Spin-Polarizabilities of the Nucleon in Chiral Soliton Model with Dispersion Relation

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Abstract

We calculate the spin-polarizabilities of the nucleon by means of dispersion relation, where the pion photoproduction amplitude predicted by chiral soliton model is utilized. We consider the \( N\pi \) and \( \Delta\pi \) channels in the pion-photoproduction amplitude, and also evaluate the contribution of the anomalous term of the \( \pi^0\gamma\gamma \) process. In a narrow decay-width limit of the \( \Delta \) particle the result coincides with that in heavy baryon chiral perturbation theory (HBChPT) except for the interference part of the electric and magnetic amplitudes. A comparison with the multipole analysis is given.

I. INTRODUCTION

The electromagnetic polarizabilities are the quantities to represent the response of the nucleon to external electromagnetic field and reflect its internal structure. The electric and magnetic polarizabilities \( \alpha \) and \( \beta \) are measured by elastic Compton scattering with unpolarized photon and nucleon, and many experiments have been already devoted to the study.

Recently experiments with polarized photon beam and proton target have been available, and it is expected that much information about the spin-dependent structure can be obtained from low energy polarized Compton scattering. The celebrated Drell-Hearn-Gerasimov sum

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rule [1] is given by the spin-dependent photoabsorption cross section $\sigma_{abs}^\lambda$ with the total helicity $\lambda$ as follows:

$$-\frac{e^2 \kappa^2}{8\pi M^2} = \frac{1}{4\pi^2} \int_{\omega_{th}}^\infty \frac{d\omega}{\omega} \left( \sigma_{abs}^{1/2}(\omega) - \sigma_{abs}^{3/2}(\omega) \right),$$

(1.1)

where $\omega_{th}$ is the threshold energy, $\kappa$ the anomalous magnetic moment of the nucleon, and $M$ the nucleon mass. The forward spin-polarizability $\gamma_0$ is given by the once-subtracting sum rule, the Gell-Mann-Goldberger-Thirring sum rule [2], with the inverse weight of $\omega^3$ as follows:

$$\gamma_0 = \frac{1}{4\pi^2} \int_{\omega_{th}}^\infty \frac{d\omega}{\omega^3} \left( \sigma_{abs}^{1/2}(\omega) - \sigma_{abs}^{3/2}(\omega) \right).$$

(1.2)

Recently, Ragusa showed [3] that there are four independent spin-polarizabilities $\gamma_i (i = 1, \ldots, 4)$ in the amplitude of $\mathcal{O}(\omega^3)$, and that the forward spin-polarizability $\gamma_0$ is given by $\gamma_0 = \gamma_1 - \gamma_2 - 2\gamma_4$. There has not yet been any experiment about the spin-polarizabilities.

Theoretical investigations on the spin-polarizabilities have been carried out within the framework of heavy-baryon chiral perturbation theory (HBChPT): The leading order terms from the $N\pi$ loops are of $1/m^2$, which show the importance of the pion cloud around the nucleon for the polarizabilities. However, the result contradicts with and has an opposite sign to that of the pion photoproduction multipole analysis [4]. Bernard et al. [5] showed that the contribution of the $\Delta$-pole terms is negative and reduces largely that of the $N\pi$-loop terms: $\gamma_0^{N\pi\text{-loop}} + \gamma_0^{\Delta\text{-pole}} = (4.44 - 3.66) \times 10^{-4} \text{fm}^4$. The sum is $0.78 \times 10^{-4} \text{fm}^4$, while the multipole analysis yields $\gamma_0 = -1.34(-0.38) \times 10^{-4} \text{fm}^4$ for the proton (neutron) [4]. Similar results are obtained by Hemmert et al. [6,7], within a small scale expansion framework, where the small scale is either of the pion mass, nonrelativistic momentum or the mass difference between the $\Delta$ particle and the nucleon, and the $\Delta$ particle is treated as explicit degrees of freedom. In the work the spin-polarizabilities $\gamma_1, \ldots, \gamma_4$ are calculated in detail and compared with the result of the multipole analysis.

In a previous paper [8] we have calculated the forward spin-polarizability $\gamma_0$ with use of dispersion relation in the chiral soliton model. This followed the calculation of the electric and magnetic polarizabilities in the same context [4]. Chiral soliton model is a QCD motivated one based on the idea of large $N_c$ and of the spontaneous breaking of chiral symmetry [10]. The electromagnetic polarizabilities are sensitive to the pion cloud around the nucleon, so that the model may be well-suited to the study of the polarizabilities. The pion photoproduction Born amplitudes obtained by the model have been shown to satisfy the low-energy theorem [11], and were employed for calculating the dispersion integrals. Further, we note that the $\Delta(1232)$ state is a rotational excited state of the soliton, so that they are naturally treated as an equal partner with each other. In our approach, the chiral soliton model is understood as an effective model to derive the pion-photoproduction amplitude. The imaginary part of the amplitude of Compton scattering can be derived with use of the unitarity condition, and the polarizabilities are then calculated with dispersion relation, which is considered to be a means to calculate loop integrals [12,13].

In Ref. [8] we found that the contributions from the electric and magnetic parts in the $N\pi$ channel coincide with those from the $N\pi$-loop and $\Delta$-pole terms calculated in HBChPT, if we take the narrow width limit of the $\Delta$ state. In this paper we study the spin-polarizabilities.
\(\gamma_1, \ldots, \gamma_4\). It is shown that the imaginary parts of the structure functions of Compton scattering amplitude, \(A_5(\omega, \theta = 0)\) and \(A_6(\omega, \theta = 0)\), turn out to be zero. Naive application of the dispersion relation results in that \(\gamma_3\) and \(\gamma_4\) vanish. We argue that the convergent dispersion integral does not necessarily mean no need of the subtraction, and discuss how to remedy the ill-defined dispersion integrals.

In section II we give the dispersion relations for the polarizabilities. For this paper to be self-contained the results on the spin-independent polarizabilities are summarized. Section III is devoted to the calculation of the imaginary parts of Compton scattering amplitude with use of the pion photoproduction amplitude to the \(N\pi\) and \(\Delta\pi\) channels. The \(\pi_0\gamma\gamma\) anomalous contribution is given in section IV. Numerical results and discussion are given in section V.

II. DISPERSION RELATIONS OF THE AMPLITUDES

The electromagnetic polarizabilities of the nucleon are defined by Compton scattering amplitude, which is represented, at the center-of-mass system, as

\[
f = A_1(\omega, \theta)\epsilon^* \cdot \epsilon + A_2(\omega, \theta)(\epsilon^* \cdot \hat{k})(\epsilon \cdot \hat{k}') + A_3(\omega, \theta) i\sigma \cdot (\epsilon^* \times \epsilon) + A_4(\omega, \theta) i\sigma \cdot (\hat{k}' \times \hat{k})(\epsilon^* \cdot \epsilon) + A_5(\omega, \theta) i\sigma \cdot \left[\left((\epsilon^* \times \hat{k})(\epsilon \cdot \hat{k}') - (\epsilon \times \hat{k}')(\epsilon^* \cdot \hat{k})\right)ight] + A_6(\omega, \theta) i\sigma \cdot \left[\left((\epsilon^* \times \hat{k}'')(\epsilon \cdot \hat{k}') - (\epsilon \times \hat{k}'')(\epsilon^* \cdot \hat{k}')\right)\right],
\]

where \(\epsilon(\epsilon')\) and \(\hat{k}(\hat{k}')\) are the polarization vector and the unit momentum of the incident(outgoing) photon, respectively. \(\sigma\) denotes the Pauli matrix of the nucleon, so that the first and second terms are spin-independent, while the others are spin-dependent. The independent structure functions \(A_i(\omega, \theta)\) with \(i = 1, \ldots, 6\) are functions of the photon energy \(\omega(= \omega')\) and the scattering angle \(\theta\).

