MAGNETIC PROPERTIES OF NEUTRINOS IN HIGH TEMPERATURE $SU(2)_L \otimes U(1)$ GAUGE THEORY

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Abstract

We calculate the finite temperature self-energy for neutrinos in the presence of a constant magnetic field in a medium in the unbroken $SU(2) \otimes U(1)$ model. We obtain the exact dispersion relation for such neutrinos and find that the thermal effective mass is modified by the magnetic field. We also find a simple analytic expression for the dispersion relation and obtain the index of refraction for large neutrino momentum.

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I. INTRODUCTION

It has been shown in the past that neutrinos in the unbroken $SU(2)\otimes U(1)$ model acquire an effective mass due to finite temperature effects \[1\]-\[3\], and it has also been shown that the dispersion relation for neutrinos in the spontaneously broken Weinberg-Salam model is significantly changed in the presence of a magnetic field \[4\]. It is therefore conceivable that finite temperature effects may modify in a significant way the propagation of massless Dirac neutrinos in a magnetic field. The purpose of this paper is to study the propagation of neutrinos in a medium at some finite temperature $T$ ($T > 250$ GeV) and in the presence of a constant and homogeneous magnetic field, which in the rest frame of the medium is $B^\mu = (0, \vec{B})$ with $\vec{B}$ parallel to the $z$-axis.

The presence of a medium introduces a special Lorentz frame: the center of mass of the heat bath. Therefore, in the presence of a magnetic field, the neutrino self-energy will be of the form

$$\Sigma(p, B) = \gamma_R(a \not{p} + b \not{u} + c \vec{B} + b' \not{u})\gamma_L$$  \hspace{1cm} (1.1)$$

where $u_\mu$ is the 4-velocity of the medium, $\gamma_R$ and $\gamma_L$ are the right-handed and left-handed projection operators respectively, and the coefficients $a$, $b$, $b'$ and $c$ are Lorentz-invariant functions. The two functions $a$ and $b$ have been calculated previously \[1\], and the purpose of our paper is to calculate the functions $b'$ and $c$, which appear only in the presence of a magnetic field. In particular $b'$ will be explicitly dependent on $B$ and will vanish for $B \rightarrow 0$.

In section II we calculate the finite temperature neutrino self-energy in the presence of a constant magnetic field for $T > 250$ Gev and $eB \ll T^2$. In section III we obtain the exact neutrino dispersion relation in a medium and in the presence of a magnetic field. The presence of the magnetic field is shown to modify the value of the neutrino thermal effective mass. We also obtain a simple analytic expression for the neutrino dispersion relation in a magnetic field and find the index of refraction in the case of large neutrino momentum limit. In section VI we comment on our results and discuss implications of our results.
II. COMPUTATION OF THE FINITE TEMPERATURE SELF-ENERGY IN A
CONSTANT MAGNETIC FIELD

We start by considering the one loop neutrino self-energy $\Sigma_0(p, B)$ in a constant, homogeneous magnetic field $\vec{B}$ (which we take to be oriented along the $z$-axis) at zero temperature. We need to consider two Feynman diagrams for this calculation, the W-lepton diagram and the scalar-lepton diagram. However, only the W-lepton diagram is relevant since the scalar-lepton can be neglected because the Yukawa coupling $f$ is much smaller than the electroweak coupling $g$. We follow the notation used in Ref. [5], except for taking $g^{\mu\nu} = \text{diag}(+1, -1, -1, -1)$, and we write the vacuum self-energy as

$$\Sigma_0(x', x'') = i \frac{g^2}{2} \gamma_R \gamma_\mu S_-(x', x'') G^\mu_\nu(x', x'') \gamma^\nu \gamma_L$$

(2.1)

where

$$\gamma_R = \frac{1 + \gamma_5}{2}, \quad \gamma_L = \frac{1 - \gamma_5}{2},$$

(2.2)

$S_-(x', x'')$ is the exact lepton propagator in a constant magnetic field [6,7] and $G^\mu_\nu(x', x'')$ is the exact W-propagator in a magnetic field [5] and they are given by

$$S_-(x', x'') = i \phi_-(x', x'') \int \frac{d^4q}{(2\pi)^4} e^{iq(x''-x')} \int_0^\infty ds_1 \exp \left[ is_1 \left( q^2_\parallel + q^2_\perp \frac{\tan z_1}{z_1} \right) \right] \left[ q^\mu_\parallel e^{-iz_1\sigma_3} + \frac{q^\mu_\perp}{\cos z_1} \right]$$

