Unified meta-theory of information, consciousness, time and the classical-quantum universe

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Abstract

As time advances in our perceived real world, existing information is preserved and new information is added to history. All the information that may ever be encoded in history must be about some fundamental, unique, atemporal and pre-physical structure: the bare world. Scientists invent model worlds to efficiently explain aspects of the real world. This paper explores the features of and relationships between the bare, real, and model worlds. Time—past, present and future—is naturally explained. Both quantum uncertainty and state reduction are needed for time to progress, since unpredictable new information must be added to history. Deterministic evolution preserves existing information. Finite, but steadily increasing, information about the bare world is jointly encoded in equally uncertain spacetime geometry and quantum matter. Because geometry holds no information independent of matter, there is no need to “quantize” gravity. At the origin of time, information goes to zero and geometry and matter fade away.

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One path only is left for us to speak of, namely, that It is. In it are very many tokens that what is, is uncreated and indestructible, alone, complete, immovable and without end. Nor was it ever, nor will it be; for now it is, all at once, a continuous one. For what kind of origin for it will you look for? In what way and from what source could it have drawn its increase? I shall not let thee say nor think that it came from what is not; for it can neither be thought nor uttered that what is not is. And, if it came from nothing, what need could have made it arise later rather than sooner? Therefore must it either be altogether or be not at all.

Parmenides of Elea, On Nature, 5th c. BCE
translated by John Burnet, 1892

Reality cannot be found except in One single source, because of the interconnection of all things with one another.

Gottfried Leibniz, Philosophical Investigations, 1670

1 Introduction

Evolution has enabled our minds to efficiently comprehend the world as a synthesis of elements and to build rational models of how those elements unite to yield the whole. From ancient times, philosophers have viewed the world in terms of elemental parts or principles; for example: earth, water, air and fire. Complex, higher-level structures, including cognitive beings like ourselves, are presumed synthesized from these elements, explained by the elements’ intrinsic properties and mutual relationships and interactions. Modern physics has adopted geometrical spacetime and numerous species of quantum matter as its primitive elements, with an action function and least action principle to relate and govern them. But the quantum measurement problem [1–4] and conceptual incompatibility of general relativity and quantum theory, especially with respect to time [5], remain critical barriers to consistent explanation of the high-level structures we directly perceive and of the universe as a whole.

The fundamental nature of the elements is axiomatic in synthetic worldviews. Individually, however, the elements are useless; any relevance of an element to the world depends on its relationships with other elements, expressed as laws of nature. Thus, in quantum field theory (QFT), physical (or dressed) properties and instances of the various species of quantum matter arise from virtual (or bare) interactions involving all species; the measured charges, masses and momenta of elementary particles are the collective result of their indiscernible virtual interactions.

Conceptual barriers seem to block consensus on how the properties of quantum matter and spacetime geometry are related, while respecting the principles of general relativity. We measure and perceive distances and times as c-number quantities, but there are no generally accepted means for a classical environment to emerge from quantum matter or for observations of quantum matter to yield c-number outcomes. Explanations based on decoherent or consistent histories [6–8], environment induced decoherence [9–11] and quantum Darwinism [12] must assume existence of the global classical environment they were trying
to avoid [4, 13, 14]. From a general relativity perspective, even claiming that a quantum system has some spatial locale or is in some spin state requires an implied but unexplained binding between position and angular momentum operators on the quantum Hilbert space and the quasi-classical matter through which the spacetime geometry and its inertial frames can be known.\footnote{We use the term \textit{quasi-classical} to refer to properties of quantum matter that have become \textit{real} by influencing the perceived world of conscious observers. While the influences on different observers’ perceptions must be mutually consistent, the representation of abstract information about the matter in terms of observable properties of the world will so entangle the bits that they are individually inaccessible (i.e., entropy).} The dynamical laws of quantum theory and general relativity, considered separately, relate the ways in which given information (or prior knowledge) is represented at different times, without adding, removing, or otherwise changing the information—they are both deterministic theories. The non-deterministic, probabilistic character usually associated with quantum theory actually enters only in the state reduction, or \textit{measurement}, process that links the quantum and classical worlds: the mysterious quantum-classical transition.

Although the synthetic approach has been fruitful, there has always been a desire to attain a fully unified view of the world, with fewer—ideally just one—primitive elements. Einstein showed how abandoning the Newtonian elements, space and time, in favour of principled analysis could reveal important new physics of unified spacetime. The ultraviolet catastrophe, implied by a unifying application of classical statistical mechanics to electromagnetic radiation, led to the discovery of quantum phenomena. Soon all matter was viewed as quantum mechanical and QFT was developed to unify quantum theory with special relativity, forcing abandonment of particles as immutable elements. The standard model of particle physics builds on a QFT framework to give a unified account of all observed elementary particles and their interactions.

In this paper we step back from the established frameworks of the synthetic worldview. We reconsider the nature and relationships of basic concepts such as information, consciousness, history and time. Our arguments are intuitive and mostly analytic:

1. I am a conscious observer, aware of my present real world which includes me and other conscious observers. I am also aware of changes, that I associate with progress of time, and their cumulative contributions to the history of my world.

2. The real world $R_A$ that a conscious observer $A$ may perceive, or that may have in some way influenced $A$’s present state of being, has exactly the same information as the complete history of $A$’s present.\footnote{\textit{Information} is always \textit{about} something, say $X$. Complete information about $X$ can be identified with $X$: $I(X) \equiv X$. If $I(Y) \subseteq I(X)$ then $Y$ is a representation of $X$. If $I(Y) = I(X)$ the representation is faithful, otherwise it is unfaithful.} History and the present real world are faithful representations of each other; they are just different conceptions of the same information.

3. Information that is part of present history will be preserved in the future even though most details of it may be inaccessible to conscious observers (e.g., as the unknowable actual microstate of a high entropy macrosystem).

4. History, and therefore $R_A$, can be identified with incomplete, but progressively accumulating, information about some particular thing $B$. Every $R_A$ must be an unfaithful representation of this same $B$. If time $t_1 < t_2$, $R_{A_{t_1}}$ must be an unfaithful...
representation of $R_{A_{\text{B}}}$. (No inference should be made here regarding a relationship of $R_A$ or $B$ to conceptual models of the physical world involving matter and geometrical spacetime.)

5. $B$, as conceived here, is ontologically prior to observers, perception, time and the real world. Therefore $B$, from which our real world emerges, must be unique and possess more information than will ever be represented by our real world. Other observers and real worlds could emerge from the same or other $B$’s, but that is irrelevant to us.

6. The advance of time can be identified with both the growth of history and the addition of information about $B$ to the observer’s perceived real world.

7. Since $A$ is included in $R_A$, $A$ cannot infer or reliably predict specific information about $B$ that is not (yet) represented in $R_A$. Therefore, for $A$, the advance of time / growth of history must be a random (unpredictable, non-deterministic) process.

8. At earlier times, $R_A$ will have represented progressively less information about $B$, with the limit of zero information corresponding to the origin of time.

Our unique $B$ is equivalent to Parmenides’ “It” or the “One single source” of Leibniz.

Taking $B$ as the bare foundation, Section 2 introduces a consistent view of the world with three, hierarchically-related levels: bare, real and model worlds. Section 3 then focuses on models, examining the implications of this analytic worldview for quantum theory, general relativity, and their integration and interpretation. Recognizing real world information as being about the bare world allows unified explanation of time, quantum uncertainty, the low entropy state of the early universe, and more. Section 4 summarizes our new worldview and its implications. A tentative proposal for the structure of $B$ is presented in Appendix A, with arguments regarding its natural fit with quantum field theory and the general properties of elementary particles.

2 Comprehending the world

We propose that the world should be considered at three levels, with hierarchical relationships and very different conceptual natures:

**W-1** The pre-physical bare world, $B$, is the unique foundation for all that might ever be perceived in the universe.

**W-2** Perceived real worlds, $R$, arise or emerge as nested, incomplete representations of $B$, in which participatory, conscious observers discover and interact with each other and the dressed matter of the physical universe.

**W-3** Mathematical and interpretational models — model worlds — are devised by conscious observers to efficiently explain, or account for, their past and future perceptions.

All that is perceived as reality, including the physical universe and all living things, evolving in time, is confined to W-2. Physics theories, such as quantum theory, general relativity, and the standard model of particle physics, appear in W-3. Time is explicitly absent in W-1, arises as an integral feature of W-2, and is represented in W-3 according to the needs of each model.
Before quantum theory, it was thought that the dynamical state of the present fully determines the states at all future times, while also allowing us to know the past. Bohr taught us otherwise: only the probabilities of specific future outcomes can be predicted, no matter how well the present state is known. Nonetheless, our common intuition is that actual perceived outcomes are real, definite properties of the physical universe — properties whose existence becomes a robust part of real world history. (Quantum Darwinism [12] shows how quantum theory, when applied to idealized subsystems, can still be consistent with this intuition.) It is also intuitive that history always accumulates. Even though the observable records of history may evolve and decay, making most information about the past inaccessible for practical reasons, it would contradict the meaning of history if the information it encodes were to be changed other than through accumulation.

