Dynamic data processing method based on generalized least square method

Zhaoxin Yang*, Zhenghua Gu, Wenqing Zhang

Facility Design and Instrumentation Institute China Aerodynamics Research and Development Center, Mianyang, Sichuan, 621000, China

*Corresponding author’s e-mail: yangzx@cardc.cn

Abstract. In this paper, a time domain-processing method is introduced for the dynamic calibration based on Hopkinson bar and shock tube. With a group of accelerometer and pressure sensor data processed in our work, the algorithm for establishing dynamic mathematical model is described in detail. "Generalized least square method for special whitening filter" and the simulation results are given. It shows that this method has the characteristics of simplicity and accuracy, and is especially suitable for the establishment of difference equation model in the data processing of dynamic calibration experiment.

1. Introduction

It is of great significance to study and improve the dynamic characteristics of the test system for the measurement effect of the dynamic signal with wide band and to ensure the accuracy of the dynamic test results. Currently, the commonly used time domain modeling methods include[1-3]: system identification method[4-5], Walsh transformation method[6-7], maximum entropy spectrum method, adaptive method and neuron method[8]. Among them, system identification modeling method is widely used. In contrast, the Walsh transformation method uses less data and the order of the model is low. It can directly obtain the coefficient of the differential equation and reduce the transformation error. However, it is not a recursive method and has strict requirements on the number of data[9]. The maximum entropy spectrum method is a time series analysis modeling method, which only needs the output data of the sensor dynamic calibration, but its modeling accuracy is not high. In long sequence modeling, adaptive method has obvious advantages over maximum entropy spectrum method. Although the neuron method has the advantages of low order and high accuracy, it is still far from practical application in the field[10].

In this paper, on the basis of studying various existing modeling methods, a sensor dynamic modeling method based on "generalized least square method of special whitened filter" is used. The method first uses "SIM algorithm which can simultaneously identify the order and parameters of linear difference equation model" to calculate the initial values of model parameters. The exact parameters of the model are calculated iteratively by the generalized least square method of a special whitening filter, and then the transient response is calculated recursively by the difference equation model. In this paper, the mathematical models of 4367 piezoelectric acceleration sensor and 6213 piezoelectric pressure sensor of Kistler company based on special whitened filter are established by means of Hopkinson bar and shock tube respectively. The established model has high accuracy and low order, and it is easy to realize the recursive algorithm, which lays a foundation for improving the dynamic characteristics of the sensor and realizing the dynamic compensation, to ensure the measurement accuracy within the allowable range.
2. System identification theory
The difference equation model of single input - single output linear system:

\[ A(d^{-1})y(k) = B(d^{-1})u(k) \]
\[ A(d^{-1}) = 1 + a_1d^{-1} + a_2d^{-2} + \ldots + a_n d^{-n} \]
\[ B(d^{-1}) = b_0 + b_1d^{-1} + b_2d^{-2} + \ldots + b_n d^{-n} \] (1)

where \( u(k) \) is the observation result of the system input, \( d^t \) is the post-shift operator.

The observation equation with noise:

\[ B(d^{-1})u(k) - A(d^{-1})z(k) = \varepsilon(k) \] (2)

where \( z(k) \) is the observation result of the system output, \( \varepsilon(k) \) is the residual error.

According to a given sequence of observations \( u(k), z(k) \), the order of the model \( \hat{n} \) and the model parameters of the least squares estimation can be acquired.

3. The procedure of dynamic calibration

3.1. Dynamic calibration of accelerometers

The impact device used in the experiment is shown in Figure 2, which is mainly composed of Hopkinson bar, digital oscilloscope and computer system.

![Figure 1. Hopkinson bar impact calibration system](image)

The projectile body with a certain taper at the end is pushed by (0.05-0.8MPa) compressed air and vertically impinges on the aluminum pad. The impact causes a right half sine compressive stress pulse generated in Hopkinson bar. The accelerometer is fixed on the end face of the mounting seat through bolts, and the axis direction of the mounting seat surface is affixed with a reflection grating, and the other end face is tightly attached to the end of the Hopkinson bar by a vacuum fixture. When the compressive stress pulse is reflected as a tension pulse at the free end of the mounting seat and the net tension appears at the interface between Hopkinson bar and the mounting sea, the mounting seat and the calibrated accelerometer will fly away, and the accelerometer will acquire accelerated motion.

