A planetary mill modelling in chaotic mode

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Abstract. Consider a random walk in a regular m-polygon inscribed in the unit circle. The movement of the point to a randomly selected vertex determines the new position of the point. The point passes a prescribed distance and stops. For such a process, there is a limit distribution of the probabilities of the position of the point. This algorithm was used for modelling different processes. Now we apply it for high-energy ball milling. The impact frequency spatial distribution calculated with the stochastic model simulation has coincided with the distribution predicted by the distinct element method. The real granulometric analysis also can be explained by the model.

1. Introduction
The planetary ball mill processing has been modelled within decades. We can point two main branches of investigations: analytical and numerical modelling. Both are aimed to study the dependence between numerous grinding parameters. In order to fully understand the grinding process, the following issues should be considered:

- the nature and quantification of the interactions between the ball and powder, including a complete description of the kinetics of the ball during grinding;
- deformation phenomena arising from the interaction of one powder particle with a ball;
- physical phenomena occurring inside the powders.

The analytical approach gives the most reliable results but small quantity of parameters can be included. The numerical methods are the power instrument of investigation of relations between parameters, but does not give clear understanding of processes. To illustrate, a numerical dynamic-mechanical modelling indicates just a correlation between the grinding parameters and the resulting microstructure of the ground material.

In work [1], supplementing the results of the EURAM project implemented by P. Le Brun [2], the kinetics of the balls was investigated. The calculated trajectories were compared with the trajectories observed with the high-speed camera. The authors introduced into consideration the angular velocity ratio $R = \omega/\Omega$ of the vials a. v. $\omega$ and the sun disc a. v. $\Omega$, which are rotating in opposite direction. It turned out that the introduced parameter determines the types of the motion of the balls in the mill. The authors identified three different modes and defined the parameter value limits for them. For $R < R_{\text{limit}}$ the mode was termed “chaotic”, for $R_{\text{limit}} < R < R_{\text{critical}}$ – “impact-friction” mode, and for $R > R_{\text{limit}}$
– “friction” one. It is for the chaotic mode that the most effective grinding process is observed. However, numerical analysis becomes impracticable. Moreover, the authors noted a strong discrepancy between the results of numerical analysis of trajectories and observed ones.

In [3] a numerical dynamic-mechanical model of a planetary ball mill was developed to study the dependence of the process efficiency on grinding parameters, such as the size and number of the ball, the jar geometry and the velocity of rotating parts. Modeling indicates a correlation between the grinding parameters and the resulting microstructure of the ground material. Analogously, the maximum efficiency of the grinding process is observed with the most disordered movement of the ball, which is achieved in a well-defined range of the parameter \( R \).

In the paper of Gy. Kakuk et al. [4] also the relationship between the ratio of the angular velocity of the sun disk and the containers and the geometrical parameters of the mill were considered. Having studied the relationship between the model created for the grinding process that occurs in a planetary ball mill and the grinding parameters that depend on the mill, and using the calculations performed, the authors obtained more reliable than the previous data on the energy transmitted to the mill.

In the article of Abdellaoui M. and Gaffet E. [5], on the basis of kinematic modeling of a planetary ball mill, kinematic equations are given that determine the velocity and acceleration of the ball. A comparison of those calculated with some experimental results documented in the literature shows that neither the impact energy nor the frequency of the impact, taken separately into account, do not affect the grinding result, but their product has the determining value.

2. Simulation

In the study of the authors [6], a new stochastic model of a planetary mill based on the concept of random walk on a polygon was proposed. This model was designed for chaotic mode of grinding.

Consider a regular \( m \)-polygon inscribed in the unit circle. Denote by \( z_0, z_1, \ldots \) the wandering points. The position of the point at each subsequent moment of time is determined by the result of the point moving to a randomly selected vertex (the probabilities of selecting vertices are the same), so that the point passes the fraction \( \alpha \) of the distance to the selected vertex and stops, etc. This recurrent algorithm is written in the form of an autoregression equation of the 1st order.

\[
z_n = (1 - \alpha)z_{n-1} + \alpha \epsilon_n, \quad n = 0, 1, \ldots
\]

For such a process, subject to the condition \(| 1 - \alpha | < 1\), there exists a limiting probability distribution of the position of a point, regardless of the initial position. For example, if we consider a regular triangle and \( \alpha = 1/2 \), then the limit set will be the Sierpinski triangle. In any case, the limit set is a fractal figure. Note that the recurrence algorithm itself allows modifications that allow it to be used for modeling many real processes, for example, in financial mathematics [7].

The described stochastic process is applicable to the description of the distribution density of grinding element impacts in a planetary mill with continuous feed.

Here are the basic assumptions:

- the movement of one grinding element is considered in projection on the base of the cylindrical body of the mill;
- we consider the movement of the element to be straightforward before the impact (stop);
- the direction after the impact will be considered a normally distributed vector in a regular polygon with a large number of vertices (in our numerical experiments, it is 1000);
- for the primary model, we assume that the path length \( \mu \) for a particle has a fixed length in fractions of the drum radius;
- the individual trajectory of the wandering of the particle will be considered common to all homogeneous grinding elements;
- the density of distribution in the drum of the number of particle stops is taken as the density of collisions of a collective of particles.
The first assumption means that we use two-dimensional model of the process, which is a typical assumption in the majority of models. The third assumption reflects the chaotic nature of the process.

