Quantum Teleportation Using Quantum Non-Demolition Technique

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We propose a new scheme and protocol for quantum teleportation of a single-mode field state, based on entanglement produced by quantum non-demolition interaction. We show that the recently attained results in QND technique allow to perform the teleportation in quantum regime. We also show that applying QND coupling to squeezed fields will significantly improve the quality of teleportation for a given degree of squeezing.

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Quantum teleportation is a transport of quantum state of a system to another similar system via a classical channel. The principles of quantum mechanics demand that for realization of such a transport, the sender and the receiver must have two ancillary quantum systems, being strongly quantum-mechanically correlated (entangled). Quantum teleportation is a fundamental phenomenon of quantum world and also one of the key procedures in the rapidly growing area of quantum information. Recently several teleportation schemes have been proposed and the successful realization of some of them has been reported: teleportation of the polarization state of a single photon, of the state of a single-mode optical field and of nuclear spin state. In this Letter we propose a new scheme for quantum teleportation of the state of single-mode optical field. In contrast to the existing schemes, where a superposition of two squeezed fields has been used as a source of entanglement, we propose to use the entanglement produced by quantum non-demolition (QND) interaction, realized in the last years for optical fields in various media, the best results being obtained for coupling two optical beams by parametric interaction in a $\chi^{(2)}$ crystal or using cold atoms in a trap.

Our scheme is depicted in Fig. 1. The nonlinear QND interaction produces the following coupling between two bright coherent fields $E_a$ and $E_b$

\[ \tilde{E}_a = E_a + \frac{g}{2} (E_b^* + E_b), \]
\[ \tilde{E}_b = E_b + \frac{g}{2} (E_a^* - E_a), \]

where $\tilde{E}_a$ and $\tilde{E}_b$ are the outgoing fields and $g$ is the QND gain. It is easy to find that the quadratures $X = E^+ + E$ and $Y = i(E^+ - E)$ of both fields are transformed as

\[ \tilde{X}_a = X_a + gX_b, \quad \tilde{Y}_a = Y_a, \]
\[ \tilde{X}_b = X_b, \quad \tilde{Y}_b = Y_b - gY_a. \]

The quadratures $X_a$ and $Y_a$ are the so-called QND variables, which are not affected by coupling. The conjugated quadratures $X_b$ and $Y_b$ become correlated with $X_b$ and $Y_a$ respectively and for infinitely large QND gain, or for infinitely squeezed $Y_b$ ($X_a$) gives a result proportional to that of measurement of $Y_a$ ($X_b$). This is the main idea of QND measurement.

In our approach this measurement is not performed, but the entanglement produced by QND coupling is used for quantum teleportation, as described below.

One of the resulting from QND interaction beams ($\tilde{E}_a$) passes to the sender Alice, and another one ($\tilde{E}_b$) passes to the receiver Bob for further reconstruction of the teleported state. The protocol of quantum teleportation is as follows. Alice superimposes the field $E_{in}$, which is to be teleported, with the field $E_a$ at a beam splitter with the transmittance $\epsilon$, and, using homodyne photodetection, measures the $X$ quadrature of the reflected (for $E_{in}$) field and the $Y$ quadrature of the transmitted field, obtaining the values

\[ x = \epsilon^{1/2} (X_a + gX_b) + (1 - \epsilon)^{1/2} X_{in}, \]
\[ y = \epsilon^{1/2} Y_{in} - (1 - \epsilon)^{1/2} Y_a, \]

where the field variables are written in the Wigner representation. The measured values are sent to Bob via a classical channel, and upon receiving them Bob prepares the bright laser beam $E_{\beta}$ by means of phase and amplitude modulation in a coherent state with the amplitude $\beta = \frac{G\epsilon}{2}$, where $\Gamma \gg 1$ and $G$ are some real gains. After that Bob superimposes the field $E_{\beta}$ with the beam $\tilde{E}_b$ at a beam-splitter with very low transmittance $\epsilon' = 1 - \epsilon^2 \ll 1$, so that the reflected field $E_{\beta}$ is just shifted by $G(x + igy)/2$, but no noise is added. The result of interference of the reflected field $\tilde{E}_b$ and the transmitted field $E_{\beta}$ is the output $E_{out}$:

\[ 2E_{out} = 2G^{-1}E_{\beta} - 2\tilde{E}_b = G(x + igy) - X_b - iy_b + igY_a \]
\[ = Ge^{1/2}X_a + Gge^{1/2}X_b + G(1 - \epsilon)^{1/2}X_{in} \]
\[ + ig \left( 1 - G(1 - \epsilon)^{1/2} \right) Y_a - iY_b + iGge^{1/2}Y_{in}. \]

Now, if the electric gain $G$ and the transmittance $\epsilon$ satisfy for a given QND gain $g$ the relations $G = (1 - \epsilon)^{1/2}$, $g = (1 - \epsilon)^{1/2}e^{-1/2}$, the output field reads as

\[ 2E_{out} = X_{in} + iY_{in} + g^{-1}X_b - iY_b, \]

that is, for $\langle X_a \rangle = \langle Y_b \rangle = 0$, it reproduces the incoming field with additional noise caused by quadratures $X_a$ and $Y_b$. For infinite QND gain $g$ and infinitely squeezed quadrature $Y_b$ the teleportation is ideal. However, even for finite QND gain and not squeezed $Y_b$ the proposed
teleportation scheme works in an essentially quantum regime.

This can be seen from comparing this scheme to classical teleportation. In the latter case Alice and Bob do not have correlated ancillary systems and all they can do in order to accomplish a true reproduction of the average values of $X_{in}$ and $Y_{in}$, is to measure this quadratures with uncertainty imposed by the Heisenberg’s principle and create a coherent field with the average values of quadratures equal to measured ones. So the protocol of classical teleportation is as follows. Alice divides the incoming field $E_{in}$ in two parts by a 50:50 beam splitter and measures the $X$ quadrature of the reflected field and the $Y$ quadrature of the transmitted field, obtaining the values

$$x = \frac{1}{\sqrt{2}} X_{in} + \frac{1}{\sqrt{2}} X_v, \quad (7)$$

$$y = \frac{1}{\sqrt{2}} Y_{in} - \frac{1}{\sqrt{2}} Y_v, \quad (8)$$

where $E_v = (X_v + iY_v)/2$ is the vacuum field at the unused port of the beam-splitter. The measured values are send to Bob who creates a coherent state

$$2E_{out} = \sqrt{2}(x + iy) + 2E_w \quad (9)$$

$$X_{in} + iY_{in} + X_v - iY_v + X_w + iY_w,$$

where $E_w = (X_w + iY_w)/2$ is the vacuum fluctuation inevitably added at the reconstruction stage (e.g., as in the previous protocol, Bob can superimpose a strong coherent field $2E_\beta = \Gamma \sqrt{2}(x + iy)$ with vacuum field $E_w$ at a low transmittance $\epsilon' = \Gamma^{-2}$ beam splitter and use the reflected (for $E_w$) field as output). Now, to compare quantum (Eq. (3)) and classical (Eq. (8)) teleportation, we must calculate a quantitative measure of teleportation success, connected with some verification procedure. Let us consider the verification scheme shown in Fig. 2, similar to that of Ref. [3]. The verifer Victor, independent on Alice and Bob, prepares an ensemble of states of field $E_0$. Victor can directly measure the average value and the variance of the field quadrature $X_0$. Or he can shift the phase of the field by random phase $\varphi(t)$ and give it to Alice who teleports the state to Bob. After receiving the state $E_{out}$ from Bob, Victor again shifts the phase, this time by $-\varphi(t)$ and measures the $X$-quadrature of the obtained field $E_T$. If the teleportation is ideal, the average value and variance of $X_T$ will be the same as for $X_0$. The proposed in this Letter teleportation scheme gives the following expression for $X_T$:

$$X_T = X_0 + g^{-1} X_a \cos \varphi - Y_b \sin \varphi. \quad (10)$$

So for $\langle X_0 \rangle = \langle Y_b \rangle = 0$ the average value of $X_T$ is equal to that of $X_0$. The variance of $X_T$ is given by

$$\langle (\delta X_T)^2 \rangle = \langle (\delta X_0)^2 \rangle + g^2 \langle (\delta X_a)^2 \rangle \cos^2 \varphi$$

$$+ \langle (\delta Y_b)^2 \rangle \sin^2 \varphi, \quad (11)$$

where $\delta X = X - \langle X \rangle$ and the bar denotes averaging over the random phase shift. If the phase is distributed homogeneously in $[0, 2\pi]$, then $\cos^2 \varphi = \sin^2 \varphi = \frac{1}{2}$, and

$$N_{add} = \frac{1}{2} \langle (\delta X_a)^2 \rangle + \frac{1}{2} \langle (\delta Y_b)^2 \rangle,$$

where $N_{add}$ is the noise added to the teleported light. We use the added noise as a measure of teleportation success instead of commonly used fidelity $F = \langle \psi_{in} | \rho_{out} | \psi_{in} \rangle$ (where $| \psi_{in} \rangle$ is the incoming state and $\rho_{out}$ is the density matrix of the output field), for two following reasons. Firstly, fidelity generally depends on the incoming state, while for the teleportation of field state, which is a linear in field transformation, the output field is always the incoming one plus some noise; so it is natural to describe the teleportation by this added noise, independent on the incoming state. Secondly, the output state can differ from the input one in two ways: in averages (displacement) and in variances of quadratures (distortion). Fidelity (as well as the parameters introduced in Ref. [7]) takes into account both these effects. However, classical teleportation always can be arranged so that the average values (of quadratures) are transported correctly. So only a measure of state distortion is necessary for characterizing the quantum nature of teleportation, and the added noise just provides such a measure.

It is easy to find from Eq. (3) that for classical teleportation

$$N_{add} = \frac{1}{2} \langle (\delta X_a)^2 \rangle + \frac{1}{2} \langle (\delta Y_b)^2 \rangle \geq 2$$

for any states of fields $E_v$ and $E_w$, which can be seen from the inequality $a^2 + b^2 \geq 2ab$ and the Heisenberg’s uncertainty principle. So, to reach the quantum regime of teleportation, one must reach added noise $0 \leq N_{add} < 2$, under condition that $\langle X_T \rangle = \langle X_0 \rangle$ for any phase $\varphi$. In the proposed scheme it can be obtained by using coherent fields $E_a$ and $E_b$ (with imaginary and real amplitudes respectively) and QND gain $g > 1/\sqrt{3}$.

The entanglement produced by a QND device (or its quantum state preparation regime) is usually characterized by conditional variance of signal given a measured value of meter, $V_c$. For transformation described by Eqs. (6), (7) and coherent input fields $E_a$ and $E_b$ this variance is given by

$$V_c = \frac{1}{2} \langle (\delta X_a)^2 \rangle = \frac{1}{1 + g^2} \quad (13)$$

and the condition $g > 1/\sqrt{3}$ corresponds to the condition $V_c < \frac{4}{9}$. In a recent QND experiment using cold atoms
in a trap as a nonlinear medium \[13\] the value \( V_e = 0.45 \) has been measured (\( V_e = 0.37 \) with corrections for optical losses). In the same time, our requirements for field phases are exactly that needed for QND interaction based on cross-phase modulation, implemented in this experiment. For the QND schemes based on \( \chi^{(2)} \) crystals, a value \( V_e = 0.65 - 0.7 \) has been achieved \[12\], without restrictions on the phases of interacting beams. These results speak about a very good possibility to reach the quantum regime of teleportation using the existing experimental technique and the protocol presented in this Letter.

