Research Article

Hybrid Decision-Making Frameworks under Complex Spherical Fuzzy $N$-Soft Sets

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1. Introduction

Multiattribute decision-making (MADM) and multiattribute group decision-making (MAGDM) methods are broad sections in the field of decision-making. Researchers and practitioners have resorted to them in order to evaluate optimal solutions among a finite number of choices under several attributes. For the purpose of improving the flexibility of the evaluations that support the decision-making process, Zadeh [1] proposed fuzzy set (FS) theory that reshaped the field of decision-making and related disciplines such as mathematical social sciences [2, 3]. In FS theory, a membership degree belongs to the interval $[0, 1]$; thus, when assigned to an object, it represents its degree of belongingness to a mathematical object (a fuzzy set); in formal logic, it means a degree of truth. This means an extension of binary valuations, which is, henceforth, referred to as crisp evaluations. Concerning its use for solving MADM and MAGDM problems in fuzzy environments, Song et al. [4] gave an algorithm based on arithmetic operators, and Chen [5] built up a theory for a fuzzy-TOPSIS method. No doubt, FS theory produced a turn of direction in the field of decision-making. However, it was not designed to look at the dissatisfaction nature of humans in decision-making. This drawback prompted Atanassov [6] to present intuitionistic fuzzy sets (IFS) in 1986. They allocate both degrees of satisfaction and dissatisfaction to an object.
Other extensions soon followed. Yager [7] further extended the IFS to Pythagorean fuzzy set ($PF_S$) in which the sums of the squares of degree of satisfaction and dissatisfaction should be within the closed unit interval. Later on, Cuong [8] introduced picture fuzzy set ($PFS$) keeping in view the existence of neutral positions under natural circumstances. For example, in case of voting systems a candidate could be either satisfied, remain neutral, and disagree with any given participant [9]. Many researchers chose this environment for solving decision-making problems, but others pointed out that PFS has the limitation that it is not applicable in situations where the sum of the degrees of satisfaction, neutrality, and dissatisfaction exceeds 1. This is the origin of spherical fuzzy sets and the spherical fuzzy TOPSIS method presented by Gundogdu and Kahraman [10] in 2019. Similarly and also motivated by spherical fuzzy sets, Kahraman et al. [11] used a spherical TOPSIS method to find the best location for hospital. Later on, Mahmood et al. [12] proposed $T$-spherical fuzzy sets, a generalization of spherical fuzzy sets, which are less restrictive. They overcame all the limitations of the existing models except in the presence of 2-dimensional problems. Such 2-dimensional problems in MADM and MAGDM can now be analyzed with the tool developed by Ramot et al. [13], who introduced complex fuzzy set in which the degree of satisfaction belongs to the complex unit circle and consists of a periodic term as well as the amplitude term which belong to the unit closed interval. Akram and Bashir [14] extended the averaging operators in the framework of complex fuzzy sets. Alkouri and Salleh [15] presented the idea of complex intuitionistic fuzzy set ($CIFS$), which describes both degree of satisfaction and dissatisfaction within the complex unit circle, where the sum of amplitude and periodic terms of the satisfaction and dissatisfaction degrees should be within the unit interval $[0,1]$.

Recently, Akram et al. [16] introduced the concept of complex spherical fuzzy set and extended the TOPSIS method to that setting. As an application, a model for the selection of best water supply strategy for Nohoor village in Iran was considered. This novel concept contains degrees of satisfaction, neutrality, and dissatisfaction which lie in the complex unit circle. They are further restricted by the condition that the sum of the squares of their amplitude and phase terms should be less than or equal to 1.

There is a widespread handicap in the aforementioned models and methods: they discard the frameworks that are characterized by the satisfaction of certain attributes or the fulfilment of properties. Soft set theory, launched in 1999, accommodates all type of parameters [17]. Alkouri and Salleh [15] introduced some new operators on soft set theory which soon found applications in the fields of operations research, game theory, stability, regularization, medicine, and obviously in decision-making. Following this trend, researchers brought up many models and methods for soft sets and its extensions, inclusive of a new decision-making method for valuation fuzzy soft sets introduced by Alcantud et al. [18]. Despite these improvements there were still problems in real life that could not be solved using the existing MADM and MAGDM methods, for example, because the objects are evaluated using a ranking system or a nonbinary scale. When we check out from hotels, hotel staff ask for our feedback, which we give, for example, in the form of 4 stars, 3 stars, 2 stars, 1 star, and big dot: 4 stars mean “outstanding,” 3 stars mean “superb,” 2 stars mean “good,” 1 star means “satisfactory,” and big dot means “unacceptable.” Similarly, nonbinary rates are given to third-party apps, whether we use a transportation service (Uber, Cabify, etc.) or online shopping facilities. As technology improved and extended, people have become accustomed to such types of ranking systems due to their ease of use and widespread utilization. For this reason, many researchers have become interested in formal models for nonbinary evaluations. The idea presented by Fatima et al. [19], namely, $N$-soft set and their decision-making methods, stirred up new decision-making methodologies. Very soon and keeping in view the possible fuzziness of the parameters, Akram et al. [20] combined the concept of $N$-soft with a fuzzy definition of the attributes thus producing fuzzy $N$-soft sets ($FNS_S$). This novel prescription involves a finite number of ordered grades as well as fuzziness in the conception of the attributes that are used for decision-making. Still another hybrid model called hesitant $N$-soft set was introduced by Akram et al. [21] in order to allow for hesitancy in the allocation of grades. Hesitant fuzzy $N$-soft sets [22] combine the features of these two models. Akram et al. [23] extended the idea of fuzzy $N$-soft set in another direction. They conceived intuitionistic fuzzy $N$-soft sets ($IFNS_S$) that describe the dissatisfactory part separately, with the usual constraint that the sums of the degrees of membership and nonmembership always belong to $[0,1]$. Finally, so far, Zhang et al. [24] extended $IFNS_S$ to Pythagorean fuzzy $N$-soft set ($PFNS_S$) which is more flexible than the existing models.

The motivation of this article depends on the following facts:

1. The existing models $IFNS_S$ and $PFNS_S$ make decisions based on degrees of membership and nonmembership; however, they are unable to incorporate a neutral part of judgement.
2. The decision-making techniques based on existing models $FNS_S$, $IFNS_S$, and $PFNS_S$ can solve only problems of the 1-dimensional type. Neither of these models can operate in the presence of a periodic term or 2-dimensional type problems.
3. Although CSFSs deal with 2-dimensional problems of real life, they are unable to describe parameterized information as well as finitely many ranked grades of association of the alternatives with the pertinent parameters.
4. These limitations motivated us to put forward a new model called $CSFNS_S$ which efficiently deals with abstention (together with degrees of satisfaction and dissatisfaction) as well as the periodic term of 2-dimensional decision-making problems. At the same time, $CSFNS_S$ competently handles the ordered grades of the alternatives according to the different attributes.
The main contributions of this article are as follows:

(1) The proposed model, CSFNS\(_f\), allows for neutral opinions in the framework of 2-dimensional problems. In this way, it can manipulate conditions on amplitude and periodic terms with more flexibility.

(2) This model establishes a modern theory that captures a new perspective of decision-making. It is based on ratings or ranking systems including ordered grades of elements according to related attributes.

(3) The algorithms and CSFNS\(_f\)-TOPSIS method defined in this article solve MADM and MAGDM problems, respectively. They apply to more general situations than the existing algorithms and TOPSIS Method. These methods for decision-making under the framework of CSFNS\(_f\) are illustrated with numerical examples.

(4) The comparative study with PFNS\(_f\) algorithms and the CSF-TOPSIS method shows their ability and significance.

The rest of the paper is organized as follows. Section 2 contains some definitions from existing models. In Section 3, we propose the novel concept of CSFNS\(_f\), which is then followed by the operations on CSFNS\(_f\) and CSFNS\(_f\)-TOPSIS. Section 3 describes three algorithms for making decisions and performs a comparison with a PFNS\(_f\) method. In Section 4, we develop a theoretical foundation for the CSFNS\(_f\)-TOPSIS method. In Section 5, we present the mathematical algorithms of these decision-making mechanisms that are applied to some numerical examples. Section 6 describes the comparison analysis with CSF-TOPSIS method. In Section 7, we conclude the paper and provide future directions for research.

2. Preliminaries

Definition 1 (see [10]). A spherical fuzzy set (SFS) \(Y\) on a universe of discourse \(U\) has the form

\[ Y = \langle u, \mu_Y (u), \eta_Y (u), \nu_Y (u) | u \in U \rangle, \]  

where \(\mu_Y (u)\), \(\eta_Y (u)\), and \(\nu_Y (u)\), which lie within the unit interval, are called the grade of the positive, neutral, and negative degree of membership, respectively. They are restricted by the conditions \(p_Y (u) \leq 1\) and \(\phi_Y (u) \leq 1\), for every \(u \in U\), where \(i = \sqrt{-1}\). The degree of refusal of \(u\) in \(U\) is defined as

\[ \Theta_Y (u) = \sqrt{1 - (p_Y (u)^2 + \nu_Y (u)^2 + \phi_Y (u)^2)}. \]  

Definition 2 (see [25]). A complex T-spherical fuzzy set (CTSF) \(Y\) on the universe \(U\) is defined as

\[ Y = \langle u, \mu_Y (u), \eta_Y (u), \nu_Y (u) | u \in U \rangle, \]  

where \(\mu_Y (u) = p_Y (u)e^{2\pi i \phi_Y (u)}, \eta_Y (u) = \nu_Y (u)e^{2\pi i \delta_Y (u)},\) and \(\nu_Y (u) = r_Y (u)e^{2\pi i \lambda_Y (u)}\), which denote the positive, neutral, and negative degree of membership, respectively. They are restricted by the conditions \(p_Y (u) \leq 1\) and \(\phi_Y (u) \leq 1\), for every \(u \in U\), where \(i = \sqrt{-1}\), and \(p_Y, \nu_Y, \eta_Y, \phi_Y, \delta_Y, \lambda_Y \in [0, 1] \). The degree of refusal of \(u\) in \(U\) is defined as

\[ \Pi_Y (u) = \frac{1}{2\pi} \sqrt{1 - (p_Y (u)^2 + \nu_Y (u)^2 + \phi_Y (u)^2)} \]  

The triplet \((\mu_Y, \eta_Y, \nu_Y) = (p_Y e^{2\pi i \phi_Y (u)}, \nu_Y e^{2\pi i \delta_Y (u)}, \phi_Y e^{2\pi i \lambda_Y (u)})\) is called CSFN.

Definition 3 (see [26]). Let \(W\) be a nonempty set and \(R\) be a set of attributes and \(Z \subseteq R\). A soft set \(S\) over \(W\) is a pair \((T, Z)\), where \(T\) is a set-value function from \(Z\) to the set of all subsets of \(W\), which is denoted as

\[ (T, Z) = \{ (z, T(z)) | z \in Z, T(z) \in 2^W \}. \]  

Definition 4. Let \(W\) be a nonempty set and \(R\) be a set of attributes, \(Z \subseteq R\). A complex spherical fuzzy soft set \((CSFSS, S)\) over \(W\) is a pair \((T, Z)\), where \(T\) is a set-value function from \(Z\) to the set of all subsets of \(CSFSS\), which is denoted as

\[ (T, Z) = \{ (z, T(z)) | z \in Z, T(z) \in CSFSS \}. \]  

where \(T(z) = \{ (w, p_z (w)e^{2\pi i \phi_z (w)}, \nu_z (w)e^{2\pi i \delta_z (w)}, \phi_z (w)e^{2\pi i \lambda_z (w)}) | w \in W \} \). The triplet \((p_z, \nu_z, \phi_z) = (p_z (w)e^{2\pi i \phi_z (w)}, \nu_z (w)e^{2\pi i \delta_z (w)}, \phi_z (w)e^{2\pi i \lambda_z (w)})\) is called CSFN.

Definition 5 (see [19]). Let \(W\) be a nonempty set and \(R\) be a set of attributes. Let \(Z \subseteq R\) and \(G = \{ 0, 1, 2, \ldots, N - 1 \}\) be a set of ordered grades with \(N \in \{ 2, 3, \ldots \}\). A triple \((F, Z, N)\) is called \(N\)-soft set \((NS, S)\) over \(W\) if \(F\) is a mapping from \(Z\) to \(2^{U \times G}\), such that for each \(z \in Z\) and \(w \in W\), there exist a unique \((w, g^w_z) \in W \times G\) such that \((w, g^w_z) \in F(z), w \in W, g^w_z \in G\).

3. Complex Spherical Fuzzy \(N\)-Soft Sets

Definition 6. Let \(W\) be a nonempty set and \(R\) be a set of attributes. Let \(Z \subseteq R\) and \(G = \{ 0, 1, 2, \ldots, N - 1 \}\) be a set of ordered grades with \(N \in \{ 2, 3, \ldots \}\). A triple \((F, Z, N)\) is called a complex spherical fuzzy \(N\)-soft set \((CSFNS, S)\) on \(Z\), when \((F, Z, N) : F : Z \rightarrow 2^{W \times G}\) is an \(NS, S\) on \(W\), if \(F : Z \rightarrow 2^{W \times G} \times CSFN\) is a mapping, which is defined as
\( (F_j, Z, N) = \{ (z, F(z), I(z)) | z \in Z, (F(z), I(z)) \in 2^W \times \text{CSFN} \} \)
\[= \{ (z, ((w, g^w_z), (\mu_z(w), \eta_z(w), \nu_z(w))) ) \} \]
\[= \{ (z, ((w, g^w_z), p_z(w)e^{2\pi \delta_z(w)}, \nu_z(w)e^{2\pi \delta_z(w)}) ) \} \}, \] (8)

where \( J: Z \rightarrow \text{CSFN}, \text{CSFN} \) denotes the collection of all complex spherical fuzzy numbers of \( W \), \( g^w_z \) denotes the level of attribute for the element \( w \) and \( p_z, \nu_z, \rho_z, \phi_z, \delta_z, \lambda_z \in [0, 1] \), restricted with conditions

\[0 \leq p_z(w)^2 + \nu_z(w)^2 + \rho_z(w)^2 \leq 1,\]
\[0 \leq \phi_z(w)^2 + \delta_z(w)^2 + \lambda_z(w)^2 \leq 1, \quad \text{for all } w \text{ belongs to } W.\]
(9)

**Definition 7.** Let \( F_j(z_i) = ((w_i, g^j_{w_i}), p_{k_j}e^{2\pi \phi_{k_j}}, \nu_{k_j}e^{2\pi \delta_{k_j}}, r_{k_j}e^{2\pi \lambda_{k_j}}) \) be a \( \text{CSFN}_jS \). Then, the complex spherical fuzzy N-soft set (\( \text{CSFN}_j, N \)) is defined as

\[Y_{k_j} = (g^j_{k_j}p_{k_j}e^{2\pi \phi_{k_j}}, v_{k_j}e^{2\pi \delta_{k_j}}, r_{k_j}e^{2\pi \lambda_{k_j}}),\]
\[\Omega_{Y_{k_j}} = \sqrt{1 - (p_{k_j}^2 + w_{k_j}^2 + r_{k_j}^2) e^{2\pi i(\phi_{k_j}^2 + \delta_{k_j}^2 + \lambda_{k_j}^2)}},\]

is the hesitancy degree, where \( p_{k_j}, v_{k_j}, r_{k_j}, \phi_{k_j}, \delta_{k_j}, \lambda_{k_j} \) represent \( p_z(w), \nu_z(w), r_z(w), \phi_z(w), \delta_z(w), \lambda_z(w) \), respectively.

**Definition 8.** Consider a \( \text{CSFN}_jN \) \( Y_{k_j} = (g^j_{k_j}p_{k_j}e^{2\pi \phi_{k_j}}, \nu_{k_j}e^{2\pi \delta_{k_j}}, r_{k_j}e^{2\pi \lambda_{k_j}}) \). The score function \( S(Y_{k_j}) \) is

\[S_{Y_{k_j}} = \left( \frac{g^j_{k_j}}{N-1} \right)^2 + (p_{k_j}^2 - w_{k_j}^2 - r_{k_j}^2) + [\phi_{k_j}^2 - \delta_{k_j}^2 - \lambda_{k_j}^2],\]

where \( S_{Y_{k_j}} \in [-2, 3] \). The accuracy function \( A(Y_{k_j}) \) is

\[A_{Y_{k_j}} = \left( \frac{g^j_{k_j}}{N-1} \right)^2 + (p_{k_j}^2 + w_{k_j}^2 + r_{k_j}^2) + [\phi_{k_j}^2 + \delta_{k_j}^2 + \lambda_{k_j}^2],\]

where \( A_{Y_{k_j}} \in [0, 3] \), respectively.

**Definition 9.** Let \( Y_{ij} = (g^j_{i}p_{i}e^{2\pi \phi_{i}}, \nu_{i}e^{2\pi \delta_{i}}, r_{i}e^{2\pi \lambda_{i}}) \) and \( Y_{kj} = (g^j_{k}p_{k}e^{2\pi \phi_{k}}, \nu_{k}e^{2\pi \delta_{k}}, r_{k}e^{2\pi \lambda_{k}}) \) be two \( \text{CSFN}_jS \):

1. If \( S_{Y_{ij}} < S_{Y_{kj}} \), then \( Y_{ij} \sim Y_{kj} \) (\( Y_{ij} \) is inferior to \( Y_{kj} \)).
2. If \( S_{Y_{ij}} > S_{Y_{kj}} \), then \( Y_{ij} \succ Y_{kj} \) (\( Y_{ij} \) is superior to \( Y_{kj} \)).
3. If \( S_{Y_{ij}} = S_{Y_{kj}} \),
   1. \( A_{Y_{ij}} < A_{Y_{kj}} \), then \( Y_{ij} \sim Y_{kj} \) (\( Y_{ij} \) is inferior to \( Y_{kj} \)).
   2. \( A_{Y_{ij}} > A_{Y_{kj}} \), then \( Y_{ij} \succ Y_{kj} \) (\( Y_{ij} \) is superior to \( Y_{kj} \)).
   3. \( A_{Y_{ij}} = A_{Y_{kj}} \), then \( Y_{ij} \sim Y_{kj} \) (\( Y_{ij} \) is equivalent to \( Y_{kj} \)).

