Effect of the Number of Work-roll Surface Division on Prediction of Contact Length in Coupled Analysis of Roll and Strip Deformation during Sheet Rolling

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In order to perform accurate three-dimensional rolling analysis of sheet deformation, it is required to predict the contact length accurately in consideration of the surface waviness due to the flattening deformation of work roll surface. Flattening deformation analysis based on theory of elasticity and three-dimensional elastic FEM have been developed, and their accuracy in predicting flattening deformation and contact length are validated in this report. As the flattening deformation of the work roll surface changes sharply at the boundary of contact area, the contact length is expected to change according to the number of work roll surface division, especially in circumferential direction, for calculation. Then, the influences of the number of division on roll profile and contact length used for numerical analysis are investigated. The following results are obtained; 1) The differences between the results obtained by three-dimensional elastic FEM and formula for roll flattening of Nakajima and Matsumoto is not significant. 2) When the number of division of work roll surface is increased, the end point of contact region moves toward the downstream side, therefore, the contact length becomes longer and it is necessary to divide the work roll surface into enough much elements. 3) As long as enough much elements are used for the calculation, solutions, such as rolling pressure distribution or thickness distribution of rolled strip, obtained by three-dimensional analysis reveal little difference from those obtained using formula for roll flattening given by Hitchcock.

KEY WORDS: rolling; numerical analysis; FEM; roll flattening.

1. Introduction

Steel industry is now encountering the severe requirement for the dimensions of hot rolled sheets, especially in width direction. Dimensional control of hot rolled sheets in width direction promotes technologies to reduce ‘body crown’ and ‘edge drop’, so that the theoretical analysis which can be applied to the accurate prediction of strip profile is strongly requested. Several three-dimensional coupled analyses of work roll deformation and sheet deformation have been developed based on elementary theory of elasticity and plasticity.1) Now more accurate analyses of sheet deformation become possible using three-dimensional rigid-plastic finite element method (FEM),1) therefore, reliable results for rolling pressure distribution can be easily obtained under imposed boundary conditions such as three-dimensional geometry of roll bite, which strongly depends the elastic deformation of work roll surface. In fact, accurate prediction of strip profile requires accurate results for transversal distribution of contact length, which influences rolling pressure distribution in contact area.

Flattening deformation distributes in transversal direction as well as the rolling direction. The length of contact arc is influenced by the flattening deformation in rolling direction, which occurs in plane perpendicular to transversal direction. The length of contact arc in this plane is usually calculated by using formula given by Hitchcock, which assumes that deformed work roll profiles keeps round shape. But, the profile of work roll does not keep round shape when it is elastically deformed by distributed rolling pressure.8) The accurate elastic analyses of work roll flattening deformation can be made by modified theory of elasticity or three-dimensional elastic FEM, without introducing the Hitchcock’s assumption.

As rolling pressure changes steeply in transversal plane at the entry and exit of roll bite, the flattening deformation is expected to exhibit abrupt change correspondingly at the entry and exit of roll bite. Only the numerical analyses could be applied to the elastic analysis of complicated deformation of roll surface, therefore, the number of division or number of integration points used for numerical analysis should be examined. Unfortunately, until now, the effect of division of work roll surface on numerical solution of work roll flattening and contact length has not been clarified. The effect of number of division, used for the elastic analysis, on work roll flattening deformation is investigated, and nu-
Numerical solution for contact length is compared with that obtained by Hitchcock’s formula. The elastic analysis used there is based on the modified theory of elasticity, as the solution obtained by this analysis and that obtained by three-dimensional elastic FEM is almost identical for the normal hot rolling condition.

2. Difference in Flattening Deformation Obtained by Three-dimensional FEM and Modified Theory of Elasticity

Distribution of roll flattening in rolling direction, as well as its transversal direction, could be analyzed by the modified theory of elasticity, by integrating the equation using rolling pressure distribution in contact area. However, until now, it is normally used in an approximated form, which is derived by integrating original form in rolling direction assuming the elliptic rolling pressure distribution in this direction. The approximated form is easier to be implemented into elementary theory of plastic deformation, as it is expressed as a relation between ‘rolling force per unit width of sheet’ and ‘flattening displacement at the exit of roll bite’, which directly influences strip profile after rolling.

