On the Possibilities of Distinguishing Majorana from Dirac Neutrinos

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Abstract

Cross sections involving massive Dirac or Majorana neutrinos usually differ only by terms which are suppressed by the smallness of the neutrino mass. The process $e^+e^- \rightarrow \nu\nu$, however, is an example in which the two differential cross sections are quite distinct even in the limit $m_\nu \rightarrow 0$ as long as $m_\nu \neq 0$. I discuss the cause of this phenomenon and comment on strategies to identify processes with quite different Majorana and Dirac neutrino signals.

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I. INTRODUCTION

The question of distinguishing Dirac from Majorana neutrinos has been frequently discussed (see e.g. [1,2]). Since the Majorana neutrino is its own anti-particle, it has in general quite distinct properties from the Dirac neutrino. Experimentally, it is nevertheless difficult to distinguish between these two kinds of neutrinos. I discuss the possibility of revealing the neutrino’s nature using neutral-current (NC) neutrino data such as measured at LEP I or by the CHARM II collaboration. Because of contradicting statements which have recently appeared [3–5] I provide explicit results for the neutrino cross sections of concern, keeping the neutrino mass dependence where necessary. In particular I illustrate and comment on the following list of important and more or less familiar properties of neutrinos:

a) If neutrinos are massless particles and if only the left-handed (or, equivalently, only the right-handed) field interacts then Majorana and Dirac neutrinos are identical, such that the theoretical distinction into “Dirac” and “Majorana” is meaningless. [6]
This is particularly true for standard model (SM) neutrinos. The proof is given in Appendix A.

b) A massless neutrino \( m_\nu = 0 \) is always a helicity eigenstate for both Majorana and Dirac neutrinos. In this case the helicity is an intrinsic property and represents a Lorentz invariant quantum number.

c) A massive Majorana neutrino, even in the small-mass limit \( m_\nu \to 0 \) with \( m_\nu \neq 0 \), is in general not an eigenstate of helicity and can have a spin pointing in an arbitrary direction. This is also true for massive Majorana neutrinos moving at relativistic velocities \( \beta \to 1, \beta \neq 1 \); see e.g. [7,8].

d) As a consequence of b) and c), observable quantities for Majorana neutrinos with small non-vanishing masses are in general different from those of massless Majorana neutrinos, and they remain distinct even in the limit of \( m_\nu \to 0, m_\nu \neq 0 \). Similarly, massless
Majorana neutrinos and highly relativistic massive Majorana neutrinos have in general distinguishable observables even in the limit $\beta \to 1$, $\beta \neq 1$. In other words, the limits $m_\nu \to 0$ and $\beta \to 1$ are not smooth for Majorana neutrinos. An example of this important and often neglected point are neutrino cross sections for LEP I which are given in Sec. II.

e) An exception of d) are processes in which neutrinos are produced via a weak charged current (neglecting here the possibility of CC neutrino pair production): If the charged current (CC) is purely left-handed (or, equivalently, purely right-handed) observables of these processes are approximately the same for light ($m_\nu \to 0$ with $m_\nu \neq 0$) and for massless ($m_\nu = 0$) neutrinos, disregarding whether the neutrinos are Majorana or Dirac particles. The limit $m_\nu \to 0$ is smooth. This feature is illustrated by an explicit calculation of neutrino cross sections for the CHARM II experiment; see Sec. III and Appendix B.

f) A massive Dirac neutrino in the small-mass limit ($m_\nu \to 0$ with $m_\nu \neq 0$) is approximately an eigenstate of helicity. Unlike in the case of Majorana neutrinos, the limit $m_\nu \to 0$ is always smooth: Physical observables calculated for massive Dirac neutrinos are identical to those of massless Dirac neutrinos when taking $m_\nu \to 0$. (Similarly, the relativistic limit $\beta \to 1$ is smooth.) This property of Dirac neutrino cross sections is apparent in the results given in Sec. II and III.

g) The neutral current coupling of a massive Majorana neutrino is pure axial-vector (assuming diagonal NCs), independent of the specific values of the vector and axial-vector couplings in the Lagrangian. This means that the vector coupling of a massive Majorana neutrino does not contribute to the neutral current, no matter what its value is. This follows from charge-conjugation properties of the free Majorana field; see Ap-

\[ \text{In these cases both light and massless neutrinos are helicity eigenstates.} \]
In contrast, the vector coupling of a (massless or massive) Dirac neutrino does contribute to the neutral current, such that the general neutral current cross sections of Dirac neutrinos consist of both vector and axial-vector contributions.

In Sec. IV, I provide a summary, focusing on the statements d) and e) given above. I point out the importance of the history of an incoming neutrino, in particular, whether it has been produced in a (chiral) charged current interaction or not. Furthermore, I comment on previous publications on the subject of distinguishing Majorana and Dirac neutrinos. The possibility of flipping the helicity of the neutrino through strong magnetic fields is also considered.

II. NEUTRINO CROSS SECTIONS AT LEP I

Neutral current neutrino pair production has been measured indirectly at LEP I. Taking the neutrino couplings to be the purely left-handed neutrino interaction of the SM Lagrangian, the angular distribution for \(e^+e^- \rightarrow \nu_f\bar{\nu}_f\) can be easily calculated. At the Z peak \([s \approx m^2_Z, s\) being the squared center-of-mass-system (CMS) energy\], the neutrinos are (almost) exclusively produced via neutral current exchange (Fig. 1). In the case of Majorana (M) neutrinos \((\bar{\nu} \equiv \nu)\) and Dirac (D) neutrinos the differential cross sections at \(s \approx m^2_Z\) are

\[
\left(\frac{d\sigma}{d\Omega}\right)_{M}^{\nu \neq 0} = \sigma_0 \left\{ \left[ (g^e)_V^2 + (g^e)_A^2 \right] (1 + \cos^2 \theta) + O \left( \frac{m^2_e}{m^2_Z} \right) + O \left( \frac{m^2_\nu}{m^2_Z} \right) \right\}, \tag{2.1}
\]

\[
\left(\frac{d\sigma}{d\Omega}\right)_{D} = \sigma_0 \left\{ \left[ (g^e)_V^2 + (g^e)_A^2 \right] (1 + \cos^2 \theta) + 4g^e_V g^e_A \cos \theta + O \left( \frac{m^2_e}{m^2_Z} \right) + O \left( \frac{m^2_\nu}{m^2_Z} \right) \right\}, \tag{2.2}
\]

for general neutrino couplings, see Appendix B.

