Atomic Size Dark Matter Pearls, Electron Signal

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Abstract

We seek to explain both the seeming observation of dark matter by the seasonal variation of the DAMA-LIBRA data and the observation of “electron recoil” events at Xenon1T in which the liquid-Xe-scintillator was excited by electrons - in excess to the expected background - by the same dark matter model. In our model the dark matter consists of bubbles of a new type of vacuum containing ordinary atomic matter, say diamond, under high pressure ensured by the surface tension of the separation surface (domain wall). This atomic matter is surrounded by a cloud of electrons extending almost out to atomic size. We also seek to explain the self interactions of dark matter suggested by astronomical studies of dwarf galaxies and the central structure of galaxy clusters. At the same time we consider the interaction with matter in the shielding responsible for slowing the dark matter down to a low terminal velocity, so that collisions with nuclei in the underground detectors have insufficient energy to be detected. Further we explain the “mysterious” X-ray line of 3.5 keV from our dark matter particles colliding with each other so that the surfaces/skins unite. Even the 3.5 keV X-ray radiation from the Tycho supernova remnant is explained as our pearls hitting cosmic rays in the remnant.

What the DAMA-LIBRA and Xenon1T experiments see is supposed to be our dark matter pearls excited during their stopping in the shielding or the air. The most remarkable support for our type of model is that both these underground experiments see events with about 3.5 keV energy, just the energy of the X-ray line.

We get a good numerical understanding of the fitted cross section over mass ratio of self interacting dark matter observed in the study of dwarf galaxies. Also the total energy of the dark matter pearls stopped in the shield is reasonably matching order of magnitudewise with the absolute observation rates of DAMA-LIBRA and Xenon1T, although the proposed explanation of their ratio requires further development.

It should be stressed that accepting that the different phases of the vacuum could be realized inside the Standard Model, our whole scheme could be realized inside the Standard Model. So then no new physics is needed for dark matter!
1 Introduction

For a long time we have worked on a dark matter model [1, 2, 3, 4, 5, 6, 7, 8], in which the dark matter consisted of cm-size pearls which were in fact bubbles of a new vacuum type surrounded by a skin caused by the surface tension of this new vacuum. This skin kept a piece of usual atomic matter highly compressed inside the bubble. In fitting data with this model the most and almost only successful fit consisted in that we fitted, with a common parameter, both the overall rate and the very 3.5 keV energy of the X-ray line originally observed in several galaxy clusters, Andromeda and the Milky Way Center [9, 10, 11, 12, 13, 14] and supposedly coming from dark matter. But now it turned out that this successful fitting relation between the 3.5 keV energy and the overall rate of the X-ray radiation only depends on the density of the pearls or equivalently the fermi momentum or energy of the electrons kept inside the pearls, but not on the absolute size of the pearls. Thus we could change the model to make the pearl sizes much smaller, as we shall do in this article, so that they are e.g. now rather of atomic size. Really we shall let the pearls be of radius $r_{\text{cloud}} = 3.3 \text{MeV} = 5 \times 10^{-12} \text{m}$. But even such small pearls get stopped to some extent by the shielding into which they must penetrate to reach the underground experiments like the DAMA-LIBRA and Xenon experiments looking for dark matter. Using an astronomical observation based model by Correa [15] especially, we shall construct a rather definite picture of our pearls from which we estimate that the pearls hitting the earth actually get stopped presumably in the atmosphere, but if not then at least in the earth shielding. The pearls thereby lose so much speed that it becomes quite understandable that the Xenon-experiments, looking for nuclei being hit by them and causing scintillation in fluid xenon, will not see any such events. However the DAMA-LIBRA experiment [16, 17] would not distinguish if it is a nucleus that is hit or some energy is released which causes the scintillator to luminesce. So only the DAMA-LIBRA experiment would be able to get a signal if the dark matter, e.g. our pearls, could be somehow excited and emit their excitation energy when they pass through the detector. In our model we shall indeed suggest that the pearls get excited and emit their energy by electron emission. That would not be easy to distinguish for DAMA-LIBRA but would still of course come with seasonal variation [6] so that it would be observed as dark matter by DAMA-LIBRA. Whether the emission is via electrons or nuclei would not matter. But for the xenon-experiments such electron emission was effectively not counted for a long time, but now rather recently the Xenon1T experiment has actually observed an excess of “electron recoil events”. So they have now in fact seen an electron emission somehow.

We shall see in section 7 that both the excess of electron recoil events in Xenon1T [19] and the events seen by DAMA-LIBRA [16, 17] have the energy of each event remarkably enough centering about the energy value 3.5 keV of the mysterious X-ray line found astronomically!

This coincidence of course strongly suggests that these events from DAMA-LIBRA and Xenon1T are related to dark matter particles that can be excited precisely by this energy 3.5 keV.

In our earlier papers [5, 6, 7] we have already connected the excitability of

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1 We note however that the ANAIS experiment has failed to see an annual modulation with NAI(Tl) scintillators and their results [18] are incompatible with the DAMA-LIBRA results at 3.3σ.
our pearls by just this energy 3.5 keV and especially the emission of photons (or here in the present work also electrons) with just this energy with a gap in the single particle electron spectrum of the pearls caused by what we call the homolumo-gap effect.

A very serious warning, which needs an explanation in order to rescue our model, is delivered by the fact that if as we now suggest the Xenon1T electron recoil event excess is coming from just the same decay of dark matter excitations as the DAMA-Libra observation, then these two experiments ought a priori to see equally many events, say per kg. However, DAMA-LIBRA sees 250 times as many events as Xenon1T sees excess events.

We shall postpone this question to a later article in detail, but the hope for now is that the Xenon1T experiment has the observed decaying pearls falling through a fluid, namely the fluid xenon, while the scintillator in DAMA-LIBRA is a solid made from NaI(Tl). The pearls are likely to form a little Xe-fluid bubble around them and flow or fall through the xenon-fluid, while they will much more easily get caught so as to almost sit still or only move much slower through the NaI scintillator. If so the pearls with their supposed excitations would spend much more time in the DAMA-LIBRA NaI than in a corresponding volume of xenon-liquid.

