Flavour Democracy in Strong Unification

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Abstract

We show that the fermion mass spectrum may naturally be understood in terms of flavour democratic fixed points in supersymmetric theories which have a large domain of attraction in the presence of “strong unification”. Our approach provides an alternative to the approximate Yukawa texture zeroes of the Froggatt-Nielsen mechanism. We discuss a particular model based on a broken gauged $SU(3)_L \times SU(3)_R$ family symmetry which illustrates our approach.

CERN-TH/98-144
September 8, 2018

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1 Introduction

Attempts to understand the pattern of fermion masses and mixing angles in supersymmetric theories are usually based on the idea of Yukawa texture zeroes, or approximate texture zeroes which result from high powers of a small expansion parameter as in the Froggatt-Nielsen approach \cite{1}. In this paper we discuss an alternative to the texture approach based on the idea of flavour democracy \cite{2}. If flavour democracy is implemented in conventional supersymmetric grand unified theories (SUSY GUTs) then the perturbations from democracy required at the GUT scale in order to account for light masses and mixing angles must be tuned to be very small. This is because the renormalisation group equations (RGEs) of the minimal supersymmetric standard model (MSSM) tend to cause any small deviations from democracy to be magnified when the Yukawa couplings are run down to low energy. In this paper we point out that in a particular class of supersymmetric unified model, namely those with “strong unification” and a particular Higgs structure, the situation is reversed and arbitrary Yukawa matrices at high energy get driven to flavour democratic fixed points at low energies. In such theories the fermion mass spectrum that we observe in low energy experiments containing all the familiar (but bizarre) hierarchies may result from rather arbitrary Yukawa matrices at high energies.

Flavour democratic fixed points are a natural consequence of “strong unification” in which there is extra matter in complete ‘ SU(5)’ representations at an intermediate mass scale \( M_I \) below the unification scale \( M_{GUT} \). The extra matter consists of \( n_5 \) copies of \((5 + \bar{5})\) plus \( n_{10} \) copies of \((10 + \bar{10})\) representations which serve to increase the beta functions above the scale \( M_I \), resulting in an increased value of the unified gauge coupling \( \alpha_{GUT} \). On the other hand the unification scale \( M_{GUT} \sim 2 \times 10^{16} \) GeV is virtually unchanged from its MSSM value due to an accurate cancellation between the two-loop and threshold effects \cite{3}. The presence of such additional matter is typical of a certain class of string model in which gauge symmetries are broken by Wilson lines \cite{4}. Moreover such extra matter is welcome since it may serve to increase the unified coupling to a value which is
large enough to solve the “dilaton runaway problem” [3], provided that the string scale $M_{\text{string}}$ is reduced down to $M_{\text{GUT}}$ as suggested in Ref. [5].

In such theories the values of the gauge couplings near the unification scale may be raised into the strong coupling region [4], effectively placing the question of the unification of the gauge couplings outside perturbation theory. At first sight this would seem to imply that all the predictive power of unification is lost. However, as shown in [7], low energy predictivity is maintained since the steeply falling gauge couplings are quickly driven to precise fixed point ratios:

$$\frac{\alpha_1}{\alpha_3} \to r_1 \equiv \frac{b_3}{b_1}, \quad \frac{\alpha_2}{\alpha_3} \to r_2 \equiv \frac{b_3}{b_2},$$

(1)

where the beta functions are

$$b_a = \left( \begin{array}{c} 33/5 + n \\ 1 + n \\ -3 + n \end{array} \right)$$

(2)

where $n = (n_5 + 3n_{10})/2$. Thus one may take the ratios $r_1, r_2$ as a boundary condition at the scale $M_I$, and use them to determine the low energy measured couplings. In this approach the scale $M_I$ is regarded as an input parameter which may, for a given value of $n$, be fixed by two of the gauge couplings (say $\alpha_1$ and $\alpha_2$). The third gauge coupling may be predicted at low energies as in the standard unification picture, and indeed leads to values of $\alpha_3(M_Z)$ in good agreement with experiment [7]. This prediction, which follows without a conventional scale $M_{\text{GUT}}$, originates from the precise boundary conditions in Eq.(1) at $M_I$. The gauge couplings become non-perturbative at a scale $M_{NP} \sim 3 \times 10^{16}$ GeV, close to the conventional GUT scale [4]. Note that $M_I$ is the mass scale in the superpotential, not the physical mass of the heavy states which receive large radiative corrections. Such radiative splitting effects decouple from the evolution equations for the couplings [8]. The key to the predictive power of this scheme is the steeply falling gauge couplings in the region $M_{NP} - M_I$, which drives the gauge couplings to their fixed point values at $M_I$ in Eq.(1) [7].

