Research on consistency check of natural storage and enhancement test for missile servo system

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Abstract. In order to check the consistency of two mechanisms for a missile servo system, we conduct two enhancement tests and utilize two sets of test data to establish the accelerated degradation models of the servo system. Based on the above accelerated degradation models and natural storage profile of the servo system, we present and demonstrate a procedure to check the consistency of two mechanisms by checking the correlation of two sets of degradation data. The results indicate that two degradation mechanisms are significantly consistent with each other.

1. Introduction
The missile servo system is a single-use product with long storage periods in its lifecycle. Natural storage temperature of the missile servo system varies with the alternations of day-nights and four seasons, so temperature change rate is regarded as the main stress that influences the storage lifetime of the servo system during storage periods. To accelerate the degradation process of the servo system, we hastened performance degradation of the servo system by reliability enhancement testing (RET) and generate large amounts of performance degradation data. Based on these degradation data, we established the accelerated degradation models to describe the relationships between degradation characteristics and applied stresses. The life or reliability under normal stress conditions can be extrapolated based on accelerated degradation models estimated from various accelerated conditions [1,2]. However, this extrapolating method is based on such a basic assumption: the degradation mechanism under accelerated stress is the same as the one under normal stress [3,4]. It is an important theory basis for reliability enhancement testing. But sometimes accelerated stresses may change degradation mechanism.

Currently, the methods of checking mechanism consistency are classified into two categories: one identified the consistency of mechanisms and identify the stress boundary before accelerated tests, and the other was based on accelerated degradation models after accelerated tests. For example, Guo [5] provided a method to rapidly identify the consistency at the preliminary stage of accelerated test. Feng [6] proposed a method by measuring the difference of the degradation path based on the Spearman rank correlation coefficient. Wang [7] investigated a method based on degradation model after accelerated test. Based on the constant principle of acceleration coefficient, Xi [8] presented an identification method of failure mechanism consistency with taking Gamma process as study object. The literatures [9,10] utilize the grey theory to identify the consistency.

The remainder of this paper is organized as follows. Section 2 presents the stress profiles and
degradation curves of these two tests. Section 3 develops the accelerated degradation models based on test data. Section 4 provides a method to check the consistency of natural storage and enhancement test. Finally, a brief summary is given in Section 5.

2. Enhancement Test and Degradation data
According to standard principles, reliability enhancement tests are usually conducted by applying several accelerated stress levels with simple profiles, such as step-temperature, rapid cyclic-temperature, step-vibration and combined stresses. We design two enhancement tests orderly: a cold step-temperature test and a rapid cyclic-temperature test. Only one sample is provided for testing. The sample goes through a cold step-temperature test and a rapid cyclic-temperature test two test orderly.

A certain performance is chosen as the degradation metric which possesses significant degradation tendency. The stress profiles and performance degradation curves of these two tests are plotted as shown in Figure 1. The solid curves represent the temperature stress profiles of enhancement tests, and the dashed represent performance degradation of the tested system. In the cold step-temperature test, as shown in figure 1(a), degradations are measured and recorded at the end of every temperature step. There is a remarkable increasing tendency for performance degradation below -35°C. In the rapid cyclic-temperature test, as shown in figure 1(b), the degradations are measured at the end of every temperature keep stage. Degradations steadily fluctuate in the first three cycles, and the amplitudes of degradation vibration become large from the forth cycle.

3. Degradation Modeling and Parameter Estimates
Let $Y_i$ be the individual performance degradation of the sample at stress level $i$, $q$ be the number of stress levels, $p_i$ be an accelerated coefficient of the sample at stress level $i$, $\Delta t_i$ be the duration of stress level $i$, then the cumulative degradation after the sample has experienced the $q$th stress level by time $t$ can be expressed by

$$W_q(t) = \sum_{i=1}^{q} p_i \Delta t_i$$

For the cold step-temperature test, we suppose that the total amount of degradation observed at stress level $i$ can be divided into two components: temporary damage and permanent damage

$$\begin{align*}
W_{T_i} &= W_{T_i}^{tem} + W_{T_i}^{per} \\
W_{T_i}^{tem} &= \gamma \cdot T_i \\
W_{T_i}^{per} &= \sum_{j=1}^{i} Y_j = \sum_{j=1}^{i} p_j^{per} \Delta t_j
\end{align*}$$

Figure 1. Stress profiles and performance degradation curves changing with time.
where $W_{tem}^i$ and $W_{per}^i$ represent the temporary and permanent damage at stress level $i$ respectively; $T_i$ is the temperature value of stress level $i$; $\gamma$ is an unknown coefficient on temporary damage; $p_{per}^i$ is an accelerated coefficient on permanent damage at stress level $i$.

