Abstract—In aerospace applications, a Ground Power Unit (GPU) has to provide balanced and sinusoidal 400 Hz phase-to-neutral voltages to unbalanced and non-linear single-phase loads. Compensation of high-order harmonics is complex, as the ratio between sampling frequency and compensated harmonics can be very small. Thus multiple superimposed resonant controllers or PI nested controllers in multiple dc frames are not good alternatives. The first approach cannot ensure stability, while the second cannot track sinusoidal zero-sequence components, typically present in unbalanced system, and unachievable high bandwidth at the inner current control loop is typically required. In this paper, a simple methodology for designing a single-loop, multiple resonant controller for simultaneous mitigation of several high-order harmonics, ensuring stability, is presented. Experimental results, based on a 6kW four-leg NPC converter, validates the proposed controller design, showing excellent steady state and transient performance.

Index Terms—Four-Leg Converters, Three level Neutral Point Clamped (NPC) inverter, Resonant Controllers.

I. INTRODUCTION

Recently the aircraft industry has faced a tremendous development in the technology used for communication, services and control systems within an aircraft [1]. Electronic devices have played a fundamental role in this growth, leading to a more complex and sophisticated electrical system within the plane, which has to fulfill stringent power quality and safety regulations [2], [3]. When the aircraft is on ground, a power converter based Ground Power Unit (GPU) is connected to it, providing 110V phase-to-neutral at 400 Hz [2].

The aircraft electrical system is four-wire unbalanced system, with many individual loads connected phase-to-neutral. Therefore, a GPU is a four-wire power supply rated at 110V (phase), 400Hz and typically between 30-180kVA (being 90kVA the most utilized power rate) [4].

Different solutions have been proposed for power electronics based GPUs [5]. Typically, three H-bridges connected in parallel, sharing the same dc-link and connected to LC filters, are used to independently control each phase-to-neutral voltage. However, the compensation of high order harmonic components is limited by the switching frequency. An alternative configuration uses transformers to add the output voltages of several inverters, generating a stepped waveform [6]. Although this configuration offers good THD with low switching frequency, it is complex to control and the high number of elements reduces its reliability. Recently new topologies such as matrix converters have been proposed for GPU applications [7], [8]. Despite advantages in terms of size and weight, this approach has a high number of switches and for unbalanced load operation produces distorted currents with low power quality in the input side of the converter.

The relatively high output frequency of the GPU is an issue for design. Standard 50/60 Hz Uninterruptible Power Supply (UPS) converters typically have sampling frequencies between 2-12kHz. The high ratio between the fundamental and the sampling frequency allows many control schemes, such as: nested $d$-$q$ controllers, resonant controllers, repetitive and predictive controllers [9]–[11] to be implemented relatively easily. However, for a 400 Hz GPU, a 2 kHz sampling frequency is not feasible, because the ratio between the fundamental and the sampling frequency is insufficient. Therefore, for a GPU, the sampling frequency typically has to be around 10-15 kHz and even higher when harmonic compensation is required. Moreover, the high bandwidth required for an inner current control loop means the use of double-loop structures for nested voltage/current controllers is not practically implementable and the use of single-loop voltage controllers have been preferred for this application. In [4] a robust single-loop strategy has been proposed to control the output voltage of a GPU and whilst it achieves good performance with linear loads, it is not capable of compensating harmonic distortion with non-linear loads.

Due to their simple and robust implementation, resonant controllers are an interesting solution for high order harmonic
compensation. They have been used for selective harmonic elimination in several applications [5], [7], [12]. However, careful consideration of stability margins and the selection of discretization method are important during the controller design to ensure stability, particularly when high order harmonics have to be compensated [13]–[15]. In [5] a resonant controller has been proposed as a solution for a standard two-level, 400 Hz GPU. The implementation includes third, fifth and seventh order harmonic compensation but cannot be reasonably extended to higher order harmonics such as 11th, 13th and 15th as the power losses of the converter limit the switching frequency and therefore the higher order harmonic compensation. Furthermore, no stability analysis has been undertaken in [5].

