Topological mass from vortex-electron interaction

Shantonu Mukherjee∗ and Amitabha Lahiri†

S N Bose National Centre for Basic Sciences,
Block JD, Sector III, Salt Lake, Kolkata 700106, India

(Dated: January 6, 2020)

Abstract

We consider the Abelian Higgs model in 3+1 dimensions with vortex lines, into which charged fermions are introduced. This could be viewed as a model of a type-II superconductor with unpaired electrons (or holes), analogous to the boson-fermion model of high-$T_c$ superconductors but one in which the bosons and fermions interact only through the electromagnetic gauge field. We investigate the dual formulation of this model, which is in terms of a massive antisymmetric tensor gauge field $B_{\mu\nu}$ mediating the interaction of the vortex lines. This field couples to the fermions through a nonlocal spin-gauge interaction term. We then calculate the quantum correction due to the fermions at one loop and show that due to the presence of this new nonlocal term a topological $B \wedge F$ interaction is induced in the effective action, leading to an increase in the mass of both the photon and the tensor gauge field. Additionally, we find a Coulomb potential between the electrons, but with a large dielectric constant generated by the one-loop effects.

∗ shantonumukherjee@bose.res.in
† amitabha@bose.res.in
I. MOTIVATION

Vortices and vortex lines appear as solutions in the field theoretic description of many physical systems, from quantized vortices in superfluid Helium [1–5] and Bose-Einstein condensates to Abrikosov lattices in type-II superconductors, all of which have been observed. They also appear in field theories which describe high energy physics, e.g. as cosmic strings which appear in many gauge theories including grand unified theories [6–9], or as color flux tubes which are conjectured to appear in non-Abelian gauge theories such as QCD, leading to color confinement [10–12]. In this paper we will be interested in the interaction of charged fermions with vortices, which appears in the description of different physical systems.

Type II superconductors allow magnetic flux to pass through in the form of lines of quantized flux [13, 14], a phenomenon which has been experimentally confirmed [15–17] including in high-$T_c$ superconductors [18–20]. The phenomenon of superconductivity at high temperature has remained mysterious since its discovery in cuprates [21], the Bardeen-Cooper-Schrieffer (BCS) description of low-temperature superconductivity [22–24] cannot explain it. The typical features of a high temperature superconductor (HTS) are expressed through a phase diagram [25–27] which looks like Fig. 1. In the absence of doping, the cuprate material remains anti-ferromagnetic (AFM) and insulating. As doping is increased, anti-ferromagnetism does not persist and at small doping concentration and below a temperature $T^*$ a gap opens up in the electronic energy spectrum, which is called a pseudo gap [28–30]. As doping is increased further, superconductivity (SC) starts to appear beyond this gap. The presence of a pseudo gap above $T_c$ and very small coherence length ($\sim 10$ Å) [31] indicates the formation of localized bosonic pairs of fermions (preformed pairs [26, 32, 33])

![Phase Diagram Of A Hole Doped High $T_c$ Superconductor](image)

FIG. 1. Phase Diagram Of A Hole Doped High $T_c$ Superconductor
below $T^*$ and their condensation below $T_c$. Based on this idea of preformed pairs, a phenomenological field-theoretic model of high temperature superconductivity was proposed by Friedberg and Lee \[34,35\]. In this field theory there are localized pairs, described by a bosonic field $\phi$ of charge $2e$ and mass $\sim 2m_e$, where $m_e$ is the mass of the electron. These bosons are unstable and decompose into pairs of electrons with opposite spins and these electrons recombine to form bosons. Thus in a large system there is always a macroscopic distribution of bosons coexisting with fermions following their respective statistical distribution laws. At temperature below $T_c$ these bosons condense, i.e. there is a large number of bosons in the zero momentum state which coexist with fermions. This type of system with bosons and fermions coexisting in thermal equilibrium is generically referred to as boson-fermion (B-F) model.

Apart from their use in models of high-$T_c$ superconductors \[36-40\], mixtures of bosons and fermions are studied in many other contexts as well, both experimental and theoretical. Experimental work on the properties of a mixture of Bose and Fermi gases include study of quantum degeneracy \[41-44\] and interactions \[45-47\]. Boson-fermion mixtures of dilute atomic and molecular gases at low temperatures are also studied theoretically in optical lattices to study their quantum phases including superfluid-insulator transition \[48-51\]. Boson-fermion systems also appear in studies of superconductor-insulator transitions \[52-54\], of BCS-BEC crossover \[55,56\], of charged Bose liquids \[57\], etc.

The boson-fermion model of Friedberg and Lee closely resembles the Abelian Higgs model \[58-61\] as a field theory, including the appearance of vortices \[35\]. These vortices carry quantized magnetic flux, as can be derived from the minimum energy condition. The Abelian Higgs model in 3+1 dimensions contains vortex lines \[62\] which are minimum energy solutions of the field equations with topologically nontrivial boundary conditions. The interaction between these lines, or strings, is mediated by a 2-form (2-index antisymmetric tensor) gauge potential $B_{\mu \nu}$ called the Kalb-Ramond field \[63\]. In the symmetry broken phase and when vortex strings are present, the Abelian Higgs model can be written in terms of the string world sheet and the 2-form gauge field $B_{\mu \nu}$ using dualization \[64\-68\]. Our goal for this work is to do this with fermions, i.e., to dualize the boson-fermion system in presence of vortices and reach what should be a useful starting point for the interaction of vortex lines with unpaired charged fermions.

Of particular interest is the coupling between the 2-form gauge field $B_{\mu \nu}$ and fermions.
In an earlier work it was proposed that the 2-form field couples nonlocally to a topologically conserved current of the electrons \[69\],

\[
\int d^4 x B_{\mu\nu} \frac{1}{\Box} J^{\mu\nu},
\]

where \(J^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} \partial_\rho (\bar{\psi} \gamma_\lambda \psi)\) is the 4-dimensional “curl” of the conserved fermion current. The nonlocal current \(\frac{1}{\Box} J^{\mu\nu}\), more specifically its \(\{0i\}\) component, contains the spin magnetic moment density as a contribution from the spin part. Then we can say that the coupling \(B_{0i} \frac{1}{\Box} J^{0i}\) corresponds to a spin-spin interaction mediated via the 2-form gauge field \(B\). If we find this interaction here, we will be able to say that we have found a local theory, namely that of the boson-fermion mixture, which has a description containing this “spin-gauge interaction”. The electrons interact via photons as well, and quantum corrections due to fermion loops give rise to an effective \(B \wedge F\) interaction. This term is central to the topological mass mechanism in 3+1 dimensions \[70\], analogous to the Chern-Simons term in 2+1 dimensions \[71–73\] which can also be generated by fermion loops \[74\]. We will be working with the Abelian Higgs model in the broken phase, in which the photon is already massive. Thus we expect that the mass of the photon will only be modified in this case. However, the \(B \wedge F\) interaction and its lower dimensional version, the mixed Chern-Simons action, have also been useful in theories of topological superconductors, topological insulators, and quantum Hall effect \[75–81\]. Our results should be useful for these systems.

The outline of our paper is as follows. In Sec. II we dualize the Abelian Higgs model in the presence of vortex lines (strings) and charged fermions which couple through electromagnetic interactions, culminating in a nonlocal dual Lagrangian involving strings and the 2-form field which mediates interstring interactions. In Sec. III we derive the effective action by taking into account 1-loop corrections due to fermion loops. This generates a \(B \wedge F\) interaction, which affects the propagators of both \(B_{\mu\nu}\) and \(A_\mu\). Then in Sec. IV we calculate the static potential between nonrelativistic fermions taking into account all the interactions as well as the 1-loop correction and end with some comments.

