Data-Driven Linear Koopman Embedding for Model-Predictive Power System Control
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Abstract—This paper presents a linear Koopman embedding for model predictive emergency voltage regulation in power systems, by way of a data-driven lifting of the system dynamics into a higher dimensional linear space over which the MPC (model predictive control) is exercised, thereby scaling as well as expediting the MPC computation for its real-time implementation for practical systems. We develop a Koopman-inspired deep neural network (KDNN) architecture for the linear embedding of the voltage dynamics subjected to reactive controls. The training of the KDNN for the purposes of linear embedding is done using the simulated voltage trajectories under a variety of applied control inputs and load conditions. The proposed framework learns the underlying system dynamics from the input/output data in the form of a triple of transforms: A Neural Network (NN)-based lifting to a higher dimension, a linear dynamics within that higher dynamics, and a NN-based projection to original space. This approach alleviates the burden of an adhoc selection of the basis functions for the purposes of lifting to higher dimensional linear space. The MPC is computed over the linear dynamics, making the control computation scalable and also real-time. We validate the efficacy of the approach via application to the standard IEEE 39 Bus systems.

Index Terms—MPC, Deep learning, Voltage control, Koopman operator, Data-driven method

I. INTRODUCTION

RAPID modernization of the power grid with massive integration of renewable generation resources, dynamic loads, and inverter-based resources has necessitated reliable, fast, and real-time controlled power infrastructures to mitigate any chances of instability following any disturbances in system operations. At the same time, owing to the system dynamics’ complex, nonlinear, high-dimensional nature, the model-based design of control in power systems has always been challenging. To this end, data-driven control approaches offer opportunities, and have started to emerge as a potential alternative to the conventional model-based control designs in cyber-physical systems (CPSs) [1], [2]. A data-driven approach replaces the need for comprehensive knowledge of the system model that can help make the control computation fast and real-time, essentially trading a small modeling accuracy for a huge gain in speed-up of control computation. The advent of deep learning technologies combined with reinforcement learning (RL) is starting to be also explored in power system controls [3]. But, those pre-dominantly appear in the feedback-loop where their training with or without the use of models is challenging for practical power systems. An appealing alternative is to employ learned models (as opposed to learned controls) and perform model-based optimizations. Inspired by Koopman operator theory, here we propose to learn the underlying system dynamics in the form of a triple of transforms: A NN-based lifting to a higher dimension, a linear embedding within that higher dimension, and a NN-based projection to the original space. We propose to employ MPC for control optimization computation directly on the linear embedding, thereby making the control computation scalable as well as real-time.

Previous works [4]–[6] have shown the potential of MPC in power system optimal regulation. But, it is well accepted that designing MPC for real-time power system application is challenging, even with the incorporation of different approximation strategies, for instance, trajectory sensitivity-based methods [5], [7]. In contrast, the design of any standard control technique, including that based on MPC, for linear systems dynamics is much simpler, scalable, and computationally efficient. Motivated by these, we present a method of linear embedding of nonlinear power system dynamics, specifically the voltage dynamics, and design its MPC over the embedded linear dynamics, making the entire setup scalable and real-time (which is a primary drawback of the otherwise promising MPC for power systems and other complex CPSs).

Our work is inspired by linear Koopman embedding of nonlinear dynamics. Recent advances in data driven methods have made Koopman operators a leading candidate for the linear representation of nonlinear systems [8]. According to Koopman theory, the nonlinear dynamical system is lifted to a higher-dimensional space using nonlinear basis functions, where the evolution of lifted states is governed by a linear operator, known as the Koopman operator, and whose projection to the original space provides an approximation to the original trajectory of the nonlinear system [9], [10]. While obtaining a linear embedding is promising, lifting nonlinear dynamics to a higher dimensional linear space is nontrivial since an appropriate set of nonlinear bases need to be identified. Standard approaches utilize some predefined dictionary of basis functions, for instance, radial or polynomial basis functions for such lifting, that are not necessarily the best choices. Building an appropriate dictionary of basis functions for an unknown dynamics is challenging, which restricts the application of Koopman-based analysis of real-world systems. To alleviate this problem, recent researchers have leveraged advances in data-driven approaches to learn the basis functions required to lift the nonlinear dynamics and also learn the linear Koopman embedding [11]–[17].

