A relativistic description of the $A(\pi^+, K^+)\Lambda A$ reaction

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Abstract
We investigate the $A(\pi^+, K^+)\Lambda A$ reaction within a covariant model. We consider those amplitudes which are described by creation, propagation and decay into relevant channel of $N^*(1650)$, $N^*(1710)$, and $N^*(1720)$ intermediate baryonic resonance states in the initial collision of the incoming pion with one of the target nucleons. The bound state nucleon and hyperon wave functions are obtained by solving the Dirac equation with appropriate scalar and vector potentials. Expressions for the reaction amplitudes are derived taking continuum particle wave function in the plane wave approximation. Numerical calculations are presented for reactions on $^{12}$C, $^{40}$Ca, $^{51}$V and $^{89}$Y target nuclei. The predictions of our model are in reasonable agreement with the available experimental data.

Keywords: strangeness production, pion-nucleus collisions, covariant model

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1. Introduction

Hypernuclei, where one or two nucleons ($N$) are replaced by hyperons ($Y$) in the bound orbits, provide a unique opportunity for studying a new form of the hadronic system which has strangeness degrees of freedom [1, 2, 3, 4]. Lambda ($\Lambda$) hypernuclei are the most familiar and extensively investigated hypernuclear systems. Since the $\Lambda$ hyperon does not suffer from Pauli blocking by other nucleons, it can penetrate deep inside the nucleus and form deeply bound hypernuclear states. Thus, hypernuclei can provide information about the nuclear states which are not accessible in ordinary nuclei. Such systems are perhaps the only tool currently available to get information about the $\Lambda-N$ interaction as $\Lambda$-nucleon scattering experiments are very difficult to perform due to short life time of the $\Lambda$ particle. During the past years, data on hypernuclear spectroscopy have been used extensively to extract information about the hyperon-nucleon interaction within a variety of theoretical approaches (see, e.g., Refs. [5, 6, 7, 8, 9, 10, 11, 12]).

A hypernuclei have been studied extensively by the stopped as well as the in-flight ($K^-, \pi^-$) reaction (see, e.g., the reviews [2, 3, 4, 13, 14, 15]) and also by the ($\pi^+, K^+$) reaction [16, 17, 18, 19]. The kinematical properties of the
The $K^-,\pi^-$ reaction allows only a small momentum transfer to the nucleus (at forward angles), thus there is a large probability of populating $\Lambda$-substitutional states ($\Lambda$ assumes the same orbital angular momentum as that of the neutron being replaced by it). On the other hand, in the $(\pi^+,K^+)$ reaction the momentum transfer is larger than the nuclear Fermi momentum. Therefore, this reaction can populate states with the configuration of an outer neutron hole and a $\Lambda$ hyperon in a series of orbits covering all the bound states having high spin natural parity configurations. The richness of the spectroscopic information on $\Lambda$ bound states in the $(\pi^+,K^+)$ reaction was demonstrated in the experiments performed at the Brookhaven National Lab and National Laboratory for High Energy Physics (KEK) (see, e.g., Ref. [4] for a comprehensive review). Furthermore, although the reaction cross section of the strangeness production, via the $(\pi^+,K^+)$ process, is smaller than that of the strangeness exchange reaction ($K^-,\pi^-$), the higher luminosity of pion beams makes experiments more feasible.

In the experimental studies reported in Refs. [18, 19], this reaction has been used to carry out the spectroscopic investigations of hypernuclei ranging from light mass $^\Lambda_\Lambda^12C$, to medium mass $^\Lambda_\Lambda^51V$ and $^\Lambda_\Lambda^89Y$ with the best resolution (~1.6–1.7 MeV) achieved in the spectrometer at KEK. This experiment has succeeded in clearly observing a characteristic fine structure in heavy systems by precisely obtaining a series of $\Lambda$ single-particle states in a wide range of excitation energies.

Most of the theoretical models used so far to describe the $(\pi^+,K^+)$ reaction employ a non-relativistic distorted wave impulse approximation (DWIA) framework [20] (see also Ref. [3] for a comprehensive review of these models). In these calculations, the $\Lambda$ bound states are generated by solving the Schrödinger equation with Woods–Saxon or harmonic oscillator potentials. However, for processes involving momentum transfers of typically 300 MeV/c or more, a non-relativistic treatment of the corresponding wave functions may not be adequate as in this region the lower component of the Dirac spinor is no longer negligible in comparison to its upper component (see, e.g., Ref. [21]).

In this paper, we study the $A(\pi^+,K^+)\Lambda A$ reaction within a fully covariant model by retaining the field theoretical structure of the interaction vertices and by treating the baryons as Dirac particles. In this model, the kaon production proceeds via the collision of the projectile pion with one of the target nucleons. This excites intermediate baryon resonance states ($N^*$) which decay into a kaon and a $\Lambda$ hyperon. The hyperon is captured in the respective nuclear orbit while the kaon rescatters onto its mass shell (see Fig. 1). A similar picture has been used to describe the $A(p,K^+)\Lambda B$ and $A(\gamma,K^+)\Lambda B$ reactions in Refs. [22, 23, 24]. In our model, the intermediate resonance states included are $N^*(1650)[{\frac{1}{2}}^+]$, $N^*(1710)[{\frac{1}{2}}^+]$, and $N^*(1720)[{\frac{3}{2}}^+]$ which have dominant branching ratios for the decay to the $K^+\Lambda$ channel [25, 26]. Terms corresponding to the interference among various resonance excitations are included in the total reaction amplitude.

In section 2 we present the details of our formalism for calculating the amplitudes corresponding to the diagrams shown in figure 1. In section 5
numerical results are presented for the \((\pi^+, K^+)\) reaction on \(^{12}\text{C}, \text{^{40}Ca}, \text{^{51}V}\) and \(\text{^{89}Y}\) targets using continuum wave functions in the plane wave approximation. Summary, conclusions and future outlook of our work are given in section 6.

2. Covariant model for the \(A(\pi^+, K^+)_{\Lambda A}\) reaction

The structure of our model for the \((\pi^+, K^+)\) reaction is described in Fig. 1. The \(N^*\) corresponds to the \(N^*(1650)\left[\frac{1}{2}^+\right], N^*(1710)\left[\frac{3}{2}^+\right],\) and \(N^*(1720)\left[\frac{3}{2}^+\right]\) baryon resonance intermediate states. Terms corresponding to interference between various amplitudes are retained. The elementary process involved in this reaction is shown in Fig. 2.

