Propagation of Scalars in Non-extremal Black Hole 
and Black $p$-brane Geometries

YUJI SATOH

Joseph Henry Laboratories, Princeton University
Princeton, NJ 08544, USA

Abstract

We discuss the propagation of scalars in a large class of non-extremal black hole 
and black $p$-brane geometries in generic dimensions. We show that the radial 
wave equation near the horizon possesses the $SL(2,R)$ structure in every case; 
approximately it takes the form of the wave equation in the $SL(2,R)(AdS_3)$ 
background and has a symmetry related to the T-duality of the string model in 
that geometry. We see a close connection to two and three dimensional black 
holes. We also find that, in some parameter region, the absorption cross-
sections by the black objects take the form expected from a conformal field 
theory. Our results indicate that some of the properties known about a certain 
class of four and five dimensional black holes hold more generally.

PACS codes: 04.70.Dy, 11.25.Hf
1 Introduction

The microscopic origin of black hole thermodynamics has been an intriguing subject of the quantum theory of gravity. In fact, a vast number of works has been devoted for the purpose of understanding the Bekenstein-Hawking entropy and the Hawking radiation from a fundamental theory. In this respect, excellent progress has recently been achieved in superstring theory. (For a review see, e.g., [1].) For a certain class of four and five dimensional extremal black holes, the entropy formula was derived by counting the degeneracy of the corresponding Bogomol’ni-Prasad-Sommerfield (BPS) states. It was soon realized that such a counting reproduces the black hole entropy for slightly non-extremal cases. Furthermore, using a string model (the effective string model) based on solitonic solutions called $D$-branes, it was shown that the decay rates of some non-BPS string states precisely agree with the Hawking radiation from the non-extremal black holes [2]-[7].

However, the relationship between the non-BPS strings and the non-extremal black holes is not completely clear: The analysis in string theory is based on $D$-branes at weak coupling whereas the semi-classical calculation in general relativity is valid in the strong coupling regime (see, e.g., [4]); Without the BPS condition, the extrapolation of the results from weak coupling to strong coupling is not justified. We do not have a first-principle derivation of the effective string model either. Moreover, subsequent detailed analysis showed that, in a generic parameter region, one does not find agreement between the decay rates from general relativity and those from the simple effective string model [9]-[11].

One outstanding feature in these arguments is that the non-extremal black holes possess properties suggestive of conformal field theory (CFT) even beyond the parameter region of the agreement [1], [12], [9]-[11], [13]-[20]. Let us see this for the absorption cross-sections of scalars. First, let us consider a quantum mechanical system of a scalar whose dynamics is described by some CFT (which is not necessarily related to black holes and string theory) and suppose that the system is in thermal equilibrium. Under this general assumption, the absorption cross-section of the scalar is given by [13, 14]

$$
\sigma_{\text{abs}} \sim \omega^{2l-1} \sinh \left( \frac{1}{2} \beta_H^\omega \right) \Gamma \left( h_L + \frac{i}{4\pi} \beta_L^\omega \right) \Gamma \left( h_R + \frac{i}{4\pi} \beta_R^\omega \right) \Bigg| \frac{1}{2} \beta_H^\omega \Bigg|^2,
$$

(1.1)

where $\omega$ is the frequency; $\beta_L^{-1}$ ($\beta_R^{-1}$) is the left-(right-) temperature; $\beta_H/2 = \beta_L + \beta_R$; $l$ is the number of derivatives acting on the scalar in the interaction term and $h_{L(R)}$ is the left-(right-) dimension of the operator coupled to the scalar. Remarkably, the absorption cross-sections of scalars by the black holes take the same form in some parameter region at low energy. On the black hole side, $l$ indicates the $l$-th partial wave, $\beta_H^{-1}$ is the Hawking temperature, $\beta_{L,R}$ are some geometric quantities associated with horizons and $h_{L,R}$ are linear functions of

---

1For the arguments regarding this issue, see [3].
The effective string model is a kind of CFT and the absorption cross-section takes the form (1.1). Furthermore, one finds the precise agreement including numerical factors with the general relativity calculation in some parameter region [3]-[7]. However, for the semi-classical black holes, the expression (1.1) is valid in a more general parameter region which the string model cannot probe, and in which disagreement is found [9]-[11], [13]-[18]. This region includes near-extremal Kerr-Newman black holes in four and five dimensions [13, 16] which are not supersymmetric even in the extremal limit.

Another interesting feature in the general relativity calculation is that the radial wave equations have an $SL(2,\mathbb{R})$ structure near the horizons; introducing auxiliary variables, they can be rewritten as an eigenvalue equation of the Laplace operator on $SL(2,\mathbb{R})$ [16, 20]. In addition, one finds that the wave equations have a symmetry similar to T-duality symmetries in string theory [16]. These facts suggest a relation between the non-extremal black holes and the CFT’s associated with $SL(2,\mathbb{R})$.

Given this state of the problem, one might expect a close connection between the non-extremal black holes and CFT’s which are more general than the effective string model. Also, it would be interesting to study how general the above properties of the wave equations and the absorption cross-sections are. We address this issue in the following. We will discuss the propagation of scalars in various non-extremal spherically symmetric black hole and black $p$-brane geometries. We will find that the above properties are very general and common to all these cases.

The organization of the paper is as follows. In section 2, we discuss the scalar propagation in the Reissner-Nordstrom black hole background in generic dimensions. This provides the simplest case in this paper. We discuss it in some detail to make the points of the later discussions clear. We find that the properties discussed above are valid also in this case. Moreover, we find that the T-duality-like symmetry corresponds to the actual T-duality of the string model on $SL(2,\mathbb{R})$. We observe a close relation to the two dimensional $SL(2,\mathbb{R})/U(1)$ black holes and three dimensional BTZ (Bñados-Teitelboim-Zanneli) black holes. We notice that the arguments for the Reissner-Nordstrom black holes do not depend on the details of the geometry. In section 3, we argue that the properties found in section 2 may hold for a general class of black objects. We confirm this in the subsequent sections. In section 4, we discuss the dyonic dilaton black holes by Gibbons and Maeda. In section 5 and 6, we discuss charged black holes in string theory and a large class of $p$-branes in generic dimensions, respectively. It turns out that the arguments are almost the same in every case. Summary and discussion are given in section 7.
2 Reissner-Nordstrom black holes

We begin our discussion with the simplest case, i.e., the Reissner-Nordstrom black hole. To make the points clear and this article self-contained, we discuss this case in some detail.

2.1 Space-time geometry

The metric of the Reissner-Nordstrom black holes in $D$-dimensional space-time is given by

$$ds_{\text{RN}}^2 = -f_{\text{RN}}(r)dt^2 + f_{\text{RN}}^{-1}(r)dr^2 + r^2d\Omega_{d+1}^2,$$

where $d = D - 3$; $d\Omega_{d+1}$ is the metric of the unit $(d + 1)$-dimensional sphere;

$$f_{\text{RN}}(r) = \left[1 - \left(\frac{r_+}{r}\right)^d\right]\left[1 - \left(\frac{r_-}{r}\right)^d\right].$$

The geometry has two regular horizons at $r = r_{\pm}$. For $D = 4$ and 5, this is a special case of the black holes extensively studied in string theory [1] and the extremal geometry becomes supersymmetric. Thus, for $D = 4$ and 5, the results in this section are included in [10, 13, 16, 18].

For later use, we introduce a parameter $\mu = r_+^d - r_-^d \equiv r_0^d$ and a new coordinate $z = (r^d - r_-^d)/\mu$. The extremal limit corresponds to $\mu \to 0$. The outer and the inner horizon are at $z = 1$ and 0 respectively. In terms of this coordinate, the metric is written as

$$ds_{\text{RN}}^2 = -z(z - 1)V_{\text{RN}}^{-\frac{d}{d+1}}(z)dt^2 + r_0^2V_{\text{RN}}^{\frac{1}{d+1}}(z)\left[\frac{1}{d^2}\frac{dz^2}{z(z - 1)} + d\Omega_{d+1}^2\right],$$

where

$$V_{\text{RN}}(z) = \left(\frac{r}{r_0}\right)^{2(d+1)} = (z + \hat{Q}_+)^{2(1+\frac{1}{d})},$$

and $\hat{Q}_\pm = r_\pm^d/\mu$.

Near the outer horizon, the geometry looks like Rindler space;

$$ds_{\text{RN}}^2 \sim -\kappa_+^2\sigma_+^2 dt^2 + d\sigma_+^2 + \cdots.$$  \hspace{1cm} (2.5)

Here $\sigma_+$ is $(2r_0/d)V_{\text{RN}}^{\frac{1}{d+1}}(1)\sqrt{z - 1}$ and $\kappa_+^2 = (2r_0/d)^2V_{\text{RN}}(1)$. The imaginary time $it$ is regarded as an angle coordinate in $(\sigma_+, it)$-plane and its period is given by

$$\beta_H^{\text{RN}} = \frac{2\pi}{\kappa_+} = \frac{4\pi}{d}r_0V_{\text{RN}}^{\frac{1}{d+1}}(1) = \frac{4\pi}{d\mu}r_+^{d+1}.$$ \hspace{1cm} (2.6)

Similarly, near the inner horizon, the metric takes the form

$$ds_{\text{RN}}^2 \sim -\kappa_-^2\sigma_-^2 dt^2 + d\sigma_-^2 + \cdots,$$ \hspace{1cm} (2.7)
with \( \sigma_- = (2r_0/d)V_{RN}^{d+1}(0)\sqrt{z} \) and \( \kappa_-^{-2} = (2r_0/d)^2V_{RN}(0) \). Then the imaginary time is again regarded as an angle coordinate, whose period is

\[
\beta_{RN}^- = \frac{2\pi}{\kappa_-} = \frac{4\pi}{d}r_0^{\frac{1}{2}}V_{RN}^{\frac{1}{2}}(0) = \frac{4\pi}{d\mu}r^{d+1}.
\]

\[ (2.8) \]

### 2.2 Absorption cross-section of scalars

Now we consider the propagation of a minimally coupled massless scalar in this geometry. Its \( l \)-th partial wave satisfies the radial equation,

\[
\left[ d^2 \left\{ z(z-1)\partial_z \right\}^2 + (r_0\omega)^2V_{RN}(z) - z(z-1)\Lambda \right] \varphi_l = 0,
\]

where \( \omega \) is the frequency and \( \Lambda = l(l+d) \). We are interested in the absorption cross-section by the black hole. The basic strategy to calculate it is the same as in the literature: we (i) solve the approximated wave equation in the near and the far region, (ii) match the two solutions, (iii) compute the fluxes at the outer horizon and at infinity and (iv) obtain the absorption cross-section.

