Charm-beauty meson bound states from $B(B^*)D(D^*)$ and $B(B^*)\bar{D}(\bar{D}^*)$ interaction

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(Dated: July 19, 2017)

We evaluate the s-wave interaction of pseudoscalar and vector mesons with both charm and beauty to investigate the possible existence of molecular $BD$, $B^*D$, $BD^*$, $B^*D^*$, $BD$, $B^*D$, $BD^*$ or $B^*D^*$ meson states. The scattering amplitude is obtained implementing unitarity starting from a tree level potential accounting for the dominant vector meson exchange. The diagrams are evaluated using suitable extensions to the heavy flavor sector of the hidden gauge symmetry Lagrangians involving vector and pseudoscalar mesons, respecting heavy quark spin symmetry. We obtain bound states at energies above 7 GeV for $BD$ ($J^P = 0^+$), $B^*D$ ($1^+$), $BD^*$ ($1^+$) and $B^*D^*$ ($0^+$, $1^+$, $2^+$), all in isospin 0. For $BD$ ($0^+$), $B^*D$ ($1^+$), $BD^*$ ($1^+$) and $B^*D^*$ ($0^+$, $1^+$, $2^+$) we also find similar bound states in $I = 0$, but much less bound, which would correspond to exotic meson states with $b$ and $c$ quarks, and for the $I = 1$ we find a repulsive interaction. We also evaluate the scattering lengths in all cases, which can be tested in current investigations of lattice QCD.

PACS numbers:

INTRODUCTION

The present situation of the mesons with one quark of type $b$ and an antiquark of type $c$, $B_c$ mesons, is still at an early beginning. There are just two states reported in the PDG (Particle data Book) $^1$, the $B_c(6275)$ and the $B_c(2S)(6842)$. This contrasts with the situation in the bottom strange sector, where we have the states $B_s(5377)$, $B_s^*(5415)$, $B_{s1}(5830)$, $B_{s2}(5840)$, $B_{s2}(5850)$ and in the charm strange sector where there are already ten $D_s$ states reported, with an average separation between the masses of about 100 MeV. By contrast, the only two $B_c$ states reported are separated by nearly 600 MeV. Lattice QCD has also made a contribution to the heavy meson sector, investigating possible tetraquarks or molecular states $^2$ $^4$, however, none of them deals with the $BD$ quantum numbers.

It is clear that many states are missing which most hopefully will be discovered in coming years. An idea of the advance made in time is the addition of three new $D_s$ states since the 2008 edition of the PDG $^7$ and one $B_c$ state. Yet, the advent of LHCB has made the prognosis brighter, one recent example being the determination of five new $\Omega_c$ states $^5$.

Although some of the states expected should correspond approximately to the $q\bar{q}$ standard structure of the mesons, the irruption of so many XYZ states $^4$, which do not fit into the traditional $q\bar{q}$ picture, motivated a large number of theoretical studies that go beyond this picture, invoking especial quark configurations $^1$ $^7$, tetraquarks $^2$ $^17$, or meson meson molecules $^16$ $^34$. Mixtures of charmonium states and molecules have also been investigated $^35$ and methods to disentangle the nature of the states have been suggested $^36$ $^40$. Reviews on these issues are available in Refs. $^41$ $^43$.

In the present work we take the case of the interaction of $B(B^*)$ and $D(D^*)$ mesons, an also the corresponding cases with $D(D^*)$. Given the analogy of the $B$ meson with a $K$ meson, the states we study have an analogy with the $DK$, $DK^*$, $D^*K$ interactions. According to Ref. $^26$ the $DK$ channel is the main building block of the $D_{s0}(2317)$, something that is corroborated by the analysis of lattice QCD results in the light-heavy sector $^44$. Similarly, the $D^*K$ component appears as the main building block of the $D_{s1}(2460)$ in Ref. $^45$, which is again corroborated by the lattice QCD study of Ref. $^44$. And in Ref. $^46$ one also finds that the $D_{s1}(2536)$ resonance is mostly formed from the $DK^*$ component. Similarly the $D^*K^*$ interaction appears as the main building block of the $D_{s2}(2573)$ in Ref. $^46$. The $D^*D^*$ interaction is also studied in Ref. $^21$ and bound states are reported there. In view of that, it is reasonable to expect bound states of the $B(B^*)D(D^*)$ systems, which we study in the present work. The formalism that we use is the local hidden gauge approach $^47$ $^50$, which combines pseudoscalar and vector mesons, properly extended to the heavy quark sector $^51$. The interaction stems from the exchange of vector mesons between the interacting mesons, and in the limit of small momentum transfers this gives rise to the chiral Lagrangians in the light quark sector. An example for the interaction of vector-pseudoscalar is given in Ref. $^51$, where it is shown that it gives rise to the chiral Lagrangian of Ref. $^51$. It is also interesting to mention that the exchange of light vector mesons between hadrons involving heavy quarks respects heavy quark symmetry $^52$ as shown in Refs. $^23$ $^53$.

