Solving the Optimal Strategy of "Crossing the Desert" Based on Dynamic Programming

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Abstract. In real life, many practical situations need to maximize the benefits under certain conditions. This paper studies the B problem of the 2020 National College Students Mathematical Modeling Contest. Through the establishment of a mathematical model, a combination of weighted undirected graphs, dynamic programming, computer simulation and multi-person non-cooperative models are used to discuss different conditions. The best decision and maximum benefit. For question one, get the maximum benefit of the first and second levels; for question two, solve the strategies that players should adopt for the third and fourth levels; for question three, it is concluded that players should use under normal circumstances Specific strategies.

Keywords: Dynamic programming; Computer Simulation; Multiplayer non-cooperative model; Empowered undirected graph; Across the desert.

1. Introduction
The main content of this article is the analysis and modeling of problem B in the 2020 National College Students Mathematical Contest in Modeling.

For problem one: we construct a weighted undirected graph to solve the shortest path between any two regions. Considering that we cannot determine when we need to go to the village to purchase materials during mining, we adopt dynamic planning to obtain the maximum benefit of the first pass is RMB, and the maximum profit of the second pass is RMB.

For question two: Based on the model in the first question, we introduced a weather model to quantitatively describe weather conditions. The binomial distribution is used to solve the expected value of the process of the shortest path with lower complexity and the maximum benefit, and the computer simulation method is used to solve the expected value of the dynamic programming process with higher complexity, and the third pass is generally adopted. The strategy should be to directly follow the shortest path from the starting point to the ending point; for the fourth level, under normal circumstances, the player will get a greater profit when using the mining strategy.

For question 3: On the basis of the second question model, introduce a multi-person non-cooperative model that includes players, strategy sets and payment functions, and use computer simulation methods to solve the expected value, and get the general situation that players should use Specific strategies.

Finally, we analyzed and summarized the advantages and disadvantages of the model, and tried to promote the model.
2. Problem Analysis
We need to establish a suitable mathematical model and solve it in the process of solving the maximum profit or optimal strategy. Below, this article will analyze the three issues raised one by one in detail based on the understanding of the topic.

2.1. Problem One Analysis
For problem 1, we can construct an undirected weighting graph, use Lingo to solve the shortest path; establish a mathematical programming model, use MATLAB for dynamic programming, and then find the maximum benefit.

Since the adjacent areas of each area are known, we can construct an undirected weighting graph to find the shortest path from the start point to the end point, and judge whether the village and the mine are located on the shortest path, so as to make the next judgment decision.

Our goal is to enable players to reach the end within the stipulated time and get the maximum benefit. Therefore, we can establish the objective function and constraint equations according to the conditions given in the title. Solving the maximum profit is divided into two situations, that is, the player chooses to go to the mine to mine or directly follow the shortest path to the destination.

2.2. Problem Two Analysis
For problem two, similar to analysis problem one, we use Lingo to solve the shortest path and MATLAB to solve the maximum benefit.

First, we construct an undirected weighting map again according to Map 2, find the shortest path and judge the relationship between the village, mine and the shortest path. Finally, the maximum profit is solved according to the situation.

In the case of adopting the shortest path strategy, due to the low complexity of the model, according to the number of days of sunny weather, the expected value obtained by the binomial stepwise comparison can be followed, and the strategy generally adopted by the player can be given according to the model constraints.

2.3. Problem Three Analysis
For question three, we introduce a multi-person non-cooperative model.

For the fifth level, since the weather conditions are all known, we can use dynamic programming to solve the maximum profit for the strategy combination of the strategy concentration.

For the sixth level, since the player only knows the weather conditions of the day, we assume that the ratio of the probability of the occurrence of sunny weather to the probability of occurrence of high-temperature weather is as follows. Discuss and use computer simulations to simulate the maximum when there is a storm or whether to go for mining. The range and expected value of income. By analyzing and comparing the value range and the expected value, the optimal strategy of the player under normal circumstances is obtained.

