No Higher Criticism of the Bianchi Corrected WMAP Data

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ABSTRACT

Motivated by the success of the Bianchi $VII_h$ model in addressing many of the anomalies observed in the WMAP data (Jaffe et al.), we present calculations in real and in wavelet space of the Higher Criticism statistic of the Bianchi corrected Wilkinson Microwave Anisotropy Probe (WMAP) first year data. At the wavelet scale of 5 degrees the Higher Criticism of the WMAP map drops from a value above the 99% c.l. to a value below the 68% c.l. when corrected by the Bianchi template. An important property of the Higher Criticism statistic is its ability to locate the pixels that account for the deviation from Gaussianity. The analysis of the uncorrected WMAP data pointed to a cold spot in the southern hemisphere centered at $l \sim 209^\circ$, $b \sim -57^\circ$. The Higher Criticism of the Bianchi corrected map indicates that this spot remains prominent, albeit at a level completely consistent with Gaussian statistics. Consequently, it is debatable how much emphasis should be placed on this residual feature, but we consider the effect of modestly increasing the scaling of the template. A factor of only 1.2 renders the spot indistinguishable from the background level, with no noticeable impact on the results published in Jaffe et al. for the low-l anomalies, large-scale power asymmetry or wavelet kurtosis. A trivial interpretation would be that the Bianchi template may require a small enhancement of power on scales corresponding to the wavelet scale of 5 degrees.

1 INTRODUCTION

The first year observations of the Wilkinson Microwave Anisotropy Probe (WMAP) satellite has provided us with a data set of unprecedented accuracy (Bennett et al. 2003). It is exciting to see that we are still far from understanding all the information that is encoded in the data. Among the many papers that have been published based on analyses of the WMAP data, a large section of them points to possible deviations from the standard model. Different so called anomalies have been detected and possible explanations have been offered by several authors: (1) Low value of the quadrupole, its alignment with the octopole and other multipole alignments (De Oliveira-Costa et al. 2004, Schwarz et al. 2004, Vale 2005, Land & Magueijo 2005a), (2) Asymmetries (Park 2004, Eriksen et al. 2004a, Eriksen et al. 2004b, Larson & Wandelt 2004, Hansen et al. 2004a, Hansen et al. 2004b, Donoghue & Donoghue 2005, Tojeiro et al. 2005, Bielewicz et al. 2005, Tomita 2005, Freeman et al. 2005), (3) Deviations from Gaussianity, in particular, detections in wavelet space pointing to a cold spot as the source of non-Gaussianity (Vielva et al. 2004, Cruz et al. 2005, Cayón et al. 2005).

Several works have explored the possibility of accounting for some or all of the anomalies observed in the WMAP data by introducing some shear and vorticity in the universe (Jaffe et al. 2005a,b, Land & Magueijo 2005b, McEwen et al. 2005b). Models introducing these anisotropic characteristics fall under the class of Bianchi $VII_h$ models (Barrow et al. 1985). In this paper we confirm the validity of the Bianchi model obtained as a best fit to the WMAP data by Jaffe et al. 2005b, as a possible explanation of all the observed anomalies. Following the same procedure as in Cayón et al. 2005 we show that the Higher Criticism $HC$ statistic (Donoho & Jin 2004) of the Bianchi corrected WMAP map...
is compatible with Gaussianity (at the 99% c.l.) at all the wavelet scales.

This paper is organized as follows. We present the formalism in Section 1. Section 2 is dedicated to introduce the analyzed data and simulations. Results are presented in Section 3 and they are discussed in Section 4 (conclusions also included in this section).

2 FORMALISM

As indicated above we here follow the formalism presented in Cayón et al. 2005. The statistical study is based on the statistic proposed in Donoho & Jin 2004, Jin 2004. For a set of individual observations \( X_i \) from a certain distribution, \( HC \) is defined as follows. The \( X_i \) observed values are first converted into \( p \)-values: \( p_i = P(|X_i| > |X_c|) \). After sorting the \( p \)-values in ascending order \( p(1) < p(2) < \ldots < p(n) \), we define the \( HC \) at each pixel \( i \) by:

\[
HC_{n,i} = \sqrt{n} \frac{|i/n - p(i)|}{\sqrt{p(i)(1 - p(i))}}.
\]

Unusually large values of \( HC_{n,i} \) imply deviations from Gaussianity. The fact that the statistic is calculated at every pixel will allow for the location of the source of any detected deviation. In order to quantify the statistical level of any detection we here follow the formalism presented in section 3 and they are discussed in Section 4 (conclusions also included in this section).

