A Role of a Spatial Dispersion of an Electromagnetic Wave Coming Through a Quantum Well

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A theory of light transmission through a quantum well (QW) in a magnetic field perpendicular to the QW plane is developed. The light wave length is supposed comparable with the QW width. The formulas for reflection, absorption and transmission take into account the spatial dispersion of the light monochromatic wave and a difference of the refraction indexes of the QW and barrier. We suppose a normal light incidence on the QW plane and consider only one excited energy level. These two factors influence mostly light reflection, since an additional reflection from the QW borders appears to the reflection due to interband transitions in the QW. The most radical changes in reflection appear when a radiative broadening of the excited energy level is small in comparison to a nonradiative broadening. Our theory is limited by the condition of existence of size-quantized energy levels which is satisfied for quite narrow QW’s.

Reflection and transmission data of an electromagnetic wave coming through a QW contain some characteristic features carrying a valuable information about electronic processes in QW’s. Most interesting results are available for electronic systems with discrete energy levels. Such a situation is realized in a QW in a quantizing magnetic field perpendicular to the QW plane or for excitonic energy levels in absence of magnetic field. Modern semiconductor technologies allow to obtain QW’s of high quality where radiative broadenings of absorption lines may be comparable with nonradiative broadenings or even exceed them. In such cases one cannot use only the linear approximation on the electron-photon interaction, but must take into account all the orders of this interaction.

Reflection, absorption and transmission of an electromagnetic wave interacting with QW discrete energy levels in an interband light frequency region had been considered in [12-18]. Both light pulses [12-17] and monochromatic radiation [18] had been considered. One [16], two [17,18] and many excited energy levels [15] had been taken into account. These results are rightful for comparatively narrow QW’s, when the inequality

\[ \kappa d << 1 \]  

is fulfilled (\( d \) is the QW’s width, and \( \kappa \) is the modulus of the wave vector \( \mathbf{k} \) of the light wave). The parameter \( \kappa d \) was equalled to zero in mentioned above articles, thus obtained reflection, absorption and reflection did not depend on the QW’s width \( d \).

Let us make use the wave length \( 0.8 \mu \) (a laser on the base of GaAs) to estimate numerically the value \( \kappa \). The energy, corresponding to this wave length, is \( h \nu_1 = 1.6 \text{eV} \). If the refraction index of the QW material is \( \nu = 3.5 \), then \( \kappa = \nu \omega_1 / c = 2.810^5 \text{cm}^{-1} \) (\( c \) is the light velocity in vacuum). For the QW width \( d = 500 \text{A} \) the parameter \( \kappa d = 1.4 \). Thus, taking into account the spatial dispersion may be essential for the wide enough QW’s.

For wide QW’s the inequality \( d \gg a_0 \) (where \( a_0 \) is the lattice constant) is very strong, thus one can apply the Maxwell equations for the continuum to describe a transmission of light wave through a QW. Properly speaking, at such approach one has to take into account a difference in refraction indexes of the QW and barrier. Then some additional reflection must appear from QW’s borders which will decrease with decreasing of the parameter \( \kappa d \), but in the region \( \kappa d \approx 1 \) it may become equal or even enhance the reflection due to interband transitions inside of the QW. Transmission of the light wave will be also changed. Thus the difference in refraction indexes of the QW and barrier must be taken into account as well as the spatial dispersion of the light wave.

Our article is devoted to a consideration of influence of both factors on reflection, transmission and absorption of an electromagnetic wave coming through a QW and exciting interband transitions in the QW.

1. THE MODEL AND MAIN RELATIONS

Let us consider a deep semiconductor QW displaced in the interval \( 0 \leq z \leq d \) between two half-infinite barriers. A constant quantizing magnetic field is directed perpendicularly to the QW’s plane (along the z axis). An external light wave penetrates along the z axis from the negative \( z \). We suppose that barriers are transparent.
for the light wave, which is absorbed in part inside of the QW, inducing interband transitions. At \( T = 0 \) (the ground state) the valence band is filled completely and the conductivity band is empty. In an excited state at least one electron is transited into the conductivity band from the valence band where one hole remained. This is true in a linear approximation on the wave amplitude. Let us consider the light frequencies which are close to the QW band gap, when a small part of the valence electrons near the band extremum takes part in absorption and the heterogeneity size (the QW in our case) is smaller or the order of the light wave length optical characteristics of such system are to be determined from a solution of Maxwell equations with current and charge densities.

Neglect an influence of other energy levels if the exciting \[19,20\].

\[
\begin{align*}
\text{Let us consider the light frequencies which are close to the QW band gap, when a small part of the valence electrons near the band extremum takes part in absorption and the heterogeneity size (the QW in our case) is smaller or the order of the light wave length optical characteristics of such system are to be determined from a solution of Maxwell equations with current and charge densities obtained with the help of the microscopic consideration [19,20].}
\end{align*}
\]

The final result will be obtained for the only discrete energy level for the electronic system in a QW. We can neglect an influence of other energy levels if the exciting light frequency \( \omega_{1} \) is close to the interband transition frequency \( \omega_{0} \). In the case \( \hbar K_{L} = 0 \) (\( \hbar K_{L} \) is the electron-hole (EHP) summary quasi-momentum in the QW plane) discrete energy levels in the QW are excitonic energy levels or the Landau levels in a quantizing magnetic field directed perpendicularly to the QW plane. We consider an EHP energy level in a quantizing magnetic field. The Coulomb interaction is a weak perturbation for strong magnetic fields and not too wide QWs and it is neglected below [2]. However the excitonic effect will not lead to principal changes of our results: it will change only the radiative broadening \( \gamma_{r} \) of the electronic excitation. This is also true for excitonic energy levels in absence of magnetic fields.

