Cosmic Ray Spectrum and Tachyonic Neutrino

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In the cosmic ray spectrum, there are two knees (abrupt changes of slopes) located around the energies $E_{th} = 10^{15.5}$ eV and $10^{17.8}$ eV respectively. Based on the pioneering work by Kostelecky and Ehrlich, we ascribe the first knee to a sudden opening of the reaction channel $\nu_\mu + p \rightarrow n + e^+$ when the proton has a velocity just exceeding a critical value and so can absorb a tachyonic neutrino $\nu_\epsilon$ in the form of an antineutrino $\bar{\nu}_\epsilon$. Similarly, the second knee is triggered by the reaction $\bar{\nu}_\mu + p \rightarrow \Lambda + \mu^+$. The fitting of these two values of $E_{th}$ gives the tachyon mass of $\nu_\epsilon$ and $\nu_\mu$ being 0.54 eV/c$^2$ and 0.48 eV/c$^2$ respectively, which are in favor of a minimal three-flavor model for tachyonic neutrino with one parameter $\delta$ being estimated to be $\delta = 0.34$ eV.

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I. INTRODUCTION

It has been known for years that the observed energy spectrum of primary cosmic rays can be well described by an inverse power law in the energy $E$ from $10^{11}$ to $10^{20}$ eV.

$$\frac{dJ}{dE} \sim E^{-\gamma}, \quad (1)$$

where $J$ is the flux in $m^{-2} s^{-1} sr^{-1}$. However, the index $\gamma$ reveals an abrupt change at around $10^{15.5}$ eV $= 3.16 \times 10^{15}$ eV $= 3.16$ PeV:

$$\gamma = \begin{cases} 2.7, & E \leq 10^{15.5} \text{ eV} \\ 3, & E > 10^{15.5} \text{ eV} \end{cases} \quad (2)$$

This sudden change in the slope of cosmic ray spectrum (CRS) is usually called the “knee”. It seems that there is a second knee at around $10^{17.8}$ eV $= 6.31 \times 10^{17}$ eV, then follows the “ankle” at around $10^{19}$ eV, at which the slope changes from $\gamma = 3.16$ to 2.78.

Among various tentative explanations for the knee in CRS, the model initiated by Kostelecky first and then elaborated by Ehrlich is the most attractive one. Their basic idea is as follows:

Corresponding to the decay of a neutron

$$n \rightarrow p + e^- + \bar{\nu}_e \quad (3)$$

the “proton decay”

$$p \rightarrow n + e^+ + \nu_e \quad (4)$$

is considered. For the decay to conserve energy in the proton rest frame [ to be referred as the $S'$ frame in which an observer Alice (A) is present ], $E_\nu < 0$ is needed. Then the threshold laboratory [ defined by the cosmic background radiation (CBR), to be referred as the $S$ frame in which another observer Alice (A) is present ] energy for proton was derived as ($\Delta = m_n + m_e - m_p$):

$$E_{th} = \frac{m_p \Delta}{|m_{\nu_e}|} = \frac{1.7 \times 10^{15}}{|m_{\nu_e}|} \text{ eV} \quad (5)$$

where $|m_{\nu_e}| = \sqrt{m^2}$ is the tachyon mass of neutrino. If setting $E$ to be the energy of knee $\sim 4.5 \times 10^{15}$ eV, they found $|m_{\nu_e}| = 0.38 \text{ eV/c}^2$.

The idea of how a “stable” proton can decay was explained by the so-called “reinterpretation principle” and the emitted $\nu_e$ with $E_\nu > 0$ in the process in the $S$ frame could be identified with the process

$$\bar{\nu}_e + p \rightarrow n + e^+ \quad (6)$$

with an absorbed $\nu_e$ with $E_\nu < 0$ from a background sea in the $S'$ frame. Furthermore, the cosmic ray nucleons on their way to Earth would lose their energy through the chain of decays $p \rightarrow n \rightarrow \Lambda \rightarrow \Lambda \rightarrow \cdots$, which not only depletes the spectrum at energies above $E_{th}$ and may also account for the existence of the “ankle” and a neutron “spike” just above $E_{th}$. Moreover, the reason why the GZK cutoff ( cosmic rays with energies much above $5 \times 10^{19}$ eV were predicted to not reach Earth from distant sources because of their interaction with photons comprising the CBR) is absent (as shown in ) is because neutrons have a much smaller interaction with the CBR.

