A phenomenological neutrino mass matrix is proposed to explain the solar and atmospheric neutrino deficits and to present the neutrino as a candidate of hot dark matter in the $3\nu_L + 3\nu_R$ framework. The realization of mixing angles which can explain the solar and atmospheric neutrino problems is taken as the first criterion in this construction. The differences among neutrino mass eigenvalues are introduced as a perturbation. In this scheme the structure of a charged lepton mass matrix is not severely constrained by the solar and atmospheric neutrino data.
In both particle physics and astrophysics there are many indications for massive neutrinos. The deficiency of solar neutrinos\(^1\) and atmospheric neutrinos\(^2\) has been suggested to be explained by \(\nu_e - \nu_x\) and \(\nu_\mu - \nu_y\) oscillations, respectively. The predicted neutrino masses and mixings from these observations for the solar neutrino problem\(^3\) are

\[
\Delta m^2_{\nu_x\nu_e} \sim (0.3 - 1.2) \times 10^{-5}\text{eV}^2, \quad \sin^2 2\theta \sim (0.4 - 1.5) \times 10^{-2}, \tag{1}
\]

and for the atmospheric neutrino problem,\(^4\)

\[
\Delta m^2_{\nu_y\nu_\mu} \sim (4 - 6) \times 10^{-3}\text{eV}^2, \quad \sin^2 2\theta \gtrsim 0.85. \tag{2}
\]

It was also recently suggested that a cold + hot dark matter model agrees well with astrophysical observations if there is one neutrino species with \(\sim 5 \text{ eV mass}\).\(^5, 6\)

An interesting feature of these indications is that they require wide range mixing angles, in particular, a maximal mixing among different neutrinos in addition to hierarchically small mass eigenvalues. Although this smallness of masses is thought to be explained by the seesaw mechanism\(^7\) in general, the hierarchy of masses and the mixing structure will be completely dependent on models.

Our aim in this paper is to propose a phenomenological neutrino mass matrix which can explain both of solar and atmospheric neutrino deficits. We additionally require neutrinos to be hot dark matter in a cold + hot dark matter scenario. In the study of such a neutrino mass matrix we believe that the realization of these mixing angles is the most important clue, and we adopt the following strategy. We first prepare a mass matrix which can realize the required mixing structure among different neutrinos in the zeroth approximation in the seesaw framework. After this, we add the mass perturbation needed to induce the hierarchical masses without disturbing this mixing structure.

In this direction the author has proposed a neutrino mass matrix in the \(3\nu_L + 2\nu_R\) framework.\(^8\) In the present study we extend this to the \(3\nu_L + 3\nu_R\) case. The motivation of this extension is based on the consideration of a relation to the charged lepton mass matrix. The mixing matrix appearing in neutrino oscillation phenomena is generally influenced by the charged lepton mass matrix. In the five neutrinos model this is the case.\(^8\) The extension to six neutrinos makes it possible to avoid this situation, as discussed later. It

* There are also the large mixing solutions. However, we do not consider these solutions in this paper.
is also encouraging for this extension that the $3\nu_L + 3\nu_R$ framework seems to be natural from the viewpoint of the generation structure of other quarks and leptons.

We assume the following effective mass terms in the $3\nu_L + 3\nu_R$ framework

$$-\mathcal{L}_{\text{mass}} = \sum_{i=2,3,4} m_i \bar{\psi}_L i N_R + \sum_{i=1,5} m_i \psi_R i N_R + \frac{1}{2} M N_R N_R + \text{h.c.}$$

and we also assume the mass hierarchy

$$M \gg m_5 \gg m_4 \sim m_3 \gg m_1 \gg m_2.$$  

Following the seesaw mechanism, a heavy right-handed neutrino decouples and a mass matrix for the five light states becomes

$$M_0 = M \begin{pmatrix} \mu_1^2 & \mu_1 \mu_2 & \mu_1 \mu_3 & \mu_1 \mu_4 & \mu_1 \mu_5 \\ \mu_1 \mu_2 & \mu_2^2 & \mu_2 \mu_3 & \mu_2 \mu_4 & \mu_2 \mu_5 \\ \mu_1 \mu_3 & \mu_2 \mu_3 & \mu_3^2 & \mu_3 \mu_4 & \mu_3 \mu_5 \\ \mu_1 \mu_4 & \mu_2 \mu_4 & \mu_3 \mu_4 & \mu_4^2 & \mu_4 \mu_5 \\ \mu_1 \mu_5 & \mu_2 \mu_5 & \mu_3 \mu_5 & \mu_4 \mu_5 & \mu_5^2 \end{pmatrix},$$

