Parameter-space-based robust control of event-triggered heterogeneous platoon

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Abstract
A parameter-space-based robust platooning controller for the event-triggered heterogeneous platoon is presented. The novel heterogeneous platoon is established with different longitudinal vehicle dynamic and driving preferences and the event-triggered communication scheme is designed to reduce the utilization of communication resources. To handle the stricter demand of the heterogeneous system and the disturbance caused by event-triggered communication scheme, the parameter space approach is adopted to optimize the gains of a robust controller. The feasible region where internal stability and \( \mathcal{L}_2 \) string stability are satisfied is then visualized in the parameters space, by extending the parameter space approach to the D-stability and frequency response magnitude specifications. Subsequently, the robust controller, which combines the feedforward loop and feedback loop, is developed by selecting the gain from the feasible solution area. The simulation results of the event-triggered heterogeneous platoon are presented and evaluated to verify the efficiency of this control algorithm. It is seen that the internal stability and string stability of the event-triggered heterogeneous platoon are guaranteed by this robust controller. Furthermore, 89.7\% of communication resources are saved by the application of event-triggered communication scheme, while the following performance is also favourable during the highway fuel economy cycle.

1 | INTRODUCTION

Limited highway capacity leads to traffic congestion with the continuously growing freight transport and private vehicles. As a result, considerable attention has been paid to developing the method to improve efficiency of traffic flow and road capacity significantly. As one of the most promising methods, intelligent transportation systems are expected to increase the roadway throughput and guarantee traffic safety by intelligent scheduling and control [1–3]. From the aspect of macro traffic flow, several traffic estimation models have been developed and used to provide the theoretical basis for traffic flow optimization [4, 5]. For example, linear regression can be used during regular headway traffic and logistic regression can be used for priority headway traffic [5]. Otherwise, the roadway throughput can also be improved by decreasing the inter-vehicle distance, which focuses on micro traffic [6–8]. Although the small following distance would be unsafe for the human driver, cooperative adaptive cruise control (CACC) possesses better capability of handling this smaller following distance. Compared with the traditional adaptive cruise control [9], CACC can gather more information about the surrounding environment and traffic via vehicular ad hoc networks [10–12]. The vehicle equipped with CACC can detect the following strategy according to the distance from its preceding vehicle as measured by the on-board sensors and the acceleration transmitted by wireless communication [13, 14]. This novel control architecture makes it possible to keep a smaller time gap. Due to this advantage, varieties of research works about CACC have been studied by both theoretical analysis and experiments [11, 12, 15, 16].

As a typical application of the networked control system (NCS), the foremost requirement of platoon control is keeping string stability, which represents the ability to attenuate the disturbance in the upstream direction [17, 18]. This

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characteristic guarantees comfort and safety when several vehicles are organized in the form of a platoon. As opposed to traditional stability definitions for dynamic systems concentrating on the evolution of system responses over time, string stability of a vehicular platoon mainly depends on the propagation of system states along the length of the platoon [19]. Consequently, string stability is interpreted as asymptotic stability of interconnected systems in [20, 21]. Various definitions of string stability have been proposed in the literature using time-domain, frequency-domain, and z-domain frameworks. The most formal approach is $\mathcal{L}_2$ string stability. It is commonly used in the analysis of the NCS and applied in platoon control [21–23]. In this approach, the string stability is defined by the amplification of signals upstream of the platoon, which can be quantified by the magnitude of transfer functions between the leading vehicle and its followers.

Recently, most works of literature focus on two fields, that is, vehicle parameters and communication. From the aspect of vehicle parameters, some early-designed platoon control systems assume that every vehicle has the same dynamic parameters and apply the homogeneous platoon for the theoretical analysis [10, 11, 24]. Nevertheless, it is impossible to guarantee that every vehicle in a platoon has the same parameters and even the same vehicle will have different dynamic performances during different operation conditions [13, 25, 26]. Consequently, a large amount of papers in the literature attempt to design a robust CACC controller for the heterogeneous platoon. The H-infinity method is applied in [27] to handle the influence of parameter uncertainty and vehicle dynamic constraint. Some optimization approaches, for example, adaptive optimal control and model predictive control are applied in [13, 28] to use the optimal feedback which guarantees string stability of the heterogeneous platoon. Different from the above two approaches, [29] adopts the parameter space approach to design the robust controller for the heterogeneous platoon and graphically represents the feasible solution area in the parameter space. However, another important feature of the heterogeneous platoon, that is, the initiative of drivers has not gotten enough attention. Although one objective of developing CACC is to improve traffic effectiveness, it should also consider the experience of drivers as each driver has his/her own driving preference [9, 30]. While considering the traffic throughput, a well-designed CACC controller should allow each driver to choose his/her own following strategy. During the real field experiment by PATH [31], different time headways were selected by drivers in the range of 0.6 to 1.1 s, which are given in Table 1. This becomes the motivation of this paper to design the robust CACC controller for the heterogeneous platoon with different time headway.

From the aspect of communication, the influence of the non-ideal communication condition, e.g., communication time delay, stochastic packet dropout is analysed in several researches [14, 15, 32]. It worth noting that most of these issues are caused by the extensive usage of wireless communication channels [33] as most platoon systems use the time-triggered communication scheme in which every vehicle transmits data periodically at all sampling instants and this leads to the excessive occupancy of the limited communication resource. Hence, it becomes an important issue to design a more reasonable communication scheme to efficiently utilize the limited communication resource which becomes another motivation of this paper.

To the best of the authors’ knowledge, a feasible solution to address this issue is applying the event-triggered communication scheme (ETCS). The ETCS has been widely applied to deal with control and filtering issues [34–36]. The core idea of the ETCS is that the information is transmitted to the neighbours only after receiving the event trigger instead of sending packages every period. The event triggers are generated based on changes in the system measurement and the predefined threshold. Different forms of ETCS were proposed during the last decades, for example, event-triggered sampling scheme (ETSS) [37, 38], self-triggered sampling scheme [39, 40], and discrete event-triggered scheme [34, 41]. The differences between these ETCS are summarized in [42]. The most commonly mentioned characteristic is minimum inter-event time (MIET) which prevents the ETCS from degrading to continuous communications. Reference [43] proposed mixed ETSS to ensure a large MIET and to guarantee a positive MIET even in the presence of disturbances. Dynamic ETSS was applied in [44] to ensure its non-negativity on average and to provide a large MIET. Since the packages are not transmitted as frequently as that with the time-triggered scheme to save limited communication resources, a significant difference always exists between the previously transmitted information and the real-time status information and this poses a great challenge to the control performance. Consequently, the stability analysis is an indispensable prerequisite step before the application of the ETCS [33, 45, 46].

