may be ruled out, since they violate standard causality [8].

  For models not ruled out on theoretical grounds, there is the same general problem as with quintessence, i.e. that no model is selected by fundamental physics and any choice of model is more or less arbitrary. Quintessence then appears to at least have the advantage of simplicity – although the $\Lambda$ model has the same advantage over quintessence.

- **Dark energy as a nonlinear effect from structure formation** [9].

  As structure forms and the matter density perturbation becomes nonlinear, there are two questions that are posed: (1) what is the backreaction effect of this nonlinear process on the background cosmology?; (2) how do we perform a covariant and gauge-invariant averaging over the inhomogeneous universe to arrive at the correct FRW background? The simplistic answers to these questions are (1) the effect is negligible since it occurs on scales too small to be cosmologically significant; (2) in light of this, the background is independent of structure formation, i.e., it is the same as in the linear regime. A quantitative analysis is needed to fully resolve both issues. However, this is very complicated because it involves the nonlinear features of general relativity in an essential way.

  There have been claims that these simplistic answers are wrong, and that, on the contrary, the effects are large enough to accelerate the universe. (Note the possibility that averaging effects could be significant, even if they do not lead to acceleration.) This would indeed be a dramatic and satisfying resolution of the coincidence problem, without the need for any dark energy field. Of course, the problem of why the vacuum does not gravitate, Eq. (16), would remain.

  However, these claims have been disputed, and it is fair to say that there is as yet no convincing demonstration that acceleration could emerge naturally from nonlinear effects of structure formation.
III. THE MODIFIED GRAVITY APPROACH: DARK GRAVITY

Late-time acceleration from nonlinear effects of structure formation is an attempt, within general relativity, to solve the coincidence problem without a dark energy field. The modified gravity approach shares the assumption that there is no dark energy field, but sources the acceleration in “dark gravity”, i.e. a weakening of gravity on the largest scales, due to a modification of general relativity itself.

Could the late-time acceleration of the universe be a gravitational effect? (Note that this would not remove the problem of explaining why the vacuum energy does not gravitate.) An historical precedent is provided by attempts to explain the anomalous precession of Mercury’s perihelion by a “dark planet”. In the end, it was discovered that a modification to Newtonian gravity was needed.

It is important to first stress that a consistent modification of general relativity requires a covariant formulation of the field equations in the general case, i.e., including inhomogeneities and anisotropies. It is not sufficient to propose ad hoc modifications of the Friedman equation, of the form

\[ f(H^2) = \frac{8\pi G}{3}\rho \quad \text{or} \quad H^2 = \frac{8\pi G}{3}g(\rho), \quad (23) \]

for some functions \( f \) or \( g \). We can compute the supernova distance/ redshift relation using this equation – but we cannot compute the density perturbations without knowing the covariant parent theory that leads to such a modified Friedman equation. And we also cannot compute the solar system predictions.

It is very difficult to produce infrared corrections to general relativity that meet all the minimum requirements:

- Theoretical consistency, without ghosts, instabilities or causality violation.
- Late-time acceleration consistent with supernova data.
- A matter-dominated era with an evolution of the scale factor \( a \) that is consistent with the requirements of structure formation.
- Density perturbations that are consistent with the observed matter power spectrum, CMB anisotropies and weak lensing power spectrum.
- Stable static spherical solutions for stars and vacuum, and consistency with terrestrial and solar system observational constraints.
- Consistency with binary pulsar period data.

General relativity has a unique status as a theory where gravity is mediated by a massless spin-2 particle, and the field equations are second order. If we introduce modifications to the Einstein-Hilbert action of the general form

\[ \int d^4x \sqrt{-g} R \rightarrow \int d^4x \sqrt{-g} f(R, R_{\mu\nu}R^{\mu\nu}, \ldots), \quad (24) \]

then the field equations become fourth-order, and gravity is carried also by a massless spin-0 field. In order to avoid ghosts, we impose \( f = f(R) \), and we assume \( f''(R) \neq 0 \). However, it turns out to be extremely difficult for this simplified class of modified theories to pass the observational and theoretical tests. An example is [10]

\[ f(R) = R - \frac{\mu}{R}, \quad (25) \]

For \( |\mu| \sim H_0^{-1} \), this model achieves late-time acceleration as the \( \mu/R \) term starts to dominate. But the model suffers from a non-standard matter era, nonlinear matter instabilities and violation of solar system constraints [11].

