The Formation of Low Surface Brightness Galaxies

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The formation of low surface brightness galaxies is an unavoidable prediction of any hierarchical clustering scenario. In these models, low surface brightness galaxies form at late times from small initial overdensities, and make up most of the faint end of the galaxy luminosity function. Because there are tremendous observational biases against finding low surface brightness galaxies, the observed faint end of the galaxy luminosity function may easily fall short of predictions, if hierarchical structure formation is correct. We calculate the number density and mass density in collapsed objects as a function of baryonic surface density and redshift, and show that the mass in recently formed low surface brightness galaxies can be comparable to the mass bound into “normal” high surface brightness galaxies. Because of their low gas surface densities, these galaxies are easily ionized by the UV background and are not expected to appear in HI surveys. Low surface brightness galaxies (LSBs) are not a special case of galaxy formation and are perhaps better viewed as a continuance of the Hubble sequence.
1. Introduction

Over the past few decades, more and more machinery has been developed to attempt to explain the distributions of galaxy luminosities and morphologies that we see today. A standard picture has developed in which galaxies form through gravitational collapse of small density perturbations. In hierarchical models, the smallest astronomical objects form early, with progressively larger objects forming at progressively later times through merging of the earlier generations of smaller objects. Galaxies form fairly late in these models, with clusters of galaxies assembling only in the most recent times. Hundreds of analytical and numerical papers have explored the predictions of hierarchical structure formation and have been able to reproduce many of the properties of normal galaxies and clusters (see for example White & Frenk 1991, Efstathiou & Silk 1983, and the recent review by White 1994). However, as discussed in these papers, hierarchical structure formation consistently predicts many more faint galaxies than are actually observed. The faint-end of the predicted luminosity function is much steeper than luminosity functions generated from catalogs of nearby galaxies.

Simultaneously, there has been increasing attention focused on the existence of an often overlooked population of low surface brightness galaxies (LSB’s). Because of the brightness of the night sky, observers are naturally biased towards detecting high-surface brightness galaxies, whose high contrast against the background makes them easily detectable. This strong selection effect, noted by Zwicky (1957) and further explored by Disney (1976), can lead to strong correlations in the properties of observed galaxies that are not intrinsic to the objects (Disney 1976, Allen & Shu 1979), most notably the universal surface brightness of spiral disks discovered by Freeman (1970). Whenever the bias against finding low surface brightness galaxies has been reduced, however, LSB’s have appeared which violate the Freeman relation (see surveys by Impey, Bothun, & Malin 1988, Bothun, Impey, & Malin 1991, Schombert et al. 1992, Schombert & Bothun 1988, Irwin, Davies, Disney, & Phillipps 1990, Turner, Phillipps, Davies, & Disney 1993, Sprayberry et al 1995, and references therein). These surveys show that LSB’s do exist and have been previously overlooked as a potentially significant species in the galaxy menagerie. The existence of elusive but omnipresent LSB’s implies that we have been attempting to solve the puzzle of galaxy formation with many pieces missing.

Thankfully, our ignorance is succumbing to a growing body of observations of low surface brightness galaxies. First, LSB’s exist at every surface brightness to which surveys have been sensitive. They have been detected down to central surface brightnesses of 26.5 mag/arcsec$^2$ in $V$ (Dalcanton 1995); fainter than this, it is difficult to separate LSB’s from true fluctuations in the optical extragalactic background due to distant clusters of galaxies.
(Shectman 1973, Dalcanton 1994). Second, LSB’s exist at every size, from minute dwarf galaxies in the Local Group, through galaxies with scale lengths typical of “normal” high surface brightness (HSB) galaxies (McGaugh & Bothun 1994), up to a handful of truly giant galaxies with scale lengths of \( \approx 50 \) kpc, typified by Malin \(^3\) (Bothun et al. 1987). Third, LSB’s are roughly a factor of two less correlated than HSB galaxies in the CfA and IRAS catalogs on scales \( > 1 \) Mpc (Mo, McGaugh, & Bothun 1994), and even less correlated on smaller scales (Bothun et al. 1993). Finally, extensive studies of LSB colors (McGaugh & Bothun 1994, Knezek 1993) and spectroscopy of HII regions (McGaugh 1994, McGaugh & Bothun 1994) show that LSB’s have rather blue colors (although with large scatter) that cannot be explained by either low metallicity or high current star formation rates; the blue colors are more readily explained by LSB’s having a relatively young mean age with a relatively long time-scale for star formation.

The properties of known LSB’s fit naturally with their likely formation scenario. Their clustering properties and long formation timescales immediately suggest that LSB’s form from the collapse of smaller amplitude overdensities than normal HSB galaxies (Mo, McGaugh, & Bothun 1994), with the exception of Malin-type LSB’s which likely form from rare, isolated \( 3\sigma \) peaks (Hoffman, Silk, & Wyse 1992). For a simple top-hat collapse of an overdense region, small amplitude peaks in the background density take longer to reach their maximum size and longer to recollapse than higher amplitude peaks, implying that galaxies that collapse from the smaller peaks will have later formation times and longer collapse times. In any theory with Gaussian fluctuations, small amplitude galaxy-sized peaks are more likely to be found in underdense regions, where their mean level is pulled down by the large scale underdensity, suggesting that objects which collapse from small amplitude peaks will be less correlated than those that collapse from larger ones (Kaiser 1984, White et al. 1987). These are exactly the properties being uncovered for LSB galaxies.

In this paper, we expand upon this idea, showing that LSB’s are a general prediction of existing hierarchical theories of structure formation. LSB’s naturally make up most of the faint end of the predicted galaxy luminosity function, which, given the severe underrepresentation of these galaxies in all existing catalogs, explains how current low

\(^3\)The exceptional giant “Malin-type” galaxies, while being fascinatingly odd, are unlikely to be the dominant type of LSB galaxy. Their large sizes imply that 2-dimensional surveys have extremely large accessible volumes for this particular type of object. The fact that so few have been found (for example, see Figure 3 of Schombert et al. (1992)) immediately suggests that their number density is small. If these objects are the result of large fluctuations in low density regions (as postulated by Hoffman, et al 1992), then they ought to be rare.
measurements of the faint end of the luminosity function can be reconciled with the theoretical predictions of large numbers of low luminosity galaxies. Furthermore, we show that the mass density in recently forming LSB’s can be comparable to the mass in “normal” galaxies, particularly in models of high bias. In §5, we conclude with a discussion the astrophysical implications of the existence of large population of LSBs, in particular considering HI surveys, Lyman-α absorbers, the faint blue galaxy excess, and the Tully-Fisher relation.

2. The Formation of LSB’s

2.1. Theoretical Assumptions

In the standard gravitational instability picture for galaxy formation, initial overdensities in the distribution of mass expand more slowly than the universe as a whole, eventually separating from the global expansion and collapsing onto themselves (Lifshitz 1946). The collapsed regions still continue to grow roughly isothermally in size and mass, as successively larger shells of material themselves collapse onto the initial overdensity and virialize, or as adjacent collapsed regions merge together. It is straightforward to track the collapse of the non-dissipative material to form a dark halo. The collapse of the baryonic matter, however, is more complicated. The baryons are dissipative, subject to pressure, heating, cooling, and feedback due to star formation. They undergo a more complicated collapse within the dark matter halo to form the stellar disks and ellipsoids.

Because the surface brightness of a galaxy will be more closely linked to the baryonic surface density than to the dark matter surface density, we must find a way to relate the easily calculated collapse of the dark matter to the collapse of the baryons. Thankfully, with a few reasonable assumptions, the structure of the dissipative baryonic matter can be simply related to the structure of the dark halo (Faber 1982). These assumptions are (i) the baryonic fraction within initial overdensities is constant and (ii) the net angular momentum that the baryonic component acquires during the collapse is ultimately responsible for halting the collapse. The first assumption is supported by the hydrodynamic simulations of Evrard, Summers, & Davis (1994), which finds a roughly constant ratio between the baryonic and dark matter for all galaxies (see their Figure 12). The second assumption appears to be valid for disk systems that are supported by rotation rather than by random motions. Although we will use the term “baryonic” interchangeably with “dissipative” for the duration of this paper, we recognize that there may be baryons which have been bound
into an early generation of stars (Population III) or black holes and thus will not participate in a dissipative collapse.

