Fast Lane-Level Intersection Estimation using Markov Chain Monte Carlo Sampling and B-Spline Refinement

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Abstract—Estimating the current scene and understanding the potential maneuvers are essential capabilities of automated vehicles. Most approaches rely heavily on the correctness of maps, but neglect the possibility of outdated information.

We present an approach that is able to estimate lanes without relying on any map prior. The estimation is based solely on the trajectories of other traffic participants and is thereby able to incorporate complex environments. In particular, we are able to estimate the scene in the presence of heavy traffic and occlusions.

The algorithm first estimates a coarse lane-level intersection model by Markov chain Monte Carlo sampling and refines it later by aligning the lane course with the measurements using a non-linear least squares formulation. We model the lanes as 1D cubic B-splines and can achieve error rates of less than 10cm within real-time.

I. INTRODUCTION

Highly accurate maps are seen as essential base for environment perception \cite{1}. Maps, that have a high precision and allow for precise localization, ease a lot of difficult perception tasks. For example, recognizing the road boundary may be simplified to efficient map lookups.

Nevertheless, maps are a static representation of a dynamically changing world. Road layouts and, thus, the routing are prone to changes due to construction sites leading to expensive and frequent map updates.

Most presented approaches are suitable for lane estimation in simple environments like almost straight roads or highways, but lack the ability to estimate the complexity of urban intersections. The focus on (simple) lane estimation tasks was mainly motivated by the TuSimple challenge, that provided a highway video dataset with lane boundary annotation\cite{2}. More complex and urban datasets currently lack sufficient and precise lane information in intersections and if anything provide rough estimates on center lines.

In the following, we present an approach that is able to estimate the intersection including the lane course precisely as depicted in Figure 1. We base the estimation on the detection and the tracking of other vehicles passing the intersection.

In order to determine the most probable estimate for an intersection at once, the maximum of the posterior distribution has to be calculated, which is highly intractable for the complexity of an intersection model. Markov chain Monte Carlo is a sampling based approach especially designed to overcome the complexity of the posterior. Thus, similar to \cite{2}, we apply a two-stage setting, facilitating Markov chain Monte Carlo (MCMC). At first, we estimate the coarse structure of the intersection leading to an initial lane-level model. Here, we examine a multitude of hypotheses on the intersection structure using MCMC sampling. Given the coarse estimate, which we assume to be correct up to the exact lane courses, the lane course estimation can be formulated as a least-squares problem.

We present our work structured as the following: At first, we examine related work in Section \textbf{II}. In Section \textbf{III} the coarse parameter estimation is presented that estimates the position of arms and the number of lanes as well as the center position of the intersection. On the basis of this, we present our B-Spline model and the lane course refinement in Section \textbf{IV}. We evaluated our approach on simulated trajectories and present the results in Section \textbf{V}.

II. RELATED WORK

In recent years, many approaches dealt with lane estimation and rough intersection estimation. Only a few approaches aim at precise, lane-level intersection estimation which is necessary for driving across.

Many approaches, for estimating lanes or intersections, used camera images and their cues. However, the image domain is not particularly suitable for huge intersections. Interesting cues like curbs and markings might be occluded in heavy traffic and the image coordinate system induces an imprecision for distant areas due to perspective distortion.
This makes it especially hard to detect road areas in huge intersections solely based on camera information, e.g. if a lane or a curbstone is represented by just a few pixels.

Thus, focusing on different sensor data or aggregated camera data is far more promising for more complex environments like urban intersections. Occlusions could be tackled by e.g. aggregating multiple frames in a topview grid, but still lack the precision in distance.

**Map Generation:** Apart from image cues, some approaches based their estimation on the trajectories of traffic participants. They mostly facilitated fleet data and thus relied on a multitude of measurements.

Roeth et al. [3], for example, determined the connection of incoming and outgoing lanes. Similar to our approach, they applied a MCMC algorithm, but only sample connecting pairs instead of geometric properties or lane courses.

As an extension, they presented an approach that is able to estimate the exact lanes inside of an intersection [4]. Since they aim at offline map generation, they do not regard any execution time limits and thus rely on complex and time consuming models. Here as well, MCMC sampling was successfully applied to overcome the complexity of intersections.

