VCRIX - a volatility index for crypto-currencies*

Alisa Kim¹, Simon Trimborn²,³, and Wolfgang Karl Härdle¹,⁴,⁵,⁶,⁷

¹Humboldt-University at Berlin, Germany
²Department of Management Sciences, City University of Hong Kong, Hong Kong
³School of Data Science, City University of Hong Kong, Hong Kong
⁴Wang Yanan Institute for Studies in Economics, Xiamen University, China
⁵Sim Kee Boon Institute for Financial Economics, Singapore Management University, Singapore
⁶Faculty of Mathematics and Physics, Charles University, Czech Republic
⁷Yushan Scholar, National Yangming Jiaotong University, Taiwan

Abstract

Public interest, explosive returns, and diversification opportunities gave stimulus to the adoption of traditional financial tools to crypto-currencies. While the CRIX offered the first scientifically-backed proxy to the crypto-market (analogous to S&P 500), measuring the forward-oriented risk in the crypto-currency market posed a challenge of a different kind. Following the intuition of the "fear index" VIX for the American stock market, the VCRIX volatility index was created to capture the investor expectations about the crypto-currency ecosystem. VCRIX is built based on CRIX and offers a

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†Corresponding author, phone: +852 3442-8587, E-Mail: trimborn.econometrics@gmail.com
forecast based on the Heterogeneous Auto-Regressive (HAR) model. The
HAR model was selected as the most suitable out of a horse race of volatility
models, with two proxies for implied volatility, namely the 30 days mean
annualized volatility and realized volatility. The model was further examined
by the simulation of VIX (resulting in a correlation of 78\% between the
actual VIX and a "VIX" version estimated with the VCRIX technology).
Trading strategies confirmed the predictive power of VCRIX and supported the
selection of the 30 days means annualized volatility proxy. The best performing
trading strategy with the use of VCRIX outperformed the benchmark strategy
for 99.8\% of the tested period and 164\% additional returns. VCRIX provides
forecasting functionality and serves as a proxy for the investors’ expectations in
the absence of a developed crypto derivatives market. These features provide
enhanced decision making capacities for market monitoring, trading strategies,
and potentially option pricing.

**Keywords:** index construction, volatility, crypto-currency, VCRIX

**JEL classification:** C51, C52, C53, G10

## 1 Introduction

Since the inception of Bitcoin (BTC) in 2008 the crypto-currency (CC) ecosystem
has seen a market capitalization explosion that reached 2525 billion USD at its
highest point on May 12, 2021 (CoinMarketCap (2021)). Apart from traditional
hedge-funds and institutional investors who are interested in diversification, the
CC ecosystem saw more than 400 crypto-funds launched during the years (next.
autonomous.com/cryptofundlist). The rapid growth of BTC price led to persistent
talks about "bubble-like" behavior and general skepticism of the market (Hafner
(2020), Cheung et al. (2015)), exposing the need for a deeper understanding of the
underlying processes driving the valuation of CCs. The interest in the underlying
process led to the development of monetary equilibrium models (Schilling and Uhlig,
2019) and factor models for the pricing of CCs (Liu et al., 2019). Traditional market
instruments (indices, ratings, investment portfolios) joined the ecosystem, including
the early efforts such as CRIX by Trimborn and Härdle (2018) and exploration
of the potential of CC as an investment tool (Petukhina et al. (2021)). Also the
informative value from experts sentiment and discussion topics was investigated for their ability to explain market movements (Trimborn and Li, 2021).

Introduction of BTC futures by the CME and Chicago Board Options Exchange (Cboe) on December 18, 2017 reinforced the positions of CC as a new asset class. The emergence of the derivatives market signaled the need for solid pricing strategies and a reliable (and stable) risk measure. The paper on pricing CCs by (Hou et al., 2020) addressed this issue by employing a Stochastic Volatility with a Correlated Jumps model (Duffie et al., 2000) and using insights on implied volatility dynamics (Fengler et al., 2003) in order to match non-stationarity and local heterogeneity phenomena of CRIX returns.

Industry demand and research revealed the necessity to explore the behavior of the CC volatility further, to provide the final ingredient - a proxy for implied volatility. In traditional markets, implied volatility is measured by volatility indices which can be considered a traditional financial tool. At the end of the 20th century, financial markets of the USA and Europe aimed to capture the global measure of volatility in the respective market, which led to the introduction of VIX or VDAX. The index providers settled on the model most appropriate for the specifics of the behavior of the corresponding derivative. Given the absence of a developed derivatives market for CCs, we have to infer the characteristics of the implied volatility from the CC market behavior only. The specifics of the latter (high volatility and low liquidity) triggered the development of new investment methods, see Trimborn et al. (2019), further justifying the need for a volatility index, that would capture the unique specifics of CC as an asset class and provide a reliable indicator for the continuously unstable market.

This study aims to develop a measure and model to capture the expectations of the CC market and map them into an index referred to as VCRIX - a volatility index for CCs. VCRIX is especially designed for markets akin to the CC ecosystem, see Subsection 3.4, capable of accurately reflecting the market risk. The goal of the proposed VCRIX is proper risk measurement for the CRIX components and delivery of market status information, analogous to implied volatility indices that capture investors expectations. The study is challenged by the absence of derivatives for most CCs (except Bitcoin), which impedes the direct application of the underlying
methodologies of VIX or VDAX which measure the implied volatility from the
derivatives and map them into an index. Therefore any study and therefore also the
methodology for VCRIX rests on achieving the following milestones:

1. Identification and estimation of a valid implied volatility proxy
2. Identification of a model to facilitate consistent predictive performance for the
   implied volatility proxy
3. Construction of VCRIX based on the implied volatility proxy as a measure for
   market uncertainty

To tackle these milestones, we develop volatility measures intended to proxy
implied volatility, namely 30 day rolling historical volatility and realized volatility.
To account for the forward-looking behaviour of implied volatility, we implement
volatility models to achieve an estimated measure catching future information. We
compare the performance of a large variety of volatility models to identify the
best performing model towards our goal: Development of a measure and model
to capture the expectations of the CC market and construction of VCRIX. The
model contestants comprise of univariate and multivariate GARCH-type model,
the machine learning approach of Long-Short Term Model (LSTM) as well as the
Heterogeneous AutoRegressive Model (HAR). The results indicate that the HAR
model is the best prediction tool for the proxies. The outperformance of the HAR
model rests on its structure which is designed to reflect the behaviour of market
participants and the resulting effect on the volatility. Corsi (2009) motivates the
HAR model from the Heterogeneous Market Hypothesis which recognizes heterogeneity
in market participants behaviour. Whereas the HAR model was not designed and
motivated for the CC market, its outstanding performance in this study gives rise
to believe that similar volatility structures exist in the CC market as in equity
markets. To evaluate the appropriateness of the selected methodology for its ability
to proxy a volatility index, we apply the methodology to the S&P500 and compare
the Approximated VIX (AVIX) against the actual VIX (Cboe, 2019). We find that
AVIX has a high correlation and matching Mean Directional Accuracy with VIX.
Finally we study the ability of VCRIX as a risk measure by employing its signals
into a long-cash and long-short trading strategy. The trading strategy based upon
VCRIX achieves a higher cumulative return than an investment into CRIX which provides evidence for VCRIX suitability as a global CC vola measure. Lastly we discuss the economic insights which VCRIX provides and how its state relates to major events in the CC market.

The paper is structured as follows. Section 2 offers an overview of the used data sets for both traditional and CC markets. Section 3 provides a detailed explanation of the methodology used, whereas Subsection 3.1 contains the details on the implied volatility proxy estimation, followed by Subsection 3.2 that clarifies VCRIX model selection and the evaluation in Subsection 3.3. A brief revision of CRIX is provided in Subsection 3.4 which was selected as an equivalent for the S&P 500, a note on the existing implied volatility indices and VIX methodology in particular (Subsection 3.5). Methodological results, details of the VIX simulation conducted to test the selected methodology and final time series are showcased in Section 4. The ability of the proposed volatility index to predict future risk is further explored in Section 5, which contains a long-short trading implementation of VCRIX as the signal to the trading strategy. Additional observations and a summary of the conducted research are provided in Sections 6 and 7.