II. DUAL LAGRANGIAN FOR BOSON-FERMION SYSTEM

We first determine the dual of the field theory describing a boson-fermion system in the presence of vortices. Even though all particles in this system move non-relativistically, we
will work with a four dimensional relativistic field theory. This is because the field theoretic
duality we consider is most conveniently constructed in four dimensions and for relativistic
theories, and also because we will be able to use several standard results from usual quantum
electrodynamics.

We start with the Abelian Higgs model where the gauge field $A_\mu$ is minimally coupled to
unpaired charged fermions in addition to the complex scalar Higgs field. The Lagrangian of
our system is thus

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_\mu \phi^\dagger D^\mu \phi + \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi - V (\phi^\dagger \phi) - e A_\mu \bar{\psi} \gamma^\mu \psi, \quad (2.1)$$

where $\phi$ is a complex scalar field of charge $q$ (2e if $\phi$ describes Cooper pairs), $\psi$ is the fermionic
field with charge $e$, $D_\mu \phi = \partial_\mu + iq A_\mu \phi$ and $V (\phi^\dagger \phi) = \frac{\lambda}{4} (\phi^\dagger \phi - v^2)^2$ is the symmetry breaking
potential.

As is well known, topologically stable structures like vortices or flux tubes can appear
in this theory because the circle on which $\phi$ lies at its minimum is mapped on a circle at
infinity. If there is a vortex in a plane, corresponding to a flux tube cutting through the
plane as we will consider, the phase of $\phi$ becomes multivalued as we go around a circle at
infinity. The vacuum condition $D_\mu \phi = 0$ then leads to quantization of magnetic flux in the
vortex,

$$\int_C A_\mu dx^\mu = -\frac{2n\pi}{q}, \quad (2.2)$$

where $C$ is a circle at infinity and $n$ is the winding number, i.e., the number of times the
phase of $\phi$ winds around the vortex. It is a topological quantum number and describes
the quantization of topological charge, while $\frac{2\pi}{q}$ is the quantum of magnetic flux passing
through the vortex. To consider vortices explicitly we express $\phi$ in polar form,

$$\phi = v f \exp(i\chi). \quad (2.3)$$

The function $f$ vanishes along the core of the flux tube and reaches $f = 1$ far from the core
region. The Lagrangian, including a gauge-fixing term, then takes the form

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} v^2 \partial_\mu f \partial^\mu f + \frac{1}{2} \frac{v^2}{2} f^2 (\partial_\mu \chi + q A_\mu) (\partial^\mu \chi + q A^\mu)
- \frac{1}{2\xi} (\partial_\mu A^\mu)^2 - \frac{\lambda}{4} (f^2 - 1)^2 + \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi - e A_\mu \bar{\psi} \gamma^\mu \psi. \quad (2.4)$$

1 The zero of $f$ on a plane is the location of a vortex. A locus of zeroes in space defines a vortex string or
a flux tube.
We shall dualize the theory, starting from the partition function

\[ Z = \int D \! A_\mu \, D f \, D \chi \, D \bar{\psi} \, D \psi \exp \left( i \int d^4 x \, \mathcal{L} \right). \] (2.5)

The duality transformation takes us from the Higgs picture above, where the degrees of freedom are adequately described by a charged Higgs minimally coupled to the electromagnetic gauge field, to an equivalent vortex picture in which vortices interact through the second rank antisymmetric tensor Kalb-Ramond field. To implement this we first linearize the term \( \frac{1}{2} v^2 f^2 (\partial_\mu \chi + qA_\mu)^2 \) by introducing an auxiliary field through a Gaussian integral into the partition function as

\[ N \int D C_\mu \exp \left( -i \int d^4 x \left[ \frac{C_\mu}{\sqrt{2} v} + \frac{v}{\sqrt{2} f} (\partial_\mu \chi + qA_\mu) \right]^2 \right) = 1. \] (2.6)

Then we can write the partition function as

\[ Z = \int D \! A_\mu \, D f \, D \chi \, D \bar{\psi} \, D \psi \, D C_\mu \exp \left( i \int d^4 x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} v^2 \partial_\mu f \partial^\mu f - \frac{C_\mu C^\mu}{2 v^2} - C^\mu f (\partial_\mu \chi + qA_\mu) \right. \right. \]
\[ \left. \left. - \frac{1}{2 \xi} (\partial_\mu A^\mu)^2 - V(f^2) + \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi - e A_\mu \bar{\psi} \gamma^\mu \psi \right) \right). \] (2.7)

As mentioned earlier, the phase \( \chi \) is multivalued around a vortex string and the value of \( \chi \) changes by \( 2n\pi \) as one goes around the vortex string \( n \) times, \( n \) being the winding number. So we decompose \( \chi = \chi^r + \chi^s \), where the superscript \( s \) indicates the singular part of \( \chi \) describing a vortex configuration and \( r \) denotes the regular part which is single valued and corresponds to fluctuations around a given vortex configuration. By doing an integration by parts on the term \( C^\mu f \partial_\mu \chi^r \) we can shift the partial derivative onto \( C^\mu f \) and then integrate over \( \chi^r \) producing a delta function. Thus we can write

\[ Z = \int D \! A_\mu \, D f \, D \chi \, D \bar{\psi} \, D \psi \, D C_\mu \delta(\partial_\mu (C^\mu f)) \]
\[ \exp \left( i \int d^4 x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} v^2 \partial_\mu f \partial^\mu f - \frac{C_\mu C^\mu}{2 v^2} - C^\mu f (\partial_\mu \chi^s + qA_\mu) \right. \right. \]
\[ \left. \left. - \frac{1}{2 \xi} (\partial_\mu A^\mu)^2 - V(f^2) + \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi - e A_\mu \bar{\psi} \gamma^\mu \psi \right) \right). \] (2.8)

The delta function can be solved by introducing an antisymmetric tensor (2-form) potential
$B_{\mu\nu}$ and setting $C^\mu = \frac{1}{2f} \epsilon^{\mu\nu\rho\lambda} \partial_\nu B_{\rho\lambda}$, which allows us to write the partition function as

$$
Z = \int \mathcal{D}A_\mu \mathcal{D}f \mathcal{D}\chi^s \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}B_{\mu\nu} \exp \left( i \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} v^2 \partial_\mu f \partial^\mu f + \frac{1}{12 v^2 f^2} H^{\nu\rho\lambda} H_{\nu\rho\lambda} - \frac{1}{2f} (\partial_\mu A^\mu)^2 \\
- \frac{1}{2} \epsilon^{\mu\nu\rho\lambda} B_{\rho\lambda} \partial_\mu \partial_\nu \chi^s - \frac{q}{2} \epsilon^{\mu\nu\rho\lambda} \partial_\nu B_{\rho\lambda} A_\mu - V(f^2) + \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - eA_\mu \bar{\psi} \gamma^\mu \psi \right) \right).
(2.9)
$$

