Bragg resonance behavior of the neutron refractive index and crystal acceleration effect

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Abstract. The energy dependence of neutron refraction index in a perfect crystal for neutron energy, close to the Bragg ones, was studied. The resonance shape of this dependence with approximately the Darwin width was found. As a result, the value of deviation from the exact Bragg condition can change during the neutron time of flight through the accelerated crystal and so the refraction index and the velocity of outgoing neutron can change as well. Such new mechanism of neutron acceleration in the accelerating perfect crystal was proposed and found experimentally. This mechanism is march more effective then known one concerning with the neutron acceleration in the accelerating usual media.

1. Introduction
The possibility to control the energy of neutron beams is of a great interest because of the wide neutron applications in various scientific fields from material science to nuclear, particle and astrophysics. The acceleration effect for neutrons scattered by exited isomeric nuclei was first predicted in 1959 [1] and was discovered experimentally in 1980 [2, 3]. The acceleration of neutrons in an inversely populated medium [4, 5] turned out to be very important in processes of stellar nucleosynthesis. In [6] acceleration of neutrons by vibrationally excited nitrogen molecules was observed.

Acceleration of neutrons in the uniform magnetic field due to spin flip by radio-frequency flipper is well known and successfully used in physical experiments (see, e.g., [7]). The phenomenon of neutron acceleration in a strong alternating magnetic field (of amplitude ∼ 0.4 T) was observed in [8]. The acceleration of neutrons in a weak alternating magnetic field (of 1–10 G) was measured using anomalous behaviour of the velocity dispersion for neutrons, moving in a crystal close to the Bragg directions [9]. The foundations of the neutron acceleration in a laser radiation field were considered in [10].

Not long ago a new interest arised to the acceleration of neutron, passing through accelerating media [11, 12]. This effect was first observed [13] and described in detail [14]. It was noticed [14] that "the observed effect was a manifestation of quite a general phenomenon – the accelerated medium effect (AME) – inherent to waves and particles of different nature". In
the acceleration and deceleration of neutrons were observed by applying a specific time-of-flight method. In some new special features of the effect for a birefringent medium were discussed with the applications to neutron spin optics and evolution of flavor states of neutrino, propagating through a free space. The acceleration of the samples in the mentioned experiments reached several tens of g units and the value of the energy transfer to a neutron was within the range of $(2 - 6) \cdot 10^{-10} \text{eV}$, so up to now AME was observed only for ultracold neutrons and by only one research group (see [14, 16]).

In present paper a new much more effective mechanism of acceleration effect is proposed, which is tested and confirmed experimentally for cold neutrons passing through the accelerated perfect crystal.

The essence of the effect is following. Neutron wave function is significantly modifying for neutrons moving through the crystal close to the Bragg conditions. As a result neutrons concentrate on "nuclear planes" or between them [17, 18] ("nuclear planes" mark the positions of maxima of periodic nuclear potential for corresponding system of crystallographic planes).

The degree of such concentration depends on the deviation from exact Bragg condition (i.e. on the crystal-neutron relative velocity) as well as the crystal refraction index for neutron. The neutrons enter into accelerated crystal with one mean potential of a neutron-crystal interaction and exit with the other, so the energy change at the crystal boundaries will be different and neutrons will be accelerated or decelerated after passage through such a crystal, energy transfer to a neutron being at the level of $\sim 4 \cdot 10^{-8} \text{eV}$.

2. Neutron crystal optics

Interest to neutron optics in perfect crystals has grown sharply at the last time. It concerns with new possibilities to study fundamental properties and interactions of neutron. For instance, strong electric fields up to $10^9 \text{V/cm}$, acting on the neutron inside the noncentrosymmetric crystals [18, 19, 20, 21] give new feasibilities to search for neutron electric dipole moment as well as CP-violating pseudomagnetic forces due to exchange of a pseudoscalar axion-like particle, using neutron optics in crystal [22, 23, 24, 25, 26, 27], what can be considered now as a most important tasks in this area.