2 Data

This research employs, crypto-currency price data, CRIX data and traditional financial data, namely S&P 500 index values and VIX, which is the volatility index of Cboe based on the S&P 500. The daily historical closing values of CRIX for the period from 2014-11-28 - the emergence of CRIX - to 2021-06-01 (2498 observations, including weekends) were sourced from thecrix.de and converted to log-returns. The CC data underlying the derivation of CRIX were obtained from the database for CRIX, kindly provided by CoinGecko. One should note that the intra-day data for CRIX and the CCs it is comprised of are only available from the 2016-06-30. Therefore parts of the study consider data from this date forward, however this still includes the CC market peaks and the cool-down periods.

The daily historical closing prices of the S&P 500 and VIX from 03.01.2000 to 31.12.2018 (4780 observations) were sourced from finance.yahoo.com. It must be
pointed out that SPY (ETF on S&P 500 index) has closer relations to VIX by design, as clarified in Subsection 3.1, however, the log-returns of S&P 500 and SPY reveal no difference and thus could be interchangeable for the conducted analysis. The S&P 500 time series were converted to log-returns, VIX values remained as is.

3 Methodology

Implied volatility became a subject of academic research with the development of the derivatives market in the last quarter of the 20th century. The Black and Scholes (1976) model yields implied volatility as a volatility measure because, by definition, the implied volatility is the future volatility expected by the market. However, the market crash of October 1987 that bent the volatility surface of index options into a skewed "volatility smile", motivated an alternative solution that would provide a more accurate fit to market conditions. Bakshi et al. (1997) provide an extensive overview of the further developments in this field, including the stochastic interest rate option models of Merton et al. (1973), the jump - diffusion/pure jump models of Bates (1991), the stochastic volatility models of Heston (1993) and others. While acknowledging the diversity of options pricing models, authors agree on the necessity of matching the selection of one to the goals at hand.

The goal of this study, developing a measure to capture the expectations of the CC market and mapping them into the index VCRIX, requires a methodology which overcomes the challenges posed by the CC market, in particular the absence of a developed derivatives market. The methodology is inspired by the construction of VIX. In simplified terms, VIX "predicts" the mean annualized volatility of the S&P 500 for the next 30 days in the future, that is in turn derived from the implied volatility extracted from the S&P 500 ETF swap prices. In this section we are describing the measures which will serve as proxies for the implied volatility. Since the actual implied volatility has intrinsic predictive power, we conduct a model comparison to find the proper volatility model to predict the proxies. Mapping the forecasted values into VCRIX, results in a forward-looking property of the index. Finally we validate the appropriateness of this methodology in a simulation study for the S&P500 market by approximating VIX with our methodology.
3.1 Implied volatility proxy

The CC market lacks a broad derivative market which challenges the measurement of implied volatility. Though a volatility index akin to VIX relies on implied volatility. As such the challenge is to find a viable proxy which leads then to VCRIX. Due to the lacking implied volatility, a model has to be identified, capable of capturing the predictive power of a traditional implied volatility index like VIX. VIX is derived from derivatives on the S&P500. Therefore the underlying, S&P500, carries already part of the information which influence the implied volatility. We investigate the dynamics of the underlying to proxy implied volatility, whereas for the CC market CRIX serves as the underlying. In particular we utilize 2 measures as a proxy for implied volatility for this study:

1. the annualized historical rolling volatility of the log-returns over 30 days

2. the realized volatility measured from the intra-day observations of the underlying.

The choice is motivated by the fact that VIX measures how much the market thinks the S&P 500 will fluctuate in the 30 days from the time of each tick, according to Cboe (2019)). Realized volatility proved to carry important information about the movements of assets (Hansen and Lunde, 2005; Patton and Sheppard, 2015), as such we include it as a second candidate for the implied volatility proxy. Equation (1) displays the rolling volatility method (\( r_t \) being a daily return of an asset on day \( t \) and \( \hat{\mu} \) an estimated mean daily return over the 30 day period):

\[
\sigma_t = \sqrt{\frac{1}{30} \sum_{i=t-30}^{t-1} (r_i - \hat{\mu})^2 \times \sqrt{365} \times 100}
\]  

(1)

In case of historical volatility, the \( \sigma_t \) would define the volatility of the last day of the month, while for forward volatility the same calculation will account for the volatility of the first day of the month. It should be pointed out that we are not using the notion of forward volatility as in Taleb (1997), namely, how implied volatility differs for related financial instruments with different maturities. In this case, the "forward" part only bears the idea of adjusting the time span of the traditional rolling volatility measure to be forward-looking (results are displayed in Figure 3).
The second proxy, the annualized realized volatility is defined as the sum of the intraday log-returns:

\[ RV_t^d = \sqrt{\sum_{i=1}^{n} r_i^2 \times \sqrt{365} \times 100} \quad (2) \]

3.2 Model selection

The two proxies (1) and (2) are by construction current and/or backward looking measures, whereas implied volatility includes future information over the investors believes about the future performance of the underlying. Therefore we have to forecast the proxies to achieve a proper underlying for forward oriented analysis. We considered the following models:

1. GARCH family (tested by Hansen and Lunde (2005), French et al. (1987), Antoniou and Holmes (1995), see also Teräsvirta (2009))
   - IGARCH
   - FGARCH
   - SGARCH
   - GJRGARCH
   - EGARCH
   - EWMA

2. Heterogeneous Auto-Regressive (HAR) model (introduced by Corsi (2009) and tested by Chiriac and Voev (2011), Busch et al. (2011), Patton and Sheppard (2015))

3. neural network-based Long short-term memory cell (LSTM) models (Hochreiter and Schmidhuber (1997))

4. Multivariate GARCH models (see Bauwens et al. (2006))
   - DCC & SGARCH
   - DCC & EWMA
The models are calibrated based on the log-returns of the CRIX, the annualized daily volatility over a 30-day rolling window and realized volatility. Hereby the models in the univariate and multivariate GARCH family are derived from the log-returns only. This setting is conducted, since the HAR model has the GARCH model as a special case. Whereas the models considered here are not exact special cases of the HAR model, it clearly outperforms these models in the model evaluations.

The LSTM represents a comparatively new approach to volatility modeling. Its architecture belongs to the Recurrent Neural Networks family and has been extensively used (together with Gated Recurrent Units) for the modeling of sequential data like text or time series. Its complex architecture provides interesting forecasting opportunities that have been explored and proven useful by Kong et al. (2017), Pichl and Kaizoji (2017), Kim and Won (2018), Luo et al. (2018).

The EWMA is a special case of a GARCH with a pre-specified decay parameter $\lambda$:

$$\sigma^2_{i,t+1} = \lambda \sigma^2_{i,t} + (1 - \lambda) r^2_{i,t},$$

where $\sigma^2_{i,t+1}$ is the variance of CRIX log-returns ($r_{i,t}$) in the next period. In this study, the decay parameter $\lambda$ was set to $\lambda = 0.96$ since it showed the best performance under this setting in the model comparison.

The LSTM model in its most general form is defined as

$$\sigma^2_{i,t+1} = f_{\hat{\theta}}(\sigma^2_{i,t}),$$

whereas $f_{\hat{\theta}}$ is a function $f$ dependent on $\hat{\theta}$, which signifies the complex set of parameters that are optimized during the training of the neural network. As for the other models in this study, 365 observations were used as the testing dataset. The training data correspond to the earlier remaining observations in the dataset. In this study, the best results were achieved with an LSTM which has 15 epochs and 3 layers of 365 neurons. The detailed explanation of the LSTM methodology with regards to financial forecasting has been provided previously in papers by Chen et al. (2015), Heaton et al. (2017), Fischer and Krauss (2018).

The HAR model is constructed based on realized volatility. Since CCs are traded also on the weekend, we amend the HAR model such that the weekly and monthly realized volatility is derived on 7 and 30 days respectively, instead of 5 and 21 as it is
common in traditional equity markets. The change of 5 (weekly) and 21 (monthly) trading frequencies to 7 and 30 days respectively is reflected in the calculation of weekly and monthly volatilities (Equations (5) and (6)).

\[ RV^w_t = \frac{1}{7}(RV^d_t + RV^d_{t-1} + \cdots + RV^d_{t-6}) \]  \hspace{1cm} (5)

\[ RV^m_t = \frac{1}{30}(RV^d_t + RV^d_{t-1} + \cdots + RV^d_{t-29}) \]  \hspace{1cm} (6)

Then, the daily realized volatility is forecasted with

\[ RV^d_{t+1} = \alpha + \beta^d RV^d_t + \beta^w RV^w_t + \beta^m RV^m_t + \omega_{t+1} \]  \hspace{1cm} (7)

For the market risk proxy of 30-days historical rolling volatility (annualized, as shown in Equation (8), the daily realized volatility, \( RV^d_t \), is proxied as follows:

\[ RV^d_t = \sigma_t = \sqrt{\frac{1}{30} \sum_{i=t-30}^{t-1} (r_i - \hat{\mu})^2 \times \sqrt{365} \times 100} \]  \hspace{1cm} (8)

Similarly to Equation (1), \( r_t \) is a daily return of CRIX on day \( t \) and \( \hat{\mu} \) an estimated mean daily return over the past 30 days, meanwhile, the number of days for annualising the realized volatility was changed to 365 for the same reason. Further on we will refer to \( \sigma^2_t \) as daily realized volatility \( RV^d_t \) to maintain the usual HAR notation, even when derived from the 30 days historical rolling volatility. The weekly and monthly realized volatility can be derived based on \( RV^d_t \) as in equations (5) and (6).

