On Brane Inflation Potentials and Black Hole Attractors

Adil Belhaj\textsuperscript{1,*}, Pablo Diaz\textsuperscript{1†}, Mohamed Naciri\textsuperscript{2,3‡}, Antonio Segui\textsuperscript{1§}

1. Departamento de Fisica Teorica, Universidad de Zaragoza, 50009-Zaragoza, Spain.
2. Lab/UFR-Physique des Hautes Energies, Faculté des Sciences (FS), Rabat, Morocco.
3. Groupement National de Physique des Hautes Energies, Siège focal: FS Rabat, Morocco.

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Abstract

We propose a new potential in brane inflation theory, which is given by the arctangent of the square of the scalar field. Then we perform an explicit computation for inflationary quantities. This potential has many nice features. In the small field approximation, it reproduces the chaotic and MSSM potentials. It allows one, in the large field approximation, to implement the attractor mechanism for bulk black holes where the geometry on the brane is de Sitter. In particular, we show, up to some assumptions, that the Friedman equation can be reinterpreted as a Schwarzschild black hole attractor equation for its mass parameter.

Key words: Inflation, Braneworld model, Attractor mechanism, Black hole.
1 Introduction

Recently, an increasing interest has been devoted to the study of inflation in connection with string theory [1, 2, 3, 4, 5, 6], black hole [7, 8] and brane physics [9, 10]. In this regard, one of the famous work is the Randall-Sundrum (RS) model which was shown to be classified into two types: A model with two branes with opposite tensions and another in which there is only one brane with positive tension. It may be thought of as sending the negative tension brane off to infinity.

The scalar potential shape turns out to be essential in inflationary models [11]. The well known examples are the chaotic inflation potential dealt with in [12, 13] and the minimal supersymmetric standard model (MSSM) inflation studied recently in [14, 15]. Besides these examples, there are several other models which have been much discussed in the literature. In particular, one has the exponential and the inverse power-law potentials [17] which may be used in the study of quintessence in brane and tachyonic inflation.

One of the aims of the present work is to contribute to this program by proposing a new potential in the brane inflation scenario. This potential is given by

\[ V(\phi) = \lambda f(\phi^{\nu}) \]

where \( f(\phi^{\nu}) = \arctg(\phi^{2}) \) and \( \lambda \) is a mass parameter specified later on. In this study \( \nu \) is fixed to one. An objective in this paper is to give an explicit computation for inflationary quantities corresponding to this potential. Then we make contact with observational results. We will see that, for a large range of values of \( \lambda \), one can get results in agreement with the observations.

On the other hand, the potential that we propose here has interesting features. In the very small field regime, it behaves as the chaotic potential. In this way, \( \lambda \) can be related to the inflaton mass. If we expand our potential at order 10, it can be identified with the one involved in the MSSM inflation. We will see that, a trivial identification of these potentials puts strong constraints on the MSSM parameters. Finally, one of the nice features of our potential is that, in the large field approximation, it allows one to implement the attractor mechanism in the presence of a black hole in the bulk. For a Schwarzschild black hole, we show, up to some assumptions, that its mass is proportional to \( \lambda \) defining the asymptotic value of the potential. In this way, the Friedman equation can be interpreted as an attractor equation involving the mass parameter of a bulk black hole when the geometry on the brane is de Sitter one. We consider the presence of two effective cosmological constants; one drives the inflationary era and the other accounts for the current observations. The attractor mechanism with a bulk black hole is the responsible for the cancellation of the inflationary cosmological constant, as observed today.

After a brief summary of brane inflation in section 2, we propose the new potential,
and then we discuss its inflationary aspects in section 3. The idea is to give a detailed computation for some inflation quantities and make contact with the observations. Section 4 concerns with a connection between the black hole in the bulk and the scale of the potential. In particular, we interpret the Friedman equation as an attractor equation for the Schwarzschild black hole in the bulk. In this sense, we find that the mass of the black hole can be related to the asymptotic value of the potential. A conclusion is presented in section 5.