The above structure functions can be divided into the Born, non-Born and anomalous terms, respectively, as follows:

\[
A_i(\omega, \theta) = A_i^B(\omega, \theta) + A_i^{nB}(\omega, \theta) + A_i^{\text{anom}}(\omega, \theta)
\]

with \(i = 1, \ldots, 6\). The anomalous term is given by the \(\pi_0\gamma\gamma\) process through the Wess-Zumino-Witten term \[\text{[14]}\]. The Born terms represent the scattering from a spin 1/2 point particle and are determined by its mass, charge, and anomalous magnetic moment, while the non-Born terms reflect its internal structure. The polarizabilities appear in the low-energy expansion of the non-Born and anomalous terms of the amplitudes. The electric and magnetic polarizabilities \(\alpha\) and \(\beta\) are defined as the coefficients of the terms of \(\mathcal{O}(\omega^2)\), while the spin-polarizabilities \(\gamma_i\) are those of \(\mathcal{O}(\omega^3)\):

\[
\begin{align*}
A_1(\omega, \theta) &= A_1^B(\omega, \theta) + (\alpha + \beta \cos \theta) \omega^2 + \mathcal{O}(\omega^3), \\
A_2(\omega, \theta) &= A_2^B(\omega, \theta) - \beta \omega^2 + \mathcal{O}(\omega^3), \\
A_3(\omega, \theta) &= A_3^B(\omega, \theta) + [\gamma_1 - (\gamma_2 + 2\gamma_4) \cos \theta] \omega^3 + \mathcal{O}(\omega^4), \\
A_4(\omega, \theta) &= A_4^B(\omega, \theta) + \gamma_2 \omega^3 + \mathcal{O}(\omega^4),
\end{align*}
\]
\[
A_5(\omega, \theta) = A^B_5(\omega, \theta) + \gamma_4 \omega^3 + O(\omega^4),
\]
\[
A_6(\omega, \theta) = A^B_6(\omega, \theta) + \gamma_3 \omega^3 + O(\omega^4).
\]

(2.3)

We now introduce the fixed-\(t\) dispersion relations for the non-Born parts of the structure functions \(A^n_{iB}(\omega, \theta)\). Then, with use of the dispersion relations the polarizabilities are calculated from the imaginary parts of the non-Born terms of the \(A^n_{iB}\) and the anomalous term as follows:

\[
\alpha + \beta = \frac{2}{\pi} \int_{\omega_{th}}^{\infty} \frac{d\omega}{\omega^3} \text{Im} A^u_{1B}(\omega, 0),
\]
\[
\beta = -\frac{2}{\pi} \int_{\omega_{th}}^{\infty} \frac{d\omega}{\omega^3} \text{Im} A^u_{2B}(\omega, 0),
\]
\[
\gamma_0 = \frac{2}{\pi} \int_{\omega_{th}}^{\infty} \frac{d\omega}{\omega^4} \text{Im} A^u_{3B}(\omega, 0),
\]
\[
\gamma_2 = \frac{2}{\pi} \int_{\omega_{th}}^{\infty} \frac{d\omega}{\omega^4} \text{Im} A^u_{4B}(\omega, 0),
\]
\[
\gamma_3 = \frac{2}{\pi} \int_{\omega_{th}}^{\infty} \frac{d\omega}{\omega^4} \text{Im} A^u_{5B}(\omega, 0) + \text{anomalous term},
\]
\[
\gamma_4 = \frac{2}{\pi} \int_{\omega_{th}}^{\infty} \frac{d\omega}{\omega^4} \text{Im} A^u_{6B}(\omega, 0) + \text{anomalous term}.
\]

(2.4)

The forward spin-polarizability \(\gamma_0\) is defined by \(\gamma_0 = \gamma_1 - \gamma_2 - 2\gamma_4\). It can be shown that the anomalous term does not contribute to the forward spin-polarizability \(\gamma_0\), and to \(\gamma_2\).

The imaginary part of the scattering amplitude is calculated from the photoabsorption amplitude of the nucleon using the unitarity condition. We consider the \(N\pi\) and \(\Delta\pi\) channels for the intermediate states. In a nonrelativistic approximation, the unitarity condition leads to

\[
\text{Im} f_{N\gamma \rightarrow N'\gamma'} = \frac{q}{4\pi} \sum_{B=N,\Delta} \int d\Omega_q f_{N'\gamma' \rightarrow B\pi} f_{N\gamma \rightarrow B\pi},
\]

(2.5)

where \(q\) is the pion momentum, and the integrals are over the angle of the pion momentum.

The anomalous contribution to the polarizabilities is calculated in section IV.

III. THE NON-BORN AMPLITUDES

We calculate the pion photoabsorption amplitudes in terms of the \(\gamma + N \rightarrow \pi + N\) and \(\gamma + N \rightarrow \pi + \Delta\) processes in the chiral soliton model. We have shown [9] that the electric and magnetic Born parts of the amplitude obtained by the chiral soliton model satisfy the low-energy theorem at the threshold except for the order \((m_\pi/M)^2\) term which was recently introduced by the effect of chiral loops [15].

A. \(N\pi\) channel

At first, we evaluate the contribution of the \(\gamma + N \rightarrow \pi + N\) channel. The pion photo-production amplitude is decomposed into three terms as
\[ f^a_N = i \epsilon_{a3b} \tau^b f^{(-)}_N + \tau^a f^{(0)}_N + \delta_{a3} f^{(+)}_N, \] (3.1)

where \( \tau^a \)'s are the isospin matrices, and each of the amplitudes \( f^{(\pm,0)} \) is given by the sum of the electric and magnetic parts.

The electric Born amplitude for the \( \gamma + N \rightarrow \pi + N \) process is given as

\[ f^{(-)}_{N,e} = \left( \frac{eG_{NN\pi}}{8\pi M} \right) \left\{ \frac{i\sigma \cdot \epsilon + 2 \frac{i\sigma \cdot (k - q) (\epsilon \cdot q)}{m^2 - (k - q)^2}} \right\}, \] (3.2)

where \( k \) and \( q \) are the incident photon and the outgoing pion 4-momenta, respectively, \( \epsilon \) the polarization vector of the incident photon, and \( G_{NN\pi} \) the \( NN\pi \) coupling constant. In Fig. 1 we show the Born graphs of the pion photoproduction represented in Eq. (3.2): (a) and (b) are the Kroll-Ruderman and the pion-pole terms, respectively. We see that \( f^{(-)}_{N,e} \) is of \( O(N^{-1/2}) \), while \( f^{(0)}_{N,e} \) are of \( O(N^{-1/2}) \) and behave as \( O(\omega) \). Therefore, the latter amplitudes do not lead to finite results without unitarization of them, and are neglected in the following.

We note that the amplitude in Eq. (3.2) satisfies the low-energy theorem, and is, therefore, model-independent. The same amplitude is known to be obtained in the leading part of the amplitude calculated in HBChPT. Furthermore, we should note that the following relativistic amplitude \[16\] reduces to Eq. (3.2) in nonrelativistic approximation if we neglect the \( t \)-dependence of \( G_A(t) \):

\[ f = \left( \frac{eG_{NN\pi}}{8\pi M} \right) \frac{[\tau_a, \tau_0]}{2} \frac{G_A(t)}{G_A(0)} \hat{u}_p' \left\{ \frac{\hat{v} + \frac{2M}{t - m^2} \epsilon \cdot (2q - k)}{2} \right\} \gamma_5 u_p. \] (3.3)

The magnetic Born amplitude is given as

\[ f^{(-)}_{N,m} = \left( \frac{eG_{NN\pi}}{8\pi M} \right) \frac{\mu_V}{2M} \left\{ \frac{-(\sigma \cdot q)(\sigma \cdot s)}{\omega} - \frac{-(\sigma \cdot s)(\sigma \cdot q)}{\omega} \right\} \]
\[ + \frac{1}{2} \left( \frac{3s \cdot q - (\sigma \cdot q)(\sigma \cdot s)}{\omega - \Delta} + \frac{1}{2} \left[ \frac{3s \cdot q - (\sigma \cdot s)(\sigma \cdot q)}{\omega + \Delta} \right] \right), \]
\[ f^{(+)}_{N,m} = \left( \frac{eG_{NN\pi}}{8\pi M} \right) \frac{\mu_V}{2M} \left\{ \frac{-(\sigma \cdot q)(\sigma \cdot s)}{\omega} + \frac{(\sigma \cdot s)(\sigma \cdot q)}{\omega} \right\} \]
\[ - \frac{3s \cdot q - (\sigma \cdot q)(\sigma \cdot s)}{\omega - \Delta} + \frac{3s \cdot q - (\sigma \cdot s)(\sigma \cdot q)}{\omega + \Delta} \}, \] (3.4)

where \( s = k \times \epsilon \), \( \Delta \) is the mass difference of the nucleon and the \( \Delta(1232) \), and \( \mu_V \) is the vector part of the nucleon magnetic moment defined by \( (\mu_p - \mu_n)/2 \) in units of the nuclear magneton. Note that we introduced the nucleon- and \( \Delta \)-pole terms, and used the relation \( \mu_V^{\Delta} = -(3/\sqrt{2}) \mu_V \) in the chiral soliton model, which is known to be correct in large \( N_c \) limit \[17\]. In Fig. 2 are shown the Born graphs in Eqs. (3.4). We also see that the \( f^{(\pm)}_{N,m} \) reduces to \( O(N^{-1/2}) \) by the cancellation among the \( N \) - and \( \Delta \)-pole terms. The amplitude \( f^{(0)}_{N,m} \) is of \( O(N^{-1/2}) \) and is neglected in the following. We rewrite the \( f^{(\pm)}_{N,m} \) as