Admixture of the waves reflected by crystallographic planes to the neutron wave function significantly changes the picture of neutron propagation in the crystal and leads to new phenomena, which manifest sharply defined resonance character with Bragg (Darvin) width. For example, small change of the neutron energy within this width ($\Delta E/E \sim 10^{-5}$ for thermal and cold neutrons) results in significant changing the neutron mean velocity in crystal (the anomalous velocity dispersion), and so the sharp energy dependence arises for the time neutron passes through the crystal [28].

In the present paper we discuss the other phenomenon related to the transformation of neutron wave function in the crystal, namely the resonance dependence of neutron refractive index (i.e. potential energy of neutron inside the crystal) on the incident neutron energy. If neutron passes through the non-absorbing perfect crystal and Bragg conditions are not satisfied for either crystallographic planes, the passage of the neutron through the crystal can be described by refractive index, which depends on the amplitude of zero harmonic $V_0$ of the neutron-crystal potential (average crystal potential). But when the energy or velocity direction of neutron approaches to the Bragg one, amplitude of the wave reflected by the corresponding plane system will sharply increase. The amplitude of this reflected wave is determined by the corresponding potential harmonic amplitude $V_g$ and the deviations from the exact Bragg condition. When this deviation more than harmonic amplitude we can use the perturbation theory [23, 29]. In this

1 Under perfect crystal we consider the crystal with the dispersion of the interplanar spacing much less than the intrinsic width of the Bragg reflection.
case if the neutron has initial energy equal to $E_0$ and wave vector $k_0$ ($E_0 = \hbar k_0^2/2m$), its wave function inside the crystal can be written as

$$\psi = e^{ikr} + \frac{V_g}{E_k - E_{k_g}}e^{ik_g r} \equiv e^{ikr} \left[ 1 - \frac{1}{\Delta_B} e^{ig r} \right],$$

(1)

where $\Delta_B = (E_k - E_{k_g})/V_g = 2(E_k - E_B)/V_g$ is the dimensionless parameter of deviation from exact Bragg condition for the plane system $g$; $k$ and $k_g = k + g$ are the wave vectors of incident and reflected waves inside the crystal with the mean refraction index taking into account; $E_k$ and $E_{k_g}$ are the unperturbed neutron kinetic energies in states $|k\rangle$ and $|k_g\rangle$ ($E_k = \hbar^2 k^2/2m = \hbar^2 k_0^2/2m - V_0$, $E_{k_g} = \hbar^2 k_g^2/2m$), $E_B = \hbar^2 g^2/(8m\sin^2 \theta_B)$ is the neutron energy that corresponds to exact Bragg condition.

The presence of reflected wave with the amplitude equal to $1/\Delta_B$ leads to concentration of neutron density in crystal on or between reflecting planes with the degree, depending on value and sign of $\Delta_B$. That in turn leads to additional changing the neutron kinetic energy $\tilde{E}_k$ and, respectively, the value of wave vector and refractive index inside the crystal depending on the deviation parameter $\Delta_B$. So averaging the periodic neutron crystal interaction potential over the wave function (1), one can get for the neutron kinetic energy in crystal

$$\tilde{E}_k = \frac{\hbar^2 k^2}{2m} = E_0 - V_0 + \frac{1}{\Delta_B}.$$

(2)

The last term in equation (2) extremely increases approaching to the Bragg condition ($E_k = E_{k_g}$), so it becomes incorrect (and the perturbation theory as well) already for $E_k - E_{k_g} \sim V_g$. More accurate calculation in two wave approach of the last term (2) gives the following result for kinetic energy of neutron after its entrance into crystal:

$$\tilde{E}_k = E_0 - V_0 + V_g \cdot \frac{\Delta_B}{\Delta_B^2 + 1}.$$

(3)

The last term in (3) describes the additional potential energy due to neutron localization. It significantly changes with small variation of neutron energy inside the Bragg reflection width $\Delta_B \simeq 1$, i.e. in the narrow energy range within $E_B - V_g < E_k < E_B + V_g$. For thermal and cold neutrons $V_g/E_B \simeq 10^{-5}$. Value $V_g$ is comparable with that of mean crystal potential $V_0$.

