Hanle effect driven by weak-localization

I. S. Lyubinskiy, V. Yu. Kachorovskii
A.F. Ioffe Physical-Technical Institute, 26 Polytechnicheskaya str., Saint Petersburg, 194021, Russia
(Dated: March 23, 2022)

The influence of weak localization on Hanle effect in a two-dimensional system with spin-split spectrum is considered. We show that weak localization drastically changes the dependence of stationary spin polarization $S$ on external magnetic field $B$. In particular, the non-analytic dependence of $S$ on $B$ is predicted for III-V-based quantum wells grown in [110] direction and for [100]-grown quantum wells having equal strengths of Dresselhaus and Bychkov-Rashba spin-orbit coupling. It is shown that in weakly localized regime the components of $S$ are discontinuous at $B = 0$. At low $B$, the magnetic field-induced rotation of the stationary polarization is determined by quantum interference effects. This implies that the Hanle effect in such systems is totally driven by weak localization.

PACS numbers: 71.70Ej, 72.25.Dc, 73.23.-b, 73.63.-b

In recent years, the spin-related phenomena in semiconductor nanostructures have been subject to intense study both in terms of fundamental physics and in view of applications in the field of spintronics [1]. The ultimate goal of spintronics is to develop novel electronic devices that exploit the spin degree of freedom. The effective manipulation of the spin in such devices requires that the characteristic spin lifetime be long compared to the device operation time. This is a challenging problem, especially for III-V-based semiconductor nanostructures where spin polarization relaxes rapidly due to Dyakonov-Perel spin relaxation mechanism [2]. This mechanism is based on the classical picture of the angular spin diffusion in random magnetic field induced by spin-orbit coupling. In two-dimensional (2D) systems, the corresponding spin-relaxation time $\tau_S$ is inversely proportional to the momentum relaxation time $\tau$. As a consequence, in high-mobility structures which are most promising for device applications, $\tau_S$ is especially short. However, in some special cases, the relaxation of one of the spin components can be rather slow even in a system with high mobility. In particular, a number of recent researches [4,5,6,7,8,9] are devoted to GaAs symmetric quantum wells (QW) grown in [110] direction. In such wells, the random field is perpendicular to the QW plane and the normal to the plane component of the spin does not relax [3] or, more precisely, relaxes very slowly. Also, the random magnetic field might be parallel to a fixed axis in an asymmetric [100]-grown QW due to the interplay between structural and bulk spin-orbit coupling [10,11,12,13,14]. Since in both cases one component of the spin relaxes slowly, these structures are especially attractive for spintronics applications.

In this Letter, we discuss the dependence of the spin polarization in such structures on external magnetic field $B$. We assume that the spin is injected into the system with a constant rate, for example, by optical excitation [15]. The stationary spin polarization $S$ is proportional to the product of injection intensity and the spin relaxation time. The Hanle effect is that the external field modifies the stationary polarization. In particular, $S(B)$ deviates from $S(0)$ by an angle $\theta$ which depends on the relation between spin precession frequency $\Omega = g\mu_B B/h$, and the spin relaxation rate (here $\mu_B = e\hbar/2m_e$ is the Bohr magneton, $m_e$ is the free electron mass and $g$ is Landé $g$ factor). We show that at low temperatures $\theta$ is very sensitive to weak localization (WL) effects. Usually such effects are discussed in context of quantum corrections to the conductivity [16,17]. Though the WL correction is small compared to the classical conductivity it has attracted much attention due to its fundamental nature and anomalous behavior with external parameters such as magnetic field. Remarkably, the influence of WL on Hanle effect can not be considered as a small correction to the classical result. We demonstrate that at low temperatures WL gives rise to a discontinuity in the dependence of $\theta$ on $B$. As a result, at low $B$ the Hanle effect is totally driven by WL.

