Design research on calculus: Students’ journey in learning definition of definite integral

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Abstract. Some researchers report that students find difficulties in learning definition of definite integration. Therefore, classroom activities designed with design research method to help university students gain better understanding on this topic. In order to support students, realistic mathematics approach was chosen. Series of instructional activities had been designed to reach mathematical goal. Thirty of first year university students in Universitas Negeri Padang were chosen to implement the classroom activities that have been designed. This paper will report students’ journey in learning definition of definite integral in our local instructional theory (LIT). The research shows that students understanding about area, function, inscribed/circumscribed polygon, limit play important role to understand the definition of definite integral. Students need to see those concepts as a unit which cannot be separated to understand the definition of definite integral in a designed classroom activity.

Keywords: Definition of Definite Integral, Design Research, RME, Local Instructional Theory.

1. Introduction
One of the cores of mathematics is Calculus. It plays an important role in secondary and tertiary education [1]. It uses as a foundation for students in university especially majoring engineering, natural science, and mathematics. It has three major topics and one of them is Integration. However, many studies such as Mahir [2], Yadav [3], Torner [4] Jones [5] and Tasman [6], [7] found that students have difficulties in understanding concept of integration in calculus. Serhan [8] and Torner [4] found in their research that students had narrow understanding on topic of definite integral. Their competencies to represent the concept in different ways were limited; they dominant knowledge is the procedural one. These situations are also happened in Universitas Negeri Padang (UNP). Lack of understanding of integration concept consider as one of the causes which make students fail in this university [6]. Most of UNP Students might be able to perform integral calculation. But they do not know about the meaning of their calculation. For instance, students are able to count the integration of $x^3$ from zero to two which give them result 4. However they do not know what the meaning of the ‘4’ that they found. This happened because most of them tend to focus on the integrating procedures without building their understanding on the concept of integration [7]. Tasman [9] in his research suggest that educators should not rush to define the procedure without a deep understanding of the concept. Therefore, it is necessary to design classroom activities to support students’ gain better understanding on definite integral.

Technology has its role in designing activity. It has been used to improve capability and success in teaching and learning [10]. The utilization of technology in learning can be represented as the use of software such as Geogebra. Geogebra can be used in learning especially learning mathematics. It
provides dynamic visualization that can be used to build students understanding. Many research show that this software is powerful software in mathematics. It is not only powerful in geometry and algebra [11], [12] but also in calculus, [13].

Learning definition of definite integral needs some concept such as Sigma, Riemann Sum, Limit and Area [14]. However, students found it is difficult for them to understand about those concept and they cannot see the relation among those concepts [6]. Therefore, we interested to design a series of integrated classroom activities using Geogebra software to create a path for students as a road for their journey in understanding the definition of definite integral in order to help them understand the connection amongst sigma, Riemann sum, limit and area in the definition of definite integral.

To support students, realistic mathematical approach (RME) was applied in this research. This approach has five tenets [15]. They are (1) constructions stimulated using concreteness which means it does not begin with formal level but it starts with a situation which is real for students. (2) Mathematical tools are developed from concreteness to abstraction. This tenet connects concreteness to a more formal level by using symbols models, or graphic. (3) Stimulating free production and reflection by students because it assumed meaningful for them. (4) Stimulating social activities of learning which create a good interaction. Classroom social activity provides students opportunity to have interaction between each other. It will trigger them to discuss ideas and to have a rich discussion to solve problems. (5) Intertwining learning strands to get mathematical idea structured. The definition of definite integral has solid relation to Sigma, Riemann Sum, Limit and Area.

2. Design and Method
Design research methodology was applied to this research. There were three reasons to chose this methodology in which in line with [16]; First, this methodology gave productive views to develop theory. Second, it had specific usefulness of its outcomes. Third, it directly made the researcher in the refinement of mathematics education. Five characteristics of design research [17] were applied to this research. Design research itself consists of three phases. They are;

2.1. Design and Preparation
This stage was focus on determining the goals of mathematical learning and it was mixed with anticipatory thought on how to reach the goal. It produced conjectures which consist of three things; Students’ learning goal, Designed instructional activities and Conjectured of learning processes that anticipates how students' thinking and understanding could evolve when instructional activities used in classroom [18].

