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between earlier results [2,3] and those presented here is almost entirely due to the change from eq. (1) to eqs. (2) and (3).

In figure 7 we show a comparison of results for $\alpha_s$ based on lattice and perturbative QCD. Since the analysis of ref. [5] is somewhat different from ours, the results labeled "NRQCD" in figure 7 are different from the values we quote but agree within errors. The quarkonium results are in good agreement with those from other experiments, with comparable uncertainties.

![Figure 7](image)

Figure 7. $\alpha_s^{(5)}(m_Z)$ from lattice QCD in comparison with perturbative results. For recent references see ref. [1].

5. CONCLUSIONS AND OUTLOOK

An important goal of lattice QCD is the determination of Standard Model parameters from experimental input. Among the first results that include all systematic errors are determinations of $\alpha_s$ from quarkonium spectroscopy. This is an important achievement, even though the calculations are not entirely based on first principles.

The phenomenological correction that accounts for the systematic effects from the omitted sea quarks has been tested in two ways. The consistency of the results from $b\bar{b}$ and $c\bar{c}$ spectra is a phenomenological test of the correction. A first principles test with dynamical fermions was provided by the KEK collaboration [10], albeit with large errors.

As already mentioned in the introduction, phenomenological corrections are a necessary evil that enter most coupling constant determinations. However, in the long run, lattice QCD will give us a truly first principles determination of $\alpha_s$ with an accuracy that cannot be matched by any other method. At that point, the MS standard should be replaced with a new one, a coupling that can be computed both in lattice QCD and perturbative QCD.

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Figure 5. The 0 and 3, 4 flavor running of $\alpha_V$ from a matching scale at $q = 700$ MeV to $q = 5$ GeV.

Figure 6. $\alpha_{\text{MS}}^{(5)}(m_Z)$ in comparison; Fermilab (□: $b\bar{b}$, ◊: $c\bar{c}$), NRQCD (△: $b\bar{b}$, ×: $c\bar{c}$), KEK (+), UK(NR)QCD (o), and UKQCD (+).

Table 3

| uncertainty | $b\bar{b}$ (%) | $c\bar{c}$ (%) |
|-------------|---------------|---------------|
| stat., $a''$ | 1 - 3 | 1 - 3 |
| perturbative | 5 | 5 |
| $n_f = 0 \rightarrow n_f = 3, 4$ | 5 | 8 |
| total | 8 | 10 |
| total at $m_Z$ | 5 | 8 |

The $c\bar{c}$ result, $\alpha_{\text{MS}}^{(5)}(m_Z) = 0.110(8)$, is in good agreement with our central result. The difference in the charmonium scale gives consistent results, albeit with large errors.

Table 3 lists all the uncertainties that enter the lattice determination of $\alpha_s$. We quote as our final result the value obtained from the $b\bar{b}$ spectrum:

$$\alpha_{\text{MS}}^{(5)}(m_Z) = 0.108 \pm 0.006 \ .$$

The $c\bar{c}$ result, $\alpha_{\text{MS}}^{(5)}(m_Z) = 0.110(8)$, is in good agreement with our central result. The difference
that was observed in $a^{-1}$ is still present. Also, the 2 flavor coupling [10] is about 10% bigger than the quenched results.

In the next section we will show how, after proper inclusion of sea quark effects, everything falls into place.

4. SEA QUARK EFFECTS

This final correction can, at this point, not be included entirely from first principles. However, it can be estimated using potential model phenomenology and perturbation theory [3,14].

The physical quantity that is used to determine $a^{-1}$ sets the “matching scale”, where by construction the coupling of the quenched lattice potential matches the coupling of the effective 3 flavor potential. The difference in the perturbative (2-loop) evolution of the 0 and 3,4 flavor couplings then causes the quenched coupling at short distances to be too small. This is illustrated in figure 5.

Different “matching scales” for $b\bar{b}$ and $c\bar{c}$ then lead to differences in the quenched coupling at short distances, as observed earlier. The matching scale is, in effect, the average gluon momentum transfer in the 'onium state in question. It can be estimated as the expectation value of the gluon momentum using potential model or lattice wave functions:

$$ q^* = \langle q \rangle_{S,1P,2S,...} $$ (5)
3. RENORMALIZED COUPLING

Within the framework of Lattice QCD the conversion from the bare to a renormalized coupling can, in principle, be made nonperturbatively. One can, for example, define a renormalized coupling from the nonperturbatively computed heavy quark potential (see the talk by Schilling [11]). In [12] a renormalized coupling is defined nonperturbatively through the Schrödinger functional; its running is computed using a finite size scaling technique. Of course, as always, the associated lattice spacing errors have to be carefully removed.

