Diffractive Photoproduction of $Z^0$

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Abstract

The two-gluon exchange model of the pomeron is used to compute the photoproduction reaction $\gamma p \rightarrow Z^0 p$. The predicted cross section is too small to be observed at HERA, but may be detectable at an eventual Next Linear Collider.
I. INTRODUCTION

The two-gluon exchange model \cite{1-7} has for a long time offered a semi-quantitative understanding of the pomeron in QCD. The model originated as a view of elastic scattering. It has been applied extensively to vector meson photoproduction, and has recently been used to predict jet production in double pomeron interactions \cite{8,9}. In this paper, we use the model to calculate exclusive $Z^0$ photoproduction at low $Q^2$.

The diagrams for $\gamma p \to Z^0 p$ are shown in Figure 1. The upper half of each diagram can be calculated in a straight-forward manner, since only known electroweak and gluonic couplings appear. The lower half of each diagram involves the non-perturbative color structure of the proton, but the required discontinuity can be obtained by parametrizing it and constraining the parameters to fit $pp$ elastic scattering. The calculation of $Z^0$ photoproduction thus introduces no new parameters. A similar calculation can be done for $\gamma p \to \Upsilon p$, where $m_\Upsilon$ provides a large momentum scale; but that calculation is not as clean because it brings in the $\Upsilon$ hadronic wave function.

Reggeization of the gluons and interactions between them (e.g., as given by ladder diagrams) must be responsible for the gradual energy dependence of pomeron exchange \cite{10}. We will ignore these effects here. From a practical standpoint, the energy dependence of $\gamma p \to Z^0 p$ will be dominated by the minimum longitudinal momentum transfer that is kinematically required when the energy is not far above threshold.

The predicted cross section is necessarily small, because the large mass $m_Z$ sets a short distance scale $\sim 1/m_Z$ for the typical transverse separation between the $q\bar{q}$ pair that form the $Z^0$. This leads to a factor $1/m_Z^2$ in the final amplitude, which is proportional to the color dipole moment of the pair. A couple of enhancement factors $\propto \ln m_Z$ arise in the calculation, as explained below; but the final predicted cross section turns out to be too small to observe at the HERA $ep$ collider in its present form.
II. CALCULATION OF $\gamma P \rightarrow Z^0 P$

The imaginary part of the amplitude is equal to $1/2i$ times the discontinuity given by the four diagrams of Figure 1. The upper half of diagram (a), for example, corresponds to a factor

$$D_{\mu\nu}^{(a)} = \int \frac{d^4 k_1}{(2\pi)^4} \frac{[2\pi \delta(k_1^2 - m^2)] [2\pi \delta(k_2^2 - m^2)]}{[(p_1 - k_1)^2 - m^2] [(p_3 - k_2)^2 - m^2]} \text{tr}_a$$

in the discontinuity, where

$$\text{tr}_a = Tr\{\gamma \cdot \epsilon(p_1) [m - i\gamma \cdot (k_1 - p_1)] \gamma_{\mu} (m + i\gamma \cdot k_2) (A + B\gamma_5) \gamma \cdot \epsilon^*(p_3) [m - i\gamma \cdot (k_1 - q_2)] \gamma_{\nu} (m - i\gamma \cdot k_1)\}.$$  \hspace{1cm} (2)

To obtain the high energy limit of the amplitude, we use light-cone coordinates $p_{\pm} = (p_0 \pm p_z)/\sqrt{2}$ where $p_{1+}$ and $p_{2-}$ are large and $s \cong p_{1+} p_{2-}$. One of the two delta functions in Eq. (1) is saved for the eventual integration over gluon momentum $q_1$, which carries no large $+ \ or -$ component as a result of this and a similar delta function from the lower half of the diagram. The other delta function reduces Eq. (1) to a three-dimensional integral over transverse momentum and light-cone momentum fraction $x = k_{1+}/p_{1+}$:

$$D_{\alpha\beta}^{(a)} = \delta(p_1 \cdot q_1 + \mathcal{O}(1)) p_{1\alpha} p_{1\beta} T_a$$

$$T_a = \frac{1}{4\pi^2 s^2} \int_0^1 dx \frac{dx}{x(1-x)} \int d^2 k_{1\perp} \frac{p_{2\mu} p_{2\nu} \text{tr}_a}{d_1 d_2},$$  \hspace{1cm} (3)

where

$$d_1 = (p_1 - k_1)^2 - m^2 \cong [(k_1 - xp_1)^2 + m^2 - x(1-x)p_1^2]/x$$

$$d_2 = (p_3 - k_2)^2 - m^2 \cong [(k_1 - q_2 - xp_3)^2 + m^2 - x(1-x)p_3^2]/(1-x).$$  \hspace{1cm} (4)