Similarly, Busch et al. [5] employed trajectory clustering for estimating the center lines of the lanes. The cluster representative was modeled with cubic 2D splines and, thus, implicitly smooth. They also used an MCMC based approach, but evaluated different clustering hypotheses without explicit intersection models.

However, approaches aiming at map generation used fleet data and assumed a huge number of trajectory detections per lane and, thus, show increased computation times. In addition, a majority of these approaches leverage complex models that still can not be applied in a real-time approach with fewer measurements.

**Online Lane Estimation:** On comparison, Geiger et al. [6] estimated the intersection geometry in an online manner using the RGB images of an ego vehicle. The approach achieved promising results because they incorporated object detections instead of the direct use of camera data. The viewing angle in the dataset, however, was limited to a single camera and lacked sensors facing to the sides. This was sufficient for the specific dataset with only small intersections. Consistent with this, the intersection model was limited and assumed only a single lane per driving direction and collinearity of the crossing arms. This might have been sufficient, but will not suffice for big, urban intersections.

In 2019, we [2] presented a first approach on estimating the lane course of intersections within real-time. Here, we used the trajectories of other traffic participants as measurements, but only required a few of them. We estimated the lanes in a two-stage approach fully based on MCMC sampling. First, we estimated the intersection parameters and, based on that, we refined the lanes using a multitude of samples similarly in the manner of MCMC. We showed that the division into two steps reduces the search space and, thus, the computation time radically. We could estimate the coarse parameters in less than 50 ms, but experienced slow convergence for the lane estimation with execution times above 100 ms. In addition the resulting lane courses lacked smoothness despite an additional smoothness term in the MCMC evaluation.

Thus, in this work we present how to overcome the drawbacks of a purely MCMC based approach and emphasize the advantages of representing lanes with B-splines.

![Fig. 2: Parametric model of an intersection $I = (c, A)$. The base is a center point $c$ with a set of arms $A$. Each arm is defined as $a = (\alpha, g, E, O, w)$ with angle $\alpha$ describing the orientation in the intersection and $g$ enabling a structural separation between driving directions on the same arm. The number of entries $E$ and exits $O$ in conjunction with the width $w$ of the lanes determine the overall layout of the intersection.](image)

**III. INTERSECTION PARAMETER ESTIMATION**

Analogous to [2]–[6], we apply MCMC sampling to the problem of estimating the coarse intersection parameters. Thus, we sample different intersection models from a Markov chain, accepting a new estimate only, if it is sufficiently close or better than the previous estimate.

**A. Intersection Parameter Model**

We use the model presented in [2], that is constructed as depicted in Figure 2. An intersection $I = (c, A)$ is based at a center point $c$. It is comprised of a set of arms $A$ with arm $a = (\alpha, g, E, O, w)$ having several parameters assigned. An arm $a$ has an angle $\alpha$ describing the orientation in the intersection according to its coordinate system. $g$ enables a structural separation between driving directions on the same arm and describes the gap between the lanes with different driving directions. The number of intersection entries $E$ and exits $O$ in conjunction with the width $w$ of the lanes determine the overall layout of an intersection.

**B. Markov chain Monte Carlo Sampling**

We start with an initial intersection model $I$ and iteratively modify a single parameter according to Algorithm 1. The step probabilities presented here, have been empirically determined and might differ for different driving scenarios.
Adding an arm to the model, requires a more sophisticated approach than the other sampling steps. A new arm is sampled by drawing from a uniform distribution spread across the gaps between existing arms while maintaining a minimum angular distance of $25^\circ$ to the existing arms.

Most of the sampling steps follow the detailed balance equation required by the Metropolis algorithm [7]. However, when modifying the number of arms of the estimate, we did not design a symmetric step. However, the Metropolis-Hastings algorithm [7] allows for incorporating imbalanced sampling probabilities. In addition, we extend (and reduce) the state space, when adding (and removing) and arm, so we need to model reversible jumps as well.

Thus, in each iteration, we evaluate the posterior probability $P(I' | Z)$ and perform a Metropolis-Hastings step [7], where the trajectory data $Z$ is evaluated against the new model $I'$.