So changing the incident neutron energy in the vicinity of $E_B$, one can observe a well-defined resonance-type energy dependence of neutron refraction index in crystal. For example, for (110) plane of quartz $V_g = 4 \cdot 10^{-8}$eV, $V_0 = 10^{-7}$eV, and $E_B = 3.2 \cdot 10^{-3}$eV for diffraction angle close to $\pi/2$.

3. Experimental setup

Experiment was carried out at the horizontal neutron beam of the WWR-M reactor (PNPI, Gatchina). The energy variations for neutrons passed through accelerated crystal close to the Bragg condition was measured.

If neutron moves through accelerated crystal, the parameter of deviation from the Bragg condition will depend on the time, and so the mean potential of interaction of neutron with the crystal too, see (3). As a result, the refractive index will vary during neutron travel in crystal. But the neutron kinetic energy (wave vectors) inside the crystal cannot change because of homogeneity of the averaged crystal potential. Therefore because of the boundary changes of the neutron kinetic energy at the entrance and exit surfaces of the crystal are different, we should observe acceleration or deceleration of the neutron after passage through such a crystal. It should

2 The neutron refraction index $n$ is determined as usual $n^2 = 1 - \tilde{E}_k/E_0$. 

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be noticed that it doesn’t matter in what way one changes the deviation parameter at the time of neutron passage through the crystal. For example, one can vary the temperature of crystal or deform it with pressure. Both these ways lead to the variation of the Bragg energy. The motion of the crystal was used due to the convenience of its realization. Numerical estimations show that for the quartz crystal plane (110) the Bragg width in velocity units is equal to $\Delta v_B \simeq 9 \text{ mm/s}$, i.e. if the crystal velocity changes by 9 mm/s during the time of neutron passage through the crystal, the deviation from the Bragg condition will vary by one Bragg width.

Scheme of experimental setup is shown in Fig. 1. Preselected neutron beam, reflected by the mosaic crystal of pyrolytic graphite (PG) with reflecting plane (002), falls on the monochromator $K_1$ made from perfect quartz crystal. Reflected by $K_1$ highly monoenergetic (within Bragg width) neutrons pass through the working crystal $K_3$ and then are reflected by the crystal-analyzer $K_2$. The second PG crystal redirects these reflected neutrons to the detector. The quartz crystals $K_1$, $K_2$ and $K_3$ with the same working reflecting planes (110) were arranged to have their plane orientations in parallel directions. The diffraction angle was close to the right one: $\theta_B = 89^0$ ($\lambda \simeq 4.9\text{Å}$). Scanning over the Bragg wave length performed by varying temperature difference $T_{21} = T_2 - T_1$ between crystals $K_2$ and $K_1$, the temperature $T_3$ of the crystal $K_3$ being a reference one. Example of a scanning curve is shown in Fig. 2, when the crystal $K_3$ is absent. We have scanned the shape of neutron reflex from the $K_1$ crystal changing $T_{21}$ and so the relative interplanar distance for the $K_2$ crystal-analyzer. The width of this convolution scanning curve is close to that calculated (solid line) for two perfect crystals.

![Figure 1. Scheme of experimental setup.](image1)

![Figure 2. Two-crystal reflection curve (the $K_3$ crystal is absent).](image2)

There was a possibility to vary the temperature of the crystal $K_3$ and so its interplanar spacing too. Also we could move it in the direction parallel to the reciprocal lattice vector $g$ for working plane. In experiment the crystal was set in harmonic motion by a piezoelectric motor. The frequency of crystal vibration was $\nu_c = 4.5 \text{ kHz}$ and the period $\tau_c = 222 \mu \text{s}$. Vibration amplitude reached 0.15 $\mu \text{m}$. The crystal length was $L = 5 \text{ cm}$ and neutron time-of-flight through the crystal was $\tau_n = 62 \mu \text{s}$ that was about a quarter of the crystal vibration period.

