FINANCIAL LIQUIDITY: AN EMERGENT PHENOMENA

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Abstract. In a complex system model we simulate runs for different strategies of economic agents to study diverse types of fluctuations. The liquidity of financial assets arises as a result of agent’s interaction and not as intrinsic properties of the assets. Small differences in the strategic rules adopted by the agents lead to divergent paths of market liquidity. Our simulation also supports the idea that the higher the maximum local allowed fluctuation the higher the path divergence.

1. Introduction. At the end of any real investment, investors expect cash payments, e.g. expected capital gains to “be realized”. While the real investment is ongoing, this mechanism is not available for all the agents. Some of them have to wait until the expected returns are realized or, alternatively support a loss. The existence of secondary markets creates the illusion that any agent has the chance to realize its benefits beforehand. A bottle neck could eventually appear. Agents know that, and; confronted with uncertainty, keep a variety of liquid marketable securities (in terms of different duration, risks and returns) to diversify they portfolios. How liquid a marketable security is, relies on everyday markets results.

Liquidity depends on a complex network of interactions. Highlighting the usefulness of the approach Allen and Babus [2] states that network analysis is a theoretical framework that allow to analyse systemic risk, freezes in the interbank market, investment decisions and corporate governance, role of networks in distributing primary issues of securities and mutual monitoring as in microfinance. For microfinance, Banerjee et al. [4] suggest that households who participate in microfinance are much more likely to inform their friends that microfinance is available than households who choose not to participate. Concepts like diffusion dynamics and key nodes appear whether we take a small part of the system like a microfinance system or as in our case, study the entire financial system respect to one topic like liquidity. Macro characteristics such as density of connections and micro characteristics like the position of the node [20] are properties of the system that defines attributes that are essential to the system and are necessary to understand its structure and dynamics.

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1Once the investment decision is made, it is irreversible [7].
2A model that shows the complexities of linking uncertainty, competition and regulation has been proposed by Huu [19].
3Economic change leads to highly interconnected systems [1].
As for the rules that govern the interactions, agent based models, Bookstaber [8, 9]; considers specific agents with simple interaction rules, exposed to external real cycles. In our approach the emergency of liquidity (the idea that fixed investments may be transformed in liquid financial assets) is the base for producing: a) two type of behaviors and two kind of agents (conservative or speculative behavior), b) a dynamic interaction based on the presumption that real cycles are part of their expectations, c) volatility as the parameter by which agents adapt their portfolios to the unknown future fluctuations of the real cycle4.

Cash is limited and at the top of the financial asset pyramid, state and bank money is the last resource in times of financial distress, becoming the King of the financial assets. But for a long period of low volatility, other forms of financial liquidity may take the portfolio control. We developed a simulation model to describe the dynamic properties of a hierarchical liquidity pyramid, that emerge from such complexity, and its fuzzy upper and lower bounds as a function of the level of uncertainty agent are perceiving.

Small differences in the strategic rules adopted by agents lead to divergent paths of market liquidity. Our simulation also supports the idea that the higher the maximum local fluctuation allowed, the higher the path divergence. Linearity is recovered only when upper and lower bound for liquidity fluctuations are imposed. Section 2 discusses the main characteristics of liquidity, in section 3 the model implications for the study of economics are analysed, section 4 describes the model and simulations performed, whereas section 5 concludes.

2. Liquidity. Liquidity can be seen as the characteristic of an asset to be bought or sold at a value that remains stable at any future time. There is no consensus in the literature on the definition of liquidity. Sometimes it focuses on specific assets (relative position in a pyramid created by the bank system, sovereign bonds, blue chip stock), some other times definitions refer to a characteristic of an agent (its ability to generate cash flow) and sometimes simply to a medium of exchange [21]. Some authors try to establish links between what they call macro liquidity (monetary flows) and micro liquidity (transactional liquidity) where monetary expansions increase liquidity in the stock market and interest rate movements impact the volatility of bonds and shares [12].