For $Q^2 = 0$ we can choose a gauge in which $\epsilon(p_1)$ has only transverse components. Then $\epsilon(p_3)$ can be assumed to have only transverse components as well, because the production of longitudinal $Z^0$ can be shown to be suppressed by a factor $\sqrt{-t}/m_Z$ in the amplitude.
Upon summing over all four diagrams, the contribution from $\gamma_5$ terms is found to vanish in the large $s$ limit. Combining the denominators in Eq. (3) using the Feynman parameter trick $\frac{1}{a_b} = \int_0^1 dy \, [ay + b(1 - y)]^{-2}$ allows the $k_{1\perp}$ integral to be performed, and leads to

$$D^{(1)}_{\alpha\beta} = \delta(p_1 \cdot q_1 + O(1)) \, p_1\alpha \, p_1\beta \, T^{(1)}$$

(5)

$$T^{(1)} = F^{(1)}(q_{2\perp}, \Delta_{\perp}) - F^{(1)}(0, \Delta_{\perp})$$

(6)

where

$$F^{(1)}(q_{\perp}, \Delta_{\perp}) = \frac{A}{\pi} \int_0^1 dx \int_0^1 dy \, \{ \epsilon_\perp(p_1) \cdot \epsilon_\perp^*(p_3) \left( [m^2 - y(1 - y)v^2]/c \right. \\
- \left. [1 - 2x(1 - x)] \ln c \right) + 4v \cdot \epsilon_\perp(p_1) \cdot \epsilon_\perp^*(p_3) \cdot x(1 - x) \cdot y(1 - y)/c \}$$

$$c = m^2 + y(1 - y)v^2 - x(1 - x) [y \, p_3^2 + (1 - y) \, p_1^2]$$

$$v = q_{\perp} + x\Delta_{\perp}.$$  

(7)

Here $p_1^2 = -Q^2$ is approximately zero, $p_3^2 = m_Z^2$, and $\Delta_{\perp} = (p_3 - p_1)_{\perp} = (q_1 - q_2)_{\perp}$ is the transverse momentum transfer.

The $F(q_{2\perp}, \Delta_{\perp})$ term in Eq. (3) comes from the two “off-diagonal” diagrams in which one gluon is attached to the quark line and the other to the anti-quark. The $F(0, \Delta_{\perp})$ term comes from the diagonal diagrams in which both gluons are attached to the same line. There is a strong cancellation between these two contributions when $q_{1\perp}$ or $q_{2\perp}$ is small, because the two-gluon system is color neutral and the $q\bar{q}$ system is spatially compact due to the large $Z^0$ mass. However, the diagonal term will dominate in the full amplitude because the range of eventual integration over $q_{\perp}$ is controlled by $m_Z$.

We can estimate the integral in Eq. (2) using the approximation $m_Z^2 \gg q_{1\perp}^2, q_{2\perp}^2, \Delta_{\perp}^2$. This limit is obtained by splitting the integration into six separate regions according to $0 < y < y_0$ or $y_0 < y < 1$ and $0 < x < x_0$, $x_0 < x < 1 - x_0$, or $1 - x_0 < x < 1$. The result is independent of $x_0$ and $y_0$ in the limit $x_0 \ll 1$, $y_0 \ll 1$ and is given by
\[
T^{(1)} = \frac{-A m^2}{\pi m_Z^2} \left\{ \epsilon_{\perp}(p_1) \cdot \epsilon^*_{\perp}(p_3) \left[ f(q_{1\perp}^2/m^2) + f(q_{2\perp}^2/m^2) - f(\Delta_{\perp}^2/m^2) \right] + [q_{1\perp} \cdot \epsilon_{\perp}(p_1) q_{2\perp} \cdot \epsilon^*_{\perp}(p_3) + q_{2\perp} \cdot \epsilon_{\perp}(p_1) q_{1\perp} \cdot \epsilon^*_{\perp}(p_3)]/m^2 \right\}
\]

\[
f(a) = a \left[ \ln(m^2/m_Z^2) - i\pi - \frac{1}{2} + b \ln \frac{b + 1}{b - 1} \right]
\]

\[
b = \sqrt{1 + 4/a}
\]

