Applications of computed Nuclear Structure Functions to Inclusive Scattering, $R$-ratios and their Moments

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Abstract

We discuss applications of previously computed nuclear structure functions (SF) to inclusive cross sections, compare predictions with recent CEBAF data and perform two scaling tests. We mention that the large $Q^2$ plateau of scaling functions may only in part be due to the asymptotic limit of SF, which prevents the extraction of the nucleon momentum distribution in a model-independent way. We show that there may be sizable discrepancies between computed and semi-heuristic estimates of SF ratios. We compute ratios of moments of nuclear SF and show these to be in reasonable agreement with data. We speculate that an effective theory may underly the model for the nuclear SF, which produces overall agreement with several observables.

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I. INCLUSIVE CROSS SECTIONS.

In the following we discuss three applications of previously computed nuclear structure functions $F^A_k$:

i) Cross sections for inclusive scattering of high-energy leptons from nuclei and associated scaling tests.

ii) Comparison of computed $R$ ratios and previously used, semi-heuristic methods aimed to isolate $F^A_2$

iii) Determination of moments of the above nuclear structure functions (SF) and comparison of their ratios with data.

We start with the cross section per nucleon of a one-photon induced process

$$\frac{d^2\sigma_{eA}(E;\theta,\nu)}{d\Omega d\nu} = \frac{2}{M}\sigma_M(E;\theta,\nu)\left[\frac{xM^2}{Q^2}F^A_2(x, Q^2) + \tan^2(\theta/2)F^A_1(x, Q^2)\right] \quad (1)$$

The inclusive, as well as the Mott cross for point-nucleons $\sigma_M$, appear above as functions of the beam energy $E$, the scattering angle $\theta$ and the energy loss $\nu$, but may also be expressed in alternative kinematic variables.

The two SF $F^A_k$ above describe the scattering of unpolarized electrons from randomly oriented targets and have been expressed as functions of the squared 4-momentum $Q^2 = q^2 - \nu^2$ and the Bjorken variable $x = Q^2/2M\nu$. We shall analyze recent data on inclusive scattering of 4.05 GeV electrons on various targets. The ranges of scattering angles $15 \lesssim \theta \lesssim 74$ and the measured energy losses $\nu$ correspond to $1 \lesssim Q^2(\text{GeV}^2) \lesssim 7$ and $0.20 \lesssim x \lesssim 4.2$, vastly extending the kinematic limits of the older NE3 SLAC and related experiments.

On the deep-inelastic side $0.2 \lesssim x \lesssim 1$ those overlap with the 'classical' EMC domain. Certainly there a description ought to include the quark-gluon sub-structure of the nucleon.

In the past we have proposed and applied a model, which relates SF’s $F^A_k$ and $F^N_k$ of a nucleus and a nucleon, by means of a SF $f^{PN}$ of a nucleus composed of point-nucleons. Disregarding virtual pions etc., one has

$$F^A_k(x, Q^2) = \int_x^A \frac{dz}{z^2-k} f^{PN}(z, Q^2) F^N_k\left(\frac{x}{z}, Q^2\right) \quad (2)$$
A similar equation for momentum fractions has been proposed before. Those quantities tend in the $Q^2 \rightarrow \infty$ limit to Bjorken variables and (2) states the approximate validity for large, finite $Q^2$: Its quality will deteriorate with decreasing $Q^2$.

A calculation of the above nuclear SF $F^A_k$ rests on two input elements. The first is the averaged SF of a nucleon $F^N_k \equiv F^{<p,n>}_k$, properly weighted with proton and neutron fractions in the nucleus. The non-perturbative model leading to (2) prescribes the $N$ to be on-mass shell, and its SF’s are therefore known.

The second element in (2) is the SF for a nucleus, composed of point-nucleons $f^{PN}$ and which accounts for nuclear dynamics. It is calculable in a relativistic extension of the non-relativistic (NR) Gersch-Rodriguez-Smith (GRS) series in $1/q$. The latter contains an asymptotic limit (AL), related to the single-nucleon momentum distribution (MD) $n(p)$, and Final State Interactions (FSI), dominated by hard binary collisions between the knocked-on nucleon and a nucleon from the core.