Similar fixed points also apply to the Yukawa couplings whose fixed points are of
the Pendleton-Ross type rather than the quasi-fixed point type. It was shown that if all the third family Yukawa couplings are assumed to be in the (large) domain of attraction of the fixed point at high energies then this leads to precise predictions for third family Yukawa couplings at the scale $M_I$ and hence to precise low energy predictions for third family masses and the ratio of Higgs vacuum expectation values $\tan \beta$ as a function of the parameter $n$. In this case $\tan \beta \approx 46 - 47$ and the top quark mass exceeds 200 GeV in all cases. However, as pointed out, these predictions for the third family Yukawa couplings are sensitive to other Yukawa couplings which are large enough to be within the domain of attraction of the fixed point and, in a particular theory of fermion masses, the presence of large Yukawa couplings involving the first and second families will affect the low energy predictions of the third family spectrum and reduce the top mass prediction to acceptable values.

In one implementation of the model of ref. there is a separate Higgs doublet for each entry of the Yukawa matrix, leading to 9 Higgs doublets $H_{U_{ij}}$ coupling to the up sector and 9 Higgs doublets $H_{D_{ij}}$ coupling to the down and lepton sector. The Yukawa matrices just above $M_I$ are of the flavour democratic form as a consequence of the fixed point structure of the theory. However the flavour democracy is destroyed by the manner in which the two low energy Higgs doublets of the MSSM $H_U$ and $H_D$ are extracted from the 18 Higgs doublets which couple to quarks and leptons. Essentially the light doublets are identified as $H_U \sim H_{U_{33}}$ and $H_D \sim H_{D_{33}}$, with all the other Higgs acquiring mass of order $M_I$. Thus, from the point of view of the MSSM, the two Higgs doublets only have large Yukawa couplings in the 33 entry and, since these Yukawa couplings are roughly equal at the fixed point, one must explain the top-bottom mass hierarchy by taking a large ratio of low energy Higgs vacuum expectation values $\tan \beta \sim m_t/m_b$. Other entries of the Yukawa matrix generated by Higgs mixing effects controlled by a gauged $U(1)_X$ family symmetry. The $U(1)_X$ has a Green-Schwarz anomaly and is assumed to be broken close to the string scale by the vacuum expectation values (VEVs) of standard

\footnote{For a discussion of the relation between these two types of fixed point see [1].}
model singlet fields $\theta$ and $\bar{\theta}$ with $U(1)_X$ charges 1 and -1 respectively [13]. In order to achieve a realistic pattern of masses the simple $X$ charges are assumed for the three families of quarks and leptons. The 9 Higgs of each type mix via Froggatt-Nielsen [1] diagrams involving insertions of the $\theta$ and $\bar{\theta}$ fields along the Higgs line, so that at low energies effective Yukawa matrices emerge. Approximate texture zeroes and hierarchies are interpreted as high powers of an expansion parameter

$$\epsilon = <\theta>/M_I = <\bar{\theta}>/M_I$$

where $\epsilon \approx 0.2$.

In the above model [12] it is clear that the flavour democracy of the high energy theory at its fixed point is not transmitted to the MSSM since it is maximally broken by the way in which the light Higgs doublets are identified. However the embedding of $H_U$ and $H_D$ in the high energy theory is certainly not unique; for example it is possible to identify $H_U \sim H_{U_{33}}$ but $H_D \sim H_X + \gamma H_{D_{33}}$ where $H_X$ is some extra Higgs doublet which does not couple to quarks and leptons and $\gamma \sim m_b/m_t$ allows values of $\tan \beta \sim 1$. Indeed one can envisage further possibilities for embedding the MSSM Higgs in the high energy theory.

In this paper we are interested in the possibility that the MSSM Higgs doublets preserve the flavour democracy of the high energy theory. In other words we shall suppose that the light Higgs doublets of the MSSM are democratic mixtures of the Higgs doublets in the high energy theory:

$$H_U = \frac{1}{3} \sum_{ij=1,3} H_{U_{ij}},$$

$$H_D = \frac{1}{3} \sum_{ij=1,3} H_{D_{ij}}.$$  \hspace{1cm} (4)

In this case the flavour democratic Yukawa couplings of the high energy theory will be preserved in matching the theory onto the MSSM, and we will have realised our goal of obtaining flavour democracy as an infrared fixed point.