The temperature at pixel \( i \), \( T(i) \) results from the ratio of the weighted sum of temperatures at pixel \( i \) at each radiometer divided by the sum of the weights of each radiometer at pixel \( i \). The radiometers Q1, Q2, V1, V2, W1, W2, W3 and W4 are sequentially numbered from 3 to 10. The weights at each pixel, for each radiometer \( w_r(i) \), are the ratio of the number of observations \( N_r(i) \) divided by the square of the receiver noise dispersion \( \sigma_{n,r} \). The resulting map is downgraded from resolution \( nside = 512 \) to resolution \( nside = 256 \) (the total number of pixels being \( 12 \times nside^2 \)).

The Bianchi template that is used in this work is the best fit found by Jaffe et al. 2005b. They considered Bianchi type \( VII_b \) models characterized by the values of \( \Omega_m \) and \( \chi = (h/(1 - \Omega_m))^1/2 \), where \( h \) is the scale on which the basis vectors change orientation. The best fit was obtained for \( \Omega_m = 0.5 \) and \( \chi = 0.62 \). We present here an analysis of the Bianchi corrected WMAP data set obtained after subtraction of this template (and multiples of it) from the original WMAP data. The final analysis is performed on a masked version of this map that includes the so called Kp0 mask (released by the WMAP team and available at the LAMBDA website). An extension of this mask is applied to the wavelet convolved maps as discussed in Vielva et al. 2004.

The confidence levels of the \( HC \) statistic corresponding to the Gaussian assumption are drawn from 5000 simulations were the power spectrum is the one that best fits the WMAP, CBI and ACBAR CMB data, plus the 2dF and Lyman-alpha data. The simulations take also into account the beam transfer functions, the number of observations and the noise dispersion for each receiver. All these are provided by the WMAP team through the LAMBDA website.

4 RESULTS

The values of the \( HC \) statistic for the WMAP and the Bianchi corrected WMAP data are presented in Figure 1. Solid, dotted-dashed and dashed lines show the 99%, 95% and 68% confidence levels respectively. These are obtained from the 5000 Gaussian simulations described above. Since the \( HC \) statistic of a map is a maximum value \( HC = \max_i (HC_{n,i}) \), the confidence regions are one-sided. As one can see, there is no detection of deviations from Gaussianity in the Bianchi corrected WMAP data (factor of 1.0 in the figure). This corroborates the results obtained by Jaffe et al. 2005b. Subtraction of the Bianchi template corrects most of the anomalies that have been observed in the first year WMAP data.

The maps of \( HC \) values for the WMAP and the Bianchi corrected WMAP data at a wavelet scale of 5 degrees are presented in Figures 2 and 3. As one can immediately see

\[ \text{http://lambda.gsfc.nasa.gov/} \]
Figure 1. Values of the $HC$ statistic for the WMAP (stars) and the Bianchi corrected WMAP (crosses) data. The bands outlined by dashed, dotted-dashed and solid lines correspond to the 68%, 95% and 99% confidence regions respectively. The Bianchi template is multiplied by different factors before subtraction. Factors 0.3, 0.5 and 1.0 are considered in the results presented in the left panel (blue, green and magenta crosses respectively). Results for factors 1.1, 1.2 and 2.0 are presented in the right panel (magenta, blue and green crosses respectively).

Figure 2. $HC$ values of the WMAP data at the wavelet scale of five degrees.

The detected non-Gaussianity in the WMAP data is dominated by the values of a ring of pixels at the spot centered at $l \sim 209^\circ$, $b \sim -57^\circ$ (note that the ring structure is likely to be caused by the convolution with the wavelet). Subtraction of the Bianchi template drops the values of the $HC$ statistic at that spot below the 68% c.l. (see Figure 1). However, the spot still appears as one of the most prominent regions. Even if the relevance of this feature is debatable we decided to see whether different normalizations of the subtracted template can influence its amplitude. A factor of 1.2 reduces the amplitude of the spot making it comparable with the surrounding values as can be seen in Figure 4. We have also studied whether this normalization factor affects the other anomalies observed in the WMAP first year data. The effect on the alignment between the quadrupole and the octopole and the planarity as defined in Oliveira-Costa et al. 2004 is presented in Table 1. These large scale anomalies are not very much affected by a change in the normalization factor of the Bianchi template. The ratio of power between two hemispheres for a certain range of $l$ was calculated as in Eriksen et al. 2004a. The probability of having a maximum power asymmetry ratio larger than that of a given map (either the WMAP or the Bianchi corrected WMAP maps) is shown in Table 2. This probability is slightly affected by the normalization factor. There is no much of a difference between the results for a factor of 1 and a factor of 1.2. However, above
5 DISCUSSION AND CONCLUSIONS

