Quintessence ghost dark energy model

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Abstract – A so-called “ghost dark energy” was recently proposed to explain the present acceleration of the universe expansion. The energy density of ghost dark energy, which originates from the Veneziano ghost of QCD, is proportional to the Hubble parameter, \( \rho_D = \alpha H \), where \( \alpha \) is a constant which is related to the QCD mass scale. In this paper, we establish the correspondence between ghost dark energy and quintessence scalar-field energy density. This connection allows us to reconstruct the potential and the dynamics of the quintessence scalar-field according to the evolution of ghost energy density.

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Introduction. – A wide range of cosmological observations, direct and indirect, provides an impressive evidence in favor of the present acceleration of the cosmic expansion. To explain this acceleration, in the context of standard cosmology, we need an anti-gravity fluid with negative pressure, usually dubbed “dark energy” in the literature. The first and simple candidate for dark-energy is the cosmological constant with equation-of-state parameter \( w = -1 \) which is located at the central position among dark-energy models both in theoretical investigation and in data analysis [1]. However, there are several difficulties with the cosmological constant. For example, it suffers from the so-called fine-tuning and cosmic coincidence problems. Besides, its origin is still a big source of doubt. Furthermore, the accurate data analysis shows that the time-varying dark energy gives a better fit than a cosmological constant and, in particular, \( w \) can cross \(-1\) around \( z \approx 0.2 \) from above to below [2]. Although the galaxy cluster gas mass fraction data do not support the time-varying \( w \) [3], an overwhelming flood of papers has appeared which attempt to understand the \( w = -1 \) crossing. Among them are a negative kinetic scalar field and a normal scalar field [4], or a single-scalar-field model [5], interacting holographic [6] and interacting agegraphic [7] dark-energy models. Other studies on the \( w = -1 \) crossing [8] and dark-energy models have been carried out in [9]. For a recent review on dark-energy models see [10]. It is worth noting that in most of these dark-energy models, the accelerated expansion is explained by introducing new degree(s) of freedom or by modifying the underlying theory of gravity.

Recently a very interesting suggestion on the origin of a dark energy is made, without introducing new degrees of freedom beyond what are already known, with the dark energy of just the right magnitude to give the observed expansion [11,12]. In this proposal, it is claimed that the cosmological constant arises from the contribution of the ghost fields which are supposed to be present in the low-energy effective theory of QCD [13–17]. The ghosts are required to exist for the resolution of the \( U(1) \) problem, but are completely decoupled from the physical field [17]. The above claim is that the ghosts are decoupled from the physical states and make no contribution in the flat Minkowski space, but once they are in the curved space or time-dependent background, the cancellation of their contribution to the vacuum energy is off-set, leaving a small energy density \( \rho \sim H \Lambda_{QCD}^3 \), where \( H \) is the Hubble parameter and \( \Lambda_{QCD} \) is the QCD mass scale of the order of \( 100 \text{MeV} \). With \( H \sim 10^{-33} \text{eV} \), this gives the right magnitude \( \sim (3 \times 10^{-3} \text{eV})^4 \) for the observed dark-energy density. This numerical coincidence is remarkable and also means that this model gets rid of the fine-tuning problem [11,12]. The advantage of this new model compared to other dark-energy models is that it is totally embedded in the standard model and general relativity; one need not introduce any new parameter, new degree

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of freedom or modify gravity. The dynamical behavior of the ghost dark-energy (GDE) model in flat [18] and non-flat [19] universe has been studied in ample details.

On the other hand, the scalar-field model can be regarded as an effective description of the underlying dark-energy theory. Scalar fields naturally arise in particle physics including supersymmetric field theories and string/M theory. Therefore, a scalar field is expected to reveal the dynamical mechanism and the nature of dark energy. However, although fundamental theories such as string/M theory do provide a number of possible candidates for scalar fields, they do not predict their potential $V(\phi)$ uniquely. Consequently, it is meaningful to reconstruct the potential $V(\phi)$ from some dark-energy models possessing some significant features of the quantum gravity theory, such as holographic and agegraphic dark-energy models. In the framework of holographic and agegraphic dark-energy models, the studies on the reconstruction of the quintessence potential $V(\phi)$ have been carried out in [20] and [21], respectively. Till now, quintessence reconstruction of ghost energy density has not been done.

