The optimal redshift for detecting ionized bubbles in H I 21-cm maps

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ABSTRACT

The detection of individual ionized bubbles in H I 21-cm maps is one of the most promising, direct probes of the epoch of reionization (EoR). At least 1000 h of observation would be required for such a detection with either the currently functioning Giant Metrewave Radio Telescope (GMRT) or the upcoming Murchison Widefield Array (MWA). Considering the large investment for telescope time, it is essential to identify the ‘optimal redshift’ where the prospects of a detection are most favourable. We find that the optimal redshift is determined by a combination of instrument dependent factors and the evolution of the neutral fraction \(x_{\text{HI}}\). We find that the redshift range \(8.1 \pm 1.1\) and \(9.8 \pm 1\) are optimum for detecting ionized bubbles with the GMRT and MWA, respectively. The prospects of a detection, we find, are more favourable in a scenario with late reionization with \(x_{\text{HI}} \approx 0.5\) at \(z \approx 7.5\) as compared to an early reionization model where \(x_{\text{HI}} \approx 0.5\) at \(z \approx 10\). In the late reionization scenario, for both instruments a \(3\sigma\) detection is possible for bubbles of comoving radius \(R_b \geq 30\) Mpc with 1000 h of observation. Future observations will either lead to the detection of ionized bubbles, or in the event of non-detection, lead to constraints on the product \(x_{\text{HI}} R_b^\gamma\) for the observational volume, where \(\gamma = 1.5\) and 2 for GMRT and MWA, respectively.

Key words: methods: data analysis – cosmology: theory – cosmology: diffuse radiation.

1 INTRODUCTION

It is currently accepted that the Universe was reionized by the growth of ionized bubbles around luminous sources in the redshift range \(z \sim 6–15\) (Choudhury & Ferrara 2006a; Fan et al. 2006; Komatsu et al. 2008). Detection of individual ionized bubbles (H II regions) in H I 21-cm maps of reionization is one of the major, important approaches that will be adopted by the present and upcoming radio experiments [Giant Metrewave Radio Telescope (GMRT), MWA, Low Frequency Array, Square Kilometer Array (SKA)] to probe the epoch of reionization (EoR). Such observations will directly probe the properties of the ionizing sources and the evolution of the surrounding intergalactic medium (IGM) (Wyithe & Loeb 2004; Wyithe, Loeb & Barnes 2005; Gei & Wyithe 2007; Maselli et al. 2007) and are expected to complement the study of reionization through the power spectrum of H I brightness temperature fluctuations. Detection of individual bubbles is a big challenge because the H I signal will be buried in strong foregrounds and system noise (Ali, Bharadwaj & Chengalur 2008).

In an earlier paper (Datta, Bharadwaj & Choudhury 2007, hereafter Paper I), we have proposed a visibility based matched filter technique to optimally combine the entire H I signal from an ionized bubble while removing the foregrounds and minimizing system noise. Using visibilities has an advantage over image based techniques, because the system noise contribution in different visibilities is independent whereas the noise in different pixels of a radio-interferometric image is not. Our investigations show that for both the GMRT and the MWA, at redshift \(z = 8.5\), it will be possible to detect ionized bubbles of comoving radius \(R_b \geq 40\) and \(> 22\) Mpc in 100 and 1000 h of observations, respectively. We also find that fluctuations in the H I outside the bubble that we are trying to detect impose a fundamental restriction on the smallest bubble that can be detected. Assuming that the H I outside the bubble traces the dark matter, we find that it will not be possible to detect bubbles with comoving radius less than 8 and 16 Mpc with the GMRT and the MWA, respectively, however large be the integration time. In a subsequent paper (Datta et al. 2008, hereafter Paper II), we have used simulations to validate our matched filter technique and assess the impact of patchy reionization outside the bubble that we are trying to detect on the bubble detection.

