Testing two-nucleon transfer reaction mechanism with elementary modes of excitation in exotic nuclei

R.A. Broglia$^{1,2}$, G.Potel$^{3,4}$, A. Idini$^5$, F. Barranco$^6$ and E. Vigezzi$^7$

$^1$Dipartimento di Fisica, Università di Milano, Via Celoria 16, I-20133 Milano, Italy
$^2$The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen, Denmark
$^3$National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA
$^4$Lawrence Livermore National Laboratory L-414, Livermore, CA 94551, USA
$^5$Department of Physics, University of Jyväskylä, FI-40014 Jyväskylä, Finland
$^6$Departamento de Física Aplicada III, Escuela Superior de Ingenieros, Universidad de Sevilla, Camino de los Descubrimientos, Sevilla, Spain and
$^7$INFN Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy

Nuclear Field Theory of structure and reactions is confronted with observations made on neutron halo dripline nuclei, resulting in the prediction of a novel (symbiotic) mode of nuclear excitation, and on the observation of the virtual effect of the halo phenomenon in the apparently non-halo nucleus $^7$Li. This effect is forced to become real by intervening the virtual process with an external (t,p) field which, combined with accurate predictive abilities concerning the absolute differential cross section, reveals an increase of a factor 2 in the cross section due to the presence of halo ground state correlations, and is essential to reproduce the value of the observed $d\sigma(7\text{Li}(t,p)9\text{Li})/d\Omega$.

I. FOREWORD

At the basis of single-particle motion, fermionic elementary modes of nuclear excitation, one finds delocalization, measured by the quantality parameter ($q \ll 1$ localisation, $q \sim 1$ delocalization $[1]$), ratio of the kinetic energy (ZPF) of confinement, and of the strength of the NN-interaction ($V_0 = -100 \text{ MeV}, a \approx 1 \text{ fm}$),

$$q = \frac{\hbar^2}{ma^2 |V_0|} \approx 0.4.$$  \hfill (1)

At the basis of BCS pairing one finds Cooper pairs and independent pair motion, in which the partner nucleons are correlated over distances of the order of

$$\xi = \frac{\hbar v_F}{2E_{corr}} \approx 20\text{fm}$$ \hfill (2)

in keeping with the value of $E_{corr} \approx 1 - 1.5 \text{ MeV}$ (see e.g. [2]) displayed by pair addition and pair subtraction modes $[3, 5, 6]$ around closed shell nuclei ($E_{corr} \approx \Delta$ in superfluid systems ($\approx 1.5 \text{ MeV}$ in $^{120}$Sn) $[4, 6]$), and the fact that, for nuclei along the stability valley, $v_F/c \approx 0.3$. The (generalised) quantality parameter associated with Cooper pairs can be redefined as

$$q' = \frac{\hbar^2}{2m\xi^2} \frac{1}{2E_{corr}} \approx 0.02,$$ \hfill (3)

implying localization. In other words, in a Cooper pair, each nucleon is solidly anchored to its partner leading to an emergent property: rigidity in gauge space. In keeping with the fact that the Cooper pair transfer cross section $\sigma \sim \sum_{\nu>0} U_{\nu}V_{\nu} = (\Delta/G)^2 \sim (N(0))^2$ is proportional to the square of of the density of levels $N(0)$, Cooper tunneling takes place essentially as successive transfer (without breaking the pair) as a particle of mass $2m$ which sets instantaneously into rotation (vibration) superfluid (normal) nuclei, in gauge space $[3]$. Adding to independent particle motion and pair addition and subtraction modes correlated particle-hole vibrations, complete the elementary modes of excitation count $[3]$ around closed shell nuclei. This basis of states is able to provide a first overall picture of the low energy spectra as probed by nuclear reactions.

However, the basis is non-orthogonal and violates Pauli principle, in keeping with the fact that all the degrees of freedom of the nucleus are exhausted by the nucleonic degrees of freedom. Pauli exchanging and orthogonalizing it with the help of NFT rules $[7]-[10]$, together with two-nucleon transfer reaction theory (second order DWBA describing simultaneous and successive transfer corrected for non-orthogonality, see refs. $[11]-[13]$ and refs. therein), one can calculate the variety of absolute cross sections and transition probabilities which can be directly compared with the experimental data.
II. PAIRING VIBRATIONS OF N=6 MAGIC NUMBER ISOTOPES