How can the democratic combinations in Eq.4 [13] be achieved in practice? One simple example is to take the $\epsilon \to 1$ limit of the above theory [12]. In this limit we would
expect the Higgs mixing to be maximal corresponding to the approximately democratic combinations in Eq.4. In the usual model the Higgs $H_{U_{ij}}$ and $H_{D_{ij}}$ are all assigned various $X$ charges consistent with their renormalisable couplings to the quarks and leptons which are assigned $X$ charges of $-4, 1, 0$ for the 1st, 2nd, 3rd families. The $H_{U_{33}}$ and $H_{D_{33}}$ are therefore assigned zero $X$ charge. Furthermore the 18 Higgs doublets above are accompanied by 16 conjugate Higgs doublets $\bar{H}_{U_{ij}}$ and $\bar{H}_{D_{ij}}$ (with $i = j = 3$ missing) which carry opposite quantum numbers and so form vector masses $M_I$ with 16 of the Higgs doublets. Since $\bar{H}_{U_{33}}$ and $\bar{H}_{D_{33}}$ are missing this implies that $H_{U_{33}}$ and $H_{D_{33}}$ remain light and so are identified as the two Higgs doublets of the MSSM. This is just the usual scheme in which a small effective Yukawa coupling in the $ij$ position of the up-matrix for example is small because the corresponding Higgs $H_{U_{ij}}$ has some $X$ charge which must be stepped down via $n$ insertions of $\theta$ fields to reach the physical Higgs $H_{U_{33}}$ which has zero $X$ charge, causing the effective Yukawa coupling to be of order $\epsilon^n$. Now if $\epsilon \approx 1$ there is no price to pay for $\theta$ field insertions, so the effective Yukawa couplings in all the entries would be expected to be approximately equal to their fixed point values in the high energy theory. In other words the Higgs would mix democratically as in Eq.4. Of course the precise nature of the mixing depends also on the full set of Yukawa couplings which control the mixing of the singlets with the Higgs, which are rather complicated [15]. In general it is rather difficult to get a “handle” on this kind of approach to the democratic Higgs, so we now introduce an alternative mechanism which departs from the $X$ symmetry completely.

2 A New Model of Democratic Higgs

The real explanation for the democratic Higgs may lie in additional fixed points of the high energy theory [15] leading to the picture described above, or it may have to do with the non-perturbative physics existing above the scale $M_{NP}$. For the moment however we would like to have a working model of flavour democracy. In this section we will therefore present a scheme in which a flavour democratic Higgs is generated in a perturbative
supergravity framework, bearing in mind that non-perturbative effects could in principle have important effects which are beyond our analysis.

In order to present a perturbative model, we are obliged to depart significantly from the model of Ref.[12]. In the example we consider in this section there is an additional $SU(3)_L \times SU(3)_R$ family symmetry at the Planck scale, under which all the fields transform;

$$Q = (\bar{3}, 1)$$
$$L = (\bar{3}, 1)$$
$$U^c = (1, 3)$$
$$D^c = (1, 3)$$
$$E^c = (1, 3)$$
$$\nu^c = (1, 3)$$
$$H_U = (3, \bar{3})$$
$$H_D = (3, \bar{3})$$
$$D_U = 8 \times (1, 1)$$
$$D_D = 8 \times (1, 1).$$

We have added the 16 extra ‘down’ superfields so that the content in addition to the MSSM falls into $5 + \bar{5}$ multiplets as required by the renormalisation group running from the intermediate scale although the symmetry we have chosen means that there can no longer be any underlying $SU(5)$ symmetry\[^3\]. We have also added a right handed neutrino to cancel any potential anomalies.

In order to obtain the democratic Higgs we need to add extra fields which are singlets under the Standard Model gauge group but which transform under the $SU(3)_L \times SU(3)_R$ family symmetry. The family symmetry is broken by Planck scale VEVs of four fields

\[^3\] The requirement is that $\alpha_s$ should be correct when we run the gauge couplings down from the boundary conditions at the intermediate scale which are dictated by the fixed points. If we are prepared to drop the ‘unification’ normalisation of the $U(1)_Y$ charges, $k_1 = 5/3$, then other solutions are possible – although there are no solutions which require no extra ‘down’ states.
which transform in the adjoint representation of each group factor (flagged by the subscripts);

\[ \Omega_L = (8, 1) \]
\[ \Omega'_L = (8, 1) \]
\[ \Omega_R = (1, 8) \]
\[ \Omega'_R = (1, 8) \]  

(6)

where

\[ \langle \Omega_L \rangle, \langle \Omega_R \rangle, \langle \Omega'_L \rangle, \langle \Omega'_R \rangle = \mathcal{O}(1) \]  

(7)

in natural units. (The VEVs must also commute, which is a mild assumption if there are non-trivial interactions in \( D \)-terms for example.) In addition we need gauge singlet fields to generate the intermediate scale, \( M_I \), and select the democratic Higgs to be the low energy (below \( M_I \)) Higgs. These are

\[ \Theta_L = (3, 1) \]
\[ \Theta'_L = (\overline{3}, 1) \]
\[ \Theta_R = (1, 3) \]
\[ \Theta'_R = (1, \overline{3}) \]
\[ \Lambda_L = (6, 1) \]
\[ \Lambda'_L = (\overline{6}, 1) \]
\[ \Lambda_R = (1, 6) \]
\[ \Lambda'_R = (1, \overline{6}) \]

\[ S_R, S_L, S'_R, S'_L = (1, 1) \]  