Here we have defined $H_{\nu\rho\lambda} = \partial_\nu B_{\rho\lambda} + \partial_\rho B_{\lambda\nu} + \partial_\lambda B_{\nu\rho}$ as the field strength of the 2-form field. The curl of the velocity of the scalar field (or supercurrent) is called vorticity, $\Sigma^{\rho\lambda} = \epsilon^{\mu\nu\rho\lambda} \partial_\mu \partial_\nu \chi^s$. Around a vortex this quantity is non-zero and in case of a straight rod like array of vortices, i.e. a vortex line or a flux tube, along the $Z$-axis, it is given by

$$
(\partial_x \partial_y - \partial_y \partial_x) \chi^s = 2n\pi \delta^2(\vec{r}).
(2.10)
$$

This expresses the location of the vortices in the $X - Y$ plane. By dualizing it we get the world sheet of the vortex line in 3+1 dimensions,

$$
\Sigma^{\rho\lambda} = \epsilon^{\mu\nu\rho\lambda} \partial_\mu \partial_\nu \chi^s = \int d\sigma_{\mu\nu} \delta(x - X).
(2.11)
$$

$X^\mu$ are the coordinates of the world sheet of the vortex line and $d\sigma_{\mu\nu} = d\tau ds \partial(X_\mu, X_\nu) / \partial(s, \tau)$ is the surface element over the world sheet. Thus we write the partition function as

$$
Z = \int \mathcal{D}A_\mu \mathcal{D}f \mathcal{D}\chi^s \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}B_{\mu\nu} \exp \left( i \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} v^2 \partial_\mu f \partial^\mu f + \frac{1}{12 v^2 f^2} H^{\nu\rho\lambda} H_{\nu\rho\lambda} - \frac{1}{2f} (\partial_\mu A^\mu)^2 \\
- \frac{1}{2} \epsilon^{\mu\nu\rho\lambda} B_{\rho\lambda} \partial_\mu \partial_\nu \chi^s - \frac{q}{2} \epsilon^{\mu\nu\rho\lambda} \partial_\nu B_{\rho\lambda} A_\mu - V(f^2) - \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - eA_\mu \bar{\psi} \gamma^\mu \psi \right) \right).
(2.12)
$$

Next we rename the currents $\frac{q}{2} \epsilon^{\mu\nu\rho\lambda} \partial_\nu B_{\rho\lambda} = J_H^\mu$ and $e\bar{\psi} \gamma^\mu \psi = J_\psi^\mu$, separate out the terms which depend on $A_\mu$ from the rest of the partition function and then integrate over $A_\mu$,

$$
\int \mathcal{D}A_\mu \exp \left( i \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2f} (\partial_\mu A^\mu)^2 + A_\mu (J_H^\mu + J_\psi^\mu) \right) \right) = \mathcal{N} \exp \left( -\frac{i}{2} \int d^4x d^4y (J_H^\mu + J_\psi^\mu) \Delta_{\mu\nu} (J_H^\nu + J_\psi^\nu) \right).
(2.13)
$$
Here $\Delta_{\mu\nu}$ is the Green function corresponding to the operator

$$\Delta^{-1}_{\mu\nu} = g^{\mu\nu} \Box - (1 - \frac{1}{\xi}) \partial^\mu \partial^\nu. \quad (2.14)$$

In momentum space it is given by

$$\Delta_{\mu\nu}(k) = -\frac{g^{\mu\nu} - (1 - \xi) \frac{k^\mu k^\nu}{k^2}}{k^2 + i\epsilon}. \quad (2.15)$$

We will suppress the $+i\epsilon$ in what follows, but it is present in each of the propagators appearing below. The integration over $A_\mu$ has produced a normalization factor $\mathcal{N}$ which does not contribute to the rest of the partition function. We can thus write the action as

$$S = \int d^4x \left( \frac{1}{2} v^2 \partial_\mu f \partial^\mu f + \frac{1}{12v^2f^2} H^{\mu\rho\lambda} H_{\nu\rho\lambda} - \frac{1}{2} B_{\rho\lambda} \Sigma^{\rho\lambda} - V(f^2) + \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \right)$$

$$- \frac{1}{2} \int d^4x d^4y \left( J^\mu_H(x) \Delta_{\mu\nu}(x,y) J^\nu_H(y) \right). \quad (2.16)$$

We can further simplify the last term. Note that since $J^\mu_H$ is a (topologically) conserved current, the second term in $\Delta_{\mu\nu}$ annihilates it, so we can write

$$\frac{1}{2} \int d^4x d^4y J^\mu_H(x) \Delta_{\mu\nu}(x,y) J^\nu_H(y) = -\int d^4x \frac{q^2}{12} H_{\nu\rho\lambda} \frac{1}{12} H^{\nu\rho\lambda}, \quad (2.17)$$

as well as

$$\int d^4x d^4y J^\mu_H(x) \Delta_{\mu\nu}(x,y) J^\nu_H(y) = \int d^4x \frac{1}{2} eq B^{\mu\nu} \varepsilon_{\mu\nu\rho\lambda} \partial^\rho \frac{1}{\Box} \bar{\psi} \gamma^\lambda \psi. \quad (2.18)$$

In order to understand the remaining part, which is quadratic in the fermion current $J^\mu_\psi$, we note that it is exactly what we would get if we integrate over $A_\mu$ ordinary quantum electrodynamics, i.e.

$$\int \mathcal{D}A_\mu \exp \left( \frac{i}{2} \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu J^\mu_\psi + \frac{1}{2\xi} (\partial_\mu A^\mu)^2 \right) \right)$$

$$= \mathcal{N}_0 \exp \left( -\frac{i}{2} \int d^4x d^4y J^\mu_\psi(x) \Delta_{\mu\nu}(x,y) J^\nu_\psi(y) \right), \quad (2.19)$$

where $\mathcal{N}_0$ is a normalization factor.

Thus after collecting all these terms, we can write the dual Lagrangian as

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - eA_\mu \bar{\psi} \gamma^\mu \psi - \frac{1}{2} eq B^{\mu\nu} \varepsilon_{\mu\nu\rho\lambda} \partial^\rho \frac{1}{\Box} \bar{\psi} \gamma^\lambda \psi + \frac{1}{2} \psi^2 \partial_\mu f \partial^\mu f$$

$$+ \frac{1}{12v^2} H^{\mu\rho\lambda} \left( \frac{1}{f^2} + \frac{q^2 v^2}{\Box} \right) H_{\nu\rho\lambda} - \frac{1}{2} B_{\rho\lambda} \Sigma^{\rho\lambda} - V(f^2), \quad (2.20)$$
where we have suppressed gauge-fixing terms for $A_\mu$ or $B_{\mu\nu}$. Thus starting from a system containing vortex strings and described by an Abelian Higgs model in the broken phase in which charged fermions are also present, we have arrived at the dual Lagrangian of Eq. (2.20) in which the Kalb-Ramond field $B_{\mu\nu}$ couples to a topologically conserved nonlocal tensor current,

$$J^{\mu\nu} = \frac{1}{2} q \varepsilon^{\mu\nu\rho\lambda} \Box^{-1} \partial_{\rho} J_{\lambda},$$

(2.21)

with $J^\mu$ being the conserved electron current. The conserved charge density $J^{0i}$ for this current can be split into orbital and spin parts, with the spin contribution for nonrelativistic electrons being the intrinsic spin density of the electron,

$$(J^{0i})_{\text{spin}}^{\text{NR}} \propto \bar{\psi} \sigma^i \psi,$$

(2.22)

up to dimensionful constants, when the charge is time-independent and cannot accumulate. Thus in other words we have a gauge theory in which the gauge potential mediating string-string interaction couples to the spin current of charged fermions.

