Does the hierarchy problem set the seesaw scale?

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We find that minimizing the number of fine tuning relations in non-supersymmetric models can determine the scales at which gauge symmetries beyond the standard model must break. We show that B-L gauge symmetry of the minimal left-right symmetric model breaks at a scale $10^{15}\text{GeV}$ or higher, determined by the hierarchy problem and small quark mass ratios, provided parameters that break chiral or $\mu-$symmetries are not fine-tuned. This provides the *raison d’être* for the seesaw scale $\sim 10^{15}\text{GeV}$ indicated by neutrino experiments.

**Introduction:** What we make of the hierarchy between the weak scale ($v_{wk} \sim 246\text{GeV}$) and the reduced Planck scale ($M_{Pl} \sim 2.4 \times 10^{18}\text{GeV}$), can determine where we expect the next scale of new physics to be. Note that radiative corrections to $v_{wk}$ of the order $h_i^2 M_{Pl}^2/(16\pi^2)$ (with top Yukawa coupling $h_t \sim 1$) would shift it towards the Planck scale unless there is fine tuning or a precise cancellation of such large terms to generate the small weak scale. This fine tuning of the standard model is the well known hierarchy problem [1–3].

In order to avoid fine tuning, new physics such as supersymmetry that solves the hierarchy problem was expected close to the weak scale. However despite direct and indirect searches, no hint for such new physics has been found. Thus the minimal supersymmetric standard model is now considered to be fine tuned to a 1% level or worse (see for example [4–6]). It appears that nature accepts fine tuning of gauge symmetry breaking scales and is it possible that there is no supersymmetry (or no SUSY till Planck scale).

However there is some evidence for a new high energy scale in nature. The neutrino mass data [7] ($|\Delta m^2_{32}| = 0.0023 \text{eV}^2$) points to a seesaw [8][11] scale $M_{ss} = v_{wk}^2/\sqrt{|\Delta m^2_{32}|} \sim 1.3 \times 10^{15}\text{GeV}$. But what is the *raison d’être* for this scale?

The standard lore is that this is near where the weak, strong and electro-magnetic forces unify. However several grand unified theories (GUTs) are tightly constrained or ruled out. For example a recent paper [12] finds that only one chain of non-supersymmetric SO(10) breaking is still consistent with data and that the constraint of unification of couplings determines the intermediate seesaw scale to be $\sim 10^{11}\text{GeV}$, with grand unification at $10^{16}\text{GeV}$. However the model suffers from a “very large level of fine-tuning” (quoting [12]) and “...the idea of an SO(10) GUT is very appealing but all its practical realizations are clumsy, more so in the non SUSY case because of the hierarchy problem....”

Historically the extended survival hypothesis [13][14] used in grand unified models such as [12], was motivated by the desire to minimize the number of fine tuning relations [14]. The hypothesis however is to minimize the Higgs multiplets that are needed at lower and intermediate mass scales, rather than the number of fine tunings.

Instead of taking the grand unified route, we ask how many fine tuning constraints are actually necessary in non-supersymmetric theories to keep the scale of gauge symmetry breaking small? In the standard model $SU(2)_L \times U(1)_Y$ breaks to $U(1)_{em}$ requiring one Higgs vacuum expectation value (VEV) that is kept small by fine-tuning. If the standard model is extended to the left-right symmetric model based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, then we can first break $SU(2)_R \times U(1)_B$ to $U(1)_Y$ at the scale $v_R \sim v_{B-L}$. In exact analogy with the standard model we expect that one fine-tuning relation is required to keep $v_R^2 / M_{Pl}^2$. Therefore, totally we require two fine tunings for gauge scales in this model - one to ensure $v_R^2 \ll M_{Pl}^2$ and the other to ensure $v_{wk}^2 \ll M_{Pl}^2$.

In this work we show that if we allow exactly two fine-tuning relations in the minimal left-right symmetric model, while $v_R^2 \ll M_{Pl}^2$ is possible, $v_R$ cannot be kept arbitrarily small and there is a meaningful lower bound $v_R \sim v_{B-L} > 10^{15}\text{GeV}$. The reason for the bound is that a chiral $\mu-$symmetry is needed along with the two fine-tuning relations to obtain the correct symmetry breaking pattern. However the $\mu-$symmetry is approximately broken by small second generation Yukawa terms to obtain the proper quark mass spectrum. The hierarchy problem then reappears to destabilize the symmetry breaking pattern, but the quadratic divergence from the Planck scale is now suppressed by second generation Yukawas, leading to a scale of $10^{15}\text{GeV}$ for $v_R \sim v_{B-L}$. Breaking of $SU(2)_R \times U(1)_B$ triggers the seesaw mechanism, and the see-saw scale $\sim 10^{15}\text{GeV}$ hinted at by neutrino mass data is thus generated by the hierarchy problem.

