On the Brieskorn (a,b)-module of an isolated hypersurface singularity.

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Abstract

We show in this note that for a germ $g$ of holomorphic function with an isolated singularity at the origin of $\mathbb{C}^n$ there is a pole for the meromorphic extension of the distribution

$$\frac{1}{\Gamma(\lambda)} \int_X |g|^{2\lambda} g^{-n} \Box$$

at $-n-\alpha$ when $\alpha$ is the smallest root in its class modulo $\mathbb{Z}$ of the reduce Bernstein-Sato polynomial of $g$. This is rather unexpected result comes from the fact that the self-duality of the Brieskorn (a,b)-module $E_g$ associated to $g$ exchanges the biggest simple pole sub-(a,b)-module of $E_g$ with the saturation of $E_g$ by $b^{-1}a$.

In the first part of this note, we prove that the biggest simple pole sub-(a,b)-module of the Brieskorn (a,b)-module $E$ of $g$ is ”geometric” in the sense that it depends only on the hypersurface germ $\{g = 0\}$ at the origin in $\mathbb{C}^n$ and not on the precise choice of the reduced equation $g$, as the poles of (*)

By duality, we deduce the same property for the saturation $\tilde{E}$ of $E$. This duality gives also the relation between the ”dual” Bernstein-Sato polynomial and the usual one, which is the key of the proof of the theorem.

Key words Isolated hypersurface singularity, Brieskorn (a,b)-module, Bernstein-Sato polynomial, dual Bernstein-Sato polynomial.

AMS Classification : 32-S-05, 32-S-25, 32-S-40.
1 Introduction.

Let $\tilde{g} : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ a germ of holomorphic function with an isolated singularity. Denote by $g : X \to D$ a Milnor representative of $\tilde{g}$. Let $b_g$ be the reduced Bernstein-Sato polynomial of $g$. Let $\alpha$ be the biggest root of $b_g$ in its class modulo $\mathbb{Z}$. A classical question is whether for $j \in \mathbb{N}$ big enough the meromorphic extension of the distribution

$$\frac{1}{\Gamma(\lambda)} \int_X |g|^{2\lambda} \bar{g}^{-j} \square$$

has a pole at $\lambda = \alpha$.

The present note gives a result which, in a sense, suggests that, may be, this question is not the good one.

Let me introduce the dual Bernstein-Sato polynomial of $g$ by the formula

$$b_g^*(z) = (-1)^q b_g(-n - z)$$

where $q := \text{deg}(b_g)$. Recall that all roots of $b_g$ (and $b_g^*$) are contained in $]-n, 0[$, see [K.76] for the inequality $< 0$, and the section 3 for the inequality $> -n$.

We shall prove the following result.

**Théorème 1.0.1** Let $\alpha$ be the smallest root of $b_g$ in its class modulo $\mathbb{Z}$, and let $d$ be its multiplicity (as a root of $b_g$). Then the meromorphic extension of the distribution

$$\frac{1}{\Gamma(\lambda)} \int_X |g|^{2\lambda} \bar{g}^{-n} \square$$

has a pôle of order $\geq d$ at $-n - \alpha$.

**Remarks.**

1. In general $b_g^* \neq b_g$ so it is not clear that $-n - \alpha$ is a root of $b_g$. But, of course, the previous theorem implies that there exists at least $d$ roots of $b_g$ (counting multiplicities) which are bigger than $-n - \alpha$. If $-n - \alpha \in [-1, 0]$ then there is no choice : $-n - \alpha$ is a root of multiplicity $\geq d$ of $b_g$.

2. This result gives, in term of the Bernstein-Sato polynomial $b_g$, a precise value where we know that a pole appears in the class $[\beta]$ modulo $\mathbb{Z}$ of a root $\beta$ of $b_g$. But the pole which is given is not at the biggest root of $b_g$ in this class but at the biggest root of $b_g^*$ in this class !

