Anomalies, horizons and Hawking radiation

Sunandan Gangopadhyay (a)
National Institute for Theoretical Physics (NIThep), Stellenbosch Institute for Advanced Study (STIAS)
7600 Stellenbosch, South Africa

received 11 November 2008; accepted 6 December 2008
published online 16 January 2009

PACS 04.70.Dy – Quantum aspects of black holes, evaporation, thermodynamics
PACS 03.65.Sq – Semiclassical theories and applications
PACS 04.62.+v – Quantum fields theory in curved spacetime

Abstract – Hawking radiation is obtained from the Reissner-Nordström black hole with a global monopole and the Garfinkle-Horowitz-Strominger black hole falling in the class of the most general spherically symmetric black holes (√−g ≠ 1), using only the chiral anomaly near the event horizon and the covariant boundary condition at the event horizon. The approach differs from the anomaly cancellation approach since apart from the covariant boundary condition, the chiral anomaly near the horizon is the only input to derive the Hawking flux.

Introduction. – On quantising matter fields in a background black-hole spacetime, Hawking radiation is obtained, a result that cannot be obtained classically. Ever since Hawking’s original paper [1,2], there have been several derivations [3,4] and all of them take the quantum effect of fields in black-hole backgrounds into account in various ways.

A few years ago, Robinson and Wilczek [5] advanced a new approach known as the anomaly cancellation approach to derive Hawking radiation from a Schwarzschild-type black hole, where diffeomorphism symmetry plays a significant role. The crucial observation that they make is that, near the horizon, the black-hole dynamics is effectively described by a two-dimensional chiral theory that breaks diffeomorphism symmetry. Hence, this chiral theory is anomalous. Requiring that the complete theory with contributions near the horizon, outside the horizon and inside the horizon be anomaly free, a condition is obtained from which the Hawking flux is identified. The method was soon extended to the case of charged black holes [6]. Further applications of this approach using the consistent form of the chiral anomaly and later with the covariant form of the chiral anomaly [7] may be found in [8–18].

However, certain problems still persist in this derivation. The universality of Hawking radiation requires that the flux gets determined only from information at the horizon. The point to be noted is that, apart from the anomalous Ward identity at the horizon, the normal Ward identity outside the horizon is also required. Furthermore, it is also necessary to interpret an additional Wess-Zumino term as a contribution from the (classically irrelevant) ingoing modes. The question is whether it is possible to derive the flux just from the information of the chiral anomaly at the horizon. The fact that this is indeed so has been shown in [19] using covariant chiral anomalies. It has been observed from the structure of the anomaly and imposing asymptotic flatness of the metric that the anomaly vanishes in the asymptotic limit (r → ∞ limit). Hence, the Hawking flux, which is measured at infinity, is given by the r → ∞ limit of the r-t component of the energy-momentum tensor near the horizon since this would correspond to the anomaly-free expression associated with the r-t component of the energy-momentum tensor outside the horizon. The Hawking flux is therefore obtained solely from the information near the horizon bypassing all the problems encountered in the anomaly cancellation approach.

In this paper, we extend this method for the most general spherically symmetric black-hole spacetime (√−g ≠ 1) which may not be asymptotically flat, e.g. the Reissner-Nordström black hole with a global monopole [20]. Further, we derive the Hawking flux using both forms of the chiral anomaly, namely the consistent and the covariant one. However, in case of the consistent anomaly, a little bit more work is required since one needs to add a local counterterm [21,22] to the consistent-current/energy-momentum tensor to obtain
the covariant-current/energy-momentum tensor which is required for imposing the covariant boundary condition at the horizon.

Examples of the Reissner-Nordström black hole with a global monopole and the Garfinkle-Horowitz-Strominger (GHS) black hole [23] are finally discussed.

New approach of deriving the Hawking radiation. – We start with the most general spherically symmetric black-hole spacetime \( \sqrt{-g} \neq 1 \) given by

\[
ds^2 = f(r)dt^2 - h(r)^{-1}dr^2 + r^2d\Omega^2.
\] (1)

With the aid of the dimensional reduction procedure one can effectively describe a theory with a metric given by the “r-t” sector of the full spacetime metric (1) near the horizon [5].

Now we divide the spacetime into two regions and concentrate only in the near horizon region to discuss the gauge/gravitational anomalies separately. We shall first derive the Hawking flux using the consistent chiral anomaly and then with the covariant chiral anomaly.

