Cosmology and the neutrino mass ordering

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Abstract. We propose a simple method to quantify a possible exclusion of the inverted neutrino mass ordering from cosmological bounds on the sum of the neutrino masses. The method is based on Bayesian inference and allows for a calculation of the posterior odds of normal versus inverted ordering. We apply the method for a specific set of current data from Planck CMB data and large-scale structure surveys, providing an upper bound on the sum of neutrino masses of 0.14 eV at 95\% CL. With this analysis we obtain posterior odds for normal versus inverted ordering of about 2:1. If cosmological data is combined with data from oscillation experiments the odds reduce to about 3:2. For an exclusion of the inverted ordering from cosmology at more than 95\% CL, an accuracy of better than 0.02 eV is needed for the sum. We demonstrate that such a value could be reached with planned observations of large scale structure by analysing artificial mock data for a EUCLID-like survey.

Keywords: neutrino masses from cosmology, neutrino properties, particle physics - cosmology connection, cosmological parameters from LSS

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1 Introduction

Current data on neutrino oscillations show a degeneracy between two possible orderings of the neutrino mass states, the normal ordering (NO) and inverted ordering (IO). Breaking this degeneracy is one of the main goals of upcoming oscillation experiments, e.g., [1–5], see [6] for an overview. On the other hand, also cosmological observations potentially may contribute to this question. Cosmological structure formation is sensitive mostly to the sum of the neutrino masses, Σ. There are subtle effects sensitive to the details of the neutrino mass spectrum beyond the sum, see e.g., [7–10]. With realistic observations in the foreseeable future those effects will be very hard to detect [10]. Focusing on the sum of masses, we can use that oscillation data determine the mass-squared differences and we have:

$$\Sigma \equiv \sum_{i=1}^{3} m_i = \left\{ \begin{array}{ll} m_0 + \sqrt{\Delta m^2_{21}} + m_0^2 + \sqrt{\Delta m^2_{31}} + m_0^2 & \text{(NO)} \\
 m_0 + \sqrt{|\Delta m^2_{32}|} + m_0^2 + \sqrt{|\Delta m^2_{21}|} - \Delta m^2_{21} + m_0^2 & \text{(IO)} \end{array} \right.$$  

(1.1)

where $m_0$ denotes the lightest neutrino mass, where by convention $m_0 \equiv m_1$ ($m_3$) for NO (IO). The mass-squared differences $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$ are determined to [11] ($1\sigma$ uncertainties):

$$\Delta m^2_{21} = 7.49^{+0.19}_{-0.17} \times 10^{-5} \text{eV}^2,$$

$$\Delta m^2_{31} = 2.484^{+0.045}_{-0.048} \times 10^{-3} \text{eV}^2 \quad \text{(NO)}$$

$$\Delta m^2_{32} = -2.467^{+0.041}_{-0.042} \times 10^{-3} \text{eV}^2 \quad \text{(IO)}.$$  

(1.2)

For a zero lightest neutrino mass ($m_0 = 0$), the predictions for the sum are ($1\sigma$ uncertainties)

$$\Sigma = \left\{ \begin{array}{ll} 58.5 \pm 0.48 \text{meV} & \text{(NO)} \\
 98.6 \pm 0.85 \text{meV} & \text{(IO)} \end{array} \right. \quad (m_0 = 0).$$  

(1.3)

Hence, if cosmological observations provide a determination of Σ significantly below 0.098 eV, the inverted mass ordering would be disfavoured.

Recent data from Planck CMB data combined with baryonic acoustic oscillations (BAO) and other observations lead to the bound $\Sigma < 0.23 \text{eV}$ at 95% CL (PlanckTT + lowP + lensing + BAO + JLA + $H_0$), see [12] for details. Depending on the used data and variations in the analysis, different authors obtain upper bounds from current data approaching the
“critical” value of 0.1 eV [13–17]. These results suggest that IO starts to get under pressure from cosmology.

In this note we want to point out that such a claim should be based on a proper statistical analysis. The question to be answered is, whether the hypothesis of IO can be rejected with some confidence against NO. For a related discussion in the context of oscillation experiments see for instance ref. [6] formulated in terms of frequentist hypothesis testing, or ref. [18] using Bayesian reasoning. Indeed, just from the numbers in eq. (1.3) one sees that it is not enough that the upper bound on $\Sigma$ is below $0.098 \text{ eV}$, but instead cosmology needs to determine $\Sigma$ with an accuracy better than about $0.02 \text{ eV}$ in order to exclude a value of $0.098 \text{ eV}$ against $0.059 \text{ eV}$ at $2\sigma$. Note that this would imply a $\gtrsim 3\sigma$ detection of a non-zero value of $\Sigma \approx 0.058 \text{ eV}$. Obviously, requirements would be even more demanding if $m_0$ turns out not to be zero. Below we are going to substantiate this simple estimate by more detailed calculations.

