Determining the orbital elements of the binary stars help us to obtain the necessary information such as the mass and the radius of stars which play the important roles during the evolutions of the stellar structures. Analyzing both the light and the radial velocity curves deducing from the photometric and the spectroscopic observations, respectively, yields to derive the orbital parameters. One of the usual methods to analyze the velocity curve is the method of Lehmann-Filhés (Smart 1990). Here we introduce a new method to derive these parameters by the nonlinear regression of the radial velocity-acceleration curves. We test our method for the three double-line spectroscopic binary systems RZ Cas, CC Cas and V1130 Tau which have the following properties.

The Algol system RZ Cas is the semi-detached eclipsing binary stars with a period \( P = 1.195 \) days. There is a mass transfer between the two components. Some evidences existed for periodicity in the period changes due to apsidal motion. RZ Cas is also a radio source and an X-ray source. See Riazi et al. (1994) and Maxted et al. (1994) and references therein. CC Cas is the massive O-type binary system with a period \( P = 3.366 \) days. This system is a detached binary with the two components within the main sequence band. It is an actively interacting system. The system shows a mass loss rate \( 3 \times 10^{-7} \) \( m_{\odot} \) yr\(^{-1} \) due to the stellar winds. Such mass loss from the system yields to a change in the orbital period by the rate of less than \( 3 \times 10^{-6} \) days per century (Hill et al. 1994). V1130 Tau is a very closed detached system including the two components with similar size and brightness. It consists of two almost identical F3 V stars on a tight orbit with a short period \( P = 0.799 \) days. (Rucinski et al. 2003).

In Sect. 2, we reduce the problem to solving an equation for the radial velocity-acceleration relation which is a nonlinear function in terms of the orbital parameters. In Sect. 3 the nonlinear regression technic for estimating the orbital elements is discussed. In Sect.4 the numerical results implemented for the three different binary systems are reported. Section 5 is devoted to concluding remarks.

1. Formulation of the problem

The radial velocity of star in a binary system is defined as follows

\[
V_r = V_{cm} + \dot{Z},
\]

where \( V_{cm} \) is the radial velocity of the center of mass of system with respect to the sun and

\[
\dot{Z} = K \left[ \cos(\theta + \omega) + e \cos \omega \right],
\]

is the radial velocity of star with reference to the center of mass of the binary (see Smart 1990). Note that the dot denotes the time derivative. In Eq. \( \text{2} \), \( \theta \), \( \omega \) and \( e \) are the angular polar coordinate, the longitude of the perihelion and the eccentricity, respectively. Also

\[
K = \frac{2\pi}{P} \frac{a \sin i}{\sqrt{1 - e^2}},
\]

where \( P \) is the period of motion and the inclination \( i \) is the angle between the line of sight and the normal of the orbital plane.
The radial acceleration of star can be obtained as
\[ \ddot{Z} = -K \sin(\theta + \omega) \dot{\theta}. \]  
(4)

Using Kepler’s second law and the relations obtaining for the orbital parameters in the inverse-square field as
\[ \dot{\theta} = \frac{h}{r^2}, \]
(5)
\[ h = \frac{2\pi}{P} a^2 \sqrt{1 - e^2}, \]
(6)
\[ r = \frac{a(1 - e^2)}{1 + e \cos \theta}. \]
(7)

the radial acceleration, Eq. (4), yields to
\[ \ddot{Z} = -\frac{2\pi K}{P(1 - e^2)^{3/2}} \sin(\theta + \omega)(1 + e \cos \theta)^2, \]
(8)
where \( r, a \) and \( h \) are the radial polar coordinate, the semi major axis of the orbit and the angular momentum per unit of mass, respectively.

Substituting for \( \theta \) from Eq. (2) in Eq. (8), the result reduces to
\[ P \ddot{Z} = \frac{-2\pi K}{(1 - e^2)^{3/2}} \sin \left( \cos^{-1}(\dot{Z}/K - e \cos \omega) \right) \times \left\{ 1 + e \cos \left( -\omega + \cos^{-1}(\dot{Z}/K - e \cos \omega) \right) \right\}^2. \]

To simplify the notation further, we let \( Y = P \ddot{Z} \) and \( X = \dot{Z} \). Hence Eq. (9) describes a nonlinear relation, \( Y = Y(X, K, e, \omega) \), in terms of the parameters \( K, e \) and \( \omega \).

