Citation: Frankfurt, L.; Strikman, M.

Perturbative QCD Core of Hadrons and Color Transparency Phenomena.
Preprints 2022, 4, 0. https://doi.org/

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Abstract: In the current paper, we argue that the ground state of a hadron contains a significant perturbative quantum chromodynamics (pQCD) core as the result of color gauge invariance and of the values of chiral and gluon vacuum condensates. The evaluation within the method of dispersion sum rules (DSR) of the vacuum matrix elements of the correlator of local currents with the proper quantum numbers leads to the value of the radius of the pQCD core of a nucleon of about 0.4–0.5 fm. The selection of the initial and final states allows to select processes in which the pQCD core of the projectile gives the dominant contribution to the process. It is explained that the transparency of nuclear matter for the propagation of a spatially small and color-neutral wave packet of quarks and gluons—a color transparency (CT) phenomenon—for a group of hard processes off nuclear targets can be derived in the form of the QCD factorization theorem accounting for the color screening phenomenon. Based on the success of the method of DSR, we argue that a pQCD core in a hadron wave function is surrounded by the layer consisting of quarks interacting with quark and gluon condensates. As a result, in the quasi-elastic processes $e^+ A \rightarrow e^+ N + (A - 1)^*$, the quasi-Feynman mechanism could be dominating in a wide range of the momentum transfer squared, $Q^2$. In this scenario, a virtual photon is absorbed by a single quark, which carries a large fraction of the momentum of the nucleon and dominates in a wide range of $Q^2$. CT should reveal itself in these processes at an extremely large $Q^2$ as the consequence of the presence of the Sudakov form factors, which squeeze a nucleon.

Keywords: quantum chromodynamics (QCD); exclusive processes; color transparency; nucleon structure

1. Introduction

The color transparency (CT) phenomenon is the suppression of the final and/or initial state interaction for the small size wave packet of quarks and gluons produced in the hard processes and propagated through a nucleus. The quantitative approach is to calculate the cross-section of the interaction of small size wave packet scattering off a target, as well as to describe some properties of the bound state hadrons specific for quantum chromodynamics (QCD).

CT has been derived (i) for the deep inelastic scattering (DIS) processes initiated by highly virtual photons; (ii) for the processes of the diffractive electroproduction of vector mesons such as $\gamma^* + A \rightarrow V + A$ for $V = \rho, \omega, \phi, J/\psi, Y$, etc.; (iii) for the processes: $\pi + A \rightarrow$ two jets $+ A$, $p + A \rightarrow$ three jets $+ A$; (iv) in the case of small parton momentum fraction, small-$x$ processes, factorization theorems can be derived for the diffractive photoproduction of the bound states of heavy quarks such as $J/\psi$ and $Y$, produced in the ultraperipheral collisions of heavy ions. and (v) CT is generalized to include effects of the leading twist gluon shadowing for small $x$ processes.

Within QCD, a hadron consists of the three overlapping layers, corresponding to two distinctive phases of the QCD matter. The outer layer is formed by the pion cloud of a hadron. Just this layer produces internucleon attraction in low-energy nuclear phenomena. The next
layer is formed by quarks interacting with chiral and gluon condensates. The existence of this layer is the basis of the success of the method of dispersion sum rules in the calculation of parameters of the ground states of hadrons. The important role of the vacuum condensate of chiral quark pairs is implied by the phenomenon of spontaneously broken chiral symmetry in QCD. The boundary between both layers can be evaluated as the boundary of the region where two-pion exchange between nucleons dominates. This value of the boundary was estimated in Ref. [1] long ago. In any case, the position of this boundary is not well defined since it fluctuates.

The second layer is relevant for the competition between soft and hard processes for the large momentum transfer behavior of hadron form factors. This competition results from the specific of the Lorentz transformation within light cone quantum mechanics. It was found that for a two-body system that large momentum transfer $Q$ is multiplied by the factor, $1 - \alpha$, in the argument of the wave function of a final state of a two-body system [2]. A similar pattern holds for many body systems. Thus, the relative contribution into form factors of configurations where one constituent carries most of the light cone (LC) fraction—$\alpha$—of the hadron is enhanced. Feynman has pointed out that this may lead to a dominance of the configurations where one parton carries practically all the momentum of the hadron. Hence, this contribution is referred to as the Feynman mechanism. In practice, there is a wide pre-asymptotic region, e.g., $\alpha \geq 0.8$, which is strongly enhanced in a wide range of $Q^2$. In what follows, this kinematics is referred to as a quasi-Feynman mechanism.

Together, these two layers describe the QCD phase of the spontaneously broken chiral symmetry. The significant probability of the perturbative QCD (pQCD) core within the wave function of a hadron follows from the analysis of the vacuum correlator of local currents in the coordinate space, i.e., effectively from the color gauge invariance and the values of the chiral and gluon condensates. Thus, the evaluation of the radius of the pQCD core of a hadron requires using a model accounting for the condensates.