The Lagrangian of Eq. (2.20) is invariant, not only with respect to the usual gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$ for arbitrary real functions $\lambda$, but also under the vector (or extended or higher or Kalb-Ramond) gauge transformation $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$, provided

$$\partial_\mu \Sigma^{\mu\nu} = 0.$$  

(2.23)

This shows that vortex lines must either form closed loops or be infinitely long (or end at the boundaries of the superconducting region) as the world-sheet current is conserved by itself. This is of course expected as magnetic field lines must either close on themselves or go out to infinity, since there are no magnetic monopoles.

III. INDUCED $B \wedge F$ TERM

In order to see the effect of the nonlocal coupling on the boson-fermion system, let us calculate the quantum corrections at one fermion loop. We will do this by first setting $f \rightarrow 1$, which corresponds to the limit of the flux tubes being very thin. We also redefine $\frac{1}{v} B_{\mu\nu}$ as $B_{\mu\nu}$ for convenience of calculations.
The partition function then becomes
\[
\mathcal{Z} = \int \mathcal{D} \chi \mathcal{D} \bar{\psi} \mathcal{D} \psi \mathcal{D} B_{\mu\nu} \mathcal{D} A_{\mu} \\
\exp \left( i \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - eA_{\mu}\bar{\psi}\gamma^\mu\psi \right) \right. \\
\left. - \frac{1}{2} eMB^{\mu\nu} \varepsilon_{\mu\nu\rho\lambda} \partial^\rho \bar{\psi}\gamma^\lambda\psi + \frac{1}{12} H_{\nu\rho\lambda}(1 + M^2 \Box^{-1}) H^{\nu\rho\lambda} - \frac{v}{2} B_{\rho\lambda} \Sigma^{\rho\lambda} \right) \right), \tag{3.1}
\]
where we have written \( M = qv \). The nonlocal term involving \( M^2 \Box^{-1} \) is a mass term for \( B_{\mu\nu} \) and is sometimes called a Meissner term for that reason \[82 \; \text{[106]}\].

The nonlocal interaction term between the \( B_{\mu\nu} \)-field and the fermion can be written as
\[
\int d^4x \frac{1}{2} eMB^{\mu\nu} \varepsilon_{\mu\nu\rho\lambda} \partial^\rho \bar{\psi}\gamma^\lambda\psi = \int d^4x \frac{1}{2} eM \varepsilon_{\mu\nu\rho\lambda} \bar{\psi}\gamma^\mu\psi \partial^\nu B^{\rho\lambda}, \tag{3.2}
\]
as can be seen directly from where it first appeared in Eq. \[2.18\]. For convenience of calculations let us now define an “effective gauge field” \( A_{\mu}^{\text{eff}} \) as \( A_{\mu}^{\text{eff}} = A_{\mu} + M\Box^{-1}F_{\mu} \) where we have written \( F_{\mu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\lambda} \partial^\nu B^{\rho\lambda} \). Quantum corrections to the action due to fermion loops are calculated in the standard textbook method \[84 \; \text{[107]}\]: We expand \( \psi = \psi_0 + \eta \), where \( \psi_0 \) is a solution of the equation of motion \( \frac{\delta S}{\delta \psi} \bigg|_{\psi=\psi_0} = 0 \), similarly for \( \bar{\psi}_0 \), then integrate over \( \eta, \bar{\eta} \) to first order in \( e^2 \) for one loop,
\[
\int \mathcal{D} \bar{\eta} \mathcal{D} \eta \exp \left( i \int d^4x \eta \left( i\gamma^\mu \partial_\mu - m - e\gamma^\mu A_{\mu}^{\text{eff}} \right) \eta \right) \\
\sim \exp \left[ -\frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \Pi(k^2) A_{\mu}^{\text{eff}}(-k) \left( g^{\mu\nu}k^2 - k^\mu k^\nu \right) A_{\nu}^{\text{eff}}(k) \right]. \tag{3.3}
\]
Here \( A_{\mu}^{\text{eff}} \) is defined as before, while \( \Pi(p^2) \) includes the effect of modes up to a cutoff \( \Lambda \) and is given by
\[
\Pi(k^2) = \frac{e^2}{2\pi^2} \int_0^1 z(1-z) \left[ \ln \left( 1 + \frac{\Lambda^2}{m^2 - k^2z(1-z)} \right) - \frac{\Lambda^2}{\Lambda^2 + m^2 - k^2z(1-z)} \right], \tag{3.4}
\]
for a cutoff \( \Lambda \). The natural cutoff scale in the system of vortices and electrons is the thickness of the vortex string. This is typically comparable to the atomic scale, so we set \( \Lambda \ll m \) (also \( |k^2| \ll m^2 \)) to find at the leading order
\[
\Pi(k^2) = \frac{e^2}{24\pi^2} \frac{\Lambda^4}{m^4} + \cdots, \tag{3.5}
\]
ignoring terms of the order of \( \frac{\Lambda^6}{m^8}, \frac{\Lambda^4k^2}{m^{10}}, \frac{\Lambda^2k^4}{m^6} \). We can now write the Lagrangian including the loop correction as
\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{12} H_{\nu\rho\lambda}(1 + M^2 \Box^{-1}) H^{\nu\rho\lambda} - \frac{v}{2} B_{\rho\lambda} \Sigma^{\rho\lambda} - \frac{e^2}{24\pi^2} \frac{\Lambda^4}{m^4} \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right), \tag{3.6}
\]
where we have suppressed terms involving $\psi_0$. Recalling the definition of $A_{\mu}^{\text{eff}}$, we can write

$$F_{\mu\nu}^{\text{eff}_\mu}F_{\rho\sigma}^{\text{eff}_\rho} = F_{\mu\nu}F_{\rho\sigma} - M\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}B_{\mu\nu} + \frac{1}{3}M^2H_{\mu\nu\lambda}\square H_{\mu\nu\lambda},$$

(3.7)

after taking into account an integration by parts. Writing \( Z = \frac{e^2}{24\pi^2}\Lambda^4/m_4 \), we rewrite the Lagrangian as

$$L = -\frac{1}{4}(1 + Z)F_{\mu\nu}F_{\rho\sigma} + \frac{1}{12}H^{\mu\rho\lambda}\left(1 + (1 - Z)\frac{M^2}{\square}\right)H_{\mu\rho\lambda} + \frac{1}{4}ZM\epsilon_{\mu\nu\rho\lambda}F_{\mu\nu}B_{\rho\lambda}$$

$$- \frac{v}{2}B_{\rho\lambda}\Sigma_{\rho\lambda} + \bar{\psi}_0(i\gamma^\mu\partial_\mu - m)\psi_0 - eA_\mu\bar{\psi}_0\gamma^\mu\psi_0 - \frac{1}{2}eMB_{\mu\nu}\epsilon_{\mu\nu\rho\lambda}\partial^\rho\bar{\psi}_0\gamma^\lambda\psi_0. \tag{3.8}$$

If we now rescale $A_\mu \to \sqrt{1 + Z}A_\mu$ and also define the “renormalized charge” $e_R^2 = e^2(1 + Z)^{-1} \simeq e^2(1 - Z)$, we obtain the Lagrangian in the form

$$L = -\frac{1}{4}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{12}H^{\mu\rho\lambda}\left(1 + \frac{M_R^2}{\square}\right)H_{\mu\rho\lambda} + \frac{1}{4}ZM_R\epsilon_{\mu\nu\rho\lambda}F_{\mu\nu}B_{\rho\lambda} - \frac{v}{2}B_{\rho\lambda}\Sigma_{\rho\lambda}$$

$$+ \bar{\psi}_0(i\gamma^\mu\partial_\mu - m)\psi_0 - e_RA_\mu\bar{\psi}_0\gamma^\mu\psi_0 - \frac{1}{2}e_RM_RB_{\mu\nu}\epsilon_{\mu\nu\rho\lambda}\partial^\rho\bar{\psi}_0\gamma^\lambda\psi_0. \tag{3.9}$$

Here we have written $M_R^2 = M^2(1 - Z) \simeq \frac{q^2v^2}{(1 + Z)}$ as the “renormalized mass” of the gauge boson. Note that we can write $M_R = q_Rv$ since all electric charges should be renormalized the same way, so that $q_R = \frac{q}{\sqrt{1 + Z}}$. There is also an induced $B \wedge F$ term with a coefficient which depends on the cutoff $\Lambda$.

