Can boson stars supplant black holes?

F. Siddhartha Guzmán
Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo. Edificio C-3, Cd. Universitaria. C. P. 58040 Morelia, Michoacán, México.
E-mail: guzman@ifm.umich.mx

Abstract. The time-like geodesics of spherically symmetric boson stars (BS) are compared to those of the Schwarzschild black hole (BH). It is shown that the compactness of the BS is the quantity that determines how similar time-like geodesics are to those of a BH with the same mass. It is also found that the self-interaction of the scalar field the BS is made of, determines how compact a stable BS can be. The combination of these two results indicates that BSs could supplant BHs better when they are stable, have strong self-interaction and high central density. If boson stars will be considered as serious toy models for astrophysical BH candidates it will be important to choose correctly the free parameters of the scalar field; here the basic guidelines are pointed out for the case of spherical symmetry.

1. Introduction
Since boson stars (BSs) were found to be stationary solutions of the Einstein-Klein-Gordon (EKG) system of equations [1, 2] there have been two aims for their study: i) considering BSs as possible existing objects made of a fundamental scalar field (with fixed parameters: mass and self-interaction of the scalar field) that eventually could be observed in the laboratory and ii) objects that have served to propose solutions to astrophysical problems requiring exotic particle constituents keeping the parameters of the field free. In the first approach we find the introduction of the self-interaction added to the scalar field in order to obtain more massive BSs, rather similar to usual compact objects [3], the efforts related to the detection of BSs through their gravitational wave signatures [4, 5, 6], the study of the quasi normal mode oscillations of BSs [7], the full perturbation of BSs as a source of gravitational waves [6] and the effects on the emission spectrum from accretion disks around BSs [8]. Within the second approach we find the models for scalar field dark matter galactic halos [9], the explanation of active galactic nuclei phenomena through the substitution of a the supermassive black hole with a supermassive BS, which also provides predictions for the possible signatures of BSs [10], BSs as a non-baryonic alternative for MACHOS [11], etc.

Aside of such effort, a third approach related to numerical relativity has started developing. Boson stars, aside of their potential existence and source of toy models, possess convenient features that help at developing numerical techniques used in simulations of perfect fluid stars and black holes [12]. Among these properties of BSs are: 1) the evolution equation of the matter field is pretty much the wave equation on a curved background, 2) BSs have no sharp surface, 3) these two properties together avoid the formation of shocks during the evolution in full 3+1 numerical relativity, 4) eventually (if the BS belongs to the unstable branch) can form a black
hole whose event horizon properties can be monitored [13].

Within this new approach it can be considered the possibility of faking compact objects by taking advantage of all the benefits and convenient features of BSs. The idea is to allow the parameters of mass and self-interaction to have any value and just choose the most adequate BSs in order to model any compact object. The extra bonus of this point of view is that eventually through the evolutions of BSs, fundamental differences with compact objects would arise and thus possible signatures particular of BSs can be proposed as predictions to be observed. This manuscript focuses on the search for the BS that better fakes a black hole in the spherically symmetric case. In order to do so, a brief review on Boson Stars is developed in the next section. In section 3 the analysis of time-like geodesics is presented and compared to that of black holes; in there, the criteria for the best faker is presented. Finally a discussion and some conclusions are drawn in section 4.

2. Background on Boson Stars
Boson stars are stationary solutions of Einstein’s equations $G_{\mu\nu} = 8\pi GT_{\mu\nu}$, where $T_{\mu\nu} = \frac{1}{2}[\partial_{\mu}\phi^*\partial_{\nu}\phi + \partial_{\mu}\phi\partial_{\nu}\phi] - \frac{1}{2}g_{\mu\nu}(\phi^{*2}\phi,\alpha + V(|\phi|^2))$ is the stress-energy tensor of a complex scalar field $\phi$, $g_{\mu\nu}$ is the metric of the space time and $V$ is the potential of the field, which for the present case is restricted to have the form $V(|\phi|) = \frac{1}{2}m^2|\phi|^2 + \frac{1}{4}|\phi|^4$ where $m$ is the mass of the field and $\lambda$ its self-interaction. Whether or not the scalar field plays the role of a fundamental spinless particle, one can always consider it appears in an effective theory with lagrangian density $L = -\frac{R}{16\pi G} + g^{\mu\nu}\partial_{\mu}\phi^{*}\partial_{\nu}\phi + V(|\phi|)$; this mean field approach helps at avoiding the formulation of quantum field theory in a curved background (for a recent review on BSs see [14]).