The discovery of heavy quarks such as $c$, $b$, etc. allows to expand CT to the number of the processes, where CT has already been observed and can be further investigated.

The understanding of the QCD structure of the wave function of a hadron and of a nucleus allows to separate a group of hard processes, where the hard interaction produces spatially small wave packets of quarks and gluons. The strength of the interaction of such wave packets with hadrons is unambiguously calculable within QCD for high-energy processes. Selecting special initial and final states is necessary for ensuring the dominance of the contribution of the pQCD core of the hadrons. For this group of phenomena, CT is one of the elements of the QCD factorization theorem. In these processes, CT was already unambiguously observed. The second group is formed by the processes, where there is no constraint for the pQCD core to dominate in the wave function of a hadron. In this case, the quasi-Feynman mechanism dominates in a wide range of momentum transfer. At an extremely large momentum transfer, the contribution of the pQCD core should dominate since the Sudakov form factors gradually squeeze the wave packet.

2. Three-Layer Structure of the Nucleon Wave Function in QCD

2.1. The Spatial Distribution of Valence Quarks in a Nucleon

The valence quark and momentum sum rules for the parton distributions within a hadron unambiguously follow from the Wilson operator expansion. For certainty, the discussion in this paper is restricted to the case of a nucleon target. The sum rules for the generalized valence quark distributions, $V_N(x, Q^2, t)$, at a non-zero momentum transfer, $\int V_N(x, Q^2, t)dx = F^V(t)$, (1)
is of prime interest to us. Here, $F_N^V(t)$ is the isotopic (SU(3)) vector form factor of a nucleon calculable in terms of the combination of the electromagnetic form factors of a proton and a neutron. The generalized valence quark sum rules follow for any $Q^2$ from the combination of the Ward identities for electroweak currents and the energy dependence of the high-energy amplitudes in QCD in non-vacuum quantum numbers in the crossed channel [3]. To investigate the pQCD core of a nucleon at moderate $Q^2$, the method of dispersion sum rules (DSR), developed in [4], is used here.

The radius of valence quark distribution follows from the sum rules and from the data on the electromagnetic form factors of a nucleon:

$$ (r_V^2)^{1/2} = 0.65 \text{ fm}. \quad (2) $$

2.2. The pQCD Core of the Wave Function of a Nucleon

The Dirac sea is an important property of relativistic quantum field theories. Ignoring the Dirac sea in the non-relativistic approximation leads to the non-conservation of baryon and electric charges of the energy—momentum tensor as defined in QCD and a related violation of probability conservation for the high-energy processes. For example, these violations lead to the so-called West correction for the structure functions of the deuteron, $\sigma_{tot}(eD) < \sigma_{tot}(eN) + \sigma_{tot}(eN)$ in the impulse approximation [5]. This calculation used the non-relativistic wave function of the deuteron. Such a correction results in the violation of the exact QCD sum rules such as the baryon charge sum rule, in the violation of the Glauber decomposition over rescatterings, etc.

The only approach known so far to account for the Dirac sea in a way consistent with the exact QCD sum rules is to use the LC mechanics of nuclei. It resembles the parton model approximation for a quantum field theory, suggested by Feynman. Imposing angular momentum conservation and the requirement of Lorentz symmetry for on-mass-shell amplitude, LC mechanics can be transformed into the instant time form for a wide range of nucleon momenta [6]. In the LC mechanics of a nucleus, the West correction disappears, as can be found in Ref. [6]. This phenomenon is important in nuclear theory, in the kinematics, where $k^2/m_N^2$ is not negligible; here, $k$ denotes nucleon momentum and $m_N$ denotes nucleon mass.

The phenomenon is also present in quantum electrodynamics (QED) in the calculation of high-order corrections to the wave functions of molecules.

To investigate the properties of an LC wave function of a nucleon, the vacuum element of the retarded commutator of local color-neutral currents, $J$, with the quantum numbers of a nucleon, Omitting Lorentz indices we can write

$$ K(y_0, \vec{y}) = \langle 0|\theta(y_0)|J(y_0, \vec{y}), J(0)|0 \rangle. \quad (3) $$

are analyzed here. $|0\rangle$ and $|0\rangle$ denote in and out states, $y$ is the difference of 4 D coordinates of the in and out currents, and $y_0$ is the zero component of $y$. $\theta$ is the Heaviside function.

The retarded commutator is equivalent to the $T$- product but allows the analytic continuation of the Fourier transform into the complex plane of energies and derives dispersion relations. This correlator in the momentum space was analyzed in Refs. [4,7] within the DSR approximation. For the aims of the study, given here, it is convenient to analyze this correlator in the coordinate space.