Then, the performance degradations per unit time at every stress level can be rewritten as

$$\dot{Y}_i = p_{per}^i + \gamma \cdot \frac{\Delta T_i}{\Delta t_i}, \quad i = 1, 2, ..., q$$

(3)

where $\Delta T_i = T_i - T_{i-1}$ denotes the temperature increment from stress level $i$ to $i$, therefore, $\frac{\Delta r_i}{\Delta t_i}$ is the temperature change rate of stress level $i$.

For the cold step-temperature test, temperature is the main stress, so we adopt a typical temperature accelerated model, i.e., the Arrhenius law. Then, the permanent degradation per unit time can be proposed as

$$p_{per}^i = a \cdot \exp\left(-\frac{b}{T_i}\right)$$

(4)

where $a$ is an unknown constant relevant to the material, shape, and machining process of the sample; and $b = E/k$, here $k$ is Boltzmann’s constant and $E$ is the activation energy of the reaction.

Substituting (4) into (3) yields the relationships between $\dot{Y}_i$ and $T_i$ as follows:

$$\dot{Y}_i = \frac{\gamma}{\Delta t_i} \cdot \Delta T_i + a \cdot \exp\left(-\frac{b}{T_i}\right) + \epsilon_i, \quad i = 1, 2, ..., q$$

(5)

where $\epsilon_i$ denotes random error of $\dot{Y}_i$, which follows a normal distribution, i.e., $\epsilon_i \sim N(0, \sigma_i^2)$. The parameters can be estimated by a non-linear fitting method based on the data $\{\dot{Y}_i, \Delta T_i, \Delta t_i, i = 1, 2, ..., q\}$.

The rapid cyclic-temperature test profile can be divided into two categories of stages: temperature keep stages and temperature change stages. It is assumed that two categories are independent of each other. By the temperature accelerated equation (5), the degradation at the high and low temperature keep stages can be written as respectively

$$\begin{cases} Y_{high} = \gamma \cdot T_{high} + a \cdot \exp\left(-\frac{b}{T_{high}}\right) \cdot \Delta t_{keep} \\ Y_{low} = \gamma \cdot T_{low} + a \cdot \exp\left(-\frac{b}{T_{low}}\right) \cdot \Delta t_{keep} \end{cases}$$

(6)

where $\Delta t_{keep}$ is the duration of temperature keep stage in the rapid cyclic-temperature test profile; $T_{high}$ and $T_{low}$ are the temperature values of high and low temperature keep stages respectively.

As a result of the main stress being temperature change rate, we eliminate the performance degradations generated at high and low temperature keeping stages from the total degradations of every temperature change cycle. The performance degradation increment due to temperature change rate can be obtained by eliminating the high and low temperature degradations, namely

$$Y_{change}^i = Y_{V_i} - Y_{high} - Y_{low}$$

(7)

where $Y_{V_i}$ is the degradation of stress level $V_i$.

Furthermore, the degradation per unit time $\dot{Y}_{change}^i$ can be obtained by

$$\dot{Y}_{change}^i = \frac{Y_{change}^i}{\Delta t_{change}^i}$$

(8)

where $\Delta t_{change}^i$ is the duration of temperature change stage at stress level $V_i$.

Then, the mean and variance of performance degradation per unit time at stress level $V_i$ are
\[
\begin{align*}
\mu_{i}^{\text{change}} &= \frac{\sum_{k=1}^{n} Y_{ik}^{\text{change}}}{n} \\
\sigma_{i}^{2\text{change}} &= \frac{\sum_{k=1}^{n} (Y_{ik}^{\text{change}} - \mu_{i}^{\text{change}})^2}{n - 1}
\end{align*}
\]

(9)

According to the characteristics of temperature change rate, we choose a power law model as the form of the temperature change rate accelerated equation, i.e.,
\[
\begin{align*}
\mu_{i}^{\text{change}} &= \alpha_1 \cdot V_i^{\beta_1} \\
\sigma_{i}^{2\text{change}} &= \alpha_2 \cdot V_i^{\beta_2}
\end{align*}
\]

(10)

where \(\alpha_1, \beta_1, \alpha_2\) and \(\beta_2\) are the unknown parameters that need to be estimated.

