Reconstruction of air-shower parameters for large-scale radio detectors using the lateral distribution

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Abstract

We investigate features of the lateral distribution function (LDF) of the radio signal emitted by cosmic ray air-showers with primary energies \( > 0.1 \text{ EeV} \) and its connection to air-shower parameters such as energy and shower maximum using CoREAS simulations made for the configuration of the Tunka-Rex antenna array. Taking into account all significant contributions to the total radio emission, such as by the geomagnetic effect, the charge excess, and the atmospheric refraction we parameterize the radio LDF. This parameterization is two-dimensional and has several free parameters. The large number of free parameters is not suitable for experiments of sparse arrays operating at low SNR (signal-to-noise ratios). Thus, exploiting symmetries, we decrease the number of free parameters and reduce the LDF to a simple one-dimensional function. The remaining parameters can be fit with a small number of points, i.e. as few as the signal from three antennas above detection threshold. Finally, we present a method for the reconstruction of air-shower parameters, in particular, energy and \( X_{\text{max}} \) (shower maximum), which can be reached with a theoretical accuracy of better than 15% and 30 g/cm², respectively.

Keywords: cosmic rays, extensive air-showers, radio detection, lateral distribution

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1. Introduction

The determination of the composition of the primary particles is one of the most interesting and complicated problems of experimental high-energy cosmic ray physics. Imaging instruments, particularly, fluorescence or Čerenkov detectors, detect cosmic ray air showers with high precision, but their duty cycle is only in the order of 10%. On the other hand, detectors with full duty cycle, such as particle detectors, until now have poor sensitivity to the shower maximum and can not provide accurate studies of the composition. A candidate to solve this dilemma is the radio detection of cosmic rays. Probably it can reach a precision comparable with air-Čerenkov measurements. However, it still has a number of important open issues such as efficiency, systematic uncertainties and precision of the energy and shower maximum reconstruction, all also depending on the detector layout.

In the present paper we perform a detailed theoretical study based on a real large-scale detector layout. We performed about 300 simulations based on the reconstruction of measured high energy Tunka-Rex [1] and Tunka-133 [2] events. To simulate air showers we used CoREAS [3], a software integrated in CORSIKA, which implements the end-point formalism for calculating radio emission from air showers. In comparison with previous works made for ideal detectors (see, for example, [4,5]), LOPES [6] and LOFAR [7], our investigations have several important differences. First, we reproduce detected events with small uncertainty, thus, our simulation could be compared with signals measured by Tunka-Rex, which, in turn, features an absolute amplitude calibration. Second, the geometry of the detector matches modern large scale setups, i.e. the spacing between antennas is about 200 m. Finally, we transform the simulated signals applying the real hardware properties of Tunka-Rex (amplifiers, antennas, etc.), and check the sensitivity of selected antennas. That means, we do a statistical study which gives realistic upper limits for the precision of the reconstruction of air shower properties. “Upper limits” because we do not include noise yet,

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the precision could be slightly worse when taking into account realistic background. Therefore, this limit could be reached in case of large signal-to-noise ratios (SNR).

The complication in describing the radio LDF originates from the interference of two complete different mechanisms of radio emission: synchrotron-like geomagnetic emission, and the Askaryan (also known as charge-excess) effect. Adding these two effects causes an asymmetric two-dimensional lateral distribution function (LDF). There are two obvious approaches to describe this lateral distribution: to use a complex two-dimensional function, or to find some symmetries and rewrite the LDF invariably. The first approach was successfully tested in [7]. It was shown that the LDF can be described with good accuracy, but the method used in this approach requires a big number of points and, thus, a dense array. Because of that we used the second, customized approach. We found an operator reducing the number of dimensions in the LDF representation to one, and converts this function to an azimuthal-symmetric one.

In additional to the simulation reproducing real Tunka-133 events, we also performed an “ideal” simulation using a symmetrical geometry and a three-dimensional dense detector. In difference to the other simulations this is not for a statistical study of air-showers with different $X_{\text{max}}$, but for performing a tomographic study of a mean air-shower (the description of this study is given in Chapter 2.2.). By this, we obtain evidence of a new feature of the radio emission, and found new connections of the parameters of the radio emission with the shower maximum.

