Telecom-heralded entanglement between multimode solid-state quantum memories

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Future quantum networks will enable the distribution of entanglement between distant locations and allow applications in quantum communication, quantum sensing and distributed quantum computation. At the core of this network lies the ability to generate and store entanglement at remote, interconnected quantum nodes. Although various remote physical systems have been successfully entangled11–12, none of these realizations encompassed all of the requirements for network operation, such as compatibility with telecommunication (telecom) wavelengths and multimode operation. Here we report the demonstration of heralded entanglement between two spatially separated quantum nodes, where the entanglement is stored in multimode solid-state quantum memories. At each node a praseodymium-doped crystal13–14 stores a photon of a correlated pair15, with the second photon at telecom wavelengths. Entanglement between quantum memories placed in different laboratories is heralded by the detection of a telecom photon at a rate up to 1.4 kilohertz, and the entanglement is stored in the crystals for a pre-determined storage time up to 25 microseconds. We also show that the generated entanglement is robust against loss in the heralding path, and demonstrate temporally multiplexed operation, with 62 temporal modes. Our realization is extendable to entanglement over longer distances and provides a viable route towards field-deployed, multiplexed quantum repeaters based on solid-state resources.

Fast developing quantum simulation and computation centres, as well as secure communication systems, will soon require a reliable network for the distribution of entanglement. A promising blueprint for such a network, based on the quantum repeater architecture29–32, relies on distributing entanglement in a heralded fashion between remote quantum nodes, where information can be stored and manipulated in stationary, matter qubits. Entanglement between separate material systems has been demonstrated with atomic ensembles5–17, single trapped ions and atoms8–10 and more recently with solid-state systems12–13,25. However, integration into a quantum network will require a high entanglement rate with heralded operation, compatibility with the telecom network, long storage times and particularly multiplexed operation, a combination that has not yet been achieved.

Among possible candidates, rare-earth-doped solids played a pivotal role in the development of quantum memories26, providing a system with a large number of atoms naturally trapped in a solid-state matrix and with excellent coherence properties. Fundamental progress has been reported in rare-earth-based systems, including high-efficiency25–28 and long-lived20 quantum storage, as well as light–matter entanglement21–24. A fundamental advantage of rare-earth-based quantum memories is the large multiplexing capability, combining temporal, spectral and spatial degrees of freedom26–28. Although entanglement between two such memories has already been demonstrated, it was either in a non-heralded fashion29 or with a technique not directly extendable to long-distance communication30.

Here we move towards the realization of the unitary element of a quantum network by demonstrating scalable, telecom-heralded matter–matter entanglement between two remote multimode solid-state quantum memories. We demonstrate a heralding rate in the kilohertz regime and a storage time up to 25 μs, resulting in the storage of 62 temporal modes. We also show that our implementation is resistant against loss, and directly extendable to operation over long distances.

Our proposal, sketched in Fig. 1, is a hybrid approach, in which a single collective atomic excitation, created by non-degenerate parametric down-conversion sources, is shared between two remote praseodymium (Pr) quantum memories29. The two sources are based on type-I cavity-enhanced spontaneous parametric down-conversion (cSPDC) that, from a continuous-wave pump laser at 426 nm, generates photon pairs: a telecom idler at 1,436 nm and a signal at 606 nm, in resonance with the 3H4–3H2 transition of Pr3+:Y2SiO5 (ref. 15). The source cavity allows for a bi-photon linewidth of 1.8 MHz, matching the memory bandwidth. To ensure joint emission of indistinguishable photon pairs, the relative length of the two cavities, each approximately 1.2 m long, was adjusted to within a few nanometres (see Supplementary Information). Two filter cavities selected the central frequency modes of the idler channels. Both idler and signal photons were coupled in
Fig. 1 | Schematics of the experiment. Each node hosts a cSPDC and a quantum memory, with the two memories placed in two labs 10 m apart, and separated by 50 m of fibre. Each signal photon is fibre-coupled and sent to a Pr-doped memory, where it is stored through the AFC protocol. Fibre-coupled idler photons are mixed at a fibre beamsplitter in the central station. Here they are detected with superconducting single-photon detectors, where a click heralds the generation of an entangled state of the two quantum memories.

