Cosmological implications of neutrinos
Subir Sarkar

Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK

Massive neutrinos were the first proposed, and remain the most natural, particle candidate for the dark matter. In the absence of firm laboratory evidence for neutrino mass, considerations of the formation of large scale structure in the universe provide a sensitive, albeit indirect, probe of this possibility. Observations of galaxy clustering and large angle anisotropy in the cosmic microwave background have been interpreted as requiring that neutrinos provide $\sim 20\%$ of the critical density. However the need for such 'hot' dark matter is removed if the primordial spectrum of density fluctuations is tilted below scale-invariance, as is often the case in physically realistic inflationary models. This question will be resolved by forthcoming precision measurements of microwave background anisotropy on small angular scales. This data will also improve the nucleosynthesis bound on the number of neutrino species and test whether decays of relic neutrinos could have ionized the intergalactic medium.

1. A BIT OF HISTORY

Several years before neutrinos had even been experimentally detected, Alpher et al. \cite{1} noted that they would have been in thermal equilibrium in the early universe "...through interactions with mesons" at temperatures above 5 MeV. Below this temperature the neutrinos "...freez-in and continue to expand and cool adiabatically as would a pure radiation gas". These authors also observed that the subsequent annihilation of $e^\pm$ pairs would heat the photons but not the decoupled neutrinos so by entropy conservation $T_\nu/T$ would decrease from its high temperature value of unity down to $(4/11)^{1/3}$ at $T \ll m_\nu$. \footnote{Thus the present density of massless relic neutrinos is}

$$\frac{n_\nu}{n_\gamma} = \left(\frac{T_\nu}{T}\right)^3 \frac{n_\nu}{n_\gamma}|_{T=T_{dec}} = \frac{4}{11} \left(\frac{3 g_\nu}{4 \pi}\right),$$

(1)

where $g_\nu = 2$ (left-handed neutrinos and right-handed antineutrinos) and the factor $3/4$ reflects Fermi versus Bose statistics. This would also be true for massive neutrinos if $m_\nu \ll T_{dec}$ so that the neutrinos are relativistic at decoupling. Thus for a present blackbody temperature $T_0 = 2.728 \pm 0.002$ K \footnote{The abundance per flavour of massless relic neutrinos is $\frac{n_\nu}{n_\gamma} = \left(\frac{T_\nu}{T}\right)^3 \frac{n_\nu}{n_\gamma}|_{T=T_{dec}} = \frac{4}{11} \left(\frac{3 g_\nu}{4 \pi}\right)$}, the abundance per flavour is $(3/11)\frac{1}{2\pi^2} = 1.13 \times 10^{-3}$ per cubic meter. While relativistic they retain a Fermi-Dirac distribution with phase-space density

$$f_\nu = \frac{g_\nu}{(2\pi)^3} \exp \left(\frac{p}{T_\nu}\right) + 1 \right)^{-1}.$$

(2)

Subsequently, Chiu and Morrison \cite{2} calculated the rate for $e^+e^- \rightarrow \nu_\mu\bar{\nu}_e$ in a plasma to be $\Gamma_\nu \approx G_F^2 T^5$ for the universal Fermi interaction and Zel’dovich \cite{3} equated this to the Hubble expansion rate in the radiation-dominated era,

$$H = \sqrt{\frac{8\pi G_N \rho}{3}}, \quad \text{with } \rho = \frac{\pi^2}{30} g^* T^4$$

(3)

(where $g^*$ counts the relativistic degrees of freedom), to obtain the decoupling temperature $T_{dec}(\nu_\mu) \approx 2$ MeV. (Neutral currents were then unknown so $T_{dec}(\nu_\mu)$ was estimated from the reaction $\mu \rightarrow e\nu_\mu\nu_\mu$ to be 12 MeV. Later De Graaf \cite{4} noted that they would keep $\nu_\mu$'s coupled to the plasma down to the same temperature as $\nu_e$'s \footnote{In fact $T_{dec}(\nu_\mu, \nu_e) \approx 3.5$ MeV while $T_{dec}(\nu_\mu) \approx 2.3$ MeV because of the additional charged current reaction \cite{5}. Actually decoupling is not an instantaneous process so the neutrinos are slightly heated by the subsequent $e^+e^-$ annihilation, increasing the number density \cite{6} by $\lesssim 1\%$ \footnote{In fact $T_{dec}(\nu_\mu, \nu_e) \approx 3.5$ MeV while $T_{dec}(\nu_\mu) \approx 2.3$ MeV because of the additional charged current reaction \cite{5}. Actually decoupling is not an instantaneous process so the neutrinos are slightly heated by the subsequent $e^+e^-$ annihilation, increasing the number density \cite{6} by $\lesssim 1\%$}}. However Zel’dovich \cite{3} and Chiu \cite{2} concluded that relic neutrinos, although nearly as numerous as the blackbody photons, cannot make an interesting contribution to the cosmological energy density since they are presumably massless.