Although axi-symmetric BS solutions have been found, we restrict here to the spherically symmetric case and choose the line element of space-time to be $ds^2 = -a^2dt^2 + a^2dr^2 + r^2d\Omega^2$ where the radial coordinate has been chosen so that it coincides with that of the usual Schwarzschild solution. Assuming the field to have a time dependence $\phi(r,t) = \phi_0(r)e^{-i\omega t}$ the stress energy tensor is time independent, which implies that the space-time is stationary and therefore that the metric functions $\alpha$ and $a$ depend only on the radial coordinate $r$. Under this assumptions the EKG equations are:

\[
\frac{a'}{a} = \frac{1 - a^2}{2r} + \frac{1}{4}\kappa r \left[ \omega^2\phi_0^2\frac{a^2}{\alpha^2} + \phi_0'^2 + a^2V \right] \quad (1)
\]
\[
\frac{\alpha'}{\alpha} = \frac{a^2 - 1}{r} + \frac{a'}{a} - \frac{1}{2}\kappa ra^2V \quad (2)
\]
\[
\phi_0'' + \phi_0' \left( \frac{2}{r} + \frac{\alpha'}{\alpha} - \frac{a'}{a} \right) + \omega^2\phi_0 a^2\frac{a^2}{\alpha^2} - a^2 \frac{dV}{d|\phi_0|^2}\phi_0 = 0 \quad (3)
\]

where $\kappa = 8\pi G$ and a prime indicates derivation with respect to $r$. System (1-3) is a set of coupled ordinary differential equations to be solved under the conditions $a(0) = 1$, $\phi_0(0)$ finite and $\phi_0'(0) = 0$ in order to guarantee regularity and spatial flatness at the origin, and $\phi_0(\infty) = \phi_0'(\infty) = 0$ in order to ensure asymptotic flatness at infinity as described in [1, 15, 16, 17]; these conditions reduce the solution of the system (1-3) to an eigenvalue problem for $\omega$. The solution was calculated numerically using finite differencing and a shooting routine that searched $\omega$. 
The standard rescaled variables $\tilde{\phi}_0 = \sqrt{\frac{4\pi G}{\omega}} \phi_0$, $\tilde{r} = m r$, $\tilde{t} = \omega t$, $\tilde{\alpha} = \frac{\omega}{\sqrt{\omega}} \alpha$ and $\Lambda = \frac{\lambda}{4\pi G m^2}$ were used, so that the coordinate time is given in units of $1/\omega$ and the distance in units of $1/m$. The results define the sequences of equilibrium configurations like those shown in Fig. 1(a). The slope of the curves shown indicates two important points for each value of $\Lambda$: i) the critical points -marked with a filled circle- indicating the threshold between the stable and unstable branches of each sequence, that is, configurations to the left of this point are stable and those to the right are unstable, and ii) the point at which the binding energy $E_B = M - N m$ is zero -marked with an inverted filled triangle- where the number of particles is $N = \int \sqrt{-g} d^3 x = \frac{1}{2} \sqrt{-g} g^{uv} \left[ \phi^* \partial_u \phi - \phi \partial_u \phi^* \right] d^3 x$ and the mass $M = (1 - 1/a^2) r/2$ evaluated at the outermost point of the numerical domain: the configurations between the instability threshold and the zero binding energy point collapse into black holes whereas those to the right disperse away as shown in [13].

