Quantum Decision Theory: A Darwinist Approach to a Centuries-old Enigma

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ABSTRACT

We live in a world brimming with uncertainty. While navigating the twists and turns of our uncertain world, the decisions we make are always clouded by a degree of uncertainty. We propose a Darwinist evolutionary approach, combined with quantum mechanics to cement our attempt in explaining a centuries long mystery - decision-making under uncertainty. And we believe that it is the unification of people’s subjective beliefs and the objective world that will solve this mystery. In this paper, we postulate a quantum decision theory to attempt to do just that. A quantum density operator as a value operator is proposed to simulate people’s subjective beliefs. Then the value operator will guide people to choose corresponding actions based on their subjective beliefs through the objective world. The value operator can be constructed from quantum gates and logic operations as a quantum decision tree. The genetic programming is applied to optimize and auto-generate quantum decision trees.

Introduction

How we make decisions is truly an enigma. Throughout the centuries until now, there have been many cracks taken at explaining the way decisions are made, starting from the ancient Greek and Chinese philosophers Socrates, Plato, Aristotle, Lao Tzu, Confucius, to Renaissance philosophers Descartes and Bacon. The first real mathematical foundation of decision-making didn’t come along until Gerolamo Cardano laid down the fundamentals of the then-new field of probability – a new way to describe the chance of how likely something was to happen¹.

Blaise Pascal and Pierre de Fermat discussed the problem of probability over the course of a 6-year exchange of letters, in which led them to developing modern-day probability theory. Over the course of their exchange, they discussed a problem posed by Chevalier de Méré, one of gambling with dice. They then proposed to use expected value to make decisions, which is to select the biggest expected value out of the all-possible selections, terming it the expected value theorem. The St. Petersburg Paradox was a game that would be proposed by Nicolaus Bernoulli to go against the expected value decision theory. Then in order to solve the problem of the St. Petersburg Paradox regarding expected value decision theory, Daniel Bernoulli would go onto propose the expected utility decision theory.

John von Neumann and Oscar Morgenstern were the first to axiomatize expected utility theory as the von Neumann-Morgenstern Utility Theorem (VNM)². von Neumann and Morgenstern showed that, under certain axioms of rational behavior, when a decision-maker is faced with risky outcomes of multiple different choices, especially ones with a high probability of happening, they will behave as if they’re maximizing the expected value of some function that’s been defined over the potential outcomes at some specified point in the future.

Leonard “Jimmie” Savage put forth his theory of subjective utility³, one that he stirred up statistics and probability along with reviving a centuries-old theory – Bayes’ theorem. Savage’s subjective expected utility theory combines two subjective concepts: a personal utility function, and a personal probability distribution, with the later based off of Bayesian probability theory. According to subjective expected utility, whatever decision that a decision-maker prefers to make will depend on their subjective beliefs.

The Allais Paradox and the Ellsberg Paradox⁴-⁵ were developed to refute the von Neumann-Morgenstern objective expected utility theorem and Savage’s subjective utility theory, respectively. Both of these paradoxes have showed that when we are faced with a choice of attaining something with more certainty over something that has a considerable
amount of risk though with a big reward, we tend to decide to take the former. Because of our tendencies to be more risk averse when faced with choosing a small-probabilistic event from happening, Israeli-American psychologist Daniel Kahneman, and his Israeli compatriot Amos Tversky, developed prospect theory that has detailing how individuals assess the chance of losing and gaining in a relative asymmetric manner.

Throughout time, the theories and hypotheses postulated from a classical approach, have all stuck with using calculus differential equations. While mathematics and probability can be a good "I told you so" way of summarizing things, it doesn’t quite hit the mark when trying to look towards the future. As Alfred Korzybski put it, "A map is not the territory it represents, but, if correct, it has a similar structure to the territory, which accounts for its usefulness." In other words, it’s difficult to find a perfect mathematical mode to rely on when assessing what will actually happen in the real world.

Isaac Newton started the mathematical "revolution" when he founded classical mechanics and calculus, while Charles Darwin kickstarted another scientific "revolution" with his theory of evolution. Ever since Newton, science has taken a path of determinism, but it is with Darwin’s theory of evolution in which we will take along with the theories of quantum mechanics to deal with indeterminism. We believe that this is the perfect combination of realistically explaining the inherently uncertain nature of the real world.

In our view, decision-making can be viewed as a two-phase process: an evaluation process and a selection process. Classical decision theory holds that the average Joe knows the utility function as well as the probability distribution through the evaluation process and then maximizes their utility by an optimal selection every time they make a decision, and all the while assuming everyone is rational. As for expected utility theory to work, information completeness and selection consistence are required, however, in the real-world information is rarely complete and consistency of selection cannot be guaranteed due to complexity and inherent uncertainty in the real world, plus people also show irrational behaviors.

In a sense, classical decision theory is a "black box"; we don’t know what really happens inside that box. A breadth of academicians from all fields of study has taken their best shot at explaining this "black box" of decision-making, yet it has eluded them all. We believe that the results of decisions are the unity of subjective beliefs and objective facts, and the value of observed results is the bridge between those two different worlds. If von Neumann’s VNM is entirely objective, and Savage’s theory is entirely subjective, then the "black box" of making decisions might just well be opened through the unification of the objective and the subjective, through the bridging of the observed value.