Physically, the non-analytic dependence of $\theta$ on $B$ is related to memory effects specific for WL. It is known that the WL is caused by interference of electron waves propagating along a closed loop in the opposite directions [17]. Such interference process can be considered as a coherent scattering (additional to the Born scattering) involving a large number of impurities [18] (see Fig. 1). The probability of the coherent scattering is proportional to the probability of the diffusive return $1/Dt$, where $t$ is the time of the electron passage along the loop and $D$ is the diffusion coefficient. In the absence of magnetic field, such scattering does not change the direction of the spin [19]. Thus, electrons keep memory about initial spin polarization during the time much larger than $\tau_S$ and the long-living tail $1/t$ in the spin polarization appears [13,20]. When the external magnetic field is applied, the electron spin rotates with a frequency $\omega_p + \Omega$, where $\omega_p$ is the momentum-dependent frequency of the spin precession in the spin-orbit-induced magnetic field. In the special case under discussion, $\omega_p$ is parallel to a
A system with a spin-split spectrum is given by the weakly localized regime. The Hamiltonian of a 2D situation, the external magnetic field leads to the spin ent segments of the loop do not commute in this case, also on the positions of impurities $\Omega$. The interference contribution to the electron spin-density matrixes, and $U$ is the Pauli matrix. In 2D systems, $\omega_p$ is linear in $p$. Since for any closed path $\int_0^1 p(t') dt' = 0$, we find $\phi = \phi' = \Omega t$. However, the electron spin path is rotated by $\Omega t$ with respect to the spin before scattering. The deviation of the spin from the initial direction is proportional to $\sin(\Omega t)$. The integration over $t$ weighted with the probability of the coherent scattering yields $\theta \sim \int dt \sin(\Omega t)/t \sim \Omega/t |\Omega|$. In other words, the rotation frequency $\omega_p$ should be multiplied by the effective rotation time $1/|\Omega|$, which is very long for small $\Omega$. Rigorous calculations (see below) give an additional factor $\lambda/\Omega$, which is related to the transport scattering time by $\gamma$. In contrast to coherent scattering, the classical rotation of spin is limited by the spin relaxation time $\tau_S$, so that the classical contribution is $\theta \sim \Omega \tau_S$. For $\Omega \tau_S \ll \lambda/|\Omega|$, this contribution can be neglected and the Hanle effect is totally driven by WL. The discontinuity at $\Omega = 0$ is smeared by inelastic scattering, which destroys phase coherence between clockwise and counterclockwise propagating paths, thus limiting a time of the coherent spin rotation: $t < \tau_\omega$, where $\tau_\omega$ is the phase breaking time. It worth noting that the above considerations do not work if $\omega_p$ changes its direction with a change of $p$. The reason is that the rotation matrixes corresponding to the different segments of the loop do not commute in this case. As a result, $\phi$ and $\phi'$ no longer equal to each other and depend not only on the total time spent on the loop but also on the positions of impurities $1, 2, \cdots, N$. In such a situation, the external magnetic field leads to the spin decay rather than to the spin relaxation $\tau_S$.

Next we develop a rigorous theory of Hanle effect in the weakly localized regime. The Hamiltonian of a 2D system with a spin-split spectrum is given by

$$H = \frac{p^2}{2m} + \frac{\hbar}{2} \omega_p + \Omega |\sigma| + U(r).$$

Here $p$ is the in-plane electron momentum, $m$ is the electron effective mass, $\sigma$ is a vector consisting of Pauli matrixes, and $U(r)$ is the impurity potential, which we assume to be short-ranged $U(r)U(r') = \gamma \delta(r - r')$ (here averaging is taken over impurity positions and the coefficient $\gamma$ is related to the transport scattering time by $\tau = \hbar^2/m\gamma$). The spin-orbit interaction is described by the term $i\omega_p \sigma/2$. It can be separated into two parts ($\omega_p = \omega_p^I + \omega_p^J$) related to so-called Dresselhaus and Bychkov-Rashba contributions. The Bychkov-Rashba coupling depends on the asymmetry of the QW confining potential. Its strength can be tuned by varying the gate voltage $\omega_p^J$. The Dresselhaus term is present in semiconductors with no bulk inversion symmetry. In 2D case the resulting spectrum splitting is linear in the electron momentum $\omega_p^I$. We assume that the random magnetic field is directed along $z$-axis

