Delay and Throughput Optimal Scheduling for OFDM Broadcast Channels

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Abstract—In this paper a scheduling policy is presented which minimizes the average delay of the users. The scheduling scheme is investigated both by analysis and simulations carried out in the context of Orthogonal Frequency Division Multiplexing (OFDM) broadcast channels (BC). First the delay optimality is obtained for a static scenario providing solutions for specific subproblems, then the analysis is carried over to the dynamic scheme. Furthermore auxiliary tools are given for proving throughput optimality. Finally simulations show the superior performance of the presented scheme.

I. INTRODUCTION

The allocation of limited resources among users is a fundamental problem in the design of next generation wireless systems. In general, resource allocation problems can be formulated as some kind of optimization problem where the objective is to maximize/minimize some system performance measure under physical layer as well as Quality of Service (QoS) constraints. One of the most important performance measures of a communication systems is the total system throughput and therefore it is often considered as the objective of the optimization problem. In a queuing system with random packet arrival, the throughput can be considered as the maximal possible offered load without violating the stability of queues. A scheduling policy is called throughput-optimal, if it can keep the queues stable whenever any other feasible scheduling scheme can stabilize the queues. It was shown that there exist several queue-length-based scheduling schemes which achieve throughput optimality [1], [2], [3], [4].

However, since the stability of a queuing system only guarantees that the queue lengths do not grow without bounds, but by no means indicate how long the queue length will be, the next step in the performance optimization is to keep the queue lengths as short as possible so that the queuing delay is minimized. It was shown in [5], [6] that the Longest-Queue-Highest-Possible-Rate (LQHPR) policy which maximizes the queue-weighted sum of rates is throughput-optimal and is strongly delay-optimal for the multiple-access channel. The necessary condition for its delay optimality is the symmetry both in the fading channels and in the packet arrival rates. However, for the BC the LQHPR is not delay-optimal even with symmetry assumptions. Seong et al. introduced in [4] another throughput-optimal scheduling called Queue Proportional Scheduling (QPS) which provides superior delay and fairness properties for the BC compared to LQHPR.

Generally, the aforementioned policies are based on the same class of optimization problems: Maximizing the sum of rates weighted with different parameters, i.e. queue length, delay, etc. The solution of the optimization problem always corresponds to some boundary point of the channel capacity region. Since the capacity region of OFDM BC is completely achieved with Costa Precoding and the optimal power allocation and precoding order for the weighted sum rate maximization problem can be efficiently solved [7], all results can also be easily extended to OFDM BC systems.

In this paper, we analyze characteristics of all throughput-optimal scheduling policies and show that they can be formulated as weighted sum rate maximization problems differing only in the choice of the weight factors. Further, the weight factors are independent of the current channel states, hence cross-layer optimization problems, which usually involve the optimization over system parameters in medium access control (MAC) layer and physical layer, can be clearly separated into two steps: 1. Finding the optimal weight factors according to the MAC layer parameters. 2. Solving the weighted sum rate maximization problem. Then we introduce an iterative algorithm to calculate the weight factors that are optimal with respect to the average delay in OFDM BC channels. Here the average delay is defined as the average waiting time of each bit in the queues.

The rest of this paper is organized as follows. Section II presents the system model. The throughput optimality is discussed in Section III. In Section IV we introduce our delay-optimal scheduling policy and evaluate results in Section V. Finally, we conclude in Section VI.

II. SYSTEM MODEL

A. Physical layer

We assume an OFDM BC with \( M \) users, \( K \) subcarriers, and a short term sum power constraint \( \bar{P} \)

\[
\sum_{m,k=1}^{M,K} E[|x_{m,k}|^2] = \sum_{m,k=1}^{M,K} p_{m,k} \leq \bar{P},
\]

where \( x_{m,k} \) is the signal transmitted to user \( m \in \mathcal{M} = \{1, ..., M\} \) on subcarrier \( k \in \mathcal{K} = \{1, ..., K\} \) with power \( p_{m,k} \) and \( E\{\cdot\} \) stands for the expectation operator. Then, the system equation for each user on each subcarrier can be written as

\[
y_{m,k} = h_{m,k} \sum_{j \in \mathcal{M}} x_{j,k} + n_{m,k}, \quad m \in \mathcal{M}, k \in \mathcal{K},
\]  