We find that all the four systems lead to bound states in $I = 0$, and in the case of $B^*D^*$ there are three spin states, degenerate in energy within the model. We also study the $B(B^*)\bar{D}(\bar{D}^*)$ systems and here we find that there is attractive interaction in $I = 0$ and repulsive in-
In the present work, it is sufficient to write the $P$ matrix in Eq. (1) in $SU(4)$ ($u, d, c$ and $b$ flavors) and is given by

$$P = \begin{pmatrix} 0 & 0 & B^+ & D^0 \\ 0 & 0 & B^0 & D^+ \\ B^- & B^0 & 0 & B_c^- \\ D^0 & D^+ & B_c^- & 0 \end{pmatrix},$$

where we do not show the light pseudoscalars which are not relevant for the present work. Analogously, for the vector mesons we have

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\phi^0}{\sqrt{2}} & \rho^+ & B^{++} & D^{*0} \\ \rho^- & \frac{\rho^0}{\sqrt{2}} - \frac{\phi^0}{\sqrt{2}} & B^{*0} & D^0 \\ B^{*+} & B^{*0} & 0 & B_c^{*-} \\ B^{*+} & D^{**} & B_c^{*-} & 0 \end{pmatrix}.$$  

Since both $D$ and $B$ are isospin $I = 1/2$ states, the total $BD$ isospin can be 0 and 1. However, the $I = 1$ interaction is very small since, from the above Lagrangians, it can be obtained that the $\rho$ and $\omega$ exchange contributions for this isospin channel have different sign and they cancel among themselves, up to the small difference between the masses squared of the $\rho$ and $\omega$. This is not the case for the $BD$ interaction in $I = 1$, where $\rho$ and $\omega$ contributions have the same sign. This can also be understood at the quark level by looking at the diagrams of Fig. 2 where one can see that for the $BD$ case in $I = 1$ it is not possible to exchange a vector meson at first order, while for $BD$ it is allowed via $u\bar{u}$ exchange.

It might look that we are making use of $SU(4)$ symmetry by using Eqs. (2) and (3), but actually writing the Lagrangian in this form is only a practical way to obtain the couplings of the heavy mesons to the light vectors, that we can also obtain in a very simple picture where the heavy quarks are spectators, in the spirit of the heavy quark formalism, and we are only making use of $SU(2)$ symmetry. Indeed, we can write the $\rho^0$, $\omega$, sources ($\phi$ is not present in our case) as

$$g \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d}) \quad \text{for } \rho^0 \text{ exchange},$$

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and taking into account the vector type coupling we would have the operator
\[ -g \frac{1}{\sqrt{2}} \left( (u \partial_\mu \bar{u} - \partial_\mu u \bar{u}) - (d \partial_\mu \bar{d} - \partial_\mu d \bar{d}) \right). \tag{5} \]

