Packet Travel Times in Wireless Relay Chains under Spatially and Temporally Dependent Interference

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Abstract—We investigate the statistics of the number of time slots $T$ that it takes a packet to travel through a chain of wireless relays. Derivations are performed assuming an interference model for which interference possesses spatiotemporal dependency properties. When using this model, results are harder to arrive at analytically, but they are more realistic than the ones obtained in many related works that are based on independent interference models.

First, we present a method for calculating the distribution of $T$. As the required computations are extensive, we also obtain simple expressions for the expected value $E[T]$ and variance $\text{Var}[T]$. Finally, we calculate the asymptotic limit of the average speed of the packet. Our numerical results show that spatiotemporal dependence has a significant impact on the statistics of the packet. In particular, we show that, with respect to the independent interference case, $E[T]$ and $\text{Var}[T]$ increase, whereas the packet speed decreases.

Index Terms—Relaying, interference, packet travelling time, Rayleigh fading, Poisson point processes.

I. INTRODUCTION

This paper studies the transport of a packet through a multi-hop chain of nodes, under interference that exhibits spatial and temporal dependency. In particular, we derive the statistics of the number of time slots $T$ required to travel across the chain, in a scenario where multiple retransmissions due to failures on each link may delay the packet reception. We consider interferers located on the plane according to a Poisson point process (PPP), and we assume that their positions do not change over time. This modeling assumption leads to temporally and spatially dependent interference, since the interference powers at different locations and times are influenced by the same set of nodes.

Our contributions are as follows:
- We provide an expression for the probability mass function (PMF) of $T$.
- We derive and study the first two moments of $T$; we compare the results for the case of having temporally and spatially dependent interference with the somewhat less realistic scenario where the interference levels at different transmissions are independent.
- We define the average speed of a packet traveling the relay chain, and we derive its asymptotic limit when the number of relays grows to infinity.
- Ultimately, we show that the model adopted for the interference significantly changes the estimated network performance; in particular, relying on the assumption that interference is independent at different times and locations leads to an underestimation of the values of $E[T]$ and, especially, $\text{Var}[T]$, as well as an overestimation of the asymptotic limit of the average speed.

The paper is organized as follows. Section II briefly summarizes the literature that is related to this work. Section III details the system under study and presents our modeling assumptions. The PMF of $T$ is derived in Section IV. Since computing the PMF is infeasible for large networks, in Sections V and VI we derive simple expressions for the expected value and variance of $T$. Section VII compares these values for temporally and spatially dependent or independent interference. Section VIII discusses the average speed of a packet traveling through the relay chain. Finally, Section IX concludes.

II. RELATED WORK

Relay chains have attracted significant research interest. The authors of [1] consider a node chain and derive the achievable coding rate under the assumption of a discrete memoryless channel. The authors of [2] derive the ergodic capacity for such chains. The diversity achieved in a multi-hop relay network is discussed in [3]. These papers, however, do not consider the influence of interference on system performance.

The influence of co-channel interference generated by nodes located according to a PPP is studied for single-hop networks in [4]–[11]. Multi-hop networks are considered in [12], where the authors derive the average progress of a packet toward its destination. Such PPP networks, where transmissions interfere with each other, have also been studied in [13] and [14], where the authors analyze the local delay of interference-limited mobile scenarios. The end-to-end delay of a Poisson point process network is discussed in [15].

III. MODELING ASSUMPTIONS

We consider a wireless scenario where a packet sent from a source $S$ toward a destination $D$ travels through a chain of $(N-1)$ relays $\{R_n\}_{n=1}^{N-1}$. The network is composed of $N$ links $\{L_n\}_{n=1}^{N}$ that connect $S$ and $D$. We let $S$, $D$, $R_n$ denote both
the nodes and their coordinates on the plane. The transmitter
and the receiver that are connected by $L_n$ are called $t_n$ and $r_n$,
respectively. This notation implies that $S = t_1$ and $D = r_N$.
Again, $t_n$ and $r_n$ denote both the nodes and their positions.
We assume that relays have known, and not random, positions.
An example network with two relays is shown in Fig. 1

Time is slotted, and the transmission sequence develops as
follows. In the first time slot, $S$ transmits the packet and $R_1$
tries to receive it. If the decoding at $R_1$ fails, $S$ retransmits
the packet in the following slot. Retransmissions continue until
$R_1$ successfully receives the packet. Then, $R_1$ transmits the
packet to $R_2$, following the same procedure. The transmission
sequence continues until the packet forwarded by $R_{N-1}$ is
correctly decoded at $D$.