The contributions of this paper can be summarized as follows:

- A simple methodology for designing a single-loop resonant controller with multiple resonances, for regulation of the fundamental 400 Hz voltage signal as well as compensation of high order harmonics components, under non-linear and unbalanced loads, is proposed and successfully validated. The methodology allows direct design of one controller with multiple resonances, avoiding the superposition of individually designed resonant controllers as presented in [16], [17]. Multiple dq frame transformations and positive-negative sequence decomposition are avoided, which is an important advantage over other published methods [18], [19].

- The proposed methodology can be used to design a robust and more stable control system when the sampling frequency is close the Shannon-Nyquist frequency (theoretical limitation). This is important for a 400Hz GPU application. An analytical design for the full discretized controller based on Nyquist response, considering delay compensation is proposed in this paper and experimentally validated. This improves stability and robustness, when a non-damped resonant LC output filter is implemented and the sampling frequency is close to the compensated harmonic components.

- A four-leg three-level Neutral Point Clamped (NPC) has been validated as a suitable solution for a 400Hz GPU application. The multi-level nature of this topology increases the equivalent frequency of the modulated waveform compared to the switching frequency of each device, which is a major advantage for the GPU, allowing higher-order harmonic compensation compared to the two-level VSI typically reported in the literature [5], [13], [16]–[18]. The fourth leg of the converter provides a path for zero-sequence current components, i.e. unbalanced loads, producing 5-levels phase-to-neutral output voltages which allows implementation of smaller output power filters with lower reactive power consumption [20].

The rest of this paper is organized as follows. In Section II the proposed control methodology is introduced and extensively analyzed. Three-dimensional modulation is discussed in Section III. Experimental results are discussed in section IV.

II. SINGLE-LOOP VOLTAGE CONTROL STRATEGY

Resonant controllers and rotating dq controllers are the most suitable alternatives for single-loop voltage control of grid connected and stand alone power converters [13], [21]–[23]. Both schemes provide good transient and steady-state performance. However, when unbalanced and harmonic compensation are required, resonant controllers are simpler to design and require less computational burden. They avoid multiple reference frames transformations and positive/negative sequence separation and give direct control over the zero-sequence present in four-wire systems.

Fig. 1 shows the proposed solution for a 400Hz, 110V GPU, where the aircraft represents a linear/non-linear, balanced/unbalanced load. A small LC filter is used at the output of the converter. A single-loop resonant controller controls the output voltage of the GPU, compensating third, fifth, seventh, ninth and eleventh harmonics. A three-dimensional SVM algorithm in αβγ coordinates is used to synthesize the output voltages [24]. Furthermore, using the redundant

Fig. 1. Control scheme for the proposed four-leg NPC converter used as GPU. An small output LC filter is used to obtain 400Hz sinusoidal output voltages. $R_f$ represents the series resistance, $L_f$ is the filter inductance and $C_f$ is the capacitor of the filter.

\[
\Delta V_c = v_{C1}(t) - v_{C2}(t)
\]

\[
\Delta V_c = 0
\]
are measured every sampling time as a unit delay and the VSI is represented as a constant gain. 

Fig. 2 presents the discrete-time control scheme and design for the controller and the plant 

A. Discrete Time Representation 

To implement the controller in a digital platform, a discrete time representation and design for the controller and the plant are required. Fig. 2 presents the discrete-time control scheme of a resonant controller implemented for a second order system. The control algorithm calculation time is represented as a unit delay and the VSI is represented as a constant gain.

For the SVM algorithm, the output voltages of the LC filter are measured every sampling time $T_s$. Furthermore, the output voltages of the converter are maintained constant during each sampling time $T_s$. Therefore, the plant can be considered as a continuous system cascaded with a Zero-Order Hold (ZOH) circuit and a sampled output. Thereby, the ZOH discrete representation (see [15]) of the second order system of (1) is given by:

$$
P_{zoh}(z) = \frac{1}{z - 1} + \frac{z - 1}{\sqrt{1 - \xi^2}} \frac{1}{z^2 - 2z\cos(\omega_n T_s) \cos(\omega_n T_s) - \cos^2(\omega_n T_s) + \cos^2(\omega_n T_s)}
$$