Direct integration of modified theory of elasticity is applied here, and the flattening deformation is compared with that obtained by three-dimensional elastic FEM. These analyses for flattening deformation are implemented into coupled analysis of sheet rolling, where sheet deformation is solved by three-dimensional rigid-plastic FEM, work roll deflection is solved by divided model proposed by Shohet, and contact displacement between work roll and back-up roll is solved by Loo’s equation. Entry and delivery points of sheets are solved by contact analysis, which is described in the following chapter in detail.

Rolling conditions employed for numerical case study are shown in Table 1. Normal hot rolling condition with work roll diameter 800 mm and sheet width 1000 mm is assumed. Thickness before and after rolling is 5 mm and 3.5 mm, respectively and strip profile before rolling is square. Work roll and sheet in the roll bite are divided into 10 elements in rolling direction and 8 elements in width direction. Four additional elements are added to sheets before rolling, and two are that after rolling. Rolling force per unit width obtained by coupled analysis of work roll deformation and sheet deformation is shown in Fig. 1. Rolling force gradually increases from the center, and, after it reaches maximum near the edge of sheet, rolling force per unit width sharply decreases due to the transversal metal flow of sheet. The methods of calculating flattening deformation, those are 3D elastic FEM and Nakajima and Matsumoto equation based on modified theory of elasticity, shows little difference on analytical results for rolling force per unit width. Figure 2 shows distribution of roll radius in rolling direction, at the center of sheet. Work roll radius just before and after roll bite is smaller than original work roll radius, and its change in the roll bite shows similar profile with rolling pressure. The maximum radius of flattened roll is observed where rolling pressure becomes its maximum, and the maximum value is bigger than the flattened roll radius obtained by Hitchcock’s equation. Very little difference due to the calculation methods of roll flattening, those are 3D

| Table 1. Rolling conditions employed for numerical case study. |
|-----------------|-----------------|
| Dia. of W.R.    | mm              |
| Dia. of W.R. Neck | mm              |
| W.R. crown      | μm/dia          |
| Length of W.R. Neck | mm            |
| WR Bender       | kN/chk          |
| Dia. of BUR     | mm              |
| Dia. of BUR Neck | mm              |
| BUR crown       | μm/dia          |
| Length of BUR Neck | mm            |
| Barrel length   | mm              |
| Young’s modulus | MPa             |
| Position’s ratio | %               |
| Entry thickness | mm              |
| Strip crown     | μm              |
| Width           | mm              |
| Reduction in height | %            |
| Velocity of roll surface | mpm |
| Flow stress     | MPa             |
| Friction of coefficient | 0.3 |
| Front tension   | MPa             |
| Back tension    | MPa             |
| Number of division of strip | Width direction N_x = 10 |
| Rolling direction N_z = (4+8+3) |
| Thickness direction N_y = 3 |

Fig. 1. Transversal distribution of rolling force per unit width.

Fig. 2. Longitudinal distribution of local work roll radius.
FEM and Nakajima–Matsumoto equation, can be observed also in Fig. 2. Transversal profile of sheet after rolling is shown in Fig. 3. Lateral distribution of contact length is shown in Fig. 4. The lateral distribution of delivery position from the plane including work roll axes is also shown in Fig. 4. From both figures, it can be concluded that the difference in flattening deformation obtained by 3D FEM and Nakajima–Matsumoto equation is quite small. Similar conclusion is obtained by Narita et al.12) From the analytical results described above, as long as the hot rolling conditions are concerned, the deformation analysis of work roll flattening obtained by modified theory of elasticity, that is Nakajima–Matsumoto equation,9) gives us the almost same results as that obtained by 3D FEM, if it is directly integrated without using Hitchcock’s assumption. Then, in the following chapter, the effect of number of divisions on contact length is examined only by the numerical results obtained by Nakajima–Matsumoto equation.

