Thermal properties of the nuclear surface

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The thermal evolution of a few thermodynamic properties of the nuclear surface like its thermodynamic potential energy, entropy and the symmetry free energy are examined for both semi-infinite nuclear matter and finite nuclei. The Thomas-Fermi model is employed. Three Skyrme interactions, namely, SkM*, SLy4 and SK255 are used for the calculations to gauge the dependence of the nuclear surface properties on the energy density functionals. For finite nuclei, the surface observables are computed from a global liquid-drop inspired fit of the energies and free energies of a host of nuclei covering the entire periodic table. The hot nuclear system is modeled in a subtracted Thomas-Fermi framework. Compared to semi-infinite nuclear matter, substantial changes in the surface symmetry energy of finite nuclei are indicated; surface thermodynamic potential energies for the two systems are, however, not too different. Analytic expressions to fit the temperature and asymmetry dependence of the surface thermodynamic potential of semi-infinite nuclear matter and the temperature dependence of the surface free energy of finite nuclei are given.

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I. INTRODUCTION

The liquid-drop model provides a sound framework [1–3] for having a good estimate of the nuclear surface energy from experimental binding energy systematics. This is the case for cold nuclei; this has helped, for instance in understanding barrier heights or saddle point configurations in nuclear fission. This estimate has also its place in the framing of effective nucleon-nucleon interactions [2, 3] by providing an important empirical input. There is a strong motivation too to examine the thermal properties of the nuclear surface. Hot nuclei, produced in multifragmentation in nuclear collisions are surrounded by nucleonic vapor, knowledge of the energy of the interface between the nuclear liquid and vapor is a crucial determinant in their mass distributions [5, 6] or in our understanding of their thermodynamic limit of existence [7, 8]. It has also important astrophysical applications. It puts significant constraints in determining the equilibrium nuclear masses, electron capture rates and level densities that play a seminal role in the dynamical evolution of neutron stars and supernovae [9, 10].

Semi-infinite nuclear matter (SINM) offers a good starting ground for exploring the nuclear surface properties. It has a simplicity coming from absence of many undesirable complications arising from shell, Coulomb and finite-size effects. Considerable effort has been directed in the past in understanding its surface properties at zero temperature, mostly in the semi-classical Thomas-Fermi (TF) framework [11–13]; studies have also been done in the quantal Hartree-Fock approach [14, 15]. In a need for applications in astrophysical scenario, Ravenhall, Pethick and Lattimer in their pioneering work [16, 17] explored the thermodynamic evolution of the surface properties of symmetric as well as asymmetric nuclear matter in the semi-classical approach using a plausible Skyrme interaction. With increasing temperature or asymmetry, the density of vapor consisting of hot or drip nucleons surrounding the liquid phase of the nuclear matter increases. The evolution of the interface energy with this change was quantitatively evaluated by them; they showed how the surface thermodynamic potential energy or the surface entropy eventually dissolves at a critical temperature when the distinction between the nuclear liquid and vapor is lost. A temperature dependence of the surface thermodynamic potential energy of the form $g(T, T_c(X))^{\alpha_1}$ with $\alpha_1 = 1.25$ was suggested by them. The function $g(T, T_c(X))$ has the form

$$g(T, T_c(X)) = \left(\frac{T_2^2(X) - T^2}{T_2^2(X) + T^2}\right),$$

where $T_c(X)$ is the critical temperature for infinite nuclear matter of isospin asymmetry $X$, defined as $X = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ where $\rho_n$ and $\rho_p$ are the neutron and proton densities (for SINM, the definition of asymmetry is somewhat more subtle and given later). Since then, numerous calculations have been done to understand properties of hot nuclear matter [5, 6, 17] with this form of temperature-dependent interface energy.

The scenario for finite nuclei is, however, different. The Coulomb interaction, coupled with the microscopic nuclear size may influence the thermal evolution of their surface differently from that of the semi-infinite matter. It was already noticed, in the course of the evaluation of the thermal dependence of volume and surface symmetry energy coefficients [18] of nuclei with a Skyrme-type KDE0 interaction and the finite-range modified Seyler-Blanchard interaction, that the surface thermodynamic potential of finite nuclear systems evolves in a slightly different way. The form of the evolution function $h(T) = g(T, T_c(X = 0))$ is the same, but the exponent $\alpha_1$ is seen to have a different value, slightly different for the two interactions. The hot nuclei undergo Coulomb instability at a limiting temperature [2], which is much lower compared to the critical temperature $T_c$. Consequently, the whole temperature range upto $T_c$ is not accessible for the finite nuclei. The limiting temperature is generally a decreasing function of the atomic number [1, 8].
On theoretical grounds, it is known that for infinite systems the surface thermodynamic energy behaves with temperature as $(T - T_c)^{\alpha_1}$, with $\alpha_1 = 1.26$. But that is near the critical point. We note that for finite nuclei only a lower temperature range can be mapped. In this case, the value of the calculated exponent is found different from that found earlier in case of SINM. We therefore intend to examine further in this article the evolution of the nuclear interface energy with temperature for semi-infinite nuclear matter, with a focus to the temperature range accessible to microscopic nuclei. Calculations are done in the TF approximation. To assess any possible range accessible to microscopic nuclei. Calculations are the nuclear interface energy with temperature for semi-infinite nuclear matter and of finite nuclei. Results and discussions are presented in Sec. III. Conclusions are drawn in Sec. IV.

