Research on lightning radiation electromagnetic field of induced thunder on the ground of perfect conductor

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Abstract. The influence of lightning on smart substation mainly includes three aspects, one of which is lightning striking the grounding body of adjacent smart substation and affecting smart components through space electromagnetic induction. When the lightning strike point of induction lightning is in the station, its radiation field is closely related to the location of the lightning strike point. It is necessary to verify whether it meets the electromagnetic compatibility level of intelligent components of lightning radiation field according to the actual situation. Firstly, the simulation model of intelligent component under the influence of induction mine is studied, and the steps of simulation modelling are given. On this basis, through derivation, the calculation method and corresponding formula of lightning current radiation field on the ground of perfect conductor are given in detail.

1. Introduction
The impact of lightning on smart substation mainly includes three aspects. First, lightning strikes the substation lightning rod, causing the potential difference on the grounding network, thus affecting the normal operation of the intelligent components. Second, the lightning strikes the transmission line of the substation, and the lightning penetrated wave passes through a device, transformer and connecting cable to enter the intelligent component. Thirdly, lightning strikes the grounding body near the smart substation, and affects the intelligent components through space electromagnetic induction.

For induction lightning, there are two main ways to produce it. One is that the lightning strike point is outside the substation, and the other is that the lightning strike point is inside the station. Although the former produces large intensity of induction lightning, the influence of this kind of induction lightning on intelligent components can be neglected because of its long distance. The latter produces low inductive lightning intensity, and its radiation field is closely related to the location of lightning strike point. It is necessary to verify whether it meets the electromagnetic compatibility level of intelligent components of lightning radiation field according to the actual situation.

The simulation model of intelligent components under the influence of induction mine is given in this paper. The simulation method is as follows: (1) Given the location of lightning strike point and intelligent component, the induction field of lightning current at the location of intelligent component is calculated by using lightning current as excitation source. (2) Consider the shielding effect of the metal structure. For AIS (air insulated switchgear) substation, only the shielding effect of smart component shell is considered. If it is a GIS (gas insulated switchgear) substation, the shielding effect of GIS pipeline should be considered. Then the radiated interference voltage is obtained when the induction field decays and enters the port of the smart component. (3) Based on the field-circuit coupling equation, the induced current on the pipeline of GIS is calculated by taking the induction field as the incident field,
and then combined with the above analysis, the external induction field is converted to the conduction coupling value.

2. Calculation of Electromagnetic Field of Lightning Radiation from Induced Thunder on the Ground of Perfect Conductor

The calculation method of lightning current radiation field on the ground of perfect conductor is given in this paper. After nearly 30 years of research, the calculation of lightning electromagnetic field has achieved a lot of results. However, due to the limitation of calculation speed, a lot of work still concentrates on the analysis of lightning electromagnetic field on the ground of pure conductor and damaged soil. In this paper, the calculation of lightning electromagnetic field on the ground of pure conductor is summarized, and the calculation formula of lightning electromagnetic field is given.

Assuming that the ground flashover channel is approximately perpendicular to the ground, the earth is regarded as a pure conductor. Let the current element be \( i(R', t) \, dl \), where \( dl \) is the length of the current element. According to the vector \( A(R, t) \) and Maxwell equation, the electric field \( E(R, t) \) and magnetic field \( B(R, t) \) at any point generated by the current element can be obtained. The calculation formula is shown in formulas 1-4.

\[
\nabla \cdot E(R, t) = \rho(R', t)/\varepsilon_0 \tag{1}
\]
\[
\nabla \cdot B(R, t) = 0 \tag{2}
\]
\[
\nabla \times E(R, t) = -\frac{\partial B(R, t)}{\partial t} \tag{3}
\]
\[
\nabla \times B(R, t) = \mu_0 i(R', t) + \frac{1}{c^2} \frac{\partial E(R, t)}{\partial t} \tag{4}
\]

The expressions of electric field and magnetic field can be obtained by using vector magnetic potential \( A(R, t) \) and scalar potential \( R, t) \).