All the information represented by the ultimate history of our real world, all the outcomes that will actually occur throughout time, must be about something, some ultimate structure or foundation, $\mathcal{B}$. Parmenides arguments are sufficient. All that may ever be perceived in our universe thus stems from $\mathcal{B}$. This does not imply that all the detailed properties of $\mathcal{B}$ will actually, or even potentially, contribute to history or perceptions — $\mathcal{B}$ might hold far more information than can ever be encoded in one history, or even many histories. If so, which we presume to be the case, the real world will always be an unfaithful representation of $\mathcal{B}$. Regardless, we are free to think of $\mathcal{B}$ as the “One single source” that determines all possible outcomes, whether or not they are realized in our real world. Borrowing language from quantum field theory, we refer to $\mathcal{B}$ as the bare world — a pre-physical world, most detailed properties of which will never be manifested in the real world.

Not only is it pre-physical, the bare world is atemporal. There is no time independent of $\mathcal{B}$, and no time embedded within $\mathcal{B}$. It is unique and unchanging. As Parmenides wrote, it just is.

What should we expect $\mathcal{B}$ to be like? To avoid ambiguity, we assume that $\mathcal{B}$ can be characterized as a member of some class of objects that may be clearly defined using the language of mathematics. It must be unique and definite since there is no external time whose progress would admit changes to $\mathcal{B}$ or transition to a different $\mathcal{B}$. Generic structural properties, that admit discrete characterization, are required for $\mathcal{B}$ to produce a perceived universe with a small number of dominant matter species whose distinct properties and governing laws seem generic across (observed) space and time. To be the source of all the actual particles in the universe and their detailed interactions throughout time, $\mathcal{B}$ must have sufficiently numerous detailed (as opposed to generic) properties. Finally, $\mathcal{B}$ must be highly interconnected to give rise to the causal relationships and entanglement that make the physical universe function as an undivided whole.

A tentative proposal, elaborated in Appendix A, is that $\mathcal{B}$ is a connected 4-dimensional topological manifold, $\mathcal{T}$, with no geometry, fields or other decoration. The properties of interest will be the generic structural properties of 4-manifolds and the detailed, very complicated, specific connectivity of $\mathcal{T}$ — nothing more.
W-2: Real worlds

If \( \mathcal{B} \) is the foundation for all potential knowledge of the real world, then the real world perceived by any observer \( A \), at her time \( t \), must be a representation, \( \mathcal{R}_{A_t} \), of \( \mathcal{B} \). All such representations, \( \mathcal{R} \), will be incomplete, or unfaithful; but as the observer’s time advances they will hold / encode / represent progressively more complete information about \( \mathcal{B} \).

Since any conscious observer \( A \) must intuitively perceive herself as part of the real world, she and all her cognitive capability, memories, and mental models must be included within each \( \mathcal{R}_{A_t} \). So also must any other observers who may exist, or have existed, within the world that \( A \) perceives at time \( t \). \( A \)'s perceptions need not, and indeed will not, reflect all properties of (or information in) \( \mathcal{R}_{A_t} \). However, all of \( A \)'s actual perceptions up to and including time \( t \) must be attributable to \( \mathcal{R}_{A_t} \), while at any later time, \( t_1 > t \), \( A \) may have new perceptions that cannot be attributed to \( \mathcal{R}_{A_t} \).

It is not necessary for every representation \( \mathcal{R} \) of \( \mathcal{B} \) to contain observers, but observers are essential to our analysis and, being part of reality, they have no existence independent of the representations. We will simply not concern ourselves with representations of \( \mathcal{B} \) that do not include conscious observers. Accommodating the observations, intuitions and actions of observers severely constrains the features and relationships of the representations \( \mathcal{R} \).

As indicated above, one should think of each \( \mathcal{R} \) as the entire real world (which we identify with its history, regardless of what details are accessible or decipherable) at some present time of some actual or hypothetical conscious observer.\(^3\) Let \( \mathcal{R}_{t_1} \) and \( \mathcal{R}_{t_2} \) correspond to \( A \)'s two times \( t_1 < t_2 \). Since history always accumulates, \( \mathcal{R}_{t_1} \) must be an unfaithful representation of \( \mathcal{R}_{t_2} \), which we denote by \( \mathcal{R}_{t_1} \prec \mathcal{R}_{t_2} \) or \( \mathcal{R}_{t_2} \succ \mathcal{R}_{t_1} \). Now let \( B \) be a second observer such that \( A \) and \( B \) have, at their respective times \( t \geq t_0 \) and \( u \geq u_0 \), memories of each other. Since \( A \) and \( B \) are contemporaries, there must exist times \( u_1 \) and \( t_1 \) such that:

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\begin{align*}
\mathcal{R}_{B_u} &< \mathcal{R}_{A_{t_0}} \quad \text{for } u \leq u_1, \quad \mathcal{R}_{B_u} \neq \mathcal{R}_{A_{t_0}} \quad \text{for } u > u_1, \\
\mathcal{R}_{A_t} &< \mathcal{R}_{B_{u_0}} \quad \text{for } t \leq t_1, \quad \mathcal{R}_{A_t} \neq \mathcal{R}_{B_{u_0}} \quad \text{for } t > t_1.
\end{align*}
\]  \( (1) \)

If \( \mathcal{R}_{A_t} \neq \mathcal{R}_{B_u} \) and \( \mathcal{R}_{B_u} \neq \mathcal{R}_{A_t} \), then \( A \)'s real world at her time \( t \) encodes information about \( \mathcal{B} \) that is not encoded in \( B \)'s real world at his time \( u \), and vice versa. In that case, \( \mathcal{R}_{A_t} \) and \( \mathcal{R}_{B_u} \) are temporally unordered.

The representations \( \mathcal{R} \) are nested and overlapping in a manner analogous to observers’ past light cones (including the interiors) in general relativity. But each \( \mathcal{R} \) is here understood, instead of as a geometrical object in spacetime, as the entire real world (not just the obviously physical aspects) at the perceived present time of an observer. All information about \( \mathcal{B} \) held in each \( \mathcal{R} \) has been woven into the history and memories whose culmination is the observer’s present. Our deep sense of time is rooted in the partial order “\( \prec \)” that organizes this information.

Progress of time corresponds, identically, to accumulation of history, including new memories for the observer. Each addition to the history of the real world perceived by \( A \) represents information about \( \mathcal{B} \) that was not represented in \( \mathcal{R}_{A_t} \) at earlier times. Although it is eternal in \( \mathcal{B} \), this is always truly new information in \( \mathcal{R}_{A_t} \), and as such it is, in princi-

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\(^3\) As cautioned in Section 1, no inference should be made regarding a relationship of \( \mathcal{R} \) to, for example, the causal past of a point in geometrical spacetime, which involves a conceptual model of the physical world.
ple, unpredictable by $A$. This unpredictability is the underlying reason for uncertainty of quantum measurement results.

The status and distinct characteristics of the past, present and future are natural consequences of this worldview. The *present* is always the complete real world $\mathcal{R}$ perceived by a conscious observer. An observer’s perceived present exists, in essence, as the mental models and memories in her mind. The *past* is contained within the present as a conceptual means to efficiently organize and store (incomplete) information about $\mathcal{B}$. The observer’s conceptual past times, $t_0$, serve to label the representations $\mathcal{R}_{t_0}$ related by the partial order “≺” to the present $\mathcal{R}_{t}$, for $t_0 < t$. This ensures that a bit of information about $\mathcal{B}$ that is first captured as part of reality in $\mathcal{R}_{t_0}$ is efficiently maintained as part of the more complete representations that are perceived as later times. The *future* is a conceptual feature of our mental models that ensures the integrity of history will always be maintained as we continue to discover and consistently incorporate more information about $\mathcal{B}$ into $\mathcal{R}$.

That reality should be subject to dynamical laws can be attributed to the organizing principle— the hierarchical organization of history — applied to efficiently encode information about $\mathcal{B}$. To maintain consistency while incorporating additional information requires the detailed representation of history to be continually adjusted. Preservation of prior information necessitates deterministic classical dynamics and deterministic evolution of quantum states (which encode prior knowledge). Our predictions of the future are perhaps best thought of as conditions the future must satisfy to consistently preserve present history. Probabilistic quantum predictions must anticipate, and be compatible with, all possible consistent outcomes and the hypothetical new information about $\mathcal{B}$ they represent. Actual future outcomes must correspond to actual information about $\mathcal{B}$. While all hypothetical outcomes must be compatible with the generic properties of $\mathcal{B}$, most of those hypothetical outcomes will be incompatible with the actual details of $\mathcal{B}$; but there is no way of knowing, in advance, which ones to exclude.

