Computation of the masses of neutrinos from the Hadron and Boson masses via the Rotating Lepton model of elementary particles

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Abstract. We use the Rotating Lepton Model (RLM) of elementary particles in conjunction with special relativity and the de Broglie equation to compute analytically the masses of neutrinos from the masses of composite particles, such as hadrons, in the structures of which neutrinos have been recently shown to participate. In this way, three distinct neutrino masses are computed which are in good agreement with the values obtained experimentally at Superkamiokande for the three neutrino flavors of the Normal Hierarchy.

1. Introduction
The pioneering work of Kajita and Mc Donald [1,2] has shown that neutrinos are not massless and that their masses correspond to three different neutrino mass states both in the Normal (Figure 1) and in the Inverted Hierarchy [3].

Figure 1. The three neutrino masses as a function of the lightest mass, $m_\nu$, for the normal hierarchy, reprinted from ref. [3] and comparison with equation (5), i.e.

$$m_0 \approx \left(\frac{m_\nu}{3}\right)^{3/2} / (3^{1/8} m^{1/2}_\text{Pl}) = 0.043723 \text{ eV/} c^2,$$

where $m_\text{n} (=939.565 \text{ MeV/} c^2)$ is the neutron mass.

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Recent work, based on Einstein’s famous special relativity formula for inertial mass [4,5], thus in view of the equivalence principle also for gravitational mass [6,7,8], has shown that, surprisingly, the gravitational masses, $\gamma^2 m_\nu$ [4,5], of neutrinos with energy above 300 MeV reach the Planck mass, i.e. the value $(\hbar c / G)^{1/2} = 1.22 \cdot 10^{28}$ eV/c$^2$, and that consequently the gravitational attraction between such neutrinos becomes as strong as the Strong Force itself [6,7]. Based on this, the Rotating Lepton Model (RLM) has been formulated [6,7,8], which allows for the computation of the hadron and boson masses with good accuracy, typically within a surprisingly small 2% deviation [8].

In the present work we utilize the RLM to derive analytical expressions for the neutrino rest masses in terms of hadron, meson and boson masses. The thus computed neutrino rest masses are in very good agreement with the values measured at the Superkamiokande [3].

2. Three rotating neutrino systems
Recent work has shown that both hadrons, such as protons and neutrons [6,7], as well as mesons, such as the pions [8], can be modeled as rotating trios or doublets of neutrinos (Figure 2).

Denoting by $m_\nu$ the rest mass of each neutrino and by $m_c$ the mass of the composite particle (e.g. hadron) formed, we write the equation of circular motion

$$F = \frac{G m_\nu \gamma^2 v^2}{\sqrt{3} \gamma v}$$

and the de Broglie equation

$$\gamma m_\nu v r = \hbar$$

Upon using the definitions of the gravitational mass, and the equivalence principle, i.e. [4,5,6,7,8]

$$m_g = m_i = m_\nu \gamma^3$$

as well as the energy conservation equation

$$E = m_c c^2 = 3 \gamma m_\nu c^2$$
\[ m_c c^2 = 3\gamma_m \gamma_c^2 \]  

we solve the system of equations (1), (2), (3) and (4) to obtain:

\[ m_c = 3^{13/12} (m_p m_0^2)^{1/3}; \quad m_0 = \frac{(m_c / 3)^{3/2}}{3^{1/8} m_p^{1/2}} \]  

and thus using \(m_c=m_p=938.272\) MeV/c\(^2\) for the proton and \(m_c=m_n=939.565\) MeV/c\(^2\) for neutron, one obtains:

\[ m_0 = 0.0437\text{ eV/c}^2, \text{ i.e. from the neutron mass, and} \]

\[ m_0 = 0.0436\text{ eV/c}^2, \text{ i.e. from the proton mass} \]  

These mass values are, amazingly, within the current uncertainty limits of the \(m_3\) mass of the Normal Hierarchy [3] as shown in Figure 3. The same value is computed also from the mass of the deuteron (d) which comprises a proton and a neutron. Since the fusion of two protons to produce a deuteron is known to lead to the generation of an electron neutrino, one may conclude that the \(m_3\) mass of the Normal Hierarchy corresponds to electron neutrinos \(\nu_e\). Thus the neutrino rest masses computed from equations (6) and (7) can be denoted \(m_{\nu_e}\)

\[ \begin{align*}
\text{Figure 3. Comparison of computed via the RLM (horizontal dotted lines) and experimental [11] neutrino flavor masses [3,11,12]; The } m_1, m_2 \text{ and } m_3 \text{ expression correspond to equations (5), (21) and (22). These equations imply that, interestingly, } m_3/m_2 &= m_2/m_1 \approx 6.3 \approx 2\pi. \text{ As shown in the Figure and in ref. [8] it is } m_{\text{MIN}} &= m_1 = m_{\nu,\mu}. \\
3. Two different rotating lepton systems \text{3a. The W boson} \\
\text{Recent work [10] has shown that W bosons can be modeled as rotating pairs of electrons or positrons with electron neutrinos. The equations of motion are}
\end{align*} \]
\[ \gamma_e m_e v_e^2 / r = \gamma_v m_v v_v^2 / r = \frac{G m_e m_v \gamma_e^3 \gamma_v^3}{4r^2} \]  

