Twist–three relations of gluonic correlators for the transversely polarized nucleon

Yoshitaka Hatta\textsuperscript{1a}, Kazuhiro Tanaka\textsuperscript{2b}, and Shinsuke Yoshida\textsuperscript{1c}

\textsuperscript{1}Faculty of Pure and Applied Sciences, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan
\textsuperscript{2}Department of Physics, Juntendo University, Inzai, Chiba 270-1695, Japan

Abstract

We derive exact relations among the polarized gluon and three–gluon distributions in the transversely polarized nucleon which are relevant to single and double spin asymmetries in various hard processes. We also discuss the partonic decomposition of the transverse nucleon spin and point out a potential problem with frame–independence.

\textsuperscript{a} E-mail: hatta@het.ph.tsukuba.ac.jp
\textsuperscript{b} E-mail: tanakak@sakura.juntendo.ac.jp
\textsuperscript{c} E-mail: yoshida@het.ph.tsukuba.ac.jp
1. INTRODUCTION

Twist–three effects in QCD spin physics have been a subject of much interest over the past decade or so, primarily due to their relevance to the single–spin asymmetry (SSA). In the collinear factorization framework, the SSA is a unique observable in QCD in which the first nonvanishing contribution comes solely from the ‘genuinely twist–three’, multi-parton correlations in the transversely polarized nucleon. Notwithstanding the presumed suppression by a hard scale, sizable SSAs have been observed in light hadron (h) productions in the semi-inclusive deep inelastic scattering (DIS) $ep^\uparrow \rightarrow ehX$ and in the $pp$–collision $pp^\uparrow \rightarrow hX$ [1]. A related observable, the transverse–longitudinal double–spin asymmetry $A_{LT}$ in Drell–Yan experiments, gives access to the polarized quark distribution $g_T(x)$ [2, 3] which is also twist–three. Its first moment is the quark helicity contribution to the transverse spin $\int dx g_T(x) = \Delta g$ which actually coincides with the usual quark polarization in the longitudinal spin. A closer analysis based on the operator product expansion (OPE) [4–8] reveals that $g_T(x)$ is actually a quantity beyond a (polarized) parton density and embodies the genuinely twist–three, quark–gluon correlations inside the nucleon.

In this paper, we discuss the polarized gluon distribution in the transversely polarized nucleon $G_{3T}(x)$ [9, 10] (defined in Eq. (1) below) which is the gluonic counterpart of $g_T(x)$, having the same spin, twist, charge–conjugation–even, and chiral–even properties. As in the case of $g_T$, $G_{3T}$ consists of multi-parton components, and we will investigate its precise twist structure using the nonlocal version of the OPE [7, 8]. The result of this is a set of exact identities which relate $G_{3T}$ to a certain integral of three–gluon light–cone correlation functions. Not only do these three–gluon correlators emerge in the QCD evolution of $g_T(x)$ and the quark–gluon correlators [4, 11–14], but already at leading order they contribute to SSAs in pion (light hadron) production, jet production, Drell–Yan and direct photon production [9, 15] at high $P_T$, and also to $A_{LT}$ with various high-$P_T$ final states [16]. As a matter of fact, there is a priori no fundamental argument that the three–gluon contribution is negligible compared to the quark–gluon contribution in these processes. Moreover, in high-$P_T$ open charm productions $pp^\uparrow \rightarrow DX$ [21, 23] the three–gluon contribution is expected to even dominate over the quark–gluon contribution, and in $ep^\uparrow \rightarrow eDX$ only the three–gluon distribution contributes to the SSA [17, 20].

\footnote{The same function was denoted by $G_3(x)$ in [9].}
Another issue we would like to clarify is the relation between the two types of three–gluon correlators. The above–mentioned ‘genuinely twist–three’ correlator is defined in terms of the matrix element \( \sim \langle PS_\perp | F^{+\perp} F^{+\perp} F^{+\perp} | PS_\perp \rangle \) with the gluon field–strength tensor \( F^{\mu\nu} \) on the light-cone \[20\]. In addition to this ‘F–type’ correlator, one may define the ‘D–type’ correlator \( \sim \langle PS_\perp | F^{+\perp} D^{\perp} F^{+\perp} | PS_\perp \rangle \), with \( D^{\perp} \) being the transverse component of the covariant derivative, which contains the twist–two as well as twist–three contributions. These two types of correlators are actually not independent, and problems may arise when trying to relate observables computed in different gauges and expressed in different types of correlators. For instance, both types of correlators appear in the cross section formula for SSA in Ref. \[9\] which works in the light–cone gauge, whereas only the F–type correlator appear in Refs. \[15, 20, 23\] working in the Feynman gauge. Our result will help establish the equivalence of such different–looking expressions. The desired relation can be derived in an analogous way as in the corresponding relation between the F–type and D–type quark–gluon correlators \[8\] relevant to \( g_T(x) \).

Finally, as a byproduct of our analysis of \( G_{3T} \), we shall discuss the decomposition of the transversely polarized nucleon spin. It is indeed relevant and timely to do so in view of the recent surge of interest in this problem (mostly in the longitudinally polarized case, but also in the transversely polarized case; see Ref. \[24\] and references therein). Just as the integral of \( g_T(x) \) gives the quark helicity \( \Delta q \), we show that the first moment of the twist–three distribution \( G_{3T}(x) \) is equal to the usual gluon helicity contribution \( \Delta G \) to the nucleon spin. We will further discuss the implication of this result through an analysis based on the Pauli–Lubansky vector \[25, 26\] and the newly proposed decomposition scheme of the nucleon spin \[27–29\].

