Understanding the $e^+e^- \rightarrow D^{(*)}+D^{(*)}$ processes observed by Belle

Kui-Yong Liu
Department of Physics, Peking University, Beijing 100871, People’s Republic of China and Department of Physics, Liaoning University, Shenyang 110036, People’s Republic of China

Zhi-Guo He and Yu-Jie Zhang
Department of Physics, Peking University, Beijing 100871, People’s Republic of China

Kuang-Ta Chao
China Center of Advanced Science and Technology (World Laboratory), Beijing 100080, People’s Republic of China and Department of Physics, Peking University, Beijing 100871, People’s Republic of China

Abstract

We calculate the production cross sections for $D^{*+}D^{*-}$, $D^{+}D^{*-}$ and $D^{+}D^{-}$ in $e^+e^-$ annihilation through one virtual photon in the framework of perturbative QCD with constituent quarks. The calculated cross sections for $D^{*+}D^{*-}$ and $D^{+}D^{*-}$ production are roughly in agreement with the recent Belle data. The helicity decomposition for $D^*$ meson production is also calculated. The fraction of the $D^+_L D^+_T$ final state in $e^+e^- \rightarrow D^{*+}D^{*-}$ process is found to be 65%. The fraction of $D^+_L$ production is 100% and $D^+_T$ is forbidden in $e^+e^-$ annihilation through one virtual photon. We further consider $e^+e^-$ annihilation through two virtual photons, and then find the fraction of $DD_T^*$ in $e^+e^- \rightarrow DD^*$ process to be about 91%.

PACS number(s): 12.40.Nn, 13.85.Ni, 14.40.Gx

Recently, heavy meson production in $e^+e^-$ annihilation has become a very interesting subject both experimentally and theoretically. For instance, for charmonium production in $e^+e^-$ annihilation at the B-factory energy $\sqrt{s} = 10.6\,\text{GeV}$, there are large differences between the experimental data [1, 2, 3] and the calculated cross sections for both exclusive processes [4] and inclusive processes [5]. Even by including the effects of $e^+e^-$ annihilation into two photons in the exclusive double-charmonium production [6] and inclusive charmonium production [7], the large discrepancies still exist [8]. Moreover, most recently the Belle Collaboration has measured [9] the charmed meson pair $D^{(*)}+D^{(*)}$ production in $e^+e^-$ annihilation and also found a large differences between observed data and theoretical predictions [10]. The measured cross sections are $\sigma(e^+e^- \rightarrow D^{*+}D^{*-}) = 0.65 \pm 0.04 \pm 0.07 \,\text{pb}$ and $\sigma(e^+e^- \rightarrow D^{+}D^{*-}) = 0.71 \pm 0.05 \pm 0.09 \,\text{pb}$, which are, however, lower than those predicted in Ref. [10] by an order of magnitude (see [9] for the comparison). The Belle measurements of the exclusive D meson pair production in $e^+e^-$ annihilation could be another challenge to the theoretical studies. In order to understand these production processes, in this letter we will present a calculation in the framework of perturbative QCD with constituent quarks. Namely, we will treat the charmed mesons $D^{(*)}$ as bound states of a constituent charmed quark and a constituent light antiquark, and the virtual photon will couple to the charm quark, which will subsequently emit a light quark pair with constituent quark masses through a virtual gluon (see the upper diagram in Fig. I). The virtuality of the virtual gluon could be large enough for the application of perturbative QCD. In addition, the case of the
virtual photon coupled to the light quark will also be considered (see the lower diagram in Fig. 1). In the following we will report our calculated results for both the total cross sections and the helicity decomposed cross sections for these charmed meson pair production in $e^+e^-$ annihilation at $\sqrt{s} = 10.6\text{GeV}$.