\[ f^{(\pm)}_{N,m} = \left( \frac{eG_{NN\pi}}{8\pi M} \right) \mu_V \left\{ t_1^{(\pm)} P_1(\hat{q}, \hat{s}) + t_3^{(\pm)} P_3(\hat{q}, \hat{s}) \right\}, \] (3.5)
where \( P_1(\hat{q}, \hat{s}) = (\sigma \cdot \hat{q})(\sigma \cdot \hat{s}) \) and \( P_3(\hat{q}, \hat{s}) = 3(\hat{q} \cdot \hat{s}) - (\sigma \cdot \hat{q})(\sigma \cdot \hat{s}) \) are the \( P \)-wave projection operators for the \( J = 1/2 \) and \( J = 3/2 \) states, respectively, and \( \hat{q} = \frac{q}{q} \) and \( \hat{s} = \frac{s}{k} \). We obtain

\[
\begin{align*}
t_1^{(+)} &= 2t_1^{(-)} = -\frac{2q}{3M} \frac{\Delta}{\omega + \Delta}, \\
t_3^{(+)} &= \frac{q\omega}{2M} \left[ -\frac{2\Delta}{\omega^2 - \Delta^2 + i\Delta \Gamma} + \frac{2}{3}\frac{\Delta}{\omega(\omega + \Delta)} \right], \\
t_3^{(-)} &= \frac{q\omega}{2M} \left[ \frac{\Delta}{\omega^2 - \Delta^2 + i\Delta \Gamma} - \frac{2}{3}\frac{\Delta}{\omega(\omega + \Delta)} \right].
\end{align*}
\]

(3.6)

Here, in order to avoid divergent dispersion integral due to the pole at \( \omega = \Delta \), we have introduced the finite width of the \( \Delta \) state given by

\[
\Gamma_\Delta = \frac{1}{6\pi} \left( \frac{G_{\Delta N\pi}}{2M_\pi} \right)^2 q^2
\]

(3.7)

with \( G_{\Delta N\pi} = -(3/\sqrt{2})G_{NN\pi} \), whose relation is also correct in large \( N_c \) limit. This is the expression given by Kokkedee without relativistic correction [10], and yields 145MeV with the experimental value of \( G_{NN\pi} \) at \( q = 227 \text{MeV} \).

The amplitudes \( f_{N,e}^{(-)} \) and \( f_{N,m}^{(+)} \) survive in the leading order of \( 1/N_c \) expansion, so that we find from Eq. (3.1)

\[
f_{N}^f f_{N} = 2f_{N,e}^{-} f_{N,e}^{-} + \left( 2f_{N,m}^{-} f_{N,m}^{-} + f_{N,m}^{+} f_{N,m}^{+} \right) + 2 \left( f_{N,e}^{-} f_{N,m}^{+} + f_{N,m}^{-} f_{N,e}^{+} \right).
\]

(3.8)

We call the first, the second and the third terms as the electric, magnetic and interference parts, respectively.

We now begin with the contribution of the electric part for calculating the imaginary part of the Compton scattering amplitude in use of the unitarity condition in Eq. (2.7). Using the integral formulas in Appendix A, we obtain

\[
\int d\Omega_q \ 2f_{N,e}^{-+} f_{N,e}^{-+} = 8\pi \left( \frac{eG_{NN\pi}}{8\pi M} \right)^2 \times \left\{ \left[ 1 - 2I_2(v) + (\hat{k} \cdot \hat{k} + v^2)J_1(v, \theta) - 2(1 + \hat{k} \cdot \hat{k})J_5(v, \theta) \right] \epsilon^* \cdot \epsilon \right. \\
+ \left[ (\hat{k} \cdot \hat{k} + v^2)J_3(v, \theta) - 2(1 + \hat{k} \cdot \hat{k})J_5(v, \theta) - 2J_6(v, \theta) \right] (\epsilon^* \cdot \hat{k}) (\epsilon \cdot \hat{k}') \\
+ (1 - 2I_2(v)) i \sigma \cdot \epsilon^* \times \epsilon \\
+ (J_1(v, \theta) - 2J_6(v, \theta)) i \sigma \cdot (\hat{k} \times \hat{k}) (\epsilon^* \cdot \epsilon) \\
- J_6(v, \theta) i \sigma \cdot \left( (\epsilon^* \times \hat{k}') (\epsilon \cdot \hat{k}') - (\epsilon \times \hat{k}') (\epsilon^* \cdot \hat{k}) \right) \\
+ J_6(v, \theta) i \sigma \cdot \left( (\epsilon^* \times \hat{k}') (\epsilon \cdot \hat{k}') - (\epsilon \times \hat{k}') (\epsilon^* \cdot \hat{k}) \right) \\
+ (J_3(v, \theta) - 2J_5(v, \theta)) i \sigma \cdot (\hat{k} \times \hat{k}) (\epsilon^* \cdot \hat{k}) (\epsilon \cdot \hat{k}') \right\},
\]

(3.9)

where \( \theta \) is the scattering angle in the center-of-mass system, and \( I_i(v) \) for \( i = 1, \ldots, 5 \) and \( J_j(v, \theta) \) for \( j = 1, \ldots, 6 \) with \( v \) the pion velocity are defined in Appendix A. It is known that
the spin factor of the last term in Eq. (3.3), \( i\sigma \cdot (\mathbf{k}' \times \mathbf{k})(\epsilon'^* \cdot \mathbf{k})(\epsilon \cdot \mathbf{k}') \), is not independent of the other spin factors; actually, we have

\[
\begin{align*}
  i\sigma \cdot (\mathbf{k}' \times \mathbf{k})(\epsilon'^* \cdot \mathbf{k})(\epsilon \cdot \mathbf{k}') &= (1 - (\mathbf{k}' \cdot \mathbf{k})^2) i\sigma \cdot \epsilon'^* \times \epsilon + (\mathbf{k}' \cdot \mathbf{k}) i\sigma \cdot [(\epsilon'^* \times \mathbf{k})(\epsilon \cdot \mathbf{k}') - (\epsilon \times \mathbf{k}')(\epsilon'^* \cdot \mathbf{k})] \\
  &\quad - i\sigma \cdot [(\epsilon'^* \times \mathbf{k}')(\epsilon \cdot \mathbf{k}') - (\epsilon \times \mathbf{k}')(\epsilon'^* \cdot \mathbf{k})] .
\end{align*}
\]

(3.10)

The last term is, therefore, redistributed into the third, the fifth, and the sixth terms in Eq. (3.3), and it turns out that \( \text{Im} A_5(\omega, 0) = \text{Im} A_6(\omega, 0) = 0 \) due to the cancellations; namely, we can read off from Eqs. (3.9) and (3.10) that \( \text{Im} A_5 \) and \( \text{Im} A_6 \) at \( \theta = 0 \) are proportional to \( J_3(v, 0) - 2J_5(v, 0) - J_6(v, 0) \), but this is identically zero, as seen from the definitions in Appendix A. Therefore, the dispersion integrals in Eqs. (3.4) yield \( \gamma_3^{N,e} = \gamma_4^{N,e} = 0 \), except for the anomalous term. This is, however, not correct: The spin-factor of the last term in Eq. (3.9) originally consists of a fourth-order product of the photon momenta as \( i\sigma \cdot (\mathbf{k}' \times \mathbf{k})(\epsilon'^* \cdot \mathbf{k})(\epsilon \cdot \mathbf{k}') \), because the real part of the amplitude must be an analytic function around at \( \omega = 0 \) but the unit vectors \( \mathbf{k} \) and \( \mathbf{k}' \) are not analytic. Consequently, the real part of the amplitude as a coefficient of the last spin-factor should be of \( O(\omega^4) \) or higher. Note that there is no possibility for the real part of the amplitude to have an extra factor \( 1/\omega^2 \) to reduce the power down to \( O(\omega^3) \) at low energies. We shall explicitly show that this argument is correct, by comparing it with Compton scattering amplitude in HBChPT in Appendix B. Therefore, we do not include the contributions from the last term of Eq. (3.9) into the dispersion integrals.\(^1\)

