Delayed - Choice Entanglement - Swapping with Vacuum-One Photon Quantum States.

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Abstract

We report the experimental realization of a recently discovered quantum information protocol by Asher Peres implying an apparent non-local quantum mechanical retrodiction effect. The demonstration is carried out by applying a novel quantum optical method by which each singlet entangled state is physically implemented by a two-dimensional subspace of Fock states of a mode of the electromagnetic field, specifically the space spanned by the vacuum and the one photon state, along lines suggested recently by E. Knill et al., *Nature* 409, 46 (2001) and by M. Duan et al., *Nature* 414, 413 (2001). The successful implementation of the new technique is expected to play an important role in modern quantum information and communication and in EPR quantum non-locality studies.
State entanglement, the most distinctive, fundamental feature of modern physics is at the heart of the essential non-locality of the quantum world, i.e. of the irremovable property of nature first discovered in 1935 by Einstein-Podolsky-Rosen (EPR) and later formally analyzed by J. S. Bell \[1\] and recently by L. Hardy \[2\]. In the context of the modern fields of quantum information and computation entanglement lies at the core of several important protocols and methods as for instance the quantum state teleportation (QST) a fundamental process that has been implemented by different experimental approaches \[3,4\]. Very recently quantum teleportation with an unprecedented large "fidelity" has been experimentally demonstrated by adoption of the new concept of "entanglement of one photon with the vacuum" by which each quantum superposition state, i.e. "qubit", is physically implemented by a two dimensional subspace of Fock states of a mode of the electromagnetic field, specifically the state spanned by the QED "vacuum" and the 1-photon state \[5\]. This method requires, as we shall see, an entirely new re-formulation of the Hilbert space framework supporting the evolution of quantum information, and then the conception of new devices and methods to implement the transformation algebra of states and operators. In view of a further clarification of the new method in the perspective of future more complex applications we investigate in the present letter the procedure called "entanglement swapping" in which the teleported state itself is entangled, i.e. where the teleported system does not even enjoy its own state \[6\].

Let us first outline the swapping process in the new perspective. It is well known that the establishment of entanglement between two (or more) distant "quantum systems" does not necessarily require, as generally believed after the original EPR approach a direct original interaction between these ones but it can be realized by merely projecting by an appropriate joint measurement the independent entangled states pertaining to the separated systems, even in absence of any previous mutual interaction. According to this scenario two separate observers, Alice (A) and Bob (B) independently prepare two sets of entangled "singlets". They perform on one "component" of each singlet an appropriate test of EPR non-locality, e.g. a standard Bell-inequality test \[1\], a Hardy’s "no-inequality ladder" test \[2,3\] or a "con-
continuous variables” homodyne detection test \([1]\). The other two components of the singlets are sent to a third party, Eve \((E)\) who performs a joint test at its choice on the components he received, one from \(A\) and one from \(B\). By doing that Eve projects (i.e. ”swaps”) the state of the two originally non-entangled distant components in the hands of \(A\) and \(B\) onto an entangled state. Recently it has been argued by Asher Peres that the swapping process could be completed by Eve non necessarily at the time at which the two distant systems were tested by \(A\) and \(B\) but at any retarded ”delayed choice” time \([8]\). Indeed, according to Eve’s choice at a later time a fourth ”verification party”, Victor \((V)\) can sort the samples already tested by \(A\) and \(B\) into subsets and can verify that each subset behaves as if it consisted of entangled pairs of distant systems that have never communicated in the past even indirectly via other systems. This may appear a paradoxical result, as we shall see.