\[
B(R, t) = \nabla \times A(R, t) \tag{5}
\]
\[
E(R, t) = -\nabla \phi(R, t) - \frac{\partial A(R, t)}{\partial t} \tag{6}
\]
\[
E(R, t) = -\frac{\partial A(R, t)}{\partial t} + c^2 \int_0^t \nabla \cdot [\nabla A(R, t)] \, dt \tag{7}
\]

The vector magnetic potential \( A(R, t) \) in the above formula is:

\[
A(R, t) = \frac{\mu_0 dl}{4\pi} \int \left( R , t - \frac{|R - R'|}{c} \right) / |R - R'| \tag{9}
\]

In the above formulas, the parameters with apostrophes are the coordinates of source points, and those without apostrophes are the coordinates of field points. Cylindrical coordinates are used to solve the problem. If the current element \( i(R', t) \, dl \) is placed at the origin of the coordinate and the direction is the same as the Z axis, then \( i(R', t) \, dl = i(R', t) \, dz' \). Considering the field of point P in space, since \( R' = 0 \), we can get from the formula:

\[
A(R, t) = \frac{\mu_0 i(0, t - R'/c)}{4\pi} \int dl \, e_z = A_z(R) e_z \tag{10}
\]

Since \( A(R, t) \) has only vertical component, formula 11 can be deduced by substituting formula 10 for formula 4.

\[
B(r, \phi, z, t) = \nabla \times A(r, \phi, z, t) = \left( \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) e_r + \left( \frac{\partial A_\phi}{\partial r} - \frac{\partial A_r}{\partial \phi} \right) e_\phi + \left[ \frac{1}{r} \frac{\partial r A_r}{\partial \phi} - \frac{\partial A_\phi}{\partial \phi} \right] e_z
\]

\[
= e_\phi \left( -\frac{\partial A_r}{\partial r} \right) = -\frac{\mu_0 dl}{4\pi} \frac{\partial}{\partial r} \left[ \frac{i(t - R'/c)}{R} \right] e_\phi \tag{11}
\]
Among them: $r^2 = x^2 + y^2$, and $R^2 = r^2 + z^2$. So there are:

$$\frac{\partial R}{\partial r} = \frac{r}{R} \quad \frac{\partial}{\partial r} i(t - R/c) = \frac{r}{cR} i'(t - R/c) \quad \frac{\partial}{\partial t} i(t - R/c) = \frac{1}{cR} i'(t - R/c)$$  \hspace{2cm} (12)

In formula 12, the derivative of an apostrophe variable is the derivative of the entire variable (t-R/c). Thus, the relationship between time derivative and space radius derivative can be obtained.

$$\frac{\partial}{\partial r} i(t - R/c) = \frac{r}{cR} \frac{\partial}{\partial t} i(t - R/c)$$  \hspace{2cm} (13)

And,

$$\frac{\partial}{\partial r} \left( \frac{1}{R} \right) = -\frac{r}{R^2}$$  \hspace{2cm} (14)

So, formula 13 is expanded as follows:

$$B(r, \phi, z,t) = -\frac{\mu_0 dz^2}{4\pi} \left[ \frac{1}{R} \frac{\partial}{\partial r} \left( R i(t - R/c) \right) + \frac{r}{cR} \frac{\partial}{\partial t} i(t - R/c) \right] e_i$$  \hspace{2cm} (15)

By using the differential relations, formula 15 can be changed to:

$$B(r, \phi, z,t) = -\frac{\mu_0 dz^2}{4\pi} \left[ \frac{r}{cR^2} \frac{\partial}{\partial t} i(t - R/c) + \frac{1}{R} \frac{\partial}{\partial r} i(t - R/c) \right] e_i$$  \hspace{2cm} (16)

Formula 16 is the expression of the magnetic field in a plane perpendicular to the current direction. Next, the expression of the electric field is determined by formula 10. Let's first determine the second term in formula 10. According to formula 12, there are:

$$\nabla \cdot A = \frac{1}{r} \frac{\partial}{\partial r} (rA_r) + \frac{1}{r \sin \phi} \frac{\partial}{\partial \phi} \left( \frac{1}{r} \frac{\partial A_r}{\partial \phi} \right) + \frac{\partial A_z}{\partial z} = \frac{\mu_0 dz^2}{4\pi} \left[ \frac{1}{R} \frac{\partial}{\partial r} i(t - R/c) \right] R$$