(2.3)

$$G^\mu_\nu(x', x'') = i \phi_+(x', x'') \int \frac{d^4k}{(2\pi)^4} e^{ik(x''-x')} \int_0^\infty ds_2 \exp \left[ is_2 \left( k^2_\parallel + k^2_\perp \frac{\tan z_2}{z_2} \right) \right] \left[ (\delta^\mu_\nu)_\parallel + (e^{2z_2'e})^\mu_\nu \right]$$

(2.4)

with

$$\phi_\pm(x', x'') = \exp \left( \pm i \frac{e}{2} x''^\mu F_{\mu\nu} x'_\nu \right)$$

(2.5)

$$z_i = eBs_i, \quad i = 1, 2; \quad B = |\vec{B}|$$

(2.6)
\[ \sigma_3 = \sigma^{12} = \frac{i}{2} [\gamma^1, \gamma^2] \]  

(2.7)

where \( F^{\mu\nu} \) is the electromagnetic field strength tensor and for any 4-vector \( a^\mu \) we define \( a_\parallel^\mu \) and \( a_\perp^\mu \) as

\[
a_\parallel^\mu = (a^0, 0, 0, a^3) \quad , \quad a_\perp^\mu = (0, a^1, a^2, 0)
\]  

(2.8)

and

\[
\left( e^{2z_2} \epsilon^\mu \right)_\perp^\nu = (\delta^\mu_\parallel, \cos 2z_2 + \epsilon^\mu_\perp \sin 2z_2
\]  

(2.9)

with \( \epsilon^1_2 = -\epsilon^2_1 = 1 \) and \( \epsilon^\mu_\nu = 0 \) for all other values of the two indices. We are working with the unbroken version of the Weinberg-Salam theory, and so in Eqs. (2.3) and (2.4) we take the \( W \)-boson mass and the lepton mass to be zero. We also take the gauge parameter in the \( W \)-propagator to be \( \xi = 1 \). Notice that the lepton propagator contains the factor \( \delta_- \), while the \( W \)-propagator contains \( \delta_+ \). This is because they are oppositely charged. Substituting the expressions for the propagators into Eq. (2.1), we write the vacuum self-energy in the \( \xi = 1 \) gauge as

\[
\Sigma_0(x', x'') = \int \frac{d^4p}{(2\pi)^4} e^{ip \cdot (x'' - x')} \Sigma_0(p, B)
\]  

(2.10)

with

\[
\Sigma_0(p, B) = i \frac{g^2}{2} \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} (2\pi)^4 \delta(p - q - k) \int_0^\infty \frac{ds_1}{\cos z_1} \int_0^\infty \frac{ds_2}{\cos z_2} \exp \left[ is_1 \left( q_1^2 + q_1^2 \tan z_1 \right) \right] \exp \left[ is_2 \left( k_2^2 + k_2^2 \tan z_2 \right) \right] \gamma_R \gamma_\mu \left( (\delta^\mu_\parallel) + (e^{2z_2} \epsilon^\mu_\perp) \right) \\
\left[ g_\parallel e^{-iz_1 \sigma_3} + \frac{g_\perp}{\cos z_1} \right] \gamma^\nu \gamma_L.
\]  

(2.11)

When we compute temperature effects, we will be considering temperatures for which the \( SU(2)_L \otimes U(1) \) symmetry is restored, i.e. \( T \geq 250 \) GeV. Under these assumptions, it is perfectly reasonable to take \( eB \ll T^2 \), and so we rewrite Eq. (2.11) keeping terms up to order \( O(eB) \)
\[\Sigma_0(p, B) = -i \frac{g^2}{2} \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} (2\pi)^4 \delta(p - q - k) \frac{1}{q^2} \left( \frac{1}{k^2} + \frac{2}{k^2} \right) eB\sigma_3 \gamma_\mu \gamma_\nu \left[ \gamma_\mu - \left( \frac{1}{q^2} + \frac{2}{k^2} \right) eB\sigma_3 \gamma_\mu \gamma_\nu \right],\]

(2.12)

where we have carried out some of straightforward \(\gamma\)-algebra and two \(s\)-integrations. We can write Eq. (2.12) as

\[\Sigma_0(p, B) = \Sigma_0(p) + \Sigma_0'(p, B),\]

(2.13)

where \(\Sigma_0(p)\) is the usual neutrino self-energy in vacuum in the absence of a magnetic field and \(\Sigma_0'(p, B)\) is the part that depends on \(B\). The finite temperature corrections \(\Sigma_T(p)\) to \(\Sigma_0(p)\) have been calculated [1] and are well-known, so we focus our attention to the calculation of the temperature corrections to \(\Sigma_0'(p, B)\).