\section{II. THE NUCLEAR SURFACE PROPERTIES: THE MODEL}

Determination of the equilibrium density distribution of the hot nuclear systems is the starting point for calculations of the thermodynamic properties of the nuclear surface. In order to describe a hot system as a stable one, it is assumed to be in thermal equilibrium with a surrounding gas representing the evaporated nucleons. Even a very asymmetric cold nuclear system may be stable beyond the nucleon drip point, the required stability being given by the drip nucleons. The description of such a nuclear liquid embedded in a gaseous environment can be given in FTTF framework. Sec. IIA describes the procedure for obtaining the equilibrium density profiles for semi-infinite nuclear matter as well as for finite nuclei. Sec. IIB and IIC give a brief glimpse of how the different surface properties are established from these density profiles.

\subsection{A. Equilibrium density profiles}

The method to obtain the equilibrium density profiles of semi-infinite matter and of finite systems is based on the existence of two solutions to the TF equations, one corresponding to the liquid phase with the surrounding gas (lg) and the other corresponding to the gas (g) alone. The two solutions are obtained from the variational equations

\begin{equation}
\frac{\delta \Omega_g}{\delta \rho_{lg}} = 0, \quad (2)
\end{equation}

and

\begin{equation}
\frac{\delta \Omega_g}{\delta \rho_{lg}} = 0, \quad (3)
\end{equation}

where $\Omega_g$ and $\Omega_g$ are the thermodynamic potentials of the said systems. These two systems have the same chemical potentials $\mu$ because of thermodynamic coexistence between the liquid plus gas system and the embedding gas (i.e., $\mu^g = \mu^g = \mu_q, q$ refers to the isospin index for neutrons or protons). The base density profile of the nuclear liquid (l) of interest is obtained by subtracting the gas density (g) from that of the liquid plus gas system (lg), i.e., $\rho^l_q = \rho^l_q - \rho^g_q$. The thermodynamic potential is given by

\begin{equation}
\Omega = F - \sum_q \mu_q N_q, \quad (4)
\end{equation}

where $F = E - TS$; $F$, $E$ and $S$ are the total free energy, energy and entropy, respectively, $T$ is the temperature and $N_q$ the number of neutrons or protons, $\mu_q$ being the corresponding chemical potentials.

\begin{table}[h]
\begin{center}
\begin{tabular}{|c|c|c|c|}
\hline
Parameters & SkM* & SLy4 & SK255 \\
\hline
$t_0$(MeV fm$^3$) & $-2645.0$ & $-2488.91$ & $-1689.35$ \\
$t_1$(MeV fm$^3$) & $410.0$ & $486.82$ & $389.30$ \\
$t_2$(MeV fm$^3$) & $-135.0$ & $-546.39$ & $-126.07$ \\
t$3$(MeV fm$^{3(\gamma+1)}$) & $15595.0$ & $13777.0$ & $10989.59$ \\
x0 & $0.09$ & $0.834$ & $-0.1461$ \\
x1 & $0.00$ & $-0.344$ & $0.116$ \\
x2 & $0.00$ & $-1.0$ & $0.0012$ \\
x3 & $0.00$ & $1.354$ & $-0.7449$ \\
\hline
\end{tabular}
\end{center}
\caption{The values of the Skyrme parameters for SkM*, SLy4 and SK255 interactions.}
\end{table}
We have calculated the total energy with Skyrme interaction energy density functionals. The energy density is

$$E(r) = \frac{\hbar^2}{2m_n} \tau_n(r) + \frac{\hbar^2}{2m_p} \tau_p(r) + E_{sky}[\rho(r)] + E_c(r), \quad (5)$$

where \( \tau \)'s are the kinetic energy density, \( E_{sky} \) is the interaction energy density and \( E_c \) is the Coulomb energy density. The Skyrme interaction energy density is given by

$$E_{sky}[\rho(r)] = \frac{1}{2} t_0 [(1 + \frac{1}{2} x_0) \rho^2 - (x_0 + \frac{1}{2}) (\rho_n^2 + \rho_p^2)]$$

$$+ \frac{1}{12} [t_4 \rho^4 \gamma (1 + \frac{3}{2}) \rho^2 - (x_3 + \frac{1}{2}) (\rho_n^2 + \rho_p^2)]$$

