Reducing the Possibility of Ruin by Maximizing the Survival Function for the Insurance Company’s Portfolio

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1. Introduction

The insurance industry is currently undergoing a fundamental transformation in terms of operations and competitiveness. Several disruptive factors in business(7,8),(993,989) have given rise to new players in the market with disruptive business models to outperform their competitors. Investment and refinancing can be used as survival approaches when insurance players consider how they should react to this major shift. With investing and refinancing, an insurance company can manage to operate much better even if it had suffered from ruin provided the investments are done properly and refinancing is done adequately and timely [1].

According to Kolm et al. [2] in investments, there is a trade-off between risks and returns. In turn, to increase the expected returns from investment, investors must be willing to tolerate greater risks. Portfolio management theory helps to model the trade-off for the given collections of several possible investments [3]. Investigation into companies that have suffered from ruin is one of the very important areas of actuarial research. Some research studies have been done to investigate portfolio optimization, most of them applying reinsurance and refinancing approaches (see, e.g., Kasumo [4], Liu, and Yang [5]). However, more research is needed on those companies that have suffered from ruin because little has been done to investigate how these companies can be managed financially to become profitable again.

Oyatoye and Arogundade [6] applied a stochastic model for predicting the optimum portfolio of insurance businesses at an acceptable risk exposure level, excluding ruin effects. They underscored the importance of this because it would guarantee the acceptable risk levels for a viable insurance company and evaluate the retention rate of the insurance portfolio at a given risk rate. But, it would also provide good
knowledge on the importance of reinsurance in risk adjustment in times of larger claims. Finally, it would examine the unbearable risk level that would require coinsurance. Risk return analysis and catastrophe exposure analysis were performed. It was observed that there is a need to revert to stochastic modelling, which canvases the use of risk, vari-ances, and expected values for mathematical computation.

Recently, considerable attention on the part of insurance companies has been given to the procedures of forming an assigned insurance portfolio because it serves as an indicator of the quality of insurance liabilities. Oliynyk [7] studied the basic methodological principles of formation and management of the insurance portfolio to achieve equilibrium and ensure that the financial stability of insurance companies is maintained. One of the stages in the company’s insurance portfolio management process is to deal with portfolio optimization. This stage was discussed by Oliynyk [7] as it leads to the reduction of risks and an increase in profitability levels. The study finally observed that the proposed scientific and methodical approach to building and managing an insurance portfolio to achieve its equilibrium based on nonlinear programming has a differentiated character. For each company, this model chooses an optimal structure of an insurance portfolio that ensures maximal profits and minimal risks.

Ma et al. [8] extended the work of Zhu et al. [9] to include defaultable securities. The insurer was given a chance of buying proportional reinsurance and put his wealth in stock, a defaultable corporate bond, and a money account. The intention was to maximize their expected utility of wealth. In their work, Ma et al. [8] chose the constant elasticity of variance (CEV) process to describe the stocks’ behaviour. The reason for selecting a CEV model was that it could also be used as an alternative model for describing the stochastic volatility behaviour of the stocks’ price. It had several empirical pieces of evidence to support it. Using stochastic control theory, they derived a Hamilton–Jacobi–Bellman (HJB) equation and later divided the original problem into two parts a predefault case and a postdefault case. Value functions and expressions of the optimal strategy were derived. Finally, they presented numerical examples as illustrations of their results. Their study did not consider converting the Volterra integro-differential equations into an ordinary differential equation.

Shareholders of insurance businesses are interested in optimizing the returns from the insurance portfolio as well as ensuring that the business remains afloat over a long-time horizon. To achieve this, the company managers have to optimally run the business to maximize returns and reduce ruin probability. Even with extra care, many times, ruin is inevitable. Most studies in the literature, for example, Schmidli [10], Kasozi et al. [11], and Kasumo et al. [12], do not consider refinancing as a measure to overcome ruin once it hits. This paper seeks to develop and analyse an insurance portfolio optimization model incorporating investments and refinancing strategies and then find the best way to maximize the insurance company’s survival function.

