Probing Quantum Aspects of Gravity

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Abstract

We emphasize that a specific aspect of quantum gravity is the absence of a super-selection rule that prevents a linear superposition of different gravitational charges. As an immediate consequence, we obtain a tiny, but observable, violation of the equivalence principle, provided, inertial and gravitational masses are not assumed to be operationally identical objects. In this framework, the cosmic gravitational environment affects local experiments. A range of terrestrial experiments, from neutron interferometry to neutrino oscillations, can serve as possible probes to study the emergent quantum aspects of gravity.

1 Introduction

It should not at all be surprising that despite the usual “forty orders of magnitude argument” against a feasible quantum gravity phenomenology certain quantum aspects of gravity can be probed terrestrially. Just as the exceedingly small cross sections for neutrinos at accessible energies, and the anticipated life time of proton which hugely exceeds the present age of the universe, have not discouraged a robust, and a highly rewarding, program of high energy phenomenology; similarly, it is now emerging that the mere smallness of the coupling associated with the gravitational interactions of elementary particles, or the smallness of the Planck length in comparison, say, with the size of atomic nuclei, does not prevent an experimental quantum-gravity phenomenology program that probes quantum gravity from its low energy limit to all the way up to Planck scale [1,2].

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In fact, about a decade ago Audretsch, Hehl, and Lämmerzahl [3] argued as to why matter wave interferometry and quantum objects are fundamental to establishing an empirically-based conceptual framework for quantum gravity. A few years later, a specific aspect of those ideas was considered by Viola and Onofrio [4]; and subsequently, in greater detail, by Delgado [5]. Concurrently, one of us has pursued this subject in the context of neutrinos, and for other systems modeled after neutrinos [6,7], while several other authors have made significant progress in understanding these effects in the context of flavor oscillation clocks and Brans-Dicke theory [8]. Independently, in early 1999, Amelino-Camelia was to falsify the impression that quantum-gravity induced space-time fluctuations [9] are beyond the scope of terrestrial probes. The Amelino-Camelia argument is fast becoming a classic and the reader is referred to Refs. [1] for its origins. The astrophysical neutrinos, and gamma ray bursts, are also being viewed as probes of certain quantum-gravity aspects of space-time [10]. The general point to made, therefore, is that experimental and observational techniques have reached a point where various quantum aspects of gravity can now be probed terrestrially. The results reported here and those found in [1], if confirmed, could lead to first experimental signatures indicating a profound difference between the classical and quantum aspects of gravity.

The abstracted thesis lies in the observations:

A. In contrast to the simplest local $U(1)$ gauge theory coupled to electrically charged spin-1/2 matter, i.e. theory of quantum electrodynamics, quantum gravity is not endowed with a super-selection rule prohibiting a linear superposition of different gravitational charges.

B. For quantum objects in a linear superposition of different mass (or, energy) eigenstates, there is an operational limit on the fractional accuracy beyond which quantum measurement theory forbids any claim on the equality of the inertial and gravitational masses.

Since we wish to stay as close to the terrestrial experiments as possible we present our arguments in the context of the weak gravitational fields. In particular, in order that a possible violation of the equivalence principle can be studied without theoretical prejudices we will treat gravitational interaction on the same footing as any other interaction. This means that any changes in the clocks and rods will be manifestly dependent on the gravitational environment. In the event there is no violation of the equivalence principle, the affect on rods and clocks would reduce to the expectations based on general relativity.
2 Weak-field limit of quantum gravity and absence of a superselection rule

The 1975 experiment of Colella, Overhauser, and Werner (COW) on the gravitationally induced phases in neutron interferometry established the weak-field limit of quantum gravity for the non-relativistic regime to be \[ -\left(\frac{\hbar^2}{2m_i}\right) \nabla^2 + m_g c^2 \Phi \] \[ \psi(\vec{x}, t) = i\hbar \frac{\partial \psi(\vec{x}, t)}{\partial t} \] (1)

