Is the low-$\ell$ microwave background cosmic?

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The large-angle ($\ell$) correlation of the Cosmic Microwave Background exhibit several statistically significant anomalies compared to the standard inflationary cosmology. We show that the quadrupole plane and the three octopole planes are far more aligned than previously thought ($99.9\%$ C.L.). Three of these planes are orthogonal to the ecliptic at $99.1\%$ C.L., and the normals to these planes are aligned at $99.6\%$ C.L. with the direction of the cosmological dipole and with the equinoxes. The remaining octopole plane is orthogonal to the supergalactic plane at $99.6\%$ C.L.

Much effort is currently being devoted to examining the cosmic microwave background (CMB) temperature anisotropies measured with the Wilkinson Microwave Anisotropy Probe (WMAP) [1–4] and other CMB experiments [5]. While the data is regarded as a dramatic confirmation of standard inflationary cosmology, anomalies exist. In particular the correlations at large angular separations, or low $\ell$, exhibit several peculiarities.

Most prominent among the “low-$\ell$ anomalies” is the near vanishing of the two-point angular correlation function $C(\theta)$ at angular separations greater than about 60 degrees. This was first measured using the Cosmic Background Explorer’s Differential Microwave Radiometer (COBE-DMR) [6] and recently confirmed by observations with WMAP [4]. This anomalous lack of large-angle correlation is connected to the low value of the quadrupole contribution, $C_2$, in a spherical harmonic expansion of the CMB sky

$$\Delta T(\theta, \phi) \equiv \sum_{\ell=1}^{\infty} T_{2\ell} \equiv \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

(with $(2\ell + 1)C_\ell = \sum_m |a_{\ell m}|^2$) – although the smallness of $C_2$ does not fully account for the shape of $C(\theta)$. The significance of the large-angle behaviour of $C(\theta)$, especially in light of the large cosmic variance in $C_2$, is a matter of some controversy. The comparisons are, moreover, confused by the fact that one author may calculate only the probability of the low value of $C_2$, others, such as the WMAP team, calculate the probability of $C(\theta)$ being as low as it is over some range of angles. The issue of what prior probabilities and estimators [7–9] to use further complicates the statistical situation.

While the overall absence of large-scale power has attracted the most attention, several other large-angle anomalies have been pointed out. De Oliveira-Costa et al. [11] have shown that the octopole is unusually planar, meaning that the hot and cold spots of the octopolar anisotropies lie nearly in a plane. The same authors found that the axes $\mathbf{n}_2$ and $\mathbf{n}_3$ about which the “angular momentum” dispersion $\sum_m m^2 |a_{\ell m}|^2$ of the quadrupole and octopole are maximized are unusually aligned, $|\mathbf{n}_2 \cdot \mathbf{n}_3| = 0.9838$. Eriksen et al. [12] found that the deficit in large-scale power is due to a systematic deficit in power between $\ell = 2$ and 40 in the north ecliptic hemisphere compared to the south ecliptic hemisphere. Some of us [13] have shown that the $\ell = 4$ to 8 multipoles exhibit an odd, but very unlikely ($\sim 1\%$ probability), correlation with $\ell = 2$ and $\ell = 3$. These low-$\ell$ anomalies (and others [14]) have all been pointed out before, but no simple connection has been made between them. Here we remedy that situation.

By far the largest signal in the CMB anisotropy is the dipole, recently measured by WMAP [1] to be $(3.346 \pm 0.017)\mu$K in the direction $l = 263.85 \pm 0.71, b = 48.25 \pm 0.04$ in galactic coordinates. This is caused by the motion of the Sun with respect to the rest frame defined by the CMB. As shown by Peebles and Wilkinson [15], the dipole induced by a velocity $v$ is $T(v/c) \cos \theta$, where $\theta$ is measured from the direction of motion. Given $T = (2.725 \pm 0.002)\mu$K [16], one infers that $v \approx 370$ km/s.