(1)
where \( y_{m,k} \) is the signal received by user \( m \) on subcarrier \( k \), \( n_{m,k} \sim \mathcal{CN}(0,\sigma^2) \) is circular symmetric additive white Gaussian noise with variance \( \sigma^2 \). Let \( h = [h_{1,1}, \ldots, h_{1,K}, h_{2,1}, \ldots, h_{M,K}]^T \) denote the stacked vector of channel coefficients. We assume that these channel coefficients are related to a standard time-varying multipath model where the channel is approximately constant over the OFDM symbol. Furthermore, we assume that Costa Preceding is performed at the base station having full non-causal knowledge of all messages to be transmitted. Let \( \pi \in \Pi \) be an arbitrary encoding order from the set of all \( M! \) possible encoding orders, such that user \( \pi(1) \) is encoded first, followed by user \( \pi(2) \) and so on. Then the rate of user \( \pi(m) \) can be expressed as

\[
\hat{r}_{\pi(m)} = \frac{\sum_{k=1}^{K} \log \left( 1 + \frac{g_{\pi(m),k}P_{\pi(m),k}}{1 + g_{\pi(m),k} \sum_{n < m} P_{\pi(n),k}} \right)}{2}
\]

with \( g_{m,k} = |h_{m,k}|^2/\sigma^2 \) being the channel gain of user \( m \) on subcarrier \( k \) and \( P_{m,k} \) being the allocated power. The instantaneous capacity region of the OFDM BC under a given sum power constraint \( \bar{P} \) is given by

\[
C(h, \bar{P}) = \bigcup_{\sum_{m,k=1}^{M,K} P_{m,k} \leq \bar{P}} \{ r : r_{\pi(m)} \leq \hat{r}_{\pi(m)}, m \in M \}
\]

where \( \hat{r}_{\pi(m)} \) is defined in equation (2) and \( r \) denotes the vector of rates. Now, the ergodic capacity region \( C_{\text{erg}}(\bar{P}) \) is defined as the set of achievable rates averaged over the channel realizations subjected to the short term sum power constraint \( \bar{P} \):

\[
C_{\text{erg}}(\bar{P}) = \bigcup_{\sum_{m,k=1}^{M,K} P_{m,k} \leq \bar{P}} \{ r : r_{\pi(m)} = \mathbb{E}_h \{ \hat{r}_{\pi(m)} \}, m \in M \}
\]

B. Medium access control layer

Assuming that the transmission is time-slotted, data packets arrive randomly at the MAC and a buffer with finite length is reserved to store the incoming data for each user \( m \in M \). Simultaneously the data is read out from the buffers according to the system state, i.e., the random fading realization and the current queue lengths. Thus, the system can be modeled as a queuing system with random processes reflecting the arrival and the departure of data packets.

Denoting the buffer state of the \( m \)-th buffer in time slot \( n \in \mathbb{N} \) by \( q_m(n) \) and arranging all buffer states in the vector \( q(n) \in \mathbb{R}^M_+ \) the evolution of the queue system can be written as

\[
q(n+1) = [q(n) - r(n)]^+ + a(n),
\]

where \( [x]^+ = \max\{0, x\} \), \( \forall m \in M \) and \( a(n) \in \mathbb{R}^M_+ \) is a random vector denoting the data arrival process. The random vector \( r(n) \in \mathbb{R}^M_+ \) describes the rates asserted to the individual users according to a specific scheduling policy. Supposing that a scheduling policy is a mapping

\[
\mathcal{P} : \mathbb{C}^{M \times K} \times \mathbb{S} \to \mathbb{R}^M_+,
\]

which decides the rate allocation depending on the current channel fading state \( h \in \mathbb{C}^{M \times K} \) and the MAC layer system state \( w \in \mathbb{S} \). \( w \) summarizes the current and past information that is acquirable at the base station and relevant for the optimization, e.g., the current queue length \( q \), the average previous arrival rate \( \bar{a} \) and average previous transmit rate \( \bar{r} \), etc. The rate allocation according to the scheduling policy \( \mathcal{P} \) is denoted as \( r^P(h, w) \).

Note that the process is reminiscent of random walk on the half line (with dependent increments) where we have rigorously used an uncountable state space formulation. Since the random variables \( a(n) \) are sampled at a given time interval \( T \) from \( M \) independent random processes they are independent. Denoting the mean of the packet arrival rate of user \( m \) as \( \lambda_m \) and the constant packet size as \( s_m \), the expected bit arrival rate for user \( m \) is given by \( \rho_m = s_m \lambda_m \). On the other hand the random vector \( r(n) \) depends on the buffer and channel state.

III. SYSTEM STABILITY AND THROUGHPUT-OPTIMAL SCHEDULING POLICIES

A. Definition of system stability and throughput optimality

First we investigate the maximum possible offered system load without violating stability. There exist several definitions of stability. In this paper we use the definition of the strong stability, which implies also weak stability and nonevanescence of the queueing system.

