EMPIRICAL ANALYSIS AND OPTIMIZATION
OF CAPITAL STRUCTURE ADJUSTMENT

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ABSTRACT. This paper analyzes capital structure’s characteristics and presents its simplified mathematical model. Panel data analysis shows that the listed companies prefer equity financing rather than debt financing. Furthermore, we propose a capital structure optimization model with uncertain equity financing constraints. We formulate the capital structure optimization problem as a two-stage stochastic optimization problem and solve it. Finally, numerical examples show that our optimization approach can improve the statistics result of capital structure adjustment.

1. Introduction. Capital structure not only affects the capital cost and the corporate value, but also affects the corporate governance structure. The capital structure of different countries, different industries and even different enterprises in the same industry is different. Here, the capital structure refers to the ratio between all liabilities and owner’s equity, which reflects the relationship between total liabilities and total assets, total liabilities and total equity, different liabilities and different rights and interests [12]. Institutional investors are specialized, organized and socialized team investors aiming to reduce information asymmetry, increase investment effectiveness, and reduce investment risk. Generalized institutional investors include all institutions involved in the investment business, namely, the corporate investors of financial institutions and non-financial institutions. In a narrow sense, institutional investors specifically refer to financial institutions providing financial investment for clients, including monetary funds organizations, insurance companies, securities companies, commercial banks, and trust and investment banks. According to the importance degree of different institutional investors in the market, they can be

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divided into six categories based on the narrow definition: securities investment funds, insurance companies, securities companies, social security funds, qualified foreign institutional investors (QFII) and private placement. The shareholding ratio of institutional investors is the sum of holding shares of the six institutional investors. As of the end of 2016, the Chinese market capitalization of the shareholding of securities investment funds, insurance companies, securities companies, social security funds, QFII and private equity institutions was 16.85% of the market capitalization of the same period, and the market weight will rise in the future.

Static trade-off theory supports the existence of the target capital structure, which is the premise of studying the optimization problem of the capital structure. The Modigliani–Miller theorem presents that the capital structure has nothing to do with the corporate value, but the capital structure will affect the corporate value due to the existence of corporate tax, financial distress cost and agency cost. Capital-structure optimization theory establishes the model that describes the corporate characteristics and dynamic adjustment of capital structure. The study shows that there are many factors that affect the enterprises’ adjustment to the target capital structure. Only when the adjustment income is higher than the adjustment cost, will the capital structure be dynamically optimized, and scholars study the capital structure from a dynamic perspective [5, 8]. With the development of econometrics, researchers used the panel model and the adjustment model to study the capital structure from different perspectives, and put forward the partial adjustment model of capital structure. Later, it became the classic model adopted most in subsequent research [6].

The optimized contents for capital structure include commercial credit, short-term liabilities, long-term liabilities, stocks internal retention, etc. Liabilities and equity optimization are adopted most frequently. Corporate capital structure adjustment is optional. When the leverage is high, enterprises will issue shares to reduce the leverage; otherwise, the leverage will be raised by repurchasing the stocks or issuing the liabilities [11]. The adjustment speed can be divided into upward adjustment and downward adjustment: upward adjustment is realized through debt financing, and downward adjustment is realized through equity financing [21, 2]. When the economic environment is favorable, enterprises are less likely to conduct debt financing. When the economic environment is poor, they tend to carry out internal retention [9]. Different adjustment ways of capital structure will produce different adjustment speed and adjustment efficiency [13]. In the early stage of investment, large-scale investment enterprises usually use equity financing. In the later stage of investment, they usually use debt financing, measure financing constraints with SA index, and build a dynamic adjustment model of financing constraints and capital structure.

