Cosmic Neutrino Last Scattering Surface

Scott Dodelson\textsuperscript{1,2,3,} and Mika Vesterinen\textsuperscript{4}
\textsuperset{1}{Center for Particle Astrophysics, Fermi National Accelerator Laboratory, Batavia, IL 60510-0500, USA}
\textsuperset{2}{Department of Astronomy & Astrophysics, The University of Chicago, Chicago, IL 60637-1433, USA}
\textsuperset{3}{Kavli Institute for Cosmological Physics, Chicago, IL 60637-1433, USA and}
\textsuperset{4}{The School of Physics and Astronomy, The University of Manchester, Manchester M13 9PL, UK}
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Neutrinos decoupled from the rest of the cosmic plasma when the Universe was less than one second old, far earlier than the photons, which decoupled at $t = 380,000$ years. Surprisingly, though, the last scattering surface of massive neutrinos is much closer to us than that of the photons. Here we calculate the properties of the last scattering surfaces of the three species of neutrinos.

I. INTRODUCTION

The standard cosmological model predicts that neutrinos were produced in the early universe and are present today with an abundance\textsuperscript{1} of 112 cm$^{-3}$ per species.\textsuperscript{2,3} Detecting this background remains a tantalizing experimental dream,\textsuperscript{2,3,4} with recent developments encouraging the optimists.\textsuperscript{5,6,7,8,9,10,11}

For the time being, the question of where the cosmic neutrinos come from remains an academic question. Yet it is of sufficient interest that, even if there were no chance of detection, the origin of the cosmic neutrino background (CNB) seems worthy of theoretical study.

The neutrino Last Scattering Surface (LSS) is typically thought of as being located a given distance from us with a small but finite width, similar to the last scattering surface of the cosmic microwave background (CMB). Neutrinos last scatter when the temperature of the universe was a few MeV and the universe was less than a second old, while the photons in the CMB last scattered much later when the temperature was 1/3 eV at $t = 380,000$ years, so it is natural to assume that the neutrino LSS is further away than that of the CMB. Calculating how much further away leads to a little surprise. For, even massless particles can travel only a very small comoving distance in the very early universe, so the CMB comes from a comoving distance about 9540 Mpc away from us (where the present expansion rate is $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$), while massless neutrinos arrive from a comoving distance 9735 Mpc away. That is, neutrinos travel only about 200 Mpc (comoving) in the first 380,000 years.

We show in this paper that this slightly surprising result evolves into another counter-intuitive result for massive neutrinos: the CNB actually reaches us from closer than the CMB. Even for neutrino masses as small as 0.05 eV (and one of the neutrinos must be at least this massive) the effect is dramatic. For neutrinos with mass of 1 eV, the effect is truly striking with most of these neutrinos arriving from only several hundred Mpc away.

\footnotetext[1]{The uncertainty on this prediction is set by the sub-percent uncertainty in the temperature of the cosmic microwave background temperature, which serves to calibrate the thermal number density. The predicted abundance is roughly one percent larger than thermal due to $e^+/e^-$ heating.}

\footnotetext[2]{The School of Physics and Astronomy, The University of Manchester, Manchester M13 9PL, UK}

II. LAST SCATTERING SURFACE OF MASSIVE NEUTRINOS

Neutrinos stopped scattering when temperatures were of order a few MeV and the universe was less than a second old. At that time each neutrino species had a Fermi-Dirac distribution for a massless particle (assuming – as we will throughout – zero chemical potential and no heating from electron-positron annihilation).

Consider first the calculation of the last scattering surface of a massless neutrino. The comoving distance travelled by a massless particle starting from $t_i = 1$ sec until today is

$$\chi = \int_{t_i}^{t_0} \frac{dt}{a(t)} = \int_{a_i}^{1} \frac{da}{a^2 H(a)}$$

Note that early on $H(a) = H_0 \Omega_r^{1/2} a^{-2}$ where $\Omega_r = 8.3 \times 10^{-5}$ is the radiation density today in units of the critical density. The integrand peaks at late times, so the contribution to the comoving distance from early times ($t_i \sim 1$ sec) is negligible; this explains our nonchalance in defining the initial time. It also explains why all three species of neutrinos share the same last scattering surface even though electron neutrinos decouple slightly later than do the other two species. The comoving distance travelled by neutrinos until the time of photon last scattering at $a_s^{-1} = 1090.5 \pm 0.95$\textsuperscript{21} is:

$$\chi_s = \frac{1}{\Omega_m^{1/2} H_0} \int_{a_i}^{a_s} \frac{da}{\sqrt{a + a_{eq}}} \sqrt{a + a_{eq} - \sqrt{a_{eq}}}$$

where $\Omega_m$ is the matter density today in units of the critical density and $a_{eq}$ is the value of the scale factor when
the densities of matter and radiation are equal. Using the standard cosmological parameters \cite{21}, this equates to the 200 $h^{-1}$ Mpc difference alluded to in the introduction.