In the following section 2 we describe how the particles making up the dark matter in our model are imagined to be bubbles of the size \( R = r_{\text{cloud}} \times 3.3 \text{MeV} = 5 \times 10^{-12} \text{m} \) with heavy atomic matter inside, which is surrounded out to a radius \( r_{\text{cloud}} \times 3.5 \text{keV} = 5 \times 10^{-11} \text{m} \) by electrons. Here the quantities 3.3 MeV and 3.5 keV in the subscripts are the numerical electric potentials felt by an electron at the distances mentioned. A special point to note in this section already present in the earlier articles about the big pearls is the homolumo-gap effect, causing a band or gap in the energy levels without any single particle electron eigenstates. The width of this gap is fitted to the 3.5 keV line in the observed X-ray spectrum from galaxy clusters, the Milky Way Center etc. [9, 10, 11].

Next in section 3 we briefly review astronomical observations and modelling of the dark matter, which suggests the idea that dark matter interacts with itself (strongly interacting dark matter SIDM). It is only when the corresponding cross section \( \sigma \) is divided by the particle mass \( M \) do we have a combination that has any chance of being observed by its effects on the atomic matter. In fact this ratio \( \frac{\sigma}{M} \) matches well with the atomic physics structure of our pearls including the cloud of electrons outside the bubble itself.

In section 4 we list a series of numerical successes of our model for the dark matter, hopefully making the reader see that there is really some reason for it being at least in some respects correct.

In section 5 we restress that our dark matter pearls get stopped and at the same time excited, mainly to emit quanta of energy 3.5 keV, in the air and/or in the shielding above the experiments. According to our best estimates they get stopped already about 53 km up in the air. It is the braking energy from this slowing down that is supposed to feed the excitations.

A special estimation, based on energy considerations, of whether the number of events seen by DAMA-LIBRA and by the Xenon1T electron recoil excess are of a reasonable order of magnitude is put forward in section 6. The success of such an estimation has to be rather limited in as far as the rates of the two observations - that should have been the same if we do not include the possibility of faster or slower motion through the detectors - deviate by a factor of 250.
In section 7 we call attention to the perhaps most remarkable fact supporting a major aspect of our model: That the energy per event for both DAMA-LIBRA and the Xenon1T-electron recoil excess centers around 3.5 keV, just the energy of the photons in the mysterious X-ray line seen in galactic clusters mentioned above! So all three effects should correspond to the emission of an electron or photon due to the same energy transition inside dark matter.

Finally in section 8 we conclude and provide a short outlook.

2 Pearl

Dark Matter Atomic Size Pearls, Electronic 3.5 keV Signal

We sketch the structure of our small dark matter pearls in Figure 1.

- In the middle is a spherical bubble of radius
  \[ R \approx r_{\text{cloud } 3.3 \text{MeV}} \approx 5 \times 10^{-12} \text{m}. \] (1)

Here \( r_{\text{cloud } 3.3 \text{MeV}} \) denotes the radius where the electron potential is 3.3 MeV, which is identified with the Fermi energy \( E_f \) of the electrons in the bulk of the pearl - i.e. inside the radius \( R \). We estimated the value \( E_f = 3.3 \text{ MeV} \) in previous papers [6, 7, 8] by fitting the overall rate of the intensity of the 3.5 keV line emitted by galactic clusters and the very frequency 3.5 keV of the radiation in our model.

- The outer radius
  \[ r_{\text{cloud } 3.5 \text{keV}} \approx 5 \times 10^{-11} \text{m} \] (2)

is where the electron potential is 3.5 keV. By our story of the “homolumo gap”: the electron density crudely goes to zero at this radius. (It gradually falls in the range between \( r_{\text{cloud } 3.3 \text{MeV}} \) and \( r_{\text{cloud } 3.5 \text{keV}} \)).
Due to an effect, we call the homolumo-gap effect \[5, 20\], the nuclei in the bubble region and the electrons themselves become arranged in such a way as to prevent there from being any levels in an interval of width 3.5 keV. So, as illustrated in Figure 2 outside the distance \( r = 3.5 \text{keV} \) from the center of the pearl at which the Coulomb potential is \( \sim 3.5 \text{keV} \) deep there are essentially (\( \sim \) in the Thomas-Fermi approximation) no more electrons in the pearl-object.

The radius \( r = 3.3 \text{MeV} \) at which the potential felt by an electron is 3.3 MeV deep, is supposed to be just the radius to which the many nuclei inside the pearl (which replace the single nucleus in ordinary atoms) reach out. So inside the bubble the potential is much more flat.

The energy difference between the zero energy line and the effective Fermi surface, above which there are no more electrons, is of order 3.5 keV, the energy so crucial in our work.

Since in the Thomas-Fermi approximation there are no electrons outside roughly the radius \( r = 3.5\text{keV} = r_{\text{cloud} 3.5\text{keV}} \), this radius will give the maximal cross section, even for very low velocity \( \sigma_{v\to0} \).

The homolumo gap effect.

Let us consider the spectrum of energy levels for the electrons in a piece of material, e.g. one of our pearls, and at first assume that the positions or distributions of the charged particles in the material are fixed.

Then the ground state is just a state built e.g. as a Slater determinant for the electrons being in the lowest single electron states, so many as are needed to have the right number of electrons.

But now, if the charged particles can be moved due to their interactions, the ground state energy could be lowered by moving them so that the filled electron state levels get lowered.
Figure 3: Explanation of Homolumo-gap effect

So we expect introducing such a "back reaction" will lower the filled states. When the filled levels get moved downwards, then the homolone = highest occupied molecular orbital level will be lowered and its distance to the next occupied molecular orbital level will be widened on the energy axis.