One might view $\mathcal{R}$ as conceptually similar to a (crystallizing or evolving) block universe [15], but there are major differences worth noting. Most importantly, $\mathcal{R}_t \prec \mathcal{R}$ does not imply that $\mathcal{R}$ is an extension (or superset) of $\mathcal{R}_t$ but, instead, that all information about $\mathcal{B}$ encoded in $\mathcal{R}_t$ is also encoded in $\mathcal{R}$. The detailed form of the encoding will generally be different, and detailed disentangling of old from new information in $\mathcal{R}$ will generally not be feasible. Whereas block universe models are focused on matter in spacetime, such physics-specific structure is not assumed for $\mathcal{R}$ but appears instead in the model worlds of W-3.

An obvious question, now, is: what caused conscious beings to arise in the representations $\mathcal{R}$ of $\mathcal{B}$? The simple answer is: because they could; because $\mathcal{B}$ has sufficient structure and complexity to have this wonderfully rich, hierarchically linked family of representations within which consciousness could emerge. Most representations of $\mathcal{B}$ will not likely be so fruitful. But we and our perceived world are all the evidence we need to conclude that consciousness arises in at least one family of representations. There might be many other real worlds, even many other $\mathcal{B}$’s, but they will have no bearing on us.

Questions regarding free will are also relevant here. Do conscious beings have free will—the ability to consciously influence their future? Or is their future fully determined by $\mathcal{B}$ and free will just an illusion? I claim that free will is both necessary and natural. As new information is acquired and consistently integrated with prior information, each new present,
\( R_t \), corresponds to \( A \)'s progressively more complete representation of \( B \). But the order in which new information about \( B \) will become part of \( A \)'s real world is not determined by \( B \), nor by \( A \)'s present. Free will is a manifestation of the ability of \( A \), and other conscious beings who are part of \( A \)'s present, to influence the order in which previously unknown information about \( B \) is consistently incorporated into their reality, including their mental models thereof. This is roughly analogous to a conscious observer’s progressive development, by shifting their viewpoint and the illumination, of a mental model of an object (\( R \)) from the information encoded in a holographic plate (\( B \)). The big differences, here, are: (i) the conscious observer is part of \( R \); and (ii) there is so much information in \( B \) that only a tiny portion of it will ever be represented in \( R \), allowing different paths (viewpoint and illumination) to yield materially different outcomes.

Acceptance of free will leads to a further consistency challenge: a willful action of \( A \) at her time \( t \) must make compatible contributions to history for all observers \( B \), at their times \( u \) such that \( R_{A_t} \prec R_{B_u} \). Otherwise, history would become inconsistent. This leads to a somewhat idealist view, in which consciousness has a global, collective aspect rather than arising completely independently in each observer. With this view, we must recognize as illusion the intuition that we, as individuals, have completely separate psychological existences. I am not suggesting that there is no independence, only that its scope is limited. Our pervasive, underlying entanglement with each other and with our perceived physical environment — the globally distributed encoding of information about \( B \) — ensures that our individual willful actions have global effect.

**W-3: Model worlds**

The perceived real world, conscious observers, history and time emerge together as an incomplete representation of \( B \). Everything real is fully dressed — as opposed to bare or virtual. The robust, causally consistent history of the perceived universe is reflected in the content and organization of observers’ memories. In imagining the future, an observer preserves history while judging, and to a limited extent influencing, how history might be extended by unpredictable new information about \( B \).

The reality described above is intuitive, yet mysterious. As intelligent observers, we invent mathematical and interpretational models to rationally, efficiently, and reliably simulate selected features of reality. These models constitute W-3. While they simulate (features of) the perceived reality of W-2, valid models must also be consistent with and meaningfully, although perhaps not obviously, reflect the properties of \( B \). Reflecting the generic and specific properties of \( B \) and \( R \), models may take the form, respectively, of: (i) mathematical objects given some interpretation and subject to constraining laws (e.g., general relativity, quantum theory, and the standard model of particle physics), or (ii) instances of specific structure (i.e. particular solutions) within the context of a generic model (e.g., the concordance model of cosmology (\( \Lambda \)CDM)).

The laws of physics are understood here as constraints that apply within observers’ models, necessitated by the generic properties of \( B \) and the generic perceived features of the representations \( R \). Since they apply only to models, the laws of physics do not, in any way, constrain \( B \) or \( R \). But if a model is based in part on conjectures regarding the generic properties of \( B \), and then found to reliably simulate certain aspects of the real world, this
will give credence to the conjectures and justify their further use to explain the real world.

Concepts of time, 3-dimensional space, associated geometry, and matter whose spatial distribution evolves in time are entwined within our mental models of physical reality. These concepts are naturally reflected in the mathematical models of Newton, Maxwell, and Einstein, which reliably simulate aspects of reality within their respective domains of applicability. These deterministic models do not, however, simulate the addition of new information about $B$ into $R$.\(^4\) Quantum models of matter provide a mechanism—state reduction—to admit such new, unpredictable information.

Models are discussed in more detail in Section 3. The key message here is that models do not govern reality. They simulate, and thus represent, aspects of reality; but the representations are always unfaithful. This implies that all models, and hence all physical laws, have limited domains of applicability, which we need to discover and accept.

3 Models—Interpretation and Implications

Models are the product of science—created by intelligent beings who exist only as emergent features of the real world. Scientists develop mathematical and interpretational models to efficiently simulate certain aspects of real or hypothetical worlds. Theoretical constructs such as geometrical spacetime and quantum matter prove useful as elements of models because they can be mathematically precise and they can be interpreted in a manner that establishes reasonable correspondence with certain perceived properties of the real world. A theory defines the generic properties of a class of model worlds, whereas a particular solution corresponds to some specific model world.

Current physics theories typically represent information about the generic structure of $B$ in terms of: the identities of fundamental matter fields; their spacetime and internal symmetry properties; and the principles and laws that govern their interaction and evolution. Applications of a model often rely on prototype solutions, such as black holes or idealized quantum subsystems (elementary particles, nuclei, atoms, etc.). These prototypes are derived as abstract solutions of the model’s underlying theory, without reference to specific real world observations; they may be consistent with the generic structure of $B$ but do not hold any specific information about $B$.

Modeling of specific properties of the real world requires particular solutions of generic models, plus an interpretation that establishes reliable correspondence between the solution and those real world properties. Such solutions can then be interpreted as encodings of the subset of information about $B$ that corresponds to the pertinent real world properties. The concordance model of cosmology serves as a good example. The parameter values and interpretation that yield the best correspondence between the model and observations encode information about $B$ that is represented in $R_t$ as specific, coarse-grained properties of the real world—properties that are preserved in history from early times and for all known observers.

Like cartoons, models provide an idealized view of certain selected aspects of the real world. Every model thus has a limited domain of sensible correspondence with the real

\(^4\)The measured isotropy of the CMB implies that the entire observable universe is causally connected—there is no part of our universe, yet to be observed, whose corresponding prior information is not already represented in our history. Models of cosmic inflation were developed to explain how this could be.
world, even though the limits may not be obvious. Stretching a model beyond its domain, or simultaneous application of two incongruous models, may produce confusing or unreasonable results — hence the difficulty of extending quantum theory and general relativity into each other’s natural domain.

3.1 Time

The new conception of time in W-2 makes it necessary to revisit our interpretation of the correspondence between the times of classical and quantum physics models and an observer’s past, present and future in the real world. In particular, our interpretation must address how models simulate preservation of information that is already included in $R$ and how they simulate addition of new information to $R$ as time progresses.

Deterministic theories provide the means to simulate the representation of given, fixed information about $B$ in terms of robust properties of the real world — properties that are redundantly expressed as discussed in [12]. A quantum state $\psi$ always represents precisely the same (prior) information at all hypothetical times — past, present and future. To sensibly associate quantum models with specific properties (e.g., idealized quantum subsystems) of the real world we implicitly assume an association of quasi-classical, time-space-matter descriptors of the real world with corresponding self-adjoint operators on Hilbert space. (In the Heisenberg representation, unitary evolution is imposed through the time-dependence of the assumed associations between the quantum operators and the quasi-classical world, independent of $\psi$.) Existence of the quasi-classical world is a corequisite for such associations to be sensibly made. The associations define a correspondence between information represented in (quasi-classical) history and information represented in quantum states and operators. No actual quantum state can be prepared or known other than through the associations discussed above and the additions to history that reflect the state’s preparation and measurement. Moreover, all prior information is quasi-classical, having been collectively and non-locally entangled in the (mostly inaccessible) details of history.