Following [13], near the outer horizon we approximate \( V_{RN}(z) \) by

\[
V_{RN}(z) \sim \tilde{V}_{RN}(z) = V_{RN}(1) + (z-1)\partial_zV_{RN}(1).
\]

(2.10)

The validity of the approximation will be discussed later in detail. Then the wave equation becomes

\[
\left[ d^2 \left\{ z(z-1)\partial_z \right\}^2 + (r_0\omega)^2\tilde{V}_{RN}(z) - z(z-1)\Lambda \right] \varphi_l = 0.
\]

(2.11)

From this expression, we find that Eq. (2.11) has three regular singular points at \( z = 0, 1 \) and \( \infty \). These correspond to the inner horizon, the outer horizon and infinity, respectively. If a homogeneous linear ordinary differential equation of the second order has only three regular singular points, it is completely characterized by the exponents (the roots of the indicial equation) at the regular singular points. This means that the solutions to (2.11) are completely determined by the information at the horizons and infinity (in an approximated sense). We denote the exponents at each singular point by \( \nu_{RN}^{'} \) for \( z = 0 \), \( \nu_+^{RN} \) for \( z = 1 \) and \( \nu_{\infty}^{RN} \) for \( z = \infty \). By the standard procedure, we get

\[
\begin{align*}
\nu_+^{RN} &= -\nu_-^{RN} = -i\frac{r_0\omega}{d}V_{RN}^{\frac{1}{2}}(1) = -i\frac{\beta^{RN}}{4\pi}H^\omega, \\
\nu_-^{RN} &= -\nu_-^{RN} = -i\frac{r_0\omega}{d}V_{RN}^{\frac{1}{2}}(0) = -i\frac{\beta_-^{RN}}{4\pi}(1 + \mathcal{O}(\hat{Q}_-^{1}))\omega, \\
\nu_{\infty}^{RN} &= 1 - \nu_+^{RN} = 1 + \frac{\beta^{RN}}{4\pi} \left( 1 + \sqrt{1 + 4\Lambda/d^2} \right) = 1 + \frac{l}{d}.
\end{align*}
\]

\[ (2.12) \]
As a check we can confirm Fuchs’ relation $\sum \nu^{RN} = 1$. Note that $\nu^{RN(\pm)}$ are essentially the periods of the imaginary time, $\beta^{RN}_{H, L}$. Then the set of the solutions is represented by the $P$-function of Riemann,

$$
P \left\{ \begin{array}{ccc} 0 & 1 & \infty \\ \nu^{RN} & \nu^{RN}_+ & \nu^{RN}_- \\ \nu^{RN'} & \nu^{RN'}_+ & \nu^{RN'}_- \\ \nu^{RN''} & \nu^{RN''}_+ & \nu^{RN''}_- \\ \end{array} \right\} = z^{\nu^{RN}} (z-1)^{\nu^{RN}_+} P \left\{ \begin{array}{ccc} 1 & 0 & \infty \\ 0 & 0 & a \\ c-a-b & 1-c & b \\ \end{array} \right\},$$

where

$$
a = \nu^-_RN + \nu^+_RN + \nu^\infty_RN = -\frac{i}{4\pi} \beta^{RN}_L \omega - j_l, \\
b = \nu^\infty_RN + \nu^+_RN + \nu^-_RN' = -\frac{i}{4\pi} \beta^{RN}_L \omega + (j_l + 1), \\
c = 1 + 2\nu^+_RN = 1 - \frac{i}{2\pi} \beta^{RN}_H \omega, $$

and

$$
\beta^{RN}_L = \beta^{RN}_H + \beta^-_RN, \quad \beta^{RN}_R = \beta^{RN}_H - \beta^-_RN, \\
j_l = -\nu^\infty_RN = - \left( 1 + \frac{l}{d} \right). $$

Since the $P$-function in the right-hand side represents the solutions to the hypergeometric equation, the mode which is purely ingoing at the outer horizon is expressed by the hypergeometric functions as

$$
\varphi^I_l = z^{\nu^-_RN} (z-1)^{\nu^+_RN} F(a, b, c; 1 - z).$$

Using the asymptotics of the hypergeometric functions, we find the asymptotic behavior of $\varphi^I_l$ for large $z$ (i.e., large $r/r_0$),

$$
\varphi^I_l \sim \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)} \left( \frac{r}{r_0} \right)^l.
$$

In the far region $r \gg r_+$ ($z \gg \hat{Q}_+$), the wave equation simplifies to

$$
\left[ (r^{d+1}\partial_r)^2 + r^{2d} (r^2\omega^2 - \Lambda) \right] \varphi^I_l = 0.
$$

This is just the wave equation in flat space-time and can be solved in terms of the Bessel functions. It turns out that the solution which matches to the near-region solution is given by

$$
\varphi^{II}_l = A \left( \frac{2}{\omega r} \right)^{d/2} J_{l+d/2}(\omega r),
$$
where $A$ is a normalization constant. For small $r\omega$, this behaves as

$$\varphi_{II}^l \sim \frac{A}{\Gamma(1 + l + d/2)} \left(\frac{\omega r}{2}\right)^l.$$  \hspace{1cm} (2.20)

Matching $\varphi_I^l$ and $\varphi_{II}^l$ in the region $\omega^{-1} \gg r \gg r_0, r_+$, we find that

$$A = \left(\frac{r_0\omega}{2}\right)^{-l} \frac{\Gamma(1 + l + d/2)\Gamma(c)\Gamma(a - b)}{\Gamma(a)\Gamma(c - b)}.$$ \hspace{1cm} (2.21)

To get the absorption cross-section, we use the conserved flux,

$$F = \frac{1}{2} \sqrt{-g} g^{rr} \left(\varphi_I^l \partial_r \varphi_I^l - \text{c.c.}\right),$$ \hspace{1cm} (2.22)

where $g_{\mu\nu}$ and $g$ are the metric and its determinant, respectively. The absorption probability is the ratio of the ingoing flux at the outer horizon, $F_H$, to the ingoing flux at infinity, $F_\infty$. Using the asymptotic forms,

$$\varphi_I^l \sim e^{\nu_z R_H \ln(z - 1)}$$ \hspace{1cm} for $z \to 1$,

$$\varphi_{II}^l \sim A \frac{2}{\sqrt{\pi}} \left(\frac{2}{\omega r}\right)^{(d+1)/2} \cos \left\{\omega r - \frac{\pi}{4}(2l + d + 1)\pi\right\}$$ \hspace{1cm} for $\omega r \to \infty$, \hspace{1cm} (2.23)

we get the absorption probability,

$$P_{\text{abs}} = \frac{F_H}{F_\infty} = d\mu \beta_{H}^{\omega |A|^{-2}} \left(\frac{\omega}{2}\right)^{d+1}.$$ \hspace{1cm} (2.24)

Finally, the absorption probability is converted to the absorption cross-section by \[15, 22\]

$$\sigma_{\text{abs}} = \frac{\pi^{d/2}}{d} \left(\frac{2}{\omega}\right)^{d+1} \left(l + \frac{d}{2}\right) \Gamma \left(1 + \frac{d}{2}\right) \left(1 + l - \frac{d - 1}{l}\right) P_{\text{abs}}.$$ \hspace{1cm} (2.25)

Putting Eqs. (2.21), (2.24) and (2.25) together, we obtain the absorption cross-section,

$$\sigma_{\text{abs}} = \pi^{d/2} \mu^{1+\frac{d}{2}} \left(l + \frac{d - 1}{l}\right) \frac{(l + d/2)\Gamma(1 + d/2)}{\Gamma(1 + l + d/2)\Gamma(1 + 2l/d)!^2} \times \left(\frac{\omega}{2}\right)^{2l-1} \sinh \left(\frac{1}{2}\beta_{H}^{\omega R_L}\right) \left|\Gamma \left(1 + \frac{l}{d} - \frac{i}{4\pi} \beta_{H}^{\omega R_L}\right)\right|^2.$$ \hspace{1cm} (2.26)

Here we have used the identity $|\Gamma(1 - i\gamma)|^2 = \pi\gamma / \sinh \pi\gamma$.

For $l = 0$ and $\omega \to 0$, $\sigma_{\text{abs}}$ reduces to the area of the outer horizon as proved in \[22\]:

$$\sigma_{\text{abs}} \to A_H.$$ \hspace{1cm} (2.27)

\[2\] The condition of the validity of the matching procedure has been discussed in \[23\].
The above form of the absorption cross-section (2.26) is the same as Eq.(1.1), which is expected from a CFT. Therefore, if the above expression gives the non-trivial frequency dependence from the hyperbolic and gamma functions under the condition of validity, the Reissner-Nordstrom black holes in generic dimensions share the CFT structure with the four and five dimensional black holes. In the next subsection, we will confirm that this is the case. We remark that, for \( D \geq 6 \), we do not have any corresponding D-brane configurations near and at extremality. We also note that, in the formal extremal limit \( r_0 \to 0 \) or low energy limit \( \omega \to 0 \), the solution to Eq.(2.9) is given by the hypergeometric functions with \( \beta_{RN}^{L,R} = 0 \) without the approximation about \( V_{RN} \).