We calculate the temperature effects on the neutrino self-energy in a magnetic field using the imaginary time method of finite temperature field theory in the rest frame of the medium, where the magnetic field is \(B^\mu = (0, \vec{B})\) with \(\vec{B}\) pointing in the \(z\)-direction. We make the following standard substitutions [8,9] in Eq. (2.12)

\[p^0 = \frac{\pi i}{\beta} (2n_p + 1), \quad q^0 = \frac{\pi i}{\beta} (2n_q + 1), \quad k^0 = \frac{\pi i}{\beta} 2n_k,\]

(2.15)

\[\int \frac{d^4q}{(2\pi)^4} \rightarrow \frac{i}{\beta} \sum_{n_q} \int \frac{d^3\vec{q}}{(2\pi)^3}, \quad \int \frac{d^4k}{(2\pi)^4} \rightarrow \frac{i}{\beta} \sum_{n_k} \int \frac{d^3\vec{k}}{(2\pi)^3},\]

(2.16)

\[(2\pi)^4 \delta(4)(p - q - k) \rightarrow -i \beta (2\pi)^3 \delta_{n_p, n_q + n_k} \delta(3)(\vec{p} - \vec{q} - \vec{k}),\]

(2.17)

where \(n_p = n_f(p)\), \(n_q = n_f(q)\), and \(n_k = n_b(k)\) are the fermion and the boson occupation numbers(given in Eq.(2.22)), \(\delta_{n_p, n_q + n_k}\) is the Kronecker delta, and \(\beta = 1/T\). After these substitutions, which are the standard procedure for going from zero temperature to finite
temperature \([\beta^\star]\), we obtain \(\Sigma'(p, B)\) which will be separated as \(\Sigma'_0(p, B) + \Sigma'_T(p, B)\) where \(\Sigma'_0\) is the vacuum self-energy and \(\Sigma'_T\) is the finite temperature correction

\[
\Sigma'(p, B) = \frac{i g^2}{2} \int \frac{d^3q}{(2\pi)^3} \int \frac{d^3k}{(2\pi)^3} (2\pi)^3 \delta(\bar{p} - \bar{q} - \bar{k}) \left( \frac{i}{\beta} \right)^2 \sum_{n_q, n_k} (-i\beta) \delta_{n_p, n_q + n_k} \left( \frac{1}{q^2 + 2} \right) \epsilon B \sigma_3 \left( \frac{q}{q^2} \right) \gamma_{\mu} \gamma_L.
\]

We first evaluate the double sum

\[
I = \left( \frac{i}{\beta} \right)^2 \sum_{n_q, n_k} (-i\beta) \delta_{n_p, n_q + n_k} \left( \frac{1}{q^2 + 2} \right) \epsilon B \sigma_3 \left( \frac{q}{q^2} \right) \gamma_{\mu} \gamma_L
\]

by doing analytic continuations and employing contour integrals. We need to use an analytic continuation of the Kronecker delta \(\delta\)

\[
\beta \delta_{n_p, n_q + n_k} = \frac{e^{\beta(k^0 - q^0)} - e^{\beta p^0}}{p^0 - q^0 - k^0},
\]

since this procedure guarantees that the normal vacuum is recovered in the limit of zero temperature \([\beta^\star]\).

