33 Liaoning University, Shenyang 110036, People’s Republic of China
34 Nanjing Normal University, Nanjing 210023, People’s Republic of China
35 Nanjing University, Nanjing 210093, People’s Republic of China
36 Nankai University, Tianjin 300071, People’s Republic of China
37 North China Electric Power University, Beijing 102206, People’s Republic of China
38 Peking University, Beijing 100871, People’s Republic of China
39 Qufu Normal University, Qufu 273165, People’s Republic of China
40 Shandong Normal University, Jinan 250014, People’s Republic of China
41 Shandong University, Jinan 250100, People’s Republic of China
42 Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China
43 Shanxi Normal University, Linfen 041004, People’s Republic of China
44 Shanxi University, Taiyuan 030006, People’s Republic of China
45 Sichuan University, Chengdu 610064, People’s Republic of China
46 Soochow University, Suzhou 215006, People’s Republic of China
47 South China Normal University, Guangzhou 510006, People’s Republic of China
48 Southeast University, Nanjing 211100, People’s Republic of China
49 State Key Laboratory of Particle Detection and Electronics, Beijing 100049, Hefei 230026, People’s Republic of China
50 Sun Yat-Sen University, Guangzhou 510275, People’s Republic of China
51 Suranaree University of Technology, University Avenue111, Nakhon Ratchasima 30000, Thailand
52 Tsinghua University, Beijing 100084, People’s Republic of China
53 Turkish Accelerator Center Particle Factory Group, (A)Istanbul Bilgi University, 34060 Eqap, Istanbul, Turkey; (B)Near East University, Nicosia, North Cyprus, Mersin 10, Turkey
54 University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
55 University of Groningen, NL-9747 AA Groningen, The Netherlands
56 University of Hawaii, Honolulu, Hawaii 96822, USA
57 University of Jinan, Jinan 250022, People’s Republic of China
58 University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom
59 University of Minnesota, Minneapolis, Minnesota 55455, USA
60 University of Muenster, Wilhelm-Klemm-Str. 9, 48149 Muenster, Germany
61 University of Oxford, Keble Rd, Oxford, UK OX13RH
62 University of Science and Technology Liaoning, Anshan 114051, People’s Republic of China
63 University of Science and Technology of China, Hefei 230026, People’s Republic of China
64 University of South China, Hengyang 421001, People’s Republic of China
65 University of the Punjab, Lahore-54590, Pakistan
66 University of Turin and INFN, (A)University of Turin, I-10125, Turin, Italy; (B)University of Eastern Piedmont, I-15121, Alessandria, Italy; (C)INFN, I-10125, Turin, Italy
67 Uppsala University, Box 516, SE-75120 Uppsala, Sweden
68 Wuhan University, Wuhan 430072, People’s Republic of China
69 Xinyang Normal University, Xinyang 464000, People’s Republic of China
70 Zhejiang University, Hangzhou 310027, People’s Republic of China
71 Zhengzhou University, Zhengzhou 450001, People’s Republic of China

