(Anti-)Chiral Superfield Approach to Interacting Abelian 1-Form Gauge Theories: Nilpotent and Absolutely Anticommuting Charges

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Abstract: We derive the off-shell nilpotent (fermionic) (anti-)BRST symmetry transformations by exploiting the (anti-)chiral superfield approach (ACSA) to Becchi-Rouet-Stora-Tyutin (BRST) formalism for the interacting Abelian 1-form gauge theories where there is a coupling between the $U(1)$ Abelian 1-form gauge field and Dirac as well as complex scalar fields. We exploit the (anti-)BRST invariant restrictions on the (anti-)chiral superfields to derive the fermionic symmetries of our present D-dimensional Abelian 1-form gauge theories. The novel observation of our present investigation is the derivation of the absolute anticommutativity of the nilpotent (anti-)BRST charges despite the fact that our ordinary D-dimensional interacting Abelian 1-form gauge theories are generalized onto the $(D, 1)$-dimensional (anti-)chiral super manifolds (of the general $(D, 2)$-dimensional supermanifold) where only the (anti-)chiral super expansions of the (anti-)chiral superfields have been taken into account. We also discuss the nilpotency of the (anti-)BRST charges and (anti-)BRST invariance of the Lagrangian densities of our present interacting Abelian 1-form gauge theories within the framework of ACSA to BRST formalism.

PACS numbers: 11.15.-q, 11.30.Pb

Keywords: Interacting $U(1)$ Abelian 1-form gauge theories; Dirac and complex scalar fields; ACSA to BRST formalism; chiral and anti-chiral superfields; (anti-)BRST invariant restrictions; conserved charges; nilpotency and absolute anticommutativity.
1 Introduction

The usual superfield approach (USFA) to Becchi-Rouet-Stora-Tyutin (BRST) formalism exploits the idea of horizontality condition (HC) for the derivations of the (anti-) BRST symmetries for the gauge and corresponding (anti-)ghost fields as well as the Curci-Ferrari condition of a non-Abelian 1-form gauge theory (see, e.g. [1-8]). However, it does not shed any light on the (anti-)BRST symmetry transformations associated with the matter fields of an interacting (non-)Abelian gauge theory where there is a coupling between the gauge and matter fields. The USFA has been systematically generalized so as to derive the nilpotent (anti-)BRST symmetry transformations for the matter, gauge and (anti-)ghost fields together. The latter superfield approach has been christened as the augmented version of superfield approach (AVSA) to BRST formalism (see, e.g. [9-12]). In the above superfield approaches [1-12], the full super expansions of the superfields have been taken into account along all possible Grassmannian directions of the (D, 2)-dimensional supermanifold on which the ordinary D-dimensional gauge theory is generalized.

In our recent works [13-15], a simpler version of the AVSA to BRST formalism has been proposed where only the (anti-)chiral superfields have been taken into account for the derivation of the (anti-)BRST symmetry transformations. In the USFA/AVSA to BRST formalism, a given D-dimensional gauge theory is generalized onto a (D, 2)-dimensional supermanifold (characterized by the superspace coordinates $Z^M = (x^\mu, \theta, \bar{\theta})$ where $x^\mu (\mu = 0, 1...D - 1)$ are the bosonic coordinates and a pair of Grassmannian variables $(\theta, \bar{\theta})$ satisfy: $\theta^2 = 0 = \bar{\theta}^2$, $\theta \bar{\theta} + \bar{\theta} \theta = 0$). On the contrary, in the (anti-)chiral superfield approach (ACSA) to BRST formalism, a given D-dimensional gauge theory is generalized onto the (D, 1)-dimensional (anti-)chiral super-submanifolds of the general (D, 2)-dimensional supermanifold. In our present endeavor, we have considered ACSA to BRST formalism and discussed various aspects of the (anti-)BRST symmetries and (anti-)BRST charges of an interacting Abelian 1-form gauge theory where there is a coupling between the $U(1)$ gauge field ($A_\mu$) and fermionic ($\psi^2 = \bar{\psi}^2 = 0, \psi \bar{\psi} + \bar{\psi} \psi$) Dirac fields ($\psi, \bar{\psi}$). We have also discussed briefly the interacting Abelian 1-form $U(1)$ gauge theory with complex scalar fields where there is a coupling between the gauge and matter fields (i.e. complex scalar fields) and have utilized the potential and power of the ACSA to BRST formalism to derive the proper (anti-)BRST symmetries (cf. Appendix A).

The key results of our present investigation are the proof of nilpotency and absolute anticommutativity of the (anti-)BRST conserved charges within the framework of ACSA to BRST formalism. The derivation of the (anti-)BRST symmetries and their nilpotency properties have been discussed in all the previous works [9-15]. However, the proof of the absolute anticommutativity of the (anti-)BRST charges is a novel result because, in our earlier works on AVSA, we have not discussed this aspect of the (anti-)BRST charges [9-12]. In fact, the discussion about the ACSA to BRST formalism and the proof of absolute anticommutativity of the nilpotent (anti-)BRST charges (within the framework of ACSA to BRST formalism) have been a set of challenging problems for us and we have resolved these issues in our present investigation in an elegant manner (despite the fact that we have taken into account only the (anti-)chiral super expansions of the superfields). We have derived the (anti-)BRST symmetries by imposing the (anti-)BRST invariant restrictions on the (anti-)chiral superfields which are the quantum analogues of the classical gauge invariant
restrictions (GIRs) that have been utilized in our earlier works [9-12].

Against the backdrop of the above discussions, it is pertinent to point out that the ACSA has also been applied to \( \mathcal{N} = 2 \) supersymmetric (SUSY) quantum mechanical systems of interest in our earlier works [16-19] where we have derived the nilpotent \( \mathcal{N} = 2 \) SUSY transformations in an elegant manner. We have also derived the conserved \( \mathcal{N} = 2 \) SUSY charges and expressed them in terms of the supervariables obtained after the appropriate \( \mathcal{N} = 2 \) SUSY invariant restrictions. However, these charges do not obey the absolute anticommutativity property\(^*\). We have been able to capture the nilpotency property of the super charges in the terminology of the (anti-)chiral supervariables. We have not been able to say anything, however, about the absolute anticommutativity property between two \( \mathcal{N} = 2 \) SUSY conserved charges. Thus, in our present endeavor, the proof of the absolute anticommutativity property between the (anti-)BRST charges is a completely novel result.

The main motivations behind our present endeavor are as follows. First of all, we demonstrate that the absolute anticommutativity of the (anti-)BRST charges is true even if we take only the (anti-)chiral super expansions of the superfields. This is a novel observation within the framework of ACSA to BRST formalism. Second, we have established a surprising observation that the anticommutativity of the BRST charge with anti-BRST charge is deeply connected with the nilpotency \((\partial^2_\theta = 0)\) property of the translational generator \((\partial_\theta)\) along the \(\theta\)-direction of the chiral super submanifold (and the anticommutativity of the anti-BRST charge with BRST charge is intimately related to the nilpotency \((\partial^2_{\bar{\theta}} = 0)\) of the translational generator \((\partial_{\bar{\theta}})\) along \(\bar{\theta}\)-direction of the anti-chiral super submanifold). Third, our present idea has been generalized to the proof of nilpotency and absolute anticommutativity of the (anti-)BRST charges of an interacting \(SU(N)\) non-Abelian gauge theory [20], too. Finally, our method of derivation supports the results that have been obtained from the mathematically precise use of HC for the self-interacting (non-)Abelian 1-form theory without any interaction with matter fields (see, e.g. [4, 5] for details).

Theoretical material of our present paper is organized as follows. In Sec. 2, we discuss the bare essentials of (anti-)BRST symmetries for the Lagrangian density of an interacting D-dimensional Abelian 1-form gauge theory in the Feynmen gauge and derive the conserved charges. Our Sec. 3 deals with the ACSA to BRST formalism where we derive the BRST symmetries using the anti-chiral superfields. Sec. 4 of our present endeavor is devoted to the derivation of anti-BRST symmetries by using the ACSA to BRST formalism where the chiral superfields are utilized. In Sec. 5, we express the conserved (anti-)BRST charges on the \((D, 1)\)-dimensional super submanifolds (of the general \((D, 2)\)-dimensional supermanifold on which our theory is generalized) and provide the proof of nilpotency and absolute anticommutativity properties of the (anti-)BRST charges within the framework of ACSA to BRST formalism. We discuss the (anti-)BRST invariance of the Lagrangian density, within the framework of ACSA to BRST formalism, in Sec. 6. Finally, we summarize our key results in Sec. 7 and point out a few theoretical directions for future investigations within the framework of superfield formalism.

In our Appendix A, we discuss the absolutely anticommuting (anti-)BRST charges for the interacting Abelian 1-form gauge theory where there is a coupling between the \(U(1)\)

\(\text{\footnotesize *In the case of } \mathcal{N} = 2 \text{ SUSY quantum mechanical models, the anticommutator of two distinct SUSY transformations on a variable leads to the time derivative on that specific variable.}\)
gauge field and complex scalar fields. The subject matter of Appendix B concerns itself with the natural and automatic proof of the absolute anticommutativity property of the (anti-)BRST symmetries (and corresponding charges) when the full super expansions of the superfields are taken into account.

