Relativistic Strain and Electromagnetic Photon-Like Objects

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Abstract

This paper aims to relate some properties of photon-like objects, considered as spatially finite time-stable physical entities with dynamical structure, to well defined properties of the corresponding electromagnetic strains defined as Lie derivatives of the Minkowski (pseudo)metric with respect to the eigen vector fields of the Maxwell-Minkowski stress-energy-momentum tensor. First we recall the geometric sense of the concept of strain, then we introduce and discuss the notion for photon-like objects (PhLO). We compute then the strains along the eigen vectors of the stress-energy-momentum tensor \( T^\nu_\mu \) and establish important correspondences with the divergence terms of \( T^\nu_\mu \) and the terms determining some internal energy-momentum exchange between the two recognizable component-fields \( F \) and \( *F \) of a vacuum electromagnetic field. The role of appropriately defined Frobenius curvature is also discussed and emphasized. Finally, equations of motion and interesting PhLO-solutions are given.

1 Introduction

In correspondence with modern theoretical view on classical fields we consider time dependent and space propagating electromagnetic fields as flows of spatially finite physical entities which have been called in the early 20th century photons. However, the efforts made during the past years to find appropriate in this respect nonlinearizations of Maxwell vacuum equations [1],[2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12],[13],[14],[15], and the seriously developed quantum theory have not resulted so far in appropriate, from our viewpoint, description of such single time stable entities of electromagnetic field nature with finite spatial support, in particular, adequate theoretical image of single photons as spatially finite physical objects with internal dynamical structure, explaining in such a way their time stability and spin nature. As an illustration to this we give here the Einstein’s confession [16] ”All these fifty years of pondering have not brought me any closer to answering the question What are light quanta”, and this happens in front of the already developed quantum field theory. In view of this, a decision to look back to the rudiments of the electromagnetic theory trying to reconsider its basic assumptions in order to come to equations giving appropriate solutions, in particular, solutions with spatially finite carrier at every moment of their existence and space-propagating as a whole, keeping, of course, their physical identity, shouldn’t look unjustified.

In our reconsideration and looking through the above cited attempt and review articles and deeper studies we found some, more or less, unexplained from theoretical viewpoint moments:

- how the static Coulomb field may be considered as acting physical object without any self-changing: the charged mass particle changes its energy, while the field enjoys ”nontouching”.

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- where in Coulomb force law written as $qE$ is the really existing field generated by the charge $q$,
- should the flows of electric and magnetic fields across imaginary and arbitrary 2-surfaces be considered as really detectable events, or the corresponding local energy-momentum flows across appropriate physical volumes should be used in writing corresponding balance relations,
- are the electric $E$ and magnetic $B$ fields are recognizable subsystems of the time-dependent and space-propagating field, and if such a view is accepted, how it corresponds to the fact that neither $E$ nor $B$ are able to carry momentum independently of each other,
- if the energy and momentum densities of the field are given by $\frac{1}{2}(E^2 + B^2)$ and $\frac{1}{c}E \times B$, then which members of the family ($f$ is a function)

\[
\Sigma_1 = E \cos(f) + B \sin(f), \quad \Sigma_2 = -E \sin(f) + B \cos(f)
\]

giving the same energy-momentum densities, should be chosen for formal images of the two subsystems,
- why the later introduced relativistic formal images $(F, *F)$ of the two recognizable subsystems should satisfy $dF = 0, d*F = 0$, which forbids internal energy-momentum exchange between these two subsystems in view of the explicit divergence of the energy-momentum tensor

\[
\nabla_\nu T^\nu_\mu = \nabla_\nu \left[ -\frac{1}{2} [F^\mu\sigma F^\nu\sigma + (*F)_{\mu\sigma} * F^\nu\sigma] \right] = \frac{1}{2} \left[ F^{\alpha\beta}(dF)_{\alpha\beta\mu} + (*F)^{\alpha\beta}(d*F)_{\alpha\beta\mu} \right].
\]

An interesting observation based on the formal identity

\[
\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \delta^\beta_\alpha = F_{\mu\alpha} F^{\mu\beta} - (*F)_{\mu\alpha} (*F)^{\mu\beta}
\]

leading to equal energy-momentum carried by $F$ and $*F$ in the free case where $F_{\mu\nu} F^{\mu\nu} = 0$: if $F$ and $*F$ exchange locally energy-momentum this exchange should respect the fact that the two subsystems carry equal energy-momentum permanently.