The coefficient of the induced $B \wedge F$ term is very small and depends on the cutoff, which is in turn determined by the properties of the system. As we shall see now, this term will increase the mass of the gauge fields $A_\mu$ or $B_{\mu\nu}$. By the mass of the gauge fields we mean the pole of the propagator of the relevant field, which can be calculated either by summing an infinite series or by taking only the part of the Lagrangian quadratic in the fields and eliminating one gauge field in favor of the other. Let us use the second method to find the poles in the propagators, starting with $B_{\mu\nu}$. We separate out the quadratic terms containing $A_\mu$,

$$Z_A = \int \mathcal{D}A_\mu \exp \left( i \int d^4x \left( \frac{1}{2}A_\mu K^{\mu\nu}_A A_\nu + \frac{1}{4}ZM_R\epsilon_{\mu\nu\rho\lambda}F_{\mu\nu}B_{\rho\lambda} \right) \right), \tag{3.10}$$

where $K_{\mu\nu}$ is the invertible operator $g_{\mu\nu}\square - (1 - \frac{1}{\xi})\partial_\mu \partial_\nu$. We complete the square and write

$$Z_A = \left( \int \mathcal{D}A'_\mu \exp \left( i \int d^4x \frac{1}{2}A'_\mu K^{\mu\nu}_A A'_\nu \right) \exp \left( i \int d^4x \left( \frac{Z^2M_R^2}{12}H^{\sigma\rho\lambda}_A \square H_{\sigma\rho\lambda} \right) \right) \right). \tag{3.11}$$
The integration over $A'_\mu$ provides a normalization factor while the second term gets added to the Lagrangian. The mass term of $B_{\mu\nu}$ becomes \( \frac{1}{12}(1 + Z^2)M_R^2 H^{\mu\rho\lambda} \Box^{-1} H_{\nu\rho\lambda} \), resulting in a shift of the coefficient of the Meissner term so that $M_B = M_R \sqrt{1 + Z^2}$ is the new mass of $B$. Thus the mass of the interstring gauge potential $B_{\mu\nu}$ increases because of quantum effects due to fermion loops.

Let us now see what happens to the propagator of $A_\mu$ if we integrate out $B_{\mu\nu}$ instead. We start from

\[
Z_B = \int \mathcal{D}B_{\mu\nu} \exp \left( -i \int d^4x \left( \frac{1}{4} B_{\mu\nu} M^{\mu\nu\rho\lambda} B_{\rho\lambda} - \frac{1}{4} Z M_R \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu} F_{\rho\lambda} \right) \right),
\]

where for convenience we have written

\[
M^{\mu\nu\rho\lambda} = (\Box + M_R^2) g^{\mu\rho} g^{\lambda\nu} + \left( 1 + M_R^2 \Box^{-1} - \frac{1}{\eta} \right) \left( g^{\mu\rho} g^{\lambda\nu} \partial_\sigma \partial^\mu - g^{\mu\rho} g^{\lambda\nu} \partial_\sigma \partial^\nu \right). \tag{3.12}
\]

We can now perform the integral by completing the square and find

\[
Z_B = N' \exp \left( -\frac{i}{4} \int d^4x d^4y Z^2 M_R^2 F_{\mu\nu} \left( \frac{1}{\Box + M_R^2} F^{\mu\nu} \right) \right). \tag{3.14}
\]

This is added to the action for $A_\mu$, so that the quadratic term in the Lagrangian becomes

\[
- \frac{1}{4} F_{\mu\nu} \left( 1 + \frac{Z^2 M_R^2}{\Box + M_R^2} \right) F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A_\mu)^2. \tag{3.15}
\]

A similar expression was found in three dimensions in the closely related context of the disorder field \[86\] in a superconductor, but in the absence of charged fermions. The conclusion there was that at high temperatures, vortex lines proliferate and Meissner effect is destroyed, leaving a long range Coulomb-like interaction with a complicated dispersion relation.

In our case, charged fermions are present and Meissner effect has not been destroyed. The propagator of the field $A_\mu$ is

\[
G_{\mu\nu} = - \left( \frac{g_{\mu\nu}}{p^2 - M_R^2} + \frac{p_\mu p_\nu}{p^4} \frac{p^2 (\xi - 1) - M_R^2 (\xi (Z^2 + 1) - 1)}{p^2 - M_R^2 (1 + Z^2)} \right). \tag{3.16}
\]

The second term in the Green function will disappear when a conserved current couples to it, while the first term can be written as

\[
G_{\mu\nu} = - \left( \frac{1}{1 + Z^2} \frac{g_{\mu\nu}}{p^2} + \frac{Z^2}{1 + Z^2} \frac{g_{\mu\nu}}{p^4} \right). \tag{3.17}
\]

Unlike the usual mechanism of topological mass generation using a $B \wedge F$ interaction where the massive $B_{\mu\nu}$ field is ‘dual’ to the massive $A_\mu$ field in the sense that the propagating degrees of freedom can be described equally well by either field, the boson-fermion system shows a distinction between the two alternative descriptions in presence of vortices.
Another window to the physics of the system is provided by the force law as experienced by the charged fermions, which we proceed to derive now. We will find it by integrating out both the gauge fields $A_\mu$ and $B_{\mu\nu}$ and then taking the nonrelativistic limit for the fermionic currents. We start by integrating out $B_{\mu\nu}$ from the Lagrangian of Eq. (3.9), which includes corrections up to one fermion loop,

$$Z_B = \int \mathcal{D}B_{\mu\nu} \exp \left(-i \int d^4x \left( \frac{1}{4} B_{\mu\nu} M^{\mu\nu\rho\lambda} B_{\rho\lambda} - \frac{1}{2} B_{\mu\nu} J^{\mu\nu} \right) \right),$$

(4.1)

where we have written $J^{\mu\nu} = \frac{1}{2} Z M R \epsilon_{\mu\nu\rho\lambda} F^{\rho\lambda} - v \Sigma_{\mu\nu} - e R M R \epsilon_{\mu\nu\rho\lambda} \partial^{\rho}(\Box^{-1}) J^{\lambda}$, with the fermion current being $J^{\lambda} = \bar{\psi}_0 \gamma^{\lambda} \psi_0$. Integrating over $B_{\mu\nu}$ we get

$$Z_B \sim \exp \left( -\frac{i}{4} \int d^4x d^4y \left(-Z^2 M^2_R F^{\mu\nu} \frac{1}{\Box + M^2_R} F^{\mu\nu} - 4 Z e R M^2_R A_{\mu} \frac{1}{\Box + M^2_R} J^{\mu} \right) \right),$$

(4.2)

