DQC\(^2\)O: Distributed Quantum Computing for Collaborative Optimization in Future Networks

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Abstract—With the advantages of high-speed parallel processing, quantum computers can efficiently solve large-scale complex optimization problems in future networks. However, due to the uncertain qubit fidelity and quantum channel noise, distributed quantum computing, which relies on quantum networks connected through entanglement, faces many challenges in exchanging information across quantum computers. In this paper, we propose an adaptive distributed quantum computing approach, called DQC\(^2\)O, to manage quantum computers and quantum networks for solving optimization tasks in future networks. Firstly, we describe the fundamentals of quantum computing and its distributed concept in quantum networks. Secondly, to address the uncertainty of future demands of collaborative optimization tasks and instability over quantum networks, we propose a quantum resource allocation scheme based on stochastic programming for minimizing quantum resource consumption. Finally, based on the proposed approach, we discuss the potential military applications of collaborative optimization in future networks, such as smart grid management, IoT cooperation, and semantic communications. Promising research directions that can lead to the design and implementation of future distributed quantum computing frameworks are also highlighted.

Index Terms—Distributed quantum computing, quantum networks, resource allocation, stochastic programming.

I. Introduction

In the quantum era, the advancement of quantum computing and communication has attracted significant interest from researchers, government organizations, and industries [1]. Quantum computers provide effective solutions to complex optimization problems in a resource-efficient manner [2], a feat not possible for classical computers to achieve. The recent adoption of quantum computing has further boosted technology innovations such as artificial intelligence (AI), improved battery chemistry, and life-saving pharmaceuticals [3]. In particular, military applications in the future viability of quantum technologies could further develop the ability to implement and improve military strategies in competition, conflict, and even war between nations [1]. By integrating quantum computers via quantum networks [4], multiple tiny quantum computers can be integrated into a single available quantum computer to enhance the collaborative capabilities of modern battle swarms on the battlefield. The proliferation of quantum computing has also evolved to accelerate computation for various applications and services in future networks, which must be designed efficiently using limited resources to perform a wide range of heterogeneous tasks [5].

By using the concept of quantum bits (i.e., qubits) in quantum computers, a number of quantum algorithms, which are superior to classical algorithms, have been devised. For example, Shor’s [6] and Grover’s [7] algorithms, two well-known quantum algorithms, are developed to solve factorization and search for unstructured data, respectively. These tasks are highly challenging for classical computers. However, scaling up quantum computers is a key challenge in deploying quantum computers in practice [1]. To date, only a small number of qubits can be implemented in a single quantum computer. Moreover, quantum tasks usually require using multiple qubits in more complex applications [8]. Due to the instability of qubits and the amount of information required, it becomes more challenging to manage and control information in quantum computers with a small number of qubits [1].

Thus, distributed quantum computing has been proposed to alleviate this problem. The concept of distributed quantum computing, which refers to multiple interconnected quantum computers, was introduced to accelerate and perform collaboratively quantum computations through quantum networks [1]. Distributed quantum computing requires quantum teleportation, in which qubits are teleported between two quantum computers [1]. In this way, a large, complex computational task can be accomplished jointly by multiple quantum computers.

Although distributed quantum computing has many advantages over a single quantum computer and classical...
II. Fundamentals of Quantum Computing

A. Quantum Computing

Three properties of quantum mechanics define quantum computing: superposition, interference, and entanglement.

1) Superposition: A superposition in quantum computing allows the encoding of qubits in a combination of two classical binary states, where the coefficients correspond to the probabilities of the qubit amplitudes. A well-known example of quantum mechanics is the tossing of a coin. When a coin is tossed in the air, the exact outcome is unknown, and its probability values reflect qubits. Due to the superposition, a qubits can store $2^n$ possible outcomes and have the same chance of being measured for each. Therefore, quantum computers can store and manipulate more information than classical computers, providing far more diverse possibilities and opportunities.

2) Interference: Qubits must be subjected to measurement to represent and store their values and results. If one intervenes in the process, it is possible to measure and see the results of the paths. When the process is interrupted, the states of the qubits collapse to classical bits, and the results appear. For example, the result of a coin toss is known definitively (e.g., heads or tails) when the coin reaches the bottom.

3) Entanglement: Two qubits can be entangled with each other as an entanglement pair [1], which means that when one qubit is measured, the other qubit can also be known because of its entangled nature. In addition, a pair of entanglement qubits are entangled maximally, also known as a Bell state, when the result of measuring one of them will certainly affect the result of measuring the other one later. The fidelity of entanglement pairs is the metric of attenuation for the entangled qubits between two quantum computers. The fidelity scale runs from 0 to 1,
where 1 indicates the best performance the entanglement can achieve.

