Simple do-it-yourself experimental set-up for electron charge $q_e$ measurement

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Received 16 April 2018, revised 9 July 2018
Accepted for publication 17 July 2018
Published 29 August 2018

Abstract

A simple experiment for the electron charge $q_e$ measurement is described. The experimental set-up contains standard electronic equipment only and can be built in every high-school lab all around the world with several days’ pocket money budget. It is concluded that it is time such a practice should be included in regular high-school education. The achieved 13% accuracy is comparable to the best student university labs. The measurement is based on Schottky noise generated by a photodiode. Using a criterion of dollar-per-accuracy for the electron charge $q_e$ measurement, this is definitely the world’s best educational experiment. An industrial replica can easily be sold across the globe.

Supplementary material for this article is available online

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1. Introduction

The electron charge $q_e$ is included in the program of high-school education in physics. Unfortunately, $q_e$ is not measured in high schools and even in many of the best universities it is determined with poor accuracy using kilodollar ($k$) equipment.

Let us recall a cursory history of the experimental method. Young Walter Schottky, a student of Max Plank, attended a lecture by Albert Einstein on fluctuations; cf. his obituary [1]. Impressed that fluctoscopy could be used to determine fundamental constants, Schottky suggested determining the electron charge $q_e$ by investigating current fluctuations [2, 3] exactly a century ago. The Schottky relation $(I^2)_f = 2q_e \langle I \rangle$ between the the averaged current $\langle I \rangle$ and the spectral density of the current noise $(I^2)_f$ will be discussed later. The experiment suggested by Schottky was performed as a research in the field of fundamental physics [4] and led to a precise determination of the electron charge [5].

For physics education, reaching a metrological accuracy is not necessary and the emphasis is to describe experimental set-ups giving illustration of physics principles. Already for half a century the determination of $q_e$ by shot noise has been an eternal theme of articles addressed to physics teachers and students laboratories, including a dozen articles [6–18], most of which are in the American Journal of Physics (AJP) and are expensive for high-school physics (several $k$ range). However, all those works give a contemporary version of the original set-up. Even a recent work requires the following [19]: a high-level electronics controller, low-level electronics controller, temperature module w/probe, break-out box, clear dewar with adjustable height support, coax cables, 45 W +/−15 V power supply, hook-up wire, resistors, transistors, diodes, photodiode in holder, light bulbs and LEDs; the commercial price of all this exceeds $7k$. The purpose of this research is to describe a sufficiently different set-up for which all those expensive accessories are not necessary, while the accuracy is the same.

The experimental set-up contains standard electronic parts, which can be delivered to every town within a week: two low-noise dual-operational amplifiers (OpAmps), one analog multiplier, a prototype board, resistors, capacitors (four of them of film layer-type), standard-type batteries, an incandescent bulb (or white LED with luminophore), a potentiometer, a photodiode and two standard multimeters. As an out-of-school physics education and for university freshmen, the construction of the described set-up is a great starting point in electronics. Every high-school student rotating the axis of the potentiometer can see a change of the photocurrent voltage in the first voltmeter and the fluctuations in the second voltmeter. The first step of education is a qualitative description of the effect. Even a person with a very basic physics background can feel with their hands how the set-up works, which is described in the next section, and reach an intuitive understanding.

2. The experimental set-up

The circuit of the set-up is depicted in figure 1. The light from a lamp creates a photocurrent through the photodiode, which passes through the connected resistor with resistance $R$. The photovoltage $U(t)$ is amplified by an instrumentation amplifier followed by an inverting amplifier.
The amplified voltage with total amplification \( y \), which we will describe shortly after
\[
U_{\text{amp}}(t) = yU(t),
\]
is applied to a multiplier giving output voltage \( U_{\text{out}}(t) = U_{\text{amp}}(t)/\bar{U} \), which is finally averaged by an averaging low-pass filter with time constant \( \tau_{\text{av}} = R_{w}C_{w} \). The voltage parameter \( \bar{U} \) is introduced later in this section. The averaged voltage \( U_{2} = \langle U_{2}(t) \rangle \) is measured by a voltmeter \( V_{2} \), a cheap commercial multimeter. Another multimeter \( V_{1} \) measures the time-averaged photovoltage
\[
U_{1} = y_{1}\langle U(t) \rangle,
\]
where \( y_{1} \) is the amplification of the high-impedance buffer
\[
y_{1} = 1 + 2R_{f}/R_{g},
\]
which contains two non-inverting amplifiers. The gain capacitors right after the buffer stop the operational amplifiers' noise and voltage offsets, as well as the time-averaged DC photovoltage \( U_{1} \). Next there is a difference amplifier with amplification \( y_{2} = -R_{f}/R_{g} \) and an inverting amplifier with amplification \( y_{3} = -R_{f}/R_{g} \). The amplifier has a total amplification

