Low-energy Theorems for Strongly Interacting $W$’s and $Z$’s in $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

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Abstract. In this work, low energy theorems for gauge boson (GB) longitudinal modes of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak gauge theory are derived. These theorems are valid for energies in the domain of applicability of the Equivalence Theorem, and hold for all left-right symmetric models assuming that there are no extra contributions from light scalars to the scattering amplitudes.

1. Introduction

Even though the standard model (SM) is extremely successful, the actual electroweak symmetry breaking (EWSB) mechanism is still unknown. Perhaps the most important goal of the LHC is to elucidate this phenomenon, and an essential piece of the puzzle is to identify the nature of GB scattering reactions $VV \rightarrow VV$. If we do observe these reactions, it means that the quanta of the symmetry breaking sector are of strong magnitude; otherwise, EWSB is accomplished by weakly coupled quanta below $O(1) \text{ TeV}$.

L-R symmetric extensions of the SM based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ have several attractive features[1]. Plenty of L-R symmetric models have been proposed with different contents of Higgs multiplets, but there also exist the possibility of strong dynamics taking place in this kind of models. In this work we find LETs for the scattering of longitudinally polarized GB’s in a general L-R context, without the contribution of any explicit Higgs. Our results hold for all $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models provided they contain no other fundamental bosons with masses below the symmetry breaking scale $M_{SB}$.

2. Effective Lagrangian

The most general $O(p^2)$ electroweak chiral Lagrangian density invariant under local $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ transformations [2] can be written as

$$
\mathcal{L}_{\text{eff}} = -\frac{f_L^2}{4} \text{Tr} \{ X_{L\mu} X_{L}^{\mu} \} - \frac{f_R^2}{4} \text{Tr} \{ X_{R\mu} X_{R}^{\mu} \} + \frac{\beta_L f_L^2}{8} \text{Tr} \{ \tau^3 X_{L}^{\mu} \}^2 + \frac{\beta_R f_R^2}{8} \text{Tr} \{ \tau^3 X_{R}^{\mu} \}^2 \\
+ \frac{\kappa f_L f_R}{2} \text{Tr} \{ X_{L\mu} X_{R}^{\mu} \} + \frac{\beta f_L f_R}{4} \text{Tr} \{ \tau^3 X_{L}^{\mu} \} \text{Tr} \{ \tau^3 X_{R}^{\mu} \} - \frac{1}{2} \text{Tr} \{ A'_{L\mu} A'_{L}^{\mu} \} - \frac{1}{2} \text{Tr} \{ A'_{R\mu} A'_{R}^{\mu} \} - \frac{1}{2} \text{Tr} \{ B'_{\mu\nu} B'_{\mu\nu} \}. 
$$

(1)
Here $A^B_A$ and $B^A$ are the electroweak gauge fields, with indices $j = 1, 2, 3$, $A = L, R$, and $X^\mu_A \equiv U^A_\mu D^\mu U_A$ with $U_A$ as the unimodular matrices that parameterize the corresponding Nambu-Goldstone bosons (NGB). Using the definitions $A^B_A = T^j A^j_A$, $B^A = T^3 B^0$, $U_A = \exp(i\tau^j \pi^j_A / f)$, and $T^j = \tau^j / 2$, the covariant derivative of $U_A$ is $D^\mu U_A = \partial^\mu U_A - ig_A A^B_A U_A + ig_1 U_A B^\mu$, where $g_A$ and $g_1$ are the gauge couplings of $SU(2)_A$ and $U(1)_{B-L}$, respectively. On the other hand, $f_L$ and $f_R$ stand for the scales of EWSB and parity symmetry breaking (PSB), while $\kappa$ is a L-R mixing parameter and the coefficients $\beta_L$, $\beta_R$ and $\beta$ are responsible for L-R mixing caused by custodial symmetry violating interactions at $\mathcal{O}(p^2)$.