where \( \Phi \) is the dimensionless gravitational potential. In the kinetic term, \( m_i \) is the inertial mass of a particle, and in the interaction term, \( m_g \) is its gravitational mass. To be precise, the 1975 COW-experiment established this equation for neutrons and verified the equality, \( m_i = m_g \) for neutrons, to an accuracy of roughly 1%. Since then these measurements have become more refined. Among the most notable confirmations of this result are by a recent atomic-interferometry experiment at Stanford [12], and by an experiment at Institute Laue-Langevin using a new type of neutron interferometer [13]. The Stanford experiment has established that a macroscopic glass object falls with the same gravitationally-induced acceleration, to within 7 parts in 10^9, as a cesium atom in linear superposition of two different energy eigenstates. The experiment at Institute Laue-Langevin indicates that a statistically significant anomaly observed in the silicon single-crystal interferometer [14] can be excluded by more than one standard-deviation. Clearly, one eagerly awaits more data from Institute Laue-Langevin to unambiguously interpret the anomaly reported in Ref. [14]. If the theoretical framework presented in this Letter turns out to be correct, we should expect the Stanford experiment with a significant improvement in its already-impressive accuracy to see a difference in the gravitationally-induced accelerations of the classical and quantum objects.

2.1 Terrestrial Gravitational Environment

From the perspective of terrestrial experiments, \( \Phi \) contains two important contributions: \( \Phi(z) \), and \( \hat{\Phi} \). The former has Earth as its source, and the latter

\[1\] In the context of this equation, a knowledgeable physicist wrote to one of us (DVA) “... quantum gravity is the theory of the quantum properties of the gravitational field, not the study of the properties of quantum matter in a gravitational field. You work in a different context than quantum gravity: the context of quantum matter interacting with classical gravity. Isn’t it?” Yes, and yet if experiments found that quantum matter interacting with classical gravity carries unexpected features, then the envisaged theory of quantum gravity cannot remain immune to such a development. It is in this realm of “quantum gravity” that this Letter is set.
arises from the cosmic distribution of matter. As a study of the planetary motions reveals, and as simple calculations confirm, \( \hat{\Phi} \) is essentially constant over the dimensions of the solar system. While in the context of the theory of general relativity it is difficult for local observations to detect \( \hat{\Phi} \), a \( \hat{\Phi} \) acquires local observability through a violation of the equivalence principle. A violation that, we shall show, is inherent to quantum gravity.

Explicitly, under the boundary condition that \( \Phi(\infty) \) vanishes at spatial infinity, \( \Phi(z) \) is given by:

\[
\Phi(z) \approx -7 \times 10^{-10} \frac{R_\oplus}{R_\oplus + z}
\] (2)

where \( R_\oplus \) is radius of the earth, and \( z \) is the vertical distance from the surface of the Earth. The remark on \( \Phi(\infty) \) also implies that we shall restrict to the “local quasi-Newtonian” neighborhood (LQNN)

\[
\text{LQNN : } R_{\text{Milky Way}} \ll R_{\text{LQNN}} \ll R_{\text{Hubble}}
\] (3)

where \( R \) refers to dimensions of the indicated region. It is not surprising that an estimate of \( \hat{\Phi} \) is a more involved procedure. Towards that end we note that a compilation of roughly three hundred galaxy clusters shows that the super-clusters they define are in a lattice-like distribution with cells 430 million light-years in size [15,16]. The whole distribution resembles a “three-dimensional chessboard” [15,17]. It is the gravitational environment created by such a structure that interests us. For laboratory regions much smaller than the lattice size, a phenomenological description of the gravitational environment contains, (a) a potential gradient that is annulled (for an observer at rest inside the laboratory) by the gravitationally induced acceleration of the laboratory under consideration, and (b) a non-acceleration inducing constant gravitational potential. To see this clearly, imagine all the LQNN-matter uniformly distributed around the region of interest with a spherical symmetry. In this approximation, there are no gradients in the gravitational potential and the effect of the local cosmic distribution of matter can be phenomenologically replaced by a constant gravitational potential. It is also to be noted that this still assumes, \( \Phi(\infty) = 0 \), and only those contributions are included in \( \hat{\Phi} \) for which the Newtonian condition \( \Phi \ll 1 \) is still valid. To obtain an estimate for the contributions to \( \hat{\Phi} \), we note that a local super-cluster known as Shapley Concentration (SC) has a mass around \( 10^{16} \) solar masses, and it is at a distance of about 700 million light-years from us. This information yields:

\[
\hat{\Phi}_{\text{SC}} \approx -2 \times 10^{-6}
\] (4)

Similarly, the Great Attractor’s (GA) contribution to \( \hat{\Phi} \) is [18]:
\[ \Phi_{GA} \approx -3 \times 10^{-5} \]  

(5)

In the presence of a non-zero cosmological constant these considerations can be modified in a well defined manner \[19\].