(8)

Finally, in order to enforce the correct form of couplings we invoke an extra discrete \( Z_N \) symmetry under which the non-zero charges are

\[ Z_{\Omega_L} = 1 \]
\[ Z_{S_L} = Z_L = Z_Q = -1, \]  

(9)
a. $Z'_N$ symmetry with non-zero charges

\[ Z'_{\Omega_R} = 1 \]
\[ Z'_{S_R} = Z'_{\nu} = Z'_{U^c} = Z'_{D^c} = -1, \tag{10} \]

a. $Z''_N$ symmetry under which the non-zero charges are

\[ Z''_{\Omega_R} = Z''_{\nu} = 1 \]
\[ Z''_{S_R} = Z''_{\nu} = -1, \tag{11} \]

and a $Z'''_N$ symmetry under which the non-zero charges are

\[ Z'''_{\Theta_L} = Z'''_{\Theta_R} = Z'''_{\Lambda_L} = Z'''_{\Lambda_R} = 1 \]
\[ Z'''_{\Theta_L'} = Z'''_{\Theta_R'} = Z'''_{\Lambda_L'} = Z'''_{\Lambda_R'} = -1. \tag{12} \]

With this set of charges the superpotential is of the correct form to give us the low energy democratic Higgs structure we require. The most general superpotential allowed by the above symmetries is

\[ W = W_{yuk} + W_{\mu} + W_D + W_S \tag{13} \]

where

\[ W_{yuk} = \lambda_u Q_O L H_U \Omega_R U^c + \lambda_d Q_O L H_D \Omega_R D^c \]
\[ + \lambda_u L \Omega_R H_U \Omega_R U^c + \lambda_d L \Omega_R H_D \Omega_R D^c + \ldots, \tag{14} \]

\[ W_{\mu} = \lambda_{\mu_R} \Theta_L \Lambda_R H_U H_D + \lambda_{\mu_L} \Theta_R' \Lambda_L' H_U H_D + \ldots \]

\[ W_D = \lambda_{D_L} \Theta_L \Theta_R' D_D U + \lambda_{D_R} \Theta_R \Theta_R' D_D U + \ldots \tag{15} \]

and

\[ W_S = S_L \Theta_L' \Omega_L \Theta_L + S_R \Theta_R' \Omega_R \Theta_R + S_L' \Theta_L' \Omega_L' \Theta_L + S_R' \Theta_R' \Omega_R' \Theta_R \]
\[ + \lambda_{\Theta_L} (\Theta_L' \Theta_L)^2 + \lambda_{\Theta_R} (\Theta_R' \Theta_R)^2 + \lambda_{\Lambda_L} (\Lambda_L' \Lambda_L)^2 + \lambda_{\Lambda_R} (\Lambda_R' \Lambda_R)^2 + \ldots \tag{16} \]

In the above the ellipses indicate higher order terms which are suppressed by at least a factor of $\Omega_L$ or $\Omega_R$ or $\Theta_{L,R}$, $\Theta_{L,R}', \Lambda_{L,R}$, $\Lambda_{L,R}'$. (All of the latter get small VEVs as we
shall shortly see. We are also assuming that there are no mass terms for these fields.) Since we assume $\langle \Omega \rangle < 1$ (in natural units) it is safe to neglect them provided that $N$ is a large number. The $\varepsilon$’s are Levi-Cevita symbols for the family symmetry with $SU(3)$ indices being suppressed; hopefully the contractions are self evident. The $\lambda$’s are single couplings.

We now go through the superpotential term by term to describe the role each piece plays. The first term, $W_{\text{yuk}}$ leads to a Yukawa coupling structure ansatz which is quite restrictive although remarkably successful; in particular we will see later that the mass matrices have two massless eigenvalues at the $M_{NP}$ scale, leading to the required CKM and mass structure by the time the model is renormalised down to $M_I$. (With this ansatz there are only 5 free parameters in the Yukawa couplings; initially there are 7, $(\lambda_u, \lambda_d, \lambda_e, \langle \Omega_L \rangle_{11}, \langle \Omega_L \rangle_{22}, \langle \Omega_R \rangle_{11}$ and $\langle \Omega_R \rangle_{22}$), but $\lambda_u$ and $\Omega_{L11}$ may be absorbed into the definition of the other parameters.) Indeed the Yukawa couplings at $M_{NP}$ can be identified as

$$
\begin{align*}
    h_{ij} &= \lambda_u \langle \Omega_L \rangle_{ii} \langle \Omega_R \rangle_{jj} \\
    k_{ij} &= \lambda_d \langle \Omega_L \rangle_{ii} \langle \Omega_R \rangle_{jj} \\
    l_{ij} &= \lambda_e \langle \Omega_L \rangle_{ii} \langle \Omega_R \rangle_{jj}
\end{align*}
$$

(17)
once we have rotated to a basis in which the adjoint VEVs are diagonal.