We believe that we can estimate the homolumo-gap $E_{H}$. Using the Thomas-Fermi approximation - or crudely just some dimensional argument where the fine structure constant has the dimension of velocity - we calculated the homolumo gap in highly compressed ordinary matter for relativistic electrons:

$$E_{H} \sim \left( \frac{\alpha \cdot c}{\sqrt{2}} \right)^{3/2} \sqrt[3]{p_{F}}$$

where $p_{F} = \text{Fermi momentum}$

$\alpha = 1/137.03$... (5)

( the $\sqrt{2}$ comes from our Thomas-Fermi calculation).

It is by requiring this homolumo-gap to be the 3.5 keV energy of the X-ray line mysteriously observed by satellites from clusters of galaxies. Andromeda, and the Milky Way Center that we estimate the Fermi-energy to be $E_{F} \approx p_{F} = 3.3$ MeV in the interior bulk of the pearl.

Brief summary of theoretical ideas underlying our dark matter pearls

- Principle: Nothing but Standard Model! (Seriously it would mean not in a BSM-workshop.)

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• **New Assumption** Several Phases of Vacuum with Same Energy Density; this is the so-called Multiple Point Principle [5, 6, 21, 22, 23, 24, 25, 26].

• **Central Part** Bubble of New Phase of Vacuum with e.g. carbon under very high pressure, surrounded by a surface with tension $S$ (= domain wall) providing the pressure.

• **Outer part** Cloud of Electrons much like an ordinary atom having a nucleus with a charge of order ten to a hundred thousand ($Z \approx 5 \times 10^4$ effectively).

### 3 Non-gravitational Interactions

The collisionless cold dark matter model provides a good description of the large scale structure of the Universe. However there are various problems at small scales [27, 28] for the hypothesis that dark matter only has gravitational interactions. Originally Spergel et al. [29] suggested that the lack of a peak or cusp in the center of galaxy clusters, as expected for cold dark matter with purely gravitational interactions, required self interacting dark matter with a relatively large cross section. The relevant parameter is in fact the cross section per mass $\sigma / M$ and for the cores in galaxy clusters, where the collision velocity is $v \sim 1000$ km/s, a value $\sigma / M \sim 0.1$ cm$^2$/g is needed. The self interaction can of course be velocity dependent and the cores in spiral galaxies where $v \sim 100$ km/s require $\sigma / M \sim 1$ cm$^2$/g. In dwarf galaxies around our Milky Way, where dark matter moves more slowly $v \sim 30$ km/s, larger cross section to mass ratios $\sigma / M \sim 50$ cm$^2$/g are needed.

Recently Correa [15] made a study of the velocity dependence of self interacting dark matter. In particular she analysed the Milky Way dwarf galaxies and her results are displayed in Figure 5. The extrapolation of Correa’s fit to the data towards zero velocity points to the ratio $\sigma / M \to 150$ cm$^2$/g. This ratio
Figure 5: Cross section per mass $\sigma/\langle m_d \rangle$ of self interacting dark matter particles as a function of the collision velocity $v$ in dwarf galaxies from reference [15].

| Name   | $M_{\text{vir}} \times 10^5 M_\odot$ | $\epsilon_{\text{vir}}$ [pc] | $\epsilon_{\text{core}}$ [pc] | $\sigma/\langle m_d \rangle$ [cm$^2$g$^{-1}$] |
|--------|-------------------------------------|-------------------------------|-------------------------------|-----------------------------------------------|
| UM     | 0.13                               | 54.2                          | 180.8                         | 40 - 50                                       |
| Draco   | 1.17                               | 26.8                          | 472.9                         | 20 - 30                                       |
| Carina  | 1.69                               | 19.1                          | 648.4                         | 40 - 50                                       |
| Sextans | 0.32                               | 20.8                          | 395.5                         | 70 - 120                                      |
| CVn    | 0.46                               | 25.7                          | 356.8                         | 50 - 80                                       |
| Sculptor | 1.65                             | 28.8                          | 553.2                         | 30 - 40                                       |
| Fornax | 2.29                               | 15.3                          | 1036.7                        | 30 - 50                                       |
| Leo II | 0.05                               | 30.6                          | 148.8                         | 90 - 150                                      |
| Leo I  | 1.17                               | 31.1                          | 490.8                         | 50 - 70                                       |

Table 2. From left to right: name of the dSph galaxy, present time virial mass, concentration parameter and core size of the subhalo hosting the dSph and range of preferred cross section values that reproduce the observed DM central densities.

Figure 6. Cross section per unit mass, $\sigma/\langle m_d \rangle$, as a function of the average collision velocity, $\langle v \rangle$, of DM particles within each subhalo's core. Symbols show the range of $\sigma/\langle m_d \rangle$, needed for the SIDM model to reproduce the central DM densities reported by Koplignah et al. (2019). The solid line corresponds to the best-fit relation given by eq. (15) to the MW dSph data.

Figure 5: Cross section per mass $\sigma/\langle m_d \rangle$ of self interacting dark matter particles as a function of the collision velocity $v$ in dwarf galaxies from reference [15].
can be taken as an experimental estimate of the impact area over the mass as seen for very soft collisions. In our model the cross section in this low velocity limit is given by the extent out to which the electrons surrounding our pearls reach. This range of extension of electrons is supposed to be given by the requirement that the electron binding energy is of the order of the homolumo gap value 3.5 keV. So we denote this radius by \( r_{\text{cloud} \, 3.5 \text{keV}} \). Similarly the radius of the bubble containing the nucleons inside our dark matter pearl corresponds to a radius \( r_{\text{cloud} \, 3.3 \text{MeV}} \) at which the potential for the electron is -3.3 MeV (= Fermi energy of the electrons). The high velocity hard collisions of our pearls, supposed to result in the unification of two pearls into a single pearl, correspond to interactions between the bubble skins with a cross section of order \( \pi r_{\text{cloud} \, 3.3 \text{MeV}}^2 \).