With these assumptions, if the non-baryonic component of a shell collapses to form an isothermal halo of radius $r_H$, then the baryonic component collapses to a radius of $r_\star$ where the collapse factor $\Lambda(\lambda)$ is defined to be

$$\Lambda^{-1}(\lambda) \equiv \frac{r_\star}{r_H} = \lambda \sqrt{\left(\frac{F}{2\lambda}\right)^2 + 2 - \left(\frac{F}{2\lambda}\right)}$$  \hspace{1cm} (1)$$

(Faber 1982). $F$ is the ratio of the dissipative baryonic mass to the total mass within the shell, and $\lambda$ is the dimensionless spin parameter

$$\lambda = \mathcal{L}|E|^{1/2} G^{-1} M^{-5/2}$$  \hspace{1cm} (2)$$

(Peebles 1969). Here, $\mathcal{L}$ is total angular momentum of the luminous and non-luminous matter, $E$ is its energy, and $M$ its mass. Because the baryonic component collapses to a fixed fraction of the size of the non-baryonic component (for a given value of $\lambda$), we may now use the properties of the dark matter halos to predict the properties of the visible matter.

If we further assume that galaxies, on average, have a similar ratio between their baryonic mass and their luminosity (i.e., a similar efficiency in converting baryons to light-producing stars), then we now have a way to relate the surface density of the dark halo to the surface brightness of the luminous galaxy. While other scenarios may argue that the low surface brightness of LSB’s is a result of baryonic physics that we do not consider in this calculation (for example, low star-formation efficiencies due to low metallicity or the absence of tidal triggering, gas loss through supernovae explosions or ionization by the UV background), we prefer to explore the straightforward assumption that a galaxy with fewer baryons per unit area will in general form fewer stars per unit area. For the purposes of this paper, we relegate this rich astrophysics to being only a perturbation to a global proportionality between baryonic surface density and surface brightness. The results will show that even with this naive assumption, the bulk properties of LSBs can be understood as the inevitable result of the properties of low surface density galaxies.

We would venture that our neglect of detailed physical processes within the galaxies causes us to systematically overestimate the surface brightness of LSBs; almost all mechanisms for reducing the star formation efficiency are likely to be more effective in low surface density galaxies than in high surface density ones. First, if star formation requires a large reservoir of neutral gas, then the extragalactic ultraviolet background, which will
completely ionize low surface density galaxies (§5.1.), may suppress or shut off the formation of stars in these galaxies (Babul & Rees 1992). Second, if star formation is associated with disks becoming Toomre unstable to the formation of spiral structure (implying that the Toomre stability parameter \( Q \propto \sigma \kappa / G \Sigma \) is smaller than some constant, where \( \sigma \) is the velocity dispersion of the gas, \( \Sigma \) is the surface density of the disk, and \( \kappa \) is the epicyclic frequency, set by the disk structural parameters (see Kennicutt 1989)), then one would expect lower surface density disks to be less unstable for star formation. Observations by van der Hulst et al. (1993) find that LSB galaxies do have HI surface densities that fall below the critical density implied by \( Q \) and are about a factor of 2 lower than the HI surface densities of HSB galaxies. Third, if star formation is enhanced by tidal interactions with nearby galaxies, then galaxies that have fewer close neighbors will have lower star formation efficiencies. We will argue that, as seen observationally and discussed by (Mo, McGaugh, & Bothun 1994), low surface density galaxies should have lower correlation amplitudes and thus fewer close neighbors, which would once again lead to lower surface brightnesses\(^4\). Fourth, low surface density, low mass galaxies are more likely to lose their gas through supernova driven winds than high surface density, high mass galaxies. This both shuts off star formation prematurely and evolves the galaxy towards larger sizes through sudden mass loss and subsequent revirialization (Dekel & Silk 1986, DeYoung & Heckman 1994) – both mechanisms which lead to lower surface brightnesses for low surface density objects.

In spite of our previous assertion to the contrary, there is one piece of baryonic physics which we cannot ignore, namely pressure. We are ultimately interested in the gravitational collapse of baryons, but a collapsing baryonic gas both the inward tug of gravity and a resistive force due to its own internal pressure. For sufficiently large masses, the self-gravity of the combination of baryons and dark matter is strong enough for the effects of pressure to be negligible. However, for small masses the pressure is sufficient to support the baryonic gas against collapse, in spite of the additional gravitational pull provided by the collapsed dark matter halo. Because low surface density galaxies are more likely to have lower total masses than high surface density galaxies, we must consider the mass scales on which the effects of pressure become important; we will do so in §3.1.

\(^4\)Note that the case for causality is a bit ambiguous. The impression that close neighbors induce higher star formation rates could be an artifact of low surface density galaxies being less correlated than high surface density ones, with no dependence of the star formation efficiency on environment.
2.2. Properties of LSBs

We now examine the conditions that lead to variations in baryonic surface density (and thus surface brightness) among galaxies. We choose to compare the properties of galaxies within a fixed radius rather than at a constant mass scale.

At constant mass, comparing LSB’s to HSB’s is a comparison between galaxies of widely different scale lengths: high surface brightness dwarf ellipticals would be compared to giant low-surface brightness galaxies. By comparing the surface brightness of galaxies at constant mass, one is saying that what distinguishes LSB’s from HSB’s is that LSB’s have abnormally large scale lengths for their mass. Instead, we will compare galaxies of different masses, but with similar scale lengths. This more closely mimics how one draws the distinction between LSB’s and HSB’s. We therefore define the effective surface density, $\bar{\Sigma}$ to be the mean surface density within some radius $r^*$,

$$\bar{\Sigma} = \frac{M(r < r^*)}{\pi r^2}.$$  

where $M(r < r^*)$ is the mass of the baryonic component within a radius $r^*$. We can relate $M(r < r^*)$ to the total mass of the halo,

$$M(r < r^*) = F M_{\text{tot}}(r < r_H) = F \frac{4\pi}{3} \rho_0 r_0^3,$$

where $\rho_0$ is the current density of the universe and assuming that the dark matter that initially was in a shell of comoving radius $r_0$ has collapsed to a virial radius $r_H$, while baryons from the same shell have collapsed and dissipated to a radius $r^*$, where they are supported by their angular momentum. Equations 3 and 4 imply that $r_0$ is determined by the choice of $\bar{\Sigma}$ and $r^*$:

$$r_0(\bar{\Sigma}, r^*) = r^* \left( \frac{3\bar{\Sigma}}{4F \rho_0 r^*_0} \right)^{1/3}.$$  

In turn, the redshift of collapse can be determined from $r_0$ and $r^*$ as follows. In a spherical symmetric top-hat collapse (Gott and Gunn 1972, Peebles 1980), the dark matter virializes at a radius of half its size at maximum expansion, the density within a shell at maximum expansion is $(3\pi/4)^2$ times the background density, and the collapse time is twice the time of maximum expansion. Therefore, for top-hat collapse in a $\Omega = 1$ universe,
where \( z_c \) is the redshift at which the shell has collapsed and virialized. Equations 4 and 7 may be substituted into equation 3 to solve for the redshift of collapse of a galaxy with mean baryonic surface density \( \bar{\Sigma} \) within \( r_* \):

\[
z_c(\bar{\Sigma}, r_*) = \left( \frac{\bar{\Sigma}}{24\pi^2 F \rho_0 r_*} \right)^{1/3} \Lambda^{-1}(\lambda) - 1.
\]

Alternatively, this equation may be rearranged to express \( \bar{\Sigma} \) in terms of \( z_c \):

\[
\bar{\Sigma} = 24\pi^2 F \rho_0 r_* \Lambda^3(\lambda) (1 + z_c)^3,
\]

which immediately implies that low surface density galaxies form at later times than high surface density galaxies with the same angular momentum. If, as we have asserted, surface density is proportional to surface brightness, then low-surface brightness galaxies have formed more recently than high-surface brightness galaxies of similar size. This also suggests that for galaxies of a given size and angular momentum, there is a minimum surface density \( \bar{\Sigma}_0 = 24\pi^2 F \rho_0 r_* \Lambda^3(\lambda) \) below which few galaxies should exist.