Let us study, as an example, the cases of \( B^0B^0 \rho^0 \), \( D^+D^+\rho^0 \) and the other cases follow directly from them. The heavy mesons are \( B^0 = \bar{b}d, D^+ = c\bar{d} \) and, since the heavy quarks are spectators, we have the matrix elements
\[ -(\bar{b}d)\frac{1}{\sqrt{2}} \left( (u \partial_\mu \bar{u} - \partial_\mu u \bar{u}) - (d \partial_\mu \bar{d} - \partial_\mu d \bar{d}) \right) |\bar{b}d \rangle = -g \frac{1}{\sqrt{2}} (-ip_\mu - ip'_\mu) \tag{6} \]
for \( B^0B^0 \rho^0 \) and
\[ -(c\bar{d})\frac{1}{\sqrt{2}} \left( (u \partial_\mu \bar{u} - \partial_\mu u \bar{u}) - (d \partial_\mu \bar{d} - \partial_\mu d \bar{d}) \right) |c\bar{d} \rangle = -g \frac{1}{\sqrt{2}} (ip_\mu + ip'_\mu) \tag{7} \]
for \( D^+D^+\rho^0 \), where \( p (p') \) is the light quark initial (final) momentum. In the limit of \( B \) at rest, \( p_\mu + p'_\mu \) will become \( 2m_q \delta_{\mu \nu} \), with \( m_q \) the mass of the light quark. Let us compare this with the coupling of \( K^0K^0 \rho^0 \) \((K^0 = \bar{s}d)\), which in the limit of the \( K^0 \) at rest gives us the same contribution \( 2m_q \delta_{\mu \nu} \). This means that in the spectator picture for the \( b \) or \( s \) quarks the matrix element for \( B^0B^0 \rho^0 \), \( K^0K^0 \rho^0 \), are the same at the microscopic quark level. However, when we write the amplitudes at macroscopic hadron level, we must take into account that the S-matrix has the field normalization factors \( \frac{1}{\sqrt{2m_q}} \) for each external hadron \( (H) \) (see Eqs. (14)-(16) of ref. \[53\]). Hence, at the macroscopic level we would have at threshold
\[ \frac{t_{B^0B^0\rho^0}}{t_{K^0K^0\rho^0}} = \frac{M_B}{M_K}. \tag{8} \]

Since
\[ -it_{K^0K^0\rho^0,\mu} = -g \frac{1}{\sqrt{2}} (-iM_K - iM_K) \delta_{\mu \nu}, \tag{9} \]
then
\[ -it_{B^0B^0\rho^0,\mu} = -g \frac{1}{\sqrt{2}} (-iM_B - iM_B) \delta_{\mu \nu}, \tag{10} \]
and in covariant form
\[ -it_{B^0B^0\rho^0,\mu} = -g \frac{1}{\sqrt{2}} (ip_\mu - ip'_\mu), \tag{11} \]
and this is what we get straightforwardly from the use of the Lagrangian of Eq. \[11\]. It is also interesting to see that the relative sign between \( B^0B^0 \rho^0 \) and \( D^+D^+\rho^0 \) comes because in \( B^0 \) we have a \( d \) quark and in \( D^+ \) we have a \( \bar{d} \) quark and we have the operator \( \bar{q} \partial_\mu q - \partial_\mu \bar{q} q \).

In order to evaluate the \( I = 0 \) \( BD \) interaction we need the \( I = 0 \) combination, with the doublets \( (B^+, B^0), (D^+, -D^0) \),
\[ (BD)_{(I=0)} = -\frac{1}{\sqrt{2}} (|B^+D^0\rangle + |B^0D^+\rangle) \tag{12} \]
and therefore the \( I = 0 \) amplitude can be written as
\[ t^{(I=0)}_{BD \rightarrow BD} = \frac{1}{2} (t_{B^+D^0 \rightarrow B^+D^0} + t_{B^0D^+ \rightarrow B^0D^+} + t_{B^0D^+ \rightarrow B^+D^0} + t_{B^0D^0 \rightarrow B^0D^+}) \tag{13} \]
The amplitudes in the bracket in the previous equation account for the diagrams in Fig. \[3\]. From the Lagrangian of Eq. \[11\], the amplitude of Eq. \[13\] can be readily calculated, leading to
\[ t^{(I=0)}_{BD \rightarrow BD} = -\frac{1}{2} (p + p')_\mu (k + k')^\mu \tag{14} \]
where we have approximated \( m_\rho^2 = m_\rho^2 \simeq m_\rho^2 \). In addition, in the derivation of Eq. \[14\], we have neglected the momentum transferred in the vector meson propagator. (The correction due to this effect will be taken into account below including form factors in the loop functions that will appear in the unitarization procedure.)

There is still one more issue that should be discussed since the heavy quark spectator hypothesis in the \( BB\rho \) coupling has been challenged by some other approaches. Indeed evaluations using the Dyson-Schwinger equation
or QCD lattice simulations lead to a value of about twice as big as $g_{KK\rho} = g_{DD}\rho$ of Ref. 63, which we are using here in Eq. 14. However, the couplings go together with a form factor leading to a stronger off-shell reduction than the one used here. A detailed discussion of this issue and the added uncertainties to our approach, extending the discussion to the $B$ sector, was done in section VII of Ref. 24, concluding that it added extra uncertainties in the bindings, increasing the binding by an amount which could be as large as 40%. Although our system here is different than those studied in 24, it still deals with heavy mesons, and the exercise done here serves to give an idea of possible uncertainties and a hint that those considerations might increase the binding that we get.

