New ideas for ring formation and reconstruction in water-Cherenkov detectors

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Abstract. Water-Cherenkov detector technology is well established as is easily seen in the almost identical designs of Kamiokande, Super- and Hyper-Kamiokande. We explore methods to further improve upon their success by extracting directional information from originally lost photons using reflectors placed in the inactive space between PMTs. The reflected photons form a secondary antipodal ring, sensitive to the vertex position and track topology. We further show how ring reconstruction algorithms can benefit from modeling variations of the Cherenkov profile due to showering. Monte Carlo based quantitative estimates are accompanied by discussions of strategies to overcome potential issues with the presented methods. A new CNN-architecture to efficiently implement the cylindrical topology and geometry is also presented.

Water-Cherenkov detectors are composed of a huge volume of water surrounded by photomultiplier tubes (PMTs), which record the hit position and timing of photons emitted from relativistic particles inside the tank. We present three ideas for improved reconstruction, one requiring hardware change, and two purely based on software.

1. Retro-reflectors
The area between PMTs is commonly covered with light-absorbing materials. Replacing these with reflective materials, some originally lost photons can be detected by PMTs on the other side of the tank, and we gain sensitivity to photon direction through the long leverage arm of reflections. Retro-reflectors are especially interesting for a few reasons: the reflection creates an antipodal ring which is easy to fit, one has no issues with reflector alignment, and finally the retro-reflectors trap multi-reflections, such that they do not cause further PMT hits. Here we report on a more realistic performance evaluation compared to our original proposal [1].

Event reconstruction was implemented in fitQun [2], a likelihood-based fitter used by SuperK and T2K experiments. The retro-reflected light is easily modeled by changing \( \cos \theta \rightarrow -\cos \theta \) when evaluating the Cherenkov profile. A simple performance evaluation\(^1\) shows that by adding retro-reflectors, transverse position can be reconstructed without the use of timing information (Fig. 1). Even with timing information, retro-reflectors reduce the error by about 25% (Fig. 2).

Challenges
Corner cubes have good retro-reflection accuracy, but at angles larger than \( \sim 30 \text{deg} \) the reflection is primarily not retro-reflective, as the photons miss one or two of the three reflection planes. Such undesired reflections might be removable using blinds to cut higher angle photons, at the loss of acceptance for retro-reflection as well. For commercial retro-reflector tapes, we benefit from the cheap price, but have to fight specular reflection.

\(^1\) Reconstruction of 500 MeV side-going electron generated at center of SuperK tank in Geant4 [3] based WCSim [4], no photon scattering or reflections. The same dataset is used for all evaluations in this work.
A possible alternative is to put quasi-retroreflectors (just two mirror planes at 90 deg) on barrel walls, and ordinary mirrors on the top and bottom faces of the tank. We will not be able to avoid multiple reflection, but since the reflections are confined inside an upright plane, it might still be manageable. Non-retro reflection can be cut by blinds to remove in- and outgoing angles > 45 deg in xy-plane, without sacrificing too much acceptance.

2. Cherenkov profile variations
For PMTs near the Cherenkov peak, variations in the Cherenkov profile (due to EM shower etc.) can cause correlations of PMT charges. Conventional fitters such as fiTQun\(^2\) or APfit \([5]\) fit events using an average Cherenkov profile, and do not account for these. We tried to include these correlations in the reconstruction model and studied the impact.

In a simple model, we integrate out the longitudinal s-dependence, Fourier-expand in the azimuthal angle \(\phi\) as \(f(\theta, \phi) = \sum_m f_m(\theta) e^{i m \phi}\), and bin by opening angle \(\cos \theta\). Rotational invariance demands \(\langle f_m \rangle = 0\) unless \(m = 0\), and \(\langle f_m f_{m'}^* \rangle = 0\) unless \(m = m'\), corresponding to a block-diagonal structure in the covariance matrix (Fig. 3). Variations are of order 30% compared to the mean flux (Fig. 4). A basic performance evaluation suggests a 10% reduction of longitudinal position error by fitting monopole and dipole variations (Fig. 5).

Further improvements can be expected from including the longitudinal s-dependence of variations. Extending the covariance matrix with \(s^n\)-weighted integrals, we see electrons have longitudinal variations strongly correlated in \(\cos \theta\) due to showering, suggesting additional power for particle identification (PID). Another improvement is expected from the inclusion of correlations between multipoles. These can be studied by dividing Fourier coefficients into magnitude and argument \(f_m = |f_m| e^{-i \alpha_m}\), where we see clear correlations between both the magnitudes and angles which were not captured in the simple model studied so far.

