On finite size corrections to the dispersion relations of giant magnon and single spike on $\gamma$-deformed $T^{1,1}$

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Abstract

In this paper we consider the finite size effects for the strings in $\beta$-deformed $AdS_5 \times T^{1,1}$ background. We analyze the finite size corrections for the cases of giant magnon and single spike string solution. The finite size corrections for the undeformed case are straightforwardly obtained sending the deformation parameter to zero.

1 Introduction

After the remarkable AdS/CFT conjecture of Maldacena [1] relating the string theory on $AdS_5 \times S^5$ to the $\mathcal{N} = 4$ super Yang-Mills the search for less supersymmetric cases has started. One possible way is to consider a stack of N D3-branes at the tip of the conifold [2] resulting in the spacetime being the product $AdS_5 \times T^{1,1}$. The dispersion relations on the string theory side plays an important role, since they correspond to the large N limit of the anomalous dimension of particular gauge theory operators. Finding the spectrum is however a complicated task and one usually considers first states for which one of the string charges is infinite. This simplification can be seen in the light-cone gauge as a decompactifying limit, since one can rescale the worldsheet coordinate, such that the theory is defined on a cylinder with circumference proportional to the light-cone momentum (see [3] and references therein). In [4] it was shown that at large $\lambda$ in the decompactifying limit a one-magnon excitation with the finite worldsheet momentum can be identified with the classical string sigma model. This string solution will have infinite momentum and infinite energy, but the difference of the two will be finite and equal to the energy of the worldsheet soliton [4, 3]. On the target space it corresponds to an open rigidly moving string with the distance of the endpoints fixed and proportional to the worldsheet momentum. This solution constructed in the conformal gauge was called the giant magnon [4]. The situation on the conifold was analyzed in [5].

One possible way of breaking the supersymmetry was found by Lunin and Maldacena in [6]. They considered a marginally deformed $\mathcal{N} = 4$ super Yang-Mills and found its gravity dual. This idea was further extended in [7, 8] and applied to other backgrounds, see [9].

Since all these considerations solve the theory on a plane, one can ask what is the difference to the physical case of a cylinder worldsheet. Such a modification can be significant, but for

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sufficiently large size of the cylinder one can use an asymptotic construction \[3\] and obtain the finite size corrections.

In this paper we extend our previous considerations \[9\] computing the finite size corrections for the case of giant magnons and single spikes on the $\gamma$-deformed $T^{1,1}$. Since the deformation is continuous we obtain the finite size correction to the undeformed case \[5\] as well, simply by setting $\tilde{\gamma} = 0$.

## 2 A review of the case of infinite charges and finite size setup

In this section we review the results of the case of infinite charges \[9\], reformulate some of the important steps of the calculations to a convenient form and make a setup for the finite size calculations. Setting $\tilde{\gamma} = 0$ one can recover the undeformed case $AdS_5 \times T^{1,1}$ and the obtained results translate to that case.

### 2.1 Review of the infinite case

The TsT transformations \[6, 8, 10\] of the standard conifold metric results in the metric and the B-field of the $\gamma$-deformed $T^{1,1}$

\[
ds^2 = ds^2_{AdS} + G \left( \frac{1}{6} \sum_{i=1}^{2} \left( G^{-1} d\theta_i^2 + \sin^2 \theta_i d\phi_i^2 \right) + \frac{1}{9} \left( d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2 \right)^2 + \tilde{\gamma} \frac{\sin^2 \theta_1 \sin^2 \theta_2}{324} d\psi^2 \right), 
\]

\[
B = \tilde{\gamma} G \left( \frac{\sin^2 \theta_1 \sin^2 \theta_2}{36} + \frac{\cos^2 \theta_1 \sin^2 \theta_2 + \cos^2 \theta_2 \sin^2 \theta_1}{54} \right) d\phi_1 \wedge d\phi_2 + \frac{\sin^2 \theta_1 \cos \theta_2}{54} d\phi_1 \wedge d\psi - \frac{\cos \theta_1 \sin^2 \theta_2}{54} d\phi_2 \wedge d\psi \right). 
\]

The standard conifold metric is the one in (1) with $\tilde{\gamma} = 0$ and the conformal factor $G$ in (1),(2) is

\[
G^{-1} = 1 + \tilde{\gamma} \left( \frac{\cos^2 \theta_1 \sin^2 \theta_2 + \cos^2 \theta_2 \sin^2 \theta_1}{54} + \frac{\sin^2 \theta_1 \sin^2 \theta_2}{36} \right).
\]

Since the complete $T^{1,1}$ background amounts to a complicated set of equations of motion we proceed with a consistent truncation to the subspace defined by

$\theta_2 = 0$, $\phi_2 = \text{const.}$

The ansatz leading to a solitonic type solution is

$\Psi = \omega_\phi \tau - \psi (y)$, $\Phi = \omega_\tau + \phi (y)$, $\theta = \theta (y)$,

where $y = c \sigma - d \tau$ and setting $R = 1$ we restrict the metric (1) and the B-field (2) to

\[
ds^2 = -dt^2 + \frac{1}{6} d\tau^2 + \frac{G}{6} \sin^2 \theta d\phi^2 + \frac{G}{9} \left( d\psi + \cos \theta d\phi \right)^2 
\]

\[
B = G \frac{\sin^2 \theta}{54} 
\]

\[
G^{-1} = 1 + \frac{\gamma \sin^2 \theta}{54}.
\]
The Virasoro constraints \[9\] read off

\[ L = t^2 + \frac{c^2 - d^2}{6} \theta'^2 + \frac{G}{9} \left( 1 + \frac{\sin^2 \theta}{2} \right) \left( - (\omega_\phi - d \psi')^2 + c^2 \psi'^2 \right) + \]

\[ \frac{2}{9} G \cos \theta \left( - (\omega_\phi - d \psi') (\omega_\phi - d \psi') + c^2 \psi' \phi' \right) + \frac{G}{9} \left( - (\omega_\phi - d \psi')^2 + c^2 \psi'^2 \right) + \]

\[ 2 G \frac{\sin^2 \theta}{54} c (\omega_\phi - d \psi') \psi' - (\omega_\psi - d \psi') \phi' \]

and the once integrated equations of motion are \((A_\phi \text{ and } A_\psi \text{ below are the integration constants})\)

\[ (c^2 - d^2) \phi' + d \omega_\phi = \frac{3 (A_\phi - A_\psi \cos \theta)}{G \sin^2 \theta} + \frac{c}{9} (\omega_\phi + \omega_\psi \cos \theta) \]

\[ (c^2 - d^2) \psi' + d \omega_\psi = \frac{3 (A_\psi - A_\phi \cos \theta)}{G \sin^2 \theta} + \frac{3 A_\psi}{2G} - \frac{c}{9} \left( \omega_\psi \cos \theta + \omega_\phi \left( 1 + \frac{\sin^2 \theta}{2} \right) \right). \]

