Analysis of students’ conceptual and procedural understanding of linear programming

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Abstract. Conceptual understanding is an understanding of theories, facts, rules, descriptions, and terminology of mathematics as well as all relevant information while procedural understanding is the procedure or set of rules (mathematical formula) to calculate or solve problems. Linear programming is a mathematical content very closely with procedural. The purpose of this study was to analyze the conceptual understanding and procedural understanding mathematics education students in the program linear material which familiar with the procedure. This research was conducted at a university by descriptive quantitative research methods using the test as an instrument for analyzing the students' abilities of in the form of a percentage. The results showed that 1) Students could graphically depict but have difficulty in language description. 2) Less than half students were able in connecting information. 3) More than half of students able to do the step by step procedures, but only two people who remember to emphasize the answer.

1. Introduction
Mathematics is one of the fields of study that must be mastered by school students. The part of mathematics taught in school, or school mathematics is basically the elements and components of mathematics that are chosen based on: (1) the meaning of education that is to develop the abilities and personality of the students; (2) Produce a tangible development of the environment that always develops along with the advancement of science and technology [1]. With mathematics, students can improve their abilities and personality. Therefore, mathematics is essential in the effort to prepare quality human resources.

Mathematics material considered requires high memory and reasoning thus making students challenging to learn it. This requirement makes students unlike mathematics as written by Ruseffendi [2]. Mathematics for children is generally an undesirable lesson. This obnoxious is due to one concept related to another that only can be used if the idea is genuinely appropriately mastered. Besides, doing math requires the execution of hundreds of different procedures, which too much to memorize as a separate piece of information. Therefore, in mathematics, two aspects cannot be separated, namely conceptual and procedural understanding.

Both of these understandings can be distinguished and the differences that facilitate ways of interpreting the learning process so that it helps to understand students’ failures and successes better.
Additionally, these understandings also have a function to understand informal mathematics acquisition such as for formal mathematics [3].

Conceptual understanding refers to an integrated and functional grasp of mathematical ideas [4]. Aspects used for conceptual understanding are: (1) to represent mathematical situations into mathematical models; (2) to represent the mathematical expression in the form of verbal or other forms; (3) able to connect the information with additional information; (4) able to distinguish the available information.

Procedural Understanding (PU) that within five strands mathematical proficiency called procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently [4]. Mathematics procedural understanding includes two types of information. First, a sort of PU is familiarity with symbols of a system and with syntactic conventions for acceptable symbol configurations. The second type of procedural understanding is a rule or procedure for solving mathematical problems [3]. This procedure is a step-by-step instruction carried out in a predetermined linear sequence. The procedural understanding indicator used in this paper is the implementation of step by step rules to solve mathematical problems.

The material chosen for this research is linear programming, a mathematical matter involving conceptual understanding also procedural. Linear programming describes the interrelations of the components of a system. [5]

2. Methods
This research is descriptive quantitative. Students were already learning linear programming before mid-test material. Then they administered the question about conceptual and procedural understanding based on chosen linear programming lesson.

2.1. Participant
The research participants were 55 sophomore students majoring in mathematics education at Syarif Hidayatullah State Islamic University Jakarta (Unversitas Islam Negeri Syarif Hidayatullah Jakarta). UIN students sourced from the results of several different admission selections whose class placement based on the time of re-enrollment without distinguishing their admission selection shape. Mathematics education student admissions consist of three channels, namely SBMPTN/Seleksi Bersama Masuk Perguruan Tinggi Negeri (joint selection entrance state colleges), UM PTKIN/Ujian Masuk Perguruan Tinggi Keagamaan Islam Negeri (selection entrance state Islamic religious colleges) and Ujian Mandiri (the Independent Selection). So, there is one class of students from the same admission selection system, and there is another class derived assorted selection system acceptance.

2.2. Instrument
The test on conceptual and procedural understanding used as research instrument. The answer from students are categorized and percentages which then described as analysis. There are five questions given for four aspects of conceptual understanding and one point of procedural understanding. First, fourth and fifth questions only have conceptual understanding aspect, second has just procedural aspect and third have both. There are thirteen indicators from the issue given.

The material chosen for this test is the linear programming on the discussion of the general form of linear programming, the use of graphics and the two-phase simplex methods to get the optimal solution, and giving examples of a specific case for linear programming problems. This selection is due to the research subject is a prospective mathematics teacher who will teach the material, at least until using the graphical method.

3. Result and Discussion
This section displays the frequency and percentage of the results of the conceptual and procedural understanding tests based on the indicators. Those indicators have been analyzed using the chi-square test, and the result is the asymp. Sig value less than 0.05 give meaning every value category which exists
is significantly different. Students’ probability in the G(Good), F(Fair), L(Less) and NA(No Answer) categories are statistically significantly different (Table 1). The providing categorizing can clearly distinguish the value of the student answers (Table 2).

