Nucleon Decay Matrix Elements from Lattice QCD

Yoshinobu Kuramashi for JLQCD Collaboration
Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan
E-mail: yoshinobu.kuramashi@kek.jp

We present a GUT-model-independent calculation of hadron matrix elements for all dimension-six operators associated with baryon number violating processes using lattice QCD. Our results cover all the matrix elements required to estimate the partial lifetimes of (proton, neutron) → (π, K, η) + (ν, e+, µ+) decay modes. We point out the necessity of disentangling two form factors that contribute to the matrix elements; previous calculations did not make the separation, which led to an underestimate of the physical matrix elements. With a correct separation, we find that the matrix elements have values 3 – 5 times larger than the smallest estimates employed in phenomenological analyses of the nucleon decays, which gives stronger constraints on GUT models. We also find that the values of the matrix elements are comparable with the tree-level predictions of chiral Lagrangian.

1 Introduction

Nucleon decay is one of the most exciting predictions of grand unified theories (GUTs) regardless of the existence of supersymmetry (SUSY). Although none of the decay modes have been detected up to now, experimental efforts over the years have pushed the lower limit on the partial lifetimes of the nucleon, which can give a constraint on (SUSY-)GUTs. Moreover, exciting plans for the next generation of the Super-Kamiokande experiment to search the nucleon decay events are proposed in this conference.

On the theoretical side predictions of the nucleon partial lifetimes suffer from various uncertainties. In general the partial lifetime of the nucleon decay process \( N \rightarrow PS + \bar{l} \), where \( N \), \( PS \) and \( \bar{l} \) denote nucleon, pseudoscalar meson and antilepton respectively, is given by

\[
\tau \propto \left| \langle PS|O^B|N \rangle \right|^{-2} \cdot |F_{\text{GUT}}|^{-2},
\]

with \( O^B \) the baryon number violating operator that appears in the low-energy effective Lagrangian of (SUSY-)GUTs. \( F_{\text{GUT}} \) denotes some function in terms of parameters defined in the (SUSY-)GUT models. Although the precise numbers of the nucleon partial lifetimes depend on the details of the theories be-
Beyond the standard model, which contain many unknown parameters, another main source of uncertainty is found in the evaluation of the hadronic part $\langle PS|O^\beta|N \rangle$ in eq. (1). The matrix elements have been estimated by employing various QCD models. Their results, however, scatter over the range whose minimum and maximum values differ by a factor of ten. Therefore, a precise determination of the nucleon decay matrix elements from the first principles using lattice QCD is of extreme importance.

Most important feature of our calculation is GUT model independence. All dimension-six operators associated with baryon number violating processes are classified into four types under the requirement of SU(3)$\times$SU(2)$\times$U(1) invariance at low-energy scales. If one specifies the decay processes of interest, namely the processes among (proton, neutron) $\rightarrow$ (π, K, η) + ($\bar{\nu}$, $\nu$, e$^+$, μ$^+$), we can list a complete set of independent matrix elements in QCD, and we calculate all the matrix elements.

We employed two methods to evaluate the nucleon decay matrix elements in lattice QCD. One is the indirect method in which the matrix element $\langle PS|O^\beta|N \rangle$ is estimated from $\langle 0|O^\beta|N \rangle$ measured on the lattice employing the tree-level results of chiral Lagrangian. The other is the direct method which directly measures $\langle PS|O^\beta|N \rangle$ on the lattice. In the previous lattice QCD studies the indirect method gave values of the matrix elements 2 or 3 times smaller than those obtained by the direct method. Recently, however, we pointed out that the nucleon decay matrix elements $\langle PS|O^\beta|N \rangle$ allow the contributions of two form factors for general lepton momentum, which should be disentangled in the direct method, and this explains the discrepancy between the direct and indirect estimations of the matrix elements found in the previous studies where the separation was not made.

In this report we first formulate the method to calculate the nucleon decay matrix elements using the lattice QCD in Sec. 2. Our results and conclusions are given in Secs. 3 and 4. More details are found in Ref. 10.