In ref. [13] it was shown that perturbation theory describes all short distance lattice quantities considered there up to a few percent provided a renormalized coupling is used. In ref. [2,3] the MS coupling was determined from the bare lattice coupling using the mean field formula proposed in ref. [13]:
\[ \alpha_{\text{MS}}^{-1}(\pi/a) = 4\pi \langle \text{Tr} U P \rangle / g_0^2 + 0.309 \].  

An alternative is to define a renormalized coupling through short distance lattice quantities, like small Wilson loops or Creutz ratios. For example, the coupling defined from the the plaquette, \( \alpha_P = -3\ln \langle \text{Tr} U P \rangle / 4\pi \), can be expanded in \( \alpha_V \) by [13]:
\[ \alpha_P = \alpha_V(q)[1 - 1.19\alpha_V(q) + O(\alpha_V^2)] \]  

or, equivalently
\[ \alpha_V^{-1}(q) = \alpha_P^{-1} - 1.19 + O(\alpha_V) \].  

where \( q = 3.41/a \) is chosen according to the prescription in ref. [13]. The MS and V couplings are related by:
\[ \alpha_{\text{MS}}^{-1}(q) = \alpha_V^{-1}(q) + 0.822 \].

The size of higher order corrections associated with the above defined coupling constants can be tested by comparing perturbative predictions for short distance lattice quantities with nonperturbative results. It was shown in ref. [13] that, while the mean field coupling (1) accounts for the bulk of the corrections, \( \alpha_V \) from the plaquette yields more accurate results. We will use both eqs. (2) and (3) for the analysis presented here, and average over the results.

In figure 3, the nonperturbative coupling from ref. [12] is compared to perturbative predictions for this coupling using eq. (3) and the 1-loop relations given in ref. [12]. Similarly, in ref. [5] the reliability of \( \alpha_V \) using eq. (2) is estimated, from the spread observed in small Wilson loops and Creutz ratios up to size 3 compared to the plaquette. The next to next to leading order corrections for these quantities have been calculated numerically. We estimate a 5% uncertainty associated with perturbation theory.

Figure 4 shows the zero-flavor MS coupling (using eq. (2)) as a function of \( a^2 \) for \( b\bar{b} \) and \( c\bar{c} \) for the different groups in comparison (with statistical errorbars only). The Fermilab results alone indicate that residual lattice spacing artifacts (higher than \( O(a) \)) are too small to resolve with the statistical errors present. The scatter between results from the different groups, using either \( b\bar{b} \) or \( c\bar{c} \) states, is within the range of statistical errors. The small difference in \( \alpha_s \) from \( b\bar{b} \) as compared to
Table 1
Comparison of various determinations of $\alpha_s$.

| process         | observable                  | theory                        | caveats                        |
|-----------------|-----------------------------|-------------------------------|--------------------------------|
| $e^+e^-$        | had. event shapes, ...      | pert. QCD (nlo, resummed)     | hadronization                   |
|                 | $R_{e^+e^-}, \ T$          | pert. QCD (nlo)               |                                |
|                 | $R_\gamma$                 | pert. QCD (nlo), analyticity  |                                |
| DIS             | $F_3$                       | pert. QCD (nlo)               | higher twist                   |
|                 | $F_2$                       | pert. QCD (nlo)               |                                |
| $p\bar{p}$      | $p\bar{p} \to W + \text{ jets}$ | pert. QCD (nlo)         | large pert. correction         |
| $b\bar{b}, c\bar{c}$ | spin averaged splittings | lattice QCD                  |                                |

2. SETTING THE SCALE

In setting the scale there are two sources of systematic error to be considered, apart, of course, from sea quark effects.

The leading lattice spacing errors of the relativistic Wilson action are spin dependent and therefore do not contribute to spin-averaged quantities. Likewise, for the non-relativistic action, spin-dependent interactions are added as higher dimensional operators; spin-averaged quantities do not depend on the additional couplings.

The scale, $a^{-1}$, depends in principle also on the other parameter which enters the calculation, the quark mass [4]. Table 2 shows that the experimentally observed mass dependence for spin-averaged splittings between the $c\bar{c}$ and $b\bar{b}$ systems is mild.

