Percolation of color sources and critical temperature

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Abstract

We argue that clustering of color sources, leading to the percolation transition, may be the way to achieve deconfinement in heavy ion collisions. The critical density for percolation is related to the effective critical temperature of the thermal bath associated to the presence of strong color fields inside the percolating cluster. We find that the temperature is rapidity, centrality and energy dependent. We emphasize the similarities of percolation of strings with color glass condensate.
Recently [1][2] it has been proposed a new thermalization scenario for heavy ion collisions which at sufficiently high energies implies the phase transition to the quark-gluon plasma. The key ingredient is the Hawking-Unruh effect [3][4]. It is well known that the black holes evaporate by quantum pair production, and behave as if they have an effective temperature of

\[ T_H = \frac{\kappa}{2\pi} \]  

where \( \kappa = (4GM)^{-1} \) is the acceleration of gravity at the surface of a black hole of mass M. The rate of pair production in the gravitational background of the black hole can be evaluated by considering the tunneling through the event horizon. The imaginary part of the action for this classically forbidden process corresponds to the exponent of the Boltzmann factor describing the thermal emission [5]. Unruh showed that a similar effect arises in a uniformly accelerated frame, where an observer detects an thermal radiation with the temperature

\[ T_U = \frac{a}{2\pi} \]  

where a is the acceleration. The event horizon in this case emerges through the existence of causally disconnected regions of space-time [6].

Let us now turn to hadronic interactions. The probability to produce states of masses M due to the chromo-electric field E and color charge g is given by the Schwinger mechanism [7]

\[ W_M \sim \exp\left(-\frac{\pi M^2}{gE}\right) \sim \exp\left(-\frac{M}{T}\right) \]  

which is similar to the Boltzmann weight in a heat bath with an effective temperature

\[ T = \frac{a}{2\pi} , \quad a = 2gE/M. \]  

The full probability P to produce any state of invariant mass M is

\[ P(M) = 2\pi W_M \rho(M) , \]  

being \( \rho \) the density of hadronic final states. In the dual resonance model, \( \rho \) is given by

\[ \rho(M) \sim \exp\left(M \sqrt{\frac{b}{6}}\right) , \]
where $b$ is related to the string tension $\sigma$ by the relation $b = 1/2\pi\sigma$. Unitarity implies that the sum of $P(M)$ over all finite states should be finite, what means

$$T = \frac{a}{2\pi} \leq \frac{1}{4\pi}\sqrt{\frac{6}{b}} \equiv T_{Hag}. \quad (7)$$

The quantity of the right hand side of (7) is the well known Hagedorn temperature, the limiting temperature of hadronic matter above which hadronic matter undergoes a phase transition into the deconfined phase [8]. So, it also can be considered as the existence of a limiting acceleration $a_0 = \sqrt{3/2b}$ corresponding to a strong color field. The existence of a limiting temperature is not exclusive of the dual resonance model, but it occurs in any system whose exponential mass spectrum has combinatorics structure [9].

Heavy ion collisions are currently described in terms of color strings stretched between the nucleons of the projectile and the target, which decay into new strings through $q-\bar{q}$ pairs production and subsequently hadronize to produce observed hadrons [10]. Color strings may be viewed as small discs in the transverse space, $\pi r_0^2$, $r_0 \simeq 0.2 fm$, filled with the color field created by the colliding partons. Particles are produced by the Schwinger mechanism, emitting $q-\bar{q}$ pairs in this field. With growing energy and/or atomic number of colliding nuclei, the number of strings grows and they start to overlap forming clusters. At a critical density a macroscopic cluster appears that marks the percolation phase transition [11]. The strong color field inside this large cluster produces an acceleration which can be seen as a thermal temperature $T$, by means of the Hawking-Unruh effect.

In this paper, we compute this temperature to check if its value is above the critical phase temperature found in lattice QCD [12] which is 170-185 MeV depending on the number of flavour (2 or 3) and for vanishing chemical potentials.

In two dimensional percolation theory the relevant variables is the density

$$\eta = N_S S_1/S_A \quad (8)$$

where $N_S$ is the number of strings formed in the collisions and $S_1$ and $S_A$ are the transverse area of one string and the nuclear overlapping area respectively. Thus, it
depends on the impact parameter \( b \) of the collision. For very central collisions \( b = 0 \) and \( S_A = \pi R_A^2 \).

The critical point for percolating is \( \eta_C = 1.18 - 1.5 \), depending on the type of the employed nuclei profile functions, homogeneous or Wood-Saxon [13].

