Strong decays of nucleon and delta resonances

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Abstract

We study the strong couplings of the nucleon and delta resonances in a collective model. In
the ensuing algebraic treatment we derive closed expressions for decay widths which are used to
analyze the experimental data for strong decays into the pion and eta channels.
1 Introduction

A large amount of experimental data was accumulated in the 1960’s and 1970’s on the spectroscopy of light hadrons. These data led first to the introduction of $SU_f(3)$ by Gell’Mann [1] and Ne’eman [2] (later enlarged to $SU_{sf}(6)$ by Gürshey and Radicati [3]), and subsequently to $SU_c(3)$ color symmetry as a gauge symmetry of strong interactions. The construction of dedicated facilities (e.g. CEBAF, MAMI, ELSA) that promise to produce new and more accurate data, have stimulated us to reexamine baryon spectroscopy.

In our reanalysis we have introduced, in addition to the basic spin-flavor-color symmetry, $SU_{sf}(6) \otimes SU_c(3)$, a new ingredient, namely a space symmetry, $G$, which was taken to be $G = U(7)$ for baryons [4]. The space symmetry makes it possible to study, in a straightforward way, several limiting situations (e.g. harmonic oscillator and collective dynamics), and to derive a set of transparent results that can be used to analyze the experimental data. This algebraic approach has been used [4, 5] to analyze the mass spectrum and electromagnetic couplings of nonstrange baryon resonances. It presents an alternative for the use of nonrelativistic or relativized Schrödinger equations [6, 7]. In addition to electromagnetic couplings, strong decays of baryons provide an important, complementary, tool to study the structure of baryons.

The strong decays have been analyzed previously in the nonrelativistic [8] and relativized quark models [9]. These models emphasize single-particle aspects of quark dynamics in which only a few low-lying configurations in the confining potential contribute significantly to the baryon wave function. In the framework of the earlier mentioned algebraic approach it is possible to study also other, more collective, types of dynamics. In this contribution, we investigate in detail the strong decays in a collective model of baryon structure.

2 Algebraic model of the nucleon

In [4] we introduced an algebraic model, in which the nucleon has the string configuration of Figure 1. Its three constituent parts are characterized by the internal degrees of freedom of spin, flavor and color and by the two relative Jacobi coordinates, $\vec{\rho}$ and $\vec{\lambda}$, and their conjugate momenta. For these six spatial degrees of freedom we suggested to use a $U(7)$ spectrum generating algebra whose building blocks are six dipole bosons, $b_{\rho,i}^\dagger$ and $b_{\lambda,i}^\dagger$ ($i = x, y, z$), and an auxiliary scalar boson, $s^\dagger$. For a system of interacting bosons the model space is spanned by the symmetric irreps $|N\rangle$ of $U(7)$, which contains the oscillator shells with $n = 0, 1, 2, \ldots, N$. Here $N$ is the conserved total number of bosons. We note that the scalar boson does not introduce a new degree of freedom, since for a given total boson number $N$ it can always be eliminated by $s \rightarrow \sqrt{N-n_{\rho}-n_{\lambda}}$ (Holstein-Primakoff realization
of $U(7)$). The full algebraic structure is obtained by combining the geometric part, $U(7)$, with the internal spin-flavor-color part, $SU_{sf}(6) \otimes SU_c(3)$.

For the nucleon (isospin $I = 1/2$) and delta ($I = 3/2$) families of resonances the three strings of Figure 1 have equal length and equal relative angles. Hence this configuration is an oblate top and has $D_{3h}$ point group symmetry. The classification under $D_{3h}$ is equivalent to the classification under permutations and parity. States are characterized by $(v_1, v_2); K, L_P, t$, where $(v_1, v_2)$ denote the vibrations (stretching, bending); $K$ denotes the projection of the rotational angular momentum $L$ on the body-fixed symmetry axis, $P$ the parity and $t$ the symmetry type of the state under $D_3$ (a subgroup of $D_{3h}$ isomorphic to the $S_3$ permutation group). The symmetry type of the geometric part must be the same as that of the spin-flavor part (the color part is antisymmetric). Therefore, one can use the representations of either $D_3$ or $SU_{sf}(6)$ to label the states: $A_1 \leftrightarrow 56, A_2 \leftrightarrow 20, E \leftrightarrow 70$. We use the latter notation and denote the total baryon wave function as

$$\left| 2S+1 \dim\{SU_{f}(3)\}_J [\dim\{SU_{sf}(6)\}_{[v_1,v_2];K} \rightangle,$$

where $S$ and $J$ are the spin and total angular momentum $\vec{J} = \vec{L} + \vec{S}$. In this collective model of the nucleon baryon resonances are interpreted as vibrations and rotations of an oblate symmetric top.