1.1. Geomagnetic coordinate system

To perform our calculations and later the reconstruction in an invariant way, we will use the so-called geomagnetic coordinate system, a special version of shower coordinates. The outstanding feature of this system is that the electrical field vector has only two non-zero projections to the axes, the third projection is always close to zero. The basis of this coordinate system takes the form

$$\mathbf{e}_x = \hat{V} \times \hat{B},$$  \hspace{1cm} (1)

$$\mathbf{e}_y = \hat{V} \times (\hat{V} \times \hat{B}),$$  \hspace{1cm} (2)

$$\mathbf{e}_z = \hat{V},$$  \hspace{1cm} (3)

where $\mathbf{V}$ and $\mathbf{B}$ are the shower axis and the Earth magnetic field (hat over vector means normalization, e.g. $\hat{B} = B/|B|$), respectively. Let us also define useful angles: the geomagnetic angle $\alpha_g = \angle(\mathbf{V}, \mathbf{B})$ and the geomagnetic azimuth $\phi_g = \angle(\mathbf{e}_x, \mathbf{e}_y)$.

1.2. One-dimensional lateral distribution function

Corresponding to Allan’s approach [9], the contribution from a single frequency $\nu$ to the total signal $\mathcal{E}$ has the form

$$\mathcal{E}_\nu(r) \sim E_{pr} \sin \alpha_g \int_{h_{\min}^\nu}^{h_{\max}^\nu} \frac{dh}{r} n_e(h) \mathrm{d}h,$$  \hspace{1cm} (4)

where $E_{pr}$ is the energy of primary particle, $r$ is the distance to the shower axis, $h$ is the height on the shower axis, $N_e(h)$ is the number of charged particles at the height $h$. The integration limits $h_{\min}^\nu(r)$ and $h_{\max}^\nu(r)$ strongly depend on the frequency $\nu$. All distances are taken in meters, frequency in MHz. Performing the calculations similar to Allan, one can obtain the following approximation for the lateral distribution function (see [5][10])

$$\mathcal{E}_\eta(r) = \mathcal{E}_\eta \sin \alpha_g \exp[\eta(r - r_0)],$$  \hspace{1cm} (5)

with parameters $\mathcal{E}_\eta$ depending on the characteristics of the specific air-shower. Renormalizing parameter $r_0$ helps to factorize the lateral distribution function for the further reconstruction of the air-shower parameters. In section 3.1.3 we show, that $\mathcal{E}_\eta(r)$, i.e. $N = 2$ (Gauss-type function), provides a good and sufficient description of the radio lateral distribution.

1.3. Simulation sets

All simulations used in the present paper, have been produced with CoREAS [3]. As hadronic interaction model we selected QGSJET-II. As detector layout we used the setup of the Tunka-Rex experiment, which is located at an
altitude of 675 m. The strength of the geomagnetic field was set to $\approx 60 \mu T$, with inclination and declination of about $72^\circ$ and $-3^\circ$, respectively. For the incoming direction and energy we used measured Tunka-133 events from 2012/2013. We selected events satisfying the condition $E_{pr} \sin \alpha_g > 0.05$ EeV Tunka-133 reconstructs only air-showers with zenith angles < 50° due to design restrictions. That way, as initial parameters we used the energy of the primary particle $E_{pr}$, the arrival direction $(\theta, \phi)$, and the core coordinates $(x, y)$ on the detector plane. As primary particle we used the two possible extreme cases for these energies: protons and iron nuclei. Due to the high resolution of the Tunka-133 instrument, we can reproduce real events with high accuracy. The most important unknown parameter in the simulation is the shower maximum. Using different random seeds and primary particles (proton and iron) we try to limit the deviation between simulation and real events to less than 30 g/cm². For the present work we selected about 300 simulated events of each primary particle, using for each event the simulated $(proton and iron)$ we try to limit the deviation between simulation and real events to less than 30 g/cm². For the present work we selected about 300 simulated events of each primary particle, using for each event the simulated shower with smallest deviation between simulated and real $X_{max}$.

Signal transformation and event selection on the detector level are made with the Offline software framework [8]. We used the pattern of the Tunka-Rex antenna type, which is close to a dipole. The frequency range is 30 – 80 MHz. The event reconstruction pipeline is similar to Tunka-Rex, except for the SNR cuts: we do not add noise to the simulations, thus, we put only a threshold on the signal amplitude to reduce the digital noise.

All plots, except Fig. 2, are obtained with the Tunka-Rex layout and Tunka-133 event set. For Fig. 2, we performed a different simulation, as explained in Section 2.2.

