Phase transitions to spin-triplet ferromagnetic superconductivity in neutron stars

Diana V. Shopova, Tsvetomir E. Tsvetkov, Dimo I. Uzunov,

CP Laboratory, G. Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences, BG-1784 Sofia, Bulgaria.

Abstract

Effects of the anisotropy of Cooper pairs in spin-triplet ferromagnetic superconductors are investigated on the basis of the Ginzburg-Landau theory. A special attention is paid to the triggering of the superconducting state by the ferromagnetic order. The ground states of these superconductors are outlined and discussed. The idea about a possible coexistence of ferromagnetism and spin-triplet superconductivity in neutron stars is introduced.

1. Introduction

The cold and dense matter in the interior of neutron stars undergoes phase transitions to both superfluid and superconducting states (see, e.g., Refs. [1, 2, 3]). Both the superfluidity due to neutron Cooper pairing and the superconductivity due to proton Cooper pairing may be of unconventional spin-triplet ($p$-wave) type [1]. The latter is known from the theory of $^3$He liquids [4, 5, 6] as well as from the theory of heavy fermion [7, 8] and high-temperature (9, 10, 11, 12) superconductors.

The possible superconducting phases in unconventional superconductors are described in the framework of the general Ginzburg-Landau (GL) effective free energy functional [11] with the help of the symmetry group theory. Thus a variety of possible superconducting orderings were predicted for different crystal structures [13, 14, 15, 16]. A detailed thermodynamic analysis [9, 14] of the homogeneous (Meissner) phases and a renormalization group investigation [9] of the superconducting phase transition up to the two-loop approximation [11] were also performed.

Recent experiments [17] at low temperatures ($T \sim 1$ K) and high pressure ($T \sim 1$ GPa) demonstrated the existence of spin triplet superconducting states in the metallic compound UGe$_2$. This superconductivity is triggered by the spontaneous magnetization ($M$) of the ferromagnetic phase ($M$-trigger effect [18]). The ferromagnetic order exists at much higher temperatures and coexists with the superconducting phase in the whole domain of existence of the latter below $T \sim 1$ K; see also experiments published in Refs. [19]. Moreover, the same phenomenon of existence of superconductivity at low temperatures and high pressure in the domain of the ($T, P$) phase diagram where the
ferromagnetic order is present has been observed in ZrZn and URhGe, too; for details, see, e.g. Ref. [22, 23, 24, 18]. Note, that the superconductivity in the metallic compounds mentioned above, always coexists with the ferromagnetic order and is enhanced by the latter. Besides, in these systems the superconductivity seems to arise from the same electrons that create the band magnetism.

A similar phenomenon of coexistence of ferromagnetism and spin-triplet superconductivity may exist also in neutron stars. In this case the Cooper pairing of fermions (protons) will be triggered by the spontaneous magnetic moment of the same proton subsystem of the nuclear star matter. The basic features of these phenomena can be described within an extension of the GL theory. The results can be applied to a new and interesting problem of coexistence of superconductivity and ferromagnetism in neutron stars. The coexistence of spin-triplet superconductivity and ferromagnetism may explain the large magnetic field of these stars. To our best knowledge the problem of a possible coexistence of ferromagnetism and superconductivity in neutron stars and asprophysics objects is introduced for the first time in our present report.

The theory allows the description of various types of phase transitions and multicritical points [11, 25]. Following notations in Ref. [18], here we summarize previous studies [22, 23, 24, 18] and present new results on the effect of the Cooper pair anisotropy on the phase diagram of spin-triplet ferromagnetic superconductors [18].

Our consideration is focussed on the ground state, namely, we are interested in uniform phases, where the order parameters (the superconducting order parameter $\psi$ and the magnetization vector $M = \{M_j, j = 1, 2, 3\}$, do not depend on the spatial vector $\vec{x} \in V$ ($V$ is the volume of the system).