Also, why so much interest to plane wave solutions should be paid in view of their quite unphysical sense: infinite 3-volume supports and corresponding infinite energy.

These and some other moments made us think that the basic equations must have energy-momentum balance sense, and these equations should be naturally consistent with the energy-momentum relations introduced by classical Maxwell-Minkowski equations. During the past years we have published various views on the problem, the interested readers may look at the final pages of [17]. In this paper we approach the problem from the viewpoint of elasticity: the equations might connect local changes of the above given Minkowski stress-energy-momentum tensor and the strain tensors on Minkowski space-time, measuring the change of the Minkowski metric $\eta$ along the eigen vector fields $Z_1, Z_2, ...$ of $T^{\mu\nu}_\eta$. In other words, outside electromagnetic field objects the metric may be assumed to be the flat Minkowski metric $\eta$, but inside such objects where $T^{\mu\nu}_\eta \neq 0$, a consideration of the internal local interaction processes should take in view the values of the Lie derivatives $L_{Z_i}(\eta), i = 1, 2, ...$, i.e., should try to look what happens inside in terms of such quite rapidly arising and vanishing strains/deformations, caused by propagating electromagnetic field objects (although on unknown media, frequently called vacuum).

2 Strain

The concept of strain is introduced in studying elastic materials/media subject to external stresses of different nature: mechanical, electromagnetic, etc. The classical strain describes mainly the abilities of the media to bear stress-actions from outside through deformation, i.e. through changing its shape, or, configuration.
The mathematical counterparts of the allowed deformations are those diffeomorphisms \( \varphi \) of a Riemannian manifold \((M, g)\), which change the metric \( g \). It is assumed that every such \( \varphi : M \rightarrow M \) generates a possible configuration of the material considered. Since the essential diffeomorphisms \( \varphi \) must transform the metric \( g \) to some new metric \( \varphi^*g \), such that \( g \neq \varphi^*g \), the naturally arising tensor field \( e = (\varphi^*g - g) \neq 0 \) appears as a measure of the physical abilities of the material to withstand external force actions. It seems reasonable to recall that these diffeomorphisms do not induce non-zero Riemannian curvature.

The continuity of the process of deformation leads to consider 1-parameter group \( \varphi_t, t \in [a, b] \subset \mathbb{R} \) of deformations, i.e. of local diffeomorphisms. Let the vector field \( X \) generate \( \varphi_t \), then the quantity

\[
\frac{1}{2} L_X g := \frac{1}{2} \lim_{t \to 0} \frac{\varphi_t^*g - g}{t},
\]

i.e. one half of the Lie derivative of \( g \) along \( X \), is called (infinitesimal) strain/deformation tensor.

**Remark.** Further in the paper we shall work with \( L_X g \), i.e. the factor \( 1/2 \) will be omitted.

Geometric approach to elasticity is given in [18], and relativistic extensions of the classical elasticity theory may be found in [19] and in the corresponding quotations therein. In our further study we shall carry the above given Lie derivative definition of strain tensor to the case of Minkowski space-time, so we shall call \( L_X \eta \), where \( \eta \) is the Minkowski (pseudo)metric and \( X \) is any eigen vector of the Minkowski stress-energy-momentum tensor \( T^\nu_\mu \), eigen strain tensor.

Clearly, the term "material" does not seem to be appropriate for photon-like objects (PhLO) because, as we suggest in the next section, these objects admit NO static situations, they are of entirely dynamical nature, so the corresponding relativistic strain tensors must take care of this.

