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Gravity Sources in a Quantum Milne Universe

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Summary

A quantum-cosmology-suited sliced Milne spacetime, located inside a ‘big-bang’ forward lightcone, comprises the interiors of a sequence of 4-dimensional slices whose invariant ‘age’ width is at Planck scale. The age of any lightcone-interior point is its Minkowski distance from the lightcone vertex, but excluded from the spacetime of a ‘quantum Milne universe’ (QMU) are 3-dimensional slice-separating, ray-carrying hyperboloids.

Each (fixed-age) slice boundary houses a Gelfand-Naimark unitarily Lorentz-transformable cosmological-Fock-space ray—a sum of tensor products of complex normed fiber-bundle ‘source’ functions. Each bundle is a unit 3-sphere (fiber) over an invariantly metricized 3-hyperboloid (base space). The evolving QMU comprises a slice-interior sequence (each interior a 4-manifold) and a ray sequence housed by the slice-separating 3-manifolds. Throughout each slice interior, the preceding ray specifies classical-field reality. Action, of elsewhere-prescribed slice-traversing Feynman paths, determines any ray after the first from its predecessor.

QMUs related by a global 7-parameter Lie-group symmetry transformation, at fixed age, are equivalent—the same quantum universe. This ‘cosmological relativity’ implies vanishing of total universe angular momentum (Mach’s principle) as well as of total universe momentum. Milne’s cosmological principle accompanies vanishing of total-universe energy. (Seven ‘Noether-conserved’ quantities separately aggregate to zero).

Any QMU ray specifies ‘immediate-future’ local reality through continua of self-adjoint-operator expectations that prescribe retarded real (classical) fields over the immediately-following (4-dimensional) slice interior. Exemplifying QMU reality is a divergenceless Lorentz symmetric-tensor conserved energy-momentum current density—the Dalembertian divided by $G$ of a retarded gravitational potential. Conserved electric-charge current density is the Dalembertian of a retarded electromagnetic 4-vector potential.
Introduction

‘Relativistic’ quantum theory, thus far, has not only lacked Dirac’s self-adjoint Hilbert-space operators whose spectra (he proposed) define ‘reality’ but has failed to recognize a global arrowed time. Further, throughout quantum theory’s first century a ‘Copenhagen’ probabilistic Hilbert-space interpretation has perplexingly associated reality’s meaning to that of ‘measurement by conscious observer’. Correction of the foregoing flaws is here achieved by identifying the ‘quantum’ content of Milne’s universe with ‘retarded’ evolution-propelling sources of gravity. The acronym QMU will here be employed for ‘quantum Milne universe’.

‘Universe age’—QMU’s anchor—enjoys a purely-classical scale-setting (see Appendix) status as a symmetry (Lie) group invariant that parallels the global (single) time of nonrelativistic Dirac quantum theory—not associating to any operator’s spectrum. Admitted by a heretofore unutilized fiber-bundle Fock space, that unitarily represents a 7-parameter symmetry group (7-SG), are self-adjoint source-time operators whose canonically-conjugate (Dirac-sense) partners are the conserved energies of individual gravity sources. Six other conserved local 7-SG generators represent source momentum and angular momentum.

Meaning for continuous-spacetime ‘reality’ is QMU-achieved through classical fields that are expectations of self-adjoint field operators with respect to a source-bundle Fock-space ray at an age ‘slightly’ earlier than the field-point age. Exemplifying the classical fields that define ‘settled local reality’ is a divergenceless second-rank symmetric Lorentz-tensor—conserved energy-momentum current density. This tensor field is proportional, through Newton’s gravitational constant, to the gravitational potential’s Dalemberian.

The reader is cautioned against confusing the Hilbert-space term ‘expectation’—a real number prescribed by the pairing of a ray and some self-adjoint operator—with this term’s ordinary-language significance. Absent from our meaning for ‘quantum-cosmological expectation’ is any probabilistic or conscious aspect.

Gelfand-Naimark (GN) unitary Hilbert-space representation of the Lorentz-group fits with Milne’s meaning for (classical, continuous) ‘spacetime’, rather than with Einstein’s meaning (either 1905 or 1915), and defines a source-interpretable bundle Fock space at each member of a sequence of exceptional universe ages. Each bundle associates a 3-sphere to every location in an invariantly metricized 3-hyperboloid. Throughout ‘SS’—a Lorentz-group-based sliced spacetime that occupies ‘much of’ but not ‘all’ the interior of a forward lightcone—the (positive) Lorentz-invariant (continuous) ‘age’ of any spacetime point is its Minkowski distance from the lightcone’s vertex. Each source bundle belongs to one member of a discrete sequence of exceptional ages.