Let us calculate the high-frequency current density in the QW induced by the exciting light. For any spatially heterogeneous system one can introduce the conductivity tensor \( \sigma_{\alpha \beta}(\mathbf{k}, \omega, \mathbf{r}) \) which links the average current density \( \mathbf{J}(\mathbf{r}, t) \) and the electric field \( \mathbf{E}(\mathbf{k}, \omega) \)

\[
\begin{align*}
J_{\alpha}(\mathbf{r}, t) &= (2\pi)^{-4} \int d\mathbf{k} \int_{0}^{\infty} d\omega \sigma_{\alpha \beta}(\mathbf{k}, \omega, \mathbf{r}) \times E_{\beta}(\mathbf{k}, \omega) \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)] + c.c., \quad (2) \\
E_{\beta}(\mathbf{k}, \omega) &= \int d\mathbf{r} \int_{-\infty}^{\infty} dt E_{\beta}(\mathbf{r}, t) \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)], \quad (3)
\end{align*}
\]

Since the temperature \( T = 0 \) the current density is averaged on the ground state of the electronic system

\[
\mathbf{J}(\mathbf{r}, t) = \langle 0 | \mathbf{J}(\mathbf{r}, t) | 0 \rangle, \quad (4)
\]

where \( \mathbf{J}(\mathbf{r}, t) \) is the current density operator in linear approximation on the external field.

We make use the following expression for the conductivity tensor \[\]

\[
\sigma_{\alpha \beta}(\mathbf{k}, \omega, \mathbf{r}) = (i/\hbar) \int d\mathbf{r}' \int_{-\infty}^{\infty} dt' \times \Theta(t') \times 0 | j_{\alpha}(\mathbf{r}, t), d_{\beta}(\mathbf{r} - \mathbf{r}', t - t') | 0 \rangle, \quad (5)
\]

where \( \Theta(t) \) is the Heaviside function, and \( j_{\alpha}(\mathbf{r}, t) \) is the projection of the current density without external magnetic field, but taking into account the constant quantizing magnetic field

\[
\mathbf{H} = \text{rot} \mathbf{A}^{(0)}(\mathbf{r}), \quad (6)
\]

where \( \mathbf{A}^{(0)}(\mathbf{r}) \) is its vector potential. This operator is as follows

\[
\begin{align*}
\hat{J}_{\alpha}(\mathbf{r}, t) &= \exp(i \mathcal{H} t/\hbar) j_{\alpha}(\mathbf{r}) \exp(-i \mathcal{H} t/\hbar), \quad (7) \\
j_{\alpha}(\mathbf{r}, t) &= (e/2) \sum_{i} [v_{i \alpha} \delta(\mathbf{r} - \mathbf{r}_{i}) + \delta(\mathbf{r} - \mathbf{r}_{i}) v_{i \alpha}], \quad (8) \\
v_{i \alpha} &= -i(\hbar/m_{0}) \partial_{\alpha} r_{i} - (e/m_{0} c) A_{\alpha}^{(0)}(\mathbf{r}_{i}), \quad (9)
\end{align*}
\]

\( \mathcal{H} \) is the Hamiltonian of the electronic system in a quantizing magnetic field (but without an external electromagnetic field),

\[
\begin{align*}
\hat{d}_{\alpha}(\mathbf{r}, t) &= \exp(i \mathcal{H} t/\hbar) \hat{d}_{\alpha}(\mathbf{r}) \exp(-i \mathcal{H} t/\hbar), \quad (10) \\
\hat{d}_{\alpha}(\mathbf{r}) &= e \sum_{i} (r_{i \alpha} - < 0 | r_{i \alpha} | 0 >) \delta(\mathbf{r} - \mathbf{r}_{i}). \quad (11)
\end{align*}
\]

Applying the effective mass approximation and taking into account the homogeneity of the system in the QW plane from Eqs. (2), (5) we obtain that

\[
\begin{align*}
\bar{J}_{\alpha}(z, t) &= \frac{i}{4\pi^{2}} \frac{(e/m_{0})^{2}}{\hbar \omega_{\gamma} a_{\mathcal{H}}} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \sum_{\chi} \Phi_{\chi}(z) \times \\
&\times \left\{ \frac{p_{\gamma}^{z} z_{\alpha \alpha} p_{\gamma}^{z}}{\omega - \omega_{\chi} + i\gamma_{\chi}/2} + \frac{p_{\gamma}^{z} z_{\alpha \alpha} p_{\gamma}^{z}}{\omega + \omega_{\chi} + i\gamma_{\chi}/2} \right\} \\
&\times \int_{-\infty}^{\infty} dz' \Phi_{\chi}(z') \times E_{\beta}(z', \omega) \quad (12)
\end{align*}
\]

*In Eq. (5) contributions containing factors of the order \( \kappa v/\omega \) are omitted (\( v \) is the electron velocity).
where \( \tilde{J}_\alpha(z,t) \) is the current density averaged on the ground state of the electronic system. An additional averaging on the elementary cell is performed (which is available under condition \( d \gg a_g \)). Following designations are used in Eq. (12): \( m_0 \) is the bare electron mass, \( \hbar \omega_g \) is the energy gap, \( a_H = (\varepsilon/h^2|e|H)^{1/2} \) is the magnetic length, \( \chi \) is the set of the indexes

\[
\chi = (j, \lambda), \quad \lambda = (n = n_c = n_v, m_c, m_v),
\]

