An Adaptive Federated Filter in Multi-source Fusion Information Navigation System

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Abstract. Aiming at the problem that the existing federated filtering algorithms are not sensitive to the abnormal measurement and poor tolerance, an improved adaptive federated filtering algorithm based on innovation is proposed, and when the navigation source is abnormal, it can automatically adjust the allocation factor. The experimental results show that the algorithm can effectively suppress the error when the navigation source is abnormal and improve the fault-tolerance of the multi-source information fusion navigation system. The implementation is simple, and the calculation is small and easy to implement.

1. Introduction
In order to solve the problem of poor real-time performance of the traditional Kalman filter algorithm, Speyer proposed the idea of decentralized filtering in 1979 [1], and Carlson proposed the federated filtering algorithm in 1988 [2]: all navigation sources are processed by parallel filtering before merging. Because of its high computing efficiency and strong real-time advantages, it is widely used in multi-source information fusion navigation systems. However, the traditional federated filtering algorithm uses a fixed information allocation method. When the navigation source is disturbed, the algorithm cannot be adjusted in time, which results in the positioning accuracy of the multi-source navigation system is reduced and the fault-tolerance is poor. To solve this problem, scholars conducted a lot of research. In document [3], an online adjusted federal filter algorithm is proposed, but it needs to store the measurement noise variance matrix for the first two fusion periods. When the dimension of the measurement vector increases, the system storage capacity will increase sharply. The dynamic optimal federated filtering (DOFF) designed by document [4] uses the error covariance matrix to calculate the allocation factor. The method is simple for engineering practice but when the navigation source is disturbed, its positioning results are not improved significantly compared to the traditional federated filtering.

In this paper, an adaptive allocation factor is designed based on the measurement innovation of each filter. When the navigation source is abnormal, the error covariance matrix and the system noise covariance matrix of each filter are adjusted by the adaptive allocation factor. On the premise of system storage and real-time performance, the fault-tolerance of the federated filtering algorithm is effectively improved.

2. Federal filtering algorithm
The basic idea of the federated filtering algorithm is to select a navigation source with full information, high output rate, and reliable reliability as a reference navigation source, and performs a pairwise
combination with other navigation sources to form a local filter, and then the Kalman filter algorithm is used to solve the local filter, and the solution is sent to the main filter. Finally, the fusion is performed according to the principle of information sharing. The algorithm structure is shown in figure 1.

![Algorithm Structure](image)

**Figure 1. Federal filter algorithm structure**

The federated filtering algorithm mainly includes four steps: information distribution, time transfer, measurement update and information fusion.

The algorithm will allocate information in local filters and main filter after initialization:

$$
P_i(k) = \beta_i^{-1}P(k), P_m(k) = \beta_m^{-1}P(k)$$  
$$Q_i(k) = \beta_i^{-1}Q(k), Q_m(k) = \beta_m^{-1}Q(k)$$  
$$\hat{x}_i(k) = \hat{x}(k), \hat{x}_m(k) = \hat{x}(k)$$

$$\sum_{i=1}^{N} \beta_i + \beta_m = 1$$  

Where $P(k), P_i(k)$ and $Q(k), Q_i(k)$ are the error covariance matrix, the allocation factor, the system noise covariance matrix and the state estimation of the local filter $i$ at $t_i$, respectively. $P(k), P_m(k), \beta_m, Q_m(k)$ and $\hat{x}_m(k)$ are the error covariance matrix, the allocation factor, the system noise covariance matrix and the state estimation of the main filter at $t$ respectively.

The common federated filter fusion mode is fusion-reset, and its information allocation factor is:

$$\beta_m = 0, \beta_1 = \cdots = \beta_N = 1 / N$$  

Where $N$ is the number of local filters. In the fusion-reset mode, the algorithm has the highest positioning accuracy and can achieve optimal estimation, but its fault-tolerance is poor. This is because the main filter in this mode feeds back the local filters and the information distribution occurs in each fusion period, so the error generated by a local filter will affect other local filters through the main filter.

After the information distribution is completed, the local filters and the main filter are time transferred respectively:

$$\hat{x}_i(k+1) = \phi(k+1,k)\hat{x}_i(k)$$  
$$P_i(k+1) = \phi(k+1,k)P_i(k)\phi(k+1,k)^T + Q_i(k)$$

$$\hat{x}_m(k+1) = \phi(k+1,k)\hat{x}_m(k)$$
\[
P_x(k+1) = \phi(k+1,k)P_x(k)\phi(k+1,k)^T + Q_x(k) \tag{9}
\]

Where \( \phi(k+1,k) \) is the transition matrix for the system from \( t_k \) to \( t_{k+1} \).

Since the main filter has no measurement information, the measurement update is only performed in the local filters:

\[
P'_i(k+1) = (I - K_i(k+1)H_i(k+1,k))P_i(k+1) \tag{10}
\]

\[
\hat{x}_i(k+1) = \hat{x}_i(k+1) + K_i(k+1)(Z_i(k+1) - H_i(k+1,k)\hat{x}_i(k+1)) \tag{11}
\]

Where \( I \) is a unit matrix, \( H_i(k+1,k) \) and \( K_i(k+1,k) \) are the measurement matrix and the Kalman gain matrix of the local filter \( i \) from \( t_k \) to \( t_{k+1} \), \( Z_i(k+1) \) is the measurement vector of the local filter \( i \) at \( t_{k+1} \).

