Spin transitions in an incompressible liquid Coulomb coupled to a quantum dot

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We report on our investigation of the low-lying energy spectra and charge density of a two-dimensional quantum Hall liquid at $\nu = \frac{2}{3}$ that is Coulomb coupled to a quantum dot. The dot contains a hole and two/three electrons. We found that any external perturbation (caused by the close proximity of the quantum dot) locally changes the spin polarization of the incompressible liquid. The effect depends crucially on the separation distance of the quantum dot from the electron plane. Electron density distribution in the quantum Hall layer indicates creation of a quasihole that is localized by the close proximity of the quantum dot. Manifestation of this effect in the photoluminescence spectroscopy is also discussed.

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The two-dimensional electron gas (2DEG) in an intense external perpendicular magnetic field and at very low temperatures has proven to be a remarkable system to explore various quantum phase transitions. Immediately after the discovery of the fractional quantum Hall effect\textsuperscript{1} and the subsequent seminal theoretical work by Laughlin\textsuperscript{2} who introduced the idea of an incompressible quantum liquid to explain the effect, it became clear that there are other interesting transitions, for example, the spin transitions at a few special filling factors, that are entirely driven by the electron correlation. Spin unpolarized ground states\textsuperscript{3,4} and excitations\textsuperscript{5} were proposed and intensely investigated theoretically\textsuperscript{6,7} and experimentally\textsuperscript{8} in various groups\textsuperscript{8}. It is now widely accepted that the $\nu = \frac{3}{5}$ filling factor is spin unpolarized in its ground state and has spin-reversed quasiholes as low-energy excitations\textsuperscript{7}. This is in contrast to the other fundamental filling factor $\nu = \frac{1}{3}$ which has a fully spin-polarized ground state and spin-polarized fractionally-charged quasiparticles and quasiholes\textsuperscript{3,7}.

The $\nu = \frac{1}{3}$ fractionally-charged quasihole was probed recently in a very interesting experiment that involved photoluminescence (PL) spectroscopy of a two-dimensional electron gas confined in a quantum well subjected to an external magnetic field. In that experiment, electron-hole pairs are excited with a single laser line, and afterwards, electrons and holes relax and form excitons. Studying the luminescence lines (when the electrons and holes recombine) as one sweeps the magnetic field, one can determine various physical properties of the system in a magnetic field. The physics in this situation is seemingly well established. When the lowest Landau level is completely occupied the observed PL lines show a nearly linear magnetic field dispersion that can be attributed to recombination of electrons from the occupied Landau levels with photocreated holes in the valence band. However, interesting things were found to happen at the intermediate density (0.9×10\textsuperscript{11} cm\textsuperscript{-2}) and a moderate electron mobility. Here, Schüller et al.\textsuperscript{9} observed that, near the 1/3 fractional quantum Hall region the PL lines exhibit a large reduction in energy (≈ 2 meV) at a very low temperature (T=0.1 K) compared to those at a much higher temperature (T=1.8 K) where the lines are as expected, linear. Further, a puzzling observation associated with the anomaly was that a very small thermal energy ($\ll$ 2 meV) is sufficient to destroy the anomaly.

The anomaly was resolved by a theoretical model\textsuperscript{10,11} where we considered a situation in which the excitons are localized but they are in close proximity to a $\nu = \frac{1}{3}$ incompressible liquid. In this model, localized exciton are represented by quantum dots (the nanometer-scale boxes for confining electrons and holes)\textsuperscript{11} and the observed anomaly is related to the properties of the combined dot-plus-liquid system (a liquid-qd complex) where the quantum dot (QD) contains a charge-neutral exciton plus an extra electron. The close proximity of the dot perturbs the incompressible electron liquid and creates fractionally-charged quasiholes in the liquid. Interestingly, the calculated energy to create those quasiholes turned out to be similar to the tiny energy that was required to destroy the observed anomaly. This result explained the observed puzzle described above: application of a small thermal energy closes the quasihole energy gap and therefore the incompressibility of the liquid, the prime reason for the existence of quasiholes, disappears and so does the anomaly\textsuperscript{9}. The experiment and its theoretical explanation clearly demonstrated that exciton spectroscopy as opposed to transport measurements might provide an important route to explore the properties of the incompressible states.

