Matter Power Spectrum Covariance Matrix from the DEUS-PUR \(\Lambda\)CDM simulations: Mass Resolution and non-Gaussian Errors

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ABSTRACT
The upcoming generation of galaxy surveys will probe the distribution of matter in the universe with unprecedented accuracy. Measurements of the matter power spectrum at different scales and redshifts will provide stringent constraints on the cosmological parameters. However, on non-linear scales this will require an accurate evaluation of the covariance matrix. Here, we compute the covariance matrix of the matter power spectrum for the concordance \(\Lambda\)CDM cosmology from an ensemble of N-body simulations of the Dark Energy Universe Simulation - Parallel Universe Runs (DEUS-PUR). This consists of 12288 realizations of a \((656 h^{-1} \text{Mpc})^3\) simulation box with 256\(^3\) particles. We combine this set with an auxiliary sample of 96 simulations of the same volume with 1024\(^3\) particles to assess the impact of non-Gaussian uncertainties due to mass resolution effects. We find this to be an important source of systematic errors at high redshift and small intermediate scales. We introduce an empirical statistical method to correct for this effect and provide an accurate determination of the covariance matrix over a wide range of scales including the Baryon Acoustic Oscillations. The large statistical sample of DEUS-PUR simulations enables us to finely sample the probability distribution of the matter power spectrum. Contrary to previous studies which have used smaller N-body simulation ensembles, we find the skewness of the distribution to significantly deviate from expectations of a Gaussian random density field at \(k \geq 0.25 h \text{Mpc}^{-1}\) and \(z < 0.5\). This suggests that in the case of finite volume surveys an unbiased estimate of the ensemble averaged band power at these scales and redshifts may require a careful assessment of non-Gaussian errors more than previously considered.

1 INTRODUCTION
Surveys of the large scale structures have been providing insightful data for more than a decade now. Observational projects such as the 2-degree Field Galaxy Redshift Survey (2dFGRS, Percival et al. 2001, Cole et al. 2005) and the Sloan Digital Sky Survey (SDSS, Tegmark et al. 2004) have yielded unprecedented measurements of the clustering of matter on the large scales. These observations have made possible the first detection of the Baryon Acoustic Oscillations (BAO) signal (Eisenstein et al. 2005, Percival et al. 2007) and provided constraints on model parameters that are complementary to those obtained from other standard cosmological probes. The success of these projects has opened the way to a new generation of survey programs.

In the years to come the Dark Energy Survey (DES), the Large Synoptic Survey Telescope (LSST) or the Euclid mission will map the distribution of galaxies in larger cosmic volumes and with higher sensitivity. Through a variety of probes, these surveys aim to achieve a few percent error on the determination of several cosmological parameters. However, multiple challenges need to be addressed. From the theoretical point of view one of the most challenging aspects concerns the availability of robust theoretical predictions of the clustering of matter at small scales, which is crucial to correctly interpret the data and infer unbiased constraints on model parameters.

At small scales and late times the gravitational collapse becomes a highly non-linear process. Thus, model predictions cannot rely on standard linear perturbation theory and require solving the complex dynamics of matter collapse through numerical simulations. The need for accurate predictions of the clustering of matter over large interval of scales has driven up the demand for large volume high-resolution N-body simulations (Angulo et al. 2012, Kim et al. 2011, Alimi et al. 2012). As an example the Baryon Oscillation Spectroscopic Survey (BOSS, Dawson et al. 2013) has recently determined the cosmic distance scale to one percent accuracy from measurements of the BAO spectrum in the range of modes \(0.01 < k [h \text{Mpc}^{-1}] < 0.30\) (Anderson et al. 2013). Future surveys will push these measurements even further. Predicting the matter power spectrum in such a large interval of scales at a few percent level requires N-body simulations that cover very large cosmic volumes to reduce cosmic variance uncertainties at low \(k\) and with sufficient mass and spatial resolution to limit numerical systematic errors at large \(k\). However, this is still not sufficient to correctly interpret the data since an unbiased statistical analysis also require knowledge of the covariance matrix.