Another refinement points to the existence of different types of liquidity. A distinction is made between central bank5, fund and market liquidity [10] including the analysis of various types of instruments (money, bonds, stocks and other financial positions) that are related in multiple aspects with heterogeneous agents (central banks, banking institutions, sovereign wealth funds, hedge funds, high net worth individuals).

In terms of dynamics, Nikolaou [28] considers liquidity as a flow. Due to the time-variability of liquidity Nikolaidi [27] proposes a definition of liquidity that varies over time.

Following these concepts we can understand that liquidity is a relationship between different assets that emerges from complex dynamics6 and not a intrinsic characteristic of any of the particular assets. In other words, the same asset (for instance, a blue chip stock) can be a perfect liquid asset for a long time, entering

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4For a model that is based on a manager of an individual portfolio see [3]
5Monetary base offer [15].
6Expanding Chordia’s cited work in terms of formation of transmission channels.
in different portfolios, but in a stress market situation may become illiquid (have to be sold accepting a big price discount).

Categorical definitions often cause paradoxical situations. For example, money issued by a central bank is liquid in times of low inflation but may lose liquidity in the face of inflation expectations, eventually being replaced by another asset (a foreign reserve currency, inflation protected bond, or some alternative commodity, such as gold).

Liquidity “fluctuates”, leading to situations that may be extreme, when for example it is said that the financial market is “dry” because only a few assets maintain the certainty of being recognized at their par value in the future and are retained in the portfolios; on the contrary it is said that the market has excess of liquidity since more assets (discounted checks, securitized debt, sovereign bonds) begin to be considered close substitutes. Liquidity is therefore an outcome, an emerging phenomena, in the interaction of agents and their perceptions about the future and not an essential property of a particular asset that is fixed over time.

To measure liquidity, it is necessary to consider the set of relationships that links and values the entire spectrum of assets that are operated in the markets, and requires knowledge of interconnection rules, the network topology that form the transactions and the dynamics that link the financial aspects with those of the real economy.

Holders of financial assets have in their hands a varied and growing number of instruments to regulate liquidity\footnote{In all its variants, central bank, funding and market.} from the monetary multiplier (secondary creation) to pass operations, via discount checks or short-term sovereign debts operated on derivative instruments (call, puts, swaps).

The existence of liquidity arises from a continuous range of interchangeable assets on par or near par, that is, with little uncertainty on their value over extended periods of time. With a perfect forecast on the physical investment cash flow, we would be living in the world of “forever” investors, where there would be no need of liquidity (and therefore no illiquidity). In this world, original owners will kept their physical investments until the end, collecting the expected return.

When forecasts includes uncertainty, liquidity is created as it becomes possible to transfer property rights to other agents. The previous forever investors may want to diversify their portfolios to diversify risks.

In the liquidity pyramid, a central bank can issue money to buy medium-term bonds, transferring liquidity to the market. In the same way, a commercial bank can transform future salaries into mortgage payments, giving liquidity to employees; investment banks will get access to future dividends of a company by buying its shares. A liquidity cascade is built; the emergent liquidity gives way to some agents to think and act as if the real economy can be anticipated and “beat” uncertainty, trading its future results.

We propose a model where liquidity is the result of a complex interaction that arises from different perceptions about the future of the real economic system.

3. **Complex system model.** In our model, liquidity is constantly fluctuating and agents attempt to “beat the market” anticipating others with respect to an expected result, can be successful or frustrated without the need to assume defined behaviours based on the characteristics of agents (rational or irrational) but of strategic rules.
In our model rules are heuristic and predefined, adaptation and learning are not considered\textsuperscript{8} and agent have minimum information about the value of the changes in the variables\textsuperscript{9}. Agents could decide increase or decrease their liquidity portfolios considering a bigger (conservative) or a smaller (speculative) protection against expected changes of the business cycle. What matters in our model is the effect of this decisions on the level of market liquidity and market volatility as emergent phenomenons. Portfolio diversification, degree of exposure, have to be re-evaluated in every period.