A question that naturally arises then is, what happens when the two-dimensional electron liquid is in the $\nu =$
and the electron layer separated by a distance $a$ that are specific for $\nu = \frac{2}{5}$. The quantum dot contains two electrons and a hole and is separated from the 2DEG by $a = 1.0 - 2.0 l_0$. In this figure, $\bullet$ represents the energies of a spin unpolarized state, and the $\circ$ corresponds to the spin partially polarized state energy.

2/5 state where the ground state, at low magnetic fields, is expected to be spin unpolarized [7], rather than in the $\nu = \frac{1}{3}$ state where the ground state is fully spin polarized. It is not immediately obvious how that state would evolve when a charged quantum dot containing one or two electrons plus an exciton is Coulomb coupled to that quantum Hall state. In this paper, we report on our investigation of the spin transitions in such a liquid-QD complex.

Details for calculation of the energy spectrum and charge densities for the liquid-QD complex are available in Refs. [9, 10]. Electrons and the hole in the QD are confined by a parabolic potential $V_{\text{conf}}(x, y) = \frac{1}{2}m^*\omega_0^2(x^2 + y^2)$, where $\omega_0$ is the confinement potential strength. In our calculations the size of the quantum dot, $l_{\text{dot}} = (\hbar/m^*\omega_0)^{\frac{1}{2}}$, was equal to 15 nm. We take into account only eight states in the QD, which can be occupied either by electrons or the hole. Electrons and the hole in the dot are treated as spinless particles. To model the electron layer we use the spherical geometry with six electrons and twelve flux quanta. This corresponds to the electron filling factor $\nu = \frac{2}{5}$. We consider the Coulomb interaction between the electrons and the hole in the dot and the electron layer separated by a distance $a$. For a charge neutral quantum dot containing one electron and one hole, the effect of the dot on the incompressible liquid is small. In this case we have an almost decoupled system of the 2D layer and the quantum dot, similar to the case of $\nu = 1/3$ observed earlier [9]. The new features that are specific for $\nu = 2/5$ appear when the quantum dot becomes negatively charged.

In Fig. 1, the energy spectrum (in units of Coulomb energy, $\epsilon^2/\epsilon l_0$, where $\epsilon$ is the background dielectric constant and $l_0^2 = \hbar c/eB$ is the magnetic length) of the system with two electrons and a hole in the quantum dot is shown. The open circles correspond to unpolarized states of the incompressible liquid, while $\circ$ describe the spin partially polarized states. In our case the partially polarized states correspond to four electrons with spin-down and two electrons with spin-up. The ground state of the unperturbed $\nu = 2/5$ liquid has zero angular momentum and is spin unpolarized [4, 7]. When we introduce the interaction between the 2D layer and the quantum dot, the spin polarization of electrons in the electron layer crucially depends on the separation between the 2D layer and the quantum dot, i.e. on the strength of interaction between electrons in the layer and electrons and hole in the dot. For a small separation, $a = 1.0 l_0$ and 1.25 $l_0$, the ground state is spin partially polarized and has a finite angular momentum, $M = 3$ (see Figs. 1a and 1b). These data illustrates that any external perturbation of the 2/5-liquid changes locally the spin polarization of the incompressible liquid. This is in correspondence with the energy spectrum of the unperturbed (without the presence of a dot) 2/5 liquid (shown as inset in Fig. 1d), where the lowest excitation of the 2D liquid is spin partially polarized. For large separation, $a = 2.0 l_0$, the ground state is again spin unpolarized at $M = 0$, as in the unperturbed case.

Figure 2 displays the energy spectra of the system containing three electrons and one hole in the dot. In this case the quantum dot effectively becomes doubly charged (negative), which should increase the interaction strength between the quantum dot and the electron liquid. This increase can be clearly seen in Fig. 2. When the separation between the quantum dot and the layer is small ($a = 1.0 l_0$) the ground state is spin partially polarized and has a large angular momentum, $M = 6$. Let us
now compare these results with those in Fig. 1a, where there are only two electrons in the dot and the angular momentum of the ground state is equal to three. The increase of the angular momentum of the ground state illustrates the effect of stronger interaction between the quantum dot and electrons in the 2D layer. When the separation between the 2D layer and quantum dot is increased, there appears a new feature which is not present in Fig. 1, namely, the ground state which in some region of \( a \) becomes unpolarized at finite angular momentum (Fig. 2c). Therefore for three electrons in the quantum dot we have a competition between the unpolarized and partially spin polarized states at finite angular momentum, \( M = 6 \). When the separation \( a \) is further increased the ground state at first becomes partially polarized at a finite angular momentum, \( M = 2 \), and then it becomes unpolarized at the zero angular momentum. This occurs at separations larger than in the case of two electrons in the dot (Fig. 1). In Fig. 2d the separation between the quantum dot and the layer is \( a = 2.25 l_0 \) and the energy spectrum is similar to that of the unperturbed 2/5 liquid.