3. The Effect of Work Roll Division on the Estimation of Contact Length in Coupled Analysis of Sheet Deformation and Flattening of Work Roll

As is shown in the following chapter, longitudinal distribution in flattening deformation changes abruptly at the entry and exit of roll bite. The division of work roll surface should be fine enough in order to reflect this change accurately to the length of contact arc. The effect of number of work roll surface division on analytical results is examined in the followings.

3.1. Analytical Model

Schematic illustration of analytical model is shown in Fig. 5. Upper figure shows the model to analyze roll deflection. Divided model proposed by Shohet10) and Loo’s equation11) are used to calculate deflection of work roll and back-up roll axes, and the contact displacement between both rolls. Lower figure shows the model to analyze roll flattening. Here, without using Hitchcock’s assumption, the vertical displacement of work roll $u_i(x,z)$ is calculated by direct integration of Nakajima–Matsumoto equation shown in the Eq. (1).

$$u_i(x,z) = \frac{1 - \nu^2}{\pi E} \times \int \left( \frac{1}{\sqrt{2} \sqrt{x-x_i}^2 + (x-x_i)^2} - \frac{1}{\sqrt{2} \sqrt{x-x_i}^2 + (D_w/e)^2} \right) p(\xi,\zeta)d\xi d\zeta$$

The Eq. (1) is derived from the modified theory of elasticity.9) The work roll is assumed to be a semi-infinite body for calculating flattening deformation because the diameter of the work roll is much bigger than the contact length. A solution for calculating elastic displacement is introduced whereby a concentrated load is applied to a semi-infinite body. Strip width, however, is the same degree compared to the diameter of the work roll and so this solution has been modified. The second term in the equation expresses the modification. $p(\xi,\zeta)$ is rolling pressure obtained by FEM.
analysis of sheet deformation. As is shown in Fig. 5, \( z \) shows the longitudinal coordinate, and \( x \) shows the transversal coordinate corresponding to width direction. \( \xi \) and \( \zeta \) show the position in the width direction and rolling direction respectively where rolling pressure is applied to. \( e \) is the basis of natural logarithm, and \( D_w \) is the diameter of work roll. Sheet deformation is analyzed by simplified three-dimensional rigid-plastic FEM, where uniform distribution in strain rate in thickness direction is assumed. After FEM analysis of strip deformation under initial geometry of roll bite, rolling pressure \( P(\xi, \zeta) \) is obtained, and it is used to estimate deflection of rolls and vertical displacement of work roll \( u(x, z) \) due to flattening deformation. Coarse division of the work roll surface in the rolling direction corresponds to the strip division, and it is divided into fine elements. It is assumed that rolling pressure \( P(\xi, \zeta) \) is constant in each fine element of the work roll surface division. Flattening displacement relative to that in the reference position (width center, exit of roll bite) and work roll deflection relative to that in the reference position are used to update the geometry of roll bite. Then the rigid-plastic FEM analysis of sheet deformation is made again under updated geometry of roll bite, to obtain new distribution of rolling pressure. These procedures are repeated until the strip profile gets sufficient convergence.

The division of the work roll surface in the width direction corresponds to the strip division, and the division at both edges of the strip is fine enough taking the lateral flow in the strip into consideration.

Analysis of roll flattening is made with sufficient number of divisions in roll bite. The regions in the same length of contact arc are added to the roll surface on entry area as well as the exit area. The intervals of divisions in these areas are the same with that in the roll bite. The exit of roll bite is assumed to be in the position of minimum roll gap in rolling direction, which is obtained by solving equivalent roll radius from every neighboring three points. The nodal points of sheet at exit are moved to be in the same position of exit of roll bite, which distributes in the transversal direction of sheet under rolling.

