$b - \tau$ Unification, $g_\mu - 2$, the $b \to s + \gamma$ Constraint and Nonuniversalities

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Abstract

An analysis is given of the $b - \tau$ Yukawa coupling unification in view of the recent result from Brookhaven on $g_\mu - 2$ and under the constraint of $b \to s + \gamma$. We explore $b - \tau$ unification under the above constraints for nonuniversal boundary conditions for the soft SUSY breaking parameters. We find new regions of the parameter space where significant negative supersymmetric contribution to the $b$ quark mass can arise and $b - \tau$ unification within SU(5) framework can occur with nonuniversal gaugino masses. Specifically we find that for the case where the gaugino mass matrix transforms like a 24 plet one finds a negative contribution to the $b$ quark mass irrespective of the sign of the Higgs mixing parameter $\mu$ when the supersymmetric contribution to $g_\mu - 2$ is positive. We exhibit regions of the parameter space where $b - \tau$ unification occurs for $\mu > 0$ satisfying the constraints of $b \to s + \gamma$ and $g_\mu - 2$. The $\mu < 0$ case is also explored. The dependence of the accuracy of $b - \tau$ unification defined by $|\lambda_b - \lambda_\tau|/\lambda_\tau$ on the parameter space is also investigated and it is shown that unification with an accuracy of a few percent can be achieved in significant regions of the parameter space. The allowed parameter space is consistent with the naturalness constraints and the corresponding sparticle spectrum is accessible at the Large Hadron Collider.
1 Introduction

The unification of $b$ and $\tau$ Yukawa couplings, along with gauge coupling unification, is traditionally been viewed as a success of supersymmetric grand unification models with grand unification group structure SU(5) and SO(10). However, a close scrutiny reveals that $b - \tau$ unification is rather sensitively dependent on the parameter space of the supersymmetric models such as $\alpha_3$ and $\tan \beta[1, 2]$, GUT threshold corrections and the effects of gravitational smearing[3]. It is also known that $b - \tau$ unification is sensitive to the sign of the Higgs mixing parameter $\mu[4, 5]$ and that it prefers the negative sign of $\mu$ in the standard $\mu$ convention[6]. However, the recent experimental results from Brookhaven[7] indicate that for a class of SUSY models the sign of $\mu$ is positive[8] and this result then makes the realization of $b - \tau$ unification more difficult. Some recent analyses have tried to address this problem. In the analysis of Ref.[9], the authors work in an SO(10) supersymmetric grand unified model and show that with the inclusion of the D-term contribution to the sfermion and Higgs masses arising from the breakdown of SO(10) it is possible to obtain Yukawa coupling unification up to about 30% with a positive value of $\mu$. The sparticle spectrum found in this analysis is close to that of an inverted hierarchy model[10]. In this scenario the first and the second generation sparticles masses are typically above a TeV and $a_\mu (a=(g-2)/2)$ is on the extreme low side of the corridor allowed by the BNL experiment. Another analysis within SO(10) is of Ref.[11] which requires an almost exact Yukawa coupling unification for positive $\mu$. The $a_\mu$ predicted in this work is also rather small, i.e., in the range $(5 - 10) \times 10^{-10}$. The scenarios of both Refs.[1, 11] require $b - t - \tau$ unification and are for large $\tan \beta$, i.e., typically $\sim 50$.

In this paper we carry out an analysis with focus on $b - \tau$ Yukawa unification within SU(5). We do a comprehensive study of this problem with inclusion of nonuniversalities. We find new regions of the parameter space where one has $b - \tau$ unification with satisfaction of the $g_\mu - 2$ and the $b \rightarrow s + \gamma$ constraints. While most of the new parameter space where the desired constraints are satisfied requires $\mu > 0$, we also find regions of the parameter space satisfying all the constraints for the $\mu < 0$ case. The outline of the rest of the paper is as follows. In Sec.II we discuss briefly the current situation on $g - 2$. In Sec.III we discuss the framework of the proposed analysis. In Sec.IV we give a discussion of the
results. Conclusions are given in Sec.V.

