Room-temperature steady-state optomechanical entanglement on a chip

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A potential experimental system, based on the Silicon Nitride (SiN) material, is proposed to generate steady-state room-temperature optomechanical entanglement. In the proposed structure, the nanostring interacts dispersively and reactively with the microdisk cavity via the evanescent field. We study the role of both dispersive and reactive coupling in generating optomechanical entanglement, and show that the room-temperature entanglement can be effectively obtained through the dispersive couplings within the reasonable experimental parameters. In particular, we find, in the high Temperature (T) and high mechanical quality factor (Qm) limit, the logarithmic entanglement depends only on the ratio T/Qm. This means that improvements in the material quantity and structure design may lead to more efficient generation of stationary high-temperature entanglement.

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Introduction.- The quantum entanglement not only represents one of the most interesting fundamental phenomena in quantum mechanics, but also is regarded as very important resource in quantum computation and quantum information processing [1]. It has been a focus in quantum mechanics since the seminal work of Einstein-Podolsky-Rosen (EPR) gedanken experiment [2]. Nowadays, quantum entanglement has been observed in various quantum systems, such as photons, atoms, and quantum dots [3]. Although it is believed that large decoherence effect would mask quantum signatures of macroscopic objects, great efforts are still dedicated to macroscopic quantum entanglement, in order to exploring the boundary between quantum and classical mechanics.

Recently, rapid progress in nanofabrication technologies offers novel opportunities to study macroscopic quantum entanglement [4]. Especially, combined with mature optical technologies, the emerging optomechanics enables precisely controlling the motion of mechanical oscillators through optical radiation force or gradient force [5]. It was expected that the mechanical vibration could be entangled with the optical field in various optomechanics systems, such as Fabry-Pérot cavity with a movable mirror [6] or nanomembranes [7]. It could also be extended to microwave cavities [8]. Unfortunately, quantum entanglement in these systems is very sensitive to the temperature, typically only valid in cryostat. Up to now quantum entanglement related to mechanical modes has not been observed in any experiment. One of the key question arises: Is it possible to entangle a light beam and macroscopic objects at room temperature?

In this paper, we propose a potential microcavity-nanostring system to generate the room-temperature optomechanical entanglement. The system is based on the Silicon Nitride material, the nanostring interacts dispersively and reactively with the microdisk cavity via the evanescent field. It is shown that entanglement can be effectively generated through dispersive couplings within the current experimental parameters, and can be preserved at room temperature or even at a high temperature. We analyzes the the dependence of entanglement on the temperature and mechanical dissipation, and find that the logarithmic entanglement is function of the ratio T/Qm in the high-Qm and high-temperature limit T. This demonstrates that the improvements in the material quantity and structure design may lead to more efficient generation of stationary room-temperature entanglement.

System.- As shown in Fig.1(a), the system consists of a nanostring oscillator and a microdisk cavity, fabricated by SiN, which can be integrated on single chip. This structure have been experimentally realized in silicon chip, but the performance is limited by low mechanical quality factor (Qm) and low optical quality factors (Qo) [6]. Similar structure with a nanostring closed to a silica toroid has also been studied by Anetsberger et al. [9], which shows displacement sensitivity beyond the standard quantum limit. However, it is a challenge to pre-
cissely control the gap between the nanostring and cavity in experiments. Compared with these studies, the experimental system proposed in this paper has the following advantages: (1) The fabrication technology of SiN device is compatible with silicon, and permits further expendability of on-chip optical components and opto-electronics elements. (2) The SiN is transparent for a wide band, and has relative high refractive index. Thus dielectric microcavity based on SiN can realize the great confinement of light, possess ultrahigh qualify factor and small mode volumes. Currently, the whispering great confinement of light, possess ultrahigh qualify factor, and holds great potential for quantum optomechanics. (3) The mechanical properties of SiN is excellent, so that we can fabricate strained nanostrings with mechanical qualify factor $Q_m > 10^6$ at room temperature. The large $Q_o$ significantly enhanced the light-matter interaction at low input intensity, and large $Q_m$ permits very long coherence time. Therefore, our system provides a unique combination of ultrahigh mechanical and optical quality factor, and holds great potential for quantum optomechanics.