Milne age—foundational to QMU—revived a Newtonian idea which a century ago, because of Einstein’s local-time emphasis, fell into disrepute. Absence of dynamical laws prevented Milne (despite his identifying the Lemaître-Hubble redshift ‘constant’ with the inverse of universe age) from persuading contemporaries of utility for Lorentz-group-founded cosmology. We resurrect Milne’s cosmology by a Planck-scale slicing of spacetime that, in concert with GN’s source-bundle Fock space, accommodates Feynman-path quantum-gravitational dynamics.

Absent slicing, Milne described the universe-encompassing forward-lightcone interior as ‘continuously filled’ with (unbounded) 3-dimensional metricized spacelike hyperboloids, the age of any hyperboloid being that of all its spacetime locations. Hyperboloid curvature is inversely proportional to hyperboloid age. Temporal tendency toward Euclid’s (‘ordinary’ 3-space) geometry—a limit approached by Milne’s geometry as age increases—classically endows
‘age arrow’ with local as well as global significance. Notice that, even without spacetime slicing, reproducibility of a ‘measurement’ (or of any local-history portion) is precluded by Milne’s perpetually-ongoing flattening of 3-space.

A QMU sequence of exceptional 3-hyperboloids of age $N\delta$, with $N$ a positive integer and $\delta$ a universal Planck-scale age interval, each house a fiber-bundle Fock-space ray. Milne relativity (MR) means that (at any age) two QMU’s related by a global 7-SG transformation are the same universe. Each ray (after the first) maintains this equivalence through invariance of the Feynman-path action that determines the ray from the (one-age-unit younger) predecessor ray.

The 7-SG group is a subgroup of the 12-parameter group of transformations of a unimodular 2x2 complex matrix by left or right multiplication by some (other) such matrix. Any left-group element commutes with any right element. (6-parameter left and right subgroups) The QMU symmetry group combines all right transformations (6-parameter subgroup) with left multiplication by real diagonal unimodular matrices (1-parameter subgroup). Energy generates the latter, momentum and angular momentum the former.

Although Milne’s hyperbolic 3-space is noncompact, QMU houses, via a periodic Hilbert-space constraint, only a finite number of gravity sources. MR, via the vanishing of total momentum and angular momentum of a finite (although currently huge) universe, encompasses Mach’s principle. Perpetuation of initially-zero total-universe energy sustains Milne’s ‘cosmological principle’.

SS comprises the interiors of a sequence of 4-dimensional slices that share a common width in age. Excluded are the 3-dimensional hyperbolic manifolds that separate these slices. SS definition further excludes spacetime locations whose age is smaller than $N_0 \delta$--the age at which the universe ‘began’. Unrecognized by Milne, $N_0$ is a huge Mersenne prime that (along with 2, 3, 7 and 127 is foundational to QMU, although in this paper introduced without raison d’être, and associates both to ‘inflation’ and to ‘universe size’. (The author expects, via Hilbert-space theory, future mathematical shrinkage of the current gap separating ‘dimensionality’-ignoring discrete number theory and topology from dimensionality-accommodating continuous Lie-group theory and geometry. QMU invokes all the foregoing domains of mathematics. Fueling our anticipation has been the discovery, within the last century, of group contraction.)

The ‘particle’ concept of discrete and stable positive-energy ‘clump’, foundational to the S matrix and to the accompanying ‘measurement’ concept, lacks a priori QMU status. QMU foundations include temporally-unstable ‘nonmaterial’ (not ‘particle’) gravity sources with both negative and positive energies. (5) (QMU total energy, remember, vanishes.) The S-matrix, prerequisite to particle-physics meaning for ‘measurement’, is a ‘dynamically-emergent’ concept associated to approximate temporal stability of certain special positive-energy source ‘clumps’. The accuracy within limited scale ranges for approximations such as that of the S matrix stems from $N_0$ hugeness. We shall employ the acronym FAPPP to characterize an approximation that, within some range of scales, is reliable ‘for all practical physics purposes’.

QMU foundation eschews notions either of ’observer’ or of ‘measurement-result probability’. An observer-independent ‘reality’, consistent with an elsewhere-detailed complex-
number-based Feynman-path quantum dynamics, will here be exposed. Within the 4-dimensional slice immediately following any global ray, ‘settled’ (real) classical fields are prescribed through expectations over this ray of self-adjoint field operators. These fields and derivatives thereof notably include locally-conserved (real) energy-momentum and electric-charge current densities—continuous local reality that manifests Planck’s constant through ‘microscopic’ correlations between electric charge, energy, momentum, and spatial location. (‘Micro’ spacetime scale is huge compared to the Planck scale of $\delta$ while tiny compared to the ‘macro’ scale of $N_0\delta$ or to the still-larger redshift-manifested ‘Lemaître-Hubble’ cosmological scale set by universe age.) Such correlations enable, FAPPP, microscale meaning for ‘particle’ as well as ordinary-language macroscale meaning for ‘object’. Strikingly absent from below-defined reality-associated self-adjoint field operators are any (zero-Dalmbertian) quantum radiation fields.