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\(\bar{\nu} = \nu\) in the case of \(f = e\), a charged current contribution exists, but can be neglected for \(s \approx m^2_Z\).
\[ \sigma_0 = \frac{G^2_F}{128\pi^2} \left| \frac{m_Z^2}{s - m_Z^2 + im_Z \Gamma_Z} \right|^2, \]  

(2.3)

\( \theta \) being the CMS angle between the momenta of the initial electron and the outgoing neutrino, and \( g^e_V \) (\( g^e_A \)) being the vector (axial-vector) coupling of the electron to the Z boson. Setting \( m_\nu = 0 \), the Dirac differential cross section is identical to the SM result for massless neutrinos. The Majorana and Dirac differential cross sections, however, remain different even in the limit \( m_\nu \to 0, m_\nu \neq 0 \). This illustrates the statement d) of the introduction. Setting \( m_\nu = 0 \), (2.1) is no longer valid because massless and massive Majorana neutrinos are completely different objects; see Appendix A. The cross section of a massless Majorana neutrino is actually equal to the massless Dirac neutrino cross section and is obtained by taking the limit \( m_\nu \to 0 \) in (2.2). In the case of generalized couplings, the massless Majorana and Dirac cross sections are not identical; see Appendix B.

Measuring the angular distribution of the neutrino pairs would clearly provide for a conclusive answer on the Dirac or Majorana nature of massive neutrinos. In particular, the forward-backward asymmetries of (2.1) and (2.2) are

\[ A_{FB}^M = 0, \]
\[ A_{FB}^D = \frac{3}{2} \frac{g^e_V g^e_A}{(g^e_V)^2 + (g^e_A)^2} \approx 0.45 \]

(2.4) \hspace{1cm} (2.5)

for Majorana and Dirac neutrinos, respectively. In a somewhat different context such asymmetries have already been considered in [7,10,11]. Even if the CC contribution to neutrino pair production was included, \( A_{FB}^M \) would remain zero and would still be significantly distinct from \( A_{FB}^D \). Hence measuring the differential cross section for neutrino pair production would determine the nature of the neutrino even at CMS energies far away from the Z peak.

Unfortunately, the measurement of the angular distribution of the outgoing neutrino-pair is not feasible at LEP I since the number of produced neutrino pairs (about 2 million) is far too small to create a sufficient number of detected neutrino-pair events in the LEP experiments.

Assuming lepton flavour universality and the existence of three light neutrino generations,
the LEP I data on the Z peak provide for an indirect measurement of the total cross section for NC neutrino-pair production. Integrating (2.1) and (2.2), the theoretical predictions of the Dirac and Majorana total cross sections are found to be identical up to terms of $O(m_\nu^2/m_Z^2)$. [The terms of $O(m_\nu^2/m_Z^2)$ in (2.1) and (2.2) are the same.] Taking into account the existing experimental upper bounds on light neutrino masses \[12\], the $O(m_\nu^2/m_Z^2)$ difference in the total cross sections is too small to be tested by the LEP I experiments. Hence LEP I is not able to distinguish between light Dirac and light Majorana neutrinos.

III. NEUTRINO CROSS SECTIONS OF THE CHARM II EXPERIMENT

Neutral current neutrino cross sections have also been measured by the CHARM II collaboration \[13\] using the processes $\nu_\mu e^- \rightarrow \nu_\mu e^-$ and $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$ \[14,15\]. The incident neutrinos are produced via a chiral CC interaction. They are therefore approximately eigenstates of helicity. This needs to be taken into account when calculating the relevant cross sections: Recall point e) of the introduction.

The CHARM II experiment proceeds as follows: Charged pions are produced and directed at a fixed target. They predominantly decay into a muon and a muon neutrino. This neutrino can interact with a target electron only via neutral current Z exchange, leading to an electron and a muon neutrino in the final state.

In the case of Dirac neutrinos, a positively (negatively) charged pion can decay into $\mu^+\nu_\mu$ ($\mu^-\bar{\nu}_\mu$). The cross sections for neutrino and anti-neutrino scattering are different.

In the case of Majorana neutrinos there is no distinction between neutrino and anti-neutrino. However, the fact that the intermediate neutrino has been produced via a chiral CC interaction is important and affects the subsequent neutral current interaction. To take this into account, I calculate the complete process

$$\pi^+(k_1) \rightarrow \mu^+(k_3) + \nu_\mu(q)$$

with the subsequent NC reaction.
\[ \nu_\mu(q) + e^-(p_1) \rightarrow \nu_\mu(k_2) + e^-(p_2) . \] (3.1)