The research on the shareholding of institutional investors mainly focuses on three aspects: corporate governance, corporate value and internal control. Institutional investors not only have positive correlation with the earnings management quality of listed companies, but also significantly reduce the financing constraints that enterprises face [7]; moreover, it urges the management to improve the quality of accounting information and the level of corporate governance, and then send positive signals to the capital market to reduce the cost of capital structure adjustment through means of financing, so institutional investors can raise the level of adjustment by improving corporate governance levels.
The greater the shareholding ratio of institutional investors is, the greater the impact on corporate financing decision-making programs is \[14\]. When business conditions are favorable, the financial leverage generated by debt financing is strong and effective for the promotion of corporate value; therefore, institutional investors prefer enterprises with high asset-liability ratio. The controlling power owned by institutional investors can significantly reduce the information asymmetry that exists in enterprises, reduce the difficulties caused by financing constraints, and reduce the dependence of enterprises on internal funds, which in turn enhances debt financing capability. The higher the shareholding ratio of institutional investors is, the greater the controlling power they have in companies, and the institutional investors are inclined to higher financial leverage. The higher the number of institutional investors is, the more significant the level of long-term liabilities of enterprises is. However, although increasing institutional investors’ shareholding can improve the ability of long-term debt financing, it will reduce the amount of short-term financing. Some scholars also tested the impact of institutional investors on the capital cost, and pointed out that the capital cost of listed companies whose shares are held by institutional investors is lower \[4\].

In this paper, we first analyze capital structure adjustment and present a simplified mathematical model of capital structure. Empirical analysis of the known commercial data shows the listed companies prefer equity financing rather than debt financing. Next, under these equity financing constraints, we consider a capital structure optimization problem with incomplete information. We formulate the optimization problem to a two-stage stochastic optimization problem and solve it accordingly.

2. Capital structure adjustment.

2.1. Background. According to the dynamic trade-off theory, as the market imperfections and corporate financing are restricted by multiple factors, it is impossible for enterprises to be in the state of target capital structure all the time in business operation process. However, enterprises are constrained by adjustment cost and agency cost in the optimization towards target capital structure. The adjustment of capital structure shows a dynamic cycle of “deviation-adience-re-deviation-re-adience”. The capital structure cannot be quickly adjusted to the optimal state, and it can only have partial and phased adjustment; moreover, the adjustment speed is affected by benefit and cost of adjustment. The dynamic capital structure is a form of capital structure that changes with the characteristics of enterprises in order to optimize the capital structure.

According to the agency theory, the higher the corporate financial leverage is, the higher the financial risks are. Enterprises can effectively limit the managers’ over-expansion motivation and reduce the agency cost of free cash flow by increasing the debts; as a result, the common stock generates positive excess returns and increases the corporate value. Institutional investors are risk seekers, who prefer debt financing, making the equity agency costs and claims agency costs balanced, so as to achieve the lowest total agency costs.

Institutional investors can also play the positive role of shareholders, supervise the interest encroachment of management and controlling shareholders, and protect the interests of small and medium investors, which will help reduce the cost of debt financing enterprises. When the equity financing is overmuch, there may be internal control or capital structure imbalance, affecting the business performance
of enterprises, so institutional investors have the motivation to influence the financing methods through vote power of the general meeting of shareholders, and the greater the shareholding ratio of institutional investors, the more likely it is for them to influence the financing program put forward by the management through the controlling power, and diversify the risks and enhance the corporate value by adopting various investment portfolio. Finally, due to the international convergence of institutional investors involving in corporate governance, institutional investors can also participate in corporate governance through public opinions, lawsuits, exercising voting rights, submitting shareholders’ proposals, and communicating with management and other effective ways, so as to pass a good signal.

2.2. Mechanism analysis. The availability of financing methods is a major factor restricting the optimization of capital structure. Different financing methods lead to different financing speed. The lower the cost of external financing is, the faster the capital structure adjustment is. The financing cost minimization theory is the basic method to determine the optimal capital structure. Since the cost of each financing method and the financing structure of different companies is different, the financing cost is different, so in order to maximize the corporate value, enterprises should select the most appropriate financing structure to minimize the total financing cost.

