SCALAR THREE BODY DECAYS AND SIGNALS
FOR NEW PHYSICS

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ABSTRACT

If massive invisible particles are pair-produced in a three-body decay, then the energy
distribution of the other (visible) product is sensitive to the mass of the invisible pair. We
use this fact in the contexts of a Higgs boson decaying into (i) a $Z$-boson and two massive
neutrinos of a fourth generation, and (ii) a $Z$ and two lightest supersymmetric particles in
the minimal supersymmetric standard model. We discuss how the $Z$-energy spectrum in
each case can reflect the values of the parameters of such models.

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It is a known fact that if a pair of invisible particles are produced in some three-body decay, then the energy distribution of the third particle is sensitive to the masses of the invisible particles. This sensitivity has been utilised earlier in the context of rare decays like $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ to study the dependence of the resulting pion spectrum on the mass of the $\tau$-neutrino [1]. Also, it has been claimed that the decay spectra in such cases are different for Dirac and Majorana neutrinos respectively, thereby suggesting a method for distinguishing between these two kinds of fermion masses [2].

The essential argument in the above works is as follows. If all the neutrinos have masses that are negligible compared to $m_\pi$, then the differential decay rate $d\Gamma/dE_\pi$ in the centre-of-mass frame will be a monotonically increasing function of $E_\pi$ over the allowed region of phase space, as can be seen from straightforward kinematics. If, on the other hand, one of the neutrino species is significantly massive, then the decay distribution for the corresponding channel attains a peak and then falls with increasing $E_\pi$, due to the unavailability of phase space. As a result, $\sum_i \frac{d\Gamma_i}{dE_\pi}$ exhibits a kink (i is the generation label). With increasingly higher mass of the invisible pair, the kink is displaced progressively to lower energy regions. However, as higher mass implies more phase space suppression for the channel under question, the consequent distortion in the decay spectrum also tends to be less and less conspicuous. In between, there is an optimal region where one expects the highest sensitivity to the mass of the invisible pair.

Since the current upper bound on the $\tau$-neutrino mass from laboratory measurements [3, 4] is as low as 35$MeV$, the idea summarised above is of little potential use in its original context; the kink in the $\pi$-spectrum in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ can barely occur at the very edge of the phase space even if $\nu_\tau$ has a mass close to its upper limit. However, because of its essentially kinematic origin, a similar effect in the decay distributions of heavier particles can also be expected. This should have interesting applications in obtaining the signatures of new particles which may be invisible in character. As an example, we consider in this note...
the decay channels of a Higgs boson into a $Z$ and two invisible fermions, and show how the
decay spectra are sensitive to the masses of such fermions. This confirms the expectation of
a kink-like behaviour even when the visible decay product is a particle with spin (whereas
the original observation concerned only spinless mesons). We illustrate such behaviour in the
contexts of two types of decays, namely (a)$H \rightarrow ZN\bar{N}$ where $N$ is a heavy sequential Dirac
neutrino $[^4]$, and (b)$H \rightarrow Z\chi^0\chi^0$ where $\chi^0$ is the lightest supersymmetric particle (LSP) in
the supersymmetric (SUSY) extension of the standard model $[^6, 7]$. Of course, we are conscious of the fact that the decays of the Higgs boson into the
above channels will have rather small branching ratios, and that, considering the unavoidable
backgrounds in hadron colliders, the observation of decay patterns of the expected type poses
practical problems. Still we find it worthwhile to undertake this study because of two main
reasons. First, as we have already mentioned, it enables one to see the effects in a general
perspective, whereby some insight might be gained about the signals of massive invisible
particles in theories beyond the standard model. In addition, such decay distributions could
be useful as cross-checks of the signatures of invisible particles whose usual search strategy is
to look for missing $p_T$ $[^8]$. Such cross-checks are particularly helpful if more than one types
of non-standard physics show up at the same time in experiments.

As the first example, we consider the decay $H \rightarrow ZN\bar{N}$, where $N$ is a heavy neutrino
belonging to, say, a fourth generation $[^5]$. As we know from the measurement of the $Z$-width
[^4], $m_N > m_Z/2$. If such a neutrino exists and happens to be lighter than its corresponding
charged lepton, then its only possible mode of decay is into a $\tau$ and a real or a virtual $W$.
But the decay is controlled by the mixing angle between the third and fourth generations.
From a purely phenomenological standpoint it is possible to have a very small value ($10^{-8}$ or
less) of this mixing angle. Under such circumstances, the decay length of the heavy neutrino
can be so large that it may pass off as invisible in a hadronic collider. The final state in
the mode considered above will then essentially consist of $Z + P_T$, similar in nature to the situation where massless, standard model neutrinos are pair-produced along with the $Z$.

