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Numerical analysis of COVID-19 model with constant fractional order and variable fractal dimension

Badr Saad T. Alkahtani a, Sonal Jain b,*

a Department of Mathematics, College of Science, King Saud University, P.O. Box 1142, Riyadh 11989, Saudi Arabia
b Department of Humanity and Sciences, Rizvi College of Engineering, Bandra West, Mumbai, Maharashtra 400050, India

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ABSTRACT

This work has considered a mathematical model describing the spread of COVID-19 in a given population. The model comprised 5 systems of equations that take into account different classes describing the impact of COVID-19 in a given population. The time differential operator was replaced with three different types of nonlocal operators. These operators are defined as the convolution of variable order fractal differential operators with different kernels including power law, exponential decay law, and Mittag-Leffler functions. We presented the well-posedness of the models for different differential operators that were presented in detail. A novel numerical scheme was used to solve numerically the system and numerical simulations were provided.

Introduction

Differential operators with non-local characters have been noticed to be able to replicate several complexities occurring in nature in the last decades. One can mention among which fractional differential and integral operators with power law, exponential decay and Mittag–Leffler kernels on the other hand differential operators defined as convolution of power law, exponential decay and Mittag–Leffler kernels with fractal differential operator, these cases are called fractal-fractional operators [1,2,5,10]. Finally fractal-fractional differential operators with variable fractal orders. These last one are considered to be adequate to modelling complex real world problems, for example real world problems with anomalous patterns could be replicated using variable order differential operators. These operators have been successfully applied in many academicks disciplines, however much attention have been devoted to epidemiologic models[3,18,15,4,12]. They could be very useful tools to modelling epidemiologic problems as the spread of some infectious diseases are anomalous therefore neither the classical differentiation nor fractional with constant can be applied in these cases. Very recently, the world have been surprised with an outbreak of a fatal disease called COVID-19, which was first observed in Wuhan, China, December 2019 [9,7,8]. From this time to 07 May 2020 the disease has infected 3836183, the total number of deaths from COVID-19 is about 265 364 of which this number is for those registered or declared by each nation around the globe. However, 1307 608 have been recovered. The exponential spread of this disease leads humans no choice than to undertake serious researchers activities in all field of science. In applied mathematics, many new mathematical models have been suggested, some including fractional differential and fractal fractional operators. In this paper, we aim to revert the model suggested by Shafiq and Atangana using the fractal-fractional with variable fractal-order[4]. One of the major concern of differential operators with variable orders is perhaps their solvability as analytical methods cannot be used to provide their exact solutions. Thus, numerical methods are adequate to providing approximate solutions to such models. In the last decades, many numerical methods have been provided all with their advantages and limitations. In the case of nonlinear equations, it is known that the Adams–Bashforth is powerful numerical scheme to provide approximate solutions of nonlinear equations [13,11,16,17,19]. However, the case with fractional differential has some limitation thus, Atangana and Toufit [20] suggested an alternative efficient numerical scheme to be used, and the method has been used in various nonlinear equations arising in many fields of science, engineering and technology. In this paper, we aim at using such scheme to solve the model of COVID-19 suggested by Altaf and Atangana [14], where the time derivative is reverted to fractal-fractional with fractal variable order. The paper is organized as follow, we start with the model description, then, the well-posedness of the model is presented in the case of fractal-fractional with variable order fractal and finally numerical analysis for different cases.

Definition 0.1. A discontinuous media can be described by fractal
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The above definition is with power law kernel. For exponential decay law, we have

\[ \varpi(t) = \exp(-\lambda t) \]

where

\[ \varpi(t) = \lim_{\tau \to 0} \frac{1}{\varpi(\tau) - \varpi(0)} \]

The above definition is with power law kernel. With exponential kernel, we have

\[ F_{\alpha}^{\beta} \varphi(t) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - \tau)^{-\alpha} \varphi(t) \, dt \]

with Mittag-Leffler law, we have

\[ F_{\alpha}^{\beta} \varphi(t) = \frac{\alpha}{\Gamma(1 - \alpha)} \int_0^t (t - \tau)^{-\alpha} \varphi(t) \, dt \]