2. TWIST–TWO AND TWIST–THREE GLUON CORRELATORS

2.1. Two–gluon correlator

Let us start with the two–gluon correlator. Up to twist–three, and using the light–cone coordinates \( x^\pm = x_+ = \frac{1}{\sqrt{2}}(x^0 \pm x^3) \), we define the matrix element of the two–gluon operator
in a polarized nucleon state as

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS| F^{+\alpha}(0) W_{\alpha\beta}(\lambda n)|PS\rangle = -\frac{1}{2} \frac{i}{2} x G(x) (P^+)^2 (g^{\alpha\beta} - P_\alpha n_\beta - P_\beta n_\alpha)$$

$$- \frac{i}{2} x \Delta G(x) P^+ \epsilon^{+\alpha\beta} S^+ - i x G_{3T}(x) P^+ \epsilon^{+\alpha\beta\gamma} S_{\perp\gamma} + \cdots ,$$

(1)

where $n^\mu = \delta^\mu_n / P^+$ is a lightlike vector. We use Greek letters for four–vector indices $\mu, \nu = \pm, 1, 2$, and Latin letters $i, j = 1, 2$ for the transverse coordinates. We shall also use the two–dimensional antisymmetric tensor $\epsilon^{ij} = \epsilon^{-+ij}, \epsilon^{12} = +1$. $P^\mu$ and $S^\mu$ are the momentum and spin vectors, respectively, normalized as $P^2 = - S^2 = M^2$. $W_{0\lambda}$ is the Wilson line along the light–cone which makes nonlocal operators gauge invariant. The unpolarized/polarized gluon distributions $G(x)/\Delta G(x)$ are standard [9, 10], whereas our focus in this paper is $G_{3T}(x)$ which is relevant to the transverse polarization $S_{\perp} \equiv \delta^\mu_n S^\mu$.

From (1), one can derive the following compact expression

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS| F^{+\alpha}(0) \bar{W}^{\mu\alpha}(\lambda n)|PS\rangle = i x \Delta G(x) S^+ P^\mu + 2 i x G_{3T}(x) P^+ S^\mu$$

$$= i x \{ (\Delta G(x) - 2 G_{3T}(x)) S^+ P^\mu + 2 G_{3T}(x) P^+ S^\mu \} ,$$

(2)

which is valid to twist–three accuracy, that is, ignoring the twist–four component corresponding to $\mu = -$. [Hereafter the subscripts on the Wilson line indicating the initial and final points will be omitted.] (2) is the gluonic counterpart to the polarized quark distributions [2]

$$P^+ \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle \bar{\psi}(0) \gamma_5 \gamma^\mu W \psi(\lambda n)|PS\rangle = 2 (g_1(x) S^+ P^\mu + g_T(x) P^+ S^\mu) ,$$

(3)

where $\gamma_5 \equiv - i \gamma^0 \gamma^1 \gamma^2 \gamma^3$. It is well known [10] that $g_T(x)$ can be written as the sum of terms related to twist–two distributions (‘Wandzura–Wilczek part’) and genuine twist–three contributions. One of our goals in this paper is to perform a similar decomposition for $G_{3T}(x)$.

Let us quickly obtain the Wandzura–Wilczek part of $G_{3T}(x)$, deferring a more detailed analysis to the next section. Take the $(n - 1)$-th moment of (2):

$$\langle PS| F^{+\alpha}(iD^+)^{n-1} \bar{W}^{\mu\alpha}|PS\rangle$$

$$= i (P^+)^{n-1} \int_{-1}^1 dx x^n \{ (\Delta G(x) - 2 G_{3T}(x)) S^+ P^\mu + 2 G_{3T}(x) P^+ S^\mu \} .$$

(4)

The composite operator on the left–hand–side can be decomposed into those with a definite
and comparing with (4), we find

\[ F^{+\alpha}(iD^+)^{n-1}\tilde{F}_\alpha^{\mu} \]

\[ = \frac{1}{n+1} \left( F^{+\alpha}(iD^+)^{n-1}\tilde{F}_\alpha^{\mu} + \sum_{k=1}^{n-1} F^{+\alpha}(iD^+)^{k-1}iD^\mu(iD^+)^{n-k-1}\tilde{F}_\alpha^{+} + F^{\mu\alpha}(iD^+)^{n-1}\tilde{F}_\alpha^{+} \right) \]

\[ + \frac{1}{n+1} \left( nF^{+\alpha}(iD^+)^{n-1}\tilde{F}_\alpha^{\mu} - \sum_{k=1}^{n-1} F^{+\alpha}(iD^+)^{k-1}iD^\mu(iD^+)^{n-k-1}\tilde{F}_\alpha^{+} - F^{\mu\alpha}(iD^+)^{n-1}\tilde{F}_\alpha^{+} \right). \]

Parameterizing the matrix elements of the first line (twist–two operator) and the second line (twist–three operator) in terms of the corresponding reduced matrix elements \(a_n\) and \(d_n\), respectively, as

\[ \langle PS|F^{+\alpha}(iD^+)^{n-1}\tilde{F}_\alpha^{\mu}|PS\rangle = \frac{2ia_n}{n+1}(S^n(P^+)^n + nS^+P^n(P^+)^{n-1}) + \frac{2i\eta d_n}{n+1}(S^n P^+ - S^+ P^n)(P^+)^{n-1}, \]

and comparing with (4), we find

\[ \frac{1}{2} \int_{-1}^{1} dx x^n \Delta G(x) = a_n, \quad \int_{-1}^{1} dx x^n G_{3T}(x) = \frac{a_n + nd_n}{n+1}. \]

This leads to

\[ \int dx G_{3T}(x) = \frac{1}{2} \int dx \Delta G(x) = a_0 = \Delta G, \]

where \(\Delta G\) is the gluon polarization. Moreover, \(G_{3T}(x)\) can be written as \((x \geq 0)\)

\[ G_{3T}(x) = \frac{1}{2} \int_{x}^{1} \frac{dz}{z} \Delta G(z) + \delta G(x). \]

The first term is the Wandzura–Wilczek part and \(\delta G(x)\) is the genuinely twist–three contribution which integrates to zero: \(\int dx \delta G(x) = 0.\)

### 2.2. F–type three–gluon correlator

Next we define the ‘F–type’ twist–three gluon correlator [9, 20]

\[ \frac{1}{(P^+)^2} \int \frac{d\lambda d\mu}{2\pi 2\pi} e^{\lambda x_1 + i\mu(x_2-x_1)} \langle PS|F^{+ij}(0)WF^{+j}(\mu n)WF^{+k}(\lambda n)|PS\rangle \]

\[ = \left( F(x_1,x_2)g^{jk}\epsilon^{+j+\lambda\mu} - F(x_2,x_2-x_1)g^{ij}\epsilon^{+k+\lambda\mu} - F(x_1,x_1-x_2)g^{jk}\epsilon^{+i+\lambda\mu} \right) S_{\mu}^{+} P_{\sigma}. \]