\( j \) is the valence band number (since the valence band in cubic crystals (considered below) is degenerated), \( n_c (n_v) \) is the Landau quantum number, \( m_c (m_v) \) is the size quantization number, \( \Phi_\lambda(z) = (2/d) \sin(\pi m_c z/d) \sin(\pi m_v z/d) \) is the product of the electron and hole wave functions depending on \( z \),

\[
E_\beta(z,\omega) = \int_{-\infty}^{\infty} dt \exp(i \omega t) E_\beta(z, t),
\]

(15)

\( \mathbf{p}_{cv}^\beta \) is the interband matrix element of the quasimomentum, corresponding to the transition between extrumums of the valence and conductivity bands,

\[
\hbar \omega_\lambda = \hbar \omega_g + \varepsilon(m_c) + \varepsilon(m_v) + \hbar \Omega_\mu (n + 1/2)
\]

(16)

is the electronic excitation energy with indexes \( \lambda \), \( \varepsilon(m_c)(\varepsilon(m_v)) \) is the size quantized energy of electrons (holes), \( \Omega_\mu = |e|H/\mu c \) is the cyclotron frequency, \( \mu = m_e/m_h/(m_e + m_h) \), \( m_e (m_h) \) is the electro (hole) effective mass, and \( \gamma_\lambda \) is the nonradiative broadening of the excited state with the quantum numbers \( \lambda \). Obtaining Eq. (12) we used the relation

\[
\mathbf{r}_{cv} = -(i/m_0 \omega_g) \mathbf{p}_{cv}.
\]

(17)

where \( \mathbf{r}_{cv} \) is the interband matrix element of the radius-vector \( \mathbf{r} \). Eq. (12) is applicable for the monochromatic exciting wave as well as for light pulses.

Further we use the model (see \[13,18\]) where the vectors \( \mathbf{p}^l_{cv} \) for two degenerated bands are as follows

\[
\mathbf{p}^l_{cv} = p_{cv}(\mathbf{e}_x - i \mathbf{e}_y)/\sqrt{2},
\]

\[
\mathbf{p}^H_{cv} = p_{cv}(\mathbf{e}_x + i \mathbf{e}_y)/\sqrt{2},
\]

(18)

where \( \mathbf{e}_x, \mathbf{e}_y \) are the unite vectors along the axis \( x \) and \( y \), and \( p_{cv} \) is the real constant. This model corresponds to heavy holes in crystals with the zinc blend structure if the axis \( z \) is directed along the fourth order axis. If the circular polarization vectors are applied

\[
\mathbf{e}_l = (\mathbf{e}_x \pm i \mathbf{e}_y)/\sqrt{2},
\]

(19)

the conservation of the polarization vector is performed

\[
\sum_{j=\ell, I} \left[ \mathbf{p}^j_{cv}(\mathbf{e}_l \mathbf{p}^j_{cv}) \right. \\
\left. \left( \omega - \omega_\lambda + i \gamma_\lambda/2 \right) + \mathbf{p}^j_{cv}(\mathbf{e}_l \mathbf{p}^j_{cv}^*) \right. \\
\left. \left( \omega + \omega_\lambda + i \gamma_\lambda/2 \right) \right],
\]

(20)

where \( \mathbf{e}_l \) is any vector of Eq. (19), what makes easy following calculations. Since for the model Eq. (18) the projection \( p_{cvz} = 0 \), the current \( \mathbf{J}(z,t) \) is transverse one, the induced charge density \( \rho(z,t) \) = 0. Then one can choose the gage \( \varphi(z,t) = 0 \), where \( \varphi(z,t) \) is the scalar potential and

\[
\mathbf{E}(z,\omega) = -(1/c)(\partial \mathbf{A}(z,t)/\partial t), \quad \mathbf{H}(z,t) = \text{rot} \mathbf{A}(z,t),
\]

(21)

where \( \mathbf{A}(z,t) \) is the vector potential of the electromagnetic wave. Applying the relation

\[
E_\alpha(z,\omega) = (i\omega/c)A_\alpha(z,\omega),
\]

(22)

we will transit in Eq. (12) to the vector potential. The result may be conveniently written

\[
\tilde{J}_\alpha(z,t) = -\frac{e_l n_c \omega_\nu}{8\pi^2} \sum_\lambda \Phi_\lambda(z) \int_{-\infty}^{\infty} \omega d\omega e^{-i\omega t}
\]

\[
\times \left[ \frac{1}{\omega - \omega_\lambda + i \gamma_\lambda/2} + \frac{1}{\omega + \omega_\lambda + i \gamma_\lambda/2} \right]
\]

\[
\times \int_0^d dz' A(z',\omega)\Phi_\lambda(z') + c.c.,
\]

(23)

\[
\gamma_r = (2e^2/h c \nu)(m_{cv}^2/m_0 \hbar \omega_g)(|e|H/m_0 c)
\]

is the radiative broadening of the EHP energy level in a magnetic field under the condition \( \kappa d = 0 \) \[13,18\], and the scalar \( A(z,\omega) \) is introduced:

\[
A(z,\omega) = e_l A(z,\omega) + e_l^* A^*(z, -\omega).
\]

(24)

II. THE ELECTRIC FIELD OF THE ELECTROMAGNETIC WAVE

We proceed below from the two assumptions. First, the plane wave is a monochromatic one with the frequency \( \omega_l \), i. e. in Eq. (24)

\[
A(z,\omega) = 2\pi \delta(\omega - \omega_l) A(z),
\]