3. Model assumptions
1. Do not consider the out-of-stock situation when purchasing resources.
2. Less sandstorm weather means that the number of days the player encounters sandstorm weather before reaching the end point is or.
3. The occurrence probability of sunny weather and high temperature weather is equal.
4. Notation Convention

| Notation | meaning |
|----------|---------|
| $w_t$    | The amount of water the player has on day $t$ |
| $f_t$    | The amount of food the player has on day $t$ |
| $t_i$    | Mining days in the i stage |
| $bw_i$   | The amount of water purchased in the village for the i time |
| $bf_i$   | The amount of food purchased in the village for the i time |
| $M_i$    | Passing through the village for the i time |
| $C_{iw}$ | Under the weather of day i, the basic water consumption |
| $C_{it}$ | Under the weather of day $t$, the basic water consumption |
| $Q_i$    | The funds that the player has when he arrives at the end |
| $p_q$    | Probability of clear weather |
| $p_g$    | Probability of hot weather |
| $q$      | Total sunny weather |
| $g$      | Total number of hot weather |
| $x_i$    | The i-th person's funds to reach the end |
| $y_i$    | I-th player |

5. Problem solving

5.1. Question one

5.1.1. Shortest path solving

1. Model building

Here we need to establish a weighted undirected graph $G = (V(G), E(G))$. Among them, the node set is $V(G) = \{a_1, a_2, \ldots, a_n\} (n = 27)$, the edge set is $E(G) = \{b_1, b_2, \ldots, b_n\}$. Node set is in $V(G)$ at $a_i (i = 1, 2, \ldots, n)$, element $e_i = (a_i, a_j) (k = 1, 2, \ldots, m)$ is in edge set $E(G)$.

As each area in the desert satisfies the same neighboring edge, it is said that the areas satisfying this condition are adjacent. So get the edge $e_i (k = 1, 2, \ldots, m)$ equal $c_i (i = 1, 2, \ldots, m)$, which is $c_1 = c_2 = L = c_n = 1$.

2. Model solving

In order to simplify the calculation of the model, we use the following strategies to optimize the model:

(1) The player’s route in the desert is divided into three parts: (1) From the starting point to the mine; (2) From the beginning of mining to the end of mining; (3) From the mine to the end;

(2) Let the weight of each side equal, which is $c_1 = c_2 = L = c_n = 1$. 
5.1.2. Maximum profit solution

1. Model building

(1) Decision variables

We introduce three 0-1 sets of variables \( y, m, k \), among them, \( y \) indicates that the player is resting or walking during the journey to the destination, \( m \) indicates that the player is resting in the mine or mining, \( k \) indicates whether the player is in the mine.

We use variables \( w_0, f_0 \) Indicates the amount of water and food purchased by the player at the starting point, Use variables \( bw_i, bf_i (i = 1, 2, 3) \) Indicates the amount of water and food purchased by the player through the village for the first time.

(2) Objective function

It can be seen from the question that our goal is to enable the player to reach the end within the specified time and obtain the maximum benefit. The final profit of the player, that is, the objective function is:

\[
\max Q = 10000 + 1000 \sum_{i=0}^{n} t_i - 10 \sum_{i=0}^{n} bw_i - 20 \sum_{i=0}^{n} bf_i - 5w_0 - 10f_0
\]

(3) Restrictions

Analyzing the meaning of the question, we can draw the following eleven constraints:

\[
\begin{align*}
5w_0 + 10f_0 & \leq 10000 \\
M_1 &= 10000 - 5w_0 - 10bf_0 - 20bw_0 \\
m_{i+1} &= m_i - 10bw_i - 20bf_i + 1000t_i \ (i = 1, 2) \ \\
10bw_i + 20bf_i & < M_i \ (i = 1, 2, 3) \\
w_i & = w_i + bw_i \ (i = 1, 2, 3) \\
s.t. f_{i+1} &= f_i + bf_i \ (i = 1, 2, 3) \\
w_i &= w_i (2yC_{1i} - (1-y)C_{yi}) \ (i) \\
f_i &= f_i (2yC_{2i} - (1-y)C_{2i}) \ (i) \\
3w_i + 2f_i & \leq 1200 \\
w_i & > 0, f_i > 0, t \leq 30 \\
y, m, k & \in \{0, 1\}
\end{align*}
\]

2. Model solving

(1) first round

① Mining strategy solution

We use MATLAB to solve the built model according to the constraints.