Here the color indices are contracted by the totally antisymmetric structure constants: \(F^{+ij}F^{+jk} = F_a^{+ij}F_b^{+jk}(T_b)^{ac}F_c^{+k} = i\epsilon a_b F_a^{+ij}F_b^{+jk}F_c^{+k}.\) The function \(F(x_1,x_2)\) at special
values of its arguments $F(x,x)$ and $F(x,0)$ is related to the gluonic contribution to the single–spin asymmetry (SSA) \cite{20}. [Note that $F(x_1, x_2)/2$ equals the corresponding correlation function $N(x_1, x_2)$ in the notation of \cite{20}.] The three–gluon correlator whose color indices are contracted by the totally symmetric $d$–symbol also contributes to SSA, but it will not be discussed here because it is associated with a $C$–odd operator and bears no relation to $G_{3T}$ and the D–type distribution defined below which are $C$–even. By PT invariance, $F$ satisfies

$$F(x_1, x_2) = F(x_2, x_1).$$

(10)

On the other hand, from symmetry under permutation of the gluon field strength tensor $F^\mu\nu$, we find

$$F(x_1, x_2) = -F(-x_2, -x_1).$$

(11)

We shall also need the F–type quark–gluon operator

$$\int \frac{d\lambda d\mu}{2\pi 2\pi} e^{i\lambda x_1 + i\mu(x_2-x_1)} \langle PS | \tilde{\psi}(0) \gamma^+ W g F^+\alpha (\mu n) W \psi(\lambda n) | PS \rangle = P^+ \epsilon^{+\alpha\rho\sigma} S_\rho P_\sigma G_F(x_1, x_2),$$

(12)

with the symmetry property $G_F(x_1, x_2) = G_F(x_2, x_1)$. $G_F$ at special values of its arguments, e.g., $G_F(x,x)$ (‘soft-gluon pole’ \cite{8, 30–32}) and $G_F(x,0)$ (‘soft-fermion pole’ \cite{33–35}), contributes to the SSA.

### 2.3. D–type three–gluon correlator

Finally we define the D–type three–gluon correlator

$$\frac{1}{P^+} \int \frac{d\lambda d\mu}{2\pi 2\pi} e^{i\lambda x_1 + i\mu(x_2-x_1)} \langle PS | F^+\alpha (\mu n) W D^j (\mu n) W F^+\kappa (\mu n) | PS \rangle$$

$$= (D_1(x_1, x_2) g^{ik} \epsilon^{+j+\rho\sigma} + D_2(x_1, x_2) g^{ij} \epsilon^{+\kappa\rho\sigma} - D_2(x_2, x_1) g^{ij} \epsilon^{+\kappa+\rho\sigma}) S_\rho P_\sigma .$$

(13)

Unlike the F–type correlator \cite{9}, we need two independent functions $D_1$ and $D_2$. Their symmetry properties are:

$$D_1(x_1, x_2) = -D_1(x_2, x_1).$$

(14)

\footnote{Ref. \cite{9} introduced four functions which later turned out to be redundant, see the discussion in \cite{20, 22}.}
\begin{equation}
D_1(x_1, x_2) = D_1(-x_1, -x_2), \quad D_2(x_1, x_2) = D_2(-x_1, -x_2). \tag{15}
\end{equation}

In Ref. [9], both the F–type and D–type correlators appear in the cross section formula for SSA computed in the light–cone gauge. On the other hand, Refs. [15, 20, 23] worked in the Feynman gauge and obtained a formula which contains only the F–type correlator, as a result of a rather sophisticated reorganization of the collinear expansion of the Feynman amplitudes using Ward identities. Thus, even in a covariant gauge, one may achieve an expression of the cross section in terms of the D–type correlators depending on the intermediate steps of the calculations.\footnote{In order to relate these seemingly different results, one has to establish relations between the D–type and F–type distributions, to which we now turn. Note that, because the SSA is a ‘genuinely twist–three’ effect as demonstrated in [15, 20, 23, 36], the twist–two parts of the D–type function (see below) must cancel in the cross section formula.}

3. RELATIONS BETWEEN GLUONIC CORRELATORS

The various quark and gluon correlators introduced in the previous section are not independent as they are related by the equations of motion. In this section we show that \( G_{3T}(x) \) and \( D_{1,2}(x_1, x_2) \) can be entirely written by the other distributions \( \Delta G(x), F(x_1, x_2) \) and \( G_F(x_1, x_2) \).

Firstly, following [8] one can derive a general relation between the F–type and D–type correlators which in the present case reads

\begin{equation}
D_1(x_1, x_2) = \mathcal{P} \frac{F(x_1, x_2)}{x_1 - x_2}, \tag{16}
\end{equation}

\begin{equation}
D_2(x_1, x_2) = -\mathcal{P} \frac{F(x_2, x_2 - x_1)}{x_1 - x_2} + \delta(x_1 - x_2)\tilde{g}(x_1), \tag{17}
\end{equation}

where \( \mathcal{P} \) denotes the principal value and \( \tilde{g} \) is defined via the following relation

\begin{equation}
\int \frac{d\lambda}{2\pi} e^{i\lambda x}(PS|F^+(0)W_0\left(D^j(\lambda n) + \frac{ig}{P_+} \int d\mu \frac{\epsilon(\mu - \lambda)}{2} W_{\lambda \mu} F^+(\mu n)W_{\mu j}\right)F^+(\lambda n)|PS⟩ = (P^+)^2 \epsilon^{ik} S^{ij}_1 \tilde{g}(x). \tag{18}
\end{equation}

\footnote{The NLO correction to the deep-inelastic-scattering structure function \( g_2 \) was obtained in terms of the D–type correlators in [22], while the same contribution was calculated in terms of the F–type correlators in [13].}
We shall later see that $\tilde{g}$ is not an independent function, but can be entirely expressed in terms of $\Delta G$, $G_F$, and $F$.

Next we derive a formula which relates $G_{3T}(x)$ to the genuine twist–three correlators, namely, we determine the function $\delta G(x)$ in (8). For this purpose, we employ the ‘nonlocal OPE’ approach of Refs. [7, 8] and consider the following matrix element

$$I \equiv z^\alpha \left( \frac{\partial}{\partial z^\alpha} z_\nu \langle PS| F^{\nu\gamma}(0) W \tilde{F}^\mu (z)|PS \rangle - (\alpha \leftrightarrow \mu) \right).$$

(19)

In (19), $z^\mu$ is generic, not necessarily proportional to $n^\mu$, and this ensures that the constraints from Lorentz invariance are fully taken into account. The covariant expansion of the nonlocal operators in powers of $z^2$ is equivalent to the OPE according to the deviation from the light-cone (i.e., the twist expansion), and one can identify exact relations among operators belonging to the same twist.