Anytime when we make a decision, there are the natural states of the world that we have to take into account, and then there are actions we can choose to take. The natural states of the world are objective, they are there no matter what, and our beliefs are influenced by nature and ultimately affect what actions we take are subjective.

Decision-making under uncertainty can be represented by Table 1. We subjectively choose an action $a_i \in \{a_1, a_2, \ldots, a_n\}$ where nature’s objective state is in $q_i \in \{q_1, q_2, \ldots, q_n\}$ when a decision is made, and the result of the decision (value matrix $v_{ij}$) depends on both the state of nature and choice of brain.

| State | q1 | ⋯ | qj | ⋯ | qn |
|-------|----|---|----|---|----|
| Action |    |   |    |   |    |
| $a_1$  | $v_{11}$ | ⋯ |     | ⋯ | $v_{1n}$ |
| ⋮      | ⋮   | ⋮ | ⋮   | ⋮ | ⋮   |
| $a_l$  | ⋮   | ⋮ |     | ⋮ | ⋮   |
| ⋮      | ⋮   | ⋮ | ⋮   | ⋮ | ⋮   |
| $a_m$  | $v_{m1}$ | ⋯ | ⋯   | ⋯ | $v_{mn}$ |

Table 1 State-action-value decision table
Using the futures market as an example, Table 2 shows the two states of $q_1$ and $q_2$ signify that the market is rising and falling, respectively. Then we have the two possible actions, $a_1$ and $a_2$ being buy and sell, respectively. All together, we have the all the possible outcomes of $v_{11}$, $v_{12}$, $v_{21}$, and $v_{22}$, which signify whether we profited or not.

| State  | $q_1$ | $q_2$ |
|--------|-------|-------|
| Action | $a_1$ | $v_{11}$ |
|        | $a_2$ | $v_{21}$ |
|        |       | $v_{12}$ |
|        |       | $v_{22}$ |

Table 2 State-action-value decision table of futures market

In the case of the market, it’s the indeterminacy of the objective world (whether the market is up or down) that causes the uncertainty of people’s subjective beliefs when making decisions (to buy or sell). Therefore, the result of decisions is uncertain (gain or loss). Basically, whenever we make decisions, we do so to maximize our value to the greatest extent. Why do we make decisions that maximize our value? Maximizing value is the equivalent to survival. At the very fundamental level, all living beings make decisions to survive. In the animal kingdom the rules are, eat or be eaten. And those fundamental decisions are usually binary. Yes or no, true or false, fight or flight. Essentially 0 or 1. When making a decision, most people, and obviously animals will definitely not use some mathematical formula and equation to calculate the probability every single time a decision is made.

Another important factor when learning from experience to make decisions is that everyone to some degree remembers and factors in past decisions when making future ones, which is in contrast to the Markov decision process where all decision-makers have amnesia. Everyone makes a decision with at least some influence of the past. Whether or not what happened in the past can be used to make a good decision, it has to be acknowledged that people make decisions with the results of what decisions they made in the past in mind.

Here we propose a quantum decision theory to tackle the decision-making enigma, one that utilizes the theories of quantum mechanics and Darwin’s theory of evolution. A quantum density operator as value operator is proposed to simulate people’s subjective beliefs. The value operator, which can be constructed from quantum gates and logic operations as a quantum decision tree, guides people to choose corresponding actions based on their subjective beliefs through the objective world to maximize their expected value. Lastly, genetic programming is applied to learning the historical data to build up the decision-maker’s experience (knowledge about the natural world) and then optimizing and generating quantum decision trees for decision-making.

**Quantum expected value (qEV)**

Building off of the futures market as an example, when a trader makes a decision, he/she subjectively chooses an action (buy or sell) $a_i \in \{a_1, a_2\}$ where the market’s objective state (rising or falling) is in $q_i \in \{q_1, q_2\}$ when a trade is made, and the result of the trade depends on both the state of the market and choice of the trader’s brain as already shown in table 2 in the above section. The state of the market describes the objective world; it can be represented by the superposition of all possible states in terms of the Hilbert state space as shown below.

$$|\psi\rangle_{\text{market}} = c_1 |q_1\rangle + c_2 |q_2\rangle \quad |c_1|^2 + |c_2|^2 = 1 \quad (1)$$

Where $|q_1\rangle$ denotes a state in which the market has gone up and $|q_2\rangle$ denotes a state in which the market has gone down. $|c_1|^2$ is the objective frequency of the rising; $|c_2|^2$ is the objective frequency of the falling market.

The market is the objective world; and the trader’s mental state is the subjective world. We postulate that when the trader is undecided in making a trade (buy or sell), it can be represented by superposition of all possible actions as follows.

$$|\phi\rangle_{\text{mental}} = \mu_1 |a_1\rangle + \mu_2 |a_2\rangle \quad |\mu_1|^2 + |\mu_2|^2 = 1 \quad (2)$$
exactly what happens when, how Aristotle stated all those years ago, the decision goes from potential to actuality.

We have the objective (state of the market) and the subjective (trader’s beliefs), and the possibilities of what could happen. But in order to take it to the next step, essentially how does the external objective world, the market, and the subjective world, the trader with their beliefs, come together into action. Basically, anytime a trader goes head-on into the market to trade, the information that they have available to them prior making their decision is limited, in a sense, basically incomplete. They don’t know whether the market will rise or fall. If they did, they wouldn’t have to make a decision. Just buy or sell according to what will happen and then happily collect money. The prior information that every trader has is not complete, which then force trader’s to essentially “guess” or “bet”, which are based off of the given prior information and their subjective beliefs. This way, prior information also influences trader’s subjective beliefs as well.

