Nonfactorization Effects in Inclusive Decay

\[ B \to J/\Psi + X_s \]

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Abstract

We discuss the nonfactorization effects in \( B \to J/\Psi + X_s \), which is similar to the nonperturbative effect found by Voloshin in the decay \( B \to \gamma + X_s \). The QCD sum rule has been used to estimate the hadron matrix elements. We find that the correction from this effect is very large and the large discrepancy between the theory and the experimental data can be reduced considerably.

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It is amazing that there is a large discrepancy between the theoretical predictions, which is based on the operator product expansion (OPE) with assumption of the factorization, and experimental data in the process $B \to J/\Psi + X_s$. The theoretical prediction of the branching ratio $B \to J/\Psi + X_s$ is 0.23%, which is only one-third of the experiment data $(0.8 \pm 0.08)\%$\cite{[1]}. In order to resolve this puzzle, some scenarios have been suggested \cite{[2]}. Besides the possible existence of new charmonium states above the $\bar{D}D$ threshold, nonfactorization effects have been widely considered. In the later case, color-octet effect seems to be the most reasonable source that may enhance the theoretical calculation. Color-octet mechanism was first used in calculating cross section of $\bar{c}c$ production, where the theoretical prediction based on color-singlet mechanism is also much smaller than the experimental data. In this mechanism, the gluon parton has been taken account so that the color-octet operators responsible for $b \to \bar{c}c + q$ may give a non-zero contribution to the decay mentioned above. However, the most important parameter, the matrix element of color-octet operators, is not calculable so far. It needs future determination both from various experiments and theoretical estimations.

Recently, a nonperturbative correction of order of $O(\Lambda_{QCD}^2/m_c^2)$ to $B \to \gamma + X_s$ is first discussed by Voloshin\cite{[3]} (a similar correction is also independently discussed by \cite{[4]}) which arises from the contribution of the gluon-photon penguin graph shown in Fig.1, and then discussed by Grant et al, Legeti et al\cite{[5]} and Buchalla et al\cite{[6]}. It is obvious that this effect also exits in $B \to J/\Psi + X_s$ as nonfactorization effects. Although this correction is not large in $B \to \gamma + X_s$, one can expect a considerable contribution from this mechanism in $B \to J/\Psi + X_s$ due to the large ratio of the Wilson Coefficients between color-octet operator and color-singlet operator responsible for $B \to J/\Psi + X_s$. QCD sum rule\cite{[7]} so far is a powerful tool to deal with hadron matrix elements. Although there is an up to 30 percent uncertainty in this method, we can still give a useful estimation of the nonfactorization effect mentioned above.

The effective weak interaction Hamiltonian at a scale $\mu$ is given by

$$H_{\text{eff}} = - \frac{4G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_{i=1}^{2} C_i(\mu) O_i(\mu),$$

where $V_{cb}$ and $V_{cs}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and $C_i(\mu)$ are the Wilson coefficients.
in the conventional notation, \( O_2 = (\bar{s}_L \gamma_\mu \frac{\lambda}{2} b_L)(\bar{c}_L \gamma^\mu \frac{\lambda}{2} c_L) \), \( O_1 \) only differs from \( O_2 \) in the way of color indices contraction, and the former is referred as color-octet operator. On the factorization assumption, the matrix element of \( O_1 \) dominates the decay \( B \to J/\Psi X_s \) while \( O_2 \) gives zero.

The computation of the inclusive \( B \to J/\Psi X_s \) decay rate from the \( O_1 \) contribution is performed by calculating the correlation function

\[
T = i \int d^4 x e^{-i q \cdot x} \langle B(v)|T[j_\mu^\dagger(x) j_\nu(0)]|B(v)\rangle (-g^{\mu\nu} + \frac{q^\mu q^\nu}{M^2_{\Psi}}),
\]

(2)

where \( j_\mu = \bar{s}_L \gamma_\mu b_L \). The corresponding decay rate is given by

\[
\frac{d\Gamma}{P_{J/\Psi} dE_{J/\Psi}} = \frac{C^2_F f_{J/\Psi}^2 |V_{ts} V_{tb}|^2 C^2_1}{2 M_B \pi^2} Im T,
\]

(3)

where \( f_{J/\Psi} \) is defined by \( \langle 0|\bar{c}_L \gamma_\mu c|J/\Psi(p, \epsilon)\rangle = i f_{J/\Psi} \epsilon_\mu \).