The corresponding Feynman diagram is shown in Fig. 2. The intermediate neutrino \( \nu_\mu(q) \) is a virtual particle. It appears as a propagator in the amplitude. For simplicity I assume \( \nu_\mu \) to be approximately a mass eigenstate (small mixing angles in the neutrino sector). The 2 \( \rightarrow 3 \) amplitudes involving a Dirac (D) or Majorana (M) muon neutrino are obtained as

\[ \mathcal{M}^{\mu^+}_D = A j^e_\lambda \bar{u}_t(k_2) \gamma^\lambda P_L (\not{q} + m_\nu) \gamma_\rho P_L v_t(k_3) \cdot k_1^\rho , \] (3.2)

\[ \mathcal{M}^{\mu^+}_M = -A j^e_\lambda \bar{u}_t(k_2) \gamma^\lambda \gamma_5 (\not{q} + m_\nu) \gamma_\rho P_L v_t(k_3) \cdot k_1^\rho , \] (3.3)

where the superscript \( \mu^+ \) indicates the detection of a positively charged muon in the final state. In the case of an incoming negatively charged pion, the three-particle final state contains a \( \mu^- \) and the amplitudes involving a Dirac or Majorana neutrino are

\[ \mathcal{M}^{\mu^-}_D = A j^e_\lambda \bar{u}_t(k_3) \gamma_\rho P_L (\not{q} + m_\nu) \gamma^\lambda v_t(k_2) k_1^\rho , \] (3.4)

\[ \mathcal{M}^{\mu^-}_M = -A j^e_\lambda \bar{u}_t(k_3) \gamma_\rho P_L (\not{q} + m_\nu) \gamma^\lambda \gamma_5 v_t(k_2) \cdot k_1^\rho . \] (3.5)

Here the following abbreviations have been used:

\[ q = k_1 - k_3 , \]

\[ j^e_\lambda = \bar{u}_s(p_2) \gamma_\lambda (g^e_\nu - g_A^e \gamma_5) u_s(p_1) , \]

\[ P_L = \frac{1}{2}(1 - \gamma_5) , \quad P_R = \frac{1}{2}(1 + \gamma_5) , \] (3.6)

\[ A = \left( \frac{g}{2 \cos \theta_W} \right)^2 \frac{g^2}{8m_W^2} \frac{1}{(p_2 - p_1)^2 - m_Z^2} \frac{1}{q^2 - m_\nu^2} f_\pi \cos \theta_1 , \]

\( f_\pi \) being the pion decay constant, and \( \theta_1 \) being the CKM mixing angle for \( \pi^\pm \) decay \[^{[16]}\].

The axial-vector coupling of the Majorana neutrino to the \( Z \) boson is apparent in (3.3) and (3.5). Splitting the amplitudes involving a Majorana neutrino into a part with left-handed and a part with right-handed \( Z\nu\nu \) vertex, they can be written as \( (\gamma_5 = P_R - P_L) \)

\[ \mathcal{M}^{\mu^\pm}_M = \mathcal{M}^{\mu^\pm}_D + \Delta \mathcal{M}^{\mu^\pm} , \] (3.7)

where I define
\[
\Delta M^{\mu^+} = -A j_{\lambda}^e \bar{u}_{t_2}(k_2) \gamma^\lambda P_R (\not{q} + m_{\nu}) \gamma_\rho P_L \bar{v}_{t_3}(k_3) k_1^\rho ,
\]
\[\Delta M^{\mu^-} = -A j_{\lambda}^e \bar{u}_{t_3}(k_3) \gamma_\rho P_L (\not{q} + m_{\nu}) \gamma^\lambda P_R \bar{v}_{t_2}(k_2) k_1^\rho .\]

In the case of general vector and axial-vector couplings, it is also possible to write the resulting Dirac and Majorana amplitudes as a combination of the above results for \(M_D^{\mu^\pm}\) and \(\Delta M^{\mu^\pm}\), the coefficients being functions of the generalized coupling constants; see Appendix B.

Using the identities

\[k_1 = k_3 + q , \quad \not{q}^2 = q^2 , \quad \not{k}_3 \bar{v}_{t_3}(k_3) = -m_\mu \bar{v}_{t_3}(k_3) ,\]

I arrive at the following expressions:

\[M_D^{\mu^+} = A j_{\lambda}^e \left( q^2 \bar{u}_{t_2}(k_2) \gamma^\lambda P_L \bar{v}_{t_3}(k_3) - m_\mu \bar{u}_{t_2}(k_2) \gamma^\lambda P_L \not{q} \bar{v}_{t_3}(k_3) \right) ,\]
\[\Delta M^{\mu^+} = A j_{\lambda}^e m_\nu \left( m_\mu \bar{u}_{t_2}(k_2) \gamma^\lambda P_R \bar{v}_{t_3}(k_3) - \bar{u}_{t_2}(k_2) \gamma^\lambda P_R \not{q} \bar{v}_{t_3}(k_3) \right) ,\]
\[M_D^{\mu^-} = A j_{\lambda}^e \left( q^2 \bar{u}_{t_3}(k_3) \gamma^\lambda P_L \bar{v}_{t_2}(k_2) + m_\mu \bar{u}_{t_3}(k_3) \not{q} \gamma^\lambda P_L \bar{v}_{t_2}(k_2) \right) ,\]
\[\Delta M^{\mu^-} = -A j_{\lambda}^e m_\nu \left( m_\mu \bar{u}_{t_3}(k_3) \gamma^\lambda P_R \bar{v}_{t_2}(k_2) + \bar{u}_{t_3}(k_3) \not{q} \gamma^\lambda P_R \bar{v}_{t_2}(k_2) \right) ,\]

Since \(\Delta M\) is proportional to the neutrino mass, one might expect the square of the Majorana amplitude, \(|M_M|^2 = |M_D + \Delta M|^2\), to be equal to the squared Dirac amplitude plus a term proportional to \(m_\nu\) plus terms proportional to higher powers of \(m_\nu\). The term proportional to a single power of \(m_\nu\), however, vanishes since the “interference term” yields

\[M_D^{\mu^+}\Delta M + \Delta M^{\mu^+}M_D = O(m_\nu^2) .\]