2.3 Validity of the approximation

We obtained the expression of the absorption cross-section (2.26). Now we discuss the range of validity of the approximation. For the time being, we focus on \( \Lambda \neq 0 \).

First, let us examine the matching procedure. In the near region, we approximated \( V_{RN} \) by \( \tilde{V}_{RN} \) and used the asymptotic form for \( z \gg 1 \). In the far region \( r \gg r_+ \), we used the asymptotic form for \( \omega r \ll 1 \). Thus, we need to require the following conditions so that both approximations are valid at the matching point \( r = r_m \):

\[
(r_0 \omega)^2 |V_{RN} - \tilde{V}_{RN}| \ll |z(z - 1)\Lambda - (r_0 \omega)^2 V_{RN}|, \quad (2.28)
\]

for \( z \gg 1 \), which means that the error is small enough compared with the true total potential term, and

\[
r \gg r_+, \quad r \omega \ll 1. \quad (2.29)
\]

For \( \Lambda \neq 0 \), these conditions reduce to

\[
r_m \gg r_+, \quad r_m \omega \ll 1. \quad (2.30)
\]

This indicates that the frequency \( \omega \) should be small enough.

For \( \omega \to 0 \), Eq. (2.26) gives \( \sigma_{abs} \sim \omega^{2l-1} \). However, we are interested in more precise \( \omega \)-dependence. To get such a non-trivial \( \omega \)-dependence, the expression (2.26) should be valid in the parameter region \( |\nu_{RN}^\pm| \sim \beta_{RN}^{L,R} \omega \sim \mathcal{O}(1) \). In fact, \( \beta_{RN}^{H} \omega \) can be of order unity when \( \hat{Q}_+ = \frac{r^4}{\mu} \) is sufficiently large. This is typically achieved near extremality. Notice that we need two scales to achieve this; one scale, say \( r_0 \omega \), should be very small from the condition (2.30), but we can make another scale \( \hat{Q}_+ \) very large independently of \( r_0 \omega \). This is impossible for geometries with only one scale such as the Schwarzschild black holes. In turn, for \( |\nu_{RN}^\pm| \sim \mathcal{O}(1) \) and \( \hat{Q}_+ \gg 1 \), it follows that \( \beta_{RN}^H \sim \beta_{RN}^L \) and hence

\[
\beta_{RN}^L = 2\beta_{RN}^H, \quad \beta_{RN}^R = 0. \quad (2.31)
\]
Thus, the absorption cross-section does not take the factorized form into the left and right part. Instead, Eq.(2.26) should be read as

\[
\sigma_{\text{abs}} = \pi \mu^{4+d+\frac{d}{2}} \left( l + d - 1 \right) \frac{(l + d/2)\Gamma(1 + d/2)\Gamma(1 + l/d)^2}{\left[ \Gamma(1 + l + d/2)\Gamma(1 + 2l/d)^2 \right]^2} \times \left( \frac{\omega}{2} \right)^{2l-1} \sinh \left( \frac{1}{2} \beta_{\text{RN}} \omega \right) \left| \Gamma \left( 1 + \frac{l}{d} - i \frac{\beta_{\text{RN}} \omega}{2\pi} \right) \right|^2. \tag{2.32}
\]

This form still retains the CFT structure and is consistent with the results in [10, 13, 16, 18].

In the following, we concentrate on the case \( \hat{Q}_+ \gg 1 \) and \( |\nu_{\text{RN}}^\pm| \sim \mathcal{O}(1) \), which we are interested in. In addition to (2.30), we need to further check the conditions of validity. In the far region, it is obvious that the fractional error is at most \( \mathcal{O}(r/m) \). However, we should be careful about the error from the near region other than the matching point, i.e., \( r < r_m \).\footnote{This type of errors was not estimated in some references. It seems that we should be careful about the range of validity of those results, in particular, far from extremality.}

Let us recall that \( \tilde{V}_{\text{RN}} \) is the first two terms of the expansion of \( V_{\text{RN}} \) around \( z = 1 \) (\( r = r_+ \)). Since

\[
\partial_z V_{\text{RN}} = \left( 1 + \frac{1}{d} \right) \frac{2V_{\text{RN}}}{z + \hat{Q}_-}, \tag{2.33}
\]

\( \tilde{V}_{\text{RN}} \) is a good approximation for \( |z| \ll \hat{Q}_- \sim \hat{Q}_+ \) whereas it is not for \( |z| \sim \hat{Q}_+ (r - r_+ \sim r_+; r \sim 0) \). In spite of that, the approximation can be valid even for \( |z| \sim \hat{Q}_+ \) if the remaining potential term \( z(z - 1)\Lambda \) is large enough. In fact, we find that the condition (2.28) is satisfied there if \( r_+ \omega \ll \Lambda \), which is already included in (2.30).

Therefore, the non-trivial \( \omega \)-dependence in (2.32) is actually obtained for

\[
r_+ \omega \ll 1, \quad \hat{Q}_\pm \gg 1, \quad |\nu_{\text{RN}}^\pm| \sim \mathcal{O}(1), \tag{2.34}
\]

and the absorption cross-section has the CFT structure in this parameter region. Although the further consideration for \( r < r_m \) did not give any additional conditions here, we will see that the errors become significant inside the matching point in some cases.

For \( \Lambda = 0 \) (\( l = 0 \)), the term \( z(z - 1)\Lambda \) vanishes and we need to require that \((r_0\omega)^2 V_{\text{RN}}, (r_0\omega)^2 \times \tilde{V}_{\text{RN}} \ll 1 \) for \( |z| \gg \hat{Q}_\pm \). This yields just (2.27), which is obtained also from (2.32) with \( l = 0 \).

Let us summarize the discussion. In order for the approximation to be valid, the frequency \( \omega \) had to be small enough compared with the size of the black hole. In spite of that, the non-trivial \( \omega \)-dependence in (2.32) could be obtained for large \( \hat{Q}_\pm \). As a result, we confirmed that the absorption cross-section took the form expected from a CFT. The approximation to \( V_{\text{RN}} \) was valid near the horizons where the other potential term \( z(z - 1)\Lambda \) was small. Although \( \tilde{V}_{\text{RN}} \) was not a good approximation far from the horizons including the matching point, \( z(z - 1)\Lambda \) grew large enough and hence the approximation was valid in total for \( \Lambda \neq 0 \).
2.4 $SL(2, R)$ structure

We saw that the absorption cross-section takes the CFT structure. Another interesting property found for the black holes in string theory is that the wave equations have an $SL(2, R)$ symmetry near the horizon [10, 21]. In this subsection, we find that this is also the case for the Reissner-Nordstrom black holes in generic dimensions. Moreover, we find a T-duality-like symmetry similar to the one discussed in [10]. We see that this corresponds to the actual T-duality of the string model on $SL(2, R)$. We also observe a close relation to the two dimensional $SL(2, R)/U(1)$ black holes [24] and the three dimensional BTZ black holes [23].

We start with a brief review of relevant properties about $SL(2, R)$. Using analogs of Euler angles, an element $g_0 \in SL(2, R)$ is parametrized as

$$g_0 = e^{θ_L σ_3/2} e^{ρσ_1/2} e^{-θ_R σ_3/2},$$

with $σ_i$ the Pauli matrices. In terms of these coordinates, the metric of $SL(2, R)$ takes the form

$$ds^2_{SL} = -\left(\left(\frac{r}{r_0}\right)^2 - 1\right) dθ_+^2 + \left[\left(\frac{r}{r_0}\right)^2 - 1\right]^{-1} dr^2 + \left(\frac{r}{r_0}\right)^2 dθ_-^2,$$

where

$$r_0θ_{L,R} = θ_± ± θ_−, \quad r^2 = r_0^2 \cosh^2(ρ/2),$$

and we have introduced a dimensionful parameter, $r_0$, with which the scalar curvature is written as $-6r_0^{-2}$. The geometry has two Rindler space-like regions; the metric becomes

$$ds^2_{SL} \sim \left[-(σ/r_0)^2 dθ_+^2 + dσ^2\right] + dθ_-^2,$$

for $r \sim r_0$ with $σ^2 = r^2 - r_0^2$ whereas

$$ds^2_{SL} \sim dθ_+^2 - \left[dr^2 - (r/r_0)^2 dθ_-^2\right],$$

for $r \sim 0$.

The translations along $θ_±$ are isometries of $SL(2, R)$. These correspond to the vector and axial symmetry of the $SL(2, R)$ WZW model, respectively. By gauging either of them, we obtain the Lorentzian two dimensional black holes [24]. The orbifolds with respect to the discrete symmetries generated by their linear combinations yield the three dimensional black holes [23, 26]. The points $r = r_0$ and 0 become the event horizon and the singularity of the two dimensional black hole whereas the outer and the inner horizon for the three dimensional black hole.

For the string model on $SL(2, R)$, there is a self-dual T-duality transformation (at semi-classical level), $T$, which exchanges $θ_+$ and $θ_−$ (i.e. $θ_R$ and $-θ_R$), or equivalently $(r/r_0)^2 - 1/2$
This corresponds to the duality transformation to the two dimensional black hole geometry which exchanges the horizon and the singularity \[^{27}\] and that to the three dimensional black hole geometry which exchanges the outer and inner horizon \[^{27}\].

