Post-buckling analysis of the shape memory polymer sandwich composite beam under dynamic temperature variation

Achche Lal1* and Kanif Markad2
1,2 Department of Mechanical Engineering, S.V.N.I.T., Surat-395007, Gujarat, India.
*E-Mail: lalachchhe@yahoo.co.in

Abstract: Post-buckling analysis of the shape memory polymer sandwich composite (SMPSC) beam under HSDT utilizing von Karman kinematics using FEM is performed in present analysis. The aim of the study is to develop a model which accurately perform the buckling analysis. The nondimensional critical buckling load (NCBL) evaluation under the action of inplane uniform load with different boundary conditions (BC), plate thickness ratio, under dynamic temperature variation for SMPSC. The study clearly revealed the differentiation between SMPC and SMPSC beam.

Keyword: HSDT, SMPSC, FEM, NCBL, glass transition temperature ($T_g$)

1. Introduction

Shape memory polymers (SMP) responds in the vicinity of various stimuli such as mechanical stress, heat, pH, moisture, light, electricity, magnetic field or solvents. It can be deformed into intermediate shape and as it comes in contact with corresponding sensation and regain its programmed shape. Temperature stimulated shape memory polymers have become popular in last three decades as an effective alternate of another widely used smart material - Shape Memory Alloy (SMA). Kumpfer et al. [1] utilized soft poly and hard metal phase for investigating shape fixing and recovery of materials. Considerable research has been done on the experimental estimation of glass transition region of SMP selected for analysis. Properties needed to perform analysis can either be obtained through experimentation or by curve-fitting of the experimental results Wang et al. [2]. Qi et al. [3] analyzed and performed the finite deformation thermo-mechanical behavior of thermally induced shape memory polymers. Lal and Markad [4, 5] performed the post buckling, deflection and stress analysis of MWCNT reinforced composite beam under HSDT utilizing FEM. Kumar and Srinivas [6] were examined the buckling investigation of functionally graded (FG) CNT beam, and also property of the composites were found either by Mori Tanaka or rule of mixture method. Many of them also analyzed the effect of beam aspect ratio, volume fraction of CNT ($V_{cnt}$) over buckling response of the beam. Karamanli and Aydogdu [7] performed the buckling analysis of composite laminated beam with the inplane variable load with different BC and beam aspect ratio. The properties of composite material is estimated through the theory of volume averaging, as per the assumption that new two phase matrix and fiber have similar deformations Shen et al. [8]. Zhang et al. [9] studied micro-buckling of fibres reinforced in Elastomeric Memory Composites (EMC) under deformation due to bending. Carbon reinforced in SMP matrix has been considered as EMC, applicable for deployable antennas subjected to high strains. Buckling phenomenon of EMC was elaborated and its calculated results were compared with Lan et al. [10], which indicated appreciable convergence to define accuracy of results.

From the studied limited literature it is observed that, no literature is observed who study the effect of post buckling analysis of shape memory polymer composite and sandwich structures under inplane UDL with HSDT utilizing FEM. In present paper, effect of dynamic temperature variation over post buckling analysis
of SMPC and SMPSCB is analyzed for different thickness ratio, BC. Study also clearly defines the effect of the core thickness variation over buckling analysis of the composite beam.

2. Mathematical formulation

2.1 Modeling of SMP effect

Material properties of the selected SMP matrix has been extensively evaluated in the prior work of Kumpfer et al. [1]. Thus the characteristics required for the numerical analysis can either be experimentally investigated or can be determined through curve fitting of experimental data Wang et al. [2]. Storage modulus $E_m(T)$ of polymers is find by Eq. (1) for wide range of temperatures across $T_g$ with a constant frequency, according to X. Wang et al. [2].

$$E_m(T) = (E_2 - E_3) \exp \left( -\frac{T}{T_g} \right) + E_3$$

The values for Eq. (1) unknown considered as $T_g = 68 \, ^\circ \text{C}$, $E_2=1053.41 \, \text{MPa}$, $E_3=44.11 \, \text{MPa}$ and $m_2 = 41$.