One of the outputs of the fibre beamsplitter is connected to a photodiode, used to lock the phase of the idler paths. The inset shows the energy levels of Pr, where the AFC transition is highlighted. At the bottom of the figure, in shaded grey, is the setup required to measure the off-diagonal elements of the entangled state.

single-mode fibres, with the latter sent to different quantum memories. These are two Pr-doped Y2SiO5 crystals that are located in two closed-cycle cryostats, placed in different laboratories, roughly 10 m apart. Quantum light is stored in the crystals through the atomic frequency comb (AFC) protocol25. The inhomogeneously broadened profile of the Pr-ion ensemble is tailored into a comb-like structure on the $±\frac{1}{2} g$ to $±\frac{3}{2} e$ transition by means of optical pumping. Light absorbed by the memory will be re-emitted after a fixed time $\tau = 1/\Delta$, where $\Delta$ is the period of the frequency comb. The signal photons were focused into the crystal to a waist of 40 μm, where they were stored as collective excitations $\sum_i c_i |g_i, \ldots, g_i, g_i\rangle$ of ground $|g\rangle$ and excited $|e\rangle$ states of about $N = 10^7$ Pr ions.

Entanglement between the two quantum memories is achieved by mixing the two idler modes on a fibre beamsplitter, thus erasing the which-path information. A detection event after the idler beamsplitter heralds the generation of an entangled state of the two quantum memories.

We first demonstrated matter–matter entanglement between the two remote quantum memories, with 2 μs of storage time. We used an intermittent-locking system to stabilize the setup, where $\Delta \phi$ was controlled adjusting a piezo-electric fibre stretcher25 (see Supplementary Information). We prepared the AFCs of both memories such that the storage times and temporal widths of the echoes would be equal, and we synchronized the preparation of the two AFCs to match the two storage periods. We measured an entanglement-heralding rate of 1.43 kHz at one output of the idler beamsplitter, with a duty cycle of 43%. This value, limited by the available pump power (see Supplementary Information), is about 40 times higher than the current record of distribution between long-lived memories, albeit with a lower quantum link efficiency26.

We quantified the matter–matter entanglement between the two nodes by mapping the atomic excitations back to photons and measuring the concurrence $C$ of the two modes27. This can be expressed as $C = \max(0, (2|d| - 2|p_{gA}p_{eB}|)/2)$, where $p_j$ are the probabilities of having $j$ excitations in mode A (mode B), $d = V(p_{gA} + p_{eB})/2$ and $V$ is the visibility of the interference between the two modes. The different probabilities $p_j$ can be inferred from photon statistics, and we mixed the two signal paths on a beamsplitter to measure $V$. To stabilize this interferometer, with a total fibre length close to 75 m, we used another intermittent-locking scheme where we periodically injected light detuned from the AFC. This allowed us to stabilize the relative phase between the two signal paths using a piezo-electric fibre stretcher (see Supplementary Information), while leaving the quantum memory undisturbed. The results are summarized in Fig. 2. We measured a high suppression of two-photon events26, $H_{\text{sup}} = p_{\text{tot}}(p_{gA}p_{eB}) = 0.036(8)$ (all errors herein are one standard deviation), and interference visibilities of 84(2)% and 84(4)% at the two outputs of the signal beamsplitter. This resulted in a total concurrence of $C_1 = 1.15(5) \times 10^{-2}$ and $C_2 = 1.15(7) \times 10^{-2}$, measured with detected events. As the AFC emission is a local operation, and both values of $C$ are above zero by more than
The measurement of the diagonal elements took 90 min. The two bottom plots represent the actual coincidence histograms for the points indicated in the fringe plot. The reconstructed density matrix for the matter–matter entangled state of the two remote quantum memories. The points indicated in the fringe plot.

Fig. 2 | Entanglement verification for a 2-μs AFC. a, Interference fringes obtained at the two outputs of the beamsplitter by recombining the two signal modes (circles) and heralding rate (squares). Each point was obtained by integrating for about 2 min, and with a coincidence window of 400 ns. The error bars represent one standard deviation; some bars are smaller than the symbol size. The two bottom plots represent the actual coincidence histograms for the points indicated in the fringe plot. b, Reconstructed density matrix for the matter–matter entangled state of the two remote quantum memories. The measurement of the diagonal elements took 90 min.