If the initial matter density field is Gaussian distributed, dur-
ing the linear regime its Fourier modes evolve independently and the covariance of the matter power spectrum has a simple diagonal form. From now on we will refer to this configuration as the Gaussian case, for which errors on the band powers are uncorrelated. In contrast at small scales and late times the covariance develops non-vanishing off-diagonal terms which account for the mode coupling caused by the non-linear regime of gravitational collapse. In such a case the errors on band powers become correlated causing a larger dispersion on power spectrum measurements (Meiksin & White 1999). Neglecting such correlations may lead to biased results as shown by several studies of weak lensing observables (see e.g. White & Hu 2000, Semboloni et al. 2002, Lee & Pettorino 2008, Kiessling, Taylor & Heavens 2011) and to a biased determination of BAO parameters (see e.g. Takahashi et al. 2011, Ngan et al. 2012). Hence, the future generation of large scale structure surveys will need estimates of the covariance matrix which require sampling the matter power spectrum from large samples of N-body simulations (Taylor, Joachimi & Kitching 2013).

Here, we use a set of simulations ($\sim 10^{12}$) of the concordance ΛCDM model from the Dark Energy Universe Simulation Parallel Universe Runs (DEUS-PUR). By combining this large ensemble with an auxiliary set of high resolution runs we estimate the impact of the mass resolution of the simulations on the power spectrum covariance matrix. We show that mass resolution effects can be an important source of uncertainty. Furthermore, our analysis clearly indicates the necessity of using large N-body datasets when studying the matter power spectrum distribution at small scales and low redshifts. In particular we find significant deviations from expectations of a Gaussian density field which were overlooked in previous studies. This has potentially important implications for inferring unbiased power spectrum measurements for future survey programs. Our work extends the analysis of Takahashi et al. (2009) to a larger statistical sample of N-body simulations with higher mass resolution and better spatial resolution.

The paper is organized as follows: in Section 2 we describe the characteristics of the DEUS-PUR simulations. In Section 3 we present the analysis of the DEUS-PUR covariance matrix, while in Section 4 we show the results of the computation of the probability distribution of the matter power spectrum. In Section 5 we discuss the effect of non-Gaussian errors on the signal-to-noise of power spectrum measurement. Finally we present our conclusion in Section 6.

### 2 N-BODY DATASET

#### 2.1 DEUS-PUR Simulations

We use the N-body simulation dataset from DEUS-PUR project. This consists of 12288 simulations of a flat ΛCDM model best-fitted to the WMAP-7yr data (Percy et al. 2007) of a cosmological volume of $(656 h^{-1} \text{ Mpc})^3$ with $256^3$ particles, for a formal mass resolution of $1.2 \times 10^{12} h^{-1} M_\odot$ (Set A), and 96 simulations of the same cosmological model and equal volume with $1024^3$ particles, corresponding to a mass resolution of $2 \times 10^{10} h^{-1} M_\odot$ (Set B). These runs have been realized with “A Multiple purpose Application for Dark Energy Universe Simulation” (AMADEUS) (Alimi et al. 2012). This workflow application includes a dynamical solver based on RAMSES (Teyssier 2002), an adaptive mesh refinement code with a tree-based data structure that allows recursive grid refinement on a cell-by-cell basis, in which particles are evolved using a particle-mesh (PM) solver, while the Poisson equation is solved using a multigrid method (Guillet & Teyssier 2011).

| Set  | $N_s$  | $L$ (Mpc $h^{-1}$) | $N_p$  | $m_p$ (M$_\odot$ $h^{-1}$) |
|------|-------|------------------|-------|--------------------------|
| A    | 12288 | 656.25           | 256$^3$ | $1.2 \times 10^{12}$   |
| B    | 96    | 656.25           | 1024$^3$ | $2 \times 10^{10}$   |
| C    | 512   | 1312.5           | 512$^3$ | $1.2 \times 10^{12}$   |

**Table 1.** DEUS-PUR simulation characteristics: $N_s$ is the number of realizations, $L$ is the box-side length, $N_p$ is the number of dark matter particles and $m_p$ the mass resolution. Taking set A as a reference, set B and C have been designed to study respectively simulation mass resolution and volume effects on large scale structure observables.

In the case of the DEUS-PUR simulations the refinement criterion is set such as to allow up to 6 levels of refinement.