### 3.2. Conditions of Analysis

Table 2 shows the number of divisions used for the analysis. Division of work roll surface in the contact area, denoted by \( N_Z^{\text{R}} \), is changed into five levels, and the same number of division \( N_Z^{\text{W}} \) is employed also in entry area and exit area. The longitudinal length of entry area and exit area is the same as the contact length throughout the analysis shown in the followings. The number of division of each area \( N_Z^{\text{R}} \) is increased from 20 to 800, denoted as \( 20+20+20 \) or \( 800+800+800 \) in Table 2, to see the influence of divisions of work roll surface on the calculated value on flattening deformation and contact length. Other condition of analysis, such as rolling conditions, is the same as is shown in Table 1. Also, the reference contact length is calculated by Hitchcock’s equation shown Eq. (2).

\[
R^* = \left[ 1 + \frac{16(1-v^2)P}{\pi E \Delta h} \right] \tag{2}
\]

Here, \( R \) is the radius of non-deformed work roll, \( R^* \) is the radius of deformed work roll, \( P \) is the rolling force per unit width and \( \Delta h \) is the decrease in sheet thickness during rolling.

### 3.3. Results of Analysis

The change in rolling force according to the increase in work roll division \( N_Z^R \) is shown in Fig. 6. The rolling force increases according to the increase in \( N_Z^R \), and it saturates after \( N_Z^R \) exceeds 400. When the smaller \( N_Z^R \) is used, the rolling force is smaller than that obtained by Hitchcock’s assumption for roll work profile. But, the saturated value of rolling force is slightly larger than that obtained under Hitchcock’s assumption. The rolling force without the analysis of roll flattening is much smaller, because the contact length is underestimated in this kind of analysis. The change in rolling torque is shown in Fig. 7, and Fig. 8 shows the change in forward slip. Both results shows that they reach saturated value after \( N_Z^R \) exceeds 400. From these figures, it can be concluded that sufficient number of divisions on work roll surface is about 400 for the rolling conditions simulated in this investigation. It also can be seen

| Number of division of roll surface | \( N_Z^R = 16 \) |
|-----------------------------------|----------------|
|                                   | \( 20+20+20 \) |
|                                   | \( 50+50+50 \) |
|                                   | \( 200+200+200 \) |
|                                   | \( 400+400+400 \) |
|                                   | \( 800+800+800 \) |

| Number of division of strip | Width direction | \( N_Z^W = 16 \) |
|-----------------------------|----------------|
|                             | Rolling direction | \( N_Z^W = (9+10+3) \) |

![Fig. 6.](image1.png)

![Fig. 7.](image2.png)
that the saturated value of them calculated without using Hitchcock's assumption is slightly larger than those with Hitchcock's assumption.

Transversal distribution of rolling force per unit width is shown in Fig. 9. Here, the rolling force per unit width using Hitchcock’s assumption is almost identical with the rolling force per unit width with sufficient number of division on work roll surface. The sheet profile after rolling is shown in Fig. 10. When the sufficient number of division is used in the integration of Eq. (1), the sheet profile is almost identical with that obtained by using Hitchcock’s formula. The sheet profile and rolling force obtained by rough division, that is \( N_z^R = 20 \), is different from other results. This difference is due to the smaller number of work roll \( N_z^R = 20 \). Also, this comes from the characteristics of analytical model for the sheet deformation, where strain rate is assumed to distribute uniformly in thickness direction. The results obtained by fully three-dimensional analysis of sheet deformation, that was shown in Fig. 3, does not give us the gradual decrease in sheet profile in transversal direction even when smaller \( N_z^R \) is used in the analysis. Figures 11, 12 and 13 show the contact characters between sheet and work roll in transversal direction of sheet. The contact length shows the maximum value at the position of 50 mm from the edge of sheet, where the rolling force per unit width becomes maximum value as is shown in Fig. 9. No clear difference between the results obtained with and without Hitchcock’s assumption can not be seen in these figures, when \( N_z^R \) exceeds 400.
Figures 14 and 15 show the roll profiles on transversal planes located at 50 mm from the edge of sheet. The result for $N^g_2=20$ is shown in Fig. 14, and that for $N^g_2=800$ is shown in Fig. 15, respectively. The deformed profile of work roll becomes closer to that with Hitchcock’s formula when the division of work roll becomes larger. It is worth noting that the profile of deformed work roll when $N^g_2=800$ is almost identical with that calculated by using Hitchcock’s assumption. The distribution of local work roll radius in the same plane is shown in Fig. 16. It is clearly shown that local work roll radius is not uniform in longitudinal direction. It is bigger than that obtained by Hitchcock’s formula around neutral point, and smaller than the non-deformed radius of work roll at entry and exit of roll bite. Smaller radius of work roll is obtained at entry and exit of work roll, which affect the length of contact shown in Figs. 11, 12 and 13.