We remark that the model leading to (2) locates weak $A$-dependence of $F^A_k(x, Q^2)$ in the neutron excess $\delta N/2A$, and in $f^{PN}$, thus

$$F^A_k(x, Q^2) \approx F_k(x, Q^2) + O(1/A); \quad A \gtrsim 12,$$

(3)

The above mentioned input allows predictions to be made for the cross sections. From a comparison with the new CEBAF data on Fe in Fig. 1 one concludes:

i) For all but the smallest $Q^2$, there is good agreement in the (deep-)inelastic region $\nu > Q^2/2M$, $x < 1$ and satisfactory correspondence on the nucleon elastic (NE) side $x \gtrsim 1$, contiguous to the quasi-elastic peak (QEP).

ii) Since $Q^2$ increases with $\theta$, and for given $\theta$ with decreasing $\nu$, one observes that discrepancies grow with decreasing $\theta$, i.e. for decreasing $Q^2$, as expected. One estimates $Q^2_c(x, \theta) \approx 1.5$ GeV$^2$, below which the representation (2) may become progressively flawed.

iii) For each $\theta$, cross sections for the lowest energy losses $\nu$ drop orders of magnitude from their maximum. Theory overestimates the data there by a factor up to 2-3.

In spite of the above, it is not at all clear that in the latter regions there is a real
discrepancy: Alternative MD $n(p)$ for Fe, produce results which range over the area of the above mentioned local discrepancies without spoiling the agreement for higher $\nu$ (see Figs. 5,6 in Ref. 8).

The NE3 experiment has also been analysed by means of versions of the Plane Wave Impulse Approximation (PWIA) in terms of a spectral function, occasionally supplemented by additional FSI, e.g. $2p - 1h$ FSI on the PWIA. The above mentioned GRS and IA approaches agree very well with data, except for the smallest $\nu$, where Ciofi and Simula somewhat underestimate intensities, while our approach overestimates those. The otherwise surprising correspondence can be understood in the light of a recent proof, that a NR version of these two, quite different theories agree order-by-order in $1/q$.

We turn to scaling analyses, previously applied to the NE3 data. We first consider ratios of inclusive cross sections for different targets under identical kinematic conditions

$$\xi_{A_1,A_2} = \left( \frac{d^2\sigma^{eA_1}}{A_1} \right) / \left( \frac{d^2\sigma^{eA_2}}{A_2} \right), \quad (4)$$

for instance using a relativistic GRS-West scaling variable suggested by Gurvitz or the related $x$

$$y_G \approx (M/q)(\nu - Q^2/2M) \approx (Mq/\nu)(1 - x) \quad (5)$$

Originally the analysis had been limited to the region $y_G < 0$ below the QEP, and universal scaling for all $A \gtrsim 12$ had been observed. However, Eq. (3) holds for all, kinematically allowed $y_G$, including the (deep-)inelastic region $y_G > 0$. In spite of 4-5 orders of magnitude variations of cross sections, one has for all $y_G$, and independent of $E, x, Q^2$, $\xi_{A_1,A_2}(E, x, Q^2) \approx 1$ within 15-20%, and frequently better. Occasional larger deviations for data with lowest intensity are readily ascribed to experimental uncertainties. Table I illustrates the above for the pairs C,Fe and Fe,Au.

Next we focus on the EMC ratio $\xi_{NE}^{A,N}$, with $A_2 \rightarrow \langle N \rangle \approx D/2$. Both the NE region, where the nucleon remains intact, and the nucleon inelastic (NI) region, describing excitation or fragmentation of the $N$, contain information on the single-nucleon MD $n(p)$, implicit in
However, simple expressions for $\xi^{A,N}$ can only be given for $y_G < 0$. Densities, MD and pair-distribution functions $g_2$ are different for the $D$ and $A \gtrsim 12$ and consequently, $\xi^{A,N}_{NE}$ is not a special case of $\xi^{A_1,A_2}$.

