Simulation Biomechanical Modeling of Human Walking

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Abstract. The purpose of this paper is simulation parameters of human walking in normal and pathological conditions. Modeling does possible to determine dynamic characteristics of motion and to optimize designs of walking mechanisms, prostheses, orthoses, implants and more. In developing the model of human walking, we used a special case of the Lagrange equation of the second kind. The experiment was conducted by video analysis of real gait of people of different ages and sex. As a result of the simulation the speeds and accelerations of all links of the model, moments in joints, and peak values of moments are determined. Simulation of human walking makes it possible to develop technical specifications for the design of walking mechanisms, prostheses, implants and more.

1. Introduction
Simulation parameters of a person walking in normal and pathological conditions makes it possible to determine the torque, support reactions, dynamic characteristics of motion, energy loss in joints of lower limbs in phases of its support or transfer. The mathematical model allows to determine the laws of control of the musculoskeletal system, analyze their violations, optimize designs of prostheses, implants, various biomechanical devices, robots, the basic data for the development of actuators, which is relevant in the Prosthetics, sports biomechanics, medicine, robotics.

2. Simulation parameters of a person walking
A mathematical model has been developed on the basis of fundamental sciences: physics, with the use of numerical methods for solving the problems of mechanics and mathematics. Musculoskeletal human system consists of three interconnected systems: bones, muscles, nervous. Mathematical model of distance should take into account the presence of all three systems. In this case, we get very cumbersome and complex model to realize that would be virtually impossible. In addition, this model will have a significant error.

Analysis of essential and nonessential factors and components of the proposed model, analysis of studies of walking parameters available in the literature was carried out [1–6].

One of the tasks of biomechanics of the lower limb is to choose the optimal drive power for each joint. To ensure the required drive power, it is necessary to take into account the elastic properties of the muscle-tendon structures of the joints of the links of the system, which significantly affect the joint moments.

In this article the planar four–link model imitates a walking human in the sagittal plane (Figure 1). Skeletal system is presented as follows: femur, tibia, upper part (body) – absolutely rigid links long l1, l2, l4 with constant mass center position C1, C2, C4, with masses m1, m2, m4 and constant moments
of inertion \( f_{z1}, f_{z2}, f_{z4} \) consequently. Side foot contour with approximated triangle weight \(-m_3\), and its position is set by changing the position link length \(-l_3\), triangle connecting center of gravity \( C_3 \) and ankle joint. The hip, knee and ankle joints pivotally interconnected planar uniaxial hinges without friction; communication between stationary units.

![Figure 1. Model of human foot.](image)

Muscular system includes three coil springs with stiffness’s \( c_1, c_2, c_3 \), simulating muscle–tendon junctions structure units of the system: body – hip, thigh – drumstick – stop. The nervous system is not considered in the model due to the complexity of its formalization. Attached to the link model moments simulating musculo–tendinous structures: from the hip – \( M_1 \), to the shin – \( M_2 \), to the foot – \( M_3 \), to the body – \( M_4 \). The uniaxial hinges (joints) applied torques (moments muscle force) equal to the sum or difference of moments \( M_1, M_2, M_3, M_4 \) depending on the position of the lower limb in the coordinate system.

The center of mass coordinates of the model has \( x_0 \) and \( y_0 \) is located in the hip (in the hinge connecting the body and the thigh). In comfortable walking vertical component of the center of mass velocity is not considered, so the model is moved at a constant horizontal velocity component \( V_c \).

The position of the model links in a plane coordinate system is given by two Cartesian coordinates and four generalized coordinates. Generalized coordinates are angles of deviation of model links from vertical axis: of the hip \( \alpha \), of the shin \( \beta \), of the body \( \gamma \), of the foot \( \varphi \). The experimental values of these angles are presented in the table 1.

To produce lower limb motion equations, we should use the special case of the Lagrange equations of the second kind:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\theta}_i} \right) - \frac{\partial T}{\partial \theta_i} = M_i - \frac{\partial \Pi}{\partial \dot{\theta}_i},
\]

where \( \theta_i \) and \( \dot{\theta}_i \) – generic \( i \)-th coordinate and speed system, \( M_i \) – actuating torque (or moment of muscular strength), acting on the \( i \)-th generalized coordinate; \( T_i \) and \( \Pi_i \) – kinetic energy and potential energy of each of the links in the mathematical model.

