Uncertainty Quantification for Robust Control of Wind Turbines using Sliding Mode Observer

Horst Schulte
Control Engineering Group, University of Applied Sciences Berlin (HTW), Berlin, Germany
E-mail: horst.schulte@htw-berlin.de

Abstract. A new quantification method of uncertain models for robust wind turbine control using sliding-mode techniques is presented with the objective to improve active load mitigation. This approach is based on the so-called equivalent output injection signal, which corresponds to the average behavior of the discontinuous switching term, establishing and maintaining a motion on a so-called sliding surface. The injection signal is directly evaluated to obtain estimates of the uncertainty bounds of external disturbances and parameter uncertainties. The applicability of the proposed method is illustrated by the quantification of a four degree-of-freedom model of the NREL 5MW reference turbine containing uncertainties.

1. Introduction
Systematic wind turbine (WT) control design with uncertain conditions is one of the key issues to fulfill future needs related to active load mitigation and grid-forming ability. There exist multiple sources of uncertainties. Generally, those are caused by unmodeled or unknown dynamics, variations due to changes in the operating conditions and time varying dynamics caused by the wear of components. The objective of robust wind turbine control is to maintain a certain desired performance in the partial and full-load region despite of the existence of those uncertainties. These uncertainties can be characterized from the control oriented viewpoint as unstructured and structured uncertainties. The unstructured uncertainties description is employed, if the only available knowledge is the loop location (such as plant input, controller output, plant output) and norm-bounded description or the frequency-dependent magnitude. On the other hand variations of particular physical model parameters such as natural frequency coefficients, equivalent stiffness and damping values, etc. are structured. Those are the so-called structured uncertainties.

In recent years, some uncertain wind turbine models with the objective of robust control are proposed: In [15] the problem of robust control of a wind turbine using μ-synthesis technique is solved. Parametric uncertainties in drivetrain stiffness, drivetrain damping, and linearization coefficients of the aerodynamic torque are considered in the uncertain model. An individual pitch controller based on the $H_{\infty}$ norm reduction is proposed in [3] to mitigate loads in the tower and to align the rotor plane in the turbine. The wind effect in the tower side-to-side first mode is regarded as unstructured model uncertainties.

In this paper, a new quantification method of structured and unstructured uncertain models for robust wind turbine control design is presented. In Section 2 the Takagi-Sugeno (TS) framework for modeling uncertain nonlinear dynamical system is introduced. A sliding-mode technique for
observer-based reconstruction of uncertainties is presented in Section 3. The applicability of the proposed method, illustrated by the quantification of an uncertain wind turbine model with four degree-of-freedom is shown in Section 4.

2. Reduced Modeling of Wind Turbines with Uncertainties

2.1. Polytopic model structure and the class of Takagi-Sugeno models

Polytopic models for controller design are made of a convex combination of linear time invariant (LTI) systems. The combination can be parameter varying and/or state varying. The parameter varying concept is originally based on a class of linear parameter varying (LPV) systems. In contrast, the state varying concept of Takagi-Sugeno (TS) model framework also allows the description of systems, that are non-linear in the states. Hence, TS models provide a useful and uniform framework for nonlinear controller and observer design. Originally introduced in the context of fuzzy systems [16], TS models can either be derived from input-output data using system identification techniques or from mathematical models of nonlinear systems. Methods based on linear matrix inequalities (LMIs) enable a systematic controller and observer design for TS models.

For TS models several approaches can be followed to represent structured and unstructured uncertainties. In the framework of LMI-based design methods it is particularly suitable to model the structured uncertainty by a norm-bounded uncertainty system matrix \( \Delta A \). The unstructured uncertainty is modeled by the norm bounded disturbance input \( \xi(t) \in \mathbb{R}^{n_d}, \| \xi(t) \| \leq \Xi \in \mathbb{R}^+ \) and a common disturbances matrices \( D \in \mathbb{R}^{n \times n_d} \). The description of a uncertain TS model structure used in this work is given as