In the present work we report the experimental demonstration of the Peres’s ”delayed choice” process by applying the new concept of ”entanglement of one photon with the vacuum” \([3,4]\). Because of its novelty let us outline here the rationale of this approach. The concept of ”non-locality of a single photon”, first introduced by Albert Einstein in 1927 \([10]\) has been thoroughly analyzed in the last decade by S. M. Tan et al. \([11]\), by L. Hardy \([12]\) and others in connection with the superposition state emerging from a beam splitter (\(BS\)) excited by a single photon at one of its input ports. In our view this state should indeed be interpreted as an entangled state by considering that in the domain of optics the modes of the electromagnetic field (e.m.) rather than the photons must be taken as the ”systems”, or ”components” to be entangled. This is consistent with the content of two recent comprehensive theoretical works by E. Knill et al. \([13]\) and by M. Duan et al. \([14]\). Thus, any single particle superposition state expressed in the form: \(\Sigma_A = (2)^{-1/2} (|1\rangle_A |0\rangle_{A'} - |0\rangle_A |1\rangle_{A'})\) must be interpreted as a ”singlet” entangling the mode pair \((k_A, k_{A'})\) which is excited by the Fock states \(|1\rangle\) and \(|0\rangle\), this last one expressing the QED vacuum state. If the same states \(|0\rangle, |1\rangle\) are interpreted respectively as the logic ”zero”, ”one” information states, the singlet \(\Sigma_A\) is viewed as an \(E\)-bit, i.e. an entangled bit of quantum information \([15]\). Of course in order to make use of the entanglement present in this picture we need to use the second
quantization procedure of creation and annihilation of particles and/or use states which are superpositions of states with different numbers of particles. Another puzzling aspect of this second quantized picture is the need to define and measure the relative phase between states with different number of photons, such as the relative phase between the vacuum and one photon states appearing in Eq. 2, below. That we can associate a relative phase between the vacuum and anything else seems most surprising, but it is less so if we recall the more familiar case of a coherent state, where the relative phase between the different photon number states in the superposition is reflected physically in the phase of the classical electric field. To be able to control these relative phases we need in general, and in analogy with classical computers, to supply all gates and all sender/receiving stations of any quantum information network with a common synchronizing clock signal, e.g. provided by an ancillary photon or by an ancillary multi-photon, Fourier transformed coherent state [16]. Optionally in simple cases, as in the present work, an ad hoc clock generator is not needed as the mutual phase information can be retrieved by a linear two mode superposition in a beam splitter (BS).

An example concerning the present experiment is illustrated by Figure 1 which shows how the non-locality implied by the quantum state of the overall system could be tested by the distant parties A and B via two coherent states $|\alpha\rangle \equiv ||\alpha| \exp i\theta\rangle$ and $|\alpha'\rangle \equiv ||\alpha| \exp i\theta'\rangle$ that can operate at the same time as clock states and as local oscillators (LO) of the corresponding homodyne detectors performing the same test. The feasibility of a similar single photon homodyne technique has been demonstrated recently [17].

Figure 1 shows the basic layout of the delayed-choice entanglement swapping experiment. Pair of photons were generated by Spontaneous Parametric Down-Conversion (SPDC) excited by a single mode UV CW argon laser in a Type I LiIO3 crystal with the same wavelengths (wl) $l = 727, 6nm$ and with the same linear polarizations ($\pi$). Each pair of photons, each of which associated with an ultrashort optical pulse characterized by a coherence time $\tau_c = 0, 1ps$, was injected into 2 equal 50:50 beam splitters, $BS_A$ and $BS_B$ characterized by equal real transmittivity and reflectivity parameters $t = r = 2^{-\frac{1}{2}}$. Precisely, each $BS$ consisted of a 45°$\pi$-rotator followed by a calcite crystal. As it is well known [17], the
product state character of each pair, $|\Phi\rangle = |1\rangle_A \otimes |1\rangle_B$ did not imply any inter-particle EPR correlation, in agreement with the data reported in Figure 2 (open circles). In other words, as far as the dynamics of the overall system is concerned, each photon pair could have been supplied equally well by any pair of distant sources. The state $|\Phi\rangle$ was transformed by the BS’s into the product of two singlets defined over the pairs of output modes $(k_A, k_{A'})$ and $(k_B, k_{B'}):$ $|\Phi\rangle = \Sigma_A \otimes \Sigma_B = \frac{1}{2} (|1\rangle_A |0\rangle_B - |0\rangle_A |1\rangle_B) \otimes (|1\rangle_B |0\rangle_A - |0\rangle_B |1\rangle_A)$. The pure state $|\Phi\rangle$ may be expressed as a sum of products of Bell states defined in the two 2-dimensional Hilbert subspaces spanned by the state eigenvectors to be measured respectively by the couple (Alice, Bob) and by Eve:

$$|\Phi\rangle = \frac{1}{2} \left[ \Phi^+ \otimes \Phi^+_E - \Phi^- \otimes \Phi^-_E - \Psi^+ \otimes \Psi^+_E + \Psi^- \otimes \Psi^-_E \right]$$

(1)

and the Bell states defined in the corresponding 2-d Hilbert sub-spaces are [5]:

$$\Phi^\pm = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B \pm |1\rangle_A |1\rangle_B), \Psi^\pm = \frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B)$$

(2)

Equation (1) shows how the original entanglement condition existing within the two separated systems $(k_A, k_{A'})$ and $(k_B, k_{B'})$ can be swapped to the ”extreme” modes $k_A$ and $k_B$ by any joint Bell type measurement made by Eve on the ”intermediate” modes $(k_{A'}, k_{B'})$. In absence of such a measurement the overall state $|\Phi\rangle$ is a superposition while the one reaching the $(A + B)$ sector is a mixed state.