\[\text{Figure 1. Experimental circuit for } q_{e} \text{ measurement. Light from the incandescent light bulb } B \text{ shines on the photodiode } D, \text{ which creates a photocurrent. The photocurrent passes through the connected resistor } R, \text{ and creates a photovoltage } U(t) \text{ at the inputs of the instrumentation amplifier followed by an inverting amplifier. The high-impedance buffer of the instrumentation amplifier amplifies the time-averaged photovoltage } U_{1} = y_{1}\langle U(t) \rangle \text{ to be measured by multimeter } V_{1}. \text{ Alongside the DC photovoltage, Schottky noise from the photodiode is amplified by the high-impedance buffer, too. The gain capacitors } C_{g} \text{ stop the operational amplifiers' offset and the voltage DC component of the amplified photovoltage } U_{1}. \text{ The amplified shot noise is further amplified by the difference and inverting amplifiers. The amplified Schottky noise } U_{\text{amp}}(t) \text{ is filtered by a low-pass filter, then is squared by an AD633 multiplier } U_{2}(t) \text{ and finally it is averaged } U_{2} = \langle U_{2}(t) \rangle \text{ by a low-pass filter with a large time constant } \tau_{\text{av}} = R_{w}C_{w} \text{ to be measured by multimeter } V_{2}. \text{ The batteries for the light bulb and their connected cables pick up electromagnetic noise and act like an effective antenna } A. \text{ The inductances } L \text{ are radial-type choke coils with ferrite core CW68-102K and they switch off the circuit for higher frequencies, while the capacitor } C_{B} \text{ short circuits the remaining AC current component that has managed to pass through the inductances. In such a manner, no AC current reaches the light bulb. Sometimes only the capacitor is enough to switch off the antenna from the lamp. The variable resistance } R_{V} \text{ is used to change the current through the light bulb, which in turn changes the luminosity of the latter, enabling measurements of } V_{1} \text{ and } V_{2} \text{ for different values of photovoltage and Schottky noise. The values of the elements used are given in table 1.} \]
\[ y = y_1 y_2 y_3 = \left( \frac{R_i + R_g/2}{R_g/2} \right)^2 \left( \frac{R_i'}{R_g'} \right)^2. \] (4)

It is possible to test each amplification \( y_1, y_2, y_3 \) by means of a simple voltage divider, using 1% accuracy resistors. After the voltage is reduced 100 times by the voltage divider, it is applied to the input of each amplification step (1 high-impedance buffer, 2 difference amplifier and 3 inverting amplifier) and the voltage at the output should be equal to its initial value before the reduction, i.e. the input voltage should be recovered.

1. Input voltage of the first step should be applied between the (+) inputs of the buffer without removing the resistor \( R \). The recovered voltage should be measured with multimeter \( V_1 \). The OpAmps participating in the high-impedance buffer are placed in module U1 in figure 1. The hidden secret of the whole circuit is the tiny distance between the (+) inputs of the dual-operational amplifier. The usage of a single OpAmp requires screening metallic boxes and BNC connecting cables.

2. For the difference amplifier, input voltage should be applied in the two points between \( C_g \) and \( R_g \), shown in figure 1, before the left U2 OpAmp. The recovered voltage should be measured between the output point and the common point of this OpAmp.

3. Input voltage of the inverting amplifier should be applied between the point between \( C_g \) and \( R_g \) and the common point of the right U2 OpAmp. The recovered voltage should be measured between the output voltage and common point of this last OpAmp.

This simple method allows for a quick verification of the expected amplification using a DC input voltage source, for instance an ordinary battery.