2 Overview on brane inflation

In this section, we shortly review some basic facts on 3-brane cosmology in five-dimensional space-time [12]. Related works can found in [9, 10, 18, 19, 20, 21, 22, 23, 24]. We assume that the universe is filled with a perfect fluid with energy density $\rho(t)$ and pressure $p(t)$. In the framework of the flat Friedmann-Robertson-Walker model, with a scale factor $a(t)$, the Friedman equation is

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_4^2} \rho \left[1 + \frac{\rho}{2T}\right] + \frac{\Lambda_4}{3} + \frac{\mu}{a^4},$$  \hspace{1cm} (2.1)$$
describing the time evolution of $a(t)$. Here $H(t) = \frac{\dot{a}}{a}$ defines the Hubble parameter and the first term of (2.1) is the responsible for inflation. $M_4$ is the 4-dimensional Planck scale which is related to the 5-dimensional one $M_5$ by $M_4 = \sqrt{\frac{3}{4\pi}} \left(\frac{M_5^2}{T}\right)^{\frac{1}{4}} M_5$ where $T$ is the 3-brane tension. $\Lambda_4$ is the current cosmological constant. $\mu$ is related to the mass of the black hole in the bulk by $\mu = M_{BH} G_N^{(5)}$ where $G_N^{(5)}$ is the Newton constant in five dimensions [18]. Consider now an inflationary theory driven by a scalar field $\phi$. The dynamics can be described by a perfect fluid with a time dependent energy density $\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi)$ and pressure $p = \frac{1}{2} \dot{\phi}^2 - V(\phi)$. $V(\phi)$ is the dominant energy contribution to inflation. The scalar field satisfies the Klein-Gordon equation, namely,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \hspace{1cm} V' = \frac{dV}{d\phi}. \hspace{1cm} (2.2)$$

The dynamics of inflation requires that the scalar field moves away from the false vacuum and slowly rolls down to the minimum of its effective potential [17]. The slow-roll condition is characterized by two parameters

$$\epsilon = \frac{M_4^2}{4\pi} \left(\frac{V'}{V}\right)^2 \left[\frac{T(T + V)}{(2T + V)^2}\right], \hspace{1cm} \eta = \frac{M_4^2}{4\pi} \left(\frac{V''}{V}\right)^2 \left[\frac{T}{2T + V}\right].$$

Inflation ends when any of the two parameters equals one. In the slow-roll approximation, these parameters are very small, namely $\max\{\epsilon, |\eta|\} \ll 1$. In this case, it is easy to compute the number of e-foldings between the beginning and the end of inflation, given
by $N = \int_{t_i}^{t_e} H dt$. If $\phi_i$ and $\phi_e$ are the values of the scalar field at the beginning and at the end of inflation respectively, then $N$ takes the following form

$$N = -8\frac{\pi}{M_4^2} \int_{\phi_i}^{\phi_e} \frac{V}{V'} \left[ 1 + \frac{V(\phi)}{2T} \right] d\phi.$$  \hfill (2.3)

The inflationary model can be tested by computing the spectrum of perturbations produced by quantum fluctuations of fields around their homogeneous background values. In the large brane tension limit, the scalar amplitude $A_s^2$ of density perturbation is given by

$$A_s^2 \sim \frac{512 \pi}{75 M_4^6} \frac{V^3}{V'^2},$$  \hfill (2.4)

and the spectral index reads as

$$n_s = 1 - 6\varepsilon + 2\eta.$$  \hfill (2.5)

The above brane-world formalism has been applied for inflationary models involving many potential forms. In the present work, we shall apply this formalism for a new potential and give a detailed computation for some inflation quantities. Then we make contact with black hole attractor.

### 3 A new inflation potential

In this section we propose a new potential for inflation. Recall that a generic single scalar field potential, which can be characterized by two independent parameters, has the following form

$$V(\phi) = \lambda f\left(\frac{\phi}{\nu}\right)$$  \hfill (3.1)

where $\lambda$ corresponds to the vacuum energy density and $\nu$ corresponds to changes in the field value $\Delta \phi$ during inflation, which will be fixed to one $[1]$. Different models can be obtained by taking different choices for the function $f$. The most famous example, which has been intensively studied, is the chaotic inflation and its phenomenological hybrid extension $[4]$. It has the following form

$$V(\phi) = \frac{1}{2} m^2 \phi^2,$$  \hfill (3.2)

where $m$ is the mass of the inflaton. Besides this example, there are several other models which have been much discussed in the literature $[4]$. As mentioned in the introduction, they involve, among others, the exponential potential $V(\phi) = V_0 exp(-\beta \phi)$ and the inverse power-law potential $V(\phi) = \frac{\nu^{\alpha+4}}{\phi^\alpha}$. 
The brane inflation potential that we propose to study is

\[ V(\phi) = \lambda f(\frac{\phi}{\nu}) = \lambda \arctg(\phi^2). \]  

(3.3)