Any Fock-space ray sums ‘tensor’ products of complex normed functions of 6 source-bundle coordinates that span a 3-dimensional compact ‘fiber space’ above each point of a 3-dimensional noncompact invariantly-metricized ‘base space’ which independently represents the Lorentz group. Schwinger’s earlier use of the term ‘source’ (6) is related to ours but was applied to an operator rather than to a Fock-space vector and was not related to gravity. We below connect our meaning to the generation of both gravity and electromagnetism. The energy (regardless of sign) of any QMU source generates gravitation; charged sources additionally generate electromagnetism.

QMU sources, despite emulating in Hilbert-space status nonrelativistic Dirac particles at some common (global) time, are neither particles nor fields. FAPPP, at GUT spacetime scales--larger than Planck scale but not ‘hugely’ larger--another paper will attach ‘elementary-particle’ meaning to a special approximately-stable and approximately-localized positive-energy single-source ray that not only unitarily but also irreducibly represents the 7-SG--a ‘unirrep’. This ray’s expectation, of the source’s self-adjoint 4-vector (lightlike although not positive) energy-momentum operator, approximates elementary-particle mass. Approximate stability of some ray (approximate age independence) we presume requires positivity for the ray’s energy expectation.

QMU meaning for the adjective ‘macroscopic’ derives from the universe’s spatial curvature at its ‘beginning’—set by $N_0\delta$. A measure of ‘universe size’ is provided by the expectation of an operator whose eigenvalues are logarithms of positive integers that specify total numbers of sources. Such size would vanish at a universe single-source beginning but, regardless of the initial ray at age $N_0\delta$, QMU’s 3-space began with macroscopic (~km) 3-space (inverse) curvature--far above ‘particle’ (microscopic) scale and even further above the Planck scale set by the SS width $\delta$. (4) For microscopic scales down to GUT scale wherever stably-clumped positive-energy matter is accompanied by small gravitational potential, local Poincaré (flat 3-space) invariance applies FAPPP.

Although the (zero-age, ‘universe-surrounding’) LSS forward-lightcone boundary admits ‘big-bang’ association, the spacetime location of the lightcone vertex is meaningless. There is no Poincaré cosmological-symmetry group. Nevertheless, individual sources provide local unitary
representation of conserved energy, momentum and angular momentum as well as of nonconserved chirality.

Locality and globality are linked by a ‘local Lorentz frame’ associated to each source location in the 6-dimensional product of fiber and base spaces. In the local frame of any such location the time component of the 4-vector spacetime displacement from Milne-lightcone vertex is equal to location age while the 3 spatial components all vanish. A source’s fiber coordinates -- three ‘Euler’ angles—specify for the local frame a spatial orientation tied to the source’s velocity direction and chirality.

Local-frame orientation is parallel-transportable along a base-space geodesic to any other location within the same hyperboloid. Iff a geodesic parallels a source’s velocity, the fiber-space location of a source moving along this geodesic remains unchanged. Two of the 3 fiber angles coordinate a ‘tangent space’ of (local-frame) source-velocity directions. Source wave-function dependence on the remaining angle is elsewhere shown to define a ‘chirality’ that distinguishes ‘bosonic’ sources from ‘fermionic’ (while excluding either ‘Higgs’ or ‘Majorana’ sources).

Milne’s use of the Lorentz group differs from that of Einstein and Poincaré--whose global-time-ignoring (‘physics-appropriate’) representation of flat 3-space and reversible local particle-time has been called ‘special relativity’. Applied to an individual source, a ‘Milne-Lorentz’ boost shifts source location in hyperbolic (base) 3-space (at fixed age), as well as changing source orientation.

At those exceptional ages where a ray is defined, the universe comprises fluctuating sets of individual sources each endowed with spatial location, velocity direction, chirality, energy (of indefinite sign), momentum and angular momentum, as well as with (nonfluctuating) ‘electric-charge number’ and ‘quark number’ (3 times baryon number). All source attributes associate in Dirac sense \(^{(1)}\) to (generally non-commuting) self-adjoint operators on the GN source fiber-bundle Hilbert space which unitarily represents the 7-SG.

Any source-velocity magnitude equals \(c\); there is no self-adjoint velocity-magnitude operator. Only the direction of source velocity is variable. Dirac attempted to represent Hilbert-space sublightlike motion through expectations over quantum-fluctuating lightlike-velocity direction—i.e., through ‘zitterbewegung’-- \(^{(1)}\) although failing because finite-dimensional Lorentz-group representations are not unitary. Via GN, we accomplish his objective. Fiber further allows Hilbert-space definition of a source (‘reversible’) local time that is independent of, while in Feynman paths dynamically consistent with, (irreversible) universe age. Conjugate to source (local) time in Dirac sense is source (local-frame) energy. A gravitational-potential operator will, three sections later, be defined by a sum of self-adjoint individual-source potentials each proportional to a source-energy operator defined two sections below.

