How to Improve Top Tagging

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In time for the first tests on LHC data we introduce a set of improvements and tests of purely kinematic top tagging algorithms. First, we show how different jet algorithms can be used for different transverse momentum regimes. Combining pruning and filtering in the reconstruction can enhance the signal over background ratio significantly, while larger jet radii only give minor improvements. Finally, bottom tagging can be added to the top tagger, but at least for the HEPTopTagger does not improve the kinematic selection algorithm.

I. INTRODUCTION

The top quark, found in 1995 [1], is the heaviest and so-far only observed fermion with a weak-scale mass. Therefore, it is expected to have strong ties to the mechanism which triggers electroweak symmetry breaking. Searches for new physics in the top sector are of high priority because they can shed additional light on the structure of the Standard Model at and above the weak scale. Many extensions of the Standard Model, like supersymmetry or little Higgs models [2], predict top partners to ameliorate the loop-induced effect of the top quark on the Higgs-boson’s mass. Typical signatures for such extended top sectors include top partners decaying to a top quark and missing energy [3–6] or heavy resonances decaying to two often strongly boosted top quarks [7, 8].

During the Tevatron’s final years D0 and CDF have measured several anomalies directly related to the top sector: in CDF’s single top analysis the ratio between the s- and t-channel production rates deviates by 2.5σ from the SM prediction [9], and both experiments measure an enhanced t\bar{t} forward-backward asymmetry compared to SM predictions [10]. Also including the excess in the di-jet invariant mass spectrum of the Wjj final state, measured (only) by CDF [11], all these anomalies show that we need an improved understanding and simulation of top production processes [12].

The top pair production rate at the LHC ranges around one million tops per inverse femtobarn of integrated luminosity. On the one hand, this means that top pair production is a very challenging background for searches relying on high multiplicity final states of jets, leptons and missing energy [3–5, 13]. On the other hand, this means that already now we can test top pair events in many different kinematic regimes. In this paper we will focus on moderately boosted top quarks in the semileptonic decay channel.

While the idea of studying the substructure of jets is already a classic [14], the potential for searches of massive Higgs and gauge bosons has only been appreciated recently [15–17]. In this paper we will focus on tagging boosted top quarks [5, 18–26]. Aside from being sensitive probes of new physics they are also the prime candidates to generally establish that fat-jet or subjet methods work at the LHC. Some very promising ATLAS results on the HEPTopTagger [5, 19] performance on data can be found in Ref. [27]. CMS has already released first search results using a top tagger [28]. Of the Tevatron anomalies listed above the top forward-backward or charge asymmetry is particularly interesting in the light of boosted top quarks; the ratio of initial state quarks vs gluons increases in the boosted regime, thereby enhancing the otherwise small asymmetry at the LHC [29].

Starting from the default purely kinematic setup of the HEPTopTagger we investigate several avenues on how to improve its performance: In Sec. II we discuss how well the momentum of subjets matches the decay partons in the default setup and which strategies for an improvement should be promising. In Sec. III we investigate the performance of different jet algorithms for the filtering and subjet reconstruction. In Sec. IV we then study the tagging performance if we include pruning in combination with filtering. The pruned top mass we use as an additional kinematic variable according to Ref. [30]. In Sec. V we investigate the possibility to enlarge the size of the fat jet to $R = 1.8$, focussing on the currently most relevant low-$\mathit{p_T}$ tops. Finally, in Sec. VI we augment the kinematic top tagger by a $b$-tag inside the fat jet [31]. Two possible
strategies are simply adding the $b$-tag at the end of the top tagging algorithm or including it in a modified extraction of the relevant subjets.

II. SUBJET-PARTON RECONSTRUCTION

Before we can suggest and test improvements to our top tagger it is crucial that we study measures for the quality of the top reconstruction. The geometrical distance between the reconstructed and the true top momenta is simply

$$\Delta R^2_{\text{top}} = \Delta R^2(p_{\text{tagged}}^{\text{top}}, p_{\text{parton}}^{\text{top}}).$$ (1)

For a more detailed study we also compute the geometric separation of the top decay products, which requires a proper definition of the parton and jet level constituents. Jet combinatorics is the main challenge, in particular for hadronic top pair production at the LHC. For example including up to two additional hard QCD jets the $t\bar{t}$ sample consists of 6 to 8 partons which we label as (1,4) for the bottom quarks, (2,5) for the harder $W$ decay partons, (3,6) for the softer $W$ decay partons, and (7,8,...) for additionally radiated partons. The partons 1-3 and 4-6 come from one top decay each. After hadronization and jet reconstruction the corresponding $b$, $W_1$, and $W_2$ subjets are defined such that $W_1$ (harder) and $W_2$ (softer) reconstruct $m_W$ best. Note that we do not apply any $b$-tagging, an issue we will look at in Sec. VI. We can then define

$$\Delta R^2_{\text{sum}} = \min_{\text{mappings}} \sum_{i=1}^{3} \Delta R^2(p_{\text{sub}}^{i}, p_{\text{parton}}^{j_i}),$$ (2)

where the label $j_i$ denotes the $i$-th hardest parton in the tagged top. The best parton-subjet mapping $\{j_i\} = \{j_1, j_2, j_3\}$ is defined by the minimum $\sum_{i=1}^{3} \Delta R^2(p_{\text{sub}}^{i}, p_{\text{parton}}^{j_i})$ value. For example, for $\{j_i\} = \{1, 5, 7\}$ the hardest subjet corresponds to a $b$-quark, the second hardest to a $W$ decay from the other top, and the softest an additional parton from jet radiations. This way we can categorize all tagged tops into three types:

- type 1: $\{j_1, j_2, j_3\}$ come from one top decay, i.e. $\{1, 2, 3\}$ or $\{4, 5, 6\}$.
- type 2: only the two hardest $\{j_1, j_2\}$ come from one top decay, $j_3$ has a different origin.
- type 3: else.

