Cryptanalysis of a Practical Quantum Key Distribution With Polarization-Entangled Photons

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Abstract

Recently, a quantum key exchange protocol has been described [6], which served as basis for securing an actual bank transaction by means of quantum cryptography [8]. Here we show, that the authentication scheme applied is insecure in the sense that an attacker can provoke a situation where initiator and responder of a key exchange end up with different keys. Moreover, it may happen that an attacker can decrypt a part of the plaintext protected with the derived encryption key.

Keywords: quantum key exchange, authentication, cryptanalysis

1 Introduction

In April 2004, in Vienna an actual bank transfer was protected by means of a one-time-pad-based encryption [8] where the one-time-pad has been derived by means of a quantum key exchange using a novel authentication scheme. However, as pointed out, e.g., by Raub et al. [7], using the “textbook version” of a one-time-pad for encrypting a bank transfer is not a suitable choice, if the plaintext involves no further integrity protection: assume, for instance, the amount of money to be transferred is represented as an ASCII string which is XORed with the one-time-pad. Then, by just flipping certain bits in the ciphertext, an attacker may change the amount of money to be transferred. Similarly, the attacker may be able to change the name of the recipient of the money. Thus, the one-time-pad encryption should be combined with (unconditionally secure) means ensuring the authenticity of the plaintext [7]. However, even a scheme modified in this sense would not provide a secure bank transfer:

In this contribution we describe an attack on the quantum key exchange scheme itself that has been used in the Vienna experiment. Due to a flaw in the classical authentication part, an attacker may gain access to a part of the plaintext later encrypted under an established key. Also she may provoke a situation where the participants of the key exchange end up with different keys without noticing this. Of course, a trivial denial-of-service-attack (“cutting all wires”) may also prevent the users from establishing a shared key; but the attack presented here is more severe in the sense that both protocol participants obtain a key which they might bring to use, as—differing from the “cut the wires approach”—the failure of the key exchange remains undetected.
2 The quantum key exchange scheme used in Vienna

The published version of the quantum key exchange protocol does not describe all details of the Vienna experiment. However, for describing our attack this is not really necessary, and it is sufficient to look at the (classical) privacy amplification and authentication part. Owing to the attacks described below, the published version of these parts of the protocol [5] deviates from the version used in the Vienna experiment; from the latter at the time of writing only a poster presentation [4] was available to us, and we are indebted to Momchil Peev for kindly providing us with further details [3]. In summary, for establishing a common key between Alice and Bob, the following steps are performed:

- A raw key between Alice and Bob is established by means of polarization-entangled photons.
- In a sifting step, parts of the raw key are discarded based on a public discussion between Alice and Bob.
- Next, the quantum bit error rate is estimated based upon which the protocol is either aborted or continued with an error correction step.
- Hereafter, privacy amplification is performed, based on a matrix sent from Alice to Bob. The result of the privacy amplification is the final key, if the subsequent authentication step succeeds.
- Finally, a protocol-log extract is formed from the messages sent throughout the protocol so far; the authenticity of this log is checked by means of a message authentication procedure. The final key from the privacy amplification phase is accepted if and only if this authentication check succeeds.

As already indicated above, for our attack only the last two steps are of importance, as only one variant of our attack interferes with the quantum part of the protocol.

Privacy amplification This part of quantum key exchange protocols has been introduced by Bennett et al. [1] and is based on a binary rectangular matrix $P_A$ with random entries. Multiplying $P_A$ with the raw key yields the shorter final key about which the adversary has only negligible information. Thus, each row of $P_A$ determines one bit of the final key.

Protocol-log extract The protocol-log extract is comprised of five parts (and has to be identical for Alice and Bob) [3]:

- the basis for each sifted bit;
- the positions of the bits disclosed in the process of error estimation;
• the estimated error rate;
• the positions of the bits corrected by the specific error correction routine;
• the last 128 bits of the jointly generated key (these are subsequently discarded).

Note that the only part of the protocol-log extract influenced by the privacy amplification matrix are the last 128 bits of the jointly generated key; the privacy amplification matrix itself is not explicitly included, and the attack described in the next section exploits this.