Consistent gauge anomaly. – Near the horizon there are only outgoing (right-handed) fields and the current becomes anomalous. The consistent form of the \( d = 2 \) Abelian anomaly satisfies [6]

\[
\nabla_\mu J^\mu_{(H)} = -\frac{ie^2}{4\pi} \epsilon^\sigma \partial_\sigma A_\tau = \frac{e^2}{4\pi \sqrt{-g}} \partial_\tau A_\tau, \tag{2}
\]

where \( \epsilon^{\mu\nu} = \epsilon^{\mu\nu}/\sqrt{-g} \) and \( \epsilon_{\mu\nu} = \sqrt{-g} \epsilon_{\mu\nu} \) are two-dimensional antisymmetric tensors for the upper and lower cases with \( \epsilon^{rt} = \epsilon^{tr} = 1 \).

It is easy to see that the above anomaly, eq. (2), leads to the following differential equation:

\[
\partial_\tau (\sqrt{-g} J^\tau_{(H)}) = \frac{e^2}{4\pi} \partial_\tau A_\tau. \tag{3}
\]

Solving (3) in the region near the horizon, we get

\[
J^\tau_{(H)}(r) = \frac{1}{\sqrt{-g}} a_H + \frac{e^2}{4\pi} \int_{r_H}^r \partial_\tau A_\tau(r) \left( \frac{h(r)}{\sqrt{-g}} \right), \tag{4}
\]

where, \( a_H \) is an integration constant. This constant \( a_H \) gets fixed by requiring that the covariant current \( J^\tau_{(H)}(r) \) vanishes at the horizon. To impose this boundary condition, we recall that the covariant and consistent currents are related by local counterterms [21]:

\[
J^\mu_{(H)} = J^\mu_{(H)} + \frac{e^2}{4\pi \sqrt{-g}} A_\tau \epsilon^{\tau\mu}, \tag{5}
\]

which using (4) leads to

\[
\sqrt{-g} J^\tau_{(H)}(r) = a_H + \frac{e^2}{2\pi} A_t(r) - \frac{e^2}{4\pi} A_t(r_H). \tag{6}
\]

Now the condition that the covariant current \( J^\tau_{(H)}(r) \) vanishes at the horizon yields

\[
a_H = -\frac{e^2}{4\pi} A_t(r_H). \tag{7}
\]

The charge flux can now be obtained by taking the asymptotic limit of (4) and is given by

\[
(\sqrt{-g} J^\tau_{(H)})(r \to \infty) = -\frac{e^2}{2\pi} A_t(r_H). \tag{8}
\]

This is precisely the charge flux obtained in [5] using the method of cancellation of consistent gauge anomaly.

Consistent gravitational anomaly. – The consistent form of the \( d = 2 \) gravitational anomaly is given by [5,6]

\[
\nabla_\mu T^\mu_{\nu(H)} = F_{\mu\nu} J^\nu_{(H)} + A_{\nu} \nabla_\nu J^\mu_{(H)} + \mathcal{A}_\nu, \tag{9}
\]

where \( \mathcal{A}_\nu \) is the consistent form of the gravitational anomaly given by

\[
\mathcal{A}_\nu = \frac{1}{96 \pi} \epsilon^\beta \partial_\beta \Gamma^\alpha_{\nu\beta}. \tag{10}
\]

For the \( \nu = t \) component, the above equation simplifies to

\[
\nabla_\mu T^\mu_{t(H)} = F_{rt} J^t_{(H)} + A_t, \tag{11}
\]

where

\[
A_t = \frac{1}{\sqrt{-g}} \partial_t N^t; \quad A_r = 0, \quad N^t = \frac{1}{192\pi} (hf'' + f'h'). \tag{12}
\]

Equation (11) finally leads to

\[
\partial_t (\sqrt{-g} T^t_{(H)H}) = \sqrt{-g} \left( F_{rt} J^t_{(H)} + A_t \right) = \sqrt{-g} F_{rt} J^t_{(H)} + \partial_t N^t = \frac{e^2}{2\pi} (A_t(r) - A_t(r_H)) \partial_t A_t + \partial_t N^t. \tag{13}
\]