The corresponding wave functions, when expressed in a harmonic oscillator basis, are spread over many shells and hence are truly collective.

### 3 Strong couplings

In this contribution we study strong decays of baryons by the emission of a pseudoscalar meson

$$B \to B' + M.$$  

(2)

In order to calculate the decay widths of this process we have to specify the form of the operator inducing the transition. Several forms have been discussed in [10]. Here we use the transition operator that has also been used in [6]

$$\mathcal{H} = \frac{1}{(2\pi)^{3/2}(2k_0)^{1/2}} \sum_{j=1}^{3} X^M_j \left[ 2g (\vec{s}_j \cdot \vec{k}) e^{-i\vec{k} \cdot \vec{r}_j} + h \vec{s}_j \cdot (\vec{p}_j e^{-i\vec{k} \cdot \vec{r}_j} + e^{-i\vec{k} \cdot \vec{r}_j} \vec{p}_j) \right],$$

(3)

where $\vec{r}_j$, $\vec{p}_j$ and $\vec{s}_j$ are the coordinate, momentum and spin of the $j$-th constituent, respectively; $k_0 = E_M = E_B - E_{B'}$ is the meson energy, and $\vec{k} = \vec{P}_M = \vec{P} - \vec{P}' = k \hat{z}$ denotes the momentum carried by the outgoing meson. Here $\vec{P} = P_z \hat{z}$ and $\vec{P}' (= P'_z \hat{z})$ are the momenta of the initial and final baryon. The flavor operator $X^M_j$ (expressed in terms of Gell-Mann matrices) corresponds to the emission of an elementary meson by the $j$-th constituent: $q_j \to q'_j + M$ (see Figure 2). The coefficients $g$ and $h$ are parameters.
Using the symmetry of the wave functions for nonstrange baryons, transforming to Jacobi coordinates and integrating over the baryon center of mass coordinate, we find

\[ H = \frac{1}{(2\pi)^{3/2}(2k_0)^{1/2}} 6X_3^M \left[ g ks_{3,z} \hat{U} - h s_{3,z}(\hat{T}_z - \frac{1}{6}(P_z + P_z')\hat{U}) - \frac{1}{2} h (s_{3,+}\hat{T}_+ + s_{3,-}\hat{T}_-) \right]. \]

(4)

The operators \( \hat{U} \) and \( \hat{T}_+ \) only act on the spatial part of the baryon wave function. In an algebraic treatment they are given by

\[ \hat{U} = e^{ik\beta \hat{D}_{\lambda,m}/X_D}, \]
\[ \hat{T}_m = -\frac{i m k_0 \beta}{2X_D} (\hat{D}_{\lambda,m} \hat{U} + \hat{U} \hat{D}_{\lambda,m}), \]

(5)

where \( \hat{D}_{\lambda,m} = (b_\lambda^\dagger \times s - s^\dagger \times \tilde{b}_\lambda)^{(1)}_m \) is a dipole operator in \( U(7) \) and has the same transformation properties as the Jacobi coordinate \( \lambda_m \). The coefficient \( X_D \) is a normalization factor and \( \beta \) represents the scale of the coordinate [4].

Since \( \hat{D}_\lambda \) is a generator of the algebra of \( U(7) \), the matrix elements of \( \hat{U} \) are representation matrix elements of \( U(7) \), i.e. generalized Wigner \( D \)-matrices. By making an appropriate basis transformation they can be obtained exactly. In addition, in the limit of \( N \to \infty \) (large model space) they can also be derived in closed form. This derivation consists of several steps. The rotational states \(|K,L,M\rangle\) belonging to the the vibrational ground state band of the oblate symmetric top with \((v_1,v_2) = (0,0)\), can be obtained by projection from an intrinsic (or coherent) state

\[ |K,L,M\rangle = \sqrt{\frac{(2L+1)}{8\pi^2}} \int d\Omega D^{(L)}_{MK}(\Omega) R(\Omega) |N,R\rangle, \]
\[ |N,R\rangle = \frac{1}{\sqrt{N(1+R^2)^N}} \left[ s^I + \frac{R}{\sqrt{2}} (b_{\lambda,x}^I + b_{\rho,y}^I) \right]^N |0\rangle. \]