In summary, all models $I'$ that can not comply with the acceptance criterion:\footnote{The Jacobian for the reversible jump is neglected here.}

$$u \leq A(I',I) = \frac{P(I' | Z) P(I \rightarrow I')} {P(I | Z) P(I' \rightarrow I)} ^{\frac{1}{\Delta}} , \quad (1)$$

are rejected from the sampling and future models are only based on the previous model $I$. Here, $u \sim U[0,1]$ is a uniformly distributed random variable. For better convergence we apply simulated annealing with $T_s$ as the annealing parameter.

By following these principles we will only accept samples, that follow the target posterior distribution of $P(I | Z)$. Thus, the advantage of MCMC is that we only need to sample from a simple prior distribution $P(I)$ and thereby estimate the maximum of the more complex posterior distribution $P(I | Z)$. This is especially necessary, if the posterior is intractable like here.

Please note, that we do not estimate the width of the lanes, but assume a fixed value $w = 2.7$ m. Since we base our estimation on trajectories and, thus, use cues related to the centerline of a lane, the width is hardly observable.

The evaluation of the model requires the calculation of the posterior probability $P(I | Z)$. Because we only need a relation between the posterior probabilities of two intersections as shown in \textbf{Equation 1} and according to the rules of Bayesian theory we only need to calculate the likelihood $P(Z | I)$ and a prior probability $P(I)$ for each intersection. Thus the acceptance criterion becomes

$$u \leq A(I',I) = \frac{P(I' | Z) P(Z | I') P(I \rightarrow I')} {P(I | Z) P(Z | I') P(I' \rightarrow I)} ^{\frac{1}{\Delta}} . \quad (2)$$

The prior $P(I)$ is modeled uninformative. Assuming independent measurements

$$P(Z | I) = \prod_z P(z | I). \quad (3)$$

For the likelihood $P(z | I)$ of each measurement $z = (x, y, \phi, t, id)$, we further assume the positions $(x, y)$ of the

\begin{algorithm}
\begin{algorithmic}
\State \begin{align*}
\omega & \leftarrow U[0,1] \\
& \quad \text{if } \omega < 0.3 \text{ then} \\
& \quad \text{rotate a random arm by } \Delta \alpha \leftarrow U[-6^\circ, 6^\circ] \\
& \quad \text{else if } \omega < 0.52 \text{ then} \\
& \quad \text{shift center by } \{\Delta c, \phi\} \leftarrow U([0 \text{ m}, 6 \text{ m}] \times [0, 2\pi]) \\
& \quad \text{else if } \omega < 0.68 \text{ then} \\
& \quad \text{change gap by } \Delta g \leftarrow U[-1.8 \text{ m}, 1.8 \text{ m}] \\
& \quad \text{else if } \omega < 0.73 \text{ then} \\
& \quad \theta \leftarrow U[0,1] \\
& \quad \text{if } \theta < 0.5 \text{ then add arm (see subsection III-B) } \\
& \quad \text{else remove arm } a \leftarrow U_D(A) \\
& \quad \text{end if} \\
& \quad \theta \leftarrow U[0,1] \\
& \quad \text{if } \theta < 0.5 \text{ then add lane to the left } \\
& \quad \text{else add lane to the right } \\
& \quad \text{end if} \\
& \quad \text{else remove lane } l \leftarrow U(E \cup O) \\
& \quad \text{end if} \\
\end{align*}
\State \end{algorithmic}
\caption{Sample new intersections by modifying a single parameter at a step.}
\end{algorithm}
trajectories to be normally distributed around the center line of the lane. The angular deviation of the orientation \( \phi \) of a vehicle in these positions and the center line is also assumed to be normally distributed.

For both measures, we only regard the closest estimated lane with the same semantic driving direction (incoming or outgoing) for the evaluation of each measurement. We assume the remaining associations to vanish and thus be neglectable in favor of a faster computation time. We also found, that modeling the distribution of the number of trajectories per lane as a multinomial distribution improves the results.

The transition probabilities between the sampling steps \( P(I \rightarrow I') \) and vice-versa can be inferred from the probabilities and distributions shown in Algorithm 1. E.g. the probability of changing the angle of an arm by \( 2^\circ \) can be formulated as

\[
P(\alpha' = 32^\circ | \alpha = 30^\circ) = P_{\text{Rot}} \cdot P_{\text{Rot}}(-6^\circ, 6^\circ)(\Delta \alpha = 2^\circ). \tag{4}
\]

\( P_{\text{Rot}} \) describes the probability of choosing the rotation step \( ^3 \).