2.2. Teaching experiment
Instructional activities were tried and data collection explained how a set of the instructional activities could work. Afterward the instructional activities were revised on daily basis to create a well-considered and empirically grounded local instructional theory (LIT).

2.3. Retrospective analysis
Data collected analyzed and the hypothetical learning trajectory (HLT) compared with students’ actual learning in the classroom. This stage formed a new cycle in the emergence of LIT.

2.4. Data Collection
Data such as video recording, students’ works, and field noted were collected during the teaching experiments. Videotape of the activities and interview with some students were taken. Then, the data were analyzed to improve our HLT.

2.5. Data Analysis
The HLT and students’ actual learning were compared using video recording. The whole video recording was watched and was looked for fragment in which students learned or did not learn what this
research conjectured in HLT. The unexpected situations were taken into consideration. Afterward, the selected fragment was registered for a better organization of the analysis. The part that not relevant with the students’ learning was ignored.

The selected fragments were transcribed and the analysis would start by looking at the short conversation and students’ gesture in order to make interpretation of students’ thinking process. The interpretation was discussed with other researchers.

Other data that also used was students’ interviews in order to improve the validity of the research (data triangulation). After that for the second opinions of the analysis, the interpretation was asked to the expert in order to analyse intensively and to improve the analysis itself. Finally, the conclusion would be drawn based on the retrospective analysis.

2.6. Validity and Reliability

The validity and reliability concern about the quality of the data collection. The data were collected throughout the learning activities. To guarantee the internal validity of this research, this research used many sources of data, namely video recording of classroom observation, calculus lecturer’ interview, students’ interview and students’ work. Having these data, allow this research to conserve the triangulation so that we could control the quality of the conclusions which made it become reliable. The research was conducted in a real classroom setting, therefore could guarantee the ecological validity – a form of validity in research study where the methods, materials and setting of the study must approximate the real-life situation that is under investigation.

3. Finding and Discussion

The learning trajectory is defined as a description of the process of students’ activities in learning that they can follow to build their understanding on definition of definite integral. It examines the goal, activities and conjecture of learning processes. This trajectory is hypothetical because interpretations, ideas and strategies of students never sure until the students really work on problems. The brief overview HLT of definition of definite integral showed in Table 1.

Table 1. Overview of HLT

| Name of Activity | Students’ Activity | Goal of Learning | Math Idea | Strategy |
|------------------|--------------------|------------------|-----------|----------|
| What is the meaning of our calculation? | Using their knowledge about Sigma, function, Area, Riemann Sum and Limit to understand definition of definite integral. | • Students understand about Area | • Uniting, Area can be counted using unit with different plane region. | • Counting area of plane region using some units by using students understanding about function. |
| | | • Students are able to apply sigma properties in counting area inscribed and circumscribed polygon. | • Relating function and Sigma properties in counting area of inscribed and circumscribed polygon | • Using sigma to count inscribed and circumscribed polygon |
| | | • Students understand about the different between inscribed polygon and circumscribed polygon. | • Differences between area inscribed polygon and circumscribed polygon | • Using geogebra software to gain insight about definition of definite integral. |
| | | • Students are able to count the area under the curve using inscribed and circumscribed polygon. | • Understand about the consequences of taking infinite | • Trying to connect formal definition |
| | | • Students are able to connect area, sigma and | | |
limit and definition of definite integral

partition of the base in counting area.

the activities.

- Counting infinite limit
- Introducing formal definition of definite integral

The lesson began with explanation about the classroom setting. RME approach was applied in our classroom in which students were given some problems on three students’ worksheets and they discussed the problems that have been design in the preparation phase in a group as shown in figure 1. Afterward the classroom discussion held. Students were very enthusiastic to discuss and shared their idea with friends in group.

Figure 1. Students discussed in their group

The journey started with understanding area. Students were asked to determine the area of some plane region as shown in figure 2.