(6)

The coefficient \( R \) appears in the mass operator and is associated with the size of the string [4]. Next we construct states with good \( D_3 \) symmetry by taking the linear combinations

\[ |\psi_1\rangle = \frac{1}{\sqrt{2(1+\delta_{K,0})}} \left[ (-)^L |K,L,M\rangle + (-)^L |-K,L,M\rangle \right], \]
\[ |\psi_2\rangle = \frac{i}{\sqrt{2(1+\delta_{K,0})}} \left[ |K,L,M\rangle - (-)^L |-K,L,M\rangle \right]. \]

(7)

For \( K(\text{mod} 3) = 0 \) the wave function \(|\psi_1\rangle\) is symmetric \((A_1 \leftrightarrow 56)\) and \(|\psi_2\rangle\) antisymmetric \((A_2 \leftrightarrow 20)\), whereas for \( K(\text{mod} 3) \neq 0 \) the wave functions \(|\psi_1\rangle\) and \(|\psi_2\rangle\) are the \( E_\lambda \) and \( E_\rho \) components, respectively, of the mixed-symmetry doublet \((E \leftrightarrow 70)\). Eq. (5) is consistent with the choice of geometry in \(|N,R\rangle\). The matrix element of \( \hat{U} \) relevant for a decay in which the final state is either the nucleon or the delta with \(|\psi_0\rangle = |K = 0, L = 0, M = 0\rangle\) is given by

\[ \langle \psi_0 | \hat{U} | \psi_1 \rangle = \delta_{M,0} F(k\beta), \]

(8)
with
\[ F(k\beta) = i^K \sqrt{\frac{2L + 1}{2(1 + \delta_{K,0})}} \sqrt{\frac{(L-K)!(L+K)!}{(L-K)!(L+K)!}} \left[ 1 + (-1)^{L-K} \right] j_L(k\beta). \] (9)

In the derivation we have used that in the large $N$ limit the intrinsic matrix element becomes diagonal in the orientation $\Omega$ of the condensate. For the the matrix elements of $\hat{T}$ we find similar expressions in terms of spherical Bessel functions
\[ \langle \psi_0 | \hat{T}_z | \psi_1 \rangle = -\delta_{M,0} m_3 k_0 \beta dF(k\beta) \],
\[ \langle \psi_0 | \hat{T}_\pm | \psi_1 \rangle = \mp \delta_{M,\mp1} m_3 k_0 \beta \sqrt{L(L+1)} \frac{F(k\beta)}{k\beta}. \] (10)

The operators $\hat{U}$ and $\hat{T}_m$ do not connect the nucleon and delta wave function $\psi_0$ with $\psi_2$ of Eq. (7). In the collective model discussed here, these spatial matrix elements are folded with a distribution function
\[ g(\beta) = \beta^2 e^{-\beta/a}/2a^3. \] (11)

With this distribution function we reproduce the observed dipole form for the electric form factor of the proton $G_E^p = 1/(1 + k^2 a^2)^2$. In Table 1 we present some collective matrix elements $F(k) = \int d\beta g(\beta) F(k\beta)$ for the decay of resonances which are interpreted in the collective model as rotational excitations and have $(v_1, v_2) = (0, 0)$. We note that the matrix elements of $\hat{U}$ and $\hat{T}$ between states belonging to the $(v_1, v_2) = (0, 0)$ ground state band do not depend on the coefficient $R$ of Eq. (6). The decays involving resonances associated with vibrationally excited states, e.g. with $(v_1, v_2) = (1, 0)$ or $(0, 1)$, show an explicit dependence on $R$. In this contribution we only consider resonances that are interpreted as rotational excitations and have $(v_1, v_2) = (0, 0)$.