(3)

$$
\omega_n = \sqrt{\frac{1}{L J f}}
$$

(4)

To ensure zero steady state error to sinusoidal reference signals, the discretization method applied to (2) must maintain the resonant frequencies $\omega_n$ unaltered, providing infinite gain at each of these frequencies [13], [14]. Accordingly, the First Order Hold (FOH) and Tustin with Prewarping (TPW) methods, setting the prewarping frequency as each $\omega_n$, are suitable approximations of the continuous system. Thus, using the FOH approximation, the discrete form of the resonant controller of (2) is given by:

$$
R_{f-foh}(z) = \sum_{n=1}^{K_n} \frac{\cos(\omega_n D_n T_s)(1 - z^{-2})(1 - \cos(\omega_n T_s))}{\omega_n^2 T_s (1 - 2z^{-2}\cos(\omega_n T_s) + z^{-2})}
$$

(6)

$$
\sin(\omega_n D_n T_s) \left[ \omega_n T_s - \sin(\omega_n T_s) + 2z^{-2}\sin(\omega_n T_s) \right]
$$

$$
\omega_n^2 T_s (1 - 2z^{-2}\cos(\omega_n T_s) + z^{-2})
$$

$$
\omega_n^2 T_s (1 - 2z^{-2}\cos(\omega_n T_s) + z^{-2})
$$

(7)

where $T_s$ is the sampling time used for the discretization and $\omega_n$ is the resonant frequency of the $n^{th}$ resonant controller. The term $D_n$ represents the number of samples required for compensating the phase-shift introduced by the plant. Equivalently to $d_n$ of (2), $D_n$ for the $n^{th}$ resonant controller is given by:

$$
D_n = \frac{-\angle P_{zoh}(z_n = e^{j\omega_n T_s}) \text{rad}}{T_s \omega_n}
$$

(7)

where $P_{zoh}(z_n = e^{j\omega_n T_s})$ represents the ZOH discrete-time representation of (3) evaluated at the resonance frequency $\omega_n$.

B. Controller Design 

Different strategies have been proposed in the literature for the design of resonant controllers employing Nyquist diagrams [13], [16]–[19]. However, those approaches usually use a first order plant or make a first order approximation, neglecting the resonant effect of a higher order filter. This allows independent design for each resonant compensator, which can be useful when the resonant effect of the filter is far from the resonant effect of each compensator. However, when high order harmonics relatively close to the resonance of the filter have to be compensated, an independent design of each compensator can lead to instability. Therefore, a design considering all the resonant compensators in a single transfer function is required and studied in this section.

Fig. 3 shows the Bode and Nyquist response for the open loop system $H_1(s) = G(s)P(s)e^{-s T_s}$, considering the
resonant controller of (2), tuned only at the fundamental 400Hz frequency with $\vartheta_1 = 0^\circ$ and the second order filter of (1). Fig. 3a shows the bode plot of $H_1(s)$ for three different gains $K_{1a,1b,1c}$ including a sampling time delay $T_s$. The later introduces a phase shift of $-10^\circ$ at $\omega_1$ and the phase of $H_1(s)$ steps from $\xi_1 = 80^\circ$ to $\xi_1 - 180^\circ$ at $\omega_1$. Fig. 3b shows the Nyquist diagram of $H_1(s)$ for the three different gains $K_{1a,1b,1c}$. The Nyquist diagram starts from the point $(0,0j)$ at $\omega = 0$ rad/s, as $\omega$ approaches from 0 to $\omega_1$, the magnitude of $H_1(s)$ increases towards infinity with an angle $\xi_1$. After $\omega > \omega_1$, the phase of the Nyquist plot turns rapidly through $180^\circ$ to appear from the bottom of the frame. Thereafter, instead of approaching directly to zero, the Nyquist plot describes a curve which approaches towards the critical point $(-1,0j)$. This curve is produced by the resonance of the second order filter $\omega_c$, defining the stability margin of the closed-loop system and representing a constraint for the gain of the controller in order to maintain closed-loop stability. From Fig. 3b, the curve $\kappa_c$ leads to an unstable system, nevertheless the phase margin of the open-loop system, before the resonance peak is $\approx 90^\circ$. This shows one of the problems of Bode diagrams to design resonant control systems, especially when multiple resonant controllers are implemented. There are many frequencies where the magnitude of $G(s)H(s)$ crosses the 0db line (e.g see Fig. 5). Therefore it is difficult to predict the performance and overall stability using a Bode diagram and phase margins, in control systems where multiple resonant controllers are implemented. For this reason Nyquist diagrams are preferred in this work, because they have significant advantages for the design of high-order controllers with multiple resonant peaks.