The rest of the paper is organized as follows: in Section 2, we derive a Volterra integro-differential equation (VIDE) corresponding to the model for maximizing the survival function for insurance companies. In Section 3, the numerical simulations were carried out using the fourth-order Runge-Kutta method after converting VIDE into a third-order ordinary differential equation that was later converted into a system of ODEs of the first order. The results are presented and discussed. In the last section, we present a conclusion for this paper.

2. Materials and Methods

2.1. Model Formulation and Analysis. For a mathematical formulation of the problem, we assume all the stochastic quantities and random variables are defined on a complete filtered probability space \((\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \in [0,T]}, \mathbb{P})\), satisfying the usual conditions. The filtration \((\mathcal{F}_t)_{t \in [0,T]}\), which represents the information available at time \(t\) and forms the basis for the decision making, is right continuous and \(\mathbb{P}\)-complete.

Due to the fact that there exist fluctuations in the real market, for example, the amount of premium income, claim arrivals, and a number of customers are not static or uniform, the model must capture these phenomena by considering the perturbed Cramér-Lundberg process \(X_t\), as in Kasumo et al. [12], given by

\[
X_t = p + c t + \sigma_X W_{X,t} - \sum_{i=1}^{N_{X,t}} Y_{X,i}, \quad t \geq 0, \tag{1}
\]

where \(p > 0\) is the initial capital, \(W_X\) is a standard Brownian motion independent of the compound Poisson process \(\sum_{i=1}^{N_{X,t}} Y_{X,i}\), and \(c\) is the premium rate per unit time which is calculated by the expected value principle; that is, \(c = (1 + \theta)\lambda_X \mu_X\) where \(\theta > 0\) is the relative safety loading of the insurer and \(\lambda_X\) is the intensity of the counting process \(N_{X,t}\) for the claims.

We proceed as in Schmidli [1] by assuming that the insurance company managers engage in the process of refinancing or capital injections, given by \(X_t^M = X_t + M_t\), where \(X_t\) is the surplus process and \(M_t\) is the capital injected. Finally, using equation (1), the insurance model with capital injections is given by

\[
X_t^M = p + c_M t + \sigma_X W_{X,t} - \sum_{i=1}^{N_{X,t}} Y_{X,i} + M_t, \quad t \geq 0, \tag{2}
\]

where \(c_M\) is the premium rate during the capital injection.

To manage its portfolio, we assume the insurance company can undertake investment in either risk-free or risky assets. We assume the risk-free price process was modelled as in Meng et al. [13], given by

\[
dB_t = r_0 B_t dt, \tag{3}
\]

where \(r_0 \geq 0\) is the constant risk-free interest rate and \(B_t\) is the price of the risk-free bond at time \(t\). Let us further assume as in Badaoui et al. [14] that the risky asset, such as the stock price process, is given by the geometric Brownian motion (GBM)
where $S_t$ is the price of a stock at time $t$, $r \geq 0$ is the expected instantaneous rate of stock return, $\sigma^2_t$ is the volatility of the stock price, and $\{W_{S_t}, t \geq 0\}$ is a standard Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

If we further assume that there are no jumps, the Paulsen et al.’s work [15] suggested that the return on investment process $R_t$, will be given by the Black-Scholes option pricing formula of the form
\[ R_t = rt + \sigma^2_t W_{R_t} \quad t \geq 0, \quad R_0 = 0, \]
where $r$ is the risk-free part of the investment. Hence, $R_t = rt$ means that one unit invested at time zero will be worth $e^{rt}$ at time $t$.