\textsuperscript{a} Also at Bogazici University, 34342 Istanbul, Turkey
\textsuperscript{b} Also at the Moscow Institute of Physics and Technology, Moscow 141700, Russia
\textsuperscript{c} Also at the Novosibirsk State University, Novosibirsk, 630090, Russia
\textsuperscript{d} Also at the NRC ”Kurchatov Institute”, PNPI, 188300, Gatchina, Russia
\textsuperscript{e} Also at Istanbul Arel University, 34295 Istanbul, Turkey
\textsuperscript{f} Also at Goethe University Frankfurt, 60323 Frankfurt am Main, Germany
\textsuperscript{g} Also at the Academy of Sciences, Institute of Nuclear Physics, Moscow 127984, Russia
\textsuperscript{h} Also at the Moscow Institute of Physics and Technology, Moscow 141700, Russia
\textsuperscript{i} Also at Istanbul Arel University, 34295 Istanbul, Turkey
\textsuperscript{j} Also at Goethe University Frankfurt, 60323 Frankfurt am Main, Germany
\textsuperscript{k} Also at Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education; Shanghai Key Laboratory for Particle Physics and Cosmology; Institute of Nuclear and Particle Physics, Shanghai 200240, People’s Republic of China
By analyzing an $e^+e^-$ annihilation data sample corresponding to an integrated luminosity of 2.93 fb$^{-1}$ collected at a center-of-mass energy of 3.773 GeV with the BESIII detector, we measure the branching fraction of the $D^0 \to \rho^+ \mu^- \nu_\mu$ decay for the first time. We obtain $B_{D^0 \to \rho^+ \mu^- \nu_\mu} = (1.35 \pm 0.09_{\text{stat}} \pm 0.09_{\text{syst}}) \times 10^{-3}$. Using the world average of $B_{D^0 \to \rho^- e^+ \nu_e}$, we find a branching fraction ratio of $B_{D^0 \to \rho^- \mu^+ \nu_\mu}/B_{D^0 \to \rho^- e^+ \nu_e} = 0.90 \pm 0.11$, which agrees with the theoretical expectation of lepton flavor universality within the uncertainty. Combining the world average of $B_{D^+ \to \rho^+ \mu^+ \nu_\mu}$ and the lifetimes of $D^0(\rho^0)$, we obtain a partial decay width ratio of $\Gamma_{D^0 \to \rho^- \mu^+ \nu_\mu}/(2\Gamma_{D^+ \to \rho^+ \mu^+ \nu_\mu}) = 0.71 \pm 0.14$, which is consistent with the isospin symmetry expectation of unity within 2.1$\sigma$. For the reported values of $B_{D^0 \to \rho^- \mu^+ \nu_\mu}/B_{D^0 \to \rho^- e^+ \nu_e}$ and $\Gamma_{D^0 \to \rho^- \mu^+ \nu_\mu}/(2\Gamma_{D^+ \to \rho^+ \mu^+ \nu_\mu}$, the uncertainty is the quadratic sum of the statistical and systematic uncertainties.

PACS numbers: 13.20.Fc, 12.15.Hh

Lepton flavor universality (LFU) is usually thought of as a basic property of the Standard Model (SM) [1–4]. It postulates that the couplings between the three families of leptons and gauge bosons do not depend on the lepton flavor. Experimental studies of semileptonic decays of pseudoscalar mesons are important to test LFU and explore possible new physics. Since 2012, tests of LFU have been carried out in several semileptonic $B$ decays at BaBar, Belle, and LHCb. The measured branching fraction ratios $R^{\rho\ell}_{\rho\ell} = B_{B \to \rho^0 \ell^+ \nu_\ell}/B_{B \to \rho^0 \ell^+ \nu_\ell}$ ($\ell = \mu, e$) [5–11] indicate a 3.1$\sigma$ deviation from the value predicted in the SM [12]. This tension stimulated development of various theoretical models [2, 13–17]. In this context, investigations of exclusive semileptonic $D$ decays give important complementary tests of LFU. In recent years, BESIII reported tests of $\mu-e$ LFU with the semileptonic decays $D \to X\ell^+\nu_\ell$ ($X = K, \pi, \omega$, and $\eta$) [18–22]. For each decay, the difference between the measured branching fraction ratio $(R^X_{\mu/e} = B_{D \to X\mu^+\nu_\mu}/B_{D \to X\ell^+\nu_\ell})$ and the corresponding SM prediction is less than 1.7$\sigma$. The decay $D^0 \to \rho^- \mu^+ \nu_\mu$, calculated using the quark potential model in 1989 [23], has not yet been measured. Observation of this decay and verification of the SM prediction for $R^{\rho\mu}_{\mu/e}$ offer a crucial LFU test.

In addition to the quark potential model work [23], the branching fraction of $D^0 \to \rho^- \mu^+ \nu_\mu$ has been calculated using QCD light-cone sum rules (LCSR) [24, 25], the light-front quark model (LFQM) [26], the covariant confined quark model (CCQM) [27, 28], the chiral unitarity approach (χUA) [29], and the relativistic quark model (RQM) [30]. The predicted branching fractions are in the range of $(1.55 - 2.01) \times 10^{-3}$. This decay also provides an opportunity to determine the $c \to d$ Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{td}|$. Furthermore, the measured branching fraction helps constrain lattice QCD calculations on the hadronic form factors of semileptonic $D$ and $B$ decays. More precise calculations of branching fractions and hadronic form factors are key inputs in the determination of CKM parameters [31–34] which allow important tests of CKM matrix unitarity.