Summing over the spins of the final fermions and averaging over the spin of the initial electron I obtain the exact final result of the squared \(\pi^\pm e^- \rightarrow \nu_\mu \mu^\pm e^-\) amplitude for the Majorana case:

\[|M_M^{\mu^\pm}|^2 = 8 |A|^2 \left\{ \left( g_v^e \right)^2 + \left( g_A^e \right)^2 \right\} \left( q^2 - m_\nu^2 \right) \left( q^2 - m_\mu^2 \right) \left[ (p_1 k_2)(p_2 k_3) + (p_1 k_3)(p_2 k_2) \right] + 2m_\mu^2 \left[ (q^2 + m_\nu^2) + 1 + \frac{m_\mu^2}{m_\nu^2} \right] \left( k_3 q \right) \left[ (p_1 k_2)(p_2 q) + (p_1 q)(p_2 k_2) \right] \right\} .\]
\[
\pm 2 g_V^e g_A^e \left\{ (q^2 + m_\nu^2)(q^2 - m_\mu^2) \left[ (p_1 k_3)(p_2 k_2) - (p_1 k_2)(p_2 k_3) \right] \\
+ 2 m_\mu^2 \left[ (q^2 - m_\mu^2) + (1 - \frac{m_\nu^2}{m_\mu^2})(k_3 q) \right] \left[ (p_1 q)(p_2 k_2) - (p_1 k_2)(p_2 q) \right] \right\} \\
- \left[ (g_V^e)^2 - (g_A^e)^2 \right] m_\nu^2 \left\{ (q^2 - m_\nu^2)(q^2 - m_\mu^2) (k_2 k_3) \\
+ 2 m_\mu^2 \left[ (q^2 + m_\nu^2) + (1 + \frac{m_\nu^2}{m_\mu^2})(k_3 q) \right] (k_2 q) \right\} \\
\pm 2 m_\nu^2 \left\{ [(g_V^e)^2 + (g_A^e)^2] \left[ (q^2 + m_\mu^2) (p_1 p_2)(k_3 q) + 2 m_\mu^2 q^2(p_1 p_2) \right] \\
\pm 2 g_V^e g_A^e (q^2 - m_\mu^2) \left[ (p_1 k_3)(p_2 q) - (p_1 q)(p_2 k_3) \right] \\
- 2 \left[ (g_V^e)^2 - (g_A^e)^2 \right] m_\nu^2 \left[ (q^2 + m_\mu^2) (k_3 q) + 2 m_\mu^2 q^2 \right] \right\}. \tag{3.16}
\]

The squared amplitude for the Dirac case, \( |\mathcal{M}_D^{\mu\nu}|^2 \), is recovered when setting \( m_\nu = 0 \) in the above result for \( |\mathcal{M}_M^{\mu\nu}|^2 \). For all of the allowed phase space, including \( q^2 = 0 \), the result for massive Majorana neutrinos can be expressed as

\[
|\mathcal{M}_M^{\mu\nu}|^2 = |\mathcal{M}_D^{\mu\nu}|^2 + O \left( \frac{m_\nu^2}{m_\mu^2} \right). \tag{3.17}
\]

In the limit of vanishing neutrino mass the Majorana cross section clearly approaches the Dirac cross section: A light Majorana neutrino produced via the CC process \( \pi^+ \rightarrow \mu^+ + \nu_\mu \) behaves approximately like a Dirac neutrino because of the left-handedness of the CC vertex. Similarly, a Majorana neutrino produced via \( \pi^- \rightarrow \mu^- + \nu_\mu \) behaves approximately like a Dirac anti-neutrino. Although the Majorana \( \nu \nu Z \) vertex is pure axial-vector, the polarization of the incoming muon neutrino leads to a suppression of the right-handed contribution to the cross section. The above results illustrate point e) of the introduction.

In the case of Dirac neutrinos one can in good approximation calculate neutrino cross sections by only considering the sub-reaction (3.1). In the case of Majorana neutrinos this, however, yields a wrong result. Instead of calculating the complete \( 2 \rightarrow 3 \) process as done above, one can evaluate the sub-reaction (3.1) taking into account a “state preparation factor” \( \text{[6]} \) for the incoming Majorana neutrino (see Appendix A). This gives a good approximation, but neglects effects of \( O(m_\nu) \).

Taking into account the upper bound for the muon neutrino mass, \( m_{\nu_\mu} < 0.17 \text{ MeV} \) at
90% confidence level [12], the difference between Dirac and Majorana cross sections is too small to be detected by the CHARM II experiment.

IV. SUMMARY AND COMMENTS

Typical neutrino experiments such as CHARM II are based on the fact that the incident neutrinos are created via purely left-handed CC processes. This makes it very difficult to experimentally distinguish between Majorana and Dirac neutrinos: The Majorana cross section smoothly approaches the Dirac result as $m_\nu \to 0$. Differences in the two cross sections are of $O(m_\nu/M)$, where $M$ is a typical energy scale of the process. So far such differences are below the detection limits of present-day experiments. A recent claim [3] that the neutrino events observed by the CHARM II experiment give strong evidence for the absence of Majorana neutrinos is incorrect. This was already noticed by Kayser [4]. The main error is the negligence of the fact that the incident neutrinos are produced via a left-handed interaction. The calculation in Sec. III and its generalization in Appendix B show this explicitly.

The effect of the left-handed production mechanism can approximately be formulated by the introduction of a state preparation factor [6]. This approximation is valid for small neutrino masses; see (A.15). Not taking into account the left-handed production mechanism, that is, neglecting the state preparation factor, one obtains incorrect Majorana amplitudes. In the case of Dirac neutrinos, the state preparation factor either leaves the amplitude approximately unchanged or leads to approximately non-interacting ("sterile") Dirac neutrinos. In the case of massless chiral neutrinos there is no distinction between Majorana and Dirac particles. Hence their cross sections are identical [6] as is pointed out in [3].