The first two terms and fifth term in the integral contribute to the action of $A_\mu$, which is now integrated over to get the force law. The integral over $A_\mu$ now reads

$$Z_A = \int \mathcal{D}A_{\mu} \exp \left( -\frac{i}{4} \int d^4x \left[ -\frac{1}{4} F^{\mu\nu} \left( 1 + \frac{Z^2 M^2_R}{\Box + M^2_R} \right) F^{\mu\nu} - \frac{1}{2s} (\partial_{\mu} A^{\mu})^2 \right. \right.

$$

$$\left. - A_{\mu} \left( e R J^{\mu} + e R Z M^2_R J^{\mu} + v Z M R \frac{1}{\Box + M^2_R} K^{\mu} \right) \right] \right),$$

(4.3)

where we have written $K^{\mu} = \frac{1}{2} \epsilon^{\mu\rho\lambda\sigma} \partial_{\rho} \Sigma_{\sigma}$. Integration over $A_\mu$ produces

$$Z_A \sim \exp \left( \frac{i}{2} \int d^4x d^4y \tilde{J}^\mu(x) G_{\mu\nu}(x-y) \tilde{J}^{\nu}(y) \right),$$

(4.4)

where $G_{\mu\nu}$ is the Green function calculated in Eq. (3.16) and $\tilde{J}^\mu = \left( 1 + \frac{Z M^2_R}{\Box + M^2_R} \right) e R J^{\mu} + v Z M R \frac{1}{\Box + M^2_R} K^{\mu}$. To get the net interaction potential between fermions we now add the third term of Eq. (4.2) to the above integral and convert the result to momentum space. Finally we are left with the effective current-current interaction

$$\frac{e^2_R}{2} \int \frac{d^4p}{(2\pi)^4} J^{\mu}(-p) \left( \frac{(1-Z)^2}{1+Z^2} \frac{1}{p^2 - M^2_R(1+Z^2)} + \frac{2Z}{1+Z^2} \frac{1}{p^2} \right) J^{\mu}(p).$$

(4.5)
This represents, for the current of non-relativistic fermions, a Yukawa potential in the leading order along with a very small Coulomb correction. The expression (4.3) also includes the vortex-vortex and vortex-fermion interaction terms and combining them with the relevant terms in $Z_B$ we get

$$-\frac{v^2}{4} \int \frac{d^4 p}{(2\pi)^4} \Sigma_{\mu\nu}(-p) \frac{1}{p^2 - M_R^2(1 + Z^2)} \Sigma_{\mu\nu}^\prime(p),$$

which gives the interaction between two vortex lines and

$$\frac{ie_R(1 - Z)M_R}{2} \epsilon_{\mu\nu\rho\lambda} \int \frac{d^4 p}{(2\pi)^4} \Sigma_{\mu\nu}(-p) \frac{1}{p^2(p^2 - M_R^2(1 + Z^2))} p_\rho J_\lambda(p),$$

which gives the vortex-fermion interaction. Since in the non-relativistic static limit we can write $\epsilon^{0ijk} \partial_j \frac{1}{\Box} J_k \sim S^i$, the spin magnetic moment density of a static electron, the above expression gives an effective vortex-spin interaction.

V. CONCLUSION

In this paper we analyzed the interaction of vortices in an Abelian Higgs model with charged fermions. This may be thought of as a field theoretic description of a type II superconductor with thin tubes of magnetic flux, in which unpaired electrons coexist with the charged pairs and interact electromagnetically through their minimal coupling with the photon. Dual formulation of the system using the four dimensional relativistic theory leads to a nonlocal interaction term between the antisymmetric tensor field and fermions, equivalent to a gauge field coupled to the spin density current of the fermions. This provides a post-facto justification of working with a relativistic formulation in four dimensions, for the spin of fermions appears naturally in it.

One motivation of this work was to see if the vortex-electron interaction could give rise, in the dual picture, to the nonlocal coupling of the two-form field with electrons proposed earlier in [69]. We found this, as an ‘emergent’ interaction involving the spin current of the electrons that does not appear in the original way of writing the model but emerges in the process of dualization. Often the dual picture of the Abelian Higgs model in the context of a type II superconductor is studied as a nonrelativistic field theory (often in two spatial dimensions), leading to the disorder field [86] [87], analogous to the antisymmetric tensor potential. We note however that since spin has to be introduced by hand in a non-
relativistic theory, this interaction with the spin current would not have emerged from the non-relativistic field theory calculations usually done for superconductors.

We have also found, as had been shown earlier, that this interaction generates a $B \wedge F$ term in one-loop effective action. This increases the mass of both gauge fields $A_\mu$ and $B_{\mu\nu}$, which should decrease the penetration depth. It was also shown earlier that the nonlocal interaction gives rise to a linear attractive potential between two non-relativistic fermions and thus spatially localized fermion pairs would appear. We will investigate this phenomenon in future work. Here we have found something else – a Coulomb potential between two charges, corresponding to a medium with a very high dielectric constant $\kappa \sim (2Z)^{-1}$. Thus we see from general principles that a material, otherwise a type II superconductor, will gain characteristics of a dielectric if unpaired electrons appear in it.

We have also found the general forms of vortex-vortex and fermion-vortex interactions for this system. It should be possible to reduce our calculations to 2+1 dimensions and find the effective vortex-fermion, fermi-fermi, and vortex-vortex interactions in planar type II superconductors with unpaired electrons. It is also possible to consider temperature dependence of the coupling constants and investigate critical phenomena in the vortex-electron system in the dual picture presented in this paper. We leave these for future work.

[1] G. W. Rayfield and F. Reif. Quantized vortex rings in superfluid helium. *Phys. Rev.*, 136:A1194, 1964.
[2] V. P. Mineyev and G. E. Volovik. Planar and linear solitons in superfluid $^3$He. *Phys. Rev. B*, 18:3197, 1978.
[3] P. J. Hakonen, O. T. Ikkala, and S. T. Islander. Experiments on vortices in rotating superfluid $^3$He-a. *Phys. Rev. Lett.*, 49:1258, 1982.
[4] M. M. Salomaa and G. E. Volovik. Quantized vortices in superfluid $^3$He. *Rev. Mod. Phys.*, 59:533, 1987.
[5] Gregory P. Bewley, Daniel P. Lathrop, and Katepalli R. Sreenivasan. Visualization of quantized vortices. *Nature*, 441:588, 2006.
[6] T. W. B. Kibble. Topology of Cosmic Domains and Strings. *J. Phys.*, A9:1387–1398, 1976.
[7] Alexander Vilenkin. Cosmic Strings and Domain Walls. *Phys. Rept.*, 121:263–315, 1985.
[8] M. B. Hindmarsh and T. W. B. Kibble. Cosmic strings. *Rept. Prog. Phys.*, 58:477–562, 1995.

[9] A. Vilenkin and E. P. S. Shellard. *Cosmic Strings and Other Topological Defects*, Cambridge University Press, 2000.

[10] Yoichiro Nambu. Strings, Monopoles and Gauge Fields, *Phys. Rev.* D10:4262, 1974. [310(1974)].

[11] Gerard ’t Hooft. Magnetic Monopoles in Unified Gauge Theories, *Nucl. Phys.* B79:276, 1974. [291(1974)].

[12] S. Mandelstam. Vortices and Quark Confinement in Nonabelian Gauge Theories, *Phys. Rept.* 23:245, 1976.

[13] A. A. Abrikosov. On the magnetic properties of superconductors of the second group, *Soviet Physics JETP* 5:1174, 1957.