The VEVs of the $\Theta$ fields are enforced by the $W_S$ term to be democratic as long as $\langle S_L \rangle = \langle S_R \rangle = \langle S'_L \rangle = \langle S'_R \rangle = 0$; the $F$-flatness condition is

$$
\begin{align*}
    F_{S_L} &= \text{Tr}\Theta'^L_L \Omega_L \Theta_L = 0 \\
    F_{S_R} &= \text{Tr}\Theta'^R_L \Omega_R \Theta_R = 0 \\
    F_{S'_L} &= \text{Tr}\Theta'^L_L \Omega'_L \Theta_L = 0 \\
    F_{S'_R} &= \text{Tr}\Theta'^R_L \Omega'_R \Theta_R = 0
\end{align*}
$$

(18) which imposes

$$
\begin{align*}
    \langle \Theta_{L1} \Theta'_{L1} \rangle &= \langle \Theta_{L2} \Theta'_{L2} \rangle = \langle \Theta_{L3} \Theta'_{L3} \rangle \\
    \langle \Theta_{R1} \Theta'_{R1} \rangle &= \langle \Theta_{R2} \Theta'_{R2} \rangle = \langle \Theta_{R3} \Theta'_{R3} \rangle
\end{align*}
$$

(19)

9
as a result of the tracelessness of the adjoint VEVs. In supergravity, the scalar potential is of the form

$$V = -e^K \left( 3|W|^2 - (K_i W + W_i) K^i j (K_j W + W_j) \right)$$

(20)

where $K$ is the Kähler potential, $i, j$ label generic fields, subscripts imply differentiation, and $K^i j = K^{-1} j i$. We look for non-trivial solutions which are $F$-flat and therefore represent minima of the potential. They are given by the solutions to

$$\langle K_i (W + W_i) \rangle = 0$$

(21)

where $\langle W \rangle \sim m_W$ is fixed by the requirement that supersymmetry breaking in the visible sector be of order $m_W$. By defining the Kähler potential to be minimal (these can be considered to be the first terms in an expansion)

$$K = \Theta_L \overline{\Theta}_L + \Theta'_L \overline{\Theta}'_L + \Theta_R \overline{\Theta}_R + \Theta'_R \overline{\Theta}'_R + \Lambda_L \overline{\Lambda}_L + \Lambda'_L \overline{\Lambda}'_L + \Lambda_R \overline{\Lambda}_R + \Lambda'_R \overline{\Lambda}'_R + \ldots$$

(22)

we see that Eq.(21) implies that

$$\langle \Theta_L \rangle = \langle \Theta'_L \rangle = \theta_L(1, 1, 1)$$

$$\langle \Theta_R \rangle = \langle \Theta'_R \rangle = \theta_R(1, 1, 1)$$

(23)

where

$$\langle \theta_L \rangle, \langle \theta_R \rangle \sim (m_W/\lambda)^{1/2}$$

(24)

in natural units, where $\lambda$ is one of $\lambda_{\Theta_L}, \lambda_{\Theta_R}$, and also that the VEVs of the $\Lambda$ fields are of the same order. (In fact this situation holds even for the most general Kähler potential.)

The $W_\mu$ term now generates mass terms for Higgs fields of order

$$M_I \sim \lambda_{\mu_R} m_W / \lambda_{\Theta_L} + \lambda_{\mu_L} m_W / \lambda_{\Theta_R}$$

(25)

for all except for the democratic Higgs which remains light; this can be seen from the fact that for example

$$\varepsilon \langle \Theta_L \rangle = \theta_L \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}.$$
This matrix has one eigenvector with zero eigenvalue – the democratic one, \((1, 1, 1)\). The first term of \(W_\mu\) gives masses of order \(M_I\) to all components of \(H_U\) and \(H_D\) which are not democratic in left indices, and the second term to all components which are not democratic in right indices. Thus the only component of the Higgs fields which does not receive a mass from the first and second terms is that which is democratic in both left and right indices – i.e. the democratic Higgs (as may easily be checked by expanding out and finding the zero eigenvector of the full \(9 \times 9\) Higgs mass matrix). This, low energy Higgs can receive mass from the higher order contributions which should be of order \(m_W\) for phenomenology. These terms can be at most of order \(\Omega_{L,R}^N\) so that we require \(\langle \Omega_{L,R} \rangle^N \sim m_W/M_I \sim \lambda_{\Theta_L, \Theta_R}\) in order to generate the conventional \(\mu\)-term of the MSSM. They also disturb the democratic Higgs thereby introducing a mixing into the CKM matrix of order \(m_W/M_I \sim \lambda_{\Theta_L, \Theta_R}\). Thus we require \(\lambda_{\Theta_L, \Theta_R} \lesssim 10^{-4}\) and hence \(M_I \gtrsim 10^6\) GeV to avoid generating significant mixing this way.

