Quantum Energy Inequality for the Massive Ising Model

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A Quantum Energy Inequality (QEI) is derived for the massive Ising model, giving a state-independent lower bound on suitable averages of the energy density; the first QEI to be established for an interacting quantum field theory with nontrivial S-matrix. It is shown that the Ising model has one-particle states with locally negative energy densities, and that the energy density operator is not additive with respect to combination of one-particle states into multi-particle configurations.

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In relativistic quantum field theory, the Hamiltonian is a positive operator in all inertial frames of reference; this is the content of the spectrum condition. By contrast, it is impossible for nontrivial local averages of the energy density to be positive operators in any quantum field theory obeying standard assumptions [1]: typically, the expectation value of the energy density at any point is unbounded from below with respect to the state [2]. This state of affairs demonstrates a fundamental incompatibility between quantum fields and the energy conditions usually assumed in classical general relativity, and which are the essential input for results such as the singularity theorems of Penrose and Hawking [3–5], the positive mass theorems [6–9], and Hawking’s chronology protection results [10], among many others.

Nonetheless, various models of quantum field theory obey local remnants of the spectrum condition called Quantum Energy Inequalities (QEIs), which provide lower bounds on expectation values of the energy density when averaged along a timelike curve or over a spacetime region. Their study originates from Ford’s insight [11] that quantum field theory could produce observable deviations from the second law of thermodynamics unless there were mechanisms to constrain negative energy densities (or fluxes). QEI bounds place severe constraints on the extent to which quantum fields can support exotic spacetime geometries [12, 13]; moreover, weakened classical energy conditions inspired by QEIs can be used to prove singularity theorems [14].

QEIs have been established for free (minimally coupled) Klein–Gordon [15–21], Dirac [22–25], Maxwell [26, 27] and Proca fields [27] in both flat and curved spacetimes, for the Rarita–Schwinger field in Minkowski space [28], and for the whole class of unitary positive-energy conformal field theories in two-dimensional Minkowski space [29] (generalising a special case [16]). In all these cases, the energy density is bounded from below on the class of physically acceptable states, if it is smeared against a positive test function over a region or curve of nonzero temporal extent. Even within the setting of free fields, however, the nonminimally coupled Klein–Gordon field provides an example where only a weaker type of QEI holds [30] – the lower bound is no longer state-independent, but exhibits dependence on the energy scale – and it is expected that this behaviour would be typical for interacting quantum field theories [31]. Analogues of these energy-dependent QEIs exist in general quantum field theories for observables arising in operator product expansions of ‘classically positive’ expressions [32]; however, a direct connection to the energy density is lacking.

In this paper, we derive a QEI for the continuum Ising model [33], a quantum field theory of interacting massless spin-0 bosons in two-dimensional Minkowski space. The Ising model is the simplest integrable quantum field theory (other than free models) with a factorizing scattering matrix – the two-particle S-matrix is $S_2 = -1$. It is closely related to other integrable models such as the sinh-Gordon theory. Our result is the first QEI to be obtained for a theory with a nontrivial S-matrix. Although the scattering theory is rather simple, the energy density of the Ising model differs in essential ways from that of the free scalar field. As we will show, there are single-particle states of the Ising model that have locally negative energy density. In a free theory one could combine such single-particle states to obtain multi-particle configurations with arbitrarily large negative energy densities (cf. [30]), thus excluding the existence of a state independent QEI. However, the energy density of the Ising model is not additive with respect to tensor products of single-particle states; a signature of the interacting nature of the theory. It is intriguing that the interaction is responsible for maintaining the QEI in this sense.

Although it is a theory of bosons, the Ising model is conveniently defined on a fermionic Fock space. Our argument will make essential use of this underlying structure, and indeed is based on arguments originally developed in the context of the free Dirac field [22, 24]. The QEI bound is given as the $(−)$ case of [8] below; the $(+)$ case applies to the free scalar field and is given for comparison.