To become more sensitive to the Majorana or Dirac nature of neutrinos, it would be ideal to avoid the presence of the "state preparation factor". An obvious example for such a process is the production of neutrino pairs. If such neutrinos are massive Majorana particles, their transverse polarization is not suppressed by the smallness of their mass. In contrast,
Dirac neutrinos are almost eigenstates of helicity. For the NC process $e^+e^- \rightarrow Z \rightarrow \nu_f \bar{\nu}_f$

I have pointed out the existence of Majorana neutrino cross sections which are not smooth as $m_\nu \rightarrow 0$: Recall the forward-backward asymmetries of (2.4) and (2.5). Such observables are ideal candidates for distinguishing Majorana and Dirac neutrinos. The existence of this possibility was not recognized in the discussion given in [6]. The challenge, of course, is to identify experiments with high enough luminosity to collect enough events.

Another possibility is to change the “state preparation factor” for CC produced neutrinos. Massive Dirac neutrinos have a radiatively induced magnetic moment (see e.g. [17]). Massive Majorana neutrinos possibly have a magnetic transition moment (see e.g. [18]), connecting two different mass eigenstates with opposite chiral preparation. Though these moments are expected to be very small, the application of extremely strong magnetic fields can lead to a state transition of the neutrino: The neutrino can lose its memory of its chiral production. Effectively, the “state preparation factor” of neutrinos is altered via magnetic fields. Taking into account the smallness of the neutrino mass, this is approximately the same as flipping the neutrino’s helicity. For example, Dirac neutrinos produced in the sun could be turned non-interacting (“sterile”) when passing through the strong solar magnetic field. This is considered as a possible solution to the solar neutrino problem (see e.g. [1]). On the other hand, Majorana neutrinos which have traveled through the same magnetic field would still be interacting via left-handed currents. In fact, CC produced Majorana neutrinos which would behave as if they were Dirac neutrinos (anti-neutrinos) are changed into a state such that they then behave as Dirac anti-neutrinos (neutrinos).

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APPENDIX A: BASIC PROPERTIES OF NEUTRINOS

The quantized wave function for a free Majorana field may be written as

\[
\nu(x) = \int \frac{d^3 p}{(2\pi)^3 2 p_0} \sum_{s=\pm} \left[ f_s(\vec{p}) \ u_s(\vec{p}) \ e^{-ipx} + \lambda \ f_s^\dagger(\vec{p}) \ v_s(\vec{p}) \ e^{ipx} \right] ,
\]  

(A.1)

\(f_s(\vec{p})\) and \(f_s^\dagger(\vec{p})\) being the annihilation and creator of a free one particle state with helicity \(s\), and \(\lambda\) being an arbitrary phase factor (see e.g. [1,2]). In general an amplitude for a certain process can be derived from the interaction Lagrangian by S-matrix expansion, using Wick’s theorem. In processes with massive Majorana particles, compared to processes with Dirac neutrinos, additional Wick contractions of field operators can appear, due to the fact that there are only two possible states: a neutrino with helicity + and a neutrino with helicity – which are created by \(f_+^\dagger\) and \(f_-^\dagger\), respectively. The two helicity states are connected by a Lorentz transformation. A detailed discussion on how the matrix element for massive Majorana neutrinos can be calculated using the charge conjugation property of the free Majorana field is given in [19].

The neutral current of a massive Majorana neutrino,

\[
j_\lambda(x) = \bar{\nu}(x) \gamma_\lambda \ (g_\nu^\gamma - g_\nu^A \gamma_5) \ \nu(x) ,
\]

(A.2)
is pure axial-vector for arbitrary vector and axial-vector couplings \(g_\nu^\gamma\) and \(g_\nu^A\) in the Lagrangian. Using the charge conjugation Dirac matrix \(C\) and taking the neutral current to be normal-ordered\(^4\), one can write:

\[
\begin{align*}
  j_\lambda(x) & = \bar{\nu}(x) \gamma_\lambda \ (g_\nu^\gamma - g_\nu^A \gamma_5) \ \nu(x) \\
          & = -\nu^T(x) \ C^{-1}C \ (g_\nu^\gamma - g_\nu^A \gamma_5) \ \gamma^T_\lambda \ C^{-1}C \ \bar{\nu}^T(x) \\
          & = \bar{\nu}(x) \ C \ (g_\nu^\gamma - g_\nu^A \gamma_5) \ \gamma^T_\lambda \ C^{-1} \ \nu(x) \\
          & = -\bar{\nu}(x) \ \gamma_\lambda \ (g_\nu^\gamma + g_\nu^A \gamma_5) \ \nu(x) ,
\end{align*}
\]

(A.3)

where the following identities have been used:

\(^4\)For the calculation of S matrix elements, only time-ordered Green’s functions are relevant. It can be shown that therefore only the normal-ordered part of the neutral current contributes.
\[ C \bar{\nu}^T = \lambda^* \nu, \quad C^T = C^{-1} = -C, \quad C \gamma_5^T C^{-1} = -\gamma_\lambda \quad \text{and} \quad C \gamma_5 \gamma_\lambda^T C^{-1} = \gamma_\lambda \gamma_5. \quad (A.4) \]

From (A.3) the pure axial-vector nature of the neutral current is apparent:

\[ j_\lambda(x) = -g_\lambda \bar{\nu}(x) \gamma_\lambda \gamma_5 \nu(x). \quad (A.5) \]

I now prove point a) of the introduction. In the massless case the Dirac equation only has two linear independent solutions:

\[ u_{L,R} \propto v_{R,L}, \quad (A.6) \]

where

\[ u_{L,R} = P_{L,R} u, \quad v_{R,L} = P_{L,R} v, \quad (A.7) \]

and \( P_{L,R} \) are the chiral projectors as defined in (3.6). In addition, chirality and helicity are the same for massless neutrinos:

\[ u_{L,R} = u_{-,+}, \quad v_{L,R} = v_{-,+}. \quad (A.8) \]