Thus with the set of multiplets and charges defined above, a democratic Higgs results in perturbative supergravity. This model has of course no other justification, but its existence, and the Standard Model like structure which (as we shall see) results, makes Higgs democracy in strong unification an avenue worth exploring. This and the model outlined in the previous section should therefore be thought of as an existence proof of the possibility of obtaining a democratic Higgs. In the following section we describe the renormalisation of the strongly unified model, and then we go on to show how the Standard Model like structure emerges during the running of the high energy theory above the scale \(M_I\).

3 Renormalisation Group Equations

In the region below the scale \(M_{NP}\), the scale at which the gauge couplings become large, but above \(M_I\), the renormalisable superpotential contains Yukawa couplings involving the 18 Higgs doublets \(H_{U_{ij}}, H_{D_{ij}}\) as well as ‘\(\mu\)-terms’ which couple the Higgs to each other and
which can generically be of order $M_I$. These are

$$W = \sum_{i,j=1}^{3} \left( h_{ij} Q_i U^c_i H_{U_{ij}} + k_{ij} Q_i D^c_i H_{D_{ij}} + l_{ij} L_i E^c_i H_{D_{ij}} 
+ \mu_{ijkl} H_{U_{ij}} H_{D_{kl}} \right). \quad (27)$$

Such a structure leads to flavour symmetric or democratic Yukawa fixed points for the Yukawa couplings $h_{ij}, k_{ij}, l_{ij}, [12].^4$

The renormalisation group equations (RGEs) for the gauge couplings are:

$$\frac{d\tilde{\alpha}_a}{dt} = -b_a \tilde{\alpha}_a^2, \quad (28)$$

where we have defined $\tilde{\alpha}_a \equiv \frac{g^2}{16\pi^2}, \quad t \equiv \ln(\frac{M^2_{NP}}{\mu^2})$ with $\mu$ being the $\overline{MS}$ scale and $b_a$ the beta functions given in Eq. (2). The RGEs for the Yukawa couplings may be factorised into a Yukawa coupling multiplied by a sum of wavefunction anomalous dimensions for the three legs:

$$\frac{dY^{h_{ij}}}{dt} = Y^{h_{ij}} (N_{Q_i} + N_{U^c_j} + N_{H_{U_{ij}}})$$

$$\frac{dY^{k_{ij}}}{dt} = Y^{k_{ij}} (N_{Q_i} + N_{D^c_j} + N_{H_{D_{ij}}})$$

$$\frac{dY^{l_{ij}}}{dt} = Y^{l_{ij}} (N_{L_i} + N_{E^c_j} + N_{H_{D_{ij}}}) \quad (29)$$

where we have defined $Y^{h_{ij}} \equiv \frac{h^2_{ij}}{16\pi^2}, \quad Y^{k_{ij}} \equiv \frac{k^2_{ij}}{16\pi^2}, \quad Y^{l_{ij}} \equiv \frac{l^2_{ij}}{16\pi^2}$. If we assume that the gauge couplings are rapidly driven to their fixed point ratios then the wavefunction anomalous dimensions, $N_i$ may be expressed in terms of the single gauge coupling $\tilde{\alpha}_3$ as:

$$N_{Q_i} = \left( \frac{8}{3} + \frac{3}{2} r_2 + \frac{1}{30} r_1 \right) \tilde{\alpha}_3 - \sum_{j=1}^{3} (Y^{h_{ji}} + Y^{k_{ji}})$$

$$N_{U^c_i} = \left( \frac{8}{3} + \frac{8}{15} r_1 \right) \tilde{\alpha}_3 - 2 \sum_{j=1}^{3} Y^{h_{ji}}$$

$$N_{D^c_i} = \left( \frac{8}{3} + \frac{2}{15} r_1 \right) \tilde{\alpha}_3 - 2 \sum_{j=1}^{3} Y^{k_{ji}}$$

$$N_{L_i} = \left( \frac{3}{2} r_2 + \frac{3}{10} r_1 \right) \tilde{\alpha}_3 - \sum_{j=1}^{3} Y^{l_{ij}}$$