Under the assumption of isospin symmetry, the partial width ratio $R^\rho_{IS} = \Gamma_{D^0 \to \rho^- \ell^+ \nu_\ell}/2\Gamma_{D^0 \to \rho^- \ell^+ \nu_\ell} = (B_{D^0 \to \rho^- \ell^+ \nu_\ell} \tau_{D^0})/(2B_{D^0 \to \rho^- \ell^+ \nu_\ell} \tau_{D^0})$ is expected to be unity. Here, $\tau_{D^0(\rho^0)}$ is the lifetime of the $D^0(\rho^0)$ meson. Using the world average values [35], one obtains $R^\rho_{IS} = 0.87 \pm 0.13$, which agrees with unity within the uncertainty. A measurement of the branching fraction of the decay $D^0 \to \rho^- \mu^+ \nu_\mu$ allows a determination of $R^\rho_{IS}$ which tests isospin symmetry in $D^{0(\rho^0)} \to \rho^- \mu^+ \nu_\mu$ decays.

Using a data sample corresponding to an integrated luminosity of 2.93 fb$^{-1}$ [36] taken at a center-of-mass energy of 3.773 GeV with the BESIII detector, we report the first observation and a branching fraction measurement of $D^0 \to \rho^- \mu^+ \nu_\mu$, a determination of $|V_{td}|$ and tests of both LFU with $D^0 \to \rho^- \ell^+ \nu_\ell$ decays and isospin symmetry in $D^{0(\rho^0)} \to \rho^- \ell^+ \nu_\ell$ decays. Throughout this Letter, charge conjugate channels are always implied and $\rho$ denotes the $\rho(770)$. 
Details about the design and performance of the BESIII detector are given in Ref. [37]. Monte Carlo (MC) simulated data samples, produced with a GEANT4-based [38] software package including the geometric description of the BESIII detector and the detector response, are used to determine the detection efficiency and to estimate the backgrounds. The simulation includes the beam-energy spread and initial-state radiation in the $e^+e^-$ annihilations modeled with the generator KKMC [39]. The inclusive MC sample consists of the production of $D\bar{D}$ pairs with consideration of quantum coherence for all neutral $D$ modes, the non-$D\bar{D}$ decays of the $\psi(3770)$, the initial-state radiation production of the $J/\psi$ and $\psi(3686)$ states, and the continuum processes. The known decay modes are modeled with EVTGEN [40] using the branching fractions taken from the Particle Data Group [35], and the remaining unknown decays from the charmonium states are modeled with LUNDCHARM [41]. Final state radiation from charged final state particles is incorporated with the PHOTOS package [42]. This analysis assumes that the same form factors are applicable even in the presence of LFU violation. The vector hadronic form factors of the meson, where

\[ E_{\text{beam}} \] and the beam-constrained mass $M_{BC} \equiv \sqrt{E_{\text{beam}}^2/c^4 - |\vec{p}_{D^0}|^2/c^2}$, where $E_{\text{beam}}$ is the beam energy, and $E_{D^0}$ and $\vec{p}_{D^0}$ are the total energy and momentum of the ST $D^0$ candidate in the $e^+e^-$ center-of-mass frame, respectively. When multiple combinations for an ST mode are present in an event, the combination with the smallest $|\Delta E|$ per tag mode per charge is retained for further analysis. The ST candidates are required to be within $\Delta E \in (-0.055, 0.040)$ GeV for $D^0 \rightarrow K^+\pi^-\pi^0$ and $\Delta E \in (-0.025, 0.025)$ GeV for $D^0 \rightarrow K^+\pi^-\pi^0$. $D^0 \rightarrow K^+\pi^-$ and $D^0 \rightarrow K^+\pi^-\pi^+$. Figure 1 shows the $M_{BC}$ distributions of the accepted ST $D^0$ candidates. For each tag mode, the yield of ST $D^0$ mesons is obtained from a maximum likelihood fit to the $M_{BC}$ distribution of the accepted candidates. In the fit, the signal and background are described by the signal shape from MC simulation and an ARGUS function [52], respectively. To compensate for offsets in calibration and resolution differences between data and MC simulation, the signal shape is convolved with a double-Gaussian function. The means, widths and relative fractions of the Gaussian components are free parameters in the fit. The resulting fits to the $M_{BC}$ distributions are also shown in Fig. 1. Candidates in the $M_{BC}$ mass window (1.859, 1.873) GeV/c$^2$ are kept for further analysis. For each tag mode, the yield of the ST $D^0$ mesons is obtained by integrating the fitted signal shape over the $M_{BC}$ mass window. The total yield of ST $D^0$ mesons is $N_{\text{ST}} = (232.1 \pm 0.2_{\text{stat}}) \times 10^4$.