20 and 15 standard deviations respectively, this is a definite confirmation that the two crystals shared an entangled state. The high contribution of \( p_{00} \) to the density matrix, visible in Fig. 2b, is mostly due to the various losses in the signal channels. It can be reduced by back-tracing the value of \( c \) to the crystal (see Methods): the concurrence is then rescaled to \( c = 7.3(5) \times 10^{-2} \) and \( c = 7.4(5) \times 10^{-2} \)—that is still limited mostly by the source heralding efficiencies \( (\eta_H = 0.11(1) \text{ and } \eta_H = 0.19(1)) \) and to a lesser extent by the imperfect visibility and memory-write efficiencies. Moreover, in a Duan–Lukin–Cirac–Zoller (DLCZ)-like quantum repeater\(^3\) with built-in purification the \( p_{00} \) term would only affect the entanglement distribution rate but not the fidelity of the repeater. To provide an estimation of how well our entangled link would perform in such a repeater we defined an effective fidelity (see Methods) considering only the one- and two-photon terms of the matrix, and we inferred values of \( F_{\text{eff}}^1 = 0.92(1) \) and \( F_{\text{eff}}^2 = 0.92(2) \).

This manner of distributing entanglement using single-photon detection is particularly suited for operation over long distances owing to its robustness against idler loss, as only one photon has to be created, transmitted and detected\(^{10,12} \). The two nodes, each containing a source and a quantum memory, could be placed several kilometres apart, with the idler mixing station placed between them and connected by telecom fibres. Although this would require stabilizing the global phase of the system\(^2\), the additional propagation losses would not affect the concurrence, since only those signals that are correlated with the detection of an idler will be considered, but only lower the heralding rate. To verify this statement we used variable attenuators to add losses in both idler channels, to mimic the increased attenuation in long optical fibres. We measured the concurrence for losses up to 6.5 dB per channel: as shown in Fig. 3a, the concurrence does not change appreciably. The point-to-point fluctuation is due to the change in visibility of the interference between the single excitation states, that are affected by imbalances between the two sources (see Supplementary Information); as expected, \( H_{\text{eff}}^2 \) is instead constant.

Even if the propagation losses would not affect the quality of the entanglement distribution, in a real demonstration a greater distance between the nodes would require a longer storage time in the quantum memories, to allow the idler photons to reach the central station and for the heralding signal to come back to the node—the so-called communication time, \( t_{\text{com}} \). We therefore increased \( r \) for the two quantum memories, and again measured the entanglement. As reported in Fig. 3b, the concurrence decreased with increasing \( r \) owing to the reduced efficiency of the storage protocol, resulting in a higher contribution of \( p_{00} \). The visibility also decreased, owing to decreasing overlap between the AFC echoes at longer storage times (see Supplementary Information). Nevertheless, even for the longest \( r \) of 25 μs—about 10\(^5\) times longer than previous demonstrations in solid-state rare-earth quantum memories\(^{10,29} \)—the positivity of the concurrence was maintained with more than 5 standard deviations, allowing heralded entanglement over 5 km of optical fibre.

Finally, we would like to emphasize the advantage that comes from the temporal multimodality of this protocol. Thanks to the nature of the AFC storage, several temporal modes can be stored in the quantum memory before the first absorbed mode has been re-emitted\(^{27} \). This means that it is not necessary to wait for the entanglement-heralding signal from the central station before storing another mode. Multimode operation then becomes particularly useful for operation over long distances, where \( t_{\text{com}} \) seriously limits the entanglement distribution rate for single-mode memories. To investigate this effect we analysed the data acquired for the longest storage, where we considered a \( t_{\text{com}} \) of 25 μs, equal to the AFC storage and simulating quantum memories 5 km apart. Considering a single temporal mode duration of 400 ns, which comprises 90% of the photon, we split \( r \) into 62 temporal modes. We then divided the whole measuring time into ‘communication trials’ of 25 μs, and sorted the idlers into the 62 modes according to their arrival time in each of these trials. Although a single-mode memory would allow storage only in the first mode (as reported in Fig. 4b), we were able to repeat our entanglement analysis for an increasing number of allowed modes per communication trial. We selected only those idlers that arrived within the first \( n \) temporal modes and used this list to calculate the concurrence and heralding rate. Owing to the low number of two-photon events, we had to estimate the values of \( p_{1} \) for each mode (see Methods for more details). The results are reported in Fig. 4c. The concurrence remained constant whereas the heralding rate increased linearly with the number of modes allowed. This increase becomes larger as longer distances and longer \( t_{\text{com}} \) are considered.