As a result of the (mainly quadrupole) dressing and Pauli exchange of the 2s_{1/2} and of the 1p_{1/2} orbitals respectively \[14\] \[15\], parity inversion takes place in an island of light nuclei at the drip line. As a result, the N = 8 closed shell dissolves, N = 6 becoming a novel magic number. This has profound effects in the associated (multipole) pairing vibrational spectrum. In particular for ^7Li, in which case one is confronted with exotic monopole and dipole pair addition modes \(^{13}\)Li(gs)>, \(^{11}\)Li(1\(^-\); 0.4 MeV) > namely the Giant Dipole Pygmy Resonance (GDPR) and with an, apparently, normal pair removal mode \(^{7}\)Li(gs)>. At the basis of the almost degenerate \(0^+\) and \(1^-\) pair addition modes one finds the fact that in \(^{10}\)Li (not bound) the s\(_{1/2}\) and p\(_{1/2}\) orbitals are both at threshold lying close in energy \(\epsilon_{s_{1/2}} \approx 0.2\) MeV, \(\epsilon_{p_{1/2}} \approx 0.5\) MeV. They are thus not available to contribute to standard nuclear Cooper pairing \(^1\)S\(_0\) short range NN-potential. Induced pairing becomes overwhelming. In keeping with the heavily dressed inverted pairing s, p orbitals, the GDPR mode \((E_x \leq 1\) MeV, \(\approx 8\%\) of TRK) exchanged between \(s^2_{1/2}(0)\) and \(p^2_{1/2}(0)\) configurations provides most of the glue binding the halo neutron Cooper pair to the core \(^9\)Li \[14\], as testified by \(^1\)H\(^{11}\)Li,\(^9\)Li(gs)\(^3\)H absolute cross section. The population of the first excited state of \(^9\)Li \((1H\(^{11}\)Li,\(^9\)Li(1/2\(^-\)))\(^3\)H) provides information on phonon induced pairing mechanism \[16\] \[18\]. This is the reason why the pair of symbiotic states under discussion are boxed in Fig. 1. They are expected to be a new (composite) elementary mode of excitation.

Turning back to the probing of this \(1^-\) mode, it could be illuminating in shedding light into its actual structure (low energy E1-vortex-like mode, i.e. a Cooper pair with angular momentum and parity 1\(^-\)), to carry out the \(^9\)Li(t,p)\(^{11}\)Li reaction. Aside from weak Q-value effects and simple geometrical factors, one will be able to relate the "intrinsic" contribution to the absolute cross sections associated with the population of the ground state and of the 1\(^-\) state\[1\]. One can then test the role ground state correlations (gsc) play in both states. To the extent that the 1\(^-\) state can be viewed as a particle-hole-like (2qp) excitation, gsc will decrease the cross section, the inverse being expected to be the case if this state is the dipole pair addition mode of \(^9\)Li (vortex-like Cooper pair). These effects should be reversed concerning the intensity of the \(\gamma\)-decay, as discussed in [19]. How these relations get qualified in the case of the exotic system under discussion is an open question, which may benefit from the analogies to be drawn concerning the situation encountered in connection with the first \(0^+\) excited state of \(^{12}\)Be.

In fact, it is posited that the pair of \(0^{++}\), 1\(^-\) (boxed) states of \(^{12}\)Be displayed in Fig. 1, are (part of ?) the corresponding symbiotic states of \(^{11}\)Li, modified by the extra binding energy provided by the fourth proton. In this case, the possibility of studying this new proposed elementary mode of excitation with a variety of probes is richer, due to the greater stability of the \(^{12}\)Be(\(0^{++}\), 2.24 MeV)> state as compared to the \(^{11}\)Li(gs)>.

It is quite suggestive the presence in \(^{12}\)Be, of a quadrupole pair addition mode almost degenerate with the halo monopole pair addition mode \(0^{++}\). One can thus expect important quadrupole dynamic deformation effects resulting from this degeneracy. Within this context, parity inversion arises because of Pauli repulsion between the p\(_{1/2}\) nucleon in \(^{10}\)Li (\(^{11}\)Be) and that participating in the quadrupole vibration of the core (\(^9\)Li, \(^{10}\)Be). The polarization self energy processes make the s\(_{1/2}\) particle heavier and thus closer to becoming bound \[20\] \[21\], see also \[22\].