The advantages of learning-based linear Koopman embedding of nonlinear dynamics have also been recognized...
Contributions: To summarize, the main contributions are:

- An end-to-end KDNN training methodology that only utilizes the trajectory data collected from simulating the nonlinear dynamics to a higher dimensional linear state space by way of employing the theory of Koopman operators.
- A Koopman-based data-driven linear embedding of an unknown nonlinear dynamics is presented, that comprises of a deep learning-based end-to-end encoder-decoder architecture, termed KDNN, for lifting any unknown nonlinear dynamics to a higher dimensional linear space.
- The history at instant $k$ is captured in a matrix form:

$$V_k^{\text{hist}} = \begin{bmatrix} V_{k,0}^1 & \cdots & V_{k,H-1}^1 \\ \vdots & \ddots & \vdots \\ V_{k,0}^n & \cdots & V_{k,H-1}^n \end{bmatrix}.$$  

In contrast, the control input $U_k$ only takes the value at the instant $k$. Henceforth, with a slight abuse of notation, we use $V_k$ to represent $V_k^{\text{hist}}$.

In a practical setting, the control implementation occurs at a larger time period $T_c$, compared to the simulation time period, $T_s$. We let $H = T_c / T_s$ denote the length of simulation history between any two control instants. The controls are held constant in between the two control instances, while the voltage dynamics continues to evolve forming a time-series of length $H$ in between any two control instants. Thus at each instant $k$, there exists the temporal correlation captured in the time-series of voltage values between the instants $k - 1$ and $k$, and we use this history in learning the voltage dynamics. The history at instant $k$ is captured in a matrix form:

$$V_k^{\text{hist}} = \begin{bmatrix} V_{k,0}^1 & \cdots & V_{k,H-1}^1 \\ \vdots & \ddots & \vdots \\ V_{k,0}^n & \cdots & V_{k,H-1}^n \end{bmatrix}.$$  

Next we discuss the lifting of the nonlinear mapping $\mathcal{T}(\cdot, \cdot)$ to a higher dimensional linear state space by way of employing the theory of Koopman operators.

### B. Koopman Operator and Lifting of Voltage Dynamics

Let’s consider a discrete time nonlinear dynamics:

$$x_{k+1} = f(x_k, u_k)$$  

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $f$ is the vector field. We adopted the lifting mechanism presented in [10] which only lifts the state variables $x$ to a higher dimension, leaving the control variables $u$ unlifted. According to koopman operator theory for finite dimensional linear approximation of $4$, there exist $N >> n$, real-valued nonlinear basis functions (lifting functions) $G_i : \mathbb{R}^n \to \mathbb{R}$ for $i = 1, \cdots, N$ forming $G = [G_1, \cdots, G_N]^T$, which lifts the original state space to the higher dimensional state-space so that in the lifted space, the system follows the linear dynamics:

$$G(x_{k+1}) = A G(x_k) + B u_k,$$

where $A \in \mathbb{R}^{N \times N}$, and $B \in \mathbb{R}^{N \times m}$ are the Koopman operators associated with the state and control spaces respectively.
Collecting $T$ tuples $\{(x_k, u_k, x_{k+1})\}^T_{k=1}$ from each trajectory data of the given nonlinear system under different initial conditions, one way to obtain the Koopman operators for a finite dimensional approximation is to utilize the method of extended dynamic mode decomposition (EDMD) [9], [10], where a predefined dictionary of a limited set of the possible bases is utilized. Instead of relying on such a predefined dictionary of basis function choices, here we explore the power of data-driven learning of the basis functions, canonically represented as deep neural networks (DNNs), that have proved to be a good choice as demonstrated in [13]. Since a linear embedding is a special case of DNN, the data-driven approach not only learns the lifting functions (mapping to higher dimension), but also the linear embedding (in the higher dimension) as well as the projections (mapping to original lower dimension).