**Challenges** Some data-driven validation or correction would be necessary not to become

\(^2\) For muons fiTQun has multi-segment fits to scattering, here we focus on electrons instead. All studies use WCSim as MC, reconstruction with modified fiTQun.

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**Figure 1.** Fit without timing information (charge only fit).

**Figure 2.** Fit with timing information.

**Figure 3.** Covariance matrix of Cherenkov profile parameters \(f_m\).

**Figure 4.** Size of different multipole variations compared to mean photon flux.

**Figure 5.** Reconstructed longitudinal vertex error when fitting variations.
dependent on the MC model. Natural calibration sources are likely insufficient for this purpose. For PID the additional latent space dimensionality introduced by variation fits might improve separation in performing unsupervised classification using high statistics atmospheric data.

3. Topology and geometry for CNN

One alternative for exploiting such profile correlations is using convolutional neural networks (CNN), which comes with two challenges: First, the correct boundary condition that captures detector topology of 2-sphere. Second, how to encode geometrical information — being in the pixel-plane next to each other has a completely different meaning for barrel and endcaps.

Topology We define an isomorphic map to a square ($X_+$, $X_-$) by cutting along open meridian:

$$X_\pm = \begin{cases} 1 - \chi_\pm & (z \geq 0) \\ \chi_\pm & (z < 0) \end{cases} \chi_\pm = W(\rho, z) \frac{\pi \phi}{2\pi}$$

$$W(\rho, z) = \sqrt{\frac{\rho^2 - 2R|z| + RH}{R^2 + RH}}$$

(3.1)

where $R$ and $H$ correspond to the radius and height of the tank, and $W(\rho, z)$ is chosen for constant surface density. The PMTs are then put on a square grid which can be fed into machine learning libraries. Prior to convolution, we pad sides identifying neighboring sides to embody the topology of 2-sphere.

Geometry Ordinary convolution uses integration with a kernel $K$ that only depends on the relative grid position, exhibiting translational invariance: $N_i^{(n+1)} = \sum_j K_{i-j} N_j^{(n)}$. For typical $3 \times 3$ convolution, this operation is essentially a weighted sum of moving average, 1st, and 2nd order discrete derivatives. Analogously, we propose a map invariant under spatial translations:

$$N^{(n+1)}(x) = (K + K^I \partial_I + \frac{1}{2} K^{IJ} \partial_I \partial_J)N^{(n)}(x)$$

(3.2)

where the Taylor expansion coefficients $K$, $K^I$, and $K^{IJ}$ (scalar, vector, tensor) are learned ($I, J$: spatial indices). This can be implemented as convolution on the 2D grid

$$N_i^{(n+1)} = \sum_j (Kw_{ij} + K^I w_{I,ij} + \frac{1}{2} K^{IJ} w_{IJ,ij})N_j^{(n)}$$

(3.3)

using precomputed weight matrices $w$, $w_I$, and $w_{IJ}$, defined to obtain spatial derivatives:

$$f_i = \sum_j w_{ij} f_j, \quad \partial_I f_i = \sum_j w_{I,ij} f_j, \quad \partial_I \partial_J f_i = \sum_j w_{IJ,ij} f_j, \quad \text{with} \quad f_i := f(x_i)$$

(3.4)

The sum over $j$ is over nearby sites only, possible because the topological map preserves locality. As such the whole calculation can be efficiently realized on GPUs by tensor operations.

Further notes Inserting batch-norm layers after the weighted sum helps to normalize scales introduced by the dimensionality of derivatives. Since $K$, $K^I$, and $K^{IJ}$ are geometric objects, they can be randomly rotated during the training to impose the $\phi$-symmetry of the tank. Dead PMTs could be removed by simply recomputing weights $w$ without much retraining, since training is in 3D-space and not in pixel-space. PMT orientation could be passed as another “spatial” dimension, which should be useful especially for multi-PMTs.

Conclusions

We presented three alternative approaches for event reconstruction in water-Cherenkov detectors. Retro-reflectors placed between PMTs for transverse position resolution, fits of Cherenkov profile variations for small improvements to longitudinal position resolution, and a new CNN architecture based on learning of Taylor expansion coefficients in 3D space. Further studies and development are ongoing. We would like to thank Yoshida T (TokyoTech), Vilela C (SBU), Wilking M (SBU), and Konaka A (TRIUMF) for very helpful discussions. This work was supported by JSPS KAKENHI Grant Number JP19J22440.

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