Quantifying stellar radial migration in a N-body simulation: blurring, churning, and the outer regions of galaxy discs

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ABSTRACT

Radial stellar migration in galactic discs has drawn a lot of attention in studies of galactic dynamics and chemical evolution, but remains a dynamical phenomenon that needs to be fully quantified. In this work, using a Tree-SPH simulation of a spiral galaxy, we quantify the effects of blurring (epicyclic excursions) and churning (change of guiding radius). We quantify migration (either blurring or churning) both in terms of flux (the number of migrators passing at a given radius), and by estimating the population of migrators at a given radius at the end of the simulation compared to non-migrators, but also by giving the distance over which the migration is effective at all radii. We confirm that the corotation of the bar is the main source of migrators by churning in a bar-dominated galaxy, its intensity being directly linked to the episode of strong bar, in the first 1-3 Gyr of the simulation. We show that within the OLR, migration is strongly dominated by churning, while blurring gains progressively more importance towards the outer disc and at later times. Most importantly, we show that the OLR acts as a fundamental barrier separating the disc in two distinct parts with no exchange, except in the transition zone delimited by the position of the OLR at the epoch of the formation of the bar, and at the final epoch. We discuss the consequences of these findings for our understanding of the structure of the Milky Way disc. Because of the Sun being situated slightly outside the OLR, we suggest that the solar vicinity may have experienced very limited churning from the inner disc.

Key words. Galaxies: formation — Galaxies: evolution — Galaxies: spiral — Galaxies: structure — Galaxy: stellar content

1. Introduction

Radial stellar migration in the galactic discs has been attracting an increasing attention, including some theoretical (Sellwood & Binney 2002 Minchev & Famaey 2010 Daniel & Wyse 2014) or numerical work (Brunetti et al. 2011 Minchev et al. 2011 2012b Di Matteo et al. 2013 Roskar et al. 2013 Vera-Ciro et al. 2014) and studies based on observations of the stars of the Milky Way (Haywood 2008 Schönherr & Binney 2009 Yu et al. 2012 etc).

In galactic discs where non-axisymmetric potential perturbations such as bars or spiral arms occur, stars can gain or lose angular momentum (Lynden-Bell & Kalnajs 1972 e.g.), leading to outward, respectively inward migration. The result of this mechanism is that stars can be found at a galactocentric radius differing significantly from their birth radius. Another reason for apparent radial migration is simply the nature of orbits in axisymmetric or nearly axisymmetric potentials: stars oscillate radially around a guiding radius, the amplitude of the oscillations increasing with the radial velocity dispersion. Sellwood & Binney (2002) studied the impact of transient spiral arms on the change in angular momentum of stars in N-body simulations. They found that stars nearly at corotation with a spiral pattern experience the largest changes in angular momentum, larger than what can be experienced by stars at the Lindblad resonances of the spiral pattern or in the rest of the disc. This prominent effect of corotation was confirmed, in particular by Roskar et al. (2012) in N-body simulations exhibiting several spiral patterns amongst which the strongest one causes large changes in angular momentum for stars around its corotation radius. Other studies have shown the influence of the bars on radial migration (Di Matteo et al. 2013 Brunetti et al. 2011 Kubryk et al. 2013 e.g.). In the case where both a bar and spiral arms are present, the bar-spiral resonance overlap studied by Minchev & Famaey (2010) Minchev et al. (2011 2012b) can produce some non-linear coupling that can generate larger changes in angular momentum. Besides, while angular momentum can occur at corotation without any increase in radial kinetic energy, resonance overlap is expected to increase radial energy: stars that were originally on nearly circular orbits can thus have significantly more eccentric orbits once they have experienced a change in angular momentum due to a resonance overlap.

As it redistributes stars over the disc of a galaxy, radial migration has been invoked as a possible explanation to a number of unanswered issues of galactic and extra-galactic stellar populations: the upturn of the mean age in the outer parts of discs (Roskar et al. 2008 and e.g. Bakos et al. 2008; Zheng et al. 2014), the formation of the Galactic thick disc (Schönherr & Binney 2009 Loebman et al. 2011), the dispersion in metallicity at a given age observed on solar vicinity stars (Haywood 2008 Schönherr & Binney 2009).
Each of these problems requires a different level of migrations. For instance, U-shaped age profiles would require that stars from the inner disc of a galaxy are massively present in the outer regions. Similarly, shifting the mean solar vicinity metallicity towards higher values, as it has been suggested to explain the flatness of the age-metallicity relation at the solar vicinity (Loebman et al. 2011), would require massive migration of stars to the solar radius. Explaining the range of metallicities in the solar vicinity is a more subtle problem, because it may require only very little migration as it depends on the local (6-10 kpc) gradient of metallicity. If the gradient is steep, the presence of high-metallicity stars found at the solar radius may be explained by blurring alone, without any significant churning. That the gradient is steep is suggested by the data, which now heavily indicates that the Sun is at the interface between an inner and an outer disc, with different chemical properties, as advocated in Haywood et al. (2013), and confirmed on more extended data by Nidever et al. (2014). The recent results found for stars at the solar radius seem thus to question the necessity of strong migration by churning to explain the metallicity distribution found.

A common prediction of N-body models of galaxy evolution is the significant migration of inner disc stars in the external parts of galaxy discs (see for example Roskar et al. 2008; Minchev et al. 2011, 2014). In the case of the outer disc of the Milky Way, which contains stars with guiding radii greater than ~10 kpc – some of them with pericentres small enough to reach the solar vicinity – there is no evidence that this extreme migration has ever occurred, at any time in the last 10 Gyr. At every age, stars of the outer disc are more metal poor than inner disc stars and more alpha-enhanced (Haywood et al. 2013), pointing out to the finding that the outer Galactic disc must have followed a different chemical evolutionary path (Snaith et al. 2014). Even without having access to age informations, large scale spectroscopic surveys like APOGEE are confirming the substantial difference between the inner and the outer Milky Way disc, as it can be evidenced by the properties of their stars in the [α/Fe]-[Fe/H] plane or by the change of the metallicity distribution function with distance from the Galaxy centre (Anders et al. 2014 Nidever et al. 2014). What is striking in this context is that, whatever the nature and formation of the outer disc of the Milky Way have been, stars from the inner thin disc have not been able to reach and pollute the outer parts of the Galactic disc, when models of radial migration would predict this as a common scenario. This casts some doubts that migration due to the present bar/spiral can be at the origin of the U-shaped age profiles observed. However, it is possible that the first pattern that was occurring in the galaxies had a speed lower than today, and that the OLR was at a larger radii, extending the churning action at a larger radii. During the mass assembly of galaxies, the concentration of mass increases, and consequently the pattern speed increases, implying a smaller OLR radius with time.

Also the characteristics of the thick disc of the Milky Way hardly require any significant migration, and they can be well explained by formation from a disc rich in gas and turbulent (Haywood et al. 2013 Lehmer et al. 2014, Haywood et al. 2015). On the theoretical side, the uncertainties are equally important, and although phenomenological radial migration is now implemented in several Galactic chemical evolution models (Schönrich & Binney 2009 Kubryk et al. 2014), the basic effects of migration are still not quantified. For example, what fraction of stars are subject to migration and on which distances? Is the migration episodic or continuous? How are the different parts of the disc affected by migration?

In this paper, we are interested in distinguishing between the effects of ‘blurring’, i.e. the radial migration that is due to epicyclic excursions around a fixed guiding radius, and ‘churning’, i.e. radial migration due to a change in this guiding radius (according to the terminology of Schönrich & Binney (2009)). In particular, we seek to quantify the fraction of the stars involved in the migration, the extent of the migration, the fraction of migrants at a given galactocentric radius and the regions affected by the process. The study is performed using a N-body+SPH simulation of a Sb type galaxy first presented in (Halle & Combes 2013). A few results on radial migration for this simulations were presented in the Appendix of Di Matteo et al. (2014). In the present paper, we first present briefly the parameters of the simulation in Section 2. In Section 3 we study the density resonances and in Sect. 4 we quantify the effects of blurring and churning, their strength and extent. Section 5 discusses the behaviour of migrants near the outer Lindblad resonance, and in particular its role as a barrier for migrants. Finally the kinematic characteristics of migrating stars are discussed in Sect. 6, and in Sect. 7 we present our conclusions.