Interestingly enough, some years earlier Pontecorvo and Smorodinski \cite{4} had discussed the bounds set on the cosmological energy density of MeV energy neutrinos (created e.g. by large-scale...}
matter-antimatter annihilation) using data from the Reines–Cowan and Davis experiments. (They even suggested searching for GeV energy neutrinos by looking for upward going muons in underground experiments!) Not surprisingly these bounds were rather weak so these authors stated somewhat prophetically that “...it is not possible to exclude a priori the possibility that the neutrino and antineutrino energy density in the Universe is comparable to or larger than the average energy density contained in the proton rest mass”. Zel’dovich and Smorodinski [11] noted that better bounds can be set by the limits on the total cosmological energy density \( \rho_0 \equiv \Omega_\rho \) following from the observed present expansion rate \( H_0 \) and age \( t_0 \) of the universe.\(^4\) Of course they were still discussing massless neutrinos. Weinberg [11] even speculated whether a degenerate sea of relic neutrinos can saturate the cosmological energy density bound and noted that such a sea may be detectable by searching for (scattering) events beyond the end-point of the Kurie plot in \( \beta \)-decay experiments!

Several years later, Gershte˘ in and Zel’dovich [12] made the connection that if relic neutrinos are massive, then a bound on the mass follows from requiring that \( m_\nu m_\bar{\nu} < \rho_0 \). Using the general relativistic constraint \( \Omega h^2 H_0^2 < (\pi / 2)^2 \), they derived \( \rho_0 < 2 \times 10^{-28} \text{gm cm}^{-3} \) (just assuming \( t_0 > 5 \text{ Gyr}, \) i.e. that the universe is older than the Earth) and inferred that \( m_{\nu_e}, m_{\bar{\nu}_e} < 400 \text{ eV} \) for a present photon temperature of 3 K. Unfortunately their calculation of the relic neutrino abundance was erroneous. They took \( q_\nu = 4 \), i.e. assumed massive neutrinos to be Dirac particles with fully populated right-handed (RH) states (although they acknowledged that according to the V − A theory such states are non-interacting and would thus not be in equilibrium at \( T_{\text{dec}} \)). Moreover they did not allow for the decrease in the neutrino temperature relative to photons due to \( e^+e^- \) annihilation. Nevertheless their bound was competitive with the best laboratory bound on \( m_{\nu_e} \) and \( 10^4 \) times better than that on \( m_{\nu_\mu} \), demonstrating the sensitivity (if not the precision!) of cosmological arguments.

A better bound of \( m_\nu < 130 \text{ eV} \) was quoted by Marx and Szalay [13] who numerically integrated the cosmological Friedmann equation from \( \nu_\mu \) decoupling down to the present epoch, subject to the condition \( t_0 > 4.5 \text{ Gyr} \). Independently Cowsk and McCleland [14] used direct limits on \( \Omega \) and \( h \) to obtain \( m_\nu < 8 \text{ eV} \), assuming that \( m_\nu = m_{\nu_e} = m_{\bar{\nu}_e} \); however they too assumed incorrectly that \( T_\nu = T \) and that RH states were fully populated. As Shapiro et al [15] first emphasized, even if massive neutrinos are Dirac rather than Majorana, the RH states have no gauge interactions so should have decoupled much earlier than the left-handed ones. Thus subsequent entropy generation by massive particle annihilations would have diluted their relic abundance to a negligible level.\(^4\) Now we arrive [17] at the modern version of the ‘Gershte˘ in-Zel’dovich bound’ [18]:

\[
\Omega_\nu h^2 \simeq \sum_i \left( \frac{m_{\nu_i}}{94 \text{ eV}} \right) \left( \frac{q_{\nu_i}}{2} \right) < 1. \quad (4)
\]

Note that \( t_0 = 2/3 H_0 \) for a critical density universe so \( t_0 > 10 \text{ Gyr} \) requires \( h \lesssim 2/3 \). For example a 30 eV neutrino would provide \( \Omega_\nu \approx 0.95 \) (allowing \( \Omega_N = 1 - \Omega_\nu \approx 0.05 \) in nucleons) if \( h \approx 0.55 \). According to a recent discussion [20], most determinations \( \Omega_\nu \) have converged on the value \( h = 0.6 \pm 0.1 \) corresponding to \( t_0 = 9.3 - 13 \text{ Gyr} \) for \( \Omega = 1 \), which is consistent with the recently revised age \( 22 \) of the oldest stars in globular clusters. Measurements of the global space-time geometry using Type I SN as ‘standard candles’

\(^3\)The critical density is \( \rho_c = 3 H_0^2 / 8 \pi G N \simeq 1.879 \times 10^{-29} h^2 \text{ gm cm}^{-3} \) where the Hubble parameter \( h \equiv H_0 / 100 \text{ km sec}^{-1} \text{ Mpc}^{-1} \), so \( H_0 = 9.778 h^{-1} \text{ Gyr} \).