According to the principle-agency theory, the higher the corporate financial leverage is, the higher the financial risks are. Enterprises can effectively limit the managers’ over-expansion motivation and reduce the agency cost of free cash flow by increasing the debts; as a result, the common stock generates positive excess returns and increases the corporate value.

Institutional investors are risk seekers, who prefer debt financing, making the equity agency costs and agency costs of creditor’s rights balanced, so as to achieve the lowest total agency costs. Institutional investors can also play the initiative of shareholders, recommend candidates for directors, make decisions on production and management, make suggestions on investment and financing decisions, regulate irregularities of listed companies, and promote the improvement of information disclosure quality of listed companies and the operating performance of enterprises. Therefore, institutional investors’ activism as shareholders can supervise the interests encroachment of management and controlling shareholders, protect the interests of small and medium-sized investors, establish the confidence of investors in enterprises, ease the principal-agent issues, reduce the principal-agency costs, and reduce the required rate of return of investors, thereby reducing the cost of debt financing of enterprises.

When the equity financing is overmuch, there may be imbalance of internal control or capital structure, affecting the business performance of enterprises, so institutional investors have the motivation to influence the financing methods through vote power of the general meeting of shareholders, and the greater the shareholding ratio of institutional investors, the more likely it is for them to influence the financing program put forward by the management through the controlling power, and diversify the risks and enhance the corporate value by adopting various investment portfolio. Through the research on institutional investors’ shareholding and corporate financing behavior, it was found that there is a significant positive correlation between the institutional investors’ holding and the debt level. When the debt level increases, the total financing cost decreases, and corporate value increases, indicating that the institutional investors have promoted the debt financing of listed companies.
Finally, due to the international convergence of institutional investors involving in corporate governance, institutional investors can also participate in corporate governance through public opinions, lawsuits, exercising voting rights, submitting shareholders’ proposals, and communicating with management and other effective ways, so as to pass a good signal, which helps reduce the information asymmetry and agency costs, reduce the bank’s credit risk and increase the willingness of banks to provide loans, thus reducing the cost of obtaining loans for enterprises, and reducing the cost of adjusting the capital structure by increasing the liabilities of enterprises.

3. Mathematical model of capital structure.

3.1. Target capital structure. The target capital structure of enterprises balances the debt tax shield and the cost of financial distress. When the company’s asset-liability ratio is low, the financial risk is low and the cost of financial distress is also low, in which case enterprises can increase the corporate value through the tax shield effect of debt financing; however, as the debt ratio increases, the financial risk increases and enterprises may face the huge loss caused by bankruptcy, and enterprises will not continue to be indebted, so the target capital structure makes the financing of enterprises reach the most ideal state, and it is the prerequisite to study the dynamic optimization of capital structure. Considering the macroeconomic environment, policy system orientation and the characteristics of enterprises, the target capital structure is set as a linear function, and the proximate matching target capital structure function is:

\[ \text{Lev}^*_{i,t} = \partial X_{i,t-1} \]  

where \( \partial \) is the regression coefficient vector, \( X_{i,t-1} \) are the variables that affect the capital structure, including the scale of enterprises, non-debt tax shield, profitability and growth of enterprises. Substituting corporate governance variables with the ownership concentration (Top10) and controlling shareholder type, and the industry characteristics are represented by the median industry capital structure of the industry in which enterprises are. Due to the weak efficiency of the market in processing information and the delay of management decision-making, the optimization of capital structure lags behind. Therefore, the target capital structure of \( t \) years is described with the data of \( t - 1 \) years.