Unfortunately the true NE part does not coincide with $y_G \leq 0$ which one wishes to investigate. As had already been realized in the analysis of early high-$E$ experiments on the lightest nuclei $D$, $^3$He, $^4$He, even on the elastic side $y_G < 0$ ($x > 1$) of the QEP (cf. Ref. 4, Fig. 4), NI and NE contributions compete. Consequently the NE part is not directly observable and a scaling analysis of $\xi^{A,N}_{NE}$ requires NI parts to be removed from the data. For instance, Eq. (2) provides a model for NI parts, which appear to be in perfect agreement with the data for $y_G > 0$: we assume the same for $y_G < 0$.

The procedure becomes impractical for $y_G \lesssim -0.25$, where both total and NI parts decrease rapidly. There one has to rely on directly calculated NE parts, using again (2), now with NE parts for $F^N_k$. The result in terms of static form factors reads

\begin{align}
F_1^{N(NE)}(x, Q^2) &= \frac{x}{2} [G_M^N(Q^2)]^2 \delta(x - 1) \\
F_2^{N(NE)}(x, Q^2) &= \frac{[G_E^N(Q^2)]^2 + \eta [G_M^N(Q^2)]^2}{1 + \eta} \delta(x - 1) \\
F_1^{A(NE)}(x, Q^2) &= \frac{1}{2} f^{PN}(x, Q^2)[G_M^N(Q^2)]^2 \\
F_2^{A(NE)}(x, Q^2) &= xf^{PN}(x, Q^2) \frac{[G_E^N(Q^2)]^2 + \eta [G_M^N(Q^2)]^2}{1 + \eta}
\end{align}

(6a)

Substitution in (1) yields expressions for the NE parts of cross sections and consequently for the corresponding parts of $\xi^{A,N}_{NE}$ ($\eta = Q^2/4M^2$)

\begin{align}
\xi^{A,N}_{NE}(x, Q^2) &= f^{PN}(x, Q^2) \left[ \frac{(x^2m^2/Q^2)[G_E^N]^2 + \eta [G_M^N]^2}{(1 + \eta)(m^2/Q^2)[G_M^N]^2} + \tan^2(\theta/2)[(G_M^N)^2 + \tan^2(\theta/2)[G_M^N]^2] \right] \\
\xi^{A,N}_{NE}(x \approx 1, Q^2) &= f^{PN}(x \approx 1, Q^2),
\end{align}

(7a)

(7b)

where arguments on $G^N$ have been dropped.

Fig. 2 displays $\xi^{F,e,N}_{NE}(y_G < 0, Q^2)$ against $Q^2$ for a number of narrowly binned $y_G$ data. Whenever possible, we give the result for the described procedures which, with the exception of $y_G = 0$, approximately agree. For all $y_G < 0$ in the kinematic range of the experiment,
$\xi_{NE}$ approaches the AL $Q^2 \to \infty$ from above, or can confidently be extrapolated to $Q^2$ beyond the observed ones. For the largest $|y_G|$, there is hardly any $Q^2$ dependence.

An observed plateau is conventionally related to the AL, from which one wishes to extract the MD $n(p)$. However, the standard argument becomes invalid, if parts of the FSI happen to be weakly $Q^2$-dependent, causing the plateau to also contain FSI parts. There are strong indications that this might be the case for the kinematic range of the CEBAF experiment. The interest in scaling analyses may well wane, if the AL cannot be separated from $Q^2$-independent FSI, thereby blocking a model-independent extraction of $n(p)$ (see Ref. [20] for details and an explanation for the seeming contradiction in the behaviour of $\xi_{NE}^{A,N}$ for low $Q^2$ as seen in Fig. 2).

II. R RATIOS.

Our second topic deals with the separation of the two nuclear SF $F^A_k$ in (1) and more specifically, with the isolation of the dominant $F^A_2$. This requires data for fixed $x, Q^2$ at different scattering angles $\theta$ or beam energies $E$, and those are not frequently available. Approximate methods start with alternative expressions for the cross section ratios

$$\frac{d^2\sigma_{eA}(E; \theta, \nu)}{d\Omega \, d\nu} / [A\sigma_M(E; \theta, \nu)] = \frac{2Mx}{Q^2} F^A_2(x, Q^2) \left[ 1 + \frac{2 \left( 1 + Q^2/4M^2x^2 \right) \tan^2(\theta/2)}{1 + R(x, Q^2)} \right]$$