Additionally, from Fig. 3b it is concluded that for an angle $\xi_1 < 0$ the Nyquist path would enclose the point $(-1,0j)$ regardless of the controller gain. This is of particular importance when high order harmonics have to be compensated, as the phase of the open-loop system rapidly decreases as a result of the phase injected by the sampling delay $P_d(s) = P(s)e^{-sT_s}$. Thereby, to increase the stability margin of the closed-loop system it is desirable to set $\xi_1$ to $90^\circ$ at the resonance frequency $\omega_1$. Thereby, to set each angle $\xi_n$ to $90^\circ$, the phase shift produced by the plant at each resonance frequency, $\vartheta_n = -\angle P_d(j\omega_n)$, has to be compensated in (2).

Fig. 4a shows the open-loop bode diagram of $H_1^c(s) = P_d(s)G_c^c(s)$ considering a resonant controller to regulate the 400Hz fundamental including the phase compensation. The phase shift is now $\pm 90^\circ$ at the resonance frequency $\omega_1$. Similarly to Fig. 3b, Fig. 4b shows the Nyquist diagram for the open-loop system considering phase compensation ($H_1^c(s)$). The introduction of the phase shift compensation allows placement of the asymptotes parallel to the $j\omega$ axis ($\xi_1 = 90^\circ$), which helps to maintain the stability of the closed-loop system when high frequencies are compensated. The paths $\vartheta_1$ and $\vartheta_2$ show the trajectory of the Nyquist response as $\omega$ approaches the resonance frequency $\omega_1$. Additionally, the magnitude of the closed-loop frequency response, i.e. M circle tangent to the Nyquist response, has been plotted in order to set the gain of the controller $K_1$ for obtaining a damping factor of $\zeta = 0.65$. The damping factor $\zeta$ and the magnitude of the closed-loop frequency response are related by [15]:

$$M \approx \frac{1}{2\sqrt{1 - \zeta^2}}$$ (8)

Fig. 5 shows the Bode and Nyquist diagram for the open loop system composed of $P_d(s)$ and a resonant voltage controller to regulate not only the fundamental 400 Hz frequency, but to eliminate all the odd harmonics up to the eleventh. As expected, Fig. 5a possesses six resonant peaks at the frequencies $\omega_{c,3,5,7,9,11}$ with $\pm 90^\circ$ phase shift around the resonance frequencies. The phase compensation places the asymptotes, for each resonance frequency $\omega_{c,1,3,5,7,9,11}$ parallel to the $j\omega$ axis in the Nyquist diagram of Fig. 5b, which reduces the possibility of enclosing the critical point $(-1,0j)$. Equivalent to Fig. 4b, from Fig. 5b a set of paths $\vartheta_1$ to $\vartheta_{12}$ can be recognized. The frequency increases starting from 0 Hz along the trajectory described by $\vartheta_1$, the magnitude of the Nyquist plot increases to infinity (with an angle of $90^\circ$) at the resonant frequency $\omega_1$, thereafter the plots moves to the bottom of the diagram with an angle of $-90^\circ$ (trajectory $\vartheta_2$). Similar trajectories are produced for each resonant frequency, thus: $\vartheta_{3-7}$, $\vartheta_{9-11}$, $\vartheta_{0}$, $\vartheta_{1-5}$, $\vartheta_{11}$, etc. Although the gain of each compensator of $G_{c,1,3,5,7,9,11}(s)$ contributes to driving the Nyquist plot towards the critical point $(−1,0j)$, the gains $K_5$ and $K_7$ have to be well limited as their associated peaks are closer to $\omega_c$. Table I summarizes the angles injected by the plant and the computational delay in order to compensate $\vartheta_n$.
Fig. 4. (a) Bode and (b) Nyquist diagram for the open-loop system $H_2(s) = G_1(s)P_d(s)$. $P_d(s) = P(s) \cdot e^{-sT_s}$, $T_s = 1/16800$ s, $P(s)$ with $R_f = 0.5$ M, $L_f = 219$ $\mu$H and $C_f = 20$ $\mu$F possesses a resonance at $\omega_c = 2\pi \cdot 2400$ rad/s. $G_1(s) = K_1 \cos(\omega_o \sin(\omega_o s))$, $\omega_1 = 2\pi \cdot 400$ rad/s, $\theta_1 = -10^\circ$, $K_1 = 1255$. In (b) Nyquist response for positive and negative frequencies (dotted lines) is shown. The tangent M circle for $M=1.012$ is centred at (-42.41,0j) with a radius of 41.91 and associated to a damping factor of $\zeta = 0.65$.

