Papadopoulos, I. P. A.; Süli, E.
Numerical analysis of a topology optimization problem for Stokes flow. (English)
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Summary: Borrvall and Petersson (2003) developed the first model for the topology optimization of fluids in Stokes flow. They proved the existence of minimizers in the infinite-dimensional setting and showed that a suitably chosen finite element method will converge in a weak\(-\ast\) sense to an unspecified solution.

In this work, we prove novel regularity results and extend their numerical analysis. In particular, given an isolated local minimizer to the infinite-dimensional problem, we show that there exists a sequence of finite element solutions, satisfying necessary first-order optimality conditions, that strongly converges to it. We also provide the first numerical investigation into convergence rates.

MSC:
76Dxx Incompressible viscous fluids
35Qxx Partial differential equations of mathematical physics and other areas of application
49Qxx Manifolds and measure-geometric topics

Keywords:
topology optimization; Stokes flow; regularity; finite element method; nonconvex variational problem; multiple solutions

Full Text: DOI

References:
[1] Adam, L.; Hintermüller, M.; Peschka, D.; Surowiec, T. M., Optimization of a multiphysics problem in semiconductor laser design, SIAM J. Appl. Math., 79, 1, 257-283 (2019) · Zbl 1412.35110
[2] Jang, I. G.; Kim, I. Y., Computational study of Wolff’s law with trabecular architecture in the human proximal femur using topology optimization, J. Biomech., 41, 11, 2353-2361 (2008)
[3] Liu, J.; Gaynor, A. T.; Chen, S.; Kang, Z.; Suresh, K.; Takizawa, A.; Li, L.; Kato, J.; Tang, J.; Wang, C. C.L.; Cheng, L.; Liang, X.; To, A. C., Current and future trends in topology optimization for additive manufacturing, Struct. Multidiscip. Optim., 57, 6, 2457-2483 (2018)
[4] Allaire, G., Shape Optimization by the Homogenization Method, Vol. 146 (2012), Springer Science & Business Media
[5] Bendsoe, M. P.; Sigmund, O., Topology Optimization (2004), Springer Berlin Heidelberg: Springer Berlin Heidelberg Berlin, Heidelberg · Zbl 1099.74001
[6] Borrvall, T.; Petersson, J., Topology optimization of fluids in Stokes flow, Internat. J. Numer. Methods Fluids, 41, 1, 77-107 (2003) · Zbl 1025.76007
[7] Alexandersen, J.; Andreassen, C. S., A review of topology optimisation for fluid-based problems, Fluids, 5, 1, 29 (2020)
[8] Alonso, D. H.; de Sá, L. F.N.; Saenz, J. S.R.; Silva, E. C.N., Topology optimization applied to the design of 2D swirl flow devices, Struct. Multidiscip. Optim., 58, 6, 2341-2364 (2018)
[9] Alonso, D. H.; Saenz, J. S.R.; Silva, E. C.N., Non-newtonian laminar 2D swirl flow design by the topology optimization method, Struct. Multidiscip. Optim., 1-23 (2020)
[10] Olesen, L. H.; Okkels, F.; Bruus, H., A high-level programming-language implementation of topology optimization applied to steady-state Navier-Stokes flow, Internat. J. Numer. Methods Engrg., 65, 7, 975-1001 (2006) · Zbl 1111.76017
[11] Gersborg-Hansen, A.; Sigmund, O.; Haber, R. B., Topology optimization of channel flow problems, Struct. Multidiscip. Optim., 30, 3, 181-192 (2005) · Zbl 1243.76034
[12] Evgrafov, A., State space Newton’s method for topology optimization, Comput. Methods Appl. Mech. Engrg., 278, 272-290 (2014) · Zbl 1243.74744
[13] Evgrafov, A., Topology optimization of slightly compressible fluids, ZAMM J. Appl. Math. Mech./Z. Angew. Math. Mech.: Appl. Math. Mech., 86, 1, 46-62 (2006) · Zbl 1176.76113
[14] Deng, Y.; Wu, Y.; Liu, Z., Topology Optimization Theory for Laminar Flow: Applications in Inverse Design of Microfluidics (2018), Springer Singapore: Springer Singapore Singapore · Zbl 1386.76004
[15] Guest, J. K.; Prévost, J. H., Topology optimization of creeping fluid flows using a Darcy-Stokes finite element, Internat. J.
[16] Aage, N.; Poulsen, T. H.; Gersborg-Hansen, A.; Sigmund, O., Topology optimization of large scale Stokes flow problems, Struct. Multidiscip. Optim., 35, 2, 175-180 (2008) · Zbl 1273.76094

[17] Kreissl, S.; Pingen, G.; Maute, K., Topology optimization for unsteady flow, Internat. J. Numer. Methods Engng., 87, 13, 1229-1253 (2011) · Zbl 1242.76052

[18] Sá, L. F. N.; Romero, J. S.; Horikawa, O.; Silva, E. C. N., Topology optimization applied to the development of small scale pump, Struct. Multidiscip. Optim., 57, 5, 2045-2059 (2018)

[19] Garcke, H.; Hecht, C., A phase field approach for shape and topology optimization in Stokes flow, (New Trends in Shape Optimization (2015), Springer), 103-115 · Zbl 1329.76160

[20] Garcke, H.; Hecht, C., Applying a phase field approach for shape optimization of a stationary Navier-Stokes flow, ESAIM Control Optim. Calc. Var., 22, 2, 309-337 (2016) · Zbl 1342.35218

[21] Garcke, H.; Hecht, C.; Hinze, M.; Kahle, C., Numerical approximation of phase field based shape and topology optimization for fluids, SIAM J. Sci. Comput., 37, 4, A1846-A1871 (2015) · Zbl 1322.35113

[22] Thore, C.-J., Topology optimization of Stokes flow with traction boundary conditions using low-order finite elements, Comput. Methods Appl. Mech. Engrg., 386, Article 114177 pp. (2021) · Zbl 07427364

[23] Evans, L. C., Partial Differential Equations (2010), American Mathematical Society

[24] Papadopoulos, I. P. A., Computing Multiple Solutions of Topology Optimization Problems (2021), University of Oxford, URL https://ora.ox.ac.uk/objects/uuid:455f5465-af68-49c7-8e11-96509ef5b6a1 · Zbl 1472.35308

[25] Ożański, W. S., The Lagrange multiplier and the stationary Stokes equations, J. Appl. Anal., 23, 2, 137-140 (2017) · Zbl 1379.49018

[26] Girault, V.; Raviart, P.-A., Finite Element Methods for Navier-Stokes Equations: Theory and Algorithms, Vol. 5 (1986), Springer-Verlag Berlin Heidelberg

[27] Petersson, J., A finite element analysis of optimal variable thickness sheets, SIAM J. Numer. Anal., 36, 6, 1759-1778 (1999) · Zbl 0938.74054

[28] Logg, A.; Mardal, K.-A.; Wells, G., Automated Solution of Differential Equations by the Finite Element Method: The FEniCS Book, Vol. 84 (2012), Springer Science \& Business Media

[29] Balay, S.; Abhyankar, S.; Adams, M.; Brown, J.; Brune, P.; Buschelman, K.; Dalcin, L.; Dener, A.; Eijkhout, V.; Gropp, W.; Kaushik, D.; Knepley, M.; Smith, B.; Swarztrauber, P.; Tuminaro, R.; Walker, K.; Zhang, H.; Zhang, H., PETSc Users ManualTech. Rep. ANL-95/11 - Revision 3.11 (2019), Argonne National Laboratory, URL http://www.mcs.anl.gov/petsc

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