Are Q-stars a serious threat for stellar-mass black hole candidates?

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ABSTRACT

We examine the status of the threat posed to stellar-mass black hole candidates by the possible existence of Q-stars (compact objects with an exotic equation of state which might have masses well above the normally-accepted maximum for standard neutron stars). We point out that Q-stars could be extremely compact (with radii less than 1.5 times the corresponding Schwarzschild radius) making it quite difficult to determine observationally that a given object is a black hole rather than a Q-star, unless there is direct evidence for the absence of a solid surface. On the other hand, in order for a Q-star to have a mass as high as that inferred for the widely-favoured black hole candidate V404 Cygni, it would be necessary for the Q-matter equation of state to apply already at densities an order of magnitude below that of nuclear matter and this might well be considered implausible on physical grounds. We also describe how rotation affects the situation and discuss the prospects for determining observationally that black hole candidates are not Q-stars.

Key words: black hole physics – equation of state – binaries: close – X-rays: stars.

1 INTRODUCTION

The search for an unequivocal demonstration of the existence of stellar-mass black holes has focussed on X-ray emitting binary systems consisting of an ordinary star together with a compact object. If the mass of the compact object (as determined from kinematical measurements) appears to be greater than the maximum possible for a neutron star $M_{\text{max}}$, then the object has been acknowledged as a black hole candidate. The value of $M_{\text{max}}$ is still not reliably known, because of uncertainties in the equation of state at high densities, but it has been widely believed that the limit of 3.2 $M_\odot$ derived by Rhoades & Ruffini (1974) (together with a possible 25% upward correction for rotation) gives a secure upper bound.

The best current stellar-mass black hole candidates are in soft X-ray transients (SXTs), a sub-class of the low mass X-ray binaries (see Charles 1996). During quiescence, the accretion disc in these systems becomes extremely faint and it is then possible to carry out detailed photometry and spectroscopy of the optical companion, which allows the mass of the compact object to be directly determined (van Paradijs & McClintock 1995). Fig. 1 shows the presently-known masses of neutron stars and black-hole candidates. The “neutron star” masses all lie within a small range of $1.4 M_\odot$, whereas the black-hole candidates seem to form a distinct grouping around ~ $10 M_\odot$. The most convincing of the black hole candidates are V404 Cyg (Shahbaz et al. 1994b) and Nova Sco (Orosz & Bailyn 1996) with masses of $12 M_\odot$ and $7 M_\odot$ respectively, well above the Rhoades/Ruffini limit.

Rhoades and Ruffini derived their result in response to the problem caused by different high-density equations of state leading to widely different values for $M_{\text{max}}$, the idea being to derive a firm upper limit for non-rotating models on the basis only of knowledge which could be considered as completely secure. However, some of the assumptions made are, in fact, distinctly questionable (see Hartle 1978, Friedman & Ipser 1987). In particular: (i) it was assumed that the equation of state can be taken as accurately known for densities up to a fiducial value $\rho_0$ which they took as $4.6 \times 10^{14} \text{ g cm}^{-3}$; (ii) they imposed a causality condition which would only be appropriate for a non-dispersive medium. If a more conservative value is taken for $\rho_0$ ($10^{14} \text{ g cm}^{-3}$), the causality condition is dropped and allowance is made for rotation, the upper bound for $M_{\text{max}}$ obtained in this way goes up to $14.3 M_\odot$ (Friedman & Ipser 1987) which is no longer useful in considering black hole candidates such as V404 Cyg. However, standard realistic neutron star equations of state do, in practice, give masses satisfying the original condition $M \lesssim 3.2 M_\odot$ even with rotation. (It should be noted that all of this discussion of the mass limit is being made within the context of general relativity which is taken to provide a correct description of
gravity within this strong-field regime. We work within this context throughout the present paper. However, while general relativity is well-accepted as our theory of gravity, we recall that it has not yet received convincing experimental verification away from the weak-field limit and so one should not overlook the possibility that this might not be the correct description.