For semileptonic top pairs the combinatorics is simplified significantly, but we can still categorize all tagged tops along the same lines. Our analysis is based on ALPGEN-PYTHIA [32, 33] samples with MLM merging [34] ($R^{\text{MLM}} = 0.4, p_T^{\text{MLM}} > 30 \text{ GeV}$) and we cluster the visible final state using FASTJET [35]. We

![Figure 1](image-url)  

Figure 1: Left: $\Delta R_{\text{sum}}$ as defined in Eq.(2); center: $\Delta R_{\text{top}}$ as defined in Eq.(1); right: $\Delta p_T^{\text{top}}/p_T$. The colors represent type 1 (red), type 2 (green), type 3 (blue) for semileptonic top pairs at a collider energy of 14 TeV.
take into account two hard jets in association with $t\bar{t}$ production and three to five hard jets for $W+$jets and QCD jets. The $t\bar{t}$ sample we re-weight to 918 pb [36]. The left panel of Fig. 1 shows $\Delta R_{\text{sum}}$ for each type. The quality of the reconstructed subjets direction is the same for all types, with the exception of long type 2 and type 3 tails. In the central and right panels of Fig. 1 we test the actual top momentum reconstruction in terms of $\Delta R_{\text{top}}$ and $\Delta p_T^{\top}/p_T^{\top} = (p_T^{\text{tagged parton}} - p_T^{\text{tagged}})/p_T^{\text{tagged}}$. Its quality depends on the different types of parton identification and the most poorly reconstructed candidate tops are of type 2 and type 3. Unlike for type 1 tops their distributions do not follow a largely Gaussian shape centered at zero but show a significant shift.

Fig. 2 shows the transverse momentum difference between each subjet and the corresponding parton $\Delta p_T^{\top}/p_T^{\top} = (p_T^{\text{subjet}} - p_T^{\text{parton}})/p_T^{\text{subjet}}$. We see that the subjet momentum reconstruction is essentially of the same quality for all subjets and for all types, with the exception of $p_T^{W1}$ which is better reconstructed than $p_T^{W2}$ because of its larger value.

In Tab. 1 we give the fraction of tagged tops for each of the three types after different requirements on the quality of the reconstruction. For type 1 tags the momentum reconstruction both for subjets and tops is almost perfect. For type 2 and type 3 tags, a large fraction of tagged tops satisfies $\Delta R_{\text{sum}} < 0.4$, which means the individual subjets are reconstructed well but the set of the partons is wrongly picked. Consequently, the top momentum reconstruction for type 3 tags becomes worse. Thanks to a correct assignment for the hardest two subjets in type 2 tags the top momentum reconstruction is not too bad because the wrong third subjet does not contribute much to the top momentum.

From the discussion above we can conclude that the individual subjet-parton momentum reconstruction works well for all types. The limitation to the top momentum reconstruction arises from events where some of the identified subjets do not correspond to a top decay product. From Tab. 1 we estimate that $\mathcal{O}(20\%)$ of type 3 tops and $\mathcal{O}(50\%)$ of type 2 tops still give the correct top momentum within 15%. The fraction of tagged tops with good momentum reconstruction within each type does not depend much on $p_T^{\text{top}}$; however,

| $p_T^{\text{tagged}}$ | all | $p_T^{\text{tagged}} > 250$ GeV |
|-----------------------|-----|---------------------------------|
| $\Delta R_{\text{sum}} < 0.4$ | 14156 (84%) | 6029 (88%) |
| $\Delta R_{\text{top}} < 0.2$ | 11004 (77%) | 5279 (88%) |
| $\Delta p_T^{\top}/p_T^{\top} < 0.15$ | 10841 (78%) | 5191 (86%) |
| $\Delta p_T^{\text{top}}/p_T^{\text{top}} < 0.15$ | 11037 (78%) | 5179 (86%) |

Table 1: Tagged top rates (in fb) in the semileptonic $t\bar{t}$ sample for 14 TeV collider energy. The percentages for the additional cuts are relative to the numbers of tagged tops in each row.
the fraction of type 2 and type 3 tags in all tagged tops decreases for higher $p_T$ and effectively leads to a better momentum reconstruction. In total, 85% of all tagged tops and up to 90% of all tags with $p_T^{\text{tagged}} > 250$ GeV reproduce the true momentum within a 15% error bar.

An assigned top tagging efficiency should describe which fraction of hadronic tops in any event sample are tagged. Such an efficiency we define step by step:

1. all decay products satisfy $R_{C/A} < 1.5$.
2. all decay products appear in a fat jet, i.e. there exist unfiltered subjets with $\Delta R(p_{\text{parton}}, p_{\text{subjet}}) < 0.4$.
3. all decay products appear in a fat jet with a top candidate fulfilling $150 < m_{jjj}^{\text{filter}} < 200$ GeV.
4. all decay products appear in a fat jet with a tagged top, i.e. after the mass plane cut.

For the first step we use $R_{C/A} = \max\{R_1, R_2\}$ based on the two C/A measures for the necessary clustering steps $R_{1,2}$. All these events are defined for the signal, so we can show them as a function of the true $p_T,t$ at parton level.