Let us calculate (19) in two ways. On one hand, we can directly evaluate $I$ in terms of the gluon distributions by generalizing the definition (2) away from the light–cone

$$z_\nu \langle PS| F^{\nu\alpha}(0) W \tilde{F}_\alpha (z)|PS \rangle = i \int dx e^{-ixPz} \left\{ \left( \Delta G(x) - 2 G_{3T}(x) \right) S \cdot z P^\mu + 2 G_{3T}(x) P \cdot z S^\mu \right\}. \quad (20)$$

Plugging this into (19), we find

$$I = i z^{-} \int dx e^{-ixPz} \left( 2x^2 \frac{\partial}{\partial x} G_{3T}(x) + x \Delta G(x) \right) \left( P^\mu S^+ - S^\mu P^+ \right), \quad (21)$$

where we set $z^\alpha = \delta^\alpha z^-$ after differentiation. On the other hand, we can apply the $z$–derivative on fields and Wilson lines, and then use the equations of motion,

$$I = z^- \langle PS| \left( F^{+\gamma} W \tilde{F}_\gamma^\mu - F^{\mu\gamma} W \tilde{F}^+_\gamma \right)$$

$$+ z^- F^{+\gamma} \left( WD^+ \tilde{F}_\gamma^\mu - WD^\mu \tilde{F}^+_\gamma + iz^- \int_0^1 dt W g F^{+\mu}(tz) W \tilde{F}^+_\gamma (z) \right)|PS \rangle, \quad (22)$$

where again we set $z^\alpha = \delta^\alpha z^-$ after differentiation. With the help of an identity

$$\epsilon^{\alpha\beta\gamma\sigma} D^\lambda F_{\lambda\sigma} = D^\alpha \tilde{F}^{\beta\gamma} + D^\beta \tilde{F}^{\gamma\alpha} + D^\gamma \tilde{F}^{\alpha\beta}, \quad (23)$$

4 We note that the operator in the parentheses on the left–hand–side can be written as $D^j(\lambda n) - ig A_{phys}^j(\lambda n) = D^j_{pure}(\lambda n)$ using (46) below with $K(\lambda) = \frac{1}{2^j}(\lambda)$. $D^\mu_{pure}$ is the covariant derivative associated with the ‘pure gauge’ part of the gluon field [29].

8
and the equations of motion, $D_\nu F^{\nu\mu} = \bar{\psi} \gamma^{\mu} \psi$, where the color matrix $T^a$ is suppressed, the second line of (22) takes the form

$$(z^-)^2 \langle PS | F^{+\gamma} \left( \epsilon^{+\nu}_{\gamma\sigma} W D_\lambda F^{\lambda\sigma} - W D_\lambda \tilde{F}^{+\nu} + iz^- \int_0^1 dt g F^{+\mu}(tz) W \tilde{F}^{+\gamma}(z) \right) | PS \rangle$$

$$= (z^-)^2 \langle PS | \epsilon^{+\mu}_{\gamma\sigma} F^{+\gamma}(0) W \bar{\psi} \gamma^{\sigma} \psi(z) - \bar{\psi} \gamma^{\nu} \psi(0) W \tilde{F}^{+\mu}(z)$$

$$- iz^- \int_0^1 dt F^{+\gamma} W g F^{+\nu} W \tilde{F}^{+\mu} + iz^- \int_0^1 dt t F^{+\gamma} W g F^{+\nu} W \tilde{F}^{+\mu} | PS \rangle.$$  

(24)

As for the first line of (22), we use the following trick (see, e.g., [13])

$$F^{+\gamma}(0) W \tilde{F}^{+\mu}_\gamma(z) - F^{\mu\gamma}(0) W \tilde{F}^{+\gamma}(z)$$

$$= \int_0^1 du \frac{d}{du} \left( F^{+\gamma}(0) W \tilde{F}^{+\mu}(uz) - F^{\mu\gamma}(0) W \tilde{F}^{+\gamma}(uz) \right).$$  

(25)

The contribution from the lower limit $u = 0$ vanishes because of the following identity

$$F^{+\gamma}(z) W \tilde{F}^{+\mu}_\gamma(z') - F^{\mu\gamma}(z) W \tilde{F}^{+\gamma}(z') = \epsilon^{+\mu\alpha\beta} F^{\nu}_{\alpha\gamma}(z) W F^{\beta\gamma}(z'),$$  

(26)

which gives zero when evaluated at equal points $z = z'$. The matrix element of the right-hand-side of (25) will depend only on $uz^-$, so we can write $u \frac{d}{du} = z^- \frac{\partial}{\partial z^-}$ and the first line of (22) becomes

$$z^- \int_0^1 du \frac{z^-}{u} \frac{\partial}{\partial z^-} \langle PS | F^{+\gamma}(0) W \tilde{F}^{+\mu}_\gamma(uz) - F^{\mu\gamma}(0) W \tilde{F}^{+\gamma}(uz) | PS \rangle$$

$$= (z^-)^2 \int_0^1 du \langle PS | \left\{ F^{+\gamma} \tilde{D}_\lambda W \tilde{F}^{+\nu} + F^{+\nu} W D_\lambda \tilde{F}^{+\gamma} + \epsilon^{+\mu\nu}_{\gamma\delta} F^{+\gamma} W D_\lambda F^{\lambda\delta}$$

$$- iz^- \int_0^1 dt \left( - F^{+\gamma}(0) W g F^{+\mu}(tuz) W \tilde{F}^{+\gamma}(uz)$$

$$+ F^{+\gamma} W g F^{+\nu} W \tilde{F}^{+\mu} + F^{+\nu} W g F^{+\gamma} W \tilde{F}^{+\mu} \right) \right\} | PS \rangle,$$  

(27)

where we again used (23) and the Bianchi identity, $D^\alpha F^{\beta\gamma} + D^\beta F^{\alpha\gamma} + D^\gamma F^{\alpha\beta} = 0$.