$$+ \frac{1}{4} [t_1 (1 + \frac{1}{2} x_1) + t_2 (1 + \frac{1}{2} x_2)] \rho \tau$$

$$+ \frac{1}{4} [t_2 (x_2 + \frac{1}{2}) - t_1 (x_1 + \frac{1}{2})] (\rho_n \rho_\tau + \tau \rho_p)$$

$$+ \frac{1}{16} [3 t_1 (1 + \frac{1}{2} x_1) - t_2 (1 + \frac{1}{2} x_2)] (\nabla \rho)^2$$

$$- \frac{1}{16} [3 t_1 (x_1 + \frac{1}{2}) + t_2 (x_2 + \frac{1}{2})] ((\nabla \rho_n)^2 + (\nabla \rho_p)^2),$$

with \( \tau = \tau_n + \tau_p \) and \( \rho = \rho_n + \rho_p \). Here the \( t_i \)'s, \( x_i \)'s and \( \gamma \) are the Skyrme parameters listed in Table I, for the three chosen interactions. The Coulomb contribution is present in Eq. (5) only for finite nuclei, they can not be treated for both homogeneous nuclear matter and semi-infinite nuclear matter.

In Eq. (6), we have not included the spin-gradient terms \[27\] as they were ignored while fitting the parameters for the Skyrme forces we have chosen.

At finite temperature, the effective kinetic energy density is \[27\]

$$\tau_q^* = \frac{2m_q^*}{\hbar^2} A_{T,q} T J_{3/2}(\eta_q), \quad (7)$$

with

$$A_{T,q} = \frac{1}{2 \pi^2} \left( \frac{2 m_q^* T}{\hbar^2} \right)^{3/2}. \quad (8)$$

In Eqs. (7) and (8), \( m_q^* \) is the nucleon effective mass,

$$m_q^* = m \left[ 1 + \frac{m}{2 \hbar^2} \left( t_1 (1 + \frac{1}{2} x_1) + t_2 (1 + \frac{1}{2} x_2) \right) \rho \right.$$

$$+ [t_2 (x_2 + \frac{1}{2}) - t_1 (x_1 + \frac{1}{2}) \rho_q] \left. \right]^{-1}, \quad (9)$$

and \( \eta_q \) the fugacity,

$$\eta_q = (\mu_q - U_q)/T. \quad (10)$$

In Eq.(10), \( U_q \) is the nucleon single-particle potential,

$$U_q = \frac{\delta (E_{sky}[\rho(r)] + E_c)}{\delta \rho_q}. \quad (11)$$

the symbol \( \delta \) referring to the functional derivative here. For neutrons, \( E_c = 0 \). The density \( \rho_q \) is obtained as,

$$\rho_q = A_{T,q} T J_{1/2}(\eta_q). \quad (12)$$

The \( J_k \)'s are the Fermi integrals. The \( T=0 \) case is a special case which can be addressed easily from Eqs.(7)-(12). The Coulomb energy density \( E_c(r) \) is the sum of the direct and exchange contributions:

$$E_c(r) = E_c^D(r) + E_c^Ex(r). \quad (13)$$

The direct term is

$$E_c^D(r) = \pi e^2 \rho_p(r) \int_0^\infty \rho_p(r') [(r + r') - |r - r'|] r' dr'. \quad (14)$$

and the exchange term is calculated from the Slater approximation as

$$E_c^Ex(r) = -\frac{3 e^2}{4 \pi} (3 \pi^2)^{1/3} \rho_p^{4/3}(r). \quad (15)$$

The entropy density \( S \) is the sum of contributions from neutrons and protons:

$$S = \sum_q S_q = \sum_q \left\{ \frac{5}{3} \frac{\hbar^2}{2 m_q^* T} \tau_q^* - \eta_q \rho_q \right\}. \quad (16)$$

From Eqs. (2) and (3), the coupled equations follow:

$$T \eta_q^q + U_q^q = \mu_q. \quad (17)$$

$$T \eta_g^q + U_g^q = \mu_q. \quad (18)$$

Solutions of the above two equations yield the required density profiles \( \rho_q^q(r) \) and \( \rho_g^q(r) \). The calculations proceed as follows: from guess densities \( \rho_q^0(r) \) and \( \rho_g^0(r) \), one calculates \( U_q \) and \( \eta_q \) from Eqs. (9)-(12), then obtains \( \mu_q \) as

$$\mu_q = \frac{1}{A_q} \left\{ \left[ T \eta_q^q + U_q^q \right] \rho_q^q d^3r - \left[ T \eta_g^q + U_g^q \right] \rho_g^q d^3r \right\}. \quad (19)$$

where \( A_q \) is the neutron or proton number of the subtracted liquid part,

$$A_q = \int \left[ \rho_q^q(r) - \rho_g^q(r) \right] d^3r. \quad (20)$$

With this \( \mu_q \), one can calculate \( \eta_q \) with the previous \( U_q \) obtaining the next stage densities \( \rho_q \) through Eq. (12) and proceed iteratively until convergence is achieved.