Low-energy theorems (LET’s) can be obtained calculating the amplitudes of NGB scattering

\[
\begin{align*}
\mathcal{L}_{\text{eff}} & = \frac{1}{2} \left[ F^2_\gamma + \sum_{\mu = 1}^3 \left( F^2_{\gamma^\mu} + 2 F_{W^{+\mu}} F_{W^{-\mu}} \right) \right], \\
F_{\gamma^\mu} & = \frac{\partial^\mu Z_{1,2}^\mu + M_{Z_{1,2}}^\mu Z_{1,2}^\mu}{\sqrt{\xi_{1,2}}}, \\
F_{W^{+\mu}} & = \frac{\partial^\mu W^{+\mu}_{1,2} + M_{W^{+\mu}_{1,2}} W^{+\mu}_{1,2}}{\sqrt{\xi_{1,2}}},
\end{align*}
\]

where $w^{+\mu}_{1,2}$, $z_{1,2}$ are the linear combinations of NGB’s that become the longitudinal degrees of freedom of each physical GB.

### 3. Low Energy Theorems

Although NGB’s are not part of the physical spectrum, one can compute $S$-matrix elements with them [3, 4]. Fixing the gauge with $\mathcal{L}_{\text{fix}}$ implies that the one-particle state $w_{1,2}^\mu |0\rangle$ has some overlap with the one-particle state $\partial_\mu W_{1,2}^+ |0\rangle$. Equivalence theorem (ET) [5] codifies the fact that two overlapping fields can be traded for each other in $S$-matrix calculations at high energy $E \gg M$:

\[
\mathcal{M} [W_1(p_1), \ldots, W_1(p_n)]_U = \mathcal{M} [w_1(p_1), \ldots, w_1(p_n)]_R + \mathcal{O}(M/E),
\]

Low-energy theorems (LET’s) can be obtained calculating the amplitudes of NGB boson scattering which follow from (1) assuming the global symmetry breaking pattern $SU(2) \times SU(2) \times \overline{U(1)/U(1)}$. Written in terms of $\pi^j_A = (\pi^j_A \mp i\pi^j_A) / \sqrt{2}$, the kinetic part of Lagrangian (1) must be diagonalized and properly normalized to satisfy the definitions of $w^{+\mu}_{1,2}$ and $z_{1,2}$. Now, parameters $f_A$, $\beta_A$, $\kappa$, and $\beta$ can be eliminated in favor of the masses and mixing angles of the GB’s. Definitive expressions for two-body scattering of the lighter NGB’s in the energy range
where both $L_{\text{fix}}$ and ET are valid, $M^2 \ll s \ll \Lambda_{SB}^2$, are:

$$
\mathcal{M}(w_i^+ w_i^- \rightarrow w_i^+ w_i^-) = \frac{e^2 u}{4M_W^4 O_{13}^2 O_{23}^2} \left\{ -4M_W^2 (O_{23}^2 \xi + O_{13}^2 \xi) \\
+ 3M_Z^2 (O_{11} O_{23}^2 + O_{21} O_{13}^2 \xi)^2 \\
+ 3M_Z^2 (O_{12} O_{23}^2 + O_{22} O_{13}^2 \xi)^2 \right\}
$$

(5)

$$
\mathcal{M}(w_i^+ w_i^- \rightarrow z_1 z_1) = \frac{e^2 s}{4M_W^4 M_{W_1}^2 M_{Z_1}^2 O_{13}^2 O_{23}^2} \left\{ M_{W_2}^2 M_{Z_1}^4 (O_{11} O_{23}^2 \xi + O_{21} O_{13}^2 \xi)^2 \\
+ s^2 c^2 \xi M_{W_1}^2 \left[ 4M_W^2 (M_{W_1}^2 - M_{W_2}^2) + (M_{Z_1}^2 + M_{W_2}^2 - M_{W_1}^2)^2 \right] O_{32}^2 \right\}
$$

(6)

where $O_{ij}$ are the elements of the neutral GB mixing matrix (3).

4. Conclusions

We have found LETs for the longitudinal modes of the lighter $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ GB’s. These theorems are valid for energies in the domain $M^2 \ll s \ll \Lambda_{SB}^2$ and hold for all L-R symmetric models assuming that there are no extra contributions from light zero-spin particles to the scattering amplitudes.

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