How much can the standard neutron star equations of state be trusted for matter at high densities? Is it correct that neutron star matter consists of a mixture of neutrons, protons, electrons, mesons, hyperons, etc., and is held together just by its own self-gravity? Quantum chromodynamics (QCD) contains the idea of confinement by the strong force which is normally thought of in terms of quarks being confined within nucleons. It was in connection with this that the strange star model was introduced (Witten 1984; Haensel, Zdunik & Schaeffer 1986; Alcock, Farhi & Olinto 1986), resting on the hypothesis that strange quark matter might be the absolute ground state of baryonic matter even at zero pressure. Strange stars would be essentially single giant nucleons with baryon number $A \sim 10^{37}$ and with the confined quarks being free to move within the false vacuum which extends throughout the interior. They could have masses and radii similar to those of standard neutron stars and have been advocated as a viable alternative model for pulsars although there is a problem over explaining glitches.

Strange stars are “safe” as far as the mass limit is concerned. Although their equation of state is very different from that for standard neutron star matter, the maximum mass is within the standard range (it is $\sim 2.0 M_\odot$ for non-rotating models with some variation depending on uncertain parameter values). However, some effective field theories of the strong force allow for it not only to confine quarks in the normal way but also to confine nucleons (neutrons and protons) at densities well below that of nuclear matter ($\rho_{\text{nom}} \sim 2.7 \times 10^{14} \text{ g cm}^{-3}$), giving an equation of state different from the standard one at densities below the values normally taken for $\rho_0$. Models based on this idea were introduced by Bahcall, Lynn & Selipsky (1989a,b, 1990), who named them Q-stars (although note that the “Q” here does not stand for “quark” but for a conserved particle number). These are not safe for the mass limit and might, in principle, have very high masses up to more than 100 $M_\odot$.

Even if one rules out Q-stars as models for pulsars (they have similar difficulties in this respect as for strange stars) there still remains the possibility that they could be an alternative model for the more massive objects in black-hole candidate systems. In connection with this, there are a number of questions which immediately present themselves. For a given mass, how small could a Q-star be? How physically reasonable are the versions of the Q-star equation of state which would allow masses as high as that of the compact object in V404 Cyg? What happens when rotation of the object is considered? These issues are considered in the next Section.

2 PROPERTIES OF Q-STAR MODELS

For our calculations, we have used the simplest form of the Q-star equation of state:

$$\rho - 3p - 4U_0 + \alpha_v (\rho - p - 2U_0)^{3/2} = 0$$

where $\rho$ is the density, $p$ is the pressure, $U_0$ is the energy density of the confining scalar field and $\alpha_v$ measures the strength of the repulsive interaction between nucleons. This represents chiral Q-matter (for which the particles have zero mass within the false vacuum) but the results obtained are only marginally different for non-chiral Q-matter. It is convenient to introduce the parameter $\zeta = (\alpha_v U_0^{1/2} \pi / \sqrt{3})$ since, when this is fixed, all results scale with $U_0$.

Fig. 2 shows representative mass/radius curves for Q-stars and for a standard neutron star equation of state (model C from the collection of Arnett & Bowers 1977) and it is clear that the two curves are quite different. The lowest-mass Q-star models, for any given $\zeta$, have an almost constant value of the density, giving $M \propto R^3$, but with increasing mass, the density profile becomes progressively more peaked towards the centre. As the central density is increased further, a maximum mass is eventually reached and this marks the end of models which are stable under radial perturbations. The maximum-mass model is the most compact stable one (i.e. $R/M$ is a minimum) and therefore the closest in size to a black hole of the same mass. In Fig. 3, we show $(R/M)_{\text{min}}$ plotted as a function of $\zeta$ for Q-stars together with corresponding values for a black hole and for the representative neutron star equation of state. (We are using the standard geometrical units of general relativity with $c = G = 1$.) It can be seen that Q-stars can be very compact with $R/M < 3$ (although note that very large values of $\zeta$ are not physically reasonable.) A stable, non-rotating 12 $M_\odot$ Q-star might have a radius as small as $\sim 52$ km as compared with $\sim 36$ km for a non-rotating black hole of the same mass. Having $R/M < 3$ means that the surface lies inside the circular photon orbit, the place where centrifugal force becomes attractive rather than repulsive (see Abramowicz & Prasanna 1990), but the models do not seem to be quite compact enough to allow for some more exotic relativistic effects such as resonance of axial gravitational wave modes (Chandrasekhar & Ferrari 1991) or production of an internal ergoregion when the object is set into rapid rotation (see Butterworth & Ipser 1976), which would lead to an instability.