Solving the above equation, we get

\[
\sqrt{-g} T^t_{(H)H} = d_H + \frac{e^2}{4\pi} \left[ A_t^2(r) - A_t^2(r_H) \right] - \frac{e^2}{2\pi} A_t(r_H) (A_t(r) - A_t(r_H)) + N^t(r) - N^t(r_H), \tag{14}
\]

where \( d_H \) is an integration constant. This constant \( d_H \) gets fixed by requiring that the covariant energy-momentum tensor \( \tilde{T}^\tau_{(H)H}(r) \) vanishes at the horizon. To impose this boundary condition, we note that the covariant and consistent energy-momentum tensors are related by local counterterms (see appendix for a derivation):

\[
\sqrt{-g} \tilde{T}^\tau_{(H)H} = \sqrt{-g} T^\tau_{(H)H} + \frac{h}{192\pi f} (ff'' - 2f'f), \tag{15}
\]
which leads to
\[
\sqrt{-g} \hat{T}_{\nu}^{\mu}(H)_H = \partial_{\nu} \left( \frac{e^2}{2\pi} \frac{\epsilon_{\nu\rho} F_{\rho\sigma}}{\epsilon_{\nu\rho} F_{\rho\sigma} + e^2 \partial_{\nu} A_t} \right) - \frac{e^2}{2\pi} A_t \left[ A_t - A_t(r_H) \right] + \frac{1}{2} \frac{e^2}{2\pi} \partial_{\nu} \left( \frac{e^2}{2\pi} \frac{\epsilon_{\nu\rho} F_{\rho\sigma}}{\epsilon_{\nu\rho} F_{\rho\sigma} + e^2 \partial_{\nu} A_t} \right)
\]
\[+ \frac{1}{2} \frac{1}{2} \frac{e^2}{2\pi} \partial_{\nu} \left( \frac{e^2}{2\pi} \frac{\epsilon_{\nu\rho} F_{\rho\sigma}}{\epsilon_{\nu\rho} F_{\rho\sigma} + e^2 \partial_{\nu} A_t} \right) \] (19)

Now the condition that the covariant current \(
\tilde{T}^\nu_{(H)_H}(r) \)
vanishes at the horizon yields
\[
d_H = \frac{1}{96\pi} f'(r_H) h'(r_H).
\]
(17)

The energy flux can now be obtained by taking the asymptotic limit of (14) and is given by
\[
(\sqrt{-g} \hat{T}^\nu_{(H)_H})(r \rightarrow \infty) = \frac{e^2}{4\pi} A_t^2(r_H) + \frac{1}{192\pi} f'(r_H) h'(r_H).
\]
(18)

This is precisely the energy flux obtained in [5] using the method of cancellation of consistent gravitational anomaly.

**Covariant gravitational anomaly.** The covariant form of the \(d = 2\) gravitational anomaly is given by [5,6]
\[
\nabla_{\mu} \tilde{J}_{(H)_H}^\mu = \frac{e^2}{4\pi} \frac{\epsilon_{\nu\rho} F_{\rho\sigma}}{\epsilon_{\nu\rho} F_{\rho\sigma} + e^2 \partial_{\nu} A_t} \frac{e^2}{2\pi} \partial_{\nu} A_t.
\]
(19)

It is easy to see that the above anomaly, eq. (19), leads to the following differential equation:
\[
\partial_{\nu} \left( \sqrt{-g} \hat{T}^\nu_{(H)_H} \right) = \frac{e^2}{2\pi} \partial_{\nu} A_t.
\]
(20)

Solving (20) in the region near the horizon, we get
\[
\hat{T}^\nu_{(H)_H}(r) = \frac{1}{\sqrt{-g}} \left( c_H + \frac{e^2}{2\pi} \int_{r_H}^r \partial_{\nu} A_t \right)
\]
\[= \frac{1}{\sqrt{-g}} \left( c_H + \frac{e^2}{2\pi} \left[ A_t(r) - A_t(r_H) \right] \right),
\]
(21)

where \(c_H\) is an integration constant. The constant \(c_H\) vanishes by requiring that the covariant current \(\hat{T}^\nu_{(H)_H}(r)\) vanishes at the horizon. The charge flux can now be obtained by taking the asymptotic limit of the above equation and is given by
\[
(\sqrt{-g} \hat{T}^\nu_{(H)_H})(r \rightarrow \infty) = -\frac{e^2}{2\pi} A_t(r_H).
\]
(22)

The above result agrees with (8) and is precisely the charge flux obtained in [14,16,18] using the method of cancellation of covariant gauge anomaly.