Figure 1 shows the mass dependence of the 1P-1S splitting in a quenched lattice calculation by the Fermilab group. Note that the splitting is about 10% smaller for $b\bar{b}$ than for $c\bar{c}$. Also shown in figure 1 is the (in-)dependence of the 1P-1S splitting on the improvement term. It is thus clear that spin-averaged splittings give the most accurate determination of the scale, $a^{-1}$.

The NRQCD collaboration fits to the entire spin-averaged spectrum to determine $a^{-1}$ [4,5].

A summary of all the results for $a^{-1}$ is given in figure 2 as a function of the gauge coupling ($\beta \equiv 6/g_0^2$). The comparison of $a^{-1}$ from the $b\bar{b}$ and $c\bar{c}$ spectra shows a small scale dependence which is already apparent in figure 1. This ambiguity can only be resolved with the proper inclusion of sea quark effects. In order to see how consistent the different results for $a^{-1}$ are with each other, we will apply in the following the same analysis in converting from $a^{-1}$ and $\alpha_{\text{lat}} \equiv 4\pi/g_0^2$ to $a_{\text{MS}}^{(5)}$ (5 GeV).

Figure 1. The 1P-1S splitting vs. kinetic mass on the $24^3 \times 48$ lattice at $\beta = 6.1$. ×: Wilson ($c = 0$) action; □: Improved ($c = 1.4$) action.

| Table 2
Spin-averaged splittings in the $\Upsilon$ and $J/\psi$ systems in comparison. |
| quantity | $c\bar{c}$ (MeV) | $b\bar{b}$ (MeV) |
|-----------------|-----------------|-----------------|
| $m(1P - 1S)$    | 456.8           | 452             |
| $m(2S - 1S)$    | 596             | 563             |
| $m(2P - 1P)$    | —               | 359.7           |
The Strong Coupling from Quarkonia

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The status of determinations of $\alpha_s$ from quarkonia using lattice QCD is reviewed. We compare the results with those obtained from perturbative QCD.

1. INTRODUCTION

At present, the QCD coupling, $\alpha_s$, is determined from many different experiments, performed at energies ranging from a few to 90 GeV [1]. In most cases perturbation theory is used to extract $\alpha_s$ from the experimental information. Experimental and theoretical progress over the last few years has made these determinations increasingly precise. However, all determinations, including those based on lattice QCD, rely on phenomenologically estimated corrections and uncertainties, from nonperturbative effects. Table 1 (adapted from S. Bethke [1]) lists the caveats associated with each determination. These effects will eventually (or already do) limit the accuracy of the coupling constant determination. In the case of the lattice determination the limiting uncertainty comes from the (total or partial) omission of sea quarks in numerical simulations. We review the progress that has been made since the first determinations of $\alpha_s$ from the lattice [2,3].

The lattice determination of the strong coupling, $\alpha_s$, requires three ingredients:

1. The experimental input to the lattice determination is usually a mass or mass splitting, from which by comparison with the corresponding lattice quantity the scale, $a^{-1}$, is determined in physical units. For this purpose, one should identify quantities that are insensitive to lattice errors.

2. In the definition of a renormalized coupling, systematic uncertainties should be controllable, and at short distances, its (perturbative) relation to other conventional definitions calculable.

3. Calculations that properly include all sea quark effects do not yet exist. If we want to make contact with the "real world", these effects have to be estimated phenomenologically. One should therefore in "setting the scale" consider physical systems where the reliability of the phenomenological correction can be quantitatively estimated.

As has been pointed out some time ago by Lepage [2], quarkonia are the ideal systems for this program, since systematic errors can be analyzed using potential models.

Several groups presented results for quarkonium spectroscopy, which are reviewed by Davies [4]. The NRQCD collaboration [5,6] has results for the $b\bar{b}$ and $c\bar{c}$ spectra using a nonrelativistic QCD action at next to leading order. The leading order NRQCD action is used by the UK(NR)QCD collaboration [7] who presented results for the spin-averaged $b\bar{b}$ spectrum. The Fermilab group [3,8] has results for both the $b\bar{b}$ and $c\bar{c}$ spectra, using an $O(a)$ improved Wilson action. The UKQCD collaboration [9], using a similar action, looked at the $c\bar{c}$ spectrum. The first calculation of the $c\bar{c}$ spectrum with dynamical ($n_f = 2$ staggered) fermions was presented by Onogi [10] for the KEK collaboration; they looked at the 1P-1S splitting using Wilson and staggered valence fermions.