We thus obtain the imaginary parts of the structure functions, \( \text{Im} A_i^{N,e}(\omega, \theta) \), for the electric part in the \( N\pi \) channel as

\[
\begin{align*}
  \text{Im} A_1^{N,e}(\omega, \theta) &= \frac{e^2 G_{NN\pi}^2}{4\pi 8\pi M^2} q \left[ 1 - 2J_2(v) + (\cos \theta + v^2)J_1(v, \theta) - 2(1 + \cos \theta)J_6(v, \theta) \right] , \\
  \text{Im} A_2^{N,e}(\omega, \theta) &= \frac{e^2 G_{NN\pi}^2}{4\pi 8\pi M^2} q \left[ (\cos \theta + v^2)J_3(v, \theta) - 2(1 + \cos \theta)J_5(v, \theta) - 2J_6(v, \theta) \right] , \\
  \text{Im} A_3^{N,e}(\omega, \theta) &= \frac{e^2 G_{NN\pi}^2}{4\pi 8\pi M^2} q \left( 1 - 2I_2(v) \right) , \\
  \text{Im} A_4^{N,e}(\omega, \theta) &= \frac{e^2 G_{NN\pi}^2}{4\pi 8\pi M^2} q \left( J_1(v, \theta) - 2J_6(v, \theta) \right) , \\
  \text{Im} A_5^{N,e}(\omega, \theta) &= -A_6^{N,e}(\omega, \theta) = -\frac{e^2 G_{NN\pi}^2}{4\pi 8\pi M^2} qJ_6(v, \theta) .
\end{align*}
\]

(3.11)

Using the imaginary parts we integrate the dispersion integrals in Eqs. (2.4), and obtain the contribution of the electric part to the polarizabilities:

\[
\alpha^{N,e} = \frac{e^2}{4\pi} \frac{10G_{NN\pi}^2}{192\pi M^2 m_\pi} , \quad \beta^{N,e} = \frac{1}{10} \alpha^{N,e} .
\]

\(^1\)L’vov already argued about this problem and states that the dispersion relation does not work in this case from the high-energy behavior of the relativistic invariant amplitudes with Regge-pole assumption [8].
\[ \gamma_{0}^{N,e} = \frac{e^2 G_{NN\pi}^2}{4\pi 24\pi^2 M^2 m_{\pi}^2}, \]
\[ \gamma_{1}^{N,e} = \gamma_{0}^{N,e}, \quad \gamma_{2}^{N,e} = \frac{1}{2} \gamma_{0}^{N,e}, \quad \gamma_{3}^{N,e} = 4 \gamma_{0}^{N,e}, \quad \gamma_{4}^{N,e} = -\frac{1}{4} \gamma_{0}^{N,e}. \] (3.12)

In terms of the Goldberger-Treiman relation, the results are shown to be the same as those of the \( N\pi \)-loops in HBChPT \[7\]. The \( 1/m_\pi \) and \( 1/m_\pi^2 \) dependences mean that these are the contributions from the pion cloud. Because the proton-neutron difference depends on the amplitude \( f_{N,e}^{(0)} \), we predict only the average between them. It is also known that the prediction of HBChPT up to chiral order \( \epsilon^3 \) yields no isospin dependence.

For the the magnetic part we obtain the imaginary parts, \( \text{Im} A_{i}^{N,m}(\omega, \theta) \), in the same way as those in the electric part

\[ \text{Im} A_{2}^{N,m}(\omega, \theta) = -\frac{e^2 G_{NN\pi}^2}{4\pi 16\pi M^2} \mu_V^2 q \left[ 2 \left( \left| t_{1}^{(-)} \right|^2 + 2 \left| t_{3}^{(-)} \right|^2 \right) + \left( \left| t_{1}^{(+)} \right|^2 + 2 \left| t_{3}^{(+)} \right|^2 \right) \right], \]
\[ \text{Im} A_{1}^{N,m}(\omega, \theta) = -\text{Im} A_{2}^{N,m}(\omega, \theta) \cos \theta, \]
\[ \text{Im} A_{4}^{N,m}(\omega, \theta) = \frac{e^2 G_{NN\pi}^2}{4\pi 16\pi M^2} \mu_V^2 q \left[ 2 \left( \left| t_{1}^{(-)} \right|^2 - \left| t_{3}^{(-)} \right|^2 \right) + \left( \left| t_{1}^{(+)} \right|^2 - \left| t_{3}^{(+)} \right|^2 \right) \right], \]
\[ \text{Im} A_{3}^{N,m}(\omega, \theta) = \text{Im} A_{4}^{N,m}(\omega, \theta) \cos \theta, \]
\[ \text{Im} A_{5}^{N,m}(\omega, \theta) = -\text{Im} A_{4}^{N,m}(\omega, \theta), \]
\[ \text{Im} A_{6}^{N,m}(\omega, \theta) = 0. \] (3.13)

Substituting these terms into the dispersion integrals we find

\[ \gamma_{2}^{N,m} = -\gamma_{4}^{N,m} = \gamma_{0}^{N,m}, \quad \alpha_{N,m}^{N,m} = \gamma_{1}^{N,m} = \gamma_{3}^{N,m} = 0, \] (3.14)

which are equal to the results of the \( \Delta \)-pole contribution in HBChPT \[7\]. At the limit of \( \Gamma_\Delta \to 0 \) we obtain

\[ \beta_{N,m}^{N,m} = \frac{e^2 \mu_V^2}{4\pi M^2 \Gamma_\Delta} \gamma_{0}^{N,m} = -\frac{e^2 \mu_V^2}{4\pi 2M^2 \Delta^2}. \] (3.15)

Identifying the coefficient of a counter term in the HBChPT, \( b_1 \), as \( \mu_V^{\Delta N}/2 \) we see that this is just the \( \Delta \)-pole contribution in HBChPT \[7\]. \( b_1 \) is numerically about \(-2.5 \pm 0.35 \) from a tree level relativistic analysis \[2\], while \( \mu_V^{\Delta N}/2 \) is \(-2.5 \) with use of the experimental value of \( \mu_V \).

The interference part of the electric and magnetic terms is given as follows, by noting that \( t_{3}^{(-)} \) are complex functions due to the \( \Delta \) width:

\[ \int d\Omega_q \left[ f_{N,e}^{(-)} f_{N,m}^{(-)} + f_{N,e}^{(-)} f_{N,m}^{(-)} \right]. \]

\[ ^2 \text{Recently, Hemmert et al. used the value } b_1^2 = 3.85 \pm 0.15 \text{ obtained within the "small scale expansion" } \[7\]. \text{ The numerical results using this value are similar to our results using the finite } \Delta \text{ width.} \]
The last term, which is anti-hermite, disappears at the narrow width limit of the \( \Delta \) state and is neglected in the following. We then get

\[
\text{Im} A_{2}^{N,i}(\omega, \theta) = -\frac{e^2 G_{NN\pi}^2}{4\pi 8\pi M^2} \mu_v \frac{2q}{v} I_2(v) \left( t_1^{(-)} - \text{Re} t_3^{(-)} \right), \\
\text{Im} A_{1}^{N,i}(\omega, \theta) = -\text{Im} A_{2}^{N,i}(\omega, \theta) \cos \theta, \\
\text{Im} A_{3}^{N,i}(\omega, \theta) = \frac{e^2 G_{NN\pi}^2}{4\pi 8\pi M^2} \mu_v \frac{2q}{v} \left[ I_2(v) t_1^{(-)} + (2 I_2(v) - 3 I_4(v)) \text{Re} t_3^{(-)} \right] \cos \theta, \\
\text{Im} A_{4}^{N,i}(\omega, \theta) = \frac{e^2 G_{NN\pi}^2}{4\pi 8\pi M^2} \mu_v \frac{2q}{v} \left[ I_2(v) t_1^{(-)} - (I_2(v) - 3 I_4(v)) \text{Re} t_3^{(-)} \right], \\
\text{Im} A_{5}^{N,i}(\omega, \theta) = -\frac{e^2 G_{NN\pi}^2}{4\pi 8\pi M^2} \mu_v \frac{q}{v} I_2(v) \left( 2t_1^{(-)} + \text{Re} t_3^{(-)} \right), \\
\text{Im} A_{6}^{N,i}(\omega, \theta) = 0. 
\]  

(3.17)

From these equations, we find

\[
\alpha_{1}^{N,i} = \gamma_{1}^{N,i} = \gamma_{3}^{N,i} = 0. 
\]  

(3.18)

As an example we show a contribution to Compton scattering from the interference part in Fig. 3 diagrammatically. This kind of diagrams is not taken into account in HBChPT.

