Dynamic characteristic and interaction analysis of synchronous generator based on amplitude–phase motion equation

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Abstract: With the increasing penetration of power electronic devices, the dynamic characteristics of devices and power system change. Thus, new models and analytical methods applied to stability analysis of power electronics dominated power system need to be put forward. This study takes synchronous generator as an example. The method of device modelling based on amplitude–phase motion equation is introduced, and the idea of interaction analysis based on concepts of self-stabilising and en-stabilising is proposed.

1 Introduction

With the widespread application of renewable generations, high-voltage direct current, and flexible AC transmission systems, power converter interfaced devices occupy a high penetration in power system nowadays, and the pattern of power electronics dominated power system is gradually formed. The time-domain method, the torque method, and the complex-frequency-domain method are the common analysis methods for small disturbance stability under an electromechanical time scale of traditional power system [1]. The time-domain method can show the state response clearly, but it just calculates the numerical solution. No analytic solution results in limitations in the analysis of the problem. Torque method divides the electromagnetic torque of a synchronous generator (SG) into two parts: synchronous torque and damping torque. The physical meaning is clear, but there is no further study on the multi-machine problem. The complex-frequency-domain method can obtain the accurate analysis result of the system at a certain operating point, but the path of the interaction of the devices cannot be seen from the result, and the process of interaction is not described.

In this paper, in order to describe the dynamic characteristics of the devices uniformly, according to the relationship between input, output, storage, dissipated power and phase and amplitude of the internal voltage, a generalised modelling method based on the amplitude–phase motion equation is proposed [2]. Under the electromechanical time scale, the grid can be described by algebraic equation without considering its dynamic characteristics, and the dynamic characteristics of devices can be described by the amplitude–phase motion equation. Thus, the system model is shown in Fig. 1.

This paper takes SG, the main generation in traditional power system as an example, to study its dynamic characteristics and stability under the electromechanical time scale. Firstly, the amplitude–phase motion model of SG under terminal voltage control is established, and its physical meaning is explained. The accuracy of the model is verified by comparing with the time-domain simulation of the non-linear original model. Then a new method based on self-stabilising and en-stabilising is proposed to analyse and quantify the interaction between devices. The influence of excitation magnification, line impedance parameters and power system stabiliser (PSS) parameters on the system damping is analysed through a clear interaction path in a single machine interfaced infinite bus (SMIB) system.

2 Modelling of SG based on amplitude–phase motion equation

2.1 Basic operating principle of SG

The basic schematic diagram of SG and its control system are shown in Fig. 2. SG can be described by the three-winding model (d, q-axis windings of stator and field winding of rotor) when small disturbance stability under the electromechanical time scale is studied.

Fast-response excitation system using thyristor is widely used in modern SGs [3]. The voltage controller magnification $K_4$ is large and the exciter time constant $T_4$ is very small. In this paper, it is assumed that the no-load electromotive force $E_{d0}$ corresponding to the excitation voltage $U_{d0}$ can be expressed directly by the following control equation:

$$E_{d0} = \frac{K_4}{1 + T_4 s}(U_r^* - U_r) \simeq K_4(U_r^* - U_r)$$

where $U_r^*$ is the command value of the excitation system. A typical PSS takes generator shaft speed $\omega$ as the input signal, the equivalent transfer function is

$$G_p(s) = K_r \frac{sT_1}{1 + sT_2} \frac{1 + sT_3}{1 + sT_4}$$

2.2 Amplitude–phase motion model of SG under the electromechanical time scale

The dynamic characteristics of devices are reflected by the amplitude and phase of the internal voltage which are driven by the unbalanced active power and reactive power. Therefore, to establish the amplitude–phase motion model of devices, firstly, the original model of the device should be clear, then active and reactive...
2.2.1 Phase motion equation: The physical state of rotor directly determines the frequency and phase of the internal voltage, so the phase motion equation can be obtained by linearising the rotor motion equation:

\[
\begin{align*}
\frac{d\Delta \omega}{dt} &= \frac{1}{M_1}(\Delta P_m - \Delta P_e - D\Delta \omega) \\
\frac{d\Delta \theta}{dt} &= \omega_0\Delta \omega
\end{align*}
\]

(3)

where \(M_1\) represents the mass of the internal voltage phase motion. In the case of small disturbances, the variation in the rotor speed can be omitted. Considering \(P = \alpha T\), the input mechanical torque \(T_m\) and the output electromagnetic torque \(T_e\) can be approximated by the input mechanical power \(P_m\) and the output active power \(P_e\) of the internal voltage.