Because they can neither add nor remove information, but only change the details of its representation at different times, deterministic theories cannot distinguish the present or identify a preferred direction of time. They are suited to modeling the evolving time-space-matter representation of already-represented information about $R$ in a manner that decouples it from the acquisition of new information and the growth of history. Such decoupling is required in all deterministic models, but it is an idealization with limited validity since it denies the growth of history. True progress of time always involves enriching $R$, and thus history, with unpredictable new information about $R$. Increasing information (and growing history) defines the preferred direction of time. New information will be manifested in the real world with an encoding that necessarily entangles new and prior information, and that no deterministic model can simulate. Only through a non-deterministic process can a model simulate the required introduction of new information.

Quantum state reduction is precisely the needed non-deterministic process. When we focus on a particular subject quantum subsystem, our working assumption is that time advances independently. But for this to be valid, some other subsystem(s) must undergo state reduction—clocks must tick—thereby progressively enriching history and increasing the algorithmic information content (or Kolmogorov complexity [16]) of the quasi-classical
quantities in terms of which the reduced subsystems and operator associations are defined. Each such state reduction will add at least one bit of information about $B$. Each new bit will correspond to one Planck unit of action ($\hbar$) that irreversibly changes the state of the quasi-classical universe. Since the Hilbert space of quantum subsystems must be defined and interpreted through reference to quasi-classical descriptors of the perceived world, the increasing complexity of the quasi-classical universe will also steadily increase the size of the total Hilbert space, of which a (now) time-dependent subspace corresponds to our subject subsystem.

By construction, any subject quantum subsystem represents certain limited prior information encoded in history at some reference present time. Attempting to assign likelihoods to different (past) histories consistent with this limited subsystem would be counterproductive, since our actual, full history, including the inaccessible details of its microstate, is what defines the complete present. (Our inability to resolve details or disentangle the present to unambiguously reconstruct the history of earlier times does not make present history any less definite.) Applying the Born rule (or, equivalently, Bayes theorem [17]) to determine the likelihood of potential future outcomes related to our subject subsystem is sensible, provided there is sufficient isolation so that the validity of the subsystem's unitary evolution will not be significantly impacted by entanglement with, and unpredictable state reduction of, other quantum subsystems. New information represented in our actual future, as it comes, will be reflected in the consistent encoding of its history; the likelihood of each potential history is identically the likelihood of the potential future to which it corresponds.

A notable prior attempt to understand physics in terms of steadily increasing information is Von Weizsäcker’s ur-theory [18–20]. Following the development of quantum information theory it was realized that urs are the same as qubits. The time operator in ur-theory is just the ur-number operator, which gives the number of bits. In our analysis, state reduction transforms qubits to Planck unit steps in action, whose random walk determines, with suitable coarse-graining, the quasi-classical action of the perceived world. The relation of the amount of accumulated information—the number of bits gained through elementary state reductions—to metrical time is monotonic, just as in ur-theory.

Interpretations of orthodox quantum theory (e.g., environment induced decoherence, quantum Darwinism, consistent histories) attempt to explain the apparently classical universe while avoiding state reduction as a distinct process. Collapse theories (see [21] and references therein) introduce state reduction as a stochastic nonlinear dynamical process that may involve gravity [22–24]. Penrose considered a possible entropic relationship between such state reduction and its influence on spacetime geometry [25]. In almost all these approaches, time is presumed to exist independent of the state reduction events; and none recognizes that state reduction events must add information that progressively accumulates. An exception is a simple model proposed by Pearle [26] in which, starting from a “nothingness” state, collapse events generate discrete time and space. Our position is that state reduction is the essential process through which time progresses and information is added to history. Since the information is nonlocally encoded in the inaccessible details of the microstate of the world, almost all state reductions are noticed only as the passage of time—something changed, enriching history.
3.2 Uncertain reality — matter and geometry

The dominant view of physicists today is that the universe is fundamentally quantum. Arguments based on environment-induced decoherence and coarse-grained histories show how an orthodox quantum model can lead to observations of matter that are apparently quasi-classical. Essentially, the environment gains sufficient information from its entanglement with a subsystem to fully describe the subsystem’s decohered, quasi-classical aspects. This information is claimed to be “somewhere in the universe” \cite{6, 27}. Of course, that claim relies on implicit correspondence between the model and the real world. Quantum Darwinism shows that, given such correspondence, this information will be accessible throughout the universe (subject to causal constraints), thereby ensuring that different observers will agree on quasi-classical outcomes — their perceived real worlds will be mutually consistent \cite{12}.

Most environment and apparatus degrees of freedom that are traced over to focus attention on a subject quantum subsystem are inaccessible to classical observers; their measurement is not feasible. The amount of inaccessible information encoded in the actual, but unmeasurable, microstate of the world associated with these degrees of freedom is referred to as entropy. Remaining subsystem degrees of freedom may be measured, with residual uncertainty, by entangling the subsystem with the apparatus and environment. The timing of such measurements depends on the apparatus; some subsystem degrees of freedom may be measured sooner than others. For example: independent measurements of the two particles of an EPR pair may localize their uncertain energy-momentum densities within limited spatial regions that have spacelike separation. Until a later spin measurement is performed on one of these particles, their carefully isolated collective spin state (which is known by virtue of the preparation process) remains bound to the pair but itself has no locale. Following the spin measurement, the reduced state represents both the prior information, encoded in the collective spin state, and new information encoded in the measured spin. Violation of the Bell inequalities is necessary because a spin measurement on the second particle must also preserve the same prior information.

General relativity establishes a mutual relationship between classical matter and spacetime geometry, with the idealization that both are characterized by smooth, real-valued tensor fields on a 4-manifold $\mathcal{M}$:

$$G_{\mu\nu}(x) = 8\pi G_N T_{\mu\nu}(x), \ x \in \mathcal{M},$$  \hspace{1cm} (2)

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor corresponding to the metric $g_{\mu\nu}$, and $G_N$ is Newton’s gravitational constant. (We use units in which $\hbar = c = k_B = 1$.) A cosmological constant term is not excluded, but left implicit in the stress-energy tensor, $T_{\mu\nu}$. Observations to date are consistent with $\mathcal{M}$ being simply connected. Equation (2) demands consistency of dynamical matter and spacetime geometry on any Cauchy surface and, when combined with deterministic equations of motion for the matter, constrains the evolution of geometry to encode precisely the same information at other times.

Even if there is no matter ($T_{\mu\nu} = 0$), use of real-valued fields implies that the information encoded in the geometry of a generic solution of the Einstein equations will be uncountably infinite. We have argued, however, that the amount of information represented by the real world equals the finite number of elementary state reductions since the origin of time. Generic spacetime geometries thus encode far too much information to be good models of
the real world. If we want a model that reflects the finite, growing information content of
the real world we must reconsider the geometrical spacetime interpretation.

Recall that $A$’s real world $\mathcal{R}_A$, for finite $t$, encodes a finite amount of information about
$\mathcal{B}$. The algorithmic information content of the corresponding model-space, expressed as
bits, should then be the number of elementary state reduction events in $A$’s entire causal past. Equation (2)
tells us that the information encoded in $G_{\mu\nu}$ (or $R_{\mu\nu}$) is identical to the information about
matter encoded in $T_{\mu\nu}$. Moreover, our “knowledge” of spacetime
geometry is entirely inferred from observations of matter, so it is reasonable to expect
that the information represented by geometry corresponds identically to all the information
represented by matter. This means that Weyl curvature components, which when combined
with Ricci curvature fully determine the metric geometry, must be non-zero only to the
extent required by causality and the Bianchi identities, $\nabla_\nu G^{\nu}_{\mu} = 0$. Any Weyl curvature
that is not required for causal consistency with the stress-energy tensor of matter sources
would encode information that has no relation to $\mathcal{B}$. Solutions of the Einstein equations
with source-free Weyl curvature, or with sources that persist to the origin of time, should
be considered non-physical. This includes primordial gravitational waves and primordial
black holes.