2. Asymmetry

Presently there are a number of mechanisms for air-shower radio emission suggested by theorists (see, for example, corresponding chapter in [11]). We will consider only two contributions, which have been proven experimentally and which are the most important and dominant ones: transverse currents (synchrotron-like geomagnetic emission) [12,13] and Askaryan effect [14,15]. The complexity of adding these two contributions rises from the different mechanisms of the emission. While the electrical field of the geomagnetic emission is obtained by integration of $N_e(h)$ and lies along the $\mathbf{v} \times \mathbf{B}$ vector, the Askaryan emission is mostly defined by the derivative $N_e'(h) = dN_e/dh$ and polarized along $\mathbf{v} - \mathbf{V}$, where $\mathbf{v}$ is the velocity of the particle. In our study we therefore assume that total polarization is a sum of two linear polarized contributions with unknown amplitudes.

2.1. Origin of asymmetry

The total electrical field at an antenna at distance $r$ can be represented as vector

$$\mathbf{E} = \mathbf{E}_g + \mathbf{E}_{ce} + \mathbf{E}_v,$$

where $\mathbf{E}_g$ is a geomagnetic contribution, $\mathbf{E}_{ce}$ is a contribution from the Askaryan effect and $\mathbf{E}_v = 0$ is a vertical contribution to the signal. We neglect the contribution from the vertical component, since the angle between the shower plane and radio wavefront is only 1–2° [16]. The signal has the following components in the introduced geomagnetic coordinate system

$$\mathbf{E}_g = (E_g, 0, 0) = (E_0 \sin \alpha_g, 0, 0),$$

$$\mathbf{E}_{ce} = (E_{ce} \cos \phi_g, E_{ce} \sin \phi_g, 0),$$

where $E_g = E_0 \sin \alpha_g \sim E_{pr} \sin \alpha_g$, and $E_{ce} \sim E_{pr}$, and $E_{pr}$ is the energy of the primary particle. We assume that the strength of the radio emission depends linearly on the energy of the electromagnetic component (and, consequently, on the total energy) of the air-shower. Thus, the total amplitude takes the form

$$\mathbf{E} = \mathbf{E}_g + \mathbf{E}_{ce} = (E_0 \sin \alpha_g + E_{ce} \cos \phi_g, E_{ce} \sin \phi_g, 0)$$

The squared amplitude has the form

$$E^2 = (E_0 \sin \alpha_g + E_{ce} \cos \phi_g)^2 + E_{ce}^2 \sin^2 \phi_g$$

$$= E_0^2 \left(\sin \alpha_g + \epsilon \cos \phi_g\right)^2 + \epsilon^2 \sin^2 \phi_g,$$

where the asymmetry is defined as $\epsilon = E_{ce}/E_0$. As we can see, the one-dimensional LDF transforms itself to a two-dimensional one when taking into account the contribution from the charge excess phenomena: $\mathbf{E}(r) \rightarrow \mathcal{E}(r, \phi_g)$. To reduce the number of dimensions back to one, we define a special operator $\hat{K}$ eliminating the azimuthal dependence

$$\hat{K}\mathcal{E}(r, \phi_g) = \mathcal{E}_{cos}(r).$$
Assuming \( \varepsilon_z = \varepsilon_r = 0 \) we can measure only two components of the electrical field

\[
\begin{align*}
\varepsilon_x &= \varepsilon_0 \sin \alpha + \varepsilon_{ce} \cos \phi_g \\
\varepsilon_y &= \varepsilon_{ce} \sin \phi_g
\end{align*}
\]

Solving this system we obtain for the asymmetry \( \varepsilon \)

\[
\varepsilon = \frac{\varepsilon_{ce}}{\varepsilon_0} = \frac{\varepsilon_y / \sin \phi_g}{\varepsilon_x - \varepsilon_y \cot \phi_g} \sin \alpha_g
\]

From Eqs. (10) and (11) it is obvious, that \( \tilde{K} \) takes the form

\[
\tilde{K} = \left( \varepsilon^2 + 2 \varepsilon \cos \phi_g \sin \alpha_g + \sin^2 \alpha_g \right)^{-\frac{1}{2}} ; \quad \tilde{K}_{e}(r, \phi_g) = \varepsilon_{corr}(r) \bigg|_{r=\text{const}} = \varepsilon_0.
\]

In the geomagnetic limit (\( \varepsilon_g \gg \varepsilon_{ce} \)) this is

\[
\tilde{K} = \frac{1}{\sin \alpha_g} ; \quad \varepsilon_{corr} = \varepsilon \sin \alpha_g.
\]

### 2.2. Asymmetry behavior

For the asymmetry reconstruction we use the formulas given in Eq. (12). The distribution of the asymmetry values for different showers is broad. Thus we created a profile distribution for the asymmetry values (see Fig. 1). The results for proton and iron are similar within about 10%.