The helicity amplitudes for strong decays of baryon resonances $B \to B' + M$ can be obtained by combining the spatial contribution with the appropriate spin-flavor matrix elements of $X_3^M s_{3,m}$. The calculation of the contribution of the spin-flavor part is straightforward. For the pseudoscalar $\eta$ mesons we introduce, as usual, a mixing angle $\theta_P$ between the octet and singlet mesons
\[ X^\eta = X^{\eta_8} \cos \theta_P - X^{\eta_1} \sin \theta_P, \quad X'^\eta = X^{\eta_8} \sin \theta_P + X^{\eta_1} \cos \theta_P. \] (12)

For decays in which the initial baryon has angular momentum $\vec{J} = \vec{L} + \vec{S}$ and in which the final baryon is either the nucleon or the delta with $L' = 0$ and thus $J' = S'$, the helicity amplitudes are
\[ A_\nu(k) = \int d\beta g(\beta) \langle \alpha', L' = 0, S', J' = S', \nu | \mathcal{H} | \alpha, L, S, J, \nu \rangle, \] (13)
where $\alpha$ denotes the quantum numbers that, in addition to $L, S, J$ and $\nu$, are needed to specify the baryon wave function. With the definition of the transition operator in Eq. (3) and the helicity
amplitudes in Eq. (13), the decay widths for a specific isospin channel are

\[ \Gamma(B \rightarrow B' + M) = 2\pi\rho_f \frac{2}{2J+1} \sum_{\nu>0} |A_\nu(k)|^2, \tag{14} \]

where \( \rho_f \) is a phase space factor. We adopt the procedure of [10] to calculate the decay widths: (i) all calculations are performed in the rest frame of the decaying resonance \( (P_z = 0 \text{ and hence } P'_z = -k) \), and (ii) for the momentum \( k \) of the emitted meson and the phase space factor \( \rho_f \) we use the relativistic expressions

\[ k^2 = -m_B^2 + \frac{(m_B^2 - m_{B'}^2 + m_M^2)^2}{4m_B^2}, \]

\[ \rho_f = \frac{4\pi E_{B'}E_Mk}{m_B}. \tag{15} \]

Here \( E_{B'} = \sqrt{m_{B'}^2 + k^2} \) and \( E_M = \sqrt{m_M^2 + k^2} \).

4 Results

For all resonances with the same value of \( (v_1, v_2, L^P) \), the expression for the decay widths of Eq. (14) can be rewritten in a more transparent form in terms of only two elementary partial wave amplitudes \( W_l(k) \) with \( l = L \pm 1 \),

\[ \Gamma(B \rightarrow B' + M) = 2\pi\rho_f \frac{1}{(2\pi)^22k_0} \sum_{l=L\pm1} c_l |W_l(k)|^2. \tag{16} \]

For this set of resonances, the amplitudes \( W_l(k) \) contain the \( k \) dependence, while the coefficients \( c_l \) depend on the specific baryon resonance involved in the decay.

In Table 2 we give the coefficients \( c_l \) for the negative parity resonances with \( (v_1, v_2, L^P) = (0, 0, 1^-) \). In the collective model with distribution given by Eq. (11) the corresponding \( S \) and \( D \) elementary partial wave amplitudes are

\[ W_0(k) = i \left[ \frac{gk + \frac{1}{6}h(P_z + P'_z)}{6} \frac{k\alpha}{(1+k^2\alpha^2)^2} + h m_3 k_0 a \frac{3 - k^2\alpha^2}{(1+k^2\alpha^2)^3} \right], \]

\[ W_2(k) = i \left[ \frac{gk + \frac{1}{6}h(P_z + P'_z)}{6} \frac{k\alpha}{(1+k^2\alpha^2)^2} - h m_3 k_0 a \frac{4k^2\alpha^2}{(1+k^2\alpha^2)^3} \right]. \tag{17} \]

Partial widths for other models of the nucleon and its resonances can be obtained by introducing the corresponding expressions for the elementary amplitudes \( W_l(k) \). For example, the relevant expressions in the harmonic oscillator quark model are

\[ W_0(k) = \frac{i}{3} \left[ \frac{gk + \frac{1}{6}h(P_z + P'_z)}{6} \right] \frac{k\beta}{(1+k^2\beta^2)^2} - \frac{3}{3} h m_3 k_0 a \frac{k^2\beta^2}{(1+k^2\beta^2)^3} \right] e^{-k^2\beta^2/6}, \]

\[ W_2(k) = \frac{i}{3} \left[ \frac{gk + \frac{1}{6}h(P_z + P'_z)}{6} \right] \frac{k\beta}{(1+k^2\beta^2)^2} - \frac{1}{3} h m_3 k_0 a \frac{k^2\beta^2}{(1+k^2\beta^2)^3} \right] e^{-k^2\beta^2/6}. \tag{18} \]
5 Analysis of experimental data