2 Preliminaries: Lagrangian Formulation

First of all, we begin with the interacting D-dimensional Abelian 1-form gauge theory where there is a coupling between the $U(1)$ gauge field ($A_\mu$) and the Dirac fields ($\bar{\psi}, \psi$). The Lagrangian density for this system, in the Feynman gauge, is as follows (see, e.g. [21])

$$\mathcal{L}_B = -\frac{1}{4} F^\mu\nu F_{\mu\nu} + \bar{\psi} (i \gamma^\mu D_\mu - m) \psi + B (\partial \cdot A) + \frac{B^2}{2} - i \partial_\mu \bar{C} \partial^\mu C,$$  

(1)

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor and $D_\mu \psi = \partial_\mu \psi + i e A_\mu \psi$ is the covariant derivative on the Dirac field $\psi$. The gauge-fixing term ($-\frac{1}{2} (\partial \cdot A)^2$) has been linearized by invoking the Nakanishi-Lautrup auxiliary field $B$, $\gamma^\mu$ (with $\mu = 0, 1, 2, \ldots D - 1$) are the $(D \times D)$ Dirac gamma matrices, $m$ is the rest mass of the Dirac particle and the fermionic ($C^2 = \bar{C}^2 = 0$, $C \bar{C} + \bar{C} C = 0$) (anti-)ghost fields ($\bar{C} C$) are needed for the validity of unitary in the theory. It is evident that the fermionic fields ($\psi, \bar{\psi}, C, \bar{C}$) anticommute among themselves and they commute with the bosonic fields $A_\mu$ and $B$ of our theory. It is also elementary to state that the bosonic fields commute among themselves.

The above Lagrangian density respects the following infinitesimal, continuous, off-shell nilpotent ($s^2_{(a)b} = 0$) and absolutely anticommuting ($s_b s_{ab} + s_{ab} s_b = 0$) (anti-)BRST transformations $s_{(a)b}$

$$s_{ab} A_\mu = \partial_\mu \bar{C}, \quad s_{ab} \bar{C} = 0, \quad s_{ab} C = -i B, \quad s_{ab} B = 0,$$
$$s_{ab} \bar{\psi} = -i e \bar{\psi} C, \quad s_{ab} \psi = -i e \bar{C} \psi, \quad s_{ab} F_{\mu\nu} = 0, \quad s_{ab} (\partial \cdot A) = \Box \bar{C},$$
$$s_b A_\mu = \partial_\mu C, \quad s_b \bar{C} = i B, \quad s_b B = 0,$$
$$s_b \bar{\psi} = -i e \bar{\psi} C, \quad s_b \psi = -i e C \psi, \quad s_b F_{\mu\nu} = 0, \quad s_b (\partial \cdot A) = \Box C,$$

(2)

because the Lagrangian density (1) transforms to the total spacetime derivatives

$$s_{ab} \mathcal{L}_B = \partial_\mu [B \partial^\mu \bar{C}], \quad s_b \mathcal{L}_B = \partial_\mu [B \partial^\mu C],$$

(3)

thereby rendering the action integral $S = \int d^D x \mathcal{L}_B$ invariant for the physically well-defined fields that vanish off at infinity. The conserved currents, due to Noether’s theorem, are:

$$J^\mu_{ab} = -F^{\mu\nu} \partial_\nu \bar{C} + B \partial^\mu \bar{C} - e \bar{\psi} \gamma^\mu \bar{C} \psi,$$
$$J^\mu_b = -F^{\mu\nu} \partial_\nu C + B \partial^\mu C - e \bar{\psi} \gamma^\mu C \psi.$$

(4)

1The background flat D-dimensional Minkowskian spacetime manifold is endowed with a metric tensor $\eta_{\mu\nu} = \text{diag} (+1, -1, -1, \ldots)$. This implies that the short-hand notations, in their explicit forms, are: $(\partial \cdot A) = \eta_{\mu\nu} \partial_\mu A^\nu \equiv \partial_\mu A^\nu \equiv \partial_0 A_0 - \partial_i A_i$ and $\Box = \eta_{\mu\nu} \partial^\mu \partial^\nu = \partial^2_0 - \partial^2_i$. Here the Greek indices $\mu, \nu, \lambda, \ldots = 0, 1, 2, \ldots D - 1$ and Latin indices $i, j, k, \ldots = 1, 2, \ldots D - 1$ correspond to the spacetime and space directions, respectively, on the flat Minkowskian spacetime manifold.
Using the following Euler-Lagrange (EL) equations of motion (EOM)
\[
\begin{align*}
\partial_\mu F^{\mu \nu} - \partial_\nu B &= e \bar{\psi} \gamma^\nu \psi, \quad \Box C = 0, \quad (i \gamma^\mu \partial_\mu - m) \psi = e \gamma^\mu A_\mu \psi; \\
i (\partial_\mu \bar{\psi}) \gamma^\mu + m \bar{\psi} &= -e \bar{\psi} \gamma^\mu A_\mu, \quad B = -(\Theta \cdot A), \quad \Box \bar{C} = 0,
\end{align*}
\]
it can be readily checked that \(\partial_\mu J^a_{(a)b} = 0\). Thus, the conserved and the off-shell nilpotent (anti-)BRST charges \(Q_{(a)b} = \int d^{D-1}x \, J^a_{(a)b}\) can be expressed (with \(\dot{B} = \frac{\partial B}{\partial t} \), \(\dot{C} = \frac{\partial C}{\partial t}\), etc.) as follows
\[
Q_{ab} = \int d^{D-1}x \left[ -F^{0i} \partial_i C + B \dot{C} - e \bar{\psi} \gamma^0 C \psi \right] \equiv \int d^{D-1}x \left[ B \dot{C} - \dot{B} C \right],
\]
\[
Q_b = \int d^{D-1}x \left[ -F^{0i} \partial_i C + B \dot{C} - e \bar{\psi} \gamma^0 C \psi \right] \equiv \int d^{D-1}x \left[ B \dot{C} - \dot{B} C \right],
\]
where we have used the EOM: \(\partial_i F^{0i} = -(\dot{B} + e \bar{\psi} \gamma^0 \psi)\) and carried out a partial integration to drop the total space derivative term due to Gauss’s divergence theorem. We have also used the convention of left-derivative while deriving the EL-EOM w.r.t. to the fermionic fields \((C, \bar{C}, \psi, \bar{\psi})\) of our present interacting D-dimensional Abelian 1-form gauge theory with Dirac’s fields \((\psi, \bar{\psi})\).

The above conserved charges are nilpotent \((Q_{(a)b}^2 = 0)\) of order two and absolutely anticommuting \((Q_b Q_{ab} + Q_{ab} Q_b = 0)\) in nature (cf. Sec. 5 below for details). In fact, the above conserved charges \((Q_{(a)b})\) in (6) and (7) are the generators of the infinitesimal, continuous and nilpotent (anti-)BRST symmetry transformations \(s_{(a)b}\). In other words, we have the following explicit relationship
\[
s_{(a)b} \Phi = \mp i [\Phi, Q_{(a)b}]\pm,
\]
where \(\Phi(= A_\mu, C, \bar{C}, B, \psi, \bar{\psi})\) is the generic field of the theory and \((\pm)\) signs, as the subscripts on the square bracket, denote the (anti)commutator for the generic field \(\Phi\) being fermionic/bosonic in nature. The \((\mp)\) signs in front of the bracket has to be chosen [22] judiciously for the derivation of nilpotent and absolutely anticommuting (anti-)BRST symmetry transformations (i.e. \(s_{(a)b} \Phi\)).

### 3 Nilpotent BRST Symmetries: Anti-Chiral Superfields and Their Super Expansions

We derive here the nilpotent BRST symmetries of Eq. (2) by applying ACSA to BRST formalism where we use the anti-chiral superfields only. To this end in mind, first of all, we generalize the ordinary fields of the Lagrangian density (1) onto (D, 1)-dimensional anti-chiral super-submanifold (of the general (D, 2)-dimensional supermanifold) as follows:
\[
\begin{align*}
A_\mu(x) &\longrightarrow B_\mu(x, \bar{\theta}) = A_\mu(x) + \bar{\theta} R_\mu(x), \quad C(x) \longrightarrow F(x, \bar{\theta}) = C(x) + i \bar{\theta} B_1(x), \\
\bar{C}(x) &\longrightarrow \bar{F}(x, \bar{\theta}) = \bar{C}(x) + i \bar{\theta} B_2(x), \quad \psi(x) \longrightarrow \psi(x, \bar{\theta}) = \psi(x) + i \bar{\theta} b_1(x), \\
\bar{\psi}(x) &\longrightarrow \bar{\psi}(x, \bar{\theta}) = \bar{\psi}(x) + i \bar{\theta} b_2(x), \quad B(x) \longrightarrow \bar{B}(x, \bar{\theta}) = B(x) + i \bar{\theta} f(x),
\end{align*}
\]
where the fields \((R_\mu, f)\) are the fermionic secondary fields and \((B_1, B_2, b_1, b_2)\) are the bosonic secondary fields that have to be determined in terms of the basic and auxiliary fields of the theory (cf. Eq. (1)) by invoking the BRST invariant restrictions\(^3\). It is straightforward to note that the \((D, 1)\)-dimensional anti-chiral super-submanifold is parameterized by \(x^\mu\) and \(\bar{\theta}\). This is why, it is called as anti-chiral.