2.2.2 Amplitude motion equation: According to the linearised model of SG, the amplitude variation of terminal voltage \(\Delta U_i\) and internal voltage \(\Delta E_q\) can be expressed as

\[
\Delta U_i = K_{pu} \Delta P_e + K_{qu} \Delta Q_e + K_{el} \Delta E_q
\]

(4)

\[
\Delta E_q = G_{eq}(s) \Delta P_e + G_{qg}(s) \Delta Q_e
\]

(5)

where \(K_{pu}, K_{qu}, \) and \(K_{el}\) represent the coupling of phase and amplitude motion. The variation of terminal voltage command value \(\Delta U_i^*\) can be regarded as 0 in the small disturbance study. In order to meet the target of terminal voltage control, according to (5), the input reactive power needed by the internal voltage of SG is

\[
\Delta Q_m = -\frac{K_{pu}}{K_{qu}} \Delta P_e - \frac{K_{el}}{K_{qu}} \Delta E_q
\]

(6)

Then, the unbalanced reactive power acting on SG is

\[
\Delta Q = \Delta Q_m - \Delta Q_e
\]

(7)

Considering that the governor does not change rapidly in the electromechanical time scale, the input mechanical power remains constant. The internal voltage amplitude expressed by the unbalanced reactive power and the variation of internal voltage frequency \(\Delta \omega\) is

\[
\Delta E_q = G_{21}(s) \Delta \omega + G_{22}(s) \Delta Q
\]

(8)

where \(G_{21}(s)/s^2\) represents the relationship between the unbalanced reactive power and the amplitude of the internal voltage, \(G_{22}(s)\) represents the effect of the internal voltage frequency on the amplitude of the internal voltage, i.e. the coupling of phase and amplitude motion. The amplitude–phase motion model of SG under electromechanical time scale is shown in Fig. 4.
When PSS is not considered,

\[ G_{21}(s) = G_{211}(s) = \frac{a_2 s + a_3}{a_1 s + a_2} \]

\( G_{211}(s) \) represents the coupling of phase and amplitude motion caused by the demagnetisation effect of the armature reaction.

If considering PSS, an additional signal \( \Delta E \) related to rotor speed \( \omega \) is superimposed on the command value of terminal voltage. PSS will affect the coupling of phase and amplitude motion. When PSS is considered

\[ G_{21}(s) = G_{211}(s) + G_{2112}(s) \]

where

\[ G_{2112}(s) = \frac{K_{0k} K_d G_0(s)}{a_1 s + a_2} \]

represents the coupling between phase and amplitude motion.

3 Dynamic interaction analysis

3.1 Concept of self-stabilising and en-stabilising

Self-stabilising refers to the ability of the studied device to maintain its own stability when the state of other device is not considered. En-stabilising refers to the ability of other device maintains the stability of the studied device when the system is disturbed. There are two dimensions of motion of device, i.e. amplitude and phase. Therefore, the self-stabilising and en-stabilising can also be used to describe the interaction between amplitude and phase motion.

In the following, the SMIB system is taken as an example to introduce the idea of using self-stabilising and en-stabilising to analyse and quantify the dynamic interaction.

3.2 Idea of using self-stabilising and en-stabilising to analyse interaction

The parameters of the SG model based on the amplitude-phase motion equation are just related to the device parameters and the steady-state operating point, which are independent of the external network and external devices.

As shown in Fig. 4, the reactive power input is not constant, which varies with the device state changing. It is because the reactive power input is based on the voltage control command. The reactive power output of internal voltage changes with the state to meet the target of voltage control. This kind of amplitude–phase motion model has a clear physical meaning.

However, the model can be further simplified when analysing self-stabilising and en-stabilising. The change of the reactive power input is equivalent to the change of the mass of amplitude motion and coupling relationship between the amplitude and phase motion. It is assumed that the reactive power input is constant. Equivalent amplitude–phase motion model of SG is as shown in Fig. 5.