2 The $g_\mu - 2$ Constraint on SUSY

We begin with a discussion of the current situation on $a_\mu$. The recent Brookhaven National Laboratory (BNL) experiment gives

$$a^{exp}_\mu = 11659203(15) \times 10^{-10}$$

while the prediction in the Standard Model where $a^{SM}_\mu = a^{qed}_\mu + a^{EW}_\mu + a^{hadronic}_\mu$ is

$$a^{SM}_\mu = 11659159.7(6.7) \times 10^{-10}$$

Essentially the entire error above arises from the hadronic correction since

$$a^{had}_{\mu} (vac.pol.) = 692.4(6.2) \times 10^{-10}$$. There is thus a 2.6σ deviation between theory and experiment,

$$a^{exp}_\mu - a^{SM}_\mu = 43(16) \times 10^{-10}.$$ 

The result above is to be compared with the electro-weak correction in the Standard Model

$$a^{EW}_\mu = 15.2(0.4) \times 10^{-10}.$$ The observed difference between experiment and theory could arise due to a variety of phenomena. Of special interest to us here is the possibility that the observed phenomenon is supersymmetry. It is well known that SUSY makes important contributions to $g_\mu - 2$. After the result of the BNL experiment became available analyses within SUSY were carried out for a variety of scenarios. We begin by summarizing here the results for mSUGRA which is characterized by the following parameters: $m_0$, $m_{1/2}$, $A_0$, $\tan \beta$ and $sign(\mu)$ where $m_0$ is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass, $A_0$ is the universal trilinear coupling, $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$ where $H_2$ gives mass to the up quark and $H_1$ gives mass to the down quark and the lepton of each generation and $\mu$ is the Higgs mixing parameter. In this case one finds that the sparticle spectrum consistent with the observed effect satisfies the constraint so that $m_0 \leq 1.5$ TeV and $m_{1/2} < 0.8$ TeV and further that the sign of $\mu$ is determined to be positive. Now positive $\mu$ is preferred by the $b \to s + \gamma$ constraint since for positive $\mu$, the parameter space consistent with the constraint is large, while for negative $\mu$ the parameter
space consistent with the constraint is small\cite{19, 20}. A positive $\mu$ is also desirable for the
direct detection of dark matter\cite{8, 21}. Within mSUGRA the sparticle spectrum consistent
with the BNL constraint lies within reach of the Large Hadron Collider\cite{22}. However,
the upper limits are significantly model dependent\cite{23, 24, 25}. Further, there is also a
significant dependence on the CP violating phases\cite{26}. Such model dependence should be
taken account of while interpreting the implications of the BNL result.

3 Framework of Analysis

The main purpose of this paper concerns the investigation of the parameter space where
$b - \tau$ unification occurs consistent with the BNL $g - 2$ constraint and the $b \rightarrow s + \gamma$
constraint by relaxing the universal boundary conditions in mSUGRA. The universality
of the soft parameters in mSUGRA model arises from the assumption of a flat Kahler
potential and the assumption that the gauge kinetic energy function $f_{\alpha\beta}$ is proportional
to $\delta_{\alpha\beta}$ where $\alpha, \beta = 1, .., 24$ for SU(5). However, the nature of Planck scale physics is
still largely unknown. Thus it is reasonable to investigate more general scenarios based
on curved Kahler potentials and with non gauge singlet gauge kinetic energy functions.
Of course, there exist strong constraints on the allowed form of nonuniversalities from
flavor changing neutral currents. In the scalar sector the flavor changing neutral current
constraints still allow for the presence of significant amounts of nonuniversalities in the
Higgs doublet sector and in the third generation sector\cite{27}. Our focus in this paper is
on nonuniversalities in the gaugino mass sector. In this sector supergravity theories in
general allow for the presence of an arbitrary gauge kinetic energy function $f_{\alpha\beta}$. In the
presence of a curved Kahler potential this then leads to a gaugino mass matrix of the
form $M_{G} = \frac{1}{4}e^{G/2}G^a(G^{-1})_b^{\gamma}(\partial f_{\alpha\gamma}^*/\partial z^b)f_{\gamma\beta}^{-1}$, where $G = -\ln[\kappa^6 WW^*] - \kappa^2 K$. Here, $W$ is
the superpotential, $K(z, z^*)$ is the Kahler potential, $z^a$ are the complex scalar fields, and
$\kappa = (8\pi G_N)^{-\frac{1}{2}} = 0.41 \times 10^{-18}$ GeV$^{-1}$ where $G_N$ is Newton’s constant. Now $f_{\alpha\beta}$ would
have a nontrivial field dependence in general involving fields which transform as a singlet
or a nonsinglet irreducible representation of the underlying gauge group. With imposition
of the $SU(3)_C \times SU(2)_L \times U(1)$ gauge invariance the gaugino masses at the GUT scale
$M_{G}(\sim 2 \times 10^{16}$ GeV) will in general have the form\cite{28}.
\[ \bar{m}_i(0) = m_\frac{1}{2} \sum_r c_r n_i^r \]  

