On the Maximum Sum-rate Capacity of Cognitive Multiple Access Channel

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Abstract—We consider the communication scenario where multiple cognitive users wish to communicate to the same receiver, in the presence of primary transmission. The cognitive transmitters are assumed to have the side information about the primary transmission. The capacity region of cognitive users is formulated under the constraint that the capacity of primary transmission is not changed as if no cognitive users exist. Moreover, the maximum sum-rate point of the capacity region is characterized, by optimally allocating the power of each cognitive user to transmit its own information.

Index Terms—Cognitive radio, multiple access, achievable rate region, maximum sum-rate.

I. INTRODUCTION

With the increasing demand of radio spectrum, Cognitive Radio (CR) [1] has drawn great attention in the world these days for its ability of operation in licensed bands without a license. In cognitive radio systems, the cognitive (unlicensed) user needs to detect the presence of the primary (licensed) users as quickly as possible and dynamically change system parameters, such as the transmitting spectrum and modulation schemes, so as to best utilize the valuable spectrum.

Recently, several papers [2],[3] have discussed the achievable rate of such a cognitive radio system from the view of information theory. Literature [3] studied the achievable rate of cognitive user under the constraint that (i) no interference is created for the primary user, and (ii) the primary encoder-decoder pair is oblivious to the presence of cognitive radio. The authors pointed that, the maximum achievable rate is achieved by a mixed strategy of dirty paper coding and cooperation with the primary user.

In this letter, we further study the scenario containing multiple cognitive users in the presence of primary transmission. These cognitive users wish to communicate with a same access point (AP). We first model this system as a cognitive multiple access channel (MAC) and characterize the capacity region of the cognitive MAC channel. Then, the problem of maximizing the sum-rate capacity is formulated as a nonlinear optimization problem. Employing the classic Lagrangian multiplier method, an iterative algorithm to achieve the maximum sum-rate capacity is proposed.

The remainder of this letter is organized as follows. Section II gives the system model. Section III abstracts the system model in Section II into an information theory model, and the capacity region of it is analyzed. In Section IV, the problem of achieving the sum-rate optimal point is discussed and the corresponding algorithm is given. Finally, we conclude the letter in Section V.

II. SYSTEM MODEL

The system model we study is the heterogeneous network depicted in Fig. 1. There is a primary user $U_p$ communicating with the Base Station (BS) in a large 3G cellular network, which uses a licensed spectrum. A small WLAN network without license of the spectrum lies in the communication range of a 3G cell, where $K$ cognitive radio users $U_i(i=1,2,...,K)$ want to access the WLAN Access Point (AP).

For the above scenario to be studied, we assume the primary user $U_p$ has message $m_p \in \{0,1,...,2^{nR_p}\}$ intended for the primary receiver $B_p$ to decode, and the cognitive user $U_i(i=1,2,...,K)$ has message $m_i \in \{0,1,...,2^{nR_i}\}$ intended for the cognitive receiver $A_c$. The average transmit powers of $U_p$ and $U_i$ are constrained by $P_p$ and $P_i(i=1,2,...,K)$, respectively. It is assumed that all cognitive users have perfect knowledge of the primary user’s codeword $m_p$.

III. CAPACITY REGION

In this section, we will model the heterogeneous network in Fig. 1 as an information theory model, and then analyze its capacity region. The information theory model of the heterogeneous network is depicted in Fig.2. The small WLAN can be viewed as a multiple-access channel with inference from the primary user, whose receive signal is expressed as follows:

$$Y^n = \sum_{k=1}^{K} h_k \cdot X_k^n + f \cdot X_p^n + Z^n,$$  \hspace{1cm} (1)
where \( h_k, (k = 1, 2, ..., K) \) is the path gain of the link from cognitive user \( U_i \) to \( A_c \), \( f \) is the path gain of the inference link from \( U_p \) to \( A_c \), and \( Z^n \) is additive white Gaussian noise with variance \( \sigma_p^2 \). For the large 3G cellular network, the received signal model is:

\[
Y^n_p = h_p X^n_p + \sum_{k=1}^{K} g_k X^n_k + Z^n_p,
\]

where \( h_p \) is the path gain of the link from primary user \( U_p \) to \( B_p \), \( g_k \), \((k = 1, 2, ..., K)\) is the path gain of the inference link from \( U_i \) to \( B_p \), and \( Z^n_p \) is additive white Gaussian noise with variance \( \sigma_p^2 \).