For a finite system, \( A_q \) refers to the actual number of neutrons (\( N \)) or protons (\( Z \)) in the nucleus one is dealing with. For semi-infinite system, one chooses for calculation a sufficiently large box size and suitable number of nucleons so that both \( \rho_q \) and \( \rho_g \) attain constant values at large and short distances, respectively (the two extreme ends of the box as shown in Fig. 1). The asymptotic constancy of \( \rho_q \) and \( \rho_g \) at \( z \to -\infty \) and \( z \to +\infty \), respectively is assured this way.
can likewise be defined from 

\[ \sigma_c(0,0) \text{(MeV fm}^{-2}) \]

where \( \sigma_c \) is the surface density of constant throughout. It may be noted that the superscript 0 referring to the corresponding bulk cylinder in the liquid is finite.

\[ A \text{ is the total free energy of the semi-infinite liquid.} \]

\[ \text{It may be pointed out that the actual calculations are performed over a finite range of } z, 0 \leq z \leq z_{\text{max}}, \text{ (see Fig. 1)} \]

so that the total number of nucleons \( A \) in the said cylinder in the liquid is finite.

From Eq. (21), one gets for \( \sigma_c \),

\[ \sigma_c = \int_0^{z_{\text{max}}} dz \left( \Omega^{l}_{g}(z) - \Omega^{g}(z) \right) - A f_{B}/A. \]

Using the expression for \( f_{B} \), one finds,

\[ \sigma_c = \int_0^{z_{\text{max}}} dz \left\{ \left( \frac{\Omega^{l}_{g}(z) - \Omega^{g}(z)}{\rho^{l}_{g}(z) - \rho^{g}(z)} \right) - \left( \frac{\rho^{l}_{g}(z) - \rho^{g}(z)}{\rho^{l}_{g}(z) - \rho^{g}(z)} \right) \right\}. \]

where we have used Eq. (20) for the evaluation of \( A \).

In Eq. (23), \( \Omega^{l}_{g}, \Omega^{g} \) etc. are the free energy densities, the superscript 0 referring to the corresponding bulk quantities. It may be noted that \( \Omega^{l}_{g}(z) = \Omega^{0}_{g} \) as the gas density is constant throughout.

The surface thermodynamic potential per unit area \( \sigma_{\mu} \) can likewise be defined from

\[ \Omega_{A} = \Omega^{l}_{B} + \sigma_{\mu} A, \]

B. Calculation of surface energy coefficients: semi-infinite matter

In the context of the subtraction method for isospin asymmetric semi-infinite nuclear system, there could be two definitions \[11, 13\] of the nuclear interface energy. The definitions differ depending on whether one calculates the change in the total free energy or that in the total thermodynamic potential of semi-infinite matter from the corresponding quantities of the bulk matter. The subtraction of the constant gas density from that of the liquid plus gas does not change the surface profile of the semi-infinite matter (see Fig. 1). Delineating the liquid density \( (l) \) one can define the surface free energy per unit area \( \sigma_{e} \) from

\[ F_{A} = A f_{B} + \sigma_{e} A, \]  

where \( F_{A} \) is the total free energy of the semi-infinite liquid \( (l) \) containing \( A \) nucleons in a cylinder of area of cross-section \( A \) normal to the liquid surface and \( f_{B} \) is the free energy per particle of the homogeneous bulk liquid. It may be pointed out that the actual calculations are performed over a finite range of \( z, 0 \leq z \leq z_{\text{max}}, \) (see Fig. 1) so that the total number of nucleons \( A \) in the said cylinder in the liquid is finite.

In Eq. (24), \( \Omega^{l}_{g} = -(P^{l}_{g} - P^{0}), A z_{\text{max}} \) where \( P^{0} \) refers to the bulk pressure. For thermodynamic equilibrium, the pressure \( P^{0} = P^{0}_{p} \), hence the bulk thermodynamic potential of the liquid \( (l) \) \( \Omega^{l}_{B} = 0 \). Further, \( \mu^{n}_{l} = \mu^{p}_{l} = \mu_{n} \) and \( \mu^{n}_{g} = \mu^{p}_{g} = \mu_{p} \).

For isospin symmetric systems, these two definitions given by Eqs. (23) and (25) yield different surface interface energies. The difference is given by \[13\]

\[ \sigma_{c} - \sigma_{\mu} = (\mu_{n} - \mu_{p})(R_{n} - R_{p}) \frac{\rho^{0}_{n} \rho^{0}_{p}}{\rho^{0}}, \]

where \( R_{n} \) and \( R_{p} \) are the equivalent sharp surface locations (in the spirit of the liquid-drop model) of the neutron and proton fluid: \( R_{l_{n}}^{0} = \rho_{l_{n}}^{0} \) are the bulk neutron density in the liquid and \( R_{l_{p}}^{0} = \rho_{l_{p}}^{0} \). For symmetric matter, the two definitions yield identical results.