Figure 2. Problems about area

In those problems we expect our students understand about two aspects. They are (1) unit to count the area, (2) approximation using any unit. Those two aspects are crucial to students. It will be used as a base for understanding area under the curve. On one hand, classroom experiment shows that some students do not understand about unit, most of them tend to answer directly with number without any unit. On the other hand, they are able to understand that they can approximate the area using any unit.
Overall, there is not difficult to students to understand the two aspects that we expect to the classroom activity.

Obstacles started with mathematical communication. The students were asked about the way how to count area under the curve of $x^2$ which are bounded with the x-axis and $x = 2$. Most of students tend to use their prior knowledge about calculating integral as shown in the figure 3. Students directly did mathematical calculation without aware with the question “how” which need explanations to discuss the steps. In this case the students are expected to relate with their knowledge about area. They expected to aware about the need to approximate with a certain unit and sum up the entire unit. Based on interviewed with some students, we found that students mastered on procedural knowledge without deep understanding about that knowledge.

![Figure 3](image1.png)

**Figure 3.** Student’ answers about the problem

In the classroom discussion students were asked whether their answers match with question ‘How’. Then suddenly they realized that they need to shows the steps to count the area. They started with approximate the area using some unit. They agreed that they used rectangle to count it. Then the next question brings the next obstacle to them.

Students were asked to divide the base (the x-axis from 0 to 2) into n equal length as shown in figure 4.

![Figure 4](image2.png)

**Figure 4.** Partition of base into n equal part

In the classroom discussion students were asked why they need to divide the base and most of them relate with the previous activity about area which shows our conjecture works. They realize that they need to make a smaller unit in order to get a good approximation. This means understanding the area activity plays an important part in their journey.

It was difficult to some students to understand about the length of one part of the segment is $\frac{2}{n}$. The following transcribe shown students difficulty that found in the group discussion.

Lecturer : From zero to two, we want to divide the segment into n equal parts. You were asked the length of one part. The length from $x_i$ into $x_{i+1}$ is?

Student : Then it must be $x_i$ minus n sir!

Lecturer : (Repeated the previous explanation)!

Student : n over two or 2 over n (Seem she confused with her answer)
To encounter that situation, the lecturer tries to give the group of students idea as shown in the following transcribe.

Lecturer: If you have a line segment that has 1 unit in length (while drawing a line with zero in initial point and one at the end point). Then I need to divide it into two equal parts. How long is the length of one part?

Students: A half

Lecturer: If I want to divide into three equal parts, four equal parts, and five equal parts, then how long is the length of one part?

Students: One third, a quarter, and one fifth

Lecturer: How do you know that? What is the one on one third and what is the three in one third?

Students: The one is the length of the line and the three is the number of parts that we want to divide.

Lecturer: How do you know the length of a line is one?

Students: The line starts from zero to one, so the length must be one minus zero!

Lecturer: So, you have to know the length of the segment and you have to know how many equal parts do you want to make.

Students: Yes

Lecturer: Ok, if a line from zero to two divide into n equal parts, How long the length of a part of that segment?

Students: Two over n

Then in the classroom discussion one of member of this group shared that idea to others that idea. He said that in order to know the length of a part of segment it must be the end point minus the initial point over n.

The students’ journey continues with understanding the difference between inscribe and circumscribed polygon. In this part, students were gave picture as shown in figure 5.

![Figure 5](image.png)

*Figure 5. Picture to understand the difference inscribe and circumscribe polygon*

Fruitful discussion happened in classroom discussion when the students are asked the meaning of the area formula. In this phase the students have to understand that the high of the rectangular is related with the function of the curve and they have to understand the different between inscribed polygons and circumscribe polygons. In the group discussion, most of students had difficulties to understand this
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figure. However in the classroom discussion, one student can share their idea about the area formula for those two rectangles. Furthermore she was able to differentiate those two rectangles. She said that for the inscribed polygon the high of the rectangle is \( f(x_{i-1}) \) while for the circumscribed polygon the high of the rectangle is \( f(x_i) \) this situation made the area of circumscribed polygon will be larger compared with the area of inscribed polygon.