where \( n_i^r \) are characteristic of the representation \( r \) and \( c_r \) are the relative weights of the various representations. Since \( f_{\alpha\beta} \) transform as \((24 \times 24)_{symm} = 1 + 24 + 75 + 200\) the representations \( r \) consist of the set \( \{1, 24, 75, 200\} \). The singlet representation leads to universality of the gaugino masses while the nonsinglet representations 24, 75 and 200 generate nonuniversalities (\( n_i^r \) for various representations are given in the Appendix). We will investigate \( b - \tau \) unification by exploring a wide range of the soft SUSY parameter space with inclusion of nonuniversalities of the type that enter in Eq.(4). The details of the analysis are as follows: We begin at the GUT scale with a prescribed set of boundary conditions and carry out a two loop renormalization group evolution (RGE) for the couplings as well as for the soft parameters. The electroweak symmetry is broken radiatively by the minimization of the Higgs potential computed to the complete one-loop level [29] at the scale \( Q \sim \sqrt{\bar{m}_{t_1}\bar{m}_{t_2}} \). In the analysis we include the supersymmetric corrections [31] to the top quark (with \( M_t = 175 \) GeV) and the bottom quark mass. For the light Higgs boson mass we have used the code \textit{FeynHiggsFast} [32].

In the analysis we impose the BNL \( g_\mu - 2 \) constraint and the \( b \to s + \gamma \) constraint which we now summarize. In the Standard Model the branching ratio for the process \( b \to s + \gamma \) is estimated to be [33] \( B(b \to s + \gamma) = (3.29 \pm 0.33) \times 10^{-4} \). The most recent experimental determination of this branching ratio gives [34] \( B(b \to s + \gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4} \) where the first error is statistical, and there are two types of systematic errors. The experimental determination of \( B(b \to s + \gamma) \) has many inherent errors and so we use a \( 2\sigma \) range around the current experiment [34] and thus take

\[ 2 \times 10^{-4} < B(b \to s + \gamma) < 4.3 \times 10^{-4} \]  

The issue of \( b - \tau \) unification is closely tied with the supersymmetric correction to the b quark mass

\[ m_b(M_Z) = \lambda_b(M_Z) \frac{v}{\sqrt{2}} \cos \beta (1 + \Delta_b) \]  

where \( \Delta_b \) is loop correction to \( m_b \). The running b quark mass \( m_b(M_b) \), where \( M_b \) is the pole mass, is computed via the RG running from \( M_Z \) to \( M_b \) using 2-loop Standard Model
renormalization group equations. The pole mass \( M_b \) is then related to the running mass \( m_b(M_b) \) by the relation \[ 7 \]

\[ M_b = (1 + \frac{4\alpha_3(M_b)}{3\pi} + 12.4\frac{\alpha_3(M_b)^2}{\pi^2})m_b(M_b) \]

Now the largest contributions to \( \Delta_b \) arise from the gluino and the chargino pieces \[ 31 \]. For the gluino one has

\[ \Delta_{\tilde{g}} = \frac{2\alpha_3 \mu M_{\tilde{g}}}{3\pi} \tan \beta I(m_{\tilde{b}_1}^2, m_{\tilde{t}_1}^2, M_{\tilde{g}}^2) \]

where

\[ I(a, b, c) = \frac{ab \ln(a/b) + bc \ln(b/c) + ac \ln(c/a)}{(a-b)(b-c)(a-c)} \]

We note that since \( I(m_{\tilde{b}_1}^2, m_{\tilde{t}_1}^2, M_{\tilde{g}}^2) \) is always positive one finds that \( \Delta_{\tilde{g}} \) is negative when \( \mu M_{\tilde{g}} \) is negative. This situation can arise when either \( \mu \) is negative and \( M_{\tilde{g}} \) positive or when \( \mu \) is positive and \( M_{\tilde{g}} \) is negative. There is a similar situation for the case of the chargino contribution which is

\[ \Delta_{\tilde{\chi}_+} = \frac{Y_t \mu A_t}{4\pi} \tan \beta I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, \mu^2) \]