The left panel of Fig. 3 shows the event fractions corresponding to all four categories normalized by the number of hadronic tops as a functions of $p_T$. First, the dotted entries show the fraction of tops with $R_{C/A} < 1.5$, while the solid entries show the fraction of top decay products inside a fat jet. For any fat jet analysis they fix an upper bound on all tagging efficiencies.

The red dotted and solid entries show the fraction of candidate and tagged top events, all constrained to type 1 tags. The difference between the two is simply given by the mass plane cuts. After these cuts roughly 30% of all hadronic tops are tagged above $p_T \sim 250$ GeV. Note that there exist type 2 and type 3 candidates and tags, i.e. hadronic tops whose decay products are included in a fat jet but the extracted subjets do not fall into type 1.

The right panel of Fig. 3 first shows the fraction of type 1 candidates and type 1 tags relative to the number of fat jets including all three top decay products, corresponding to the second category in the above list. Type 2 and type 3 are shown in green and blue. The difference between all top decay products inside a fat jet and type 1 candidates is about 25% and almost constant for $200 < p_T < 500$ GeV. It is partly (at most 10%) due to the existence of type 2 or three candidates, which means the tagger wrongly selects subjets even though the fat jet does include all decay products. Alternatively, there might be overlapping of subjets such that any three subjets are inconsistent with the top mass constraint. The difference between candidates and tags corresponds to the mass plane cuts, where some signal loss is inevitable for rejecting QCD and $W$+jets backgrounds. The numbers of type 2 and type 3 tags are negligible; most of the type 2 and type 3 tags shown in Tab. I come from fat jets not including a top. The mass plane cuts efficiently

![Figure 3](image-url)

Figure 3: Left: tagging efficiencies normalized to the number of hadronic tops as a function of $p_T$. Right: tagging efficiencies relative to the number of hadronic tops included in a fat jet. The different curves are discussed in the text.
remove such contributions. Hence, we see that once the top decay products are captured in a fat jet, 50% to 60% of the tops are tagged.

The rapid drop of efficiency below $p_T \sim 250$ GeV in the left panel of Fig. 3 is simply due to the rapid drop of the fraction of decay products within $R_{C/A} < 1.5$ and inside the fat jet. The ratio of type 1 tags to decay products within a fat jet is not small for $150 < p_T < 250$ GeV. Therefore, we will test an increased $R$ value in Sec. V.

An obvious question from the right panel of Fig. 3 is why the tagging efficiency degrades for larger $p_T$, where in principle the tagging performance should tend to increase. This effect of the mass plane cut is caused by a mis-reclustering after filtering and is more pronounced for the C/A algorithm. The C/A algorithm relies on the $R$ distance exclusively, therefore the softest two of the five filtered subjets are not necessarily combined according to their shower history. As a result, we find unbalanced invariant masses from the three re-clustered subjets. This happens more frequently in the high-$p_T$ regime where all five subjets are not well separated, so the rejection probability by the mass plane cut increases with $p_T$. This tendency and the possibility of changing the underlying jet algorithm and its effect on mass plane cuts we will discuss in Sec. III.

To include backgrounds and mis-tagging efficiencies we need to define a slightly different set of scenarios, namely as a function of the fat jet $p_T$. We define four scenarios similar to the ones before, but now in terms of fat jets:

1. fat jets with three subjets or more after the mass drop criteria
2. fat jets where all distances between top decay products and their closest subjets are less than 0.4.
3. fat jets with a top candidate
4. fat jets with a top tag

The left panel of Fig. 4 shows all corresponding fractions relative to the number of fat jets as a function of $p_T$. The black dotted and solid symbols show the fraction of events with at least three subjets and of all top decay products included. More than half of the fat jet events with at least three subjets do not include the top decay products, even for the semileptonic $t\bar{t}$ sample. The red entries show the fraction of candidates (dotted) and tags (solid). Because all efficiencies are shown as a function of the fat jet $p_T$, we can also show the backgrounds in blue and green. On the plateau we find tagged tops in roughly 20% of the fat jets for semileptonic top pairs and 2-4% for $W+\text{jets}$ and QCD jets.

The candidate histogram in the left panel of Fig. 4 exceeds the numbers for top decay products in the fat jet because there exist fat jets whose three main subjets are not from a top decay and accidentally give $m_t$. The central panel of Fig. 4 shows the composition of each type separately for candidates (dotted) and tags (solid). Indeed, a considerable fraction of candidates are of type 3. On the other hand, type 3 and type 2

![Figure 4: Left: efficiencies $\epsilon_{t\bar{t}}$, $\epsilon_{W+\text{jets}}$, $\epsilon_{QCD}$ as functions of the fat jet $p_T$. Center: fraction of tagged tops for type 1 (red), type 2 (green) and type 3 (blue). The dotted lines show the corresponding candidate fractions. Right: fraction of type 1, type 2 and type 3 only in for fat jets including a top.](image-url)
tags are effectively rejected by the mass plane cut, so most of the tagged tops are of type 1. The fraction of
type 2 and type 3 tags ranges around 2-4\%, similar to QCD and W+jets backgrounds.

The right panel shows the fractions of candidate and tagged fat jets including a hadronic top, so they are
also constrained to belong to the second category. Compared with the central panel we are now less likely to
encounter type 2 or type 3 candidates. Most tagged tops are now of type 1 and most type 2 and type 3 tags
correspond to fat jets which do not include a hadronic top. Consequently, we find that there is not much
room to improve our algorithm in selecting subjets after applying the mass drop criterion.