Transmissions are subject to co-channel interference. Interferers
are distributed on the plane according to a PPP $\Phi$ of intensity $\lambda$ [5]. Let $\Phi$ also denote the locations of the interferers.
Each interferer accesses the channel at each time slot using slotted ALOHA, i.e., it transmits at each time slot
with probability $p$. We denote the set of active interferers in
a time slot with $\Phi_t$.

Two scenarios are compared: the dependent interference
one, where the positions of the interferers remain the same
for the whole transmission sequence, and the independent
interference one, where the positions change at every time
slot according to a new realization of the PPP. As opposed to
assuming independent interference, the dependent interference
model is a somewhat more realistic model for the temporal and
spatial dependency of interference under the given assumptions.

The channel connecting $t_n$ and $r_n$ is modeled using a
distance-dependent path loss combined with Rayleigh fading.
In particular, the power that is received at the receiver from
the transmitter is

$$P_n = h(n)g(t_n, r_n),$$

where $h(n)$ models Rayleigh fading and is an exponentially
distributed random variable having both mean and variance
equal to unity. The parameter $g(t_n, r_n)$ represents path loss.
In our model, the path loss parameter captures the variance of
the received power due to the distance between two stations,
while the Rayleigh fading accounts for micro-mobility. We
also denote with $h_u(n)$ the fading coefficient of the link
connecting the interferer $u \in \Phi$ to $r_n$, and with $g(u, r_n)$ the
corresponding path loss value. We assume that the values of
$h(n)$ and of $h_u(n)$ at different time slots are independent.
For simplicity, we do not explicitly mention the time indices
of fading values. Furthermore, we assume that the fading
values of two links connecting two different node pairs are
independent, even when they refer to the same time slot and
when the two pairs share a single common node.

Our modeling assumptions can be classified according to
the $(i, j, k)$ notation introduced in [7] for the nodes’ positions,
the channel evolution and the traffic behavior, respectively. In
particular, our dependent interference model has coordinates
$(2, 1, 1)$, since we assume static but unknown positions of interferers $(i = 2)$, a channel that changes randomly every slot $(j = 1)$, and ALOHA $(k = 1)$. The independent interference model has coordinates $(1, 1, 1)$, since the positions of interferers change randomly every slot $(i = 1)$, and the same channel and traffic models are used.

We consider an interference limited scenario, hence we
neglect the effects of noise. The signal-to-interference ratio
(SIR) at $r_n$ at the considered time slot can be expressed as

$$\rho_n = \frac{P_n}{I_n} = \frac{h(n)g(t_n, r_n)}{\sum_{u \in \Theta} h_u(n)g(u, r_n)1[u \in \Phi_t]}.$$  \hspace{1cm} (2)

where $I_n = \sum_{u \in \Theta} h_u(n)g(u, r_n)1[u \in \Phi_t]$ is the total
interference power received at $r_n$.

Finally, we assume that the transmission technology adopted
is such that $r_n$ correctly decodes the received packet if and
only if $\rho_n$ is higher than a threshold $\theta$. The ratio between
the success threshold and the path loss between $t_n$ and $r_n$ is

$$\theta_n = \frac{\theta}{g(t_n, r_n)},$$  \hspace{1cm} (3)

IV. PMF OF PACKET TRAVEL TIME

The random variable representing the number of time slots
required to traverse the chain is called $T$. We can express $T$
as

$$T = \sum_{n=1}^{N} T_n,$$  \hspace{1cm} (4)

where $T_n$ is the number of time slots required to traverse link
$L_n$. We are interested in calculating the PMF of $T$. We have

$$P[T = t] = \sum_{T_1, T_2, \ldots, T_N = t} P[T_1 = t_1, \ldots, T_N = t_N],$$  \hspace{1cm} (5)

where the positive integer $t_n \geq 1$ represents the number of
transmissions on link $L_n$.