### 3 The Concept of photon-like object(s)

We introduce the following notion of Photon-like object(s) (we shall use the abbreviation "PhLO" for "Photon-like object(s)") :

**PhLO are real massless time-stable physical objects with an intrinsically compatible time-stable translational-rotational dynamical structure.**

We give now some explanatory comments, beginning with the term real. **First** we emphasize that this term means that we consider PhLO as really existing physical objects, not as appropriate and helpful but imaginary (theoretical) entities. Accordingly, PhLO **necessarily carry energy-momentum**, otherwise, they could hardly be detected. **Second**, PhLO can undoubtedly be created and destroyed, so, no point-like and infinite models are reasonable: point-like objects are assumed to have no structure, so they can not be destroyed since there is no available structure to be destroyed; creation of infinite physical objects (e.g. plane waves) requires infinite quantity of energy to be transformed from one kind to another for finite time-period, which seems also unreasonable. Accordingly, PhLO are **spatially finite** and have to be modeled like such ones, which is the only possibility to be consistent with their "created-destroyed" nature. It seems hardly reasonable to believe that PhLO can not be created and destroyed, and that spatially infinite and indestructible physical objects may exist at all. **Third**, "spatially finite" implies that PhLO may carry only finite values of physical (conservative or non-conservative) quantities. In particular, the most universal physical quantity seems to be the energy-momentum, so the model must allow finite integral values of energy-momentum to be carried by the corresponding solutions. **Fourth**, "spatially finite" means also that PhLO **propagate**, i.e. they do not "move" like classical particles along trajectories, therefore, partial differential equations should be used to describe their evolution in time.
The term "massless" characterizes physically the way of propagation in terms of appropriate dynamical quantities: the integral 4-momentum $p$ of a PhLO should satisfy the relation $p_{\mu}p^{\mu} = 0$, meaning that its integral energy-momentum vector must be isotropic, i.e. to have zero module with respect to Lorentz-Minkowski (pseudo)metric in $\mathbb{R}^4$. If the object considered has spatial and time-stable structure, so that the translational velocity of every point where the corresponding field functions are different from zero must be equal to $c$, we have in fact null direction in the space-time intrinsically determined by a PhLO. Such a direction is formally defined by a null vector field $\zeta$, $\zeta^2 = 0$. The integral trajectories of this vector field are isotropic (or null) straight lines as is traditionally assumed in physics. It follows that with every PhLO a null straight line direction is necessarily associated, so, canonical coordinates $(x^1, x^2, x^3, x^4) = (x, y, z, \xi = ct)$ on $\mathbb{R}^4$ may be chosen such that in the corresponding coordinate frame $\zeta$ to have only two non-zero components of magnitude 1: $\zeta^\mu = (0, 0, -\varepsilon, 1)$, where $\varepsilon = \pm 1$ accounts for the two directions along the coordinate $z$ (further such a coordinate system will be called $\zeta$-adapted and will be of main usage). Our PhLO propagates as a whole along the $\zeta$-direction, so the corresponding energy-momentum tensor $T_{\mu\nu}$ of the model must satisfy the corresponding local isotropy (null) condition, namely, $T_{\mu\nu}T^{\mu\nu} = 0$ (summation over the repeated indices is throughout used).

The term "translational-rotational" means that besides translational component along $\zeta$, the propagation necessarily demonstrates some rotational (in the general sense of this concept) component in such a way that both components exist simultaneously and consistently with each other. It seems reasonable to expect that such kind of behavior should be consistent only with some distinguished spatial shapes. Moreover, if the Planck relation $E = h\nu$ must be respected throughout the evolution, the rotational component of propagation should have time-periodical nature with time period $T = \nu^{-1} = h/E = \text{const}$, and one of the two possible, left or right, orientations. It seems reasonable also to expect periodicity in the spatial shape of PhLO, which somehow to be related to the time periodicity.

The term "dynamical structure" means that the propagation is supposed to be necessarily accompanied by an internal energy-momentum redistribution, which may be considered in the model as energy-momentum exchange between (or among) some appropriately defined subsystems. It could also mean that PhLO live in a dynamical harmony with the outside world, i.e. any outside directed energy-momentum flow should be accompanied by a parallel inside directed energy-momentum flow.