Comparing (21) with (24) and (27), and noting that $D_\gamma \tilde{F}^{+\gamma} = 0$ from the Bianchi identity,
we find
\[
i \int dx \, e^{-ix^{j+P}z} \left( 2x^2 \frac{\partial}{\partial x} \mathcal{G}_{3T}(x) + x \Delta G(x) \right) (P^\mu S^+ - S^\mu P^+) \]
\[
= z^{-}\langle PS | e^{+^\mu \gamma^\sigma} F^{+\gamma}(0) \tilde{W} \tilde{\bar{\psi}} \gamma^\sigma \psi(z) - g \tilde{\bar{\psi}} \gamma^\mu \psi(0) \tilde{W} \tilde{F}^{+^\mu}(z) \rangle
\]
\[
- i z^{-} \int_0^1 dt F^{+\gamma} \tilde{W} g \tilde{F}^{+^\gamma} \tilde{F}^{+^\mu} + i z^{-} \int_0^1 dt t F^{+\gamma} \tilde{W} g \tilde{F}^{+^\mu} \tilde{W}^{+^\gamma} |PS \rangle
\]
\[
+ z^{-} \int_0^1 du \langle PS | \left\{ -g \tilde{\bar{\psi}} \gamma^\mu \psi(0) \tilde{W} \tilde{F}^{+^\mu}(uz) + e^{+^\mu \gamma^\delta} F^{+^\gamma}(0) \tilde{W} g \tilde{\bar{\psi}} \gamma^\delta \psi(uz)
\]
\[
- i u z^{-} \int_0^1 dt \left( -F^{+\gamma}(0) \tilde{W} g \tilde{F}^{+^\mu}(tuz) \tilde{W} \tilde{F}^{+^\gamma}(uz) \right)
\]
\[
+ F^{+\gamma} \tilde{W} g \tilde{F}^{+^\mu} \tilde{W}^{+^\gamma} + F^{+^\mu} \tilde{W} g \tilde{\bar{\psi}} \gamma^\delta \psi(uz) \right) \rangle |PS \rangle.
\] (28)

The matrix elements on the right–hand–side can be evaluated by the F–type correlators $F(x_1, x_2)$ and $G_F(x_1, x_2)$ (cf., [9], [12]). After very tedious calculations, we get

\[
2x^2 \frac{\partial}{\partial x} \mathcal{G}_{3T}(x) + x \Delta G(x) = \int dX \left( -2 \frac{\partial}{\partial x} G_F(X, x) + 2 \frac{x}{x} G_F(X, x) \right)
\]
\[
+ \int dx' \mathcal{P} \frac{1}{x-x'} \left[ \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial x'} \right) \left( 2F(x, x') - 3F(x', x' - x) - 3F(x, x - x') \right) \right.
\]
\[
- \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) \left( F(x', x' - x) - F(x, x - x') \right) \right]
\]
\[
+ 4 \int dx' \mathcal{P} \frac{1}{x(x-x')} \left( F(x', x' - x) + F(x, x - x') \right),
\] (29)

where we used the symmetry relations (10) and (11) and switched to the notation $X \equiv \frac{x_1 + x_2}{2}$ and $x \equiv x_1 - x_2$ in the arguments of $G_F$. [Note that $G_F(X, x) = G_F(X, -x).$] (29) can be solved for $\mathcal{G}_{3T}(x)$ with the boundary condition $\mathcal{G}_{3T}(\pm 1) = 0.$

\[
\mathcal{G}_{3T}(x) = \frac{1}{2} \int_x^{e(x)} dx' \Delta G(x') + \int_x^{e(x)} dx' \int_x^{e(x)} dx'' \int dX \left( x' \frac{\partial}{\partial x'} G_F(X, x') - G_F(X, x') \right)
\]
\[
- \frac{1}{2} \int_x^{e(x)} dx'' \int dx' \mathcal{P} \frac{1}{x'^2} \left( 2F''(x', x') - 3F(x', x' - x'') - 3F(x'', x' - x') \right)
\]
\[
- \left( \frac{\partial}{\partial x''} - \frac{\partial}{\partial x'} \right) \left( F(x', x' - x'') - F(x'', x' - x'') \right) \] 
\[
-2 \int_x^{e(x)} dx'' \int dx' \mathcal{P} \frac{1}{x'^2} \left( F(x', x' - x'') + F(x'', x' - x') \right),
\] (30)
where \( \epsilon(x) = x/|x| \). The first term is the Wandzura–Wilczek part which agrees with (8). The rest is the genuinely twist–three function of \( G_F \) and \( F \) which was denoted by \( \delta G(x) \) in (8).

Let us also show the moments of (30):

\[
\int_{-1}^{1} dx x^{n-1} G_{3T}(x) = \frac{1}{2n} \int_{-1}^{1} dx x^{n-1} \Delta G(x) + \frac{1}{n} \int_{-1}^{1} dx x^{n-1} \left( \frac{1}{x} \frac{\partial}{\partial x} G_F(X, x) - \frac{1}{x^2} G_F(X, x) \right) \]

\[
- \frac{1}{2n} \int_{-1}^{1} dx dx' \mathcal{P} \left( \frac{x^{n-1}}{x(x-x')} \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial x'} \right) \left( 2F(x, x') - 3F(x', x' - x) - 3F(x, x - x') \right) \right) \]

\[
- \frac{2}{n} \int_{-1}^{1} dx dx' \mathcal{P} \left( \frac{x^{n-1}}{x^2(x-x')} \left( F(x', x' - x) + F(x, x - x') \right) \right). \tag{31}
\]

When \( n = 1 \), the \( G_F \) terms vanish and we get, after integration by parts,

\[
\int_{-1}^{1} dx G_{3T}(x) = \Delta G + \int dx dx' \left\{ \mathcal{P} \left( \frac{1}{x(x-x')} \right) \left( F(x', x' - x) - F(x, x - x') \right) \right. \]

\[
\left. - \mathcal{P} \left( \frac{1}{x^2(x-x')} \right) \left( F(x, x') + F(x, x - x') \right) \right\}. \tag{32}
\]

By the change of variables and the symmetry property \( F(x, x') = F(x', x) \), it is easy to check that the two terms in the curly brackets are equal and they both vanish:

\[
\int dx dx' \mathcal{P} \left( \frac{1}{x(x-x')} \right) \left( F(x', x' - x) - F(x, x - x') \right) = \int dx dx' \mathcal{P} \left( \frac{1}{x^2(x-x')} \right) \left( F(x, x') + F(x, x - x') \right) = 0. \tag{33}
\]

We therefore recovered (7).

