Old ideas and new twists in string cosmology

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Abstract

Some of the phenomenological implications of string cosmological models are reviewed, with particular attention to the spectra of the tensor, scalar and vector modes of the geometry. A class of self-dual string cosmological models is presented. These solutions provide an effective description of cold bounces, where a phase of accelerated contraction smoothly evolves into an epoch of decelerated Friedmann–Robertson–Walker expansion dominated by the dilaton. Some of the general problems of the scenario (continuity of the perturbations, reheating, dilaton stabilization,...) can be successfully discussed in this framework.

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1 Introduction

Heeding observations, the large-scale temperature fluctuations detected in the microwave sky are compatible with a quasi-flat spectrum of curvature inhomogeneities. Quasi-flat means, in the present context, that the Fourier transform of the two-point function of the scalar fluctuations of the geometry depends on the comoving momentum $k$ as $k^{n_s-1}$, with $n_s \simeq 1$. Taking only the WMAP determination of $n_s$ [1, 2], the scalar spectral index lies in a rather narrow range, $0.95 \leq n_s \leq 1.03$. For later convenience, if $n_s \geq 1$ the spectra are said to be blue, if $n_s < 1$ the spectra are said to be red and, finally, if $n_s \gg 1$ the power spectra are called violet.

From the observations at smaller angular scales, it is by now established that the temperature fluctuations exhibit oscillations (the so-called Sakharov or Doppler oscillations) as a function of the sound horizon at decoupling. From the typical structure of these oscillations it is possible to argue that the curvature fluctuations present outside the horizon after equality (but before decoupling) were also constant. If the curvature fluctuations are constant, the solutions of the evolution of the density contrasts and of the peculiar velocities for the various species present in the plasma imply that the fluctuations in the total entropy density vanish at large-distance scales. These initial conditions for the evolution of the CMB anisotropies are often named adiabatic.

If a single field drives the conventional inflationary dynamics, the scalar fluctuations of the geometry naturally have a quasi-flat spectrum and are also constant at large-distance scales after matter–radiation equality. The quasi-flatness of the spectrum is related, in these models, to the quasi-constancy of the Hubble expansion rate and of the Ricci scalar during the inflationary stage. More precisely, in the context of single-field inflationary models, the curvature scale has a monotonic behaviour as a function of (cosmic or conformal) time coordinate, and it is always (slowly) decreasing. Since the curvature scale decreases, it can be argued, on a rigorous basis, that a true physical singularity is present in the far past [3]. However, the dynamics of the initial singularity is screened by the long period of inflation, during which the possible gradients arising in the matter fields are diluted and eventually erased if the duration of inflation exceeds 65-efolds (see for instance [4]).

In the context of string cosmological models the conventional inflationary
scenario seems quite difficult to obtain and therefore, during the last fifteen years, various cosmological models inspired by string theory have been explored. One of the features of these models is that the curvature scale is far from being constant but it is rather steeply increasing, at least during a sizeable portion of the early history of the Universe. In these models a singularity is often encountered just after the phase of growing curvature and gauge coupling. This problem is not an easy one to address be it technically or conceptually. Owing to the mentioned phase of growing curvature, the perturbation spectra obtained in string cosmological models are far from being quasi-flat. They are, indeed, rather violet.

There are by now several variations on this pre-big bang theme. Besides the original pre-big bang (PBB) scenario [5, 6], based on the duality symmetries of string cosmology, new models incorporating brane and M-theory ideas have been proposed under the generic name of ekpyrotic (EKP) scenarios [7, 8]. The various proposals differ in the way the scale factor behaves during the growing-curvature phase. However, they all share the feature of describing a bounce in the space-time curvature. A common theoretical challenge to all these models is that of being able to describe the transition between the two regimes.