Following the Koopman operator theory that approximates (4) as (5), we can write the voltage dynamics (2) in the lifted linear space as:

$$G(V_{k+1}) = A G(V_k) + B U_k$$  \hspace{1cm} (6)

The encoder-decoder KDNN architecture to realize (6) is presented in Fig. 1, where the structure of the input $V_k$ in the form of a matrix is as provided in (3), with its dimension being $n \times H$, and which can also be flattened to form an input of length $n \times H$ at each control instant $k$. Since this input length can be long increasing the complexity of the network to train, while the relevant temporal relation may only be captured in a condensed history, we extract the relevant temporal correlation in a condensed form by first training an LSTM (a type of recurrent neural network that can learn the long-term dependencies in a given time series), before the lifting functions. Subsequent to the LSTM layer, fully connected NN (FCNN) layers are used to train for the lifting, embedding linear dynamics, and projection (see Fig. 1). LSTMs capture the time dependencies of the data using three gates, namely input gate, forget gate, and output gate as detailed in [19]. Note together the operation of the LSTM layer and the initial FCNN layer define the lifting function $G(\cdot)$ as shown in Fig. 1. Once lifted to a suitable higher dimension, the linear transformation of $G(V_k)$ by the Koopman operator $A(\cdot)$ is an instance of another FCNN (with identity activation function) to be trained as shown in Fig. 1. Similarly, the input $U_k$ is also processed through the Koopman operator $B(\cdot)$, which is yet another instance of a FCNN with identity activation function (see Fig. 1). The output of these two operation is added in the lifted space to obtain $A G(V_k) + B U_k$, which equals $G(V_{k+1})$ as per (6). Finally to project to the original state-space, we utilize a decoder architecture shown in Fig. 2. The decoder, denoted $G^{-1}(\cdot)$, does the inverse operation of the encoder $G(\cdot)$, and takes $G(V_{k+1}) = A G(V_k) + B U_k$ as input and maps it to predicted $\hat{V}_{k+1}$. Since we also have the signal $G(V_k)$, the same decoder is also used to obtain the estimated version $\hat{V}_k$ so as to compared against the the ground truth $V_k$ and thereby ensure the accuracy of the decoder. The output layer is also a cascade of a FCNN and a LSTM layer as the input layer, but placed in a reverse order.

To summarize, the complete end-to-end architecture of the proposed KDNN is as shown in Fig. 3. The end-to-end KDNN is parameterized by $\theta$, and can be expressed as $F(\cdot, \theta)$. The training input, output for the end-to-end KDNN are: $X = \{V_k, U_k\}$, $Y = \{\hat{V}_{k+1}, V_k\}$, respectively, while the predicted output of the KDNN are $\hat{Y} = \{\hat{V}_{k+1}, \hat{V}_k\} = F(X, \theta)$. We used mini-batch stochastic gradient descent to train $F(\cdot, \theta)$. The loss function corresponds to the reconstruction error of both $\hat{V}_{k+1}$ and $V_k$: $L(\theta) = \frac{1}{T} \sum_{i=0}^{T-1} ||Y_i - \hat{Y}_i||_2$, where $L$ denotes the batch size.
C. Formulation of MPC problem

The objective of the control design is to minimize the post-disturbance voltage deviations with respect to a user-defined reference value $V_{ref}$. For this we employ MPC, which by design computes, at each control instant, an optimal sequence of control inputs for the remaining control horizon by optimizing a predicted future behavior of the underlying system, implements the first control action of the computed sequence, and then repeats the same process with the new measurements at the next control instant [5]. This iterative control computation, employing new state measurements, helps to correct the effects of modeling error as compensated by the new measurement taken. Using the trained KDNN architecture, we can extract the functions $G(\cdot), A(\cdot),$ and $B(\cdot)$ that are then utilized in the computation of the MPC in the lifted linear space as formulated next.

