RESEARCH ARTICLE

Traffic flow cellular automaton model with bi-directional information in an open boundary condition

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Abstract

With Connected Vehicle Technologies being popular, drivers not only perceive downstream traffic information but also get upstream information by routinely checking backward traffic conditions, and the backward-looking frequency or probability is usually affected by prevailing traffic conditions. Meanwhile, the bi-directional perception range of drivers is expected to significantly increase, which results in more informed and coordinated driving behaviours. So, we propose a traffic flow bi-directional CA model with two perception ranges, and perform the numerical simulations with the field data collected from a one-lane highway in Richmond, California, USA as the benchmark data. Numerical results show that the CA model can effectively reproduce the oscillation of relatively congested traffic and the traffic hysteresis phenomenon. When adjusting the backward-looking probability and the perception range, the CA model can well simulate the travel times of all vehicles, and the generation and dissolution of traffic jams under various scenarios.

Keywords: traffic flow; cellular automaton; backward-looking; perception range; hysteresis phenomenon

1. Introduction

Traffic flow can be considered as multi-body systems with strongly interacting elements (vehicles) and complex behaviours. Existing traffic flow models include macroscopic ones such as hydrodynamic [1–3], kinetic [4–9] and microscopic models, e.g. car-following models [10–17] and cellular automaton (CA) models [18–23]. One of the most recent and representative car-following models is the optimal velocity (OV) model originally proposed by Bando et al. [10] in 1995. Modifications to the OV model include the incorporation
of single and multiple spatial headways with the preceding vehicles [13, 24–26], the relative speed [11, 25, 27] and also the spatial headway of the following vehicles [28–31]. The benefits of incorporating the following vehicle information into the OV model are the better description of actual traffic conditions, especially with the development of Connected Vehicle Technologies.

Different from car-following models, CA models are conceptually simpler, and can be easily implemented on computers for numerical simulations. Since the first traffic flow CA model was proposed by Nagel and Schreckenberg (NS model) [18] in 1992, many researchers have contributed to the improvement of the NS model by incorporating several important characteristics in reality. For example, Benjamin et al. [19] looked at braking behaviours. Li et al. [32] modelled the virtual velocity of the preceding vehicle. Larraga et al. [33] described the safe distance. Chen et al. [34] incorporated the expected moving distance of the preceding vehicle. Ge et al. [35] studied the cooperative behaviour of the three nearest preceding vehicles. Knope et al. [36] considered the driving smoothness and comfort, i.e. comfortable driving (CD). Jiang et al. [37] captured driving smoothness and comfort by considering synchronized flow, i.e. modified CD (MCD). Obviously, existing modifications to the NS model do not consider the influence of the following vehicles.

However, the extended OV models [28–31] with the bi-directional traffic information perform better than those with only forward-looking or backward-looking traffic information. Moreover, the following vehicle information helps to improve the traffic flow stability and then to provide a safer and more comfortable driving environment. Inspired by these extended OV models, Zheng et al. [38] proposed a modified MCD (MMCD) model with a honk stimulation term to represent the impact of the following vehicles, and numerical results showed the better performance of the MMCD model than other CA models without the following vehicle information. The limit of the MMCD model is that honking usually occurs in extreme conditions, e.g. spatial headway within safe distance. In reality, drivers can still get the following traffic conditions via rear mirrors, or via Connected Vehicle Technologies [39, 40] which are being incorporated into future vehicles under non-hazardous conditions. Hence, previous CA models can be further improved by handling the back-looking phenomenon more realistically for normal traffic conditions. The remainder of this study is organized as follows. First, Section 2 proposes the CA model with the bi-directional traffic information. Section 3 carries out the numerical experiments with the field headway data as the benchmark data. Finally, some important conclusions are drawn.