C. Calculation of surface energy coefficients: finite nuclei

Unlike SINM, isospin asymmetry does not fully define the surface characteristics of atomic nuclei. With the same isospin asymmetry, there may be nuclei with

![Graph showing density profiles](Image)
different neutron and proton numbers, the surface properties of nuclei may thus be somewhat different. In that sense, one can talk meaningfully only about average surface properties of nuclei. To calculate the surface interface energy of hot finite systems, we limit ourselves in the liquid-drop framework. In that model, the total free energy of a nucleus is given by

$$F(A, Z, T) = f_v(T)A + f_s(T)A^{2/3} + E_c(A, Z, T) + (f^{sym}_v(T) - f^{sym}_s(T)A^{-1/3})AX^2 + \text{(27)}$$

where $f_v$ and $f_s$ are the volume and the surface free energy coefficients for symmetric matter, $E_c$ is the total Coulomb energy of the nucleus and $f^{sym}_v$ and $f^{sym}_s$ are the volume and surface symmetry energy coefficients. Here $X = (N - Z)/A$ is the isospin asymmetry of the nucleus. Referring to our discussion in the previous subsection, one wonders whether $(f_s - f^{sym}_s X^2)$ should be connected with $\sigma_v$ or $\sigma_{\mu}$. It has been argued in Ref. [11] that in the context of the liquid-drop model, it should be connected with $\sigma_{\mu}$, the surface thermodynamic potential. We follow the prescription. To calculate the thermal dependence of this surface interface energy of finite nuclear systems, we evaluate, in the subtraction scheme, the free energies of a host of spherical or near-spherical nuclei, sixty nine in number (the list of nuclei is taken from Ref. [28]), covering almost the entire periodic table ($34 \leq A \leq 218; 14 \leq Z \leq 92$) at a finite temperature and make a least-squares fit of the calculated free energies with $f_v$, $f_s$ etc. as free parameters. In actual calculations, we fitted $F(A, Z, T) - E_c(A, Z, T)$, i.e., the nuclear part of the free energy. The four parameters, namely, $f_v$, $f_s$, $f^{sym}_v$ and $f^{sym}_s$, so fitted, reflect their desired temperature dependence.

**TABLE III**: Values of exponents $\alpha_1$ and $\alpha_2$ for symmetric SINM and for finite nuclei in the temperature range $T = 0$ to $7.5$ ($\sim T_{lim}$) MeV.

| Interaction | Symmetric SINM | Finite Nuclei |
|-------------|----------------|---------------|
|             | $\alpha_1$    | $\alpha_2$    | $\alpha_1$    | $\alpha_2$    | $\alpha_1$    | $\alpha_2$    |
| SkM*        | 0.966         | -0.076        | 0.814         | 1.486         | -0.222        | 1.042         |
| SLy4        | 0.919         | -0.069        | 0.781         | 1.404         | -0.226        | 0.952         |
| Sk255       | 1.019         | -0.077        | 0.855         | 1.427         | -0.208        | 1.011         |

**FIG. 2**: (Color online) Thermal evolution of the surface thermodynamic potential $\sigma_{\mu}$ of SINM with the SLy4, SkM* and SK255 interactions. Panels (a), (b) and (c) display results for $X = 0.0$, 0.3 and 0.6, respectively.

**FIG. 3**: (Color online) The surface thermodynamic potential $\sigma_{\mu}$ of SINM with the three interactions plotted as a function of isospin asymmetry. The three panels show results at temperatures $T = 0.0$, 5.0 and 10.0 MeV.

**FIG. 4**: (Color online) Thermal evolution of the surface symmetry free energy coefficient $f^{sym}_s$ of SINM for the interactions SLy4, SkM* and SK255.
III. RESULTS AND DISCUSSIONS

The calculations for semi-infinite nuclear matter (SINM) and for finite nuclei are done with three Skyrme interactions, SkM*, SLy4 and SK255. The actual calculations for SINM are done in a reasonably large box size. In Fig. 1, we display a typical density profile for symmetric SINM at a temperature \( T = 10 \) MeV with SLy4 interaction. As seen in the figure, with a box size of 35 fm, the gas density on the right side of the box and the liquid plus gas density on the left side attain constant asymptotic values. The liquid density \( \rho_l \) shown by the purple shaded region is obtained after subtracting the constant gas density \( \rho_g \) (shown by the green shaded region) from the liquid plus gas density \( \rho_{lg} \). The quantities \( \rho_{lg}^0 \) and \( \rho_g^0 \) represent the bulk values for the liquid plus gas and gas densities, respectively. As seen in the figure, the choice of 35 fm for the maximum value of \( z (z_{\max}) \) suffices, provided suitable value for the number of nucleons per unit area is chosen (in the present calculation, it is 2.5 per \( \text{fm}^2 \)). For asymmetric SINM, the neutron and proton fractions in the liquid part and those in the gas part may be different. For the system in phase equilibrium, definition of global asymmetry of the whole system is then not practical, it depends on the size of the box. For numerical convenience, as in Refs. [13, 16], we therefore define asymmetry as that of the denser side of SINM, i.e., \( X = (\rho_{lg,n} - \rho_{lg,p})/(\rho_{lg,n} + \rho_{lg,p}) \).

