A PRACTICAL APPROACH TO CORONAL MAGNETIC FIELD EXTRAPOLATION BASED ON THE PRINCIPLE OF MINIMUM DISSIPATION RATE

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ABSTRACT

We present a newly developed approach to solar coronal magnetic field extrapolation from vector magnetograms, based on the principle of minimum dissipation rate (MDR). The MDR system was derived from a variational problem that is more suitable for an open and externally driven system, like the solar corona. The resulting magnetic field equation is more general than force-free. Its solution can be expressed as the superposition of two linear (constant-α) force-free fields (LFFFs) with distinct α parameters, and one potential field. Thus, the original extrapolation problem is decomposed into three LFFF extrapolations, utilizing boundary data. The full MDR-based approach requires two layers of vector magnetograph measurements on the solar surface, while a slightly modified practical approach only requires one. We test both approaches against three-dimensional MHD simulation data in a finite volume. Both yield quantitatively good results. The errors in the magnetic energy estimate are within a few percent. In particular, the main features of relatively strong perpendicular current density structures, representative of the non-force-freeness of the solution, are well recovered.

Subject headings: methods: data analysis — MHD — Sun: corona — Sun: magnetic fields

1. INTRODUCTION

Plasma relaxation processes are ubiquitous in astrophysical and laboratory plasmas. One well-known approach is to invoke the variational principle, minimizing the total magnetic field energy subject to the constraint of constant total magnetic helicity (Taylor 1974). Such an approach yields the magnetic field equation for a relaxed plasma state, in which the Lorentz force vanishes, i.e., the so-called linear force-free field (LFFF) with a constant-α parameter,

\[ \nabla \times \mathbf{B} = \alpha \mathbf{B}. \] (1)

Although the above equation (1) has been widely applied in modeling magnetic field configurations in laboratory plasma devices and in the solar atmosphere, it is found inadequate in certain situations (e.g., Gary 2001; Metcalf et al. 1995). In particular, Amari & Luciani (2000) showed by three-dimensional (3D) numerical MHD simulation that in certain solar physics situations, after the initial helicity drive, the final “relaxed state is far from the constant-α linear force-free field that would be predicted by Taylor’s conjecture” with helicity conserved. They suggested deriving alternative variational problem.

An alternative approach, also based on the variational principle, was recently developed for an open system with external drive and applied to theoretical investigation of the solar arcade structures with flow (Bhattacharyya & Janaki 2004; Bhattacharyya et al. 2007). It was later applied to develop a new approach to the extrapolation of solar coronal magnetic field in a non-force-free state (Hu & Dasgupta 2006, 2008; Hu et al. 2007). The core of the approach is to construct a variational problem based on the principle of minimum dissipation rate (MDR), which states that the “steady state of an irreversible process is characterized by a minimum value of the rate of entropy production” (Prigogine 1947). In most cases, the entropy production rate is equivalent to the energy dissipation rate. The basis is the set of generalized momentum balance equations, and the system, with flow, is always in a broader defined force-balanced dynamic equilibrium, satisfying the MHD equations (Bhattacharyya & Janaki 2004). Prior works based on MDR (Montgomery & Phillips 1988; Dasgupta et al. 1998, 2002) had successfully shown that it was able to yield a pressure-balanced configuration, supporting a finite plasma pressure gradient found in plasma confinement devices. Recently, a proof of MDR was rigorously sought by 3D numerical simulations (Shaikh et al. 2008). We refer the readers to the above-referenced literature for detailed descriptions and justifications of the theoretical basis of MDR.