The remote matter–matter entanglement demonstrated in this work represents a crucial step towards a viable realization of a quantum repeater, to our knowledge showing for the first time telecom-heralded

Fig. 3 | Concurrence for different experimental configurations. a, Increasing losses in the idler channels. Although the heralding rate decreases, the concurrence remains constant. The error bars represent one standard deviation. The solid bands represent a ±1σ variation from the mean values of visibility, \( K_{\text{eff}}^2 \) and concurrence.

b, Increasing storage time. We increased \( r \) up to 25 μs, and measured positive concurrence for all cases. The empty dark circles are the expected values for visibilities, and the error bars represent one standard deviation.
entanglement between two multimode quantum memories. Our herading limit was set by the available cSPDC pump power, and could thus be boosted up to 27 kHz while maintaining positive concurrence (see Supplementary Information). We showed that our scheme is scalable for long distances, and integration into field-deployed quantum networks could soon be achieved thanks to the depth of possibilities available to these systems. First, entanglement between fully decoupled setups could be realized by employing a remote verification scheme and an active stabilization of the global phase of the system. 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Methods

Phase locking

The relative phase between the idler photons interfering at the fibre beamsplitter in the central station needs to be locked to achieve consistent heralding. In addition, since the entanglement of the heralded state is verified by recombining the signal photons on another beamsplitter, their relative phase has to be controlled as well.

We achieved phase locking by alternating between stages of locking and measuring. During the locking stage, we injected classical fields in the setup as a reference. To lock the relative phase between the idler modes we used a classical beam at 1,436 nm. This field is naturally generated by a difference-frequency-generation process in the same periodically poled lithium niobate (PPLN) crystals that we use for the cavity-enhanced photon-pair generation. It is a valid reference as it has the same wavelength as the idler photons and it shares the same optical path. In the case of the signal photons, we used light 300 MHz detuned from the AFC frequency to avoid disturbing the comb spectrum. See Supplementary Information for additional details.

For both idler and signal stabilization, we detected the interference between the classical reference fields at the two beamsplitters with a photodiode, which we use as the input of a PID (proportional–integral–derivative) controller. We fed back the output of the PID on home-made fibre stretchersthat we built by rolling a portion of the optical fibres around cylindrical piezoelectric actuators.

Limit on the fringe visibilities

There are three main factors that limited the visibilities that we measured and that we account for in Fig. 3b.

Phase noise in optical fibres. As mentioned in the previous section, we alternated between locking stages and measuring stages. During the latter there is no control over the phase evolution in the fibres, as the classical reference fields are blocked. Despite the good passive stability that we achieved in our fibres, the contribution of phase noise in the fibres carrying the signal photons is still considerable and is the main factor that limits the visibility that we measured.

Indistinguishability of the idler modes. We quantified the overlap between the idler modes generated by the two sources by performing a Hong–Ou–Mandel experiment. After correcting for accidental counts we estimated an overlap of $\eta_{\text{ind}} = 90(7)$%.

Overlap of the retrieved signal modes. The state tomography that we used required a recombination of the signal modes after storage in the AFC. Therefore, we adjusted the two AFCs to best match the wavepackets of the two retrieved signal photons. However, because of the different experimental conditions of the two memories, placed in different laboratories and prepared from different setups, this overlap was not perfect. For each case we measured the intensity, width and position of each echo, and calculated the maximum interference visibility with these parameters. The final expected visibility was calculated as the product of these three factors.

Effective fidelity

The density matrix of the reconstructed entangled state has a high $p_{00}$ contribution, as visible in Fig. 2b. In a DLCZ-like quantum repeater the vacuum components will affect the rate of the experiment, but not the fidelity of the distributed entanglement. This is because the DLCZ scheme includes built-in purification by using two chains of entangled memories.

