On the influence of the method for estimating the parameters of the pole tide on the amplitude of the steady motion of the Earth's pole

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Abstract. The steady-state Chandler wobble of the Earth's pole is investigated for various choices of the pole tide model. The aim of the work is to estimate the parameters of the stationary Chandler wobble. The steady motion of the Earth's pole is considered taking into account the pole tide, which depends both on the position and on the speed of the pole. The Chandler motion of the Earth's pole is studied on the basis of the Euler-Liouville equations with a variable tensor of inertia. It is shown that both the choice of the pole tide model and the method of estimating the parameters of the chosen model significantly affect the parameters of the steady oscillatory process of the earth's pole and the amplitude of the necessary disturbance to excite the observed Chandler wobble.

1. Introduction

Modern methods of gravimetry, geophysics and space geodesy make it possible to measure with high accuracy the temporal variations of the decomposition coefficients of the geopotential and the corresponding small radial oscillations of the Earth's surface [1-3]. The pole tide - the response of the deformable layer of the Earth to the displacement of the Earth's pole - depends on the visco-elastic properties of the Earth [4,5]. The model may differ in amplitude and phase due to the different rheological model of the deformable Earth. Usually, a model of a pole tide is considered, which depends on the position of the pole and does not depend on its velocity according to conventions of the International Earth Rotation and Reference Systems Service (IERS) [1].

The study of the properties of the pole tide [1,6] of the deformable medium of the mantle of the visco-elastic Earth [7-9] is of significant interest for constructing a model of the Earth's motion relative to the center of mass [10-13] and, in particular, for constructing a model of the Earth's pole motion [13-14]. One of the main conditions for solving the problem of the excitation and maintenance of the Chandler wobble [15-18] is taking into account the pole tide in the model of its motion. The decisive factor for the steady motion is the accuracy of the pole tide model. So, for models of a deformable Earth with different rheology, the steady-state oscillatory process of the Earth's pole can differ significantly under the same external impact [19-21]. Or, if we are talking about restoring the disturbing function on the basis of observational data, then from the latter it follows that with different rheology of the Earth's mantle, the external disturbance required to excite the observed pole oscillation will be different.

If the Earth's mantle has only elastic properties and there is no energy dissipation, then variations in the Earth's centrifugal moments of inertia arising from the pole tide will be coherent with the Earth's
pole vibrations. In this case, the amplitudes of the pole tide and the pole movement will be in phase [6]. A consequence of the viscosity of the Earth's mantle [11] will be a small displacement of the pole tide and a shift of oscillation phase of the coefficients of the tesseral harmonic of the geopotential relative to oscillations of the Earth's pole.

In this paper, the steady-state mode of oscillations of the Earth's pole is investigated on the basis of the dynamic Euler-Liouville equations, taking into account the terms caused by tidal deformations of the Earth's mantle. It is shown that the choice of the pole tide model significantly affects the parameters of the steady oscillatory process of the Earth's pole and the amplitude of the necessary disturbance with the Chandler frequency to excite the observed Chandler pole oscillation [21].

2. Equations of motion of the Earth's pole

According to [22,23], the differential equations of the model of motion of the earth's pole are obtained from the dynamic Euler-Liouville equations with a variable tensor of inertia:

$$ \dot{J} \omega + \omega \times J \dot{\omega} = M - J \omega $$

where $J$ is the matrix of variable tensor of inertia, $M$ is the vector of all disturbances, $\omega$ is the vector of instantaneous angular velocity of the Earth.

In the first approximation, the equations of motion of the pole can be reduced to the form [11]:

$$ \dot{x}_p - N_x y_p = J_{pr}^0 + \mu_x, \quad x_p(t_0) = x_0, $$

$$ \dot{y}_p + N_y x_p = J_{qr}^0 + \mu_y, \quad y_p(t_0) = y_0, $$

where the tidal "ledges" $J_{pr}^0$, $J_{qr}^0$ are proportional to the centrifugal moments of inertia $J_{pr}$, $J_{qr}$ (they determine the dissipative terms of the model of motion of the Earth's pole), values $\mu_x$, $\mu_y$ are external disturbance with annual and Chandler frequencies, leading to the observed motion of the pole [6,7], and $N_x \approx N_y$ is the natural vibration frequency.