The VCRIX is constructed as follows:

\[ VCRIX_{i,t} = \frac{RV^d_{i,t+1}}{Divisor} \]  \hspace{1cm} (9)

whereas \( RV^d_{i,t+1} \) indicates the volatility forecast derived from the method \( i \). The final version of VCRIX is forward-looking and re-estimated daily based on the realized daily volatility and mean annualized daily volatility for the next 30 days respectively.

### 3.3 Model evaluation

The Table 1 compares the results of the model contestants over the time period 2016-06-30 to 2020-05-05 (some earlier data reduction was necessary to align with
high-frequency data and to estimate rolling volatility). The initial models were estimated over the time period 2016-06-30 until 2019-05-05 and the prediction performance was derived over 2019-05-06 to 2020-05-05 (365 days). The remaining data will be excluded here and utilized in trading strategy evaluation for a out-of-sample analysis from the model evaluation. The models are constructed as described above. Due to the usage of the 30-days rolling volatility as a proxy for the implied volatility and therefore the market risk, the correlation of the estimated volatility is high for all models. Though EWMA, HAR and LSTM, which are constructed based on respective data, stand out. Also in terms of Mean Squared Error (MSE) and Mean Absolute Error (MAE), they clearly outperform the other models. For the univariate GARCH models, the MSE and MAE are not too different from each other, though they perform better than the multivariate ones. This is an interesting observation, since the DCC based models are based on the individual CC return series, which comprise the CRIX. Therefore this models carry more information, however the large dimensionality makes them also subject to convergence problems. The results allow for the interpretation, that for the 30-days rolling volatility measure, the multivariate GARCH models are not appropriate.

As for the three best performing models, EWMA, HAR and LSTM, the HAR model clearly outperforms the other ones. It is remarkable, that HAR even outperforms an LSTM model since deep learning models are designed to model complex underlying structures, which a linear model like HAR would miss. However it appears, that the market behavioural foundation of the HAR model gives it an edge in this study.

The Table 3 compares the results of the model contestants over the time period 2016-06-30 to 2020-05-05 against the realized volatility as a proxy for the market risk and implied volatility. The observations from the case of 30-days rolling volatility also hold for this comparison. The HAR model again performs best. One observes that the correlation is much lower now, which is explainable due to the fact that the realized volatility does not contain the overlapping information any longer. Interestingly, the multivariate GARCH model give now results close to the one for the HAR model. In terms of MAE, they even perform slightly better, whereas the MSE is smaller for the HAR. This indicates that the DCC models at times have larger gaps between the true and estimated volatility, whereas on average it is lower.
Table 1: Evaluation of the one-day-ahead predicted values of 30-day annualized rolling volatility of log-returns on CRIX (daily re-estimation)

| Model        | MSE    | MAE    | MDA    | Correlation |
|--------------|--------|--------|--------|-------------|
| IGARCH       | 0.5620 | 19.5582| 0.5220 | 0.6655      |
| EGARCH       | 0.6677 | 18.9042| 0.5220 | 0.5904      |
| FGARCH       | 0.5190 | 18.3144| 0.5165 | 0.6681      |
| GJR GARCH    | 0.6190 | 18.7798| 0.5247 | 0.6368      |
| SGARCH       | 0.5190 | 18.3145| 0.5192 | 0.6681      |
| EWMA         | 0.1275 | 10.3847| 0.5247 | 0.9049      |
| HAR          | 0.0370 | 3.4319 | 0.4835 | 0.9736      |
| LSTM         | 0.0757 | 5.8136 | 0.9942 | 0.9470      |
| DCC (SGARCH) | 3.4701 | 73.8863| 0.4725 | 0.4791      |
| DCC (EWMA)   | 3.5207 | 74.3934| 0.5082 | 0.8868      |

than for the HAR. Since larger deviations from the mean are dissatisfactory and the HAR model also has a higher correlation with the true values than the DCC models, we evaluate the HAR model as the best performing model again. For a robustness analysis of our results, we also investigate the performance with both proxies for a seven-day-ahead prediction window, see Tables 2 and 4. Due to the underperformance of the multivariate models in the one-day-ahead prediction, we do not consider them for the robustness analysis. The results support the previous analysis, the HAR model outperforms all other model contestants.

The HAR model outperformed other contestants for both market risk proxies and therefore proved to be an appropriate model to predict respective implied volatility proxies. Thus we chose HAR for the modeling of VCRIX and will further investigate the VCRIX with the HAR model in this study.

After introducing the implied volatility proxies for the VCRIX and the model contestants to predict respective proxies, we will review the underlying for VCRIX, the CRIX, and introduce the construction of VCRIX.
| Model   | MSE   | MAE   | MDA   | Correlation |
|---------|-------|-------|-------|-------------|
| IGARCH  | 0.5453| 19.4341| 0.5220| 0.6703      |
| EGARCH  | 0.4639| 17.7627| 0.5275| 0.6439      |
| FGARCH  | 0.4388| 17.6930| 0.5220| 0.6899      |
| GJR-GARCH| 0.4770| 17.8253| 0.5275| 0.6694      |
| SGARCH  | 0.4388| 17.6925| 0.5220| 0.6900      |
| EWMA    | 0.1241| 10.1381| 0.5385| 0.9110      |
| HAR     | 0.0370| 3.4319 | 0.4835| 0.9736      |
| LSTM    | 0.1666| 10.1449| 0.6476| 0.8800      |

Table 2: Evaluation of the seven-day-ahead predicted values of 30-day annualized rolling volatility of log-returns on CRIX (daily re-estimation)

| Model       | MSE   | MAE   | MDA   | Correlation |
|-------------|-------|-------|-------|-------------|
| IGARCH      | 5.9491| 67.6552| 0.4203| 0.1922      |
| EGARCH      | 5.9380| 65.9107| 0.4038| 0.1714      |
| FGARCH      | 5.8133| 65.5942| 0.4258| 0.1877      |
| GJR-GARCH   | 5.8485| 65.1610| 0.4231| 0.1928      |
| SGARCH      | 5.8133| 65.5943| 0.4231| 0.1877      |
| EWMA        | 5.7368| 64.8740| 0.4121| 0.1218      |
| HAR         | 0.0469| 2.8473 | 0.4203| 0.1864      |
| LSTM        | 0.0517| 3.4841 | 0.5170| 0.0809      |
| DCC (SGARCH)| 0.0524| 2.9107 | 0.4203| 0.1615      |
| DCC (EWMA)  | 0.0461| 2.8106 | 0.4396| 0.1278      |

Table 3: Evaluation of the one-day-ahead predicted values of realized volatility of log-returns on CRIX (daily re-estimation)
| Model       | MSE     | MAE   | MDA   | Correlation |
|-------------|---------|-------|-------|-------------|
| IGARCH      | 5.9329  | 67.6364 | 0.4203 | 0.1928      |
| EGARCH      | 5.6790  | 65.2067 | 0.3984 | 0.1818      |
| FGARCH      | 5.7134  | 65.2755 | 0.4203 | 0.1894      |
| GJRGARCH    | 5.6726  | 64.6395 | 0.4258 | 0.1975      |
| SGARCH      | 5.7134  | 65.2761 | 0.4203 | 0.1894      |
| EWMA        | 5.7169  | 64.6605 | 0.4038 | 0.1142      |
| HAR         | 0.0459  | 3.2440  | 0.4066 | 0.1998      |
| LSTM        | 0.1621  | 7.0423  | 0.5016 | 0.0439      |

Table 4: Evaluation of the seven-day-ahead predicted values of realized volatility of log-returns on CRIX (daily re-estimation)