**Covariant gravitational anomaly.** The covariant form of the \(d = 2\) gravitational anomaly is given by [5,6]
\[
\nabla_{\mu} \tilde{J}_{(H)_H}^\mu = \frac{1}{96\pi} \epsilon_{\nu\rho} \partial^\nu R = F_{\mu\nu} \hat{J}_{(H)_H}^\nu + \hat{A}_\nu.
\]
(23)

It is easy to check that for the metric 1, the two-dimensional Ricci scalar \(R\) is given by
\[
R = \frac{h f'}{f} + \frac{f' h'}{2f^2} - \frac{f'^2 h}{2f^2}
\]
(24)

and the anomaly is purely timelike with
\[
\hat{A}_t = 0,
\]
\[
\hat{A}_t = \frac{1}{\sqrt{-g}} \partial_{\nu} \hat{N}^\nu_t,
\]
(25)

where
\[
\hat{N}^\nu_t = \frac{1}{96\pi} \left( h f' + \frac{f' h'}{2} - \frac{f'^2 h}{2f^2} \right).
\]
(26)

We now solve the anomaly equation (23) for the \(\nu = t\) component and this leads to the following differential equation for the \(r\)-component of the near horizon covariant energy-momentum tensor:
\[
\partial_{r} \left( \sqrt{-g} \hat{T}^r_{(H)_H} \right) = \sqrt{-g} F_{rt} \hat{J}^r_{(H)_H} + \hat{A}_r \hat{T}^r_{(H)_H} + \hat{A}_t \hat{N}^r_t
\]
\[= \frac{1}{96\pi} \left( h f' + \frac{f' h'}{2} - \frac{f'^2 h}{2f^2} \right).
\]
(27)

where we have used (21) in the second line and set \(c_H = 0\) in the last line of the above equation. Integration of the above equation leads to
\[
\hat{T}^r_{(H)_H}(r) = \frac{1}{\sqrt{-g}} \left( b_H + \int_{r_H}^r \partial_{r} \left( \frac{e^2}{2\pi} \frac{\epsilon_{\nu\rho} F_{\rho\sigma}}{\epsilon_{\nu\rho} F_{\rho\sigma} + e^2 \partial_{\nu} A_t} \right) \right)
\]
\[= \frac{1}{\sqrt{-g}} \left( b_H + \frac{e^2}{2\pi} \left[ A_t^2 + A_t^2(r_H) \right] \right) - \frac{e^2}{2\pi} A_t \left( A_t(r_H) + \frac{1}{2} \hat{N}^r_t(r_H) \right),
\]
(28)

where \(b_H\) is an integration constant. The integration constant \(b_H\) can be fixed by imposing that the covariant energy-momentum tensor vanishes at the horizon. From (28), this gives \(b_H = 0\). Hence the total flux of the energy-momentum tensor is given by
\[
(\sqrt{-g} \hat{T}^r_{(H)_H})(r \rightarrow \infty) = \frac{e^2}{4\pi} A_t^2(r_H) - \hat{N}^r_t(r_H)
\]
\[= \frac{e^2}{4\pi} A_t^2(r_H) + \frac{1}{192\pi} f'(r_H) h'(r_H).
\]
(29)
The above result agrees with (18) and is precisely the Hawking flux obtained in [14,16,18] using the method of cancellation of covariant gravitational anomaly.

Examples:

**Hawking radiation from Reissner-Nordström black hole with a global monopole**

The metric of a general non-extremal Reissner-Nordström black hole with a global monopole $O(3)$ is given by [20]

$$ds^2_{\text{string}} = p(r)dt^2 - \frac{1}{h(r)}dr^2 - r^2d\Omega^2,$$  \hspace{1cm} (30)

where

$$A = \frac{q}{r}dt, \quad p(r) = h(r) = 1 - \eta^2 - \frac{2m}{r} + \frac{q^2}{r^2},$$  \hspace{1cm} (31)

with $m$ being the mass parameter of the black hole and $\eta$ is related to the symmetry breaking scale when the global monopole is formed during the early universe soon after the Big-Bang [24]. The event horizon for the above black hole is situated at

$$r_H = (1 - \eta^2)^{-1}[m + \sqrt{m^2 - (1 - \eta^2)q^2}].$$  \hspace{1cm} (32)

Now it has been argued in [12] that the metric (30) has to be rewritten in the form (1) with

$$f(r) = (1 - \eta^2)h(r),$$

$$h(r) = 1 - \eta^2 - \frac{2m}{r} - \frac{q^2}{r^2},$$  \hspace{1cm} (33)

in order to get the correct Hawking temperature for the metric (30) by the anomaly cancellation approach. We shall take this form of the metric (33) to derive the Hawking flux. Note that the determinant of the above metric $\sqrt{-g}(\neq 1$).