We consider strong decays of nucleon and delta resonances into the $\pi$ and $\eta$ channels. In Table 3 we show a comparison between the experimental widths, which are extracted from the most recent compilation by the Particle Data Group [11], and the results of our calculation. The calculated values depend on the two parameters $g$ and $h$ in the transition operator of Eq. (3), and on the strong interaction radius $a$ (see Eq. (11)). The values of $g$, $h$ and $a$ are determined in a least square fit to the $N\pi$ partial widths (which are relatively well known). The $S_{11}$ resonances were excluded from the fit, since for these resonances the situation is not clear due to possible mixing of $N(1535)S_{11}$ and $N(1650)S_{11}$ and the possible existence of a third $S_{11}$ resonance [12]. As a result we find that the value of the strong interaction radius $a$ is the same as the electromagnetic value $a = 0.232$ fm [5]. Furthermore $g = 1.164$ GeV$^{-1}$ and $h = -0.094$ GeV$^{-1}$. The relative sign is consistent with a previous analysis of the strong decay of mesons [13] and with a derivation from the axial-vector coupling (see e.g. [10]). We note that $g$ and $h$ have the same value for all resonances and all decay channels. In comparing with previous calculations, it should be noted that in the calculation in the nonrelativistic quark model of [8] the decay widths are parametrized by four reduced partial wave amplitudes instead of the two elementary amplitudes $g$ and $h$. Furthermore the momentum dependence of these reduced amplitudes are represented by constants. The calculation in the relativized quark model of [9] was done using a pair-creation model for the decay and involved a different assumption on the phase space factor. Both the nonrelativistic and relativized quark model calculations include the effects of mixing induced by the hyperfine interaction, which in the present calculation are not taken into account.

The calculation of decay widths into the $N\pi$ channel, as shown for the 3 and 4 star resonances in Table 3, is in fair agreement with experiment. This is emphasized in Figures 3 and 4, where we plot the $N\pi$ and $\Delta\pi$ decay widths of the negative parity resonances of Table 2. The results are to a large extent a consequence of spin-flavor symmetry. The use of collective form factors improves somewhat the results when compared with harmonic oscillator calculations. This is shown in Table 4 where the decay of a $\Delta$ Regge trajectory into $N\pi$ is analyzed and compared with the calculations of [10], which are based on the harmonic oscillator model discussed in [14]. We also include the results of more recent calculations in the nonrelativistic quark model [8] and in the relativized quark model [9].

Contrary to the decays into $\pi$, the decay widths into $\eta$ have some unusual properties. Our calculation gives systematically small values for these widths. This is due to a combination of phase space factors and the structure of the transition operator. The spin-flavor part is approximately the same for $N\pi$ and $N\eta$, since $\pi$ and $\eta$ are in the same $SU_f(3)$ multiplet. The $\eta$ decays are suppressed relative to the $\pi$ decays because of phase space (due to the difference between the $\eta$ and $\pi$ masses). Moreover, for small values of $ka$ the elementary amplitudes of Eqs. (17) are dominated by the $S$ wave.
amplitude, $W_0(k) \approx 3hm_jk_0a$, whose contribution to the $\eta$ decay width is suppressed by the small value of $h = -0.094$ GeV$^{-1}$. We emphasize here, that the structure of the transition operator was determined by fitting the coefficients $g$ and $h$ to the $N\pi$ decays of the 3 and 4 star resonances. Hence the $\eta$ decays are calculated without introducing any further parameters.

The experimental situation is unclear. The previous PDG compilation gave systematically small widths ($\sim 1$ MeV) for all resonances except $N(1535)S_{11}$. The latest compilation deletes all $\eta$ widths with the exception of $N(1535)S_{11}$. The results of our analysis suggest that the large $\eta$ width for $N(1535)S_{11}$ is not due to a conventional $q^3$ state. One possible explanation is the presence of another state in the same mass region, e.g. a quasi-bound meson-baryon $S$ wave resonance, just below or around threshold. Recently it was suggested that a quasi-bound $K\Sigma-K\Lambda$ state with properties remarkably similar to the $N(1535)S_{11}$ resonance could be responsible for the large $N\eta$ branch of the $N(1535)S_{11}$ resonance.