To start, we fix our conventions for the quantum Ising model of mass $\mu > 0$, in a notation that makes parallels to the two-dimensional scalar free field clear. We work on the single particle space $\mathcal{K} = L^2(\mathbb{R}, d\theta)$, where the rapidity $\theta$ is related to two-momentum $p$ by...
The conventions, we define quantum fields following Lechner (though with slightly different conventions), we define quantised fields $\phi(\theta)$, $a_\pm(\theta)$, which fulfill canonical (anti-)commutation relations,

$$a_\pm(\theta) a^\dagger_\mp(\theta) + a^\dagger_\pm(\theta) a_\mp(\theta) = \delta(\theta - \eta) I.$$  

Spacetime symmetries, i.e., translations $x = (t, x)$, boosts $\lambda$, and the space-time reflection $j$, act on $H_{\pm}$ by

$$U_{\pm}(x, \lambda) a_\pm(\theta_1) \cdots a_\pm(\theta_n) \Omega = e^{i(\vec{p}(\theta_1) + \cdots + \vec{p}(\theta_n)) \cdot \vec{x}} a_\pm(\theta_1 + \lambda) \cdots a_\pm(\theta_n + \lambda) \Omega, \quad (1)$$

$$U_{\pm}(j) a_\pm(\theta_1) \cdots a_\pm(\theta_n) \Omega = a^\dagger_\pm(\theta_1) \cdots a^\dagger_\pm(\theta_n) \Omega, \quad (2)$$

with $U_{\pm}(j)$, the PCT operator, extended antilinearly.

We now describe the basic observables of the model. Following Lechner (though with slightly different conventions), we define quantum fields $\phi_\pm$, $\phi'_\pm$ as

$$\phi_\pm(x) = \frac{1}{\sqrt{4\pi}} \int d\theta \left( e^{i\vec{p}(\theta) \cdot \vec{x}} a_\pm(\theta) + e^{-i\vec{p}(\theta) \cdot \vec{x}} a_\pm^\dagger(\theta) \right),$$

$$\phi'_\pm(x) = U_{\pm}(j) \phi_\pm(-x) U_{\pm}(j).$$

These are covariant under the symmetry operations (1), with space-time reflection exchanging $\phi_\pm$ and $\phi'_\pm$. Of course, $\phi_\pm = \phi'_\pm$ is the usual local free scalar field.

The energy density $\rho_\pm(x)$, $\rho'_\pm(y)$ is nonnegative in $\text{vol}$. Our approach is to consider the local observables $\phi_\pm$.

Let us recall the scattering theory of our models. The case $\phi_\pm$ is trivial of course. As mentioned, $\phi_\pm$ defines the (interacting) massive Ising model with nontrivial scattering matrix [34]. Both the incoming and outgoing states are bosonic Fock states, i.e., we can identify $H_\text{in}$ and $H_\text{out}$ with $H_\pm$. The incoming Møller operator is then given by $V_{\text{in}} : H_\pm \rightarrow H_\pm$.

$$V_{\text{in}} a_\pm(\theta_1) \cdots a_\pm(\theta_n) \Omega = \left( \prod_{i<j} \epsilon(\theta_i - \theta_j) \right) a_\pm^\dagger(\theta_1) \cdots a_\pm^\dagger(\theta_n) \Omega$$

and the outgoing Møller operator $V_{\text{out}}$ by a similar formula, but with the argument of each $\epsilon$ negated. The $S$-matrix is $S = V_{\text{in}} V_{\text{out}} = (-1)^{N_+ (N_+ - 1)/2}$, where $N_+$ is the bosonic number operator on $H_\pm$. The free field and interacting Ising model are defined to be free.

A crucial observation, for our purposes, is that a free Majorana field $\psi = (\psi_1, \psi_2)^T$ can be defined on the fermionic Fock space $H_\pm$ by

$$\psi_{1,2}(x) = \sqrt{\frac{\mu}{4\pi}} \int d\theta e^{i\vec{p}(\theta) \cdot \vec{x}} \epsilon(\theta) \cdot \vec{F}(\theta) \cdot \vec{a}_\pm^\dagger(\theta) + \text{h.c.}$$