III. ALTERNATIVE JET ALGORITHMS

The combination of the Cambridge–Aachen clustering algorithm \cite{37} with a mass drop criterion \cite{15} is a
core feature of the HEPTopTagger and hence not negotiable. However, in the mass reconstruction after
filtering the C/A algorithm should be compared to alternative jet algorithms, like $k_T$ \cite{38} or anti-$k_T$ \cite{39}.
After identifying three subjets based on the mass drop criteria there are two steps left to extract the $b$, $W_1$
and $W_2$ subjets: filtering \cite{15} and reclustering. We find that the choice of jet algorithm for the filtering does
not have a visible effect on the efficiency at particle level, while for the reclustering step it does. In our
explanations we therefore focus on effects of this reclustering which combines five filtered subjets into three
top decay subjets while we always use the same jet algorithm for filtering and for reclustering.

Our first result is that the anti-$k_T$ algorithm fails to reliably identify the three hard top decay products.
It tends to first recombine a pairing with large transverse momentum, such that of the three reclustered
subjets one is very hard and two are very soft. Applying our $W$ and top mass cuts will typically reject such
unbalanced combinations.

Using the C/A algorithm a similar problem arises, but only for very large $p_{T,t}$, where the five filtered
subjets are close. According to QCD the two softest filtered subjets should then be merged into the main
three subjets, which the C/A algorithm achieves as long as the three main subjets are geometrically well

![Figure 5: Mass plane for top candidates in $t\bar{t}$, $W$+jets and QCD events (left to right). The upper three panels use C/A reclustering while the lower three panels use $k_T$ reclustering.](image)
Figure 6: Left: signal efficiencies relative to the number of hadronic tops for the $k_T$ (red), C/A (black) and anti-$k_T$ (with C/A clustering, blue) algorithms. Center: $k_T$ tagging efficiency relative to the number of tops included in a fat jet for type 1 (red), type 2 (green), and type 3 (blue) tags. Right: mis-tag rates for QCD (blue) and $W$+jets (green) for the C/A (dotted) and $k_T$ algorithm (solid).

The $k_T$ algorithm recombines soft filtered subjets most reliably, so it can resolve the three main top decay products best. We can see this in the lower panels of Fig. 5 where hardly any signal events migrate to $m_{23} \sim 0$. As a result, the signal efficiency of passing the mass plane cut increases. This tagging efficiency we show in Fig. 6. For both cases the filtering and reclustering algorithms are the same. In the low-$p_T$ regime the difference between the two algorithm is indeed small. In the central panel we see that unlike the C/A results in Fig. 3 the $k_T$ efficiency hardly decreases towards high $p_T,t$, i.e. the efficiency of the mass plane cut is essentially constant. The fractions of type 2 and type 3 tags which include a top also decrease because the subjet momentum reconstruction improves. If the subjets are better matched to the hard top decay products more tagged tops are categorized as type 1.

Going beyond the signal, the same feature of fewer events with $m_{23} \sim 0$ also appears for the backgrounds. The mass plane cut then leads to a less efficient rejection in particular for soft masses. This increase of mis-tag probabilities we also show in Fig. 6. The solid crosses show the results for the $k_T$ algorithm while the dotted crosses show those for the C/A algorithm. Quantitatively, the mis-tag efficiencies increase by a slightly larger factor than the tagging efficiency, so switching to the $k_T$ algorithm does not improve $S/B$ but can improve $S/\sqrt{B}$ slightly. In addition, these results are obtained without detector simulation and pileup. Switching to the $k_T$ algorithm might have additional implications from both of them, so only a detailed experimental analysis can determine which jet algorithm to use in the HEPTopTagger framework, and its result might well be process dependent.

IV. PRUNING

From the Higgs tagger based on the C/A algorithm and a mass drop criterion \[15\] we know that it can be advantageous to combine filtering and pruning in the tagging procedure \[30\]. Pruning removes soft radiation while clustering the fat jet \[20, 21\]: first, a sequential jet algorithm combines unfiltered subjets until no pair of constituents is geometrically closer than $R_{\text{cut}}$, representing an effective subjet cone size usually associated with an intrinsic fatjet scale. We choose $R_{\text{cut}} = m_j/p_{T,j}$. This cutoff can act differently for different underlying jet algorithms. After this recombination the unfiltered subjet merging continues, but with the
additional restriction that each combined pair of subjets has to be sufficiently hard,
\[
    z = \min(p_{T,i}, p_{T,j}) > z_{\text{cut}},
\]
where we choose \( z_{\text{cut}} = 0.1 \). Otherwise, the constituents \( i \) and \( j \) are not combined and the one with smaller transverse momentum is discarded. This algorithm continues until all constituents have been combined or eliminated.

There are two ways we can include pruning in our top tagger. First, we can prune the fat jet before we run the C/A algorithm extracting the relevant splittings using the mass drop criterion. Alternatively, we can use the pruning procedure in parallel to filtering procedure and combine the two pieces of information. This approach ensures that the additional pruning step does not affect the performance of the rest of the tagging algorithm, so we investigate it in this section.