How this is denoted is that, since the value of the trader’s decision (profit or loss) is uncertain, it is expressed by a mixed state’s density operator as a value operator, as shown below. A value operator is a sum of projection operators that project the trader’s degrees of beliefs onto an action of choice (buy or sell).

$$\hat{V} = p_1 |a_1\rangle (a_1| + p_2 |a_2\rangle (a_2|\quad p_1 + p_2 = 1$$  \hspace{1cm} (3)

Where $p_1$ is the subjective probability of choosing $a_1$ (buying), and $p_2$ is the subjective probability of choosing $a_2$ (selling), $|a_1\rangle (a_1|$ and $|a_2\rangle (a_2|$ being the von Neumann projection operators (matrixes) which projects trader’s actions to buy or sell.

In quantum mechanics, all observable variables are expressed by operators. The value is expressed by an operator because value is an observable variable. Classical decision theory uses value functions to evaluate people’s decisions (gains or losses). In our quantum decision theory, we use value operators to evaluate people’s decisions.

We’ve seen the objective, subjective, and the probabilities of the actions that can be taken and the outcomes that could happen, as well as how these are all expressed. From here we can take the step of how all of this becomes one and exactly what happens when, how Aristotle stated all those years ago, the decision goes from potential to actuality.

Before a trader makes a decision, his/her mental state is in a pure state, a state in which they can decide whether to buy and sell at the same time. It’s kind of like being on the fence about whether or not you should buy something new. But in reality, you can’t make a decision to buy and sell simultaneously; it’s just not possible. Thus, this pure state is when the states of buy and sell are superposed in the trader’s mind. Then when the trader makes the decision, their mental state is then transformed from that pure state into a mixed state, which is when they decide to either buy or sell, do one or the other, with certain degrees of belief (subjective probabilities). Basically, this transformation is the brain choosing from one of the available actions, in this case buy or sell, with action $a_1$ being buy with probability $p_1$ and action $a_2$ being sell with probability $p_2$, as shown below.

$$D: \hat{\rho} = |\phi\rangle \langle \phi| \rightarrow \hat{V} = p_1 |a_1\rangle (a_1| + p_2 |a_2\rangle (a_2|\quad \text{decision} \quad |a_1\rangle (a_1|, i = 1,2$$  \hspace{1cm} (4)

Given the information that the brain has before the trade is made as reference, there’s only the possibility of selecting an action with a subjective probability. And by making the decision, choosing an action, is what transforms from a potential possibility into reality; just like how Aristotle put it: ”matter exists potentially, because it may attain to the form; but when it exists actually, it is then in the form”.

Throughout the decision-making process, a trader’s mental state can be simulated by the continuous evolution of the value operator according to the environment (information). Information is the essence of people’s subjective beliefs just like energy is the essence of the objective world. Claude Shannon said, “Information is the resolution of uncertainty,” we believe that it is not just information that resolves uncertainty, but valuable information is what reduces it. While valuable information reduces uncertainty, it does not come by easily. Though the subjective will always be influenced by the objective to certain degree, it is the unification of the two that will bring along much needed valuable information when
making a decision. And it is here that we now take the two and unify them into one, as quantum expected value. Quantum expected value (qEV) can be represented below:

$$\text{qEV} = \langle \psi | \mathcal{V} | \psi \rangle = (c_1(q_1) + c_2(q_2))(p_1|a_1\rangle(a_1|q_1) + p_2|a_2\rangle(a_2|q_2))$$

$$= p_1\omega_1(a_1|q_1)^2 + p_2\omega_2(a_1|q_2)^2 + p_1\omega_1(a_2|q_1)^2 + p_2\omega_2(a_2|q_2)^2$$

$$= \sum_{i=1,2} p_i \sum_{j=1,2} \omega_j r_{ij}$$

(5)

Where $p_i$ is a trader’s subjective probability in choosing an action $a_i$, subjective probability represents the trader’s degrees of belief in a single trade; $\omega_j$ is the objective frequency at which state of the futures market is in $q_j$, objective frequency represents the statistical results of multiple occurrences of objective states; matrix $v_{ij}$ is the value when the trade was made, in which the trader chose an action $a_i$ where state is in $q_j$ as shown below.

$$v_{ij} = (|a_i|q_j)|^2 = \begin{cases} 1, & i = j \\ -1, & i \neq j \end{cases}$$

(6)

When the trader “guesses” the direction of the market correctly ($i = j$), then he/she profits. This could be if the market goes up and they buy or if the market goes down and they sell. If the trader doesn’t “guess” correctly ($i \neq j$), then he/she suffer a loss. This would be if the market goes up but the trader sells, or if the market goes down and the trader buys. The different actions that the trader’s took lead to different value (gain or loss); in other words, the value is “created” based on both traders’ subjective beliefs and objective natural states.