The Virasoro constraints \[9\] read off

\[ \frac{c^2 + d^2}{6} \theta'^2 + \frac{G}{9} \left( 1 + \frac{\sin^2 \theta}{2} \right) \left( (\omega_\phi - d \psi')^2 + c^2 \phi'^2 \right) + 2 \frac{G}{9} \cos \theta ((\omega_\phi - d \psi') (\omega_\phi - d \phi') + \]

\[ c^2 \psi' \phi') + \frac{G}{9} \left( (\omega_\phi - d \psi')^2 + c^2 \psi'^2 \right) = \kappa^2, \]

\[ \frac{1}{6} \theta'^2 + \frac{G}{9} \left( 1 + \frac{\sin^2 \theta}{2} \right) \left( \phi'^2 - \frac{\omega_\phi \phi'}{d} \right) + \frac{G}{9} \left( \psi'^2 - \frac{\omega_\psi \psi'}{d} \right) + \]

\[ \frac{G}{9} \cos \theta \left( 2 \phi' \phi' - \frac{\omega_\phi \phi' + \omega_\psi \psi'}{d} \right) = 0 \]

and the restriction on the parameters is just

\[ 2 d \kappa^2 = A_\phi \omega_\phi + A_\psi \omega_\psi. \quad (4) \]

Using the second Virasoro constraint we obtain the equation for \(\theta\)

\[ \theta'^2 + \frac{1}{3 (c^2 - d^2) \sin^2 \theta} \left( \frac{2 c \omega_\phi - \tilde{\gamma} A_\psi}{2} \right)^2 \cos^4 \theta - \frac{1}{3 (c^2 - d^2) \sin^2 \theta} \left( \frac{2 c \omega_\phi - \tilde{\gamma} A_\psi}{2} \right)^2 \cos^2 \theta - \]

\[ \frac{1}{2} \left( 2 c \omega_\phi - \tilde{\gamma} A_\psi \right)^2 + 2 \left( 2 c \omega_\phi - \tilde{\gamma} A_\psi \right)^2 + 27 A_\phi^2 - \frac{18}{d} (c^2 + d^2) \left( A_\phi \omega_\phi + A_\psi \omega_\psi \right) \cos^2 \theta + \]

\[ \left( 2 c \omega_\phi - \tilde{\gamma} A_\psi \right)^2 + \frac{1}{2} \left( 2 c \omega_\phi - \tilde{\gamma} A_\psi \right)^2 + \frac{3}{2} \left( 2 c \omega_\phi - \tilde{\gamma} A_\psi \right)^2 + \]

\[ 27 \left( 3 A_\phi^2 + 2 A_\psi^2 \right) - \frac{18}{d} (c^2 + d^2) \left( A_\phi \omega_\phi + A_\psi \omega_\psi \right) = 0. \quad (5) \]

### 2.2 Finite size analysis setup

In \[9\] we considered the case of infinite charges. In contrast to this case the finite size corrections originate from the expression

\[ \int_0^{\omega_{\max}} d\sigma = \int_0^{\omega_{\max}} \frac{dy}{c} = \frac{1}{c} \int_{\omega(0)}^{\omega(\omega_{\max})} \frac{du}{u^2}. \quad (6) \]
where \( u = \cos^2 \left( \frac{\theta(y)}{2} \right) \) and the worldsheet coordinate \( \sigma \) is in the range \((-w_{\text{max}}, w_{\text{max}})\) with \( w_{\text{max}} \) finite. Rewriting the equation of motion (5) in \( u(y) \) we obtain an equation accounting to the finite size

\[
u^2 = -f^2 (u - u_0)(u - u_1)(u - u_2)(u - u_3) = P_4(u).
\]

The quartic polynomial \( P_4(u) \) on the right hand side of (7) has in general four different roots \( u_i \). In order to have a well defined derivative \( u' \) in the worldsheet integral (5) the integration has to be taken in the positive region of the \( P_4(u) \). We are looking for certain string profiles, namely magnons and spikes, which have one turning point. This condition in the \( u \) coordinate is simply \( u(w_{\text{max}}) = u_{\text{turn}} = u_j \) i.e. the turning point is one of the roots \( u_i \). Our aim is to find finite size corrections to the dispersion relations obtained in the infinite case, therefore the limit \( w_{\text{max}} \to \infty \) amounts to setting the turning point to \( u_{\text{turn}} = 0 \). Looking at the following relation

\[
y = \int_{u(0)}^{u} \frac{du}{u'} = F[u] - F[u(0)]
\]

and the form of \( u' \) (7) it is clear that demanding \( y \to \infty \) for \( u_{\text{turn}} \to 0 \) we actually demand \( F[u] \to \infty \). The latter is possible if there is another root \( u_k \) that equals \( u_{\text{turn}} \) at \( w_{\text{max}} = \infty \). In order to have finite \( y \) for any \( u \neq 0 \) we demand one of the roots, say \( u_0 \) to be \( u_0 = 0 \). The polynomial \( P_4(u) \) then changes to

\[
P_4(u) = -f^2 u (u - u_1) (u - u_2) (u - u_3),
\]

which results in the condition \( A_\phi = -A_\psi \). Using the notations of [9]

\[
B_\phi = 2c\omega_\phi + \hat{\gamma}A_\phi, \quad B_\psi = 2c\omega_\psi + \hat{\gamma}A_\psi
\]

we find \( P_4(u) \) in the following form

\[
P_4(u) = -\frac{1}{3(c^2 - d^2)} \left( B_\phi^2 u^4 - 2B_\phi (B_\phi + B_\psi) u^3 + \frac{1}{2} \left( B_\phi^2 - B_\psi^2 + 6B_\phi B_\psi - 27A_\phi^2 + \frac{18}{d} (c^2 + d^2) A_\phi (\omega_\phi - \omega_\psi) \right) u^2 \right) + \frac{1}{2} \left( B_\psi - B_\phi \right)^2 + 81A_\phi^2 - \frac{18}{d} (c^2 + d^2) A_\phi (\omega_\phi - \omega_\psi) \right) u.
\]

The prefactor \( f^2 \) is thus

\[
f^2 = \frac{B_\phi^2}{3(c^2 - d^2)^2}.
\]

The solution to the equation (7) written in the implicit form is

\[
\int_{u_3 - |f| \sqrt{\hat{u}(u - u_1)(u - u_2)(u_3 - \hat{u})}}^{u} \frac{d\hat{u}}{f} = y,
\]

where we set \( u_2 \) to be the turning point and \( u_3 > u_2 > 0 \). Analyzing the polynomial \( P_4(u) \) one finds that \( u_1 < 0 \). In order to work with positive parameters \( u_i \), it is convenient to define \( u_{11} = -u_1 \).