Massive neutrinos slow down once they become non-relativistic, so the integral determining the distance to the last scattering surface generalizes to:

$$\chi = \int_{t_i}^{t_0} \frac{dt}{a(t)} \frac{p_0/a}{\sqrt{(p_0/a)^2 + m_\nu^2}} \quad (3)$$

where the second term in the integrand is the redshifted velocity $p/E$, with $p_0$ the current neutrino momentum. The neutrino temperature today is $T_\nu = 1.95 \times 10^{-4}$ eV, so there will be a range of $p_0$’s drawn from a Fermi-Dirac distribution, each of which will be associated with a different distance to the LSS. Fig. 1 plots this distance as a function of neutrino mass for two different values of the present day neutrino momentum.

Since neutrinos with different momenta arrive from different distances\ cite{22}, the last scattering surface of the CNB is quite broad compared to that of the CMB. To quantify this, we can define the probability that a neutrino last scattered a distance $\chi$ away from us, or the visibility function:

$$\frac{dP}{d\chi} = \frac{dP}{dp_0} \left( \frac{d\chi}{dp_0} \right)^{-1} \quad (4)$$

where the equality uses the chain rule; the first differential probability is given by the massless Fermi-Dirac distribution \cite{23}:

$$\frac{dP}{dp_0} = \frac{2}{3\zeta(3)T_\nu^3} \frac{p_0^2}{e^{p_0/T_\nu} + 1}; \quad (5)$$

and the second term on the right is obtained by differentiating Eq. (3).

Fig. 2 displays the probability that a neutrino with a given mass arrived from a distance $\chi$. Note for all masses above $10^{-4}$ eV, the spread in arrival distances is much larger than the spread in the CMB last scattering surface. The width of the CNB and CMB last scattering surfaces have different origins: the CMB does not have an infinitely thin last scattering surface because the process of recombination, and therefore decoupling, extends over a finite time period. The CNB last scattering surface is thick, reflecting the different velocities of the different momenta in the Fermi-Dirac distributions.

One might ask about the last scattering surface of heavier particles, such as sterile neutrinos or ordinary cold dark matter particles. Those that become non-relativistic before equality travel a distance of order the

\footnote{Ref. 22 also mentioned this feature of neutrinos and proposed to exploit it to test the Copernican Principle.}
comoving horizon at equality (136 Mpc) times the comoving velocity at equality \([p_0/m]/a_{\text{EQ}}\). A keV sterile neutrino would therefore have a last scattering surface of order a Mpc away. For particles more massive than this – e.g., Cold Dark Matter – their LSS is so close that the distance traveled in a straight line (before gravity in our halo started moving them around) was negligible. So in some sense, the question ceases to make much sense for masses above a keV. Some of these considerations apply even to light neutrinos in our Galaxy: to determine what has happened to these neutrinos recently, one must carry out simulations along the lines of those presented in \cite{24}.

### III. OSCILLATIONS

Neutrinos are produced in flavor eigenstates but then propagate as mass eigenstates. Therefore, the simple notion of a neutrino with fixed mass having its own last scattering surface is a bit too naive. As emphasized in \cite{25}, the number density of neutrinos in mass eigenstate \(i\) is:

\[
\frac{d\nu_i}{d\chi} = \frac{3}{\alpha} \left| U_{\alpha i} \right|^2 \frac{d\nu_\alpha}{d\chi} \tag{6}
\]

where \(U_{\alpha i}\) are the elements of the unitary matrix which transforms from the mass basis to the flavor basis, and the sum is over all three flavors.