![Averaged asymmetry profile of the radio lateral distribution for showers initiated by protons and iron nuclei.](image)

Figure 1: Averaged asymmetry profile of the radio lateral distribution for showers initiated by protons and iron nuclei.

From this picture we can conclude that the strength of the asymmetry varies not only from shower to shower, but also with distance to the shower axis. Similar results were obtained in Ref. [17]. However, de Vries gives his own explanation of this effect. He states that the behavior in the near region (up to 150 m from shower axis) is mostly caused by Čerenkov-like effects. His statement does not explain the peak appearing in this region. We give an alternative explanation of the near-distance behavior. The explanation could be the existence of two sources for the charge excess contribution: intensive particle production and inelastic scattering close to the shower axis and...
particle absorption at distances far from the shower axis (the latter generally agrees with de Vries). The behavior of
the production and absorption regions could also depend on the distance to the shower maximum.

To test this statement we study a simulated radio profile of an air-shower in detail: we simulated a vertical air-
shower induced by a proton with initial energy $10^{17}$ eV. The geomagnetic field was set to a strength of about 60 $\mu$T
(similar to the strength at the Tunka valley) with geomagnetic angle $\alpha_g = 45^\circ$. The CORSIKA simulated shower
has its maximum at an atmospheric depth of $636 \ g/cm^2$. We put several detector planes at different observation
levels from 800 to 1000 $g/cm^2$ with steps of 10 $g/cm^2$. Each layer consists of concentric rings with radii from 20 to
300 m with steps of 10 m. Each ring consists of 36 antennas placed with azimuthal steps of 10°. By this we obtain a
tomographic picture of the shower development in the region after $X_{\text{max}}$.

The obtained results show that the behavior of the asymmetry in the absorption region does not depend on the
shower maximum. But the position and height of the peak in the production region has a clear correlation with
distance to the shower maximum (see Fig. 2). As we can see, it has the opposite behavior than expected from the
Čerenkov-like explanation given by de Vries [17]. In case of a Čerenkov-like nature, the peak should move further
away from the shower axis with increasing observation level, but we obtain the opposite trend. Due to attenuation
of the particle production and re-scattering with distance, the region with maximal energy density cools and moves
closer to the axis.

![Figure 2](image.png)

Figure 2: Correlation between height and position of the asymmetry peak in the production region as function of
the observation level of the shower. The results are obtained for a vertical CoREAS shower with a primary energy of
0.1 EeV. The lines connect the points to guide the eye.

Since the production region depends on the shower geometry we neglect it, and can approximate the asymmetry
with a polynomial function $\epsilon_p(r)$ with fixed point $\epsilon_p(0) = 0$

$$\epsilon_p(r) = \sum_{k=0}^{4} a_k r^k.$$ (16)

For simplification we set $a_{4,r} = 0$ and performed a global fit for both types of initial particles. The fit values are given
in Table 1. One can find the mean asymmetry value by solving the equation $\epsilon''_p(r) = 0$. The asymmetry obtained in
this way at $r \approx 150$ m is $\epsilon_{\text{mean}} = 0.11 \pm 0.02$. This value is in agreement with previous observations [18–20].

The next question we studied was: what is the simplest function for description of the asymmetry $\epsilon$ sufficient
for a satisfactory description of the LDF after correction with $\hat{K}(\epsilon)$? To test the quality of the correction we use a
chi-square test. We define the goodness of the correction by the quantity $N(\chi^2)$: the fraction of events passing
the cut $\chi^2/\text{NDF} \leq Q$ when the LDF is fitted with $\hat{K}(\epsilon)\epsilon_0(r)$ (for why we selected $\epsilon_0(r)$, see next section). We start
with the simplest function, a constant value of the asymmetry. To find the optimal value $\epsilon_{\text{mean}}$ we solve the simple
The numerical solution of this equation gives the following values: $\epsilon_{\text{proton}}^{\text{mean}} \approx 0.095$ and $\epsilon_{\text{iron}}^{\text{mean}} \approx 0.075$. If we compare the goodness of the correction made with this constant asymmetry and the parameterized one (see Fig. 3), we can see that the simple constant function gives a better result. This can be explained by neglecting the peak close to the shower axis and big deviations at distances far from the shower axis (a quantitative overestimation of the asymmetry can even decrease the goodness of the correction). Consequently, we use a correction by a constant value $\epsilon = 0.085$ for the further analysis.