(8)

and the de Broglie equations are

\[ \gamma_e m_e v_e = \gamma_v m_v v_v r = \hbar \]  

(9)

For \( v = c \) equations (6) and (7) become

\[ 4 \hbar c = G m_e m_v \gamma_e^3 \gamma_v^3 \]  

(10)

The synchronization condition, requires equal electron and neutrino velocities, thus equal \( \gamma \) values, i.e.

\[ v_e \approx v_v \approx c \quad ; \quad \text{thus} \]  

(11)

\[ \gamma_e \approx \gamma_v \]  

(12)

which gives

\[ (\gamma_e \gamma_v)^3 = \frac{4 \hbar c}{G m_e m_v} = 4 \frac{m_{Pl}^2}{m_e m_v} \]  

(13)

Denoting \( \gamma = (\gamma_e \gamma_v)^{1/2} \) and \( m = (m_e m_v)^{1/2} \) equation (13) becomes

\[ \gamma^6 = 4 \frac{m_{Pl}^2}{m^2} \]  

(14)

and thus

\[ \gamma^2 = 4^{1/3} \frac{m_{Pl}^{2/3}}{(m_e m_v)^{1/3}} \quad ; \quad \gamma = 2^{1/3} \frac{m_{Pl}^{1/3}}{(m_e m_v)^{1/6}} \]  

(15)

thus

\[ m_W = (\gamma_e \gamma_v)^{1/2} (m_e m_v)^{1/2} = \gamma m \]  

(16)

therefore

\[ m_W = 2^{1/3} \frac{m_{Pl}^{1/3}}{(m_e m_v)^{1/6}} (m_e m_v)^{1/2} = 2^{1/3} (m_{Pl} m_e m_v)^{1/3} \]  

(17)

Solving equation (15) for \( m \), one obtain

\[ m_{ve} = \frac{m_W}{2m_{Pl} m_e} \]  

(18)

Substituting \( m_{Pl} = 1.22 \cdot 10^{28} \text{eV} / c^2, m_e = 0.511 \cdot 10^6 \text{eV} / c^2, m_W = 80.42 \text{GeV} / c^2 \) we obtain

\[ m_{ve} = 0.0417 \text{ eV} / c^2 \]  

(19)

which differs less than 4\% from the value of the masses \( m_{ve} \) obtained from the neutron and the proton. It thus follows that the neutrinos forming the proton and neutron have the same flavor with those forming the W boson, i.e. they are electron neutrinos, \( v_e \).

3b. The muon

Recent work [8,11] has shown that the muon can be modeled as a rotating \( v_e v_\mu \) pair rotating around an electron. This model may also explain the muon magnetic moment anomaly [9]. Thus application of equation (18) and subsequent use of equation (5) gives
\[ m_{\nu}\mu = \frac{m_\mu^3}{2m_p m_{\nu e}} = \frac{3^{1/8} m_\mu^3}{2(m_\mu / 3)^{3/2}} m_{pl}^{1/2} = 1.106 \cdot 10^{-3} \text{ eV} / c^2 \]  

(20)

which coincides with the lowest mass \( m_1 \) of the normal hierarchy (Fig. 3).

3c. The pion

This is a three rotating neutrino system [8] and the mass of each of its two equal detected decay fragments [8,11] can be computed from equation [5] divided by two, i.e.

\[ m_\pi = (1/2)3^{13/12}(m_p m_{\nu e} m_{\nu e})^{1/3} = (1/2)3^{13/12}(m_p m_{\nu e}^2)^{1/3} \]  

(21)

where \( m_{\nu e} = (m_{\nu e} m_{\nu e})^{1/2} \), which gives

\[ m_{\nu e} = \frac{3^{1/4} m_\mu^3}{2^{1/2} 3^{13/4} m_p m_{\nu e}} = \frac{(4m_\pi^2 / 3m_p)^{3/2}}{3^{1/8} m_{pl}^{1/2}} = 1.101 \cdot 10^{-3} \text{ eV} / c^2 \]  

(22)

where the second expression is obtained using equation (5), which is in excellent agreement with the mass \( m_1 \) of the normal hierarchy (Fig. 3) [3] and with the value computed in equation (20).

