The evolution of potential energy and contactopy two dimension granular materials in the case of compaction inhibited Brazil-nut effect

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Abstract. Compaction inhibit Brazil nut effect experiment by using acrylic material as bed and intruder has performed. There are five diameter of granular bed (cm): 0.68, 0.53, 0.47, 0.42, and 0.375 and single intruder with diameter 2.4 cm used in this experiment. Vibration taken under Γ = 3 and frequency (Hz): 14, 15, 16, 17, 18, and 19. Discontinue vibration technique is used in this experiment. Vibrating system is done around one second then stop for a moment before next vibration. This technique used to make easier the process photo taken in each second. These photos then processed to gain contactopy and height of system. Based on the data, contactopy of all system observed are increasing. The average of contactopy without intruder system toward ratio of granular bed diameter and the width of shaking place giving logarithmic equation

\[ \Delta C = -210.5 \ln \left( \frac{d}{L} \right) - 443.56 \] .

This equation show the value of contactopy without intruder system under certain ratio of granular bed diameter and the width of shaking place. Potential energy change for both system without intruder and with intruder are decreasing as the height system is decreasing. These result show that compaction inhibit Brazil nut effect phenomena still obey the minimum energy principle (MEP). The whole results of this research confirm that increasing contactopy and decreasing potential energy not only valid for Brazil nut effect, but also for compaction inhibit Brazil nut effect phenomenon.

1. Introduction

Granular material is a material which often we find in daily life, such as coffee, sugar, salt, rice, corn, and sand. Granular material consist of small dimensional grain in the large number. The granular material can be a solid, liquid, or gas. In certain system, these three characters of substance can appear together [1]. One of the most interesting phenomena to learn about granular material is Brazil nut effect. Brazil nut effect occur when there are two type of granular material with different mass and size are vibrated. Large granular (intruder) which is at the beginning in the beneath of small granular (bed) will move to the top and against the gravitation when vibrated [2].

Besides mass and granular diameter [3], another factor influence Brazil nut effect are the roughness of intruder surface [4] and the roughness of shaker base [5]. The two factors assumed to give impact in forming hexagonal closed packed structure in bed granular of the system. When the configuration of hexagonal closed packed is reached, intruder will be locked and cannot move upward [6-8].

The result of experiment before [6, 9, 10] show that in Brazil nut effect, potential energy is decreasing while contactopy is increasing. In this research, the change of potential energy and
contactopy in the system which compaction is inhibiting Brazil nut effect will be studied. The system that will be studied is the system when the intruder has locked in certain place caused by hexagonal closed packed configuration of bed granular.

2. Theory

Brazil nut effect can be caused by percolation, convection, and condensation [3]. By using molecular dynamic simulation, the research before [3] show that when percolation effect larger than condensation, the Brazil nut effect is occur. And vice versa, when condensation is larger than percolation, the reverse Brazil nut effect occur to the system. The result is reinforced by another research [12] which is based on experiment. Besides ratio of mass and diameter of granular material [3], another factor influence Brazil nut effect are the roughness of intruder surface [4] and the roughness of shaker base [5]. External energy is also give important contribution to the behaviour of granular material motion. The influence of amplitude and frequency is given by $\Gamma \left( \frac{a}{g} \right)$.

2.1. Compaction

The formation of 2-D granular material in disk form is said to be compact if every single disk touch other disk without overlapping each other. Based on Thue Theorem (Axel Thue), hexagonal configuration is the most possibility configuration to form compaction with highest density for disk shape of material [7]. 2-D hexagonal closed packed is the nucleation that give smallest cluster and geometrically more precise as used in 2-D crystal system [8]. Hexagonal configuration consist of one centre of mass surrounded by six others centre of mass as shown in Fig. 1 below.