After projecting over s-wave, Eq. (14) reads

$$t_{BD\rightarrow BD}^{(1=0,\text{ s-wave})} = -\frac{1}{4f^2} \left[ 3s - 2(m_B^2 + m_D^2) - \frac{(m_B^2 - m_D^2)^2}{s} \right]$$

(15)

For the other channels we are considering in the present work, $B^*D$, $BD^*$, $B^*D^*$, $BD$, $B^*D$, $BD^*$ or $B^*D^*$ the formalism is analogous to the $BD$ case with the following particular features and considerations:

- $B^*D$ and $BD^*$: The $SU(4)$ matrices have the same structure as in the previous case since the quark content of $B^*$ is the same as $B$, and $D^*$ the same as $D$. Furthermore in Ref. 22 it was justified by using HQSS that in the heavy sector the vector-pseudoscalar interaction is the same as pseudoscalar-pseudoscalar at leading order in the inverse of the heavy quark mass. Therefore, the only difference with the $BD$ case is the vector character of the $B^*$ and $D^*$ which implies that, neglecting terms of order $q^2/m_V^2$, one has to add an $\hat{c} \cdot \hat{c}'$ factor in Eq. (15) (where $\hat{c}(\hat{c}')$ is the initial(final) vector polarization vector) and the masses must be replaced by $m_{B^*}$ or $m_{D^*}$ accordingly.

- $B^*D^*$: Again the flavor structure is the same and analogous arguments than before apply. In addition, a contact $VVV$ term from the HGS Lagrangian $\mathcal{L}_{VVV} = \frac{g}{2} (\bar{V}_\mu V^\mu + \bar{V}_\mu V'^{\mu})$ would be present but is subdominant and can thus be neglected. Furthermore, all four external particles are now vector mesons and thus it turns out that we can use the same expression as Eq. (15) but adding a factor $\hat{c}_{B^*} \cdot \hat{c}_{D^*}$, and replacing $m_B$ and $m_D$ by $m_{B^*}$ and $m_{D^*}$.

- $BD$, $B^*D$, $BD^*$ or $B^*D^*$: We can analogously calculate the same interactions as before but substituting $D$ and $D^*$ by $\bar{D}$ and $\bar{D}^*$.

The kernels $t_{BD\rightarrow BD}^{(1=0,\text{ s-wave})}$, $t_{BD\rightarrow BD}^{(1=0,\text{ s-wave})}$, and $t_{BD\rightarrow BD}^{(1=1,\text{ s-wave})}$ as functions of $\sqrt{s}$ are given in Fig. 4 which shows the attractive nature of the $BD$ and $BD^*$ ($I = 0$) interactions and the repulsive one of $BD$ ($I = 1$).

We can think of other elements that could be exchanged, apart from the vector mesons considered. A thorough investigation of other possible mechanisms was made in Refs. 65, 66 in the study of the $DD^*$ interaction with $I = 1$ and its relationship to the $Z_{b}(3900)$ (or $Z_{c}(3885)$) state in one case and in the study of $BB^*$, $B^*B^*$ interaction with $I = 1$ and its relationship to the $Z_{b}(10610)$, $Z_{b}(10650)$ states in the other case. This was done because in these cases there is no light vector exchange and then one could only exchange $J/\psi$ in one case and $Y$ in the other, which made the vector exchange very small and gave chances to other mechanisms to contribute. One of the possible mechanisms was the exchange of two pions, uncorrelated (non interacting) or correlated (interacting). The case of two pion exchange with interacting pions gives rise to a “n” exchange in this picture, as was shown in Ref. 67. The conclusion of Ref. 67 was that the two pion exchange still gave a factor of four smaller contribution than the $DD^*$ or $DD^* \rightarrow \eta_\rho$ or $DD^* \rightarrow \pi J/\psi$ transitions that involve a $D^*$ exchange. Considering that the light vector exchange potential gives an $m_\rho^2$ dependence rather than $m_{B^*}^2$ in the $DD^* \rightarrow \eta_\rho$ transition.
transitions, this gives a suppression of a factor about 30 of the two pion exchange with respect to the light vector exchange when it is allowed, as in the present case. Similar conclusions can be reached from the results in the B sector when one increases the $B^*$ exchange potential in terms like $BB^* \rightarrow \rho \Upsilon$ by the ratio $m_{B^*}^2/m_{\rho}^2$ to compare the two pion exchange with an allowed light vector transition.