**Definition 1**: The queueing system is strongly stable, if

\[
\limsup_{n \to +\infty} \mathbb{E} \{ q_\pi(n) \} < +\infty, \forall \pi \in \Pi.
\]

In the sense of the stability definition, we call the set of expected arrival rates \( \rho \) stabilizable by a specific scheduler the throughput region of the scheduling policy \( \mathcal{P} \). A scheduling policy is throughput-optimal if it stabilizes the system whenever any other scheduling policies can stabilize the system. If the arrival rate \( \rho \notin C_{\text{erg}}(\bar{P}) \) and the fading gains can be practically upperbounded by some constants, then it is impossible to stabilize the system, even if the policy is non-stationary and it has knowledge of the future events [7]. Therefore, we can define a throughput-optimal scheduling policy as a policy, which keeps the system stable for any arrival rate whose expected value \( \rho \) lies in the ergodic capacity region. For example, using Lyapunov drift technique the LQHPR scheduler can be proven to be throughput optimal.

B. Characterization of throughput-optimal scheduling policies

For a given channel state, the rate vector allocated with a throughput-optimal policy is always a boundary point of the instantaneous capacity region \( C(h, \bar{P}) \). Therefore, any throughput-optimal scheduling policy can be formulated as the optimization problem

\[
r^P(h, w) = \arg\max_{r \in C(h, \bar{P})} \mu^r, \quad \mu \in \mathbb{R}^M_+.
\]
BC channel is continuous and differentiable, the rate allocation \( r^P(h, w) \) can be uniquely characterized with the normal vector \( \mu \) on the capacity region.

Furthermore, for any given \( \mu \) the power and rate allocation problem \((7)\) can be efficiently solved \cite{7}. Hence, we can use the vector \( \mu \), which is also called weight vector in the optimization problem \((7)\), to characterize the scheduling decision, instead of using power and rate allocation directly.

In the following we show some properties of the weight vector \( \mu \).

**Theorem 1:** The weight vector \( \mu \) which characterizes a throughput-optimal scheduling policy is independent of the current fading state \( h \).

**Proof:** We choose arbitrarily a weight vector \( \mu^* \) corresponding to a fixed boundary point of the ergodic capacity region, hence \( \mu^* \) is independent of the instantaneous channel state. Then we denote \( \mu_h \), the weight vector determined by a scheduling policy \( P \). We have

\[
\mu^*^T \cdot \mathbb{E}_h \left\{ r^P(h, w) \right\} = \mu^*^T \cdot \mathbb{E}_h \left\{ \arg \max_{r \in C(h, \bar{P})} \mu_h^T \cdot r \right\} \\
= \mathbb{E}_h \left\{ \mu^*^T \cdot \arg \max_{r \in C(h, \bar{P})} \mu_h^T \cdot r \right\} \\
\leq \mathbb{E}_h \left\{ \max_{r \in C(h, \bar{P})} \mu^*^T \cdot r \right\},
\]

The equality holds only if \( \mu_h = \mu^* \) and the boundary point is achieved by the corresponding scheduler, otherwise the scheduling policy gives a rate vector in the interior of the ergodic capacity region. If the expected rates of the arrival process equals the rates on the boundary point, there must be some user \( i \) who has \( \mathbb{E}_h \left\{ r^P_i(h, w) \right\} < \rho_i \) and its queue expands infinitely.

Following the result in Theorem 1, we can define the weight vector \( \mu^P(w) \) of a throughput-optimal policy as a function only determined by the MAC layer state \( w \). In this way, the classical cross-layer optimization problem can be separated into two parts: Finding the optimal weight vector \( \mu \) according to the MAC layer parameters; solving the rate and power allocation problem \((7)\) on the physical layer with the giving weight vector. Since the second part can be efficiently solved on the physical layer, the scheduling design problem reduced to find the optimal weight vector for the optimization problem.

**IV. DELAY-OPTIMAL SCHEDULING POLICY**

So far we characterize the class of throughput-optimal scheduling policies. Since the stability definition doesn’t restrict the explicit length of the queues, even throughput-optimal policies have different delay performance. In the following we study the scheduling policy minimizing the average bit delay \( \bar{D} \), which is defined as:

\[
\bar{D} = \frac{1}{M} \sum_{i=1}^{M} D_i = \frac{1}{M} \sum_{n=1}^{N} \sum_{i=1}^{M} \frac{q_i(n)}{\bar{a}_i}.
\]

It can be regarded as the extension of the common definition of the queueing delay in \cite{8}. \( N \) is the length of the observation time window and \( \bar{a}_i \) is the average bit arrival rate for the user \( i \) in the time window.

