Correcting 100 years of misunderstanding: electric fields in superconductors, hole superconductivity, and the Meissner effect

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From the outset of superconductivity research it was assumed that no electrostatic fields could exist inside superconductors, and this assumption was incorporated into conventional London electrodynamics. Yet the London brothers themselves initially (in 1935) had proposed an electrodynamic theory of superconductors that allowed for static electric fields in their interior, which they unfortunately discarded a year later. I argue that the Meissner effect in superconductors necessitates the existence of an electrostatic field in their interior, originating in the expulsion of negative charge from the interior to the surface when a metal becomes superconducting. The theory of hole superconductivity predicts this physics, and associated with it a macroscopic spin current in the ground state of superconductors (“Spin Meissner effect”), qualitatively different from what is predicted by conventional BCS-London theory. A new London-like electrodynamic description of superconductors is proposed to describe this physics. Within this theory superconductivity is driven by lowering of quantum kinetic energy, the fact that the Coulomb repulsion strongly depends on the character of the charge carriers, namely whether electron- or hole-like, and the spin-orbit interaction. The electron-phonon interaction does not play a significant role, yet the existence of an isotope effect in many superconductors is easily understood. In the strong coupling regime the theory appears to favor local charge inhomogeneity. The theory is proposed to apply to all superconducting materials, from the elements to the high $T_c$ cuprates and pnictides, is highly falsifiable, and explains a wide variety of experimental observations.

PACS numbers:

I. INTRODUCTION

The eminent experimental physicist K. Mendelssohn wrote in 1966[1] “To the layman it may come as a disappointment that the explanation of such a striking phenomenon as superconductivity should, on the atomistic scale, have been revealed as nothing more exciting than a foiling small interaction between electrons and lattice vibrations. This feeling was shared by many physicists who had hoped that superconductivity might reveal some new fundamental principle of nature”, undoubtedly including himself in that physicists’ group. The work discussed in this paper suggests that Mendelssohn’s disappointment may have been premature.

In the conventional theory of superconductivity the speed of light $c$ plays no role, as pointed out by Alexandrov[2]. Whether it is 300,000km/s or 3mm/s would not change the magnitude of $T_c$ = 4.153K for mercury, as discovered by Kammerlingh Onnes 100 years ago[3], nor the isotope coefficient $\alpha$ = 0.5 of mercury ‘discovered’ theoretically by Fröhlich 60 years ago[4].

However, E.U. Condon wrote a paper in 1949 (just before the discovery of the isotope effect) entitled “Superconductivity and the Bohr magneton”[5]. He pointed out in that paper that the maximum magnetization measured in superconducting cylinders of various materials appears to have an empirical relationship with the magnetization that would result from the conduction electrons intrinsic magnetic moment due to spin, all alligned. This suggests an intrinsic relationship between the Bohr magneton, which contains the speed of light, and the superconducting state.

Indeed, consider the following expression for the lower critical field of type II superconductors:

$$H_{c1} = -\frac{\hbar c}{4e\lambda_L^2}$$  (1)

with $e$ the electron charge and the London penetration depth $\lambda_L$ given by the usual expression[6]

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m_e c^2}$$  (2)

with $n_s$ the number of superconducting electrons of mass $m_e$ per unit volume. Eq. (1) follows if the flux through an area of radius $2\lambda_L$ is the flux quantum $\phi_0 = \hbar c/2e$. The usual expression for $H_{c1}$ derived from Ginzburg Landau theory has an extra logarithmic factor of order 1). The magnetization of a cylindrical sample in the presence of an external magnetic field of magnitude just below $H_{c1}$ is

$$M_e = \frac{H_{c1}}{4\pi} = \frac{1}{2} n_s \mu_B$$  (3)

with $\mu_B = e\hbar/2m_e c$ the intrinsic magnetic moment of the electron (Eq. (2) was used in obtaining Eq. (3)), $(1/2)n_s$ is the number of superconducting electrons of each spin.

Thus, we can think of the superconductor in the absence of applied field as having magnetization zero arising from the cancellation of $+M_e$ and $-M_e$ originating in the intrinsic magnetic moments of the conduction electrons with spin down and up, as shown schematically in Fig. 1(a). When an external magnetic field is applied, the superconductor develops a magnetization due to the
and may as well be zero (as would be the case for the associated magnetic moment plays absolutely no role). It made the bold proposition that the ‘perfect conductor’ equation for the supercurrent was the first electrodynamic theory of the London brothers\cite{7}. It made the bold proposition that the ground state within a London penetration depth of the interior of the superconductor (except for a single point) was homogeneous and an electric field exists everywhere in the interior of the superconductor (except for a single point) pointing towards the nearest surface\cite{10}. It also results in the existence of a macroscopic spin current flowing in the ground state within a London penetration depth of the surface, a kind of zero-point motion of the superfluid\cite{14}. This results in a superconducting ground state where the charge distribution is macroscopically inhomogeneous and an electric field exists everywhere in the interior of the superconductor (except for a single point) pointing towards the nearest surface\cite{10}. It also results in the existence of a macroscopic spin current flowing in the ground state within a London penetration depth of the surface, a kind of zero-point motion of the superfluid\cite{14}. This physics is shown schematically in Fig. 2.