\[
\dot{x} = \sum_{i=1}^{N_r} h_i(x)(A_i + \Delta A)x + Bu + D \xi_d, \quad y = C x, \tag{1}
\]

where \( N_r \) denotes the number of submodels, \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^m \) is the input vector, \( y \in \mathbb{R}^p \) is the output vector and \( A_i, B \) and \( C \) are the system, input and output matrices with appropriate dimensions. The vector \( z = (x, u, \chi) \) may comprise states \( x_k \), inputs \( u_k \), and external variables \( \chi \). The functions \( h_i \) in (1) fulfill the convex sum condition: \( \sum_{i=1}^{N_r} h_i(x) = 1, \ h_i(x) \geq 0, \ i = 1, \ldots, N_r \). It is assumed, that a given uncertain nonlinear model can be represented by (1) in a compact set. To reconstruct the TS model from the given nonlinear model of the plant, the so-called "sector nonlinearity approach" [17] is used in this work.

2.2. Derivation of a reduced nominal wind turbine model in polytopic form

The nominal control-oriented wind turbine model used in the following consideration consists of a mechanical structure with four degrees of freedom, the aerodynamics, as well as the dynamics of the pitch drive and the generator-converter system. A detailed derivation of the WT model is given in [6] and corresponds to the model presented in [1], yet originates from [2]. The mechanical submodel includes the following generalized coordinates: fore-aft tower top displacement \( y_T \), flap-wise blade tip displacement \( y_B \), rotor rotational angle \( \theta_r \) and generator rotational angle \( \theta_g \). The equations of motion are derived by means of Lagrangian dynamics:

\[
(m_T + Nm_B) \ddot{y}_T + Nm_B \ddot{y}_B + d_T \dot{y}_T + k_T y_T = F_T
\]

\[
N m_B \ddot{y}_T + N m_B \ddot{y}_B + N d_B \dot{y}_B + N k_B y_B = F_T
\]

\[
J_r \ddot{\theta}_r + d_s (\omega_r - \omega_g) + k_s (\theta_r - \theta_g) = T_a
\]

\[
J_g \ddot{\theta}_g - d_s (\omega_r - \omega_g) - k_s (\theta_r - \theta_g) = -T_g
\]
with \( \omega_r = \dot{\theta}_r \) and \( \omega_g = \dot{\theta}_g \), where \( \theta_s = \theta_r - \theta_g \) denotes the shaft torsion angle, \( T_g \) denotes the applied generator torque, \( J_r \) and \( J_g \) denote the rotor and generator inertia, \( m_T \) and \( m_B \) denote the effective mass of the blade-tip and nacelle-tower motion, \( N \) is the number of blades, \( k_T \), \( k_B \), \( k_S \) denote the effective stiffness parameter of the tower (fore-aft displacement), blade (collective flap-wise displacement) and drivetrain. The parameters \( d_T \), \( d_B \), \( d_S \) denote the damping parameter of the tower, blade and drivetrain. The generalized force \( F_T \) in (2) is the rotor thrust force, which is calculated from the wind speed and from the characteristic curve of the thrust coefficient:

\[
F_T = \frac{\rho \pi R^2}{2} C_T(\lambda, \beta) \nu^2, \tag{3}
\]

with the rotor radius \( R \), the air density \( \rho \) and the undisturbed wind speed in front of the rotor \( \nu \). The function \( C_T(\lambda, \beta) \) denotes the aerodynamic rotor thrust coefficient as a function of the collective pitch angle \( \beta \) and the tip speed ratio \( \lambda = \omega R / \nu \). The generalized force \( T_a \) is given by the aerodynamic rotor torque, which is calculated from the wind speed and from the characteristic curve of the torque coefficient \( C_Q \)

\[
T_a = \frac{\rho \pi R^3}{2} C_Q(\lambda, \beta) \nu^2. \tag{4}
\]

