Muon g–2 and Neutrino Mass in a New Minimal Extension of the MSSM

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Abstract

If the one term $(\tilde{\nu}_e\tilde{\mu} - \tilde{e}\tilde{\nu}_\mu)\tilde{\tau}^c$ is added to the MSSM (Minimal Supersymmetric Standard Model) superpotential, the recently observed muon g–2 (anomalous magnetic moment) excess can be explained very simply by a light $\tilde{\nu}_e$. If the soft symmetry-breaking terms $\tilde{\nu}_\alpha\tilde{h}_2^0 - \tilde{l}_\alpha\tilde{h}_2^+$ are also added, realistic neutrino masses (with bimaximal mixing) are generated as well.

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1 Lepton numbers in supersymmetry

The particle content of the Standard Model with two Higgs doublets as required in supersymmetry is given by

\[ L = (\nu, l)_L \sim (1, 2, -1/2), \quad \bar{E} = l^c_L \sim (1, 1, 1), \]

\[ Q = (u, d)_L \sim (3, 2, 1/6), \quad \bar{U} = u^c_L \sim (3^*, 1, -2/3), \quad \bar{D} = d^c_L \sim (3^*, 1, 1/3), \]

\[ \Phi_1 = (\phi^0_1, \phi^-_1)_L \sim (1, 2, -1/2), \quad \Phi_2 = (\phi^+_2, \phi^0_2)_L \sim (1, 2, 1/2), \]

under \(SU(3) \times SU(2)_L \times U(1)_Y\). In the Standard Model without supersymmetry, the global quantum numbers \(L_e, L_\mu, L_\tau\) and \(B\) are separately conserved automatically. In the Minimal Supersymmetric Standard Model (MSSM), they are conserved by assumption, i.e. by the removal of the allowed terms \(\lambda LL\bar{E}, \lambda' LQ\bar{D}, \lambda'' \bar{U}\bar{D}\bar{D}\), and \(\mu'L\Phi_2\). In that case, the well-known \(R\) parity, i.e. \(R \equiv (-1)^{3B+L+2J}\), is conserved.

On the other hand, these terms need not all be forbidden. In particular, the really important restriction is only for the product \(\lambda'\lambda''\) to be very small or zero to prevent rapid proton decay. Hence a class of \(R\)-parity violating models has been widely discussed in the literature which assumes \(\lambda'' = 0\), but allows nonzero values of \(\lambda, \lambda',\) and \(\mu'\). This means that \(B\) is conserved, but \(L\) is not, so that there can be neutrino mass and lepton-flavor violation, etc.

Actually, there are 17 well-defined models of lepton numbers in supersymmetry, as pointed out already many years ago [1]. Consider the following sets of terms in the superpotential:

\[ W^{(1)} = h_1 \Phi_1 L_i E_i + h_{ij} \Phi_1 Q_i D_j + h''_{ij} \Phi_2 Q_i \bar{U}_j + \mu_0 \Phi_1 \Phi_2, \]

\[ W^{(2)} = f_\epsilon L_3 L_1 \bar{E}_1 + f_\mu L_3 L_2 \bar{E}_2 + \mu_3 L_3 \Phi_2 + f_{ij} L_3 Q_i \bar{D}_j, \]

\[ W^{(3)} = f_{e\mu\tau} L_1 L_2 \bar{E}_3, \]
\[ W^{(4)} = f_{e\mu} L_3 L_1 \bar{E}_2 + f_{\mu e} L_3 L_2 \bar{E}_1, \]  
\[ W^{(5)} = f'_e L_2 L_1 \bar{E}_1 + f'_\mu L_2 L_3 \bar{E}_3 + \mu_2 L_2 \Phi_2 + f'_{ij} L_2 Q_i \bar{D}_j. \]  

Five models can then be defined with lepton numbers for \((e, \mu, \tau)\) as shown below.

\[ \text{Model 1 : } W = W^{(1)} + W^{(2)}, \; \left[ (1, 0), \; (0, 1), \; (0, 0) \right] \]  
\[ \text{Model 2 : } W = W^{(1)} + W^{(3)}, \; \left[ (1, 0), \; (0, 1), \; (1, 1) \right] \]  
\[ \text{Model 3 : } W = W^{(1)} + W^{(2)} + W^{(3)}, \; \left[ 1, \; -1, \; 0 \right] \]  
\[ \text{Model 4 : } W = W^{(1)} + W^{(2)} + W^{(4)}, \; \left[ 1, \; 1, \; 0 \right] \]  
\[ \text{Model 5 : } W = W^{(1)} + W^{(2)} + W^{(5)}, \; \left[ 1, \; 0, \; 0 \right]. \]

Models 1 and 2 have two conserved lepton numbers. Models 3, 4, and 5 have one conserved lepton number. Each model has also 3 permutations, hence there are \(1 + 5 \times 3 + 1 = 17\) models, ranging from the MSSM with conserved \(R\) parity to the most general \(R\) parity violating model which conserves \(B\).