The purpose of this paper is to further justify the above model by refining the theory of tachyon (superluminal particle, i.e., faster-than-light particle) for neutrinos. We will take both energy and momentum conservation laws into account to rigorously derive the relation between $E_{th}$ and the tachyon mass $m$ ( real and positive ) of neutrinos. If setting $E_{th} = 3.16 \times 10^{15}$ eV, we will find the value of $m$ for $\nu_e$ being 0.54 eV/c$^2$. Moreover, we consider a similar process in the $S'$ frame to create the neutral hyperon $\Lambda$ (with quark content $uds$ ) and a muon:

$$\bar{\nu}_\mu + p \rightarrow \Lambda + \mu^+. \quad (7)$$

Fitting the energy at the second knee $E_{th}^{(2)} = 6.31 \times 10^{17}$
eV, we find the tachyon mass of $\nu_\mu$ around $0.48\;\text{eV}/c^2$. The implications of our theory will be discussed below.

II. THE LORENTZ TRANSFORMATION AND A SOLUTION TO SUPERLUMINAL PARADOX

The reason why many physicists do not believe in tachyons goes back to a strange puzzle involving tachyon motion. See Fig. II.10. For clarity, we only consider its motion in a one dimensional space.

A tachyon (P) is moving along the $x$ axis with velocity $u > c$ in the $S$ frame. Bob takes another $S'$ frame moving relative to $S$ with velocity $v$. Then if $v > c^2/u$, the time coordinate of P in the $S'$ frame will become negative:

$$t' < 0 \quad (u > c, \; v > c^2/u)$$

(8)

which was regarded as the “tachyon traveling backward in time” or “a violation of causality” [8].

In our opinion, the above puzzle can be better displayed in an alternative way. From the well known Lorentz transformation (LT), we have the addition law for velocities as:

$$u' = \frac{u - v}{1 - uv/c^2}$$

(9)

where $u'$ is the velocity of tachyon in the $S'$ frame. As shown in the Fig. II.11, there is a pole at $uv/c^2 = 1$.

For a fixed $u$, when $v$ increases across the singularity $c^2/u$, Bob will see that $u'$ leaps abruptly from $\infty \to -\infty$:

$$u' < -c. \quad (u > c^2/v \; \text{or} \; v > c^2/u)$$

(10)

However, Eq. (10) still remains as a puzzle. According to LT, the momentum $p'$ and energy $E'$ of tachyon in the $S'$
frame are related to $p$ and $E$ in the $S$ frame as follows:

$$p' = \frac{p - ve/c^2}{\sqrt{1 - v^2/c^2}}, \quad E' = \frac{E - vp}{\sqrt{1 - v^2/c^2}}. \tag{11}$$

with

$$p = \frac{mu}{\sqrt{u^2/c^2 - 1}} > 0, \quad E = \frac{mc^2}{\sqrt{u^2/c^2 - 1}} > 0. \tag{12}$$

Here $m$ is the tachyon mass of a particle with kinematical relation as;

$$E^2 = p^2 c^2 - m^2 c^4, \quad u = \frac{dE}{dp} = \frac{pc^2}{E} > c. \tag{13}$$

Combining (10) with (11) leads to:

$$p' = \frac{m}{\sqrt{1 - v^2/c^2}} (u - v) > mc > 0, \tag{14}$$

$$E' = \frac{m}{\sqrt{1 - v^2/c^2}} (c^2 - uv) < 0, \quad (u > \frac{c^2}{v} \text{ or } v > \frac{c^2}{u})$$

Now the puzzle arises: How can a particle have $u' < 0$ ($u > c^2/v$) whereas its $p' > 0$? How can it have energy $E' < 0$ whereas $E > 0$? All of the above puzzles from $S$, $S'$ comprise the “superluminal paradox”.