\[ B_{D^0 \rightarrow \rho^-\mu^+\nu_\mu} = N_{\text{ST}}/\langle N_{\text{DT}} \rangle, \epsilon_{D^0 \rightarrow \rho^-\mu^+\nu_\mu}, \] (1)

where $N_{\text{ST}}$ and $N_{\text{DT}}$ are the yields of the ST and DT candidates in data, respectively. Here, $\epsilon_{D^0 \rightarrow \rho^-\mu^+\nu_\mu} = \Sigma_i [\langle \epsilon_{\text{DT}} \cdot N_{\text{ST}} \rangle]/\langle \epsilon_{\text{ST}} \cdot N_{\text{ST}} \rangle]$ is the effective signal efficiency of finding $D\bar{D} \rightarrow \rho^-\mu^+\nu_\mu$ in the presence of the ST $D^0$ meson, where $\epsilon_{\text{ST}}$ and $\epsilon_{\text{DT}}$ are the detection efficiencies of the ST and DT candidates, respectively, and $i$ labels the ST modes.

In this analysis, the selection criteria for $K^\pm$, $\pi^\pm$, $\gamma$, and $\pi^0$ candidates follow those employed in Refs. [19–22, 44–50]. For the $D^0 \rightarrow K^+\pi^-$ tag mode, backgrounds related to cosmic rays and Bhabha scattering events are vetoed by using the requirements described in Ref. [51]. To distinguish the ST $D^0$ mesons from combinatorial backgrounds, we define the energy difference $\Delta E \equiv E_{\text{D}0} - E_{\text{beam}}$ and the beam-constrained mass $M_{BC} \equiv \sqrt{E_{\text{beam}}^2/c^4 - |\vec{p}_{D^0}|^2/c^2}$, where $E_{\text{beam}}$ is the
for various particle hypotheses (CLυ, CLμ, CLπ, and CLK) are calculated. Charged tracks satisfying CLμ > 0.001, CLμ > CLυ, and CLμ > CLK are identified as muons. In muon identification, no requirement of CLμ > CLπ is applied because of inefficient separation between muon and pion due to their very close masses.

Also, no muon counter information is used because most of muons in D0 → ρ⁺μ⁺νμ have momenta lower than 0.6 GeV/c, which are too low to leave effective information in muon counter. To reduce misidentification of hadrons as muons, the deposited energy in the EMC of the muon candidate (Eµ,EMC) is required to be in the range (0.125, 0.275) GeV. This requirement suppresses about 40% of total background.

The signal yield of the D0 → ρ⁺μ⁺νμ decay is determined by a kinematic quantity defined as M2miss ≡ |pmiss|^2/c^2, which is expected to peak around zero for correctly reconstructed signal events. Here, Emiss ≡ Ebeam − Eρ− − Eμ+ and pmiss ≡ −pρ− − pμ+. are the missing energy and momentum of the DT event in the e⁺e− center-of-mass frame, in which Eμ−(μ+) and pμ−(μ+) are the energy and momentum of the ρ−(μ+) candidates. The M2miss resolution is improved using pD0 ≡ −pD0 = Ebeam/c^2 − mD0c^2, where pD0 is the unit vector in the momentum direction of the ST D0. and mD0 is the D0 nominal mass [35].

The selected sample is contaminated by background events with correctly reconstructed ST mesons but mis-reconstructed signal decays which can peak in the M2miss distribution. Residual backgrounds are mainly due to misidentification between charged pion and muon. They are dominated by few peaking background sources with a fraction of about 75% in total. In order to reject the peaking background from the hadronic decays D0 → K0 S(→ π+π0)π−π− and D0 → K0 S(→ π+π−)π0(π0), the mass recoiling against the D0π−π−π− system and the invariant mass of the πµ→π−π− combination are required to be outside (0.458, 0.538) GeV/c^2 and (0.468, 0.528) GeV/c^2, respectively, where πµ→π denotes a track identified as a muon candidate whose mass has been replaced by the πµ mass. To reduce the peaking background from D0 → π⁺π⁻π⁰, the invariant mass of the ρµ±μ± combination (Mρµ±μ±) is required to be less than 1.5 GeV/c^2. To suppress the peaking background from D0 → π⁺π⁻π⁰, the maximum energy of any photon that is not used in the DT selection (Eextraγmax) is required to be less than 0.25 GeV. With these requirements, about 88% of D0 → π⁺π⁻π⁰ background are rejected and more than 99% of the other backgrounds aforementioned are vetoed.