2. The proposed CA model

Empirically, drivers not only look ahead continuously to avoid collisions with front vehicles, but also look backwards with a certain frequency so as to prevent rear-end accidents. Here, the backward-looking behaviours are realized by Connected Vehicle Technologies, rearview mirror, or honking communication. Moreover, the backward-looking frequency or probability is positively correlated to traffic density. The proposed CA model starts by formulating the impact of the following vehicles. The gap and relative velocity between two successive vehicles are taken into account to determine whether following vehicles pose a potential threat to their predecessors. If the gap is smaller than the security gap (i.e. gapafe), and the velocity of vehicle \( n \) exceeds that of its front vehicle \( n-1 \), then vehicle \( n \) is a potential threat to vehicle \( n-1 \), denoted as \( rs_n(t) = 1 \). Otherwise, it does not, that is, \( rs_n(t) = 0 \). So, this rule is expressed as follows.

\[
\text{if } [d_n(t) <= \text{gapafe}] \text{ and } [v_n(t) > v_{n-1}(t)] \\
\quad \text{then} : rs_n(t) = 1; \text{else then} : rs_n(t) = 0. 
\]

where \( d_n(t) = x_{n-1}(t) - x_n(t) - 1 \) is the gap between two successive vehicles \( n \) and \( n-1 \).

Meanwhile, only if the nearest following vehicle is in the back perception range of the driver (i.e. \( L_b \)) is it meaningful for the driver to judge the gap and relative velocity of the follower. So, if there is a vehicle that is a potential threat to the driver in his/her backward perception range, the driver would receive the threat stimulation from the follower. Otherwise, there is no threat stimulation. This rule is expressed as follows (see Fig. 1).

\[
\text{if } [x_n(t) - L_b \leq x_{n+1}(t) < x_n(t)] \text{ and } [rs_{n+1}(t) = 1] \\
\quad \text{then} : s_n(t) = 1; \\
\text{if } [x_n(t) - L_b \leq x_{n+1}(t) < x_n(t)] \text{ and } [rs_{n+1}(t) = 0] \\
\quad \text{then} : s_n(t) = 0; \\
\text{if } x_{n+1}(t) \notin [x_n(t) - L_b, x_n(t)] \text{ then} : s_n(t) = 0. 
\]

where \( s_n(t) = 1 \) indicates that driver \( n \) receives the potential threat from the follower \( n+1 \) in his/her
backward perception range. Otherwise, driver $n$ does not.

Similarly, each driver can only obtain the gap with and the braking light of the nearest front vehicle in the front perception range $L_f$ (see Fig. 1), and then adjust his/her driving behaviour. If a cellular between $x_n(t)$ and $x_n(t) + L_f$ is occupied by the vehicle $n-1$, whose braking light is on (i.e. $b_{n-1}(t) = 1$), vehicle $n$ may brake accordingly. In this condition, vehicle $n$ receives the stimulation of the brake light within the front perception range. Otherwise, there is no braking stimulation to vehicle $n$ from downstream. The rule is given as follows.

$$\begin{align*}
\text{if } [x_n(t) < x_{n-1}(t) \leq x_n(t) + L_f \text{ and } b_{n-1}(t) = 1] & \quad \text{then : } b_{sn}(t) = 1; \\
\text{if } [x_n(t) < x_{n-1}(t) \leq x_n(t) + L_f \text{ and } b_{n-1}(t) = 0] & \quad \text{then : } b_{sn}(t) = 0; \\
\text{if } x_{n-1}(t) \neq (x_n(t), x_n(t) + L_f) & \quad \text{then : } b_{sn}(t) = 0.
\end{align*}$$

where $b_{n-1}(t)$ is the brake light state of vehicle $n-1$ at time step $t$ (i.e. $b_{n-1}(t) = 0$, when on; $b_{n-1}(t) = 1$, otherwise), and $b_{sn}(t)$ denotes whether vehicle $n$ receives the brake light effect in response to the nearest vehicle within the front perception range at time step $t$ ($b_{sn}(t) = 1$ if applied, and $b_{sn}(t) = 0$, otherwise).