The paradox disappears in a reasonable quantum theory (developed from the “reinterpretation principle”) as follows: According to Bob’s point of view, the tachyon behaves in the $S'$ frame (with $v > c^2/u$) just like an antitachyon moving at a velocity $u'$. So its momentum and energy should be measured as:

$$p'_c = -p' < 0, \quad E'_c = -E' > 0. \tag{15}$$

This is because the well known operators in quantum mechanics:

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}, \quad \hat{E} = i\hbar \frac{\partial}{\partial t} \tag{16}$$

are valid only for a particle. For its antiparticle, we should use instead:

$$\hat{p}_c = i\hbar \frac{\partial}{\partial x}, \quad \hat{E}_c = -i\hbar \frac{\partial}{\partial t}, \tag{17}$$

(where the subscript $c$ refers to an antiparticle) which are just the essence of special relativity (SR) [11].

Note that, however, the distinction between (10) and (16) is merely relative, not absolute. For example, the energy of positron $e^+$ (which is the antiparticle of electron) in the process [6] is always positive like that of neutron $n$. But once a neutrino has energy $E > 0$ in the $S$ frame [see (15) below] but has $E' < 0$ in the $S'$ frame, it behaves just like an antineutrino in the $S'$ frame. Then the relations (16) and (17) must be taken into account when dealing with the LT. For further discussion, see the Appendix.

III. THE THRESHOLD ENERGY OF PROTONS AND TACYHON MASS OF NEUTRINOS

According to the present knowledge of particle physics, a free proton is stable in the vacuum. It will never decay no matter how high its energy is (principle of relativity). However, according to the theory of modern cosmology, low energy neutrinos including all three flavors ($\nu_e$, $\nu_\mu$ and $\nu_\tau$) and antineutrinos ($\bar{\nu}_e$, $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$) are spreading through out space isotropically. (There may also be considerable amount of high energy neutrinos directly related to observable distant sources). So the process (4) could be induced for either the subluminal or superluminal antineutrino if the proton has enough energy but still well below $10^{15}$ eV, no knee will be seen. Hence, to explain the appearance of a knee, which implies a sudden opening of a reaction channel, a new mechanism of tachyonic neutrinos must be considered.

Now let us consider a process:

$$\nu_e + p \rightarrow n + e^+, \tag{18}$$

which is strictly forbidden at energy $E_p < E_{ih}$ due to the different lepton quantum numbers on opposing sides. However, once the proton velocity $v$ exceeds a critical value to be calculated below, a low energy neutrino $\nu_e$ (with $E_\nu \sim 0$ in the $S$ frame) suddenly transforms into an antineutrino with sufficiently high energy $E'_p$ in the $S'$ frame as discussed in the previous section. Then the process (4) suddenly occurs as an exotic realization of (15) in the $S$ frame and contributes to the abrupt change of the slope in CRS as shown in [4].

Denoting the rest masses (and velocities in the $S'$ frame) of protons, neutrons and positrons by $m_p$, $m_n$ and $m_e$ ($v'_p$, $v'_n$ and $v'_e$) respectively, Bob can write down the conservation laws of energy and momentum in the $S'$ frame as follows:

$$m_p + \frac{m}{\sqrt{u^2/c^2 - 1}} = \frac{m_n}{\sqrt{1 - v'^2_n/c^2}} + \frac{m_e}{\sqrt{1 - v'^2_e/c^2}}, \tag{19}$$

$$\frac{m_{v'}}{\sqrt{u^2/c^2 - 1}} = \frac{m_nv'_n}{\sqrt{1 - v'^2_n/c^2}} + \frac{m_ev'_e}{\sqrt{1 - v'^2_e/c^2}}, \tag{20}$$