The remaining peaking background events are mainly from D0 decays into π⁺π⁻π⁰π⁰ final states, including D0 → K0 S(→ π0π0)π⁺π−, D0 → K0 S(→ π⁺π⁻)π⁰π⁰, D0 → K⁻(→ π−π⁰)π⁺π⁰, and D0 → π⁺π⁻π⁰π⁰ non-KS. Since there is little difference in their M2miss shape, these four components are combined together, and will be called D0 → π⁺π⁻π⁰π⁰. The remaining background events from D0 → K0 S(→ π⁺π⁻)π⁰, and D0 → π⁺π⁻π⁰ are negligible and have been combined into the combinatorial background in further analysis.

To suppress the background from D0 → K⁺(892)−(→ K−π⁰)µ⁺νµ, the candidate events are further required not to be within the range |M2missπ−π−K| < 0.05 GeV^2/c^4, where M2missπ−π−K is the M2miss value calculated by replacing the mass of the charged pion candidate with the kaon mass in the calculation of M2miss.

Figure 2 shows the M2miss distribution of the accepted DT events in data. The semileptonic decay yield is obtained from an unbinned maximum likelihood fit to the M2miss distribution. In the fit, the semileptonic signal is modeled by the MC-simulated shape convolved with a Gaussian function describing differences in resolution and calibration between data and MC simulation. The parameters of this Gaussian function are fixed to the values obtained from a similar fit to D0 → ρ⁺e⁺νe candidate events which have much cleaner environment and comparable momentum resolution. The peaking background of D0 → π⁺π⁻π⁰π⁰ is modeled by the M2miss shape derived from the D0 → π⁺π⁻π⁰π⁰ control sample in data, in which one π⁰ is removed and the π⁺ mass is replaced by the µ⁺ mass. The non-peaking backgrounds, including the contribution from wrongly reconstructed ST candidates, are described by the MC-simulated shape obtained from the inclusive MC sample. The yields of the signal, peaking background, and non-peaking backgrounds are free parameters in the fit. The fit result is also shown in Fig. 2. From the fit, we obtain the signal yield of D0 → ρ⁺μ⁺νμ to be NDT = 570 ± 40stat and the yield of the peaking background of D0 → π⁺π⁻π⁰π⁰ to be 373 ± 36. The statistical significance, calculated by √−2ln(L0/Lmax), is greater than 10σ. Here, Lmax and L0 are the maximum likelihoods of the fits with and without the signal component, respectively, and the difference in the number of fit parameters is one.

The tag-related values NDT, εDT, and εDT are summarized in Table 1. The average efficiency of detecting D0 → ρ⁺μ⁺νμ decays is εD0→ρ⁺μ⁺νμ = (18.22 ± 0.13stat)% which includes the branching fraction of D0 → γγ. The kinematic distributions of the D0 → ρ⁺μ⁺νμ candidate events agree well between data and MC simulation, as shown in Fig. 3.

| D0 mode | NDT | εDT (%) |
|---------|------|---------|
| K⁺π⁻   | 516971 ± 746 | 64.28 ± 0.09 | 12.87 ± 0.11 |
| K⁺π⁻π⁰  | 1099361 ± 1327 | 36.35 ± 0.04 | 6.95 ± 0.08 |
| K⁺π⁻π⁻π⁺ | 704677 ± 1094 | 40.26 ± 0.07 | 6.25 ± 0.08 |

Table 1. The ST D0 yields in data (NDT), the ST efficiencies (εDT) and the DT efficiencies (εDT). The uncertainties are statistical only.
Fig. 2. Fit to the $M_{\text{miss}}^2$ distribution of the accepted candidate events for $D^+ \rightarrow \rho^- \mu^+ \nu_\mu$ in data (points with error bars). The solid blue curve is the fit result, the solid black curve is the semileptonic signal, the dashed pink curve is the peaking background (Peaking BKG) of $D^0 \rightarrow \pi^+ \pi^- \pi^0 \pi^0$, and the dashed red curve is the fitted combinatorial background (Fitted CBKG). The filled green histogram is the simulated combinatorial background (Simulated CBKG) from inclusive MC sample.

