Spontaneous strain and magnetization in doped topological insulators with nematic and chiral superconductivity

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We show that a spontaneous strain and spontaneous magnetization can arise in the doped topological insulators with a two-component superconducting vector order parameter. The details of the effects crucially depend on the symmetry of the superconducting order parameter, whether it is nematic or chiral. The transition from the nematic state to the chiral one can be performed by application of a magnetic field while the transition from the chiral state to the nematic one is tuned by the external strain. These transitions associated with a jump of the magnetic susceptibility and mechanical stiffness. Possible experimental observations of the predicted effects are discussed.

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I. INTRODUCTION

Topological superconductivity in doped topological insulators, such as A2Bi3Se3, where A=Cu, Nb, Sr, possess unusual properties. In particular, cooper pairs in such material have a triplet pairing, and measurements of the magnetoresistance show a two-fold symmetry of the second critical field despite three-fold crystal symmetry. These properties are well-described in the framework of the theory of nematic superconductivity. The nematic superconductor has a two-component order parameter with \( E_u \) symmetry, which can be presented in the form of the real-valued vector. The nematic superconductivity could be accompanied by such intriguing properties as surface Andreev bound states, vestigial order, unconventional Higgs modes, and Majorana fermions.

An alternative superconducting state with \( E_u \) symmetry is a chiral phase. In this phase, the time-reversal symmetry is broken and the order parameter is a two-component complex-valued vector. It has been argued that the chiral phase can be realized in thin films of doped topological insulators. Recent experiments show fingerprints that can be attributed to the existence of the chiral superconductivity in doped topological insulators.

One of the distinct features of the nematic superconductivity is a non-trivial coupling with strain. This coupling leads to the two-fold symmetry of the second critical field. Really, in Refs. 4 and 5 this effect was observed. The X-ray studies reveal that in most samples initial strain about \( \delta l/l \sim 10^{-5} \) is presented at room temperature. However, in some samples, the deformation at room temperature is absent up to the experimental accuracy, while a two-fold symmetry of the second critical field remains. In Ref. 6 experiment on the magnetostriction reveals that the crystal lattice is deformed in the superconducting state. The strain decreases with an increase in temperature and vanishes in the normal state. These experiments demonstrate that either spontaneous or initial strain probably exists in the crystals of doped topological insulators in the superconducting state.

In Ref. 17 the existence of a non-zero magnetization in the superconducting state of Sr0.25Bi2Se3 has been found. The magnetization exists in the superconducting state and vanishes in the normal state. The origin of such magnetization is yet to be clarified. DFT calculations show that this magnetization can be attributed to the existence of the free spins in the intercalated Sr adatoms in the normal state.

In Refs. 12 and 7 the Ginzburg-Landau (GL) functional was derived for the superconducting state in the topological insulator with vector order parameter \( \vec{\eta} = (\eta_1, \eta_2) \). The nematic state corresponds to the real order parameter \( \vec{\eta} = \eta \cos \alpha, \sin \alpha \), while the chiral state is described by the solution with the complex vector \( \vec{\eta} = \eta (1, \pm i) \). This GL theory successfully describes macroscopic properties of the superconductivity in the doped topological insulators including in-plane anisotropy of \( H_{c2} \).

In general, multicomponent structure of the superconducting order parameter allows emergence of the subsidiary order parameters. In the case of \( D_{3d} \) point group symmetry of topological insulators, such as Bi2Se3, the subsidiary point groups have \( E_2 \) and \( A_2g \) symmetries. Corresponding bilinear forms are

\[
E_g \to (N_1, N_2) = (|\eta_1|^2 - |\eta_2|^2, \eta_1^* \eta_2 + \eta_1 \eta_2^*),
A_{2g} \to M_0 = \eta_1^* \eta_2 - \eta_1 \eta_2^*.
\]

As we can see, the nematic state corresponds to \( E_g \) subsidiary order with \( M_0 = 0 \), while the chiral state corresponds to \( A_{2g} \) bilinear with \( (N_1, N_2) = (0, 0) \). It was pointed out in Ref. 7 that the existence of the subsidiary order parameters gives rise to a non-trivial coupling of the superconductivity to the magnetization and strain. Corresponding contribution to the GL free energy has a structure \( 2iM_2 M_0 + (u_{xx} - u_{yy}) N_1 + 2u_{xy} N_2 \). Thus, the nematic state couples with the strain degrees of freedom, while chiral state couples with the magnetization. We demonstrate below that these symmetric properties of
the vector order parameter leads to emergence of a spontaneous deformation in the nematic state and a spontaneous magnetization in the chiral state.