TABLE I

| $R_1(s)$ | $R_2(s)$ | $\omega_c$ rad/s | $-\angle P(j\omega_c)$ | $-\angle e^{-j\mu_n\pi}$ | $\sigma_n$ |
|----------|----------|-------------------|------------------------|---------------------------|--------|
| $R_1(s)\{\sigma_1, \sigma_2\}$ | $2\pi \cdot 400$ | 1.35$^\circ$ | 8.57$^\circ$ | 10.05$^\circ$ |   |
| $R_1(s)\{\sigma_3, \sigma_4\}$ | $2\pi \cdot 1200$ | 5.73$^\circ$ | 25.71$^\circ$ | 31.44$^\circ$ |   |
| $R_1(s)\{\sigma_5, \sigma_6\}$ | $2\pi \cdot 2000$ | 22.17$^\circ$ | 42.85$^\circ$ | 65.03$^\circ$ |   |
| $R_1(s)\{\sigma_7, \sigma_8\}$ | $2\pi \cdot 2800$ | 153.68$^\circ$ | 60$^\circ$ | 213.68$^\circ$ |   |
| $R_1(s)\{\sigma_9, \sigma_{10}\}$ | $2\pi \cdot 3600$ | 169.67$^\circ$ | 77.14$^\circ$ | 246.81$^\circ$ |   |
| $R_1(s)\{\sigma_{11}, \sigma_{12}\}$ | $2\pi \cdot 4400$ | 173.28$^\circ$ | 94.28$^\circ$ | 267.56$^\circ$ |   |

in each compensator. Additionally, the path $\sigma_n$ of Fig. 5 related to each resonance frequency is also depicted.

III. THREE-DIMENSIONAL SPACE VECTOR MODULATION

To generate the voltage demanded by the multiple resonant controllers, the three-dimensional SVM presented in [24], [25] has been used. Fig. 6 shows the three-dimensional modulation space, where the reference vector can be modulated. This space is formed by 65 different vectors, where 14 vectors possess one redundancy and the zero vector possesses two redundancies, making up the $3^4 = 81$ switching states of the converter. For simplicity, only 14 vectors, that shape the modulation region, are represented in Fig. 6. The reference vector $v^*_{\alpha\beta\gamma}$ is also depicted inside the modulation space. To obtain the modulation space of the converter, each of the 81 switching states are transformed using the Clarke transform (see Appendix) as presented in (9)-(11). The voltages used in (10) are referred to Fig. 1.

$$v_{\alpha\beta\gamma} = T_{abc}v_{abc}$$  \hspace{1cm} (9)

$$v_{abc} = \begin{bmatrix} v_{a}^f \\ v_{b}^f \\ v_{c}^f \end{bmatrix}$$  \hspace{1cm} (10)

$$v_{\alpha\beta\gamma} = T_{abc}v_{abc}$$  \hspace{1cm} (11)