2 Tensor, scalar and vector modes in string cosmological models

String cosmological models are naturally formulated in more than four dimensions. This occurrence implies that the fluctuations of a higher dimensional geometry may be more complicated than a simple four-dimensional space-time. However, in order to simplify the discussion, let us consider, as was done in the past, the dimensionally reduced low-energy string effective action, which can be written as

\[ S_{\text{eff}} = \int d^4x \sqrt{-G} e^{-\phi} \left[ \frac{1}{2\lambda^2} \left( R + G^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V(\phi) - \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} \right) \right. \]

\[ \left. + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \ldots \right] \]  

(2.1)

which is the typical outcome of the compactification of ten-dimensional superstrings on a six-dimensional torus. A few specifications are in order about this expression
• $\lambda_s$ is the string length scale, related to the Planck length scale by $l_p = e^{\varphi/2} \lambda_s$;

• $\varphi = \Phi_{10} - \ln V_6$ is the four-dimensional dilaton field, which can be expressed in terms of the ten-dimensional dilaton $\Phi_{10}$ and in terms of the volume of the six-dimensional torus;

• $V(\varphi)$ is the four-dimensional dilaton potential;

• $H^{\mu\nu\alpha}$ is the antisymmetric tensor field, related, in four dimensions, to a pseudo-scalar field $\sigma$ by $H^{\mu\nu\alpha} = e^{\varphi} e^{\mu\nu\alpha\beta} / \sqrt{-G} \partial_\beta \sigma$;

• $F_{\mu\nu}$ is a generic Abelian gauge field;

• the ellipses stand for other fields (other gauge fields, both Abelian and non-Abelian, chiral fermions, ...) and for the corrections, which can be both of higher order in $\lambda_s^2 \partial^2$ (higher derivatives expansion producing quadratic corrections to the Einstein–Hilbert action) and of higher order in $e^\varphi$ (loop expansion).

Equation (2.1) is written in the so-called string frame metric where the Ricci scalar $R$ is coupled to the four-dimensional dilaton. Other frames can be employed by appropriately redefining the metric and the dilaton. A particularly useful frame is the Einstein frame, where the Ricci scalar is not directly coupled to $\varphi$.

As already mentioned, the evolution equations for the metric and the dilaton can lead to singular solutions. This situation is, however, not generic, since also non-singular solutions can be found [9, 10, 11] and an example is reported Fig. 2.1. The solutions illustrated in there can be derived in the presence of a dilaton potential, that depends directly, not on $\varphi$, but on $\varphi' = \varphi - \log \sqrt{-G}$, i.e. the shifted dilaton usually defined in the context of the $O(d, d)$-covariant description of the low-energy string effective action [12]. From the point of view of the Einstein frame dynamics, these solutions describe a phase of accelerated contraction, evolving smoothly into an epoch of decelerated expansion [11]. The two regimes of the solution are connected, by scale-factor duality [5].
Figure 2.1: The evolution of the Hubble expansion rate $H$, of its derivative, and of the scale factor $a(\tau)$ in the cold bounce models. The variable $\tau = t/t_0$ is the cosmic time coordinate (in the string frame) rescaled by the typical time scale of the bounce, $t_0$.

### 2.1 Tensors

The spectrum of the tensor modes arising from solutions where the Hubble expansion rate is increasing has been computed in various steps [13]. The amplified tensor modes of the geometry lead to a stochastic background of gravitational waves (GW) with violet spectrum both in the GW amplitude and energy density. This expectation is confirmed also in the context of the models illustrated in Fig. 2.1 as well as in the context of other non-singular models. In Fig. 2.2 the GW signal is parametrized in terms of the logarithm of $\Omega_{GW} = \rho_{GW}/\rho_c$, i.e. the fraction of critical energy density present (today) in GW. On the horizontal axis of Fig. 2.1 the logarithm of the present (physical) frequency $\nu$ is reported. In conventional inflationary models, for $\nu \geq 10^{-16}$ Hz, $\Omega_{GW}$, is constant (or slightly decreasing) as a function of the present frequency. In the case of string cosmological models $\Omega_{GW} \propto \nu^3 \ln \nu$, which also implies a steeply increasing power spectrum. This possibility spurred various experimental groups to analyse possible direct limits on the scenario arising from specific instruments such as resonant mass detectors [15] and microwave cavities [16, 17]. These attempts are
justified since the signal of pre-big bang models may be rather strong at high frequencies and, anyway, much stronger than the conventional inflationary prediction.