(here $+$ for the case 1 and $-$ for the case 2). This Majorana field is covariant under $U_{\pm}(\theta, \lambda)$ as defined above, but not under $U_{\pm}(j)$; its associated PCT operator is fundamentally different. By analogy with [34], Sec. 6, $\psi$ fulfills

$$\{ \phi_\pm(x), \psi(x) \} = 0 = [\psi(y), \phi_\pm(y)]$$

if $y$ is to the left of $x$. That is, $\psi(x)$ is not a local observable of the interacting theory (cf. (3)). However, from (4), all even polynomials in $\psi(x)$ have this property. This applies in particular to the energy density

$$T_{00}(x) = \frac{i}{4} \phi(x) \partial_\lambda \psi(x) - (\partial_\lambda \psi(x))^T \psi(y).$$

Since the energy-momentum operators of the Majorana field coincide with those on $H_\pm$ by (1). $T_{00}$ is also the energy density of the interacting Ising model; nonetheless, we emphasize that the Ising model is distinct from the free Majorana theory.

For completeness, we recall that the energy density of the free Bosonic scalar field $\phi_\pm$ on $H_\pm$ is

$$T_{00}(x) = \frac{1}{2} (\partial_\lambda \phi_\pm(x))^2 + (\partial_\lambda \phi_\pm(x))^2 + \mu^2 \phi_\pm(x)^2.$$  

After some computation, the energy densities of both the free and interacting Bose models take the form

$$T_{00}(x) = \frac{1}{2} \int d\eta d\theta \left( F_\pm(\theta, \eta, x) a_\pm^\dagger(\theta) a_\pm^\dagger(\eta) \right) + 2 F_\pm(\theta, \eta + \pi i, x) a_\pm^\dagger(\theta) a_\pm(\eta)$$

$$+ F_\pm(\theta + \pi i, \eta + \pi i, x) a_\pm^\dagger(\theta) a_\pm(\eta).$$

with

$$F_+(\xi, x) = \frac{\mu^2}{2\pi} \sin^2 \frac{\xi_1 + \xi_2}{2} e^{i(\vec{p}(\xi_2) + \vec{p}(\xi_3)) \cdot \vec{x}},$$

$$F_-(\xi, x) = i \sinh \frac{\xi_1 - \xi_2}{2} F_+(\xi, x).$$

(See [38] for the general theory of these expansions.) Using methods from [37], Sec. 9.1, one can see from the analyticity structure of $F_\pm$ that $T_{00}(h) = \int dt h(t) T_{00}(t, x)$ is a closable operator, and a local observable, for any $x \in \mathbb{R}$ and Schwartz test function $h$.

In the Bosonic free field situation, it is well known (see e.g., [35]) that $T_{00}$ has nonnegative expectation value in all single particle states, or more generally, states with sharp particle number, while more general superpositions (such as that of the vacuum and a two-particle state) can yield negative expectation values (see, e.g., [1]). By contrast, we will now exhibit single-particle states in the
Ising model with negative energy density at the origin, reminiscent of the situation for free Dirac fields \[35\].

These states are essentially superpositions of two plane waves. More specifically, let us choose a nonnegative real-valued Schwartz function \( h \) with \( \int h(\theta) d\theta = 1 \). We set \( \Phi := a^\dagger(\varphi_{\alpha, \beta, \gamma}) \Omega = \int d\theta \varphi_{\alpha, \beta, \gamma}(\theta) a^\dagger(\theta) \Omega \), where

\[
\varphi_{\alpha, \beta, \gamma}(\theta) := c_{\alpha, \beta, \gamma}(h_\alpha(\theta) + \beta h_\alpha(\theta - \gamma)),
\]

\[
h_\alpha(\theta) := \alpha^{-1} h(\alpha^{-1} \theta),
\]

with parameters \( \alpha > 0, \beta, \gamma \in \mathbb{R} \) and normalization constant \( c_{\alpha, \beta, \gamma} > 0 \). We will show that \( \langle \Phi, T_\alpha^{00}(0) \Phi \rangle < 0 \) for a suitable choice of the parameters. To that end, we compute from \[35\],

\[
\langle \Phi, T_\alpha^{00}(0) \Phi \rangle = \frac{\mu^2}{2\pi} c^2_{\alpha, \beta, \gamma} (I_\alpha + J_{\alpha, \gamma} \beta + K_{\alpha, \gamma} \beta^2),
\]

where we denoted

\[
I_\alpha = \int d\theta d\eta h_\alpha(\theta) h_\alpha(\eta) \cosh^2 \frac{\theta + \eta}{2} \cosh \frac{\theta - \eta}{2},
\]

\[
J_{\alpha, \gamma} = 2 \int d\theta d\eta h_\alpha(\theta) h_\alpha(\eta) \cosh^2 \frac{\theta + \eta + \gamma}{2} \cosh \frac{\theta - \eta}{2},
\]

\[
K_{\alpha, \gamma} = \int d\theta d\eta h_\alpha(\theta) h_\alpha(\eta) \cosh^2 \frac{\theta + \eta + 2\gamma}{2} \cosh \frac{\theta - \eta}{2}.
\]