(25)

and the vector \( \mathbf{A}(z,t) \) takes the form

\[
\mathbf{A}(z,t) = e_l \exp(-i \omega_l t) A(z) + c.c.
\]

(26)

Second, the only excited energy level in the QW is taken into account. We assume that rest energy levels are disposed far away from this level and their influence is negligible. The equation for the scalar amplitude \( A(z) \) of the vector potential is
\[
d^2 A/dz^2 + \kappa_1^2 A = 0, \quad \kappa_1 = \nu_1 \omega_1/c, \quad z \leq 0, \quad z \geq d, \quad (27)
\]
where \( \nu_1 \) is the barrier refraction index. In the QW region \( 0 \leq z \leq d \) we have the equation
\[
d^2 A/dz^2 + \kappa_2^2 A = -(4\pi/c) \bar{J}(z), \quad (28)
\]
where the scalar amplitude of the current density \( \bar{J}(z) \) for the case of the only energy level according Eqs. (22), (25) is as follows
\[
\bar{J}(z) = -(\gamma_r \nu_1/4\pi) \Phi(z) \int_0^d dz' A(z') \Phi(z') \\
\times [\omega_1 - \omega_0 + i\gamma_2/2]^{-1} (\omega_1 + \omega_0 + i\gamma_2/2]^{-1} + c.c. \quad (29)
\]
For the sake of simplicity we have introduced designations
\[
\Phi_\lambda(z) = \Phi(z), \quad \omega_\lambda = \omega_0, \quad \gamma_\lambda = \gamma. \quad (30)
\]
In the vicinity of the resonance \( \omega_1 = \omega_0 \) the term proportional to \( (\omega_1 + \omega_0 + i\gamma_2/2)^{-1} \) in Eq. (29) is omitted. Eq. (29) is integro-differential one. If one represents the solution of Eq. (29) as a sum of the general solution of the homogeneous equation and the partial solution of the non-homogeneous equation, then one obtains the Fredholm integral equation of the second type
\[
A(z) = C_1 e^{i\nu_1 z} + C_2 e^{-i\nu_1 z} - \frac{i(\gamma_r/2)F(z)}{\omega_1 - \omega_0 + i\gamma_2/2} \\
\times \int_0^d dz' A(z') \Phi(z'). \quad (31)
\]
C\(_1\) and C\(_2\) are the arbitrary constants determined from boundary conditions in the planes \( z = 0 \) and \( z = d \), and the function \( F(z) \) is determined as
\[
F(z) = \exp(i\nu_1 z) \int_0^z \int_0^{z'} d z'' \exp(-i\nu_1 z'') \Phi(z'') + \exp(-i\nu_1 z) \int_0^d \int_z^d d z'' \exp(i\nu_1 z'') \Phi(z''). \quad (32)
\]
If \( \gamma_r \ll \gamma_1 \), the integral term in Eq. (31) is a small perturbation which may be counted in the first approximation only. If \( \gamma_r \geq \gamma_1 \), one has to take into account the whole iterative sequence. Representing the desirable function \( A(z) \) as a series
\[
A(z) = A_0(z) + A_1(z) + A_2(z) + ..., \\
A_0(z) = C_1 \exp(i\nu_1 z) + C_2 \exp(-i\nu_1 z) \quad (33)
\]
and substituting it in Eq. (31), we obtain the recurrent relation
\[
A_j(z) = sF(z) \int_0^d dz' \Phi(z') A_{j-1}(z'), \quad j = 1, 2, 3...
\]
Making use this relation, one can reduce Eq. (33) to the geometric progression
\[
A(z) = A_0(z) - h s F(z)(1 - s \varepsilon + s^2 \varepsilon^2 - ...)
\]
\[
= A_0(z) - h s F(z)/(1 + s \varepsilon), \quad (34)
\]
where for the sake of simplicity we have designated
\[
\int_0^d dz \Phi(z) A_0 = h, \\
i(\gamma_r/2)/(\omega_1 - \omega_0 + i\gamma_2/2) = s
\]
and have introduced the complex function
\[
\varepsilon = \varepsilon' + i\varepsilon'' = \int_0^d dz' \Phi(z') F(z'). \quad (35)
\]
Substituting h, s and \( \varepsilon \) in Eq. (34) we obtain the solution
\[
A(z) = C_1 e^{i\nu_1 z} + C_2 e^{-i\nu_1 z} - \frac{i(\gamma_r/2)F(z)}{\omega_1 - \omega_0 + i(\gamma + \gamma_r \varepsilon)/2} \\
\times \int_0^d dz' (C_1 e^{i\nu_1 z'} + C_2 e^{-i\nu_1 z'}) \Phi(z'). \quad (36)
\]
The complex value \( \varepsilon \) determines broadening and the level shift which appear due to the spatial dispersion of the light wave. As it follows from the definition Eq. (35), in the limiting case \( \kappa d = 0 \) \( \varepsilon = \delta_{m_c m_v} \), and the integral in the RHS of Eq. (36) equals to \( (C_1 + C_2)\delta_{m_c m_v} \), i.e. only permitted transitions \( m_c = m_v \) contribute into the current. If \( \kappa d \neq 0 \), the forbidden transition \( m_c \neq m_v \) provides the interband current and the appearance in the denominator of Eq. (36) the value \( \varepsilon \), however this \( \varepsilon \to 0 \) if \( \kappa d \to 0 \). Let us also note that the function Eq. (32) \( F(z) = \delta_{m_c m_v} \) if \( \kappa d \to 0 \). Below we consider only the permitted transitions.