By employing Eq. (2.20) the double sum \(I\) can be evaluated in a closed form

\[
I = -\frac{i}{4q^2} n'_f(q) \left[ \frac{-\gamma^0 q + \gamma^3 q^3}{(p^0 - q)^2 - k^2} + \frac{\gamma^0 q + \gamma^3 q^3}{(p^0 + q)^2 - k^2} \right] - \frac{i}{2k^2} \left( n'_b(k) - \frac{n_b(k)}{k} \right) \left[ \frac{\gamma^0 (p^0 - k) - \gamma^3 q^3}{(p^0 - k)^2 - q^2} + \frac{\gamma^0 (p^0 + k) - \gamma^3 q^3}{(p^0 + k)^2 - q^2} \right] + \frac{i}{4q^2} n_f(q) \left\{ -2(p^0 + q) \left[ \frac{-\gamma^0 q + \gamma^3 q^3}{(p^0 + q)^2 - k^2} \right] + \gamma^3 q^3 \right\} + \frac{1}{q^2 (p^0 - q)^2 - k^2} \left[ \frac{\gamma^0 (p^0 + k) - \gamma^3 q^3}{(p^0 + k)^2 - q^2} \right] + \frac{1}{q^2 (p^0 - k)^2 - q^2} \left[ \frac{\gamma^0 (p^0 - k) - \gamma^3 q^3}{(p^0 - k)^2 - q^2} \right] + \gamma^0 (p^0 + k) - \gamma^3 q^3 \right\} + \text{vacuum terms}
\]

where the vacuum terms do not depend on the boson and fermion occupation numbers

\[
n_b(k) = \frac{1}{e^{\beta k} - 1}, \quad n_f(q) = \frac{1}{e^{\beta|q|} + 1}
\]

and we have used the notation \(q = |\bar{q}|, k = |\bar{k}|,\)

\[\text{(2.22)}\]
\[ n'_b(k) = \frac{dn_b(k)}{dk}, \quad n'_f(q) = \frac{dn_f(q)}{dq} \]  

(2.23)

and these notations will be used hereafter in this article.

We now substitute the expression of \( I \) in Eq.(2.21) into Eq.(2.18) for \( \Sigma'(p, B) \), and then perform the \( \vec{q} \)-integration. After the \( \vec{q} \)-integration is done, we change \( \vec{p} - \vec{k} \) to \( \vec{k} \) in the \( \vec{k} \)-integration of the terms proportional to \( n_f \) and \( n'_f \), and obtain

\[
\Sigma'_T(p, B) = \frac{g^2}{2} \int \frac{d^3k}{(2\pi)^3} eB \gamma \mu \sigma_3 \left[ -\frac{i}{2k^2} n'_b(k) \left( \frac{\gamma_0 k + \gamma_3 k^3}{A_-} + \frac{\phi_0 + \gamma_3 k^3}{A_+} \right) 
- \frac{i}{4k^3} n'_f(k) \left( -\frac{\gamma_0 k + \gamma_3 k^3}{A_-} + \frac{\gamma_0 k + \gamma_3 k^3}{A_+} \right) + \frac{i}{2k^3} n_b(k) \left( \frac{\phi_0 + \gamma_3 k^3}{A_-} \right) \left( \frac{1}{A_-} + \frac{1}{A_+} \right) 
+ \frac{i}{4k^3} n_f(k) \gamma^3 k^3 \left( \frac{1}{A_-} + \frac{1}{A_+} \right) 
- \frac{i}{2k^3} n_b(k) \left( 2p^0 - k \right) \left( \frac{\phi_0 + \gamma_3 k^3}{A_-} \right) 
- \frac{i}{2k^3} n_f(k) \left( p^0 + k \right) \left( -\gamma_0 k + \gamma_3 k^3 \right) \right] \gamma_\mu \gamma_L, 
\]

(2.24)

where we have used the notations \( p^\mu = (\omega, \vec{p}) \), \( A_\pm = \omega - |\vec{p}|^2 \pm 2k \omega + 2\vec{k} \cdot \vec{p} \), and have written \( \Sigma'_T \) instead of \( \Sigma' \) because the vacuum terms were dropped from the sum \( I \). After the integration over the angles and an integration by parts, Eq.(2.24) reduces to

\[
\Sigma'_T(p, B) = -\frac{g^2}{2} \int_0^\infty \frac{k \, dk}{\omega^2 - |\vec{p}|^2} \frac{eB \gamma \mu \sigma_3}{(4\pi)^2} \left( \gamma^0 \frac{\gamma^3 p^3}{|\vec{p}|} \right) \omega_+ \left( \frac{n_b(k)}{\omega^2 - k^2} - \frac{n_f(k)}{\omega^2 - k^2} \right) 
- \left( -\gamma_0 + \frac{\gamma^3 p^3}{|\vec{p}|} \right) \omega_+ \left( \frac{n_b(k)}{\omega^2 - k^2} - \frac{n_f(k)}{\omega^2 - k^2} \right) + \frac{4 \beta^2}{k^2} n_b(k) \frac{1}{\omega^2 - k^2} \left( \frac{n_b(k)}{\omega^2 - k^2} - \frac{n_f(k)}{\omega^2 - k^2} \right) 
+ \left( n_b(k) - n_f(k) \right) \frac{\gamma^3 p^3}{|\vec{p}|} \omega_+ \frac{|\vec{p}|}{4k^2} \ln \left( \frac{\omega_+ - k \omega_+ + k}{\omega_+ + k \omega_+ - k} \right) \gamma_\mu \gamma_L 
\]