Inserting $N_{DT}$, $\xi_{D^0 \rightarrow \rho^- \mu^+ \nu_\mu}$, and $N^{\text{stat}}_{SST}$ into Eq. (1), we obtain

$$B_{D^0 \rightarrow \rho^- \mu^+ \nu_\mu} = (1.35 \pm 0.09 \pm 0.09) \times 10^{-4},$$

where the first uncertainty is statistical and the second is systematic as discussed below.

In the branching fraction measurement with the DT method, most uncertainties related to the ST selection cancel. Systematic uncertainties arise from the following sources. The uncertainty in the total yield of ST $D^0$ mesons has been studied in Refs. [19, 20, 44] and is 0.5%. The systematic uncertainties originating from the tracking and PID efficiencies of $\pi^\pm$ are 0.3% and 0.2% per pion, respectively, based on an analysis of DT $D \bar{D}$ hadronic events [53]. The muon tracking and PID efficiencies are studied by analyzing $e^+e^- \rightarrow \gamma \mu^+\mu^-$ events. Here, the muon identification efficiencies include the $E_{\mu,EMC}$ requirement. Using this control sample, data-MC differences are studied in the two-dimensional momentum versus cos $\theta$ plane. We re-weight using the obtained data-MC differences, accounting for the different distribution of events in momentum versus cos $\theta$ for the $D^0 \rightarrow \rho^- \mu^+ \nu_\mu$ signal decays. Systematic uncertainties are obtained as the integral over the re-weighted two-dimensional distribution, giving 0.2% and 0.2% per muon for the muon tracking and PID efficiencies, respectively. The uncertainty of the $\pi^0$ reconstruction is studied with DT $D \bar{D}$ hadronic decays of $D^0 \rightarrow K^-\pi^+, K^-\pi^+\pi^-\pi^-$ versus $D^0 \rightarrow K^+\pi^-\pi^0$, $K_S^0\pi^0$ [19, 44] and is found to be 0.6%. The uncertainty of the combined $E_{\text{extra}}^\text{max}$ and $N_{\text{extra}} \pi^0$ requirements is estimated to be 1.3% by analyzing the DT candidate events of $D^0 \rightarrow \pi^-\pi^0e^+\nu_e$. The uncertainty of the $M_{\text{miss}}^2$ fit is found to be 6.6% by examining the branching fraction changes with an alternative signal shape without Gaussian smearing of the MC-simulated signal shape (0.9%), an MC-simulated shape of the peaking background of $D^0 \rightarrow \pi^+\pi^-\pi^0\pi^0$ (5.3%), and combinatorial background shapes after varying the quoted branching fractions by $\pm1\sigma$ for the two main combinatorial components of $D^0 \rightarrow K_S^0\pi^+\pi^-\pi^0$ and $D^0 \rightarrow K^*(892)^-\mu^+\nu_\mu$ (3.8%). The uncertainty arising from the finite MC statistics used to determine the efficiencies is 0.7%. The uncertainty due to the signal MC model is 0.3%, determined by the difference between our nominal DT efficiency and that determined by varying the input form factors by $\pm1\sigma$. Systematic uncertainties from other selection criteria are found to be negligible. Adding these uncertainties in quadrature yields a total systematic uncertainty of 6.8%.

In summary, the semileptonic decay $D^0 \rightarrow \rho^- \mu^+ \nu_\mu$ has been observed for the first time. The absolute branching fraction of this decay is determined to be $B_{D^0 \rightarrow \rho^- \mu^+ \nu_\mu} = (1.35 \pm 0.09\text{stat} \pm 0.09\text{syst}) \times 10^{-3}$, Table 2 shows comparisons of the measured and predicted branching fractions for $D^0 \rightarrow \rho^- \mu^+ \nu_\mu$. Using the world average value of $B_{D^0 \rightarrow \rho^- e^+\nu_e} = (1.50 \pm 0.12) \times 10^{-3}$ [35], we obtain the branching fraction ratio $R_{\mu/\ell} = B_{D^0 \rightarrow \rho^- \mu^+ \nu_\mu}/B_{D^0 \rightarrow \rho^- e^+\nu_e} = 0.90 \pm 0.11$. This result...
Table 2. Comparison of the measured and predicted branching fractions for $D^0 \rightarrow \rho^- \mu^+ \nu_\mu$. The differences include both experimental and theoretical uncertainties for the LCSR and LFQM models; only experimental uncertainties are used for the other models.