Thus without using any grand unification constraints, we can obtain meaningful bounds on gauge symmetry breaking scales and a *raison d’être* for the seesaw scale. The idea of minimizing the number of fine-tunings and using the hierarchy problem and chiral symmetries to provide limits on gauge symmetry breaking scales can be generalized to other groups.

**Hierarchy problem in left-right models:** We consider the minimal Left-Right symmetric model [15][18] based on $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with triplets $\Delta_R$ (1, 1, 3, -2) and $\Delta_L$ (1, 3, 1, -2), and biquark doublet $\phi$ (1, 2, 2, 0). We impose parity (P) under which, as is usual, $\phi \rightarrow \phi^*$ and subscripts $L \leftrightarrow R$ for all other
fields. The scalar fields have the form
\[
\phi = \left( \begin{array}{c} \phi_1^0 \\ \phi_1^- \\ \phi_2^+ \\ \phi_2^0 \end{array} \right), \quad \Delta_{L,R} = \left( \begin{array}{cc} \delta_{L,R}^+ & \sqrt{2} \\ \delta_{L,R}^- & -\sqrt{2} \end{array} \right).
\]

The VEV \(<\delta_R^+>\) breaks \(SU(2)_R \times U(1)_{B-L}\) to \(U(1)_Y\), while \(<\phi_1^+>\) cause electro-weak symmetry breaking, and \(\delta_L^+\) is a much smaller induced VEV. We designate these by
\[
<\delta_R^+> = \frac{v_R}{\sqrt{2}}, \quad <\phi_1^+> = \frac{k_1}{\sqrt{2}}, \quad <\phi_2^-> = \frac{k_2}{\sqrt{2}}, \quad <\delta_L^-> = \frac{v_L}{\sqrt{2}} \tag{2}
\]
with \(v_{ew}^2 = |k_1|^2 + |k_2|^2\) and study the fine-tuning implications to obtain the hierarchy \(v_{ew}^2\ll v_R^2\ll M_{Pl}^2\), where the cut-off scale of the theory is taken to be \(M_{Pl}\). Note that the fine tuning of the weak scale from \(v_R\) scale in left-right models was discussed in [18]. However fine tuning issues of weak and \(v_R\) scales due to quadratically divergent radiative corrections from a cut-off scale (such as \(M_{Pl}\)) much greater than \(v_R\) were not studied.

To simplify calculations, without loss of generality, we take all VEVs to be real. In fact all parameters of the Higgs potential are real due to parity, except for one that is discussed in the comments at the end.

Relevant Higgs potential terms (using standard notation, see for example [19] [20]) that involve only \(\Delta_R\) were discussed in [18]. However fine tuning issues of weak and \(v_R\) scales due to quadratically divergent radiative corrections from a cut-off scale (such as \(M_{Pl}\)) much greater than \(v_R\) were not studied.

We impose a chiral \(\mu\)-symmetry under which \(\phi \to e^{i\beta}\phi\) (we provide transformations of other fields when they appear later) and write the terms involving \(\phi\) (but ignore terms involving \(\Delta_L\) until later) in the Higgs potential responsible for electroweak symmetry breaking:
\[
-\mu_3^2\text{Tr}(\phi^\dagger\phi) + \lambda_1[\text{Tr}(\phi^\dagger\phi)]^2 + \lambda_3\text{Tr}(\phi^\dagger\phi)\text{Tr}(\tilde{\phi}\phi) + \alpha_1\text{Tr}(\phi^\dagger\phi)\text{Tr}(\Delta^\dagger_R\Delta_R) + \alpha_3\text{Tr}(\phi^\dagger\phi\Delta_R\Delta_R) \tag{3}
\]
where \(\tilde{\phi} = \tau_2\phi^\dagger\tau_2\). Substituting for the VEVs from equation (2) we can rewrite the above as
\[
(-\mu_3^2 + \frac{\alpha_1}{2}v_{ew}^2) \frac{k_1^4}{2} + (-\mu_1^2 + \frac{\alpha_1}{2}v_R^2 + \frac{\alpha_3}{2}v_{ew}^2) \frac{k_2^2}{2} + \frac{\lambda_1}{4}(k_1^4 + k_2^4) + \frac{\lambda_3}{2}(k_1^2k_2^2) \tag{4}
\]
It is easy to see that if \(\mu_3^2\) is fine tuned so that the quantity in the brackets of the first term of equation (4) is of the order of the \(-v_{ew}^2\), minimization with respect to \(k_2\) and \(k_1\) leads to \(k_2 = 0\) and \(v_{ew} \approx k_1 = \sqrt{(\mu_3^2 - \alpha_1v_R^2/2)/\lambda_1}\). This is the usual fine tuning that needs to be done to keep the weak scale small. Note that we have assumed \(\alpha_3 > 0\) and the quantity in brackets of second term of eq. (4) can be rewritten as \((\alpha_3/2)v_R^2 + O(v_{ew}^2)\sim(\alpha_3/2)v_R^2\). This implies that the second Higgs doublet (that is fields with subscript 2) in matrix representing bidoublet \(\phi\) in equation (1) has a mass \(m_{H^2} = (\sqrt{\alpha_3/2})v_R\).