The solution of Eq. (27) is
\[
A^I(z) = A_0 \exp(i\nu_1 z) + C_R \exp(-i\nu_1 z), \quad z \leq 0, \\
A^T(z) = C_T \exp(i\nu_1 z), \quad z \geq d, \quad (37)
\]
\( C_R, C_T \) determine the amplitude of the reflected and transmitted wave, respectively. On boundaries \( z = 0 \) and \( z = d \) the continuity of the magnetic field of the wave leads to the continuity of \( dA/dz \), the continuity of the tangential projections of the electric field leads to the continuity of \( A(z) \). As a result we obtain following expressions for the coefficients \( C_1, C_2, C_R \) and \( C_T \):

\footnote{The similar equation was considered in \cite{24} for the inversion layer.}
\[ C_i = A_0 C_i \quad (i = 1, 2), \quad C_{R(T)} = A_0 C_{R(T)}, \]
\[ C_1 = (2/\Delta) \exp(-i \kappa d) \left[ 1 + \zeta + (1 - \zeta)N \right], \]
\[ C_2 = -\frac{1}{2} \Delta (1 - \zeta) \left[ \exp(i \kappa d) + N \right], \]
\[ C_T = 4 \kappa \exp(-i \kappa d) \left[ 1 + \exp(-i \kappa d)N \right], \]

\[ \Delta = (\zeta + 1)^2 \exp(-i \kappa d) \]
\[ -\zeta \exp(i \kappa d) \]
\[ -2 \Delta (1 - \zeta) \left[ \exp(-i \kappa d) - \zeta - 1 \right], \]
\[ \rho = 2i \zeta \left( \zeta - 1 \right) \sin \kappa d \]
\[ + 2 \left[ (\zeta^2 + 1) \exp(-i \kappa d) + \zeta^2 - 1 \right] N \]

In Eqs. (38), (39) we have introduced the determinations
\[ \zeta = \frac{\kappa}{\kappa_1} = \frac{\nu}{\nu_1}, \]

\[ N = -s F^2 (0) \]
\[ = -i(\gamma_r/2) F^2 (0)/\omega_0 + i(\gamma + \gamma_r \varepsilon)/2. \]

Returning to the time-representation and transiting in Eq. (37) from \( A(z) \) to the electric fields to the left \( (E^l(z,t)) \) and to the right \( (E^r(z,t)) \) of the QW we obtain that
\[ E^l(z,t) = e_t E_0 e^{-i \omega t} \left[ e^{i \kappa_1 z} + C_R e^{-i \kappa_1 z} \right] + c.c., \]
\[ E^r(z,t) = e_t E_0 C_T e^{-i(\omega (t-\kappa z))} + c.c. \cdot \]

The electric field inside of the QW is determined by Eq. (36), if to substitute \( A(z) \) by \( E(z) \) and to transit to the time-representation. Expressions for fields contain \( \varepsilon \) and \( F(0) \), and the field inside of the QW contains additionally \( F(z) \), which contributes into the space dependence of the field. For the case \( m_e = m_v = \frac{1}{2} F(z) \) and \( \varepsilon \) are as follows
\[ F(z) = iB \{ 2 - \exp(i \kappa z) - \exp[ik (d - z)] \}
\[ - (\kappa d/\pi m)^2 \sin^2 (\pi m z/d) \}, \]
\[ F(0) = F(d) = iB \{ 1 - \exp(i \kappa d) \}, \]
\[ B = \frac{4\pi^2 m^2}{\kappa d(4\pi^2 m^2 - (\kappa d)^2)}. \]

\[ \varepsilon' = F^2 (0) \exp(-i \kappa d) = 4B^2 \sin^2 (\kappa d/2), \]
\[ \varepsilon'' = 2B \{ 1 - B \sin \kappa d - 3(\kappa d)^2/8\pi^2 m^2 \}. \]

The functions \( \varepsilon' \) and \( \varepsilon'' \) from the parameter \( \kappa d \) are represented in Fig.1.

In the limiting case of an homogeneous medium \( (\kappa_1 = \kappa) \) we obtain

\[ \Delta E^l(z,t) = e_t E_0 C_R e^{-i(\omega (t+\kappa z))} + c.c.. \]

The value \( \gamma_r \) coincides with EHP radiative broadenings in a quantizing magnetic field calculated in \([13,18]\) at \( K_\perp \) = 0 and for the arbitrary value \( \kappa d \). Comparing Eqs. (47), (48) to corresponding expressions in \([18]\) for fields to the left and right of the QW we find that in the case \( \kappa d \neq 0 \) the value \( \gamma_r \) is substituted by \( \tilde{\gamma}_r \), the shift on the value \( \gamma_r \varepsilon' \) appears and the additional factor \( \exp(i \kappa d) \) appears in the expression for the induced wave to the left from the QW. One can make sure that the induced field to the left of the QW coincides with the induced field to the right of the QW if the value \( d - z \) is substituted by \( z \). It is seen from Eq. (46) and in Fig. 1 that \( \tilde{\gamma}_r \) decreases with growing \( \kappa d \). At \( \kappa d \gg 1 \tilde{\gamma}_r \to 0 \), what corresponds to the transition from the QW to the bulk crystal. In such a case the contribution of the only energy level into induced fields and, consequently, into absorption and reflection approaches to zero.

In the limiting case \( \gamma_r = 0 \) from Eqs. (42),(43) one obtains the well known solution for a monochromatic wave spreading in a medium containing a transparent layer of another matter \([2]\).