\(^4\)Although spin-flip scattering (at a rate \( \propto (m_\nu / T)^2 \) can generate RH states, this can be neglected for \( m_\nu \ll 1 \text{ MeV} \). If RH neutrinos have new (superweak) interactions, as in the \( SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \) model, then corresponding bounds on their masses follow [14].

\(^5\)If the neutrinos are non-relativistic at decoupling, then they drop out of chemical equilibrium with an abundance inversely proportional to their self-annihilation cross-section so \( \Omega_\nu h^2 \approx (m_\nu / 2 \text{ GeV})^{-2} \) for \( m_\nu \ll m_Z \). Thus neutrinos with a mass of \( O(\text{GeV}) \) can also account for the dark matter; however LEP has ruled out such (4th generation) neutrinos. Conversely \( \Omega_\nu h^2 > 1 \) for the mass range \( \sim 100 \text{ eV} - 2 \text{ GeV} \), which is thus forbidden for any stable neutrino having only electroweak interactions.
are also consistent with a critical density universe \[ \Omega \approx 0.3 \] [23], although local dynamical measurements indicate a smaller value of \( \Omega \approx 0.7 \) [24]. This has led some cosmologists to consider an open universe while others introduce a cosmological constant (\( \Lambda = 1 - \Omega \approx 0.7 \)) to maintain a flat geometry, notwithstanding the extreme fine-tuning of initial conditions implied in either case.

The above bound assumes conservatively that neutrinos constitute all of the (dark) matter permitted by the global dynamics of the universe. Further constraints must be satisfied if they are to cluster on a specified scale (e.g. galactic halos or galaxy clusters) and provide the dark matter whose presence is inferred from dynamical measurements. Cowell and McClelland [25] first suggested that neutrinos with a mass of a few eV could naturally be the ‘missing mass’ in clusters of galaxies. This follows from the relation \( m_\nu \propto 1/G_N \rho(r_{cl})^{3} \) (reflecting the Pauli principle) which they obtained by modeling a cluster of mass \( M_{cl} \) as a square potential well of core radius \( r_{cl} \) filled with a Fermi-Dirac gas of neutrinos at zero temperature. Subsequently Tremaine and Gunn [26] noted that this provides a lower bound on the neutrino mass. Although the microscopic phase-space density is conserved for collisionless particles, the ‘coarse-grained’ phase-space density in bound objects can decrease below its maximum value of \( g_{\nu}/2(2\pi)^{3} \) during structure formation. Modeling the bound system as an isothermal sphere with velocity dispersion \( \sigma \) and core radius \( r_{cl}^{2} = 9\sigma^{2}/4\pi G_{N}\rho(r_{cl}) \) then gives

\[
m_\nu > 120 \text{ eV} \left( \frac{\sigma}{100 \text{ km sec}^{-1}} \right)^{-1/4} \left( \frac{r_{cl}}{\text{kpc}} \right)^{-1/2} .\tag{5}
\]

This is consistent with the cosmological upper bound \( \left[ \text{six}\right] \) down to the scale of galaxies. Moreover since neutrinos would cluster more efficiently in larger potential wells, there should be a trend of increasing mass-to-light ratio with scale, as seemed observational to be the case [29].

6 However, there is a conflict for smaller objects, viz. dwarf galaxies [27]. In fact the central phase space density of observed dark matter cores in these structures drops rapidly with increasing core radius, rather than being constant as would be expected for neutrinos [28].

7 This too proved to be a problem later when it was recog-

2. THE RISE AND FALL OF HOT DARK MATTER

Such cosmological arguments became of particular interest in the eighties after the ITEP tritium \( \beta \)-decay experiment claimed a \( \approx 30 \text{ eV} \) mass for the electron neutrino. The attention of cosmologists now focussed on how the large-scale structure (LSS) of galaxies, clusters and superclusters [31] would have formed if the universe is dominated by massive neutrinos. The basic picture is that structure grows through gravitational instability from primordial density perturbations [32]; these perturbations have now been detected [33] (see Figure 1) via the temperature fluctuations they induce [34] in the cosmic microwave background (CMB). On large scales ( \( > 30 \text{ Mpc} \)) the universe approaches spatial homogeneity and gravitational dynamics is linear, while on smaller scales structure formation is complicated by non-linear gravitational clustering as well as non-gravitational (gas dynamic) processes. So although we lack a standard model of galaxy formation [35], the physics of large-scale structure is sufficiently well understood as to provide a reliable probe of the nature of the dark matter [24].