This unification of the traders’ beliefs (subjective) and the state of the market (objective), the quantum expected value (qEV), can be seen in two distinct parts – an “average of average” like situation. This is simply taking the subjective and the objective then assigning them to two respective parts of the full qEV, resulting in the unification of the two.

$$\text{oEV}_i = \sum_{j=1,2} \omega_j v_{ij}$$

(7)

Breaking it up, we have the first part, or first average, which is the objective expected value, which we will term as in equation (7). This is the part of the objective world, the external info that we take in, using from which we will call the outside brain. So, the outside brain takes in information from the external world and turns that info into the oEV, while making an evaluation, the first evaluation in the qEV average of average process.

$$\text{sEV}_i = p_i \text{oEV}_i$$

(8)

Next, we have the second part, or average of the oEV, which is the subjective expected value as in equation (8). But just this sEV$_i$ alone is not enough; it doesn’t get us where we want to go. In order to get the average of average, the unified qEV, we take the two, the oEV, and the sEV, and then we average it, or mathematically, $qEV = \sum_{i=1,2} sEV_i$. All together this becomes the subjective expected value of the objective expected value (average of average), which is the unification of the objective natural world and the subjective human beliefs.

We postulate that there are two “brains”, one outside and one inside, that work together in harmony to convert the external world and the info from it. The objective world is taken in by the outside brain. Then the inside brain converts what the outside brain collected to the qEV. By maximizing the qEV, then the brain becomes a middle state between the objective external world and the subjective “inside” world, which is what we call the qOS. The q is the qEV, the O is Objectivity, and the S is Subjectivity. The qOS is the reflection of what the brain thinks the real world is. Or some people call it an “illusion” (subjective model of objective nature). This state of qOS can be signified by the Venn diagram below.
The final qEV, the objective and subjective states that are then unified together to the qOS middle state in the brain, which is the unification of the external world’s external info and the “inside” world’s subjective beliefs.

From the great classical theories of von Neumann and Savage, their theories are the two extremes: von Neumann is all objective, while Savage is all subjective. And thus, ours is the unified both objective and subjective, qOS. And no, this is not a quantum operating system. It’s not going to power a smartphone. But it may power our brain. Because what an operating system does is, it connects the hardware and software, and so, if we believe that the brain is a quantum-like computer, then the qOS is the “quantum operating system” that powers it: it connects the outside world (hardware), and the subjectivity of what the user wants to do (software).

Quantum decision tree (qDT)

The qDT, which is the value operator \( \hat{V} \), is constructed with the logic operations (the non-leaf nodes represented by operation set \( F \)), and the basic quantum gates \(^{10}\) (the leaf nodes represented by data set \( T \)), both shown in (9) and (10).

\[
F = \{ + (\text{ADD}), \quad \ast (\text{MULTIPLY}), \quad \text{//(OR)} \} \tag{9}
\]

\[
T = \{ H, X, Y, Z, S, D, T, I \} \tag{10a}
\]

\[
\begin{align*}
H &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, & X &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, & Y &= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, & Z &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
S &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & D &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, & T &= \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}, & I &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} 
\end{align*} \tag{10b}
\]

qDT will have branches and nodes, which will be further categorized as leaf nodes and non-leaf nodes. The actual construction process of a qDT is by randomly selecting a logic symbol from the operation set \( F \), the non-leaf nodes, which are the “branches” that grow corresponding accordingly to the nature of the operation symbol and so on until a leaf node is reached. And just how like living trees can’t grow more leaves from a leaf; a qDT will stop growing more “leaves” once it reaches a leaf node. Thus, a qDT starts off with one of the operation logic symbols from \( F \), and can continually grow until a leaf node is reached. The final product that is reached is the quantum gates from data set \( T \) that is in the final leaf nodes. A fully grown qDT has the non-leaf nodes as circles and the leaf nodes as squares. A simple qDT is in Figure 2 (a). This qDT1 is constructed like this. The + in the circle (non-leaf nodes) on top is the operation symbol, in this case addition from the operation set \( F \). According to the rules of addition, it now must “grow” two “branches” to add together. Thus, we get the two squares (leaf nodes), with \( H \) and \( X \) inside, which are the quantum gates from data set \( T \). Since \( H \) and \( X \) are leaf nodes, this qDT ends there.
The qDT\(_1\) can be mathematically represented as below:

\[
qDT_1 = (H + X)
\]

\[
S_1 = (H + X) \rightarrow \bar{V} = 0.50|a_1\rangle\langle a_1| + 0.50|a_2\rangle\langle a_2|
\]

Where \(S_1\) denotes the strategy available for the trader to select from. What this strategy tells us is that the trader has an exactly 50% degree of belief of buying and selling, respectively. This strategy’s subjective probability is basically the equivalent of tossing a coin to choose between one or another. And in that case, this qDT gives us no valuable information whatsoever, hence, that’s the reason why it won’t fare well if employed in a real trading environment.

