Representations of the fundamental group of a surface in $\text{PU}(p,q)$ and holomorphic triples

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Abstract. We count the connected components in the moduli space of $\text{PU}(p,q)$-representations of the fundamental group for a closed oriented surface. The components are labelled by pairs of integers which arise as topological invariants of the flat bundles associated to the representations. Our results show that for each allowed value of these invariants, which are bounded by a Milnor–Wood type inequality, there is a unique non-empty connected component. Interpreting the moduli space of representations as a moduli space of Higgs bundles, we take a Morse theoretic approach using a certain smooth proper function on the Higgs moduli space. A key step is the identification of the function’s local minima as moduli spaces of triples. We prove that these moduli spaces of triples are non-empty and irreducible.

Représentations du groupe fondamental d’une surface dans $\text{PU}(p,q)$ et triplés holomorphes

Résumé. Nous comptons le nombre de composantes connexes de l’espace des modules de représentations du groupe fondamental d’une surface compacte orientée dans $\text{PU}(p,q)$. Les invariants topologiques du fibré plat correspondant permettent d’associer à une telle représentation deux entiers, dont la valeur est bornée par une inégalité du type Milnor–Wood. Nous montrons que pour n’importe quelle valeur autorisée de ces invariants, il existe une unique composante connexe non vide dans l’espace des modules. Notre méthode utilise l’interprétation de l’espace des modules de représentations comme un espace des modules de fibrés de Higgs. L’analyse des minimum locaux d’une certaine fonction propre définie sur l’espace des modules est un élément clé de la démonstration : nous identifions ces ensembles avec l’espace des modules de triplets holomorphes et démontrons que ces espaces de modules sont non vides et irréductibles.

Version française abrégée

Soit $X$ une surface compacte de genre $g \geq 2$ et soit $G$ un groupe de Lie connexe. L’ensemble des représentations réductives $\text{Hom}^+(\pi_1 X, G)$ du groupe fondamental de $X$ dans $G$ est une variété analytique réelle et par conséquent un espace topologique avec une topologie analytique. On a une action de conjugaison de $G$ sur $\text{Hom}^+(\pi_1 X, G)$ et on munit l’espace des modules

$$\mathcal{M}_G = \text{Hom}^+(\pi_1 X, G)/G$$

de la topologie quotient; c’est un espace de Hausdorff parce que nous ne considérons que des représentations réductives. Il est bien connu que $\mathcal{M}_G$ peut également être interprété comme l’espace des modules de $G$-fibrés plats sur $X$. Cet espace qui est une source d’intérêt tant en mathématiques qu’en physique a été étudié largement du point de vue de la topologie, des théories de jauge et de la géométrie algébrique. L’objet de cet article est de déterminer le nombre de composantes connexes de $\mathcal{M}_G$ dans le cas où...
G = PU(p,q). Des résultats antérieurs concernant ce problème pour des groupes réels non compacts G sont dus à Goldman [7], Gothen [8], Hitchin [9], Markman et Xia [11], et Xia [12, 13, 14, 15].

Afin d’enoncer notre résultat, nous devons faire quelques remarques préliminaires. Soit \([\rho]\) \(\in \mathcal{M}_{\text{PU}(p,q)}\), alors le \(\text{PU}(p,q)\)-fibré plat correspondant peut être relevé en un \(U(p,q)\)-fibré qui admet de plus une réduction à son sous groupe compact maximal \(U(p) \times U(q)\). Soit \(V \oplus W\) le fibré vectoriel complexe de rang \(p + q\) correspondant, alors sa classe d’isomorphisme est indépendante des choix que nous avons faits et le couple d’entier \((d_V, d_W) = (\deg(V), \deg(W))\) est un invariant \([\rho]\) bien défini. D’après les résultats de Domic et Toledo [3] (améliorant une borne obtenue par Dupont [6] dans le cas où \(G = SU(p,q)\)), on a l’inégalité du type Milnor-Wood

\[
\frac{|qd_V - pd_W|}{p + q} \leq \min\{p, q\}(g - 1),
\]

donnant des bornes sur les valeurs que l’invariant topologique \((d_V, d_W)\) peut prendre. Définissons

\[
\mathcal{M}(d_V, d_W) = \{[\rho] : (\deg(V), \deg(W)) = (d_V, d_W)\},
\]

où \(V \oplus W\) est le fibré vectoriel de rang \(p + q\) associé à \([\rho]\) comme ci-dessus. Nous pouvons maintenant énoncer notre résultat principal.