The more general scalar-tensor theories [12], which may be motivated via low-energy string theory, have an action of the form

\[ \int d^4x \sqrt{-g} \left[F(\psi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - U(\psi) \right], \quad (26) \]

where \( \psi \) is the spin-0 field supplementing the spin-2 graviton. In the context of late-time acceleration, these models are also known as “extended quintessence”. The extra degree of freedom relative to the \( f(R) \) theories allows for greater flexibility in meeting the observational and theoretical tests. However, the price we pay is additional complexity – and arbitrariness. The \( f(R) \) theories have one arbitrary function, and here there are two, \( F(\psi) \) and \( U(\psi) \). There is no preferred choice of these functions from fundamental theory.
In summary, modifications of the Einstein-Hilbert action, which lead to fourth-order field equations, either fail the minimum tests in the simplest cases, or contain more complexity and arbitrary choices than quintessence models in general relativity. Therefore, none of these models appears to be a serious competitor to quintessence in general relativity.

We turn now to a class of brane-world models whose background is no more complicated than that of LCDM, offering the promise of a serious dark gravity contender. However, there are hidden complexities and problems, as we will explain below.

IV. DGP BRANE-WORLDS: SELF-ACCELERATING COSMOLOGIES

An infra-red modification to general relativity could emerge within the framework of quantum gravity, in addition to the ultraviolet modification that must arise at high energies in the very early universe. The leading candidate for a quantum gravity theory, string theory, is able to remove the infinities of quantum field theory and unify the fundamental interactions, including gravity. But there is a price – the theory is only consistent in 9 space dimensions. Branes are extended objects of higher dimension than strings, and play a fundamental role in the theory, especially D-branes, on which open strings can end. Roughly speaking, open strings, which describe the non-gravitational sector, are attached at their endpoints to branes, while the closed strings of the gravitational sector can move freely in the higher-dimensional “bulk” spacetime. Classically, this is realised via the localization of matter and radiation fields on the brane, with gravity propagating in the bulk (see Fig. 2).

\[ e^- \quad e^+ \]
\[ \gamma \]
\[ \text{FIG. 2: The confinement of matter to the brane, while gravity propagates in the bulk (from [13]).} \]

The implementation of string theory in cosmology is extremely difficult, given the complexity of the theory. This motivates the development of phenomenology, as an intermediary between observations and fundamental theory. (Indeed, the development of inflationary cosmology has been a very valuable exercise in phenomenology.) Brane-world cosmological models inherit key aspects of string theory, but do not attempt to impose the full machinery of the theory. Instead, simplifications are introduced in order to be able to construct cosmological models that can be used to compute observational predictions (see [14] for reviews in this spirit). Cosmological data can then be used to constrain the brane-world models, and hopefully thus provide constraints on string theory, as well as pointers for the further development of string theory.

It turns out that even the simplest brane-world models are remarkably rich – and the computation of their cosmological perturbations is remarkably complicated, and still incomplete. A key reason for this is that the higher-dimensional graviton produces a tower of 4-dimensional massive spin-2 modes on the brane, in addition to the standard massless spin-2 mode on the brane (or in some case, instead of the massless mode). Here I will describe brane-world model of Dvali-Gabadadze-Porrati (DGP) type [15], which was generalized to cosmology by Deffayet [16]. These are 5-dimensional models, with an infinite extra dimension. (We effectively assume that 5 of the extra dimensions in the “parent” string theory may be ignored at low energies.)
Most brane-world models modify general relativity at high energies. The main examples are those of Randall-Sundrum (RS) type [17], where a FRW brane is embedded in an anti de Sitter bulk, with curvature radius $\ell$. At low energies $H\ell \ll 1$, the zero-mode of the graviton dominates on the brane, and general relativity is recovered to a good approximation. At high energies, $H\ell \gg 1$, the massive modes of the graviton dominate over the zero mode, and gravity on the brane behaves increasingly in a 5D way. On the brane, the standard conservation equation holds,

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (27)$$

but the Friedmann equation is modified by an ultraviolet correction:

$$H^2 = \frac{8\pi G}{3}\rho \left(1 + \frac{2\pi G \ell^2}{3}\rho\right) + \frac{\Lambda}{3}. \quad (28)$$

The $\rho^2$ term is the ultraviolet term. At low energies, this term is negligible, and we recover $H^2 \propto \rho + \Lambda/8\pi G$. At high energies, gravity “leaks” off the brane and $H^2 \propto \rho^2$. This 5D behaviour means that a given energy density produces a greater rate of expansion than it would in general relativity. As a consequence, inflation in the early universe is modified in interesting ways [14].