The chances of liquidity entering the system is represented by the appearance of coalitions (concentrations of decisions favourable to delivering more liquidity to the system). Agents’ response to a specific business climate may or may not (probabilistically) result in speculative bubbles.

The idea of a complex network emphasizes some important issues: a) agents need to evaluate their portfolios looking at others portfolios (value is a relative concept, since no financial assets has a value “per se”), b) as prices and stocks transactions are known at closing, all agents are only aware of their final liquidity position simultaneously at closing time, c) agents operate with rules based on the desired liquidity of their portfolios, which may be different compared to the value of the portfolio finally achieved in each period, d) the main driver is not market stability but the level of volatility.

The interactions are deterministic as to the rule the agents follow, but the results for the set are stochastic. There is no Walrasian actioner and no representative agent that synthesize aggregate behaviour in different markets, such as in DGSE (Dynamic general stochastic equilibrium) models, but rather interactions at an intermediate level between micro and macro results.

The characterization of a “complex system” shows that there is a network topology and that a single asset is not capable of constantly preserving a certain degree of liquidity defined a priori. Portfolios value is a result related to all the others portfolios, since value is a relative concept in relation to all other assets.

The complex system model also allows to reflect the interconnectivity between the “real” sector (physical investments represented by the matrix) and the financial sector (virtual investments that mirror portions of the matrix in the form of clusters). The propagation of shocks that connect one sector to the other, allows to highlight the cases where the existence of systemic risk can be verified\textsuperscript{10}.

With regard to the assets price, financial operators are usually divided by the adoption of two practical analysis: a) those who use projections of future cash flows (dividends, bond yields etc.) discounted at an adequate interest rates (the so-called fundamental analysis) and those that analyse various curves of moving averages and trends (technical analysis). Both formulations allow us to emphasize that neither of these methods uses the current or historical prices as the sole source.

\textsuperscript{8}This justified by persistence and overconfidence as pointed out by the behavioural literature\cite{29,5}.

\textsuperscript{9}These are the bases for assuming that the variations are distributed with uniform distribution and that each variation is independent of the others.

\textsuperscript{10}This type of risk is highlighted by the Basel banking supervision committee report in the Basel III documents\cite{6} that argues that: “While procyclicality amplified shocks over the time dimension, excessive interconnectedness among systemically important banks also transmitted shocks across the financial system and economy.” Systemic risk is generated largely via: “an array of complex transactions”.
of decisions, since what is relevant to decide is based always on the future expected price. Historic prices are relevant information in the formalization here proposed, agents use at least immediate recent ranges of past prices since they reflect the liquidity degree of the portfolio assets in terms of its volatility¹¹.

The liquidity of portfolios is a central indicator for agents. For conservatives because it is a bridge between the uncertain and long-term results that the real economy may provide. It is often said that in conservative portfolios “cash is king”, as they tent to maintain a high proportion of liquid assets. For speculators, liquidity is the instrument to wait for the moment to enter the market, incorporate less liquid assets and then sell them to make a profit without waiting for the real investment maturity.

3.1. The structure of the model. We distinguish two types of agents¹²: financial investors (rentiers), who decide to increase or reduce the liquidity of their portfolios through placements (deposits) in specialized financial institutions (professional investors and speculators).

Examples of rentier are large family fortunes, sovereign wealth funds or pension funds. Examples of institutions are investment banks, mutual funds, brokers, hedge funds, etc.

The model simulates the transition from an initial liquidity situation to another after agents use information available in the market leading to fluctuations with the link of liquidity and uncertainty as seen in section 2. The complex system framework emphasises the idea of heterogeneous agents interaction exposed in section 2 and the first part of this section.