**Remark 1.** We see that

(1) For \( N = 2 \), \( \text{CSFN}_jS \) becomes complex spherical fuzzy soft set

(2) When \( |Z| = 1 \), \( \text{CSFN}_jS \) becomes complex spherical fuzzy set

(3) When \( \phi_z = \delta_z = \lambda_z = 0 \), \( \text{CSFN}_jS \) becomes spherical fuzzy N-soft set

**Example 1.** In a city, a parent wants to choose the best school for their child. It is necessary to go after the advice of experts, for the selection of a school based on rankings and ratings. Let \( W = \{w_1, w_2, w_3\} \) be the family of three schools under consideration and \( Z = \{z_1 = \text{size of school}, z_2 = \text{location}, z_3 = \text{academic performance}, z_4 = \text{services} \} \) be the attributes which are used to assign rankings to schools by the experts. In a relation to these parameters, a 5-soft set is given in Table 1, where

Four diamonds means “Outstanding”

Three diamonds means “Super”

Two diamonds means “Good”

One diamond means “Satisfactory”

Big dot means “Acceptable”

This level assessment by diamonds can be represented by numbers as \( G = \{0, 1, 2, 3, 4\} \), where

0 means “\( \bullet \)”
1 means “\( \bigcirc \)”
2 means “\( \bigcirc \bigcirc \)”
3 means “\( \bigcirc \bigcirc \bigcirc \)”
4 means “\( \bigcirc \bigcirc \bigcirc \bigcirc \)”

Table 2 can be adopted as natural convention of 5-soft set model.

By Definition 6, when the data is vague and uncertain, we need \( \text{CSFN}_jS \) which provides us information on how these grades are given to schools. The evaluation of schools by experts follows the following grading:

\[\begin{align*}
\text{when } g^w_z &= 0, & -2.00 &\leq S_j &< -1.85, \\
\text{when } g^w_z &= 1, & -1.85 &\leq S_j &< -1.30, \\
\text{when } g^w_z &= 2, & -1.30 &\leq S_j &< -0.15, \\
\text{when } g^w_z &= 3, & 0.15 &\leq S_j &< 1.30, \\
\text{when } g^w_z &= 4, & 1.30 &\leq S_j &< 2.00.
\end{align*}\]
(13)

According to the above criteria, we can obtain Table 3. At last, \( \text{CSFN}_jS \) is defined as
Table 1: Evaluation data provided by the experts.

| W/Z | z_1  | z_2  | z_3  | z_4  |
|-----|------|------|------|------|
| w_1 | ☆     |      |      |      |
| w_2 |      |      |      |      |
| w_3 |      |      |      |      |

Table 2: Tabular representation of 5-soft set.

| W/Z | z_1 | z_2 | z_3 | z_4 |
|-----|-----|-----|-----|-----|
| w_1 | 2   | 3   | 0   | 2   |
| w_2 | 0   | 3   | 1   | 1   |
| w_3 | 1   | 3   | 4   | 2   |

Table 3: Grading criteria.

| g_w^F | Positive membership | Neutral membership | Negative membership |
|-------|---------------------|--------------------|---------------------|
| Δz    | 2πφ_z              | 2πδ_z             | 2πλ_z              |
| Δz    | [0, 0.15]           | [0, 0.03π]        | [0, 0.03π]         |
| Δz    | [0.15, 0.35]        | [0.03π, 0.7π]     | [0.03π, 0.2062π]   |
| Δz    | [0.35, 0.65]        | [0.7π, 1.3π]      | [0.03π, 0.2062π]   |
| Δz    | [0.65, 0.85]        | [1.3π, 1.7π]      | [0.03π, 0.2062π]   |
| Δz    | [0.85, 1]           | [1.7π, 2π]        | [0.03π, 0.2062π]   |

\[
\left( \mu_{z'}, \eta_{z'}, \nu_{z'} \right) = \left( (w_1, 2), (0.4e^{-0.82\pi}, 0.017e^{0.0356\pi}, 0.6e^{1.122\pi}), (w_2, 0), (0.02e^{-0.06\pi}, 0.012e^{-0.026\pi}, 0.98e^{-1.962\pi}) \right),
\]

\[
\left( (w_3, 4), (1, 0, 0) \right),
\]

... (14)...

The tabular representation of CSF5S_jS is shown by Table 4.

Definition 10. A CSFNS_jS (F_j, Z, N) over a nonempty set W is said to be efficient, where (F, Z, N) is an NS_jS if F_j(x) = (w, N - 1), 1, 0, 0 for some z \in Z, w \in W.

Example 2. Let (F_j, Z, 5) be CSF5S_jS, as in Example 1. It is easy to check from Table 4 that F_j(z) = (w_3, 4, 1, 0, 0), i.e., Example 1 is efficient.

Definition 11. Let (F_j, Z, N_1) and (H_j, B, N_2) be two CSFNS_jSs on a universe of discourse W. Then, they are said to be equal if and only if F = H, J = A, Z = B, and N_1 = N_2.

Example 3. Let (F_j, Z, 5) be CSF5S_jS, as in Example 1. The weak complement (F'_j, Z, N) is given in Table 5.

Definition 12. Let (F_j, Z, N) be CSFNS_jS on W. The weak complement of CSFNS_jS is defined as the weak complement of the N-soft set (F, Z, N), that is, any N-soft set such that F' \cap F = \emptyset for all z \in Z. The weak complement of CSFNS_jS of (F_j, Z, N) is represented as (F'_j, Z, N).

Definition 13. Let (F_j, Z, N) be CSFNS_jS on W. The complex spherical fuzzy complement of CSFNS_jS is denoted as (F_j, Z, N) and is defined as
| \((F_j, Z, S)\) | \(z_1\) | \(z_2\) | \(z_3\) | \(z_4\) |
|-----------------|--------|--------|--------|--------|
| \(w_1\)        | \((2, (0.4e^{0.82\pi}, 0.017e^{0.036\pi}, 0.6e^{0.22\pi})))\) | \((3, (0.65e^{0.32\pi}, 0.018e^{0.038\pi}, 0.28e^{0.58\pi})))\) | \((0, (0.1e^{0.24\pi}, 0.012e^{0.022\pi}, 0.985e^{1.964\pi})))\) | \((2, (0.5e^{1.1\pi}, 0.1e^{0.38\pi}, 0.59e^{1.28\pi})))\) |
| \(w_2\)        | \((0, (0.02e^{0.06\pi}, 0.012e^{0.026\pi}, 0.988e^{0.962\pi})))\) | \((3, (0.7e^{0.42\pi}, 0.019e^{0.041\pi}, 0.3e^{0.56\pi})))\) | \((1, (0.2e^{0.30\pi}, 0.027e^{0.031\pi}, 0.91e^{1.82\pi})))\) | \((1, (0.3e^{0.58\pi}, 0.019e^{0.042\pi}, 0.885e^{1.73\pi})))\) |
| \(w_3\)        | \((4, (1, 0, 0))\) | \((1, (0.16e^{0.34\pi}, 0.1e^{0.204\pi}, 0.89e^{1.784\pi})))\) | \((3, (0.69e^{1.384\pi}, 0.101e^{0.204\pi}, 0.32e^{0.682\pi})))\) | \((2, (0.45e^{0.88\pi}, 0.015e^{0.022\pi}, 0.78e^{1.568\pi})))\) |

**Table 4:** Tabular representation of the CSFSS\(F_j, Z, S\).
Table 5: A weak complement of the CSFS $S(F,J,Z)$ in Example 1.

| $(F,J,Z,5)$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ |
|-------------|------|------|------|------|
| $w_1$       | $(3, (0.4e^{i0.82\pi}, 0.017e^{i0.0356\pi}, 0.6e^{i2.23\pi}))$ | $(0, (0.65e^{i1.32\pi}, 0.018e^{i0.0384\pi}, 0.28e^{i0.58\pi}))$ | $(1, (0.1e^{i0.24\pi}, 0.012e^{i0.022\pi}, 0.985e^{i1.964\pi}))$ | $(3, (0.5e^{i1.48\pi}, 0.1e^{i0.38\pi}, 0.59e^{i1.28\pi}))$ |
| $w_2$       | $(3, (0.02e^{i0.96\pi}, 0.012e^{i0.026\pi}, 0.986e^{i1.962\pi}))$ | $(1, (0.7e^{i1.42\pi}, 0.019e^{i0.047\pi}, 0.3e^{i0.56\pi}))$ | $(4, (0.69e^{i1.34\pi}, 0.101e^{i0.204\pi}, 0.32e^{i0.062\pi}))$ | $(2, (0.3e^{i0.56\pi}, 0.019e^{i0.042\pi}, 0.885e^{i1.72\pi}))$ |
| $w_3$       | $(2, (1, 0, 0))$ | $(3, (0.16e^{i0.34\pi}, 0.1e^{i0.204\pi}, 0.89e^{i1.744\pi}))$ | $(1, (0.69e^{i1.34\pi}, 0.101e^{i0.204\pi}, 0.32e^{i0.062\pi}))$ | $(4, (0.45e^{i0.86\pi}, 0.015e^{i0.022\pi}, 0.78e^{i1.56\pi}))$ |
\[ F_r(z) = \langle v_z(w), \eta_z(w), \mu_z(w) \rangle = \langle (w, \delta^w_r) \rangle \text{ if and only if } (F_r, Z, N) \text{ is a weak complement of } (F_r, Z, N). \]

\[ \text{Definition 15. Let } (F_r, Z, N) \text{ be a CSFNS_S on } W; \text{ then the top weak complex spherical fuzzy complement } (F_{r^+}, Z, N) \text{ is defined as } \]

\[ (F_{r^+}, Z, N) = \left\{ \begin{array}{ll}
F_j(z_k) = \left( w_j, N - 1 \right), & \text{if } g_k^j < N - 1, \\
F_j(z_k) = \left( w_j, 0 \right), & \text{if } g_k^j = N - 1.
\end{array} \right. \]

\[ \text{Definition 16. Let } (F_r, Z, N) \text{ be a CSFNS_S on } W; \text{ then the bottom weak complex spherical fuzzy complement } (F_{r^-}, Z, N) \text{ is defined as } \]

\[ (F_{r^-}, Z, N) = \left\{ \begin{array}{ll}
F_j(z_k) = \left( w_j, N - 1 \right), & \text{if } g_k^j > 0, \\
F_j(z_k) = \left( w_j, 0 \right), & \text{if } g_k^j = 0.
\end{array} \right. \]

\[ \text{Definition 17. Let } W \text{ be a nonempty set and } (F_r, Z, N_1) \text{ and } (H_A, B, N_2) \text{ be CSFNS_Ss and CSFNS_Ss on } W, \text{ respectively, and their restricted intersection is defined as } (K_L, M, S) = (F_r, Z, N_1) \cap (H_A, B, N_2), \text{ where } K_L = F_r \cap H_A, \text{ and } M = Z \cap B, \text{ and } S = \min(N_1, N_2), \text{ i.e., for } \forall u_k \in M \text{ and } w_j \in W, (g_k^j, (\mu_j, \eta_j, \nu_j)) \in K_L(u_k), g_k^j = \min(g_k^j, g_k^j), \mu_j(u_k) = \min(\mu_j(u_k), \mu_j(u_k)), \nu_j(u_k) = \max(\nu_j(u_k), \nu_j(u_k)), \text{ where } (\mu_j(u_k), \eta_j(u_k), \nu_j(u_k)) \in (\mu_{H_A}(u_k), \eta_{H_A}(u_k), \nu_{H_A}(u_k)), \text{ with } u_k \in Z \text{ and } u_k \in B. \]

\[ \text{Example 8. Let } (E_p, Z, 5) \text{ and } (H_A, B, 6) \text{ be two CSF55S_S and CSF66S_S, given in Tables 10 and 11, respectively. Their restricted intersection is shown in Table 12.} \]

\[ \text{Definition 18. Let } W \text{ be a nonempty set and } (F_r, Z, N_1) \text{ and } (H_A, B, N_2) \text{ be two CSFNS_Ss on } W; \text{ their extended intersection is defined as } \varnothing_D(C, Y) = (F_r, Z, N_1) \cap E (H_A, B, N_2), \text{ where } \varnothing_D = F_r \cap E H_A, C = Z \cup B, \text{ and } Y = \max(N_1, N_2), \text{ that is, } \forall u_k \in C \text{ and } w_j \in W, (g_k^j, (\mu_j, \eta_j, \nu_j)) \in \varnothing_D(u_k), \text{ with } \]

\[ \text{Example 5. Let } (F_r, Z, 5) \text{ be CSF55S_S, as in Example 1. The weak complex spherical fuzzy complement } (F_{r^+}, Z, N) \text{ is given in Table 7.} \]
| $(F_{j'}, Z, S)$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ |
|----------------|-------|-------|-------|-------|
| $w_1$          | $(2, (0.6e^{1.12π}, 0.017e^{0.0356π}, 0.4e^{0.82π}))$ | $(3, (0.28e^{0.88π}, 0.018e^{0.34π}, 0.65e^{1.32π}))$ | $(0, (0.985e^{1.964π}, 0.012e^{0.022π}, 0.1e^{0.246π}))$ | $(2, (0.59e^{1.288π}, 0.1e^{0.18π}, 0.5e^{1.18π}))$ |
| $w_2$          | $(0, (0.98e^{1.962π}, 0.012e^{0.026π}, 0.02e^{0.008π}))$ | $(3, (0.3e^{0.56π}, 0.019e^{0.64π}, 0.7e^{1.42π}))$ | $(1, (0.32e^{0.62π}, 0.101e^{0.204π}, 0.69e^{1.384π}))$ | $(1, (0.885e^{1.72π}, 0.019e^{0.042π}, 0.3e^{0.56π}))$ |
| $w_3$          | $(4, (0, 0, 1))$ | $(1, (0.89e^{1.78π}, 0.1e^{0.204π}, 0.16e^{0.34π}))$ | $(3, (0.32e^{0.62π}, 0.101e^{0.204π}, 0.69e^{1.384π}))$ | $(2, (0.78e^{0.568π}, 0.015e^{0.022π}, 0.45e^{0.86π}))$ |

Table 6: The complex spherical fuzzy complement $(F_{j'}, Z, S)$ of the CSFS $S$ in Example 1.
Table 7: The weak complex spherical fuzzy complement \((P'_x, Z, S)\) of the CSF in Example 1.

| \((P'_x, Z, S)\) | \(z_1\) | \(z_2\) | \(z_3\) | \(z_4\) |
|------------------|--------|--------|--------|--------|
| \(w_1\) (3, \((0.6^{1.122\pi}, 0.017e^{0.0368\pi}, 0.4e^{0.82\pi})\)) | (0, \((0.28e^{0.038\pi}, 0.018e^{0.038\pi}, 0.65e^{1.123\pi})\)) | (1, \((0.985e^{1.964\pi}, 0.012e^{0.022\pi}, 0.1e^{0.24\pi})\)) | (3, \((0.59e^{1.28\pi}, 0.1e^{0.18\pi}, 0.5e^{1.1\pi})\)) |
| \(w_2\) (3, \((0.98e^{1.962\pi}, 0.012e^{0.026\pi}, 0.02e^{0.209\pi})\)) | (1, \((0.3e^{0.56\pi}, 0.019e^{0.04\pi}, 0.7e^{1.42\pi})\)) | (4, \((0.32e^{0.62\pi}, 0.101e^{0.204\pi}, 0.69e^{3.384\pi})\)) | (2, \((0.885e^{1.72\pi}, 0.019e^{0.042\pi}, 0.3e^{0.56\pi})\)) |
| \(w_3\) (2, \((0, 0, 1)\)) | (3, \((0.89e^{1.78\pi}, 0.1e^{0.204\pi}, 0.16e^{0.34\pi})\)) | (1, \((0.32e^{0.62\pi}, 0.101e^{0.204\pi}, 0.69e^{3.384\pi})\)) | (4, \((0.78e^{1.76\pi}, 0.015e^{0.02\pi}, 0.45e^{0.88\pi})\)) |
**Table 8:** The top weak complex spherical fuzzy complement \((F'_J, Z, N)\) of the CSF5S\(_J\)S set in Example 1.

| \((F'_J, Z, S)\) | \(z_1\) | \(z_2\) | \(z_3\) | \(z_4\) |
|------------------|--------|--------|--------|--------|
| \(\omega_1\)    | (4, (0.6e\(^{0.32}\pi, 0.017e\(^{0.03}\pi, 0.4e\(^{0.82}\pi)))\) | (4, (0.28e\(^{0.38}\pi, 0.018e\(^{0.38}\pi, 0.65e\(^{0.32}\pi)))\) | (4, (0.98e\(^{1.96}\pi, 0.012e\(^{0.02}\pi, 0.1e\(^{0.24}\pi)))\) | (4, (0.59e\(^{1.28}\pi, 0.1e\(^{0.18}\pi, 0.5e\(^{0.1}i\pi)))\) |
| \(\omega_2\)    | (4, (0.98e\(^{1.96}\pi, 0.012e\(^{0.02}\pi, 0.02e\(^{0.06}\pi)))\) | (4, (0.3e\(^{0.3}\pi, 0.019e\(^{0.94}\pi, 0.7e\(^{1.42}\pi)))\) | (4, (0.32e\(^{0.06}\pi, 0.101e\(^{0.20}\pi, 0.69e\(^{1.84}\pi)))\) | (4, (0.88e\(^{1.72}\pi, 0.019e\(^{0.04}\pi, 0.3e\(^{0.56}\pi)))\) |
| \(\omega_3\)    | (0, (0, 0)) | (4, (0.89e\(^{1.78}\pi, 0.1e\(^{0.20}\pi, 0.16e\(^{0.34}\pi)))\) | (4, (0.32e\(^{0.62}\pi, 0.101e\(^{0.20}\pi, 0.69e\(^{1.84}\pi)))\) | (4, (0.78e\(^{1.56}\pi, 0.015e\(^{0.02}\pi, 0.45e\(^{0.88}\pi)))\) |
Table 9: The bottom weak complex spherical fuzzy complement \((F^c, Z, N)\) of the \(CSFS_S\) in Example 1.

