Abstract This paper presents a method to calculate vertical overlap probability based on the position error caused by pilot control. Firstly, a fuzzy correlation model is established and combined with Quick Access Recorder (QAR) data. The correlation rules between QAR parameters and vertical position error are mined, and the relevance is calculated according to the rules. The results show that the correlation between the control column position and the vertical position error is the strongest. According to the flight dynamics principle, the Vertical Attitude Adjustment Model (V AAM) of an aircraft during the approach process is then established. The Pilot Operating Model (POM), which describes different actions taken by pilots when they encounter different position errors, is also created by employing the concepts of the Markov decision process. The V AAM can restore the process of changing the attitude of the aircraft by the pilot operating column and can simulate the position error of the aircraft in the approach process when combined with the POM. Based on the previous simulated result and considering the overlap condition of the Interval Management (IM) aircraft overlap, the vertical overlap process in the paired approach can then be simulated. We employ MATLAB to carry out numerous simulations. Changing the accuracy of pilot operation in the simulations show that the lower the accuracy, the greater the overlap probability and frequency. However, when the accuracy is between 61% and 73% (61%–73% is the range of operational accuracy based on QAR data analysis), the vertical overlap probability and frequency are maintained at about 5% and 20 flights per hour, respectively.

Index Terms Air traffic management, closely spaced parallel runways, fuzzy correlation model, paired approach, QAR, pilot operation.

I. INTRODUCTION

The concept of the paired approach was first presented at a workshop held at NASA Langley Research Center in 1996 [1] and can effectively improve approach efficiency. Four years later, Hammer [2] proposed a 3° offset approach program, which added the Interval Management (IM) aircraft approach offset angle into the process and improved the operational feasibility. Both the straight-line and the 3° offset approaches have strict requirements for the dynamic position of the IM aircraft, not only to ensure a safe distance from the target aircraft but also to avoid the wake generated by the target aircraft. As both conditions will result in operational risks that are different from the traditional approach, many scholars have studied the safety of closely spaced parallel runway paired approaches. Existing research results of paired approach safety can be divided into two categories: the study of wake safety interval and the study of collision risk.

In 2005, Teo et al. [3] proposed a real-time dangerous area calculation method to ensure the safety of the paired approach, which could warn an aircraft to take evasive action when a wrong approach occurred, thus avoiding collision. Madden [4] analyzed the effect of two aircraft’ speed on safety from the point of view of kinematics and obtained the longitudinal safety interval between the paired aircraft. Guerreiro et al. [5], [6] used the Monte-Carlo method to...
perform preliminary characterization of the wake safety zone. In a follow-up study, a simulation experiment of wake movement in the horizontal plane was carried out to evaluate the safety interval. Lu et al. [7], [8] evaluated the lateral and longitudinal collision risk of the paired approach by analyzing the positioning error distribution and the operation process of the paired approach. In their research on wake collision using the paired approach, Geyer et al. [9] established a system error model based on Regional Navigation (RNAV) and Global Positioning System (GPS) navigation. The model added lateral navigation and flight technology errors, which had not been included in previous research. The research results of Torrespomales et al. [10] showed that when using the paired approach, assigning different glide paths to two aircraft could improve the safety of operations. Lv et al. [11] combined acceleration error and navigation error distribution to establish a collision model and calculated the longitudinal collision risk of the paired approach. Mori and Fujita [12] proposes mathematical modifications to the calculation of shadowing to address the risk overestimation to estimate collision probability to ground obstacles during Instrument Landing System (ILS) approach. Priess [13] calculated the safety interval of a paired approach aircraft based on Automatic Dependent Surveillance-Broadcast (ADS-B) data, and Gu et al. [14] established a kinematic model to compare and analyze the impact of changes in the initial interval and other related parameters on the collision risk. Williams et al. [15] used the fast-time simulation method to analyze the collision risk during the paired approach when yawing of the IM aircraft occurred. The analysis results suggested that the speed difference between two aircraft and the distance between the runway centerline and runway approach alignment affected the collision risk. Lu et al. [16], [17] established the moment balance equation under the premise that the aircraft could withstand a certain induced rolling moment, and a collision model was established based on the error distribution to evaluate the safety interval between the target aircraft and the IM aircraft. This concept increased the safety zone of the paired approach.

The paired approach requires the rear aircraft to stay ahead of the front aircraft’s wake to avoid it. Therefore, if two aircraft come into contact by airframe or the rear aircraft enters the wake of the front aircraft, it is regarded as a collision. When implementing the paired approach procedure, the longitudinal spacing between two aircraft must comply with the following requirements:

1. Avoid airframe contact: the rear aircraft must be kept far enough away from the front aircraft to prevent collision;
2. Avoid wake contact: the rear aircraft shall keep close enough to the front aircraft to avoid the wake of the front aircraft.

Because the assessment of collision risk can be regarded as the premise of calculating the safety interval, an accurate calculation method is needed to evaluate the collision risk. Collision risk is usually calculated from three dimensions [18], [20], the basic parameters of which are vertical, longitudinal, and lateral overlap probability and frequency. The overlap in the vertical dimension is caused by the vertical position error. The aircraft that deviates from the glide slope to a certain extent may collide with the paired aircraft in the vertical direction. The conflict process is shown in Figure 1.

Previous research has tended to analyze the position error directly from the point of the error distribution without considering the effect of pilot operation.