It is clear that our model has only \(s\)-channel resonance contributions. In principle, Born terms and the resonance contributions in \(u\)- and \(t\)-channels should also be included in description of both the processes depicted by Figs. 1 and 2. These graphs constitute the non-resonant background terms. Their magnitudes depend on particular models used to calculate them and also the parameters used in those models. This can be seen in Ref. [27] where the effect of the background terms is studied for \(S_{11}\) phase shifts in the pion-nucleon interaction. Whereas the contributions of the background terms are about 15–20\% of the resonance terms within a coupled-channel \(K\) matrix model, they are limited to less than 10\% in the model of Ref. [28] for the energies of our interest (\(\approx 1\) GeV/nucleon; the corresponding invariant mass is about 1.7 GeV) (see Fig. 15 of [27]). The effect of the background terms on the total production cross sections of the \(\pi^- p \rightarrow K^0\Lambda\) reaction at our beam energies can be indirectly inferred from the calculations reported in Ref. [29]. It is seen in Fig. 2 of this reference that the total production cross sections at invariant mass around 1.7 GeV are dominated by the contributions of \(S_{11}\) and \(P_{11}\) resonance terms. Therefore, the magnitudes of the background terms are likely to be limited to about 10–20\% of those of the resonance terms. Our results reported later on
in this paper may be uncertain by 10–20% due to omission of the background contributions.

2.1. Interaction Lagrangians

For the interaction terms of the spin-1/2 resonances, we have vertices of pseudoscalar (PS) or pseudovector (PV) form. The pseudovector coupling is consistent with the chiral symmetry requirement of the fundamental theory of strong interactions (quantum chromodynamics (QCD)). In contrast to that, the pseudoscalar one does not have this property, but it is easier to calculate. The couplings are in both cases fixed in such a way that they are equal on-shell; for off-shell cases, their difference is suppressed due to the denominator of the resonance propagator. It is, therefore, arguable which Lagrangian to use. The best approach would be to introduce a mixing parameter, which was investigated in Refs. [30, 24].

To avoid the introduction of additional parameters in our model due to a PS-PV mixing for the interaction Lagrangians of Refs. [30, 24], we use the convention of either choosing the PS or the PV couplings for these vertices as done in Refs. [27, 31]. This is in line with the studies reported in Refs. [32, 33].

The pseudoscalar interaction Lagrangians for the spin-1/2 resonances are given by

\[ \mathcal{L}^{PS}_{\pi NN'} = -g_{\pi NN'} \bar{\psi}_N \Gamma(\tau \cdot \phi_\pi) \psi_N + h. c. , \]  
(1a)

\[ \mathcal{L}^{PS}_{N^*1/2K} = -g_{N^*1/2K} \bar{\psi}_N \Gamma \psi_N + h. c. , \]  
(1b)

where the \( \Gamma \) takes care of parity conservation. We use

\[ \Gamma = \begin{cases} 1 & \text{for odd parity} \\ i\gamma^5 & \text{for even parity} \end{cases} , \]

and h. c. in Eqs. (1) denotes the hermitian conjugate.

The pseudovector Lagrangians involve the derivative of the pion wave function rather than the wave function itself. This introduces an additional mass dimension, which is taken care of by a “rescaling” of the coupling constant. It also ensures the matching of the on-shell behaviour the two types of Lagrangians. The pseudovector Lagrangians are given by

\[ \mathcal{L}^{PV}_{\pi NN'} = - \frac{g_{\pi NN'}}{m_N} \bar{\psi}_N \gamma^\mu \Gamma \partial_\mu (\tau \cdot \phi_\pi) \psi_N + h. c. , \]  
(2a)

\[ \mathcal{L}^{PV}_{N^*1/2K} = - \frac{g_{N^*1/2K}}{m_N} \bar{\psi}_N \gamma^\mu \Gamma \partial_\mu \phi_K \psi_N + h. c. , \]  
(2b)
Table 1: Coupling constants for various vertices used in the calculations.

| vertex               | coupling constant (g) |
|----------------------|-----------------------|
| $N^*(1650)N\pi$      | 0.81                  |
| $N^*(1650)\Lambda K^+$ | 0.76                  |
| $N^*(1710)N\pi$      | 1.04                  |
| $N^*(1710)\Lambda K^+$ | 6.12                  |
| $N^*(1720)N\pi$      | 0.21                  |
| $N^*(1720)\Lambda K^+$ | 0.87                  |

where $\Gamma$ is given by

$$
\Gamma = \begin{cases} 
  i & \text{for odd parity} \\
  \gamma^5 & \text{for even parity ,}
\end{cases}
$$

and the upper and lower signs are used for even and odd parity resonances, respectively.

The spin-$3/2$ resonance Lagrangians are given by

\[ L_{\pi N N^*_{3/2}} = \frac{g_{\pi NN^*}}{m_\pi} \bar{\psi}_N \tau \cdot \phi \psi_N + \text{h.c.} , \quad (3a) \]

\[ L_{N^*_{3/2}\Lambda K} = \frac{g_{N^*\Lambda K}}{m_K} \bar{\psi}_\Lambda \partial_\mu \phi \psi_N + \text{h.c.} . \quad (3b) \]

Such a form of the coupling was used for studying the hypernuclear production also in other reactions [21, 25]. The values and signs of the various coupling constants have been taken from Refs. [25, 22] and are shown in Table 1. These parameters describe well the associated $K^+\Lambda$ production in proton-proton collisions within a similar resonance picture. All the pion-resonance-kaon vertices that are of interest in this paper are involved in this reaction. Thus the vertex parameters used by us inherently describe the elementary process shown in Fig. 2.