2 Formulation of the method

2.1 Independent matrix elements for nucleon decays

The low energy effective theory in the baryon number violating processes is described in terms of SU(3)$\times$SU(2)$\times$U(1) gauge symmetry based on the strong and the electroweak interactions, which enables us to make a GUT-model-independent analysis. Our interest is focused on the dimension-six operators which are the lowest dimensional ones in the low energy effective Hamiltonian: operators must contain at least three quark fields to form SU(3) color singlet, and then an additional lepton field is required to construct a Lorentz scalar.
Higher-dimensional operators are suppressed by inverse powers of heavy particle mass characterized by the theory beyond the standard model.

All dimension-six operators are classified into the four types under the requirement of SU(3) × SU(2) × U(1) invariance:

\begin{align*}
O_{abcd}^{(1)} &= (\bar{D}^c_{iaR} U^b_{jbR})(\bar{Q}^c_{\alpha kcL} L^d_{\beta dL}) \epsilon_{ij} \epsilon_{\alpha \beta}, \\
O_{abcd}^{(2)} &= (\bar{Q}^c_{\alpha iaL} Q^\beta_{\beta jL})(\bar{U}^c_{\gamma kcR} L^d_{\delta dL}) \epsilon_{ij} \epsilon_{\alpha \gamma}, \\
O_{abcd}^{(3)} &= (\bar{Q}^c_{\alpha iaL} Q^\beta_{\beta jL})(\bar{Q}^c_{\gamma kcL} L^d_{\delta dL}) \epsilon_{ij} \epsilon_{\alpha \delta}, \\
O_{abcd}^{(4)} &= (\bar{D}^c_{iaR} U^b_{jbR})(\bar{U}^c_{\gamma kcR} L^d_{\delta dL}) \epsilon_{ij},
\end{align*}

where \( \bar{\psi} = \psi^T C \) with \( C = \gamma_4 \gamma_2 \) the charge conjugation matrix; \( i, j \) and \( k \) are SU(3) color indices; \( \alpha, \beta, \gamma \) and \( \delta \) are SU(2) indices; \( a, b, c \) and \( d \) are generation indices; \( L_L \) and \( Q_L \) are generic lepton and quark SU(2) doublets with the left-handed projection \( P_L = (1 - \gamma_5)/2 \); \( L_R, U_R \) and \( D_R \) are generic charged lepton and quark SU(2) singlets with the right-handed projection \( P_R = (1 + \gamma_5)/2 \). Fierz transformations are used to eliminate all the vector and tensor Dirac structures in eqs. (2)–(5).

Our interest exists in the decay processes from the nucleon to one pseudoscalar meson: (proton, neutron) \( \rightarrow (\pi, K, \eta) + (\bar{\nu}, e^+, \mu^+) \). Once these decay modes are specified, we can list the set of independent matrix elements in QCD from the operators of eqs. (2)–(4):

\begin{align*}
\langle \pi^0 | \epsilon_{ijk} (u^T C P_{R,L} d^j) P_L u^k | p \rangle, \\
\langle \pi^+ | \epsilon_{ijk} (u^T C P_{R,L} d^j) P_L d^k | p \rangle, \\
\langle K^0 | \epsilon_{ijk} (u^T C P_{R,L} s^j) P_L u^k | p \rangle, \\
\langle K^+ | \epsilon_{ijk} (u^T C P_{R,L} s^j) P_L d^k | p \rangle, \\
\langle K^+ | \epsilon_{ijk} (u^T C P_{R,L} d^j) P_L s^k | p \rangle, \\
\langle K^0 | \epsilon_{ijk} (u^T C P_{R,L} s^j) P_L s^k | n \rangle, \\
\langle \eta | \epsilon_{ijk} (u^T C P_{R,L} d^j) P_L u^k | p \rangle,
\end{align*}

where we assume SU(2) isospin symmetry \( m_u = m_d \) and use the relations

\begin{align*}
\langle PS | O_{LR} | N \rangle &= \langle PS | O_{RL} | N \rangle, \\
\langle PS | O_{RR} | N \rangle &= \langle PS | O_{LL} | N \rangle,
\end{align*}

due to the parity invariance. All we have to calculate in lattice QCD are these 14 matrix elements. Other matrix elements can be obtained by using the exchange of the up and down quarks and the relations of eqs. (13) and (14).
2.2 Form factors in nucleon decay matrix elements