### 2.2. Risk Process with Refinancing and Investment

This section obtains the insurance process compounded with refinancing and investment. It is a careful combination of equations (2) and (5). In the case of reinsurance and investment, the process has been extensively studied for ultimate ruin probability in studies such as Kasozi et al. [16], Paulsen et al. [15], Paulsen and Gjessing [17], and Paulsen and Rasmussen [18] among others. This paper follows a similar approach as in Kasozi et al. [16]. The process $P^M = \{p_t^M, t \in [0, \infty)\}$, which represents the insurance portfolio, is given by
\[ p_t^M = X_t^M + \int_0^t p^M(s^-) dR(s), \]
which is the solution of the stochastic differential equation
\[ dp_t^M = dX_t^M + p^M(t^-) dR(t), \]
where $P_0^M > 0$ is the initial capital of the insurance company same as in equation (2), $X_t^M$ is the primary insurance process given in equation (2), $R(t)$ is the return on investment process in equation (5), and $P^M(t^-)$ stands for the insurer’s surplus just before time $t$.

### 2.3. Maximizing Survival Function or Minimizing Probability of Ruin

Let us consider equation (6) to maximize survival function or minimize the probability of ruin for the insurance company. Since both $X$ and $R$ have stationary independent increments, $P$ is a homogeneous robust Markov process. By using Itô’s formula, the infinitesimal generator for $P$ can be given by
\[ \mathcal{L} g(p) = \frac{1}{2} \left( \sigma^2_p + \sigma^2_t \right) g''(p) + (r_p + c_M) g'(p) + \lambda g(p - (\lambda M)) - g(p) dF_X(y). \]

The integro-differential operator presented in equation (8) is quite complicated, and explicit analytical computations are hard to perform. However, Paulsen and Gjessing [17] have introduced and proved the following beneficial results.

Let $\tau_\rho = \inf\{t: P_t < 0\}$ be the time of ruin where $\tau_\rho = \infty$ means ruin never occurs, and then, let $\psi(p) = P(\tau_\rho < \infty)$ be the probability of eventual ruin to occur.

Assuming that $\psi(p)$ is bounded and twice continuously differentiable on $p \geq 0$ with a bounded first derivative there, where at $p = 0$ is meant the right-hand derivative, and that $\psi$ solves $\mathcal{L} \psi(p) = 0$ on $p > 0$ together with the boundary conditions
\[ \psi(p) = 1 \quad \text{on} \quad p < 0, \]
\[ \psi(0) = 1 \quad \text{if} \quad \sigma^2_t > 0, \]
\[ \lim_{p \to \infty} \psi(p) = 0. \]

Paulsen and Gjessing [17] have shown that
\[ \psi(p) = P(\tau_\rho < \infty). \]

Assuming that $q_a(p)$ is bounded and twice continuously differentiable on $p \geq 0$ with a bounded first derivative there, where at $p = 0$ is meant the right-hand derivative, and Paulsen and Gjessing [17] have also shown that if $q_a$ solves $\mathcal{L} q_a(p) = 0$ on $p > 0$ together with the boundary conditions
\[ q_a(p) = 1 \quad \text{on} \quad p < 0, \]
\[ q_a(0) = 1 \quad \text{if} \quad \sigma^2_t > 0, \]
\[ \lim_{p \to \infty} q_a(p) = 0, \]
then
\[ q_a(p) = E[e^{\tau_\rho r}]. \]

Now, using Paulsen et al. [15] idea, we replace the first part of the theorem with the survival function $\phi(p) = 1 - \psi(p)$ with boundary conditions given as follows:
\[ \phi(p) = 0 \quad \text{on} \quad p < 0, \]
\[ \phi(0) = 0 \quad \text{if} \quad \sigma^2_t > 0, \]
\[ \lim_{p \to \infty} \phi(p) = 1. \]

