remote information concentration by GHZ state and by bound entangled state

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We compare remote information concentration by a maximally entangled GHZ state with by an unlockable bound entangled state. We find that the bound entangled state is as useful as the GHZ state, even do better than the GHZ state in the context of communication security.

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I. INTRODUCTION

As a peculiar feature of quantum formalism without classical counterpart, quantum entanglement has already been at the heart of the emerging quantum information theory. The two-qubit maximally entangled state has produced highly nonintuitive effects such as quantum teleportation \cite{1}, quantum dense coding \cite{2}, quantum cryptography \cite{3}. In practice, one usually deals with noisy entanglement represented by mixed state of a composite system. All the noisy entanglement of two-qubit system can be distilled to the singlet form \cite{4}. But, for the mixed entangled states of more-than-two-qubit systems, there are two qualitatively different kinds of entanglement: free entanglement which can always be distilled and bound one which cannot be brought to the singlet form only by local quantum operations and classical communication (LOCC) \cite{5}. In the last few years, the bound entangled states of the multipartite systems is under extensive research because it arouses a deeper understanding of the entanglement and the nonlocality of quantum states \cite{6,7,8,9}.

On the other hand, the bound entanglement seems to be useless for quantum information work such as reliable transmission of quantum data via teleportation \cite{5,10}. However, it is not the case. Recent research has shown that in a sense many copies of a bound entangled state can be pumped into a free entangled state \cite{11}. And it has been found that entanglement can be distilled from two copies of a bound entangled state distributed to different parties \cite{12}. Namely, even a small amount of bound entanglement can be activated \cite{13} and used to process quantum information \cite{14}. More recently, it has been demonstrated that a single copy of a bound entangled state can perform information work better than classically correlated states \cite{15}. That is, a single bound entangled state alone is useful for quantum information processing. Here we give an example to demonstrate that in a sense a single bound entangled state performs as good as the maximally entangled GHZ state.

In this paper we consider that quantum information originally from a single qubit, but now asymmetrically distributed into three spatially separated qubits, is remote concentrated back to a single qubit by an initially shared four-partite GHZ state without performing any global operations. This quantum information work can also be achieved by an initially shared four-partite unlockable bound entangled state \cite{16}. We discuss remote information concentration in detail by GHZ state in Sec.II and by the bound entangled state in Sec.III, then give our discussion and conclusion in Sec.IV.

II. REMOTE INFORMATION CONCENTRATION BY GHZ STATE

For simplicity, we focus on the protocol of three parties.

Supposing that three separate parties Alice, Bob and Charlie hold three qubits A, B and C, respectively. The three qubits are in an asymmetric telecloning state

$$\vert \psi \rangle_{ABC} = \alpha \vert \phi_0 \rangle_{ABC} + \beta \vert \phi_1 \rangle_{ABC} ,$$  \hspace{1cm} (1)

where $\alpha$ and $\beta$ are real numbers and satisfies $\alpha^2 + \beta^2 = 1$. The states $\vert \phi_0 \rangle_{ABC}$ and $\vert \phi_1 \rangle_{ABC}$ is defined as

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where \( q = 1-p, p > q, N \) is a normalization factor given by \( N = 1 + p^2 + q^2 \). That is, the information of an unknown state \( |\chi\rangle = \alpha |0\rangle + \beta |1\rangle \) is diluted to a composite system consisting of qubits A, B and C by an asymmetric telecloning process \[4\]. Now the task is to concentrate the information back to a single qubit without the collective operations between the three qubits. The schematic picture of the remote information concentration is illustrated in Fig.1.

