Kramers-Kronig relation based direct detection for unrepeated long-haul lightwave transmission

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Abstract: We numerically study transmission limitations of an unrepeated optical link using Kramers-Kronig (KK) relation-based direct detection. The transmission distance is limited by the thermal noise of the optical front-end and the signal distortions caused by fiber nonlinearity in optical amplifier-free transmission and by amplified spontaneous emission (ASE) noise and fiber nonlinearities when pre-amplification is enabled at the receiver. The transmission limits of QPSK, 16-QAM, and 64-QAM modulation formats in optical amplifier-free 25-GBaud transmission can be extended from 109, 80, and 52 km to 186, 134, and 90 km by enabling pre-amplification at the receiver, respectively.

Keywords: optical fiber communication, fiber transmission, self-coherent heterodyne, modulation, direct-detection

Classification: Optical fiber for communications

References

[1] P. R. Trischitta, and D. T. S. Chen, "Repeaterless undersea lightwave systems," \textit{IEEE Communications Magazine}, vol. 27, no. 3, pp. 16-21, March 1989, DOI:10.1109/35.20263

[2] M. Li, and T. Hayashi, “Advances in low-loss, large-area, and multicore fibers” \textit{Optical Fiber Telecommunications VII}, ed. A. E. Willner, pp. 3-50, Academic Press, 2020.

[3] J. Xu, S. Sun, Q. Hu, J. Yu, J. Liu, Q. Luo, W. Wang, L. Huang, M. Xiang, J. Wu, F. Zheng, W. Li, L. Deng, H. Zhou, L. Zhang, S. Jia, X. Zhang, and H. Chen, "Unrepeatered Transmission Over 670.64 km of 50G BPSK, 653.35 km of 100G PS-QPSK, 601.93 km of 200G 8QAM, and 502.13 km of 400G 64QAM," \textit{Journal of Lightwave Technology}, vol. 38, no. 2, pp. 522-530, January 2020, DOI:10.1109/JLT.2019.2939842.

[4] E. Tsukui, K. Toba, T. Fujita, K. I. A. Sampath, and J. Maeda, “Availability of Kramers-Kronig relation based direct detection for unrepeated long haul light wave transmission,” Proc. International Conference on Emerging Technologies for Communications (ICETC) 2020, no. J1-3, December 2020.
1 Introduction

Transmission over the longest possible distance without repeaters was an important issue for optical communication since the early commercial deployments [1]. Recent advancements of low-loss fibers with large-effective areas have enabled unrepeated transmission over 600km [2]. Phase modulation formats such as binary phase-shift keying combined with the coherent receiver have enabled the maximum reach of 670.64 km in a repeater-less link [3].

In terms of field recovery, both the coherent receiver and the recently introduced KK relation-based receiver perform equally despite the single photodiode structure of the KK receiver. Extending our previous report of [4], we numerically investigate the transmission limits of the KK receiver when applied to unrepeated transmission links. In this paper, we compare the performance of the KK receiver in long-haul unrepeated links in the presence and absence of the pre-amplification at the receiving end.

2 Field Recovery using Kramers-Kroning Relation

In self-coherent heterodyne transmission, the phase information of the received signal can be recovered using the KK relation. To use the KK relation, the received optical signal should be a minimum phase signal and its spectrum should be in the form of a single-sideband. For such a signal, the received photocurrent $I$ can be expressed as in Eq. (1).

$$ I = |A|^2 + A \cdot s(t)e^{j2\pi f_s t} + A^* \cdot s(t^*)e^{-j2\pi f_s t} + |s(t)|^2 \quad (1) $$