In short, analogous to the principle of minimum energy, but for a more complex, open system like the solar corona, the MDR yields the following equations for the magnetic field \( \mathbf{B} \) and flow vorticity \( \mathbf{\omega} = \nabla \times \mathbf{v} \), \( \mathbf{v} \) being the plasma flow velocity (Bhattacharyya & Janaki 2004; Bhattacharyya et al. 2007; Hu & Dasgupta 2008):

\[ \nabla \times \nabla \times \mathbf{B} + a_1 \nabla \times \mathbf{B} + b_1 \mathbf{B} = \nabla \psi, \] (2)

\[ \nabla \times \nabla \times \mathbf{\omega} + a_2 \nabla \times \mathbf{\omega} + b_2 \mathbf{\omega} = \nabla \chi, \] (3)

where coefficients \( a \) and \( b \) are constants and involve the parameters of the system. The right-hand sides of both equations are arbitrary, undetermined functions. In the present work, we concentrate on the magnetic field, equation (2), only. A complete description of the plasma state, including the pressure profile, is
important and requires a completely resolved flow field, from equation (3), as well. We plan to pursue this study in future work.

In what follows, we first briefly provide a heuristic justification of the MDR in § 2, based on a simple but standard analysis. We then present a brief but comprehensive description of the full MDR-based coronal magnetic field extrapolation approach in § 3, including a new test case study utilizing 3D MHD simulation data of a bright point region, highlighting the non-force-free features of the solution. In § 4 we develop a practical approach that requires only one single-layer vector magnetogram and show that it achieves the same satisfactory results as the full approach for the same test case. In § 5 we conclude and discuss the significance and limitations of our approach.

2. HEURISTIC justIFICATION OF THE MDR

In a generally resistive plasma, the Lundquist number, $S$, is defined as the ratio of the resistive timescale over the Alfvén timescale, $S = \tau_R/\tau_A$ (e.g., Ortolani & Schnack 1993). The Lundquist number scales with $L_0/\eta$, the ratio of the characteristic length to the resistivity of the system. Therefore, the number $S$ can be very large when $L_0$ is large and/or plasma is highly conductive ($\eta_0$ is small). For example, it can reach $\sim 10^6-10^8$ in a hot fusion plasma, while for solar coronal structures, it often exceeds $10^{12}$.

From standard procedures (e.g., Ortolani & Schnack 1993), the decay rates of global magnetic helicity, $K$, and magnetic energy, $W$, are given as $K = \sum_k b_k \exp(ik \cdot r)$ and $W = -\frac{2\eta}{S} \int_V j \cdot B \, dV$, where $\eta$ is the resistivity.

Taylor envisioned the relaxation process occurring as a result of small-scale turbulence, with $S \gg 1$. Following Ortolani & Schnack (1993), at scale lengths for which the Fourier decomposition wavenumber $k$ is $\sim S^{-1/2}$, the energy decay $dW/dt$ is $\sim O(1)$. But for such a scale length, the helicity decay rate $dK/dt$ is $\sim O(S^{-1/2}) \ll 1$. Thus, from this heuristic argument, we may expect that small-scale turbulence dissipates energy at a greater rate than helicity. In the same way, we can show that $dR/dt \sim -(2\eta^2/S) \sum_k k^2 b_k^2$ goes at a much faster rate than $dK/dt$. Furthermore, a recent 3D time-dependent simulation of MHD fluids proves this argument (Shaih et al. 2008). It is shown that the energy dissipation rate, $\int \eta j^2 dV$, decays the fastest toward a minimum state. We again refer readers to Shaih et al. (2008) for a complete description of the 3D numerical simulation results. Further simulation work tailored toward the general situation in the solar corona is underway.

In addition, based on the MDR theory, Bhattacharyya et al. (2007) derived coronal arcade structures and flow patterns that are supported by observations and certain numerical simulations. We also believe that, in turn, the demonstration of the success of the application of MDR to practical cases, including the aforementioned ones of plasma confinement devices and the one to be presented below, provides an additional means of validating the theory.

3. THE FULL MDR-BASED APPROACH TO CORONAL MAGNETIC FIELD EXTRAPOLATION

The full MDR-based approach to coronal magnetic field extrapolation has been developed and tested against analytic models in Hu & Dasgupta (2008) and Hu et al. (2007), based on the general equation yielded from MDR. The following equations results by taking an extra curl on both sides of equation (2),

$$\nabla \times \nabla \times \nabla \times B + a_1 \nabla \times \nabla \times B + b_1 \nabla \times B = 0. \quad (4)$$

The aim is to solve the above equation in a finite volume utilizing boundary data. The key is that one exact solution to equation (4) exists and can be expressed as the superposition of three LFFFs. Each satisfies equation (1) with distinct $\alpha$ parameters.