In Fig. 3 the electron density distribution in the two-dimensional electron layer at \( \nu = \frac{2}{5} \) with the QD located at the origin is shown. The dot contains either two electrons and one hole (Fig. 3a), or three electrons and a hole (Fig. 3b). The separations between the electron layer and the quantum dot are \( a = 2.0 l_0 \) and \( a = 2.25 l_0 \) in Fig. 3a and Fig. 3b, respectively. These data correspond to the states presented in Fig. 1d and Fig. 2d respectively. The charge distribution is shown for the unpolarized ground state (solid line) and for the first three excited states at \( M = 1, 2, \) and 3 (broken lines). These excited states are partially polarized. Compared to Fig. 3a, the charge distribution for the system with three electrons in the quantum dot (Fig. 3b) has a plateau-like structure at a distance \( r \approx 2 l_0 \) from the center of the quantum dot. This structure is a direct manifestation of the effectively larger size of the quantum dot, as compared to that in Fig. 3a. For the fully spin polarized electron fluid Coulomb coupled to a singly-charged quantum dot, or to a singly-charged impurity, the fractionally-charged quasihole generated by the ionization process as discussed earlier in the literature moves away from the QD, located at the origin, as \( M \) is increased. This was indicated by the minimum in the charge density being shifted to large \( r \) for increasing \( M \). It should be pointed out that the position of the local minimum at different angular momenta of the charge density correspond to the orbit radius of the quasihole. In the present case of spin-reversed ground state and the spin-reversed excitations, the minima in the charge densities remain near the same position for all values of \( M \). This behavior of the charge densities could perhaps be a consequence of the spin-reversed quasiholes being localized entirely due to correlations. We have also calculated the quasihole creation energy at \( \nu = \frac{2}{5} \) to be \( \sim 0.20 \) meV, which can also be measured in exciton spectroscopy, as reported earlier for \( \nu = \frac{1}{3} \). PL experiments to detect the localized \( e/5 \) charged spin-reversed quasihole would be very exciting.

PL experiments in the FQH regime reported as yet, can be roughly divided into three main categories: (i) remotely acceptor-doped heterojunctions, (ii) single heterojunctions, and (iii) single quantum wells. In (i) and (ii), influence of the photoexcited valence band holes on the electron liquid is mostly neglected since the hole is expected to be located at a relatively large distance \( a \) from the electron layer. The situation is different for the case (iii). There, in a quantum well \( a \) is at the maximum of the quantum well width which is typically 10-25 nm. The Coulomb interaction between the electron liquid and the hole in the valence band then plays a crucial role, and at high magnetic fields, charged excitons can form. These are bound states of two electrons plus a hole. In high quality samples, the charged excitons move as quasi free particles in the 2DEG, and the strong Coulomb interaction between electrons and holes hinders the formation of fractionally-charged objects at fractional filling factors. There are also theoretical predictions that at a separation of electrons and valence band holes larger than \( l_0 \), fractionally charged excitons...
charged exciton can coexist \[9\]. The Coulomb coupling and very low temperature, an electron liquid and localized charged exciton can coexist \[9\]. The Coulomb coupling of the localized charged exciton to the electron liquid at \( \nu = \frac{1}{3} \) leads to a reduction in the ground state energy of the coupled system, which was manifested as an energetic anomaly in the experiment. From Fig. 1 we can see that also at \( \nu = \frac{2}{5} \) we would expect a lowering of the ground state energy of the coupled system as compared to the uncoupled liquid (cf. inset of Fig. 1d). The magnetic field range where the anomaly appears in Fig. 2 of Ref. \[9\] is relatively broad. In this experiment, \( \nu = \frac{2}{5} \) would be roughly at \( B = 9 \) T, which is just at the onset of the anomaly. Therefore, it is possible that already in this experiment, there is a signature of the spin-reversed quasihole at \( \nu = \frac{2}{5} \) present. To study this in more detail, it would be highly desirable to have a better control over the distance, \( a \), between the localized charged excitons and the electron liquid. We propose therefore experiments employing a special sample structure. The sample should consist of a layer of self-organized InGaAs quantum dots and a GaAs quantum well, containing a high mobility electron system which is separated from the QD layer by a distance \( a \). In such a structure, the distance between the charged dots and the electron liquid can be very precisely controlled by the vertical growth of the sample. The QD can be charged with single electrons via the external gates. Under these conditions, with a well defined distance \( a \), we would expect much sharper anomalies in the photoluminescence recombination energies of excitons in the quantum dots at fractional fillings. Details will be published elsewhere.