Poisson’s ratio is also temperature dependent property which can be calculated through phase transition model as per (Qi et al. 2008),

$$\mu_m = \mu_f f_g + \mu_v (1 - f_g) \text{ where, } f_g = 1 - \frac{1}{1 + \exp[-(T - T_m) / Z]}$$

Where reference temperature defined by $T_m$ and $f_g$ is the frozen phase volume fraction

2.2 Estimation of carbon fiber reinforced SMPC properties

Based on Eqs. (3-7), properties of laminated SMPC has been calculated and applied in the subsequent sections.

$$E_{c1} = E_{f1} v_f + E_w v_w$$

$$E_{c2} = (1 - C) E_{c1} + C E_{c2}$$

$$\mu_{c21} = (1 - C) \mu_{c21} + C \mu_{c21}$$

$$\mu_{c12} = \mu_{c21} \frac{E_{c2}}{E_{c1}}$$

$$G_{c12} = (1 - C) G_{c12} + C G_{c12}$$

Where $E_{c1}$ and $E_{c2}$ are modulus of composites along longitudinal and transverse directions respectively. $E_{f1}$ and $E_{f2}$ are Young’s modulus of fiber along longitudinal and transverse directions respectively. $G_{f12}$ is shear modulus along plane 1-2. $G_{c12}$ is shear modulus of composite in 1-2 plane. C is transverse contact coefficient between fibers. $\mu_f$ is Poisson’s ratio of carbon fiber. $\mu_{c12}$ and $\mu_{c21}$ indicate Poisson’s ratio of composites along 1-2 and 2-1 plane respectively.

Table 1: Material properties of reinforcement [8]

| Parameters | $E_{f1}$ | $E_{f2}$ | $G_{f12}$ | $\mu_f$ | C |
|------------|----------|----------|------------|---------|---|
| Values     | $2.3 \times 10^8 \, \text{MPa}$ | $8.2 \times 10^8 \, \text{MPa}$ | $2.73 \times 10^8 \, \text{MPa}$ | 0.25 | 0.2 |

2.3 Displacement field model
In present analysis one dimensional composite beam is utilized for the buckling and vibrational analysis under variable inplane nonuniform loading. The composite beam is having length, width and thickness is \( a, b \) and \( t \) respectively, as shown in the figure 2.

![Figure 1: Influence of variation of temperature on modulus: (a) longitudinal, (b) transverse (c) shear modulus and (d) Poissons ratio of SMPC](image)

\[ \bar{u}(x, z) = u_0 + \left( z - \frac{4}{3} h^2 z^3 \right) \psi_x + \left( -C_x z^2 \right) \phi_x; \quad \bar{w}(x, z) = w_0 \]  

(8)

where \( u_0, w_0, \) and \( \phi = \partial w / \partial x \) are displacement along mid-plane axis, displacement along transverse direction, rotation of normal to the mid-plane along ‘y’- axis and slope along ‘x’- axis, respectively.

Displacement vector for the modified C0 continuous model can be expressed as,

\[ \{ q \} = \begin{bmatrix} u & w & \phi_x & \psi_x \end{bmatrix}^T \]  

(9)

2.4 Stress, Strain and displacement relation

Here, total strain is the combination of linear \( \{ \varepsilon^L \} \) and nonlinear strain vector \( \{ \varepsilon^{NL} \} \) with von Karman nonlinearity expressed as, Lal and Markad [4, 5],

\[ \{ \varepsilon \} = [B] \{ q \} + \frac{1}{2} [A_{nl}] \{ \phi_{nl} \} \]  

(10)

Similarly, plane stress and total strain relationship expressed as, Lal and Markad [4, 5],
\[
\{\sigma\} = [Q_{ij}]{\varepsilon}
\]  \hspace{1cm} (11)