Since pruning is originally targeted at removing soft radiation, its impact is similarly to that of filtering \[15\]. To quantify the difference we apply pruning to the constituents of the three subjets which we obtain after the usual mass drop criterion. These three subjets are selected such that they give the best filtered top mass among all combinations. The difference between the two algorithms is illustrated by the variable
\[
    \Delta m^{\text{prune}} = m^{\text{prune}} - m^{\text{filter}},
\]

| \( \Delta m^{\text{prune}} > 15 \) | 20 | 30 | \( \Delta m^{\text{unfilter}} > 15 \) | 20 | 30 |
|---|---|---|---|---|---|
| \( t\bar{t} \) [fb] | 14156 | 7773 (55%) | 9072 (64%) | 10875 (77%) | 6237 (44%) | 7926 (56%) | 10505 (74%) |
| type 1 | 10318 | 6255 (61%) | 7152 (69%) | 8316 (81%) | 5141 (50%) | 6377 (56%) | 847 (63%) |
| type 2 | 1336 | 551 (41%) | 693 (52%) | 893 (67%) | 403 (30%) | 570 (43%) | 847 (63%) |
| type 3 | 2503 | 967 (39%) | 1227 (49%) | 1666 (67%) | 693 (28%) | 979 (39%) | 1529 (61%) |
| \( W + \text{jet} \) [fb] | 6590 | 2716 (41%) | 3373 (51%) | 4459 (68%) | 2052 (31%) | 2797 (42%) | 4162 (63%) |
| \( \epsilon_{S/B} \) | 1 | 1.33 | 1.25 | 1.14 | 1.41 | 1.32 | 1.18 |
| \( \epsilon_{S/\sqrt{B}} \) | 1 | 0.86 | 0.9 | 0.93 | 0.79 | 0.86 | 0.93 |
| \( QCD \) [pb] | 1229 | 359 (29%) | 474 (39%) | 719 (59%) | 207 (17%) | 331 (27%) | 609 (50%) |
| \( \epsilon_{S/B} \) | 1 | 1.88 | 1.66 | 1.31 | 2.62 | 2.08 | 1.5 |
| \( \epsilon_{S/\sqrt{B}} \) | 1 | 1.02 | 1.03 | 1 | 1.07 | 1.08 | 1.05 |

Table II: Tagged top rates (in fb for \( t\bar{t} \) and \( W + \text{jets} \) and pb for QCD jets) after cuts on \( \Delta m^{\text{prune}} \) or \( \Delta m^{\text{unfilter}} \). The percentages are relative to the numbers without pruned or unfiltered mass cut in each category. \( \epsilon_{S/B} \), \( \epsilon_{S/\sqrt{B}} \) denote improvement factors relative to no cuts.
where \( m_{\text{filter}} \) is the filtered mass for the selected three subjets, and \( m_{\text{prune}} \) is the jet mass of the pruned jet. The left panel in Fig. 7 shows \( \Delta m_{\text{prune}} \) for tagged tops in \( t\bar{t} \) (type 1), QCD jets, and \( W+jets \) events. We find that \( \Delta m_{\text{prune}} \) is larger for background events than for signal events. Pruning generally collects more constituents than filtering, which discards some of the filtered subjets, so the pruned mass increases in a busy jet environment. Background events rely on such busy events to mimic the generically hard top decay products. Therefore, selecting tags with small \( \Delta m_{\text{prune}} \) effectively rejects the backgrounds. Even though we do not show them, type 2 and type 3 tags behave similar to the backgrounds samples, because QCD jet radiation partly contributes to these tags. Thus, pruning also purifies the type 1 fraction for all tagged tops.

The event numbers after imposing

\[-10 \text{ GeV} < \Delta m_{\text{prune}} < \{15, 20, 30\} \text{ GeV}\]

we show in the left half of Tab. II. The percentages are relative to the number of tagged tops in each row. The different efficiencies we can translate into improvement factors for \( S/B \) and for \( S/\sqrt{B} \). For example, we can improve \( S/B \) by roughly a factor two without loss of \( S/\sqrt{B} \). In addition, the quality of the momentum reconstruction improves with the increased fraction of type 1 tags.

This additional cut becomes more important for larger thresholds of the subjet mass in the top tagger. For example, compared with the default choice \( m_{\text{subjet}} > 50 \text{ GeV} \) a reduction to 30 GeV makes the cut on \( \Delta m_{\text{prune}} \) less efficient. This arises because with a larger \( m_{\text{subjet}} \) threshold more constituents can contribute in the background case.

According to the above discussion the difference in the pruned mass distribution for signal and backgrounds is due to the subjet multiplicity inside the fat jet. A simpler intuitive measure for this feature is the fat jet mass before filtering. Algorithmically, we select the relevant three subjets only after filtering, but the original mass of the unfiltered subjets includes additional information:

\[ \Delta m_{\text{unfilter}} = m_{\text{unfilter}} - m_{\text{filter}}. \]  

In Fig. 8 we show this distribution for signal and backgrounds. Indeed, the left panel is very similar to the \( \Delta m_{\text{prune}} \) distribution of Fig. 7. The two-dimensional correlation confirms that almost all events for signal and background lie on the central diagonal of the \( \Delta m_{\text{prune}} \) vs \( \Delta m_{\text{unfilter}} \) plane.

In the right half of Tab. II we show the corresponding efficiencies after cutting on \( \Delta m_{\text{unfilter}} \). From both variables we can obtain significant improvements on \( S/B \), and it remains an experimental question which of them is more stable once we include detector effects and pile-up.

V. FATTER JETS

In the Standard Model the cross section for top pairs falls very steeply with increasing transverse momentum. The fraction of top pair event above different \( p_{T,T}^{\text{min}} \) values at a 14 TeV LHC we calculated using
Figure 9: Left: tagging efficiency (solid red) and the fraction of tops included in the fat jet for $R = 1.8$ (solid) and for $R = 1.5$ (dotted) as a function of $p_{T,t}$ for semileptonic $t\bar{t}$ events. Center: tagging candidates and tags relative to all tops included in a fat jet. Type 2 and type 3 tags are shown in green and blue. Right: mis-tag rate as a function of fat jet $p_T$ for QCD (blue) and $W$+jets (green).

