Splitting electronic spins with a Kondo double dot device

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We present a simple de made of two small capacitively coupled quantum dots in parallel. This set-up can be used as an efficient "Stern-Gerlach" spin filter, able to simultaneously produce, from a normal metallic lead, two oppositely spin-polarized currents when submitted to a local magnetic field. This proposal is based on the realization of a Kondo effect where spin and orbital degrees of freedom are entangled, allowing a spatial separation between the two spin polarized currents. In the low temperature Kondo regime, the efficiency is very high and the de conductance reaches the unitary limit, \( \frac{e^2}{h} \) per spin branch.

Controlling the electron spin in electronic circuits is the challenge of the new emerging field called spintronics. Part of current research aims at the injection of spin-polarized electrons into mesoscopic structures. For instance, control of single spins, owing to long decoherence times in semiconductor nanostructures, opens the way to spin-based quantum information processing.\[1\] One of the major goals is the production of efficient spin filters, with the following requirements: i) high polarization, especially for very demanding tests of quantum entanglement;\[2\] ii) bidirectional spin filtering, e.g. filtering at will "up" and "down" spins; iii) low impedance, to allow unperturbed transport (conductance and noise) measurements on a variety of devices. Several set-ups fulfilling part, but not all the above constraints, have been proposed or tested to inject spins or create spin filters. They rely either on ferromagnetic materials,\[3\] external magnetic fields,\[4\]\[5\]\[6\]\[7\], or spin-orbit coupling.\[8\] Recher et al.\[4\] have in particular considered a quantum dot weakly coupled to current leads, in the sequential tunneling regime. They have shown that in the presence of a local magnetic field it can act as an efficient spin filter whose spin direction can be controlled by energy filtering, with the help of the dot plunger gate voltage: given a single electron level in the dot, with occupancy \( n \), transitions between \( n = 0, 1 \) states or between \( n = 1, 2 \) states respectively involve opposite spins. Another interesting possibility developed by Borda et al.\[6\] is to use a double quantum dot (DD) system with strong capacitive interdot coupling. When an external magnetic field is applied to such a system, these authors showed that the low energy physics can be described by a purely orbital Kondo effect where spin flip processes are suppressed and only charge fluctuations are allowed between the dots (the latter representing the orbital degrees of freedom). A major consequence of the Kondo effect is the reach of the unitary limit at \( T \ll T_K \) where \( T_K \) is the Kondo temperature.\[9\] In this limit the DD proposed by Borda et al.\[6\] thereby acts as an almost perfect unidirectional spin filter (with high conductance \( e^2/h \)), provided the temperature is low enough.

In this Letter, we go one step further and propose a simple, robust and efficient "Stern-Gerlach" spin splitter, able to simultaneously produce from a normal metallic lead two oppositely spin-polarized currents, using non-magnetic semiconductor materials. Realization of such a spin splitter, used as a source or an analyzer of polarized electrons, opens the way to many experiments, including Bell correlations of entangled states,\[2\] or spin-resolved shot noise measurements.\[10\] Our proposal is schematized in Figure 1. Spin filtering is achieved by energy filtering, as in Ref.\[4\], selecting each of the spin directions in either dot 1 or 2. Our set-up does not work in the sequential regime, but in the Kondo regime, as in Ref.\[6\], the two small quantum dots being strongly coupled in a capacitive way. Rather than coupling each dot to two independent reservoirs, here each dot is connected to a common source kept at the chemical potential \( \mu_L \) and to a distinct current lead at chemical potential \( \mu_R \). The two outgoing spin-polarized currents of opposite polarizations emerge from these two separate leads.

![FIG. 1: Schematic representation of the proposed setup: two small quantum dots coupled by a capacity \( C_0 \) and connected to a common source. Each dot 1 and 2 is also connected to an extra lead from which the two spin polarized currents will emerge. Depending on how the gate voltages are tuned, the upper lead can be polarized in the up direction and the lower lead in the opposite direction or vice versa.](image-url)