The initial conditions of the simulations have been generated using the code MPFR.ACIC (Prunet et al. 2008), which convolves a white noise with the square root of the input power spectrum. More specifically, the white noise of the initial conditions for Set A have been drawn from sub-boxes in the initial conditions of $10.5 h^{-1}$ Gpc box-side ΛCDM simulation with 4096$^3$ particles (i.e. same mass resolution) for three different phases. Two of these $(10.5 h^{-1}$ Gpc)$^3$ volume realizations were generated in the context of the DEUS-FUR project (see e.g. Alimi et al. 2012, Rasera et al. 2014). On the other hand the white noise of the simulations in Set B has been specifically generated. The initial redshift has been set to $z_i \approx 105$ for Set A and $z_i \approx 190$ for Set B. Such large values guarantee that transient effects (Scoccimarro 1998, Crocce et al. 2006) are negligible.

Table 1 summarizes the characteristics of the DEUS-PUR simulations. For comparison Takahashi et al. (2009) have used 5000 simulations of a standard ΛCDM model with a $(1000 h^{-1} \text{ Mpc})^3$ volume and a mass resolution of $4.1 \times 10^{10} h^{-1} M_\odot$, realized using a Particle Mesh (PM) solver with no spatial refinement and initial redshift $z_i = 20$. A third sample of simulations (Set C) listed in Table 1 consists of 512 realizations with mass resolution identical to those of Set B but 8 times the volume. This set is not used in the analysis presented here.

The workflow of the AMADEUS application has been automated to generate a large number of N-body simulations. An external script has been coded to monitor in real time the job-queue and submit new simulations as soon as other simulations have terminated. For each simulation the initial conditions, the dynamic evolution, the data reduction and measurements of the matter power spectrum and the halo mass function are controlled through the same script. A final check on the file content has been implemented to detect any error due to unexpected machine failure.

#### 2.2 Power Spectrum & Covariance Matrix Estimators

We compute the matter power spectrum using the code POWERGRID (Prunet et al. 2008). This estimates the power spectrum from the Fourier transform of the matter density field in band powers of size $\Delta k = 2\pi/L$, where $L$ is the simulation boxlength. We correct the measured spectrum for the effect of smoothing due to the Coulomb-In-Cell (CIC) algorithm, that is used to estimate the density contrast field from the particle distribution. We do not correct for aliasing, since varying the size of the CIC grid we find that aliasing effects are negligible below half the Nyquist frequency of the CIC grid. Since our CIC grid is two times finer than the coarse grid of the simulation, its Nyquist frequency is given by $k_N = 2(\sqrt{N_p} \pi/L)$, where $N_p$ is the number of particles. Thus the range of modes in which we compute the power spectrum is...
given by \( k_{\text{min}} = 2\pi/L \) and \( k_{\text{max}} = k_N \). More specifically \( k_{\text{min}} \approx 0.01 \text{ h Mpc}^{-1} \) for both sets A and B, while \( k_{\text{max}} \approx 1.22 \text{ h Mpc}^{-1} \) for set A and \( k_{\text{max}} \approx 5.9 \text{ h Mpc}^{-1} \) for set B. To be conservative we restrict our analysis to Fourier modes up to \( k \approx 1 \text{ h Mpc}^{-1} \).

The covariance matrix is computed using the unbiased sample covariance estimator:

\[
\hat{\text{cov}}(k_1, k_2) = \frac{1}{N_s - 1} \sum_{i=1}^{N_s} [\hat{P}(k_1) - \bar{P}(k_1)] [\hat{P}(k_2) - \bar{P}(k_2)],
\]

where \( N_s \) is the number of independent realizations and \( \bar{P}(k) = \sum_{i=1}^{N_s} \hat{P}(k)/N_s \) is the sample mean, with \( \bar{P}(k) \) the matter power spectrum estimation of the \( i \)-th realization.