First, we choose to purchase resources only when we go to the mine and pass through the village at the end. It is found that when the profit is maximized, the player exceeds the upper limit of load, so we know that the player will definitely go to the village to purchase resources during mining.

After preliminary analysis, the player will set off to the village on the 14th day to avoid insufficient resources. Since food is more expensive than water, we ask players to buy as much food as possible at the starting point, and only the amount of water that can support the player's next visit to the village is reserved.

In this case, after calculation, it can be known that the player will not have insufficient resources, the maximum profit generated is 10220 RMB. And finally the player has left boxes of food, so the maximum profit is 10230 RMB.

② Shortest path strategy solution

Then consider the situation where the player does not mine but goes directly to the end point. The player needs to pass through three sides, that is, it takes at least three days to reach the end. Finally, the maximum benefit is solved, and the maximum benefit generated is 9140 RMB.

Since 9410RMB \leq 10230RMB, the optimal route taken by the player is shown in Table 2.
Table 2. The best route for the first pass

| Date | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| Area | 25| 24| 23| 23| 22| 9 | 9 | 15| 14| 12 |
| Date | 11| 12| 13| 14| 15| 16| 17| 18| 19| 20 |
| Area | 12| 12| 12| 12| 14| 15| 15| 15| 14| 12 |
| Date | 21| 22| 23| 24| 25| 26| 27| 28| 29| 30 |
| Area | 12| 12| 12| 12| 14| 15| 9 | 21| 27|    |

(2) Second level
In the same way as solving the first pass, we use MATLAB to solve it according to the constraints.

① Mining strategy solution
We have analyzed in 5.1.1 that the player will go to the mine in area 30 and the village in area 39. Now select the player to purchase resources only on the way to the destination. It is found that when the profit is maximized, the player exceeds the upper limit of the load, so the player will go to the village to purchase resources on the way to mining. Consider buying as much food as possible at the starting point, and get the maximum profit of 11990RMB.

② Shortest path strategy solution
Then consider the situation where the player does not mine but goes directly to the end point. We use Lingo to solve the shortest path, and we need to pass through at least 11 areas. In this case, the player reaches the end on the first day, and his maximum profit is 7390 yuan.

Since $7390 \leq 11990$, the optimal route plan adopted by the player is shown in Table 3.

Table 3. The best route for the second pass

| Date | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| Area | 2 | 3 | 4 | 4 | 5 | 13| 13| 22| 30| 39 |
| Date | 11| 12| 13| 14| 15| 16| 17| 18| 19| 20 |
| Area | 30| 30| 30| 30| 30| 30| 30| 30| 30| 39 |
| Date | 21| 22| 23| 24| 25| 26| 27| 28| 29| 30 |
| Area | 30| 30| 30| 30| 30| 30| 39| 47| 56| 64 |

5.2. Question two

5.2.1. Shortest path model establishment
In order to solve the shortest path between any two points in the third and fourth level maps, we construct a weighted undirected graph to solve the problem, and the weights $c_i (i = 1, 2, \ldots, m)$ of edge $e_i (k = 1, 2, \ldots, m)$ are equal.

5.2.2. Maximum profit model establishment
Since the third level has no villages, mines, and players only know the weather of the day and will not have sandstorms, and the fourth level has villages and mines, players only know the weather of the day and rarely have sandstorms. We built a model for 5.1.2 Make modifications and optimizations.

(1) Third pass
Objective function:

$$\max Q = 10000 + 200t_i - 5w_0 - 10f_0 + \frac{1}{2}(5w_i + 10f_i)$$

Constraint model:
0, 0, 10
\{0,1 \}
\begin{align*}
\text{(2) Fourth stage weather simulation model} \\
\text{(1)The third stage weather simulation model} \\
\text{In the third level, players only know the weather conditions of the day, and there will be no} \\
sandstorms before the deadline. We assume that in ten days, the probability of sunny weather and high} \\
temperature weather is the same, that is \( p_s = p_g = 0.5 \).
\end{align*}

Then in ten days, the probability of the total number of sunny weather is obeying the binomial distribution, which is \( P = C_{10}^0 (0.5)^0 (0.5)^{10} = C_{10}^0 (0.5)^0 \) \[1\].