Because maximizing the survival function influences the minimization of the probability of ruin which increases the probability of survival for the insurance company, the goal is now to maximize the survival function $\phi(p)$. Therefore, the value function, in this case, is defined by
\[ V(p) = \sup_{M_t \in [0, \infty)} \phi^M(p), \]
and if we exist, we determine the corresponding refinancing strategy $M_t \in [0, \infty)$ that will satisfy the objective function. Therefore, we are interested in finding the optimal refinancing strategy in the presence of investments in risky and risk-free assets. We refer to this strategy as optimal because it maximizes survival function, which is the same as minimizing the probability of ultimate ruin. In other words, the survival function is the objective function, and the refinancing strategy $M_t$ is the control variable to be adjusted such that the objective function is maximized.
2.4. Hamilton–Jacobi–Bellman Equation and Integro-Differential Equation. In this section, the HJB equation for the value function given by equation (14) is derived and solved. Then, the integro-differential equation for the survival function is formulated and solved too. In the literature, several HJB equations of a similar kind have been used, and for example, the reader may refer to Schmidli [10], Paulsen et al. [15], Kasozi et al. [11], and Kasumo et al. [12] for more details.

2.4.1. Hamilton–Jacobi–Bellman Equation. To derive the HJB equation for the value function given by equation (14), we follow a similar approach as in Kasozi et al. [11]. Let \( \tau \) be a small interval and suppose that for each surplus \( p(h) > 0 \) at time \( h \) we have a refinancing strategy \( M^\epsilon \) such that \( \delta M^\epsilon (p(h)) > \delta (p(h)) - \epsilon \). Let also that \( M = M \in [0, \infty) \) for \( t \leq h \). Then, as in Kasozi et al. [11], by Markov property, one has the following equation:

\[
\phi(p) \geq \phi^M(p) = \mathbb{E}[\phi^M(p^M(h)); \tau_p > h] = \mathbb{E}\left[\phi^M(p^M(\tau_p \land h)) \right] \geq \mathbb{E}\left[\phi^M(p^M(\tau_p \land h)) \right] - \epsilon,
\]

where \( \epsilon \in (-\infty, \infty) \) one can choose \( \epsilon = 0 \) to get

\[
\phi(p) \geq \mathbb{E}\left[\phi^M(p^M(\tau_p \land h)) \right].
\]

Let us assume that \( \phi(p) \) is twice continuously differentiable; by using Ito’s formula, we obtain

\[
\phi(p^M(\tau_p \land h)) = \phi(p) + \int_0^{\tau_p \land h} \left( (r p + c_M) \phi'(p^M(s)) + \frac{1}{2} (\sigma^2_R p^2 + \sigma^2_X) \phi''(p^M(s)) + \lambda^X \int_0^s \phi(p^M(s) - (y \land M)) dF_X(y) - \phi(p) \right) ds,
\]

where \( y \land M = \max(M, Y) \) denote the retained amount to the insurance company.

Now, put (17) into (16) to get

\[
\mathbb{E}\left[\int_0^{\tau_p \land h} \left( (r p + c_M) \phi'(p^M(s)) + \frac{1}{2} (\sigma^2_R p^2 + \sigma^2_X) \phi''(p^M(s)) \right) + \lambda^X \int_0^s \phi(p^M(s) - \max(M, Y)) dF_X(y) - \phi(p^M(s)) \right] ds \leq 0.
\]

Provided the limit and expectation can be interchanged, then dividing the later equation by \( h \) and letting \( h \longrightarrow 0 \) gives the following equation:

\[
(r p + c_M) \phi'(p) + \frac{1}{2} (\sigma^2_R p^2 + \sigma^2_X) \phi''(p) + \lambda^X \int_0^p \phi(p - \max(M, Y)) dF_X(y) - \phi(p) \leq 0.
\]

This equation (19) must hold for all \( M > 0 \), that is, to write

\[
\sup_{M>0} \left[ (r p + c_M) \phi'(p) + \frac{1}{2} (\sigma^2_R p^2 + \sigma^2_X) \phi''(p) + \lambda^X \int_0^p \phi(p - \max(M, Y)) dF_X(y) - \phi(p) \right] \leq 0.
\]

Suppose that there is an optimal strategy \( M \in [0, \infty) \) such that \( \lim_{\tau \rightarrow \infty} M(t) = M(0) \). Then, using a similar approach, we have
Finally, this gives the HJB equation
\[
\sup_{M>0}\left[(rp + c_M)\phi'(p) + \frac{1}{2}(\sigma^2 p^2 + \sigma^2_X)\phi''(p) + \lambda_X \int_0^p \phi(p - \max(M, Y))dF_X(y) - \phi(p)\right] = 0.
\]

whose boundary conditions are \(\phi(p) = 0\) on \(p < 0\) and \(\lim_{p \to -\infty} \phi(p) = 1\).