We will now consider the electric potential for our pearl using the Thomas-Fermi approximation for a heavy atom \[30, 31, 32\]. In this approximation the Coulomb potential of the “nuclear” charge \( Z \) is multiplied by the Thomas-Fermi screening function \( \chi(r/b) \) where

\[
b = 0.88 \frac{a_0}{Z^{1/3}}
\]  

and \( a_0 \) is the Bohr radius. The skin of the bubble or “nucleus” of the pearl mainly acts on the nucleons or rather nuclei. So the electrons spread out and an appreciable part, say half of them, are outside the central part of the pearl inside the skin. Therefore the effective charge \( Z \) of the central part of the pearl or bubble of the new phase is e.g. one half of the number of protons inside the skin. Assuming also that there are about equally many neutrons and protons inside the central part, the mass of the pearl is then given order of magnitudewise by

\[
M = 4m_N * Z,
\]

where \( m_N \) is the nucleon mass.

In the Thomas-Fermi approach we are then led to the following equations for \( r_{\text{cloud} \, 3.5 \text{keV}} \) and \( r_{\text{cloud} \, 3.3 \text{MeV}} \):

\[
\alpha * Z \frac{r_{\text{cloud} \, 3.5 \text{keV}}}{\chi(r_{\text{cloud} \, 3.5 \text{keV}}/b)} = 3.5 \text{ keV}
\]  

\[
\alpha * Z \frac{r_{\text{cloud} \, 3.3 \text{MeV}}}{\chi(r_{\text{cloud} \, 3.3 \text{MeV}}/b)} = 3.3 \text{ MeV}
\]

\[
b = 0.88 * \frac{a_0}{Z^{1/3}}
\]

We identify \( r_{\text{cloud} \, 3.5 \text{keV}} \) with the radius of the electron cloud and \( r_{\text{cloud} \, 3.3 \text{MeV}} \) with the skin radius \( R \) of the pearl.

It is going to be an important success of our model that we get a similar value for \( R \approx r_{\text{cloud} \, 3.3 \text{MeV}} \) using another method to calculate it. We shall use

\[
\left. \frac{\sigma}{M} \right|_{v \to 0} = 150 \text{ cm}^2/\text{g}
\]

and

\[
\sigma = \pi * r_{\text{cloud} \, 3.5 \text{keV}}^2
\]

to determine the mass \( M \). Then using the formula for the mass of a pearl in terms of the radius \( R \) and the Fermi momentum \[7, 8\]

\[
M/m_N = \frac{8}{9\pi} * (R * p_f)^3,
\]

where
we can calculate another value for $R$.

In our updated contribution to the Bled Proceedings from last year [8] we estimated a pearl mass of $M \sim 10^5$ GeV. So we take here $Z = 5.3 \times 10^4$ as a typical charge in the central part of the pearl, for which then $b = 1.24 \times 10^{-12} m$. Using numerical values for the Thomas-Fermi screening function in the paper [33], we obtain from (7) the radius of the electron cloud to be

$$r_{\text{cloud} \ 3.5keV} = 4.96 \times 10^{-11} m$$

(14)

Then assuming the low velocity ratio $\frac{\sigma M}{v} = 150 \ cm^2/g$ we obtain

$$M = \frac{\pi \cdot (4.96 \times 10^{-11} m)^2}{150 \ cm^2/g} = 5.2 \times 10^{-19} g \quad (15)$$

$$= 3.1 \times 10^5 \ m_N$$

As a side remark notice that, using our proposed rule of taking $Z$ to be a quarter of the number $M/m_N$, we would get $Z = 8 \times 10^4$ to be compared with our input here $5.3 \times 10^4$, which is very well consistent within a factor 2.

Next using (13) with $p_f = 3.3 \ MeV = 1.6 \times 10^{13} m^{-1}$

$$(R \cdot p_f)^3 = 3.1 \times 10^5 \cdot \frac{9\pi}{8}$$

(18)

$$= 10.9 \times 10^5$$

$$R \cdot 1.6 \times 10^{13} m^{-1} = \sqrt[3]{10.9 \times 10^5} = 102$$

(19)

(20)

giving

$$R = \frac{102}{1.6 \times 10^{13} m^{-1}} = 6.4 \times 10^{-12} m.$$ \quad (21)

(22)

This is to be compared with the Thomas-Fermi value obtained from (8) using the numerical values for $\chi(r/b)$ in [33]

$$R = r_{\text{cloud} \ 3.3MeV} = 3.66 \times 10^{-12} m.$$ \quad (23)

These two different estimates of the radius $r_{\text{cloud} \ 3.3MeV}$ at which the potential is 3.3 MeV essentially coincide to the accuracy of our calculation; they deviate by a factor of order unity $6.4/3.7 = 1.7$. So we could claim that formally our model is able to predict the low velocity limit $\frac{\sigma M}{v} \ \big|_{v \rightarrow 0}$ in agreement with the value $150 \ cm^2/g$ estimated from the study of dwarf galaxies around the Milky Way.

We shall take the average of the two values (22) and (23) as our best estimate of the bubble skin radius:

$$r_{\text{cloud} \ 3.3MeV} = 5.0 \times 10^{-12} m$$

(24)

and from (14) we have the radius of the electron cloud

$$r_{\text{cloud} \ 3.5keV} = 5.0 \times 10^{-11} m.$$ \quad (25)

We note that these two radii differ by an order of magnitude, which means that the quantity $\frac{\sigma M}{\chi}$ for our pearls should differ by two orders of magnitude between low velocities and high velocities, as astronomical observations indicate is the case for self interacting dark matter [15].
4 Achievements

- **Low velocity \( \sigma_{M|v \to 0} \) cross section to mass ratio.** The a priori story, that dark matter has only gravitational interactions seems not to work perfectly: Especially in dwarf galaxies (around our Milky Way) where dark matter moves relatively slowly an appreciable self interaction cross section to mass ratio \( \sigma_{M|v \to 0} \) is needed. According to the fits in [15] this ratio has the low velocity limit \( \sigma_{M|v \to 0} = 150 \text{ cm}^2/\text{g} \). We may say our pearl-model “predicts” this ratio in order of magnitude.

