Stochastic Field evolution of disoriented chiral condensates

Luís M. A. Bettencourt

aCenter for Theoretical Physics, Massachusetts Institute of Technology,
77 Massachusetts Avenue, Cambridge MA 02139, USA

I present a summary of recent work [1] where we describe the time-evolution of a region of disoriented chiral condensate via Langevin field equations for the linear \( \sigma \) model. We analyze the model in equilibrium, paying attention to subtracting ultraviolet divergent classical terms and replacing them by their finite quantum counterparts. We use results from lattice gauge theory and chiral perturbation theory to fix nonuniversal constants. The result is a ultraviolet cutoff independent theory that reproduces quantitatively the expected equilibrium behavior of pion and \( \sigma \) quantum fields. We also estimate the viscosity \( \eta(T) \), which controls the dynamical timescale in the Langevin equation, so that the near equilibrium dynamical response agrees with theoretical expectations.

The relativistic heavy ion collider (RHIC) is currently smashing nuclei together to produce the most extreme states of nuclear matter ever achieved in a controlled environment. Their cooling dynamics may then reveal collective properties of hot nuclear matter \textit{viz.} its equation of state and the nature of the confining and chiral symmetry breaking transitions.

Extracting this information requires dynamical extrapolation from the final products that hit particle detectors, across possible critical points, up to the initial hot phase. This is practically impossible in general but the theory of critical dynamics, a byproduct of the renormalization group, tells us how it may be achievable by modeling the dynamics of light degrees of freedom as stochastic (classical) fields. In nuclear Physics these are the pion and \( \sigma \) fields and the associated equations of motion are stochastic versions of the Euler-Lagrange equations for the linear \( \sigma \)-model.

The simplest Langevin prescription is to add stochastic sources \( \xi_a(x,t) \) and dissipation terms to the classical field equations for \( \phi_a = (\sigma, \vec{\pi}) \)

\[
\frac{\partial^2 \phi_a}{\partial t^2} - \nabla^2 \phi_a + \lambda \left( \phi_a^2 - \nu^2 \right) \phi_a - H \delta_{a0} = -\eta_{ab} \frac{\partial \phi_b}{\partial t} + \xi_a .
\]

The stochastic fields \( \xi_a \) are taken to obey the fluctuation-dissipation relations

\[
\langle \xi_a(x)\xi_b(y) \rangle = 2\eta_{ab}(x)T \delta^4(x-y) ,
\]

with \( \langle \xi_a(x) \rangle = 0 \). Here, we shall always choose \( \eta_{ab}(x) = \eta(T)\delta_{ab} \). This dynamics reduces to the classical microcanonical relativistic evolution for the pions and sigma as \( \eta \to 0 \).

In the limit of vanishing masses the long wavelength modes of the fields are effectively overdamped, and suffer critical slowing down [2]. Then our equations correspond to Model A, in the classification of dynamical stochastic theories of Hohenberg and Halperin [3]. Although Model A will be adequate for our purposes, a complete analysis of the universal \( O(4) \) dynamics in the chiral limit of QCD requires coupling the order parameter (which is not conserved) to other conserved quantities in the theory, as in Hohenberg and Halperin’s Model G [4].

The long time Langevin dynamics takes the system to its (classical) canonical ensemble, at temperature \( T \). This results in divergences of certain field expectation values as the lattice spacing \( a_s \) is taken to zero. To make physical sense of the model in this limit requires a \( T \) dependent renormalization scheme. This is achieved by making the replacement in the bare Eq. (1),

\[
-\lambda \nu^2 \to -\lambda \nu^2 + \Delta I_{\text{hadpole}} + \Delta I_{\text{sunset}},
\]
where
\[
\Delta I_{\text{tadpole}} \equiv I^{B-E}_{\text{tadpole}} - I^{cl}_{\text{tadpole}},
\]
\[
I^{cl}_{\text{tadpole}} = \frac{N + 2}{2\pi^2} \lambda T \Lambda \to 0.25 \frac{(N + 2)\lambda T}{a_s},
\]
\[
I^{B-E}_{\text{tadpole}} = \frac{N + 2}{12} \frac{\lambda T^2}{h},
\]
\[
\Delta I_{\text{sunet}} \to \left[0.014 \ln\left(\frac{T a_s}{h}\right) - 0.037\right] (N + 2)\lambda^2 T^2.
\]
This removes linear and logarithmic divergences in the self energy of pion and \(\sigma\) fields.

![Figure 1. The order parameter \(\langle \phi \rangle\) vs. temperature \(T\), for \(H = 0\) (circles) and \(H\) nonzero (triangles). Open and filled symbols show results obtained with \(a_s = 1\) on a \(26^3\) lattice and \(a_s = 0.4\) on a \(64^3\) lattice respectively, with counterterms as described in the text. The line shows the prediction from chiral perturbation theory, Eq. (4).](image)

In Fig. 1, we required that the order parameter \(\langle \phi \rangle\) agrees with the expectations from chiral perturbation theory at low temperature (4).

\[
\langle \phi \rangle = f_\pi \left(1 - \frac{T^2}{12f_\pi^2}\right), \quad T \ll f_\pi .
\] (4)

In particular, we have enforced the absence of linear \(T\)-dependence at small \(T\) by the addition of a further mass counterterm. This counterterm is not divergent as \(a_s \to 0\), but it does vary with \(a_s\). We find that the linear \(T\)-dependence at small \(T\) is removed by \(-\lambda v^2 \to -\lambda v^2 + b_1 T\), with \(b_1 = 0.425\) in dimensionless units. Next, we fixed the finite counterterm \(\propto T^2\). We note that in the absence of any finite counterterm proportional to \(T^2\) the second order phase transition (for \(H = 0\)) occurs at \(T_c \simeq 130\) MeV, which is somewhat lower than that expected in QCD (4).