where \( Y_t = \lambda_t^2/4\pi \). In this case \( \Delta_{\tilde{\chi}_+} \) is negative when either \( \mu \) is negative and \( A_t \) is positive or when \( \mu \) is positive and \( A_t \) is negative. We note that typically satisfaction of the \( g_\mu - 2 \) constraint requires that the sign of \( \mu m_2 \) be positive. For universal boundary conditions this would lead to a positive \( \Delta_{\tilde{g}} \) which is not preferred by \( b - \tau \) unification. We note, however, that for the case when the gaugino masses arise from a 24 plet term in Eq.(4) one has that the signs of the \( \tilde{m}_3 \) and \( \tilde{m}_2 \) are necessarily opposite. Because of this one finds that a positive \( a^{SUSY}_\mu \) implies a negative \( \Delta_{\tilde{g}} \) which is what is preferred by \( b - \tau \) unification. This anti-correlation between the positivity of \( a^{SUSY}_\mu \) and the negativity of \( \Delta_{\tilde{g}} \) occurs independent of the sign of \( \mu \). However, since the satisfaction of \( b \to s + \gamma \) typically prefers a positive \( \mu \) the positive \( \mu \) sign would still be preferred but solutions with negative \( \mu \) are not necessarily excluded just because of the BNL constraint in this case. (The fact that a negative \( \mu \) can still give a positive supersymmetric contribution to \( g_\mu - 2 \) in the 24 plet case was also noticed in Ref. [24]). Typically one expects the gluino contribution of Eq.(8) to be the largest supersymmetric contribution. An enhancement of \( \Delta_b \) can occur when both \( \Delta_{\tilde{g}} \) and \( \Delta_{\tilde{\chi}_+} \) are negative. For \( \mu > 0 \) this occurs when \( C_{24} < 0 \) and \( A_t < 0 \) and for \( \mu < 0 \) this occurs when \( C_{24} > 0 \) and \( A_t > 0 \).
4 Discussion of Results

We give now details of the numerical analysis with focus on the gaugino mass nonuniversalities as discussed above. We find that the most favorable situation arises for the case when $c_{24}$ is negative while cases with nonvanishing $c_{75}$ and $c_{200}$ do not produce $b - \tau$ unification consistent with $g_\mu - 2$ and $b \to s + \gamma$ constraints. It is indeed the $g_\mu - 2$ constraint which is not satisfied in the latter two scenarios. The result of the analysis is given in Figs. 1-10 which we now discuss in detail. In analyzing $b - \tau$ unification it is useful to define the parameter $\delta_{b\tau}$ which prescribes the accuracy with which $b - \tau$ unification is achieved where

$$\delta_{b\tau} = \frac{|\lambda_b - \lambda_{\tau}|}{\lambda_{\tau}}$$  \hspace{1cm} (11)

In Fig.(1a) we plot $\delta_{b\tau}$ vs $\tan \beta$ for the following range of parameters: $0 < m_0 < 2000$ GeV, $-1000$ GeV $< c_{24} m_1^2 < 1000$ GeV, $-6000$ GeV $< A_0 < 6000$ GeV and $\mu > 0$. The dotted points satisfy $b - \tau$ unification at the level shown, the crosses additionally satisfy $b \to s + \gamma$ constraint and the filled circles satisfy all the constraints, i.e., $b - \tau$ unification, $b \to s + \gamma$ and $g_\mu - 2$ constraints. If one uses a criterion that $b - \tau$ unification be satisfied to an accuracy of 30% then Fig.(1a) exhibits that there are significant regions of the parameter space with $\delta_{b\tau} \leq 0.3$. However, Fig.(1a) also shows that there are appreciable regions in the parameter space where $b - \tau$ unification can be satisfied to 20%, 10% and even less than 5% level. An interesting phenomenon we see in Fig.(1a) is that the values of $\tan \beta$ consistent with $b - \tau$ unification lie in a wide range depending on the accuracy of the $b - \tau$ unification that is desired. Thus for a $\delta_{b\tau} \leq 0.3$ one finds that the allowed values of $\tan \beta$ can be as low as 15 – 20 and as high as 40 – 45. However, a $b - \tau$ unification with a few percent accuracy would require a value of $\tan \beta$ in the vicinity of 30 or above. An analysis similar to that of Fig.(1a) but with a plot of $\delta_{b\tau}$ vs $m_0$ is given in Fig.(1b) where the parameter range is as in Fig.(1a) and $\tan \beta$ lies in the range $2 < \tan \beta < 55$. We note that the bulk of $m_0$ values satisfying all the desired constraints are such that $m_0 \leq 1$ TeV. An analysis of $\delta_{b\tau}$ vs $C_{24} \cdot m_1^2$ is given in Fig.(1c). Here one finds quite remarkably that the allowed points in the parameter space which satisfy the desired constraints all appear to lie in a rather narrow strip $-200$ GeV $< C_{24} \cdot m_1^2 < 0$, where the narrow strip specifically comes from the $g_\mu - 2$ constraint. Finally in Fig.(1d) a plot of $\delta_{b\tau}$ vs $A_0$ reveals that the
bulk of the allowed $A_0$ values consistent with all the desired constraints are negative. In most of the parameter space the dominant supersymmetric correction to the $b$ quark mass arises from the gluino exchange term. These results are consistent with our discussion at the end of Sec.3.