3.4 Cryptocurrency Index & VCRIX settings

S&P 500 and DAX serve as indicators of the current state of American and German markets by aggregating the weighted performance of the most significant listed companies. CRIX, developed by Trimborn and Härdle (2018), plays a similar role for the CC market, providing a statistically-backed market measure, which distinguishes it from other CC indices like Crypto20, CCi30, WorldCoinIndex. At the core of CRIX lies the idea that a fixed number of constituents (as in case of S&P 500) may be a good approach for relatively stable markets, however, with the ever-growing number of CC, practical implementation would demand a filter that keeps out the noise, while preserving the information about the market dynamics. CRIX employs Akaike Information Criterion (AIC, Akaike (1987)) that determine the number of constituents quarterly according to the explanatory power each CC has over the market movements. CRIX was used as a proxy to the CC market before in research papers by Elendner et al. (2018), Klein et al. (2018), Mihoci et al. (2020), and was adopted as a benchmark by commercial projects like Smarter Than Crypto, Crypto20, F5 Crypto Index, and also used by the European Central Bank, Euro Area Statistics, as a market indicator in the report dedicated to understanding the "crypto-asset phenomenon" (Chimenti et al., 2019). These use cases confirm the applicability of CRIX as an appropriate basis for VCRIX.
Consequently, the index rules will have a significant impact on the behavior of VCRIX. The initial paper by Härdle and Trimborn (2015) defines CRIX as a Laspeyres index, taking the value of a $k$ asset basket and comparing it against the base period, as indicated in Equation (10):

$$CRIX_t(k) = \frac{\sum_{i=1}^{k} P_i Q_{i,t_l}}{\text{Divisor}(k)_{t_l}}$$

with $P_i$ the price of asset $i$ at time $t$ and $Q_{i,t_l}$ the quantity of asset $i$ at time $t_l$ (the last time point when $Q_{i,t_l}$ was updated). Monthly re-balancing accounts for the changes in the market capitalization of a CC and the number of index components, the Divisor ensures that this procedure does not affect the value of CRIX, rather only price changes in its constituents shall be of effect. The Divisor adjusts the nominator of the equation (10) such that the last and first value of CRIX before and after the re-adjustment do not differ. This is necessary, since the in- or exclusion of CCs may alter the value of CRIX whereas the index has to be invariant to such an operation, which is ensured by the Divisor. For more details on the precise construction of the Divisor, we refer to Trimborn and Härdle (2018).

The initial value of VCRIX is set to 1000, following the convention set by CRIX. A Divisor is introduced in order to account for the jumps that might occur due
to the change in the number of constituents every month. The *Divisor* is set to a certain value on the first day to transform the estimated volatility to 1000 points of VCRIX. *Divisor* remains the same over the month. Every month the constituents can change. In this case, the value of VCRIX from the last day of the month will be transferred to the first day of the next month, after that the *Divisor* will be reevaluated in order to reflect the value for transformation.

### 3.5 Implied volatility indices

Consideration of the existing volatility indices would constitute a logical step towards the selection of the appropriate solution. As observed by Siriopoulos and Fassas (2009) recent decades saw the rise of the model-free indices (based on model-free implied volatility (MFIV)) that were made possible by highly liquid options markets and readily available model-free implied variances (France, Germany, Japan, Switzerland, the U.K., and the U.S). Major alternatives to the "model-free" approaches are the Black-Scholes (BS) implied volatility and statistical models such as GARCH (Bollerslev, 1986). While MFIV is extracted from the corresponding set of current option prices without the need to assume any specific pricing model, this approach comes along with a range of methodological issues. For example, Biktimirov and Wang (2017) tested both approaches on the subject of forecasting accuracy, and BS implied volatility came out superior both in terms of in-sample "encompassing" models that include several forecasts in the same combined specification and also in out-of-sample forecasting. We consider model-free and model-based methodologies given the available data and above mentioned empirical results.

Introduction of XBT-Cboe BTC Futures by the Cboe in 2017 became the first step in the establishment of the CC derivatives market, thus approaching the possibility of the model-free implied volatility index construction. However BTC futures were not considered for this research due to several reasons: officially listed (Cboe and CME Group) futures do not provide insight into implied volatility of the underlying like option prices do by design; existing data for options is so far only available for BTC from commercial providers like Deribit (2019), not for the broader CC market. Most importantly, the goal of the VCRIX is to grasp the investors' expectations of the whole CC market. As Figure 2 shows, the weight of BTC in CRIX has been
remaining below 0.6 most of the time, and thus BTC and its options cannot be considered sufficiently representative.

Figure 2: Weight of BTC as a constituent of the CRIX over time

Given the outlined limitations of the CC derivatives market, we settle for a model-based index, that is capable of capturing the predictive power of a traditional volatility index. The VIX by Cboe for the US market was selected as a guidance and benchmark. VIX is acknowledged by the established CC players as a standard for the implied volatility modeling: in 2019 one of the biggest CC derivative trading platforms Ledger X - a US company regulated by CFTC (United States Commodity Futures Trading Commission) - introduced an implied volatility index for BTC called LXVX (Cointelegraph, 2019), announcing its inheritance to VIX (LXVX, 2019). Unfortunately the LXVX is no longer available, which leaves CC market participants without the means of a market risk measure akin to VIX. The development further drives the importance of the introduction of VCRIX.

The current VIX methodology was developed based on the pioneering research of Whaley (1993), Neuberger (1994), Madan et al. (1998), Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000) among others. It estimates the implied volatility of option prices on the S&P 500 by taking strikes and option prices as inputs. With exchange-traded S&P 500 variance swap rate as its underlying, VIX
became a proxy for market volatility (Cboe, 2019):

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[ \frac{F}{K_0} - 1 \right]^2$$  \hspace{1cm} (11)

$$VIX = \sigma \times 100,$$  \hspace{1cm} (12)

where $T$ is time to expiration, $F$ is a forward index level from index option prices, $K_0$ is a first strike price below $F$, $K_i$ is a strike price of the $i$th OTM option (on average the range of $i$ is between 1 and 500, reflecting the composition of the S&P 500), $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike $K_i$, $\Delta K_i$ is an interval between strike prices (half the difference between the strike on either side of $K_i$) and $R$ the risk-free interest rate to expiration.

## 4 Simulation and assessment

In model back-testing, the HAR model won the horse race; for the setup of the model competition and the evaluation of the results, see the previous sections 3.1 - 3.3. The results are presented in Table 1 for the 30-days rolling volatility and Table 3 for the realized volatility. It should be specified that the original HAR model, Corsi (2009), is built on the premise that traders conduct their activities according to the strategies based on different frequencies (high-frequency trading, daily traders, weekly, monthly), which in turn affects the overall market volatility at certain points in time. As the CC market is young and presumably still dominated by sporadic non-expert traders (due to the pseudo-anonymity of most CC, justification of this assumptions remains challenging), presenting an informed judgment at this stage is rendered impossible by the implicit anonymity of most CC and its users. The recent analysis for potential herding behavior by Bouri et al. (2018) and Gama Silva et al. (2019) touches on this topic, without providing actual analysis of the traders’ practices.

In the absence of data on CC traders’ behavior, we have made the assumption that the traditional practices could potentially be applied for the CC case. Recall that CCs are traded 7 days a week and not 5 days a week like equities. We made two adjustments for the length of a week and month (see equations (5) and (6)) to
the original HAR model, such that we account for the 2 additional trading days of CCs per week.

Notably we use 2 kinds of measures as proxies for the implied volatility and market risk, the realized volatility and 30-days rolling historical volatility. Whereas realized volatility is a widely studied measure for market risk and proved to carry important information about the market state (Hansen and Lunde, 2005; Patton and Sheppard, 2015), the latter is not an established measure. As such it requires additional justification for the selection of this measure as a viable proxy for market risk. To perform this task, we construct a version of the VIX with the methodology described in this study, which we will refer to as Approximated VIX (AVIX). It comprises of the application of the selected HAR model to the log-returns of the S&P 500 instead of CRIX. From the S&P 500 log-returns, we derive the 30-days rolling historical volatility and derive the AVIX. Then, we compare the AVIX to the actual values of VIX, which allows for a justification of the suitability of the 30-days rolling historical volatility and HAR model as a proxy for a volatility index.