The thermal evolution of the surface thermodynamic potential per unit area \( \sigma_\mu \) of semi-infinite nuclear matter is shown in Fig. 2 for the three chosen interactions. The panels (a), (b) and (c) display calculated results for different asymmetries. The general feature seen is that \( \sigma_\mu \) monotonically decreases with temperature reaching zero at the critical temperature. The value of the critical temperature depends on the choice of interactions and asymmetries. That \( \sigma_\mu \) depends on the choice of interactions is evident. This dependence is quite weak for symmetric matter, becoming pronounced with increasing asymmetry.

As shown in Ref. [16], near critical temperature, the surface thermodynamic potential \( \sigma_\mu(T,X) \) goes as \( (T_c(X) - T)^{\alpha_1} \) with \( \alpha_1 \approx 1.26 \). Keeping this in mind, the commonly used temperature dependence of \( \sigma_\mu(T,X) \) for SINM over the whole temperature range for all asymmetries has been taken as [16]

\[
\sigma_\mu(T,X) = \sigma_\mu(0,X)[g(T,T_c(X))]^{\alpha_1},
\]

(28)

where \( g(T,T_c(X)) \) is given by Eq. (1) and \( \alpha_1 = 5/4 \). The asymmetry dependence of \( \sigma_\mu(0,X) \) is taken as

\[
\sigma_\mu(0,X) = \sigma_\mu(0,0)(16 + C_0)/[y^{-3} + C_0 + (1 - y)^{-3}],
\]

(29)

where \( y = (1 - X)/2 \) and \( C_0 \) a parameter. The plausibility of the dependence of \( \sigma_\mu(0,X) \) on \( X^{\beta} \) has its origin on the phase equilibrium conditions [16]. The critical temperature is asymmetry dependent.

From our calculation, we find that an algebraic expression of \( \sigma_\mu(T,X) \) of the form

\[
\sigma_\mu(T,X) = \sigma_\mu(0,0)[g(T,T_c(X))]^{\alpha_1} \times \frac{16 + C_0[g(T,T_c(X))]^{\beta}}{y^{-3} + C_0[g(T,T_c(X))]^{\beta} + (1 - y)^{-3}}
\]

(30)

gives an extremely good fit to the calculated values for \( X \leq 0.7 \) and \( T \leq T_c(X = 0.7) \approx 7.5 \) MeV. Upto a value of \( X = 0.7 \), the asymmetry dependence of \( T_c \) can be well described by a polynomial of the form \( T_c(X) = T_c(X = 0)[1 + aX^2 + bX^4] \). The values of \( a \)

FIG. 5: (Color online) The surface entropy per unit area of SINM shown as a function of temperature for the three interactions at asymmetries \( X = 0.0, 0.3 \) and 0.6.

FIG. 6: (Color online) The surface entropy per unit area of SINM shown as a function of asymmetry for the three interactions at \( T = 4.0 \) and 8.0 MeV.
are \(-0.9238, -0.8126\) and \(-1.2654\), the values of \(b\) are \(-0.3529, -0.4193\) and \(0.1158\) for SkM*, SLy4 and SK255 interactions, respectively. The values of \(\sigma_0(0,0), \alpha_1, C_0\) and \(\beta\) are given in Table II for the three chosen interactions. The value of \(\alpha_1\) is seen to be \(\approx 0.9\) in this temperature range. It has been checked that this value gradually rises to the canonical value of \(\approx 1.26\) in a narrow temperature window near the critical temperature. The critical temperatures \(T_c(\Sigma = 0)\) are 14.61, 14.53 and 15.98 MeV for SkM*, SLy4 and SK255 interactions, respectively. In Fig. 3, the asymmetry dependence of \(\sigma_n\) is displayed for the same interactions at three temperatures, namely, at \(T = 0, 5\) and 10 MeV in the the panels (a), (b) and (c), respectively. The surface potential \(\sigma_n(T, X)\) decreases both with temperature and asymmetry reaching zero at \(T_c(X)\). One could see that the temperature and asymmetry dependent \(\sigma_n(T, X)\) has a non-negligible dependence on the interactions one chooses to describe the semi-infinite nuclear matter.