## 3 Finite size corrections

In this section we proceed with the conserved charges. Referring to [9] for further details, we start with the relations for conserved charges in the finite case

\[
J_i = 2 \int_{0}^{w_{\text{max}}} \frac{d\hat{y}}{c} P_i = \frac{2}{c} \int_{u_3}^{w_2} \frac{du}{u'} P_i,
\]

\[
J_i = 2 \int_{0}^{w_{\text{max}}} \frac{d\hat{y}}{c} P_i.
\]
Using the notations
\[ I_1 = \int_{u_3}^{u_2} \frac{du}{u}; \quad I_2 = \int_{u_3}^{u_2} \frac{du}{u}; \quad I_3 = \int_{u_3}^{u_2} \frac{du}{u}; \quad I_4 = \int_{u_3}^{u_2} \frac{1}{1-u} \frac{du}{u} \]
we write the general expression for conserved charges
\[ cE = 2T \kappa I_1 \]
\[ 9c (c^2 - d^2) J_\psi = T (9dA_\phi - c (B_\phi - B_\psi)) I_1 + 2cTB_\phi I_2 \]
\[ 9c (c^2 - d^2) J_\phi = T (c (B_\phi - B_\psi) - 9dA_\phi) I_1 + 2cT (B_\phi + B_\psi) I_2 - 2cTB_\phi I_3 \]
\[ 9c (c^2 - d^2) \Delta = \frac{9}{2} (3cA_\phi - d (B_\phi - B_\psi)) I_1 + \gamma c ((2B_\phi + B_\psi) I_2 - B_\phi I_3) + 27cA_\phi I_4. \]  

The integrals \( I_i \) are expressed in terms of complete elliptic integrals and thus the above set of equations \([11]\) becomes transcendental one. From the Virasoro constraints \([12]\) we have
\[ \kappa = \sqrt{\frac{A_\phi (\omega_\phi - \omega_\psi)}{2d}}, \]
which results in
\[ I_1 = \frac{c\sqrt{dE}}{T \sqrt{2A_\phi (\omega_\phi - \omega_\psi)}}. \]  

In order to proceed we assume that our turning point \( u_2 \) is very small i.e. \( u_2 \rightarrow \epsilon \rightarrow 0 \) which is equivalent to having large \( w_{\text{max}} \) and so large, but finite charges. In the rest of this paper we expand our calculations in \( \epsilon \equiv u_2 \). The turning point condition \( u' (\epsilon) = 0 + O (\epsilon^2) \) gives again two relations for the parameter \( A_\phi \) defining the magnon and the spike solution
\[ A_\phi \begin{cases} 
\frac{2}{9} d (\omega_\phi - \omega_\psi) + \frac{1}{243 (c^2 - d^2) (\omega_\phi - \omega_\psi)} & \text{magnon} \\
4d \left( (27c^2 + 12dc_\gamma + d^2 (\gamma_2 + 9)) \omega_\phi^2 + d \left( d (\gamma_2 + 9) - 6c_\gamma \right) \omega_\psi^2 + 2 \left( 27c^2 - 3dc_\gamma - d^2 (\gamma_2 + 9) \right) \omega_\phi \omega_\psi \right) e + O (\epsilon^2) & \\
2 \left( 27c^2 - 3dc_\gamma - d^2 (\gamma_2 + 9) \right) \omega_\phi \omega_\psi + \frac{1}{9d} (\omega_\phi - \omega_\psi) + \frac{1}{243 d (c^2 - d^2) (\omega_\phi - \omega_\psi)} \right) & \text{spike} \\
4c^2 \left( (27c^2 + 12dc_\gamma + c^2 (\gamma_2 + 9)) \omega_\phi^2 + c \left( d (\gamma_2 + 9) - 6d_\gamma \right) \omega_\psi^2 + 2 \left( 27d^2 - 3dc_\gamma - d^2 (\gamma_2 + 9) \right) \omega_\phi \omega_\psi \right) e + O (\epsilon^2) 
\end{cases} \]  

Since \( A_\phi \) receives \( \epsilon \)-correction so does \( B_\phi, B_\psi \), therefore for convenience we write them as
\[ B_{\phi, \psi} = B_{0, \phi, \psi} + \epsilon B_{1, \phi, \psi} + O (\epsilon^2). \]  

Using the above form of parameters \( B_{\phi, \psi} \) \([14]\) and the expansions of the elliptic integrals \([35]\) we
One see that the first integral $I_1$:

\[
I_1 = \frac{\sqrt{3}(c^2 - d^2)}{Bo_\phi \sqrt{u_{11}u_3}} \left( \log \left( \frac{16u_{11}u_3}{(u_{11} + u_3) \epsilon} \right) + \frac{1}{4u_{11}u_3} \right) 
\]

\[
\left( 1 + \frac{4u_{11}u_3}{(u_{11} + u_3) \epsilon} \right) - 2(u_{11} - u_3) \right) \epsilon + O(\epsilon^2)
\]

\[I_2 = \frac{\sqrt{3}(c^2 - d^2)}{Bo_\phi} \left( \arctan \left( \frac{u_3}{u_{11}} \right) + \frac{1}{2\sqrt{3}(c^2 - d^2)} \arctan \left( \frac{u_3}{u_{11}} \right) \right) \epsilon + O(\epsilon^2)
\]

\[I_3 = \frac{\sqrt{3}(c^2 - d^2)}{Bo_\phi} \left( \frac{u_3}{u_{11}u_3} + (u_{11} - u_3) \arctan \left( \frac{u_3}{u_{11}} \right) \right) \epsilon + O(\epsilon^2)
\]

\[I_4 = \frac{6(c^2 - d^2)}{\sqrt{3}Bo_\phi \sqrt{(1 + u_{11})(1 - u_3)}} \arctan \left( \frac{(1 + u_{11})u_3}{u_{11}(1 - u_3)} \right) + \left( \frac{1}{2}I_1 + \frac{\sqrt{3}(c^2 - d^2)}{2Bo_\phi \sqrt{u_{11}u_3}} \right)
\]

\[\epsilon + O(\epsilon^2)
\]