Then, a backward-looking probability $p_{lb}$ is introduced to describe the possible backward-looking frequency in response to traffic density. Fig. 2 describes the relationship between the backward-looking probability versus the traffic density. In simple terms, the probability is relatively large when in congested traffic, but relatively small when in free flow. It is formulated as $p_{lb} = \min(w \cdot \rho, 1)$, where $w = 1/\rho_c$ is a critical spacing, i.e. the inverse of a traffic density threshold $\rho_c$. When $0 < \rho \leq \rho_c$, the probability increases linearly as the traffic density increases. Once the traffic density reaches $\rho_c$, $p_{lb}$ is always 1, which implies that drivers are very careful and look backwards at each time step. The selection of the piecewise linear curve is to reflect the general trend regarding the relationship between backward-looking probability and traffic density.

Finally, the proposed CA model with the bi-directional traffic information is summarized as follows.

Rule 1: Judge the potential threat between two successive vehicles.

$$\begin{align*}
\text{if } [d_n(t) <= ga_{eff}] \text{ and } [v_{n}(t) > v_{n-1}(t)] & \quad \text{then : } r_{sn}(t) = 1; \text{ else then : } r_{sn}(t) = 0.
\end{align*}$$

Rule 2: Determine the potential influence within the backward perception range.

$$\begin{align*}
\text{if } [x_n(t) - L_b \leq x_{n+1}(t) < x_n(t)] \text{ and } [r_{sn+1}(t) = 1] & \quad \text{then : } s_{n}(t) = 1; \\
\text{if } [x_n(t) - L_b \leq x_{n+1}(t) < x_n(t)] \text{ and } [r_{sn+1}(t) = 0] & \quad \text{then : } s_{n}(t) = 0; \\
\text{if } x_{n+1}(t) \neq [x_n(t) - L_b, x_n(t)] & \quad \text{then : } s_{n}(t) = 0.
\end{align*}$$

Rule 3: The brake light effect from the front perception range.

$$\begin{align*}
\text{if } [x_n(t) < x_{n-1}(t) \leq x_n(t) + L_f] \text{ and } [b_{n-1}(t) = 1] & \quad \text{then : } b_{sn}(t) = 1; \\
\text{if } [x_n(t) < x_{n-1}(t) \leq x_n(t) + L_f] \text{ and } [b_{n-1}(t) = 0] & \quad \text{then : } b_{sn}(t) = 0; \\
\text{if } x_{n-1}(t) \neq (x_n(t), x_n(t) + L_f) & \quad \text{then : } b_{sn}(t) = 0.
\end{align*}$$

Rule 4: Determine the backward-looking probability.

$$p_{lb} = \min(w \cdot \rho, 1) \text{ for } 0 \leq \rho \leq 1.$$
Rule 5: Determine whether a driver looks backwards.

\[
\begin{align*}
  &\text{if } [\text{rand}() < p_{lb}] \text{ then: } l_{bn}(t) = 1; \\
  &\text{else } l_{bn}(t) = 0.
\end{align*}
\]

where \(l_{bn}(t)\) denotes whether driver \(n\) looks backwards at time step \(t\) (\(l_{bn}(t) = 1\) if it always; otherwise, \(l_{bn}(t) = 0\)).

Rule 6: Acceleration.

\[
\begin{align*}
  &\text{if } [(l_{bn}(t) = 1) \text{ and } (s_n(t) = 1) \text{ and } [b_{sn}(t) = 0]] \\
  &\text{then: } v_{n}(t + 1) = \min(v_{n}(t) + d_{n}^{eff}(t), v_{max}); \\
  &\text{else if } [(l_{bn}(t) = 1) \text{ and } (s_n(t) = 0) \text{ and } [b_{sn}(t) = 0]] \\
  &\text{then: } v_{n}(t + 1) = \min(v_{n}(t) + d_{n}(t), v_{max}); \\
  &\text{else: } v_{n}(t + 1) = \min(v_{n}(t) + 1, v_{max}).
\end{align*}
\]