The fact that one is now able to accurately calculate two-nucleon transfer absolute differential cross sections \[13\] opens a number of possibilities, in particular to find new elementary modes of excitation in exotic nuclei. A simple, but nonetheless instructive example of the consistency of the physics and associated accuracy of the results which is at the basis of clothed, physical elementary modes of excitation as building blocks of the nuclear spectrum, is provided by the \(^{3}\)Li(t,p)\(^{4}\)Li (gs) absolute differential cross section. As seen from Figs. 1 and 3, theory provides an accurate account of the experimental findings \[18\] \[23\]. The two-nucleon spectroscopic amplitudes were calculated by solving the \(\alpha = -2\) monopole dispersion relation \[3\] \[18\] in the RPA. The results are shown in Fig. 2. Eliminating ground state correlations theory underpredicts experiment by about 50\% (cf. Fig. 3). In other words, even the ground state of an apparent "normal" nucleus like \(^7\)Li (\(S_{2n}\) = 12.91 MeV), resents of the properties displayed by the exotic nucleus \(^{11}\)Li(gs). In fact, the population of the pair removal mode through ground state correlation proceeds by the pick-up of s, p parity-inverted orbits, typical of the neutron halo pair addition mode.

Within this context one expects that much insight on the interplay between the GPDR and the monopole neutron halo pair addition modes emerges from the systematic study of the reactions \(^{10}\)Be(p,t)\(^{8}\)Be(gs), \(^8\)Be(t,p)\(^{10}\)Be, \(^{10}\)Be(t,p)\(^{12}\)Be, \(^{12}\)Be(p,t)\(^{12}\)Be as well as those associated with (p,p2n) knockout reactions and eventually 2n-transfer induced by heavy ions (e.g. \(^{18}\)O,\(^{16}\)O) (Fig. 1). An important example of such insight is provided by the fact that while the cross sections associated with the ground state and two-phonon (normal) monopole pairing vibrational states \((E_x \approx 4.8\) MeV in \(^{10}\)Be), i.e. \(d\sigma^{(8}\Be(t,p)\^{10}\Be(gs))/d\Omega\) and \(d\sigma^{(8}\Be(t,p)\^{10}\Be(0^+; 4.8\) MeV))/d\Omega are expected to have the same order of magnitude (cf. Fig. 13 of \[13\]), that associated with the \(0^{++}\) state in \(^{12}\)Be is predicted to be much smaller (observable?), reflecting

---

1 These two states paradigmatically represent the competition between paired and aligned coupling schemes, which play such an important role in defining e.g. quadrupole shape transitions (see ref. \[3\] and refs. therein).
FIG. 1. Monopole pairing vibrational modes associated with $N = 6$ parity inverted closed shell isotopes, together with low-energy E1-strength modes. The levels are displayed as a function of the two-neutron separation energies $S(2n)$. These quantities are shown in parenthesis on each level, the excitation energies with respect to the ground state are quoted in MeV. Absolute differential cross sections from selected (t,p) and (p,t) reactions calculated as described in the text (cf. [17, 18]), in comparison with the experimental data [23, 24].

FIG. 2. RPA wavefunction of the pair removal mode $|gs(7Li)\rangle$ of the closed shell $N = 6$ parity inverted system $^9Li$ obtained solving the dispersion relation graphically displayed in the upper part of the figure.
FIG. 3. Absolute differential cross section associated with the reaction $^7\text{Li}(t,p)^9\text{Li}(g.s.)$, $E_t = 15$ MeV calculated making use of the forward-going and backwards going amplitudes displayed in Fig. 2. The dashed curve corresponds to the result obtained by neglecting the backwards going amplitudes, normalising the X’s to 1 (TD approximation). In the inset the variety of contributions (successive, simultaneous, non-orthogonality) to the cross section are shown.

the poor overlap between halo and core nucleons [21] (within this context see Table 3 of ref. [18] and associated discussion).

Arguably, one would be able to state that a real understanding of the neutron halo pair addition pattern displayed in Fig. 1 has been obtained, once the two-nucleon transfer predictions are tested, supplemented with one-particle and $\gamma$–decay data, worked out making use of microscopically calculated optical (polarization) potentials, with the help of the same physical modes to be probed.