6 Conclusions

We have presented a calculation of the strong decay widths of nucleon and delta resonances into the $\pi$ and $\eta$ channel. By exploiting the symmetry of the problem, both in its spin-flavor-color part, $SU_{sf}(6) \otimes SU_c(3)$, and in its space part, $U(7)$, we have been able to express the results in a transparent analytic way. The analysis of experimental data shows that, while the decays into $\pi$ follow the expected pattern, the decays into $\eta$ have some unusual features. Our calculations do not show any indication for a large $\eta$ width, as is observed for the $N(1535)S_{11}$ resonance. The observed large $\eta$ width indicates the presence of another configuration, which is outside the present model space. A possible candidate for such a configuration is a quasi-bound meson-baryon $S$ wave resonance ($K\Sigma-K\Lambda$) [10]. Experiments at the new electron facilities (CEBAF, MAMI, ELSA) could help to elucidate this point.

Other decay channels, such as $\Lambda K$ and $\Sigma K$, are presently being included in our calculations. This work forms part of a study to incorporate strange resonances as well in our model.

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References

[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[2] Y. Ne’eman, Nucl. Phys. **26**, 222 (1961).

[3] F. Gürsey and L.A. Radicati, Phys. Rev. Lett. **13**, 173 (1964).

[4] R. Bijker and A. Leviatan, Rev. Mex. Fís. **39**, Suplemento 2, 7 (1993);
  R. Bijker, F. Iachello and A. Leviatan, Ann. Phys. (N.Y.) **236**, 69 (1994).

[5] R. Bijker and A. Leviatan, Rev. Mex. Fís. **41**, Suplemento 1, 62 (1995);
  R. Bijker, F. Iachello and A. Leviatan, preprint [nucl-th/9510001]. submitted.

[6] N. Isgur and G. Karl, Phys. Rev. **D18**, 4187 (1978); **D19**, 2653 (1979); **D20**, 1191 (1979).

[7] S. Capstick and N. Isgur, Phys. Rev. **D34**, 2809 (1986).

[8] R. Koniuk and N. Isgur, Phys. Rev. Lett. **44**, 845 (1980); Phys. Rev. **D21**, 1868 (1980).

[9] S. Capstick and W. Roberts, Phys. Rev. **D47**, 1994 (1993); *ibid.* **D49**, 4570 (1994).

[10] A. Le Yaouanc, L. Oliver, O. Pène and J.-C. Raynal, ‘Hadron transitions in the quark model’;
  Gordon and Breach (1988).

[11] Particle Data Group, Phys. Rev. **D50**, 1173 (1994).

[12] Z. Li and R. Workman, preprint [nucl-th/9511041].

[13] C. Gobbi, F. Iachello and D. Kusnezov, Phys. Rev. **D50**, 2048 (1994).

[14] R.H. Dalitz, in ‘Quarks and Hadronic Structure’, Ed. M. Morpurgo, Plenum (1977).

[15] Particle Data Group, Phys. Rev. **D45**, S1 (1992).

[16] N. Kaiser, P.B. Siegel and W. Weise, Phys. Lett. **B 362**, 23 (1995).
Table 1: Matrix elements $F(k)$ in the collective model for $N \to \infty$ (large model space). The final state is $[56, 0^+](0,0,0)$.

| Initial state | $F(k)$ |
|---------------|---------|
| $[56, 0^+](0,0,0)$ | $\frac{1}{(1+k^2a^2)^{\frac{3}{2}}}$ |
| $[20, 1^+](0,0,0)$ | 0 |
| $[70, 1^-](0,0,1)$ | $i \sqrt{3} \frac{ka}{(1+k^2a^2)^{\frac{3}{2}}}$ |
| $[56, 2^+](0,0,0)$ | $\frac{1}{2} \sqrt{3} \left[ \frac{-1}{(1+k^2a^2)^{\frac{3}{2}}} + \frac{3}{2k^2a^2} \left( \arctan ka - \frac{ka}{1+k^2a^2} \right) \right]$ |
| $[70, 2^-](0,0,1)$ | 0 |
| $[70, 2^+](0,0,2)$ | $-\frac{1}{2} \sqrt{15} \left[ \frac{-1}{(1+k^2a^2)^{\frac{3}{2}}} + \frac{3}{2k^2a^2} \left( \arctan ka - \frac{ka}{1+k^2a^2} \right) \right]$ |
Table 2: Coefficients $c_i$ of Eq. (16) for the strong decay widths $\Gamma(B \to B' + M)$ of the negative parity resonances with $(v_1, v_2), L^P = (0, 0), 1^-$. The final state is $^2S_{1/2}[56, 0^+]_{(0,0),0}$ for the nucleon ($B' = N$) and $^4D_{3/2}[56, 0^+]_{(0,0),0}$ for the delta ($B' = \Delta$). The coefficient $\xi = (\cos \theta_P - \sqrt{2} \sin \theta_P)/\sqrt{3}$ for $M = \eta$ and $\xi = (\sin \theta_P + \sqrt{2} \cos \theta_P)/\sqrt{3}$ for $M = \eta'$.