In well-equipped labs it is possible to use AC voltages to test two steps sequentially and even all three steps of the amplifier. Only several centimeters of wire connected at the input of the amplifier creates ringing and parasitic capacitance between amplifier input and output. Therefore extreme care is needed. It is necessary to use triaxial cable and for the external shielding to be connected to the common point of the amplifier. Parallel to the resistor \( R \) one 50 \( \Omega \) resistor should be connected to minimize reflected waves. This triaxial cable should be connected to the circuit in a way to ensure the absence of ringing. Only after that, one million time reduced AC voltage in a frequency range between the lower cutoff frequency \( f_g \) and \( f_A \) in table 2 from a signal generator should be applied to the internal wire and the first shield of the triaxial cable. In this way the reduced input signal will be amplified by the amplifier to recover the original signal from the signal generator. We emphasize that recovery of signals after one million times amplification is a difficult task. In case of external noise one can use a lock-in amplifier for measurement of the amplified voltage. Changing the frequency, one can investigate the frequency dependence of the transmission of the amplifier. This procedure is standard for an electronics lab but it is far beyond the educational measurement of a school lab. That is why we recommend only the DC measurement of the different steps of the circuit.

Do not hesitate to write to the authors if you try repeating the experiment.

Usually specialists in electronics give the advice that care is needed to construct high-gain and high-bandwidth amplifiers. This is absolutely true for the standard circuits used for the investigation of Schottky and Johnson noise, and up to now screening metallic boxes and BNC connectors were indispensable. However, for the instrumentation amplifier with dual OpAmps in the buffer parasitic capacitances are very small and ringing can come only by a large battery (which is also an effective antenna for radio signals) of the photodiode. To ensure circuit stability, care needs to be taken to minimize electrostatic coupling between the photodiode battery (for instance, using a small 3 V lithium battery) and million times
amplified photovoltage fluctuations. There are no screening boxes and no BNC connectors in our set-up, which is an advantage from a pedagogical point of view.

After the amplification of \( y \) times, the signal passes through a low-pass filter with transmission coefficient

\[
y_{\text{LPF}} = \frac{1}{1 + j\omega \tau_{\text{LPF}}}, \quad \tau_{\text{LPF}} = R_L C_L,
\]

and bandwidth

\[
B_{\text{LPF}} = \int_{0}^{\infty} \frac{1}{1 + (\omega \tau_{\text{LPF}})^2} \frac{d\omega}{2\pi} = \frac{1}{4\tau_{\text{LPF}}}.
\]

This filter reduces the total noise and its purpose is to fix exactly the value of the pass bandwidth. The values of \( R_L \) and \( C_L \) can be measured with 3-digit accuracy with an inexpensive LCR multimeter, while in university labs [17] commercial Butterworth filters have an accuracy of 2%. In such a way, the bandwidth \( B_{\text{LPF}} \) of the simple low-pass filter can be evaluated with higher accuracy.

The standard operation of the multiplier with the voltages at its input terminals is described by the equation [20] \( W = (X_1 - X_2)(Y_1 - Y_2)/U_m + Z \). For our circuit both \( X_2 = Y_2 = 0 \) and \( X_1 = Y_1 = U_{\text{amp}}(t) \). In such a way, for the output voltage we have

\[
U_o(t) = W(t) = \frac{U_{\text{amp}}^2(t)}{U}, \quad \bar{U} \equiv \frac{U_m}{R_1 + R_2},
\]

where \( U_m \) is a constant of the multiplier. Finally, the squared voltage \( U_o(t) \) is averaged by the low-pass filter \( U_2 = \langle U_o(t) \rangle \) and measured by the voltmeter \( V_2 \). Substitution of equation (1) in equation (7) gives

\[
\bar{U}_2 = \frac{(\delta U)^2}{U^2}, \quad \bar{U}^2 = \frac{\bar{U}}{y^2}, \quad (\delta U)^2 \equiv \langle (U - \langle U \rangle)^2 \rangle.
\]

Here it is taken into account that the DC voltage \( \langle U \rangle \) is stopped by the first pair of gain capacitors \( C_g \) immediately after the buffer.

According to the theory of Schottky noise, there is a linear dependence between the measured voltages, which is described in the next section, and the slope determines \( q_e \).

In the experimental set-up we use two dual low-noise operational amplifiers (ADA4898-2) [21], a cheap multiplier (AD633) [20], a photodiode (BPW34) [22] (with small capacitance \( C_{ph} \)) powered by a 3 V (CR1220) Li battery in reverse bias, a 3V 80 mA incandescent light bulb (T303 T3) powered by three AA 1.5 V batteries and four metal layer WIMA capacitors [23].