Under the requirement of Lorentz and parity invariance, the matrix elements between the nucleon and the pseudoscalar meson in eqs. (12) can have two form factors:

\[ \langle PS(\vec{p}) | O_L^B | N(s) \rangle = P_L \left( W_0(q^2) - W_q(q^2)i\hat{q} \right) u(s), \tag{15} \]

where \( O_L^B \) represents the three-quark operator projected to the left-handed chiral state, \( u(s) \) denotes the Dirac spinor for nucleon with either the up (\( s = 1 \)) or down (\( s = 2 \)) spin state, and \( q^2 \) is the momentum squared of the outgoing antilepton. The contribution of the \( W_q \) term in eq. (15) is negligible in the physical decay amplitude, because its contribution is of the order of the lepton mass \( m_l \) after the multiplication with antilepton spinor. However, since the relative magnitude of the two form factors \( W_0 \) and \( W_q \) is a priori not known, we have to disentangle these two form factors in the lattice QCD calculation. Hereafter we refer to \( W_0 \) and \( W_q \) as relevant and irrelevant form factor respectively.

In our lattice QCD calculation we choose \( \vec{k} = \vec{0} \) for the nucleon spatial momentum and \( \vec{p} = \vec{k} - \vec{q} \neq \vec{0} \) for the PS meson. In this case the Dirac structure of the right hand side in eq. (15) is given by

\[ \begin{pmatrix} W_0 - W_q i\hat{q} & \cdot \sigma \end{pmatrix} \]

(16)

\[ \begin{pmatrix} W_0 - i\hat{q} W_q & -W_q i\hat{q} \cdot \sigma \\ W_q i\hat{q} \cdot \sigma & W_0 + i\hat{q} W_q \end{pmatrix} \]

where \( W_0 - W_q i\hat{q} \) is expressed by a 2x2 block notation; \( \hat{\sigma} \) are the Pauli matrices, and \( u(s)^T = (1, 0, 0, 0) \) or \((0, 1, 0, 0)\). Using the \((1,1)\) and \((2,1)\) components in the 2x2 block notation of eq. (14), where the other components vanish, we can extract the relevant form factor \( W_0 \).

The need for the separation of the contribution of the irrelevant form factor was not recognized in the previous studies with the direct method. The values found in these studies correspond to \( W_0 - i\hat{q} W_q \) instead of \( W_0 \). We examine how much this affects the estimate of the matrix elements in Sec. 3.

2.3 Calculational methods

The nucleon decay matrix elements of eq. (15) are calculated with two methods referred to as the direct and the indirect ones. The former is to extract the
matrix elements from the three-point function of the nucleon, the PS meson and the baryon number violating operator. The latter is to estimate them with the aid of chiral Lagrangian, where we have two unknown parameters to be determined by the lattice QCD calculation.

In the direct method we calculate the following ratio of the hadron three-point function divided by the propagators of the pseudoscalar meson and the nucleon:

\[
R(t, t') = \frac{\sum\vec{x},\vec{x}' e^{i\vec{p} \cdot (\vec{x}' - \vec{x})} \langle J_{PS}(\vec{x}', t') \hat{O}_{L,\gamma}^B (\vec{x}, t) \bar{J}_{N,s}(0) \rangle}{\sum\vec{x},\vec{x}' e^{i\vec{p} \cdot (\vec{x}' - \vec{x})} \langle J_{PS}(\vec{x}', t') J_{PS}(\vec{x}, t) \rangle \sum\vec{x} \langle J_{N,s}(\vec{x}, t) \bar{J}_{N,s}(0) \rangle} \sqrt{Z_{PS}} \sqrt{Z_N} 
\]

\[
\rightarrow \frac{1}{L_x L_y L_z} \langle PS(\vec{p}) | \hat{O}_{L,\gamma}^B | N^{(s)}(\vec{k} = 0) \rangle \quad t' \gg t \gg 0. \quad (17)
\]

Here \(\hat{O}_{L,\gamma}^B\) denotes the renormalized operator in the naive dimensional regularization (NDR) with the \(\overline{MS}\) subtraction scheme, and \(J_{PS}\) and \(J_{N,s}\) are interpolating fields for the PS meson and the nucleon, respectively. The subscripts \(\gamma\) and \(s\) are spinor indices; we can specify the spin state of the initial nucleon at rest by choosing \(s = 1\) or \(2\). \(Z_{PS}\) and \(Z_N\) represent the residues of the PS meson propagator and the nucleon propagator. \(L_x L_y L_z\) is the spatial volume of the lattice. In lattice QCD calculation we evaluate the Green functions numerically by the Monte Carlo method with supercomputer. We move the baryon number violating operator \(\hat{O}_{L,\gamma}^B\) in terms of \(t\) between the nucleon source placed at \(t = 0\) and the PS meson sink fixed at some \(t'\) well separated from \(t = 0\). Under the condition that the baryon number violating operator is sufficiently distanced from both the PS meson field and the nucleon field, which is required to avoid excited state contaminations, we can extract the matrix elements normalized by the spatial volume of the lattice.