3 DEUS-PUR COVARIANCE MATRIX

Let us consider the formal expression of the matter power spectrum covariance matrix (see e.g. Scoccimarro, Zaldarriaga & Hui 1999):

\[
\text{cov}(k_1, k_2) = \frac{2}{N_k} P^2(k_1) \delta_3(k_1, k_2) + \frac{1}{V} \int \Delta_{k_1} \int \Delta_{k_2} \left[ T(\vec{k}_1 - \vec{k}_1', \vec{k}_2, -\vec{k}_2') / V \right],
\]

where \( P(k) \) is the matter power spectrum, \( N_k \approx k^2 \Delta V/(2\pi^2) \) is the number of \( k \)-modes in the volume \( V \), \( \Delta_k \) is the band power integration interval centered on the mode \( k \), and \( V \) is the integration volume in Fourier space; the integrand \( T(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) \) is the trispectrum (the fourth order connected moment in Fourier space) of the density fluctuation field. The first term in Eq. (1) represents the Gaussian contribution to the covariance. As already mentioned, for an initial Gaussian density field, during the linear regime the Fourier modes evolve independently and the power spectrum covariance is diagonal with amplitude \( 2P^2(k)/N_k \). The second term in Eq. (2) represents the contribution of non-Gaussianity arising during the non-linear regime of gravitational collapse at small scales.

Non-linearities induce mode couplings which source a non-vanishing trispectrum of the density fluctuation field. Since this cannot be computed exactly then the covariance matrix must be estimated by sampling the matter power spectrum from a large ensemble of numerical N-body simulations. This computation is not exempt of systematic uncertainties. For instance, the finite volume of simulations is source of non-Gaussian errors (Rimes & Hamilton 2006) and as shown in Takahashi et al. (2009) this can introduce large uncertainties even on weakly non-linear scales. We leave a detailed study of this effect to a forthcoming work. In the following, we focus on systematic errors due to the mass resolution of the simulations, which have been neglected in previous studies.

3.1 Numerical Simulation Mass Resolution Errors

In Fig. [1] we plot the diagonal elements of the matter power spectrum covariance matrix normalized to the linear Gaussian amplitude \( 2P^2(k)/N_k \) for Set A and B at \( z = 0, 0.3, 0.5, 1 \) and 2. The curves corresponding to Set A have negligible noise due to the large size of the simulation sample. This is not the case of Set B for which the covariance estimates are characterized by a higher level of noise. As expected, the onset of the non-linear regime causes deviations from the Gaussian prediction which occur at large \( k \)-values and shift towards smaller ones at lower redshifts. For instance, a deviation of a factor \( \sim 5 \) at \( z = 1 \) occurs at \( k \sim 0.55 \text{ h Mpc}^{-1} \), while at \( z = 0 \) the same deviation occurs at \( k \sim 0.30 \text{ h Mpc}^{-1} \). The effect of such deviations is to increase the statistical errors on the power spectrum measurements at non-linear scales. Despite the higher level of statistical noise associated to Set B, it is evident that there is a systematic down shift of the variance of lower resolution simulations. In the bottom panel of Fig. [1] we can see that such a discrepancy exceeds the statistical noise of Set B at redshifts \( z > 0.5 \) in the range of modes \( 0.20 \leq k \leq 0.80 \text{ h Mpc}^{-1} \) with an amplitude that on average can be as large as \( \sim 40 \) per cent. This is a direct consequence of the suppression of the matter power spectrum at large/intermediate \( k \) for lower mass resolution simulations.

The mass resolution of numerical simulations is a known source of systematic errors and a generic feature of simulations which rely on the PM method (Joyce et al. 2009; Heitmann et al. 2010; Rasera et al. 2014). The suppression of power at small scales for lower mass resolution simulations is due to the combination of the precision of force calculation, which is based on grid discretization of the gravitational field, and the particle sampling of the matter density field. In a PM code the number of coarse grids is usually set equal to the number of particles, therefore, before any refinement is triggered, particles in under-dense or over-dense regions will experience a reduced force by the close environment compared to higher resolution simulations. Knebe et al. (2001) showed that for PM based codes the ideal configuration is to have 8 times more grid points than particles, but in a cosmological simulation this requirement may be too expensive from the computational point of view. None the less, the artificial suppression of power is mitigated at low redshifts and/or higher \( k \) when the local density of particles in the simulations triggers the AMR grid refinement, which explains why this systematic effect shown in Fig. [1] fades away across the whole interval at \( z < 0.5 \). Rasera et al. (2014) corrected the BAO spectrum for the mass effects.
resolution effect by combining the cosmic variance limited power spectrum from DEUS-FUR with that of smaller volume and higher resolution simulations. Here, we opt for a similar strategy. In fact, the covariance is obtained by sampling the matter power spectrum of independent realizations, thus we can correct the lower resolution power spectra to also account for the mass resolution effect on the off-diagonal components.