The maximum mass which a Q-star could have is related to the threshold minimum density, $\rho_{\text{nom}}$, above which the Q-matter equation of state is taken to apply. (This is the density at which the pressure goes to zero.) The relation is shown in Fig. 4 for the limiting cases $\zeta = 0$ and $\zeta \to \infty$ and it can be seen that varying $\zeta$ over the entire range in between makes only a small difference. In order to have a non-rotating Q-star with as high a mass as that inferred for the compact object in V404 Cyg (marked with the horizontal dashed line), it would be necessary for the threshold density to be below $\rho_{\text{nom}}$ (marked with the vertical dashed line) by about a factor of 10 or more. Rotation makes little difference to this. Even if one admits the idea of Q-matter in principle for some range of densities around $\rho_{\text{nom}}$, such a very low value of $\rho_{\text{nom}}$ might seem implausible.

Compact objects accreting matter in binary systems will be spun up by the angular momentum of the accreted material and so it is important to ask what effect this would have for the comparison between the black hole and Q-star pictures. Fig. 5 shows the locations of the equatorial radii of a black hole, an extreme Q-star ($\zeta \to \infty$, maximum compactness) and a representative neutron star, as functions of
$a/M$ where $a$ is the angular momentum per unit mass and all quantities are again in geometrical units. All of the radii are calculated in the same coordinate-independent way, by taking the proper distance round the circumference of the circle concerned and dividing by $2\pi$. Doing this, the equator of the black hole event horizon is always at $r = 2M$ irrespective of the value of $a/M$. (We emphasize that this is in contrast with the situation for the commonly-used measure in Boyer-Lindquist coordinates for which the equator is at $r = 2M$ when $a = 0$ but $r \rightarrow M$ as $a \rightarrow M$. Other frequently-quoted quantities such as observed rotational frequencies and binding energies are usually calculated in an invariant way.) The formulae used for the Q-star and neutron star models are calculated within the slow-rotation approximation, correct to second order in the rotational velocity $\Omega$ and comparing rotating and non-rotating models with equal central density. Also shown in Fig. 5 is the location of the marginally stable orbit (corresponding to the inner edge of a Keplerian accretion disc) which, for a given $a/M$, is the same for all of the objects within the slow rotation approximation. While bearing in mind that results obtained with the slow-rotation approximation should only be regarded as indicative for the higher values of $a/M$, several interesting conclusions can be drawn from this figure. As $a/M$ is increased, the $r/M$ of the marginally stable orbit decreases and eventually it reaches the equator of the neutron star which expands out to meet it. For the extreme Q-star, $r/M$ of the equator actually decreases slightly and the marginally stable orbit would reach it only for larger $a/M$.

3 DISCUSSION AND CONCLUSIONS

How can one be sure that a high-mass compact object such as that in V404 Cygni is a black hole and not a Q-star? Any evidence for the existence of a solid surface (e.g. by observing a Type I X-ray burst) would, of course, immediately rule out the possibility of the object being a black hole but in the absence of such evidence, what can be done? Narayan, McClintock & Yi (1996) have proposed a model for some SXTs in quiescence which is in agreement with available observational data and depends on the compact object in question being a black hole. However, this is still an indirect argument. What one would really like is to have direct observational evidence of accreting material at radii smaller than would be possible with a Q-star. With satellite experiments such as the Rossi X-ray Timing Experiment (RXTE) and the Unconventional Stellar Aspect experiment (USA) being capable of time resolution down to the order of 1 sec, there is a possibility of detecting the location of the inner edge of the accretion disc (if, indeed, there is a well-defined inner edge) by seeing a corresponding frequency cut-off in the power spectrum, by seeing dying pulse trains from bright features as they reach the inner edge and fall in (Stoeger 1980) or from analysis of quasi-periodic oscillations (Miller, Lamb & Psaltis 1997; see Zhang et al. 1997 for discussion of a possible detection of the inner edge of the disc by this means for binaries with compact components in the neutron star mass range). For non-rotating compact objects, the marginally stable orbit is at $r = 6M$, much larger than the radius of the most interesting Q-star models, but with increasing rotation of the compact object, it moves inwards and if the compact object were a Q-star, it would eventually meet the surface. If evidence were to be found for an accretion flow extending further inwards than would be possible with a Q-star, this would then point very strongly towards the compact object being a black hole, since the Q-star model probably presents the last realistic possibility for avoiding that conclusion within the context of general relativity.