The external gas surrounding nuclear drops in clustered nuclear matter in astrophysical environment has its origin on both temperature and asymmetry. Even at \(T = 0\), nuclei may exist embedded in a nucleon gas in astrophysical environment [29, 30] leading to the modification of the nuclear properties. In Ref. [31], Papakonstantinou et al. find an increase in surface energy with increase in asymmetry for cold nuclei in contradiction to what we find for SINM from Eq. (30) with \(T = 0\). It may possibly be attributed to different definitions of the surface energy (\(\sigma_e\) is known to increase initially with asymmetry [13]). Part of the reason may also lie in the definition of the asymmetry parameter; \(X\) in our case is the asymmetry of the denser part of the liquid-gas system, \(\Sigma\) for the SK255 force compared to the SKM* and SLy4 forces. The large value of \(f_{\text{sym}}^*(T = 0)\) for SINM with SK255 interaction can be qualitatively understood as being directly related to the corresponding large value of \(L (\approx 95\text{ MeV}\) for SK255 as compared to \(\approx 45\text{ MeV}\) for SLy4 or SkM*).

For a heavy nucleus of mass number \(A\), \(f_{\text{sym}}^*(T = 0)\) is \(\approx A^{1/3}[L_{\text{FA}} - K_{\text{sym}}/2]\) [29], where \(K_{\text{sym}}\) is the symmetry incompressibility and \(\epsilon_A = (\rho_0^2 - \rho_A)/(3\rho_0^2), \rho_A\) being the equivalent density for the nucleus. The quantity \(\rho_0^2 - \rho_A\) can be parametrized as \(\rho_0^2 - \rho_A = 4\rho_0^2(1 + cA^{1/3})\) [31] (c is around 0.28 for terrestrial nuclei) so that for a hypothetical charge less large nuclear drop of mass \(A\), \(\epsilon_A\) is \(\approx A^{-1/3}/3c\). For SINM, \(f_{\text{sym}}^*(T = 0)\) is then \(\approx L/(3c)\).

Entropy per unit area of the surface of semi-infinite matter is discussed in association with the following two figures. The surface entropy per unit area is defined as [16]

$$s_{\text{surf}} = -\frac{\partial \sigma_n(T, X)}{\partial T} \bigg|_{X=0}$$

$$= -\frac{\partial \sigma_n(T, X)}{\partial T} \bigg|_{X} + \frac{\partial \sigma_n}{\partial X} \bigg|_{T} \frac{\partial \mu}{\partial X} \bigg|_{T}.$$  \(33\)

In Fig. 5, the temperature dependence of \(s_{\text{surf}}\) is shown for three asymmetries (\(X = 0.0, 0.3\) and 0.6). The thermal evolution of \(s_{\text{surf}}\) for all the three interactions shows nearly the same behavior for all asymmetries; temperature raises the surface entropy as is expected, but after a maximum is reached, the entropy falls sharply as the interface and the energy associated with it dissolve near the critical point. In Fig. 6, the asymmetry dependence of the surface entropy is displayed at two temperatures, \(T = 4.0\) and 8.0 MeV. No marked sensitivity on either the interaction or on \(X\) except at large values of asymmetry is noticed.

As discussed in Sec. IIC, we have calculated the total free energies of the nuclei as a function of temperature in the subtracted FTTF procedure and then found the values of the parameters \(f_e(T), f_s(T), f_{\text{sym}}^*(T)\) and \(f_{\text{sym}}^*(T)\) from a least-squares fit. To be in concordance with the liquid-drop model of finite nuclei, a connection of \(f_s(T)\) with \(\sigma_n(T, X = 0)\) for symmetric semi-infinite nuclear matter is looked for, which can be established as

$$f_s(T) = 4\pi r_0^2(T)\sigma_n(T).$$  \(34\)
For finite nuclei, \( r_0(T) \) can be defined from \( r_0(T) = R_0(T)/A^{1/3} \), where \( R_0(T) \) is its sharp surface radius. For semi-infinite matter, \( r_0(T) \) is already defined in connection with Eq. (31). One can see that \( f_s(T=0) \sim 18 \text{ MeV} \), with \( r_0(T=0) \sim 1.2 \text{ fm} \) 

