On the Significance of the Fifth Coordinate in Wesson’s Version of Kaluza-Klein Theory

N. Redington
Building 14S-100
Massachusetts Institute of Technology
Cambridge, MA 02139, USA

ABSTRACT:
It is argued that the fifth coordinate should correspond to an intensive parameter rather than to rest-mass as originally proposed by Wesson.

Traditionally, the fifth coordinate in Kaluza-Klein theories has been considered unobservable. This “cylindricity condition” has been justified on a number of grounds, especially on that of dimensional compactification. In the last decade, however, Wesson and his colleagues have argued that five-dimensional general relativity without matter and without the cylindricity condition is equivalent to normal four-dimensional relativity if all the explicitly dependent terms are equated to the four-dimensional stress tensor. Although this theory does not require any specific interpretation of $x^5$, Wesson et al. tentatively identify the fifth dimension as rest mass.

This suggestion, which was also made independently by Neacsu in 1981, seems to me to face an insuperable difficulty. Mass is additive almost by definition. In Newtonian mechanics, two separated objects may be regarded as a single entity at some average location, with a mass equal to the sum of the constituent masses. This property is retained, at least in the classical limit, both in relativity and in quantum mechanics.

By contrast, space and time coordinates are not additive for separated events: a mountain range cannot be replaced by a single peak as high as a hundred mountains, nor are two events in the Middle Ages conflatable into one event now! Any physical quantity which is to be identified with a coordinate must not, in the classical limit, behave like an extensive variable. This would seem to eliminate both the rest-mass interpretation and the proposal of Ruder that $x^5$ in the compactified Kaluza theory be identified with action.

If $x^5$ is not an extensive variable, it is natural to ask whether any intensive variable suggests itself as a candidate. One fairly obvious possibility is mass density, which would make mass into a kind of 4-volume and therefore additive.
In such an interpretation, point masses of density \( \rho = m \delta(x^4 - x^0) \) would appear rather like long tears in the fabric of space time.

Such an identification makes the relationship between the time-time component of the stress tensor and the fifth dimension slightly more tractable than in the rest-mass interpretation. For a pure fluid, the trace of the five-dimensional Ricci tensor in Wesson’s theory is\(^7\):

\[
(8\pi G/c^4)(\rho c^2 - 3P) = \left(1/4g_{55}\right)(\partial_5 g^{\alpha\beta} \partial_5 g_{\alpha\beta} + (g^{\alpha\beta} \partial_5 g_{\alpha\beta})^2)
\]

In the weak field limit at low pressure, this reduces to

\[
\eta^{\alpha\beta} h_{\alpha\beta;55} = (32\pi G/c^2) \rho
\]

So if \( x^5 \) is proportional to \( \rho \),

\[
h_5^\alpha \sim (16\pi G/3c^2) \rho^3 + b(x^\mu) \rho + a(x^\mu)
\]

Of course, there is no reason to prefer density over any other intensive variable. Temperature, for instance, would work just as well; such an interpretation might provide hope for a geometrization of thermodynamics. Many of these possibilities are moreover experimentally testable; unlike rest-mass, both density and temperature can be varied in the laboratory. It is interesting that a number of unusual and unexplained phenomena, such as sonoluminescence\(^8\), involve sudden large changes in temperature and/or density.

Finally, it is perhaps worth recalling that, in the original Kaluza theory, electric charge is the fifth component of momentum. The coordinate \( x^5 \) should then in that theory be identified with the quantity conjugate to charge in the uncertainty relations. That quantity is magnetic flux.\(^9\)

BIBLIOGRAPHY

1. P. S. Wesson 1990 Gen. Rel. & Grav. 22(707)
2. P. S. Wesson & J. Ponce de Leon 1992 J. Math. Phys. 33(3883)
3. P. S. Wesson et al. 1996 Int. J. Mod. Phys. A11(3247)
4. M. A. Neacsu 1981 M.S. Thesis, Texas Tech University.
5. Yu. B. Rumer 1949 Zh. Eksp. Teor. Fiz. 19(86 and 207)
6. Yu. B. Rumer 1959 Zh. Eksp. Teor. Fiz. 36(1894) [Sov. Phys. JETP 36(1348)]
7. J. M. Overduin & P. S. Wesson 1997 Phys. Rep., (forthcoming); [http://astro.uwaterloo.ca/~wesson/ PUB](http://astro.uwaterloo.ca/~wesson/ PUB)
8. L. A. Crum and R. A. Roy 1994 Science 266(233)
9. A. Widom et al. 1985 Phys. Rev. B 31(6588)