The question ‘What is the optimal redshift for bubble detection?’ is particularly important when planning future observations. Estimates show (Papers I and II) that at least 1000 h of observation will be required for a detection with either the GMRT or the MWA. Considering the large investment in observing time, it is important to target the redshift where the prospect of a detection is most favourable. In addition, it is important to have a clear picture of different factors that contribute towards deciding the most optimal...
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2 THE MATCHED FILTER TECHNIQUE FOR DETECTING IONIZED BUBBLES IN REDShiftED 21-CM MAPS

The visibility recorded in a radio-interferometric observation of an ionized bubble can be written as

\[ V(U, v) = S(U, v) + HF(U, v) + N(U, v) + F(U, v). \]  (1)

Here, we refer to \( U = d/\lambda \) as a baseline, \( d \) being the physical separation between a pair of antennae projected on the plane perpendicular to the line of sight and \( \lambda \) is wavelength corresponding to the observed frequency \( v \). In equation (1), \( S(U, v) \) is the \( H_I \) signal from the ionized bubble, \( HF(U, v) \) is the contribution from fluctuations in the \( H_I \) outside the target bubble, \( N(U, v) \) is the system noise and \( F(U, v) \) is the contribution from other astrophysical foregrounds. The contributions \( HF(U, v), N(U, v) \) and \( F(U, v) \) are all assumed to be random variables with zero mean, whereby \( \langle V(U, v) \rangle = \langle S(U, v) \rangle \). The angular brackets here denote average with respect to different realizations of the \( H_I \) fluctuations, system noise and foregrounds.

We consider a spherical ionized bubble of comoving radius \( R_b \) centred at redshift \( z_c \), located at the centre of the field of view (FoV) the bubble is assumed to be embedded in a uniform IGM with neutral hydrogen fraction \( x_{HI} \). A bubble of comoving radius \( R_b \) will be seen as a circular disc in each of the frequency channels that cut through the bubble. At a frequency channel \( v \), the angular radius of the disc is \( \theta_e = (R_b/r_b) \sqrt{1 - (\Delta v/v_c)^2} \), where \( \Delta v = v_c - v \) is the distance from the bubble’s centre \( v_c = 1420 \text{ MHz}/(1 + z_c) \) and \( \Delta v_b = R_b/r_b \) is the bubble’s radius in frequency space. Here, \( r_b \), is the comoving distance corresponding to \( z = (1420 \text{ MHz}/v) - 1 \) and \( r_b' = dr_b/dv \). The expected visibility signal \( S(U, v) \) in each frequency channel is the Fourier transform of a circular disc which can be expressed in terms of \( J_1(2\pi v U \theta_e) \), the first order Bessel function (Paper I). In each channel, the signal has a peak value \( |S(0, v)| = \pi x_{HI} L_I \rho_I \), where \( L_I = 2.5 \times 10^7 \text{ cm}^3/\text{s} \) is the background \( H_I \) specific intensity expected from completely neutral medium. The signal is largely contained within baselines \( U \leq U_0 = 0.61/\theta_e \), where the Bessel function has its first zero crossing, and the signal is much smaller at larger baselines. The signal \( S(U, v) \) picks up an extra phase if the bubble is shifted from the centre of the FoV. The amplitude of the signal also falls because of the telescope’s primary beam pattern (Paper I), and in this paper we restrict our analysis to the most favourable situation where the bubble is at the centre of the FoV. The terms \( x_{HI}, L_I, \theta_e \) and \( \Delta v_b \) are all redshift dependent, and hence the signal too is strongly redshift dependent. In Section 3, we will discuss the combined effect of all these factors on bubble detection.