Motivated by the aforementioned issues, this paper aims to design the robust CACC controller for the event-triggered heterogeneous platoon. The main contributions of this paper are summarized as:

1. Modelling the heterogeneous platoon with different longitudinal dynamics and driving preference, and applying the parameter space approach to analyse the $D$-stability with the influence of parametric uncertainty
2. Proposing a decentralized ETCS to efficiently utilize the limited communication resource. The MIET and steady-state errors are fully considered during the design process of the event-triggered strategy
3. Analysing the $\mathcal{L}_2$ string stability of the event-triggered scheme. The feasible region of $\mathcal{L}_2$ string stability is mapped in the parameter space considering parametric uncertainty and the disturbance caused by ETCS

The remainder of this paper is organized as follows. Section 2 illustrates the framework of the CACC controller and
proposes ETCS. Section 3 introduces the $L_2$ string stability for the NCS. Section 4 analyses the internal stability and string stability of the event-triggered system using the parameter space approach where the feasible region is mapped in the parameter space. In Section 5, several numerical simulations were run to test the robustness of the parameter-space designed controller in the event-triggered approach. The paper ends with conclusions in Section 6.

2 | PLATOON DYNAMICS

2.1 | Heterogeneous platoon

In Figure 1, a schematic of the heterogeneous platoon is illustrated with wireless communication between the vehicles. The predecessor-following topological structure is applied in this platoon. This means that each vehicle calculates the desired acceleration only based on the information of the preceding vehicle and self-state. As shown in the schematic, each vehicle detects the distance between the preceding vehicle accurately by its on-board sensor(s) and has access to the acceleration profile of the preceding vehicle by wireless communication. This framework combines the advantages of autonomous technology and connected technology well since the ego vehicle can get the distance without communication time delay and receive the acceleration profile of the preceding vehicle for feedforward control.

For the heterogeneous platoon proposed in Figure 1, it should be noted that: (i) every vehicle has different dynamic parameters, for example, time lag; (ii) each driver chooses their favourite following time headway.

2.2 | Cooperative adaptive cruise control controller

The core objective of the platoon control is to keep a suitable distance between vehicles. The design of this desired distance should fully consider the safety factor and traffic flow efficiency. Three distance strategies are commonly applied in most works of literature, i.e. constant headway [12, 47], constant time headway [21, 22] and nonlinear time headway [15, 48]. Because the constant time headway has a quite simple structure and adjusts the distance for varying velocity, this distancing strategy is applied in this paper to design the CACC controller. Thus, the desired following distance $d_i^*$ between vehicle $i$ and its preceding vehicle is expressed as

$$d_i^* = r + h_i \ast n_i.$$  \hfill (1)

where $n_i$ is the velocity of the vehicle $i$; $r$ is the desired standstill distance when the vehicle is stationary; $h_i$ is the desired time headway of the vehicle $i$. The time headway varies along the length of the platoons due to the different driving preferences of different drivers.

Then, the following error $e_i$ of the vehicle $i$ is expressed as

$$e_i = d_i - d_i^* = d_i - (r + h_i \ast n_i).$$  \hfill (2)

in which $d_i$ is the distance detected by the on-board sensor.

Remark 1. The control objective can be defined as guaranteeing $\lim_{t \to \infty} e_i (t) = 0$ for every single vehicle.

To simplify the CACC controller design process, the longitudinal vehicle dynamics is simplified as a delayed third-order state space with time lag, which is commonly used for theoretical analysis, and is given by

$$\begin{cases}
\dot{p} (t) = v (t) \\
\dot{v} (t) = a (t) \\
\dot{a} (t) = \frac{1}{\tau} u (t - \beta) - \frac{1}{\tau^2} a (t)
\end{cases}$$  \hfill (3)

in which, $p, v, a$ denote the position, the velocity, and the acceleration, respectively; $\tau$ is the time lag; $u$ expresses the desired acceleration; $\beta$ is signal transmission delay ($\beta = \beta_{sc} + \beta_{ca}$), in which $\beta_{sc}$ is the sensor-to-controller delay and $\beta_{ca}$ is the controller-to-actuator delay. Based on Equation (3), the
longitudinal dynamics in the Laplace domain can be described as

\[ G_i(s) = \frac{a_i(s)}{u_i(s)} = \frac{1}{(\tau_i s + 1)} e^{-\tilde{\tau}_i}. \] (4)

Due to the heterogeneous assumption, the longitudinal dynamic varies along the platoon appeared by different \( \tau_i \).

For the sake of following the state of the preceding vehicle and keeping the desire distance, the desired acceleration of the ego vehicle is determined by using the acceleration of the preceding vehicle as feedforward and applying the following error as feedback. Then the control strategy is described as

\[ u_i(t) = k_p e_i(t) + k_v \dot{e}_i(t) + k_a a_{i-1}(t - \sigma(t)). \] (5)
in which \( \sigma(t) \) is the random wireless communication time delay.

### 2.3 Event-triggered communication scheme

As shown in the Equation (5), the CACC controller requires the acceleration of the preceding vehicle every sampling period. Nevertheless, this occupies a variety of limited communication resources, and it may not be necessary to transmit the acceleration profile when the acceleration does not change sharply. Consequently, the ETCS is proposed in this paper to achieve platoon control with the limitation of communication bandwidth.

The distributed event-triggered communication framework of the heterogeneous platoon is proposed in Figure 1, where the sampling period of the communication system is \( \zeta \). A time-driven sampler is applied to sample the acceleration of the ego vehicle at the constant sampling period \( \zeta \). Different from the time-triggered communication scheme, whether the acceleration should be submitted to the succeeding vehicles at the sampling instant \( k \zeta \) is dependent on whether the event-triggering condition is satisfied. To introduce the event-triggering condition, several variables should be defined first.

**Definition 1.** Let \( i_{N+1} \zeta \) denote the sequence of the data transmission time instants of the vehicle \( i \), where \( N \) counts the number of transmissions starting from the initial time 0.

Then the acceleration profile that is transmitted to the succeeding vehicle is expressed as

\[ \hat{a}_i(t) = a_i(i_{N+1} \zeta) < t < i_{N+1} \zeta, \] (6)
in which \( \hat{a}_i \) is the acceleration used by the feedforward controller (FFC) of the succeeding vehicle. This means that if the succeeding vehicle does not receive the acceleration message, it will use the previous message to calculate the desired acceleration until the next package.

The difference between the actual acceleration and the previously transmitted acceleration is defined as

\[ \delta_i(t) = \hat{a}_i(t) - a_i(t). \] (7)

Furthermore, the next transmission instant \( i_{N+1} \zeta \) is determined by the Event-triggering mechanism (ETM) with the designed threshold as

\[ i_{N+1} \zeta = i_N \zeta + \min_{\rho \in \mathbb{N}} \left\{ \rho \zeta \left\| \delta_i \left( i_N \zeta + \rho \right) \right\| \gamma \left\| a_i \left( i_N \zeta \right) \right\|^2 + \varepsilon \right\}. \] (8)

where \( \gamma \) is a class \( \mathcal{K} \) function, and the scalar \( \varepsilon \geq 0 \).

