Robust Quantum Communication Using A Polarization-Entangled Photon Pair

J.-C. Boileau, R. Laflamme, M. Laforest, C. R. Myers

Institute for Quantum Computing, University of Waterloo, Waterloo, ON, N2L 3G1, Canada.

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Noise and imperfection of realistic devices are major obstacles for implementing quantum cryptography. In particular, birefringence in optical fibres leads to decoherence of qubits encoded in photon polarization. We show how to overcome this problem by doing single qubit quantum communication without a shared spatial reference frame and precise timing. Quantum information will be encoded in pairs of photons using “tag” operations which corresponds to the time delay of one of the polarization modes. This method is robust against the phase instability of the interferometers despite the use of time-hobs. Moreover synchronized clocks are not required in the ideal no photon loss case as they are only necessary to label the different encoded qubits.

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Quantum mechanics allows the distribution of cryptographic keys whose security is based on the laws of physics instead of the difficulty of solving mathematical problems \cite{1,2}. Turning this idea into practical technologies brings exciting challenges. The first prototype for quantum cryptography was built more than ten years ago over a distance of 30 cm in free space \cite{3} and used the photons' polarization as qubits of information. Since, many quantum key distribution (QKD) experiments have been realized through air and optic fibres \cite{4}. One of the obstacles to improve the fibre based prototypes is the birefringence effects due to geometric asymmetries and tension fluctuations which are a major impediment for polarization based-coding experiments \cite{4}. When the coherence time of the photon is large compared to the delay caused by polarization mode dispersion, the birefringence can be represented by a time dependent unitary transformation $U(t)$ that acts on the polarization space. The time dependence comes from the mechanical variations in the fibre over time and its rate varies with the environmental conditions.

A possible solution to this problem is the application of active feedback \cite{6}. Tomography on some predetermined polarization states could be used to approximate $U$ for a certain time interval \cite{6,15}. By applying his approximation of $U^\dagger$ before his measurements, Bob (the receiver) could recover the states sent by Alice (the sender). However, this technique is practical only if the rate of change of $U$ is relatively low. For this reason, the most successful QKD experiments were not based on polarization coding, such as the phase based experiment proposed by Bennett et al. using an unbalanced interferometer \cite{5,7,10,11}. However, a good control of the polarization modes is necessary to obtain a better visibility since some components like phase modulators are polarization dependent and the temperature of the interferometers must be stabilized since very small fluctuations between the two arms cause phase shifts that corrupt the quantum states.

Another very important example of a successful QKD protocol is the plug-and-play set-up \cite{12,13}. Using a Faraday mirror \cite{14}, the photons sent by Bob are reflected back in the fibre by Alice, who in turn encodes information in their phase. By travelling back in the fibre, the birefringence is reversed and, as it can be shown, the polarization state received by Bob are orthogonal to the original one. Since Bob controls the polarization state of the photon, he can make use of a polarized beamsplitter which increases the interference visibility. Although the plug-and-play set-up has very interesting characteristics, it is not compatible with a non-Poissonian source which could get rid of the multi-photons per pulse problem. Another disadvantage is that the use of two-way quantum cryptography is more vulnerable to a certain kind of eavesdropping strategy: the Trojan attack. An eavesdropper (i.e. Eve) could send photons in Alice’s lab, catch them after they were reflected by the Faraday mirror and get some information about Alice’s set-up without being detected.

To circumvent the threat of the Trojan attack and the instability of the interferometers, Walton et al. \cite{15} proposed a one-way protocol based on decoherence free-subspaces in which each qubit is encoded in the time and phase of a pair of photons. In this Letter, we propose a new way to protect qubits encoded in polarization states of a photon pair from birefringence effects in optical fibre.

The idea is to take advantage of the fact that birefringence can be well approximated by a collective error model as long as the photons travel inside a time window small compared to the variation of the birefringence. Thus, if the effect of birefringence on one photon is $U(t)$, on $n$ photons it is $U(t)^\otimes n$. This latter operator can be interpreted as a rotation of the reference frame axis and our protocol reduces to the problem of developing a strategy to do quantum communication without a shared reference frame.

In a recent paper \cite{16}, Bartlett et al. showed it should be possible to “communicate with perfect fidelity without a shared reference frame at a rate that asymptotically approaches one encoded qubit per transmitted qubit.” In particular, they proposed a method to encode a qubit using four photons in a decoherence-free-subspace of the collective noise model. However this required having full control of the states of qubits. This is out of reach of today’s technology. More recently, two realistic QKD
protocols that do not require any shared reference frame have been proposed \[1\]. These protocols do not require a general state of a qubit but only a set of non-orthogonal states. It encodes qubits in both three and four photon states, which makes the protocol more sensitive to photon loss. For these reasons, we will describe a two photon protocol robust against phase instability of the interferometer without the need for a shared spatial reference frame or synchronized clocks. If we neglect dispersion and discard relativistic situations then we are close to having no need for a shared reference frame at all.\(^1\)

To explain our protocol we need to introduce the “tag” operation \(T_t\) which delay the photons in the state \(|i\rangle\) by a specific amount of time. Experimentally it can be implemented using a polarized beamsplitter to separate polarization modes in arms of different length before recombination in the same optical path.