2 Quantifying the evidence against inverted ordering

In this section we provide a simple recipe to quantify possible evidence against inverted ordering from cosmology. Note that as long as only $\Sigma$ is the dominating observable, it will never be possible to reject NO. We will use Bayesian statistics, following closely [18]. Similar methods have been used in [19] in the context of the mass ordering in cosmology. Bayesian methods are especially suitable for our problem, since we are interested in a region close to a physical boundary implied by $m_0 \geq 0$. Indeed, the mechanism to exclude IO is based on the fact that the data may prefer a value of $\Sigma$ outside the physical domain accessible in the case of IO. Such a situation is easily incorporated in Bayesian statistics. In a frequentist approach, the relevant distribution of a test statistics needs to be obtained by Monte Carlo simulations, since one expects non-Gaussian behaviour close to a physical boundary.

We consider the likelihood function $L(D|\theta,m_0,O)$, of some set of cosmological data $D$, with the theoretical model depending on a set of cosmological parameters $\theta$, the lightest neutrino mass, $m_0$, and the discrete parameter $O$ describing the mass ordering, $O = N,I$. Using Bayes theorem, we easily obtain the probability for a mass ordering given data $D$:

$$ p_O \equiv p(O|D) = \frac{\pi(O)}{\pi(D)} \int d\theta \int dm_0 L(D|\theta,m_0,O) \pi(\theta) \pi(m_0), \quad (2.1) $$

where the $\pi$ denote prior probabilities. Defining the likelihood marginalized over cosmological parameters as $L(D|m_0,O) \equiv \int d\theta L(D|\theta,m_0,O) \pi(\theta)$ and adopting a flat prior for $m_0 \geq 0$ we obtain

$$ p_O = \frac{\pi(O) \int_0^\infty dm_0 L(D|m_0,O)}{\pi(N) \int_0^\infty dm_0 L(D|m_0,N) + \pi(I) \int_0^\infty dm_0 L(D|m_0,I)} \quad (2.2) $$

with $p_N + p_I = 1$. If no prior information on the mass ordering is available an obvious choice is to assume that NO and IO are equally likely a priori: $\pi(N) = \pi(I) = 1/2$. However, using $\pi(O)$ it is straightforward to include possible prior information on the ordering from oscillation data. The Bayesian analysis of [20] gives for present oscillation data a posterior probability for IO of 0.55 (i.e., very close to equal probabilities for NO and IO). However, this may improve in the near future by upcoming oscillation data.

Using eq. (2.2), one can then consider for instance the ratio $p_I/p_N$ to define the posterior odds of IO versus NO [18]. Alternatively one can report $p_I$ to quantify how likely an inverted
mass ordering is for given data. Values of $p_I \ll 1$ will provide exclusion of IO at a confidence level of $(1 - p_I)$.

3 Analysis of cosmological data

3.1 Current data

In any parameter estimation analysis of cosmological data a model has to be specified. A larger parameter space inevitably leads to less stringent bounds on the neutrino mass (and other cosmological parameters). In the standard $\Lambda$CDM model data from the Planck mission provides an upper bound $\Sigma < 0.72\text{eV}$ at 95% CL (“PlanckTT + lowP”); with the addition of large scale structure data this upper bound improves to $\Sigma < 0.23\text{eV}$ (“PlanckTT + lowP + lensing + BAO + JLA + $H_0$”), see [12] for details of the used data.

However, the parameter space used in this model is quite restrictive and fits data using only 6 parameters in addition to the sum of neutrino masses: $\Omega_b h^2$, the physical baryon density, $\Omega_c h^2$, the physical cold dark density, $H_0$, the Hubble parameter, $A_s$, the amplitude of the primordial scalar fluctuation spectrum, $n_s$, the spectral tilt of the primordial spectrum, and $\tau$, the optical depth to reionization. In more general parameter spaces these bounds can be relaxed significantly (see e.g. [21–23] for recent examples of extended models). Since our goal here is not so much to see to what extent the neutrino mass bound can be relaxed, but rather a study of the sensitivity to reject the IO already based on current data we will use the restricted parameter space defined by the 6 parameter $\Lambda$CDM model with the addition of $\Sigma$.

