Mixed Hardening Characteristics of the Anisotropic Coal under Cyclic Loading

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Abstract. Pulse fracturing has been used to increase permeability and weaken the strength of the coal seam, making the coal fracture under cyclic loading. During the cyclic loading, the rock-like materials tend to present mixed hardening (mixed mode of isotropic and kinematic hardening) from initial yielding to failure (critical yielding). At present, understanding of the mixed hardening characteristics in anisotropic coal involving massive cleats remains challenging and crucial. In this paper, the cylinder specimens (diameter: 50 mm; length: 100 mm) of the coal were tested under cyclic uniaxial loading (loading rate: 900 N/s), and acoustic emission (AE) was employed to characterize the hardening process. The samples were drilled at the angle (α) of 0°, 45° and 90° with the coal surface cleat respectively. The upper-stress limit (P\text{max}) increases by 2 kN at each loading cycle, and the lower-stress limit (P\text{min}) was kept at 1 kN. Several findings were obtained based on the experimental results. (1) Uniaxial compression strength and the cyclic number increase with α, presenting isotropy. (2) The remarkable accumulation of AE energy is the feature of identifying plastic hardening. Both the isotropic and kinematic hardening processes are significant for the specimen with a dip angle (α) of 90°, validated by the dramatically increased isotropic and kinematic hardening indexes. However, the coal presents a slight isotropic and kinematic hardening, with α ranging from 0° to 45°. (3) The remarkably mixed hardening (α=90°) corresponding to a complex fragmentation of the coal, which is supposed to be caused by the continuous weakening of coal matrix (including the butt cleat). In contrast, at α of 0° and 45°, the mixed hardening characteristics are slight. Accordingly, the final fracture surfaces of coal tend to be relatively single, roughly along with the surface cleats. Thus, we can infer that the slightly mixed hardening is due to the weakening of surface cleats. Based on the mixed hardening characteristics of anisotropic coals, conducting pulse fracturing to apply cyclic loading perpendicular to the cleat surface is supposed to be beneficial for generating more complex fractures, to improve coal permeability.

Keywords: mixed hardening, coal, anisotropic, cyclic loading

1. Introduction

Pulse fracturing, fracturing the rock with the cyclic injection of high-pressure fluid, is a significant approach to increase permeability and weaken the strength of coal seam [1-4]. During pulse fracturing, the coal surrounding the hydraulic fracture bears cyclic loading and unloading, and the coal tends to
follow the mixed hardening rule, i.e. the combination of isotropic and kinematic hardening [5-9]. Given the remarkable transverse anisotropy of coal due to cleats, understanding of the mixed hardening characteristics in anisotropic coal is fundamental to pulse fracturing.

Based on the previous investigations, the mechanical properties of rock under cyclic loading have been validated to be significantly different from those under monotonic loading [10-14]. The compressive strength and elastic modulus present different sensitivities to the cyclic loading. For example, the compressive strength of salt decreases with the loading cycles, whereas the elastic modulus decreases slightly during the first few loading cycles and remain constant until failure [10]. The continuous damage with cyclic loading can be characterized by energy dissipation [15-16]. For coal, the accumulated dissipated energy shows an exponential growth with the maximum principal stress [17]. The coal fractures once the accumulated dissipated energy attains to the critical value [18]. Under cyclic loading, due to cleat distributions in coal, the evolution of energy dissipation and mechanical properties presents anisotropy. Based on the present experimental investigations, many researchers modified the damage and elastic-plastic constitutive model [19-23]. However, in view of the significant anisotropy of coal with massive cleats, the anisotropic mixed (isotropic and kinematic) plastic hardening rule remains unclear, which restricts the application-optimization of pulse fracturing in coal seams.

In this paper, the cylinder coal specimens were tested under cyclic uniaxial loading, with the characterization of acoustic emission (AE). The loading direction is designed as 0°, 45°, and 90° with the surface cleat surface, to perform the anisotropic mixed hardening in coal. The isotropic and kinematic hardening characteristics corresponding to the stress-increment and stress-decrement were identified, with the integrated characterization of AE energy and loading curves. Then the coal fragmentation is compared with the mixed hardening rule of the anisotropic coal.

2. Experimental Procedure

2.1. Testing Material and Specimen Dimensions
A set of cylinder specimens of coal from the Ordos, China, were tested to investigate the anisotropic mixed hardening rule under cyclic uniaxial compression. As shown in figure 1(a), three groups of cylinder specimens (diameter: 50 mm; length: 100 mm) were drilled from the same coal, at the angle (α) of 0°, 45° and 90° with the coal surface cleat. For clarity, in the next context, type-0, type-45 and type-90 represent the angle (α) 0°, 45° and 90°, respectively.

![Figure 1](image-url)  
**Figure 1.** Specimen photos and loading system. (a) the specimen drilled at different directions; (b) the loading system and AE monitor.

