A novel reliability index approach and applied it to cushioning packaging design

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Abstract
This paper is aimed at proposing a new approach to Reliability-based Design Optimization (RBDO) and applying it to cushioning packaging design with a highly nonlinear system. The problem is formulated as an RBDO problem, which is included a minimizing cost function and probabilistic constraints. Here, the thickness of the cushion material is dealt with uncertainty and uncontrolled parameters. The traditional reliability index approach (RIA) has evolved as a powerful tool to solve the RBDO problem; however, due to its convergence problem, the modified reliability index approach (MRIA) is proposed. Although the MRIA method solves the problems of the traditional RIA, it inherits the low efficiency of searching for the most probability point (MPP). Thus, we developed a novel RIA based on MRIA to improve the efficiency and robustness during the RBDO process. The innovation active set strategy is developed in reliability assessment, which is a strict inequality to determine whether the current constraint is active or inactive. An application example is presented, and the results are compared with MRIA to assess cost-effectiveness and efficiency. Results indicate the proposed method is feasible to solve the uncertainty problem of packaging materials in the processing process and is also an efficient RBDO method.

Keywords
Uncertainties, reliability-based design optimization, cushioning packaging design

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Introduction
During handling and transportation, a lot of goods are damaged due to various uncontrolled uncertainties, including cushion material properties, drop height, and storage temperature.\textsuperscript{1,2} Therefore, many methods have been well developed and widely applied to protect the product from the risk of damage.

One of the most well-known ways to protect products from damage is to provide a more reasonable packaging based on the cushion curves of a certain material, which is established by ASTM D 1596.\textsuperscript{3} The simple cushion curves are depicted in Figure 1, which provide a lot of information, including the G-value (the fragility of the product) vs statics stress, the drop heights (h), and the thickness of the cushion (t). However, it takes a lot of experimental cost and time to generate all the curve information, and the cushion designed in this way is often over-designed. Although it protects the products in the circulation environment, it is at the cost of a lot of materials. Those drawbacks prompted many researchers to establish a simplified
cushion-curve method based on the stress–strain curve of the material. However, this simplified method of establishing the cushion curves is performed well for closed-cell cushion materials and has certain limitations for open-cell packaging materials. Moreover, the stress–strain characteristics of packaging materials are not willing to be provided by the manufacturer.

Another way to protect the product from damage is to make the impact peak acceleration of the product from vibration less than the allowable G-value. Generally, the drop model of the packaging system, like a single-degree-of-freedom or two-degree-of-freedom spring system, is abstracted to assess the damage to the product. Besides, the Transport Packaging Laboratory of Kobe University has conducted a large number of various drop tests and simulations to predict whether the product is damaged or not based on the comparison of the measured impact acceleration with the G-value of the product. Although many efforts have been made to protect products from being destroyed, the uncertainties in the transportation process are still one of the great challenges faced by designers. Every year, the product damage and commercial value caused by uncertainties in the circulation environment and is highly due to uncontrolled manufacturing parameters of the cushion are still immeasurable. Therefore, the uncertainties deserve special attention, especially the uncontrolled manufacturing parameters of the cushion.

To date, reliability-based design optimization (RBDO) has been derived as a dominant model to consider uncertainties and is widely utilized in various engineering fields. Traditionally, two different approaches, which mainly include the reliability index approach (RIA) and the performance measure approach (PMA) are widely utilized for assessing the probabilistic constraints in RBDO. RIA is developed focused on the concept of the first-order second moment (FORM), which is widely applied to solve RBDO problems. Conceived António proposed a gradient algorithm for RIA, based on the Hasofer-Lind method, and then applied it to the RBDO problem of composite laminates. Tu et al. emphasized that RIA has the disadvantages of convergence problems and numerical singularities, which promotes PMA as a more efficient and robust choice to wisely deal with RBDO problems. Ting Lin et al. proposed the modified reliability index approach (MRIA) to overcome the disadvantage of RIA by a new definition of reliability index. Although MRIA could converge the optimal solution efficiently and stably in assessing the active probabilistic constraints, it also inherits the low efficiency of searching for the most probability point (MPP); hence, the hybrid reliability method is presented. das Neves Carneiro and António developed a prominent reliability method based on the RIA by introducing the genetic algorithm (GA’s) with elite strategy into the reliability assessment process to improve the efficiency and convergence of RIA. Although these approaches have been presented to enhance the efficiency of evaluating the failure probability of RIA, the computational cost is still huge in the process of solving the RBDO problem.