**III. REFLECTION, ABSORPTION AND TRANSMISSION OF AN ELECTROMAGNETIC WAVE**

Thus according Eq. (42) the electric field vector of the reflected wave \( \Delta E^l(z,t) \) and of the circular polarization is as follows

\[ \Delta E^l(z,t) = e_t E_0 C_R e^{-i(\omega(t+\kappa z))} + c.c.. \]

According Eq. (43) the electric field vector of the transmitted wave is
\[ \mathbf{E}^\prime (z, t) = e_I E_0 C_T e^{-i(\omega t - \kappa z)} + c.c. \] (52)

Analogically to [18] let us introduce the part of the reflected energy \( \mathcal{R} \) which is defined as the relation of the reflected flux energy module to the incident energy flux module, i. e.

\[ \mathcal{R} = |\mathcal{C}_R|^2. \] (53)

The part of the transmitted energy \( \mathcal{T} \) equals

\[ \mathcal{T} = |\mathcal{C}_T|^2, \] (54)

and the part of the absorbed energy \( \mathcal{A} \) is defined as

\[ \mathcal{A} = 1 - \mathcal{R} - \mathcal{T}. \] (55)

At first let us consider an influence of the spatial dispersion on a frequency dependence of reflection of a homogeneous medium. From Eqs. (38), (39) and (53)-(55) we obtain

\[ \mathcal{R} = \frac{(\tilde{\gamma}_r/2)^2}{\Omega^2 + \Gamma^2/4}, \quad \mathcal{A} = \frac{\gamma \tilde{\gamma}_r/2}{\Omega^2 + \Gamma^2/4}, \quad \mathcal{T} = \frac{\Omega^2 + \gamma^2/4}{\Omega^2 + \Gamma^2/4}. \] (56)

These expressions coincide with those without the spatial dispersion. The difference is in the substitution of the function \( \tilde{\gamma}_r \) \((\tilde{\gamma}_r \rightarrow \gamma_r \kappa d \rightarrow 0)\) instead of the constant \( \gamma_r \) and the appearance of the function \( \varepsilon'' \) which determines the shift of the extremum of the corresponding curve and disappears in the limit \( \kappa d = 0 \).

The spatial dispersion is demonstrated more stronger in reflection in the case \( \gamma \gg \gamma_r \). Indeed, if \( \gamma \ll \gamma_r \) the maximum of the reflection curve from Eq. (56) \( \mathcal{R}_{\max} \equiv 1 \) and does not practically depend of \( \kappa d \). If \( \gamma \gg \gamma_r \), then \( \tilde{\gamma}_r \) in the denominator of Eq. (56) gives a small contribution in the dependence from \( \kappa d \) and this dependence is determined by the function \( \tilde{\gamma}_r \) in the numerator. But however \( \mathcal{R}_{\max} = (\tilde{\gamma}_r/\gamma)^2 \ll 1 \). Just the contrary picture is realized for transmission: at \( \gamma \ll \gamma_r \) of the transmission curve \( \mathcal{T}_{\max} = (\gamma/\tilde{\gamma}_r)^2 \ll 1 \) and depends noticeably from \( \kappa d \) growing with increasing \( \kappa d \). When \( \gamma \gg \gamma_r \) then \( \mathcal{T} \equiv 1 \) and is weakly dependent on \( \kappa d \). The maximum of the absorption peak in these limiting cases is equal \( \mathcal{A}_{\max} = 2\gamma/\tilde{\gamma}_r \) \((\gamma \ll \gamma_r)\) and \( \mathcal{A}_{\max} = 2\tilde{\gamma}_r/\gamma (\gamma \gg \gamma_r) \), in both limiting cases \( \mathcal{A}_{\max} \ll 1 \), \( \mathcal{A}_{\max} \gg \mathcal{R}_{\max} \) \((\gamma \gg \gamma_r)\) \( \mathcal{A}_{\max} \gg \mathcal{T}_{\min} \) \((\gamma \ll \gamma_r)\). The frequency dependence \( \mathcal{R}, \mathcal{A}, \mathcal{T} \) for the limiting cases \( \gamma \gg \gamma_r \) \( \gamma \ll \gamma_r \) is represented in Figs. 2-4. It is seen in Fig. 2 how the spatial dispersion influences the height and width of the reflection peak which decrease with growing \( \kappa d \). The peak frequency shift is inobservable since \( \varepsilon'' \gamma_r \ll \gamma \). Vice verse, in Fig. 3 the spatial dispersion leads to the shift of the reflection peak without changing its form. Fig. 3 demonstrates also transmission \( \mathcal{T} \) for the case \( \gamma \ll \gamma_r \). There are both the shift \( \mathcal{T}_{\min} \) with growing \( \kappa d \) and its increasing which is almost inobservable due to the chosen scale on the ordinate axis. In Fig. 4 absorption \( \mathcal{A} \) is demonstrated for two limiting cases. On the narrow peak set (Fig. 4b), the case \( \gamma \ll \gamma_r \) one can see growing of \( \mathcal{A}_{\max} \), as well as its shift. The shift appearance (as well as in Fig. 2 for \( \mathcal{T} \)) is due to that \( \mathcal{A}_{\max} \sim \gamma_{\varepsilon''}^{-1} \) and \( \varepsilon'' \gamma_r \ll \gamma_r \). The curves in Fig. 4 correspond to the case \( \gamma \gg \gamma_r \), the shift is small and \( \mathcal{A}_{\max} \) decreases with growing \( \kappa d \).