### 4 The Electromagnetic eigen strain tensors

From formal point of view the standard relativistic formulation of classical electrodynamics in vacuum (zero charge density) is based on the following assumptions. The configuration space is the Minkowski space-time $M = (\mathbb{R}^4, \eta)$ where $\eta$ is the pseudometric with $\text{sign}(\eta) = (-, -, -, +)$ with the corresponding volume 4-form $\omega_\alpha = dx \wedge dy \wedge dz \wedge d\xi$ and Hodge star $\ast$ defined by $\alpha \wedge \ast \beta = (-1)^{\text{ind} \eta}(\alpha, \beta)\omega_\alpha$, $\alpha$ and $\beta$ are forms of the same degree. The electromagnetic field is described by two closed 2-forms $(F, \ast F) : df = 0$, $d \ast F = 0$. The physical characteristics of the field are deduced from the following stress-energy-momentum tensor field

$$T_{\mu\nu}(F, \ast F) = -\frac{1}{2} [F_{\mu\sigma}F^{\nu\sigma} + (\ast F)_{\mu\sigma}(\ast F^{\nu\sigma})].$$

Note that there is NO interaction stress-energy term between $F$ and $\ast F$. In the non-vacuum case the allowed energy-momentum exchange with other physical systems is given in general by the divergence

$$\nabla_\nu T'_{\mu\nu} = \frac{1}{2} \left[F^{\alpha\beta}(dF)_{\alpha\beta\mu} + (\ast F)^{\alpha\beta}(d \ast F)_{\alpha\beta\mu}\right] = F_{\mu\nu}(\delta F)^{\nu} + (\ast F)_{\mu\nu}(\delta \ast F)^{\nu},$$

where $\delta = d\ast$ is the co-derivative. Since $dF = 0$, $d \ast F = 0$, this divergence is obviously equal to zero on the vacuum solutions: its both terms are zero. Therefore, energy-momentum exchange between
the two subsystem-fields $F$ and $*F$, as suggested by the above divergence terms, should be expressed by

$$(*F)^{\alpha \beta} (dF)_{\alpha \beta \mu}, \quad F^{\alpha \beta} (d * F)_{\alpha \beta \mu},$$

summation over $\alpha < \beta$, is NOT allowed on the solutions of $dF = 0, d * F = 0$. This shows that the widely used 4-potential approach (even if two 4-potential 1-forms $U, U^*$ are introduced so that $dU = F, dU^* = *F$ locally) to these equations forbids real (coordinate free) changes, and excludes any possibility to individualize two energy-momentum exchanging time-recognizable subsystems of the field that are mathematically represented by $F$ and $*F$.

According to the above introduced and discussed concept of PhLO we have to impose the condition $T_{\mu \nu} T^{\mu \nu} = 0$ on the energy tensor (1). As is well known [20] this is equivalent to zero eigen values of $T_{\mu \nu}$, which implies zero invariants: $I_1 = \frac{1}{2} F_{\mu \nu} F^{\mu \nu} = \frac{1}{2} F_{\mu \nu} (*F)^{\mu \nu} = 0$, and that $T_{\mu \nu}$ admits just one null eigen direction defined by the vector field $\zeta, \zeta^2 = 0$, determining a null straight-line direction along which the energy density propagates translationally. This isotropic nature of the field leads [20] to the following structure of $F$ and $*F$

$$F = A \wedge \zeta, \quad *F = A^* \wedge \zeta,$$

where $\zeta_\mu = \eta_{\mu \nu} \zeta^\nu$, $A$ and $A^*$ are the spatial-like electric and magnetic 1-form components of the field: $A^2 < 0, (A^*)^2 < 0, \eta(A, A^*) = 0$, moreover, $(A, A^*)$ are eigen vectors of $T_{\mu \nu}$, and $\eta(A, \zeta) = \eta(A^*, \zeta) = 0$. Under these conditions it is possible to find, and convenient to use, a canonical coordinate system $(x, y, z, \xi = ct)$ such that $\zeta = \varepsilon dz + d\xi, A = u dx + p dy, A^* = \varepsilon p dx - \varepsilon u dy$, where $(u, p)$ are two functions, and $\varepsilon = \pm 1$ carries information about the direction of propagation along the coordinate $z$ of our PhLO.

**Remark** All further identifications of tangent and cotangent objects will be made by $\eta$, and if $A$ is a tangent coobject we shall denote by $A$ the $\eta$-corresponding tangent object.