Following the method in Ref. [11], the amplitude for producing the heavy quark ($Q$)-light antiquark ($\bar{q}$) bound-state ($Q\bar{q}$) is given by

$$A(P) = \sum_{L,Z,S_Z} \int \frac{d^3k}{(2\pi)^3} \Psi_{L,L_Z}(k) \langle LL_Z; SS_Z | JJ_{Z} \rangle M(P, k),$$

where

$$M(P, k) = \mathcal{O}_R \Gamma_{SS_Z}(P, k),$$

and $\mathcal{O}_R$ represents the short-distance interaction producing the $Q$ and $\bar{q}$ in a specific bound-state. $k$ is the relative momentum between $Q$ and $\bar{q}$. $\Psi_{L,L_Z}(k)$ is the Bethe-Salpeter bound state wave-function. $\Gamma_{SS_Z}(P, k)$, up to second order in $k$, is given by

$$\Gamma_{SS_Z}(P, k) = \sqrt{\frac{m_Q + m_q}{2m_Q m_q}} \sum_{s, s'} \langle \frac{1}{2} \frac{1}{2} | SS_Z \rangle v(r P - k, s) \bar{u}(r P + k, s),$$

$$\approx \sqrt{m_Q + m_q} \left( \frac{r}{2m_q} (P - F - m_q) \right) \left( - \gamma^5 \right) \left( \frac{r}{2m_Q} (P - F + m_Q) \right)$$

where $\gamma^5$ is for spin-singlet state and $- \gamma(P, S_Z)$ is for spin-triplet state.

For the S-wave state, we have $M(P, k) = M(P, 0)$. After integrating over $k$, the amplitude $A(P)$ is related to the origin of the radial wave-function $R_S(0)$

$$\int \frac{d^3k}{(2\pi)^3} \Psi_{00} = \frac{R_S(0)}{\sqrt{4\pi}}.$$ (4)

Using Eq. (4), the calculation of the cross section for the process showed in Fig. 1 is straightforward. We get the cross section as follows

$$d\sigma = \frac{\pi \alpha^2 \alpha_s^2 R_s(0)^4}{8s} \sqrt{1 - \frac{4m^2}{s}} | \tilde{M} |^2 dx,$$ (5)

where $x = \cos \theta$, $\theta$ is the angle between $p_1$ and $p_3$. The meson mass $m$ equals $m_c + m_d$ in the leading order non-relativistic approximation. For the $e^+e^- \rightarrow D^{*+}D^{*-}$ process, $| \tilde{M} |^2$ reads

$$| \tilde{M} |^2 = \frac{128(m_c + m_d)^2(4m_c^2 + 8m_cm_d + 4m_d^2 - s)}{81m_c^2 m_d^2 s^5} \times [-48m_c^{12} - 192m_c^{11}m_d - 288m_c^{10}m_d^2 - 192m_c^9m_d^3 - 96m_c^8m_d^4 - 192m_c^7m_d^5 - 288m_c^6m_d^6 - 192m_c^5m_d^7 - 60m_c^4m_d^8 - 48m_c^3m_d^9 - 72m_c^2m_d^{10} - 48m_cm_d^{11} - 12m_d^{12} - 8m_c^{10}s - 48m_c^9m_d^2s - 80m_c^8m_d^3s - 64m_c^7m_d^4s - 56m_c^6m_d^5s - 64m_c^5m_d^6s - 50m_c^4m_d^7s - 28m_c^3m_d^8s - 20m_c^2m_d^9s - 12m_cm_d^{10}s - 2m_d^{11}s]$$

2
\[-4m_c^6m_d^2s^2 - 4m_c^4m_d^4s^2 - m_c^8m_d^6s^2 + (48m_c^{12} + 192m_c^{11}m_d + 288m_c^{10}m_d^2
+ 192m_c^9m_d^3 + 96m_c^7m_d^4 + 192m_c^5m_d^5 + 288m_c^6m_d^6 + 192m_c^4m_d^7 + 60m_c^4m_d^8
+ 48m_c^3m_d^9 + 72m_c^2m_d^{10} + 48m_c^1m_d^{11} + 12m_d^{12} - 8m_c^4s - 16m_c^3m_d^4s - 16m_c^2m_d^5s
- 16m_c^3m_d^7s - 4m_c^4m_d^8s - 4m_c^5m_d^9s - 4m_c^6m_d^{10}s - 4m_c^7m_d^{11}s
- 4m_c^8m_d^{12}s - 4m_c^9m_d^{13}s - 2m_c^{10}s^2 + 4m_c^6m_d^2s^2 + 4m_c^4m_d^4s^2 + m_c^2m_d^6s^2)4x^2].
\]