where $\mu_i = m_i / M (\ll 1)$. As is easily checked, $M_0$ is diagonalized as $U^{(\nu)} M_0 U^{(\nu)T}$ by using the matrix

$$U^{(\nu)} = \begin{pmatrix} 0 & \mathcal{O} \\ \mathcal{O} & 1 \end{pmatrix} \begin{pmatrix} \frac{\mu_2}{\xi_1} & -\frac{\mu_1}{\xi_1} & 0 & 0 & 0 \\ \frac{\mu_1 \mu_2}{\xi_1 \xi_2} & \frac{\mu_2^2}{\xi_1 \xi_2} & -\frac{\xi_1}{\xi_2} & 0 & 0 \\ \frac{\mu_1 \mu_3}{\xi_1 \xi_3} & \frac{\mu_2 \mu_3}{\xi_1 \xi_3} & \frac{\mu_3^2}{\xi_1 \xi_3} & -\frac{\xi_1}{\xi_3} & 0 \\ \frac{\mu_1 \mu_4}{\xi_1 \xi_4} & \frac{\mu_2 \mu_4}{\xi_1 \xi_4} & \frac{\mu_3 \mu_4}{\xi_1 \xi_4} & \frac{\mu_4^2}{\xi_1 \xi_4} & -\frac{\xi_1}{\xi_4} \\ \frac{\mu_1}{\xi_1} & \frac{\mu_2}{\xi_1} & \frac{\mu_3}{\xi_1} & \frac{\mu_4}{\xi_1} & \frac{\mu_5}{\xi_1} \end{pmatrix},$$

where $\xi_n^2 = \sum_{i=1}^{n+1} \mu_i^2$, and $\mathcal{O}$ is an arbitrary $4 \times 4$ orthogonal matrix.

In order to investigate a consistent explanation for the neutrino mixings (1) and (2), we need to identify these five states with the physical neutrino states. In this context the constraint from the standard big bang nucleosynthesis (BBN)\[^{[3]}\] is useful. The BBN predicts that the effective neutrino species during the primordial nucleosynthesis should be less than 3.3. This fact severely constrains the mixing angle $\theta$ and squared mass difference $\Delta m^2$ for a sterile neutrino ($\nu_s$) mixing with left-handed active neutrinos.\[^{[10]}\] These

\[^{[1]}\]This arbitrariness remains because of the degeneracy of mass eigenvalues. The author thanks M. Tanimoto for pointing this out.
constraints rule out the large mixing MSW solution of the solar neutrino problem due to $\nu_e \rightarrow \nu_s$ and also the explanation of the atmospheric neutrino problem by $\nu_\mu \rightarrow \nu_s$. In this study we consider the possibility that $\psi_1$ and $\psi_5$ are right-handed sterile neutrinos and $\psi_2, \psi_3$ and $\psi_4$ are $\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$. The solar and atmospheric neutrino deficits are explained by the small mixing MSW solution due to $\nu_e \rightarrow \nu_s$ and the $\nu_\mu - \nu_\tau$ oscillation, respectively.

Here it is useful to remind ourselves that the transition probability for $\nu_i \rightarrow \nu_j$ during the time interval $t$ in the vacuum is expressed as

$$P_{\nu_i \rightarrow \nu_j}(t) = -4 \sum_{k \neq k'} V_{ik}^{(l)} V_{jk}^{(l)} V_{ik'}^{(l)} V_{jk'}^{(l)} \sin^2 \left( \frac{\Delta m_{kk'}^2 t}{4E} \right),$$

where $V^{(l)}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix for the lepton sector, which can be written as $V^{(l)} = U^{(\nu)} U^{(l)\dagger}$ by using the diagonalization matrix $U^{(l)}$ of the charged lepton mass matrix: $M_{\text{diag}} = U^{(l)\dagger} M^{(l)} U^{(l)\dagger}$.