For our analysis we use the Planck 2015 data, including polarisation [12]. We furthermore include BAO data from a variety of different surveys: 6dFGS [24], SDSS-MGS [25], BOSS-LOWZ [26] and CMASS-DR11 [27]. Finally, we also include the recent local universe measurement of the Hubble parameter, $H_0 = 73.02 \pm 1.79 \text{km s}^{-1} \text{Mpc}^{-1}$ [28].\footnote{Within the minimal $\Lambda$CDM model this local value for $H_0$ is more than $3\sigma$ away from the global result from Planck, see e.g., [21] for a discussion. We have checked that our constraints for $\Sigma$ are not sensitive to this tension in the data, and we obtain indistinguishable results for $\Sigma$ as well as for the probability of IO without using the local $H_0$ prior. We note that this would have been different in an extended analysis with more free parameters. In that case the $H_0$ measurement is important for breaking e.g. the degeneracy between $\Sigma$ and $N_{\text{eff}}$.} To perform parameter estimation and derive constraints we have used the publicly available CosmoMC code [29].

In this relatively restricted model we find an upper bound of $\Sigma < 0.14\text{eV}$ (95% CL). This is comparable to other recent estimates using somewhat different data sets and model assumptions [13–17]. Since our purpose here is mainly to discuss what claims can be made about the neutrino mass ordering given a constraint on $\Sigma$ in this range we will not explore how the bound changes with the use of different data and model assumptions (this has been discussed in many other recent papers). In order to check the approximation that current data is sensitive only to the sum of neutrino masses we have performed three analyses with the following assumptions: (i) two massless and one massive neutrino, (ii) one massless and two degenerate massive neutrinos, and (iii) three degenerate massive neutrinos. Note that none of these scenarios actually corresponds to the realistic cases of NO or IO with mass-squared differences constrained by oscillations. However, the spread in the results will be indicative for our assumption that cosmology is sensitive only to $\Sigma$. Indeed we confirm that within the numerical accuracy all three models lead to an upper bound of $0.14\text{eV}$ (95% CL).
The posterior likelihood function is shown in figure 1. The left panel shows the likelihood as a function of $\Sigma$, and we indicate the predicted values for NO and IO assuming $m_0 = 0$, as well as the 95% CL upper bound on $\Sigma$, assuming a flat prior in $\Sigma \geq 0$. Note that the region of largest likelihood, for $\Sigma < 59$ meV, is actually unphysical, since such small values for the sum of the neutrino masses are inconsistent with neutrino oscillation data. Hence, this region will be cut away once the sum is expressed using eq. (1.1) and imposing the physical requirement of $m_0 \geq 0$.

In order to apply eq. (2.2) to calculate the probability of IO vs NO we translate the likelihood into a posterior likelihood as a function of $m_0$ by using eq. (1.1). The resulting likelihoods are shown in the right panel of figure 1. The posterior odds for NO versus IO are given by the ratio of the integrals over those two curves weighted by the prior probabilities for the orderings. Assuming equal prior probabilities for NO and IO, eq. (2.2) leads to a probability for IO of $p_I = 0.35$, which corresponds to posterior odds for NO versus IO of about 1.9:1. Clearly, using even quite restrictive assumptions about the cosmological model current data is not sufficient to distinguish between the NO and the IO at a statistically significant level.

If instead of equal priors for the two orderings we use the prior probabilities from oscillation data [20], $\pi(I) = 0.55$, $\pi(N) = 0.45$, we obtain a posterior probability of $p_I = 0.392$ or equivalently, posterior odds of 1.55:1 for NO vs IO. Again this result shows that present data from neutrino oscillations and cosmology are not sensitive enough to reach a significant conclusion. However, this exercise does illustrate the power of the method to combine information from oscillations and cosmology which is expected to be very useful in the near future.

The different curves in figure 1 (dashed, dot-dashed, solid) correspond to the three different assumptions about how $\Sigma$ is shared between the three neutrinos (1 massive, 2 massive, 3 massive). We neglect the uncertainty induced by the uncertainty on the mass-squared differences from oscillation data. For an accuracy on $\Sigma$ larger than 0.01 eV this is an excellent approximation, see also section 4.
3 massive, respectively). We see that the differences are small, and the MO analysis gives identical results within the numerical accuracy. This justifies our approximation that the likelihood depends on $\Sigma$ only when converting $\mathcal{L}(D|\Sigma)$ into $\mathcal{L}(D|m_0,O)$ by using eq. (1.1).