Similarly, the probability of a hot weather is \( P = C_{10}^0 (0.5)^0 (0.5)^{10} = C_{10}^0 (0.5)^0 \).

\( 5.2.4. \ \text{Model solving} \)
\( 5.2.3. \ \text{Weather simulation model} \)
\( 5.2.4. \ \text{Model solving} \)
\( 5.2.4. \ \text{Model solving} \)
\begin{align*}
\text{(1) Solving the third level model} \\
\text{① Mining strategy solution} \\
\text{We consider that the player will go to the mine for mining, and use MATLAB to solve the problem} \\
according to the constraints. In order to ensure that the player will not fail during the game, we need to \\
make the amount of resources initially purchased by the player meet the amount of resources consumed \\
under extreme conditions of high temperature for 10 days. After calculation, the quantity of initially \\
purchased water is 98 boxes, and the quantity of food is 464 boxes.
\end{align*}

In this case, according to the weather simulation model built in 5.2.3 and the maximum return model 
built in 5.2.2, the maximum return \( \max Q \) is obtained and the value satisfies \( 6490 \leq \max Q \leq 8570 \).

\( 5.2.4. \ \text{Model solving} \)
\begin{align*}
\text{(2) Shortest path strategy solution}
\end{align*}

\begin{align*}
\begin{cases}
5w_0 + 10f_0 & \leq 10000 \\
3w_0 + 2f_0 & \leq 1200 \\
\text{s.t.:} \\
w_{i+1} = w_i (2yC_{i+1} - (1-y)C_{i+1})k - (3mC_{i+1} - (1-m)C_{i+1})(1-k) \\
f_{i+1} = f_i (2yC_{i+1} - (1-y)C_{i+1})k - (3mC_{i+1} - (1-m)C_{i+1})(1-k) \\
w_i > 0, f_i > 0, t \leq 10 \\
y, m, k \in \{0,1 \}
\end{cases}
\end{align*}

\begin{align*}
\text{(2) Fourth level} \\
\text{Objective function:} \\
\max Q = 10000 + 10000 \sum_{i=1}^{n} f_i - 10\sum_{i=0}^{n} bw_i - 20\sum_{i=0}^{n} bf_i - 5w_0 - 10f_0 + \frac{1}{2}(5w_0 + 10f_0)
\end{align*}

\begin{align*}
\text{Constraint model:} \\
5w_0 + 10f_0 & \leq 10000 \\
M_t & = 10000 - 5w_0 - 10f_0 - 10bw_i - 20bf_i, t = 1, 2, L \\
M_i & = M_t - 10bw_i - 20bf_i + 1000, t = 1, 2, L \\
w_{i+1} & = w_i + bw_i, t = 1, 2, L \\
w_{i+1} & = w_i(2yC_{i+1} - (1-y)C_{i+1})k - (3mC_{i+1} - (1-m)C_{i+1})(1-k) \\
f_{i+1} & = f_i(2yC_{i+1} - (1-y)C_{i+1})k - (3mC_{i+1} - (1-m)C_{i+1})(1-k) \\
3w_0 + 2f_0 & \leq 1200 \\
w_i & > 0, f_i > 0, t \leq 30 \\
y, m, k \in \{0,1 \}
\end{cases}
\end{align*}
Consider that instead of mining, the player will go directly to the destination based on the shortest path. In this case, the maximum return value satisfies $9190 \leq \max Q \leq 9310$, the formula for solving the expected value is as follows:

$$E = \sum_{i=1}^{n} \max Q, p_i$$

Get $E = 9310$.

Analyzing the conclusions of the above two situations, the general strategy adopted in the third pass is to directly follow the shortest path from the start point to the end point, and the specific path is $1 \rightarrow 5 \rightarrow 6 \rightarrow 13$.

(2) Solving the fourth level model

① Mining strategy solution

Assuming that the player does not go to the village for supplies, it will consume 738 boxes of food and boxes of water, which exceeds the upper limit of weight. Therefore, the player must go to the village to purchase resources. Due to the complexity of this situation, we adopt the method of multiple simulations to find that the maximum return value satisfies $9713 \leq \max Q \leq 16052.5$, and its expected value is 11922.