| \((F^c, Z, S)\) | \(z_1\) | \(z_2\) | \(z_3\) | \(z_4\) |
|-----------------|---------|---------|---------|---------|
| \(w_1\) | \((0, 0.6e^{0.122i}, 0.017e^{0.033i}, 0.4e^{0.82i})\) | \((0, 0.28e^{0.038i}, 0.018e^{0.034i}, 0.65e^{0.132i})\) | \((4, 0.985e^{1.964i}, 0.012e^{0.022i}, 0.1e^{0.242i})\) | \((0, 0.598e^{1.228i}, 0.1e^{0.182i}, 0.5e^{1.152i})\) |
| \(w_2\) | \((4, 0.986e^{1.962i}, 0.012e^{0.032i}, 0.022e^{0.160i})\) | \((0, 0.3e^{0.034i}, 0.019e^{0.042i}, 0.7e^{0.414i})\) | \((0, 0.321e^{0.204i}, 0.101e^{0.138i}, 0.697e^{1.384i})\) | \((0, 0.885e^{0.372i}, 0.019e^{0.042i}, 0.3e^{0.562i})\) |
| \(w_3\) | \((0, 0.898e^{1.784i}, 0.1e^{0.204i}, 0.16e^{0.34i})\) | \((0, 0.321e^{0.204i}, 0.101e^{0.138i}, 0.697e^{1.384i})\) | \((0, 0.785e^{1.566i}, 0.015e^{0.022i}, 0.45e^{0.860i})\) | \((0, 0.785e^{1.566i}, 0.015e^{0.022i}, 0.45e^{0.860i})\) |
Example 9. Let \((E_P, Z, 5)\) and \((H_A, B, 6)\) be two CSF5S\(_J\)S and CSF6S\(_J\)S, given in Tables 10 and 11, respectively. Their extended intersection \((E_D, C, Y) = (E_P, Z, N_1) \cap E (H_A, B, N_2)\) is shown in Table 13.

Definition 19. Let \((E_P, Z, 5)\) and \((H_A, B, 6)\) be two CSFNS\(_J\)S on \(W\); their restricted union is defined as \((\mathcal{R}_T, M, S) = (F_J, Z, N_1) \cup R (H_A, B, N_2)\), where \(\mathcal{R}_T = F_J \cup R H_A, M = Z \cap B, S = max (N_1, N_2)\), i.e., \(w_j \in M\) and \(w_j \in W\). Let \(g^1_k (\mu_{k, j}^1, \eta_{k, j}^1, v_{k, j}^1)\) be the extended union, \(g^2_k (\mu_{k, j}^2, \eta_{k, j}^2, v_{k, j}^2)\) be the extended intersection, \(\mu_{k, j}^1 (v_{k, j}^1) = \max \{v_{k, j}^1 (u_k), v_{k, j}^1 (u_k')\}\), \(\mu_{k, j}^2 (v_{k, j}^2) = \max \{v_{k, j}^2 (u_k), v_{k, j}^2 (u_k')\}\), and \(\eta_{k, j} (v_{k, j}) = \min \{v_{k, j} (u_k), v_{k, j} (u_k')\}\), where \(v_{k, j} (u_k) = \min (r_{k, j} (u_k), r_{k, j} (u_k'))\) and \(v_{k, j} (u_k') = \min (r_{k, j} (u_k), r_{k, j} (u_k'))\) for \(j = 1, 2\), \(k = 1, 2\), \(\mathcal{R}_T\) is the restricted union, and \(\mathcal{R}_S\) is the restricted intersection.

Example 10. Let \((E_P, Z, 5)\) and \((H_A, B, 6)\) be two CSF5S\(_J\)S and CSF6S\(_J\)S, given in Tables 10 and 11, respectively. Their restricted union \((E_D, C, Y) = (E_P, Z, N_1) \cap E (H_A, B, N_2)\) is shown in Table 14.

Definition 20. Let \(W\) be a nonempty set \(F_J, Z, N_1\) and \((H_A, B, N_2)\) be two CSFNS\(_J\)S on \(W\); their extended union is defined as \((\Sigma_X, C, Y) = (F_J, Z, N_1) \cup \Sigma (H_A, B, N_2)\), where \(\Sigma_X = F_J \cup \Sigma H_A, C = Z \cap B, Y = max (N_1, N_2)\), that is, \(\forall v_k \in C\) and \(w_j \in W\), \((g^1_k (\mu_{k, j}^1, \eta_{k, j}^1, v_{k, j}^1) = \min (r_{k, j} (u_k), r_{k, j} (u_k'))\) and \(\eta_{k, j} (v_{k, j}) = \min (r_{k, j} (u_k), r_{k, j} (u_k'))\) for \(j = 1, 2\), \(k = 1, 2\), \(\mathcal{R}_T\) is the extended union, and \(\mathcal{R}_S\) is the extended intersection.
| (\(\eta_D, C, 6\)) | \(z_1\) | \(z_2\) | \(z_3\) | \(z_6\) |
|----------------|----------|----------|----------|----------|
| \(w_1\) | (1, (0.23 e^{0.48\pi}, 0.019 e^{0.036\pi}, 0.92 e^{1.85\pi})) | (3, (0.65 e^{1.32\pi}, 0.03 e^{0.062\pi}, 0.28 e^{0.58\pi})) | (0, (1.1 e^{0.24\pi}, 0.04 e^{0.084\pi}, 0.985 e^{1.96\pi})) | (3, (0.69 e^{1.39\pi}, 0.04 e^{0.084\pi}, 0.65 e^{1.36\pi})) |
| \(w_2\) | (0, (0.02 e^{0.08\pi}, 0.015 e^{0.032\pi}, 0.983 e^{1.968\pi})) | (2, (0.35 e^{0.66\pi}, 0.019 e^{0.048\pi}, 0.8 e^{0.68\pi})) | (1, (0.2 e^{0.04\pi}, 0.027 e^{0.082\pi}, 0.91 e^{1.83\pi})) | (4, (0.87 e^{1.72\pi}, 0.035 e^{0.068\pi}, 0.933 e^{1.862\pi})) |
| \(w_3\) | (2, (0.4 e^{0.82\pi}, 0.012 e^{0.026\pi}, 0.77 e^{1.52\pi})) | (1, (0.16 e^{0.34\pi}, 0.1 e^{0.204\pi}, 0.89 e^{1.784\pi})) | (3, (0.69 e^{1.384\pi}, 0.101 e^{0.204\pi}, 0.32 e^{0.682\pi})) | (1, (0.17 e^{0.344\pi}, 0.035 e^{0.068\pi}, 0.933 e^{1.862\pi})) |
Table 14: Tabular representation of restricted union \((R, M, 6)\).

| \((R, M, 6)\) | \(z_1\) | \(z_2\) |
|--------------|------|------|
| \(w_1\) | \((2, (0.4e^{0.10x}, 0.017e^{0.035x}, 0.6e^{1.212x}))\) | \((5, (0.9e^{1.88x}, 0.018e^{0.038x}, 0.14e^{2.2xy}))\) |
| \(w_2\) | \((0, (0.03e^{0.12x}, 0.012e^{0.026x}, 0.98e^{1.962x}))\) | \((3, (0.7e^{1.42x}, 0.014e^{0.029x}, 0.3e^{0.56x}))\) |
| \(w_3\) | \((4, (1, 0, 0))\) | \((3, (0.6e^{1.22x}, 0.02e^{0.06x}, 0.49e^{1.2xy}))\) |

Example 11. Let \((E_p, Z, S)\) and \((H_A, B, 6)\) be two CSFNSs over the same universe \(W\) and \(R\), respectively. Their restricted union \((Q_D, C, Y) = (E_p, Z, N_1) \cap_E (H_A, B, N_2)\) is shown in Table 15.

We state the following properties without their proofs.

Theorem 1. Let \((F, Z, N)\) be CSFNSs over a nonempty set \(W\). Then,

1. \((F, Z, N) \cap (F, Z, N) = (F, Z, N)\)
2. \((F, Z, N) \cap (F, Z, N) = (F, Z, N)\)
3. \((F, Z, N) \cap (F, Z, N) = (F, Z, N)\)
4. \((F, Z, N) \cup (F, Z, N) = (F, Z, N)\)

We state the following properties without their proofs.

Theorem 2. Let \((F, Z, N_1)\) and \((H_A, B, N_2)\) be two CSFNSs over the same universe \(W\); then, the absorption properties hold:

1. \(((F, Z, N_1) \cup (H_A, B, N_2)) \cap (F, Z, N_1) = (F, Z, N_1)\)
2. \(((F, Z, N_1) \cup (H_A, B, N_2)) \cap (F, Z, N_1) = (F, Z, N_1)\)
3. \(((F, Z, N_1) \cap (H_A, B, N_2)) \cup (F, Z, N_1) = (F, Z, N_1)\)
4. \(((F, Z, N_1) \cap (H_A, B, N_2)) \cup (F, Z, N_1) = (F, Z, N_1)\)

We state the following properties without their proofs.

Theorem 3. Let \((F, Z, N_1), \ (H_A, B, N_2)\) and \((D_p, R, N_3)\) be any three CSFNSs over the same universe \(W\); then, the following properties hold:

1. \((F, Z, N_1) \cup_E (H_A, B, N_2) = (H_A, B, N_2) \cup_E (F, Z, N_1)\)
2. \((F, Z, N_1) \cup_R (H_A, B, N_2) = (H_A, B, N_2) \cup_R (F, Z, N_1)\)
3. \((F, Z, N_1) \cap_E (H_A, B, N_2) = (H_A, B, N_2) \cap_E (F, Z, N_1)\)
4. \((F, Z, N_1) \cap_R (H_A, B, N_2) = (H_A, B, N_2) \cap_R (F, Z, N_1)\)
5. \((F, Z, N_1) \cup_E (H_A, B, N_2) \cap_E (D_p, R, N_3) = (F, Z, N_1) \cup_E (H_A, B, N_2) \cup_E (D_p, R, N_3)\)
6. \((F, Z, N_1) \cup_R (H_A, B, N_2) \cup_R (D_p, R, N_3) = (F, Z, N_1) \cup_R (H_A, B, N_2) \cup_R (D_p, R, N_3)\)
7. \((F, Z, N_1) \cap_E (H_A, B, N_2) \cap_E (D_p, R, N_3) = (F, Z, N_1) \cap_E (H_A, B, N_2) \cap_E (D_p, R, N_3)\)
8. \((F, Z, N_1) \cap_R (H_A, B, N_2) \cap_R (D_p, R, N_3) = (F, Z, N_1) \cap_R (H_A, B, N_2) \cap_R (D_p, R, N_3)\)
9. \((F, Z, N_1) \cup_E (H_A, B, N_2) \cap_R (D_p, R, N_3) = (F, Z, N_1) \cup_E (H_A, B, N_2) \cap_R (D_p, R, N_3)\)
10. \((F, Z, N_1) \cup_R (H_A, B, N_2) \cap_R (D_p, R, N_3) = (F, Z, N_1) \cup_R (H_A, B, N_2) \cap_R (D_p, R, N_3)\)
Table 15: Tabular representation of extended union \((\Phi_X, C, 6)\).

| \((\Phi_X, C, 6)\) | \(z_1\) | \(z_2\) | \(z_3\) | \(z_6\) |
|---------------------|--------|--------|--------|--------|
| \(w_1\)            | (2, \((0.4e^{0.82\pi}, 0.017e^{0.036\pi}, 0.6e^{1.22\pi})\)) | (5, \((0.95e^{1.88\pi}, 0.018e^{0.033\pi}, 0.14e^{0.29\pi})\)) | (0, \((0.1e^{0.24\pi}, 0.04e^{0.084\pi}, 0.985e^{1.964\pi})\)) | (3, \((0.69e^{1.39\pi}, 0.04e^{0.084\pi}, 0.65e^{1.36\pi})\)) |
| \(w_2\)            | (0, \((0.03e^{1.1\pi}, 0.012e^{0.026\pi}, 0.98e^{1.962\pi})\)) | (3, \((0.7e^{1.42\pi}, 0.014e^{0.029\pi}, 0.3e^{0.56\pi})\)) | (1, \((0.2e^{0.36\pi}, 0.027e^{0.039\pi}, 0.91e^{1.824\pi})\)) | (4, \((0.87e^{1.72\pi}, 0.035e^{0.068\pi}, 0.93e^{1.862\pi})\)) |
| \(w_3\)            | (4, \((1, 0, 0)\)) | (3, \((0.6e^{1.22\pi}, 0.02e^{0.06\pi}, 0.49e^{2\pi})\)) | (3, \((0.69e^{1.84\pi}, 0.101e^{0.204\pi}, 0.32e^{0.62\pi})\)) | (1, \((0.17e^{0.344\pi}, 0.035e^{0.068\pi}, 0.93e^{1.862\pi})\)) |
Alternatives by the experts. Let \( x'_h \), and \( x'_e \) be the results of the top alternative and the best alternative, respectively.

According to the needs of MAGDM problems, set of alternatives is presented the corresponding set of a best alternative using both the positive ideal solution and the negative ideal solution. The elements and steps of an environment under such methodology are shown in Tables 16–19, respectively.

\[ F_{ij}(z) = \begin{cases} (\mu_{kj}(z), \eta_{kj}(z), \nu_{kj}(z)), \\ (0, 0, 1), \end{cases} \]

Example 12. Let \((E_j, Z, N)\) be a CSFS\(_f\)S given in Table 10. Then, CSFS\(_f\)S associated with the thresholds 1, 2, 3, and 4 are shown in Tables 16–19, respectively.

Definition 22. Let \((F, Z, N)\) be CSFS\(_f\)S, where \((F, Z, N)\) is NS\(_f\)S over the universe \(W\). Let \(0 < L < N\) and \(\alpha \in [1, 2, 3]\) be two thresholds. \(S_j\)S over \(W\) associated with \((F, Z, N)\) and \((L, \alpha)\), denoted by \((F^{(L, \alpha)}, Z)\), is defined as follows:

\[
\begin{align*}
\sigma_{T_{ij}} &= \left( g_j^i, \left[ 1 - (1 - p_{ij}^2)^\alpha \right] e^{2\pi \left[ 1 - \left( 1 - \delta_j^2 \right)^\alpha \right] ^\alpha}, v_{ij} e^{\lambda_1 \delta_j}, r_{ij} e^{\lambda_2 \delta_j} \right), \\
T_{ij}^\alpha &= \left( g_j^i, p_{ij}^\alpha e^{\lambda_1 \delta_j}, \left[ 1 - (1 - v_{ij}^2)^\alpha \right] e^{2\pi \left[ 1 - \left( 1 - \delta_j \right)^\alpha \right] ^\alpha}, \left[ 1 - (1 - r_{ij}^2)^\alpha \right] e^{2\pi \left[ 1 - \left( 1 - \delta_j \right)^\alpha \right] ^\alpha} \right), \\
\sigma_{T_j \otimes T_{kj}} &= \left( \max(g_j^i, g_k^j), \sqrt{\frac{p_{ij}^2 + p_{kj}^2}{p_{ij}^2}}, \left[ 1 - \left( 1 - v_{ij}^2 \right)^\alpha \right] e^{2\pi \left[ 1 - \left( 1 - \delta_j \right)^\alpha \right] ^\alpha}, v_{ij} e^{\lambda_1 \delta_j}, r_{ij} e^{\lambda_2 \delta_j} \right), \\
T_j \otimes T_{kj} &= \left( \min( g_j^i, g_k^j), p_{ij} p_{kj} e^{\lambda_1 \delta_j \delta_k}, \left[ 1 - \left( 1 - v_{ij}^2 \right)^\alpha \right] e^{2\pi \left[ 1 - \left( 1 - \delta_j \right)^\alpha \right] ^\alpha} v_{ij}, v_{kj} e^{\lambda_1 \delta_j \delta_k}, r_{ij} e^{\lambda_2 \delta_j \delta_k} \right).
\end{align*}
\]

4. CSFS\(_f\)-TOPSIS Method for MAGDM

In this section, we combine CSFS\(_f\)S with the TOPSIS method. The main idea of this methodology is the selection of a best alternative using both the positive ideal solution (PIS) and the negative ideal solution (NIS). Therefore, we present the corresponding CSFS\(_f\)-TOPSIS method in order to solve MAGDM problems in a CSFS\(_f\) environment under such methodology. The elements and steps of this algorithm for MAGDM are as follows.