Density perturbations in a medium composed of relativistic collisionless particles are subject to a form of Landau damping (viz. phase-mixing through free streaming of particles from high to low density regions) which effectively erases perturbations on scales smaller than the free-streaming length \( \approx 41 \text{ Mpc}(m_{\nu}/30 \text{ eV})^{-1} \) [36]. This is essentially the (comoving) distance traversed by a neutrino from the big bang until it becomes non-relativistic, and corresponds to the scale of superclusters of galaxies. Thus huge neutrino condensations (generically in the shape of ‘pancakes’), containing a mass \( \approx 3 \times 10^{15}(m_{\nu}/30 \text{ eV})^{-2} M_{\odot} \), would have begun growing at a redshift \( z_{\text{eq}} \approx 7 \times 10^{3}(m_{\nu}/30 \text{ eV}) \) when the universe becomes matter-dominated and gravitational instability sets in. This is well before (re)combination (at \( z_{\text{rec}} \approx 10^{3} \)) so the baryons were still closely coupled to the photons, while the neutrinos were mildly relativized that the actual increase is less than expected [30].
Figure 1. COBE [33] maps of the cosmic microwave background, showing the monopole, the dipole due to our motion relative to the frame in which the CMB is isotropic, and higher multipoles due to gravitational potential perturbations on the last scattering surface. The horizontal band is synchrotron emission from our Galaxy. (Courtesy of the COBE Science Working Group)

The gross features of such a ‘top-down’ model for structure formation are compatible with several observed features of LSS, in particular the distinctive ‘voids’ and ‘filaments’ seen in large galaxy surveys. It was also noted that since primordial density perturbations can begin growing earlier than in an purely baryonic universe, their initial amplitude must have been smaller, consistent with extant limits on the isotropy of the microwave background. Detailed studies [37] found however that galaxies form late through the breakup of the pancakes, at a redshift $z \lesssim 1$, counter to observations of galaxies, in particular quasars at $z > 4$. (Another way of saying this is that galaxies should have formed last in an HDM universe, whereas our Galaxy is in fact dynamically much older than the local group [38].) There are other difficulties such as too large ‘peculiar’ (non-Hubble) velocities [39], excessive X-ray emission from baryons which accrete onto neutrino clusters [40], and too large voids [41] (although detailed simulations [42] showed later that some of these problems had been exaggerated).

Therefore cosmologists soon abandoned HDM and turned, with considerably more success, to cold dark matter (CDM) [43], i.e. particles which were non-relativistic at the epoch of matter-domination. Detailed studies of CDM universes gave excellent agreement with observations of galaxy clustering [44] and even led to progress in the understanding of galaxy formation [30,35].

Thus a ‘standard CDM model’ for large-scale structure formation was established, viz. a critical density CDM dominated universe with an initially scale-invariant spectrum of density perturbations. Moreover particle physicists provided plausible candidate particles, notably the neutralino in supersymmetric models with conserved $R$-parity which naturally has a relic abundance of order the critical density [45].

3. COBE AND THE ADVENT OF MIXED DARK MATTER

Although the underlying physics is well known, cosmological structure formation is a complex subject and some implicit assumptions must necessarily be made in order to make progress. The key one concerns the nature of the primordial density perturbations. Cosmologists usually assume these to have a power spectrum of the scale-invariant ‘Harrison-Zel’dovich’ form:

$$P(k) = \langle |\delta_k|^2 \rangle = Ak^n, \quad \text{with } n = 1, \quad (6)$$

where $\delta_k \equiv \int \frac{\delta \rho(x)}{\rho} e^{-ik \cdot x} d^3x$ is the Fourier transform of spatial fluctuations in the density.
field (of wavelength $\lambda = 2\pi/k$). Moreover the perturbations are assumed to be gaussian (i.e. different phases in the plane-wave expansion are uncorrelated) and to be ‘adiabatic’ (i.e. matter and radiation fluctuate together). Concurrent with the above studies concerning the nature of the dark matter, powerful support for this conjecture came from the development of the ‘inflationary universe’ model \cite{18,46}. Here the perturbations arise from quantum fluctuations of a scalar field $\phi$, the vacuum energy of which drives a period of accelerated expansion in the early universe. The corresponding classical density perturbations have a spectrum determined by the ‘inflaton’ potential $V(\phi)$, with a power-law index \cite{47}:

$$n(k) = 1 - 3M^2 \left( \frac{V'}{V} \right)^2 + 2M^2 \left( \frac{V''}{V} \right) \star$$

