QCD RENORMALONS AND HIGHER TWIST EFFECTS

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Abstract
I give a short review of the relation of infrared renormalons in QCD and higher twist effects, with the emphasis on possible applications. In particular, I present estimates of renormalon-induced uncertainties in deep inelastic sum rules and explain how the renormalons can potentially be used to unravel the structure of nonperturbative effects in complicated situations and to indicate possible systematic sources of large perturbative corrections.

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The modern precise data on “hard” processes in QCD require a theoretical description to power-like accuracy. A clear example is provided by the Gross-Llewellyn Smith sum rule in the deep inelastic scattering, where the higher-twist (HT) \(1/Q^2\) correction produces a major uncertainty in determination of \(\alpha_s\) [1]. For a generic physical observable dominated by short distances one expects a theoretical prediction of the form, schematically,

\[
R(Q) = R_{\text{tree}} \left[ 1 + r_1 \frac{\alpha_s}{\pi} + r_2 \left( \frac{\alpha_s}{\pi} \right)^2 + \ldots \right] + \sum_s \frac{H_s}{(Q^2)^s}, \tag{1}
\]

where \(\alpha_s = \alpha_s(Q)\) and \(H_s\)'s are dimensionful nonperturbative parameters (with dimension 2\(s > 0\)) describing the HT corrections. A conceptual problem, which I am going to review in this talk, is that the discrimination between perturbative corrections and HT contributions is ambiguous. This will imply that it is not possible to attribute a fully quantitative meaning to power-suppressed corrections, unless some prescription is used to sum the perturbative series. On the other hand, ambiguities in summation of the perturbative series can serve to indicate which powers of \(1/Q\) are required in the sum in (1).

Assuming for simplicity a (euclidian) quantity dominated by a large scale \(Q\), the leading-order correction involves the gluon exchange with the large virtuality of order \(Q\). Progressing to a higher order \(n\) the average gluon virtuality still remains proportional to \(Q\), \(k \sim a_n Q\) where \(a_n\) is a certain coefficient, simply because there are no more dimensionful parameters. However, the \(a_n\) can (and do) decrease with \(n\), so that in very high orders \(n\) such that \(a_n \sim Q/\Lambda_{\text{QCD}}\) the perturbative calculation fails. An inspection shows that the most dangerous Feynman diagrams are those related to running of the QCD coupling, in which the average virtuality decreases exponentially \(a_n \sim \exp[-n/s]\), where \(s\) is some number. It is possible to show that this intervention of infrared (IR) regions reveals itself in a rapid – factorial – increase of perturbative coefficients in high orders [2]:

\[
R(Q) = R_{\text{tree}} \sum_n r_n \alpha_s^n(Q); \quad r_n \sim (\beta_0/s)^n n! \tag{2}
\]

where \(\beta_0 = (11 - 2/3n_f)/(4\pi)\). The physical origin of the large coefficients is simple: the gluon exchange with virtuality \(k\) involves the QCD coupling at this scale \(\alpha_s(k)\). However, the perturbative expansion [3] is assumed to be in powers of \(\alpha_s(Q)\) at the scale of the external momenta. Thus, we get large coefficients simply by reexpressing \(\alpha_s(k \sim \exp[-n/s]Q)\) in terms of \(\alpha_s(Q)\).

A factorial growth of perturbative coefficients means that the perturbative series is at best an asymptotic series: the fixed order contributions \(r_n \alpha_s^n\) decrease at small \(n\), reach a certain minimum value at \(n = n_0 \sim 1/\alpha_s\), but then again start to increase and blow up. This means that a perturbative calculation only makes sense up to the order \(n_0\), and the accuracy of this calculation, or, equivalently, the effect of the “tail” with \(n > n_0\) is of the order of the minimum term

\[
\sum_{n=n_0}^{\infty} r_n \alpha_s^n(Q) \sim r_{n_0} \alpha_s^{n_0}(Q) \sim \exp[-s/(\beta_0 \alpha_s(Q))] \sim \left( \frac{\Lambda_{\text{QCD}}^2}{Q^2} \right)^s, \tag{3}
\]

where I used the asymptotic form of the coefficients [2] and the one-loop formula \(\alpha_s(Q) = 1/(\beta_0 \ln(Q^2/\Lambda^2))\). In the common jargon, the divergences of perturbative expansions are called “renormalons” [4], and I will refer to the ambiguity in the summation of the series [3] as to the “renormalon ambiguity”.