2. Numerical simulation

In this paper, we use one of the simulations presented in Halle & Combes (2013). We briefly sum up the main characteristics of the simulations. They were performed with the N-body SPH code Gadget-2 (Springel 2005) and include stochastic star formation following a Schmidt law, kinetic feedback from core-collapse supernovae and some detailed cooling of the gas down to 100 K, with the possibility of including cooling by molecular hydrogen, whose local mass fraction is computed based on a semi-analytic recipe from Krumholz et al. (2008, 2009). We briefly sum up the main characteristics of the simulations. They were performed with the N-body SPH code Gadget-2 (Springel 2005) and include stochastic star formation following a Schmidt law, kinetic feedback from core-collapse supernovae and some detailed cooling of the gas down to 100 K, with the possibility of including cooling by molecular hydrogen, whose local mass fraction is computed based on a semi-analytic recipe from Krumholz et al. (2008, 2009). We briefly sum up the main characteristics of the simulations. They were performed with the N-body SPH code Gadget-2 (Springel 2005) and include stochastic star formation following a Schmidt law, kinetic feedback from core-collapse supernovae and some detailed cooling of the gas down to 100 K, with the possibility of including cooling by molecular hydrogen, whose local mass fraction is computed based on a semi-analytic recipe from Krumholz et al. (2008, 2009). We briefly sum up the main characteristics of the simulations. They were performed with the N-body SPH code Gadget-2 (Springel 2005) and include stochastic star formation following a Schmidt law, kinetic feedback from core-collapse supernovae and some detailed cooling of the gas down to 100 K, with the possibility of including cooling by molecular hydrogen, whose local mass fraction is computed based on a semi-analytic recipe from Krumholz et al. (2008, 2009). We briefly sum up the main characteristics of the simulations. They were performed with the N-body SPH code Gadget-2 (Springel 2005) and include stochastic star formation following a Schmidt law, kinetic feedback from core-collapse supernovae and some detailed cooling of the gas down to 100 K, with the possibility of including cooling by molecular hydrogen, whose local mass fraction is computed based on a semi-analytic recipe from Krumholz et al. (2008, 2009).
Table 1. Sb galaxy parameters

| Sb | M_b | M_d | M_b | M_d | T | a_d | T | a_g | T | h_b | T | a_g | T | h_g |
|----|-----|-----|-----|-----|---|-----|---|-----|---|-----|---|-----|---|-----|
| 1.7 \times 10^{10} | 4.5 \times 10^{10} | 1.1 \times 10^{10} | 0.9 \times 10^{10} | 12 | 5 | 0.5 | 1 | 11.8 | 0.2 |

Table 2. Resolution of the simulation

| \epsilon | h | m_{DM} | m_\star | m_g |
|---------|---|-------|-------|-----|
| 100 \geq \epsilon | 3.7 \times 10^5 | 1.4 \times 10^5 | 2.5 \times 10^4 |

The masses of the different components and density profiles parameters are shown in Tables 1. There are initially 400,000 particles of each component with particle masses of \( m_{DM} \), \( m_\star \) and \( m_g \) for, respectively, the dark matter, stars and gas, shown in Table 2. The softening length \( \epsilon \) used in the gravitational force computation and the smoothing length \( h \) used for hydrodynamics are also described in Table 2. Particles velocities have been assigned in the following way (see Halle & Combes (2013) for details). For gaseous and stellar disc particles, we computed circular velocities from gravitational accelerations and applied an analytic asymmetric drift correction to have a more realistic profile. Radial velocity dispersions were derived as

\[
\sigma_r(r) = \frac{3.36G\rho(r)}{\kappa(r)} \]

with \( \kappa(r) \) and \( \rho(r) \), respectively, the epicyclic frequency and the surface density at radius \( r \), and adopting a Toomre parameter \( Q \) equal to 1. The azimuthal velocity dispersions \( \sigma_\theta(r) \) are computed from the radial velocity dispersions \( \sigma_r(r) \), using the epicycle approximation, and the vertical velocity dispersion \( \sigma_z(r) \) is derived by assuming isothermal equilibrium for the discs. For the spheroidal components, the velocity dispersion is isotropic and derived from the second moment of the Jeans equation.

The simulated time-span used in this paper is of 9 Gyr, which is similar to the estimated time-span during which the Milky Way has been evolving with no major mergers (Haywood et al. 2013; Hammer et al. 2007, e.g.). Figure 1 shows the surface density maps of different components of the disc (gas, all stars, and new stars formed during the simulation). A bar is formed during the first Gyr. In this simulation, the feedback efficiency is of 10% and there is no molecular hydrogen cooling. With these parameters, the surface density of gas and stars are relatively smooth because this feedback efficiency and the absence of molecular hydrogen cooling prevent gas and stellar clumps from forming. Note that, compared to many previous studies (for example Minchev et al. 2011; Di Matteo et al. 2013), the stellar and gaseous discs are very extended (initial conditions are generated with an initial cut at \( R = 36 \) kpc for both gas and stars). This choice allows to study at all times also regions outside the outer Lindblad resonance (hereafter OLR), which in this model is located between 11 and 23 kpc from the centre (see next sections). Figure 2 shows the azimuthally averaged surface density of the simulated disc components as a function of the galactocentric radius. The old stellar disc dominates the surface density at all radii throughout the simulated time-span. It can be noted also that its surface density profile is very stable, with only mild departures from the initial conditions, due to the presence of the bar and spiral arms formed during the simulation.

3. Resonances in the disc

The disc forms a central bar that persists during the whole simulated time-span. Transient spiral arms are also present. We determine the pattern speeds through a classic Fourier method:

- At times spaced by a constant interval of 10 Myr, we perform a spatial Fourier transform of the surface density \( \Sigma(R,\theta) \) of the stars to obtain the dominant azimuthal modes in the different radial bins.

\[
S_m(R) \propto \int_0^{2\pi} \Sigma(R,\theta)e^{im\theta} d\theta
\]  \hspace{1cm} (3)

- We then perform a time Fourier analysis of the different modes in each radius bin to study their azimuthal speed.

\[
T_m(R,\omega) \propto \int_t^{t+\tau} S_m(R)e^{i\omega t} dt
\]  \hspace{1cm} (4)