Suppose Alice inputs a two-photon state of the form \(\alpha|HV\rangle + \beta|VH\rangle\) where \(H\) and \(V\) correspond to the horizontal and vertical polarization state of a photon. The time delay between the two photons \(\Delta t_p\), must be fixed by Alice and known by Bob. It must be large enough such that Bob’s apparatus can differentiate between the two photons and that “tag” operation will never change their order of arrival. If Alice applies the “tag” operation \(T_T\) on the initial state then she will have \(\alpha|HV_T\rangle + \beta|V_T H\rangle\), where subscript \(T\) denotes the delay. Suppose some collective noise \(U^\otimes 2\) (that includes a change of reference frame) is applied to this state when it travels to Bob and suppose also that Bob applies the “tag” operation \(T_{H'}\) when he receives it. Up to a global phase, the state is then mapped to

\[
\begin{align*}
\frac{\alpha}{2}(|H'V'_T\rangle - |V'H'_T\rangle + |V'H'_T\rangle + |V'H'_T\rangle) \\
+ \frac{\beta}{2}(|V'_T H'\rangle - |H'_T V'_V\rangle + |H'_T V'_V\rangle + |H'_T V'_V\rangle) \\
+ \delta_1(|H'_T V'_T\rangle + |V'_T H'_T\rangle) + \delta_3(|H'_T V'_T\rangle - |V'_T H'_T\rangle) \\
+ \delta_2(|H'_T V'_T\rangle + |V'_T H'_T\rangle) + \delta_4(|H'_T V'_T\rangle - |V'_T H'_T\rangle)
\end{align*}
\]

where \(|H'\rangle\) and \(|V'\rangle\) notation is used since the state is now defined in Bob’s reference frame. We used the fact that the anti-symmetric state \(|\Psi^-\rangle = \frac{1}{\sqrt{3}}(|HV\rangle - |VH\rangle)\) is invariant under collective noise and that \(|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle)\) will be mapped to a superposition of the triplet Bell states for which the \(\delta\)’s represent the relative weights and phases and follow the equality \(||\delta_1||^2 + ||\delta_2||^2 + ||\delta_3||^2 + ||\delta_4||^2 = 1\). For later convenience, we define \(|\Phi^\pm\rangle = \frac{1}{\sqrt{3}}(|HV\rangle \pm |VH\rangle)\) and we will drop the apostrophe notation for simplicity.

The last operation is to project onto the states subspace in which the photons are separated in time by exactly \(\Delta t_p\), i.e. both have been subjected to one tag operation. This operation does not require synchronized clocks, since Bob just needs to compare the arrival time of both photons. If the interval of time between a pair of photons is not \(\Delta t_p\), then he discards these qubits, which happens \(1 - ||\frac{1+\delta_1}{2}||^2\) of the time if we neglect photon loss. Otherwise, Bob will obtain Alice’s initial state \(\alpha|HV_T\rangle + \beta|V_T H\rangle\) with certainty. As it could have been showed using simple calculations, the final result is independent of the phase coherence instability between both arms of the interferometer in a way similar to the qubits encoded in the Walton et al. protocol \[12\].

To check if the communication is efficient, \(||\frac{1+\delta_1}{2}||^2\) must be estimated. If the collective noise is averaged uniformly\(^2\) over all possible values of \(U(t)\otimes 2\), then \(\langle\langle\frac{1+\delta_1}{2}\rangle\rangle = \frac{1}{3}\), which means Bob will obtain Alice’s state with a probability of \(\frac{1}{3}\). Yet, this result supposes that the unitary matrix \(U\) will average uniformly over all possible values during the communication time. To make the protocol independent of the environment, Bob could apply a random unitary matrix \(B\otimes 2\) on the photon polarization states just before making his “tag” operation\(^3\).