Since the three vector fields $(\zeta, A, A^*)$ are eigen vectors of $T_{\mu \nu}$, we compute the corresponding three eigen strain tensors.

**Proposition.** The following relations hold:

$$L_\zeta \eta = 0, \quad (L_A \eta)_{\mu \nu} \equiv D_{\mu \nu} = \left| \begin{array}{cccc}
2u_x & u_y + p_x & u_z & u_\xi \\
u_y + p_x & 2p_y & p_z & p_\xi \\
u_z & p_z & 0 & 0 \\
u_\xi & p_\xi & 0 & 0 \\
\end{array} \right|,$$

$$L_{A^*} \eta_{\mu \nu} \equiv D^*_{\mu \nu} = \left| \begin{array}{cccc}
-2\varepsilon p_x & -\varepsilon(p_y + u_x) & -\varepsilon p_z & -\varepsilon p_\xi \\
-\varepsilon(p_y + u_x) & 2\varepsilon u_y & \varepsilon u_z & \varepsilon u_\xi \\
-\varepsilon p_z & \varepsilon u_z & 0 & 0 \\
-\varepsilon p_\xi & \varepsilon u_\xi & 0 & 0 \\
\end{array} \right|.$$

**Proof.** Immediately verified.

Let $[X, Y]$ denote the Lie bracket of vector fields on $M$, and $\phi^2 = u^2 + p^2, \psi = \arctg(p/u)$, i.e. $u = \phi \cos \psi, p = \phi \sin \psi$. We note that the local conservation law $\nabla_{\nu} T^\nu_{\mu} = 0$ reduces in our case to $L_\zeta \phi^2 = 0$, so $(u, p)$ would be running waves if $L_\zeta \psi = 0$ too.
We give now some important from our viewpoint relations.

\[ D(\zeta, \zeta) = D^* (\zeta, \zeta) = 0, \]
\[ D(\zeta) \equiv D(\zeta)_\mu dx^\mu \equiv D_{\mu\nu} \zeta^\nu dx^\mu = (u_\xi - \varepsilon u_z) dx + (p_\xi - \varepsilon p_z) dy, \]
\[ D(\zeta) \equiv D(\zeta)_\mu \frac{\partial}{\partial x^\mu} \equiv D_{\mu\nu} \zeta^\nu \frac{\partial}{\partial x^\mu} = -(u_\xi - \varepsilon u_z) \frac{\partial}{\partial x} - (p_\xi - \varepsilon p_z) \frac{\partial}{\partial y} = -[\bar{A}, \zeta], \]
\[ D_{\mu\nu} \bar{A}^\mu \zeta^\nu = -\frac{1}{2} \left[ (u^2 + p^2)_\xi - \varepsilon(u^2 + p^2)_z \right] = -\frac{1}{2} L_\zeta \phi^2, \]
\[ D_{\mu\nu} \bar{A}^\mu \zeta^\nu = -\varepsilon \left[ u(p_\xi - \varepsilon p_z) - p(u_\xi - \varepsilon u_z) \right] = -\varepsilon R = -\varepsilon \phi^2 L_\zeta \psi, \]

where \( R \equiv u(p_\xi - \varepsilon p_z) - p(u_\xi - \varepsilon u_z) \). We also have:

\[ D^*(\zeta) \equiv \varepsilon \left[ -(p_\xi - \varepsilon p_z) dx + (u_\xi - \varepsilon u_z) dy \right], \]
\[ D^*(\zeta) \equiv (D^*)_\mu \zeta^\nu \frac{\partial}{\partial x^\mu} = -(p_\xi - \varepsilon p_z) \frac{\partial}{\partial x} + (u_\xi - \varepsilon u_z) \frac{\partial}{\partial y} = [\bar{A}^*, \zeta], \]
\[ D^*_{\mu\nu} \bar{A}^{*\mu} \zeta^\nu = -\frac{1}{2} \left[ (u^2 + p^2)_\xi - \varepsilon(u^2 + p^2)_z \right] = -\frac{1}{2} L_\zeta \phi^2, \]
\[ D^*_{\mu\nu} \bar{A}^{*\mu} \zeta^\nu = \varepsilon \left[ u(p_\xi - \varepsilon p_z) - p(u_\xi - \varepsilon u_z) \right] = \varepsilon R = \varepsilon \phi^2 L_\zeta \psi. \]