Figure 2(b) is a somewhat more complex-looking qDT\(_2\). We start off with the tree’s “root”, in this case is the multiplication operation in the top circle. By rules of multiplication, it “grows” two branches, the \(I\) quantum gate in the square and another multiplication operated in a circle. Since the quantum gate \(I\) is a leaf node, it ends there. Since the second multiplication is a non-leaf node, meaning it will keep growing from that one. It keeps growing, resulting in two more operation symbols, an OR (\(/\)) and an ADD (+). From here, they both go onto branch off to more nodes. The OR grows another multiplication operator and an \(X\) quantum gate leaf node. The addition grows two leaf nodes, therefore ending it with an \(X\) quantum gate and a \(I\) quantum gate. The multiplication that arose from the OR, then continues to grow branches, while the \(X\) quantum gate produced by the OR ends. And finally, we get to the two “leaves” produced by the * from the \(/\), a \(T\) and \(Y\) quantum gates, completing this qDT\(_2\). Mathematically this second qDT\(_2\) is expressed as follows:

\[
qDT_2 = \left( I \ast \left( ((T \ast Y) /X) \ast (X + I) \right) \right)
\]

\[
S_1 = \left( I \ast (X \ast (X + I)) \right) \rightarrow \bar{V} = |a_1\rangle\langle a_1|
\]
\[ S_2 = \left( I * ((T * Y) * (X + 1)) \right) \rightarrow \Psi = 0.04|a_1\rangle \langle a_1| + 0.96|a_2\rangle \langle a_2| \tag{12c} \]

According to this second qDT, what it tells us is that there are two different strategies that we can take: strategy \( S_1 \) is 100% certain to take action \( a_1 \) (buy); and strategy \( S_2 \) is 96% degree of belief to take action \( a_2 \) (sell) and a 4% degree of belief to take action \( a_1 \) (buy). This shows that the two strategies respective subjective probabilities of this qDT are almost completely close to unity, resonating almost maximum information. With this qDT, any trader will gain way more valuable information than the first one. Just a quick elaboration on why this qDT produces two strategies. Since there is an OR operation symbol, that means that the two branches under the OR have a 50/50 chance of being used, one of which is the X gate and the other is the \( (T * Y) \) gate. Thus, that's why are two different strategies produced by this qDT.

In the words of Ray Solomonoff in his paper "Does Algorithmic Probability Solve the Problem of Induction?", "We are uncertain of the probabilities needed for the decision and we cannot express this uncertainty probabilistically." Hence, this is the exact reason why we use qDT's, because we first select one strategy from a group of strategies, and then select one action with subjective probability to deal with the uncertainty need of the decision. The nest of hierarchy structure of qDT attempts to solve the problem of "we cannot express this uncertainty probabilistically." qDT's alone will not aid us into making the best decisions, the next step is to find a way to optimize qDT's with a group of satisfactory strategies to guide our decisions.

**Quantum Generic Programming (qGP)**

To optimize anything, there needs to be: first, a selection of a good evaluation function and two, how to acquire an optimal solution. First off, in our model, any decision-maker will try to maximize their qEV (quantum expected value) when making any decision. Thus, we need to evaluate how "fit" the result (profit or deficit) of the trader's decision, which we can do so by using qEV as a fitness function to optimize qDT's by evolving them (using qGP).

What the fitness function essentially is a particular type of function that is used to summarize, as a single figure of merit, how close a given design solution is to achieving the set aims. Fitness functions are used in genetic programming\(^{11-12}\) to guide simulations towards optimal design solutions. In order to reach the optimal solution, the qGP algorithm implements a continuous evolution process through selection, crossover, and mutation. The whole idea of having qGP go through an iterative evolution loop is to find a satisfactory qDT by means of learning historical data to obtain the most optimal solution. The learning rules are as follows:

a) If the futures market is up \( (q_1) \)
   i. If the trader bets the market is up \( (a_1) \), the trader profits \( (v_{11} = 1) \);
   ii. If the trader bets the market is down \( (a_2) \), the trader deficits \( (v_{21} = -1) \);

b) If the market is down \( (q_2) \)
   i. If the trader bets the market is down \( (a_2) \), the trader profits \( (v_{22} = 1) \);
   ii. If the trader bets the market is up \( (a_1) \), the trader deficits \( (v_{12} = -1) \).

The result (qEV) of a kth trade as follows:

\[
< \text{qEV}_k > = \begin{cases} 
 p_1 \omega_1, & |\phi\rangle_{\text{mental}} = |a_1\rangle \text{ and } |\psi\rangle_{\text{market}} = |q_1\rangle \\
-p_1 \omega_2, & |\phi\rangle_{\text{mental}} = |a_1\rangle \text{ and } |\psi\rangle_{\text{market}} = |q_2\rangle \\
-p_2 \omega_1, & |\phi\rangle_{\text{mental}} = |a_2\rangle \text{ and } |\psi\rangle_{\text{market}} = |q_1\rangle \\
 p_2 \omega_2, & |\phi\rangle_{\text{mental}} = |a_2\rangle \text{ and } |\psi\rangle_{\text{market}} = |q_2\rangle 
\end{cases} \tag{13} 
\]
It is critical to note here that the above four possible outcomes, only one of those can actually happen. Because the market can only be up or down, and a trader’s choices are only to buy or sell, only one of those combinations of whether the market is up or down and if the trader will buy or sell can occur in reality. \( \omega_1 \) and \( \omega_2 \) (the frequencies of whether the market is up or down) are known variables, which are approximately 50/50. Now, there are a couple of things that we don’t know. One, what action a trader will take at any given time, since everyone is different. Two, we also don’t know how “confident” a trader is in their choice when they take their action, which is either buy or sell based off of the info and beliefs they have. This is represented by \( p_1 \) and \( p_2 \), which are their beliefs that the market will go up or down, respectively. The whole point of optimizing all of this is for two reasons: one, getting the trader to make a decision as correct as possible, and two, to do so with the highest degree of belief, which would mean either \( p_1 \) or \( p_2 \) is equal to 1. Essentially this is what qDT is trying to do: get the trader to make the best decision every single time, and by doing so the chosen strategy needs to be “selected” correctly every time with the greatest degree of belief. The fitness function represents the sum of all the qEV\(_k\) by learning all the historical data as shown below.

\[
f_{\text{fitness}} = \sum_{k=0}^{N} qEV_k
\]  

(14)

The optimal solution is implemented by a qGP algorithm, below.