To properly model the nesting of real worlds discussed in Section 2, the field $T_{\mu\nu}(x)$, for
all points $x$ on and within the past lightcone of $A$’s perceived spacetime location $x_A(t)$, must
encode (a subset of) the information represented in $\mathcal{R}_A$. Since a given (nearly) isolated
quantum subsystem must have little influence on geometry relative to the the influence of
the rest of the universe, we adopt the decoherence approach and expand the stress-energy
tensor of equation (2) as:

$$T_{\mu\nu}(x) = T_{e\mu\nu}(x) + \langle \hat{T}_{\mu\nu}(x) \rangle_s,$$  

where $T_{e\mu\nu}(x)$ is the perceived quasi-classical stress-energy tensor of the environment (the
rest of the universe), excluding the quantum subsystem of interest, and the expectation
value:

$$\langle \hat{T}_{\mu\nu}(x) \rangle_s = \langle \psi_s | \hat{T}_{\mu\nu}(x) | \psi_s \rangle$$

(4)

corresponds to $A$’s prior knowledge of the subsystem’s stress-energy operator $\hat{T}_{\mu\nu}(x)$ and
state $\psi_s$.

The environment-subsystem split indicated in (3) can be freely chosen to treat as quantum
as much of the universe as can be effectively isolated (with minimal decoherence of its
quantum properties by the environment). No matter where the split is chosen, the combined
environment and subsystem will always represent exactly the same prior knowledge
and the same total stress-energy and geometry. In practice, however, for any subsystem
that can be effectively isolated the geometry will be overwhelmingly determined by $T_{e\mu\nu}$.
For all practical purposes, this dominance of the environment allows the needed association
to be made between $x$ and $\partial/\partial x$ in $\hat{T}_{\mu\nu}(x)$ and spacetime coordinates and inertial frames
defined by reference to quasi-classical matter of the environment.

In our new worldview, emergent geometry derives all its information from non-unitary
state reduction of quantum matter. There are no independent geometrical degrees of freedom.
State reduction causes $T_{\mu\nu}$ to evolve in a stochastic manner, but the fluctuations are
imperceptible because of the vast amount of information already encoded in the observable
universe. Existing information (prior knowledge) encoded in $T_{e\mu\nu}(x)$ is always preserved,
even though it does not fully determine the future. With only finite information, the representation of stress-energy and geometry in terms of real valued fields on a 4-manifold must be intrinsically uncertain, which is consistent with the quantum uncertainty of \( \langle \hat{T}_{\mu\nu}(x) \rangle \) that has been studied in the context of stochastic semi-classical gravity [28].

A Feynman sum-over-histories approach can be used to predict the likelihoods of future outcomes, but our new interpretation imposes some new restrictions:

- Allowed histories must have monotonically increasing information content. This includes both accessible and inaccessible (entropy) information.
- Each consistent history must be determined solely by its matter degrees of freedom and associated state reduction events, with the geometry implicitly matching the matter. Equation (2) allows determination of mutually consistent \( T_{\mu\nu} \) and metric, \( g_{\mu\nu} \), for each potential history.

For every allowable history, \( c \)-number descriptions of both matter and geometry are uncertain, due to the finite information content, but the uncertainties are correlated; moreover the number of Planck unit steps in the action between initial and final times (i.e., between the corresponding past light cones) should precisely equal the information added to history.

An exact correspondence between uncertainties of matter and geometry should carry over to perturbative calculations in quantum field theory. In particular, the compatibility between matter and geometry imposed by equation (2) should ensure that classical uncertainty of spacetime curvature, due to its finite information content, effectively cuts off the momenta of loop integrals to naturally eliminate ultraviolet divergences. This same effect, together with the exclusion of source-free Weyl curvature modes, should naturally limit the Casimir energy. Finiteness of information in the causal past of any spacetime point will also impose a natural infrared cutoff.

Traditionally, semi-classical coupling of geometry to quantum matter has assumed a \( c \)-number stress-energy tensor given by \( T_{\mu\nu}(x) = \langle \psi | \hat{T}_{\mu\nu}(x) | \psi \rangle \), where \( \psi \) is the pure, complete Heisenberg state of the world. But that formulation suffers from the measurement problem because it fails to explain the \( x \) dependence of the stress-energy operator. In the decoherence approach advocated here, the quasi-classical domain, now with finite information, is taken to represent the present real world. The quantum domain represents all potential futures allowed by the generic structure of \( \mathcal{B} \).

## 3.3 Information, entropy and entanglement

Modern studies of quantum information (quantum computing, quantum cryptography, etc.) rely on the assumption that quantum theory is the foundation of a fundamentally correct model of physical reality. Linkage, within that model, between quantum systems and spacetime geometry remains unexplained. The obstacle may be that quantum information theory treats quantum information as properties of real physical systems, within independently given space and time. Our perspective is that real physical systems, space, and time are secondary; the bare world, \( \mathcal{B} \), is primary. Dressed physical systems and corresponding spacetime geometry encode incomplete information about \( \mathcal{B} \) in a manner that compels progressively more faithful representation of \( \mathcal{B} \) and corresponding advance of perceived time.
Time, space and matter provide the mode of representation — they are subservient to the
information.

New information enters $\mathcal{R}$ as an apparently random event. This adds to the prior
information, causing a projective reduction in the space of potential futures. In a time-
space-matter model, the mode in which the new information is encoded expands from the
event at the speed of light. Therefore, for this information to be preserved over time its
encoding, in terms of mutually consistent geometry and matter fields, must be maximally
non-local.

Encodings of information from different events very quickly become highly entangled
in the time-space-matter model. With their non-linear coupling through equation (2), the
contributions of a vast number of modes give spacetime geometry its seemingly definite,
though still uncertain, form. Almost all matter degrees of freedom are inaccessible to direct
observation, and are thus identified as entropy; but their specific microstate determines the
details of spacetime geometry. Entropy associated with spacetime geometry, such as the
Bekenstein-Hawking entropy [31], should then be identified with the global entanglement
entropy of the corresponding matter. See [32] for an alternative argument leading to the
same conclusion.

As an example, consider the gravitational collapse of a star. The world of a distant
observer, $A$, just after witnessing the rapid collapse to a “dark star” will encode only
slightly more information than it did just before the collapse. The corresponding spacetime
geometry of $A$’s past will be comparably uncertain before and after; the total entropy of
$A$’s world will thus also grow just a little. Over a very long time, from $A$’s perspective,
progressive state reduction of the matter fields throughout the universe will increase the
common information content of the matter and geometry. As the geometry near the dark
star approaches that of an idealized Schwarzschild black hole the information and entropy
attributable to it will grow toward the limiting value, $S_{BH} = 4\pi G N M^2$. Since we have
argued that there are no primordial black holes, the information content of astrophysical
black holes, such as Sgr A*, must come primarily from the baryonic matter that collapsed
to form them.$^5$ The corresponding entropy will be more than 20 orders of magnitude less
than $S_{BH}$ for a similar mass [33, 34]. This contrasts with the naïve notion that the entropy
suddenly increases to $S_{BH}$ during the collapse process. Of course, the difference is due to
the horizon and interior of a black hole being forever in the unknowable future of all distant
observers. (Even an observer who falls into a black hole will never see most of the event
horizon or detect a burst of entropy at the horizon.)

Given information is accessible to multiple observers only if its encoding takes the form
of quasi-classical records with associated pointer observables. Of necessity, many different
perspectives of the same detailed configuration will yield compatible quasi-classical obser-
vations by different observers and at different times. Highly redundant encoding of pointer
observables [12] makes this possible, at the cost of much more information being inac-
cessible — i.e., entropy. Advancing time increases both accessible information and entropy.
Projecting backward, one must conclude that the universe at earlier times had progressively

$^5$Since information is encoded in the global entanglement of all matter and the corresponding global geom-
etry, it is formally incorrect to say that information (or entropy) are contained in a star or other matter.
Nonetheless, we can attribute the information to the star by imagining that removing the star and the past
influences of all its matter and radiation would yield another universe with this much less information.
less information and entropy, with both going to zero at the origin of time.

Studying the mutual entanglement of the modes that encode information associated with a quantum subsystem becomes practical only when the entanglement of those modes with the environment (including any apparatus) is relatively insignificant. Given such isolation from the environment, spatial location is excluded from consideration as a property of the quantum subsystem. Entanglement of the subsystem modes is then also freed from the nonlinearity imposed by coupling to spacetime geometry. The internally-relevant subsystem properties represent prior knowledge of the observer about the nature of the subsystem, such as the number and type(s) of particles and their total momentum and angular momentum. This knowledge is obtained entirely through the quasi-classical processes used to prepare or otherwise identify the subsystem.