The first protocol we proposed is to share in advance a four-partite GHZ state

\[
|\varphi\rangle_{DEFG} = \frac{1}{\sqrt{2}}(|0\rangle_D |0\rangle_E |0\rangle_F |0\rangle_G + |1\rangle_D |1\rangle_E |1\rangle_F |1\rangle_G)
\]

between three parties Alice, Bob and Charlie, who hold the qubits E, F and G respectively. Let David hold the qubit D. Then the three parties Alice, Bob and Charlie perform the Bell-state measurements (BSMs) on the respective pairs of qubits, and inform David of the results of the measurements. Each of the three parties gets one of four possible outputs \( \{\Phi^i_(i=0,1,2,3)\} \) of the BSM, where \( \Phi^i_(i=0,1,2,3) \) represent the four Bell states \( |\Phi^0\rangle = (|00\rangle \pm |11\rangle) / \sqrt{2} \), \( |\Phi^2\rangle = (|01\rangle \pm |10\rangle) / \sqrt{2} \). Each of the four possible outputs is associated with the corresponding Pauli operators \( \{\sigma^i_(i=0,1,2,3)\} \), where \( \sigma_0 = I, \sigma_1 = \sigma_x, \sigma_2 = \sigma_x, \sigma_3 = \sigma_y \). According to their results of the measurements David rotates his qubit D, finally obtains the state \( |\chi\rangle \) on the qubit D.

The information concentration can be understood in a formular way. We write the states \( |\psi\rangle_{ABC} \) and \( |\varphi\rangle_{DEFG} \) together:

\[
|\psi\rangle_{ABC} |\varphi\rangle_{DEFG} = (\alpha |\phi_0\rangle_{ABC} + \beta |\phi_1\rangle_{ABC}) \otimes \frac{1}{\sqrt{2}}(|0\rangle_D |0\rangle_E |0\rangle_F |0\rangle_G + |1\rangle_D |1\rangle_E |1\rangle_F |1\rangle_G)
\]

A lengthy but straightforward calculation gives that if the combining result of the three BSMs is one of the set

\[
\{\Phi^0_{AB}, \Phi^0_{BC}, \Phi^0_{CG}, \Phi^0_{AE}, \Phi^0_{BF}, \Phi^0_{CG}, \Phi^1_{AB}, \Phi^1_{BC}, \Phi^1_{CG}, \Phi^1_{AE}, \Phi^1_{BF}, \Phi^1_{CG}, \Phi^2_{AB}, \Phi^2_{BC}, \Phi^2_{CG}, \Phi^2_{AE}, \Phi^2_{BF}, \Phi^2_{CG}, \Phi^3_{AB}, \Phi^3_{BC}, \Phi^3_{CG}, \Phi^3_{AE}, \Phi^3_{BF}, \Phi^3_{CG}, \Phi^4_{AB}, \Phi^4_{BC}, \Phi^4_{CG}, \Phi^4_{AE}, \Phi^4_{BF}, \Phi^4_{CG}\},
\]

which are associated with the Pauli operator \( \sigma_0 = I \), the state of the qubit D is projected to the state \( |\chi\rangle = \alpha |0\rangle + \beta |1\rangle \). Analogously, there is a Pauli operator \( \sigma^j \) pertaining to the combining result of the three BSMs from each of other sets. The Pauli operator \( \sigma^j \) is the product (up to a global phase) of the respective Pauli operators pertaining to the three BSMs. That is, up a global phase factor, \( \sigma^j \) is equal to \( \sigma^j_{AE, \sigma^j_{BF}, \sigma^j_{CG}} \), where \( l, j, k \) are 0, 1, 2, 3 and the subscripts denote the BSMs on the corresponding pairs of qubits. Then David performs the operator \( \sigma^j \) on his qubit D to retrieve the state \( |\chi\rangle \). So the protocol achieves the task: \( |\psi\rangle_{ABC} \rightarrow |\chi\rangle \) with certainty by pre-sharing a four-partite GHZ state. The task of remote information concentration in \[4\] is the special case when \( p = q = \frac{1}{2} \).