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FIGURE CAPTIONS

**Figure 1:** Mass distribution of neutron stars and black holes for which masses have been directly measured (Gies & Bolton 1986; Nagase 1989; Shahbaz, Naylor & Charles 1993; Thorsett et al. 1993; Shahbaz, Naylor & Charles 1994a; Shahbaz et al. 1994b; van Paradijs & McClintock 1995; Beekman et al. 1996; Orosz & Bailyn 1996; Beekman et al. 1997; Remillard et al. 1997; Shahbaz, Naylor & Charles 1997). The components of the binary pulsars are marked with with P (pulsar) and C (companion). Also shown are the X-ray pulsars. The neutron star systems lie at $\sim 1.4 M_\odot$, whereas the black-hole candidates seem to cluster around $\sim 10 M_\odot$. The strongest black hole candidates are V404 Cyg and J1655-40. The vertical hashed line represents the Rhoades/Ruffini limit of $3.2 M_\odot$.

**Figure 2:** Representative mass/radius relations for (a) Q-stars and (b) standard neutron stars. Note that the scaling and precise form of the Q-star curve depend on parameter values which uncertain.

**Figure 3:** $(R/M)_{\text{min}}$, the minimum value of $(R/M)$ for stable non-rotating Q-star models, is plotted as a function of $\zeta$. Also shown, for comparison, are values for a representative neutron star equation of state and for a black hole. For the Q-star, $(R/M)_{\text{min}}$ tends towards a constant value of 2.8 as $\zeta \to \infty$: Q-stars can be very compact.

**Figure 4:** The maximum mass of non-rotating Q-star models is plotted as a function of the threshold density for Q-matter $\rho_{\text{min}}$. Curves are drawn for the limiting cases $\zeta = 0$ and $\zeta \to \infty$. The horizontal dashed line corresponds to the mass of the compact object in V404 Cyg; the vertical dashed line marks the nuclear matter density $\rho_{\text{nm}}$. To allow for a Q-star with a mass as high as $12 M_\odot$, the threshold density would have to be about an order of magnitude below nuclear matter density, or less.

**Figure 5:** Equatorial radii of a black hole, an extreme Q-star, a typical compact neutron star and the location of the marginally stable orbit are plotted as functions of $a/M$. These are (coordinate independent) circumferential proper-distance radii calculated consistently within the slow rotation approximation. The equatorial radius $R_{eq}$ of the black hole measured in this way is constant, independent of $a/M$. $R_{eq}/M$ for the most extreme Q-stars actually decreases very slightly as $a/M$ is increased within the slow rotation regime, which is in contrast with the behaviour of the neutron star.
Figure 1

Neutron Stars

PSR2127+11C (C)
PSR2127+11C (P)
PSR2302+46 (C)
PSR2302+46 (P)
PSR1802-07 (P)
PSR1855+09 (P)
PSR1534+12 (C)
PSR1534+12 (P)
PSR1913+16 (C)
PSR1913+16 (P)
4U1700–37
4U1538–52
Her X–1
Vela X–1
SMC X–1
LMC X–1
Cen X–3
Cen X–4

Black Holes

1.4 $M_\odot$
10 $M_\odot$

10^{-1} 10^{0} 10^{1} 10^{2}
Compact object mass in solar masses

J0422+32
J1655–40
H1705–25
GS1124–68
V404 Cyg
GS2000+25
A0620–00
Cyg X–1
Figure 2b
Figure 3
Figure 5