Projected future states $\psi_s(t)$, obtained by unitary evolution of the prepared state $\psi_s = \psi_s(t_0)$, encode exactly the same prior information, with the implicit assumption that the environment evolves in a similar unitary manner. But we contend that advancing real-world time is always associated with new information and corresponding non-unitary changes to the environment. Without those changes, time would not advance (as in the problem of time [35]). Because no subsystem can be completely isolated, new information about $B$ will eventually (depending on the degree of isolation) be encoded as an event that involves significant entanglement between the subsystem and its environment. The result is state reduction of $\psi_s$, non-linear encoding of the new information in the quasi-classical environment, and new prior knowledge to allow identification of a modified subsystem $\psi_s'$. Although the new knowledge, and the reduced state $\psi_s'$, will be consistent with the possible outcomes based on $\psi_s(t)$, the latter has no influence on the actual outcome.

Since, conceptually, the subsystem can be expanded to treat as much as needed in a quantum manner, our interpretation should not lead to conflict with any current experimental tests of quantum theory. However, our claim of finite information content and our epistemic interpretation of quantum states may have significant implications for quantum computing and other aspects of quantum information technology.

### 3.4 Cosmology

Inflationary cosmology models, commonly involving scalar “inflaton” fields and designer potentials, were developed to explain how the present universe could have become so flat, homogeneous and isotropic, without extraordinary fine tuning of initial conditions. Although nothing more than a sketch can be provided here, it is expected that our new worldview will point to a more natural replacement model for inflation that produces the required conditions, at the end of the “formative” period, for subsequent astrophysical evolution and observations.

We have argued that our real world, which we model as spacetime and the matter within it, is nothing but a representation of information about $B$. It does not objectively exist. It started with no initial data — zero total information, including zero entropy. We should thus expect a cosmological model of the early universe to have geometry and matter that emerge from nothing (physical or real). With no a priori physical laws (independent of

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6Complete disentanglement from the environment is not possible. Indeed, some continued coupling to the environment is essential in order to allow preparation and observation of the subsystem.
the mutual relationship of geometry and matter suitable for representing $\mathcal{B}$ will need to be determined in a generalized Machian manner. All physical “constants”, such as $G_N$, the masses of elementary particles, and the strengths of interactions, should obtain values based on the generic properties of $\mathcal{B}$ and the specific information encoded in $\mathcal{R}$.

Time and the amount of information about $\mathcal{B}$ represented by $\mathcal{R}_t$ increase in unison; so, as was done by Von Weizsäcker, the (non-metrical) parameter $t$ can be chosen equal to the number of bits of information. With that choice, $\mathcal{R}_1$ must be the origin of the real world. But with just one bit, there would be no quasi-classical spacetime geometry or matter; such concepts require sufficient information to support the environment-subsystem decomposition discussed above. To be viable, the environment must incur only minor perturbation while adapting to the state reduction that encodes each new bit.

The mutual uncertainty of geometry and matter, discussed in Section (3.2), must have been extreme at very early times. An initial formative period was thus essential, at the end of which the information represented by $\mathcal{R}_t$ was sufficient for: (a) environment-subsystem decomposition to be viable; (b) subsystems to be modeled using quantum theory; and (c) quasi-classical spacetime and matter to be related by equations (2) and (3). Of course, there were no conscious observers during the formative period, so any characterization of it is based solely on the requirement of consistency with presently accessible historical records. Looking back in time from the present, spacetime geometry does not reach a singular origin; instead, geometry becomes so intrinsically uncertain that its utility fades to nil.

Since the information in $\mathcal{R}_t$ vanishes as $t \to 0$, the notion of fine tuning never arises. We cannot say the initial geometry was flat, homogeneous, isotropic, or otherwise; it was simply nondescript. Addition of information to $\mathcal{R}$ is manifested in physical models as state reduction and progress of time. Since state reduction events are stochastic, they should appear as random perturbations of the emerging, but still highly uncertain, geometric environment. (Individual perturbations will be masked by the geometric uncertainty, thus never discernable.) Most information will contribute to entropy, with coarse-grain properties yielding the observables associated with a quasi-classical “environment”. Each emergent mode requires a larger scale environment (matter and geometry) within which it can be meaningfully defined, so emergence must start at a global scale and proceed iteratively to smaller scales, eventually resulting in an initially scale-invariant spectrum of uncorrelated modes. Ultimately, the non-linear coupling of quasi-classical modes will cause growth of density perturbations, indicating substantial completion of formation of the geometrical environment and the matter it contains.

In parallel with geometry, matter will emerge progressively. Species, masses, coupling constants, modes within species, and collective modes will acquire dressed physical status according to the generic and specific structure of $\mathcal{B}$. The non-zero stress-energy density of quasi-classical degrees of freedom, which makes them observable, is exactly reflected (in value and uncertainty, through equation (2)) in the spacetime geometry.

The formative period produced initial populations of many identical particles of each significant species. Given these populations, further evolution encodes new information about $\mathcal{B}$ in terms of more detailed interactions and distribution of the given particle fields — the formation of nuclei, atoms, molecules, dust, stars, galaxies, etc. Since information and entropy can only ever increase, the universe will never return to its initial state. Should $\mathcal{B}$ have finite information then the ability to improve its time-space-matter representation
may eventually be exhausted, resulting in an effective end of time.

3.5 Unification

For over fifty years, physicists have presumed the quantum world is fundamental and have attempted to derive the classical world from it. The position advocated here is that neither quantum nor classical has priority; a unified model world must have complementary, mutually supportive classical and quantum aspects. Their common foundation is the bare world $\mathcal{B}$. The real world that emerges is quantum and classical.

The formal mathematical frameworks of both quantum theory (including quantum field theory) and general relativity are idealizations, each of which ignores the other. The remarkable success of the idealized, independent theories cannot be disputed. However, the puzzles, paradoxes and infinities associated with them may well arise from failure to respect their complementarity.

Theoretical predictions derived from the mathematical formalisms of idealized theories must be considered with caution. The enormous entropy of Schwarzschild black holes may have no relevance to any conceivable observations or testable predictions. The dramatic speed-up of quantum computers may disappear when the limits of finite information content and the reality of state reduction are imposed.

Our new worldview, with $\mathcal{B}$ as the foundation, reveals the origin and complementary nature of classical and quantum. It explains why the universe started with zero entropy, and why the coupling of geometry and matter must be semi-classical. Semi-classical coupling forces departure from the linearity of quantum theory, demands examination of real physical states rather than abstract states, and requires real state reduction in order to enrich history. Deciphering the generic structure of $\mathcal{B}$ (with $\mathcal{T}$ as a tentative proposal — see Appendix A) may reveal why observed matter can be modeled by the fields and interactions of the standard model of particle physics. It may even reveal the nature of dark matter and dark energy.

4 Summary

Intuitive arguments regarding information, conscious perception, and history have led us to consider the world at three-levels: W-1 — the bare world $\mathcal{B}$ that is the unique atemporal, pregeometric foundation of everything that may ever be perceived; W-2 — real worlds $\mathcal{R}$ in which we perceive ourselves and everything that seems real at each present time; and W-3 — model worlds invented by scientists to efficiently simulate aspects and relations of the real worlds.

Progressive inclusion of information about $\mathcal{B}$ into its emerging, hierarchically nested representations $\mathcal{R}$ generates consistent history and gives rise to the concept of time. Dynamical evolution of the time-space-matter encoding of history must accommodate new information while preserving all prior information. The present always corresponds to all of accumulated history, with the past implicit in its encoding. Possible futures must preserve information encoded in the present, while adding unpredictable new information. Acquisition of new information necessitates both quantum uncertainty and state reduction.
The bare world must have sufficiently complicated generic and specific structures to account for both the generic properties of matter and the large number of particles and interactions in the observed universe. To consistently preserve history, the quasi-classical, geometric structure of the perceived real world must encode most information about $\mathcal{B}$ in a form so globally entangled that the bits become individually inaccessible (entropy). While most entanglement remains implicit, because observation of separate degrees of freedom is not viable, entanglement can become explicit when studying quantum subsystems whose few degrees of freedom are entangled more strongly with each other than with the environment.

As models of $\mathcal{R}$, general relativity and quantum theory are both idealizations whose limits must be understood in order to avoid nonsensical conclusions. Their integration requires adaptation of both. Adopting a decoherence approach to semi-classical coupling allows both matter and geometry to have exactly corresponding uncertainty as they jointly represent finite information about $\mathcal{B}$. Geometry can have no information, or degrees of freedom, independent of matter. Because the information is finite: geometry cannot be resolved at very early times and at very small distances; path integrals become finite sums; and calculations should give finite results in both ultra-violet and infra-red limits.

The fields, symmetries and interactions of observed elementary particles must have their origin in $\mathcal{B}$. To illustrate with a specific example, it is conjectured, in Appendix A, that $\mathcal{B}$ is a complicated 4-manifold $\mathcal{T}$ — the topoverse — with no geometry or other decoration. The ability to represent $\mathcal{T}$ as a labeled graph, with edges corresponding to prime 3-manifolds and vertices to elementary cobordisms, points to correspondence with Feynman graphs. This makes the conjecture plausible.