**Implementation of unitarity**

Using the techniques of the coupled channels unitary approach, exact unitarity can be implemented into the $BD$ interaction, which can be carried out by means of the Bethe-Salpeter equation: (equivalent to the $N/D$ \[70, 71\] or IAM \[72, 73\] methods)

$$T = [1 - VG]^{-1} V,$$  

(18)

where $V$ is the potential, or kernel of the unitarization procedure, provided by Eq. (15) and $G$ is the $BD$ loop function:

$$G = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_B^2 + i\epsilon} \frac{1}{(q - p)^2 - m_D^2 + i\epsilon},$$

(19)

for a total initial four momentum $P$. The regularization of the loop function $G$, which is logarithmically divergent, has been usually done in the chiral unitary approach by means of dimensional regularization or with a three-momentum cutoff, $q_{\text{max}}$, and both usually provide equivalent results. However, it was justified in Refs. \[22, 74, 75\] that the cutoff method is more convenient in the heavy flavor sector and, therefore, this is the regularization method we will use in the present work. In terms of a three-momentum cutoff, the loop function reads

$$G = \int_{0}^{q_{\text{max}}} \frac{d^4q}{(2\pi)^4} \frac{\omega_D + \omega_B}{2\omega_D\omega_B} \frac{1}{(P^0)^2 - (\omega_D + \omega_B)^2 + i\epsilon},$$

(20)

with $\omega_{D(B)} = \sqrt{m_{D(B)}^2 + q^2}$. It was shown in ref. \[74\] that in order to respect heavy quark symmetry in the unitarized hadron-hadron interaction a special $G$ function could be used, which was, however, equivalent to stating that in the cutoff method the same cutoff, independent of heavy flavor, should be used. The same conclusion, with different arguments, was reached in \[22\]. Hence we use values of the same order as those used in $BB$. Therefore, we will consider values $q_{\text{max}} \in [400, 600]$ MeV, where the differences in the results by varying the cutoff within this range can be considered as an estimation of the uncertainty in our calculation. In Eq. (15), $V$ is factorized out of the loop function since the momentum in the propagator of the exchanged vector meson is neglected. However the running momentum inside the loop can reach values comparable to the exchanged vector meson mass. In Ref. \[72\] it was justified that this effect can be taken into account by including a factor $f^2(q^2)$ in the integrand of Eq. (20), where $f(q)$ is the form factor

$$f(q) = \frac{m_V^2}{q^2 + m_V^2}. \quad (21)$$

The factor corresponds to the propagator of the exchanged vector neglecting the energy exchange $q^0$, which is zero in the on shell diagonal transitions, $BD \rightarrow BD$ for instance, and we also take it zero in the propagator of the exchanged vector in the loops, following the on shell factorization of the potential as discussed in Refs. \[71, 74\]. Eq. (21) has assumed the external meson momentum to be zero. We can improve upon that, by considering an average initial momentum of the order of $p = \sqrt{2\mu B}$ where $B$ is the binding energy of the molecule and $\mu$ the reduced mass of its two components. Then $q$ in Eq. (21) has to be replaced by $(\vec{p} - \vec{q})$. After projecting over s-wave, for the new Eq. (21) we obtain the factor

$$f(q) = \frac{m_V^2}{4pq} \ln \left[ \frac{(p + q)^2 + m_V^2}{(p - q)^2 + m_V^2} \right]. \quad (22)$$

Note that this factor is never singular because $p$ is real, as it corresponds to an average over the momentum distribution of the molecular state.

On the other hand, in the evaluation of the $B^* D(\bar{D})$, $BD^*(\bar{D}^*)$ and $B^* D^*(\bar{D}^*)$ interaction, there are vector mesons in the loop function whose polarization vectors should be carefully treated in the resummation implicit in the unitarization procedure. For the general vector-pseudoscalar interaction this was done in Ref. \[74\], where it was shown that, using the $\varepsilon \cdot \varepsilon'$ structure in the potential, the same Bethe-Salpeter equation could be used, up to a correction in the loop function of $q^2/(3m_V^2)$ which we can safely neglect. Furthermore, the masses in the loop function must be changed to $m_D$- and/or $m_{B^*}$ accordingly for the corresponding channels.