| State  | $N\pi$ | $N\eta$ | $\Delta\pi$ | $\Delta\eta$ |
|--------|--------|---------|-------------|-------------|
|        | $c_0$  | $c_2$  | $c_0$  | $c_2$  | $c_0$  | $c_2$  | $c_0$  | $c_2$  |
| $S_{11}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ |
|        | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ |
| $D_{13}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ |
| $S_{11}$ | $^4S_{1/2}[70, 1^-]_{(0,0):1}$ | $^4S_{1/2}[70, 1^-]_{(0,0):1}$ | $^4S_{1/2}[70, 1^-]_{(0,0):1}$ | $^4S_{1/2}[70, 1^-]_{(0,0):1}$ | $^4S_{1/2}[70, 1^-]_{(0,0):1}$ | $^4S_{1/2}[70, 1^-]_{(0,0):1}$ | $^4S_{1/2}[70, 1^-]_{(0,0):1}$ | $^4S_{1/2}[70, 1^-]_{(0,0):1}$ |
| $D_{13}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ | $^4S_{3/2}[70, 1^-]_{(0,0):1}$ |
| $D_{15}$ | $^4S_{5/2}[70, 1^-]_{(0,0):1}$ | $^4S_{5/2}[70, 1^-]_{(0,0):1}$ | $^4S_{5/2}[70, 1^-]_{(0,0):1}$ | $^4S_{5/2}[70, 1^-]_{(0,0):1}$ | $^4S_{5/2}[70, 1^-]_{(0,0):1}$ | $^4S_{5/2}[70, 1^-]_{(0,0):1}$ | $^4S_{5/2}[70, 1^-]_{(0,0):1}$ | $^4S_{5/2}[70, 1^-]_{(0,0):1}$ |
| $S_{31}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ | $^2S_{1/2}[70, 1^-]_{(0,0):1}$ |
| $D_{33}$ | $^2S_{3/2}[70, 1^-]_{(0,0):1}$ | $^2S_{3/2}[70, 1^-]_{(0,0):1}$ | $^2S_{3/2}[70, 1^-]_{(0,0):1}$ | $^2S_{3/2}[70, 1^-]_{(0,0):1}$ | $^2S_{3/2}[70, 1^-]_{(0,0):1}$ | $^2S_{3/2}[70, 1^-]_{(0,0):1}$ | $^2S_{3/2}[70, 1^-]_{(0,0):1}$ | $^2S_{3/2}[70, 1^-]_{(0,0):1}$ |
Table 3: $N\pi$, $N\eta$ and $\Delta\eta$ decay widths of (3 and 4 star) nucleon and delta resonances in MeV. The experimental values are taken from [11]. The mixing angle for the $\eta$ mesons is $\theta_P = -23^\circ$ [13].