In order to detect an ionized bubble whose expected signal is \( S(U, v) \), we use the matched filter \( \hat{S}(U, v) \) defined as

\[ \hat{S}(U, v) = \left( \frac{v_c}{v_b} \right)^2 \left[ S(U, v) - \Theta \left( 1 - 2\frac{|v - v_b|}{B'} \right) \right] \times \frac{1}{B'} \int_{v_b - B'/2}^{v_b + B'/2} S(U, v') \, dv'. \]  (2)

Note that the filter is constructed using the signal that we are trying to detect. The term \( \left( v/v_b \right)^2 \) accounts the frequency dependent \( U \) distribution for a given array. The function \( \Theta \) is the Heaviside step function. The second term in the square brackets serves to remove the foregrounds within the frequency range \( v_b - B'/2 \) to \( v_b + B'/2 \). Here, \( B' = 4\Delta v_b \) is the frequency width that we use to estimate and subtract out a frequency independent foreground contribution. This, we have seen in Paper I, is adequate to remove the foregrounds such that the residuals are considerably smaller than the signal. Further, we have assumed that \( B' \) is smaller than the total observational bandwidth \( B \). The filter \( \hat{S}(U, v) \) depends on \([R_b, z_c, \theta_e, \theta_b]\) the comoving radius, redshift and angular position of the target bubble that we are trying to detect.

Bubble detection is carried out by combining the entire observed visibility signal weighted with the filter. The estimator \( \hat{E} \) is defined as

\[ \hat{E} = \left[ \sum_{a,b} S_a^*(U_a, v_b) \hat{S}(U_a, v_b) \right] / \left[ \sum_{a,b} 1 \right], \]  (3)

where the sum is over all frequency channels and baselines. The expectation value \( \langle \hat{E} \rangle \) is non-zero only if an ionized bubble is present, and it is zero if there is no bubble in the FoV.

The system noise (NS), \( H_I \) fluctuations (HF) and the foregrounds (FG) all contribute to the variance of the estimator \( \langle (\Delta \hat{E})^2 \rangle = \langle (\Delta E)^2 \rangle_{\text{NS}} + \langle (\Delta E)^2 \rangle_{\text{HF}} + \langle (\Delta E)^2 \rangle_{\text{FG}} \).  (4)

A 3σ detection is possible only if \( \langle \hat{E} \rangle > 3\sqrt{\langle (\Delta \hat{E})^2 \rangle_{\text{NS}}} \). In a situation where this condition is satisfied, the observed value \( E_0 \) may be interpreted as a detection if \( E_0 > 3\sqrt{\langle (\Delta \hat{E})^2 \rangle_{\text{NS}}} \).

Because of our choice of the matched filter, the contribution from the residuals after foreground subtraction \( \langle (\Delta \hat{E})^2 \rangle_{\text{NS}} \) is expected to be smaller than the signal (Paper I) and we do not consider it in the subsequent analysis. The contribution \( \langle (\Delta \hat{E})^2 \rangle_{\text{HF}} \) which arises from the \( H_I \) fluctuations outside the target bubble imposes a fundamental restriction on bubble detection. It is not possible to detect an ionized bubbles for which \( \langle \hat{E} \rangle \lesssim 3\sqrt{\langle (\Delta \hat{E})^2 \rangle_{\text{HF}}} \). Bubble detection is meaningful only in situations where the contribution from \( H_I \) fluctuations is considerably smaller than the expected signal. Once this condition is satisfied, it is the SNR defined as

\[ \text{SNR} = \langle \hat{E} \rangle / \sqrt{\langle (\Delta \hat{E})^2 \rangle_{\text{NS}}}, \]  (5)

which is important for bubble detection. The value of SNR peaks when the parameters of the filter exactly match the bubble that is actually present in the observation, and decreases from its peak value if there is a mismatch (Paper II). In the subsequent analysis,