If only the left-handed part\(^5\) of the massless Majorana field interacts, only the left-handed chiral projection of (A.1) is relevant. Because of (A.8) I immediately obtain

\[ \nu_L(x) = \frac{1}{2} (1 - \gamma_5) \nu(x) = \int \frac{d^3p}{(2\pi)^3 2p_0} \left[ f_-(\vec{p}) \ u_-(\vec{p}) \ e^{-ipx} + \lambda \ f_+^\dagger(\vec{p}) \ v_+(\vec{p}) \ e^{ipx} \right]. \quad (A.9) \]

Since \( f_- \) and \( f_+ \) are independent operators obeying the anti-commutation relations of the Dirac algebra \( \nu_L(x) \) has the well-known form of the quantized field of a left-handed massless Dirac neutrino.\(^6\) Therefore a massless Majorana neutrino behaves in the same way as a massless Dirac neutrino if only left-handed (or only right-handed) interactions are present. End of proof. Please notice that the two different helicity states corresponding to \( f_- \) and \( f_+ \)

\(^5\)Of course the same discussion could be made for the right-handed part.

\(^6\)The phase factor \( \lambda \) can be absorbed by redefining \( f_+ \).
are the same in all Lorentz frames because the massless neutrino is traveling with the speed of light. This is not the case as soon as the neutrino has a non-zero mass. In particular, there is no smooth limit for restoring the Lorentz invariance of the helicity as $m_\nu \to 0$ or $\beta \to 1$. There is an additional fundamental difference between massless Majorana neutrinos on one hand and light or relativistic Majorana neutrinos on the other hand: Only in the massless case chirality is a good quantum number, being then identical to helicity.

Next I show that the neutral current for a massless Majorana neutrino is chiral. For both Dirac and Majorana neutrinos the neutral current of (A.2) can be rewritten as

$$ j_\lambda(x) = (g^\nu_V + g^\nu_A) \bar{\nu}_L(x) \gamma_\lambda \nu_L(x) + (g^\nu_V - g^\nu_A) \bar{\nu}_R(x) \gamma_\lambda \nu_R(x) . $$  \hspace{1cm} (A.10)

Because massless chiral Majorana fields satisfy $C\bar{\nu}^T_{L,R}(x) = \lambda^* \nu_{R,L}(x)$, the following identity is obtained:

$$ \bar{\nu}_L(x) \gamma_\lambda \nu_L(x) = -\bar{\nu}_R(x) \gamma_\lambda \nu_R(x) $$  \hspace{1cm} (A.11)

Inserting this result in (A.10) I find that even for arbitrary vector and axial-vector couplings the neutral current for massless Majorana neutrinos is chiral:

$$ j_\lambda(x) = 2g^\nu_A \bar{\nu}_L(x) \gamma_\lambda \nu_L(x) = -2g^\nu_A \bar{\nu}_R(x) \gamma_\lambda \nu_R(x) . $$  \hspace{1cm} (A.12)

As was proven above, the massless chiral Majorana field $\nu_L (\nu_R)$ can equivalently be treated as a massless chiral Dirac field if only $\nu_L (\nu_R)$ interacts. For Dirac neutrinos the current $j_\lambda$ in (A.10) is only chiral if

$$ g^\nu_V = \pm g^\nu_A . $$  \hspace{1cm} (A.13)

Comparison of (A.10) with (A.12) shows that massless Dirac and Majorana neutrinos are the same if and only if (A.13) is valid.

A comparison of (A.5) with (A.12) confirms the statement d) of the introduction that for Majorana neutrinos the small-mass limit is in general not approaching the massless case: While the $\nu\nu Z$ vertex involving massive Majorana neutrinos is always axial-vector,
it is always chiral in the case of massless Majorana neutrinos. Additionally, $g^\nu$ cannot be measured if neutrinos are Majorana particles, disregarding whether they are massive or massless.

Next I consider neutrinos which are produced in a chiral interaction such as in a CC interaction. Because of its very small mass, a chirally produced neutrino behaves in good approximation as if the initial neutrino state contains a preparation factor $\frac{1}{2}(1 \mp \gamma_5)$. For Dirac neutrinos the state preparation factor can be left away if neutrinos exclusively couple left-handedly (or, equivalently, only right-handedly) and if terms of $O(m_\nu)$ are neglected. However for Majorana neutrinos the state preparation factor plays an important role. It was shown above that the NC Majorana neutrino vertex is pure axial vector, hence [using $j_\lambda(x)$ in (A.2)]:

$$
\langle \nu(p_f, s_f) | \bar{\nu}(x) \gamma_\lambda (g^\nu - g_A^\nu \gamma_5) \nu(x) | \nu(p_i, s_i) \rangle = -2g_A^\nu \bar{u}(\vec{p}_f, s_f) \gamma_\lambda \gamma_5 u(\vec{p}_i, s_i) \times e^{-i(p_i-p_f)x}.
$$

(A.14)

In contrast a calculation for a chirally prepared initial Majorana state yields

$$
\langle \nu(p_f, s_f) | \bar{\nu}(x) \gamma_\lambda (g^\nu - g_A^\nu \gamma_5) \nu(x) | \nu_{L,R}(p_i, s_i) \rangle = -2g_A^\nu \bar{u}(\vec{p}_f, s_f) \gamma_\lambda P_{L,R} u(\vec{p}_i, s_i) \times e^{-i(p_i-p_f)x} + O(m_\nu) = -2g_A^\nu \bar{v}(\vec{p}_i, s_i) \gamma_\lambda P_{R,L} v(\vec{p}_f, s_f) e^{-i(p_i-p_f)x} + O(m_\nu).
$$

(A.15)

The chirally prepared initial Majorana state $|\nu_{L,R}(p, s)\rangle$ in (A.15) is defined as

$$
|\nu_{L,R}(p, s)\rangle = f_{L,R}^\dagger(p, s)|0\rangle, 
$$

(A.16)

$$
f_{L,R}^\dagger(p, s) = \int d^3x \bar{v}(x) \gamma_0 P_{L,R} u(p, s) e^{-ipx} = \int d^3x \bar{v}(p, s) \gamma_0 P_{R,L} v(x) e^{-ipx}.
$$