To derive equation 9, we have ignored infall due to the collapse of shells larger than \( r_0 \) after \( z_c \). This is not a bad assumption for the dark matter; the larger shells virialize at larger radii, and thus the increase in mass occurs primarily at large radii. Baryons are dissipative, however, and can collapse until halted by their angular momentum. Including the effects of infall at late times would only exacerbate the surface brightness distinction between early-forming HSB’s and late-forming LSB’s, as HSB’s would have more time to accrete mass and further increase their surface brightness. Thus, late-time infall likely enhances rather than erases the correlation between formation time and surface brightness. Simulations of Evrard et al. (1994) justify our neglect of late time infall as they find that the rate at which galaxies accrete mass slows dramatically at late times.

It is revealing to note that we have implicitly made the assumption that there is a linear correspondence between a galaxy’s mean surface brightness and its luminosity (eq. 3, assuming a single mass-to-light ratio). This is an artifact from associating a fixed physical scale \( r_* \) with all galaxies; for example, if instead one were to consider the luminosity within some isophotal level, the linearity between surface brightness and luminosity would break
down. There are also additional sources of scatter in the relationship which we have ignored: variations in angular momentum, variations in collapse time due to changes in the mean local overdensity, variations in the mass-to-light ratio with mass, and deviations from spherical collapse. Regardless of these complications, it is difficult to conceive of eq. \( \lambda \) not being a reasonable first-order approximation to the relationship between surface density (i.e. brightness) and mass (i.e. luminosity); certainly the trend is displayed when considering spirals and ellipticals. The relationship between surface brightness and luminosity has immediate implications for redshift surveys. If a redshift survey has a limiting surface brightness for spectroscopy, set either by design or by the limits of the spectrograph, then there will be an associated limiting luminosity to the galaxies that are observed. Decreasing the magnitude limit of a spectroscopic survey without decreasing the surface brightness limit will lead one to pick up galaxies of the same luminosity as were observed previously, only further away. This will inevitably lead to underestimates in the derived faint-end slope of the luminosity function, as well as variations among different surveys which have different, often unstated, surface brightness limits.

### 2.3. Angular Momentum and Profiles of LSBs

In addition to suggesting that galaxies that form late are more likely to be LSB’s, Equation \( \lambda \) also implies that LSBs could be galaxies whose baryons have not collapsed much further beyond the non-baryonic dark halo. Galaxies whose baryons have collapsed very little must have either acquired large amounts of angular momentum during their formation, or have been inefficient at transporting angular momentum outwards during their dissipative collapse. Both these traits can be associated with galaxies that form at lower redshifts.

First, galaxies that formed late are unlikely to have extremely low angular momenta. Analytic calculations and numerical simulations have shown that galaxies acquire angular momentum through tidal torquing. While systems that form very early, such as globular clusters and (presumably) most ellipticals, may have collapsed so quickly that they had little time to acquire angular momentum, late forming galaxies have had ample time to be torqued by nearby galaxies and by the shear in the global gravitational field. There have been suggestions of this trend in numerical work, but it has not yet been well quantified, particularly for the low amplitude peaks that are associated with LSBs. Eisenstein & Loeb (1994, private communication) find that, on a given mass scale, the value of \( \lambda \) is anti-correlated with peak height, leading to a correlation between angular momentum and
collapse times (see eq. 10). They suggest that the relation arises because the separate axes of an object with a small overdensity collapse at very different times, giving a larger quadrupole moment and making the object more susceptible to external torques.

Second, the trend from ellipticals to spirals suggests that early forming galaxies have been particularly efficient at shedding angular momentum. While ellipticals appear to have collapsed dramatically, they have very low angular momenta, suggesting that large amounts were transported outwards during collapse. Spiral disks, which are in general younger systems than ellipticals, have large angular momenta, suggesting that the dramatic angular momentum transport that was operating during the epoch of elliptical formation was much less spectacular during the formation of spiral disks. This trend between angular momentum transport and formation epoch may have several different origins. It could reflect changes in the galaxy environment with time (e.g. higher external pressure, more gravitational shear, evolution in the triaxiality of halos) or it could reflect processes within the galaxy halo that depend upon the duration of collapse time (e.g. feedback from star formation, interactions between the halo and baryons). However, regardless of the origins of the trend, it can be extrapolated to the present day to suggest that any galaxies that are currently forming are more likely to have inefficient angular momentum transport. If this were not true, then one might expect a large population of very young, blue elliptical galaxies to be forming today. Except for the very smallest dwarf galaxies ($M_B > -14$), the young blue galaxies that are observed tend to be irregular galaxies and be supported by rotation (Lo, Sargent, & Freeman 1993, Gallagher & Hunter 1984, and references therein). We note that classical ellipticals are exempted from much of the discussion in this paper. Because they have obviously not conserved angular momentum during their collapse, they do not obey equation 1 or any equation that follows from it.

When angular momentum transport is inefficient, the distribution of angular momentum per unit mass is likely to be constant during collapse, which leads to the formation of exponential disks (Gunn 1982). Thus, if LSBs formed late, as suggested by equation 3 and observed stellar populations (Knezek 1993, McGaugh 1994, McGaugh & Bothun 1994), then they should be disk systems with exponential profiles. For non-dwarf LSBs, exponential profiles are an excellent fit to the stellar distribution in both infrared and visible bands (Davies, Phillips, & Disney 1990, McGaugh & Bothun 1994, James 1994, McGaugh et al 1995, Dalcanton 1995). Alternatively, this argument can be inverted and the ubiquity of exponential disks in LSBs can be used to argue for inefficient angular momentum transport and large values of $\lambda$.

With this view of LSB’s, one can interpret LSB’s as being an extension of the Hubble sequence. At one end of the sequence are ellipticals, consisting entirely of highly collapsed,
low angular momentum systems with short formation timescales. In the middle of the sequence are spiral galaxies, with an increasing fraction of high angular momentum disk stars, an increasing timescale for star formation, and lower masses. LSB’s could be the obvious next step in the sequence, with even longer star formation times, higher angular momenta, and smaller masses and bulges. In the absence of strong arguments for why disks should have stopped forming past the Sc end of the Hubble sequence, and in the undeniable presence of rapidly rotating low surface brightness disks (Schombert et al 1992, Sprayberry et al 1995), it is difficult to argue that the Hubble sequence truly ends at its standard terminus.

### 2.4. Correlations of LSBs

If LSBs formed at late times from low amplitude fluctuations, then the LSBs should be less correlated than HSBs, which formed from earlier from higher amplitude fluctuations. Thus, LSBs extend the morphology-density relationship seen within elliptical and spirals: high overdensities implies early formation and high correlation, which in turn implies high surface brightness. Recasting equation \[\delta = \delta \rho / \rho\] in terms of \[\delta = \delta / \rho\], the initial overdensity from which the galaxy collapsed,

\[
\Sigma \propto F r_\ast \Lambda^3(\lambda) \delta^3.
\] (10)

Here, we assume for simplicity, \[\Omega = 1\] and thus, \[\delta \propto (1 + z)^{-1}\]. In a Gaussian theory, the probability of finding two peaks of amplitudes between \(\nu\sigma\) and \((\nu + \epsilon)\sigma\) within a sphere of radius of \(r\) is:

\[
P_2 = \frac{1}{2\pi} \frac{\epsilon^2 \sigma^2}{\sqrt{\xi(0)^2 - \xi(r)^2}} \exp \left[ -\frac{\nu^2 \sigma^2}{\xi(0) + \xi(r)} \right],
\] (11)

where \(\epsilon \ll \nu\) and \(\sigma^2\) is the variance of the Gaussian fluctuations (Kaiser 1984). Recall \(\xi(0) = \sigma^2\) and that this assumption of Gaussian fluctuations ignores the non-linear evolution of the density field. As the probability of finding a single peak within the sphere, \(P_1\) is \((\epsilon/\sqrt{2\pi\sigma}) \exp(-\nu^2/2)\), the correlation of peaks of amplitude \(\nu\) is

\[
\xi_\nu(r) = \frac{P_2}{P_1^2} - 1 = \frac{\xi(0)^2}{\sqrt{\xi(0)^2 - \xi(r)^2}} \exp \left[ \frac{\nu^2 \xi(r)}{\xi(0) + \xi(r)} \right] - 1
\] (12)
At large separation, where $\xi(r) \ll \xi(0)$, then

$$\xi_\nu(r) = \frac{\nu^2}{\sigma^2} \xi(r)$$  \hspace{1cm} (13)$$

Thus, the proportionality between fluctuation amplitude and correlation strength is expected not only for the high amplitude peaks that form clusters (Kaiser 1984), but for all collapsed objects. Equations 13 and 10 predict that LSBs are less correlated than HSBs, consistent with the Mo, McGaugh, & Bothun (1994) analysis of observed LSB and HSB samples. Further evidence of this reduced correlation may be manifested in weak correlation of faint galaxies (Koo & Szalay 1984, Stevenson et al. 1985, Efstathiou et al. 1991, Pritchett & Infante 1992, Bernstein et al. 1993), given that the deep observations are more sensitive to LSBs than local surveys (Phillips, Davies, & Disney 1990, McGaugh 1994, Ferguson & McGaugh 1995).