The intermediate states in the correlator with the quantum numbers of a nucleon are accounted for as the full system of eigen-states of the QCD Hamiltonian:

$$ K(y_0, \vec{y}) = \sum_n \langle 0|J(n)|n \rangle^2 \exp(i(E_n - E_0)y_0 - i(p_n \cdot \vec{y}))\theta(y_0)d\tau_n. \quad (4) $$

Here, $E_n$ and $E_0$ are ..., respectively; $p_n$ is ..., and $\tau_n$ is ...
In the Euclidian domain, in the limit \( i(E_0 - E_n)y_0 \to \infty \), only the contribution of the state with the minimal mass \((n = 0)\) survives. To improve the convergence with the DSR approach, one subtracts the contribution of the intermediate states with the masses significantly larger than the nucleon mass. The subtracted contribution is calculable in pQCD [7].

The initial condition for the space–time evolution for the valence quark component of the wave function of a hadron is

\[
\phi_N(t = 0, \vec{y} = 0) = \langle 0 | J(0) | N \rangle \bigg/ (E_N - E_0). \tag{5}
\]

The current, \( J \), is local in the coordinate space to ensure the conservation of the color gauge invariance. At the starting point of the space–time evolution, the pQCD wave packet has the zero size. The hierarchy of the initial conditions is regulated by the asymptotic freedom combined with the approximate conformal invariance of QCD. In the actual calculations, one choses the current, \( J \), following Ref. [7], where the spatially local and color-neutral current \( J = f^3q \) was used. The currents, containing a larger number of quark–gluon fields, were used for evaluating the gluon distribution in the nucleon; see discussion in Ref. [8].

The investigation of the space–time evolution of the nucleon wave function with the increase in the relative distance \( y \) between constituents allows to identify the coexistence of the three distinctive layers in the wave function.

The radius of the pQCD core of a nucleon can be estimated based on the analysis of the correlator, \( K \), in the coordinate space performed in Ref. [9]. The analysis uses the standard values of the quark and gluon condensates and indicates that the correlator is close to the free correlator for the radius \( r \leq r_c \), where

\[
r_c \approx (0.4 - 0.5) \text{ fm}. \tag{6}
\]

The pQCD core of the radius \( r_c \) is surrounded by a layer of nonperturbative QCD phase where bare quarks and antiquarks interact with the vacuum condensates and obtain masses. As a result, the properties of this layer resemble the constituent quark model of a nucleon. The thickness, \( \Delta_{n.p.} \), of the nonperturbative layer can be estimated from

\[
\langle r \rangle_N = \Delta_{n.p.} + r_c, \tag{7}
\]

where \( \langle r \rangle_N \) is the radius of a nucleon. For illustration, let us choose \( \langle r \rangle = 0.85 \text{ fm} \). Thus, \( \Delta_{n.p.} \approx (0.35-0.45) \text{ fm} \). It is natural to separate the pion cloud of a nucleon from the nonperturbative layer. \( \Delta_\pi \approx 0.2 \text{ fm} \) is the thickness of the pion cloud calculated from the contribution of the two-pion state in the electromagnetic form factors of a nucleon [1]. Thus, \( \Delta_{n.p.} - \Delta_\pi \approx 0.2 \text{ fm} \).

To conclude, the derived form of the nucleon wave function follows in the weak coupling regime of QCD from the local color gauge invariance, asymptotic freedom, and from the small density of the vacuum condensate of the chiral pairs.

### 2.3. Lattice QCD as the Tool to Probe Nucleon Wave Function

The lattice evaluation of vacuum correlators \( K \) (4) may help improve the evaluation of the nonperturbative QCD phenomena beyond the mean field approximation used in this paper. The three-layer structure of a nucleon reveals itself in the distinctive QCD phenomena:

(i) The generation of the running masses of constituents due to their interaction with vacuum condensates was discussed in Ref. [10] but without a pQCD core. As a result, the second layer resembles the constituent quark model. The educated guess for the thickness of this phase is between 0.4 fm and 0.6 fm.
(ii) The investigation of the transitions between the nonperturbative and pQCD phase would provide a new probe of the role of the spontaneous violation of the chiral symmetry in the structure of the ground states of hadrons.

2.4. The Proof of the Presence of pQCD Core within a Hadron

The very existence of the pQCD phase within a nucleon differs from the expectations of the nonrelativistic nuclear theory at the scale comparable to the scale observed in the low-energy nuclear phenomena. Thus, an additional theoretical reasoning is presented below.

The existence of the pQCD core follows directly from the QCD factorization theorems derived in the momentum space representation. Let us consider the example of the process involving a pion.

(i) The dominant high-momentum quark-antiquark, \( q \bar{q} \), component of the pion wave function is given in QCD by the one gluon exchange:

\[
\psi_{\pi}(k) = \alpha_s f_{\pi} C_{\pi} / k^2.
\]  

Here, \( f_{\pi} \) is the constant, determined from the pion \( \beta \) decay, \( C_{\pi} \) is a constant calculable in pQCD, \( \alpha_s \) is the running coupling constant, and \( k \) is the momentum of quark (antiquark). The product \( f_{\pi} C_{\pi} \) is calculable in the approximation of the partial conservation of the axial current. The Fourier transform of this wave function into coordinate space gives \( \psi_{\pi}(r = 0) \). The high momentum tail of the wave function of pion in the momentum space has been measured at FNAL (Fermi National Accelerator Laboratory, Batavia, IL, USA) in the experiment, where CT was also discovered in the reaction \( \pi + A \to \) two jets + A [11] with the pattern, predicted in Ref. [12]. A similar behavior is expected for the scattering of any meson. The only difference is the dependence of the probability of the tail on the type of a meson.