Finally in this section, we determine the function \( \tilde{g}(x) \) defined in (17). Recall the definition (11),

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS| F^{+i}(0)W F^{-j}(\lambda n)|PS \rangle = -i x G_{3T}(x) P^+ \epsilon^{ij} S_j. \tag{34}
\]
After integration by parts, the left–hand–side can be written as

\[
\frac{i}{P^+ x} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | F^{i+}(0) W D^+ F^{-}(\lambda n) | PS \rangle
\]

\[
= -\frac{i}{P^+ x} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | F^{i+}(0) W (D_j F^{i+j}(\lambda n) + g \bar{\psi}(\lambda n) \gamma^+ \psi(\lambda n)) | PS \rangle
\]

\[
= iP^+ \left( \int dx' (D_1(x', x) + D_2(x', x) - 2D_2(x, x')) + \int dX G_F(X, x) \right) \epsilon^{ij} S_j ,
\]  

(35)

where in the second line we used the equations of motion, \( D_\nu F^{\mu\nu} = -g \bar{\psi} \gamma^\mu \psi \), and in the third line we substituted (12) and (13). We thus find, using (16) and (17),

\[
\tilde{g}(x) = x^2 G_{3T}(x) + \int dX G_F(X, x)
\]

\[
+ \int dx' \mathcal{P} \frac{1}{x' - x} (F(x', x) - F(x, x - x') - 2F(x', x' - x)) .
\]  

(36)

Eliminating \( G_{3T} \) via (30) and doing integrations by parts, we arrive at

\[
\tilde{g}(x) = x^2 \int_{\epsilon(x)} x' \frac{d\epsilon}{x^3} \Delta G(x') + x^2 \int_{\epsilon(x)} x' \frac{d\epsilon}{x^3} \int dX G_F(X, x')
\]

\[
+ x^2 \int_{\epsilon(x)} x' \int d\epsilon' \left\{ \mathcal{P} \frac{2}{x' - x} (F(x', x') - F(x', x' - x'))
\]

\[
+ \mathcal{P} \frac{1}{x'^2 (x'' - x')^2} (F(x', x' - x'') - F(x'', x'' - x')) \right\} .
\]  

(37)

4. GLUON HELICITY CONTRIBUTION TO THE TRANSVERSE SPIN

Due to the property (17), it is natural to identify \( \Delta G = \int G_{3T}(x) \) as the total gluon helicity contribution to the transversely polarized nucleon spin. In this section we show that this expectation is consistent with the newly developed framework of spin decomposition in QCD [27, 29].

Recently there has been a lot of debate (and confusion) about the proper decomposition of the transversely polarized nucleon spin [26, 37–39]. We first note that it is important to distinguish two ‘transverse spin operators’ considered in the literature. One is the QCD angular momentum tensor itself [37–39]

\[
J^{\mu\nu} = \int d^3 x M^{+\mu\nu} ,
\]  

(38)

where \( d^3 x = dx^- d^2 x_\perp \) in the light–front form. The other is the Pauli–Lubanski vector [25, 26]

\[
W^\mu = \frac{1}{2} \epsilon_{\nu\rho\sigma} P^\nu \int d^3 x M^{+\rho\sigma} .
\]  

(39)
In the longitudinally polarized case, the distinction is irrelevant because only one component of $M$ is involved

$$W^+ = \frac{1}{2} \varepsilon^{ij} P^+ \int d^3 x M^+_{ij} \propto J^{12}. \quad (40)$$

However, in the transversely polarized case it is expected on general grounds that any transverse spin sum rule based on $\langle J^{\mu\nu} \rangle$ has frame–dependence [39, 40]. Following [26], in this paper we only consider the expectation value of the Pauli–Lubanski vector $\langle W^i \rangle$ where

$$W^i = \varepsilon^{ij} \left( P^- \int d^3 x M^{++}_{-j} - P^+ \int d^3 x M^{+-}_{-j} \right), \quad (41)$$

in pursuit of a frame–independent sum rule.

According to (41), the partonic decomposition of $W^i$ boils down to that of $M^{++}_{ij}$ and $M^{+-}_{ij}$. The latter is apparently of higher twist, but we shall shortly see that it nevertheless gives equally important contributions as the $M^{++}_{ij}$ term. In fact, the helicity contributions of quarks and gluons entirely come from $M^{+-}_{ij}$. In the quark case, we have

$$M_{q-\text{spin}}^{\mu\nu\lambda} = -\frac{1}{2} \varepsilon^{\mu\nu\lambda\sigma} \bar{\psi} \gamma_5 \gamma_\sigma \psi, \quad (42)$$

so that $M_{q-\text{spin}}^{++} = 0$ and

$$M_{q-\text{spin}}^{+-} = \frac{1}{2} \varepsilon^{ij} \bar{\psi} \gamma_5 \gamma_j \psi. \quad (43)$$

This immediately gives the usual quark helicity contribution $\Delta \Sigma$

$$\frac{\langle PS|W^i_{q-\text{spin}}|PS \rangle}{2P^+(2\pi)^3 \delta^3(0)} = \frac{1}{4} \frac{\langle PS| \int d^3 x \bar{\psi} \gamma_5 \gamma^j \psi|PS \rangle}{(2\pi)^3 \delta^3(0)} = \frac{1}{2} \int dx g_T(x) S^i = \frac{1}{2} \Delta \Sigma S^i, \quad (44)$$

where $g_T(x)$ is defined in (3). We thus concur with [26] that the quark helicity relevant to the nucleon spin sum rule is the first moment of the twist–three distribution $g_T(x)$, and not that of the twist–two transversity distribution $h_1(x)$ [38]. This is indeed a direct consequence of rotational invariance in the rest frame of the nucleon.