where \(d_{n}^{eff}(t) = d_{n}(t) + \max(v_{n}(t) + 1 - ga_{p_{safe}}, 0)\) is the effective gap, and \(v_{n}(t)\) is the expected velocity of the preceding vehicle in the next time step. When a driver observes the safety threats from the following vehicle in the back perception range rather than the preceding vehicle in the front perception range, they take an effective acceleration value of \(d_{n}^{eff}(t)\). The driver will choose the acceleration value of \(d_{n}(t)\) that is not larger than \(d_{n}^{eff}(t)\) when perceiving no threats within \(L_{f}\) and \(L_{b}\). At all other conditions, the velocity of vehicle \(n\) increases by 1.

Rule 7: Deceleration.

\[
\begin{align*}
  &\text{if } [(l_{bn}(t) = 1) \text{ and } (s_n(t) = 1)] \\
  &\text{then: } v_{n}(t + 1) = \min(d_{n}^{eff}(t), v_{n}(t) + 1)); \\
  &\text{else: } v_{n}(t + 1) = \min(d_{n}(t), v_{n}(t) + 1));
\end{align*}
\]

If driver \(n\) perceives the safety threats through backward looking, he/she should have a small deceleration to avoid influencing the acceleration or the current speed of the following vehicle \(n + 1\). Thus, the velocity of vehicle \(n\) becomes \(\min(d_{n}^{eff}(t), v_{n}(t) + 1))\) after braking. Otherwise, the updated velocity of vehicle \(n\) is \(\min(d_{n}(t), v_{n}(t) + 1))\).

Rule 8: Randomization of braking.

\[
\begin{align*}
  &\text{if } [\text{rand}() < p] \text{ then: } v_{n}(t + 1) = \max(v_{n}(t) + 1, 0]
\end{align*}
\]

where \(\text{rand}()\) returns a random number between 0 and 1, and \(p\) is the randomization probability.

Rule 9: Determination of brake light state.

\[
\begin{align*}
  &\text{if } [v_{n}(t) < v_{n}(t)] \text{ then: } b_{n}(t + 1) = 1; \\
  &\text{if } [v_{n}(t) > v_{n}(t)] \text{ then: } b_{n}(t + 1) = 0; \\
  &\text{if } [v_{n}(t) = v_{n}(t)] \text{ then: } b_{n}(t + 1) = b_{n}(t);
\end{align*}
\]

That is, vehicle \(n\) switches on the brake light if its speed decreases, and switches it off if it increases. Otherwise, the braking light state is unchanged.

Rule 10: Car motion.

\[
x_{n}(t + 1) = x_{n}(t) + v_{n}(t + 1)
\]

Obviously, rules 1, 2, 4, 5 and 7 describe the backward-looking behaviours, i.e. the responses of a driver to the potential threat from upstream. Rule 3 is based on the brake light effect from downstream. Rule 6 involves the bi-directional traffic information. Thus, the proposed CA model takes into account the bi-directional information that can be observed in real traffic, and may lead to a better description of traffic flow dynamics.

3. Numerical experiments

To validate how well the proposed CA model can reproduce the driving processes, we select the San Pablo Dam Road Data recorded on a single-lane road in Richmond, California (http://www.ce.berkeley.edu/~daganz/spdr.html) as the benchmark data. The data collection process is described as follows. From about 6:45 AM to 9:00 AM on Tuesday, 18 November 1997, eight human observers were positioned on one side of the road at equal distances from each other with a traffic light at the end of the road (see Fig. 3(a)). Each observer clicked a key on a laptop each time a vehicle passed them. The data sets therefore consisted of the arrival times of all vehicles passing the observers. In this way, we could capture the travel times between the observers for each vehicle. Usually, the traffic congestion was caused by the traffic light, and then propagated backwards.