In this paper we are interested, if we assume the GDE scenario as the underlying theory of dark energy, in how the low-energy effective scalar-field model can be used to describe it. In this direction, we can establish the correspondence between the GDE and the quintessence scalar field, and describe GDE in this case effectively by making use of quintessence. We shall reconstruct the quintessence potential and the dynamics of the scalar field in the light of the GDE.

**Quintessence ghost dark energy.** – We assume the GDE is accommodated in a flat Friedmann-Robertson-Walker (FRW) universe whose dynamics is governed by the Friedmann equation

$$H^2 = \frac{1}{3M_p^2}(\rho_m + \rho_D),$$

(1)

where $\rho_m$ and $\rho_D$ are the energy densities of pressureless matter and GDE, respectively. We define the dimensionless density parameters as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}}, \quad \Omega_D = \frac{\rho_D}{\rho_{cr}},$$

(2)

where the critical energy density is $\rho_{cr} = 3H^2M_p^2$. Thus, the Friedmann equation can be rewritten as

$$\Omega_m + \Omega_D = 1.$$  

(3)

The conservation equations read

$$\dot{\rho}_m + 3H\rho_m = 0,$$

(4)

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = 0.$$  

(5)

The ghost energy density is proportional to the Hubble parameter [12,18]

$$\rho_D = \alpha H,$$

(6)

where $\alpha$ is a constant of order $\Lambda_{QCD}^3$ and $\Lambda_{QCD} \sim 100$ MeV is QCD mass scale. Taking the time derivative of relation (6) and using Friedmann equation (1) we find

$$\dot{\rho}_D = -\frac{\alpha}{2M_p^2}\rho_D(1 + u + w_D),$$

where $u = \rho_m/\rho_D$ is the energy density ratio. Inserting this relation into the continuity equation (5) and using eq. (3) we find

$$w_D = -\frac{1}{2 - \Omega_D}.$$  

(8)

At the early time where $\Omega_D \ll 1$, we have $w_D = -1/2$, while at the late time where $\Omega_D \to 1$, the GDE mimics a cosmological constant, namely $w_D = -1$. In fig. 1 we have plotted the evolution of $w_D$ vs. the scale factor $a$. From this figure we see that $w_D$ of the GDE model cannot cross the phantom divide and the universe has a de Sitter phase at late time.

Now we are in a position to establish the correspondence between GDE and the quintessence scalar field. To do this, we assume that the quintessence scalar-field model of dark energy is the effective underlying theory. The energy density and pressure of the quintessence scalar field are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$

(9)

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$  

(10)

Thus, the potential and the kinetic energy term can be written as

$$V(\phi) = \frac{1 - w_\phi}{2}\rho_\phi,$$

(11)

$$\dot{\phi}^2 = (1 + w_\phi)\rho_\phi.$$  

(12)

In order to implement the correspondence between GDE and the quintessence scalar field, we identify $\rho_\phi = \rho_D$ and $w_\phi = w_D$. Therefore, the potential and the kinetic energy term can be written as

$$V(\phi) = \frac{1 - w_D}{2}\rho_D,$$

(13)

$$\dot{\phi}^2 = (1 + w_D)\rho_D.$$  

(14)

Fig. 1: (Colour on-line) The evolution of $w_D$ for GDE.
\[ w_{\phi} = w_D. \] Using eqs. (6) and (8) as well as relation \( \dot{\phi} = H \frac{d\phi}{d\ln a} \), we obtain the scalar potential and the dynamics of scalar field as

\[
V(\phi) = \frac{\alpha^2}{6M_p^2} \times \frac{3 - \Omega_D}{\Omega_D(2 - \Omega_D)}, \tag{13}
\]

\[
\frac{d\phi}{d\ln a} = \sqrt{3}M_p \sqrt{\frac{\Omega_D(1 - \Omega_D)}{2 - \Omega_D}}. \tag{14}
\]