We assume that a cluster of \( N \) strings behave as a single string with energy momentum that corresponds to the sum of the energy-momentum of the overlapping strings and with a stronger color field resulting from the vectorial sum of the color charges \( \vec{Q}_i \) of each individual string. The obtained color field covers the area \( S_N \) of the cluster. As the individual string colors are random oriented in color space, the average \( \langle \vec{Q}_1 \cdot \vec{Q}_j \rangle \) is zero, such that if \( S_N = S_1 \) then \( Q_N^2 = N Q_1^2 \) or \( Q_N = \frac{1}{\sqrt{N}} N Q_1 \) with a color reduction factor \( 1/\sqrt{N} \). In general, \( S_N \geq S_1 \) and the color reduction becomes \( \sqrt{\frac{S_N}{S_1}} \) [14].

In the framework of the Schwinger model extended to color fields, the particle density \( \mu_N \) is proportional to the color charge \( Q_N \) and the \( <p_T^2>_N \) is proportional to the string tension \( \sigma_N, \mu_N \sim Q_N \), and \( <p_T^2>_N \sim \sigma_N \). On the other hand, from Gauss theorem

\[
Q_N \sim S_N \sigma_N
\]  

we thus obtain

\[
\mu_N = \sqrt{\frac{1}{N} \frac{S_N}{S_1}} N \mu_1 \quad \text{and} \quad <p_T^2>_N = \sqrt{\frac{N \frac{S_1}{S_N}}{S_1}} <p_T^2>_1
\]  

The ratio of the two equations (10) gives rise to the scaling law

\[
<p_T^2>_N/\mu_N = <p_T^2>_1 S_1/\mu_1
\]  

valid for all projectiles, targets and energies. This scaling law is in reasonable agreement with data [15]. A similar scaling is found in the Color Glass Condensate (CGC) framework [16].

Asymptotically, for a random distribution of \( N_S \) strings,

\[
\frac{1}{N} \frac{S_N}{S_1} \equiv F^2(\eta)
\]  

with

\[
F(\eta) = \sqrt{\frac{1 - e^{-\eta}}{\eta}}
\]
being the color reduction factor. It follows that

\[ \mu = F(\eta)N_S\mu_1 \quad \text{and} \quad < p_T^2 > = < p_T^2 >_1 / F(\eta) \] (14)

Note that \( \sigma_N = \sigma_1 / F(\eta) \).

The tension of the macroscopic cluster fluctuates around its mean value because the chromoelectric field is not strictly constant. Assuming a gaussian form for these fluctuations [17] we have for the transverse momentum distributions

\[ \frac{dn}{d^2 p} \sim \sqrt{\frac{2}{< x^2 >}} \int_0^\infty dx \exp(-x^2/2 < x^2 >) \exp(-\pi p_{T1}^2 / x^2) \] (15)

which gives rise to the thermal distribution

\[ \frac{dn}{d^2 p} \sim \exp\left(-p_{T1} \sqrt{\frac{2\pi}{< x^2 >}}\right) \] (16)

with

\[ < x^2 > = \pi < p_T^2 >_1 / F(\eta) \] (17)

Therefore,

\[ T = \sqrt{< p_T^2 >_1 / 2F(\eta)} \] (18)

If we identify the percolation transition temperature to the Hagedorn temperature, we obtain

\[ T_H = \sqrt{< p_T^2 >_1 / 2F(\eta_C)} \] (19)

which gives for \( \sqrt{< p_T^2 >_1} \approx 250\text{MeV} \), and with \( \eta \approx 1.18 - 1.5 \), \( T_C \approx 200 - 250\text{MeV} \).

Similar value has been obtained previously in the framework of the CGC [1],[18]. In this case \( T = Q_S / 2\pi \) where \( Q_S \) is the saturation momentum. As it has been pointed out, the role of the scale \( Q_S \) in the transverse momentum distributions is played by \( < p_T >_1 / F(\eta) \) in percolation. Similar phenomenology can be obtained in both schemes [19].

Most of the considerations presented in the framework of the CGC [18] related to a rapid thermalization can be applied to our approach. Hydrodynamics models [20] have sucessfully described the collective flow of the produced hadrons and the low \( p_T \) single particle spectra. This succes has led to the conclusion that the created matter behaves
like an ideal liquid with almost negligible viscosity [20][21]. The existing microscopic approaches are not able to explain a fast thermalization. For instance, a transport model based on independent strings [22] does not present early thermalization although the corresponding energy density profiles at proper time $\tau = 1 \, fm/c$ compare well with hydrodynamics assumptions for initial energy density distributions. At this point, the interaction among string, forming a large cluster with strong color field inside helps to reach a fast thermalization.