In general, the above methods are mainly divided into two strategies, the double-loop strategy (DLS), the single-loop strategy (SLS), followed by the decouple-loop strategy, and the hybrid loop strategy are carried out based on the DLS and the SLS, the former is simpler and more stable, although DLS has a nested nature. In general, those strategies for assessing the probabilistic constraints lead to performing first-order or second-order approximate expansion at the MPP, that is, the first-order reliability method (FORM) and the second-order reliability method (SORM) to convert probability constraints into deterministic constraints. The FORM has been widely used in RBDO procedures due to its simplicity and efficiency. Besides, another method of probability assessment is Monte Carlo simulations (MCS), which is regarded as a robust reliability analysis and often is applied as an auxiliary tool to verify the accuracy of the optimal solution. Nevertheless, if the performance function has a highly nonlinear system, MCS may need excessively
classical RBDO mathematical model is the target failure probability. \( P_f \) is the target failure probability. \( P_f | g_i(\vec{X}) \geq 0 \) describes the \( i \)-th probability constraints of the system while \( g_i(\vec{X}) \) is greater than zero denotes the failure region. \( P_f \) is the target failure probability.

### Reliability index approach

The system failure probability, \( P_f \), is estimated in the statistical model as

\[
P_f = P[g_i(\vec{X}) \geq 0] = \int_{g_i(\vec{X}) \geq 0} \cdots \int f_{\vec{X}}(\vec{x})d\vec{x}_1 \cdots d\vec{x}_n
\]

where \( f_{\vec{X}}(\vec{x}) \) represents the joint probability density function (JPDF) of \( \vec{X} \). To obtain the failure probability of the structure, multi-dimensional integration is often required. However, for most practical highly nonlinear problems, it is very complicated to solve multiple integrals.

Thus, the reliability index \( \beta_{HL} \) is reported by Hasofer and Lind\(^{30} \) to obtain the failure probability of the system instead of solving multidimensional integrals. The graphical of \( \beta_{HL} \) is depicted in Figure 2, where \( \beta_{HL} \) is the minimum distance from the origin to the performance function in the standard normal space. Based on the above definition, the RIA is formulated as

\[
\min \beta_{HL} = \left\| \frac{\partial \ln f_{\vec{X}}}{\partial \vec{X}}(\vec{u}_p) \right\| \\
\text{s.t.} \quad g_i(\vec{u}) = 0
\]

where \( \vec{u}_p \) denotes the MPP.

Equation (3) is a sub-optimization to solve MPP and the corresponding reliability index. In equation (3), the reliability index obtained is all positive. If the design point falls within the failure region, then the currently defined reliability index is invalid and will not converge to obtain a true MPP, here, the convergence problem will occur.

### Modified reliability index approach

Li et al.\(^{35} \) proposed a modified reliability index expression to solve the convergence problems of RIA, \( \beta_M \) as

\[
\min \beta_M = \vec{u}_p^T \cdot \frac{\nabla g(\vec{u}_p)}{\|\nabla g(\vec{u}_p)\|} \\
\text{s.t.} \quad g_i(\vec{u}) = 0
\]

where \( \nabla g(\vec{u}_p) \) denotes the gradient vector of \( g(\vec{u}) \) at the point \( \vec{u}_p \), as \( \nabla g(\vec{u}_p) = [\frac{\partial g}{\partial u_1}, \frac{\partial g}{\partial u_2}, \ldots, \frac{\partial g}{\partial u_n}]^T \) and its module \( \|\nabla g(\vec{u}_p)\| \).

This definition takes full advantage of the property that the collinear relationship between vector \( \vec{u}_p \) and gradient \( \nabla g(\vec{u}_p) \) to distinguish whether the current
design point is within the failure area or not. Although MRIA can accurately and stably converge to the optimal solution regardless of the location of the initial design. However, when strong nonlinear probabilistic constraints are involved, MRIA inherits some inefficient features of the MPP search.26,27

Construct the search region of MPP

In this section, the EMRIA is presented, which inherits the robustness of the MRIA while improving the search efficiency of the MPP at the same time.