Then, the experimental site is modelled by cellular automaton with the open boundary condition (see Fig. 3(b)). The entire single-lane road is divided into 815 cells (i.e. \(D = 815\)) of length 7.5 m, which can either be empty or occupied by just one vehicle at each time step. Meanwhile, the time is scattered into time steps of one second. Based on the arrival times for all vehicles at observer ‘1’ and at the traffic light at the end of the
For the road, the open boundary condition is set simply as follows. The inflow condition is that one vehicle is fed into the first cellular if the arrival time is reached and the first cellular is empty. The outflow condition is that only if the traffic light turns green can the vehicles near the end of the road leave the road.

Besides, other parameters are set as follows: the traffic signal cycle is $C = 60 \text{ s}$, the green signal ratio is $0.5$, the maximum velocity is $v_{\text{max}} = 5 \text{ cells/s}$ (i.e. $v_{\text{max}} = 135 \text{ km/h}$) and $\text{gap}_{\text{safety}} = 1$. Density is $\rho(t) = N(t)/D$, where $N(t)$ is the number of vehicles at $t$th time step. The mean velocity at $t$th time step is $v(t) = (1/N) \sum v_j(t)$. The mean flux is $J(t) = \rho(t) \cdot v(t)$. Here, $L_b = L_f = L$.

### 3.1 Analysis of flux-density diagrams

During the numerical simulations, we record the traffic flux and density at each time step and then depict the flux-density diagrams in Fig. 4(a, c, e, g). It is obviously known that vehicles are distributed sparsely in the road and that the traffic state is free flow at the beginning (i.e. at about 6:45 AM). As more and more vehicles enter, the traffic state has some random oscillation and the traffic flux fluctuates around a certain value. In particular, when the vehicle density is in the range $(0.3, 0.4)$, the traffic flux value is scattered randomly. That is also to say, the flux and density do not have a one-to-one relationship. Through auto-covariance $[41]$ of the traffic flow time series (see Fig. 4(b, d, f, h)), it is easily seen that the traffic flow time series has no long-range correlation, which validates the random volatility of the traffic flow when the vehicle density is relatively large.

Moreover, Fig. 4(a, c, e, g) show that the flux-density diagram has two branches due to the change in direction of vehicle density in the open boundary condition. As the vehicles are fed into the road by the actual time headways, the vehicle density gradually grows, and results in traffic flux increase and traffic flux fluctuation in a certain region. This can be depicted by the upper branch of the flux-density diagram. Later, the vehicle density is relatively large and begins to decrease, that is, the traffic flow evolution in the next stage starts from the traffic jam state. In this condition, the flux-density diagram goes along the lower branch rather than returning to the upper branch. In conclusion, the proposed CA model in the open boundary condition is able to reproduce the hysteresis phenomenon.

### 3.2 Fitting analysis of travel time

This section aims to study how well the proposed CA model can simulate the vehicle movements in the single-lane road given the actual input data, and why there is different fitting accuracy about travel times with the different perception ranges and looking backward probability set in the CA model. Here, the arrival times at observer ‘1’ and observer ‘8’ for all vehicles are used to calculate the actual travel times (ATTs) for all vehicles that pass the entire road. The entry and departure times for each vehicle during simulation are utilized to compute the simulated travel times. The numerical results in Fig. 5 show that the actual and simulated travel times for all vehicles are close to each other. That is, the proposed CA model with bi-directional traffic information is able to reproduce the vehicle movements at a high level of accuracy, although the perception range and looking backward probability are set as different values.