In view of this, this paper employs pilot operation process and operation process recording Quick Access Recorder (QAR) data to determine the operating parameters that have the greatest correlation with the vertical position error (glide deviation in the QAR parameters) during an approach. The Vertical Attitude Adjustment Model (VAAM) is established according to the parameters, and the Pilot Operating Model (POM) is established using the concept of the Markov decision process based on the parameter statistics. Finally, according to the motion characteristics of the wake and the vertical overlap condition, a simulation model of paired approach vertical overlap is obtained. By running the simulation model, the vertical overlap time and overlap times are counted, and then the overlap probability and frequency are calculated. The technical roadmap for this article is shown in Figure 2. The differences between the research process of this paper and the general methods are shown in Table. 1.

In this paper, the influence of 20 related factors on vertical error was comprehensively analyzed using the fuzzy correlation method to find the most relevant parameters, and the internal mechanism of the vertical error was analyzed. Moreover, on the basis of considering human factors, a pilot
TABLE 1. Innovation of research methods.

| Perspective          | Other methods                                                                 | The proposed method                                                                 |
|----------------------|-------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| Operation error      | It is assumed that it is subject to normal distribution or double exponential distribution, but the actual aircraft position error statistics are not fully considered. | From the viewpoint of actual data and pilot’s operation, the operation model is established considering the pilot’s operation error, and the overlap probability and operation error are calculated accordingly. |
| Influencing factor   | Most studies default to and mainly consider the influence of a major error factor or generally attribute it to human factors. There is no systematic analysis of the influencing factors of aircraft position error, nor analysis of the impact mechanism between these influencing factors on collision risk. | In this paper, the influence of all factors on vertical overlap is systematically analyzed with the fuzzy association model, and the factors with the greatest influence are extracted. |
| Simulation content   | In terms of verification, usually only an example is taken to evaluate the collision risk of paired approach. | Based on the pilot’s operation model, this paper simulates the change of the glide deviation of the final approach, which is more convincing than the calculation example. |

II. CORRELATION ANALYSIS OF QAR PARAMETERS

The QAR parameters related to the errors can be identified by analyzing the correction process of the vertical position errors in the paired approach segment. A fuzzy correlation model is then established to mine the correlation rules between the QAR parameters and position operation model was established to simulate and analyze the glide deviation during flight, we count and calculate the frequency of vertical overlap, provide theoretical and research basis for vertical collision risk assessment of paired approach.

FIGURE 2. Research process.

errors, and the correlation rules are used to quantify the correlation.

A. PARAMETER IDENTIFICATION

In the approach segment, the direct cause of the vertical overlap of the paired aircraft is the vertical position error of the target aircraft and IM aircraft; that is, the vertical position deviation from the nominal track. While the position error is caused by pilot operation, it is also influenced by navigation equipment and the environment. A QAR can record the operation state of every system completely, accurately recording
The data of pilot operation, navigation, and atmosphere environment. The factors affecting the vertical position error of the aircraft can be obtained from the vertical position error correction point of the aircraft in the approach segment and the flight data recorded by QAR. The process of error generation and correction is shown in Figure 3.

The parameters associated with the above process are identified by combining the QAR data. The summarized parameters are shown in Table 2.

| Serial number | QAR parameter         | Serial number | QAR parameter         |
|---------------|-----------------------|---------------|-----------------------|
| A1            | Radio altitude        | A11           | Glide deviation       |
| A2            | angle of attack       | A12           | Vertical load         |
| A3            | control column position| A13           | Air speed             |
| A4            | Pitch trim            | A14           | Ground speed          |
| A5            | Left angle of attack  | A15           | Left engine N1        |
| A6            | Right angle of attack | A16           | Right engine N1       |
| A7            | Left elevator         | A17           | Left throttle         |
| A8            | Right elevator        | A18           | Right Throttle        |
| A9            | Roll angle            | A19           | Wind direction        |
| A10           | Decline rate          | A20           | Wind speed            |

FCM [21] gives weight to the data and the clusters, which indicates the degree to which the data belongs to a cluster. The specific algorithm of FCM is:

1. Input the dataset \( X = [x_i] \) and initialize the membership matrix \( U^{(0)} \)

\[
U^{(0)} = \begin{bmatrix}
    u_{11}^{(0)} & u_{12}^{(0)} & \cdots & u_{1q}^{(0)} \\
    u_{21}^{(0)} & \cdots & \cdots & \cdots \\
    \vdots & \vdots & \ddots & \vdots \\
    u_{p1}^{(0)} & \cdots & \cdots & u_{pq}^{(0)}
\end{bmatrix}
\] (1)

In the formula, \( u_{ij}^{(0)} \) represents the initial membership degree of data \( x_i \) belongs to class \( j \), \( i = 1, 2, \ldots, p \), \( j = 1, 2, \ldots, q \), \( p \) is the number of data, and \( q \) is the number of clusters.

2. Compute the Cluster Center \( c_j^{(k)} \), where

\[
c_j^{(k)} = \frac{\sum_{i=1}^{p} \left( u_{ij}^{(k)} \right)^m x_i}{\sum_{i=1}^{p} \left( u_{ij}^{(k)} \right)^m}
\] (2)

here, \( k \) represents the \( k \)-th iteration, and \( m \) is the weighted index of the membership matrix.

3. The iteration of the membership matrix \( U^{(k)} \) and \( U^{(k+1)} \) is

\[
u_{ij}^{(k+1)} = \frac{1}{q} \sum_{s=1}^{q} \left( \frac{\left\| x_i - c_j^{(k)} \right\|}{\left\| x_i - c_g^{(k)} \right\|} \right)^{2(m-1)}
\] (3)

where \( U^{(k)} \) is the membership matrix after the \( k \)-th iteration, and \( \| \cdot \| \) is the measure of distance.