4. Discussions

As is shown in the previous chapter, length of contact arc is underestimated when smaller number of division $N^g_2$ is used in the analysis. General characteristics of flattening deformation tends to be in convergence if $N^g_2$ is increased, but it is not easy to judge that they are in full convergence even when $N^g_2$ is increased up to 800. As the coupling analysis of roll deformation and sheet deformation is conducted here, rolling pressure distribution obtained by deformation analysis of strip is closely related to the geometry of roll bite. The roll bite geometry is calculated as a function of rolling pressure, therefore, the results presented in the following chapter includes the effect of sheet deformation as well as the elastic deformation of work roll, which is influenced by the number of division on work roll surface. In order to examine only the effect of work roll division more in detail, the elastic analysis of work roll deformation is conducted using the fixed value of rolling pressure distribution as a boundary condition. The change in contact length, profile of work roll and local radius of work roll is calculated and discussed according the change in division of work roll surface $N^g_2$. The scheme of analysis is as follows.

(1) The flattening deformation is calculated directly from Eq. (1), for the initial length of contact arc $L_0$. From the result, local radius of deformed work roll is calculated by the coordinate of neighboring three points. Also the gradient of each point is calculates by these fitting circles, including the gradient at the exit of contact arc $g^{Exit}$.

(2) The work roll is rotated around the exit of contact arc, to sacrifice $g^{Exit}$. Then the modified roll profile, with the zero gradient at the exit of contact arc, is obtained.

(3) From the modified roll profile geometry, entrance of contact arc is calculated. Then the corrected value of contact length $L_{i+1}$ is calculated.

(4) The difference between $L_{i+1}$ and $L_i$ is evaluated. If their difference is bigger than 20 μm, the above-mentioned procedure is repeated again.

Rolling pressure distribution used as a boundary condition is that obtained by rigid-plastic FEM. The precise analysis of contact arc necessitate that the rolling pressure should continuously distribute in the contact arc, and it should be zero at the entrance and exit of contact arc. As the rolling pressure obtained by rigid-plastic FEM can not be zero at the entry and exit of roll bite, it is corrected to become zero by adding the small area with the length $L_0$ and assuming that rolling pressure linearly drops down to zero in $L_0$, as is shown in Fig. 17. $L_0$ is assumed to be 1/400 of contact length. Figure 18 shows the calculated contact length, with the correction of pressure distribution (bottom of Fig. 18) and without correction (top of Fig. 18). Contact length, calculated by using corrected pressure distribution, increases according to the increase in $N^g_2$, and it perfectly saturates when $N^g_2$ exceeds 500. However, it does no saturate when the correction of pressure distribution, which is shown in Fig. 17, is not conducted.

Figure 19 shows roll profile near exit of roll bite. Figure 20 shows the distribution of local radius along the contact arc. The calculated results using $N^g_2=10$ and $N^g_2=2000$ are shown in these figures, to emphasize the change in flattening deformation according to the increase in the division of
These results are obtained by using the corrected pressure distribution shown in Fig. 17. From Fig. 19, it can be seen that delivery position moves downstream side when \( N^2 \) is increased. This means that the contact length is underestimated when smaller \( N^2 \) is used for the analysis, because the flattened work roll radius is underestimated in this case, as is shown by circle symbols in Fig. 20.