One see that the first integral $I_1$ diverges for $\epsilon \to 0$ and behaves as $I_1|_{\epsilon \to 0} \sim -\log(\epsilon)$. Using this fact we get from the integral $I_1$ (15)

\[I_1 \sqrt{u_{01}u_0}Bo_\phi \frac{1}{\sqrt{3}(c^2 - d^2)} + \log \left( \frac{(u_{01} + u_0)}{16u_{01}u_0} \epsilon \right) + O(\epsilon^1) = 0.
\]

Substituting $I_1$ from (12) and solving (16) we find

\[\epsilon \approx \frac{16\epsilon X}{u_{011}u_0} + X = \frac{B_\phi \epsilon \sqrt{d\sqrt{u_{11}u_3}}}{\sqrt{6(c^2 - d^2)A_\phi(\omega_\phi - \omega_0)}}, \quad \epsilon = E/T.
\]

We can now return to the calculation of $u_{11}, u_3$ from (9). In order to expand in the $\epsilon$ parameter first we write

\[u_{11,3} = u_{011,3} + \epsilon u_{11,3}.
\]

The turning point condition simplifies $P_4(u) = 0 (\text{10})$ to the form

\[u (27A0_\phi^2 + (u - 1)Bo_\phi (uBo_\phi - B_0 - 2Bo_\phi)) + \epsilon (54uA0_\phi A1_\phi + 27(u - 1)A0_\phi^2 + (u - 1)
\]

\[(2u^2B_0^2B1_\phi - 2u (B_0^2B1_\phi + B_0^2 (11_\phi + 2B_0)) + B_0^2 (B_0^2 + 2B_0)) + O(\epsilon^2) = 0,
\]

where we expanded the parameter $A_\phi$ in $\epsilon$ as $A_\phi = A0_\phi + \epsilon A1_\phi$. The solution to (12) at the zeroth order in $\epsilon$, which we call $u_{011,3}$, coincides with the infinite case results in (9). Thus we have

\[u_3 = 1 + \frac{Bo_\phi}{Bo_\phi} \frac{B_0^2 - 27A0_\phi^2}{Bo_\phi} + \epsilon u_{13}, -u_1 \approx u_{11} = -1 - \frac{Bo_\phi}{Bo_\phi} \frac{B_0^2 - 27A0_\phi^2}{Bo_\phi} + \epsilon u_{111}
\]
with
\[
\begin{align*}
    u_{13} &= \frac{-1}{2B_0^2 \sqrt{B_0^2 - 27A_0^2}} 
    (27A_0^2 (B_0 - 2B_1) + 54A_0B_0A_1 + 
    (2B_0B_1 - 2B_0B_1 + B_0^2) \left( \sqrt{B_0^2 - 27A_0^2} + B_0 \right) 
    
    u_{11} &= \frac{-1}{2B_0^2 \sqrt{B_0^2 - 27A_0^2}} 
    (27A_0^2 (B_0 - 2B_1) + 54A_0B_0A_1 - 
    (2B_0B_1 - 2B_0B_1 + B_0^2) \left( \sqrt{B_0^2 - 27A_0^2} - B_0 \right) .
\end{align*}
\]

In the further analysis we consider the different cases of giant magnon and single spike.

### 3.1 Giant magnon

We start with the value of \( A_\phi \) specifying the magnon solution. In order to find the dispersion relation we will substitute the integrals \( I_i \) from (15), (12) to the charges (11), use (18) as well as the relations

\[
\begin{align*}
    Bm_0 & = - \frac{1}{2} (u_{011} - u_{013} + 2) Bm_0 
    Bm_1 & = \frac{1}{2} ((-u_{011} + u_{013} - 2) Bm_1 + (-u_{111} + u_{113} + 1) Bm_0) 
    Bm_\phi & = \frac{d\gamma}{d^2 (u_{011} - u_{013} + 4)^2 - 12c^2 (u_{011} - u_{013} + 1))} \left( \frac{27c^2 - d^2}{u_{011} - u_{013} + 4} \right) + O(\epsilon) .
\end{align*}
\]

The parameters \( Bm_{I_0} \) above are just \( B_{0\phi, \psi} \), \( B_{1\phi, \psi} \) from (14) with the value of \( A_\phi \) defining the magnon case, i.e. \( Bm_{I_0} = B_{I_0} |_{A_\phi=\phi_\text{magnon}} \), \( I \in \{0, 1\} \), \( \alpha \in \{\phi, \psi\} \). Thus we obtain relations for \( J_\phi \), \( J_\psi \) accounting for finite size effects

\[
\begin{align*}
    \frac{4}{\sqrt{3}} \arctan \left( \frac{u_{013}}{u_{011}} \right) &= \mathcal{E} + 3\mathcal{J_\phi} + \epsilon \left( \frac{2(u_{011}u_{013} - u_{014}u_{011})}{\sqrt{6}\sqrt{u_{011}u_{013}(u_{011} + u_{013})}} + \frac{1}{\sqrt{3}u_{011}u_{013}} \right) + d^2 \mathcal{E} \left( \frac{d^2 (u_{011}^2 + (2 - 14u_{013})u_{011} + u_{013}^2 - 2u_{013} - 8)}{36(c^2 - d^2)^2 (u_{011} + 1) (u_{013} - 1)} \right) + O(\epsilon^2) .
\end{align*}
\]

and

\[
\begin{align*}
    \frac{2\sqrt{u_{011}u_{013}}}{\sqrt{3}} &= (\mathcal{E} - 3\mathcal{J_\phi}) + \frac{\epsilon}{36((c^2 - d^2)^2 u_{011} (u_{011} + 1) (u_{013} - 1) u_{014})} \left( \mathcal{E} u_{011} u_{013} d^2 (3c^2 ((u_{011} - u_{013})^2 - 4) - 2d^2 (u_{011}^2 + (4u_{013} + 2) u_{011} + (u_{013} - 4) (u_{013} + 2))) - 6\sqrt{3}(c^2 - d^2)^2 (u_{011} + 1) (u_{013} - 1) \right) \right) + O(\epsilon^2) .
\end{align*}
\]