2.2. Loading Setups and AE measurements
Cyclic uniaxial compression tests were conducted on the cylinder specimens with a closed-loop, servo-hydraulic MTS load system (figure 2b). Three groups of cyclic uniaxial compression tests were conducted under cyclic loading and unloading control at a rate of 900 N/s. The upper-stress limit \( P_{\text{max}} \) increases by 2 kN at each loading cycle, and the lower-stress limit \( P_{\text{min}} \) was kept at 1 kN. Eight AE sensors were employed to characterize the plastic hardening process (figure 2b), with receiving the elastic waveform released from the onset of microcracks. The sampling frequency and the threshold level are 5000 kHz and 30 dB, respectively. Note that the mixed hardening rule of the anisotropic coal (with different dip angles: 0°, 45° and 90°) can be investigated with the combined responses of load and AE energy.

3. Global Responses of the Cyclic loading on the Anisotropic Coal

For the specimens of type-0, type-45 and type-90, the stress-strain responses and the AE energy evolutions are shown in figures 2-3, respectively. Figures 2-3 indicate that the uniaxial stress strength and the cyclic number increase with the dip angle between the loading direction and surface cleat, as in table 1. Table 1 delineates the increased anti-pressure ability with \( \alpha \) is one of the anisotropic mechanical properties of coal.

| specimen type | \( \alpha \) | uniaxial compression strength (MPa) | cyclic number |
|---------------|--------------|-----------------------------------|--------------|
| type-0        | 0°           | 15.8                              | 15           |
| type-45       | 45°          | 20.3                              | 19           |
| type-90       | 90°          | 37.4                              | 37           |

As shown in figure 3, AE energy remarkably accumulated at both the stress-increment and stress-decrement stage. Since AE is the elastic wave released from the onset of microcracks, the accumulation of AE energy indicates the plastic development in coal specimens. Therefore, AE-identified plastic development occurs both in the stress-increment and stress-decrement stages (figures 3). The accumulation of AE energy is more and more remarkable as the specimen close to failure. During uniaxial compression, the developments of plasticity in the stress-decrement stage are usually due to two categories: the contract (plastic softening) and movement (kinematic hardening) of the yield surface. During plastic loading, the increasing load peak (point P in figure 4b) at every loading cycle indicates the specimen can bear higher stresses, which means that the yield surface has not reached the critical state, i.e., yield surface cannot contract. Therefore, the plastic mechanical behaviours in both the stress-increment and stress-decrement stage are plastic hardening rather than plastic softening. Note that the plastic development in the stress-decrement stage cannot be the softening, due to the continuous increment of load peak. Therefore, the generation of plasticity in the stress-decrement stage is supposed
to be the movement of yield surface, i.e. the kinematic hardening. In the next context, we will discuss the isotropic and kinematic hardening (i.e. the mixed hardening) rule of the anisotropic coal, in detail.

Figure 3. Stress-time responses and AE energy evolutions of different specimens: (a) type-0; (b) type-45; (c) type-90.

4. The Mixed Hardening Rule of the Anisotropic Coal Based on the Integrated Analysis of Stress and AE Energy

4.1. Identifications of Isotropic and Kinematic Hardening during Cyclic Loading

To better identify the stress state corresponding to isotropic and kinematic hardening, we replot the AE hit with disk overlapping the stress-time curve of cyclic loading, as shown in figure 4. Note that both the size and opacity of disks are proportional to the logarithm of AE energy. On the stress-time curve (figure 4), the cluster of AE energy indicates plastic hardening. More specially, as in figure 4(b), segment AP and segment PB indicate the isotropic and kinematic hardening, respectively.

As we just perform the uniaxial loading on the coal specimen, the geometry of yield surface applicable to the coal cannot be delineated. However, in the deviatoric plane (i.e. \( \pi \)-plane), the uniaxial stress state can be regarded as a point on the maximum principal stress axis, such as point U in figure 5. The expansion and movement of the yield surface, i.e. the isotropic and kinematic hardening, can be delineated by the offset of point U (figure 5), whatever the geometry of yield surface is. Taking the
Drucker-Prager yield function (a circle in the deviatoric plane as in figure 5) as an example, the mixed hardening involving isotropic hardening and kinematic hardening can be expressed as in Eq. (1).

\[
f(\sigma_{ij}, K_I, K_k) = F(\sigma_{ij} + K_k) + K_I
\]

where \( f \) is the hardening yield function, \( F \) the initial yield function, \( K_I \) the isotropic hardening parameter, and \( K_k \) the kinematic hardening parameter.

\[\text{(1)}\]

**Figure 4.** AE energy distributed along the stress-time curve: (a) type-0; (b) type-45; (c) type-90.

**Figure 5.** Schematic diagram of the mixed-mode of isotropic and kinematic hardening, i.e. the mixed hardening.

Identification of points A, B and P in the stress-time curve is fundamental to determining the plastic developing process, i.e., segment AP and PB. In figure 4, the remarkable accumulation of AE energy indicates the generation of massive microcracks, representing the plastic deformation developing in coal.
specimens. In figure 4(b), point A corresponds to the beginning of AE energy remarkable accumulation, indicating stress state attains to plasticity. The feature point B is in accordance with the vanishment of AE energy accumulation, i.e., the termination of kinematic hardening. Based on the isotropic and kinematic hardening mechanisms of uniaxial compressions, in figure 4(b), point A (σI) indicates the initial yield state of isotropic hardening and point B (σk) indicates the termination of yield surface movement, in a loading cycle. Therefore, segments PA (ΔσI) and PB (Δσk) represent the isotropic and kinematic hardening amplitudes (ΔσI and Δσk) with cyclic loading, which provide bases for characterizing the mixed hardening rule in Section 4.2.

