Probing reheating via direct detection of the inflationary gravitational wave background and its implications for dark matter candidate

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Abstract. Future direct detection gravitational wave experiments would provide vital information on the early Universe. It may enable us to determine the reheating temperature by detecting a signature of reheating on the spectrum of the inflationary gravitational wave background. We investigate the detectability of the reheating signature and discuss its implications for particle physics, especially on the scenario where gravitinos are dark matter candidates.

1. Introduction
Inflation generically predicts the cosmological gravitational wave background originating from quantum fluctuations in the spacetime metric. If it is directly detected by future experiments such as DECIGO/BBO [1, 2, 3], it will be a powerful tool to extract information about the early Universe. In particular, information on the Hubble expansion rate is imprinted on the shape of the gravitational wave background spectrum and it may enable us to observe the epoch of reheating [4, 5, 6, 7, 8].

The effect of reheating is seen as a change of the frequency dependence of the spectrum, which is induced by the transition of the expansion rate at the end of reheating. We estimate the detectability of the reheating signature, by examining whether the experiment can distinguish the change of the characteristic frequency dependence. Also, we discuss the implications of a possible lower bound of the reheating temperature for particle physics, which is expected from the future direct detection by DECIGO/BBO.

2. Gravitational wave background from inflation
Gravitational waves in the expanding Universe can be described as a transverse-traceless part of the metric perturbation in a Friedmann Robertson-Walker background, \( ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j \). Applying the metric of this space-time to the perturbed Einstein equation, \( \delta G_{ij} = 8\pi G \delta T_{ij} \), and Fourier transforming the equation, we obtain the equation of motion for the gravitational wave of the mode \( k \)

\[
\ddot{h}_k + 3H\dot{h}_k + \frac{k^2}{a^2} h_k = \Pi_k.
\]

(1)
where the over-dot describes the time derivative, $H$ is the Hubble parameter, and $\Pi_k$ is the Fourier transform of the transverse-traceless part of the anisotropic stress. The effect of reheating appears through the second term via the Hubble expansion rate.

If we assume reheating is proceeded by perturbative decay of the inflaton field into light fermions, the background evolution of the Universe during inflation and reheating can be described by the following equations,

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0,$$

$$\dot{\rho}_r + 4H\rho_r = \Gamma \rho_\phi,$$

$$H^2 = \frac{8\pi}{3m_{Pl}^4}(\rho_\phi + \rho_r),$$

where $\Gamma$ is the decay rate of the scalar field into radiation, $V(\phi)$ is the potential of the inflaton scalar field, $\rho_r$ is the energy density of the radiation, and the energy density of the scalar field is given as $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$. The reheating temperature $T_R$, defined at the end of reheating, can be related to the decay rate $\Gamma$ as

$$T_R \simeq g_*^{-1/4} \left( \frac{45}{8\pi^2} \right)^{1/4} (m_{Pl}\Gamma)^{1/2},$$

where $g_* \sim 100$ is the effective number of relativistic degrees of freedom at the end of reheating and $m_{Pl} = G^{-1/2}$ denotes the Planck mass.

From Eqs. (2), (3) and (4), one can find the Hubble expansion rate behaves like matter-dominated Universe during reheating and becomes radiation-dominated after reheating. This transition gives rise to change in frequency dependencies of the spectrum. In Fig 1, the spectrum is shown in terms of the density parameter per logarithmic wave number, defined as $\Omega_{GW} \equiv (d\rho_{GW}/d\ln k)/\rho_c$, where the critical density of the Universe is defined as $\rho_c \equiv 3H^2/8\pi G$ and $\rho_{GW}$ denotes the energy density of the gravitational waves, $\rho_{GW} = \langle (\partial_\tau h_{ij})^2 + (\nabla h_{ij})^2 \rangle / (64\pi G a^2)$. The matter-dominated Universe gives the dependence of $\Omega_{GW} \propto f^{-2}$, and the radiation-dominated Universe gives $f^0$. As seen from the figure, if the reheating temperature is $\sim 10^7 \text{GeV}$, future gravitational wave experiments would be able to detect the knee-like feature in the spectrum. The change of the frequency dependence arises around the frequency of

$$f_R \simeq 0.26 \left( \frac{T_R}{10^7 \text{GeV}} \right) \left( \frac{g_*}{110} \right)^{1/2} \left( \frac{g_{ss}}{110} \right)^{-1/3} \text{Hz},$$

where $g_{ss}$ is the effective number of relativistic degrees of freedom for the entropy density. Since the signature arises at different frequency depending on the reheating temperature, it may even be possible to constraint the reheating temperature by measuring the position where the change of the frequency dependence arises.