This potential has many nice features, we will just quote some of them. For very small values of the field, the potential behaves as the chaotic one. Indeed, when \( \phi^2 \) is very close to zero, at second order, one gets a chaotic inflation scenario with

\[ V(\phi) = \lambda \phi^2, \]  

(3.4)

where now \( \lambda \) is related to the mass of the inflaton as

\[ \lambda = \frac{1}{2} m^2. \]  

(3.5)

On the other hand, it has been proposed in [15, 16] a potential, corresponding to soft supersymmetry breaking, with the form

\[ V(\phi) = \frac{1}{2} m^2 \phi^2 - A \frac{\alpha_6 \phi^6}{pM_p^{p-3}} + \alpha_6^2 \frac{\phi^{2(p-1)}}{pM_p^{2(p-3)}}, \]  

(3.6)

to study the MSSM inflation. In particular, it has been suggested in [14] that we are dealing with one of the flat directions of the MSSM. In particular, for \( p = 6 \) we have

\[ V(\phi) = \frac{1}{2} m^2 \phi^2 - A \frac{\alpha_6 \phi^6}{6M_6^3} + \alpha_6^2 \frac{\phi^{10}}{6M_6^6}. \]  

(3.7)

This is a new type of inflation model which might work with any flat direction generating an A-term. More details can be found in [14, 15, 16]. It is easy to see that (3.7) can be rederived from (3.3). In the small field approximation, the Taylor expansion at order 10 of the function (3.3) gives the following potential

\[ V(\phi) = \lambda (\phi^2 - \frac{1}{3} \phi^6 + \frac{1}{5} \phi^{10}) \]  

(3.8)

which has a form similar to the MSSM potential given in (3.7). A trivial identification constrains the MSSM parameters:

\[ \frac{1}{2} m^2 = \frac{A \alpha_6}{2M_6^3} = \frac{5 \alpha_6^2}{6M_6^6}. \]  

(3.9)

Lastly we mention that the computation of some inflation quantities is in agreement with the observable results [12]. Let us discuss this feature in detail. Within the slow-roll regime, we shall compute the spectral index which is an essential quantity in inflation theory. To obtain it, we need to calculate the value of the scalar field \( \phi_i \) at the beginning of inflation. A way to get this value is by using the formula for the number of e-foldings
and then make appropriate assumptions. Imposing that the scalar field $\phi$ is very large, $N$, in our model, can be written as

\[ N \simeq 4 \frac{\pi^2}{M_4^4} (1 + \frac{\lambda \pi}{4T}) \int_{\phi_e}^{\phi_i} \frac{1 + \phi^4}{\phi} d\phi. \]  

(3.10)

Identifying $\phi_i$ with $\phi_{\text{cobe}}$ and getting $\phi_e$ from the end of inflation, we can find an expression for $N$. Performing the integral, within the slow roll regime and at high energy physics ($V \gg T$) we get

\[ N \simeq \frac{\lambda \pi^3}{T M_4^2} \left[ \ln \frac{\phi_{\text{cobe}}}{\phi_{\text{end}}} + \frac{1}{4} \left( \phi_{\text{cobe}}^4 - \phi_{\text{end}}^4 \right) \right]. \]  

(3.11)

Note that, after neglecting the first term, this equation is similar to the usual chaotic inflation one, which is given by

\[ N \simeq m^2 \left( \phi_{\text{cobe}}^4 - \phi_{\text{end}}^4 \right). \]  

(3.12)

Setting $\varepsilon_{\text{end}} \simeq 1$, which defines the end of inflation, and taking the limit $\lambda \pi \gg 2T$, we obtain

\[ \phi_{\text{end}}^4 \simeq \frac{3}{32 \lambda \pi^5} M_5^6. \]  

(3.13)

In order to estimate the value of $\phi_{\text{cobe}}$, we assume that the second term on the right of (3.11) dominates. Then, we can deduce the expression of $\phi_{\text{cobe}}$ in terms of the number of e-foldings $N$

\[ \phi_{\text{cobe}}^4 \simeq \frac{3N}{\lambda \pi^4} M_5^6. \]  

(3.14)

As we show below, to test our inflation model we have to compute the spectrum of perturbations. Indeed, a simple calculation from (2.4) reveals that the spectrum of perturbations takes the following form

\[ A_s^2 \simeq \frac{\pi^7 \lambda^6}{300 T^3 M_4^4} \phi_{\text{cobe}}^6. \]  

(3.15)