(A.17)

In comparison, for chirally prepared Dirac neutrinos (anti-neutrinos) one obtains

$$
\langle \nu(p_f, s_f) | \bar{\nu}(x) \gamma_\lambda (g^\nu - g_A^\nu \gamma_5) \nu(x) | \nu_L(p_i, s_i) \rangle = (g^\nu + g_A^\nu) \bar{u}(\vec{p}_f, s_f) \gamma_\lambda P_L u(\vec{p}_i, s_i) \times e^{-i(p_i-p_f)x} + O(m_\nu),
$$

(A.18)

$$
\langle \bar{\nu}(p_f, s_f) | \bar{\nu}(x) \gamma_\lambda (g^\nu - g_A^\nu \gamma_5) \nu(x) | \bar{\nu}_L(p_i, s_i) \rangle = -(g^\nu - g_A^\nu) \bar{u}(\vec{p}_f, s_f) \gamma_\lambda P_L u(\vec{p}_i, s_i)
$$
\[ (\nu(p_f, s_f) | \bar{\nu}(x) \gamma_\lambda (g_{\nu}^\nu - g_{\nu}^A \gamma_5) \nu(x) | \nu_R(p_i, s_i)) = (g_{\nu}^\nu - g_{\nu}^A) \bar{\nu}(\bar{p}_i, s_i) \gamma_\lambda P_L v(\bar{p}_f, s_f) \times e^{-i(p_i - p_f)x} + O(m_\nu) , \quad (A.19) \]

\[ \langle \bar{\nu}(p_f, s_f) | \bar{\nu}(x) \gamma_\lambda (g_{\nu}^\nu - g_{\nu}^A \gamma_5) \nu(x) | \bar{\nu}_R(p_i, s_i) \rangle = -(g_{\nu}^V + g_{\nu}^A) \bar{v}(\bar{p}_i, s_i) \gamma_\lambda P_L v(\bar{p}_f, s_f) \times e^{-i(p_i - p_f)x} + O(m_\nu) . \quad (A.20) \]

From (A.15), (A.18) and (A.21) it is apparent that for SM couplings \((g_{\nu}^\nu = g_{\nu}^A)\) a left-handedly prepared Majorana neutrino behaves like a Dirac neutrino while a right-handedly prepared Majorana neutrino behaves like a Dirac anti-neutrino, up to terms of \(O(m_\nu)\). In contrast a right-handedly prepared Dirac neutrino as well as a left-handedly prepared Dirac anti-neutrino would be “sterile” in respect to left-handed NC \(^7\) interactions, because (A.19) and (A.20) are of \(O(m_\nu)\). (A.18)-(A.21) furthermore show that in the case of a chiral NC the state preparation factor may be left away for Dirac neutrinos if terms of \(O(m_\nu)\) are neglected.

**APPENDIX B: CROSS SECTIONS FOR GENERALIZED COUPLINGS**

The case of arbitrary vector and axial-vector coupling \(g_{\nu}^V\) and \(g_{\nu}^A\) of the neutral neutrino current is considered. I discuss the restrictions on the general couplings \(g_{\nu}^V\) and \(g_{\nu}^A\) which can be obtained from the two neutral current experiments LEP I and CHARM II.

In the case of LEP I \((s \approx m_Z^2)\), the generalized differential cross sections for neutrino pair production for the different cases of Majorana and Dirac neutrinos with or without a mass are

\[
\left( \frac{d\sigma}{d\Omega} \right)_{M}^{m_\nu \neq 0} = 4 \sigma_0 \left( g_{\nu}^V \right)^2 \left\{ \left[ (g_{\nu}^e)^2 + (g_{A}^e)^2 \right] \left( 1 + \cos^2 \theta \right) + 4 \left[ (g_{V}^e)^2 - (g_{A}^e)^2 \right] \frac{m_e^2}{s} + O \left( \frac{m_e^2}{s} \right) \right\} , \quad (B.1) \]

\(^7\)It is easy to show that this is also the case for left-handed CC interactions.
\[
\left( \frac{d\sigma}{d\Omega} \right)_M^{m_\nu=0} = 4 \sigma_0 \left( g_A^\nu \right)^2 \left\{ \left[ (g_V^\nu)^2 + (g_A^\nu)^2 \right] \left( 1 + \cos^2 \theta \right) + 4 g_V^\nu g_A^\nu \cos \theta \right. \\
\left. + 4 \left[ (g_V^\nu)^2 - (g_A^\nu)^2 \right] \frac{m_e^2}{s} \right\}, \quad (B.2)
\]

\[
\left( \frac{d\sigma}{d\Omega} \right)_D^{m_\nu \neq 0} = 2 \sigma_0 \left[ (g_V^\nu)^2 + (g_A^\nu)^2 \right] \left\{ \left[ (g_V^\nu)^2 + (g_A^\nu)^2 \right] \left( 1 + \cos^2 \theta \right) \\
+ 8 \frac{g_V^\nu g_A^\nu g_V^\nu}{(g_V^\nu)^2 + (g_A^\nu)^2} \cos \theta + 4 \left[ (g_V^\nu)^2 - (g_A^\nu)^2 \right] \frac{m_e^2}{s} + O \left( \frac{m_\nu^2}{s} \right) \right\}, \quad (B.3)
\]

and \((d\sigma/d\Omega)_B^{m_\nu=0}\) is obtained from (B.3) by setting \(m_\nu = 0\). The quantity \(\sigma_0\) is defined in (2.3). At LEP I the sum of the total cross sections for neutrino pair production has been measured indirectly and is (assuming three light neutrino generations)

\[
\sigma_{\text{LEP}} = \sum_{f=e,\mu,\tau} \sigma(e^+e^- \rightarrow \nu_f\bar{\nu}_f) \\
= \frac{16}{3} \sigma_0 \left[ (g_V^\nu)^2 + (g_A^\nu)^2 \right] \sum_{f=e,\mu,\tau} \left\{ \left[ (g_V^\nu)^2 + (g_A^\nu)^2 \right] : \text{for Dirac,} \right. \\
\left. \left[ 2 (g_V^\nu)^2 \right] : \text{for Majorana.} \right\} \quad (B.4)
\]