A clear reason for that is given in the proof: the dual Bernstein-Sato polynomial is the minimal polynomial of $-b^{-1}a$ acting on $F/b.F$ where $F$ is the biggest simple pole sub-(a,b)-module of the Brieskorn (a,b)-module $E$ associated to $g$. So it lies in the lattice given by holomorphic forms.

\[1\text{recall that we are dealing with negative numbers.}\]
On the contrary, \( b_g \) is the minimal polynomial of \( -b^{-1}a \) acting on \( \bar{E}/b\bar{E} \) where \( \bar{E} \) is the saturation of \( E \) by \( b^{-1}a \), or, in other words, the minimal simple pole \((a,b)\)-module containing \( E \). So, if \( E \) is not a simple pole \((a,b)\)-module, elements in \( \bar{E} \) are not always representable in the holomorphic lattice, and so we may need some power of \( g \) as denominators. And this may introduce integral shifts for the poles.

3. The case where \( E \) is a simple pole \((a,b)\)-module (that is to say when we have \( F = E = \bar{E} \)) corresponds to a quasi-homogeneous \( g \), with a suitable choice of coordinates. In this case we have \( b_g^* = b_g \), so \( -n - \alpha \) is the smallest root of \( b_g \) in its class modulo \( \mathbb{Z} \).

In the first part of this note, we prove that the biggest simple pole sub-(a,b)-module of the Brieskorn \((a,b)\)-module \( E \) of \( g \) is "geometric" in the sense that it depends only on the hypersurface germ \( \{ g = 0 \} \) at the origin in \( \mathbb{C}^n \) and not on the precise choice of the reduced equation \( g \).

Remark that the poles of the meromorphic distributions \( \frac{1}{\pi(\lambda)} \int_X |g|^{2\lambda} \bar{g}^{-j} \Box \) are also "geometric" in the sense above.

By duality, we deduce the same property for the saturation \( \bar{E} \) of \( E \). This duality gives also the relation between the dual Bernstein-Sato polynomial and the usual one, which is the key of the proof of the theorem.

2 Changing the reduced equation.

Let \( g : X \to D \) be a Milnor representative of a germ of an holomorphic function with an isolated singularity at the origin of \( \mathbb{C}^n, n \geq 2 \). We define the function
\[
f(t, x) := e^t g(x) \quad \text{where} \quad f : \mathbb{C} \times X \to \mathbb{C}
\]
and we denote by \( \pi : \mathbb{C} \times \mathbb{C} \times X \) the projection defined by \( \pi(\lambda, t, x) = (t, x) \). We shall denote by \( F \) the function \( \pi^*(f) \). Its critical locus is \( S := \mathbb{C} \times \mathbb{C} \times \{0\} \).

We consider on \( Y = \{ F = 0 \} \), as in [B.05], the complex of sheaves \( ((\text{Ker} dF)^*, d^*) \). The following theorem is an easy generalization of [B.05] th.2.2 (case LII).

**Théorème 2.0.2** In the situation describe above, the \( n \)-th cohomology sheaf of the complex \( ((\text{Ker} dF)^*, d^*) \) is a constant sheaf whose fiber is \( F_g \) the biggest simple pole sub-(a,b)-module of the Brieskorn \((a,b)\)-module \( E_g \) associated to the function \( g \).

It is easy to deduce from the previous theorem the following corollary.

**Corollaire 2.0.3** Let \( g \) be a germ of an holomorphic function with an isolated singularity at the origin of \( \mathbb{C}^n \). Let \( h \) be any invertible holomorphic germ at the origin. Then the biggest simple pole sub-(a,b)-module of the Brieskorn \((a,b)\)-module associated to the function \( h.g \) does not depend on the choice of \( h \) up to isomorphism.
More precisely, if the holomorphic invertible function depends holomorphically on some parameter $\lambda$ in a complex manifold $\Lambda$, the subsheaf of the sheaf on $\Lambda$ defined by the Brieskorn $(a, b)$-modules of the fibers, which is given in each fiber by the biggest simple pole sub-$(a, b)$-module of the Brieskorn $(a, b)$-module, is a locally constant sheaf on $\Lambda$.