| State | Mass | Resonance | $\Gamma(N\pi)$ | $\Gamma(N\eta)$ | $\Gamma(\Delta\eta)$ |
|-------|------|-----------|----------------|----------------|---------------------|
|       |      |           | $\text{th}$ | $\text{exp}$ | $\text{th}$ | $\text{exp}$ | $\text{th}$ | $\text{exp}$ |
| $S_{11}$ | $N(1535)$ | $^2S_{1/2}[70,1^-],[0,0];1$ | 85 | 79±38 | 0.1 | 86±85 |
| $S_{11}$ | $N(1650)$ | $^4S_{1/2}[70,1^-],[0,0];1$ | 35 | 117±23 | 8 |
| $P_{13}$ | $N(1720)$ | $^2S_{3/2}[56,2^+],[0,0];0$ | 31 | 22±11 | 0.2 |
| $D_{13}$ | $N(1520)$ | $^2S_{3/2}[70,1^-],[0,0];1$ | 115 | 67±9 | 0.6 |
| $D_{13}$ | $N(1700)$ | $^4S_{3/2}[70,1^-],[0,0];1$ | 5 | 10±7 | 4 |
| $D_{15}$ | $N(1675)$ | $^4S_{5/2}[70,1^-],[0,0];1$ | 31 | 72±12 | 17 |
| $F_{15}$ | $N(1680)$ | $^2S_{5/2}[56,2^+],[0,0];0$ | 41 | 84±9 | 0.5 |
| $G_{17}$ | $N(2190)$ | $^2S_{7/2}[70,3^-],[0,0];1$ | 34 | 67±27 | 11 |
| $G_{19}$ | $N(2250)$ | $^4S_{9/2}[70,3^-],[0,0];1$ | 7 | 38±21 | 9 |
| $H_{19}$ | $N(2220)$ | $^2S_{9/2}[56,4^+],[0,0];0$ | 15 | 65±28 | 0.7 |
| $I_{1,11}$ | $N(2600)$ | $^2S_{11/2}[70,5^-],[0,0];1$ | 9 | 49±20 | 3 |
| $S_{31}$ | $\Delta(1620)$ | $^2P_{1/2}[70,1^-],[0,0];1$ | 16 | 37±11 | – | – |
| $P_{31}$ | $\Delta(1910)$ | $^4P_{1/2}[56,2^+],[0,0];0$ | 42 | 52±19 | 0.0 |
| $P_{33}$ | $\Delta(1232)$ | $^4P_{3/2}[56,0^+],[0,0];0$ | 116 | 119±5 | – | – |
| $P_{33}$ | $\Delta(1920)$ | $^4P_{3/2}[56,2^+],[0,0];0$ | 22 | 28±19 | 0.5 |
| $D_{33}$ | $\Delta(1700)$ | $^2P_{3/2}[70,1^-],[0,0];1$ | 27 | 45±21 | – | – |
| $D_{35}$ | $\Delta(1930)$ | $^2P_{5/2}[70,2^-],[0,0];1$ | 0 | 52±23 | 0 |
| $F_{35}$ | $\Delta(1905)$ | $^4P_{5/2}[56,2^+],[0,0];0$ | 9 | 36±20 | 1 |
| $F_{37}$ | $\Delta(1950)$ | $^4P_{7/2}[56,2^+],[0,0];2$ | 45 | 120±14 | 2 |
| $H_{3,11}$ | $\Delta(2420)$ | $^4P_{11/2}[56,4^+],[0,0];0$ | 12 | 40±22 | 2 |
Table 4: Strong decay widths for $\Delta^* \rightarrow N + \pi$ and $N^* \rightarrow N + \pi$ in MeV. Experimental values are from [11].

| Resonance  | $L$ | $\Gamma$(th) | $\Gamma$(exp) |
|------------|-----|--------------|---------------|
|            |     | Ref. [10]    | Ref. [8]      | Ref. [9]      | Present |
| $\Delta(1232)P_{33}$ | 0   | 70           | 121           | 108           | 116     | 119 ± 5 |
| $\Delta(1950)F_{37}$  | 2   | 27           | 56            | 50            | 45      | 120 ± 14 |
| $\Delta(2420)H_{3,11}$ | 4   | 4            | 8             | 12            | 40 ± 22 |
| $\Delta(2950)K_{3,15}$ | 6   | 1            | 3             | 5             | 13 ± 8  |
| $N(1520)D_{13}$       | 1   | 85           | 74            | 115           | 67 ± 9  |
| $N(2190)G_{17}$       | 3   | 48           | 34            | 67 ± 27       |
| $N(2600)I_{1,11}$     | 5   | 11           | 9             | 49 ± 20       |
Figure 1: Collective model of baryons.
Figure 2: Elementary meson emission.
Figure 3: $N\pi$ and $\Delta\pi$ decay widths of negative parity nucleon resonances with $(v_1, v_2), L^P = (0,0), 1^-$. The theoretical values are in parenthesis. All values in MeV.
Figure 4: $N\pi$ and $\Delta\pi$ decay widths of negative parity delta resonances with $(v_1, v_2), L^P = (0, 0), 1^-$. The theoretical values are in parenthesis. All values in MeV.