Quantum circuit model proceeds with implementations on qubits and involves an ordered sequence of quantum gates that permit logical interaction between qubits. Specifically, the measurement of qubits must occur near the end of the quantum circuit. A quantum processor is a small quantum computer that can execute quantum gates on a small number of qubits and allows the entanglement of qubits inside.

B. Distributed Quantum Computing

The concept of distributed quantum computing relies on the following principles:

1) Quantum Networks through Entanglement: Due to the non-cloning theorem in quantum mechanics, qubits cannot be duplicated or cloned across quantum computers. Thus, two quantum computers must exchange or transfer qubits through quantum teleportation. In quantum teleportation, a pair of qubits from the source to the destination quantum computer is transferred for local operations and measurements before decoherence, which is the loss of the shared entangled qubits that causes disturbances and collapse superposition. In detail, decoherence may produce errors in quantum information; while receiving processed information, there must be loose interactions between qubits. After quantum teleportation, Bell-state measurement is used to determine the quantum state of a teleported qubit occurring between the two quantum computers [1], as shown in Fig. 2b. The results of the Bell-state measurement are generated and distributed between the source and destination quantum computers. Then, a conventional communication channel is used to transmit the two classical bits. Specifically, errors and failures incur in quantum teleportation while preserving the entangled qubits. An entanglement distillation procedure is developed to iteratively increase fidelity or the probability that the qubit will be transferred without changing its state. However, quantum teleportation requires a lot of iterations, time, and resources to achieve the Bell state of a pair of qubits.

2) Distributed Quantum Circuits: Required quantum circuits can be split and distributed among multiple quantum processors of quantum computers, with each quantum computer executing a fragment of quantum circuits. The architecture of the quantum circuits executes non-local quantum gates for the shared entangled qubits. A logically identical set of instructions is used to coordinate the additional operations required for the non-local operation to replace the multiple qubit operations. In particular, a partitioning of quantum circuits to achieve the least number of qubits among quantum processors must be determined remotely by exchanging non-local operations with other quantum gates.

C. Quantum Algorithms and Distributed Quantum Algorithms

The quantum algorithms to be executed in quantum computers can be of several types depending on the computational paradigm. The key issues on quantum algorithms and distributed quantum algorithms are as follows.

1) Quantum Algorithm: Numerous studies have focused on quantum algorithms operating at exponentially faster speeds on quantum computers implemented on quantum circuits [13]. We discuss the most prevalent quantum algorithms as follows:

- Shor’s algorithm is devised to factor prime numbers in polynomial time, which cannot be done polynomially by classical algorithms. The factoring problem is reformulated as the finding-period problem and solved through quantum phase estimation, which estimates phases or eigenvalues of eigenvectors of unitary operators.
- Grover’s algorithm solves a searching problem for an unstructured database, which provides quadratically
speed-up computation. An equal superposition of all possible solutions is employed as inputs that result in the same amplitudes. The inputs are passed and processed through the oracle diffuser to boost the amplitudes and then reflect the solution in terms of the greatest amplitudes.

Grover’s algorithm can be used as a generic algorithm to address various problems due to the independence of the algorithm and the internal structure of lists. As a result, executing Grover’s algorithm gives many classical problems a quadratic speed-up computation.

2) Distributed Quantum Algorithm: Although quantum computing is significantly more advanced than classical computing, only small quantum computers (with a limited number of qubits) have so far been constructed due to the noise and depth of quantum circuits. As an alternative to large-scale monolithic designs, a distributed grid of small quantum computers has been proposed to help advance quantum computing. In particular, as distributed quantum computing necessitates sharing the superposition state among quantum computers, quantum networks through entanglement and swap gates need to be implemented. We discuss the most prevalent distributed quantum algorithms as follows:

- Distributed Shor’s algorithm is implemented to solve factorization problems by small-capacity quantum computers. The architecture of quantum circuits to perform collaboratively is designed to handle quantum teleportation and simulate a large-capacity quantum computer [6]. The complexity of distributed Shor’s algorithm is \( O((\log N)^2) \), while Shor’s algorithm requires \( O((\log N)^2(\log \log N)(\log \log \log N)) \), where \( N \) denotes the integer to be factored.

- Distributed Grover’s algorithm is developed to reduce query times of solving the unstructured search problem [7]. Functions that need to be computed can be divided into \( 2^k \) subfunctions by the distributed Grover’s algorithm, which then computes one of the usable subfunctions to find the solution to the original function. A procedure of Grover’s algorithm solving a four-qubit quantum task for quantum computing and distributed quantum computing is shown in Fig. 2.

Among distributed quantum algorithms, the grid of distributed quantum circuits has to be well-designed to best utilize the constrained resources in quantum computers. Therefore, it becomes increasingly important to address resource allocation in distributed quantum computing to solve resource constraints given quantum computing environments.