In the spatial Fourier transforms, the \( m = 2 \) mode usually dominates, which corresponds to \( \pi \)-periodic patterns: the central bar or two-armed spirals. The contours of the obtained power in the \( R-\Omega \) plane for this mode are shown on Figure 3 for time Fourier integration performed on total time intervals of 9 Gyr (first panel) and of 1 Gyr centred on 1, 3, 5, 7 and 9 Gyr. \( \Omega = \frac{\omega}{m} \), with \( m = 2 \) here, is the pattern...
Fig. 1. Surface density maps of the gas (left), stars formed during the simulation (middle) and all the stars (right). Face-on view boxes are [40 kpc × 40 kpc] large and edge-on views are [40 kpc × 10 kpc] large. The colour scale, shown in the top right corner, is the same for all the views.
Fig. 3. Spectrograms of the $m = 2$ Fourier mode. Top left plot: integration on 9 Gyr from 0.5 Gyr to 9.5 Gyr. Other plots: integrations on 1 Gyr centred on the times specified on the plots. The black curves are $\Omega(R)$ (solid), $\Omega(R) - \frac{\kappa(R)}{2}$ (dot-dashed), $\Omega(R) + \frac{\kappa(R)}{2}$ (dashed). The horizontal line is the estimation of the bar pattern speed. The vertical lines are the estimations of the bar ILR (blue), corotation (red) and OLR (green) radii. Speed. The integration on 9 Gyr shows that the contours at low radii, which correspond to the bar, fill a $\Omega$ range from $\simeq 10$ to $\simeq 30$ km/s/kpc. The time interval of 1 Gyr on which the time Fourier transform is then performed around specific times is chosen so as to optimise the determination of the pattern speeds: for a given time resolution, a too short integration time prevents from correctly determining small pattern speeds and yields a poor resolution in frequency, while a too large one leads to a determination of overly averaged pattern speeds if they change during the integration time. It can indeed be seen on Figure 3 that the bar slows down with time. The bar transfers angular momentum to the rest of the disc and to the stellar bulge and dark matter halo, as already discussed, among others, by Debattista & Sellwood (1998); Athanassoula (2002); Martinez-Valpuesta et al. (2006); Saha et al. (2012); Saha & Naab (2013); Di Matteo et al. (2014). The angular momentum transfer between the different components of the galaxy is detailed on Fig. 4. At the beginning of the simulation, the angular momentum is contained in the disc because the DM halo and stellar bulge have no initial rotation. In 9 Gyr, the disc loses 8% of its angular momentum that is transferred to the DM halo, while a slight fraction is given to the bulge.

The contours of Fig. 3 show the main peaks of power in the $\Omega$-$R$ plane. The bar (power peak at low radii) is clearly the strongest $\pi$-periodic surface density perturbation with a rigid-body rotation on the integration periods of 1 Gyr, but spiral arms (peaks at larger radii) are visible too. For example, the panel corresponding to $t = 3$ Gyr suggests the presence of spiral patterns rotating at $\simeq 18$ km/s/kpc and $\simeq 7$ km/s/kpc. These resonant modes with pattern speeds such that some of the resonance radii coincide might be sustained by non-linear mode coupling (Tagger et al. 1987 e.g.). Some higher $m$-periodic features are also present ($m = 3, 4, \ldots$) with various pattern speeds, but their strength is smaller than the one of the bar in the period from 1 to 9 Gyr.

By determining the angular speeds corresponding to the main contours, we obtain the radii at which stars on circular orbits are in resonance with the main patterns. We compute the angular speed $\Omega$ from the potential by $\Omega^2 = \frac{1}{R} \frac{d^2 \phi}{dR^2} |_{z=0}$ and the epicyclic frequency $\kappa$ by $\kappa^2 = \left( \frac{d^2 \phi}{dR^2} + \frac{3}{R} \frac{d \phi}{dR} \right) |_{z=0}$. The corotation resonance (CR) occurs for $\Omega = \Omega_P$, the inner Lindblad resonance (ILR) for $\Omega - \Omega_P = \frac{\kappa}{2}$ and the outer Lindblad resonance (OLR) for $\Omega - \Omega_P = -\frac{\kappa}{2}$. By a number of time Fourier transforms on 1 Gyr around times separated by 100 Myr, we estimate the values of the ILR, CR and OLR radii of the bar as a function of time (see Fig. 3). The bar slow down makes the bar corotation radius larger and allows for the bar to become more elongated. A measure of the bar strength using the same spatial Fourier decomposition of coefficients $A_m$ at each time is shown on Fig. 5. The bar strength increases until $t = 3$ Gyr when it drops before increasing again due to angular momentum exchange with the DM halo (Athanassoula 2002).
4. Blurring and churning

In this section, we quantify different types of radial stellar migration. We examine:

- migration in terms of galactocentric radius (hereafter simply radius), i.e. the difference of radius between two different times. Blurring and churning can both cause a change in galactocentric radius of a stars, thus this definition captures both types of migrators and can be considered as the overall migration experienced by disc stars. The comparison between radii at two different times during disc evolution is widely used in the literature, see for example Roškar et al. (2008); Brunetti et al. (2011); Loebman et al. (2011); Bird et al. (2012); Di Matteo et al. (2013); Kopylov et al. (2013); Minchev et al. (2012b, 2014).

- migration in terms of guiding radius, or ‘churning’, i.e. the difference of guiding radius between two different times. This follows the nomenclature of Schönrich & Binney (2009). Stars which increase their radial amplitude over time but do not change their guiding radius, i.e. stars that experience blurring only, are not considered as migrators according to this definition.

We caution the reader that our model is not intended to reproduce neither the characteristics of the Milky Way disc – as an example, the pattern speed of our simulated bar is lower than the pattern speed measured for the Galaxy (Gerhard 2011) and as a consequence, the main resonances are located at much larger distances from the centre than those measured for the Milky Way – nor of any other specific galaxy. As a result, the reader should not take the exact values of spatial or kinematic variations experienced
by stars in the model as directly applicable to any galaxy. However, they can take the effects described below as representative of those that any typical barred galaxy should experience.

4.1. Determination of guiding radii and amplitude of radial oscillations

A star radius oscillates around a guiding radius. In the case of small eccentricities, it is possible to determine this guiding radius by finding the radius $R_i$ at which a star has the same vertical component of the angular momentum and a circular trajectory: $L_z = R_i v_{\text{circ}}(R_i)$, where $v_{\text{circ}}$ is the circular velocity obtained from the potential. The amplitude of the radial oscillations around the guiding radius should scale as $\frac{\sigma_R}{\kappa}$, where $\sigma_R$ is the radial velocity dispersion, and with $\sigma_R$ and the epicyclic frequency $\kappa$ being both functions of the radius $R$.

The output snapshots of our simulation are separated by 10 Myr, which is small enough to allow to compute the guiding radius at any time $t$ by a simple method:

- determination of the relative minima and maxima of the oscillatory evolution of the radius.
- use of a linear fit between the relative minima on the one hand and the relative maxima on the other to obtain a local minimum radius $R_{\min}(t)$ and a local maximum radius $R_{\max}(t)$ at time $t$.
- definition of the guiding radius at $t$ by the average of these two radii: $\langle R(t) \rangle = \frac{R_{\min}(t) + R_{\max}(t)}{2}$.

Fig. 7 shows examples of this determination of the guiding radius for two random stellar particles. The positions of the particles are centred on the centre of mass of the whole galaxy at each time-step for this analysis. This method provides directly the guiding radius and also the amplitude of the radial oscillations at any time $t$.

![Fig. 7](image_url) Determination of the guiding radius for two random stellar particles. Blue lines: galactocentric radii $R(t)$. Red lines: $R_{\min}(t)$. Green lines: $R_{\max}(t)$. Black lines: guiding radii $\langle R(t) \rangle = \frac{R_{\min}(t) + R_{\max}(t)}{2}$.

4.2. Overall migration compared with churning alone

Our aim is to compare the overall population of migrating stars, by blurring and churning, with that of migrators by churning alone. For this, we first examine the distributions of the variations of radius ($\langle R(t) \rangle$) as a function of the initial radius ($\langle R_0 \rangle$), for different time-intervals between 1 and 9 Gyr of evolution. The top set of plots of Fig. 8 shows the difference of radii of stellar particles at final time $t_f$ and initial time $t_i$ as a function of the radius at $t_i$, while the bottom set shows the difference of guiding radii as a function of the guiding radius at $t_i$, where $t_i$ and $t_f$ are given a range of values. Thus, in the top set of plots the whole population of migrators is shown, whilst in the bottom plots only migrators by churning are selected. The considered stellar components are the old stellar disc and the new stars that are formed before $t_i$. The RMS of the variations are indicated for each time interval.