The time series (Figure 3) analysis shows the correlation of 0.89 between VIX and 30-days rolling historical volatility, which is already a strong sign for the applicability, while the correlation between VIX and the 30 days rolling volatility measured 30-days in the future (forward-looking) was 0.78. These two measures indicate, that the 30-days rolling historical volatility is already a viable measure to approximate VIX (89% correlation is fairly high) and also shows that VIX predicts the volatility in 30 days (78% correlation). However the fit is worse within crisis periods (as it can be seen for 2009, compare Figure 3) but improves again during market cool-down.
Figure 3: Difference between VIX and historical and forward-looking volatilities (30 calendar days) for the time period 2000-01-03 to 2018-12-31

| Days of lag | Correlation | MDA |
|------------|-------------|-----|
| Day-on-day | 0.89        | 51% |
| 21 days    | 0.89        | 64% |
| 42 days    | 0.87        | 73% |

Table 5: Evaluation of the simulation of VIX using VCRIX methodology, comparison of true and simulated values

Further comparing the AVIX against VIX, Table 5 shows the correlations and MDA between the two time series with different lags between the measures. The AVIX exhibited correlation of 89% and a Mean Directional Accuracy (MDA) of 51% rising to 64% in case lag of 21 days is considered, as indicated in Table 5. Figure 4 and Figure 5 showcase the difference between the estimated values and actual VIX. The analysis unveils, that the VIX can be approximated with the 30-days rolling historical volatility, however there is room for improvement. This underlines on the one hand the strength of the methodology which was selected for this study, but also showcases the importance to measure implied volatility. These results led us to believe that the chosen methodology of the HAR model in combination with the 30 days rolling historical volatility does indeed provide a solid estimation of the implied volatility in the absence of the derivatives market.
Figure 4: VIX estimated with HAR (AVIX) model on scaled daily volatility of SPY log-returns, VIX estimated with HAR (AVIX) with 21 days lag and true VIX values from 2000 to 2019

Figure 5: Difference between VIX and AVIX, values from 2000 to 2019. One can observe that the proposed model lags in catching the big spikes but performs well when market volatility is lower.

5 Trading implementation

As a final validation between the 2 proxies for implied volatility and the resulting volatility indices, we test their ability to predict market movements and utilize the signal in two trading strategies. This will indicate if the proxies and the construction of VCRIX actually give a forward-oriented look on the CC market movements. Certainly, there is no perfect prediction tool, however it will provide
an indication about its suitability. The implementation also shows how VCRIX can become increasingly employed in trading strategies as the CC market develops and new financial instruments based on CC appear. As one of the examples, an inverse volatility ETF is a financial product that allows investors to gain exposure to volatility, and thus hedge against portfolio risk, without having to buy options. The trading strategy, which we employ here, is an inverse volatility strategy. In such a strategy, one goes long in an asset if the volatility is low and goes short (or sells the position) if the volatility is high. We compare the strategies over 2 time periods. Once over the full time period from 2016-10-20 until 2021-06-01 and once from 2018-02-09 until 2021-06-01. The second period starts after the 2017 peak of the CC market and therefore excludes the exorbitant rise in market value of many CCs, whereas the first period includes this time period. Though it does include the 2021 peak and will provide insights on how the VCRIX signal evolves from a relatively longer calmer market period into one which is highly volatile.

Regardless of the absence of the above mentioned derivative instruments, volatility-based trading strategies may still be employed and tested. Conventional short-term reversal strategies have been explored and perfected by scholars and industry practitioners (Lehmann (1990), Jegadeesh (1990), Blitz et al. (2013)) over the years. As an input, we employ VCRIX with either the 30 day historical volatility or realized volatility and derive the Moving Average (MA) of respective VCRIX over various time horizons, namely 1, 3, 7 and 14 days. Notably a MA over 1 day is just the VCRIX value of the day itself.

The strategies get their signals from the Moving Average-smoothed mean of VCRIX. The trading strategy, Algorithm 1, dictates to go long in cash when the volatility measured by VCRIX is high and go long in an ETF on CRIX when the volatility measured by VCRIX is low. Respectively the trading strategy, Algorithm 2, signals one to go short in an ETF on CRIX when the volatility measured by VCRIX is high. We compare if the volatility is high or low by the MA-smoothed mean of VCRIX. A MA with a longer time span (7 or 14 days) gives a long term average for VCRIX, whereas a MA with smaller time span (1 or 3 days) gives the short term average. In particular, we go Long in a CRIX ETF when the short term volatility is low compared to the long term one, $MA_i \geq MA_j$, and vice versa go
Long in cash (Short in CRIX ETF) when the short term volatility is comparably high, \( MA_i < MA_j \), see Algorithms 1 and 2.

**Algorithm 1 : Long-Cash Trading strategy**

**Set:** \( i, j \in \{3, 7, 14\}, \ i > j \)

**Input:** MA\(_i\), MA\(_j\), CRIX ETF

**Output:** Investment product \( y \)

1. if \( MA_i \geq MA_j \) then
2. \( y = \) CRIX ETF
3. else \( MA_i < MA_j \)
4. \( y = \) Cash
5. end if

**Algorithm 2 : Long-Short Trading strategy**

**Set:** \( i, j \in \{3, 7, 14\}, \ i > j \)

**Input:** MA\(_i\), MA\(_j\), CRIX ETF

**Output:** Investment product \( y \)

1. if \( MA_i \geq MA_j \) then
2. \( y = \) CRIX ETF
3. else \( MA_i < MA_j \)
4. \( y = \) Short CRIX ETF
5. end if

Figures 6 and 7 provide an illustration of two trading strategies over the two time periods that are based on long-cash and long-short signals respectively, whereas the best performing strategy with either of the two proxies in the analysis is displayed in terms of cumulative return. The red line is the best performing realized volatility based strategy and the blue one shows the 30-days rolling historical volatility based strategy. The upper graph for the full time span shows RealizedVola\(_{14,1}\) (red) and Vola30day\(_{7,3}\) (blue), which are generated by the relationships between the daily VCRIX value based on realized volatility as well as 30-days rolling historical volatility and its two MA curves (1 and 14 days, 3 and 7 days respectively). The lower graph for the subset shows RealizedVola\(_{7,3}\) and Vola30day\(_{7,3}\), which is comprised of the daily VCRIX value based on the two proxies and its two MA curves (3 and 7 days).
days). In further notation we indicate days with the subscripts, as in $\text{RealizedVol}_{14,1}$ whereas the subscript indicates that the 14-days MA curve was compared to the 1-days curve and taken as a trading signal. The respective trading strategy, see Algorithm 1, works in this case in the following way: We go long in a CRIX ETF when $MA_{14} \geq MA_1$ and long in cash when $MA_{14} < MA_1$. For the long-short trading strategy, it holds that we go long in a CRIX ETF when $MA_{14} \geq MA_1$ and short in a CRIX ETF when $MA_{14} < MA_1$, see Algorithm 2. We observe that the long-cash and long-short trading strategies do not perform well in the year of 2017, see Figures 6 and 7. The trading strategies are inverse-volatility and in this period the volatility was often high though the market went up. As a result, a trading strategy, which protects ones portfolio against high volatility underperforms. However we observe that from 2018 the difference between the cumulative return of the CRIX ETF and the trading strategy becomes ever thinner, regardless of the choice of the long-cash/long-short strategy or the proxy. The portfolio driven by the trading strategy based on realized volatility even overtakes a CRIX ETF at some point. This is a clear indicator, that the signals from VCRIX worked well after the CC market peak. The second graph in Figures 6 and 7 shows the performance of the same trading strategies (but different MA signals involved) when initiated after the market peak in early 2018. Whereas the CRIX ETF diminishes in value and only returns to reflect a gain in October 2020, the long-cash and long-short trading strategies with either proxy generate a substantial return for the trader long before and hold the portfolio value mostly in the positive domain. The best performing long-cash strategy based on the VCRIX signals generates a cumulative return of 174% and long-short of 256% versus 92% from the CRIX ETF, which clearly indicates the ability of VCRIX to predict future market movements.
Figure 6: Cumulative returns of the best performing combination of Moving Averages (upper RealizedVola$_{14,1}$, Vola$_{30}$day$_{7,3}$ and lower RealizedVola$_{7,3}$, Vola$_{30}$day$_{7,3}$) for the long-cash trading strategy versus the cumulative returns on CRIX over the full time series and a subset from 2018-02-09 (starting after the CC market peak)
Figure 7: Cumulative returns of the best performing combination of Moving Averages (upper $\text{RealizedVol}_{14,1}$, $\text{Vola}_{30\text{day}7,3}$ and lower $\text{RealizedVol}_{7,3}$, $\text{Vola}_{30\text{day}7,3}$) for the long-short trading strategy versus the cumulative returns on CRIX over the full time series and a subset from 2018-02-09 (starting after the CC market peak)

By construction the choice of the time span of MA is critical for the performance of the trading strategy. The Figures 6 and 7 showed the best performing strategies of either proxy in respective periods by cumulative return, $\text{RealizedVol}_{14,1}$, $\text{RealizedVol}_{7,3}$ and $\text{Vola}_{30\text{day}7,3}$, which compares $MA_1$ to the $MA_{14}$ and $MA_3$ to $MA_7$ respectively. We construct the MA for the spans 1, 3, 7, 14 days, and compare the results with the following measures:

1. cumul.returns: the aggregate gain over the observed time period up to the final day of trading.
2. **mean.returns**: the mean of the daily trading strategy returns.