Here, $A$ is the amplitude of the carrier light, $s(t)$ is the information signal, and $f_s$ is the frequency difference between the center frequency of the information signal and the frequency of the carrier light, hereinafter called the shift frequency. The second and third terms on the right-hand side of Eq. (1) contain the amplitude and phase information of the signal. On the other hand, the fourth term is the component called signal-to-beat interference (SSBI). If the SSBI component is in-band, it leads to distortions of the received signal. However, despite SSBI, the phase information of $s(t)$ can be recovered from $I$ according to the KK relation using the receiver side digital signal processing (DSP) [5].
The KK relation connects the real and imaginary parts of the frequency response of linear systems using the Hilbert transform. In KK relation-based direct-detection, this mathematical relation is used to recover the field of the transmitted signal using detected intensity. A block diagram of the field recovery process is shown in the bottom inset of Fig. 1. The intensity of the signal obtained by direct detection is expressed as in Eq. (2). 

\[ |E(t)|^2 = |A + s(t)e^{j2\pi f_s t}|^2 \]  

(2)

According to the KK relation, phase \( \varphi(t) \) of the received signal is given by Eq. (3). 

\[ \varphi(t) = 0.5 \cdot H \{ \ln |E(t)|^2 \} \]  

(3)

Here, \( H \) represents the Hilbert transform. The recovered field can be expressed as in Eq. (4) using the intensity and phase calculated in Eq. 2 and 3, respectively. 

\[ s(t) = \left[ |E(t)|e^{j\varphi} - A \right] \cdot e^{-j2\pi f_s t} \]  

(4)

**Fig. 1.** Schematic of assumed transmission system. Top inset: spectra at corresponding points, bottom inset: field-recovery block diagram using the KK relation.

### 3 Simulation Outline

Figure 1 shows the assumed transmission system in our study. A 25-GBaud signal is generated at the transmitter, which is spectrally shaped by a root-raised cosine (RRC) filter with a roll-off ratio of 0.05, and then D/A converted. One of the two optical tones generated at the frequency comb is I/Q modulated. The modulated light is combined with the unmodulated tone whose frequency difference (\( f_s \)) is 13.3 GHz. Here, the \( f_s \) is chosen to satisfy the minimum phase condition. The carrier to signal power ratio (CSPR) of the fiber input is set to 10 dB.

The transmission link is assumed to be a standard single-mode fiber (SSMF). The evolution of the light wave during transmission is calculated by solving the nonlinear Schrodinger equation (for single-mode and single-polarization) using the
split-step Fourier method. The following values are used as the fiber parameters: dispersion +17 ps/km/nm, loss 0.2 dB/km, nonlinear parameter 1.27 W⁻¹/km.

Assuming unrepeated transmission, we investigate the transmission performance of our system with and without pre-amplification at the receiving end. In the case of pre-amplification at the receiver, we assumed the use of an optical pre-amplifier with a maximum gain of 30 dB.

At the receiver, the transmitted signal is direct-detected using a photodetector with a sensitivity of 0.5 A/W. We neglect the effect of sampling and quantization at the analog-to-digital converter (ADC). We consider thermal noise of the optical front-end and ASE as the source of noise in our system. The voltage spectral density of the thermal noise ($S_T$) generated in the receiver front-end is modeled as in Eq. 5.

$$S_T = 4kTR$$  \hspace{1cm} (5)

Here, $k$ is the Boltzmann’s constant, and $T$ is the temperature of the load resistor $R$. Noise having the power spectral density of Eq. (6) is added to the real and imaginary parts of the optical field as ASE.

$$S_{ASE} = 0.5n_{sp}h

In Eq. (6), $G$ denotes amplifier gain, $n_{sp}$ is the spontaneous emission coefficient, $h$ is Planck's constant, and $\nu_0$ is the optical frequency of the signal to be amplified.

### 4 Repeater-less Transmission using KK Receiver

First, we investigate the degradation of the received signal quality in an optical amplifier-free transmission link. The left column of Fig. 2 depicts the received constellations of the 16-quadrature amplitude modulation (16-QAM) signal after 20 and 80-km transmissions with fiber input powers of +4 and +8 dBm. The clearest constellation is observed for the transmission distance of 20 km with a transmission power of +4 dBm. This is because of the weak effect of fiber nonlinearity at lower transmission powers. Whereas, for 80-km transmission, better signal quality is found for transmission power of +8 dBm. This is because of the increased signal-to-noise ratio (SNR) due to increased optical power.