**A. Delay-optimal scheduling policy for a static channel**

We consider first the delay optimization problem for a static channel \( h \) and the initial buffer states \( q(n=1) \). We denote the previous average arrival rates as \( \bar{a} \) and assume there is no packet arriving after \( n = 0 \). Further we choose the length of observation time window \( N \) with \( q_i(N) = 0 \), \( \forall i \in M \) so that the buffers are completely emptied within the time window.

Thus the delay-optimal scheduling policy can be written as the solution of the optimization problem

\[
\min \sum_{n=1}^{N} \left( \sum_{i=1}^{M} q_i^r(n) - \bar{a}_i \right) \\
\text{s.t.} \quad \forall i \in M, n \in [1, ..., N],
\]

where \( q_i^r(n) \), \( r_i^r(n) \) denote the queue length and transmit rate of user \( i \) in time slot \( n \). For convenience we also use the superscript to denote the time slot in the following. Extending the problem \((13)\) in each queue state \( q^n \) we have the equivalent optimization problem

\[
\min \sum_{n=1}^{N} \left( \sum_{i=1}^{M} q_i^r(n) - \bar{a}_i \right) \\
\text{s.t.} \quad \forall i \in M, n \in [1, ..., N]
\]

The Lagrangian function is

\[
L(r^n, \lambda^n) = \sum_{n=1}^{N} \sum_{i=1}^{M} q_i^r(n) - \bar{a}_i - \sum_{n=1}^{N} (N - n) \frac{q_i^r(n)}{\bar{a}_i} - \sum_{n=1}^{N} \sum_{i=1}^{M} \lambda_i(n) \left( q_i^r(n) - \bar{a}_i \right)
\]

Denote \( \eta_i^* = N - \bar{a}_i \), we get the optimal \( \mu^n_i \) with

\[
\mu^n_i = \begin{cases} \frac{q_i^r(n)}{\bar{a}_i} & n \leq \eta_i^* \\ 0 & n > \eta_i^* \end{cases}
\]

and the delay-optimization problem is transformed into

\[
\max \sum_{n=1}^{N} \sum_{i=1}^{M} \mu^n_i^T \cdot r \\
\text{s.t.} \quad r^n \in \bar{C}(h, \bar{P})
\]

which can be solved easily.
The $\eta^*_i$ in (16) can be obtained with a iterative approach given in Algorithm 1.

**Algorithm 1 Idle State Prediction Algorithm**

1. Set $\mu_i^{(0)} = \frac{1}{a_i}$ and calculate $r^{(0)} = \arg \max_{r \in C_n} \mu^{(0)T} \cdot r$.
2. Initialize the length of non-idle state $\eta_i^{(0)} = \min_{i \in M} \eta_i^{*(0)}$.
3. Set the order $\pi$ so that $\frac{q_{\pi(1)}^{(0)}}{\eta_i^{(0)}} \geq \frac{q_{\pi(2)}}{\eta_i^{(0)}} \geq \ldots \geq \frac{q_{\pi(M)}}{\eta_i^{(0)}}$.
4. Set $t = 0$.

 repeat
   (5.1) Set $\eta_i^{(t+1)} = \eta_i^{(t)}$
   for $i = 1$ to $M$
   (5.2.1) $\eta^* = \eta_i^{(t+1)}$
   repeat
     (5.2.2.1) Increase $\eta_i^{*}$.
     Solve the maximization problem (17) and calculate the evolution of the queue state.
     if $q_{\pi(i)}^{(t+1)} \geq 0$ then
     $\eta_i^{(t+1)} = \eta_i^{*}$
     end if
     until $q_{\pi(i)}^{(t+1)} < 0$
   end for
   until $t = t + 1$
   (3) $\eta^* = \eta_i^{(t)}$
\end{algorithm}

$[\eta]$ denotes the smallest integer larger than $\eta$.
\[
\epsilon \text{ is the predefined error tolerance of } \eta.
\]

B. Delay-optimal scheduling for dynamic channels

It is worth noting that if channel state $h$ varies over time and the base station has the knowledge of each channel state in advance, the algorithm in previous subsection can also be used in this case with some modification. However, in reality the base station has usually only the current channel state information and the statistical knowledge of the channel.