We have pushed \(T_c\) up to \(T_c \simeq 150\) MeV by introducing \(-\lambda v^2 \to -\lambda v^2 + b_1 T + b_2 T^2\), with \(b_2 = -0.066\) in dimensionless units. These values of \(b_1\) and \(b_2\) were obtained with a lattice spacing \(a_s = 0.4\) on a \(64^3\) lattice, as shown by the filled symbols in Fig. 1. On a \(26^3\) lattice with \(a_s = 1\) (with the same physical volume) we find that with \(b_2 = 0.084\) and \(b_1\) unchanged from above, the order parameter as a function of \(T\) is the same as that for \(a_s = 0.4\) within error bars (open symbols in Fig. 1). We have also verified that the \(a_s\)-dependence of \(b_2\) vanishes for small \(\lambda\) and is non-divergent in the \(a_s \to 0\) limit.

Next we measured the \(T\) dependence of the masses and decay rates of the pion and \(\sigma\) fields. These quantities follow from the dynamical response to small amplitude perturbations around equilibrium. Then the response takes the form
\[
\langle \phi_a \rangle = \langle \phi_a (t = 0) \rangle \exp[-t/\tau] \cos(\omega t),
\] (5)
with \(\omega = \sqrt{m^2(T) - \tau^{-2}}\), for the particular case of a spatially homogeneous perturbation. Results are shown in Fig. 2 together with the chiral perturbation theory prediction for the pion mass (4)

\[
m^2_\pi(T) = m^2_\pi(0) \left(1 + \frac{T^2}{24f_\pi^2}\right), \quad T \ll f_\pi .
\] (6)

The \(\sigma\) becomes lighter in medium as a result of symmetry restoration at high \(T\). \(m_\sigma\) has a minimum at \(T_{\text{cross}} \simeq 180\) MeV, which defines the crossover temperature. At \(T_{\text{cross}}\), \(m_\sigma \simeq 220\) MeV and \(m_\sigma \simeq 280\) MeV. The two masses are equal within error bars for \(T \gtrsim 220\) MeV.

The timescale \(\tau(T)\) for the decay of a region of disoriented chiral condensate in contact with an equilibrated gas of hadrons has been computed previously by Steele and Koch (8), using a hadron gas model and perturbatively (at 2-loops) by Rischke (8). We seek to choose \(\eta(T)\) so that the \(\tau(T)\) we measure reproduces their calculations.

A difficulty is that the classical thermal field theory already leads to long wave length decay.
The thermal masses of the $\sigma$ and $\pi$ fields vs. $T$. Error bars denote statistical and best fit uncertainties. The line shows the prediction from chiral perturbation theory, Eq. (6). The minimum of $m_\sigma$ is reached at $T_{\text{cross}} \approx 180$ MeV. Calculations were done with $a_s = 0.4$ on a $64^3$ lattice.

The associated time scale is much shorter (larger dissipation) than the target quantum $\tau$ at low $T$, because of the absence of Bose-Einstein suppression of short wave lengths in the classical case.

Because $\eta \geq 0$ in the Langevin equation (1) we find that our model can only be used to describe the long wavelength dynamics in the presence of a heat bath with $T \gtrsim 145$ MeV, corresponding to about 80% of $T_{\text{cross}}$. Above this temperature we can adjust our input bare $\eta$ to reproduce the DCC decay time computed by Steele and Koch, see Fig. 3. However, $T \sim 145$ MeV is large enough that the assumptions in the calculation of $\tau$ as in Ref. [5] may be starting to break down, which means that our estimate of the limit of validity of our analysis may have a little play in it. The same considerations apply to the perturbative 2-loop calculation, with the additional caveat that at $\lambda = 20$ it may be unreliable.

In spite of these difficulties we are currently studying the non-equilibrium dynamics of the system induced by the combined effect of the cooling of the thermal bath and by volume expansion. The latter becomes the principal means of cooling at low $T$, which may help mitigate the problems associated with the choice of $\eta$.

REFERENCES

1. L. M. Bettencourt, K. Rajagopal and J. V. Steele, Nucl. Phys. A 693, 825 (2001).
2. L. M. A. Bettencourt, Phys. Rev. D 63, 045020 (2001).
3. P.C. Hohenberg and B.I. Halperin, Rev. Mod. Phys. 49, 435 (1977).
4. K. Rajagopal and F. Wilczek, Nucl. Phys. B 399, 395 (1993).
5. S. Gottlieb et al., Phys. Rev. D 55, 6852 (1997).
6. See F. Karsch, Nucl. Phys. Proc. Suppl. 83, 14 (2000). E. Laermann, Nucl. Phys. Proc. Suppl. 63, 114 (1998); and A. Ukawa, Nucl. Phys. Proc. Suppl. 53, 106 (1997).
7. P. Gerber and H. Leutwyler, Nucl. Phys. B 321, 387 (1989); R. D. Pisarski and M. Tytgat, Phys. Rev. D 54, 2989 (1996).
8. J.V. Steele, and V. Koch, Phys. Rev. Lett. 81, 4096 (1998).
9. D. H. Rischke, Phys. Rev. C 58, 2331 (1998).