Research and Application of Seismic data Denoising and Reconstruction method based on Compressed Sensing

Haitao Yang¹, Qiong Li*
¹ College of Geophysics, Chengdu University of Technology, Chengdu, Sichuan Province, China
* College of Geophysics, Chengdu University of Technology, Chengdu, Sichuan Province, China
*Corresponding author’s e-mail: liqiong@cdut.cn

Abstract. The quality of seismic data has always been an important factor restricting the results of data processing and interpretation. With the development of new technologies, the amount of collected data is also increasing, and the existence of noise is inevitable. Therefore, denoising has become a major problem. Therefore, the compressed sensing theory is used in the article, and the K-SVD learning dictionary is used as a sparse base to perform denoising tests on the model data of different layers, and then the actual seismic profile of a certain work area is processed. The results show that the K-SVD learning dictionary can better denoise the signal and reconstruct the original signal, which lays a good foundation for the subsequent interpretation work.

1. Introduction
Signal-to-noise ratio, Resolution, and Fidelity are the three major evaluation criteria for seismic data quality, among which high signal-to-noise ratio is the basis of those (high signal-to-noise ratio, high resolution, and high fidelity) [1]. Methods include: Discrete cosine transform [2], F-K [3], Wavelet transform, Radon transform. The denoising based on compressed sensing is to use the corresponding sparse matrix in the sparse domain to transform the actual data into a coefficient matrix, and then reconstruct the seismic trace through the coefficient matrix, and then process the coefficient matrix to finally achieve the desired denoising effect.

The emergence of compressed sensing theory [4][5] broke the previous limitation of sampling interval and provided new directions in sparseness and reconstruction. Many scholars have proposed a series of sparse reconstruction algorithms and dictionary learning methods, such as: BP algorithm, MP algorithm, DCT dictionary and K-SVD dictionary, etc. Afterwards, many scholars have optimized the OMP algorithm and sparse K-SVD dictionary. In this paper, the OMP algorithm is used to solve the iterative problem of K-SVD dictionary to denoise and reconstruct seismic data.

2. Method

2.1. Compression sensing, CS
Compressed sensing changes the sampling premise of the previous sampling theorem-the signal x(t) has a limited bandwidth, and the highest frequency of its spectrum is fc". The premise of compressed sensing sampling is that "The signal x(t) can be sparsely represented in a transform domain, and its..."
sparsity is K", Compared with before, CS is sampling while compressing, transforming simulation information into AIC, which greatly reduces the amount of seismic data collected. Compressed sensing can be divided into three steps as shown in Figure 1(b): Sparse, Observation and Refactor.

![Figure 1. Signal sampling compression process](image)

The mathematical model of compressed sensing can be expressed as:

\[ y = Cf \]  

Where the sampling result is of length m, that is, the Observation Signal, \( C \in \mathbb{R}^{m \times n} \) is the \( m \times n \) dimensional sampling matrix, and \( f \) is the length of the signal to be sampled as n. It can be seen that the amount of sampled data required for compressed sensing is greatly reduced compared to before.

However, the seismic data we collect are generally non-sparse signals in the time domain, so the signal needs a sparse representation to meet the premise of CS sampling the signal is K sparse in a certain transform domain. Commonly used as sparse bases are Wavelet, Cufferlet, Seislet, Radon, Discrete Cosine, etc. According to the sparse representation of the signal, most seismic signals can be sparsely represented in a certain transform domain, so it can be expressed as:

\[ f = \psi x \]  

Substituting (1) into (2) to get:

\[ y = Cf = C\psi x \]  

We can combine \( C\psi \) into a matrix and get:

\[ y = Bx \]  

Since matrix B is an \( m \times n \) dimensional matrix and \( m \ll n \), the equation system is an underdetermined equation system. Therefore, Candes et al. proposed Restricted Isometry Principle (RIP), namely: zero-space characteristics, restricted isometry and low degree of coherence [6]. From the above theory, it can be known that (constant matrix) \( \delta_{k} \in (0,1) \) is to satisfy:

\[ (1-\delta_{k})||x||_{2}^{2} \leq ||y||_{2}^{2} \leq (1+\delta_{k})||x||_{2}^{2} \]  

The observation matrix generally uses the Gauss matrix, because the random Gaussian matrix is irrelevant to most fixed sparse base matrices. Observe the seismic data, so when the orthogonal basis is selected as the transformation basis, B satisfies the RIP property [7].