Further the packet arrival process is non-ergodic and cannot be predicted. In order to avoid the possible infinite queuing delay, the delay-optimal policy must also be throughput-optimal, so that the queue state is kept stable for any expected arrival rate $\rho$ inside the ergodic capacity region.

If no new packet arrives after the time slot $n = 0$, the expected delay for a given policy $P$ is

\[
E \left\{ \sum_{n=1}^{N} \sum_{i=1}^{M} D_{ij}^{n} \right\}
= \frac{\sum_{n=1}^{N} \sum_{i=1}^{M} q_{ij}^{n} - \sum_{n=1}^{M} (N - n) r_{ij}^{Pn} \bar{a}_{ij}}{\bar{a}_{ij}},
\]

where $r_{ij}^{Pn}$ is the rate allocated by the policy $P$ for the $i$-th user at $n$-th time slot. From Theorem 1 we know that if $P$ is a throughput-optimal policy, then

\[
E \left\{ r^P \right\} = \arg \max_{r \in C_{err}(P)} \mu^{PT} \cdot r,
\]

where $\mu^P$ is independent of the current channel state. Hence the optimization problem is equivalent to

\[
\min_{\mathcal{P}} \sum_{n=1}^{N} \left( \sum_{i=1}^{M} \frac{q_{ij}^{n}}{\bar{a}_{ij}} - \sum_{n=1}^{M} (N - n) \frac{r_{ij}^{Pn}}{\bar{a}_{ij}} \right)
\]

s.t.

\[
q_{ij}^{n} - \sum_{i=1}^{n} r_{ij}^{n} \geq 0, \quad \forall i \in M, n \in \{1, \ldots, N\}.
\]

Then the optimization problem can be solved using algorithm 2.

In the system with new packet arrival, the weight vector $\tilde{\mu}^1$ should be recalculated according to the new queue state and the rate allocation is determined with $\tilde{\mu}^1$ and current channel state $h$.

**Theorem 3:** The proposed scheduling policy keeps the queue lengths finite for all arrival processes with expected arrival rates inside the capacity region.

**Sketch of the proof:** It is easy to show that $\tilde{\mu}^1$ monotonically increases from 0 to infinity if $q_i$ grows from 0 to infinity, for all $i \in M$. Therefore, the expected transmit rate vector $E\{r^*\}$ converges to a boundary point $\tilde{r}$ where $\tilde{q}_i = \frac{\mu^* - r_i}{\rho_i - r_i}$, $\forall i, j \in M$. Since $\rho_i$ lies inside the ergodic region, we have $\tilde{r}_i \geq \rho_i$, $\forall i \in M$ and the theorem follows.
Algorithm 2 Delay Optimal Scheduling

for each time slot $n$ do
1 Calculate the previous average arrival rate $\bar{a}$
2 Calculate $\eta^*$ according to $\bar{a}$ and current queue state $q$ using algorithm 1, where the static channel region $\mathcal{C}(h, P)$ is replaced with the ergodic capacity region $\mathcal{C}_\text{erg}(P)$.
3 Calculate the current weight vector $\tilde{\mu}_1$ according to equation (16).
4 Calculate the current rate allocation $r^* = \arg \max_{r \in \mathcal{C}(h_n, \bar{P})} (\tilde{\mu}_1)^T \cdot r$, \hspace{1cm} (22)
where $h_n$ is the current channel state.
end for

V. NUMERICAL EVALUATIONS

We compare our scheduler with the LQHPR scheduler in [1] and the QPS in [4] in a two-user scenario. The OFDM system has 250 subcarriers and an entire bandwidth of 2.5MHz. The multipath channel has i.i.d block fading model and the length of fading block $T$ was assumed to be 0.1ms. For an average transmit SNR of 15dB the ergodic capacity region was shown in Fig.1. Having chosen $\rho_1 = [1.5, 2, 2.5, 3, 3.5, 4, 4.5]$ Mbits/s and $\rho_2 = [3, 4, 5, 6, 7, 8, 10]$ Mbits/s the average bit delay is shown in Fig.2. It can be seen that for the arrival rate outside the capacity region, the queueing delay becomes extremely long. For the arrival rate inside the capacity region, the introduced scheduling policy has superior delay performance compared to the other two scheduling policies.

VI. CONCLUSION

We have provided a throughput and delay optimal scheduling policy for OFDM BC channels. Simulation results show that the average bit delay is significantly reduced with the introduced scheduling policies.

Since the weighted sum rate maximization problem can be solved for any systems with convex capacity regions, i.e., MIMO and OFDM uplink/downlink channel, most of results presented in this paper can also be applied to these systems.

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