(7)

where $M \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the normalized Plank mass and $\star$ denotes that this is to be evaluated when a mode of wavenumber $k$ crosses the ‘Hubble radius’ $H^{-1}$. Thus we see that for a sufficiently ‘flat potential’ (as is necessary to achieve sufficient e-folds of inflation to solve the problems of the standard cosmology), the spectrum indeed has $n \simeq 1$.

As mentioned earlier, gravitational instability only sets in when the universe becomes matter-dominated and this modifies the spectrum on length scales smaller than the Hubble radius at this epoch, viz. for $k > k_{\text{eq}} \simeq 80h^{-1}$ Mpc. Thus the characteristics of the dark matter can be encoded into a ‘transfer function’ $T(k)$ which modulates the primordial spectrum; for HDM this is an exponentially dropping function while for CDM it is a more gradual power-law. Now the power spectrum inferred from observations may be compared with theoretical models but another problem arises concerning how we are to normalize the amplitude of the primordial density perturbations, particularly since these are in the dark matter and may differ significantly (i.e. be ‘biased’) from the observable fluctuations in the density of visible galaxies. Fortunately, the primordial perturbations have another unique observational signature. As mentioned earlier, they induce temperature fluctuations in the CMB through the ‘Sachs-Wolfe effect’ (gravitational red/blue shifts) on large angular scales ($\gtrsim 2^\circ$), corresponding to spatial scales larger than the Hubble radius on the last scattering surface. It was the COBE detection \cite{33} of such fluctuations in 1992 that began the modern era of cosmological structure formation studies.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The matter power spectrum as inferred from LSS and CMB data compared with theoretical models \cite{48}. As seen top left, the excess small-scale power in the COBE-normalized standard CDM model ($n = 1, \Omega_N = 0.03, h=0.5$) is reduced in the MDM model which has $\Omega_\nu = 0.3$. This can also be achieved in a CDM model with a tilted spectrum ($n = 0.9$) and higher nucleon density ($\Omega_N = 0.1, h = 0.45$) as shown top right. The other panels show the expectations in an open universe (middle) and in a flat universe with a cosmological constant (bottom).}
\end{figure}

\footnote{Such fluctuations cannot have a causal origin in the standard cosmological evolution and imply that the mechanism which created them must have operated during a period of superluminal expansion in the past, viz. inflation!}
The quadrupole anisotropy in the CMB measured by COBE [33] allows a determination of the fluctuation amplitude at the scale, \(H_0^{-1} \simeq 3000 \, h^{-1} \text{Mpc}\), corresponding roughly to the present ‘size’ of the universe. With this normalization it became clear that a \(\Omega_\nu \simeq 1\) HDM universe indeed had too little power on small-scales for adequate galaxy formation. However it also became apparent (see Figure 2) that the ‘standard CDM model’ when normalized to COBE had too much power on small-scales! It was thus a logical step to invoke a suitable mixture of CDM and HDM to match the power spectrum to the data on galaxy clustering and motions [51].

Figure 3. The expected scale-dependence of mass fluctuations in supersymmetric inflationary MDM models compared with observations [54].

In fact the possibility that the dark matter may have both a hot and a cold component had been discussed several years earlier, motivated by theoretical considerations of SUSY GUTs, and the consequent advantages for large-scale structure noted [52]. In the post-COBRA era, a number of detailed studies of mixed dark matter (MDM or CHDM) universes have been performed and a neutrino fraction of about 20% found to give the best match with observations [53]. In Figure 3 we show a fit to observational data for a MDM model which also incorporates a (globally) supersymmetric mechanism for inflation [54]. The implied neutrino mass is about 5 eV and the expectation (in the ‘see-saw’ model for neutrino masses) is that this is the \(\nu_\tau\). Therefore CHORUS/NOMAD will provide a test of this possibility (assuming that the CKM mixing with the lighter neutrinos is of the same order as in the quark sector). More baroque schemes in which two neutrinos have comparable masses may be constructed (e.g. \(m_{\nu_\mu} \approx m_{\nu_\tau} \approx 2.5 \text{eV}\)) if one wishes to reconcile the LSND report of neutrino oscillations with the indications of atmospheric neutrino oscillations [55]. In this case, both KAR-MEN and the forthcoming long-baseline experiments will provide crucial tests.

