The strong decays of the light scalar mesons $f_0(500)$ and $f_0(980)$

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The partial width of the decay channels $f_0(500) \to \pi \pi$, $f_0(980) \to \pi \pi$ and $f_0(980) \to K\bar{K}$ are calculated using QCD light-cone sum rules method and a technique of the soft meson approximation. The scalar particles are treated as mixtures of the heavy $|H⟩ = \{[su]  |\bar{s}u⟩ + [sd]  |\bar{s}d⟩\}/\sqrt{2}$ and light $|L⟩ = [ud]  |\bar{u}d⟩$ scalar diquark-antidiquark components. Obtained results for the full width of the $f_0(500)$ meson $Γ_{th} = 434.7 \pm 72.3$ MeV and for the $f_0(980)$ meson $Γ_{th} = 42.12 \pm 6.70$ MeV are compared with the world averages for these parameters, and a reasonable agreement between them is found.

1. Light scalar mesons with masses $m < 1$ GeV form a family of particles, structure and properties of which remain unclear till now and give rise to different models and theories. The standard model of the mesons and baryons that considers mesons as bound states of quarks and antiquarks could not correctly describe the mass hierarchy of these particles. Therefore, the scalars especially $f_0(500)$ and $f_0(980)$ mesons have already been in the spotlight of unconventional theories claiming to solve relevant problems. In most of existing models the light scalar mesons are treated as multi-quark states: These particles were considered as four-quark states $q^2 \bar{q}^2$ [1], or analyzed as meson-meson molecules [2, 3]. Experimental investigation of the light scalars also meets with difficulties. Their masses and widths are known with large uncertainties, which generate additional problems for theoretical studies. Indeed, for example, the mass and full width of the $f_0(500)$ meson is $m = 400 − 550$ MeV and $Γ = 400 − 700$ MeV [4], respectively. The experimental data of this quality almost do not restrict suggested models. The contemporary physics of the light scalars embraces variety of ideas, models and theories, information on which can be found in the reviews [5–8].

The diquark-antidiquark model of the light scalar mesons [1, 9, 10] opened new opportunities for their theoretical studies. This model was used to calculate the spectroscopic parameters and width of the scalar mesons in the context of various computational schemes [11–20]. Because within some of these approaches pure diquark-antidiquark states did not lead to desired predictions for the parameters of the mesons different mixing schemes were introduced to evade emerged discrepancies. In these studies the physical particles were considered as superpositions of diquark-antidiquarks with different flavor structures [17], or as mixtures of diquark-antidiquarks and conventional $qq$ mesons [18–20].

Recently, a suggestion was made to treat the scalar mesons by grouping them into two nonets with masses below and above 1 GeV [21]. In this work the possible mixing of the flavor octet and singlet states inside of each nonet, as well as mixing between states from the different nonets was systematically elaborated. In our work [22] we treated the mesons $f_0(500)$ and $f_0(980)$ from the first nonet of the scalar particles by taking into account the mixing of flavor octet and singlet diquark-antidiquarks by neglecting, at the same time, their possible mixing with tetraquarks composed of the spin-1 diquarks. To this end, we used the heavy-light basis

$$|H⟩ = \frac{1}{\sqrt{2}} \{[su]  |\bar{s}u⟩ + [sd]  |\bar{s}d⟩\} , \quad |L⟩ = [ud]  |\bar{u}d⟩,$$  \hspace{1cm} (1)

and introduced the two-angle mixing scheme to get the physical mesons

$$|f⟩ = U(φ_H, φ_L) |H⟩ , \quad |f'⟩ = U(φ_H, φ_L) |L⟩,$$  \hspace{1cm} (2)

For simplicity in Eq. (2), and in what follows we use the notations $f = f_0(500)$ and $f' = f_0(980)$.