**Input:**

- Historical data set \( \{d_k = (q_k, x_k), k = 0, \cdots , N\} \).
- Setting
  1) Operation set \( F = \{+, \ast, \oslash\} \)
  2) Data set \( T = \{H, X, Y, Z, S, D, T, I\} \), eight basic quantum gates
  3) Crossover probability = 75%; Mutation probability = 5%.

**Initialization:**

- Population: randomly create 300 qDTs.

**Evolution:**

- for \( i = 0 \) to \( n \) (\( n=50\text{~to}100 \) generations)
  a) Calculate fitness for each qDT based on historical data set.
  b) According to the quality of fitness:
    i. Selection: selecting parent qDTs.
    ii. Crossover: generate a new offspring using the roulette algorithm based on crossover probability.
    iii. Mutation: randomly modify parent qDT based on mutation probability.

**Output:**

- A qDT of the best fitness.

There are four parts that are in sequential order when the algorithm is applied: Input, Initialization, Evolution, and Output. In the input phase, historical data is gathered to learn from (\( q_k \) denotes state of equity and \( x_k \) denotes closing price). Then the qDT’s that will be constructed by the algorithm are “set”, with the operation and the quantum gates. For the crossover and mutation probabilities, what that basically means is that the qDT’s have a 75% chance of producing an offspring, or child, and a 5% of having one of its own branches modified. Then the initialization phase, 300 qDT’s are randomly created as population. The evolution stage is pretty much self-explanatory; it’s basically a survival of the fittest of the generated qDT’s, with the strongest surviving and passing down their “genes” to their “children”. Output is then the best qDT’s that have evolved.
Results

The k-data of rebar rb1901 (transaction cycle is day from 2018/1/16 to 2018/12/7) traded on the Shanghai Futures Exchange is used as the historical data for optimization. It is the 218 closing prices throughout the duration of the contract. Just like any human would go about entering the world of trading, most wouldn’t go into trading blind, it would probably be best if they look at past historical trading data to formulate their trading strategies. By following the learning rules, the qGP algorithm will use this historical data as training data to search for satisfactory qDTs by maximizing qEV. One qDT includes a group of strategies that guide a trader to take actions which allows him/her to make the greatest possible profit.

Below is one satisfying qDT (Figure 4) that’s been evolved by qGP:

$$\text{qDT} = \left( \frac{D \ast \left( Y \ast \left( \left( \frac{I}{(I + Y) + T} \right) + 1 \right) \right)}{I} \right) \left( \frac{I}{(I + Y) + T} \right)$$

This qDT can be broken down by the 8 strategies it produces:

- $S_1 = \left( \left( \frac{I}{(I + Y) + T} \right) + 1 \right) \rightarrow \hat{V} = 0.88|a_2|\langle a_1 \rangle + 0.12|a_2|\langle a_2 \rangle$ (88% belief to buy, 12% belief to sell)
- $S_2 = \left( \left( \frac{I}{I + Y} + T \right) + 1 \right) \rightarrow \hat{V} = 0.83|a_1|\langle a_1 \rangle + 0.17|a_2|\langle a_2 \rangle$ (83% belief to buy, 17% belief to sell)
- $S_3 = \left( \left( \frac{I}{I + Y} + H \right) + 1 \right) \rightarrow \hat{V} = 0.97|a_1|\langle a_1 \rangle + 0.03|a_2|\langle a_2 \rangle$ (97% belief to buy, 3% belief to sell)
- $S_4 = \left( \left( \frac{I}{I + Y} + H \right) + T \right) \rightarrow \hat{V} = 0.5|a_1|\langle a_1 \rangle + 0.5|a_2|\langle a_2 \rangle$ (50% belief to buy, 50% belief to sell)
- $S_5 = \left( D \ast \left( Y + (X + T) \right) \right) \rightarrow \hat{V} = 0.43|a_2|\langle a_1 \rangle + 0.57|a_2|\langle a_2 \rangle$ (43% belief to buy, 57% belief to sell)
- $S_6 = \left( D \ast \left( Y + (I + T) \right) \right) \rightarrow \hat{V} = 0.68|a_1|\langle a_1 \rangle + 0.32|a_2|\langle a_2 \rangle$ (68% belief to buy, 32% belief to sell)
- $S_7 = \left( D \ast \left( Y + \left( \frac{(D + (S + Z)) + T}{} \right) \right) \right) \rightarrow \hat{V} = 0.16|a_1|\langle a_1 \rangle + 0.84|a_2|\langle a_2 \rangle$ (16% belief to buy, 84% belief to sell)
- $S_8 = \left( D \ast \left( Y + \left( (T + H) + T \right) \right) \right) \rightarrow \hat{V} = 0.24|a_1|\langle a_1 \rangle + 0.76|a_2|\langle a_2 \rangle$ (24% belief to buy, 76% belief to sell)

Based on the degrees of beliefs they can be further grouped into 4 groups:

- Group One: the trader is close to evenly split (50/50) of buy and sell, which includes strategies $S_4$ and $S_5$
- Group Two: the trader is heavily committed to buying, which includes strategies $S_1$, $S_2$, and $S_3$
- Group Three: the trader is heavily committed to selling, which includes strategies $S_1$, and $S_8$
- Group Four: the trader is leaning towards buying but is not that fully committed yet, in this case is only strategy $S_6$
This qDT has a mixed strategy comprising of the eight strategies to choose from. If a strategy from group one is chosen, with its close to 50/50 probability of either buy or sell is selected, then in this case no valuable information will be obtained because it’s totally uncertain; or if one of the strategies from the group two is selected with almost 100% surety to buy, then maximum information is obtained; or if a strategy from group three is selected with close to 100% surety to sell, then maximum information is also obtained; and since strategy $S_6$ is the lone strategy of not fully committed to a decision, though leaning towards one, the best information that it can provide is to just follow your instinct to choose. One thing that is clear from these groups of strategies is that, if the belief of buying or selling is high then it is best to follow through on that belief. If it is hovering between 50/50 it’s best to just wait (don’t do anything), or just flip a coin to decide. It’s just like how the old sayings go, “trust your gut”, and “when in doubt, hit the luck.” qDT can only pick one of the eight strategies to actually execute. It can’t pick two, three, or all eight strategies at once, it has to choose one and one only. Once it selects that one strategy, it is still with a certain degree of belief of choosing whether to buy or sell with the chosen strategy. For example, if it chooses strategy $S_1$, then there’s an 88% degree of belief to buy, and a 12% degree of belief to sell.

| Market State | Strategy | Action | Degree of Belief | Gain or Loss |
|--------------|----------|--------|-----------------|--------------|
| down         | $S_1$    | buy    | 88%             | -8.0         |
| up           | $S_1$    | buy    | 88%             | 10.0         |
| up           | $S_8$    | buy    | 24%             | 2.0          |
| up           | $S_3$    | buy    | 97%             | 3.0          |
| up           | $S_5$    | sell   | 57%             | -5.0         |
| up           | $S_8$    | buy    | 24%             | 3.0          |
| up           | $S_1$    | buy    | 88%             | 16.0         |
| down         | $S_7$    | sell   | 84%             | 2.0          |

Table 3 Details of the first eight transactions.
The table above illustrates the first 8 transactions our qDT made and which strategies it took. In the table, in the first trade that it completed, strategy $S_1$ was chosen, with an 88% degree of belief to buy, but since the state of the market was trending downwards at the time of the transaction, it resulting in a deficit of -8. And in the second trade, strategy $S_1$ was also chosen, but this time the market was trending upwards, which resulted in a profit of 10. Trade number 3 saw our qDT take strategy $S_6$, with only a 24% degree of belief to buy, and though the market was trending up at the time, there was a gain of 2. Following this logic, the rest of the trades are pretty much self-explanatory, with the qDT ending off with five “victories” out of the first eight transactions, as illustrated by the full table above. For the total 218 trades that the qDT made, the odds are around 60% in favor, which is up from the expected 50/50 chance of winning or losing in the market.

**Discussion**

$$ \text{market} = \{ \text{state}_i, \text{price}_i \mid i = 1, 2, \cdots, N \} \quad (16) $$

The state, depicted is the dynamic state of the market, basically if it’s up or down at any given time, but at set intervals, i.e., 1 day, 2 days, 3 days. And the price that is depicted above is the closing price that is determined by all the traders participating in the market, their trading, whether it be buy or sell, will determine the sequence of the price, $\{ \text{price}_i \mid i = 1, 2, \cdots, N \}$, which represents the “path” of the market, where we see it as the market’s volatility.

Thus, the traditional way is to use stochastic differential equations to describe the market’s volatility and treat it as a stochastic process. Economists have used this “classical” way of formulating a mathematical model of the market and how traders make decisions under uncertainty. But this stochastic math model of the market comes short in some ways, the most notably being that it can’t really describe what actually happens in the real world because of all the uncertainty at play. In the words of statistician George Box, “All models are wrong, but some are useful.” Or Alan Watts’ way, “The menu is not the meal.” In a sense it kind of only shows the “tip of the iceberg”.

Actually, traders face the constant problem of decision-making under the influence of an uncertain market, in which it can be seen as a game between trader(s) and the market, a game of question and answer, with the market "asking" the questions, in which the trader responds (answers) through their actions. This can be described as:

The “questions” posed by the market: $\{ \text{state}_i, \text{price}_i \mid i = 1, 2, \cdots, N \}$

The “answers” by the trader using yes/no logic in quantum decision trees: $\{ \text{qDT}_j \mid j = 1, 2, \cdots, M \}$