Finally, the local filters transmit the updated values to the main filter for information fusion:

\[
P(k+1) = (\sum_{i=1}^{N} P_i^{-1}(k+1) + P_w^{-1}(k+1))^{-1} \tag{12}
\]

\[
\hat{x}(k+1) = P(k+1)\sum_{i=1}^{N} P_i^{-1}(k+1)\hat{x}_i(k+1) + P_w^{-1}(k+1)\hat{x}_w(k+1) \tag{13}
\]

### 3. Adaptive allocation factor

In the multi-source information fusion navigation system, the navigation source may be disturbed, malfunction or abnormal measurement. If the anomaly measurement is not processed, the positioning accuracy of the system will be affected. From the previous analysis, it is known that the allocation factor is the key to the federated filtering, and the fixed information allocation method cannot adapt to the fusion system under the disturbance of the navigation source, so it can improve the system fault-tolerance by adjusting the allocation factor in real time.

Measurement innovation is the difference between the actual measurement of the navigation source \( Z(k) \) and the measurement estimated by the filter \( \hat{Z}(k) \) at \( t_k \). It is expressed as:

\[
\delta_k = Z(k) - \hat{Z}(k) = Z(k) - H(k)\hat{x}(k-1) \tag{14}
\]

It can be seen from equation (14) that the measurement innovation can reflect the error of the actual measurement. Therefore, it can construct the allocation factor by measurement innovation, and adjust the error covariance matrix and the system noise covariance matrix of each filter.

The theoretical covariance matrix of the measurement innovation is:

\[
\tilde{C}(k) = H(k)P(k-1)H(k)^T + R(k) \tag{15}
\]

Where \( R(k) \) is the measurement noise covariance matrix of filter at \( t_k \).

And the actual covariance matrix of the measurement innovation is [5]:

\[
C(k) = \delta_k \ast \delta_k^T \tag{16}
\]

Then, the adaptive allocation factor can be constructed by equation (15) and equation (16):

\[
\beta(k) = \text{tr}(C(k)) / \text{tr}(\tilde{C}(k)) \tag{17}
\]

Applying equation (17) to the fusion-reset federated filtering algorithm, there are:

\[
\beta_n(k) = \text{tr}(C_n(k)) / \text{tr}(\tilde{C}(k)), \quad \beta_n(k) = 0 \tag{18}
\]
Where $C_i(k)$ and $\hat{C}_i(k)$ are the actual covariance matrix and the theoretical covariance matrix of the measurement innovation of the local filter $i$ at $t_k$.

Based on the law of conservation of information, the adaptive allocation factor is normalized:

$$\bar{\beta}_i(k) = \beta_i(k) / \sum_{j=1}^N \beta_j(k)$$  \hspace{1cm} (19)

From equations (18) and (19), the information distribution method in the federated filtering algorithm using the adaptive allocation factor is:

$$P_i(k) = \bar{\beta}_i(k) \cdot P(k), P_\epsilon(k) = 0$$  \hspace{1cm} (20)

$$Q_i(k) = \bar{\beta}_i(k) \cdot Q(k), Q_\epsilon(k) = 0$$  \hspace{1cm} (21)

4. Experiment and simulation

A multi-source information fusion navigation system consisting of a global navigation satellites system (GNSS), an inertial navigation system (INS) and a wheel speed sensor (WSS) is commonly used for in-vehicle navigation. The application of the proposed adaptive federated filtering algorithm to the GNSS/INS/WSS fusion navigation system is shown in the following figure:

Figure 2. GNSS/INS/WSS fusion navigation system

The state vectors of the local filters all adopt the error state of INS, including attitude error and horizontal velocity error:

$$x = [\delta \phi, \delta \theta, \delta \psi, \delta v_N, \delta v_E]^T$$  \hspace{1cm} (22)

Where $\delta \phi$, $\delta \theta$ and $\delta \psi$ are the roll, pitch and heading angle error respectively, $\delta v_N$ and $\delta v_E$ are the north and east velocity error.

The state equation of each filter is:

$$x(k) = \phi(k, k-1) x(k-1) + W(k)$$  \hspace{1cm} (23)

Where $W(k)$ is the input noise vector at $t_k$.

The GNSS position difference can be used to obtain the northward and eastward velocity of the carrier, so that the velocity error of the INS can be obtained at the same time point, and the velocity error of the INS is used as a measure of the filter, and the measurement vector of the INS/GNSS filter is:
\[
Z_i = \begin{bmatrix}
v_{N,i} - v_{\text{GNS},N} \\
v_{E,i} - v_{\text{GNS},E}
\end{bmatrix} = \begin{bmatrix}
\delta v_{N} \\
\delta v_{E}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\delta \phi \\
\delta \theta \\
\delta \psi \\
\delta v_{N} \\
\delta v_{E}
\end{bmatrix} = H_i x_i 
\]

Where \( \tilde{v}_{N} \) and \( \tilde{v}_{E} \) are the north and east velocity measured by INS.