From the results described in this and previous chapters, it can be concluded that the sufficient number of work roll division \( N^2 \) should be used to get accurate solution for flattened work roll profile at the exit of roll bite, which influences the delivery position and length of contact arc. At least, division around the exit of roll bite should be fine enough. For example, the hot strip rolling condition shown in Table 1, the division of work roll surface should be smaller than 1/400 of contact length in this area.

5. Conclusion

Regarding the coupled analysis of roll deformation and sheet deformation in hot strip rolling, the analysis of roll flattening without introducing Hitchcock’s assumption is conducted. The influence of number of divisions on work roll surface is shown and discussed, referring the result obtained by Hitchcock’s equation, and the following results are obtained.

1. Roll radius along the contact arc in sheet rolling does not keep the uniform radius, which is assumed by the Hitchcock’s equation. Local radius of work roll is smaller than non-deformed radius of work roll at the entry and the exit of roll bite, and its maximum value is observed around the neutral point. Although the local radius of deformed work roll distributes in a wide range along the contact arc, deformation characteristics of sheet and work roll, such as strip profile, rolling pressure distribution and transversal distribution of contact length, shows slight difference from those obtained by using Hitchcock’s assumption, when the work roll division is fine enough.

2. The increase in the number of work roll surface division results in the shift of delivery position in the downstream side. Therefore, the contact length and rolling force become larger if the number of division is increased. In order to reflect the abrupt change in local work roll radius of deformed work roll at the exit of roll bite, finer mesh on
work roll should be adopted, especially in the area around delivery position. For the strip rolling condition used in this investigation, the division in this area should be smaller than 1/400 of contact length.

(3) The forward slip is underestimated when the Hitchcock’s assumption is introduced in the analysis of work roll flattening.

This investigation deals with the analysis of only the hot strip rolling. As the local work roll distribution along the contact arc may distribute much wider in the cold rolling of thinner sheet and skin-pass rolling, accurate analysis of roll flattening may be needed. In addition, the direct analysis of flattening equation based on the modified theory of elasticity may be necessary when the accurate estimation in forward slip is requested, even in the hot strip rolling.

REFERENCES

1) Y. Tozawa, M. Nakamura and T. Ishikawa: J. Jpn. Soc. Technol. Plast., 17 (1976), No. 180, 37.
2) H. Matsumoto: ISIJ Int., 31 (1991), No. 6, 550.
3) K. Mori and K. Osakada: Trans. JSME, 56 (1990), No. 525, 268.
4) J. Yanagimoto, T. Sasaki, M. Kiuchi and T. Kono: J. Jpn. Soc. Technol. Plast., 33 (1992), No. 383, 1406.
5) K. Yamada, S. Ogawa and M. Ataka: Proc. 41st Japanese Joint Conf. Techn. Plast., JSTP, Tokyo, (1990), 63.
6) H. Furumoto, K. Morimoto, K. Hayashi and K. Osakada: J. Jpn. Soc. Technol. Plast., 37 (1996), No. 429, 1059.
7) Roll Neck Bearings, Appendix I, ed. by J. H. Hitchcock, ASME, N.Y. (1935), 33.
8) H. Matsumoto and Y. Uehori: J. Jpn. Soc. Technol. Plast., 29 (1988), No. 331, 851.
9) H. Matsumoto, K. Nakajima, T. Kikuma and Y. Uehori: J. Jpn. Soc. Technol. Plast., 23 (1982), No. 263, 1201.
10) K. N. Shohet and N. A. Townsend: J. Iron Steel Inst., 206 (1968), No. 11, 1088.
11) T. T. Loo: J. Appl. Mech., 25 (1958), No. 1, 122.
12) K. Narita, K. Yasuda and S. Shida: J. Jpn. Soc. Technol. Plast., 36 (1995), No. 418, 1228.