The voltages $V_k$, known from the measurements at control instant $k$ can be lifted to $Z_k = G(V_k)$. For the optimization formulation, we also lift the reference voltage $V_{ref}$ to obtain $Z_{ref} = G(V_{ref})$. Then at any control instant $k$, the model-predictive optimization problem that needs to be solved is an instance of LQR (linear quadratic regulator):

$$
\min_{U_{k},\ldots,U_{k+Nk-1}} \sum_{i=0}^{Nk-1} \left[ (Z_{k+i+1} - Z_{ref})^T Q (Z_{k+i+1} - Z_{ref}) + U_{k+i}^T R U_{k+i} \right] \tag{7a}
$$

subject to:

$$Z_{k+i+1} = A Z_{k+i} + B U_{k+i}, \quad \forall i \in [0, Nk - 1], \tag{7b}$$

$$U_{\min} \leq U_{k+i} \leq U_{\max}, \quad \forall i \in [0, Nk - 1], \tag{7c}$$

where $N_k$ is the number of control instants at instant $k$, $U_{\max}$ and $U_{\min}$ are the control bounds, and $Q \in \mathbb{R}^{N \times N}$ and $R \in \mathbb{R}^{m \times m}$ are the designer specified weight matrices (with $N$ being the dimension of lifted space, and $m$ being the dimension of the control space).

III. Simulation Results

Our proposed methodology is validated through application to the IEEE-39 Bus system to generate a closed loop MPC-based voltage stabilization policy following a 3-phase fault, with the objective of keeping the voltage trajectories close to a reference value $V_{ref} = 1.00$ p.u. under minimal control effort. The critical part of this design is the training the KDNN architecture and learning the linear embedding for the voltage dynamics.

To formulate the problem, we consider a fault at bus 15, which gets cleared by tripping a line between bus 15 and 16 within six cycles of system operation. Following the bus fault, voltages in the vicinity of the fault bus drop below the desired level almost immediately, and needs stabilization to avoid any potential collapse. Since the tripping of line between the buses 15 and 16 creates a disconnect between the region of Fig. 4 marked by the dashed red lines and the rest of the system, the voltages of the buses in the marked region are most affected, namely the voltages in the set:

$$S = \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}.$$ It then suffices to provide reactive compensation in the marked region, implying that the number of buses of interest is $n = |S| = 12$. For the control of this region, we consider reactive control inputs at buses 4, 5, 7, 8, and 15, that can provide fast reactive support of up to 0.25 p.u. in each control step. To stabilize the voltage within 15 sec, the control horizon is set as 15 secs, divided into 5 segments of 3 secs each, i.e., $T_s = 3$ and there are a total of $15/3 = 5$ control instant. Note the sampling duration is smaller by a factor of 4, i.e., $T_c = T_s/4 = 3/4$ sec, which implies the length of time-series between two control samples is $H = T_c/T_s = 4$.

For training the proposed KDNN, we created a large pool of training data by simulating the system voltage trajectory under various load conditions within ±10% of the nominal load, and also under different possible length-5 sequences of controls for each load condition, where the controls at each instant were chosen randomly in the range $[0, 0.25\text{p.u.}]$. In total, we created 2500 random load conditions, 3 different controls at each of the 5 control steps, collecting a total of $2500 \times 5 \times 5 = 37500$ data samples in the form of $\{(V_k, U_k, V_{k+1})\}$ triples. We divided the data into 70 : 30 ratio to create the training versus the test data sets.