Because of the cylindrical symmetry of the field distribution, the gradient operator is:

$$\nabla = e_r \frac{\partial}{\partial r} + e_\phi \frac{\partial}{\partial \phi} + e_z \frac{\partial}{\partial z}$$  \hspace{2cm} (18)

By substituting formula 17 and 18 into the second item of formula 10, we can get:

$$\nabla \left( \nabla \cdot A \right) = \mu_0 dz^2 \left[ \frac{1}{4\pi} \frac{1}{R} \frac{\partial}{\partial r} \left( R i(t - R/c) \right) - \frac{z}{R^3} i(t - R/c) \right] e_r +$$

$$\mu_0 dz^2 \left[ \frac{1}{4\pi} \frac{\partial}{\partial z} \left( \frac{1}{R} \frac{\partial}{\partial r} i(t - R/c) \right) - \frac{z}{R^3} i(t - R/c) \right] e_z$$  \hspace{2cm} (19)

After expanding formula 19, we can get:

$$\frac{\nabla (\nabla \cdot A)}{\mu_0 dz^2 / 4\pi} = \left[ \frac{1}{R} \frac{\partial}{\partial r} \left( \frac{R i(t - R/c)}{R} \right) - \frac{3zR}{R^3} i(t - R/c) + \frac{1}{R} \frac{\partial}{\partial z} i(t - R/c) - \frac{z}{R^3} i(t - R/c) \right] e_r +$$

$$\left[ \frac{z}{R^3} \frac{\partial}{\partial z} i(t - R/c) - \frac{1}{R} \frac{\partial}{\partial r} \left( \frac{R z^2}{R} \right) i(t - R/c) \right] e_z$$  \hspace{2cm} (20)

According to the differential relation, there are:

$$\frac{\partial}{\partial z} \left[ i(t - R/c) \right] = \frac{z}{cR} \frac{\partial}{\partial t} \left[ i(t - R/c) \right] = -\frac{r}{cR} \frac{\partial}{\partial r} \left[ i(t - R/c) \right]$$  \hspace{2cm} (21)

The derivatives of the above formulas are as follows:

$$\frac{\partial}{\partial z} \left( \frac{z}{cR} \frac{\partial}{\partial t} i(t - R/c) \right) = -\frac{1}{cR} \frac{\partial}{\partial t} i(t - R/c) + \frac{z}{cR^3} \frac{\partial}{\partial t} i(t - R/c) - \frac{z}{cR} \frac{\partial}{\partial z} i(t - R/c)$$

$$= \frac{1}{cR} \frac{\partial}{\partial t} i(t - R/c) + \frac{z^2}{cR^3} \frac{\partial}{\partial t} i(t - R/c) + \frac{z}{c^2 R^2} \frac{\partial}{\partial z} i(t - R/c)$$  \hspace{2cm} (22)
Similarly, there are:

\[ \frac{\partial^2 i(t - R/c)}{\partial z \partial r} = \frac{\partial}{\partial r} \left( -z \frac{\partial i(t - R/c)}{\partial t} \right) - \frac{zr}{cR^3} \frac{\partial^2 i(t - R/c)}{\partial t^2} \]

\[ = \frac{zr}{cR^3} \frac{\partial i(t - R/c)}{\partial t} + \frac{zr}{c^2 R^2} \frac{\partial^2 i(t - R/c)}{\partial t^2} \]  

(23)

In the substitution of formula 12 and formula 20 - 22 into formula 19, there are:

\[ \frac{V(V \cdot A)}{\mu_0 c^2/4\pi} = \left[ \frac{3z^2}{R^3} i(t - R/c) + \frac{3z}{R^3} \frac{\partial i(t - R/c)}{\partial t} + \frac{rz}{cR^3} \frac{\partial^2 i(t - R/c)}{\partial t^2} \right] e_z \]

\[ + \left[ \frac{3z^2}{R^3} - \frac{1}{R^2} \right] i(t - R/c) + \left[ \frac{3z^2}{cR^3} - \frac{1}{cR^2} \right] \frac{\partial i(t - R/c)}{\partial t} + \frac{r^2}{c^2 R^3} \frac{\partial^2 i(t - R/c)}{\partial t^2} \right] e_z \]  