(2.25)

where, following the notation introduced by Weldon in Ref. [1],

\[ \omega_\pm = \frac{\omega + |\vec{p}|}{2}. \]

(2.26)

Now the last integration is performed by using the following

\[ \int_0^\infty \frac{k \, dk}{\omega^2 - k^2} n_b(k) = -\frac{1}{2} \ln \left( \frac{\beta \omega_\pm}{2\pi} \right) - \frac{C_E}{2} + \frac{\beta^2 \omega_\pm^2}{8\pi^2} \zeta(3) + O(\beta^4) \]  

(2.27)

\[ \int_0^\infty \frac{k \, dk}{\omega^2 - k^2} n_f(k) = \frac{1}{2} \ln \left( \frac{2\beta \omega_\pm}{\pi} \right) + \frac{C_E}{2} - \frac{7\beta^2 \omega_\pm^2}{8\pi^2} \zeta(3) + O(\beta^4) \]  

(2.28)
where \( C_E = 0.5772 \) is the Euler-Mascheroni constant,

\[
\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3}.
\]  

(2.29)

In the above, the terms of order \( O(\beta^4) \) or higher have been neglected because \( T > 250 \text{ GeV} \).

We finally obtain the value of the finite temperature self-energy in a magnetic field

\[
\Sigma_T'(p, B) = -\frac{g^2}{(4\pi)^2} \frac{eB\gamma_R\gamma^\mu\sigma_3}{\omega^2 - |\vec{p}|^2} \left\{ 2 \gamma_\parallel \left[ -\frac{1}{2} \ln \frac{\beta^2(\omega^2 - |\vec{p}|^2)}{4\pi^2} - C_E + \frac{\beta^2}{8\pi^2} \zeta(3)(7\omega_+^2 + 7\omega_-^2)

-3\omega_+\omega_- \right] + \gamma_\parallel^3 \frac{\omega^2 - |\vec{p}|^2}{4|\vec{p}|^2} \left[ \frac{\omega}{|\vec{p}|} \ln \frac{\omega_-}{\omega_+} + 2 - \frac{\beta^2}{12\pi^2} \zeta(3)(-31\omega_+^2 - 31\omega_-^2)

+80\omega_+\omega_- \right] + 4 \gamma_\parallel \int_0^\infty \frac{n_b(k)}{k} dk \right\} \gamma_\mu \gamma_L .
\]  

(2.30)

Notice that the last term in Eq. (2.30) is an infrared divergence. It turns out that the infrared divergence will be canceled if we include the neutrino bremsstrahlung (soft \( Z \) boson emitting process) integrated in a range \( E < \Delta E \) with a typical energy scale \( \Delta E \), as is well-known in quantum electrodynamics \([11]\), and therefore we can neglect it. We can further simplify the expression of \( \Sigma_T'(p, B) \) by using the following identity \([4]\), which is valid for any \( 4 \times 4 \) matrix \( A \)

\[
\gamma^\mu \gamma_L A \gamma_\mu \gamma_L = - (\text{Tr} A \gamma^\mu \gamma_L) \gamma_\mu \gamma_L
\]  

(2.31)

and find

\[
\Sigma_T'(p, B) = -\frac{g^2}{(4\pi)^2} \left\{ 2\gamma_R(e\omega \vec{B} + e\vec{B} \cdot \vec{\gamma}_\parallel \gamma_\parallel) \gamma_L \left[ -\ln \frac{\beta^2(\omega^2 - |\vec{p}|^2)}{4\pi^2} - 2C_E

+\frac{\beta^2}{4\pi^2} \zeta(3)(7\omega_+^2 + 7\omega_-^2 - 3\omega_+\omega_-) \right] - \gamma_R \vec{B} \cdot \vec{\gamma}_\parallel \gamma_\parallel \gamma_L \left[ \frac{\omega}{|\vec{p}|} \ln \frac{\omega_-}{\omega_+}