In the DGP case the bulk is 5D Minkowski spacetime. Unlike the AdS bulk of the RS model, the Minkowski bulk has infinite volume. Consequently, there is no normalizable zero-mode of the graviton in the DGP brane-world. Gravity leaks off the 4D brane into the bulk at large scales. At small scales, gravity is effectively bound to the brane and 4D dynamics is recovered to a good approximation. The transition from 4- to 5D behaviour is governed by a crossover scale $r_c$; the weak-field gravitational potential behaves as

$$\Psi \sim \begin{cases} r^{-1} & \text{for } r \ll r_c \\ r^{-2} & \text{for } r \gg r_c \end{cases} \quad (29)$$

Gravity leakage at late times initiates acceleration – not due to any negative pressure field, but due to the weakening of gravity on the brane. 4D gravity is recovered at high energy via the lightest massive modes of the 5D graviton, effectively via an ultralight metastable graviton.

![Confidence contours for supernova data in the DGP density parameter plane. The blue (solid) contours are for SNLS data, and the brown (dashed) contours are for the Gold data. The red (dotted) curve defines the flat models, the black (dot-dashed) curve defines zero acceleration today, and the shaded region contains models without a big bang. (From [18]).](image)

The energy conservation equation remains the same as in general relativity, but the Friedmann equation is modified:

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (30)$$

$$H^2 = \frac{H}{r_c} = \frac{8\pi G}{3}\rho. \quad (31)$$
This shows that at early times, i.e., $H r_c \gg 1$, the general relativistic Friedman equation is recovered. By contrast, at late times in a CDM universe, with $\rho \propto a^{-3} \to 0$, we have

$$H \to H_\infty = \frac{1}{r_c},$$

so that expansion accelerates and is asymptotically de Sitter. Since $H_0 > H_\infty$, in order to achieve self-acceleration at late times, we require

$$r_c \gtrsim H_0^{-1},$$

and this is confirmed by fitting supernova observations, as shown in Fig. 3. The dimensionless cross-over parameter is

$$\Omega_{r_c} = \frac{1}{4(H_0 r_c)^2},$$

and the LCDM relation,

$$\Omega_m + \Omega_\Lambda + \Omega_K = 1,$$

is modified to

$$\Omega_m + 2\sqrt{\Omega_{r_c}}\sqrt{1-\Omega_K} + \Omega_K = 1.$$

It should be emphasized that the DGP Friedman equation (31) is derived covariantly from a 5D gravitational action,

$$\int_{\text{bulk}} d^5x \sqrt{-g^{(5)}} R^{(5)} + r_c \int_{\text{brane}} d^4x \sqrt{-g} R.$$

LCDM and DGP can both account for the supernova observations, with the fine-tuned values $\Lambda \sim H_0^2$ and $r_c \sim H_0^{-1}$ respectively. If we include the further contraints on the expansion history provided by baryon acoustic oscillations and the CMB shift parameter, then DGP is still compatible with the data, although LCDM provides a better fit [18], as illustrated in Fig. 4. The degeneracy may be broken by observations based on structure formation, since the two models suppress the growth of density perturbations in different ways [19]. The distance-based observations draw only upon the background 4D Friedman equation (31) in DGP models -- and therefore there are quintessence models in general relativity that can produce precisely the same supernova distances as DGP [20]. By contrast, structure formation observations require the 5D perturbations in DGP, and one cannot find equivalent general relativity models [21].
FIG. 5: The growth factor $g(a) = \Delta(a)/a$ for LCDM (long dashed) and DGP (solid, thick), as well as for a dark energy model with the same expansion history as DGP (solid, thick). DGP-4D (solid, thin) shows the incorrect result in which the 5D effects are set to zero. (From [21].)