The capacity region of the cognitive multiple access channel is defined as the set of achievable rates of cognitive users under the constraints that (i) no interference is created for the primary user, and (ii) the primary encoder-decoder pair is oblivious to the presence of cognitive radio. A simpler scenario which contains only one cognitive user is studied in [3]. The largest rate of the cognitive user is achieved by a mixed strategy of dirty paper coding and cooperation with the primary user. The optimal coding strategy for the cognitive user \( U_k \) is:

\[
X^n_k = \hat{X}^n_k + \gamma_k \sqrt{\frac{P_k}{P_p}} X^n_p, \quad k = 1, 2, ..., K,
\]

where the first term \( \hat{X}^n_k \) denotes that the user \( U_k \)’s codeword \( m_k \) is dirty-paper coded based on the primary user’s codeword \( m_p \) (so it is called the dirty-paper code part), and the second term, the cooperation part, is the duplicated information of the primary user, which is combined with the information from \( U_p \) in order to assure the primary user’s rate. In addition, \( \gamma_k \) denotes the power ratio of the cooperation part.

For the receiver of the AP, it can decode each cognitive user’s signal \( X^n_k (k = 1, 2, ..., K) \) as if there is no interference from the primary user due to the dirty paper coding. Therefore, we can regard this model as a Gaussian multiple-access channel, and its capacity region is written as

\[
C_{mac}(\mathbf{h}, \mathbf{P}, \gamma) = \{ \mathbf{R} : \mathbf{R}(S) \leq \frac{1}{2} \log \left( 1 + \frac{\sum_{k \in S} h_k^2 (1 - \gamma_k^2) P_k}{\sigma_c^2} \right), \quad \forall S \subset \{1, 2, ..., K\} \}
\]

where \( \mathbf{h} = (h_1, h_2, ..., h_K)^T \) and \( \mathbf{P} = (P_1, P_2, ..., P_K)^T \) are constant, but \( \gamma = (\gamma_1, \gamma_2, ..., \gamma_K)^T \) is a variable vector. For the receiver of \( B_p \), its rate can be expressed as follows:

\[
R_p = \frac{1}{2} \log \left( 1 + \frac{(h_p \sqrt{P_p} + \sum_{k=1}^{K} g_k \gamma_k \sqrt{T_k})^2}{\sigma_p^2 + \sum_{k=1}^{K} g_k^2 (1 - \gamma_k^2) P_k} \right),
\]

which should be equal to the original rate as if there is no interference from the cognitive users, i.e., the following equation must be satisfied:

\[
\frac{1}{2} \log \left( 1 + \frac{\gamma_h P_p}{\sigma_p^2} \right) = \frac{1}{2} \log \left( 1 + \frac{(h_p \sqrt{P_p} + \sum_{k=1}^{K} g_k \gamma_k \sqrt{T_k})^2}{\sigma_p^2 + \sum_{k=1}^{K} g_k^2 (1 - \gamma_k^2) P_k} \right).
\]

Define the set of feasible power allocation policies \( \mathcal{F} \) as

\[
\mathcal{F} = \left\{ \gamma : \frac{h_p^2 P_p}{\sigma_p^2} = \frac{(h_p \sqrt{P_p} + \sum_{k=1}^{K} g_k \gamma_k \sqrt{T_k})^2}{\sigma_p^2 + \sum_{k=1}^{K} g_k^2 (1 - \gamma_k^2) P_k} \right\},
\]

then the capacity region of the cognitive MAC channel is the convex hull of all capacity regions for every \( \gamma \in \mathcal{F} \). That is,

\[
C = \text{Co}(\bigcup_{\gamma \in \mathcal{F}} C_{mac}(\mathbf{h}, \mathbf{P}, \gamma))
\]

An example of capacity region of a two-user cognitive MAC channel is shown in Fig. 3.