$^4$The soft mass RGEs corresponding to this theory have recently been studied in [16].
\[ N_{E_i} = \left( \frac{6}{5} r_1 \right) \bar{\alpha}_3 - 2 \sum_{j=1}^{3} Y^{l_j i} \]
\[ N_{H_{U,ij}} = \left( \frac{3}{2} r_2 + \frac{3}{10} r_1 \right) \bar{\alpha}_3 - 3 Y^{h_{ij}} \]
\[ N_{H_{D,ij}} = \left( \frac{3}{2} r_2 + \frac{3}{10} r_1 \right) \bar{\alpha}_3 - 3 Y^{k_{ij}} - Y^{l_{ij}} \]  

(30)

The Yukawa RGEs are flavour independent and are driven to the flavour independent infra-red stable fixed points (IRSFPs) of eq(29)

\[ R^{h*} = \left( \frac{232}{3} + 45 r_2 + \frac{232}{15} r_1 + 15 b_3 \right) / 219 \]
\[ R^{k*} = \left( 80 + 39 r_2 + \frac{21}{15} r_1 + 13 b_3 \right) / 219 \]
\[ R^{l*} = \left( -24 + 54 r_2 + 39 r_1 + 18 b_3 \right) / 219 \]  

(31)

where \( R^{h*} \equiv \frac{Y^{h*}}{\bar{\alpha}_3} \), \( R^{k*} \equiv \frac{Y^{k*}}{\bar{\alpha}_3} \), \( R^{l*} \equiv \frac{Y^{l*}}{\bar{\alpha}_3} \), where \( Y^{h*} \equiv Y^{h_{ij}*} \), \( Y^{k*} \equiv Y^{k_{ij}*} \), \( Y^{l*} \equiv Y^{l_{ij}*} \), \( \forall i, j \). For example for \( n = 6 \) we find \( b_3 = 3, r_1 = 0.238, r_2 = 0.428, R^{h*} = 0.663, R^{k*} = 0.621, R^{l*} = 0.285 \). (Note the approximate isospin symmetry in the IRSFPs.)

4 Numerical Results

In this section we examine the mass hierarchies and mixings which result from the democracy in the fixed points. We assume that the matrices \( h_{ij}, k_{ij} \) and \( l_{ij} \) in Eq.(27) correspond directly to the Yukawa couplings of the low energy MSSM below the scale \( M_I \), and that the fermion mass matrices are therefore given by

\[ m_U = h_{ij} v_2 \]
\[ m_D = k_{ij} v_1 \]
\[ m_E = l_{ij} v_1 \]  

(32)

where \( v_1 \) and \( v_2 \) are the VEVs of the two light MSSM Higgs.

We begin by making an ansatz for the mass matrices at the high \( M_{NP} \) scale based on the \( SU(3)_L \times SU(3)_R \) model – i.e. the matrices are parameterized by 5 parameters;

\[ h_{ij} = (\delta_{i1} + a \delta_{i2} - (1 + a) \delta_{i3})(b_1 \delta_{ji} + b_2 \delta_{j2} - (b_1 + b_2) \delta_{j3}) \]
We shall be ignoring the question of CP violation here, although it could arise purely in
the soft supersymmetry breaking terms as in Ref.[17]. This form is the same as Eq.[17] in
which the tracelessness arose because the Yukawa couplings were generated by the VEVs
of adjoint fields. This structure can be thought of as a kind of texture since the matrices
have zero determinant at the $M_{NP}$ scale and in fact have rank 1 – i.e. two zero eigenvalues
apiece. Generally this type of ansatz will always be required. The reason is that, although
the RGEs in strong unified models can produce the third/second generation hierarchy, we
still need to explain the smallness of the first generation.

At this stage, one could be forgiven for thinking that we have not gained anything
beyond what can be achieved with conventional texture models. However in conventional
texture models (based on the MSSM) the hierarchy we observe in the quark and lepton
masses requires additional small parameters. This is because the RGEs of the MSSM
dictate that the rank of the mass matrices is the same at all energy scales – in this case,
in the MSSM, there would always be six zero eigenvalues unless we invoked the Froggat–
Nielsen mechanism. In strong unification however this is not the case because, although
the Yukawa couplings are drawn towards IRFPs which also have two zero eigenvalues,
these are not the same eigenvectors. Hence there is a period during the running of the
renormalisation group equations, before the Yukawa couplings have become significantly
focused, in which the light quark masses receive small contributions. This is the origin of
our small parameters and quite arbitrary values of the mass matrices at $M_{NP}$ result in
masses and mixings in line with the observed pattern.