**Théorème 1.** Pour chaque \((d_V, d_W) \in \mathbb{Z}^2\) satisfaisant [11] l’espace \(\mathcal{M}(d_V, d_W)\) est non vide et c’est une composante connexe de l’espace des modules \(\mathcal{M}_{\text{PU}(p,q)}\).

Il est plus naturel si on adopte le point de vue des représentations du groupe fondamental et des fibrés plats de travailler avec la forme associée \(\text{PU}(p,q)\) de \(U(p,q)\). Pourtant on peut tout de même considérer l’espace des modules \(\mathcal{M}_{\text{U}(p,q)}\) des fibrés munis d’une connexion réductive avec courbure centrale constante; en terme de représentations du groupe fondamental ceci correspond aux représentations d’une extension centrale universelle de \(\pi_1 X\) dans \(U(p,q)\). On a une application naturelle \(\mathcal{M}_{\text{U}(p,q)} \to \mathcal{M}_{\text{PU}(p,q)}\) dont les fibres s’identifient à \(\text{Hom}(\pi_1 X, U(1))\). Puisque le centre \(U(1)\) de \(U(p,q)\) est connexe, cette application induit un isomorphisme entre les ensembles de composantes connexes des espaces de modules. Par conséquent, nous pourrons également considérer l’espace \(\mathcal{M}_{\text{U}(p,q)}\). Ceci est finalement plus naturel du point de vue des techniques sur les fibrés holomorphes que nous emploierons. La démonstration repose sur un approche holomorphe et nécessite de donner à \(X\) une structure de surface de Riemann. La théorie de fibrés de Higgs et leurs liens avec les fibrés plats (Corlette [2], Donaldson [4], Hitchin [9, 10], Simpson [12, 13, 14, 15]) ainsi que les triplés holomorphes ([1, 3]) interviennent de façon essentielle.

1. Introduction

Let \(X\) be a closed oriented surface of genus \(g \geq 2\) and let \(G\) be a connected Lie group. The set of reductive representations \(\text{Hom}^+(\pi_1 X, G)\) of the fundamental group of \(X\) in \(G\) is a real analytic variety, and hence it is a topological space with the analytic topology. There is an action of \(G\) on \(\text{Hom}^+(\pi_1 X, G)\) by conjugation and we give the moduli space

\[
\mathcal{M}_G = \text{Hom}^+(\pi_1 X, G)/G
\]

the quotient topology; this is Hausdorff because we have restricted attention to reductive representations. It is well known that \(\mathcal{M}_G\) can also be viewed as the moduli space of flat \(G\)-bundles on \(X\). This space has been the object of much interest both in mathematics and physics and it has been extensively studied from the points of view of topology, gauge theory and algebraic geometry. The purpose of the present note is to determine the number of connected components of \(\mathcal{M}_G\) in the case of \(G = \text{PU}(p,q)\). Previous results on this problem for non-compact real groups \(G\) include those of Goldman [7], Gothen [8], Hitchin [9, 10], Markman et Xia [11], et Xia [17, 18, 19].

In order to state our result we need some preliminary observations. Let \([\rho]\) \(\in \mathcal{M}_{\text{PU}(p,q)}\), then the corresponding flat \(\text{PU}(p,q)\)-bundle can be lifted to a \(U(p,q)\)-bundle which, in turn, has a reduction to its maximal compact subgroup \(U(p) \times U(q)\). Let \(V \oplus W\) be the corresponding complex rank \(p + q\) vector bundle, then its isomorphism class is independent of the choices made and hence the pair of integers \((d_V, d_W) = (\deg(V), \deg(W))\) is a well defined invariant of \([\rho]\). It follows from results of Domic and
Toledo [3] (improving on a bound obtained by Dupont [3] in the case $G = SU(p,q)$), that there is the Milnor-Wood type inequality

$$\frac{|qdV - pdW|}{p+q} \leq \min\{p,q\}(g-1),$$

(1)
giving bounds on the possible values of the topological invariants $(d_V, d_W)$. Define

$$\mathcal{M}(d_V, d_W) = \{[\rho] : (\deg(V), \deg(W)) = (d_V, d_W)\},$$

where $V \oplus W$ is the rank $p + q$ vector bundle associated to $[\rho]$ as above. We can now state our main result.