**Remark 2.** The event-triggered condition shown in Equation (8) combines the state-dependent threshold \( \| \delta_i \left( i_N \zeta + \rho \right) \| \gamma \left\| a_i \left( i_N \zeta \right) \right\|^2 \), and state-independent threshold \( \varepsilon \) that ensures a larger MIET. Moreover, the application of \( \varepsilon \) also overcomes the limitation of small MIET during the steady-state response.

Therefore, the logic of the ETCS can be summarized as:

(i) The time-driven sampler samples the acceleration at the constant sampling period \( \zeta \).
(ii) The ETM determines whether the threshold is satisfied.
(iii) When the trigger condition is met, ETM sends a signal to activate the event trigger, and the sampled acceleration package is transmitted to the succeeding vehicle by wireless communication.
(iv) ETM updates the previously transmitted acceleration \( a_i(i_{N+1} \zeta) \) to detect the new threshold, which will be used until the next transmission instant \( i_{N+1} \zeta \).
(v) The zero-order holder is applied when the vehicle \( i \) does not receive an acceleration package from its preceding vehicle. This operation guarantees that the CACC controller has the acceleration of the preceding vehicle for FFC every sampling period.

Then control strategy with ETCS can be expressed as

\[ u_i(t) = k_p e_i(t) + k_v \dot{e}_i(t) + k_a a_{i-1}(t - \sigma(i_{N+1} \zeta)) \]
\[ = k_p e_i(t) + k_v \dot{e}_i(t) + k_a a_{i-1}(t - \sigma(i_{N+1} \zeta)) + k_a \delta_{i-1}(t - \sigma(i_{N+1} \zeta)) \] (9)

where \( \delta_{i-1}(t - \sigma(i_{N+1} \zeta)) \) is the difference between the actual acceleration and the previously transmitted acceleration.

**Assumption 1.** The wireless communication time delay is random in the real application, which is dependent on the amount of data in the communication channel. According to the (SAE) J2735 DSRC Message Set, the frequency of sending the acceleration profile is recommended to be set as
described as dropout. We assume that the acceleration profile can be sent and received within one communication period and no package dropout.

Then, the random wireless communication time delay can be described as $\delta \leq \xi$. To keep the effectiveness of the CACC controller for the random wireless communication delay, the maximum time delay is applied in the following analysis. Then, the control strategy expressed in Equation (9) is rewritten as Equation (10) and the overall control block diagram of vehicle $i$ is given in Figure 2.

$$a_i(t) = k_p \dot{e}_i(t) + k_v \delta(t) + k_a \ddot{\delta}_{i-1} (t - \xi) = k_p \dot{e}_i(t) + k_v \delta(t) + k_a \ddot{\delta}_{i-1} (t - \xi) + k_3 \delta_{i-1} (t - \xi).$$ (10)

In Figure 2, $D = e^{-\xi}$, $G_B = k_p + k_v$, $H_i = b_i s + 1$, $M = 1/s^2$, and $G_1$ is the feedforward gain which is designed in Section 4.

**Remark 3.** As is explained in Remark 2, the application of $\varepsilon$ is quite necessary for the steady-state response. Nevertheless, it leads to the oversized inter-event time when the system changes form unsteady-state to steady-state, which causes steady-state error. On the other hand, when the state $a_{i-1}(\xi_i^{-1})$ is relatively large, the state-dependent threshold may be too large to guarantee the following accuracy. Consequently, the maximum inter-event time (MAXIET) is proposed in the ETCS as

$$\varphi \leq 10$$ (11)

Figure 3 shows the acceleration which will be used in further simulation as an example. It shows that there is a steady-state error from 25 to 30 s and after 55 s if the MAXIET is not defined. The transmitted acceleration profile exhibits a big difference with the actual acceleration when the acceleration is relatively large as shown by the results between 40 and 50 s. After applying the MAXIET, the difference between the transmitted acceleration profile and the actual acceleration profile is bounded in a reasonable region and the steady-state error is also corrected.

**3 | $L_2$ STRING STABILITY**

As a typical application of the NCS, a well-designed platoon not only requires the following accuracy performance of every single vehicle, but also has a high demand for the overall performance, that is, string stability. The string stability represents how well the disturbance of the leading vehicle is attenuated along the platoon. To guarantee this performance, $L_2$ string stability is introduced in this paper.

The heterogeneous platoon shown in Figure 1 can be formulated as

$$\dot{x}_0 = f_1(x_0, u_c),$$
$$\dot{x}_i = f_i(x_i, x_{i-1}), i \in S_m,$$
$$\gamma_i = h(x_i), i \in S_m,$$

where $u_c$ is the external input of the platoon (the desired acceleration of the leader vehicle); $x_0 = (x_0^T, x_1^T, \ldots, x_m^T)^T$, $x_i = (x_i^1, x_i^2, \ldots, x_i^m)^T$, $y_i = a_i$, $S_m = \{i \in \mathbb{N} | 1 \leq i \leq m\}$, $m$ is the number of the vehicles in the platoon.

**Definition 2.** ($L_2$ string stability) Consider the interconnected system expressed in Equation (12). Let $x = (x_0^T, x_1^T, \ldots, x_m^T)^T$ be the lumped state vector and let $\bar{x} = (\bar{x}_0^T, \bar{x}_1^T, \ldots, \bar{x}_m^T)^T$ denote the constant equilibrium solution of Equation (12) for $u_c = 0$. The system is $L_2$ string stable if there exist class $K$ function $\alpha$ and $\beta$ such that for any initial state $x(0) \in \mathbb{R}^{3 \times (m+1)}$ and any $u_c \in L_2$

$$\|y_i(t)\|_{L_2} \leq \alpha\left(\|u_c\|_{L_2}\right) + \beta\left(\|x(0) - \bar{x}\|\right),$$

$$\forall i \in S_m \text{ and } \forall m \in \mathbb{N}.$$ (13)

If, in addition, with $x(0) = \bar{x}$ the following also holds

$$\|y_i(t)\|_{L_2} \leq \alpha\left(\|u_c\|_{L_2}\right),$$

$$\forall i \in S_m \text{ and } \forall m \in \mathbb{N}.$$ (14)
Lemma 1. (L₂ string stability) Consider the interconnected system in Equation (12) and assume that the unstable and marginally stable modes are unobservable, then it is strict L₂ string stability, if the following inequality holds

\[ \| \Gamma_i(j\omega) \|_{\mathcal{H}_\infty} \leq 1 \ \forall \ i \in S_w. \]  

(15)

where \( \Gamma_i = y_i(i)/y_{i-1}(i) \).

Proof: The output \( y_i \) can be formulated in the Laplace domain as

\[ y_i(i) = P_i(i) u_i(i). \]  

(16)

where \( P_i(i) = y_i(i)/u_i(i) \).