![Diagram of sandwich composite beam](image)

**Figure 2:** Geometrical configuration of sandwich composite beam

### 2.5 Finite element analysis of the composite beam

In the present work, a \(C^0\) 1-D Hermitian beam element with four degrees of freedom per node is employed, [Lal and Markad 2018], rewritten as this type of beam element geometry and the displacement vector.

\[
\{q\} = \sum_{i=1}^{NN} N_i \{q\}_i; \quad x = \sum_{i=1}^{NN} N_i x_i;
\]  \hspace{1cm} (12)

In order to calculate transverse displacement and slope, linear interpolation for axial displacement and rotation of normal and Hermite cubic interpolation functions are considered.

\[
\Pi_1 = \sum_{e=1}^{NE} \Pi_a^{(e)} = \sum_{e=1}^{NE} \left( U_L^{(e)} + U_{NL}^{(e)} \right)
\]  \hspace{1cm} (13)

Where, \(NE\) and \((e)\) denote the number of elements and elemental, respectively.

Eq. (13) can be further evaluated as,

\[
\Pi_1 = \frac{1}{2} \sum_{e=1}^{NE} \left[ (q)^T[K_i + K_m]q^{(e)} \right] = (q)^T[K_i + K_m](q)
\]  \hspace{1cm} (14)

Work done due to inplane variable nonuniform loading can be represented as follows, [5]

\[
\Pi_2 = \frac{1}{2} \int_A N_n^{(e)} \left( \frac{\partial u}{\partial x} \right)^2 dA \quad \text{where,} \quad N_n^{(e)} = N
\]  \hspace{1cm} (15)

\(N\) is the inplane uniformly distributed load (UDL) as shown in the figure 3.

Mechanically induced buckling and vibrational analysis of the inplane variable hydrostatic loading over laminated composite beam evaluated by the minimization of first variation of \((\Pi_1-\Pi_2)\) with respect to generalized displacement vector is given by, Lal and Markad [4, 5]

\[
\frac{\partial}{\partial \{q\}} (\Pi_1 - \Pi_2) = 0
\]  \hspace{1cm} (16)

\[
\{[K_i + K_m] - \lambda[K_G]\}\{q\} = 0
\]  \hspace{1cm} (17)

\[
[K]\{q\} = \lambda[M]\{q\}
\]  \hspace{1cm} (18)

\([K] = \{[K_i + K_m] - [K_G]\}\)

Where, \([K_G]\) is the geometric stiffness matrix, \([M]\) is the global consistent mass matrix [27] and \(\{q\}\) is the transverse deflection. The solution of the above buckling standard equation obtained by Newton Raphson
method which is the most popular solution method due to its quick convergence. Eq. (17) and Eq. (18) 
corresponds to buckling and vibration analysis of the considered structure.

3. Results and discussion

User defined MATLAB program is developed based on defined formulation to find the post buckling of 
the SMP sandwich composite beam under dynamic temperature variation. One dimensional composite 
beam under uniformly distributed load is considered for the present simulation. Temperature dependent 
polymer is considered and its effect over analysis is studied over a wide range of temperature that includes 
glass transition region of SMPSC so that the buckling phenomenon with respect to uniform variation of 
temperature can be understood.

Table 2 shows the convergence study to select the proper number of element for further investigation 
without diluting the solutions. For the convergence analysis laminated composite beam is considered with 
ply orientation of [0/90/0], beam aspect ratio of 5 and 

$$E_1 / E_2 = 40, \ E_2 = E_4, \ G_{12} = G_{13} = 0.5E_2, \ G_{23} = 0.2E_2, \ \nu_{12} = \nu_{13} = \nu_{23} = 0.25, \ a / h = 5 \text{ under inplane UDL.}$$

For the further analysis 30 number of element is considered as from neL=20 it starts to converge.

