We develop a new method for estimating the redshift of galaxy clusters through resolved images of the Sunyaev-Zel’dovich effect (SZE). Our method is based on morphological observables which can be measured by actual and future SZE experiments. The method is tested using a set of high resolution hydrodynamical simulations of galaxy clusters at different redshifts. The method combines the observables in a principal component analysis. We show how this can give an estimate of the redshift of the galaxy clusters. Although the uncertainty in the redshift estimation is large, the method should be useful for future SZE surveys where hundreds of clusters are expected to be detected. A first preselection of the high redshift candidates could be done using our proposed morphological redshift estimator.

1 Introduction

The advent of new experiments dedicated to the observation of the Sunyaev-Zel’dovich effect (Sunyaev & Zel’dovich, 1972) (SZE hereafter), demands the development of new techniques to best analyze these new and exciting data. Through the SZE it is possible to trace the hot plasma in the galaxy clusters which distorts the spectrum of the cosmic background radiation. This distortion is redshift independent and it is proportional to the temperature of the plasma and its electron density ($n_e$). This quality ($z$-independent, and $\propto n_e$) makes the SZE effect an ideal way to explore the high redshift population of galaxy clusters. However, the fact that the distortion induced by the cluster in the CMB is independent of the redshift of the cluster, makes the determination of the cluster redshift from SZE observations a challenging task.

Redshifts can be easily measured for relatively nearby clusters but for distant clusters one should use other approaches, for instance photometric redshifts. However, photometric redshifts for clusters above redshift $\approx 0.5$ require large telescopes. Ongoing and future SZE experiments
will detect hundreds or thousands of clusters. The redshift estimation of all these clusters using 10-m class telescopes is unfeasible. An optimal solution could be to combine small, and medium-sized telescopes to determine the redshifts of the low and intermediate $z$ clusters respectively. Then, we could leave the estimation of the redshifts of the most distant clusters to the large telescopes. However, to select the low, intermediate and high $z$ clusters we need an estimate of their redshift. The motivation of this work is to show how it is possible to make this preselection of the low, intermediate and high redshift clusters using SZE data alone. Then, this preselection can be used to determine more accurately the redshifts of the clusters combining small (for the low $z$ clusters) medium-sized (for the intermediate $z$ clusters) and large (for the high $z$ clusters) telescopes.

Our method is based only on observed quantities of the SZE. These quantities are associated with the observed shapes of the 2D surface brightness profile of the clusters which does have some dependence on the redshift. The observed size of a particular cluster, for instance, will decrease with increasing redshift. So, given an observed size we could, in principle, get a probability for the cluster to be at a given redshift. However, the size of the cluster will also depend on its total mass. This means that two clusters with different redshifts and masses could have the same apparent size provided the most distant cluster had a larger mass that compensates the decrease in the apparent size due to the increase in redshift. There are, therefore, degeneracies between the redshift of the cluster and its mass. The question now is, can we break this degeneracy by including more information in our analysis? A resolved SZE image of a cluster provides information, not only about its size, but also about the shape of the entire profile. The total observed flux of the cluster, for instance, will depend on the total mass of the cluster (and its redshift and temperature). The central SZE decrement will depend on the core radius and electron central density but not on the redshift. By adding these and other additional observables, it should be possible to break the degeneracies between the mass and redshift.

Our method will have, however, one limitation: it works with resolved SZE images. Therefore it should be useful for sub-arcmin experiments but not for experiments like Planck where the best resolution will be 5 arcmin.

2 Morphological redshifts

The idea behind morphological redshift estimation is that by combining many observables taken from the 2D SZE cluster profile it is possible to divide the clusters in different groups, each one for a different redshift interval.

The first two obvious observables are the total flux and size of the cluster. However, in real experiments it will be difficult to measure these two quantities due to the instrumental noise and the limited sensitivity of the detectors. In order to get the total flux and size of the cluster, one should extrapolate the observed 2D profile below the noise level. The easiest approach to solve this problem is just to consider the isophotal size and isophotal flux instead of the total flux and total size of the cluster. Isophotal fluxes and sizes are direct observed quantities. Like the total flux and total size, their isophotal equivalents also show strong scaling relations in galaxy clusters (see Mohr & Evrard 1997, Mohr et al. 2000, Verde et al. 2000 for a relation between isophotal size and temperature).