② Shortest path strategy solution

When the player experiences a sandstorm before reaching the end, the maximum profit value meets $7690 \leq \max Q \leq 8330$, The expected value is 8010.

Analyzing the above calculation results, it can be seen that the player obtains a greater profit when adopting the mining strategy, so the specific strategy adopted by the player in general is as follows:

I. When the sandstorm weather is not considered, if the remaining time is enough to return to the end, it is calculated every day whether the remaining resources are sufficient for the next day's mining and whether it is enough to reach the end. If not, then go to the village to purchase resources.

II. When considering the sandstorm weather, if the remaining time is enough to return to the end point, calculate whether the remaining resources are sufficient for the next day's mining and whether it is enough to reach the end point. If not, go to the village to purchase resources.

5.3. Question three

5.3.1. Multiplayer non-cooperative model

Since the sixth level is similar to the fifth level strategy model, only the fifth level strategy model is analyzed here. We establish the following strategy model [2]:

1. Middleman

The two players in this level are the players in the game, they are $x_1, x_2$.

2. Strategy set

(1) Shortest path strategy

After solving the shortest path of the weighted undirected graph, we get the shortest path from the start point to the end point as:

When the player does not choose to mine:

① $1 \rightarrow 5 \rightarrow 6 \rightarrow 13$ ② $1 \rightarrow 4 \rightarrow 6 \rightarrow 13$

When the player does not choose to mine:

① $1 \rightarrow 2 \rightarrow 3 \rightarrow 9 \rightarrow 11 \rightarrow 13$ ② $1 \rightarrow 4 \rightarrow 3 \rightarrow 9 \rightarrow 11 \rightarrow 13$

Below, we will propose and reduce the set of strategies for $x_1$ and $x_2$ based on the shortest path.

(2) $x_1, x_2$'s strategy

When the player does not choose to mine:

a. The path is $1 \rightarrow 5 \rightarrow 6 \rightarrow 13$

b. The path is $1 \rightarrow 4 \rightarrow 6 \rightarrow 13$

c. The path is $1 \rightarrow 4 \rightarrow 7 \rightarrow 12 \rightarrow 13$
d. The path is $1 \rightarrow 4 \rightarrow 7 \rightarrow 6 \rightarrow 13$

e. $x_1, x_2$ can choose to stay or walk

When the player does not choose to mine:

a. $1 \rightarrow 2 \rightarrow 3 \rightarrow 9 \rightarrow 11 \rightarrow 13$

b. $1 \rightarrow 4 \rightarrow 3 \rightarrow 9 \rightarrow 11 \rightarrow 13$

c. $1 \rightarrow 2 \rightarrow 3 \rightarrow 9 \rightarrow 10 \rightarrow 13$

d. $1 \rightarrow 4 \rightarrow 3 \rightarrow 9 \rightarrow 10 \rightarrow 13$

e. $x_1, x_2$ can choose to stay or walk

3. Payment function

The combination of strategies selected by each player in the game is called the situation, which is recorded as $S = (S_1, S_2, \ldots, S_n)$. The payout function is the winning number of players in each situation.

5.3.2. Mathematical programming model

The objective function is:

$$\max Q = \sum_{i=1}^{n} x_i$$

Meet the following constraints:

$$\begin{align*}
    f_t &= 10000 - 5w_t - 10f_{t-1} \\
    x_t &= f_t + 200(1 - j)\Delta t_t + 100j\Delta t_t \\
    w_t &= w_{t-1} - (1 - a)[2(1 - j)(C_{t-1} + 2kC_{t-2})] - 3aC_{t-1} \\
    f_t &= f_{t-1} - (1 - a)[2(1 - j)(C_{t-1} + 2kC_{t-2})] - 3aC_{t-2} \\
    w_t &> 0, f_t > 0 \\
    3w_t + 2f_t &\leq 1200 \\
    5w_t + 10f_t &\leq 10000 \\
    f, a &\in \{0, 1\}
\end{align*}$$

5.3.3. Multi-person non-cooperative model solution

A. Solve the fifth level

We solve the fifth level based on the established multi-person non-cooperative model and mathematical programming model.