The relationship between $\bar{\Sigma}$ and $\delta$ given in equation 10 also explains why differences in the correlation function of LSB’s and HSB’s have only recently become apparent with the development of new samples of truly low-surface brightness galaxies. Because surface density is a strong power of the initial overdensity, only a sample with a large range of surface densities would manifest properties that depend on a weaker power of $\delta$. The amplitude of the correlation function traced by galaxies is proportional to $\delta^2$ (eq. 13), and thus proportional to $\bar{\Sigma}^{2/3}/\Lambda^2(\lambda)$. This suggests that to detect a 50% difference between the correlation amplitudes for HSB’s and for LSB’s, as was seen in Mo, McGaugh, & Bothun (1994), one needs the LSB sample to be cleanly separated in surface brightness from the HSB sample by at least one magnitude per square arcsecond, and possibly more if low surface brightness galaxies have smaller values of the collapse factor $\Lambda(\lambda)$. The required range of surface brightness did not exist in earlier work comparing the CfA galaxies to galaxies drawn from the UGC catalog (Thuan, Alimi, & Gott 1991), nor in the work of Bothun et al. (1986) where the larger range in surface brightness was swamped by large uncertainties in the measurement of the surface brightnesses. Only by pressing surveys for LSB’s to the lowest possible surface brightnesses would one hope to uncover a population of galaxies encroaching upon the voids.

3. The Surface Brightness Distribution of Galaxies

With the formulas developed above, we are now in a position to calculate the number density of galaxies as a function of surface density and redshift. We will use the
Press-Schechter formalism, which uses linear theory to calculate the number density of regions of initial radius \( r_0 \) whose extrapolated linear overdensities are sufficiently large that the region has in fact undergone a non-linear collapse to form a virialized object at redshift \( z_c \) (Press & Schechter 1974). The formalism assumes that the initial fluctuations are Gaussian and that the overdense regions undergo a spherical top-hat collapse which stops when the systems virialize at a radius of half the radius at maximum expansion (see Peacock & Heavens 1989, Bond et al. 1991, & Bower et al 1991 for discussions of this formalism). With these assumptions, the number density of objects with initial radius \( r_0 \) that have just collapsed at \( z_c \) is,

\[
n_{PS}(r_0, z_c) = -\left(\frac{2}{\pi}\right)^{1/2} \left[ \frac{\delta_c (1 + z_c)}{\Delta^2(r_0)} \frac{\partial \Delta(r_0)}{\partial r_0} e^{-\frac{\delta_c^2 (1+z_c)^2}{2\Delta^2(r_0)}} \right] \frac{\rho_0}{M(r_0)},
\]

(14)

where \( M(r_0) \) is the mass within an initial radius \( r_0 \) and \( \Delta(r_0) \) is the variance in density within shells of radius \( r_0 \) for a power spectrum of fluctuations \( P(k) \), defined as

\[
\Delta^2(r_0) = \int_0^\infty 4\pi k^2 \, dk \, P(k) \, W^2(kr_0)
\]

(15)

where \( W(x) = 3(\sin x - x \cos x)/x^3 \) and \( \delta_c \) is the extrapolated linear overdensity that the clump would have had at \( z_c \) if it had not collapsed, taken to be 1.68 to agree with numerical simulations. For a CDM power-spectrum, we use

\[
P(k) = 1.94 \times 10^4 b^{-2} k (1 + 6.8k + 72k^{3/2} + 16k^2)^{-2} \text{Mpc}^3,
\]

(16)

for \( H_0 = 50 \text{ km/s/Mpc} \), taken from Davis et al. (1985). The bias parameter, \( b \) is defined to be the ratio between the variances of the galaxy and the mass fluctuations within 16 Mpc radius spheres. This particular approximation is accurate to 10% for scales between 0.05 Mpc and 40 Mpc, and is too high on small scales.

The collapsed objects traced by equation (14) never stop increasing their mass; they continue to accrete matter from progressively larger shells which continue to collapse around and merge into the object. Therefore, something that we might identify as a single object is associated with a different mass and different \( r_0 \), depending upon the redshift at which we choose to identify it. If we associate these “collapsed objects” with the more prosaic term “galaxy”, we immediately see how difficult it is to define exactly when a galaxy has formed. In the absence of any physics beyond gravity, there is no particular scale that naturally selects the criteria for labelling a galaxy.
As motivated by our discussion of surface brightness above, we will choose a radius criterion for deciding when a galaxy forms; we will say that a galaxy has formed when the baryons originating in some particular shell collapse to a final size $r_\ast$. For a given surface density at the time of formation, the choice of $r_\ast$ fully specifies both the redshift at which the galaxy formed (eq. 8), and the initial size of the shell from which the galaxy formed (eq. 5). Equation 7 can be used with equation 14 to calculate the number density of galaxies that have already collapsed by redshift $z$ from an initial radius $r_0$ to a final radius $r_\ast$, with spin parameter $\lambda$:

\[ n_r(r_0, \lambda|z) = n_{PS}(r_0, z_c(r_0, \lambda)) p(\lambda|z_c(r_0, \lambda)) \left| \frac{\partial z_c}{\partial r_0} \right| \Theta(z_c(r_0, \lambda) - z), \]

where $p(\lambda|z)$ is the probability that a galaxy forming at redshift $z$ has a spin parameter $\lambda$, and where $\Theta(x) = 1$ if $x > 0$, and equals zero otherwise. For the sake of notational simplicity, we do not make the dependence on $r_\ast$ explicit.

Transferring variables from $r_0$ to $\bar{\Sigma}$ (eq. 5), and integrating over $\lambda$ gives the total number density of galaxies that have formed by $z$ with a given surface density $\bar{\Sigma}$:

\[ n(\bar{\Sigma}|z) = \int_0^\infty n_r\left( r_0(\bar{\Sigma}), \lambda|z < z_c\left( r_0(\bar{\Sigma}), \lambda\right) \right) \left| \frac{\partial r_0}{\partial \bar{\Sigma}} \right| d\lambda, \]

where $z_c\left( r_0(\bar{\Sigma}), \lambda\right)$ reduces to equation 8.