In this paper, we show that either the spontaneous strain or the spontaneous transverse magnetization arises in the topological superconductor depending on the system parameters. The spontaneous deformation is observed in the case of nematic symmetry of the superconducting order parameter, \( \vec{\eta} = \eta(\cos \alpha, \sin \alpha) \), while the spontaneous magnetization exists in the case of the chiral solution, \( \vec{\eta} = \eta(1, \pm i) \). We also study the effects of initial strain and applied magnetic field on the order parameter symmetry. We show that the growth of the applied magnetic field gives rise to a transition of the nematic order to the chiral one, while with the growth of the initial strain, the chiral state transits to the nematic one. We found that the magnetic susceptibility and the stiffness experience jump at the transition points. We discuss the relation of the obtained theoretical results with recent experimental observations.

The paper is organized as follows. In Section II we outline the main features of the GL theory for topological superconductivity in the doped topological insulators. In Section III we calculate the spontaneous strain and magnetization, in Sections IV and V we analyze the effect of the applied magnetic field and strain on the symmetry of the superconducting order parameter. The relations of the obtained results with experiments are discussed in Section VI.

II. GL FREE ENERGY

We study here the superconducting state in the doped topological insulator assuming that the system is spatially uniform and the order parameter is independent of coordinates. This means, in particular, that we do not consider Abrikosov vortexes or other possible local current or magnetic structures. In this case, the GL free energy of the system can be expressed in the form:

\[
F_0 = A(|\eta_1|^2 + |\eta_2|^2) + B_1(|\eta_1|^2 + |\eta_2|^2)^2 + B_2|\eta_1^* \eta_2 - \eta_1 \eta_2^*|^2, 
\]

where GL coefficients \( A \propto T - T_c < 0 \), \( B_1 > 0 \), and \( B_2 \) either positive or negative. Minimization of this GL free energy predicts the existence of two different superconducting states depending on the sign of \( B_2 \):

\[
|\eta_1|^2 + |\eta_2|^2 = -\frac{A}{2B_1}, \quad \text{Im}(\eta_1, \eta_2) = 0, \quad F_0 = -\frac{A^2}{4B_1}, \quad B_2 > 0, \\
|\eta_1|^2 = \frac{-A}{4(B_1+B_2)}, \quad \eta_1 = i\eta_2, \quad F_0 = -\frac{A^2}{4(B_1+B_2)}, \quad B_2 < 0. 
\]

The state with \( B_2 > 0 \) is usually referred to as nematic. This state has a real order parameter \( \vec{\eta} = \eta(\cos \alpha, \sin \alpha) \). The state with \( B_2 < 0 \) is commonly called chiral. In the chiral state, the time-reversal symmetry is broken spontaneously, which is related to the complex order parameter \( \vec{\eta} = \eta(1, \pm i) \).

The total GL free energy is a sum

\[
F_{\text{GL}} = F_0 + F_u + F_M, 
\]

where \( F_0 \) is the bare superconducting part given by Eq. (3). \( F_u \) and \( F_M \) are the symmetry-breaking terms arising due to coupling of the superconducting order with the strain and the magnetization, respectively.

The term \( F_u \) for the system with \( D_{3d} \) group symmetry can be presented as:

\[
F_u = g_N(u_{xx} - u_{yy})(|\eta_1|^2 - |\eta_2|^2) + 2g_N u_{xy}(\eta_1^* \eta_2 + \eta_1 \eta_2^*) + \lambda_1 [(u_{xx} - u_{yy})^2 + 4u_{xy}^2] + \lambda_2 (u_{xx} + u_{yy})^2, 
\]

where \( u_{ik} \) are components of the strain tensor, \( u_{xx} \) and \( u_{yy} \) are uniaxial strains components, \( u_{xy} = u_{yx} \) is a shear strain, and \( g_N \) is a GL coupling constant between superconducting order and strain. Two last terms in Eq. (5) correspond to the self energy of the elastic deformation and \( \lambda_1, \lambda_2 > 0 \) are corresponding elastic modules.

The term with the transverse (z-axis) magnetization reads

\[
F_M = -2ig_M \lambda_z (\eta_1 \eta_2^* - \eta_1^* \eta_2) + aM_z^2, 
\]

where \( M_z \) is a Zeeman magnetization, \( g_M \) is a GL coupling constant between superconducting order and the magnetization, and an empirical coefficient \( a > 0 \). The last term in this equation accounts for the free energy loss due to magnetization in a non-magnetic phase.