An optimal strategy is obtained from the solution set \((\phi(p), M^*(p))\) of equation (22), in which \(M^*(p)\) is a point at which the supremum in (22) is obtained. The insurance company has a nonnegative net premium income if \(c > (1 + \eta)\lambda_X E[(M - Y)^+]\).

Let \(M\) be the value where the equality holds, that is, \(c = (1 + \eta)\lambda_X E[(M - Y)^+]\), but the aim is to find a nondecreasing solution to equation (22), and let us write it as follows:

\[
\sup_{M>0}\left[(rp + c_M)\phi'(p) + \frac{1}{2}(\sigma^2 p^2 + \sigma^2_X)\phi''(p) + \lambda_X \int_0^p \phi(p - \max(M, Y))dF_X(y) - \phi(p)\right] = 0. \tag{22}
\]

\[\lambda_5 = 5\]
\[\lambda_7 = 7\]
\[\lambda_{10} = 10\]
whose boundary conditions are $\phi(p) = 0$ on $p < 0$ and \( \lim_{p \to -\infty} \phi(p) = 1 \).

According to Hipp and Plum [19], the function $\phi(p)$ will satisfy equation (23), only if $\phi(p)$ is strictly increasing, strictly concave, twice continuously differentiable, and it satisfies the second condition; that is, $\lim_{p \to -\infty} \phi(p) = 1$.

Now, let us assume that $\phi(p)$ solves the HJB equation (22), according to Hipp and Vogt [20] if $\phi(p)$ is a smooth solution of the HJB equation (22), then the supremum over $M > M$ is either attained at $M = 0$ when there is no refinancing for small claims or at $M = p$ or $M < M < p$.

2.4.3. Converting VIDE into ODEs. From the HJB equation (22), the integro-differential equation for the survival function $\phi(p)$ takes the following form:

$$
\mathcal{L}\phi(p) = 0, \quad p \geq 0,
$$

where $\mathcal{L}$ is the infinitesimal generator defined by equation (8) for the underlying risk process with refinancing and investment given by equation (6). Thus, from the HJB equation (22), the integro-differential equation for the survival function is given by

$$
(rp + c_M)\phi'(p) + \frac{1}{2} \left( \sigma_p^2 p^2 + \sigma_x^2 \right) \phi''(p) + \lambda_X \int_0^p \phi(p - \max(M, Y)) dF_X(y) - \lambda_X \phi(p) = 0,
$$

for $0 < p \leq \infty$.

Equation (25) is a second-order integro-differential equation of Volterra type (VIDE). In this paper, the VIDE given by equation (25) is converted into an ordinary differential equation (ODE) that can be solved numerically to determine the optimal strategies.

2.4.2. Integro-Differential Equation. From the HJB equation (22), the integro-differential equation for the survival function $\phi(p)$ takes the following form:

$$
\sup_{M > M} \left[ (rp + c_M)\phi'(p) + \frac{1}{2} \left( \sigma_p^2 p^2 + \sigma_x^2 \right) \phi''(p) + \lambda_X \int_0^p \phi(p - \max(M, Y)) dF_X(y) - \phi(p) \right] = 0,
$$

(23)

that the claims are exponentially distributed. If $\sigma_p = 0$ and $r = 0$, then there is no investment. For this case, the analytical solution to a similar problem is given by Belhaj [21], and if $\lambda_X = 0$, a similar case was solved analytically by Paulsen and Giessing [17]; however, when $\lambda_X \neq 0$, $\sigma_p \neq 0$, and $r \neq 0$, equation (25) has no analytical solution.