**Theorem 1.** For each $(d_V, d_W) \in \mathbb{Z}^2$ satisfying (1) the space $\mathcal{M}(d_V, d_W)$ is non-empty and it is a connected component of the moduli space $\mathcal{M}_{PU(p,q)}$.

From the points of view of representations of the fundamental group and flat bundles it is most natural to work with the adjoint form $PU(p,q)$ of $U(p,q)$. However one can also consider the moduli space $\mathcal{M}_{U(p,q)}$ of bundles with a reductive connection with constant central curvature; in terms of representations of the fundamental group this corresponds to representations of a universal central extension of $\pi_1 X$ in $U(p,q)$. There is a natural map $\mathcal{M}_{U(p,q)} \to \mathcal{M}_{PU(p,q)}$ whose fibres can be identified with $\text{Hom}(\pi_1 X, U(1))$. Since the centre $U(1)$ of $U(p,q)$ is connected, this map induces an isomorphism between the sets of connected components of the moduli spaces. Thus we could equally well consider the space $\mathcal{M}_{U(p,q)}$. This is actually more natural from the point of view of the holomorphic vector bundle methods which we shall employ, and we shall do this from now on.

2. Outline of proof of Theorem [1]

The proof rests on a holomorphic approach. This requires giving $X$ the structure of a Riemann surface. The two basic ingredients are Higgs bundles and holomorphic triples.

A Higgs bundle is a pair $(E, \Phi)$, where $E$ is a holomorphic bundle on $X$ and $\Phi \in H^0(X; \text{End}(E) \otimes K)$ is a holomorphic endomorphism of $E$ twisted by the canonical bundle $K$ of $X$. The slope of a Higgs bundle is by definition the slope of the underlying vector bundle, $\mu(E) = \deg(E)/\text{rk}(E)$. There is a stability condition, generalizing the usual slope stability for holomorphic vector bundles: a Higgs bundle is called stable if $\mu(F) < \mu(E)$ for any proper subbundle $F$ of $E$ preserved by $\Phi$, and one has the corresponding notions of semi-stability and poly-stability.

By results of Corlette [2], Donaldson [4], Hitchin [8, 10], and Simpson [12, 13, 14, 15] there is a homeomorphism between the moduli space $\mathcal{M}_{U(p,q)}$ and the moduli space of poly-stable Higgs bundles $(E, \Phi)$ of the form

$$E = V \oplus W, \quad \Phi = (\begin{smallmatrix} 0 & b \\ -b & 0 \end{smallmatrix})$$

where $V$ and $W$ are holomorphic vector bundles on $X$ of rank $p$ and $q$ respectively, $b \in H^0(\text{Hom}(W, V) \otimes K)$, and $c \in H^0(\text{Hom}(V, W) \otimes K)$. We shall call such a Higgs bundle a $U(p,q)$-Higgs bundle.

The homeomorphism above comes via an interpretation of $\mathcal{M}_{U(p,q)}$ as a gauge theory moduli space: it is possible to find a preferred (harmonic) metric in a reductive flat bundle and also to find a preferred metric in a poly-stable Higgs bundle. Both of these constructions lead to a solution to a set of gauge theoretic equations known as Hitchin’s equations [8]. Thus $\mathcal{M}_{U(p,q)}$ can also be seen as the gauge theory moduli space of solutions to Hitchin’s equations modulo gauge equivalence. This point of view allows one to define the function

$$f(E, \Phi) = \int_X |\Phi|^2$$

on $\mathcal{M}_{U(p,q)}$, which from Uhlenbeck’s weak compactness theorem is known to be proper [3].

Clearly each $\mathcal{M}(d_V, d_W)$ is a union of connected components. Hence all we need to prove is that each of these spaces is non-empty and connected. Since $f$ is proper it is sufficient to show connectedness of the subspace $\mathcal{N}(d_V, d_W)$ of local minima of $f$ restricted to $\mathcal{M}(d_V, d_W)$. Thus the first main step in the proof of Theorem [3] is the following characterization of the local minima of $f$.