For the ADA4898-2 operational amplifiers used, the \( 1/f \) noise dominates below the corner frequency \( f_c \) on the order of 10 Hz. Such low frequencies and DC offsets are stopped by the gain capacitors, which together with gain resistors form a lower cutoff frequency of the described amplifier \( f_c = 1/2\pi R_g C_g = 159 \) Hz. In other words, the \( 1/f \) noise of the OpAmps is negligible for our circuit depicted in figure 1.

The other elements are ordinary 1% accuracy resistors and the rest of the capacitors are within 20% accuracy. The exact value of \( C_g \) is essential and therefore has to be measured with high accuracy. The values of the elements used are given in table 1.

To take into account the non-ideal effects of the operational amplifiers, a small correction factor \( Z = 1 - \epsilon \) has to be introduced in the denominator of equation (17). This correction requires frequency-dependent analysis of the circuit and will be presented in an unabridged version of the present work that will be published in arXiv later. For the element values used
in the described experiment here, the correction is $\epsilon = 6.74\%$ and it is included in the obtained value for the electron charge. This correction takes into account the finite value of the crossover frequency $f_0$ of the operational amplifier. The inclusion of this correction is necessary only if we wish to reach 1% accuracy of the measurement of $q_e$.

### 3. Recalling shot noise theory

When the current through the photodiode consists of separate $\delta$-shaped electron impulses at arbitrary times $t_i$

$$I(t) = \sum_i q_e \delta(t - t_i),$$

then between the average current

$$\langle I \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T I(t) \frac{dt}{T}$$

and the low-frequency spectral density $(I^2)_f$ current, which parameterizes the current dispersion

$$(\delta I)^2 = \langle (I - \langle I \rangle)^2 \rangle = \int_0^\infty (I^2)_f \frac{d\omega}{2\pi}$$

the Schottky law [2] is in effect:

$$(I^2)_f = 2q_e \langle I \rangle.$$  

The theory of shot noise is given in appendix A of the current work.

In such a way for the spectral density of the voltage we obtain

$$(U^2)_f = 2q_e \langle I \rangle R^2 + 4k_nTR + e^2_n + R^2i_n^2,$$

where the second term describes the thermal noise of the resistor $4k_nTR$, the third $e^2_n$ gives the contribution of the voltage noise of the operational amplifier and the last term expresses the current noise $i_n^2$ of the operational amplifier. The detailed theory of Johnson noise will be given in another paper.

Taking into account the bandwidth $B_{\text{LPF}}$ of the low-pass filter, we obtain for the voltage dispersion

\[
\begin{array}{|c|c|}
\hline
\text{Circuit element} & \text{Value} \\
\hline
R & 200 \Omega \\
C_B & 10 \mu F \\
C_L & 47 \text{nF} \\
R_L & 510 \Omega \\
r_g & 20 \Omega \\
R_f & 1 \text{k}\Omega \\
C_f & 10 \text{pF} \\
C_g & 10 \mu F \\
R_g & 100 \Omega \\
R_f & 10 \text{k}\Omega \\
\hline
\text{Circuit element} & \text{Value} \\
\hline
V & 0 - 1 \text{k}\Omega \\
V_1 & 2 \text{k}\Omega \\
R_2 & 18 \text{k}\Omega \\
R_{av} & 1.5 \text{M}\Omega \\
C_{av} & 10 \mu F \\
V_{CC} & +9 \text{ V} \\
V_{EE} & -9 \text{ V} \\
U_m & 10 \text{ V} \\
L & 1 \text{ mH} \\
C_{ph} & \leq 40 \text{ pF} \\
\hline
\end{array}
\]
\[
(\delta U)^2 = \langle (U - \langle U \rangle)^2 \rangle = \int_0^\infty (U^2) \frac{d\omega}{2\pi} = \frac{q^2 R}{2C_1 R_0} \langle U \rangle.
\]

where \( \langle U \rangle = R (I) \) is the time-averaged photovoltage of the resistor. In this formula for the Schottky noise we express \( \langle U \rangle \) and \( \delta U \) by the experimentally measurable \( U_1 \) from equation (2) and \( U_2 \) from equation (8) and obtain

\[
U_2 = \alpha U_1 + v, \quad \alpha = \frac{\Delta U_2}{\Delta U_1} = \frac{q}{2C_1 y_1 U^*}, \quad v = \text{const}.
\]