One can show that the adopted spectroscopic elements are related to the orbital parameters. First according to the definition of the center of mass, the mass ratio in the system is obtained as
\[ \frac{m_p}{m_s} = \frac{a_s \sin i}{a_p \sin i}, \]
(10)
where subscripts \( p \) and \( s \) return to the primary and the secondary components of the system.

Then from Kepler’s third law and Eq. (10), the result reduces to
\[ m_p \sin^3 i = a_s \sin i \left( \frac{\dot{\theta}_p \sin i + a_s \sin i}{P} \right)^2, \]
(11)
where \( a, P \) and \( m \) are expressed in AU, years and solar mass, respectively. The similar relation is obtained for the secondary component only by replacing \( p \) to \( s \) and vise versa, in Eq. (11). Note that in Eqs. (10) and (11) the parameter \( a \sin i \) is related to the orbital parameters by the aid of Eq. (3).

2. Nonlinear regression of the radial velocity-acceleration curve

To obtain the orbital parameters \( K, e \) and \( \omega \) in Eq. (9), we use the nonlinear regression method. In this approach, the sum of squares of errors (SSE) for the number of \( N \) measured data is calculated as
\[ SSE = \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^{N} [Y_i - Y(X_i, K, e, \omega)]^2, \]
(12)
where \( Y_i \) and \( \hat{Y}_i \) are the real and the predicted values, respectively. To obtain the model parameters, the \( SSE \) should be minimized in terms of \( K \), \( e \) and \( \omega \) as

\[
\frac{\partial SSE}{\partial K} = \frac{\partial SSE}{\partial e} = \frac{\partial SSE}{\partial \omega} = 0.
\]  
(13)

To solve Eq. (13), we use the SAS (Statistical Analysis System) software. Note that the nonlinear models are more difficult to specify and estimate than linear models. For instance, in contrast to the linear regression, the nonlinear models are very sensitive to the initial guesses for the parameters. Because in practice, \( SSE \) may has to be minimized in several points in the three dimensional parametric space including \( K \), \( e \) and \( \omega \). However the final goal is finding the absolute minimum. Hence choosing the relevant initial parameters yields to the absolute minimum of \( SSE \) which is also stationary. That means if one change the initial guesses slightly, then the result reduces to the previous values for the parameters. But if the regression converged at the local minimum, the model would not be stationary. See Sen & Srivastava (1990) and Christensen (1996). To avoid the problem, two things can help us, first by taking into account the bound intervals for the parameters which causes the model to converge quickly. For instance one can put \((0 \leq e \leq 1)\) for the binary stars which have closed orbits. The last one is checking the differences between the observational data and the fitted diagrams using Eq. (9).

3. Numerical results

Here we test our new method to derive the orbital and the combined elements of the binary stars. To do this we consider the three different double line spectroscopic systems, RZ Cas, CC Cas and V1130 Tau. Using the measured experimental data for the radial velocities of two components of these systems obtained by Maxted et al. (1994) for RZ Cas, Hill et al. (1994) for CC Cas and Rucinski et al. (2003) for V1130 Tau, the suitable fitted velocity curves are plotted in terms of the photometric phase in Figs. 1, 4 and 7. The best fits are obtained by the polynomials of the intermediate degrees, e.g., sixth and seventh degrees. Because the low degrees do not suitably cover the data and high degrees take the fluctuations of the data which yields to the divergency in the nonlinear regression of Eq. (9). The velocity of the center of mass of the stars are obtained by calculating the areas up and down of the radial velocity curves. Everywhere these areas become equal to each other then the velocity of the center of mass is obtained.

The radial acceleration data corresponding with the radial velocity data are obtained by taking the derivative of the adopted radial velocity curves. Figures 2, 5, 6, 8, and 9 show the radial acceleration scaled by the period versus the radial velocity data for the primary and the secondary components of RZ cas, CC Cas and V1130 Tau, respectively. The solid closed curves are the result of the nonlinear regression of Eq. (9), which their good coincidence with the observational data yields to derive the optimized parameters \( K \), \( e \) and \( \omega \).

The orbital parameters obtaining from nonlinear least squares of Eq. (12) for RZ Cas, CC Cas and V1130 Tau are tabulated in Tables 1, 3, and 5, respectively. Table 1 shows that for RZ Cas: 1) \( V_{cm} \) and \( K_p \) for the primary component are in good agreement with the those
obtained by Maxted et al. (1994) and Duerbeck & Hänel (1979). 2) $e$ and $\omega$ are closed to the results obtained by Horak (1952). 3) $V_{cm}$ and $K_s$ for the secondary component are coincide with the results derived by Maxted et al. (1994). Whereas for $e$ and $\omega$, we had no any references for comparing.