(8a)

$$\frac{d^2\sigma_{eA}(E; \theta, \nu)}{d\Omega \, d\nu} / [A\sigma_M(E; \theta, \nu)] = \frac{2Mx}{Q^2} F^A_2(x, Q^2) \left[ 1 + \frac{Q^2}{2M^2x^2} \kappa(x, Q^2) \tan^2(\theta/2) \right],$$

(8b)

where

$$R^A = \frac{d^2\sigma_L}{d^2\sigma_T} = \left( 1 + \frac{4M^2x^2}{Q^2} \right) \frac{1}{\kappa^A(x, Q^2)} - 1$$

(9a)

$$\kappa^A(x, Q^2) = \frac{2xF^A_1(x, Q^2)}{F^A_2(x, Q^2)}$$

(9b)

$R$ is the ratio of cross sections for the scattering of longitudinal and transverse photons. It is related to the ratio of the two SF in $\kappa^A(x, Q^2)$, Eq. (13), which we call the nuclear Callen-Gross (CG) function. Its dependence on $A \gtrsim 12$ follows from [3] and reads
\[ \kappa^A = \kappa^{(N)} + \mathcal{O}(1/A) \approx \kappa^D(x, Q^2) + \mathcal{O}(1/A) \]
\[ R^A(x, Q^2) \approx R(x, Q^2) + \mathcal{O}(1/A), \]

which agrees with data. Recalling the CG relation for nucleons

\[ \epsilon^N_{CG} = \lim_{Q^2 \to \infty} \kappa^N(x, Q^2) = 1, \]

one finds from (13) and (14) its nuclear analog

\[ \epsilon^A_{CG} = \lim_{Q^2 \to \infty} \kappa^A(x, Q^2) = 1 + \mathcal{O}(1/A) \]

The latter relation can also be proven directly from (2), using (??), whereas the equality of nuclear and nucleonic CG functions (15) and (16) is compatible with (2), but does not follow from it.

First we mention an observation for the computed CG functions in the range (0.2-0.3) \( \lesssim x \lesssim (0.7 - 0.75) \); \( Q^2 \geq 5\text{GeV}^2 \)

\[ |\kappa^A(x, Q^2) - 1| \approx (0.11 - 0.12), \]

i.e. CG functions in those ranges are close to their asymptotic limit, the nuclear CG relation (12). Without apparent cause, theory predicts a sign change in \( \kappa - 1 \) at \( x_s \approx 0.5 - 0.6 \), which is in agreement with data from high energy \( \nu, \bar{\nu} \) inclusive scattering (see Fig. 18 in Ref. 24).

The following remarks relate to computed CG functions (13):

i) Disregard of other than valence quarks requires smoothing of \( F_k^N \) for \( x \lesssim 0.15-0.20 \), which entails the same for \( F_k^A \).

ii) Eq. (2) shows that beyond \( x \approx 1 \), \( f^{PN} \) draws on an ever smaller support of dwindling intensity and accuracy, rendering unreliable \( F_k^A(x, Q^2) \), and thus \( \kappa(x, Q^2) \), for \( x \gtrsim 1.3 \).

We briefly mention approximations \( R_n \) for \( R^A \approx R \), or for the CG function \( \kappa_n \). For those one has from (11)

\[ R(x, Q^2) = \beta_n(x, Q^2) R_n(x, Q^2) + \left( \beta_n(x, Q^2) - 1 \right) \]

\[ R(x, Q^2) = \beta_n(x, Q^2) R_n(x, Q^2) + \left( \beta_n(x, Q^2) - 1 \right) \]
Deviations of $\beta_n(x, Q^2) = \kappa_n(x, Q^2)/\kappa(x, Q^2)$ from 1 determine the quality of the approximation.

A) High-$Q^2$ approximation for $1 \lesssim x \lesssim 0.6$: $\kappa_L = [\beta_L]^{-1} = 1$,

\begin{align}
R^{\text{comp}}(x, Q^2) &= \beta_L(x, Q^2)R_L(x, Q^2) + \left(\beta_L(x, Q^2) - 1\right) \quad (15a) \\
&\approx R_L(x, Q^2) + \left(\beta_L(x, Q^2) - 1\right) \quad (15b) \\
R^{(1)}_L(x, Q^2) &= \frac{4M^2x^2}{Q^2} + \left(\beta_L(x, Q^2) - 1\right) \quad (15c) \\
R^{(2)}_L(x, Q^2) &= \frac{4M^2x^2}{Q^2}, \quad (15d)
\end{align}

with (19) the result, computed from Eqs. (2).