Source Fiber-Bundle Hilbert Space

Source Hilbert-space vectors are functions of the coordinates of a 6-dimensional SL(2,c) manifold. Roughly speaking, three dimensions spatially locate a source, two specify its velocity
denoting Ray

product of single

maximum determined by

the number of sources

source coordinates

Invariant

invariant (finite) norm,

\[ d \Re \xi \times d \Im \xi \]

analogous left transformation.

right

operators from

the reader of Dirac’s quantum

individual

Source

coordinate. An example is the symbol

coordinate

focus

nature of a source

conjugate matrix pair,

\( \sigma \), each has a unit off-diagonal element. Although the fiber-bundle

nature of a source is better displayed by coordinates alternative (while equivalent) to \( s, y, z \), we focus first on \( GN \)’s coordinate set as we proceed toward definition of classical-field reality.

We employ Dirac’s shorthand \(^1\) of denoting, by a single symbol, both a (real classical) coordinate and a self adjoint operator whose spectrum comprises the possible values of this coordinate. An example is the symbol \( \Re s^\sigma \)--linearly related (we shall see) to the ‘local time of

Source \( \sigma \).’ The symbol \( E^\sigma \) will denote Source-\( \sigma \) local-frame energy. ‘Canonically-conjugate’

individual-source operators for local-time and local-frame-energy do not commute.

A discrete \( \sigma \) superscript serves not only to designate an ‘individual source’ but to remind

the reader of Dirac’s quantum-classical dualism. Context will here distinguish self-adjoint operators from their classical counterparts.

The 6-dimensional ‘volume element’ (Haar measure),

\[ da = ds dy dz, \]

is invariant under \( a \rightarrow a^\Gamma = a \Gamma^{-1}, \) with \( \Gamma \) a \( 2 \times 2 \) unimodular matrix representing a (Milne-sense)

right Lorentz transformation of the coordinate \( a \). The measure (2) is also invariant under

analogous left transformation. The volume-element symbol \( d \xi \), with \( \xi \) complex means

\( d \Re \xi \times d \Im \xi \).

Any universe ray has age \( N \delta \) with \( N \) a (positive) integer. An \( N \)-Hilbert-space (single) \( \sigma \)-labeled ‘gravity-source’ vector is a complex differentiable and periodic function \( \psi^N(a^\sigma) \) with invariant (finite) norm,

\[ \int da^\sigma \left| \psi^N(a^\sigma) \right|^2. \]

Invariant periodicity is addressed in an Appendix.

The Fock-space \( N \) ray \( \psi^N \) is a sum of normed functions of differing numbers, \( n \), of the source coordinates \( a^\sigma \):

\[ \psi^N = \sum_n \psi^N_n (a^1 \ldots a^n), \]

the number of sources \( n \) running between 1 and, for Feynman-path reasons exposed elsewhere, a maximum determined by \( N - N_0 \). ‘Vacuum’—i.e., \( n = 0 \)—is absent from \( QMU \) Fock space.

The norm-defining volume element for functions of \( n \) \( \sigma \)-labeled source coordinates is a product of single-source volume elements each having the form (2). With a notation \( \{ O \}_N \) denoting Ray-\( N \) expectation of a self-adjoint operator \( O \), the non-negative bounded
dimensionless expectation, \( \{ \ln n \}_N \), defines a ‘universe size’ that, at any human-history age, is huge even though finite.

A superscript \( \sigma \), as in (3), associated to a ray-labeling age integer, \( N \ge N_0 \), designates a ‘source’ within some set of \( n \) sources, as in (4). Understanding the meaning of \( \sigma \) to include some value of \( n \), the present paper henceforth will dispense with the \( n \) symbol. We shall use the symbol \( \tau \) to designate age (global time), whether a discrete ray age—\( \tau = N \delta \)—or a continuous \( SS \)-interior age.

A ‘Milne-Lorentz’ \( SL(2,c) \) transformation specified by the \( 2 \times 2 \) complex unimodular right-acting matrix \( \Gamma \) is unitarily Source-\( \sigma \) Hilbert-space represented by

\[
\Psi^N(\mathbf{a}^\sigma) \rightarrow \Psi^N(\mathbf{a}^\sigma \Gamma^{-1}).
\]  

Calculation shows \( \mathbf{a} \Gamma^{-1} \) (here omitting the source-label \( \sigma \)) to be equivalent to

\[
z^F = (\Gamma_{22}z - \Gamma_{21})/(\Gamma_{11} - \Gamma_{12}z),
\]  
\[
y^F = (\Gamma_{11} - \Gamma_{12}z)(\Gamma_{11} - \Gamma_{12}z)y - \Gamma_{12},
\]  
\[
s^F = s + \ln(\Gamma_{11} - \Gamma_{12}z). \tag{8}
\]

Notice that the 2-dimensional volume element \( ds \) within the Haar measure is separately invariant (implying invariance also of the 4-dimensional volume element \( dy \, dz \)). This invariant factorizability of measure dovetails with source energy and chirality definition and the Appendix-prescribed periodicity constraint on source Hilbert space.