Kinetic and potential energy links limb:

\[
T = 0.5 \sum_{k=1}^{4} f_{z_k} \dot{\theta}_i^2 + 0.5 \sum_{k=1}^{4} m_i v_{c_i}^2,
\]

\[
\Pi = \sum_{k=1}^{4} \Pi_i = \sum_{k=1}^{4} m_i g y_i
\]

where \( y_i \) – value of the ordinate of the center of mass of each of the links in the mathematical model. The coordinates of the system units’ mass center: the hip, shin, feet and body, respectively.
\[ \begin{align*}
x_1 &= x_0 + 0.5l_1 \sin \alpha; \hspace{1em} y_1 = -0.5l_1 \cos \alpha \\
x_2 &= x_0 + l_1 \sin \alpha + 0.5l_2 \sin \beta; \hspace{1em} y_2 = -l_1 \cos \alpha - 0.5l_2 \cos \beta \\
x_3 &= x_0 + l_1 \sin \alpha + l_2 \sin \beta + l_3 \sin \gamma; \hspace{1em} y_3 = -l_1 \cos \alpha - l_2 \cos \beta - l_3 \cos \varphi \\
x_4 &= x_0 + 0.5l_4 \sin \gamma; \hspace{1em} y_4 = 0.5l_4 \cos \gamma 
\end{align*} \]

Centers of mass movement speed units:
\[ V_c = \sqrt{\dot{x}_c^2 + \dot{y}_c^2} \]

Considering expressions (2)–(5), the kinetic and potential energy of the lower limb movement are equal:
\[ \begin{align*}
T &= 0.5V_c^2(m_1 + m_2 + m_3 + m_4) + 0.5\ddot{\alpha}^2[l_1^2(0.25m_1 + m_2 + m_3) + J_{z_1}] \\
&+ 0.5\ddot{\beta}^2[l_2^2(0.25m_2 + m_3) + J_{z_2}] + 0.5\ddot{\gamma}^2(m_3l_3^2 + J_{z_3}) \\
&+ 0.5\dot{\theta}^2(0.25m_4l_4^2 + J_{z_4}) + \ddot{\alpha} \cos \alpha V_c l_1(0.5m_1 + m_2 + m_3) \\
&+ \ddot{\beta} \cos \beta [V_c l_2(0.5m_2 + m_3)] + \ddot{\gamma} V_c l_4 m_4 \cos \gamma \\
&+ \dddot{\alpha} \ddot{\beta} l_1 l_2 (0.5m_2 + m_3)(\cos(\alpha - \beta)) + \dddot{\alpha} \ddot{\gamma} l_1 l_3 m_3 \cos(\alpha - \varphi) \\
&+ \dddot{\beta} \ddot{\gamma} l_2 l_3 \cos(\beta - \varphi); \\
\Pi &= -0.5m_1 l_1 g \cos \alpha - m_2 g (l_1 \cos \alpha + 0.5l_2 \cos \beta) \\
&- m_3 (g l_1 \cos \alpha + 0.5l_2 \cos \beta + l_4 \cos \gamma) + 0.5m_4 l_4 \cos \gamma \\
&+ c_1(\alpha + \gamma)^2 + c_2(\alpha + \beta)^2 + c_3(\beta - \varphi)^2
\end{align*} \]
where \( \dddot{\alpha}, \dddot{\beta}, \dddot{\gamma}, \dddot{\phi} \) - angular velocities of the centers of mass of the model links.

Articular moments in the hip, knee and ankle joints are determined by the expressions:
\[ \begin{align*}
M_{14} &= M_1 - M_4 \\
M_{12} &= M_1 - M_2 \\
M_{23} &= M_2 - M_3
\end{align*} \]