Adopting the L₂ signal norm for the output, one can obtain

\[ \left\| P_i(j\omega) \right\|_{\mathcal{L}_2} \leq \left\| P_i(j\omega) \right\|_{\mathcal{H}_\infty} \left\| u_i \right\|_{\mathcal{L}_2}. \]

(17)

The existence of the max\( \in S_w \left\| P_i(j\omega) \right\|_{\mathcal{H}_\infty} \) for all \( m \in \mathbb{N} \) becomes the sufficient and necessary condition. Hence, definition 2 can be transferred to: the interconnected system in Equation (12) is L₂ string stable if and only if \( \sup_{i \in S_w} \left\| P_i(j\omega) \right\|_{\mathcal{H}_\infty} \) exists. The class K function \( \alpha \) can be selected as

\[ \alpha \left( \left\| u_i \right\|_{\mathcal{L}_2} \right) = \left( \sup_{i \in S_w} \left\| P_i(j\omega) \right\|_{\mathcal{H}_\infty} \right) \left\| u_i \right\|_{\mathcal{L}_2}. \]  

(18)

As \( P_i(s) \) can be expressed by

\[ P_i(s) = \left( \prod_{k=1}^{i} \Gamma_k(s) \right) P_0(s). \]  

(19)

The sub multiplicative property results in

\[ \left\| P_i(j\omega) \right\|_{\mathcal{H}_\infty} \leq \left( \prod_{k=1}^{i} \left\| \Gamma_k(j\omega) \right\|_{\mathcal{H}_\infty} \right) \left\| P_0(j\omega) \right\|_{\mathcal{H}_\infty}. \]  

(20)

Consequently, the necessary and sufficient condition of L₂ string stability can be expressed as: \( \left\| P_i(j\omega) \right\|_{\mathcal{H}_\infty} \) exists and \( \prod_{k=1}^{i} \left\| \Gamma_k(j\omega) \right\|_{\mathcal{H}_\infty} < 1 \) because \( P_0(s) = \frac{1}{\tau_0 + 1} \), so \( \left\| P_i(j\omega) \right\|_{\mathcal{H}_\infty} \) always exists.

For the heterogeneous assumption, the value of \( \left\| \Gamma_i(j\omega) \right\|_{\mathcal{H}_\infty} \) directly depends on \( i \). And one sufficient condition can be obtained as \( \left\| \Gamma_i(j\omega) \right\|_{\mathcal{H}_\infty} \leq 1 \ \forall \ i \in \mathbb{N} \setminus \{1\} \).

### 4 ROBUST CACC CONTROLLER DESIGN

For the heterogeneous platoon, each vehicle has different dynamic performance and desired time headway. It is complicated and a source-waste to design different control parameters for each vehicle. In this paper, we design the robust CACC controller for every possible vehicle dynamic and time headway.

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**FIGURE 4** The equivalent model of event-triggered heterogeneous platoon

The overall model of the heterogeneous platoon with different time headway can be transferred to one vehicle with parameter uncertainty and varying time headway for further analysis. The control block is proposed in Figure 4, where

\[ G = \frac{1}{(\tau s + 1)} e^{-\beta t}, \quad H = bs + 1. \]

And \( \tau, b \) are uncertain and bounded, which are expressed as

\[ \tau \in \left[ \min \tau_m, \max \tau_i \right], \quad b \in \left[ \min b_i, \max b_i \right]. \]

### 4.1 Internal stability

As is mentioned in Section 2, the most important objective of the CACC control is to keep \( \lim_{t \rightarrow \infty} e_i(t) = 0, \ \forall \ i \in S_w \). This requirement can be reformulated as keeping the following error \( e(t) \) of the system described in Figure 4 converging to zero under the influence of the parametric uncertainty. This objective can be achieved by designing the characteristic of the transfer function \( J \).

\[ J(s) = \frac{e(s)}{a_{in}(s)} = \frac{M - G_H D G_H M}{1 + G_H D G_H H}. \]

(21)

**Remark 4.** To keep every vehicle following the preceding vehicle with the small following error, the transfer function \( J \) should meet the following two requirements:

(i) Choosing suitable feedforward gain to keep the numerator close to zero
(ii) Designing the feedback gain to make the roots of the denominator (characteristic polynomial) located in the left half s-plane

The feedforward loop gain is designed to reduce the model matching error.

\[ G_f = \frac{1}{G_H D} = \frac{\tau s + 1}{bs + 1} e^{(\beta + \varphi) t}. \]

(22)
As the time lag and time headway of each vehicle is unknown in the design process and they vary between vehicles, we attempt to use the same feedforward loop gain for each vehicle to improve the flexibility of the CACC controller.

\[ G_{ff} = \frac{\tau_0 r + 1}{h_0 r + 1}, \]

where \( \tau_0 = (\min \tau) + \frac{1}{2} \) and \( h_0 \) is the most popular time headway as \( h_0 = 0.6 \) s.

After ignoring the influence of time delay [29], the feedforward gain is selected as

\[ G_{ff} = \frac{\tau_0 r + 1}{h_0 r + 1}. \]

To further control the convergence performance, \( D \)-stability is proposed in this paper, and the left half s-plane in the condition (ii) of Remark 4 is replaced by the region \( D \) for \( D \)-stability.

The specified region \( D \) for the pole of the characteristic polynomial is surrounded by its boundary \( \partial D \), which consists of one or more contours described by [49].

\[ \partial D := \{ f s = \sigma (\alpha) + j \omega (\alpha), \alpha \in [\alpha^- , \alpha^+] \}. \]

The boundary \( \partial D \) shown in the Figure 5 can be divided into two types: the real root boundary (RRB) that crosses region \( D \) on the real axis at \( s = \sigma \) and the complex root boundary (CRB) that crosses region \( D \) at \( s = \sigma \pm j \omega \). More specifically, the boundary \( \partial \hat{D}_1 \) corresponds to the desired settling time and the boundary \( \partial \hat{D}_2 \) corresponds to the minimum value of damping.

**Definition 3.** The set \( q \) denotes the plant parametric uncertainty and the control parameter, \( q^* \) is the parameter of interest, \( \tilde{q}^* \) is the set of parametric uncertainty. \( \Omega \) denotes the operating domain, \( p(s, q) \) is the closed-loop characteristic polynomial.

\[ q = [\tau, b, k_p, k_v]. \]

\[ q^* = [k_p, k_v]. \]

\[ Q = \{ q \mid q \in [q^-, q^+] \}. \]

\[ \tilde{q}^* = [\tau, b]. \]

The parameter space approach is applied to map the region \( D \) in the \( q \)-plane. As the roots of \( p(s, q) \) are continuous in \( q \), that is, it is impossible to step out of the region \( D \) without crossing the boundary \( \partial D \), the boundary-crossing theorem for pole region \( D \) is defined as

**Lemma 2.** Boundary-crossing theorem [50]

The family of polynomials \( P(s, \Omega) \) guarantees robust \( D \)-stability, if and only if:

(i) There is a \( D \)-stable polynomial \( p(s, q) \in P(s, \Omega) \)