(8)

We want the limit of high energy at small momentum transfer, where the helicity non-flip amplitude dominates. We can therefore take \(\epsilon_{\perp}(p_3) \approx \epsilon_{\perp}(p_1) = \frac{1}{\sqrt{2}}(1, -i, 0, 0)\) to obtain

\[
\epsilon_{\perp}(p_1) \cdot \epsilon^*_{\perp}(p_3) \approx 1,
\]

\[
q_{1\perp} \cdot \epsilon_{\perp}(p_1) q_{2\perp} \cdot \epsilon^*_{\perp}(p_3) + q_{2\perp} \cdot \epsilon_{\perp}(p_1) q_{1\perp} \cdot \epsilon^*_{\perp}(p_3) \approx q_{1\perp} \cdot q_{2\perp}.
\]

(9)

It is a good approximation to neglect the quark mass \(m\), since Eq. (8) is not singular in the limit \(m \to 0\). This approximation is reasonably good numerically even for quark masses up to the charm quark mass, in the important regions of \(q_{1\perp}^2, q_{2\perp}^2, \) and \(\Delta_{\perp}^2\), because the corrections to it do not contain the large factor \(\ln m_Z^2\). In this way, we find

\[
T^{(1)} \approx \frac{-A}{\pi m_Z^2} \left[ g(q_{1\perp}^2) + g(q_{2\perp}^2) - g(\Delta_{\perp}^2) \right]
\]

\[
g(q^2) = q^2 \left[ \ln q^2/m_Z^2 - i\pi \right].
\]

(10)

Eq. (10) was derived for the region where \(q_{1\perp}^2, q_{2\perp}^2, \) and \(\Delta_{\perp}^2 = (q_1 - q_2)^2\) are small compared to \(m_Z^2\). We can provide it with approximately correct behavior when these quantities become comparable to \(m_Z^2\) by inserting an additional factor \([1 + (q_{1\perp}^2 + q_{2\perp}^2)/m_Z^2]^{-1}\) into Eq. (10). The adequacy of the resulting approximation to \(T^{(1)}\), for the purpose of calculating the full amplitude, has been checked by numerical integration.

The coefficient \(A\) in Eq. (10) is given by

\[
A = \frac{4\pi^2 \alpha \alpha_s}{\sin \theta_w \cos \theta_w} \left( 1 - \frac{20}{9} \sin^2 \theta_w \right).
\]

(11)
This includes a factor 2 from the sum over quark flavors $u + d$ and $s + c$, and a factor $1/2$ from the sum over quark colors. I take $\alpha_s \cong 0.25$ for the strong coupling at the low average momentum transfer scale that occurs here. This is slightly optimistic, since the amplitude receives contributions from a spectrum of gluon transverse momenta extending all the way up to $\sim m_Z$. It leads to $A = 0.083$.

A reasonable parametrization for the discontinuity of the gluon-proton amplitude represented by the bottom half of Figure 1 has been given in Ref. [8]:

$$D_{\mu\nu} = \delta(p_2 \cdot q_1 + O(1)) p_{2\mu} p_{2\nu} T^{(2)} \quad (12)$$

where

$$T^{(2)} = F^{(2)}(q_2, \Delta) - F^{(2)}(0, \Delta)$$

$$F^{(2)}(q_\perp, \Delta) = N \sqrt{\frac{2\pi ab}{\beta}} \left( a + b \right)^{-2} e^{-(\beta/2)[(a+b)^2-\Delta^2_\perp]}, \quad (13)$$

with $a = \sqrt{q_\perp^2 + 4m_q^2}$ and $b = \sqrt{(\Delta - q)_\perp^2 + 4m_q^2}$. A selection of reasonable parameter choices that have been tuned to fit $pp$ elastic scattering is given in Table I. Dependence of the results on the choice of parameter set provides an estimate of systematic errors.
TABLES