Table 3 shows that for CC Cas: 1) $V_{cm}$ and $K$ for the primary component are in good analogy with the results obtained by Hill et al. (1994). For the secondary component, $K$ is in relatively well coincidence with the those derived by Pearce (1927) and Hill et al. (1994). But for $V_{cm}$ is not, however its interval is compatible with the result of Hill et al. (1994). The difference may be caused from the existence of high scattering for the data around the maximum and minimum of the radial velocity curve (see again Fig. 4).

Table 5 shows that for V1130 Tau, 1) for both components, $K$ is in very good concord with the result estimated by Rucinski et al. (2003). 2) For $V_{cm}$, its average value for the two components is in good agreement with the results of Rucinski et al. (2003).

The combined spectroscopic elements including $m_p \sin^3 i$, $m_s \sin^3 i$, $(a_p + a_s) \sin i$ and $m_p/m_s$ are calculated by substituting the estimated parameters $e$, $K$ and $\omega$ in Eqs. (3), (10) and (11). The results obtained for the same three previous systems are tabulated in Tables 2, 4, 6. Tables show that our results are in good agreement with the those obtained by Maxted et al. (1994), Hill et al. (1994) and Rucinski et al. (2003) for RZ Cas, CC Cas and V1130 Tau, respectively.

4. Concluding remarks

A new method to derive the orbital elements of the spectroscopic binary stars is introduced. These parameters are obtained from the nonlinear regression of the radial velocity-acceleration curve of the system. Using the measured experimental data for radial velocities of RZ Cas, CC Cas and V1130 Tau obtained by Maxted et al. (1994), Hill et al. (1994) and Rucinski et al. (2003), we find the orbital elements of these systems by the mentioned method. Our numerical results show that the results obtained for the orbital elements and the combined spectroscopic parameters are in good agreement with the those obtained by others via the method of Lehmann-Filhés. In a subsequent paper we intend to test numerically our method for the other different systems.

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Fig. 1. The radial velocities of the primary and secondary components of RZ Cas plotted against photometric phase. The observational data have been measured by Maxted et al. (1994). Legend is given in the right hand up caption.
Fig. 2. The radial acceleration scaled by the period versus the radial velocity of the primary component of RZ Cas. The solid curve is obtained from the nonlinear regression of Eq. (9). The plus points are the experimental data.
Fig. 3. Same as Fig. 2 for the secondary component of RZ Cas.
Fig. 4. Same as Fig. 1 for CC Cas. The observational data are belong to Hill et al. (1994).
Fig. 5. Same as Fig. 2 for the primary component of CC Cas.
Fig. 6. Same as Fig. 2 for the secondary component of CC Cas.
Fig. 7. Same as Fig. 1 for V1130 Tau. The observational data have been deduced from Rucinski et al. (2003).
Fig. 8. Same as Fig. 2 for the primary component of V1130 Tau.
Fig. 9. Same as Fig. 2 for the secondary component of V1130 Tau.
Table 1. Spectroscopic orbit of RZ Cas. Values bold type are assumed.

| Parameter | This Paper | Maxted et al. (1994) | Hork (1952) | Duerbeck & Hänel (1979) |
|-----------|------------|----------------------|-------------|--------------------------|
| Primary   |            |                      |             |                          |
| $V_{cm}$  | $-45.08 \pm 0.93$ | $-45.5 \pm 0.7$  | $40 \pm 2$ | $-45.5 \pm 1.1$          |
| $K_p$     | $70.4 \pm 0.34$    | $70.9 \pm 0.8$   | $68 \pm 2$ | $70.5 \pm 1.4$           |
| $e$       | $0.013 \pm 0.002$  | $0.000$          | $0.01 \pm 0.03$ | $0.024 \pm 0.023$        |
| $\omega(°)$ | $250 \pm 11.5$ | $-$               | $240 \pm 160$ | $87 \pm 2$               |
| Secondary |            |                      |             |                          |
| $V_{cm}$  | $-40.68 \pm 3.69$ | $-41 \pm 4$      | $-$         | $-$                      |
| $K_s$     | $213.35 \pm 0.45$ | $213 \pm 4$     | $-$         | $-$                      |
| $e$       | $0.008 \pm 0.001$  | $0.000$          | $-$         | $-$                      |
| $\omega(°)$ | $88.84 \pm 7$   | $-$               | $-$         | $-$                      |