The factor \((g_V^\nu)^2 + (g_A^\nu)^2\) can be determined by the measurement of the partial width of \(e^+e^-\) production at LEP. The calculated cross sections are consistent with the measured data if the couplings satisfy

\[
\sum_{f=e,\mu,\tau} \left[ (g_V^\nu)^2 + (g_A^\nu)^2 \right] = \frac{3}{2} \quad \text{for Dirac neutrinos} \quad (B.5)
\]

or

\[
\sum_{f=e,\mu,\tau} (g_A^\nu)^2 = \frac{3}{4} \quad \text{for Majorana neutrinos} \quad (g_V^\nu \text{ being arbitrary}) \quad (B.6)
\]

The amplitudes of Sec. III for NC \(\nu_e e^-\) scattering need also to be modified for arbitrary couplings \(g_V^\nu\) and \(g_A^\nu\). The new amplitudes \(\tilde{M}_D\) and \(\tilde{M}_M\) can be written as combinations of the amplitudes \(M_D\) and \(\Delta M\) defined in (3.2), (3.4), (3.8) and (3.9):

\[
\tilde{M}_M^{m_\nu \neq 0} = 2 g_A^\nu \cdot (M_D + \Delta M), \quad (B.7)
\]

\[
\tilde{M}_M^{m_\nu=0} = 2 g_A^\nu \cdot M_D, \quad (B.8)
\]

\[
\tilde{M}_D^{m_\nu \neq 0} = \left( g_V^\nu + g_A^\nu \right) \cdot M_D + \left( g_V^\nu - g_A^\nu \right) \cdot \Delta M, \quad (B.9)
\]

\[
\tilde{M}_D^{m_\nu=0} = \left( g_V^\nu + g_A^\nu \right) \cdot M_D. \quad (B.10)
\]
Summing over final spins and averaging over the initial electron spin the squared matrix elements are

\[
|\tilde{M}_M|_\pm^2 = 4(g^\nu_A)^2 |M_D|^2 + O \left( \frac{m_\nu}{m_\mu} \right),
\]

(B.11)

\[
|\tilde{M}_M|_0^2 = 4(g^\nu_A)^2 |M_D|^2.
\]

(B.12)

\[
|\tilde{M}_D|_\pm^2 = (g^\nu_V + g^\nu_A)^2 |M_D|^2 + \left[ \left( g^\nu_V \right)^2 - \left( g^\nu_A \right)^2 \right] (M_D^* \Delta M + \Delta M^* M_D)
\]

\[
+ (g^\nu_V - g^\nu_A)^2 |\Delta M|^2
\]

\[
= (g^\nu_V + g^\nu_A)^2 |M_D|^2 + O \left( \frac{m_\nu}{m_\mu} \right),
\]

(B.13)

\[
|\tilde{M}_D|_0^2 = (g^\nu_V + g^\nu_A)^2 |M_D|^2.
\]

(B.14)

The CHARM II collaboration has also measured $\nu_e e^-$ scattering. This channel includes a CC contribution.

Again the cross sections can be used to determine the couplings $g^\nu_f$ and $g^\nu_A$ ($f = e, \mu$). The experimental data are consistent with the conditions

\[
(g^\nu_V + g^\nu_A)^2 = 1 \quad \text{for Dirac neutrinos},
\]

(B.15)

\[
(g^\nu_A)^2 = \frac{1}{4} \quad \text{for Majorana neutrinos (} g^\nu_A \text{ being arbitrary)}.
\]

(B.16)

Combining the LEP I results in (B.5) (where the existence of three light neutrino generations has been assumed) and the CHARM II results in (B.15) (assuming that this result is also true for tau Dirac neutrinos) the numerical values of $g^\nu_{V,A}$ are determined for Dirac neutrinos:

\[
g^\nu_V = g^\nu_A = \frac{1}{2}, \quad f = e, \mu, \tau.
\]

(B.17)

Under the above assumptions, the neutral Dirac neutrino current has to be left-handed and flavour universality is obtained. These results are in complete agreement with the standard model.

Correspondingly, the LEP I and CHARM II results can also be combined assuming Majorana neutrinos. From (B.6) (again assuming three light neutrino generations) and (B.16) (without any assumptions regarding the tau Majorana neutrino) it follows that
\( g_A^{\nu} = \frac{1}{2}, \quad g_V^{\nu} \) being arbitrary, \( f = e, \mu, \tau. \) \hspace{1cm} (B.18)

Under the assumption of SM couplings and flavour universality for all light neutrino flavours, the LEP I measurements have determined the number of light (Dirac or Majorana) neutrinos to be three. Clearly, the CHARM II experiment has confirmed the assumption of flavour universality for two neutrino flavours, namely for the electron and muon family, independent of the fact whether neutrinos are Dirac or Majorana particles. Presently, there is no experimental proof that the tau neutrino has the same couplings as the electron neutrino and muon neutrino.
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FIGURE CAPTIONS

1. The Feynman diagram of the process $e^+e^- \rightarrow Z \rightarrow \nu\nu$. The neutrinos can be either Dirac or Majorana particles.

2. The Feynman diagram of the $2 \rightarrow 3$ process $\pi^+e^- \rightarrow \nu_\mu\mu^+e^-$. The neutrino can be either a Dirac or Majorana particle.
FIGURES

FIG. 1.

FIG. 2.