On each of the panels of Fig. 8, the shaded areas shows the span of the values of the ILR (blue), CR (red) and OLR (green) radii of the bar determined from Fig. 3 at $t_i$ and $t_f$. The vertical solid line is the average of the corotation radius and the diagonal line equation is $y = R_{\text{CR}} - R_i$ for the radii plots, $y = R_{\text{CR}} - (R_i)$ for the guiding radii plots. The regions between the diagonal and vertical lines allow to estimate the migrators that cross the corotation radius, by blurring+churning or by churning alone:

- In the region corresponding to the dots-filled region of the schematic diagram, the stars in the radii plots have $R_i < R_{\text{CR}}$ and $R_f > R_{\text{CR}}$, and those on the guiding radii plots have $\langle R_i \rangle < R_{\text{CR}}$ and $\langle R_f \rangle > R_{\text{CR}}$, and those on the guiding radii plots have $\langle R_i \rangle > R_{\text{CR}}$ and $\langle R_f \rangle < R_{\text{CR}}$.

Analogously, in each panel, the vertical dashed line shows the position of the OLR at $t = t_f$, and the dashed diagonal line equation is $y = R_{\text{OLR}} - R_i$ for the radii plots, $y = R_{\text{OLR}} - (R_i)$ for the guiding radii plots. The regions between the diagonal and vertical dashed lines thus allow to estimate the migrators that cross the OLR, at its final position in the time interval considered, by blurring+churning or by churning alone. We will make use of these criteria in Sects. 4.3 and 5.

The results presented in Fig. 8 can be summarised as follows:

1. As a general behaviour, the distribution for the whole population and for migrators by churning alone shows some features appearing as diagonal structures. These features are seeded by several resonances due to the presence of the bar and of the spiral arms. The most prominent diagonal features occur around the corotation of the bar, the main source of migrators in the stellar disc, as in Minchev & Famaey (2010); Minchev et al. (2011); Brunetti et al. (2011).

2. Quantifying radial migration by means of the instantaneous difference between radii at two different times leads to overestimate churning both in terms of maximum extent and RMS of the variations and in terms of fraction of migrators (see further discussion on Fig. 9).

As an example, the RMS values for the 1 to 3 Gyr...
Fig. 8. Distributions of the variations of galactocentric radii (top half) and guiding radii (bottom half). The colour of each bin codes for the stellar mass in the bin and the colour scale shown on the bottom of the figure applies everywhere. The shaded areas represent the bar ILR (blue), CR (red) and OLR (green) radii variation in the time-span of each plot. The red vertical lines are the average of the bar CR radius during the time-span of a plot. The diagonal red lines help identify the migrators that cross the bar CR radius from lower radii (dots-shaded part of the schematic plot) or from larger radii (grid-shaded part).
time-interval is of 2.5 kpc for the variations of radius, while it is only of 1.8 kpc for the variations of guiding radius. These differences are due to the radial excursions that can make a stellar particle have a radius between \( R(t) - A(t) \) and \( R(t) + A(t) \), where \( A(t) = R_{\text{max}}(t) - R(t) \) is the semi-amplitude of the radial oscillations at time \( t \). The radius \( R(t_0) \) at time \( t_0 \) can be expressed as:

\[
R(t_0) = (R(t_0) + r(t_0))
\]

where \( r(t_0) \) is in the interval \([-A(t_0), A(t_0)]\). The variation in radius \( R \) between time \( t_1 \) and \( t_f \) is thus:

\[
R(t_f) - R(t_i) = (R(t_f) - R(t_i)) + r(t_f) - r(t_i)
\]

and \( r(t_f) - r(t_i) \) is in the interval \([-A(t_f) + A(t_i)], A(t_f) + A(t_i)]\).

3. The migration in terms of change of radius (=blurring+churning) keeps approximately the same distribution and RMS value on time-intervals of the same length although the amplitude and RMS of the migration in terms of change of guiding radius (=churning alone) are smaller at late times. For example for time-intervals of 2 Gyr, the RMS value of the radius variation varies only from 2.2 to 2.5 kpc, while the amplitude and RMS value of the guiding radius variation decreases from 1.8 kpc in a time interval of 2 Gyr from \( t = 1 \) to 3 Gyr to 1.1 kpc between \( t = 5 \) and 7 Gyr or \( t = 7 \) and 9 Gyr. This difference is due to the increase, with time, of the amplitude of radial oscillations, i.e. blurring, as quantified in Table 3, where the RMS of the distribution is given for the same time intervals shown in Fig. 8. In particular, from the comparison of the values given in Table 3 with those of churning alone given in the bottom panels of Fig. 9, it can be noted that, in the early phase of disc evolution, the spatial variations induced by churning overwhelm those due to blurring. This trend in reversed already after 3 Gyr of disc evolution, when the extent of the radial variations by blurring dominates over the variations induced by churning alone.

4. The disc experiences its most intense phase of migration by churning in the very early phase of its evolution, when the stellar bar forms from an axisymmetric potential and it is thin and strong. After the buckling instability, and the formation of the boxy/peanut-shaped bulge at \( t=3 \) Gyr, the amplitude of the variation of the guiding radii of stars migrated by churning alone diminishes. Note that the increase in the RMS variation of guiding radii as the time-span increases (cf., for example, the RMS variations of guiding radii in the time interval 1-3 Gyr versus 1-9 Gyr) does not imply necessarily that the number of migrators by churning increases with time, but only that migrators by churning have time to reach larger distances on larger timescales. We comment more on this point in the following part of this section.

To emphasize the difference between migration simply defined as a change in the instantaneous radius and migration due to a change of guiding radius, the decrease of

\[
\Delta R = R_{\text{max}}(t) - R(t)
\]

with time and to quantify the fraction of stars involved in blurring and/or churning in this simulation, we represent on the top panels of Fig. 8 the mass fraction of the stars that migrate by more than \( x \) kpc in terms of radius or guiding radius from 1 to 3 Gyr. Figures 10 and 11 then show the same fraction, but for stars migrating in the time interval between 5 and 7 Gyr, and over the whole interval 1-9 Gyr, respectively. Fig. 9 shows for example that, between 1 and 3 Gyr, \( \sim 40\% \) of the ensemble of stars (with a mass-weighting accounting for the different masses of stellar particles) change their instantaneous radius by more than 2 kpc, while this fraction reduces to only \( \sim 20\% \) for a change of the guiding radius of the same amplitude. The plotted fraction decreases more rapidly with \( x \) for the variation of guiding radius and it extends to a lower maximum value. The second line plots show the detail of the inward and outward migration. On the negative \( x \) left part of the plot, the curves represent the fraction of stars migrating inwards of more than \( |x| \) kpc, while on the right positive \( x \) part, the curves represent the fraction of stars migrating outwards of more than \( x \) kpc. The curves are almost symmetric with respect to the \( x = 0 \) vertical axis. It can be seen that the migration has a lower amplitude in terms of guiding radius than radius for both inwards and outwards migration. The lower panels show the same study for stars in specific initial radius or guiding radius bins. More specifically, in these last two rows we quantify the stellar mass migrating of more than \( \Delta R \) (showing both the absolute and the real value of the variation) from some initial radii \( R_i \), normalizing this stellar mass to the mass initially contained at \( R = R_i \). In particular, we have chosen to analyse the fraction of migrating stars with respect to three different initial radii, around \( R_i = 5 \) kpc, \( R_i = 10 \) kpc and \( R_i = 20 \) kpc, corresponding, respectively, to radii between the ILR and the CR, the CR and the OLR, and outside the OLR. From inspection of these plots, we can deduce the following:

1. Over the time period 1-3 Gyr, the whole population of migrators moves limitedly with respect to its initial radius, independently of the location of this initial radius with respect to the bar’s resonances. As an example, the fraction of stars that has a variation of its radius, in absolute values, greater than \( |\Delta R| = 4 \) kpc is about 10% at \( R_i = 5 \) kpc, and 20% at \( R_i = 10 \) kpc and \( R_i = 20 \) kpc. These fractions decrease when considering stars that experience churning, the fraction of stars that change their guiding radii by more than 4 kpc being less than 10% for all values of the initial radius inspected.