Proof of the theorem. Let us first consider the case of an holomorphic function $f$ on a complex manifold $Z$ and let the holomorphic function $F$ be $F := \pi^*(f)$ on $\mathbb{C} \times Z$ where $\pi: \mathbb{C} \times Z \to Z$ is the projection.

In this situation we have the following description of $(\hat{\operatorname{Ker}} dF)^p$:

$$(\hat{\operatorname{Ker}} dF)^p = \pi^*((\hat{\operatorname{Ker}} df)^p) \oplus d\lambda \wedge \pi^*((\hat{\operatorname{Ker}} df)^{p-1}).$$

Then $\alpha \oplus d\lambda \wedge \beta \in (\hat{\operatorname{Ker}} dF)^p$ is $d-$closed iff it satisfies:

$$d/\alpha = 0 \text{ and } \frac{\partial \alpha}{\partial \lambda} = d/\beta$$

where $\frac{\partial \alpha}{\partial \lambda}$ is defined by the equation $d\alpha = d/\alpha + d\lambda \wedge \frac{\partial \alpha}{\partial \lambda}$.

Lemme 2.0.4 In the situation above set $Y = \{f = 0\}$; we have the short exact sequence of complex of sheaves on $\mathbb{C} \times Y$:

$$0 \to (\hat{\operatorname{Ker}} df^\bullet, d^\bullet) \to (\pi^*(\hat{\operatorname{Ker}} df^\bullet), d^\bullet) \xrightarrow{i} (\pi^*(\hat{\operatorname{Ker}} df^\bullet), d^\bullet) \to 0.$$

So if the sheaf $\hat{\mathcal{H}}^{p-1}_f$ is 0 on $Z$ for $p \geq 3$ or is isomorphic to $E_1 \otimes \mathbb{C}_Y$ for $p = 2$, then we have for $p \geq 2$ the exact sequence of sheaves on $\mathbb{C} \times Y$:

$$0 \to \hat{\mathcal{H}}^p_f \to \pi^*(\hat{\mathcal{H}}^p_f) \xrightarrow{\partial/\partial \lambda} \pi^*(\hat{\mathcal{H}}^p_f).$$

Proof. Here the sheaf $\pi^*(\hat{\mathcal{H}}^p_f)$ is defined via $\lambda-$relative holomorphic forms. On this complex we have a derivation $\partial/\partial \lambda$ commuting with the product by the function $F$, the wedge product with $dF$ and the $\lambda-$relative de Rham differential denoted by $d/\lambda$. Remark also that we have $d/\lambda = dF$.

The exactness of the short exact sequence of complexes is obvious and the associated long exact cohomology sequence is enough to conclude for $p \geq 3$. For the $p = 2$ case, we have only to check the injectivity of the map $i$.

Let $\alpha \oplus d\lambda \wedge \beta \in (\hat{\operatorname{Ker}} df)^p \cap \overline{\operatorname{Ker}} d$; its image by $i$ is the class $[\alpha]$. If it vanishes

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2we define this sheaf via the cohomology of the formal completion of the de Rham complex of $\Lambda-$relative holomorphic forms annihilated by $\wedge dF$.

3recall that $E_1 := \mathbb{C}[[b]].e_1$ where $a.e_1 = b.e_1$. 
3. The dual Bernstein-Sato polynomial.

We shall now consider an (a,b)-module E such that

\[ \pi \ast (\hat{H}_p f) \]

we can find \( \gamma \in \pi \ast \left( (\hat{K}_{\text{ker} df})^{-1} \right) \) such that \( d/\gamma = \alpha \). Differentiating with respect to \( \lambda \) gives, using the relation \( \partial \beta \partial = d/\beta \),

\[ d/\left( \beta - \partial \gamma \partial \right) = 0. \]

But as \( \beta - \partial \gamma \partial \in \pi \ast \left( (\hat{K}_{\text{ker} df})^{-1} \right) \) this form induces a class in \( \pi \ast (\hat{H}_p f)^{-1} \). So we can write

\[ \beta = \partial \gamma \partial + \phi(\lambda, f) df, \]

where \( \phi \in \pi \ast (\text{C}[\lambda]) \). We obtain, if \( \partial \psi \partial = \phi(\lambda, f) df \),

\[ d(\beta - \partial \gamma \partial) = 0. \]

The corollary follows, because we can always join two invertible functions to one another (the restriction of a constant sheaf is a constant sheaf).