Two explicit models for the pitch drive and generator/converter dynamics can be optionally added to the submodels (2), (3) and (4): The pitch model, which describes the electromechanical pitch system within the linear range (without limiting the pitch angle and speed) is given as

\[
\dot{\beta} = -\frac{1}{\tau} \beta + \frac{1}{\tau} \beta_d, \tag{5}
\]

where \( \beta \) and \( \beta_d \) denote the physical and the demanded pitch angle, respectively, and \( \tau \) denotes the delay time constant. For the purpose of robust and fault tolerant control a commonly used first order delay model is introduced

\[
\dot{T}_g = -\frac{1}{\tau_g} T_g + \frac{1}{\tau_g} T_{g,d}, \tag{6}
\]

where \( T_{g,d} \) denotes the demanded generator torque and \( \tau_g \) the delay time constant. The previously presented submodels (2) - (5) are combined to an overall WT model in quasi-linear matrix form by introducing the state vector \( \mathbf{x} = (y_T \ y_B \ \dot{\theta}_s \ \dot{\theta}_B \ \omega_r \ \omega_g \ \beta \ T_g)^T \) and the input vector \( \mathbf{u} = (\beta_d \ T_{g,d})^T \):

\[
\ddot{\mathbf{x}} = \mathbf{A}(\mathbf{x}, \nu) \mathbf{x} + \mathbf{B} \mathbf{u} \tag{7}
\]

with

\[
\mathbf{A}(\mathbf{x}, \nu) = \begin{pmatrix}
-k_T/m_T & \frac{N k_B}{m_T} & 0 & -d_T/m_T & \frac{N d_B}{m_T} & 0 & 0 & 0 & 0 \\
-k_r/m_T & -k_B/m_T & N k_B/m_T & 0 & d_r/m_T & -d_B/m_T & \frac{N d_B}{m_T} & f_1(\mathbf{x}, \nu) & 0 & 0 \\
0 & 0 & -k_B/m_B & 0 & 0 & 0 & \frac{d_B}{\tau_B} & 0 & \frac{d_B}{\tau_B} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{d_B}{\tau_B} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau} \\
\end{pmatrix},
\]

\[
\mathbf{B} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.
\]
\[ L_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0_{7 \times 2} \\ \frac{1}{7} \\ 0 \\ \frac{1}{s} \end{pmatrix}. \]

The wind speed \( v \) in (8) is treated as a disturbance input and will be estimated for control purpose by TS observer or Kalman filter [5]. The nonlinear functions \( f_1(x, v) \) and \( f_2(x, v) \) in (8) are given by

\[
\begin{align*}
f_1(x, v) &= \frac{1}{N m_B} F_T(x, v) \frac{1}{x_6} (x_6 := \omega_r > 0), \\
f_2(x, v) &= \frac{1}{J_r} T_a(x, v) \frac{1}{x_6} (x_6 := \omega_r > 0). 
\end{align*}
\]

In order to obtain the TS structure the sector nonlinearity approach [17] is used, which yields an exact representation of (7) in the compact set \( f_1 \in [f_{11}, f_{12}] \) and \( f_2 \in [f_{21}, f_{22}] \). The following identities hold (in this case with two nonlinearities):

\[
\begin{align*}
f_1(x, v) &= w_{11}(x, v) f_{11} + w_{12}(x, v) f_{12}, \\
f_2(x, v) &= w_{21}(x, v) f_{21} + w_{22}(x, v) f_{22}, 
\end{align*}
\]

where the so-called weighting function \( \omega_{jk} \) are given by

\[
\begin{align*}
w_{j1}(x, v) &= \frac{J_j - f_j(x, v)}{J_j - f_j}, \quad w_{j2}(x, v) := \frac{J_j - f_j(x, v)}{J_j - f_j}, \quad j \in \{1, 2\}. 
\end{align*}
\]

The functions satisfy the convex sum condition \( w_{j1} + w_{j2} = 1, \ j \in \{1, 2\} \). Consequently, \( f_1 \) and \( f_2 \) can be written as a combinations of the minimum and maximum values of the convex set:

\[
\begin{align*}
f_1 &= \left( w_{11} f_{11} + w_{12} f_{12} \right) (w_{21} + w_{22}) = h_1 f_{11} + h_2 f_{11} + h_3 f_{11} + h_4 f_{11}, \\
f_2 &= \left( w_{21} f_{21} + w_{22} f_{22} \right) (w_{11} + w_{12}) = h_1 f_{22} + h_2 f_{22} + h_3 f_{22} + h_4 f_{22}.
\end{align*}
\]