For the $e^+e^- \to D^+D^{*-}$ process, one should replace $\sqrt{1 - 4m_x^2}$ with $\sqrt{(s+m_x^2-m_x(\gamma^*)^2-m_s^2)} - 4m_x^2$ in the phase space integration. $|\tilde{M}|^2$ is given by

\[|\tilde{M}|^2 = -\frac{128(m_c + m_d)^6(2m_c^3 - m_d^3)^2(4m_c^2 + 8m_c m_d + 4m_d^2 - s)(1 + x^2)}{81m_c^6m_d^6s^4}.
\]

For the $e^+e^- \to D^+D^-$ process, $|\tilde{M}|^2$ is as follows

\[|\tilde{M}|^2 = -\frac{128(m_c + m_d)^2(4m_c^2 + 8m_c m_d + 4m_d^2 - s)(1 - x^2)}{81m_c^6m_d^6s^4} \times (4m_c^6 + 8m_c^5m_d + 4m_c^4m_d^2 + 2m_c^3m_d^3 + 4m_c^2m_d^4 + 4m_c m_d^5 + 2m_d^6 - 2m_c^2m_d s - m_c m_d^3 s).
\]

The Mandelstam variables are defined as

\[t = (p_3 - p_1)^2 = m^2 - \frac{s}{2} \left(1 - \sqrt{1 - \frac{4m^2}{s}x}\right),
\]
\[u = (p_3 - p_2)^2 = m^2 - \frac{s}{2} \left(1 + \sqrt{1 + \frac{4m^2}{s}x}\right).
\]

We notify that for $e^+e^- \to D^+D^{*-}$ process, the formula of Mandelstam variables t and u are a little different from Eq. (9) and Eq. (10) because of the mass difference between $D^+$ and $D^{*-}$. As the difference is small, the change for the cross section is negligible. For simplicity of the module of amplitude, we still use the definition in Eq. (9) and Eq. (10).

In the numerical calculation, the input parameters are as follows

\[\alpha = 1/137, \quad \alpha_s = 0.26.
\]

For the Coulomb-plus-linear potential case, the value of the wave function at the origin for charmonium can be found e.g. in [12]. With $m_c = 1.84$ GeV, for the S-wave charmonium it could be given by

\[|RS(0)|^2 = 1.454 \text{ GeV}^3.
\]

We then use the potential scaling rules to get a rough estimate for wave function at the origin for the charmed meson. We fix the constituent c-quark mass at 1.6 GeV. The cross sections for $D^{(+)}D^{(-)}$, $D^+D^{(*)-}$ and $D^+D^-$ are found to be

\[\sigma(e^+e^- \to \gamma^* \to D^{++}D^{*-}) = 0.532 \text{ pb},
\]

\[\sigma(e^+e^- \to \gamma^* \to D^+D^{(*)-}) = 0.850 \text{ pb},
\]

\[\sigma(e^+e^- \to \gamma^* \to D^+D^-) = 1.225 \text{ pb}.
\]
\[ \sigma(e^+e^- \rightarrow \gamma^* \rightarrow D^+D^-) = 0.699 \text{ pb}, \] (14)

\[ \sigma(e^+e^- \rightarrow \gamma^* \rightarrow D^+D^-) = 0.098 \text{ pb}. \] (15)

We also calculate the polarized \( D^{(s)+}D^{(s)-} \) and \( D^+D^{(s)-} \) production. The polarized cross section can be calculated by defining the longitudinal polarization vector as follows [13]

\[ \epsilon_L^\mu(p) = \frac{p^\mu}{M} - \frac{M n^\mu}{n \cdot p}, \] (16)

where \( p^2 = M^2 \) and \( n^\mu = (1, -\vec{p}/|\vec{p}|). \)