2. Ginzburg-Landau free energy

Consider the GL free energy $F(\psi, M) = V f(\psi, M)$, where the free energy density $f(\psi, M)$ (for short hereafter called “free energy”) of a spin-triplet ferromagnetic superconductor is given by [18]

$$f(\psi, M) = a_s |\psi|^2 + \frac{b_s}{2} |\psi|^4 + \frac{u_s}{2} |\psi^2|^2 + \frac{v_s}{2} \sum_{j=1}^{3} |\psi_j|^4 + a_f M^2 + \frac{b_f}{2} M^4$$

$$+ i \gamma_0 M \cdot (\psi \times \psi^*) + \delta M^2 |\psi|^2.$$  \(1\)

In Eq. (1), $\psi = \{\psi_j; j = 1, 2, 3\}$ is the three-dimensional complex vector ($\psi_j = \psi_j^\prime + i \psi_j^\prime\prime$) describing the unconventional (spin-triplet) superconducting order and $B = (H + 4\pi M) = \nabla \times A$ is the magnetic induction; $H = \{H_j; j = 1, 2, 3\}$ is the external magnetic field, $A = \{A_j; j = 1, 2, 3\}$ is the magnetic vector potential ($\nabla \cdot A = 0$). In Eq. (1), $b_s > 0$, $b_f > 0$, $a_f = \alpha_f (T - T_f)$ is given by the positive material parameter $\alpha_f$ and the ferromagnetic critical temperature $T_f$ corresponding to a simple superconductor ($M \equiv 0$), and $a_s = \alpha_s (T - T_s)$, where $\alpha_s$ is another positive material parameter and
$T_s$ is the critical temperature of a standard second order phase transition which may occur at $|H| = M = 0; \mathcal{M} = |M|$. The parameter $u_s$ describes the anisotropy of the spin-triplet Cooper pair whereas the crystal anisotropy is described by the parameter $v_s$ [9, 14].

The two orders – the magnetization vector $M = \{M_j\}$ and $\psi = \{A_j\}$, interact through the last two terms in (1). The $\gamma_0$-term [21] ensures the triggering of the superconductivity by the ferromagnetic order ($\gamma_0 > 0$) whereas the $\delta$-term makes the model more realistic in the strong coupling limit [20]. Both $\psi M$-interaction terms included in (1) are important for a correct description of the temperature-pressure ($T, P$) phase diagram of the ferromagnetic superconductor [18]. In general, the parameter $\delta$ for ferromagnetic superconductors may take both positive and negative values.

The values of the material parameters ($T_s, T_f, \alpha_s, \alpha_f, b_s, u_s, v_s, b_f, K_j, \gamma_0$ and $\delta$) depend on the choice of the concrete substance and on intensive thermodynamic parameters, such as the temperature $T$ and the pressure $P$. One may assume that the general form (1) of the free energy may describe the general features of the uniform orders in neutron stars provided one makes a suitable choice of parameters ($T_s, T_f, \alpha_s, ...$).

As we are interested in the ground state properties, we set the external magnetic field equal to zero ($H = 0$). Besides, we emphasize that the magnetization vector $M$ may produce vortex superconducting phase [11, 14] in case of type II superconductivity. The investigation of nonuniform (vortex) states can be made with the help of gradient terms in the free energy [18] which take into account the spatial variations of the order parameter field $\psi$. This task is beyond our present consideration. Rather we investigate the basic problem about the possible stable uniform (Meissner) superconducting phases which may coexist with uniform ferromagnetic order. For this aim the free energy (1) is quite convenient.

In case of a strong easy axis type of magnetic anisotropy, as is in UGe$_2$ [17], the overall complexity of mean-field analysis of the free energy $f(\psi, M)$ can be avoided by performing an “Ising-like” description: $M = (0, 0, M)$. Further, because of the equivalence of the “up” and “down” physical states ($\pm M$) the thermodynamic analysis can be performed within the “gauge” $M \geq 0$. But this stage of consideration can also be achieved without the help of crystal anisotropy arguments. When the magnetic order has a continuous symmetry one may take advantage of the symmetry of the total free energy $f(\psi, M)$ and avoid the consideration of equivalent thermodynamic states that occur as a result of the respective symmetry breaking at the phase transition point but have no effect on thermodynamics of the system. In the isotropic system one may again choose a gauge, in which the magnetization vector has the same direction as $z$-axis ($|M| = M_z = \mathcal{M}$) and this will not influence the generality of thermodynamic analysis. Here we shall prefer the alternative description within which the ferromagnetic state may appear through two equivalent “up” and “down” domains with magnetization $\mathcal{M}$ and $-\mathcal{M}$, respectively.
For our aims we use notations in which the number of parameters is reduced. Introducing the parameter

