A Composite Twin Higgs Model

Puneet Batra\textsuperscript{1} and Z. Chacko\textsuperscript{2}

\textsuperscript{1}Department of Physics, Columbia University, 538 W. 120th St., NYC, NY 10027
\textsuperscript{2}Department of Physics, University of Maryland, College Park, MD, 20742

Twin Higgs models are economical extensions of the Standard Model that stabilize the electroweak scale. In these theories the Higgs field is a pseudo Nambu-Goldstone boson that is protected against radiative corrections up to scales of order 5 TeV by a discrete parity symmetry. We construct, for the first time, a class of composite twin Higgs models based on confining QCD-like dynamics. These theories naturally incorporate a custodial isospin symmetry and predict a rich spectrum of particles with masses of order a TeV that will be accessible at the LHC.

I. INTRODUCTION

Quantum corrections to the Higgs mass parameter in the Standard Model are quadratically divergent. Stabilizing the weak-scale against these divergences generally requires a phenomenologically rich spectrum of new particles near a TeV, associated with the existence of a new symmetry of nature. One appealing idea is that the Higgs sector of the Standard Model (SM) might actually be the non-linear sigma model of some larger, dynamically generated pattern of symmetry breaking \cite{1, 2}. In such theories, the Higgs behaves much like a pion in QCD; the Higgs is a composite pseudo Nambu-Goldstone boson (pNGB) which is protected from the worst class of quadratic divergences that usually afflict scalar bosons. Precision electroweak measurements currently constrain the compositeness scale to lie above 5 TeV. The fact that the SM gauge couplings, top Yukawa coupling and Higgs self-coupling necessarily break any global symmetry with order one strength then implies that the mass parameter of the composite Higgs needs additional protection, if the theory is to be natural.

Little Higgs theories \cite{3, 4, 5, 6}, for reviews see \cite{7} are a class of non-linear sigma models which realize the Higgs as a protected pNGB. The underlying concept behind little Higgs theories is the idea of ‘collective symmetry breaking’ - the global symmetry of which the Higgs is the pNGB is broken only when two or more couplings in the Lagrangian are non-vanishing. This is a significant restriction on the form of the quantum corrections to the pNGB potential, which can be used to engineer natural electroweak symmetry breaking. These theories stabilize the weak scale to about 5-10 TeV.

Since in general little Higgs and twin Higgs theories have been formulated only as non-linear sigma models, above 5-10 TeV these theories require ultra-violet completions to maintain unitarity. Weakly coupled ultra-violet completions using supersymmetry have been constructed for both the little Higgs \cite{8, 9} and the twin Higgs \cite{10, 11}, in the context of the supersymmetric little hierarchy problem. In the little Higgs case, non-supersymmetric ultra-violet completions have also been constructed \cite{12, 13, 14, 15}. There has also been some work on the difficult problem of realizing the little Higgs as a strongly coupled composite \cite{16, 17, 18, 19}, furthering the analogy to QCD. However, in the twin Higgs case the corresponding problem has not been addressed.

A significant challenge in dynamically realizing a composite twin Higgs is to ensure that the strong dynamics respects a custodial SU(2) symmetry. For this to happen, the custodial symmetry must be contained in the non-linearly realized global symmetry of the Higgs sector. In the twin Higgs models currently in the literature this global symmetry is either SU(4), which is spontaneously broken to SU(3), or O(8), spontaneously broken to O(7). While the breaking of SU(4) to SU(3) is fairly straightforward to realize through QCD-like strong dynamics \cite{10}, this pattern does not admit a custodial SU(2). On the other hand, while the O(8) → O(7) pattern does preserve a custodial symmetry, this pattern is significantly more complicated to realize through strong dynamics.

In this paper we identify an alternative pattern of symmetry breaking for twin Higgs models which naturally incorporates a custodial isospin symmetry. We then show how this pattern can be realized through QCD-like dynamics, and apply these ideas to construct a class of composite twin Higgs models with left-right symmetry. These theories predict a rich spectrum of new particles at the TeV scale that will be accessible at the LHC.