The right-hand side of \( \langle \Phi, T_\alpha^{00}(0) \Phi \rangle \) is negative for some \( \beta \) if the polynomial \( I_\alpha + J_{\alpha, \gamma} \beta + K_{\alpha, \gamma} \beta^2 \) has two real zeros, that is, if \( J^2_{\alpha, \gamma} > 4I_\alpha K_{\alpha, \gamma} \). This inequality holds for small \( \alpha \) if it holds in the limit \( \alpha \to 0 \). Noting that \( h_\alpha(\theta) \to \delta(\theta) \) in this limit, we obtain

\[
I_0 = 1, \quad J_{0, \gamma} = 2 \cosh^3 \frac{\gamma}{2}, \quad K_{0, \gamma} = \cosh^2 \gamma.
\]

The condition \( J^2_{0, \gamma} > 4I_0 K_{0, \gamma} \) then becomes

\[
(1 + \cosh \gamma)^3 > 8 \cosh^2 \gamma,
\]

which is in fact fulfilled for sufficiently large \( \gamma \). With these choices, we achieve \( \langle \Phi, T_\alpha^{00}(0) \Phi \rangle < 0 \). For an example see Fig. 1 (solid line), where \( \langle \Phi, T_\alpha^{00}(t, 0) \Phi \rangle \) is plotted for a suitable choice of the parameters.

Given any normalized single-particle state wave function \( \varphi \in \mathcal{K} \), we may form multi-particle states by taking tensor products in the ‘in’ Hilbert space and applying the inverse Møller operator. For the free model, this yields \( n \)-particle vectors \( \Phi_{n, \pm} := (n!)^{-1/2} a_{\pm}^\dagger(\varphi)^n \Omega \in \mathcal{H}_n \), while the corresponding states in the Ising model are

\( \Phi_{n, \pm} := V^*_{\pm} \Phi_{n, \pm} \in \mathcal{H}_n \).

Now, the total energy, given by the Hamiltonian \( H_\pm \) (the generator of time translations), is additive in the sense that \( \langle \Phi_{n, \pm}, H_{\pm} \Phi_{n, \pm} \rangle = n \langle \Phi_{1, \pm}, H_{\pm} \Phi_{1, \pm} \rangle \). (For \( H_+ \) this is evident from \[36\], and for \( H_- \) it follows since \( V_{\text{in}} H_- V^{\dagger}_{\text{in}} = H_+ \)). Furthermore, the energy density in the free model is also additive: \( \langle \Phi_{n, \pm}, T_{\pm}^{00}(x) \Phi_{n, \pm} \rangle = \n(\Phi_{1, \pm}, T_{\pm}^{00}(x) \Phi_{1, \pm}) \).

However, the same relation does not hold in the interacting situation: In general,

\[
\langle \Phi_{n, \pm}, T_{\pm}^{00}(x) \Phi_{n, \pm} \rangle \neq n \langle \Phi_{1, \pm}, T_{\pm}^{00}(x) \Phi_{1, \pm} \rangle.
\]

We can deduce this from our other results: We saw above that \( \langle \Phi_{1, \pm}, T_{\pm}^{00}(x) \Phi_{1, \pm} \rangle \) is negative in some examples, and therefore at large \( n \) equality in \( \text{7} \) would be in contradiction to the QEI \( \text{8} \) that we will establish below.