(ii) The factorization theorem of QCD for two-body processes see Equation (8) is expressed through the wave function of the quark–gluon core of a pion. The existence of the quark–gluon core of a nucleon unambiguously follows from the QCD factorization theorem. The direct experimental observation of the quark–gluon core of a nucleon in the process \( p + A \to \) three jets + A is much more difficult than that for the pion since the pQCD contribution into a hard process is significantly smaller—\( \propto \alpha_s^2 \). Additionally, the absolute value of the high-momentum tail of a parton wave function of a nucleon is the subject of modeling.

2.5. Pion Field in QCD

Internucleon attraction in low-energy phenomena is mostly due to the interaction of the pion clouds of the interacting nucleons. The pion clouds are located at the nucleon outer layer. Internucleon repulsion is dominated by the nucleon core. The long-range pion field of a nucleon has been derived within QCD from the phenomenon of a spontaneously broken chiral symmetry. The pion mass was calculated in QCD in terms of the chiral vacuum condensate, \( \langle 0 | \bar{q} q | 0 \rangle \) [13]. The pion mass arises due to the explicit violation of chiral symmetry by the term in the Lagrangian containing non-zero masses of up (\( u \)) and down (\( d \)) quarks [13]:

\[
m_{\pi}^2 = (m_u + m_d) \frac{\langle 0 | \bar{q} q | 0 \rangle}{f_{\pi}^2}.
\]  

The concept of the pion cloud of a nucleon is valid within the restricted kinematical region where a virtual pion is close to the pion mass shell because a pion is a collective mode. The experimental restriction is that the antiquark distribution in a nucleon is rapidly decreasing in a nucleon as \( \bar{q}(x, Q^2) \propto (1 - x)^m \). Global parton distribution fits typically give \( n(Q^2) \approx 7 \). Thus, the distribution of antiquarks mostly originate from the second layer where antiquarks obtain masses as a result of the interaction with the chiral condensate. The role of the second
layer, for the antiquark distribution within a nucleon is decreasing with increase in nuclear density. This pattern is the opposite to the expectation of the models where pion condensate is present in the cores of neutron stars. Within QCD, the pion field in a hadron is reduced with a significant decrease in the overall size of the hadron due to effects of color screening and color gauge invariance. This phenomenon is the property of the weak coupling regime where the interaction is decreasing with the decrease of the overall size of a hadron; see discussion below. This QCD phenomenon is manifested, for example, in the decays of the excited state of charmonium and bottomonium and may be tested by lattice QCD methods. This phenomenon is opposite to the experience of the low-energy nuclear physics and it is absent in the bag models. This is because the bag models of a hadron—in contrast to the full QCD—do not include color screening and asymptotic freedom phenomena. Therefore, in the bag models, a position of the hadron bag surface does not depend on the distribution of quarks and gluons. This feature also results in the violation of causality. Thus, QCD implies that the pion field is not a universal property of a hadron. One of the striking experimental confirmations was the discovery that for \( J/\psi \) and other onium mesons, interaction with pion field is suppressed.

Thus, the preQCD models of nucleon–pion interactions differ from QCD. This difference reveals itself for the large pion momenta, as can be found in Ref. [14] (and references therein). For the distances within a nucleon that are significantly smaller than the thickness of a pion cloud, different degrees of freedom dominate within QCD.

Thus, the difference between the preQCD models of nucleon–pion interactions and the expectations of QCD reveals itself for the large pion momenta and large pion virtualities. For the distance scale within a nucleon that is significantly smaller than the thickness of the pion cloud, different degrees of freedom dominate in QCD. The found properties of the wave function of a nucleon are sufficient to resolve the zero charge puzzle: in a preQCD quantum field theory, ultraviolet divergencies nullify the interaction of hadrons [15]. In QCD, ultraviolet divergencies present in the preQCD quantum field theories disappear due to the composite structure of a hadron. the wave function of a hadron contains in its central region the quark–gluon pQCD core instead of meson fields characteristic for preQCD field theories as the consequence of the color gauge invariance, asymptotic freedom, and the small density of vacuum chiral condensates. The scale where the nonperturbative QCD surface effects are transformed into the quark–gluon core of a hadron plays the role of cutoff within preQCD field models within effective field theory (EFT).

Let us note that the related phenomena in hot nuclear matter are beyond the scope of this paper.

3. The Properties of QCD Which Lead to CT Phenomena

The wave packet, produced in the hard processes, is built of highly virtual quarks and gluons, which weakly interact with the nuclear environment [17].