4. Iterative judgment. If \( \| U^{(k+1)} - U^{(k)} \| < \varepsilon \), the iteration ends; otherwise, the result will be substituted back into step 2, where \( \varepsilon \) is the error threshold.

B. FUZZY CORRELATION MODELS

The model first uses FCM to cluster the identified QAR parameters fuzzily and then inputs the fuzzy clustering results into the Apriori algorithm. The model uses the membership matrix of the clustering results to calculate the support and confidence of the item set, mine fuzzy frequent item sets, and output correlation rules between parameters.
The QAR parameter dataset is recorded as $B = [b_{x,y}]_{N \times n}$, where $b_{x,y}$ is the value of the $x$-th sample on the attribute $z_1, z_2, \ldots, z_n$; the fuzzy membership degree is represented by $u_{x,y} = [R_{x,1}, R_{x,2}, \ldots, R_{x,l}]$; $R$ is the membership function; $l$ is the number of membership functions. Dataset $B$ can then be transformed into fuzzy dataset $C$, $C = \{a_1, a_2, \ldots, a_N\}$, $a_x = [u_{x,1}, u_{x,2}, \ldots, u_{x,n}]$, $(x = 1, 2, \ldots, N)$. A fuzzy term is defined as $\langle z_r \mid R_{r,w} \rangle$, and a set of fuzzy terms [22] is defined as

$$\langle Z \mid R \rangle = \left\{ \langle z_{r_1} \mid R_{r_1,w} \rangle \cup \langle z_{r_2} \mid R_{r_2,w} \rangle \cup \cdots \cup \langle z_{r_\lambda} \mid R_{r_\lambda,w} \rangle \right\},$$

$$\lambda \leq n, \quad w \leq l \quad (5)$$

The support of the fuzzy term sets $Sup(\langle Z \mid R \rangle)$ is

$$Sup(\langle Z \mid R \rangle) = \sum_{x=1}^{N} \prod_{\{z_r \mid R_{r,w}\} \in \langle Z \mid R \rangle} a_x(z_r) / N \quad (6)$$

In the formula, $a_x(z_r)$ is the corresponding membership degree of $x$-th data on the attribute $z_r$. If $Sup(\langle Z \mid R \rangle)$ is greater than the minimum set of support, $\langle Z \mid R \rangle$ is called the fuzzy frequent item set.

The support for correlation rules is

$$Sup(\langle Z \mid R \rangle \Rightarrow \langle Q \mid R \rangle) = Sup(\langle Z \mid R \rangle \cup \langle Q \mid R \rangle) \quad (7)$$

The confidence of correlation rules is

$$Conf(\langle Z \mid R \rangle \Rightarrow \langle Q \mid R \rangle) = \frac{Sup(\langle Z \mid R \rangle \cup \langle Q \mid R \rangle)}{Sup(\langle Z \mid R \rangle)} \quad (8)$$

The support indicates the importance of the rules, and the confidence indicates the reliability of the rules. When the support and the confidence of the correlation rules satisfy both the minimum set of support and confidence, the correlation rules are called strong rules.

The process of mining correlation rules using the Apriori algorithm [9] is shown in Figure 4. Firstly, the frequency of occurrence of a fuzzy item set $C_1$ is counted, the support is calculated, the fuzzy item sets smaller than the minimum set of support are deleted, and a frequent fuzzy set is obtained as $L_1$. The next operation is the connection, where $L_1$ and $L_1$ are connected to obtain the candidate two fuzzy sets $C_2$. The candidate fuzzy sets with subsets that are infrequent items in $C_2$ are deleted. If the result of the deleted is an empty set, then $L_1$ is the largest fuzzy frequent item set; otherwise, the frequent fuzzy set $L_2$ with two items is derived. The above steps are looped until the result is an empty set. Finally, the confidence of the correlation rule is calculated by Equation (8) from the fuzzy frequent set.

The correlation between parameters can be quantified by Equation (9), as shown at the bottom of the next page, with the help of the support and confidence correlation rules, where $q$ is the number of clusters.

The QAR data of the same B737-800 aircraft landing at the same airport within a week was selected as the research data set. The approach segment studied was from the time the aircraft intercepted the glide path to the landing gear touching the ground, and a total of 1694 data sets were obtained. According to the 20 QAR parameters related to vertical position errors identified in the previous paper, the FCM algorithm was used to operate fuzzy pre-processing.
The number of clusters was 3, which corresponded to small, medium, and large fuzzy terms, respectively; the weighted index of the membership matrix \( m \) was set to 2. The clustering results are shown in Table 3.

The clustering results were input into the Apriori algorithm, and the confidence was set to 60%. When the minimum support was 40%, a total of 6,635 correlation rules were generated, which were used as the research object. The results of the correlation rules related to the glide deviation are shown in Table 4.

Taking the first correlation rule in the table as an example, the support of the correlation rule is 55.29%, and the confidence was set to 60%. When the minimum support was 40%, a total of 6,635 correlation rules were generated, which were used as the research object. The results of the correlation rules related to the glide deviation are shown in Table 4.