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we shall use this to assess the redshift that is optimal for bubble detection. It is possible to analytically estimate \( \langle \dot{E} \rangle, \langle (\Delta \dot{E})^2 \rangle_{\text{NS}} \) and \( \langle (\Delta \dot{E})^2 \rangle_{\text{HF}} \) in the continuum limit (Paper I). We have

\[
\langle \dot{E} \rangle = \int d^2U \int dv \rho_0(U, v) S_\nu^*(U, v) S(U, v),
\]

\[
\langle (\Delta \dot{E})^2 \rangle_{\text{NS}} = \sigma^2 \int d^2U \int dv \rho_0(U, v) |S_\nu(U, v)|^2
\]

and

\[
\langle (\Delta \dot{E})^2 \rangle_{\text{HF}} = \int d^2U \int dv_1 \int dv_2 \left( \frac{dB_\nu}{dT} \frac{dB_\nu}{dT} \right) \times \rho_0(U, v_1) \rho_0(U, v_2) S_\nu^*(U, v_1) S(U, v_2) \times C_{2\nu/[v_1, v_2]}, \]

where \( \rho_0 \) is the specific intensity of blackbody radiation (which can be approximated as \( 2k_B T^5/c^2 \) in the Rayleigh–Jeans regime) and \( \rho_0(U, v) \) is the normalized baseline distribution function defined, so that \( \int d^2U \int dv \rho_0(U, v) = 1 \). For a given observation, \( d^2U \int dv \rho_0(U, v) \) is the fraction of visibilities in the interval \( d^2U \int dv \) of baselines and frequency channels. Further, we expect \( \rho_0(U, v) \propto v^{-2} \) for a uniform distribution of the antenna separations \( d \).

The term \( \sigma \) in equation (7) is the rms noise expected in an image made using the radio-interferometric observation being analysed. Assuming observations at two polarizations, we have

\[
\sigma = \frac{k_B T_{\text{sys}}}{A_{\text{eff}} \sqrt{N_{\text{obs}} B}},
\]

where \( k_B \) is the Boltzmann constant, \( T_{\text{sys}} \) the system temperature, \( A_{\text{eff}} \) the effective collecting area of an individual antenna in the array, \( N_{\text{obs}} \) the number of baselines, \( t_{\text{obs}} \) the total observing time and \( B \) the observing bandwidth. 

The contribution from H I fluctuations \( \langle (\Delta \dot{E})^2 \rangle_{\text{HF}} \) is calculated using \( \langle dB_\nu/(dT) \rangle \), the conversion factor from temperature to specific intensity at frequency \( \nu \), and \( C_{2\nu/[v_1, v_2]} \) the multifrequency angular power spectrum (MAPS; Datta, Choudhury & Bharadwaj 2007). The H I distribution during the EoR is highly uncertain. The value of \( \langle (\Delta \dot{E})^2 \rangle_{\text{HF}} \) is sensitive to the size and clustering of the ionized patches outside the target bubble (Paper II). Given the lack of information, we make the simplifying assumption that the H I outside the target bubble exactly traces the dark matter. This gives the most optimistic constraints on bubble detection, the constraints are more severe if patchy reionization is included.

This completely quantifies the dependence of the SNR on observation time, system temperature, neutral fraction, bubble radius and the background expansion history. In principle, measurements of the SNR will provide a unique and independent way to probe the source properties (through \( R_\nu \); Yu 2005), IGM and the background cosmology during the EoR. For the redshift range of our interest, it is reasonable to assume \( r_\nu \propto (1 + z)^{0.25} r'_\nu \propto (1 + z)^{0.5} \) and \( H(z) \propto (1 + z)^{2.5} \). Further, for the frequency range of our interest, the system temperature is dominated by the sky temperature which scales as \( T_{\text{sky}} \propto \nu^{-\beta} \) with \( \beta \sim 2.6 \) which implies \( T_{\text{sky}} \propto (1 + z)^{\beta} \). The effective collecting area is nearly constant for dish antennae like the GMRT, whereas it scales as \( A_{\text{eff}} \propto \nu^{-2} \) for dipoles (e.g. MWA). Combining all of these factors, we determine the scaling of the SNR with redshift

\[
\text{SNR} \propto x_{\text{HI}}(1 + z)^\alpha,
\]

where \( \alpha = -\beta - 1 \) or \( -\beta + 1 \) for dish antennae or dipoles, respectively. While \( x_{\text{HI}} \) increases with \( z \), the other term \( (1 + z)^\alpha \) has the opposite behaviour. These two competing effects decide the redshift where the SNR peaks, which is the optimal redshift for bubble detection.