The deficiency of the perturbation theory has a profound reason, indicating that calculation of physical quantities to power-like accuracy requires taking into account nonperturbative
effects. The renormalon ambiguities must be compensated by ambiguities in the HT corrections complementing truncated perturbative expansions. Hence, the required values of \( s \) in (2), (3) have to be in one-to-one correspondence to the required HT corrections in (1). This implies that the required powers of \( 1/Q \) can in principle be determined from purely perturbative calculations. In practice, already a simple approximation (referring to the \( 1/N_f \) expansion) usually recognizes all power-suppressed corrections which are required by the Operator Product Expansion (OPE).

2. The relation of IR renormalons and the OPE has been studied in much detail [3, 4, 5, 6] for the polarization operator of vector currents. In this case the perturbative series is complemented by the contribution of the gluon condensate [7]

\[
Q^2 \frac{d}{dQ^2} \Pi(Q^2) = 1 + \frac{\alpha_s(Q)}{\pi} + \ldots - \frac{1}{6Q^2} \langle g^2 G^2 \rangle + O(1/Q^6) \tag{4}
\]

By an explicit calculation [6] it has been shown that the perturbative series diverges, producing an ambiguity \( O(1/Q^4) \) \((s=2,3,\ldots \) in (3)). On the other hand, numerical value of the gluon condensate cannot be determined to better accuracy than of order \( \Lambda_{QCD}^4 \) because of the quartic power divergence. These uncertainties must mutually cancel, since they only arise because of our (illegal) attempt to separate perturbative and nonperturbative effects, and an immediate question is whether one can organize the expansion in such a way that this problem does not appear. A possible solution is suggested by the Wilson OPE, which in fact is not designed to separate perturbative and nonperturbative effects, but rather to separate contributions of small and large distances. Following this logic literally, we would subtract from the perturbative answer the contribution of small virtualities \( k^2 < \mu^2 \) (where \( \mu \) is of order 1 GeV), and add it to the gluon condensate as a perturbative contribution of order \( \mu^4 \). The premium is that the separation between the subtracted perturbative answer and the power suppressed contribution of the condensate is now unambiguous: since the IR region is deleted, the perturbative expansion is not plagued by factorially large coefficients and the cancellation of ambiguities between perturbative and nonperturbative contributions to the condensate becomes implicit. The price to pay, however, is that both perturbative and condensate contributions depend explicitly of the scale \( \mu \). This dependence may be strong; elimination of the renormalon ambiguity of order \( \Lambda_{QCD}^4 \) requires reshuffling of a much bigger contribution of order \( \mu^4 \). Thus, usefulness of this rearrangement should be judged by the practical gain: if renormalon ambiguities are numerically much smaller than the phenomenological estimates for the condensates, it is hardly reasonable to eliminate the ambiguity at the cost of a large \( \mu \) dependence. In fact, it is easy to see that the whole idea to add HT corrections to the perturbative expansions, in which we only know a few first terms and do not see any sign of the factorial divergence (and thus expect that we have not yet reached the minimum term), implicitly implies that the “true nonperturbative” HT corrections are much bigger than the minimum term in the perturbative series, and thus much bigger than the uncertainty in its summation.

In this respect it is important that semiquantitative estimates of ambiguities in the summation of the perturbative series can be worked out. For the particular case of the polarization operator using the results of [3] I get

\[
Q^2 \frac{d}{dQ^2} \Pi(Q^2) = \left[ 1 + \frac{\alpha_s(Q)}{\pi} + \ldots + \frac{0.002 - 0.02 \text{ GeV}^4}{Q^4} \right] - \frac{(0.08 - 0.12) \text{ GeV}^4}{Q^4} + O(1/Q^6) \tag{5}
\]

where the first number (with an error bar) is an estimate of the renormalon uncertainty\(^1\). The simplest way to make this estimate is by the minimum term in the perturbative expansion. I use a somewhat more analytic method and give an imaginary part (divided by \( \pi \)) of the Borel transform.
and the second number refers to the phenomenological value of the gluon condensate $\langle g^2 G \rangle$. The large range of values for the renormalon ambiguity is due to the fact that it is proportional to the fourth power of $\Lambda_{QCD}$ and the uncertainty in the latter is amplified. It is seen that the ambiguity is in fact insignificant compared to a 50% error in the phenomenological value of $\langle g^2 G \rangle$. This fact was recognized long ago (see, e.g. [8]) and is one of the starting points of the QCD sum rules, where the QCD renormalons are ignored as being (supposedly) numerically insignificant.