It is worth noting that the distribution probability of the combining results of the three BSMs reveals some information of the state \( |\psi\rangle_{ABC} \). For instance, while we get \( \Phi^0_{AE} \Phi^0_{BF} \Phi^0_{CG} \) with probability of \( \frac{1}{16N} \), we obtain \( \Phi^2_{AE} \Phi^2_{BF} \Phi^2_{CG} \) with the probability of \( \frac{p^2}{16N} \) and \( \Phi^4_{AE} \Phi^4_{BF} \Phi^4_{CG} \) with the probability of \( \frac{q^2}{16N} \).

Consequently, by utilizing a pre-sharing four-partite GHZ state a remote information concentration by LOCC can be achieved with certainty. And the distribution of the combining results of the three BSMs reveals some information of the state \( |\psi\rangle_{ABC} \).

### III. REMOTE INFORMATION CONCENTRATION BY BOUND ENTANGLED STATE

In the above discussion a remote information concentration is achieved by sharing a GHZ state and LOCC. In the following we demonstrate that the information work can also be implemented by an unlockable bound entangled state and LOCC.
In order to concentrate the information diluted in the state $|\psi\rangle_{ABC}$ back to the state $|\chi\rangle$ on a single qubit, a four-partite unlockable bound entangled state

$$\rho_{DEFG}^b = \frac{1}{4} \sum_{i=0}^{3} |\Phi_i\rangle_{DE} \langle \Phi_i| \otimes |\Phi_i\rangle_{FG} \langle \Phi_i|,$$

(5)

where $|\Phi_i\rangle$ is defined as before, is pre-shared between Alice, Bob, Charlie and David. In the same way as above, the qubit E is sent to Alice, the qubit F to Bob, the qubit G to Charlie and the qubit D to David. No joint operations between qubits belonging to different parties is allowed. The three parties Alice, Bob, and Charlie perform the Bell-state measurements on their respective pairs of qubits in hand. Likewise, each of them obtains one of the possible outputs $\{\{\Phi_i\}_{i=0,1,2,3}\}$ of the BSM, which is associated with a corresponding Pauli operator in the set $\{\sigma^i_{i=0,1,2,3}\}$, and communicates the result with David, respectively. David determines a Pauli operator $\sigma^i$ on his qubit D for retrieving the state $|\chi\rangle$ on the qubit D, according to the product of the three Pauli operators pertaining to each one of the three BSMs. Finally, the pure state $|\chi\rangle$ comes out on the qubit D.

The process also can be expressed in a formular way. We joint the state $|\psi\rangle_{ABC}$ with $\rho_{DEFG}^b$:

$$|\psi\rangle_{ABC} \otimes \rho_{DEFG}^b \otimes_{ABC} \langle \psi| = (\alpha |\phi_0\rangle_{ABC} + \beta |\phi_1\rangle_{ABC})\rho_{DEFG}^b (\alpha |\phi_0\rangle + \beta |\phi_1\rangle)$$

$$= \frac{1}{64} \left\{ (|I\rangle I)_{AE, BF, CG} \sigma_0 |\alpha\rangle_D \beta |\beta\rangle_D \right\} \sigma_0$$

$$+ \left\{ (|II\rangle II)_{AE, BF, CG} \sigma_1 |\alpha\rangle_D \beta |\beta\rangle_D \right\} \sigma_1$$

$$+ \left\{ (|III\rangle III)_{AE, BF, CG} \sigma_2 |\alpha\rangle_D \beta |\beta\rangle_D \right\} \sigma_2$$

$$+ \left\{ (|IV\rangle IV)_{AE, BF, CG} \sigma_3 |\alpha\rangle_D \beta |\beta\rangle_D \right\} \sigma_3.$$