where the antineutrino velocity $u'$ is given by (9) with $v = v_p > 0$ and $u > 0$. Any one of these velocities can take either positive [along the $x(x')$ axis] or negative [along the $-x(−x')$ axis] value automatically. We introduce notations used in the theory of SR:

$$\beta_i = \frac{v_i}{c} = \tanh \zeta_i, \quad (i = p, n, e) \tag{21}$$

$$\gamma_i = \frac{1}{\sqrt{1 - \beta_i^2}} = \cosh \zeta_i, \quad \beta_i \gamma_i = \sinh \zeta_i,$$

where $\zeta_i$ is called the rapidity of particle $i$. Then (19)
Here a dimensionless parameter: 
\[ \xi \]
Since the hyperbolic function cosh \( \xi \) \( \geq 1 \), we find the condition for existence of a solution to (24) being:
\[ \cosh(\xi' - \xi) = \frac{1}{2m_m^2} \langle m_p^2 + 2m_m^2 \rangle - m^2 - m_-^2 - m_+^2 \].
Since the hyperbolic function cosh \( \xi \) \( \geq 1 \), we find the condition for the occurrence of process (18) with \( \nu_\mu \rightarrow p + \nu_\mu \), we get:
\[ \cos(\xi' - \xi') = \frac{1}{2m_m^2} \langle m_p^2 + 2m_m^2 \rangle - m^2 - m_-^2 - m_+^2 \].

\[ \begin{align*}
\frac{2m_pm_m^2}{\sqrt{\beta_\nu^2 - 1}} & > (m_n + m_e)^2 + m^2 - m_p^2, \\
\text{or} & \\
\frac{1}{\sqrt{\beta_\nu^2 - 1}} & > \frac{1}{\eta} \gg 1.
\end{align*} \]

Here a dimensionless parameter:
\[ \eta = \frac{2m_pm_m^2}{(m_n + m_e)^2 + m^2 - m_p^2} \]
is defined. Rewriting (22) as \( (\beta_\nu = u/c, \beta_p = v/c) \):
\[ \beta_p' = \beta_p - \frac{\beta_p - \beta_\nu}{1 - \beta_\nu \beta_p} < 0. \]
we obtain the condition for the occurrence of process in the \( S' \) frame as:
\[ \beta_p > \frac{\beta_p + \sqrt{1 + \eta^2}}{\sqrt{1 + \eta^2 \beta_\nu + 1}}, \]
\[ \frac{1}{\sqrt{1 - \beta_p^2}} > \frac{\sqrt{1 + \eta^2 \beta_\nu + 1}}{\eta \sqrt{\beta_\nu^2 - 1}}. \]

Since \( \eta \ll 1 \), within a good approximation, we find the corresponding condition for proton energy in the \( S' \) frame as \( (c = 1) \):
\[ E_p = \frac{m_p}{\sqrt{1 - \beta_p^2}} > \frac{m_p}{\eta} \sqrt{\frac{\beta_\nu + 1}{\beta_\nu - 1}} = \frac{m_p}{\eta} \sqrt{\frac{p_\nu + E_\nu}{p_\nu - E_\nu}}. \]

As discussed at the beginning of this section, the first knee in CRS should be the threshold value of \( E_p \) in (31) with \( \beta_p \rightarrow \infty \) (i.e., \( E_\nu \rightarrow 0 \)):
\[ E_{th} = E^{(1)}_{th} = \frac{1}{2m} \langle (m_n + m_e)^2 + m^2 - m_p^2 \rangle \]
\[ \simeq 1.695 \times 10^{15} \text{ eV}. \]

where the value of tachyon mass of neutrino is in unit of eV/c². If we adopt the value of \( E^{(1)}_{th} = 3.16 \times 10^{15} \text{ eV} \) as in Ref. 2, we find the tachyon mass:
\[ m = m(\nu_\nu) = 0.54 \text{ eV}/c^2. \]

