A Method to include Detector Effects in Estimators sensitive to the Trilinear Gauge Couplings

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Abstract
This paper describes the use of weighted Monte Carlo events to accurately approximate integrals of functions of the experimentally measured kinematical vectors and their dependence on physical parameters. This technique is demonstrated in estimating the evolution of cross sections, efficiencies, measured kinematical distributions and mean values as functions of the Trilinear Gauge Couplings.
1 Introduction

The accurate estimation of the ElectroWeak parameters (in the following just parameters) at LEPII demands attention to the definition of the probability density functions (p.d.f.). Measurement effects such as the detector resolution and the selection efficiency have to be convolved with the physics functions in order to describe the distribution of the observed kinematical variables. Due to the fact that detector effects are not easily parametrized, one usually employs Monte Carlo (M.C.) integration techniques. The disadvantage of this method, though, is that the integrated quantities are in principle by themselves functions of the parameters (e.g. the cross section or the efficiency corresponding to a part of the phase space) and consequently several M.C. sets of events must be used to cover the parametric space.

This paper describes a reweighting technique which covers a wide range of values of the parameters (extrapolation area) whilst using a single M.C. set of events. Furthermore this technique can be extended by combining several sets of M.C. events, produced with different parameter values, and thus enlarging the extrapolation area in addition to minimizing the statistical error.

The general principles of reweighting are demonstrated in calculating the cross sections in multidimensional bins of the observed kinematical variables and their dependence on the parameter values. This particular example has important applications in the shape definition of the kinematical distributions which are used in measuring the Trilinear Gauge Couplings (TGC’s). Other estimating procedures compare the average of certain functions of the observed kinematical vectors with the phenomenological expectations convolved with the resolution and efficiency functions. It is shown, in this paper, that by employing the reweighting technique this convolution is performed accurately and a very efficient estimation of the TGC’s can be made. This work finally concludes with a demonstration of the accuracy and consistency of the technique by numerical results.

2 Reweighting a Single Set of M.C. events

Throughout this analysis the symbol $V = V_1, \ldots, V_k$ denotes the k-dimensional kinematical vector which defines completely a M.C. event at generation (generation vector) whilst $\Omega = \Omega_1, \ldots, \Omega_k$ stands for the reconstructed equivalent of $V$ (reconstructed or observed vector). The following symbols are also defined:

$\bar{\alpha} = \alpha_1, \ldots, \alpha_\rho$ denoting the $\rho$ parameters which are required to define the p.d.f.

$d\sigma(V, \bar{\alpha})/dV$ denoting the differential cross section as a function of the generated vector.

$R(V, \Omega)$ denoting the resolution function, i.e. the probability of an event generated with $V$ to be observed with $\Omega$ kinematical vector. Obviously

$$\int R(V, \Omega)d\Omega = 1$$

$\epsilon(V)$ denoting the probability of an event generated with $V$ to be observed.

\[1\]

Where the limits of an integration are not explicitly stated an integration all over the phase space is meant.
\( d\tilde{\sigma}(\Omega, \bar{\alpha})/d\Omega \) denoting the differential observed cross section as a function of the reconstructed vector which is defined as:

\[
\frac{d\tilde{\sigma}(\Omega, \bar{\alpha})}{d\Omega} = \int \frac{d\sigma(V, \bar{\alpha})}{dV} \cdot \epsilon(V) \cdot R(V, \Omega) dV
\]  

(2)

\( \sigma_{\text{obs}}(\bar{\alpha}) \) denoting the total observed cross section.