III. A Case Study: Resource Allocation in Distributed Quantum Computing

As shown in Fig. 1, we consider a system model, where the quantum computer operator aims to reduce the total cost of using quantum computers and on-demand quantum computers to perform computational tasks. Different computational tasks may require different numbers of qubits. Moreover, the links between two interconnected quantum computers are aligned and have static and limited capacity. Therefore, allocating quantum computers and quantum network resources must provide a sufficient number of quantum computers to perform computational tasks in distributed quantum computing.

A. System Model, Decisions, and Costs

In distributed quantum computing, a quantum computer operator distributes computational tasks among one or multiple quantum computers in performing quantum computation. However, the deployed quantum computers may not be sufficient to accomplish a large-scale computational task. Therefore, the quantum computer operator can request the deployment of new quantum computers, or the utilization of on-demand quantum computing may be required. Shortly, quantum computing from multiple organizations can be shared in which one organization can borrow or buy quantum computing time from another. Such a scenario is inspired by the present conventional cloud computing paradigm. For example, Amazon Braket offers on-demand quantum cloud computing services (https://aws.amazon.com/braket/). For this option, the quantum computer operator installs and configures the on-demand quantum computers to work collaboratively with the deployed quantum computers. Utilizing the deployed quantum computer will cost the computing power of the quantum computer and the Bell
pair of the two connected quantum computers. However, using an on-demand quantum computer is more expensive.

B. Uncertainty of Distributed Quantum Computing

While determining the best resource allocation in distributed quantum computing, uncertainties occur, which can be classified into three types as follows:

- First, the actual demands are unknown when the option of deploying the quantum computer is made. Different applications such as minimization, dimensional reduction, and machine learning (ML) problems may request various qubits [8]. For example, around 100 qubits are required depending on the complexity of optimization tasks [8].

- Second, the precise availability of the quantum computer and its qubits is uncertain since they might be reserved for other purposes or because the quantum computer’s backend may not support all of them (https://www.ibm.com/quantum). For example, only 5 out of 10 quantum computers are available to compute quantum tasks.

- Third, the fidelity of the entangled qubits is also not known exactly due to the degradation of the entangled qubits in distributed quantum computing [13]. For example, 0.5 of the fidelity means that 50% efficiency of the entangled qubits can be achieved.

Therefore, an adaptive resource allocation approach is required to efficiently provision quantum computers under the aforementioned uncertainty.

C. The Proposed Approach

In the deterministic resource allocation for computing quantum tasks in distributed quantum computing, the quantum computer operator precisely knows the required demand of quantum tasks, the computing power of quantum computers, and the fidelity of the entangled qubits. Therefore, quantum computers can certainly be deployed, and on-demand quantum computer deployment is unnecessary. However, due to the aforementioned challenges and uncertain environments, the deterministic resource allocation approach is not applicable, and on-demand deployment will be the solution. Therefore, we propose the adaptive distributed quantum computing approach based on the two-stage stochastic programming model [14]. The stochastic programming model is used to make optimal resource allocation decisions in distributed quantum computing under uncertainty. It is effective in minimizing the total deployment cost while exploiting uncertainty. As shown in Fig. 1, in the first stage, the quantum computer operator decides to deploy quantum computers before observing the computational task requirements, the availability of quantum computers, and the instability of quantum networks. In the second stage, after observing the actual computational task requirements, the availability of quantum computers, and the instability of quantum networks, the quantum computer operator decides to deploy quantum computers on demand when the deployed quantum computers are not sufficient. The stochastic programming model can be formulated as minimizing the total deployment cost consisting of two-stage costs. The first-stage cost is the cost of deploying quantum computers. The second-stage cost is the expectation of the costs for using the deployed quantum computers, using the Bell pair that connects the quantum computers, and deploying on-demand quantum computers over the set of possible realized requests, the availability of quantum computers, and the fidelity of quantum networks. The constraints of the stochastic programming model that accounts for uncertainty are the choice of deployed quantum computers, the ability of both deployed and on-demand quantum computers to fully execute the computational tasks over distributed quantum computers, and the computational power of the deployed quantum computers being below the ability of the Bell pair to interleave qubits between the deployed quantum computers.

D. Experimental Results

We consider the quantum computer operator, which consists of 10 deployed quantum computers. We set the cost values, measured in normalized monetary, for using the deployed quantum computer, qubits, and Bell pairs to be 5000, 1000, and 450, respectively. All quantum computers have 257 qubit capacities and identical costs. The cost and computing power of on-demand quantum computers are 25000 and 127, respectively. Costs for both on-demand and quantum computers are determined based on the quantum cloud computing services. We consider two scenarios for the stochastic model. The first scenario is that the demand of the quantum task is 10, the computing power of the quantum computers is 127 qubits, and the fidelity of the entangled qubits is 1 (i.e., the best performance). The second scenario is no demand, no availability of qubits, and zero fidelity (i.e., the worst performance). We assume the default probability values with 0.8 and 0.2, respectively. We use the General Algebraic Modeling Language (GAMS) script to implement the stochastic programming model and solve it with the CPLEX solver. In the experiments, we first study the cost breakdown and then compare the models to illustrate the advantages of the proposed stochastic programming.