**Table III:** Numbers of tagged tops with $R = 1.8$ with several cuts on $\Delta m^{prune}$ or $\Delta m^{unfilter}$. The percentages are relative to the numbers of tagged tops without pruned or unfiltered mass cut in each category. $\epsilon_{S/B}$ and $\epsilon_{S/\sqrt{B}}$ denote improvement factors relative to the $R = 1.5$ numbers with no cuts as shown in Tab. II.
The central panel shows the fractions of type 1 candidates and type 1 tags relative to all hadronic tops included in a fat jet as functions of hadronic top $p_T$. All the way down to $p_{T,t} = 100$ GeV the efficiencies are flat, which means we have a fair chance to collect very moderately boosted tops. Type 2 and type 3 fractions we show at the bottom of the figure. While the fraction of type 2 and type 3 tags increases the fraction of type 1 candidates and tags does not drastically change compared to $R = 1.5$ as shown in the right panel of Fig. 3.

Finally, increasing $R$ also increases the mis-tag rate significantly. The right panel of Fig. 9 shows the mis-tag efficiency as functions of the fat jet $p_T$ for QCD and $W$+jets events. We observe a larger increase for these background processes than for the signal, which means we will not improve $S/B$ through larger jet sizes. However, we might improve the statistical significance measure $S/\sqrt{B}$.

In Tab. III we show the improvements in $S/B$ and $S/\sqrt{B}$ relative to $R = 1.5$ case. We see that roughly twice the number of tops get tagged, mainly at low transverse momenta. However, $S/B$ still decreases by a factor $1/2$, while $S/\sqrt{B}$ slightly improves as long as QCD is the main background.

To compensate for the increased backgrounds we can also apply pruning for $R = 1.8$. The corresponding efficiencies for different pruned mass cuts we also include in Tab. III. Adding pruning shifts back the performance to a similar level as our $R = 1.5$ results, and there is no obvious advantage in combining it with larger fat jets, unless there should be a specific reason to target the low-$p_{T,t}$ regime.

VI. BOTTOM TAGGING

A major difference in the background rejection between the C/A based Higgs tagger [15] and corresponding top taggers are additional $b$-tags. Only based on kinematic conditions it appears unlikely to achieve a QCD or $W$+jets rejection of more than a factor 1/100. However, we can gain a significant improvement by requiring a $b$-tag for one of the the top decay jets. At the same time, for moderately boosted top quarks the kinematic tagging algorithm might benefit from the identification of the $b$-jet, so we can first ensure that it is captured and second use this information in the kinematic reconstruction.

Our first attempt to improve top tagging through an additional $b$-tag will leave the kinematic top tagging algorithm unchanged and will instead focus on the selection of the subjet which should be $b$-tagged. All of the usual top taggers treat three subjets democratically, i.e. without any $b$-tagging information. We label the $b$, $W_1$ and $W_2$ subjets such that $W_1$ and $W_2$-subjets (ordered by hardness) reconstruct $m_W$ best; $j_b$, $j_{W1}$, and $j_{W2}$ are the corresponding parton labels from Monte Carlo truth, as defined in Sec. II. An obvious question is for what fraction of all tags the subjet labeled $b$ really points to the bottom quark $b = j_b$. For type 1 tags by definition one of the subjets corresponds to a bottom, so $b = j_b$ implies that all subjets are correctly assigned. For such tags a $W$ decay angle analysis [23] will work well.

In the first column of Tab. IV we summarize the probabilities to correctly assign the label $b$ in the kinematics-based top tagging. The percentages are defined relative to the number of tagged tops in each category. For type 1 tags there will be exactly one $b$-parton in the tagged top while for type 2 and type 3 tags there can be any number of $b$-partons. This means that only for type 1 tags all three fractions sum to 100%. Almost 80% of type 1 tags correctly assigns $b = j_b$, so they make a good test sample for the question if identifying $b = j_b$ through a $b$-tag leads to an efficient rejection for $W$+jets and QCD events. From Tab. IV we also see that the second-most likely subjet to be kinematically identified as a $b$ really is $j_{W1}$. However, this probability is less than a third of that for $b = j_b$. To rely on an additional $b$-tag for the case $b = j_{W1}$ does not even improve $S/\sqrt{B}$, so the only way to increase $S/B$ is to apply a $b$-tag only to the kinematically identified $j_b$.

As we show in Fig. 5 a significant fraction of events passing the mass plane cut have two pairs of subjets consistent with the $W$ mass. This happens because the upper bound $m_{T,j}^2 < m_t^2 - m_W^2$ is numerically close to $m_t^2$. We expect that tagged tops where only one pair of subjets is consistent with $m_W$ should more reliably give $b = j_b$. To test this, we count the number of subjet pairs consistent with $m_W$ as $n_W = 2$ when $\Delta m_2 - \Delta m_1 < 10\% \times m_W$ where $\Delta m = |m_{jj} - m_W|$ for each pairing. Because $n_W = 3$ essentially never appears we find $n_W = 1$ for the other top tags.

The distributions for kinematically identified $b$-subjets for different $n_W$ we also show in Tab. IV. The fractions of tagged tops with $n_W = 1$ is almost the same ($\sim 57\%$) for $t\bar{t}$ and QCD, $W$+jets. Type 1 tagged
Figure 10: Modified mass plane distribution for signal, W+jets and QCD events. The mass plane is now defined as $\arctan(m_{b_2}/m_{b_1})$ vs. $m_{12}/m_{123}$.

tops with $n_W = 1$ are more likely to give $b = j_b$ than those with $n_W = 2$, while for type 2 and type 3 we do not observe a significant difference.