Let us consider a single link $L_n$, and denote the event that its
$i$-th transmission is successful as $S_{i,n}$. In the case of dependent
interference, the locations $\Phi$ of interferers remain the same
for all transmissions. Since these locations are random, the
path loss values $g(u, r_n)$ are random variables. Moreover, the
path loss values at different transmissions are the same, since
the positions of interferers do not change. Hence, the success
events of two transmissions on link $L_n$ are dependent, since
they are a function of the path loss values. However, if we
consider a given realization of $\Phi$, that is if we fix the positions
of the interferers, the path loss values are not random. Hence,
given a realization of the PPP, two transmissions $i$ and $j$ at
different time slots are independent, since the only remaining
random contributions are from the fading values $h(n)$ and
$h_u(n), \forall u \in \Phi$, which we assumed to be independent at
We also note that (8) does not depend on the particular slot having independent fading values from interferers and from the slotted ALOHA access and by using the complementary cumulative distribution function of $\Phi$ and conditioning on the fading values of interferers and on $\tau_{n-1}$-th transmission was unsuccessful. This expression is valid for both dependent and independent interference. Hence, the probability that $\tau_{n-1}$-th transmission was unsuccessful.

The probability $P[\tau_n = t_n]$ is successful after exactly $t_n$ transmissions is

$$P[\tau_n = t_n] = E_\Phi \left[ \prod_{u \in \Phi} \left( \frac{p}{1 + \theta_n g(u, r_n)} + (1-p) \right) \right] \cdot \left( \prod_{i=0}^{t_n-1} \frac{p}{1 + \theta_i g(x, r_i)} + (1-p) \right) \cdot \left( \prod_{i=0}^{t_n-1} \frac{p}{1 + \theta_N g(x, r_N)} + (1-p) \right).$$

By substituting (8) into (7) we have

$$P[\tau_n = t_n] = E_\Phi \left[ \prod_{u \in \Phi} \left( \frac{p}{1 + \theta_n g(u, r_n)} + (1-p) \right) \right] \cdot \left( \prod_{i=0}^{t_n-1} \frac{p}{1 + \theta_i g(x, r_i)} + (1-p) \right) \cdot \left( \prod_{i=0}^{t_n-1} \frac{p}{1 + \theta_N g(x, r_N)} + (1-p) \right).$$

We finally calculate $P[T_1 = t_1, \ldots, T_N = t_N]$ as shown in (6) at the top of this page, where (a) follows from applying the probability generating functional of $\Phi$ (see (4.8)). By substituting (6) into (5) we can obtain the PMF of $T$. However, we can only calculate it for small values of $t$ and $N$ because of computational complexity, since $\sum_{n=1}^{N-1}$ values calculated according to (6) are summed in (5), and each of those terms requires numerically evaluating $\prod_{n=1}^{N-1} t_n$ integrals.

Since it is computationally infeasible to obtain the PMF of $T$ for large values of $t$ and $N$, in the rest of the paper we characterize the relay chain by calculating the expected value and the variance of $T$ using a computationally tractable method.

V. Expected Packet Travel Time

In this section, we obtain a closed form expression for the expected value of $T$. We note that

$$E[T] = E \left[ \sum_{n=1}^{N} T_n \right] = \sum_{n=1}^{N} E[T_n],$$

hence $E[T]$ is completely characterized once having found the expected number of time slots that it takes to travel a single link. This expression is valid for both dependent and independent interference.

We remember from the previous section that, in the case of dependent interference, the success events of single transmissions are independent once given a realization of $\Phi$. The expected value of $T$ for the case of dependent interference is obtained by computing $E[T] = E \left[ \sum_{n=1}^{N} T_n \right]$ as shown in (6) at the top of this page, where (a) follows from applying the probability generating functional of $\Phi$ (see (4.8)). By substituting (6) into (5) we can obtain the PMF of $T$. However, we can only calculate it for small values of $t$ and $N$ because of computational complexity, since $\sum_{n=1}^{N-1}$ values calculated according to (6) are summed in (5), and each of those terms requires numerically evaluating $\prod_{n=1}^{N-1} t_n$ integrals.

Since it is computationally infeasible to obtain the PMF of $T$ for large values of $t$ and $N$, in the rest of the paper we characterize the relay chain by calculating the expected value and the variance of $T$ using a computationally tractable method.
\[ E[T_m T_n] = E_\Phi [E[T_m | \Psi] \cdot E[T_n | \Psi)] = E_\Phi [E[T_m | \Psi] \cdot E[T_n | \Phi]] = E_\Phi \left[ \frac{1}{P[S_m | \Phi]} \cdot \frac{1}{P[S_n | \Phi]} \right] \]

\[ = E_\Phi \left[ \prod_{u \in \Phi} \left( \frac{p}{1 + \theta_m g(u, r_m)} + (1 - p) \right)^{-1} \left( \frac{p}{1 + \theta_n g(u, r_n)} + (1 - p) \right)^{-1} \right] \]

\[ = \exp \left( -\lambda \int_{\mathbb{R}^2} \left[ 1 - \left( \frac{p}{1 + \theta_m g(x, r_m)} + (1 - p) \right) \right] dx \right). \quad (11) \]