In the above scenario

\[ |\hat{\Phi}| = |\hat{\Phi}_{GA}| + |\hat{\Phi}_{SC}| + \cdots \gtrsim 3.2 \times 10^{-5} \]  

(6)

where the unspecified contributions contain the influence of the entire LQNN. The total \( \hat{\Phi} \) must necessarily involve general-relativistically defined and calculated contributions from the entire cosmic matter. The purpose of expression (6) is only to serve as an existence argument for, and a lower bound on, \( |\hat{\Phi}| \).

It is to be noted that

\[ |\hat{\Phi}| / |\Phi(z = 0)| \gtrsim 0.5 \times 10^5 \]  

(7)

Despite the fact that \( |\hat{\Phi}| \) exceeds \( |\Phi(z = 0)| \) by five orders of magnitude, the observability of \( \Phi(z) \), as seen for example through the lunar orbit, arises because

\[ |\vec{\nabla}\Phi(z)| \gg |\vec{\nabla}\hat{\Phi}| \]  

(8)

The insensitivity of the planetary orbits to \( \hat{\Phi} \) resides in its essential constancy over the solar system and in the validity of the equivalence principle for classical objects to a high precision \[20,21\]. The \( \hat{\Phi} \) does contribute to galactic motion.

In the context of the solar neutrino anomaly, and a violation of the equivalence principle, we shall show that \( \hat{\Phi} \) acquires a local observability. In general, in any quantum gravity phenomenology which does not \textit{a priori} exclude a violation of the equivalence principle (VEP) as a possibility, \( \hat{\Phi} \) can carry significant physical consequences. In fact, we shall show that if the inertial and gravitational masses are considered as operationally distinct objects, quantum gravity carries an inherent quantum induced violation of the equivalence principle (qVEP).

2.2 \textit{Flavor-oscillation clocks as probes of qVEP and \( \hat{\Phi} \)}

Having once defined the gravitational environment of interest, we now remind the reader that the standard text-book \[22\] understanding of the neutron interferometry is based on the Schrödinger equation (1) with \( \Phi \) replaced by
\[ \Phi(z) \]. Such a treatment completely ignores all possible effects due to \( \Phi \). This is entirely justified for a single mass eigenstate in a non-interferometry setting (and without a VEP). However, for states that are in linear superposition of different masses, or, more generally, energies, a quantum-gravity phenomenology that wishes to study any possible violations of the equivalence principle cannot \textit{a priori} ignore \( \Phi \) \cite{23,24}.

In particular, if one allows for a violation of the equivalence principle, under the correction\(^2\)

\[ \Phi(z) \rightarrow \Phi(z) + \hat{\Phi}, \]  

equation (1) is not invariant for states that are in a linear superposition of different energy, or different mass, eigenstates. The argument that establishes this is simple, but not trivial. Therefore, the reader’s attention is invited to the details. We construct two flavor states at a time \( t = 0 \)

\[
|\alpha\rangle = \cos \theta |m_1\rangle + \sin \theta |m_2\rangle \\
|\beta\rangle = -\sin \theta |m_1\rangle + \cos \theta |m_2\rangle
\]

Let each of the mass eigenstates, \( |m_1\rangle \) and \( |m_2\rangle \), be at rest. The idealized frame of observations is chosen to be such that the only gravitational potential present is \( \hat{\Phi} \). Then, the probability amplitude for a flavor oscillation from the state \( |\alpha\rangle \) to the state \( |\beta\rangle \) at a later time, \( t > 0 \), is

\[ A_{\alpha \rightarrow \beta}(t) = \langle \beta | \left\{ \cos \theta \exp \left[ -i \frac{(m_1^2 + m_2^2 \Phi) c^2 t}{\hbar} \right] |m_1\rangle \\
+ \sin \theta \exp \left[ -i \frac{(m_1^2 + m_2^2 \Phi) c^2 t}{\hbar} \right] |m_2\rangle \right\} \]

(11)

The superscripts “i” and “g” on “\( m \)” refer respectively to inertial and gravitational masses.