According to the basic tenets of ACSA/AVSA to BRST formalism, the BRST invariant quantiles must remain independent of the Grassmannian variable (\(\bar{\theta}\)) when they are generalized onto the \((D, 1)\)-dimensional anti-chiral super submanifold (see, e.g. [9-15]). Such useful and interesting BRST invariant quantities are:

\[
\begin{align*}
& s_b C = 0, \quad s_b B = 0, \quad s_b (\bar{\psi} \bar{\psi}) = 0, \quad s_b (\bar{\psi} D_\mu \psi) = 0, \quad s_b (A^\mu \partial_\mu C) = 0, \\
& s_b [A^\mu \partial_\mu B + i \partial_\mu \bar{C} \partial^\mu C] = 0, \quad s_b (C \psi) = 0, \quad s_b (\bar{\psi} C) = 0.
\end{align*}
\]

(10)

The above quantities are to be generalized onto the \((D, 1)\)-dimensional anti-chiral super-submanifold with the following restrictions:

\[
\begin{align*}
F(x, \bar{\theta}) &= C(x), \quad \bar{\Psi}(x, \bar{\theta}) F(x, \bar{\theta}) = \bar{\psi}(x) C(x), \quad \bar{\Psi}(x, \bar{\theta}) \bar{\Psi}(x, \bar{\theta}) = \bar{\psi}(x) \bar{\psi}(x) \\
B^\mu(x, \bar{\theta}) \partial_\mu F(x, \bar{\theta}) &= A^\mu(x) \partial_\mu C(x), \quad F(x, \bar{\theta}) \bar{\Psi}(x, \bar{\theta}) = C(x) \bar{\psi}(x), \\
\bar{B}(x, \bar{\theta}) &= B(x), \quad \bar{\Psi}(x, \bar{\theta}) \bar{\partial}_\mu \Psi(x, \bar{\theta}) + i e \bar{\psi}(x, \bar{\theta}) B_\mu(x, \bar{\theta}) \Psi(x, \bar{\theta}) \\
&= \bar{\psi}(x) \bar{\partial}_\mu \psi(x) + i e \bar{\psi}(x) A_\mu(x) \psi(x), \quad B^\mu(x, \bar{\theta}) \partial_\mu \bar{B}(x, \bar{\theta}) + i \partial_\mu \bar{F}(x, \bar{\theta}) \partial^\mu F(x, \bar{\theta}) \\
&= A^\mu(x) \partial_\mu B(x) + i \partial_\mu \bar{C}(x) \partial^\mu C(x).
\end{align*}
\]

(11)

These restrictions lead to the derivation of the expressions for the secondary fields (of the super expansions (9)) in terms of the basic and auxiliary fields as:

\[
\begin{align*}
R_\mu &= \partial_\mu C, \quad B_1(x) = 0, \quad B_2(x) = B(x), \\
b_1 &= - e C \psi, \quad b_2 = - e \bar{\psi} C, \quad f(x) = 0.
\end{align*}
\]

(12)

We elaborate on a few of the above derivations here. It is evident that the first entry and the restriction \(\bar{B}(x, \bar{\theta}) = B(x)\) in (11) produce the following results (with \(f(x) = 0, B_1(x) = 0\), namely;

\[
\begin{align*}
F^{(b)}(x, \bar{\theta}) &= C(x) + \bar{\theta} (0) \equiv C(x) + \bar{\theta} (s_b C(x)), \\
\bar{B}^{(b)}(x, \bar{\theta}) &= B(x) + \bar{\theta} (0) \equiv B(x) + \bar{\theta} (s_b B(x)),
\end{align*}
\]

(13)

where the superscript \((b)\) stands for the super expansions of the anti-chiral superfields that have been derived after the BRST invariant restrictions (11) and which lead to: \(s_b C = 0, s_b B = 0\). We use (13) now in

\[
\begin{align*}
B^\mu(x, \bar{\theta}) \partial_\mu F^{(b)}(x, \bar{\theta}) &= A^\mu(x) \partial_\mu C(x), \\
B^\mu(x, \bar{\theta}) \partial_\mu \bar{B}^{(b)}(x, \bar{\theta}) + i \partial_\mu \bar{F}(x, \bar{\theta}) \partial^\mu F^{(b)}(x, \bar{\theta}) \\
&= A^\mu(x) \partial_\mu B(x) + i \partial_\mu \bar{C}(x) \partial^\mu C(x),
\end{align*}
\]

(14)

\(^3\)The BRST and anti-BRST invariant restrictions are at the quantum level and these are the analogues of the classical gauge invariant restrictions (GIRs) where we demand that the physical (i.e. gauge invariant) quantities should be independent of the “soul” coordinates \((\theta, \bar{\theta})\).
leading to the following relationships:

\[ R^\mu(x) \partial_\mu C(x) = 0 \implies R_\mu(x) \propto \partial_\mu C(x), \]
\[ R^\mu(x) \partial_\mu B(x) - \partial_\mu B_2(x) \partial^\mu C(x) = 0 \implies R_\mu = \partial_\mu C(x), \quad B_2(x) = B(x). \quad (15) \]

We have chosen, for the shake of brevity: \( R_\mu = \partial_\mu \), which implies that \( B_2(x) = B(x) \).

These inputs lead to the following:

\[ B^{(b)}_\mu(x, \bar{\theta}) = A_\mu(x) + \bar{\theta} (\partial_\mu C(x)) \equiv A_\mu(x) + \bar{\theta} (s_b A_\mu(x)), \]
\[ F^{(b)}(x, \bar{\theta}) = C(x) + \bar{\theta} (i B) \equiv C(x) + \bar{\theta} (s_b C(x)). \quad (16) \]

Thus far, we have derived the BRST symmetry transformations for the gauge field and corresponding (anti-)ghost fields of our present D-dimensional interacting Abelian theory with Dirac’s fields which are usually derived by exploiting the theoretical power and potential of HC (see, e.g. [4, 5]).

We are in the position now to derive the BRST symmetry transformations for the matter fields \((\bar{\psi}, \psi)\). Towards this goal in mind, we note that the restrictions, corresponding to the BRST invariances \( s_b(C \psi) = 0 \) and \( s_b(\bar{\psi} C) = 0 \) in (11), imply: \( C(x) b_1(x) = 0 \) and \( b_2(x) C(x) = 0 \). In other words, the secondary fields \( b_1(x) \) and \( b_2(x) \) are proportional to the ghost field \( C(x) \). The condition \( \bar{\Psi}(x, \bar{\theta}) \Psi(x, \bar{\theta}) = \bar{\psi}(x) \psi(x) \) leads us to conclude that \( b_2 \psi = \bar{\psi} b_1 \). With these inputs, we write down the final restriction of Eq. (11) as:

\[ \bar{\Psi}(x, \bar{\theta}) \partial_\mu \Psi(x, \bar{\theta}) + i e \bar{\Psi}(x, \bar{\theta}) B^{(b)}_\mu(x, \bar{\theta}) \Psi(x, \bar{\theta}) = \bar{\psi}(x) \partial_\mu \psi(x) + i e \bar{\psi}(x) A_\mu(x) \psi(x), \quad (17) \]

where the exact form of \( B^{(b)}_\mu(x, \bar{\theta}) \) has been illustrated in Eq. (16). The above equality, ultimately, leads to the following:

\[ i e A_\mu (b_2 \psi - \bar{\psi} b_1) - \bar{\psi} \partial_\mu b_1 + b_2 \partial_\mu \psi - e \bar{\psi} \partial_\mu C \psi = 0. \quad (18) \]

The first term is zero because \( b_2 \psi = \bar{\psi} b_1 \) which has been discussed in the paragraph above Eq. (17). In fact, the restriction \( \bar{\Psi}(x, \bar{\theta}) \Psi(x, \bar{\theta}) = \bar{\psi}(x) \psi(x) \) leads to it. The rest of the terms (with inputs \( b_1 \propto C(x) \) and \( b_2 \propto C(x) \)) are satisfied by the following choices:

\[ b_1 = -e C \psi, \quad b_2 = -e \bar{\psi} C \implies b_2 \psi = \bar{\psi} b_1. \quad (19) \]

Thus, finally, we have obtained the following super expansion\(^5\) for all the superfields of our theory (on the anti-chiral (D, 1)-dimensional super submanifold), namely;

\[ B^{(b)}_\mu(x, \bar{\theta}) = A_\mu(x) + \bar{\theta} (\partial_\mu C) \equiv A_\mu(x) + \bar{\theta} (s_b A_\mu(x)), \]
\[ F^{(b)}(x, \bar{\theta}) = C(x) + \bar{\theta} (0) \equiv C(x) + \bar{\theta} (s_b C(x)), \]
\[ F^{(b)}(x, \bar{\theta}) = C(x) + \bar{\theta} (i B) \equiv C(x) + \bar{\theta} (s_b C(x)), \]
\[ \Psi^{(b)}(x, \bar{\theta}) = \bar{\psi}(x) + \bar{\theta} (- i e C \psi) \equiv \bar{\psi}(x) + \bar{\theta} (s_b \bar{\psi}(x)), \]
\[ \bar{\Psi}^{(b)}(x, \bar{\theta}) = \bar{\psi}(x) + \bar{\theta} (- i e \bar{\psi} C) \equiv \bar{\psi}(x) + \bar{\theta} (s_b \bar{\psi}(x)), \]
\[ \bar{B}^{(b)}(x, \bar{\theta}) = B(x) + \bar{\theta} (0) \equiv B(x) + \bar{\theta} (s_b B(x)). \quad (20) \]

\(^5\)We have taken the coefficient of \( \bar{\theta} \) as the BRST transformation on an ordinary field because it has already been proven that \( s_b \leftrightarrow \partial_\theta \) (see, e.g. [1-15] for details).
where the superscript \((b)\) on the anti-chiral superfields denotes the fact that these superfields have been obtained after the use of BRST invariant quantities \((10)\) (which lead to their explicit form in Eq. \((11)\)). It is also evident that all the relationships listed in \((12)\) are correct and the coefficients of \(\tilde{\theta}\) in \((20)\) are nothing but the BRST symmetry transformations\(^\dagger\) quoted in Eq. \((2)\) for the Lagrangian density \((1)\). Hence, we have the mapping between the BRST symmetry transformation and partial derivative w.r.t. Grassmannian variable on the anti-chiral super-submanifold as: \(s_b \longleftrightarrow \partial_b\) \([1-15]\).