The scaling relation between the tension and the mass of the inflaton which is consistent with the observations is obtained from (3.14) and (3.15) and reads

\[ A_s \sim \frac{\lambda^3}{T^{3/2} M_4^4} \phi_{\text{cobe}}^3. \]  

(3.16)

The analogous result was obtained in [12]

\[ A_s \sim \frac{\lambda^2}{T^{3/2} M_4^4} \phi_{\text{cobe}}^5. \]  

(3.17)

and plotted in figure 1. We can now make use of numerical results. We assume that the number of e-foldings before the end of inflation, at which observable perturbations are generated, corresponds to $N \simeq 55$ and we take $A_s = 2.10^{-5}$. Varying $\phi_{\text{cobe}}$ in terms of
λ and taking different values of $M_5 < 10^{17}$ GeV, we can see that $\phi_{cobe}$ is always less than $M_4$ which is consistent with the observations.

Using the above results and the same method used in the literature [12], one can easily obtain the spectral index for our model. A straightforward computation from (2.5) shows that

$$1 - n_s \simeq \frac{\pi}{N\lambda}.$$  \hspace{1cm} (3.18)

Fixing $N$, the spectral index will depend only on the parameter $\lambda$. When $\lambda \to \infty$, the spectral index is driven towards the Harrison-Zeldovich spectrum, $n_s = 1$.

As we have seen at the beginning of this section, in the small field regime, $\lambda$ is related to the mass of the inflaton in the chaotic scenario. This can be seen directly from the formula for the number of e-foldings (3.12). One might ask the following question: Is there any interpretation of $\lambda$ in the large field limit? This question will be addressed in the next section.

## 4 Large field approximation and the attractor mechanism in inflation

In this section we consider the large field approximation for the potential proposed in the previous section and then make contact with the attractor mechanism in the presence of a black hole in the bulk. Motivated by the result of the attractor mechanism given in [25, 26], we will interpret the Friedman equation (2.1) as an attractor equation involving the mass of the black hole. To do this, let us consider nonzero values for $\Lambda_4$ and $\mu$. Recall that the parameter $\mu$ is the mass of the black hole in the bulk. For simplicity, we consider the case of a Schwarzschild black hole which is characterized only by its mass.

Let us take the large field approximation. In this regime, one has

$$V(\phi) \sim \lambda.$$  \hspace{1cm} (4.1)

In the vanishing limit $\mu = 0$, it follows, from the Friedman equation, that $H(t)$ should be constant, defining a de Sitter geometry. In this case the Universe expands exponentially and so the scale factor is given by

$$a(t) = a_0 \exp(Ht),$$  \hspace{1cm} (4.2)

where $a_0$ is a free constant parameter, which can be fixed to one. By integration (2.2), one can derive the time evolution of the scalar field which is consistent with (1.2). In the above approximation, (2.2) can be simplified as

$$\ddot{\phi} + 3H\dot{\phi} = 0.$$  \hspace{1cm} (4.3)
Using (4.2), the last equation is solved by

\[ \phi(t) = \xi + \exp(-3Ht), \]  

(4.4)

where \( \xi \) is a constant field. At this point, there are some comments. The first one is that, in the case of \( \xi = 0 \), the field solution becomes

\[ \phi(t) = \exp(-3Ht). \]  

(4.5)

and goes rapidly to zero so that the potential tends to the chaotic limit. The second one is that, we may interpret \( \xi \) as the value of the scalar field when \( t \) goes to infinity and so we can write

\[ \phi(t) = \phi_\infty + \phi_0 \exp(-3Ht). \]  

(4.6)

The large field approximation requires that \( \phi_\infty \) should be very large. In this way, the scalar field is almost constant and tends to a very large value as \( t \) tends to infinity.

Take now a generic non zero value for \( \mu \). In this way \( H \) is no longer constant. However, a constant value for \( H \) requires a particular form for the mass of the black hole in the bulk. A simple inspection shows that \( \mu \) should take the following form

\[ \mu \sim a^4. \]  

(4.7)

With this condition at hand we will interpret the Friedman equation as an attractor equation. To do so, we will restrict ourself to a de Sitter geometry corresponding to a specific constraint on \( H \)

\[ 3H^2 = \Lambda_4. \]  

(4.8)

For a large value of the brane tension, we can show that

\[ \mu(t) \sim -\lambda e^{A H t}. \]  

(4.9)

In the case where \( H \) takes small values, that is in the limit \( Ht \ll 1 \), the mass of the black hole becomes

\[ \mu(t) = \mu \sim -\lambda. \]  

(4.10)