2.2. Resonance propagators

The two interaction vertices of Figure 2 are connected by a resonance propagator. For the spin-$1/2$ and spin-$3/2$ resonances the propagators are given by

\[ D_{1/2} = i \frac{\gamma^\mu p^\mu + m}{p^2 - (m - i\Gamma N^*/2)^2} \]

and

\[ D_{3/2}^{\mu\nu} = -i \frac{\gamma^\lambda p^\lambda + m}{p^2 - (m - i\Gamma N^*/2)^2} P^{\mu\nu} , \]

respectively. In Eq. 5 we have defined

\[ P^{\mu\nu} = \eta^{\mu\nu} - \frac{1}{3} \gamma^{\mu} \gamma^{\nu} - \frac{2}{3m^2} p^{\mu} p^{\nu} + \frac{1}{3m} (p^{\mu} \gamma^{\nu} - p^{\nu} \gamma^{\mu}) . \]

5
\( \Gamma_N \) in Eq. (4) and Eq. (5) is the total width of the resonance. It is introduced in the denominator term to account for the finite life time of the resonances for decays into various channels. This is a function of the centre of mass momentum of the decay channel, and it is taken to be the sum of the widths for pion and rho decay (the other decay channels are considered only implicitly by adding their branching ratios to that of the pion channel) [23]. We have not introduced any correction in the resonance propagators to account for the nuclear medium effects as no major change is expected in our results due to these effects. As pointed out in Refs. [34, 35], the medium correction effects on the widths of the s- and p-wave resonances, which make the dominant contribution to the cross sections investigated here, are not substantial. The reason for this is that resonances occur only as intermediate states which implies an integration over their respective spectral distributions.

3. Nuclear model

The spinors for the final bound hypernuclear state (corresponding to momentum \( p_\Lambda \)) and for the intermediate nucleonic state (corresponding to momenta \( p_N \)) are required to perform numerical calculations of various amplitudes. We assume these states to be of pure-single particle or single-hole configurations with the core remaining inert. In experimental measurements, however, core excited states have also been detected (see, e.g., [4]). A covariant description of the core polarisation can, in principle, be achieved by following the method discussed, e.g., in Ref. [36]. This procedure is somewhat tedious and is out of the scope of our present study. Therefore, in this paper we concentrate on those transitions which involve pure single-particle and single-hole states.

The spinors in momentum space are obtained by the Fourier transformation of the corresponding coordinate space spinors which are solutions of the Dirac equation with potential fields consisting of an attractive scalar part \( (V_s) \) and a repulsive vector part \( (V_v) \) having a Woods–Saxon form. This choice appears justified as the Dirac–Hartree–Fock calculations in Refs. [37, 38] suggest that these potentials tend to follow the nuclear shape. The same potential form has also been used in the relativistic one-nucleon model [39, 40] and two-nucleon model calculations [21] of the \((p, \pi)\) reaction.

3.1. Nucleon bound states

In our approach for describing the nucleon and hyperon wave functions, we begin with the Dirac equation which is modified by introducing a scalar and a time-like vector potential. The modified Dirac equation with potentials is written as

\[
(i\gamma^\mu \partial_\mu - m - \gamma^0 V_s - V_v) \psi = 0.
\]  

(7)

The solution, transformed to momentum space, can be written as in Ref. [23]

\[
\hat{\psi}(\hat{p}) = \delta(p_0 - E) \left( \hat{f}_{n,j}(k) \hat{\chi}^{jm,j}_{\ell s}(\hat{p}) \right.
\]

\[
\left. - i\hat{g}_{n,j}(k) \hat{\gamma}^{jm,j}_{\ell s}(\hat{p}) \right),
\]

(8)
Table 2: Potential parameters of the nuclear vector and scalar potentials. For $^{12}$C and $^{40}$Ca, the potentials are fitted to the experimental data on the charge radius, the nucleon separation energy, and the first diffraction minimum of the charge form factor for the nucleon (Ref. [41]). For $^{51}$V and $^{89}$Y, the potential depths are fitted to the neutron separation energies with radial parameters being the same as those of $^{40}$Ca.

| nucleus | $V_\nu$ [MeV] | $r_{0\nu}$ [fm] | $a_\nu$ [fm] | $V_s$ [MeV] | $r_{0s}$ [fm] | $a_s$ [fm] |
|---------|----------------|----------------|-------------|-------------|----------------|-------------|
| $^{12}$C | 385.7          | 1.056          | 0.427       | -470.4      | 1.056          | 0.447       |
| $^{40}$Ca | 348.1          | 1.149          | 0.476       | -424.5      | 1.149          | 0.506       |
| $^{51}$V | 309.7          | 1.149          | 0.476       | -382.3      | 1.149          | 0.506       |
| $^{89}$Y | 317.8          | 1.149          | 0.476       | -392.3      | 1.149          | 0.506       |

where ‘$p$’ is the four momentum vector of the particle. ‘$p$’ represents the corresponding three momentum; its magnitude $|p|$ is denoted by $k$ and its direction by $\hat{p}$. Since we work in momentum space throughout this work, we will drop the hats from the Fourier-transformed functions $\psi$, $f$, and $g$.

The spin-spherical harmonics $Y_{j\ell s}^{m}(\hat{x})$ are given by

$$Y_{j\ell s}^{m}(\hat{x}) = \sum_{m_\ell, m_s} \langle \ell, m_\ell, s, m_s | j, m_j \rangle Y_{\ell m_\ell}(\hat{x}) \chi_{s, m_s}, \quad (9)$$

where $\chi$ is the usual two-dimensional Pauli-spinor and $Y_{\ell m_\ell}$ are the spherical harmonics of the first kind, $\ell' = 2j - \ell$, and $m_\ell + m_s = m_j$, and both are coupled to good total angular momentum.

The potentials in Eq. (7) have radial shapes of the Woods–Saxon type. The depths of the potential $V_{0\alpha}$, the radius $R_\alpha = r_{0\alpha} A^{1/3}$, and the diffuseness $a_\alpha$ are treated as free parameters which are fixed by fitting to the experimental data of the charge radius, the nucleon separation energy, and the first diffraction minimum of the charge form factor for the nucleon (see Ref. [41]). Table 2 lists the potential parameters used for the nuclear bound states.

3.2. Hyperon bound states

The $\Lambda$, in contrast to a nucleon, can occupy any bound state since it is not subject to the Pauli exclusion principle with respect to the nucleons. This property makes it an excellent probe for single bound states in a nuclear potential.