Figure 3: Comparison between different methods of correction. The fraction of events $N_x(Q_x, \epsilon)$ is calculated for different forms of LDF $\tilde{K}(\epsilon)\epsilon_x(r)$: uncorrected ($\epsilon = 0$), constant correction ($\epsilon = \epsilon_{\text{mean}} = 0.085$), correction with parameterization ($\epsilon = \epsilon(r)$). Using a constant value for the correction of the azimuthal asymmetry provides the best quality when fitting a one-dimensional LDF.

### 3. Lateral distribution and its connection to the shower parameters

Using simple considerations (see Ref. [10], Chapter 2.3.1) one can state that at the large distances the lateral distribution falls slowly, and the amplitude decreases below any detection threshold. This means, that the far-distance region is not useful for the reconstruction of the shower maximum. The slope function $f_\eta$ would have no complicated features in the near-distance region [4], if we would not take into account refraction in the atmosphere (put refractive index $n_r = 1$). But after introducing the refractive index we immediately obtain Čerenkov-like
For the geometry defined in our simulations, the radius of the Čerenkov ring is at 100-150 m (see Fig. 5). This radius can be calculated from Eq. (5) solving the equation

$$\frac{d}{dr}e_N(r) = 0, \quad r = r(r_0, a_k)$$

(18)

A solution can be found already with \( N = 2 \)

$$r_e = r_0 - \frac{a_1}{2a_2}.$$  

(19)

We make a comparison between different parameterizations and found that for the selected (Tunka-Rex) geometry a Gaussian (i.e \( N = 2 \)) parameterization after asymmetry correction fits almost all events. For \( N = 2 \), the goodness of fit \( N_\chi^2(\chi^2/NDF, 0) > 70\% \), while for a simple exponential LDF the goodness \( N_\chi^2(\chi^2/NDF, 0) \approx 35\% \) (see Fig. 4).

Figure 4: Distribution of the quality of LDF fits for different parameterizations (Eq. 5): exponential (\( N = 1 \)) and Gaussian (\( N = 2 \)). The fraction of events with LDFs corrected \( N_\chi^2(\chi^2/NDF, \epsilon_{\text{mean}}) \) and uncorrected \( N_\chi^2(\chi^2/NDF, 0) \) for the asymmetry is averaged for both primaries (proton and iron).

The properties of the Gaussian LDF such as mean (\( \mu \)) and width (\( \sigma \)) are connected to the distance to the shower maximum (Fig. 5)

$$\mu = r_e = r_0 - \frac{a_1}{2a_2}$$

(20)

$$\sigma = \frac{1}{\sqrt{-2a_2}}$$

(21)

Now, that we found a good description for the lateral distribution valid for our detector, we tested methods for the reconstruction of air-shower parameters. We will follow the ideas developed for optical air-Čerenkov emission, because after asymmetry correction the radio emission behaves similarly.

The energy can be reconstructed by probing the signal amplitude at a defined distance \( r_e \). Theoretical predictions for the optimal distance are about 50-150 m depending on the mass composition and geometry (see Ref. [9] and later). Therefore, in general, the energy and LDF are connected by the following phenomenological relation

$$E_{pr} = \kappa \left( \frac{e(r_0 = r_e)}{\mu V/m} \right)^b,$$

(22)

where \( \kappa \) is an amplitude slope parameter and \( b \) is a power coefficient. To find \( r_e \) we look simultaneously at the correlation between logarithms of energy and amplitude at different \( r_e \) and at the precision of the energy reconstruction using this formula (see Fig. 6). The maximum of the correlation points to the distance optimal for the
energy reconstruction. Since the relative difference between optimal distances $r_{\text{proton}}$ and $r_{\text{iron}}$, and slopes $\kappa_{\text{proton}}$ and $\kappa_{\text{iron}}$ is about 10% only, we selected medium values for the energy reconstruction

$$r_\text{e} = 120 \text{ m},$$
$$\kappa = \frac{422 \text{ EeV}}{\mu\text{V/m}},$$
$$b = 0.93.$$  

The precision of the energy reconstruction using these averaged parameters is less than 10% for both particle types (see Fig. 8).