In such a way the electron charge \( q_e \) can be expressed by the dimensionless slope \( \alpha \) of the linear regression

\[
q_e = 2\alpha y_1 C_1 U^*.
\]

Substituting here \( y_1 \) from equation (3) and \( U^* \) from equation (8) and \( \tilde{U} \) from equation (7), finally the electron charge is expressed by the experimentally measurable values

\[
q_e = 2 \frac{\Delta U_2}{\Delta U_1} \left( \frac{r_g}{2R_l + r_g} \right)^2 \left( \frac{R_g}{R_l} \right)^4 \frac{R_1 C_1 U_{in}}{R} \frac{R_1}{R_1 + R_2} \frac{R_V + R_{av}}{R_V}.
\]

Here the last multiplier \( R_g/(R_V + R_{av}) \) describes the voltage divider created by the finite internal resistance of the multimeter \( R_g \) (1 M\( \Omega \) in our case) and the averaging low-pass filter resistance \( R_{av} \). In this formula the several percent correction related to the final cutoff frequency of the operational amplifier is not included.

In the next section we will describe how this idea for the determination of \( q_e \) is realized.

4. Experiment

For fixed light intensity the averaging time is on the order of one minute. The voltage proportional to the dispersion of the shot noise \( U_2 \) has more significant fluctuations than amplified DC voltage \( U_1 \). The measurements made over an hour are presented in table 3 and graphically depicted in figure 2; the obtained value for the electron charge \( q_e \) from these measurements is \((1.811 \pm 13\% ) \times 10^{-19} \text{ C} \), which is 13\% higher than its true value.

The accuracy can be increased by increasing the time-averaging constant \( \tau_{av} \), screening the set-up from external noises, different battery supplies for different operational amplifiers, taking non-ideal effects of the operational amplifier described by the master equation for the OpAmps

\[
\left( G^{-1} + \tau_0 \frac{d}{dt} \right) U_{\text{output}} = U_+ - U_-, \quad (18)
\]

giving the relation between the inputs \( U_+ \) and \( U_- \) and the output \( U_{\text{output}} \) of the OpAmp; for the numerical values of the parameters see table 2, precise measurement of the values of all components, etc. The master equation of the OpAmps, which apparently has never been published before (the authors will very much appreciate receiving an appropriate reference with the equation present in it), is described in our recent eprint [24] and the results of its application can be found in many technical applications. See, for example, the formulae for the frequency dependence of the inverting amplifier and non-inverting amplifiers in the specification of ADA4817 [25].

In such a way, we can achieve the accuracy of the good university students labs [6–18] using standard electronics with low-noise pre-amplifiers and Butterworth filters with 2\% accuracy. But the purpose of the present work is in a different direction.

The amplification of our set-up is on the order of that used by the Habicht [26] brothers trying to measure the Boltzmann constant \( k_B \) as it was suggested by Einstein [27]. For the electronic
Figure 2. $U_2$ versus $U_1$ plot. Linear regression from the electron charge $q_e$ measurement data given in table 3. According to equation (17) the slope $\alpha$ of the linear regression $U_2 = \alpha U_1 + \nu$ determines the electron charge $q_e$. The experimental data procession gives $\alpha = 3.145 \times 10^{-3}$ with correlation coefficient $\rho = 0.995$. The abscissa voltage $U_1 \propto \langle I \rangle$ is proportional to the time-averaged photocurrent $\langle I \rangle$ while the ordinate voltage $U_2 \propto (\delta U)^2$ is proportional to the dispersion of the shot noise. The calculated electron charge $q_e$ with the obtained slope $\alpha$ is $(1.811 \pm 13\%) \times 10^{-19}$ C, which is approximately $13\%$ larger than the real value $1.602 \times 10^{-19}$ C. The constant $\nu \approx 0.025$ V of the linear regression is the voltage offset that includes the thermal noise of $R$ and the voltage noise of the first couple operational amplifiers mainly. The large value of $\nu$ (approximately 5 times the difference between the minimal and maximal values of $U_2$) shows how crucial the usage of low-noise operational amplifiers is. The inset shows the segment of the linear dependence with 1% error bars of $U_2$ corresponding to statistical evaluation of many similar experiments and deviation of $\rho$ from one.