Clearly, \( D(\zeta) \) and \( D^*(\zeta) \) are linearly independent in general:

\[ D(\zeta) \wedge D^*(\zeta) = \varepsilon \left[(u_\xi - \varepsilon u_z)^2 + (p_\xi - \varepsilon p_z)^2\right] dx \wedge dy = \varepsilon \phi^2 (\psi_\xi - \varepsilon \psi_z)^2 dx \wedge dy \neq 0. \]

We readily obtain now

\[ D(\zeta) \wedge F = D^*(\zeta) \wedge (\ast F) = D(\zeta) \wedge A \wedge \zeta = D^*(\zeta) \wedge A^* \wedge \zeta \]
\[ = -\varepsilon \left[ u(p_\xi - \varepsilon p_z) - p(u_\xi - \varepsilon u_z) \right] dx \wedge dy \wedge dz \]
\[ - \left[ u(p_\xi - \varepsilon p_z) - p(u_\xi - \varepsilon u_z) \right] dx \wedge dy \wedge d\xi \]
\[ = -\phi^2 L_\zeta \psi (\varepsilon dx \wedge dy \wedge dz + dx \wedge dy \wedge d\xi) = -R (\varepsilon dx \wedge dy \wedge dz + dx \wedge dy \wedge d\xi), \]
\[ D(\zeta) \wedge (\ast F) = -D^*(\zeta) \wedge F = D(\zeta) \wedge A^* \wedge \zeta = -D^*(\zeta) \wedge A \wedge \zeta \]
\[ = \frac{1}{2} \left[ (u^2 + p^2)_\xi - \varepsilon(u^2 + p^2)_z \right] (dx \wedge dy \wedge dz + \varepsilon dx \wedge dy \wedge d\xi). \]

Thus we get

\[ \ast \left[ D(\zeta) \wedge A \wedge \zeta \right] = \ast \left[ D^*(\zeta) \wedge A^* \wedge \zeta \right] = -\varepsilon R \zeta = -i(\ast F) dF = i(F) d(\ast F), \]
\[ \ast \left[ D(\zeta) \wedge A^* \wedge \zeta \right] = -\ast \left[ D^*(\zeta) \wedge A \wedge \zeta \right] = \frac{1}{2} L_\zeta \phi^2 \zeta = i(F) dF = i(\ast F) d(\ast F), \]

where, if \((K, G)\) are respectively a 2-form and a 3-form, we have denoted

\[ i(\bar{K}) G = K^{\mu\nu} G_{\mu\sigma} dx^\sigma, \quad \mu < \nu. \]
5 Discussion

The above relations show various dynamical aspects of the energy-momentum redistribution during evolution of our PhLO in terms of the *intrinsically* defined electromagnetic strains.

Relation (8) shows how the local conservation law $\nabla_\nu T^\nu_F = 0$, being equivalent to $L_\phi^2 = 0$, and meaning that the energy density $\phi^2$ propagates just translationally along $\phi$, can be expressed in terms of the eigen strain components. On the other hand the phase function $\psi$ would satisfy $L_\phi^2 \psi = 0$ ONLY if the introduced quantity $R$ is equal to zero: $R = 0$. Now, what is the sense of $R$ reveals the following

**Proposition.** The following relations hold:

\[
\begin{align*}
&\text{d}A \wedge A \wedge A^* = 0; \quad \text{d}A^* \wedge A^* \wedge A = 0; \\
&\text{d}A \wedge A \wedge \zeta = \varepsilon [u(p_\xi - \varepsilon p_z) - p(u_\xi - \varepsilon u_z)] \omega_o; \\
&\text{d}A^* \wedge A^* \wedge \zeta = \varepsilon [u(p_\xi - \varepsilon p_z) - p(u_\xi - \varepsilon u_z)] \omega_o.
\end{align*}
\]

**Proof.** Immediately verified.

These relations say that the 2-dimensional Pfaff system $(A, A^*)$ is completely integrable for any choice of the two functions $(u, p)$, while the two 2-dimensional Pfaff systems $(A, \zeta)$ and $(A^*, \zeta)$ are NOT completely integrable in general, and the same curvature factor

\[
R = u(p_\xi - \varepsilon p_z) - p(u_\xi - \varepsilon u_z)
\]

determines their nonintegrability. Hence, the condition $R \neq 0$ allows in principle rotational component of propagation, i.e. $u$ and $p$ NOT to be plane waves, so compatibility between the translational and rotational components of propagation is possible in principle.