In Ref [17], it was assumed that in the elastic \( \pi N \) large angle collisions, the pion and nucleon are squeezed leading to the cross-section of a quasi-elastic reaction off nuclei \( \pi A \rightarrow \pi N(A - 1)^{\ast} \) equal to \( A \) times the elementary cross-section. There are experimental indications for a moderate increase in the transparency of nuclear matter with an increase in \( Q^2 \) for the electroproduction of pions and \( \rho \) mesons, as can be found in review [18].

The experiments, designed to search for the CT in the quasi-elastic processes, \( e + A \rightarrow e' + N + (A - 1) \), did not observe an increase in transparency up to \( Q^2 = 14 \text{GeV}^2 \) [19]. This result is qualitatively consistent with the analyses of the data on electromagnetic form factors at the intermediate \( Q^2 \) which indicate that the dominant contribution to the nucleon [7] and pion [20] form factors is the electron scattering off configurations in which the virtual photon, \( \gamma^\ast \), is absorbed by a leading quark which gained its momentum distribution in the interaction
with vacuum condensates of quarks and gluons. Thus, the data are inconsistent with the
democratic chain approximation, suggested in Ref. Ref. [21] at achievable \(Q^2\).

3.1. The Conditions for the Validity of CT

The direct observation of the CT phenomena in the hard high-energy processes requires
the validity of two conditions:

(a) The interaction between hadrons depends on the value of the region occupied by color
within the interacting hadrons. This feature of QCD is proved for hard processes in the form
of QCD evolution equations. If the transverse component of the radius of a hard projectile, \(r_t\),
perpendicular to the wave packet momentum is sufficiently small, the forward amplitudes are
proportional to \(r_t^2\):

\[
\sigma_{\text{tot}}(a + b) = c r^2_t, \tag{10}
\]

where \(c\) is the speed of light.

Equation (10) is another form of the Bjorken scaling for the total cross-sections of DIS
which follows from the approximate conformal invariance of QCD [22]. Equation (10) has
been confirmed in many high-energy experiments. In particular, it serves as a basis for the
description of exclusive and inclusive experiments and can be formulated in the form of QCD
factorization theorems for these processes. Note that the account of the QCD evolution in \(x\)
and \(Q^2\) leads to the somewhat weaker dependence on \(r_t\). However, the physics relevant for this
extreme kinematics is beyond the scope of this paper. Thus, the first condition of the validity of
CT is the presence of the trigger, which selects a squeezed quark–gluon configuration within a
hadron, and a part of the proof is the derivation of the dipole model [12,23,24]. The spatial size
of the produced wave packet of quarks and gluons should be significantly smaller than the
radius of a hadron.

(b) The second requirement is that the lifetime, \(L_c\), i.e., the distance propagated by suffi-
ciently energetic but squeezed quark–gluon wave packet, should be sufficiently large to avoid
expansion while traversing the nucleus. Thus, Einstein time dilation plays an important role in
the CT. The coherence length was derived from the theoretical analyses of the Fourier transform
of the structure functions of a target nucleus/nucleon in coordinate space. In the rest frame of
the nuclear target, at large \(Q^2\) [25,26]:

\[
z = L_c = (1/2m_Nx). \tag{11}\]

Here, \(Q^2 \approx (m_q^2 + k_t^2)/z(1 - z)\) is the resolution of the investigated configuration, \(m_q\) is the
quark mass. \(z\) is a fraction of photon momentum carried by the produced quark, and \(k_t\) is its
transverse momentum. This equation follows from the uncertainty principle for the transition
\(\gamma^* \rightarrow q\bar{q}\) and Einstein time dilation. Numerical calculations show that Equation (11) is also
valid within the parton model for moderately large \(x\). Thus, the pQCD core for a longitudinally
polarized photon together with the color-screening phenomenon guarantee the validity of
CT if the lifetime for a wave packet describing the pQCD core is sufficiently large to traverse
a nucleus.

3.2. Discovery of CT

The search for the CT allowed to establish at what resolution scale \(Q^2\) the basic degrees
of freedom characterizing the wave function of a nucleon of a nucleus are quarks and gluons.
This would allow to determine an upper bound for the range of virtualities for which EFT
approaches can be used.

The discovery of a small cross-section of a diffractive photo-production of \(J/\psi\) meson
was the first observation of CT. The value of the \(J/\psi-N\) cross-section, extracted within the
vector dominance model (VDM) of \(\sigma(J/\psi-N) \approx 1\) mb, was much smaller than the genuine
cross-section $\sigma_{\text{tot}}(J/\psi-N) \approx 4$ mb, extracted from the $A$-dependence of $J/\psi$ quasi-elastic photo-production. The difference originates from the production of $J/\psi$ in a small size configuration, whose size is about $1/m_c$ [27], where $m_c$ is the c-quark mass. The phenomenon is especially striking when one compares $J/\psi-N$ and $\psi'-N$ cross-sections, extracted from the data using VDM. VDM leads to $\sigma_{\text{tot}}(\psi'-N) < \sigma_{\text{tot}}(\psi-N)$, while in QCD, the opposite trend is expected since the transverse size of $\psi'$ is exceeds twice that of $J/\psi$.