(ii) For points on the \( D \) stability boundary with \( \sigma (\alpha) \pm j \omega (\alpha) \notin \text{Roots}[P(s, \Omega)] \) for all \( \alpha \in [\alpha^- , \alpha^+] \)

\[ \text{Roots}[P(s, \Omega)] \text{ denotes the set of all roots of all } p(s, q) \text{ for all } q \in Q. \]

Hence, the following two hypersurfaces in the \( q \) domain can represent the boundary surfaces around a nominal point \( q \).

\[ Q_{\text{CRB}} (\alpha) := \{ q \mid p (\sigma (\alpha) + j \omega (\alpha), q) = 0, \alpha \in [\alpha^- , \alpha^+] \}. \]

\[ \partial Q_{\text{RRB}} := \{ q \mid p (\sigma_0, q) = 0 \} \]

When the region \( D \) is determined as: no roots can be closer than \(-0.1\pi\) in real part, which corresponds to \( \hat{D}_1 \) in Figure 5; the damping ratio should be smaller than \( \zeta = 0.707 \) which is expressed by \( \theta = 45^\circ \) as \( \hat{D}_2 \). The extreme cases defined by the vertices of the uncertainty area are chosen to reflect the influence of parametric uncertainty. The uncertainty range used in the paper are given in Table 2.

**Table 2.** The range of parameter uncertainty

| Parameters | Uncertainty range       |
|------------|-------------------------|
| \( \tau \)  | [0.2 s, 0.5 s]          |
| \( b \)     | [0.6 s, 1.1 s]          |

Figure 6 gives the boundary \( \partial D \) in the \( q \)-plane for the four extreme cases

\[ \tilde{q}^* = \{ [0.2, 0.6], [0.2, 1.1], [0.5, 0.6], [0.5, 1.1] \}. \]

The solid lines respect the RRB and CRB of \( \hat{D}_1 \), respectively. The dotted lines denote the CRB of \( \hat{D}_2 \) and the colored region
in the small zoomed picture in Figure 6 is the feasible region where D-stability is guaranteed for all possible time lag \( \tau \) and time headway \( b \).

4.2 String stability

Based on the CACC control strategy Equation (10) and the control block proposed in Figure 4, the desired acceleration is decided by

\[
(1 + HM_G) a_{out} = (DG_G + MG_G) a_{in} + DG_G \delta.
\]

and it can be proven that the transmitted error \( \delta \) caused by the event triggering is bounded by Equation (32), which is proved in Appendix A.

\[
\|\delta(t)\| \leq \frac{(y + \sqrt{\gamma})}{1 - \gamma} \|a_{in}(t)\| + \sqrt{\frac{\varepsilon}{1 - \gamma}} \tag{32}
\]

Applying the biggest transmitted error into Equation (31), the transfer function from \( a_{in} \) to \( a_{out} \) is formulated as

\[
\Gamma(s) = \frac{a_{out}(s)}{a_{in}(s)} = \frac{(1 + \varphi) D(s) G_G(s) G(s) + M(s) G_B(s) G(s)}{1 + H(s) M(s) G_B(s) G(s)} \tag{33}
\]

where \( \varphi = \frac{y + \sqrt{\gamma}}{1 - \gamma} \).

Remark 5. It is proven in many works of literature that the larger time headway is more likely to guarantee string stability. As a consequence, we only need to analyse the string stability with the smallest time headway and set \( H(s) = (\min_{i \in S_y} h_i) s + 1 \).

Remark 6. As the vehicle dynamics \( G(s) \) also contains non-parameter uncertainty, the feedback perturbation is applied to describe this non-parametric uncertainty as shown in Fig. 7.

\[
G(s) = \frac{G_0(s)}{1 + W_\Delta(s) \Delta(s)}, \tag{34}
\]

where \( G_0(s) = \frac{1}{(\tau_1 + 1)} e^{-\beta t} \), \( \Delta(s) \) denotes the multiplicative perturbation which satisfies \( \|\Delta(s)\|_\infty < 1 \). The multiplicative uncertainty bound \( W_\Delta(s) \) is determined by the boundary of parameter uncertainty and assumed that the plant uncertainty is less than \( |\| W_\Delta(j\omega) \| | \).

Thus, the transfer function becomes

\[
\Gamma(s) = \frac{(1 + \varphi) D(s) G_G(s) G(s) + M(s) G_B(s) G(s)}{1 + W_\Delta(s) \Delta(s) + H(s) M(s) G_B(s) G(s)} \tag{35}
\]

The frequency response magnitude specifications are applied to analyse the string stability. One sufficient condition to satisfy the string stability can be summarized as

\[
\left\| \frac{(1 + \varphi) D(s) G_G(s) G(s) + M(s) G_B(s) G(s)}{1 + W_\Delta(s) \Delta(s) + H(s) M(s) G_B(s) G(s)} \right\|_{H_\infty} \leq 1. \tag{36}
\]

To further simplify Equation (36) and eliminate \( \Delta(s) \), Equation (36) is rewritten as Equation (37), and the process is explained in Appendix B.

\[
\left\| \frac{(1 + \varphi) D(s) G_G(s) G(s) + M(s) G_B(s) G(s)}{1 + H(s) M(s) G_B(s) G(s)} - |W_\Delta(j\omega)| \right\|_{H_\infty} \leq 1 \forall \omega \tag{37}
\]

While \( G_0(s) \) is selected as \( \frac{1}{(\tau_1 + 1)} e^{-\beta t} \), the boundary of parameter uncertainty can be designed as

\[
|W_\Delta(j\omega)| = \max_{i \in S_y} \left\{ \left| \frac{\tau_1 - \tau_1}{(\tau_1, \tau_1 + 1)} \right| \right\} i \in S_y, \forall \omega \tag{38}
\]

Figure 8 depicts the string stability region in the \( q \)-plane when ETCS is designed as

\[
q_{\gamma - 1} = q_{\gamma - 1} + \min_{i \in \mathbb{N}} \left| \phi \right| \left\| \delta_{\gamma - 1} \left( \left| q_{\gamma - 1} + \phi \right| \right\| \right\| \geq 0.01 \star \left\| \delta_{\gamma - 1} \left( \left| q_{\gamma - 1} + \phi \right| \right\| \right\| + 0.01 \tag{39}
\]

The grey region is where the string stability is guaranteed for the platoon with ETCS. When the feedback gains are chosen...
from the colourful area, the string stability is not satisfied during several frequencies, which are reflected in different colours.

5 NUMERICAL SIMULATION

In this section, the numerical experiments are simulated to verify the effectiveness of the proposed robust CACC algorithm for the event-triggered heterogeneous platoon. The platoon system in Figure 1 is used as an example of a six-vehicle platoon with radar and wireless communication.