As shown in Fig. 1, the lifting function $G(\cdot)$ is built using an LSTM layer followed by an FCNN layer, where the number of neurons of the FCNN layer decides the dimension $N$ of the lifted space. Given that $n = 12$, we tested $N = 64 > 5n$. The input and output layers of the KDNN employs non-linearity in the form of the tanh(·) activation function for the LSTM as well as the FCNN layers, whereas the matrices $A$ and $B$ are represented as single layer FCNNs without any activation function. The optimizer chosen for the training is ADAM, with gradient momentum $\beta_1 = 0.9/0.95$ and RMS momentum $\beta_2 = 0.999/0.95$. The loss function, batch size, learning rate, and performance metric are: mean squared error (MSE) loss, 32, $10^{-3}$, and mean absolute error (MAE), respectively. To facilitate fast and efficient learning and avoid over-fitting, we adopted the standard practice of pre-processing the input and
output data of the KDNN: Firstly, \( V_{\text{ref}} = 1.00 \) is subtracted from the voltage values \( V_k \) and \( V_{k+1} \) of the stored data. Next, this modified data is normalized in the range \([0, 1]\) with respect to this modified data set’s minimum and maximum value. The control values \( U_k \) in the range of \([0, 0.25]\) is normalized between \([-1, 1]\).

The training and testing performances of reconstruction of \( V_{k+1} \) (output-1) and \( V_k \) (output-2) are shown in Fig. 5.A in terms of MAE, and in Fig. 4.B in terms of the coefficient of determination or \( R^2 \in [0, 1] \) score. (A \( R^2 \) value of 1 indicates an exact fit.) It should be noted that both training and testing performances are satisfactory, hence we utilized the trained KDNN to extract the functions \( G(\cdot), A(\cdot), \) and \( B(\cdot) \) for control computation in the lifted linear space.

Considering the objective of minimizing the voltage deviations from a reference, we choose \( Q = I_{N \times N} \) and \( R = 0 \). For validating the robustness of the proposed control design, we considered 5 different load levels 90%, 95%, 100%, 105%, and 110% of the nominal load. In addition to the fault at Bus-15, we also considered faults at Bus-4, Bus-6, and Bus-7. These faults get cleared by tripping the transmission lines between Bus-3 and Bus-4, Bus-6 and Bus-7, and Bus-7 and Bus-8, respectively.

The MPC computations were done in the linear embedded state-space solving the LQR of (7a). Next the computed controls were applied to the original nonlinear system. The voltage profiles for each of the above cases are shown in Fig. 6 and Fig. 7, validating that the proposed scheme successfully achieved the desired voltage performance under different operating conditions, also confirming the effectiveness and robustness of the proposed methodology of designing controls using a lifted linear embedding of the nonlinear dynamics.

We also computed the control actions at each control instants, and plotted the accumulated control actions for the 5 different load cases in Fig. 8 and Fig. 9. The trend suggests that with increase of load, the amount of controls introduced increased, which is to be expected.

**IV. CONCLUSIONS**

The paper proposed and implemented a framework for the data-driven linear embedding of the implicit voltage dynamics of a power system (whose DAE model does not provide the voltage dynamics in an explicit manner), paving the way for
designing an MPC-based voltage control strategy in the lifted linear state-space for the first time. We combined the concept of Koopman operator theory for lifting a nonlinear dynamics into a higher dimensional linear dynamics with the power of deep learning to learn the lifting and the projection functions (of the encoder decoder respectively), together with the linear high dimensional embedding. This data-driven learning of the basis functions removed the burden of selecting an appropriate set of basis functions (traditionally taken to be polynomials or radial bases arbitrarily). The test results applied to the IEEE 39-bus system validated the performance of the newly proposed scheme in terms of efficacy, robustness against load variations as well as different fault conditions. We also validated that our approach is superior in performance compared to the standard controls.

Our promising technique of unraveling the implicit nonlinear dynamics combining Koopman theory and deep learning methods is generic that opens up a new direction of control study for any complex nonlinear systems.

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