In the gluon case we use the recently developed gauge invariant framework of spin decomposition in which the gluon helicity component reads [27, 29]

$$M_{g-\text{spin}}^{\mu\nu\lambda} = F^{\mu\nu\lambda} A^\nu_{\text{phys}} - F^{\mu\nu} A^\lambda_{\text{phys}}, \quad (45)$$

where $A^\mu_{\text{phys}}$ is the ‘physical part’ of the gauge field [29]

$$A^\alpha_{\text{phys}}(\lambda n) = -\int d\zeta K(\zeta - \lambda) W_{\lambda \zeta} n_\mu F^{\mu\alpha}(\zeta n), \quad (46)$$
with $\mathcal{K}(\lambda)$ being either $\frac{1}{2}\epsilon(\lambda)$, or $\pm\theta(\pm\lambda)$. The matrix element of (45) is related to $\Delta G$, both for the transverse and longitudinal polarizations. Indeed, dividing (1) by $x$ and integrating over $x$ we find, using $\int_{-1}^{1} dx G(x) = 0$,

$$\langle PS|F^{+\alpha}(0)A^{\beta}_{phys}(0)|PS\rangle = \frac{1}{2} \epsilon^{+\alpha\beta} S^+ \int dx \Delta G(x) + \epsilon^{+\alpha\beta j} S_j \int dx \mathcal{G}_{ST}(x)$$

$$= \epsilon^{+\alpha\beta\mu} S_{\mu} \Delta G \, . \quad (47)$$

As seen in (47), the matrix element of operators involving $A^{\mu}_{phys}(0)$, despite being nonlocal, does not depend on the noncovariant vector $n^{\mu}$. This is because the scale transformation $n^{\mu} \rightarrow cn^{\mu}$ applied to (46) with $\lambda = 0$ can be absorbed by the rescaling $\zeta \rightarrow \zeta/c$ of the integration variable. Noticing that $A^+_{phys} = 0$ and therefore

$$M^{++i}_{g-spin} = 0 \, , \quad M^{-i}_{g-spin} = F^{+i}A^-_{phys} - F^{++}A^i_{phys} \, , \quad (48)$$

we obtain

$$\langle PS|W^{i}_{g-spin}|PS\rangle = S^i \int dx \mathcal{G}_{ST}(x) = S^i \Delta G \, . \quad (49)$$

We thus see that $\Delta G$ is the gluon helicity contribution also for the transverse polarization, due to the contribution of the Wandzura–Wilczek part of $\mathcal{G}_{ST}(x)$ as fully given by (30).

**Complete transverse spin decomposition?**

Now that we have seen the helicity components, (42) and (45), of the decomposition scheme [27, 29] works also for the transverse polarization, it seems a straightforward task to demonstrate the complete decomposition of the transverse spin including the *canonical* orbital angular momentum of quarks and gluons

$$M_{q, orbit}^{\mu \nu \lambda} = \bar{\psi}\gamma^{\mu}(x^\nu iD^\lambda_{pure} - x^\lambda iD^\mu_{pure})\psi \, , \quad M_{g, orbit}^{\mu \nu \lambda} = F^{\mu \alpha}(x^\lambda D^\nu_{pure} - x^\nu D^\lambda_{pure})A^\alpha_{phys}$$

where $D^\mu_{pure} = D^\mu - igA^\mu_{phys}$, as has been done in the longitudinally polarized case [41]. In doing so, however, we encountered an unexpected difficulty which actually already arises in the more conventional decomposition scheme by Ji [42] recently revisited in [26]. In Ji’s scheme, the angular momentum tensor is given by

$$M_{q,g}^{\lambda \mu \nu} = x^\mu T_{q,g}^{\lambda \nu} - x^\nu T_{q,g}^{\lambda \mu} \, , \quad (51)$$

---

5 When integrating over $x$, one has to specify the $ie-$prescription for the pole $1/x$. Different prescriptions correspond to the different kernels $\mathcal{K}$ in (46).
where $T_{q,g}$ is the (Belinfante–improved) energy momentum tensor of quarks/gluons whose matrix element is parameterized as

$$
\langle P'S'|T_{q,g}^{\mu\nu}|PS\rangle = \bar{u}(P'S') \left[ A_{q,g} \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g} \frac{\bar{P}^{(\mu} i\sigma^{\nu)} \alpha}{2M} \Delta^\alpha + C_{q,g} g^{\mu\nu} \right] u(PS),
$$

where $\bar{P}^{\mu} = \frac{1}{2} (P^{\mu} + P'^{\mu})$, $\Delta^{\mu} = P'^{\mu} - P^{\mu}$. Ref. [26] concludes that the sum rule based on the Pauli–Lubanski vector is nothing but the Ji sum rule

$$
\langle PS|W^i_{q,g}|PS\rangle = \frac{1}{2}(A_{q,g} + B_{q,g}) S^i,
$$

where $J_q + J_g = \frac{1}{2}$ is the total nucleon spin.

Although (53) seems appealing, as its frame–independent form exemplifies the advantage of using the Pauli–Lubanski vector, it should be corrected as follows: As noted already, the transverse angular momentum receives a contribution from the twist–four matrix element $\langle M_{q,g}^{+,-i} \rangle \sim \langle x^i T_{q,g}^{+-} \rangle \sim i \frac{\partial}{\partial \Delta} \langle T_{q,g}^{+-} \rangle$ which involves the derivative with respect to the momentum transfer $\Delta$. The problem is that the nonforward spinor product

$$
\bar{u}(P'S') u(PS) \approx 2M + i \frac{\bar{P}^3}{M(P^0 + M)} \epsilon^{ij} \Delta_j S_i,
$$

contains an order $O(\Delta)$, and manifestly frame–dependent (velocity–dependent) term in the transversely polarized case, as first observed in [38]. There are two sources of $\bar{u}(P'S') u(PS)$ in (52). The first term of $\langle T_{q,g}^{+-} \rangle$ (proportional to $A_{q,g}$) contains it as one can see from the Gordon identity

$$
\bar{u}(P'S') \gamma^{\mu} u(PS) = \frac{\bar{P}^\mu}{M} \bar{u}(P'S') u(PS) + \bar{u}(P'S') \frac{i\sigma^{\mu\alpha} \Delta^\alpha}{2M} u(PS).
$$

Fortunately, this unwanted term cancels against a similar term from $\langle T_{q,g}^{++} \rangle$ and does not contribute to $\langle W^i_{q,g} \rangle$ [26]. However, the fourth term

$$
\bar{C}_{q,g} M g^{+-} \bar{u}(P'S') u(PS),
$$

which is related to the QCD trace anomaly and has no compensating term from $\langle T_{q,g}^{++} \rangle$, leads to an additional frame–dependent contribution to $J_{q,g}$

$$
J_{q,g} = \frac{1}{2}(A_{q,g} + B_{q,g}) + \frac{P^3}{2(P^0 + M)} \bar{C}_{q,g},
$$

which is parametrically unsuppressed if $\bar{C}_{q,g} \sim O(1)$ and the motion is relativistic $P^3 \sim O(P^0)$. It should be noted that the sum vanishes $\bar{C}_q + \bar{C}_g = 0$ due to the conservation of
the total energy momentum tensor $T^{\mu\nu} = T^{\mu\nu}_q + T^{\mu\nu}_g$, so they cancel in the total angular momentum $\frac{1}{2} = J_q + J_g$. However, they do affect the decomposition into the quark and gluon contributions.