Hence, for a given realization of \( \Phi \), the random variable \( T_n | \Phi \) is geometrically distributed with parameter \( P[S_n | \Phi] \). The parameter \( P[S_n | \Phi] \) represents the probability of having a successful transmission on link \( \mathcal{L}_n \) at any time slot conditioned on interferers’ positions. Recalling that the expected value of the geometric distribution with parameter \( \psi \) is \( \psi^{-1} \), we get

\[ E[T_n] = E_\Phi [E[T_n | \Phi]] = E_\Phi \left[ \frac{1}{P[S_n | \Phi]} \right], \quad (12) \]

where \( P[S_n | \Phi] \) is obtained in (8) for the special case of the \( i \)-th transmission.

By combining (12) and (8) and by applying the probability generating functional of \( \Phi \) (see [17], (4.8)), we finally get

\[ E[T_n] = E_\Phi \left[ \prod_{u \in \Phi} \left( \frac{1}{1 + \theta_n g(u, r_n)} + (1 - p) \right) \right] \]

\[ = \exp \left( -\lambda \int_{\mathbb{R}^2} \left[ 1 - \left( \frac{1}{1 + \theta_n g(x, r_n)} + (1 - p) \right) \right] dx \right). \quad (13) \]

If we have independent interference, the single retransmissions on \( \mathcal{L}_n \) are independent, since the positions of interferers at two time slots are independent. Hence, \( T_n \) is geometrically distributed with parameter \( P[S_n] \), leading to

\[ E[T_n] = \frac{1}{P[S_n]} = \frac{1}{E_\Phi [P[S_n | \Phi]]} \]

\[ = \frac{E_\Phi \left[ \prod_{u \in \Phi} \left( \frac{p}{1 + \theta_n g(u, r_n)} + (1 - p) \right) \right]}{E_\Phi \left[ \prod_{u \in \Phi} \left( \frac{p}{1 + \theta_m g(u, r_m)} + (1 - p) \right) \right]} \]

\[ = \exp \left( \lambda \int_{\mathbb{R}^2} \left[ 1 - \left( \frac{p}{1 + \theta_n g(x, r_n)} + (1 - p) \right) \right] dx \right). \quad (14) \]

where, again, (a) follows from applying the probability generating functional of \( \Phi \) (see [17], (4.8)).

VI. VARIANCE OF PACKET TRAVEL TIME

We now calculate the variance of \( T \). We have

\[ \text{Var}[T] = E[T^2] - (E[T])^2. \quad (15) \]

The value of \( E[T] \) was obtained in the previous section in (10), with \( E[T_n] \) given by either (13) or (14), and we can calculate \( E[T^2] \) as

\[ E[T^2] = E \left[ \left( \sum_{n=1}^{N} T_n \right)^2 \right] = E \left[ \sum_{m=1}^{N} T_m \sum_{n=1}^{N} T_n \right] \]

\[ = \sum_{m=1}^{N} E[T_m T_n] \cdot \sum_{n=1}^{N} E[T_n] \quad (16) \]

For obtaining \( E[T_m T_n] \) we distinguish two cases, namely the case \( m \neq n \) and the case \( m = n \). Consider at first the case where \( m \neq n \). We noted in Section IV that for a given realization of the PPP the success events of two transmissions are independent, which implies that \( T_m | \Phi \) and \( T_n | \Phi \) are independent. Hence, we have the expressions for \( E[T_m T_n] \) shown in (11) at the top of this page.

If we have independent interference, the single retransmissions are independent, and so are \( T_m \) and \( T_n \). Hence, we obtain

\[ E[T_m T_n] = E[T_m] \cdot E[T_n], \quad (17) \]

where, again, \( E[T_m] \) and \( E[T_n] \) are obtained according to (14).