Moreover, Fig. 6 illustrates the average relative errors (AREs) for travel time (see equation (1))
Fig. 4: Flux-density diagrams and the autocorrelation of flux set under different parameters: (a) $L = 1, w = 1$; (b) $L = 1, w = 1$; (c) $L = 1, w = 5$; (d) $L = 1, w = 5$; (e) $L = 5, w = 1$; (f) $L = 5, w = 1$; (g) $L = 5, w = 5$; (h) $L = 5, w = 5$

under various $L$ and $w$ values. When $L$ is fixed at 1 or 5, ARE decreases as $w$ (i.e. the backward-looking probability) increases. Empirically, the larger backward-looking possibility would result in the more frequently and reasonably adjusting behaviours, which can avoid the potential extreme conditions, e.g. emergency braking. This is more consistent with reality.

$$\text{ARE} = \frac{1}{M} \sum_{i=1}^{M} \left| T_{\text{sim}}^{i} - T_{\text{obs}}^{i} \right|$$

where $T_{\text{sim}}^{i}$ is the simulated travel time of vehicle $i$, $T_{\text{obs}}^{i}$ is the observed travel time of vehicle $i$ and $M$ is the total number of vehicles observed.

Theoretically, when $L$ increases, the driver is able to notice the braking behaviours of more distant vehicles, which would cause a larger probability of braking. Meanwhile, when a driver looks backwards at some time steps, the security gap $g_{\text{safe}}$ is a critical factor in the proper choice of driving operations. When $L$ is larger than $g_{\text{safe}} + 1$, and the nearest follower is not in the range $[0, g_{\text{safe}} + 1]$ but in the perception range, there is no safety threat from the follower according to rules 1 and 2. Thus, as $L$ increases, the brake light effect from downstream is strengthened and
Fig. 7: Spatial-temporal evolution of the vehicle velocity under different conditions: (a) $L = 1, w = 1, t = 1500:2000$; (b) $L = 1, w = 5, t = 1500:2000$; (c) $L = 5, w = 5, t = 1500:2000$; (d) $L = 1, w = 1, t = 7000:7500$; (e) $L = 1, w = 5, t = 7000:7500$; (f) $L = 5, w = 5, t = 7000:7500$

causes more braking behaviours, but the potential threat from upstream is only reinforced to some extent and results in greater acceleration and less deceleration. The two contradicting impacts lead to the more stable and homogeneous traffic flow.

However, the numerical results in Fig. 6 show that ARE drops a little for $w = 1$ but grows for $w = 5$, as $L$ increases from 1 to 5. That is, the numerical results do not agree well with the theoretical analyses. The reason can be analysed as follows. The data set collected in 1997 by Daganzo does not involve Connected Vehicle Technologies. In that period, drivers could not obtain and deal with bi-directional information in the long perception range, and could only perform driving operations with back and front vehicle information in the short range. In other words, the proposed CA model with a larger $L$ does not accord with the reality in that period. However, with the development of Connected Vehicle Technologies, drivers can easily receive bi-directional information in the long perception range. From this perspective, the proposed CA model would give some enlightenment for the development of Connected Vehicle Technologies so as to reach a safe and comfortable traffic state.

### 3.3 Efficiency of congestion dissolution

According to the outflow condition, vehicles near the end of the road would depart only if the traffic light turns green. This would result in periodical traffic jams at the end of the road, which meanwhile propagate backwards at a certain speed. The spatial-temporal velocity diagrams in Figs. 7(a-c) illustrate the small time headway between two successive vehicles in the downstream section during the morning peak period (about 7:10 AM to 7:18 AM). This in fact results from the large inflow rate and the relatively stable outflow rate determined by the signal cycle. Once the inflow rate exceeds the outflow rate, the traffic jams at the traffic light would propagate backwards and the propagating speed depends on the difference between inflow and outflow rates. So, Figs. 7(a-c)
show the formulation and propagation of traffic jams.

On the other hand, Figs. 7(d-f) illustrate that the time headway between two successive vehicles becomes relatively large at the end of morning peak period (about 8:41 AM to 8:49 AM). This results from the inflow rate dropping to be smaller than the unchanged outflow rate. In this condition, traffic jams dissolve gradually and the direction of the shock wave becomes forwards. Moreover, when \( L \) is fixed at 1, the dissolution efficiency of traffic jams becomes higher as the backward-looking probability (represented by \( w \)) increases (see Figs. 7(d, e)). The reason for this can be analysed as follows. Usually, drivers look backwards to keep a safe and comfortable driving state, and the larger backward-looking probability benefits the traffic congestion dissolution. However, when \( w \) is fixed at 5, such dissolution efficiency does not continue to improve and even performs worse, although \( L \) grows from 1 to 5. The reason for this can be seen in the theoretical analyses in Section 3.2.