**B. \( \Delta \pi \) channel**

Next we examine the \( \gamma + N \rightarrow \pi + \Delta \) contribution. The amplitude is also decomposed into three terms as

\[
f_\Delta f = i \epsilon_{a3b} T^b f_\Delta^{(-)} + T^a f_\Delta^{(0)} + T^a_{a3} f_\Delta^{(+)}, 
\]  

(3.19)

where \( T^a \) is the transition isospin matrix from \( N \) to \( \Delta \), and \( T^a_{a3} = T^a_{\frac{1}{2}} \tau^3 + \frac{1}{2} T^3_{\Delta} T^a \). The electric part is obtained by replacing \( \sigma \) and \( G_{NN\pi} \) in Eq. (3.2) by the transition spin operator \( S_{\Delta N} \) and \( G_{\Delta N\pi} \), respectively. The magnetic part is given by

\[
f_\Delta f^{(-)} = \left( \frac{\epsilon G_{\Delta N\pi}}{8\pi M} \right) \left( \frac{\mu_v}{2M} \right) \left\{ -\frac{(S_{\Delta N} \cdot q)(\sigma \cdot s)}{\omega} - \frac{4}{5} \frac{(S_{\Delta \Delta} \cdot q)(S_{\Delta N} \cdot s)}{\omega q} \right. \\
+ 2 \frac{(S_{\Delta N} \cdot s)(\sigma \cdot q)}{\omega q} - \frac{1}{5} \frac{(S_{\Delta \Delta} \cdot s)(S_{\Delta N} \cdot q)}{\omega} \right\},
\]
\[ f_{\Delta,m}^{(+)} = \left( \frac{eG_{N\pi}}{8\pi M} \right) \left( \frac{\mu_N}{2M} \right) \frac{1}{\omega_q} \left\{ -\frac{(S_{\Delta\pi \cdot q})(\sigma \cdot s)}{\omega} - \frac{1}{5} \frac{(S_{\Delta\Delta \cdot q})(S_{\Delta\pi \cdot q})}{\omega_q} \right\} \]
\[ + \frac{(S_{\Delta\pi \cdot s})(\sigma \cdot q)}{\omega_q} + \frac{1}{5} \frac{(S_{\Delta\Delta \cdot s})(S_{\Delta\pi \cdot q})}{\omega} \right\}, \quad (3.20) \]

where \( S_{\Delta\Delta} \) is the spin matrix for the \( \Delta \) state, and \( \omega_q \) is the energy of pion. From Eq. (3.19) we get in the leading order of \( 1/N_c \) expansion

\[ f_{\Delta} = \frac{4}{3} f_{\Delta,e}^{(-)} f_{\Delta,e}^{(-)} + \frac{4}{3} f_{\Delta,m}^{(-)} f_{\Delta,m}^{(-)} - 2 \left( f_{\Delta,m}^{(-)} f_{\Delta,m}^{(+)} + f_{\Delta,m}^{(+)} f_{\Delta,m}^{(-)} \right) + \frac{14}{3} f_{\Delta,e}^{(+)} f_{\Delta,e}^{(-)} \]
\[ + \frac{4}{3} \left( f_{\Delta,e}^{(-)} f_{\Delta,m}^{(-)} + f_{\Delta,m}^{(-)} f_{\Delta,e}^{(-)} \right) - 2 \left( f_{\Delta,e}^{(+)} f_{\Delta,m}^{(+)} + f_{\Delta,m}^{(+)} f_{\Delta,e}^{(-)} \right). \quad (3.21) \]

The results are as follows: The electric contributions are given by

\[ \text{Im} A_1^{\Delta,e}(\omega, \theta) = \frac{e^2}{4\pi} \frac{G_{N\pi}}{18\pi M^2} \frac{q}{b} \left[ b - 2I_2(v) + b \left( \cos \theta + \frac{v^2}{b^2} \right) J_1(v, \theta) - 2(1 + \cos \theta) J_6(v, \theta) \right], \]
\[ \text{Im} A_2^{\Delta,e}(\omega, \theta) = \frac{e^2}{4\pi} \frac{G_{N\pi}}{18\pi M^2} \frac{q}{b} \left[ b \left( \cos \theta + \frac{v^2}{b^2} \right) J_3(v, \theta) - 2(1 + \cos \theta) J_5(v, \theta) - 2J_6(v, \theta) \right], \]
\[ \text{Im} A_3^{\Delta,e}(\omega, \theta) = -\frac{e^2}{4\pi} \frac{G_{N\pi}}{36\pi M^2} \frac{q}{b} \left[ b - 2I_2(v) \right], \]
\[ \text{Im} A_4^{\Delta,e}(\omega, \theta) = -\frac{e^2}{4\pi} \frac{G_{N\pi}}{36\pi M^2} \frac{q}{b} \left[ bJ_1(v, \theta) - 2J_6(v, \theta) \right], \]
\[ \text{Im} A_5^{\Delta,e}(\omega, \theta) = -\text{Im} A_6^{\Delta,e}(\omega, \theta) = \frac{e^2}{4\pi} \frac{G_{N\pi}}{36\pi M^2} \frac{q}{b} J_6(v, \theta), \quad (3.22) \]

where \( b = \omega/\omega_q \). The dispersion integrals with the above amplitudes yield

\[ \alpha^{\Delta,e} = \frac{e^2}{4\pi} \frac{G_{N\pi}}{216\pi^2 M^2} \left[ \frac{9\Delta}{\Delta^2 - m_\pi^2} + \frac{\Delta}{m_\pi^2} + \frac{\Delta^2 - 10m_\pi^2}{(\Delta^2 - m_\pi^2)^{3/2}} \ln R \right], \]
\[ \beta^{\Delta,e} = \frac{e^2}{4\pi} \frac{G_{N\pi}}{216\pi^2 M^2} \left[ -\frac{\Delta}{m_\pi^2} + \frac{\ln R}{(\Delta^2 - m_\pi^2)^{1/2}} \right], \]
\[ \gamma_1^{\Delta,e} = -\frac{e^2}{4\pi} \frac{G_{N\pi}}{216\pi^2 M^2} \left[ \frac{\Delta^2 + 2m_\pi^2}{(\Delta^2 - m_\pi^2)^2} - \frac{3m_\pi^2 \Delta \ln R}{(\Delta^2 - m_\pi^2)^{5/2}} \right], \]
\[ \gamma_2^{\Delta,e} = -\frac{e^2}{4\pi} \frac{G_{N\pi}}{216\pi^2 M^2} \left[ \frac{1}{\Delta^2 - m_\pi^2} - \frac{\Delta \ln R}{(\Delta^2 - m_\pi^2)^{3/2}} \right], \]
\[ \gamma_3^{\Delta,e} = -\gamma_4^{\Delta,e} = \frac{1}{2} \gamma_2^{\Delta,e} \quad (3.23) \]

\(^3\)In a previous paper \[8\], we have not taken into account the energy transfer in the pion propagator in Eq. (3.2), because the energy transfer is of higher order in the \( 1/N_c \) expansion. This leads to different expressions to the amplitudes depending on the parameter \( a = (1 + b^2)/2 \) as shown in the paper. However, we note that \( a = b + \mathcal{O}(1/N_c^2) \). Then, if we put \( a = b \) by neglecting the \( 1/N_c^2 \) term, we obtain the same results as those by the pion propagator with the energy transfer.
with
\[
R = \frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}.
\] (3.24)

The results for the spin-polarizabilities are the same as the results of the \(\Delta\pi\)-loops in HBChPT, but there are a little difference for the spin-independent polarizabilities: we see that the sum \(\alpha + \beta\) is the same as that in [6], but there is no \(\Delta/m_\pi^2\) term in [6]. The magnetic terms are

\[
\text{Im} A_{\Delta,m}^m(\omega, \theta) = -\frac{e^2}{4\pi} \frac{G_{\Delta N\pi}^2}{54\pi M^4} \mu_V q v^2,
\]
\[
\text{Im} A_{\Delta,m}^m(\omega, \theta) = -\text{Im} A_{\Delta,m}^m(\omega, \theta) \cos \theta,
\]
\[
\text{Im} A_{\Delta,m}^m(\omega, \theta) = \frac{1}{4} \text{Im} A_{\Delta,m}^m(\omega, \theta),
\]
\[
\text{Im} A_{\Delta,m}^m(\omega, \theta) = -\text{Im} A_{\Delta,m}^m(\omega, \theta) = -\frac{1}{4} \text{Im} A_{\Delta,m}^m(\omega, \theta),
\]
\[
\text{Im} A_{\Delta,m}^m(\omega, \theta) = 0.
\] (3.25)