**Results**

Since we are evaluating the interaction in s-wave, the possible quantum numbers of the different channels are, $J^P = 0^+$ for $BD$; $1^+$ for $B^* D$ and $BD^*$ and degenerate $0^+, 1^+, 2^+$ for $B^* D^*$. (All in isospin $I = 0$ as explained below Eq. (3)). For $BD$, $B^* D$, $BD^*$ and $B^* D^*$ the spin-parities are the same as for the $B(B^*)D(D^*)$ case but now the isospin can be 0 or 1 (see Eqs. (10) and (17)).

By looking for poles in the second Riemann sheet of the unitarized amplitudes, Eq. (15), for $Re(\sqrt{s})$ above the threshold or in the physical sheet below, we can see whether the interaction is strong enough to generate dynamically a resonance in the former case or a bound state.
TABLE I: Positions of the bound states (√s_ρ), binding energies (B) and couplings (g) of the different channels. The first number in the last four columns represents the result for q_{max} = 400 MeV and the second for 600 MeV. All units are in MeV except the scattering lengths, a, which are in fm.

| Channel | I(J^P) | √s_ρ | B (MeV) | g | a [fm] |
|---------|-------|------|--------|---|-------|
| BD      | 0(0^+) | 7133/711 | 15/38 | 33484 | 49867 | -1.78|-1.45 |
| B^*D    | 0(1^+) | 7179/7156 | 15/38 | 33742 | 50243 | -1.78|-1.45 |
| BD^*    | 0(1^+) | 7270/7247 | 16/39 | 35171 | 52262 | -1.75|-1.45 |
| B^*D^*  | 0(0^+, 1^+, 2^+) | 7316/7293 | 16/39 | 35438 | 52652 | -1.75|-1.45 |
| BD      | 0(0^+) | 7146/7140 | 17/8.4 | 33225 | 23296 | -3.77|-1.93 |
| B^*D    | 0(1^+) | 7192/7186 | 17/8.4 | 33537 | 23494 | -3.74|-1.93 |
| BD^*    | 0(1^+) | 7284/7277 | 21/9.5 | 14539 | 24915 | -3.32|-1.83 |
| B^*D^*  | 0(0^+, 1^+, 2^+) | 7330/7322 | 21/9.5 | 14678 | 25123 | -3.31|-1.83 |

In the latter one. In the present case, for all the channels with I = 0, we find poles in the physical sheet below the threshold which thus correspond to bound states. The position of these poles, √s_ρ, can be identified as the mass of the generated bound state and are shown in table I for different values of the regularization cutoff, the first number in the last four columns stands for the result for q_{max} = 400 MeV and the second one for 600 MeV. The difference in the values obtained for both cutoffs should be regarded as the main uncertainty in our model. In the table we also show the corresponding binding energies B = √s_{threshold} − √s_ρ and the values of the couplings of the different poles to the corresponding channels, which are defined considering that close to the pole

\[ T \simeq \frac{g^2}{s - s_p}, \]  

and they can be obtained by evaluating the residue of T at the pole position.

We find bound states, poles below threshold, for all the channels with I = 0 at energies ranging a few hundred MeV above 7 GeV and binding energies of about 15-40 MeV for BD, B^*D, BD^* and B^*D^* interactions and about 2-10 MeV for B^*D, B^*D, BD^* and B^*D^*. For the latter channels, in I = 1 the interaction is repulsive and, thus, no poles are found. Note that, despite the large uncertainty in the binding energy, stemming from the cutoff dependence, the absolute size of the binding energy is small compared to the mass of the system and is of the same order of magnitude as in other heavy flavor systems [22, 23, 72]. Note also that the binding energies are almost degenerate for all the channels. This is a manifestation of the independence of the binding energy on the heavy quark mass as a consequence of the HQSS [23, 76, 78]. For the BD, B^*D, BD^* and B^*D^* channels in I = 0 the binding energy is very small, therefore the claim of their correspondence to actual mesons should be taken cautiously since further refinements of the model could make the pole disappear. However, the fact that we find poles for all the range of the cutoff considered is a point in favor of their actual existence.