In the SMIB system, the relationship between amplitude and phase of SG is

\[ \frac{\Delta E}{\Delta \theta} = -\frac{k_1 G_2'(s)/s^2 + k_11 G_21(s)/(M_i s + D)}{k_22 G_2'(s)/s^2 + k_11 G_21(s)/(M_i s + D) + 1} \]

Thus, the active power output of SG is

\[ \Delta P_e = \Delta P_{e1} + \Delta P_{e2} = k_1 \Delta \theta + k_{12} \Delta E \]

\[ H_{12}(s) = -k_{12} \frac{k_1 G_2'(s)/s^2 + k_11 G_21(s)/(M_i s + D)}{k_22 G_2'(s)/s^2 + k_11 G_21(s)/(M_i s + D) + 1} \]

It can be considered that the active power output from the internal voltage consists of two parts. The first part is the effect of the phase motion on the output through the network without considering the amplitude change. The coefficient \( k_{12} \) reflects the phase motion self-stabilising. The second part is the effect of the amplitude motion on the output through the network. \( H_{12}(s) \) is a transfer-function, which reflects the en-stabilising that amplitude motion providing to phase motion.

Fig. 6 shows the self-stabilising and en-stabilising model to analyse the interaction. The synchronous torque and damping torque are studied under a certain mode, so the synchronous and damping power are studied in the same way. Assuming that the electromechanical oscillation mode of the system is \( \lambda_{k_1} = \sigma + j \omega_p \), substituting \( s = j \omega_p \) in (10), we obtain

\[ \Delta P_e' = (k_{11} + Re(H_{12}(j \omega_p))) + j \text{Im}(H_{12}(j \omega_p)) \Delta \theta^\phi \]

Since

\[ \omega_0 \Delta \omega = \sigma \Delta \theta \Rightarrow \Delta \theta^\phi = \frac{\omega_0}{j \omega_p} \Delta \omega^\phi, \]

Substituting it in (12)

\[ \Delta P_e' = (k_{11} + \Delta M_s) \Delta \theta^\phi + \Delta M_d \Delta \omega^\phi \]

\( \Delta M_s \) and \( \Delta M_d \) denote the en-stabilising synchronous and damping coefficient that the amplitude motion provides to the phase motion, respectively. The specific expression is as follows:

\[ \Delta M_s = \text{Re}(H_{12}(j \omega_p)) \]

\[ \Delta M_d = \frac{\omega_0}{\omega_p} \text{Im}(H_{12}(j \omega_p)) \]
Therefore, the en-stabilising power of the amplitude motion providing to the phase motion can be divided into two parts, i.e. the synchronous power and the damping power. By studying the characteristics of $H_{12}(s)$ in the electromechanical oscillation mode, the effect of the amplitude motion on the phase motion can be understood deeply.

4 Simulation verification and case analysis

4.1 Verification of SG model based on amplitude-phase motion equation

The correctness of the amplitude-phase motion model is proved from two aspects, i.e. eigenvalue and time-domain simulation.

(i) Eigenvalue comparison: Using the example of the SMIB system in the Appendix, the electromechanical oscillation eigenvalues of the original model and the amplitude-phase motion model is shown in Table 1.

(ii) Time domain simulation comparison: The infinite bus is disturbed at $t = 3$ s. Considering PSS, the dynamic response of the original model and the amplitude-phase motion model are compared as shown in Fig. 7.

The comparison results of eigenvalue and time-domain simulation verify the correctness of the amplitude-phase motion model.

4.2 Simulation analysis of en-stabilising

As can be seen in Section 3.2, by analysing the transfer-function $H_{12}(s)$, we can quantitatively analyse the interaction between the amplitude motion and the phase motion. In this subsection, the effect of excitation magnification, line impedance and PSS parameters on en-stabilising is studied.

4.2.1 Effect of excitation magnification $K_A$: According to the example given in the Appendix, the electromechanical oscillation eigenvalues change with $K_A$. As shown in Table 2, the effect of $K_A$ variation on the electromechanical oscillation frequency $\omega_d$ can be negligible, but with $K_A$ increasing, the damping in the oscillation mode becomes smaller.