Suppose that one of the two detectors $D_j$ of the Eve sector ”clicks”, i.e. measures the state $\Psi^-_E$, say. A sudden state reduction occurs that projects the overall system onto the corresponding entangled Bell state: $|\Phi\rangle \Rightarrow \Psi^-$. The Eve’s apparatus consisting of a 50:50 beam splitter and of a $\varphi$ - phase shifter is apt to perform this task with a 50 % efficiency. Indeed it can be easily found by applying the standard BS theory [5] that the realization of the 1-photon Bell state $\Psi^-_E$ (or $\Psi^+_E$) over the input modes $k_{A'}, k_{B'}$ determines a click by $D_1$ (or $D_2$ ). It is also well known that the states $\Phi^+_E$ corresponding to a 2 photon excitation of the Eve’s sector cannot be discriminated by any linear device [9].
the present experiment is noise free since a 2-photon excitation of Eve’s sector implies no detections by the \((A + B)\) sector, an event easily discarded by the electronic apparatus. An additional degree of freedom under Eve’s control, indeed an optional “delayed choice”, was provided by the micrometric displacement \(\Delta X\) of the mirror \(M\), activated by a piezoelectric transducer. This one induced a corresponding phase shift \(\Delta \varphi = (2)^{3/2} \pi \lambda^{-1} \Delta X\) between the modes \((k_{A'}, k_{B'})\). Optionally, the same task can be accomplished by fast Electro Optic (EO) phase modulator, as we shall see. The 4 detectors adopted in the experiment were equal Si-avalanche EG&G-SPCM200 modules with quantum efficiencies: \(QE \approx 0.45\).

Suppose that a complete EPR non-locality test is performed by Alice and Bob by means of the two optical homodyne devices shown in Fig. 1, according to the scheme by Tan et al. \([11]\). Assume that the eigenvalues of the clock-LO coherent states are: \(\alpha = |\alpha| \exp \theta\), \(\alpha' = |\alpha| \exp \theta'\). By a simple extension of a previous analysis \([11]\) it can be shown that if Eve’s detector \(D_1\) clicks, i.e. \(\Psi^-\) is realized, the probability of a coincidence involving the detectors \(D_A'\) and \(D_B\) is:

\[
\langle \Psi^- | I_{A'} I_B | \Psi^- \rangle = \frac{1}{4} |\alpha|^2 \left\{ |\alpha|^2 + [1 + \cos (\theta' - \theta + \varphi)] \right\}
\]

(3)

Rather than performing the difficult double homodyne experiment, in our case Alice and Bob carried out an equally significant EPR non-locality test by mixing the modes \((k_A, k_B)\) by a 50:50 BS coupled to the detector pair \(D_1^*, D_2^*\): Figure 2, inset. In analogy with Eve’s apparatus, this device may be thought to perform a test on the Bell states \(\Psi^\pm\) spanning the Hilbert subspace pertaining to the 2-d manifold \((k_A, k_B)\). At the same time it also provides the necessary synchronizing clock effect, as said. Consider for instance the photo-detection by \(D_1^*\). Note first that the coincidence probability of simultaneous clicks by \(D_1^*\) and \(D_1\) is found by standard theory to be expressed by \([5]\):

\[
\langle \Psi^- | I_D I_{D'} | \Psi^- \rangle = \frac{1}{2} [1 + \cos \varphi]
\]