(24)

In the derivation of the above formula, the velocity relation expression of electromagnetic wave in the upper space needs to replace \( z \) in the expression (z = z'). At this time, the electric field has only \( e_r \) direction and \( e_\theta \) direction components, so the expression of electric field can be written as follows:

\[ c^2 = 1/\mu_0 \varepsilon_0 \]  

(25)

By formula 8, there are:

\[ -\frac{\partial A(R, t)}{\partial t} = -\frac{dc^2}{4\pi \varepsilon_0} \frac{\partial i(0, t - R/c)}{\partial t} e_z \]  

(26)

Now we can get the complete expression of the electric field. From the above discussion, we can see that the electric field has only \( e_r \) direction and \( e_\theta \) direction components, so the expression of electric field can be written as follows:

\[ \mathbf{E} = E_r \mathbf{e}_r + E_\theta \mathbf{e}_\theta \]  

(27)

In the substitution of formula 22 and formula 23 into formula 17, there are:

\[ E_r(r, \phi, z, t) = \frac{dc^2}{4\pi \varepsilon_0} \left[ \frac{3z^2}{R^3} i(t - R/c) d\tau + \frac{3z}{cR^3} \frac{\partial i(t - R/c)}{\partial t} - \frac{r^2}{c^2 R^3} \frac{\partial^2 i(t - R/c)}{\partial t^2} \right] \]  

(28)

\[ E_\theta(r, \phi, z, t) = \frac{dc^2}{4\pi \varepsilon_0} \left[ \frac{2z^2 - r^2}{R^3} i(t - R/c) + \frac{2z^2 - r^2}{cR^3} \frac{\partial i(t - R/c)}{\partial t} - \frac{r^2}{c^2 R^3} \frac{\partial^2 i(t - R/c)}{\partial t^2} \right] \]  

(29)

The current element i (R', t) dz' is placed at any point on the \( z \) axis (R' = z'). The calculation of the electromagnetic field in the upper space needs to replace \( z \) in the expression \( (z' - z') \). At this time, the expression of the electromagnetic field is as follows:

\[ dB_\phi(r, \phi, z, t) = \frac{\mu_0 dz}{4\pi} \left[ \frac{r}{cR^2} \frac{\partial i(z', t - R/c)}{\partial t} + \frac{r}{R^2} i(z', t - R/c) \right] \]  

(30)

\[ dB_r(r, \phi, z, t) = \frac{dz}{4\pi \varepsilon_0} \left[ \frac{3r(z - z')}{R^3} i(z', \tau - R/c) d\tau + \frac{3r(z - z')}{cR^3} i(z', t - R/c) \right] \]  

(31)

\[ dE_r(r, \phi, z, t) = \frac{dz}{4\pi \varepsilon_0} \left[ \frac{2(z - z')^2 - r^2}{R^3} i(z', \tau - R/c) d\tau + \frac{2(z - z')^2 - r^2}{cR^3} i(z', t - R/c) \right] \]  

(32)

The first item on the right side of formula 29 and formula 30 is the so-called electrostatic field, the second is the induction field or the intermediate field, and the third is the radiation field or the far field. The first term on the right side of formula 28 is the induction field and the second term is the radiation field. When the earth is regarded as an ideal conductor, the electromagnetic field reflected by the earth should also be considered. This part of the reflection field can be obtained by the mirror principle, and
then superimposed on the three formulas above to integrate \( z' \), then the electromagnetic field produced by lightning current \( i(z', t) \) can be obtained.

\[
B_y(r, \phi, z, t) = \frac{\mu_0}{4\pi} \left[ \frac{r}{R_0} i(z', t - R_0/c) + \frac{r}{c R_0^2} \frac{\partial i(z', t - R_0/c)}{\partial t} \right] dz' \\
+ \frac{n}{\eta} \left[ \frac{r}{R_1} \left( i(z', t - R_1/c) + \frac{r}{c R_1^2} \frac{\partial i(z', t - R_1/c)}{\partial t} \right) \right] dz' 
\]