The reference vector $v^*_{\alpha\beta\gamma}$, presented in Fig. 6, can be placed anywhere inside the modulation region and is generated by selecting the four nearest vectors at each sampling time. Thus, the first step to achieve modulation is to identify the four vectors that form the tetrahedron that enclose the reference vector. This can be achieved by the following simple steps:

$$v_{0\alpha\gamma} = T_{abc}v_{abc}$$  \hspace{1cm} (12)

$$v_{0\alpha\gamma} = v_{0\alpha\gamma} + [0, 0, 1]$$  \hspace{1cm} (13)

$$\hat{v}_{\alpha\beta\gamma} = v^*_{\alpha\beta\gamma} - v_{0\alpha\gamma} = \begin{bmatrix} \hat{v}_\alpha \\ \hat{v}_\beta \\ \hat{v}_\gamma \end{bmatrix}$$  \hspace{1cm} (14)

$$\phi = \tan^{-1}\left( \frac{\hat{v}_\beta}{\hat{v}_\alpha} \right)$$  \hspace{1cm} (15)

In (12) and (13) the first two vectors of the converter are selected, then, and similarly to the SVM algorithm for a two-level converter, based on the angle calculated on (15), the final two active vectors are selected, which are separated by $60^\circ$ and
have a constant amplitude. Thereafter, the duty cycles for each vector can be calculated as:

$$
\begin{pmatrix}
    d_2 \\
    d_3 \\
    d_4 \\
\end{pmatrix} = D_n
\begin{bmatrix}
    \hat{\beta} \\
    \hat{\beta} \\
    \hat{\gamma} \\
\end{bmatrix}
$$

(16)

where $d_1$ to $d_4$ represent the duty cycle of each of the four vectors that modulate the reference vector and the matrix $D_n$ takes six different values depending on the sector according to (15) (see Appendix).

A. Switching Sequence and Power Losses

After the stationary vectors and their dwell times have been obtained, a switching pattern that arrange these vectors during the sampling time $T_s$ has to be defined. The selection of this pattern, also known as the switching sequence, is always a trade-off between the number of commutations, i.e. switching power losses, and the accuracy of tracking a reference signal, i.e. current or voltage ripple. Therefore, the selection between different patterns mostly depends on the application. In this paper a single redundancy switching pattern is defined and implemented. This pattern uses only one of all the available redundant vectors at each sampling time. This reduces the switching frequency of the devices and also distributes the power losses uniformly among the devices. Fig. 7 shows an example of the commutation sequence for the modulation of a reference vector inside a tetrahedron formed by the vectors: $v_4$, $v_{15}$, $v_1$, $v_{10}$. Although vectors $v_1$, $v_4$ and $v_{10}$ posses one redundancy, only the redundancy of the vector $v_4$ has been used.

As shown in Fig. 7, regardless of the path described by the reference vector in Fig. 6, always one commutation takes place at every sampling time. Thereby, considering that $ac$ signals are being modulated, the switching frequency per device over a fundamental cycle in a four-leg NPC converter, is given by:

$$
d_1 = 1 - d_2 - d_3 - d_4
$$

(17)

where $f_{dev}$ represents the sampling frequency and $f_{dev}$ is the average switching frequency of each of the four switching devices of each leg of the converter. Notice that the equivalent switching frequency of the output voltage is still $f_s$. This is an important advantage compared with a two-level inverter, where the switching frequency of each device is equal to the equivalent frequency of the modulated waveform, i.e. $f_s$.

IV. EXPERIMENTAL VALIDATION

The experimental rig used to validate the proposed controller in a four-leg NPC is illustrated in Fig. 8 (load is not shown). The control system hardware is composed of a Pentium-System board (3.2GHz Pentium processor, 2Gb RAM host PC, based on the Arch-Linux operating system) and an FPGA board connected via an ISA-bus. This platform runs the algorithm using the Real Time Application Interface (RTAI) for Linux. The FPGA board manages the ADCs, implements the over-voltage and over-current protections, performs the dead time and generates the IGBT switching signals. Optical fibers are used to transmit the IGBT gate signals from the FPGA output buffer to the NPC converter.