Calculations performed in Ref. [22] using QCD two-point sum rules approach led to the following results for the mixing angles

$$φ_H = −28°.87 ± 0°.42 , \quad φ_L = −27°.66 ± 0°.31.$$  \hspace{1cm} (3)

For masses of the scalar particles we obtained

$$m_f = (518 ± 74) \text{ MeV} , \quad m_{f'} = (996 ± 130) \text{ MeV},$$  \hspace{1cm} (4)

which are in reasonable agreement with the experimental data.
Apart from the masses of the mesons we defined also their couplings
\[
\langle 0| J^i | f(p) \rangle = F^i J m_f, \quad \langle 0| J^i | f'(p) \rangle = F^i J' m_{f'}, \quad i = H, L,
\]
and suggested that they follow the pattern of state mixing
\[
\begin{pmatrix}
F^H_f & F^L_f \\
F^H_{f'} & F^L_{f'}
\end{pmatrix} = U(\varphi_H, \varphi_L) \begin{pmatrix}
F_H & 0 \\
0 & F_L
\end{pmatrix}.
\]
Here \(F_H\) and \(F_L\) can be formally interpreted as couplings of the “particles” \(|H\rangle\) and \(|L\rangle\). Calculations using QCD two-point sum rules allowed us to evaluate them and find
\[
F_H = (1.35 \pm 0.34) \cdot 10^{-3} \text{ GeV}^4, \quad F_L = (0.68 \pm 0.17) \cdot 10^{-3} \text{ GeV}^4.
\]

In the present Letter we extend our investigation of the \(f_0(500)\) and \(f_0(980)\) mesons by analyzing a mechanism of their strong decays and calculate corresponding partial widths. To this end, we use an information on the \(f - f'\) system’s parameters, i. e. on the masses, mixing angles and coupling constants, which were extracted from analysis of the two-point sum rules in Ref. [22] and are not subject to any adjustments. In investigations we employ QCD light-cone sum rule (LCSR) method [23] and technical tools of the soft-meson approximation [24]. It is worth noting that these methods were adapted in Ref. [25] to study strong vertices composed of tetraquarks and two conventional mesons.

2. The dominant strong decay channels of the \(f_0(500)\) and \(f_0(980)\) mesons are the processes \(f_0(500) \rightarrow \pi\pi\) and \(f_0(980) \rightarrow \pi\pi\). The decay \(f_0(980) \rightarrow K\overline{K}\) was also observed and investigated in experiments [4]. Suggestion on the structure of these scalar particles as superpositions of the \(|H\rangle\) and \(|L\rangle\) diquark-antidiquark states has important consequences for analysis of their decays. Indeed, ignoring the mixing phenomenon and assuming that \(f_0(500)\) and \(f_0(980)\) mesons are pure \(|L\rangle\) and \(|H\rangle\) four-quark states one has to introduce different mechanisms to describe decays \(f_0(980) \rightarrow K\overline{K}\) and \(f_0(980) \rightarrow \pi\pi\) : If the first channel runs through the superallowed Okubo-Zweig-Iizuka (OZI) mechanism, the second one can proceeds due to one gluon exchange [14]. The mixing of the \(|H\rangle\) and \(|L\rangle\) states to form the physical particles allows one to treat all of these strong decays on the same footing using the superallowed OZI mechanism. It is known that the full width of the mesons \(f_0(500)\) and \(f_0(980)\), which amount to \(\Gamma = 400 - 700\) MeV and \(\Gamma = 10 - 100\) MeV [4], respectively, suffer from large uncertainties and differ from each other considerably. In the mixing framework this difference finds its natural explanation: As we shall see below the dependence of the strong couplings on the mixing angle \(\varphi_L\) alongside with other parameters that enter to sum rules generates a gap in the partial widths of the scalar particles.