\( g(V, \bar{\alpha}) \) denoting the p.d.f. defined as:

\[
g(V, \bar{\alpha}) = \frac{(d\sigma(V, \bar{\alpha})/dV) \cdot \epsilon(V)}{\sigma_{\text{obs}}(\bar{\alpha})}
\]  

(3)

Following the above definitions the joint distribution of the observed kinematical vectors is described by:

\[
G(\Omega, \bar{\alpha}) = \int g(V, \bar{\alpha}) \cdot R(V, \Omega) dV
\]  

(4)

2.1 The Number of Expected Events in a Part of the Phase Space.

Let \( \Delta\Omega \) be a part of the observed phase space and let the integral I to be the corresponding cross section

\[
I = \int_{\Delta\Omega} \frac{d\tilde{\sigma}(\Omega, \bar{\alpha})}{d\Omega} d\Omega
\]  

(5)

for a set \( \bar{\alpha} \) of parameter values. The number of the expected events, \( \mu(\bar{\alpha}, \Delta\Omega) \), in the \( \Delta\Omega \) part of the phase space for an accumulated luminosity \( L \) is:

\[
\mu(\bar{\alpha}, \Delta\Omega) = I \cdot L
\]  

(6)

Let us assume that there is a set of \( N_0 \) M.C. events generated with the p.d.f. \( g(V, \bar{\alpha}_g) \) with \( \bar{\alpha}_g \) parameter values. Let also \( N \) be the number of the above events which have survived the acceptance, reconstruction and selection criteria after full detector simulation whilst \( n_{\Delta\Omega} \) of them lie inside \( \Delta\Omega \). By multiplying and dividing the integrant of (5) by the properly normalized p.d.f.

\[
g(V, \bar{\alpha}_g) \\
D(\Delta\Omega, \bar{\alpha}_g)
\]

(7)

the integral I is written as:

\[
I = D(\Delta\Omega, \bar{\alpha}_g) \cdot \int_{\Delta\Omega} \int \frac{d\sigma(V, \bar{\alpha})}{dV} \cdot \frac{\epsilon(V)}{g(V, \bar{\alpha}_g)} \cdot \frac{g(V, \bar{\alpha}_g) \cdot R(V, \Omega)}{D(\Delta\Omega, \bar{\alpha}_g)} dV d\Omega
\]  

(8)

where

\[
D(\Delta\Omega, \bar{\alpha}_g) = \int_{\Delta\Omega} \int g(V, \bar{\alpha}_g) \cdot R(V, \Omega) dV d\Omega
\]  

(9)

Equation (8) can be written in a simpler form, by expressing the differential cross section \( d\sigma(V, \bar{\alpha})/dV \) as a product of the matrix element squared \( \mathcal{M}(V, \bar{\alpha}) \) and of the Lorentz Invariant Phase Space \( d\text{LIPS}(V) \). Specifically:

\[
I = D(\Delta\Omega, \bar{\alpha}_g) \cdot \sigma_{\text{obs}}(\bar{\alpha}_g) \cdot <w(V, \bar{\alpha}, \bar{\alpha}_g)>
\]  

(10)
\[ w(V, \bar{\alpha}, \bar{\alpha}_g) = \frac{\mathcal{M}(V, \bar{\alpha})}{\mathcal{M}(V, \bar{\alpha}_g)} \]  

(11)

\[ < w(V, \bar{\alpha}, \bar{\alpha}_g) >= \int_{\Delta\Omega} \int w(V, \bar{\alpha}, \bar{\alpha}_g) \cdot \frac{g(V, \bar{\alpha}_g) \cdot R(V, \Omega)}{D(\Delta\Omega, \bar{\alpha}_g)} dV d\Omega \]  

(12)

Then, the integral (5) can be approximated by using the set of the \( n_{\Delta\Omega} \) M.C. events as:

\[ I \simeq \frac{D(\Delta\Omega, \bar{\alpha}_g)}{n_{\Delta\Omega}} \cdot \frac{\sigma_{\text{obs}}(\bar{\alpha}_g)}{n_{\Delta\Omega}} \sum_{i=1}^{n_{\Delta\Omega}} w(V_i, \bar{\alpha}, \bar{\alpha}_g) \]  

(13)

Obviously the function \( D(\Delta\Omega, \bar{\alpha}_g) \) can be estimated as:

\[ D(\Delta\Omega, \bar{\alpha}_g) \simeq \frac{n_{\Delta\Omega}}{N} \]  