For the time being, we confine our attention to $U^{(\nu)}$ with $O = 1$, assuming that the charged lepton mass matrix is diagonal. Under this assumption, taking account of (6) and (7), the desired mixing angles in (1) and (2) can be obtained by setting

$$16 \lesssim \frac{\mu_1}{\mu_2} \lesssim 32, \quad 0.44 \lesssim \frac{\mu_3}{\mu_4} \lesssim 2.3.$$  \hfill (8)

Although the mixing in the neutrino sector can take a suitable pattern by imposing this mass hierarchy (8), the rank of $M_0$ is 1, and the required hierarchical mass pattern is not realized. To remedy this situation and remove the arbitrariness $O$ in Eq. (6), we need to add a mass perturbation to yield the hierarchical mass eigenvalues without disturbing the mixing structure $U^{(\nu)}$. As such a mass perturbation, we consider the simplest example,

$$M_{\text{per}} \simeq \begin{pmatrix}
A \mu_1^2 & B \mu_1 \mu_2 & D \mu_1 \mu_3 & E \mu_1 \mu_4 & F \mu_1 \mu_5 \\
B \mu_1 \mu_2 & C \mu_2^2 & D \mu_2 \mu_3 & E \mu_2 \mu_4 & F \mu_2 \mu_5 \\
D \mu_1 \mu_3 & D \mu_2 \mu_3 & D \mu_3^2 & E \mu_3 \mu_4 & F \mu_3 \mu_5 \\
E \mu_1 \mu_4 & E \mu_2 \mu_4 & E \mu_3 \mu_4 & E \mu_4^2 & F \mu_4 \mu_5 \\
F \mu_1 \mu_5 & F \mu_2 \mu_5 & F \mu_3 \mu_5 & F \mu_4 \mu_5 & 2F \mu_5^2
\end{pmatrix},$$

\hfill (9)

\footnote{There is another possibility that $\psi_1$ is identified with $\nu_{eL}$. However, in that case $\psi_2$ plays no role and it is reduced to the model considered in Ref. 8).}

\footnote{Here $V^{(l)}$ is defined as $\nu = V^{(l)\dagger} \tilde{\nu}$, where $\tilde{\nu}$ are the mass eigenstates. The bases $\nu$ are chosen so that the leptonic charged current and the charged lepton mass matrix are diagonal.}
where $A \sim F$ are parameters which satisfy

$$
\frac{A - D}{B - D} = \frac{B - D}{C - D} = -\frac{\mu_2^2}{\mu_1^2}, \quad \frac{F}{D + E} = -\frac{\mu_4^2}{\mu_5^2}.
$$

(10)

This matrix is also diagonalized by $U^{(\nu)}$ with $O = 1$, and the mass eigenvalues of $M_0 + M_{\text{per}}$ is obtained as

$$
M_1 = (C - D)\mu_2^2, \quad M_2 = 0, \quad M_3 = (D - E)\mu_3^2, \quad M_4 = (D + E)\mu_4^2, \quad M_5 = M\mu_5^2.
$$

(11)

In order to realize the desired masses $M_1 \sim 10^{-2.5}$ eV, $M_4 \sim 10^{-1.2}$ eV and $M_5 \gtrsim 10$ eV for the neutrino deficits and the hot dark matter\footnote{The heavier right-handed neutrino can be a hot dark matter candidate. The constraint on $M_5$ comes from the consistency with the BBN.\cite{11,8} which is somehow larger than one of the usual predictions.\cite{5,6}} in a manner consistent with (8), we must introduce at least three new parameters, $C$, $D$ and $E$.\footnote{For the model defined by (12), mass parameters $C$ and $D \simeq E$ are related by $C \sim 10^{3.4}D$ and $D$ should be larger than $M$ by two orders of magnitude.} Then we can settle our phenomenological mass matrix by taking, for example,

$$
m_1 \sim 10^{-1 - a}, \quad m_2 \sim 10^{-2.4 - a}, \quad m_3 \lesssim m_4 \sim 1, \quad m_5 \sim 10^2, \quad M \sim 10^{12},
$$

(12)

where $a(>)$ is a free parameter, and we use GeV units.\footnote{There are no strong quantitative constraints on $\mu_4/\mu_5$ and $\mu_1/\mu_3$ as long as these are sufficiently small.}