We begin by constructing an alternative realization of the twin Higgs model, in its left-right symmetric incarnation. Consider a scalar field $H$ which transforms as a fundamental under an Sp(4) global symmetry, and which is also charged under a global U(1). If $H$ acquires a VEV such that $\langle H \rangle = (0, 0, 0, f)$, the Sp(4)×U(1) global symmetry is spontaneously broken to SU(2)×U(1), and
there are 7 Goldstone bosons. We now break the
global Sp(4) explicitly by gauging an SU(2)_L × SU(2)_R
subgroup. The overall U(1), which is to be identified with
U(1)_{B−L}, is also gauged. The overall gauge structure is
therefore that of a left-right symmetric model [2].

Under gauge transformations the field \( H \) decomposes into
\( (H_L, H_R) \) where \( H_L \) is a doublet under SU(2)_L
and \( H_R \) as a doublet under SU(2)_R. If the VEV of
\( \langle H \rangle \) points along a direction which breaks SU(2)_R
but preserves SU(2)_L, the surviving gauge symmetry is the
familiar SU(2)_L × U(1)_Y of the SM. Of the 7 Goldstone
bosons, 3 are eaten. The remaining 4 Goldstone bosons,
which are contained in \( H_L \), are to be identified with
the SM Higgs. If the discrete parity symmetry, which
interchanges SU(2)_L and SU(2)_R, is exact, \( H_L \) is pro-
tected against quadratic divergences by the twin Higgs
mechanism. The key observation is that the discrete
symmetry ensures that any quadratically divergent con-
tribution to the scalar potential has an Sp(4) invariant
form, and therefore cannot contribute to the mass of the
Goldstones.

Yukawa interactions can take the same form as in the
original left-right twin Higgs model, since they are only
required to respect the gauge and parity symmetries,
which are identical in both models. Although these
couplings violate the global symmetry with order one
strength, the discrete parity symmetry again ensures that
quadratic divergences are absent. From this we infer that
\( \text{Sp}(4) \times U(1) / \text{SU}(2) \times U(1) \) constitutes an alternative
symmetry breaking pattern which allows the realization
of a twin Higgs model with left-right symmetry.

Although this construction is extremely simple, it does
not admit a custodial SU(2) symmetry. Furthermore, it
is not clear whether such a pattern of symmetry breaking
can arise from strong dynamics. In the next section we
show that a natural generalization of this model exists
which addresses the first problem. We then go on to
discuss how the required symmetry breaking pattern can
be realized through the condensation of strongly coupled
fermions, in analogy with QCD.

II. A CUSTODIAL SYMMETRY FOR THE
TWIN HIGGS

Consider a theory with an \( \text{Sp}(4) \times \text{Sp}(4) \) global symme-
try, which is spontaneously broken down to the diagonal
\( \text{Sp}(4) \). We label the 10 resulting NGBs that are produced
by \( \pi^A \), and define

\[
X = f \exp \left( 2i \pi^A T^A / f \right). \tag{1}
\]

Here the matrices \( T^A \) are the generators of Sp(4), and
correspond to the matrices

\[
\begin{pmatrix}
\sigma^a & 0 \\
0 & \sigma^a
\end{pmatrix}
\begin{pmatrix}
0 & iI \\
-iI & 0
\end{pmatrix}
\begin{pmatrix}
0 & \sigma^a \\
\sigma^a & 0
\end{pmatrix}. \tag{2}
\]