The fiber-bundle character of a source is exhibited by a factorization of the 6-dimensional single-source manifold which is alternative to that of \( GN \). The \( 2 \times 2 \) unimodular matrix \( \mathbf{a} \) equals the product \( \mathbf{uh} \) of a 3-parameter unitary unimodular matrix \( \mathbf{u} \) and a 3-parameter positive hermitian unimodular matrix \( \mathbf{h} \). The matrix \( \mathbf{u} \) may be parameterized by 3 real (‘Euler’) angles, \( 0 < \varphi < 4\pi, 0 < \theta < \pi, 0 < \varphi < 2\pi \), such that

\[
\mathbf{u} = \exp(i\sigma_3 \varphi/2) \exp(i\sigma_1 \theta/2) \exp(i\sigma_3 \varphi/2), \tag{9}
\]

and the matrix \( \mathbf{h} \) by a real 3-vector \( \mathbf{\beta} \) such that

\[
\mathbf{h} = \exp(-\mathbf{\sigma} \cdot \mathbf{\beta}/2), \tag{10}
\]

with the \( \cdot \) symbol in (10) denoting the rotationally-invariant inner product of two 3-vectors.

Calculation shows that in source local frame, where \( \mathbf{\beta} \) vanishes, \( Im \, s = -\varphi'/2 \),

\[
Re \, s = -\frac{1}{2} \ln \left[ 1 + \left| \frac{y}{z} \right|^2 \right], \quad y = -\frac{z^*}{(1 + \left| z \right|^2)}, \tag{11}
\]

while \( z = i \, e^{i\varphi} \tan \theta/2 \). Generally, as elaborated below, \( \mathbf{\beta} \) specifies source spatial (base-space) location while \( \mathbf{u} \) specifies local-frame source orientation—i.e., location in fiber space. The 2-dimensional fiber subspace coordinated by \( \theta, \varphi \), we refer to as the base space’s ‘tangent’ subspace or, alternatively, as source local-frame ‘velocity space’. The circle doubly spanned by \( \varphi' \) might be called ‘source-chirality’ space.
The Haar measure (2) equals a product of two separately-invariant ‘volume elements’: fiber volume, $du = dp' \sin \theta \, d\theta \, d\phi$, and base volume, $d\Omega = \sinh \beta \, d\beta \, d\Omega$, where $\beta = \beta_n$ with $\beta > 0$ and $n$ a unit 3-vector. The symbol $d\Omega$ denotes an infinitesimal solid angle at the direction $n$. The ‘base’ 3-space coordinated by $\beta$ enjoys an invariant ‘dimensionful’ (age-related) metric while the compact fiber 3-space coordinated by $u$ does not. (Lorentz transformation of location in fiber space is ‘entangled’ with location in base space although the converse is not true. The fiber’s Haar measure is a product of two separately 7-SG invariant—velocity-direction and chirality-volume elements.)

**Self-Adjoint Single-Source Operators**

A base-space geodesic is spanned by the 1-parameter group of Haar-measure-preserving left transformations, $a^\rho \rightarrow [\exp (-\sigma d\alpha)] a^\rho$ with $d\alpha$ real and positive. Such a left transformation commutes with all Milne-Lorentz (right) transformations—increasing $Re s^\rho$ by $d\alpha$ at fixed $Im s^\rho$, $y^\rho$ and $z^\rho$. Because a source is thereby spatially displaced in its velocity direction, (5) a ‘source-time’ increase becomes defined.

A local-frame source-$\sigma$ Hilbert-space energy operator emerges from Fourier transformation of Source-$\sigma$ wave-function dependence on $Re s^\sigma$. A self-adjoint operator $E^{N,\sigma}$, representing the Source-$\sigma$ local-frame energy, is definable in the ‘spacetime’ $(s, y, z)$ basis by

$$E^{N,\sigma} = \frac{i}{\hbar} (N\delta) \partial / \partial s\, 2Re s^\sigma,$$

(11)

the partial derivative being at fixed $Im s^\rho, y^\rho$, and $z^\rho$ as well as at fixed $a^\rho$ for $\rho \neq \sigma$. The indefinite-sign-spectrum of $E^{N,\sigma}$ is 7-SG invariant. In Dirac sense $E^{N,\sigma}$ is conjugate to the self-adjoint source-time operator $t^{N,\sigma} \equiv N\delta [2Re s^\sigma + \ln (1 + |z^\sigma|^2)]$, that vanishes in the source’s local frame and whose increase another paper will coordinate with age increase along a Feynman path.