The indirect method uses the tree-level results of chiral Lagrangian, which are given by

\[
\langle \pi^0 | (ud_R) u_L | p \rangle = \alpha_P u_p [1 + D + F] / (\sqrt{2} f), \quad (18)
\]

\[
\langle \pi^0 | (ud_L) u_L | p \rangle = \beta_P u_p [1 + D + F] / (\sqrt{2} f), \quad (19)
\]

\[
\langle \pi^+ | (ud_R) d_L | p \rangle = \alpha_P u_p [1 + D + F] / f, \quad (20)
\]

\[
\langle \pi^+ | (ud_L) d_L | p \rangle = \beta_P u_p [1 + D + F] / f, \quad (21)
\]

\[
\langle K^0 | (us_R) u_L | p \rangle = \alpha_P u_p [-1 - (D - F)m_{N/B}] / f, \quad (22)
\]

\[
\langle K^0 | (us_L) u_L | p \rangle = \beta_P u_p [1 - (D - F)m_{N/B}] / f, \quad (23)
\]

\[
\langle K^+ | (us_R) d_L | p \rangle = \alpha_P u_p [2Dm_{N/B}] / (3f), \quad (24)
\]
\[(K^+ | (u s_L) d_L | p) = \beta P_L u_p \left[ 2 D m_{N/B} \right] / (3 f), \]  
\[(K^+ | (u d_R) s_L | p) = \alpha P_L u_p \left[ 3 + (D + 3 F) m_{N/B} \right] / (3 f), \]  
\[(K^+ | (u d_L) s_L | p) = \beta P_L u_p \left[ 3 + (D + 3 F) m_{N/B} \right] / (3 f), \]  
\[(K^0 | (u s_R) d_L | n) = \alpha P_L u_n \left[ -3 - (D - 3 F) m_{N/B} \right] / (3 f), \]  
\[(K^0 | (u s_L) d_R | n) = \beta P_L u_n \left[ 3 - (D - 3 F) m_{N/B} \right] / (3 f), \]  
\[\langle \eta | (u d_R) u_R | p \rangle = \alpha P_L u_p \left[ -1 - (D - 3 F) \right] / (\sqrt{6} f), \]  
\[\langle \eta | (u d_L) u_L | p \rangle = \beta P_L u_p \left[ 3 - (D - 3 F) \right] / (\sqrt{6} f), \]

where $f$ is the pion decay constant; $F$ and $D$ parameters are determined from experimental results of the semileptonic baryon decays; $m_{N/B} = m_N/m_B$ with $m_B \equiv m_{\Sigma} \approx m_{\Lambda}$. We use $\langle PS|(\psi_1 \bar{\psi}_{2R,L})\psi_{3L}|N\rangle$ as a shortened form of $\langle PS|\epsilon_{ijk}(\psi_1^T C P_{R,L} \psi_2^j) P_{L} \psi_3^k |N\rangle$.

The expressions in eqs. (18)–(31) contain two unknown coefficients $\alpha$ and $\beta$, which are to be determined by lattice QCD calculation. The definitions of $\alpha$ and $\beta$ parameters are given by

\[\langle 0 | \epsilon_{ijk}(u^T C P_R d^j) P_L u^k | p^{(s)} \rangle = \alpha P_L u^{(s)}, \]  
\[\langle 0 | \epsilon_{ijk}(u^T C P_L d^j) P_L u^k | p^{(s)} \rangle = \beta P_L u^{(s)}, \]

where operators are renormalized in the NDR scheme. $p$ denotes the proton state. These matrix elements are obtained from the ratio of two-point functions:

\[R^{\alpha \beta}(t) = \frac{\sum_x \langle \epsilon_{ijk}(u^T C P_{R,L} d^j) P_L u^k(x, t) | J_{p,s}(0) \rangle}{\sum_x \langle J_{p,s}(x, t) | J_{p,s}(0) \rangle} \sqrt{Z_N} \rightarrow \langle 0 | \epsilon_{ijk}(u^T C P_{R,L} d^j) P_L u^k | p^{(s)} \rangle \quad t \gg 0. \]  

Incorporating the $\alpha$ and $\beta$ values determined by the lattice calculation into the tree-level results of chiral Lagrangian, we can obtain the values of the nucleon decay matrix elements in eqs. (6)–(12).