B) NE approximation for $x \approx 1$: Using (7) and exploiting in (8) the approximate scaling of static electro-magnetic form factors, one has $1/[\left(\mu_M^p\right)^2 + \left(\mu_M^n\right)^2] = 0.0874^{25}$ and thus

\begin{equation}
\kappa_{NE}^A = 2xF_1^{A(NE)}/F_2^{A(NE)} \\
\approx (0.0874 + \eta)/(1 + \eta) \quad (16)
\end{equation}

Inserting (16) into (18) gives ($Q^2$ in GeV$^2$)

\begin{align}
R(x, Q^2) &= \beta_{NE}(x, Q^2)R_{NE}(x, Q^2) + \left(\beta_{NE}(x, Q^2) - 1\right) \quad (17a) \\
R^{(1)}_{NE}(x, Q^2) &= \frac{0.31}{Q^2} + \left(\frac{0.31}{Q^2} + 1\right) \left(\frac{x^2 - 1}{1 + \eta}\right), \quad (17b) \\
R^{(2)}_{NE}(x, Q^2) &\approx \frac{0.31}{Q^2}, \quad (17c)
\end{align}

Eq. (17) is the result in Ref. 25 for $x \approx 1$, while Eq. (17) contains corrections for $x \neq 1$.

C) Empirical estimate, taken to be independent of $x$ (and $A^{25,26}$):

\begin{equation}
R_C(x, Q^2) \approx \frac{\delta}{Q^2} \ ; 0.2 \lesssim \delta \lesssim 0.5, \quad (18)
\end{equation}

The above, and the approximations (15), (17) for $x \approx 1$, yield $R \propto 1/Q^2$, while A) and B) for $x \neq 1$ prescribe $x$ dependence. It is likely that the range of extracted $\delta$ values in (18) actually hides some $x$-dependence.
Were it not for the listed uncertainties in $\kappa^{\text{comp}}$, the expressions (12), (13) or (19) would provide a standard, against which one could test the approximate $R$ ratios A)-C). In such a comparison, one occasionally finds substantial differences which clearly reflect on the extracted $F_2^A$ (See Ref. 27 for details).

III. MOMENTS OF STRUCTURE FUNCTIONS AND THEIR RELATION.

Our last topic regards moments of various SF

$$
M_k^A(m; Q^2) = \int_0^A dx x^m F_k^A(x, Q^2)
$$

$$
M_k^N(m; Q^2) = \int_0^1 dx x^m F_k^N(x, Q^2)
$$

$$
\mu^A(m; Q^2) = \int_0^A dx x^m f^{PN}(x, Q^2)
$$

The moments $M_k^N$ measure higher twist corrections in SF of nucleons and the same holds for their nuclear counterparts, had those been calculated in QCD. Our interest here lies in their sensitivity for large $x$ and consequently the trust in calculated $F_k^A$ for that range. One derives from (2)

$$
F_k^A(0, Q^2) = \mu^A(-2 + k; Q^2) F_k^N(0, Q^2)
$$

$$
M_k^A(m; Q^2) = \mu^A(m + 1; Q^2) M_k^N(m; Q^2)
$$

$$
\mu^A(m + 1; Q^2) = \frac{M_k^A(m + 1; Q^2)}{M_k^N(m + 1; Q^2)} = \frac{M_k^A(m; Q^2)}{M_k^N(m; Q^2)}
$$