$$\omega_p = (p\alpha)\hat{z}. \quad (2)$$

Here $\hat{z}$ is the unit vector along the direction of the random field and $\alpha$ is a constant in-plane vector. Equation (4) implies that the spectrum splitting depends only on one component of momentum $p_\alpha$. This happens in symmetric [110]-grown QWs. In this case, $\hat{z}$ is normal to the QW plane. For asymmetric [100] wells Eq. (2) can also take place if Bychkov-Rashba and Dresselhaus couplings have equal strengths [111311]. For such wells vector $\hat{z}$ lies in the QW plane. The temperature is assumed to be low, so that $\tau_\omega$ be large compared to $\tau_S$.

To describe spin dynamics in the weakly localized regime we use the kinetic equation [11]. If the spin polarization is uniform in space, this equation looks as follows

$$\frac{\partial s}{\partial t} = [\omega_p + \Omega] \times s - \frac{s_\alpha}{\tau} + \delta Js + I, \quad (3)$$

where $s = s(p)$ is the spin density in the momentum space, related to the averaged spin by $S = \int s d^2p/(2\pi)^2$, $s_\alpha = s - \langle s \rangle$ is the anisotropic part of the spin density, $s_\alpha = \langle s \rangle$ is its isotropic part (here $\langle \ldots \rangle$ stand for averaging over directions of the electron momentum), $I$ is the constant source, which we assume to be monoenergetic, $I \sim \delta(E - E_0)$, and $\delta J$ is the WL-induced correction to the Boltzmann collision integral [1224]. For $E = E_0$, $\tau = \hbar^2/m\gamma$. We let us consider the simplest case, when $\Omega$ also lies along $z$ axis. In this case, spin rotation matrixes, describing rotation of electron spin on the different segments of the closed loop commutes with each other and the spin rotation angles for clockwise and counterclockwise propagating waves are simply given by

$$\phi = \Omega t + \int_0^t \omega_p dt' \quad \text{and} \quad \phi' = \Omega t + \int_0^t \omega_p dt'.$$

The initial electron spinor $|\chi_0\rangle$ is transformed to $|\chi\rangle = e^{-i\phi'/2}|\chi_0\rangle$ and $|\chi'\rangle = e^{-i\phi'/2}|\chi_0\rangle$ for clockwise and counterclockwise paths, respectively. Here $\hat{z}$ is the Pauli matrix.
this correction reads (see Eq. (24) in Ref. 19)
\[
\delta J_2 = -\frac{\lambda}{\pi r^2} \int_{-\infty}^{t} dt' \hat{W}(t-t') s_{\mu}(p, t').
\]
(4)
Here \( \lambda = 2\pi\hbar/\sqrt{2mE_0} \) is the time-nonlocal scattering kernel \( \hat{W}(t) \) and \( s_{\mu} \) is the spin relaxation tensor (tensor of inverse relaxation times) given by \( \hat{\Gamma} \).