[14] A. A. Abrikosov. The magnetic properties of superconducting alloys, *Journal of Physics and Chemistry of Solids* 2:199, 1957.

[15] J Sutton. Electron tunnelling evidence for fluxon entry into thin type II superconducting films, *Proceedings of the Physical Society* 87:791, 1966.

[16] U. Essmann and H. Truble, The direct observation of individual flux lines in type II superconductors, *Physics Letters A* 24:526, 1967.

[17] H. F. Hess, R. B. Robinson, R. C. Dynes, J. M. Valles, and J. V. Waszczak, Scanning-tunneling-microscope observation of the Abrikosov flux lattice and the density of states near and inside a fluxoid, *Phys. Rev. Lett.* 62:214, 1989.

[18] G. Blatter, M. V. Feigel’man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Vortices in high-temperature superconductors, *Rev. Mod. Phys.* 66:1125, 1994.

[19] P. L. Gammel, D. J. Bishop, G. J. Dolan, J. R. Kwo, C. A. Murray, L. F. Schneemeyer, and J. V. Waszczak, Observation of hexagonally correlated flux quanta in YBa2Cu3O7, *Phys. Rev. Lett.* 59:2592, 1987.

[20] L.Ya. Vinnikov, L.A. Gurevich, G.A. Yemelchenko, and Yu.A. Ossipyan, Direct observation of the lattice of Abrikosov vortices in high-$T_c$ superconductor YBa2Cu3Ox single crystals, *Solid State Communications* 67:421, 1988.

[21] J. G. Bednorz and K. A. Muller. Possible high $T_c$ superconductivity in the Ba-La-Cu-O system. *Z. Phys.* B64:189, 1986.

[22] J. Bardeen, L. N. Cooper, and J. R. Schrieffer. Theory of superconductivity. *Phys. Rev.*
[23] J. Bardeen, L. N. Cooper, and J. R. Schrieffer. Microscopic theory of superconductivity. *Phys. Rev.* 106:162, 1957.

[24] Leon N. Cooper, Bound electron pairs in a degenerate Fermi gas, *Phys. Rev.*, 104:1189, 1956.

[25] D. J. Scalapino, The cuprate pairing mechanism, *Science* 284:1282, 1999.

[26] Q. Chen, J. Stajic, S. Tan, and K. Levin, BCSBEC crossover: From high temperature superconductors to ultracold superfluids, *Physics Reports* 412:1, 2005.

[27] Samuel Bieri. Resonating-valence-bond approaches to high-temperature superconductivity. *EPFL thesis*, 2008.

[28] Mohit Randeria and Nandini Trivedi. Pairing correlations above tc and pseudogaps in under-doped cuprates. *Journal of Physics and Chemistry of Solids*, 59(10):1754 – 1758, 1998.

[29] Tofik Mamedov and Manuel de Llano. Superconducting pseudogap in a bosonfermion model. *Journal of the Physical Society of Japan*, 79(4):044706, 2010.

[30] Bin-Quan Luan, Jian-Xin Li, and Chang-De Gong. Preformed pairs induced pseudogap behavior in high-$T_c$ cuprates. *Phys. Rev. B*, 64:064503, Jul 2001.

[31] P. Chaudhari, R. T. Collins, P. Freitas, R. J. Gambino, J. R. Kirtley, R. H. Koch, R. B. Laibowitz, F. K. LeGoues, T. R. McGuire, T. Penney, Z. Schlesinger, Armin P. Segmüller, S. Foner, and E. J. McNiff. Properties of epitaxial films of YBa$_2$Cu$_3$O$_{7-\delta}$. *Phys. Rev. B*, 36:8903–8906, Dec 1987.

[32] V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin. Superconductivity in a system with preformed pairs. *Phys. Rev. B*, 55:3173–3180, Feb 1997.

[33] Y. I. Seo, W. J. Choi, Shin-ichi Kimura, and Yong Seung Kwon. Evidence for a preformed Cooper pair model in the pseudogap spectra of a Ca$_{10}$(Pt$_4$As$_8$)(Fe$_2$As$_2$)$_5$ single crystal with a nodal superconducting gap. *Scientific Reports*, 9:3987, Mar 2019.

[34] R. Friedberg and T. D. Lee, Gap energy and long-range order in the boson-fermion model of superconductivity. *Phys. Rev. B*, 40:6745, 1989.

[35] R. Friedberg, T. D. Lee, and H. C. Ren, Coherence length and vortex filament in the boson-fermion model of superconductivity. *Phys. Rev. B*, 42:4122, 1990.

[36] J. Ranninger and J. M. Robin, The boson-fermion model of high-$T_c$ superconductivity. Doping dependence, *Physica C: Superconductivity* 253:279, 1995.

[37] E. Piegari and S. Caprara, Superconducting transition in a mixture of bosons and fermions.
[38] R. Micnas, S. Robaszkiewicz, and A. Bussmann-Holder, Anisotropic superconductivity in systems with coexisting electrons and local pairs, *Phys. Rev. B* 66:104516, 2002.

[39] G. Pawłowski, R. Micnas, and S. Robaszkiewicz, Effects of disorder on superconductivity of systems with coexisting itinerant electrons and local pairs. *Phys. Rev. B*, 81:064514, 2010.

[40] P. Salas, M. Fortes, M.A. Sols, and F.J. Sevilla, Specific heat of underdoped cuprate superconductors from a phenomenological layered bosonfermion model. *Physica C: Superconductivity and its Applications*, 524:37 – 43, 2016.

[41] F. Schreck, L. Khaykovich, K. L. Corwin, G. Ferrari, T. Bourdel, J. Cubizolles, and C. Salomon, Quasipure Bose-Einstein Condensate Immersed in a Fermi Sea, *Phys. Rev. Lett.* 87:080403, 2001.

[42] A. G. Truscott, K. E. Strecker, W. I. McAlexander, G. B. Partridge, and R. G. Hulet, Observation of Fermi Pressure in a Gas of Trapped Atoms, *Science*, 291:2570, 2001.

[43] Z. Hadzibabic, C. A. Stan, K. Dieckmann, S. Gupta, M. W. Zwierlein, A. Görlitz, and W. Ketterle, Two-Species Mixture of Quantum Degenerate Bose and Fermi Gases, *Phys. Rev. Lett.*, 88:160401, 2002.

[44] G. Roati, F. Riboli, G. Modugno, and M. Inguscio, Fermi-Bose Quantum Degenerate $^{40}$K-$^{87}$Rb Mixture with Attractive Interaction *Phys. Rev. Lett.*, 59:150403, 2002.

[45] J. Goldwin, S. Inouye, M. L. Olsen, B. Newman, B. D. DePaola, and D. S. Jin, Measurement of the interaction strength in a Bose-Fermi mixture with $^{87}$Rb and $^{40}$K, *Phys. Rev. A* 70:021601(R), 2004.

[46] C.-H. Wu, I. Santiago, J. W. Park, P. Ahmadi, and M. W. Zwierlein, Strongly interacting isotopic Bose-Fermi mixture immersed in a Fermi sea, *Phys. Rev. A* 84:011601(R), 2011.

[47] J. W. Park, C.-H. Wu, I. Santiago, T. G. Tiecke, S. Will, P. Ahmadi, and M. W. Zwierlein Quantum degenerate Bose-Fermi mixture of chemically different atomic species with widely tunable interactions *Phys. Rev. A* 85:051602(R), 2012.