**Lemma 2.** A poly-stable $U(p,q)$-Higgs bundle is a local minimum of $f$ if and only if at least one of the sections $b$ and $c$ vanishes.
Proof (sketch). Assume that \((E, \Phi)\) is a stable \(U(p,q)\)-Higgs bundle. This means that it represents a smooth point of the moduli space (an extra argument, similar to the one given by Hitchin in [10], is required to deal with the case of non-smooth points of the moduli space). It is known from [10] that \((E, \Phi)\) represents a critical point of \(f\) on \(\mathcal{M}_{U(p,q)}\) if and only if it is a variation of Hodge structure. In our case this means that it is a \(U(p,q)\)-Higgs bundle of the form

\[ E = F_1 \oplus \cdots \oplus F_m, \]

where \(\Phi\) maps \(F_k\) to \(F_{k+1} \otimes K\) and each \(F_k\) is contained in either \(V\) or \(W\). Define

\[ U_k = \bigoplus_{i-j=k} \text{Hom}(F_j, F_i). \]

It follows from the results of [10] that the eigenvalues of the Hessian of \(f\) at \((E, \Phi)\) are all even integers and that its \(-2k\)-eigenspace is isomorphic to the first hypercohomology of the complex

\[ C_{2k}^\bullet: U_{2k} \xrightarrow{\text{ad}(\Phi)} U_{2k+1} \otimes K. \]

The key ingredient in the proof is then the following vanishing criterion: the first hypercohomology \(H^1(C_{2k}^\bullet)\) vanishes if and only if \(\text{ad}(\Phi): U_{2k} \to U_{2k+1} \otimes K\) is an isomorphism. This is proved using the fact that \((\text{End}(E), \text{ad}(\Phi))\) is a stable Higgs bundle. Using this criterion it is then a matter of elementary linear algebra to show that \(H^1(C_{m-1}^\bullet) \neq 0\). It follows that \((E, \Phi)\) is not a local minimum of \(f\) whenever \(m \geq 3\), thus concluding the proof.

Remark 3. We can actually be a bit more specific in the statement of this Lemma: it is easy to see that a \(U(p,q)\)-Higgs bundle with \(d_V/p < d_W/q\) is a local minimum if and only if \(c = 0\), that a \(U(p,q)\)-Higgs bundle with \(d_V/p > d_W/q\) is a local minimum if and only if \(b = 0\), and that a \(U(p,q)\)-Higgs bundle with \(d_V/p = d_W/q\) is a local minimum if and only if \(b = c = 0\).

Remark 4. The vanishing criterion for hypercohomology described in the above proof can also be formulated for groups \(G\) other than \((U(p,q))\) and it should prove useful for the analysis of the connected components of \(\mathcal{M}_G\) in these cases.

Lemma 3 allows us to reinterpret \(\mathcal{N}(d_V,d_W)\) as a moduli space of so-called holomorphic triples. These objects were studied in [3] and [5]; we briefly recall the relevant definitions. A holomorphic triple on \(X\), \(T = (E_1, E_2, \phi)\) consists of two holomorphic vector bundles \(E_1\) and \(E_2\) on \(X\) and a holomorphic map \(\phi: E_2 \to E_1\). For any \(\alpha \in \mathbb{R}\) the \(\alpha\)-slope of \(T\) is defined to be

\[ \mu_\alpha(T) = \mu(E_1 \oplus E_2) + \alpha \frac{\text{rk}(E_2)}{\text{rk}(E_1) + \text{rk}(E_2)}. \]

There is an obvious notion of subtriple and, using the \(\alpha\)-slope, one can define the notions of stability, semi-stability and poly-stability in the standard way. The existence of moduli spaces of \(\alpha\)-stable triples was proved in [3]. Denote by

\[ \mathcal{M}_\alpha(n_1, n_2, d_1, d_2) \]

the moduli space of \(\alpha\)-poly-stable triples \(T\) with \(\text{rk}(E_i) = n_i\) and \(\deg(E_i) = d_i\) for \(i = 1, 2\).

Let \((E, \Phi)\) be a \(U(p,q)\)-Higgs bundle with \(c = 0\). We can then define a holomorphic triple \((E_1, E_2, \phi)\) by setting

\[ E_1 = V \otimes K, \quad E_2 = W, \quad \phi = b; \]

and, conversely, given a holomorphic triple we can define an associated \(U(p,q)\)-Higgs bundle with \(c = 0\). Analogously, there is a bijective correspondence between \(U(p,q)\)-Higgs bundles with \(b = 0\) and holomorphic triples. Of course \((n_1, n_2, d_1, d_2)\) can be expressed in terms of \((p,q,d_V,d_W)\), and vice-versa.