Using Eqs. (3) and (6), we find a closed equation for the isotropic spin density in the stationary case
\[
\hat{\Gamma}(1 - \hat{\Lambda}) s_i - \Omega \times s_i = \mathbf{I}.
\]
(5)
Here \( \hat{\Gamma} = \hat{\tau}_S^{-1} \) is the spin relaxation tensor and \( \hat{\Lambda} \) is the tensor \( \hat{\Gamma} \) has two nonzero components:
\[
\Gamma_{ik} = [\delta_{ik} (\omega_p^2) - \langle \omega_p \omega_p \rangle] \tau,
\]
(6)
and the matrix
\[
\hat{\Lambda} = \frac{\lambda}{4\pi^2} \int_{-\infty}^{t} dt \hat{W}(t).
\]
(7)
describes WL correction \( \hat{\Lambda} \). As follows from Eqs. (3) and (6), tensor \( \hat{\Gamma} \) has two nonzero components:
\[
\hat{\Gamma}_{xx} = \hat{\Gamma}_{yy} = \Gamma = mE_0\alpha^2 T
\]
(8)
We will assume that there exists a slow spin relaxation of the \( x \) component of the spin \( \hat{\Gamma}_{xx} = \epsilon \ll \Gamma \). Such relaxation arises due to the cubic in terms, neglected in the Hamiltonian \( \hat{\Gamma} \), or due to other mechanisms of the spin relaxation. In the absence of the external magnetic field the scattering kernel is given by \( W_{\mu \nu} = -\delta_{\mu \nu} / 4\pi D t \). Therefore, \( \hat{\Gamma}_{xx} = (\lambda/2\pi^2) \delta_{x x} \ln(\tau_s/\tau_r) \), where phase breaking time \( \tau_r \) is taken as an upper cut-off of the integral in Eq. (6).

Next we consider the dependence of \( \hat{\Lambda} \) on the magnetic field for the case when \( \mathbf{B} \) is parallel to \( z \)-axis. We start with discussing of the QW grown in [100] direction. Since in this case \( \hat{\Lambda} \) lies in the QW plane, the external field does not affect the orbital motion of the electrons and the Zeeman term \( \varphi \Omega \sigma_z/2 \) commutes with the Hamiltonian. As a consequence, the solution of Eq. (3) with \( I = 0 \) and \( B \neq 0 \) can be obtained from solution at zero field: \( s_B(t) = \hat{T}(\Omega t) s_B(0) \), where \( \hat{T} = 3 \times 3 \) matrix, describing rotation around \( z \)-axis by the angle \( \Omega t \). In order that \( s_B(t) \) obeys Eq. (6), the scattering kernel has to be as follows:
\[
\hat{W}(t) = \hat{T}(\Omega t) \hat{W}(0) \hat{T}(\Omega t)^{-1}.
\]
(9)
Equations (8), (7) and (6) allows us to find the isotropic spin polarization. If \( \mathbf{I} \) is perpendicular to the \( z \)-axis, we get
\[
\hat{\Lambda} = \chi \hat{\Lambda}_0 + (\hat{\Lambda}_s + \Omega / \Gamma) \hat{\Lambda}_0
\]
(10)
Here \( \chi \) is the angle between \( \mathbf{S} \) and \( \mathbf{S}_0 \) is
\[
\hat{\Lambda}_0 \approx \frac{\chi}{2\pi^2} \int_{-\infty}^{t} dt' \hat{W}(t-t') s_{\mu}(p, t'),
\]
(11)
and the function \( \hat{\Lambda}_0 \) obeys Eq. (10).

For \( \Omega / \Gamma \ll 1 \), the angle between \( \mathbf{S} \) and \( \mathbf{S}_0 \) is
\[
\theta(\Omega) \approx \frac{\chi}{\Gamma} + \frac{\lambda}{4\pi^2} \left[ \frac{\pi}{2} |\Omega| \right] \theta(\chi).
\]
(12)
In this equation, \( \Omega / \Gamma \) stands for the classical contribution, while the second term is due to WL effect. At small fields the classical contribution can be neglected. We see that WL contribution to \( \theta(\Omega) \) has a discontinuity
\[
\theta(\chi) - \theta(\chi_0) = \lambda / 2\pi l.
\]
(13)
In deriving Eq. (10), we neglected inelastic scattering. Such scattering destroys phase coherence, thus suppressing WL contribution. In order to take it into account, we introduce a factor \( \exp(-t/\tau_r) \) in the integrand in Eq. (10). As a result, we find
\[
\theta(\chi) \approx \frac{\chi}{\Gamma} + \frac{\lambda}{2\pi^2} \left[ \frac{\pi}{2} |\Omega| \right] \theta(\chi).
\]
(14)
This equation shows that for low temperatures, when \( 1/\tau_r \ll \chi / \Gamma \), the Hanle effect is still totally driven by WL at small \( \Omega \). With increasing the temperature, the WL-induced discontinuity is smeared out.