Above we used the charge densities \( \mathcal{J}_I = J_I/T \) instead of charges. From the relations (22), (23) we find at the zeroth order approximation in \( \epsilon \)

\[
\begin{align*}
    u_{011} &= \frac{\sqrt{3}}{2} (\mathcal{E} - 3\mathcal{J_\phi}) \cot \left( \frac{\sqrt{3}}{4} (\mathcal{E} + 3\mathcal{J_\psi}) \right), \quad u_{013} = \frac{\sqrt{3}}{2} (\mathcal{E} - 3\mathcal{J_\phi}) \tan \left( \frac{\sqrt{3}}{4} (\mathcal{E} + 3\mathcal{J_\psi}) \right) .
\end{align*}
\]

Substituting \( u_{011,3} \) (24) to the angle difference equation we recover the infinite case dispersion relation at the zeroth order. At the first order we find the corrections \( u_{11,3} \) which can be found in the appendix (37). The angle difference equation written in a concise form is

\[
\begin{align*}
    2 \arctan \left( \frac{(u_{011} + 1) u_{013}}{u_{011} - u_{011} u_{013}} \right) &= \Delta - \frac{1}{2} (\mathcal{J_\phi} + \mathcal{J_\psi}) \gamma + \frac{27\Delta}{c(c^2 - d^2)} + O(\epsilon^2) ,
\end{align*}
\]
where $\Delta_\epsilon = \mathcal{E} \Delta_\epsilon + \Delta_1$ and the parts $\Delta_\epsilon, \Delta_1$ are given in the appendix (see (35)). Since we assume $\epsilon \ll 1$ and thus the energy density $\mathcal{E}$ being very large, we see that the coefficient $\Delta_\epsilon$ produces the leading part of the correction and the $\Delta_1$ the subleading one. Expressing $\Delta - \frac{1}{2} (J_\phi + J_\psi) \gamma$ from (25) in a concise form [9] we obtain

$$
cos \left( \Delta - \frac{1}{2} (J_\phi + J_\psi) \gamma \right) = \frac{u_{011} - u_{03} - 2u_{03}u_{011}}{u_{011} + u_{03}} + \frac{108\Delta_\epsilon u_{011}u_{03} (1 + u_{011}) (1 - u_{03}) \epsilon}{\epsilon (c^2 - d^2) (u_{011} + u_{03})^2 \sin \left( \Delta - \frac{1}{2} (J_\phi + J_\psi) \gamma \right) + O(\epsilon^2)}. \quad (26)
$$

Restricted to the first term it is just the infinite case dispersion relation.

In order to find the dispersion relation in terms of conserved charges we must substitute the integrals $I_i$ from (13) to the relations for conserved charges (11) and use the relations:

$$
BsI_\phi = -\frac{1}{2} (u_{011} - u_{03} + 2) Bs \phi
$$

$$
BsI_\psi = \frac{1}{2} ((-u_{011} + u_{03} - 2) Bs \phi + (-u_{11} + u_{13} + 1) Bs \phi)
$$

$$
BsJ_\phi = \frac{c^3}{27d(c^2 - d^2) (u_{011} - u_{03} + 4)} + O(\epsilon).
$$

Here $BsI_j = B|A_\phi = 0 = A_\alpha, \alpha \neq 1$. Introducing the charge densities $J_i = J_i / T$ we obtain the following expressions for the conserved charges $J_\phi, J_\psi$ (11):

$$
\frac{4}{\sqrt{3}} \arctan \left( \sqrt{\frac{u_{03}}{u_{011}}} \right) = 3J_\phi + \epsilon \left( \frac{c^3 \mathcal{E} ((u_{011} - u_{03} + 1) (u_{011} - u_{03} + 4) c^2 + 9d^2 (u_{01} - u_{11}))}{18d(c^2 - d^2)^2 (u_{011} + 1) (1 - u_{03})} \right. + \left. \frac{\sqrt{u_{011}u_{03} (-2u_{13}u_{011} + u_{011} + 2u_{11}u_{03} + u_{03})}}{\sqrt{3u_{011}u_{03} (u_{011} + u_{03})}} \right) + O(\epsilon^2). \quad (28)
$$

3.2 Single spike

For the case of a single spike defined by the value of $A_\phi$ (13) we use the same procedure as in the magnon case. In order to find the dispersion relation including the finite size correction we substitute the integrals $I_i$ from (13), (12) to the relations for conserved charges (11) and use the relations:

$$
BsI_\phi = -\frac{1}{2} (u_{011} - u_{03} + 2) Bs \phi
$$

$$
BsI_\psi = \frac{1}{2} ((-u_{011} + u_{03} - 2) Bs \phi + (-u_{11} + u_{13} + 1) Bs \phi)
$$

$$
BsJ_\phi = \frac{c^3}{27d(c^2 - d^2) (u_{011} - u_{03} + 4)} + O(\epsilon).
$$

Introducing the charge densities $J_i = J_i / T$ we obtain the following expressions for the conserved charges $J_\phi, J_\psi$ (11):

$$
\frac{4}{\sqrt{3}} \arctan \left( \sqrt{\frac{u_{03}}{u_{011}}} \right) = 3J_\phi + \epsilon \left( \frac{c^3 \mathcal{E} ((u_{011} - u_{03} + 1) (u_{011} - u_{03} + 4) c^2 + 9d^2 (u_{01} - u_{11}))}{18d(c^2 - d^2)^2 (u_{011} + 1) (1 - u_{03})} \right. + \left. \frac{\sqrt{u_{011}u_{03} (-2u_{13}u_{011} + u_{011} + 2u_{11}u_{03} + u_{03})}}{\sqrt{3u_{011}u_{03} (u_{011} + u_{03})}} \right) + O(\epsilon^2). \quad (28)
$$
and

\[ J_\phi = \frac{2\sqrt{3}u_{011}u_{03}}{3\sqrt{3}} - \frac{1}{108} \epsilon \left( \frac{6\sqrt{3}(2u_{111}u_{03} + u_{03} + u_{011}(2u_{13} - 1))}{\sqrt{u_{011}u_{03}}} \right) e^3 \epsilon ^{\frac{3}{2} \frac{(c^2 - d^2)^2 (u_{011} + 1)(1 - u_{03})}{(u_{011} + u_{03})^2}} \]

\[ \frac{2\sqrt{3}e(u_{011} - u_{03} - 8)\sqrt{(u_{011} + 1)(1 - u_{03})}}{3d(u_{011}^2 - 2(u_{03} + 2)u_{011} + u_{03}^2 + 4u_{03} - 8))} + O \left( \epsilon^2 \right) \]