In this case too, the spinors in the momentum space were obtained by Fourier transforming the corresponding coordinate space spinors which are solutions of the Dirac equation with potential fields consisting of an attractive scalar part and a repulsive vector part having a Woods-Saxon form. With a fixed set of the geometry parameters (reduced radii $r_s$ and $r_v$ and diffusenesses $a_s$ and $a_v$), the depths of the potentials were searched in order to reproduce the $\Lambda$ separation energy of the particular state (the corresponding values are given in table 3 for $^{12}$C and $^{51}$V, and in table 5 for $^{89}$Y). We use the same geometry for the scalar and vector potentials. The depths of potentials $V_s$ and $V_v$ were further constrained by requiring that their ratios are equal to $-0.81$ as suggested in Ref. [42]. Experimental inputs have been used for the configurations and the single baryon
Table 3: Potential parameters of the Λ vector and scalar potentials of the $^{12}\Lambda\,^{12}C$ and the $\Lambda\,^{51}V$ hypernuclei. The $^{12}\Lambda\,^{12}C$ geometry is fixed to $r_{0v} = 1.1486\,\text{fm}$, $a_{0v} = 0.3960\,\text{fm}$ and $r_{0s} = 1.1207\,\text{fm}$, $a_{0s} = 0.4764\,\text{fm}$. The $\Lambda\,^{51}V$ geometry is fixed to $r_{0v} = 0.9827\,\text{fm}$, $a_{0v} = 0.5779\,\text{fm}$ and $r_{0s} = 0.9825\,\text{fm}$, $a_{0s} = 0.6064\,\text{fm}$.

| orbital          | $E_{\text{bind}}$ [MeV] | $V_{0v}$ [MeV] | $V_{0s}$ [MeV] |
|------------------|--------------------------|----------------|----------------|
| $^{12}\Lambda\,^{12}C(s_{1/2})$ | $10.79 \pm 0.11$ | $171.5230$ | $-211.7654$ |
| $^{12}\Lambda\,^{12}C(p_{3/2})$ | $0.10 \pm 0.04$ | $171.5230$ | $-211.7654$ |
| $\Lambda\,^{51}V(s_{1/2})$ | $19.75$ | $151.2006$ | $-186.6674$ |
| $\Lambda\,^{51}V(p_{3/2})$ | $11.75$ | $171.2202$ | $-211.3830$ |
| $\Lambda\,^{51}V(d_{5/2})$ | $3.75$ | $197.5942$ | $-243.9435$ |

binding energies of the states in hypernuclei $^{12}\Lambda\,^{12}C$ in Refs. [3, 13, 14], $\Lambda\,^{51}V$ in Ref. [3], and $^{89}\Lambda\,^{89}Y$ in Ref. [3]. However, the single baryon binding energies for states in $^{40}\Lambda\,^{40}\text{Ca}$ hypernucleus were taken from the density dependent relativistic hadron field (DDRH) theory predictions of Refs. [11, 12], which reproduce the corresponding experimental binding energies reasonably well. In this case, we have compared, for each state, the spinors calculated by our well-depth search method with those calculated within the DDRH theory (see Refs. [11, 12]) and find an excellent agreement between the two.

The upper and lower components of the Dirac spinors for the Λ bound states in momentum space are shown in figures 3–5 for $^{12}\Lambda\,^{12}C$, $\Lambda\,^{51}V$, and $^{89}\Lambda\,^{89}Y$, respectively. We note that in each case, only for momenta $q < 2\,\text{fm}^{-1}$ is the lower component of the spinor substantially smaller than the upper component. In the region of momentum transfer pertinent to exclusive kaon production in pion-nucleus collisions, the lower components of the spinors are not negligible as compared to the upper component which clearly demonstrates that a fully relativistic
Table 4: Potential parameters of the Λ vector and scalar potentials of the $^{89}\Lambda$Y hypernucleus. The geometry is fixed to $r_{0v} = 0.9827$ fm, $a_{0v} = 0.5779$ fm and $r_{0s} = 0.9825$ fm, $a_{0s} = 0.6064$ fm.

| orbital $^{89}\Lambda$Y | $E_{\text{bind}}$ [MeV] | $V_0v$ [MeV] | $V_0s$ [MeV] |
|------------------------|------------------------|---------------|---------------|
| $s_{1/2}$              | 23.1                   | 149.3054      | -184.3276     |
| $p_{1/2}$              | 16.5                   | 171.1863      | -211.3411     |
| $p_{3/2}$              | 16.5                   | 165.3949      | -204.1913     |
| $d_{3/2}$              | 10.0                   | 208.0682      | -256.8744     |
| $d_{5/2}$              | 10.0                   | 190.7247      | -235.4626     |
| $f_{5/2}$              | 2.3                    | 253.8945      | -313.454      |
| $f_{7/2}$              | 2.3                    | 214.0216      | -264.2242     |

approach is better for an accurate description of this reaction.

4. Calculation of the cross section

We use the subscripts $\pi$, $K$, $A$, and $B$ to denote the quantities of the incoming pion, the outgoing kaon, the target nucleus, and the final hypernucleus, respectively. The differential cross section for the ($\pi^+, K^+$) reaction is given by

$$d\sigma = \frac{1}{(2\pi)^2} \frac{d^3p_K}{2E_K} \frac{d^3p_B}{2E_B} \frac{m_A m_B}{|p_s| \sqrt{s}} \left| \sum_{R_i} M_{R_i} \right|^2 \delta^{(4)} (p_s + p_A - (p_K + p_B)) \ . \ (10)$$

The summation is carried out over initial ($m_i$) and final ($m_f$) spin states; $\sum_{R_i}$ indicates the summation over all three resonances.
After having established the effective Lagrangians and the coupling constants, one can write down, by following the well known Feynman rules, the amplitudes for the graph shown in figure 2. The isospin part is treated separately which gives rise to a constant factor for each graph. Thus, the matrix element for our process is given by

$$M = \int \frac{d^4k_N}{(2\pi)^4} \int \frac{d^4k_A}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4} \phi^*_K(p - k_A) \bar{\psi}_A(k_A) \Gamma_\alpha$$
$$\times i^\gamma_{\mu} p^\mu + m_N \cdot \Gamma_\beta \phi_\pi(p - k) \psi_N(k_N).$$

(11)

In Eq. (11), the factors $\Gamma_\alpha$ and $\Gamma_\beta$ are given by the interaction Lagrangians from Eqs. (1)–(3). The $\phi$s denotes the meson wave functions, and the $\psi$s are the solutions of the in-medium single-particle Dirac equations given in sections 3.1 and 3.2.