As we mentioned above it is possible the translational and rotational components of the energy-momentum redistribution to be represented in form depending on the $\zeta$-directed strains $D(\zeta) \wedge A$, and $D^*(\zeta) \wedge A$ in our PhLO. So, the local translational changes of the energy-momentum carried by the two components $F$ and $*F$ of our PhLO are given by the two 1-forms $[D(\zeta) \wedge A]$ and $[D(\zeta) \wedge A]$, and the local rotational ones are given by the 1-forms $[D^*(\zeta) \wedge A \wedge \zeta]$. In fact, the 1-form $[D(\zeta) \wedge A \wedge \zeta]$ determines the strain that tends to "leave" the 2-plane defined by $(A, \zeta)$ and the 1-form $[D^*(\zeta) \wedge A^* \wedge \zeta]$ determines the strain that tends to "leave" the 2-plane defined by $(A^*, \zeta)$. So, the particular kind of nonintegrability of $(A, \zeta)$ and $(A^*, \zeta)$ mathematically guarantees some physical interaction through establishing local dynamical equilibrium (eq.(7)) between two subsystems of the field, that are mathematically represented by $F$ and $*F$.

Since the PhLO is free, i.e. no energy-momentum is lost or gained, this means that the two (null-field) components $F$ and $*F$ exchange locally *equal* energy-momentum quantities:

\[
\ast [D(\zeta) \wedge A \wedge \zeta] = \ast [D^*(\zeta) \wedge A^* \wedge \zeta].
\]

Now, the local energy-momentum conservation law

\[
\nabla_\nu [F_{\mu \sigma} F^{\nu \sigma} + (*F)_{\mu \sigma} (\ast F)^{\nu \sigma}] = 0,
\]

i.e. $L_\phi^2 = 0$, requires the corresponding eigen strain-fluxes $[D(\zeta) \wedge A \wedge \zeta]$ and $[D(\zeta) \wedge A^* \wedge \zeta]$ to become zero:

\[
\ast [D(\zeta) \wedge A^* \wedge \zeta] = - \ast [D^*(\zeta) \wedge A \wedge \zeta] = \frac{1}{2} L_\phi^2 \zeta = i(\bar{F})dF = i(\ast F)d(\ast F) = 0,
\]

and the local dynamical balance between $F$ and $*F$ to hold:

\[
\ast [D(\zeta) \wedge A \wedge \zeta] = \ast [D^*(\zeta) \wedge A^* \wedge \zeta] = -\varepsilon R \zeta = -i(\ast F)dF = i(\bar{F})d(\ast F).
\]
It seems important to note that, only differential relation between the energy-momentum change and natural strain fluxes exists, so NO analog of the assumed in elasticity theory generalized Hooke law, i.e. linear relation between the stress tensor and the strain tensor, seems to exist here. This clearly goes along with the fully dynamical nature of PhLO, i.e. linear relations exist between the divergence terms of our stress tensor
\[
\frac{1}{2} \left[ -F_{\mu\sigma} F^{\mu\sigma} - \ast (\ast F)_{\mu\sigma} (\ast F)^{\mu\sigma} \right]
\]
and the correspondingly directed eigen strain fluxes as given by equations (2,7,8).