We draw attention to the fact that each of the mass eigenstates picks up a different, mass-dependent, gravitationally induced phase from \( \hat{\Phi} \). In consequence, these relative phases become experimentally observable. This observability is in the usual sense in which one measures red shifts as, e.g., done by a comparison of clocks located in regions characterized by a different \( \hat{\Phi} \). However, if the principle of equivalence is violated, then \( \Phi \) acquires a local observability.

\(^2\) This does not constitute a transformation of changing \( \Phi(\infty) \) by a constant. Both \( \Phi(z) \) and \( \hat{\Phi} \) have been calculated with \( \Phi(\infty) = 0 \).

\(^3\) See remarks contained in the last paragraph at the end of Sec. 1.
It is precisely the absence of a super-selection prohibiting the flavor states, simplest of which are given by $|\alpha\rangle$ and $|\beta\rangle$, that allows for emergence of the $\hat{\Phi}$-dependent relative phases. Had a super-selection rule prohibited the existence of flavor states, i.e. demanded $\theta = 0$, then the emergent $\hat{\Phi}$-dependent phase would have been a global one, and thus it would have carried no physical observability.

Incidently, as it emerges from the above discussion, a freely falling frame, contrary to the usual assertions, is not necessarily a frame devoid of a gravitational field. Vanishing of the curvature tensor is a necessary, but not sufficient, condition for the absence of gravity.

The probability amplitude (11) yields probability for the flavor oscillation from the state $|\alpha\rangle$ to the state $|\beta\rangle$:

$$P_{\alpha \rightarrow \beta}(t) = A_{\alpha \rightarrow \beta}(t) A_{\alpha \rightarrow \beta}^*(t) = \sin^2(2\theta) \sin^2(\omega_{\alpha \rightarrow \beta}t)$$

where the angular frequency for the flavor oscillations is,

$$\omega_{\alpha \rightarrow \beta} = \frac{c^2}{2\hbar} \left( m_2^i - m_1^i \right) + \frac{c^2}{2\hbar} \left( m_2^g \hat{\Phi} - m_1^g \hat{\Phi} \right)$$

The flavor-oscillation frequency, $\omega_{\alpha \rightarrow \beta}$, consists of two parts. The first is the kinetic part, and the second is the $\hat{\Phi}$ contribution. In a distant planetary system, where the local gravitational environment can be significantly different, the second part would be different. The existence of time-periodicity via $\omega_{\alpha \rightarrow \beta}$ allows us to interpret the system of flavor states as a clock. The flavor-oscillation clocks red-shift via the $\hat{\Phi}$-contribution to $\omega_{\alpha \rightarrow \beta}$. On assuming the equality of the inertial and gravitational masses in this setting, the general-relativistic predicted red shift is reproduced. This can be measured by comparing this clock with another embedded in a distant region with a different $\hat{\Phi}$. So far all we have established is that red-shift of flavor oscillations clocks shows a deep consistency in the quantum mechanical evolution and the principle of equivalence as embedded in the equality of the inertial and gravitational masses.

We now turn to a fundamentally quantum mechanical source of a VEP. In what follows we shall take the view that the inertial and gravitational masses are two independent objects, a view compatible with Lyre’s recent critique of the subject [25]. This view has the further support in that the very operational procedures that define inertial and gravitational masses are profoundly different, and it is more so in the quantum context.

The essential physical idea is that if any one of the two quantities $A$ and $B$ carries an intrinsic quantum uncertainty, $\Delta$, then from an operational point
of view the equality of $A$ and $B$ cannot be claimed beyond the fractional accuracy

$$\frac{\Delta}{(A + B)/2}$$

(14)

We shall consider this general statement in the context of the equality of the inertial and gravitational masses to establish a quantum-induced violation of the equivalence principle (qVEP):

$$m_g = (1 + f) m_i$$

(15)

where $f$ vanishes for objects with a classical counterpart and is non-zero for objects with no classical counterpart. Coupled with result (13), this implies that flavor oscillation clocks carry a local observability for $\hat{\Phi}$. That is, while in the absence of a qVEP (or, even in the absence of a VEP) $\hat{\Phi}$ has no local observability. A qVEP/VEP can make $\hat{\Phi}$ observable locally. This is an important implication of the violation of equivalence principle. Quantum systems are particularly sensitive to a $\hat{\Phi}$ as can be seen by referring to Eqs. (7) and (8), and on taking note of the observations: (a) Classically important gravitationally-induced force in being proportional to $\vec{\nabla}\hat{\Phi}$ carries relatively little physical significance in the present context, and (b) Quantum mechanically important gravitationally-induced relative phases are proportional, apart from the difference in masses of the underlying mass eigenstates, to $f\hat{\Phi}$.