Firstly, the innovative active set (A∗) is defined using inequality to construct a region for the MPP search. In the innovative active strategy, we introduced the concept of “6-sigma” to efficiently and robustly solve MPP sub-optimization, which is shown in Figure 3. If the MPP is in the defined set of A∗, then the corresponding constraint is active and keeps it, as the point u∗ j.

Thus, the innovative active set is suggested as:

\[ A^* = \{ \tilde{u}_j^* | |\tilde{u}_j^*| - 6\sigma \leq 0, \tilde{u}_j^* \in \mathbb{R} \} \]

(5)

where \( \sigma \) denotes the standard deviation. A new reliability index, \( \hat{\beta}_j^* \), is defined as

\[
\hat{\beta}_j^* = \begin{cases} 
\frac{\partial g_j(\tilde{u}_j^*)}{\partial \tilde{u}_j^*} & \text{for } \tilde{u}_j^* \in A^* \\
\frac{\partial g_j(\tilde{u}_{j+1}^*)}{\partial \tilde{u}_{j+1}^*} & \text{otherwise inactive}
\end{cases}
\]

(6)

In which \( \tilde{u}_j^* \) states the effective MPP of the \( j \) th active constraint. \( \tilde{u}_{j+1}^* (\tilde{d}) \) is the MPP of the next constraint.

It is worth mentioning that the design and quality of “6-sigma” products have been recognized by the global market because of their high reliability and confidence level.44,45 The “6-sigma” design level is equivalent to a 99.9999998% confidence level, which is significantly more reliable than the 3-sigma design (99.73%). Therefore, we adopt a “6-sigma” design level to assess the reliability of the system.

EMRIA in RBDO

The proposed EMRIA in this paper solves the RBDO problem in equation (1), which mainly adopts DLS. In this section, we will introduce the functions of the inner loop and the outer loop in detail.

FORM for constraint conversion

As mentioned before, a sub-optimization iteration will be executed in the inner loop, which is utilized to solve the MPP \( \tilde{u}_j^* \) and reliability index \( \hat{\beta}_j^* \) in the standard normal space (U-space). Thus, the basic independent random variables are in the original space (X-space) should be transformed into independent U-space using the mapping factors \( \hat{x} = \hat{d} + \hat{u} \sigma \), as illustrated in Figure 4.

Then, the sub-optimization scheme is normally expressed as:

\[
\min_{\hat{u}} \hat{\beta}_j^* = \left| \sqrt{(\tilde{u}_j^*)^T \tilde{u}_j^*} \right| \\
\text{s.t. } g_j(\tilde{u}_j^* + \hat{u} \sigma) = 0
\]

(7)

where \( g_j(u) \) represents the \( j \) th active performance function. With the help of the MPP \( \tilde{u}_j^* \) and \( \hat{\beta}_j^* \), the probability constraints of RBDO are converted into the deterministic constraints by the Rosenblatt46 transformation, which is expressed as:

\[
F_G(g_j) = \Phi( -\hat{\beta}_j^* (\hat{d}) )
\]

(8)

where \( \Phi \) represents the standard cumulative distribution function (CDF). \( F_G(g_j) \) is the cumulative
distribution function. Using the following two inverse Gaussian transformation\textsuperscript{47} to complete the transformation of constraint:

\[
\hat{\beta}_{s_j}(\bar{d}) = \Phi^{-1}(F_G(s_j))
\]

\[
g(\hat{\beta}_{s_j}(\bar{d})) = - F_G^{-1}(\Phi(- \hat{\beta}_{s_j}(\bar{d})))
\]

(9)

where \( \Phi^{-1} \) stands for the inverse cumulative distribution function of the standard normal distribution. \( F_G^{-1} \) is the inverse cumulative distribution function of the performance function. According to equation (9), the probability constraints of equation (1) are re-expressed as the deterministic formulation:

\[
\min \ z(\bar{d})
\]

\[
s.t. \quad -\hat{\beta}_{s_j}(\bar{d}) \leq -\beta^*_j
\]

(10)

where \( \hat{\beta}_{s_j}(\bar{d}) \) is the function of \( \bar{d} \) and \( \bar{u}_{s_j}^* \) while \( \beta^*_j \) is the target reliability index.