Analytical calculations and numerical simulations find that large protogalaxies typically form with values of $\lambda \sim 0.05 \pm 0.05$ (Peebles 1969, Barnes & Efstathiou 1987, Warren et al. 1992, Steinmetz & Bartelmann 1994, Eisenstein & Loeb 1994). Instead of choosing one particular, highly uncertain model for $p(\lambda|z)$, we find it more illustrative to assume single fixed values for $\lambda$: $p(\lambda|z) = \delta(\lambda - \lambda_0)$. In §2.3 we argued that LSBs can be treated as late-forming, high angular momentum extensions to the Hubble sequence, and as such we should consider values of $\lambda$ that are appropriate for the end of the Hubble sequence and beyond. Following an argument given by Faber (1982), the measured fraction of baryonic mass within the optical radius ($\equiv X$), can be used to estimate the factor by which the baryons have collapsed. For collapse within a fixed, non-dissipative, isothermal halo, the baryonic collapse factor is

\[ \Lambda = \left[ \frac{X}{1 - X} \right] \left[ \frac{1 - F}{F} \right] \]
For the values of $X$ given in Faber’s Table 3 for Sa, Sc, and Irr galaxies\(^5\), the collapse factors are roughly 100 for the Sa’s, 10-15 for the Sc’s, and upper limits of 7-11 for the Irr’s, which corresponds to $\lambda = 0.02, 0.06 - 0.09, > (0.09 - 0.12)$, assuming $F = 0.05$. Based on this, we examine cases where $\lambda = 0.075, 0.15$, which are the 90% and 99% percentiles of the distribution of $\lambda$ found numerically by Eisenstein & Loeb (1994) for $> 2.5\sigma$ peaks with $M = 10^{12} \text{M}_{\odot}$. Note that types Sc and later make up 30% of the number density of galaxies with $L/L_* > 0.1$ (Marzke et al 1994), a larger fraction than would be implied by the distribution of $\lambda$ in Eisenstein & Loeb (1994). This suggests that numerical simulations may either underestimate the fraction of high-spin halos or that baryons acquire more specific angular momentum during their collapse than does the dark matter. It could also be taken as evidence that galaxies with types later than Sc form smaller overdensities than were assumed in Eisenstein & Loeb (1994), and thus form in greater numbers and with larger values of $\lambda$.

The resulting distributions are plotted in the first two columns of Figures 1 & 2 for $b = 1 & 2.5$, $z = 0 - 5$ (light to dark) with $\lambda = 0.03, 0.075, 0.15$ from the top row to the bottom. To interpret the distributions, it is necessary to first establish some tie with “normal” galaxies (by which we mean high surface brightness, nearby, cataloged galaxies with $L/L_* > 0.1$) through their redshift of formation, angular momenta, surface densities, and number densities. First, we choose to associate normal galaxies with formation times of $z = 2$ or earlier. The lookback time to $z = 2$ is $7 - 8 h_{75}^{-1}$ Gyr for $\Omega = 0.2 - 1$ and $h_{75} = H_0/75 \text{km/s/Mpc}$. In the Milky Way, the ages of F and G dwarfs show that a significant population of stars in the disk were formed over 8 Gyr ago from high metallicity gas, suggesting that the disk of the galaxy had already settled into place to some degree by $z = 2$ (Barry 1988, Carlberg et al. 1985, Twarog 1980), bolstering our identification of this epoch as being the one by which the mass of galaxy disks had been assembled. Second, we chose $\lambda = 0.075$ to be the fiducial case for identifying “normal” galaxies because it roughly demarks the maximum spin angular momentum of the most numerous classes of galaxies (Sd and earlier). Galaxies that form with smaller values of $\lambda$ will all form with larger surface densities, and thus a galaxy that forms with $\lambda = 0.075$ can be used to define the limit where normal surface brightnesses end, and low surface brightnesses begin. Furthermore, by choosing a value of $\lambda$ that corresponds to galaxies with very small spheroidal components, we hopefully avoid the need to determine the fraction of baryons that wind up in the disk rather than the bulge.

We use the Milky Way to estimate the surface density associated with normal galaxies.

\(^5\) drawn from Faber & Gallagher (1979), Thuan & Seitzer (1979), and Roberts (1969)
If the baryonic surface density in the solar neighborhood is $75 \, M_\odot/\text{pc}^2$, the mean baryonic surface density of the Milky Way is roughly $200 - 300 \, M_\odot/\text{pc}^2$ for $r_* = 7.5 - 10 \, \text{kpc}$, including a 20% correction for the mass in the bulge. (For a disk mass-to-light ratio of 5, this gives the correct Freeman disk central surface brightness.) In Figures 1 & 2 we have chosen $r_*$ such that for $\lambda = 0.075$ the integrated number density of galaxies that have collapsed by $z = 2$ is roughly the total number density of normal galaxies measured today (corresponding to the horizontal dashed and dotted lines in the second columns of Figures 1& 2. Note that this choice of $r_*$ automatically gives surface densities for the galaxies forming at $z = 2$ that are in good agreement with the Milky Way value. Galaxies that form before $z = 2$ with larger surface densities are only a small fraction of the number density at $z = 2$; 75-99% of the galaxies that form by $z = 2$ (for $b = 1 - 2.5$) have collapsed between $z = 2$ and $z = 3$.

The particular form of $n(\bar{\Sigma}|z)$ seen in Figures 1 & 2 arises because, one, we’ve assumed that the surface density of a galaxy doesn’t change after it forms, and two, galaxies of a given surface density all form at the same redshift. With these two assumptions, the only change in the distribution of surface densities with redshift is the creation of new galaxies at increasingly lower surface brightnesses (eq. [3]). Because low surface density galaxies form from low amplitude peaks, which are more common than the high amplitude peaks which are thought to form normal galaxies (Eq. [10]), there is a dramatic increase in the number density of galaxies with decreasing surface density. Depending on the choice of bias, there are $10 - 100$ times more young low surface density galaxies than normal galaxies, assuming that all galaxies form with $\lambda = .075$.

If instead we had assumed that all galaxies form with a much smaller value of $\lambda$, say $\lambda = 0.03$, then the total number density of galaxies formed by $z = 0$ would have been a factor of 10 below the observed number density of normal galaxies. Because galaxies with low angular momenta have very large collapse factors, they must have collapsed from very large shells ($r_0 >> r_*$). Such shells are rare and have long collapse times, causing the reduced number density seen if Figures 1& 2.

This points to a hidden assumption in the calculation. By using the Press-Schechter formalism, we are expressly counting galaxies at the time when their halo collapses and virializes. Thus, there is an implicit assumption that the collapse of the baryons occurs simultaneously with the collapse of the halo. For the collapse of the baryons and the dark matter to be asynchronous, the baryons must collapse significantly faster than the free-fall time from the radius of maximum expansion ($= 2\Lambda r_*$). Such large amounts of dissipation could only occur after gravity had organized the baryons into a dense enough system for radiative shocks and star formation to take place, in other words, after a gravitational
collapse time; this suggests that asynchronous collapse of the baryons and halo is not a problem for the Press-Schechter system of accounting. On the other hand, a fragmentary collapse, where baryons clump during infall, could speed both dissipation and angular momentum loss; this may be the formation pathway for spheroids, perhaps leaving globular clusters as debris.

While from the first two columns of Figures 1 & 2 it is clear that galaxies of very low surface densities form in large numbers, it is ambiguous whether or not the mass (or luminosity) of the galaxies has any leverage when weighed against the mass in normal galaxies. The baryonic mass density, \( \rho(\Sigma|z) \), may be calculated by multiplying \( n(\Sigma|z) \) by the baryonic mass of the galaxies formed with surface density \( \Sigma \) (see equation (3)). Integrating \( \rho(\Sigma|z) \) to get the cumulative distribution of baryonic density, \( \Gamma(\Sigma|z) \):

\[
\Gamma(\Sigma|z) \equiv \int_0^\Sigma \rho(\Sigma'|z)/F \rho_c d\Sigma' \tag{20}
\]

\[
= \int_0^\Sigma n(\Sigma'|z) * M(\Sigma'|)/F \rho_c d\Sigma'. \tag{21}
\]

The resulting distributions for \( \rho(\Sigma|z) \) and \( \Gamma(\Sigma|z) \) are plotted in the last two columns of Figures 1 & 2. Note that the cumulative density \( \Gamma(\Sigma|z) \) does not always integrate to 1 because \( \rho(\Sigma|z) \) does not include mass from shells that collapse to radii outside of \( r_* \) or that have not yet collapsed to \( r_* \). For small \( \Lambda \) (short collapse times), the cumulative density is much larger, as there has been ample time for most overdensities to collapse. The small normalization problem for low \( \Lambda \) models reflects the inaccuracy of the approximation to the CDM power spectrum at small \( r_0 \).

