Alternative linearisation methodology for aero-elastic Floating Offshore Wind Turbine non-linear models

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Abstract. Software capability for generating linear models from non-linear systems is a very powerful resource for floating offshore wind turbine (FOWT) systems. In particular, this linear models allow the application of the model design approach for system control design to improve and optimise the wind generator performance. However, systems with high non-linearities can difficult the linearisation process, where a good accuracy of the linear model is essential for the achievement of the desired control performance. So that, an alternative linearisation procedure based on the application of a chirp signal to the FOWT full non-linear model to provide high accuracy SISO linear models is presented in this work. Furthermore, this and FAST provided two linear models are analysed and compared, where frequency domain discrepancies are discussed. Time domain simulations with the FOWT full non-linear model are carried out for the linear model validation.

1. Introduction

FOWT are catching wind turbine market attention due to the good quality of the sea wind resource, low visual and noise impact, and due to the onshore wind farm saturation. However floating systems present poor pitch and roll stability in comparison to land-based or offshore bottom-fixed wind turbines. This affects to the control tuning process directly because of the well known phenomenon called negative platform damping \cite{1}\cite{2}\cite{3}. Then, a great engineering effort is necessary to design control solutions able to achieve admissible system stability and performance for FOWT systems. One of the first FOWT dynamic analysis was done in \cite{2} with the 5MW wind turbine model mounted on the ITI Energy barge platform and carried out with FAST software developed by NREL.

On the other hand, model based control design method is a very successful design paradigms which requires good system models, usually non-linear. The control design process normally starts with the achievement of the system linear models of those full non-linear models around a steady state solution. FAST provides the necessary tools for applying this procedure using its linearisation features. However, platforms with low hydrodynamic stiffness can difficult the steady state of the system in above rated wind speed presenting linearisation convergence issues for the desired operating point. Thus, based on the NREL 5MW wind turbine mounted on the ITI Energy barge, FAST provided linear model accuracy is discussed in this work. Two
different ways for linearising with FAST in above rated wind speed are presented, showing several discrepancies between them. Time domain simulations are carried out for the validation of this linear models accuracy. Furthermore, an alternative FOWT full non-linear model linearisation methodology based on the chirp signal [4] analysis is presented in this work. This methodology provided high accuracy SISO linear model and those achieved with FAST have been compared.

2. FAST linear models
In this work, the linearised models of the NREL 5MW baseline wind turbine mounted on the ITI Energy barge are obtained for control design purposes. Conventional generator variable speed and blade-pitch to feather turbine configurations are used. It is focus on the blade-pitch control region (Region 3), above rated wind speed, where the main aim is to regulate the generator speed in the face of wind speed variations via modification of the blade pitch angle.

The linearised models are obtained around a periodic steady state solution in the desired operating point at constant wind speed and still water conditions. The operating point is defined when the FOWT reaches the generator nominal speed and torque for the selected constant wind speed. This operating point can be manually found changing the blade pitch angle in an open loop configuration. Table 1 shows the operating point at 13m/s constant wind speed.

| Wind speed | Generator torque | Generator speed | Pitch angle |
|------------|------------------|-----------------|-------------|
| 13m/s      | 43093.55Nm       | 1173.7rpm       | 6.52°       |

FAST also provides the capability for computing this steady state operating point and numerically linearising the FOWT model. In this way, a first linear model is obtained following the guidelines of the FAST User’s Guide [5] for the Region 3. Collective blade-pitch angle is trimmed (TrimCase 3) while the generator torque is fixed to the nominal value (see Table 1). However, FAST does not converge to the desired operation point due to the oscillations of the rotor speed and blade-pitch angle caused by the platform-pitch motions. DispTol and VelTol convergence tolerances were increased to achieve the convergence in the nearest operation point. In this way, the obtained pitch angle after convergence is relatively far from the desired operating point as can be seen in Table 2.

Due to this convergence issue the linearisation process has been repeated to obtain a second linear model. In this case, the generator torque is trimmed instead of the pitch angle (TrimCase 2). For this purpose, initial blade pitch angle has been adjusted to achieve the nominal generator torque in steady state. The linear model converged at lower tolerances and closest to the desired operating point as can be seen in Table 2.

| Convergence method               | Generator torque | Generator speed | Pitch angle |
|----------------------------------|------------------|-----------------|-------------|
| Blade-pitch trimmed              | 43093Nm          | 1173rpm         | 4.45°       |
| Generator torque trimmed         | 43099Nm          | 1173rpm         | 6.52°       |
Figure 1 shows the Bode and Nyquist diagrams of the two linear transfer functions, from collective blade-pitch angle to the generator speed, obtained through the aforementioned linearisation processes. Analysing the frequency response Bode diagram of both linear systems, a discrepancy in the magnitude whereas in the frequency around platform surge mode (0.07 rad/s) is shown. This discrepancy is also shown in the Nyquist diagram. The blade-pitch trimmed model shows two potentially closed loop unstable frequencies at 0.524 rad/s and 3.75 rad/s, with a maximum gain ($K_{limit}$) to ensure system stability of 0.0013. The generator torque trimmed model shows the same two potentially closed loop unstable frequencies, but also depicts another slower one at 0.0627 rad/s. Then, the $K_{limit}$ to ensure system stability becomes smaller, 0.00094 (see Table 3).