In this case only events with one photon at each side will be considered, therefore removing the vacuum component of the density matrix. We then infer the effective fidelity $F_{\text{eff}}$ as $F_{\text{eff}} = (\rho^* \bar{\rho} \rho^* \bar{\rho})^{1/2}$, where $\rho$ is the reconstructed state where we fix the $|00\rangle\langle00|$ component to zero, and $\bar{\rho}$ is the ideal state $|\psi\rangle\langle\psi| = 1/2(|10\rangle\langle10| + |10\rangle\langle01| + |01\rangle\langle10| + |01\rangle\langle01|)$.

In the notation with $p_{ij}$ the fidelity can be expressed as:

$$F_{\text{eff}} = \frac{1}{2} \left( \frac{p_{00}^2 + p_{01}^2 + p_{10}^2 + p_{11}^2}{p_{00} + p_{01} + p_{10} + p_{11}} \right).$$

where the components $p_{ij}$ are defined in equations (3)–(6) below. The fidelity reported in the main text was calculated using values of $p_{ij}$ back-traced to the crystal, and the error was calculated with standard error propagation.

Calculation of the concurrence in the crystal

As presented in the main text, the concurrence can be calculated from the photon statistics of the two signal modes and from the visibility of the recombination fringe. Note that the exact expression for the concurrence can be easily calculated from the eigenvalues of the matrix $\rho_{\psi}^\dagger \rho_{\psi}$, where $\rho$ is the density matrix of the entangled state $\psi = (\sigma_{-} \otimes \sigma_{+}) \rho \sigma_{+}^\dagger (\sigma_{-} \otimes \sigma_{+})$ is the‘spin-flipped’ state of $\rho$. $\sigma_\pm$ is its complex conjugate, $\sigma_i$ is the Pauli matrix and $\otimes$ is the tensor product. Note that in our analysis we assume that all the photonic modes have, at most, one photon. This assumption is enough to prove entanglement.

The value obtained is an estimation of the concurrence of the two signal modes at the detection stage, and it therefore includes the detector efficiency $\eta_{\text{det}}$, the transmission $\eta_{\text{meas}}$ between the memory and the detectors, and the efficiency $\eta_{\text{read}}$, with which the collective excitation stored in the AFC is mapped back into a photon. By correcting for all of these losses it is possible to estimate the concurrence of the entangled state of the two quantum memories. The correction changes the values of the components of the density matrix $p_{ij}$ to $\tilde{p}_{ij}$ according to:

$$\tilde{p}_{11} = \frac{1}{\eta_{\text{det}}} p_{11},$$

$$\tilde{p}_{10} = \frac{1}{\eta_{\text{det}}} p_{10} - \frac{1 - \eta_{\text{det}}}{\eta_{\text{meas}}} p_{11},$$

$$\tilde{p}_{01} = \frac{1}{\eta_{\text{meas}}} p_{01} - \frac{1 - \eta_{\text{meas}}}{\eta_{\text{det}}} p_{11},$$

$$\tilde{p}_{00} = p_{00} - \frac{1 - \eta_{\text{det}}}{\eta_{\text{meas}}} p_{10} - \frac{1 - \eta_{\text{meas}}}{\eta_{\text{det}}} p_{01} + \frac{(1 - \eta_{\text{det}})(1 - \eta_{\text{meas}})}{\eta_{\text{det}} \eta_{\text{meas}}} p_{11}.$$

where $\eta_{\text{det}}$ and $\eta_{\text{meas}}$ are the efficiencies of nodes A and B. As mentioned these are calculated from $\eta = \eta_{\text{det}} \times \eta_{\text{meas}} / \eta_{\text{read}}$. $\eta_{\text{det}}$ was measured with an attenuated laser beam of known power. $\eta_{\text{meas}}$ was measured with classical light as the ratio between the power of the beam after the memory and the power after the final fibre output before the detector. These values were taken at the beginning and at the end of the measurement, taking the average between the two as the final value. $\eta_{\text{read}}$ was calculated as $C/(A - B)$, where $A$ is the rate of coincidences of photons transmitted through a pit in the quantum memory, corrected for the residual pit background optical depth absorption; $B$ is the rate of coincidences of photons that are transmitted through the AFC, and not absorbed; and $C$ is the rate of coincidences of the AFC echo. $\eta_{\text{read}}$ is not corrected to include those photons that are absorbed by the AFC but not re-emitted; however, the background optical depth of the AFC is very small, and can therefore be neglected. The values for all efficiencies are reported in Extended Data Table 1, where we have also included the heralding efficiency $\eta_{\text{her}}$, (calculated to right before the quantum memory) and the AFC storage efficiencies.