## 2 Muon anomalous magnetic moment

In the presence of supersymmetric particles, there are certainly additional contributions \cite{2} to the muon anomalous magnetic moment \cite{3}. To obtain an excess \(\Delta a_\mu \sim 10^{-9}\), light \(\tilde{\nu}_\mu, \tilde{\mu}, \) and large \(\tan \beta\) are required, thereby restricting the MSSM parameter space. In this talk I will present a new minimal extension \cite{4} of the MSSM based on Model 2 of the last section, such that \(\Delta a_\mu\) is explained entirely by the single new term we add to the superpotential, i.e.

\[ \Delta \hat{W} = h(\hat{\nu}_e \hat{\mu} - \hat{e} \hat{\nu}_\mu) \bar{\tau}^c. \]  

The new interaction terms of the resulting Lagrangian are then given by

\[ \mathcal{L}_{int} = h(\nu_e \mu - e \nu_\mu) \bar{\tau}^c + h(\nu_e \tau^c \tilde{\mu} - e \tau^c \tilde{\nu}_\mu) + h(\mu \tau^c \tilde{\nu}_e - \nu_\mu \tau^c \tilde{e}) + \text{H.c.} \]
Hence there are 2 contributions to the muon anomalous magnetic moment from $\tilde{\nu}_e$ and $\tilde{\tau}^c$ exchange. They are easily evaluated \[5\] and we obtain

$$\Delta a_{\mu} = \frac{h^2 m_e^2}{96\pi^2} \left( \frac{2}{m_{\tilde{\nu}_e}^2} - \frac{1}{m_{\tilde{\tau}^c}^2} \right). \tag{16}$$

Similarly,

$$\Delta a_e = \frac{h^2 m_e^2}{96\pi^2} \left( \frac{2}{m_{\tilde{\mu}_\mu}^2} - \frac{1}{m_{\tilde{\tau}^c}^2} \right), \tag{17}$$

$$\Delta a_\tau = \frac{h^2 m_\tau^2}{96\pi^2} \left( \frac{2}{m_{\tilde{\nu}_e}^2} + \frac{2}{m_{\tilde{\nu}_\mu}^2} - \frac{1}{m_{\tilde{\tau}^c}^2} - \frac{1}{m_{\tilde{\mu}_\mu}^2} \right). \tag{18}$$

Of all the possible effective four-fermion interactions which can be derived from Eq. (15), only two are easily accessible experimentally: $\mu \to e\nu_\mu\bar{\nu}_e$ through $\tilde{\tau}^c$ exchange \[3\] and $e^+e^- \to \tau^+\tau^-$ through $\tilde{\nu}_\mu$ exchange. For simplicity, both $\tilde{\tau}^c$ and $\tilde{\nu}_\mu$ may be assumed to be heavy, say a few TeV, then the coupling $h$ is allowed to be of order unity in Eq. (15). To obtain $\Delta a_\mu \sim 10^{-9}$ in Eq. (16) to account for the possible discrepancy of the experimental value \[3\] with the standard-model expectation \[7\], we need $\tilde{\nu}_e$ to be relatively light, say around 200 GeV.

### 3 Collider signatures

Since $\tilde{\nu}_e$ is required to be light, $\tilde{e}$ must also be light, because of the well-known relationship

$$m_{\tilde{e}}^2 = m_{\tilde{\nu}_e}^2 - M_W^2 \cos 2\beta. \tag{19}$$

Now both $\tilde{\nu}_e$ and $\tilde{e}$ can be produced by electroweak interactions, such as $Z \to \tilde{\nu}_e^*\tilde{\nu}_e$ and $W^- \to \tilde{\nu}_e^*\tilde{e}$. They must then decay according to Eq. (15), i.e.

$$\tilde{\nu}_e \to \mu^+\tau^-, \quad \tilde{e} \to \tilde{\nu}_\mu\tau^- \tag{20}.$$ 

These are very distinctive signatures and if observed, the two masses may be reconstructed and the value of $\beta$ determined by Eq. (19).
If the MSSM neutralinos $\tilde{\chi}_i^0$ and charginos $\tilde{\chi}_i^+$ are produced, as decay products of squarks for example, then the decays

$$\tilde{\chi}_i^0 \rightarrow \tilde{\nu}_e \tilde{\nu}_e (\tilde{\nu}_e^* \nu_e), \quad \tilde{\nu}_e (\tilde{\nu}_e^* e^-), \quad \tilde{\chi}_i^+ \rightarrow \tilde{\nu}_e e^+, \quad \tilde{e}^* \nu_e$$

are possible. The subsequent decays of Eq. (20) would again be indicative of our model. In a future muon collider, the process

$$\mu^+ \mu^- \rightarrow \tilde{\nu}_e^* \tilde{\nu}_e$$

(through $\tau$ exchange) is predicted, by which the $\tilde{\nu}_e$ decay of Eq. (20) could be studied with precision.