Finally, the accepted models converge to the maximum of the posterior distribution of the intersection. With this estimate, the most probable intersection model, given the measured trajectories is determined. The resulting model, represents the intersection already very precisely, but provides no information on the connectivity of lanes and the lane courses.

IV. LANE COURSE REFINEMENT

Using the coarse intersection parameters and the trajectories (cf. Figure 3a), an initial estimate on the connectivity of the lane stubs can be made by connecting all stubs passed by the same vehicle (cf. Figure 3b). When estimating a connection between the lane stubs, we get an initial lane model, that reduces the refinement of the center lines to a least-squares problem. When modeled as 1D cubic B-splines, that reduces the refinement of the center lines to a least-squares problem. When estimated as 1D cubic B-splines, the corridor connecting two associated lane stubs is initialized straight. Since the estimated entry and exit lane stubs of the parameter estimation only have an angle \( \alpha \) the initial model can only assume straight arms that will be refined.

C. Uniform Cubic B-Splines

For the lane refinement, we represent each center line by an 1D cubic B-spline with equidistantly placed knots \( [8] \). A cubic B-spline is a piece-wise polynomial function defined in variable \( x \).

The B-spline curve is defined as a linear combination of \( n+1 \) control points \( c_i \) and B-spline basis functions \( N_{i,k}(x) \) with

\[
f_i(x) = \sum_{i=0}^{n} c_i N_{i,k}(x). \tag{5}
\]

Thus, for each part of the spline, \( k \) control points influence the positioning and by changing these control points the spline curve can be fitted to any function of interest. In addition, the resulting spline model is implicitly smooth up to the \((k-2)\)th derivative, which is sufficient for lane modeling with \( k = 4 \) [5, 9].

For easing the later fitting of the lanes, we define the spline models in rotated coordinate systems as depicted in Figure 4. We take the set of all assigned trajectories and set the x-axis of the lane-specific coordinate system (depicted in green and red) to the average direction of all assigned trajectories. The origin is set to the origin of the fixed world coordinate system (depicted in black).

For representing a lane, we use an uniform cubic 1D B-spline. We define the spline as \( f_1 : x \rightarrow y \) and thus can only adjust the y-coordinate of the lanes (see Figure 4). However, with the rotated coordinate system modifying only one dimension of the problem is sufficient for fitting the lanes to the trajectories while maintaining low complexity of the optimization problem.

D. Least Squares Formulation

We estimate the lane course and thus implicitly the intersection by constructing a least squares problem using the previously defined lane splines. As stated above the parameters of the cost functor are the 1D control points \( g_i \). Using the trajectories \( Z_l \), we can optimize the spline parameters.

Given a the point \( p = (x,y) \) of a trajectory \( z \), we can project the point into the coordinate system of the assigned spline \( l \). Given its x-coordinate \( p_l \) we can calculate the

\[ 4 \text{Here, we use 20 control points for each lane.} \]
distance along the y-axis to the closest spline point \( f_i(p^y_B) \) given with **Equation 5**. Thus, for each trajectory point we construct the residual as

\[
e_1 = (p^y_i - f_i(p^y_B))^2. \tag{6}
\]

After 10 iterations, we add a second residual to the problem that specifically takes into account that neighboring lanes should have the same border points. We found it to be beneficial for the quality of the results to first optimize only the lane course and later on regard the bounds. For an equidistantly sampled set of evaluation points \( a \) on a lane spline \( A \), we calculate the distance to all lanes \( B \), that were estimated as neighbor by the parameter estimation (Section III).

We project the evaluation point \( a_A \) into the coordinate system of spline \( B \). Given the x-coordinate \( a^x_B \), we get the point on \( B \) with \( f_B(a^x_B) \).