### 3 Results

#### 3.1 Details of numerical simulation

Our calculation is carried out with the Wilson quark action in quenched QCD at $\beta = 6.0$ on a $28^3 \times 48 \times 80$ lattice. Gauge configurations are generated with the single plaquette action separated by 2000 pseudo heat-bath sweeps.
We analyzed 100 configurations for the calculation of the nucleon decay matrix elements after the thermalization of 22000 sweeps. The four hopping parameters $K = 0.15620, 0.15568, 0.15516$ and 0.15464 are adopted such that the physical point for the $K$ meson can be interpolated. The critical hopping parameter $K_c = 0.15714(1)$ is determined by extrapolating the results of $m_π^2$ at the four hopping parameters linearly in $1/2K$ to $m_π^2 = 0$. The $\rho$ meson mass at the chiral limit is used to determine the inverse lattice spacing $a^{-1} = 2.30(4)$GeV with $m_\rho = 770$MeV as input. The strange quark mass $m_s = 0.0464(16)(K_s = 0.15488(7))$, which is estimated from the experimental ratio $m_K/m_\rho = 0.644$, is in the middle of $K = 0.15516$ and $K = 0.15464$.

We calculate the ratio of eq. (17) with the nucleon field fixed at $t = 0$ and the PS meson field at $t = 29$. Four spatial momenta $\vec{p}_a = (0,0,0), (\pi/14,0,0), (0,\pi/14,0)$ and $(0,0,\pi/24)$ are imposed on the PS meson in the final state. For the $\vec{p} \neq 0$ cases we provide different quark masses for the valence quark connecting $J_{PS}$ and $G_{L,\gamma}^R$ and the other valence quarks: $m_2$ is for the former and $m_1$ for the latter. By this assignment we can distinguish the strange quark mass from the up and down quark mass.

### 3.2 Nucleon decay matrix elements with direct and indirect methods

We first present the results of the nucleon decay matrix elements obtained by the direct method. In Fig. 1 we show time dependence of $R(t, t' = 29)$ with $|\vec{p}|a = \pi/14$ for the matrix element $\langle \pi^0 | \epsilon_{ijk} (u^T C P_R d^j )^T P_L u_k | p \rangle$ as a representative case. The result of constant fit is depicted by the set of three horizontal lines. We choose the fitting range to be $8 \leq t \leq 16$ for all the matrix elements.
of eqs. (6)−(12) such that the excited state contaminations in the nucleon and PS meson states can be avoided simultaneously.

Figure 2 shows $-q^2a^2$ dependence of the relevant form factor $W_0(q^2)$ in the matrix element $\langle \pi^0 | \epsilon^{ijk} (u^T C P d^j) P_L u^k | p \rangle$, where the operators are renormalized with the NDR scheme at $\mu = 1/a$. The fitting result in Fig. 2 corresponds to the data at $-q^2a^2 = 0.0259(2)$. The combination $W_0 - iqW_q$ is also plotted in Fig. 2 for comparison, which is obtained by following the method in Ref. 8. The magnitude of $W_0(q^2)$ is more than two times larger than that of $W_0(q^2) - iqW_q(q^2)$.

To interpolate the relevant form factor $W_0$ to $q^2 = 0$, which is the on-shell point of outgoing antilepton, we employ the following fitting function:

$$c_0 + c_1 \cdot (-q^2) + c_2 \cdot (-q^2)^2 + c_3 \cdot m_1 + c_4 \cdot m_2,$$

where we assume that the form factor could have the $m_1$ and $m_2$ dependences through the nucleon and PS meson masses. We extrapolate $m_1$ and $m_2$ to the chiral limit for the matrix elements of eqs. (6), (7) and (12), while $m_2$ is interpolated to the physical strange quark mass with $m_1$ taken to the chiral limit for the matrix elements of eqs. (8)−(11). The solid line and the open circle at $-q^2a^2 = 0$ in Fig. 2 denote the fitting result of the data employing the function of eq. (35), where we find that the charged lepton masses $m_2a^2 = 4.9 \times 10^{-8}$ and $m_1a^2 = 2.1 \times 10^{-3}$ are negligible in the current numerical statistics. The solid line expresses the function $c_0 + c_1 \cdot (-q^2) + c_2 \cdot (-q^2)^2$ employing the fitting results of $c_0$, $c_1$ and $c_2$.