**DLS combining the EMRIA**

In the outer loop, \( \hat{\beta}_{s_j}(\bar{d}) \) is expanded by the first-order approximate Taylor series at the current design point \( \bar{d}_k \). For the \( k \) th iteration, \( \hat{\beta}_{s_j}(\bar{d}) \) is approximated as:

\[
\hat{\beta}_{s_j}(\bar{d}) \approx \hat{\beta}_{s_j}(\bar{d}_k) + (\bar{d} - \bar{d}_k) \frac{\partial \hat{\beta}_{s_j}(\bar{d}_k)}{\partial \bar{d}}
\]

(11)

where \( \hat{\beta}_{s_j}(\bar{d}) \) is obtained equation (10). In equation (6), if \( \bar{u}_{s_j}^* \in \mathcal{A}^* \), \( \hat{\beta}_{s_j}(\bar{d}_k) \) takes the partial derivative of \( \bar{d} \) is written as:

\[
\frac{\partial \hat{\beta}_{s_j}(\bar{d}_k)}{\partial \bar{d}} = \frac{\partial \bar{u}_{s_j}^*}{\partial \bar{d}} \cdot \frac{\partial g(\bar{u}_{s_j}^*)}{\partial \bar{d}} \left\| \frac{\partial g(\bar{u}_{s_j}^*)}{\partial \bar{d}} \right\|^{-1}
\]

(12)

Combining equations (12), (10), and (6), the optimized iterative scheme of the outer loop is:

**Min**: \( z(\bar{d}) \)

\[
\begin{cases}
\text{for } \bar{u}_{s_j}^* \in \mathcal{A}^* \cup \beta_{s_j} \leq \beta^*_j \\
(1) = -\bar{u}_{s_j}^* \frac{\partial g(\bar{u}_{s_j}^*)}{\partial \bar{d}} + (\bar{d} - \bar{d}_k) \frac{\partial \hat{\beta}_{s_j}(\bar{d}_k)}{\partial \bar{d}} \leq -\beta^*_j \frac{\partial g(\bar{u}_{s_j}^*)}{\partial \bar{d}}
\end{cases}
\]

otherwise

\[
(1) = -\bar{u}_{s_j}^* \frac{\partial g(\bar{u}_{s_j}^*)}{\partial \bar{d}} + (\bar{d} - \bar{d}_k) \frac{\partial \hat{\beta}_{s_j}(\bar{d}_k)}{\partial \bar{d}} \leq -\beta^*_j \frac{\partial g(\bar{u}_{s_j}^*)}{\partial \bar{d}}
\]

(13)

where \( c(\cdot) \) is expressed as the constraint.

In the beginning, the initial values, \( \bar{d}^{(0)} \) and \( \bar{u}_{s_j}^{(0)} \), are given arbitrarily. Equation (7) is applied to perform the inner loop and update the MPP, \( \bar{u}_{s_j}^* \) and reliability index, \( \hat{\beta}_{s_j}(\bar{d}) \). When \( \bar{u}_{s_j}^* \in \mathcal{A}^* \), the current constraint is active and goes to the out loop of equation (13).

**Application**

To demonstrate the ability of the proposed method in RBDO, a practical problem is discussed here. Comparing the convergence results of EMRIA with that of both MRIA and RIA. Among them, function evaluations (FEs) are used as a prominent indicator to evaluate the efficiency of both methods in the entire solution process.

**A drop packaging system**

A package is depicted in Figure 5, where the product and cushioning materials are idealized as a nonlinear mass-spring system with a stiffness coefficient, \( k \). Here, the outer corrugated carton is ignored, and the mass of the product is termed by \( m \).