### IV. Maximum Sum-rate Point

In this section, our interests turn to the maximum sum rate point on the capacity region \( C \). For a fixed \( \gamma \in \mathcal{F} \), the sum-rate capacity of the multiple cognitive radio access network is expressed as follows:

\[
R_{opt}(\gamma) = \frac{1}{2} \log \left( 1 + \frac{\sum_{k=1}^{K} (1 - \gamma_k^2) h_k^2 P_k}{\sigma_c^2} \right).
\]
Now, the problem is how to optimally choose the $\gamma_k (k = 1, 2, ..., K)$ so as to maximize the the sum-rate capacity, which can be formulated as a nonlinear optimization problem:

$$\text{maximize } \sum_{k=1}^{K} (1 - \gamma_k^2) h_k^2 P_k,$$

subject to

$$\frac{h_k^2 P_p}{\sigma_p^2} = \left( \frac{h_p \sqrt{P_p} + \sum_{k=1}^{K} g_k \gamma_k \sqrt{P_k}}{\sigma_p^2 + \sum_{k=1}^{K} g_k^2 (1 - \gamma_k^2) P_k} \right)^2$$

$$0 \leq \gamma_k \leq 1, k = 1, 2, ..., K$$

We can see that, the optimization object is quadratic and concave, and meanwhile, the constraint is quadratic and convex, but not affine. So we can’t solve the above optimization by classical convex optimization algorithms [4]. Therefore, we will give an iterative algorithm to solve the above problem. First, a Lagrangian multiplier is defined as

$$J(\gamma) = \sum_{k=1}^{K} (1 - \gamma_k^2) h_k^2 P_k + \lambda \left[ \sigma_p^2 \left( \frac{h_p \sqrt{P_p} + \sum_{k=1}^{K} g_k \gamma_k \sqrt{P_k}}{\sigma_p^2 + \sum_{k=1}^{K} g_k^2 (1 - \gamma_k^2) P_k} \right)^2 \right] - h_p^2 \sigma_p^2 \left[ \sigma_p^2 + \sum_{k=1}^{K} g_k^2 (1 - \gamma_k^2) P_k \right],$$

(12)

If there is no constraint of $0 \leq \gamma_k \leq 1$, a unique maximizer of $J(\gamma)$ exists when

$$\frac{dJ}{d\gamma_k} = -2 h_k^2 P_k \gamma_k + 2 \lambda \sigma_p^2 X g_k \sqrt{P_k} \gamma_k + 2 h_p^2 \sigma_p^2 P_k \gamma_k = 0,$$

(13)

where

$$X = h_p \sqrt{P_p} + \sum_{k=1}^{K} g_k \gamma_k \sqrt{P_k}.$$

(14)

Taking into account the restrict of $0 \leq \gamma_k \leq 1$, we obtain

$$\gamma_k = \begin{cases} \frac{\lambda \sigma_p^2 X}{(\beta_k^2 - \lambda h_p^2 \gamma_k) g_k \sqrt{P_k}} & \text{if } 0 \leq \gamma_k < 1, \\ 1 & \text{else,} \end{cases}$$

(15)

where $\beta_k = h_k / g_k$. Define the set $\mathcal{S}$ as

$$\mathcal{S} = \{ k : \gamma_k < 1, \forall k \},$$

(16)

and substitute (15) into (14). $X$ can be determined as

$$X = \frac{h_p \sqrt{P_p} + \sum_{k \in \mathcal{S}} g_k \sqrt{P_k}}{1 - \lambda \sigma_p^2 \sum_{k \in \mathcal{S}} (\beta_k^2 - \lambda h_p^2 \gamma_k)^{-1}}.$$  

(17)

Fig. 4. Flow chart of the proposed algorithm

that some $\gamma_k$ turns to one, $\mathcal{S}$ should be updated, so are $X$ and $\gamma_k$.

The flow chart of the proposed algorithm is depicted in Fig.4 where $\Delta \lambda$ is a step size. The algorithm starts searching from $\lambda = 0$ and $\mathcal{S} = \{ 1, 2, \cdots, K \}$. For each $\lambda$, the algorithm first computes $X$ by (17) using the former $\mathcal{S}$, and computes $\gamma_k (k = 1, 2, \ldots, K)$ by (15). If $\mathcal{S}$ is changed, $X$ and $\gamma_k$ should be calculated according to the updated $\mathcal{S}$. The algorithm converges when $\gamma_k$ satisfies (11).

V. Conclusion

In this letter, we first study the capacity region of cognitive multiple access channel under the constraint that the transmission of primary user is not interfered by cognitive users. Then, we investigate the optimization of sum-rate capacity on the capacity region. The optimization is formulated as a nonlinear problem and an iterative algorithm is introduced.

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