The steady motion of the pole is determined by variations in the centrifugal moments of inertia $J_{pr}$, $J_{qr}$ caused by the pole tide. Let us consider how the structure of dissipative terms in (2) affects the steady-state mode of Chandler oscillations. Expressions for the coefficients of the tesseral harmonic of the geopotential due to the displacement of the instantaneous axis of rotation, recommended by the International Earth Rotation and Reference Systems Service, depend on the coordinates of the Earth's pole $x_p, y_p$ [1]:

$$ \left[ \begin{array}{c} \delta c_{21} \\ \delta s_{21} \end{array} \right] = 1.333 \times 10^{-9} \left[ \begin{array}{c} -x_p \\ y_p \end{array} \right] + 0.0115 \left[ \begin{array}{c} y_p \\ x_p \end{array} \right]. $$

Variations in the coefficients $\delta c_{21}$, $\delta s_{21}$ are associated with centrifugal moments of inertia $J_{pr}$, $J_{qr}$ as follows:

$$ \delta c_{21} = \frac{J_{pr}}{m_E R_E^2}, \quad \delta s_{21} = \frac{J_{qr}}{m_E R_E^2}. $$

Here $m_E$, $R_E$ is Earth's mass and radius.

Expressions for $J_{pr}$, $J_{qr}$ and $J_{pr}^0$, $J_{qr}^0$ due to the influence of the pole tide, can be written from (3), (4). We represent them in the form:

$$ \dot{J}_{pr} = a \omega_1 - c \omega_2, \quad \dot{J}_{qr} = a \omega_2 + c \omega_1, \quad a < 0, \quad c > 0, $$

$$ J_{pr}^0 = s_1 x_p - s_2 y_p, \quad J_{qr}^0 = -s_1 y_p - s_2 x_p, \quad s_1 > 0, \quad s_2 > 0. $$

The values of the parameters $s_1, s_2$ in (5) are set from the correspondence with (3).

In [6] a model of the pole tide of the viscoelastic Earth, which depends on the velocity of the pole, was considered. According to this model, expressions for $J_{pr}^0$, $J_{qr}^0$ can be represented as:
The expressions for $J_{pr}^0$, $J_{qr}^0$ in (6) and (5) differ in terms with the coefficients $s_2$ and $s_3$, due to dissipative properties of the Earth's mantle. The dissipative terms in (6) lead both to a "lag" of the pole tide and to the presence of a small component incoherent to the pole oscillation, while the corresponding terms in (5) lead only to a "lag" of the pole tide.

Let's take a closer look at the differences. For simplicity, we will take into account only the main motion of the pole - the time dependence of the components $\omega_1$, $\omega_2$ of the instantaneous angular velocity vector in the form:

$$\omega_1 = a_{\chi} \sin \alpha_{\chi} + a_h \sin \alpha_h,$$

$$\omega_2 = a_{\chi} \sin \alpha_{\chi} + a_h \sin \alpha_h.$$

Here $a_{\chi}, a_h$ are the amplitudes of the Chandler and annual oscillations, respectively, and $a_{\chi}, a_h$ - their phases, which correspond to the Chandler ($N=0.843$ cycle per year) and annual (1 cycle per year) frequencies.

The amplitude of the resulting movement of the Earth's pole is obtained from (18) and has the form:

$$A = \sqrt{a_{\chi}^2 + a_h^2 + 2a_{\chi}a_h \cos(\alpha_{\chi} - \alpha_h)}.$$

If $\alpha_{\chi} = 2\pi Nt + \alpha_{\chi}^0$, $\alpha_h = 2\pi t + \alpha_h^0$, then the constant component of the amplitude modulation phase is $\alpha_{\chi}^0 - \alpha_h^0$.

3. Comparative analysis of models of polar tide

The phase of the amplitude modulation of the pole tide according to (5) coincides with the phase of the amplitude modulation of the pole oscillations:

$$\sqrt{(J_{13})^2 + (J_{23})^2} = \sqrt{a^2 + c^2} A.$$

Now let us calculate the amplitude of the pole tide from expressions (6):

$$\sqrt{(J_{13})^2 + (J_{23})^2} \approx \left[ a^2 + 4ab\pi(N+1)^2 \right]^{1/2}$$

As it follows from (7), the amplitude modulation of the pole tide should be phase shifted, which is physically natural, in contrast to the pole tide model, which is coherent with the pole motion. To determine the phase shift of the amplitude modulation, it is necessary to estimate the coefficient $b$ included in (7).