Highly compressed encoding of accessible features of history as the mental models of perceptive creatures is central to their consciousness (at whatever level they function). It is the conjunction of time, history and mental models that distinguishes the representations $\mathcal{R}$ from other, uninteresting representations of $\mathcal{B}$.

Acquisition of information and development of mental models is an interactive process. We actively participate in our perceived reality. We control our bodies and the world we inhabit sufficiently to collectively investigate, formulate, record, and debate fundamental ideas regarding the world. We refine our models by acting on the world and sensing its response. Within bounds — consistent with our limited present knowledge, a prohibition on revising history, and severely limited scope and capacity — we are able to consciously influence our future.

Contrasting with the perception that we can act independently on the world is the obvious requirement for mutual consistency of the perceptions of all observers. Such consistency can be understood only if consciousness is actually a single, global phenomenon.\(^7\) The perception of individuality is then a robust illusion, just as quasi-classical particles seem independent even though their apparent localization relies on their global entanglement with the rest of the world.

Although I have assumed that $\mathcal{B}$ has definite mathematical structure, it is possible that no mathematical model can fully simulate the real world. At a minimum, mathematical formalism must be devised / found to describe geometry and fields whose uncertainty reflects their finite information content and the unpredictability of new information that must

\(^7\)Reiner Hedrich has proffered the technical term monistic panpsychism for this collective consciousness.
accompany progress of time. Applying such formalism should then enable resolution of the major puzzles of quantum physics, general relativity, and their integration.

Reality, as Leibniz insisted, can indeed be found in “One single source”. The “interconnection of all things with one another” originates in that source and is maintained in the unity of history, the alignment of perceptions, and the global integration of classical and quantum models of the world.

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A Topology as foundation?

The belief that gravity must be a quantum phenomenon, just like matter fields, led Wheeler to propose that vacuum fluctuations of the geometry of space could imply topology changes at the Planck scale, and foam-like paths through superspace [36, 37]. Exciton-like superpositions involving many different topologies might be manifested as emergent particles of matter; field lines trapped in the topology could yield charges [38]. Although the original proposal involved geometry changes driving topology changes, Wheeler presented a broader vision of some unknown pregeometry from which the geometrical world would emerge. The history and numerous proposals regarding pregeometry are reviewed in [39]. Careful analysis of pregeometry proposals reveals, however, that rather than emerging from some fundamentally distinct structure, geometry and time have generally been built-in [40].

The bare world, $\mathcal{B}$, of W-1, seems a better fit for Wheeler’s notional pregeometry. Unlike earlier proposals, the perceived geometry of space and time are not built-in to $\mathcal{B}$; instead they are genuinely emergent in W-2. As first suggested in [41], a particularly simple proposal for the structure of $\mathcal{B}$, inspired by spacetime foam, is as follows:

**Conjecture 1 (Topoverse):** The bare world, $\mathcal{B}$, is a unique, connected, smooth 4-manifold, $\mathcal{T}$, with complicated global topology but no inherent geometry, fields, or other decoration.

To emphasize its topological nature and that its identification with $\mathcal{B}$ is only tentative, I shall call $\mathcal{T}$ the topoverse. If the conjecture is true then $\mathcal{B} \equiv \mathcal{T}$.

As the pregeometric foundation, the topoverse has several novel features: (i) $\mathcal{T}$ is unique, whereas other proposals (e.g. [42, 43]) consider Feynman sums over all or large classes of 4-manifold topologies; (ii) the topological connectivity of $\mathcal{T}$ serves as the complete foundation for the real world throughout time, whereas other proposals assume additional structure, such as geometry, time, fields, group representations or causal structure, while generally ignoring the global topological information; and (iii) particles and quantum phenomena emerge naturally from the structure of $\mathcal{T}$, as we shall see below.
A.1 The topoverse as a labeled graph

Points and local neighbourhoods in $\mathcal{T}$, being all equivalent, contribute no information to the real worlds of $W-2$. But the vast quantity of non-local, relational information residing in the global structure of $\mathcal{T}$ underlies the representations $R$ and their partial order “≺”. If we can discover an invariant way of building up the global structure of $\mathcal{T}$ from elementary components then that will reveal generic properties of its global connectivity that repeatedly occur in the specific detailed structure of $\mathcal{T}$. We approach this by first finding an invariant decomposition of 3-manifolds, and then considering how $\mathcal{T}$ can be ( quasi-locally) decomposed as a stack of compact 3-manifolds (and their elementary cobordisms).

The connected sum, $M = M_1 \# M_2$, of any two $n$-manifolds is constructed by cutting an open cell out of each of $M_1$ and $M_2$ and identifying the resultant $(n-1)$-sphere $(S^{n-1})$ boundaries via a homeomorphism. The $n$-manifold $M$ is independent of the choice of open cells. The connected sum operation is commutative and associative.

An $n$-manifold is said to be prime if $M = M_1 \# M_2$ implies that at least one of $M_1$, $M_2$ is homeomorphic to $S^n$. The only compact, prime 2-manifolds are $S^2$, $T^2$ (the torus), and $P^2$ (the real projective plane). By contrast, there is a countably infinity variety of compact, prime 3-manifolds. Progress on their classification, in terms of orientability, spinoriality, chirality and other properties, is reported in [44, 45]. The invariant decomposition of 3-manifolds that we need is provided by the following result.\footnote{We shall be interested only in those compact 3-manifolds that are also closed, with no boundary.}

**Theorem (3-Manifold Prime Decomposition):** Each connected compact 3-manifold can be uniquely expressed as a connected sum of a finite number of prime factors. [46, 47]

Counter-examples demonstrate that not all 4-manifolds admit a prime decomposition, although a weaker result has been obtained for connected compact oriented 4-manifolds [48]. For that restricted subset, decomposition is unique only up to connected sum with any number of closed, simply connected 4-manifolds. With its unrestricted, complicated topology, $\mathcal{T}$ will, by default, be non-orientable and have no prime decomposition. But the construction below shows how the prime decomposition of 3-manifolds can be leveraged to represent $\mathcal{T}$ as a labeled graph.

Let $g$ be any smooth, positive-definite Riemannian metric on $\mathcal{T}$ and use $g$ to define $d : \mathcal{T} \times \mathcal{T} \rightarrow \mathbb{R}$ as the minimum metrical distance between each pair of points $x, y \in \mathcal{T}$. Select an arbitrary point $O \in \mathcal{T}$ and define

$$f(x) = d(O, x)$$

$$\Sigma_r = \{ x \in \mathcal{T} | f(x) = r, \ r > 0 \}. \quad (A-1a)$$

Choose $r_0$ such that $\Sigma_{r_0}$ is a 3-sphere bounding a solid ball containing $O$. Because $\mathcal{T}$ has complicated global topology, there must be values $r_i > r_{i-1}$, $i \in \mathbb{N}$, such that every $\Sigma_{r_i}$ is a compact 3-manifold but $\Sigma_{r_i}$ is not homeomorphic to $\Sigma_{r_{i-1}}$. Adjust $g$, if necessary, so that critical levels $r$ at which $\Sigma_r$ becomes singular are nondegenerate and $f$ is a Morse function on $\mathcal{T}$ [49, 50]. Furthermore, adjust $g$ such that there is only one critical level $r^c_i$ of $f$ between each $r_i$ and $r_{i-1}$, and all non-essential critical points are eliminated. Each region

$$\mathcal{T}_i = \{ x \in \mathcal{T} | r_{i-1}^c + \epsilon < f(x) < r_i^c - \epsilon \}, \quad (A-2)$$
where $\epsilon$ is infinitesimal, now has the product topology $\Sigma_r \times \mathbb{R}$.

Finally, adjust $g$ so that the prime summands of each $\Sigma_r$, $r$ not a critical level, are separated from each other and confined within small diameter cylinders in each $T_i$ and, where possible, the cylinders extend smoothly across the critical levels. At each critical level the topology change from $\Sigma_{r_{i-1}}$ to $\Sigma_r$ will involve only a small number of prime summands, and the enclosing cylinders for these will naturally meet at the critical level. This leaves a geometrical representation of $T$ that has the form of a graph, $G_T$. Edges of $G_T$ are the above cylinders, shrunk to infinitesimal radius and labeled according to the enclosed prime 3-manifold summand. Nodes or vertices of $G_T$ are the critical points at which topology changes.