With this formulation, the nonlinearities have been shifted to the membership functions \( h_i \), which in the considered case are given by

\[
h_1 := w_{11} w_{21}, \quad h_2 := w_{11} w_{22}, \quad h_3 := w_{12} w_{21}, \quad h_4 := w_{12} w_{22}.
\]

By substituting the function \( f_1, f_2 \) in (8) with (12), we obtain the state differential equation of the TS model

\[
\dot{x} = \sum_{i=1}^{N_r=4} h_i(z) \left( A_i + B u \right)
\]

with \( z = (x, v) = (\omega_r, \beta, v) \). The system matrices \( A_i \) have the same values \( a_{ij} \) as \( A(x, v) \) in (8) with the exception that the minimum and maximum values of \( f_1 \) and \( f_2 \) are at the matrix
can be observed from Figure 1 in the shaft torsion angle $\theta$ system and generator/converter dynamics. Significant difference between the simulated states are the same caused by the same system input based on the same submodels of the pitch m/s are illustrated in Figure 1. As one can see the simulation of the generator torque and pitch form (13) in comparison with aeroelastic FAST model [11] with IEC wind speed gusts around 11

The eigenfrequency of the drivetrain $k$ parameter is excited by the step change of the generator torque at $t = 45$ s caused by the transition from the partial to full load region of the wind turbine. The discrepancy is caused by the neglected edgewise blade deflection in the reduced model. A common approach uses a modified stiffness parameter $k_S(f_{shaft1}, f_{B,edge})$ for representing the impact of the blade eigenfrequency $f_{B,edge}$ on the eigenfrequency of the drivetrain $f_{shaft1}$ that is described in [10]. However, this approach does not consider that as the blades are deflected their inertia and stiffness with respect to the blade root coordinate system changes, hence the natural frequencies change as a function of deflection [14]. Both parametric uncertainties are reflected by the expanded coefficients

$$a_{63} = a_{63} + \Delta a_{63}, \quad a_{73} = a_{73} + \Delta a_{73}$$ (16)
in $A(x, v)$ respectively $A_i$. The second and third parameter uncertainties are directly connected to the underlying blade bending model considered in the motion equation (2), where only one collective blade degree of freedom is considered. The blade bending caused by the rotor thrust force (3) is not modeled by means of bending beam models. Instead, the flap-wise blade tip translational displacement is considered, where the bending stiffness parameters are transformed to equivalent translational stiffness parameter $k_B$ and damping parameter $d_B$. This leads for the illustrated simulated load case (see Figure 1) to a less stiff blade displacement. A further difference in the simulation results of the blade motion is given by the influence from the tower shadow which is not addressed by the collective blade degree of freedom. Note, a detailed general analysis is difficult but one can assume that all possible deflection are described by motion equations with the following unknown but bounded equivalent parameters

$$\tilde{a}_{42} = a_{42} + \Delta a_{42} , \quad \tilde{a}_{52} = a_{52} + \Delta a_{52}$$

in $A(x, v)$ respectively $A_i$. All parametric uncertainties in (16) and (17) are summarized in the
uncertainty system matrix

\[
\Delta A = \begin{pmatrix}
0_{3 \times 3} & 0_{3 \times 4} & 0_{3 \times 2} \\
0 & \Delta a_{42} & 0 & 0 & 0 & 0 \\
0 & \Delta a_{52} & 0 & \Delta a_{63} & 0 & 0 \\
0 & 0 & \Delta a_{73} & 0 & 0 & 0 \\
0_{2 \times 3} & 0 & 0 & 0 & 0 & 0 & 0_{2 \times 6}
\end{pmatrix}
\] .