Similarly, based on a known semileptonic decay mode of hyperon \( \Lambda \):
\[ \Lambda \rightarrow p + \mu^- + \nu_\mu, \]
we may consider the process (17) induced in the \( S' \) frame right at the threshold energy of the second knee in CRS, \( E^{(2)}_{th} = 6.31 \times 10^{17} \text{ eV} \), for the exotic reaction \( \nu_\mu + p \rightarrow \Lambda + \mu^- \) triggered in the \( S \) frame. Thus we find \( (m_\Lambda = 1115.6 \text{ MeV}/c^2, m_\mu = 105.7 \text{ MeV}/c^2) \):
\[ E^{(2)}_{th} = \frac{m_p}{\eta'} = \frac{1}{2m'} \langle (m_\Lambda + m_\mu)^2 + m^2 - m_p^2 \rangle \]
\[ \approx 3.056 \times 10^{17} \text{ eV}, \]
\[ m' = m(\nu_\nu) = 0.48 \text{ eV}/c^2. \]

IV. NEUTRINO OSCILLATION AND A MINIMAL THREE-FLAVOR MODEL

As shown in the particle table published in 2000 [13], the mass square of electron neutrino defined by
\[ E^2 = p^2 + m^2(\nu_e)c^4 \]
seems negative in tritium beta decay:
\[ m^2(\nu_e) = -2.5 \pm 3.3 \text{ eV}^2 \]
(see also [3]). Due to difficulties in experiments and theoretical analysis [14], physicists often think the present data is not accurate enough to fix the value of \( m(\nu_e) \). However, the first reason why its uncertainty is so large lies in the fact that a neutrino is oscillating among three flavors as verified by the Kamiokande [15] and SNO [16] experimental groups. The oscillation implies that neutrinos are staying (at least) in two mass eigenstates. For instance, in a minimal three-flavor model for tachyonic neutrino [17], an equation containing only one parameter \( \delta \) is proposed (\( h = c = 1 \)):
\[ \begin{align*}
i\dot{\xi}_e & = i\vec{\sigma} \cdot \vec{\nabla} \xi_e - \delta(\eta_\mu + \eta_\tau) \\
i\dot{\eta}_e & = -i\vec{\sigma} \cdot \vec{\nabla} \eta_e + \delta(\xi_\mu + \xi_\tau) \\
i\dot{\xi}_\mu & = i\vec{\sigma} \cdot \vec{\nabla} \xi_\mu - \delta(\eta_\tau + \eta_\rho) \\
i\dot{\eta}_\mu & = -i\vec{\sigma} \cdot \vec{\nabla} \eta_\mu + \delta(\xi_\tau + \xi_\rho) \\
i\dot{\xi}_\tau & = i\vec{\sigma} \cdot \vec{\nabla} \xi_\tau - \delta(\eta_\mu + \eta_\rho) \\
i\dot{\eta}_\tau & = -i\vec{\sigma} \cdot \vec{\nabla} \eta_\tau + \delta(\xi_\mu + \xi_\rho)
\end{align*} \]
where $\xi_i (i = e, \mu, \tau)$ and $\eta_i$ are the left-handed and right-handed chiral states of flavor $i$ for a neutrino ($\sigma$ are Pauli matrices). The neutrino is oscillating among three mass eigenstates of energy square being:

$$E_j = p^2 - m_j^2, \quad (j = 1, 2, 3)$$  \hspace{1cm} (40)

$$m_1^2 = 4\delta^2, \quad m_2 = m_3^2 = \delta^2.$$  \hspace{1cm} (41)

In this model, however, different flavors all have the same mass. The fitting values of $\nu_e$ and $\nu_\mu$ from $^{38}$ and $^{39}$ seem to favor the predictions above. And the large uncertainty in $^{38}$ is primarily due to the existence of oscillation between two mass eigenvalues:

$$m_1 = 2\delta, \quad m_2 = m_3 = \delta.$$  \hspace{1cm} (42)