For the GLS and Bjorken sum rules, assuming the range of values $\alpha_s(Q^2 = 3 \text{ GeV}^2) = 0.24 \pm 0.29$, as suggested by the new CCFR data [1], I get

\[
\begin{align*}
\text{GLS} &= 3 \left\{ \left[ 1 - \frac{\alpha_s(Q)}{\pi} + \ldots \pm \frac{0.02 - 0.07 \text{ GeV}^2}{Q^2} \right] - \frac{(0.1 \pm 0.03) \text{ GeV}^2}{Q^2} + O(1/Q^4) \right\} \\
\text{Bj} &= \frac{1}{6} \left\{ g_A \left[ 1 - \frac{\alpha_s(Q)}{\pi} + \ldots \pm \frac{0.02 - 0.07 \text{ GeV}^2}{Q^2} \right] - \frac{(0.09 \pm 0.06) \text{ GeV}^2}{Q^2} + O(1/Q^4) \right\}
\end{align*}
\]

where the numerical values of the HT $1/Q^2$ corrections are taken from the QCD sum rule calculations [9]. In this case the renormalon ambiguities are roughly factor two smaller than the “true” HT effects. Note that the renormalon ambiguities are determined by divergences in the coefficient functions in the OPE and are universal in the sense that they do not depend on the particular target, while “genuine” HT corrections reflect correlations between partons in the target (nucleon) and are process-dependent.

Finally, it has been shown [10, 11] that the pole mass of a heavy quark is affected by IR renormalons already at the level of $1/m$ corrections, which do not allow to determine it from the relation to the mass defined at short distances to the accuracy better than

\[
m_{\text{pole}} = m(m) \left[ 1 + \frac{4}{3} \frac{\alpha_s(m)}{\pi} + \ldots \right] \pm 100 \text{ MeV}
\]

This number should be compared to the “true” nonperturbative mass difference between the heavy meson and the heavy quark, which is estimated to be of order $\bar{\Lambda} \equiv m_B - m_b \sim 300 - 500$ MeV [12].

To summarize, the “renormalon problem” of the separation of perturbative and nonperturbative contribution can be more or less important in each practical case, depending on which method is used to estimate the HT contribution, and what accuracy is claimed. Existing calculations of the HT corrections do not pretend to an accuracy better than of order 30-50% which is larger (or of order) of the estimated renormalon uncertainty, so that the latter can be ignored. However, to improve these predictions one has to treat the renormalon problem explicitly.

3. The IR renormalons have received much attention recently because of their potential to expose power-like corrections: the $\sim 1/Q^{2s}$ uncertainty in summation of the perturbative series should be interpreted as indicating presence of the $\sim 1/Q^{2s}$ nonperturbative correction in the particular physical observable. It should be stressed that this argument cannot compete with the OPE if the latter is applicable: attempts to “check” or to “disprove” OPE using the renormalon arguments are essentially meaningless. It is precisely the agreement with the OPE in its traditional domain which serves as main justification of the hope that the IR renormalons may provide a nontrivial information about nonperturbative effects in a more general situation, since this technique relies on a purely perturbative analysis and can be relatively easily implemented.
Despite the complicated terminology, the idea of this application is simple, and is that IR renormalons in fact probe IR sensitivity of perturbative diagrams to power-like accuracy. This can be made explicit using the result of Refs. [13, 14]: for IR-safe inclusive quantities there is a one-to-one correspondence between IR renormalons and nonanalytic terms in the expansion of corresponding Feynman diagrams in powers of some IR regulator like a small gluon mass \( \lambda \). In particular, existence of the \( \sqrt{\lambda^2} \) term in this expansion signals existence of the \( 1/Q \) uncertainty in summation of the perturbative series, and this necessitates the \( 1/Q \) nonperturbative correction. The \( \lambda^2 \ln \lambda^2 \) term implies the \( 1/Q^2 \) correction, etc. For illustration, consider the following result [14] for the polarization operator (4) calculated with a small gluon mass:

\[
Q^2 \frac{d}{dQ^2} \Pi(Q^2) = 1 + \frac{\alpha_s}{\pi} \left\{ 1 - \left[ \frac{32}{3} - 8\zeta(3) \right] \frac{\lambda^2}{Q^2} - \left[ 2 \ln(Q^2/\lambda^2) + \frac{20}{3} - 8\zeta(3) \right] \frac{\lambda^4}{Q^4} \right\} + O \left( \frac{\lambda^6}{m^2} \frac{\lambda^2}{Q^2} \right). \tag{9}
\]