Each investor decide on their own portfolio, but the results of aggregate decisions are only known at the end of each period and nobody can anticipate the aggregate result. The information used in each period takes into account what happened in previous periods. The rules followed by the agents reflect two aspects recognized by the operators: a) what the market is doing (results review of several recent previous periods) and b) the existence of uncertainty (certain sequence of consecutive results of the same sign can amplify the reviews on the liquidity injected to the system).

The rules of section 4 are defined as 1) for rentiers there is a $w$ proportion that determines how much of their wealth is deposited in specialized financial entities and 2) for institutions there is a $b$ proportion of the deposits received, which are dedicated to purchase financial assets (always backed by real assets - mainly stocks and bonds).

The larger the $w$, the more confidence in the financial system and the greater the proportion of the deposited wealth.

Proportion $b$ represents the aggregate liquidity position that results from the decisions of all financial institutions. The higher $b$, the greater the liquidity that penetrates the matrix. For example, banks can expand or reduce the liquidity received in deposits, applying reserve requirements or taking advances from the Central Bank; other specialized institutions can maintain a conservative position or conversely get leverage (take debt) and buy more financial assets than they are allowed by the original deposits.

In this way the proportion $p$ can be defined as,

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¹¹The point of the significance of past prices was suggest by professor Hartwell.

¹²A single agent can’t react at the same time with different rules for different part of his portfolio if these parts are not differentiated by some characteristic.
Variables $w$, $b$, and $p$ have a minimum value of 0 and a maximum of 1. In figure 1 we represent the case for a physical and a financial good at square matrix levels.

The concept of network is a necessary condition assume a minimum of interconnections between the agents as we have seen in section 1, since we must have at least the latent [17, 18] possibility that liquidity circulate. Loepfe et. al. [23] shows that interconnection can not be defined only by density of connections and system size; modularity plays a key role. Further, Cont [13] suggest heavy tail distributions of connexions numbers and exposure sizes, asymmetry between incoming and outgoing links and that concentration is a key feature. On the other hand, a network give also a natural representation for the flow of assets and liquidity between agents.

4. Rules definitions and formalism. In this section we describe the rules, the formalism and show the numerical results of the simulations.

4.1. Rules definitions. The $\Delta w$ variation of the $w$ proportion of their wealth that rentiers deposit in financial institutions is modelled as a random variation with a maximum threshold.

The variation $\Delta b$ of the proportion $b$ representing the aggregate decisions of the financial institutions on their degree of aggregate liquidity is also modelled as a random variation. The randomness of the variation represent the fact that the new information is valuable [31], cannot be known in advance and produce revaluation of all portfolios at the same time. In this sense, this work departs from the hypothesis in which all the economically relevant information about an asset is contained in its price [16]. In the context of this work, each individual rentier or institution cannot control the confidence of the market as a whole or become totally independent of the strategy followed by the others.

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13Giving that the different authors point out to different characteristics that a financial network posses and that we are not aware of a single definition that have consensus we assume that a network sufficiently connected (without a minimum of connections asset cannot circulate and liquidity is not created) exist and we will work directly with agents proportions using the two rules.

14For a representation of the complexities of a factual economic network see Vitali [32].

15Distributed uniformly to calculate its absolute value, a proportion $k$ of these are negative variations allowing a maximum of uncertainty over the changes in the range indicating that agents have minimum information and is conceptually linked to the uncertainty connected to liquidity as exposed in section 2.

16Idem footnote 15.
Because of the system’s reliance on payment promises mentioned in subsection 3.1, two aspects are to be considered: a) uncertainty of future results and b) the level of liquidity adjusted every time new information is available.

We assume scenarios where one every \(i\) periods (in random terms) is negative to reflect the fact that recessive periods are usually scarcer. In other words, given that agents know that physical assets have a long-term temporary structure and that the liquidity attributed to them may or may not translate into real results, that is, they know that they are exposed to situations of uncertainty, we assume that one every \(i\) periods (in random terms) there is an expectation adjustment of the type explained throughout section 2.