3.1. Procedures

The exact solution to equation (4) is written

$$B = B_1 + B_2 + B_3, \quad (5)$$

with $\nabla \times B_i = \alpha_i B_i$ and $i = 1, 2, 3$. Subsequently, one obtains (taking curls on both sides of eq. [5])

$$\begin{pmatrix} B \\ \nabla \times B \\ \nabla \times \nabla \times B \end{pmatrix} = \mathbf{V}^{-1} \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix}, \quad (6)$$

where the matrix $\mathbf{V}$ is a (transposed) Vandermonde matrix composed of constant elements $\alpha_i^{-1}$, with $i, j = 1, 2, 3$. It is guaranteed invertible provided that the parameters $\alpha_i$, for $i = 1, 2, 3$, are distinct. Therefore, it follows that

$$\begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \mathbf{V}^{-1} \begin{pmatrix} B \\ \nabla \times B \\ \nabla \times \nabla \times B \end{pmatrix}.$$
where \( M = N^2 \), the total number of grids on the transverse plane, and the optimal pair \((\alpha_1, \alpha_3)_{\text{opt}}\) is chosen such that the corresponding \( E_{n_{\text{opt}}} \) is at its minimum.

To summarize, the procedures of the full MDR-based approach to coronal magnetic field extrapolation constitute the following two steps.

1. Set \( \alpha_2 = 0 \) and search through LFFF parameters, \( \alpha_1 \) and \( \alpha_3 \) (\( \alpha_1 > \alpha_3 \), due to the interchangeability of subscripts 1 and 3). Apply an LFFF solver to calculate the transverse components \( b_{2i} \), utilizing the normal component \( B_n \) (given by eqs. [6]), \( i = 1, 2, 3 \), at the bottom boundary only. A pair of optimal \( \alpha_i, i = 1, 3 \), is found, for which the \( E_{n_i} \), given by equation (7), is at its minimum. A plot of the \( E_n \) distribution over the \((\alpha_1, \alpha_3)\) parameter space is prepared to show the goodness and uniqueness of the solution.

2. Solve for \( B_1 \) and \( B_3 \) in a finite volume above the bottom boundary, for the optimal \( \alpha_1 \) and \( \alpha_3 \) found in step 1, respectively, and the potential field \( B_2 \) as well. Obtain \( B = B_1 + B_2 + B_3 \) (eq. [5]).

The run time of the approach based on the fast Fourier transform algorithm solving each LFFF is approximately on the order of \( P N^2 \log_2 N \), largely dependent on the number of search grids, \( P \), on \((\alpha_1, \alpha_3)\) in step 1. However, this process can be easily parallelized by simply dividing the search domain into individual pieces and assigning each piece to a separate processor. One can also start with a coarse grid, then allocate a small but finer grid around a minimum; thus, the total number of iterations can be effectively reduced.

3.2. A Test Case Study Using Numerical Simulation Data

Several analytic solutions have been utilized to test the full approach outlined in § 3.1, and the results showed the approach was able to recover the solution in a finite volume to a certain degree of satisfactory accuracy for several cases (Hu & Dasgupta 2008; Hu et al. 2007). Here, in order to simulate a case of real magnetograph measurements as closely as possible, we employ 3D MHD simulation data surrounding a region of an X-ray bright point (BP; Brown et al. 2001) from Büchner & Nikutowski (2005; see also Otto et al. 2007; Büchner 2005, 2006). The simulation started with actual solar magnetic field measurements and was continuously driven by photospheric motion inferred from solar observations. The initial state was obtained by LFFF extrapolation from photospheric magnetic field observation (Otto et al. 2007).