The charge electrodynamics of superconductors is described by the four-dimensional equation\cite{16}

\begin{equation}
J - J_0 = -\frac{c}{4\pi\lambda_L} (A - A_0)
\end{equation}

with the four-vectors

\begin{equation}
A = (A, i\phi); J = (J, ic\rho)
\end{equation}

\begin{equation}
A_0 = (0, i\phi_0); J_0 = (0, ic\rho_0)
\end{equation}

The potential $\phi_0$ results from the existence of a uniform positive charge density $\rho_0$ in the interior of the supercon-
ductor

\[ \phi_0(r) = \int_V d^3r' \frac{\rho_0}{|r - r'|} \] (7)

and the magnitude of \( \rho_0 \) is fixed by the requirement that the average electric field near the surface is

\[ E_m = -\frac{\hbar c}{4e\lambda_L^2} \] (8)

i.e. the same as \( H_c1, \text{ Eq. (1)} \) (in c.g.s. units). For a spherical or cylindrical sample this results in a uniform negative charge density \( \rho_- \) within a London penetration depth of the surface of magnitude

\[ \rho_- = -\frac{E_m}{4\pi\lambda_L} \] (9)

and the interior charge density \( \rho_0 \) is given by

\[ \rho_0 = -\frac{2\lambda_L}{R} \rho_- ; \rho_0 = -\frac{3\lambda_L}{R} \rho_- \] (10)

for a cylindrical and spherical sample respectively.

The electric potential in the interior of the superconductor satisfies the differential equation

\[ \phi(r) = \lambda_L^2 \nabla^2 \phi(R) + \phi_0(r) \] (11)

and can be found analytically for simple geometries (sphere, cylinder, plane)[17] and numerically for other cases, to find the charge distribution and electric field both inside and outside the superconductor. Boundary conditions assumed are that \( \phi \) and its normal derivative are continuous at the surface of the superconductor, so we assume that no two-dimensional surface charge exists. For a spherical geometry, infinite cylinder and infinite plane, no electric fields exist outside the superconductor. For other geometries however electric field lines do exist outside the superconductor, implying that the surface is not equipotential.

Figure 3 shows a representative example of electric field lines obtained by numerical solution of the differential equation for the case of an ellipsoidal shape[18]. The field is quadrupolar in nature, and it can be seen that electric field lines come out of the region of low surface curvature and come in in the region of high surface curvature. This is understood qualitatively as follows: electrons in the spin current near the surface can move faster when the surface has less curvature, just like a racing car, hence they have higher kinetic energy and consequently lower potential energy, than slower-moving electrons in regions of higher surface curvature. Lower potential energy for the electron corresponds to higher electric potential, hence electric field lines come out of region of low electric potential. Thus, electrons in the superconductor, a macroscopic quantum system, keep their total energy constant throughout the superconductor by adjusting their potential and kinetic energy, giving rise to a macroscopically inhomogeneous charge distribution, in contrast to a classical system or normal metal that has a macroscopically uniform charge distribution that gives lowest macroscopic potential energy.

In Fig. 4 we show an example of the electric field arising for a body with an egg-like geometry, composed of half an oblate and half a prolate ellipsoid fused together. Here the electric field distribution is more complicated, however it can be seen that it follows the general pattern discussed in the previous paragraph, with electric field lines coming out of regions of lower curvature and coming into regions of higher curvature. It is interesting to note that we find numerically that the electric field configuration for samples of this shape has no net electric dipole moment, which would be allowed by symmetry in
the sample shown in Fig. 4 (not in the sample of Fig. 3). We have not been able to find an analytic explanation of this finding.

These electric fields around superconductors should be experimentally detectable in small (μ-size) superconducting samples. High quality smooth surfaces are required to prevent trapping of charge in localized states and imperfections and to allow the electric field lines to extend away from the surface. The magnitude of the expected electric fields near the surface is of order thousands of Volts/cm[18], and these shape-dependent electric fields should only exist when the sample is superconducting. ‘Conventional’ superconductors such as Pb and Nb would be the best systems to check this prediction[17]. Indirect evidence that these electric fields exist is seen in the Tao experiment[19, 20].

Associated with the inhomogeneous charge distribution our theory predicts that electrons move in orbits of radius \(2\lambda_L\) and carry orbital angular momentum \(\hbar/2\) in direction opposite to the spin angular momentum[14]. From this results a macroscopic spin current flowing within a London penetration depth of the surface of superconductors[17]. The speed of the carriers is given by

\[
v^0_\sigma = -\frac{\hbar}{4m_e\lambda_L} \hat{\sigma} \times \hat{n}
\]

with \(\hat{n}\) the outward pointing normal to the surface. The current within \(\lambda_L\) of the surface for electrons of spin \(\sigma\) is

\[
J_\sigma = n_\sigma e v^0_\sigma = \frac{n_\sigma}{2} \mu_B
\]

with \(n_\sigma = n_s/2\), and the resulting magnetization is

\[
M_\sigma = \frac{J_\sigma \lambda_L}{c} = \frac{n_\sigma}{2} \mu_B = \frac{M_s}{2}
\]

pointing in opposite direction to the magnetization due to the electron spin, which is twice as large because of the gyromagnetic factor \(g = 2\). Thus, the situation depicted in Fig. 1 gets modified to what is shown in Fig. 5. The initial magnetization for spin \(\sigma\), \(M_\sigma/2\), results from the compensation of spin magnetization \(M_c\) and orbital magnetization \(M_\sigma/2\) in opposite directions. The external field \(H_{c1}\) generates magnetization \(M_c\) in one direction, and in the process brings the orbital motion of one of the spin components to a stop[14], at which point the system goes normal.

The Meissner effect in this theory is explained as resulting from the orbital expansion and associated charge expulsion in the transition to superconductivity[13]. A distinctive property of superconductors within this theory is that they exhibit macroscopic quantum zero-point motion, and the same is proposed to be true in superfluid \(^3\)He[21].

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