The feedback gains should be selected from the intersection of the feasible region in Figure 8, which is the intersection of region $D$ in Figure 6 and the string stability region represented by the grey region in Figure 8. Then, the feedback gain is selected as $k_p = 1.6$, $k_v = 1.7$ to guarantee the $D$-stability of each vehicle and keep the string stability of the whole platoon, which is marked in Figure 8. Afterwards, the string stability of the event-triggered heterogeneous platoon is verified by gridding the time lag $\tau$ and time headway $h$. Figure 9 presents the magnitude of $\Gamma(s)$ for 24 different cases. It is obvious that the string stability is ensured in all frequency ranges as the magnitude is below unity.

Furthermore, the heterogeneous platoon is modelled by Simulink and the time lag and the desired time headway are chosen as $\tau = [0.35, 0.44, 0.27, 0.50, 0.37, 0.23]$ and $h = [0.6, 0.9, 0.7, 1.1, 0.8]$, respectively. It is assumed that there is no following error at the initial moment and that all vehicles have the same velocity $v_i = 10 \text{ m/s}$. The external input of the leader vehicle $u_1$ is given as the red line in Figure 3.

Figures 10 and 11 display the simulation results of the event-triggered heterogeneous platoon and time-triggered heterogeneous platoon, respectively. According to the comparison of these results, the most obvious difference between these two platoons is that the acceleration and the following error of the platoon with ETCS are not as smooth as that of no ETCS. This is because the acceleration profile transmitted by ETCS is discrete and stepwise, which directly leads to the steps in the desired acceleration. Although the acceleration profiles are not very smooth, each vehicle in the platoon is following the desired acceleration profile and the velocity profile of the preceding vehicle accurately. The string stability of the heterogeneous platoon is also guaranteed, which is shown by the phenomenon that the fluctuation of acceleration is attenuated along the platoon and the peak values of acceleration given in Table 3 also decrease.

As is shown in Figure 10(d), the controller of vehicle 1 only sends 87 acceleration messages during the whole 60 s. The MIET is 0.3 s and the average inter-event time is 0.6897 s, while the vehicles in the time-triggered platoon send a message per 0.1 s. It means the event-triggered platoon only uses 14.5% communication resource compared with the time triggered case. As for the following error shown in Figure 10(c), the following error of each vehicle converges to zero quickly at the end of the driving scenario, despite the convergence speeds being different. It well seen that the robust CACC controller guarantees the following objective for vehicles with different time lag and desired time headway while it must be acknowledged that ETCS has a considerable influence on the following accuracy. Figure 12 shows the maximum following error during the 60 s drive scenario. FE $i$ responses show the following error of vehicle $i$. It is evident that the following error of the event-triggered platoon is larger than that of the time-triggered platoon and the following error of the event-triggered platoon reaches the peak at 26.09 cm of FE 1.

To sum up, the robust CACC controller is suitable for vehicles with different time lag and desired time headway. The accurate vehicle following objective and string stability are satisfied, in spite of the influence of the parametric uncertainty and the disturbance caused by ETCS. ETCS saves communication resources while sacrificing part of the following accuracy.

To further verify the rationality and effectiveness of ETCS, the highway fuel economy cycle (HWFET) driving cycle is simulated. It is recognized that highway driving is the most suitable condition to apply V2X technology and the HWFET driving cycle reflects the real-world scenario. Figure 13 proposes the inter-event time of vehicle_1 during the HWFET driving cycle. There are only 827 packages transmitted during the entire 800s cycle, in the sense that 89.7% of communication resources are
FIGURE 10  The results of the event-triggered platoon. (a) The acceleration of the event-triggered platoon. (b) The velocity of the event-triggered platoon. (c) The following error of the event-triggered platoon. (d) The inter-event time of the event-triggered platoon

FIGURE 11  The results of the time-triggered platoon

TABLE 3  The peak value of the acceleration of the event-triggered heterogeneous platoon

|    | 0 | 1 | 2 | 3 | 4 | 5 |
|----|---|---|---|---|---|---|
| \(a_i\) (m s\(^{-1}\)) | Max | 1.98 | 1.96 | 1.94 | 1.93 | 1.90 | 1.89 |
|        | Min | -2.97 | -2.95 | -2.92 | -2.89 | -2.85 | -2.83 |

FIGURE 12  The maximum following error of event-triggered and time-triggered heterogeneous platoon

FIGURE 13  The inter-event time of the event-triggered heterogeneous platoon
saved by the application of ETCS. Otherwise, 88.9% of these packages are transmitted after reaching the MAXIET. This is strong proof that the time-triggered platoon sends a lot of unnecessary information.

Figure 14 shows the maximum value and the root mean square value (RMS) of the following error during the HWFET driving cycle for the event-triggered heterogeneous platoon. The largest following error occurs in vehicle 1 which has the smallest desired time headway. On the contrary, vehicle 4 behaves with the best following accuracy. This phenomenon shows that the larger time headway may lead to better vehicle following performance. Overall, the following performance of each vehicle in the event-triggered heterogeneous platoon is satisfactory, as the biggest following error is limited to 12.31 cm and the RMS error values ranges from 1.045 to 2.206 cm. This demonstrates that the heterogeneous platoon has superior following effect with the robust CACC controller even in the complex driving scenario. It can be summarized that although the event-triggered strategy makes the following performance worse to some extent, it still keeps the adverse effects within a reasonable range.

Please note that system safety and risk of collision within the platoon was not specifically addressed in this paper for the sake of brevity. There are both positive and negative acceleration limits in CACC platooning to ensure driving comfort as platooning is not an extreme manoeuvre but rather a continuous and routine driving operation. Full braking is only needed in an emergency manoeuvre in which case the AEB (automatic emergency brake) system in each vehicle would take over due to the deceleration limits in CACC not being sufficient. That is why full braking is not treated in this paper. Time headway values of 0.6 s to more than 1 s were used in real life applications (like the field experiments in the PATH program) without any collision safety problems.

6 CONCLUSION

In this paper, a robust CACC control method was adopted for the heterogeneous platoon with different longitudinal dynamics and driving preferences. The ETCS method was developed to effectively utilize the limited communication resource. The mixed ETSS, combining the state-dependent and state-independent thresholds, is applied to ensure larger MIET and avoid Zeno behaviour. To reduce the negative effect of the heterogeneous platoon and ETCS, the internal stability and $L^2$ string stability were analysed by applying the parameter space approach. Based on the boundary-crossing theorem, the feasible region where the $D$-stability and $L^2$ string stability are guaranteed were mapped in the $q^*$-plane.

The simulation results of a six-vehicle heterogeneous platoon with ETCS were presented to evaluate the effectiveness of the designed robust CACC controller. The simulation results show that the acceleration profile is not smooth enough and is affected by the disturbance of ETCS. However, the string stability of the heterogeneous platoon with ETCS is still guaranteed as the fluctuation of acceleration is attenuated along the platoon and the following error quickly converged to zero. To further verify the rationality and utility of the ETCS, the simulation results of the time-triggered heterogeneous platoon were presented for comparison. The comparison demonstrated that a very good trade-off between platoon control performance and communication resource utilization was achieved by the ETCS approach as the event-triggered platooning used only 10.3% of the communication resource while the maximum following error during the HWFET driving circle was limited to 12.31 cm.