End of the proof of the theorem.
follows: we define on the $\mathbb{C}[[b]]$-module $\text{Hom}_{\mathbb{C}[[b]]}(E, F)$, which is free and of finite rank, an action of $a$ by the formula:

$$(a.\varphi)(x) = a_F \varphi(x) - \varphi(a_E x), \quad \forall x \in E.$$ 

Of course, we have to check that $a.\varphi$, defined in this way, is $\mathbb{C}[[b]]$-linear and that we have $a.b.\varphi - b.a.\varphi = b^2.\varphi$. It is not difficult to check also that $\text{Hom}_{a,b}(E, F)$ is regular when $E$ and $F$ are regular (see [B.95]).

Recall also that the Brieskorn $(a,b)$-module of a germ of holomorphic function with an isolated singularity in $\mathbb{C}^n$ satisfies properties i) and ii) above with $\delta = n$, see [Be.01].

**Proposition 3.0.5** Under hypotheses i) and ii) above, let $F$ be the biggest simple pole sub-$(a,b)$-module in $E$, and let $\tilde{E}$ the saturation of $E$ for $b^{-1}a$. Then we have natural isomorphisms of $(a,b)$-modules deduced from $\kappa$:

$$\kappa' : \tilde{E} \to \text{Hom}_{a,b}(F, E_{\delta}) \quad \text{and} \quad \kappa'' : F \to \text{Hom}_{a,b}(\tilde{E}, E_{\delta}).$$

In the proof of this proposition we shall use the following lemmas.

**Lemme 3.0.6** Let $E$ and $F$ be simple pole $(a,b)$-modules. Then $\text{Hom}_{a,b}(E, F)$ is also a simple pole $(a,b)$-module.

**Proof.** Fix an element $\varphi \in \text{Hom}_{a,b}(E, F)$. Then define $\theta : E \to F$ by the formula $\theta(x) := b^{-1}.a.\varphi(x) - b^{-1}.\varphi(a.x)$ for all $x \in E$. As $E$ has a simple pole, we have $a.x \in b.E$ and so $\varphi(a.x) \in b.F$ from $b$-linearity of $\varphi$. But $F$ has also a simple pole, so $b^{-1}.a : F \to F$ is well defined.

Now $\theta$ is $b$-linear:

$$\theta(b.y) = b^{-1}.a.\varphi(b.y) - b^{-1}.\varphi(a.b.y) = (a + b).\varphi(y) - \varphi((a + b).y)$$

$$= a.\varphi(y) - \varphi(a.y) = b.\theta(y).$$

But we have $a.\varphi = b.\theta$ in $\text{Hom}_{a,b}(E, F)$. Therefore $\text{Hom}_{a,b}(E, F)$ is a simple pole $(a,b)$-module. ■

**Lemme 3.0.7** Let $E$ be a regular $(a,b)$-module and let $\delta$ be any complex number. Then we have a canonical $(a,b)$-module isomorphism

$$\tau : E \to \text{Hom}_{a,b}(\text{Hom}_{a,b}(E, E_{\delta}), E_{\delta}).$$

**Proof.** The map $\tau$ is defined by $x \to \tau(x)[\varphi] = \varphi(x)$. It is obviously a $b$-linear isomorphism. So we have only to check the $a$-linearity. But, with the notation $\theta = \tau(x)$, we have:

$$(a.\theta)[\varphi] = a.(\theta[\varphi]) - \theta[a.\varphi] = a.\varphi(x) - (a.\varphi(x) - \varphi(a.x)) = \tau(a.x)[\varphi].$$