3.2. Model design of capital structure. Enterprises generally use business credit, short-term liabilities, long-term liabilities, stocks, internal retention, etc., whose financing costs are different as the duration, restrictions, and interest rates are different. The availability of financing method is an important factor in restricting the adjustment of capital structure. Different financing methods lead to different financing speed. The lower the cost of external financing is, the faster the adjustment of capital structure is. In the previous literature, as for the measurement of the adjustment method, the internal financing, equity financing and debt financing are all measured by certain index separately, and then the correlation between the explanatory variables and the measured factor is analyzed. This method can only explain the relationship between the variables and equity financing or debt financing, but it cannot explain the adjustment method chosen by explanatory variables. People usually set the ratio of the debt financing net to total assets as a limit: when the value is greater than 5%, it shows that the enterprise will choose the debt
method to adjust the capital structure; when the value is less than 5% it shows that
the enterprise will use equity financing to adjust the capital structure.
Therefore, a simplified model without each year’s effect is formulated as

\[
\text{Deb} = \partial_0 + \beta_1 \text{IIshare}_{i,t} + \beta_2 \text{Dev}_{i,t} + \beta_3 \text{Roa}_{i,t} + \beta_4 \text{Grow}_{i,t} + \beta_5 \text{NDTS}_{i,t}. \tag{2}
\]

Let Debtissue be a variable to describe whether enterprises choose debt financing. Then Debtissue will be decided by Deb. When the value of the Deb is over 5%, the enterprises have chosen debt financing, and Debtissue=1, otherwise Debtissue=0. The net debt financing value is the total principal amount gained by issuing bonds and borrowing minus the debt repayments. IIshare represents the annual sum of the shareholding ratio of institutional investors. Institutional investors in this paper include the six major institutional investors such as public fund, insurance companies, securities companies, social security funds, QFII and private placement. The classification of institutional investors is based on the ratio of their shareholdings. Dev = Lev\text{*}_t - Lev_{t-1} is used to indicate the company’s liabilities level. When Dev is greater than zero the target capital structure is greater than the actual capital structure, and the liabilities are deemed insufficient; on the contrary, it is regarded as excess liabilities. By judging the liabilities level, we analyze its influence on the dynamic optimization of capital structure. Roa means profitability, expressed as earnings before interest and tax (EBIT) divided by total assets. According to the Pecking Order Theory, enterprises will choose internal financing first, and then choose debt financing. Because of the asymmetric information, enterprises will have adverse costs by issuing equity, so that the cost of equity financing is higher than the cost of debt financing. Therefore, the enterprises will finally consider equity financing; enterprises with higher profitable abilities are more likely to conduct internal financing, showing that the profitability is negatively correlated with the financial leverage. Grow represents the growth abilities of enterprises, replaced by the main business revenue growth rate. According to the trade-off theory, growth is positively related to bankruptcy costs, as enterprises with high growth have high agency costs and great shareholder flexibility, so they often prefer equity financing. The trade-off theory also holds that enterprises with high growth have under-investment issues and generally prefer equity financing. In addition, NDTS represents the non-debt tax shield, expressed as accumulated depreciation divided by total assets. According to trade-off theory, enterprises prefer to use interest to reduce taxes and fees, so when the corporate tax rate is high, the business tends to debt financing. However, excessive debt issuance will produce “debt crowding-out effect”. Non-debt tax shields, such as fixed assets depreciation and investment taxes, can also produce the same result, so non-debt tax shields can replace debt tax shields thus reducing the amount of debt financing.

4. Optimization model. Next, we consider a capital structure optimization problem to optimize the decision variables from $\beta_1$ to $\beta_5$ under uncertain equity financing constraints. All these parameters must be between -1 and 1. For the capital structure decision problem with uncertainty, people often decide the decision variable $\beta_1$ and $\beta_2$ at the first stage and then the rest decision variables $\beta_3$, $\beta_4$, and $\beta_5$. It is because the decision variable $\beta_1$ and $\beta_2$ are two most important decision variables for the equity financing performance. The objective is to optimally adjust these two group decision variables to maximize relative debt financing amount under the equity financing constraints. Letting $c = -[\text{IIshare}, \text{Dev}]^T$, \( x = [\beta_1, \beta_2]^T \),
$d = [\text{RoA, Grow, NDTS}]^T$, $y = [\beta_3, \beta_4, \beta_5]^T$, we formulate debt financing decision problem as a two-stage stochastic programming problem with fixed recourse:

$$\min_{x \in X} \left\{ c^T x + \mathbb{E}_P[Q(x, \tilde{z})] \right\}$$ (3)

where the $T$ represents the transpose operator and

$$Q(x, z) = \min_y d^T y$$

s.t. $A(z)x + Dy = b(z)$,

$y \geq 0$,

where $x$ is the first-stage decision vectorial variables subject to the feasible region $X = [-1, 1] \times [-1, 1] \subseteq \mathbb{R}^2$. In addition, $d \in \mathbb{R}^3$, $b(z) \in \mathbb{R}^l$, $A(z) \in \mathbb{R}^{l \times 2}$ are second-stage data and $D \in \mathbb{R}^{l \times 3}$ is the fixed recourse matrix. The random vector $\tilde{z}$ belongs to $\Omega \subset \mathbb{R}^m$, $A(\tilde{z})$ and $b(\tilde{z})$ are the associated uncertain data, and $\mathbb{P}$ is the probability measure of $\tilde{z}$. In our framework, $y \in \mathbb{R}^k$ denotes the second-stage decision variables with respect to a realization $z$ of $\tilde{z}$. The optimization problem (3) can be solved by a linear programming method for the case of given scenarios. Many related algorithms are referred to [3, 1, 16, 19, 10, 20, 18, 17, 15].

5. Empirical analysis. This paper took 1715 listed companies in China as samples to carry out comprehensive panel data analysis through selecting from listed companies with continuous financial data of A-shares in Shenzhen and Shanghai stock markets of China from 2013 to 2017, excluding companies in financial industry, ST companies and PT companies, and companies with listed years less than 5 years and incomplete data. The sample data are mostly from CSMAR Database and Wind Database and parts of the data are derived from sorting.

| Trade                  | Asset-Liability Ratio | IIShare |
|------------------------|-----------------------|---------|
| Mining                 | 48.19%                | 18.39%  |
| Electric Heating Water | 60.33%                | 28.55%  |
| Real Estate            | 62.77%                | 21.12%  |
| Construction           | 70.69%                | 16.66%  |
| Traffic                | 49.62%                | 21.01%  |
| Agriculture            | 41.71%                | 17.6%   |
| Wholesale and Retail   | 56.37%                | 18.1%   |
| Entertainment          | 46.19%                | 13.15%  |
| Information Technology | 37.02%                | 18.16%  |
| Manufacturing          | 29.86%                | 16.88%  |
|                       | 52.21%                | 17.19%  |

The data of Table 1 is collected from Cathay Pacific Database; the asset-liability ratio and institutional investor Shareholding ratio are the average for different industries in 2017. As can be seen from Table 1, the asset-liability ratio of companies in different industries is different, which can reflect the characteristic difference of different industries to a great extent. The real estate industry and construction industry are typical industries that rely on debt management, and the asset-liability
ratio of the two is the highest, of which, the real estate industry has a high asset-liability ratio because its investment in projects is large and the payback period is long. The intensity of competition in the product market can also affect the capital structure of companies. In general, the companies with higher competition intensity always have higher asset-liability ratio. Among the listed companies, the companies in more competitive industries such as wholesale and retail industry, manufacturing industry, etc., have higher asset-liability ratio than companies in other industries. Companies that produce or provide products with few substitute products or stronger innovation have smaller capital structure; the asset-liability ratio of diversified and integrated listed companies is relatively high because with large scale, high growth and diversified strategy, these companies are easier to be in debt and they always bear high debt. The mining industry, agriculture, forest industry, animal husbandry, fishery industry and manufacturing industry are more traditional industries, therefore, companies in these industries have small operating risk and their asset-liability ratio can also remain at normal levels, of which, companies in manufacturing industry have the lowest asset-liability ratio. It can be seen from that the different industries that the companies in can not only lead to different asset-liability ratios, but also have different impacts on the optimization of the companies’ capital structure; therefore, the industries shall be controlled in the study on optimization of capital structure.