Let us now consider the system with symmetric QW grown in [110] direction, assuming again that \( \Omega \parallel \mathbf{z} \) and \( \mathbf{I} \perp \mathbf{z} \). The above considerations can be applied with a slight modification of Eq. (10). Since in this case \( \hat{\Lambda} \) is normal to the QW plane, the external magnetic field also affects orbital motion of the electrons leading to the suppression of WL. Including into the integral in Eq. (10) an additional factor \( \gamma/t \), which accounts for destruction of WL by external field (here \( \gamma = 2eBD/\hbar c \)), we obtain
\[
\theta(\chi) \approx \frac{\chi}{\Gamma} + \frac{\lambda}{4\pi^2} \left[ \frac{\pi}{2} |\Omega| \right] \theta(\chi).
\]
(15)
Next we find the stationary polarization for arbitrary directions of \( \mathbf{B} \) and \( \mathbf{I} \). We choose \( x, y \) axes in such a way that \( \hat{\Lambda}_x = 0 \). The classical solution (\( \hat{\Lambda} = 0 \)) reads
\[
\hat{S} \approx \left( \begin{array}{ccc}
\frac{1}{\Omega \pi / 2} & -\Omega \pi / 2 & \Omega \pi / 2 \\
\Omega \pi / 2 & \Omega \pi / 2 & \Omega \pi / 2 \\
-\Omega \pi / 2 & \Omega \pi / 2 & \Omega \pi / 2 \\
\end{array} \right) \hat{S}_0.
\]
(16)
(here we assumed \( \Omega \ll \Gamma \)). To find \( \hat{\Lambda} \) we write
\[
\hat{W}_{ij}(t) = \sum_{\beta \gamma \alpha} \langle \sigma_i | \hat{\beta} \rangle \hat{W}_{\beta \gamma \alpha \theta}(0, t) \langle \alpha | \hat{\gamma} \rangle | \sigma_j \rangle / 2,
\]
where function \( \hat{W}_{\beta \gamma \alpha \theta}(r, t) \) obeys
\[
\hat{K}_{\beta \gamma \alpha \theta}(r, t) = \delta(t) \delta_{\alpha \beta} \delta_{\gamma \theta},
\]
\[
\hat{K} = \frac{\partial}{\partial t} - eA \frac{\partial}{\partial r} + im \xi \hat{\sigma}^{(1)} \hat{\sigma}^{(2)} / 2.
\]
(17)
Here $\sigma_{\beta \gamma}^{(1)} = \langle \beta | \sigma | \gamma \rangle \delta_{\theta \theta}$, $\sigma_{\beta \gamma}^{(2)} = \delta_{\beta \gamma} \langle \theta | \sigma | \theta \rangle$ and $A$ is the vector potential of external field. Using Eqs. 10 and 11, one can find

$$\hat{\Lambda} = \frac{\lambda}{2\pi^2t} \int_0^\infty dt \frac{\gamma'}{\sinh(\gamma' t)} \hat{T}(\Omega z t). \tag{18}$$

Here $\gamma' = 2eBnD/hc$ and $n$ is the unit vector normal to the QW plane. It is worth noting that the rotation matrix, entering Eq. (18), depends only on $z$-component of the external field. Accounting for WL leads to the following replacement in Eq. (15):

$$\Gamma \rightarrow \Gamma (1 - \Lambda_c), \quad \Omega_z \rightarrow \Omega_z + \Gamma A_s, \quad \epsilon \rightarrow \epsilon (1 - \Lambda_0), \tag{19}$$