Let \(W = \{w_1, w_2, w_3, \ldots, w_m\}\) denote the set of alternatives that are evaluated by \(s\) experts \(E_1, E_2, E_3, \ldots, E_s\). According to the needs of MAGDM problems, set of \(m\) attributes \(Z = \{z_1, z_2, z_3, \ldots, z_m\}\) are assigned to these alternatives by the experts. Let \(\sigma = (\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_s)^T\) be the weight vector, which represents the weightage of experts such that \(\sum_{d=1}^s \sigma_d = 1\), where \(\sigma_d \in [0, 1]\). The step by step Algorithm 1 of CSFS\(_f\)-TOPSIS method is presented in Section 5.1, and its theoretical description is as follows:

Step 1: According to the MAGDM problem and attributes related to the alternatives, each expert assigns ratings to them. There is a linguistic term corresponding with each rating, which could be a number of stars (such as “three stars,” “two stars,” and “one star” in MAGDM), numerical labels (such as 3 as a label for “high,” 2 for “medium,” and 0 for “low”). In such a way, \(NS_j S\) (\(F^d, Z, N\)) is found corresponding to each expert \(E_d\) with \(G = \{0, 1, 2, 3, \ldots, N - 1\}\) as the set of grades, where \(N \in \{1, 2, 3, \ldots, s\}\) and \(d \in \{1, 2, 3, \ldots, s\}\). Now, \(CSFS_j S\) is assigned by the \(d\)th expert \(E_d\), corresponding to each rank in the \(NS_j S\) (\(F^d, Z, N\)), according to the grading criteria defined for the MAGDM problem. Similarly, we get \(s\)CSFS\(_f\)Ss by \(s\) experts, respectively. The complex spherical fuzzy \(N\)-soft decision matrix \(CSFS_j DM\) of the \(d\)th expert \(E_d\) is as follows:
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\((E^1, Z, 5)\) & \(z_1\) & \(z_2\) & \(z_3\) \\
\hline
\(\omega_1\) & \(0.4e^{0.828}\) & \(0.017e^{0.0356}\) & \(0.6e^{1.222}\) \\
\(\omega_2\) & \((0, 0, 1)\) & \((0.7e^{1.424}, 0.019e^{0.046}, 0.3e^{0.565})\) & \((0.2e^{0.363}, 0.027e^{0.057}, 0.91e^{1.824})\) \\
\(\omega_3\) & \((1, 0, 0)\) & \((0.16e^{0.344}, 0.1e^{0.204}, 0.89e^{1.784})\) & \((0.69e^{1.384}, 0.101e^{0.204}, 0.32e^{0.628})\) \\
\hline
\end{tabular}
\caption{CSFS\(_f\)S related with \((E, Z, 5)\) and threshold 1.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\((E^2, Z, 5)\) & \(z_1\) & \(z_2\) & \(z_3\) \\
\hline
\(\omega_1\) & \(0.4e^{0.828}\) & \(0.017e^{0.0356}\) & \(0.6e^{1.222}\) \\
\(\omega_2\) & \((0, 0, 1)\) & \((0.7e^{1.424}, 0.019e^{0.046}, 0.3e^{0.565})\) & \((0.2e^{0.363}, 0.027e^{0.057}, 0.91e^{1.824})\) \\
\(\omega_3\) & \((1, 0, 0)\) & \((0, 0, 1)\) & \((0, 0, 1)\) \\
\hline
\end{tabular}
\caption{CSFS\(_f\)S related with \((E, Z, 5)\) and threshold 2.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\((E^3, Z, 5)\) & \(z_1\) & \(z_2\) & \(z_3\) \\
\hline
\(\omega_1\) & \(0, 0, 1\) & \((0.65e^{1.324}, 0.018e^{0.0368}, 0.28e^{0.586})\) & \((0, 0, 1)\) \\
\(\omega_2\) & \(0, 0, 1\) & \((0.7e^{1.424}, 0.019e^{0.046}, 0.3e^{0.565})\) & \((0, 0, 1)\) \\
\(\omega_3\) & \(1, 0, 0\) & \((0, 0, 1)\) & \((0, 0, 1)\) \\
\hline
\end{tabular}
\caption{CSFS\(_f\)S related with \((E, Z, 5)\) and threshold 3.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\((E^4, Z, 5)\) & \(z_1\) & \(z_2\) & \(z_3\) \\
\hline
\(\omega_1\) & \(0, 0, 1\) & \((0, 0, 1)\) & \((0, 0, 1)\) \\
\(\omega_2\) & \(0, 0, 1\) & \((0, 0, 1)\) & \((0, 0, 1)\) \\
\(\omega_3\) & \(1, 0, 0\) & \((0, 0, 1)\) & \((0, 0, 1)\) \\
\hline
\end{tabular}
\caption{CSFS\(_f\)S related with \((E, Z, 5)\) and threshold 4.}
\end{table}

\begin{equation}
\mathcal{P}^{(d)} = \begin{pmatrix}
(g_{11}^{1(d)}, p_{11}, \eta_{11}, \gamma_{11}) & (g_{12}^{1(d)}, p_{12}, \eta_{12}, \gamma_{12}) & \cdots & (g_{1m}^{1(d)}, p_{1m}, \eta_{1m}, \gamma_{1m}) \\
(g_{21}^{2(d)}, p_{21}, \eta_{21}, \gamma_{21}) & (g_{22}^{2(d)}, p_{22}, \eta_{22}, \gamma_{22}) & \cdots & (g_{2m}^{2(d)}, p_{2m}, \eta_{2m}, \gamma_{2m}) \\
\vdots & \vdots & \ddots & \vdots \\
(g_{q1}^{q(d)}, p_{q1}, \eta_{q1}, \gamma_{q1}) & (g_{q2}^{q(d)}, p_{q2}, \eta_{q2}, \gamma_{q2}) & \cdots & (g_{qm}^{q(d)}, p_{qm}, \eta_{qm}, \gamma_{qm}) 
\end{pmatrix}
\end{equation}

where \(\mathcal{P}^{(d)} = (g_{jk}^{(d)}, p_{jk}, \eta_{jk}, \gamma_{jk}) = (g_{j}^{(d)}, p_{j}^{e^{2\pi jk}}, \nu_{j}^{e^{2\pi jk}}, r_{jk}^{e^{2\pi jk}}, \cdots, \) \(j = 1, 2, 3, \ldots, q, \) \(k = 1, 2, 3, \ldots, m, \) and \(d = 1, 2, 3, \ldots, s.\)

Step 2: to formulate the aggregate complex spherical fuzzy \(N\)-soft decision matrix (ACSFS\(_f\)SDM), \(\mathcal{P} = (\mathcal{P}_{jk})_{q\times m},\) CSFS\(_f\)SDM of all experts are assembled with the help of CSFS\(_f\)WA operator. For ACSFS\(_f\)SDM, CSFS\(_f\)WA operator is defined as

\begin{equation}
\begin{aligned}
\mathcal{P}_{jk} &= \text{CSFS}_{j}^{WA} \left( \mathcal{P}_{jk}^{(1)}, \mathcal{P}_{jk}^{(2)}, \ldots, \mathcal{P}_{jk}^{(s)} \right) \\
&= \sigma_{j} \mathcal{P}_{jk}^{(1)} + \sigma_{j} \mathcal{P}_{jk}^{(2)} + \cdots + \sigma_{j} \mathcal{P}_{jk}^{(s)} \\
&= \left( \max_{\mathcal{P}^{(d)}} \left( \left( g_{j}^{(d)} \right), \sqrt{1 - \prod_{d}^{e^{2\pi d}} \left( 1 - \left( p_{j}^{e^{2\pi d}} \right) \right)^{\sigma_{d}}} e^{\frac{1}{2\pi} \int_{0}^{2\pi} \left( 1 - \left( q_{j}^{(d)} \right) \right) d\theta_{j}} \right), \prod_{d}^{e^{2\pi d}} p_{jk}^{e^{2\pi d}}, \prod_{d}^{e^{2\pi d}} \nu_{jk}^{e^{2\pi d}}, \prod_{d}^{e^{2\pi d}} r_{jk}^{e^{2\pi d}} \right)
\end{aligned}
\end{equation}
Using these entities, we can form ACSFNS/DM as

\[ \mathcal{P} = \begin{pmatrix}
(g_1, \mu_{11}, \eta_{11}, \gamma_{11}) & (g_2, \mu_{12}, \eta_{12}, \gamma_{12}) & \cdots & (g_m, \mu_{1m}, \eta_{1m}, \gamma_{1m}) \\
(g_1, \mu_{21}, \eta_{21}, \gamma_{21}) & (g_2, \mu_{22}, \eta_{22}, \gamma_{22}) & \cdots & (g_m, \mu_{2m}, \eta_{2m}, \gamma_{2m}) \\
\vdots & \vdots & \ddots & \vdots \\
(g_1, \mu_{q1}, \eta_{q1}, \gamma_{q1}) & (g_2, \mu_{q2}, \eta_{q2}, \gamma_{q2}) & \cdots & (g_m, \mu_{qm}, \eta_{qm}, \gamma_{qm})
\end{pmatrix}. \tag{25} \]

Step 3: in MAGDM problem, each attribute has its own worth. Therefore, each expert \( E_d \) assigns rank as weightage of each attribute \( z_k \) relative to their importance in MAGDM problem. Furthermore, CSFNS/\( N \)s are assigned to the weights, according to the grading criteria, by the experts. Let \( \chi_k = (g_k^{(d)}, P_k^{(d)} e^{2\pi n_k^{(d)}}, v_k^{(d)} e^{i2\pi k^{(d)}}, r_k^{(d)} e^{2\pi l_k^{(d)}}) \) be the weightage of \( k \)th attribute given by the \( d \)th expert. To find out the weight vector \( \chi = (\chi_1, \chi_2, \ldots, \chi_m)^T \), we aggregated them, as follows:

\[ \chi_k = \text{CSFNS}_{WA}(\chi_1^{(1)}, \chi_2^{(2)}, \ldots, \chi_m^{(s)}) \\
= \sigma_1 \chi_1^1 \varphi_1^1 \chi_2^2 \cdots \sigma_s \chi_s^s \\
= \left( \max_{d=1}^D \left( g_k^{(d)} \right) \right) \sqrt{1 - \prod_{i=1}^{d-1} \left( 1 - (p_k^{(d)})^2 \right)} e^{i2\pi \left( 1 - (\gamma_k^{(d)})^2 \right) / D} \prod_{i=1}^{d-1} \left( 1 - (\gamma_k^{(d)})^2 \right) / D, \prod_{i=1}^{d-1} \left( \pi \chi_k^{(d)} \right) \prod_{i=1}^{d-1} \left( \pi \chi_k^{(d)} \right) / D, \prod_{i=1}^{d-1} \left( \pi \chi_k^{(d)} \right) / D, \prod_{i=1}^{d-1} \left( \pi \chi_k^{(d)} \right) / D \right) \tag{26} \]

Step 4: calculate the aggregated weighted complex spherical fuzzy \( N \)-soft decision matrix (AWCSFNS/DM) using ACSFNS/DM \( \mathcal{P}_{jk} \) and the weight vector of attribute \( \chi_k \) as follows:

\[ \mathcal{P}_{jk} = \mathcal{P}_{jk} \otimes \chi_k \\
= \left( \min \left( g_k^{(d)}, g_k^{(d)} \right), P_k^{(d)} P_k^{(d)} e^{2\pi n_k^{(d)}}, v_k^{(d)} e^{i2\pi k^{(d)}}, r_k^{(d)} e^{i2\pi l_k^{(d)}} \right) \tag{27} \]

Using these entities, we can form AWCSFNS/DM as

\[ \overline{\mathcal{P}} = \begin{pmatrix}
(g_1, \mu_{11}, \eta_{11}, \gamma_{11}) & (g_1, \mu_{12}, \eta_{12}, \gamma_{12}) & \cdots & (g_1, \mu_{1m}, \eta_{1m}, \gamma_{1m}) \\
(g_2, \mu_{21}, \eta_{21}, \gamma_{21}) & (g_2, \mu_{22}, \eta_{22}, \gamma_{22}) & \cdots & (g_2, \mu_{2m}, \eta_{2m}, \gamma_{2m}) \\
\vdots & \vdots & \ddots & \vdots \\
(g_m, \mu_{q1}, \eta_{q1}, \gamma_{q1}) & (g_m, \mu_{q2}, \eta_{q2}, \gamma_{q2}) & \cdots & (g_m, \mu_{qm}, \eta_{qm}, \gamma_{qm})
\end{pmatrix}. \tag{28} \]

Step 5: let \( \mathcal{P}_B \) and \( \mathcal{P}_C \) be the collection of benefit-type attribute and cost-type attribute, respectively. CSFNS/\( P \)S related to the attribute \( z_k \) can be taken as follows:

\[ \tilde{\mathcal{P}}_k = \begin{cases}
\max_{j=1}^D \overline{\mathcal{P}}_{jk}, & \text{if } z_k \in \mathcal{P}_B, \\
\min_{j=1}^D \overline{\mathcal{P}}_{jk}, & \text{if } z_k \in \mathcal{P}_C. \tag{29}
\end{cases} \]
Now, CSFNS$_f$-NIS related to the attribute $z_k$ can be taken as follows:

$$\hat{\wp}_k = \begin{cases} \max_{j=1}^d \mathcal{F}_{jk}, & \text{if } z_k \in \mathcal{P}_C, \\ \min_{j=1}^d \mathcal{F}_{jk}, & \text{if } z_k \in \mathcal{P}_B. \end{cases}$$  \quad (30)

To evaluate $\max \mathcal{F}_{jk}$ and $\min \mathcal{F}_{jk}$, we use the score value and accuracy value of CSFNS$_f$-NIS. CSFNS$_f$-PIS and CSFNS$_f$-NIS are denoted as follows: $\hat{\wp}_k = (\hat{g}_k, \hat{\mu}_k, \hat{\eta}_k, \hat{v}_k) = (\hat{g}_k, \hat{p}_ke^{2\alpha \eta_k}, \hat{v}_ke^{2\alpha \eta_k}, \hat{r}_ke^{2\alpha \eta_k})$ and $\hat{\wp}_k = (\hat{g}_k, \hat{p}_k, \hat{\eta}_k, \hat{v}_k) = (\hat{g}_k, \hat{p}_ke^{2\alpha \eta_k}, \hat{v}_ke^{2\alpha \eta_k}, \hat{r}_ke^{2\alpha \eta_k})$, respectively.

Step 6: calculate the normalized Euclidean distance of each alternative $w_j$ from CSFNS$_f$-PIS and CSFNS$_f$-NIS. In this way, we get the best alternative that is nearer to CSFNS$_f$-PIS and far from CSFNS$_f$-NIS. The normalized Euclidean distance between CSFNS$_f$-PIS and any of the alternative $w_j$ can be formulated as follows:

$$d(\hat{\wp}_k, w_j) = \left( \frac{1}{4k} \sum_{k=1}^m \left[ \left( \frac{\hat{g}_k}{N-1} \right)^2 - \left( \frac{\mathcal{F}_{jk}}{N-1} \right)^2 + (\hat{p}_k - \mathcal{F}_{jk})^2 + (\hat{v}_k - \mathcal{F}_{jk})^2 + (\hat{r}_k - \mathcal{F}_{jk})^2 \right] \right)^{1/2}.$$  \quad (31)

Similarly, the normalized Euclidean distance between CSFNS$_f$-NIS and any of the alternative $w_j$, can be formulated as follows:

$$d(\hat{\wp}_k, w_j) = \left( \frac{1}{4k} \sum_{k=1}^m \left[ \left( \frac{\hat{g}_k}{N-1} \right)^2 - \left( \frac{\mathcal{F}_{jk}}{N-1} \right)^2 + (\hat{p}_k - \mathcal{F}_{jk})^2 + (\hat{v}_k - \mathcal{F}_{jk})^2 + (\hat{r}_k - \mathcal{F}_{jk})^2 \right] \right)^{1/2}.$$  \quad (32)

Step 7: to choose one of the most appropriate alternative, we have to use some ranking index. For this purpose, the revised closeness index corresponding to the alternative $w_k$ is evaluated using the formula [10]

$$\mathfrak{F}(w_j) = \frac{d(\hat{\wp}_k, w_j)}{\max_k d(\hat{\wp}_k, w_j)} - \frac{d(\hat{\wp}_k, w_j)}{\min_k d(\hat{\wp}_k, w_j)},$$  \quad (33)

where $k = 1, 2, \ldots, m$.

Step 8: the alternative with the minimum value of revised closeness index would be the best solution for the MAGDM problem. Therefore, the ascending order of the revised closeness index gives the ranking of the alternatives.

5. Development of Algorithms and Numerical Examples

In this section, we describe multiattribute decision-making (MADM) methods that work on models to identify the best alternative. Therefore, we characterize respective algorithms for the MADM problems in CSFNS$_f$ environment, as well as we present Algorithm 1 for CSFNS$_f$-TOPSIS method described in Section 4. Let $W = \{w_1, w_2, w_3, \ldots, w_m\}$ be a set, representing the available alternatives with a set of attributes $Z = \{z_1, z_2, z_3, \ldots, z_m\}$ having weight vector $\sigma = (\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_m)^T$ describing the worth of attributes according to the MADM problem, where $\sum_{k=1}^m \sigma_k = 1$ and $\sigma_k \in [0, 1]$.

The algorithm for CSFNS$_f$-TOPSIS method is described in Algorithm 1.

Let us now introduce some explicit MADM and MAGDM problems and solve them using Algorithms 1–4, respectively. We apply Algorithms 2–4 to solve the MADM problem defined in Section 5.1 and Algorithm 1 is used to solve the MAGDM problem defined in Section 5.2 which show their importance and feasibility in the field of decision-making.

5.1. Selection of Best Third-Party App of the Year. A third-party app is a software application made by someone other than the manufacturer of a mobile device or its operating system. This world is full of gadgets and gadgets are full of apps. We can access the world if we have these apps. Therefore, selecting one of the best third-party app of the
To find out the best app of the year, we will use http://www.trustraduis.com regarding to each third-party app. Keeping in view the priorities of people is a very difficult task. For this purpose, the data has been collected from the websites http://www.makeawebsitethub.com and http://www.trustraduis.com regarding to each third-party app. To find out the best app of the year, we will use CSFNS$_f$S.