One may read off the density contributed by galaxies in a particular range of surface density; the change in \( \Gamma(\Sigma = \infty|z) \) between any \( z_1 \) and \( z_2 \) is the mass density in galaxies between the corresponding minimum surface densities at those redshifts, \( \Sigma_0(z_1) \) and \( \Sigma_0(z_2) \). Therefore, we may read off the mass density in low surface density galaxies as \( P(\infty|z = 0) - P(\infty|z = 2) \), and compare it to the mass density in normal galaxies \( P(\infty|z = 2) \). The mass density of galaxies with “sub-normal” surface density is comparable to or significantly greater than the mass density in normal galaxies. In particular, for models with high bias the fraction of the total mass density that is tied up in LSB’s is dramatically large. This suggests that there can easily be as much mass tied up in a population of low surface brightness galaxies as there are in normal galaxies! Thus, LSBs have all the properties that a theorist could hope for: they are almost entirely unconstrained by observations although they are known to exist, and they may contain a large fraction of mass of the universe.
To develop an appreciation for the impressively low surface brightness implied by Figures 1 & 2, consider the spread in the surface brightnesses of recently formed galaxies. If $350 \, M_\odot/pc^2$ is the baryonic surface density for normal spirals, then Figures 1 & 2 show that galaxies formed between $z = 2$ and the present may have surface densities that are several hundred times smaller than normal galaxies. Assuming a linear relationship between surface density and surface brightness, this corresponds to a $6$ mag/arcsec$^2$ range in surface brightness, implying a cosmologically significant population of galaxies with central surface brightnesses in $V$ of $27$ mag/arcsec$^2$! This range in surface brightness is being probed in a survey which should soon yield interesting measures of the density of LSB galaxies (Dalcanton 1995). Unfortunately, at much lower surface brightnesses the signal from LSBs may easily be swamped by fluctuations from very distant cosmologically dimmed clusters of galaxies (Dalcanton 1995); this will strongly limit the ability of observations to reveal galaxies with surface brightnesses much fainter than $V = 27$ mag/arcsec$^2$.

3.1. On the Effects of Pressure

In the preceding calculation of the number density of galaxies as a function of surface density and formation time, we made an implicit assumption that the baryons will collapse along with the dark matter halos. However, unlike the dark matter (presumably), a baryonic gas experiences pressure forces in addition to gravitational forces. For small masses, internal pressure may support the gas against collapse, in spite of the inward gravitational pull of the collapsed dark matter halo (Jeans 1929).

The question arises, then, of whether or not low-surface brightness galaxies (which we postulate to have low mass) are capable of collapsing to form stars at all. This question may be addressed by considering the detailed interaction between pressure, gravitational collapse, and the numbers of collapsed objects, effectively extending the Press-Schechter formalism to include the effects of pressure (Babul 1987, Shapiro et al 1994). However, this level of complication may be avoided by rephrasing the above question to ask: what is the range of surface densities for which the gas is bound to the dark matter halo? The gas will be bound if the gravitational binding energy is greater than its thermal energy. Assuming that the gas is fully ionized, it will be gravitationally bound to the dark halo if

$$\frac{m_p V_c^2}{2} > kT,$$

where $V_c$ is the circular velocity of the dark matter halo, $m_p$ is the proton mass, and $T$ is
the temperature of the gas. The circular velocity is independent of radius or mass scale if the dark matter collapses to form an isothermal sphere, and can be expressed as

\[ V_c^2 = \frac{(18\pi^2)^{1/3}}{2} \left( 1 + z_c \right) H_0^2 r_0^2 \]  

(Narayan & White 1988). Using Eqns. 5 & 9 to eliminate \( z_c \) and \( r_0 \) in favor of \( \Sigma \) and \( r_\ast \), we can express the condition for collapse as a baryonic surface density threshold:

\[ \Sigma > \frac{2kTF\Lambda}{\pi m_p r_\ast G} \]  

where \( G \) is the gravitational constant. Note that the resulting threshold surface density has no dependence on \( z_c \) or \( r_0 \). This reflects that the gravitational potential well of a dark matter halo depends only on its internal size scale and mass, which are fully determined by \( \Sigma \), \( \Lambda \), and \( r_\ast \).

The temperature of the gas in the final halo depends on its ability to cool during collapse. If the cooling time of the gas is shorter than the dynamical time of the system, then the gas cools to roughly \( T \approx 10^4 \) K at which point cooling becomes highly inefficient. Using Figure 3 of Blumenthal et al (1984), we can examine where forming LSBs are likely to lie with regard to the “cooling curve” – the locus on the temperature-density plane that delineates the region of parameter space where cooling is rapid compared to the dynamical time (Rees & Ostriker 1978). Blumenthal et al plot the equilibrium positions of collapsed, non-dissipative structures (i.e. dark matter halos) on this plane, as a function of the amplitude of the initial overdensity (0.5\( \sigma \) - 3\( \sigma \)). Even for the small amplitude overdensities which are the likely LSB precursors, the gas in halos with masses between roughly a few times \( 10^8 \, M_\odot \) and \( 10^{12} \, M_\odot \) is capable of rapid cooling to \( T \approx 10^4 \) K. Associating normal galaxies with halo masses of \( > 10^{10} \, M_\odot \), and taking a naive proportionality between mass and surface brightness (Eqn. 3), there is a factor of roughly 100 in surface density over which cooling is efficient. For a direct proportionality between surface density and surface brightness, this corresponds to a disk central surface brightness of 5 magnitudes below the Freeman value, or \( \mu_0(V) = 26.5 \, \text{mag/arcsec}^2 \). Therefore, over an enormous range of surface brightnesses, we may assume that the baryonic gas in the halo cools to \( T \approx 10^4 \) K.

Substituting this temperature into Eqn. 24, we find a baryonic surface density threshold of
\[
\Sigma > 0.61 \text{ M}_\odot/\text{pc}^2 \left( \frac{\Lambda}{10} \right) \left( \frac{F}{0.05} \right) \left( \frac{10 \text{ kpc}}{r_\ast} \right) \left( \frac{T}{10^4 \text{ K}} \right).
\]

(25)

Comparing to Figures 1&2, this threshold lies below the lowest surface density expected to be collapsing today and therefore, none of the baryons in these galaxies should be pressure supported against collapse. There is a slight problem with self-consistency, however, in our assumption of a $10^4 \text{ K}$ gas temperature throughout this range of surface density. The assumption of efficient cooling breaks down for surface densities that are roughly a factor of 100 below the surface density of normal galaxies. In \S\ 3, we associated normal galaxies with surface densities of a few times $10^2 \text{ M}_\odot/\text{pc}^2$, and thus the regime of efficient cooling breaks down at a few $\text{ M}_\odot/\text{pc}^2$. Thus, the likely cut-off in the distribution of galaxy surface densities is somewhat higher than given in Eqn. 25, and thus about a factor of three higher than the smallest surface density found in Figures 1&2. Note, however, that this slight change in cut-off hardly changes the conclusions of the previous section; there are still enormous numbers of low surface density galaxies for every normal galaxy, as well as substantial masses tied up in these galaxies.

4. The Role of Dwarf Galaxies

The discussion above has focussed on large galaxies. The choice of $r_\ast \approx 10 \text{ kpc}$ has led us to consider only galaxies with scales that are typical of normal spiral and elliptical galaxies. However, there are also many smaller dwarf galaxies of both high and low surface brightnesses which exist at the present day. We now turn our discussion to these galaxies. Although galaxies exist with a continuum of sizes, we will use the term “dwarf galaxy” to refer to galaxies with sizes typical of the sub-$0.1L_\ast$ galaxies in the local group, effectively the Large Magellanic Cloud and smaller.

The formation and subsequent evolution of dwarf galaxies is necessarily more complicated than for large galaxies. First, star-formation in dwarf galaxies is more subject to interruptions due to supernovae, ionization, or ram-pressure stripping than their more massive counterparts (see Dekel & Silk 1986, DeYoung & Heckman 1994, Efstathiou 1992). Secondly, there are other mechanisms besides gravitational collapse which are capable of producing dwarf galaxies, for example, clumping in tidal debris during mergers (Barnes & Hernquist 1992, Mirabel, Dottori, & Lutz 1992, Mirabel, Lutz, & Maza 1991, Elmegreen, Kaufman, Thomasson 1993), and compression of the intergalactic medium (Silk, Wyse, & Shields 1987). Finally, if structure grows hierarchically, then many of the dwarfs that exist
at early times merge together and are assimilated into larger galaxies by the present time. There is a wealth of papers that treat the formation and evolution of dwarf galaxies in more detail than we are capable of doing justice to here. Recognizing our limitation in treating only the gravitational physics of dwarf galaxies, we restrict ourselves to a general discussion of how dwarf galaxies fit into our scheme of early-forming HSB’s and late-forming LSB’s.