We then obtain

\[
\alpha_{\Delta,m}^m = 0,
\]
\[
\beta_{\Delta,m}^m = \frac{e^2}{4\pi} \frac{G_{\Delta N\pi}^2}{54\pi^2 M^4} \mu_V \left[ \frac{\Delta^3}{\Delta} - 3\pi m_\pi^2 - \frac{3m_\pi^2(\Delta^2 - 2m_\pi^2)}{\Delta^2 \sqrt{\Delta^2 - m_\pi^2}} \ln \frac{\Delta}{m_\pi} \right],
\]
\[
\gamma_{\Delta,m}^m = \gamma_{\Delta,m}^m = 0,
\]
\[
\gamma_{\Delta,m}^m = -\gamma_{\Delta,m}^m = -\frac{e^2}{4\pi} \frac{G_{\Delta N\pi}^2}{108\pi^2 M^4} \frac{m_\pi^2}{\Delta^3} \left[ \frac{\Delta(24m_\pi^4 - 20m_\pi^2\Delta^2 - \Delta^4)}{6m_\pi^2(\Delta^2 - m_\pi^2)} \ln \frac{\Delta}{m_\pi} \right] + 2\pi + \frac{8m_\pi^4 - 12m_\pi^2\Delta^2 + 3\Delta^4}{2m_\pi(\Delta^2 - m_\pi^2)^{3/2}} \ln \frac{\Delta}{m_\pi}.
\] (3.26)

The interference part is calculated to be

\[
\text{Im} A_{\Delta,i}^m(\omega, \theta) = \frac{e^2}{4\pi} \frac{G_{\Delta N\pi}^2}{18\pi M^3} \mu_V q I_2(v),
\]
\[
\text{Im} A_{\Delta,i}^m(\omega, \theta) = \text{Im} A_{\Delta,i}^m(\omega, \theta) \cos \theta,
\]
\[
\text{Im} A_{\Delta,i}^m(\omega, \theta) = \frac{e^2}{4\pi} \frac{G_{\Delta N\pi}^2}{36\pi M^3} \mu_V \frac{q}{b} I_4(v) \cos \theta,
\]
\[
\text{Im} A_{\Delta,i}^m(\omega, \theta) = \frac{e^2}{4\pi} \frac{G_{\Delta N\pi}^2}{36\pi M^3} \mu_V q \left( I_2(v) - \frac{1}{b} I_4(v) \right),
\]
\[
\text{Im} A_{\Delta,i}^m(\omega, \theta) = -\frac{1}{4} \text{Im} A_{\Delta,i}^m(\omega, \theta),
\]
\[
\text{Im} A_{\Delta,i}^m(\omega, \theta) = 0.
\] (3.27)

We note that the relations of the polarizabilities (3.14) are also realized in the \(\Delta\pi\) channel.
IV. CONTRIBUTION OF THE ANOMALOUS PART

The anomalous part is described as the contribution from the Wess-Zumino-Witten term \([14]\), whose Lagrangian is given by

\[
L_{WZW} = e^2 \int d^3x \, \epsilon^{\mu
u\rho\sigma} \partial_\mu A_\nu(x)A_\rho(x)W_\sigma(x) \tag{4.1}
\]

with

\[
W_\sigma(x) = -\frac{1}{8\pi^2 f_\pi^2} [\Phi_0(x) \partial_\sigma \Phi_3(x) - \Phi_3(x) \partial_\sigma \Phi_0(x)] , \tag{4.2}
\]

where \(\Phi_a(x)\) and \(A_\mu(x)\) are the pion and photon fields, respectively.

Following the method in Ref. \([9]\), we obtain the anomalous part as the following seagull term:

\[
f_{\text{seagull}} = \epsilon_i^i \epsilon_j^j \frac{i}{4\pi} \int d^3 y \, e^{-ik'y} \langle N(p') | \{ [A_i(y), J_j(0)] - i\omega [A_i(y), J_j(0)] \} | N(p) \rangle . \tag{4.3}\]

Here, the interaction current \(J_i\) is calculated from the Wess-Zumino-Witten Lagrangian as

\[
J_i = 2e^2 \epsilon_ijk \pi^k , \tag{4.4}
\]

where \(\pi^k\) denotes the momentum field of photon. We then obtain

\[
f_{\text{seagull}} = 2i\omega \epsilon_i^i \epsilon_j^j \frac{e^2}{4\pi} \langle N(p') | W_k | N(p) \rangle . \tag{4.5}\]

The pion fields between baryon states are reduced to the classical soliton fields as follows:

\[
\Phi_0(x) = \hat{\phi}_0(x - X(t)), \Phi_a(x) = R_{ai} \hat{\Phi}_a(x - X(t)) \text{ with } \hat{\phi}_0(r) = f_\pi \cos F(r) \text{ and } \hat{\phi}_a(r) = f_\pi \hat{r}_i \sin F(r), \]

where \(F(r)\) is the profile function, \(R_{ai}\) the orthogonal rotation matrix, and \(X(t)\) the center of the soliton. The matrix element is then given by

\[
\langle N(p') | W_k | N(p) \rangle = -\frac{1}{8\pi^2 \Lambda_{NN} \tau_3 \sigma_l} \frac{1}{\omega_q^2} F_{lk}(q^2) , \tag{4.6}\]

where \(\Lambda_{NN}\) is defined to be \(-1/3\), \(q = k' - k = p - p'\) is the pion momentum, and

\[
F_{lk}(q^2) = \frac{\omega_q^2}{f_\pi^2} \int d^3r \, e^{iqr} \left[ \hat{\phi}_0(r) \partial_k \hat{\phi}_l(r) - \hat{\phi}_l(r) \partial_k \hat{\phi}_0(r) \right] . \tag{4.7}\]

\(F_{lk}(q^2)\) is calculated to be

\[
F_{lk}(q^2) = -i q_k q_l \omega_q^2 \int d^3r \, \hat{j}_l(qr) \frac{\cos F(r) \sin F(r) + \mathcal{O}(\omega_q^2)}{q} \rightarrow \frac{q_k q_l \, G_{NN\pi}}{\Lambda_{NN} 2M f_\pi} , \tag{4.8}\]

where we have taken the limit of \(\omega_q^2 \rightarrow 0\) \([11]\). Neglecting the \(q^2\) dependence of \(F_{lk}(q^2)\) we finally obtain
\[
f_{\text{seagull}} = -\frac{e^2 \omega G_{NN\pi}}{4\pi^2 m^2_\pi 2M f_\pi} i\mathbf{q} \cdot \epsilon' \times \epsilon (\mathbf{\sigma} \cdot \mathbf{q}) \tau_3. \tag{4.9}
\]

In Fig. 4 we show the diagram of the anomalous part of Compton scattering. The spin-polarizabilities from the anomalous part can be read off as follows:

\[
\begin{align*}
\gamma'^\text{anom}_1 &= -\frac{e^2 G_{NN\pi}}{4\pi^2 M f_\pi m^2_\pi} \tau_3, \\
\gamma'^\text{anom}_2 &= 0, \quad \gamma'^\text{anom}_3 = -\gamma'^\text{anom}_4 = -\frac{1}{2} \gamma'^\text{anom}_1. \tag{4.10}
\end{align*}
\]

These results are also the same as those in HBChPT. We see that there is no contribution to the forward spin-polarizability \(\gamma_0\).

\textbf{V. RESULTS AND DISCUSSION}

Numerical results of the polarizabilities for the non-Born part are given in Table I, where empirical values of constants in the formulas are used; namely, \(f_\pi = 93\text{MeV}, M = 939\text{MeV}, \Delta = 293\text{MeV}, G_{\pi NN} = 13.5\), and \(m_\pi = 138\text{MeV}\). The results of HBChPT [7] are also given in the table. In the results of HBChPT we give two cases for the \(\Delta\)-pole terms: the upper ones are obtained using the \(\pi N\Delta\) and \(\gamma N\Delta\) coupling constants determined by the “small scale expansion” itself, while the lower in the parentheses are by a tree-level relativistic analysis.

We note that, although the electric part of the polarizabilities is the same as the \(N\pi\) and \(\Delta\pi\) loops in HBChPT, the numerical values are slightly different, because we did not used the Goldberger-Treiman relation. For the \(\Delta\pi\) loops the values in the parentheses are by a tree-level relativistic analysis.

The magnetic part of the \(N\pi\) channel with the narrow-width limit in the chiral soliton model, which is shown in the parentheses, is the same as those of the \(\Delta\)-poles in HBChPT. In the chiral soliton model the finite-width effect of the \(\Delta\) particle reduces the magnetic contribution in the same as for the magnetic polarizability \(\beta\) [5]. In HBChPT the numerical results with the parameters which are determined by the “small scale expansion” are similar to the ones with the finite width in the chiral soliton model.

As shown in section III, no interference part of the electric and magnetic amplitudes is calculated in HBChPT, because these terms are of higher orders in the heavy baryon expansions. We see that the interference part contributes to \(\gamma_2\) and \(\gamma_4\), and that their values are small in \(\gamma_2\), but considerably large in \(\gamma_4\). For the forward spin-polarizability \(\gamma_0\) the contribution of the magnetic part becomes smaller by the effect of the finite width of the \(\Delta\) state, but that of the interference part becomes large; as a result, the sum of them is nearly the same as that at the narrow-width limit of the \(\Delta\) state.