Table 2. Combined spectroscopic orbit of RZ Cas.

| Parameter | This Paper | Maxted et al. (1994) |
|-----------|------------|----------------------|
| $m_p \sin^3 i/M_\odot$ | $2.126 \pm 0.002$ | $2.16 \pm 0.07$ |
| $m_s \sin^3 i/M_\odot$ | $0.701 \pm 0.005$ | $0.715 \pm 0.02$ |
| $(a_p + a_s) \sin i/10^6 km$ | $4.663 \pm 0.007$ | $4.68 \pm 0.07$ |
| $m_p/m_s$ | $3.03 \pm 0.02$ | $3.02 \pm 0.07$ |

Table 3. Same as Table 1 for CC Cas.

| Parameter | This Paper | Hill et al. (1994) | Pearce (1927) |
|-----------|------------|--------------------|--------------|
| Primary   |            |                    |              |
| $V_{cm}$  | $-7.23 \pm 2.68$ | $-7.3 \pm 4.2$  | $-9.9 \pm 2.4$ |
| $K_p$     | $117.87 \pm 0.51$ | $116 \pm 1.8$   | $138 \pm 4$  |
| $e$       | $0.0025 \pm 0.0005$ | $-$               | $-$          |
| $\omega(°)$ | $330 \pm 2$ | $-$               | $-$          |
| Secondary |            |                    |              |
| $V_{cm}$  | $-8.63 \pm 7.31$ | $-12.3 \pm 6$  | $3.2 \pm 7$  |
| $K_s$     | $285.63 \pm 1.14$ | $279.4 \pm 2.3$ | $288 \pm 9$  |
| $e$       | $0.023 \pm 0.005$  | $-$               | $-$          |
| $\omega(°)$ | $54 \pm 13$    | $-$               | $-$          |

Table 4. Same as Table 2 for CC Cas.

| Parameter | This Paper | Hill et al. (1994) | Pearce (1927) |
|-----------|------------|--------------------|--------------|
| $m_p \sin^3 i/M_\odot$ | $16.2 \pm 0.2$ | $15.3 \pm 0.3$  | $18.3 \pm 1.4$ |
| $m_s \sin^3 i/M_\odot$ | $6.69 \pm 0.08$ | $6.3 \pm 0.2$   | $8.8 \pm 0.6$  |
| $(a_p + a_s) \sin i/R_\odot$ | $26.83 \pm 0.08$ | $26.2 \pm 0.2$ | $28.3 \pm 0.6$ |
| $m_p/m_s$ | $2.42 \pm 0.02$ | $2.41 \pm 0.04$ | $2.09 \pm 0.09$ |
Table 5. Same as Table 1 for V1130 Tau. Number in parenthesis are errors in the final digits.

| Parameter | This Paper | Rucinski et al. (2003) |
|-----------|------------|------------------------|
| \( V_{cm} \ (km/s) \) | \(-14.45 \pm 0.89\) | \(-12.74(0.46)\) |
| \( K_p \ (km/s) \) | \(146.61 \pm 1.01\) | \(147.21(0.63)\) |
| \( e \) | \(0.005 \pm 0.003\) | — |
| \( \omega(\degree) \) | \(126 \pm 42\) | — |

**Secondary**

| Parameter | This Paper | Rucinski et al. (2003) |
|-----------|------------|------------------------|
| \( V_{cm} \ (km/s) \) | \(-10.9 \pm 0.9\) | \(-12.74(0.46)\) |
| \( K_s \ (km/s) \) | \(159.36 \pm 0.61\) | \(160.11(0.74)\) |
| \( e \) | \(0.004 \pm 0.004\) | — |
| \( \omega(\degree) \) | \(292 \pm 7\) | — |

Table 6. Same as Table 2 for V1130 Tau.

| Parameter | This paper | Rucinski et al. (2003) |
|-----------|------------|------------------------|
| \( m_p \sin^3 i/M_\odot \) | \(1.23 \pm 0.02\) | — |
| \( m_s \sin^3 i/M_\odot \) | \(1.13 \pm 0.2\) | — |
| \((a_p + a_s) \sin i/R_\odot \) | \(4.82 \pm 0.02\) | — |
| \( m_p/m_s \) | \(0.919 \pm 0.009\) | \(0.919(7)\) |
| \((m_p + m_s) \sin^3 i/M_\odot \) | \(2.37 \pm 0.02\) | \(2.408(32)\) |