Given that, the spline distance \( d_\perp \) in point \( a_A \) is defined as the dot product of the distance between the two points \( a^x_B \) and \( f_B(a^x_B) \) and the normal \( n_a \) of the spline \( A \) in point \( a_A \)

\[
d_\perp = n_a \cdot (a^y_B - f_B(a^x_B)). \tag{7}
\]

Since we create an initial lane for all combinations of traveled exits and entries (see Section IV-B) a single entry (or exit) might have multiple overlapping lanes, that have different destinations (or origins). Lane \( A \) and \( C \) in Figure 4 share an exit but have different entries. Thus, we construct a cost functor \( e_2 \), that has two minima, so lanes either fully overlap and thus have a distance \( d_\perp = 0 \) or be adjacent with \( d_\perp = w \). This way, we can have multiple splines representing the same entry, but diverge into multiple exits and vice-versa with

\[
e_2 = ((d_\perp - 0) \cdot (d_\perp - w))^2. \tag{8}
\]

For the residual, we only regard points, where the distance \( d_\perp \) is less than a threshold \( \delta > w \).

In a postprocessing step, we combine all estimated lane courses into lanelets and merge lane estimates, that converged onto the same center line, into a single lanelet. This way we end up with a consistent, map-like representation.

**V. Evaluation**

The presented approach has been evaluated on 1000 simulated intersections and 14 real-world intersection geometries from Karlsruhe, Germany. For all these intersections we simulated trajectories traveling along each lane. Each lane generated randomly between three and five trajectories. The individual measurement points of the trajectories were equidistantly initialized every \( 1 \) m and distorted by white Gaussian noise \( \Delta z_i \sim N^2(0 \text{ m}, 1 \text{ m}) \).

The coarse parameter estimation has not been quantitatively evaluated as it did not change significantly compared to \( \mathbf{2} \).

For the evaluation of the lane estimation we measure the average orthogonal distance between the estimated lane center line and the center line of the ground truth lanes.

As described in \( \mathbf{2} \) we can freely weigh the precision against the execution time of the parameter estimation, neglecting convergence quality. Thus, we base our evaluation of the lane estimation on two settings for the parameter estimation. The presented results here are initialized with either the results after 5000 or 10000 sampling steps in the parameter estimation. The calculation of these samples takes \( 49 \text{ ms} \) and \( 94 \text{ ms} \), respectively.

With the estimated parameter model, we can refine the lane courses within \( 47 \text{ ms} \) independent of the quality of the parameter estimation. With the 5000-sample parameter model, we achieve an average center line deviation of only \( 7 \) cm. If we generate 10000 samples in the parameter estimation, we can further improve the result of the lane estimation by \( 1 \) cm. \( \mathbf{2} \) achieved an average deviation of \( 14 \) cm after more than \( 100 \text{ ms} \). Thus, we could halve both the error and the execution time, here.

For real-world geometries we can show, that our approach is able to estimate the lanes with an average deviation of \( 10 \) cm after \( 34 \text{ ms} \). Example results for the real-world geometries are depicted in Figure 5. We added aerial images for visualization, but only used the geometries for generation of the trajectories and evaluation.

**VI. Discussion and Future Work**

We showed an approach that is able to estimate the lane course of intersections up to an average error of only a few centimeters based on trajectories of other traffic participants. Further we could achieve faster execution times of less than \( 50 \text{ ms} \) compared to a fully MCMC approach by leveraging a least-squares formulation.

Using the trajectories as measurement, we robustify the estimation for heavy traffic situations where visual cues like markings and curbs might be occluded. Additionally, the calculation of trajectories is ideally based on multiple sensors.
or at least a sensor like lidar or radar with a far field of view. As discussed in Section II an estimation purely based on camera information is not sufficient due to distortion induced imprecision.

Our approach is evaluated with simulated data. This is due to the fact, that no dataset exists that contains precise lane information in intersections combined with sensor data from a vehicle. In the future, we would like to evaluate the approach further on data taken by a measurement vehicle.

Since the model is highly flexible, we can easily add new cues on the road and lane layout. Each measurement has to be represented in a probabilistic manner for the MCMC and one has to build a suitable residual for the refinement.

Thus, including markings or curbstones, will be an easy task. When including those or measurements related to the lane width or border, the model can easily be extended to a width estimation both in the parameter estimation for a lane as a whole as well as in the lane refinement for each section of the spline individually.

We believe that this builds a solid foundation for further work in the direction of lane-level intersection estimation for the purpose of mapless driving, map verification and map updates, all in a real-time feasible manner while driving.

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