Let $A = \{a_1 = \text{Facebook}, a_2 = \text{Skype}, a_3 = \text{Viber}, a_4 = \text{Twitter}, a_5 = \text{Whatsapp}\}$ be universe of third-party apps and $Z = \{z_1 = \text{telecom framework}, z_2 = \text{reliability}, z_3 = \text{worldwide contact}, z_4 = \text{data usage}\}$ be the attributes. According to these attributes, a 6-soft set is modeled in Table 20, where...
Algorithm 4: The algorithm of \( L \)-choice values of \( CSFNS/\Sigma \).

5.1.1. Choice Values of \( CSFNS/\Sigma \). The choice values of \( CSFNS/\Sigma \) is evaluated using the steps defined in Algorithm 2. Table 23 presents the calculated choice values of \( CSFNS/\Sigma \) for the selection of the third-party app. We can observe from Table 23 that, according to the choice values, the ranking of third-party apps is as follows: \( a_1 \) > \( a_4 \) > \( a_3 \) > \( a_2 \) > \( a_5 \), which shows that \( a_1 \) = Facebook has maximum choice value. Therefore, Facebook is selected as best third-party app of the year.

5.1.2. Weighted Choice Values of \( CSFNS/\Sigma \). Let \( \sigma_1 = 0.4, \sigma_2 = 0.3, \sigma_3 = 0.2, \) and \( \sigma_4 = 0.1 \) be the weights for each attribute \( z_k, k = 1, 2, 3, 4 \). Using these weights in Algorithm 3, we can compute weighted choice values of \( CSFNS/\Sigma \), which are given by Table 24.

It is clear from Table 24 that \( G_6 \) has maximum score; therefore, \( a_4 \) = Facebook is selected as best third-party app of the year. According to the weighted choice values, ranking of third-party apps is as follows: \( a_1 \) > \( a_4 \) > \( a_3 \) > \( a_2 \) > \( a_5 \).

5.1.3. \( L \)-Choice Values of \( CSFNS/\Sigma \). The \( L \)-choice values of \( CSFNS/\Sigma \) are evaluated using Algorithm 4 to find out the best alternative for the proposed MADM problem. Let \( L = 4 \) be threshold; then, \( 4 \)-choice values of \( CSFNS/\Sigma \) is shown in Table 25. We can observe that, from Table 25, the ranking of third-party apps according to \( 4 \)-choice values is as follows: \( a_1 \) > \( a_4 \) > \( a_5 \) > \( a_2 \) > \( a_3 \), which shows that \( a_1 \) = Facebook has maximum choice value so that Facebook is selected as the best third-party app of the year.

5.2. Selection of the Best Physiotherapist Doctor of Mayo Hospital in Lahore. Physiotherapy helps to restore movement and function when people are affected by injury or disability. A physiotherapist treats such kind of people and helps them through exercise, manual therapy, education, and advice. A physiotherapist is very helpful in maintaining the health of people of all ages as well as encourages them for happy life. A physiotherapist must have patience, communication skills, and ability to establish a good relationship with patients and their families. The motive of this study is to select the best physiotherapist doctor in Lahore relative to their attributes under the environment of \( CSFNS/\Sigma \). For this purpose, the data has been collected from the students of Mayo Hospital, Lahore, enact here as experts \( E_1, E_2, E_3 \), and \( E_4 \) whose weight vectors are \( \sigma = (0.4, 0.2, 0.1, 0.3)^T \). The following physiotherapists of Mayo Hospital are treated as alternatives in this MAGDM problem:

\[
\begin{align*}
\text{Dr. Amna} & : w_1 \\
\text{Dr. Rizwan} & : w_2 \\
\text{Dr. Akmal} & : w_3 \\
\text{Dr. Sidra} & : w_4 \\
\text{Dr. Saleem} & : w_5
\end{align*}
\]

Five attributes considered as key factors for a physiotherapist are as follows:

\[
\begin{align*}
z_1 : \text{knowledge and experience} & \\
z_2 : \text{behavioral (positivity, patience, and humbleness)} & \\
z_3 : \text{availability and flexibility} & \\
z_4 : \text{communication skills} & \\
z_5 : \text{ability to establish a good relationship} & 
\end{align*}
\]
We solve this MAGDM problem by following the CSFNS\_TOPSIS method.

Step 1: according to these attributes, each expert model 6-soft set is in Table 26, where

| Attribute | \( z_1 \) | \( z_2 \) | \( z_3 \) | \( z_4 \) |
|-----------|-----------|-----------|-----------|-----------|
| a_1       | 5         | 4         | 4         | 2         |
| a_2       | 3         | 2         | 3         | 3         |
| a_3       | 2         | 2         | 2         | 3         |
| a_4       | 2         | 3         | 5         | 4         |
| a_5       | 3         | 4         | 3         | 2         |

\( z_4 \): master of skills (communication, organizational, or problem-solving skills).

\( z_5 \): session fee.

Step 2: using equation (24), we can put together the opinions of all experts. ACSFNS\_DM formed by aggregation is given in Table 31.

Step 3: to demonstrate the importance of attributes in the MAGDM problem, experts rank them and associate CSFNS\_N to each attribute which are arranged in Table 32. We cumulated the weights given by experts using equation (26) to form CSFNS\_ weight vector \( \chi \) of attributes, i.e.,

\[
\chi = \begin{pmatrix}
4, (0.89e^{1.72x}, 0.017e^{0.034x}, 0.23e^{0.58x}) \\
4, (0.91e^{1.86x}, 0.016e^{0.034x}, 0.09e^{0.22x}) \\
3, (0.62e^{1.24x}, 0.016e^{0.028x}, 0.53e^{1.12x}) \\
3, (0.53e^{0.06x}, 0.02e^{0.04x}, 0.73e^{1.42x}) \\
2, (0.55e^{1.14x}, 0.019e^{0.042x}, 0.67e^{1.34x})
\end{pmatrix}

(34)

\[\text{Step 4: by utilizing } \text{ACSFNS}\_\text{DM and weight vector } \chi \text{ of attribute in equation (34), } \text{AWCSFNS}\_\text{DM is evaluated and summarized in Table 33.}\]

\[\text{Step 5: in the MAGDM problem, all the attributes' knowledge and experience, behavior, availability and flexibility, and master of skills are benefit-type attributes except the session fee, which is a cost-type attribute. According to the nature of attributes and applying equation (29) and (30), } \text{CSFNS}\_\text{PIS and } \text{CSFNS}\_\text{NIS are evaluated and arranged in Table 34.}\]

\[\text{Step 6: Table 35 represents the normalized Euclidean distance from each alternative to } \text{CSFNS}\_\text{PIS and } \text{CSFNS}\_\text{NIS using equations (31) and (32), respectively.}\]

\[\text{Step 7: the revised closeness index of each alternative is calculated by utilizing equation (33) and given in Table 36.}\]

\[\text{Step 8: since } w_1 \text{ has least revised closeness index, therefore, Dr. Amna is the best physiotherapist in Mayo Hospital, Lahore. The ranking of alternatives is shown in Table 37.}\]

### 6. Comparative Analysis

We now compare our proposed model with Pythagorean fuzzy N-soft set \( \text{PFNS}_N \) that was discussed by Zhang et al. [24].

(1) Table 38 represents the ratings of MADM problem as shown in Section 5.1. in \( \text{PFNS}_N \).

(2) Table 39 presents the calculated choice values of \( \text{PF6S}_N \) using the algorithm defined in [24] for the selection of the third-party app. Clearly, from Table 39, \( a_1 \) is the best choice, and the ranking of third-party apps is as follows: \( a_1 > a_4 > a_5 > a_2 > a_3 \).

(3) Let \( L = 4 \) be threshold; then, 4-choice values of \( \text{PFS}_N \) are shown in Table 40. The ranking of third-party apps according to 4-choice values is as follows:
Table 22: Tabular representation of the CSF6's (F, Z, 6).

| \( (F, Z, 6) \) | \( z_1 \) | \( z_2 \) | \( z_3 \) | \( z_4 \) |
|----------------|----------|----------|----------|----------|
| \( a_1 \)    | \((5, 0.95e^{1.88e^{0.024}}, 0.13e^{0.036}, 0.12e^{0.028})\) | \((4, 0.87e^{1.76e^{0.024}}, 0.01e^{0.024}, 0.21e^{0.38e^{0.024}})\) | \((4, 0.88e^{1.78e^{0.024}}, 0.012e^{0.026}, 0.2e^{0.026})\) | \((2, 0.48e^{0.98e^{0.024}}, 0.012e^{0.024}, 0.77e^{1.52e^{0.024}})\) |
| \( a_2 \)    | \((3, 0.76e^{0.04e^{0.024}}, 0.0173e^{0.034}, 0.49e^{0.034})\) | \((2, 0.32e^{0.06e^{0.024}}, 0.015e^{0.034}, 0.9e^{0.034})\) | \((3, 0.66e^{0.14e^{0.024}}, 0.081e^{0.034}, 0.69e^{0.13e^{0.024}})\) | \((3, 0.62e^{0.13e^{0.024}}, 0.079e^{0.013e^{0.024}}, 0.73e^{1.48e^{0.024}})\) |
| \( a_3 \)    | \((2, 0.47e^{0.04e^{0.024}}, 0.014e^{0.034}, 0.07e^{0.034})\) | \((2, 0.33e^{0.04e^{0.024}}, 0.016e^{0.028}, 0.89e^{0.028})\) | \((2, 0.35e^{0.07e^{0.024}}, 0.016e^{0.033}, 0.8e^{0.17e^{0.024}})\) | \((3, 0.53e^{0.11e^{0.024}}, 0.063e^{0.011e^{0.024}}, 0.67e^{0.38e^{0.024}})\) |
| \( a_4 \)    | \((2, 0.47e^{0.04e^{0.024}}, 0.016e^{0.034}, 0.09e^{0.034})\) | \((3, 0.72e^{0.14e^{0.024}}, 0.05e^{0.088e^{0.024}}, 0.59e^{0.088e^{0.024}})\) | \((5, 0.9e^{0.18e^{0.024}}, 0.08e^{0.158e^{0.024}}, 0.18e^{0.158e^{0.024}})\) | \((4, 0.77e^{0.13e^{0.024}}, 0.017e^{0.013e^{0.024}}, 0.35e^{0.66e^{0.024}})\) |
| \( a_5 \)    | \((3, 0.71e^{0.44e^{0.024}}, 0.0174e^{0.034}, 0.51e^{0.034})\) | \((4, 0.89e^{0.78e^{0.024}}, 0.013e^{0.027e^{0.024}}, 0.145e^{0.027e^{0.024}})\) | \((3, 0.55e^{0.12e^{0.024}}, 0.06e^{0.11e^{0.024}}, 0.7e^{0.14e^{0.024}})\) | \((2, 0.49e^{0.96e^{0.024}}, 0.011e^{0.024}, 0.76e^{0.54e^{0.024}})\) |
Table 23: Tabular representation of the choice values of \((F_j, Z, 6)\).

| \((F_j, Z, 6)\) | \(z_1\) | \(z_2\) | \(z_3\) | \(X_j\) | \(S_j\) |
|------------------|--------|--------|--------|--------|--------|
| \(a_1\)         | (5.092e+03, 0.0172e+04, 0.13e+04) | (4.082e+03, 0.015e+04, 0.21e+04) | (4.082e+03, 0.015e+04, 0.21e+04) | (5.092e+03, 0.0172e+04, 0.13e+04) | (5.092e+03, 0.0172e+04, 0.13e+04) |
| \(a_2\)         | (3.07e+03, 0.0172e+04, 0.49e+04) | (2.032e+03, 0.015e+04, 0.91e+04) | (3.07e+03, 0.0172e+04, 0.49e+04) | (2.032e+03, 0.015e+04, 0.91e+04) | (3.07e+03, 0.0172e+04, 0.49e+04) |
| \(a_3\)         | (2.041e+03, 0.014e+04, 0.772e+04) | (2.033e+03, 0.016e+04, 0.89e+04) | (2.033e+03, 0.016e+04, 0.89e+04) | (2.033e+03, 0.016e+04, 0.89e+04) | (2.033e+03, 0.016e+04, 0.89e+04) |
| \(a_4\)         | (2.047e+03, 0.016e+04, 0.936e+04) | (2.032e+03, 0.015e+04, 0.91e+04) | (2.032e+03, 0.015e+04, 0.91e+04) | (2.032e+03, 0.015e+04, 0.91e+04) | (2.032e+03, 0.015e+04, 0.91e+04) |
| \(a_5\)         | (3.072e+03, 0.0174e+04, 0.52e+04) | (4.082e+03, 0.015e+04, 0.14e+04) | (3.055e+03, 0.015e+04, 0.76e+04) | (4.082e+03, 0.015e+04, 0.14e+04) | (4.082e+03, 0.015e+04, 0.14e+04) |
Table 24: Tabular representation of the weighted choice values of \((F_j, Z, 6)\).

| \((F_j, Z, 6)\) | \(z_1\) | \(z_2\) | \(z_3\) | \(z_4\) | \(X_j\) | \(S_{X_j}\) |
|---------------|--------|--------|--------|--------|--------|----------|
| \(a_1\)      | (5, 0.95\(d_{1.36}\), 0.0172\(d_{0.0268}\), 0.13\(d_{0.78}\)) | (4, 0.88\(d_{1.36}\), 0.012\(d_{0.0268}\), 0.38\(d_{0.78}\)) | (2, 0.48\(d_{1.36}\), 0.012\(d_{0.0268}\), 0.77\(d_{0.78}\)) | (5, 0.013\(d_{1.36}\), 0.012\(d_{0.0268}\), 0.77\(d_{0.78}\)) | 2.2482  |
| \(a_2\)      | (3, 0.72\(d_{1.36}\), 0.0172\(d_{0.0268}\), 0.49\(d_{0.78}\)) | (4, 0.32\(d_{1.36}\), 0.015\(d_{0.0268}\), 0.96\(d_{0.78}\)) | (3, 0.62\(d_{1.36}\), 0.015\(d_{0.0268}\), 0.79\(d_{0.78}\)) | (5, 0.596\(d_{1.36}\), 0.026\(d_{0.0268}\), 0.65\(d_{0.78}\)) | 0.081   |
| \(a_3\)      | (2, 0.41\(d_{1.36}\), 0.014\(d_{0.0268}\), 0.77\(d_{0.78}\)) | (2, 0.35\(d_{1.36}\), 0.015\(d_{0.0268}\), 0.84\(d_{0.78}\)) | (3, 0.53\(d_{1.36}\), 0.015\(d_{0.0268}\), 0.67\(d_{0.78}\)) | (3, 0.392\(d_{1.36}\), 0.015\(d_{0.0268}\), 0.80\(d_{0.78}\)) | -0.7378 |
| \(a_4\)      | (3, 0.72\(d_{1.36}\), 0.016\(d_{0.0268}\), 0.94\(d_{0.78}\)) | (3, 0.72\(d_{1.36}\), 0.016\(d_{0.0268}\), 0.94\(d_{0.78}\)) | (5, 0.72\(d_{1.36}\), 0.017\(d_{0.0268}\), 0.72\(d_{0.78}\)) | (5, 0.72\(d_{1.36}\), 0.017\(d_{0.0268}\), 0.72\(d_{0.78}\)) | 1.185   |
| \(a_5\)      | (3, 0.72\(d_{1.36}\), 0.0174\(d_{0.0268}\), 0.51\(d_{0.78}\)) | (4, 0.88\(d_{1.36}\), 0.013\(d_{0.0268}\), 0.13\(d_{0.78}\)) | (3, 0.5\(d_{1.36}\), 0.011\(d_{0.0268}\), 0.76\(d_{0.78}\)) | (4, 0.757\(d_{1.36}\), 0.019\(d_{0.0268}\), 0.55\(d_{0.78}\)) | 1.123   |
Table 25: Tabular representation of the 4-choice values of \((F_j, Z, 6)\).

| \((F_j, Z, 6)\) | \(z_1\) | \(z_2\) | \(z_3\) | \(z_4\) | \(X_j\) | \(S_j\) |
|----------------|--------|--------|--------|--------|--------|--------|
| \(a_1\)       | \(0.95 e^{0.88 e^{0.017964}}\) | \(0.87 e^{0.10234 e^{0.11612 e^{0.89}}}\) | \(0.88 e^{0.10234 e^{0.11612 e^{0.89}}}\) | \(0.01\) | \(0.01\) | \(0.997 e^{0.19964 e^{0.11612 e^{0.89}}}\) | \(2.664 \times 10^{-5} e^{0.10234 e^{0.11612 e^{0.89}}}\) | \(5.46 \times 10^{-7} e^{0.10234 e^{0.11612 e^{0.89}}}\) | 1.989 |
| \(a_2\)       | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(-1\) |
| \(a_3\)       | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(-1\) |
| \(a_4\)       | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(-1\) |
| \(a_5\)       | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(0.01\) | \(-1\) |


Table 26: Experts’ rating according to attributes.

| Attributes | Alternatives | $\hat{E}_1$ | $\hat{E}_2$ | $\hat{E}_3$ | $\hat{E}_4$ |
|------------|--------------|-------------|-------------|-------------|-------------|
| $w_1$      | $**** = 4$   | $*** = 3$  | $*** = 3$  | $**** = 4$  | $**** = 4$  |
| $w_2$      | $*** = 3$   | $*** = 3$  | $*** = 3$  | $*** = 3$  | $*** = 3$  |
| $w_3$      | $* = 1$     | $* = 1$    | $* = 1$    | $* = 1$    | $* = 1$    |
| $w_4$      | $** = 2$    | $*** = 3$  | $** = 2$   | $*** = 4$  | $** = 2$   |
| $w_5$      | $** = 2$    | $* = 1$    | $** = 2$   | $* = 1$    | $** = 2$   |
| $w_6$      | $** = 3$    | $** = 2$   | $* = 1$    | $*** = 4$  | $*** = 3$  |
| $z_1$      | $* = 1$     | $* = 1$    | $* = 1$    | $* = 1$    | $* = 1$    |
| $z_2$      | $* = 1$     | $* = 1$    | $* = 1$    | $*** = 4$  | $*** = 4$  |
| $z_3$      | $* = 1$     | $* = 1$    | $* = 1$    | $*** = 4$  | $*** = 4$  |
| $z_4$      | $* = 1$     | $* = 1$    | $* = 1$    | $*** = 4$  | $*** = 4$  |
| $z_5$      | $* = 1$     | $* = 1$    | $* = 1$    | $*** = 4$  | $*** = 4$  |

$a_1 > a_4 > a_5 > a_2 > a_3$, which further shows that $a_1$ - Facebook has maximum choice value.