(29)

Again, at the zeroth order in \( \epsilon \) we find for the parameters \( u_{011,3} \)

\[ u_{011} = -\frac{3}{2} \sqrt{3}J_\phi \cot \left( \frac{3\sqrt{3}}{4} J_\phi \right), \quad u_{03} = -\frac{3}{2} \sqrt{3}J_\phi \tan \left( \frac{3\sqrt{3}}{4} J_\phi \right) \]

(30)

which substituted to the remaining conserved quantity, the angle difference \( \Delta \), gives the infinite case dispersion relation. From the equations (28), (29) we calculate the corrections (31). The other convenient form of (31) reads off

\[ 2 \arctan \left( \frac{u_{011} + 1}{u_{011} - u_{011}u_{03}} \right) = \Delta - \frac{3}{2} \epsilon - \frac{1}{2} (J_\phi + J_\phi) \gamma + \frac{27\Delta}{c(c^2 - d^2)} \epsilon + O \left( \epsilon^2 \right), \]

(31)

with \( \Delta_\epsilon = \epsilon \Delta + \Delta_1 \). The leading \( \Delta_\epsilon \) and the subleading \( \Delta_1 \) part of \( \Delta_\epsilon \) are in the appendix 42. The other convenient form of (31) reads off

\[ \cos \left( \Delta - \frac{3}{2} \epsilon - \frac{1}{2} (J_\phi + J_\phi) \gamma - \frac{108\Delta u_{011}u_{03}(1 + u_{011})(1 - u_{03})}{c(c^2 - d^2)}(J_\phi + J_\phi) \gamma \right) = \frac{u_{011} - u_{03} - 2u_{03}u_{011}}{u_{011} + u_{03}} + \frac{108\Delta u_{011}u_{03}(1 + u_{011})(1 - u_{03})}{c(c^2 - d^2)}(J_\phi + J_\phi) \gamma + O \left( \epsilon^2 \right), \]

(32)

with its zero-order being just the infinite case dispersion relation for single spike 49.

In order to find the explicit form of the corrections we substitute for \( \epsilon \) from (17) and for \( u_{011,3} \) from (30) and obtain

\[ \cos \Delta \delta = \frac{3\sqrt{3}}{2} \left( J_\phi \sin (R_\phi) + \frac{2 \cos (R_\phi)}{3\sqrt{3}} \right) \left( 1 + C_1 \epsilon \epsilon X \phi + D_1 \epsilon \epsilon X \phi + O \left( \epsilon \epsilon X \phi \right)^2 \right). \]

(33)

Above we used \( R_\phi = \frac{3}{2} \sqrt{3}J_\phi \), \( \Delta \delta = \Delta - \frac{3}{2} \epsilon - \frac{1}{2} (J_\phi + J_\phi) \gamma \).

The expressions for \( C_1, D_1 \) can be found in the appendix 43, 44. The exponential factor \( X \) is of the form

\[ X = -\sin^2 \left( R_\phi \right) \left( J_\phi ^9 \sqrt{12 - 81J_\phi ^2} - 81 \sqrt{3}J_\phi \cot (R_\phi) \right) \left( 2 + 27J_\phi ^2, \sin^2 (R_\phi) + 3J_\phi \left( 9J_\phi + 2 \sqrt{3} \sin (2R_\phi) \right) \right) < 0. \]

Since in this case \( J_\phi < 0 \), negativity of the exponent is better recognizable from its equivalent form

\[ X = \frac{6cJ_\phi \epsilon \epsilon X}{(c^2 - d^2) \left( 3 - 2 \sqrt{3}J_\phi \cot (R_\phi) \right)}, \]

and we have again exponentially suppressed contribution.
4 Conclusion

In this paper we calculated the leading finite size corrections to the dispersion relations of giant magnon and single spike living on the $\gamma$-deformed Sasaki-Einstein manifold $T^{1,1}$. For the special case $\gamma = 0$ we get the finite size corrections to the undeformed case discussed in [3]. The result in this case has the same structure, with the difference in the $R_\Delta$, $\Delta \delta$ parameters for the magnon and the spike case respectively. An interesting property of the results we obtained (27), (33) is the leading $\sim e^{E_X}$ and the subleading $\sim e^{E_X}$ part of the first order correction. Our results differs from the finite size correction for giant magnons in the conformal gauge on $\mathbb{R} \times S^2$ discussed in [3], which is natural from the reduced symmetry point of view. The authors of [3] considered finite size corrections of a one spin giant magnon, whereas we are discussing a two spin case. If we even restrict ourselves to the one spin case, which is nontrivial limit of the results, we will obtain a correction for the consistent subsector of the $AdS_5 \times T^{1,1}$ which is quite different space than the maximally supersymmetric $AdS_5 \times S^5$. We restricted ourselves to the first order correction since higher orders behave extremely complicated. In principle however, one can carefully repeat the same steps and find corrections of higher order.

A Solution to the integrals and expansions

Using the following convenient notation $k = \frac{(u_0 - u_1)(u_0 - u_2)}{(u_1 - u_2)(u_0 - u_3)}$, $m = \frac{u_1 - u_2}{u_1 - u_3}$, we obtain the explicit solutions to the integrals $I_i$:

$$ I_1 = -\frac{2K(k)}{\sqrt{(u_1 - u_2) (u_0 - u_3)}} $$

$$ I_2 = -2 \frac{1}{\sqrt{(u_1 - u_2) (u_0 - u_3)}} \left( u_1 K(k) + (u_3 - u_1) PI(m,k) \right) $$

$$ I_3 = \frac{1}{\sqrt{(u_1 - u_2) (u_0 - u_3)}} \left( - (u_1 (u_1 + u_2) + (u_1 - u_2) u_3) K(k) - (u_1 - u_2) (u_0 - u_3) E(k) + (u_1 - u_3) (u_0 + u_1 + u_2 + u_3) PI(m,k) \right) $$

$$ I_4 = -2 \frac{u_1 - u_2}{u_0 - u_3} \frac{(u_3 - 1) K(k) + (u_1 - u_3) PI \left( \frac{1-u_3}{1-u_1} m, k \right)}{(u_1 - 1) (u_1 - u_2) (u_3 - 1)} $$