4 Nilpotent Anti-BRST Symmetries: Chiral Superfields and Their Super Expansions

We derive, in this section, the nilpotent anti-BRST symmetry transformations for the Lagrangian density \((1)\) which are listed in Eq. \((2)\). To accomplish this goal precisely, we have to generalize the ordinary fields of Lagrangian density \((1)\) onto the \((D, 1)\)-dimensional chiral super submanifold (of the \((D, 2)\)-dimensional supermanifold) as

\[
A_\mu(x) \longrightarrow B_\mu(x, \theta) = A_\mu(x) + i \theta \tilde{R}_\mu(x), \quad C(x) \longrightarrow F(x, \theta) = C(x) + i \theta \tilde{B}_1(x),
\]
\[
\tilde{C}(x) \longrightarrow \tilde{F}(x, \theta) = \tilde{C}(x) + i \theta \tilde{B}_2(x), \quad \psi(x) \longrightarrow \Psi(x, \theta) = \psi(x) + i \theta \tilde{b}_1(x),
\]
\[
\bar{\psi}(x) \longrightarrow \bar{\Psi}(x, \theta) = \bar{\psi}(x) + i \theta \tilde{b}_2(x), \quad B(x) \longrightarrow \tilde{B}(x, \theta) = B(x) + i \theta \tilde{f}(x),
\]

where the pair of secondary fields \((\tilde{R}_\mu, \tilde{f})\) are fermionic in nature in contrast to the secondary fields \((\tilde{B}_1, \tilde{B}_2, \tilde{b}_1, \tilde{b}_2)\) which are bosonic. These secondary fields would be determined in terms of the basic and auxiliary fields of the Lagrangian density \((1)\) by exploiting the strength of ACSA/AVSA to BRST formalism where the anti-BRST invariant quantities would be required to be independent of the Grassmannian variable \(\theta\) (which characterizes the chiral super-submanifold along with the bosonic coordinates \(x^\mu\)).

In the above connection, we have the following:

\[
s_{ab} \bar{C} = 0, \quad s_{ab} B = 0, \quad s_{ab} (\bar{\psi} \psi) = 0, \quad s_{ab} (\bar{\psi} D_\mu \psi) = 0, \quad s_{ab} (A^\mu \partial_\mu \bar{C}) = 0,
\]
\[
s_{ab} (\bar{C} \psi) = 0, \quad s_{ab} (\bar{\psi} \bar{C}) = 0, \quad s_{ab} (A^\mu \partial_\mu B + i \partial_\mu \bar{C} \partial^\mu \bar{C}) = 0.
\]

Thus, the above quantities are anti-BRST invariant and, therefore, they should be independent of \(\theta\) when these are generalized onto the chiral super-submanifold according to the basic tenets of ACSA to BRST formalism. In other words, we have the following equalities:

\[
\bar{F}(x, \theta) = \bar{C}(x), \quad \bar{\Psi}(x, \theta) \frac{\partial}{\partial \theta} \Psi(x, \theta) = \bar{\psi}(x) \bar{\psi}(x), \quad \bar{\Psi}(x, \theta) \bar{F}(x, \theta) = \bar{\psi}(x) \bar{C}(x),
\]
\[
+i \partial_\mu \bar{\psi}(x) A_\mu(x) \psi(x), \quad B^\mu(x, \theta) \partial_\mu \bar{F}(x, \theta) = A^\mu(x) \partial_\mu \bar{C}(x),
\]
\[
\bar{F}(x, \theta) \frac{\partial}{\partial \theta} \Psi(x, \theta) = \bar{C}(x) \psi(x), \quad \bar{B}(x, \theta) = B(x), \quad B^\mu(x, \theta) \partial_\mu \bar{B}(x, \theta)
\]
\[
= i \partial_\mu \bar{F}(x, \theta) \partial^\mu \bar{F}(x, \theta) = A^\mu(x) \partial_\mu B(x) + i \partial_\mu \bar{C}(x) \partial^\mu \bar{C}(x).
\]

\(^\dagger\) It is worth pointing out here that the mathematical power of HC leads to the derivation of (anti-) BRST symmetry transformations for the basic fields \((A_\mu, C, \bar{C})\) of the theory (cf. Eqs. \((13)\) and \((16)\)). However, it does not shed any light on the (anti-)BRST transformations for the matter fields \((\psi, \bar{\psi})\) which have been derived in Eq. \((20)\) due to the BRST invariant restrictions (cf. Eqs. \((10)\) and \((11)\)) that have been considered and utilized in our present endeavor.
The above restrictions lead to the derivation of secondary fields in terms of the basic and auxiliary fields of the Lagrangian density (1) as follows:

\[
\bar{R}_\mu = \partial_\mu \bar{C}, \quad \bar{b}_1 = - e \bar{C} \bar{\psi}, \quad \bar{b}_2 = - e \bar{\psi} \bar{C}, \\
\hat{f}(x) = 0, \quad \bar{B}_1 = - B, \quad \bar{B}_2 = 0.
\] (24)

The process of derivation is same as the one that has been adopted and used in the case of BRST symmetries where we have exploited the BRST invariant restrictions on the anti-chiral superfields (cf. Sec 3). The substitution of the above expressions for the secondary fields (cf. (24)) into the super expansions (21) yields the following expressions

\[
B^{(ab)}(x, \theta) = A_\mu(x) + \theta (\partial_\mu \bar{C}) \equiv A_\mu(x) + \theta (s_{ab} A_\mu(x)), \\
F^{(ab)}(x, \theta) = C(x) + \theta (- i B) \equiv C(x) + \theta (s_{ab} C(x)), \\
\bar{F}^{(ab)}(x, \theta) = \bar{C}(x) + \theta (0) \equiv \bar{C}(x) + \theta (s_{ab} \bar{C}(x)), \\
\Psi^{(ab)}(x, \theta) = \psi(x) + \theta (- i e \bar{C} \psi) \equiv \psi(x) + \theta (s_{ab} \psi(x)), \\
\bar{\Psi}^{(ab)}(x, \theta) = \bar{\psi}(x) + \theta (- i e \bar{\psi} \bar{C}) \equiv \bar{\psi}(x) + \theta (s_{ab} \bar{\psi}(x)), \\
\tilde{B}^{(ab)}(x, \theta) = B(x) + \theta (0) \equiv B(x) + \theta (s_{ab} B(x)),
\] (25)

where the superscript \((ab)\) denotes the super expansions of the chiral superfields after the application of anti-BRST invariant restrictions in Eq. (23).

We would like to end this section with the following remarks. First, we have derived the anti-BRST symmetry transformations of (2) by invoking the anti-BRST invariant restrictions on the superfields (cf. Eqs. (22), (23)). Second, the anti-BRST symmetry of an ordinary field corresponds to the translation of corresponding chiral superfield (obtained after the application of anti-BRST invariant restrictions) along \(\theta\)-direction of the \((D, 1)\)-dimensional chiral super-submanifold (of the general \((D, 2)\)-dimensional supermanifold). Finally, the nilpotency \((s_{ab}^2 = 0)\) of the anti-BRST symmetry transformation is intimately connected with the nilpotency \((\partial_\theta^2 = 0)\) of the translation generators \((\partial_\theta)\) along \(\theta\)-direction of the chiral super-submanifold. Similar kinds of observations can be made and stated for the BRST symmetries, too, in the language of ACSA to BRST formalism (cf. Sec. 3).

5 Conserved (Anti-)BRST Charges: Nilpotency and Absolute Anicommutativity Properties

In this section, we shall capture the properties of nilpotency and absolute anticommutativity of the conserved (anti-)BRST charges in the language of ACSA to BRST formalism. It is straightforward to express the BRST charge \((Q_b = \int d^{D-1}x \left[ B \bar{C} - B \bar{C} \right])\) in terms of the
anti-chiral superfields and partial derivative $\partial_{\bar{\theta}}$ and/or differential $d\bar{\theta}$ as

$$Q_b = \frac{\partial}{\partial \bar{\theta}} \int d^{D-1}x \left[ i \hat{F}^{(b)}(x, \bar{\theta}) F^{(b)}(x, \bar{\theta}) - i \hat{\bar{F}}^{(b)}(x, \bar{\theta}) \hat{F}^{(b)}(x, \bar{\theta}) \right]$$

$$\equiv \int d\bar{\theta} \int d^{D-1}x \left[ i \hat{F}^{(b)}(x, \bar{\theta}) F^{(b)}(x, \bar{\theta}) - i \hat{\bar{F}}^{(b)}(x, \bar{\theta}) \hat{F}^{(b)}(x, \bar{\theta}) \right],$$

where the superscript $(b)$ stands for the anti-chiral superfields that have been obtained after the application of the BRST invariant restrictions (11). Thus, the nilpotency ($\partial^2_{\bar{\theta}} = 0$) of the translational generator $\partial_{\bar{\theta}}$ implies that we have

$$\partial_{\bar{\theta}} Q_b = 0 \iff \partial^2_{\bar{\theta}} = 0 \iff s_b Q_b = -i \{Q_b, Q_b\} = 0,$$

which implies the nilpotency ($Q^2_b = 0$) of the BRST charge $Q_b$. Thus, we have shown that there is a deep connection between the nilpotency ($\partial^2_{\bar{\theta}} = 0$) of the translational generator ($\partial_{\bar{\theta}}$) and the nilpotency (i.e. $Q^2_b = 0$) of the BRST charge ($Q_b$).