To study the interaction between the microcavity and nanostring, we plot the mode field distribution of microdisk in Fig.1(b), which with diameter $\Phi = 10\mu m$ and thickness $t = 1 \mu m$. It is shown that the WG modes is well confined in the microdisk, with radiation related $Q > 10^8$. A small portion of energy outside the cavity form the evanescent waves, which is exponentially decay with the distance from the dielectric interface $(x)$, and can be approximate describe by $E_c(x) = E_c(0)e^{-x/l_0}$, where $l_0$ is the decay length. In Fig.1(c), we plot the normalized field $|E|^2$ distribution at $z = 0$, and find that the evanescent field is fit to a exponential decay curve with decay length $l_0 = 100nm$. In addition, as shown in Fig.1(a), the nanostring, as a waveguide, transfers light into and out from the microcavity. When the nanostring is placed in the evanescent-field of the WG mode, the mechanical oscillator will be attracted to the cavity through the gradient force, and the displacement of the nanostring also gives a backaction on the WG mode, and changes both the resonance frequency (dispersive coupling, DC) and energy decay rate (reactive coupling, RC).

From the coupling mode theory, the frequency shift $\delta \omega$ to the cavity resonance and the coupling strength $\kappa_1$ between the cavity mode and waveguiding mode can be expressed as $\delta \omega(x) \approx \delta \omega(0)e^{-2x/l_0}$, and $\kappa_1(x) \approx \kappa_1(0)e^{-2x/l_0}$. Thus, for a small displacement $x$ around steady position, $\omega_c(x) \approx \omega_c(0) - dx$, and $\kappa_1(x) \approx \kappa_1(0) + r_0x$, where $d$ and $r$ describe the dispersive and reactive coupling strengths between the mechanical and optical modes, respectively, and both of them could be controlled experimentally by adjusting the size of disk and nanostring and gap between them.

Model. - The Hamiltonian of the coupled microcavity and nanostring system is given by

$$ H = \hbar \omega_m a^\dagger a + \frac{1}{2} \hbar \omega_m (p^2 + q^2) - \hbar D a^\dagger q $$

$$ + i\hbar (\sqrt{2\kappa_1(x)} + Rq/\sqrt{2\kappa_1(x)})E(e^{-i\omega t}a^\dagger - e^{i\omega t}a), $$

where Bose operators $a$ and $a^\dagger$ represent the annihilation and creation operators of the cavity mode with frequency $\omega_c$ and intrinsic loss $\kappa$. For the nanostring, we define dimensionless position and momentum operators $q = \sqrt{m\omega_m/\hbar}x$, $p = \sqrt{m/\omega_m}x$, where $m$ stands for effective mass of the nanostring with a resonant frequency $\omega_m$. The corresponding dispersive and reactive coupling strength are normalized by zero point fluctuation $\sqrt{\hbar/\omega_m}$; $D = d/\sqrt{\omega_m/\hbar}$ and $R = r/\sqrt{\omega_m/\hbar}$. The system is driven by a coherent laser with frequency $\omega_l$ and power $P$, corresponding to the driving field $E = P/\hbar \omega_l$.

By including the noise on both the mechanical $(\xi(t))$ and optical $(a_{in}(t))$ modes, we obtain the quantum Langevin equations (QLE) in the reference frame rotating with frequency $\omega_l$,

$$ \dot{q} = -\omega_m p + G a^\dagger a - iR/\sqrt{2\kappa_1}E(a^\dagger - a) $$

$$ - i\hbar \sqrt{2\kappa_1}(a^\dagger a_{in} - a_{in} a^\dagger) + \xi(t), $$

$$ \dot{a} = -i\Delta a - (\kappa + Rq) a + iGq a $$

$$ + (\sqrt{2\kappa_1} + Rq/\sqrt{2\kappa_1})E + \sqrt{2\kappa_{in}}a_{in}(t), $$