Table 2. Table of the calculated parameters necessary for the analysis of the circuit (time constants, frequencies, voltages and linear amplification). $f_0$ is the $-3$ dB bandwidth of the ADA4898, $\tau_0 \equiv 1/2\pi f_0$, $f_\Lambda \equiv f_0/\tau_1$, $\tau_\Lambda \equiv \tau_0/\tau_1$, $\tau_{\text{LPF}} = R_f C_L$, $\tau_\tau = R_t C_t$, $\tau_\tau = R_f C_f$, $\tau_{\text{av}} = R_{\text{av}} C_{\text{av}}$, and $\tau_0 \ll \tau_1$, $\tau_\Lambda \ll \tau_{\text{LPF}} \ll \tau_\tau \ll \tau_{\text{av}}$.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $\tau = R C_{\text{ph}}$ | $\leq 8$ ns | $y_1$ | 101 |
| $f_0$ | 65 MHz [21] | $y_2$ | $-100$ |
| $\tau_0$ | 2.45 ns | $y_3$ | $-100$ |
| $\tau_\Lambda$ | 247 ns | $y$ | $1.01 \times 10^6$ |
| $f_\Lambda$ | 669 kHz | $U_m$ | 10 V [20] |
| $\tau_{\text{LPF}}$ | 23.97 $\mu$s | $\tilde{U}$ | 1 V |
| $\tau_f$ | 10 ns | $U^*$ | 891 fV |
| $\tau_{\text{av}}$ | 1 ms | $G^2$ | 103 dB [21] |
| $f_0$ | 159 Hz | $G$ | $10^5$ |
| $\tau_f$ | 100 ns | Input voltage noise | 0.9 nV/$\sqrt{\text{Hz}}$ [21] |
| $\tau_{\text{av}}$ | 15 s | CMRR (typical) | $-126$ dB [21] |
amplifier the problem of the floating of the zero was solved by the gain capacitors. For our set-up the calibration of the amplifier should be checked by determination of the Boltzmann constant.

Having removed the photodiode, we have to change the resistor \( R \) with several values to perform a linear regression in the plot \( U_2 \) versus \( R \). The slope of the linear regression then gives \( k_B \).

5. Discussion and conclusions

The goal of our work is not a competition with the best student laboratories but to distribute the set-up in all universities and to consider even high-school labs. Our approach does not require specialized (in that sense expensive) laboratory equipment. A soldering station and two multimeters can be found in every high-school physics lab. No such opportunity to build cheap and widely accessible experimental set-ups is offered by any of the published works on Johnson and Schottky noise.

Working in extremely noisy environments, for instance in the presence of luminescent light, it is better to use electromagnetic screening. A small fridge is absolutely enough, and moreover cooling significantly improves the operation of the photodiode. Our set-up works very well in temperatures lower than 15 °C. And vice versa, a hair dryer can destabilise the work of the set-up and give spurious and different values for the electron charge \( q_e \).

It is extremely stimulating to measure a fundamental constant with a do-it-yourself set-up. But we can consider the next step. Industrial production of the present set-up using printed circuit boards (PCBs) and automated soldering of elements could lead to a price which is comparable to that of a scientific calculator, acceptable for every high-school around the world. The electron charge \( q_e \) is included in high-school education programs all around the globe. It is time to include its measurement as a standard lab practice. The electron and electronics affect the lives of everyone, and it is time that people using electronic devices remembered how they measured \( q_e \) whilst as teenagers. Last but not least, this experiment is fun.

Acknowledgments

The authors are grateful to Vasil Yordanov for the friendship and his contribution at the early stages of the present research [28] (for an extended bibliography and history of the problem see
this unabridged version), to Alexander Petkov for making the first measurements, to Nikolay Zografov for introducing order in the lab, to Andreana Andreeva for animation of the spirit in the lab and to Petar Todorov for the interest and assistance in the current research.

The present measurement of the electron charge \( q_e \) is one of a series of set-ups created by the educational laboratory for measurements of fundamental constants. Here we wish to mention the set-ups for measurement of the Planck constant \( \hbar \) [29], the speed of light \( c \) [30], and most recently the Boltzmann constant \( k_B \) [31].

TMM is thankful to Genka Dinekova and Pancho Cholakov for recommending ADA4898 in the beginning of the present work and for a valuable bit of advice related to the integrated circuits tested in the development of our set-up. The University of Sofia St. Clement of Ohrid received many free-of-charge operational amplifiers from Analog Devices, which is an important help to our university research and education.