(14)

with a binomial error \( \sqrt{n_{\Delta\Omega} \cdot (N - n_{\Delta\Omega})/N^3} \), which for \( N \gg n_{\Delta\Omega} \) becomes \( \sqrt{n_{\Delta\Omega}}/N \). The global term in (13) \( \sigma_{\text{obs}}(\bar{\alpha}_g) \) (independent of the particular \( \Delta\Omega \) interval), can be written as a fraction of the total cross section \( \sigma_{\text{tot}}(\bar{\alpha}_g) \) in the form:

\[ \sigma_{\text{obs}}(\bar{\alpha}_g) \simeq \sigma_{\text{tot}}(\bar{\alpha}_g) \cdot \frac{N}{N_0} \]  

(15)

with a fractional error of \( \sqrt{N \cdot (N_0 - N)}/N_0^2 \) which for \( N_0 \gg N \) becomes \( \sqrt{N}/N_0 \). Finally by substituting (14) and (13) into (13) the integral I is approximated as:

\[ I \simeq \frac{\sigma_{\text{tot}}(\bar{\alpha}_g)}{N_0} \sum_{i=1}^{n_{\Delta\Omega}} w(V_i, \bar{\alpha}, \bar{\alpha}_g) \]  

(16)

The accuracy of this approximation can be quantified in terms of the variance of \( w(V_i, \bar{\alpha}, \bar{\alpha}_g) \) and the statistical errors of (14) and (13). Thus the error in (16) is:

\[ \delta_I = \frac{\sigma_{\text{tot}}(\bar{\alpha}_g)}{N_0} \cdot [< w(V, \bar{\alpha}, \bar{\alpha}_g) >^2 \cdot (n_{\Delta\Omega} + \frac{n_{\Delta\Omega}^2}{N}) + n_{\Delta\Omega} \cdot (< w^2(V, \bar{\alpha}, \bar{\alpha}_g) > - < w(V, \bar{\alpha}, \bar{\alpha}_g) >^2)]^{1/2} \]  

(17)

The above approximation (16) and its error estimation (17) are valid for every value of \( \bar{\alpha} \) and \( \bar{\alpha}_g \) as long as the p.d.f. defined in (3) is not zero inside the \( \Delta\Omega \) interval.

### 2.2 The Mean Value of a Function of the Observed Kinematical Vectors

It has been shown [1] that there are functions of the true kinematical vectors, \( \mathcal{O}_k(V; \bar{\alpha}_0) \) with \( k = 1, \ldots, \rho \) which locally (around \( \bar{\alpha}_0 \)) carry the whole information concerning the \( k^{th} \) parameter. This is easily proven by expanding the p.d.f. in a Taylor series around an initial set of values \( \bar{\alpha}_0 \) of the parameters and keeping only the linear terms. Furthermore the mean values of these functions, usually called Optimal Observables, for any \( \bar{\alpha} \) around \( \bar{\alpha}_0 \), can be expressed as linear functions of the parameters with known functions as slopes and intercepts. The situation becomes more complicated when one has to take into account detector effects. However, it can be shown [2] that in this case too, there exist
functions of the observed kinematical vectors which retain the same information content as the Likelihood estimators. In general this function \( \omega(\Omega; \bar{\alpha}_0) \) is defined as:

\[
\omega(\Omega; \bar{\alpha}_0) = \int \mathcal{O}(V; \bar{\alpha}_0) \cdot \frac{g(V, \bar{\alpha}_0) \cdot R(V, \Omega)}{\int g(V, \bar{\alpha}_0) \cdot R(V, \Omega) dV} dV
\] (18)