An improved version of the scheme exploiting some partial knowledge of the shared reference frame to modify the transformation \(B\) to approximate the transformation \(U(T)\) would increased the ratio of useful encoded qubits. Depending on the efficiency of the active feedback mechanism and the rate of change of \(U(t)\), the ratio could converge to 1.

\[\text{FIG. 1:} \text{ After receiving the two photons and applying his “tag” operation, Bob can use this circuit to measure the qubit } \alpha|HV\rangle + \beta|VH\rangle \text{ in any basis by adjusting the gate } M \text{ with a success probability of at least } \frac{1}{3}. \text{ We refer to the text for more details.} \]

To measure the qubit in a particular basis, Bob could use a normal symmetric beamsplitter and consider the result when each photon goes through a different branch, as shown in figure \[\text{II}\]. Define \(p\) such that \(p = 0\) if the

\[\text{\textsuperscript{2}We assume that the randomness of the birefringence is such that the distribution of } U \text{ over a large amount of time is uniform. The Haar measure over the space of unitary matrices is then used to calculate the average } \langle\langle\psi|U^\otimes 2T_1|\psi\rangle\rangle \text{ which equals } \frac{1}{3} \text{ independently of } |\psi\rangle. \text{ Consequently, } ||\frac{1+\delta_1}{2}||^2 = \frac{1}{3}. \]

\[\text{\textsuperscript{3}The distribution of the operator } B \text{ should correspond to the normalized Haar measure. Experimentally, } B \text{ could be implemented with Pockels cells the same way as Franson and Jacobs in their 1995 experiment.} \]

\[\text{\textsuperscript{1}For reasons we will explain later, Bob needs to know the relative rate of time flow in Alice’s reference frame.}\]
The first photon goes through branch b1 and 1 if it is the second photon. Remark that the two photons arrive at the beamsplitter at different times and that Bob can differentiate them. At the end of branch b1, Bob measures in his diagonal \{ (+), (−) \} polarization basis. Define k such that \( k = 0 \) if the outcome is \( + \) and 1 if it is \( − \). The photon on the other branch b2 must then be in the state \( X^p Z^q (\alpha |H\rangle + \beta |V\rangle) \) where X and Z are the corresponding Pauli operators. Using Fockels cells (M) on the second branch and a polarized beam splitter, Bob can measure the qubit in any specific basis with a chance of success reduced by a factor of at most 8, since at the very least the measurement is successful when each photon exits from a different branch and \( p = k = 0 \). Measurement in some bases will be successful more often than others.

We have described a technique to encode a robust qubit against collective noise and to measure it in any basis. We now show how this could be useful for a realistic QKD implementation. First, we describe the well known QKD protocol BB84. This protocol uses a set of four quantum states consisting of two maximally conjugate basis states \( |0\rangle, |1\rangle \) and \( |±\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle) \). Alice randomly chooses which basis she will use to encode qubits to send Bob, who, upon arrival of a qubit, also chooses at random in which of the two bases he will perform a measurement. After repeating the protocol for a string of random bits, they publicly share what basis they used for each qubit. The bits for which they have used the same basis is used to build the sifted key. Since Eve has no prior knowledge of which basis Alice and Bob will use, any attempt of eavesdropping will disturb the states and induce errors in the sifted key with high probability. A portion of the sifted key is used to detect possible eavesdropping. If the error rate is lower than some given threshold, the left overs will be transformed to the final secret key by using error correction and privacy amplification.

To implement a protocol similar to BB84, Alice needs to encode the states \( |HV\rangle, |VV\rangle, |VT_H\rangle, |V_T_H\rangle \) and \( \frac{1}{\sqrt{2}} (|HV\rangle + |V_T_H\rangle) \) or \( \frac{1}{\sqrt{2}} (|HV\rangle − |V_T_H\rangle) \) using parametric down conversions, filters and polarized beamsplitters as shown in figure 2. We have to note that the measurement procedure described earlier works only if the state received by Bob after post-selection was of the form \( \gamma_1 |HV\rangle + \gamma_2 |VH\rangle \) where \( \gamma_i \in \mathbb{C} \) respecting a normalizing condition. This condition may no longer be true if sources of noise other than collective noise are considered or if we suppose that Eve altered the state sent to Bob. In the latter case, Bob’s state after post-selection would look like \( \gamma_1 |HV\rangle + \gamma_2 |VH\rangle + \gamma_3 |VV\rangle + \gamma_4 |HH\rangle \). To implement the provenly secure BB84 protocol, Bob must be able to project that state into the subspace in which Alice has encoded her space i.e. the space spanned by \( |HV\rangle \) and \( |VH\rangle \). If Bob wants to measure in the computational basis \( \{ |V\rangle, |H\rangle \} \), then immediately after his “tag” operation he simply needs to measure the \( |H\rangle \) or \( |V\rangle \) polarization of each photon. In this case, he will also distinguish and be able to discard the states \( |HH\rangle \) and \( |VV\rangle \). The measurement in the diagonal basis \( |\Psi^\pm\rangle \) is not as straightforward. Suppose Bob applies an extra Hadamard gate on both photons before measuring the polarization states. If \( \gamma_3 = \gamma_4 = 0 \), then he measures \( |\Psi^\pm\rangle \) if both photons have the same polarization and \( |\Psi^-\rangle \) if they have different polarization. In general, \( \gamma_3 = \gamma_4 \neq 0 \), but the uniformly distributed random rotation \( B \) performed by Bob (unknown to Eve) when he received the state will destroy any phase coherence between the states \( \gamma_1 |HV\rangle + \gamma_2 |VH\rangle, |HH\rangle \) and \( |VV\rangle \) from Eve’s perspective. Intuitively, this means if Eve used the space spanned by \( \{ |V\rangle, |HH\rangle \} \) it would be the same as if she randomly sent one of \( |\Psi^-\rangle \) or \( |\Psi^+\rangle \) to Bob, giving her no advantage. The complexity of the QKD security proof which includes coherent attacks restrains our argument, but the authors conjecture that our protocol is unconditionally secure with the same error threshold as BB84. As a last remark, we note that only the qubits that have survived the post-selection are used to build the sifted key to estimate the error rate and construct the final secret key.