Several explanations have been discussed in the literature to account for the different anomalies observed in the first year of WMAP data (Schwartz et al. 2004, Vale 2005, Freeman et al. 2005). This work has concentrated on explaining the anomalous low-l structure. Jaffe et al. however, serendipitously discovered that at least part of the large angular scale structure of the WMAP data was described by a particular Bianchi VIIh model, and that correcting the data for this contribution provided a potential resolution for each of the above anomalies. We here confirm their results by estimating the effect of subtracting the proposed Bianchi template from the WMAP data, on the HC statistic. A detection of non-Gaussianity localized in a cold spot at $l \sim 209^\circ$, $b \sim -57^\circ$ was found by the estimation of the HC statistic of the WMAP data (Cayón et al. 2005). The Bianchi template renders the WMAP data statistically compatible with the expected levels based on Gaussian simulations. The map of HC values for the Bianchi corrected WMAP data (see Figure 3) still shows the cold spot in the southern hemisphere as one of the most prominent regions. Although no special consideration should now be given to the spot on a statistical basis, it remains interesting given to the spot on a statistical basis, it remains interesting.

The results presented here, however, may be interpreted as implying that the Bianchi template may require a small enhancement of power on scales corresponding to the wavelet scale of 5 degrees. One could create a Bianchi template with more power on small scales by reducing the matter density, but this would also affect the large-scale structure and therefore the significance of the fit to the data. The most pragmatic interpretation of Jaffe et al. - that the best-fit Bianchi model provides a template temperature pattern which alternative models would need to reproduce in order to resolve the observed anomalous anisotropy structure - remains valid.

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Table 1. Alignment and planarity of low order multipoles. The normalization $F$ of the Bianchi template is included in the first column. Second and third columns correspond to the direction of $l = 2$ (Galactic longitude and latitude in degrees). The direction of $l = 3$ is shown in the fourth and fifth columns ($l$ and $b$ in degrees). The angle $\alpha$ (degrees) corresponding to the alignment between the quadrupole and the octopole is shown in column 6. The probability of finding a weaker alignment is presented in column 7. Values in the last three columns correspond to the probability of finding a more planar multipole for $l = 3, 5, 6$ respectively.

| $F$ | $l_2$ | $b_2$ | $l_3$ | $b_3$ | $\alpha$ | $P$ | $P_3$ | $P_5$ | $P_6$ |
|-----|-------|-------|-------|-------|----------|-----|-------|-------|-------|
| 0.0 | 61.9  | 247.8 | 63.4  | 232.8 | 7.0      | 0.992| 0.89  | 0.001 | 0.98  |
| 0.5 | 8.3   | 57.1  | 73.7  | 253.3 | 82.7     | 0.127| 0.73  | 0.003 | 0.99  |
| 1.0 | 3.3   | 44.5  | 56.8  | 323.5 | 82.3     | 0.134| 0.55  | 0.05  | 0.98  |
| 1.2 | 0.3   | 40.6  | 45.6  | 327.2 | 78.3     | 0.203| 0.62  | 0.10  | 0.97  |
| 2.0 | 9.0   | 212.2 | 26.9  | 317.3 | 80.9     | 0.159| 0.86  | 0.37  | 0.92  |

Table 2. Probability of the observed asymmetry in the power spectrum between the two hemispheres. The normalization factor $F$ is given in column 1. The probability for $l = 2, 20$ is presented in column 2. Values in column 3 correspond to the probability for $l = 2, 40$.

| $F$ | $P(l = 2, 20)(\%)$ | $P(l = 2, 40)(\%)$ |
|-----|--------------------|--------------------|
| 0.0 | 1.0                | 0.7                |
| 0.5 | 9.8                | 6.0                |
| 1.0 | 17.0               | 22.0               |
| 1.2 | 15.9               | 28.9               |
| 2.0 | 4.3                | 13.6               |

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Figure 3. $HC$ values of the Bianchi corrected WMAP data at the wavelet scale of five degrees.

Figure 4. $HC$ values of the WMAP data minus 1.2 times the Bianchi template, at the wavelet scale of five degrees.