(18)

Finally, the unstructured uncertainty in (1) addresses the mismatch of the aerodynamic models (3) and (4). Based on the structure, particular the position of \(f_1\) and \(f_2\) in the nominal system matrix \(A_i\), the uncertainty distribution matrix of (1) is defined as

\[
D_\xi = \begin{pmatrix}
0_{1 \times 1} & 0_{1 \times 1} \\
\Delta d_1 & 0 \\
0 & \Delta d_2 \\
0_{3 \times 1} & 0_{3 \times 1}
\end{pmatrix}
\] (19)

with the uncertainty \(\xi_d = (\xi_{d,1}, \xi_{d,2})^T\).

3. Observer-based Framework of Uncertainty Quantification for Robust Control

3.1. Observer model

For preparing the design, the observer is based on the Takagi-Sugeno structure

\[
\dot{x} = \sum_{i=1}^{N_r} h_i(z) A_i x + B u + D_\Delta \xi_\Delta + D_\xi \xi_d, \quad y = C x,
\] (20)

with two uncertainty vectors \(\xi_\Delta\) and \(\xi_d\) as unknown but bounded inputs. A comparison of (20) with (1) shows that the structured uncertainty term in \(\Delta A x\) can be circumscribed as follows:

\[
\Delta A x = \begin{pmatrix}
0_{3 \times 1} \\
\Delta a_{42} x_2 \\
\Delta a_{52} x_2 \\
\Delta a_{63} x_3 \\
\Delta a_{73} x_3 \\
0_{2 \times 1}
\end{pmatrix} \equiv D_\Delta \xi_\Delta,
\]

\[
\Rightarrow D_\Delta = \begin{pmatrix}
0_{4 \times 4} \\
I_{4 \times 4} \\
0_{2 \times 4}
\end{pmatrix}, \quad \xi_\Delta = \begin{pmatrix}
\Delta a_{42} x_2 \\
\Delta a_{52} x_2 \\
\Delta a_{63} x_3 \\
\Delta a_{73} x_3
\end{pmatrix}
\]

Note, the basic idea to estimate parameter variations in the framework of an observer-based scheme with disturbance terms was inspirit by [8]. In this work, however, the parameter variations are caused by model uncertainties rather than by faults as proposed in [8].

3.2. Estimation of Uncertainties by Sliding Mode Observer

The sliding mode observer approach involves the design of a discontinuous feedback ensuring that a sliding surface, a subspace of the error space, is reached in finite time and that a sliding
motion is maintained for which the output error is zero. A special form, the Takagi-Sugeno sliding mode observer (TS SMO), initially proposed in [9] for fault reconstruction of nonlinear systems, is given by

\[
\dot{\hat{x}} = \sum_{i=1}^{N_r} h_i(\hat{z})(A_i \hat{x} + B u - G_{l,i} e_y + G_{n,i} \nu), \quad \dot{y} = C \hat{x},
\]

(21)

with \( \hat{z} = (\omega_r, \beta, \hat{v}) \) where

\[
C = \begin{pmatrix} 0_{6 \times 3} & I_6 \end{pmatrix}.
\]

In (21) \( G_{l,i} \) and \( G_{n,i} \) denote the linear and nonlinear observer gain matrices, \( e_y := \hat{y} - y \) denotes the output error and \( \nu \) the necessary discontinuous switching term to maintain a sliding motion:

\[
\nu = \begin{cases} -\rho \frac{P_2 e_y}{\|P_2 e_y\|} & \text{if } e_y \neq 0 \\ 0 & \text{otherwise} \end{cases}, \quad \nu_{eq} = \begin{cases} -\rho \frac{P_2 e_y}{\|P_2 e_y\| + \delta} & \text{if } e_y \neq 0 \\ 0 & \text{otherwise} \end{cases}.
\]

(22)

The average of the discontinuous switching term is a measure for the effort to maintain the sliding motion on the sliding surface [4] and can be approximated to arbitrary precision by introducing a small positive scalar \( \delta \) into the discontinuous term, see \( \nu_{eq} \) in (22). Once a sliding mode has been established (\( e_y \approx 0 \)), it can be shown that the equivalent output injection signal is given by

\[
\nu_{eq} = \sum_{i=1}^{N_r} h_i(\hat{z})(D_{\Delta,2,i} D_{\xi,2,i}) \begin{pmatrix} \xi_\Delta \\ \xi_d \end{pmatrix}.
\]