Single production of $\tilde{\nu}_e$ and $\tilde{e}$ is also possible in an $e^+ e^-$ collider. There are 4 different final states: $\tau^+ \mu^- \tilde{\nu}_e$, $\tau^+ \nu_\mu \tilde{e}$, and their conjugates. With the subsequent decays given by Eq. (20), the experimental signatures are 4 charged leptons ($\tau^+ \tau^- \mu^+ \mu^-$) and 2 charged taus + missing energy ($\tau^+ \tau^- \nu_\mu \nu_\mu$). The absence of such events at LEP up to 207 GeV constrains $h$ and $m_{\tilde{\nu}_e}$. Although a quantitative analysis is not available at present, we estimate the likely mass bound (on the basis that it would be similar to that of single scalar leptoquark production) to be around 180 GeV for $h = 1$. With such a large mass, we will need a larger $h$ to get $\Delta a_\mu \sim 10^{-9}$, so we have chosen $h = 2$ and $m_{\tilde{\nu}_e} = 200$ GeV (which puts $\tilde{\nu}_e$ beyond the production capability of LEP) as representative values.

4 Neutrino masses

Our model as it stands forbids neutrino masses because it conserves $L_e$ and $L_\mu$ (with $L_\tau = L_e + L_\mu$). Consider now the soft breaking of these lepton numbers by the terms

$$\mu_\alpha (\hat{l}_i \hat{h}_2^+ - \hat{\nu}_i \hat{h}_2^0)$$

(23)
in the superpotential, i.e. the so-called bilinear $R$-parity violation [8]. In that case, the $4 \times 4$ neutralino mass matrix of the MSSM must be expanded to include the 3 neutrinos as well to form a $7 \times 7$ mass matrix. It is well-known that one tree-level mass, corresponding to a linear combination of $\nu_e$, $\nu_\mu$, and $\nu_\tau$ is now obtained. In this scenario, the scalar neutrinos also acquire nonzero vacuum expectation values [9] and one-loop radiative neutrino masses are possible [10]. To fit the present data on atmospheric [11] and solar [12] neutrino oscillations, restrictions on the parameters of the MSSM are implied.

In our model there is another, unrestricted source of radiative neutrino mass, which gives a contribution only to the off-diagonal $\nu_e\nu_\mu$ term through $\tilde{\tau}^c - h^c_1$ mixing. Hence our effective $3 \times 3$ neutrino mass matrix in the basis $(\nu_e, \nu_\mu, \nu_\tau)$ is of the form

$$M_\nu = \begin{pmatrix}
a_1^2 & a_1 a_2 + b & a_1 a_3 \\
a_1 a_2 + b & a_2^2 & a_2 a_3 \\
a_1 a_3 & a_2 a_3 & a_3^2
\end{pmatrix},$$

(24)

where we have assumed that the usual one-loop contributions from bilinear $R$-parity violation [10] are actually negligible, which is the case for most of the MSSM parameter space. This matrix has 4 parameters and yields 3 eigenvalues and 3 mixing angles. Consider for example $a_3 = a_2$ and define $x \equiv 1 + (b/a_1 a_2)$, we then have

$$M_\nu = \begin{pmatrix}
a_1^2 & xa_1 a_2 & a_1 a_2 \\
xa_1 a_2 & a_2^2 & a_2^2 \\
a_1 a_2 & a_2^2 & a_2^2
\end{pmatrix},$$

(25)

Assuming that $a_1$ and $xa_1$ are much smaller than $a_2$, the eigenvalues are easily determined to be

$$m_{1,2} = \pm \frac{(1-x) a_1 a_2}{\sqrt{2}} + \frac{(1-x)(3+x)a_1^2}{8},$$

(26)

$$m_3 = 2a_2^2 + \frac{(1 + x)^2 a_1^2}{4},$$

(27)
corresponding to the eigenstates

\[
\begin{bmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{bmatrix} = \begin{bmatrix}
1/\sqrt{2} & 1/2 & -1/2 \\
1/\sqrt{2} & -1/2 & 1/2 \\
0 & 1/\sqrt{2} & 1/\sqrt{2}
\end{bmatrix} \begin{bmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{bmatrix}, \tag{28}
\]

to order \(a_1/a_2\), which is of course very near the case of bimaximal mixing. Atmospheric neutrino oscillations are thus explained by \(\nu_\mu \to \nu_\tau\) with \(\sin^2 2\theta \simeq 1\) and

\[
\Delta m_{23}^2 \simeq \Delta m_{13}^2 \simeq 4a_2^4 + \frac{1}{2}(1 + 6x + x^2)a_1^2 a_2^2, \tag{29}
\]

and solar neutrino oscillations by \(\nu_e \to (\nu_\mu - \nu_\tau)/\sqrt{2}\) with \(\sin^2 2\theta \simeq 1\) and

\[
\Delta m_{12}^2 \simeq \frac{(1 - x)^2(3 + x)}{2\sqrt{2}}a_1 a_2. \tag{30}
\]