In fact, below the percolation threshold we have clusters with a few number of strings and different occupied areas what give rise to different temperatures. Above the threshold, due to the high density, there are strong interactions among the strings forming essentially only one large cluster. The fragmentation of this cluster give rise to a thermal distribution of the produced particles. The interaction among these large number of strings can be considered as an strong interaction at partonic level which give rise to a color composition and the formation of one cluster.

Similarly to what happens in the CGC framework, we can ask for the charasteristic time over which the chromo-electric field changes. In our case, it would be proportional to the momentum scale, i.e. $<p_T>_1 / F(\eta)$, which implies a lifetime of the cluster, $\tau = 1/T \simeq 0.7 - 1 \, fm$ for $\eta \geq \eta_C$. As the string density increases $\tau$ decreases. Similar conclusions can be obtained, in a more detailed model as it is discussed in ref [23].

In the CGC the temperature depends on the rapidity provides that $Q_S$ depends on $y$. This dependence may trigger instability of the system and speed up thermalization process. In percolation, the temperature (average transverse momentum) depends also on the center of mass rapidity $y$. In fact, the first equation in (14), making $\mu \rightarrow \frac{dn}{dy}$ and $\eta_i \rightarrow \bar{n}$, can be rewritten in the form

$$\frac{dn}{dy} = F(\eta)N_S(\Delta, y_b)\bar{n} \tag{20}$$

where $\Delta = y - y_b, \frac{\sqrt{s}}{2} \sim exp(y_b)$ and $\eta \equiv \left(\frac{\bar{n}}{N}\right)^2 N_S(\Delta, y_b)$. Note that, in general, $N_S$ is a function of $y$ and $y_b$ while $\bar{n}$ is not.

For $\Delta \simeq 0$, the rapidity distribution reflects the scaling behaviour of the parton structure functions of the valence strings, their number being proportional to the num-
ber of nucleon participants, $N_{\text{part}} = 2N_A$:

$$\Delta \simeq 0 \quad \mathcal{N}_S(0, y_b) \rightarrow \mathcal{N}_S(0) \simeq N^P_S(0)N_A$$  \hspace{1cm} (21)$$

where $N^P_S$ is the number of strings in $pp$ collisions.

For $\Delta \simeq -y_b$, we are in the domain of sea partons and sea string contribution, their number being proportional to the number of nucleon-nucleon collisions, $N_A^{4/3}$:

$$\Delta \simeq -y_b \quad \mathcal{N}_S(-y_b, y_b) \rightarrow \mathcal{N}_S(y_b) \simeq N^P_S(y_b)N_A^{4/3}$$  \hspace{1cm} (22)$$

From equations (20),(21)and (22) it follows

$$\Delta \simeq 0 : \frac{dn}{dy} \sim N_A \frac{1}{N_A^{1/6}} \text{ independent of } y_b$$  \hspace{1cm} (23)$$

$$\Delta \simeq -y_b : \frac{dn}{dy} \sim N_A, \text{ increasing with energy}$$  \hspace{1cm} (24)$$

Regarding the $p_T$ behaviour we have

$$\Delta \simeq 0 : < p_T > \simeq \eta^{1/4} \simeq N_A^{1/12}, \text{ independent of } y_b$$  \hspace{1cm} (25)$$

$$\Delta \simeq -y_b : < p_T > \simeq \eta^{1/4} \simeq N_A^{1/6}, \text{ increasing with } y_b$$  \hspace{1cm} (26)$$

In this way, the temperature is larger in central rapidity than in the extremes in a factor $N_A^{1/12}$, which for central $Au - Au$ collisions is around 1.6. This difference increases with energy. This considerable difference of temperature in different rapidity slices generates a viscosity, which is very small. Following [18], the shear viscosity can be estimated by

$$\frac{\eta_S}{n} = < p_T > \lambda \sim < p_T > \frac{1}{nS_1} = \frac{< p_T >_1 L}{\eta F(\eta)}$$  \hspace{1cm} (27)$$

where we use for $n$, the number of strings per unit of volume, $n = \frac{N_S}{\pi R_A^2 L}$ being $L$ the longitudinal extension, $L \simeq 1 fm$. For $\eta \geq \eta_C$, we obtain a value around 1 decreasing with the density.

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