The baseline distribution, in general, does not uniformly sample all baselines. Typically, the sampling falls at larger baselines and we do not expect the scaling relations discussed here to be exactly valid. The deviations from the scaling relations depend on the bubble size and the array configuration, and, in the next section, we discuss these for the GMRT and the MWA.

### 3 Scaling Relations

The scaling of the expectation value of the estimator with various parameters can be estimated from equation (6) whereby

\[
\langle \dot{E} \rangle \propto U_0^2 A_{\text{d}} B \nu^{-2} \left| S(0, v_0) \right|^2.
\]

Here, we have assumed that \( B \) is larger than the frequency extent of the bubble \( A_{\text{d}} \), and that the baselines in the array extend well beyond \( U_0 \). Further, it is assumed that the array configuration is such that the antenna separations \( d \) are uniformly sampled, whereby \( \rho(U, v) \propto v^{-2} \). Considering the noise contribution next, it also follows from equation (7) that \( \langle (\Delta \dot{E})^2 \rangle \propto \sigma^2 \langle \dot{E} \rangle \). We use these and the relations from the previous section to determine that the SNR scales as

\[
\text{SNR} \propto A_{\text{eff}} \sqrt{N_{\text{obs}} t_{\text{obs}}} \frac{1}{T_{\text{sys}}} \chi_{\text{HI}} \left( \frac{R_\nu^2}{r'_\nu r'_\nu} \right) \frac{1}{H(z)}.
\]

3.1 Evolution of neutral fraction with redshift

In this work, we consider two physically motivated models of reionization, namely, the early reionization (ER) and the late reionization (LR) scenario. These models are constructed using the semi-analytical formalism (Choudhury & Ferrara 2005; Choudhury & Ferrara 2006b) which implements most of the relevant physics governing the thermal and ionization history of the IGM, such as the inhomogeneous IGM density distribution, three different classes of ionizing photon sources (massive Pop III stars, Pop II stars and quasi-stellar objects), radiative feedback inhibiting star formation in low-mass galaxies and chemical feedback for transition from Pop III to Pop II stars. The models are consistent with various observational data, namely, the redshift evolution of Lyman-limit absorption systems (Storrie-Lombardi et al. 1994), the Gunn–Peterson effect (Songaila 2004), electron scattering optical depths (Kogut et al. 2003), temperature of the IGM (Schaye et al. 1999) and cosmic star formation history (Nagamine et al. 2005). We assume that these two models ‘bracket’ the range of models which are consistent with available data.

In ER scenario, hydrogen reionization starts around \( z \approx 16 \) driven by metal-free (Pop III) stars, and it is 50 per cent complete by \( z \approx 7.5 \). The contribution of Pop III stars decreases below this redshift because of the combined action of radiative and chemical feedback. As a result, reionization is extended considerably competing only at \( z \approx 6 \) (Fig. 1). In LR scenario, the contribution from the metal-free stars is ignored, which makes reionization start much later and is only 50 per cent complete only around \( z \approx 7.5 \). The main difference between the ER and LR models is in their predictions for the electron scattering optical depth (which is 0.12 and 0.06 for the ER and LR scenarios, respectively).
4 RESULTS AND CONCLUSIONS

We consider two possible definitions of the ‘optimal redshift’ for bubble detection. The first is the redshift where, for a fixed observing time and bubble radius $R_b$, the SNR is maximum. Another possibility is, for a fixed observing time and SNR, the redshift where a bubble of the smallest size can be detected. While the two definitions are the same if the instrument has uniform baseline coverage, we do not expect this to be true in general.