Eq. (20c) for $m=-1$

$$
\mu^A(0, Q^2) = \int_0^A dx f^{PN}(x, Q^2) = \int_0^A dx f^{as}(x) = 1,
$$

expresses unitarity, whereas the relations (20a)-(20b) for finite $Q^2$ rest on the representation (2) and embody effects of the binding medium on moments of $F_k^N$ through $\mu(n, Q^2)$. For instance, the deviation of $\mu^A(2, Q^2)$ from 1 measures the difference of the momentum fraction at given $Q^2$ of a quark in a nucleus and in the nucleon.
We have calculated the lowest moments $\mathcal{M}_k$ and their ratios $\mu$ from computed $F^A_k$, $f^{PN}$ and parametrized $F^N_k$. With inaccuracies in $F^A_k$ growing with $x$ (say for $x \gtrsim 1.2$) one expects moments to get less trustworthy for increasing order $m$. Yet we found consistency between ratios of moments of $F^A, F^N$, Eq. (??), up to $m = 4$ and and their corresponding ratios $\mu$, Eq. (??). Fig. 3 shows reasonable agreement with available Fe data. We note in particular the rendition of the observed $Q^2$-dependence. Cothran et al, also used the generalized convolution with $f^{PN}$ in (2) in the $Q^2$-independent PWIA, and naturally find the same results for $\mu(m)$. Estimates for off-shell nucleons produced far too small moment ratios with, moreover incorrect $Q^2$ behaviour.11

Modifications of the nucleon SF in a binding medium as expressed by (??)-(??), are reminiscent of proposals to attribute discrepancies between data and computed results for relatively low-$q$, longitudinal responses $S_L$ as well as for the integral of the latter, the Coulomb sumrule31,32. Those have occasionally been ascribed to the influence of the binding medium on the size of a nucleon, i.e. on the second moment of the static charge density (see 33 for possible conventional explanations). One notes that Eqs. (2) and (??)-(??) relate to (moments of) dynamical SF and not to static form factors, from which one obtains the rms radii of nucleons.

In summary, we applied model calculations of nuclear structure functions $F^A_k(x, Q^2)$ at relatively high $Q^2$ to a number of observables, as are inclusive cross sections, $R$-ratios and moments of SF. Those observables for variable $Q^2$ are sensitive to quite different $x$-ranges, not all of which can be computed with comparable accuracy. It is gratifying to note good agreement between predictions and data for these observables. Of course, that agreement is no proof for the basic conjecture (2) but it certainly supports it.

The fact that the computations avoid any element of QCD, while manifestly being related to those, makes one wonder whether the above relation results from an effective theory. We have as yet no answer to this intriguing question.
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Table I

| $\langle y_G \rangle$ (GeV) | $\theta$ | $x$ | $Q^2$ (GeV$^2$) | $\xi_{C,Fe}$ | $\xi_{Fe,Au}$ |
|-----------------------------|---------|-----|-----------------|-------------|-------------|
| 23                          | 2.30    | 2.26| 0.81            | 1.03        |
| -0.4                        | 30      | 1.95| 3.38            | 0.70        | 0.84        |
|                             | 45      | 1.67| 5.46            | 0.97        | -           |
| 15                          | 2.49    | 1.05| 0.82            | 1.00        |
| -0.2                        | 30      | 1.37| 3.09            | 0.98        | 1.19        |
|                             | 55      | 1.30| 5.78            | 0.87        | 1.24        |
| 15                          | 1.02    | 0.97| 1.18            | 1.05        |
| 0.0                         | 30      | 1.01| 2.79            | 1.04        | 1.16        |
|                             | 74      | 1.01| 5.77            | 1.28        | 0.84        |
| 15                          | 0.65    | 0.91| 0.97            | 1.02        |
| 0.2                         | 30      | 0.72| 2.43            | 1.00        | 1.10        |
|                             | 74      | 0.74| 4.54            | 1.08        | -           |
| 0.4                         | 15      | 0.43| 0.83            | 1.00        | 1.03        |

Selection of cross section ratios $\xi^{A_1,A_2}$, Eq. (5). For each selected, narrowly-binned $\langle y_G \rangle$, available data of the ratios are given for the smallest, some medium and largest ($x, Q^2$) in the data sets.

Figure captions

Fig. 1. Data and predictions for the CEBAF 89-008 experiment.

Fig. 2. The NE part of the GRS-type scaling function $\xi^{Fe,N(NE)}(y_G) \leq 0, Q^2$ as function of $Q^2$. Dots connect extracted values, drawn lines data, with calculated NI parts removed.

Fig. 3. Second, third and fourth moments $\mu(m, Q^2)$, Eq. (20d).
Computed with (6)