2. If one considers the sign of the displacement (bottom panels in Fig. 9) – with positive \( \Delta R \) corresponding to outward migration and negative \( \Delta R \) to inward migra-

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Table 3. RMS values of the variations of radius induced by blurring alone, at different times. See Fig. 8 for a comparison with the variations obtained for the whole population of migrators, and for those migrated by churning alone.

| Time Interval | RMS Value |
|--------------|-----------|
| \( t_i = 1Gyr \) | 1.73 kpc |
| \( t_i = 3Gyr \) | 1.84 kpc |
| \( t_i = 5Gyr \) | 1.90 kpc |
| \( t_i = 7Gyr \) | 2.05 kpc |
| \( t_i = 9Gyr \) | 2.13 kpc |
Fig. 9. Migration between 1 and 3 Gyr. First line: Mass fraction of stars that migrate by more than $x$ kpc inwards or outwards in terms of radius (solid line) or guiding radius (dashed line). Second line: Mass fraction of stars that migrate outwards (right half of the plot) or inwards (left part) by more than $x$ kpc in terms of radius (solid) or guiding radius (dashed). Third and fourth lines: Same study but only for stars having a radius (solid lines) or a guiding radius (dashed lines) around $R_i$.

Migration – one sees that migration is, in general, not symmetric. For radii between the ILR and the CR, there is an excess of stars migrating outwards, $\Delta R = -R_i$ being the largest possible extent of inwards migration for migrators originating at $R = R_i$. Between the CR and the OLR, there is an excess of inward migrators, both in terms of blurring+churning that of churning alone.

Note in particular that outward migrators by churning do not cross the OLR at $t=3$ Gyr, the fraction of stars with $\Delta R > 4$ kpc – this value being the minimal displacement required to cross the OLR during this time interval – is indeed null. A limited number of stars orig-
Outward migration

\[ \frac{M(\Delta R > x)}{M_{\text{disc}}} \]

Inward migration

\[ \frac{M(\Delta R > |x|)}{M_{\text{disc}}} \]

Fig. 11. Migration between 1 and 9 Gyr. First line: Mass fraction of stars that migrate by more than \( x \) kpc inwards or outwards in terms of radius (solid line) or guiding radius (dashed line). Second line: Mass fraction of stars that migrate outwards (right half of the plot) or inwards (left part) by more than \( x \) kpc in terms of radius (solid) or guiding radius (dashed). The yellow lines on these two plots are the corresponding migration from 1 to 3 Gyr (identical to the curves of Fig. 8). Third and fourth lines: Same study but only for stars having a radius (solid lines) or a guiding radius (dashed lines) around \( R_i \).

The results found in Fig. 9 are confirmed when analysing migration over the same time duration (2 Gyr), but for a different time interval (5-7 Gyr). Figure 10 shows indeed the same study as Fig. 9 for the time period from 5 to 7 Gyr, with the bottom panels detailing the migration for initial radii located again between the ILR and the CR (\( R_i = 7 \) kpc), between the CR and the OLR (\( R_i = 15 \) kpc) and beyond the OLR (\( R_i = 25 \) kpc). Note that, because of the slowing down of the bar with respect to earlier times, the initial radii \( R_i \) are now much more external than those chosen for the analysis shown in Fig. 9. The striking difference between migration between 5 and 7 Gyr, compared to the time interval 1-3 Gyr (Fig. 9), is the even smaller change in guiding radius experienced by the stars at these late times: there are no migrators by churning with \( \Delta R > 4 \) kpc, most of the stars (80\% for the region between the CR and the OLR, almost 100\% for stars originating inside the CR or outside the OLR) experiencing a change in their guiding radii smaller than 2 kpc. The strength of churning, in terms of spatial extent covered by the process, is thus decreasing. We confirm also what already noted in Figs. 8 and 9, that is migrators by churning originated in the inner disc, i.e. inside the OLR, do not cross this resonance: the maximum displacement of outward migrators by churning whose \( R_i = 15 \) kpc at \( t_i = 5 \) Gyr is about 3.5 kpc, which is not enough to cross the OLR, whose location is at \( R_{\text{OLR}} = 20 \) kpc at time \( t = 7 \) Gyr.

It is not obvious to identify the reason of the decrease of the importance of churning with time that is observed in our modelled galaxy. It may be due to a concomitance of factors, that may be difficult to separate. This decrease with time can be explained by the evolution of the bar, and of its strength, the influence of other resonances in the disc, and the kinetic state of the disc. As an example, Fig. 6 shows that after an abrupt decline, the bar takes time to regain strength in the final Gyr. It is then as concentrated in the galactic plane (it has a peanut shape) and the tangential force it exerts on stars in the plane is thus less important than at earlier times. In the period from 1 Gyr to about 3 Gyr, there is also a phenomenon of resonance overlap that can increase the radial migration (see Minchev & Famaey (2010)). The disc also becomes gradually hotter (as will be shown in Section 6, see especially Figs. 14 and 19) and stars become thus less responsive to potential perturbations.

Finally, when analysing migration over the whole time interval 1-9 Gyr, as done in Fig. 11, one can see that the maximum displacement of migrators has increased, this because stars have had time to cross a larger portion of the disc. For example, 5\% of migrators by churning migrated by more than \( \Delta R = 6 \) kpc, whilst this fraction was null between 1 and 3 Gyr. But when looking at the details of migration, it is still valid that the amount of the displacement depends both on its sign (inward versus outward migrators) and on the location of the initial radius \( R_i \). Note in particular that the largest displacement of outward migrators are experienced in the region between the ILR and the CR (\( R_i = 6 \) kpc and \( R_i = 9 \) kpc, respectively), but that even over such a large time interval, these outward migrators do not cross the final OLR radius. It should be mentioned also that there is a significant contribution of blurring in shaping the outer disc – compare for example the fraction

\[ \frac{\Delta R}{|x|} \]
of outwards migrators in the outer disc ($R_i = 30$ kpc) by churning+blurring versus churning alone.

4.3. Local flux of migrators

We now attempt to determine which migration effect dominates at some radius (migration due only to epicyclic excursions away from a guiding radius or change of guiding radius), before computing the fraction of different kinds of migrators at a final radius.

A quantification of a ‘migration flux’ as a function of the radius can be obtained by computing the mass fraction of the disc that crosses a given radius. We represent on the top plots of Fig. 12 as a function of the galactocentric radius $R$:

- the ‘whole migration flux’, i.e. the mass fraction of the stellar disc that crosses $R$, the crossing of $R$ being quantified in terms of radius, that is: $R_i < R < R_t > R$ from time $t_i$ to time $t_f$ for outward migration (solid lines) and $R_t > R$, $R_i < R$ for inward migration (dashed lines)
- the ‘migration flux by churning alone’, i.e. the mass fraction of the stellar disc that crosses $R$ in terms of radius and also of guiding radius: $R_i < R_i < R_t > R$ and $(R_i) < R_i < (R_t) > R$ for outward migration (solid lines) and $R_t > R$, $R_t < R$ and $(R_i) > R_i, (R_t) < R$ for inward migration (dashed lines).

Note that both fluxes in the top panels of Fig. 12 are normalized to the total stellar mass in the disc.