We have used equations (6)–(8) to calculate the SNR and determine the constraints from $\text{H} I$ fluctuations. The baseline distribution function $\rho_N(U, v)$, which we assume to be circularly symmetric $[\rho_N(U, v) = \rho_N(U)]$, has been calculated in Paper I for both the GMRT and the MWA. In both cases, $\rho_N(U)$ falls off with increasing $U$. For the GMRT, $\rho_N(U)$ is roughly constant for antenna separations $d < 1$ km and it extends out to large baselines $d \sim 25$ km. For the MWA, we have assumed that the antennae are distributed over a circular region of diameter 1.5 km, with the number density of antennae falling as $1/r^2$ with the distance from the centre.

We first consider, for a fixed bubble radius and observing time, how the SNR varies with $z$. Assuming $x_{\text{HI}} = 1$ and uniform baseline coverage, we expect that $\text{SNR} \propto (1 + z)^\alpha$ with $\alpha = -3.6$ and $-1.6$ for GMRT and MWA, respectively. For the GMRT, we find (Fig. 2) that the predicted scaling holds for large bubbles $R_b \geq 50$ Mpc where the entire signal lies within a small baseline range which is nearly uniformly sampled. For smaller bubbles, a significant amount of signal spreads over to larger baselines which are not uniformly sampled. We find that $\alpha$ changes, approximately linearly, from $-3.6$ to $-4.1$ as $R_b$ is varied from 50 to 20 Mpc. For the MWA, the non-uniform baseline coverage makes the scaling steeper than $-1.6$ for all values of $R_b$, and we find $\alpha = -2.4$ and $-2.5$ for $R_b = 50$ and 20 Mpc, respectively. We combine these findings with earlier results at $z = 8.3$ (Paper I) as to how the SNR scales with $R_b$ to obtain

$$\text{SNR} = x_{\text{HI}} K \left( \frac{t}{1000 \, \text{h}} \right)^{0.5} \left( \frac{1 + z}{10} \right)^\alpha \left( \frac{R_b}{50 \, \text{Mpc}} \right)^\gamma,$$  \hspace{1cm} (13)

where $t$ is the observing time, and $\alpha$, $\gamma$ and $K$ are parameters whose values are listed in Table 1. This expression is found to match the numerically computed SNR to within 20 per cent, which is quite adequate given the large uncertainty in $x_{\text{HI}}$.

Considering Fig. 3, which shows the SNR for the two reionization models, we find that it increases monotonically as $z$ decreases when $x_{\text{HI}} \approx 1$ and thereafter declines rapidly once $x_{\text{HI}} \leq 0.5$. The peak SNR, the corresponding optimal redshift $z_0$ and the $z$ range where the SNR is within 80 per cent of the peak value are tabulated in Table 2. Results have been shown only for $R_b = 50$ Mpc and $t = 1000$ h of observation, these can be easily scaled to other $R_b$ and $t$ values using equation (13). The $z$ dependence is not very different for smaller bubbles in the range $50 > R_b > 20$ Mpc.

The effective collecting area of the individual MWA antennae increases with wavelength as $\lambda^2$. This reduces the noise at higher redshifts, and puts the MWA at an advantage over the GMRT in detecting bubbles at high redshifts. This also pushes the optimal redshift for MWA to a higher value as compared to the GMRT (Table 2). The MWA is also at an advantage over the GMRT in

![Figure 1](https://academic.oup.com/mnrasl/article-abstract/399/1/L132/1203102)

Figure 1. The evolution of the mean neutral fraction $x_{\text{HI}}$ with redshift for the two different reionization models discussed in the text.