Now let us consider the eigenvalue equation of the Laplace operator on \(SL(2, R)\),

\[
\left[ \frac{1}{\sqrt{-g}} \partial_{\mu} \sqrt{-g} g^{\mu \nu} \partial_{\nu} - 4 j(j + 1) r_0^2 \right] \phi = 0 ,
\]

where \(-j(j + 1)\) is the Casimir. Physically, this is the Klein-Gordon equation on \(SL(2, R)\) and the Casimir is interpreted as mass or a coupling to the scalar curvature.\(^{27}\) Here, we rescale \(\theta_\pm\) as \(\tilde{\theta}_\pm = (\beta_\pm / 2 \pi r_0) \theta_\pm\) so that their imaginary periods associated with the Rindler space-like regions become \(\beta_\pm\), and make a separation of variables \(\phi = \exp(i \omega_+ \tilde{\theta}_+ + i \omega_- \tilde{\theta}_-) \varphi(r)\). The above equation then reduces to

\[
\left\{ z(z - 1) \partial_z \right\}^2 + z(z - 1) \left\{ \frac{\nu_+^2}{z - 1} - \frac{\nu_-^2}{z} - j(j + 1) \right\} \varphi = 0,
\]

with \(z = (r/r_0)^2\) and

\[
\nu_\pm = - \frac{i}{4 \pi} \beta_\pm \omega_\pm.
\]

We find that the equation \((2.41)\) is the same as the radial wave equation \((2.11)\) under the identifications

\[
\nu_\pm \leftrightarrow \nu_\pm^{RN}, \quad \text{i.e.,} \quad \beta_\pm \leftrightarrow \beta_{H_\pm}^{RN},
\]

\[
j \leftrightarrow j_l, \quad |\omega_\pm| \leftrightarrow |\omega|.
\]

This shows that the radial wave equation for the Reissner-Nordstrom black hole takes the \(SL(2, R)\) structure as we mentioned and that the scalar propagation near the horizon is formally the same as that in \(SL(2, R)\).

The Laplacian on \(SL(2, R)\) is expressed as \(J_{L,R}^j J_{L,R}^j \eta_{ij}\) using the left- or the right-\(sl(2, R)\) currents. So, we can rewrite \((2.41)\) and hence \((2.11)\) in terms of the \(sl(2, R)\) currents as shown for the four and the five dimensional black holes \[^{16, 20}\]. In this expression, the \(SL(2, R)\) structure becomes manifest. This holds even for the Schwarzschild black hole.

Furthermore, the wave equations of tachyons or scalars in the two and the three dimensional black hole background also take the form \((2.41)\) in the entire geometries \[^{28, 29}\]. This means that, for all the black holes discussed so far, the propagation of scalars (or tachyons) near the horizon is governed by essentially the same equation, namely, that for the \(SL(2, R)\) geometry.\(^{4}\)

\[^{4}\text{This is the case also for the three dimensional black string geometry.}^{30}\]

\[^{5}\text{The three dimensional black hole is not asymptotically flat. Because of this, the calculation of the absorption cross-section is different. However, by a certain definition of the asymptotic states at infinity, the CFT structure}^{11}\text{is obtained.}^{31}\]
The solutions are then given by the hypergeometric functions in all the cases. They are characterized by the geometric quantities associated with the Rindler-space regions, the horizons, the singularities and/or infinity.

Next, we turn to the action of the T-duality transformation $\mathcal{T}$ on the equation (2.41). Note that $(r/r_0)^2 - 1/2 \rightarrow 1/2 - (r/r_0)^2$ corresponds to $z \rightarrow 1 - z$, and $\theta_R \rightarrow -\theta_R$ to $\beta_R \equiv \nu_+ - \nu_- \rightarrow -\beta_R$ with $\beta_L \equiv \nu_+ + \nu_-$ fixed. Thus, from $\mathcal{T}^2 = 1$, (2.41) should be invariant under

$$
z \rightarrow 1 - z,
\beta_L \rightarrow \beta_L, \quad \beta_R \rightarrow -\beta_R.
$$

We easily check that this is the case. Since the wave equations near the horizons are the same, this kind of symmetry is also common to all the cases we have discussed so far. Moreover, in every case, this symmetry exchanges the outer horizon and the inner horizon (or the singularity).

For the four and five dimensional black holes, this symmetry is nothing but the T-duality-like symmetry discussed in [16]. Thus, we find that it corresponds to the actual T-duality of the string model on $SL(2, R)$.

### 3 General argument

In the previous section, we discussed properties of the scalar propagation for the Reissner-Nordstrom and other black holes: (i) The wave equations near the horizons have the $SL(2, R)$ structure. (ii) They also have a symmetry related to the T-duality on $SL(2, R)$. (iii) Their solutions are given by the hypergeometric functions and characterized by the information at the outer horizons, the inner horizons (or the singularities) and infinity. Typically, the periods of the imaginary time of the corresponding Rindler space appeared. (iv) In some parameter region, the absorption cross-sections at low energy take the form expected from a CFT.

Then, a natural question is: how general are these properties? In the subsequent sections, we will show that they are common also to a large class of other spherically symmetric non-extremal black holes and black $p$-branes. Furthermore, it turns out that most of the arguments does not depend on the details of the geometries. Therefore, before moving on to explicit examples, it would be useful to discuss general features about the wave equations for spherically symmetric black objects. Propagation of scalars in a general class of spherically symmetric geometries has been discussed in [32].

Here, we consider a spherically symmetric $p$-brane geometry in $D$-dimensions,

$$
ds^2 = H(r) \left[ -f(r) dt^2 + dy^i dy^i \right] + R(r) \left[ h(r) dr^2 + r^2 d\Omega_{d+1}^2 \right],
$$

with $i = 1, \cdots, p$ and $D = d + p + 3$. We will deal with the asymptotically flat case in the following. This means that $f, h, H, R \rightarrow 1$ as $r \rightarrow \infty$. Since the geometries of black objects are
usually expressed naturally by the harmonic functions $1/r^d$, we introduce a coordinate $x \equiv r^d$. In terms of $x$, the radial equation of a scalar independent of $y^i$ becomes

$$\left( (dK \partial_x)^2 + U(x; \omega) \right) \varphi = 0 ,$$

(3.2)

where

$$K(x) = x^2 H^2 R^{d+1} \left( -g_{tt}/g_{rr} \right)^{1/2} , \quad U(x; \omega) = x^2 H^p R^d \left( R x^4 \omega^2 - f H \Lambda \right) .$$

(3.3)

First, we note that the kinetic term vanishes where $K = 0$ and the differential equation becomes singular there. Since $(dr/dt)^2 = (-g_{tt}/g_{rr})$ for the light-like geodesics, this typically occurs at the horizons and singularities of the geometry.

Next, as is the case for the geometries discussed in section 2, suppose that $K(x)$ has the form $\gamma (x - x_+)(x - x_-)$ with $\gamma$ a non-vanishing constant. Then, we find that $x = x_\pm$ become regular singular points if $U(x; \omega)$ is analytic there with respect to $x$. Introducing a new coordinate $y = 1/x$, the equation (3.2) becomes

$$\left( \gamma d(1 - x_+ y) (1 - x_- y) \partial_y \right)^2 + \frac{1}{y^2} H^p R^d \left( R y^{-2/d} \omega^2 - f H \Lambda \right) \varphi = 0 .$$

(3.4)

Taking the asymptotic flatness into account, we see that $y = 0$ ($x = \infty$) is also a singular point but it is irregular because of the term $y^{-2/d}$.

From the above observation, we find that, by any approximation which makes the potential term $U(x; \omega)$ an at most quadratic polynomial of $x$, the wave equation (3.2) becomes a differential equation which has only three regular singular points. The approximated solutions are then expressed by the hypergeometric functions. We can always make this kind of approximation at least in the neighborhood of a point by expanding $U(x; \omega)$ if $U(x; \omega)$ is regular there.

Almost all the approximations in the literature which lead to the absorption cross-sections of the form (1.1) fall into this category. Note that $d = 1$ and 2 cases ($D = 4$ and 5 for black hole geometries) are special in that $r^2 = x^{2/d}$ in $U(x; \omega)$ is a monomial of $x$. Because of this fact, one can use approximations of the type used, e.g., in [4, 9], which are impossible for $d > 3$.

The hypergeometric functions are characteristic of $SL(2, R)$ symmetry; they are the matrix elements of $SL(2, R)$ representations. So, we expect that the $SL(2, R)$ structure discussed in section 2 also appear even in the above general situation. In fact, we can confirm this as follows. Let us denote the approximated potential by

$$\tilde{U}(x; \omega) = (x - x_+)(x - x_-) \left[ \frac{\tilde{U}_\infty}{\mu^2} + \frac{\tilde{U}(x_+)}{\mu(x - x_+)} - \frac{\tilde{U}(x_-)}{\mu(x - x_-)} \right] ,$$

(3.5)

with $\mu \equiv x_+ - x_-$. Then, in terms of $z = (x - x_-)/\mu$, the wave equation becomes

$$\left[ \left\{ z(z - 1) \partial_z \right\}^2 + z(z - 1) \left\{ \frac{\tilde{\nu}_+^2}{z - 1} - \frac{\tilde{\nu}_-^2}{z} + \frac{\tilde{U}_\infty}{(d\gamma^2)} \right\} \right] \varphi = 0 ,$$

(3.6)
where $\tilde{\nu}_\pm^2 = -\tilde{U}^2(x_\pm)/(d\gamma \mu)^2$. This is the same as the equation \(2.41\) under appropriate identifications of the parameters. Therefore, we find that the above equation indeed possesses the $SL(2, R)$ structure and the T-duality-like symmetry discussed in section 2.4. We remark that a differential equation with three regular singular points does not always take the form \(2.41\); for this to be the case, the exponents associated with two singular points, $\nu_{1(2)}$ and $\nu'_{1(2)}$, need to satisfy $\nu_{1(2)} = -\nu'_{1(2)}$.