The incident pion and outgoing kaon fields are given by

$$\phi^{(\pm)}_\pi(p'_\pi) = \delta(p'_\pi - E_\pi - r_{\pi 0}) \sum_{\ell_{\pi m_\pi}} (-1)^{\ell_{\pi}} Y_{\ell_{\pi m_\pi}}^*(\hat{p}_\pi) Y_{\ell_{\pi m_\pi}}(\hat{p}'_\pi)$$
$$\times f_{\ell_{\pi}}(k'_\pi, k_\pi),$$

$$\phi^{(-)}_K(p'_K) = \delta(p'_K - E_K) \sum_{\ell_{K m_K}} (-1)^{\ell_{K}} Y_{\ell_{K m_K}}(\hat{p}_K) Y_{\ell_{K m_K}}^ *(\hat{p}'_K)$$
$$\times f_{\ell_{K}}(k'_K, k_K).$$

(12)

Figure 5: The $^{89}\Lambda Y$ bound states in momentum space.
where $E_{\pi}$ and $E_{K}$ represents the energies of the incident pion and outgoing kaon, respectively; $p_{K}$ and $p_{\pi}$ denote the meson on-shell momenta. The functions $f_{\ell}$ are given by

$$f_{\ell}(k') = \frac{1}{2\pi^2} \int_{0}^{\infty} j_{\ell}(kr)f_{\ell}^{C}(r)r^2 dr,$$

where the wave function $f_{\ell}^{C}$ is the coordinate space solution of the Klein–Gordon equation with a meson-nucleus optical potential (see, e.g., Refs. [43, 44]).

It should be mentioned here that due to the oscillatory nature of the wave functions in the asymptotic region, the integrals involved in Eq. (12) and in Eq. (13), and in a similar fashion the radial components of the bound state wave functions in Eq. (8), converge poorly. Such integrals can, however, be calculated very accurately by using a contour integration method as in Ref. [45].

In the plane wave approximation, the wave functions $\phi_{\pi}^{(+)}$ and $\phi_{K}^{(-)*}$ are given by

$$\phi_{\pi}^{(+)}(p'_{\pi}) = \delta^4(p'_{\pi} - p_{\pi})$$

and

$$\phi_{K}^{(-)*}(p'_{K}) = \delta^4(p'_{K} - p_{K}).$$

Thus, the integration over $k_{N}$ and $k_{\Lambda}$ become redundant. This not only reduces the dimensionality of the integrations by a factor of eight, but also removes the requirement of partial wave summations altogether.

For a fully dynamical description of the production of hypernuclei, however, we have to consider the interactions of the incoming and outgoing particles with the nucleus. In this exploratory study, however, we would like to make calculations for very many cases involving a variety of targets and beam energies in order to understand the basic mechanism of this reaction and to get the relative estimates of the order of magnitudes of various cross sections within a covariant model. Therefore, we restrict ourselves to the plane wave treatment of the scattering states in the initial and final channels. The weak mutual interaction (kaon-nucleus) in the final channel and higher energies of the projectile in the initial channels do provide support to this choice. Indeed, it has been shown in Ref. [46] that in the non-relativistic impulse approximation calculations the pattern of the spectra look similar in the PW and DW calculations—only the absolute magnitudes of the cross sections are affected by the initial and final channel meson-nucleus interactions. Indeed, distortion effects from an optical potential $U_{\text{opt}}$ should contribute in the leading order by $O(|U_{\text{opt}}/E|)$, which is well below unity at the energies around $1.0\text{ GeV}$ considered in this work.

Nevertheless, we have made an estimate of the influence of the initial state interaction (pion-nucleus channel) on the magnitudes of the $(\pi^+, K^+)$ cross sections within an eikonal approximation.

### 4.1. The eikonal approximation for the pion-nucleus interaction

The eikonal approximation (see Refs. [47, 48]) has been quite successful in describing the scattering of pions on nuclei at higher incident energies [49, 50].
In this approximation, the relative motion wave function is given by (see for example Ref. [48])

\[
\phi(x) = \exp \left\{ i k \cdot x - \frac{i}{v} \int_{-\infty}^{z} V(b, z') \, dz' \right\},
\]

(17)

where \( k \) is the incident (asymptotic) momentum of the particle, \( v = |k|/m \) is the magnitude of the incident velocity, and \( b \) is the (two-dimensional) impact vector in cylindrical coordinates. The optical potential is, in general, a complex function \( V = U - iW \) which results in a phase factor from the real part \( U \) and an amplitude reduction from the imaginary part \( W \).

Since at higher energies several effects are suppressed, a simple form for the optical potential can be used. For example, the \( tp \)-approximation (see, e.g., Ref. [48]) relates the potential to the free-space single-particle scattering amplitudes (or to the total cross section) and the density. The optical potential in this case is given by

\[
V_{\text{opt}}(x) = -\frac{4\pi}{2E_{\text{lab}}} \left[ f_{mp} \rho_p(x) + f_{mn} \rho_n(x) \right]
\]

\[
= -\frac{k}{2E_{\text{lab}}} \left[ i\sigma_{\text{tot}}^{mp} (1 - i\gamma_{mp}) \frac{Z}{A} + i\sigma_{\text{tot}}^{mn} (1 - i\gamma_{mn}) \frac{N}{A} \right] \rho(x),
\]

(18)

where \( f_{mp} \) and \( f_{mn} \) are the elementary free-space meson-proton and meson-neutron scattering amplitudes, respectively. Applying the optical theorem, they can be substituted by the total cross sections \( \sigma_{\text{tot}}^{mp} \) and \( \sigma_{\text{tot}}^{mn} \). The respective ratios of the real to the imaginary part of the scattering amplitudes are denoted by \( \gamma_{mp} := \Im f_{mp}/\Re f_{mp} \) and \( \gamma_{mn} := \Im f_{mn}/\Re f_{mn} \). We have also separated the neutron and proton contributions to the potential, whereby \( Z \) is the proton number, \( N \) is the neutron number, and \( A \) is the total number of nucleons.