(4) We conclude the same results from both choice values and $L$-choice values of $PFNS_s$ [24], which shows the reliability of our proposed method, and it can be applied to any MADM problem.

(5) The data arranged in Table 23 is able to handle more real-life problems compared to Pythagorean $N$-soft set and intuitionistic $N$-soft set as it includes the neutral membership degree as well as it could deal with 2-dimensional data.

(6) The proposed model would provide the same results under spherical fuzzy $N$-soft environment by taking the periodic terms equal to zero.

6.1. Comparison with Complex Spherical Fuzzy TOPSIS Method. In this section, we solve the MAGDM problem “selection of best physiotherapist doctor of Mayo Hospital in Lahore” by complex spherical fuzzy TOPSIS method, proposed by Akram et al. [16], to demonstrate the importance and superiority of the proposed model. The solution by the complex spherical fuzzy TOPSIS method is as follows:

Step 1: the linguistic term corresponding to each rank assessed by the experts are the same as given in Table 26. To apply the CSF TOPSIS method, the grading part is excluded from $CSFNS_s N$ and CSFNs are assigned by each expert $\hat{E}_1$, $\hat{E}_2$, $\hat{E}_3$, and $\hat{E}_4$, which are arranged in Tables 41–44, respectively, according to the grading criteria defined in Table 3.

Step 2: using the weight vector of experts $\sigma = [0.4, 0.2, 0.1, 0.3]^T$ and complex spherical fuzzy weighted average (CSFWA) operator [16], we can calculate the aggregated complex spherical fuzzy decision matrix (ACSFDM), whose entries are evaluated by the formula defined as follows [16]:

$$P_{jk} = \left( \prod_{i=1}^{d_{ws}} \left( 1 - \left( p_{ik}^{(d)} \right)^2 \right) \right)^{1/2} \prod_{i=1}^{d_{ws}} \left( 1 - \left( q_{ik}^{(d)} \right)^2 \right) \prod_{i=1}^{d_{ws}} \exp \left( - \frac{1}{2\pi} \sum_{i=1}^{d_{ws}} \left( p_{ik}^{(d)} \right)^2 \right) \prod_{i=1}^{d_{ws}} \delta_{ik} \exp \left( - \frac{1}{2\pi} \sum_{i=1}^{d_{ws}} \left( q_{ik}^{(d)} \right)^2 \right) \prod_{i=1}^{d_{ws}} \lambda_{ik} \exp \left( - \frac{1}{2\pi} \sum_{i=1}^{d_{ws}} \left( r_{ik}^{(d)} \right)^2 \right) \prod_{i=1}^{d_{ws}} \delta_{ik} \exp \left( - \frac{1}{2\pi} \sum_{i=1}^{d_{ws}} \left( s_{ik}^{(d)} \right)^2 \right). \right)$$

ACSFDM is summarized in Table 45.

Step 3: the experts’ opinion about the importance of attributes are given in Table 46. The experts’ opinion are combined using (CSFWA) operator [16], to formulate
Table 27: Tabular representation of CSFNS,DM of expert $\bar{E}_1$.

| $(F_i^{[1]}, Z, S)$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ |
|----------------------|-------|-------|-------|-------|-------|
| $w_1$                | (4, (0.98e^{0.38}, 0.01e^{0.52}, 0.02e^{0.09})) | (4, (0.98e^{0.94}, 0.012e^{0.02}, 0.1e^{0.98})) | (4, (0.97e^{0.90}, 0.013e^{0.02}, 0.05e^{0.08})) | (3, (0.84e^{0.06}, 0.019e^{0.04}, 0.29e^{0.08})) | (4, (0.98e^{0.12}, 0.012e^{0.02}, 0.03e^{0.07})) |
| $w_2$                | (3, (0.84e^{0.66}, 0.02e^{0.02}, 0.29e^{0.02})) | (2, (0.37e^{0.78}, 0.015e^{0.02}, 0.87e^{0.17})) | (1, (0.16e^{0.34}, 0.018e^{0.08}, 0.89e^{0.78})) | (4, (0.96e^{0.94}, 0.013e^{0.02}, 0.06e^{0.06})) | (2, (0.36e^{0.74}, 0.016e^{0.02}, 0.59e^{0.17})) |
| $w_3$                | (1, (0.17e^{0.56}, 0.02e^{0.01}, 0.91e^{0.01})) | (1, (0.19e^{0.59}, 0.021e^{0.01}, 0.93e^{0.02})) | (2, (0.59e^{0.27}, 0.0155e^{0.02}, 0.82e^{0.09})) | (1, (0.2e^{0.38}, 0.022e^{0.02}, 0.92e^{0.08})) | (1, (0.21e^{0.01}, 0.024e^{0.01}, 0.93e^{0.08})) |
| $w_4$                | (2, (0.58e^{0.12}, 0.015e^{0.02}, 0.82e^{0.05})) | (3, (0.66e^{0.14}, 0.1e^{0.38}, 0.56e^{0.16})) | (3, (0.67e^{0.30}, 0.021e^{0.04}, 0.31e^{0.16})) | (2, (0.55e^{0.14}, 0.016e^{0.02}, 0.8e^{0.02})) | (3, (0.67e^{0.38}, 0.025e^{0.04}, 0.33e^{0.08})) |
| $w_5$                | (2, (0.52e^{0.12}, 0.012e^{0.02}, 0.74e^{0.15})) | (1, (0.21e^{0.44}, 0.02e^{0.06}, 0.95e^{0.98})) | (1, (0.7e^{0.30}, 0.026e^{0.02}, 0.35e^{0.68})) | (1, (0.16e^{0.42}, 0.018e^{0.03}, 0.89e^{0.98})) | (2, (0.58e^{0.14}, 0.016e^{0.02}, 0.82e^{0.33})) |
Table 28: Tabular representation of CSFNS DM of expert $\tilde{E}_2$.

| $\bar{P}_{21}$, $\bar{Z}_5$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ |
|---------------------------|-------|-------|-------|-------|-------|
| $w_1$ (3, 0.63, 0.11, 0.79) | (4, 0.96, 0.13, 0.01) | (4, 0.98, 0.13, 0.02) | (4, 0.98, 0.13, 0.02) | (4, 0.98, 0.13, 0.02) |
| $w_2$ (3, 0.72, 0.30, 0.16) | (3, 0.72, 0.30, 0.16) | (2, 0.57, 0.30, 0.16) | (2, 0.57, 0.30, 0.16) |
| $w_3$ (3, 0.21, 0.30, 0.16) | (3, 0.21, 0.30, 0.16) | (3, 0.21, 0.30, 0.16) |
| $w_4$ (3, 0.54, 0.30, 0.16) | (3, 0.54, 0.30, 0.16) |
| $w_5$ (1, 0.31, 0.30, 0.16) | (2, 0.52, 0.30, 0.16) | (1, 0.23, 0.30, 0.16) | (1, 0.23, 0.30, 0.16) |

Note: The table entries are in the form $(\text{Column 1}, \text{Column 2}, \text{Column 3}, \text{Column 4}, \text{Column 5})$. The values represent the weights assigned to different attributes or criteria under the expert's decision-making process.
Table 29: Tabular representation of $CSFNS_{j}DM$ of expert $E_{3}$.

|                                                                 | $z_{1}$                      | $z_{2}$                      | $z_{3}$                      | $z_{4}$                      | $z_{5}$                      |
|-----------------------------------------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|
| $w_{1}$                                                         | $(3, 0.84e^{1.068}, 0.019e^{0.008}, 0.29e^{0.089})$ | $(4, 0.97e^{1.088}, 0.013e^{0.208}, 0.14e^{0.028})$ | $(4, 0.96e^{1.088}, 0.014e^{0.098}, 0.04e^{0.019})$ | $(2, 0.5e^{1.928}, 0.014e^{0.039}, 0.74e^{1.908})$ | $(3, 0.8e^{1.628}, 0.018e^{0.388}, 0.29e^{0.018})$ |
| $w_{2}$                                                         | $(3, 0.72e^{1.069}, 0.036e^{0.088}, 0.41e^{0.089})$ | $(2, 0.49e^{1.488}, 0.013e^{0.028}, 0.71e^{1.488})$ | $(2, 0.46e^{0.098}, 0.015e^{0.013}, 0.7e^{0.138})$ | $(3, 0.74e^{1.488}, 0.034e^{0.107}, 0.42e^{0.028})$ | $(3, 0.75e^{1.488}, 0.035e^{0.078}, 0.43e^{0.028})$ |
| $w_{3}$                                                         | $(2, 0.47e^{1.068}, 0.015e^{0.034}, 0.72e^{1.036})$ | $(1, 0.27e^{1.538}, 0.034e^{0.088}, 0.94e^{0.927})$ | $(2, 0.46e^{0.928}, 0.014e^{0.013}, 0.48e^{1.308})$ | $(4, 0.88e^{1.768}, 0.016e^{0.028}, 0.25e^{0.918})$ | $(1, 0.29e^{0.568}, 0.04e^{0.078}, 0.96e^{1.497})$ |
| $w_{4}$                                                         | $(4, 0.98e^{1.948}, 0.015e^{0.034}, 0.1e^{2.334})$  | $(3, 0.76e^{1.358}, 0.036e^{0.074}, 0.44e^{0.908})$ | $(3, 0.77e^{0.128}, 0.015e^{0.013}, 0.98e^{0.927})$ | $(3, 0.77e^{0.128}, 0.038e^{0.028}, 0.45e^{0.818})$ | $(1, 0.3e^{0.588}, 0.042e^{0.028}, 0.925e^{0.907})$ |
| $w_{5}$                                                         | $(2, 0.44e^{1.928}, 0.012e^{0.028}, 0.66e^{1.138})$ | $(2, 0.43e^{0.188}, 0.015e^{0.208}, 0.66e^{1.138})$ | $(4, 0.87e^{1.388}, 0.016e^{0.028}, 0.21e^{0.438})$ | $(2, 0.42e^{0.988}, 0.012e^{0.012}, 0.65e^{1.128})$ | $(3, 0.78e^{1.548}, 0.04e^{0.078}, 0.46e^{0.918})$ |
| $(\mathcal{F}_{i}^{0}, \mathcal{Z}, \mathcal{S})$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ |
|----------------|------|------|------|------|------|
| $w_1$           | $(4, 0.97 e^{1.94}, 0.01 e^{0.024}, 0.13 e^{0.39})$ | $(3, 0.8 e^{1.62}, 0.018 e^{0.08}, 0.31 e^{0.69})$ | $(4, 0.97 e^{1.94}, 0.01 e^{0.024}, 0.13 e^{0.39})$ | $(2, 0.63 e^{1.29}, 0.012 e^{0.024}, 0.64 e^{1.18})$ | $(2, 0.64 e^{1.29}, 0.012 e^{0.024}, 0.59 e^{1.08})$ |
| $w_2$           | $(3, 0.77 e^{1.58}, 0.041 e^{0.08}, 0.47 e^{0.32})$ | $(1, 0.31 e^{0.64}, 0.022 e^{0.08}, 0.89 e^{1.48})$ | $(0, 0.06 e^{0.14}, 0.012 e^{0.024}, 0.98 e^{0.97})$ | $(3, 0.8 e^{1.04}, 0.05 e^{0.024}, 0.48 e^{0.94})$ | $(2, 0.37 e^{1.78}, 0.014 e^{0.024}, 0.86 e^{1.74})$ |
| $w_3$           | $(1, 0.3 e^{0.64}, 0.05 e^{0.024}, 0.91 e^{1.18})$ | $(2, 0.41 e^{0.41}, 0.01 e^{0.001}, 0.64 e^{1.13})$ | $(2, 0.4 e^{0.41}, 0.01 e^{0.001}, 0.63 e^{1.20})$ | $(2, 0.3 e^{0.41}, 0.01 e^{0.001}, 0.62 e^{1.20})$ | $(1, 0.3 e^{0.41}, 0.01 e^{0.001}, 0.93 e^{1.08})$ |
| $w_4$           | $(2, 0.38 e^{0.28}, 0.01 e^{0.024}, 0.61 e^{1.28})$ | $(4, 0.9 e^{1.84}, 0.014 e^{0.024}, 0.19 e^{0.97})$ | $(2, 0.3 e^{0.64}, 0.012 e^{0.024}, 0.6 e^{1.22})$ | $(3, 0.8 e^{1.28}, 0.05 e^{0.024}, 0.48 e^{0.85})$ | $(0, 0.1 e^{0.28}, 0.01 e^{0.024}, 0.98 e^{1.08})$ |
| $w_5$           | $(0, 0.07 e^{0.12}, 0.01 e^{0.024}, 0.99 e^{0.88})$ | $(1, 0.3 e^{0.64}, 0.06 e^{0.024}, 0.95 e^{1.23})$ | $(1, 0.3 e^{0.64}, 0.01 e^{0.024}, 0.95 e^{1.23})$ | $(1, 0.3 e^{0.64}, 0.01 e^{0.024}, 0.95 e^{1.23})$ | $(1, 0.2 e^{0.64}, 0.02 e^{0.024}, 0.94 e^{1.09})$ |
Table 31: Tabular representation of $ACSFNS_fDM$.

| $(R_f^{[1]}; Z, S)$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ |
|---------------------|-------|-------|-------|-------|-------|
| $w_1$               | $(4, 0.90e+00, 0.013e+00, 0.07e+00)$ | $(4, 0.97e+00, 0.013e+00, 0.15e+00)$ | $(4, 0.97e+00, 0.013e+00, 0.05e+00)$ | $(3, 0.77e+00, 0.016e+00, 0.04e+00)$ | $(4, 0.91e+00, 0.013e+00, 0.15e+00)$ |
| $w_2$               | $(3, 0.79e+00, 0.028e+00, 0.36e+00)$ | $(3, 0.48e+00, 0.019e+00, 0.75e+00)$ | $(2, 0.33e+00, 0.013e+00, 0.88e+00)$ | $(4, 0.98e+00, 0.020e+00, 0.15e+00)$ | $(3, 0.47e+00, 0.016e+00, 0.68e+00)$ |
| $w_3$               | $(2, 0.25e+00, 0.029e+00, 0.95e+00)$ | $(2, 0.29e+00, 0.018e+00, 0.83e+00)$ | $(2, 0.55e+00, 0.020e+00, 0.72e+00)$ | $(4, 0.55e+00, 0.020e+00, 0.68e+00)$ | $(2, 0.35e+00, 0.020e+00, 0.90e+00)$ |
| $w_4$               | $(4, 0.76e+00, 0.016e+00, 0.63e+00)$ | $(4, 0.77e+00, 0.034e+00, 0.42e+00)$ | $(3, 0.51e+00, 0.013e+00, 0.53e+00)$ | $(3, 0.71e+00, 0.026e+00, 0.55e+00)$ | $(3, 0.52e+00, 0.019e+00, 0.66e+00)$ |
| $w_5$               | $(2, 0.35e+00, 0.015e+00, 0.83e+00)$ | $(2, 0.35e+00, 0.025e+00, 0.87e+00)$ | $(4, 0.63e+00, 0.020e+00, 0.51e+00)$ | $(2, 0.29e+00, 0.016e+00, 0.76e+00)$ | $(3, 0.48e+00, 0.013e+00, 0.83e+00)$ |
Table 32: Experts opinion related to each attribute.

| $(F_j, Z, 6)$ | $E_1$ | $E_2$ | $E_3$ | $E_4$ |
|---------------|-------|-------|-------|-------|
| $z_1$         | $(3, (0.85e^{1.68\pi}, 0.019e^{0.042\pi}, 0.29e^{0.61\pi}))$ | $(4, (0.99e^{1.94\pi}, 0.012e^{0.021\pi}, 0.1e^{0.26\pi}))$ | $(4, (0.88e^{1.77\pi}, 0.015e^{0.032\pi}, 0.25e^{0.46\pi}))$ | $(3, (0.67e^{1.36\pi}, 0.021e^{0.044\pi}, 0.31e^{0.64\pi}))$ |
| $z_2$         | $(3, (0.66e^{1.34\pi}, 0.021e^{0.044\pi}, 0.33e^{0.61\pi}))$ | $(4, (0.96e^{1.94\pi}, 0.013e^{0.024\pi}, 0.04e^{0.066\pi}))$ | $(3, (0.78e^{1.54\pi}, 0.04e^{0.076\pi}, 0.46e^{0.96\pi}))$ | $(4, (0.98e^{1.98\pi}, 0.01e^{0.034\pi}, 0.02e^{0.066\pi}))$ |
| $z_3$         | $(2, (0.63e^{1.28\pi}, 0.012e^{0.032\pi}, 0.59e^{1.37\pi}))$ | $(3, (0.72e^{1.42\pi}, 0.03e^{0.066\pi}, 0.38e^{0.78\pi}))$ | $(3, (0.75e^{1.48\pi}, 0.035e^{0.072\pi}, 0.43e^{0.82\pi}))$ | $(2, (0.42e^{0.86\pi}, 0.012e^{0.032\pi}, 0.65e^{1.32\pi}))$ |
| $z_4$         | $(1, (0.29e^{0.68\pi}, 0.014e^{0.031\pi}, 0.94e^{1.86\pi}))$ | $(1, (0.21e^{0.46\pi}, 0.024e^{0.052\pi}, 0.94e^{1.86\pi}))$ | $(2, (0.37e^{0.76\pi}, 0.012e^{0.026\pi}, 0.66e^{1.22\pi}))$ | $(3, (0.78e^{1.54\pi}, 0.04e^{0.076\pi}, 0.46e^{0.96\pi}))$ |
| $z_5$         | $(2, (0.62e^{1.28\pi}, 0.01e^{0.024\pi}, 0.58e^{1.2\pi}))$ | $(2, (0.66e^{1.34\pi}, 0.1e^{0.18\pi}, 0.56e^{1.14\pi}))$ | $(2, (0.64e^{1.26\pi}, 0.01e^{0.024\pi}, 0.59e^{1.2\pi}))$ | $(1, (0.21e^{0.41\pi}, 0.02e^{0.046\pi}, 0.95e^{1.88\pi}))$ |
### Table 33: Tabular representation of $AWCSFNS_{DM}$.