Since the mass of the configurations in Fig. 1(a) scales with $m$, the self-interaction $\Lambda$ was introduced in order to allow bigger masses even if the mass parameter $m$ was fixed [3] and thus BS configurations similar -gravitationally- to compact objects like neutron stars were designed. In the present approach neither $m$ nor $\Lambda$ will be considered related to any type of possibly existing particle, instead they are thought of as free parameters that permit faking compact objects. We prefer to focus on another property of BSs: the compactness. In Fig. 1(b) the compactness of equilibrium configurations is shown. Provided BSs have no defined surface we consider that the radius containing 95% ($R_{95}$) of the total particle number (following [15, 16]) is a reasonable place where to measure the gravitational field of the star; thus the compactness plotted in Fig. 1(b) is defined as $N_{95}/R_{95}$ where $N_{95}$ is the number of particles integrated up to $R_{95}$.

Figure 1. (a) Sequences of equilibrium configurations for different values of $\Lambda$ are shown as a function of the central value of the scalar field $\phi_0(0)$. The circles indicate the critical point that divides the stable from the unstable solutions. The inverted triangles indicate the point at which the binding energy is zero. Those configurations between the circles and the triangles (along each sequence) collapse into black holes even under infinitesimal perturbations (see [13] for the full track of the formation of an event horizon out of an unstable boson star). Configurations to the right of the triangles disperse away. (b) The compactness of each solution is shown. The critical point is also marked, since the interesting fakers would be those boson stars that are stable (to the left of the circles). It is therefore important noticing that stable solutions with small $\Lambda$ do not reach the compactness of those with bigger $\Lambda$. 
3. Time-like geodesics

Given the line element $ds^2 = -\alpha(r)^2 dt^2 + a(r)^2 dr^2 + r^2 d\Omega^2$ the equation for time-like geodesics followed by test particles of mass $m_p$ reads:

$$\dot{r}^2 + \frac{1}{a^2} \left( 1 + \frac{L^2}{r^2} \right) = \frac{E^2}{\alpha^2 a^2},$$  \hspace{1cm} (4)

where $L^2 = r^2 \dot{\varphi}^2 / m_p^2$ and $E^2 = -\dot{t}^2 / m_p^2$ are the squared angular momentum and energy at spatial infinity per unit of mass, which are the conserved quantities related to the independence on the azimuthal angle $\varphi$ and $t$ of the space-time respectively; it has been assumed the particle moves in the equatorial plane; an overdot indicates derivative with respect to the proper time of the test particle. The geodesics for a Schwarzschild BH are given by (4) with the values $\alpha^2 = a^{-2} = (1 - \frac{2M}{r})$. Since for BSs the equation (2) for $\alpha$ is linear, we rescale this function so that at infinity $\alpha(r \to \infty) = 1 / a(r \to \infty)$, thus at infinity $\alpha^2 = a^{-2}$, which implies the coefficient of $E^2$ in (4) is one for BSs too. The second term in (4) is usually interpreted as an effective potential $V_{eff}^2 = \frac{1}{a^2} (1 + L^2 / r^2)$, which determines the behavior and restrictions of the motion of the particle.

For a Schwarzschild BH we know there is a double minimum of $V_{eff}^2$ at $L^2 = 12M^2$ that marks the point for the innermost stable circular orbit of material particles. We choose this point to be the one indicating how much a particle behaves in the same way in a BS or in a BH. In Fig. 2(a) the different types of effective potentials obtained for different values of $L^2$ for a BS equilibrium configuration with $m = 1, \Lambda = 100, \phi(0) = 0.0555$ and $N_{0S}/R_{0S} = 0.222$ is shown. It can be noticed that when $L = 0$ the particle can have enough energy to pass the barrier at $r = 0$ because we demanded $a(0) = 0$; however the potential diverges at $r = 0$ for $L \neq 0$ and shows a barrier that cannot be passed over by any particle despite its energy. The curves corresponding to the black hole are constructed assuming the mass of the hole is that of the boson star, so that for an observer at infinity the mass cannot be specifically associated to a BH or to a BS.