For the reconstruction of the atmospheric depth of the shower maximum ($X_{\text{max}}$) we use a slope parameter defined as

$$\eta = \frac{df_\text{0}}{dr} = \frac{E'}{E}.$$  

To get rid of the dependence $\eta = \eta(r)$, we normalize $r_\text{0} = r_\text{x}$, thus

$$\eta \bigg|_{r=r_x} = a_1.$$  

This quantity is simply connected to slope method parameters from Refs. [4,17] by the following relation

$$f_{\text{flat}}(r_\text{flat} + r_\text{x}) - f_{\text{steep}}(r_\text{steep} + r_\text{x}) = a_1.$$  

For $X_{\text{max}}$ reconstruction we use the parameterization suggested in Ref. [2]

$$X_{\text{max}} = X_\text{0} / \cos \theta - (A + B \log(\eta + \bar{b})).$$  

This formula is more complicated than the one chosen for energy reconstruction. It has two free parameters $A$ and $B$ which will be obtained from a fit to the simulated showers, one distance-dependent parameter $\eta(r_\text{x})$, and one correction parameter $\bar{b}$. We followed the same procedure as for the energy reconstruction: finding the best correlation between the reconstructed and true shower maximum depending on the point $(r_\text{x}, \bar{b})$ in the two-dimensional space. One can see the correlation in a contour plot (Fig. 7).
Figure 6: Correlation between logarithms of amplitude and energy and precision of the energy reconstruction using Eq. 22 at distances from 40 to 160 m.

Figure 7: Contour plot of the correlation between LDF slope $\eta$ and true $X_{\text{max}}$ depending on the LDF parameter $r_x$ and the free parameter $\bar{b}$. In the chosen range of this two-parametric space, the correlation function behaves analytically and converges around $(b = 0.003, \ r_x = 195)$ and $(b = 0.008, \ r_x = 165)$ for iron and proton primaries, respectively. We have chosen the average point $(b = 0.005, \ r_x = 180)$ for further analysis, marked as star on the plot.

As for the energy reconstruction, parameters in Eq. 26 have about 10% dependence on the particle type. After averaging the parameters and applying the formula, the relative difference between true (simulated) and reconstructed
\( X_{\text{max}} \) values is smaller than 30 g/cm\(^2\) (see Fig. 8). The averaged parameters are

\[
\begin{align*}
    r_x &= 180 \text{ m}, \\
    A &= -1864 \text{ g/cm}^2, \\
    B &= -566 \text{ g/cm}^2, \\
    \bar{b} &= 0.005 \text{ m}^{-1}.
\end{align*}
\]

\[\text{(30)}\]
\[\text{(31)}\]
\[\text{(32)}\]

4. Conclusion

We found a way to describe the lateral distribution of the radio emission for a large-scale detector with antenna spacing in the order of 200 m. Our investigation of the azimuthal asymmetry has shown, that in spite of the complex structure of interference between two different contributions to the radio emission (geomagnetic and Askaryan effects), a simple LDF is sufficient without introducing a large number of arbitrary parameters. After correction for the asymmetry we need only a one-dimensional LDF with 3 parameters to describe Čerenkov-like effects. It was shown that for a non-dense radio detector array (with spacing in the order of hundred meters) using a Gaussian LDF is sufficient for primary energies in the EeV range. For the reconstruction of the energy and shower maximum we used formulas developed for the air-Čerenkov detector Tunka-133. The free parameters of these formulas were fit to CoREAS simulations made for Tunka-Rex events. The comparison between the true and reconstructed values of the simulations can be seen in Fig. 8.

![Figure 8: Comparison between true and reconstructed shower parameters of the CoREAS simulations.](image)

Our results are comparable with the precision of the Tunka-133 host experiment: the precision is better than 15% for the energy reconstruction and better than 30 g/cm\(^2\) for the shower maximum. The precision can be even improved by applying quality cuts. Let us note that about 20% of the uncertainty comes from the unknown chemical composition of the primary cosmic rays (i.e. the precisions for a fixed particle types are about 12% and 25 g/cm\(^2\) for energy and shower maximum, respectively). The reason could be a different distribution of \(X_{\text{max}}\) for the proton and iron primaries, or different shower developments. The developed methods will be used for the data analysis of Tunka-Rex. In future, our methods will be further optimized by exploiting information on the polarization, which is connected to the asymmetry. It is worth noticing that there is no evidence of restriction to our methods to be applied for more inclined events or events with higher energies, or other detectors with similar antenna spacing and frequency range.

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