+2 - \frac{\beta^2}{12\pi^2} \zeta(3)(-31\omega_+^2 - 31\omega_-^2 + 80\omega_+\omega_-) \right] \right\}
\]  

(2.32)

where we have used \( Bp^3 = \vec{B} \cdot \vec{p} \), \( B\gamma_3 = \vec{B} \) and \( \gamma_0 = \gamma_\parallel \), which are true in the rest frame of the medium. It is clear that for very high temperature (i.e. \( \beta \ll 1 \)) the term proportional to \( \ln \beta^2 \) is dominant compared with terms proportional to \( \beta^2 \) or independent of \( \beta \), and therefore the first logarithmic term in Eq. (2.32) is dominant, and we obtain

\[
\Sigma_T'(p, B) \simeq -\frac{2g^2}{(4\pi)^2} \frac{\gamma_R(e\omega \vec{B} + e\vec{B} \cdot \vec{\gamma}_\parallel \gamma_\parallel) \gamma_L \ln \frac{\beta^2(\omega^2 - |\vec{p}|^2)}{4\pi^2}}{\omega^2 - |\vec{p}|^2}
\]  

(2.33)
III. DISPERSION RELATION IN MATTER IN THE PRESENCE OF A MAGNETIC FIELD

In the previous section we have calculated the $B$-dependent part of the finite temperature neutrino self-energy in a medium in the presence of an external magnetic field. We now cast the result in a form given in Eq. (1.1)

$$
\Sigma_T(p, B) = \gamma_R(a \not p + b \not u + c \not B + b' \not b)\gamma_L
\tag{3.1}
$$

where $u_\mu$ is the 4-velocity of the medium (in the rest frame of the medium $u_\mu = (1, 0)$), and the coefficients $a$ and $b$ are well known:

$$
a = \frac{M^2}{|\vec{p}|^2} \left( 1 - \frac{\omega}{2|\vec{p}|} \ln \frac{\omega_+}{\omega_-} \right) \tag{3.2}
$$

$$
b = \frac{M^2}{|\vec{p}|} \left[ -\frac{\omega}{|\vec{p}|} + \left( \frac{\omega^2}{|\vec{p}|^2} - 1 \right) \frac{1}{2} \ln \frac{\omega_+}{\omega_-} \right] \tag{3.3}
$$

with $\omega_\pm$ defined in Eq. (2.26) and the thermal effective mass $M$ for neutrinos in the unbroken $SU(2)_L \otimes U(1)$ model given by $M^2 = \frac{3}{32} g^2 T^2$. The other two coefficients $b'$ and $c$ are immediately obtained from Eq. (2.33) of our calculation as

$$
b' = -\frac{g^2}{8\pi^2 \omega^2 - |\vec{p}|^2} e \vec{B} \cdot \vec{p} \ln \frac{(2\pi T)^2}{\omega^2 - |\vec{p}|^2} \tag{3.4}
$$

$$
c = -\frac{g^2}{8\pi^2 \omega^2 - |\vec{p}|^2} e \omega \ln \frac{(2\pi T)^2}{\omega^2 - |\vec{p}|^2} . \tag{3.5}
$$

Starting from the inverse of the full neutrino propagator in matter in a magnetic field $S^{-1} = \gamma_R \not p' \gamma_L + \Sigma_T(p, B)$, with $\Sigma_T(p, B)$ given by Eq. (3.1), and following the same procedure as in Refs. [1] [3], we square $S^{-1}$ and set it equal to zero to find

$$
[\omega(1 + a) + b + b']^2 = |\vec{p}(1 + a) + c\vec{B}|^2 \tag{3.6}
$$

from which the dispersion relation is easily obtained. The positive-energy solution of Eq. (3.6) occurs when $\omega$ and $\vec{p}$ satisfy
\[ \omega(1 + a) + b = \left[ (1 + a)^2 |\vec{p}|^2 + c^2 B^2 - 2bb' - b'^2 \right]^{1/2}. \] 

(3.7)

The above equation is easily derived from Eq.(3.4) using the relation \( \omega b' = c\vec{p} \cdot \vec{B} \) and is the exact finite temperature dispersion relation for neutrinos in a magnetic field. The last three terms on the right-hand side of Eq.(3.7) are due to the presence of the magnetic field, and vanish for \( B \to 0 \).