The dropping process of a system is built with the nonlinear mass-spring system, as shown in Figure 6. Here, \( h \) denotes the drop height while \( t \) is the thickness of the cushion. In the collision with the ground, the...
cushion material is compressed and shows a dynamic compression process.\(^8\)

In this study, closed-cell foam is used as cushion material in the presented analysis due to its good cushioning property. The relationship between stress and strain is fitted by Matlab and the data from Instron 5566 static compressor, as shown in Figure 7, where the cushion material is placed between two parallel plates. The fitting analytical formula is:

\[
\sigma(e) = 6.7578e - 43.02144e^2 + 119.4211e^3 - 147.45467e^4 + 68.45325e^5
\]  

where \(\sigma\) and \(e\) are the stress and strain of the foam, respectively.

During the impact, we assume the impact energy is consumed by the cushioning material (without energy loss), which makes the cushion material has maximum deformation. Thus, apply the energy balance as:\(^4\)

\[
\frac{mgh}{At} = \int_0^{e_m} \sigma(e)de
\]  

where \(g\) denotes gravity acceleration. \(A\) and \(e_m\) indicate the bearing area of the cushion. \(e_m\) is the peak strain. The integral expression of equation (15) can be better understood from a geometric point of view regarding Figure 8, here, the marked area is related to energy consumed by the cushion \(\int_0^{e_m} \sigma(e)de\), which corresponds to the numerical value \(mg\text{h/At}\).

Thus, impact acceleration is achieved:

\[
a_{\text{max}} = \frac{\sigma(e_m)A}{m}
\]  

It is worth emphasizing that the maximum impact acceleration is compared with the fragility value of the product to assess the damaged possibility of a shock to the product.\(^4\) The fragility value is the critical acceleration of product damage.

**Formulated RBDO**

A packaged product with a mass \((m)\) of 2.5 kg falls freely from the height \((h)\) of 0.6096 m. The cost function is the volume of the cushion should be minimized while the random uncertainty parameter is the thickness \((t)\) of the cushion. In the reliability assessment, the uncertainty based on cushion thickness plays a vital role in protecting the product from damage during product transportation. Its uncertainty is related to the manufacturing of the material itself and has superior importance on system reliability than the other design parameters.

Assuming the random variable follows a normal distribution \(t \sim N(\bar{d}, \sigma^2)\), where the mean value \(\bar{d}\) is the design variable of the system and the standard deviation value is kept as constant during the RBDO. The parameters are detailed in Table 1.

| Physical meaning (unit) | \(m\) (kg) | \(h\) (m) | \(A\) (m\(^2\)) | \(\sigma\) |
|------------------------|------------|-----------|----------------|---------|
| Value                  | 2.5        | 0.6096    | 0.007          | \(3 \times 10^{-5}\) |
The RBDO model contains two highly nonlinear performance functions. One constraint is that the maximum impact acceleration generated during the impact should be less than the G-value of 110 g. The second is that the maximum strain of the cushion cannot exceed the given 0.2. In these two failure modes, the failure probability of the system shall not exceed the target of 3%. In addition, the iteration convergence criterion is set to $10^{-2}$

$$
\text{Min} : z = 0.007 \times \tilde{d}
$$

$$
\text{s.t.} : \quad P(g_1 = a_{\text{max}} - 110g \geq 0) \leq 3% \\
\quad P(g_2 = e_m - 0.2 \geq 0) \leq 3% \\
\quad 0.02 \leq \tilde{d} \leq 0.1, \sigma = 0.00003
$$

The results of RIA, MRIA, and EMRIA are tabulated in Table 2 for this RBDO problem. From Table 2, all approaches presented the same optimal design point as 0.0365. Meanwhile, the minimum objective function or cost function obtained is also the same as [2.5526e-04].

Table 2. Comparative results in MRIA and EMRIA.

|        | Initial (m) | Cost (m$^3$) | Optimal (m) | Iter. | FEs  | Failure pro. (%)$^a$ |
|--------|-------------|--------------|-------------|-------|------|----------------------|
| RIA    | 0.04        | 2.5551e-04   | 0.0365      | 35    | 423  | 0/2.316              |
| MRIA   | 0.04        | 2.5526e-04   | 0.0365      | 17    | 264  | 0/2.528              |
| EMRIA  | 0.04        | 2.5526e-04   | 0.0365      | 4     | 158  | 0/2.762              |

$^a$Probability evaluated by MCS (1st/2nd constraints).