For spherically symmetric nuclei the density, and hence the potential, depends only on the magnitude of \( x \), \( r := |x| \). In these cases we can integrate Eq. (17) by rewriting the argument of the potential as

\[
V(r) = V(b, z) = V \left( \sqrt{b^2 + z^2} \right).
\]

(19)

We approximate the density by relatively simple parameterisations which are easier and faster to compute numerically. For light nuclei \( (A \leq 16) \), a modified Gaussian shape has been used,

\[
\rho_{\text{G}}(r) = \frac{1}{(\sqrt{\pi}R_{\text{G}})^2} \left[ 4 + \frac{2(A - 4)}{3} \frac{r^2}{R_{\text{G}}^2} \right] e^{-\frac{r^2}{R_{\text{G}}^2}},
\]

(20)

with \( R_{\text{G}} \) being the radial parameter. For heavier nuclei \( (A > 16) \), a Woods–Saxon shaped density with the radial parameter \( R \) and the \textit{diffuseness} parameter \( a \) has been used,

\[
\rho_{\text{WS}}(r) = \frac{\rho_0}{1 + \exp \left\{ \frac{r - R}{a} \right\}}.
\]

(21)
Table 5: Density parameters for the eikonal approximation for $^{12}$C and $^{40}$Ca, fitted to the nucleon wave functions and the elastic scattering cross sections.

|       | $A$ | $Z$ | Woods–Saxon | Gauss |
|-------|-----|-----|-------------|-------|
|       | $R$ [fm] | $a$ [fm] | $R_G$ [fm] |
| $^{12}$C | 12 | 6 | 2.24 | 0.46 | 1.6 |
| $^{40}$Ca | 40 | 20 | 3.49 | 0.547 | 2.08 |

The densities are normalised to the total nucleon number, such that

$$\int_{R^3} \varrho(x) \, d^3x = A,$$  \hspace{1cm} (22)

which is already fulfilled in the Gaussian case. In the Woods–Saxon case this determines $\varrho_0$, which is given by Refs. [51, 52]

$$\varrho_0 = \frac{3A}{4\pi R^3 1 + \left(\frac{\pi a}{R}\right)^2}.$$  \hspace{1cm} (23)

We fit the parameters $R_G$, $R$ and $a$ to the radial densities. The fitted values for both density approximations for $^{12}$C and $^{40}$Ca are given in table 5.

5. Results and discussion

5.1. Cross sections for the $A(\pi^+, K^+)\Lambda A$ reaction

First, we present our results for the $^{12}$C($\pi^+, K^+)^{12}$C reaction at an incident pion lab momentum ($p_{\text{lab}}$) of 1.05 GeV. In the left panel of Fig. 6 we show a comparison of our calculations with the experimental data (taken from Ref. [19]) for the $K^+$ angular distribution of the $^{12}$C($\pi^+, K^+)^{12}$C reaction where the hypernuclear state has the $[(p_{3/2}^{-1/2})N, (s_{1/2})\Lambda]$ configuration. The leading contribution to this transition comes from the $1^-$ ground state of $^{12}$C corresponding to the first peak seen in the hypernucleus spectrum. Also shown in this figure are the contributions of the individual baryon resonances. We note that while the contributions of the $N^*(1650)$ and $N^*(1710)$ resonances are almost identical, those of the $N^*(1720)$ state is weaker by factors of 3–5. The dominant contribution of the two spin-$1/2$ resonances was also noted in case of the hypernuclear production reactions studied via ($\pi^+, K^+$) and ($\gamma, K^+$) reactions in Refs. [22, 53] within a model similar to that employed in this paper. We also note that interference effects of the resonances are important as their individual contributions do not sum up to the total cross sections shown by the solid lines. We see that the agreement between the calculations and the experimental data is better at forward angles while at higher angles the calculation overestimate the data. This trend was also noted in the non-relativistic distorted wave impulse approximation (DWIA) calculations as shown in Ref. [19]. This indicates that at larger momentum transfers the simple no-core excitation picture may
Figure 6: The differential cross section for the $\pi^+ + ^{12}\text{C} \rightarrow K^+ + ^{12}\text{C}$ reaction leading to states of the hypernucleus $^{12}\text{C}$ with $[(p_{3/2})_N, (s_{1/2})_\Lambda]$ (left panel) and $[(p_{3/2})_N, (p_{3/2})_\Lambda]$ (right panel) configurations. The lab momentum of the incoming pion ($p_{\text{lab}}$) is 1.05 GeV.

not be adequate. Indeed, in Ref. [4] it is shown that agreement of the DWIA calculations with the data improves if contributions from the core excited states are also included in the calculated cross sections. There could also be a weak contribution (about 10%) from the $2^-$ member of the multiplet.

The next $\Lambda$-bound state in $^{12}\text{C}$ has the configuration $[(p_{3/2})_N, (p)_{\Lambda}]$ which according to Ref. [19] is just bound by about 0.1 MeV. In the right panel of Fig. 6 we compare the calculated differential cross section for this transition with the data (set 5 of Ref. [19]). The theoretical results correspond to the natural parity $2^+$ state of the hypernucleus. In this case our calculations underestimate the data at very forward angles. It should, however, be noted that the data could also have contributions from other members of the $[(p_{3/2})_N, (p)_{\Lambda}]$ configuration and also from the core excited configurations. Nevertheless, the DWIA calculations, which include contributions from such configurations, overestimate the data for this transition [19] in the entire angular range.

In Fig. 7 we show the contributions of various baryonic resonances to the angular distribution of the $\pi^+ + ^{40}\text{Ca} \rightarrow K^+ + ^{40}\text{Ca}$ reaction corresponding to the $2^+$ state of $^{40}\text{Ca}$ having a $[(d_{3/2})_N, (s_{1/2})_\Lambda]$ configuration. The corresponding binding energy was taken to be 20 MeV which is consistent with the experimental value reported in Ref. [54]. We note that like the $^{12}\text{C}$ case the two spin-1/2 resonances contribute almost equally to the cross sections and both are larger than the contribution of the spin-3/2 one. Detailed experimental data are not available for this case. The single point reported in Ref. [54] corresponds to the differential cross section at zero angle which is $\sim 9$ mb/sr. Our result for this case is close to this value.