The importance of supersymmetry for $b-\tau$ unification is exhibited in Fig.(2) which gives a plot of $\delta_{b\tau}$ vs $\Delta_b$ where $\Delta_b$ is the supersymmetric contribution to the $b$ quark mass. Here the dots refer to the points satisfying $b-\tau$ unification, and filled squares represent points which additionally satisfy the $b \to s + \gamma$ and the $g_\mu - 2$ constraints. From Fig.(2) one finds that the more accurate the $b-\tau$ unification the larger is the fraction of the supersymmetric contribution and that $b-\tau$ unification within a few percent accuracy requires a SUSY correction to the $b$ quark mass which can be as large as 30% to 40%. Next we discuss $b-\tau$ unification with the stricter constraint that the unification hold to within 10%. In Fig.(3a) a plot of $\Delta_b$ is given as a function of $\tan \beta$ with all other parameters the same as in Fig.(2) but with the additional constraint that $\delta_{b\tau} \leq 0.1$. Here one finds that the SUSY corrections must be at least 15-20% of the $b$ quark mass but could be as large as 40%. A similar analysis but with $\Delta_b$ vs $m_0$ is given in Fig.(3b), with $\Delta_b$ vs $C_{24}m_{1/2}$ in Fig.(3c), and with $\Delta_b$ vs $A_0$ in Fig.(3d). We note that for the case of Fig.(3d) the allowed values of $A_0$ are all negative as anticipated in our discussion at the end of Sec.3 for the $\mu > 0$ case.

Motivated by the fact that $\tan \beta \sim 35$ is the most favored zone for $b-\tau$ unification (see Fig. II(a)) we compare the sparticle mass spectra in mSUGRA with the one arising in the 24-plet nonuniversal case in Figs.III(a) and III(b). Since the $SU(3)$ gauge sector produces important renormalization group effects on the sparticle masses we have chosen identical $m_3(M_G)$ values in Figs.III(a) and III(b) for a direct comparison. Thus for $m_0$ and $m_3(M_G)$ fixed we find that the squark and slepton masses undergo a relatively small change as we go from Fig.(4a) to Fig.(4b) but a relatively large change is seen in the lightest neutralino mass due to the very different ratio of the $U(1)$ and $SU(3)$ gaugino masses for the 24-plet case compared to the mSUGRA case. Having identified the signature of the 24-plet nonuniversal scenario on the mass spectra as given by Figs.(4a) and (4b) we now study the sparticle spectrum in the most favored $b-\tau$ unification region under the $b \to s + \gamma$ constraint and the $g_\mu - 2$ limits. The result is exhibited in Fig.IV(c). The choice of parameters in Fig.(4c) is based on the fact that the requirement of $b-\tau$ unification along
with the $b \rightarrow s + \gamma$ constraint prefers the parameters $\tan \beta = 35$, $A_0 = -1$ TeV and $c_{24}m_{1/2} < 0$. The choice $c_{24}m_{1/2} = -100$ GeV (see Fig.1(a)) is guided by the fact that the $g_\mu - 2$ constraint is most easily satisfied for this value of $c_{24}m_{1/2}$. In Fig.(4c) $\delta_{b\tau}$ ranges between 0.2 to 0.4. Regarding the sparticle spectra we note that as in Fig.(4b) the lightest neutralino mass is much smaller relative to the mSUGRA case of Fig.(4a). Another interesting phenomenon is that because of the large $A_0$ value in this case there is a large L-R mixing in the stop mixing matrix resulting in significantly smaller $m_{\tilde{t}_1}$ here relative to that in Fig.(4b). More generally the third generation scalar sector is relatively lighter. Finally, most importantly, while considering $m_0$ between 400 to 800 GeV which is the favored region for satisfying all the constraints (see Fig.1(b)) we see that the spectrum in Fig.4(c) is well below a TeV. Thus the naturalness requirement is preserved (see, e.g., Ref.[30]).