Table 2. Institutional investor's shareholding ratio

| Trade                      | Public Offering Funds | Securities Trader | Insurance Funds | Social Insurance Funds | Other | Sum  |
|----------------------------|-----------------------|-------------------|----------------|------------------------|-------|------|
| Mining                     | 4.71%                 | 3.23%             | 4.03%          | 3.76%                  | 2.66% | 18.39% |
| Electric Heating Water     | 6.31%                 | 3.17%             | 3.48%          | 4.76%                  | 10.83%| 28.55% |
| Real Estate                | 5.08%                 | 3.23%             | 5.21%          | 2.66%                  | 4.94% | 21.12% |
| Construction               | 5.73%                 | 2.16%             | 2.33%          | 3.09%                  | 3.35% | 16.66% |
| Traffic                    | 3.14%                 | 2.64%             | 3.72%          | 3.53%                  | 7.98% | 21.01% |
| Agriculture                | 5.32%                 | 2.26%             | 3.14%          | 3.05%                  | 3.83% | 17.6%  |
| Wholesale and Retail       | 6.62%                 | 1.79%             | 2.93%          | 2.77%                  | 3.99% | 18.1%  |
| Entertainment              | 2.33%                 | 0.74%             | 1.67%          | 1.24%                  | 7.17% | 13.15% |
| Information Technology     | 8.91%                 | 2.27%             | 2.71%          | 2.73%                  | 1.54% | 18.16% |
| Manufacturing              | 6.24%                 | 2.36%             | 2.7%           | 2.87%                  | 2.71% | 16.88% |
| Synthesize                 | 4.26%                 | 1.93%             | 3.65%          | 3.28%                  | 4.07% | 17.19% |

As can be seen from Table 2, collected from Cathay Pacific database, the distribution of different institutional investors in different industries is relatively similar. Among them, the shareholding ratio of public fund is 14.9% to 49% and its highest proportion of investment is made in the information technology industry because the innovation ability of the information technology industry is strong, which can bring about a large return on assets. The investment of public fund in the transportation industry is relatively low, which has a great relationship with the relatively high risk accident and single profit model in this industry. The shareholding ratio of securities companies is about 5.6% to 17.6%, which relates to the issuance ways that listed companies adopt. The shareholding ratio of securities companies that adopt consignment sale is small and that of securities companies adopt exclusive sale is high. The shareholding ratio of insurance companies is about 18% on average, of which its highest proportion of investment is 17.6%, made in the real estate
industry, and the lowest proportion of investment is 5.6%, made in the culture and entertainment industry. The investment of social security fund in different industries is relatively stable with an average of 15%, which is related to the nature of social security fund. In order to ensure the rate of return and reduce the possibility of loss, the social security fund usually carries out risk hedging for industries with high, medium and low risk, which provides a reference model for insurance funds in entering the market. When summarizing the rest of the institutional investors, the author found that there is a big difference between different industries. It is mainly because these investors have different preferences. In particular, the difference between trusts is large. As can be seen from Table 3, collected from Cathay Pacific Database and through Stata Statistics, the variance of shareholding ratio mortgage ability, debt tax shield and profitability of institutional investors is relatively small, indicating that these indicators of different companies are relatively stable. However, the variance of growth is particularly obvious which is 235.6; although the industries and growth stages that different companies are not the same, but too big variance of growth may affect the significance of the empirical results. It can be obtained through specific analysis that the average shareholding ratio of institutional investors is 18.1%, of which some companies do not have institutional investors and some other companies are controlled by institutional investors; the variance of mortgage ability of institutional investors is only 0.179, with an average of 23.9%, indicating that the overall mortgage ability of companies in different industries is good but not high, which is conducive to raising the operation competency of funds. The ratio of net debt financing to total assets is 0.0321 on average, which is less than 5%, indicating that these listed companies generally have the preference for equity financing.