A 4-vector with Lorentz index $\mu = 0, 1, 2, 3$ is equivalent to a hermitian 2×2 matrix $\hat{f}$ through the formula $1/2tr \sigma_\mu \hat{f}$. By $\sigma_0$ is meant a unit matrix. Omitting for a few paragraphs the source label, two (commuting, classical or operator) positive $(tr \hat{f} > 0)$ 4-vectors, for $\tau = N\delta$,

$$x^\tau \equiv \tau a^\dagger a \ [= \tau h^2 = \tau \exp (-\sigma \beta)],$$

(12)

$$= \tau [\sigma_0 \cosh \beta - \sigma \cdot n \sinh \beta], \ (\beta = \beta_n, n \cdot n = 1, \beta \geq 0),$$

(12')

and

$$v \equiv a^\dagger (\sigma_0 - \sigma_3) a,$$

(13)

satisfying three Milne-Lorentz-covariant inner-product (homogeneous-quadratic) constraints, are equivalent to either quantum or classical meanings for the coordinate quintet $Re s, y, z$, of spatial-location and velocity. I.e., this quintet is a set of 5 commuting self-adjoint operators equivalent to the location-velocity 4-vector operator pair $x^\tau, v$. The (missing, sixth) coordinate-operator, $Im s$, associates to source chirality (to fermionic-source helicity).

The trio of 4-vector constraints, built into (12) and (13), are $x^\tau \cdot x^\tau = \tau^2, x^\tau \cdot v = \tau, v \cdot v = 0$, where the symbol $\cdot$ denotes a Lorentz inner product of 4-vectors. These constraints (applying both to real coordinates and to commuting self-adjoint operators) complicate expression through the 4-vector pair of the 5-dimensional (chirality-ignoring) Haar-measure.
The positive-timelike 4-vector \( x^i \) specifies a source’s spacetime location with respect to Milne-lightcone vertex while the positive-lightlike 4-vector \( v \) specifies the source’s velocity (in \( c \) units). In source local frame (i.e., locating Source-\( \sigma \) at base-space ‘origin’) the location 4-vector components, which generally are \( \tau (cosh \beta, n \sinh \beta) \), become \((\tau, 0, 0, 0)\) while the (‘tangent-space’) velocity-4-vector components become \((1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)\).

Dependence on the operator \( Re_s \) by both \( x^i \) and \( v \) [revealed by Formulas (12) and (13)], means from (11) that neither the source-location operator nor the source-velocity operator commutes with the (invariant) source local-frame-energy operator (11). We nevertheless may define a self-adjoint indefinite-sign single-source energy-momentum lightlike 4-vector operator,

\[
p^{N\sigma} \equiv \frac{i}{2} (E^{N\sigma}v^\sigma + v^\sigma E^{N\sigma}),
\]

whose (real) 4-vector expectation, \( \{p^{N\sigma}\}_N \), need be neither positive lightlike nor timelike. A source’s ‘mass squared’—defined by \( \{p^{N\sigma}\}_N \cdot \{p^{N\sigma}\}_N \)—generally may be either positive or negative, as also may be its energy. Only when \( \{p^{N\sigma}\}_N \) and \( \{p^{N\sigma}\}_N \cdot \{p^{N\sigma}\}_N \) are both non-negative may a single-source wave function associate to ‘matter’.

The 3-vector components of the dimensionful 4-vector (14) are different from the (Appendix-addressed) 3-vector dimensionful generators of the Milne-Lorentz group (components of a 6-vector). Special QMU single-source wave functions with approximately equal ‘lab-frame’ expectations for the two different momentum operators are particle candidates.

Expectation 4-vectors of the operator (14) provide (approximate) QMU contact with the Poincaré-group-representing (momentum-basis) \( S \) matrix. Although QMU ‘classical reality’ (following section), fails to single out individual sources, we expect the (‘standard’) \( S \)-matrix meaning for ‘massive elementary particle’ to connect with certain real-mass, positive-energy single-source wave functions that are approximately stable—i.e., approximately \( N \) independent.

As proved true for Dirac’s ‘relativistic electron, \(^{(1)}\) the QMU ratio of stable-source momentum and energy expectations we anticipate will equal (approximately) the expectation of source velocity. (‘Dark matter’ we associate to positive-energy gravity sources so unstable as to be inaccessible to particle physics—i.e., to be \( S \)-matrix indescribable.)

**Classical-Field Spacetime Reality**

We postulate a retarded Source-\( \sigma \)-generated (‘Lienard-Wiechert’—LW) divergenceless gravitational potential that is a self-adjoint field operator—a function of the Source-\( \sigma \) operators (11), (12) and (13) which depends also on a classical ‘sink’ location in spacetime. Our ‘Newton-LW-Dirac-Milne’ gravitational-potential operator transforms as a symmetric second-rank Lorentz tensor of zero invariant trace. A sum over all \( N \) sources then defines a global operator whose expectation over that ray is the (classical) gravitational potential throughout the immediately-following \( SS \). This potential’s Dalembertian (when divided by \( G \)) prescribes within the slice the conserved current density of energy-momentum.