Integrating yields

\[
\phi(a) - \phi(a_0) = \sqrt{3}M_p \int_{a_0}^{a} \frac{da}{a} \sqrt{\frac{\Omega_D(1 - \Omega_D)}{2 - \Omega_D}}, \tag{15}
\]

where we have set \( a_0 = 1 \) for the present value of the scale factor. The analytical form of the potential in terms of the ghost quintessence field cannot be determined due to the complexity of the equations involved. However, we can obtain it numerically. The evolutionary form of the field and the reconstructed quintessence potential \( V(\phi) \) are plotted in figs. 2 and 3, where we have taken \( \phi(a_0 = 1) = 0 \) for simplicity. From fig. 2 we can see the dynamics of the scalar field explicitly. Obviously, the scalar field \( \phi \) rolls down the potential with the kinetic energy \( \dot{\phi}^2 \) gradually decreasing. In other words, the amplitude of \( \phi \) decreases with time in the past.

**Interacting quintessence ghost dark energy.**

Next we generalize our discussion to the interacting case. Although at this point the interaction may look purely phenomenological, different Lagrangians have been proposed in support of it (see [22] and references therein). Besides, in the absence of a symmetry that forbids the interaction, there is nothing, in principle, against it. In addition, given the unknown nature of both dark energy and dark matter, which are two major contents of the universe, one might argue that an entirely independent behavior of dark energy is very special [23,24]. Thus, microphysics seems to allow enough room for the coupling; however, this point is not fully settled and should be further investigated. The difficulty lies, among other things, in the fact that the very nature of both dark energy and dark matter remains unknown, whence the detailed form of the coupling cannot be elucidated at this stage. Since we consider the interaction between dark matter and dark energy, \( \rho_m \) and \( \rho_D \) are not conserved separately; they must rather enter the energy balances [24],

\[
\dot{\rho}_m + 3H \rho_m = Q, \tag{16}
\]

\[
\dot{\rho}_D + 3H \rho_D (1 + w_D) = -Q, \tag{17}
\]

where \( Q \) represents the interaction term and we take it as

\[
Q = 3b^2 H (\rho_m + \rho_D) = 3b^2 H \rho_D (1 + u), \tag{18}
\]

with \( b^2 \) being a coupling constant. Inserting eqs. (7) and (18) in eq. (17) we find

\[
w_D = -\frac{1}{2 - \Omega_D} \left( 1 + \frac{2b^2}{\Omega_D} \right). \tag{19}
\]

One can easily check that in the late time where \( \Omega_D \to 1 \), the equation-of-state parameter of interacting GDE necessary crosses the phantom line, namely, \( w_D = -(1 + 2b^2) < -1 \), independent of the value of the coupling constant \( b^2 \). For the present time, taking \( \Omega_D = 0.72 \), the phantom crossing can be achieved provided \( b^2 > 0.1 \), which is consistent with recent observations [23]. It is worth mentioning that the continuity equations (24) and (25) imply that the interaction term should be a
function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor $H$) multiplied by the energy density. Therefore, the interaction term could be in any of the following forms: i) $Q \propto H \rho_D$, ii) $Q \propto H \rho_m$, or iii) $Q \propto H (\rho_m + \rho_D)$. We can present the above three choices in one expression as $Q = \Gamma \rho_D$, where

$$
\Gamma = 3b^2 H \quad \text{for} \quad Q \propto H \rho_D,
\Gamma = 3b^2 Hu \quad \text{for} \quad Q \propto H \rho_m,
\Gamma = 3b^2 H (1 + u) \quad \text{for} \quad Q \propto H (\rho_m + \rho_D).
$$

It should be noted that the ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on a purely dimensional basis for choosing an interaction $Q$. To be more general in this work we choose expression iii) for the interaction term. The coupling $b^2$ is taken in the range $[0, 1]$ [25]. Note that if $b^2 = 0$ then it represents the non-interacting case while $b^2 = 1$ yields complete transfer of energy from dark energy to matter ($Q > 0$). Although, in principle, there is now reason to take $Q > 0$, one may take $Q < 0$, which means that dark matter transfers to dark energy; however, as we will see below this is not the case. It is easy to show that for $Q < 0$, eq. (19) becomes

$$
w_D = -\frac{1}{2 - \Omega_D} \left( 1 - \frac{2b^2}{\Omega_D} \right). \quad (21)
$$

In the late time where $\Omega_D \to 1$, we have $w_D = -\left( 1 - 2b^2 \right)$, which for $b^2 > 1/3$ leads to $w_D > -1/3$. This implies that in the late time where dark energy dominates we have no acceleration at least for some value of the coupling parameter. For the present time if we take $\Omega_D = 0.72$, from eq. (21) we have $w_D = -0.78 + 2.2b^2$. Again for $b^2 > 0.20$ we have $w_D > -1/3$ for the present time. This means that universe is in the deceleration phase at the present time which is ruled out by recent observations.