**Figure 4.** Effect of the mass of the composite particle, \( m_c \) (c=p, n, d, W, Z or H) on the computed via eqs. (5), (18) constituent \( m_3(\nu_e) \) neutrino mass value.

4. Constancy of the computed \( m_1, m_2 \) and \( m_3 \) values

Figure 4 presents the dependence of the computed \( m_3 \) value (i.e. of the mass value of the \( \nu_e \) neutrinos) on the composite particle mass of p, n, d, W, Z and H. The computed \( m_3 \) values are all within the uncertainty limits of the Superkamiokade measurements and vary with \( m_c \) less than 3% as the latter varies by a factor of 100. As shown in Figure 5, the same applies for the computed \( m_1 \) and \( m_2 \) values. The fact that the \( m_3 \) value computed from bosons (W, Z and H) are 2-3% smaller than those computed from hadrons may be due to more important role of electrostatic attraction in the case of bosons (quark charges or charge-induced dipole forces [10]).

5. Mass ratios from RLM and from experiment

On the basis of equations (5), (18), (20) and (22) the following four mass ratio predictions can be made regarding the masses of muons, pions [11], neutrons [6], protons [6], W [10], Z [12] and Higgs bosons [13]:
\[ \frac{m_\pi}{m_\mu} = \frac{(1/2)^{\frac{13}{12}}}{2^{\frac{1}{3}}} = 2^{-\frac{4}{3} \times \frac{13}{12}} = 1.305 \]  

This prediction is in excellent agreement with the experimental observations, i.e.

\[ \left( \frac{m_\pi}{m_\mu} \right)_{\text{exp}} = 1.304 \]  

which is the experimental value [14].

Figure 5. Effect of the mass of the composite particle on the computed values of the neutrino masses \( m_1, m_2 \) and \( m_3 \). Shaded areas show the uncertainty limits of Superkamiokande measurements [3].

\[ \frac{m_Z}{m_W} = \frac{2^{1/2}}{2^{1/3}} = 2^{1/6} = 1.122 \]  

This prediction is also in very good agreement with the experimental value [14]

\[ \left( \frac{m_Z}{m_W} \right)_{\text{exp}} = 1.1134 \]
c. *Higgs and W boson mass ratio* [13]

\[
\frac{m_H}{m_W} = 2 \left( \frac{1 - \alpha / 4}{2^{1/2} + 2^{-1}} \right)^{1/6} = \frac{2(0.897)}{2^{1/3}} = 1.423
\]  

(27)

where \(\alpha\) is the fine structure constant (\(\alpha=1/137.035\)). The value computed in eq. (27) is in qualitative agreement with the experimental value [14]

\[
\left( \frac{m_H}{m_W} \right)_{\text{exp}} = 1.556
\]

(28)

d. *W boson and proton mass ratio*

\[
\frac{m_W}{m_p} = \frac{2^{1/3}}{3^{13/12}} \left( \frac{m_e}{m_\nu} \right)^{1/3} = \frac{1.2598}{3.2876} = 86.93
\]

(29)

which is in very good agreement with the experimental value [14]

\[
\left( \frac{m_W}{m_p} \right)_{\text{exp}} = 85.71
\]

(30)

The good agreement of these four predictions with experiment is shown in Fig. 6.

![Figure 6](image_url)

**Figure 6.** Computed and experimentally measured values of the mass ratios of the pairs pion and muon (\(\pi/\mu\)), Z and W boson (Z/W), Higgs and W boson (H/W), and W boson and proton (W/p).

Experimental mass values: p: 938.272 MeV/c\(^2\), \(\pi^+\): 139.57 MeV/c\(^2\), \(\mu\): 105.7 MeV/c\(^2\), W: 80.4 GeV/c\(^2\).

6. **Conclusions**

The present analysis, and in particular Figures 3, 4, 5 and 6 show that the RLM can be used efficiently to compute the rest masses of neutrinos from the rest masses of hadrons, bosons and mesons with an
accuracy of at least 5%. This is quite significant in view of the extremely demanding and expensive nature of neutrino mass measurements [1,2].

Acknowledgements
We thank Professor Ilan Riess from the Physics Department at the Technion for helpful discussion.

This research is co-financed by the State of Greece and the European Union (European Social Fund-ESF) through the Operational Programme «Human Resources Development, Education and Lifelong Learning» in the context of the project “Reinforcement of Postdoctoral Researchers - 2nd Cycle” (MIS-5033021), implemented by the State Scholarships Foundation (IKY).

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