Take logarithm on both sides of the equations (10), and we obtain the linear forms as follows:
\[
\begin{align*}
\log \mu_{i}^{\text{change}} &= \log \alpha_1 + \beta_1 \cdot \log V_i \\
\log \sigma_{i}^{2\text{change}} &= \log \alpha_2 + \beta_2 \cdot \log V_i
\end{align*}
\]

(11)

The estimates \(\hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_1, \hat{\beta}_2\) can be obtained by the least square estimation (LSE) method based on the data \(\{\mu_{i}^{\text{change}}, \sigma_{i}^{2\text{change}}, V_i\}\) and the linear equations (11).

4. Consistency Check of Natural Storage and Enhancement Test

In the natural storage profile of missile servo system, there are 11 months in which temperature varies within the range of 15°C~25°C and 1 months in which temperature varies within the range of 5°C~40°C. Therefore, there are 11 months of temperature change rate \(\frac{10^\circ C}{12h\times60\text{min}} = 0.0139^\circ C/\text{min}\) and 1 months of temperature change rate \(\frac{35^\circ C}{12h\times60\text{min}} = 0.0486^\circ C/\text{min}\). The annual mean of temperature change rate is \(\frac{0.0139 \times 11 + 0.0486 \times 1}{12} = 0.0168^\circ C/\text{min}\).

In practice, the missile servo system is tested every half a year and the period of storage is 5 years. The performance degradation data under natural storage condition as shown in table 1.

| Measurements | Cumulative test time (a) | Critical performance degradation |
|--------------|--------------------------|---------------------------------|
| 1            | 0                        | 0                               |
| 2            | 0.5                      | 0.0006                          |
| 3            | 1                        | 0.0013                          |
| 4            | 1.5                      | 0.0020                          |
| 5            | 2                        | 0.0026                          |
| 6            | 2.5                      | 0.0032                          |
| 7            | 3                        | 0.0038                          |
| 8            | 3.5                      | 0.0044                          |
| 9            | 4                        | 0.0052                          |
| 10           | 4.5                      | 0.0058                          |
| 11           | 5                        | 0.0065                          |

We propose a procedure to check the consistency of two degradation mechanisms. The detailed procedure is recommended as follows:

**Step 1:** The performance degradation characteristics are obtained by the method presented in section 3.

The estimates \(\hat{\gamma}, \hat{a}, \hat{b}\) and \(\hat{\sigma}_i^2\) of the formula(5) are obtained by based on the cold step-temperature test data are listed in table 2.
Table 2. The parameter estimations of the cold step-temperature test

| $\hat{\gamma}$ | $\hat{a}$ | $\hat{b}$ | $\sigma_{\hat{\gamma}}^2$ |
|----------------|---------|---------|-------------------------|
| $2.1386 \cdot 10^{-6}$ | $1.1806 \cdot 10^{-5}$ | -0.9957 | $2.5054 \cdot 10^{-10}$ |

The mean $\mu_i^{\text{change}}$ and variance $\sigma_i^{\text{2,change}}$ of degradation per unit time at three levels of temperature change rates can be calculated by the formulas (10). The results are listed in Table 3.

Table 3. Means and variances of degradations per unit time due to temperature change rates

| Temperature change rate ($^\circ$C/min) | $\mu_i^{\text{change}}$ | $\sigma_i^{\text{2,change}}$ |
|--------------------------------------|-------------------------|-------------------------|
| 20 $^\circ$C/min                     | $0.0850 \cdot 10^{-3}$  | $1.4452 \cdot 10^{-8}$  |
| 25 $^\circ$C/min                     | $0.1233 \cdot 10^{-3}$  | $2.0702 \cdot 10^{-8}$  |
| 30 $^\circ$C/min                     | $0.1548 \cdot 10^{-3}$  | $4.7916 \cdot 10^{-8}$  |

Step 2: The accelerated degradation equations are established by using the method presented in section 3.

Based on the data listed in Table 3, the parameter estimations $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_1, \hat{\beta}_2$ are obtained by LSE as shown in Table 4.