As discussed in $^{17}$, the expectation value of mass square of $\nu_e$ just created from beta decay should be:

$$m^2(\nu_e) = \frac{8}{5} \delta^2 + \frac{6}{5} \delta^2$$  \hspace{1cm} (43)

in comparison with $^{38}$. On the other hand, the two knees in CRS should be fitted by one expectation value of tachyon mass for a neutrino in flight:

$$\bar{m}(\nu_e) = \bar{m}(\nu_\mu) = \frac{3}{2} \delta + \frac{1}{2} \delta.$$  \hspace{1cm} (44)

Comparing it with the average of $^{38}$ and $^{39}$, 0.51 eV/c$^2$, we find the value of $\delta$ being approximately:

$$\delta = 0.34 \text{ eV}.$$  \hspace{1cm} (45)

However, we suggest that if experimental physicists can treat their data in a two-center fitting as shown by $^{41}$ or $^{42}$ in 4 : 1 or 2 : 1 ratio (with statistical weight ratio 1 : 4 or 1 : 1) for the case of $^{38}$ or $^{41}$ respectively, better results could be obtained.

V. SUMMARY AND DISCUSSION

(a) Following Kostelecky and Ehrlich, we elaborate a model of tachyonic neutrinos to explain two knees in the CRS by two Eqs. $^{29}$ and $^{35}$ respectively with one tachyon mass of neutrino ($\nu_e$ or $\nu_\mu$) being estimated to be around 0.51 eV/c$^2$ regardless of the flavor. This result favors a minimal three-flavor model for tachyonic neutrinos containing only one coupling parameter $\delta = 0.34 \text{ eV}$.

(b) The tachyon theory for neutrinos is a natural extension of the theory of special relativity (SR) in combination with the theory of quantum mechanics (QM). Especially, the basic operator relations $^{16}$ for particles should be supplemented by $^{17}$ for antiparticles while the addition law for velocities in SR, Eq. $^{9}$, remains valid for both subluminal and superluminal motions.

(c) We dare not discuss the explanation of the ankle in CRS before further calculation could be made in detail which might be related to the distribution of distant sources for both cosmic rays and neutrinos. However, one thing can supplement the interpretation of the evasion of GZK cutoff $^{2, 3, 4, 5, 6, 7}$. In the long chain of decays: $p \rightarrow n \rightarrow p \rightarrow n \rightarrow \cdots$ (or $p \rightarrow \Lambda \rightarrow p \rightarrow \Lambda \rightarrow \cdots$), the lifetime of $n$ ($\Lambda$) may be different for different polarizations: while the right-handed $n$ ($\Lambda$) has lifetime $\tau_r$, the left-handed one has $\tau_L$ $^{20, 21}$.

$$\tau_r = \frac{\tau}{1 - \beta^2}, \quad \tau_L = \frac{\tau}{1 + \beta},$$  \hspace{1cm} (46)

where $\tau = \tau_0/\sqrt{1 - \beta^2}$ ($\beta = v/c$). The faster the speed $v$ of $n$ ($\Lambda$) is, the larger $\tau_r$ will be. Therefore, we expect that most nucleons (fermions) in the cosmic ray should be right-handed polarized. Future experiments will pose a serious test on the mechanism of the knees, ankle and the evasion of GZK cutoff as well as the prediction of $^{40}$ — a phenomenon of parity violation in the beta decay.

(d) A particle is always impure in the sense of having two contradictory fields, $\varphi$ and $\chi$. They obey the basic symmetry of space-time inversion ($x \rightarrow -x, t \rightarrow -t$):

$$\varphi(-x, -t) \rightarrow \chi(x, t), \quad \chi(-x, -t) \rightarrow \varphi(x, t).$$  \hspace{1cm} (47)

