Scaling Behaviors and Novel Creep Motion of Flux Lines under AC Driving

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We performed Langevin dynamics simulations for the ac driven flux lines in a type II superconductor with random point-like pinning centers. Scaling properties of flux-line velocity with respect to instantaneous driving force of small frequency and around the critical dc depinning force are revealed successfully, which provides precise estimates on dynamic critical exponents. From the scaling function we derive a creep law associated with the activation by the regular shaking. The effective energy barrier vanishes at the critical dc depinning point in a square-root way when the instantaneous driving force increases. The frequency plays a similar role of temperature in conventional creep motions, but in a nontrivial way governed by the critical exponents. We have also performed systematic finite-size scaling analysis for flux-line velocity in transient processes with dc driving, which provides estimates on critical exponents in good agreement with those derived with ac driving. The scaling law is checked successfully.

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Introduction. – Dynamics of elastic manifolds driven through a random medium is one of the rich paradigms in condensed matter physics. Important examples range from ferromagnetic or ferroelectric domains, charge-density waves, frontiers of fluids in porous media, Wigner crystal, and flux lines in type II superconductors [1–3]. The notion that the depinning transition by a dc driving at zero temperature in these systems exhibits properties similar to the critical phenomena of a second-order phase transition at thermal equilibrium has been verified successfully in many systems in this class [4], while a possibility was raised recently on the absence of divergent correlation length below the depinning point [5].

Special attentions have been paid to the dynamic phenomena of flux lines in bulk superconductors with randomly distributed defects [6]. It is because, first of all, the system is important for applications of superconductivity, such as transport of large current and generation of strong magnetic field. Flux lines are also unique with respect to the depinning transition due to the two-dimensional (2D) dynamic degrees of freedom under a driving transverse to the direction of magnetic field, which hampers an analytic treatment based on the functional renormalization group [7]. Computer simulations based on Langevin dynamics [8–14] were performed in order to fill the gap.

While the critical depinning properties of flux lines under dc driving were addressed quite satisfactorily (see for example [14]), behaviors under ac driving have not yet been fully explored so far, despite of its importance and convenience as an experimental probe. Fortunately, a scaling theory for small frequencies around the critical dc depinning point has been formulated based on analysis on an elastic string embedded in 2D random medium [15], which provides a facet into this problem and motivates our present study.

We performed computer simulations for the ac driven flux lines in presence of quenched random point-like pins based on the Langevin dynamics. Scaling properties of flux-line velocity with respect to instantaneous driving force of small frequency and around the critical dc depinning force are revealed successfully, which provides precise estimates on dynamic critical exponents. From the scaling function we derive a creep law associated with the activation by the regular shaking. The effective energy barrier vanishes at the critical dc depinning point in a square-root way when the instantaneous driving force increases. The frequency plays a similar role of temperature in conventional creep motions, but in a nontrivial way governed by the critical exponents. We have also performed systematic finite-size scaling analysis for flux-line velocity in transient processes with dc driving, which provides estimates on critical exponents in good agreement with those derived with ac driving. The scaling law is checked successfully.

Model and simulation technique. – The model system is a stack of superconducting layers with a perpendicular magnetic field, same as that used in Ref. [14]. The overdamped equation of motion of the i-th vortex at position $r_i$ is

$$\eta \ddot{r}_i = - \sum_{j \neq i} \nabla r_{ij} U_{VV}(r_{ij}) - \sum_p \nabla r_{ip} U_{VP}(r_{ip}) + \mathbf{F} + \mathbf{F}_{th}, \quad (1)$$

with $\eta$ the viscosity coefficient. The intraplane vortex repulsion is given by the modified Bessel function, the interplane vortex attraction is approximated by a spring potential between two vortices belonging to the same flux line and sitting on adjacent planes, and pinning centers are modeled by Gaussian potentials with typical size $R_p$ distributed at random positions but with constant pinning range and dimensionless pinning strength $\alpha$; $\mathbf{F}$ is the Lorentz force uniform over the system but varies with time as $F(t) = A \sin \omega t$, and $\mathbf{F}_{th}$ is the thermal
noise force. In this work, the units for length, energy, temperature, force, time, and velocity are taken as \( \lambda_{ab}, \) 
\( \Delta q, \) 
\( \Delta q/k_B, \) 
\( \Delta q/\lambda_{ab}, \) 
\( \eta \lambda_{ab}^2/\Delta q (\equiv \tau_0), \) 
and \( \Delta q/\eta_{ab} \), respectively, where \( \tau_0 = \phi_0^2/2\pi \mu_0 \lambda_{ab}^2 \) \( \lambda_{ab} \) the magnetic penetration depth, and \( d \) the thickness of the superconducting layer. The results will be given in dimensionless units hereafter. The dynamic equation (1) is integrated with the 2nd order Rumen-Kutta algorithm with \( \Delta t = 0.01 \sim 0.02. \) More details about our model system can be found in Ref. [13].