In summary, we have investigated the ground state and low-energy excitations of a quantum Hall liquid at a filling factor \( \nu = \frac{2}{5} \) which is Coulomb coupled to a quantum dot (a liquid-qd complex) separated from the liquid by a distance \( a \). The dot consists of two or three electrons and a hole. It was found that the presence of the dot changes locally the spin polarization of the electron liquid that depends crucially on the separation distance. In addition, the electron density distribution in the quantum Hall layer indicates creation of a quasihole that seems to have been localized in the electron plane due to the close proximity of the quantum dot containing interacting electrons. The energy required to create the quasihole has been calculated. We have also outlined a proposal to detect the quasihole properties at \( \nu = \frac{2}{5} \) via PL experiments.

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[1] D. Tsui, H.L. Störmer, and A.C. Gossard, Phys. Rev. Lett. 48, 1559 (1982).
[2] R.B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983); Surf. Sci. 142, 163 (1984).
[3] B.I. Halperin, Helv. Phys. Acta 56, 75 (1983).
[4] T. Chakraborty, Adv. Phys. 49, 959 (2000); T. Chakraborty and F.C. Zhang, Phys. Rev. B 29, 7032(R) (1984); F.C. Zhang and T. Chakraborty, ibid. B 30, 7320 (R) (1984).
[5] T. Chakraborty, P. Pietiläinen, and F.C. Zhang, Phys. Rev. Lett. 57, 130 (1986).
[6] P.A. Maksym, J. Phys.: Condens. Matter 1, 6299 (1989); X.C. Xie, Yin Guo, and F.C. Zhang, Phys. Rev. B 40, 3487 (1989).
[7] T. Chakraborty and P. Pietiläinen, The Quantum Hall Effects (Springer, 1995), 2nd Edition; T. Chakraborty, in Handbook on Semiconductors, vol. 1, edited by P.T. Landsberg (Elsevier, New York, 1992), Ch. 17.
[8] See, for example, G.S. Boebinger et al., Phys. Rev. Lett. 55, 1606 (1985); R.G. Clark et al., ibid. 62, 1536 (1989); J.P. Eisenstein et al., ibid. 62, 1540 (1989); R.G. Clark and P.A. Maksym, Phys. World 2 (9), 39 (1989).
[9] C. Schüller, K.-B. Broocks, P. Schröter, Ch. Heyn, D. Heitmann, M. Bichler, W. Wegscheider, T. Chakraborty, and V.M. Apalkov, Phys. Rev. Lett. 91, 116403 (2003); Adv. Solid State Physics 44, 81 (2004); Physica E 22, 131 (2004).
[10] V.M. Apalkov and T. Chakraborty, Physica E 14, 289 (2002).
[11] P.A. Maksym and T. Chakraborty, Phys. Rev. Lett. 65, 108 (1990); T. Chakraborty, Comments Condens. Matter Phys. 16, 35 (1992); T. Chakraborty, Quantum Dots (North-Holland, Amsterdam, 1999).
[12] E.H. Rezayi and F.D.M. Haldane, Phys. Rev. B 32, 6924 (1985).
[13] See, for example, I.V. Kukushkin et al., Phys. Rev. Lett. 82, 3665 (1999); ibid. 85, 3688 (2000).
[14] See, for example, A.J. Turberfield et al., Phys. Rev. Lett. 65, 637 (1990).
[15] G. Yusa et al., Phys. Rev. Lett. 87, 216402 (2001).
[16] A. Wojs et al., Phys. Rev. B 62, 4630 (2000).