### Table 1. Test statistics.

|        | CUA1a  | CUA1b  | CUA2a  | CUA2b  | CUA2c  | CUA3a  | CUA4a  |
|--------|--------|--------|--------|--------|--------|--------|--------|
| Chi-Square | 16.636⁹ | 107.836⁹ | 127.473⁹ | 37.436⁹ | 65.364⁹ | 11.109⁹ | 20.709⁹ |
| df     | 3      | 3      | 3      | 3      | 3      | 3      | 3      |
| Asymp. Sig. | .001   | .000   | .000   | .000   | .000   | .011   | .000   |

### Table 2. Test statistics.

|        | CUA4b  | PUA5a  | PUA5b  | PUA5c  | PUA5d  | PUA5e  |
|--------|--------|--------|--------|--------|--------|--------|
| Chi-Square | 36.709⁹ | 42.673⁹ | 15.473⁹ | 19.345⁹ | 27.982⁹ | 27.545⁹ |
| df     | 3      | 3      | 3      | 2      | 3      | 3      |
| Asymp. Sig. | .000   | .000   | .001   | .000   | .000   | .000   |

³. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 13.8.

³b. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 18.3.

### 3.1. Conceptual Understanding

The first aspect of this part is to represent mathematical situations into mathematical models (Table 3).

### Table 3. Represent mathematical expression.

|                        | G   | F   | L   | NA  |
|------------------------|-----|-----|-----|-----|
| f | %  | f  | %  | f  | %  | f  | %  |
| Can represent mathematical expressions of a given situation | 26  | 47  | 7  | 13 | 14 | 25 | 8  | 15 |
| Can present a mathematical model for the constraints of the general form of a linear program | 1   | 2   | 2  | 4  | 47 | 85 | 5  | 9  |

There are two indicators for the first aspect, each one given one question. First, ask students to represent mathematical expressions of a given situation which they already know the form and second asking them to represent a mathematical model to unknown equality or inequality. Table 1 shows that 47% of the students can represent mathematical expressions of a given situation. On the contrary, they cannot perform mathematical model which they have to build their own.

The second aspect for this part is to represent the mathematical expression in the form of verbal or other shapes (Table 4).

### Table 4. Represent the mathematical expression in the form of verbal or other forms.

| Indicator                                           | G   | F   | L   | NA  |
|-----------------------------------------------------|-----|-----|-----|-----|
| f | %  | f  | %  | f  | %  | f  | %  | f  | %  |
| Able to present graphical forms of a particular case | 50  | 90  | 2  | 4  | 1  | 2  | 2  | 4  | 2  | 4  |
| Able to offer reasons for a particular case graph  | 27  | 49  | 23 | 42 | 2  | 4  | 3  | 5  | 3  | 5  |
| Able to make verbal forms a mathematical model following the symbols and signs available | 11  | 20  | 39 | 71 | 2  | 4  | 3  | 5  | 3  | 5  |
There are three indicators for the second aspect. The first indicator is presenting a graphical case of a particular case of linear programming which they have to build themselves. There are fifty students able to create it correctly. The second indicator is giving the reason why their graphics show a particular case and half more students able to provide the reason. The third indicator is making the verbal form from providing a mathematical model and one-fifth of the students able to do that.

The third aspect is students able to connect the information with additional information (Table 5).

| Indicator | G | F | L | NA |
|-----------|---|---|---|----|
|           | f | % | f | % | f | % | f | % |
| Capable of connecting one or some of the information with other information. | 20 | 36 | 18 | 33 | 13 | 24 | 4 | 7 |

The indicator for this aspect is to connect the information with another information in case of social arithmetic. Table 3 shows that more than half students able to connect the information correctly.

The fourth aspect of conceptual understanding can distinguish the available information (Table 6).

| Indicator | G | F | L | NA |
|-----------|---|---|---|----|
|           | f | % | f | % | f | % | f | % |
| Able to distinguish and determine the value of existing basic variables. | 2 | 4 | 24 | 44 | 10 | 17 | 19 | 35 |
| Able to distinguish and determine the optimal value available. | 2 | 4 | 20 | 36 | 4 | 7 | 29 | 53 |

There are one questions for each indicator. Students are asked to identify and determine the value of existing basic variables then the optimal value which they have from the result of the two-phase simplex method. Based on Table 4, less than fifty percent of the student able to do those things.