\[ b = (b_s + u_s + v_s) > 0 \]  

we redefine the order parameters and the other parameters in the following way:

\[ \varphi_j = b^{1/4} \psi_j = \phi_j e^{\theta_j}, \quad M = b_f^{1/4} M, \]

\[ r = \frac{a_s}{\sqrt{b}}, \quad t = \frac{a_f}{\sqrt{b_f}}, \quad w = \frac{u_s}{b}, \quad v = \frac{v_s}{b}, \]

\[ \gamma = \frac{\gamma_0}{b^{1/2} b_f^{1/4}}, \quad \gamma_1 = \frac{\delta}{(bb_f)^{1/2}}. \]

Having in mind our approximation of uniform \( \psi \) and \( M \) and the notations (2) - (3), the free energy density \( f(\psi, M) \) can be written in the form

\[ f(\psi, M) = r \phi^2 + \frac{1}{2} \phi^4 + 2 \gamma \phi_1 \phi_2 M \sin(\theta_2 - \theta_1) + \gamma_1 \phi^2 M^2 + t M^2 + \frac{1}{2} M^4 \]

\[ -2w [\phi_1^2 \phi_2^2 \sin^2(\theta_2 - \theta_1) + \phi_1^2 \phi_3^2 \sin^2(\theta_1 - \theta_3) + \phi_2^2 \phi_3^2 \sin^2(\theta_2 - \theta_3)] \]

\[ -v [\phi_1^2 \phi_2^2 + \phi_1^2 \phi_3^2 + \phi_2^2 \phi_3^2]. \]

In this free energy the order parameters \( \psi \) and \( M \) are defined per unit volume.

In contrast to the situation in superconducting compounds, for the case of neutron stars, the crystal field anisotropy represented by the \( v_s \)-terms in (1) – (4) can be ignored, and for this reason we shall discuss the case \( v_s \equiv 0 \). We assume that \( T_f > T_s \). This is the case when the superconductivity is triggered by the magnetic order. Besides we shall discuss the stable phases in the temperature region \( T > T_s \). The case \( T_f < T_s \) may also present interest for neutron stars and, hence, it needs a special investigation. As mentioned in Ref. [18], the case \( T_s \sim T_f \) allows for a quite simple analytical treatment. All these cases may be of interest to the description of ferromagnetic superconductivity in stellar objects whereas in condensed matter only cases of \( T_f \gg T_s \) have been observed so far.

Our consideration is performed within the framework of the standard mean-field analysis [11]. The stable phases correspond to global minima of the GL energy (1). The equilibrium phase transition line separating two phases is defined by the thermodynamic states, where the respective GL free energies are equal.

### 3. Phases and phase diagram

The calculations show that for temperatures \( T > T_s \), i.e., for \( r > 0 \), we have three stable phases. Two of them are quite simple: the normal (N-) phase (\( \psi = M = 0 \)) with existence and stability domains given by \( t > 0 \) and \( r > 0 \), and the ferromagnetic phase (FM) given by \( \psi = 0 \) and \( M^2 = -t \) which has the existence condition \( t < 0 \), and a stability domain defined by the inequalities \( r > \gamma_1 t \) and

\[ r > \gamma_1 t + \gamma \sqrt{-t}. \]
The third stable phase is a phase of coexistence of superconductivity and ferromagnetism (hereafter referred to as FS). It is given the following equations:

\[ \phi_1 = \phi_2 = \frac{\phi}{\sqrt{2}}, \quad \phi_3 = 0, \quad (6) \]

\[ \phi^2 = (\pm \gamma M - r - \gamma_1 M^2), \quad (7) \]

\[ (1 - \gamma_1^2)M^3 \pm \frac{3}{2} \gamma \gamma_1 M^2 + \left(t - \frac{\gamma^2}{2} - \gamma_1 r\right)M \pm \frac{\gamma r}{2} = 0, \quad (8) \]

and

\[ (\theta_2 - \theta_1) = \pm \frac{\pi}{2} + 2\pi k, \quad (9) \]