However another way to reconcile a CDM universe with the small-scale observations is to relax the underlying assumption that the primordial spectrum is strictly scale-invariant. As shown in Figure 2, a mildly ‘tilted’ spectrum with index \(n \approx 0.9\) also gives a good fit to the data [56]. At first sight this might strike one as simply introducing an additional parameter (although this is arguably no worse than introducing an additional form of dark matter). However one should really ask why the spectrum should be assumed to be exactly scale-invariant in the first place!

As we saw earlier, the spectral index is determined by the slope and curvature of the scalar potential at the epoch when the fluctuation on a specified scale crosses the Hubble radius. The corresponding number of e-folds before the end of inflation is just

\[
N_e(k) \simeq 51 + \ln \left(\frac{k^{-1}}{3000 h^{-1} \text{Mpc}}\right)
\]

(for typical choices of the inflationary scale, reheat temperature etc). We see that fluctuations

\footnote{To save HDM would require new sources of small-scale fluctuations, e.g. relic topological defects [49], or isocurvature primordial perturbations [41].}
on the scales probed by LSS and CMB observations (\(\sim 1 - 3000 \text{ Mpc}\)) are generated just 40 - 50 e-folds before the end of inflation. It would be not unnatural to expect the inflaton potential to begin curving significantly as the end of inflation is approached (e.g. in ‘new inflation’ models). There are certainly attractive models of inflation in which the spectrum is significantly tilted in this region \[57\]. In a successful inflationary model based on \(N \equiv 1\) supergravity \[58\], the spectral index is simply given by \(n(k) \simeq (N_* - 2)/(N_* + 2)\) so is naturally \(\approx 0.9\) at these scales. In Figure 4 we compare the power spectrum for such a tilted spectrum in a CDM universe (TCDM) with data from the APM galaxy survey. It is seen that even the effects of non-linear evolution (which generate a ‘shoulder’ in the power spectrum) at small scales can be successfully reproduced. Indeed even MDM models \[53\] now allow for the possibility that the primordial spectrum may be tilted in order to achieve better fits to the data. Figure 5 indicates schematically how the HDM fraction required decreases as the tilt is increased \[59\]. There are certainly differences in detail between MDM and TCDM models and new observational constraints, e.g. the epoch of quasar formation or the abundance of primordial Lyman-\(\alpha\) clouds, may serve to distinguish between them. \[20\] However a more powerful and unambiguous discriminator is provided by the angular power spectrum of CMB anisotropy.

\[ T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_l^m Y_l^m(\theta, \phi), \]  

where the \(l^{th}\) multipole corresponds to an angle \(\approx 200/\theta\) and probes spatial scales around \(k^{-1} \approx 6000h^{-1} \text{ Mpc}\). In inflationary theories, the fluctuations are gaussian so the coefficients \(a_l^m\) are independent stochastic variables with zero mean and variance \(C_l = \langle |a_l^m|^2 \rangle\); each \(C_l\) has a \(\chi^2\) distribution with \((2l + 1)\) degrees of freedom

\[11\] Such studies have not however allowed for the possible scale-dependence of the tilt, as is crucial in this context.
For an assumed set of cosmological parameters \((\Omega, H_0, \Omega_N)\) and given the primordial density perturbation spectrum, the \(C_l\)'s can be determined by solution of the Einstein-Boltzmann equations which describe how the different components (photons, ions, electrons, dark matter particles . . . ) evolve \[61\]. Thus theoretical estimates of the power \((2l+1)C_l/2\pi\) at each multipole can be compared with observations. The low multipoles (large spatial scales) are sensitive to the primordial spectrum alone,\[12\] but the COBE measurement (of the first \(\sim 20\) multipoles) finds \(n = 1.2 \pm 0.3\) \[33\] so cannot discriminate between a scale-invariant and a mildly tilted spectrum. However forthcoming measurements with angular resolution far superior to COBE’s \(\sim 70\) will measure the power at higher multipoles. The dominant features in the power spectrum here are the ‘acoustic peaks’, the most prominent at \(l \approx 200\), arising from oscillations of the coupled plasma-photon fluids at last scattering \[34\].

As seen in Figure 6, the expectations for CMB anisotropy in a MDM universe do not differ significantly from a CDM universe having the same initial perturbation spectrum. However if the primordial spectrum is tilted, there is a significant suppression of the acoustic peaks at high multipoles (see Figure 7). Whereas present observations of small-angle anisotropy have a large scatter \[53\], forthcoming measurements, in particular by the MAP \[55\] and PLANCK \[56\] satellite missions, will enable a definitive test. These observations will also determine all the cosmological parameters \((\Omega, H_0, \Omega_N)\) to an accuracy of a few percent \[67\], opening up a new era in cosmology.