where $\Lambda_c = (\lambda/2\pi^2 t) \ln(t_0/\tau)$, $\Lambda_0 = (\lambda/2\pi^2 t) \ln(t_0/\tau)$ and $\Lambda_s = (\lambda/4\pi t) \tanh(\lambda\Omega n/8m_0 |\Omega n|)$ (here $t_0^{-1} \sim \max[|\gamma' \rangle, |\tau']$, $t_1^{-1} \sim \max[|\gamma' \rangle, |\Omega_z|]$). Eqs. (15) and (19) are valid both for [110] and [100] orientations provided that $\Omega_z \gg \Delta \Omega_z \sim \max[|\Omega^n/\Gamma, 1/\tau_e|]$. One can show that the discontinuity of $\Lambda_s$ (at the point $\Omega = 0$) is smeared over the frequency interval $\Delta \Omega_z$.

A case when $I$ is parallel to $y$ axis is of particular interest. For $\Omega_y \gg \epsilon$ and $\Omega_z \ll \Gamma A_s$ we find

$$\theta = \arctan \left( \frac{\Omega_y \Omega_s}{\epsilon \Gamma + \Omega_y^2} \right). \tag{20}$$

We see that $\theta \gg \Lambda_s$ due to the factor $\Omega_y/\Gamma (\epsilon + \Omega_y^2) \gg 1$. For $\Gamma y_0/\epsilon (\Gamma x + \Omega_y) \gg 1/\lambda$ the $z$-component of the spin will be larger than $y$-component. Hence, the quantum effects might lead to rotation of spin by a large angle.

In the above calculations we assumed that $I$ is homogeneous. For slowly varying $I$, the derived equations relate $S(r)$ with $I(r)$ provided that the spatial scale of inhomogeneity $L$ is large compared to $\sqrt{D/\Gamma} = 1/ma$ [21]. One can show that in the opposite case $L \ll 1/m \alpha$, these equations also valid relating $\int dr S(r)$ with $\int dr I(r)$.

Finally, we briefly discuss a possible experimental realization of the predicted effect. One of the most efficient ways of the spin injection is the optical excitation of interband transitions with circular polarized light [17]. The observation of the weakly localized regime requires that $\tau_e \gg \tau_\sigma$. In optical experiments, the phase breaking is due to both inelastic scattering (caused by electron-phonon and electron-electron interactions) and recombination of electrons with holes. The recombination of spin-polarized electrons with holes is suppressed in a $n$-type highly doped QW excited by low-intensity light. In this case, the number of spin polarized electrons is small compared to their total number and the holes are more probably to recombine with unpolarized electrons. Therefore, the stationary amount of holes is small and the recombination of polarized electrons with the holes can be neglected. The characteristic time of inelastic scattering will be large at low temperatures if the electrons are excited close to the Fermi level. In such a situation, the emission of the optical phonons is forbidden and the phase breaking is due to electron-electron collisions. Since the rate of such collisions decreases with approaching to the Fermi level, $\tau_e$ can be tuned to be much longer than $\tau_\sigma$.

To conclude, the theory of Hanle effect in a 2D system is developed for the weakly localized regime. At low external magnetic fields the Hanle effect is totally driven by quantum interference effects. In the absence of inelastic scattering, the components of the spin polarizations are discontinuous as functions of the external field.

We are grateful to K. V. Kavokin for useful discussions. This work has been supported by RFBR, a grant of the RAS, a grant of the Russian Scientific School 2192.2003.2, and a grant of the foundation "Dynasty"-ICFPM.
proportional to the rotation matrix (see Eqs. (5), (13)).

[27] S. Chakravarty and A. Schmid, Physics Reports, 140, 195 (1986)

[28] W. Knap et al, Phys. Rev. B 53 3912 (1996)

[29] S.V. Iordanskii, Yu.B. Lyanda-Geller, and G.E. Pikus, JETP Letters 60, 206 (1994).

[30] Note that $1/m\alpha$ is a characteristic scale of non-decaying spin oscillations which might exist in the system under discussion (see Ref. [13]).