We discuss now briefly the 24 plet case for $\mu < 0$. The results are exhibited in Fig.(5) where we have used the range of parameters $0 < m_0 < 2TeV$, $-1TeV < c_{24}m_{1/2} < 1TeV$, and $-6TeV < A_0 < 6TeV$ and imposed the unification criterion $\delta_{b\tau} \leq 0.2$. The analysis of Fig.(5) shows that there are significant regions of the parameter space where all the constraint, i.e., $b \rightarrow s + \gamma$, $g_\mu - 2$ and the $b - \tau$ unification are consistently satisfied. In Fig.(6) we exhibit the allowed and the disallowed regions due to various constraints in the $m_0 - m_3(M_Z)$ plane. Dotted points satisfy $b - \tau$ constraint with $\delta_{b\tau} < 0.3$, the blue crosses are the points which are additionally allowed by the $g_\mu - 2$ constraint and the black filled circles satisfy the $g_\mu - 2$ constraint, the $b \rightarrow s + \gamma$ constraint, and the constraint $\delta_{b\tau} < 0.3$. Fig.(6a) gives the analysis for $\mu > 0$ and Fig.(6b) for $\mu < 0$. We note that the density of points where all the constraints are satisfied is quite significant for both the $\mu > 0$ and the $\mu < 0$ cases.

Next we discuss cases where the gaugino masses transform like the 75 and the 200 plet representations. We start by exhibiting in Fig.(7) the gaugino masses at the scale $M_Z$ for the universal (mSUGRA) case (Fig.(7a)) and for the nonuniversal (24,75 and 100 plet) cases (Figs.(7b-7d)). We note that the ratio of gaugino masses for the 75 plet and the 200 plet cases are drastically different from each other and from the mSUGRA and the 24 plet cases. This is of course what we expect from the nature of the boundary conditions for the four cases as given in the Appendix. In Fig.(8) we exhibit the mass spectra for the universal and the nonuniversal cases as a function of $m_0$. One notices the
drastic modification of the pattern of sparticle masses as one goes from the universal case of Fig.(8a) to the nonuniversal cases of Figs.(8b-8d). A similar display with respect to $m_2$ for the universal case and with respect to $m_3(M_G)$ for the nonuniversal cases is given in Fig.(9) where the gray areas represent the excluded regions because of LEP chargino mass limits[35] and the absence of radiative electroweak symmetry breaking. Again one finds some drastic modifications of the sparticle spectrum when one compares the four cases exhibited in Fig.(9). In Fig.(10) we exhibit the points in the parameter space which satisfy $b - \tau$ unification for 75 and 200 plet cases. One finds that there exist significant regions of the parameter space in Fig.(10a) and Fig.(10c) satisfying $b - \tau$ unification which also additionally satisfy the $b \to s + \gamma$ constraint for $\mu > 0$. Both the requirements of $b - \tau$ unification as well as the $b \to s + \gamma$ constraint reduces the allowed parameter space significantly for $\mu < 0$, but there still exist significant regions of the parameter space in this case which are allowed by the constraints as shown in Fig.(10b) and Fig.(10d). However, none of the allowed points in Figs.(10a-10d) which satisfy the $b - \tau$ unification and the $b \to s + \gamma$ constraints satisfy the $g_\mu - 2$ constraint.