Table 3: Fuzzy C-means clustering results.

| QAR parameter       | Cluster Center | QAR parameter       | Cluster Center |
|---------------------|----------------|---------------------|----------------|
|                     | Sm | Md | Lg | Sm | Md | Lg |
| Radio altitude      | 518.169 | 329 | 75 | 6.69 | 3.66 |
| Angle of attack     | - | - | 0.53 | 1.29 | 2.40 |
| Control column position | 0.68 | 0.01 | 1.94 | 2.54 |
| Pitch trim          | 8.5 | 6.49 | 7.56 |
| Left attack angle   | 0.18 | 2.58 | 3.84 |
| Right attack angle  | 0.01 | 2.69 | 3.90 |
| Left elevator       | 2.68 | 3.82 | 5.22 |
| Right elevator      | 2.67 | 3.98 | 5.59 |
| Roll angle          | 1.54 | 0.18 | 1.47 |
| Decline rate        | 1465 | 825 | 587 | 85 | 40 | 03 |

III. VERTICAL ATTITUDE ADJUSTMENT MODEL

In order to study the effect of the control column on the vertical position error, the mapping relationship between the control column and the pitching attitude of the aircraft must first be determined. This is because, in the approach process, the change of aircraft pitch attitude will cause the change of glide deviation, which affects the calculation of overlap probability.

A. DIRECT CORRESPONDENCE RULE

1. The push/pull of the control column causes the elevator to experience a downward/upward deflection;
2. The downward/upward deflection of the elevator causes an increase/decrease of the horizontal tail lift;
3. The change of the horizontal tail lift will produce a pitching moment to the center of gravity of the aircraft, which will change the pitch angle of the aircraft.

In Process (1), there is a corresponding relationship between the control column and the elevator: \( \delta_{ele} = f(d) \). When the aircraft is stationary, an angle of the control column corresponds to an angle of the elevator. This rule is called the direct correspondence rule of the elevator and control column.

Figure 5 shows the corresponding rule fitted from the QAR data. It can be seen that the control column angle is positive and the elevator angle is negative when the pilot advances the control column. The elevator angle is about 4.5° when the control column position is 0°. This is because the moment generated by the engine thrust is the lift moment when the aircraft is in a smooth configuration and is also due to the
position of the center of gravity. Therefore, it is necessary to set the elevator at a smaller angle of about 4.5° to balance the longitudinal moment of the aircraft.

**B. PITCH MOMENT AND AIRCRAFT ATTITUDE**

The elevator yaw creates an additional aerodynamic force on the horizontal fin. When the pilot backward the control stick back, the elevator is raised, and the downward aerodynamic force generated by the horizontal tail overcomes the damping torque and makes the aircraft rotate around the center of gravity. This will cause the horizontal tail damping moment $M_2$ and the stabilizing moment $M_3$ to increase, both of which will prevent the aircraft from rotating further. The damping torque first increases and then decreases until it disappears. At the same time, the aircraft stops rotating and reaches a steady-state. The torque on the aircraft changes from the moment the pilot operates the stick until the aircraft stops turning to reach a steady state, which is shown in Figure 6.

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**TABLE 4. Correlation rules.**

| Correlation rule                                                                 | Support | Confidence |
|---------------------------------------------------------------------------------|---------|------------|
| Small vertical load 1 > Medium glide deviation                                  | 55.29%  | 83.31%     |
| Small radio altitude, medium control column position > Medium glide deviation    | 42.99%  | 90.37%     |
| Small radio altitude, medium roll angle > Medium glide deviation                | 42.96%  | 90.80%     |
| Small radio altitude, small wind direction > Medium glide deviation              | 41.01%  | 96.06%     |
| Small vertical load 1, small airspeed > Medium glide deviation                  | 45.17%  | 90.58%     |
| Small vertical load 1, Small wind direction > Medium glide deviation            | 41.58%  | 99.00%     |
| Small air speed, medium roll angle > Medium glide deviation                     | 46.77%  | 88.83%     |
| Small left engine N1, Medium control column position > Medium glide deviation   | 43.26%  | 97.50%     |
| Small left engine N1 > Medium glide deviation                                    | 45.09%  | 95.86%     |
| Small wind direction, medium control column position > Medium glide deviation   | 46.12%  | 96.79%     |
| Small airspeed > Medium glide deviation                                          | 54.07%  | 83.74%     |
| Small ground speed > Medium glide deviation                                      | 50.63%  | 72.74%     |
| Small wind direction > Medium glide deviation                                    | 50.36%  | 94.62%     |
| Medium glide deviation > Medium control column position                          | 66.09%  | 87.29%     |

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FIGURE 5. Direct correspondence rule.

FIGURE 6. Process of pitch moment change.

FIGURE 7. Horizontal tail model.
where the constraints are \( \lambda \leq n, w \leq l; M_1 \) is the moment generated by the elevator; \( k \) is the speed retardation coefficient; \( k < 1 \); \( \rho \) is the atmospheric density; \( V \) is the flight speed; \( L_{\text{hor}} \) is the distance from the horizontal tail to the center of gravity; \( C_{y_{\text{hor}}} \) is the lift coefficient of the horizontal tail when the elevator deflects. Additionally, \( S_{\text{hor}} \) is the horizontal tail horizontal aircraft area; \( \omega_c \) is the angular speed of the aircraft pitch; \( M_2 \) is the damping moment generated by the horizontal tail; \( a_{\text{hor}} \) is the slope of the horizontal tail lift coefficient curve; \( M_3 \) is the vertical stabilizing moment; \( M_{\text{wing}} \) is the moment of the aerodynamic force acting on the wing around the focal point, whose magnitude does not change with the attack angle. \( x_G \) is the distance from the center of gravity of the aircraft to the average leading edge of the aerodynamic chord; \( x_F \) is the distance from the focus of the wing to the average leading edge of the aerodynamic chord; \( Y \) is the aerodynamic force acting on the focus; \( \delta_{\text{ele}} \) is the angle of the elevator, which is positive when there is a downward deviation; \( f(d) \) is the direct correspondence law; \( \theta_{\text{hor}} \) is the effective angle of attack of the horizontal tail; \( n \) is the elevator efficiency; \( I \) is the moment of inertia; \( m \) is the horizontal tail mass; \( \alpha_c \) is the longitudinal angular acceleration generated by the resultant moment.