[48] M. Lewenstein, L. Santos, M. A. Baranov, and H. Fehrmann, Atomic Bose-Fermi Mixtures in an Optical Lattice, *Phys. Rev. Lett.*, 92:050401, 2004.

[49] M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen(De), and U. Sen, Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond, *Advances in Physics*, 56:2, 243-379, 2007.
[50] F. Illuminati and A. Albus, High-Temperature Atomic Superfluidity in Lattice Bose-Fermi Mixtures *Phys. Rev. Lett.* 93:090406, 2004.

[51] K. Yang, Superfluid-insulator transition and fermion pairing in Bose-Fermi mixtures, *Phys. Rev. B* 77:085115, 2008.

[52] M. Cuoco and J. Ranninger, Superconductor-insulator transition driven by local dephasing *Phys. Rev.* B70:104509, 2004.

[53] Y. Dubi, Y. Meir, and Y. Avishai, Nature of the superconductor-insulator transition in disordered superconductors, *Nature* 449:876, 2007.

[54] Y. L. Loh, M. Randeria, N. Trivedi, C.-C. Chang, Chia-Chen and R. Scalettar, Superconductor-Insulator Transition and Fermi-Bose Crossovers, *Phys. Rev. X* 6:021029, 2016.

[55] Jian Deng, Andreas Schmitt, and Qun Wang, Relativistic BCS-BEC crossover in a boson-fermion model. *Phys. Rev. D*, 76:034013, 2007.

[56] M. Maska and N. Trivedi, Temperature-driven BCS-BEC crossover in a coupled boson-fermion system, arxiv: 1706.04197.

[57] V. V. Kabanov and A. S. Alexandrov, Vortex matter in the charged Bose liquid at absolute zero, *Phys. Rev. B* 71:132511, 2005.

[58] Peter W. Higgs. Broken Symmetries and the Masses of Gauge Bosons. *Phys. Rev. Lett.,* 13:508–509, 1964. [,160(1964)].

[59] Peter W. Higgs. Spontaneous Symmetry Breakdown without Massless Bosons. *Phys. Rev.,* 145:1156–1163, 1966.

[60] F. Englert and R. Brout. Broken Symmetry and the Mass of Gauge Vector Mesons. *Phys. Rev. Lett.,* 13:321–323, 1964. [,157(1964)].

[61] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble. Global Conservation Laws and Massless Particles. *Phys. Rev. Lett.,* 13:585–587, 1964. [,162(1964)].

[62] H.B. Nielsen and P. Olesen. Vortex-line models for dual strings. *Nuclear Physics B,* 61:45 – 61, 1973.

[63] Michael Kalb and P. Ramond. Classical direct interstring action. *Phys. Rev. D,* 9:2273–2284, Apr 1974.

[64] Kimyeong Lee. Dual formulation of cosmic strings and vortices. *Phys. Rev. D,* 48:2493–2498, Sep 1993.

[65] Chandrasekhar Chatterjee and Amitabha Lahiri. Monopole confinement by flux tube. *Euro-
phys. Lett., 76:1068–1073, 2006.

[66] M Franz. Vortex-boson duality in four space-time dimensions. *Europhysics Letters (EPL)*, 77(4):47005, Feb 2007.

[67] R. O. Ramos, J. F. Medeiros Neto, D. G. Barci and C. A. Linhares, “Abelian Higgs model effective potential in the presence of vortices,” Phys. Rev. D 72, 103524 (2005)

[68] A. J. Beekman, D. Sadri, and J. Zaanen. Condensing Nielsen-Olesen strings and the vortex-boson duality in 3+1 and higher dimensions. *New Journal of Physics*, 13(3):033004, March 2011.

[69] Ishita D. Choudhury, M. Cristina Diamantini, Giuseppe Guarnaccia, Amitabha Lahiri, and Carlo A. Trugenberger. 4D Topological Mass by Gauging Spin. *JHEP*, 06:081, 2015.

[70] Theodore J. Allen, Mark J. Bowick, and Amitabha Lahiri. Topological mass generation in (3+1)-dimensions. *Mod. Phys. Lett.*, A6:559–572, 1991.

[71] Jonathan F. Schonfeld. A Mass Term for Three-Dimensional Gauge Fields. *Nucl. Phys.*, B185:157–171, 1981.

[72] Stanley Deser, R. Jackiw, and S. Templeton. Three-Dimensional Massive Gauge Theories. *Phys. Rev. Lett.*, 48:975–978, 1982.

[73] Stanley Deser, R. Jackiw, and S. Templeton. Topologically Massive Gauge Theories. *Annals Phys.*, 140:372–411, 1982. [Annals Phys.281,409(2000)].

[74] N. Dorey and N. E. Mavromatos. QED in three-dimension and two-dimensional superconductivity without parity violation. *Nucl. Phys.*, B386:614–680, 1992.

[75] Gil Young Cho and Joel E. Moore. Topological BF field theory description of topological insulators. *Annals of Physics*, 326(6):1515 – 1535, 2011.

[76] M. Cristina Diamantini and Carlo A. Trugenberger. Higgsless superconductivity from topological defects in compact BF terms. *Nucl. Phys.*, B891:401–419, 2015.

[77] Apoorv Tiwari, Xiao Chen, Titus Neupert, Luiz H. Santos, Shinsei Ryu, Claudio Chamon, and Christopher Mudry. Topological BF theory of the quantum hydrodynamics of incompressible polar fluids. *Phys. Rev. B*, 90:235118, Dec 2014.

[78] M. Cristina Diamantini and Carlo A. Trugenberger. Spin-charge soldering from tensor higgs mechanism. *Phys. Rev. D*, 89:107702, May 2014.

[79] Joseph Maciejko, Victor Chua, and Gregory A. Fiete. Topological order in a correlated three-dimensional topological insulator. *Phys. Rev. Lett.*, 112:016404, Jan 2014.
[80] Peng Ye and Zheng-Cheng Gu. Vortex-line condensation in three dimensions: A physical mechanism for bosonic topological insulators. *Phys. Rev. X*, 5:021029, Jun 2015.

[81] Yuji Hirono and Yuya Tanizaki. Effective gauge theories of superfluidity with topological order. *Journal of High Energy Physics*, 2019(7):62, Jul 2019.

[82] Aron J. Beekman, Jaakko Nissinen, Kai Wu, and Jan Zaanen. Dual gauge field theory of quantum liquid crystals in three dimensions. *Phys. Rev. B*, 96(16):165115, Oct 2017.

[83] Aron J. Beekman and Jan Zaanen. Electrodynamics of abrikosov vortices: the field theoretical formulation. *Frontiers of Physics*, 6(4):357–369, Dec 2011.

[84] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to quantum field theory*. Addison-Wesley, Reading, USA, 1995.

[85] Pierre Ramond. Field Theory. A Modern Primer. *Front. Phys.*, 51:1–397, 1981. [Front. Phys.74,1(1989)].

[86] Hagen Kleinert. Multivalued fields in condensed matter, electromagnetism, and gravitation. Singapore, Singapore: World Scientific (2008), 2008.

[87] H. Kleinert. Disorder version of the Abelian Higgs model and the order of the superconductive phase transition. *Lett. Nuovo Cim.*, 35:405–412, 1982.

[88] Chandrasekhar Chatterjee, Ishita Dutta Choudhury, and Amitabha Lahiri. Meissner effect and a stringlike interaction. *Eur. Phys. J.*, C77(5):300, 2017.