We can in principle provide a small VEV to \(k_2\) without any more fine tuning by breaking the \(\mu\)-symmetry softly,
\[
V_{\text{break}} = -\mu_2^2\text{Tr}(\tilde{\phi}\phi) + \text{h.c.} \tag{5}
\]
This term adds \(-2\mu_2^2k_1k_2\) to equation (4). Minimizing with respect to \(k_2\) now gives to the lowest order
\[
k_2 = \left[ \frac{4\mu_2^2}{\alpha_3v_R^2} \right]k_1 \tag{6}
\]

Ignoring the quantity in brackets of the first term of (4) that has been fine tuned to be at weak scale, note that \(\mu_2^2 < \alpha_3v_R^2/8\) must be satisfied to get the right symmetry breaking pattern. Otherwise due to the \(\mu_2^2\) term, there will be a saddle point in the \(k_1 = k_2\) direction that can provide VEVs \(\gtrsim v_R\) to \(k_1\) and \(k_2\), breaking \(G_{LR} \to U(1)_{J_L + J_R} \times U(1)_{B-L}\) rather than to standard model.

As long as the \(\mu\)-symmetry breaking is soft, \(\mu_2^2\) does not receive quadratically divergent radiative correction from the Planck scale and can be naturally small. However the \(\mu\)-symmetric Yukawa terms (with \(\{Q_L, \phi\} \to e^{i\beta}\{Q_L, \phi\}\) and \(Q_{IR}\) invariant under \(\mu\)-symmetry) which provide masses to quarks are of the form
\[
\sum_{i,j=1,3} h_{ij}Q_{iL}\phi Q_{jR} + \text{h.c.} \tag{7}
\]
where \(Q_{UL} \equiv (u_{UL}, d_{UL})^T\) and \(Q_{IR} \equiv (u_{IR}, d_{IR})^T\) are the left and right handed Quark doublets of the \(i^{th}\) generation and are represented by \(2 \times 1\) column vectors in isospace. Substituting for the VEVs of \(\phi\) we can see that the up and down quark mass matrices are proportional to each other. Therefore they can be simultaneously diagonalized and hereafter we work in the basis where \(h_{ij}\) is diagonal. We can now obtain the quark masses in terms of the Yukawas. Writing these explicitly we get,
\[
\frac{h_{33}k_1}{\sqrt{2}} = m_t, \quad \frac{h_{33}k_2}{\sqrt{2}} = m_b, \quad \frac{h_{22}k_1}{\sqrt{2}} = m_c \tag{8}
\]
From the above we obtain \(k_2/k_1 = m_t/m_b\). However the mass of the strange quark turns out to be too low, \(m_s = h_{22}k_2/\sqrt{2} = (m_b/m_t)m_c\). Also the Cabibo-Kobayashi-Maskawa (CKM) mixing angles vanish. Therefore we must allow for approximate breaking of the \(\mu\)-symmetry by Yukawa terms of the kind
\[
\tilde{h}_{22}Q_{2L}\tilde{\phi}Q_{2R} + \tilde{h}_{23}Q_{2L}\tilde{\phi}Q_{3R} + \tilde{h}_{23}^*Q_{3L}\tilde{\phi}Q_{2R} + \text{h.c.} \ldots \tag{9}
\]
so that we now have
\[
m_s \approx \frac{\tilde{h}_{22}k_1}{\sqrt{2}}, \quad V_{ts} \approx \frac{\tilde{h}_{23}k_1}{\sqrt{2}m_b} \tag{10}
\]
Note that the occurrence of $\tilde{h}_{33}^3 (= \tilde{h}_{32}^3)$ in equation (9) is because Yukawa matrices involving the bi-doublet are Hermitian due to parity, as is well known in left-right symmetric models.