At the leading order in the operator product expansion (OPE), \( Im T \) has the form

\[
Im T = \pi \delta(m_b^2 + M^2_{J/\Psi} - 2m_b E_{J/\Psi}) M_B \frac{2 E^2_{J/\Psi} m_b - 3 E_{J/\Psi} M^2_{J/\Psi} + M^2_{J/\Psi} m_b}{M^2_{J/\Psi}}.
\]

(4)

It is well known that the next to leading order contribution in OPE is suppressed by \( O(\Lambda^2_{QCD}/m^2_b) \). Therefore, on the factorization approximation, \( (3) \) almost gives the total decay rate of \( B \to J/\Psi X_s \). The main non-factorization contribution arises from the interference terms of \( O_1 \) and \( O_2 \) as shown in Fig.2, which is of order of \( \Lambda^2_{QCD}/m^2_b \). The coefficient ratio of operator \( O_2 \) over \( O_1 \) is \( C_2/C_1 \sim 20 \). Therefore, it is reasonable to expect a large enhancement when contribution of Fig.2 is taken into account.

The decay rate from Fig.2 is obtained as

\[
\frac{d\Gamma}{P_{J/\Psi} dE_{J/\Psi}} = \frac{2 C^2_F |V_{ts}^* V_{tb}|^2 C_1 \ast C_2}{2 M_B} [Im I^T_{\nu\mu\beta\alpha} T^\nu_{\mu\beta\alpha} + Im I'^T_{\nu\mu\beta\alpha} T'^\nu_{\mu\beta\alpha}] \frac{d^4 p}{(2\pi)^4}.
\]

(5)
where \( I_{\nu\mu\beta\alpha} \) and \( T^{\nu\mu\beta\alpha} \) are defined as

\[
I_{\nu\mu\beta\alpha} = \int d^4x \frac{x^\beta}{16\pi} \delta(p^2 - M^2_{J/\Psi}) \langle 0 | j^\nu_{\mu} | J/\Psi(p) \rangle \langle J/\Psi(p) | j^\alpha_{\beta}(x) j^{a5}_{\mu}(0) | 0 \rangle \\
= Im \int d^4x \frac{x^\beta}{16} d^4 ye^{ipy_i} \langle 0 | T j_{\nu}(y) j^\alpha_{\beta}(x) j^{a5}_{\mu}(0) | 0 \rangle \\
= Im - i \frac{\partial}{\partial y_i} \left\{ \int d^4x \frac{x^\beta}{16} d^4 ye^{ipy_i} \langle 0 | T j_{\nu}(y) j^\alpha_{\beta}(x) j^{a5}_{\mu}(0) | 0 \rangle \right\}_{y=0} \\
= ImK_{\nu\mu\beta\alpha}(p), \\
T^{\nu\mu\beta\alpha} = \int d^4x e^{-ipx} i \langle B | \bar{b}_L(x) \gamma^\nu S(x, 0) \gamma^\mu (-ig_\lambda G^{\beta\alpha}) b_L(0) | B \rangle , \\
\]

In equation (6), we have assumed that other resonances contributions to \( I \) and \( I' \) are small compared with \( J/\Psi \). \( S_s(x, y) \) is the propagator of \( s \) quark, \( j_{\mu} = \bar{c} \gamma_{\mu} c \), \( j^{a}_{\mu} = \bar{c} \gamma_{\mu} \frac{\lambda^a}{2} c \) and \( j^{a5}_{\mu} = \bar{c} \gamma_5 \gamma_{\mu} \frac{\lambda^a}{2} c \). We only keep the leading order terms in OPE. \( Im \) in the definition of \( I \) and \( I' \) should be understood as an operator which selects the imaginary part of the scalar form factors which factorize the followed matrix element.

In this paper, \( I \) and \( I' \) are estimated by QCD sum rule. The matrix elements \( K_{\nu\mu\beta\alpha} \) \( K'_{\nu\mu\beta\alpha} \) in equation (6) are obtained by calculating the diagrams shown in Fig.3 and Fig.4,

\[
K_{\nu\mu\beta\alpha} = K^0_{\nu\mu\beta\alpha} + 2K^a_{\nu\mu\beta\alpha} + 2K^b_{\nu\mu\beta\alpha} + 2K^c_{\nu\mu\beta\alpha}, \\
K'_{\nu\mu\beta\alpha} = 2K^d_{\nu\mu\beta\alpha},
\]

where \( K^0 \) is from Fig.3 and others are from Fig.4 and correspond to the diagrams with the indices \( a, b, c \) and \( d \) respectively.