A four-leg NPC converter has been designed based on the Microsemi IGBT-APTGL60TL120T3G rated at 60A and 1200V. The nominal power of the experimental prototype is 6kVA. The experimental results presented in this Section utilize step impacts and steady state connection of linear/non-linear unbalanced loads in order to verify the performance of the prototype and its capacity to fulfill the standards of [2].

A. Experimental Results

In order to validate the proposed topology and control design, the aircraft of Fig. 1 is replaced by linear/non-linear,
balanced/unbalanced loads in the experimental rig. The steady-state and transient performance is investigated in the following sections. Table II gives the common parameters used for the experimental results.

### TABLE II
**General Parameters of the Implemented Systems**

| Parameter       | Value     | Parameter       | Value     |
|-----------------|-----------|-----------------|-----------|
| $C_{f1}$        | 3300 uF   | $Z_{a2}$        | 10 Ω ; 0.8 mH |
| $C_{f2}$        | 3300 uF   | $Z_{b2}$        | 10 Ω ; 0.8 mH |
| $V_{dc}$        | 325 V     | $Z_{c2}$        | 14 Ω ; 0.8 mH |
| $C_{s1}$        | 20 μF     | $Z_{a}$         | 18 Ω ; 0.8 mH |
| $L_{f}$         | 219 mH    | Single/Three-phase Full Wave Diode Bridge Rectifier | 220 uF ; 57 Ω |
| $f_{sw}$        | 16.8 kHz  |                 |           |

1) *Steady-State Performance:* Fig. 9 shows the performance of the proposed system of Fig. 1 under unbalanced operation, using $Z_{a2}$, $Z_{b2}$ and $Z_{c2}$. The power consumption of each branch is: 1.21kW, 0.86kW and 0.67kW respectively. As depicted in Fig. 1, the controller compensates the different voltage drops on each phase of the filter and maintains the output voltages at 110V, 400Hz. Note that the neutral current has a reduced harmonic distortion due to cancellation of some of the harmonic components in the line currents. The THDs of each output voltage are 0.87%, 0.92%, 1.10% respectively, which easily meets the maximum of 5% required by the standard [2].

In order to evaluate the performance of the proposed topology and control strategy when feeding non-linear loads, a three-phase and a single-phase full wave diode bridge rectifier, both with a $RC$ load at the dc side, are connected at the output of the GPU (see Table II). Fig. 11 shows the reference voltage generated by the controller, which is then synthesized by the 3D-SVM [24], [25]. Clearly, the reference voltage is far from an ideal sinusoidal shape. However, this is the required waveform to compensate the harmonic voltage drop produced by the distorted current circulating through the filter inductance $L_f$ to subsequently obtain a sinusoidal voltage at the load. To highlight the effectiveness of the high-order harmonics compensation of the proposed method, output voltages and currents of the proposed GPU, considering only compensation of 400Hz fundamental waveform, are showed in Fig.12. As result, highly distorted output voltages are obtained, with a THDv of 9.8% on each phase approximately.

Finally, Fig. 13 shows the performance of the proposed GPU for a test using a 0.42kW single-phase full wave diode bridge rectifier and a 2.8kW linear-unbalanced load ($Z_{a2}$, $Z_{b2}$, $Z_{c2}$). As the single-phase full wave diode bridge rectifier introduces a distorted zero sequence component, the neutral current is not sinusoidal and contains the harmonics generated by the rectifier. This current also contains some even harmonics which are not compensated by the controller. Hence, although the THD is within the required limit for each phase (3.07%, 1.2% and 1.2% respectively), the phase where the single-phase rectifier is connected has the highest harmonic distortion.

2) *Transient Performance:* Fig. 14 illustrates a transient from unbalanced operation to a full balanced load, while Fig. 15 shows a load impact from nominal balanced load to open circuit (The parameters of the load step are given in Table II). In both cases the controller achieves a good dynamic response, compensating the transient after approximately 1-2 cycles (2.5ms-5ms), easily meeting the transient requirements.
unbalanced load (Fig. 13. Performance of the proposed GPU under a 2.8kW linear-wave diode bridge rectifier (57Ω and 220 uF). Sampling frequency of $f_s=16.8$kHz. (a) line current $I_A$, (b) reference voltage for the converter $V_{af-ref}$ and (c) GPU output voltage $V_{af}$.