The polarized cross sections are

\[ \sigma(e^+e^- \rightarrow \gamma^* \rightarrow D^{(s)+}D^{(s)-}) = 0.347 \text{ pb}, \] (18)

Using the results in Eq. (13) and (18), we get

\[ \frac{\sigma(e^+e^- \rightarrow D^{(s)+}D^{(s)-})}{\sigma(e^+e^- \rightarrow D^{(s)+}D^{(s)-})} = 65.2\%. \] (19)

For \( D^+D^{(s)-} \) case, the longitudinal cross section for one photon process must be zero, since it is forbidden by parity conservation and angular momentum conservation. The effective \( \gamma DD^* \) vertex in Fig 1 only can have the form as

\[ \mathcal{L}_{\text{int}} \sim \epsilon^{\mu \nu \rho \sigma} F_{\mu \nu}(q) G_{\rho \sigma}(p_4) \phi(p_3), \] (20)

where \( F_{\mu \nu}(q) \) represents the field of the virtual photon, \( G_{\rho \sigma}(p_4) \) represents the field of the \( D^{(s)-} \) meson and \( \phi(p_3) \) represents the field of the \( D^+ \) meson. In the momentum space, the effective vertex is as follow

\[ \mathcal{L}_{\text{int}} \sim \epsilon^{\mu \nu \rho \sigma} q_\mu \epsilon_\nu^\lambda(q) p_{4\rho} \epsilon_\sigma^\lambda(p_4). \] (21)

If we choose the center-of-mass frame of the \( e^+e^- \), the formula in Eq. (21) becomes

\[ \mathcal{L}_{\text{int}} \sim \epsilon^{0 \nu \rho \sigma} q_0 \epsilon_\nu^\lambda(q) p_{4\rho} \epsilon_\sigma^\lambda(p_4) \sim q_0 (\vec{p}_4 \times \vec{e}(q)) \cdot \vec{e}(q). \] (22)

Because the space component of the longitudinal polarization vector of the \( D^* \) meson is parallel to the momentum of the \( D^* \) meson, from Eq. (22), one knows that the longitudinally polarized \( D^* \) meson cannot contribute to the process listed in Fig. 1. The longitudinal \( D^* \) only comes from two photons process, as listed in Fig. 2.
The modules of amplitude for \(e^+e^- \rightarrow 2\gamma^* \rightarrow \bar D^+D^{(*)-}\) and \(e^+e^- \rightarrow 2\gamma^* \rightarrow D^+D_T^{(*)-}\) are listed, respectively, in Eq. (23) and Eq. (24).

\[
|\mathcal{M}| = \frac{8(m_c - m_d)^2(m_c + m_d)^8(m_c^2 + 3m_c m_d + m_d^2)^2(4(m_c + m_d)^2(1 - x^2) + s(1 - x^2))}{81m_c^6m_d^6(2m_c + m_d)^2(m_c + 2m_d)^2s^3},
\]  
(23)

\[
|\mathcal{M}_T|^2 = \frac{32(m_c - m_d)^2(m_c + m_d)^{10}(mc_2 + 3m_c m_d + m_d^2)^2(1 + x^2)}{81m_c^6m_d^6s^3(2m_c + m_d)^2(m_c + 2m_d)^2}.
\]  
(24)

The corresponding cross sections are

\[
\sigma(e^+e^- \rightarrow 2\gamma^* \rightarrow \bar D^+D^{(*)-}) = 0.091 \text{ pb}
\]  
(25)

\[
\sigma(e^+e^- \rightarrow 2\gamma^* \rightarrow D^+D_T^{(*)-}) = 0.018 \text{ pb}.
\]  
(26)

Then one can get the ratio of cross section for \(D_T^+\) to total cross section as follows

\[
\frac{\sigma(e^+e^- \rightarrow D^+D_T^{(*)-})}{\sigma(e^+e^- \rightarrow D^+D^{(*)-})} = 90.8\%.
\]  
(27)