\((k = 0, \pm 1, \ldots)\). The upper sign in Eqs. (7) – (9) corresponds to a domain in which \(\sin(\theta_2 - \theta_1) = -1\) and the lower sign corresponds to a second domain which may be referred to as FS\(^*\); in the latter, \(\sin(\theta_2 - \theta_1) = 1\). These two domains are equivalent and describe the same ordering. We shall focus on the upper sign in (7) – (9), i.e. on FS.

The phase diagram \((t, r)\) is outlined in Figs. 1 and 2 for different values of the anisotropy parameter \(w\). The phase transition lines for \(w > 0\) and \(w < 0\) shown in Figs. 1 and 2, respectively, have qualitatively the same shape as the phase transition lines corresponding to \(w = 0\) [FS], but there are essential quantitative differences between these cases. We shall discuss them in the next section.
Figure 2: Phase diagram in the \((t, r)\) plane for \(\gamma = 1.2, \gamma_1 = 0.8,\) and \(w = -2.\) The various lines are explained in the text.

Note, that the phase diagrams in Figs. 1 and 2 exhibit two types of phase transitions. The dashed curves indicate second order phase transitions of type N-FM, FM-FS and N-FS, whereas the curve AC and the straight line BC indicate the first order phase transitions N-FS and FM-FS, respectively. The points \(A\) and \(B\) are tricritical whereas \(C\) is a triple point, where N, FM and FS coexist \([11]\). The negative values of the parameter \(r\) are restricted by \(r(T) \geq r(T = 0)\). Having in mind this condition as well as the shape of the phase diagram (Figs. 1 and 2) we easily conclude that both FM and FS ground states (at \(T = 0\)) are possible in systems described by the model (1). This is the case for the itinerant magnets, mentioned above, and one may speculate that this situation may occur in neutron stars, too. Whether FM and FS ground state will occur depends on the particular values of the material parameters \((T_s, T_f, \alpha_s, \ldots)\).

The final aim of the phase diagram investigation is the outline of the \((T, P)\) diagram. Important conclusions about the shape of the \((T, P)\) diagram can be made from the form of the \((t, r)\) diagram without an additional information about the values of the relevant material parameters \((a_s, a_f, \ldots)\) and their dependence on the pressure \(P\). For example, the equilibrium temperature \(T_{FS}\) of the phase transitions to FS phase varies with the variation of the system parameters \((\alpha_s, \alpha_f, \ldots)\) from values which are much higher than the characteristic temperature \(T_s\) up to zero temperature.

4. Anisotropy effects

Our analysis demonstrates that when the anisotropy of the Cooper pairs is taken in consideration, there will be not drastic changes in the shape the phase diagram for
\( r > 0 \) and the order of the respective phase transitions. Of course, there will be some changes in the size of the phase domains and the formulae for the thermodynamic quantities. This is seen from Figs. 1 and 2 which are shown for the first time in the present report. Besides, it is readily seen from Figs. 1 and 2 that the temperature domain of first order phase transitions and the temperature domain of stability of FS above \( T_s \) essentially vary with the variations of the anisotropy parameter \( w \). The parameter \( w \) will also insert changes in the values of the thermodynamic quantities like the magnetic susceptibility and the entropy and specific heat jumps at the phase transition points \[21\].

Besides, and this seems to be the main anisotropy effect, the \( w \)- and \( v \)-terms in the free energy lead to a stabilization of the order along the main crystal directions which, in other words, means that the degeneration of the possible ground states is considerably reduced. This means also a smaller number of marginally stable states.

