Missile Control Design for Moving Target using Model Predictive Control

Tahiyatul Asfihani, Mirzaq Khoirul Mufidah, Subchan Subchan, Dieky Adzkiya
Department of Mathematics, Institut Teknologi Sepuluh Nopember, Kampus ITS
Sukolilo-Surabaya 60111, Indonesia
E-mail: subchan@matematika.its.ac.id

Abstract. In this paper, Model Predictive Control (MPC) is proposed to control a missile such that the missile reaches the moving target in minimum time. The dynamics of missile is a non-linear model. MPC is applied to linearized missile dynamics. In this study, we conduct some simulations, namely with state constraints and without state constraints. We also observe the influence of weights and prediction horizon to the time needed to reach the target. The simulation results show that the fastest time to reach the target is 20 seconds. This is achieved when the prediction horizon is 10 and the state constraints are present.

1. Introduction
There have been many approaches in the literature for missile guidance. Acho proposed iterative learning control of missile guidance for moving target [1]. Pan, et al. conducted the estimation of acceleration for unknown targets on the dynamics of two-dimensional nonlinear missiles using Extended Kalman Filter Unknown Inputs Without Direct Feedthrough (EKF-UI-WDF) [2]. Subchan and Asfihani employed minimum time control of missile using the Pontryagin minimum principle. The estimated motion of the target is obtained using EKF-UI-WDF [3]. Makena and Omwoma proposed non-linear $H_{\infty}$ guided missile for moving target [4].

Model Predictive Control (MPC) is a controller design concept based on the process model. This method is used to minimize an objective function [5]. Model Predictive Control (MPC) can be applied to complex models, for example, multi-input multi-output models, state, input and output constraints. The mathematical model is used to predict the value of future output. Then, the input can adjust to changes based on given predictions [6]. Model Predictive Control is proposed by Hu and Chen for autopilot missile design in a non-linear form [7]. Li, et al. presented the application of Model Predictive Control (MPC) to solve missile interception problem [8].

In this paper we proposed Model Predictive Control for missile guidance with moving target. The input of the model is a acceleration of missiles. The objective is minimizing the distance between missile and target. Since the model is non-linear, it is necessary to linearize and discretize the model before applying the Model Predictive Control (MPC) algorithm.
2. Missile Dynamical Model

The mathematical model of the missile is given in the following equations [9]:

\[
\begin{align*}
\dot{\lambda} &= \frac{(V_T \sin(\theta_T - \lambda) - V_M \sin(\theta_M - \lambda))}{R} \\
\dot{R} &= V_T \cos(\theta_T - \lambda) - V_M \cos(\theta_M - \lambda) \\
\dot{\theta}_T &= a_T/V_T \\
\dot{\theta}_M &= a_M/V_M,
\end{align*}
\]

where \( R \) is distance between missile and target, \( \lambda \) is line of sight (LOS) angle, \( V_T \) is tangential velocity of moving target, \( V_M \) is tangential velocity of missile, \( \theta_T \) is flight path angle of the target, \( \theta_M \) is flight path angle of the target and the missile, \( a_T \) is the normal acceleration of the target, \( a_M \) is the normal acceleration of the missile, \( \tilde{a}_T \) is the tangential accelerations of the missile, \( \tilde{a}_M \) is the tangential accelerations of the target.

Figure 1 shows a two-dimensional scenario of missile dynamics:

Furthermore, MPC can be applied if the model is discrete and linear. Thus, we first linearize the model and then discretize the model. The model is linearized by using Jacobian around \( \lambda = 0, R = 100000, \theta_T = 2\pi/3, \theta_M = \pi/3 \). We obtain the following linear system:

\[
\dot{x} = Ax + Bu,
\]

where \( x = [\lambda, R, \theta_T, \theta_M]^T \), \( u = a_M \),

\[
A = \begin{bmatrix}
3.2 \times 10^{-4} & 3.8 \times 10^{-13} & -1.5 \times 10^{-4} & 1.7 \times 10^{-4} \\
-3.89 & 0 & -25.98 & 29.87 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
1/V_M
\end{bmatrix}.
\]

After that, the model is discretized using forward difference method. If we use \( \Delta t = 1 \), we obtain the following equation:

\[
x(k+1) = Ax(k) + Bu(k),
\]
where
\[
A = \begin{bmatrix}
3.2 \times 10^{-4} & 3.8 \times 10^{-13} & -1.5 \times 10^{-4} & 1.7 \times 10^{-4} \\
-3.89 & 0 & -25.98 & 29.87 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
0 \\
0.02
\end{bmatrix}.
\] (7)

Since the objective is to minimize the distance between missile and target, the discrete-time measurement equation is defined as follows:
\[
y(k) = Cx(k),
\]
where \(C = [0 \ 1 \ 0 \ 0]\).