(33)

\[
dE_x(r, \phi, z, t) = \frac{d\varepsilon}{4\pi \varepsilon_0} \left[ \frac{3r(z - z')}{R_0^3} \int_0^t i(z', \tau - R_0/c) d\tau + \frac{3r(z - z')}{c R_0^4} i(z', t - R_0/c) \right] \\
+ \frac{r(z - z')}{c^2 R_0^3} \frac{\partial i(z', t - R_0/c)}{\partial t} dz' 
\]

(34)

\[
E_y(r, \phi, z, t) = \frac{1}{4\pi \varepsilon_0} \left[ \frac{2(z - z')^2 - r^2}{R_0} \int_0^t \left( i(z', \tau - R_0/c) d\tau + \frac{2(z - z')^2 - r^2}{c R_0^4} i(z', t - R_0/c) \right) \right] \\
- \frac{r^2}{c^2 R_0^3} \frac{\partial i(z', t - R_0/c)}{\partial t} dz' 
\]

(35)

\[
E_z(r, \phi, z, t) = \frac{1}{4\pi \varepsilon_0} \left[ \frac{2(z + z')^2 - r^2}{R_0} \int_0^t \left( i(z', \tau - R_0/c) d\tau + \frac{2(z + z')^2 - r^2}{c R_0^4} i(z', t - R_0/c) \right) \right] \\
+ \frac{r^2}{c^2 R_0^3} \frac{\partial i(z', t - R_0/c)}{\partial t} dz' 
\]

(36)

In the formula, \( R_1 \) is the distance from the field point to the mirror point and \( R_0 \) is the distance from the field point to the source point. Lightning current flows on the lightning channel as on the transmission line. The current of the lightning channel is expressed by the current at the bottom of the channel. Its current conforms to the following equation:

\[
i(z', t) = i(t - z'/v) 
\]

(37)

By substituting formula 34 into formulas 31, 32 and 33, the space field generated by lightning current can be obtained.

Considering the following properties of Fourier transform:

Integral characteristics:

\[
F \left[ \int_0^t i(z', \tau) d\tau \right] = \frac{1}{j \omega} I(z', \omega) 
\]

Differential properties:

\[
F \left[ \frac{d}{d\tau} i(z', \tau) \right] = j \omega I(z', \omega) 
\]

Time-lapse property:

\[
F \left[ i(z', \tau \pm t_0) \right] = e^{\pm j \omega t_0} I(z', \omega) 
\]

Using the above properties of Fourier transform, the frequency domain formulas of space electromagnetic field can be obtained.
$$B_{y}(r, \phi, z, t) = \frac{\mu_{0}}{4\pi} \left[ \int_{0}^{\mu} \frac{r}{R_{0}^{2}} I(z', \omega) \left[ 1 + jk R_{0} e^{-jk R_{0} t} \right] dz' \right] + \left[ \int_{0}^{\mu} \frac{r}{R_{0}^{2}} I(z', \omega) \left[ 1 + jk R_{0} e^{-jk R_{0} t} \right] dz' \right]$$

$$E_{r}(r, \phi, z, t) = \frac{1}{4\pi\varepsilon_{0}} \left[ \int_{0}^{\mu} \frac{r}{R_{0}^{2}} I(z', \omega) e^{-j\omega R_{0} t/c} \right] + \left[ \int_{0}^{\mu} \frac{r}{R_{0}^{2}} I(z', \omega) e^{-j\omega R_{0} t/c} \right]$$

$$E_{z}(r, \phi, z, t) = \frac{1}{4\pi\varepsilon_{0}} \left[ \int_{0}^{\mu} \frac{r}{R_{0}^{2}} I(z', \omega) e^{-j\omega R_{0} t/c} \right] + \left[ \int_{0}^{\mu} \frac{r}{R_{0}^{2}} I(z', \omega) e^{-j\omega R_{0} t/c} \right]$$

3. Conclusion

When the lightning strike point of induction lightning is in the station, its radiation field is closely related to the location of the lightning strike point. It is necessary to verify whether it meets the electromagnetic compatibility level of intelligent components of lightning radiation field according to the actual situation.

Firstly, the simulation model of intelligent component under the influence of induction mine is studied, and the steps of simulation modelling are given. On this basis, through derivation, the calculation method and corresponding formula of lightning current radiation field on the ground of perfect conductor are given in detail.

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