![Nyquist Diagram](image)

**Table 3.** Gain limit for system stability of each linearised model.

| Linearisation method                  | $K_{limit}$ | Freq.(rad/s) |
|---------------------------------------|-------------|--------------|
| Trimming collective blade pitch       | 0.0013      | 0.524        |
| Trimming generator torque             | 0.00094     | 0.0627       |

These $K_{limits}$ to ensure system stability were checked with the FOWT full non-linear model. Closed loop generator speed ($W_g$) feedback configuration was used with a purely proportional gain (K) as shown in Figure 2. Reference generator speed ($W_g^*$), generator torque ($T_q$) and wind speed ($W_s$) of Table 1 were set-up. Blade-pitch angle is represented by beta ($\beta$) symbol.

Generator speed time domain simulations are shown in Figure 3. The system response presents an unstable behaviour showing an oscillation around 0.524 rad/s (platform-pitch mode) which grows with the time if the proportional control gain is larger than the blade-pitch trimmed $K_{limit}$ ($K > 0.0013$), as expected by both linear models. On the other hand, the platform-pitch mode oscillations are correctly damped if the proportional gain is smaller than the blade-pitch trimmed $K_{limit}$ and larger than the generator torque trimmed $K_{limit}$ ($0.0013 > K > 0.00094$). However, a slower platform oscillation corresponding to the platform-surge mode is still shown.
Figure 2. Generator speed closed loop scheme for proportional gain stability testing.

This oscillation is only predicted by the Nyquist diagram corresponding to the generator torque trimmed linear model. Where the frequency of this oscillation is about 0.0627 rad/s with a $K_{\text{limit}}$ to ensure system stability of 0.00094. However, a time domain simulation with a lower gain value than that $K_{\text{limit}}$ ($K < 0.00094$) does not damp the platform-surge oscillation. Note that the generator speed output is not around the nominal value due to a permanent error introduced by the implementation of the purely proportional controller.

Figure 3. Generator speed closed loop response with proportional pitch control.

Therefore, discrepancies are shown between both linearised models. Firstly, the generator torque trimmed linear model identifies three unstable frequencies whereas the collective blade-pitch trimmed linear model only two, as shown in Figure 1. Moreover, the $K_{\text{limit}}$ to ensure system stability is also different for both models as it is summarised in Table 3. In summary, the generator torque trimmed linear model is slightly more accurate than the blade-pitch trimmed one because it predicts the platform-surge mode. However, the $K_{\text{limit}}$ provided by the generator torque trimmed linear model neither does it ensure the overall system stability, meaning an accuracy lack in such a linear model. Thus, these linear model uncertainties motivate us to propose an alternative methodology for linear model validation.
3. Chirp signal based linearisation method

Industrial machinery and tools sector commonly uses methodology based on the impact-hammer test, among others, which is applied to the mechanical systems for mode shape analysis [6]. It is based on the application of an impulse signal for exciting simultaneously all system modes around an operating point. Other methodologies provide the feasibility of exciting the frequencies one after another in a determined range, for instance the chirp signal analysis [4]. The chirp signal is based on a sweep sine signal where the instantaneous frequency $f(t)$ varies linearly with time, from the starting frequency $f_0$ until the final frequency $f_1$, given as

$$f(t) = f_0 + kt$$  \hspace{1cm} (1)

$$k = (f_1 - f_0)/T$$  \hspace{1cm} (2)

where $T$ denotes the necessary time interval so that the frequency changes from $f_0$ to $f_1$.

Adequate chirp signal frequency range for our study should go from $f_0 = 0.0063\text{rad/s}$ to $f_1 = 3.1\text{rad/s}$. This is because the frequency range of interest is around the platform-surge mode (0.0627rad/s), where discrepancies between the presented both linear models have been shown in Section 2. The amplitude of the perturbation signal must be small enough to perform the analysis close to the desired operating point. For this purpose, the chirp signal amplitude of 0.001° is chosen as shown in Figure 4. Simulation time depends on the system stability as well as chirp analysis time. In this sense, the simulation has to be large enough to get operating in steady state before applying the chirp signal. In this case 3000s were enough to achieve steady state, and $T = 5000\text{s}$ was used for the chirp analysis.