We now gauge an SU(2)_L × SU(2)_R subgroup of the
first Sp(4), and an SU(2)_L' × U(1)_R' subgroup of the
second Sp(4). Here U(1)_R' is the diagonal generator of
the SU(2)_R contained in the second Sp(4). We label
the gauge coupling constants of these four groups as
\( g_L, g_R, g'_L \) and \( g'_R \) respectively. The unbroken gauge
symmetry is then SU(2) × U(1), which is identified with
the electroweak gauge group of the SM. Note that this
symmetry breaking pattern is similar to that of the little
Higgs model of Chang and Wacker [3]. Of the original 10
Goldstone bosons, 6 are eaten. The remaining 4 pseudo-
Goldstone bosons are identified with the SM Higgs, and
correspond to the generators \( T^a \),

\[
\{ T^a \} = \left\{ \begin{pmatrix} 0 & iI \\ -iI & 0 \end{pmatrix}, \begin{pmatrix} 0 & \sigma^a \\ \sigma^a & 0 \end{pmatrix} \right\}. \tag{3}
\]

We can write an effective field theory for the pNGBs
which is valid at low momenta. This takes the form of
a non-linear sigma model. In general the Lagrangian
for this theory will contain all operators consistent with the
non-linearly realized Sp(4) × Sp(4) global symmetry. Non-renormalizable operators
are suppressed by the cutoff \( \Lambda \) of the non-linear sigma
model, and their coefficients are determined by the
specific ultra-violet completion. The cutoff \( \Lambda \) must be
less than about \( 4\pi f \), where the upper bound corresponds
to strong coupling.

In this low-energy theory, the masses of the pseudo-
Goldstones are protected against one loop quadratic
divergences from gauge interactions. This can be un-
derstood as a consequence of the little Higgs mechanism.
The theory has an exact Sp(4) global symmetry in the
limit that \( g_L \) and \( g_R \) are zero, and also in the limit
that \( g'_L \) and \( g'_R \) are zero. Any diagram that results
in a quadratic divergence must therefore involve both
these sets of couplings. The leading contributions to
the pseudo-Goldstone masses arise at order \( g^4 \), and
are therefore necessarily suppressed by at least two loop
factors.

If the low-energy effective theory is weakly coupled
at the scale \( \Lambda \), in the special case that the SU(2)_L and
SU(2)_R of the first Sp(4) are related by a discrete inter-
change symmetry, so that \( g_L = g_R \), there is an alternative
way of understanding this cancellation based on the twin
Higgs mechanism. This discrete symmetry ensures that
at quadratic order in \( X \) all radiative corrections to the
pseudo-Goldstone potential are invariant under the first
Sp(4), and therefore must simply vanish. To see this
let us consider all possible operators consistent with the
gauge symmetry at quadratic order in \( X \) in the non-linear
sigma model. At one loop these terms are the only ones
generated with a quadratically divergent coefficient in the
effective potential. Schematically these operators include

\[
X_{LL} X^\dagger_{LL'} X_{RL} X^\dagger_{RL'} X_{LL'} \mathcal{L}_{E} X_{LL'} X_{LL'} X_{RL} X^\dagger_{RL'}
\]

and also (suppressing hermitian conjugates)

\[
\mathcal{L}_{E} = \left( X_{LL'} X_{LL'} X_{RL} X^\dagger_{RL'} X_{LL'} X_{LL'} X_{RL} X^\dagger_{RL'} \right).
\]
\[ \epsilon_{LL}X_{L3}X_{L4} \quad \epsilon_{RR}X_{R3}X_{R4} \]  

Here \( L \) and \( L' \) take values 1 and 2, while \( R \) takes values 3 and 4. The discrete \( L \leftrightarrow R \) symmetry ensures that in the Lagrangian operators on the same line above necessarily have the same coefficient. Then it is clear that at quadratic order in \( X \) the Lagrangian is actually invariant under the first global \( \text{Sp}(4) \) symmetry. This symmetry is only broken at quartic order in \( X \), and therefore corrections to the pNGB mass are loop suppressed, and at most logarithmically divergent.