Since $\mu$–symmetry is approximately broken by Yukawa terms, $\mu^2$ receives a quadratically divergent radiative contribution at 1-loop level from diagrams involving the second generation such as the one in Figure 1. We evaluate such diagrams by providing a cut-off at the reduced Planck scale and find the radiative correction at one-loop level,

$$\mu^2 \sim \left(6 \over 2\right) \left[ M^2_{Pl} \right] h_{22} \tilde{h}_{22}$$

(11)

The factor of 6 in the first numerator is to account for the 3 colors of the strange quark, and because there is also an equal contribution from a similar diagram with the charm quark in the loop (with $h_{22}$ and $\tilde{h}_{22}$ interchanged). The 2 in the denominator accounts for the trace in eq. [5].

Since

$$\mu^2 = \mu^2_{(bare)} + \mu^2_{(rad)}$$

(12)

if we do not allow fine tuning of equation (12), so that there is no precise cancellation between the bare and radiative terms, we obtain the bound

$$\mu^2 \geq \mu^2_{(rad)}$$

(13)

This bound on $\mu^2$ translates to a bound on $v_R^2$ through equation (6). Combining equations (13), (11) and (6), and using (8) and (10) to express the Yukawas in terms of quark masses (with $k_1 \approx v_{wk}$), we get the following lower bound on $v_R$

$$v_R \sim v_{B-L} \geq \left[ M_{Pl} \over 2\pi \right] \left[ \sqrt{6m_s m_c} \over \alpha_3 m_t m_L \right] \left( m_t \over v_{wk} \right).$$

(14)

Several papers have results on the running Yukawa couplings in the standard model. We use the results of Das and Parida [21] (updated recently in [22]) for the quark masses in the standard model evaluated at a scale of $2 \times 10^{16} GeV$ to evaluate the above. From their work we have at the high scale, $m_s = 20.4 MeV, m_c = 0.22 GeV, m_t = 0.93 GeV, m_t = 70.5 GeV$ and $v_{wk} = 155 GeV \sqrt{2} = 219 GeV$.

Substituting in (14), we get the bound

$$\sqrt{\alpha_3 v_R} \geq 1.04 \times 10^{-3} M_{Pl} \approx 2.5 \times 10^{15} GeV. \quad (15)$$

where we used $M_{Pl} = 2.4 \times 10^{18} GeV$. Note that the left hand side of (15) is $\sqrt{2}$ times the mass of the second Higgs doublet. Since we expect in the perturbative regime, $\alpha_3 \approx 1$, equation (15) evaluated with $\alpha_3 = 1$ also provides a lower bound on $v_R$. That is $v_R \geq 2.5 \times 10^{15} GeV$.

Note that instead of the scale $2 \times 10^{16} GeV$, if we use the Yukawa couplings evaluated at the scale $10 TeV$ (from a recent paper by Antusch and Maurer [23]), we get $\sqrt{\alpha_3 v_R} \geq 1.8 \times 10^{-3} M_{Pl}$. This shows that the scale or the method used to evaluate the Yukawas does not make much of a difference to the bound.

The dots in equation (9) represent other terms that approximately violate $\mu$–symmetry, that can potentially be there. However for the purposes of our lower bound calculation, the strength of their couplings can be neglected without resorting to fine-tuning. For example the term $h_{33} Q_L \phi Q_R$, can be radiatively generated at one-loop with the strength (up to logarithmic factors) $h_{33}^2 \sim \left[1/(16\pi^2)\right](h_{32}^2 h_{23}^2) \sim 10^{-13}$. Since $h_{ij}$ is diagonal there is no contribution to $\tilde{h}_{33}^2$ from the combination of Yukawas $(h_{33}^2 h_{32}^2 h_{33}^3)$. Therefore we can choose $h_{33} \leq h_{32}^2 h_{23}^2$ without fine tuning, so that its contribution to $\mu^2_{(rad)}$ is at most of the same order of magnitude as already present in equation (11).