We now immediately note that the very quantum construct that defines the flavor eigenstates does not allow them to carry a definite mass. Therefore, within the stated framework, the equality of the inertial and gravitational masses looses any operational meaning beyond the flavor-dependent fractional accuracy defined as

$$f_\eta \equiv \sqrt{\langle \nu_\eta | \hat{m}^2 | \nu_\eta \rangle - \langle \nu_\eta | \hat{m} | \nu_\eta \rangle^2 \over \langle \nu_\eta | \hat{m} | \nu_\eta \rangle}$$

(16)

Here $\hat{m}$ is the mass operator: $\hat{m}|m_j\rangle = m_j|m_j\rangle$. Explicitly, this yields:

$$f_\alpha = \frac{\sin(2\theta)\delta m}{2 \left( m_1 + \sin^2(\theta)\delta m \right)}$$

$$f_\beta = \frac{\sin(2\theta)\delta m}{2 \left( m_1 + \cos^2(\theta)\delta m \right)}$$

(17)

where $\delta m \equiv m_2 - m_1$. For $\delta m \ll m_i$, $i = 1, 2$, we have the flavor-independent relation
\[ f = \frac{\sin(2\theta)\delta m}{2\langle m \rangle} \]  

(18)

Only for some very specific values of \( \theta \), i.e. for \( 2\theta = n\pi, \) \( n = 0, 1, 2, \ldots \), do the states \( |\alpha\rangle \) and \( |\beta\rangle \) coincide with states characterized by well-defined masses. For such states alone \( f \) vanishes. Otherwise, \( f \) remains non-vanishing.

Once the equality of the inertial and gravitational masses has been compromised, the potential \( \hat{\Phi} \) carries not only observability only in comparison with “distant” systems but it also becomes observable locally.

3 Illustrative Experimental Settings

We now explore how the above obtained qVEP can be studied in some realistic experimental settings.

3.1 Atomic interferometry

Consider two sets, differentiated by the index “\( i \),” of “flavors” for Cesium atoms:

\[ \begin{bmatrix} |^\alpha Ce\rangle_{\xi_i} \\ |^\beta Ce\rangle_{\xi_i} \end{bmatrix} = \begin{bmatrix} \cos(\xi_i) & \sin(\xi_i) \\ -\sin(\xi_i) & \cos(\xi_i) \end{bmatrix} \begin{bmatrix} |E_1 Ce\rangle \\ |E_2 Ce\rangle \end{bmatrix}, \quad i = a, b \]  

(19)

Here, \( |E_1 Ce\rangle \) and \( |E_2 Ce\rangle \) represent two different energy eigenstates of the Cesium atom. The “flavor” states, \( |^\alpha Ce\rangle_{\xi_i} \) and \( |^\beta Ce\rangle_{\xi_i} \), are linear superposition of the energy eigenstates and are characterized by the flavor indices \( \{\alpha, \beta\} \), and by the mixing angle \( \xi_i \). That is, we have two copies of the flavor states similar to the ones introduced in Eqs. (10). Each of the two copies is defined by a different mixing angle.

As observed in identical free fall experiments by a stationary observer on Earth, the flavor-dependent qVEP predicts a fractional difference in the spread in their accelerations to be:

\[ \frac{\Delta a_{\ell\xi_b} - \Delta a_{\ell\xi_a}}{g} \approx \left\{ \left| \frac{\sin(2\xi_b)}{\langle E_{\ell\xi_b} \rangle} \right| - \left| \frac{\sin(2\xi_a)}{\langle E_{\ell\xi_a} \rangle} \right| \right\} \frac{\delta E}{2}, \quad \ell = \alpha, \beta \]  