4. Conclusions

This study proposes a CA model with two important factors, that is, the bi-directional perception range and the backward-looking probability of drivers. Based on the data set recorded by Daganzo’s research team, the simulation experiments are performed by the proposed CA model under the open boundary condition. Then, the numerical results are used to analyse the characteristics of the flux-density diagrams and the impact of the above two important factors on the fitting accuracy of travel time and the dissolution efficiency of traffic jams. Finally, we draw some important findings as follows. 1) The proposed CA model under the open boundary condition is able to reproduce the scattered points in the flux-density plane and the hysteresis phenomenon. 2) The backward-looking probability has a positive effect in the fitting of actual travel times, while the bi-directional perception range does not. 3) The proposed CA model is able to reproduce the generation of traffic jams and the backward-propagating shock wave at the beginning of the morning peak hours, and also the dissolution of traffic jams and the forward-propagating shock wave once the inflow rate falls below the outflow rate. 4) The backward-looking probability produces a positive effect in the dissolution of traffic congestion, but the bi-directional perception range does not.

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References

1. Lee H, Lee H, Kim D. Origin of synchronized traffic flow on highways and its dynamic phase transitions. Phys Rev Lett 1998; 81:1130–3.
2. Lee H, Lee H, Kim D. Dynamic states of a continuum traffic equation with on-ramp. Phys Rev E 1999; 59:5101–11.
3. Zhu C, Zhong S, Li G, et al. New control strategy for the lattice hydrodynamic model of traffic flow. Physica A 2017; 468:445–53.
4. Helbing D, Hennecke A, Shvetsov V, et al. MASTER: macroscopic traffic simulation based on a gas-kinetic, non-local traffic model. Transportation Res B-Meth 2001; 35:183–211.
5. Lighthill M, Whitham G. On kinematic waves II. A theory of traffic flow on long crowded roads. Proc R Soc A-Math Phys Eng Sci 1955; 229:317–45.
6. Aw A, Rascle M. Resurrection of “second order” models of traffic flow. SIAM J on Appl Math 2000; 60:916–38.
7. Jiang R, Wu Q, Zhu Z. A new dynamics model for traffic flow. Sci Bull 2001; 46:345–8.
8. Jiang R, Wu Q, Zhu Z. A new continuum model for traffic flow and numerical tests. Transportation Res B-Meth 2002; 36:405–19.
9. Zyryanov V, Kocherga V, Topilin I, et al. Investigation of dependencies between parameters of two-component models of the kinetic theory of traffic flow and traffic characteristics. Transportation Research Procedia 2017; 20:746–50.
10. Bando M, Hasebe K, Nakayama A, et al. Dynamical model of traffic congestion and numerical simulation. Phys Rev E 1995; 51:1035–42.
11. Helbing D, Tilch B. Generalized Force Model of Traffic Dynamics. Phys Rev E 1998; 58:133–8.
12. Jiang R, Wu Q, Zhu Z. Full velocity difference model for a car-following theory. Phys Rev E 2001; 64:017101.
13. Tomer E, Safonov LA, Havlin S. Presence of many stable non-homogeneous states in an inertial car-following model. Phys Rev Lett 2000; 84:382–5.
14. Liang Z, Jin J, Huang H, et al. A vehicle type-dependent visual imaging model for analyzing the heterogeneous car-following dynamics. Transportation B 2016; 4:68–85.
15. Xu T, Laval JA. Statistical inference for two-regime stochastic car-following models. Transportation Res B-Meth 2020; 134:210–28.