Our postulate for the retarded gravitational-potential operator \( \Phi_{N\mu\nu}^{\sigma}(x) \), associated to Source-\( \sigma \) of age \( N\delta \) and to sink spacetime location \( x \) (the ‘field-point’), is
\[ \Phi_{\mu \nu}^N(x) = G c^{-3} \left[ E_{\nu}^{N \sigma} V_{\mu \nu}^{N \sigma}(x) + V_{\nu \mu}^{N \sigma}(x) E_{\nu}^{N \sigma} \right], \]  
(15)

where

\[ V^{N \sigma}_{\mu \nu}(x) \equiv \Theta_{\nu \mu}(x, a^\sigma) v^\sigma \mu \nu / v^\sigma \bullet (x - x^{N \sigma}), \]  
(16)

with the retardation step function \( \Theta_{\nu \mu}(x, a^\sigma) \) to be defined two paragraphs below. We suppose the field-point location \( x \) to lie within the \( N \)-ray immediate future, where

\[ (N\delta)^2 \times x \times x < [(N+1)\delta]^2. \]  
(17)

The \( x \) dependence of our potential is seen to reside in the invariant \( LW \)-denominator operator, \( v^\sigma \bullet (x - x^{N \sigma}) \). Because \( v^\sigma \bullet v^\sigma = 0 \), this classical denominator-polynomial has the same value at all ‘potential-source’ spacetime locations (not only those of age \( N\delta \)) along the lightlike trajectory with Source-\( \sigma \) velocity that passes through \( x^{N \delta} \). If the \( a^\sigma \) trajectory intersects the \( x \) backward lightcone we choose, in classical language, to call that intersection’s location the spacetime location of the ‘retarded source’ for \( \Phi_{\mu \nu}^N(x) \). Sources ‘classically located’ in the ‘distant past’ of \( x \) then associate to Age-\( N \)-ray (‘near past’) Hilbert-space sources whose spatial location is far from that of the field point \( x \).

The symbol \( \Theta_{\nu \mu}(x, a^\sigma) \) in (16) denotes a step function equal to 1 iff the \( a^\sigma \) trajectory (passing with velocity \( v^\sigma \) through the spacetime location \( x^{N \delta} \)) intersects the \( x \) backward lightcone. Otherwise \( \Theta_{\nu \mu}(x, a^\sigma) \) vanishes. (Any lightlike trajectory not located on the \( x \) lightcone intersects the \( x \) forward-backward lightcone exactly once.) Summed over all sources, the Ray-\( N \) expectation of (15) prescribes the classical gravitational tensor potential \( \Phi_{\mu \nu}^N(x) \) within the \( N \) immediate future.

Because the source-velocity 4-vectors \( v^\sigma \) are lightlike, the Lorentz-divergence of \( \Phi_{\mu \nu}^N(x) \) vanishes. When divided by \( G \), the Dalembertian of \( \Phi_{\mu \nu}^N(x) \) prescribes the \( N \) immediate future (without Heisenberg uncertainty) the 4-vector current density of conserved energy-momentum—‘gravitational reality’.

Although a symbol \( \Phi_{\mu \nu}(x) \), without superscript \( N \), conveniently designates the gravitational retarded potential almost everywhere in Milne spacetime, exclusion must be remembered of ray ages where \( \tau = t \delta \). Classical fields are not defined on the exceptional hyperboloids that house rays. Differential equations for (classical) gravitational fields and energy-momentum current densities, meaningful inside any \( SS \), only approximately extrapolate classical fields from one slice to the next. (In humanity-occupied universe such approximation often suffices \textit{FAPP}.) Feynman paths determine universe evolution quantum-mechanically—via action-specified phases of complex numbers.

Paralleling Formulas (15) and (16) we define (within the \( SS \) following Ray-\( N \)) a divergenceless electromagnetic Source-\( \sigma \) retarded vector potential operator whose dimensionality is that of action divided by time:

\[ A^{N \sigma}(x) \equiv \Theta_{\nu \mu}(x, a^\sigma) Q^\sigma v^\sigma / v^\sigma \bullet (x - x^{N \delta}). \]  
(18)

Here the symbol \( Q^\sigma \) designates source electric charge. The Ray-\( N \) expectation of \( A^{N \sigma}(x) \), summed over all sources, then yields the (classical) divergenceless electromagnetic vector
potential \( A(x) \) within Ray, \( N \)’s immediate future. (Represented by this expectation, when applied to the FAPP-P-stable source wave function of a single charged electron, is an effect QED calls ‘charge renormalization’. ) With an appropriate factor the Dalembertian of \( A(x) \) specifies ‘electromagnetic reality’ as a locally conserved electric-charge current density. Maxwell’s (classical) equations for electric and magnetic fields apply inside each spacetime slice.