Seesaw scale – The bound value $v_R \sim 2.5 \times 10^{15} GeV$ we obtained is close to the seesaw scale hinted by neutrino experiments namely $M_{\nu} \sim v^2_{wk} \sqrt{\Delta m^2_{31}} \sim 1.3 \times 10^{15} GeV$, which suggests that the seesaw scale is determined by the hierarchy problem and small quark mass ratios. Since we have imposed an approximate $\mu$–symmetry, we will now verify that the terms necessary for seesaw mechanism are not suppressed.

As before under $\mu$–symmetry, $\{ \phi, Q_L, L_L \} \rightarrow e^{i\beta} \{ \phi, Q_L, L_L \}$, and $\Delta_L \rightarrow e^{-2i\beta} \Delta_L$, with $\Delta_R, Q_R$ and $L_R$ being invariant. There is only one $P$ and $\mu$–symmetric quartic term in the Higgs potential that contains all three fields - $\beta_2 \left[ T_R \left( \phi \Delta_R \phi^\dagger \Delta_L \right) + T_L \left( \phi \Delta_L \phi^\dagger \Delta_R \right) \right]$. The remaining $\mu$–symmetric terms containing $\Delta_L$ relevant for providing it a VEV are $-\mu^2_2 T_R (\Delta_R \Delta_L^\dagger) + \rho_3 T_R (\Delta_R \Delta_L^\dagger) T_R (\Delta_R \Delta_R)$. Rewriting the Higgs potential in terms of the VEVs using equation (2), recalling that $v_R^2 = \mu^2 / \rho_3$, and minimizing with respect to $v_L$, we get, $v_L \sim (\beta_2 / (\rho_3 - 2\rho_1)) (v^2_{wk} / v_R)$, with $\rho_3 > 2\rho_1 > 0$.

Noting that the usual Yukawa terms that give rise to Majorana masses $f_{ij} L_i^c \tau_3 \Delta_L L_J + h.c.$ (and terms with subscript $L \rightarrow R$), are permitted by $\mu$–symmetry we find that $v_L$ contributes to the largest light neutrino mass $m^\nu_L$

$$m^\nu_L \sim \sqrt{2} f_{ii} v_L \sim \sqrt{2} \left( f_{ii} \beta_2 \right) \left( v^2_{wk} / v_R \right).$$

(16)
with \( i = 3 \) (or \( i = 2 \)) depending on whether it is natural (or inverted) hierarchy. Substituting our bound value, \( v_R \sim 2.5 \times 10^{15} \text{GeV} \) and \( v_{ew} \sim 246 \text{GeV} \), we see that if the quantity in the square brackets has a natural value around 1.4 for \( i = 3 \) (or \( i = 2 \)), we obtain the observed \( |m^{2}_{3} - m^{2}_{2}| \sim |\Delta m^{2}_{32}| \sim 0.0023 \text{eV}^2 \). With \( v_R \sim 2.5 \times 10^{15} \text{GeV} \), the full seesaw mechanism can proceed either as Type II or Type I seesaw, or a hybrid of the two.

A few comments –

- If the hierarchy problem is solved at a scale \( \Lambda \), then \( M_{Pl}^2 \) in equations of this work will be replaced by \( \Lambda^2 \).
- The lower end of the bound in equation (14), that is \( v_R \sim 10^{15} \text{GeV} \), corresponds to the minimum violation of \( \mu \)-symmetry needed to obtain the quark mass spectrum. It also corresponds to not introducing an additional scale through the term \( \mu^2 \text{Tr}(\phi^4) \) – that is if \( \mu^2 = 0 \) in equation (12) then only the radiative correction \( \mu^2 \text{rad} \) determines \( \sqrt{\alpha_2} v_R \) to be at the lower end of the bound.

- If we add a second bi-doublet \( \phi' \), without any additional fine tuning, its mass will naturally be of the order \( M_\phi \gtrsim M_{Pl}/(4\pi) \). Though with two bi-doublets the \( \mu \)-symmetry can be broken only softly, the soft symmetry breaking mass term involving \( \phi' \) such as \( \text{Tr}\phi^4 \phi' \) must be at a large scale \( \sim (m_4/m_3) M_{Pl}^2 \) in order to obtain the needed VEVs for \( \phi' \). Thus the \( \mu \)-symmetry is once again broken at a large scale, which in turn radiatively generates the \( \mu^2 \) term of first bi-doublet \( \phi \) and results in a significant lower bound on \( v_R \) as before. Depending on choice of parameters, the exact bound value will change. The effective theory below \( M_\phi \) is the minimal model.