Consequently, the best strategy to improve $S/B$ in top tagging is to target $n_W = 1$ events and check the kinematically identified $b$-subjet with a $b$-tag. In terms of a tagging efficiency $\varepsilon_b$ and a mis-identification rate $\varepsilon_b^{\text{mis}}$ for light flavors (we ignore c quarks in this simple estimate) we show the possible improvements in Tab. V. We expect an enhancement by $0.66\varepsilon_b/\varepsilon_b^{\text{mis}} (0.73\varepsilon_b/\varepsilon_b^{\text{mis}}$ for selecting $n_W = 1$) for $S/B$, so assuming $\varepsilon_b = 50\%$ and $\varepsilon_b^{\text{mis}} = 2\%$ we find that $S/B$ improves by 16.5 (18 for selecting $n_W = 1$). Similarly, we find an improvement in $S/\sqrt{B}$ around a factor 2.

As an alternative to an external $b$-tag, we can use the $b$-tagging information during the kinematic top tagging algorithm. The selection of the three decay subjets after the mass drop criteria we modified in four steps:

1. group the subjets into either $b$-tagged or non-$b$-tagged subjets.
2. take all possible triplets of one $b$-tagged and two non-$b$-tagged subjets and select the one with the best filtered top mass.

|                  | all $n_W$ | $n_W = 1$ | $n_W = 2$ |
|------------------|-----------|-----------|-----------|
| $tt$ [fb]        | 14156     | 8058      | 6099      |
| $b = j_b$        | 9325 (66%)| 5882 (73%)| 3442 (56%)|
| $b = j_W$        | 2971 (21%)| 1242 (15%)| 1728 (28%)|
| $b = j_W$        | 1666 (12%)| 833 (10%) | 833 (14%) |
| type 1           | 10318     | 5808      | 4509      |
| $b = j_b$        | 7917 (77%)| 5044 (87%)| 2874 (64%)|
| $b = j_W$        | 1695 (16%)| 502 (9%)  | 1193 (26%)|
| $b = j_W$        | 706 (7%)  | 263 (4%)  | 443 (10%) |
| type 2           | 1336      | 781       | 555       |
| $b = j_b$        | 565 (42%) | 341 (44%) | 224 (40%) |
| $b = j_W$        | 499 (37%) | 294 (38%) | 205 (37%) |
| $b = j_W$        | 392 (29%) | 226 (29%) | 166 (30%) |
| type 3           | 2503      | 1468      | 1035      |
| $b = j_b$        | 842 (34%) | 498 (34%) | 344 (33%) |
| $b = j_W$        | 777 (31%) | 447 (30%) | 331 (32%) |
| $b = j_W$        | 568 (23%) | 344 (23%) | 224 (22%) |
| W+jets [fb]      | 6590      | 3733      | 2857      |
| QCD [pb]         | 1229      | 713       | 516       |

Table IV: Kinematic identification probabilities ($b$) for all three top decay partons for different types of top tags and for different numbers of subjet pairings consistent with $m_W (n_W)$. 
3. check if this $m^{\text{filter}}$ satisfies our criterion.

4. apply modified mass plane cuts.

Since we require exactly one $b$-tagged subjet in step 2, we adapt the labels $b, W_1, W_2$ to reflect this. In step 4, we should change the invariant masses in terms of $(p_1,p_2,p_3)$ into $(p_b,p_W^1,p_W^2)$. Our two-dimensional mass plane becomes $\arctan(m_{W_3}/m_{b_1})$ vs. $m_{12}/m_{123}$, where $m_{b_1}^2 = (p_b + p_{W_1})^2$ and $m_{12}^2 = (p_{W_1} + p_{W_2})^2$. Fig. 10 shows the two-dimensional distribution of top candidates in the modified mass plane for semi-leptonic $t\bar{t}$ events, $W$+jets, and QCD events. Unlike before, the signal events now show a clear $W$ mass peak only for $m_{12}$. To see how well this new algorithm might do we assume a 100% $b$-tagging efficiency of perfect purity. Because for the backgrounds all three subjets could equally likely be mis-tagged we simply reweighting each of the possibilities by $\varepsilon_b^{\text{mis}}$. Following these plots we can apply stricter mass plane cut on the modified mass plane than before. For our test we use

$$\left| \frac{m_{12}}{m_{123}} - \frac{m_W}{m_t} \right| < 15\% \quad \text{and} \quad 0.2 < \arctan \left( \frac{m_{b_1}}{m_{12}} \right) < 1.3 \quad . \quad (7)$$

Based on this modified algorithm the left panel of Fig. 11 shows the signal efficiency as a function of $p_{T,t}$. Again, for the signal we assume a perfect $b$-tag. We can simply multiply by $\varepsilon_b$ to compute the final top tagging efficiency, ignoring the small effect of $\varepsilon_b^{\text{mis}}$. Similarly, we ignore cases with two $b$-subjets in the tagged top as subleading by a factor $\varepsilon_b(1-\varepsilon_b)$ and only appearing for type 2 or type 3.

Taking into account the probability of 77% with which the default top tagger correctly assigns $b = j_b$, the modified algorithm slightly increases the number of tagged tops if we assume perfect $b$-tagging. It returns almost the same tagging efficiency as the purely kinematic tagger before adding the factor $\varepsilon_b$. This is because the default top tagger identifies the crucial type 1 tops even without $b$-tagging information while type 2 or type 3 configurations are comparably rare. The use of $b$-tagging really only helps to identify which subjet of a type 1 tag corresponds to the bottom and to reject type 2 and type 3 tags. To confirm this, in the central panel of Fig. 11 we show the efficiency as a function of the fat jet’s $p_T$. Compared to $p_{T,t}$ this effectively removes some type 2 and type 3 tops.