(23)

The observer gain matrices \( G_{l,i} \) and \( G_{n,i} \) are obtained from the transformed observer using a series of coordinate transformations with the resulting matrix \( T_i \). The underlying TS system is transformed into \( \tilde{x} = [x^T_1 \ y^T]^T \), where the measurable states \( y \in \mathbb{R}^p \) and the non measurable states \( x_1 \in \mathbb{R}^{(n-p)} \) are separated such that \( \xi_\Delta \) and \( \xi_d \) only influence the change of the measurable states. Applying the coordinate transformations the matrices in (23) are determined with

\[
D_{\Delta,i} = \begin{pmatrix} 0 \\ D_{\Delta,2,i} \end{pmatrix} = T_i D_\Delta, \quad D_{\xi,i} = \begin{pmatrix} 0 \\ D_{\xi,2,i} \end{pmatrix} = T_i D_\xi.
\]

(24)

From (23), the uncertainties can be estimated using the relation

\[
\begin{pmatrix} \xi_\Delta \\ \xi_d \end{pmatrix} = \left[ \sum_{i=1}^{N_r} h_i(\hat{z})(D_{\Delta,2,i} D_{\xi,2,i}) \right]^+ \nu_{eq},
\]

(25)

where \( [\cdot]^+ \) denotes the pseudo-inverse of the convex combination of the matrices \( (D_{\Delta,2,i} \ D_{\xi,2,i}) \). A pseudo-inverse of the convex combination of matrices exists if the full rank characterization is satisfied by the theorem proposed in [13].

4. Simulation results for uncertainty estimation

The proposed method is validated by the reconstruction of the uncertainty parameter \( \Delta a_{63} \) for the previously presented load case (see Figure 1) of the considered NREL FAST 5 MW wind turbine. Note, the change of natural frequencies as function of deflection described above can not simulated because FAST uses a linear modal structural model during simulation, so natural
Figure 2. 1. Upper sub-figures: NREAL FAST simulation of the wind speed $v$ and shaft torsion angle $\theta_s$ (blue solid curves) and the estimated wind speed $\hat{v}$ and shaft torsion angle $\hat{\theta}_s$ (green solid curves), 2. Lower sub-figures: Reconstructed uncertainty term $\Delta a_{63} \cdot x_3$ in $\frac{N}{kg \cdot m}$ and calculated uncertain parameter $\Delta a_{63}$ in $\frac{N}{kg \cdot m \cdot rad}$

frequencies are constant. This means that only the second source of uncertainty, the impact of the blade eigenfrequency $f_{B,edge}$ on the eigenfrequency of the drivetrain $f_{shaft}$ is considered. The simulation results of the estimated effective wind speed $\hat{v}$ based on an Takagi-Sugeno disturbance observer [5]. The estimated torsion angle $\hat{\theta}_s$, the reconstructed uncertainty term $\Delta a_{63} \cdot x_3$ and uncertain parameter $\Delta a_{63}$ based on the presented sliding mode observer (21) - (25) are illustrated in Figure 2. The uncertainty term $\Delta a_{63} \cdot x_3$ is reconstructed as described in the previous sub-section. The uncertain parameter $\Delta a_{63}$ is calculated as

$$\Delta a_{63} = \frac{1}{x_3} (\Delta a_{63} \cdot x_3), \quad |x_3| > \varepsilon_{x_3}.$$  (26)