where $\kappa = \kappa_0 + \kappa_1$, $\Delta = \omega_c - \omega_l$, and the noise correlation function are $\langle a_{in}(t)a_{in}^\dagger(t') \rangle = \delta (t - t')$, and $\langle \xi(t)\xi(t') \rangle = \gamma_m(2n+1)\delta (t - t')$. Here $n = 1/\exp (\hbar \omega_m/k_BT)$ is the mean thermal phonon number, where $k_B$ and $T$ denote Boltzmann constant and temperature. By linearizing operators around the steady state values ($\delta = a_{o} + \delta \delta$, $a$ is a system operator), we obtain the QLE for the fluctuation operators

$$ \hat{f}(t) = M f(t) + n(t), $$

where $f = (q,p,X,Y)^T$ is the vector of fluctuation operators, $n$ is corresponding noises and the matrix

$$ M = \left( \begin{array}{ccccc}
0 & \omega_m & 0 & 0 & 0 \\
-\omega_m & -\gamma_m & D X_s & -\sqrt{D} & 0 \\
0 & -\gamma_m & 0 & -\kappa_s & \Delta_s \\
0 & 0 & -\kappa_s & -\kappa_s & -\kappa_s \\
0 & 0 & 0 & 0 & 0 \\
\end{array} \right) $$

Here we introduce the cavity field quadratures $X = (a + a^\dagger)/\sqrt{2}$ and $Y = i(a^\dagger - a)/\sqrt{2}$. By carefully choosing the phase of driving light, we set $\text{Im} a_\delta = 0$, then $X_s = \sqrt{2} a_s$ and $Y_s = 0$. And $\Delta_s = \Delta - D q_s$, $\kappa_s = \kappa + R q_s$ the detuning and decay rate of cavity mode at steady state.

When stability condition is fulfilled, which can be derived by employing the Routh-Hurwitz criterion.
we can solve the stochastic differential equation for the steady-state correlation matrix (V)
\[
M \cdot V + V \cdot M^T = -I,
\]
where V is defined as \(V_{ij} = (f_i^* f_i + f_j^* f_j) / 2\), and \(I = \text{Diag} [\gamma_m (2n + 1) + (R/\sqrt{2\kappa})^2 X_s / 2, \kappa, \kappa]\). We represent V in the 2 × 2 block form,
\[
V = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix},
\]
then the entanglement of between optical field and mechanics oscillation can be quantified by the logarithmic negativity \(E_N\):
\[
E_N = \max[0, -\frac{1}{2} \ln 2(\Sigma - \sqrt{\Sigma^2 - 4 \det V})],
\]
where \(\Sigma = \det A + \det B - 2 \det C\).

**Results and Discussion.** To model the quantum entanglement between light field and motion of nanostring, we adapt the parameters close to recent experiments. The microdisk with \(Q_o = 4 \times 10^6\) is working at wavelength \(\lambda = 850\) nm, which have been demonstrated experimentally \[11\]. The nanostring is chosen with \(\omega_m = 15\) MHz, \(m = 2\) pg, and \(Q_m = 10^6\) at room temperature, which is achievable by current technology \[12\]. We set the coupling strength \(r = 2\kappa_1 / l_0\) with \(l_0 = 100\) nm, and \(g = 50\) MHz/mm. All calculations of \(E_N\) are performed under the stability condition.

In Fig. 2(a), we calculated the dependence of \(E_N\) on the cavity mode detuning \(\Delta_e\) and the coupling condition \(\kappa_1 / \kappa_0\) at low temperature (\(T = 0.05\) K). It can be seen that entanglement appears at blue detuning, and there exist a optimal coupling strength \(\kappa_1 / \kappa_0 \approx 0.3\) for entanglement. This can be understood as follows: From the Eq.(4), we see that the entanglement depends on effective opto-mechanics coupling strength \(\zeta = D \sqrt{2\kappa_0 \epsilon} E / (\Delta_e + \kappa_1 + \kappa_0)\) and \(\eta = R \cdot E / \sqrt{\kappa_1}\). If the stationary condition is fulfilled, we could say, the entanglement is enhanced by increasing \(\zeta\) and \(\eta\). Thus, there is a optimal coupling strength as a result of the trad-off between the energy dissipation and transferring introduced by the waveguide.