Under appropriate conditions on the parameters, the original wave equation \(3.2\) can be simplified to the Bessel-type equation like \(2.18\) near infinity. So, the calculation of the absorption cross-section can be performed similarly to section 2.2. In this picture, the matching procedure is formally the same as the matching of the wave function on $SL(2, R)$ and that in flat Minkowski space-time. We would then find the CFT structure of the cross-section in the parameter region where the non-trivial $\omega$-dependence is obtained.

In this section, we saw that (i) the wave equation for a black object may have singular points typically at the horizons, singularities and infinity, (ii) if the function $K(x)$ in the kinetic term takes the form $\gamma(x-x_+)(x-x_-)$, the wave equation can be approximated by the equation of the form \(2.41\), (iii) then, the properties discussed in section 2 may hold even in this general situation.

In the following sections, we will confirm that these are actually realized for a large class of black objects. Moreover, we will find that the potential terms proportional to $\Lambda$ always become of the form $(x-x_+)(x-x_-)\Lambda$. Consequently the only difference is encoded in the potential term corresponding to $V_{RN}$.

### 4 Dyonic dilaton black holes

The Reissner-Nordstrom black holes belong to a class of black holes with two regular horizons. In this section, we deal with another class of such black holes, namely, the dyonic dilaton black holes by Gibbons and Maeda \[33\]. In almost the same way as in section 2, we will find that the properties of the scalar propagation discussed so far hold for these black holes.

#### 4.1 Space-time geometry

The metric of the dyonic dilaton black holes in $D$-dimensions is given by

$$ds^2_{GM} = - f_{GM}(r) \lambda_{GM}^d(r) dt^2 + \lambda_{GM}^{-1}(r) \left[ f_{GM}^{-1}(r) dr^2 + r^2 d\Omega_{d+1}^2 \right],$$

where $d = D - 3$;

$$f_{GM}(r) = \left( 1 - \frac{\eta \eta_0}{r^d} \right) \left( 1 + \frac{\eta \eta_0}{r^d} \right), \quad \lambda_{GM}^{-d} = \left( 1 + \frac{Q_1}{r^d} \right)^{\alpha_1} \left( 1 + \frac{Q_2}{r^d} \right)^{\alpha_2},$$

$$\alpha_1 = \frac{2g_2}{g_2 + g_{D-2}}, \quad \alpha_2 = \frac{2g_{D-2}}{g_2 + g_{D-2}}, \quad g_2g_{D-2} = d \quad \text{(4.2)}$$

---

6 Our notation is somewhat different from that in \[33\]. The relation is: $\eta \leftrightarrow r^d, \eta_1,3 \leftrightarrow Q_{1,2}$. 

13
$g_{2(D-2)}$ is a parameter related to the coupling between the dilaton and the two $(D-2)$-form. Without loss of generality, we can set $\alpha_1 \geq \alpha_2$. Also, we concentrate on the case $Q_{1,2} > \eta_0 > 0$ in which the geometry has two regular horizons at the points $r^d = \pm \eta_0$ and the singularity at $r^d = -\min\{Q_1, Q_2\}$. When $Q_1 = Q_2$, the geometry reduces to that of the Reissner-Nordstrom black hole. Similarly to section 2, we introduce $\zeta \equiv (r^d + \eta_0)/\mu$; $\mu = 2\eta_0 \equiv \eta_0^2$. The extremal limit corresponds to $\mu \to 0$. In terms of $\zeta$, the metric becomes of the form (2.3) with

$$V_{GM}(z) = \left(\frac{r}{r_0}\right)^{2(d+1)} \lambda_{GM}^{-2(d+1)} = \left\{\left(z - \frac{1}{2} + \hat{Q}_1\right)^{\alpha_1} \left(z - \frac{1}{2} + \hat{Q}_2\right)^{\alpha_2}\right\}^{1 + \frac{1}{d}} , \quad (4.3)$$

instead of $V_{RN}$, and $\hat{Q}_{1,2} = Q_{1,2}/\mu$. Thus we readily obtain the periods of the imaginary time associated with the near horizon geometries,

$$\beta_{GM}^+ = \frac{4\pi}{d} r_0 V_{GM}^+(1) = \frac{4\pi}{d} r_0 \left\{\left(\hat{Q}_1 + \frac{1}{2}\right)^{\alpha_1} \left(\hat{Q}_2 + \frac{1}{2}\right)^{\alpha_2}\right\}^{\frac{d+1}{2d}} , \quad (4.4)$$

$$\beta_{GM}^- = \frac{4\pi}{d} r_0 V_{GM}^-(0) = \frac{4\pi}{d} r_0 \left\{\left(\hat{Q}_1 - \frac{1}{2}\right)^{\alpha_1} \left(\hat{Q}_2 - \frac{1}{2}\right)^{\alpha_2}\right\}^{\frac{d+1}{2d}} . \quad (4.4)$$

### 4.2 Wave equation and absorption cross-section

Now we turn to the discussion on the propagation of a minimally coupled massless scalar. It is easy to find that the wave equation is obtained from (2.9) simply by the replacement $V_{RN} \to V_{GM}$. We see that the $V_{GM}$ (or $\lambda_{GM}$)-dependence disappears in the kinetic term and the potential term proportional to $\Lambda$. Thus the wave equation reduces to the form discussed in section 3.

We then examine the wave equation near the horizon. In this region, we can approximate $V_{GM}$ by its value at the outer horizon:

$$V_{GM}(z) \sim \tilde{V}_{GM}(z) \equiv V_{GM}(1) . \quad (4.5)$$

Since

$$\partial_z V_{GM} = - \left(1 + \frac{1}{d}\right) V_{GM}(z) \left[ \frac{\alpha_1}{(z - 1/2) + \hat{Q}_1} + \frac{\alpha_2}{(z - 1/2) + \hat{Q}_2} \right] , \quad (4.6)$$

$\tilde{V}_{GM}$ is a good approximation for $|z - 1| \ll \min\{\hat{Q}_1, \hat{Q}_2\}$. By this approximation, the total potential term becomes a quadratic polynomial of $z$ and the wave equation again takes the form

\footnote{It turns out that, even if we include the linear term in the expansion, it is irrelevant to the final result of the cross-section as in section 2.}
form (2.41). As a result, we find that it possesses the $SL(2, R)$ structure and the T-duality-like symmetry. In this case, the exponents associated with the singular points are

$$
\nu_{\pm}^{GM} = -\nu_{\pm}^{GM'} = -\frac{r_0 \omega}{d} \tilde{V}_{GM}^{\frac{1}{2}} = -\frac{i}{4\pi} \beta_{GM}^{GM} \omega, \\
\nu_{\infty}^{GM} = 1 - \nu_{\infty}^{GM'} = -j_f. 
$$

(4.7)

The wave equation in the far region $r^d \gg Q_{1,2}$ simplifies to (2.18). Therefore, the calculation of the absorption cross-section is identical with that in section 2.2, and hence the cross-section is given by Eq.(2.32) with $\beta_{GM}^{GM}$ instead of $\beta_{RN}^{GM}$.

As in the previous section, we still need to check the range of validity of the result. Since the procedure is similar to that in section 2.3, we will just give the points. First, we focus on $\Lambda \neq 0$. For the approximation to be valid at the matching point $r = r_m$, we require that $r_m^d \gg Q_{1,2}, \eta_0$ and $r_m \omega \ll 1$ should hold. Under these conditions, $|\nu_{\pm}^{GM}| \sim \beta_{GM}^{GM} \omega$ can be of order unity if $\hat{Q}_{1,2}$ are large enough, and then the non-trivial $\omega$-dependence appears. Although, in some parameter region, $|\nu_{\pm}^{GM}| \sim O(1)$ even when one of $\hat{Q}_{1,2}$ is small, let us concentrate on the case $\hat{Q}_1 \geq \hat{Q}_2 \gg 1$ and $|\nu_{\pm}^{GM}| \sim O(1)$ for simplicity. For $|z - 1| \approx \hat{Q}_2$, $\tilde{V}_{GM}$ is not a good approximation to $V_{GM}$. So, we need to require that $z(z - 1)\Lambda$ term dominates $(r_0 \omega)^2 V_{GM}$ and $(r_0 \omega)^2 \tilde{V}_{GM}$ there as in (2.28). After some algebra, we find that this is satisfied without any further conditions. Therefore, we confirm that the absorption cross-section has the CFT structure for

$$
Q_{i}^{1/d} \omega \ll 1, \quad \hat{Q}_i \gg 1, \quad |\nu_{\pm}^{GM}| \sim O(1). 
$$

(4.8)

Finally, $\Lambda = 0$ case gives (2.27) as before.

5 Charged black holes in string theory

The black holes in section 2 and 4 had two regular horizons. In this section, we discuss charged black holes in string theory which have only one horizon in a generic case. We will find that the propagation of scalars is similar to the previous cases; the wave equations near the horizons have the properties associated with $SL(2, R)$ and the absorption cross-sections take the CFT structure under certain conditions of validity.