The data were provided in a finite Cartesian volume, \( x \times y \times z = 128 \times 128 \times 63 \), with \( z \) being the vertical dimension along the normal direction, at one chosen moment during the 3D dynamic MHD simulation. The bottom two layers of the vector magnetic field data are utilized to generate the bottom boundary \((z = 0)\) conditions by equations (6). The two steps are carried out as described in § 3.1. The distribution of \( E_n \) in \((\alpha_1, \alpha_3)\) parameter space is shown in Figure 1. The minimum value is \( E_{n_{\text{opt}}} \approx 0.30 \), with corresponding \((\alpha_1, \alpha_3)_{\text{opt}} \approx (0.00156, 0.000779)\) (dimensionless), as denoted by the plus sign in Figure 1. A fairly significant amount of uncertainty associated with this optimal pair exists. However, further analysis shows that choosing any other pair of \((\alpha_1, \alpha_3)\) within the innermost contour does not significantly change the extrapolation results. A similar pattern and small-\( \alpha \) values were also observed in Hu & Dasgupta (2008).

The corresponding optimal transverse magnetic field vectors \( b_i \) at \( z = 0 \) are shown in Figure 2, together with the “exact” solution \( B \), from the simulation data and the corresponding potential field results using the normal component \( B_2 \) only (Venkatakrishnan & Gary 1989). Our solution agrees very well with the “exact” one, whereas the potential field result exhibits apparent deviations.

The final extrapolation results are shown in Figure 3, in the form of 3D field line plots. The base image shows the normal
component distribution at \( z = 0 \), with two major polarities in the center corresponding to the location of a BP. The field lines from our result demonstrate a good deal of similarity (nearly identical in the top left part) to the “exact” solution, indicating good quality recovery of the magnetic field in the finite volume from the bottom two layers of data. This judgment is supported by the set of quantitative measures, so-called figures of merit, adopted from Schrijver et al. (2006), given in Table 1. They are used to evaluate, quantitatively, the agreement between the exact vector field \( \mathbf{B} \) and the corresponding extrapolated field \( \mathbf{b} \) in a volume (second through fifth columns). The second row in Table 1 lists the numbers for this calculation (result 1), and the bottom two rows list the values for a result identical to the exact solution and a potential field extrapolation, respectively. The rightmost two columns show the energy estimates, the magnetic energy ratio of the extrapolated field \( b \) to the exact field \( B \), i.e., \( \epsilon = b^2/B^2 \), and the energy ratio, \( \epsilon_p \), of \( b \) to the corresponding potential field. For this case, several figures from the potential field extrapolation are comparable with our results. However, the energy estimates clearly mark the distinction; ours only differs from the exact ones by \( \sim -1\% \), whereas the potential field results differ by as much as \( \sim -16\% \).

To further characterize the non-force-freeness of the solution, which cannot be captured by either the potential or the force-free extrapolation, and to better visualize the results, we calculate the field-line-integrated current density, \( J = \int J\, dl \), along individual field lines (e.g., Büchner 2006). Figure 4 shows this quantity separated into the field-aligned (\( \parallel \mathbf{B} \)) component, \( J_\parallel \), and the component perpendicular to \( \mathbf{B} \), \( J_\perp \), for the “exact” solution. A non-vanishing \( J_\perp \) indicates the field is non-force-free. Two footpoints of one single field line, both rooted on the bottom boundary, have the same value. Figure 5 shows the corresponding \( J_\parallel \) and \( J_\perp \) for our result 1 discussed above. The distribution of \( J_\parallel \) is well recovered with concentrations at the two major polarities around the center. The major features of \( J_\parallel \) at the same locations are well recovered as well, but most of the remaining weak structures are lacking, compared with the right panel in Figure 4. However, both \( J_\parallel \) and \( J_\perp \) enhancements near the lower boundary tracing back to one of the strong polarities to the right, remain. It is indicative of strong currents, implying the possible locations of current sheets that are important for magnetic reconnection (Büchner 2005, 2006).