Fig. 11. Performance of the proposed GPU under a 2.8kW linear-unbalanced load ($Z_{a2}$, $Z_{b2}$, $Z_{c2}$) and a 1.27kW three-phase full wave diode bridge rectifier (57Ω and 220 uF). Compensation of only fundamental 400Hz component ($f_s=16.8$kHz). (a) Output voltages $V_{af}$, $V_{bf}$ and $V_{cf}$ and (b) output currents $I_A$, $I_B$, $I_C$. $THD_V \approx 9.8\%$.

Fig. 12. Performance of the proposed GPU under a 2.5kW linear-balanced load ([2]) and a 1.27kW three-phase full wave diode bridge rectifier (57Ω and 220 uF). Compensation of only fundamental 400Hz component ($f_s=16.8$kHz). (a) Output voltages $V_{af}$, $V_{bf}$ and $V_{cf}$ and (b) output currents $I_A$, $I_B$, $I_C$.

Fig. 14. Transient response of the proposed GPU for a load step: from a 2.8kW unbalanced load to a 3.6kW balanced load. (a) output voltages $V_{af}$, $V_{bf}$, $V_{cf}$, (b) line currents $I_A$, $I_B$, $I_C$ and (c) the neutral wire current $I_f$.

Fig. 15. Transient response of the proposed GPU for a load step: from 3.6kW balanced load to open circuit. (a) output voltages $V_{af}$, $V_{bf}$, $V_{cf}$ and (b) the line currents $I_A$, $I_B$, $I_C$.

in [2]. Fig 16 shows the regulation of the voltages on the dc-link capacitors after a load impact: from no-load condition to the 2.8kW linear-unbalanced load. The voltage deviation is lower than 5V, converging to the reference after around 60 ms. This validates the effectiveness of the voltage balance algorithm used on the 3D-SVM and the appropriate design of the PI controller utilized for balancing the voltage on each capacitor [24], [25].

Fig. 17 presents the transient response for a 1.27kW load-impact connected at the output of a three-phase full wave diode bridge rectifier. Even considering that this is a large load impact, the controller can reject this disturbance after approximately 10 ms (3-4 cycles). As the capacitor is not charged before the load impact, it behaves as a short circuit and the voltage is not controllable for around 1 cycle. Additionally, the upper and lower transient boundaries of the standard are also depicted in Fig. 17a to highlight that the transient response easily meets the required standard [2], which allows an over-voltage of 118V after 87.5 ms.

V. CONCLUSION

This paper has proposed a simple methodology to design multiple resonant controllers for high-order harmonics compensation when small ratio between sampling frequency and harmonic compensated frequencies is required. Unlike
previous methodologies, the work proposed in this paper does not assume decoupled operation of the resonant controllers, taking in consideration the full dynamic of the plant and controllers. The method is based on Nyquist diagrams and it also allows the design of multiple resonant controllers when a second-order output filter is used at the GPU output. This methodology overcome typical difficulties associated with Bode design methods for high-order controllers with multiple resonant peaks. In addition, it overcome the problem of designing resonant controllers when the resonance frequency is close to the filter resonance frequency.

In order to achieve high-order harmonics compensation at 400Hz fundamental waveform, a 4-leg NPC converter is proposed as solution. Thus, a double equivalent output frequency compared to the switching frequency per device is obtained. This allows the use of smaller power filters, lower switching losses and higher harmonics compensation. A resonant controller designed to regulate a fundamental signal of 400Hz and eliminate the 3rd, 5th, 7th, 9th and 11th harmonics from the load voltage were successfully designed and experimentally validated.

Experimental results have been conducted in a 6kW prototype to validate the proposed methodology in a GPU application. The converter has been tested under unbalanced and non-linear loads performing excellent results maintaining sinusoidal output voltages under transient and steady state operation. Furthermore, it is also demonstrated that dc-link voltage balancing can be also achieved even in the presence of severely unloaded non-linear load.
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