5.1 Space-time geometry

A class of $D$-dimensional charged black holes in string theory has the Einstein metric [1, 31, 32],

$$
ds_{CB}^2 = -f_{CB}(r)\lambda_{CB}^d(r)dt^2 + \lambda_{CB}^{-1}(r) \left[ f_{CB}^{-1}(r)dr^2 + r^2 d\Omega_{d+1}^2 \right], 
$$

(5.1)
where \( d = D - 3 \) and

\[
\lambda_{CB}^{-d+1}(r) = \prod_{i=1}^{n} \left( 1 + \frac{Q_i}{r^d} \right), \quad f_{CB}(r) = 1 - \frac{\mu}{r^d}.
\] (5.2)

For later use, we define a parameter

\[
\xi_{CB} = 1 + \frac{1}{d} - \frac{n}{2}.
\] (5.3)

\( \xi_{CB} = 0 \) cases, i.e., \( D = 5, n = 3 \) and \( D = 4, n = 4 \), correspond to the five and four dimensional black holes extensively studied in relation to \( D \)-branes. \( n = 2 \) cases include the black holes closely related to fundamental strings \([1]\). The scalar propagation for \( \xi_{CB} \neq 0 \) has been discussed in \([36, 37]\). The geometry has a horizon at \( r^d = \mu \equiv r_0^d \) and, in a generic case \( \xi_{CB} \neq 0 \), a singularity develops at \( r = 0 \). \( \mu \to 0 \) is the extremal limit. Introducing \( z = \frac{r}{\mu} \), the metric takes the form (2.3) with

\[
V_{CB}(z) = \left( \frac{r}{r_0} \right)^{2(d+1)} \lambda_{CB}^{-d+1} = z^{2\xi_{CB}} \prod_{i=1}^{n} \left( z + \hat{Q}_i \right)
\] (5.4)

instead of \( V_{RN} \), and \( \hat{Q}_i = Q_i / \mu \). The period of the imaginary time associated with the near-horizon geometry is \([35]\)

\[
\beta_{H}^{CB} = \frac{4\pi}{d} r_0 V_{CB}^{\frac{1}{2}}(1) = \frac{4\pi}{d} r_0 \prod_{i=1}^{n} (1 + \hat{Q}_i)^{\frac{1}{2}}.
\] (5.5)

When \( \xi_{CB} = 0 \), \( r = 0 \) becomes a regular horizon and the corresponding period \( \beta_{H}^{CB} \) is obtained by the replacement \( V_{CB}(1) \to V_{CB}(0) \). For \( \xi_{CB} \neq 0 \), we do not have \( \beta_{H}^{CB} \) because \( r = 0 \) is a singular point. We can see this also from the fact that \( V_{CB}(0) \) is not a non-vanishing constant.

### 5.2 Wave equation and absorption cross-section

Now we consider the propagation of a minimally coupled massless scalar. We readily find that the radial wave equation takes the form (2.9) with \( V_{CB} \) instead of \( V_{RN} \). The \( V_{CB} \) (or \( \lambda_{CB} \))-dependence again disappears in the kinetic term and the potential term proportional to \( \Lambda \). The kinetic term vanishes at \( z = 1 \) and \( z = 0 \). \( z = 1 \) corresponds to the horizon as in the previous cases, but in a generic case \( z = 0 \) corresponds to the singularity instead of the inner horizon.

First, we study the wave equation near the horizon. To make the discussion definite, we will concentrate on the case \( \xi_{CB} > 0 \) in what follows. Approximating \( V_{CB} \) by the expansion around the horizon, we find that the approximated equation possesses the \( SL(2,R) \) structure and the T-duality-like symmetry. However, it turns out that this approximation does not give the non-trivial frequency dependence of the absorption cross-section; under the condition of
validity, the cross-section reduces to the leading term in (2.26). This is due to the singular nature at \( r = 0 \). In fact, since
\[
\partial_z V_{CB} = V_{CB} \left( \frac{2\xi_{CB}}{z} + \sum_i \frac{1}{z + Q_i} \right),
\]
the expansion gives a good approximation only for \( |z - 1| \ll 1 \) when \( \xi_{CB} \neq 0 \). Thus, in the region \( z \sim 0 \) \((r \sim 0)\) where the other potential term \( z(z - 1)\Lambda \) vanishes, the error becomes significant so that the validity of the approximation is not assured in the parameter region we are interested in.

Therefore, we will take a different approximation here;
\[
V_{CB}(z) \sim \tilde{V}_{CB}(z) = z \prod_i (1 + \hat{Q}_i). \tag{5.7}
\]
This gives a good approximation where \( z(z - 1)\Lambda \) becomes vanishing. Moreover, for \( \xi_{CB} = 1/2 \), the approximation is valid even for \( |z| \ll \min\{\hat{Q}_i\} \) because the error \( V_{CB} - \tilde{V}_{CB} \) is of order \( z/\hat{Q}_i \).

The wave equation near the horizon then takes the form (2.41) with the exponents
\[
\nu_{CB}^+ = -\nu_{CB}' = -i \frac{r_0 \omega}{d} \tilde{V}_{CB}^+(1) = -i \frac{r_0 \omega}{4\pi} \beta_{CB}^H \omega,
\]
\[
\nu_{CB}^- = -\nu_{CB}'' = -i \frac{r_0 \omega}{d} \tilde{V}_{CB}^-(0) = 0,
\]
\[
\nu_{CB}^\infty = 1 - \nu_{CB}' = -j_l.
\]
Thus we confirm that the wave equation has the properties related to \( SL(2, R) \).

The calculation of the absorption cross-section is the same as in the previous cases. The wave equation in the far region \( r \gg Q_i, \mu \) reduces to the Bessel-type equation and the cross-section is expressed by (2.26) with
\[
\beta_{CB}^L = \beta_{CB}^R = \beta_{CB}^H,
\]
instead of \( \beta_{RN}^{L,R,H} \).

We then turn to the discussion on the range of validity. Without loss of generality we set \( \hat{Q}_1 = \max\{\hat{Q}_i\} \) and, for simplicity, we deal with \( \hat{Q}_i \gg 1 \). We also set the matching point \( r = r_m \) \((z = z_m)\) large enough so that \( V_{CB} \gg \tilde{V}_{CB} \) there.\footnote{For \( \xi_{CB} < 1/2 \), we can choose the matching point so that \( \tilde{V}_{CB} \gg V_{CB} \).} This yields the condition
\[
z_m^{1+2/d} \gg \prod \hat{Q}_i. \tag{5.10}
\]
First, we discuss \( \Lambda \neq 0 \) case. For the approximation to be valid at the matching point, we require that \( r_m^d \gg Q_i, \mu; r_m \omega \ll 1 \) and (2.28) with \( V_{CB} \) instead of \( V_{RN} \) should hold. This gives
\[
z_m \gg \hat{Q}_1 \gg 1, \quad r_m \omega = r_0 \omega z_m^{1/d} \ll 1. \tag{5.11}
\]
Let \( r_0 \omega = \epsilon z_m^{-1/d} \) and \( z_m = L \hat{Q}_1 \) with \( L, \epsilon^{-1} \gg 1 \) so as to satisfy these conditions. Then we have

\[
|\nu_+^{CB}| \sim \beta_H^{CB} \omega \sim r_0 \omega \prod \hat{Q}_i^{1/2} \sim \epsilon L^{-1/d} \hat{Q}_1^{-1/d} \prod \hat{Q}_i^{1/2},
\]

(5.12)

This indicates that, for \( \xi_{CB} < 1 \), \( |\nu_+^{CB}| \) can be of order unity if the charges are sufficiently large, but it is impossible for \( \xi_{CB} \geq 1 \) in this approximation.

Next, we examine the error from \( z < z_m \). We consider the case \( |\nu_+^{CB}| \sim O(1) \). For \( \xi_{CB} \neq 1/2 \), \( \hat{V}_{CB} \) well approximates \( V_{CB} \) near \( z = 0, 1 \). Thus the requirement for the validity is that the condition (2.28) with \( V_{CB} \) holds except near these points, namely, at the scales \( z \sim \hat{Q}_i \), \( z \sim O(1) \) and \( z \sim -1/2 \). This is satisfied if

\[
\hat{Q}_i \gg 1, \quad d^2 |\nu_+^{CB}|^2 \hat{Q}_i^{-2(1-\xi_{CB})} \prod_k^n \left( 1 + \hat{Q}_i/\hat{Q}_k \right) \ll \Lambda \quad (i = 1, \cdots, n),
\]

(5.13)

\[
\Lambda \gg d^2 |\nu_+^{CB}|^2.
\]

(5.14)

For \( \xi_{CB} = 1/2 \), the approximation is better and the requirement is that (2.28) with \( V_{CB} \) holds for \( z \sim \hat{Q}_i \). This gives the condition (5.13). Therefore, the non-trivial frequency dependence in (2.28) with \( \beta_{L,R,H}^{CB} \) is obtained in the parameter regions (i) (5.10), (5.11), (5.13) and \( |\nu_+^{CB}| \sim O(1) \) for \( \xi_{CB} = 1/2 \), (ii) (i) plus (5.14) for \( 0 < \xi_{CB} < 1, \xi_{CB} \neq 1/2 \). In these parameter regions, the absorption cross-section has the CFT structure.

For \( \Lambda = 0 \), we obtain (2.27).

As discussed in section 3, \( r^2 \propto z^{2/d} \) becomes a positive integral power of \( z \) for \( D = 4 \) and 5 and one can utilize more precise approximations of the type used in, e.g., [4, 9]. In fact, the results for \( \xi_{CB} = 1/2 \) (\( D = 4, n = 3 \) and \( D = 5, n = 2 \)) in this section can be compared with [4] and we find agreement; our results of the absorption cross-sections reduce to those in [9] in the corresponding parameter region with the smallest charge vanishing. This agreement holds even for \( \Lambda = 0 \). This suggests that more elaborated approximations may also give (2.28) with \( \beta_{L,R,H}^{CB} \) and the range of its validity may be wider than discussed in this section.

6 Black p-branes

We have seen that the propagation of scalars possesses interesting features for various black hole geometries. As a final example, we discuss a class of non-extremal black p-branes. We will find that the properties discussed so far hold also in this case.