![Hexagonal closed packed configuration.](image)

2.2. Potential energy

Potential energy of granular material is a gravitational potential energy of system. The change of potential energy can be determined by the centre of mass height change which can be calculated by Eq. 2

$$\Delta E_p = m \times g \times (h_f - h_i)$$

with $\Delta E_p$ is the change of gravitational potential energy (J), $m$ is a mass of overall system (kg) that is mass of one grain multiple by the number of grains, $h_i$ is the centre of mass height in initial condition (m) and $h_f$ is the centre of mass height in final condition (m). Centre of mass of the system is a half of the height intruder and bed in shaker. Therefore, by determining the height of system, the centre of mass can be easily determined. Figure 2 below show the centre of mass of initial and final condition.
2.3. Contactopy

Contactopy is contact point between granular in a system. The number of contact point is influenced by the granular configuration in a certain system. Random structure and hexagonal close packed structure give different contact point [6]. Table 1 here show the number of contact point for random and hexagonal configuration.

| System Configuration                      | C (contactopy) |
|------------------------------------------|----------------|
| Random                                   | 18             |
| Hexagonal close packed (2-d HCP)         | 26             |

3. Experiment

Tools and materials of experiment are arranged as in Fig 3 (a) below. Granular material is placed on acrylic box as in Fig. 3 (b). The acrylic box is placed on the top of speaker that used as vibration tool. Intruder and granular bed are made from acrylic. Figure 3 (b) below show system with 2,4 cm diameter of intruder and 0,68 cm diameter of granular bed.
The observation of compaction inhibiting Brazil nut phenomena with intruder is done by placing acrylic contains of intruder and granular bed until 9.5 cm of height from the bottom. Intruder is placed on centre base. Tripod and black camera is placed in front of shaker. Black screen is placed behind camera to avoid the reflection. The technique of photo taken has changed several times before suitable position found. The technique of granular material photo taken has reported in Simposium Nasional Inovasi dan Pembelajaran Sains (SNIPS 2015). The procedure of vibration is not done continuously, but discontinuously every one second. Photo of observed system is taken when vibration is off. Compaction occurs when granular bed has formed dense configuration. Hence the intruder cannot move upward. The system keep on vibrated until 60 second since intruder locked in its position to ensure that compaction is occur.

4. Results and discussion
Data processing in this research are using 5 frame over all photo taken. The 5 frames consist of initial condition, final condition and 3 frames between it. These frames are not in same time series because the time of system to reach compaction is different. Therefore, the horizontal axis of the graph in this paper are not showing time variable directly, but the conversion from time series of one frame to maximum time to reach compaction that is 1.

3.1. Contactopy
Over all, contactopy of a system is increasing for both system with intruder and system without intruder. Increasing contactopy caused by the configuration of granular bed after vibrated. Compaction configuration enable to give 6 contact point for each centre of mass. Therefore, the number of contact point in hexagonal configuration is larger than in initial condition which has random configuration. Figure 5 below are the graph of the change contactopy at each diameter of granular bed.
Based on Fig. 5 above, it can be observed that contactopy of system is increasing by the decrease of diameter granular bed used. It happens because the number of small granular bed is more than the large granular bed. It is also valid for system without intruder. Contactopy of system with intruder acquired at frequency 16 Hz and 17 Hz with diameter of granular bed are 0.68 cm, 0.47 cm, 0.42 cm and 0.375 cm. Granular bed with diameter 0.53 cm is not showing compaction or it can be said that in this size of granular bed, Brazil nut effect always play a role. In other frequency vibration, compaction is rarely observed. Figure 6 here show the change of contactopy system with intruder.
Figure 6. Graph of contactopy change for system with intruder at bed diameter 0.68 cm (●), 0.47 cm (■), 0.42 cm (▲), and 0.375 cm (◆) by frequency vibration 16 Hz (left) and frequency vibration 17 Hz (right).

The average of contactopy change from initial frame and final frame for the six frequency vibration at each granular bed diameter is given by Fig. 7 below.

Figure 7. Graph of the average contactopy change from initial and final condition of system without intruder (■) and system with intruder (◆) toward ratio of granular bed diameter and the width of place vibration.

The spread data of system without intruder in Fig. 7 form a logarithmic except at d/L = 0.053 (red mark). Contactopy at this granular bed diameter is larger than diameter bed 0.68 cm. It is because the granular bed material easily change configuration when vibrated and also at this bed diameter, Brazil nut effect always occur. Based on Fig.7 also can be observed that the larger ratio of granular bed diameter and width of place vibration, the smaller contactopy change resulted. If ratio of granular bed diameter and width of place vibration is increased until reach maximum value that is 1, the change of contactopy will be 0. The result will be same for ratio with minimum granular bed diameter larger or equal to a half of the width shaker. Figure 8 is given to illustrate the system which is not has contactopy change after vibration.