The dimensionless anisotropy parameter \( w \) can be either positive or negative depending on the sign of \( u_s \). Obviously when \( u_s > 0 \), the parameter \( w \) will be positive too (\( 0 < w < 1 \)). We shall illustrate the influence of Cooper-pair anisotropy in this case. The order parameters \((M, \phi_j, \theta_j)\) are given by Eqs. (6), (9),

\[
\phi^2 = \pm \gamma M - r - \gamma_1 M^2 \geq 0 ,
\]

and

\[
(1-w-\gamma_1^2)M^3 \pm \frac{3}{2} \gamma \gamma_1 M^2 + \left[ t(1-w) - \frac{\gamma^2}{2} - \gamma_1 r \right] M \pm \frac{\gamma r}{2} = 0 ,
\]

where the meaning of the upper and lower sign is the same as explained just below Eq. (9). We consider the FS domain corresponding to the upper sign in the Eq. (10) and (11). The stability conditions for FS read,

\[
\frac{(2-w)\gamma M - r - \gamma_1 M^2}{1-w} \geq 0 ,
\]

and

\[
\gamma M - w r - w \gamma_1 M^2 > 0 ,
\]

and

\[
\frac{1}{1-w} \left[ 3(1-w-\gamma_1^2)M^2 + 3\gamma \gamma_1 M + t(1-w) - \frac{\gamma^2}{2} - \gamma_1 r \right] \geq 0 .
\]

For \( M \neq (\gamma/2\gamma_1) \) we can express the function \( r(M) \) defined by Eq. (10), substitute the obtained expression for \( r(M) \) in the existence and stability conditions (10)-(14) and do the analysis in the same way as for \( w = 0 \) \[18\]. The most substantial qualitative difference between the cases \( w > 0 \) and \( w < 0 \) is that for \( w < 0 \) the stability of FS is limited for \( r < 0 \). This is seen from Fig. 2 where FS is stable above the straight dotted line for \( r < 0 \) and \( t < 0 \). This includes into consideration also purely superconducting (Meissner) phases as ground states.

5. Final remarks
We have done an investigation of the M-trigger effect in unconventional ferromagnetic superconductors. This effect due to the $M\psi_1\psi_2$-coupling term in the GL free energy consists of bringing into existence of superconductivity in a domain of the phase diagram of the system that is entirely in the region of existence of the ferromagnetic phase. This form of coexistence of unconventional superconductivity and ferromagnetic order is possible for temperatures above and below the critical temperature $T_s$, which corresponds to the standard phase transition of second order from normal to Meissner phase – usual uniform superconductivity in a zero external magnetic field, which appears outside the domain of existence of ferromagnetic order. Our investigation has been mainly intended to clarify the thermodynamic behaviour at temperatures $T_s < T < T_f$, where the superconductivity cannot appear without the mechanism of M-triggering. We have described the possible ordered phases (FM and FS) in this most interesting temperature interval.

The Cooper pair and crystal anisotropies have also been investigated and their main effects on the thermodynamics of the triggered phase of coexistence have been established. In discussions of concrete real material one should take into account the respective crystal symmetry but the variation of the essential thermodynamic properties with the change of the type of this symmetry is not substantial when the low symmetry and low order (in both $M$ and $\psi$) $\gamma$-term is present in the free energy.

Below the superconducting critical temperature $T_s$ a variety of pure superconducting and mixed phases of coexistence of superconductivity and ferromagnetism exists and the thermodynamic behavior at these relatively low temperatures is more complex than in known cases of improper ferroelectrics; see. e.g., Ref. [25]. The case $T_f < T_s$ also needs a special investigation.

Our results are referred to the possible uniform superconducting and ferromagnetic states. Vortex and other nonuniform phases need a separate study.

More experimental information about the values of the material parameters ($a_s, a_f,$...) included in the free energy (1) is required in order to outline the thermodynamic behavior and the phase diagram in terms of thermodynamic parameters $T$ and $P$. In particular, a reliable knowledge about the dependence of the parameters $a_s$ and $a_f$ on the pressure $P$, the value of the characteristic temperature $T_s$ and the ratio $a_s/a_f$ at zero temperature are of primary interest.