3 Cryptanalysis of the scheme

By construction of the protocol, only 128 rows of the privacy amplification matrix \( P_A \) affect the protocol-log transcript. The remaining rows of \( P_A \) remain unauthenticated and hence can be modified by the attacker at will. E.g., she can

• replace all (but the last 128) rows of \( P_A \) by random vectors. Consequently, the receiver Bob of \( P_A \) will end up with a key different from Alice’s, but neither Alice nor Bob is aware of this fact. In particular, later bringing this key to use may result in the failure of an application—even when the attacker is not interfering with that application;

• flip an individual entry in the \( i \)-th row of \( P_A \). Then with a success probability of ca. 0.5 (namely, if the corresponding bit in the raw key is set) she can flip the \( i \)-th bit of the key derived by Bob.

Now suppose that the attacker succeeds in measuring a small number of qubits—logarithmic in the total number of qubits sent—and assume further that few qubits of the sifted key after error correction are known to the attacker (this happens with polynomial probability). Then the attacker may proceed as follows: She replaces one row of the privacy amplification matrix with a binary vector containing ones only at positions corresponding to bits of the sifted and error-corrected key she knows. In this way she learns a bit of the final key derived by Bob. Consequently, if the key later is used to encrypt a message from Bob to Alice by a bit-wise XOR, then the attacker immediately learns the respective plaintext bit. In fact, in the proposed form of the key exchange protocol, the attacker may use a trivial method for learning the complete key derived by Bob: even replacing—up to the last 128—all rows of the privacy amplification matrix by zero vectors remains undetected and results in the all-zero key for Bob.
4 Including the privacy amplification matrix into the protocol-log extract

From the above discussion it may be tempting to conclude that including the complete privacy amplification matrix into the protocol-log extract is sufficient for securing the protocol. However, to show that this approach does not offer acceptable cryptographic security let us consider a variant of the above protocol in which the complete matrix $P_A$ is included in the protocol-log extract and authenticated. (We stress that this variant of the protocol has not been proposed or used for the Vienna experiment [6]).

Let us recall the authentication procedure applied in the Vienna experiment: For authenticating the protocol-log extract $M$, first it is compressed by a publically known cryptographic hash function $H_0$ like SHA-256, and for all subsequent computations $M$ is identified with its hash value under $H_0$. However, in the presence of an unlimited adversary such an identification does not rule out the following attack:

- The attacker impersonates Bob and follows the quantum key exchange up to the point where Alice sends the authenticated hash $H_0(M_A)$ of her protocol-log extract $M_A$. Here the attacker aborts the protocol with Alice.

- Now, the attacker impersonates Alice and initiates a quantum key exchange with Bob. The attacker follows the protocol up to the point where the privacy amplification matrix $P_A$ is to be chosen.

- Instead of choosing a random $P_A$, she makes an exhaustive search over all possible matrices of the appropriate size to find a matrix $P'_A$ which, when included in the protocol-log extract, yields the same hash value $H_0(M_A)$ as obtained from Alice. Such a $P'_A$ exists with overwhelming probability, if we model $H_0(\cdot)$ as a random oracle. As there are significantly more degrees of freedom in the privacy amplification matrix than in the typical output of a cryptographic hash function (like, e.g., SHA-256), the existence of such a $P'_A$ is plausible.

  For actually performing this exhaustive search, the attacker exploits that up to the last 128 bits of the final key (which only depend on the data collected so far and the privacy amplification matrix), the protocol-log extract is completely known.

- The privacy amplification matrix $P'_A$ along with the authenticated hash $H_0(M_A)$ obtained from Alice are sent to Bob, who will accept this as a valid authentication.

- The subsequent authentication information from Bob is ignored, and the attacker can impersonate Alice in the subsequent use of the final key.

To avoid the above attacks and ensure that the privacy amplification matrix is identical for Alice and Bob, Peev et al. [6] make use of a scheme of Gilbert and
Hamrick [2] where the privacy amplification matrix is not sent over the public channel but derived from previously authenticated data.

5 Conclusions

The above discussion shows that in the original form the quantum key exchange scheme used in the Vienna protocol [4, 6] does not offer acceptable cryptographic security. Similarly as the “malleability problem” pointed out by Raub et al. [7], our attacks focus on the classical parts of the protocol and provide evidence of the importance of classical cryptographic aspects in quantum cryptography.

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