In other words, the Optimal Observable in the most general case is the mean value of \( \mathcal{O}(V; \bar{\alpha}_0) \) where the distribution of the vectors \( V \)'s corresponds to \( \bar{\alpha}_0 \) parameter values under the condition that the observed kinematical vectors are equal to \( \Omega \). Equation (18) can be further simplified if expressed in terms of \( \mathcal{O}(\Omega) \) as:

\[
\omega(\Omega; \bar{\alpha}_0) = \mathcal{O}(\Omega; \bar{\alpha}_0) \cdot r(\Omega; \bar{\alpha}_0)
\] (19)

where \( r(\Omega; \bar{\alpha}_0) \) is the mean value of the Optimal Observables \( \mathcal{O}(V; \bar{\alpha}_0) \) (as defined in (18)) expressed in units of \( \mathcal{O}(\Omega; \bar{\alpha}_0) \). The function \( r(\Omega; \bar{\alpha}_0) \) plays the role of a correction function which has to be calculated from the M.C. In practice (at least for the case of the TGC’s) a very good approximation is:

\[
\omega(\Omega; \bar{\alpha}_0) \simeq \mathcal{O}(\Omega; \bar{\alpha}_0)
\] (20)

The mean value of \( \mathcal{O}(\Omega; \bar{\alpha}_0) \) (in the following Modified Observable or M.O.)

\[
< \mathcal{O}(\Omega; \bar{\alpha}_0) >_{\bar{\alpha}} = \int \mathcal{O}(\Omega; \bar{\alpha}_0) \int g(V, \bar{\alpha}) \cdot R(V, \Omega) dV d\Omega
\] (21)

in a region around \( \bar{\alpha}_0 \) is, as before, a linear function of the couplings \( \bar{\alpha} \) but in this case the slopes and intercepts are convolutions of known physics functions (matrix elements and phase space) and the resolution and efficiency functions. Nevertheless it is simpler to calculate the mean value \( < \mathcal{O}(\Omega; \bar{\alpha}_0) >_{\bar{\alpha}} \) as a function of \( \bar{\alpha} \) (calibration curve) by using M.C. events with proper weights corresponding to the \( \bar{\alpha} \) parameter values. In parallel the set of the \( N_{\text{meas.}} \) vectors, \( \Omega_{\text{meas.}} \), accumulated in the real experiment is used to measure the mean value of the Modified Observable as:

\[
< \mathcal{O}(\Omega; \bar{\alpha}_0) >_{\text{meas.}} \simeq \frac{1}{N_{\text{meas.}}} \sum_{i=1}^{N_{\text{meas.}}} \mathcal{O}(\Omega_{\text{meas.}}^i; \bar{\alpha}_0)
\] (22)

Finally an estimation of the couplings is achieved by comparing the experimental value (22) with the calibration curve.

The reweighting procedure needed for the calculation of the calibration curve follows the same general principles as the method described in the previous subsection. As before, by multiplying and dividing the integrant of (21) by \( g(V, \bar{\alpha}_g) \) (\( \bar{\alpha}_g \) being the parameters used for the production of the M.C. set of events) and expressing the p.d.f.’s in terms of total cross sections, squared matrix elements and phase space factors, eq. (21) is transformed to:

\[
< \mathcal{O}(\Omega; \bar{\alpha}_0) >_{\bar{\alpha}} = \frac{\sigma_{\text{obs}}(\bar{\alpha}_g)}{\sigma_{\text{obs}}(\bar{\alpha})} \cdot \int \int \mathcal{O}(\Omega; \bar{\alpha}_0) \cdot w(V, \bar{\alpha}, \bar{\alpha}_g) \cdot g(V, \bar{\alpha}_g) \cdot R(V, \Omega) dV d\Omega
\] (23)

\(^2\) In the following the index \( k \) is dropped to simplify the expressions.

\(^3\) It must be emphasized that although the method of defining the calibration curve is valid for every \( \bar{\alpha} \), the Modified Observable is Optimal only in a narrow region around \( \bar{\alpha}_0 \). The estimation technique which extends the optimality to all the points of the parametric space is discussed elsewhere.