But let us give a direct argument for \( \text{7} \). It is useful to note that \( V_{\text{in}} a_{\pm}^\dagger(\theta)V^{\dagger}_{\text{in}} = a_{\mp}^\dagger(\theta)M(\theta) \), where \( M(\theta) \) is the multiplication operator

\[
M(\theta) a^\dagger_\pm(\eta_1) \cdots a^\dagger_\pm(\eta_n) \Omega = \left( \prod_{j=1}^n (\epsilon(\theta - \eta_j)) a^\dagger_\pm(\eta_1) \cdots a^\dagger_\pm(\eta_n) \Omega \right).
\]

Using this relation and its adjoint, it is straightforward to compute from \[35\] that

\[
\langle \Phi_{n, \pm}, T_{\pm}^{00}(x) \Phi_{n, \pm} \rangle = n \int \frac{\mu^2}{2\pi} d\theta d\eta \cosh^2 \frac{\theta + \eta}{2} \cosh \frac{\theta - \eta}{2} \times \frac{\varphi(\theta) \varphi(\eta)}{\varphi(\theta) \varphi(\eta) L_\psi(\theta, \eta)} n^{-1} e^{i(\theta - \eta) x},
\]

where

\[
L_\psi(\theta, \eta) := \int d\lambda |\varphi(\lambda)|^2 e^{i(\theta - \lambda) \epsilon} e^{i(\eta - \lambda) \epsilon}.
\]

The factor \( L_\psi \), which does not occur in the free case, prevents additivity of the energy density. This nonlinearity is also apparent in Fig. 1 where the energy density per particle, \( \rho(t) := \frac{1}{n} \langle \Phi_{n, \pm}, T_{\pm}^{00}(t, 0) \Phi_{n, \pm} \rangle \), is plotted along the time axis for the same choice of \( \varphi = \varphi_{\alpha, \beta, \gamma} \) as above. This behaviour reflects the interaction in the Ising model.
Finally, we turn to the derivation of the QEI bounds on the energy density. In fact, the free scalar field (+) and the Ising model (−) obey closely related QEI, namely

$$\int dt \, g(t)^2 \langle \Phi, T_{\pm}^{00}(t, x) \Phi \rangle \geq -\frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\omega \omega^2 |\tilde{g}(\omega)|^2 Q_{\pm}(\omega, \mu),$$

(8)

for any real-valued smooth compactly supported function $g$ and all sufficiently regular normalized states $\Phi$, where the dimensionless functions $Q_{\pm} : [1, \infty) \to \mathbb{R}^+$ are

$$Q_{\pm}(u) = \sqrt{1 - u^2} \pm \sqrt{u^2 - 1},$$

(9)

and obey $Q_{\pm}(1) = 0$, $\lim_{u \to \pm \infty} Q_{\pm}(u) = 1$, while the Fourier transform is $\tilde{g}(\omega) = \int dt \, g(t)e^{i\omega t}$.

The QEI for the free Bose field (+) has been known for some time [17] and holds rigorously for all Hadamard states $\Phi$ [21]; our goal is to establish the QEI for the Ising model (−). As we have already seen that its energy density coincides with that of the free Majorana field, the QEI is exactly the same as for the latter theory. This has not been computed before, but the argument is sufficiently similar to treatments in [23, 24] that we only sketch it here. For $\nu \in \mathbb{R}$, let $R_{\nu}$ and $S_{\nu}$ be continuous one-parameter families in $\mathcal{K} = L^2(\mathbb{R}, d\theta)$, and define for each $\nu$ the (bounded) operator

$$\mathcal{O}_{\nu} = \int d\theta \left( R_{\nu}(\theta) a_{\nu}(\theta) + S_{\nu}(\theta) a_{\nu}^\dagger(\theta) \right)$$

on $\mathcal{H}_{\nu}$. Elementary use of the CARs shows that

$$\mathcal{O}^\dagger_{\nu} \mathcal{O}_{\nu} - \|S_{\nu}\|^2 \mathbf{1} = -\mathcal{O}_{\nu} \mathcal{O}^\dagger_{\nu} + \|R_{\nu}\|^2 \mathbf{1} =: X_{\nu},$$

where $\|\cdot\|$ is the norm in $\mathcal{K}$. As $\nu \mathcal{O}^\dagger_{\nu} \mathcal{O}_{\nu}$ (resp., $-\nu \mathcal{O}_{\nu} \mathcal{O}^\dagger_{\nu}$) is positive semidefinite for $\nu \geq 0$ (resp., $\nu \leq 0$), we have