- **Can make the Dark Matter Underground Searches get Electron Recoil Events.** Most underground experiments are designed to look for dark matter particles hitting the nuclei in the experimental apparatus, which is then scintillating so that such hits presumed to be on nuclei can be seen. But our pearls are excited in such a way that they send out energetic electrons (rather than nuclei) and this does not match with what is looked for, except in the DAMA-LIBRA experiment. In this experiment the only signal for events coming from dark matter is a seasonal variation due to the Earth running towards or away from the dark matter flow.

- **The Intensity of 3.5 keV X-rays from Clusters etc.** We fit the very photon-energy 3.5 keV and the overall intensity from a series of clusters, a galaxy, and the Milky Way Center [8] with one parameter \( \xi^{1/4} f_S \Delta V = 0.6 \text{ MeV}^{-1} \).

- **3.5 keV Radiation from the Tycho Supernova Remnant.** Jeltema and Profumo [34] discovered the 3.5 KeV X-ray radiation coming from the remnant of Tycho Brahe’s supernova, which was unexpected for such a small source. We have a scenario giving the correct order of magnitude for the observed intensity in our pearl model: supposedly our pearls are getting excited by the high intensity of cosmic rays in the supernova remnant [8].

Even though we can use only the one parameter \( \xi_{fS} f_S \Delta V = 2 \), it is nice to know the notation:

\[
\Delta V = \text{“difference in potential for a nucleon between the inside and the outside of the central part of the pearl”}
\]

\[
\approx 2.5 \text{ MeV}
\]

\[
\xi_{fS} = \frac{R}{R_{crit}} \text{ estimated to be } \approx 5
\]

where \( R = \text{“actual radius of the new vacuum part”} \)

\[
\approx \tau_{\text{cloud}} 3.3 \text{MeV}
\]

and \( R_{crit} = \text{“Radius when pressure is so high that nucleons are just about being spit out”} \)

The subscript \( fS \) on the parameter \( \xi_{fS} \) indicates that the surface tension \( S \) is fixed independent of the radius \( R \).
• **DAMA-rate** Estimating observation rate of DAMA-LIBRA from kinetic energy of the incoming dark matter as known from astronomy.

• **Xenon1T Electron recoil rate** Same for the *electron recoil excess* observed by the Xenon1T experiment.

In order to explain these last calculational estimates it is necessary to know how we imagine the dark matter to interact and get slowed down in the air and the earth shielding; also how the dark matter particles get excited and emit 3.5 keV radiation or electrons.

**About the Xenon1T and DAMA-rates:**

• **Absolute rates very crudely** Our estimate of the absolute rates for the two experiments are very very crude, because we assume that the dark matter particles - in our model small macroscopic systems with ten thousands of nuclei inside them - can have an exceedingly smooth distribution of lifetimes on a logarithmic scale. These calculations are discussed in section 6.

• **The ratio of rates** The ratio of the rates in the two experiments - Xenon1T electron recoil excess and DAMA - should in principle be very accurately predicted in our model, because they are supposed to see exactly the same effect just in two different detectors in the same underground laboratory below the Gran Sasso mountain! One would therefore expect the rates to be the same, but the Xenon1T rate is 250 times smaller than the DAMA rate. We briefly refer to a possible resolution of this problem, which needs further study, in section 6.

## 5 Impact

**Illustration of Interacting and Excitable Dark Matter Pearls**

The dark matter pearls come in with high speed (galactic velocity), but get stopped down to much lower speed by interaction with the air and the shielding mountains, whereby they also get excited to emit 3.5 keV X-rays or *electrons*.

**Pearls Stopping and getting Excited in Earth Shield**

What happens when the dark matter pearls in our model of less than atomic size hit the earth shielding above the experimental halls of e.g. DAMA?

• **Stopping** Taking it that the pearls stop in the earth: The pearls are stopped in about $5 \times 10^{-6}$s from their galactic speed of about 300 km/s
down to a speed 49 km/s below which collisions with nuclei can no longer excite the 3.5 keV excitations. The stopping length, modulo a logarithmic factor, is $\frac{1}{4}m$.

But taking it that they stop in the air, which is more likely: They are stopped over a range of about 7 km - as the atmosphere density goes up with a factor $e = 2.71$.. over such a range in about $2 \times 10^{-2}s$.

- **Excitation** As long as the velocity is still over the ca. 49 km/s collisions with nuclei in the shielding can excite the electrons inside the pearl by 3.5 kev or more and make pairs of quasi electrons and holes say. We expect that often the creation of (as well as the decay of) such excitations require electrons to pass through a (quantum) tunnel and that consequently there will be decay half lives of very different sizes. We hope even up to many hours or days...

- **Slowly sinking:** After being stopped in of the order of $\frac{1}{4}m$ of the shielding, the pearls continue with a much lower velocity driven by the gravitational attraction of the Earth. After say about 26 hours a pearl reaches the 1400 m down to the laboratories. Most of the pearls have returned to their ground states, but some exceptionally long living excitations survive.

Note that the slowly sinking velocity is so low that collisions with nuclei cannot give such nuclei enough speed to excite the scintillation counters neither in DAMA nor in Xenon-experiments.

- **Electron or $\gamma$ emission** Typically the decay of an excitation could be that a hole in the Fermi sea of the electron cloud of the pearl gets filled by an outside electron under emission of another electron by an Auger-effect. The electron must tunnel into the pearl center. This can make the decay lifetime become very long and very different from case to case.