In Fig. 8 we show the differential cross section for the same reaction as that in the left panel of Fig. 6 as function of the magnitude of the momentum transfer to the nucleus ($q = p_\pi - p_K$) for several beam energies. We see that shapes of the differential cross sections are qualitatively different from each other at different beam energies. However, their absolute magnitudes differ at forward angles which is more prominent as the beam energy is increased from 1.0 GeV.
to 1.4 GeV. However, for beam energies > 1.4 GeV this difference is small. At larger momentum transfers the cross sections are too small to be amenable to measurements. Thus to get the larger cross section (i.e. counting rates) forward angles experiments (where $q$ is smaller) with beam energies around 1 GeV appear to be favourable.

In Fig. 9 we show the dependence of the total cross section on $p_{lab}$ for the reactions studied in Figs. 6 and 7. We note that in both the cases, as beam energy increases beyond the threshold, the cross sections first increases rapidly and after peaking around a given value starts decreasing slowly. The beam momentum where the cross section peaks is around 1.0 GeV. This observation was also made in the DWIA study of this reaction in Ref. [20]. This further highlights the point made in Fig. 8.

Next we examine the role of the bound state relativistic effects on the cross sections. We would like to point out that since our calculations have been performed in momentum space, there is no need to introduce any local approximation to the propagators appearing in the production amplitudes—most of the non-relativistic calculations necessarily make such approximation. In Fig. 10 we show the role of the lower component of the Dirac bound state wave functions ($g$) on the angular distributions of the ($\pi^+, K^+$) reaction on the $^{12}$C and $^{89}$Y targets for the transitions as depicted in this figure. We see that effect of $g$ is more pronounced at larger angles. It should, however, be noted that the results with upper components only can not be directly equated with those of the conventional approaches that employ the Schrödinger equation to describe the bound state wave functions. One requires the matrix elements and particularly the operator that is given in terms of the Dirac matrices to undergo a

![Figure 7: The differential cross section for $\pi^+ + ^{40}$Ca $\rightarrow K^+ + ^{40}$Ca at a pion incoming momentum of $p_{lab} = 1.05$ GeV as a function of the kaon angle $\theta$.](image-url)
Figure 8: The differential cross section for the same reaction as in the left panel of Fig. 6 for some selected values of the incoming pion momentum as a function of the momentum transfer $q$.

Figure 9: The total cross section for the $[\left(\frac{1}{2}\right)_N, (\frac{1}{2})_\Lambda]$ transition in the $\pi^+ + ^{12}\text{C} \rightarrow K^+ + ^{13}\text{C}$ reaction (left panel), and the $[\left(\frac{1}{2}\right)_N, (\frac{3}{2})_\Lambda]$ transition in the $\pi^+ + ^{40}\text{Ca} \rightarrow K^+ + ^{40}\text{Ca}$ reaction (right panel) as a function of the incident pion momentum $p_{\text{lab}}$. 

Figure 10: The differential cross section for the same reaction as in the left panel of Fig. 6 and for the \([\|(g_{9/2})_N, (f_{7/2})_\Lambda\|]\) configuration in the \(\pi^+ + ^{89}Y \rightarrow K^+ + ^{89}\Lambda Y\) reaction. Shown are the results including the full relativistic spinors (solid lines) and without the lower components (dotted lines) as a function of the kaon angle \(\theta\).

non-relativistic reduction. Such a reduction has been carried out in Refs [55, 56] where it is shown that for the hypernuclear production by the \((\gamma, K^+)\) reaction, the difference between the relativistic and non-relativistic cross sections could be about 20% even in the forward directions.

5.1.1. Spectral distributions for \(^{51}_\Lambda V\) and \(^{89}_\Lambda Y\) hypernuclei

In Figs. 11 and 12 we show the comparison of the calculated spectral distributions for the \(^{51}_\Lambda V\) and \(^{89}_\Lambda Y\) hypernuclei produced in the \((\pi^+, K^+)\) reaction on \(^{51}V\) and \(^{89}Y\) targets at the beam momentum of 1.05 GeV. In drawing the smooth spectral distributions, each level is a convolution with the appropriate Gaussian width \(\Gamma\) depending on its character in the experimental data. We used only a single Gaussian for all the levels in contrast to Ref. [19], where more than one Gaussian were used for some levels.

In \(^{51}_\Lambda V\) case the series of levels are obtained by the following configurations and binding energies \((E_{\text{bind}})\): \([f_{7/2}^{-1}_N, (s_{1/2})_\Lambda]\) \((3^-, E_{\text{bind}} = 19.75 \text{ MeV})\), \([f_{7/2}^{-1}_N, p_\Lambda]\) \((4^+, E_{\text{bind}} = 11.75 \text{ MeV})\), and \([f_{7/2}^{-1}_N, d_\Lambda]\) \((5^-, E_{\text{bind}} = 3.75 \text{ MeV})\). These states make the largest contributions to the corresponding cross sections. On the other hand, the levels in \(^{89}_\Lambda Y\) are obtained with the configuration where the neutron hole state corresponds to the \(g_{9/2}\) orbit with the \(\Lambda\) in the \(0s, 0p, 0d\) and \(0f\) orbitals. The configurations of these levels are: \([\bar{g}_{9/2}^{-1}_N, (0s)_\Lambda]\) \((4^+, E_{\text{bind}} = 23.6 \text{ MeV})\), \([\bar{g}_{9/2}^{-1}_N, (0p)_\Lambda]\) \((5^-, E_{\text{bind}} = 16.5 \text{ MeV})\), \([\bar{g}_{9/2}^{-1}_N, (0d)_\Lambda]\) \((6^+, E_{\text{bind}} = 10.0 \text{ MeV})\), \([\bar{g}_{9/2}^{-1}_N, (0f)_\Lambda]\) \((7^-, E_{\text{bind}} = 2.3 \text{ MeV})\).

We see that for both nuclei the calculated spectral distributions reproduce the overall global trends of the data of Ref. [19] reasonably well. Our calculations
Figure 11: The spectral distribution for the $^5\Lambda V$ hypernucleus.