**IV. PRESENTATION OF RESULTS**

As the pilot’s operation is only related to the current aircraft state (glide deviation), the process is Markovian. Thus, we build a pilot operation model based on the concept of the Markov process.

The flight state set of the aircraft is \( S = \{s_1, s_2, \ldots, s_n\} \), the pilot action set is \( D = \{d_1, d_2, \ldots, d_m\} \), and the probability that the pilot will take action \( d_m \) under the condition of the aircraft state \( s_n \) is \( P_{nm} = P(d_m | s_n) \). The state-action matrix \( P \) [24] is then defined as:

\[
P = \begin{bmatrix}
P_{11} & \cdots & P_{1m} \\
\vdots & \ddots & \vdots \\
P_{nm} & \cdots & P_{nn}
\end{bmatrix}
\]

For any \( i \in (1, n) \), \( \sum_{k=1}^{m} P_{ik} = 1 \). The state at a certain time \( t \) of the paired approach is \( S_t \), and then the state at time \( t + 1 \) is \( s_{t+1} = s_t + \Delta s \). Among them,

\[
\Delta s = (V + \dot{a}_t) \left[ \cos(\delta_j - \theta_c) - \cos \delta_j \right]
\]

\[
\theta_c = \int \omega_c dt
\]

where \( \delta_j \) is the glide-path angle set by the flight procedure. When calculating \( \omega_c \), the control column angle is \( d \in D \).

The POM and the whole approach process are described in Figure 8.
pilot taking the correct action when the aircraft is in a certain state.

\[ P_{Correct} = \frac{C_{Correct}}{C_A} \]  

(14)

where \( C_{Correct} \) is the number of times the pilot has taken the correct action, and \( C_A \) is the total number of actions taken for the pilot. The criteria for judging the correct action are:

\[
\begin{align*}
  a > 0, \quad s < 0 \\
  a = 0, \quad s = 0 \\
  a < 0, \quad s > 0
\end{align*}
\]  

(15)

V. PAIRED APPROACH VERTICAL OVERLAP SIMULATION MODEL

One factor that cannot be neglected in the paired approach is the motion of the wake. According to the characteristics of the diffusion [25] and sinking [26] of the wake, a model of the wake position relationship between the IM aircraft and the target aircraft is established in Equation (16).

\[
\begin{align*}
  h(t) &= h_0(t) + V_{z}(t) + \frac{1}{2}a(z)(t)^2 + \lambda_c + 2\epsilon(t) \\
  S_0 &= V_x(t)b + \frac{1}{2}a_x(t)b^2 \\
  S_0 - V_x(t) - \frac{1}{2}a_x(t)^2 &= \left( V_{x0}(p)(\Delta t - t) + \frac{1}{2}a_{x0}(p)(\Delta t - t)^2 \right), \quad t < t_h \\
  S_0 + (V_x(t) - V_{x0}(p)t + \frac{1}{2}(a_x(t) - a_{x0}(p))^2) &= \left( V_{x0}(p) + a_{x0}(p)^2 \Delta t - \frac{1}{2}a_{x0}(p)^2 \Delta t^2 \right), \quad t > t_h \\
  L_{c1}(t) &= h_0(t) + V_{z}(t) + \frac{1}{2}a(z(t))^2 + 2\lambda_c \\
  L_{c2}(t) &= h_0(t) + V_{z}(t) + \frac{1}{2}a(z(t))^2 + (v(k) + v(j))\Delta t + \frac{1}{2}\lambda_c \\
  (v(k) + v(j))\Delta t &> \frac{3}{2}\lambda_c \\
  L_{c2}(t) &= h_0(t) + V_{z}(t) + \frac{1}{2}a(z)^2 \\
  (v(k) + v(j))\Delta t &\leq \frac{3}{2}\lambda_c
\end{align*}
\]

where \( h(t) \) is the altitude of the IM aircraft, and \( e(t) \) is the glide deviation of the aircraft at time \( t \). It is assumed that the operational quality of the target aircraft and IM aircraft is the same as that of the pilot. The target aircraft is used as a reference, and the IM aircraft runs with twice the deviation. Here, \( h_0(t) \) and \( h_0(t) \) are the initial heights of the IM aircraft and target aircraft in the initial approach segment, respectively; \( V_{z}(t) \) and \( V_{z}(t) \) are the initial vertical speeds of the IM aircraft and target aircraft in the final approach segment, respectively; \( a(z(t)) \) and \( a(z(t)) \) are the initial vertical acceleration of the IM aircraft and target aircraft in the initial approach segment, respectively. The speed and acceleration are vector parameters, and \( \lambda_c \) is the average value of the height of the two aircraft fuselages.

Additionally, \( S_0 \) is the initial longitudinal interval distance between two aircraft; \( V_{z}(t) \) and \( V_{z}(t) \) are the initial longitudinal speeds of the IM aircraft and target aircraft in the final approach segment, respectively; \( a_{z}(t) \) and \( a_{z}(p) \) are the longitudinal accelerations of the IM aircraft and target aircraft, respectively; \( t_h \) is the time the IM aircraft spends flying over the initial interval \( S_0 \).