The decay of the \(f_0(500)\) meson to a pair of pions can proceed through the processes \(f_0(500) \rightarrow \pi^+\pi^-\) and \(f_0(500) \rightarrow \pi^0\pi^0\). Let us concentrate on investigation of the mode \(f_0(500) \rightarrow \pi^+\pi^-\). In order to calculate the strong coupling \(g_{f\pi\pi}\) we employ QCD light-cone sum rule method and begin from analysis of the correlation function
\[
\Pi(p, q) = i \int d^4x e^{ipx} \langle \pi^+(q)| T\{J^\pi(x)J^\pi(x')\} |0\rangle,
\]
where \(J^f(x)\) and \(J^\pi(x)\) are the interpolating currents for the \(f\) and \(\pi\) mesons, respectively. In the two-mixing angle scheme \(J^f(x)\) is given by the formula
\[
J^f(x) = J^H(x) \cos \varphi_H - J^L(x) \sin \varphi_L.
\]
Here \(J^H(x)\) and \(J^L(x)\) are the interpolating currents of the scalar mesons’ heavy and light components, respectively. They are defined by means of the following expressions
\[
J^H(x) = \frac{\epsilon_{dabc} \epsilon_{dece}}{\sqrt{2}} \left\{ \left[ u^T_a(x) C \gamma_5 s_b(x) \right] \left[ \overline{u}_c(x) \gamma_5 C \overline{s}_e(x) \right] + \left[ d^T_a(x) C \gamma_5 s_b(x) \right] \left[ \overline{d}_c(x) \gamma_5 C \overline{s}_e(x) \right] \right\},
\]
and
\[
J^L(x) = \epsilon_{dabc} \epsilon_{dece} \left[ u^T_a(x) C \gamma_5 s_b(x) \right] \left[ \overline{u}_c(x) \gamma_5 C \overline{d}_e(x) \right] .
\]
In Eqs. (10) and (11) \(a, b, c, d, e\) are color indices, whereas \(C\) is the charge conjugation operator. We interpolate the pion by means of the pseudoscalar current
\[
J^\pi(x) = \overline{\pi}(x) i \gamma_5 d(x),
\]
with the matrix element defined as
\[ \langle 0 | J^\tau | \pi^- (p) \rangle = f_\pi \mu_\pi, \quad \mu_\pi = -\frac{2 \langle \overline{q} q \rangle}{f_\pi} \]
In Eq. (13) \( f_\pi \) and \( \langle \overline{q} q \rangle \) are the pion decay constant and the quark vacuum condensate, respectively.

The required LCSR can be derived after standard operations: One has to calculate the correlation function employing physical parameters of the involved mesons and equate it to an expression of \( \Pi(p, q) \) obtained in terms of the quark-gluon degrees of freedom. We start from the physical representation of the correlation function \( \Pi(p, q) \) that is given by the formula
\[ \Pi^{\text{phys}}(p, q) = \frac{\langle 0 | J^\pi | \pi^- (p) \rangle}{p^2 - m_\pi^2} \langle \pi^- (p) \pi^+ (q) | f(p') | \langle f(p') | J^f | 0 \rangle + \ldots, \]
where \( p', p \) and \( q \) are four-momenta of the \( f, \pi^- \) and \( \pi^+ \) mesons, respectively. The contribution of the exited states and continuum is denoted in Eq. (14) by dots. The matrix element of the pion that enters to this expression is well known. The element \( \langle f(p') | J^f | 0 \rangle \) can be found by taking into account the structure of the current \( J^f (x) \) and the fact that only its light component contributes to this matrix element \( \langle f(p') | J^f | 0 \rangle = F_L m_f \sin^2 \varphi_L \). We define the matrix element corresponding to the strong vertex in the following manner
\[ \langle \pi^- (p) \pi^+ (q) | f(p') \rangle = g_{f\pi\pi} p \cdot p'. \]
When applying the LCSR method to vertices composed of a tetraquark and two conventional mesons one has to use a technique of the soft-meson approximation [25]. The reason is that the tetraquark contains four valence quarks and contraction with two quark fields from a meson leads to local matrix elements of the remaining light meson. Then the conservation of the four-momentum at the vertex requires fulfilment of the equality \( q = 0 \) (or \( p' = p \)). In other words, in the case of the tetraquark-meson-meson vertex the soft-meson approximation is only way to calculate the corresponding correlation function. For vertices of conventional mesons the correlation function can be expressed in terms of a meson’s distribution amplitudes. This is the full LCSR approach within of which one may employ the soft approximation, as well. For our purposes a decisive fact is the observation made in Ref. [24]: the soft-meson approximation and full LCSR treatment of the conventional mesons’ vertices leads for strong couplings to results that are numerically very close to each other.

In the soft-meson approximation we have to use the one-variable Borel transformation and subtract unsuppressed terms in the physical side of the sum rules. We neglect also the mass one of the final mesons in \( \Pi^{\text{phys}}(p, q = 0) \) and \( \Pi^{\text{OPF}}(p, q = 0) \). Detailed studies of mass effects in exclusive processes prove that they induce only twist-4 contributions to physical quantities under consideration [26]. Hence, in the soft-meson approximation the mass effects are also subleading corrections.