3.1.1. Establish dynamic model of accelerometer

The dynamic calibration experiment of 4367 piezoelectric acceleration sensor from B&K Company is carried out, and the sampling frequency was 1MHz, and the experimental data were obtained. Curve 1 in Fig. 2 is the recorded curve of the experimental impact response, and curve 2 is the calculated curve of the model. From this group of experimental data, the difference equation (third-order model) can be established:

\[ G(s) = \frac{0.036s^3 + 2.124 \times 10^5 s^2 + 9.1902 \times 10^7 s + 2.337 \times 10^{17}}{1.533s^3 + 1.228 \times 10^5 s^2 + 1.174 \times 10^{12} + 2.795 \times 10^{17}} \] (3)
Figure 2. The shock response curve of the piezoelectric acceleration sensor

The experimental FFT frequency characteristic curve is shown in Fig. 3, and the characteristic curve calculated by the model is shown in Fig. 4.

Figure 3. Logarithmic frequency characteristic curve of piezoelectric accelerometer with FFT results

Figure 4. Logarithmic frequency characteristic curve of piezoelectric accelerometer with modelling results

3.1.2. Dynamic performance index

According to the frequency characteristic curve in Fig. 4, the dynamic performance indexes of the piezoelectric pressure sensor in the frequency domain can be obtained as follows: the operating band of the ±10% amplitude error is \( \omega_{g1} = 6.387 \text{kHz} \), the operating band of the ±5% amplitude error is \( \omega_{g2} = 4.391 \text{kHz} \), the resonant frequency is \( \omega_r = 23.15 \text{kHz} \), and the resonant peak is \( A_m = 7.457 \).
3.2. Dynamic calibration of pressure sensor

Figure 5. Shock tube dynamic pressure calibration system

At present, among all dynamic pressure calibration devices, shock tube is regarded as the best calibration device for pressure sensor. When shock tube is used for dynamic calibration of pressure sensor, the shock tube system generates a step pressure as a standard signal and adds it to the pressure sensor under test. The pressure sensor is calibrated by analyzing its output response, and the actual working performance of the pressure sensor is analyzed. The shock tube is here the electrical equivalent of a signal source, providing an ideal step signal. Figure 6 is a shock tube dynamic pressure calibration device, which generates a typical step pressure signal. In general, the response of the first pulse on the pressure sensor is used to calibrate and analyze the characteristics of the pressure sensor.

3.2.1. Establish the dynamic model of pressure sensor

The 6213 piezoelectric pressure sensor of Kistler Company was selected to conduct the shock tube experiment to obtain the experimental data with a sampling frequency of 1MHz. Curve 1 in Figure 6 is the recorded curve of the experimental impact response, and curve 2 is the model calculation curve. From this group of experimental data, the difference equation can be established:

\[
G(s) = \frac{-3.211s^3 + 6.008 \times 10^6s^2 + 2.043 \times 10^{12}s + 5.395 \times 10^{16}}{7.219s^3 + 6.690 \times 10^3s^2 + 1.761 \times 10^{12} + 5.384 \times 10^{16}}
\]  (4)
Figure 6. Step response of piezoelectric pressure sensor

The experimental FFT frequency characteristic curve is shown in figure. 7, and the characteristic curve calculated by the model is shown in figure. 8.

Figure 7. Logarithmic frequency characteristic curve of piezoelectric pressure sensor with FFT results

Figure 8. Logarithmic frequency characteristic curve of piezoelectric pressure sensor with modeling results

3.2.2. Dynamic performance index

According to the frequency characteristic curve in Figure. 8, the dynamic performance indexes of the piezoelectric pressure sensor in the frequency domain can be obtained as follows: the operating band of the ±10% amplitude error is $\omega_g=18.7$kHz, the operating band of the ±5% amplitude error is $\omega_g=16.3$kHz, the resonant frequency is $\omega_x=77.4$kHz, and the resonant peak is $A_m=42.51$. 
4. Conclusion

Based on the dynamic response of Hopkinson bar and shock tube calibration system to acceleration and pressure sensor, a dynamic model of sensor based test system is established in this paper. Through many simulation experiments, various structural factors affecting identification accuracy are analysed and some rules are summarized. This method is suitable for the case where the input interference is negligible and the output measurement noise is almost white noise. The conditions of dynamic calibration experiment are close to this situation. Generally, the excitation signal of dynamic calibration is better, and its interference can be ignored. The output signal of the calibrated system can be approximated as white noise. Therefore, this method is especially suitable for the establishment of difference equation model in the data processing of dynamic calibration experiment.

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