In the last column of table I we also show the values of the s-wave scattering lengths

\[ a = -\frac{1}{8\pi \sqrt{s}_{th}} T(\sqrt{s}_{th}), \]

with \( \sqrt{s}_{th} \) the energy of the corresponding threshold, (and where we have used the scattering length sign convention \( p \cot \delta = \frac{1}{\rho} + \frac{1}{2}\pi \rho^2 \)).

It is worth stressing that the dynamics used here for the interaction, based on the HGS approach, stems from vector exchange. One can see that the source of attraction from this source in systems of this type is much bigger than the one obtained from pion exchange, via two step processes like BD \( \rightarrow \) B^*D^* \( \rightarrow \) BD [53, 79]. In view of this it is not surprising that in [80] no bound state for the BD system was found using one pion exchange. We would like to note here that the exchange of vector mesons has also been introduced in quark models with the name of extended chiral quark model, [81, 85] and its effects have been found to be important.

The states found in the present work could in practice correspond to actual resonances with a narrow width which would come from subdominant channels with thresholds below the pole positions. It is worth mentioning that, according to the particle data table (PDG [4]), no mesons with both charm and beauty (in addition to the \( B_c^+(6275)(0^-) \) and the \( B_c(2S)^+(6842)(0^-) \)) have been experimentally discovered. It is also worth noting that the poles in I = 0 for the BD, B^*D, BD^* and B^*D^* would correspond to exotic mesons since they would contain a \( b \) and \( c \) quark at the same time. The findings in the present work are an indication that there is still much room to improve the so far scarce experimental evidence of mesons with charm and beauty which would help understand the dynamics of the heavy flavor sector.

**SUMMARY AND CONCLUSIONS**

We have done a theoretical study of the BD, B^*D, BD^*, B^*D^*, BD, B^*D, BD^* and B^*D^* interaction to try to see the possible dynamical generation of mesons with both charm and beauty flavors. We evaluate the interaction starting from a tree level elementary process obtained from suitable extensions of the hidden gauge symmetry Lagrangians to heavy flavor, compatible with the heavy quark spin symmetry of QCD, in order to evaluate the dominant mechanisms with a vector meson ex-
change. We made a derivation of the Lagrangians in the heavy sector based on the hypothesis of having the heavy quarks as spectators. We find an attractive and sizable potential for the interaction in isospin $I = 0$ for all the interactions. These potentials are used as the kernel of the unitarization procedure using the techniques of the coupled channels unitary approach which only depends on one free regularization parameter. The dependence on the model on this parameter, a three-momentum cutoff, represents the main source of uncertainty of the model. By looking for poles of the unitarized amplitudes we find poles below the thresholds of the different channels with $I = 0$ which thus correspond to bound states with quantum numbers $J^P = 0^+$ for $BD$; $1^+$ for $B^*D$ and $BD^*$ and degenerate $0^+$, $1^+$, $2^+$ for $B^*D^*$, at energies slightly above 7 GeV and with binding energies of about 20-60 MeV. Similarly, for the $BDD^*$ ($0^+$), $B^*D$ ($1^+$), $BD^*$ ($1^+$) and $B^*D^*$ ($0^+$, $1^+$, $2^+$) interaction we also find bound states in $I = 0$ but the interaction is repulsive in $I = 1$. These latter bound states would correspond to exotic mesons with $b$ and $\bar{c}$ quarks.

We find several states of the type $B_c$ which do not correspond to the only two $B_c$ states so far reported in the PDG as the ground state $B_c$ and $B_c(2S)$. They are predictions that find an analogy with many states already found in the $D_s$ sector. On the other hand, we also find six new states of $B(B^*)D(D^*)$ type, with $I = 0$, which are clearly exotic since they contain a $b\bar{c}$ pair of heavy quark, and are not of the $q\bar{q}$ type. The results obtained here and the similarity of the states found to some already observed in the $D_s$ states should stimulate the experimental search of these states that should shed valuable light on hadron dynamics.

Acknowledgments

This work is partly supported by the Spanish Ministerio de Economía y Competitividad and European FEDER funds under the contract number FIS2011-28853-C02-01, FIS2011-28853-C02-02, FIS2014-57026-REDT, FIS2014-51948-C2-1-P, and FIS2014-51948-C2-2-P, and the Generalitat Valenciana in the program Prometeo II-2014/068 (EO).

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