Now we dwell a bit on the absolute anticommutativity property of the BRST charge ($Q_b$) with the anti-BRST charge ($Q_{ab}$). It can be readily checked that the BRST charge $Q_b$ can also be expressed in terms of the chiral superfields as

$$Q_b = \frac{\partial}{\partial \theta} \int d^{D-1}x \left[ i F^{(ab)}(x, \theta) \hat{F}^{(ab)}(x, \theta) \right]$$

$$\equiv \int d\theta \int d^{D-1}x \left[ i F^{(ab)}(x, \theta) \hat{F}^{(ab)}(x, \theta) \right],$$

where the superscript $(ab)$ denotes the chiral superfields that have been obtained after the application of the anti-BRST invariant restrictions in Eq. (23). It is now straightforward to check that we have the following:

$$\partial_{\theta} Q_b = 0 \iff \partial^2_{\theta} = 0 \iff s_{ab} Q_b = -i \{Q_b, Q_{ab}\} = 0,$$

which proves the absolute anticommutativity of the BRST charge with anti-BRST charge. It is interesting to point out that the above anticommutativity is connected with the nilpotency of the translational generator ($\partial_{\theta}$). Thus, we observe that, for the BRST charge ($Q_b$), the nilpotency ($Q^2_b = 0$) is connected with the nilpotency ($\partial^2_{\theta} = 0$) of the translational generator ($\partial_{\theta}$) along the anti-chiral super-submanifold but the absolute anticommutativity of the BRST charge ($Q_b$) with the anti-BRST charge ($Q_{ab}$) is intimately related to the nilpotency ($\partial^2_{\theta} = 0$) of the translational generator ($\partial_{\theta}$) along the $\theta$-direction of the chiral (D, 1)-dimensional super-submanifold. These are completely novel and interesting observations.

We concentrate now on the off-shell nilpotency and absolute anticommutativity of the anti-BRST charge. The nilpotency can be expressed in the language of the chiral superfields and the Grassmannian partial derivative ($\partial_{\theta}$) as well as the differential ($d\theta$) as:

$$Q_{ab} = \frac{\partial}{\partial \theta} \int d^{D-1}x \left[ i \hat{F}^{(ab)}(x, \theta) \hat{\bar{F}}^{(ab)}(x, \theta) - i \hat{\bar{F}}^{(ab)}(x, \theta) F^{(ab)}(x, \theta) \right]$$

$$\equiv \int d\theta \int d^{D-1}x \left[ i \hat{F}^{(ab)}(x, \theta) \hat{\bar{F}}^{(ab)}(x, \theta) - i \hat{\bar{F}}^{(ab)}(x, \theta) F^{(ab)}(x, \theta) \right].$$

**From the super expansions (20) and (25), it is evident that we have established, in our present endeavor, a relationship between the (anti-)BRST symmetry transformations and the translational generators ($\partial_{\theta}, \partial_{\bar{\theta}}$) along the Grassmannian directions ($\theta, \bar{\theta}$) of the chiral and anti-chiral (D, 1)-dimensional super-submanifolds (of the general (D, 2)-dimensional supermanifold), respectively.
where the chiral superfields with superscript \((ab)\) have been expressed in Eq. (25). It is straightforward to note that the following connections are true, namely;

\[
\partial_{\theta} Q_{ab} = 0 \iff \partial_{\theta}^{2} = 0 \iff s_{ab} Q_{ab} = -i \{ Q_{ab}, Q_{ab} \} = 0. \tag{31}
\]

The last entry, in the above equation, implies the nilpotency \(Q_{ab}^{2} = 0\) of the anti-BRST charge \(Q_{ab}\). It is also evident that the nilpotency of the translational generators \(\partial_{\theta}\) is deeply connected with the nilpotency of the anti-BRST charge. The absolute anticommutativity of the anti-BRST charge \((Q_{ab})\) with the BRST charge \((Q_{b})\) can be written as

\[
Q_{ab} = \frac{\partial}{\partial \theta} \int d^{D-1}x \left[ - \hat{F}^{(b)}(x, \tilde{\theta}) \hat{\dot{F}}^{(b)}(x, \tilde{\theta}) \right] \\
\equiv \int d\theta \int d^{D-1}x \left[ - F^{(b)}(x, \bar{\theta}) \dot{F}^{(b)}(x, \bar{\theta}) \right], \tag{32}
\]

where the anti-chiral superfields with superscript \((b)\) are written in (20). It is obvious that the following mapping is true, namely;

\[
\partial_{\theta} Q_{ab} = 0 \iff \partial_{\theta}^{2} = 0 \iff s_{b} Q_{ab} = -i \{ Q_{ab}, Q_{b} \} = 0, \tag{33}
\]

which establishes the absolute anticommutativity \(\{ Q_{ab}, Q_{b} \} = 0\) of the (anti-)BRST charges \(Q_{(a)b}\). From our discussions, it is clear that the nilpotency \(Q_{ab}^{2} = 0\) of the anti-BRST charge \((Q_{ab})\) is connected with the nilpotency \(\partial_{\theta}^{2} = 0\) of the translational generator \(\partial_{\theta}\) along \(\theta\)-direction of the chiral super-submanifold but the absolute anticommutativity of the anti-BRST charge with the BRST charge is intimately related with the nilpotency \(\partial_{\theta}^{2} = 0\) of the translational generator \(\partial_{\theta}\) along \(\theta\)-direction of the anti-chiral super-submanifold of the \((D, 2)\)-dimensional supermanifold on which our ordinary theory is generalized.

The above properties of the nilpotency and absolute anticommutativity of the (anti-)BRST charges \(Q_{(a)b}\), discussed within the framework of ACSA to BRST formalism, can be expressed in the ordinary space in terms of the (anti-)BRST symmetry transformations \(s_{(a)b}\) (due to their connection with \(\partial_{\theta}\) and \(\partial_{\theta}\)). In other words, these aspects (i.e. nilpotency and absolute anticommutativity) of the conserved (anti-)BRST charges can be easily proven due to the following (anti-)BRST exact forms of them, namely;

\[
Q_{b} = s_{b} \int d^{D-1}x \left[ i \hat{C} \hat{C} - i \hat{\dot{C}} \hat{\dot{C}} \right], \quad Q_{b} = s_{ab} \int d^{D-1}x \left( i C \hat{C} \right),
\]

\[
Q_{ab} = s_{ab} \int d^{D-1}x \left[ i \hat{C} \hat{\dot{C}} - i \hat{\dot{C}} \hat{C} \right], \quad Q_{ab} = s_{b} \int d^{D-1}x \left( - i \hat{C} \hat{\dot{C}} \right). \tag{34}
\]

Applying the symmetry principle on the fermionic operators (cf. Eq. (8)), we obtain:

\[
\begin{align*}
& s_{b} Q_{b} = -i \{ Q_{b}, Q_{b} \} = 0 \iff Q_{b}^{2} = 0 \iff s_{b}^{2} = 0, \\
& s_{ab} Q_{ab} = -i \{ Q_{ab}, Q_{ab} \} = 0 \iff Q_{ab}^{2} = 0 \iff s_{ab}^{2} = 0, \\
& s_{b} Q_{ab} = -i \{ Q_{ab}, Q_{b} \} = 0 \iff \{ Q_{ab}, Q_{b} \} = 0 \iff s_{b}^{2} = 0, \\
& s_{ab} Q_{b} = -i \{ Q_{b}, Q_{ab} \} = 0 \iff \{ Q_{b}, Q_{ab} \} = 0 \iff s_{ab}^{2} = 0. \tag{35}
\end{align*}
\]

Thus, we note that, because of the (anti-)BRST exact forms in (34), we are able to prove the off-shell nilpotency as well as absolute anticommutativity of the conserved (anti-)BRST
charges \( Q^{(a)b} \). The key and crucial role, in the above proof, is played by the concept behind the continuous symmetries and their generators (as the conserved Noether charges). This idea has been backed and bolstered by the nilpotency \((\gamma^{2}_{(a)b} = 0)\) of the (anti-)BRST symmetry transformations as is evident from Eq. (35). We would like to lay emphasis on the fact that Eqs. (34) and (35) have been derived due to our knowledge of ACSA to BRST formalism because the key equations (26), (28), (30) and (32) are responsible for their derivations.

6 (Anti-)BRST Invariance: Superfield Approach

We have seen that the Lagrangian density (1) of our Sec. 2 respects the infinitesimal, continuous and nilpotent (anti-)BRST symmetries because this Lagrangian density transforms to the total spacetime derivatives under the above symmetries. As a consequence, the action integral (corresponding to this Lagrangian density) respects the (anti-)BRST symmetries in a precise and perfect manner. In our present section, we briefly capture the (anti-)BRST invariance (cf. Eq. (3)) of the Lagrangian density (1) in the language of ACSA to BRST formalism.