If one takes into account a heterogeneity of the medium, i. e. \( \zeta \neq 1 \) \((\nu \neq \nu_1)\), and neglects the spatial dispersion then instead of Eq. (56) one obtains the expressions

\[ \mathcal{R} = \frac{\zeta^2 (\gamma_r/2)^2}{\Omega^2 + (\gamma + \zeta \gamma_r)^2/4}, \quad \mathcal{A} = \frac{\zeta \gamma \gamma_r/2}{\Omega^2 + (\gamma + \zeta \gamma_r)^2/4}, \quad \mathcal{T} = \frac{\Omega^2 + \gamma^2/4}{\Omega^2 + (\gamma + \zeta \gamma_r)^2/4}. \] (57)

It is seen from Eq. (57) that in the limiting case instead of \( \gamma_r \) the value \( \zeta \gamma_r \) \((\text{see Eq. (23)})\) figures in which the refractive index \( \nu_1 \) refers to the barrier matter. This coincide with the result of [18].

### IV. THE GENERAL CASE

In this section the general situation is considered when the matter is heterogeneous and the spatial dispersion is essential. Making use Eqs. (38), (39) and (41) we reduce reflection to the expression

\[ \mathcal{R} = \frac{(\tilde{\gamma}_r/2)^2 X_1 + v_1 - (\tilde{\gamma}_r/2)(\nu_1 \Omega + Z + \Gamma/2)}{\Omega^2 + \Gamma^2/4}, \] (58)

\[ |\Delta|^2 = v + \frac{(\tilde{\gamma}_r/2)^2 X - (\tilde{\gamma}_r/2)(Y \Omega + Z + \Gamma/2)}{\Omega^2 + \Gamma^2/4}, \] (59)

where

\[ v = 4 \zeta^2 \cos^2 \kappa d + (\zeta^2 + 1)^2 \sin^2 \kappa d, \]
\[ v_1 = (\zeta^2 - 1)^2 \sin^2 \kappa d, \] (60)

\[ X = 2(\zeta - 1)^2 [\zeta^2 + 1 + (\zeta^2 - 1) \cos \kappa d], \]
\[ X_1 = 2[\zeta^4 + 1 + (\zeta^4 - 1) \cos \kappa d], \] (61)

\[ Y = 2(\zeta - 1)^2 (\zeta + 1) \times [((\zeta + 1) \cos \kappa d + \zeta - 1) \sin \kappa d, \]
\[ Y_1 = 2 \zeta^2 (\zeta^2 - 1) \sin \kappa d, \] (62)
\[ Z = 2(\zeta - 1)(\zeta + 1)^2 \sin^2 \kappa d \]
\[ -2\zeta [(\zeta + 1) \cos \kappa d + \zeta - 1], \]
\[ Z_1 = 2(\zeta - 1)^3(\zeta + 1) \sin \kappa d. \]  

(63)

In the denominator of the function \( R \), determined by Eq. (59), the function \( v(v \gg 1) \) gives the main contribution, the rest terms may be neglected. One exclusion is the term \( \sim \Gamma \), which contributes essentially in the case \( \gamma \ll \tilde{\gamma}_r \). In the numerator \( R \) the function \( v_1 \) determines reflection from the QW borders. This reflection does not depend from the light frequency in the frequency interval corresponding to the peak width and disappears, as it is seen from Eq. (60), in the limiting cases \( \kappa d \to 0 \) and \( \zeta \to 0 \). At \( \gamma \ll \tilde{\gamma}_r \) the first term in the numerator of Eq. (58) dominates in reflection, such a case \( R \approx 1 \). If \( \gamma \gg \tilde{\gamma}_r \), the first term becomes small and the functions \( v_1 \) are essential, as well as the term \( \sim \Omega \), which produces some asymmetry of the reflection peak.

The frequency dependence of reflection is represented in Fig. 5, one can see the sharp asymmetry of peaks and the non-monotonic dependence \( R_{\text{max}} \) from \( \kappa d \). The non-monotone is determined by the value and sign of the function \( Y_1 \) (62). For instance, \( Y_1 \) changes from \( Y_1 = 0.507 \) (the curve \( \kappa d = 1.5 \)) in Fig.5 up to \( Y_1 = 0.072 \) (the curve \( \kappa d = 3 \)). In the last case the contribution \( Y_1 \Omega \) in the peak form is small, the asymmetry is indistinguishable, since the first term in Eq. (58) dominates.

One can see the same in Fig.5b, but there \( Y_1 < 0 \) and \( \Omega > 0 \) corresponds to the peak maximum. In the case \( \gamma_r \to 0 \)

\[ R = \frac{v_1}{v} = \frac{(\zeta^2 - 1)^2 \sin^2 \kappa d}{4 \zeta^2 \cos^2 \kappa d + (\zeta^2 + 1^2) \sin^2 \kappa d} \]

(64)

and corresponds to reflection from a plane transparent layer inserted in the matter with the different refraction index. Comparing Fig. 2 to Fig. 5 one can conclude that the medium heterogeneity leads to the more sharp dependence of reflection from the parameter \( \kappa d \).

Absorption \( A \) and transmission \( T \) are expressed as

\[ A = \frac{4 \zeta [(\zeta^2 + 1 + (\zeta^2 - 1) \cos \kappa d)] \gamma \tilde{\gamma}_r}{|\Delta|^2 (\Omega^2 + \Gamma^2/4)}, \]

\[ T = \frac{4 \zeta^2 (\Omega^2 + \gamma^2/4)}{|\Delta|^2 (\Omega^2 + \Gamma^2/4)}, \]

(65)

which transmit into Eq. (56) and Eq. (57) in the limiting cases \( \zeta = 1 \) \( \kappa d = 0 \).