Since there are two different kinds of interactions (DC and RC) in our system, we should investigate the role of them in generating entanglement. As seen from Fig.3(a), we compares the \(E_N\) in three different conditions: (1) both DC and RC are existing, (2) only DC and (3) only RC. The results demonstrated that, the RC do not influence or generate the entanglement in the microdisk-nanostring system proposed in this paper. However, if we can increase the RC coupling strength \(r\), as shown in Fig.3(b), the entanglement appears. This means that the RC could also lead to entanglement, which would be significant in other systems which have strong RC coupling.

In Fig. 2(b), we study the temperature dependence of \(E_N\). Here, we choose \(Q_m = 10^6\) at room temperature, which have been demonstrated in experiments \[12\]. It can be seen that at the optimal coupling condition, with increasing the temperature (\(T\)), the \(E_N\) decreases and the working area is also shrunk. It is surprised that, the entanglement is robust against the temperature, even at high temperature, because when the environment temperature largely exceeds the energy of the one quantum, i.e. \(k_B T > \hbar \omega_m\), the quantum signature would be masked by the thermal noise. This counterintuitive phenomena can be understood as follow: The system’s interaction with the thermal bath is governed by the mechanical dissipation rate \(\gamma_m = \omega_m / Q_m\). When the interaction between the light and oscillator is much bigger than the rate of relaxation to thermal equilibrium, the quantum entanglement could be survived. If we assume \(\gamma_m = 0\) (\(Q_m = \infty\)), the quantum system is total isolated from
the thermal bath, and the quantum behavior could be preserved at any temperature.

To study the dependence of entanglement on $Q_m$, in Fig. 4(a), we further plot $E_N$ against $T$ with different $Q_m$, by assuming all parameters do not change with temperature. Benefitting from large $Q_m = 10^6$ of SiN nanostring in our system, the entanglement vanished at a very high critical temperature $T_C$. However, for lower $Q_m$, $T_C$ is decreased, which corresponds to former results that entanglement only survived in cryostat [4]. It is not difficult to find in Fig. 4(a) that, all curves are the same shape for different $Q_m$.

In order to get a better physical picture, we analyze Eq.(4) and Eq.(5) qualitatively. When $Q_m$ is very large, $\gamma_m/\omega_m = 1/Q_m \ll 1$, and $\gamma_m/k \ll 1$, we can approximately omit the $\gamma_m$ in matrix $M$ of Eq.(4). In this simplified model, from equation (5), we can obtain the dependence of solution of correlation matrix $V$ on the temperature $T$, which is only related to $I$, and depends on the parameter $U(Q_m, T) = \gamma_m (2n + 1) \approx 2(\exp(\omega_m/k T) - 1) Q_m^m$. At low temperature $T \approx 0$, $U(Q_m, 0) \approx 0$, we can see from Fig. 4(a) that $E_N$ is not related to the $Q_m$. At high temperature, i.e. $T \gg 1$, $U(Q_m, T) \approx k n T / Q_m$, then $E_N$ is a function with $T/Q_m$. In Fig. 4(b), the dependence of $T_C$ of entanglement on the $Q_m$ shows great agreement with the relationship $T/Q_m = \text{const}$ when $Q_m > 100$.

Conclusion.- A potential experimental system has been proposed to generate stationary room-temperature optomechanical entanglement. The system consists of a high-Q nanostring oscillator and a microdisk cavity, fabricated by SiN, which can be integrated on single chip. The role of both dispersive and reactive coupling in generating optomechanical entanglement is studied. We find that in the system proposed in this paper, the room-temperature entanglement is effectively obtained through the dispersive couplings. In particular, we find, in the high temperature ($T$) and high mechanical quality factor ($Q_m$) limit, the logarithmic entanglement depends only on the ratio $T/Q_m$. This means that improvements in material quality, and optimization in the structure design may enable both $Q_o$ and $Q_m$ greater, and lead to more efficient generation of stationary entanglement. This system can be applied as fundamental elements on photon chips [13, 20]. With a rapid development in material science and fabrication technology, the scheme proposed here is expected to be realized in the near future, and to explore the high-temperature entanglement of macroscopic objects [21].

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