In simulations there are totally \( N_v = 180 \) flux lines in the system of lateral size \( L \times L = 30 \times 30 \) and 20 layers in the \( c \) axis, with periodic boundary conditions. In-plane vortex-vortex repulsions are cut off at \( r_{cut} = 6. \) Each layer contains \( N_p = 900 \) randomly distributed point pins with \( R_p = 0.22 \) and \( \alpha = 0.2, \) associated with a vortex glass at thermal equilibrium [14]. For different system sizes, the densities of the flux lines and the point pins are fixed. In the present work we concentrate on zero temperature.

**ac driving.** − We first investigate motions of flux lines under an ac driving. Double hysteresis loops [13] are observed as displayed in the inset of Fig. 1(a) for \( \omega = 0.01 \pi. \) It is clear that for a finite \( \Delta \) the velocity of the system is nonzero even when the instantaneous driving force is below the critical dc depinning force \( F_{co} \) derived from motions of flux lines under dc driving [14]. We therefore refer to \( F_{co} \) simply as critical depinning force hereafter.

As the frequency \( \omega \) approaches zero, the double loops shrink to the depinning curve of the dc driving [14], as seen in the main panel of Fig. 1(a) where the instantaneous force dependence of the velocity is shown around \( F_{co} \) in the increasing branch. According to the scaling theory [15], the behavior of the system for \( \omega < \omega_p = F_{co}/\eta h \simeq 0.08\pi \) is governed by the critical properties of the depinning fixed point

\[
v(t) = \omega^{\beta/\nu_z} \phi \left[ (F(t)/F_{co} - 1)\omega^{-1/\nu_z} \right]; \tag{2}\]

here the critical exponents are defined by the onset of the velocity \( v \sim (F/F_{co} - 1)^{\beta} \) under dc driving, the divergent correlation length \( \xi \sim |F/F_{co} - 1|^{\nu}, \) and the growth of correlation length with time \( \xi \sim t^{1/\nu_z} \) at \( F = F_{co}. \) It is requested that \( \phi(x) \sim x^\beta \) for \( x \to \infty \) in order to recover the steady depinning behavior.

The scaling plot according to Eq. (2) sees a good collapsing with \( \beta/\nu_z = 0.40 \pm 0.01 \) and \( 1/\nu_z = 0.53 \pm 0.02, \) and \( F_{co} = 0.232 \pm 0.002 \) as displayed in the main panel of Fig. 1(b). The estimates on the exponent \( \beta = 0.76 \pm 0.02 \) and \( F_{co} \) agree well with the previous results based on dc driving at low but finite temperatures (\( \beta = 0.75 \pm 0.01 \) and \( F_{co} = 0.232 \pm 0.001 \)) [14].

A clear asymptotic behavior is observed for the scaling function \( \phi(x) \) as \( x \to -\infty \) in the main panel of Fig. 1(b). As replotted in the inset of Fig. 1(b), the asymptote is well described by

\[
v(t) \sim \omega^{\beta/\nu_z} \exp \left[ -0.35\sqrt{1 - F(t)/F_{co}} \right] \omega^{1/2\nu_z}, \tag{3}\]

for \( F(t) < F_{co}. \) The motion of flux lines is creep like, with an effective energy barrier \( 0.35 \sqrt{1 - F(t)/F_{co}} \) which vanishes when the instantaneous driving force approaches \( F_{co} \) from below. The creep motions of flux lines here are caused by the regular shaking of the ac driving, instead of the thermal activations at finite temperature [14][16]. Because of the random pinning potential landscape, a regular shaking contributes to activation of flux lines over pinning barriers in a complex way governed by the critical properties, both steady and dynamic, of the system, which is captured by the frequency dependence in the creep law [14]. A theory on the square-root suppression
of the energy barrier is not available at this moment.