CT is an unambiguous prediction of QCD for the phenomena, where QCD factorization theorems are applicable:

(i) DIS processes off a nuclear target:

$$\sigma_{\text{inel}}(\gamma^* + A) \rightarrow A\sigma_{\text{inel}}(\gamma^* + N),$$

where $\sigma_{\text{inel}}$ denotes the inelastic cross-section, and $\gamma^*$ stands for the longitudinally polarized photon.

At small $x$, CT is valid as soon as the leading twist nuclear shadowing is taken into account [22].

(ii) The cross-section of the diffractive electroproduction of longitudinally polarized vector meson off nucleon and nuclear targets has been predicted in Refs. [28,29]. The derived QCD factorization theorem is also applicable to a nuclear target. For the invariant momentum transfer, $t = 0$, and the large $Q^2$ cross-section of the diffractive electroproduction of the vector meson, $V = \rho, \omega, \phi$, CT unambiguously follows from the QCD factorization theorem [29].

(iii) In the case of the diffractive photo-production of a heavy quarkonium, a heavy mass of $c$ and $b$ quarks guarantees the applicability of the QCD factorization theorem. These processes were observed at HERA facility (at German Electron Synchrotron DESY, Hamburg, Germany) and in ultraperipheral heavy-ion collisions at the LHC (Large Hadron Collider at the European Organization for Nuclear Research CERN, Geneva, Switzerland).

For $t = 0$,

$$d\sigma(\gamma + A \rightarrow V + A)/dt = A^2 d\sigma(\gamma + N \rightarrow V + N)/dt,$$

where $V = J/\psi$ or $Y$.

One can employ the observation from quarkonium models that the radii of the onium states are much smaller than those for light vector mesons and pions: $r_{L/\psi} = 0.2$ fm and $r_{\gamma} = 0.1$ fm. Note here that for a small $x$, expression (13) has an additional factor—the square of the ratio of gluon densities in the nucleus (per nucleon) and in the nucleon.

(iv) Diffractive production by the pion projectile of two jets off a nuclear target, $\pi + A \rightarrow 2$ jets + $A$. The process was observed at FNAL [11] and predictions [12] for the longitudinal and transverse momentum of jets and for the $A$-dependence of the cross-section were confirmed. The same effect is expected for kaon beams and for the fragmentation of a proton into three jets.

The restriction on the region of applicability of CT, given by Equation (11), can be somewhat weakened by accounting for the phenomenon of quantum diffusion in the space–time evolution of small size wave packet [22,30]:

$$r^2_l(l) = \left[r_l(0) + \left(\langle r^2 \rangle - r_l(0)^2\right)(l/L_c)\right]\theta(L_c - l) + \langle r^2 \rangle \theta(l - L_c).$$

Here, $r_l(l)$ is the transverse size of the wave packet which evolves with distance $l$ from the interaction point. Relativistic kinematics is considered where $l$ is equal to the distance from the interaction point; $\langle r^2 \rangle$ is a nonperturbative transverse radius squared of the wave packet. The region of applicability of the above formulae is that $\sqrt{r^2}$ should be significantly smaller than the radius of a hadron.
3.3. On the Quasi Feynman Mechanism in Quasi-Elastic Processes at Achievable \( Q^2 \)

In the relativistic theory, due to specifics of the Lorentz transformation, the quasi-Feynman mechanism may dominate in the nucleon and meson form factors up to large \( Q^2 \) (which are likely to grow with the increase in the number of constituents in a hadron). In this mechanism, the dominant contribution to the form factor originates from the configurations, in which one parton carries nearly all the LC fraction of the momentum of the nucleon. In these configurations, squeezing is moderate while the correlation length in a wide range of \( Q^2 \) is comparable to the internucleon distance. As a result, the expected CT effect is rather mild up to large \( Q^2 \).

To visualize a relativistic description, let us choose the transferred momentum, \( q_t \), to be orthogonal to the quantization axis. This condition can be achieved by choosing the center-of-mass of the \( eN \) system and setting the electron momentum, \( P \rightarrow \infty \) [31]. The analysis of the energy denominators of the LC mechanics shows that \( q_t \) is multiplied in the argument of the wave function of the final nucleon by the factor \( (1-\alpha) \) [2]. Here, \( \alpha \) is the fraction of a nucleon momentum, carried by the interacting constituent.

Let us consider the two-body system [2]:

\[
F(q_t^2) = \int \psi\left(\frac{m^2 + k_t^2}{\alpha (1-\alpha)} - 4m^2\right) \psi\left(\frac{m^2 + |k_t + q_t (1-\alpha)|^2}{\alpha (1-\alpha)} - 4m^2\right) S(k_t^2 / q_t^2) \, d\alpha d^2k_t \tag{15}
\]

Here, \( \psi \) is the wave function of two body system \( m \) denotes the constituent mass, \( S(Q_t^2 / k_t^2) \) is the Sudakov form factor, which squeezes the wave function of a nucleon, and the production of \( NN \) pairs from the vacuum is neglected. The coherence length \( (11) \) accounts for the space–time evolution of the produced wave package through an uncertainty principle. A different definition of the coherence length was used in Ref. [32].