And so $a.\tau(x) = \tau(a.x)$. ■
Lemme 3.0.8 Let $E$ and $F$ be two $(a,b)$-modules. Then we have a canonical isomorphism

$$\text{Hom}_{a,b}(E,F) \rightarrow \text{Hom}_{a,b}(\tilde{E},\tilde{F}).$$

Proof. It is clear that $\text{Hom}_{a,b}(\tilde{E},\tilde{F})$ is the same complexe vector space than $\text{Hom}_{a,b}(E,F)$ and that the action of $b$ on it is given by $-b$. The fact that the action of $a$ is the opposite of the action of $a$ on $\text{Hom}_{a,b}(E,F)$ follows also directly from the definition of $\text{Hom}_{a,b}$. $\blacksquare$

Proof of proposition 3.0.5. The functor $\text{Hom}_{a,b}(-,E_\delta)$ applied to the inclusion of $E$ in $\tilde{E}$ gives an $(a,b)$-linear injection

$$\text{Hom}_{a,b}(\tilde{E},E_\delta) \hookrightarrow \text{Hom}_{a,b}(E,E_\delta) \simeq \tilde{E}.$$

As $\text{Hom}_{a,b}(\tilde{E},E_\delta)$ has a simple pole by lemma 3.0.6 it is contained in $\tilde{F}$, by definition of $F$. Apply now the functor $\text{Hom}_{a,b}(-,E_\delta)$ to the inclusions

$$\text{Hom}_{a,b}(\tilde{E},E_\delta) \hookrightarrow \tilde{F} \hookrightarrow \tilde{E}$$

This gives $(a,b)$-linear injections

$$\text{Hom}_{a,b}(\tilde{E},E_\delta) \hookrightarrow \text{Hom}_{a,b}(\tilde{F},E_\delta) \hookrightarrow \tilde{E}$$

using lemma 3.0.7. But, as $\tilde{E}_\delta$ is canonically isomorphic to $E_\delta$, so we have isomorphims

$$\text{Hom}_{a,b}(\tilde{E},E_\delta) \simeq \text{Hom}_{a,b}(\tilde{E},\tilde{E}_\delta) \simeq \text{Hom}_{a,b}(E,E_\delta)^\sim \simeq \tilde{E} \simeq E$$

using lemma 3.0.8 and our hypothesis on $E$. So the simple pole $(a,b)$-module $\text{Hom}_{a,b}(\tilde{F},E_\delta)$ which lies between $E$ and $\tilde{E}$ is equal to $\tilde{E}$. We conclude using again the canonical isomorphism between $E_\delta$ and $\tilde{E}_\delta$ and the lemma 3.0.7. $\blacksquare$

Remark.

In the situation of the proposition 3.0.5 the non-degenerate $(a,b)$-bilinear pairing

$$h : \tilde{E} \times E \rightarrow E_\delta$$

deduced from $\kappa$ via the formula $h(x,y) := \kappa(x)[y]$, gives also non-degenerate $(a,b)$-bilinear pairings

$$h' : \tilde{E} \times F \rightarrow E_\delta \quad \text{and} \quad h'' : \tilde{F} \times \tilde{E} \rightarrow E_\delta$$

deduced from $\kappa'$ and $\kappa''$ via the formulas $h'(x,y) := \kappa'(x)[y]$ and $h''(u,v) = \kappa''(u)[v]$.

An obvious consequence of proposition 3.0.5 is the following corollary of the theorem 2.0.2.
Corollaire 3.0.9 Let $g$ be a germ of an holomorphic function having an isolated singularity at the origin in $\mathbb{C}^n$ where $n \geq 2$. For any holomorphic invertible germ $h$ at the origin, the saturation by $b^{-1}a$ of the Brieskorn $(a,b)$-module of the germ $h.g$ is independant, up to an isomorphism of $(a,b)$-module, of the choice of $h$. If the invertible $h$ depends holomorphically of a parameter $\lambda$ in a complex manifold $\Lambda$, the sheaf on $\Lambda$ defined by the saturations of the Brieskorn $(a,b)$-modules of the germs $h_\lambda.g$ is a locally constant sheaf on $\Lambda$.