In Fig. 2 we show the diagonal elements of the covariance matrix from the corrected spectra of Set A (dot-dashed line). Middle panel: relative difference of the uncorrected variance of Set A (central) with respect to the corrected one. Corrections can be as large as 40 per cent at $z = 2$, 20 per cent at $z = 1$ and less than 10 per cent at lower redshifts. Bottom panel: relative difference of the variance from the higher resolution simulations Set B with respect to the correct variance. The residuals show no systematic shift indicating that the correction efficiently accounts for the mass resolution effect.

Figure 2. Top panel: as in Fig. 1 including the diagonal components of the covariance matrix from the corrected spectra of Set A (dot-dashed line). Middle panel: relative difference of the uncorrected variance of Set A (central) with respect to the corrected one. Corrections can be as large as 40 per cent at $z = 2$, 20 per cent at $z = 1$ and less than 10 per cent at lower redshifts. Bottom panel: relative difference of the variance from the higher resolution simulations Set B with respect to the correct variance. The residuals show no systematic shift indicating that the correction sufficiently accounts for the mass resolution effect.

In Fig. 3 we plot $r(k_1, k_2)$ in the interval $0.03 < k/[h\text{ Mpc}^{-1}] < 1.00$ (which includes the BAO range) at $z = 2$ (left panels), 1 (central panels) and 0 (right panels) for Set A (top panels) and for Set B (bottom panels). We do not show the correlation coefficient inferred from the corrected spectra of Set A since this coincides with the uncorrected one to very good approximation. This is because the mass resolution effect discussed in the previous section scales approximately linearly with the power spectrum affecting the covariance matrix amplitude. Since correlation coefficient is given by the covariance matrix normalized by the root-square of the product of its diagonal elements the effect cancels out in the ratio.

Non-vanishing off-diagonal elements are already present at $z = 2$ at large $k$ values. The amplitude of the correlations increases as function of $k$ and extends towards smaller $k$ values at lower redshifts as the non-linear evolution reaches larger physical scales. The comparison between the two sets shows the importance of having a large set of simulations, in order to reduce the impact of noise. In fact, the structure of correlations is much clearer for Set A than for Set B, for which at $z = 2$ the signal is hard to distinguish from the statistical noise. It is worth noticing that in the BAO range the correlation in the off-diagonal components can reach a level up to 30–35 per cent between redshift 1 and 0, which confirms the necessity of disposing of an accurate covariance matrix estimation in BAO data analyses.

3.2 Fourier Mode Correlations

In order to quantify the correlation between pairs of Fourier modes it is useful to introduce the correlation coefficient

$$r(k_1, k_2) = \frac{\text{cov}(k_1, k_2)}{\sqrt{\text{cov}(k_1, k_1) \text{cov}(k_2, k_2)}},$$

which varies between 1 (maximum correlation) and $-1$ (maximum anti-correlation), and is 0 when modes are uncorrelated. In linear regime the correlation coefficient is the identity matrix.

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4 Probabilistic Distribution of the Power Spectrum

We now focus on the probability distribution function (PDF) of the matter power spectrum estimator.

In the case of Gaussian initial conditions, during the linear regime of gravitational collapse the matter power spectrum at a given wave-number $k$ is distributed as a $\chi^2$ with $N_k$ degrees-of-freedom (see e.g. Fisher et al. 1993). In the large $N_k$ limit, which corresponds to sufficiently large volumes and high wave-numbers, the PDF tends to a Gaussian. However, at high $k$ the non-linear evolution of matter clustering is expected to introduce non-Gaussianities (i.e. departures from a $\chi^2$-distribution in the large $N_k$ limit).

The large sample of simulations from DEUS-PUR allows us to finely sample such a distribution and test for non-Gaussianities.
To this end it is convenient to rescale the power spectrum estimator as \( \sqrt{N_k/2}(\hat{P}/P - 1) \), such that in the large \( N_k \) limit and in linear regime the distribution is a Gaussian with zero mean and unity variance.