The radius parameter \( r_0(T) \) for both finite nuclei and symmetric SINM is seen to be fitted extremely well in the temperature range \( T \sim 0 \) to \( T_{lim} \sim 7.5 \text{ MeV} \) as defined for finite nuclei with the function \( h(T) = g(T, T_s(X = 0)) \) as

\[
r_0(T) = r_0(0)[h(T)]^\alpha,
\]

so that \( f_s(T) \) can be defined as

\[
f_s(T) = 4\pi r_0^2(0)\sigma_\rho(0)[h(T)]^\alpha,
\]

where \( \alpha = \alpha_1 + 2\alpha_2 \). The values for \( \alpha_1, \alpha_2 \) and \( \alpha \) are listed in Table III for symmetric SINM as well as for finite nuclei for the three energy density functionals. From the table, one sees that the exponent \( \alpha_1 \) governing the temperature dependence of the interface energy per unit area \( \sigma_\rho(T) \) is significantly large for finite systems as compared to that of SINM. One further sees that the radius parameter \( r_0(T) \) increases faster for finite systems. Overall, one finds that \( f_s(T) \) for symmetric SINM and for finite nuclei are not qualitatively very different though \( f_s(T) \) seems to fall somewhat faster for finite nuclei. In Fig. 7, this general thermal behavior of \( f_s(T) \) is displayed. The values of \( \alpha_1, \alpha_2 \) and \( \alpha \) are seen to be nearly independent of energy density functionals. One may note that there are some differences in the values of \( \alpha_1 \) for SINM in Tables II and III. This is so as \( \alpha_1 \) for SINM in Table III pertains to a small subset of data used for fitting (symmetric SINM). The thermal evolution of the surface symmetry free energy coefficient \( f_s^{sym} \) for finite nuclei is displayed for the three interactions in the panels (a), (b) and (c) in Fig. 8 and compared with that of semi-infinite nuclear matter. Comparatively, the sensitivity of \( f_s^{sym} \) to temperature for finite nuclei is seen to be weaker. Strikingly, \( f_s^{sym} \) for semi-infinite matter is found to be much larger for all the three interactions. This appears to be a finite-size effect. The Coulomb effect on \( f_s^{sym} \) is found to be nominal. Switching off the Coulomb interaction, we have tested that as the nuclear size increases, \( f_s^{sym} \) approaches the asymptotic value for semi-infinite matter. We take a set of nuclei at a fixed temperature (say \( A \sim 500, T = 0 \)) with different isospin asymmetries, calculate their free energies with Coulomb switched off, and from a least-squares fit, find the parameters \( f_s, f_{s^*}, f_s^{sym} \) and \( f_{s^*}^{sym} \) (cf. eq. 27) and then repeat the calculations at the same temperature for several other different sets of larger masses. It is found that there is only a marginal change in \( f_s, f_{s^*}, f_{s^{sym}} \) but \( f_{s^{sym}} \) increases with the mass of the nuclear set, tending asymptotically towards the SINM value. The same conclusion emerges again even if \( f_s, f_{s^*}, f_{s^{sym}} \) are kept constant to the values specific for the interaction. It may be noted that from double difference of experimental symmetry energies of finite nuclei, the value of \( f_{s^{sym}}(T = 0) \) is empirically found to be \( 58.91 \pm 1.08 \text{ MeV} \) [33]. As is seen from Fig. 8, the value obtained from SK255 interaction is in consonance with this empirical value. Those from the other two interactions are somewhat lower. The latter values, however, agree closely with the value of \( f_{s^{sym}} \) obtained from fitting of nuclear masses [34].
IV. CONCLUSIONS

The thermal evolution of the surface properties of two-component semi-infinite nuclear matter and of finite nuclei has been investigated in the present article. Calculations have been performed in the finite-temperature Thomas-Fermi framework; stability to the seemingly unstable hot nuclear systems is achieved in the subtraction procedure. Three Skyrme-class interactions, namely SkM*, SLy4 and SK255, designed to reproduce the bulk properties of cold nuclei have been employed. The dependence of the hot nuclear surface properties on the energy density functionals are thereby explored.

The combined effect of temperature and asymmetry on the nuclear surface has important bearing in astrophysics and heavy ion collisions; in that context, for ready use, analytic expressions that fit the calculated data for SINM well over a wide range of temperatures and asymmetries are given. For hot atomic nuclei, liquid-drop model acted as a framework for obtaining the desired thermal evolution of their surface. For applications in asymmetric systems, the need to properly match the definition of the surface energy to the volume energy has been stressed earlier [11]. Due care has been taken in this work for its implementation in both finite nuclei and in semi-infinite nuclear matter; to be clear, propriety demands the evaluation of the surface thermodynamic potential which we have done.

The dependence of the surface thermodynamic potential on temperature is seen to be of the form \[g(T, T_c(X))\alpha_1;\] for SINM, \(\alpha_1\) has somewhat different values in different temperature ranges, rising slowly from \(\sim 1.0\) at low temperature to \(\sim 1.26\) near the critical temperature. For different interactions, \(\alpha_1\) is seen to have nearly the same value. For finite nuclei, the functional form of \(g(T, T_c(X = 0))\) remains the same, but \(\alpha_1\) is much larger, in the neighborhood of \(\sim 1.45\) for the three interactions that we have chosen. These are finite size effects, they leave their imprints on the surface symmetry free energy coefficient too; for finite systems, the surface symmetry energy is comparatively much smaller, rising slowly to the asymptotic value for semi-infinite nuclear matter with increasing size.

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