Next we proceed to the constraint on the charged lepton mass matrix in the present framework. The charged lepton mass matrix is also related to neutrino oscillation phenomena through the mixing matrix $V^{(l)}$, as is shown in Eq. (7). We use a Fritzsch mass matrix\footnote{For the model defined by (12), mass parameters $C$ and $D \simeq E$ are related by $C \sim 10^{3.4}D$ and $D$ should be larger than $M$ by two orders of magnitude.} for the charged lepton sector here. Using a well-known formula in the diagonalization of the Fritzsch mass matrix for $U^{(l)}$, we can obtain the CKM matrix elements $V^{(l)}_{ij}$ as

$$
V^{(l)}_{\nu e} \sim \frac{\mu_2}{\mu_1} - \frac{\mu_1}{\mu_3} \sqrt{m_e} e^{i\sigma}, \quad V^{(l)}_{\nu \mu} \sim -\frac{\mu_2}{\mu_1} \sqrt{m_\mu} - e^{i\sigma} \frac{\mu_1}{\mu_3} \sqrt{m_\mu}, \quad V^{(l)}_{\nu \tau} \sim -e^{i\sigma} \frac{\mu_1}{\mu_3} \sqrt{m_\tau},
$$

$$
V^{(l)}_{\nu e} \sim \frac{1}{\sqrt{2}} \frac{\mu_2}{\mu_3} + \frac{e^{i\sigma}}{\sqrt{2}} \sqrt{m_e} \frac{m_e}{m_\mu} + \frac{e^{i\sigma}}{\sqrt{2}} \sqrt{m_e} \frac{m_e}{m_\tau}; \quad V^{(l)}_{\nu \mu} \sim \frac{e^{i\sigma}}{\sqrt{2}}, \quad V^{(l)}_{\nu \tau} \sim -\frac{e^{i\sigma}}{\sqrt{2}},
$$

$$
V^{(l)}_{\tau e} \sim \frac{1}{\sqrt{2}} \frac{\mu_2}{\mu_3} + \frac{e^{i\sigma}}{\sqrt{2}} \sqrt{m_e} \frac{m_e}{m_\mu} - \frac{e^{i\sigma}}{\sqrt{2}} \sqrt{m_e} \frac{m_e}{m_\tau}; \quad V^{(l)}_{\tau \mu} \sim \frac{e^{i\sigma}}{\sqrt{2}}, \quad V^{(l)}_{\tau \tau} \sim \frac{e^{i\sigma}}{\sqrt{2}}.
$$

(13)

where $m_e, m_\mu$ and $m_\tau$ are charged lepton mass eigenvalues. Here it should be noted that the charged lepton mass matrix has only a negligible effect on $V^{(l)}_{\nu e}$ which is relevant to
the $\nu_e$-$\nu_s$ oscillation. If we set $\mu_1/\mu_2$ to appropriate values such as those in Eq. (8), the small mixing MSW solution of the solar neutrino problem works without dependence on the charged lepton sector. This is very different from the three$^{[13, 14]}$ and four$^{[8]}$ light neutrinos schemes, where the charged lepton sector crucially affects the $\nu_e$-$\nu_{\mu}$ mixing. In the case of the Fritzsch-type mass matrix, this mixing becomes too large to explain the solar neutrino deficit by $\nu_e \rightarrow \nu_\mu$ unless the phase is taken to be a suitable value.$^{[13, 14]}$ In the present scheme this situation can be avoided, and the Fritzsch mass matrix is also applicable to the charged lepton sector without any assumption on the phases. This is a direct result of the fact that the solar neutrino deficit is explained by the $\nu_e$-$\nu_s$ oscillation due to the extension to the five light neutrinos. As long as $U^{(d)}$ is approximately diagonal, like the case of Fritzsch mass matrix, our scenario is always applicable, independent of the details of the charged lepton mass matrix.

Finally, we briefly comment on possible underlying theories which may realize the present scenario. Such models will not be usual grand unified models, in which all the right-handed neutrinos are required to be heavy. One promising possibility is a superstring inspired $E_6$ model, in which the group theoretical constraints on the Yukawa couplings become very weak. Usually it is not easy to obtain small neutrino masses and to induce neutrino oscillations without bringing other phenomenological difficulties in the framework.$^{[15]}$ However, if we introduce unconventional field assignments under suitable conditions in the model, it is possible to show that similar structure, at least to the mixing which is discussed here, can be realized. A study along this line can be found in Ref. 16).

In summary we proposed the phenomenological neutrino mass matrix which could explain the solar and atmospheric neutrino deficits and offered a hot dark matter candidate in the $3\nu_L + 3\nu_R$ framework. In this construction we took the viewpoint that the mixing structure was the essential ingredient. The resulting mass matrix can explain the neutrino oscillation phenomena without constraining the charged lepton sector. The relation of this model to a LSND result$^{[17]}$ will be presented elsewhere.

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