The constancy of the translational propagation and the required translational-rotational compatibility suggest also constancy of of the rotational component of propagation, i.e. \( L \tilde{\zeta} \psi = \text{const} \). So, our equations
\[
\left[ D(\tilde{\zeta}) \wedge A^* \wedge \zeta \right] = - \left[ D^*(\tilde{\zeta}) \wedge A \wedge \zeta \right], \quad \left[ D(\tilde{\zeta}) \wedge A \wedge \zeta \right] = \left[ D^*(\tilde{\zeta}) \wedge A^* \wedge \zeta \right],
\]
or in terms of \( F \):
\[
i(\bar{F})dF = i(\ast F)d(\ast F) = 0, \quad i(\ast F)dF + i(\bar{F})d(\ast F) = 0
\]
reduce to
\[
L \tilde{\zeta} \phi^2 = 0, \quad L \tilde{\zeta} \psi = \frac{\kappa}{l_o},
\]
where \( \kappa = \pm 1 \) and \( l_o \) is a parameter of dimension of length. Among the nonlinear solutions of these equations we find out the following:
\[
\begin{align*}
u(x, y, z, \xi) &= \phi(x, y, \xi + \varepsilon z) \cos(-\varepsilon \kappa \frac{z}{l_o} + \text{const}), \\
p(x, y, z, \xi) &= \phi(x, y, \xi + \varepsilon z) \sin(-\varepsilon \kappa \frac{z}{l_o} + \text{const}),
\end{align*}
\]
where \( \phi(x, y, \xi + \varepsilon z) \) is an arbitrary smooth function, so it can be chosen to have 3d-finite spatial support inside an appropriate 3-region such, that at every moment our PhLO to be localized inside an one-step long helical cylinder wrapped up around the \( z \)-axis.

On the two figures below are given two theoretical examples with \( \kappa = -1 \) and \( \kappa = 1 \) respectively, at a fixed moment \( t \). For \( t \in (-\infty, +\infty) \), the amplitude function \( \phi \) fills in a smoothed out tube around a circular helix of height \( 2\pi L_o \) and pitch \( L_o \), and phase function \( \varphi = \cos(\kappa z/L_o) \). The solutions propagate left-to-right along the euclidean coordinate \( z \).

![Figure 1: Theoretical example with \( \kappa = -1 \). The translational propagation is directed left-to-right.](image1)

![Figure 2: Theoretical example with \( \kappa = 1 \). The translational propagation is directed left-to-right.](image2)

### 6 Conclusion

We introduced and discussed a concept of photon-like object(s) (PhLO) as real, massless time stable physical object(s) with an intinsically compatible translational-rotational dynamical structure. So,
PhLO are spatially finite, with every PhLO a straight-line null direction is necessarily associated, and the corresponding stress-energy-momentum tensor must be isotropic: $T_{\mu\nu}T^{\mu\nu} = 0$. We showed that in the frame of the relativistic formulation of vacuum electrodynamics two intrinsically defined strain tensors $D$ and $D^*$ can be introduced as corresponding Lie derivatives of the flat Minkowski metric along eigen vectors of $T_{\nu\mu}$. In terms of the components of these strain tensors can be defined important characteristics of the internal dynamics of PhLO, in particular, internal energy-momentum exchange between the $F$-component and $^*F$-component of the free field was explicitly obtained. Definite relations of the projections of $D$ and $D^*$ along the null direction of translational propagation to the Lie brackets of the electric and magnetic components were also given. We defined amplitude $\phi$ and phase $\psi$ of PhLO and showed that the plane wave character of $\phi$ is admissible and corresponds to the local energy conservation. The NON plane wave character of $\psi$ guarantees the availability of rotational component of propagation, which property turned out to be well defined in terms of corresponding Frobenius curvature. The physical understanding of the Frobenius nonintegrability of some subcodistributions in this context should read: there is internal energy-momentum exchange between two individualized subsystems of PhLO mathematically represented in our case by $F$ and $^*F$.

Assuming constant nature of the rotation we come to an extension of the Riemann-Clifford-Einstein idea for linear relation between energy-density and Riemannian curvature to linear relation between energy-density flow and Frobenius curvature as a measure of nonintegrability.

Finally we note that integrating any of the two 4-forms $\frac{2\pi l_0}{c} dA \wedge A \wedge \zeta$ and $\frac{2\pi l_0}{c} dA^* \wedge A^* \wedge \zeta$ over the 4-volume $\mathbb{R}^3 \times l_0$ gives $\pm E.T$, where $T = \nu^{-1} = \frac{2\pi l_0}{c}$ and $E$ is the integral energy of the solution. This is naturally interpreted as action for one period, and represents an analog of the famous Planck formula $E.T = h$.

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