The sensitivity of a pair of VIRGO detectors to string cosmological gravitons has been specifically analysed [14] with the conclusion that a VIRGO pair, in its upgraded stage, can certainly probe wide regions of the parameter space of these models. If we maximize the overlap between the two detectors [14] or if we reduce (selectively) the pendulum and pendulum’s internal modes contribution to the thermal noise of the instruments, the visible region (after one year of observation and with SNR = 1) of the parameter space will get even larger. Unfortunately, as in the case of the advanced LIGO detectors, the sensitivity to a flat $\Omega_{GW}$ will be irrelevant for ordinary inflationary models also with the advanced VIRGO detector. It is worth mentioning that growing energy spectra of relic gravitons can also arise in the context of quintessential inflationary models [18, 19]. In this case $\Omega_{GW} \propto \nu \ln \nu$ (see [19] for a full discussion).

In order to gauge carefully our theoretical expectations it is relevant to notice that direct experimental limits on stochastic GW backgrounds are rather far from the interesting region of the parameter space of a possible cosmological signal. In particular, form various instruments (resonant mass
Figure 2.3: The Fourier transform of the two-point function of $\mathcal{R}$ computed for different comoving momenta as a function of the cosmic time coordinate.

detectors, interferometers) $h_0^2 \Omega_{GW} < \mathcal{O}(10)$. From Fig. 2.2 it can be easily appreciated that a cosmological signal must satisfy $h_0^2 \Omega_{GW} < \mathcal{O}(10^{-4})$ as implied by the bound on extra-relativistic species at big-bang nucleosynthesis. This constraint is, however, model-dependent and it can be slightly relaxed in unconventional models of big-bang nucleosynthesis [20] where more extra-relativistic species are allowed since the presence of matter–antimatter domains allows an independent reduction of the $^4$He abundance (which is led to increase by the presence of extra-relativistic species).

2.2 Scalars

The spectrum of the tensor modes of the geometry is not controversial because the tensor fluctuations of the geometry are defined as a rank-2 (divergenceless and traceless) tensor in three dimensions. Consequently they are invariant under infinitesimal coordinate transformations. Scalar perturbations, in contrast, do depend on the specific coordinate system and are described, in four dimensions, by a single propagating degree of freedom. This problem is only partially alleviated by the possibility of defining vari-
ables that are invariant under coordinate transformations. In fact, different choices are equally allowed such as:

- Bardeen potentials (curvature perturbations on shear free hypersurfaces), usually denoted by $\Psi$,
- curvature perturbations on comoving hypersurfaces, usually denoted by $R$,
- curvature perturbations on constant-density hypersurfaces, usually denoted by $\zeta$,

and as well as other choices. The variables listed above are related by specific differential relations: once one of them is reliably computed, all the other follow.

Using various descriptions (both gauge-invariant and gauge-dependent), it was argued in [21] that the spectra of scalar fluctuations are also violet, with a scalar spectral index $n_s = 4$. The same analysis, applied to the case of ekpyrotic models, would lead to $n_s = 3$. The pre-big bang estimates [21] have been questioned on various grounds. The bottom line of these arguments would be that, in single field pre-big bang or ekpyrotic models, the spectrum of the tensor modes is violet but the spectrum of the scalar modes may be flat or even red, i.e. increasing at large-distance scales. The analysis of the models illustrated in Fig. 2.1 seems to give an unambiguous answer: while the evolution of the Bardeen potential is rather complicated around the bounce the time dependence of both $R$ and $\zeta$ is rather smooth. Furthermore, not only the spectrum of $R$ and $\zeta$ is, as expected, violet, but it is also in agreement with the analytical estimate. The results for the evolution of $R$ is illustrated in Fig. 2.3. The value of the comoving momentum increases from bottom to top. Hence, the spectrum is increasing as a function of $k$, as expected. An accurate numerical determination discussed in [10], also shows that $\delta_R \sim k^{3/2}$ with specific logarithmic corrections. The spectrum of the Bardeen potential has also been computed accurately in [10], with the result that $\delta_\Psi \sim k^{-1/2}$ as expected from the analytical estimates.