(20)

where \( \delta E \equiv E_2 - E_1 \), and \( g \) refers to the magnitude of the acceleration due to gravity for a classical object. One may for instance take \( \xi_a = 0, \) and \( \xi_b = \pi/4 \). Then the above expression reduces to
\[
\frac{\left| \Delta a_{\xi_b=\pi/4} \right|}{g} \approx \frac{\delta E}{2 \langle E_{\xi_b} \rangle}
\]  

(21)

where the \( \ell \) dependence has dropped because to the lowest order \( f_\ell \) is same for both flavors [not both sets, see Eq. (18)]. For flavor states of Cesium atoms prepared with \( \delta E \approx 1 \text{ eV} \), this difference is of the order of a few parts in \( 10^{12} \), and should be observable in refined versions of experiment reported in Ref. [12]. How difficult this refinement in techniques at Stanford and NIST would be is not fully known to us. However, the extraordinary accuracy in similar experiments and the already achieved absolute uncertainty of \( \Delta g/g \approx 3 \times 10^{-9} \), representing a million fold increase compared with previous experiments, makes us cautiously optimistic about observing qVEP in atomic interferometry experiments pioneered by the group of Steven Chu at Stanford.

At the same time, a definitive null result for qVEP would establish that the inertial and gravitational mass, despite their different operational definitions, are physically identical objects.

3.2 Solar Neutrino Anomaly

In recent times it has been conjectured that the solar neutrino anomaly may be related to a flavor-dependent violation of the equivalence principle [26–29]. The suggestion, in fact, originally came from Maurizio Gasperini in the late 1980’s [23,24]. After appropriate generalization, and on interpreting the flavor index \( \ell \) to represent the three neutrino flavors (\( \nu_e \), \( \nu_\mu \) and \( \nu_\tau \), the non-relativistic arguments presented so far can be readily extended to the relativistic system of neutrino oscillations. Here we simply provide an outline of this argument.

To estimate the qVEP effects it suffices to restrict to a two-state neutrino oscillation framework. A simple calculation shows that the difference in fractional measure of qVEP turns out to be exceedingly small [30]:

\[
\Delta f_{\ell \ell'} \equiv f_\ell - f_{\ell'} = 6.25 \times 10^{-26} \left[ \frac{(\Delta m^2)^2}{\text{eV}^4} \right] \left[ \frac{\text{MeV}^4}{E^4} \right] \sin(4\theta_V)
\]  

(22)

where \( \ell \) (say, \( \nu_e \)) and \( \ell' \) (say, \( \nu_\mu \)) refer to two different neutrino flavors, and \( \theta_V \) is the vacuum mixing angle between the underlying mass eigenstates (whose superposition leads to different flavors of neutrinos). The difference in the squares of the underlying mass eigenstates, \( m_2^2 - m_1^2 \), has been represented by \( \Delta m^2 \) (in \( \text{eV}^2 \)); and \( E \) is the expectation value of the neutrino energy (in MeV). The qVEP-induced oscillation length is found to be [30]
\[
\lambda_{q\text{VEP}}^{\text{osc}} = \left[ 0.66 \times 10^2 \frac{E^3}{(\Delta m^2)^2 \sin(4\theta_V)\Phi} \right] \times \lambda_{\odot}\tag{23}
\]

where \(\lambda_{\odot} \simeq 1.5 \times 10^8\) km is the mean Earth-Sun distance.

Assuming that the mass and gravitational eigenstates coincide, the oscillation length that enters the neutrino-oscillation probability is not \(\lambda_{q\text{VEP}}^{\text{osc}}\) but \[24\]

\[
\lambda^{\text{osc}} = \frac{\lambda_0^{\text{osc}} \lambda_{q\text{VEP}}^{\text{osc}}}{\lambda_0^{\text{osc}} + \lambda_{q\text{VEP}}^{\text{osc}}}\tag{24}
\]

where \(\lambda_0^{\text{osc}}\) is the usual kinematically induced oscillations length

\[
\lambda_0^{\text{osc}} = \left[ \frac{2\pi}{1.27} \right] \left[ \frac{E}{\text{MeV}} \right] \left[ \frac{eV^2}{\Delta m^2} \right]\tag{25}
\]

For \(\Delta m^2\), the LSND excess event anomaly [31] sets \(\Delta m^2 \simeq 0.4\) eV\(^2\), while the Super-Kamiokande [32] evidence on atmospheric neutrino oscillations suggests another \(\Delta m^2 \simeq 3 \times 10^{-3}\) eV\(^2\). For both of these mass-squared differences, for any set of reasonable parameters, \(\lambda_{q\text{VEP}}^{\text{osc}} \gg \lambda_0^{\text{osc}}\). Consequently, \(\lambda^{\text{osc}} \simeq \lambda_0^{\text{osc}}\), and the qVEP-induced effects is suppressed by the kinematic term.