When you choose to walk in the desert, no matter how you choose a countermeasure a, b, c, d. There will be situations where a certain route of two players overlaps, or a player walks one area more than the shortest path. Since the player can choose to rest, and the cost of taking a rest day is less than the cost of walking for a day and the cost of two people passing through the same area at the same time, it can be inferred that when the objective function takes the maximum value, a player will stay for one day on the first day. The next day he started walking on the same shortest path as the other player.

The total daily income of one person mining and two people's mining is the same, but two people's mining will double the overall consumption. Therefore, when the overall situation is maximized, one person goes directly to the end point, and one person goes to mine, and does not mine on hot days.

(1) Don’t choose mining

When the player does not choose to mine, it is found that when the objective function is the largest, its value is:

$$\max Q = \sum_{i=1}^{n} y_i = 9535 + 9480 = 19015$$

Table 4. Strategies generally adopted by a player in the fifth level (no mining)

| Date | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| Area | 5 | 5 | 6 | 13| 13| 13| 13| 13| 13| 13 |
Table 5. General strategy adopted by another player in the fifth level 5(no mining)

| Date | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| Area | 1 | 1 | 5 | 6 | 13| 13| 13| 13| 13| 13 |

(2) Choose mining
When the player chooses to mine, when the objective function is maximized, its value is:

\[ \max Q = \sum_{i=1}^{x} \gamma_{i} = 9535 + 8800 = 18335 \]

Table 6. The general strategy adopted by a player in the fifth level (mining)

| Date | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| Area | 5 | 5 | 6 | 13| 13| 13| 13| 13| 13| 13 |

Table 7. The general strategy adopted by a player in the fifth level (mining)

| Dare | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|
| Area | 4 | 3 | 9 | 9 | 9 | 11| 13| 13| 13| 13 |

In summary, it can be seen that when the player does not choose to mine, the objective function value is larger, so the strategy that the player should generally adopt is to follow the shortest path to the end, rest in hot weather, and walk in sunny weather. The specific route planning is shown in the table. 4.

B. Solve the sixth level
Unlike solving the fifth level, here we only know the weather conditions that day. We assume that within 30 days, the probability of sunny weather and hot weather is the same, that is \( P_{s} = P_{h} = 0.5 \). Here is a computer simulation solution for the occurrence of a storm:

(1) Mining strategy solution
When a storm occurs, the basic consumption of \( x_{1}, x_{2}, x_{3} \) needs to be added to the consumption on the day of the storm, and the time after the storm is delayed by one day. Other conditions are the same as the mining strategy when the storm does not occur. The objective function satisfies \( 15922.5 \leq \max Q \leq 28134 \) and the expected value is \( E = 21855.4 \).

(2) Shortest path strategy solution
When a storm occurs, The additional consideration is that when a storm occurs, the basic consumption of is added to the consumption on the day of the storm, and the time after the storm is delayed by one day. The objective function satisfies \( 7787.5 \leq \max Q \leq 8027.5 \), expected value \( E = 7905 \).

Analyzing the above calculation results, it can be seen that the player obtains a greater profit when using the mining strategy, so the specific strategy used by the player in general is: \( x_{1} \) directly go through the shortest path to the end point, directly go through the shortest path to the mine, \( x_{2} \) go to the village and wait for mining mine.

6. Conclusion
This model is also suitable for problems such as paving roads and multi-person collaborative work. This model still has shortcomings for some complex problems. If it can be further optimized based on this algorithm, it will make the model universal.
References
[1] JIANG Q Y, XIE J X, YE J. Mathematical model (third edition) [M]. Beijing: Higher Education Press.
[2] WANG S L. Research on Fuzzy Multi-Objective Multi-person Cooperation Game and Its Solution [D]. Chongqing University, 2009.
[3] JIANG Q Y, XIE J X, YE J. Mathematical model (third edition) [M]. Beijing: Higher Education Press.
[4] WANG S L. Research on Fuzzy Multi-Objective Multi-person Cooperation Game and Its Solution [D]. Chongqing University, 2009.