Consider now the case where \( m = n \). We showed in the previous section that the variable \( T_n | \Phi \) is geometrically distributed with parameter \( P[S_n] \), once having conditioned on a given PPP realization. Recalling that, for a geometrically distributed random variable \( \Psi \) with parameter \( \psi \), \( E[\Psi^2] = \frac{1}{\psi^2} - \frac{1}{\psi} \), we obtain the value of \( E[T_n^2] \) shown in (19) at the top of the next page.

If we have independent interference, \( T_n \) is geometrically distributed with parameter \( P[S_n] \). Hence, we obtain

\[ E[T_n^2] = \frac{2}{P[S_n]^2} - \frac{1}{P[S_n]} \]

\[ = \left( E_\Phi [P[S_n | \Phi]] \right)^2 - E_\Phi [P[S_n | \Phi]] \]

\[ = 2 \exp \left( 2\lambda \int_{\mathbb{R}^2} \left( 1 - \frac{p}{1 + \theta_n g(x, r_n)} - (1 - p) \right) dx \right) - \exp \left( \lambda \int_{\mathbb{R}^2} \left( 1 - \frac{p}{1 + \theta_n g(x, r_n)} - (1 - p) \right) dx \right). \quad (18) \]

VII. COMPARING THE PACKET TRAVEL TIMES WITH DEPENDENT AND INDEPENDENT INTERFERENCE

This section compares \( E[T] \) and \( \text{Var}[T] \) for the dependent and independent interference cases.

At first, note that the singular path loss of [5]

\[ g(t_n, r_n) = \|t_n - r_n\|^{-\alpha}, \quad (20) \]
produces two drastically different behaviors in the independent and dependent interference cases, when the value \( p = 1 \), i.e., when interferers transmit in every slot. In particular, we find that, in the case of dependent interference, \( \mathbb{E}[T] \) and \( \text{Var}[T] \) are infinite, while they have finite values for the independent interference case. This can be seen by looking at \( \mathbb{E}[T_n] \) for the dependent interference case. By substituting \( p = 1 \) and (20) into (13), we have

\[
\mathbb{E}[T_n] = \exp \left( \lambda \int_{\mathbb{R}^2} \theta_n \|x - r_n\|^{-\alpha} \, dx \right)
= \exp \left( 2\pi \lambda \int_0^{\infty} \theta_n \rho^{-\alpha+1} \, d\rho \right),
\]

which is infinite for all values of \( \alpha \). By contrast, following the same procedure for the independent interference case by substituting \( p = 1 \) and (20) into (14) we find a finite value for \( \mathbb{E}[T_n] \) and \( \text{Var}[T_n] \).

This behavior is an artifact of the path loss assumption in (20). For obtaining meaningful results, we consider the non-singular path loss model

\[
g(t_n, r_n) = \min \left( 1, \|t_n - r_n\|^{-\alpha} \right).
\]

The value of \( \mathbb{E}[T] \) for a scenario where \( S \) is located at \((0, 0)\), \( D \) at \((1, 0)\) and the \( N \) relays are equally spaced on the \([0, 1]\) segment is shown in Fig. 2 for an increasing value of \( N \) and different values of \( \lambda \).

From Fig. 2 we can see that dependent interference increases the average number of slots required to successfully deliver the packet to \( D \). The difference, albeit modest, is more significant when \( \lambda \) increases, i.e., when the density of interferers increases. Fig. 3 shows \( \text{Var}[T] \). It can be seen again that the value for dependent interference is significantly higher than the one for independent interference.

Results for \( \text{Var}[T] \) for dependent and independent interference when all links have a length of \( L \), and hence the total distance between \( S \) and \( D \) is \( NL \), are shown in Fig. 4.
From Figs. 5 and 4, one can notice that another difference between the dependent and independent interference cases is the relation between $\text{Var}[T]$ and $N$. In particular, we observe that in the dependent interference case $\text{Var}[T]$ grows faster than linearly with $N$, while in the independent interference case it grows linearly with $N$, as in this case $\text{Var}[T] = \sum_{n=1}^{N} \text{Var}[T_n]$. However, one can observe that the growth of $\text{Var}[T]$ with $N$ becomes almost linear when the number of relays increases while maintaining the link length constant. This is because the covariance between the interference powers at two links $L_m$ and $L_n$ decreases when the distance between the links increases, as shown in Fig. 5. Hence, two distant links are almost independent, and $\text{E}[T_m T_n]$ is very close to $\text{E}[T_m] \text{E}[T_n]$. These two terms cancel out in the expression of the variance, and thus the only superlinear contribution is given by links that are close, in the sense that their receptions are dependent. However, when having many relays located at a fixed distance, each node is close enough, in this sense, to only a small fraction of the rest of the nodes, and their quadratic contribution is not significant. Hence, the value of $\text{Var}[T]$ is very close to being linear with respect to $N$. This result is discussed in more depth in the next section.