The neutrino is no exception to this rule. Indeed, Eq. $^{39}$ with relations:

$$\varphi_i = \frac{1}{\sqrt{2}}(\xi_i + \eta_i), \quad \chi_i = \frac{1}{\sqrt{2}}(\xi_i - \eta_i)$$  \hspace{1cm} (48)

remains invariant under the transformation $^{41}$ with subscripts added. However, uniquely, neutrinos have another two symmetries: Eq. $^{39}$ is also invariant under the pure time inversion ($x \rightarrow x, t \rightarrow -t$):

$$\xi_i(x, -t) \rightarrow \eta_i(x, t), \quad \eta_i(x, -t) \rightarrow \xi_i(x, t).$$  \hspace{1cm} (49)

This is why Bob will see a neutrino (in the $S$ frame) transforming into an antineutrino in the $S'$ frame. On the other hand, the left-right symmetry (parity) is violated to maximum, since Eq. $^{39}$ is no longer invariant under the pure space inversion ($x \rightarrow -x, t \rightarrow t$):

$$\xi_i(-x, t) \rightarrow \eta_i(x, t), \quad \eta_i(-x, t) \rightarrow \xi_i(x, t).$$  \hspace{1cm} (50)

in contrast to the case of the Dirac equation. Interestingly enough, a massive neutrino (antineutrino) can preserve its permanent left-handed (right-handed) polarization because its velocity $u$ exceeds the speed of light $c$.

(e) The mass $m$ and energy $E$ of every particle or antiparticle (regardless of it being a tachyon or not) is real and positive in the strict sense that they are measured in certain experiments. However, for a theory capable of treating the particle and antiparticle on an equal footing, it must be invariant under symmetry transformation: $m \rightarrow -m$ $^{18}$. Hence, from the theoretical point of view, by using Eq. $^{10}$ only, we may say that the antiparticle state is the negative-energy state of a particle,
e.g., the wavefunction $\text{A.12}$ versus $\text{A.11}$ in the Appendix. There is an interesting question relevant to the theory of cosmology: Where the enormous energy of our universe (comprising mainly of matter) comes from? In other words, is the energy conserved in the bigbang? We try to answer the above question by assuming that during the bigbang, equal amounts of matter and antimatter were created simultaneously $\text{A.13}$. Hence, in some sense, we may regard the entire universe really as an ultimate free lunch $\text{A.14}$.

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**APPENDIX: A TACHYONIC NEUTRINO AS A MICROSCOPIC SCHröDINGER’S CAT**

Bob never observed in the $S'$ frame a neutrino moving backward in time (because “time” is a conception endowed by the observer, not by the particle) but an antineutrino flying toward him with high energy $E'_\nu > 0$ and momentum $p'_\nu = \beta'_\nu E'_\nu < 0$. So he writes down the conservation laws of energy and momentum in the form of $\text{A.10}$ and $\text{A.11}$, i.e.,

\[
E'_p + E'_e = E'_n + E'_\nu, \quad (\text{A.1})
\]

\[
p'_0 = p'_n + p'_e. \quad (\text{A.2})
\]

If Alice wishes to find corresponding laws in the $S$ frame, she first resorts to the LT $[\text{like (11)}]$ for the neutron and positron, yielding:

\[
E_n + E_e = \left(\frac{E'_n + E'_e}{\sqrt{1 - \beta^2}}\right) + \beta (p'_n + p'_e), \quad (\text{A.3})
\]

\[
p_n + p_e = \left(p'_n + p'_e\right) + \beta (E'_n + E'_e). \quad (\text{A.4})
\]

Substituting $\text{A.1}$ and $\text{A.2}$ into the right sides of $\text{A.3}$ and $\text{A.4}$ and noticing from $\text{28}$ that

\[
\frac{1}{\sqrt{\beta^2 - 1}} = \frac{\beta \sqrt{\beta^2 - 1}}{\sqrt{\beta^2 - 1} \sqrt{1 - \beta^2}} > 0, \quad (\text{A.5})
\]

\[
E'_0 = \frac{p_0 \beta - E'_\nu}{\sqrt{1 - \beta^2}}, \quad p'_0 = \frac{E_\nu p_\nu - p_\nu}{\sqrt{1 - \beta^2}} < 0. \quad (\text{A.6})
\]