If the velocity of the working crystal $K_3$ is depend on time as $v(t) = v_0 \sin \omega t$, the deviation

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3 Mentioned above the accelerated medium effect [14] in this case is negligible.
from the Bragg condition for neutrons moving through that crystal will also depend on time in the same way

\[ \Delta B(t) = \Delta B_0 + 4 \frac{1}{v_n} \frac{E_B}{V_g} v(t), \quad \Delta B_0 = 4 \frac{E_B}{V_g} \kappa T_{13}, \quad (4) \]

where \( v_n \) is the velocity of incident neutrons, \( \Delta B_0 \) is the deviation from the Bragg condition for resting crystal at \( v(t) = 0, \kappa \sim 1.3 \times 10^{-5} \) is the thermal expansion coefficient for a quartz crystal in the direction perpendicular to crystallographic planes. The deviation \( \Delta B_0 \) is determined by the difference of temperatures (interplanar distances) between \( K_1 \) and \( K_3 \) crystals \( T_{13} = T_1 - T_3 \). So in further we will use \( T_{13} \) as a parameter of deviation from the Bragg condition for neutrons passing the resting \( K_3 \) crystal.

The effect of the neutron energy change after passage through the crystal boundaries is determined by variation of the crystal velocity and so averaged potential \( (3) \) during the neutron time-of-flight through the accelerated crystal:

\[ \Delta E(t_0) = V_g \left( \frac{\Delta B(t_2)}{\Delta B(t_2)^2 + 1} - \frac{\Delta B(t_1)}{\Delta B(t_1)^2 + 1} \right) = \frac{1 - \Delta^2 B_0}{1 + \Delta^2 B_0} \frac{4E_B}{v_n} \Delta v(t_0), \quad (5) \]

where \( \Delta v(t_0) = v(t_0) - v(t_0 + \tau_n) \), \( t_0 \) is the time of neutron entrance into the crystal, \( \tau_n \) is the neutron time-of-flight through the crystal.

This change of neutron energy (and wavelength too) after accelerated crystal results in shift of the maximum of the scanning curve (Fig. 2). This maximum will be found for some other temperature difference \( T_{21} \) of the \( K_2 \) and \( K_1 \) crystals. Such variations of the scanning curve, depending on temperature and movement of the \( K_3 \) crystal, were studied to find that crystal acceleration effect. Time-of-flight technique was used for this purpose.

The main systematic error of this experiment associates with the dependence of the neutron transmission through the \( K_3 \) crystal on the deviation from the Bragg condition, that results in the spectrum distortion for neutrons passed through the crystal. Therefore, the position and shape of the scanning curve can change for neutrons passed even through the resting \( K_3 \) crystal, when the acceleration effect is absent. Examples of the neutron intensity distribution over the wave length after such passage through the resting \( K_3 \) crystal with different deviations \( T_{31} \) from the Bragg condition are shown in Fig. 4. It is evident that both intensity of transmitted neutrons and maximum position of the scanning curve can change in different ways. In particular for \( T_{31} = 0 \) the neutrons after reflection from \( K_1 \)-crystal cannot penetrate into \( K_3 \)-crystal. They will be completely reflected (due to exact Bragg condition) and cannot reach the \( K_2 \)-crystal. So \( K_2 \)-crystal will reflect only background neutrons. In the other case for \( T_{31} \gg 5K \) \( K_3 \)-crystal behaves as a homogeneous medium and practically does not distort the spectrum. In an intermediate case the spectrum will be distorted, because the \( K_3 \)-crystal reflectivity (and so its transmittance) sharply depends on the neutron wave length.

However, unlike the sought crystal acceleration effect \( (5) \), the systematic one is determined only by the deviation from the exact Bragg condition at the time moment \( t_0 \) of neutron entry to crystal, but not variation of the deviation during the time-of-flight through the crystal.

So the position \( E_s(t_0) \) of the maximum and the maximum intensity \( N(t_0) \) of the scanning curve (Fig. 2) in the absence of the crystal acceleration effect will be some functions of deviation \( \Delta B(t_0) \), depending on the crystal velocity \( v(t_0) \)

\[ E_s(t_0) = F(\Delta B(t_0)), \quad (6) \]
\[ N(t_0) = G(\Delta B(t_0)). \quad (7) \]
For further consideration and comparison with the experimental results expressions (6) and (7) can be expanded in Taylor series over \( v(t_0) \) about point \( v(t_0) = 0 \) (i.e. \( \Delta p(t_0) = \Delta B_0 \)). Taking into account that the speed of the crystal was significantly less than the typical Bragg widths it is enough to leave expansion terms up to second order over \( v(t_0) \)

\[
E_s(t_0) = A + B \cdot v(t_0) + C \cdot v(t_0)^2,
\]

\[
N(t_0) = N_0 + N_1 \cdot v(t_0) + N_2 \cdot v(t_0)^2,
\]

where \( A, B, C, N_0, N_1 \) and \( N_2 \) are free parameters depending on \( \Delta B_0 \) to be found from experiment.