5 Conclusions

In this paper we have analyzed $b - \tau$ unification within SU(5) with emphasis on nonuniversalities in the gaugino sector. We find that there exist regions of the parameter space with nonuniversalities in the gaugino mass sector which lead to $b - \tau$ unification consistent with the Brookhaven result on $g_\mu - 2$ and with satisfaction of the $b \to s + \gamma$ constraint. We discussed the 24 plet, the 75 plet and the 200 plet types of nonuniversality in the gaugino sector. We found that while $b - \tau$ unification and the $b \to s + \gamma$ constraint can be satisfied for all three types of nonuniversalities, but it is only the 24 plet case where the additional BNL $g_\mu - 2$ constraint can be satisfied. An important result that was observed for the 24 plet case is that in this case there exists an anti-correlation between the positivity of $a_\mu^{SUSY}$ and the negativity of $\Delta_\tilde{g}$ and hence of $\Delta_b$ since $\Delta_\tilde{g}$ is the dominant contribution to $\Delta_b$ over most of the parameter space of the model. Quite remarkably the above anti-correlation holds irrespective of the sign of $\mu$. The above result implies that in the 24 plet case the experimental BNL constraint that $a_\mu^{SUSY} > 0$ automatically leads to a negative contribution from $\Delta_\tilde{g}$ resulting in a negative $\Delta_b$ which is what is needed
for $b - \tau$ unification. We investigated both the $\mu > 0$ and the $\mu < 0$ branches and find satisfactory solutions for both cases. In both cases we find new regions of the parameter space which give negative SUSY contribution to the $b$ quark mass, satisfy the $g_{\mu} - 2$ as well as $b \rightarrow s + \gamma$ constraints and lead to a satisfactory $b - \tau$ unification. The new regions of the parameter space are also consistent with the naturalness criteria and further the corresponding sparticle spectrum which arises in this parameter space lies within the reach of the Large Hadron Collider. It would be interesting to discuss the details of the supersymmetric signals such as the trileptonic signal in this part of the parameter space. However, this analysis is outside the scope of this paper. Further, we have not discussed in this paper the topic of proton stability which involves the Higgs triplet sector of the theory. The most recent experimental data from SuperKamiokande appears to put a rather stringent constraint on the Higgs triplet coupling in the simplest supersymmetric grand unification models. However, it is possible to relieve this constraint in non minimal models. Further work is required in this area but again this analysis is outside the scope of this paper. While this paper was under preparation the work of Ref. appeared where the issue of $b - \tau$ unification with gaugino mass nonuniversality was also briefly discussed.

Appendix

We record here the numerical values of $n_{r}^{i}$ for the universal and the nonuniversal cases. For the universal case one has $n_{r}^{i}(1) = 1$ for all $i$. The cases $r=24, 75, 200$ are nonuniversal. For $r=24$ one has $n_{24}^{1} = -1; n_{24}^{2} = -3; n_{24}^{3} = 2$. For $r= 75$ one has $n_{75}^{1} = -5; n_{75}^{2} = 3; n_{75}^{3} = 1$. Finally, for $r=200$ one has $n_{200}^{1} = 10; n_{200}^{2} = 2; n_{200}^{3} = 1$.

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Figure 1: Plots of $\lambda_b - \lambda_\tau$ unification parameter $\delta_{b\tau}$ vs model inputs for the 24-plet case when $\tan\beta < 55$, $0 < m_0 < 2$ TeV, $-1$ TeV $< C_{24} m_{1/2} < 1$ TeV, $-6$ TeV $< A_0 < 6$ TeV and $\mu > 0$. The dotted points satisfy unification at the level shown, (blue) crosses additionally satisfy $b \rightarrow s + \gamma$ limits and (black) filled circles satisfy all the constraints, i.e., $b - \tau$ unification at the level shown, the $b \rightarrow s + \gamma$ constraint and the muon $g - 2$ constraint.
Figure 2: Plot of $\lambda_b - \lambda_\tau$ unification parameter $\delta_{b\tau}$ vs the supersymmetric correction $\Delta_6$ to the b-quark mass for the 24-plet case, when $\tan \beta < 55$, $0 < m_0 < 2$ TeV, $-1$ TeV < $C_{24}m_{1/2} < 1$ TeV, $-6$ TeV < $A_0 < 6$ TeV and $\mu > 0$. The dots refer to $b - \tau$ unification at the shown level and filled (blue) squares additionally represent points which satisfy both the $b \to s + \gamma$ and the muon $g - 2$ constraints.
Figure 3: Plots of the supersymmetric correction $\Delta_b$ to the b-quark mass vs model inputs for the 24-plet case when $\delta_{b\tau} < 10\%$. Here $\tan\beta < 55$, $0 < m_0 < 2$ TeV, $-1$ TeV $< C_{24} m_{1/2} < 1$ TeV, $-6$ TeV $< A_0 < 6$ TeV and $\mu > 0$. The (red) dots refer to points satisfying $b - \tau$ unification with $\delta_{b\tau} < 10\%$ and filled (blue) squares additionally represent points which satisfy both the $b \rightarrow s + \gamma$ and the muon $g - 2$ constraints.
Figure 4: (a): Sparticle masses in mSUGRA vs $m_0$ for the case $\mu > 0$, $\tan \beta = 35$, $A_0 = 0$, and $m_{1/2} = 300$ GeV. (b): Sparticle masses vs $m_0$ in the 24 plet case of nonuniversality. The value of $m_3(M_G)$ and other parameters are chosen to be identical with that of (a) for comparison. (c): Similar to (b) except that $A_0 = -1$ TeV and $C_{24} m_{1/2} = -100$ GeV. The parameters chosen in Fig.(4b) and Fig.(4c) favor $b - \tau$ unification as may be seen from Figs. 1(a) to 1(d).
Figure 5: The superymmetric correction $\Delta b$ to the b-quark mass vs $\tan \beta$ for the 24-plet case for $\mu < 0$ when $\delta_{b\tau} < 20\%$. Here $0 < m_0 < 2$ TeV, $-1$ TeV $< C_{24}m_{1/2} < 1$ TeV, $-6$ TeV $< A_0 < 6$ TeV. The dots refer to points satisfying $b-\tau$ unification with $\delta_{b\tau} < 20\%$. The (blue) crosses additionally satisfy $b \rightarrow s + \gamma$ limits. The (black) filled circles satisfy in addition the muon $g - 2$ constraint corresponding to the points in the parameter space where $m_2\mu > 0$, typically with $10\% < \delta_{b\tau} < 20\%$. 