The building blocks of topology change in $G_T$ are elementary 3-manifold cobordisms involving only a small number of prime summands. Selection rules that determine what combinations of primes may meet at a critical point arise from the simple requirement that $T$ be a 4-manifold. The abstract graph, with its labeled edges, provides a representation of $T$. The graph, or finite connected subsets of it, can be embedded in $\mathbb{R}^4$; almost every embedding will be a regular embedding, such that the inclusion map for each edge is $C^\infty$. The following seems plausible, and would lend credibility to Conjecture 1:

**Conjecture 2:** The graph $G_T$, or some reduced graph constructed by grouping the elementary cobordisms into equivalence classes, is uniquely determined by $T$.

If, for one or more $i > 1$, $\Sigma_{r_i} \equiv S^3$, then $G_T$ may have multiple disconnected subgraphs. In that case, our real world will represent just one highly complicated, connected subgraph while other subgraphs may correspond to other real worlds. We shall not concern ourselves with the possibility of many bare/virtual worlds, and will thus assume that $G_T$ has just one connected component.

Although the above construction was facilitated by an assumed metric, the graphical representation of $T$ carries only topological information and will exist independent of any geometry. The topology of $\Sigma_r$ can change only in a discrete manner at critical points. But there is nothing inherent in the topology to allow those critical points to be localized in $T$—interesting topological features are fundamentally non-local. Within any connected subgraph, however, one can define the topological separation of any two edges $\alpha, \beta$ as the minimum number of essential cobordisms between $\alpha$ and $\beta$.

A theorem by Rohlin [53] establishes that each closed 3-manifold is the boundary of some 4-manifold. This also follows from proofs, for both orientable and non-orientable cases, that any closed 3-manifold may be obtained by doing Dehn surgery (see [54, 55]) on a link in either $S^3$ or $P^2 \times S^1$ [56–58]. A key implication is that, given any two closed 3-manifolds $S_1, S_2$, the connected sum $M_1 \# M_2$ of the two 4-manifolds they bound is a cobordism between them. Thus all closed 3-manifolds are in just one cobordism class. It is important to recognize, however, that even the simplest cobordism between $S_1$ and $S_2$ may be a composition of many elementary cobordisms.

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9 Most cobordisms involving $S^3$, which is the identity under $\#$, will be non-essential, and are thus eliminated in the construction of $G_T$ by suitably deforming the geometry.
10 Aspects of this problem are addressed in [51], with the focus on lower dimensions. Konstantinov [52] applied Morse theory in 4-dimensions.
A.2 Quantum matter and geometrical spacetime

The ability to represent the topoverse as a graph makes plausible the hypothesis that prime 3-manifolds and their elementary cobordisms (or equivalence classes thereof) give rise to corresponding real world matter fields and their interactions. Distinct matter fields represent distinct prime 3-manifolds (or equivalence classes); interaction vertices correspond to the allowed elementary cobordisms (or equivalence classes). The discrete properties and interactions of the various matter fields reflect the generic properties of 4-manifolds, while the specific matter content and history of the observed universe correspond to the actual detailed structure of \( \mathcal{T} \) (and \( \mathcal{G}_T \)).

With such correspondence, \( \mathcal{G}_T \) can be thought of as the complete, pregeometric, bare or virtual Feynman graph of the world or, equivalently, the bare Heisenberg state of the world. Quantum field theory and general relativity provide the mathematical frameworks through which the correspondence is transformed into a time-space-matter model of the physical aspects of the representations \( \mathcal{R} \) of \( \mathcal{T} \). With this model, the mutually-consistent properties of the dressed or physical matter we perceive provide, in terms of geometry and fields on a topologically simple manifold (e.g. \( \mathbb{R}^+ \otimes \mathbb{R}^3 \)), a very coarse-grained representation of the complicated global connectivity of \( \mathcal{T} \).

Dressed quantum states of identifiable physical subsystems are projections of \( \mathcal{G}_T \) into our present real world, with observation-based encoding in terms of geometrical spacetime and physical matter. But these projections (and their associated Hilbert space and operators) exclude most of the information in \( \mathcal{G}_T \). The excluded information, since it has not yet been incorporated into \( \mathcal{R} \), can have no location in our time-space-matter models of \( \mathcal{R} \). Progressive incorporation into \( \mathcal{R} \), and our models, of this previously un-represented information involves consistent updating of the spacetime geometry and physical matter. Through this process, information about \( \mathcal{T} \), which might be thought of as quantum information, becomes represented as part of history and preserved by causal evolution. As the states of quantum subsystems are reduced, real world history is enriched. This, in turn, makes new dimensions available for the Hilbert space of physical subsystems.

Our models employ spacetime geometry to provide the conceptual (and mathematical) arena within which quantum matter fields and their interactions can be placed into sensible correspondence with perceived reality. While geometry and matter emerge and evolve in a mutually consistent manner, all their information content originates from the topology of \( \mathcal{T} \) and hence the quantum matter. Spacetime geometry has no relevance without matter, and no quantum characteristics beyond those inherited from matter.

My conjecture that \( \mathcal{B} \equiv \mathcal{T} \) was largely inspired by the naïve similarity of \( \mathcal{G}_T \) to a Feynman graph. But classification of prime 3-manifolds, determination of their elementary cobordisms, and making suitable associations with observed matter fields remain major challenges for future work to validate the conjecture. Whether the conjecture holds up or not, \( \mathcal{T} \) and \( \mathcal{G}_T \) illustrate well the global connectivity, and the kinds of generic and specific properties, that \( \mathcal{B} \) must have.

A.3 Holographic emergence of the real world

The countably infinite variety of distinct prime 3-manifolds and of elementary 3-manifold cobordisms allows \( \mathcal{T} \) (and \( \mathcal{B} \)) to have sufficient generic and detailed structural information.
Any observer’s evolving real world, \( \mathcal{R}_t \), will encode only a small fraction of the details. Like the progressive development of the image obtained from a hologram, the real world will start as a very fuzzy representation of \( \mathcal{T} \) as a whole. Development of history adds progressively more information about \( \mathcal{T} \), as finer detail, at later times \( t \).

Simplistic geometrical representations of \( \mathcal{T} \) can be obtained by mapping the graph \( \mathcal{G}_T \) into Minkowski space. But, due to the complicated connectivity of \( \mathcal{G}_T \), for all such detailed mappings the images of the graph edges will necessarily have no consistent relationship to the local null-cone structure; and labels will be needed to indicate which prime 3-manifold to associate with each edge.

By contrast, our view of the real world at large scale has physical matter — radiation, dust, stars, galaxies, clusters — tracing out timelike or null geodesic paths in a spacetime whose curvature reflects the stress-energy-momentum of the matter, in accordance with the Einstein equations. At somewhat smaller scales, direct (non-inertial) interactions — for example, electromagnetism — cause geodesic deviation that can be rationally explained in terms of intrinsic properties of the matter.

To achieve such a rational relationship between matter and geometry, our geometry-and-field representation of \( \mathcal{G}_T \) must employ a coarse-grained view, suppressing most details of the bare structure. The macroscopic geometry of the perceived world then provides a locally-stable arena within which finer details of \( \mathcal{G}_T \) can be encoded as the observed small-scale states and collective dynamics of matter. But these representations will always be limited — most details of \( \mathcal{G}_T \) will remain at the bare level, collectively influencing real world history yet resistant to explicit, rational geometrical representation and thus inaccessible to direct perception.

The interference pattern, recorded on the photographic plate of an optical hologram, holds the primary information that gives rise to the the 3-d virtual image perceived by a viewer. In our proposed worldview, the bare world \( \mathcal{B} \equiv \mathcal{T} \) takes the place of the interference pattern; the virtual holographic image is replaced by the real world \( \mathcal{R} \). Holographic emergence of the real world occurs on two levels. First is emergence of the universe and its coarse-grained / large scale geometry, differentiation of matter species, and formation of structure. Second is the ongoing development of history through quantum evolution. Models for each of these were discussed in Section 3.

The first level is like formation of an image by viewing a holographic plate through a low-pass (i.e. long wavelength) spatial filter (e.g., a defocused lens). The viewer will be unable to discern image details at scales shorter than the filter’s cutoff wavelength. If that wavelength is initially comparable to the width of the holographic plate, then the image will be a formless blur. Shifting the cut-off to shorter wavelengths (by gradually focusing the lens) makes the image progressively sharper and reveals previously unseen structural details. Once all details of the interference fringes in the holographic plate are resolvable, the viewer can develop a more complete perspective of the 3-d image by varying her vantage point and the illumination — this corresponds to the second level of emergence.

The above analogy is useful, but not perfect. Unlike the external viewer of a hologram, we construct our collective view of the real world from within. Since \( \mathcal{B} \) has topological information only, not only does \( \mathcal{R} \) gain detail as information about \( \mathcal{B} \) is added, the entire geometrical time-space-matter mode of representation is completely emergent.
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