We use the following expansions for small $\epsilon$:

$$ K(1 - \epsilon) = \left( 2 \log (2) - \frac{\log (\epsilon)}{2} \right) + \frac{1}{8} \left( - \log (\epsilon) + 4 \log (2) - 2 \right) \epsilon - \frac{21 \epsilon^2}{128} + \frac{185 \epsilon^3}{1536} + O (\epsilon^4) $$

$$ E(1 - \epsilon) = 1 + 1 \left( - \log (\epsilon) + 2 \log (4) - 1 \right) \epsilon + \frac{1}{64} \left( - 6 \log (\epsilon) + 24 \log (2) - 13 \right) \epsilon^2 + \frac{3}{256} \left( - 5 \log (\epsilon) + 20 \log (2) - 12 \right) \epsilon^3 + O (\epsilon^4) $$

$$ PI(m, 1 - \epsilon) = 2 \frac{\sqrt{\text{arctanh} (\sqrt{m})} + \log \left( \frac{16}{\epsilon} \right)}{2m - 2} + \frac{-4 \sqrt{\text{arctanh} (\sqrt{m})} + (m + 1) \log \left( \frac{16}{\epsilon} \right) - 2 \epsilon}{8 (m - 1)^2} $$

$$ + \frac{(12 - 5m) m + 3 (m - 6) \log \left( \frac{16}{\epsilon} \right) m + 48 \sqrt{\text{arctanh} (\sqrt{m})} \sqrt{m} + 9 \log (\epsilon) - 36 \log (2) + 21 \epsilon^2}{128 (m - 1)^3} + O (\epsilon^4) $$

\footnote{One must carefully take the limit $\omega_0 \to 0$ and not just turn off the charge $J_\phi$.}
B Results and useful relations

B.1 The giant magnon case

Using the convenient notation \( \{X_1 = \sqrt{u_{011}u_{03}}, \ X_2 = \sqrt{u_{03}/u_{011}}\} \) we write the corrections \( u_{11,3} \) as

\[
(c^2 - d^2)^2 u_{11} = \frac{\mathcal{E}c^2d^2 (-X_1X_2^2 + X_1 - 2X_2)}{4\sqrt{3}X_2(X_1X_2 - 1)} - \frac{c^4 + c^2d^2 - d^4}{2}
\]

\[
d^4\mathcal{E} (-16X_1X_2^4 + 2X_1^2(X_2^4 + 8X_2^2 + 3)X_2 + X_1^3(X_2^6 - 13X_2^4 - 13X_2^2 + 1) - 32X_2^3) / 48\sqrt{3}X_2(X_1 + X_2)(X_1X_2 - 1)
\]

\[
(c^2 - d^2)^2 u_{13} = \frac{c^2d^2 (2X_2 (\mathcal{E}X_2 - 2\sqrt{3}) + X_1 (\mathcal{E}X_2 (X_2^2 - 1) - 4\sqrt{3}))}{4\sqrt{3}(X_1 + X_2)} + \frac{c^4 + d^4}{2}
\]

\[
d^4\mathcal{E} (X_1^3 (X_2^6 - 13X_2^4 - 13X_2^2 + 1) - 2X_1^2 (3X_2^5 + 8X_2^3 + X_2) - 16X_1X_2^2 + 32X_2^3) / 48\sqrt{3}X_2(X_1 + X_2)(X_1X_2 - 1)
\]

(37)

The correction term \( \Delta_c = \mathcal{E}\Delta_c + \Delta_1 \) in (35) consists of

\[
\Delta_c = \frac{d^3Z\sqrt{u_{011}u_{03}}}{324((u_{011} + 1)(1 - u_{03})))^{3/2}(u_{011}^2 + 2(5u_{03} - 2)u_{011} + (u_{03} + 2)^2)}
\]

\[
((u_{011}^4 + (8 - 12u_{03})u_{011}^3 + (26u_{03}^2 + 8u_{03} - 36)u_{011}^2 - 4(3u_{03}^3 + 2u_{03}^2 + 26u_{03} - 4)u_{011} + (u_{03} + 2)^2(u_{03}^2 - 12u_{03} + 8)))
\]

\[
\Delta_1 = \frac{1}{324(u_{011} + 1)^2(u_{03}^3 - 1)}
\]

\[
(2Z(u_{011} + 1)(c^2(u_{011}(9 - 6u_{03}) - 9u_{03}) - 12))
\]

\[
cd^2(-11u_{03} + u_{011}(2u_{03} + 11) + 20 + 4\sqrt{3}d^3\sqrt{-(u_{011} + 1)(u_{03} - 1)} + \sqrt{3d(d^2 - c^2)}((u_{011} - u_{03} + 5)(u_{011}u_{03})^{3/2} + 4\sqrt{u_{011}u_{03}}^3 + \sqrt{u_{011}u_{03}}^3))}
\]

\[
Z = \arctan\left(\sqrt{\frac{(u_{011} + 1)u_{03}}{u_{011} - u_{011}u_{03}}}\right).
\]

The coefficients of the leading \((C_1)\) and the subleading \((D_1)\) part of the correction take the form:

\[
C_1 = 8\sqrt{3}R_{\Delta} \csc (R_{\Delta}) R_2^3 \sin^3 (R_{\psi}) \left(-2R_{\phi} \cos (R_{\psi}) - (1 - R_{\phi}^2) \sin (R_{\phi})\right) (3 - 21R_{\phi}^2 - 4R_2^3 +
\]

\[
(-4 + 22R_{\phi}^2 + 8R_2^3) \cos (2R_{\psi}) + (1 - R_{\phi}^2) \cos (4R_{\psi}) + 8R_{\phi} \cos (R_{\phi}) \sin^3 (R_{\psi}) + 8R_{\phi}^3 \sin (2R_{\phi})) / \left((-1 - 4R_{\phi}^2 + (1 + 2R_{\phi}^2) \cos (2R_{\psi}) + 2R_{\phi} \sin (2R_{\psi}))^2\right)
\]

\[
(R_{\phi} - 3R_{\phi} \cos (2R_{\psi}) + (-2 + R_{\phi}^2) \sin (2R_{\psi}))
\]

(39)
\[ D_1 = \frac{4 R_\phi \csc (R_\Delta) \sin^2 (R_\phi)}{-4Y (R_\phi \cos (R_\psi) + 2 \sin (R_\psi)) (2R_\phi \sin (2R_\psi) + (2R_\phi^2 + 1) \cos (2R_\psi) - 4R_\phi^2 - 1)} \]

\[ (R_\phi \sin (R_\psi) - \cos (R_\psi))^{-1} (R_\phi^2 \sin (2R_\psi) (-22Y R_\Delta R_\phi - 8R_\phi^4 + 38R_\phi^2 + 2)) + 4 \cos (2R_\phi) (R_\phi (R_\phi (-5Y R_\Delta R_\phi + 8R_\phi^2 - 4) + 2Y R_\Delta) - 2) - 2Y R_\Delta \]

\[ \cos (4R_\phi) (Y R_\Delta (8R_\phi^4 + R_\phi^2 + 2) + 2R_\phi (-5R_\phi^4 + 5R_\phi^2 + 1)) + 36Y R_\Delta R_\phi^4 - 9Y R_\Delta R_\phi^2 + 6Y R_\Delta - 6R_\phi^5 + 6R_\phi^3 + 6R_\phi \]

with \( Y = \sqrt{2R_\phi \cot (R_\psi) - R_\phi^4 + 1} \).