At the present time, multiple (\( \geq 2 \)) layers of vector magnetograms are hardly available (but see, e.g., Metcalf et al. 1995). Only the photospheric vector magnetic fields are routinely observed. Sometimes, the chromospheric magnetic field line-of-sight (LOS) components were inferred (e.g., Choudhary et al. 2001). When the observations are near the disk center, the LOS component can be used as the normal component \( B_z \), then the term \( \nabla \times \nabla \times \mathbf{B} \) of the observer may be approximated by (denoting the distance between the chromosphere and the photosphere by \( \Delta z \))

\[
\nabla^2 B_z \approx \nabla^2 B_z + 2 \left[ B_z^{\text{ch}} - B_z^{\text{pho}} - \left( \frac{\partial B_z}{\partial z} \right) \Delta z \right] / \Delta z^2. \tag{8}
\]

The first-order derivative \( \partial B_z/\partial z \) is approximated by the divergence-free condition using transverse magnetic field vectors at the photosphere. The third row of Table 1 (result 2) lists the figures of merit of the extrapolation result using the bottom boundary magnetic field vectors and only the \( B_z \) component at one level immediately above to approximate \( \nabla^2 B_z \) by equation (8). It shows

\[
\begin{array}{cccccc}
& C_{\text{vec}} & C_{\text{CS}} & E'_{\epsilon} & E'_{\epsilon_{\text{p}}} & \epsilon & \epsilon_p \\
\text{Our result 1*} & 0.97 & 0.91 & 0.74 & 0.62 & 0.99 & 1.18 \\
\text{Our result 2*} & 0.91 & 0.76 & 0.53 & 0.27 & 1.12 & 1.34 \\
\text{Our result 3*} & 0.97 & 0.93 & 0.76 & 0.64 & 0.95 & 1.14 \\
\text{Exact*} & 1 & 1 & 1 & 1 & 1 & 1.20 \\
\text{Potential*} & 0.95 & 0.92 & 0.70 & 0.63 & 0.84 & 1.00 \\
\end{array}
\]

\textbf{TABLE 1}

\textbf{Figures of Merit}

\textit{Note}.—See Schrijver et al. (2006) and text for definitions.

* Results obtained by the full MDR approach (see § 3), where two bottom layers of vector magnetic field data are utilized to derive the bottom boundary conditions.

* Results obtained by the reduced bottom boundary data (see § 3.2), where only the vector magnetic field on the bottom layer and the normal magnetic field component on the layer immediately above are utilized.

* Results obtained by the practical approach (see § 4), where only the magnetic field vectors on the bottom boundary are required.
somewhat deteriorated results, and the error in the energy estimate is $\sim +12\%$.

4. A PRACTICAL APPROACH UTILIZING SINGLE-LAYER VECTOR MAGNETOGRAM

As mentioned above, the multilayer vector magnetograms are not routinely available. On the other hand, the high-quality photospheric vector magnetograms become increasingly available. In order to apply our method to actual measurements currently available, we adapt our approach to employ only one single-layer vector magnetogram, at the expense of less general, expected limited applications, to be described below.

From equation (2), considering the arbitrary nature of $\psi$, one may write one exact solution to equation (2),

$$B = B_1 + B_3 + cB_{pot},$$

where a constant, $c$, and the potential field, $B_{pot}$, obtained from the known normal field $B_z$ at the bottom boundary, are introduced. Then it follows that $\nabla \psi = b c B_{pot}$. This is equivalent to the full approach in §3, except that the potential field $B_2$ is given a special form $cB_{pot}$, and is largely known, subject to a undetermined constant.

With an additional unknown parameter, $c$, the procedures are similar to those outlined in §3.1, but for a reduced, second-order system. The following equations, in place of equation (6), now provide the bottom boundary conditions for each LFFF from one single-layer vector magnetogram,

$$\left(\alpha_3 - \alpha_1\right)B_{1z} = \alpha_3 B'_z - (\nabla \times B)_z,$$

$$\left(\alpha_1 - \alpha_3\right)B_{3z} = \alpha_1 B'_z - (\nabla \times B)_z,$$

with $B'_z = B_z - cB_{pot,z}$. In addition to the $(\alpha_1, \alpha_3)$ search domain in step 1, another dimension in $c$ is added. A desirable choice of allowable $c$ values is $c \in [-1, 1]$, to limit the dominance of the potential field. The approach first reported in Hu & Dasgupta (2006) represents the special case $c = 0$.