The argument above does not carry over to the case where the low-energy theory is strongly coupled at the cutoff \( \Lambda \), because now the quartic terms in \( X \), though still loop suppressed, need not be small. The reason is that the quartic terms can now be generated at order \( g^2 \) by loops involving operators which are strongly coupled at the cutoff, and this could potentially compensate for the loop suppression. However, the little Higgs mechanism still ensures that any such term is invariant under the second \( \text{Sp}(4) \), and so does not contribute to the pNGB potential. Therefore the leading terms which contribute to the pNGB only arise at order \( g^2 g^2 \), and are suppressed by an additional loop factor.

This construction ensures that the strong dynamics does not violate the custodial SU(2) symmetry. To see this explicitly, note that we can write

\[ 2\pi^a T^a = \begin{pmatrix} 0 & \phi \\ \phi^\dagger & 0 \end{pmatrix} \]  

where \( \phi = (i\sigma_2 h_L^*, h_L) \). The full expression for \( X \) is

\[ \cos \left( \frac{|h_L|}{f} \right) f + i \frac{f}{|h_L|} \sin \left( \frac{|h_L|}{f} \right) \begin{pmatrix} 0 & \phi \\ \phi^\dagger & 0 \end{pmatrix} \]  

The Lagrangian written as a function of \( X \) preserves the \( \text{SU}(2)_L \times \text{SU}(2)_R \) subgroup of the diagonal \( \text{Sp}(4) \), under which \( \phi \rightarrow U_{\phi} \phi^\dagger U_{\phi}^\dagger \), and \( |h_L| \rightarrow |h_L| \). After electroweak symmetry breaking, \( |h_L| = (0, v) \), and the diagonal SU(2) symmetry is preserved. This is precisely the custodial symmetry we are looking for.

In order to write down Yukawa couplings, first make the identification

\[ X_{4i} = H_i = (H_L, H_R). \]  

Yukawa couplings can be written down exactly as in the original left-right twin Higgs model, in terms of \( H_L \) and \( H_R \), so that the discrete \( L \leftrightarrow R \) symmetry is preserved. The twin Higgs mechanism then ensures that quadratic divergences from the fermion sector preserve the first global \( \text{Sp}(4) \) symmetry and vanish from the pseudo-Goldstone potential, just as in the gauge sector.

The fermionic content of the theory then contains three generations of

\[ Q_L = (u, d)_L = [2, 1, 1/3] \quad L_L = (\nu, e)_L = [2, 1, -1] \]
\[ Q_R = (u, d)_R = [1, 2, 1/3] \quad L_R = (\nu, e)_R = [1, 2, -1] \]  

where the square brackets indicate the quantum numbers of the corresponding field under \( \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \). We identify \( \text{U}(1)_Y \) with \( \text{U}(1)_{(B-L)}/2 \). As dictated by left-right symmetry the theory includes right-handed neutrinos in addition to the SM fermions.

The Higgs fields have quantum numbers

\[ H_L = [2, 1, 1] \quad H_R = [1, 2, 1] \]  

under \( \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \). The down-type Yukawa couplings of the SM arise from non-renormalizable couplings of the form

\[ \left\{ \frac{\overline{Q}_R H_R H_L^\dagger}{\Lambda} \right\} + \text{h.c.} \]  

Here \( \Lambda \) is an ultra-violet cutoff, which we take to be about 10 TeV, the limit of validity of the non-linear sigma model. Similarly, the up-type Yukawa couplings of the SM emerge from

\[ \left\{ \frac{\overline{Q}_R H_R^\dagger H_L Q_L}{\Lambda} + \text{h.c.} \right\} \]  

The top Yukawa coupling is too large to be naturally obtained from a non-renormalizable operator. As in the original left-right twin Higgs model, we therefore introduce a pair of vector-like quarks \( T_L \) and \( T_R \) which have the quantum numbers

\[ T_L = [1, 1, 4/3] \quad T_R = [1, 1, 4/3] \]  

under \( \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \). We can then write the Yukawa coupling

\[ 
\left( y \overline{Q}_R H_R^\dagger T_L + y \overline{Q}_L H^\dagger_R T_R + M \overline{T}_L T_R \right) + \text{h.c.}
\]  

Here \( Q_L \) and \( Q_R \) are the usual left and right-handed third generation quark doublets of the left-right model.