The ‘whole outward migration flux’ at the corotation radius is thus the portion of stars that belongs to the dotted-filled area of the schematic diagram of Fig. 8, in the $R_i - R_t$ vs $R_i$ plots, and the ‘whole outward migration flux’ at a radius $R$ different from the corotation radius is simply obtained by shifting the threshold radius and the diagonal line accordingly. Similarly, the ‘whole inward migration’ corresponds to the grid-filled of the schematic diagram. The ‘migration flux by churning alone’ is obtained by selecting stars that are both in the shaded zones of the radii plots and of the guiding radii plots (top and bottom panels in Fig. 8 respectively). The ‘migration flux by churning alone’ is thus always inferior to the ‘whole migration flux’ as it is a fraction of it. The inner kpc inward migrators (both in terms of change in their instantaneous radius or in their guiding radius) are stars captured by the bar. Some other much smaller peaks are observed at large radii, corresponding to spiral patterns. However, in agreement with previous results (see Introduction), it is near the corotation of the bar that the most significant part of the migration flux occurs, this because the bar is the strongest potential perturbation in our simulation and its corotation radius is located in a high surface density region of the disc. In discussing these plots we wish to emphasize two points:

- The finding that the corotation is the locus of the strongest flux of migrators by churning can be particularly appreciated looking at time intervals of different lengths, for example comparing the time interval 1-3 Gyr with those at 1-5 Gyr, 1-7 Gyr and 1-9 Gyr. The corotation spans an increasingly larger radial extent as the duration of the time interval grows. As a result, the flux through the corotation region, which is mostly a thin spike around the CR for short time intervals (cfr 1-3 Gyr), transforms into a sort of large plateau for longer time intervals, whose extent corresponds to the spatial extent spanned by the corotation during the corresponding time.
- The dominant role of churning near corotation can also be appreciated in the bottom panels of Fig. 12, where the fraction of stars migrated by churning with respect to the whole sample of migrators crossing a given radius $R$ is shown. While more than 60% of migrators crossing the corotation region are migrating by churning, this fraction diminishes significantly for other regions of the disc. In particular, it can be appreciated that outside corotation, the fraction of migrators by churning diminishes for later times. This is a confirmation of the growing importance of radial heating (i.e. blurring) as time increases (see also discussion in the previous section).

4.4. Sources and sinks: How many migrators are there at a given radius? And how far from it did they originate?

In the previous sections, we have quantified the main sources of migration in the disc, and the flux of migrating stars as a function of distance from the galaxy centre.

We now want to discuss where migrators are redistributed in the disc, and in particular how many migrators can be expected at different radii throughout the disc. To this aim, we represent on Fig. 13 the fraction of stars of a radius bin around $R_i$ at $t_f$ that have migrated by more than $n$ kpc ($n = 2, 4, 6$) in radius since $t_i$ (top half of the figure) or whose guiding radius has changed by more than $n$ kpc since $t_i$ (bottom half). These fractions peak on both sides of the CR radius of the bar, consistently with the migrating fluxes of Fig. 12 peaking at the bar CR radius: radii $R_i > R_{CR}$ receive stars migrating outwards from $R_i < R_{CR}$, while radii $R_i < R_{CR}$ receive stars migrating inwards from $R_i > R_{CR}$. However, as already discussed previously (see also next section) inward and outward migrators cannot cross the whole disc, and in particular outward migrators originated around the CR cannot reach the outer ($R > R_{OLR}$) regions.

At a first sight, the fraction of the whole population of migrators (churning+blurring) seems very significant at certain radii (see top panels in Fig. 13): as an example, in the time interval 1-9 Gyr, 40% of the stars in the CR region are stars that have reached this region by migrating of more than 2 kpc from some inner radii by blurring or churning; in the OLR region between 40% and 50% of stars are outward 2 kpc–migrators; outside the OLR, the contribution of these stars to the local population increases nearly monotonically up to 80%. This high fraction is due to the exponentially declining stellar surface density that makes inwards migrators easily constitute a large part of the stars at an external final radius. If we look at the population of more extreme migrators –those that have migrated by more than 6 kpc outwards by blurring or churning– they are still important contributors to the local ($= at a given R_i$) stellar population: the fraction has a local maximum in the OLR region, at radii $< R_{OLR}(t = t_f)$, where it peaks at ~ 25%, it decreases to about 10% at $R_i = R_{OLR}$, and finally increase again nearly monotonically in the outer disc. However, when we examine the contribution of migrators by churning alone at a given radius (bottom panels of Fig. 13), we can notice some different trends and absolute values. In particular, extreme ($\Delta R > 6$ kpc) outward migrators by churning never exceeds 20% of the local stellar population, with this value reached in the OLR region, at
Fig. 12. Top half: Mass fraction of the stellar disc crossing a radius $R$ in terms of radius (‘apparent migration’) or both radius and guiding radius (‘true migration’). Bottom half: Ratios of the true to apparent migrators. The shaded areas represent the bar ILR (blue), CR (red) and OLR (green) radii variation in the time-span of each plot.
Fig. 13. Fraction of migrators as a function of radius (top half) or guiding radius (bottom half) at the end of the time-span specified on each plot. The shaded areas represent the bar ILR (blue), CR (red) and OLR (green) radii variation in the time-span of each plot.
radii below the final OLR radius. After this maximum, the fraction diminishes to a value of about 5% at the final OLR, and stays constant in the outer disc. For inward migrants, the maximal contribution is reached at radii just beyond the ILR region, with the contribution of long-distance migrants by churning being very limited.

5. The OLR: a barrier for migrators

In the previous section, we have pointed out the finding that migrants by churning do not cross the OLR. In this section we develop this point further.

As previously described, the bottom panels of Fig. 8 show the variation $\Delta R = R_{\text{OLR}}(t_f) - R_i$ of guiding radii as a function of their initial guiding radius $R_i$ in the time interval $[t_i, t_f]$. The dashed vertical line indicates the position of the OLR at $t_f$, that is, at the end of the time interval under consideration, and the diagonal dashed line the equation $R_{\text{OLR}}(t_f) - R_i = f$. It is evident from these figures that, at all times, stars with $R_f < R_{\text{OLR}}(t_f)$ do not cross the position of the OLR at the final time $t_f$, these stars being always below the line $\Delta R = R_{\text{OLR}}(t_f) - R_i$. Migrants in the OLR region in the time interval under consideration, that is with $R_{\text{OLR}}(t_i) \leq R_i \leq R_{\text{OLR}}(t_f)$, can cross the OLR, but these are stars that at least at $t = t_i$ had guiding radii all beyond the initial OLR position $R_{\text{OLR}}(t_i)$. In other words, when looking at a barred galaxy at a given time, the only migrants by churning that can be found beyond the OLR are stars migrating from a region between the OLR at the initial time of bar formation, and the current OLR position. The larger are the variations of the pattern speed of the bar over time, and consequently the larger are the variations $\Delta R_{\text{OLR}} = R_{\text{OLR}}(t_f) - R_{\text{OLR}}(t_i)$ of the OLR position, the larger will be the region of the inner disc where migrants crossing the OLR can originate from.

In no case, however, stars born at corotation can cross the final OLR. Analogously, if we drew a diagonal line of equation $R = R_{\text{OLR}}(t_i) - R_i$ in the bottom panels of Fig. 8 this line separating stars that can cross the initial position of the OLR, we would find that no stars with $R_i \geq R_{\text{OLR}}(t_i)$ would migrate inward crossing the radius $R = R_{\text{OLR}}(t_i)$. This suggests that the OLR region, that is the region between the initial (i.e. at the epoch of bar formation) and final (i.e. current) position of the OLR is a transition region for a disc galaxy, the only region where migrants by churning can be exchanged between the inner and the outer disc. But neither stars with $R_i \leq R_{\text{OLR}}(t_i)$ can cross the OLR, nor, in the opposite sense, stars with $R_i \geq R_{\text{OLR}}(t_i)$ can penetrate below the radius $R_i = R_{\text{OLR}}(t_i)$. Note that some more contamination of the regions around the OLR can be operated by stars migrating by blurring (top panels of Fig. 8), even if, in general, these polluters originate in regions outside those spanned by the CR.

To our knowledge, this is a result not pointed out before, that recalls us the similar barriers encountered by the gas in a barred galaxy (Schwarz 1981; Combes 1988): similarly to stars also gas in the CR-OLR region can gain angular momentum to reach at most the OLR position. Similarly to stars (Fig. 13 bottom panels), gas accumulates in the OLR region, but differently from the stellar component, gas is then able to dissipate part of its energy, shocks and forms rings of concentrated material at the OLR. Similarly, gas outside the OLR is not able to penetrate this resonance and access the inner region of the disc, until the barrier is removed (because of a significant change in the bar properties, see for example Combes (2011)).