This equation relates the mass of the black hole in the bulk with the asymptotic value of the potential. If we forget, for a while, about the negative sign, it can be interpreted as an attractor equation for a Schwarzschild black hole. Note that (4.10) is slightly different from the one used in the attractor mechanism of \( N = 2 \) supersymmetric extremal black hole embedded in string theory compactified on Calabi-Yau threefolds [25]. In our case

\^1Note that this equation may be interpreted as the 4-dimensional Einstein equations on the brane in the presence of a positive cosmological constant.
we obtain a relation involving the mass of the black hole instead of the mass square as usually appears in the $N = 2$ attractor equations. The mass parameter, in our case, seems to behave as the charge in the attractor mechanism, although it is fixed at infinity and not at the horizon. We believe that this difference is due to the absence of supersymmetry in this study. It has been discovered a sort of susy (pseudo-susy) in a relation between domain wall and cosmological solutions [27]; the deviation of our analysis from the standard result might be related to this fact. Recently a relation between the wrapped D3 and D5-branes on cycles of the resolved conifold and inflation has been reported in [28, 29]. Our model can be understood by the supersymmetric ones when susy is broken by the presence of D3-D5 brane system and fluxes; a sort of pseudo-susy would be behind the attractor mechanism we comment before. In this way, the above scalar field appearing in our potential could be identified with a scalar mode of the $R-R B$-field on the two-cycle of the small resolution of the conifold. In connection with non supersymmetric Calabi-Yau black hole attractor [30], it should be interesting to determine the entropy function using flux compactification on the resolved conifold. We hope some new results in this direction can be presented in the future [31].

As usually the instability corresponding to negative mass can be solved by introducing a brane with negative tension. Indeed, using the Friedman equation and (4.8), we have

$$
\mu = -\frac{2\pi^2}{3M_4^2} \lambda a_0^4 \left[ 1 + \frac{\pi \lambda}{2|T|} \right].
$$

(4.11)

This equation indicates that the mass of the black hole in the bulk is related to the brane tension and the value of the potential at infinity. In the high energy limit, $\pi \lambda >> 2|T|$, (4.11) gets reduced to

$$
\mu = \frac{\pi^3}{3|T|M_4^2} \lambda^2 a_0^4.
$$

(4.12)

From this relation it is clear that in order to interpret it as an attractor equation $\mu \simeq \frac{\pi^2 \lambda}{2a_0^4}$, a constraint on the brane tension is required. This is given by

$$
|T| = \frac{\pi^2 \lambda}{3M_4^2 a_0^4}.
$$

(4.13)

We interpret this result in the following way. The black hole in the bulk is placed near the second brane with negative tension ($T < 0$). The radiation of this black hole contributes to the Friedman equation (2.1) with the term $\frac{\mu}{a^4}$.

5 Conclusion

In this paper, we have proposed a new potential in the brane inflation scenario. After computing the main inflation parameters and check that they agree with the observational results we have implemented the attractor mechanism using a black hole in
the bulk in a particular way. The discussion of the fourth section is easily extended to more general potentials. It is only necessary a constant positive value of the potential at infinity. This kind of potentials are, in general, consistent with the slow roll conditions. Our potential has an extra bonus. In the small field regime, it reproduces the chaotic model as well as the MSSM inflation potential.

The result may be summarized as follows:

(1) First, we have achieved a quantitative study and computed inflationary quantities. In order to make contact with the observational data, several bounds can be imposed to our potential so that the results are in agreement.

(2) In the large field approximation, we have made contact with the black hole in the bulk. For a Schwarzschild black hole solution, we have shown that its mass is proportional to $\lambda$ defining the asymptotic value of the potential. This has been done with the help of the Friedman equation, which may be reinterpreted as an attractor equation in the de Sitter background.

Our work opens up for further studies. In connection with inflation in string theory, one may consider models with several scalar fields $\phi_i$. These fields can identified with the $R$-$R$ $B$-field on 2-cycles of the Calabi-Yau manifolds. In this way, the total potential takes the following form

$$V(\phi_i) = \sum_i \lambda_i \arctg(\phi_i^2), \quad i = 1, ..., h^{1,1}$$

(5.1)

where $h^{1,1}$ is the number of the Kahler deformations of the Calabi-Yau manifold. Another natural extension of the present results includes other type of black hole solutions. We think that the relation between the inflationary potential and the attractor mechanism in the Calabi-Yau black hole physics deserves a better understanding. We hope to report elsewhere on these open questions.

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