(6)

where $|I\rangle I$ marks a set of the combining results of the three BSMs as

$$|I\rangle I = |\Phi_0\rangle_{AE} \langle \Phi_0| \otimes |\Phi_0\rangle_{BF} \langle \Phi_0| \otimes |\Phi_0\rangle_{CG} \langle \Phi_0| + |\Phi_0\rangle_{AE} \langle \Phi_1| \otimes |\Phi_0\rangle_{BF} \langle \Phi_1| \otimes |\Phi_1\rangle_{CG} \langle \Phi_1|$$

$$+ |\Phi_1\rangle_{AE} \langle \Phi_1| \otimes |\Phi_0\rangle_{BF} \langle \Phi_0| \otimes |\Phi_1\rangle_{CG} \langle \Phi_1| + |\Phi_1\rangle_{AE} \langle \Phi_1| \otimes |\Phi_0\rangle_{BF} \langle \Phi_1| \otimes |\Phi_1\rangle_{CG} \langle \Phi_1|$$

$$+ |\Phi_0\rangle_{AE} \langle \Phi_0| \otimes |\Phi_0\rangle_{BF} \langle \Phi_0| \otimes |\Phi_0\rangle_{CG} \langle \Phi_0| + |\Phi_0\rangle_{AE} \langle \Phi_1| \otimes |\Phi_0\rangle_{BF} \langle \Phi_1| \otimes |\Phi_0\rangle_{CG} \langle \Phi_1|$$

It is clear that the combining results of the three BSMs $|I\rangle I$ correspond to the case that the product $\sigma_0^{AE, BF, CG} \sigma_0^{I, k=0,1,2,3}$ of the three Pauli operators pertaining to each one of the three BSMs amounts to $\sigma^0$. So the state of the qubit D is projected to the state $|\chi\rangle$. The rest may be deduced by analogy that $|II\rangle II$ corresponds to the product of $\sigma_1$, $|III\rangle III$ to the product of $\sigma_2$, $|IV\rangle IV$ to the product of $\sigma_3$. Correspondingly, David does a Pauli operator $\sigma^i$, $\sigma^j$ and $\sigma^k$ on his qubit D in order to get the correct state $|\chi\rangle$, respectively. In the end, David gets a qubit D in the state $|\chi\rangle$ with certainty.

Hence, with exploiting an unlockable bound entangled state the same remote information concentration is achieved only by LOCC as with a maximally entangled GHZ state.

However, in the protocol with the bound entangled state, the distribution of the combining results do not reveal any information of the state $|\psi\rangle_{ABC}$. A complicated calculation displays that each of all combining results of the three BSMs comes with the equal probability of $\frac{1}{16}$. That is, taken for example, the combining result $\Phi^0_{AE} \Phi^0_{BF} \Phi^0_{CG}$ comes out with the probability of $\frac{1}{16}$, the other combining results $\Phi^2_{AE} \Phi^0_{BF} \Phi^0_{CG}$ and $\Phi^0_{AE} \Phi^2_{BF} \Phi^0_{CG}$ do so. Allowing for communication security, the unlockable bound entangled state is more suitable for the remote information concentration than the maximally entangled GHZ state.

IV. CONCLUSION

In the paper, we consider a quantum information work, where the quantum information initially in the state $|\chi\rangle$ of a single qubit, but now distributed into three spatially separated qubits, is remotely concentrated back in the state $|\chi\rangle$
of a single qubit. It is found that a maximally entangled GHZ state and an unlockable bound entangled state all can do the work with LOCC. It gives a demonstration that in a sense, a single copy of a bound entangled state is still as useful in quantum information processing as a maximally entangled GHZ state. Even in the view of communication security, the bound entangled state does the work better than the GHZ state, because for the former the distribution of the combining results of the three local BSMs does not reveal any information about the input state $|\psi\rangle_{ABC}$.

As analyzed in [15], it is the entanglement existing in the state $|\psi\rangle_{ABC}$ that liberates the bound entangled state for transmitting the quantum information. The entanglement in the error correction state $|\psi\rangle_{ABC}$ is crucial for the remote information concentration with the bound entangled state and LOCC. It evokes another question about to what extent the bound entangled state cannot work for this information work. The consideration is helpful for deep understanding the relation between the free entanglement and the bound one, and quantum entanglement itself. We hope that our work will stimulate more research into the nature of entanglement and its applications in the quantum communication.

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Fig. 1 The schematic picture for remote information concentration