Why is this result important? For the Milky Way at least for two reasons:

1. It may explain why the inner and the outer discs of the Galaxy have been able to maintain two different stellar populations over a time interval of $\sim 10$ Gyr, corresponding probably to the whole interval of secular evolution experienced by the Galaxy. We recall here the results of Haywood et al. (2013), that have extensively shown and discussed that outer disc stars ($R \geq 10$ kpc) have chemical properties significantly different from those of stars of the inner disc having similar ages. These results have been confirmed by Anders et al. (2014), and Nidever et al. (2014) who also show that the chemical properties of the outer disc are significantly different from those of the inner disc (see also Bensby et al. 2012, for a high resolution spectroscopy study of the inner and outer disc). The current position of the OLR in the Milky Way is estimated to be slightly inside the solar radius (Dehnen 2000). This may thus explain why the Sun appears to be in a transition region (Haywood et al. 2013), and why the outer disc has not been polluted by stars originating in the inner disc ($R \leq 6-7$ kpc).

2. Depending on the exact location of the Sun with respect to the OLR, the solar vicinity may have been strongly polluted by stars migrating by churning from the inner disc (see Fig. 13 bottom panels, for stars inside the final OLR position), or not at all. If the Sun is outside the OLR position, and the bar is the main source of asymmetric perturbations in the Milky Way, then the impact of churning at the solar vicinity may have been significantly overestimated (see Fig. 13 bottom panels, for stars beyond the OLR region). In particular, stars migrated by churning at the solar vicinity may well originate in regions much closer to the solar radius than previously thought, without the possibility to invoke substantial migration from the corotation and the inner disc.

This result may also lead to question the interpretation of U-turn in age profiles, or inversion in colour-profiles found in the outer disc of external galaxies. For barred galaxies, when these inversions are observed outside the OLR position, it is difficult to explain them in terms of strong migration from the inner disc, since, according to our results, no strong migration from the corotation is expected to contaminate the outer disc, unless another pattern with lower speed existed before the present one, with an OLR extending to outer radii.

6. Migration and cooling/heating

6.1. Vertical heating/cooling

The effect of radial migration on the vertical structure of discs has been studied in a number of works (Minchev et al. 2012a; Solway et al. 2012; Roškar et al. 2013; Vera-Ciro et al. 2014 e.g.). The assumption that outward migration would help create a thick disc because of the higher vertical velocity dispersion of the outward migrants originating
from the inner hotter disc has been debated because of the vertical cooling outward disc should undergo if the vertical action of a stellar orbit is conserved (Minchev et al. 2012a, Solway et al. 2012; Roskar et al. 2013).

We observe that the vertical velocity dispersion of the stellar disc increases slightly with time at all radii. On Fig. 14 we plot the vertical velocity distribution of the old stellar disc, the stellar disc of stars formed during the simulation, and the gas disc. The velocity dispersion of the new stars, formed from the colder gas disc, is generally lower than the velocity dispersion of the old stellar disc. The mass of new stars is however only a small fraction of the total stellar mass (see Fig. 2), so star formation has no net cooling effect on the total stellar disc.

We would like to know if the stars that are going to migrate are a special sub-population in terms of kinetics of the stars having the same initial guiding radius, if the migration affects their kinetic state, and if the migrants affect the population of stars at their final guiding radius. Vera-Ciro et al. (2014) find a ‘provenance bias’ of migrants in terms of kinetic state. They find that radial migration driven by spiral arms (seeded in their simulations by massive perturbers) affect preferentially stars with a lower velocity dispersion than the average velocity dispersion at the initial radius. We find a similar trend as shown on Fig. 15 where the ratio of the vertical velocity dispersion of stars in bins in \( R_i \) and \( \langle R_f \rangle - \langle R_i \rangle \) (we use only bins containing at least the mass of 10 old disc stellar particles) to the vertical velocity dispersion of all stars in a bin at \( \langle R_f \rangle \) (a column in the plots) is represented. The stars migrating the most from an initial guiding radius tend to be colder in the z-direction than the average for all the stars at this initial guiding radius.

We next investigate whether the migrants can be subject to heating (or cooling) when they reach hotter (respectively colder) regions. On Fig. 16 we represent the distribution of the ratio of the final vertical velocity dispersion to the initial one for stars that have migrated by \( \Delta R = \langle R_f \rangle - \langle R_i \rangle \) from their initial radius \( R_i \) in a time interval \([t_i, t_f]\). We observe that, especially in the inner regions of the disc, i.e. inside the OLR, outward migrants tend to lower their velocity dispersion, while inward migrants tend to increase it. The effect increases with the amplitude of the variation (it is more visible for the extreme cases of migration).

We now discuss how the final vertical velocity dispersion of migrants compares to that of stars with the same final guiding radii \( \langle R_i \rangle \). On Fig. 17 we represent the distribution of the ratio of the velocity dispersion of migrants in a bin in \( \langle R_f \rangle \) and \( \langle R_i \rangle - \langle R_i \rangle \) to the velocity dispersion of all stars of guiding radius \( \langle R_i \rangle \). We observe different trends, depending among others, on the location of the radius \( R_i \) under consideration. We start by discussing what happens inside the OLR at the final time. We observe that in the very inner regions, stars migrating inwards to reach a guiding radius \( \langle R_f \rangle \) have a higher velocity dispersion than the whole population of stars at \( \langle R_i \rangle \). These stars must be taking part in the peanut-shaped bar with a high vertical velocity dispersion. This is in agreement to what presented by Matteo et al. (2014), who found that outside-in migrants participating to a box/peanut shaped structure tend to be kinematically hotter than in situ-stars. At higher radii, however, the inward migrants tend to have lower velocity dispersions than the average at their final guiding radius. Outward migrants tend to have a slightly higher (\( \sim 20\% \)) vertical velocity dispersion than all stars at their final guiding radii, except for the most extreme migrants – those with the highest \( \Delta R \) variations, found between the CR and the OLR region. These extreme migrants can be significantly hotter (up to 50% more) than the whole population at the same final guiding radius. Note that this effect is evident only when a selection on the amplitude of the migration \( \Delta R \) is done. When the overall population of (inward or outward) migrants is analysed as a whole, the final vertical velocity dispersions of migrants and non-migrants are much more similar, this because apart from the extreme cases of migration, many stars that have migrated by small \( \Delta R \) tend to have a final kinematic similar to that of the overall population. We can see this on Fig. 18. In this figure, we have
Fig. 15. Ratio of vertical velocity dispersion of stars in bins in $\langle R_i \rangle$ and $\langle R_f \rangle - \langle R_i \rangle$ to the vertical velocity dispersion of all stars in radial bin centred on $\langle R_i \rangle$. The shaded areas represent the bar ILR (blue), CR (red) and OLR (green) radii variation in the time-span of each plot. The red vertical lines are the average of the bar CR radius during the time-span of a plot.