Regarding signal reconstruction, it can be roughly divided into two categories: one is convex programming algorithm, and the other is greedy iterative algorithm.

2.2. Objective evaluation of signal quality

There are roughly three methods for objective evaluation of signal quality: Signal to Noise Ratio(SNR), Mean Square Error(MSE), Peak Signal-to-Noise Ratio(PSNR)[8].

\[ SNR = \frac{1}{M \times N} \sum_{i=1}^{N} (x_i - y_i)^2 \]  

PSNR:
The principle of the orthogonal matching tracking algorithm is based on the principle of the MP algorithm. It uses the atoms with the greatest correlation with the error to be selected, and the linear combination of atoms is used to reconstruct the optimal approximate signal \(^{(2)}\); and the specific flow of the OMP algorithm is shown in the figure. 2 shown:

2.4. K-SVD dictionary learning

K-SVD uses sparse constraint conditions to restrict, and then uses the singular value decomposition of the difference between the approximate signal and the original signal to continuously iteratively update. The solution has two stages, namely sparse coding and dictionary learning \(^{(5)}\). Here, the OMP algorithm is used to solve the iterative problem of singular values. The specific process is shown in Figure 3, and the model dictionary shown in Figure 4 can be obtained.

\[
PSNR = 10 \log \frac{255^2}{MSE}
\]  

(7)

Where MSE is the mean square error.

**Figure 2. Algorithm flowchart of OMP**

**Figure 3. Algorithm flowchart of K-SVD**

**Figure 4. K-SVD Learning Dictionary**
3. Results & Discussion

3.1. Model Test

In order to prove the denoising effect of the K-SVD learning dictionary, the paper chooses the 30Hz rake wavelet, and designs the horizontal stratum model with different layers as shown in Figure 5(a) and Figure 5(b). Among them, the three-layer horizontal stratum is designed with a total length of 2560m and the track spacing is 5m, and the six-layer horizontal stratum is designed with a total length of 5120m and the track spacing is 10m. Fig. 6(a) and Fig. 6(b) are the formation models with random Gaussian white noise.

![Figure 5. The original model and its f-k filter](image)

![Figure 6. Noise model and its f-k filters](image)

In order to see the presence of noise more clearly, it is f-k filtered to get the results shown in Figure 6 (c), and Figure 6 (d). In order to better display the denoising effect, zoom in the seismic trace to get the display as shown in the figure 7. In terms of peak signal-to-noise ratio, the peak signal-to-noise ratio (PSNR) of the learning dictionary decreases with the increase of the stratum. The peak signal-to-noise ratio can be seen in the title of the denoising effect map in Figure 7. We can see from Figure 7
that it can reconstruct the original data well, but because the dictionary is constantly updated through singular value decomposition in the sparse coding stage of the dictionary, in the case of high noise intensity, the denoising effect has been significantly improved, and it has stronger stability; the disadvantage is that due to the number of iterations, the running speed is slower.

3.2. The actual data
The actual seismic data is the post-stack data of a certain work area as shown in Figure 8. You can see the presence of random noise in the figure. The K-SVD dictionary is used in the text to denoise and get 10(a). In order to see the effect more clearly. Therefore, output a residual plot like 9(b).

Comparing the effect diagram of Figure 10(a) with Figure 9, it can be seen that K-SVD has a better denoising effect and can remove a large amount of random noise well. From the perspective of reconstructed information, as far as the degree of seismic energy damage is concerned, Figure 9(b) hardly damages the original information, and the signal-to-noise ratio becomes higher.

Only the noise points are displayed in the residual image, and the quality of the original signal is not damaged. Although the noise is not completely removed, the comprehensive analysis shows that the K-SVD denoising accuracy is high, and the damage to the texture details is less.
4. Conclusion

Denoising and Reconstruction study the effect of the text learning dictionary by K-SVD, indicates that it can efficiently reconstruct seismic data, K-SVD combines match-aware compression in tracing algorithm can remove a lot of noise, Since K-SVD needs to constantly update the dictionary, Therefore, it has stronger stability; it lays the foundation for future interpretation work, and its disadvantage is that it runs slowly.

For the problem of slow running speed in the K-SVD dictionary, you can improve the K-SVD dictionary to see if it can be combined with the implicit dictionary to make it have the advantage of high computational efficiency.

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