Since the top Yukawa gives the largest contribution to the Higgs potential, let us understand the cancellation of quadratic divergences in this case. As in the gauge case, the discrete \( L \leftrightarrow R \) symmetry ensures that terms quadratic in \( X \) are invariant under the first \( \text{Sp}(4) \), and do not contribute to the potential for the pNGB. Terms quartic and higher order in \( X \) that violate \( \text{Sp}(4) \) are only generated at order \( y^4 \), and not at order \( y^2 \), and are therefore suppressed by one loop factor, even in the limit that the non-linear sigma model is strongly coupled at the cutoff.

* Whether a quartic term is generated at order \( g^2 \) in a general twin Higgs model in the limit of strong coupling depends on the pattern of symmetry breaking. For example, if the symmetry breaking pattern is \( O(8) \rightarrow O(7) \), a quartic is only generated at order \( g^4 \), and is therefore always loop suppressed. [ citation needed ]
It is also possible to generate the smaller Yukawa couplings from renormalizable interactions \( \text{[21]} \), (see also \( \text{[22]} \)). To do this we introduce three generations of vector-like fermions with the following charge assignments.

\[
\begin{align*}
U_L &= [1, 1, 4/3] \\
U_R &= [1, 1, 4/3] \\
D_L &= [1, 1, -2/3] \\
D_R &= [1, 1, -2/3] \\
E_L &= [1, 1, -2] \\
E_R &= [1, 1, -2]
\end{align*}
\] (15)

Then the Yukawa couplings for the lighter fermions can be written down in analogy with that for the top. For example, the charged lepton Yukawa couplings arise from the interactions

\[
\{ \overline{L}_R H_R E_L + \overline{L}_L H_L E_R + M \overline{E}_L E_R \} + \text{h.c.} \quad (16)
\]

We choose the mass parameter \( M \) to be of order several TeV. On integrating out \( E_L \) and \( E_R \) we get back exactly the same non-renormalizable operator that earlier generated the charged lepton masses.

**III. A TWIN HIGGS MODEL FROM STRONG DYNAMICS**

We now explain how the symmetry breaking pattern \( \text{Sp}(4) \times \text{Sp}(4) \rightarrow \text{Sp}(4) \) may be obtained from QCD-like strong dynamics. Our discussion will closely follow that of \( \text{[18]} \), where the same problem was considered in the context of a dynamical realization of the little Higgs model of Chang and Wacker \( \text{[3]} \). Consider an \( \text{SU}(N_c) \) gauge group, with a set of four fermions, \( \chi_{\alpha i} \), in the fundamental representation. Here \( \alpha \) represents an \( \text{SU}(N_c) \) gauge index and \( i \) labels the fermions from 1 through 4. We also add a set of four right-handed fermions \( \psi_{\alpha i} \). When the \( \text{SU}(N_c) \) theory gets strong, a condensate \( \langle \chi_i \psi_j \rangle \propto \delta_{ij} \) forms and breaks the \( \text{SU}(4)^2 \) flavor symmetry to the diagonal \( \text{SU}(4) \). We label the 15 surviving NGBs that are produced by \( \pi^A \), and define \( X = f \exp(2\pi^A T^A / f) \), where the matrices \( T^A \) are generators of \( \text{SU}(4) \). We also add to the theory a non-renormalizable term

\[
\frac{m^2}{(4\pi f^2)^2} \text{Tr} \left[ (\chi \psi) J (\chi \psi)^T J \right] \sim m^2 \text{Tr} \left[ XJX^T J \right]
\] (17)

where \( J \) is the matrix

\[
J = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix}
\] (18)

The effect of this term is to explicitly break the global \( \text{SU}(4)^2 \) symmetry in fact \( \text{Sp}(4)^2 \rightarrow \text{Sp}(4) \), which accounts for the 10 surviving NGBs. The unbroken global symmetry, the diagonal \( \text{Sp}(4) \), contains the custodial \( \text{SU}(2) \) symmetry we desire.