III. SPONTANEOUS STRAIN AND MAGNETIZATION

Now, we have to minimize the GL free energy with respect to \( \vec{\eta}, u_{ik}, \) and \( M_z \). In this way, it is convenient to introduce following notations \( \eta_1 = \eta \cos \alpha \exp(i\varphi_1), \eta_2 = \eta \sin \alpha \exp(i\varphi_2), \) and \( \varphi = \varphi_1 - \varphi_2 \). In these terms we have

\[
F_0 = A\eta^2 + B_1\eta^4 + B_2\eta^4 \sin^2 2\alpha \sin^2 \varphi, 
\]

\[
F_M = 2g_M \lambda_z \eta^2 \sin 2\alpha \sin \varphi + aM_z^2, 
\]

We consider a uniform 2D strain in the system with hexagonal symmetry, which is characterized by three independent values \( u_{xx}, u_{yy}, \) and \( u_{xy} \). Following Ref. [19].
we use their linear combination as new independent variables. We can divide the strain tensor \( u_{ik} \) into two parts, a vector \( \vec{u} = (u_{xx} - u_{yy}, 2u_{xy}) = u \cos 2\beta \sin 2\beta \) and a scalar \( Sp \ u_{ik} = u_{xx} + u_{yy} \). In these notations Eq. (8) is rewritten as

\[
F_u = g_N \eta^2 u (\cos 2\alpha \cos 2\beta + \cos \varphi \sin 2\alpha \sin 2\beta) + \lambda_1 u^2 + \lambda_2 (u_{xx} + u_{yy})^2.
\]

(9)

To calculate the spontaneous strain and transverse magnetization, we should minimize the total free energy with respect to new independent degrees of freedom: the magnitude of the order parameter \( \eta \), the direction of the nematicity \( \alpha \), the phase difference between components of the order parameter \( \varphi \), the magnetization \( M_z \), the amplitude of the strain \( u \), the direction of \( \vec{u} \), that is, angle \( \beta \), and the trace of the deformation tensor \( Sp \ u_{ik} \). The minimization of the free energy by the latter variable is trivial: minimum of \( F_{GL} \) attains if \( Sp \ u_{ik} = 0 \) since this value does not couple with superconducting order parameter. Thus, \( u_{xx} = -u_{yy} \) and the spontaneous strain occurs without change of the sample volume.

The minimization of \( F_{GL} \) with respect to the magnetization means that \( \partial F_M / \partial M_z = 0 \) and from Eq (3) we derive

\[
M_z = -\frac{g_M \eta^2}{a} \sin 2\alpha \sin \varphi.
\]

(10)

The spontaneous magnetization, which arises after normal to superconductor transition, exists only if \( \sin 2\alpha \) and \( \sin \varphi \neq 0 \), that is, when both \( \eta_1 \) and \( \eta_2 \) are non-zero and the order parameter is complex.

Minimization by other degrees of freedom and solution of the obtained system of equation is straightforward but cumbersome. We present only final results. We obtain that the GL free energy has two possible ground states, nematic and chiral, as in the case when we neglect a part of the GL free energy as 

\[
\partial F_{GL} / \partial M_z = 0
\]

and \( \eta \) is non-zero and \( \eta \) is also non-zero. In so doing, we derive

\[
M_z = -\frac{g_M \eta^2}{a^2} \sin 2\alpha \sin \varphi.
\]

(11)

\[
\eta^2 = \frac{-A}{2(B_1 - g_N^2/4\lambda_1)} \text{,} \quad F_{nem} = \frac{-A^2}{4(B_1 - g_N^2/4\lambda_1)}.
\]

The condition \( B_1 - g_N^2/4\lambda_1 > 0 \) is necessary for the stability of the nematic state. In the case of nematic state the spontaneous magnetization is zero, while the strain is non-zero and \( u = u_{sp}(-1)^{n+1} \), where

\[
u_{sp} = \frac{g_M |A|}{4B_1 \lambda_1 - g_N^2}, \quad \beta = \alpha + \frac{\pi n}{2}, \quad M_z = 0,
\]

(12)
and \( n \) is integer. The deformation vector is parallel to the nematicity direction if \( g_N > 0 \)

\[
\vec{u} = (u_{xx} - u_{yy}, 2u_{xy}) = |u|(\cos 2\alpha, \sin 2\alpha)
\]

(13)
and anti-parallel if \( g_N < 0 \). The nematic phase is infinitely degenerate with respect to the angle \( \alpha \), and the ground state free energy is the same for any direction of the nematic order parameter \( \eta = \eta(\cos \alpha, \sin \alpha) \).

spontaneous deformation, \( u_{sp} \propto A \propto (T_c - T) \), decreases with the increase of temperature and vanishes at \( T = T_c \).