| $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ |
|-------|-------|-------|-------|-------|
| $(4.085 e^{1.665} , 0.021 e^{0.024} , 0.23 e^{0.325})$ | $(4.088 e^{1.765} , 0.021 e^{0.024} , 0.174 e^{0.385})$ | $(4.069 e^{1.225} , 0.019 e^{0.345} , 0.054 e^{1.115})$ | $(3.042 e^{0.305} , 0.025 e^{0.054} , 0.77 e^{1.545})$ | $(3.062 e^{1.685} , 0.025 e^{0.054} , 0.678 e^{1.305})$ |
| $(3.07 e^{1.345} , 0.03 e^{0.065} , 0.41 e^{0.375})$ | $(3.048 e^{0.915} , 0.02 e^{0.034} , 0.73 e^{1.465})$ | $(3.021 e^{0.345} , 0.02 e^{0.034} , 0.91 e^{1.865})$ | $(3.042 e^{0.035} , 0.028 e^{0.054} , 0.73 e^{1.425})$ | $(3.025 e^{0.035} , 0.025 e^{0.054} , 0.83 e^{1.065})$ |
| $(2.022 e^{0.445} , 0.03 e^{0.065} , 0.91 e^{1.345})$ | $(2.026 e^{0.545} , 0.02 e^{0.034} , 0.83 e^{1.345})$ | $(2.032 e^{0.245} , 0.02 e^{0.034} , 0.91 e^{1.625})$ | $(2.029 e^{0.035} , 0.035 e^{0.054} , 0.73 e^{1.425})$ | $(2.029 e^{0.035} , 0.035 e^{0.054} , 0.93 e^{1.065})$ |
| $(4.062 e^{0.025} , 0.03 e^{0.065} , 0.66 e^{1.345})$ | $(4.07 e^{0.045} , 0.037 e^{0.025} , 0.43 e^{1.345})$ | $(3.032 e^{0.005} , 0.019 e^{0.004} , 0.66 e^{1.425})$ | $(3.07 e^{0.035} , 0.03 e^{0.065} , 0.82 e^{1.625})$ | $(2.029 e^{0.035} , 0.027 e^{0.065} , 0.8 e^{1.625})$ |
| $(2.035 e^{0.625} , 0.02 e^{0.085} , 0.83 e^{1.75})$ | $(2.031 e^{0.055} , 0.029 e^{0.065} , 0.87 e^{1.75})$ | $(3.03 e^{0.075} , 0.02 e^{0.095} , 0.68 e^{0.38})$ | $(2.015 e^{0.315} , 0.025 e^{0.035} , 0.89 e^{1.85})$ | $(2.027 e^{0.355} , 0.022 e^{0.025} , 0.91 e^{1.85})$ |
the weight vector \( \chi \) for the attributes, and are defined as follows:

\[
\chi_k = \left( \frac{1}{2\pi} \int_{\Delta_{d+1}} \frac{1}{\Delta_{d+1}} \prod_{i=1}^{d+1} \left( 1 - \left( \mathcal{P}_{k}^{(d)} \right)^{1/\sigma_d} \right)^{1/\sigma_d} \right) \right. 
\]

Thus, we have

\[
\chi = \left( \frac{0.89e^{i1.72\pi}}{0.91e^{i1.86\pi}}, 0.017e^{i0.034\pi}, 0.23e^{i0.51\pi} \right) 
\]

Step 4: the aggregated weighted complex spherical decision matrix (AWCSDFM) is arranged in Table 47, where the entries of AWCSDFM are calculated using the formula [16]

\[
\overline{\mathcal{P}}_{jk} = \left( p_{jk} p_k e^{i2\pi \theta_{jk}}, \sqrt{2} v_{jk} + v_k - v_{jk} v_k e^{i2\pi \theta_{jk} \delta_{jk} \delta_k}, \sqrt{2} v_{jk} + v_k - v_{jk} v_k e^{i2\pi \theta_{jk} \delta_{jk} \delta_k} \right). 
\]

Step 5: to compute the complex spherical fuzzy positive ideal solution (CSF-PIS) and negative ideal solution (CSF-NIS), we evaluate the score degree of all CSFNs in AWCSDFM, using the formula

\[
\text{Sc}(\overline{\mathcal{P}}_{jk}) = \left( p_{jk}^2 - w_{jk}^2 - r_{jk}^2 \right) + \left[ \delta_{jk}^2 - \delta_{jk} - \lambda_{jk}^3 \right]. \tag{39}
\]
### Table 38: Tabular representation of the $PF6S_j$S defined from the problem proposed in Section 5.1.

| $z_1$  | $z_2$  | $z_3$  | $z_4$  |
|-------|-------|-------|-------|
| $\alpha_1$ | (5, (0.95, 0.13)) | (4, (0.87, 0.21)) | (4, (0.88, 0.2)) | (2, (0.48, 0.77)) |
| $\alpha_2$ | (3, (0.7, 0.49)) | (2, (0.32, 0.9)) | (3, (0.6, 0.69)) | (3, (0.62, 0.73)) |
| $\alpha_3$ | (2, (0.41, 0.77)) | (2, (0.35, 0.89)) | (2, (0.35, 0.84)) | (3, (0.53, 0.67)) |
| $\alpha_4$ | (2, (0.4, 0.9)) | (3, (0.72, 0.59)) | (5, (0.9, 0.18)) | (4, (0.77, 0.35)) |
| $\alpha_5$ | (3, (0.71, 0.51)) | (4, (0.89, 0.145)) | (3, (0.55, 0.7)) | (2, (0.49, 0.76)) |

### Table 39: Tabular representation of Choice value of $PF6S_j$S in Section 5.1.

| $z_1$  | $z_2$  | $z_3$  | $z_4$  | $H_i$ |
|-------|-------|-------|-------|-------|
| $\alpha_1$ | (5, (0.95, 0.13)) | (4, (0.87, 0.21)) | (4, (0.88, 0.2)) | (2, (0.48, 0.77)) | (15, 2.83162) |
| $\alpha_2$ | (3, (0.7, 0.49)) | (2, (0.32, 0.9)) | (3, (0.6, 0.69)) | (3, (0.62, 0.73)) | (11, 1.6754702) |
| $\alpha_3$ | (2, (0.41, 0.77)) | (2, (0.33, 0.89)) | (2, (0.35, 0.84)) | (3, (0.53, 0.67)) | (9, 1.3498273) |
| $\alpha_4$ | (2, (0.4, 0.9)) | (3, (0.72, 0.59)) | (5, (0.9, 0.18)) | (4, (0.77, 0.35)) | (14, 2.30656384) |
| $\alpha_5$ | (3, (0.71, 0.51)) | (4, (0.89, 0.145)) | (3, (0.55, 0.7)) | (2, (0.49, 0.76)) | (12, 2.231012) |

### Table 40: Tabular representation of 4 choice value of $PF6S_j$S Section 5.1.

| $z_1$  | $z_2$  | $z_3$  | $z_4$  | $H_i$ |
|-------|-------|-------|-------|-------|
| $\alpha_1$ | (0.95, 0.13) | (0.87, 0.21) | (0.88, 0.2) | (0.1) | 0.33532 |
| $\alpha_2$ | (0, 0.5) | (0, 1) | (0.5) | (0.5) | -0.4375 |
| $\alpha_3$ | (0, 1) | (0, 1) | (0, 1) | (0.5) | -0.8125 |
| $\alpha_4$ | (0, 1) | (0, 0.5) | (0.9, 0.18) | (0.77, 0.35) | -0.0005 |
| $\alpha_5$ | (0, 0.5) | (0.89, 0.145) | (0.5) | (0.1) | -0.18223125 |

The normalized Euclidean distance between the CSF-NIS and any of the alternative $w_j$ can be formulated as follows:

$$d(\overline{P}_k, w_j) = \left( \frac{1}{3k} \sum_{k=1}^{m} \left[ (\overline{p}_k - \overline{v}_j)^2 + (\overline{r}_k - \overline{v}_j)^2 + (\overline{\phi}_k - \overline{\phi}_j)^2 + (\overline{\delta}_k - \overline{\delta}_j)^2 + (\overline{\lambda}_k - \overline{\lambda}_j)^2 \right] \right)^{1/2}.$$  

Similarly, the normalized Euclidean distance between the CSF-NIS and any of the alternative $w_j$ can be formulated as follows:

$$d(\overline{P}_k, w_j) = \left( \frac{1}{3k} \sum_{k=1}^{m} \left[ (\overline{p}_k - \overline{v}_j)^2 + (\overline{r}_k - \overline{v}_j)^2 + (\overline{\phi}_k - \overline{\phi}_j)^2 + (\overline{\delta}_k - \overline{\delta}_j)^2 + (\overline{\lambda}_k - \overline{\lambda}_j)^2 \right] \right)^{1/2}.$$  

Step 7: Equation (33) is used to calculate the revised closeness index of each alternative, as given in Table 50.

Step 8: revised closeness index in Table 50 reveals that $w_1$ is the best alternative within the ranking $w_1 > w_4 > w_2 > w_5 > w_3$.

### 6.1.1 Discussion

(1) We now compare the proposed model CSFNS$_f$-TOPSIS method with the existing technique CSF-TOPSIS method to evaluate the accuracy of the result. The same result concludes from both methods as well as the ranking of the alternatives also same.

(2) We also apply technique on SF-TOPSIS methods [10, 11], to select the most appropriate physiotherapist. The same results including the ranking and best solution are organized in Table 51, which enhance the credibility of the proposed method.

(3) The proposed model deals not only with 2-dimensional uncertainties but also with the level of attribute for the alternative. The existing models are unable to handle MAGDM problems, but the proposed model has the ability to tackle those real-life DM problems having ranking system and parameterized information.

(4) The proposed model, CSFNS$_f$-TOPSIS method, could be efficiently applied to the environment of...
| $\omega_1$ | 0.98e$^{1.98}$ | 0.01e$^{0.02}$ | 0.02e$^{0.06}$ | (0.99e$^{1.94}$ | 0.012e$^{0.02}$ | 0.1e$^{0.26}$) | (0.97e$^{1.96}$ | 0.013e$^{0.02}$ | 0.05e$^{0.08}$) | (0.84e$^{1.66}$ | 0.019e$^{0.04}$ | 0.29e$^{0.66}$) | (0.98e$^{1.92}$ | 0.012e$^{0.03}$ | 0.03e$^{0.01}$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\omega_2$ | 0.84e$^{1.66}$ | 0.02e$^{0.04}$ | 0.29e$^{0.62}$ | (0.37e$^{0.78}$ | 0.015e$^{0.02}$ | 0.87e$^{1.72}$) | (0.16e$^{0.34}$ | 0.018e$^{0.03}$ | 0.89e$^{1.78}$) | (0.96e$^{1.94}$ | 0.013e$^{0.02}$ | 0.04e$^{0.06}$) | (0.36e$^{0.74}$ | 0.016e$^{0.03}$ | 0.59e$^{1.12}$) |
| $\omega_3$ | 0.17e$^{0.36}$ | 0.06e$^{0.03}$ | 0.09e$^{0.14}$ | (0.19e$^{0.36}$ | 0.021e$^{0.04}$ | 0.93e$^{1.42}$) | (0.59e$^{0.12}$ | 0.015e$^{0.03}$ | 0.82e$^{1.68}$) | (0.2e$^{0.38}$ | 0.022e$^{0.01}$ | 0.92e$^{1.86}$) | (0.21e$^{0.44}$ | 0.024e$^{0.05}$ | 0.93e$^{1.88}$) |
| $\omega_4$ | 0.58e$^{1.12}$ | 0.015e$^{0.05}$ | 0.82e$^{1.66}$ | (0.66e$^{1.34}$ | 0.1e$^{0.18}$ | 0.56e$^{1.18}$) | (0.67e$^{1.36}$ | 0.021e$^{0.04}$ | 0.31e$^{0.64}$) | (0.55e$^{0.14}$ | 0.014e$^{0.02}$ | 0.8e$^{1.62}$) | (0.67e$^{1.38}$ | 0.025e$^{0.04}$ | 0.33e$^{0.68}$) |
| $\omega_5$ | 0.52e$^{1.02}$ | 0.012e$^{0.02}$ | 0.74e$^{1.5}$ | (0.21e$^{0.84}$ | 0.021e$^{0.04}$ | 0.95e$^{0.88}$) | (0.7e$^{1.36}$ | 0.026e$^{0.05}$ | 0.35e$^{0.68}$) | (0.16e$^{0.42}$ | 0.018e$^{0.03}$ | 0.89e$^{1.88}$) | (0.58e$^{1.14}$ | 0.012e$^{0.02}$ | 0.82e$^{1.62}$) |

Table 41: Tabular representation of CSFD of expert $E_1$. 
| $w_i$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ |
|------|------|------|------|------|------|
| $w_1$ | (0.83e0.644, 0.019e0.036, 0.3xe0.584) | (0.98e1.984, 0.01e0.024, 0.1xe0.264) | (0.98e1.984, 0.01e0.024, 0.1xe0.264) | (0.84e1.664, 0.02e0.036, 0.31e0.634) | (0.84e1.664, 0.02e0.036, 0.31e0.634) |
| $w_2$ | (0.71e1.384, 0.028e0.058, 0.37e0.724) | (0.72e1.424, 0.03e1.024, 0.38e0.764) | (0.5e1.024, 0.01e0.024, 0.7e1.484) | (0.86e1.246, 0.0169e0.036, 0.027e0.024) | (2, 0.56e1.116, 0.0150e0.024, 0.8e1.384) |
| $w_3$ | (0.55e0.016, 0.01e0.064, 0.98e1.972) | (0.21e0.394, 0.024e0.521, 0.94e1.864) | (0.5e1.024, 0.01e0.024, 0.7e1.484) | (0.73e1.486, 0.032e0.024, 0.39e0.566) | (2, 0.55e1.068, 0.01e0.024, 0.79e1.566) |
| $w_4$ | (0.74e1.54, 0.035e0.006, 0.4e0.784) | (0.54e1.34, 0.015e0.024, 0.78e0.846) | (0.71e1.538, 0.028e0.024, 0.37e0.522) | (2, 0.53e1.040, 0.01e0.024, 0.77e1.522) | (2, 0.53e1.104, 0.01e0.024, 0.77e1.522) |
| $w_5$ | (0.26e0.625, 0.028e0.052, 0.91e0.846) | (0.51e1.094, 0.01e0.064, 0.76e1.574) | (0.52e1.026, 0.013e0.016, 0.75e1.520) | (0.23e0.55, 0.035e0.006, 0.89e0.826) | (0.1e0.518, 0.012e0.026, 0.99e0.872) |
| $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ |
|-------|-------|-------|-------|-------|
| $w_1$ | $(0.84 e^{1.66 \pi}, 0.019 e^{0.04 i}, 0.29 e^{0.66 \pi})$ | $(0.97 e^{0.98 i}, 0.013 e^{0.025 i}, 0.14 e^{0.36 i})$ | $(0.96 e^{0.98 i}, 0.016 e^{0.04 i}, 0.04 e^{0.04 i})$ | $(0.52 e^{1.02 i}, 0.014 e^{0.026 i}, 0.74 e^{1.46 i})$ | $(0.5 e^{0.62 i}, 0.018 e^{0.038 i}, 0.29 e^{0.46 i})$ |
| $w_2$ | $(0.72 e^{1.46 \pi}, 0.036 e^{0.068 i}, 0.41 e^{0.88 \pi})$ | $(0.49 e^{0.98 i}, 0.013 e^{0.028 i}, 0.71 e^{1.46 i})$ | $(0.48 e^{0.98 i}, 0.015 e^{0.032 i}, 0.7 e^{1.36 i})$ | $(0.74 e^{1.46 i}, 0.034 e^{0.076 i}, 0.42 e^{0.82 i})$ | $(0.75 e^{1.48 i}, 0.035 e^{0.072 i}, 0.43 e^{0.83 i})$ |
| $w_3$ | $(0.47 e^{0.98 \pi}, 0.015 e^{0.032 i}, 0.7 e^{1.36 i})$ | $(0.27 e^{0.53 i}, 0.034 e^{0.066 i}, 0.94 e^{1.92 i})$ | $(0.46 e^{0.94 i}, 0.014 e^{0.033 i}, 0.68 e^{3.88 i})$ | $(0.88 e^{1.78 i}, 0.015 e^{0.032 i}, 0.25 e^{0.86 i})$ | $(0.29 e^{0.56 i}, 0.04 e^{0.078 i}, 0.96 e^{1.97 i})$ |
| $w_4$ | $(0.98 e^{0.98 \pi}, 0.013 e^{0.065 i}, 0.1 e^{0.28 i})$ | $(0.76 e^{1.5 i}, 0.036 e^{0.074 i}, 0.44 e^{0.86 i})$ | $(0.13 e^{0.65 i}, 0.014 e^{0.03 i}, 0.98 e^{1.97 i})$ | $(0.77 e^{1.52 i}, 0.036 e^{0.078 i}, 0.45 e^{0.88 i})$ | $(0.3 e^{0.38 i}, 0.042 e^{0.082 i}, 0.925 e^{1.86 i})$ |
| $w_5$ | $(0.44 e^{0.92 \pi}, 0.012 e^{0.025 i}, 0.66 e^{1.34 i})$ | $(0.43 e^{0.88 i}, 0.015 e^{0.025 i}, 0.66 e^{1.34 i})$ | $(0.87 e^{1.78 i}, 0.016 e^{0.025 i}, 0.21 e^{0.41 i})$ | $(0.42 e^{0.86 i}, 0.012 e^{0.02 i}, 0.65 e^{1.32 i})$ | $(0.78 e^{1.54 i}, 0.04 e^{0.076 i}, 0.46 e^{0.92 i})$ |
Table 44: Tabular representation of CSFD of expert $\tilde{E}_4$.