After substituting expressions (6) and (7) into (8) and further mathematical transformations have
\[ \begin{align*}
&\dddot{\alpha} l_1 l_2(0.5m_2 + m_3) \sin(\alpha - \beta) + \dddot{\beta} l_1 l_2(0.5m_2 + m_3) \sin(\alpha - \varphi) \\
&+ m_3 l_3 \dddot{\alpha} l_3 \sin(\alpha - \varphi) + g l_1(0.5m_1 + m_2 + m_3) \cos(\alpha - \beta) + \dddot{\alpha} l_2 l_3 (0.5m_2 + m_3) \sin(\alpha - \varphi) \\
&+ \dddot{\beta} l_2 l_3 (0.5m_2 + m_3) \sin(\alpha - \beta) + \dddot{\gamma} l_2 l_3 (0.5m_2 + m_3) \sin(\beta - \varphi) + \dddot{\gamma} l_2 l_3 (0.5m_2 + m_3) \sin(\beta - \gamma) \\
&+ g l_2 \sin \beta (0.5m_2 + m_3) + 2(c_1(\alpha + \gamma) + c_2(\alpha + \beta) + c_3(\beta - \varphi)) = -M_1; \\
&\dddot{\gamma} l_2 l_3 (0.5m_2 + m_3) \sin(\alpha - \beta) + \dddot{\alpha} l_2 l_3 (0.5m_2 + m_3) \sin(\alpha - \varphi) + \dddot{\beta} l_2 l_3 (0.5m_2 + m_3) \sin(\beta - \gamma) \\
&+ \dddot{\gamma} l_2 l_3 (0.5m_2 + m_3) \sin(\beta - \gamma) + 2(c_1(\alpha + \gamma) + c_2(\alpha + \beta) + c_3(\beta - \varphi)) = -M_2; \\
&\dddot{\gamma} l_2 l_3 (0.5m_2 + m_3) \sin(\alpha - \beta) + \dddot{\alpha} l_2 l_3 (0.5m_2 + m_3) \sin(\alpha - \varphi) + \dddot{\beta} l_2 l_3 (0.5m_2 + m_3) \sin(\beta - \gamma) \\
&+ \dddot{\gamma} l_2 l_3 (0.5m_2 + m_3) \sin(\beta - \gamma) + 2(c_1(\alpha + \gamma) + c_2(\alpha + \beta) + c_3(\beta - \varphi)) = -M_3; \\
&\dddot{\gamma} l_2 l_3 (0.5m_2 + m_3) \sin(\alpha - \beta) + \dddot{\alpha} l_2 l_3 (0.5m_2 + m_3) \sin(\alpha - \varphi) + \dddot{\beta} l_2 l_3 (0.5m_2 + m_3) \sin(\beta - \gamma) \\
&+ \dddot{\gamma} l_2 l_3 (0.5m_2 + m_3) \sin(\beta - \gamma) + 2(c_1(\alpha + \gamma) + c_2(\alpha + \beta) + c_3(\beta - \varphi)) = -M_4.
\end{align*} \]

where \( \dddot{\alpha}, \dddot{\beta}, \dddot{\gamma}, \dddot{\phi} \) – angular accelerations of the centers of mass of the model links.

3. Results of research

During the experiment 50 people of different age and sex were examined by video analysis, average period of limb movements are determined as \( T = 3.33 \) seconds discrete values and twelve units of rotation angles \( \alpha, \beta, \varphi, \gamma \) in a time interval equal to the period, and the weight, length, and the moments of inertia units. Graphs \( \alpha = f(t), \beta = f(t), \varphi = f(t), \gamma = f(t) \) are presented in Figure 2. The law of motion of the lower limb in the sagittal plane does not contradict the results of the studies presented in [1–3].
After approximations of discrete angles of rotation units $\alpha, \beta, \varphi, \gamma$ polynomials of the 10-th order [6] with accuracy 0.0001, we obtain the law of motion of the lower limb in the sagittal plane:

$$\alpha = 0.0068t^{10} - 0.1560t^9 + 1.5072t^8 - 8.00706t^7 + 25.5535t^6 - 50.18496t^5 + 59.4617t^4 - 39.3583t^3 + 11.7655t^2 - 0.7437t + 0.1657$$

$$\beta = -0.0233t^{10} + 0.4289t^9 - 3.2895t^8 + 13.6072t^7 - 32.9975t^6 + 47.9860t^5 - 41.3880t^4 + 20.1798t^3 - 4.7934t^2 + 0.0460t + 0.1593$$

$$\gamma = 0.0003t^{10} - 0.0029t^9 - 0.0033t^8 + 0.1766t^7 - 1.0529t^6 + 2.9727t^5 - 4.4385t^4 + 3.3188t^3 - 0.9779t^2 + 0.0310t + 0.0182$$

$$\varphi = -0.0133t^{10} + 0.2579t^9 - 2.0874t^8 + 9.1756t^7 - 23.7036t^6 + 36.1632t^5 - 30.2759t^4 + 9.9073t^3 + 2.69130t^2 - 2.0907t - 0.0592$$

(11)

After differentiation of discrete values of the angles $\alpha, \beta, \varphi, \gamma$ (11) in a time interval equal to the period of the motion, the values of $\dot{\alpha}, \ddot{\alpha}, \dot{\beta}, \ddot{\beta}, \dot{\varphi}, \ddot{\varphi}, \dot{\gamma}, \ddot{\gamma}$ are determined.