|                | BESIII | LCSR [24] | LCSR [25] | LFQM [26] | CCQM [27] | CCQM [28] | \(\chi_{UA} [29]\) | RQM [30] |
|----------------|--------|-----------|-----------|-----------|-----------|-----------|-----------------|---------|
| \(B_{D^0 \rightarrow \rho^- \mu^+ \nu_\mu}\) \(\times 10^{-3}\) | 1.35 ± 0.09 ± 0.09 | 1.73^{+0.17}_{-0.13} | 1.65 ± 0.23 | 1.7 ± 0.2 | 2.01 | 1.55 | 1.84 | 1.88 |
| Difference (δ) | 2.1 | 1.1 | 1.5 | 5.2 | 1.6 | 3.8 | 4.2 |

agrees with the SM predictions 0.93 – 0.96 [24, 26–30]. Our result is consistent with LFU in $D^0 \rightarrow \rho^- \ell^+ \nu_\ell$ decays. Combining the world averages of $B_{D^+ \rightarrow \rho^+ \ell^+ \nu_\ell}$, $\tau_{D^0}$, and $\tau_{D^+}$ [35], we determine $R_{\ell \ell}^{\rho \mu} = 0.71 ± 0.14$. This ratio deviates from unity based on isospin symmetry at the level of 2.1σ. Improved measurements of $D^0 \rightarrow \rho^- \mu^+ \nu_\mu$ and $D^+ \rightarrow \rho^0 \mu^+ \nu_\mu$ with larger data samples [56, 57] in the near future will be crucial to clarify this tension.

The BESIII collaboration thanks the staff of BEPCII and the IHEP computing center for their strong support. This work is supported in part by National Key Research and Development Program of China under Contracts Nos. 2020YFA0406400, 2020YFA0406300: National Natural Science Foundation of China (NSFC) under Contracts Nos. 11775230, 11625523, 11635010, 11735014, 11822506, 11835012, 11935015, 11935016, 11935018, 1196114010; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; Joint Large-Scale Scientific Facility Funds of the NSFC and CAS under Contracts Nos. U1932102, U1732263, U1832207; CAS Key Research Program of Frontier Sciences under Contracts Nos. QYZDJ-SSW-SLH003, QYZDJ-SSW-SLH040; 100 Talents Program of CAS; INPAC and Shanghai Key Laboratory for Particle Physics and Cosmology; ERC under Contract No. 758462; European Union Horizon 2020 research and innovation programme under Contract No. Marie Sklodowska-Curie grant agreement No 894790; German Research Foundation DFG under Contracts Nos. 443159800, Collaborative Research Center CRC 1044, FOR 2359, GRK 214; Istituto Nazionale di Fisica Nucleare, Italy; Ministry of Development of Turkey under Contract No. DPT2006K-120470; National Science and Technology fund; Olle Engkvist Foundation under Contract No. 200-0605: STFC (United Kingdom); The Knut and Alice Wallenberg Foundation (Sweden) under Contract No. 2016.0157; The Royal Society, UK under Contracts Nos. DH140054, DH160214; The Swedish Research Council; U. S. Department of Energy under Contracts Nos. DE-FG02-05ER41374, DE-SC-0012069.