The electric part of the \(\Delta\pi\) channel is rather small in agreement with the result for the \(\Delta\pi\) loops in HBChPT. The magnetic and the interference parts in the \(\Delta\pi\) channel are almost negligibly small. It is expected that the contributions of the \(\Delta\pi\) channel are small compared with those of the \(N\pi\) channel because of the factor \(\omega^4\) in the denominator of the dispersion relation. We infer that the effect of the higher resonances other than the \(\Delta\) state is very small.
In Table II we show the calculated results of the spin-polarizabilities with the anomalous part, and compare those with the results of HBChPT [7] and of the multipole analysis, where HDT and SAID refer [19] and [20], respectively. We see good agreement with the results of HBChPT, but the value of the $\tau_3$-independent part of $\gamma_4$ is large compared with that of HBChPT, and seems to be close to the results of the multipole analysis. This is due to the effect of the interference part between the electric and magnetic amplitudes. As a result, the forward spin-polarizability $\gamma_0$ is also close to those of the multipole analysis. Note that we cannot evaluate the proton and neutron difference, since we did not consider the amplitude $f^{(0)}$ as it is of higher orders. It is known that the proton and neutron difference also cannot be predicted in HBChPT up to the small scale expansion of $O(\epsilon^3)$.

Here, let us mention about the relativistic amplitude in Eq. (3.3). The relativistic one-loop calculation of the $N\pi$ loops has been shown by Bernard et al. [5], and yields the forward spin-polarizability $\gamma_0 = 2.2(3.2) \times 10^{-4}$fm$^4$ for the proton (neutron). The amplitude in Eq. (3.3) is found to yield $\gamma_0 = 3.3 \times 10^{-4}$fm$^4$ with dispersion relation for relativistic invariant amplitude. The value is considerably smaller than the nonrelativistic value $\gamma_0 = 5.1 \times 10^{-4}$fm$^4$. Note that we must calculate the nucleon-pole terms [16], in order to predict the proton-neutron difference. The spin-polarizabilities are calculated to be $\gamma_1 = 4.5, \gamma_2 = 1.6, \gamma_3 = 0.5, \gamma_4 = -0.2$ in units of $10^{-4}$fm$^4$, respectively. These results are compared with the nonrelativistic values of the electric part of the $N\pi$ channel in Table I, and may show the importance of relativistic approach.

In conclusion we have calculated the spin-polarizabilities of the nucleon, where the dispersion relation was used with the imaginary part of Compton scattering amplitude constructed through the unitarity condition from the pion photoproduction amplitude in the chiral soliton model. The pion photoproduction amplitude is given by the electric and magnetic ones, and both the $N\pi$ and $\Delta\pi$ channels are taken into account. We have shown that the electric and magnetic parts in the chiral soliton model agree with the results of the $N\pi$-loops, $\Delta$-poles and $\Delta\pi$-loops calculated in HBChPT. The numerical results are also similar with each other and qualitatively agree with those of the multipole analysis. The interference part of the electric and magnetic amplitudes in the $N\pi$ channel, which is not considered in HBChPT as higher orders, is, however, large especially in $\gamma_4$, and the resulting values of the polarizabilities are closer to those of the multipole analysis. The next-to-leading-order calculation with the $f^{(0)}$ amplitudes is necessary to evaluate the difference of proton and neutron. In this respect it is interesting to note that a large contribution to the proton and neutron difference arises from kaon loops [21].

We showed that the approach with dispersion relation is very powerful in going further in the chiral soliton model. A various application of such an approach is expected to follow.

**APPENDIX A: INTEGRAL FORMULAS**

We give angular integral formulas which are necessary to calculate the polarizabilities:

$$\frac{1}{4\pi} \int d\Omega q \frac{v\hat{q}_i}{1 - v\hat{k} \cdot \hat{q}} = \hat{k}_i I_1(v),$$

$$\frac{1}{4\pi} \int d\Omega q v^2 \hat{q}_i \hat{q}_j \frac{\delta_{ij}}{1 - v\hat{k} \cdot \hat{q}} = \delta_{ij} I_2(v) + \hat{k}_i \hat{k}_j I_3(v),$$
The functions $I_i$ for $i = 1, \ldots, 5$ depend on the pion velocity $v$ as follows:

\[
I_1(v) = -1 + \frac{1}{2v} \ln \left( \frac{1+v}{1-v} \right), \\
I_2(v) = \frac{1}{2} - \frac{1-v^2}{4v} \ln \left( \frac{1+v}{1-v} \right), \\
I_3(v) = -\frac{3}{2} + \frac{3-v^2}{4v} \ln \left( \frac{1+v}{1-v} \right), \\
I_4(v) = -\frac{5}{3}v^2 + \frac{1}{2} - \frac{1-v^2}{4v} \ln \left( \frac{1+v}{1-v} \right), \\
I_5(v) = \frac{2}{3}v^2 - \frac{5}{2}v + \frac{5-3v^2}{4v} \ln \left( \frac{1+v}{1-v} \right). 
\] (A2)

The functions $J_i$ for $i = 1, \ldots, 6$ depend on the pion velocity $v$ and the photon scattering angle $\theta$. We need the value for the forward scattering only, so give them at $\theta = 0$:

\[
J_1(v,0) = -1 + \frac{1}{2v} \ln \left( \frac{1+v}{1-v} \right), \\
J_2(v,0) = \frac{2}{3} + \frac{1}{3(1-v^2)} - \frac{1}{2v} \ln \left( \frac{1+v}{1-v} \right), \\
J_3(v,0) = \frac{1}{3} + \frac{1}{6(1-v^2)} - \frac{1}{4v} \ln \left( \frac{1+v}{1-v} \right), \\
J_4(v,0) = \frac{13}{8}v + \frac{1}{4(1-v^2)} - \frac{15-3v^2}{16v} \ln \left( \frac{1+v}{1-v} \right), \\
J_5(v,0) = \frac{13}{24}v + \frac{1}{12(1-v^2)} - \frac{5-v^2}{16v} \ln \left( \frac{1+v}{1-v} \right), \\
J_6(v,0) = -\frac{3}{4} + \frac{3-v^2}{8v} \ln \left( \frac{1+v}{1-v} \right). 
\] (A3)

\(^4\) L’vov et al. derived the same formulas, but their definitions are different from ours, and theirs are also valid for $\theta \neq 0$ \(^22\).
APPENDIX B: DISPERSION RELATION FOR $N\pi$ ELECTRIC PART

Let $f(\omega)$ be analytic in the upper half plane of complex $\omega$ and be an odd function of $\omega$; then, we find

\[ \text{pRe} \, g(\omega) = \frac{2}{\pi} \text{P} \int_{0}^{\omega_{\text{max}}} d\omega' \frac{\omega'}{\omega'^{2} - \omega^{2}} \text{Im} \, g(\omega') \quad \text{and} \quad \frac{1}{\pi} \text{Im} \int_{C} d\omega' \frac{\omega'}{\omega'^{2} - \omega^{2}} g(\omega'), \]  

(B1)

where the path $C$ denotes the upper semi-circle path in the complex plane. We have derived the dispersion relations in Eqs. (2.4) at the limit of $\omega_{\text{max}} \to \infty$ and by neglecting the semi-circle integrals. We will show that this is erroneous in the case of the electric part for the $N\pi$ channel.

The non-Born part of the structure function of the forward scattering amplitude, $A_{5}(\omega, \theta = 0)$, which is equal to $-A_{6}(\omega, \theta = 0)$, in HBChPT [23] is given by two terms as follows:

\[ A_{51}(\omega, 0) = -\frac{g_{A}^{2}m_{\pi}}{8\pi^{2}F_{\pi}^{2}} u^{2} \int_{0}^{1} dx \frac{(1 - x)^{2} \sin^{-1} ux}{\sqrt{1 - u^{2}x^{2}}}, \]  

(B2)

and

\[ A_{52}(\omega, 0) = \frac{g_{A}^{2}m_{\pi}}{8\pi^{2}F_{\pi}^{2}} u^{4} \int_{0}^{1} dx \frac{x(1 - x)^{3}}{3(1 - u^{2}x^{2})^{3/2}} \left( \sin^{-1} ux + xu\sqrt{1 - u^{2}x^{2}} \right), \]  

(B3)

with $u = \omega/m_{\pi}$. The integration on $x$ can be carried out and lead to

\[ A_{51}(\omega, 0) = -\frac{g_{A}^{2}m_{\pi}}{8\pi^{2}F_{\pi}^{2}} \left( \frac{u^{3}}{12} + \frac{u^{5}}{315} + O(u^{7}) \right), \]  

(B4)

and

\[ A_{52}(\omega, 0) = \frac{g_{A}^{2}m_{\pi}}{8\pi^{2}F_{\pi}^{2}} \left( \frac{u^{5}}{315} + O(u^{7}) \right), \]  