In a conventional way, the fixed point gauge \( x \cdot A(x) = 0 \) is taken. At the first order of the expansion in term of \( x^\mu \) is,

\[
A_\mu(k) = -i(2\pi)^4 \frac{1}{4} G_{\nu\mu}(0) \frac{\partial}{\partial k^\nu} \delta^4(k).
\]
We obtain

\[ K_{\nu,\beta\alpha}^0 = i4 \int \frac{d^Dk}{(2\pi)^D} \frac{\partial}{\partial q^\beta} \left\{ \frac{i}{k - m_c} \frac{i}{k - \hat{q} - m_c} \frac{i}{k - \hat{q} - m_c} \frac{i}{k - \hat{q} - m_c} \frac{i}{k - \hat{q} - m_c} \frac{i}{k - \hat{q} - m_c} \right\} |q = 0, \]

\[ K_{\nu,\beta\alpha}^1 = -\frac{i}{96} \left\{ \delta_{\rho\sigma} \delta_{\rho'\sigma'} - \delta_{\rho\sigma} \delta_{\rho'\sigma'} \right\} (g_s^2 G^2) \left( \frac{2 \lambda^2}{2} \right) \int \frac{d^Dk}{(2\pi)^D} \frac{\partial}{\partial q^\beta} \left\{ \frac{i}{k - m_c} \frac{i}{k - \hat{q} - m_c} \frac{i}{k - \hat{q} - m_c} \right\} |q = k_1 = k_2 = 0, \]

\[ K_{\nu,\beta\alpha}^2 = -\frac{i}{96} \left\{ \delta_{\rho\sigma} \delta_{\rho'\sigma'} - \delta_{\rho\sigma} \delta_{\rho'\sigma'} \right\} (g_s^2 G^2) \left( \frac{2 \lambda^2}{2} \right) \int \frac{d^Dk}{(2\pi)^D} \frac{\partial}{\partial q^\beta} \left\{ \frac{i}{k - m_c} \frac{i}{k - \hat{q} - m_c} \frac{i}{k - \hat{q} - m_c} \right\} |q = k_1 = k_2 = 0, \]

\[ K_{\nu,\beta\alpha}^3 = -\frac{1}{48} \left\{ \delta_{\sigma\beta} \delta_{\rho\alpha} - \delta_{\sigma\alpha} \delta_{\rho\beta} \right\} (g_s^2 G^2) \left( \frac{2 \lambda^2}{2} \right) \int \frac{d^Dk}{(2\pi)^D} \frac{\partial}{\partial q^\beta} \left\{ \frac{i}{k - m_c} \frac{i}{k - \hat{q} - m_c} \frac{i}{k - \hat{q} - m_c} \right\} |q = 0. \]

The computation of \[ ] is tedious, we will not display the procedure. Instead, we give the final result at the end of this paper.

Using the dispersion relation

\[ K_{\nu,\beta\alpha}(p) = \frac{1}{\pi} \int \frac{ImK_{\nu,\beta\alpha}(s)}{s - p^2} ds, \]

\[ K'_{\nu,\beta\alpha}(p) = \frac{1}{\pi} \int \frac{ImK'_{\nu,\beta\alpha}(s)}{s - p^2} ds, \]

and the fact that \( I_{\nu,\beta\alpha} \) and \( I'_{\nu,\beta\alpha} \) are proportional to \( \delta(p^2 - M_{J/\Psi}) \), we get two sum rules

\[ I_{\nu,\beta\alpha}(p) = \pi \delta(p^2 - M_{J/\Psi}) M_{J/\Psi}^4 \left[ \frac{d}{dp^2} K_{\nu,\beta\alpha}(p) \right]_{p^2 = 0}, \]

\[ I'_{\nu,\beta\alpha}(p) = \pi \delta(p^2 - M_{J/\Psi}) M_{J/\Psi}^4 \left[ \frac{d}{dp^2} K'_{\nu,\beta\alpha}(p) \right]_{p^2 = 0}, \]

where \( P^2 = -p^2 \). \( \frac{d}{dp^2} K_{\nu,\beta\alpha}(p) |_{p^2 = 0} \) must be understood that only derivatives of the scalar form factors of the matrix element are set at \( p^2 = 0 \). To choose \( p^2 = 0 \) is for the sake of convenience. In general, we can set \( p^2 \) at any point in the range \(-p^2 + m_c^2 >> \Lambda_{QCD}^2\) to get a sum rule. In order to get rid of the dependence on the subtraction in the loop calculation, we have used the sum rules for derivatives of \( K_{\nu,\beta\alpha} \) and \( K'_{\nu,\beta\alpha} \) instead of themselves.
The calculations of $T_{\nu \mu \beta \alpha}$ and $T_{\nu \mu \beta \alpha}$ are performed, in which we need the identities