In Fig. 4 we plot the estimated PDF from Set A at \( z = 0 \) and \( k = 0.05, 0.20, 0.40, 0.60 \) and \( 1.00 \) h Mpc\(^{-1} \) respectively. At \( k \leq 0.20 \) h Mpc\(^{-1} \) we can see that the PDF is consistent with a Gaussian distribution, while at higher \( k \) we can clearly see deviations from Gaussianity which we quantify in terms of the skewness and kurtosis defined as:

\[
S_3(k) = \frac{\langle (\hat{P}(k) - \bar{P}(k))^3 \rangle}{\langle (\bar{P}(k) - P(k))^2 \rangle^{3/2}},
\]

\[
S_4(k) = \frac{\langle (\hat{P}(k) - \bar{P}(k))^4 \rangle}{\langle (\bar{P}(k) - P(k))^2 \rangle^2} - 3.
\]

In the case of a \( \chi^2 \)-distribution with \( N_k \) degree-of-freedom these can be computed exactly resulting in \( S_3(k) = \sqrt{8/N_k} \) and \( S_4(k) = 12/N_k \) (see e.g. Takahashi et al. 2009).

In Fig. 5 we plot \( S_3(k) \) (top panels) and \( S_4(k) \) (bottom panels) from Set A normalized to the corresponding \( \chi^2 \) expected values and estimated at \( z = 105 \) and \( z = 0.3, 0.5 \) (where mass resolution effects are subdominant) respectively. We may notice that at \( z = 105 \) the values of \( S_3(k) \) are consistent with those from a \( \chi^2 \)-distribution, while for \( z < 0.5 \) and \( k \geq 0.25 \) h Mpc\(^{-1} \) we can clearly see increasing deviations as function of \( k \). In the case of the kurtosis this is consistent with predictions from the \( \chi^2 \)-distribution at \( z > 0.5 \), while for lower redshifts the ratio \( S_4(k)/(12/N_k) \) tends to systematically shift toward large positive values at \( k \geq 0.40 \) h Mpc\(^{-1} \), although due the large noise fluctuations the departure of the kurtosis from the Gaussian still remains uncertain.

Previous studies have determined the power spectrum distribution using smaller simulation ensembles and at low redshifts found no statistically significant deviation of the skewness from expectations of a Gaussian random density field (see e.g. Takahashi et al. 2009). This stresses the necessity of using very large samples of simulations. We believe that such a result can have important observational implications which warrant further investigation. At large \( k \) and \( z < 0.5 \) the ratio \( S_3(k)/\sqrt{8/N_k} \gtrsim 2 \), hence unbiased measurements of the band power from observables of the clustering of matter such as weak lensing observations (see also Sato et al. 2009) may require prior knowledge of the \( \bar{P}(k) \) distribution. At lower \( k \) the departure from a \( \chi^2 \)-distribution is at most of a factor 2 for \( k \lesssim 0.30 \) h Mpc\(^{-1} \). Thus, measurements of the BAO may still be performed using only covariance matrix information, though aiming at sub-percent accuracy may require a more detailed study to elucidate the full impact of the non-Gaussian distribution of \( P(k) \).
Figure 4. Probability distribution of the rescaled power spectrum estimator $\sqrt{N_b/2}(P/P - 1)$ estimated from the 12288 realizations of Set A for $k = 0.05$ (green star) 0.20 (blue cross) 0.40 (red diamond), 0.60 (light-blue triangle) and 1.00 $h$ Mpc$^{-1}$ (magenta circle) respectively. The solid line curves show the Gaussian distribution with sample mean and variance of the same set at the corresponding values of $k$.