2.3 Vectors

Vector modes of the metric are not excited in the context of conventional inflationary models. If the background geometry has more than four di-
dimensions, on the other hand vector modes are expected [22]. It is also possible to envisage the situation where rotational modes of the geometry are excited by the fluctuations of the velocity field [23]. The cold-bounce solutions illustrated in Fig. 2.1 can be generalized to include fluid sources [11] as well as internal (contracting) dimensions [24]. It was recently argued that vector modes of the geometry can be produced in pre-big bang and ekpyrotic/cyclic scenarios [23]. In particular, it was argued that the vector modes of the geometry may lead to a growing mode prior to the occurrence of the bounce. This expectation has been verified in [24] but it has also been shown that, in four dimensions, the growing vector mode present before the bounce turns into a decaying mode after the bounce. This result has been achieved in specific models of smooth evolution similar to the ones presented in Fig. 2.1 but including also fluid sources. It will be interesting to analyse more precisely this problem in different both singular and non-singular models. Going beyond four dimensions, the vector modes of the geometry are copiously produced [24]. The higher-dimensional examples provided in [24] support the evidence that, in semi-realistic models these spectra may be red.

2.4 Heating up the cold bounce

The cold-bounce solutions discussed in [9] and [10] and illustrated in Fig. 2.1 certainly have realistic features. However there are two less realistic aspects

- after the bounce the Universe is cold and dominated by the dilaton field;
- the dilaton field is not stabilized in the sense that it does not reach constant value.

These two problems may be solved if the back-reaction of the various massless fields (i.e. massless gauge bosons, for instance) is included. In [11] the back-reaction effects of high-frequency photons has been included and the following results have been illustrated:

- an accurate numerical method for the calculation of the amplification of the primordial photons has been developed;
- taking into account the back-reaction of the primordial photons, the cold-bounce solution can be consistently heated;
• the transition to radiation can occur for sub-Planckian curvature scales;

• solutions have been presented where the dilaton goes to a constant value and the asymptotic value of the gauge coupling is always smaller than 1.

3 A consistent phenomenological framework

In the PBB case it was admitted early on that the tensor [13] and adiabatic-curvature perturbations [21] had too large a spectral index to be of any relevance at cosmologically interesting scales (while being possibly important for gravitational-waves searches [14]). Isocurvature perturbations (related to the Kalb–Ramond two-form) can instead be produced with an interestingly flat spectrum [25], but have to be converted into adiabatic-curvature perturbations through the so-called curvaton mechanism [26] (see also [27, 28]) before they can provide a viable scenario for large-scale anisotropies in the pre-big bang context [29]. Proponents of the ekpyrotic scenario, while agreeing with the PBB result of a steep spectrum of tensor perturbations, have also repeatedly claimed [30] to obtain “naturally” an almost scale-invariant spectrum of adiabatic-curvature perturbations, very much as in ordinary models of slow-roll inflation. These claims have generated a debate (see for instance [31]), with many arguments given in favour of the phenomenological viability of EKP scenarios in the absence of a curvaton’s help or against it. The reasons for the disagreement can be ultimately traced back to the fact that the curvature bounce is put in by hand (see for instance [32]), rather than derived from an underlying action. This leaves different authors to make different assumptions on how to smoothly connect perturbations across the bounce itself, which results in completely different physical predictions.

A specific class of models has been illustrated [9, 10]. In these the evolution of the background geometry and of the dilaton coupling is regular and specific testable predictions are possible. On the basis of these semi-analytical investigations it can be argued that in single-field pre-big bang models the spectrum of the scalar and tensor modes of the geometry is, as expected from previous estimates, violet. The situation changes when the evolution of the fluctuations of the Kalb-Ramond field is consistently
included. In this case a flat spectrum of curvature fluctuations can be obtained and compared with the observed anisotropies in the CMB (see [29] for a complete discussion). The large-scale microwave anisotropy probes can then be used to constrain the various parameters of the model.

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