\(^4\) Assuming that \( g(V, \bar{\alpha}_g) > 0, \forall V \).
This integral can be approximated as:

\[
< \mathcal{O}(\Omega; \bar{\alpha}_0) > \bar{\alpha} \simeq \frac{1}{\sum_{i=1}^{N} w(V_i, \bar{\alpha}, \bar{\alpha}_g)} \cdot \sum_{i=1}^{N} w(V_i, \bar{\alpha}, \bar{\alpha}_g) \cdot \mathcal{O}(\Omega_i; \bar{\alpha}_0)
\]  

(24)

where the \( \sigma_{\text{obs}}(\bar{\alpha}) \) was expressed as

\[
\sigma_{\text{obs}}(\bar{\alpha}) \simeq \sigma_{\text{obs}}(\bar{\alpha}_g) \cdot \frac{N}{\sum_{i=1}^{N} w(V_i, \bar{\alpha}, \bar{\alpha}_g)}
\]  

(25)

The variance of the approximation (24) can be estimated in the standard way in terms of the variance, covariance and partial derivatives of (24) with respect to the "random" variables \( w(V_i, \bar{\alpha}, \bar{\alpha}_g) \).

3 Combining Several Sets of M.C. events

In practice, the use of a limited size M.C. set of events has the disadvantage that although the p.d.f. could be non zero, there are phase space regions where very few (or none) events have been generated. The use of such a set to estimate integrals or mean values, by reweighting, could induce systematical biases. These biases are important when the particular not-covered phase space region contributes significantly to the cross section corresponding to the extrapolated parameter values. Thus the use of several M.C. sets produced at different points of the parametric space and properly combined together is preferable.

Let \( \mathcal{Q}(\bar{\alpha}) \) be the integral of a function of the observed kinematical vectors (e.g. cross section or mean value). Let \( S_j \) \((m = 1, \ldots , m)\) be \( m \) sets of M.C. events produced with parameter values \( \bar{\alpha}_1, \ldots , \bar{\alpha}_m \). Let then \( \hat{\mathcal{Q}}_i(\bar{\alpha}) \) be an estimation of \( \mathcal{Q}(\bar{\alpha}) \) with variance \( \mathcal{V}_i(\bar{\alpha}) \) where the \( i^{th} \) set of M.C. events has been used as it is described in the previous section. Then a better estimation of \( \mathcal{Q}(\bar{\alpha}) \) can be found as a linear combination of the \( \hat{\mathcal{Q}}_i(\bar{\alpha}) \)'s, i.e.

\[
\hat{\mathcal{Q}}_{\text{comb.}}(\bar{\alpha}) = \sum_{i=1}^{m} \gamma_i \cdot \hat{\mathcal{Q}}_i(\bar{\alpha})
\]  

(26)

\[
\sum_{i=1}^{m} \gamma_i = 1
\]  

(27)

such as the variance of \( \hat{\mathcal{Q}}_{\text{comb.}}(\bar{\alpha}) \) to be minimum. It is easy to show that this is equivalent to the minimization of the least square sum

\[
\chi^2 = \sum_{i=1}^{m} \left( \hat{\mathcal{Q}}_{\text{comb.}}(\bar{\alpha}) - \hat{\mathcal{Q}}_i(\bar{\alpha}) \right)^2 / \mathcal{V}_i(\bar{\alpha})
\]  

(28)

which results to:

\[
\hat{\mathcal{Q}}_{\text{comb.}}(\bar{\alpha}) = \sum_{i=1}^{m} \left( \hat{\mathcal{Q}}_i(\bar{\alpha}) / \mathcal{V}_i(\bar{\alpha}) \right) / \left( \sum_{i=1}^{m} \frac{1}{\mathcal{V}_i(\bar{\alpha})} \right)
\]  

(29)

with a variance of:

\[
V(\hat{\mathcal{Q}}_{\text{comb.}}(\bar{\alpha})) = 1 / \left( \sum_{i=1}^{m} \frac{1}{\mathcal{V}_i(\bar{\alpha})} \right)
\]  