|   | $z_1$      | $z_2$      | $z_3$      | $z_4$      | $z_5$      |
|---|------------|------------|------------|------------|------------|
| $w_1$ | $0.95e^{0.194\pi}, 0.012e^{0.022\pi}, 0.13e^{0.2\pi}$ | $0.8e^{1.02\pi}, 0.018e^{0.003\pi}, 0.31e^{0.6\pi}$ | $0.97e^{1.96\pi}, 0.012e^{0.021\pi}, 0.15e^{0.32\pi}$ | $0.63e^{1.28\pi}, 0.012e^{0.020\pi}, 0.6e^{1.18\pi}$ | $0.64e^{1.20\pi}, 0.012e^{0.036\pi}, 0.59e^{1.12\pi}$ |
| $w_2$ | $0.77e^{1.56\pi}, 0.014e^{0.028\pi}, 0.47e^{0.92\pi}$ | $0.31e^{0.64\pi}, 0.022e^{0.04\pi}, 0.89e^{1.8\pi}$ | $0.06e^{0.14\pi}, 0.012e^{0.021\pi}, 0.986e^{1.97\pi}$ | $0.8e^{1.58\pi}, 0.05e^{0.102\pi}, 0.48e^{0.94\pi}$ | $0.37e^{1.78\pi}, 0.014e^{0.024\pi}, 0.86e^{1.74\pi}$ |
| $w_3$ | $0.3e^{0.64\pi}, 0.05e^{0.021\pi}, 0.91e^{1.81\pi}$ | $0.41e^{0.84\pi}, 0.01e^{0.06\pi}, 0.64e^{1.13\pi}$ | $0.4e^{0.84\pi}, 0.015e^{0.021\pi}, 0.63e^{1.28\pi}$ | $0.39e^{0.85\pi}, 0.014e^{0.035\pi}, 0.62e^{1.26\pi}$ | $1.03e^{0.85\pi}, 0.014e^{0.035\pi}, 0.93e^{1.81\pi}$ |
| $w_4$ | $0.38e^{0.78\pi}, 0.013e^{0.023\pi}, 0.61e^{1.24\pi}$ | $0.9e^{1.98\pi}, 0.014e^{0.03\pi}, 0.19e^{0.36\pi}$ | $0.37e^{0.76\pi}, 0.012e^{0.028\pi}, 0.6e^{1.22\pi}$ | $0.81e^{1.06\pi}, 0.05e^{0.104\pi}, 0.49e^{1.83\pi}$ | $0.13e^{0.28\pi}, 0.014e^{0.026\pi}, 0.982e^{1.96\pi}$ |
| $w_5$ | $0.07e^{0.022\pi}, 0.016e^{0.001\pi}, 0.991e^{1.84\pi}$ | $0.34e^{0.64\pi}, 0.065e^{0.132\pi}, 0.95e^{1.92\pi}$ | $0.3e^{0.58\pi}, 0.014e^{0.031\pi}, 0.93e^{0.38\pi}$ | $0.36e^{0.74\pi}, 0.012e^{0.021\pi}, 0.59e^{1.21\pi}$ | $0.21e^{0.66\pi}, 0.024e^{0.039\pi}, 0.94e^{0.88\pi}$ |
Table 45: Tabular representation of ACSFD.

| $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ |
|-------|-------|-------|-------|-------|
| $(0.95e^{1.93\pi}, 0.013e^{0.026\pi}, 0.07e^{0.18\pi})$ | $(0.97e^{1.93\pi}, 0.013e^{0.026\pi}, 0.15e^{0.31\pi})$ | $(0.97e^{1.93\pi}, 0.013e^{0.026\pi}, 0.05e^{0.18\pi})$ | $(0.77e^{1.54\pi}, 0.016e^{0.035\pi}, 0.4e^{0.82\pi})$ | $(0.91e^{1.76\pi}, 0.013e^{0.035\pi}, 0.15e^{0.36\pi})$ |
| $(0.79e^{1.93\pi}, 0.028e^{0.038\pi}, 0.36e^{0.74\pi})$ | $(0.48e^{0.96\pi}, 0.019e^{0.038\pi}, 0.73e^{1.46\pi})$ | $(0.33e^{0.66\pi}, 0.01e^{0.02\pi}, 0.88e^{1.76\pi})$ | $(0.9e^{1.82\pi}, 0.02e^{0.04\pi}, 0.15e^{0.26\pi})$ | $(0.47e^{0.96\pi}, 0.016e^{0.035\pi}, 0.68e^{1.39\pi})$ |
| $(0.25e^{0.52\pi}, 0.022e^{0.048\pi}, 0.96e^{1.74\pi})$ | $(0.29e^{0.58\pi}, 0.018e^{0.038\pi}, 0.83e^{1.66\pi})$ | $(0.5e^{1.04\pi}, 0.013e^{0.028\pi}, 0.72e^{1.42\pi})$ | $(0.55e^{1.12\pi}, 0.029e^{0.038\pi}, 0.6e^{1.23\pi})$ | $(0.35e^{0.73\pi}, 0.029e^{0.054\pi}, 0.9e^{1.82\pi})$ |
| $(0.7e^{1.81\pi}, 0.016e^{0.041\pi}, 0.63e^{1.28\pi})$ | $(0.77e^{1.56\pi}, 0.034e^{0.064\pi}, 0.42e^{0.84\pi})$ | $(0.51e^{1.02\pi}, 0.01e^{0.02\pi}, 0.53e^{1.06\pi})$ | $(0.71e^{1.42\pi}, 0.026e^{0.054\pi}, 0.55e^{1.12\pi})$ | $(0.52e^{1.06\pi}, 0.019e^{0.038\pi}, 0.6e^{1.22\pi})$ |
| $(0.39e^{0.78\pi}, 0.015e^{0.032\pi}, 0.83e^{1.68\pi})$ | $(0.35e^{0.78\pi}, 0.025e^{0.052\pi}, 0.87e^{1.75\pi})$ | $(0.63e^{1.26\pi}, 0.02e^{0.04\pi}, 0.51e^{1.02\pi})$ | $(0.28e^{0.66\pi}, 0.016e^{0.035\pi}, 0.76e^{1.58\pi})$ | $(0.49e^{0.96\pi}, 0.015e^{0.034\pi}, 0.83e^{1.9\pi})$ |
Table 46: Experts opinion related to each attribute.

|      | $E_1$                  | $E_2$                  | $E_3$                  | $E_4$                  |
|------|------------------------|------------------------|------------------------|------------------------|
| $z_1$| (0.85e^{0.66π}, 0.019e^{0.042π}, 0.29e^{0.6π}) | (0.99e^{1.94π}, 0.012e^{0.02π}, 0.1e^{0.26π}) | (0.88e^{1.78π}, 0.015e^{0.03π}, 0.25e^{0.46π}) | (0.67e^{1.36π}, 0.021e^{0.044π}, 0.31e^{0.64π}) |
| $z_2$| (0.66e^{0.34π}, 0.021e^{0.044π}, 0.31e^{0.64π}) | (0.96e^{1.94π}, 0.013e^{0.024π}, 0.04e^{0.06π}) | (0.78e^{1.54π}, 0.04e^{0.076π}, 0.46e^{0.9π}) | (0.98e^{1.98π}, 0.016e^{0.024π}, 0.02e^{0.066π}) |
| $z_3$| (0.63e^{0.12π}, 0.012e^{0.024π}, 0.59e^{0.12π}) | (0.72e^{1.42π}, 0.03e^{0.06π}, 0.38e^{0.78π}) | (0.75e^{1.48π}, 0.035e^{0.072π}, 0.43e^{0.82π}) | (0.42e^{0.88π}, 0.012e^{0.02π}, 0.65e^{0.32π}) |
| $z_4$| (0.29e^{0.66π}, 0.014e^{0.03π}, 0.94e^{1.86π}) | (0.21e^{0.44π}, 0.024e^{0.032π}, 0.94e^{1.86π}) | (0.37e^{0.76π}, 0.012e^{0.026π}, 0.6e^{0.22π}) | (0.78e^{1.34π}, 0.04e^{0.076π}, 0.46e^{0.92π}) |
| $z_5$| (0.62e^{0.12π}, 0.01e^{0.024π}, 0.58e^{1.12π}) | (0.66e^{0.34π}, 0.1e^{0.18π}, 0.56e^{0.14π}) | (0.64e^{1.36π}, 0.01e^{0.024π}, 0.59e^{1.2π}) | (0.21e^{0.64π}, 0.021e^{0.046π}, 0.95e^{1.88π}) |
| $w_1$ | $(0.85 e^{0.166 \pi}, 0.21 e^{0.042 \pi}, 0.23 e^{0.52 \pi})$ | $(0.88 e^{0.76 \pi}, 0.021 e^{0.042 \pi}, 0.174 e^{0.38 \pi})$ | $(0.601 e^{1.22 \pi}, 0.019 e^{0.044 \pi}, 0.053 e^{0.15 \pi})$ | $(0.41 e^{0.32 \pi}, 0.025 e^{0.045 \pi}, 0.77 e^{0.34 \pi})$ | $(0.5 e^{0.05 \pi}, 0.023 e^{0.035 \pi}, 0.67 e^{0.36 \pi})$ |
| $w_2$ | $(0.7 e^{1.34 \pi}, 0.03 e^{0.068 \pi}, 0.41 e^{0.86 \pi})$ | $(0.43 e^{0.9 \pi}, 0.025 e^{0.048 \pi}, 0.73 e^{0.36 \pi})$ | $(0.2 e^{0.4 \pi}, 0.019 e^{0.034 \pi}, 0.91 e^{0.32 \pi})$ | $(0.47 e^{0.96 \pi}, 0.028 e^{0.056 \pi}, 0.73 e^{0.42 \pi})$ | $(0.25 e^{0.54 \pi}, 0.025 e^{0.063 \pi}, 0.83 e^{0.66 \pi})$ |
| $w_3$ | $(0.22 e^{0.34 \pi}, 0.027 e^{0.038 \pi}, 0.91 e^{0.74 \pi})$ | $(0.26 e^{0.54 \pi}, 0.024 e^{0.072 \pi}, 0.83 e^{0.30 \pi})$ | $(0.32 e^{0.64 \pi}, 0.021 e^{0.034 \pi}, 0.8 e^{0.32 \pi})$ | $(0.29 e^{0.38 \pi}, 0.035 e^{0.065 \pi}, 0.73 e^{0.42 \pi})$ | $(0.19 e^{0.34 \pi}, 0.035 e^{0.068 \pi}, 0.95 e^{0.36 \pi})$ |
| $w_4$ | $(0.62 e^{0.62 \pi}, 0.023 e^{0.068 \pi}, 0.65 e^{0.13 \pi})$ | $(0.7 e^{0.46 \pi}, 0.037 e^{0.072 \pi}, 0.43 e^{0.36 \pi})$ | $(0.32 e^{0.64 \pi}, 0.019 e^{0.034 \pi}, 0.69 e^{0.42 \pi})$ | $(0.37 e^{0.74 \pi}, 0.033 e^{0.065 \pi}, 0.82 e^{0.42 \pi})$ | $(0.28 e^{0.68 \pi}, 0.027 e^{0.056 \pi}, 0.8 e^{0.62 \pi})$ |
| $w_5$ | $(2.05 e^{0.62 \pi}, 0.022 e^{0.068 \pi}, 0.83 e^{0.17 \pi})$ | $(0.31 e^{0.66 \pi}, 0.029 e^{0.068 \pi}, 0.87 e^{0.34 \pi})$ | $(0.39 e^{0.78 \pi}, 0.025 e^{0.044 \pi}, 0.68 e^{0.13 \pi})$ | $(0.15 e^{0.318 \pi}, 0.025 e^{0.032 \pi}, 0.89 e^{0.18 \pi})$ | $(0.27 e^{0.51 \pi}, 0.024 e^{0.031 \pi}, 0.91 e^{0.82 \pi})$ |
MADM and MAGDM problems. The model of environment that have a large ability of solving real-life concept has allowed us to propose techniques in a wide families of both fuzzy sets and

Table 48: Tabular representation of CSF-PIS and CSF-NIS.

| Attribute | CSF-PIS                          | CSF-NIS                          |
|-----------|----------------------------------|----------------------------------|
| z₁        | (0.85e^{0.66}, 0.021e^{0.042}, 0.23e^{0.52}) | (0.22e^{0.44}, 0.027e^{0.058}, 0.91e^{1.17}) |
| z₂        | (0.88e^{1.76}, 0.021e^{0.042}, 0.174e^{0.036}) | (0.31e^{0.66}, 0.029e^{0.067}, 0.87e^{1.74}) |
| z₃        | (0.601e^{1.22}, 0.019e^{0.034}, 0.53e^{1.15}) | (0.2e^{0.44}, 0.019e^{0.034}, 0.91e^{1.82}) |
| z₄        | (0.47e^{0.96}, 0.038e^{0.006}, 0.73e^{1.42}) | (0.15e^{0.318}, 0.025e^{0.026}, 0.89e^{1.8}) |
| z₅        | (0.19e^{0.38}, 0.035e^{0.068}, 0.95e^{1.90}) | (0.5e^{0.23}, 0.023e^{0.046}, 0.67e^{1.36}) |

Table 49: Normalized Euclidean distance from ideal solution.

| Alternative | $d(\bar{P}_k, w_j)$ | $d(\bar{P}_k, w_i)$ |
|-------------|----------------------|----------------------|
| w₁         | 0.182947             | 0.565764             |
| w₂         | 0.383839             | 0.330875             |
| w₃         | 0.5388422            | 0.200388             |
| w₄         | 0.266370             | 0.349150             |
| w₅         | 0.510455             | 0.208933             |

Table 50: Revised closeness index of each alternative.

| Alternative | $\mathcal{S}(w_i)$ |
|-------------|--------------------|
| w₁         | 0                  |
| w₂         | 1.5132             |
| w₃         | 2.5911             |
| w₄         | 0.838853           |
| w₅         | 2.42088            |

Table 51: Comparison.

| Method          | Ranking | Best physiotherapist |
|-----------------|---------|----------------------|
| CSFNS₁-TOPSIS   | w₁ > w₂ > w₃ > w₄ > w₅   | w₁                  |
| CSF-TOPSIS [16] | w₁ > w₂ > w₃ > w₄ > w₅   | w₁                  |
| SF-TOPSIS [10]  | w₁ > w₂ > w₃ > w₄ > w₅   | w₁                  |
| SF-TOPSIS [11]  | w₁ > w₂ > w₃ > w₄ > w₅   | w₁                  |

SFNS₁, CSFS₁, and SF₂, by substituting periodic terms equal to zero and $N = 2$.

7. Conclusions

Complex spherical fuzzy N-soft sets (CSFNS₁,S) broaden the families of both fuzzy sets and N-soft sets. This novel concept has allowed us to propose techniques in a wide environment that have a large ability of solving real-life MADM and MAGDM problems. The model of CSFNS₁,S described in this paper copes with 2-dimensional fuzziness, parameterized information, and ordinal ranking systems. In addition to the notion of CSFNS₁,S, we have defined score and accuracy functions for the purpose of comparing two CSFNS₁,Ns. We have defined useful operations on CSFNS₁,S and given relevant examples. We developed three direct algorithms and, furthermore, a CSFNS₁-TOPSIS Method to solve decision-making problems. We compared them with existing methods for Pythagorean fuzzy N-soft sets and with the complex spherical fuzzy TOPSIS Method, respectively. For the purpose of extending the theoretical background of TOPSIS methods to the new CSFNS₁-TOPSIS method, we have defined complex spherical fuzzy N-soft weighted averaging operator CSFNS₁-WA which produces an aggregate complex spherical fuzzy N-soft decision matrix ACSFNS₁-WA and aggregates the weight vectors of attributes given by experts. Similarly, we have defined a normalized Euclidean distance in CSFNS₁ environment that simultaneously evaluates the distances of alternatives from CSFNS₁-PIS and CSFNS₁-NIS. This is required to find a revised closeness index. The ascending order of such an index gives us a ranking of the alternatives, where the smallest revised closeness index indicates a best solution. In the future, we intend to pursue the formalization of other methods (ELECTRE I, II, and III and VIKOR methodologies), under the framework of CSFNS₁. We can also extend this theory to accommodate T-spherical fuzzy soft sets, T-spherical fuzzy N-soft sets, and complex T-spherical fuzzy N-soft sets.

Data Availability

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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