Alice finds from $\text{A.3}$ and $\text{A.4}$ that ($E_\nu > 0$, $p_\nu = \beta_\nu E_\nu > 0$):

\[
E_n + E_e = E_p - E_\nu, \quad (\text{A.7})
\]

\[
p_n + p_e = p_p - p_\nu. \quad (\text{A.8})
\]

At first sight, $\text{A.7}$ and $\text{A.8}$. Alice insists on writing conservation laws in the $S$ frame as usual:

\[
E_n + E_e = E_p + E_\nu, \quad (\text{A.9})
\]

\[
p_n + p_e = p_p + p_\nu. \quad (\text{A.10})
\]

Then after measurements and calculation, Alice has to admit that both $E_\nu$ and $p_\nu$ here turn to negative values as shown by $\text{A.7}$ and $\text{A.8}$. When doing so, Alice is tacitly assuming that before the neutrino interacts with the proton, $E_\nu(> 0)$ and $p_\nu(> 0)$ are already existing. Hence she is worried that the energy (momentum) conservation law seems to be violated in the process $\text{A.13}$.

The above discussion reminds us of a remarkable experiment on SQUID $\text{22}$, showing a macroscopic Schrödinger’s cat puzzle—in a superconducting ring carrying clockwise current, the “hidden” antielectrical current was measured by the absorption of microwave radiation. (For its discussion, see $\text{22}$, also $\text{10}$).

Now a tachyonic neutrino is also a Schrödinger’s cat but on a microscopic scale. As shown in $\text{39}$, a plane wavefunction (WF) contains all six fields $\xi_i$ and $\eta_i (i = e, \mu, \tau)$:

\[
\xi_i \sim \eta_i \sim \exp\left[\frac{i}{\hbar}(p_\nu x - E_\nu t)\right], \quad (|\xi_i| > |\eta_i|) \quad (\text{A.11})
\]

and describes a neutrino with 100 % left-handed polarization. We may regard $\xi_i$ being the “alive-cat” state and $\eta_i$ the “dead-cat” state. The latter is in a subordinate status and so doesn’t appear as an explicit right-handed antineutrino ingredient. But once the neutrino is absorbed by a proton, the “hidden” $\eta_i$ is suddenly activated and dominates the $\xi_i$—together they show up as an antineutrino with 100 % right-handed polarization in the $S'$ frame as described by the WF :

\[
\eta'_i \sim \xi'_i \sim \exp\left[\frac{i}{\hbar}(p'_\nu x' - E'_\nu t')\right], \quad (|\eta'_i| > |\xi'_i|) \quad (\text{A.12})
\]

In our point of view, the momentum $p'_\nu$ and energy $E'_\nu$ do not exist until the antineutrino is absorbed by the proton. Similarly, before the neutrino is absorbed in the $S$ frame, the values of $p_\nu$ and $E_\nu$ also do not exist. We need not worry about conservation laws like $\text{A.9}$ and $\text{A.10}$ since all quantities written there are absent until created during the occurrence of process $\text{A.13}$. We should not interpret the quantum state and WF too materially. The WF is merely a probability amplitude of “fictitious measurement”, not real measurement.
A similar situation happened in another remarkable Which-Way (WW) experiment performed on an atom interferometer \[21, 22\]. Some physicists were worried that the uncertainty relation
\[\Delta p \Delta x \geq \frac{\hbar}{2}\]  \hspace{1cm} (A.13)
might be invalid for the momentum \(p\) and position \(x\) of the atom’s center-of-mass. But actually, \(p\) and \(x\) had not been measured in the motion of atom and so Eq. (A.13) has nothing to do with the WW experiment \[20\]. What is measured is the WW information of atomic internal states gained from the absorption of microwave pulses. In fact, the authors themselves already correctly wrote down a complementary relation for the distinguishability (of WW information) \(D\) and the fringe visibility \(V\) as \[25\]:
\[D^2 + V^2 \leq 1.\]  \hspace{1cm} (A.14)

For further discussion, see \[10\].

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