6.1 Space-time geometry

It turns out that we can deal with all the p-brane solutions in [38, 39] in the same way. However, to make the discussion definite and concise, we will concentrate on the class of the
$D$-dimensional solutions with the Einstein metric \[38\]

\[
ds^2_{pB} = H^\alpha(r) \left( H^{-N}(r) \left[-f_{pB}(r)dt^2 + dy^i dy^i\right] + f_{pB}^{-1}(r)dr^2 + r^2 d\Omega^2_{d+1} \right),
\]

\[
H(r) = 1 + \frac{Q}{r^d}, \quad f_{pB}(r) = 1 - \frac{\mu}{r^d},
\]

\[
\alpha = \frac{p+1}{d+1}N, \quad N = 4 \left[ g_{D-2}^2 + 2d^2 \frac{p+1}{d+1} \right]^{-1},
\]

where $i = 1, \ldots, p$ and $D = d + p + 3$. $g_{D-2}$ is a parameter related to the coupling between the dilaton and the $(D - 2)$-form. This includes the $p$-brane solutions in \[41\]. $g_{D-2} = 0$ corresponds to the constant dilaton. In this (non-dilatonic) case, the near extremal entropy scales as that of the massless ideal gas in $(p + 1)$-dimensional world volume \[40\]. The geometry has a horizon at $r^d = \mu \equiv r_0^d$. When $\mu = 0$, the geometry becomes extremal. In a generic case, $r = 0$ is the location of the singularity. When $N$ is an integer, the extremal solution becomes supersymmetric.

Introducing $z = r^d/\mu$, the metric becomes

\[
ds^2_{pB} = H^{\frac{d \alpha}{d+1(\nu+1)}} \left\{ V_{pB}^{-\frac{d \alpha}{d+1}} \left[-z(z-1)dt^2 + z^2 dy^i dy^i\right] \right. \\
\left. + r_0^2 V_{pB}^{-\frac{d \alpha}{d+1}} \left[ \frac{1}{d^2} \frac{dz^2}{z(z-1)} + d\Omega^2_{d+1} \right] \right\},
\]

with

\[
V_{pB}(z) = \left( \frac{r}{r_0} \right)^{2(d+1)} F^N = z^{2\xi_{pB}} \left( z + \dot{Q} \right)^N.
\]

Here we have defined $\dot{Q} = Q/\mu$ and

\[
\xi_{pB} = 1 + \frac{1}{d} - \frac{N}{2} \geq 0.
\]

Except for the overall factor and the $p$-brane part, $z^2 dy^i dy^i$, the above metric is the same as \[2.3\] with $V_{pB}$ instead of $V_{RN}$. Then the period of the imaginary time associated with the horizon is \[38, 40\]

\[
\beta_H^{pB} = \frac{4\pi}{d} r_0 V_{pB}^\frac{4}{d} (1) = \frac{4\pi}{d} r_0 (1 + \dot{Q})^\frac{4}{d}.
\]

If $r = 0$ is an inner horizon, we get the corresponding period similarly.

6.2 Wave equation and absorption cross-section

The $p$-brane geometry \[6.2\] is very similar to the previous black hole cases, and so is the discussion on the propagation of scalars. After some calculation, we find that, in the radial

---

\footnote{We basically follow the notation in \[40\].}
wave equation, the $H$-dependence cancels in the kinetic term and the term proportional to $\Lambda$. The wave equation is then given by (2.9) with $V_{pB}$. Comparing $V_{pB}$ with $V_{CB}$, we find that, when $N$ is an integer, the wave equation for $V_{pB}$ is the same as the equation for $V_{CB}$ with $\hat{Q}_i = \hat{Q}$ and $n = N$.

Let us move on to the detail. First, we discuss the special case $\xi_{pB} = 0$. In this case, the entropy is non-vanishing in the extremal limit \[38, 40\], and the geometry for $p = 0$ becomes that of the Reissner-Nordstrom black hole. We see that, only when $D = 4$ and $5$, $N$ is an integer and the extremal Reissner-Nordstrom black holes become supersymmetric.

The discussion for an integral $N$ reduces to that for $\xi_{CB} = 0$ case with the equal charges in the previous section. In this case, we can use the analysis in the literature or in section 2. Furthermore, because

$$ \partial_z V_{pB} = V_{pB} \left( \frac{2\xi_{pB}}{z} + \frac{N}{z + \hat{Q}} \right), \quad (6.6) $$

the expansion of $V_{pB}$ gives a good approximation in the region $|z| \ll \hat{Q}$ even for a non-integral $N$. This is analogous to the black holes with regular inner horizons in section 2 and 4, and similar analysis is possible. Consequently, the near-horizon wave equation has the properties related to $SL(2, R)$ and the absorption cross-section has the CFT structure in every case with $\xi_{pB} = 0$.

Next, we consider $\xi_{pB} \neq 0$ case. For the same reason as in section 5, we approximate the wave equation by

$$ V_{pB}(z) \sim \tilde{V}_{pB}(z) = z(1 + \hat{Q})^N. \quad (6.7) $$

Although the possible values of $\xi_{CB}$ and $\xi_{pB}$ are different, it then turns out that the discussion on the scalar propagation is almost the same. We can obtain the results for the $p$-branes from those in the previous section simply by the replacement $\xi_{CB} \rightarrow \xi_{pB}$, $\hat{Q}_i \rightarrow \hat{Q}$, $n \rightarrow N$ and so on.

Explicitly, the approximated wave equation near the horizon is given by (2.41) with the exponents

$$ \nu_+^{pB} = -\nu_+^{pB'} = -i \frac{\nu_0 \omega}{d} V_{pB}(1) = -i \frac{1}{4\pi} \beta_{pB}^B \omega, $$

$$ \nu_-^{pB'} = 0, \quad \nu_\infty^{pB} = 1 - \nu_\infty^{pB'} = -j_l. \quad (6.8) $$

We can confirm the properties associated with $SL(2, R)$ regarding this wave equation.

As for the absorption cross-section, it is obtained from (2.26) with

$$ \beta_L^{pB} = \beta_R^{pB} = \beta_H^{pB}, \quad (6.9) $$

The kinetic term of the wave equation takes the same form also for the $p$-brane solutions in \[12\]
instead of $\beta_{L,R,H}^{RN}$. The range of validity is examined in the same way. We choose the matching point so that $V_{pB} \gtrsim \tilde{V}_{pB}$ there. This yields the condition $z_m^{1+2/d} \geq Q^N$. For $\Lambda \neq 0$, the matching procedure is valid if $z_m \gg \hat{Q}$ and $r_m \omega = r_0 \omega z_m^{1/d} \ll 1$. Thus $|\nu_+^{pB}|$ can be of order unity for sufficiently large $\hat{Q}$ and $\xi_{pB} < 1$. The estimation of the error from $\nu < z_m$ leads to the conditions

(i) $d^2 |\nu_+^{pB}|^2 \gg \Lambda; \hat{Q} \gg 1$ for $\xi_{pB} \neq 1/2$ and (ii) $\hat{Q} \gg 1$ for $\xi_{pB} = 1/2$. Therefore, the non-trivial $\omega$-dependence is obtained in the parameter regions

$$
\begin{align*}
\xi_{pB} &< 1, \frac{1}{2}; \quad Q^{1/d} \omega \ll 1, \quad \hat{Q}, \Lambda \gg 1, \quad |\nu_+^{pB}| \sim O(1), \\
\xi_{pB} &= \frac{1}{2}; \quad Q^{1/d} \omega \ll 1, \quad \hat{Q} \gg 1, \quad |\nu_+^{pB}| \sim O(1).
\end{align*}
$$

(6.10)

The absorption cross-section has the CFT structure there.

### 7 Summary and discussion

In this paper, we discussed the propagation of minimally coupled massless scalars in various non-extremal black hole and $p$-brane geometries. We showed that some of the properties known about a certain class of non-extremal four and five dimensional black holes hold very generally: (i) The radial wave equations near the horizons approximately take the same form as the eigenvalue equation of the Laplace operator on $SL(2,R)$. (ii) The solutions there are characterized by the information at the outer horizons, the inner horizons (or the singularities) and infinity and they are expressed by the hypergeometric functions. Typically, the periods of the imaginary time associated with the near-horizon geometries appear in those solutions. (iii) The wave equations have a symmetry related to the T-duality of the string model on $SL(2,R)$. (iv) The absorption cross-sections at very low energy take the form expected form a CFT in some parameter region. We saw that the above features of the near-horizon wave equations are valid irrespectively of extremality and supersymmetry. The properties (i)-(iii) were also common to the two dimensional $SL(2,R)/U(1)$ black holes and the three dimensional BTZ black holes.

For the four and five dimensional black holes, the above properties hold also in the rotating cases [13, 16, 17] and for particles with higher spins [14, 19, 20]. Taking this into account, we expect that it is possible to extend the argument in this paper to those cases. Moreover, the structures of the geometries were very similar in terms of the quantities corresponding to $V_{RN}$ and the metrics could be written as (2.3) in all the cases. This suggests a possibility that, under quite general assumptions, black objects should possess the properties such as (i)-(iv). Also, by more elaborated approximations, it may be possible to extend the parameter regions in which the non-trivial frequency dependence of the cross-sections is obtained. For example, $V_{CB[pB]}$ is well approximated near the horizon by $z^2 \Pi(1+\hat{Q}) [z^2 (1+\hat{Q})^N]$ for $\xi_{CB[pB]} = 1$. This may be used for this purpose with appropriate modification about the far-region analysis.
As for the absorption cross-section of a minimally coupled massless scalar, it should become the area of the horizon in the low energy limit\cite{22}. Our result about the absorption cross-sections indicates that this kind of universality holds in a more detailed form for a class of black objects. One explanation is as follows. In every case we discussed, the near-horizon wave equation reduced to the form like (2.9) and the difference among various geometries was encoded in the term corresponding to $V_{RN}$. Since such a term is multiplied by the frequency as $(r_0\omega)^2V_{RN}$, the difference is becoming irrelevant and the universality appears as $\omega \rightarrow 0$.