The above problem fortunately does not arise in the longitudinally polarized case where the product $\bar{u}'u$ does not contain an $O(\Delta)$ term. Both the Ji decomposition and the complete decomposition \cite{41} stand unaffected in this case. However, in the transversely polarized case the Ji decomposition is afflicted by the (nominally) twist–four non-covariant terms, and the same problem seems to persist in the expectation value of the canonical orbital angular momenta $\langle W_{q-orbit}^i \rangle$ and $\langle W_{g-orbit}^i \rangle$ computed from (50) \cite{43}. This fact, together with \cite{44} and \cite{49}, implies that in the transversely polarized case we may only achieve the decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta \Sigma + \Delta G + L,$$

where the canonical orbital angular momentum $L$ cannot be separated into those of quarks and gluons in a frame–independent way.

ACKNOWLEDGEMENTS

We thank Feng Yuan for discussions and for sending us a copy of \cite{26} before it became publicly available. We also thank Elliot Leader for discussions. The work of K. T. is supported in part by the Grant-in-Aid for Scientific Research (Nos. 23540292 and 24540284). The work of S. Y. is supported by the Grant-in-Aid for Scientific Research (No. 21340049).

Note added: A few days after the submission of this work on arXiv, a preprint \cite{44} appeared which pointed out essentially the same non-covariant term as in (57).

\begin{thebibliography}{99}

[1] V. Barone, F. Bradamante and A. Martin, Prog. Part. Nucl. Phys. 65, 267 (2010) [arXiv:1011.0909 [hep-ph]].

[2] R. L. Jaffe and X. -D. Ji, Nucl. Phys. B 375, 527 (1992).

6 Incidentally, we note that the extra term in (57) is unrelated to the frame–dependent term asserted in \cite{39} which grows unboundedly with increasing energy.

16
[3] Y. Koike, K. Tanaka and S. Yoshida, Phys. Lett. B 668, 286 (2008) [arXiv:0805.2289 [hep-ph]].
[4] A. P. Bukhvostov, E. A. Kuraev and L. N. Lipatov, Sov. Phys. JETP 60, 22 (1984) [Zh. Eksp. Teor. Fiz. 87, 37 (1984)].
[5] X. -D. Ji and C. -h. Chou, Phys. Rev. D 42, 3637 (1990).
[6] J. Kodaira, Y. Yasui, K. Tanaka and T. Uematsu, Phys. Lett. B 387, 855 (1996) [hep-ph/9603377].
[7] I. I. Balitsky and V. M. Braun, Nucl. Phys. B 311, 541 (1989).
[8] H. Eguchi, Y. Koike and K. Tanaka, Nucl. Phys. B 752, 1 (2006) [hep-ph/0604003].
[9] X. -D. Ji, Phys. Lett. B 289, 137 (1992).
[10] J. Kodaira and K. Tanaka, Prog. Theor. Phys. 101, 191 (1999) [hep-ph/9812449].
[11] D. Mueller, Phys. Lett. B 407, 314 (1997) [hep-ph/9701338].
[12] J. Kodaira, T. Nasuno, H. Tochimura, K. Tanaka and Y. Yasui, Prog. Theor. Phys. 99, 315 (1998) [hep-ph/9712395].
[13] V. M. Braun, G. P. Korchemsky and A. N. Manashov, Nucl. Phys. B 597, 370 (2001) [hep-ph/0010128].
[14] V. M. Braun, A. N. Manashov and B. Pirnay, Phys. Rev. D 80, 114002 (2009) [arXiv:0909.3410 [hep-ph]].
[15] Y. Koike and S. Yoshida, Phys. Rev. D 85, 034030 (2012) [arXiv:1112.1161 [hep-ph]].
[16] A. Metz, D. Pitonyak, A. Schaefer and J. Zhou, [arXiv:1210.6555 [hep-ph]].
[17] Z. -B. Kang and J. -W. Qiu, Phys. Rev. D 78, 034005 (2008) [arXiv:0806.1970 [hep-ph]].
[18] Y. Koike, K. Tanaka and S. Yoshida, Phys. Rev. D 83, 114014 (2011) [arXiv:1104.0798 [hep-ph]].
[19] H. Beppu, Y. Koike, K. Tanaka and S. Yoshida, Phys. Rev. D 85, 114026 (2012) [arXiv:1204.1592 [hep-ph]].
[20] H. Beppu, Y. Koike, K. Tanaka and S. Yoshida, Phys. Rev. D 82, 054005 (2010) [arXiv:1007.2034 [hep-ph]].
[21] Z. -B. Kang, J. -W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D 78, 114013 (2008) [arXiv:0810.3333 [hep-ph]].
[22] A. V. Belitsky, X. -D. Ji, W. Lu and J. Osborne, Phys. Rev. D 63, 094012 (2001) [hep-ph/0007305].
[23] Y. Koike and S. Yoshida, Phys. Rev. D 84, 014026 (2011) [arXiv:1104.3943 [hep-ph]].
[24] C. Lorce, arXiv:1205.6483 [hep-ph].

[25] A. Harindranath, A. Mukherjee and R. Ratabole, Phys. Lett. B 476, 471 (2000) [hep-ph/9908424].

[26] X. Ji, X. Xiong and F. Yuan, Phys. Lett. B 717, 214 (2012) arXiv:1209.3246 [hep-ph].

[27] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[28] X. Ji, X. Xiong and F. Yuan, Phys. Lett. B 717, 214 (2012) arXiv:1209.3246 [hep-ph].

[29] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[30] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[31] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[32] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[33] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[34] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[35] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[36] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[37] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[38] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[39] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[40] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[41] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[42] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[43] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].

[44] X. S. Chen, X. F. Lu, W. M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100, 232002 (2008) arXiv:0806.3166 [hep-ph].