In principle the number densities of the three neutrinos flavors \(d\nu_\alpha\) could differ if, e.g., each had a different chemical potential \cite{24}. Here we assume that the chemical potentials are very small so all the \(d\nu_\alpha\) are for all practical purposes identical. In that case, they can be removed from the sum, and then the sum over the unitary matrix elements squared simply gives unity. So

\[
\frac{d\nu_i}{d\chi} = \frac{d\nu_\alpha}{d\chi} = \frac{dp_0}{2\pi^2 e_{p_0/T_\nu}^2 + 1} \tag{7}
\]

independent of mass eigenstate. So we expect the calculation of the previous section to reflect the different last scattering surfaces of the different mass eigenstates.

Detection, however, will take place in flavor space, with electron neutrinos and anti-neutrinos. To compute the last scattering surface of these eigenstates, we must transform back to flavor space, weighting each mass eigenstate by its own visibility function:

\[
\frac{dP}{d\chi} |_{\nu_e} = \frac{dP}{dp_0} \sum_i |U_{\alpha i}|^2 \left( \frac{d\chi}{dp_0} \right)_{\nu_i}. \tag{8}
\]

Fig. \ref{fig:oscillations} shows the resulting probability for an electron neutrino with two possible mass schemes assuming the tri-bimaximal mixing matrix \cite{27, 28} \((U_{e1} = \sqrt{2/3}, U_{e2} = 1/\sqrt{3}, \text{and } U_{e3} = 0)\). Note the interesting double peaked structure in the normal case, a signature of the quantum mechanical oscillations which dictate that there is roughly a 2/3 probability the electron neutrino propagates with very small mass \(m_1\) and 1/3 with mass \(m_2\).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{oscillations.png}
\caption{Probability for the distance to the last scattering surface for an electron neutrino with normal hierarchy \((m_2 = 0.009 \text{ eV}; m_1 = 10^{-4} \text{ eV}; \text{sin} \theta_{13} = 0)\) and degenerate masses \((m_1 = m_2 = m_3 = 0.2 \text{ eV})\).}
\end{figure}

### IV. CONCLUSIONS

The last scattering surface of the cosmic neutrino background is much broader and much closer than that of the cosmic microwave background. Indeed, as depicted in Fig. \ref{fig:oscillations} the last scattering “surface” of the electron neutrinos and anti-neutrinos has a very rich structure due to oscillations.

Are there any observable consequences of these distances to the last scattering surface? We can think of three potentially interesting follow-ups:

- **Neutrino Acoustic Oscillations**

The well-known *Baryon Acoustic Oscillations* (BAO) arise because the photon-baryon gas travels a distance equal to the sound horizon at decoupling, after which the baryons stop. In real space, this leads to an overdensity of baryons in a spherical shell surrounding an initial overdensity, and ultimately to a bump in the correlation function. Massive neutrinos would seem to share many of the same features: adiabatic perturbations lead to neutrinos initially being overdense at the same places as the baryons; neutrinos travel a finite distance since last scattering; and this distance might show up as a feature in the power spectrum. Unfortunately, we see no evidence for this feature when
running the Boltzmann code \cite{29} which solves for the linear evolution of perturbations (and by tracking different neutrino momenta implicitly encodes all of the relevant physics). The difference between neutrinos and the baryons are that the neutrino last scattering surface is very broad due to the Fermi-Dirac distribution; this breadth smooths out any (very small) feature rendering it undetectable. Searching for a feature might still be useful as a way of constraining the distribution function of neutrinos in the CNB.

- **Galactic Distribution function**

For direct detection, it is important to know the distribution function of neutrinos in a Milky-Way sized galaxy. Are neutrinos overdense? Has their momentum distribution changed due to virialization? If capture has occurred, the distance to the LSS would likewise be modified. Indeed, it is possible that there would be a directional effect: neutrinos from the center of the Galaxy might be captured and hence have relatively small distances to the LSS, while those from the poles maintaining their primordial distribution functions and distances to LSS.

- **Anisotropies**

If experimentalists do succeed in detecting the CNB, in the far future one might imagine maps of the anisotropy of this background \cite{30}. If the neutrinos have been sloshing in our Galaxy for a number of orbits, this angular information might be lost. But some of the neutrinos – the lightest ones or those arriving away from the Galactic Center – will arrive undeflected (see, e.g., the right panel in Fig. 2 in Ref. \cite{31} which suggests very little trapping of neutrinos with masses less than 1 eV in a Galaxy of our size). These would provide information about overdensities at a very early time from locations about which we will already have much information from galaxy surveys. The galaxy surveys probe the same locations but at much later times. This would be an almost unique opportunity to probe the evolution of structure in given regions.

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