In order to compute the tetraquark-meson-meson vertex we use the one-variable Borel transformation, which for the \( \Pi^{\text{phys}}(p) \) (we use \( \Pi(p) \equiv \Pi(p, 0) \)) leads to the following result
\[ B \Pi^{\text{phys}}(p) = g_{f\pi\pi} f_\pi F_L m_f m_\pi^2 \sin^2 \varphi_L \frac{e^{-m^2/M^2}}{M^2} + \ldots, \]
where \( m^2 = (m^2_f + m_\pi^2)/2 \) and \( M^2 \) is the Borel parameter. In Eq. (14) the dots stand for the contribution of the excited and continuum states, among of which there exist terms that in the soft limit even after the Borel transformation remain unsuppressed relative to the ground-state’s contribution [24]. In the case under consideration we are interested only in the ground-state term therefore these unsuppressed contributions should be removed from Eq. (14). But before performing necessary operations we calculate the \( \Pi^{\text{OPF}}(p) \) and find
\[ \Pi^{\text{OPF}}(p) = \sin \varphi_L \int d^4 x e^{ip \cdot x} \epsilon^{abc} \epsilon_{\alpha\beta} \gamma_5 \delta_{\alpha}^a (x) \gamma_5 \delta^b_\alpha (-x) \gamma_5 \delta^c_\beta \langle \pi^- \pi^+ | 0 \rangle | d^4 \phi_\text{a}(0) d^4 \phi_\text{b}(0) | 0 \rangle. \]
Computations of \( \Pi^{\text{OPF}}(p) \) using the pion local matrix elements in accordance with prescriptions explained in rather detailed form in Ref. [26], and the Borel transformation of the obtained result give
\[ \Pi(M^2) = \frac{f_\pi \mu_\pi}{16 \pi^2} \sin \varphi_L \int_0^\infty ds e^{-s/M^2} s + \frac{\alpha_s G^2}{\pi} \sin \varphi_L \frac{f_\pi \mu_\pi}{16}. \]
In order to perform the continuum subtraction in Eq. (13) one has to remove the unsuppressed terms from the \( B \Pi^{\text{phys}}(p) \) which can be fulfilled by applying the operator [27]
\[ \mathcal{P}(M^2, m^2) = \left( 1 - M^2 \frac{d}{dM^2} \right) M^2 e^{m^2/M^2}. \]
Then for the strong coupling \( g_{f\pi\pi} \) we get

\[
g_{f\pi\pi} = \frac{1}{\sin \varphi_L} \frac{1}{f_{F_{L\mu\pi}} m_{f} m_{\pi}} \mathcal{P}(M^2, m^2) \widetilde{\Pi}(M^2, s_0),
\]

where

\[
\widetilde{\Pi}(M^2, s_0) = \frac{f_{F_{L\mu\pi}}}{16 \pi^2} \int_0^{s_0} ds e^{-s/M^2} s + \left( \frac{\alpha_s G^2}{\pi} \right) \frac{f_{\pi \mu \pi}}{16}.
\]

The analysis of the process \( f_0(500) \to \pi^0 \pi^0 \) does not differ considerably from calculations presented above the difference being encoded in the current of the \( \pi^0 \) meson.

3. The decays of the meson \( f_0(980) \) to \( \pi \pi \) and \( K \overline{K} \) pair proceed by the same superallowed OZI mechanism. In the case of the process \( f_0(980) \to \pi \pi \) the \( (L) \) component of \( f_0(980) \) determines the decays \( f_0(980) \to \pi^+ \pi^- \) and \( f_0(980) \to \pi^0 \pi^0 \). For these channels a situation does not differ from the decays \( f_0(500) \to \pi \pi \): One needs to replace in Eq. \( (20) \) \( \sin \varphi_L \to -\cos \varphi_L \), \( m_f \to m_f' \), and set \( m^2 = (m_f^2 + m_s^2)/2 \). This modifications and properly chosen parameters \( M^2 \) and \( s_0 \) are enough to perform numerical analysis of the decay channels \( f_0(980) \to \pi^+ \pi^- \) and \( f_0(980) \to \pi^0 \pi^0 \), and find their partial widths.