$\rho_{\text{ini}}$ model and scaling with number of temporal modes

In Fig. 4c we analyse the dependence of the concurrence with the number of temporal modes that we use. This means that for the first
points of the plot we are artificially reducing the count rate by a factor of 62, 31, and so on. This is not a problem for the visibility values nor for $p_{00}$, $p_{10}$ and $p_{01}$. It is, however, for $p_{11}$, as it is a much rarer event. This is why, only for the case of Fig. 4c of the main text, we introduce a simple model that relates the value of $p_{11}$ and the cross-correlation.

In general, $p_{11}$ can be seen as the probability that, once an idler gives a click after the central beamsplitter, we also measure a click after each of the signal detectors. In our case, this can happen for two different reasons: 1. Both signal detectors fire an accidental click, with a probability $p_{\text{acc}}^{(j)}$ with $j = \{A, B\}$; and 2. There is a correlated signal photon at detector $j$ plus an accidental click at detector $i$, where $i, j = \{A, B\}$.

We can write this down as:

$$p_{11} = p_{11}^{\text{acc}}(A) + p_{11}^{\text{acc}}(B) + p_{11}^{(A)} + p_{11}^{(B)}.$$  

If now we use the expression for the second-order cross-correlation function of each source we have that $g_{11}^{(2)} = P_{11}^{(A)}/P_{\text{acc}}^{(A)}$ and $g_{11}^{(2)} = P_{11}^{(B)}/P_{\text{acc}}^{(B)}$, and we can derive the final model:

$$P_{11} = 4 \frac{P_{10}P_{01}}{P_{11}^{\text{acc}}} \left[ 1 + \frac{g_{11}^{(2)}}{2} + \frac{g_{11}^{(2)}}{2} \right].$$  (7)

The factor $\frac{1}{2}$ in $g_{11}^{(2)}$ comes from the fact that during the experiment the actual value of $g_{11}^{(2)}$ will be halved, since half of the heralding events come from a different source and therefore the accidentals are duplicated.

In Extended Data Fig. 1 we used this model to confirm that the value of $p_{11}$ for every allowed mode (brown circles) and the value that we actually measured (shaded area) are compatible.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request. Source data are provided with this paper.

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Author contributions

D.L.-R. built and operated the SPDC sources, J.V.R., A.S. and S.G. assembled and operated the solid-state quantum memory setups. D.L.-R. and S.G. designed and built the phase lock for the entanglement measurement. The experiment was conducted by D.L.-R., S.G., J.V.R. and A.S., who also jointly analysed the data. D.L.-R., S.G. and H.d.R. wrote the paper, with input from all co-authors. H.d.R. conceived the experiment and supervised the project.

Competing interests

The authors declare no competing interests.

Additional information

Supplementary information

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Peer review information

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Extended Data Fig. 1 | Comparison between experimental and modelled values for $p_{11}$. The brown dots show the scaling of $p_{11}$ using the model derived from equation (7). The shaded area corresponds to the value of $p_{11}$ that we measured experimentally considering one standard deviation for the error.
Extended Data Table 1 | Values of efficiencies for nodes A and B

|       | $\eta_{\text{det}}$ | $\eta_{\text{loss}}$ | $\eta_{\text{read}}$ | $\eta_{\text{write}}$ | $\eta_{\text{AFC}}$ | $\eta_{H}$ |
|-------|---------------------|-----------------------|-----------------------|-----------------------|---------------------|-----------|
| Node A| 0.61(3)             | 0.81(1)               | 0.35(1)               | 0.72(2)               | 0.252(4)            | 0.11(1)   |
| Node B| 0.61(3)             | 0.65(3)               | 0.33(1)               | 0.83(3)               | 0.273(4)            | 0.19(1)   |

$\eta_{\text{det}}$, detection efficiency; $\eta_{\text{loss}}$, transmission between the memory and the detectors; $\eta_{\text{read}}$, AFC read efficiency; $\eta_{\text{write}}$, AFC write efficiency; $\eta_{\text{AFC}}$, AFC storage efficiency; $\eta_{H}$, heralding efficiency.