Because dwarf galaxies in principle collapse from smaller initial radii $r_0$ than do normal galaxies, they in general collapse at earlier times. Similarly to large galaxies, dwarfs that collapse from high-amplitude peaks will collapse at earlier times and to higher surface densities than those dwarfs which collapse from low-amplitude peaks. However, while we were somewhat justified in neglecting merging of large galaxies, by no means are we justified in doing so for dwarfs. Some large fraction of the dwarfs that exist at large redshifts will merge together and have disappeared by the present day. The high surface density dwarfs have the earliest formation times, the largest amplitude correlation function and thus the largest probability of being absorbed into the large galaxy population. Late-forming, weakly correlated, low surface density dwarfs have the smallest chance of being absorbed. This will tend to deplete the distribution of dwarf surface brightnesses at the high surface brightness end. Dense dwarfs are less easily disrupted than tenuous low surface density dwarfs, however, which may help counteract the depletion of high surface density dwarfs. Obviously even the most simple treatment dwarf galaxies is complicated, and any quantitative discussion lies far outside of the scope of this paper.

At first blush it appears that our scenario as presented is overruled by the presence of some old dwarf galaxies with low surface brightnesses in the Local Group (see Ferguson & Binggeli (1994) and references therein). However, by assuming the Press-Schechter distribution function, we are implicitly concerning ourselves with the average number density of collapsed galaxies, independent of environment. A much more detailed treatment of the conditional, environmentally dependent number density by Bower (1991) shows that galaxies which exist in groups today collapse earlier than do galaxies in less dense environments. While our treatment effectively assumes perfectly synchronized formation times for galaxies of a particular surface brightness, the Bower formalism shows how this co-evality breaks down when one considers the range of environments in which galaxies form. Therefore, we are not bothered by the presence of genuinely old low surface brightness dwarfs in the Local Group. Furthermore, even in our simple picture, dwarf galaxies which are three orders of magnitude lower surface brightness than their early forming high surface brightness counterparts can still form at $z = 2$, a high enough redshift for their stellar populations to label them as “old” systems.
5. Relevance to Other Astrophysical Issues

We have postulated that hierarchical structure formation models naturally lead to a large population of low surface brightness galaxies. Such a pervasive population of LSB’s must manifest itself in many astrophysical contexts; we consider several of these here.

5.1. HI Surveys

It has often been considered a failing of hierarchical structure formation scenarios that deep HI surveys have failed to uncover a significant population of dwarf galaxies. The few uncataloged dwarfs that are discovered are preferentially found near bright galaxies (see van Gorkum 1993 for a recent review). There does not seem to be a large highly uncorrelated population of gas rich dwarfs.

However, in light of recent work showing a sharp cutoff in HI disks at column densities of $10^{19} \text{cm}^{-2}$ (van Gorkum et al. 1993, Corbelli, Schneider, & Salpeter 1989), the paucity of HI dwarfs is not surprising. Recent work by Maloney (1993) and Corbelli & Salpeter (1993) convincingly demonstrates that ionization of the HI by the UV background accounts for the sharp cutoff in HI disks. As we have shown that low-mass galaxies tend to have low surface densities, these galaxies will be prone to having their hydrogen ionized, reducing their detectable HI masses well below their total hydrogen masses. Taking Maloney’s scaling for the critical column density $N_{cr} \propto V_c^{0.5} \Sigma_H^{-0.6}$, (where $V_c \propto \Sigma_0^{1/3}$ is the halo circular velocity, $\Sigma_H \propto \Sigma_0$ is the halo surface density, and $\Sigma_0$ is the central HI surface density), a galaxy’s hydrogen will be completely ionized for $\Sigma_0 \lesssim 4 \times 10^{19} \text{cm}^{-2}$. Field spirals have central HI column densities of roughly $10^{20} - 10^{21} \text{cm}^{-2}$ (Cayatte et al. 1993), so galaxies that have surface densities roughly one-tenth below normal are likely to be highly ionized and thus have extremely low detectable HI masses. The observable HI mass can be expressed as:

$$M_{HI} = 2\pi \Sigma_0 \alpha^2 \left[ 1 - 0.01 \left( \frac{10^{21}}{\Sigma_0} \right)^{1.43} \left( 5.605 + 1.43 \ln \left( \frac{\Sigma_0}{10^{21}} \right) \right) \right]$$

A galaxy with an exponential scale length of $\alpha = 5 \text{kpc}$ and a central HI surface density of $10^{21} \text{cm}^{-2}$ has an HI mass of $10^9 M_\odot$. Another galaxy with a central surface density of $10^{20} \text{cm}^{-2}$ has an HI mass of $4 \times 10^7 M_\odot$, two and a half times lower than expected based
on its total hydrogen surface density and, more importantly, well below most limits of HI surveys. If the central HI surface density were to be dropped by another factor of two, the HI mass would fall by a factor of sixteen ($\sim 3 \times 10^6 M_\odot$).

Low surface density galaxies are therefore likely to suffer from strong biases against their detection in HI surveys, not just in optical surveys. A large population of LSB's could easily have been overlooked by existing surveys. The dwarfs that have been detected in HI surveys must have higher surface densities in general, and thus are more likely to have collapsed earlier from larger overdensities. This would explain why these dwarfs are found to trace the bright galaxy population.

The absence of neutral gas in low surface density galaxies is likely to suppress the star formation in these galaxies, or at least to drive it through different channels than in the Milky Way. It is possible that star formation in low surface density galaxies takes place within small self-shielding clumps, embedded in the diffuse ionized background; the calculation of the critical hydrogen surface density (Maloney 1993) does not take strong clumping into account.

Finally, we note two mitigating circumstances that may improve the prospects for detecting HI in LSBs. First, if the star formation efficiency is depressed in LSB galaxies, they may have a higher gas fraction than normal galaxies, and thus larger column densities than one might expect from their surface brightness alone. Second, because LSBs are more likely to be young systems, they have had less time to convert gas into stars, also increasing their gas fraction. With a higher gas fraction, LSBs would have larger hydrogen masses and lower ionization fractions than one would derive from naively scaling the properties of spiral disks to LSB disks.

5.2. Lyman-α Absorbers

Given that LSBs are gaseous and numerous, they must contribute to the Lyman-α forest. They are similar to minihalos in that they are gravitationally-confined systems that have collapsed from small overdensities. However they differ from minihalos in many important respects. LSBs are disklike systems supported by rotation, whereas minihalos are assumed to be spherical and supported against gravitational collapse only by their thermal pressure. Press & Rybicki (1993) have shown that the observed line widths of the Lyman-α forest are too large to be explained in a model where the minihalos are in thermal equilibrium with the UV background. They argue that the best explanation for
the large line widths is not thermal processes but bulk motion of the gas within the cloud. While other workers have considered collapse of a spherical cloud as a source of these additional velocities, we believe that internal rotation is an equally plausible, well-motivated explanation. We will be addressing this idea in a later paper (Spergel & Dalcanton 1995).

5.3. Faint Blue Galaxies

Because of their blue colors, weak correlations, and underrepresentation in catalogs of nearby catalogs, LSBs are a natural candidate for the “excess” faint blue galaxies seen in deep galaxy surveys (McGaugh 1994 – see references within for exhaustive listings of the body of work on the faint blue galaxy excess). The late formation times suggested by observations and by the arguments in this paper are not in conflict with the possibility that LSBs make up the excess galaxies seen at moderate redshifts; while LSBs as they have been defined in this paper are formed later than spirals, there are sufficient numbers of them at the moderate redshifts ($z \lesssim 0.7$) where the brighter of the faint blue galaxies are found.