When the target aircraft does not reach the final approach fix point, it flies with acceleration \( a_{z0}(p) \) in the longitudinal dimension. Its longitudinal speed at the final approach fix point position of the IM aircraft is \( V_{z0}(p) \), where \( \Delta t \) is the dynamic time interval between the IM aircraft and the target aircraft.

\[ L_{c1}(t) \) is the height of the upper boundary of the vertical overlap area, \( L_{c2}(t) \) is the height of the lower boundary of the vertical overlap area, \( v(k) < 0 \) is the diffusion speed of the wake of the target aircraft, and \( v(j) < 0 \) is the drift speed of the wake of the target aircraft.

In the case of the paired approach, vertical overlap is considered to have occurred when the IM aircraft enters the wake of the target aircraft in the vertical dimension or overlaps with the fuselage of the target aircraft. The probability of vertical overlap \( P_z \) is shown in Equation (17):

\[ P_z = \frac{T_{Lc1(t)>h(t)>Lc2(t)}}{T} \]

(17)

In the formula, \( T_{Lc1(t)>h(t)>Lc2(t)} \) represents the time that the IM aircraft is in the vertical overlap area during the approach segment, and \( T \) is the total time required for the flight in the final approach segment.

The vertical overlap frequency \( N_z \) is represented as:

\[ N_z = \frac{C_{Lc1(t)>h(t)>Lc2(t)}}{T} \]

(18)

where \( C_{Lc1(t)>h(t)>Lc2(t)} \) is the number of times the IM aircraft enters the vertical overlap area during the approach segment.

VI. SIMULATION ANALYSIS

The horizontal area of the horizontal tail of a certain type of aircraft is about 14 m², and the unit area weight of the horizontal tail [23] is generally 25 kg/m². Thus, the mass of the horizontal tail is 350 kg. The distance from the horizontal tail to the center of gravity is 19.3 m, \( L = 20.8 \) m, \( l = 18.8 \) m, the effective angle of attack of the horizontal tail is \( 0^\circ \), and the elevator efficiency is 0.8. The speed retardation coefficient \( k \) is 0.9, the atmospheric density is 1.293 kg/m³, and the slope of the wing lift coefficient curve is generally 0.1 [23]. Since the horizontal tail is a symmetrical airfoil, the slope of the horizontal tail lift coefficient curve is 0.03. The glide-path angle of a flight program design of an airport is 3°. Since the calculation of \( M_3 \) involves undisclosed aircraft design parameters that cannot be obtained, for the convenience of calculation during the simulation of the calculation example, \( M_3 \) is the same as \( M_1 \) when the control column is \( 0^\circ \) and the direction is opposite to \( M_1 \). The state-action matrix \( P \) is obtained by calculating the QAR data of 30 flights of the
same aircraft landing at the same airport. The direct correspondence law of the control column is:

$$
\delta_{ele} = -0.0415d^3 - 0.1349d^2 - 1.8065d + 4.4962
$$

(19)

Substitute the above parameters into the VAAM and POM, the final approach flight time of the aircraft is set to 200 s, and the deviation when the aircraft starts to cut into the glide path is 7 m.

The correct rate is the probability that the pilot operates the aircraft correctly. If the operation is right, it is considered that the action will allow the aircraft to more effectively lock the glide path, while the wrong operation will cause the aircraft to deviate from the glide path. According to statistics on the accuracy of 30 flights, the minimum value is 61%, and the maximum value is 73%. Therefore, the accuracy in the range of 61%–73% is defined as the normal value of the pilot’s accuracy. In the simulation, the pilot operation accuracies are set to 55%, 61%, 64%, 67%, 70%, and 73%, respectively, of which 55% is the control group with common accuracy. We performed 100 simulations on each of the six accuracies. The simulation results are shown in Figures. 9–14.

In the simulation results. Each line represents the glide slope error in each simulation. Some lines have sudden upward increase, which is due to the increase of glide slope error caused by the pilot’s wrong operation. The black thick line in the figure is the average value, representing the overall trend. The amplitude and frequency of the glide deviations that are greater than 0 are larger than those less than 0. This same rule is reflected in Figure. 15. This phenomenon occurs because the pilots often draw the control column in the final approach segment to avoid unsafe incidents caused by a low descending height.

Figure 15 shows the glide deviation data recorded by the QAR of a certain aircraft at the Tianjin airport when cutting into the final approach segment. We find the average deviation value, which is indicated by the thick black line. With the same method, the glide slope deviation data of Shanghai Hongqiao Airport is obtained. And compared with the simulation results with 73% accuracy and the actual data of Tianjin Binhai Airport, shown as Figure.16. The simulation results are basically consistent with the actual operation results. The initial glide deviation is set to 7 m due to
Therefore, the initial error is larger than the simulation during the actual operation (The average initial error of Shanghai Airport is about 15 meters, and that of Tianjin airport is about 16.7 meters). The adjustment time is slightly longer. Comparing the simulation result with the actual data in Figure 16, the simulation results show the same trend as the actual running data, which suggests that the simulation is effective and the established model is credible.