Consider exponential distribution given by

$$
\begin{align*}
F_X(y) &= \beta e^{-\gamma y}, \\
F_X(y) &= 1 - e^{-\gamma y}, \\
dF_X(y) &= \beta e^{-\gamma y} dy.
\end{align*}
$$

Then, equation (25) become

$$
(rp + c_M)\phi'(p) + \frac{1}{2} \left( \sigma_p^2 p^2 + \sigma_x^2 \right) \phi''(p)
$$

$$
+ \lambda_X \int_0^p \phi(p - \max(M, Y)) \beta e^{-\gamma y} dy - \lambda_X \phi(p) = 0,
$$

for $0 < p \leq \infty$.

Differentiating the above equation with respect to $p$ and simplifications give

$$
\frac{1}{2} \left( \sigma_p^2 p^2 + \sigma_x^2 \right) \phi''(p) + (rp + c_M + \sigma_p^2 \beta) \phi''(p)
$$

$$
+ (r - \lambda_X)\phi'(p) - \lambda_X \beta e^{-\gamma p} \phi(p - \max(M, Y)) = 0,
$$

(28)

for $0 < p \leq \infty$.

Equation (28) is an ODE that will be solved numerically.

3. Numerical Results and Discussion

We transform the third-order ODE given by equation (28) into a system of first-order ODEs that will be solved numerically by using the Runge–Kutta method. Letting

$$
Z_1(x) = \phi(p), \quad Z_2(x) = \phi'(p) = Z_1'(x), \quad \text{and} \quad Z_3(x) = \phi''(p) = Z_2'(x),
$$

then by using equation (28), the following system of first-order ODE is obtained.

$$
\begin{align*}
Z_1' &= Z_2, \\
Z_2' &= Z_3, \\
Z_3' &= \frac{2}{\sigma_p^2 p^2 + \sigma_x^2} \left[ \lambda_X \beta e^{-\gamma p} Z_1(p - \max(M, Y)) - (r - \lambda_X) Z_2 - (rp + c_M + \sigma_p^2 \beta) Z_3 \right].
\end{align*}
$$

(29)

In this section, system (29) of first-order ODEs is solved numerically using the fourth-order Runge–Kutta method, implemented using MATLAB codes, and results are discussed. Simulations and graphics were performed using MATLAB R2020a. Values of the parameters used are presented in Table 1.
In Figure 1, we observe an increase in survival function in the fourth year. The survival function is observed to increase as the intensity of the counting process increases. This is because as the counting process increases, the insurance company services its clients much faster. As a result, it becomes healthier, and hence, its probability of ruin is reduced.

In Figure 2, we observe that an insurance company has an increasing survival function due to capital injection. Still, we further note that when the instantaneous rate of stock return increases, the survival function decreases since the risk is much higher. These results are comparable to those of Gajek and Zagrodny [25], who studied reinsurance arrangements to maximize insurers’ survival probability, similar to minimizing ruin.

Since volatility measures the dispersion of returns for a given security, we observe in Figure 3 that an insurance company has an inverse relationship with the return volatility. We note that when returning volatility increases just by 0.02, the survival function decreases very quickly because the higher the volatility, the riskier the security. These results show that the survival function is extremely sensitive to the return volatility and support the earlier observations on the instantaneous rate of stock return. As a result, this paper suggests that more investments should be made in risk-free assets rather than in risky assets when this situation occurs.

4. Conclusion

In this paper, we have used the basic Cramér-Lundberg model to derive the Volterra integro-differential equation (VIDE) for the survival function of the insurance company. After some conversions, the VIDE was solved numerically by the Runge-Kutta method of order four. The results indicate that it is possible to maximize the survival function of the insurance company’s portfolio which helps the company reduce the possibility of ruin. The study further established that increasing the claim intensity has a positive effect in terms of increasing the survival function and reducing the probability of ruin. Therefore, it is recommended that insurance companies increase their claim intensities. This paper has also concluded that insurance companies should invest more in risk-free assets when the instantaneous rate of stock return and return volatility is much higher or increased.

Data Availability

No data were used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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