3. **takeover.days**: the percentage of days when the cumul.returns are higher for the trading strategy than for CRIX.

4. **Sharpe.ratio**: compares the mean of the returns of the trading strategy over the standard deviation of the returns of the trading strategy, reflecting extra return per unit of increase in risk.

The results are presented in Tables 6, 7, 8 and 9. The rows are named by the time span of the two MA-smoothed means involved and the respective proxy for implied volatility (30-days historical rolling window or realized volatility) in the trading strategy. CRIX returns are offered for reference.

|                  | cumul.returns | mean.returns | takeover.days | sharpe.ratio |
|------------------|---------------|--------------|---------------|--------------|
| CRIX             | 4.5139        | 0.2677       | 0.0000        | 0.0628       |
| Vola30day3,1     | 1.4206        | 0.0843       | 0.0006        | 0.0283       |
| Vola30day7,1     | 2.4384        | 0.1446       | 0.0012        | 0.0555       |
| Vola30day14,1    | 1.9041        | 0.1129       | 0.0006        | 0.0430       |
| Vola30day7,3     | 2.7349        | 0.1622       | 0.0006        | 0.0626       |
| Vola30day14,3    | 1.4430        | 0.0856       | 0.0000        | 0.0324       |
| Vola30day14,7    | 1.3970        | 0.0829       | 0.0000        | 0.0302       |
| RealizedVola3,1  | 2.6542        | 0.1574       | 0.0024        | 0.0476       |
| RealizedVola7,1  | 3.3097        | 0.1963       | 0.0018        | 0.0607       |
| RealizedVola14,1 | **3.9710**    | **0.2355**   | **0.0641**    | **0.0753**   |
| RealizedVola7,3  | 3.9197        | 0.2325       | 0.0000        | **0.0797**   |
| RealizedVola14,3 | 3.9235        | 0.2327       | 0.0297        | 0.0779       |
| RealizedVola14,7 | 3.8657        | 0.2293       | 0.0635        | 0.0730       |

Table 6: Comparison of long-cash trading strategies over the full time period with several MovingAverage-smoothed means of VCRIX.

We observe in Tables 6, 7 that the visual analysis from Figure 6 gets supported, hence none of the strategies outperform a CRIX ETF over the entire time period. Though we already stated that these results are due to the impressive gain in market value in 2017 and the trading strategies perform very good after the market
Table 7: Comparison of long-short trading strategies over the full time period with several MovingAverage-smoothed means of VCRIX.

|          | cumul.returns | mean.returns | takeover.days | sharpe.ratio |
|----------|----------------|--------------|---------------|--------------|
| CRIX     | 4.5139         | 0.2677       | 0.0000        | 0.0628       |
| Vola30day3,1 | -1.6727       | -0.0992      | 0.0006        | -0.0232      |
| Vola30day7,1 | 0.3628         | 0.0215       | 0.0012        | 0.0050       |
| Vola30day14,1 | -0.7056        | -0.0419      | 0.0006        | -0.0098      |
| Vola30day7,3 | 0.9558         | 0.0567       | 0.0006        | 0.0133       |
| Vola30day14,3 | -1.6279        | -0.0966      | 0.0000        | -0.0226      |
| Vola30day14,7 | -1.7198        | -0.1020      | 0.0000        | -0.0239      |
| RealizedVola3,1 | 0.7946        | 0.0471       | 0.0024        | 0.0110       |
| RealizedVola7,1 | 2.1054        | 0.1249       | 0.0018        | 0.0292       |
| RealizedVola14,1 | 3.4281        | 0.2033       | 0.0641        | 0.0476       |
| RealizedVola7,3 | 3.3254        | 0.1972       | 0.0000        | 0.0462       |
| RealizedVola14,3 | 3.3331        | 0.1977       | 0.0297        | 0.0463       |
| RealizedVola14,7 | 3.2174        | 0.1908       | 0.0635        | 0.0447       |

peak. We observe that the signals from the Realized Volatility provide better performing portfolios than the 30-days rolling volatility. For the long-short strategy, the overall return even turns negative for some of the 30-days rolling volatility based portfolio strategies. However, for the long-cash strategy also 30-days rolling volatility VCRIX provides good signals, though is still outperformed by the Realized Volatility VCRIX. Even though the cumulative return never outperforms a CRIX ETF, the Sharpe ratio is considerably better for the trading strategy with Realized Volatility VCRIX signals, see Table 6. This results on the full time period already indicate the predictive ability of VCRIX for market risk as well as that the VCRIX constructed with realized volatility as proxy for implied volatility is outperforming the 30-days rolling historical volatility when considering the challenging 2017 period which was comprised of a significant increase in CC market value.

The time period from the peak of the CC market provides even more insights into the performance of the trading strategies. Tables 8 and 9 show that the best performing trading strategy, $MA_3$ and $MA_7$ as signals derived from the VCRIX
with realized volatility and 30-days rolling historical volatility, outperforms other strategies and most notably clearly outperforms an investment in a CRIX ETF. Markedly the strategy RealizedVola_{7,3} performs best though Vola_{30day_{7,3}} overtakes a CRIX ETF clearly as well. A huge difference between VCRIX derived from 30-days rolling historical volatility compared to realized volatility is a tremendously higher variation of the latter. Due to occasional large intraday swings in the observed price series, the daily VCRIX based on these data can become quite large whereas the former provides an implicit smoothing into the VCRIX curve. The trading strategy based on realized volatility with the MA_{3} and MA_{7} combination outperforms the 30-days historical rolling volatility VCRIX measure in terms of all the reported measures, but Figures 6 and 7 show that the 30-days historical rolling volatility VCRIX tracks the realized volatility one closely. In 2019 and 2020 the gap becomes ever more narrow and just the 2021 market increase resulted in a clear gap between the two strategies. The results for the long-cash strategy and long-short strategy convey the same interpretation however long-short provides even more cumulative

|                | cumul.returns | mean.returns | takeover.days | sharpe.ratio |
|----------------|---------------|--------------|---------------|--------------|
| CRIX           | 0.9215        | 0.0750       | 0.0000        | 0.0180       |
| Vola_{30day_{3,1}} | 0.0171        | 0.0014       | 0.5883        | 0.0004       |
| Vola_{30day_{7,1}} | 0.7733        | 0.0629       | 0.7022        | 0.0229       |
| Vola_{30day_{14,1}} | 0.8360        | 0.0680       | 0.6111        | 0.0251       |
| Vola_{30day_{7,3}} | 1.0403        | 0.0846       | 0.7494        | 0.0313       |
| Vola_{30day_{14,3}} | 0.5680        | 0.0462       | 0.5460        | 0.0169       |
| Vola_{30day_{14,7}} | 0.3209        | 0.0261       | 0.0399        | 0.0092       |
| RealizedVola_{3,1} | 0.1772        | 0.0144       | 0.6957        | 0.0046       |
| RealizedVola_{7,1} | 0.4746        | 0.0386       | 0.8210        | 0.0126       |
| RealizedVola_{14,1} | 1.3361        | 0.1087       | 0.9146        | 0.0362       |
| RealizedVola_{7,3} | **1.7410**    | **0.1417**   | **0.9984**    | **0.0493**   |
| RealizedVola_{14,3} | 1.7375        | 0.1414       | 0.9601        | 0.0486       |
| RealizedVola_{14,7} | 1.2192        | 0.0992       | 0.8934        | 0.0315       |

Table 8: Comparison of long-cash trading strategies over the time period from 2018-01-06 with several MovingAverage-smoothed means of VCRIX.
return in this time period compared to long-cash. It is worth pointing out that the number of days when the respective trading strategies were outperforming the CRIX ETF is considerably high. For both trading strategies the two best performing Moving Average combination outperforms CRIX on 99.8% and 74.9% of the days. This is a clear indicator that the trading strategies do not only outperform in the end of the observation period, instead even an earlier market exit would have given the investor a higher return than an investment in a CRIX ETF.