4 Poles of $\int_X |g|^{2.\lambda}$.

We shall begin by a simple definition.

Démonstration 4.0.10 Let $E$ be a regular $(a,b)$-module. We shall call dual Bernstein polynomial of $E$, denoted by $b^*_{\delta E}$, the minimal polynomial of the linear endomorphism $-b^{-1}.a$ acting on the (finite dimensional) vector space $F/b.F$ where $F$ is the biggest simple pole sub-(a,b)-module of $E$. Recall that the Bernstein-Sato polynomial of $E$ is the minimal polynomial of the action of $-b^{-1}.a$ on the (finite dimensional) vector space $\tilde{E}/b.\tilde{E}$, where $\tilde{E}$, as before, is the saturation of $E$ by $b^{-1}.a$. In other words, $\tilde{E}$ is the smallest simple pole $(a,b)$-module which contains $E$. This can be understood in two ways. Either you look in $E[b^{-1}]$ for the smallest simple pole $(a,b)$-module containing $E$. The other way is to consider the inclusion $E \to \tilde{E}$ as the initial element for inclusions of $E$ in simple poles $(a,b)$-modules.

Remark.

Let $\delta$ a given complex number, and assume that the $(a,b)$-module $E$ is equipped with an $(a,b)$-linear isomorphism

$$\kappa : \tilde{E} \to \text{Hom}_{a,b}(E, E_{\delta}).$$

Then we have $b^*_{\delta E}(z) = (-1)^r.b_{\delta E}(-\delta - z)$ where $r := \text{deg}(b_{\delta E})$, since $b^{-1}a$ acts on the same way on $E$ and $\tilde{E}$.

So, for the Brieskorn $(a,b)$-module of a germ of an holomorphic function $g$ with an isolated singularity at the origin of $\mathbb{C}^n$ the dual Bernstein polynomial is given by

$$b^*_g(z) = (-1)^r b_g(-n - z).$$

Using Malgrange positivity theorem it is easy to show that the roots of $b^*_g$ are strictly negative. This gives, using [K.76], the fact that the roots of $b_g$ are contained in $]-n, 0[$.
Proof of the theorem \[1.0.1\] The only new point for this proof, compared to [B.84 a] and [B.84 b], is the following:

In a simple pole \((a,b)\)-module \(F\), if a spectral value \(\beta\) of multiplicity \(d\) for the action of \(b^{-1}.a\) on \(F/bF\), is minimal in its class modulo \(\mathbb{Z}\), there exists elements \(e_1, \ldots, e_d\) in \(F\), giving a Jordan block of size \(d\) for \(b^{-1}a\) acting on \(F/bF\), and such that they satisfy in \(F\) the relations

\[ a.e_j = \beta.b.e_j + b.e_{j-1}, \quad \forall j \in [1, d] \]

with the convention \(e_0 = 0\) (see [B.93]).

This enable us, using the standard technics of [B.84 a], to build up \((n-1)\)-holomorphic forms \(\omega_1, \ldots, \omega_d\) in a neighbourhood of the origin in \(\mathbb{C}^n\), such that

\[ d\omega_j = \beta.\frac{dg}{g} \wedge \omega_j + \frac{dg}{g} \wedge \omega_{j-1}, \quad \forall j \in [1, d] \]

with the convention \(\omega_0 = 0\), which induce a Jordan block of size \(d\) in \(H^{n-1}(F, \mathbb{C})\) where \(F\) is the Milnor fiber of \(g\), for the eigenvalue \(exp(-2i\pi . \beta)\) of the monodromy.

So we avoid in this way the integral shifts coming from the use of a lattice which may be not contained in the one given by holomorphic forms and we can realize the pole of our statement for \(\lambda = -\beta\), using the same strategy than in [B.84a] for eigenvalues \(\neq 1\) and [B.84 b] for the eigenvalue 1. \(\blacksquare\)
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