The numbers of electrons in the dots are controlled by two plunger gate voltages at potential \( V_{g_1} \) and \( V_{g_2} \). We label the lowest-lying charging states by the numbers \((n_1, n_2)\) of extra electron in dots 1, 2. We consider the regime where the gate voltages are adjusted such as the two lowest-lying and almost degenerate charging states are \((1, 1)\) and \((0, 2)\), instead of states \((1, 0)\) and \((0, 1)\) as in Ref.\[4\]. A schematic stability diagram is depicted in Fig. 2 showing the different possible charging states. At energies lower than the charging energy of
the dot $E_C = \min(E(0, 1) - E(1, 1); E(1, 2) - E(1, 1))$, the charge dynamics is restricted to states $(1, 1)$ and $(0, 2)$, states $(0, 1)$ and $(1, 2)$ appearing only as virtual states. Let us label the capacitances (chosen symmetric in the dot indices for simplicity) as $C_L$ (left), $C_R$ (right), $C_g$, $C_0$ (coupling the two dots), define $C = C_L + C_R + C_g$, and the external charges $C_g V_g1, C_g V_g2$ from a reference state with even occupation numbers. Then the intradot and interdot charging energies are respectively $U = \frac{x^2 (C + C_0)}{x (C + C_0)}$ and $V = x U$ with $x = \frac{e^2}{\epsilon C_0}$. The condition for degeneracy reads $V_g = V_g1 + \frac{e}{\epsilon C_0}$ and the excitation energies are $E(0, 1) - E(1, 1) = U[(1 + x) \frac{C_g V_g1}{\epsilon} - \frac{1}{2}]$, $E(1, 2) - E(1, 1) = U[\frac{1}{2} + x - (1 + x) \frac{C_g V_g2}{\epsilon}]$. The optimum regime is reached at the symmetric point $O$ (corresponding to the thick black dot in Fig. 2) where the two excitation energies are equal to $E_c = U \frac{x^2}{2}$.

![Diagram](image)

**FIG. 2:** Sketch of the operation region in the stability diagram showing stable charge states as function of gate voltages $V_g1$ and $V_g2$. The thick black dot $O$ corresponds to the ideal operating point in the middle of the degeneracy line.

The isolated DD system is described at low energy by

$$H_{dot} = -\delta ET^z - tT^x - g\mu_B BS^z,$$

where we have defined the orbital pseudospin $T^z = (n_1 - n_2 + 1)/2 = \pm 1/2$. Here $\delta E = E(0, 2) - E(1, 1)$ is zero when the two lowest-lying charge states are exactly degenerate. The second term in Eq. (11) represents a direct tunneling amplitude between the dots and the last term expresses the Zeeman splitting when a local magnetic field is applied in the $z$ direction.

In the following we assume that the Zeeman energy is large enough such that spin-flip scattering is suppressed. An evaluation of the typical value of the required magnetic field and other experimental parameters is provided at the end of the Letter. The spin states of the two degenerate ground states are therefore $|\uparrow, \uparrow\rangle$ and $|0, s\rangle$ where $s$ stands for singlet state. Triplet states $|0, t\rangle$ can be discarded if the level splitting $\delta z$ in each dot is large enough. Defining the total spin operator by $S^2 = S^z_1 + S^z_2$, it is crucial that $T^z = S^z - \frac{1}{2}$. This means that the spin of the electron added to the “empty state” $|0, \uparrow\rangle$ is entangled with the orbital pseudospin virtual charge fluctuations on dot 1 (resp. 2) involve spin-up (resp. spin-down) electrons exclusively, and an orbital pseudo-spin flip (between states $|\uparrow, \uparrow\rangle$ and $|0, s\rangle$) is locked to a genuine spin flip. Therefore the Kondo screening of the spin involves spin-up electrons in the upper right lead and spin-down electrons in the lower right one, as well as spin-up and spin-down electrons altogether in the common left lead. This is contrary to the set-up of Borda et al. where the real spin is quenched by the applied magnetic field and only the orbital pseudo-spin survives. This is a crucial difference which makes possible the realization of a spin splitter from our proposal. Before turning to a more quantitative analysis, we also emphasize that the tunneling term, being spin independent, no longer connects the two degenerate states, as opposed to the situation occurring in (4). We can therefore neglect it provided $t \ll g\mu_B B$. In practice, this makes a strong capacitive coupling between the dots easier to achieve than in Ref. (4) where $t \ll T_K$ is instead required.