It is obvious from the above six pictures that when the operation accuracy is 70% or 73%, the pilot can generally adjust the vertical error of the aircraft to around 0 m and stabilize it in about 40 seconds. When the accuracy is 61%, 64%, or 67%, this time will increase to about 50 seconds. When the accuracy is 55%, the vertical error is also adjusted to around 0 m in about 50 seconds, but it finally stabilizes at −2 m at about 70 seconds. Moreover, with the improvement of operation accuracy, it can be clearly observed that the figure shows a trend of regularization, which is due to the sudden reduction of glide slope error caused by wrong operation. The simulation results show that the correction accuracy of the glide path error is positively correlated with the speed and the pilot’s operation accuracy during the approach. The paired approach procedure stipulates that the height and the glide-path angle of the IM aircraft at the beginning of the pairing are greater than that of the target aircraft, and the landing time of the two aircraft is similar, which makes the speed and acceleration of the IM aircraft greater than those of the target aircraft in numerical value. According to the statistical QAR data results, the speed of the aircraft at the final approach fix point is usually 80 m/s. Here, the initial speed of the target aircraft is 78 m/s, the initial longitudinal interval between the IM aircraft and the target aircraft is 2000 m, and the time interval is 25 s. The calculated operating parameters of the two machines are shown in Table 5.

Taking the simulation results as an example, the vertical overlap situation is shown in Figure 17. It can be seen that due to the large vertical distance between two aircraft in the early stage, both aircraft are still far from the overlap area and stay within safe limits even though the IM aircraft is
running with twice the glide deviation. It is not until the last 11 seconds of operation that the IM aircraft enters the vertical overlap zone. This is due to the fact that the paired approach procedure requires both aircraft to land at a similar time and glide deviation, and the altitude difference between the IM aircraft and the target aircraft is small during the 11 seconds before entering the runway.

After several simulations, 500,000 incidents are randomly selected for statistics. The total running time of the pairing is 100 million seconds, which is about 27,778 hours. When the pilot’s operational accuracy is 55%, the time of the IM aircraft entering the vertical overlapping area is 6,848,146 seconds. As a result, the probability of vertical overlap is 0.068481.

The simulation results with 55% accuracy, and the other five groups of common accuracies are shown in Figure 18. From the figure, it can be seen that when the accuracy is in the range of 61%–73%, the probability of vertical overlap of the paired approach is kept around 0.052, overlapping frequencies are stable at around 20 flights/hour, with no significant deviation. When the operation accuracy is 55%, the overlap frequency and probability increase dramatically.

VII. DISCUSSION

The simulation results show that when the pilot operation accuracy is lower than the normal value, the glide deviation will be affected first, and then the vertical overlap probability and frequency will be influenced. The increase in accuracy can not only shorten the time to correct the aircraft to the glide path but also stabilizes the aircraft near the glide path. When the accuracy is obviously lower than 61%, the probability of vertical overlap will increase significantly. When the accuracy is within the constant range, the probability and the frequency of vertical overlap have little fluctuation and almost no difference.

Previous research results usually focus on the collision risk, where the order of magnitude of the evaluation results...
is about $10^{-9}$–$10^{-8}$. However, the difference in the order of magnitude between the simulation results in this paper and the results of previous studies is due to the different concepts of overlap probability and collision risk. The collision risk $CR$ is calculated as follows:

$$CR = N_x P_x P_z + N_y P_y P_z + N_z P_z P_y$$ (20)

In the formula, $x$, $y$, and $z$ represent longitudinal, lateral, and vertical dimensions, respectively; $P_i (i = x, y, z)$ represents the overlap probability; $N_i (i = x, y, z)$ represents the overlap frequency. If the collision risk needs to be calculated, subsequent calculations of the probability and frequency of the overlap of the lateral and longitudinal dimensions are also required.

Because the paired approach program has not been used formally and there are few data available, calculation of the paired approach vertical overlap probability and frequency has been avoided. Compared with the evaluation results of parallel routes in the same altitude layer, the probability of vertical overlap of the paired approach in this paper is about 1/10 of that in parallel routes, which is due to the difference
in the initial altitude of the paired approach. As there is a vertical interval between the two aircraft at the beginning of the approach, the probability of vertical overlap between the two aircraft is much less than that of the two aircraft at the same altitude level.

VIII. CONCLUSION
(1) By establishing the fuzzy correlation model, the influence degree of 20 factors on approach vertical error is comprehensively evaluated. We found that the QAR parameter that had the biggest correlation with the vertical position error was the control column position.

(2) Compared with the actual operating data, the simulation results showed similar trends, and VAAM and POM could simulate the approach process well. In the simulation process, because of the requirement of the paired approach procedure, the two aircraft had an initial vertical interval, and the time difference between the two aircraft entering the runway entrance was short. This caused the vertical overlap to occur in the last 10 seconds of the whole process.

(3) The accuracy of pilot operation affected the time to correct the aircraft to the glide path, where the higher the accuracy, the shorter the time. And the whole adjustment process has more stable process. A lower accuracy kept the aircraft on the wrong glide path and increased the probability and frequency of vertical overlap. When the pilot’s operational accuracy was between 61% and 73%, the vertical overlap probability and frequency were almost constant.

REFERENCES
[1] M. C. Waller and C. H. Scanlon, “Proceedings of the NASA workshop on flight deck centered parallel runway approaches in instrument meteorological conditions,” NASA, Tech. Rep. NAS 1.55:10191, 1996.
[2] J. Hammer, “Case study of paired approach procedure to closely spaced parallel runways,” Air Traffic Control Quart., vol. 8, no. 3, pp. 223–252, 2000.
[3] R. Teo, J. S. Jang, and C. Tomlin, “Flight demonstration of provably safe closely spaced parallel approaches,” in Proc. AIAA Guid., Navigat., Control Conf. Exhibit, Aug. 2005, p. 6197.