proportional to the expression (3) obtained for the homodyne devices by setting \(\theta = \theta'\) and \(|\alpha|^2 << 1\). Similar results are found for the other three coincidence combinations involving
Let Alice and Bob carry an experiment aimed at the measurement of the rate of detection by $D_i^*$: Figure 2. Since the two systems to be tested $(k_A, k_B)$ lack of any original non-local character, it is natural to expect a total insensitivity to any change of local parameters acting on remote parts of the apparatus, as for instance the phase shift $\Delta \varphi$. This is indeed shown by the experimental data (open circles) given in Figure 2. However, had the ”verification party”, Victor kept the record of the individual outcomes of both pairs $(D_1, D_2)$ and $(D_1^*, D_2^*)$, at a later time he could sort into two subsets the already tested samples detected by Alice and Bob. Figures 2 and 3 show that indeed each subset behaves as if it consisted of entangled pairs of distant systems. Note that these ones have never communicated in the past even indirectly via other systems. Furthermore, as pointed out by Peres, after Alice and Bob have recorded the results of all their measurement, Eve has still the freedom of deciding which experiment she will perform [8]. This one may consist of a standard Bell measurement, or a joint measurement with a $\Delta \varphi$ shift, or a POVM measurement [18] or one of the exotic, interesting single-photon non-locality tests suggested by L. Hardy [12]. Indeed in the present experiment, owing to a spatial displacement of the corresponding detector sets, Eve’s action could take place with a time delay $\Delta \tau \approx 3 \text{ns} \gg \tau_c$ respect to the time of the state reduction event determined by the test performed by Alice and Bob. In other words, since in our case $\Delta \tau$ was about $3 \times 10^3$ larger than $\tau_c$, the photon ”coherence time”, the ”swapping” process was completed by the Eve’s apparatus long after the complete annihilation of the particle measured by $(A + B)$. In order to offer a even more convincing demonstration, a sophisticated $\Delta \tau = 20 \text{ns}$ delay apparatus has been realized allowing delayed fast $\Delta \varphi$ changes by a randomly driven EO Phase Modulator (Inrad 621-040 with: $\Delta \varphi = \pi \equiv \Delta \varphi_{\lambda/2}$ driven by 400V rectangular pulses) triggered by the $(A + B)$ detection apparatus, i.e. long after the completion of the A+B test. Note in Figure 2 that shifts $\pm \Delta \varphi_{\lambda/2}$ correspond to the detection interchanges: $\Psi^{-} \rightleftharpoons \Psi^{+}$. This makes the original Peres’s argument, conceived for standard Bell-inequality tests of $(2 \otimes 2)$-d Hilbert photon $\pi$-states, fully consistent with the present experiment [18].

How could then Eve’s delayed choice determine data already irrevocably recorded?
According to Peres, it is meaningless to assert that two quantum systems are entangled without specifying their state, or to assert that a system is in a pure state without specifying that state or to attribute an objective meaning to the quantum state of a single system. If these prescriptions are forgotten one may encounter paradoxes as the one seen here: a past event may sometimes appear to be determined by future actions \[8\]. For better clarification it is perhaps worth reminding here that: "any phenomenon is not a phenomenon until is a measured phenomenon" (J. Wheeler) asserting the inanity of any intellectual speculation involving mental modelling of the inner evolution of a quantum superposition process. Furthermore, as pointed out by Richard Jozsa \[15\], any apparent retrodictive process, e.g. associated with the quantum evolution in presence of the EPR nonlocality in a teleportation process, cannot finally lead to paradoxes or contradictions of causality because of the inherent inaccessibility of the quantum information.

In conclusion, we have illustrated experimentally an enlightening aspect of quantum EPR non-locality. We have accomplished that by implementing a new method of photon quantum entanglement that is expected to play a significant role in the field of modern quantum information as well in future studies on EPR quantum non-locality. For instance the application of our new methods of high "fidelity" quantum teleportation and entanglement swapping to modern "quantum repeaters" will certainly improve in the near future the technology of quantum communication at large distances \[14,20\].

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FIG. 1. Layout of an experimental demonstration of the delayed-choice entanglement-swapping process. In the actual experiment the 2-homodyne apparatus was replaced by the 2-detector set shown in Figure 2, inset.

FIG. 2. Experimental results of the measurement of the count rate by detector $D_1^*$ as function of delayed settings of the phase $\varphi$ determined by micrometric displacements $X$ of the mirror $M$ (open circles). A verification party, Victor can sort at a later time the recorded pattern in two subsets showing two sinusoidal fringe patterns with opposite phases corresponding to the Bell states $\Psi^\pm$, for $\varphi = 0$. The visibility of the fringe patterns is $V = 91 \pm 2\%$. The inset shows the 2-detector apparatus that has been adopted to perform experimentally the EPR non-locality test and that for that purpose replaces, in a fully equivalent fashion the double homodyne apparatus shown in Figure 1.

FIG. 3. Histograms showing the measured detection count rates by $D_1^*$ and $D_2^*$ and the accuracy affecting the experimental determination of the Bell states $\Psi^\pm$ obtained by the delayed coincidence rates involving all detector pairs: $D_i^* - D_j(i, j = 1, 2)$, for $\varphi = 0$. 
$\Psi^+ \Psi^- = \phi = 0$

Count rate $\times 10^{-2} (s^{-1})$

Mirror position $X$ ($\mu$m)