The phenomenological GL model (1) is quite general and can be reliably used in considerations of a possible coexistence of ferromagnetism and unconventional superconductivity in stellar objects. Recent investigations [2][3] of superconductivity in neutron stars can be related with the present consideration.

**Acknowledgments:**

One of us (T.E.T.) thanks the hospitality of Dr. V. Celebonovic and the Organizers of the Workshop on Equation of State and Phase Transition Issues in Models of Ordinary Astrophysical Matter (Lorentz Center, Leiden University, 2-11 June 2004). Financial
support by SCENET (Parma) and JINR (Dubna) is also acknowledged.

References

[1] Tilley, D. R., Tilley, J.: 1974, Superfluidity and Superconductivity, Van Norstrand Reinhold Company, New York.

[2] Link, B.: 2003, Phys.Rev.Lett., 91, 101101.

[3] Buckey, K. B. W., Metlitski, M. A. and A. R. Zhitnitsky, A. R.: 2004, Phys.Rev.Lett., 92, 151102.

[4] Leggett, A.J.: 1975, Rev. Mod. Phys., 47, 331.

[5] Vollhardt, D. and Wölfle, P.: 1990, The Superfluid Phases of Helium 3, Taylor&Francis, London.

[6] Volovik, G.E.: 2003, The Universe in a Helium Droplet, Oxford University Press, Oxford.

[7] Stewart, G. R.: 1984, Rev. Mod. Phys., 56, 755.

[8] Sigrist, M. and Ueda, K.: 1991, Rev.Mod.Phys., 63, 239.

[9] Blagoeva, E.J., Busiello, G., De Cesare, L., Millev, Y.T., Rabuffo, I. and Uzunov, D.I.: 1990, Phys.Rev., B42, 6124.

[10] Uzunov, D.I., in: 1990, Advances in Theoretical Physics, Ed. by Caianiello, E., World Scientific, Singapore, p. 96.

[11] Uzunov, D.I.: 1993, Theory of Critical Phenomena, World Scientific, Singapore.

[12] Annett, J. F.: 1995, Contemp.Physics, 36, 323.

[13] Volovik, G. E. and Gor’kov, L.P.: 1984, JETP Lett., 39, 674 [1984, Pis’ma Zh. Eksp. Teor. Fiz., 39, 550].

[14] Volovik, G. E. and Gor’kov, L.P.: 1985, Sov. Phys. JETP, 61, 843 [1985, Zh. Eksp. Teor. Fiz., 88, 1412].

[15] Ueda, K. and Rice, T.M.: 1985, Phys. Rev., B 31, 7114.

[16] Blount, E.I.: 1985, Phys. Rev., B 32, 2935.

[17] Saxena, S. S., Agarwal, P., Ahilan, K. et al.: 2000, Nature, 406, 587.

[18] Shopova, D.V. and Uzunov, D.I., in: 2004, Progress in Ferromagnetism Research, Nova, New York (in press); see also [cond-mat/0404261]
[19] Huxley, A., Sheikin, I., Ressouche, E. et al.: 2001, Phys. Rev., B63, 144519.

[20] Machida, K. and Ohmi, T.: 2001, Phys. Rev. Lett., 86, 850.

[21] Walker, M. B. and Samokhin, K. V.: 2002, Phys. Rev. Lett., 88, 207001.

[22] Shopova, D. V. and D. I. Uzunov, D. I.: 2003, Phys. Lett., A 313, 139.

[23] Shopova, D.V. and Uzunov, D. I.: 2003, J. Phys. Studies, 7, 426.

[24] Shopova, D. V. and D. I. Uzunov, D.I.: 2003, Compt. Rend. Acad. Bulg. Sci., 56, 35; see also cond-mat/0310016.

[25] J-C. Tolédano, J-C. and P. Tolédano,P.: 1987, The Landau Theory of Phase Transitions, World Scientific, Singapore.