Earlier we discussed the possibility of using a feedback mechanism to increase the success rate of the post-selection. It could also be used in the QKD implementation discussion above, but Bob must be careful with whatever mechanism he uses since he must ensure the phase coherence between the three states \( \gamma_1 |HV\rangle + \gamma_2 |VH\rangle, |HH\rangle \) and \( |VV\rangle \) be lost from Eve’s perspective. A random final phase gate would be enough since it does not affect the success probability of the post selection, but will destroy the coherence between these states.

The advantages of our protocol over the plug-and-play one are that this protocol is one-way, so there is no need to be as worried with the Trojan attack. Moreover, it does not require interferometer stability like in the Walton et al. protocol (by using decoherence-free subspace). Although our protocol has similarities to the latter protocol, it is distinct for the following reasons:

First, synchronized clocks are necessary in our protocol only to label the different photon pairs. In the Walton et al. protocol, Bob must be able to distinguish between photons that have been delayed once, twice and not at all. Our protocol just needs to compare the delay between the two photons and not their particular time of arrival. Consequently, it requires a much smaller order of timing precision. For example, parametric down-conversion sources with long pulse length no longer induce errors caused by uncertainty in the emission time since both photons are always created simultaneously. Remark that if the number of events in which simultaneous dark counts on different detectors occur is negligible, extra timing precision would not help Alice and Bob to reduce the noise caused by the detector’s dark counts and is therefore not necessary to our protocol.

Second, in the Walton et al. protocol, there is a \( \frac{1}{4} \) chance, independent of the birefringence, that the photons will be measured in the phase basis and a \( \frac{1}{4} \) chance of measuring in the time basis. However, the optimal ef-
ficiency for the ideal implementation of BB84 is a probability of measurement equal to $\frac{1}{2}$ in each basis. For this reason, Walton et al. indicate that the intrinsic efficiency of their scheme was $\frac{1}{2}$. In the case where $B$ is chosen from a uniform distribution, our protocol would have an intrinsic efficiency ratio of $\frac{1}{4}$ since only a third of the photon pairs is not discarded. However, depending on the feedback mechanism, the intrinsic efficiency ratio could be higher than $\frac{1}{4}$, up to $\frac{1}{2}$.

Third, the final state Bob uses is encoded in polarization, not in time and phase. A good control of the polarization states allows Bob to get ride of the noise caused by the polarization dependence of some experimental components, like phase modulators.

In this paper, we have given a realistic robust scheme to do single qubit communication using two-photon states per encoded qubit. This technique goes around the problem of birefringence in optical fibre, the requirement of high precision synchronized timing and also the interferometer phase coherence instability. The protocol could be slightly modified to exploit partial information about a spatial reference frame to increase the bit rate by using active feedback. We also explained how to implement a slightly modified version of BB84 using the previously mentioned methods.

We would like to conclude with some problems that could make an experimental implementation of our schemes more difficult. Depolarization could be a serious distance limitation for our protocol forcing us to use sources with longer coherence times $\frac{1}{2}^2$. To prevent chromatic dispersion from affecting the time delays between the photons, the average wavelength of the photons should be chosen according to the zero chromatic dispersion of the optical fiber $\frac{1}{2}, \frac{1}{4}$. Finally, since our protocol encoded each qubit with two photons, attenuation and detector’s inefficiencies have a more significant affect on its efficiency compared to one-photon protocols. Nevertheless our proposal is in reach of experimental implementation and provides an elegant solution to the problem of birefringence in optical fibres.

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