Note that the path length, of course, depends on many factors – the density of the material being dispersed, the mass of grinding elements, the size of the mill, the speed of rotation of the mill, etc. After constructing the primary density distribution model, an adjustment should be made to the law that determines the path length of the particle.

3. Result and analysis

The simulation of the stochastic process described above was carried out using the “FractalDemo” program created by O.V. Rusakov.

We set the following parameters: \( m = 1000 \), the number of steps for calculating densities is from 8 to 30 million, depending on the path length \( \mu \). The duration of the numerical experiment did not exceed 1 minute. Below are the results of a numerical experiment for calculating the particle impact frequency distribution in the cross section of a mill drum (figure 1). The color scale is from low (green) to large (red) frequency.

As you can see, even with a path length of 0.4 radii and above, a uniform distribution of particle collisions over the sectional area is observed. Apparently, this is the threshold beyond which the model describes the behavior of the grinding elements when they are low in the substance being dispersed.

For small runs, a fundamentally different picture of the density distribution is observed: symmetrically, with increasing density towards the center of the drum. With a path length less than 0.1 radius, the simulation gives almost zero density at the edge of the drum, which corresponds to the friction-mode.

If we introduce into the model the dependence of the path length on the density of the grinding elements in the cross section of the drum, then this effect of uneven frequency distribution will increase significantly, because the length of the run (before impact) outside the red zone will be longer than in the red one. Thus, it can be assumed that due to the limited residence time of the dispersible substance in the attritor, a part of the substance will experience less impact from the grinding elements. The particle size analysis of the material obtained as a result of grinding shows the presence of two pronounced peaks in the histogram of the distribution of particle sizes \([6]\). What can be explained by the model we built.

To overcome this effect, an interesting innovative jar design has been proposed in work \([8]\). A flat wall portion was inserted halfway between the original curved wall and the axis of the jar, resulting in an half-moon (HM) cross-section. HM jar produces an homogeneous size distribution powder for a wider range of parameter \( R \).

The DEM (distinct element method) modelling has been used in \([9]\) to analyze the planetary ball mill. The fact that DEM modelling is suited to the analysis was first realized by Mishra et al \([10]\) and was used in horizontal tumbling mills (see also \([11]\)). The computer simulation incorporated a modified Kelvin viscoelastic spring/damper system. The collision spatial distribution was analyzed. The modeling was conducted for different values of the filling fraction.
\[ n_v = \frac{N_b}{N_{b,\text{max}}}, \]

where \( N_b \) is the number of balls included in the vial and \( N_{b,\text{max}} \) is the maximum number of balls that could be included in the vial. Figure 2 shows the spatial distribution of impact frequency for a range of vial filling fractions \( (n_v = 0.26, 0.42, 0.63, 0.84) \). The computational simulation in [9] were correlated with experimental measurements. Comparing the results of numerical analysis and the impact frequency distribution obtained by us, we can conclude that stochastic analysis effectively simulates the chaotic regime of a planetary mill.

The apparent discrepancy in the structure of the collision density distribution is explained by the initial choice of a fixed impact-free path length \( \mu \). The next step in the computational procedure should be the adjustment of this parameter \( \mu \) depending on the obtained distribution. This process converges and is likely to produce a result similar to [9].

4. Conclusions

The analytical solution to the problem of a collision of a large number of particles is difficult. The DEM method requires huge resources for a relatively small number of particles. Real 3D simulation of collisions becomes quite difficult if the number of particles exceeds a thousand. In our proposed stochastic model, we completely remove restrictions of this kind, replacing the collective behavior of particles with the chaotic motion of one particle, setting only the parameters of the random walk.

One of the difficult technical problems, the solution of which has not been found up to the present, is the scaling of existing samples of attritors. The production of scale models is not justified in terms of costs. Mathematical modeling actually remains the only available method for solving this problem. One of the characteristics for a continuous type mill is the ratio of the length of the drum and its radius. If the effect of the separation of a dispersible substance found by us is stable along the drum, then this problem should be solved by the method of separation rather than by changing this parameter. The question of stability, in turn, requires the construction of an additional model in projection onto the generator cylinder.

We list only a few large unsolved problems: controllability of grinding parameters and the problem of design scaling. It is assumed that both problems can be solved if an adequate model of the grinding process as a whole is created. This task is far from being solved today. Significant results were achieved neither on the path of analytical, nor on the path of computer modeling. A fundamental obstacle to the practical verification of many of the obtained theoretical extrapolating results is the high cost of the industrial design.

Our approach may perhaps complement existing models. We obtained results that are in good agreement with the numerical analysis in other models. By varying the path of particles between impacts, we can also simulate different modes of operation of the mill. The integration with other numerical modelling procedures may be essential to achieve a more complete understanding of the processes.
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