Using either of eqs. (8), (22), the charge flux is given by

$$\left(-\sqrt{-g}T^r_{(H)}\right)(r \to \infty) = \left(-\sqrt{-g}\tilde{T}^r_{(H)}\right)(r \to \infty) = \frac{e^2q}{2\pi r_H},$$  \hspace{1cm} (34)

which agrees with [18].

The energy flux can be obtained by using either of eqs. (18), (29):

$$\left(-\sqrt{-g}g^T_{(H)}\right)(r \to \infty) = \left(-\sqrt{-g}\tilde{T}^T_{(H)}\right)(r \to \infty) = \frac{e^2q^2}{4\pi r_H^2} + \frac{1}{192\pi} f'^2(r_H) = \frac{e^2q^2}{4\pi r_H^2} + \frac{1}{192\pi} \frac{1}{(1 - \eta^2)^2}. $$  \hspace{1cm} (35)

This is precisely the Hawking flux obtained in [18] using the anomaly cancellation as well as the effective action approach.

**Garfinkle-Horowitz-Strominger black hole**

The metric of the GHS black hole [23] is of the form (1) where

$$f(r) = \left(1 - \frac{2Me^{\phi_0}}{r}\right)\left(1 - \frac{Q^2e^{3\phi_0}}{Mr}\right)^{-1},$$

$$h(r) = \left(1 - \frac{2Me^{\phi_0}}{r}\right)\left(1 - \frac{Q^2e^{3\phi_0}}{Mr}\right).$$  \hspace{1cm} (36)

with $\phi_0$ being the asymptotic constant value of the dilaton field. We consider the case when $Q^2 < 2e^{-2\phi_0}M^2$ for which the above metric describes a black hole with an event horizon situated at

$$r_H = 2Me^{\phi_0}. $$  \hspace{1cm} (37)

Here, we need to consider only the chiral gravitational anomaly since the charge sector is absent. The energy flux can be obtained by using either of eqs. (18), (29):

$$(\sqrt{-g}\tilde{T}^T_{(H)}(r \to \infty) = (\sqrt{-g}T^T_{(H)})(r \to \infty) = \frac{1}{192\pi} f'J_H' + \frac{\pi}{12(8\pi Me^{\phi_0})^2}. $$  \hspace{1cm} (38)

This is precisely the Hawking flux obtained in [16,17,25] using the anomaly cancellation and effective action approach.

**Discussions.** – In this paper, we studied the problem of Hawking radiation from the Reissner-Nordström black hole with a global monopole and GHS black hole using both forms (consistent and covariant) of the near horizon chiral anomaly. An important advantage of this procedure in contrast to the anomaly cancellation technique is that the chiral anomaly near the horizon is the only ingredient (apart from the imposition of the covariant boundary condition near the horizon) to compute the Hawking flux.

**APPENDIX**

In this appendix, we shall derive the form of the local counterterm connecting the $r$-$t$ component of the near horizon covariant energy-momentum tensor $\tilde{T}^r_{(H)}$, with the $r$-$t$ component of the near horizon consistent energy-momentum tensor $T^r_{(H)}$ (15).

To do so, we note that the covariant and the consistent energy-momentum tensors are related by local counterterms [22]:

$$\tilde{T}^r_{(H)\mu\nu} = T^r_{(H)\mu\nu} + P_{\mu\nu}, $$  \hspace{1cm} (A.1)

where

$$\nabla^\mu P_{\mu\nu} = -\frac{1}{96\pi} (\epsilon_{\mu\nu\rho\sigma} R - \epsilon_{\mu\nu} \partial_\rho \partial_\sigma \Gamma^\sigma_{\rho\sigma}). $$  \hspace{1cm} (A.2)