If we only use the isophotal flux and isophotal size and we take into account that the scaling relations in galaxy clusters have an intrinsic scatter, we find that the redshift estimation is quite poor. In order to improve the result, more observables should be combined.

In this work we will include 5 additional observables. The central amplitude, second derivative in the center, and 3 mexican hat wavelet coefficients at the center.

The central amplitude does not depend on the redshift but depends on the total projected mass
along the line of sight. This will make this quantity very useful to break the degeneracy between
the mass and the redshift when combined with other observables which do depend on the red-
shift. One such observable is the second derivative at the center. It gives an idea of how cuspy
the cluster is. If we move a cluster of fixed mass back in redshift, we will see that the central
amplitude does not change with redshift but the cluster becomes more and more cuspy. Related
to the second derivative is the mexican hat wavelet. By changing the scale of this wavelet we
can trace the 2D SZE profile at different radii. Although in this work we will present only the
preliminary results obtained with these 7 observables, in a subsequent work (Diego et al.) we
will show how it is possible to improve the result by including more observables.
In the next section we will show how to combine all these observables and use that combination
to get an estimate of the redshift.

2.1 Principal Component Analysis

Principal component analysis (PCA hereafter) has been widely used in the last years as a pow-
erful classifier of data sets. For our particular case, PCA has several desirable advantages which
can be briefly summarized as follows.

• PCA produces an optimal linear combination of the observables. It is optimal in the sense
that it maximizes the variance of the linear combination (or projection).

• There is no limit in the number of observables. One can include as many observables as wanted.

• PCA is a non-parametric method. This is a key point since no assumptions about the cluster
scaling relations need to be done. If there are intrinsic scaling relations between some of the
observables, PCA will find them.

If we write our data set in matrix notation, \( X_{Nm} \) (\( N \) clusters observed and \( m \) observables
per cluster), then the principal components are given by the projections along the directions of
the eigenvectors of the matrix \( S = X^T X \).
The matrix \( S \) has as many eigenvectors as observables (7 in our case). One quality of these
eigenvectors is that they are independent one from each other. So, when projecting the data
set, \( X \), along the directions given by the eigenvectors we have a representation of the original
data set in an orthogonal system. Another quality of these new representation of the data is
that the projection along the direction of the eigenvector with the highest associated eigenvalue
retains the highest percentage of information of the original data set. The eigenvector associated
with the second highest eigenvalue retains the second largest percentage of information and so
on. Therefore we can reduce the dimensionality of the problem by considering only the \( p \) first
directions (eigenvectors) such that the percentage of information retained by these \( p \) directions
is above certain threshold (typically the percentage should be > 95%).

3 Results

We have applied the previous method to hydrodynamical N-body simulations of the SZE. We
have filtered the images with a Gaussian of FWHM = 15 arcsec simulating the effect of an an-
tenna and we have set a threshold in the maps which simulates the sensitivity of our instrument.
After computing the principal components we keep only the first three principal components
since they will retain \( \approx 95\% \) of the variance of the original data set. In Figure [3] we show the
result. As can be seen, different redshifts are grouped in different regions in this 3D space. This
suggests that it will be possible to discriminate between low, intermediate and high redshift clusters. Our method should be useful for future sub-arcmin SZE surveys where hundreds of clusters should be detected and a preselection of high-intermediate-low redshift clusters should be very useful to optimize the optical follow up.

Figure 1: First three principal components. Blue points are clusters at redshift $\approx 0$, yellow at $z = 0.5$, grey at $z = 1$ and black at $z = 2.3$.

In a subsequent work (Diego et al.) we will apply the method to better high resolution N-body simulations and we will include more observables in the analysis. Finally, we will recover the redshifts of our simulated clusters using a Bayesian approach and we will give the error of the recovered redshifts as a function of $z$.

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