- \( \alpha_2 \), the only complex parameter in the Higgs potential of the LR model is naturally small, since the term \( \alpha_2 \text{Tr}(\phi^4) \text{Tr}(\Delta L^2 \Delta_R) \) (with its parity counterpart) breaks \( \mu \)-symmetry. Thus this term is under control for the purposes of our calculation. However in LR models it is the source of the strong CP problem which can be solved without requiring an axion as shown in [24][25]. If a family of vectorlike quarks needed for the strong CP solution are at the Planck scale, the theory below it is our minimal LR model. The predictions of this ultra-violet completion are a measurable strong CP phase (neutron electric dipole moment (EDM)) that is radiatively generated in a large region of the parameter space, no electron EDM due to the high scale of new physics; and in the minimal version an absence of all lepton CP phases [24][25]. If \( v_R \sim 10^{15} \text{GeV} \) is the next scale of new physics then these predictions maybe the only window for more evidence on left-right symmetry.

Conclusion – In non supersymmetric theories fine-tuning of gauge symmetry breaking scales is necessary. By imposing a very reasonable condition that only VEVs that are actually needed to break gauge symmetries are fine tuned, while parameters that break chiral or \( \mu \)-symmetries are not fine tuned (since their smallness ought to be natural due to the chiral symmetry), we have shown that we can obtain meaningful bounds on gauge symmetry breaking scales, that depend on the hierarchy problem and small quark mass ratios that break chiral symmetries. In particular we find that the \( B - L \) breaking scale in minimal left right symmetric model \( v_{B-L} \sim v_R \gtrsim 10^{15} \text{GeV} \). Since \( B - L \) breaking triggers the seesaw mechanism, it leads to the understanding of seesaw scale \( \sim 10^{15} \text{GeV} \) indicated by neutrino experiments, without need for grand unification constraints.

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[1] E. Gildener, Phys. Rev. D 14, 1667 (1976).
[2] S. Weinberg, Phys. Rev. D 19, 1277 (1979).
[3] L. Susskind, Phys. Rev. D 20, 2619 (1979).
[4] A. Arvanitaki, M. Baryakhtar, X. Huang, K. Van Tilburg, and G. Villadoro, (2013), arXiv:1309.3568 [hep-ph].
[5] E. Bertuzzo, EPJ Web Conf. 60, 18001 (2013), arXiv:1307.0318 [hep-ph].
[6] I. Antoniadis, 1Xth Rencontres du Vietnam: Windows on the Universe, Quy Nhon Vietnam (11-18 August 2013).
[7] J. Beringer et al. (Particle Data Group), Phys.Rev. D86, 010001 (2012).
[8] P. Minkowski, Phys.Lett. B67, 421 (1977).
[9] M. Gell-Mann, P. Ramond, and R. Slansky, Supergravity, ed. by D. Freedman and P. Van Nieuwenhuizen, North Holland, Amsterdam, 315-321 (1979).
[10] T. Yanagida, Progress of Theoretical Physics 64, 1103 (1980).
[11] R. N. Mohapatra and G. Senjanovic, Phys.Rev.Lett. 44, 912 (1980).
[12] G. Altarelli and D. Meloni, JHEP 1308, 021 (2013), arXiv:1305.1001 [hep-ph].
[13] F. del Aguila and L. E. Ibanez, Nucl.Phys. B177, 60 (1981).
[14] S. Dimopoulos and H. Georgi, Phys.Lett. B140, 67 (1984).
[15] R. N. Mohapatra and G. Senjanovic, Phys. Rev. D 27, 1601 (1983).
[16] J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974).
[17] R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 566 (1975).
[18] G. Senjanovic and R. N. Mohapatra, Phys. Rev. D12, 1502 (1975).
[19] G. Barenboim, M. Gorbahn, U. Nierste, and M. Raidal, Phys. Rev. D 65, 095003 (2002).
[20] P. Duka, J. Gluza, and M. Zralek, Annals Phys. 336 (2000), arXiv:hep-ph/9910279.
[21] C. Das and M. Parida, Eur.Phys.J. C20, 121 (2001), arXiv:hep-ph/0010004 [hep-ph].
[22] K. Bora, (2012), arXiv:1206.5909 [hep-ph].
[23] S. Antusch and V. Maurer, JHEP 1311, 115 (2013), arXiv:1306.6879 [hep-ph].
[24] R. Kuchimanchi, Phys. Rev. D82, 116008 (2010), arXiv:1009.5961 [hep-ph].
[25] R. Kuchimanchi, Phys.Rev. D86, 036002 (2012), arXiv:1203.2772 [hep-ph].