Let us turn to the results obtained by the indirect method, which uses the tree-level results of chiral Lagrangian in eqs. (18)−(31). The $\alpha$ and $\beta$ parameters at each hopping parameter are extracted from a constant fit of the ratio of eq. (34). Applying linear fits to the data as a function of quark mass $m_qa = (1/K - 1/K_c)/2$, we obtain $\alpha(NDR, 1/a) = -0.015(1)$ GeV$^3$ and $\beta(NDR, 1/a) = 0.014(1)$ GeV$^3$ in the chiral limit with the use of $a^{-1} = 2.30(4)$ GeV.

In phenomenological GUT model analyses of the nucleon decays, the values $|\alpha| = |\beta| = 0.003$ GeV$^3$ are conservatively taken as these are the smallest estimate among various QCD model calculations. The previous lattice calculations, however, indicated values of these parameters considerably larger than the minimum model estimate above. Our results, significantly improved over the previous ones due to the use of higher statistics, larger spatial size, lighter quark masses and smaller lattice spacing, have confirmed this trend: the values we obtained are about five times larger than $|\alpha| = |\beta| = 0.003$ GeV$^3$.

In Fig. 3 we compare the nucleon decay matrix elements obtained by the direct method with those by the indirect one using the tree-level results of
Figure 3: Comparison of relevant form factors with tree-level predictions of ChPT. Crosses denote the ChPT results with $|\alpha| = |\beta| = 0.003$GeV. For numerical values see Ref. 10.

chiral Lagrangian (squares), where we employ the expressions of eqs. (18)–(31) with $\alpha(\text{NDR}, 1/a) = -0.015(1)$GeV, $\beta(\text{NDR}, 1/a) = 0.014(1)$GeV, $f_\pi = 0.131$GeV, $m_N = 0.94$GeV, $m_B = 1.15$GeV, $D = 0.80$ and $F = 0.42$. We observe that the two set of results are roughly comparable. This leads us to consider that the large discrepancy between the results of the two methods found in Refs. 8, 9 is mainly due to the neglect of the $W_q(q^2)$ term in eq. (15).

It is also intriguing to compare our results with the tree-level predictions of chiral Lagrangian with $|\alpha| = |\beta| = 0.003$GeV (crosses). Our results with the direct method are 3–5 times larger than the smallest estimates except $\langle \eta|(ud_R)u_L|p\rangle$. Hence they are expected to give stronger constraints on the parameters of GUT models.

4 Conclusions

In this article we have reported progress in the lattice study of the nucleon decay matrix elements. In order to enable a GUT-model-independent analysis of the nucleon decay, we have extracted the form factors of all the independent matrix elements relevant for the (proton, neutron)→($\pi, K, \eta$)+($\bar{\nu}, e^+, \mu^+$) decay processes without invoking chiral Lagrangian.

We have also pointed out the necessity of separating out the contribution
of an irrelevant form factor in lattice calculations for a correct estimate of the
matrix elements at the physical point. With this separation, the matrix ele-
ments obtained from the three-point functions are roughly comparable with
the tree-level predictions of chiral Lagrangian with the $\alpha$ and $\beta$ parameters
determined on the same lattice. The magnitude of the matrix elements, however,
are 3 to 5 times larger than those with the smallest estimate of $\alpha$ and $\beta$ among
various QCD model calculations. Our results would stimulate phenomenologi-
cal interests as the larger values of the nucleon decay matrix elements can give
more stringent constraints on GUT models.

Major systematic errors conceivably affecting our present results are the
scaling violations and the quenching effects. The former can be investigated by
repeating the simulation at several lattice spacings; the latter is eliminated once
configurations are generated with dynamical quarks, where it is straightforward
to apply our method. We leave these points to future studies.

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