In this context, we note that the ordinary Lagrangian density \( L_B \) can be generalized onto the (anti-)chiral super-submanifolds in terms of the (anti-)chiral superfields as

\[
\mathcal{L}_B \longrightarrow \bar{\mathcal{L}}^{(ac)}_B = -\frac{1}{4} \tilde{F}^{\mu\nu}(x, \bar{\theta}) \tilde{F}^{\mu\nu}(x, \bar{\theta}) + \bar{\Psi}^{(b)}(x, \bar{\theta}) (i \gamma^\mu \partial_\mu - m) \Psi^{(b)}(x, \bar{\theta}) \\
+ e \bar{\Psi}^{(b)}(x, \bar{\theta}) \gamma^\mu B^{(b)}(x, \bar{\theta}) \Psi^{(b)}(x, \bar{\theta}) + \bar{B}^{(b)}(x, \bar{\theta})(\partial_\mu B^{(b)}(x, \bar{\theta})) \\
+ \frac{1}{2} B^{(b)}(x, \bar{\theta}) \tilde{B}^{(b)}(x, \bar{\theta}) - i \partial_\mu \tilde{F}^{(b)}(x, \bar{\theta}) \partial^\mu F^{(b)}(x, \bar{\theta}),
\]

where \( \tilde{B}^{(b)}(x, \theta) = B(x) \equiv \tilde{B}^{(b)}(x, \bar{\theta}), \ F^{(b)}(x, \bar{\theta}) = C(x), \ \tilde{F}^{(ab)}(x, \theta) = \tilde{C}(x), \ \tilde{F}^{\mu\nu}(x, \bar{\theta}) = F^{\mu\nu}(x) = \tilde{F}^{\mu\nu(b)}(x, \bar{\theta}) \) because of the fact that all these quantities are (anti-)BRST invariant (i.e. \( s_{(a)b}B = 0, \ s_bC = 0, \ s_{ab}C = 0, \ s_{(a)b}F^{\mu\nu} = 0 \)). Thus, the above anti-chiral super Lagrangian density \( \tilde{L}^{(ac)}_B \) and chiral super Lagrangian density \( \tilde{L}^{(c)}_B \) can be re-written as:

\[
\tilde{L}^{(ac)}_B = -\frac{1}{4} F^{\mu\nu}(x) F_{\mu\nu}(x) + \bar{\Psi}^{(b)}(x, \bar{\theta}) (i \gamma^\mu \partial_\mu - m) \Psi^{(b)}(x, \bar{\theta}) \\
- e \bar{\Psi}^{(b)}(x, \bar{\theta}) \gamma^\mu B^{(b)}(x, \bar{\theta}) \Psi^{(b)}(x, \bar{\theta}) + B(x)(\partial_\mu B^{(b)}(x, \bar{\theta})) \\
+ \frac{1}{2} B^{2}(x) - i \partial_\mu \tilde{F}^{(b)}(x, \bar{\theta}) \partial^\mu C(x),
\]

\(^{11}\)In other words, first of all, we express the Lagrangian density (1) in the language of the (anti-) chiral superfields (that are defined on the (anti-)chiral super-submanifolds) and study their key properties by applying the translational generators \((\partial_\theta, \partial_{\bar{\theta}})\) on them.
\[
\tilde{L}_B^{(c)} = -\frac{1}{4} F^{\mu\nu}(x) F_{\mu\nu}(x) + \bar{\Psi}^{(ab)}(x, \theta) (i \gamma^\mu \partial_\mu - m) \Psi^{(ab)}(x, \theta) \\
- e \bar{\Psi}^{(b)}(x, \theta) \gamma^\mu B^{(ab)}_\mu(x, \theta) \Psi^{(ab)}(x, \theta) + B(x)(\partial_\mu B^{(ab)}_\mu(x, \theta)) \\
+ \frac{1}{2} B^2(x) - i \partial_\mu \bar{C}(x) \partial^\mu \tilde{F}^{(ab)}(x, \theta).
\]

(38)

It is worthwhile to mention here that the following are true, namely;

\[
\frac{\partial}{\partial \bar{\theta}} \left[ \bar{\Psi}^{(b)}(x, \theta) (i \gamma^\mu \partial_\mu - m) \Psi^{(b)}(x, \theta) - e \bar{\Psi}^{(b)}(x, \bar{\theta}) \gamma^\mu B^{(b)}_\mu(x, \bar{\theta}) \Psi^{(b)}(x, \bar{\theta}) \right] = 0
\]

\[
\Rightarrow s_b \left[ \tilde{\psi} (i \gamma^\mu D_\mu - m) \psi \right] = 0 \quad \text{and} \quad \frac{\partial}{\partial \bar{\theta}} \left[ \bar{\Psi}^{(ab)}(x, \theta) (i \gamma^\mu \partial_\mu - m) \Psi^{(ab)}(x, \theta) - e \bar{\Psi}^{(ab)}(x, \theta) \gamma^\mu B^{(ab)}_\mu(x, \theta) \Psi^{(ab)}(x, \theta) \right] = 0
\]

\[
\Rightarrow s_{ab} \left[ \tilde{\psi} (i \gamma^\mu D_\mu - m) \psi \right] = 0,
\]

(39)

due to the fact that 

\[
s_{(a)b} \left[ \tilde{\psi} (i \gamma^\mu D_\mu - m) \psi \right] = 0.
\]

Now we are in the position to state the (anti-)BRST invariance (cf. Eq. (3)) of the Lagrangian density (1) in the language of ACSA to BRST formalism. Taking the inputs from (39), we derive the following interesting results, namely;

\[
\frac{\partial}{\partial \bar{\theta}} \left[ \tilde{L}_B^{(c)} \right] = \partial_\mu (B \partial^\mu \bar{C}), \quad \frac{\partial}{\partial \bar{\theta}} \left[ \tilde{L}_B^{(ac)} \right] = \partial_\mu (B \partial^\mu C),
\]

(40)

which establish the geometrical interpretation of the (anti-)BRST invariance, quoted in Eq. (3), in the following manner. The translation of the super Lagrangian density \( \tilde{L}_B^{(ac)} \) along \( \bar{\theta} \)-direction of the (D, 1)-dimensional anti-chiral super-submanifold produces the total spacetime derivative in the ordinary space. Similarly, the translation of the super Lagrangian density \( \tilde{L}_B^{(c)} \) along \( \theta \)-direction of the (D, 1)-dimensional chiral super-submanifold leads to the derivation of a total spacetime derivative term (cf. (40)) thereby rendering the action integral invariant in the ordinary space for our present interacting Abelian theory with Dirac’s fields \((\psi, \bar{\psi})\).

7 Conclusions

In our present investigation, we have discussed the nilpotency and absolute anticommutativity properties of the conserved (anti-)BRST charges of the ordinary D-dimensional interacting Abelian 1-form gauge theory (where there is a coupling between the gauge field and the Dirac fields) in the language of ACSA to BRST formalism. The novel observation of our present endeavor is the proof of the absolute anticommutativity of the (anti-)BRST charges despite the fact that we have taken into account only the (anti-)chiral super expansions of the superfields (cf. Sec. 5). We have shown that these observations/results are also true when there is a coupling between the \( U(1) \) gauge field and the complex scalar fields (cf. Appendix A). The nilpotency and absolute anticommutativity of the above charges and corresponding continuous (anti-)BRST symmetries are obvious when the full super expansion of the superfields is taken into account (cf. Appendix B).
It is interesting to note that the nilpotency of the BRST and anti-BRST charges is connected with the nilpotency of the translational generators $\partial_{\theta}$ and $\partial_{\bar{\theta}}$, respectively. However, we have established (cf. Sec. 5) that the absolute anticommutativity of the BRST charge with anti-BRST charge is connected with the nilpotency ($\partial_{\theta}^2 = 0$) of the translational generator ($\partial_{\theta}$) along the $\theta$-direction of the chiral super-submanifold. On the contrary, the absolute anticommutativity of the anti-BRST charge with BRST charge is connected with the nilpotency ($\partial_{\bar{\theta}}^2 = 0$) of the translational generator ($\partial_{\bar{\theta}}$) along $\bar{\theta}$-direction of the anti-chiral super-submanifold. These observations are completely novel as far as various forms of superfield approaches to BRST formalism are concerned (see, e.g. [1-15]). These observations can be stated in the language of the nilpotency ($s^2_{(a)b} = 0$) of the (anti-)BRST symmetry transformations ($s_{(a)b}$) in a straightforward fashion (cf. Sec. 5).

We envisage the extension of our present idea in the context of D-dimensional non-Abelian 1-form gauge theory where there would be interaction between SU$(N)$ non-Abelian gauge field and the matter fields (i.e. Dirac fields) in any arbitrary dimension of spacetime [20] where the celebrated Curci-Ferrari condition [23] would play very important role. Furthermore, for the 2D non-Abelian gauge theory, we have shown the existence of nilpotent (anti-)co-BRST charges (see, e.g. [24]) in addition to the (anti-)BRST charges. We would like to capture the nilpotency and absolute anticommutativity of the (anti-)co-BRST charges within the framework of ACSA to BRST formalism as we have done for the (anti-)BRST charges in the case of our present interacting Abelian 1-form theory with Dirac’s fields ($\psi, \bar{\psi}$). These are the issues that would be discussed in our future investigations [25].

**Acknowledgments:** B. Chauhan and S. Kumar are grateful to the DST-INSPIRE and BHU fellowships for financial support under which the present work has been carried out.