Future work should pay more attention to the influence of non-ideal communication conditions, that is, cyber-attack, topological communication structure etc. We will also design the bottom-layer control for the heterogeneous platoon without inverse model and design the HIL test.

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REFERENCES

1. Zheng, Y., et al.: Research on cooperative vehicle intersection control scheme without using traffic lights under the connected vehicles environment. Adv. Mech. Eng. 9(8), 1–13 (2017)
2. Yao, W., et al.: Best response game of traffic on road network of non-signalized intersections. Physica A 490, 386–401 (2018)
3. Scarinci, R., et al.: Analysis of traffic performance of a merging assistant strategy using cooperative vehicles. IEEE Trans. Intell. Transp. Syst. 16(4), 2094–2103 (2015)
4. Wang, W., et al.: The prediction of freeway traffic conditions for logistics systems. IEEE Access 7, 138056–138061 (2019)
5. Apronti, D., et al.: Estimating traffic volume on Wyoming low volume roads using linear and logistic regression methods. J. Traffic Transp. Eng. 3(6), 493–506 (2016)
6. van Arem, B., et al.: The impact of cooperative adaptive cruise control on traffic-flow characteristics. IEEE Trans. Intell. Transp. Syst. 7(4), 429–436 (2006)
7. Santanakrishnan, K., Rajamani, R.: On spacing policies for highway vehicle automation. IEEE Trans. Intell. Transp. Syst. 4(1), 198–204 (2003)
8. Liu, H., et al.: Impact of cooperative adaptive cruise control on multilane freeway merge capacity. J. Intel. Transp. Syst. 22(3), 263–275 (2018)
9. Bolduc, A.P., et al.: Multimodel approach to personalized autonomous adaptive cruise control. IEEE Trans. Intell. Veh. 4(2), 321–330 (2019)
10. Kianfar, R., et al.: A control matching model predictive control approach to string stable vehicle platooning. Control Eng. Pract. 45, 163–175 (2015)
11. Oncu, S., et al.: Cooperative adaptive cruise control: Network-aware analysis of string stability. IEEE Trans. Intell. Transp. Syst. 15(4), 1527–1537 (2014)
12. Li, S.E., et al.: Distributed platoon control under topologies with complex eigenvalues: Stability analysis and controller synthesis. IEEE Trans. Control Syst. Technol. 27(1), 206–220 (2017)
13. Zha, Y., et al.: Adaptive optimal control of heterogeneous CACC system with uncertain dynamics. IEEE Trans. Control Syst. Technol. 27(4), 1772–1779 (2018)
14. Ge, J.L., Otros, G.: Optimal control of connected vehicle systems with communication delay and driver reaction time. IEEE Trans. Intell. Transp. Syst. 18(8), 2056–2070 (2017)
15. Qin, W.B., et al.: Stability and frequency response under stochastic communication delays with applications to connected control design. IEEE Trans. Intell. Transp. Syst. 18(2), 388–403 (2017)
16. Milanes, V., et al.: Cooperative adaptive cruise control in real traffic situations. IEEE Trans. Intell. Transp. Syst. 15(1), 296–305 (2014)
17. Naas, G.J., et al.: String stable cascaded control and experimental validation: A frequency-domain approach. IEEE Trans. Veh. Technol. 59(9), 4268–4279 (2010)
18. Rodionov, G.: An adaptive spacing policy guaranteeing string stability in multi-brand Ad Hoc platoons. IEEE Trans. Intell. Transp. Syst. 19(6), 1902–1912 (2018)
19. Firooznia, A., et al.: Co-design of controller and communication topology for vehicular platooning. IEEE Trans. Intell. Transp. Syst. 18(10), 2728–2739 (2017)
20. Yadlapalli, S.K., et al.: Information flow and its relation to stability of the motion of vehicles in a rigid formation. IEEE Trans. Autom. Control 51(8), 1315–1319 (2006)
21. Bloq, J., et al.: Lp string stability of cascaded systems: Application to vehi- cle platooning. IEEE Trans. Control Syst. Technol. 22(2), 786–793 (2014)
22. Bloq, J., et al.: Controller synthesis for string stability of vehicle platoons. IEEE Trans. Intell. Transp. Syst. 15(2), 835–846 (2014)
23. Wang, X., Lemmon, M.D.: Event-triggering in distributed networked control systems. IEEE Trans. Autom. Control 56(3), 586–601 (2011)
24. Bloq, J., et al.: Graceful degradation of cooperative adaptive cruise control. IEEE Trans. Intell. Transp. Syst. 16(1), 488–497 (2015)
25. Zheng, Y., et al.: Distributed model predictive control for heterogeneous vehicle platoons under unidirectional topologies. IEEE Trans. Control Syst. Technol. 25(3), 899–910 (2017)
26. Zhai, C., et al.: A switched control strategy of heterogeneous vehicle platoon for multiple objectives with state constraints. IEEE Trans. Intell. Transp. Syst. 20(5), 1883–1896 (2018)
27. Li, S.E., et al.: Robust longitudinal control of multi-vehicle systems—A distributed H-infinity method. IEEE Trans. Intell. Transp. Syst. 19(9), 2779–2788 (2018)
28. Chen, N., et al.: A robust longitudinal control strategy of platoons under model uncertainties and time delays. J. Adv. Transp. 2018, 1–13 (2018)
29. Emirler, M.T., et al.: Design and evaluation of robust cooperative adaptive cruise control systems in parameter space. Int. J. Automot. Technol. 19(2), 359 (2018)
30. de Gelder, E., et al.: Towards personalised automated driving prediction of preferred ACC behaviour based on manual driving. In: IEEE Intelligent Vehicles Symposium, Gothenburg, Sweden, June 2016 pp. 1211–1216
31. Shladover, S.E., et al.: Impacts of cooperative adaptive cruise control on freeway traffic flow. In: Transportation Research Board Annual Meeting, Washington, USA, January 2012 pp. 1–17
32. Bahreini, M., et al.: Robust finite-time stochastic stabilization and fault-tolerant control for uncertain networked control systems considering random delays and probabilistic actuator faults. Trans. Inst. Meas. Control 41(12), 3550–3561 (2019)
33. Liu, Z., et al.: Cooperative platoon control of heterogeneous vehicles under a novel event-triggered communication strategy. IEEE Access 7, 41172–41182 (2019)
34. Zhang, X.M., Han, Q.L.: A decentralized event-triggered dissipative control scheme for systems with multiple sensors to sample the system outputs. IEEE Trans. Cybern. 46(12), 2745–2757 (2016)
35. Wen, S., et al.: Event-triggering load frequency control for multiarea power systems with communication delays. IEEE Trans. Ind. Electron. 63(2), 1308–1317 (2016)
36. Peng, C., Han, Q.L.: A novel event-triggered transmission scheme and L2 control co-design for sampled-data control systems. IEEE Trans. Autom. Control 58(10), 2620–2626 (2013)
37. Mazo, M., Tabuada, P.: Special issue technical notes and correspondence: Decentralized event-triggered control over wireless sensor/actuator networks. IEEE Trans. Autom. Control 56(10), 2456–2461 (2011)
38. Tabuada, P.: Event-triggered real-time scheduling of controlling control tasks. IEEE Trans. Autom. Control 52(9), 1680–1685 (2007)
39. Mazo, M., et al.: An iss self-triggered implementation of linear controllers. Automatica 46(8), 1310–1314 (2010)
40. Wang, X., Lemmon, M.D.: Self-triggering under state-independent disturbances. IEEE Trans. Autom. Control 55(6), 1494–1500 (2010)
41. Yue, D., et al.: A delay system method for designing event-triggered controllers of networked control systems. IEEE Trans. Autom. Control 58(2), 475–481 (2013)
42. Peng, C., Li, F.: A survey on recent advances in event-triggered communication and control. Inf. Sci. 457–458, 113–125 (2018)
43. Borgers, D.P., Heemels, W.P.M.H.: Event-separation properties of event-triggered control systems. IEEE Trans. Autom. Control 59(10), 2644–2656 (2014)
44. Dolk, V.S., et al.: Output-based and decentralized dynamic event-triggered control with guaranteed L∞-gain performance and zero-freeness. IEEE Trans. Autom. Control 62(1), 34–49 (2017)
45. Wen, S., et al.: Event-triggered cooperative control of vehicle platoons in vehicular Ad Hoc networks. Inf. Sci. 459, 341–353 (2018)
46. Li, Z., et al.: String stability analysis for vehicle platooning under unreliable communication links with event-triggered strategy. IEEE Trans. Veh. Technol. 68(3), 2152–2164 (2019)
47. Zheng, Y., et al.: Stability and scalability of homogeneous vehicular platoon: Study on the influence of information flow topologies. IEEE Trans. Intell. Transp. Syst. 17(1), 14–26 (2016)
48. Gao, W., et al.: Data-driven adaptive optimal control of connected vehicles. IEEE Trans. Intell. Transp. Syst. 18(5), 1122–1133 (2017)
49. Guvene, L., et al.: Control of Mechatronic Systems, 1st ed. London: IET Press (2017)
50. Ackermann, J., et al.: Robust Control the Parameter Space Approach, 1st ed. London: Springer Press (2002)