$$\int_{-\infty}^{\infty} \frac{d\nu}{\pi} \nu \langle \Phi, X_{\nu} \Phi \rangle \geq -\int_{0}^{\infty} \frac{d\nu}{\pi} \nu \left( \|R_{-\nu}\|^2 + \|S_{\nu}\|^2 \right)$$

for all normalized quantum states $\Phi$. For our application,

$$R_{\nu}(\theta) = \sqrt{\frac{\mu}{4\pi}} \tilde{g}(\nu - \mu \cosh \theta) \cosh \frac{\theta}{2},$$

$$S_{\nu}(\theta) = -i \sqrt{\frac{\mu}{4\pi}} \tilde{g}(\nu + \mu \cosh \theta) \sinh \frac{\theta}{2},$$

for any real-valued $g \in C_0^\infty(\mathbb{R})$. Using the identity

$$(\omega + \omega') \tilde{g}^2(\omega' - \omega) = -\int_{-\infty}^{\infty} \frac{d\nu}{\pi} \nu \tilde{g}(\nu + \omega) \tilde{g}(\nu + \omega')$$

(a mild rewriting of Eq. (2.17) in [23]), the left-hand side of (10) becomes, after a computation,

$$\int_{-\infty}^{\infty} \frac{d\nu}{\pi} \nu \langle \Phi, X_{\nu} \Phi \rangle = \int dt \, g(t)^2 \langle \Phi, T_{\pm}^{00}(t, 0) \Phi \rangle.$$

It is expected that this is rigorously valid at least for all $\Phi \in \mathcal{H}_{\nu}$ that are Hadamard states of the Majorana field (cf. the treatment of the Dirac equation in four dimensional curved spacetimes [25]). We also compute

$$\text{RHS of (10)} = -\frac{\mu}{4\pi^2} \int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\theta \cosh \theta |\tilde{g}(\mu \cosh \theta + \nu)|^2$$

$$= -\frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\omega \omega^2 |\tilde{g}(\omega)|^2 Q_-(\omega/\mu),$$

(11)

where $Q_-$ was defined in (9). In more detail, the second equality in (11) is obtained by using evenness of the integrand to alter the inner integration region to $[0, \infty)$, then changing variables from $(\nu, \theta)$ to $(\omega, \theta)$ where $\omega = \nu + \mu \cosh \theta$ (with a consequent change of integration region) and finally evaluating the $\theta$ integral. For $x = 0$, the energy density of the Majorana and Ising models thus satisfies the QEI given as the (−) case of (8), for all real-valued test functions $g$ and for a large domain of $\Phi \in \mathcal{H}_{\nu}$; the result for general $x$ follows by translation invariance. This bound is precisely half of the bound for the free massive Dirac field in two dimensions obtained in [24] using similar arguments, as might be expected.

It is also worth considering the limit of these QEI as $\mu \to 0$, for fixed $g$, corresponding to the short-distance scaling limit of the theory [29]. In both cases,

$$\text{RHS of (9)} \to \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dt |g(t)|^2,$$

where we have used the fact that $|\tilde{g}(\omega)|$ is even, and the Plancherel theorem. But both the massless free scalar field and the massless Majorana field are conformal field theories, and obey sharp QEI [29, Eq. (4.25)]

$$\int dt \, g(t)^2 \langle \Phi, T_{\pm}^{00}(t, x) \Phi \rangle \geq -\frac{C_{\pm}}{6\pi} \int_{-\infty}^{\infty} dt |g(t)|^2$$

for suitable normalized $\Phi$, where $C_{\pm}$ are the central charges of the left- and right-moving components: $C_+ = 1$ (free scalar field) and $C_- = \frac{3}{2}$ (free Majorana), so the sharp bound is therefore tighter by a factor of $3/2$, respectively 3, in these two cases. Accordingly, we do not expect our QEI to be sharp for $\mu > 0$.

In summary, for the first time, a QEI has been derived for a quantum field theory with nontrivial S-matrix. We hope this will prove to be a foundation for similar results on other integrable theories in two spacetime dimensions. Intriguingly, the interaction conspires to yield a QEI with a state-independent lower bound; it will be interesting to see whether this persists in other models.

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