Note that there are no terms \( \sim \lambda^2 \ln \lambda^2 \) (analytic terms like \( \sim \lambda^2 \) are not related to the IR region) and the first nonanalytic correction is of order \( \sim \lambda^4 \ln \lambda^2 \), indicating a potential nonperturbative contribution of order \( 1/Q^4 \), in agreement with the OPE [8]. Note that absence of terms \( \sim \ln \lambda^2 \) is the result of celebrated Bloch-Nordsieck cancellations between contributions of real and virtual emission, and absence of certain power-like corrections (alias renormalon ambiguities) can be formulated as extension of Bloch-Nordsieck cancellations to power-like accuracy [13].

The search of nonperturbative effects using IR renormalons has been been most fruitful in heavy quark decays [10, 11, 13]. One finds for the b-quark pole mass

\[
m_{b,\text{pole}} = m_b \left[ 1 + \frac{4}{3} \frac{\alpha_s(m)}{\pi} - \frac{2}{3} \frac{\sqrt{\lambda^2}}{m_b} + \ldots \right]. \tag{10}
\]

The term \( \sim \sqrt{\lambda^2} \) indicates presence of an \( 1/m \) nonperturbative correction (which is not related to matrix element of any local operator and cannot be found using the usual OPE). On the other hand, the B-meson total semileptonic inclusive width equals to the same accuracy (for simplicity I give the answer for \( b \to u e \nu \) transitions, that is for massless quark in the final state)

\[
\Gamma(B \to X_u e \nu) = \frac{G_F^2}{192\pi^3} \left( m_{b,\text{pole}} \right)^5 \left[ 1 - 2.41 \frac{\alpha_s(m)}{\pi} + \frac{10}{3} \frac{\sqrt{\lambda^2}}{m_b} + \ldots \right]. \tag{11}
\]

The \( \sim \sqrt{\lambda^2} \) correction is again present, but is cancelled exactly if the pole mass of the b-quark is eliminated in terms of the \( \overline{\text{MS}} \) running mass using (10). This cancellation presents the result of a prime physical importance: inclusive decay widths of heavy particles do not contain nonperturbative corrections of order \( 1/m \) if they are expressed in terms of mass parameters defined at short distances [10, 13].

Further applications of IR renormalons to the study of nonperturbative effects in resummed cross sections will be reviewed by G. Korchemsky [13].

4. One more idea which has emerged from studies of the QCD renormalons is that the Feynman diagrams related to running of the strong coupling, whose low-momentum regions produce renormalons, may give dominant contributions to perturbative coefficients in
intermediate orders and can be identified and resummed. This can be considered as a natural generalization of the proposal by Brodsky-Lepage-Mackenzie (BLM) to eliminate all dependence on the QCD β-function from coefficients of the perturbation theory by adjusting the scales of the coupling separately in each order. Several examples show that the second-order perturbative coefficient is significantly reduced by this rearrangement. In high orders one again expects a considerable reduction of coefficients since IR renormalons are made implicit (they are hidden in uncertainties of the BLM scales). Thus, it is natural to speculate that the BLM-improved perturbation theory has smaller coefficients to all orders.

The corresponding resummation of running coupling effects to all orders has been proposed in [17, 18, 14]. Formally, this approach allows to calculate all perturbative corrections of order \( \beta_0^n \alpha_s^{n+1} \) and \( \beta_1 \beta_0^{n-2} \alpha_s^{n+1} \) which can be traced by contributions of fermion bubble insertions into the single gluon line. The resummation of running coupling effects is relatively simple and is probably phenomenologically relevant, although the dominance of these correctons is a conjecture, which can only be justified \textit{a posteriori} by comparing to exact calculations. At present the corresponding calculations have been done for the heavy quark pole mass, the Adler function and the τ-lepton hadronic width, and for the exclusive and inclusive B-decays. Our results for the τ decay show that the value of \( \alpha_s(m_\tau) \) extracted from these data is probably overestimated, and also give some indication that the commonly used resummation of \( \pi^2 \) corrections is disfavoured in high orders, see [14] for details. The resummation of \( \beta_0^n \alpha_s^{n+1} \) corrections in B-decays and its relevance for the extraction of \( V_{cb} \) will be addressed in her talk by P. Ball [20].