Using \(a_2 = 0.16\) eV, \(a_1 = 0.05\) eV, and \(x = -1\), we find \(\Delta m_{\text{atm}}^2 \simeq 2.5 \times 10^{-3}\) eV, and \(\Delta m_{\text{sol}}^2 \simeq 5.7 \times 10^{-5}\) eV, in very good agreement with data.

The parameter \(b\), i.e. the radiative \(\nu_e\nu_\mu\) mass, is given by

\[
b = \frac{G_F m_\mu^2}{4\pi^2 \sqrt{2}} \frac{h A m_\tau \langle \tilde{\nu}_\tau \rangle}{m_{\text{eff}}^2 \cos^2 \beta}, \tag{31}
\]

where \(m_{\text{eff}}\) is a function of \(m_{\tilde{\tau}e}\) and \(m_{h\pm}\). Using \(h = 2\) and \(m_{\text{eff}}^2/A = 1\) TeV, we find that in order to obtain \(b = -2a_1a_2 \simeq 0.016\) eV, we need \(\langle \tilde{\nu}_\tau \rangle \simeq 1.93 \cos^2 \beta\) GeV. This relatively small value is negligible compared to \(v = (2\sqrt{2}G_F)^{-1/2} = 174\) GeV (especially for large values of \(\tan \beta\)), and consistent with all present low-energy phenomenology.

5 Lepton flavor violation

Lepton-flavor violating processes are very much suppressed in our model, because they have to be proportional to the small parameters \(\mu_\alpha\) in Eq. (23) or the small vacuum expectation values \(\langle \tilde{\nu}_\alpha \rangle\). For example, the rare decay \(\tau \to e\gamma\) proceeds in one-loop order through \(\tilde{\nu}_e\)
exchange and the mixing of $\mu_L$ with $\tilde{w}^-$, and through $\tilde{e}$ exchange and the mixing of $\nu_\mu$ with $\tilde{B}$ and $\tilde{w}^0$. This amplitude and the parameter $a_2$ in Eq. (24) are both proportional to

$$\left(\frac{\mu_\mu}{\mu_0} - \frac{\langle \tilde{\nu}_\mu \rangle}{v \cos \beta}\right), \tag{32}$$

where $\mu_0$ is the coefficient of the $(\hat{h}_1^+ \hat{h}_2^+ - \hat{h}_1^0 \hat{h}_2^0)$ term in the superpotential of the MSSM. Hence

$$\frac{B(\tau \to e\gamma)}{B(\tau \to e\nu\bar{\nu})} \propto \frac{a_2^2 M_W^4}{m_\tau^2 m_{\text{eff}}^3} \tag{33}$$

Let $m_{\text{eff}} = 200 \text{ GeV}$, then $B(\tau \to e\gamma) \sim 10^{-13}$, which is many orders of magnitude below the experimental upper bound of $2.7 \times 10^{-6}$.

The $\mu \to e\gamma$ rate is even more suppressed because it has to violate both $L_\mu$ and $L_\tau$, whereas $\tau \to e\gamma$ only needs to violate $L_\mu$. We note that if we had chosen the extra term in Eq. (14) to be $h(\hat{\nu}_e \hat{\tau} - \hat{e} \hat{\nu}_\tau)\hat{\mu}^c$ or $h(\hat{\nu}_\mu \hat{\tau} - \hat{\mu} \hat{\nu}_\tau)\hat{e}^c$, then $\mu \to e\gamma$ would not be doubly suppressed and would have a branching fraction of about $4 \times 10^{-10}$, in contradiction with the present experimental bound $[13]$ of $1.2 \times 10^{-11}$.

6 Conclusion

In conclusion, we have shown how a novel minimal extension of the MSSM with $L_\tau = L_e + L_\mu$ allows it to have a large contribution to the muon anomalous magnetic moment without otherwise constraining the usual MSSM parameter space. With the soft and spontaneous breaking of this lepton symmetry, realistic neutrino masses (with bimaximal mixing) are generated for a natural explanation of atmospheric and solar neutrino oscillations. The scalar electron doublet ($\tilde{\nu}_e, \tilde{e}$) is predicted to be light (perhaps around 200 GeV) and has distinctive experimental signatures.
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