The next check point for students’ journey in learning definition of definite integral is summing up all rectangles. They have to understand that the length of rectangle was represent by \( f(x_i) \) is equal to \( \frac{2i}{n} \).

They have to use their knowledge about sigma and connect the sigma property. For some students, it was difficult. Figure 6 has shown that students cannot connect the sigma property in inscribed polygon shown in figure 6.

![Figure 6. Student calculation on counting area of inscribed polygon](image)

The student was directly using the property of sigma for \( i^2 \). The student answer on figure 6 was false because the sigma was started from 1 to \( n \), while in this situation starts from zero which is meaningless to \( n-1 \). Therefore, students need to change the \( n \) with \( n-1 \) which give them the correct answered like the red hand writing in figure 6. (The red handwriting is the answer of the student after the classroom discussion happened and the black handwriting is the student’ answer in his/her group discussion)

After students are able to count the total rectangle of inscribed and circumscribed polygon, they need to understand the relation between the formula in term of \( n \) with the area of rectangles. For example, if the \( n=2 \) that means the area of two rectangles. This situation is happen in classroom discussion in which the lecturer asked the students the meaning of their answer. Figure 7 shown student explanation which shown that she was able to understand the relation between the formula and the area of rectangles.

Students understanding about area makes them realize that their approximation will be match with the area under the curve if they made infinite rectangles. This situation provokes them to take the limit approaching infinity of \( n \).

When the lecturer asked the students questions “why the inscribe polygon never be larger than area under the curve and the circumscribed polygon never be less than area under the curve”. Figure 8 shown student answer about that question. Students realize this phenomenon because they understand that the high of polygon is a function which means it will always stop on the curve.
Figure 7. Student explanation about the meaning of her calculation when the $n$ is equal to two in inscribed polygon

The student stated that it will be impossible to inscribed polygon pass the area under the curve

Figure 8. Student realize that the infinite rectangles will be equal with area under the curve

The student stated that it will be impossible to circumscribed polygon less than the area under the curve

The designed activity made students understand about some phenomena. The phenomena are (1) the area of rectangle on inscribed polygon gets closer to the area under the curve. There will be an increase of area of inscribe polygon when the $n$ increase but the increment never pass the area under the curve. (2) the area of rectangle on circumscribed polygon gets closer to the area under the curve. There will be a decrease of area of circumscribed polygon when the $n$ increase but the decrement never lower than the area under the curve. These understanding are an important part for understanding the definition of definite integral. The visualization from geogebra software supports students to gain more insight about this phenomenon.

At the end of designed activity, the students are shown the following definition as shown in figure 9.

**Definition: Definite Integral**

Let $f$ be a function that is defined on the closed interval $[a, b]$. If

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \Delta x_i$$

exists, we say $f$ is integrable on $[a, b]$. Moreover,

$$\int_{a}^{b} f(x) \, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \Delta x_i$$

is called the definite integral (or Riemann integral) of $f$ from $a$ to $b$.
Figure 9. Formal definition of definite integral

We asked the students to look to the definition and related with their activity. Then, most students were understood that definite integral means the area under the curve by doing the designed activities.

4. Conclusion
The research shows that the journey to understand the definition of definite integral is quite long. There are some stepping stone for students to understand this definition. First, Students have to understand about the notion of area. This knowledge brings students idea to make partition of unknown area, sum up all or the partition using approximation. This includes some mathematical concepts such as function and sigma. Second, students have to able to differentiate between inscribed and circumscribed polygon. This is difficult to the students, because students have to able to use their knowledge about sigma property in counting the inscribe polygon and circumscribed polygon after they are able to determine the high and the base of rectangle and they also had to be able to adapt with the sigma property. Third, students have to able to use their knowledge about limit approaching infinity to count the inscribed and circumscribed polygon which means the area under the curve. This idea can be implemented after the students gain insight after getting brief visualization from geogebra software therefore we suggest educators also used technology to support their students about certain concept. This research shows that this software can be used to build students understanding about definite integral because it provides dynamic visualization. This research confirm the finding of [19] [20] [21] [22] about integrating learning with ICT will give good impact on learning.

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