(B5)

where the $u^{5}$ and the higher terms are the same except for the signs in both functions. Therefore, the sum is given by

\[ A_{5}(\omega, 0) = A_{51}(\omega, 0) + A_{52}(\omega, 0) = -\frac{g_{A}^{2}m_{\pi}u^{3}}{8\pi^{2}F_{\pi}^{2}12}, \]  

(B6)

because of complete cancellation among the higher terms. This shows that there is no branch cut on the real axis in the complex $\omega$ plane; namely, there is no imaginary part of the structure function $A_{5}(\omega, 0)$. However, each of $A_{51}(\omega, 0)$ and $A_{52}(\omega, 0)$ has an imaginary part: The analytic continuation to $\omega > m_{\pi}$ gives

\[ \text{Im} \, A_{52}(\omega, 0) = -\text{Im} \, A_{51}(\omega, 0) = \frac{g_{A}^{2}m_{\pi}}{8\pi^{2}F_{\pi}^{2}8u} \left( -6u\sqrt{u^{2} - 1} + (1 + 2u^{2}) \ln \frac{u + \sqrt{u^{2} - 1}}{u - \sqrt{u^{2} - 1}} \right). \]  

(B7)
We see that \( \text{Im} \, A_5(\omega, 0) \) and \( \text{Im} \, A_5(\omega, 0) \) come from the last term in Eq. (B.3) by means of the identity of Eq. (B10). Therefore, there is no contradiction with the HBChPT approach, where the imaginary parts of \( A_5(\omega, 0) \) and \( A_6(\omega, 0) \) are zero, if we include the last term in Eq. (B.3).

Now, let us consider the dispersion relations for \( A_{51}(\omega, 0) \) and \( A_{52}(\omega, 0) \),

\[
\text{Re} \frac{A_{51}(\omega, 0)}{\omega^3} \bigg|_{\omega=0} = \frac{2}{\pi} \text{P} \int_0^{\omega_{\text{max}}} d\omega' \frac{\text{Im} \, A_{51}(\omega', 0)}{\omega'^4} + \frac{1}{\pi} \text{Im} \int_C d\omega' \frac{A_{51}(\omega', 0)}{\omega'^4}, \tag{B8}
\]

\[
\text{Re} \frac{A_{52}(\omega, 0)}{\omega^3} \bigg|_{\omega=0} = \frac{2}{\pi} \text{P} \int_0^{\omega_{\text{max}}} d\omega' \frac{\text{Im} \, A_{52}(\omega', 0)}{\omega'^4} + \frac{1}{\pi} \text{Im} \int_C d\omega' \frac{A_{52}(\omega', 0)}{\omega'^4}. \tag{B9}
\]

From Eq. (B4) the left-hand side (LHS) in Eq. (B8) is

\[
-\frac{g_A^2}{8\pi^2 f_{\pi}^2} \frac{1}{12m_{\pi}^2}. \tag{B10}
\]

In the right-hand side (RHS) the dispersion integral with the imaginary part in Eq. (B7) is found to be equal to the LHS with the vanishing semi-circle integral. On the other hand, the LHS in Eq. (B3) is zero, and the second (semi-circle) integral in the RHS cancels the dispersion integral in the first term. Therefore, the dispersion relation of \( A_5(\omega, 0) \) can be written as

\[
\text{Re} \frac{A_5(\omega, 0)}{\omega^3} \bigg|_{\omega=0} = \frac{1}{\pi} \text{Im} \int_C d\omega' \frac{A_5(\omega', 0)}{\omega'^4} = -\frac{g_A^2}{8\pi^2 f_{\pi}^2} \frac{1}{12m_{\pi}^2} \frac{1}{\pi} \text{Im} \int_C d\omega' \frac{1}{\omega'}, \tag{B10}
\]

where we used the imaginary part of \( A_5(\omega, 0) \) to be zero on the real axis and used Eq. (B6).

The above consideration shows that the semi-circle integral cannot be generally neglected, even though an integral on the real axis is convergent (the integrand is zero on the real axis in the above case). Considering a once-subtracted dispersion relation we may neglect the integral on the semi-circle, but we cannot determine the polarizability in this case. This situation leads to an idea that the dispersion relation cannot be used to obtain the polarizability, which is a coefficient of \( \omega^3 \) term. However, \( A_{52}(\omega, 0) \) is of \( \mathcal{O}(\omega^5) \), as seen in Eq. (B5), so that the amplitude \( A_{52}(\omega, 0) \) does not participate in the game, and we consider only the dispersion relation in Eq. (B8), where the semicircle integral can be dropped. Such an \( \omega \) dependence can be inferred from the construction as noted in section III. However, a careful treatment is necessary in such a case as multipole analysis.
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FIG. 1. Born graphs of the electric part of the pion photoproduction in Eq. (3.2). The solid, dashed and wavy lines denote the nucleon, pion and photon, respectively. See text for the details.

FIG. 2. Born graphs of the magnetic part of the pion photoproduction in Eqs. (3.4). The double solid line denote the $\Delta$ particle. The others are the same as in Fig. 1. (a) and (b) are the direct and crossed terms of the nucleon poles, respectively. (c) and (d) are the same as (a) and (b), respectively, but with the $\Delta$ poles.

FIG. 3. The interference between the electric amplitude in Fig. 1(a) and the magnetic one in Fig. 2(c). The vertical dotted line denotes the on-shell.
FIG. 4. The anomalous $\pi_0\gamma\gamma$ term.
TABLE I. Calculated spin-polarizabilities of the nucleon in the chiral soliton model without the anomalous term due to the $\pi_0\gamma\gamma$ process. For a comparison those in HBChPT are also shown. Parameters in the chiral soliton model are taken to be empirical ones, except for the $N\Delta$ transition parameters predicted in the soliton model. For the chiral soliton model $E$, $M$ and $I$ denote the contributions of the electric, magnetic and interference parts, respectively. In the parentheses the values at the narrow-width limit of the $\Delta$ particle are given in the chiral soliton model. For the results of HBChPT the numbers in the parentheses are the values with the old estimation of $g_{\pi N\Delta}$ and $b_1$. All values are in unit of $10^{-4}$fm$^4$.

|          | Chiral Soliton model | HBChPT [7] |
|----------|----------------------|------------|
|          | $N\pi$ channel       | $\Delta\pi$ channel |
|          | $E$ | $M$ | $I$ | $E$ | $M$ | $I$ | $N\pi$-loop | $\Delta$-pole | $\Delta\pi$-loop |
| $\gamma_1$ | 5.1 | 0.0 | 0.0 | -0.4 | 0.0 | 0.0 | 4.56 | 0 | -0.21 (0.4) |
| $\gamma_2$ | 2.5 | -2.5 | -0.1 | -0.4 | 0.1 | 0.5 | 2.28 | -2.40 | -0.23 (0.5) |
| $\gamma_3$ | 1.3 | 0.0 | 0.0 | -0.2 | 0.0 | 0.0 | 1.14 | 0 | -0.12 (0.2) |
| $\gamma_4$ | -1.3 | 2.5 | 1.2 | 0.2 | -0.1 | -0.1 | -1.14 | 2.40 | 0.12 (0.2) |
| $\gamma_0$ | 5.1 | -2.5 | -2.4 | -0.4 | 0.1 | -0.3 | 4.5 | -2.4 | -0.2 (0.4) |

TABLE II. Spin-polarizabilities of the nucleon in the chiral soliton model with the anomalous term. The results in HBChPT and of the multipole analysis are also shown. All values are in unit of $10^{-4}$fm$^4$.

|          | Chiral Soliton | HBChPT [7] | HDT [19] | SAID [20] | multipole analysis |
|----------|----------------|------------|----------|-----------|-------------------|
|          | p | n | p | n | $\pi^0$ exch. |
| $\gamma_1$ | 4.7$-22.8\tau_3$ | 4.4$-21.7\tau_3$ | 5.1 | 6.1 | 3.1 | 6.3 | -22.5$\tau_3$ |
| $\gamma_2$ | 0.1 | -0.3 | -1.1 | -0.8 | -0.8 | -0.9 |
| $\gamma_3$ | 1.1$+11.4\tau_3$ | 1.1$+10.9\tau_3$ | -0.6 | -0.6 | 0.3 | -0.7 | 11.2$\tau_3$ |
| $\gamma_4$ | 2.5$-11.4\tau_3$ | 1.3$-10.9\tau_3$ | 3.4 | 3.4 | 2.7 | 3.8 | -11.2$\tau_3$ |
| $\gamma_0$ | -0.1 | 2.0 | -0.6 | -0.2 | -1.5 | -0.4 |