A New Hybrid Meta-Heuristics Algorithms to Solve APP Problems

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Abstract

In this paper, a new hybrid algorithm for linear programming model based on Aggregate production planning problems is proposed. The new hybrid algorithm of a simulated annealing (SA) and particle swarm optimization (PSO) algorithms. PSO algorithm employed for a good balance between exploration and exploitation in SA in order to be effective and efficient (speed and quality) for solving linear programming model. Finding results show that the proposed approach is achieving within a reasonable computational time comparing with PSO and SA algorithms.

Introduction

Aggregate production planning (APP) APP is a prominent method that gathers all the information related to production before determining the best way to satisfy the predicted demand through the use of available physical resources. Aggregating the information being processed is vital because it is usually impossible to take into consideration every detail related to the production process while still maintaining a long planning horizon. APP is considered a planning technique with medium-term capacity.

APP able to identify the optimum level for production, inventory, workforce, backlog, and subcontracting to meet requirements of the fluctuating demand over a specified time frame that ranges from 2 to 18 even with limited resources and capacity [1,2].

Using a successful and efficient APP can even help in maintaining the effectiveness and responsiveness of supply chains. Since the 1950s, launched numerous aggregate production planning (APP) models and techniques involved different levels of complexity. [3] presented the traditional approaches used to solve these problems. Such methods can be categorized according to six classifications: linear decision rule, transportation method, linear programming, simulation, search decision rule and management coefficient approach.

In recent decades, large-scale APP problems acquired characteristics of high complexity and NP-hard problems, which could not be tackled with mathematical programming solvers. Thus, research
community use metaheuristic algorithms to solve complex problems [5,6]. Despite successful use of
metaheuristic algorithms in attacking complex APP problems, we cannot adopt one algorithm for all real-
world APP problems. Real-world APP problems are complex and diverse.

Thus, planning process requires modern methods that are consistent with technological innovation over
time and with complexity of dynamic movement of market and competition. Consequently, all APP
problems cannot be solved by using single method. On the other hand, no standard algorithm cover all
problems based on No-Free-Lunch theorem $ (NFL) $[7]. Therefore, as APPs are large-scale problems and
strongly NP-hard [8], hybrid algorithms were proposed by combining two or more algorithms to get
better efficiencies search. numerous researchers sought to take the features of individual algorithms for a
higher purpose [9] such as [10,11].

As the scope of this study involves hybrid algorithms for linear programming model to solve APP
problems involving multiple products and periods. A hybrid optimizing algorithmic procedure that
combines simulated annealing (SA) [12], and the particle swarm optimization (PSO)[13] algorithm
called “PSOSA” is proposed to solve the model.

The remainder of the paper is outlined as follows: In section 2, describes the mathematical model for
objective linear programming based on APP. Then, sections 3 offers the procedural of hybrid algorithms
based on SA and PSO. Section 4 presents the results. In Section 5, the conclusion is summarized.

The Mathematical Model

The linear programming for APP is presented as mathematical model in this section. Presumed that the
manufacturing company makes $ n $ kinds of products to fulfill market demand as described below:

\[
\begin{align*}
    n & \text{ quantity of production, } n = 1, 2, \ldots, N. \\
    t & \text{ number of periods in the planning horizon, } t = 1, 2, \ldots, T. \\
    c_{nt} & \text{ cost of production for each ton of production } n \text{ for period } t. \\
    l_{nt} & \text{ inventory-carrying cost for each ton of product } t. \\
    h_{t} & \text{ cost of hiring each worker in period } t. \\
    f_{t} & \text{ cost firing each worker in period } t. \\
    o_{t} & \text{ cost for each man-hour of overtime labour for period } t. \\
    w_{t} & \text{ cost of regular labour per period } t. \\
    D_{nt} & \text{ forecasted demand for product } n \text{ per period } t. \\
    P_{nt} & \text{ quantity of production } n \text{ for period } t. \\
    I_{nt} & \text{ level of production inventory } n \text{ for period } t. \\
    O_{t} & \text{ man-hours of overtime labour per period } t. \\
    W_{t} & \text{ workforce level for period } t. \\
    H_{t} & \text{ workers hired for period } t. \\
    F_{t} & \text{ workers fired for period } t. \\
    M_{nt} & \text{ hours required to produce one ton of product } n. \\
    AR & \text{ regular working hours for period } t. \\
    AO & \text{ overtime working hours allowed during for period } t. \\
    K_{nt} & \text{ hours required for one ton of production } n \text{ for each worker. }
\end{align*}
\]

the objective functions is considered to minimize total workforce and production costs as following:

\[
\text{Min } Z_{t} = \sum_{n=1}^{N} \sum_{t=1}^{T} \left( c_{nt} P_{nt} + i_{nt} I_{nt} + \sum_{t=1}^{T} w_{t} W_{t} + h_{t} H_{t} + f_{t} F_{t} + o_{t} O_{t} \right) 
\] (1)
Subject to Constraint:

\[ P_{nt} + I_{nt-1} = D_{nt} \quad \forall n, \forall t \]  
\[ \sum_{n=1}^{10} M_n P_{nt} - AR \times W_t - O_t \leq 0 \quad \forall t \]  
\[ O_t - AO \times W_t \leq 0 \quad \forall t \]  
\[ F_t - H_t + W_{t-1} = 0 \quad \forall t \]  
\[ P_{nt}, I_{nt}, O_t, H_t, F_t, W_t \geq 0 \quad \forall t, \forall n \]  