In order to recreate the low energy structure of the model of the previous section we gauge the subgroups

\[
[\text{SU}(2)_L \times \text{SU}(2)_R] \times [\text{SU}(2)_{L'} \times \text{U}(1)_{R'}] \subset \text{Sp}(4)^2
\] (19)

as shown in Figure 1. After symmetry breaking, this gauge symmetry is broken down to the \( \text{SU}(2) \times \text{U}(1)_Y \) gauge symmetry of the SM, where \( \text{SU}(2) \) is the diagonal subgroup of \( \text{SU}(2)_L \times \text{SU}(2)_R \), while \( \text{U}(1)_Y \) is the unbroken linear combination of the diagonal generator of \( \text{SU}(2)_R \) and \( \text{U}(1)_{R'} \). Of the 10 surviving NGBs, 6 are eaten by the broken gauge symmetries, while the 4 which are left over precisely constitute the SM Higgs.

In order to write down the Higgs couplings to fermions we simply make the replacement

\[
H_i \rightarrow \frac{\chi_i \psi^4}{4\pi f^2}
\] (20)

in the Yukawa couplings of the previous section. For example, the left-right symmetric top Yukawa couplings become

\[
\begin{align*}
\left\{ y \overline{Q}_R \left( \frac{\chi R \psi^4}{4\pi f^2} \right) T_L + y \overline{Q}_L \left( \frac{\chi L \psi^4}{4\pi f^2} \right)^\dagger T_R \right\}
\end{align*}
\] (21)

These interactions are non-renormalizable, and therefore require additional new physics to generate them. We leave the question of the ultra-violet origin of these operators for future work.

We briefly consider the precision electroweak constraints on this theory. In general, bounds from the \( S \)-parameter on any composite Higgs force the compositeness scale \( \Lambda \sim 4\pi f \) to be larger than or of order 5 TeV. Another source of corrections to precision electroweak observables arises from higher order operators in the expansion of the kinetic term for \( X \) in terms of the \( \pi \) fields that contribute to the \( \rho \) parameter. Since the sum over the \( T^A \) in \( X = f \exp(2\pi^A T^A / f) \) now runs over all \( \text{SU}(4)_c \) generators, and not just the generators of \( \text{Sp}(4) \), there are fields with mass below the compositeness scale that correspond to the generators of \( \text{SU}(4)/\text{Sp}(4) \). These fields, which we denote by \( H'_L \), have exactly the same gauge quantum numbers as the light Higgs. The non-renormalizable terms that arise in the expansion of \( X \) in terms of the \( \pi \) fields involve custodial \( \text{SU}(2) \) violating...
couplings of $H'_L$ to the light Higgs $H_L$, and thereby contribute to the $\rho$ parameter.

The effect of the non-renormalizable term in Eq. (17) is to give a mass $m$ to the fields in $H'_L$, and to thereby decouple them from the low-energy spectrum. The precision electroweak constraints therefore translate into a lower bound on the parameter $m$. A quick estimate of the size of the correction to $\rho$ yields

$$\delta m^2_Z \frac{\langle H'_L \rangle^2}{m^2_Z}$$

where

$$\langle H'_L \rangle \sim \frac{f}{4\pi m} v$$

Here $v$ is the electroweak VEV. From these formulas we estimate that the precision electroweak constraints on deviations of the $\rho$ parameter from its SM value are comfortably satisfied provided that $m$ is greater than or of order 500 GeV.

As in any general two Higgs doublet model, the presence of a second Higgs doublet can also lead to a lower bound on $m$ somewhat weaker than the one we have already found.