Table 3. Double-check the optimal result.

|        | Failure pro. (%)$^a$ |
|--------|----------------------|
| $\tilde{d}$ | 0/2.762              |
| $0.995 \times \tilde{d}$ | 0/8.264              |

$^a$Probability evaluated by MCS (1st/2nd constraints).

The RBDO model is constructed as:

$$
\text{Min} : z = 0.007 \times \tilde{d}
$$

$$
\text{s.t.} : \quad P(g_1 = a_{\text{max}} - 110g \geq 0) \leq 3% \\
\quad P(g_2 = e_m - 0.2 \geq 0) \leq 3% \\
\quad 0.02 \leq \tilde{d} \leq 0.1, \sigma = 0.00003
$$

The results of RIA, MRIA, and EMRIA are tabulated in Table 2 for this RBDO problem. From Table 2, all approaches presented the same optimal design point as 0.0365. Meanwhile, the minimum objective function or cost function obtained is also the same as [2.5526e-04] under the same initial setting. However, the converged results obtained by the proposed EMRIA method require 4 iterations (Iter.) and 158 function evaluations (FEs), followed by the MRIA method with 17 iterations (Iter.) and 264 function evaluations (FEs), and finally, RIA 35 iterations (Iter.) and 423 FEs required. It is obvious to conclude that the proposed EMRIA is more efficient and faster than MRIA in the whole solution process. Figure 9 presents the iteration history of both methods in this problem. In addition, the failure probability (Failure Pro.%) of two constraints is evaluated by MCS at the optimal solution based on 10$^6$ samples. The outcome results are 0/2.762%, which is almost identical to the allowable failure probability of 3%. It also exhibited the constraint of $g_1$ is inactive while $g_2$ is active.

In Table 3, the design variable (optimal solution) is reduced by 0.5% which causes the failure probability of the system by nearly 10%. It shows that the obtained result is the optimal and correct solution. Therefore, analysis based on the above results, EMRIA could find the correct optimal solution and show good efficiency and accuracy.

**Conclusion**

A novel reliability analysis method of cushioning packaging design named enhanced modified reliability index method (EMRIA) is proposed based on MRIA. The problem is formulated as an RBDO model. The key step of MRIA is to construct the region for MPP search stably and robustly in the inner loop using an innovative active set strategy. This strategy is a strict inequality, which is used to determine whether the current constraint is active or inactive. The active constraint will go into the outer loop to complete structural optimization design analysis, while inactive ones will be discarded. This process is continued to iterate until it stably converges to the optimal solution.

An example of cushioning packaging design was studied. The thickness of the cushion is an uncertain and random parameter, which is related to the manufacturing of the cushion. The results show that EMRIA
could converge the same optimal solution as both RIA and MRIA but has higher computational efficiency than RIA and MRIA. In addition, the optimal solution is verified by MCS based on $10^6$ samples, which shows an agreement with the target reliability. The studied cases prove that the developed method not only has the capacity to solve the uncertainty in the cushioning packaging design but also has a better performance than MRIA, in such of efficiency, stability, and accuracy.

**Further work**

In this work, we demonstrated that the proposed EMRIA has the following advantages:

1. The proposed EMRIA can be applied to solve the RBDO problems in practical engineering, which can achieve a balance between cost and quality.
2. The proposed EMRIA is suitable for testing highly nonlinear systems and performs well.
3. The proposed EMRIA has a prominent characteristic in both efficiency and stability in comparison with MRIA.

The disadvantage of the proposed method is that the conclusion is not robust. A small disturbance of the design variables will cause a greater failure probability of the system. In Table 3, the design variable (optimal solution) is reduced by 0.5%, the Failure Pro.% is close to 10%. It can only show that the obtained result is the optimal and correct solution but not robust. In response to this important issue, we are currently investigating further.

The future work of this paper is as follows,

1. The research will focus on large and complex engineering field designs of RBDO with multiple design variables and constraints to test the proposed method.
2. The research will further explore the efficiency of the proposed method compared with the other reliability method-based RIA and PMA.
3. The research will further verify whether the proposed EMRIA applies to other cushioning materials such as open-cell cushion materials.

**Declaration of conflicting interests**

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**Data availability statement**

Data will be made available on reasonable request.

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