The results of the eigenvalue analysis only show the effect of $K_A$ on the damping of SG, but cannot show the contribution of the amplitude and phase motion to maintain the stability of the power angle. By analysing the bode diagram of $H_{12}(s)$ as shown in Fig. 8, the above conclusions can be further explained. It can be seen from Table 2 that the electromechanical oscillation frequency of the studied system is about 1.114 Hz. According to (15), under this electromechanical oscillation mode, the damping of phase motion provided by amplitude motion is as shown in Fig. 9. The en-stabilising damping coefficient $M_D$ decreases while $K_A$ increases. This agrees with the conclusion obtained by eigenvalue analysis.

4.2.2 Effect of line impedance $x_l$: Line impedance $x_l$ affects the network coupling between phase and amplitude motion, and also changes the operating point of system, thus affecting the dynamic characteristics of devices.

When amplitude motion provides positive damping to phase motion, as shown in Fig. 10, it is obvious that en-stabilising damping coefficient decreases when $x_l$ increases. It is because the

| $K_A$ | Eigenvalues $\lambda_{1,2}$ |
|-------|---------------------------|
| 50    | $-0.2175 \pm 7.024i$      |
| 100   | $-0.2008 \pm 6.969i$      |
| 300   | $-0.1645 \pm 6.985i$      |

**Table 1** Comparison of eigenvalues of two models

**Table 2** Electromechanical oscillation eigenvalues

**Fig. 7** Dynamic response of the original model and amplitude-phase motion model

**Fig. 8** Bode diagram of $H_{12}(s)$ while $K_A$ changes

**Fig. 9** En-stabilising damping coefficient $M_D$

**Fig. 10** Bode diagram of $H_{12}(s)$ while $x_l$ changes (positive damping)
smaller the line impedance, the stronger the coupling of the network, and the effect of the amplitude motion on the phase motion through the network in the corresponding condition will be amplified.

With the line impedance $x_1$ further increasing, the amplitude motion provides an increasing negative damping to the phase motion, as shown in Fig. 11. Since the phase motion itself also has a certain damping power (provided by the damping coefficient $D$), only when the negative damping provided by the amplitude motion to the phase motion counteracts this part, the system becomes unstable.

4.2.3 Effect of PSS configuration parameters: Taking the influence of PSS into account, the coupling between phase and amplitude motion adds an additional part of (16). Assuming that the change in the amplitude of the internal voltage caused by PSS is $\Delta E_2$, we obtain

$$G'_{212}(s) = \frac{\Delta E}{\Delta \omega} = G_E(s)G_P(s)$$

Equation (16) is the product of two parts, the first part is

$$G_E(s) = \frac{K_d}{sT_{\alpha0} + 1 + K_dK_{EU} + (x_d - x_1)K_{EU}}$$

It reveals the inertia of excitation system and field winding. The second part $G_P(s)$ reveals the characteristics of PSS. By adjusting the parameters of PSS to make $\Delta E_2$ and $\Delta \omega$ in the same phase, PSS can provide maximum positive damping. Specific verification is shown in Fig. 12.

5 Conclusion

In this paper, firstly, the amplitude–phase motion model of SG under terminal voltage control is established, and the correctness of the model is verified. It turns out that considering PSS or not just affects the coupling between the phase and amplitude motion.

More importantly, the concept and idea of self-stabilising and en-stabilising are proposed. In a SMIB system, with the increasing of $K_d$ and $x_1$, the damping of phase motion provided by amplitude motion will decrease. It is proved that the analysis using eigenvalues and self/en-stabilising can get the same results. Moreover, a new method of configuring PSS parameters through a clear interaction path is presented at last. If $G_P(s)$ can counteract the phase lag of $G_E(s)$, PSS can provide proper positive damping to the phase motion.

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7 References

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8 Appendix

An example of the SMIB system is given in Fig. 13

$$x_d = 1.18, \quad x_q = 1.0, \quad x'_{d} = 0.295, \quad T_j = 7 \text{ s},$$

$$T'_{\alpha0} = 5.004 \text{ s}, \quad D = 2$$

$$x_1 = 0.3, \quad K_d = 50, \quad T_I = 0, \quad U_h = 1.0, \quad U_l = 1.05, \quad P_{\alpha0} = 0.5$$

Fig. 13 SMIB system