5. To summarize, I repeat that the QCD perturbation theory is divergent and does not allow to give quantitative predictions to power-like accuracy, unless it is complemented by explicit nonperturbative (HT) corrections. In turn, the HT corrections are by themselves ill-defined. The corresponding ambiguities have to be in one-to-one correspondence to the ambiguities of perturbation theory and must cancel in the sum. For practical cases of the GLS and Bj sum rules the corresponding ambiguities are probably a factor 2 below the “true” HT corrections. The increasing interest in IR renormalons is trigged by hopes that they can help to investigate the structure of nonperturbative corrections in rather general situations, and to find physical observables with extended IR stability (to power accuracy), which are less sensitive to nonperturbative effects. Another hope is to get estimates for higher-orders of perturbation theory, combining the information about the calculated low orders and about the renormalons in very high orders. Both directions are interesting and worth efforts.

As a final remark, let me say that nonperturbative effects in QCD are not reduced to renormalons. In particular, not all nonperturbative effects can be traced by divergences of the perturbation theory, and also by no means the renormalons can be used to define QCD \textit{nonperturbatively} in the region where the coupling becomes strong. Equally, my talk cannot pretend to cover all aspects of QCD renormalons — for example, I ignored ultraviolet renormalons which deserve a special discussion. I refer the readers to the review [21] and original papers for the discussion of issues which I was not able to touch here. I thank the organizers of this conference for the invitation, and gratefully acknowledge a very rewarding collaboration with P. Ball, M. Beneke and V. Zakharov on the subjects related to this talk.

References

\footnote{For this purpose one also needs to consider the so-called ultraviolet renormalons, which are related to factorial divergences of perturbative series arising from contributions of momenta \( k \gg Q \) in Feynman diagrams and which I did not discuss in this talk.}
[1] J. Chyla, A. Kataev, Phys. Lett. B297 (1992) 385; D. Harris, these Proceedings.

[2] G. ’t Hooft, in The Whys Of Subnuclear Physics, Erice 1977, ed. A. Zichichi (Plenum, New York, 1977), p. 943; B. Lautrup, Phys. Lett. 69B (1977) 109; G. Parisi, Phys. Lett. 76B (1978) 65; Nucl. Phys. B150 (1979) 163.

[3] A. Mueller, Nucl. Phys. B250 (1985) 327.

[4] F. David, Nucl. Phys. B234 (1984) 237; Nucl. Phys. B263 (1986) 637.

[5] V.I. Zakharov, Nucl. Phys. B385 (1992) 452; M. Beneke and V.I. Zakharov, Phys. Rev. Lett. 69 (1992) 2472.

[6] M. Beneke, Nucl. Phys. B405 (1993) 424.

[7] M. Shifman, A. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448.

[8] M.A. Shifman, in ‘QCD – Twenty Years Later’, Proc. Int. Conf., Aachen, 1992, eds. P.Zerwas and H.Kastrup, vol.1, (World Scientific, Singapore, 1993)

[9] V.M. Braun and A.V. Kolesnichenko, Nucl. Phys. B283 (1987) 723; I.I. Balitsky, V.M. Braun and A.V. Kolesnichenko, Phys. Lett. B242 (1990) 245; (E) ibid. B318 (1993) 648.

[10] I. Bigi et al., Phys. Rev. D50 (1994) 2234.

[11] M. Beneke and V.M. Braun, Nucl. Phys. B426 (1994) 301.

[12] E. Bagan et al., Phys. Lett. B278 (1992) 457; M. Neubert, Phys. Rev. D45 (1992) 2451.

[13] M. Beneke, V.M. Braun and V.I. Zakharov, Phys. Rev. Lett. 73 (1994) 3058.

[14] P. Ball, M. Beneke and V.M. Braun, Preprint CERN–TH/95–26 [hep-ph/9502300].

[15] G. Korchemsky, these proceedings.

[16] S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, Phys. Rev. D28 (1983) 228.

[17] M. Beneke and V.M. Braun, Phys. Lett. B348 (1995) 513.

[18] M. Neubert, Preprints CERN–TH.7487/94 [hep-ph/9412265]; CERN–TH.7524/94 [hep-ph/9502264].

[19] P. Ball, M. Beneke and V.M. Braun, Preprint CERN–TH/95–65 [hep-ph/9503492].

[20] P. Ball, these proceedings.

[21] A.H. Mueller, in ‘QCD – Twenty Years Later’, Proc. Int. Conf., Aachen, 1992, eds. P.Zerwas and H.Kastrup, vol.1, (World Scientific, Singapore, 1993)