Table I
Parameters and predicted cross section $\sigma$ and slope $B$ for $\gamma p \to Z^0 p$.

| $m_g$ | $m_q$ | $\beta$ | $N$     | $\sigma$[pb] | $B$[GeV$^{-2}$] |
|-------|-------|---------|---------|--------------|-----------------|
| 0.14  | 0.5   | 5.797   | $2.65 \times 10^7$ | 0.014         | 7.0             |
| 0.14  | 0.3   | 4.944   | $4.89 \times 10^3$ | 0.017         | 6.4             |
| 0.30  | 0.5   | 6.659   | $2.13 \times 10^8$ | 0.018         | 7.5             |
| 0.30  | 0.3   | 5.901   | $1.40 \times 10^4$ | 0.020         | 6.9             |
| 1.0   | 0.5   | 7.638   | $4.06 \times 10^9$ | 0.045         | 7.9             |
| 1.0   | 0.3   | 7.098   | $9.04 \times 10^4$ | 0.047         | 7.5             |
The full amplitude for $\gamma p \to Z^0 p$ in the two gluon exchange picture of Figure 1 (including a factor 8 from the sum over gluon colors) is given by

$$\mathcal{M} = \frac{i s}{8 \pi^4} \int \frac{d^2 q_\perp T^{(1)}(q_\perp, \Delta_\perp) T^{(2)}(q_\perp, \Delta_\perp)}{(q_\perp^2 + m_g^2)[(\Delta_\perp - q_\perp)^2 + m_g^2]}$$  \hspace{1cm} (14)

with $\Delta_\perp^2 = -(p_1 - p_3)^2 = -t$. A finite gluon mass $m_g$ is included in the propagators to suppress contributions from long distance, as an approximation to color confinement \cite{1,2}. In elastic scattering, this is necessary to avoid unphysical behavior of the elastic slope in the limit $t \to 0$. The $q_\perp$ integral can be done numerically at each momentum transfer $\Delta_\perp$. The result is enhanced by two powers of $\ln m_Z^2$: one coming directly from $T^{(1)}$ (see Eq. \hspace{1cm} (10)) and the other coming from the integration over $q_\perp$, since the integrand behaves as $dq_\perp^2 / q_\perp^2$ for large $q_\perp^2$ up to $\sim m_Z^2$).

The differential cross section is given by

$$\frac{d\sigma}{dt} = \frac{1}{16 \pi s^2} |\mathcal{M}|^2$$  \hspace{1cm} (15)

in the high energy limit. It is independent of $s$ in that limit. The predicted integrated cross section $\sigma = \int \frac{d\sigma}{dt} dt$ is listed in Table I for each choice of parameters. The result is approximately 0.025 pb, with an uncertainty of about a factor of 2 coming from the range of parameters that provide a plausible description of elastic scattering. (It is interesting to note that this calculated result is not far from the crude estimate of $\approx 0.08$ pb that can be made by scaling the observed cross section for $\gamma p \to J/\psi p$ at HERA \cite{11} by the dipole size factor $(m_{J/\psi}/m_Z)^4$.)

The dependence of $\frac{d\sigma}{dt}$ on momentum transfer $t$ is roughly exponential, so we can characterize it by a slope parameter $B$ such that $\frac{d\sigma}{dt} \propto e^{Bt}$. The value of $B$, defined by reproducing $\sigma$ and $\frac{d\sigma}{dt}$ at $t = 0$, is also listed in Table I. It is approximately equal to the one half the slope of elastic scattering, since it arises essentially from the wave function effect associated with the proton remaining intact, while in $pp$ elastic scattering, both protons must remain intact. Equivalently, one may say that the extent of the interaction in impact parameter space is set by that of the proton.
III. PREDICTION FOR $EP \rightarrow EZ^0 P$