**B.2 The single spike case**

We find the following form for \( u_{11,3} \)

\[ u_{11} = -\frac{cd\Phi u_{011}}{2\sqrt{3}d^2 (c^2 - d^2)^2 (u_{011} - u_{03} + 4)^2 \sqrt{u_{011} u_{03}}} \left( c^2 (-u_{03}^2 + u_{011} (u_{011} + 2) - 8) 
\begin{equation}
\langle u_{011} - u_{03} + 4 \rangle + 6d^2 (-2u_{011}^2 + (u_{03} + 2) u_{011} + (u_{03} - 2) u_{03} + 4) \right) - \frac{1}{2} \]

\[ u_{13} = \frac{cd\Phi u_{013}}{2\sqrt{3}d^2 (c^2 - d^2)^2 (u_{011} - u_{03} + 4)^2 \sqrt{u_{011} u_{03}}} \left( c^2 (-u_{03}^2 + u_{011} + 2u_{03} + 8) 
\begin{equation}
\langle u_{011} - u_{03} + 4 \rangle - 6d^2 (u_{011} - u_{03} (u_{011} + 2u_{03} + 2) - 24d^2) + \frac{1}{2} \right) \]

The parameters \( \Delta_\phi, \Delta_1 \) read of

\[ \Delta_\phi = \frac{-4d^2 Z \sqrt{u_{011} u_{03}}}{3\sqrt{3} (u_{011} - u_{03} + 4) (u_{011}^2 + 2(5u_{03} - 2) u_{011} + (u_{03} + 2)^2) \left( (4u_{03} - 3) u_{011}^4 + (4u_{03}^2 + 4u_{03} + 2) u_{011}^4 + 2 (-2u_{03}^4 + 7u_{03}^2 + u_{03} + 12) u_{011}^2 - 2 (2u_{03}^4 - 2u_{03}^2 + u_{03} + 40u_{03} + 12) u_{011} - (u_{03} + 2)^2 (3u_{03}^2 - 10u_{03} + 4) \right) \}
\begin{equation}
\Delta_1 = \frac{1}{27 (u_{011} - u_{03})^2} \left( \sqrt{3d} (d^2 - c^2) \sqrt{u_{011} u_{03}} (u_{011} - u_{03} + 4) + \frac{2Z}{c} (2c^2 (u_{011} - u_{03} + 4) - c^2 d^2 (-11u_{03} + u_{011} (2u_{03} + 11) + 20) + \]

\[ 3d^2 (u_{011} - 2u_{03} - 3u_{03} + 4)) \right) \]

\[ Z = \arctan \left( \frac{(u_{011} + 1) u_{03}}{u_{011} - u_{011} u_{03}} \right) \].

The correction coefficients \( C_1, D_1 \) for the single spike dispersion relation are of the form

\[ C_1 = 162J_2^2 \Delta \delta \csc (\Delta \delta) \sqrt{4 - 27J_2^2 - 12\sqrt{3}J_2 \cot (R_\psi) \sin^4 (R_\psi) \left( 12 - 810J_2^2 + 1458J_2^4 + (-16 + 864J_2^2 + 2916J_2^4) \cos (2R_\psi) + \sqrt{3} (-36J_2^4 + 162J_2^6 + 2187J_2^8) \sin (2R_\psi) \right) 
\begin{equation}
+ (4 - 54J_2^2) \cos (4R_\psi) + 18\sqrt{3}J_2 \sin (4R_\psi) \right) \right) \left( \frac{3\sqrt{3}J_2 \cos (R_\psi) - 4 \sin (R_\psi)}{2 \cos (R_\psi) + 3\sqrt{3}J_2 \sin (R_\psi)} \right) \right) \left( 2 + 54J_2^2 - 2R_\psi^2 - 27J_2^2 \cos (2R_\psi) + 6\sqrt{3}J_2 \sin (2R_\psi) \right)^2 \]
\[
D_1 = \frac{-3J_\phi \csc(\Delta \delta) \sin^2 (R_\psi) \left(3\sqrt{3}J_\phi \sin (R_\psi) + 2 \cos (R_\psi)\right)^{-1}}{4 \left(3\sqrt{3}J_\phi \cos (R_\psi) - 4 \sin (R_\psi)\right) \left(6\sqrt{3}J_\phi \sin (2R_\psi) - (27J_\phi^2 + 2) \cos (2R_\psi) + 54J_\phi^2 + 2\right)} \\
\left(8 \left(\sqrt{3}\Delta \delta \left(-3645J_\phi^4 + 216J_\phi^2 - 32\right) + 9Y J_\phi (81J_\phi^2 + 4)\right) \cos (2R_\psi) + \\
2 \left(4\sqrt{3}\Delta \delta \left(27 (54J_\phi^4 + J_\phi^2) + 8\right) - 9Y J_\phi (81J_\phi^2 + 4)\right) \cos (4R_\psi) + 6 \left(27J_\phi^2 \sin (2R_\psi) \left(\sqrt{3}Y (2 - 9J_\phi^2) \cos (2R_\psi) + 18J_\phi^2 - 2\right) + 6\Delta \delta J_\phi \left((27J_\phi^2 - 14) \cos (2R_\psi) + 22\right) - 9Y (81J_\phi^2 + 4) J_\phi + 4\sqrt{3}\Delta \delta \left(2187J_\phi^4 - 81J_\phi^2 + 8\right)\right) \right)
\]

where we used the notation \( Y = \sqrt{-12\sqrt{3}J_\phi \cot (R_\psi) - 27J_\phi^2 + 4}. \)

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