Investigation of the strong decays \( f_0(980) \to K \overline{K} \) actually implies analysis of the following two decay modes \( f_0(980) \to K^+ K^- \) and \( f_0(980) \to K^0 \overline{K}^0 \). Naturally, all of these channels run through decays of the \( f_0(980) \) meson’s heavy component \( (H)\). Let us consider in some details the process \( f_0(980) \to K^+ K^- \). The correlation function necessary to study this decay is

\[
\Pi_K(p, q) = i \int d^4 x e^{ipx} \langle K^+(q) | \mathcal{T} \{ J^K(x) J^{\dagger}(0) \} | 0 \rangle,
\]

where the interpolating current for the \( f_0(980) \) meson is

\[
J^K(x) = J^H(x) \sin \varphi_H + J^L(x) \cos \varphi_L.
\]

For \( K \) mesons we use the pseudoscalar current

\[
J^K(x) = \pi(x) i \gamma_5 s(x),
\]

with the matrix element

\[
\langle 0 | J^K | K^-(p) \rangle = \frac{f_K m_K^2}{m_s + m_H}.
\]

Skipping details of calculations that are similar to ones presented above we write down final expressions: Thus, for \( \Pi^{OPE}_K(p, q) \) we get

\[
\Pi^{OPE}_K(p, q) = -\sin \varphi_H \int d^4 x e^{ipx} \frac{\epsilon_{abc} \epsilon_{dec}}{\sqrt{2}} \left[ \gamma_5 \tilde{S}_5(x) \gamma_5 \tilde{S}_a(-x) \right]_{\alpha \beta} \langle K^+(q) | \mathcal{T} \{ J^K_0(x) J^K_0(0) \} | 0 \rangle.
\]

The final expression for the strong coupling \( g_{f'KK} \) is

\[
g_{f'KK} = -\frac{1}{\sin \varphi_H f_K f_H m_K^2 m_f' m_f^2} \mathcal{P}(M^2, m^2) \Pi_K(M^2, s_0),
\]

where \( m^2 = (m_f^2 + m_K^2)/2 \) and

\[
\Pi_K(M^2, s_0) = \frac{f_K m_K^2}{16 \sqrt{2} m_s \pi^2} \int_0^{s_0} ds e^{-s/M^2} s - \left( \frac{2 \langle \bar{s}s \rangle - \langle \bar{u}u \rangle} {12 \sqrt{2}} \right) f_K m_K^2 + \left( \frac{\alpha_s G^2}{\pi} \right) \frac{f_K m_K^2}{16 \sqrt{2} m_s}.
\]

The strong couplings \( g_{f'KK} \) and \( g_{f'K^0 \overline{K}^0} \) provide necessary information for computing the \( f_0(980) \to K^+ K^- \) and \( f_0(980) \to K^0 \overline{K}^0 \) decays’ widths.

4. In calculations we utilize the light quark propagator (see, Ref. \( (22) \)) and use for the quark and gluon condensates the following values: \( \langle \bar{q}q \rangle = -0.24 \pm 0.01 \) \( \) GeV\(^3\), \( \langle \bar{s}s \rangle = 0.8 \) \( \) GeV\(^3\), \( \langle \bar{g}g \rangle = 0.012 \pm 0.004 \) \( \) GeV\(^3\). Apart from these parameters we also employ the masses of the light quarks \( m_u = m_d = 0 \) and \( m_s = 128 \pm 10 \) MeV, as well as the masses and decay constants of the \( \pi \) and \( K \) mesons: for the pion \( m_{\pi^0} = 139.57061 \pm 0.00024 \) MeV,
For the couplings and partial decay widths we find
\[ M^2 = (0.7 - 1.2) \text{ GeV}^2, \quad s_0 = (0.9 - 1.1) \text{ GeV}^2. \] (29)

Calculations of the strong couplings lead to the predictions
\[ g_{f\pi\pi} = 33.94 \pm 3.86 \text{ GeV}^{-1}, \quad |g_{f\pi^0\pi^0}| = 32.76 \pm 3.56 \text{ GeV}^{-1}. \] (30)

As a result, for the partial decay width of the processes \( f_0(500) \to \pi^+\pi^- \) and \( f_0(500) \to \pi^0\pi^0 \) we find
\[ \Gamma \left[ f_0(500) \to \pi^+\pi^- \right] = 223.5 \pm 53.7 \text{ MeV}, \quad \Gamma \left[ f_0(500) \to \pi^0\pi^0 \right] = 211.2 \pm 48.4 \text{ MeV}. \] (31)

The full width of the meson \( f_0(500) \) is formed almost entirely due to the decay channel \( f_0(500) \to \pi\pi \) because the width of the mode \( f_0(500) \to \gamma\gamma \) is very small. It seems reasonable to compare \( \Gamma_{\text{th}} = 434.7 \pm 72.3 \text{ MeV} \) which is the sum of two partial decay widths [31] with the available information on \( \Gamma = 400 - 700 \text{ MeV} \) noting existence of an overlapping region of these results. As we have pointed out, data for the full width of the light scalar mesons suffer from large uncertainties. Therefore, we can only state that our theoretical prediction is compatible with experimental data.