The effective potential in (4) is the same from certain radius on for BSs and BHs. However we focus on the difference between the position of the minima for both systems when $L^2 = 12M^2$; in Fig. 2(a) the location of such minimum can be observed: the innermost stable circular orbit (ISCO) for the BH at $r/M = 6$ and the location of the minimum for the BS at $r/M \sim 10.4$. The ISCO is associated with the inner edge of the accretion disc around a black hole that is considered to be the source of electromagnetic radiation in black hole candidates; thus what matters here is the difference in the location of the minima (let us call it $\Delta_{ISCO}$) and for this we choose the case $L^2 = 12M^2$. In Fig. 2(b) such difference is plotted versus the compactness of the BS, the details are as follows: from Fig. 1(b) we know that for the four values of $\Lambda$ there are stable configurations with compactness 0.1 and 0.2 so that we can define $\Delta_{ISCO}$ for the four configurations and found to be $\sim 12M$ and $\sim 5M$ respectively; when we choose the compactness to be 0.3 there are stable configurations for only $\Lambda = 50, 100$ among the values used here, and therefore only for these two cases $\Delta_{ISCO}$ can be defined; the resulting value is $\sim 2.3$.

What we learn from Fig. 2(b) are two results: i) despite the value of $\Lambda$ the compactness is the property that determines how similar the geodesics of a BS are to those of a BH of the same mass and ii) combined with the results in Fig. 1(b) we know that the higher the value of $\Lambda$ of a stable BSs the bigger its compactness and therefore the smaller our $\Delta_{ISCO}$. 

Figure 2. (a) Behavior of $V_{eff}^2$ for the configuration $\Lambda = 100$, $\phi_0(0) = 0.0555$, and various values of $L^2$. (b) Difference between the location of the ISCO (in units of $M$) for BSs and BHs. This quantity is shown for different values of the compactness and $\Lambda$ of BSs. We see that despite the value of $\Lambda$ the compactness is the property that determines the location of the ISCO. The relevance of the self-interaction consists in allowing configurations of BS with a bigger compactness (as shown in Fig. 1(b)) in the stable branch. It is evident that compactness $\sim 0.3$ can only be achieved by solutions with $\Lambda = 50, 100$ (among the cases presented here).

4. Discussion
For an observer sufficiently far away from the region of a BSs the important quantity is the mass, so that the observer does not care too much whether the object in there is a boson star, a black hole or any other system possibly with an exotic equation of state. Going further, we are now in the next regime: the study of the behavior of test particles, whose velocity dispersion is assumed to be the source of electromagnetic radiation observed near compact objects, which indicates whether it is a black hole candidate or not [18]. For instance, a reason to consider there is a supermassive BH in active galactic nuclei has to do with indirect inferences implying high concentrations of mass in a little chunk of space, however nothing definitive about the nature of the object (like the presence of horizons [19]) is known$^1$.

Because boson stars are horizonless objects, the matter inside certain radius will be inevitably disrupted, and their massive constituents, instead of being sucked by a hole, they would be thrown out by the potential barrier (provided a non-zero angular momentum) or pass over the center if the trajectories are radial. Therefore the mass of the BS would remain constant and nothing like a growing event horizon would hold as expected to happen in black holes. This is a chance to distinguish between a boson star and a black hole from a geodesic analysis, since the luminosity in the x-ray range would be higher when there is a boson star since the particles are not falling inside a horizon, but they are bouncing out. A separate analysis on the emission spectrum of from accretion discs around BSs similar to that in [8] will restrict the ideal paremeters of BSs that are good candidate to supplant BHs [21].

Further studies in numerical relativity, will be related to the ring-down comparison between the stabilization of a BS and a BH. The emission of gravitational radiation coming out of perturbed boson stars (like the analysis in [6]) versus the emission of distorted black holes (see

$^1$ In this sense it is particularly remarkable the effort to indicate how the gravitational wave signals would be due to the accretion of a stellar mass object onto a -horizonless- supermassive BS [5].
The problem for a general perturbation of a BS is much more subtle, since it is expected radiation not only through gravitational waves, but also through scalar field radiation, and restrictions on the boson star parameters would come out [21].

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