6. Numerical examples. In this example, based on Table 3’s data, we assume $\delta_0 = 0$ and select a capital structure setting with the following four scenarios.

| Scenario | IShare | Dev  | Deb  | Probability |
|----------|--------|------|------|-------------|
| 1        | 0.182  | -0.016 | 0.051 | 0.25        |
| 2        | 0.180  | -0.022 | 0.042 | 0.25        |
| 3        | 0.181  | 0.006  | 0.047 | 0.25        |
| 4        | 0.180  | 0.008  | 0.048 | 0.25        |
The profitability, growth abilities and the non-debt tax shield of enterprises are 1.216, 5.712 and 0.031. Hence, we formulate the capital structure problem as one in the framework of (3) and derive the following linear programming problem.

\[
\begin{align*}
\min & \quad -0.181\beta_1 + 0.006\beta_2 - 1.216\beta_3 - 5.712\beta_4 - 0.031\beta_5 \\
\text{s.t.} & \quad -0.182\beta_1 + 0.016\beta_2 - 1.216\beta_3 - 5.712\beta_4 - 0.031\beta_5 \\
& \quad \geq -0.051, \text{(Scenario 1 constraints)} \\
& \quad 0.18\beta_1 - 0.016\beta_2 + 1.216\beta_3 + 5.712\beta_4 + 0.031\beta_5 \geq 0, \\
& \quad -0.18\beta_1 + 0.022\beta_2 - 1.216\beta_3 - 5.712\beta_4 - 0.031\beta_5 \\
& \quad \geq -0.042, \text{(Scenario 2 constraints)} \\
& \quad 0.18\beta_1 - 0.022\beta_2 + 1.216\beta_3 + 5.712\beta_4 + 0.031\beta_5 \geq 0, \\
& \quad -0.18\beta_1 - 0.006\beta_2 - 1.216\beta_3 - 5.712\beta_4 - 0.031\beta_5 \\
& \quad \geq -0.047, \text{(Scenario 3 constraints)} \\
& \quad 0.18\beta_1 + 0.006\beta_2 + 1.216\beta_3 + 5.712\beta_4 + 0.031\beta_5 \geq 0, \\
& \quad -0.18\beta_1 - 0.008\beta_2 - 1.216\beta_3 - 5.712\beta_4 - 0.031\beta_5 \\
& \quad \geq -0.048, \text{(Scenario 4 constraints)} \\
& \quad 0.18\beta_1 + 0.008\beta_2 + 1.216\beta_3 + 5.712\beta_4 + 0.031\beta_5 \geq 0, \\
& \quad -1 \leq \beta_1, \ldots, \beta_5 \leq 1.
\end{align*}
\]

By solving the linear programming problem, we obtain the optimal solution of the first-stage decision variables \(\beta_1 = 0.1703\) and \(\beta_2 = 0.3921\) and the second-stage decision variables \(\beta_3 = 0.0023, \beta_4 = 0.0027\) and \(\beta_5 = 0.0001\). The corresponding relative debt financing percentage is 4.7%, which makes much improvement comparing the statistics result Deb = 3.2% in Table 3.

7. **Conclusion.** Capital structure plays an important role in the development of enterprises. In this paper, we have discussed the characteristics of capital structure adjustmnet, and then provided a simplified mathematical model of capital structure. Empirical analysis shows equity financing is more preferable for the listed companies. Furthermore, under equity financing constraints, a capital structure optimization problem with uncertainty is studied and formulated as a two-stage stochastic optimization problem. Finally, numerical examples are given to show the effectiveness of the optimization approach.

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