We define two types of rules: with rule \(\gamma\), agents take into account the last \(n\) variations \(\Delta w\) and/or \(\Delta b\) depending on whether they are rentiers or banks respectively. If these are all negative then they multiply the last variation by \(j\). The \(\beta\) rule causes agents to react by multiplying their variation by \(j\), if a single \(\Delta w\) or \(\Delta b\) variation is negative \((n = 1)\). In all cases \(j\) includes the current period. We use \(n\) to denote the number of negative periods after which the agents react with \(j\).

In this way we have two types of agents, those who follow the \(\gamma\) rule wait more time having more confidence on their positions. Agents following the \(\beta\) rule have less confidence in their expectations and change them quickly and expose themselves less.

Rentiers only take information on variations of \(w\) and react with \(j\Delta w\) if the rule applies. In this case, financial institutions apply the same rule \((j(-)\Delta b)\) (\(n\) periods of \(\Delta w\) negative). Financial institutions additionally follow the rules \(\beta\) or \(\gamma\) also with respect to \(\Delta b\), so they use additional information that can lead to \(j\Delta b\).

4.2. The formalism. In this section we present the variables and the relations between them. We take equation 1 and assign a time index \(t\) for each variable.

\[
p_t = w_t \times b_t \tag{2}
\]

4.2.1. Definition of variables and dynamics. The rules of evolution for \(w\) and \(b\) are given by

\[
w_t = w_{t-1} + \Delta w \land b_t = b_{t-1} + \Delta b \tag{3}
\]

Differencing between the cases for rules \(\beta\) and \(\gamma\) by the subscript, in each period \(\Delta w\) is defined as,

\[
\Delta w_\beta = \begin{cases} 
\frac{t-1}{t}U(0, m) \\
\frac{1}{t}(-)jU(0, m)
\end{cases} \quad \Delta w_\gamma = \begin{cases} 
\frac{t-1}{t}U(0, m) \\
\frac{t^n-1}{t^n}(-)U(0, m) \\
\frac{1}{t^n}(-)jU(0, m)
\end{cases}
\]

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17 A heard behaviour[14] flight to liquid asset appears.
18 Expectations are embedded in the rules, agent using beta expect negative economic results when liquidity shrinks one period, whereas agents using gamma expect a recovery unless three negative consecutive periods of shrinking liquidity appear.
19 In this case the variation of \(b\) is always taken as negative and the rule is applied.
20 In this way a coordination phenomenon occurs.
Table 1. Rules and Scenarios of Simulations

| Rules | Scenario | Stable Path | Unstable Path |
|-------|----------|-------------|---------------|
| Rule β | one period | every 10 periods | max normal 1% | max normal 2% |
| Rule γ | three periods | multiplies by 10 ng | multiplies by 5 ng |

where the quotient of each line refers to the proportion of \( i \) cases that fall under the hypothesis whereas the second part tells that they are distributed with a uniform distribution.

As the difference \( \Delta w \) may differ for agents using rules \( \beta \) and \( \gamma \), we have to take \( \Delta w \) as a sum of \( \Delta w_\beta \) and \( \Delta w_\gamma \); with \( 0 \leq \lambda \leq 1 \) as the proportional factor that denote the use of rule \( \gamma \).

\[
\Delta w = \lambda \Delta w_\gamma + (1 - \lambda) \Delta w_\beta \land \Delta b = \lambda \Delta b_\gamma + (1 - \lambda) \Delta b_\beta \tag{5}
\]

The change in \( p \) reads,

\[
\Delta p = p_t - p_{t-1} \tag{6}
\]

Using equation 2,

\[
\Delta p = w_{t-1} \Delta b + b_{t-1} \Delta w + \Delta b \Delta w \tag{7}
\]

which is an expression of the general rule for the difference of a multiplication.