16. Zheng L, He Z. A new car following model from the perspective of visual imaging. Int J Mod Phys C 2015; 26:1550090.
18. Nagel K, Schreckenberg M. A cellular automaton model for freeway traffic. *J Phys I* 1992; 2:2221–9.
19. Benjamin S, Johnson N, Hui P. Cellular automata models of traffic flow along a highway containing a junction. *Journal of Physics A* 1996; 29:3119–27.
20. Barlovic R, Santen L, Schadschneider A, et al. Metastable states in cellular automata for traffic flow. *Eur Phys J B* 1998; 5:793–800.
21. Ruan X, Zhou J, Tu H, et al. An improved cellular automaton with axis information for microscopic traffic simulation. *Transportation Res C-Emer* 2017; 78:63–77.
22. Zhao H, Liu X, Chen X, et al. Cellular automata model for traffic flow at intersections in internet of vehicles. *Physica A* 2018; 494:40–51.
23. Regragui Y, Moussa N. A cellular automata model for urban traffic with multiple roundabouts. *Chinese J Phys* 2018; 56:1273–85.
24. Lenz H, Wagner CK, Sollacher R. Multi-anticipative car-following model. *Eur Phys J B* 1999; 7:331–5.
25. Wang T, Gao Z, Zhao X. Multiple velocity difference model and its stability analysis. *Acta Phys Sin* 2006; 55:634–40.
26. Zhang J, Wang B, Li S, et al. Modeling and application analysis of car-following model with predictive headway variation. *Physica A* 2020; 540:123171.
27. Sun Y, Ge H, Cheng R, et al. An extended car-following model considering driver’s memory and average speed of preceding vehicles with control strategy. *Physica A* 2019; 521:752–61.
28. Hasebe K, Nakayama A, Sugiyama Y. Dynamical model of a cooperative driving system for freeway traffic. *Phys Rev E* 2003; 68:026102.
29. Hasebe K, Nakayama A, Sugiyama Y. Equivalence of linear response among extended optimal velocity models. *Phys Rev E* 2004; 69:017103.
30. Ge H, Zhu H, Dai S. Effect of looking backward on traffic flow in a cooperative driving car following model. *Eur Phys J B* 2006; 54:503–7.
31. Sun D, Liao X, Peng G. Effect of looking backward on traffic flow in an extended multiple car-following model. *Physica A* 2011; 390:631–5.
32. Li X, Wu Q, Jiang R. Cellular automaton model considering the velocity effect of a car on the successive car. *Phys Rev E* 2001; 64:066128.
33. Larraga M, Rio J, Alvarezlcaza L. Cellular automata for one-lane traffic flow modeling. *Transportation Res C* 2005; 13:63–74.
34. Chen S, Zhu L, Kong L, et al. The effect of Noise-First and anticipation headway on traffic flow. *Acta Phys Sin* 2007; 56:2517–22.
35. Ge H, Zhu H, Dai S. Cellular automaton traffic flow model considering intelligent transportation system. *Acta Phys Sin* 2005; 54:4621–6.
36. Knospe W, Santen L, Schadschneider A, et al. Towards a realistic microscopic description of highway traffic. *J Phys A* 2000; 33:477–85.
37. Jiang R, Wu Q. Cellular automata models for synchronized traffic flow. *J Phys A* 2003; 36:381–90.
38. Zheng L, Ma S, Zhong S. Analysis of honk effect on the traffic flow in a cellular automaton model. *Physica A* 2011; 390:1072–84.
39. Zheng L, Ran B, Huang H. Safety evaluation for driving behaviours under bi-directional looking context. *J Intell Transport S* 2017; 21:255–70.
40. Zheng L, Zhu C, He Z, et al. Safety Rule-Based Cellular Automaton Modeling and Simulation under V2V Environment. *Transportmetrica A*, 2018, 10.1080/23249935.2018.1517135.
41. Neubert L, Santen L, Schadschneider A, et al. Single-vehicle data of highway traffic: a statistical analysis. *Phys Rev E* 1999; 60:6480–90.