The strong decays of the \( f_0(980) \) meson can be analyzed in the same manner. The differences between the channels \( f_0(500) \to \pi\pi \) and \( f_0(980) \to \pi\pi \) appear due to the spectroscopic parameters of the involved mesons, and regions chosen for the Borel parameter and continuum threshold. In the case of the \( f_0(980) \) meson’s decays we use
\[ M^2 = (1.1 - 1.5) \text{ GeV}^2, \quad s_0 = (1.3 - 1.5) \text{ GeV}^2. \] (32)

Then for the couplings and partial decay widths we find
\[ g_{f\pi\pi} = 3.02 \pm 0.35 \text{ GeV}^{-1}, \quad g_{f\pi^0\pi^0} = 3.75 \pm 0.45 \text{ GeV}^{-1}, \]
\[ |g_{fKK}| = 4.29 \pm 0.75 \text{ GeV}^{-1}, \quad |g_{f\pi^0\pi^0}| = 4.97 \pm 0.98 \text{ GeV}^{-1}, \] (33)

and
\[ \Gamma \left[ f_0(980) \to \pi^+\pi^- \right] = 14.36 \pm 3.31 \text{ MeV}, \quad \Gamma \left[ f_0(980) \to \pi^0\pi^0 \right] = 22.19 \pm 5.64 \text{ MeV}, \]
\[ \Gamma \left[ f_0(980) \to K^+K^- \right] = 3.98 \pm 1.04 \text{ MeV}, \quad \Gamma \left[ f_0(980) \to K^0\bar{K}' \right] = 1.59 \pm 0.47 \text{ MeV}. \] (34)

In calculations we have utilized the different working regions for the Borel parameter \( M^2 \) and continuum threshold \( s_0 \). We have chosen these regions using standard requirements of the sum rules computations. It is known that a stability of the obtained results on \( M^2 \) and \( s_0 \) is one of the important constraints imposed on these auxiliary parameters. We demonstrate in Fig. [3] as a sample the variation of the coupling \( |g_{fKK}| \) on the \( M^2 \) and \( s_0 \). One can see that \( |g_{fKK}| \) depends on \( M^2 \) and \( s_0 \), which is a main source of uncertainties of the evaluated quantities. It is also clear that these ambiguities are less than 30% of the central values which is acceptable for the sum rule computations.

It is remarkable that there are valuable experimental information and independent theoretical predictions for the coupling \( g_{fKK} \). It was extracted from different processes, and calculated by means of numerous methods. Thus, from analysis of the radiative decay \( \phi \to f_0\gamma \) the CMD-2 and SND collaborations found \( g_{fKK} = 4.3 \pm 0.5 \text{ GeV} \) and \( 5.6 \pm 0.8 \text{ GeV} \) [28, 29], respectively. The KLOE Collaboration used the same process and from two different fits extracted the following values \( g_{fKK} = 4.0 \pm 0.2 \text{ GeV} \) and \( 5.9 \pm 0.1 \text{ GeV} \) [30]. Our result for \( g_{fKK} \) can be easily converted to a form suitable for comparison with these experimental data, and is equal to \( 4.12 \pm 0.72 \text{ GeV} \). As is seen, our prediction for the strong coupling \( g_{fKK} \) is in a reasonable agreement with this experimental information. At the same time, it overshoots experimental data extracted from other processes such as \( D_1^+ \to \pi^-\pi^+\pi^+ \) decay and \( pp \) interactions, where the coupling \( g_{fKK} \) was found equal to \( 0.5 \pm 0.6 \text{ GeV} \) and \( 2.2 \pm 0.2 \text{ GeV} \) (see, Refs. [31] and [32]), respectively.