4. UNSTABLE NEUTRINOS?

The cosmological bound \[4\] is respected by the electron neutrino for which the Particle Data Group \[54\] now quotes \(m_{\nu_e} < 15\) eV. Kinematic bounds on the masses of the other neutrinos are much weaker \((m_{\nu_\mu} < 170\) keV, \(m_{\nu_\tau} < 24\) MeV) so in principle they may have masses in the cosmologically forbidden range.

\[12\] In principle, primordial gravitational waves can also make a contribution here but this is expected to be negligible in realistic inflationary models \[52\].

There have been various suggestions (many motivated by the now-withdrawn 17 keV neutrino discovery) that neutrinos may have new interactions which enable them to decay or annihilate sufficiently rapidly such that their relic abundance is reduced below the cosmological limit. In general this can be ruled out if the decays (or annihilations) create ‘visible’ Standard Model particles, e.g. \(\nu' \rightarrow \nu\gamma\), \(\nu' \rightarrow \nu_e e^+ e^-\) \[68\]. These processes have not been seen in laboratory experiments \[69\] (which probe short lifetimes), and would have affected cosmological observables such as light element abundances \[70\] or the radiation backgrounds \[18\] (which are sensitive to long lifetimes). The only possibility is to have such decays (or annihilations) create hypothetical ‘invisible’ particles, e.g. Majorons (Goldstone bosons associated with lepton number violation) \[71\]. These are fertile grounds for speculation, as there are often no experimental constraints (by construction!) on such hypotheses.
Many such proposals which *can* be experimentally tested have already been falsified. For example, if tau neutrinos have a large magnetic moment ($\sim 10^{-6} \mu_B$) their self-annihilations are sufficiently boosted through $\gamma$ exchange that $\nu_{\tau}$'s with mass of $O(\text{MeV})$ may constitute the dark matter [72]. However this was ruled out from the absence of anomalous $\nu_{\tau}$ interactions in the BEBC beam dump experiment [73]. Another suggestion was that flavour-changing neutral currents may allow the decay $\nu' \rightarrow \nu \nu \bar{\nu}$ to be sufficiently fast [74]. However the resultant breaking of the GIM mechanism implies that the branching fraction of $\nu' \rightarrow \nu \gamma$ is smaller only by a factor of $O(\alpha)$ and this is observationally ruled out [74]. Majoron models for neutrino mass in which neutrinos may annihilate sufficiently rapidly through Higgs exchange are ruled out by the LEP measurement of the ‘invisible’ width of the $Z^0$; only rather contrived singlet-Majoron models survive and even these are severely constrained by the non-observation of spectral features due to Majoron emission in neutrinoless $\beta \beta$ decay [88]. So although exotic decays of e.g. the $\nu_{\tau}$ into singlet Majorons, cannot be definitively excluded, it seems unlikely that it can thus evade the cosmological bound and have a mass in the MeV range.

For a neutrino mass subject to the cosmological bound [4], limits on the UV radiation background require the lifetime for $\nu' \rightarrow \nu \gamma$ to be far longer than the age of the universe. Apart from large-scale structure this is another cosmological context where there may be an observational signal for hot dark matter. Sciama [76] has argued that much of the ionized hydrogen in both our Galaxy and in the intergalactic medium cannot be accounted for in terms of conventional sources of UV photons (of energy $\geq 13.6 \text{ eV}$), e.g. hot stars, supernovae or quasars. He proposes that all such observations may be consistently understood if the universe has $\Omega_\nu \approx 1$ in neutrinos of mass $27.4 \pm 0.2 \text{ eV}$ decaying radiatively with a lifetime of $\sim 2 \pm 1 \times 10^{23} \text{ sec}$. (Such a lifetime is smaller than expected in most extensions of the Standard Model [74] but *can* arise in SUSY models with broken $R$-parity [7].) Again a decisive test of this theory is provided by CMB observations. Following (re)combination the universe will soon be reionized again due to the decaying neutrinos, thus washing out the CMB anisotropy on small angular scales [78]. As seen in Figure 8, the acoustic peaks in the power spectrum are thus severely damped [83], a prediction that is already being tested by ongoing experiments.
5. THE BBN LIMIT ON $N_\nu$