The right panel of Fig. 11 shows the mis-tag rate of the modified algorithm. Here we simply assume that one of the three subjets found by the kinematic top tagger is mis-identified. These candidate tops have to be multiplied by $3 \times \varepsilon_b^{\text{mis}}$ before imposing the modified mass plane cut. The effect of mis-identifying $b$-subjets among others than the three subjets selected by the kinematic tagger we neglect. It should be less than 10% given that more than 90% of the fat jets in which three subjets fulfill the top mass criteria only have one such combination.

The main question is if the restricted mass plane cuts now reduce the backgrounds more efficiently. We find that the cuts from Eq. (7) efficiently drop tagged tops from background samples as compared to democratic mass plane cuts, but they do not compensate for the combinatorial factor 3 in the mis-tagging probability.

Up to this point we have only considered semi-leptonic $t\bar{t}$ events. In particular for the possible improvement through $b$-tagging we should consider the more combinatorics prone purely hadronic decays. The left panel of Fig. 12 shows the tagging efficiencies as a function of $p_{T,t}$ for the fully hadronic $t\bar{t}$ sample. Compared to

| $\varepsilon_b$ | $\varepsilon_b^{\text{mis}}$ |
|----------------|-----------------------------|
| 0.5            | 0.02                        |
|                |                             |

Table V: Efficiencies of a $b$-tag after top tagging and for the modified tagger with $b$-tagging. We assume $\varepsilon_b = 0.5$ and $\varepsilon_b^{\text{mis}} = 0.02$ and quote the improvement factors $\varepsilon_{S/B}, \varepsilon_{S/\sqrt{\bar{b}}}$ against the QCD background.
Fig. 3, the tagger works with almost the same efficiency, provided we normalize the number of tagged events to all hadronic tops.

The central panel of Fig. 12 shows the default top tagger efficiencies as a function of the transverse momentum of the fat jet, again for hadronic decays. The tagging efficiency and the fraction of candidate tops we show for type 1, type 2 and type 3 tags. Since the number of fat jets including a hadronic top shown as the black solid line increases compared to semi-leptonic events the resulting candidate and tagged efficiency becomes larger. The number of type 2 and type 3 tags increases simply with the jet multiplicities. In the right panel of Fig. 12 we show the same fractions as in the central panel but requiring that all fat jets include a hadronic top. For such fat jets we find type 2 or type 3 almost as rarely as in the semi-leptonic sample, i.e. type 2 and type 3 candidates contribute at most 30% relative to type 1 candidates. This means that even for the fully hadronic $t\bar{t}$ sample a modified algorithm including $b$-tagging will not provide enough of an improvement to compensate an expected factor 50% increase in the mis-tag rate.

In summary, using $b$-tagging when selecting the relevant three subjets in a fat jet does not enhance $S/B$. The purely kinematic HEPTopTagger selects the correct set of subjets too reliably to gain a significant improvement as long as the hadronic top is fully captured in a fat jet. The $b$-subjet selected based on kinematics is usually identified correctly. Relying on $b$-tagging inside the kinematic algorithm is hurt by the combinatorial mis-tagging efficiency of $3 \times \epsilon_{\text{mis}}^b$ while there is no such factor 3 for the signal. This disadvantage is hard to compensate by improved mass plane cuts. To improve $S/B$, the best approach is to use the $b$-tag only for the most probable $b$-subjet and simply add it to the kinematic tagging algorithm.

VII. OUTLOOK

In this paper we have proposed and tested several modifications to kinematic top tagging, as implemented in the HEPTopTagger. As a starting point we have shown that provided the top quark is boosted enough to be collected inside a fat jet the usual kinematic criteria, i.e. a search for mass drops in the clustering history and the reconstruction of three independent invariant mass variables from the suspected top decay subjets, do not exhibit obvious shortcomings.

1. One possible improvement, curing for example the decreasing efficiency of C/A based taggers towards larger boost is a switch to the $k_T$ algorithm once we identified the main subjets using mass drops. It keeps the tagging efficiency relative to the number of tops caught inside a fat jet on a plateau over the entire range $p_{T,t} \sim 150 - 600 \text{ GeV}$.

2. Using pruning in combination with the usual filtering procedure we gain an additional kinematic variable. Cutting on it can almost double $S/B$ relative to pure QCD backgrounds. However, this

![Figure 11](image-url)
improvement should be taken with a grain of salt until it can be confirmed by a proper detector simulation in the presence or pile-up.

3. To extend the top tagger to lower boost we can increase the original size of the fat jet from $R = 1.5$ to $R = 1.8$. Indeed, the efficiency for low transverse momenta increases, but as well does the background combinatorics. We find that neither $S/B$ nor $S/\sqrt{B}$ strongly benefit from this modification.

4. Including a $b$-tag as an additional step in the top tagging procedure can very significantly enhance the background rejection. This is well known from Higgs tagging. We also find that for the HEPTopTagger including such a $b$-tag in a modified algorithm is not promising.

At a time when we can expect the first officially tagged top quarks at the LHC these studies can guide us towards possible modifications and improvements of top taggers with different analyses in mind. They show that for example in the case of the HEPTopTagger there is still room for adjustment but that top taggers in general have within a few years reached an impressive level of maturity and reliability.

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