$$\frac{1}{2M_B} \langle B(v) | \bar{b} \Gamma g_s G_{\alpha \beta} b | B(v) \rangle = -\frac{\mu_g^2}{24} Tr \{ \Gamma(1 + \hat{v}) \sigma_{\alpha \beta}(1 + \hat{v}) \},$$

$$\langle B(v) | \bar{b} \Gamma b | B(v) \rangle = \frac{M_B}{4} Tr \{ (1 + \hat{v}) \Gamma(1 + \hat{v}) \},$$

where $\Gamma$ is any kind of Dirac structure and $\mu_g^2$ is the value of the strength of the chromo-magnetic interaction of the $b$-quark inside in $B$ hadron,

$$\mu_g^2 = \frac{1}{2M_B} \langle B(v) | \bar{b} \sigma_{\alpha \beta} G^{\alpha \beta}_{a} \frac{\lambda^a}{2} b | B(v) \rangle = \frac{3}{4} (M_B^* - M_B^2) \approx 0.4 GeV^2.$$  \hspace{1cm} (16)

After a tedious calculation, we arrive at

$$Im I_{\nu \mu \alpha \beta} T_{\nu \mu \alpha \beta} = \frac{\mu_g^2}{16} M_B^2 \frac{\pi^2 \delta(m_b^2 + M_{J/\Psi}^2 - 2m_b E_{J/\Psi}) \delta(p^2 - M_{J/\Psi}^2)}{(4m_b E_{J/\Psi}^2 - m_b^2 - 3m_c^2 E_{J/\Psi}^2) - 44m_b E_{J/\Psi}^2 - 33m_b m_c^2 - 11m_c^2 E_{J/\Psi}^2} \langle \alpha_s G^2 \rangle \left\{ \frac{\alpha_s G^2}{24\pi} \right\}$$

$$Im I_{\nu \mu \alpha \beta} T_{\nu \mu \alpha \beta} = \frac{M_{J/\Psi}^4}{12} \frac{\pi^2 \delta(m_c^2 + M_{J/\Psi}^2 - 2m_b E_{J/\Psi}) \delta(p^2 - M_{J/\Psi}^2)}{m_b^2 + 61m_c^2 - 6m_b E_{J/\Psi}^2} \langle \alpha_s G^2 \rangle \left\{ \frac{\alpha_s G^2}{6\pi} \right\}.$$  \hspace{1cm} (17)

In the second equation of (17), we have already used partial integration. Using the standard numerical values

$$\langle \alpha_s G^2 \rangle = 0.04 GeV^4, \quad m_c = 1.3 GeV, \quad m_b = 4.5 GeV$$

$$C_1(m_b) = (2C_+(m_b) - C_-(m_b))/3 = 0.133, \quad C_2(m_b) = C_+(m_b) + C_-(m_b) = 2.21,$$

$$f_{J/\Psi} = 0.38 GeV,$$

we obtain the nonfactorization contribution arisen from color-octet amplitude as large as

$$\delta \Gamma(B \rightarrow J/\Psi + X_s) \approx 1.25 \Gamma_0(B \rightarrow J/\Psi + X_s),$$  \hspace{1cm} (19)

where $\Gamma_0$ is the result with factorization assumption. This result is sensitive to the value of $m_c$, if we choose $m_c = 1.5 GeV$, the ratio would be $\sim 0.7$. The non-perturbative effect from the the gluon condense is much smaller compared with the perturbative diagram, so, the result is not sensitive to the parameter $\langle \alpha_s G^2 \rangle$. However, since the decay rate in (8) is independent of $m_c$, the non-factorization contribution is very large any way. One may notice that we have only taken account of the interference term shown in fig.2. If the non-factorization
effects from pure $O_2$ contribution are taken into account, which are very complicated to be calculated and will be presented in a separate paper, the enhancement will be hopeful to explain the discrepancy between the theoretical predictions and the experimental data.

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Figure Captions

Fig.1 Nonperturbative effect in $B \rightarrow X_s + \gamma$

Fig.2 Nonfactorization effect in $B \rightarrow X_s + J/\Psi$

Fig.3 The perturbative diagrams contribute to $K^{\nu\mu\beta\alpha}$

Fig.4 The gluon condense diagrams contribute to $K^{\nu\mu}\bar{\beta}\alpha$ and $K^{\nu\mu\beta\alpha}$
Fig. 1
Fig. 2
Fig3
Fig. 4