**Emission as electrons or photons makes Xenon-experiments not see events, except...**

That the decay energy is released most often as electron energy means that such events are discarded by most of the Xenon-experiments, which only expect the nucleus recoils to be dark matter events. This would explain the long standing controversy consisting in DAMA seeing dark matter with a much bigger rate than the upper limits from the other experiments.

Rather recently though Xenon1T looked for potential excess events among the electron recoil events and found 16 events/year/tonne/keV in the lowest keV-bands over a background of the order of $(76 \pm 2)$ events/year/tonne/keV.

In our model this rate should be compatible with the DAMA event rate. However they deviate by a factor of 250. It therefore appears that we need the pearls to run much faster through the xenon-apparatus than through the DAMA one.
6 Numerical Rates for DAMA and Xenon1T-electron-recoil-excess

6.1 The Kinetic Energy Flux from Dark Matter

The dark matter density \( D_{\text{sol}} \) in our part of the Milky Way and its velocity \( v \) are of the orders of magnitude

\[
D_{\text{sol}} = 0.3 \text{ GeV}/\text{cm}^3
\]

\[
= 5.34 \times 10^{-22} \text{ kg/m}^3
\]

\[
v = 300 \text{ km/s} \quad \text{(relative to solar system)}
\]

This gives a kinetic energy density

\[
D_{\text{kin energy}} = \frac{1}{2} v^2 D_{\text{sol}}
\]

\[
= 0.5 \times (10^{-3})^2 c^2 \times 5.34 \times 10^{-22} \text{ kg/m}^3
\]

\[
= 2.40 \times 10^{-11} \text{ J/m}^3
\]

meaning an influx of kinetic energy

\[
\text{“power per } m^2\text{”} = v D_{\text{kin energy}}
\]

\[
= \frac{1}{2} v^3 D_{\text{sol}}
\]

\[
= 3 \times 10^5 \text{ m/s} \times 2.40 \times 10^{-11} \text{ J/m}^3
\]

\[
= 7.2 \times 10^{-6} \text{ W/m}^2
\]

Distributing this energy rate over the amount of matter down to the depth 1400 m with density 3000 kg/m³ we obtain the energy rate per kg

\[
\text{“power to deposit”} = \frac{7.2 \times 10^{-6} \text{ W/m}^2}{1400 \text{ m} \times 3000 \text{ kg/m}^3}
\]

\[
= 1.7 \times 10^{-12} \text{ W/kg.}
\]

However, assuming that all the events from the dark matter - as given by the modulated part of the signal found by DAMA-LIBRA - are just due to decays with the decay energy 3.5 keV, the rate of energy deposition per kg observed by DAMA-LIBRA [17] is

\[
\text{“deposited rate”} = \frac{0.0412 \text{ cpd/kg} \times 3.5 \text{ keV}}{86400 \text{ s/day}}
\]

\[
= \frac{0.0412 \text{ cpd/kg} \times 3.5 \times 1.6 \times 10^{-16} \text{ J}}{86400 \text{ s/day}}
\]

\[
= 2.7 \times 10^{-22} \text{ W/kg},
\]

which is

\[
\frac{2.7 \times 10^{-22} \text{ W/m}^2}{1.7 \times 10^{-12} \text{ W/m}^2} = 1.6 \times 10^{-10} \text{ times as much}.
\]

We can express this by saying that there is a need for a suppression factor suppression being \( 1.6 \times 10^{-10} \) for the DAMA-LIBRA rate. For the excess of the
electronic recoil events found by Xenon1T the corresponding suppression factor must be the 250 times smaller number. This is because the event rate of these excess electron recoil events is 250 times smaller than that of the modulation part of the DAMA rate and the depth of the experiment under the earth is the same 1400 m. Thus we summarize the experimentally determined suppression factors:

\[
\begin{align*}
\text{suppression}_{\text{DAMA}} &= 1.6 \times 10^{-10} \quad (46) \\
\text{suppression}_{\text{Xenon1T}} &= 1.6 \times 10^{-10} \times \frac{1}{250} = 6.4 \times 10^{-13}. \quad (47)
\end{align*}
\]

### 6.2 Estimating “suppression” theoretically

The idea for obtaining theoretical estimates of these suppression factors is to say that the observed events come from excitations of our pearls with a lifetime of the order of the time it takes for the pearl, after its excitation under its stopping in the air or in the stone above the experiments, to reach down to the experimental detectors. We here assume the scattering cross section of dark matter on ordinary matter to be similar to that on dark matter. So we estimate the passage time of the pearl down to the detectors as being of the order of 26 hours, by using the low velocity value for the cross section over mass ratio

\[
\frac{\sigma}{M} = 150 \, \text{cm}^2/g \quad (48)
\]

Once the pearl has been stopped so much that its velocity is only upheld by the gravitational field with the acceleration \( g = 9.8 \, \text{m/s}^2 \), the terminal velocity will be obtained formally from the drag-equation:

\[
\text{Drag force } D = gM = 0.5 \times C_d \times \sigma \times \rho v^2. \quad (49)
\]

Here \( \rho \) is the density of the material passed through and the drag coefficient \( C_d \) is of order unity (so the 0.5 is hardly relevant). That is to say the terminal velocity becomes:

\[
\begin{align*}
\nu_{\text{terminal}} &\approx \sqrt{\frac{g}{\frac{\sigma}{M} \times \rho}} \\
&\approx \sqrt{\frac{9.8 \, \text{m/s}^2}{150 \, \text{cm}^2/g \times 3 \, \text{g/cm}^3}} \quad (51) \\
&= \sqrt{2.2 \, \text{cm}^2/\text{s}^2} = 1.5 \, \text{cm/s}, \quad (52)
\end{align*}
\]

which allows a pearl to pass through 1400 m in

\[
\text{“passage time”} = \frac{140000 \, \text{cm}}{1.5 \, \text{cm/s}} = 93000 \, \text{s} = 26 \, \text{hours}. \quad (54)
\]