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**APPENDIX A**

The next acceleration message of the preceding vehicle is transmitted once the triggered condition is met, one can obtain from Equation (8) that

\[ \| \delta_{i-1} (t) \|^2 \leq \gamma \| \hat{a}_{i-1} (t) \|^2 + \varepsilon. \]  

(A.1)

Since \( \hat{a}_{i-1} (t) = a_{i-1} (t) + \delta_{i-1} (t) \), one can have

\[ \| \hat{a}_{i-1} (t) \|^2 \leq \| a_{i-1} (t) + \delta_{i-1} (t) \|^2 \]

(A.2)

\[ \leq (\| a_{i-1} (t) \| + \| \delta_{i-1} (t) \|)^2. \]

Combining Equations (A.1) and (A.2), one can get

\[ (1 - \gamma) \| \delta_{i-1} (t) \|^2 - 2 \gamma \| \delta_{i-1} (t) \| \| a_{i-1} (t) \| - \gamma \| \hat{a}_{i-1} (t) \|^2 - \varepsilon \leq 0. \]

(A.3)

Solving the above quadratic inequality about \( \| \delta_{i-1} (t) \| \) and using \( 0 < \gamma < 1 \), one obtains

\[ \| \delta_{i-1} (t) \| \leq \frac{\gamma \| a_{i-1} (t) \| + \sqrt{\gamma^2 \| a_{i-1} (t) \|^2 + (1 - \gamma) \varepsilon}}{1 - \gamma}. \]

(A.4)

\[ \leq \frac{(\gamma + \sqrt{\gamma})}{1 - \gamma} \| a_{i-1} (t) \| + \sqrt{\frac{\varepsilon}{1 - \gamma}}. \]

For the system demonstrated in Figure 4, Equation (A.4) is rewritten as

\[ \| \delta (t) \| \leq \frac{(\gamma + \sqrt{\gamma})}{1 - \gamma} \| a_{i-1} (t) \| + \sqrt{\frac{\varepsilon}{1 - \gamma}}. \]

(A.5)

**APPENDIX B**

Firstly, Equation (36) can be rewritten as

\[ \left( 1 + \varepsilon \right) D (j \omega) G_{i} (j \omega) G_{i} (j \omega) + M (j \omega) G_{H} (j \omega) G_{N} (j \omega) \]

\[ \leq 1 \forall \omega. \]  

(B.1)

Using the sub multiplicative property, we have

\[ 1 + G (j \omega) = \left| 1 + G (j \omega) + W_{\Delta} (j \omega) \Delta (j \omega) - W_{\Delta} (j \omega) \Delta (j \omega) \right| \]

\[ \leq 1 + G (j \omega) + W_{\Delta} (j \omega) \Delta (j \omega) + \left| W_{\Delta} (j \omega) \Delta (j \omega) \right|. \]

(B.2)

Equation (B.2) leads to

\[ \left| 1 + G (j \omega) + W_{\Delta} (j \omega) \Delta (j \omega) \right| \geq \left| 1 + G (j \omega) - \left| W_{\Delta} (j \omega) \Delta (j \omega) \right| \right|. \]

(B.3)

where \( G (j \omega) = H (j \omega) M (j \omega) G_{H} (j \omega) G_{N} (j \omega) \).

As \( \left\| \Delta (j \omega) \right\|_{\infty} < 1 \), one can obtain

\[ \left| W_{\Delta} (j \omega) \Delta (j \omega) \right| \leq \left| W_{\Delta} (j \omega) \right| \times \left| \Delta (j \omega) \right| \]

\[ \leq \left| W_{\Delta} (j \omega) \right| \].

(B.4)

Combining Equations (B.1–B.4), one can get

\[ \left[ 1 + \varepsilon \right) D (j \omega) G_{i} (j \omega) G_{i} (j \omega) + M (j \omega) G_{H} (j \omega) G_{N} (j \omega) \]

\[ \leq 1 + \left[ 1 + H (j \omega) M (j \omega) G_{H} (j \omega) G_{N} (j \omega) \right] \]

\[ \left| W_{\Delta} (j \omega) \right| - \left| W_{\Delta} (j \omega) \right|. \]

(B.5)

Therefore, one sufficient condition for Equation (36) or Equation (B.1) can be formulated as

\[ \left[ 1 + \varepsilon \right) D (j \omega) G_{i} (j \omega) G_{i} (j \omega) + M (j \omega) G_{H} (j \omega) G_{N} (j \omega) \]

\[ \leq 1 \forall \omega. \]  

(B.6)