Assuming standard model couplings and keeping the mass of the neutrino, the differential decay width for $H \rightarrow Z \nu_i \bar{\nu}_i$ in the rest frame of the decaying $H$ is given by

$$
\frac{d\Gamma_i}{dE_Z} = \left( \frac{g_w^4 \lambda_i}{64 m_Z^2 m_H \cos \theta_W^3 \pi^3} \right) \left[ \frac{1}{(m_H^2 - 2 m_H E_Z)^2 + m_Z^2 \Gamma_Z^2} \right] \left\{ \begin{array}{l}
2 m_Z^8 + m_Z^6 m_H^2 \\
- m_H^4 m_Z^4 - 2 m_Z^2 m_H^2 - 2 m_Z^6 m_H^2 + 2 E_Z m_Z^2 m_H (-2 m_Z^4 + 2 m_H^2 m_{\nu_i}) \\
- 2 E_Z^2 m_H^2 (m_Z^2 m_{\nu_i}^2 + 3 m_H^2 m_{\nu_i}) + 4 E_Z^3 m_H^3 m_{\nu_i}^2 + m_Z^4 m_H^4 + m_Z^4 m_H E_Z^2 - m_H^4 E_Z^2 + 2 m_Z^6 - 2 m_Z^4 m_{\nu_i}^2 - 2 m_Z^2 m_H^2 - 2 m_H^2 m_{\nu_i}^2 E_Z^2 - m_H^4 E_Z^2 \right\} \times \frac{m_{\nu_i}^2}{m_Z^2 + 4 m_{\nu_i}^2 p_{Z}^2/q^2} \left[ - m_{\nu_i}^4 + m_H^2 m_{\nu_i}^2 + 2 (m_Z^2 m_{\nu_i}^2 - m_Z^4 m_{\nu_i})/m_H^2 \right] + m_{\nu_i}^2 \omega_i \left[ m_Z E_Z - 2 m_Z m_{\nu_i}/m_H \right]/\lambda_i + m_{\nu_i}^2 (-m_Z^2 + m_H^2 - 2 m_H E_Z - 2 m_{\nu_i}^2)/2 \right\}
\right]
$$

(1)

with

$$
q^2 = m_Z^2 + m_H^2 - 2 m_H E_Z
$$

(2a)

$$
p_Z^2 = E_Z^2 - m_Z^2
$$

(2b)

$$
\lambda_i = p_Z (1 - 4 m_{\nu_i}^2/q^2)^{1/2}
$$

(2c)

$$
\omega_i = \ln \left( \frac{E_Z + \lambda_i}{E_Z - \lambda_i} \right)
$$

(2d)

where $i$ is the generation label, $E_Z$ is the $Z$ energy in the rest frame of decaying Higgs and the kinematical constraint on $Z$ -energy is

$$
m_Z \leq E_Z \leq (m_H^2 + m_Z^2 - 4 m_{\nu_i}^2)/2m_H
$$

(3)

The contributions come from three Feynman graphs, there being a non-negligible $H \nu \nu$ interaction if $\nu$ is massive. For $i = 1 - 3$, we obtain the appropriately simplified expression by
setting \( m_{\nu_i} = 0 \). With a massive neutrino, \( i \) runs from 1 to 4, with \( \nu_4 = N \). The differential rate for the entire \( Z + \text{invisible} \) channel is obtained by adding the contributions from the massless as well as massive species.

Figure 1 illustrates the patterns expected for the decay of a Higgs of mass 500 GeV, with three masses for the heavy neutrino. The kink due to the presence of the heavy species begins to become perceptible as \( m_N \) approaches 100 GeV. With higher \( m_N \), the kink appears, as expected, for lesser values of \( E_Z \). However, in contrast with the results presented in reference [1], the distortion in the curve for \( m_N = 150 \) GeV is more pronounced than that for \( m_N = 100 \) GeV. This is due to the fact that unlike in the case of meson decays, here we have a resonant contribution to \( H \rightarrow Z \nu_i \bar{\nu}_i, i = 1 - 3 \) from real Z-bosons. This resonance (not shown in the figures) occurs at \( E_Z = m_H/2 \). The kink for \( m_N = 100 \) GeV occurs sufficiently near the resonance to be partially washed away by the sharp rise that ensues. That is why the fall for \( m_N = 150 \) GeV is somewhat more marked, although for even higher masses the kink again starts losing visibility.

Let us now turn our attention to the minimal SUSY standard model where invisible Higgs decay mode [9, 10] could offer signal for new physics. We consider here a decay mode \( H \rightarrow Z + \text{invisible} \) which can occur in a less restricted region of the parameter space. The invisible particle in this case is the lightest supersymmetric particle which can decay no further because of the conservation of R-parity [11]. In most theories, the favoured candidate for LSP is the lightest of the neutralinos, the physical states obtained upon diagonalisation of the mass matrix consisting of the photino, the Zino and two neutral Higgsinos [7, 12]. As is well-known, this scenario contains two complex scalar doublets which ultimately give rise to two neutral scalars (along with one pseudoscalar). Of these, the upper limit on the mass of the lighter scalar is a function of the top quark mass [13], and cannot be appreciably above 150 GeV or so. Under the circumstances, its decay into \( Z \chi^0 \chi^0 \) (\( \chi^0 \) being the LSP) does not
have much leeway kinematically. The heavier scalar, on the other hand, is more suitable for demonstrating the decay behaviour we are interested in. In what follows we present the sensitivity to the $\chi^0$-mass in the differential decay rate of this mode plotted against $Z$-energy. It may perhaps be said that if more than one scalar particles are experimentally discovered while direct evidences for SUSY (from squark and gluino production) are still not available, then such indirect signals through scalar decays may be useful in resolving the issue in favour of SUSY or otherwise.