Fig. 16. Distribution of the ratio of final to initial vertical velocity dispersion of stars migrated by $\Delta R = \langle R_f \rangle - \langle R_i \rangle$ from their initial guiding radius $\langle R_i \rangle$ in the time interval $[t_i, t_f]$. The shaded areas represent the bar ILR (blue), CR (red) and OLR (green) radii variation in the time-span of each plot. The red vertical lines are the average of the bar CR radius during the time-span of a plot.

plotted the vertical velocity dispersion of all stars as a function of radius at $t = 3$ Gyr, and the velocity dispersion of inward and outward migrators that are in a radius bin and that have migrated by more than 2 kpc in terms of radius
Fig. 17. Distribution of the ratio of vertical velocity dispersion of stars in bins in $\langle R_f \rangle$ and $\langle R_f \rangle - \langle R_i \rangle$ to the vertical velocity dispersion of all stars with guiding radii centred on the final value $\langle R_f \rangle$. The shaded areas represent the bar ILR (blue), CR (red) and OLR (green) radii variation in the time-span of each plot. The red vertical lines are the average of the bar CR radius during the time-span of a plot.

or guiding radius since $t = 1$ Gyr, and the ‘non-migrants’ defined as the stars of the radius bin that have not changed their radius or guiding radius by more than 2 kpc. We see that the inward migrators have a slightly lower vertical velocity dispersion, that the outward migrators have a slightly higher velocity dispersion, but the total velocity dispersion is very close to the one of the ‘non-migrants’, indicating a small effect of the overall migration on the local velocity dispersion. We insist however on the fact that the most extreme migrators, in particular the most extreme outward migrators, can have dispersions significantly different (50%) from the average. Thus while we agree with the results by Minchev et al. (2012a) that migration does overall little to disc thickening, extreme migrators can depart from the average small increase of the heating found in Fig. 18 and found also by Minchev et al. (2012a), contributing with significantly higher vertical velocity dispersions to the whole sample of migrating stars ending up at the same final guiding radius.

6.2. Radial heating/cooling

The radial velocity dispersion also increases on average with time, as can be seen on Fig. 19. We can follow the amplitude of the radial oscillations as well as the eccentricity of the trajectories of the stellar particles defined as:

$$e = \frac{R_{\text{max}} - R_{\text{min}}}{R_{\text{max}} + R_{\text{min}}}$$

The eccentricity varies between 0 for circular orbits and 1 for radial orbits. The mean eccentricity of the disc stars increase with time. This increase is mainly caused by the growth of the bar: More and more stars are gradually trapped by the bar and gain eccentricity as their orbits become elongated. The evolution of the amplitude of radial oscillations $2A(t)$ where $A(t)$ is the semi-amplitude introduced in section 4.2 and eccentricity $e$ as a function of radius can be seen on Fig. 20. The bar growth is visible from the increase of eccentricity and amplitude with time in the inner kpcs. The amplitude of radial oscillations increases on average with time at all radii, which is consistent with the increase of radial velocity dispersion shown on Fig. 19 and

Fig. 19. Time evolution of the radial velocity dispersion of the old stellar disc without the bulge (solid), the young stellar disc (dashed) and the gas disc (dot-dashed).
with the growing role of blurring with time, as discussed in Sect. 4.2. The eccentricity also slightly increases at all radii.

It is interesting to investigate whether radial migration changes the radial amplitude and the eccentricity of the stars trajectories. On Fig. 21 we represent the change in eccentricity as a function of the change in guiding radius between \( t = 1 \) and 3 Gyr on the top panel, and between \( t = 5 \) and 7 Gyr on the bottom panel. The distribution is broad. The majority of stars change only slightly their eccentricity, but on average (black line) it can be seen that stars migrating outwards (with positive \( \langle R_f \rangle - \langle R_i \rangle \)) tend to decrease their eccentricity while the opposite stands for stars migrating inwards. The inward effect increase of the eccentricity (especially visible in the bottom panel of Fig. 21 from 5 to 7 Gyr) is again dominated by the capture of stars by the central bar. The decrease of eccentricity does not necessarily mean that the orbits are circularised in terms of decrease in amplitude of radial oscillations, because of the denominator in the expression of the eccentricity, that can increase in the case of outwards migration. On Fig. 22 we thus represent the variation of the amplitude of radial oscillations and separate in the middle and right columns the disc in an ‘inner disc’ and an ‘outer disc’ by a cut between the CR and OLR bar radii, at 10 kpc for the first time-interval and at 15 kpc for the second one, so that the effect of the bar growth on the stellar orbits is not present in the ‘outer disc’ plots. On the inner disc plot of the time-span from 5 to 7 Gyr, a clear trend of increase of the amplitude of radial oscillations is visible for inward migration, corresponding to stars migrating inwards and contributing to the bar. All the distributions are again broad, with the largest variations of amplitude occurring for stars that do not significantly migrate. Stars that migrate outwards the most increase their radial oscillations amplitude on average, but the possible increase is limited compared with the rest of the distribution.

7. Conclusions

We have studied stellar radial migration in a simulation of a Sb type extended galactic disc. We confirm the main role of corotations of density resonances as seeds for radial migration in terms of both blurring and churning, and observe
We do not know yet whether our Galaxy has the U-profile colour-profiles found in the outer disc of external galaxies. The interpretation of U-shape in age profiles, or inversion in the pattern speed measured for the Galaxy and as a consequence, the main resonances are located at much larger distances from the centre than those measured for the Milky Way – we think that the previous result can possibly help understand the puzzling nature of the outer Galactic disc, and its significantly different stellar populations. It has been recently shown that the Milky Way outer disc followed a different chemical evolution history compared with the inner disc (Haywood et al. 2013; Smith et al. 2014). Whatever the mechanism of formation of the outer regions of the Galaxy has been, stars there have been able to evolve independently of the inner disc. Our model suggests that the Galactic OLR and its location (estimated slightly inside the solar radius, see Dehnen 2000) have a major role in explaining this finding. The OLR indeed acts as a barrier for gas (Combes 1988), but - as we show in this paper - for stars as well, inhibiting migration from the inner to the outer disc and vice versa. This may explain why stars with inner thin disc chemistry are not observed where some pollution between the inner and the outer disc is allowed.

Even if our model is not intended to reproduce the Milky Way – the pattern speed of our simulated bar is lower than the pattern speed measured for the Galaxy (Gerhard 2011) and as a consequence, the main resonances are located at much larger distances from the centre than those measured for the Milky Way – we think that the previous result can possibly help understand the puzzling nature of the outer Galactic disc, and its significantly different stellar populations. It has been recently shown that the Milky Way outer disc followed a different chemical evolution history compared with the inner disc (Haywood et al. 2013; Smith et al. 2014). Whatever the mechanism of formation of the outer regions of the Galaxy has been, stars there have been able to evolve independently of the inner disc. Our model suggests that the Galactic OLR and its location (estimated slightly inside the solar radius, see Dehnen 2000) have a major role in explaining this finding. The OLR indeed acts as a barrier for gas (Combes 1988), but - as we show in this paper - for stars as well, inhibiting migration from the inner to the outer disc and vice versa. This may explain why stars with inner thin disc chemistry are not observed where some pollution between the inner and the outer disc is allowed.

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in the stellar ages which is sometimes observed in external galaxies like M33. For barred galaxies, when these inver-
sions are observed outside the OLR position, it is difficult
to explain them in terms of strong migration from the in-
ner disc, with the observed bar/spiral pattern speed, since
they occur beyond their OLR. However, it is possible that
previous bar/spiral waves have developed with lower pat-
tern speeds, implying OLR and migration at larger radii.

Another possible explanation is that the settlement of the
Galactic disc in the outer regions permited the formation of
a significant number of old stars in situ, as proposed for
the Milky Way in [Haywood et al. (2013)] and Snaith et al.
(2014).

Finally we have analysed the kinematics of migrating
stars. We confirm the results found by [Vera-Ciro et al. (2014)]
for spiral galaxies. Similarly to that case, also when migration
is mainly induced by a stellar bar, there is a ‘provenance
bias’ of migrating stars in terms of their kinetic state. The stars
migrating the most from an initial guiding radius tend to
be colder in the z-direction than the average of all the stars
at the same initial guiding radius. We also confirm the results by
Minchev et al. (2012b) that migration does little to disc thin-
ing, but we point out that there is a trend of increasing vertical velocity disper-
sion with the extent of migration: the most extreme outward
migrants which end up at a given final radius tend to have also the highest velocity dispersions when compared
to the velocity dispersion of all the stars found at that fi-
nal radius. Thus, while the overall effect of heating at a
given radius is weak, we suggest that, at a given radius,
extreme outward migrants from the inner disc are iden-
tifiable as stars that have the highest velocity dispersions
among those measured for stars of the same age, at the
same radius.

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