The solar motion also implies [15, 17, 18] the presence of a kinematically induced Doppler quadrupole (DQ). To second order in $\beta \equiv v/c \approx 10^{-3}$, an observer moving with respect to the CMB rest-frame sees the usual monopole term with a black-body spectrum, a dipolar term $\beta \cos \theta$ with a dipole spectrum and a quadrupolar term $\beta^2 (3 \cos^2 \theta - 1)$ with a quadrupole spectrum. Higher multipoles are induced only at higher order in $\beta$ and so can be neglected. To first approximation the quadrupole spectrum differs very little from the dipole spectrum across the frequency range probed by WMAP. The DQ is itself a small contribution to the quadrupole. It has a total band-power of only $3.6\mu K^2$ compared to $(154 \pm 70)\mu K^2$ from the cut-sky WMAP analysis [3], $195.1\mu K^2$ extracted [11] from the WMAP Internal Linear Combination (ILC) full-sky map [19], or $201.6\mu K^2$ from the Tegmark et al. full-sky map [10] (henceforward the Tegmark map). Therefore, it is a good approximation to treat the Doppler-quadrupole as having a dipole spectrum plus a small spectral distortion which we shall ignore. We can then readily subtract the DQ from the ILC or Tegmark map. (The ILC and Tegmark maps differ in the amount of spatial filtering used to produce them.)
These are the oriented areas $w^{(\ell,i,j)} = \pm (\hat{v}^{(\ell,i)} \times \hat{v}^{(\ell,j)})$. The overall signs of the area vectors are again unphysical (we take them to point in the north galactic hemisphere), however their magnitudes are not. The area vectors for $\ell = 2, 3$ for the Tegmark map (cf. Fig. 1) are

$$w^{(2,1,2)} = 0.9900(-105^\circ73, 56^\circ62),$$
$$w^{(3,1,2)} = 0.9017(-78^\circ38, 49^\circ76),$$
$$w^{(3,2,3)} = 0.9072(-141^\circ56, 38^\circ96),$$
$$w^{(3,3,1)} = 0.9184(173^\circ77, 79^\circ54).$$

The directions of $w^{(2,1,2)}$ and of de Oliveira-Costa et al.’s $\mathbf{n}_2$ are mathematically equivalent. The small difference is due to the removal of the DQ here in $w^{(2,1,2)}$.

Finally, the magnitudes of the dot products between $w^{(2,1,2)}$ and $w^{(3,1,3)}$ ordered from largest to smallest are:

$$A_1 \equiv |w^{(2,1,2)} \cdot w^{(3,1,3)}| = 0.8509,$$
$$A_2 \equiv |w^{(2,1,2)} \cdot w^{(3,2,3)}| = 0.7829,$$
$$A_3 \equiv |w^{(2,1,2)} \cdot w^{(3,3,1)}| = 0.7616.$$

Using instead the normal vectors $\mathbf{\hat{n}}^{(\ell,i,j)} = w^{(\ell,i,j)}/|w^{(\ell,i,j)}|$, the dot products are:

$$D_1 = 0.9531, \quad D_2 = 0.8719, \quad D_3 = 0.8377.$$

One can see from the large values of all three of the $D_i$ that the quadrupole and octopole are aligned, and that the octopole is unusually planar.

The $A_i$ retain information about both the magnitudes and orientations of the $w^{(\ell,i,j)}$. We have compared their values against $10^5$ Monte Carlos (MCs) of Gaussian random statistically isotropic skies with pixel noise (as in [13]). The probability that the largest of these dot products is at least $A_1$, the 2nd largest at least $A_2$, and the 3rd largest at least $A_3$ is 0.021%. It is 0.11% without the DQ-correction, supporting that this is not just a statistical accident. (The ILC map yields 0.025% and 0.12%.)

Ordering dot products does not induce a well-defined ordering relation for the MC maps [20]. A robust and more conservative statistic is the sum of the dot products, $S = \sum_i A_i$. We find that only 0.128% of the MC maps have a larger $S$ than the DQ-corrected Tegmark map. Thus the quadrupole-octopole correlation is excluded from being a chance occurrence in a gaussian random statistically isotropic sky at $> 99.87\%$ C.L. This result is statistically independent of (though perhaps not physically unrelated to) the lack of power at large angular scales since all the information about the power is contained in the $A^{(\ell)}$, and not in the multipole vectors.