Now we analyze the effect of the magnetic field on the neutrino thermal effective mass and obtain a simple analytic form of the dispersion relation for \(|\vec{p}| \gg M\). Weldon [1] showed that the solution of the thermal dispersion relation with \(|\vec{p}| = 0\) is not \( \omega = 0 \), but \( \omega = M \), proving that \( M \) is an effective mass due to thermal effects. We take Eq.(3.7) in the limit for \( \vec{p} \to 0 \), and obtain

\[ \omega - \frac{M^2}{\omega} = \frac{g^2 eB}{4\pi^2} \ln \frac{2\pi T}{\omega} \] 

whence after solving for \( \omega \) with the assumption that \( eB \ll T^2 \), we find

\[ \omega = M + \frac{g^2 eB}{8\pi^2} \ln \frac{2\pi T}{M}. \] 

(3.9)

This shows that the thermal effective mass \( M \) of the neutrino is modified, in the presence of a magnetic field, to

\[ M_B = M + \frac{g^2 eB}{8\pi^2} \ln \frac{2\pi T}{M}. \] 

(3.10)

The analytic form of the thermal dispersion relation without magnetic field was obtained by Weldon [1]

\[ \omega = |\vec{p}| + \frac{M^2}{|\vec{p}|} - \frac{M^4}{2|\vec{p}|^3} \ln \left( \frac{2|\vec{p}|^2}{M^2} \right) + \cdots \quad (|\vec{p}| \gg M). \] 

(3.11)

In this article with a magnetic field we consider only the case \(|\vec{p}| \gg M\) because \( T \geq 250 \) GeV. Solving Eq.(3.7) for \(|\vec{p}| \gg M\), we find the corrections to Eq.(3.11) due to the presence of a magnetic field

\[ \omega = |\vec{p}| + \frac{M^2}{|\vec{p}|} - \frac{M^4}{2|\vec{p}|^3} \ln \left( \frac{2|\vec{p}|^2}{M^2} \right) - \frac{g^2 eB \cdot \hat{p}}{4\pi^2 |\vec{p}|} \ln \left( \frac{\sqrt{2\pi} T}{M} \right) + \cdots \quad (|\vec{p}| \gg M) \] 

(3.12)
where \( \hat{p} = \vec{p}/|\vec{p}| \) and the term proportional to \( \vec{B} \cdot \hat{p} \) is the leading order magnetic field correction. From Eq.(3.12) we obtain the index of refraction \( n = |\vec{p}|/\omega \) for neutrino in a magnetic field

\[
n = 1 - \frac{M^2}{|\vec{p}|^2} + \frac{g^2 e\vec{B} \cdot \hat{p}}{4\pi^2 |\vec{p}|^2} \ln \left( \frac{\sqrt{2}\pi T}{M} \right) + \cdots \quad (|\vec{p}| \gg M) .
\]  (3.13)

It is very interesting to notice that for neutrinos moving in different directions we obtain different dispersion relations and therefore different speeds.

**IV. DISCUSSION**

We have shown that in the presence of a magnetic field \( eB \ll T^2 \), the thermal effective mass \( M \) for neutrinos in the unbroken Weinberg-Salam model increases by

\[
\frac{g^2 eB}{8\pi^2 M} \ln \frac{2\pi T}{M} .
\]  (4.1)

The thermal dispersion relation is also modified by

\[
- \frac{g^2 e\vec{B} \cdot \hat{p}}{4\pi^2 |\vec{p}|} \ln \left( \frac{\sqrt{2}\pi T}{M} \right) , \quad (|\vec{p}| \gg M) \]  (4.2)

which is the leading order magnetic field corrections and the index of refraction is

\[
n = 1 - \frac{M^2}{|\vec{p}|^2} + \frac{g^2 e\vec{B} \cdot \hat{p}}{4\pi^2 |\vec{p}|^2} \ln \left( \frac{\sqrt{2}\pi T}{M} \right) + \cdots \quad (|\vec{p}| \gg M) .
\]  (4.3)

The above results show that neutrinos propagating in different directions have different dispersion relations. Our findings might have interesting applications in the field of the early cosmology, since our calculations are valid for \( eB \ll T^2 \), which implies magnetic fields \( B \ll 10^{24} \) Gauss, and therefore primordial magnetic fields in the very early universe might have affected the propagation of neutrinos in the way we have calculated.

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