Table 4. Parameter estimations of the temperature change rate accelerated equations

| $\hat{\alpha}_1$ | $\hat{\beta}_1$ | $\hat{\alpha}_2$ | $\hat{\beta}_2$ |
|-----------------|-----------------|-----------------|-----------------|
| $1.0067 \cdot 10^{-6}$ | 1.4850 | $2.1852 \cdot 10^{-12}$ | 2.9065 |

Therefore, the temperature change rate accelerated equations for the missile servo system can be written as:

$$
\begin{align*}
\mu_i^{\text{change}}(t) &= 1.0067 \cdot 10^{-6} \cdot V_{1.4850} \cdot t \\
\sigma_{\mu_i}^{\text{2,change}}(t) &= 2.1852 \cdot 10^{-12} \cdot V_{2.9065} \cdot t 
\end{align*}
$$

(12)

Step 3: According to the natural storage profile of the servo system, the mean and variance of performance degradation per half a year can be obtained by the accelerated equations (12). By the B-S cumulative damage theory [11], $\hat{V}_{\text{halfyear}}$ follows a normal distribution, so $\hat{V}_{\text{halfyear}}$ can be obtained by simulating the distribution. The cumulative degradation estimations under natural storage condition can be obtained by accumulating $\hat{V}_{\text{halfyear}}$.

The transformed results from enhancement test to natural storage are shown in Table 5.

Table 5. Transformed results from enhancement test to natural storage

| Measurements | Cumulative test time (a) | Transformed critical performance degradation |
|--------------|--------------------------|---------------------------------------------|
| 1            | 0                        | 0                                           |
| 2            | 0.5                      | 0.0007                                      |
| 3            | 1                        | 0.0013                                      |
| 4            | 1.5                      | 0.0020                                      |
| 5            | 2                        | 0.0026                                      |
| 6            | 2.5                      | 0.0033                                      |
| 7            | 3                        | 0.0040                                      |
| 8            | 3.5                      | 0.0046                                      |
| 9            | 4                        | 0.0053                                      |
| 10           | 4.5                      | 0.0060                                      |
| 11           | 5                        | 0.0066                                      |
Step 4: Compare the above transformed data \( \{ \bar{W}_{01}, \bar{W}_{02}, ..., \bar{W}_{0n_0} \} \) with the actual degradation data under natural storage \( \{ W_{01}, W_{02}, ..., W_{0n_0} \} \), and calculate the Pearson correlation coefficient of two sets of data

\[
\hat{\rho} = \frac{\sum_{j=1}^{n_0} (W_{0j} - \bar{W}_0)(\bar{W}_0 - \bar{W})}{\sqrt{\sum_{j=1}^{n_0} (W_{0j} - \bar{W}_0)^2 \cdot \sum_{j=1}^{n_0} (\bar{W}_0 - \bar{W})^2}}
\]

(13)

where \( \bar{W}_0 = \frac{1}{n_0} \sum_{j=1}^{n_0} W_{0j}, \bar{W} = \frac{1}{n_0} \sum_{j=1}^{n_0} \bar{W}_0 j. \)

According to two sets of data from Table 1 and Table 5, Pearson correlation coefficient during period of 5 years \( \hat{\rho}_5 \) = 0.9996.

Step 5: Determine the correlation criteria. If \( \hat{\rho} \geq 0.7 \), then two sets of data is strongly correlated; If \( \hat{\rho} \leq 0.4 \), then two sets of data have no apparent linear relationship, which means that some new mechanisms may be interpolated due to accelerated stress, hereby we need to review accelerated stress conditions; If \( 0.4 \leq \hat{\rho} \leq 0.7 \), we are unable to draw final conclusions immediately and need to apply other methods to make further determination.

According to the correlation criterion, \( \hat{\rho}_5 \) is greater than 0.7, then there are strong correlations between two degradation mechanisms. Therefore, we can draw a conclusion that two degradation mechanisms are consistent with each other.

5. Concluding Remarks

In this paper, we utilize the degradation data obtained from the rapid cyclic-temperature test as well as the step-temperature test to develop the performance degradation models based on the theory of probabilistic physics, and apply a method based on Pearson correlation coefficient and differences analysis to check the consistency of degradation mechanisms of the missile servo system under the natural storage condition and enhancement test condition. The paper provides a way to validate the effectiveness of accelerated equations and the consistency check of various degradation mechanisms.

6. References

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