[1] B.R. Mottelson, Elementary features of nuclear structure, in Trends in nuclear physics, 100 years later, Proc. of Les Houches summer school on theoretical physics, Session LXVI, eds. H. Nifenecker, J.-P. Blaizot, G. F. Bertsch, W. Weise, F. David, Elsevier (Amsterdam), p. 25 (1998)
[2] R.A. Broglia, V. Paar and D.R. Bès, Diagramatic perturbation treatment of the effective interaction between two-phonon states in closed shell nuclei: the $J^{π} = 0^{+}$ states in $^{208}\text{Pb}$, Phys. Lett. B 37 (1971) 159
[3] A. Bohr, Elementary modes of nuclear excitation and their coupling, in Comptes Rendus du Congrès International de Physique Nucléaire, Vol. I, P. Gugenberger ed., Editions du Centre Nationale de la Recherche Scientifique, Paris (1964), p. 487
[4] A. Bohr and B.R. Mottelson, Nuclear structure, Vol. II, Benjamin, New York (1975)
[5] D. R. Bes and R.A.Broglia, Paarring vibrations, Nucl. Phys. 80, 289 (1966)
[6] A. Bohr and B.R. Mottelson, Nuclear structure, Vol. I , Benjamin, New York (1969)
[7] D. R. Bès, R.A.Broglia, G. G. Dussel, R. J. Liotta and H. M. Sofia, The nuclear field treatment of some exactly soluble models, Nucl. Phys. A260, 1 (1976)
[8] D. R. Bès, R.A.Broglia, G. G. Dussel, R. J. Liotta and H. M. Sofia, Application of the nuclear field theory to monopole interactions which include all the vertices of a general force, Nucl. Phys. A260, 27 (1976)
[9] D. R. Bès, R.A.Broglia, G. G. Dussel, R. J. Liotta and R. J. Perazzo, On the many-body foundation of the nuclear field theory, Nucl. Phys. A260, 77 (1976)
[10] P. F. Bortignon, R.A. Broglia, D. R. Bès and R. Liotta, *Nuclear field theory*, Phys. Rep. **30C**, 305 (1977)
[11] R.A. Broglia and A. Winther, *Heavy ion reactions*, Addison-Wesley, Menlo Park (1999)
[12] R.A. Broglia, O. Hansen and C. Riedel, *Two-neutron transfer reactions and the pairing model*, Adv. Nucl. Phys. **6** (1973) 287 (see www.mi.infn.it/vigezzi/BHR/BrogliaHansenRiedel.pdf)
[13] G. Potel, A. Idini, F. Barranco, E. Vigezzi and R.A. Broglia, *Cooper pair transfer in nuclei*, Rep. Prog. Phys. **76**, 106301 (2013)
[14] F. Barranco, P.F. Bortignon, R.A. Broglia, G. Colò and E. Vigezzi, *The halo of the exotic nucleus $^{11}\text{Li}$: a single Cooper pair*, Eur. Phys. J. A **11**, 385 (2001)
[15] H. Sagawa, B.A. Brown and H. Eabensen, *Parity inversion in the $N=7$ isotones and the pairing blocking effect*, Phys. Lett. B **309** (1993) 1
[16] I. Tanihata et al., *Measurement of the two-halo neutron transfer reaction $^1\text{H}(^{11}\text{Li},^9\text{Li})^3\text{H}$ at 3 MeV*, Phys. Rev. Lett. **100** (2008) 192502
[17] G. Potel, F. Barranco, E. Vigezzi and R.A. Broglia, *Evidence for phonon mediated pairing interaction in the halo nucleus $^{11}\text{Li}$*, Phys. Rev. Lett. **105** (2010) 172502
[18] G. Potel, A. Idini, F. Barranco, E. Vigezzi and R.A. Broglia, *Nuclear field theory predictions for $^{11}\text{Li}$ and $^{12}\text{Be}$: shedding light on the origin of pairing in nuclei*, Phys. At. Nucl. **77** (2014) 941
[19] R.A. Broglia, C. Riedel and T. Udagawa, *Coherence properties of two-neutron transfer reactions and their relation to inelastic scattering*, Nucl. Phys. A **169** 225 (1971)
[20] F. Barranco, P.F. Bortignon, R.A. Broglia, G. Colò and E. Vigezzi, *The halo of the exotic nucleus $^{11}\text{Li}$: a single Cooper pair*, Eur. Phys. J. A **11**, 305 (2001)
[21] G. Gori, F. Barranco, E. Vigezzi and R.A. Broglia, *Parity inversion and breakdown of shell closure in Be isotopes*, Phys. Rev. C **69**, 041302(R) (2004)
[22] I. Hamamoto and S. Shimoura, *Properties of $^{12}\text{Be}$ and $^{11}\text{Be}$ in terms of single-particle motion in deformed potential*, J. Phys. G **34**, 2715 (2007)
[23] P.G. Young and R.H. Stokes, *New states in $^9\text{Li}$ from the reaction $^7\text{Li}(t,p)^9\text{Li}$*, Phys. Rev. C **4** (1971) 1597
[24] H. T. Fortune, G.B. Liu and D.E. Alburger, *$(sd)^2$ states in $^{12}\text{Be}$*, Phys. Rev. C **50** (1994) 1355