We note that a constraint exists on the amount of “missing” LSBs needed to resolve the discrepancy in the number counts. Dalcanton (1993) found that the rest frame $B$ luminosity density at $z \approx 0.4$ is 3 – 5 times higher than the local luminosity density as measured in surveys of nearby galaxies. If this discrepancy is due entirely to underestimating the contribution of LSBs to the local luminosity density, then there must be more than a factor of two greater luminosity density in uncataloged LSBs than in cataloged high surface brightness galaxies. The results of the over-simplified model presented in §3 do not rule out this possibility.

5.4. Tully-Fisher

The Tully-Fisher relationship will change as galaxy samples are extended to lower surface brightness. The Tully-Fisher relation is an artifact of the limited range of surface brightnesses sampled in large galaxy catalogs. $\Sigma \propto M/R^2$ and $V_c \propto M^{1/2}/R^{1/2}$ imply: $\Sigma M \sim V^4$. This expression reduces to the Tully-Fisher relation,

$$L \propto \frac{V^4}{(M/L) \Sigma}$$

(27)
where

\[
M/L = \frac{M_{\text{tot}}(r < r_*)}{M_{\text{baryons}}} \times T \times \frac{L}{M_*}
\]  

(28)

and where \(M_{\text{baryons}}\) is the total baryonic mass of the galaxy, \(M_*\) is the mass converted into stars, and \(M_{\text{total}}\) is the total mass within the region of HI emission (i.e. within the maximum extent of the baryons). Taking the definition of the collapse factor in Eq. [1],

\[
\frac{M_{\text{tot}}(r < r_*)}{M_{\text{baryons}}} = \frac{1}{F \Lambda}
\]  

(30)

for an isothermal dark matter halo. Referring to Eq. 9, note that Eq. 30 slightly reduces the dependence of Eq. 27 on \(\bar{\Sigma}\).

The equations above imply that low surface brightness galaxies will in general follow the slope of the Tully-Fisher relationship, but may be offset from the track followed by normal spirals. The lower surface density suggests that LSB’s will tend to be overluminous when compared to normal spirals with the same size and circular velocity. However, a concomitant reduction in the star formation efficiency, as might be expected with decreasing surface density, would pull the luminosity of LSB’s downwards. Variations in the baryonic collapse factor will also contribute to variations in \(M/L\); galaxies which have undergone very little collapse will have a smaller baryonic mass fraction (i.e. large \(M_{\text{tot}}/M_{\text{baryons}}\)), and thus larger mass-to-light ratios. The combination of lower surface density and reduced star formation efficiency may conspire to leave LSB’s on the same relationship followed by normal spirals. This would be a most fortunate coincidence for extending Tully-Fisher to larger distances. Recent work by Sprayberry et al (1995a) is beginning to shed light on these issues.

We expect much more scatter in the Tully-Fisher relation for LSB galaxies. Because of their reduced halo masses and low surface densities, the disks of LSBs will be much puffier than the disks of normal spirals, which will lead to much greater uncertainty in the inclination correction, especially for smaller galaxies. The characteristic disk scale height, \(Z_0\), is proportional to \(\sigma_{vz} V_c / 4G \bar{\Sigma}\) where \(\sigma_{vz}\) is the vertical velocity dispersion, \(V_c\) is the circular velocity at infinity, and \(\bar{\Sigma}\) is the surface density. Because \(V_c^2 \propto M\) and \(M \propto \bar{\Sigma}\),

\[
z_0 \sim \frac{\sigma_{vz}}{\Sigma^{1/2}},
\]  

(31)
suggesting that LSBs will have larger scale heights than normal galaxies with the same radial extent (modulo the uncertainty of dependence of vertical temperature on surface density). The resulting uncertainty in the inclination correction will be largest for dwarf LSBs which, while flattened, may hardly look disk-like at all.

6. Summary

Hierarchical models of galaxy formation provide a qualitatively and quantitatively reasonable explanation for many of the global properties of $L_*$ galaxies. Analytical models and N-body simulations can correctly predict the number and luminosity of bright galaxies as well as their kinematics. However, these models have consistently had difficulty in matching the observed faint end of the luminosity function. We have shown in this paper that low mass galaxies tend to form naturally with low surface densities. Low surface density galaxies can be assumed to have low surface brightnesses as well, unless they also have particularly high star formation efficiencies – a situation we consider unlikely. As such, the faint galaxies predicted by hierarchical clustering models are likely to have very low surface brightnesses, possibly hundreds of times fainter than the surface brightnesses of normal spiral disks. The observed distribution of central surface brightnesses for spirals suggests that there are selection effects which have lead to dramatic underestimates of the numbers low surface brightness galaxies; possibly only one tenth of the galaxies with surface brightnesses of half of “normal” have been cataloged (Disney 1976, Allen & Shu 1979). We have argued that the undercounted galaxies are preferentially low mass, and therefore the failure of models to match observations of the faint end of the luminosity function should not be surprising; low mass galaxies are hidden not only by their low total luminosity, but by their low surface brightnesses as well. Furthermore, these galaxies should be as difficult to detect in the radio as they are in the optical. The gas in galaxies with extremely low surface densities may be easily ionized by the UV background, effectively making these galaxies invisible in HI.

In the scenario which we have developed, LSB’s collapse slowly and at late times from small initial overdensities (Mo et al. 1994). We argue that galaxies formed from small overdensities are likely to have larger spin parameters $\lambda$ and thus smaller collapse factors, leading to lower baryonic surface densities. Because of the longer formation timescales, increased spin parameters, decreased correlations, and decreasing surface brightness, LSBs may be interpreted as a natural extension of the Hubble sequence, from Sc’s to Sd’s and beyond. However, one would expect LSBs to appear scruffier than the galaxies which define
the bulk of the Hubble sequence. The decreased surface density implies that LSBs will tend have thicker disks than normal galaxies, and thus a decreased baryonic density within the galaxy. This in turn reduces the efficiency of star formation as well as the likelihood that the galaxy becomes unstable to spiral structure. The low surface density and low overall mass also increase the likelihood that feedback from star formation through supernova-driven winds can affect the apparent morphology of LSB galaxies. All of these processes may lead to the diffuse, chaotic galaxies which are becoming associated with the extremes of galaxy surface brightness (Dalcanton 1995, Schombert et al 1992).

Although LSBs are hardly impressive members of the galactic community when viewed as individuals, their cumulative properties are impressive. Both the number density and mass density of the LSB population are substantial, with LSBs possibly contributing as much (or more) mass to the universe as normal galaxies. (Although we have derived this result specifically assuming a CDM spectrum of initial fluctuations and $\Omega = 1$, our qualitative results should be little changed by assuming a different cosmological model, as long as structure formation proceeds hierarchically.) Given this possibility, an accurate measure of the number density and mass density of the LSB population becomes an important goal for the coming years. While LSBs may be overlooked as individuals, they are potentially too important to be overlooked as a class.

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7. References

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Figure 1. The number and mass density of galaxies as a function of surface brightness and redshift. The lightest line in each frame corresponds to $z = 0$, with each successively darker line corresponding to $z = 1$, $z = 2$, etc, up to $z = 5$. Each row of plots corresponds to a single value of $\lambda$: 0.15 in the top row, 0.075 in the middle, and 0.03 in the bottom row. The leftmost column shows the number density of galaxies with surface brightness $\bar{\Sigma}$ in units of $\#$/Mpc$^3$. The second column is the cumulative distribution of the number density shown in the first column. The horizontal dashed lines are the integrated number density in normal galaxies with $L/L_\ast > 0.1$ for all galaxies (long dash), for spirals only (short dash), and for spirals Sc and later (dotted line), taken from Marzke et al. (1994). The third column is the baryonic mass density in galaxies of surface brightness $\bar{\Sigma}$, given in units of $1/F \rho_c$, the total baryonic mass density. The rightmost column is the integrated mass density. The 1–10% error in normalization in the integrated mass density for $\lambda = 0.15$ reflects the failure of the approximation to the CDM power spectrum at $r_0 < 50$ kpc. As discussed in the text, normal disk galaxies have $\bar{\Sigma} \approx 250 - 400$ M$_\odot$/pc$^2$. The value of $r_\ast$ was chosen to give the correct number density in galaxies formed before $z = 2$ for $\lambda = 0.075$, the spin parameter appropriate for spiral galaxies.

Figure 2. Same as Figure 1, except that $b = 2.5$ and $r_\ast = 7.5$ kpc.