\footnote{Strictly speaking this equation is only valid if the pearl velocity is greater than the thermal velocity of the nuclei in the shielding and so needs further study.}
6.3 Equally hard to excite and to de-excite

In order that there can be any de-excitations of the pearls after such 26 hours it is of course needed that an appreciable part of the possible 3.5 keV excitations of our pearls have lifetimes of this order of magnitude. A priori these excitations are excitons for which the electron and hole can be close by and decay rapidly or it is possible that one of the partners is outside in the electron cloud and long lived. By arguing that some tunnelling of electrons in or out or around in the pearl may be needed for some (de-)excitations, we can claim that the lifetimes for the various excitation possibilities are smoothly distributed over a wide range in the logarithm of the lifetime; then there will be some pearl-excitations with the appropriate lifetime, although somewhat suppressed by a factor of the order of $1/\text{width}$ where the width here is the width of the logarithmic distribution. We shall take this width to be of order $\text{suppression}_{\text{DAMA}} \sim 23$. But more importantly: If a certain excitation is long-lived, it is also hard to produce. So we shall talk about an effective “stopping” or “filling time” for a pearl passing into the Earth, and imagine that during this “stopping” or “filling time” the excitations of the pearls have to be created. So the probability for excitation or suppression would be expected to be

$$\text{suppression} \approx \frac{\text{“filling time”}}{\text{“lifetime”}}.$$  \hspace{1cm} (55)

If the excitation happens to be of sufficiently long lifetime - say of order 26 hours - then we can expect it to have a sensible chance of de-exciting just in the experimental detectors in Gran Sasso, DAMA or Xenon1T say.

But what shall we take for this “stopping” or “filling time”? A relatively simple idea, which is presumably right, is to say that the stopping takes place high in the atmosphere because a pearl entering the Earth's atmosphere with galactic speed will be slowed down in the high air with a $\sigma_M \sim 2 \text{ cm}^2/g$. Now the density of the atmosphere rises by a factor $e = 2.718...$ per about 7 km. So as the slowing down begins it will, because of this rising density, essentially stop again after 7 km. Thus the time during which the pearl is truly slowing down in speed and forming 3.5 keV excitations is of the order of the time it takes for it to run 7 km. With the pearl velocity of about 300 km/s (essentially the escape velocity for the galaxy) we then have

$$\text{“stopping time”} \approx \frac{7 \text{ km}}{300 \text{ km/s}} \approx 0.023 \text{ s}.$$ \hspace{1cm} (56)

The crucial factor, which we believe to be most important, is that in order to excite an excitation with a lifetime of the order 93000 s it would a priori need 93000 s so that, if we only have 0.023 s, then there will be a suppression:

$$\text{suppression} = \frac{\text{“stopping time”}}{\text{“lifetime”}} \approx \frac{0.023 \text{ s}}{93000 \text{ s}} = 2.5 \times 10^{-7}.$$ \hspace{1cm} (61)
This crudest estimate has to be compared with the experimental suppressions given above

\[
\frac{\text{suppression}_{\text{DAMA}}}{\text{suppression}_{\text{theory}}} = \frac{1.6 \times 10^{-10}}{2.5 \times 10^{-7}} = 1.6 \times 10^{3} \quad (62)
\]

\[
\Rightarrow \frac{\text{suppression}_{\text{Xenon1T}}}{\text{suppression}_{\text{DAMA}}} = \frac{1.6 \times 10^{-10}}{250} = 6.4 \times 10^{-13} \quad (63)
\]

\[
\Rightarrow \frac{\text{suppression}_{\text{Xenon1T}}}{\text{suppression}_{\text{theory}}} = \frac{2.5 \times 10^{-7}}{6.4 \times 10^{-13}} = 3.9 \times 10^{5} \quad (64)
\]

But here can be several corrections to \text{suppression}_{\text{theory}}, at least we should correct by the width in logarithm of the supposed distribution of the lifetimes among the different excitations. Above we suggested a factor 23, which would bring the DAMA rate to only deviate by about a factor 100. Our estimate is of course extremely uncertain.

We can never get the DAMA rate and the electron recoil excess rate from Xenon1T agree with the same estimate in as far as they deviate by a factor 250. Our only chance is in a later paper to justify say the story that, because the scintillator in which the Xenon1T events are observed is \textit{fluid} while the NaI in the Dama experiment is solid, the pearls pass much faster through the Xenon1T apparatus than they pass through the DAMA instrument. Imagine say that the pearls partly hang and get stuck in the DAMA experiment, but that they cannot avoid flowing down all the time while they are in the fluid Xe in the Xenon1T scintillator.

7 \textbf{3.5keV}

Order of magnitudewise we see 3.5 keV in 3 different places.

\begin{itemize}
  \item X-ray galaxy cl.
  \item Xenon1T Elec. R.
  \item DAMA-LIBRA
\end{itemize}

The energy level difference of about 3.5 keV occurring in 3 different places
is important evidence motivating our model of dark matter particles being excitable by 3.5 keV:

- **The line** From places in outer space with a lot of dark matter, galaxy clusters, Andromeda and the Milky Way Center, an unexpected X-ray line with photon energies of 3.5 keV (to be corrected for Hubble expansion...) was seen.

- **Xenon1T** The Dark matter experiment Xenon1T did not find the standard nuclei-recoil dark matter, but found an excess of electron-recoil events with energies concentrated crudely around 3.5 keV.

- **DAMA** The seasonally varying component of their events lie in energy between 2 keV and 6 keV, not far from centering around 3.5 keV.

We take it seriously and not as an accident that both DAMA and Xenon1T see events with energies of the order of the controversial astronomical 3.5 keV X-ray line. We are thereby driven towards the hypothesis that the energies for the events in these underground experiments are determined from a decay of an excited particle, rather than from a collision with a particle in the scintillator material. It would namely be a pure accident, if a collision energy should just coincide with the dark matter excitation energy observed astronomically.