In the chiral state \( \cos \varphi = 0 \) and \( \alpha = \pi/4 + \pi l/2 \), where \( l \) is zero or integer. In so doing, we derive

\[
\eta^2 = -\frac{A}{2(B_1 + B_2 - g_M^2/a)}, \quad F_{ch} = \frac{-A^2}{4(B_1 + B_2 - g_M^2/a)}.
\]

(14)

The system is stable if \( B_1 + B_2 - g_M^2/a > 0 \). In the case of chiral phase, the spontaneous strain is zero, while the spontaneous magnetization is non-zero and \( M_z = M_{sp}(-1)^{l+1} \), where

\[
M_{sp} = \frac{g_M |A|}{2(B_1 + B_2 - g_M^2/a)}, \quad u = 0.
\]

(15)

The chiral state is degenerate with respect to the sign of chirality, \( \eta = \eta(1, \pm l) \), and sign of the magnetization \( M_z \).

The spontaneous magnetization, \( M_z \propto A \propto (T_c - T) \), decreases with the increase of temperature and vanishes at \( T = T_c \).

We compare the free energy in the nematic state, Eq. (11), and in the chiral state, Eq. (14), and conclude that the nematic phase is the ground state, \( F_{nem} < F_{ch} \), if

\[
B_2 + \frac{g_N^2}{4A_1} - g_M^2/a > 0.
\]

(16)

Otherwise, the superconductor is in the chiral phase.

As we can see from Eq. (16), coupling the superconductivity with the strain shifts the system toward the nematic state, while the coupling with the magnetization drives the system to the chiral state. Below, we show how we can switch off the system from one phase to another by application of a magnetic field or an external strain.

IV. EFFECT OF THE APPLIED MAGNETIC FIELD

Here we assume that a uniform transverse magnetic field \( H = (0, 0, H) \) exists in the sample volume. Such a situation could be realized in definite cases. For example, if a corresponding size of the sample smaller than the London penetration depth. In this case, the field \( H \) is simply an external magnetic field. The treatment below is valid if this field is much smaller than the upper critical magnetic field \( H_{c2} \) when we can neglect the Landau quantization and, consequently, disregard spatial modulations of the order parameter. In other words, we consider external Zeeman magnetization. We choose \( z \)-axis directed along the applied field and, hence, \( H > 0 \). In this section, we neglect a possible existence of the spontaneous strain since it does not affect the main result but makes the calculations much cumbersome.

In the case under study we rewrite Eq. (3) for magnetic part of the GL free energy as

\[
F_M = 2g_M M_z \eta^2 \sin 2\alpha \sin \varphi + a M_z^2 - HM_z.
\]

(17)
The minimization of $F_M$ with respect to $M_z$ gives
$$M_z = \frac{1}{2a} \left( H - 2g_M \eta^2 \sin 2\alpha \sin \varphi \right).$$ (18)

After minimization of $F_{GL}$ by $\eta^2$ we obtain
$$\eta^2 = -\frac{A + g_M H \sin 2\alpha \sin \varphi / a}{2 [B_1 + (B_2 - g_M^2 / a)^2] \sin 2\alpha \sin \varphi^2}. \quad \text{(19)}$$

We substitute expressions for $\eta^2$ and $M_z$ in Eqs. (16) and (17) and derive for the total GL free energy
$$F_{GL}(t) = -\frac{H^2}{4a} - \frac{(A + g_M H / a)^2}{4 [B_1 + (B_2 - g_M^2 / a)^2]}.$$ (20)

where $t = \sin 2\alpha \sin \varphi$ and $|t| \leq 1$. Edge values $t = \pm 1$ correspond to the chiral phase $\varphi = \pm \pi / 2$, $\alpha = \pm \pi / 4$, and the first minimum of the free energy is attained if $t = -1$. As a result, we have that in the chiral phase:
$$F_{ch}(H) = -\frac{H^2}{4a} - \frac{(A - g_M H / a)^2}{4 [B_1 + (B_2 - g_M^2 / a)^2]}, \quad \text{at} \quad t = 0.$$ (21)

and magnetization
$$M_z = \frac{H}{2a} \left[ 1 + \frac{g_M^2}{a (B_1 + B_2 - g_M^2 / a)} \right] + M_{sp}. \quad \text{(22)}$$