Figure 6: The allowed and the disallowed regions for the 24-plet case in the $m_0 - m_3(M_G)$ plane with various constraints. The dotted points satisfy $b - \tau$ unification with $\delta_{b-\tau} < 0.3$ but do not satisfy the $b \rightarrow s + \gamma$ constraint or the muon $g - 2$ constraint. Blue crosses are the allowed points which satisfy the $b - \tau$ unification with $\delta_{b \tau} < 0.3$ and the muon $g - 2$ constraint. Black filled circles are the allowed points which satisfy all the constraints, i.e., the $b - \tau$ unification with $\delta_{b-\tau} < 0.3$, the muon $g - 2$ constraint, and the $b \rightarrow s + \gamma$ constraint. (a) is for the case when $\mu > 0$, $\tan \beta = 35$, and $A_0 = 1$ TeV and (b) is for the case when $\mu < 0$, $\tan \beta = 35$ and $A_0 = -1$ TeV.
Figure 7: (a): Gaugino masses in mSUGRA vs $m_{1/2}$ when $\mu > 0$, $\tan \beta = 5$, $m_0 = 400$ GeV, and $A_0 = 0$. (b): gaugino masses for the 24 plet nonuniversal case vs $m_3(M_G)$ for the same set of parameters as in (a). (c): Same as (b) except for the 75-plet case. (d): Same as (b) except for the 200-plet case.
Figure 8: (a): Sparticle masses in mSUGRA vs $m_0$ for $\mu > 0$ when $\tan \beta = 5$, $A_0 = 0$, and $m_{1/2} = 300$ GeV. (b): Sparticle masses for the 24-plet nonuniversal case for $\mu > 0$, when $\tan \beta = 5$, $A_0 = 0$, and $m_3(M_G) = 300$ GeV. Values of $m_3(M_G)$ and of other parameters are chosen to be identical with that of (a) for a direct comparison. (c): Same as (b) except for the 75-plet nonuniversal case. (d): Same as (b) except for the 200-plet nonuniversal case.
Figure 9:  (a): Sparticle masses in mSUGRA vs $m_{1/2}$ for the case $\mu > 0$, $\tan \beta = 5$, $m_0 = 400$ GeV, and $A_0 = 0$. (b): Similar plots of mass spectra vs $m_3(M_G)$ for the 24-plet nonuniversal case. (c): Same as (b) except for the 75-plet nonuniversal case. (d): Same as (b) except for the 200-plet nonuniversal case. The gray areas represent the disallowed regions because of chargino mass limits and the absence of radiative electroweak symmetry breaking.
Figure 10: (a): Plot of $\lambda_b - \lambda_\tau$ unification parameter $\delta_{b\tau}$ vs $C_{75}m_{1/2}$ for the 75-plet case when $\mu > 0$ and the other parameters are varied so that $\tan \beta < 55$, $0 < m_0 < 2$ TeV, and the third generation trilinear parameters are varied independently in the range $-6$ TeV to $6$ TeV. The dotted points satisfy $b - \tau$ unification at the level shown and the (blue) crosses additionally obey the $b \rightarrow s + \gamma$ constraint. There are no parameter points consistent with the muon $g - 2$ constraint. (b) Same as (a) except that $\mu < 0$. (c) Same as (a) except that the plot is for $\delta_{b\tau}$ vs $C_{200}m_{1/2}$. (d) Same as (c) except that $\mu < 0$. 