The generally accepted expressions (5) will be considered as an approximation of the pole tide model (6). That is, if we assume that the parameter estimation in (5) is optimal in the mean-square sense, then expressions (5) approximate the pole tide of model (6) the best way (in the same sense), and to find the estimate of the coefficient $b$, one can use the condition:

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T (J_{13} - \ddot{J}_{13})^2 dt \to \min_b$$

Let us make some explanations related to condition (8). Expressions (6), using $\omega_1$, $\omega_2$ from (7), can be reduced to a sum containing a linear combination $\omega_1$, $\omega_2$ of the form (5), and periodic terms. This presentation is not unique and can be written in various ways. For example, for three special cases they will be written in the form:
Various forms of expressions (9) show that the estimate of the coefficient \( b \) will depend on the averaging interval \( T \). However, such representations are infinitely many and in the general case it can be written:

\[
J_{13} = a\omega_1 - b\pi(N+1)\omega_2 - b\pi(N-1)[a_{ch}\sin\alpha_{ch} - a_h\sin\alpha_h] = \\
= a\omega_1 - 2b\pi\omega_2 + 2b\pi(N-1)a_h\sin\alpha_h = \\
= a\omega_1 - 2bN\pi\omega_2 - 2b\pi(N-1)a_{ch}\sin\alpha_{ch} \\
J_{23} = a\omega_2 + b\pi(N+1)\omega_1 + b\pi(N-1)[a_{ch}\cos\alpha_{ch} - a_h\cos\alpha_h] = \\
= a\omega_2 + 2b\pi\omega_1 - 2b\pi(N-1)a_h\cos\alpha_h = \\
= a\omega_2 + 2bN\pi\omega_1 + 2b\pi(N-1)a_{ch}\cos\alpha_{ch}
\]  

(10)

Three special cases written out in (10) are obtained from (9) for \( \xi = 1, 0, -1 \). Expression (10) allows us to replace minimization with respect to \( b \) by minimization with respect to \( \xi \), since the coefficient at \( \omega_2 \) must be equal to \( c \). The parameter \( \xi \) was introduced for convenience, and expression (10) does not depend on \( \xi \), but integrand (8) contains \( \xi \).

Let us denote by \( \sigma(T,t) \) standard deviation of \( \hat{J}_{13} \) from \( J_{13} \) (for \( J_{23} \) similarly) [18]:

\[
\sigma(T,t) = \frac{1}{T} \int_t^{t+T} (J_{13} - \hat{J}_{13})^2 \, dt.
\]

Now the coefficient \( b \) can be found from the correspondence of the coefficients at \( \omega_2 \) in expressions (10) and (5), which leads to the equality \( c = b\pi((1+\xi)N+1-\xi) \) and condition (11), which we rewrite as:

\[
\sigma = \lim_{T \to \infty} \sigma(T,t) \to \min_{\xi}
\]

(12)

Indeed, finding the coefficient \( b \) from condition (8) for the minimum for \( b \) due to the one-to-one correspondence \( \xi \) and \( b \) can be replaced by the condition (12) of the minimum for \( \xi \). The minimization condition in (12) means that the minimum is sought for \( \xi \) for the functions

\[
(b\pi(N-1)[(\xi+1)a_{ch}\sin\alpha_{ch}-(\xi-1)a_h\sin\alpha_h])^2, \quad (b\pi(N-1)[(\xi+1)a_{ch}\cos\alpha_{ch}-(\xi-1)a_h\cos\alpha_h])^2
\]

and determines the best in the mean-square sense approximation of expressions (10), and hence (6), by expressions of model (5).

In addition, it is obvious that condition (12) is also valid when solving the inverse problem – approximation \( \hat{J}_{13} \) to \( J_{13} \), that is, finding the coefficient \( c \) by the coefficient \( b \). It is precisely due to the one-to-one correspondence of these coefficients in the work that it is assumed that \( \hat{J}_{13} \) is an approximation of \( J_{13} \), but the dissipative coefficient \( b \), contained in \( J_{13} \), is the subject to assessment.

The value \( \sigma \) after passing to the limit in (12) turns out to be independent of the parameters \( t, T \), and \( \xi \) will be found from the minimization condition \( \sigma_\xi = \min_{\xi} \sigma \). Thus, the equation \( \sigma_\xi = \lim_{T \to \infty} \sigma(T,t) \) is equivalent to the equation \( c = b\pi((1+\xi)N+1-\xi) \) for the found \( \xi \) and will contain only one unknown \( b \).
Substituting into condition (8) expressions $J_{i3}$ from (6) and $\hat{J}_{i3}$ from (5) considering $c = b\pi[(1 + \xi)N + 1 - \xi]$. Function $\sigma$ turns out to be quadratic in the parameter $\xi$. In figure 1 shows a graph of the normalized value $\overline{\sigma}(\xi)$, related to $a^2$. Normalization is performed according to its minimum value. In figure 2 shows a graph of the values of the ratio $b/a$ depending on $\xi$.