Now for the static black-hole background (1), the above eq. (A.2) simplifies to

$$\partial_r (\sqrt{-g}P_{rt}) = \frac{1}{96\pi} \partial_r \left(hf'' - \frac{h}{f} f'^2 \right). $$  \hspace{1cm} (A.3)

Solving the above equation, we have

$$\sqrt{-g}P_{rt} = \frac{1}{96\pi} \left(hf'' - \frac{h}{f} f'^2 \right) + C, $$  \hspace{1cm} (A.4)

where $C$ is an integration constant. To determine this constant, we take the asymptotic limit ($r \to \infty$) limit of
the above equation. In this limit, $P^r_t$ vanishes since the $r$-$t$ components of the near horizon covariant and consistent energy-momentum tensors $\tilde{T}^r_{(H)t}$ and $\tilde{T}^r_{(O)t}$ coincides with the usual anomaly-free $r$-$t$ component of the energy-momentum tensor outside the horizon ($T^r_{(O)t}$) in the $r \to \infty$ limit. Hence, the integration constant $C$ vanishes since the first term on the right-hand side of (A.4) vanishes in the $r \to \infty$ limit. This leads to

$$P^r_t = \frac{h}{192\pi f \sqrt{-g}} \left( f f'' - 2 f'^2 \right), \quad (A.5)$$

which yields the required result (15) connecting the $r$-$t$ components of the near horizon covariant and consistent energy-momentum tensors $\tilde{T}^r_{(H)t}$ and $\tilde{T}^r_{(H)t}$.

REFERENCES

[1] Hawking S., Commun. Math. Phys., 43 (1975) 199.
[2] Hawking S., Nature (London), 248 (1974) 30.
[3] Christensen S. and Fulling S., Phys. Rev. D, 15 (1977) 2088.
[4] Parikh M. and Wilczek F., Phys. Rev. Lett., 85 (2000) 5042.
[5] Robinson S. P. and Wilczek F., Phys. Rev. Lett., 95 (2005) 011303 [gr-qc/0502074].
[6] Iso S., Umetsu H. and Wilczek F., Phys. Rev. Lett., 96 (2006) 151302 [hep-th/0602146].
[7] Bertlmann R., Anomalies in Quantum Field Theory (Oxford Sciences, Oxford) 2000.
[8] Murata K. and Soda J., Phys. Rev. D, 74 (2006) 044018 [hep-th/0606069].
[9] Setare M. R., Eur. Phys. J. C, 49 (2007) 865 [hep-th/0608080].
[10] Iso S., Morita T. and Umetsu H., JHEP, 0704 (2007) 068 [hep-th/0612286].
[11] Jiang Q. Q. and Wu S. Q., Phys. Lett. B, 647 (2007) 200 [hep-th/0701002].
[12] Wu S. Q. and Peng J. J., Class. Quantum Grav., 24 (2007) 5123.
[13] Iso S., Morita T. and Umetsu H., Phys. Rev. D, 76 (2007) 064015, arXiv:0705.3494 [hep-th].
[14] Banerjee R. and Kulkarni S., Phys. Rev. D., 77 (2008) 024018, arXiv:0707.2449 [hep-th].
[15] Banerjee R. and Kulkarni S., Phys. Lett. B., 659 (2008) 827, arXiv:0709.3916 [hep-th].
[16] Gangopadhyay S. and Kulkarni S., Phys. Rev. D., 77 (2008) 024038, arXiv:0710.0974 [hep-th].
[17] Gangopadhyay S., Phys. Rev. D, 78 (2008) 064027, arXiv:0712.3095 [hep-th].
[18] Gangopadhyay S., Phys. Rev. D, 78 (2008) 044026, 0803.3492 [hep-th].
[19] Banerjee R., arXiv:0807.4637 [hep-th].
[20] Barral M. and Vilenkin A., Phys. Rev. Lett., 63 (1989) 341.
[21] Bardeen W. A. and Zumino B., Nucl. Phys. B, 244 (1984) 421.
[22] Bertlmann R. and Kohlprath E., Ann. Phys. (N.Y.), 288 (2001) 137.
[23] Garfinkle D., Horowitz G. T. and Strominger A., Phys. Rev. D., 43 (1991) 3140; 45 (1992) 3888 (E).
[24] Preskill J. P., Phys. Rev. Lett., 43 (1979) 1365.
[25] Vagenas E. C. and Das S., JHEP, 0610 (2006) 025 [hep-th/0606077].