Hybrid Algorithms Based on SA and PSO

A hybrid meta-heuristics technique is an effective for difficulties to optimization problems. This technique aims to incorporate and combine different meta-heuristics with each other to enhance the search capabilities. It has been commonly accepted that finding optimality to NP-hard problems. The superiority of any meta-heuristics algorithm is its ability of performing wide exploration and deeply exploitations procedure, these two processes have been mention in plentiful meta-heuristics works among various strategies. For any algorithm to be speed and efficiency and quality, it must be able to effectively explore the entire search space, whilst it increases its search on every side of the neighborhood for a near-optimal or optimal solution.

Yang (2014), stated that to ensures achieving global optimality, must mixed an effective of these two major components.

if the algorithm converges very slowly with solutions jumping around some potentially optimal solutions, the diversification will be very powerful. On the other hand, there is a risk of being trapped in a local optimum, if the intensification is too strong since only a fraction of local space might be visited, and (yang2009).

Therefore, PSO will try to diversify the solutions and SA will try to converge the solutions.

The various stages of the recommended PSOSA are described in the following steps.

Step 1: Initialize n solutions such as \( x_1, x_2, ..., x_n \), and set initial high, where \( x_1, x_2, ..., x_n \) are the each and every objective function values; give initial value for generation \( W, R_1, R_2, c_1, \) and \( c_2 \):

Step 2: Calculate the fitness function for each \( x_i, i = 1, 2, ..., n \).

Step 3: Sort the results of fitness function such as \( f(x_1) \leq f(x_2), ..., \leq f(x_n) \), where \( f(x_1) \) better Solution than others.

Step 4: Generate a new swarm for each \( x_i \) according to the two following Equations;

\[ V(t + 1) = W(t) + c_1 R_1 (P_{best}(t)X(t)) + c_2 R_2 (g_{best}(t)X(t)), \]
\[ X(t + 1) = X(t) + V(t + 1), \]
then find new solution $g_{\text{best}}, f(x_i) = g_{\text{best}}$

Step 5: If $f(x'_i) \leq f(x_i)$ then $x_i = x'_i$ and $f(x_i) = f(x'_i)$ and go to step 7.

Step 6: If $p = e(-f(x'_i) - f(x_i)) = T$, and $p \leq z, z \in (0; 1)$ then $x_{s1} = x_{1}, f(x_{s1}) = f(x_1)$

Step 7: Reduce the parameter and if the convergence criteria are satisfied then end, if not then go to 4.

### Case Study

In this study, general company for vegetable oils industry is used to exhibit the proposed model. The production costs, inventory, and hours required to produce one ton for each product were represented in Table [1] for six months. Table [2] showed the prediction demand for per product. The initialize inventory for liquid oil, Detergent powder, Shaving cream, and Shampoo are 333, 105, 1.8, 0.25 tons, respectively. Per month needed 140 hours worked, and 500 $ /man as costs for each regular worker. In addition, the hiring overtime, and firing costs were 775$, 5.357$, and 581 for each worker in each hour, respectively. The hours of worker and overtime hours per employee for each month are 140 and 60 hours. The initial worker level is 3313 workers.

**Table 1: operating costs and data**

|          | Detergent | Liquid det | Vegetable liquid | Toilet s | Bay sc | Chlorine | Shaving | Shampoo | Toothpipe |
|----------|-----------|------------|-------------------|----------|--------|----------|---------|---------|-----------|
| c        | 328       | 385        | 451               | 1006     | 801    | 487      | 449     | 1007    | 496       | 739       |
| x        | 38        | 47         | 534               | 36       | 35     | 25       | 38      | 38      | 59        | 37        |
| M        | 92        | 53         | 69                | 64       | 50     | 121      | 42      | 607     | 172       | 692       |

**Table 2: forecast demand for all products**

| Period | Detergent | Liquid det | Vegetable liquid | Toilet s | Bay sc | Chlorine | Shaving | Shampoo | Toothpipe |
|--------|-----------|------------|-------------------|----------|--------|----------|---------|---------|-----------|
| 1      | 3049      | 54         | 341               | 100      | 606    | 23       | 2       | 1       | 3         | 1         |
| 2      | 1664      | 51         | 708               | 152      | 483    | 26       | 3       | 2       | 2         | 1         |
| 3      | 1236      | 35         | 700               | 138      | 497    | 15       | 7       | 2       | 2         | 0         |
| 4      | 783       | 43         | 650               | 77       | 430    | 25       | 9       | 3       | 3         | 1         |
| 5      | 914       | 27         | 439               | 56       | 325    | 15       | 22      | 2.4     | 2         | 2         |
| 6      | 653       | 38         | 619               | 50       | 653    | 12.4     | 29      | 1       | 3         | 1         |