3. Prospects for detectability of the reheating signature

Using the Fisher matrix formalism, we present an example of the expected future constraints on reheating temperature in Fig. 2, where the fiducial parameters are chosen as $r = 0.1$ and $T_R = 10^7 \text{GeV}$. Each ellipse represents the 2$\sigma$ errors on the reheating temperature $T_R$ and the tensor-to-scalar ratio $r$, expected from 1, 3 and 10 years of observation with the sensitivity of BBO. Here, the amplitude of the gravitational wave background is parametrized in terms of the tensor-to-scalar ratio, defined as the ratio of the amplitudes of the scalar and tensor power spectra $r \equiv \Delta^2_{R,\text{prim}}(k_0)/\Delta^2_{\text{prim}}(k_0)$, where $k_0 = 0.002 \text{Mpc}^{-1}$ is usually taken to be the scale of the CMB.
Figure 1. Spectra of the inflationary gravitational wave background for different inflation models [8]: chaotic \( m^2 \phi^2 \), \( \lambda \phi^4 \), natural and hybrid inflation for typical values of parameters. The sensitivity curves of FP-DECIGO (dotted) and BBO (solid) experiments are also shown. The spectra are calculated assuming \( T_R = 10^{6} \text{GeV} \), \( 10^{7} \text{GeV} \) and \( 10^{8} \text{GeV} \) for the \( m^2 \phi^2 \) model, respectively.

Figure 2. The 2\( \sigma \) confidence level contours in the \( T_R - r \) plane for 1-year (dotted line), 3-year (dashed line) and 10-year (solid line) observation by BBO [9]. The fiducial parameters are set as \( r = 0.1 \) and \( T_R = 10^7 \text{GeV} \), which is shown by a cross mark.

In Fig. 3, we show the accessible parameter space where the reheating signature is detected at greater than 2\( \sigma \) level with 3-year observation by BBO. As seen from the figure, if \( T_R \sim 10^7 \text{GeV} \), the reheating temperature can be determined with more than 2\( \sigma \) accuracy. We also show the region where the inflationary gravitational wave background can be detected with a signal-to-noise ratio higher than 5. However, in this case, the reheating signature is outside the frequency band of sensitivity so we can only obtain a lower bound of the reheating temperature, such as \( T_R > 10^7 \text{GeV} \).

4. Implication for dark matter candidate
As shown in the previous section, direct detection of the inflationary gravitational wave background may be able to provide constraints on reheating temperature. In the following, we consider the implication of the lower bound of the reheating temperature, on gravitino cosmology in the context of chaotic/natural inflation [8].
In the case where the gravitinos are stable and produced by the thermal scatterings at the reheating, the present-day abundance of the gravitinos is given by [10]

$$\Omega_{\chi h^2} \simeq 0.27 \left( \frac{T_R}{10^8 \text{GeV}} \right) \left( \frac{M_{3/2}}{1 \text{GeV}} \right)^{-1},$$

where we have set the gluino mass $M_g = 1\text{TeV}$. Since the total abundance should satisfy the bound $\Omega_{\chi h^2} = \Omega_{\chi H} h^2 + \Omega_{\chi X} h^2 \leq \Omega_{\chi dm} h^2 \simeq 0.134$, if $T_R > T_R^{gw}$ is indicated from gravitational wave experiments, we get the lower bound on the gravitino mass,

$$M_{3/2} > 0.21 \text{GeV} \left( \frac{T_R^{gw}}{10^8 \text{GeV}} \right) \left( \frac{\Omega_{dm} h^2}{0.134} \right)^{-1}.$$  

If the gravitinos are produced nonthermally via the decay of heavy scalar fields $X$, the abundance of the gravitinos produced by the $X$ decay is given by

$$\Omega_{\chi X h^2} \simeq 6.8 \times 10^{1} d_{3/2}^2 \left( \frac{M_X}{10^{10} \text{GeV}} \right)^{1/2} \left( \frac{M_{3/2}}{1 \text{GeV}} \right),$$

where $s_0$ is the present entropy density. In this case, the lower bound on the reheating temperature from gravitational wave experiments, $T_R > T_R^{gw}$, places the upper bound on the mass of the $X$ particle,

$$M_X < 6 \times 10^4 \text{GeV} \left( \frac{T_R^{gw}}{10^8 \text{GeV}} \right)^{-2} \left( \frac{\Omega_{dm} h^2}{0.134} \right)^4 d_{3/2}^{-4}.$$  

5. Conclusion

Gravitational waves generated during inflation have potential to be a powerful observational tool to probe the early Universe. If detected, they surely provide generous information not only on inflation but also reheating. We considered the case of $T_R \sim 10^7 \text{GeV}$, where the
characteristic feature produced at the end of reheating is observed in the spectrum as a dip at $\sim 10^{-1}$ Hz. If inflation generates large amplitude of the spectrum, the reheating temperature can be determined by future space-based gravitational wave detectors with more than $2\sigma$ accuracy. We also considered the case of $T_R > 10^7$ GeV, which would be able to confirmed by detection of the inflationary gravitational background. Taking into account of both thermal and nonthermal production of gravitinos, we showed that lower bound on the reheating temperature provides a lower bound on the gravitino mass and an upper bound on the moduli mass. These constraints could be a unique source of information for dark matter physics and complement with astrophysical and particle collider experiments.

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