The actual computation of the decay rate for $H \rightarrow Z\chi^0\chi^0$ involves evaluation of the contributions from ten Feynman diagrams, having as propagators the four neutralinos (with their crossed diagrams), the Z and the pseudoscalar particle and the corresponding Feynman rules are given in references [7, 9, 12]. The large number of parameters involved can be simplified if one considers a SUSY scenario inspired by Grand Unified Theories (GUT) [14]. There, all the neutralino masses and their mixing angles (which occur in the couplings that we require to know) can be obtained using as inputs the gluino mass ($m_{\tilde{g}}$), $\tan \beta = v_2/v_1$ where $v_2(v_1)$ is the vacuum expectation value of the Higgs doublet that gives mass to the up(down)-type quarks, and $\mu$, the Higgsino mass parameter [15]. Also, in the Higgs sector in the minimal SUSY model, $\tan \beta$ and the mass of one physical scalar suffice to fix all masses as well as $\alpha$, the mixing angle between the two doublets. As a net outcome, it is possible to compute the decay rates mentioned above by specifying $m_{\tilde{g}}$, $\tan \beta$, $\mu$ and the mass of H, the decaying particle. A further constraint on the allowed combination of these parameters comes from $Z$-decay measurements at the Large Electron Positron (LEP) collider [16].

In addition to the SUSY contribution, one has also to add the contributions from the three massless neutrinos in order to have the net observable distributions of the $Z+\text{invisible}$ final states. The neutrino contribution is given by equation (1), with $i = 1 − 3$, $m_{\nu_i} = 0$ and an overall multiplicative factor of $\cos^2(\beta − \alpha)$. The expression for the SUSY contribution
being extremely long and cumbersome, we refrain from presenting it here.

We present some of our results in figures 2 and 3, drawn for similar values of all other parameters but with opposite signs for $\mu$. The choice of a Higgs mass of $500 GeV$ is because in this region (upto an interval of about $50 GeV$) the distortion to the decay spectrum is somewhat optimal. A large fluctuation due to the LSP is observed in each curve which otherwise would have had a uniform rise due to the neutrino contributions alone. The several orders of enhancement caused by the neutralinos over the neutrinos can be understood from the fact that for the massless neutrinos, the only contributions to this mode can be mediated by a transverse Z-boson. For both the figures here we use the value $\tan \beta = 2$; for larger values of $\tan \beta$ the fall is sharper but, owing to a stronger suppression caused by the factor $\cos^2(\beta - \alpha)$ in this case, the subsequently rising part from the neutrino contributions is extremely small. Also, for a lower $m_H$, the SUSY contribution tends to become smaller compared to the standard model one, so that the distortion begins to disappear.

It is to be noted that in the SUSY case we are in a kinematic region where the decay spectrum is not plagued by resonances, so that a clean signature of the mass of the LSP can be observed. Here one can see a close resemblance to the curves of reference [1], the kink (which in this case is trully a “hump”) being progressively in the region of smaller $E_Z$ and at the same time being smaller in size as the invisible superparticle is more massive.

The production cross-section of a $500 GeV$ Higgs at the LHC is about $2 - 3 pb$ [17]. This means that about $10^6$ Higgs bosons can be produced per year. Since the branching ratios for the three-body decays under question are on the order of $10^{-4}$, a few hundreds of such events in a year of run are possible. In the SUSY scenario, as one see from figures 2 and 3, a large contribution to the observed events are expected from that part of the Z-energy spectrum where the distortion due to the massive LSP is visible. This makes the SUSY scenario more interesting for demonstrating the effects we are trying to show and highlights the possibility
of indeed observing the signals of the massive LSP’s in this manner. In the case of massive neutrinos, the number of events in the distorted region are possibly too small to make the effect experimentally interesting.

We conclude by re-iterating that the predictions made here are mainly aimed to bring forth into a bigger perspective the issue of obtaining signals of invisible particles from the observed energy spectra of visible ones. Detailed studies on some possible applications to B-factories as well as to the LEP-II are currently in progress [18].

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Figure Captions

Figure 1:

The differential decay distributions for $H \rightarrow Z + nothing$ in the presence of a massive but invisible fourth neutrino $N$. The three curves correspond to different values of $M_N (\text{in GeV})$.

Figure 2:

The differential decay distributions (modulo an overall multiplicative factor) for $H \rightarrow Z + nothing$ in the minimal supersymmetric standard model. The three curves correspond to different masses of the LSP ($\text{in GeV}$), with $\mu = 250 GeV$, $\tan \beta = 2$.

Figure 3:

The differential decay distributions (modulo an overall multiplicative factor) for $H \rightarrow Z + nothing$ in the minimal supersymmetric standard model. The three curves correspond to different masses of the LSP ($\text{in GeV}$), with $\mu = -250 GeV$, $\tan \beta = 2$. 
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