(30)
There are however cases, where integrals of more than one functions have to be estimated (e.g. in the case of two-T.G.C. simultaneous estimation, two M.O. have to be evaluated as functions of the two couplings) by integration using the same sets of M.C. events. The generalization of (28) for n integrals reads:

\[ \chi^2 = \sum_{i=1}^{m} (\hat{Q}_{\text{comb.}}(\bar{\alpha}) - \hat{Q}_i(\bar{\alpha}))^T \cdot M_i^{-1} \cdot (\hat{Q}_{\text{comb.}}(\bar{\alpha}) - \hat{Q}_i(\bar{\alpha})) \]  

(31)

where the vectors \( \hat{Q}_{\text{comb.}}(\bar{\alpha}) \) and \( \hat{Q}_i(\bar{\alpha}) \) have elements corresponding to each of the n integrated functions, whilst the subscript \( i \) stands for the set of the M.C. events used in the integration. The matrices \( M_i \) represent the covariant matrices between the elements of the vector \( \hat{Q}_i(\bar{\alpha}) \).

4 Numerical Results

For any practical application of the reweighting technique, one needs only the means to calculate the weight \( w(V, \bar{\alpha}, \alpha_g) \) as it is defined in (11). Concentrating on the physics analysis concerning the determination of the TGC’s there are several four fermion M.C. generators which include the relevant physics processes [3]. Among them, the ERATO generator [4] has been chosen and was modified keeping only the parts necessary for the calculation of the squared matrix element (M.E.). In case that an ISR photon (with or without \( P_T \)) had been emitted, the four fermions kinematical vectors were transformed to their rest frame before calling the ERATO M.E. calculating routines. Here the ISR effect is assumed to be factorizable and thus independent of the variables. Thus, as it has been shown previously with the phase space factors, the ISR factors in the weight definitions are canceled out.

In general lines one should a) define the values of the physics constants to be used for the matrix element calculation, b) pass the kinematical and event type information and c) call the relevant software twice to calculate the ratio of the M.E.’s as in (11).

In the following, the reweighting method is demonstrated through applications concerning the evaluation of physical quantities needed in the estimation of the TGC’s, using four fermion (specifically the lepton-neutrino-two hadronic jets) final state events as those produced at LEPII at 172 GeV centre of mass energy.

The available M.C. events undergone full detector simulation (DELSIM [5]) are grouped into two main categories.

- Events produced with PYTHIA [6] with the TGC’s set to their Standard Model (S.M.) values whilst six different W mass (79.35, 79.85, 80.23, 80.35, 80.85 and 81.35 GeV/c\(^2\)) values were used. These six subcategories contain in all 3280 and 3390 events having an electron (electron type) or a muon (muon type) in the final state respectively.

- Events produced with EXCALIBUR [7] with the W mass set to 80.35 GeV/c\(^2\) but divided into three groups according to the \( \alpha_{W\phi} \) values used in the production whilst the other couplings are related according to the \( W\phi \) model [8]. Namely the \( \alpha_{W\phi} = 0 \) subgroup contains 790 electron type and 900 muon type events, the \( \alpha_{W\phi} = -2 \) subgroup contains 770 electron type and 900 muon type events and the \( \alpha_{W\phi} = 2 \) subgroup contains 810 electron type and 900 muon type events.
Identical selection criteria have been applied on both generation and reconstruction level to all the M.C. set of events. The PYTHIA physics generator employs only the resonant Feynman graphs (CC03) whilst the EXCALIBUR generator includes all the four fermion (CC10 for muon type and CC20 for electron type) relevant processes. For all the applications to follow the extrapolated parameter variables correspond to the full four fermion production mechanisms and the extrapolated W mass value was equal to 80.35 GeV/c² whilst the estimated quantities are considered as functions of the $\alpha_{W\phi}$ coupling.