**Conclusion**

This paper has quantized Milne’s cosmology so as to represent gravity while maintaining Dirac’s principles as well as that of Mach. A 7-parameter symmetry group (with a 6-parameter Lorentz subgroup) is accompanied by 7 (Noether) conserved self-adjoint group-generating operators—momentum, angular momentum and energy—whose expectations separately aggregate to zero for the universe as a whole. Classical reality resides in electromagnetic and gravitational fields within invariant spacetime slices of Planck-scale width. Each universe ray is separated from its successor by such a slice. Any ray (after the first) is determined from its predecessor by the actions of Feynman paths that traverse the preceding slice. Gravitational and electromagnetic line-integral path action, determined by the potentials here defined, are specified in a separate paper. Another paper in preparation proposes at path ‘branchings’ an event action that imitates GUT quantum field theory while maintaining gravity and Dirac-Feynman principles. Avoided are (Standard-Model) arbitrary parameters to specify scales, elementary-particle masses and fermion-generation mixing.

**Appendix: Momentum Operators; Dimensionality; Hilbert-Space Periodicity**

A trio of mutually-noncommuting self-adjoint source-momentum operators, components of a Lorentz 6-vector (not a 4-vector), transform under rotations as a 3-vector whose components commute with Formula (11)’s source-energy operator. The latter generates a Hilbert-space unitarily-represented single-parameter Lie group comprising displacements in source time at fixed universe age—3-space velocity-directed displacements along some base-space geodesic.

Formula (A.1) below is a self-adjoint superposition of (partial) first derivatives, different from (11), that represents the component of source momentum in an externally-fixed direction. The standard notation for Pauli matrices suggests calling the latter (not the source’s-velocity direction) the ‘3-direction’; the 3-direction within hyperboloids may arbitrarily be assigned. Formulas (6), (7) and (8) prescribe the 3-direction source-momentum operator to be

\[
P_3^{N\alpha} = i(\hbar/cN\delta) \left[ \partial/\partial 2Re s^\alpha + \partial/\partial \ln|y^\alpha| - \partial/\partial \ln|z^\alpha| \right]. \tag{A.1}
\]

Held fixed in (A.1) are the coordinates \( \arg y^\alpha, \arg z^\alpha \) and \( 2Im s^\alpha \). Partial derivatives with respect to the latter coordinate trio appear in a representation, paralleling (A.1), of 3-direction source angular momentum, \( J^\alpha_3 \). Four companion (by rotation) operators \( P_1^{N\alpha}, P_2^{N\alpha}, J^\alpha_1, J^\alpha_2 \) complete a 6-generator nonabelian Lorentz-group algebra.

Distinction is useful between ‘dimensionful’ and ‘dimensionless’ generators. ‘Dimensionality’ relates both to the dilation group and to the nonabelian Lorentz-group algebra.
In an infinite-age limit, the Milne-Lorentz group ‘contracts’ to the Euclidean group, whose spatial-displacement generators familiarly enjoy ‘momentum’ interpretation.

Units may be chosen such that $\hbar = c = 1$. Energy and momentum then have the same dimensionality, as do time and spatial displacement, while the latter pair’s (shared) dimensionality is the negative of that of the former. Any self-adjoint QMU operator enjoys some dimensionality $\eta$, where $\eta$ is an integer. Action, angular momentum, velocity and electric charge are all ‘dimensionless’—i.e., these operators and their expectations have $\eta = 0$.

So are products of energy and time displacement, as well as products of momentum and spatial displacement. [‘Opposite values of $\eta$ are generally carried by the two members of any Dirac-sense ‘canonically-conjugate’ pair of self-adjoint operators.] The momentum generators of the Milne-Lorentz group are dimensionful (because the slice width $\delta$ has $\eta = 1$), in contrast to the angular-momentum generators—which are dimensionless.

Because Newton’s gravitational constant has $\eta = 2$, the retarded single-source gravitational-potential self-adjoint operator (15), is dimensionless, as is the expectation of the corresponding sum over all sources—i.e., the classical gravitational potential, $\Phi_{\mu\nu}(x)$.

Although Milne’s hyperbolic classical 3-space is unbounded, we ‘compactify’ QMU source Hilbert space by requiring
\[
\psi^N[a^\delta] = \psi^N[exp(-\frac{1}{2}\sigma_3\Delta)a^\delta].
\]

The foregoing periodicity in $2Re \; s^\delta$, with a (dimensionless) period $\Delta$ that is common to all $\sigma$, we accompany by similar periodicity in all $\ln |y^\sigma z^\delta|$. Iff $\Delta$ is large, the universe is ‘large on Lemaître-Hubble scale’. The spectra of both source (local-frame) energy and source momentum are thereby universally discretized in units of $\hbar/\Delta\mathcal{N}\delta$. Compactification allows an initial condition at $\mathcal{N} = \mathcal{N}_0$ to specify zero total universe energy, momentum and angular momentum—a condition perpetuated by the Feynman-path action that propagates each ray to its successor.

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