As this is valid for agents using \( \beta \) and \( \gamma \) rules we sum the results of both by the proportion \( \lambda \).

\[
\Delta p = \lambda \Delta p_\gamma + (1 - \lambda) \Delta p_\beta \tag{8}
\]

The proportion \( b \) follows exactly the same set of rules except that it copies the reaction of \( w \) in the case that \( \Delta w \) is multiplied by \( j \).

\[
\text{if } j \Delta w \implies j(-)|\Delta b|
\]

4.3. Simulations. For each graph we have run 30 series of 100 periods each,21 series of the value \( p \) are shown. We take an initial value of \( p \) of 0.2 for all series.22 In the case of combination of rules, the proportion of financial institutions and rentiers using the same rule are equal.

First, we can assume a financial region with a stable23 expansion path where the amount of negative periods is relatively small (1 every 10, \( i = 10 \)) and the fluctuations of proportions are usually not wide (in our case we take 1%) which we display in figure 2. When the \( \beta \) or \( \gamma \) rule is applied, the fluctuation becomes 10 \((j = 10)\) times more negative. The percentages in table 1 refer to the maximum

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21For this we develop a program in C++, the obtained series are analysed with Excel.
22The value 0.2 represents moderate financial deepening. We adapt the argument from Kiyotaki and Moore [22, 26].
23We identify stability with low volatility, more stable paths or scenarios are those with lower volatility.
fluctuation in the normal variation band\textsuperscript{24}. For all the graphs an average series is calculated (highlighted in black)\textsuperscript{25}.

These parameters can be compared to a financial region with a more unstable expansion path (negative periods 1 every 5, $i = 5$) and wider fluctuations (in our case we take 2\%) in figure 3. When either $\beta$ or $\gamma$ rules are applied, the fluctuation becomes 5 ($j = 5$) times more negative\textsuperscript{26}.

\textsuperscript{24}To these variations rules $\beta$ and $\gamma$ can be applied if the conditions are met.

\textsuperscript{25}The statistical metrics provided are only measures of descriptive statistics; they only show sufficient conditions that the probability of the outcomes is different of zero giving our assumptions. They are not adequate to make quantitative inferences about the whole process. The simulations does not have a known distribution, the true distribution may be a power law without average or variance, we need also assume that this is not the case. Note that in the presence of heterogeneous agents interacting in networks that need not be completely connected, as in complex networks, averages are not an informative measure for central tendency, so we need to assume that the network have the attributes to make the average informative, including that it is sufficient connected. We are grateful to anonymous referees that show us the importance of these points.

\textsuperscript{26}We refer as stability the reduction of (or low) volatility. Stable and unstable refers to the different scenarios where the permissible fluctuations of the variations are lower in one case (stable) than in the other (unstable).
Table 2. Average and Standard Deviation of final values of series

|                | Stable Path |           | Unstable Path |           |
|----------------|-------------|-----------|---------------|-----------|
| Rule           | Average     | Std dev   | Average       | Std dev   |
| β alone        | 0.0349      | 0.0561    | 0.0053        | 0.0080    |
| γ alone        | 0.777       | 0.0492    | 0.9049        | 0.1379    |
| β               | 56%         | 0.2044    | 45%           | 0.1080    |
| γ               |             | 0.1970    |               | 0.1179    |

Table 2 shows average values of the series for the different hypotheses. The first part of the table shows the values obtained using the $\beta$ and $\gamma$ rules separately for each hypothesis. The second part refers to the results obtained by combining the rules, with the $\beta$ rule used by 56% of agents in the stable case and by 45% in the unstable case. The proportion of the $\beta$ rule used is shown, the complement with the $\gamma$ rule adds up to 100%. In the case of a combination of rules, we show the combination that leaves the initial value of the series invariant.

Figure 4. 30 Series of Evolution of rules as a whole, stable and unstable paths

Figure 4 shows the temporal evolution of the results displayed in table 2. Finally, we present the graphs of the proportional relationships of the rules in cases of unstable and stable paths in figure 5.