The measurement vector of the INS/WSS filter is:

\[
z_2 = \begin{bmatrix}
(\tilde{v}^u_{N} - \bar{v}^u_{\text{wss},N})_N \\
(\tilde{v}^u_{E} - \bar{v}^u_{\text{wss},E})_E
\end{bmatrix} = \begin{bmatrix}
0 & -v^u_{\text{wss},E} & 1 & 0 \\
0 & -v^u_{\text{wss},N} & 0 & 1
\end{bmatrix} \begin{bmatrix}
\delta \phi \\
\delta \theta \\
\delta \psi \\
\delta v_{N} \\
\delta v_{E}
\end{bmatrix} = H_2 x_2 
\]

Where \( v^u_{\text{wss},N} \), \( v^u_{\text{wss},E} \) and \( v^u_{\text{wss},D} \) are the north, east and heading velocity measured by WSS.

In this paper, the measured data of GNSS/INS/WSS fusion navigation system are collected for 4.32 hours. At the beginning, the vehicle stopped moving, but the body had a certain interference slushing. At this time, the INS started the initial alignment. After 10 minutes, the vehicle was running, and the time homologous data of the INS, WSS and GNSS were collected. The sampling frequency of INS and WSS is 100Hz, and the sampling frequency of GNSS is 1Hz.

The calibration parameters of INS (front - right - bottom) are as follows:

\[
K_a = \begin{bmatrix}
2.6*10^{-5} & -7.4*10^{-5} & 2.7*10^{-5} \\
5.2*10^{-5} & 2.9*10^{-2} & -7.2*10^{-6} \\
-3.8*10^{-5} & 1.0*10^{-5} & 2.8*10^{-2}
\end{bmatrix} (m / s^2 / pulse)
\]

\[
K_g = \begin{bmatrix}
2.3*10^{-6} & 5.2*10^{-10} & -6.8*10^{-10} \\
1.4*10^{-9} & 2.3*10^{-6} & -1.0*10^{-9} \\
-1.6*10^{-9} & -3.4*10^{-10} & 2.3*10^{-6}
\end{bmatrix} (m / s^2 / pulse)
\]

\[
f_{b0} = \begin{bmatrix}
0.0504 & 0.0149 & 0.0003
\end{bmatrix} (m / s^2)
\]

\[
\omega_{b0}^x = \begin{bmatrix}
-0.0018 & 0.0011 & 0.0013
\end{bmatrix} (pulse / 0.01s)
\]

Where \( K_a \) is the accelerometer scale factor matrix, \( K_g \) is the gyroscope scale factor matrix, \( f_{b0}^x \) is the accelerometer bias, \( \omega_{b0}^x \) is the gyroscope bias.

Using the traditional fusion-reset federated filtering (TFF) algorithm, the dynamic optimal federated filtering algorithm and the proposed adaptive federated filtering (AFF) algorithm to fuse the GNSS, INS and WSS data. The period of the fusion solution is 1s, and the interference is added to the GNSS signal in the 7990s~8000s. The positioning results are shown in the following figure:
Figure 3. Northward position error (left) and eastward position error (right)

From Figure 3, it can be seen that after the navigation source is disturbed, the traditional federated filtering algorithm will cause the positioning error to increase. Using dynamic optimal federated filtering algorithm can improve the fault-tolerance of the system to a certain extent, but the effect is not obvious. The adaptive federated filtering algorithm proposed in this paper can suppress the influence of abnormal measurements and minimize the error.

In order to intuitively reflect the positioning accuracy of the three algorithms, the root mean square error (RMSE) is used to quantitatively analyse the location results, as shown in Table 1:

| Algorithms | RMSE in north direction /m | RMSE in east direction /m |
|------------|-----------------------------|---------------------------|
| TFF        | 31.2270                     | 26.0578                   |
| DOFF       | 24.8910                     | 25.6255                   |
| AFF        | 19.6092                     | 24.0879                   |

From Table 1, it can be seen that when the GNSS signal is disturbed, the RMSE of the AFF is significantly less than DOFF and TFF in the north direction and the position result of the AFF in the east is also superior to the other two algorithms.

5. Conclusion
In this paper, an adaptive federated filtering algorithm is designed. Based on the measurement innovation, an adaptive allocation factor is constructed, and the error covariance matrix and the system noise covariance matrix of each local filter are adjusted in real time by the allocation factor. And the GNSS/INS/WSS fusion navigation system is verified by measured data. The experimental results show that the adaptive federated filtering algorithm only needs to calculate the theoretical covariance matrix and the actual covariance matrix of the filter observing the innovation at the current time, and does not need to store the previous measurements at the time, the resource consumption is low, the calculation is simple and the engineering is easy to implement. In the case of GNSS interference, the positioning error can be effectively suppressed, and the fault-tolerance of the multi-source information fusion navigation system is improved.

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