3.2. Procedural Understanding

Procedural understanding consists of one aspect which is the implementation of step by step rules to solve mathematical problems (Table 7).

| Indicator | G | F | L | NA |
|-----------|---|---|---|----|
|           | f | % | f | % | f | % | f | % |
| Graph the line equation in sequence: 1) use two points; 2) determine the existing intersection; 3) determine the area of the feasible solution | 33 | 60 | 15 | 27 | 5 | 9 | 2 | 4 |
| Able to determine corner points, test them and inform the value of the basic variable and its optimum value | 22 | 40 | 15 | 27 | 16 | 29 | 2 | 4 |
| Able to change the form of the equation from inequality, substitute in r form, entry into the table, determine the initial feasible solution, specify key column and key row, transform key row and not key row, until r equal zero | 33 | 60 | 15 | 27 | 7 | 13 | 0 | 0 |
| Transform to stage two by leaving artificial variables. | 28 | 51 | 17 | 31 | 2 | 4 | 8 | 14 |
| Able to determine key columns and key rows as well as rows transformations at stage two to obtain optimization with correct calculations | 9 | 16 | 27 | 49 | 1 | 2 | 18 | 33 |
There are five indicators for this aspect. First and second are belong to graphical form, third to fifth are belong to the two-phase simplex method. Based on Table 5, students able to do the initial step of the procedure from each case, it is shown that from sixty percent or more than half students able to perform the procedures. The more extended procedures make less student able to finish it.

3.3. Discussion

3.3.1. Determine the decision variables in the modeling and non-negativity requirement. In linear programming, students are required to identify the representation and decision variables and complete the model by writing the non-negativity statement at the end of the model. Based on the results of the research are only 85% of the 55 students who are considering doing so. However, more than half of 85% that can do correctly but others are still making mistakes. This result indicates that more student has not been able to determine precisely which decision variables will look for its value and incomplete in restricting a problem. So, will likely hinder the determination of the optimal value of a solution or causes the solution can spread far from expected.

3.3.2. The connection between the variables and represent as a mathematical model. A variable can depend on other variables. It will look apparent when it in mathematics form. However, if it is still in words, it takes time to recognize it. Research results showed the difficulty of students in shaping the relationships between variables. It seen from only one student can formulate these relationships correctly and appropriately. The rest make a mistake or don't make it at all.

3.3.3. The concept of Social Arithmetic that is searching for profit. It turned out that the student is still awkward to determine the idea of social arithmetic which is related to the benefit obtained by finding the difference between the selling price and the cost that must spend. Although some are capable of modeling for this problem of social arithmetic, students are still struggling to connect it to the concept of a linear programming model consisting of goals and constraints. It categorized by eighty percent of students who cannot make this concept as the goal of the model in question.

3.3.4. The ability to model a particular case in graphical form and provide its interpretation. It is an aspect of representing mathematical expression in the way of verbal or other. Most of the students able to give an example of a linear programming particular case in the form of a graph, but only 49% can describe what is in the picture (Figure 1). On the interpretation of the students, does not specify at all about the meaning of horizontal lines or vertical, they cannot differ between constraints and objective function line.

Figure 1. A graph and its interpretation.
3.3.5. The conformity of the story with the mark on the given mathematical model. Although the story made by the students use quantities by the coefficients present in the model, the use of activity variables is not yet accurate. Only 20% of students make stories appropriate with mathematical models.

3.3.6. Ability to make graphics according to the sequence. Figure 2 shows that the student working on the problem to create the graph begins by looking for two points for each equation or inequality. After that create lines, give shading and do algebraic calculations to get the value of the existing intersection point. Then determine the corner points to be tested followed by testing those points. After getting the lowest value z then the minimum expected value is determined. Although this procedure is a common practice, there are still students who do not make this graph and immediately solve it with algebraic calculations.

![Figure 2. Graphing sequence](image)

During the completion of the problem, there are still many students who do not use rulers. They rely on student cards or identity cards to create a straight line. Although this is not prohibited, the inaccuracy in scale will affect the image results.

3.3.7. Determine corner points, inform the last values of basic variables and its optimal value. Note the Figure 2, is shown that the student has gained the lowest value for z, and he informs the values that in the question. Despite 67% of students did the calculations correctly, only 40% of students give the values of \( x_1 \), \( x_2 \), and \( z \) as asked. More than half student did not emphasize the answer.

3.3.8. Ability to solve problems that given through specific procedural. Students present their best efforts to get the correct answer. After doing the calculation, then checked back, when there is an error they will revise. This effort shows that through the mathematics lessons can make the students conscientious in their work and not easy give up to get the truth.
4. Conclusion
The research indicates one aspect of conceptual understanding which represents the mathematical expression in the form of verbal or other arrangements, have a contrary result. A lot of students able make the mathematical expression in graphical shape but cannot make it in verbal one. Students can graphically depict but have difficulty in language description. To represent mathematical situations into mathematical models aspect, student able to make it if one line and one information, but if the information spread in a few lines they start to get troubled. Less than half students able in connecting information and it has to consider to strengthen the ability. More than half of students able to do the step by step procedures, but only two who remember to emphasize the answer. It signs that student drowns in the calculation but forget to answer the question. The other problems are errors in inaccuracies and algebraic calculations which significantly affect outcomes.

5. References
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