1) Impact of Probability of Scenarios: We vary the probability of the first scenario with having the demand, the availability of the quantum task, and the best performance for the entangled qubits. The cost breakdown is shown in Fig. 3b. When the probability of the scenario is equal to or less than 0.2, the on-demand quantum computer should be deployed. The deployed quantum computer is utilized over the on-demand quantum computer when the probability of the scenario is higher as it has the demand of the quantum task, computing power of quantum computers, and fidelity.
from classical computing, quantum computing can be used to enhance computational approaches that support decision-making in smart grids, as discussed in [9]. The current smart grid issues also involve power systems that operate on several timescales and dimensions and must be resolved immediately. To address upcoming smart grid concerns, distributed quantum computing is more feasible and promising than quantum computing.

2) Future Internet of Things: Emerging future IoT devices can monitor remote locations for data collection in various battlefield services like military surveillance and monitoring. Inspired by acquiring Internet of Things (IoT) data and improving data accuracy analysis, the article [5] presents a novel quantum computing-inspired optimization to optimize IoT-sensor space by using quantum formalization based on quantum mechanics. According to the increased amount of IoT-connected devices, the scalability of utilizing one quantum computer needs to be extended to enhance the overall deployment using the properties of distributed quantum computers.

3) Future Semantic Communications: Semantic communications are necessary to convey the meaning of messages, rather than being a simple reconstruction process of raw data, to facilitate the transmission of military data, which requires security and reliability. In [15], quantum semantic representations are proposed to extract and transmit the contextual meaning of data with minimal communication resources and errors. However, issues remain to be addressed, such as formulating and optimizing a general resource allocation problem for different task-oriented quantum semantic systems.

These military applications can all be categorized as large-scale problems in future networks, which are still challenging because of the numerous computational resources and processes required. A solution to these problems could be provided by the paradigm of distributed quantum computing, which can improve performance, scalability, and cost efficiency by having each quantum computer in a pool handle different parts of a computational task simultaneously. Furthermore, classical and quantum computers will still coexist to execute computational tasks. Hybrid computing, combining quantum and classical computing, is required to significantly reduce energy consumption and costs.

B. Challenges in Distributed Quantum Computing

1) Efficient Routing for Collaborative Quantum Computers: Quantum repeaters have been introduced as an intermediate for communicating between quantum computers with long-distance links. The quantum router architecture is designed to sustain entanglement over quantum networks consisting of quantum memories coupled through photons [11]. In particular, distributed quantum computing relies on quantum teleportation, which is susceptible to decoherence issues, making it different from routing in traditional distributed computing. Open
challenges remain to design efficient routing to control and improve entanglement fidelity, including long-distance links in distributed quantum computing.

2) Coexistence of Multiple Distributed Quantum Algorithms: Quantum algorithms required for various optimization problems in future networks will differ from the number of qubits to the type of quantum gates. For a single optimization problem, distributed quantum computing can divide the problem into multiple homogeneous subproblems and deploy them on interconnected quantum computers for cooperative solutions. However, to meet the multi-task and multi-algorithm requirements, distributed quantum computing must be equipped to handle heterogeneous tasks. Therefore, standardized protocols and efficient collaboration will be indispensable in distributed quantum computing in future optimization problems, e.g., critical and secure communication in military environments.

3) Distributed Quantum Machine Learning: Quantum ML is proposed using quantum algorithms to carry out ML more rapidly. Multiple paradigms of the existing ML algorithms, encoded in a superposition state, are used to improve ML solutions [10]. For example, quantum principal component analysis (PCA) is exponentially more effective than traditional PCA [10]. More exciting issues exist in various ML applications, such as quantum federated learning, in which multiple clients (or quantum computers) can work collaboratively and securely with distributed quantum algorithms.

V. Conclusions

In this paper, we have focused on distributed quantum computing for collaborative optimization, i.e., DQCQ2O. It relies on quantum networks connected through entanglement and the architecture of distributed quantum circuits to perform complex computational tasks collaboratively. To efficiently utilize quantum resources at scale and in the face of uncertainties, we have proposed an adaptive resource allocation approach to achieve cost efficiency in distributed quantum computing. The experimental results show that the proposed model can minimize the total cost under uncertain distributed quantum computing environments. Finally, we have discussed the opportunities and challenges of distributed quantum computing in future networks. Future research will consider other key factors, such as the uncertainty in task execution time of distributed quantum computing and the required deadline of military applications in future networks.

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