The leads are described by $H_{leads} = \sum_{k,\alpha,\sigma} \varepsilon_k c^\dagger_{k,\alpha,\sigma} c_{k,\alpha,\sigma}$, where $c^\dagger_{k,\alpha,\sigma}$ creates an electron with energy $\varepsilon_k$ in the lead $\alpha$ and spin $\sigma$. Indeed, Zeeman splitting in the reservoirs can be made much smaller than in the dots. Since the Coulomb energy $E_C$ is one of the largest energy scales, only cotunneling processes where the numbers of initial and final electrons in the DD are equal are allowed. Therefore we need only to consider virtual excitations towards states with $n_1 + n_2 = 1$ and 3 in the DD. Using second-order perturbation theory in the tunneling amplitude between the dots and the leads, we obtain a Kondo effective Hamiltonian $H_{eff} = H_K + H_{tun}$ with

$$H_K = \sum_{k,k',\alpha,\beta} \left[ J_{k,k',\alpha,\beta} (c^\dagger_{k,\alpha,\downarrow} T^+ c_{k',\beta,\uparrow} + \text{h.c.}) \right]$$

and

$$H_{tun} = \sum_{k,k',\alpha,\beta,\sigma} \left[ V_{k,k',\alpha,\beta} (c^\dagger_{k,\alpha,\sigma} c_{k',\beta,\sigma} + \text{h.c.}) \right]$$

where $J_{k,k',\alpha,\beta} \sim J_{\alpha,\beta} \sim \sqrt{\gamma_\alpha \gamma_\beta} / E_C$ and $V_{k,k',\alpha,\beta} \sim J_{\alpha,\beta} / 4$, with $\gamma_\alpha$ the tunneling rate from/to lead $\alpha$. There are other cotunneling terms with smaller amplitudes involving for example higher energy processes like $EC(2, 1) - EC(1, 1) \gg E_C$. These terms may a priori pollute spin filtering. Nevertheless the strength of the Kondo effect is to renormalize the Kondo couplings toward strong coupling at low energy as opposed to direct potential scattering terms that do not renormalize. Therefore terms like those involved in Eq. (6) or other higher energy potential scattering terms can be dropped out in the low temperature regime $T \ll T_K = D exp(-1/\rho_0 (J_{LL} + J_{RR}))$, where a constant density of states $\rho_0$ has been assumed in the leads. This corresponds to the unitary limit where the spin and orbital pseudospin are completely screened and a singlet is formed together with spin-up/down electrons in the left lead, spin-up
electrons in the upper right lead and spin-down electrons in the lower right lead.

Transport across the double dot can now be described, applying a small voltage \( eV = \mu_L - \mu_R \). The conductance of each right lead is given by \( G_{L,R} = G_0 \sin^2 \delta \) and \( G_{L,R}^{\downarrow} = G_0 \sin^2 \delta_\downarrow \), both tending towards \( G_0 = \frac{G_0}{(2n+1)\pi} \) for \( T \ll T_K \) where the phase shifts \( \delta, \delta_\downarrow \) are equal to \( \pi/2 \). The conductances reach \( e^2/h \) at \( T = 0 \) for symmetric tunneling amplitudes. Notice that in the unitary limit, the polarization of the currents in the right leads is almost perfect, e.g. \( G_{L,R} = G_{L,R}^{\uparrow} = 0 \).

The ground state in the unitary limit is a Fermi liquid which is usually stable toward various perturbations. Let us analyze applying a small voltage \( V \) to the lower right lead.

The resistance of each right lead is given by \( \frac{\delta \epsilon}{2} \) where the phase shifts \( \delta \), \( \delta_\downarrow \) are equal to \( \pi/2 \). The conductances reach \( e^2/h \) at \( T = 0 \) for symmetric tunneling amplitudes. Notice that in the unitary limit, the polarization of the currents in the right leads is almost perfect, e.g. \( G_{L,R} = G_{L,R}^{\uparrow} = G_{L,R}^{\downarrow} \).

Let us now estimate the experimental requirements to realize our proposal. First, in a large enough magnetic field, the above set-up should exactly map on a single-dot Kondo problem. This requires, as in Ref. [6], that \( g_\mu B > T_K \), which is the Kondo temperature of the system without a magnetic field whose behavior is also expected to be described at low energy by a SU(4) Kondo problem with four (orbital and spin) degenerate states.

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