The additional SU(2)$_R$, U(1)$_{B-L}$ and SU(2)$_L'$ gauge bosons also contribute to the precision electroweak observables. In general these force $f$ to be of order 1500 GeV or larger, reintroducing fine-tuning. However, $f$ can be as low as 500 GeV if, as in the original left-right twin Higgs model, there is a second field $X$ with exactly the same quantum numbers as $X$ that exhibits exactly the same pattern of symmetry breaking, but where the decay constant $f$ somewhat larger than $f$. Then, provided that $f$ is greater than about 1500 GeV the precision electroweak constraints from the new gauge bosons are satisfied, and the fine-tuning is under control. The field $X$ can also be used to generate neutrino masses and dark matter, as in the original left-right twin Higgs model.

Much of the heavy spectrum of particles predicted by this theory will be accessible at the LHC. The new fields in the gauge and top sector must have mass of order the TeV scale if they are to be relevant for stabilizing the electroweak scale. Production and decay of the heavy top-partner, as well as the massive gauge bosons associated with SU(2)$_R$ and U(1)$_{B-L}$, have been studied in the context of the left-right twin Higgs model. The heavy electroweak doublet of scalars, from the explicit breaking of the global SU(4)$^2$ symmetry to Sp(4)$^2$ in Eq. (17), and the massive gauge bosons that constitute the linear combination of SU(2)$_L$ and SU(2)$_L'$ that is orthogonal to SU(2) of the SM, are key predictions of the underlying composite structure of this model. While the electroweak doublet decays primarily into third generation quarks and anti-quarks, the new gauge bosons can decay either into SM fermions, or into electroweak gauge bosons.

In summary we have identified a new class of left-right twin Higgs models which naturally incorporate a custodial SU(2) symmetry, and shown how the relevant pattern of symmetry breaking can be realized through QCD-like strong dynamics. This constitutes an important first step in the construction of completely realistic twin Higgs models from strong dynamics.

Acknowledgments

PB is supported by the DOE under contract DE-FG02-92ER. ZC is supported by the NSF under grant PHY-0801323.

[1] H. Georgi and A. Pais, Phys. Rev. D 10, 539 (1974); Phys. Rev. D 12, 508 (1975).

[2] D. B. Kaplan and H. Georgi, Phys. Lett. B 136, 183 (1984); D. B. Kaplan, H. Georgi and S. Dimopoulos, Phys. Lett. B 136, 187 (1984); H. Georgi and D. B. Kaplan, Phys. Lett. B 145, 216 (1984).

[3] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001).

[4] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, JHEP 2008, 021 (2002); N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 2007, 034 (2002); T. Gregoire and J. G. Wacker, JHEP 2008, 019 (2002); I. Low, W. Skiba and D. Smith, Phys. Rev. D 66, 072001 (2002) D. E. Kaplan and M. Schmaltz, JHEP 0310, 039 (2003).

[5] S. Chang and J. G. Wacker, Phys. Rev. D 69, 035002 (2004) [arXiv:hep-ph/0303001].

[6] M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. 55, 229 (2005) [arXiv:hep-ph/0502182]; M. Perelstein, Prog. Part. Nucl. Phys. 58, 247 (2007) [arXiv:hep-ph/0512128].

[7] Z. Chacko, H. S. Goh and R. Harnik, Phys. Rev. Lett. 96, 231802 (2006) [arXiv:hep-ph/0506256]; R. Barbieri, T. Gregoire and L. J. Hall, arXiv:hep-ph/0509242; Z. Chacko, Y. Nomura, M. Papucci and G. Perez, JHEP 0601, 126 (2006) [arXiv:hep-ph/0510273].

[8] Z. Chacko, H. S. Goh and R. Harnik, JHEP 0601, 108 (2006) [arXiv:hep-ph/0512088]; H. S. Goh and C. A. Krenke, Phys. Rev. D 76, 115018 (2007) [arXiv:0707.3650 [hep-ph]].