5 SIGNAL-TO-NOISE

We estimate the effect of the correlation between band power errors on the signal-to-noise of the matter power spectrum

$$\left(\frac{S}{N}\right)^2 = \sum_{k_1,k_2 < k_{\text{max}}} P(k_1) \psi(k_1,k_2) P(k_2), \quad (7)$$

where $\psi = \text{cov}^{-1}(k_1,k_2)$ is the inverse of covariance matrix, also known as precision matrix. Since we estimate the covariance from a finite ensemble of independent realizations, there is a statistical error associated to the sample covariance. Thus, inverting the sample covariance gives a biased estimate of the precision matrix. For a Gaussian random density field the unbiased estimator of the precision matrix is given by (see e.g. Hartlap, Simon & Schneider 2007; Taylor, Joachimi & Kitching 2013):

$$\psi(k_1,k_2) = \frac{N_b - N_0 - 2}{N_b - 1} \text{cov}^{-1}(k_1,k_2), \quad (8)$$

where $\text{cov}$ is the covariance estimator defined in Eq. (1), $N_b$ is the number of realizations and $N_0$ is the number of band power bins. This estimator is defined only for $N_b > N_0 + 2$. In any case, for $N_b < N_0 + 1$ the values of the sample covariance are not positive definite and its inverse is not defined (Hartlap, Simon & Schneider 2007).

In evaluating the signal-to-noise we set the power spectrum in Eq. (4) to the average of the corrected spectra from Set A and compute the signal-to-noise using the precision matrix defined by Eq. (8) for the corrected and uncorrected spectra of Set A respectively.

In Fig. 5 we plot the resulting signal-to-noise estimates at $z = 0, 0.3, 0.5, 1$ and 2 respectively as function of $k_{\text{max}}$. We can see that the effect of mass resolution errors is to artificially enhance the signal-to-noise. As discussed in Section 3.1 this is because lower mass resolution simulations underestimate the covariance matrix. This results into a greater amplitude of the precision matrix components and consequently in a larger signal-to-noise compared to higher mass resolution estimates. As we can see in Fig. 6 the signal-to-noise from the corrected Set A is up to $\sim 15$ per cent smaller at $k_{\text{max}} \geq 0.30 h$ Mpc$^{-1}$, while at lower redshift (where the mass resolution effect is negligible) the S/N from the corrected and uncorrected Set A agree within a few percent. For comparison, we also plot the expected S/N in the Gaussian case. As already noted by Angulo et al. (2008), the signal-to-noise saturates above the signal-to-noise. As discussed in Section 3.1 this is because lower mass resolution simulations underestimate the covariance matrix. This results into a greater amplitude of the precision matrix components and consequently in a larger signal-to-noise compared to higher mass resolution estimates. As we can see in Fig. 6 the signal-to-noise from the corrected Set A is up to $\sim 15$ per cent smaller at $k_{\text{max}} \geq 0.30 h$ Mpc$^{-1}$, while at lower redshift (where the mass resolution effect is negligible) the S/N from the corrected and uncorrected Set A agree within a few percent. For comparison, we also plot the expected S/N in the Gaussian case. As already noted by Angulo et al. (2008), the signal-to-noise saturates above...
a redshift dependent scale (see also Smith 2009; Takahashi et al. 2009).

In Fig. 7 we plot the signal-to-noise at $k_{\text{max}} = 0.40 \, h \, \text{Mpc}^{-1}$ (a scale on the plateau of the S/N) as function of the number of realizations at different redshifts. As we can see the signal-to-noise converges for $N_s > 4000$ at all the redshifts within a few percent.

Here, it is worth noticing that the signal-to-noise has been estimated using an unbiased estimator of the precision matrix, Eq. (8). However, this is only valid in the Gaussian case for which the probability distribution of the inverse Wishart distribution is given by the inverse-Wishart distribution (Press 1982). As we have shown in the previous Section, at low redshift and high $k$ the statistics of the matter power spectrum deviates from that of Gaussian random density field. Hence, one may wonder whether such deviations may also imply a departure of the precision matrix from the inverse-Wishart distribution. This is certainly plausible, nevertheless we think that it has very little effect on our estimate of the signal-to-noise. In fact, the S/N is a cumulative quantity that for low redshifts ($z < 1$) saturates at $k_{\text{max}} \approx 0.25 \, h \, \text{Mpc}^{-1}$, while the non-Gaussian deviations grow significantly at larger $k$, where there is little contribution to the overall signal-to-noise. In contrast deviations from the inverse-Wishart distribution may be relevant for cosmological model parameter estimations and studies along the line of the analysis by (Taylor, Joachimi & Kitching 2013, Taylor & Joachimi 2014) using the DEUS-PUR simulations can be very informative. A study which we leave to future work.