We can see why (heuristically) by examining the form of the RGEs and approximating
their solution by iteration. At the first step, i.e.

$$k_{ij} = ch_{ij}$$
$$l_{ij} = dh_{ij}$$

(33)
we see that there is still one zero eigenvalue for each $Y$. Hence the third/second and second/first generation mass-hierarchies are initially of order $\Delta t \sim \text{few}/16\pi^2$. Successive iterations then tend to reduce the first and second generation masses reflecting the focussing effect of the fixed point. Further analytic proof that the Standard-Model-like structure is natural is difficult because the couplings are not immediately focussed.

Henceforth we adopt an ‘empirical’ approach: we begin at the unification scale with $\alpha_i = 1$ but with the 5 remaining Yukawa parameters being chosen randomly around the central value,

$$a = b_1 = b_2 = c = d = 0$$

The parameters were varied randomly by a factor $g \times g_X$ (where $g_X = \sqrt{4\pi\alpha}$ is the gauge coupling at the unification scale) about these values, with $g$ going from 0 to 2. The running was stopped at $M_U (\approx 10^8 \text{GeV for } n = 8)$. In figures 1, 2, and 3 we show the generated masses of the charm/up, strange/down and mu/electron for random values of the 5 parameters normalised to the running third generation masses at $m_t$. (For proper comparison with the MSSM the parameters should of course be run from $M_U$ down to $m_t$ using the MSSM RGEs. Since the point here is simply to show that they are within the right range, this would not be relevant.) In figure 4 we show the CKM parameters, $V_{us}$ and $V_{ub}$, and in figure 5 we show $V_{cb}$. (Since there is no CP violation in these Yukawa couplings, the corresponding CKM matrix depends on only these three parameters.)

The point of these diagrams is not to claim that we have a definite prediction for the fermion masses and mixings (clearly we do not) but rather to show that the hierarchies observed arise naturally in the context of strong unification with a democratic Higgs. In other words, without invoking any small parameters, Standard Model like hierarchies arise from rather ordinary choices of initial parameters. And indeed, all the results from our randomly selected initial parameters are concentrated to within an order of magnitude about the correct physical values. In this sense our approach is very different from the usual Froggatt-Nielsen picture, in which the hierarchies are ‘predicted’ by the scale of
breaking of some underlying symmetry. It is also not difficult to find choices of parameters which give the physical values quoted by the Particle Data Group (modulo the MSSM running between $M_I$ and the weak scale).

5 Conclusion

In this paper we have investigated the possibility that strong unification may be responsible for the generation of fermion mass and mixing hierarchies. Given a democratic Higgs, the observed pattern appears to arise quite naturally. Some kind of ansatz is needed in order to obtain sufficiently light first family masses, as in Eq.33 which follows from Eq.[7]. After making this simple ansatz, which corresponds to zero determinant and two zero eigenvalues, our picture is very different from the usual Froggatt-Nielsen ‘texture’ approach in which further modification of the Yukawa matrices is controlled by powers of some expansion parameter. The power of our approach was demonstrated by the fact that, given this starting point, the remaining free parameters, randomly chosen, automatically give rise to a spectrum of quark and lepton masses and mixing angles, which has the character of the experimentally observed spectrum, as seen in Figs.1-5. The success of this scheme is due to the fact that the Yukawa matrices are strongly attracted towards the infrared fixed point consisting of democratic matrices with equal entries in every position. Thus even though the matrices start out at high energies as semi-random, they end up at low energies as quite accurately democratic. Although this fixed point is quite rapidly approached, due to the quickly falling gauge couplings of strong unification, if we were to choose completely random matrices we would not arrive at a spectrum resembling what is observed, and so something like the above ansatz seems a necessary additional requirement.

Having achieved a set of approximately democratic Yukawa matrices at the intermediate scale, we also demand that the two MSSM Higgs doublets be extracted from the set of 18 high energy Higgs doublets in a democratic way. We presented two examples in which such a democratic Higgs may occur. The first has the virtue that it is based on the
strong unification scenario which already exists in the literature and has some motivation. Unfortunately its complexity precludes a detailed analysis. Therefore we introduced a second model which gives some control over the generation of the democratic Higgs, but in a perturbative framework which is clearly subject to possible non-perturbative corrections beyond our control. Finally we comment that other models, perhaps based on the more recent work of Ref.[18], might be worth exploring.

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Figure 1: Charm and up masses at the intermediate scale normalised to $m_t(M_I) = 160$ GeV.

Figure 2: Strange and down masses at the intermediate scale, $M_I$, normalised to $m_b(M_I) = 2.75$ GeV.
Figure 3: Mu and electron masses at the intermediate scale, $M_I$, normalised to $m_\tau(M_I) = 1.7$ GeV.

Figure 4: CKM phases $V_{us}, V_{ub}$ at the intermediate scale.
Figure 5: CKM phase $V_{cb}$ at the intermediate scale.