Table 2: Convergence of nondimensional buckling load of composite beam under various boundary conditions

| a/h | Number of element (neL) | CF | CH | HH | CC |
|-----|------------------------|----|----|----|----|
| 5   | 16                     | 4.24 | 9.143 | 8.60 | 10.90 |
|     | 20                     | 4.24 | 9.142 | 8.51 | 10.81 |
|     | 24                     | 4.24 | 9.142 | 8.45 | 10.76 |
|     | 30                     | 4.24 | 9.142 | 8.40 | 10.72 |
| 10  | 16                     | 5.81 | 24.53 | 18.95 | 34.55 |
|     | 20                     | 5.81 | 24.44 | 18.55 | 34.06 |
|     | 24                     | 5.80 | 24.37 | 18.28 | 33.77 |
|     | 30                     | 5.80 | 24.30 | 18.10 | 33.50 |

The compatibility and accuracy of the utilized analytical model is verified by validating it with Ref. [7] for 
laminated composite and sandwich beam under inplane UDL considering different number of layups in 
Table 3. For the validation purpose beam aspect ratio considered as 5 and properties of facesheets are,

$$E_1 / E_2 = 25, \ E_2 = E_4, \ G_{12} = G_{13} = 0.7E_2, \ G_{23} = 0.1E_2, \ \nu_{12} = 0.25, \ E_{1c} = m_{1c} * E_{2c}, \ G_{12c} = m_{2c} * E_{2c};$$

$$m_{1c} = 1; \ G_{13c} = G_{12c}; \ G_{23c} = m_{3c} * E_{2c}; \ \nu_{12c} = \nu_{13c} = \nu_{23c} = 0.25; \ E_{2c} / E_{1c}; \ \rho_x = 1; \ NCBL = \frac{N_{cr} a^2}{E_{bh}}$$

Table 3: Validation of nondimensional buckling load under various boundary conditions

| Reference | No. of layer | CC | SS | CS | CF |
|-----------|--------------|----|----|----|----|
| Ref. [7] HBT [0/90/0] | 5.395 | 3.756 | 5.338 | 3.390 |
| Present   | 5.982 | 4.431 | 4.941 | 2.224 |
| Ref. [7] HBT [0/90]    | 4.571 | 2.224 | 4.289 | 1.255 |
| Present   | 4.917 | 3.212 | 3.815 | 1.360 |
| Ref. [7] HBT [0/C/0]   | 6.150 | 4.366 | 6.118 | 3.594 |
| Present   | 6.576 | 4.901 | 5.506 | 2.355 |

Figure 3 shows the variation occurred in nondimensional critical load (NCBL) with respect to beam 
thickness ratio under different BC. Analysis is carried out at \(T_g\) for the sandwich composite beam [0/C/0] 
whose facesheet and bottom sheet made from SMPC. From the figure it is observed that, maximum NCBL
observed with clamped BC whereas minimum is at CF boundary condition. With increasing thickness ratio, NCBL also get raised because of reducing overall thickness of the sandwich composite beam.

Figure 4 shows the effect of dynamic temperature variation in facesheet, bottom sheet in sandwich and SMPC over NCBL at thickness ratio of 5 with ply orientation of [0/C/0] under clamped boundary condition. Two important conclusions are drawn from this study. The very first one is the differentiation between sandwich and layered structure and second one is the effect of the core thickness over NCBL of sandwich structure. As compared to layered SMPC, maximum buckling load is observed in sandwich structure and is still increases with core thickness.

![Figure 3: Effect of thickness ratio and BC over NCBL](image1)

![Figure 4: Effect of dynamic temperature variation over NCBL](image2)

4. Conclusion

The present study deals with the buckling analysis of 1-D SMPC and SMPSC beam through a simplified $C^0$ continuity FEM based on HSDT. This work highlights the effect of dynamic temperature variation along
with glass transition temperature on the NCBL of beam subjected to uniformly distributed inplane loading. Analysis identified the effect of beam thickness ratio, BC and core thickness variation over NCBL of sandwich composite beam.

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