There is a link between the stability conditions for holomorphic triples and \(U(p,q)\)-Higgs bundles: one can show (see [3]) that a \(U(p,q)\)-Higgs bundle \((E, \phi)\) with \(b = 0\) or \(c = 0\) is (semi-)stable if and only if the corresponding holomorphic triple \((E_1, E_2, \phi)\) is \(\alpha\)-(semi-)stable for \(\alpha = 2g - 2\). Lemma 3 then implies the following result.
Lemma 5. The subspace $\mathcal{N}(d_V, d_W)$ of local minima of $f$ on $\mathcal{M}(d_V, d_W)$ is isomorphic to the moduli space $\mathcal{M}_\alpha(n_1, n_2, d_1, d_2)$ of $\alpha$-poly-stable triples for $\alpha = 2g - 2$ (and suitable values of $(n_1, n_2, d_1, d_2)$).

Thus Theorem 1 follows from our second main result:

Theorem 6. The moduli space $\mathcal{M}_\alpha(n_1, n_2, d_1, d_2)$ of $\alpha$-poly-stable triples is non-empty and irreducible for $\alpha \geq 2g - 2$ and the values of the topological invariants $(n_1, n_2, d_1, d_2)$ allowed by (1).

Proof (sketch). The proof of this Theorem applies the strategy used by Thaddeus in his proof of the Verlinde formula. First one obtains a relatively simple description of the moduli space $\mathcal{M}_\alpha$ for an extreme value of the parameter $\alpha$. Next one studies the variation of the moduli spaces $\mathcal{M}_\alpha$ as $\alpha$ varies, in order to obtain information about $\mathcal{M}_\alpha$ for the value of $\alpha$ in which one is interested.

We note that it is sufficient to consider the case $n_1 \geq n_2$, since the case $n_1 \leq n_2$ can be dealt with via duality of triples.

Via a careful analysis involving the stability condition for triples we obtain a bound on the values of $\alpha$: for $n_1 > n_2$ the moduli space $\mathcal{M}_\alpha$ is empty, unless

$$0 \leq \alpha \leq \alpha_M = \frac{2n_1}{n_1 - n_2} (\mu(E_1) - \mu(E_2)).$$

For $n_1 = n_2$ there is no upper bound on $\alpha$, however, the moduli spaces $\alpha$ stabilizes for $\alpha$ sufficiently large. Thus it makes sense to consider the “large $\alpha$ moduli space”, $\mathcal{M}_\infty$, in both cases. Analysis of $\mathcal{M}_\infty$ shows that it is non-empty and irreducible—for $n_1 = n_2$ this is the main result of Markman and Xia.

There is a finite number of so-called critical values of the parameter $\alpha$, these are values for which strict $\alpha$-semi-stability is possible. The $\alpha$-stability condition remains the same between critical values. Thus we need to study how $\mathcal{M}_\alpha$ varies as $\alpha$ crosses a critical value and, in particular, show that it remains non-empty and irreducible. The locus where $\mathcal{M}_\alpha$ changes consists of triples which are strictly $\alpha$-semi-stable for the critical value of $\alpha$, and what we need to show is that this locus has strictly positive codimension.

The category of triples is an Abelian category and we study strictly semi-stable triples via their Jordan-Hölder filtration by stable triples. This involves developing the theory of extensions of triples; in particular we show that the set of extensions of a triple $T'' = (E_1'', E_2'', \phi'')$ by a triple $T' = (E_1', E_2', \phi')$ is isomorphic to the first hypercohomology of the complex

$$E_1'' \otimes E_1' \oplus E_2'' \otimes E_2' \rightarrow E_2'' \otimes E_1' \quad (\psi_1, \psi_2) \mapsto \phi' \psi_2 - \psi_1 \phi''.$$

One can show that the codimension of the locus where $\mathcal{M}_\alpha$ changes as $\alpha$ crosses a critical value is strictly positive if this first hypercohomology group is non-vanishing for any extension. An important fact which we prove in the course of these arguments is that for $\alpha \geq 2g - 2$ any $\alpha$-stable triple is a smooth point of the moduli space.

Finally the non-vanishing of the above first hypercohomology is proved using a vanishing criterion which is reminiscent of the one described in the proof of Lemma 3 above.

Detailed proofs of these results will appear elsewhere.

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