For LCDM, the analysis of density perturbations is well understood. For DGP it is much more subtle and complicated. Although matter is confined to the 4D brane, gravity is fundamentally 5D, and the bulk gravitational field responds to and backreacts on density perturbations. The evolution of density perturbations requires an analysis based on the 5D nature of gravity. In particular, the 5D gravitational field produces an anisotropic stress on the 4D universe. If one neglects this stress and all 5D effects, and simply treats the perturbations as 4D perturbations with a modified background Hubble rate – then as a consequence, the 4D Bianchi identity on the brane is violated, i.e., $\nabla^\nu G_{\mu\nu} \neq 0$, and the results are inconsistent.

When the 5D effects are incorporated [21], the 4D Bianchi identity is satisfied. (The results of [21] confirm and generalize those of [19].) The consistent modified evolution equation for density perturbations on sub-Hubble scales is

$$\ddot{\Delta} + 2H \dot{\Delta} = 4\pi G \left\{ 1 - \frac{2Hr_c - 1}{3[2Hr_c^2 - 2Hr_c + 1]} \right\} \rho \Delta, \quad (38)$$

where the term in braces encodes the 5D correction. The linear growth factor, $g(a) = \Delta(a)/a$ (i.e., normalized to the flat CDM case, $\Delta \propto \alpha$), is shown in Fig. 5.

It must be emphasized that these results apply on subhorizon scales. On superhorizon scales, where the 5D effects are strongest, the problem has yet to be solved. This solution is necessary before one can compute the large-angle CMB anisotropies. Thus the cosmological tests of DGP remain incomplete until the superhorizon evolution of perturbations has been computed (and also until the correct matching of the linear to nonlinear regimes, involving the modified nonlinear Poisson equation [22], is developed).

However, in addition to the complexity of the cosmological perturbations, a deeper problem is posed by the fact that the late-time asymptotic de Sitter solution in DGP cosmological models has a ghost [23]. This ghost in the gravitational sector is more serious than the ghost in a phantom scalar field. But it is not yet known whether the DGP ghost, like the phantom ghost, is classically stable. If it is classically stable, then one can continue to use the DGP model as a viable dark gravity model, where we effectively assume that the ghost problem will be solved by a quantum gravity ultraviolet completion of the model.

V. CONCLUSION

The evidence for a late-time acceleration of the universe continues to mount, as the number of experiments and the quality of data grow – dark energy or dark gravity appear to be an unavoidable reality of the cosmos. This revolutionary discovery by observational cosmology, confronts theoretical cosmology with a major crisis – how to explain the origin of the acceleration. The core of this problem may be “handed over” to particle physics, since we require at the most fundamental level, an explanation for why the vacuum energy either has an incredibly small and fine-tuned value, or is exactly zero. Both options violently disagree with estimates of the vacuum energy.
If one accepts that the vacuum energy is indeed nonzero, then the dark energy is described by \( \Lambda \), and the LCDM model is the best current model. This also applies if \( \Lambda \) is treated as a geometrical constant, with the vacuum energy exactly zero. In both cases, the cosmological model requires completion via developments in particle physics that will explain the value of the vacuum energy.

In many ways, this is the best that we can do currently, since the alternatives to LCDM, within and beyond general relativity, do not resolve the vacuum energy crisis, and furthermore have no convincing theoretical motivation. None of the contenders appears any better than LCDM. Perhaps the simplest and most appealing contender is the DGP braneworld model. However, the simplicity of its Friedmann equation is deceptive, and the complexity of its cosmological perturbations, together with the problem of its ghost, seriously detract from the model.

Thus at the theoretical level, there is as yet no serious challenger to LCDM. It remains worthwhile to continue investigating alternative dark energy and dark gravity models, in order better to understand the space of possibilities, the variety of cosmological properties, and the observational strategies needed to distinguish them.

At the same time, it is in principle possible that cosmological observations, having discovered dark energy/ dark gravity, could rule out LCDM, by showing, to some acceptable level of statistical confidence, that \( w \neq -1 \).

Finally, the theoretical crisis does not have only negative implications: dark energy/ dark gravity in the cosmos provides exciting challenges for theory and observations.

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