The simplest demonstration of the reweighting technique deals with the estimation of the total cross section as a function of the coupling. This in accordance to (16) can be estimated by using the $j^{th}$ set of the available M.C. events as:

$$I_j \simeq \frac{\sigma_{tot}(\alpha^{\prime}_{W\phi})}{N^j_0} \cdot \sum_{i=1}^{N^j_0} w(V_i, \alpha_{W\phi}, \alpha^{\prime}_{W\phi})$$  (32)

with a statistical error equal to:

$$\delta_j = \frac{\sigma_{tot}(\alpha^{\prime}_{W\phi})}{\sqrt{N^j_0}} \cdot [< w^2(V, \alpha_{W\phi}, \alpha^{\prime}_{W\phi}) > - < w(V, \alpha_{W\phi}, \alpha^{\prime}_{W\phi}) >^2]^{1/2}$$  (33)

In the following several M.C. set of events are combined as in (29) to increase the statistical accuracy of this approximation. Figure 1a and 1b presents the estimated dependence of the number of produced events for a 9.6 $pb^{-1}$ accumulated luminosity on the value of $\alpha_{W\phi}$ for electronic and muonic final states when only the PYTHIA M.C. sets have been reweighted. This estimation of the reweighting procedure agrees, for coupling values between -2 and 2, within statistical errors with the prediction (solid line) of the ERATO generator containing the full set of four fermion production diagrams. However, this extrapolation suffers from the fact that the PYTHIA generated events do not cover adequately all the phase space regions which are contributing significantly to the four fermion cross section for larger absolute values of the couplings. An enlargement of the extrapolation region with a simultaneous improvement of the accuracy is achieved when the EXCALIBUR M.C. sets are also included as it can be seen in figures 1c and 1d.

The second example is dealing with integrations with respect to the observed kinematical vectors. In this case the efficiency is estimated as the fraction of the number of the produced events for a 9.6 $pb^{-1}$ accumulated luminosity on the value of $\alpha_{W\phi}$ for electronic and muonic final states when only the PYTHIA M.C. sets have been reweighted. This estimation of the reweighting procedure agrees, for coupling values between -2 and 2, within statistical errors with the prediction (solid line) of the ERATO generator containing the full set of four fermion production diagrams. However, this extrapolation suffers from the fact that the PYTHIA generated events do not cover adequately all the phase space regions which are contributing significantly to the four fermion cross section for larger absolute values of the couplings. An enlargement of the extrapolation region with a simultaneous improvement of the accuracy is achieved when the EXCALIBUR M.C. sets are also included as it can be seen in figures 1c and 1d.

The success of the reweighting technique to perform integrations with respect to the observed kinematical vectors can be also demonstrated by comparing estimations of the number of events to be observed in a special phase space region, by reweighting and by direct M.C. integration. In this example only the PYTHIA M.C. sets were used to estimate these differential quantities as functions of $\alpha_{W\phi}$ by employing reweighting. In parallel the EXCALIBUR M.C. sets were used for a direct estimation at the three available values of $\alpha_{W\phi}$. These integrals represent the number of the expected events for a total luminosity of 9.6 $pb^{-1}$, in bins either of $\cos \theta_W$ (Fig 3 to Fig 5) ($\theta_W$ being the polar angle of the hadronic system) or of the Modified Observable (Fig 6 to Fig 8). As a measure of the level of agreement in this comparison, the relative difference of the two estimations was chosen (relative deviation) but the estimated numbers of events are also presented.
as insets in the previous figures. The remarkable agreement between the two type of estimations justifies the use of the reweighting technique in producing the distribution shapes for a wide area of the $\alpha_{W\phi}$ to be used in fitting the TGC’s.

As extensively discussed previously, it is possible to define a Modified Observable such as its mean value is an estimator of the $\alpha_{W\phi}$ which carries the same information as the likelihood. It has been also shown that (independent of the optimality) the evolution of this quantity as a function of the coupling can be accurately approximated by the reweighting procedure. Indeed, as it is shown in Fig. 9, the mean value of the Modified Observables defined for $\alpha_{W\phi} = 0$ and estimated by reweighting is in a very good agreement with the direct estimations by using the EXCALIBUR M.C. sets. However the optimality of this method is restricted to the linear part around $\alpha_{W\phi} = 0$. The extention of this optimality is achieved by expanding the p.d.f. in a Taylor series around another value of the coupling and reformulating the Modified Observables. This procedure for optimal definition has been employed around the two extreme values of $\alpha_{W\phi} = \pm 2$. The results are shown in Fig 10 where also for comparison the directly estimated values with the EXCALIBUR M.C. sets are presented.