**Figure 1.** Dependency graph $\overline{\sigma}(\xi)/a^2$. Blue and red areas correspond to the allowable spread within the selected ratio accuracy $c/a$.

**Figure 2.** Ratio graph $b/a$ depending on the parameter $\xi$. Blue and red areas correspond to the allowable spread within the selected ratio accuracy $c/a$.

To illustrate how the behavior of $\sigma(T,t)$ changes with a change in the averaging interval $T$ and the parameter value $\xi$, in figure 3 we give a comparison of dependencies of $\sigma(T,0)/a^2$ and their limiting values $\sigma/a^2 = \lim_{t\rightarrow\infty} \frac{\sigma(T,0)}{a^2}$ for $\xi = 1$, $\xi = 0$, $\xi = -1$.

As follows from the calculations and the graph in figure 1, condition (8) is satisfied at the value $\xi = 0.18$. However, if we fix the ratio $b/a = c/\pi((1 + 0.18)N + 1 - 0.18)a$ and then take $\xi = 0$, then the
ratio of the coefficients \( \frac{c}{a} = b\pi((1+\xi)N+N-1-\xi)/a \) will change by a value of the order of \( 2\times10^{-4} \), which corresponds to a change in the ratio \( \frac{c}{a} = 0.0115 \) in the fourth digit. That is, a change in \( \xi \) by a relatively small value in the vicinity of the minimum of the function \( \sigma(\xi)/a^2 \) leads to a relatively small change in the coefficients. We will disregard this change further, taking \( \xi = 0 \), thereby reducing the accuracy of specifying the coefficients, but simplifying some expressions. The assumption made will not affect the conclusions obtained below.

Now, you can roughly define the ratio \( b/a = 0.002 \). Using this value, it is easy to find the phase shift of the amplitude modulation, which is delayed by about 18 h.

Note that in practice, when using the generally accepted pole tide model of the form (5) corresponding to (3) and estimating the dissipation coefficient \( c \) based on the indirect measurement data of the limiting transition at \( T \to \infty \), described above, it is impossible to perform. In this case, choosing a finite averaging interval \( T(\xi) \) from the condition \( \int_{t}^{t+T(\xi)} (J_{13} - \hat{J}_{13}) dt = 0 \), we will use \( \sigma(T, t) \to \min_{\xi} \), to arrive at a similar result – close to the best estimation of the coefficient \( c \) at \( \xi = 0 \) within the accuracy of specifying the ratio \( \frac{c}{a} \). Thus, on the interval \( T(0) \) the estimation of the dissipative coefficient of the generally accepted pole tide model (5) will be close to the best in the sense of mean square.

4. The steady motion of the Earth's pole

Finally, let us consider how the steady-state Chandler pole motion described by Eqs. (2) will change when two types of dissipative functions are taken into account:

\[ f_{p}^0 = s_{x}x_{p} - s_{y}y_{p}, \quad f_{q}^0 = -s_{y}y_{p} - s_{x}x_{p} \quad (13) \]

and

\[ f_{p}^\theta = s_{xy}x_{p} - s_{y}y_{p}, \quad f_{q}^\theta = -s_{y}y_{p} + s_{x}y_{p}. \quad (14) \]
In the second case, after transforming the equations of motion of the pole (2), the functions
\[ f^0_{x_\mu} = s_1 x_p - s_3 \dot{x}_p, \quad f^0_{y_\mu} = -s_1 y_p + s_3 \dot{y}_p \]
can be formally replaced by the functions
\[ f^0_{x_\mu} = s_1 x_p - \tilde{s}_3 y_p, \quad f^0_{y_\mu} = -s_1 y_p - \tilde{s}_3 x_p \]
with the relation:
\[ \tilde{s}_3 = \frac{s_1 (N - s_1)}{1 + s_3^2}. \]