3.2 Prospective data including a EUCLID-like survey

Let us now address the question of how this situation will change quantitatively in the future. In the coming years a whole range of new cosmological surveys will start operating, including for example the LSST survey [30] and the EUCLID satellite mission [31]. When combined with CMB data these surveys have the potential to bring the sensitivity to $\Sigma$ down to the 0.02 eV level (see e.g. [10, 32–37]).

Using the CosmoMC-based forecasting tool described in [10, 33, 38] we have generated artificial EUCLID-like data and used it to constrain cosmological parameters including $\Sigma$. Specifically we have used a EUCLID-like data set consisting of weak lensing and photometric galaxy survey components. We have used synthetic data equivalent to the “csg” case described in [33], which includes synthetic cosmic shear (s) and galaxy (g) data, as well as CMB data (c) roughly equivalent to Planck data in precision. As the cosmological model we have used a minimal $\Lambda$CDM model with $\Sigma$ and the number of relativistic degrees of freedom, $N_{\text{eff}}$, as additional parameters.

The fiducial model we use has one massive neutrino with $m_\nu = 0.06$ eV and 2.046 massless neutrinos. Note that this is not equivalent to the real physical prediction of the NO. However, from a cosmological parameter estimation point of view disentangling this model from the case with 1.015 neutrinos with mass 0.05 eV, 1.015 with mass 0.01 eV, and 1.015 massless requires much higher precision than what is projected for EUCLID [10] (see e.g. [39] for a recent treatment). Therefore this slightly simplified model is more than adequate for the purpose of this paper.

The resulting posterior likelihood as a function of $\Sigma$ is shown in the left panel of figure 2 and we obtain the formal parameter constraint $\Sigma = 0.060 \pm 0.021$ eV (68% CL). Note that here we marginalize also over $N_{\text{eff}}$ in addition to the 6 parameters of the $\Lambda$CDM model. Similar as above we transform the likelihood now into a likelihood for $m_0$ assuming either NO or IO, see right panel. We ignore the small effects of the different orderings of the neutrino masses and use the same likelihood to describe both normal and inverted orderings. As mentioned above this should be an excellent approximation for the used data set. The relative posterior likelihood for NO and IO is given by the ratio of the areas under the two curves. Assuming equal prior probabilities for NO and IO we obtain a probability for IO according to eq. (2.2) of 8%, which corresponds to posterior odds of NO versus IO of approximately 12:1.

4 Sensitivity estimates with a Gaussian toy likelihood

From figure 2 one can see that the likelihood function as a function of $\Sigma$ is close to Gaussian. This is certainly true for the simulated EUCLID data, but holds approximately also for present data. To estimate the required accuracy needed on $\Sigma$ to exclude IO we assume therefore that the likelihood function from cosmology can be approximated by

$$
\mathcal{L}_{\text{obs}}(\Sigma|m_0,O) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left[ -\frac{(\Sigma_{\text{obs}} - \Sigma(m_0,O))^2}{2\sigma^2} \right]
$$

(4.1)

where $\Sigma(m_0,O)$ is given in eq. (1.1), and $\sigma^2 = \sigma_{\text{osc}}^2 + \sigma_{\text{obs}}^2$, with $\sigma_{\text{osc}}(m_0,O)$ being the error on $\Sigma$ induced by the uncertainty on the mass-squared differences according to eq. (1.2), and
Figure 2. Posterior likelihood function from simulated future data (EUCLID+Planck CMB). The left panel shows the posterior likelihood function for $\Sigma$ for a fiducial model with one massive neutrino with $m_{\nu} = 0.06\, \text{eV}$ and two massless neutrinos. We indicate the predicted values for NO and IO in the case of $m_0 = 0$; the width of the lines corresponds to $\pm 2\sigma$ uncertainty due to current oscillation data. The gray shaded region indicates the one-sided upper bound on $\Sigma$ at 95% CL (flat prior in $\Sigma$). The right panel shows the posterior likelihood as a function of $m_0$ for NO and IO with appropriate relative normalization.

$\sigma_{\text{obs}}$ is the accuracy on $\Sigma$ assumed for the cosmological data. From eq. (1.3) we see that $\sigma_{\text{osc}}$ is below 1 meV for both orderings and $m_0 = 0$. For non-zero $m_0$, $\sigma_{\text{osc}}$ is even smaller. Hence, for $\sigma_{\text{obs}} \gtrsim 0.01\, \text{eV}$, the uncertainty on $\Sigma$ from oscillation data is negligible.