[1] A. Salam and J. C. Ward, Phys. Lett. 13, 168 (1964).
[2] S. Fajfer, J. F. Kamenik, I. Nisandzic, and U. Rojec, Phys. Rev. Lett. 109, 161801 (2012).
[3] S. Fajfer, I. Nisandzic, and U. Rojec, Phys. Rev. D 91, 094009 (2015).
[4] X. D. Guo, X. Q. Hao, H. W. Ke, M. G. Zhao, and X. Q. Li, Chin. Phys. C 41, 093107 (2017).
[5] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. Lett. 109, 101802 (2012).
[6] J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 88, 072012 (2013).
[7] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 115, 111803 (2015).
[8] M. Huschle et al. (Belle Collaboration), Phys. Rev. D 92, 072014 (2015).
[9] Y. Sato et al. (Belle Collaboration), Phys. Rev. D 94, 072007 (2016).
[10] R. Aaij et al. (LHCb Collaboration), Phys. Rev. D 97, 072013 (2018).
[11] A. Abdesselam et al. (Belle Collaboration), arXiv: 1904.08794.
[12] Y. Amhis et al. (Heavy Flavor Averaging Group), Eur. Phys. J. C 81, 226 (2021).
[13] S. Fajfer, J. F. Kamenik, and I. Nisandzic, Phys. Rev. D 85, 094025 (2012).
[14] A. Celis et al., J. High Energy Phys. 1301, 054 (2013).
[15] A. Crivellin, G. ’Ambrosio and J. Heeck, Phys. Rev. Lett. 114, 151801 (2015).
[16] A. Crivellin, J. Heeck, and P. Stoffer, Phys. Rev. Lett. 116, 081801 (2016).
[17] M. Bauer and M. Neubert, Phys. Rev. Lett. 116, 141802 (2016).
[18] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 122, 011804 (2019).
[19] M. Ablikim et al. (BESIII Collaboration), Eur. Phys. J. C 76, 369 (2016).
[20] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 121, 171803 (2018).
[21] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 101, 072005 (2020).
[22] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 124, 231801 (2020).
[23] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D 39, 799 (1989).
[24] Y. L. Wu, M. Zhong, and Y. B. Zuo, Int. J. Mod. Phys. A 21, 6125 (2006).
[25] X. Leng, X. L. Mu, Z. T. Zou, and Ying Li, Chin. Phys. C 45, 063107 (2021).
[26] H. Y. Cheng and X. W. Kang, Eur. Phys. J. C 77, 587 (2017); 77, 863(E) (2017).
[27] N. R. Soni, M. A. Ivanov, J. G. Körner, J. N. Pandya, P. Santorelli, and C. T. Tran, Phys. Rev. D 98, 114031 (2018).
[28] M. A. Ivanov, J. G. Körner, J. N. Pandya, P. Santorelli, N. R. Soni, and C. T. Tran, Front. Phys. (Beijing) 14, 64401 (2019).
[29] T. Sekihara and E. Oset, Phys. Rev. D 92, 054038 (2015).
[30] R. N. Faustov, O. V. Galkin, and X. W. Kang, Phys.
Rev. D 101, 013004 (2020).

[31] J. Koponen, C. T. H. Davies, and G. Donald (HPQCD Collaboration), arXiv:1208.6242.

[32] J. Koponen, C. T. H. Davies, G. C. Donald, E. Follana, G. P. Lepage, H. Na, and J. Shigemitsu (HPQCD Collaboration), arXiv:1305.1462.

[33] N. Brambilla et al., Eur. Phys. J. C 74, 2981 (2014).

[34] J. A. Bailey et al. (Fermilab Lattice and MILC Collaborations), Phys. Rev. D 85, 114502 (2012); 86, 039904(E) (2012).

[35] P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

[36] M. Ablikim et al. (BESIII Collaboration), Chin. Phys. C 37, 123001 (2013); Phys. Lett. B 753, 629 (2016).

[37] M. Ablikim et al. (BESIII Collaboration), Nucl. Instr. Method A 614, 345 (2010).

[38] S. Agostinelli et al. (GEANT4 Collaboration), Nucl. Instr. Method A 506, 250 (2003).

[39] S. Jadach, B. F. L. Ward, and Z. Was, Phys. Rev. D 63, 113009 (2001); Comput. Phys. Commun. 130, 260 (2000).

[40] D. J. Lange, Nucl. Instrum. Meth. A 462, 152 (2001); R. G. Ping, Chin. Phys. C 32, 599 (2008).

[41] J. C. Chen, G. S. Huang, X. R. Qi, D. H. Zhang, and Y. S. Zhu, Phys. Rev. D 62, 034003 (2000).

[42] E. Richter-Was, Phys. Lett. B 303, 163 (1993).

[43] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett 122, 062001 (2019).

[44] M. Ablikim et al. (BESIII Collaboration), Chin. Phys. C 40, 113001 (2016).

[45] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 123, 231801 (2019).

[46] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 101, 052009 (2020).

[47] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett 124, 241803 (2020).

[48] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett 125, 141802 (2020).

[49] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 102, 052006 (2020).

[50] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 102, 112005 (2020).

[51] M. Ablikim et al. (BESIII Collaboration), Phys. Lett. B 734, 227 (2014).

[52] H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. B 241, 278 (1990).

[53] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 97, 072004 (2020).

[54] N. Cabibbo and A. Maksymowicz, Phys. Rev. 137, B438 (1965).

[55] P. del Amo Sanchez et al. (BABAR Collaboration), Phys. Rev. D 83, 072001 (2011).

[56] M. Ablikim et al. (BESIII Collaboration), Chin. Phys. C 44, 040001 (2020).

[57] E. Kou et al. (Belle II Collaboration), PTEP 2019, 123C01 (2019); 2020, 029201(E) (2020).