The change made \( f^0_{x_\mu} = s_1 x_p - s_3 \dot{x}_p, \quad f^0_{y_\mu} = -s_1 y_p + s_3 \dot{y}_p \) will not be an approximation of functions \( f^0_{x_\mu} = s_1 x_p - s_3 \dot{x}_p, \quad f^0_{y_\mu} = -s_1 y_p + s_3 \dot{y}_p \), but a formal re-designation of quantities \( f^0_{x_\mu}, f^0_{y_\mu} \) after transformation of differential equations (2). To determine the coefficients \( s_3 \) and \( \tilde{s}_3 \), you can use the estimate discussed above. On the modulation interval of the Chandler and annual harmonics, we find:
\[ s_3 = s_3(N + 1), \quad \tilde{s}_3 = \frac{s_1 (N - s_1)}{1 + s_3^2} \approx 0.914 s_2. \]

It follows from (14) that for a generally accepted pole tide model of the form (3), the amplitude of the stationary mode of Chandler oscillations can differ significantly from the real one, and this depends not only on the structure of the pole tide model, but also on the duration of the interval for estimating the dissipation coefficient, if the dissipation coefficient is determined based on the processing of indirect measurement data.

In figures 4 and 5, a comparison is made of the steady motions of the pole obtained as a result of integrating equations (2) taking into account the expressions for the pole tide (13) and (14). The values \( \mu_x, \mu_y \) were found from equations (2) using the approximation of the IERS observational data of the pole motion using expressions of the form (7) on the time interval from 2010 to 2017. Ранее было показано, что the difference between the solutions \( \Delta x_p, \Delta y_p \) is shown in the upper graph in figure 4. Comparing the top graph in figure 4 with the lower one, which shows the Chandler oscillations of the pole \( x_{ch}, y_{ch} \) of the approximation of coordinates \( x_p, y_p \), it follows that the differences \( \Delta x_p, \Delta y_p \) are mainly caused by the difference in the amplitudes of the Chandler component of the solutions. If you make a replacement
\[ f^0_{x_\mu} = s_1 x_p - s_3 \dot{x}_p, \quad f^0_{y_\mu} = -s_1 y_p - s_3 \dot{y}_p \]
taking into account (14) instead of
\[ f^0_{x_\mu} = s_1 x_p - s_3 \dot{x}_p, \quad f^0_{y_\mu} = -s_1 y_p + s_3 \dot{y}_p \]
then only the annual fluctuation (figure 5) will remain in the difference \( \Delta x_p, \Delta y_p \). Due to the fact that the difference between the Chandler and annual frequencies is not small, the amplitude of the annual oscillation in \( \Delta x_p, \Delta y_p \) turns out to be small. Moreover, as shown in figure 4, the difference in the amplitude of the Chandler oscillation in the two models, taking into account the expressions for the pole tide (13) and (14), reaches 10% with respect to the amplitude of the Chandler oscillation, which is very significant. At the same time, the discrepancy of oscillations calculated by the models taking into account (14) and (15) contains only an annual component whose amplitude does not exceed 0.5% with respect to the amplitude of the annual pole oscillation. Thus, if the dissipative terms of the pole tide are determined not by the position of the pole, but by its velocity, then the optimal approximation of the parameters of the pole tide does not lead to an optimal approximation of the parameters of the steady pole oscillation.
Figure 4. Difference $\Delta x_p, \Delta y_p$ between steady-state fluctuations calculated based on the models taking into account the expressions for the pole tide (13) and (14) (upper graph); Chandler oscillation $x_{ch}, y_{ch}$, obtained as a result of approximating data from 2010 to 2017 and extrapolation six years forward and backward.

Figure 5. The difference $\Delta x_p, \Delta y_p$ between the steady-state oscillations calculated from the models taking into account the expressions for the pole tide (14) and (15) (upper graph); annual oscillation $x_h, y_h$, obtained as a result of data fit from 2010 to 2017 and extrapolation six years forward and backward.
5. Conclusion
For the model of a viscoelastic Earth, the expressions for the variations in the tesseral harmonic coefficients of the geopotential caused by the pole tide contain terms incoherent to the pole oscillations. These terms are determined not by the position of the pole, but by the speed of its movement. As a consequence, the amplitude modulation of the pole tide should be phase shifted, which is physically natural, in contrast to the pole tide model, which is coherent with the pole motion.

It is shown that even if the generally accepted model of the pole tide is an optimal approximation of the observed tide, then the neglect of the terms depending on the velocity of the pole movement leads to a significant distortion of the parameters of the steady-state Chandler oscillation.

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