Figure 8. System which is not has contactopy change at granular bed diameter equal to the
width of place vibration (left) and larger than a half of the width place vibration (right).

These two facts, they are the experiment result of contactopy change which give logarithmic equation and conformity with the fact that contactopy is not change when ratio of granular bed diameter and width of place vibration is equal, hence we can form suitable model to explain the phenomena as in Fig. 8 below.

![Graph of contactopy change for system without intruder](image)

**Figure 9.** A modelled graph of contactopy change for system without intruder.

Logarithmic equation based on the data above are as follows

$$\Delta C = -210.5 \ln\left(\frac{d}{L}\right) - 443.56$$  \hspace{1cm} (3)

The equation give the value of contactopy change when ratio of granular bed diameter and width of place vibration greater than 0 until less than 0.12155. While, when ratio of granular bed diameter and width of place vibration less than 1 until greater or equal to 0.5, the contactopy change will be 0. Mathematically can be written as follows

$$\Delta C = \begin{cases} 
-210.5 \ln\left(\frac{d}{L}\right) - 443.56, & \text{for } 0 < \frac{d}{L} < 0.12155 \\
0, & \text{for } 0.12155 < \frac{d}{L} < 0.5 \\
0.5 \frac{d}{L}, & \text{for } \frac{d}{L} \geq 0.5
\end{cases}$$  \hspace{1cm} (4)

The experiment result in this research can accommodate first and third criteria of Eq.3 above. While for ratio d/L between 0.12155 until 0.5 cm cannot be determined yet. It can be seen of Fig. 9 that 3 of 5 data is precisely on the modelled line. 2 data out of the line is the anomaly data which occur for granular bed diameter 0.53 cm and 0.375 cm. Both of these data give contactopy change of system with intruder differently. Figure 7 show that contactopy change for system with intruder always less than system without intruder, except for these two granular bed diameter.

3.2. Potential energy
Potential energy change is determined by the change of potential energy between two frames in a series. Potential energy is determined from Eq. 2. The height of system is in pixel because it is from photo taken during experiment, hence the height must be converted to meter. Decreasing the height of system indirectly indicate that potential energy also decreasing. Figure 10 and 11 below are contains the graph that represented potential energy change in a system without intruder and in a system with intruder at each granular bed diameter.

![Figure 10](image)

**Figure 10.** Graph of potential energy change on system without intruder at frequency vibration 14 Hz (△), 15 Hz (●), 16 Hz (○), 17 Hz (+), 18 Hz (▲), and 19 Hz (□) with a) bed diameter 0.68 cm, b) bed diameter 0.53 cm, c) bed diameter 0.47 cm, d) bed diameter 0.42 cm, e) bed diameter 0.375 cm.

These 5 graphs above show that potential energy is decreasing. The same result is occur for system with intruder as shown in Fig. 11 below.
Figure 11. Graph of potential energy change for system with intruder at granular bed diameter 0.68 cm (○), 0.47 cm (■), 0.42 cm (▲), and 0.375 cm (◆) at frequency 16 Hz (left) and 17 Hz (right).

The whole result of potential energy change show that compaction inhibit Brazil nut effect is obey minimum energy principle (MEP). All of this result of research confirm that increasing contactopy and decreasing potential energy is not only valid for Brazil nut affect but also valid for system with compaction inhibiting Brazil nut effect.

5. Conclusion
Entropy, which is in general the degree of regularity, cannot be used in Brazil-nut effect (BNE) phenomenon, since it decreases for this spontaneous phenomenon. Therefore, new definition of quantity called contactopy is required, which increases as expected for BNE. Minimum energy principle (MEP) is still valid for compaction inhibit Brazil-nut effect phenomenon.

Acknowledgments
This work is supported by research scheme Penelitian Unggulan Perguruan Tinggi (Desentralisasi DIKTI) in year 2015 with contract number 310/i1.C01/PL/2015.

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