Figure 12: The total averaged cross section for the $^{89}\Lambda Y$ hypernucleus.
reproduce the experimentally observed three and four clear peaks in the $^{51}\Lambda V$ and $^{89}\Lambda Y$ spectra, respectively. However, looking more closely we note that the theory overestimates the experimental cross sections for the $(0s)_\Lambda$ orbitals somewhat in both cases. For these states, the corresponding binding energies ($E_{\text{bind}}$) are quite large and for these strongly mismatched cases, distortion effects could play a more significant role.

In both cases, one observed some minor peaks which fill up the gaps between the major peaks. These peaks correspond to core excited states and the states corresponding to other, less dominant, members of the configurations mentioned above. We have not made any attempt to fit to these peaks in this exploratory study. Clearly, a quantitative description of the spectral shapes of the heavier mass $\Lambda$ hypernuclei requires proper consideration of the core excitation and mixing of states of different parity.

5.2. Effects of initial and final state interactions

The calculations presented thus far have been done in a plane wave approximation, where the pion-nucleus and kaon-nucleus interactions are ignored. While the essential features of the high momentum transfer reaction ($\pi^+, K^+$) can be understood in this approach, the nuclear interactions may have some consequences. They produce both absorptive and dispersive effects Ref. [20]. However, for the large incident energies considered in this calculation, the absorption effects are likely to be the more important.

One should also note that in the distorted wave treatment the continuum wave functions are no longer associated with sharp momenta but are states with a momentum distribution, see Eqs. (12), and (13). This leads to a redistribution of the momentum transfer differently from what is allowed in the plane wave approximation. It could shift the sensitivity of the model to even lower momenta leading to enhanced cross sections. The competition between this effect and the absorption effect would ultimately decide the role of distortions in these reactions.

We have estimated the role of the distortion effects in the initial channel on the absolute magnitude of the cross sections. For this purpose, we make use of the eikonal approximation as discussed in section 4.1. Cross sections have been calculated in both PW and eikonal approximation at one most forward angle for $^{12}\Lambda C(\pi^+, K^+)^{12}\Lambda C$, $^{40}\Lambda Ca(\pi^+, K^+)^{40}\Lambda Ca$ and $^{89}\Lambda Y(\pi^+, K^+)^{89}\Lambda Y$ reactions. The results are shown in table 6.

We see that the distortion effects in the incident channel lead to the reduction of the peak cross sections by factors of 1.2 to 12 as the target mass varies from 12 to 89. Our result for the $^{12}\Lambda C$ target is in contrast to that of the non-relativistic impulse model of Ref. [20], where the distortion effects were found to reduce the differential cross section by about an order of magnitude for this target. The distortion effects play an increasingly important role with increasing mass of the target nucleus. This result is in agreement with those of Refs [23], and [56].
Table 6: The differential cross sections at selected angles for the plane wave approximation and the distorted wave calculations for the pion in the eikonal approximation.

| nucleus  | transition          | $d\sigma/d\Omega$ [µb/sr] | plane wave | eikonal |
|----------|---------------------|---------------------------|------------|---------|
| $^{12}$C ($\theta_K = 1^\circ$) | $np_{3/2} \rightarrow \Lambda s_{1/2}$ | 16.55 | 13.21 |
|          | $np_{3/2} \rightarrow \Lambda p_{3/2}$ | 10.16 | 7.10  |
| $^{40}$Ca ($\theta_K = 4^\circ$) | $nd_{3/2} \rightarrow \Lambda s_{1/2}$ | 15.55 | 3.18  |
| $^{89}$Y ($\theta_K = 4^\circ$) | $ng_{9/2} \rightarrow \Lambda s_{1/2}$ | 7.43  | 0.642 |

6. Summary and conclusions

In summary, we studied the $A(\pi^+, K^+)\Lambda A$ reaction on $^{12}$C, $^{40}$Ca, $^{51}$V and $^{89}$Y targets within a fully covariant model, where in the initial collision of incident pion with one of the target nucleons the $N^*(1650)$, $N^*(1710)$, and $N^*(1720)$ intermediate baryon resonance states are excited which subsequently propagate and decay into the relevant channel. We have retained the full field theoretic forms of various interaction vertices and obtained the baryon bound states by solving the Dirac equation with appropriate scalar and vector potentials. The vertex constants were taken to be the same as those determined in previous studies. We have ignored, for the time being, the distortion effects in the incident and outgoing channels as our main aim in this paper has been to establish a fully covariant model for this reaction which has so far been described only within a non-relativistic distorted wave impulse approximation picture. The plane wave approximation facilitates the application of this novel approach to very many cases without requiring lengthy and cumbersome computations which are necessarily involved in the distorted wave methods.

We find that excitations of $N^*(1650)$, and $N^*(1710)$ resonant states dominate the cross sections for the $(\pi^+, K^+)$ reaction for beam energies below 2 GeV. Our model describes well the shapes of the experimental angular distributions for two states which corresponds to the prominent peaks in the $^{12}\Lambda C$ spectrum. The total cross sections show a substantial dependence on the beam energy with a distinct peak around 1 GeV for reactions on both $^{12}$C and $^{40}$Ca targets. The differential cross sections peak near zero degrees for both light mass as well as heavier targets. Thus, measurements at forward angles and at beam energies around 1.0 GeV are expected to have high yields.

The characteristic bump structures reflecting the $\Lambda$ major shell orbits as seen in the spectra of the $^{5}\Lambda V$ and $^{89}\Lambda Y$ hypernuclei were reproduced reasonably well by our model. However, for a quantitative description of the spectra, more realistic calculations with configuration mixed $\Lambda$ particle, neutron hole states are required. Mixing of the different parity states within a given particle-hole configuration may also be needed.

Our work shows that it is indeed feasible to have a fully covariant description of the hypernuclear production via the $(\pi^+, K^+)$ reaction. In future studies, the
structure aspects of the model should be improved. Also, alternative covariant bound state wave functions should be used in order to check the validity of the method used in this work to determine them. Distortion effects in the initial and final channels are required in order to have more reliable predictions of the absolute magnitudes of the cross sections.

7. ACKNOWLEDGEMENTS

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