3. Model Predictive Control

MPC is a control method that is used to minimize an objective function such that the state, input and output satisfy the given constraints. This control method uses receding horizon principle, namely the optimization is solved for a given prediction horizon and the first element of the input is applied to the system. The process is repeated until the simulation finished.

The objective function of MPC is given by
\[
J(k) = \sum_{j=1}^{N_p} ||y_r(k+j) - y(k+j)||_Q^2 + \sum_{j=0}^{N_c-1} ||u(k+j)||_P^2,
\] (8)

where \(y_r\) is the reference, \(N_p\) is the prediction horizon, \(N_c\) is the control horizon, \(Q\) is the weighting matrix for state, \(P\) is the weighting matrix for input.

The input, state and output constraints are as follows
\[
x(k+j+1|k) = Ax(k+j|k) + Bu(k+j|k),
\] (9)
\[
x(k|k) = x(k),
\] (10)
\[
Dx(k+j+1|k) \leq E,
\] (11)
\[
Su(k+j|k) \leq T,
\] (12)
for \(j = 0, 1, 2, \ldots, N_p - 1\).

The optimization problem for MPC can be formulated as a quadratic programming problem, which can be solved efficiently [10].

4. Simulation Results

In this simulation, we use the model in (6). There are 4 state variables, 1 input variable and 1 output variable. The reference \(y_r = 0\) because the objective is to minimize the distance between missile and target. The control horizon is defined to be equal to prediction horizon, namely \(N_p = N_c\). The input constraint is \(-100 \leq a_M \leq 100\).

First, we define \(Q = 1\) and \(P = 0.1\). We run MPC algorithm for prediction horizon equals 10, 20, 30. The simulation results are shown in Fig. 2.

Observe that at some time the distance between the missile and target becomes negative, which does not make sense. In order to address this issue, we introduce a new state constraint \(R > 0\). The simulation results for prediction horizon \(N_p = 10, 20, 30\) can be seen in Fig. 3. Notice that the distance between missile and target is always positive. In this scenario, the time to reach the target for each prediction horizon is given in Table 1. Prediction horizon \(N_p = 10\) provides the fastest time to reach the target compared to the others.

In the next scenario, we define \(Q = 0.1\) and \(P = 1\). The simulation results are shown in Fig. 4. We run some experiments by varying the prediction horizon \(N_p = 10, 20, 30\). The results are written in Table 2. The fastest time is achieved when prediction horizon equals 20.
Figure 2: Missile acceleration and distance between missile and target for $Q = 1$, $P = 0.1$, prediction horizon $N_p = 10, 20, 30$ without any state constraint.

Figure 3: Missile acceleration and distance between missile and target for $Q = 1$, $P = 0.1$, prediction horizon $N_p = 10, 20, 30$ and state constraint $R > 0$.

Table 1: The time required to reach the target after adding state constraint $R > 0$ for $Q = 1$ and $P = 0.1$.

| Prediction horizon | 10  | 20  | 30  |
|--------------------|-----|-----|-----|
| Time to reach the target | 20  | 22  | 25  |

Table 2: The time required to reach the target for $Q = 0.1$ and $P = 1$.

| Prediction horizon | 10  | 20  | 30  |
|--------------------|-----|-----|-----|
| Time to reach the target | 34  | 23  | 33  |
5. Conclusions

We have designed a controller for missile with moving target by using Model Predictive Control (MPC) method. The missile model is linearized and discretized so that MPC can be applied. In this paper, we conduct two scenarios. The first scenario is $Q = 1$, $P = 0.1$ and the second scenario is $Q = 0.1$, $P = 1$. The fastest time to reach the target occurs at the first scenario for prediction horizon $N_P = 10$.

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