6 CONCLUSION

The covariance matrix of the matter power spectrum is an essential ingredient to infer unbiased cosmological parameter constraints from measurements of the clustering of matter in the universe. The next generation of galaxy surveys will precisely measure the matter power spectrum across a wide range of scales. This demands for an accurate estimation of the covariance matrix. Here, we have tackled this task using a large ensemble of numerical N-body simulations from the DEUS-PUR project. First, we have assessed the impact of numerical systematic uncertainties due to mass resolution effects using a reduced sample of higher resolution simulations. We have provided an empirical statistical method to correct for this source of non-Gaussian error on the covariance matrix. By taking advantage of the large statistics of the DEUS-PUR simulations we have finely sampled the power spectrum probability distribution. We have found a non-vanishing skewness which deviates from the expectations of a Gaussian density field at low redshifts ($z \leq 0.5$) with the amplitude of the deviations increasing on scales $k \gtrsim 0.25 \, h \, \text{Mpc}^{-1}$. The non-Gaussian errors resulting from the non-linear regime of gravitational collapse mildly affect the spectrum on BAO scale, in contrast they become important at smaller scales. This suggests that an unbiased estimate of the ensemble averaged band power from finite volume surveys at these scales and redshifts may require the full probability distribution of the matter power spectrum. The ensemble of N-body simulations from the DEUS-PUR project can provide a valuable support to these future analyses.

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The goal here is to find a correction that maps each of the spectra estimated from Set A. Here, we derive the mass resolution correction to the matter power spectrum.

APPENDIX A: MASS RESOLUTION CORRECTION TO MATTER POWER SPECTRUM

Here, we derive the mass resolution correction to the matter power spectra estimated from Set A.

We assume that the corrected power spectrum estimator, $\hat{P}_\Lambda^{\text{corr}}$, and that of the lower resolution simulations, $\bar{P}_\Lambda$, are related by a simple linear transformation:

$$\hat{P}_\Lambda^{\text{corr}} = a \bar{P}_\Lambda + b.$$  \hspace{1cm} (A1)

The goal here is to find a correction that maps each of the $\bar{P}_\Lambda$ from the pdf of Set A, $f(\bar{P}_\Lambda)$, into the one of Set B, $f(\bar{P}_B)$. Since the proposed correction has two parameters, we only need the first two moments of $f(\bar{P}_B)$ to correct the spectra of Set A. In principle one can assume higher order corrections, but then higher moments of $f(\bar{P}_B)$ are needed for the computation, and the statistics of our sample is not sufficient to resolve them.

We determine the coefficients $a$ and $b$ by imposing that the average $\bar{P}_\Lambda^{\text{corr}} = \bar{P}_B$ and the variance $\sigma_\Lambda^{2 \text{corr}} = \sigma_B^2$. Those conditions translate in the system:

$$\begin{align*}
\bar{P}_B &= a \bar{P}_\Lambda + b, \\
\sigma_B^2 &= a^2 \sigma_\Lambda^2.
\end{align*}$$  \hspace{1cm} (A2, A3)

We plot $\sigma_B/\sigma_\Lambda$ and the best-fitting smoothing function at $z = 0, 0.3, 0.5, 1$ and 2 respectively. The best-fitting values of the parameters are quoted in Table A1.

![Figure A1](image_url)

**Figure A1.** Ratio of the standard deviation of the spectra from Set B and A, $\sigma_B/\sigma_\Lambda$, as function of $k$ at $z = 0, 0.3, 0.5, 1$ and 2 respectively. The solid lines are the best-fitting smoothing functions.
### Table A1. Best-fitting values for the parameters $\alpha$, $\beta$, $\gamma$ and $\delta$.  

| $z$ | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|-----|----------|---------|----------|---------|
| 0   | 0.156    | 0.569   | -0.442   | 0.146   |
| 0.3 | 0.098    | -0.039  | -0.290   | 0.188   |
| 0.5 | -0.410   | 0.695   | -0.895   | 0.343   |
| 1   | 0.266    | 1.026   | 0.280    | 0.252   |
| 2   | 2.240    | -5.272  | 2.924    | -0.051  |