The value of squeezing depends on the minimal \( k_t \) when radiation is allowed. Here, \( k_t^2 \approx (\Lambda_{QCD})^2 \) is used as it is popular in the description of hard processes [33]. Thus, at \( Q_t^2 \rightarrow \infty \), the two-body system is squeezed. For the realistic \( q_t \), maximal contribution arises from the region, where \( \alpha \) is quite close to one. This resembles the Feynman mechanism where the leading parton carries the whole momentum of a hadron. In the mean field approximation, the wave function of a nucleon is effectively modeled as a two-dimensional harmonic oscillator, as can be found in Ref. [32] (and references therein). Thus, in these models, the quasi-Feynman mechanism dominates in a wide range of the momentum transfer (actually for any \( Q^2 \)) and can explain [32] the lack of CT effect, reported in the JLab (Jefferson Laboratory, Newport News, VA, USA) experiment [19]. In a general case, a wave function contains the short-range correlation between quarks. This correlation is moderately squeezed by the Sudakov form factor accounting for the lack of radiation in the process considered.

Let us analyze the \( e + A \rightarrow e' + p + (A-1) \) process. The kinematical restriction is \( x = Q^2 / 2q_0 m_N = 1 \), where \( q_0 \) the virtual photon energy. Let \( M \) be the mass of a state within a nucleon corresponding to the Feynman mechanism. To evaluate \( M \) for the [3q] (three-quarks) system, the average transverse momentum of a leading parton within a nucleon is set to \( k_t = 0.3(0.4) \text{ GeV} / c \). Thus,

\[
M^2 \approx q_t^2 (1-\alpha) / \alpha + \sum_i k_{i}^2 / \alpha_i \approx q_t^2 (1-\alpha) / \alpha + 4k_t^2 / \left(1-\frac{1}{\alpha}\right). \tag{16}
\]

Here, the running masses of quarks are ignored in order to simplify the description and also consider \( q\bar{q} \) as a state rather than a three-quark state, approximating two recoil quarks by one massless parton (accounting for a finite mass of the recoil, two quarks would further increase
For a fixed $k^2, Q^2$, the minimal value of $M^2$ is reached for $(1 - \alpha)/\alpha = \sqrt{4k^2/Q^2}$. Thus, a produced state should have a minimal mass squared around

$$M^2 \geq 2\sqrt{Q^2k^2}$$  \hspace{1cm} (17)$$

For $Q^2 = 7(14) \text{ GeV}^2$, one finds $M^2 \approx 1.6$ to $2.1$ (2 to 3) GeV$^2$.

The coherent length

$$L_c = 2q_0/(M^2 - m^2) = Q^2x_{mN}(M^2 - m^2)$$  \hspace{1cm} (18)$$

Thus, for $Q^2 = 14 \text{ GeV}^2$, Equation (18) leads to $L_c \approx 2$ to $3$ fm. $L_c$ may appear even smaller since here it is ignored that gluons carry a fraction of the nucleon momentum, as well as the finite mass of the recoil two -quark state. Thus, the produced wave packet rapidly expands to the nucleon size. Note also that due to the presence of the growth of $M^2 \propto \sqrt{Q^2}$ with increase in $Q^2$, the increase in $L_c$ is slower than found in Ref [30]. This corresponds to a relatively slow transition to the hard regime accompanied by a CT phenomenon. Experiments at TJNAF (Thomas Jefferson National Accelerator Facility, JLab) did not find a signal of CT in the quasi-elastic processes $e + A \rightarrow e' + p + (A - 1)$ at $Q^2 = 14 \text{ GeV}^2$ [19] which may be due to both the slow increase in squeezing and a small and slow increase with the energy correlation length. Originally, in Ref. [30], the coherence length was estimated using a nonrelativistic quark model, leading to $\Delta M^2 = M^2 - m^2 \sim 0.6 \text{ GeV}^2$. Taking $M \sim 2 \text{ GeV}$, based on the mass spectrum of baryons with the nucleon quantum numbers, would reduce $L_c$ by a factor of five and completely wipe out the CT effect at $14 \text{ GeV}^2$. Hence, the current $14 \text{ GeV}^2$ data cannot a priori distinguish a short coherence length scenario and the Feynman mechanism scenario.

Also it is worth noting that further analysis of the data [19] with realistic spectral functions is necessary since the transparency for scattering off carbon differs by 40% for the $s$- and $p$-shells [34], while the short-range correlations, which constitute 15 to 20% of the spectral function, are not included at all. In addition, the EMC (European Muon Collaboration, CERN) effect leads to a small but non-negligible reduction in transparency, somewhat masking the increase in transparency due to CT [35].