As it was mentioned the function \( v_1 \), which is connected with reflection of the QW boundaries, and the term \( \sim \Omega \) in the numerator are essential in forming of the reflection curves. They determine the strong shift of the peak maximum and the minimum appearance. On the other hand the values \( A \) from Eq. (65) and \( T \) from Eq. (67) coincide in their form with those for the case of the homogenous medium: the difference is an appearance of the factors which do not depend on \( \Omega \) and weakly depend on \( \kappa d \). Therefore the medium heterogeneity influences absorption and transmission more strongly than reflection.

On the base of our analysis one can make a general conclusion that the spatial dispersion of the electromagnetic waves and the medium heterogeneities influence reflection most strongly, changing radically the peak shape. These changes are most observable in the limiting case \( \gamma \gg \tilde{\gamma}_r \). When \( R_{\text{max}} \approx (\tilde{\gamma}_r/\gamma)^2 \). This is since the function \( v_1 \) from Eq. (60) and the term linear on \( \Omega \) in Eq. (58) are small and influence the first term only when it is small. In another limiting case \( \gamma \ll \tilde{\gamma}_r \) \( R_{\text{max}} \approx 1 \) and their influence is practically inobservabel. If one takes into account only the spatial dispersion or only the medium heterogeneity reflection changes comparatively weakly, since in these limiting cases \( v_1 = Y = Y_1 = 0 \). The spatial dispersion and the medium heterogeneity influence weakly the values \( A \ T \) at \( \gamma \gg \tilde{\gamma}_r \). Indeed, as it follows from Eq. (65) in this case \( T \approx 1 \) and strong change of the small value \( R_{\text{max}} \) influence weakly on \( T \). The same is true for \( A_{\text{max}} \gg R_{\text{max}} \).

The dependence \( R \), \( A \ T \) on the parameter \( \kappa d \), characterizing the wave spatial dispersion in a QW, is obtained for the rectangular QWs and infinitely high barriers. In real semiconductor heterostructures impurity electrons of barriers, overflowing into the QW, distort its rectangular form near the boundaries. Therefore our theory is applicable for clean matters and wide QW’s, when sizes of distorted regions are small in comparison to the QW width. Besides the theory is applicable for deep QW’s in which positions of energy levels and corresponding wave functions weakly differ on energy levels and wave functions of the infinitely deep QW. Our one-level approximation supposes, that the energy distance between neighbor levels is much more than the level broadening. That leads to the restriction for the QW width. For example, for \( d = 500 A \) and \( m_e = 0.06m_0 \) the energy distance of the lowest size quantized energy levels is \( \simeq 10^{-3} \) eV.

Our results are applicable in the case of a weak influence of the Coulomb interaction on the energy spectrum of light created EHPs. These corrections are small [21][25] if

\[ a_{e\text{exc}}^2 \gg a_H^2, \quad a_{e\text{exc}} \gg d, \]

(67)

where \( a_H \) is the magnetic length, and the Wannier-Mott exciton radius in absence of magnetic field is \( a_{e\text{exc}} = h^2 / e_0 / \mu_e \). The first inequality of Eq. (67) is satisfied in strong enough magnetic fields, and the second is satisfied better for QW’s with large values of the dielectric function and small reduced effective masses \( \mu \) of the electron and hole. The second condition of Eq. (67) for GaAs is satisfied at \( d \leq 150 A \), when the spatial dispersion and matter heterogeneities influence comparatively weakly on
investigated values. That is seen in Fig. 5a where the curve $kd = 0.175$ corresponds to the GaAs QW with the width $d \approx 62\,\text{Å}$. The second inequality is approximately satisfied, but the shift and the reflection peak asymmetry are small. If the second inequality of Eq. (67) is not satisfied, the dependence of the wave function on $z$ cannot be represented by Eq. (14). However the excitonic effect does not change principally our results: it influence only the radiative broadening $\gamma_r$ of the electronic excitation. The same is true for excitonic energy levels in absence of magnetic field.

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FIG. 1. The functions $\varepsilon'$ and $\varepsilon''$, determining the width changes and the peak shifts of reflection, transmission and absorption in the homogeneous matter. The spatial dispersion is taken into account. $m_e$ ($m_v$) are the size quantization quantum numbers of electrons (holes).

FIG. 2. Influence of the spatial dispersion on the frequency dependence of reflection $R$ for the homogeneous medium. $\zeta = 1$, $\gamma/\gamma_r = 10$, $\gamma_r = 10^{-4}\,\text{eV}$, $m_e = m_v = 1$.

FIG. 3. Influence of the spatial dispersion on the frequency dependence of reflection $R$ (curves 1-3) and transmission $T$ (curves 4-6) for the homogeneous medium $\zeta = 1$, $\gamma/\gamma_r = 0.1$, $\gamma_r = 10^{-4}\,\text{eV}$, $m_e = m_v = 1$. The curves $1, 4 - k_d = 0$; $2, 5 - k_d = 1.5$; $3, 6 - k_d = 3$.

FIG. 4. Influence of the spatial dispersion on the frequency dependence of absorption $A$ for the homogeneous medium. $\zeta = 1$, $\gamma_r = 10^{-4}\,\text{eV}$, $m_e = m_v = 1$, $a - \gamma \gg \gamma_r$, $b - \gamma \ll \gamma_r$.

FIG. 5. The frequency dependence of reflection $R$. The light spatial dispersion and the medium heterogeneities are taken into account. $\gamma \gg \gamma_r$, $\gamma_r = 10^{-4}$, $m_e = m_v = 1$, $a - \zeta > 1$, the curve $kd = 0.175$ corresponds to GaAs, $b - \zeta < 1$. 
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