The results based on this toy model for the likelihood are shown in figure 3. Solid curves in the plot show the probability of IO, $p_I$, as a function of $\Sigma_{\text{obs}}$ for assumed values for $\sigma_{\text{obs}}$ ranging from 0.07 eV (corresponding approximately to current data) down to 0.01 eV. Clearly, with an accuracy of order $\gtrsim 0.05\, \text{eV}$ no meaningful statement can be made about the validity of IO. We find that in order to reject IO with a confidence greater than 95% (i.e., $p_I < 0.05$) accuracies of cosmological data of $\sigma_{\text{obs}} \lesssim 0.02\, \text{eV}$ are needed, in agreement with the simple estimate provided in the introduction. If $\sigma_{\text{obs}} = 0.03\, \text{eV}$, the probability of observing a value of $\Sigma$ such that $p_I < 0.05$ is less than 10%, if $m_0 = 0$ (for non-zero $m_0$ the probability is smaller).

The star and the triangle in the plot indicate approximately the cases corresponding to present data and EUCLID-like data, respectively, as considered above. We observe that the Gaussian toy-likelihood reproduces quite accurately the results for $p_I$ obtained for those two cases in section 3, justifying the use of this model to estimate the required sensitivity.

5 Conclusions

If the neutrino mass ordering is normal and the spectrum is hierarchical ($m_0 \ll \sqrt{\Delta m_{31}^2}$) cosmological data has the potential to reject the hypothesis of inverted ordering by constraining the sum of the neutrino masses sufficiently well. We apply Bayes theorem to quantify possible evidence against inverted ordering using present cosmological data as well as simulated data from a future EUCLID-like mission. Our method provides a straightforward way to combine cosmology with oscillation data by including a possible preference for an ordering from oscillation data into the prior probabilities for NO and IO.
Figure 3. Illustration of the potential to exclude IO for a Gaussian toy likelihood. Solid curves show the probability of inverted ordering being correct as a function of the observed value of $\Sigma$ for different assumptions about the obtained accuracy from cosmology, $\sigma_{\text{obs}}$, according to the legend (values in eV). We assume equal prior probabilities for NO and IO. The dashed curves show the probability of observing a value of $\Sigma$ equal or less than the one shown on the horizontal axis assuming that the true ordering is normal and $m_0 = 0$ for the assumed accuracy on $\Sigma$. The thin vertical line indicates the median value for $\Sigma$ for NO and $m_0 = 0$. The star and the triangle show approximately the cases of current and prospective data, respectively, as analysed in section 3.

For present cosmological data, we adopt a particular analysis of Planck CMB + BAO data within the minimal $\Lambda$CDM model (6 parameters + $\Sigma$), which leads to the constraint $\Sigma < 0.14 \text{ eV}$ at 95% CL. For this analysis, our recipe gives posterior odds for normal versus inverted ordering of about 2:1 (the posterior probability for IO is 35%). Combining current cosmological and oscillation data [20] we obtain posterior odds of about 3:2 (the posterior probability for IO is 39%). As expected those results show that current cosmological as well as oscillation data are not sensitive to the mass ordering. However, this analysis provides an example of how to quantify possible evidence against IO from cosmological observation, and how to combine it with information from oscillation data. Both of them are expected to become more sensitive in the near future.

To illustrate this, we generate artificial data for a EUCLID-like survey, assuming a fiducial model with NO and vanishing lightest neutrino mass. Combined with CMB data, this data set would obtain an accuracy to $\Sigma$ of about 0.021 at $1\sigma$, sufficient to disfavour IO at the 92% CL (corresponding to posterior odds of NO to IO of about 12:1). Hence, we conclude that being able to exclude IO with cosmology with significant confidence requires an accuracy on the sum of neutrino masses of better than 0.02 eV.

We emphasize that statements about excluding the inverted ordering with cosmology should be based on a proper statistical analysis. The method we propose in section 2 is based on Bayesian statistics. Usually Bayesian methods are applied for the analysis of cosmological data, and hence our method proposed here for the mass ordering test fits consistently in this framework. Moreover, we can deal in a straight forward way with the boundary implied
by the physics requirement that neutrino masses have to be non-negative. Let us emphasize, however, that the method proposed here is certainly not unique, and it is possible to design alternative ways to report cosmological information on the neutrino mass ordering. Hypothesis tests based on frequentist statistics most likely will require numerical simulations of the relevant test statistics because of the physical boundary. In any case, we would like to encourage the community to report possible evidence against the inverted mass ordering using well defined statistical tools.

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