In summary, motivated by the measurement of the \(e^+e^- \rightarrow D^{(*)+}D^{(*)-}\) cross-sections \cite{9}, we calculate the cross-sections for \(D^{(*)+}D^{(*)-}\), \(D^+D^{(*)-}\) and \(D^+D^-\) in \(e^+e^-\) annihilation through one photon. The calculated cross-sections are roughly in agreement with the experimental data. The fraction of the \(D_L^{(*)+}D_T^{(*)-}\) final state in the \(e^+e^- \rightarrow D^{(*)+}D^{(*)-}\) reaction is also calculated, the ratio of 65\% is in some deviation from the Belle data, which is \((97 \pm 5)\%\) \cite{10}. Moreover, for the \(D^+D^{(*)-}\) production case, in Ref. \cite{10}, it is claimed that \(e^+e^- \rightarrow D^+D^{(*)-}\) is saturated by \(D_T^+\) final state \((95.8 \pm 5.6)\%\), in good agreement with the predictions of Ref. \cite{10}. Under our analysis, the \(D_T^+\) final state is forbidden by parity conservation and angular momentum conservation. If the \(e^+e^- \rightarrow 2\gamma^* \rightarrow D^+D^{(*)-}\) process is further considered, the fraction of the \(D_T^+\) final state in \(e^+e^- \rightarrow D^+D^{(*)-}\) process is 90.8\%.

**Acknowledgments**

The authors thank J.P.Ma, C.Meng and Z.Z.Song for useful discussions. This work was supported in part by the National Natural Science Foundation of China, and the Education Ministry of China.

**References**

[1] BaBar Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. 87, 162002 (2001).

[2] Belle Collaboration, K. Abe *et al.*, Phys. Rev. Lett. 88, 052001 (2002).
[3] Belle Colleboration, K. Abe, et al., Phys. Rev. Lett. 89, 142001(2002).

[4] E.Braaten and Jungil Lee, Phys. Rev. D 67 (2003)054007; K.Y.Liu, Z.G.He and K.T.Chao, Phys. Lett. B 557, (2003)45; K.Hagiwara, E.Kou and C.F.Qiao, Phys. Lett. B570, 39(2003) .

[5] P. Cho and K. Leibovich, Phys. Rev. D 54, 6990 (1996); F. Yuan, C.F. Qiao, and K.T. Chao, Phys. Rev. D 56, 321 (1997); ibid, 1663 (1997); S. Baek, P. Ko, J. Lee, and H.S. Song, J. Korean Phys. Soc. 33, 97 (1998); K.Y.Liu, Z.G.He and K.T.Chao, hep-ph/0301218.

[6] G.T.Bodwin, J.Lee and E.Braaten, Phys. Rev. Lett. 90 162001(2003).

[7] K.Y.Liu, Z.G.He and K.T.Chao, Phys. Rev. D 68, 031501 (2003).

[8] Belle Collaboration, K. Abe et al., hep-ex/0306015.

[9] Belle Collaboration, K. Abe et al., hep-ex/0307084.

[10] A.G.Grozin and M.Neubert, Phys. Rev. D 55, 272(1997).

[11] B.Guberina, J.H.Kühn, R.D.Peccei, and R.Ruckl, Nucl. Phys. B 174, (1980)317; K.Cheung and T.C.Yuan, Phys. Rev. D 53, (1996)3591.

[12] E.J.Eichten and C.Quigg, Phys. Rev. D 52, (1995)1726.

[13] K.Cheung and T.C.Yuan, Phys. Rev. D 50 (1994)3181.
Figure 1: Feynman diagrams for $e^+e^- \rightarrow \gamma^* \rightarrow D^{(*)+}D^{(*)-}$.
Figure 2: Feynman diagrams for $e^+ e^- \rightarrow 2\gamma^* \rightarrow D^{(*)+} D^{(*)-}$.