5 Conclusions

Following very simple calculus, it has been shown that a M.C. set of events generated at specific values of the relevant physical parameters can be used to approximate physical quantities in a very wide region of the parametric space. Furthermore, a simple procedure has been demonstrated which combines several M.C. samples, independent of the specific values of the physical parameters at their generation. The quantification of the accuracy of this approximation has been expressed analytically and demonstrated by numerical examples. The applicability of this technique to produce the shapes of kinematical quantities to be used in extracting the values of physical parameters was proven.

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Figure 1: The total Cross Section as a function of the $\alpha_{W\phi}$: a) and b) only the PYTHIA, c) and d) the PYTHIA and the EXCALIBUR Monte Carlo sets are included in the reweighting.
Figure 2: The Efficiency as a function of the $\alpha_{W\phi}$ estimated by reweighting only the PYTHIA Monte Carlo sets. The squares represent the efficiency values estimated directly from the EXCALIBUR samples.
Figure 3: Comparison of the differential cross section with respect to the $\cos \theta_W$ as it is estimated by reweighting the PYTHIA sets with the distribution of unweighted Monte Carlo events produced with EXCALIBUR at $\alpha_{W\phi} = 0$. The solid lines and the black points in the inset figures correspond to the reweighting estimation whilst the open circles stand for the EXCALIBUR prediction.
Figure 4: Comparison of the differential cross section with respect to the $\cos \theta_W$ as it is estimated by reweighting the PYTHIA sets with the distribution of unweighted Monte Carlo events produced with EXCALIBUR at $\alpha_{W\phi} = -2$. The solid lines and the black points in the inset figures correspond to the reweighting estimation whilst the open circles stand for the EXCALIBUR prediction.
Figure 5: Comparison of the differential cross section with respect to the $\cos \theta_W$ as it is estimated by reweighting the PYTHIA sets with the distribution of unweighted Monte Carlo events produced with EXCALIBUR at $\alpha_{W\phi} = +2$. The solid lines and the black points in the inset figures correspond to the reweighting estimation whilst the open circles stand for the EXCALIBUR prediction.
Figure 6: Comparison of the differential cross section with respect to the Modified Observables (defined at $\alpha_{W\phi} = 0$) as it is estimated by reweighting the PYTHIA sets with the distribution of unweighted Monte Carlo events produced with EXCALIBUR at $\alpha_{W\phi} = 0$. The solid lines and the black points in the inset figures correspond to the reweighting estimation whilst the open circles stand for the EXCALIBUR prediction.
Figure 7: Comparison of the differential cross section with respect to the Modified Observables (defined at $\alpha_{W\phi} = 0$) as it is estimated by reweighting the PYTHIA sets with the distribution of unweighted Monte Carlo events produced with EXCALIBUR at $\alpha_{W\phi} = -2$. The solid lines and the black points in the inset figures correspond to the reweighting estimation whilst the open circles stand for the EXCALIBUR prediction.
Figure 8: Comparison of the differential cross section with respect to the Modified Observables (defined at $\alpha_{W\phi} = 0$) as it is estimated by reweighting the PYTHIA sets with the distribution of unweighted Monte Carlo events produced with EXCALIBUR at $\alpha_{W\phi} = +2$. The solid lines and the black points in the inset figures correspond to the reweighting estimation whilst the open circles stand for the EXCALIBUR prediction.
Figure 9: The Mean Value of the Modified Observable defined at $\alpha_{W\phi} = 0$, as a function of the $\alpha_{W\phi}$. The squares correspond to unweighted EXCALIBUR events.
Figure 10: The Mean Value of the Modified Observable defined at (a and b) $\alpha_{W\phi}$ equal to -2 and (c and d) at 2 as a function of the $\alpha_{W\phi}$. The squares correspond to unweighted EXCALIBUR events.