[9] A. Birkedal, Z. Chacko and M. K. Gaillard, problem,” JHEP 0410, 036 (2004) [arXiv:hep-ph/0404197]; P. H. Chankowski, A. Falkowski, S. Pokorski and J. Wagner, scalar Phys. Lett. B 598, 252 (2004) [arXiv:hep-
[10] Z. Berezhiani, P. H. Chankowski, A. Falkowski and S. Pokorski, Phys. Rev. Lett. 96, 031801 (2006) [arXiv:hep-ph/0509311]. T. Roy and M. Schmaltz, JHEP 0601, 149 (2006) [arXiv:hep-ph/0509357]. C. Csaki, G. Marandella, Y. Shirman and A. Strumia, Phys. Rev. D 73, 035006 (2006) [arXiv:hep-ph/0510294]. B. Bellazzini, S. Pokorski, V. S. Rychkov and A. Varagnolo, arXiv:0805.2107 [hep-ph].

[11] A. Falkowski, S. Pokorski and M. Schmaltz, Phys. Rev. D 74, 035003 (2006) [arXiv:hep-ph/0604066]. S. Chang, L. J. Hall and N. Weiner, Phys. Rev. D 75, 035009 (2007) [arXiv:hep-ph/0604076].

[12] D. E. Kaplan, M. Schmaltz and W. Skiba, Phys. Rev. D 70, 075009 (2004) [arXiv:hep-ph/0405257].

[13] P. Batra and D. E. Kaplan, JHEP 0503, 028 (2005) [arXiv:hep-ph/0412267].

[14] C. Csaki, J. Heinonen, M. Perelstein and C. Spethmann, arXiv:0804.0622 [hep-ph].

[15] E. Katz, J. Y. Lee, A. E. Nelson and D. G. E. Walker, JHEP 0510, 088 (2005) [arXiv:hep-ph/0312287].

[16] J. Thaler, JHEP 0507, 024 (2005) [arXiv:hep-ph/0502175]. C. Cheung and J. Thaler, JHEP 0608, 016 (2006) [arXiv:hep-ph/0604259].

[17] C. T. Hill and R. J. Hill, Phys. Rev. D 75, 115009 (2007) [arXiv:hep-ph/0701044]. C. T. Hill and R. J. Hill, Phys. Rev. D 76, 115014 (2007) [arXiv:0705.0697 [hep-ph]].

[18] P. Batra and Z. Chacko, Phys. Rev. D 77, 055015 (2008) [arXiv:0710.0333 [hep-ph]].

[19] R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B 671, 148 (2003) [arXiv:hep-ph/0306259]. K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719, 165 (2005) [arXiv:hep-ph/0412089]. J. Thaler and I. Yavin, JHEP 0508, 022 (2005) [arXiv:hep-ph/0501036].

[20] J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); R. N. Mohapatra and J. C. Pati, Violation," Phys. Rev. D 11, 566 (1975); R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 2558 (1975).

[21] A. Davidson and K. C. Wali, Phys. Rev. Lett. 59, 393 (1987); S. Rajpoot, Mod. Phys. Lett. A 2, 307 (1987) [Erratum-ibid. A 2, 541 (1987 PHLTA,B191,122.1987)].

[22] D. Chang and R. N. Mohapatra, Phys. Rev. Lett. 58, 1600 (1987).

[23] A. Abada and I. Hidalgo, Phys. Rev. D 77, 113013 (2008) [arXiv:0711.1238 [hep-ph]].

[24] E. M. Dolle and S. Su, Phys. Rev. D 77, 075013 (2008) [arXiv:0712.1234 [hep-ph]].

[25] H. S. Goh and S. Su, Phys. Rev. D 75, 075010 (2007) [arXiv:hep-ph/0611015]. Y. B. Liu, H. M. Han and Y. H. Cao, arXiv:hep-ph/0703268.