![Figure 4](image_url)  

**Figure 4.** Representation of the first 1000s of the chirp signal.

This procedure can be easily implemented to the FOWT non-linear model in the simulation carried out by Matlab/Simulink. All DOFs of the non-linear FOWT model are enabled. The chirp signal is applied to the input reference pitch angle ($\beta^*$) signal to perturb the steady state operating point and analyse the excited frequencies subsequently. A simple scheme is required for the implementation of this chirp analysis as shown in Figure 5.
Two open loop simulations are carried out, with and without the perturbation chirp signal. For both simulations, generator torque ($T_q$), wind speed ($W_s$) and reference blade-pitch angle ($\beta^*$) are defined for the desired operating point (see Table 1). The perturbation effect is calculated subtracting from the generator speed results obtained with the chirp signal simulation the result of the simulation without the perturbation. After the time domain simulation process, a SISO frequency response can be calculated using the Fast Fourier Transformation (FFT) tool as shown in the following equation:

$$G(w) = \frac{\text{FFT}(\text{Generator speed with chirp (t)} - \text{Generator speed without chirp (t)})}{\text{FFT}((\text{Chirp signal (t)})}}$$  \hfill (3)

Note that more developed FFT with signal convolution [7] is not required, due to the lack of external signal noise perturbation. Bode and Nyquist frequency domain results of the linear model obtained with this chirp signal analysis, and that previously achieved for the generator torque trimmed linear model, are shown in Figure 6.

There, qualitatively similarities are found between both linear models in terms of mode identification and Bode diagram shapes. However, quantitatively discrepancies are shown in the platform-surge mode frequency and in the $K_{\text{limit}}$ to ensure system stability. Generator torque trimmed model shows a platform-surge mode frequency at 0.0627 rad/s and the $K_{\text{limit}}$ to ensure...
system stability is 0.00094 as mentioned in Section 2. Whereas the linear model achieved with the chirp signal analysis shows a platform-surge mode at 0.057rad/s and the $K_{\text{limit}}$ to ensure system stability is 0.00027. Both $K_{\text{limit}}$s are summarised in Table 4.

**Table 4.** Gain limit for system stability of each linearised model.

| Linearisation method        | $K_{\text{limit}}$ | Freq.(rad/s) |
|-----------------------------|--------------------|--------------|
| Trimming generator torque   | 0.00094            | 0.0627       |
| Chirp analysis              | 0.00027            | 0.057        |

The two linear model $K_{\text{limit}}$s of Table 4 to ensure system stability were checked with the FOWT full non-linear model in the time domain simulations as shown in Figure 7. Such simulations were carried out at the same operating point of Table 1. The same generator speed feedback configuration of Figure 2 have been used. These time domain simulation results show a great generator speed stability disagreement between the generator torque trimmed $K_{\text{limit}}$ and the achieved by the chirp analysis. When the proportional gain is slightly lower ($0.00094 > K > 0.00027$) than the $K_{\text{limit}}$ provided by the generator torque trimmed linear model, the simulation shows a permanent generator speed oscillation amplitude of about 40rpm. However, when the proportional gain is slightly lower ($K < 0.00027$) than the $K_{\text{limit}}$ provided by the chirp analysis linear model, the simulation shows a great generator speed regulation where the oscillations decay with time before 1500s. Note that the generator speed is not around the nominal value due to a permanent error introduced by the implementation of the purely proportional controller.

![Figure 7. Generator speed closed loop response with proportional pitch control.](image)

This results confirm that here proposed chirp analysis provided linear model shows the highest accuracy in comparison with those achieved with the FAST software: blade-pitch and generator torque trimmed linear models. Furthermore, this linear model not only provides the highest $K_{\text{limit}}$ accuracy to ensure system stability but also provides most accurate frequency mode system identification.
4. Conclusions

FAST linearisation tool is very useful for the analysis and control design of wind turbine systems around different periodic steady state solutions. Nevertheless, some FOWT systems with low hydrodynamic stiffness platforms present a convergence issue in the desired operating point when blade-pitch trimming linearisation process is used. On the other hand, this convergence issue is avoided if the linearisation process is done from trimming the generator torque. However, although a good system mode identification is achieved from trimming the generator torque, the $K_{limi}$ to ensure system stability is not accurate enough. Thus, a small blade-pitch perturbation analysis based on the use of the chirp signal method to the FOWT non-linear model is proposed in this work. This methodology provides high accuracy SISO linear models avoiding the convergence issue and providing a $K_{limi}$ which ensures overall system stability. An example is presented: the analysis of the linearisation of the NREL 5MW FOWT mounted on the ITI Energy barge at 13m/s constant wind speed and still water conditions. The most accurate linear model has been achieved with the chirp signal analysis methodology as has been validated by means of time domain simulations of the FOWT non-linear model. Furthermore, this methodology has been expected useful not only for plant system identification but also for more developed multi-loop control system identification.

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