[4] M. M. Madden, “Kinematic modeling of separation compression for paired approaches to closely-spaced parallel runways,” in Proc. 14th AIAA Aviation Technol., Intel., Oper. Conf., Jun. 2014, p. 3150.

[5] N. Guerreiro, K. Neitzke, S. Johnson, H. Stough, B. McKissick, and H. Syed, “Characterizing a wake-free safe zone for the simplified aircraft-based paired approach concept,” in Proc. AIAA Atmos. Space Environ. Conf., Aug. 2010, p. 7681.

[6] N. Guerreiro and K. Neitzke, “Simulated wake characteristics for closely spaced parallel runway operations analysis,” in Proc. 12th AIAA Aviation Technol., Integ., Oper. (ATIO) Conf. 14th AIAA/ISSMO Multidisciplinary Anal. Optim. Conf., Sep. 2012, p. 5642.

[7] F. Lu, Z. N. Zhang, Z. Q. Wei, and B. L. Liu, “Longitudinal collision risk safety assessment of paired approach to close spaced parallel runways,” China Saf. Sci. J., vol. 23, no. 8, p. 108, 2013.

[8] F. Lu, N. Zhu, S. Yang, Z. Zhang, and B. Liu, “Assessment of lateral collision risk in closely spaced parallel runways paired approach,” China Saf. Sci. J., vol. 26, no. 11, pp. 87–92, 2016.

[9] M. Geyer, M. Soares, S. Barnes, A. Hoff, and S. Mackey, “RNAV GPS total system error models for use in wake encounter risk analysis of dependent paired approaches to closely-spaced parallel runways,” Global Positioning Syst., Tech. Rep. DOT-VNTSC-FAA-14-05, Feb. 2014.

[10] W. Torres-Pomales, M. M. Madden, R. W. Butler, and R. B. Perry, “Analysis and simulation of the simplified aircraft-based paired approach concept with the ALAS alerting algorithm in conjunction with echelon and offset strategies,” NASA, Tech. Rep. NASA/TM-2014-218151, 2014.

[11] Z. Lu, Z. Zhang, and X. Niu, “Collision risk safety assessment of paired approach based on velocity error and positioning error,” Aeronaut. Comput. Technique, vol. 45, no. 6, pp. 36–40, 2015.

[12] R. Mori and M. Fujita, “Accurate estimation of ground obstacle collision probability during ILS approach,” IEEE Access, vol. 8, pp. 66662–66671, 2020, doi: 10.1109/ACCESS.2020.2985688.

[13] S. Priess, “Analysis of an ADS-B in method for calculating the interval management paired approach collision safety limit,” in Proc. IEEE/AIAA 36th Digit. Avionics Syst. Conf. (DASC), Sep. 2017, pp. 1–8.

[14] G. U. Runping, W. U. Jun, and L. U. Fei, “Research on paired approach procedure to closed spaced parallel runways and its collision risk,” J. Henan Univ. Sci. Technol., Natural Sci., 2019.

[15] M. L. Williams, L. C. Wood, and B. J. Nelson, “Safety study of closely spaced parallel operations utilizing paired approach,” in Proc. IEEE/AIAA 38th Digit. Avionics Syst. Conf. (DASC), Sep. 2019, pp. 1–10.

[16] L. U. Fei, T. E. Jingjie, W. U. Jun, Z. H. Zhaoxing, and Z. H. Zonglu, “Lateral collision risk of CSPRs paired approach under wake impact,” China Saf. Sci. J., vol. 30, no. 2, pp. 99–105, 2020.

[17] L. U. Fei, T. E. Jingjie, W. U. Jun, Z. H. Zhaoxing, and Z. H. Zonglu, “Lateral collision dynamics of CSPRs paired approach under influence of wake vortex field,” China Saf. Sci. J., vol. 30, no. 4, pp. 7–21, 2020.

[18] P. G. Reich, “Analysis of long-range air traffic systems: Separation standards-I,” J. Navigat., vol. 19, no. 1, pp. 88–98, 1966.

[19] P. G. Reich, “Analysis of long-range air traffic systems: Separation standards-II,” J. Navigat., vol. 19, no. 2, pp. 169–186, 1966.

[20] P. G. Reich, “Analysis of long-range air traffic systems: Separation standards-III,” J. Navigat., vol. 19, no. 3, pp. 332–347, 1966.

[21] M. Zhang and Y. U. Jian, “Fuzzy partitional clustering algorithms,” J. Softw., vol. 15, no. 6, pp. 858–868, 2004.

[22] L. Jianjiang and Z. Yafei, Song Zilin Research and Application of Fuzzy Association Rules. Beijing, China: Science Press, 2008.

[23] L. R. Jenkinson, P. Simpkin, D. Rhodes, L. R. Jenkison, and R. Royce, Civil Jet Aircraft Design. London, U.K.: Arnold, 1999.

[24] S. Renjie, Basis of Markov Chain and Its Application. Xi’an, China: Xidian Univ. Publishing House, 1992.

[25] V. Rossow and L. Meyn, “On data scatter in measured linking times for lift-generated vortex pairs,” in Proc. 46th AIAA Aerosp. Sci. Meeting Exhibit, Jan. 2008, p. 338.

[26] F. Holzäpfel, “Probabilistic two-phase wake vortex decay and transport model,” J. Aircr., vol. 40, no. 2, pp. 323–331, Mar. 2003.

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