Appendix A. Pedagogical rederivation of Schottky formula for shot noise

Let us analyze the current impulse of a single electron through a resistor

\[
J(t) = q_e \delta(t - t_0),
\]

\[
\delta(t) \equiv \frac{\theta(t)}{\tau} \exp(-t/\tau), \quad \tau = RC_{ph}
\]  

(A1)

created by the quantum transition of one electron with charge \( q_e \) in the photodiode D with capacity \( C_{ph} \); see figure A1. One can imagine a real photodiode as a parallel-connected ideal photodiode and a small capacitor with capacitance \( C_{ph} \). This capacitor together with the shunting resistor \( R \) creates a circuit with time constant \( \tau \) and corresponding characteristic frequency

\[
f_{ph} = \frac{1}{2\pi \tau} = \frac{1}{2\pi RC_{ph}}.
\]  

(A2)

Supposing that after some time \( M \) (say one hour) the current is repeated \( I(t + M) = I(t) \), we can use a Fourier representation:

\[
\langle J \rangle = \int_0^M J(t) \frac{dt}{M}, \quad \omega = \frac{2\pi}{M} n, \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots,
\]  

(A3)

where in our case

\[
J_\omega = \int_0^M J(t) \exp(i\omega t) \frac{dt}{M} = \frac{q_e}{M} \frac{\exp(i\omega t)}{1 - i\omega \tau},
\]

\[
|\tilde{I}_{ph}(\omega)|^2 \equiv \frac{M^2}{q_e^2} |J_\omega|^2 = \frac{1}{1 + (\omega \tau)^2} = \frac{1}{1 + (f/f_{ph})^2}.
\]  

(A4)

Substituting in \( M \rightarrow \infty \) the Perceval’s theorem

\[
\langle J^2 \rangle = \int_0^M J^2(t) \frac{dt}{M} = \langle J \rangle^2 + \sum_{\omega \neq 0} |J_\omega|^2,
\]  

(A5)
substituting summation with integration
\[
\sum_{\omega} \approx M \int \frac{d\omega}{2\pi},
\]
we express the fluctuation \(\delta J \equiv \sqrt{\langle (J - \langle J \rangle)^2 \rangle}\) by the spectral density \((J^2)_f \equiv 2 |J_0|^2\):
\[
(\delta J)^2 = \langle (J - \langle J \rangle)^2 \rangle = M \int_0^\infty (J^2)_f \frac{d\omega}{2\pi}.
\]
\(\text{(A7)}\)

For real \(J(t)\) the spectral density is even: \((J^2)_f(\omega) = (J^2)_f(-\omega)\).

If the shot noise in time interval \(M\) is created by \(N\) incoherent and independent quantum transitions:
\[
I(t) = \sum_{a=1}^N q_e \delta_{\tau}(t - t_a), \quad 0 < t_1 < t_2 < \ldots < t_N < M,
\]
\(\text{(A8)}\)
\[
\langle I \rangle = \frac{q_e N}{M}, \quad N = \frac{\langle I \rangle M}{q_e}.
\]
\(\text{(A9)}\)

For the fluctuation of the current, its dispersion and spectral density we have just to multiply by the number of the different quantum transitions:
\[
(I^2)_f = N (J^2)_f.
\]
\(\text{(A10)}\)

We suppose that the moments \(t_a\) of the different quantum transitions are completely independent. Then substituting equation \((\text{A4})\) in equation \((\text{A7})\) we arrive to the Schottky formula for the spectral density of the current noise:
\[
\langle (I - \langle I \rangle)^2 \rangle = \int_0^\infty (I^2)_f \frac{d\omega}{2\pi},
\]
\(\text{(A11)}\)
\[
(I^2)_f = 2 q_e \langle I \rangle |\Upsilon_{\text{ph}}(\omega)|^2,
\]
\(\text{(A12)}\)
\[
|\Upsilon_{\text{ph}}(\omega)|^2 \approx 1, \quad \text{for } f \ll f_{\text{ph}}.
\]
\(\text{(A13)}\)