Here the first term is an induced magnetization and the second term is the spontaneous magnetization as in Eq. (13) for chiral phase at $H = 0$. We find the second minimum of the free energy from the condition $\partial F_{GL}(t) / \partial t = 0$ at
$$t = \frac{g_M H B_1}{a A (B_2 - g_M^2 / a)}. \quad \text{(23)}$$

The latter solution exists only if $t < 1$ or $H < |A| ((a B_2 - g_M^2) / g_M B_1)$. This minimum corresponds to the nematic state and $\sin \varphi = 0$ if $H = 0$. In the nematic state, the order parameter is independent of $H$ and only induced magnetization is observed:
$$\eta^2 = -\frac{A}{2B_1}, \quad M_z = \frac{H}{2a} \left( 1 + \frac{g_M^2}{a (B_2 - g_M^2)} \right). \quad \text{(24)}$$

The GL energy in the nematic phase is
$$F_{nem}(H) = -\frac{H^2}{4a} \left( 1 + \frac{g_M^2}{a (B_2 - g_M^2)} \right) - \frac{A^2}{4B_1}. \quad \text{(25)}$$

If we assume that inequality (16) is fulfilled, then, the nematic phase is the ground state at $H = 0$. The applied magnetic field induces a non-zero phase difference $\sin \varphi = t / \sin 2\alpha$ between the components of the order parameter and drives the nematic state to the chiral one. Using Eq. (23) and comparing the GL free energies in Eqs. (21) and (25), we conclude that the value of $|t|$ increases with $H$ and attains its maximum $|t| = 1$ at which $F_{nem}(H) = F_{ch}(H)$ when
$$H = H^* = |A| ((a B_2 - g_M^2) / g_M B_1). \quad \text{(26)}$$

With further increase of the magnetic field, the nematic state disappears, the superconducting state becomes chiral. Correspondingly, a spontaneous magnetization arises at $H > H^*$. However, at this point the magnetization $M_Z(H)$ is continuous, while the magnetic susceptibility, $\chi = \partial M_z / \partial H$ exhibits a jump:
$$\chi_{nem} = \frac{1}{2a} \left( 1 + \frac{g_M^2}{a B_2 - g_M^2} \right), \quad H < H^*, \quad \text{(27)}$$
$$\chi_{ch} = \frac{1}{2a} \left[ 1 + \frac{g_M^2}{a (B_1 + B_2 - g_M^2 / a)} \right], \quad H > H^*.$$ (28)

Thus, the transition from the nematic to chiral phase at $H = H^*$ is a typical type-II phase transition.

V. SYSTEM WITH INITIAL STRAIN

In this section we assume that some initial strain $\bar{u}_0 = u_0 (\cos 2\beta_0, \sin 2\beta_0)$ exist in the system. According to X-ray measurements, this strain arises in the process of the crystal growth and had a characteristic value of about $u_0 \approx 10^{-5}$ Ref. 12 and 13. This strain is two orders of magnitude larger than that observed due to normal to superconductor transition in Ref. 16. Thus, it is reasonable to neglect here the spontaneous deformation.

As above, we have to minimize the total GL free energy $F_{GL} = F_0 + F_M + F_{nem}(u_0)$, see Eqs. (1), (5), and (9). We can neglect the terms proportional to $u_0^2$ in $F_0$ since they are constant and, for definiteness, we assume that $g_N u_0 > 0$. From the minimization condition $\partial F / \partial \eta = 0$, we obtain
$$\eta^2 = \frac{A + g_N u_0 (\cos 2\alpha \cos 2\beta_0 + \cos \varphi \sin 2\alpha \sin 2\beta_0)}{2 [B_1 + (B_2 - g_M^2 / \gamma_M) \sin^2 2\alpha \sin^2 \varphi]}, \quad \text{at} \quad u_0 |t| = 0.$$ (29)

The free energy $F$ has two minima. The first of them corresponds to the nematic state, $\sin \varphi = 0$. In this state the spontaneous magnetization is absent and the free energy is
$$F_{nem}(u_0) = -\frac{[A + g_N u_0 \cos 2(\alpha - \beta_0)]^2}{4B_1}. \quad \text{(30)}$$