VIII. SPEED OF A PACKET TRAVELING A UNIFORM CHAIN

In the previous section we showed that the covariance $\text{Cov} [T_m, T_n] = \text{E}[T_m T_n] - \text{E}[T_m] \text{E}[T_n]$ decreases rapidly with the distance between two links in a uniform chain scenario, i.e., when all the links have the same length $L$. Motivated by Fig. 5, we can approximate the covariance value by assuming that $\text{Cov} [T_m, T_{n+k}] = 0$ if $k > K$, that is, assuming that two links that have at least $K$ other links between them are uncorrelated. We also have $\text{Cov} [T_n, T_{n+k}] \leq \lambda N \text{Var}[T_k]$, where we denote the bounding value of the covariance with $C$. We also use $T(\cdot)$ for representing the number of slots required to travel a single link, since it does not depend on the link index, given that all links have the same length $L$. We can then rewrite $\text{Var}[T]$ as

$$\text{Var}[T] = \sum_{n=1}^{N} \text{Var}[T_n] + 2 \sum_{n=1}^{N} \sum_{k=1}^{\min(N-n,K)} \text{Cov}[T_nT_{n+k}],$$

hence, one can see from (23) that $\text{Var}[T]$ grows linearly with $N$ when distant links are uncorrelated.

We define the random variable $V_p$ representing the average speed of a packet as

$$V_p = \frac{NL}{T},$$

which is the ratio between the distance that the packet travels and the time it takes the packet to reach its destination. One can see that, in order to obtain the PMF of the average speed of the packet, we need to calculate the PMF of $T$. However, we can simplify the calculation when the number of relays is infinite, and the covariance between two links is zero when they are sufficiently separated, so that (23) holds.

In particular, let us consider $V_p^{-1} = \frac{T}{NL}$. We have

$$\text{E} [V_p^{-1}] = \text{E} \left[ \frac{T}{NL} \right] = \frac{\sum_{n=1}^{N} \text{E}[T_n]}{NL} = \frac{\text{E}[T(\cdot)]}{L},$$

$$\text{Var} [V_p^{-1}] = \text{Var} \left[ \frac{T}{NL} \right] = \left( \frac{1}{NL} \right)^2 \text{Var}[T].$$
Using Chebyshev’s inequality, we have, for some $\epsilon > 0$,

$$P\left[ \left| \frac{V_p^{-1} - E[V_p^{-1}]}{L} \right| > \epsilon \right] \leq \frac{\text{Var}[V_p^{-1}]}{\epsilon^2} \tag{27}$$

and by substituting (25) and (26) into (27) we obtain

$$P\left[ \left| \frac{V_p^{-1} - E[T_{(1)}]}{L} \right| > \epsilon \right] \leq \frac{\text{Var}[T]}{N^2L^2\epsilon^2} \tag{28}$$

We now consider the asymptotic scenario where the number of relays is infinite, leading to

$$\lim_{N \to \infty} P\left[ \left| \frac{V_p^{-1} - E[T_{(1)}]}{L} \right| > \epsilon \right] \leq \lim_{N \to \infty} \frac{\text{Var}[T]}{N^2L^2\epsilon^2} (a) \leq 0 \tag{29}$$

where (a) follows from recalling that $\text{Var}[T]$ grows linearly with $N$. Hence, in the asymptotic case, the average speed of a packet converges in probability to the value $\frac{E[T]}{L}$ (see [13], Theorem 1.10). The value of $V_p$ is shown in Fig. 6 and we observe that in the dependent interference scenario the packet travels the chain with a lower average speed.

IX. CONCLUSIONS

This paper analyzed the time it takes a packet to traverse a chain of relays with surrounding interferers. Derivations assume Rayleigh fading, Poisson distributed interferers, slotted ALOHA, and infinite packet retransmissions for each hop. The spatiotemporal dependence of interference has significant impact on the travel time. In particular, if the interferers’ positions remain fixed for the entire duration, a packet is on average slower as if they are positioned anew in each slot. Also the variance of the packet travel time is higher in the first case. These facts should be taken into consideration when analyzing packet propagation in wireless multi-hop networks.

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