A promising way to investigate the Feynman mechanism as the mechanism of the onset of CT is to investigate the high-$Q^2$ process $e + d \rightarrow e + p + n$ in the kinematics, where the effective distances between nucleons in the deuteron are at approximately 1–1.5 fm. This can be achieved by choosing the momentum of the nucleon spectator to be approximately 200 MeV/c and to investigate the dependence of the cross-section on $Q^2$ at $x \approx 1$. The cross-section has a minimum and maximum as a function of the momentum of the nucleon spectator, which can be used as the effective tool to search for CT [36].

We argued above that the pion field is presumably already suppressed in the hard processes at $Q^2$ of about few GeV$^2$. This should result into the suppression of pion exchange reactions such as $ed \rightarrow e + \Delta^- + \text{slow} \Delta^{++}$ [37].

4. QCD and the Internucleon Interaction

The chiral QCD dynamics modified by including $\Delta$ isobars is used in the calculation of two-pion exchange potential, as can be found, e.g., in [38]. This is an example of how the inner structure of a nucleon enters nuclear theory. The lack of collapse of heavy nuclei and the form of the dominance of the repulsion in $NN$ interaction at short distances in the phenomenological potential of $NN$ interaction suggests the important role of strong internucleon repulsion at the distances $r_{NN} \leq 0.5 \text{ fm}$. The observation of the high momentum tail of the nucleon momentum distribution allows to restrict the form of internucleon interaction [39,40].
A pion is an not elementary particle but a pseudo-Goldstone boson of QCD. With the nuclear density increase, an interacting nucleon loses the pion cloud because the pQCD phase has no pseudo-Goldstone bosons. Thus, the chiral symmetry restoration phase transition starts at the nuclear densities as a factor of $\geq (1.4 \text{ fm}/2r_c)^3$, approximately three times larger than the saturation nuclear density.

Intermediate range internucleon forces are due to exchange by constituent quarks between nucleons. This is because the chiral condensate which gives mass to light quarks is mostly due to the instanton field of a quark; cf. Ref. [7].

The pQCD core within a nucleon has important implications for the theory of super-dense nuclear matter, i.e., for the inner cores of neutron stars. At the nucleon densities where the internucleon repulsion dominates, $r_{NN} \leq 0.5 - 0.4 \text{ fm}$, the pressure from adjacent nucleons leads to the squeezing of a nucleon, i.e., to increase $x_i$ and $k_{it}$ compared to those at the saturation of nuclear density. At these densities, only the pQCD cores of nucleons survives. If the density further increases to $(1.4 \text{ fm}/(0.5-0.7) \text{ fm})^3$, approximately 8 to 22 times the nuclear density, the chiral restoration phase transition should take place. This is obviously a rather rough estimate, which is given here merely for illustration.

The interaction between a quark–gluon core and another nucleon becomes repulsive with the increase in nuclear density. When the internucleon distances itself from $r_{NN} \ll 2r_c \approx 1 \text{ fm}$, the interaction becomes of strong repulsion.

It was suggested [41] that the repulsion is mainly due to the Pauli principle for quarks belonging to the different nucleons.

5. Conclusions

We argued that a nucleon in QCD (quantum chromodynamics) contains three coexisting layers of two QCD phases. An external layer of a nucleon is formed by the pion cloud of a nucleon and dominates in the low-energy nuclear phenomena in the form of the attractive potential of nucleon–nucleon interaction. The perturbative QCD (pQCD) core of a nucleon leads to the existence of a large group of hard processes off nuclear targets, where the onset of the color transparency (CT) phenomena is rapid, as described within the concept of QCD factorization theorems. The nonperturbative phase of spontaneously broken chiral symmetry surrounding the pQCD core of a nucleon reveals itself in the slow onset of a CT phenomenon for quasi-elastic processes off nuclear targets. The account of the Sudakov form factors accelerates the onset of CT but only slightly. Thus, the three-layer structure of a nucleon leads to the dominance of hadron degrees of freedom at the average internucleon distances in nuclei, at the saturation nuclear density, and to the restoration of the chiral symmetry with a further increase in the nuclear density.

Systematic experimental studies of possible CT effects in two-body processes should continue employing the projectiles and produced hadrons of different sizes (direct photons [42], pions, kaons, etc.). One should also study $2 \rightarrow 3$ high-energy processes such as $\pi^- + A \rightarrow \pi^- \pi^- (A - 1)^+$ with back-to-back pions with transverse momentum, $p_t > 1.5 \text{ GeV}/c$ [42] which could be measured in the COMPASS experiment at CERN. In this kinematics, the projectile pion and two final pions remain frozen over distances exceeding by far the size of heavy nucleus, namely, with the coherence length, $L_c \sim 60 \text{ fm}$ [43].

**Author Contributions:** The authors contributed equally to this work. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was supported by the U.S. Department of Energy grant DE-FG02-93ER40771 and by the Binational Science Foundation United States–Israel grant 202115.

**Acknowledgments:** We thank Misak Sargsyan for organizing the volume.
Conflicts of Interest: Authors declare no conflict of interest.

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