One can hope to look for $Z^0$ photoproduction in electroproduction $ep \rightarrow eZ^0 p$, which will be dominated by low $Q^2$ transverse photons producing $Z^0$'s with the same helicity as the photon. The cross section is given by

$$\frac{d\sigma(ep \rightarrow eZ^0 p)}{dy dQ^2} = f_{\gamma/e}(y, Q^2) \sigma_{\gamma^* p}(W),$$

where

$$f_{\gamma/e}(y, Q^2) = \frac{\alpha^2}{2\pi Q^2} \left[ 1 + \frac{(1 - y)^2}{y} - \frac{2m_e^2 y}{Q^2} \right].$$

is the flux of transverse photons. Defining the four-momenta in $ep \rightarrow eZ^0 p$ as $k, p, k', q', p'$ respectively, the usual kinematic variables are $s = (k + p)^2$, $Q^2 = -(k' - k)^2$, and $y = (k - k') \cdot p / k \cdot p$. The role of $s$ in Sect. II is now played by $W^2 = (k - k' + p)^2 \cong y s$. The minimum value of $Q^2$ is given by $Q^2_{\text{min}} = m_e^2 y^2 / (1 - y)$. The maximum $Q^2$ occurs when the electron is scattered through $180^\circ$, but most of the cross section comes from the region of very small $Q^2$. (We have assumed the limit $Q^2 \rightarrow 0$ in calculating the cross section for $\gamma p \rightarrow Z^0 p$ in Sect. II, but if that condition is relaxed, the model shows no dramatic dependence on $Q^2$.)

We assume that the dependence on $W^2$ is dominated by the effect of the minimum momentum transfer $|t_{\text{min}}|$. Specifically, we assume the approximate dependence $\sim e^{Bt}$ with $B \approx 7 \text{ GeV}^{-2}$, which was found in the large $W$ limit, to be approximately valid at finite $W$ where the minimum value of $t = (p - p')^2$ is different from 0 because of the longitudinal momentum transfer necessary to produce the high-mass state. This assumption might appear optimistic because producing the on-shell intermediate states in Figure 1 involves an increase in mass of the states at both ends of each gluon line, which requires a noticeably larger minimum longitudinal momentum transfer. However, the scale of transverse momentum transfer for each gluon exchange extends all the way to $\mathcal{O}(m_Z)$, so the longitudinal momentum transfer associated with the individual gluon exchanges need not be dominant.
Integrating Eq. (16) at the HERA energy $\sqrt{s} = 300$ GeV leads to $\sigma_{ep \rightarrow eZ^0p}/\sigma_{\gamma p \rightarrow Z^0p} = 0.032$. Combining this with the results listed in Table I yields a predicted cross section for diffractive $Z^0$ production at HERA of approximately 1 femtobarn. A realistic experimental search would have to rely on one of the clean decay modes $Z^0 \rightarrow e^+e^-$ or $Z^0 \rightarrow \mu^+\mu^-$, which would suppress the rate by a branching fraction 0.067. The resulting predicted rate is too small by $2-3$ orders of magnitude to be observed at the HERA $ep$ collider in its present form.

IV. CONCLUSION

The quasi-elastic photoproduction process $\gamma p \rightarrow Z^0p$ has been calculated using the two-gluon exchange model of the pomeron. The calculation is rather clean because the electroweak part of it is completely well defined and the soft proton part can be normalized to elastic scattering.

The process would lead to $Z^0$ production in $ep$ collisions, which would have a clean and dramatic signature: one $e^+e^-$ or $\mu^+\mu^-$ pair with invariant mass equal to the $Z^0$ mass, and hence with very large individual transverse momenta; and nothing else in the detector, since the scattered electron and proton would generally disappear down the beam pipes. Unfortunately, the predicted rate is too small to be observed with the luminosity currently available at HERA. Possible upgrades to HERA that would increase its luminosity by $2-3$ orders of magnitude could make the process observable.

Because of the higher available energy, the related diffractive process $\gamma\gamma \rightarrow Z^0\rho^0$ should be observable with a $\gamma\gamma$ option at a $\mathcal{O}(1\,\text{TeV})$ Next Linear Collider [12]. The cross section can be estimated from Sect. [1] and Regge factorization as $\sim 0.3$ femtobarns for $\gamma\gamma$ energies far above threshold.
ACKNOWLEDGMENTS

This work was supported in part by U.S. National Science Foundation grant number PHY-9507683.
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FIG. 1. Two gluon exchange model for $\gamma p \rightarrow Z^0 p$. The dashed line denotes an s-channel discontinuity.