Next, we study the transmission performance of the studied system with an optical amplifier at the receiver acting as a pre-amplifier. The received constellation of the 16-QAM signal is shown in the right column of Fig. 2. For the transmission distance of 20 km, the constellation of +4 dBm transmission is clearer than the one of +8 dBm as is the case without the use of pre-amplification. However, in contrast to the optical amplifier-free transmission, the constellation of +4 dBm transmission is clearer than that of the +8 dBm in 80-km transmission. This observation can be explained as follows; since +4 dBm transmission is less affected by the fiber nonlinearity, pre-amplification compensates the fiber loss consequently resulting in a clear constellation. However, pre-amplification cannot improve the distorted constellation due to fiber nonlinearities at the transmission power of +8 dBm.

Variations of the error vector magnitude (EVM) of 16-QAM signal in optical amplifier-free transmission are illustrated in Fig. 3 (a) along with the transmission distance. When the fiber input power is below +6 dBm, the EVM increases.
monotonically with the transmission distance. This is caused by the decrease in SNR due to fiber loss. On the other hand, when the fiber input power is increased to +12 dBm, the EVM of 0 ~ 40 km increases rapidly. This is due to the effect of the fiber nonlinearity during the transmission. The increase in EVM due to the fiber nonlinearity is weakened as the transmission distance increases. This can be explained as the decrease in optical signal power caused by the fiber loss. However, the EVM starts to increase monotonically again with the distance due to the decrease of SNR. It is necessary to select an appropriate transmission power to extend the transmittable distance.

Figure 3 (b) shows the EVM of transmitted 16-QAM signal with a pre-amplification at the receiver. Compared with Fig. 3(a), the EVM of the 16-QAM signal decreases for longer transmission distances. This can be explained as the improvement of SNR by loss compensation.

We carry out similar simulations and investigate the transmission limits of quadrature phase-shift keying (QPSK) and 64-QAM. In Fig. 3 (c), we compare the maximum transmission distances of each modulation format. Assuming the use of second-generation forward error correction (FEC), we define the maximum transmission distance as the distance where the bit-error-rate (BER) of $10^{-3}$ is achieved. Referring to [6], FEC limit is calculated using the corresponding EVMs.
as follows; QPSK 32.6 %, 16QAM 14.8 %, 64QAM 7.6 %.

In optical-amplifier-free QPSK transmission, the SNR improves with the fiber input power subsequently increasing the maximum transmittable distance.

Meanwhile, with the increasing transmission power, signal distortions due to fiber nonlinearity increase which limits the reach for transmission power above +11 dBm. The maximum transmission distance of QPSK is estimated to be 109 km at a transmission power of +11 dBm. For higher modulation formats, the maximum transmission distance becomes shorter due to lower noise tolerance. Maximum transmission distances are estimated as 80 km and 52 km for 16-QAM and 64-QAM signals, respectively.

Compared with the case without preamplification, the maximum transmission distance is extended as a consequence of loss compensation. The maximum transmission distances are estimated as 186 km for QPSK, 134 km for 16-QAM, and 90 km for 64-QAM signals. These results are in good agreement with the reported experimental results of [7].

5 Conclusion
In this paper, we numerically investigate the transmission limits of an optical communication system using the KK relation-based direct detection for unrepeated long-haul transmission. In the case of no pre-amplification, the system is limited by the fiber nonlinearity-induced signal impairments and the thermal noise of the optical front-end. When pre-amplification is enabled at the receiver, ASE noise and fiber nonlinearities become the dominant factors of transmission limitation.

The transmission limit of a 25-GBaud signal is predicted to be 109, 80, and 52 km for QPSK, 16-QAM, and 64-QAM in optical amplifier-free transmission, respectively. When pre-amplification enabled at the receiver, transmission limits could be extended up to 186, 134, and 90 km for the three modulation formats, respectively.