We again test this practical approach against the same data set utilized in §3, but only use the vector magnetic field data on the bottom boundary, $z = 0$. Only a limited number of $c$ values are chosen, $c \in \{-1, -0.5, 0, 0.5, 1\}$. The $E_n$ distribution at $c_{min} = -1.0$, which yields the minimum, $E_n^{opt} \approx 0.22$, is shown in Figure 6. Compared with Figure 1, the uniqueness of $(\alpha_1, \alpha_3)^{opt}$ is much improved. The corresponding $b_z$ at $z = 0$ and the 3D
field line plots are shown in Figures 7 and 8, respectively. The corresponding figures of merit are given in the fourth row (result 3) of Table 1. Good agreement with the exact solution is achieved. The energy estimate has a $\sim -5\%$ error. The field-line-integrated current densities shown in Figure 9 are essentially the same as the previous ones obtained by the full approach. All the main features discussed in § 3.2 are retained.

5. CONCLUSIONS AND DISCUSSION

In conclusion, we develop an approach to extrapolate the coronal magnetic field from vector magnetograms based on the principle of minimum dissipation rate (MDR). Analogous to, but yet opposed to the principle of minimum energy, the MDR yields a generally non-force-free magnetic field through a different variational approach. The full MDR-based approach requires two layers of vector magnetograms, while a practical approach, representing a class of special solutions to the MDR system, requires only one layer, which is more amenable for practical applications to currently available data. A test case study using numerical simulation data shows that both approaches recover the solution to a good degree of accuracy, as measured by a set of quantitative measures. The errors in the energy estimate are both within a few percent. Moreover, the non-force-free features of the solution, mainly the strong perpendicular current density concentrations, are well retained in the extrapolation results as well. Such results from the analysis indicate that, for this particular numerical simulation, the results are close to an MDR state, which is primarily a non-force-free state.

We hereby provide an alternative approach that is fast, is easy to implement, and allows one to extrapolate coronal magnetic field in a more general non-force-free state with manageable effort in a solo work. For instance, all the calculations reported herein (128 $\times$ 128 $\times$ 63 grid) were performed on a single-processor 2.8 GHz PC in IDL within a reasonable time frame. It potentially can be made much faster. It is apparent that equation (4) includes solutions to LFFF ($\nabla \times \mathbf{B} = \alpha \mathbf{B}$), for $-\alpha_1 = \alpha_2 = \alpha$. It is not explicit whether this is also the case for the nonlinear force-free field (NLFFF) when the parameter $\alpha$ is allowed to vary, although several numerical experiments indicated that this might be so (Hu & Dasgupta 2008) sometimes.

We are fully aware of the limitations of the method, due to the fact that the currently employed LFFF solver originates from an ill-posed problem. The nonuniqueness and unphysical behavior of the LFFF solution are well known (e.g., Chiu & Hilton 1977). It is adversely affected by the increasing resolution of vector magnetograms, since $\alpha_{\text{max}}$ is inversely proportional to N. This severely limits the search domain of $\alpha_j$ that will yield unique physical solutions (e.g., Alissandrakis 1981; Gary 1989). Recent progress in the algorithm for NLFFF extrapolations (e.g., Song et al. 2006), in addition to the existing LFFF algorithm (e.g., Abramenko & Yurchishin 1996), which modified the problem into a well-posed one, look promising and may be adapted.

It is worth noting that our approach is wholly applicable to extrapolating the plasma flow field by simply replacing magnetic field $\mathbf{B}$ with flow vorticity $\mathbf{\omega}$. However, the boundary conditions are probably harder to obtain, since it is the vorticity that is involved. A good start is to utilize the flow field data contained in the same 3D MHD simulation test case. It is not straightforward to obtain the plasma state, since the flow field has to be resolved as well, and to be consistent, the full set of MHD equations has to be considered (see, e.g., Montgomery & Phillips 1988). One may also foresee that extension of this approach to the whole solar surface data is of great significance. Further theoretical investigation is underway.

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