On the other hand, there is yet no independent confirmation for the LSND result, and the possibility remains open as to whether two of the three underlying mass eigenstates are massless. In that event, the relevant oscillation length associated with qVEP is:

\[
\lambda_{q\text{VEP}}^{\text{osc}} = \frac{2\pi c h}{\Phi (\zeta_{\ell\ell}/E) E} = \frac{2\pi c h}{\zeta_{\ell\ell} \Phi} \sim \left( \frac{\pi}{\zeta_{\ell\ell} \Phi} \right) \times 10^{-33}\text{ km}\tag{26}
\]

where, in the last term, \(\zeta_{\ell\ell}\) is measured in eV, and is defined as:

\[
\zeta_{\ell\ell} \equiv \left| \sqrt{\langle \nu_{\ell}|H^2|\nu_{\ell} \rangle} - \langle \nu_{\ell}|H|\nu_{\ell} \rangle^2 - \sqrt{\langle \nu_{\ell'}|H^2|\nu_{\ell'} \rangle} - \langle \nu_{\ell'}|H|\nu_{\ell'} \rangle^2 \right|\tag{27}
\]

This oscillation length has no explicit energy dependence, except that carried via \(\zeta_{\ell\ell}\). The \(\zeta_{\ell\ell}\) measures the difference in the energy-widths of the wave functions of the the two involved flavors.

If this was to be a solution to the solar neutrino anomaly, it requires that \(\lambda_{q\text{VEP}}^{\text{osc}} \sim \lambda_{\odot}\), i.e.

\[
\zeta_{\ell\ell} \Phi \sim 2 \times 10^{-11}\text{ eV}\tag{28}
\]
This requirement tells us that the wave functions of the two involved flavors carry the same width to a very high precision.

This solution to the solar neutrino anomaly requires a non-zero $\hat{\Phi}$, and shows this to be locally observable.

In terms of the $E^n$ dependence, the kinematic oscillation length carries $n = 1$, the Gasperini-conjectured VEP for massless neutrinos has $n = -1$, and the qVEP-induced oscillation length for the massless neutrinos has $n = 0$ (cf. $n=3$, for massive qVEP).

Therefore, as more data on the solar neutrino flux becomes available we should be able to distinguish between the various mechanisms. Of course, a possibility that more than one mechanism is at play in reality should not be ignored.

4 Concluding Remarks

Several new theoretical insights, and an outline for new experimental proposals, emerge in this Letter. First, we noted that the super-selection rule that prohibits the linear superposition of states with different electric charges, has no counterpart in the realm of gravitational interactions. The absence of this super-selection rule endows the cosmic gravitational potentials in quantum gravity with a much more visible physical status, without altering the classical results. Under the assumption that the inertial and gravitational masses are operationally different objects, we showed that since flavor states carry an inherent quantum uncertainty in their masses (or energies) the equality of the their inertial and gravitational masses loses operational meaning beyond certain fractional accuracy. We used this fact to make a prediction on the free fall of states that are in a linear superposition of different energy eigenstates. That prediction can be tested in the new generation of atomic interferometry experiments. Furthermore, we established that a qVEP can also have significant implications for the solar neutrino anomaly. Elsewhere we shall sketch how the Schrödinger’s SQUID [35]., i.e. a superconducting quantum interference device with a linear superposition of macroscopic counter-propagating supercurrents, could serve as a sensitive probe of Earth’s gravitomagnetic field.

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the dominant contribution is helicity independent. Whereas for photons, a
similar calculation yields a helicity dependent effect [33]. It appears to one of
us (DVA) that these apparently different helicity dependences arise from lack
of realization by the authors of this work that Majorana spinors form a bi-orthonormal set of spinors and that their usual norm is identically zero (even for massive particles) while their “bi-ortho norm” is imaginary definite. These aspects of Majorana spinors can be found in ref. [34].

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