Hoyle and Taylor [80] as well as Peebles [81] had emphasized many years ago that new types of neutrinos (beyond the $\nu_e$ and $\nu_\mu$ then known) would boost the relativistic energy density hence the expansion rate (3) during big bang nucleosynthesis (BBN), thus increasing the yield of $^4$He. Shvartsman [82] noted that new superweakly interacting particles would have a similar effect. Subsequently this argument was refined quantitatively by Steigman, Schramm and collaborators [83]. In the pre-LEP era when the laboratory bound on the number of neutrino species was not very restrictive [84], the BBN constraint already indicated that at most one new family was allowed [85], albeit with rather uncertain systematics [86]. Although LEP now finds $N_\nu = 2.991 \pm 0.016$ [64], the cosmological bound is still important since it is sensitive to any new light particle, not just $SU(2)_L$ doublet neutrinos, so is a particularly valuable probe of new physics. (The energy density of new light fermions $i$ is equivalent to an effective number $\Delta N_\nu = \sum_i (g_i/2)(T_i/T_\nu)^4$ of additional doublet neutrinos, where $T_i/T_\nu$ follows from considerations of their (earlier) decoupling.)

The primordial mass fraction $Y_p(^4\text{He})$ increases as $\approx 0.012\Delta N_\nu$ but it also increases logarithmically with the nucleon density (usually parameterized as $\eta = n_N/n_\gamma = 2.728 \times 10^{-8} \Omega_N h^2$). Thus to obtain a bound on $N_\nu$ requires an upper limit on $Y_p$ and a lower limit on $\eta$. The latter is poorly determined from direct observations of luminous matter so must be derived from the abundances of the other synthesized light elements, D, $^3$He and $^7$Li, which are power-law functions of $\eta$. The complication is that these abundances are substantially altered in a non-trivial manner during the chemical evolution of the galaxy, unlike $Y_p(^4\text{He})$ which just increases by a few percent due to stellar production. (This can be tagged via the correlated abundance of oxygen and nitrogen which are made only in stars.)

Even so, some cosmologists have used chemical evolution arguments to limit the primordial abundances of D and $^3$He and thus derived increasingly severe bounds on $N_\nu$ [83] culminating in a recent one below 3 [88]. However a more conservative view [89] is that there is no crisis with BBN if we recognize that such arguments are rather dubious and consider only direct measurements [90] of light element abundances, as shown in Figure 9. The $^4$He mass fraction is obtained from observations of metal-poor blue compact galaxies by linear extrapolation to zero nitrogen/oxygen abundance [91]; the upper limit is reliable, the lower one less so. At present there are two conflicting measurements of the D abundance in quasar absorption systems [92,93]; the higher value [92] is interpreted as an upper limit. Also shown is the abundance in the interstellar medium [94] which provides a reliable lower limit. The $^7$Li abundance as measured in the hottest, most metal-poor halo stars [95] as well as in disk stars [96] is shown and interpreted as providing, respectively, reliable lower and upper limits on
its primordial value. Given these uncertainties, standard BBN is consistent with observations for \( \eta \approx 2 - 9 \times 10^{-10} \). Adopting the reliable limits, \( Y_p(^4\text{He}) < 0.25 \), \( D/H > 1.1 \times 10^{-5} \) and \( ^7\text{Li}/H < 1.5 \times 10^{-9} \), and taking into account uncertainties in nuclear cross-sections and the neutron lifetime by Monte Carlo, we obtain

\[
N^\text{max}_\nu = 3.75 + 78 \left( Y_p^\text{max} - 0.24 \right),
\]

i.e. up to 1.5 additional (equivalent) neutrino species are allowed for \( \eta \) at its lowest allowed value. Other workers have applied Bayesian likelihood methods to their adopted abundances (not limits as above) to obtain \( N_\nu < 4 - 5 \) [87]. It is clear that the restrictions on new physics are less severe than had been reported previously [57].

6. CONCLUSIONS

Neutrino (hot) dark matter (with \( \Omega_\nu \sim 0.2 \)) is consistent with but not required by our present understanding of large-scale structure. Forthcoming CMB anisotropy measurements (in particular by MAP and PLANCK) will resolve this question, as well as provide precision determinations of cosmological parameters such as \( \Omega \) and \( H_0 \). The thermal history since (re)combination will also be determined, enabling a test of the decaying neutrino theory.

Big bang nucleosynthesis permits at least one new neutrino, for example a right-handed singlet neutrino which mixes with the left-handed doublets. Again, CMB anisotropy observations will provide an independent precise measurement of the nucleon density parameter \( \Omega_N \), so this bound will become a measurement.

As I have tried to indicate, the relationship between neutrinos and cosmology has had a long history but with no definitive resolution as yet. So far, intriguing hints from laboratory experiments have largely driven the quest for cosmological consequences. With the renaissance of observational cosmology, in particular studies of the cosmic microwave background, the tables may well be turned in future.

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