The behaviour of the equation-of-state parameter of interacting GDE is shown in fig. 4 for different values of the coupling parameter. In the presence of interaction, the evolution of GDE is governed by the following equation [19]:

$$
\frac{d\Omega_D}{d\ln a} = \frac{3}{2} \Omega_D \frac{\Omega_D - 1}{\Omega_D - 2} \left( 1 + 2 \frac{2b^2}{\Omega_D} \right). \quad (22)
$$

Figure 5 shows that at the early time $\Omega_D \to 0$, while at the late time $\Omega_D \to 1$, that is, the ghost dark energy dominates as expected. Now we implement a connection between interacting GDE and the quintessence scalar field. In this case the potential and scalar field are obtained as

$$
V(\phi) = \frac{a^2}{6M_p^2} \times \frac{1}{\Omega_D(2 - \Omega_D)} \left( 3 - \Omega_D + \frac{2b^2}{\Omega_D} \right), \quad (23)
$$

$$
\frac{d\phi}{d\ln a} = \sqrt{3M_p} \left( \frac{\Omega_D}{2 - \Omega_D} - \frac{1}{\Omega_D - 2} \frac{2b^2}{\Omega_D} \right). \quad (24)
$$

Finally we obtain the evolutionary form of the field by integrating the above equation. The result is

$$
\phi(a) - \phi(a_0) = \sqrt{3M_p} \times \int_{a_0}^a \frac{da}{a} \sqrt{\Omega_D(2 - \Omega_D) - \frac{2b^2}{\Omega_D}}. \quad (25)
$$

where $\Omega_D$ is now given by eq. (22). The evolutionary form of the field and the reconstructed quintessence potential $V(\phi)$ are plotted in figs. 6 and 7, where again we have taken $\phi(a_0) = 1 = 0$ for the present time. Selected curves are plotted for different values of the coupling parameter $b^2$. From these figures we find out that $\phi$ increases with time, while the potential $V(\phi)$ becomes steeper with increasing $b^2$.  

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Conclusion. – Considering the quintessence scalar-field dark-energy model as an effective description of the underlying theory of dark energy, and assuming the ghost vacuum energy scenario as pointing in the same direction, it is interesting to study how the quintessence scalar-field model can be used to describe the ghost energy density. The quintessence scalar-field is specified to an ordinary scalar field minimally coupled to gravity, namely the canonical scalar field. It is remarkable that the resulting model with the reconstructed potential is the unique canonical single-scalar model that can reproduce the GDE evolution of the universe. In this paper, we established a connection between the GDE scenario and the quintessence scalar-field model. The GDE model is a new attempt to explain the origin of dark energy within the framework of the Veneziano ghost of QCD [12]. If we regard the quintessence scalar-field model as an effective description of GDE, we should be capable of using the scalar-field model to mimic the evolving behavior of the dynamical ghost energy and of reconstructing this scalar-field model according to the evolutionary behavior of GDE. With this strategy, we reconstructed the potential of the ghost quintessence and the dynamics of the field according to the evolution of ghost energy density.

Finally we would like to mention that the aforementioned discussion in this paper can be easily generalized to other non-canonical scalar fields, such as K-essence and tachyon. It can also be extended to the non-flat FRW universe.

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Fig. 6: (Colour on-line) The evolutionary form of the scalar field $\phi(a)$ for interacting quintessence GDE, where $\phi$ is in unit of $\sqrt{3}M_p$.

Fig. 7: (Colour on-line) The reconstructed potential $V(\phi)$ for interacting quintessence GDE, where $V(\phi)$ is in unit of $(\alpha^2/6M_p^2)$.
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