In the ground state we have $\alpha = \beta_0 + \pi (n + 1 / 2)$ and $F_{nem} = -(|A| - g_N u_0)^2 / 4B_1$. The second minimum of the free energy corresponds to the chiral state. In this state we get that
$$\cos 2\alpha = -\frac{u_0 \cos 2\beta_0}{u^*}, \quad \cos \varphi = -\frac{u_0 \sin 2\beta_0}{u^* \sin 2\alpha}, \quad u^* = \frac{|A| (g_M^2 - B_2 a)}{g_N (B_1 + B_2 a - g_M^2 / a)}. \quad \text{(29)}$$

The order parameter in the chiral state is the same as in the case $u_0 = 0$, Eq. (14). For the free energy and
In the case of a pure shear strain, \( \beta \) is fulfilled. However, changing the strain \( u_0 \) from \( u_0 > u^* \) to \( u_0 < u^* \) we get \( \alpha = \pi/4 + \pi(n + 1/2) \) and \( \eta = \eta(1, \pm 1) / \sqrt{2} \). If the condition in Eq. (10) is violated, then the ground state of the system at \( u_0 = 0 \) is chiral with non-zero spontaneous magnetization. The applied strain drives the chiral phase to the nematic one. The comparison of the free energy in Eqs. (28) and (30) reveals that the transition vanishes in the normal state. Such a spontaneous deformation has been observed in the measurements of the lattice strain \( \delta l/l \sim 7 \times 10^{-7} \) reported in Ref. 16. In some samples, no deformation has been found up to the experimental accuracy. However, in all samples a two-fold in-plane anisotropy of the upper critical field \( H_{c2} \) was observed. These experiments indicate that both cases, either dominant of the initial or spontaneous strain, are possible. Initial deformation \( u_0 \) increases the critical temperature \( T_c = T_{c0} \propto n \sqrt{u/A_0} \), where \( T_{c0} \) is the critical temperature for the superconducting state. This deformation vanishes in the normal state. Such a spontaneous deformation has been observed in the measurements of the magnetostriiction \( \delta l/l = u \sim 10^{-7} \) arises in the superconducting state, decreases when the system is driven to the superconducting to normal metal transition and vanishes in the normal state, which confirms our prediction.

We argue that a transverse spontaneous magnetization occurs in the chiral state. This magnetization is tied to the superconductivity and vanishes in the normal state. The finite magnetization in the doped topological insulator \( \text{Nb}_x\text{Bi}_2\text{Se}_3 \) have been measured in Ref. 17. This magnetization decreases with an increase in temperature and vanishes in the normal state, which is in agreement with our results.

We also consider the effects of the applied magnetic field and the initial strain (produced by some external impact). We show that the application of the magnetic field drives the initially nematic superconductor to the chiral state, while the growth of the initial strain derives the chiral superconductor to the nematic state. When the applied field (initial strain) exceeds some threshold value the nematic (chiral) state is changed by the chiral (nematic) phase.

In Refs. 4 and 5 the strain has been measured by the X-ray technique. In most samples, it has been found the initial deformation \( u_0 \sim 10^{-5} \) at room temperature, which is much larger than the spontaneous deformation \( u \sim 10^{-7} \) reported in Ref. 16. In some samples, no deformation has been found up to the experimental accuracy. However, in all samples a two-fold in-plane anisotropy of the upper critical field \( H_{c2} \) was observed. These experiments indicate that both cases, either dominant of the initial or spontaneous strain, are possible. Initial deformation \( u_0 \) increases the critical temperature \( T_c = T_{c0} \propto n \sqrt{u/A_0} \), where \( T_{c0} \) is the critical temperature without strain, \( A = A_0(T_{c0} - T) \). The spontaneous strain, \( u \propto A_0(T_{c0} - T) \), vanishes at \( T = T_{c0} \) and has no effect on the critical temperature.

VI. DISCUSSION

In the framework of the GL approach, we analyzed the symmetry breaking phenomena in the topological superconductors. We predict that in the nematic state a spontaneous strain of the crystal lattice occurs due to a non-trivial coupling of the vector of superconducting order parameter and the strain degrees of freedom. This deformation vanishes in the normal state. Such a spontaneous deformation has been observed in the measurements of the magnetostriiction \( \delta l/l = u \sim 10^{-7} \)

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