As it follows from (5) the crystal acceleration effect contains a term phase-shifted with respect to the false effect (8) by the value \( \omega \tau_n / 2 \). This shift is approximately equal to \( \pi / 4 \) for our experimental conditions. Furthermore, the presence of the acceleration effect does not change the intensity of the line, but only gives its additional shift. Thus there is a phase shift between time dependencies of \( N_{\text{exp}}(t_0) = N(t_0) \) and \( E_{\text{exp}}(t_0) = E_s(t_0) + \Delta E(t_0) \) that represents the crystal acceleration effect.

4. Results

An example of experimental dependence of the line position on its maximal intensity \( E_{\text{exp}}(N_{\text{exp}}) \) is shown in Fig. 3. In the absence of the acceleration effect it should be observed a bijection between the maximum position and intensity shown by a dotted line. The presence of the neutron energy change after passage through the accelerating crystal leads to dependence \( E_{\text{exp}}(N_{\text{exp}}) \) described with a closed curve like Lissajous figure, where the figure square is determined by the crystal acceleration effect. Curved arrows in Fig. 3 show the sweep direction over time. The relation between a line shift in units of the crystal temperature and change of neutron energy is given by the following expression:

\[
\Delta E = 2E_B \cdot \omega \Delta T.
\]

The splitting marked by arrows in Fig. 3 corresponds to \( \Delta E_{\text{exp}} \approx 5 \) neV.

Dependence of the maximum value for the neutron energy variation (5) due to the acceleration effect on the deviation from the neutron Bragg energy for working crystal (temperature difference \( T_{13} \)) is shown in Fig. 5. Measurements were carried out at two different crystal oscillation amplitudes, corresponding to \( v_0 \approx 3 \) mm/s and \( v_0 \approx 1.5 \) mm/s. Curves show the results of approximating the experimental points by the theoretical curve (5). Thus one can see that the neutron energy change after passage through accelerating crystal can reach \( \sim 20 \) neV.

The mean potential energy of a neutron-crystal interaction, see (3), can be obtained from the experimental dependence shown in Fig. 5, because it is actually a derivative of (3), see (5). One should take into account that far from the Bragg condition the correction to the mean interaction potential due to the presence of \( g \)-harmonic \( V_g \) tends to zero, see (2), and so neutron refraction will be determined only by the average potential \( V_0 \). The result of the interaction potential reconstruction for neutrons moving in a crystal with energies close to the Bragg one is shown in Fig. 6. It is easy to see that the relative change of the incident neutron energy by several units of \( 10^{-5} \) leads to the variation of the interaction neutron-crystal potential by \( \pm 20\% \).

5. Conclusion

The features of refraction of a neutron, moving in a crystal close to the Bragg condition, has been studied. It is shown that the energy dependence of refractive index has evident resonance shape in a vicinity of the Bragg energy with the corresponding Bragg (Darwin) width (for thermal and cold neutrons \( \Delta E/E \approx 10^{-5} \)). The variation of the interaction potential of the neutron with the crystal in this energy range can reach about \( \pm 20\% \).
The resonance behaviour of the neutron-crystal interaction potential results in one more new phenomenon. That is neutron acceleration, which is found experimentally for neutrons passed through the accelerated perfect crystal for neutron energies, close to the Bragg one. The effect arises due to change of the parameter of deviation from the exact Bragg condition during the neutron time-of-flight through the accelerated crystal. As a result the refraction index for neutron changes too and so the velocity of the outgoing neutron.

This crystal acceleration effect was observed for the first time. One should take this phenomenon into account in precision neutron optical experiments such as mentioned above,
because the neutron refraction index is determined not only by averaged crystal potential, but also by its harmonics, which have the same order of value as the average potential itself.

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