The analysis shows that VCRIX based on either base of volatility (30-days historical rolling volatility and realized volatility) provide a good prediction of the future movements. In terms of the trading strategies the realized volatility gave more stable results over various combinations of MA, which is driven by the stronger signals provided by VCRIX due to its larger variation.

The inverse volatility trading strategies, which are basically wealth-protective strategies, illustrate the applicability of VCRIX to predict future risk and therefore mark its applicability as a tool to measure implied volatility in the CC market. It
remains to discuss which of the two candidate VCRIX outperforms as a result of this study.

6 Discussion

Both measures which serve as a proxy to implied volatility in this study, 30-days historical rolling volatility and realized volatility, predicted by the HAR model, showed better performance than competing measures, Section 3.3, and the resulting VCRIX from these volatility measures showed very good performance as signals for inverse-trading strategies, Section 5. VCRIX from realized volatility gave good signals for various combinations of MAs. The trading strategy for VCRIX from 30-days historical rolling volatility closely tracks the realized volatility VCRIX one though the overall performance was in favour of realized volatility. However the realized volatility VCRIX has very high variation in its values whereas 30-days historical rolling volatility VCRIX is more stable in comparison. An index should not vary too much on a daily basis, otherwise the evaluation of its state is challenged. A further argument in favour of 30-days historical rolling volatility is its better data availability. Intraday data for all CCs are still difficult to collect whereas provision of daily closing data is well established. Even though these are practical issues which can be circumvented, it challenges the applicability of the methodology. Further a methodology which does not rely on intraday data is better transferable to other markets. As a result of both arguments which are supported by the results of this study, we decide in favour of the 30-days historical rolling volatility VCRIX. The index as described in this study is displayed on the website thecrix.de. In the remainder of this discussion, we will discuss the economic and financial implications which can be inferred from VCRIX.

From the beginning, one of the biggest complexities in crypto-trading came from the absence of clear pricing strategies: what is BTC worth? How do we estimate the value of new coins? Are coins under- or over-appreciated? (Yermack, 2015). While mechanics and potential implications of CC in financial economics are being explored (Härdle et al., 2020), there is still no established consensus over the evaluation methods. Also an investigation into CC experts sentiment, revealed that
their sentiment does not predict the market (Trimborn and Li, 2021). Nowadays agents are often left with nothing but the information on the overall market "feeling" about the CC, which is communicated by the rise and fall of the price, in other words, its volatility.

VCRIX captures the volatility jumps that correspond to the development of the CC-ecosystem and can tell a story of the CC adoption (Figure 9). Note that VCRIX is displayed back to November 2014, which is possible due to the availability of closing data for this time period. Realized volatility, which depends on intraday data, would only allow for an analysis from 2017 onwards. We observe spikes of interest in BTC in 2015, winter and summer of 2016 when BTC was slowly making its way to the attention of the general public. The large scale swings in price would not constitute a significant shock in absolute values, but when something that was still considered a digital maverick rose in value from roughly 400 USD to 1000 USD within a year (Business Insider, 2016 ("Bitcoin is still storming higher")), investors noticed. VCRIX further captures the beginning of the first massive growth wave (also captured well by the CRIX in Figure 8) and development of altcoins (ETH, LTH, and others).

![Figure 8: CRIX and VCRIX](image)

2017 became the year of massive volatility (VCRIX showcases the values that can be interpreted as daily volatility of 140%). These levels of uncertainty were
largely caused by the major legislative shifts that were happening in countries—juggernauts of CC movement: China, Korea, Japan, and the USA. Additionally, BTC was going through the heated debates on the SegWit (Segregated Witness) fork that was supposed to improve the speed and cost of BTC transactions. The fork was implemented in August, 2017 and led to the emergence of BTC Cash due to a certain number of big miners disagreeing with the implementation. These volatility spikes yet proved to be minor in comparison with the major market meltdown that happened at the beginning of 2018, when prices of most currencies on average suffered an 80% drop (CoinMarketCap (2021)). 2018 was considered to be a stabilization period when governments and financial corporations were getting onboard, however, the end of 2018 saw another volatility spike, majorly driven by the "holiday race" and uncertainty driven by "Constantinople fork" that was expected from Ethereum at the beginning of 2019.

![Figure 9: VCRIX interpretation](image)

7 Conclusion

We have set the goal of capturing the expectations on the CC market (represented by CRIX) through the construction of an implied volatility proxy in the absence of the derivatives for the majority of CC. Following the intuition of the "fear index" VIX for the American stock market, the VCRIX volatility index was created to capture the investor expectations about the crypto-currency ecosystem. We defined
2 proxy candidates for the implied volatility due to the absence of a developed derivatives market in the CC market. Based upon the proxies, namely 30-days mean annualized historical rolling volatility and realized volatility, we performed a model comparison for the predictive power to ensure the proxies have a forward-looking nature like VIX. The model comparison between univariate and multivariate GARCH-type models, EWMA, LSTM and HAR led to the selection of the HAR model. The model was further examined by simulating the VIX with the VCRIX technology (AVIX), which resulted in a correlation of 78% between VIX and AVIX. The high correlation confirms the applicability of the model. We further investigate the volatility-predictive information value of VCRIX by creating trading strategies based upon VCRIX signals and investigate their performance on 2 time horizons against each other and a CRIX ETF. In the period after the December 2017 peak, the best performing trading strategy with the use of VCRIX outperformed the benchmark strategy even for 99.8% of the tested period and generated 164% additional cumulative return.

The study showed that VCRIX provides forecasting functionality and serves as a proxy for the investors’ expectations in the absence of a developed crypto derivatives market. These features provide enhanced decision making capacities for market monitoring, trading strategies, and potentially option pricing. Authors intend to conduct further research to capture the observed excessive volatility that is captured by derivative-based indices like VIX and presumably stems from the behavioral component of option pricing.
References

Akaike, H. (1987). “Factor analysis and AIC”. Selected Papers of Hirotugu Akaike. Springer, pp. 371–386.

Antoniou, A. and P. Holmes (1995). “Futures trading, information and spot price volatility: evidence for the FTSE-100 stock index futures contract using GARCH”. Journal of Banking & Finance 19.1, pp. 117–129.

Bakshi, G., C. Cao, and Z. Chen (1997). “Empirical performance of alternative option pricing models”. The Journal of Finance 52.5, pp. 2003–2049.

Bates, D. S. (1991). “The Crash of 1987: Was It Expected? The Evidence from Options Markets”. The Journal of Finance 46.3, pp. 1009–1044.

Bauwens, L., S. Laurent, and J. V. K. Rombouts (2006). “Multivariate GARCH models: a survey”. Journal of Applied Econometrics 21.1, pp. 79–109. DOI: 10.1002/jae.842.

Biktimirov, E. N. and C. Wang (2017). “Model-Based versus Model-Free Implied Volatility: Evidence from North American, European, and Asian Index Option Markets”. The Journal of Derivatives 24.3, pp. 42–68.

Black, F. and M. Scholes (1976). “Taxes and the Pricing of Options”. The Journal of Finance 31.2, pp. 319–332.

Blitz, D., J. Huij, S. Lansdorp, and M. Verbeek (2013). “Short-term residual reversal”. Journal of Financial Markets 16.3, pp. 477–504.

Bollerslev, T. (1986). “Generalized autoregressive conditional heteroskedasticity”. Journal of Econometrics 31.3, pp. 307–327.

Bouri, E., R. Gupta, and D. Roubaud (2018). “Herding behaviour in cryptocurrencies”. Finance Research Letters.

Britten-Jones, M. and A. Neuberger (2000). “Option prices, implied price processes, and stochastic volatility”. The Journal of Finance 55.2, pp. 839–866.

Busch, T., B. J. Christensen, and M. Ø. Nielsen (2011). “The role of implied volatility in forecasting future realized volatility and jumps in foreign exchange, stock, and bond markets”. Journal of Econometrics 160.1, pp. 48–57.

Business Insider, 2016. "Bitcoin is still storming higher".

Cboe (2019). “Cboe volatility index-VIX”. White Paper, pp. 1–19.
Chen, K., Y. Zhou, and F. Dai (2015). “A LSTM-based method for stock returns prediction: A case study of China stock market”. 2015 IEEE International Conference on Big Data (Big Data). IEEE, pp. 2823–2824.

Cheung, A., E. Roca, and J.-J. Su (2015). “Crypto-currency bubbles: an application of the Phillips–Shi–Yu (2013) methodology on Mt. Gox bitcoin prices”. Applied Economics 47.23, pp. 2348–2358.