**Appendix A: Interacting Abelian 1-Form Theory with Complex Scalar Fields: ACSA to BRST formalism**

Here we discuss about the D-dimensional interacting Abelian 1-form gauge theory where there is a coupling between the $U(1)$ gauge field ($A_{\mu}$) and the complex scalar fields ($\phi, \phi^*$). Our objective is to exploit the beauty and strength of the (anti-)chiral superfield approach to derive the (anti-)BRST symmetry transformations (for this interacting gauge theory). Towards this goal in mind, we begin with the dynamically closed system of ($A_{\mu}$) and ($\phi, \phi^*$) fields which is described by the following (anti-)BRST invariant Lagrangian density (with $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$)

$$
L_b = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + (D_{\mu}\phi)^*(D^{\mu}\phi) - m^2 \phi^* \phi + B (\partial \cdot A) + \frac{1}{2} B^2 - i \partial_{\mu} \bar{C} \partial^{\mu} C,
$$

(A.1)

where the covariant derivatives: $(D_{\mu}\phi)^* \equiv \bar{D}_{\mu}\phi^* = (\partial_{\mu} - ieA_{\mu}) \phi^*$ and $D_{\mu}\phi = (\partial_{\mu} + ieA_{\mu}) \phi$ are defined for fields ($\phi^*, \phi$). Here $m$ is the rest mass of the complex scalar fields and B is the Nakanishi Lautrup auxiliary field and $(\bar{C})C$ are the fermionic ($C^2 = \bar{C}^2 = 0, \bar{C}C = 0$) (anti-)ghost fields. It is elementary to check that the following infinitesimal, continuous, nilpotent ($s^2_{(a)b} = 0$) and absolutely anticommuting ($s_b s_{ab} + s_{ab} s_b = 0$) (anti-)BRST transformations ($s_{(a)b}$)

$$
s_{ab} A_{\mu} = \partial_{\mu} \bar{C}, \quad s_{ab} \bar{C} = 0, \quad s_{ab} C = -i B, \quad s_{ab} B = 0,
$$
conserved currents: given in Eq. (3) (i.e. transforms, under the above nilpotent (anti-)BRST symmetry transformations, derived from Lagrangian density \( L \). The conservation law \( \partial \) leave the action integral \( S \) corresponding to the (anti-)BRST currents (cf. Eq. (A.3)) as:

\[
J_{\mu}^a = -F^{\mu\nu}\partial_\nu \tilde{C} + B \partial^\mu \tilde{C} - i e \tilde{C} \phi (D^\mu \phi)^* + i e \tilde{C} \phi^* D^\mu \phi,
\]

\[
J_\mu^b = -F^{\mu\nu}\partial_\nu \phi + B \partial^\mu \phi - i e \phi (D^\mu \phi)^* + i e \phi^* D^\mu \phi. \tag{A.3}
\]

The conservation law \( \partial_\mu J_{\mu}^{a(\mu)} = 0 \) can be proven due to the following EL-EOMs that are derived from Lagrangian density \( L_b \):

\[
\partial_\mu F^{\mu\nu} - \partial^\nu B - i e \phi^* (D^\nu \phi) - i e (D^\nu \phi)^* \phi = 0, \quad \Box C = 0 = \Box \tilde{C},
\]

\[
\bar{D}_\mu (D^\mu \phi)^* = -m^2 \phi^*, \quad D_\mu (D^\mu \phi) = -m^2 \phi, \quad B = -(\partial \cdot A). \tag{A.4}
\]

According to, once again, the Noether’s theorem, we have the following expressions for the conserved charges:

\[
Q_{ab} = \int d^{D-1}x \ J_{ab}^0 = \int d^{D-1}x \ [-F^{0\mu}\partial_\mu C + B \partial_\mu \tilde{C} + i e C \phi (D^0 \phi)^* + i e \phi^* D^0 \phi],
\]

\[
Q_b = \int d^{D-1}x \ J_b^0 = \int d^{D-1}x \ [-F^{0\mu}\partial_\mu C + B \partial_\mu \tilde{C} - i e C \phi (D^0 \phi)^* + i e \phi^* D^0 \phi]. \tag{A.5}
\]

Using the EL-EOMs from (A.4) (i.e. \( \partial_\mu F^{0\mu} = -(\tilde{B} + i e \phi^* D^0 \phi - i e (D^0 \phi)^* \phi) \)) and exploiting the beauty of Gauss’s divergence theorem, we obtain the expressions for the conserved (anti-)BRST charges \( Q_{a(\mu)} \) which are exactly same as the final expressions for them in Eqs. (6) and (7) (i.e. \( Q_{ab} = \int d^{D-1}x \ (B \tilde{C} - \tilde{B} C) \)) and \( Q_b = \int d^{D-1}x \ (B \tilde{C} - \tilde{B} C) \).

To derive the (anti-)BRST symmetry transformations (A.2), we exploit the (anti-)chiral super expansions (9) and (21) except that now the matter fields \( (\phi, \phi^*) \) would have the following generalizations and super expansions

\[
\phi(x) \longrightarrow \Phi(x, \bar{\theta}) = \phi(x) + i \bar{\theta} P_1(x), \quad \phi(x) \longrightarrow \Phi(x, \theta) = \phi(x) + i \theta \bar{P}_1(x),
\]

\[
\phi^*(x) \longrightarrow \Phi^*(x, \bar{\theta}) = \phi^*(x) + i \bar{\theta} \bar{P}_2(x), \quad \phi^*(x) \longrightarrow \Phi^*(x, \theta) = \phi^*(x) + i \theta \bar{P}_2(x). \tag{A.6}
\]

where \( (P_1(x), P_2(x), \bar{P}_1(x), \bar{P}_2(x)) \) are the secondary fields which are to be determined by the (anti-)BRST invariant restrictions. In this context, we note that the following are the useful and interesting (anti-)BRST invariant quantities:

\[
s_{ab} B = 0, \quad s_{ab}(\tilde{C} \phi) = 0, \quad s_{ab}(\phi^* \tilde{C}) = 0, \quad s_{ab}(\phi^* \phi) = 0,
\]

\[
s_{ab}(\phi^* D_\mu \phi) = 0, \quad s_{ab}(\phi D_\mu \phi^*) = 0, \quad s_{ab} \tilde{C} = 0, \quad s_{ab}(\bar{D}_\mu \phi^* D^\mu \phi) = 0,
\]

\[
s_{ab} \phi = -i e \tilde{C} \phi, \quad s_{ab} \phi^* = +i e \phi^* \tilde{C}, \quad s_{ab} F_{\mu \nu} = 0, \quad s_{ab}(\partial \cdot A) = \Box \tilde{C},
\]

\[
s_b A_\mu = \partial_\mu C, \quad s_b C = 0, \quad s_b \tilde{C} = i B, \quad s_b B = 0,
\]

\[
s_b \phi = -i e \ C \phi, \quad s_b \phi^* = +i e \phi^* \ C \phi^* D^\mu \phi,
\]

\[
s_b F_{\mu \nu} = 0, \quad s_b(\partial \cdot A) = \Box C, \tag{A.2}
\]

leave the action integral \( S = \int d^D x \ L_b \) invariant because the Lagrangian density \( (L_b) \) transforms, under the above nilpotent (anti-)BRST symmetry transformations, exactly as given in Eq. (3) (i.e. \( s_b L_b = \partial_\mu (B \partial^\mu C), \quad s_b L_b = \partial_\mu (B \partial^\mu \tilde{C}) \)).
Using the basic tenets of augmented (anti-)chiral superfield approach, we shall obtain the super expansions of (anti-)chiral superfields corresponding to the auxiliary, gauge and fermionic (anti-)ghost fields as given in Eqs. (20) and (25). We shall exploit these expansions to determine the secondary fields ($P_1(x), P_2(x), \tilde{P}_1(x), \tilde{P}_2(x)$) so that we could obtain the (anti-)BRST symmetry transformations for the matter fields (i.e. complex scalar fields). Towards this objective in mind, we observe that the following are true (if we take into account the expansions of Eqs. (20) and (25)):

$$
\begin{align*}
s_{ab}(A^\mu \partial_\mu \tilde{C}) &= 0, \quad s_{ab}[A^\mu \partial_\mu B + i \partial_\mu \tilde{C} \partial^\mu C] = 0, \\
s_b(B) &= 0, \quad s_b(C \phi) = 0, \quad s_b(\phi^* C) = 0, \quad s_b(\phi^* \phi) = 0, \\
s_b(\phi^* D_\mu \phi) &= 0, \quad s_b(\phi \tilde{D}_\mu \phi^*) = 0, \quad s_b(\tilde{D}_\mu \phi^* D^\mu \phi) = 0, \\
s_b(A^\mu \partial_\mu C) &= 0, \quad s_b[A^\mu \partial_\mu B + i \partial_\mu \tilde{C} \partial^\mu C] = 0.
\end{align*}
$$

(A.7)

The above restrictions imply that the non-trivial solutions are: $P_1 \propto C, P_2 \propto C, \tilde{P}_1 \propto \tilde{C}, \tilde{P}_2 \propto \tilde{C}$. To obtain the explicit forms of ($P_1, P_2, \tilde{P}_1, \tilde{P}_2$), we have to exploit some of the other key restrictions of (A.7). For instance, we shall exploit now $s_{(a)b}(\phi^* D_\mu \phi) = 0, s_{(a)b}(\phi \tilde{D}_\mu \phi^*) = 0$ which imply the following restrictions

$$
\begin{align*}
\Phi^*(x, \bar{\theta}) \partial_\mu \Phi(x, \bar{\theta}) + i e \Phi^*(x, \bar{\theta}) B_\mu^{(b)}(x, \bar{\theta}) \Phi(x, \bar{\theta}) &= \phi^*(x) D_\mu \phi(x), \\
\Phi(x, \bar{\theta}) \partial_\mu \Phi^*(x, \bar{\theta}) - i e \Phi(x, \bar{\theta}) B_\mu^{(b)}(x, \bar{\theta}) \Phi^*(x, \bar{\theta}) &= \phi(x) \tilde{D}_\mu \phi^*(x), \\
\Phi^*(x, \theta) \partial_\mu \Phi(x, \theta) + i e \Phi^*(x, \theta) B_\mu^{(ab)}(x, \theta) \Phi(x, \theta) &= \phi^*(x) D_\mu \phi(x), \\
\Phi(x, \theta) \partial_\mu \Phi^*(x, \theta) - i e \Phi(x, \theta) B_\mu^{(ab)}(x, \theta) \Phi^*(x, \theta) &= \phi(x) \tilde{D}_\mu \phi^*(x).
\end{align*}
$$

(A.9)

Taking the helps from (20) and (25), where the explicit forms of $B_\mu^{(b)}(x, \bar{\theta})$ and $B_\mu^{(ab)}(x, \theta)$ are given, we obtain the exact expressions for the secondary fields in terms of the (anti-)ghost fields and complex scalar fields of our present interacting Abelian 1-form gauge theory.