The theoretical predictions for \( g_{fKK} \) appear to vary within wide limits and depend on a model accepted for \( f_0(980) \) and on methods used in investigations. For example, in Ref. [33] it was found equal to \( g_{fKK} = 3.8 \text{ GeV} \), whereas in Ref. [34] the authors predicted \( 6.2 \leq g_{fKK} \leq 7.8 \text{ GeV} \). The latter estimation was obtained in the context of the full LCSR method by modeling \( f_0(980) \) as a scalar meson with a \( \bar{s}s \) component. As it was emphasized by the authors, their result is larger than previous determinations. It is also larger than our prediction for \( g_{fKK} \) the reason being connected presumably with a mixing factor of the \( \bar{s}s \) component neglected in computations. Information on other theoretical studies and references to corresponding articles can be found in Ref. [34].
Using results presented in Eq. (34) we are able to evaluate the width of the decays \( \Gamma [f_0(980) \rightarrow \pi \pi] = 36.55 \pm 6.54 \) MeV and \( \Gamma [f_0(980) \rightarrow K\bar{K}] = 5.57 \pm 1.48 \) MeV. By neglecting the contribution \( \Gamma [f_0(980) \rightarrow \gamma \gamma] \) for the full width of the meson \( f_0(980) \) we find \( \Gamma_{\text{th}} = 42.12 \pm 6.70 \) MeV, which is in accord with the experimental data.

5. The partial and full widths of the scalar mesons \( f_0(500) \) and \( f_0(980) \) obtained in the present work by treating them as the mixtures of the different diquark-antidiquark components seem are in reasonable agreement with existing experimental data. Because there are great discrepancies between results of different experiments, we compare our predictions with the world average for these parameters presented by the Particle Data Group in Ref. [4]. Thus, the full width of the \( f_0(500) \) meson is slightly larger than the lower bound of the experimental data: There is small overlap region between the theoretical and experimental results. For the \( f_0(980) \) meson we have found \( \Gamma_{\text{th}} = 55 \pm 6 \) MeV, which is in a nice agreement with the data. Another parameter \( R_T = \Gamma(\pi\pi)/[\Gamma(\pi\pi) + \Gamma(K\bar{K})] = 0.87^{+0.06}_{-0.08} \) provides an information on partial decay widths of the meson \( f_0(980) \) and on its strange and non-strange components. The prediction for \( R_T \) agrees with the upper limit for this parameter from Ref. [4].

As is seen, the model of the light scalar mesons \( f_0(500) \) and \( f_0(980) \) based on the mixing of the diquark-antidiquark states leads to the results that are in agreement with the world averages for their full widths. Nevertheless, some effects which have been neglected in the present investigation, namely possible mixing with the mesons from the second (heavier) scalar nonet, as well as \( f_0(980) - a_0(980) \) mixing may improve our predictions.

The strange and non-strange quark contents of the \( f_0(500) \) and \( f_0(980) \) mesons also need additional investigations. In fact, the model accepted here implies that both the mesons \( f_0(500) \) and \( f_0(980) \) have the strange and non-strange components. The existence of sizeable non-strange content in the \( f_0(980) \) meson does not contradict to experimental measurements. But the strange component of the \( f_0(500) \) meson, as it was pointed out in Ref. [35], may cause difficulties in interpretation of existing data. In fact, in Ref. [35] the \( f_0(500) \) and \( f_0(980) \) mesons were modeled as mixtures of strange \( \bar{s}s \) and non-strange \( (\bar{u}d)/\sqrt{2} \) parts. In this model the ratio \( \Gamma[D^+_s \rightarrow f_0(500)\pi^+] / \Gamma[D^+_s \rightarrow f_0(980)\pi^+] \) depends on the mixing angle that has to be extracted from experimental measurements. But the E791 Collaboration did not observe a contribution of the process \( D^+_s \rightarrow f_0(500)\pi^+ \) to the decay \( D^+_s \rightarrow \pi^-\pi^+\pi^+ \) [31], which contradicts to the theoretical assumption on the strange component of the meson \( f_0(500) \). This experiment predicted for the strong coupling \( g_{\pi^+K\bar{K}} = 0.5 \pm 0.6 \) GeV, which contradicts also to all other measurements. The model used in the present work differs from the framework introduced in Ref. [35]. Therefore, to clarify a situation with \( f_0(500) \) meson’s strange component the decays \( D^+_s \rightarrow f_0(500)\pi^+ \) and \( D^+_s \rightarrow f_0(980)\pi^+ \) should be studied within this new model. For comparison to theoretical predictions more precise experimental data are required, as well.

There are no doubts, that the light scalar mesons as unusual particles deserve further detailed theoretical and experimental studies.

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