In order to exclude large peaks in the reconstruction of $\Delta a_{63}$ when $x_3$ is close to zero, a threshold $\varepsilon_{x_3} = 1 \cdot 10^{-6} > 0$ is defined and equation (26) is only calculated when $|x_3| > \varepsilon_{x_3}$. The results can be interpreted as follows: During the transition regime ($t = [42, 60]$ s) the observer is not able to reconstruct the structured uncertainty caused by the first edgewise blade eigenfrequency. The reconstructed waveform arises rather from a superposition of unstructured uncertainties caused by the unmodelled degree of freedom of the reduced compared to the 24 DOF FAST model. Nevertheless, the absolute maximum value can be used for a robust control design approach. In the quasi-steady state ($t > 60$ s), the average of the uncertainty $\Delta a_{63}$ is negative. This corresponds to the fact that the stiffness factor $k_S$ decreases due to the edgewise blade deflection. Note, the edgewise blade deflection can be taken into account as an additional spring $c_2$ in the mechanical model of the drivetrain with a torsional spring $c_1$. This is shown
Figure 3. Two-mass spring-damper representation of the drivetrain with modified stiffness schematically in Figure 3, where the modified stiffness parameter is given as $c = \frac{c_1 c_2}{c_1 + c_2}$, which can be straightforward deduced by the equation of motion.

5. Conclusion

In this paper, a observer-based quantification of uncertain models for robust wind turbine control design was presented. For this purpose an uncertain reduced-order wind turbine model in polytopic form was derived. To define the structured uncertainties the model mismatch between the reduced model and the nonlinear high-order aeroelastic model implemented in FAST is carried out by numerical simulation. As a first validation step, the applicability of the proposed method is considered by the reconstruction of one uncertainty parameter caused by the neglected edgewise blade deflection in the reduced model. In further studies, the simultaneous reconstruction of unstructured and structured uncertainties is considered.

References

[1] Bianchi, F D and De Battista, H and Mantz, R J 2007, Springer-Verlag, London Limited
[2] Bindner H, Riso-R-920(EN), Risø National Laboratory, Roskilde, Denmark 1999
[3] Díaz de Corcuera A, Pujana-Arrese A, Ezquerra J M, Segurola E, and Landaluze J 2012, The Science of Making Torque from Wind
[4] Edwards C, Spurgeon S K, and Patton R J 2000, Automatica 36, 541-553
[5] Gauterin E, Kammerer P, Kühn M, and Schulte H 2016 ISA Transactions 62, 60–72
[6] Georg S, Schulte H, and Aschemann H 2012, IEEE International Conference on Fuzzy Systems 1737–1744
[7] Georg S 2015 Fault diagnosis and fault-tolerant control of wind turbines nonlinear Takagi-Sugeno and sliding mode techniques Phd Thesis
[8] Georg S, and H Schulte 2014 In J. Korbicz and M. Kowal (Eds.), Intelligent Systems in Technical and Medical Diagnostics, Volume 230 of Advances in Intelligent Systems and Computing 29-41
[9] Gerland P , Groß, D, Schulte H and Kroll A 2010, IEEE Conference on Decision and Control, Atlanta, USA, 4373-4378
[10] Hansen M H, Hansen A, Larsen T J, Oeye S, Sorensen P E, and Fuglsang P 2005, Riso-R-1500(EN), Riso National Laboratory, Roskilde, Denmark
[11] Jonkman J M, Buhl Jr, and Marshall L 2005 NREL/EL-500-38230, National Renewable Energy Laboratory, Golden, Colorado
[12] Jonkman J, Butterfield S, Musial W and Scott G 2009, Definition of a 5-MW Reference Wind Turbine for Offshore System Development Tech rep. NREL/TP-500-38060, National Renewable Energy Laboratory, Golden, Colorado
[13] Kolodziejczak B, and Szulc T, 1999, Linear Algebra and its Applications, 287, 215–222
[14] Larsen T J, Hansen A M, and Buhl T 2004, Aeroelastic effects of large blade deflections for wind turbines. In Proceedings, Delft: Delft University of Technology.
[15] Mirzaei M, Niemann H H, and Poulsen N K 2011, IEEE Conference on Decision and Control and European Control Conference (CDC-ECC), 645 – 650
[16] Takagi, T and Sugeno, M 1985, IEEE Transactions on Systems, Man, and Cybernetics 15(1), 116–132
[17] Tanaka, K and Sano, M 1994, IEEE Transactions on Fuzzy Systems, 2(2), 119–134