The values of the rotation angles of the model links, angular velocities and angular accelerations of the centers of mass of the model links are presented in table 1.

Units specified average parameters for adolescence $m_1 = 9.84\ kg$, $m_2 = 4.37\ kg$, $m_3 = 1.6\ kg$, $m_4 = 21\ kg$, $l_1 = 0.44\ m$, $l_2 = 0.37\ m$, $l_3 = 0.044\ m$, $l_4 = 0.32\ m$, $J_{z1} = 0.37\ kg\cdot m^2$, $J_{z2} = 0.132\ kg\cdot m^2$, $J_{z3} = 0.0099\ kg\cdot m^2$, $J_{z4} = 10.5\ kg\cdot m^2$, after substituting them for (10) and (9), the joint points are defined.

Graphs changes articular moments $M_{14} = f(t)$, $M_{12} = f(t)$, $M_{23} = f(t)$ are described in Figure 3.

![Graphs](image_url)

**Figure 2.** Graphs of the dependence of generalized coordinates on time.
Table 1. The values of the rotation angles of the model links, angular velocities and angular accelerations of the centers of mass of the model links.

| t, s  | 0     | 0.33  | 0.66  | 0.99  | 1.32  | 1.65  | 1.98  | 2.31  | 2.64  | 2.97  | 3.33  |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| α, rad| 0.157 | 0.267 | 0.241 | 0.007 | -0.077 | -0.100 | -0.169 | -0.143 | 0.098 | 0.198 | 0.157 |
| α, rad/s | -0.558 | 0.210 | -0.705 | -0.360 | -0.152 | -0.215 | -0.047 | 0.391 | 0.543 | 0.184 | -0.558 |
| α, rad/s² | 19.873 | -4.856 | -0.030 | 1.275 | -0.024 | -0.006 | 1.125 | 1.166 | -0.474 | -1.313 | 19.873 |
| β, rad | 0.017 | -0.005 | 0.021 | 0.042 | 0.024 | 0.012 | -0.005 | 0.021 | 0.042 | 0.024 | 0.017 |
| β, rad/s | -0.028 | -0.279 | -0.202 | -0.041 | -0.042 | -0.775 | -1.387 | -0.564 | 1.390 | 1.943 | -0.028 |
| β, rad/s² | -7.723 | 0.925 | 0.069 | 0.682 | -1.148 | -2.623 | -0.123 | 5.148 | 5.090 | -2.940 | -7.723 |
| γ, rad | 0.016 | -0.008 | 0.102 | 0.007 | -0.080 | -0.050 | 0.035 | 0.068 | 0.016 | -0.042 | 0.016 |
| γ, rad/s | 1.641 | -0.602 | -0.063 | -0.386 | -0.094 | 0.248 | 0.227 | -0.056 | -0.219 | -0.084 | 1.641 |
| γ, rad/s² | -0.070 | -0.384 | -0.087 | -0.044 | -0.017 | 0.122 | 0.052 | -0.174 | 0.035 | -0.070 |
| φ, rad | -2.043 | 0.283 | 0.552 | 0.280 | 0.195 | 0.039 | -0.308 | -0.295 | 0.277 | 0.447 | -2.043 |
| φ, rad/s | 6.261 | 3.843 | -1.161 | -0.416 | -0.023 | -1.005 | -0.831 | 1.135 | 1.762 | -1.414 | 6.261 |

Figure 3. Graphs of the dependence of articular moments on time.

Thus, as a result of simulation walk in different age groups for both men and women can be drawn the following conclusions:
- the moments in joints have a maximum value at the time of flexion and extension of the lower limb;
- the knee — the highest point;
- every joint must have an individual actuator located near the axis of rotation of the joint;
- simulation method is a universal method that can analyze the dynamics of human movement;
- according to the simulation results, the use of hydraulic drive with electric motors is recommended for the lower limb;
- the mathematical model is adequate, does not contradict the literature and is suitable for use in prosthetics.

References
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