Let us briefly compare our scheme to that using squeezed states \[10, 11\]. In classical teleportation the output field is a sum of the input one and four additional noises (quantum duties), as shown by Eq. \[10\], and the noise added to a random quadrature is always more than 2. Quantum teleportation can be realized by using correlated ancillary fields \( E_a \) and \( E_w \), produced by mixing two squeezed fields \( E_1 \) and \( E_2 \) at a 50:50 beam-splitter, so that in Eq. \(10\) \( X_v + X_w = \sqrt{2}X_1 \) and \( Y_v - Y_w = \sqrt{2}Y_2 \), and the added noise is given by

\[
N_{add}^{sq} = V_1 + V_2,
\]

where \( V_1 \) and \( V_2 \) are variances of quadratures \( X_1 \) and \( Y_2 \) respectively. When the quadratures \( X_1 \) and \( Y_2 \) are squeezed, the added noise becomes less than the classical value of 2. In our approach the ancillary fields are correlated via a QND interaction. Two QND variables \( (X_b \) and \( Y_b) \) are automatically cancelled in the output field due to QND correlations. One of the remaining noises \( (X_a) \) can be made as small as desired by increasing the QND gain and/or by squeezing this quadrature. The noise of \( Y_b \) can be reduced by squeezing this quadrature only. The added noise in this case reads as \( (V_c < 1) \)

\[
N_{add}^{QND} = \frac{1}{2} \frac{V_c}{1 - V_c} V_a + \frac{1}{2} V_b,
\]

where \( V_a \) and \( V_b \) are variances of quadratures \( X_a \) and \( Y_b \) respectively. Comparing Eq. \[13\] with Eq. \[14\] we see that with decreasing conditional variance the QND coupling decreases significantly the requirements for the degree of squeezing of ancillary beams (so that for \( V_c < \frac{1}{4} \) they can be not squeezed at all).

It should be noted that a principal possibility to use QND coupling for quantum teleportation has been pointed in Ref. \[13\]. However this paper considers only QND gain \( g = 1 \) and two ideally squeezed ancillary beams, in which case no benefits from QND coupling can be seen, compared to mixing these beams at a beam-splitter.

In summary, we have described a protocol for quantum teleportation of a single mode optical field using QND coupling as a source of entanglement of ancillary laser beams. In our scheme the QND device is used in a quantum state preparation regime, and the recent achievements for realizing this regime satisfy very well the requirements for quantum teleportation. From the other side, the existing schemes for quantum teleportation of single-mode field state are limited by the restricted degree of squeezing, available at present, and using QND coupling for squeezed ancillary beams can provide further improvement of quantum teleportation technique.

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FIG. 1. Schematic of quantum teleportation arrangement. Two entangled beams \( E_a \) and \( E_b \) are produced by the QND device. The incoming field \( E_{in} \) is mixed by Alice with one of the entangled beams and the quadratures of the resulting fields are measured by two homodyne detectors, using purely real local oscillator field \( E_{LO} \) and a purely imaginary one \( iE_{LO} \). The measurement results are sent via a classical channel to Bob who prepares a laser beam in a proper coherent state by means of amplitude and phase modulator \( M \). This coherent field is superimposed with the second entangled beam, resulting in the output field. The state of the latter reproduces closely the state of the incoming field.
FIG. 2. The verification procedure. The verifier, completely independent on Alice and Bob, prepares an ensemble of states for field $E_0$. This field can be directly sent to the homodyne detector (dashed line) for measuring the distribution of its real quadrature. Either it can be sent to Alice for teleportation with the following measurement of the field received from Bob (solid line). The difference in noise level between two paths is a measure of the teleportation success. The phase shifts, introduced by phase modulators $PM$ allow to control the noise added to a corresponding quadrature of the teleported field.
Teleportation

ALICE

PM

$e^{i\varphi(t)}$

$E_0$

BOB

PM

$e^{-i\varphi(t)}$

$E_T$

$\mathcal{E}_{LO}$

$X_T$