We can look at the game between trader and market in this way: there is a sequence of “choices” made that are posed as “questions” by market, and the traders’ must “answer” those “questions” by selecting a sequence of actions guided by optimized strategies to “beat” the market. The mathematical way to explain the real world is not very ideal, especially in the context of a “game” between trader and market. But what is, and may be the ideal solution to explain this Q&A “game” is Darwin’s theory of evolution. And we can do this by proposing a hypothetical robot “trader” that will take place in playing this game of art and strategy with the market, one that will have been "created" according to the concepts of qEV, qDT, and qGP. How it will do it, is that it’ll learn the historical data that were posed in question form by the market. Then utilizing the yes/no logic of qDT (nest of hierarchy), which is then applied to evolve the natural dynamic rules to formulate satisfying strategies to guide traders in the “game” with the market. The description of utilizing a computer simulation process to answer the market’s question, instead of the traditional stochastic differential equations approach is as follows:

```
historical data (market) \begin{array}{c}
\xrightarrow{\text{learn}} \\
\xrightarrow{\text{computational simulation (robot "trader")}}
\end{array} \begin{array}{c}
\xrightarrow{\text{evolve}} \\
\xrightarrow{\text{market dynamics rules}}
\end{array} \begin{array}{c}
\xrightarrow{\text{reconstruct}} \\
\xrightarrow{\text{market price "path"}}
\end{array}
```

The measured data of the market (the questions posed by nature): {$\{q_1, x_1\}, \{q_2, x_2\}, \ldots, \{q_n, x_n\}$}, in which $q_n$ denotes the state of the market (up or down), and $x_n$ denotes the market’s price. Then we get to the robot trader’s learning and optimization process, in which it “computes” a sequence of data that will be “matched” with the market’s state in order to
see if it “answered” correctly: \( \{(a_1, y_1), (a_2, y_2), \ldots, (a_n, y_n)\} \), in which \( a_n \) denotes the action (buy or sell) taken by the robot, and \( y_n \) is the final price that is calculated based on the action, \( a_n \), that the robot “trader” took.

This process works like this: utilizing the qDT that produces a group of strategies to choose from, each one of those strategies presents two actions to take, which in turn each have a probability degree of subjective belief, and the robot “trader” can only choose one of the actions of one of the strategies, and one only. It is then with that, then the price that the trader will buy or sell at is computed that is denoted by \( y_n \). Throughout the process of learning, the robot “trader” is able to compute a price that will be consistent with the market price, which will mean that the \((q_n, x_n)\) and \((a_n, y_n)\) will “match” each other, then the trader will have “answered” the “questions” posed by nature.

Here we define the absolute deviation function as:

\[
D_i = |y_i - x_i|
\]

(17)

Where \( x_i \) is the market actual price, \( y_i \) is the computed price of the market by the robot “trader”. We take the absolute deviation as negative feedback to fine tune the fitness function as below:

\[
f_{fitness} = \sum_{i=1}^{N} qEV_i / D_i
\]

(18)

By maximizing \( qEV_i \) (quantum expected value) to “guess” the current dynamic state of the market which quantum decision tree (qDT) is utilized to assess the market’s future state. And \( D_i \) (negative feedback), which is utilized to adjust the “calculated market price” based on assessing the trader’s past experience to match the market’s actual price.

| trade | market state | qDT\(_1\) | \ldots | qDT\(_{18}\) | \ldots | qDT\(_{25}\) |
|-------|--------------|----------|--------|----------|--------|----------|
| 1     | 1            | 0        | \ldots | 0        | \ldots | 1        |
| \vdots | \vdots      | \vdots   | \ldots | \vdots   | \ldots | \vdots   |
| 100   | 0            | 0        | \ldots | 0        | \ldots | 0        |
| \vdots | \vdots      | \vdots   | \ldots | \vdots   | \ldots | \vdots   |
| N=218 | 0            | 0        | \ldots | 0        | \ldots | 0        |

Table 4 Details of the trades made by robot.

The above table is the 218 trades, in which the market state of up (0) or down (1) is presented as historical data to the robot “trader” to learn from, labeled in the second column. Then using the fitness function that was just defined above, the robot “trader” is “trained” by 25 times that produces 25 quantum decision trees’ (qDT), labeled as qDT\(_1\)~qDT\(_{25}\). Each of those qDT’s then becomes the trader’s “experience” regarding the market through learning the market’s historical data. Altogether, the 25 qDT’s are the trader’s entire experience (knowledge) about the market.

Over the course of the 218 trades, each qDT’s chance of winning is around 60%, which equates to approximately 130 times win. Just like how we rely on our entire past experiences to make decisions, our robot also needs to rely on all of its past experiences, which is the experience it gained from all of the qDT’s as a whole. According to the table above, by
applying majority rules, we can let our robot trader determine the market’s state based on the experience obtained from 25 qDTs. The rules are as follows:

1. If more than half of the qDTs decide that the market is up, then the robot trader believes the market is up.

2. If more than half of the qDTs decide that the market is down, then the robot trader believes the market is down.

Since there is always going to a different percentage of which if more favored, it could be 60-40 one time, or 70-30 another time, and because in a vote, the minority has to give into the majority, thus majority rules, by applying it above, the robot trader now has its chance of winning increased to about 90%, equating to around 195 wins. This is reflected in the graph below.

In the graph above the real market’s data curve is the red line, and the robot trader’s trades is the blue line. And as we can see, the robot trader’s trades were almost in complete sync with the actual market. So, this shows that our robot trader seems to have “decoded” the market’s “enigma” through the learning of historical data, training by the evolution algorithm, and executing its actions with the quantum decision trees. If the market acts the exact same way according to its historical data in the future, then our robot trader can definitely continue to beat the market, but in reality, the market can have an infinite number of paths it can take, and if what path it takes is completely random, it is still very hard, if not impossible to predict the future of the market. There are infinite possible paths for us to take, but we can only certainly live to see what will happen in only one of them because we cannot have prior information of market.

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