4.2. Mixed Hardening Rules of the Anisotropic Coal

In this section, the mixed (isotropic and kinematic) hardening rules of anisotropic coals, i.e. the coal specimens of type-0, type-45 and type-90, were characterized with the evolution of ∆σI and ∆σk, as shown in figure 6. Figure 6(a) indicates the specimens of type-0 and type-45 follow a consistent isotropic hardening rule, i.e., ∆σI increases steadily with the loading cycles. Besides, the specimens of type-0 and type-45 present the unremarkable kinematic hardening, which can be verified by the slightly increased ∆σk during cyclic loading (figure 6b). For the specimen of type-90, when the cyclic loading number is less than 30, the AE energy is extremely weak, implying both the isotropic and kinematic plastic hardening can be neglected (figure 6). However, as the cyclic loading number over 30, the specimen of type-90 exposes significant isotropic and kinematic hardening processes, such as the explosively increased ∆σI and ∆σk (figure 6). Therefore, the mixed hardening is sensitive to the dip angle between the loading direction and surface cleat, which present remarkable anisotropic mixed hardening characteristics for the coal.

![Figure 6](image)

**Figure 6.** The isotropic and kinematic hardening indexes of different specimens under cyclic loading: (a) ∆σI vs loading cycles; (b) ∆σk vs loading cycles.

4.3. Fragmented Geometries of Anisotropic Coals related to Mixed Hardening Characteristics

The coal fragmentations are related to mixed hardening characteristics. Plastic mechanical characteristics occurring in rock-like materials is due to the generation of massive microcracks, i.e. the weakening of rock-like materials. Therefore, significant plastic development represents remarkable nucleations and coalescences of microcracks. As shown in figure 7, the plastic hardening of type-90 is more prominent than that of type-0 and type-45, which indicates that the nucleation and coalescence of microcracks in type-90 are more obvious before the final fracturing. For the coal specimen of type-90, the nucleation and coalescence of microcracks in type-90 have no dominant fracture path, compared with type-0 and type-45; however, the specimen of type-90 almost becomes the powder. Based on this
phenomenon, when dip angle $\alpha$ is ranging from 0° to 45°, we can infer that the slight isotropic and kinematic hardening are due to the continuously tensile and shear weakening on the surface cleat. Consequently, the specimens of type-0 and type-45 have several large scale fracture surfaces connecting the surface cleats. On the contrary, when the uniaxial loading is perpendicular to the surface cleat, the cleat surface bears compression stress and tends to close. The coal matrix (involving butt cleats) is destroyed under cyclic loading. Therefore, the specimen of type-90 can resist higher cyclic stress. Once the coal matrix reaches the initial plastic condition, most of the microcracks would be generated in the matrix, increasing the potential fracture surface in type-90. The coal matrix presents significantly isotropic and kinematic hardening characteristics, which aggravate the fragmentation degree of the coal.

Above all, the mixed hardening characteristics are sensitive to the dip angle between the uniaxial loading and surface cleat, i.e., mixed hardening characteristics present anisotropy, which is related to the fragmentation of coal. In this work, the remarkably mixed plastic hardening of type-90 is a significant mechanical characteristic of generating complex major fracture network. Therefore, performing the pulse fracturing the cyclic stress perpendicular to the bedding is supposed to improve the amount and complexity of the major fractures in coal, to enhance the coal permeability.

Figure 7. Fragmented geometries of anisotropic coals with different dip angles between the loading direction and surface cleat.

5. Conclusions

- Under cyclic loading, the uniaxial compression strength and the cyclic number present isotropy, increasing with the dip angle between the loading direction and surface cleat. Namely, the critical yield state is enhanced with the dip angle.
- During uniaxial cyclic loading, the isotropic and kinematic hardening processes were identified with the remarkable accumulation of AE energy. The specimen of type-90 exposes significant isotropic and kinematic hardening processes, validated by the sharply increased isotropic and kinematic hardening indexes $\Delta\sigma_I$ and $\Delta\sigma_k$. However, specimens of type-0 and type-45 present a slight isotropic and kinematic hardening.
The coal fragmentation degree depends on the mixed hardening characteristics. The final fracture geometries of the coal are remarkably complex if the coal presents a significant mixed hardening behaviour, such as the type 90, which is supposed to be caused by the continuous weakening of coal matrix (including the butt cleat). In contrast, the coal fragmentation tends to be less complex, if both the isotropic and kinematic hardenings are slight. The less complex fracture surfaces were generated mainly due to the weakening of surface cleat under cyclic loading, for the dip angles of 0° and 45°.

Based on the mixed hardening characteristics of the anisotropic coal revealed in this work, conducting pulse fracturing to apply cyclic loading perpendicular to the cleat surface is supposed to be beneficial for generating more complex fractures, to improve coal permeability.

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