Chimienti, M. T., U. Kochanska, and A. Pinna (2019). “Understanding the crypto-asset phenomenon, its risks and measurement issues”. Economic Bulletin Articles 5.

Chiriac, R. and V. Voev (2011). “Modelling and forecasting multivariate realized volatility”. Journal of Applied Econometrics 26.6, pp. 922–947.

CoinMarketCap (2021). Charts. coinmarketcap.com/charts/.

Cointelegraph (2019). Crypto Asset Manager LedgerX Launches Bitcoin Volatility Index by Zmudzinski, Adrian. cointelegraph.com.

Corsi, F. (2009). “A simple approximate long-memory model of realized volatility”. Journal of Financial Econometrics 7.2, pp. 174–196.

Demeterfi, K., E. Derman, M. Kamal, and J. Zou (1999). “A guide to volatility and variance swaps”. The Journal of Derivatives 6.4, pp. 9–32.

Deribit (2019). Deribit: Bitcoin Futures and Options Exchange.

Duffie, D., J. Pan, and K. Singleton (2000). “Transform analysis and asset pricing for affine jump-diffusions”. Econometrica 68.6, pp. 1343–1376.

Elendner, H., S. Trimborn, B. Ong, and T. M. Lee (2018). “The Cross-Section of Crypto-Currencies as Financial Assets: Investing in Crypto-Currencies Beyond Bitcoin”. Handbook of Blockchain, Digital Finance, and Inclusion, Volume 1, pp. 145–173.

Fengler, M. R., W. K. Härdle, and C. Villa (2003). “The Dynamics of Implied Volatilities: A Common Principal Components Approach”. Review of Derivatives Research 6.3, pp. 179–202. DOI: 10.1023/B:REDR.0000004823.77464.2d.

Fischer, T. and C. Krauss (2018). “Deep learning with long short-term memory networks for financial market predictions”. European Journal of Operational Research 270.2, pp. 654–669.
French, K. R., G. W. Schwert, and R. F. Stambaugh (1987). “Expected stock returns and volatility”. *Journal of Financial Economics* 19.1, pp. 3–29.

Gama Silva, P. V. J. da, M. C. Klotzle, A. C. F. Pinto, and L. L. Gomes (2019). “Herding behavior and contagion in the cryptocurrency market”. *Journal of Behavioral and Experimental Finance* 22, pp. 41–50.

Hafner, C. (2020). “Testing for Bubbles in Cryptocurrencies with Time-Varying Volatility”. *Journal of Financial Econometrics* 18 (2), pp. 233–249. DOI: 10.1093/jjfinec/nby023.

Hansen, P. R. and A. Lunde (2005). “A forecast comparison of volatility models: does anything beat a GARCH (1, 1)?” *Journal of Applied Econometrics* 20.7, pp. 873–889.

Härdle, W. K., C. R. Harvey, and R. C. Reule (2020). “Understanding cryptocurrencies”. *Journal of Financial Econometrics* 18 (2), pp. 181–208. DOI: 10.1093/jjfinec/nbz033.

Härdle, W. K. and S. Trimborn (2015). “CRIX or evaluating blockchain based currencies”. *Mathematisches Forschungsinstitut Oberwolfach* Report No. 42/2015, pp. 17–20. DOI: 10.4171/OWR/2015/42.

Heaton, J. B., N. G. Polson, and J. H. Witte (2017). “Deep learning for finance: deep portfolios”. *Applied Stochastic Models in Business and Industry* 33.1, pp. 3–12. DOI: 10.1002/asmb.2209.

Heston, S. L. (1993). “A closed-form solution for options with stochastic volatility with applications to bond and currency options”. *The Review of Financial Studies* 6.2, pp. 327–343.

Hochreiter, S. and J. Schmidhuber (1997). “Long short-term memory”. *Neural Computation* 9.8, pp. 1735–1780.

Hou, A. J., W. Wang, C. Y. Chen, and W. K. Härdle (2020). “Pricing Cryptocurrency options: the case of CRIX and Bitcoin”. *Journal of Financial Econometrics* 18 (2), pp. 250–279. DOI: 10.1093/jjfinec/nbaa006.

Jegadeesh, N. (1990). “Evidence of predictable behavior of security returns”. *The Journal of Finance* 45.3, pp. 881–898.
Kim, H. Y. and C. H. Won (2018). “Forecasting the volatility of stock price index: A hybrid model integrating LSTM with multiple GARCH-type models”. Expert Systems with Applications 103, pp. 25–37.

Klein, T., H. P. Thu, and T. Walther (2018). “Bitcoin is not the New Gold—A comparison of volatility, correlation, and portfolio performance”. International Review of Financial Analysis 59, pp. 105–116.

Kong, W., Z. Y. Dong, Y. Jia, D. J. Hill, Y. Xu, and Y. Zhang (2017). “Short-term residential load forecasting based on LSTM recurrent neural network”. IEEE Transactions on Smart Grid.

Lehmann, B. N. (1990). “Fads, martingales, and market efficiency”. The Quarterly Journal of Economics 105.1, pp. 1–28.

Liu, Y., A. Tsyvinski, and X. Wu (2019). “Common Risk Factors in Cryptocurrency”. Working Paper Series 25882. DOI: 10.3386/w25882.

Luo, R., W. Zhang, X. Xu, and J. Wang (2018). “A neural stochastic volatility model”. Thirty-Second AAAI Conference on Artificial Intelligence.

LXVX (2019). Ledger X. ledgerx.com/lxvx/.

Madan, D. B., P. P. Carr, and E. C. Chang (1998). “The variance gamma process and option pricing”. Review of Finance 2.1, pp. 79–105.

Merton, R. C. et al. (1973). “Theory of rational option pricing”. Theory of Valuation, pp. 229–288.

Mihoici, A., M. Althof, C. Y.-H. Chen, and W. K. Härdle (2020). “FRM Financial Risk Meter”. Empirical Economics.

Neuberger, A. (1994). “The log contract, The new instrument to hedge volatility”. Journal of Portfolio Management 20.2.

Patton, A. J. and K. Sheppard (2015). “Good volatility, bad volatility: Signed jumps and the persistence of volatility”. Review of Economics and Statistics 97.3, pp. 683–697.

Petukhina, A., S. Trimborn, W. K. Härdle, and H. Elendner (2021). “Investing with Cryptocurrencies – evaluating their potential for portfolio allocation strategies”. Quantitative Finance. DOI: 10.1080/14697688.2021.1880023.

Pichl, L. and T. Kaizoji (2017). “Volatility Analysis of Bitcoin Price Time Series”. Quantitative Finance and Economics 1.QFE-01-00474, p. 474.
Schilling, L. and H. Uhlig (2019). “Some simple bitcoin economics”. *Journal of Monetary Economics* 106, pp. 16–26. DOI: 10.1016/j.jmoneco.2019.07.002.

Siriopoulos, C. and A. Fassas (2009). “Implied volatility indices—a review”.

Taleb, N. (1997). *Dynamic hedging: managing vanilla and exotic options*. Vol. 64. John Wiley & Sons.

Teräsvirta, T. (2009). “An Introduction to Univariate GARCH Models”. *Handbook of Financial Time Series*. Ed. by T. Mikosch, J.-P. Kreiß, R. A. Davis, and T. G. Andersen. Springer, Berlin, Heidelberg, pp. 17–42. DOI: 10.1007/978-3-540-71297-8_1.

Trimborn, S. and W. K. Härdle (2018). “CRIX an Index for cryptocurrencies”. *Journal of Empirical Finance* 49, pp. 107–122. DOI: 10.1016/j.jempfin.2018.08.004.

Trimborn, S., M. Li, and W. K. Härdle (2019). “Investing with Cryptocurrencies - A Liquidty Constrained Investment Approach”. *Journal of Financial Econometrics* 18 (2), pp. 280–306. DOI: 10.1093/jjfinec/nbz016.

Trimborn, S. and Y. Li (2021). “Informative Effects of Expert Sentiment on the Return Predictability of Cryptocurrency”. SSRN Working Paper Series. DOI: 10.2139/ssrn.3834279.

Whaley, R. E. (1993). “Derivatives on market volatility: Hedging tools long overdue”. *The Journal of Derivatives* 1.1, pp. 71–84.

Yermack, D. (2015). “Is Bitcoin a real currency? An economic appraisal”. *Handbook of Digital Currency*, pp. 31–43.