![Figure 2](https://academic.oup.com/mnrasl/article-abstract/399/1/L132/1203102)

Figure 2. Assuming $x_{\text{HI}} = 1$, the dashed lines show the predicted scaling of the SNR for uniform baseline coverage (equation 12), the solid lines are calculated numerically incorporating non-uniform baseline coverage. For both GMRT (upper panel) and MWA (lower panel), the upper curves are for $R_b = 50$ Mpc with 1000 h observation, and the lower curves for $R_b = 20$ Mpc with 4000 h.

![Figure 3](https://academic.oup.com/mnrasl/article-abstract/399/1/L132/1203102)

Figure 3. The SNR for $R_b = 50$ Mpc and 1000 h observation. Results are shown for both GMRT and MWA using the two different reionization models (ER and LR) discussed in the text.

| $K$     | $\alpha$ | $\gamma$ |
|---------|----------|----------|
| GMRT    | 9.1      | -3.6     | 1.5 |
| MWA     | 13.4     | -2.4     | 2.0 |

Table 1. Dimensionless parameters required to calculate the SNR using equation (13). The values of $R_b$ are restricted to the range $50$ Mpc $\geq R_b \geq 20$ Mpc.
differently with $R$ signal being smaller than the HI fluctuations in the MW A (Paper I).

$10 \text{ Mpc}$ where a detection is possible only with the GMRT, the LR scenarios, respectively. The shaded region is ruled out due to the HI fluctuations. We do not expect this to hold for smaller bubbles.

$\sim b$ for the two instruments (Table 1), and the ad-

$d$ on the product $x$ either lead to the detection of ionized $\gamma$ bubbles or lead to constraints on the product $x \gamma R^2$ for the observational volume in the event of non-detection.

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**Table 2.** For $R_b = 50 \text{ Mpc}$ and 1000 h of observation, the optimal redshift $z_o$ where the SNR peaks, the peak value and the $z$ range where the SNR is within 80 per cent of the peak value.  

| Instrument | Scenario | $z_o$ | Peak SNR | 80 per cent $z$ range |
|------------|----------|-------|----------|-----------------------|
| GMRT       | ER       | 9.2   | 3.6      | 7–12                  |
|            | LR       | 7.6   | 8.4      | 6.8–9.2               |
| MWA        | ER       | 11.0  | 6.59     | 8.8–14                |
|            | LR       | 8.4   | 11       | 7.1–10.8              |

**Figure 4.** SNR contours as a function of the redshift $z$ and comoving bubble radius $R_b$, considering 1000 h of observation with the GMRT (upper panels) and MWA (lower). The left-hand and right-hand panels show the ER and the LR scenarios, respectively. The shaded region is ruled out due to the HI fluctuations.

detecting large bubbles ($R_b \sim 50 \text{ Mpc}$; Fig. 3). The SNR scales differently with $R_b$ for the two instruments (Table 1), and the advantage that the MWA has for large bubbles balances out as the bubble size is reduced. GMRT and MWA have nearly comparable SNR for $R_b = 30 \text{ Mpc}$.

We next consider the other definition of the optimal redshift where for a fixed observing time and SNR, we determine $z_o$ where a bubble of the smallest size can be detected. Considering the constant SNR contours in Fig. 4, we find that the $z_o$ values are roughly consistent with those in Table 2. This shows that for both the GMRT and the MWA, for $50 > R_b > 20 \text{ Mpc}$ the two definitions predict the same optimal redshift which is approximately independent of the bubble size. We do not expect this to hold for smaller bubbles $R_b \sim 10 \text{ Mpc}$ where a detection is possible only with the GMRT, the signal being smaller than the HI fluctuations in the MWA (Paper I).

Given the lack of knowledge about the reionization history, it would be most judicious to choose a redshift where a high SNR is predicted for both the ER and LR models. We find that the redshift range 7–9.2 and 8.8–10.8 are most appropriate for the GMRT and MWA, respectively. For both instruments, the prospects of a detection are considerably improved in the late reionization sce-