**Experimental Results:**

To verify and evaluate the PSOSA algorithm in solving multi-objective APP problem, case study data was considered. The PSOSA algorithm is compared with SA and PSO algorithms. Then, PSOSA yields a better objective function value than SA and PSO in the smallest time as shown in Table 4.
Table 3: The Results of objective function for each algorithm

| Algorithms | Z1      | Time |
|------------|---------|------|
| SA         | 12798621| 47s  |
| PSO        | 12798619| 106s |
| PSOSA      | 12798610| 15s  |

Due to PSOSA gave the best results from PSO and SA in the idiom of cost and time, So, PSOAS was adopted to resolve the company model as appearing in the following Tables:

Table4: Production yield

| product         | P1  | P2  | P3  | P4  | P5  | P6  |
|-----------------|-----|-----|-----|-----|-----|-----|
| Detergent pow.  | 2945| 1663| 1238| 782 | 914 | 651 |
| Liquid deterge  | 55  | 51  | 35  | 40  | 27  | 37  |
| Vegetable ghe.  | 341 | 708 | 700 | 650 | 439 | 619 |
| liquid oil      | 100 | 152 | 138 | 77  | 56  | 50  |
| Toilet soap     | 144 | 482 | 496 | 429.9| 325 | 653 |
| Bay soap        | 26  | 23  | 14  | 25  | 14  | 13  |
| Chlorine bleac. | 1.0934 | 2  | 1.7 | 2.5 | 3.2532 | 1  |
| Shaving cream   | 29.1 | 0.7477 | 2  | 1.7 | 2.5 | 2.4 |
| Shampoo         | 3.2686 | 1.8 | 2.3 | 2.9 | 2.1 | 2.7 |
| Toothpaste      | 0.74 | 1.11 | 0.471 | 0.6914 | 2.601 | 0.13 |
### Table 5: Production yield

| Product            | P1  | P2  | P3  | P4  | P5  | P6  |
|--------------------|-----|-----|-----|-----|-----|-----|
| Detergent powder   | 2945| 1663| 1238| 782 | 914 | 651 |
| Liquid detergent   | 55  | 51  | 35  | 40  | 27  | 37  |
| Vegetable ghee     | 341 | 708 | 700 | 650 | 439 | 619 |
| Liquid oil         | 100 | 152 | 138 | 77  | 56  | 50  |
| Toilet soap        | 144 | 482 | 496 | 429.9|325 | 653 |
| Bay soap           | 26  | 23  | 14  | 25  | 14  | 13  |
| Chlorine bleach    | 1.0934|2  | 1.7 | 2.5 | 3.2532|1 |
| Shaving cream      | 29.1| 0.7477|2  | 1.7 | 2.5 | 2.4 |
| Shampoo            | 3.2686|1.8 | 2.3 | 2.9 | 2.1 | 2.7 |
| Toothpaste         | 0.74| 1.11| 0.471|0.6914|2.601|0.13 |

### Table 6: Inventory levels

| Product            | P1   | P2   | P3   | P4   | P5   | P6   |
|--------------------|------|------|------|------|------|------|
| Detergent powder   | 0    | 0    | 0    | 0    | 0    | 0    |
| Liquid detergent   | 0    | 0    | 0    | 0    | 0    | 0    |
| Vegetable ghee     | 0    | 0    | 0    | 0    | 0    | 0    |
| Liquid oil         | 0    | 0    | 0    | 0    | 0    | 0    |
| Toilet soap        | 0.1612|0.1612|0.1612|0.1612|0.1612|0.1612|
| Bay soap           | 0    | 0    | 0    | 0    | 0.2308|0    |
| Chlorine bleach    | 0    | 0    | 0    | 0    | 0    | 0    |
| Shaving cream      | 0.1434|0.1434|0.1434|0.1434|0.1946|0.1263|
| Shampoo            | 1.9686|1.9686|1.9686|1.9686|1.9686|1.9686|
| Toothpaste         | 0.004| 0    | 0    | 0    | 0.5142|0    |
Table 7: The rate of work force level

| product | P1   | P2   | P3   | P4   | P5   | P6   |
|---------|------|------|------|------|------|------|
| Wt      | 1905 | 1682 | 1453 | 1147 | 1081 | 1142 |
| Ht      | 0    | 0    | 0    | 0    | 0    | 1    |
| Ft      | 1400 | 234  | 229  | 306  | 106  | 0    |
| Ot      | 4.2335 | 8.9316 | 9    | 0    | 0    | 0    |

Conclusion

A new algorithm PSOSA was proposed to solve the linear programming model based on APP problems in this paper. PSO algorithm was used to give a balance among exploration and exploitation for the SA method, that way providing an effective converging velocity in improving the APP problem. Real data was presented to verify the validity of the presented method. We can apply this algorithm in conditions and unspecified vague of scheduling problems and real production planning. We compared to the proposed PSOSA method with PSO and SA. The results were showed that the SAPSO algorithm more accurate than PSO and SA for solving APP problems.

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