Figure 5. Average of the final value of 30 Series of joint Evolution with standard deviation for each point, 100 periods each series

In neither experiment the relationship is linear as seen in figure 5. In the case of a stable path, the first derivative and the second derivative of the relationship
curve are positive, so $p$ grows at increasing rates within the definition range. The standard deviation is proportionally greater in the middle range of the graph.

In the case of an unstable path, the curve suggests a positive first derivative for the entire range, but the second derivative takes positive first and negative values at values close to $\gamma = 1$. Standard deviations are markedly higher in the region with a high proportion of the $\gamma$ rule.

From an economic point of view, financial markets connect conservative and speculative agents, producing the relevant emergent information for the rules to be applied. Figure 5 shows the increase of the probability of liquidity creation under this interaction. With more conservative agents liquidity increase, but a critical mass of conservative agents in needed in order that the economy is in a range where liquidity is easily available and the risk of liquidity disappearance is low. For the middle range, with more liquidity, speculative agent have also a higher chance to retrieve liquidity from the system. For the unstable case, the influence of the speculative agents is greater and last longer than in the stable case because they have a system where more liquidity is at their disposal to retrieve. In this way a greater proportion of conservative agents does not guarantee stable liquidity growth. The exact proportion of agents using the rules is also significant\footnote{Where a increase in the proportion of conservative agents occurs.} together with the range of volatility of the system.

\subsection*{4.4. Lower and upper bounds.} In this subsection we look at the consequences of imposing lower and upper bounds to the system.

As for the lower bound we use the rule that $j$ out of $n$ crisis liquidity is injected into the system by a monetary authority. The amount is equal to the drop. This rule operate over the institutional (bank) parameter $b$. As for the upper bound $j$ out of $n$ times the parameter $b$ exceed 0.8 the value is set at 0.8 and the remaining value is accumulated in a reserve. This rule also operate only over banks. For the unstable scenario and for any combination of rules we have an average reserve accumulation of 0.3402 with a standard deviation of 0.1014. Whereas for the stable case we have 0.0996 with a standard deviation of 0.0521.

The stochastic character of the process of bound impositions may be interpreted as that the bound is not imposed all times when the condition is triggered by the mechanism designated or alternatively, that some interventions may be not successful\footnote{When liquidity level surpass some previously defined range of fluctuation (bounds), additional liquidity may be injected or retrieved by a Central Authority. This intervention only have a probabilistic chance to be successful. Financial authorities, typically a central bank, have the ability to provide or retrieve liquidity to the system. These interventions may not be successful if the financial agents have a different expectation about the future state of the real economy.}. 

\[
\text{\footnotesize\textsuperscript{11}}
\]
With the injections of liquidity the agents which follow $\beta$ has an average increase of liquidity in contrast with a non intervention scenario. The cost of a non intervention is an elevated standard deviation as seen in table 3 and figure 6.

At the same time, seeing figure 6 and comparing tables 3 and 2, with upper limits the average falls about 0.1 with a lower standard deviation than in the unbounded case. The fall of 0.07 for the stable path for the $\gamma$ rule considered alone produce no alteration in the standard deviation.

**Table 3.** Average and Standard Deviation of final values of series

|                  | Stable Path | Unstable Path |
|------------------|-------------|---------------|
|                  | Average     | Std dev       | Average     | Std dev       |
| Rule $\beta$ alone | 0.2699      | 0.1445        | 0.1655      | 0.1212        |
| Rule $\gamma$ alone | 0.7024      | 0.0488        | 0.7837      | 0.0845        |
| Rule $\beta$ & $\beta$ & $\beta$ & $\beta$ & 92% & 0.2001 & 0.1312 |

It is interesting to note that liquidity injections have an average of 0.467 with a standard deviation of 0.1605. Therefore the average injections of liquidity are higher than the average liquidity at the end of the period if there are only agents which follows rule $\beta$.