Given the CFT structure of the absorption cross-sections, one may be interested in its microscopic origin. For some parameter region of the four and five dimensional black holes, we have the description using $D$-branes. However, the corresponding microscopic theory is not known in general. Our results include the cases which, as in \cite{13,16,17}, are not supersymmetric even in the extremal limit and whose entropies near extremality do not have the massless ideal-gas scaling related to $p$-brane world volume \cite{10}. Thus the possible underlying theory should be more general than the simple $p$-brane theory. The $SL(2,R)$ structure of the near-horizon wave equations indicates a connection to a microscopic theory associated with $SL(2,R)$. Probably, this is the string theory on $SL(2,R)$ as mentioned in \cite{16}. Indeed, at extremality, the near-horizon regions of the four and five dimensional black holes are described using the $SL(2,R)$ WZW model \cite{13}. Although a simple extension to the non-extremal cases is impossible \cite{14}, a relation via duality transformations has been discussed between the above black holes and the BTZ black holes (which are locally $SL(2,R)(AdS_3)$) \cite{13,16}. Also, for black holes whose near-horizon geometries are related to $SL(2,R)$, the entropy counting has been done using a CFT associated with $SL(2,R)$ or the BTZ black holes \cite{16,17}. These are suggestive of a connection between non-extremal black holes and $SL(2,R)$.

The string theory on $SL(2,R)$ is not fully understood because its target space is a non-compact group. However, there are recent proposals for the sensible spectrum of the strings on $SL(2,R)$ \cite{13,19} and the BTZ black hole geometry \cite{19}. It would be interesting to further investigate the relationship between non-extremal black holes and the string theory on $SL(2,R)$.

\section*{Acknowledgements}

I would like to thank C.G. Callan, I.R. Klebanov, R. von Unge and D. Waldram for useful discussions, and S.P. de Alwis for correspondence. This work was supported in part by JSPS Postdoctoral Fellowships for Research Abroad.
References

[1] J.M. Maldacena, *Black holes in string theory*, hep-th/9607235.

[2] C. Callan and J. Maldacena, Nucl. Phys. **B472**, 591 (1996) hep-th/9602043;  
A. Dhar, G. Mandal and S. Wadia, Phys. Lett. **B388**, 51 (1996) hep-th/9605234.

[3] S. Das and S. Mathur, Nucl. Phys. **B478**, 561 (1996) hep-th/9606185;  
S.S. Gubser and I.R. Klebanov, Nucl. Phys. **B482**, 173 (1996) hep-th/9608108.

[4] J. Maldacena and A. Strominger, Phys. Rev. **D55**, 861 (1996) hep-th/9609026.

[5] S.S. Gubser and I.R. Klebanov, Phys. Rev. Lett. **77**, 4491 (1996) hep-th/9609079.

[6] C.G. Callan, S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. **B489**, 65 (1997) hep-th/9610172;  
M. Krasnitz and I.R. Klebanov, Phys. Rev. **D55**, 2173 (1997) hep-th/9612051.

[7] I.R. Klebanov, A. Rajaraman and A.A. Tseytlin, Nucl. Phys. **B503**, 157 (1997) hep-th/9704112.

[8] G.T. Horowitz and A. Strominger, Phys. Rev. Lett. **77**, 2368 (1996) hep-th/9602051;  
J.M. Maldacena, Phys. Rev. **D55**, 7645 (1997) hep-th/9611125.

[9] I.R. Klebanov and S. Mathur, Nucl. Phys. **B500**, 115 (1997) hep-th/9701187.

[10] S.W. Hawking and M.M. Taylor-Robinson, Phys. Rev. **D55**, 7680 (1997) hep-th/9702045;  
F. Dowker, D. Kastor and J. Traschen, preprint, hep-th/9702109.

[11] M. Krasnitz and I.R. Klebanov, Phys. Rev. **D56**, 3250 (1997) hep-th/9703216;  
M.M. Taylor-Robinson, preprint, hep-th/9704172;  
H.W. Lee, Y.S. Myung and J.Y. Kim, preprint, hep-th/9708099.

[12] F. Larsen, Phys. Rev. **D56**, 1005 (1997) hep-th/9702153.

[13] J. Maldacena and A. Strominger, Phys. Rev. **D56**, 4975 (1997) hep-th/9702015.

[14] S.S. Gubser, Phys. Rev. **D56**, 7854 (1997) hep-th/9706100.

[15] S. Mathur, preprint, hep-th/9704156;  
S.S. Gubser, Phys. Rev. **D56**, 4984 (1997) hep-th/9704195.
[16] M. Cvetič and F. Larsen, Phys. Rev. D56, 4994 (1997) [hep-th/9705192]; Nucl. Phys. B506, 107 (1997) [hep-th/9706071].

[17] A.A. Starobinsky and S.M. Churilov, Sov. Phys. JETP 38, 1 (1974); S.A. Teukolsky and W.H. Press, Astrophys. J. 193, 443 (1974); D.N. Page, Phys. Rev. D13, 198 (1976).

[18] G.W. Gibbons, Commun. Math. Phys. 44, 245 (1975).

[19] S. Das, A. Dasgupta, P. Majumdar and T. Sarkar, preprint, [hep-th/9707124]; M. Cvetič and F. Larsen, preprint, [hep-th/9712118].

[20] K. Hosomichi, preprint, [hep-th/9711072].

[21] F.R. Tangherlini, Nouvo Cimento 27, 636 (1963); R.C. Myers and M.J. Perry, Ann. Phys. (N.Y.) 172, 304 (1986).

[22] S. Das, G. Gibbons and S. Mathur, Phys. Rev. Lett. 78, 417 (1997) [hep-th/9609052].

[23] D. Kastor and J. Traschen, preprint, [hep-th/9707157].

[24] E. Witten, Phys. Rev. D44 314 (1991); G. Mandal, A. Sengupta and S. Wadia, Mod. Phys. Lett. A6, 1685 (1991).

[25] M. Bañados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. 69, 1849 (1992) [hep-th/9204099]; M. Bañados, M. Henneaux, C. Teitelboim and J. Zanelli, Phys. Rev. D48, 1506 (1993) [gr-qc/9302012].

[26] G.T. Horowitz and D.L. Welch, Phys. Rev. Lett. 71, 328 (1993) [hep-th/9302126]; N. Kaloper, Phys. Rev. D48, 2598 (1993) [hep-th/9303007]; A. Ali and A. Kumar, Mod. Phys. Lett. A8, 2045 (1993) [hep-th/9303032].

[27] M. Natsuume and Y. Satoh, Int. J. Mod. Phys. A13, 1229 (1998) [hep-th/9611041].

[28] R. Dijkgraaf, E. Verlinde and H. Verlinde, Nucl. Phys. B371 (1992) 269.

[29] K. Ghoroku and A.L. Larsen, Phys. Lett. B328, 28 (1994) [hep-th/9403005]; I. Ichinose and Y. Satoh, Nucl. Phys. B447, 340 (1995) [hep-th/9412144].

[30] J.H. Horne and G.T. Horowitz, Nucl. Phys. B368, 444 (1992) [hep-th/9108001]; K. Sfetsos, Nucl. Phys. B389, 424 (1993) [hep-th/9206048].

[31] D. Birmingham, I. Sachs and S. Sen, preprint, [hep-th/9707188].
[32] R. Emparan, preprint, hep-th/9706204.

[33] G.W. Gibbons and K. Maeda, Nucl. Phys. B298, 741 (1988).

[34] D. Youm, Black holes and solitons in string theory, hep-th/9710046.

[35] M. Cvetič and A.A. Tseytlin, Nucl. Phys. B478, 181 (1996) hep-th/9604089.

[36] S.P. de Alwis and K. Sato, Phys. Rev. D55, 6181 (1997) hep-th/9611189.

[37] R. Emparan, Phys. Rev. D56, 3591 (1997) hep-th/9704201.

[38] M.J. Duff, H. Lü and C.N. Pope, Phys. Lett. B382, 73 (1996) hep-th/9604052.

[39] R. Güven, Phys. Lett. B276, 49 (1995).

[40] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. B475, 164 (1996) hep-th/9604089.

[41] G.T. Horowitz and A. Strominger, Nucl. Phys. B360, 197 (1991);
    M.J. Duff and H. Lü, Nucl. Phys. B416, 301 (1994) hep-th/9306052;
    G.W. Gibbons, G.T. Horowitz and P.K. Townsend, Class. Quantum Grav. 12, 297 (1995) hep-th/9410073.

[42] H. Lü, C.N. Pope and K.W. Xu, Mod. Phys. Lett. A11, 1785 (1996) hep-th/9604058.

[43] M. Cvetič and A.A. Tseytlin, Phys. Lett. B366, 95 (1996) hep-th/9510097;
    A.A. Tseytlin, Nucl. Phys. B477, 431 (1996) hep-th/9605091.

[44] J.G. Russo, Nucl. Phys. B481, 743 (1996) hep-th/9606031.

[45] S. Hyun, preprint, hep-th/9704005.

[46] K. Sfetsos and K. Skenderis, preprint, hep-th/9711138.

[47] A. Strominger, preprint, hep-th/9712251;
    D. Birmingham, I. Sachs and S. Sen, preprint, hep-th/9801019.

[48] I. Bars, Phys. Rev. D53, 3308 (1996) hep-th/9503203.

[49] Y. Satoh, Nucl. Phys. B513, 213 (1998) hep-th/9705208.