As a rule the capacity of the photodiode \(C_{\text{ph}}\) is negligible and the corresponding frequency \(f_{\text{ph}}\) is much higher than the working frequencies of the rest of the electronics. In this case, we have the well-known formula for the frequency-independent spectral density of the shot noise:
\[
(I^2)_f \approx 2 q_e \langle I \rangle, \quad \text{for } f \ll f_{\text{ph}}.
\]
\(\text{(A14)}\)
For the spectral density of the voltage, we analogously have the spectral density of the voltage for the Schotky shot noise:

\[(U^2)_f = 2R^2q_e \langle I \rangle.\]  

(A15)

This white-noise approximation corresponds for \(\tau \to 0\) to \(\delta\)-function approximation in the time representation

\[\delta_i(t) = \frac{\theta(t)}{\tau} \exp(-t/\tau) \approx \delta(t), \quad \delta(t) \equiv \frac{d\theta(t)}{dt}.\]  

(A16)

Even Oliver Heaviside knew how the derivative of his \(\theta\)-function would look [32]. Many years later, defending himself against a mathematician, Dirac said, ‘Every electrical engineer knows the term impulse, and this function just represents it in a mathematical form’ (p. 78 Jemmer [32]). In our case the \(\delta\)-function approximation of the uncorrelated current impulses of the different photoelectrons is applicable if the corresponding time constant is much smaller than all other time constants of the circuit: \(\tau = RC_{ph} \ll \tau_{ex} \ll \tau_{A}\). No doubt the frequency dependence of \(\Upsilon_{ph}\) from equation (A4) and equation (A14) can be easily programmed, but it will not take into account the accuracy of a student laboratory.

Returning back to the work of our set-up after one million times amplification, the photocurrent transformed into voltage can be seen in the photo of two oscilloscopes, shown in figure A2.
Here we will allow ourselves a lateral speculation on the current noise of the OpAmps. For OpAmps, the input bias current $I_B$ is always small and obviously related to incoherent quantum transitions through an insulating barrier.

For example, tunneling from the gate to source-drain channel in a field effect transistor through the narrow insulating barrier. It is not always specified on data sheets, but the spectral density of the input current noise may be calculated in cases like simple BJT or JFETs, where all the bias current flows in the input junction, because in these cases it is simply the Schottky noise of the bias current $i_n^2 \approx 2q_e I_B$ [33].

According to our interpretation, for electrometer operational amplifiers current noise is just the spectral density of the noise $i_n^2 = \langle I^2 \rangle_f$, the average current is the input bias current $\langle I \rangle = I_B$ and these parameters are related by the Schottky formula.

For a verification with logarithmic accuracy we can even use data sheets and plot a linear regression $\ln i_n$ versus $\ln I_B$:

$$2 \ln i_n \approx \ln I_B + \ln(2 q_e).$$

If we use electrometer OpAmp AD549 [34, 35] with an extremely low bias current of $I_B = 200$ fA, the whole bias current is related with incoherent processes of quantum tunneling and we can use thermal evaporation of the charge carriers from the gate as a perfect source of shot noise for determination of $q_e$ without a photocurrent using only integrated circuits.

In conclusion, the electron charge can be determined by measuring the input bias current of a good electrometer operational amplifier. We suggest a new experiment to be performed with this method. After all, the electron charge is the beginning of electronics.

**Appendix B. Guide for building of the set-up**

The double ADA4898-2 operational amplifier used is available only in a small $4 \times 4$ mm surface mount device (SMD) package. It is strongly believed that soldering of SMD components requires specific equipment and personal skills. Here the authors have to add some observations on this point: in the video abstract of this article is exhibited how the doyen of the authors, a professor in theoretical and mathematical physics, can solder the ADA4898-2 operational amplifiers on a small out-line integrated circuit (SOIC)-to-dual in-line package (DIP) printed circuit board (PCB) adapter. All the other co-authors are much better at soldering with 10 times quicker speed. One full professor in theoretical physics gives a good approximation for a motivated but awkward high-school student. For training it is possible to start with a cheap operational amplifier with the same pin orientations, for example a TL072.

The main circuit can be prepared by a prototype PCB or in a specially designed PCB if we need to prepare more than 10 set-ups. To encourage the reproduction of the set-up we here supply photos for the SOIC-to-DIP adapter, shown in figure B1, a SMD OpAmp mounted on a removable pedestal to the main circuit, shown in figure B2, and the main set-up, shown in figure B3. The Geber files of the PCBs are available as supplementary material to the article is available online at stacks.iop.org/EJP/39/065202/mmedia.
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