To end this part we notice that, as seen in the scaling theory \([2]\), the critical exponents \(\nu\) and \(z\) appear in the form of product in the dynamics under ac driving, which leaves a full description of the critical dynamics unavailable. On the other hand, because of the hysteretic response of flux lines to ac driving, it is still hard to make any serious analysis on finite-size effects on the accuracy of the critical exponents with the computing resources available at this moment. These issues are addressed in what follows.

Onset of collective pinning. – Let us now investigate a process associated with the onset of collective pinning, which involves only the exponent \(\nu\). We lay at \(t = 0\) a perfect triangular lattice of flux lines on the random medium, and start to drive the flux lines by a dc force. The flux-line lattice deforms during traveling when the individual flux lines adapt to the random potential landscape. As the result, the dragging caused by random pinnings becomes stronger due to the collective pinning mechanism \([3]\). When the dc driving force is below the critical depinning one, the flux lines should be stopped as a whole after traveling a certain distance. This traveling distance depends on how far the driving force is from the critical point, and is determined by the critical properties of the depinning fixed point.

We have simulated this process in several finite systems, and the results are displayed in the main panel of Fig. 2(a). It is clear that, for a given finite system, the traveling distance diverges (since a periodic boundary condition is adopted) as the driving force approaches a critical value given by the system size. The critical force decreases as the system size increases. The critical behavior for an infinite system is given by

\[
D \sim (1 - F/F_{c0})^{-\nu_D},
\]

with an exponent \(\nu_D\), and the convergence from finite systems to the infinite system should be described by the finite-size scaling theory \([19]\):

\[
D = L^{\nu_D/\nu}S \left[(F/F_{c0} - 1)\right]^{1/\nu}.
\]

The scaling plot according to Eq. (5) is shown in the inset of Fig. 2(a). From the successful scaling plot, we obtain \(\nu_D/\nu = 1.00 \pm 0.05, \nu = 1.06 \pm 0.04\), and \(F_{c0} = 0.2365 \pm 0.0005\). The present estimate on the critical depinning force agrees with the above one derived for ac driving and that in the previous work for dc driving \([14]\), but with the highest precision since finite-size effects are taken into account here.

Two features of the scaling function \(S(x)\) are observed as follows: (i) As shown in Fig. 2(b), \(S(x) \sim (x) - x^{-\nu_D}\) for \(x \to -\infty\) as requested by the scaling theory, which achieves the critical behavior \([4]\). (ii) As shown in Fig. 2(c), \(S(x) \simeq a(x_c - x)^{-p}\) for \(x \to x_c\), with \(a \simeq 0.53, x_c \simeq 0.89\), and \(p \simeq 1.86\). This relation gives the system-size dependence of the critical depinning force, and the divergence of the traveling distance in any finite system with an exponent \(p\), different from that in an infinite system.

The successful finite-size scaling plots in Fig. 2 including the exponent \(\nu\) imply the divergent correlation length both above and below the critical depinning force with the same exponent. In contrast, absence of diverging correlation length below the critical depinning force was discussed for an elastic string embedded in a 2D random potential landscape \([3]\). The reason for this discrepancy remains as an interesting problem for future work.

Critical slow down. – As a second transient process associated with dc driving, we study the critical slowing down of flux lines, which provides a way to estimate separately the exponent \(z\). Here a steady state of flux
The velocity of the flux lines then decreases with time according to $v \sim t^{-\beta/\nu z}$ \cite{20}. We have simulated the critical slowing down for several finite systems, and the results are depicted in Fig. 3(a). The velocity decreases quickly in a small system since the corresponding critical force is large, as seen in the onset process of collective pinning in Fig. 2(a). The finite-size scaling behavior at $F = F_{c0}$ is described by \cite{20,21}

$$v(t, L) = b^{\beta/\nu} v(b^z t, bL).$$  

(6)

The scaling plot is performed successfully as shown in Fig. 3(b), and we obtain $\beta/\nu = 0.72 \pm 0.02$ and $z = 1.80 \pm 0.03$. The slope of the scaling function at small argument is given by $\beta/\nu z \approx 0.40$ as required by the scaling theory on short-time dynamics \cite{20,21}.

**Discussions.**—The critical exponents estimated in the present study satisfy the scaling law $\beta + \nu(2 - z) = 1$ \cite{17}. The roughness exponent $\zeta$ can be evaluated by the scaling relation $\nu = 1/(2 - \zeta)$ \cite{17} as $\zeta = 1.06 \pm 0.04$.

In conclusion, with the aid of computer simulation and scaling theory, a comprehensive picture has been obtained for the depinning transition of current driven flux lines in type II superconductors with random point-like pins.

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