So we ought to have decays rather than collisions! *How then can the dark matter particles get excited?*

You can think of the dark matter pearls in our model hitting electrons and/or nuclei on their way into the shielding:

- **Electrons** Electrons moving with the speed of the dark matter of the order of 300 km/s toward the pearls in the pearl frame will have kinetic energy of the order

\[
E_e \approx \frac{1}{2} \times 0.5 \text{ MeV} \times \left( \frac{300 \text{ km/s}}{3 \times 10^5 \text{km/s}} \right)^2 = 0.25 \text{ eV}. \quad (68)
\]

- **Nuclei** If the nuclei are say Si, the energy in the collision will be 28*1900 times larger \( \sim 5 \times 10^4 \times 0.25 \text{ eV} \approx 10 \text{ keV} \). That would allow a 3.5 keV excitation.

To deliver such \( \approx 10 \text{ keV} \) energy the nucleus should hit something harder than just an electron inside the pearls. It should preferably hit a nucleus, e.g. C, inside the pearl.

8 **Conclusion**

- We have described a seemingly viable model for dark matter consisting of atomic size but macroscopic pearls. These pearls consist of a bubble of a new speculated type of vacuum containing some normal material - presumably carbon - under the high pressure of the skin (surface tension). They each contain about a hundred thousand nucleons in the bubble of radius about \( 5 \times 10^{-12} \text{m} \).
• The electrons in a pearl have partly been pushed out of the genuine bubble of the new vacuum phase, out to a distance of about $5 \times 10^{-11} \text{m}$.

We have compared the model or attempted to fit:

• Astronomical suggestions for the self interaction of dark matter in addition to pure gravity.

• The astronomical $3.5 \text{ keV}$ X-ray emission line found by satellites, supposedly from dark matter.

• The underground dark matter searches.

We list below the quantities we have crudely estimated:

1. The low velocity cross section divided by mass.

2. That the signal from Xenon1T and Dama should agree except that the pearls may run with different velocities through the scintillator materials, because the xenon-instruments use fluid xenon, while the DAMA-LIBRA experiment uses the solid NaI.

3. The absolute rate of the two underground experiments. (But unfortunately unless we explain the ratio of the rates for the two experiments as say due to the different velocities through the scintillator materials, we can of course never predict the absolute rate to be better than deviating by about a factor of 250 with at least one of them.)

4. The rate of emission of the $3.5 \text{ keV}$ X-ray line from the Tycho supernova remnant due to the excitation of our pearls by cosmic rays [8].

5. Relation between the frequency $3.5 \text{ keV}$ and the overall emission rate of this X-ray line observed from galaxy clusters etc.

6. We also previously predicted the ratio of dark matter to atomic matter (=“usual” matter) in the Universe to be of order 5 by consideration of the binding energies per nucleon in helium and heavier nuclei, assuming that the atomic matter at some time about 1 s after the Big Bang was spit out from the pearls under a fusion explosion from He fusing into say C [1].

8.1 Outlook

At the end we want to mention a few ideas which we hope will be developed as a continuation of the present model:

• Speculative Phase from QCD. QCD and even more QCD with Nambu-Jona-Lasinio type spontaneous symmetry breaking is sufficiently complicated, that possibly a new phase appropriate for our pearls could be hiding there. There is already an extremely interesting observation [35].

• Relative Rates of DAMA and Xenon1T. A crucial test for our model is to reproduce the relative event rates in DAMA and in the excess of electron recoils in Xenon1T. This requires a careful study of the viscosity of fluid xenon and the properties of our pearls.
• **Walls in the Cosmos.** With the usual expectations for the density per area or equivalently tension $S$, cosmology would be so severely changed by such domain walls that models with say $S^{1/3} \geq 10$ MeV are phenomenologically not tenable. However, with our fit to a surprisingly small tension with $S^{1/3} \approx 2.2$ MeV it just barely becomes possible to have astronomically extended domain walls, e.g. walls around the large voids between the bands of galaxies; so that these voids could be say formally huge dark matter pearls, though with much smaller density. In fact a series of domain walls with our fitted $S = 2.2^3$ MeV$^3$ with distances between them of the order of 13 milliard light years would have a density not much different for that of the universe we know.

• **New Experiments?** According to our estimates the observed rate of decays of our dark matter pearls should be larger the less shielding they pass through. So an obvious test of our model would be to make a DAMA-like experiment closer to the earth surface where we would expect a larger absolute rate than in DAMA, although there might of course be more background. Actually such an experiment is already being performed by the ANAIS group [13], but they have so far failed to see an annual modulation in their event rate.

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Figure 6. Cross section per unit mass, $\sigma/m_v$, as a function of the average collision velocity, $\langle v \rangle$, of DM particles within each subhalo's core. Symbols show the range of $\sigma/m_v$ needed for the SIDM model to reproduce the central DM densities reported by Kaplinghat et al. (2019). The solid line corresponds to the best-fit relation given by eq. (15) to the MW dSph data.
FIG. 3. Velocity-dependence of self-interactions, given in terms of mean velocity-weighted cross section per unit mass $\langle \sigma v \rangle / m$ versus mean scattering velocity $\langle v \rangle$. Blue (red) points are from our joint fits to galaxy groups (clusters). Closed (open) circles correspond to SIDM fits without (with) AC. Gray points are from joint fits to mock observations from SIDM-plus-baryons simulations with 1 cm$^2$/g [37], assuming different slit orientations. Shaded area is preferred SIDM range for solving core-cusp problem on dwarf scales. Solid line shows $\langle \sigma v \rangle / m$ for 15 GeV mass dark matter with self-interactions mediated by an 11 MeV dark photon, consistent with constraints across all scales.