Under the assumptions of this work we see that the type of rule does not alone determines a result. For the unstable path, the cost of the average 0.1655 for the system where only rule $\beta$ is used, are injections of 0.8543.
(a) Unstable path
(b) Stable path

**Figure 7.** Average of the final value of 30 Series of joint Evolution with standard deviation for each point, 100 periods each series

With a stable scenario we have a near linear relation of $p$ with the proportion of $\gamma$ respect to $\beta$, that may be related to the classical outcomes of usual models. For the unstable scenario the relative stabilization\(^{29}\), (in the sense) that the relation is near linear and contained in the band $0.25 - 0.75$, the results come at the cost of relative significant standard deviation over the entire path.

The results may be relevant for the discussion of rules vs discretion. As the rules are not applied automatically, we see that even rules utilized with discretion may be sufficient to reduce volatility. There are therefore some instances in which rules applied with discretion may improve the control of the system if the objective is linearization; at a cost of a higher standard deviation.

The costs of stabilization, that is the accumulation of liquidity injections, is only balanced when at least a proportion of agents 0.6 (reserves of 0.34\(^{30}\)) use rule $\gamma$ for the unstable case. For the stable case, only a proportion of 0.8 (reserves of 0.1) of agents using rule $\gamma$ produce accumulation of reserves enough for compensate the injections.

\(^{29}\)The shape of the curve is nearer to a linear path, but it still maintain some curvature.

\(^{30}\)by applying upper bound limit.

As we see in figure 8 imposing lower bounds leads to the need of few conservative agent to produce an average trajectory to maintain the initial value.
5. Conclusions. We have presented a model that describes the formation of liquidity based on interactions between agents that increase or retract their financial exposure to the expected results of the real sector. To model these circumstances we implement two rules that describe possible strategies to follow when agents receive new information.

Unlike traditional models [16], we do not consider current prices as the only relevant and almost exclusive variable for making decisions. Historical prices are a result of the complex interaction of portfolio strategies. Negative expectations contract liquidity (many assets that were previously liquid lost that quality) and positive expectations extend it (there is a greater propensity to get into and stay tied to the results of the real economy).

The randomness of the liquidity variation expose the fact that new information, produced by multiple agents at the same time, is valuable [31] but cannot be anticipated with certainty in its aggregate result. In the context of this work, each individual rentier or institution cannot anticipate how much liquidity (market confidence) will result from the strategies that other agents are following.

The existence of liquidity presupposes markets where the assets can be operated, but it is not necessary for the assets to be traded in a market to lose liquidity. This approach allows to consider a wider range of underlying economic phenomena that can be incorporated into these types of models, such as divergences of expected returns with respect to real returns on investments.

The key point of the study is that slightly different decision rules (such as those defined for $\beta$ and $\gamma$) generate very different paths of liquidity expansion.

Under special conditions (stability and bounds), a linear framework may be recovered. Loosing either of the conditions leads to a standard deviation that has an increasing range for low proportion of conservative agents and a decreasing range for high proportion of conservative agents. When the proportion of conservative agents is low$^{31}$, an increase of this proportion may result in a feedback effect, as more liquidity injected into the system allow speculative agents to eventually retrieve it in a higher magnitude.

The results suggest that high proportions of liquidity in the economic system, can be detrimental to stability because, in principle, every financial asset that deviates from calculated expectations has the ability to infect the system and turn it volatile.

On the other hand, the existence of a certain proportion of financial assets can encourage the transfer of resources to the best projects [30, 25].

From the point of view of active policies, considering these kinds of theories may not be seen as a sign of inevitable disorder and the impossibility of analysis, but rather as a fertile ground where new types of interventions may be explored [11] to more effectively stabilize the system.

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$^{31}$The effect of more conservative agents has the consequence that more speculative agents has liquidity to withdraw, which is not case when the proportion of speculative agents is very high, as liquidity withdrawals by speculators keeps liquidity low any time (see figures 2, 3, 5 and 7).
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