So far we have looked only at the correlation between the CMB quadrupole and octopole. Motivated by the results of Eriksen et al. [12], we next ask whether the quadrupole and octopole correlate with the ecliptic or the galaxy. We notice (see Fig. 1) that three of the four $\mathbf{\hat{n}}^{(\ell,i,j)}$ seem to lie near the ecliptic plane. Their dot products with the north ecliptic pole are (for the Tegmark

Meanwhile, some of us showed [13] that the $\ell$-th multipole, $T_{\ell}$, can instead be written uniquely in terms of a scalar $A^{(\ell)}$ which depends only on the power in this multipole (i.e. on $C_{\ell}$) and $\ell$ unit vectors $\{\hat{v}^{(\ell,i)}|i = 1, ..., \ell\}$. These “multipole vectors” are entirely independent of $C_{\ell}$, and instead encode all the information about the phase relationships of the $a_{\ell m}$. Heuristically,

$$T_{\ell} \approx A^{(\ell)} \prod_{i=1}^{\ell} \hat{v}^{(\ell,i)} \cdot \hat{e},$$

where $\hat{v}^{(\ell,i)}$ is the $i$th multipole vector of the $\ell$th multipole. (In fact the right hand side contains terms with “angular momentum” $\ell-2, \ell-4, ...$. These are subtracted by taking the appropriate traceless symmetric combination, as described in [13].) Note that the signs of all the vectors can be absorbed into the sign of $A^{(\ell)}$. For convenience we take the vectors to point in the north galactic hemisphere. The multipole vectors for $\ell = 2$ and 3 for the DQ-corrected Tegmark map are [in galactic $(l,b)$]

$$\hat{v}^{(2,1)} = (11^\circ26, 16^\circ36),$$
$$\hat{v}^{(2,2)} = (118^\circ87, 25^\circ13),$$
$$\hat{v}^{(3,1)} = (22^\circ63, 9^\circ18),$$
$$\hat{v}^{(3,2)} = (86^\circ94, 39^\circ30),$$
$$\hat{v}^{(3,3)} = (-44^\circ92, 8^\circ20).$$

(A similar analysis has been done for the ILC map. Results are quoted where instructive.)

As described in [13] there are several ways to compare the multipole vectors, however most striking is to compute for each $\ell$ the $\ell(\ell-1)/2$ independent cross-products.
FIG. 2: Cosmic quadrupole plus octopole, $T_{2+3}^{DS}$ (from Tegmark map). Coordinates are as in Fig. 1, background color is 0μK. The ecliptic (dashed line) avoids all extrema.

FIG. 3: Left to right: Cosmic (Doppler-subtracted) quadrupole, octopole, and $T_{2+3}^{DS} + T_4$ from the Tegmark map. The coordinate system and color scale are as in Fig. 2. The ecliptic (dashed line) is noticeably less correlated with these maps than with the quadrupole-octopole map of Fig. 2.
perpendicular to the supergalactic plane. Each of these correlations is unlikely at $\geq 90\%$ C.L., and at least two of them are statistically independent. We have also seen that the ecliptic threads between a hot and a cold spot of the combined Doppler-subtracted-quadrupole and octopole map – following a node line across about 1/3 of the sky, and separating the three strong extrema from the three weak extrema of the map.

We find it hard to believe that these correlations are just statistical fluctuations around standard inflationary cosmology’s prediction of statistically isotropic Gaussian random $a_{lm}$s. That the quadrupole-octopole correlation just happened to increase by $\sim 5$ when the quadrupole was Doppler-corrected seems particularly unlikely. The correlation of the normals with the ecliptic poles suggest an unknown source or sink of CMB radiation or an unrecognized systematic. If it is a physical source or sink in the inner-solar system it would cause an annual modulation in the time-ordered data. An outer solar-system origin (beyond 100 A.U.) might avoid such a signal. It seems likely that a local source (sink) would also show up in polarization maps, especially there would be no reason for B-modes being significantly suppressed with respect to E-modes. Physical correlation of the CMB with the equinoxes is difficult to imagine, since the WMAP satellite does not show up in the difference maps and their power spectra are consistent with the predictions of standard cosmology, then one must reconsider all CMB results within the standard paradigm which rely on low-$\ell$’s, including: the high temperature-polarization correlation $C_{T}^{TE}$ measured by WMAP [1] at very low $\ell$’s (and hence the inferred redshift of reionization); the normalization of the primordial fluctuations (which relies on the extraction of the optical depth $\tau$ from low $\ell$’s); and the running $dn_{s}/d\log k$ of the spectral index of scalar perturbations (which, as noted in [21], depends on the absence of low-$\ell$ TT power).

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