To corroborate the above statement, we explicitly compute the first two lines of restrictions that are quoted in Eq. (A.9). We obtain the following relationships from these restrictions (if we take the helps from Eq. (A.6) and Eq. (20)):

$$
\begin{align*}
i \phi^* \partial_\mu P_1 + i e \phi^* (\partial_\mu C) \phi + i P_2 \partial_\mu \phi - e A_\mu (\phi^* P_1 + P_2 \phi) &= 0, \\
i \phi \partial_\mu P_2 - i e \phi (\partial_\mu C) \phi^* + i P_1 \partial_\mu \phi^* + e A_\mu (\phi P_2 + P_1 \phi^*) &= 0.
\end{align*}
$$

(A.10)

It is straightforward to note that the following choices of the secondary fields ($P_1(x), P_2(x)$) satisfy the above relations (because the first three terms and the coefficient of $e A_\mu$ should vanish separately and independently), namely;

$$
P_1 = - e C \phi, \quad P_2 = + e \phi^* C.
$$

(A.11)
It is pertinent to point out that the \((\pm)\) signs in 2 and 1 are fixed. The choices of \((\pm)\) signs in (A.11) are further fixed by the requirement \(s_b(\bar{D}_\mu\phi^* D^\mu\phi) = 0\) which amounts to the following restrictions on the superfields:

\[
(\partial_\mu - i e B^{(b)}_\mu(x, \bar{\theta})) \Phi^*(x, \bar{\theta}) (\partial^\mu + i e B^{(b)}(x, \bar{\theta})) \Phi(x, \bar{\theta})
\]

\[
= (\partial_\mu - i e A_\mu(x)) \phi^*(x) (\partial^\mu + i e A^\mu(x)) \phi(x).
\]  

(A.12)

The substitutions of expansions from (A.6) and (20) lead to:

\[
i e^2 A_\mu A^\mu (\phi^* P_1 + P_2 \phi) + i \partial_\mu \phi^* [\partial^\mu P_1 + e (\partial^\mu C) \phi] + i \partial_\mu \phi [\partial^\mu P_2 - e (\partial^\mu C) \phi^*]
\]

\[
+ e A_\mu \left[ \phi^* \partial^\mu P_1 + P_2 \partial^\mu \phi - (\partial^\mu \phi^*) P_1 - (\partial^\mu P_2) \phi + e \phi^* (\partial^\mu C) \phi + e \phi^* (\partial^\mu C) \phi^* \right].
\]  

(A.13)

It is crystal clear that the choices of \((P_1, P_2)\) (that have been pointed out in (A.11)) satisfy the above relationships, too. Thus, the expressions in (A.11) are precise and perfect. We would like to mention here (without giving all the algebraic details) that the last two restrictions of (A.9), with the substitutions from (A.6) and (25), lead to the following expressions for the secondary fields:

\[
P_1 = - e \bar{C} \phi, \quad P_2 = + e \phi^* \bar{C}.
\]  

(A.14)

The above choices are correct because these can be further confirmed by the requirement \(s_{ab}(\bar{D}_\mu\phi^* D^\mu\phi) = 0\) and corresponding restriction on the superfields (as has been done in Eqs. (A.12) and (A.13)). Hence, we obtain the explicit expressions of the super expansions of the matter superfields (i.e. super complex scalar fields), after the application of (anti-)BRST invariant restrictions (A.7) and (A.8), as

\[
\Phi^{(b)}(x, \bar{\theta}) = \phi(x) + \bar{\theta} (-i e C \phi(x)) \equiv \phi(x) + \bar{\theta} (s_b \phi),
\]

\[
\Phi^{*(b)}(x, \bar{\theta}) = \phi^*(x) + \bar{\theta} (+i e \phi^*(x) C) \equiv \phi^*(x) + \bar{\theta} (s_b \phi^*),
\]

\[
\Phi^{(ab)}(x, \theta) = \phi(x) + \theta (-i e \bar{C} \phi(x)) \equiv \phi(x) + \theta (s_{ab} \phi),
\]

\[
\Phi^{*(ab)}(x, \theta) = \phi^*(x) + \theta (+i e \phi^*(x) \bar{C}) \equiv \phi^*(x) + \theta (s_{ab} \phi^*),
\]  

(A.15)

where the superscripts \((b)\) and \((ab)\) denote the expansions for the superfields after application of the BRST and anti-BRST invariant restrictions (cf. (A.7) and (A.8)). It is crystal clear that we have derived all the proper (anti-)BRST transformations for all the fields of our interacting Abelian 1-form gauge theory where there is a coupling between the \(U(1)\) gauge field \((A_\mu)\) and matter fields \((\phi, \phi^*)\) which are nothing but the complex scalar fields.

We end this the Appendix with the remarks that the final expressions for the nilpotent and conserved (anti-)BRST charges \((Q_{(ab)}\) for the Abelian 1-form interacting gauge theories with Dirac fields \((\psi, \bar{\psi})\) and complex scalar fields \((\phi, \phi^*)\) are one and the same (cf. Eqs. (6) and (7)). Thus, we note that the proof of their nilpotency and absolute anticommutativity properties would go along the same lines (as far as the (anti-)chiral superfields approach to BRST formalism is concerned). In other words, the discussions between the Eqs. (26) and (35) would be same for the interacting Abelian 1-form gauge theory with complex scalar fields. Furthermore, invariance of the Lagrangian density (A.1) would be
same as discussed between the Eqs. (36) and (40) within the framework of augmented version of (anti-)chiral superfields approach to BRST formalism. Thus, we shall not discuss these aspects, once again, here.

Appendix B: On Full Super-Expansion and Absolute Anticommutativity

In order to corroborate and establish the novelty of our present investigation with (anti-)chiral superfields and their super expansions, we show here that the absolute anticommutativity of the (anti-)BRST symmetries and corresponding conserved charges is very natural and automatic when we consider the full super expansions of the superfields. For instance, let us take a generic superfield \( \Sigma(x, \theta, \bar{\theta}) \), defined on a general \((D,2)\)-dimensional supermanifold, with the following super expansion

\[
\Sigma(x, \theta, \bar{\theta}) = \sigma(x) + \theta \ M(x) + \bar{\theta} \ M(x) + i \theta \bar{\theta} \ N(x),
\]

where \( \sigma(x) \) is an ordinary D-dimensional field defined on a given D-dimensional ordinary Minkowskian spacetime manifold for a given BRST invariant gauge theory. If \( \sigma(x) \) were fermionic, the superfield \( \Sigma(x, \theta, \bar{\theta}) \) would also be fermionic which implies that the pair \( (M(x), \bar{M}(x)) \) would be bosonic and \( N(x) \) would be fermionic. On the other hand, if \( \sigma(x) \) were bosonic, the corresponding superfield \( \Sigma(x, \theta, \bar{\theta}) \) would be bosonic, too, and the pair \( (M(x), \bar{M}(x)) \) would be fermionic and \( N(x) \) would be bosonic. These conclusions are straightforward due to the fact that the Grassmannian variables \( (\theta, \bar{\theta}) \) are fermionic (i.e. \( \theta^2 = \bar{\theta}^2 = 0, \theta \bar{\theta} + \bar{\theta} \theta = 0 \)) in nature. It is straightforward to note that the following are true, namely;

\[
\frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \Sigma(x, \theta, \bar{\theta}) = -i \ N(x), \quad \frac{\partial}{\partial \bar{\theta}} \frac{\partial}{\partial \theta} \Sigma(x, \theta, \bar{\theta}) = +i \ N(x).
\]

Taking into account the fact that \( \partial_\theta \leftrightarrow s_{ab} \) and \( \partial_{\bar{\theta}} \leftrightarrow s_b \), we observe that the relationship \( (\partial_\theta \partial_\theta + \partial_{\bar{\theta}} \partial_{\bar{\theta}}) \Sigma(x, \theta, \bar{\theta}) = 0 \), in its operator form, implies the following:

\[
(\partial_\theta \partial_\bar{\theta} + \partial_{\bar{\theta}} \partial_\theta) = 0 \iff s_{ab} s_b + s_b s_{ab} = 0.
\]

Thus, it is crystal clear that the (anti-)BRST symmetries \( s_{(a)b} \) are absolutely anticommuting (i.e. \( s_{ab} s_b + s_b s_{ab} = 0 \)) in nature within the framework of superfield approach to BRST formalism if we take into account the full super expansions of the superfields along the \((\theta, \bar{\theta})\)-directions of \((D,2)\)-dimensional supermanifold. The relationship (B.3) is not guaranteed when we take into account only the (anti-)chiral super expansions of the superfields (as is the case in our present investigation) and in the discussion of \( \mathcal{N} = 2 \) SUSY quantum mechanical models [16-19] where the SUSY transformations are nilpotent (i.e. fermionic) in nature but not absolutely anticommuting (i.e. not linearly independent of each-other).

We end this Appendix with the remark that the generators of the infinitesimal and continuous (anti-)BRST symmetry transformations are the conserved (anti-) BRST charges and both are connected (cf. Sec. 2) by the relationship (8). As a consequence, we infer that the conserved (anti-)BRST charges \( (Q_{(a)b}) \) would also obey the absolute anticommutativity property (i.e. \( Q_b Q_{ab} + Q_{ab} Q_b = 0 \)) when the full super expansions of the superfields are...
taken into account along the \((\theta, \bar{\theta})\)-directions of \((D, 2)\)-dimensional supermanifold. The novel observation, in our present investigation, is the fact that the absolute anticommutativity property of the (anti-)BRST charges has been proven despite the fact that we have considered only the (anti-)chiral super expansions of the (anti-)chiral superfields (defined on the \((D, 1)\)-dimensional super-submanifolds of the general \((D, 2)\)-dimensional supermanifold on which our D-dimensional ordinary gauge theory is generalized).

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