Corollary 0.1. \[ \frac{d}{dt} \left( \frac{f(t)}{\xi(t)} \right) = \frac{1}{\xi(t)} \ln \frac{\xi(t)}{\xi(t) - \xi(t_1)} \]

Definition 0.4. \[ F_{\alpha}^{\beta} f(t) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - \tau)^{-\alpha} f(\tau) \left( \frac{\xi(\tau)}{\xi(t)} \right) \, d\tau \]

For exponential decay law, we have

\[ F_{\alpha}^{\beta} f(t) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - \tau)^{-\alpha} f(\tau) \left( \frac{\xi(\tau)}{\xi(t)} \right) \, d\tau \]

For Mittag-Leffler law we have

\[ F_{\alpha}^{\beta} f(t) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - \tau)^{-\alpha} f(\tau) \left( \frac{\xi(\tau)}{\xi(t)} \right) \, d\tau \]

Model description

The total population of people is denoted by \( N_p \) which is classified further into five subgroups such as \( S_p, E_p, I_p, A_p \) and \( R_p \) which represent respectively, the susceptible, exposed, infected (symptomatic), asymptotically infected, and the recovered or the removed people. Considering that the 2019-nCoV can be imported in short time to the seafood market with enough source of virus and thus, without loss of generality, ignoring of the interaction among bats and hosts, then model can be represent to the system below:

\[ \frac{dS_p}{dt} = \beta P S_p - \mu S_p - \eta S_p I_p \]

\[ \frac{dE_p}{dt} = \eta S_p I_p - (\mu + \rho) E_p \]

\[ \frac{dI_p}{dt} = (1 - \theta) \rho E_p - (\theta + \mu) I_p \]

\[ \frac{dA_p}{dt} = \theta \rho E_p - (\theta + \mu) A_p \]

\[ \frac{dR_p}{dt} = \tau \rho A_p - \mu R_p \]

\[ \frac{dM}{dt} = \omega I_p + \eta R_p - M \]

with the initial conditions

\[ S_p(0) = S_p(0) \geq 0, E_p(0) = E_p(0) \geq 0, I_p(0) = I_p(0) \geq 0, A_p(0) = A_p(0) \geq 0, R_p(0) = R_p(0) \geq 0, M(0) = M(0) \geq 0. \]

The total dynamics of the people can be obtained by adding the first five equations of the model 2.1, given by

\[ \frac{dN_p}{dt} = \beta P S_p - \mu S_p N_p \]

The feasible reason of the model 2.1 is given by

\[ \omega = \left\{ \left( S_p(t), E_p(t), I_p(t), A_p(t), R_p(t) \right) \in \mathbb{R}_+^5 : N_p(t) \leq M, M \in \mathbb{R}_+ \right\} \]

\[ \frac{\beta}{\omega} \frac{\theta P}{\rho P} \frac{\mu}{\theta} \]

The birth and natural death rate of the people is given by the parameters \( \sigma \) and \( \mu \), respectively. The susceptible people \( S_p \) will be infected through sufficient contacts with the infected people \( I_p \) through the term given by \( \eta S_p A_p \) where the \( \eta \) is the disease transmission coefficient\[14\]. The transmission among the asymptotically infected people with health people could take place at term \( \psi S_p A_p \), where the \( \psi \) is the transmissibility multiplicity of \( A_p \) to that \( I_p \) and \( \psi \in [0, 1] \), when \( \psi = 1 \), no transmissibility multiplicity will exists and hence vanish, and if \( \psi = 1 \), then the same will take place like \( I_p \) infection. The parameter \( \theta P \) is the proportion of asymptomatic infection. The parameters \( \omega P \) and \( \rho P \) respectively represent the transmission rate after completing the incubation period and becomes infected, joining the class \( I_p \) and \( A_p \). The people in the symptomatic class \( I_p \) and asymptomatic class \( A_p \) joining these class \( R_p \) with the removal or recovery rate respectively by \( \tau P \) and \( \rho P \). The class \( M \) which is denoted be the reservoir or the seafood place or market. The susceptible people infected after the interaction with \( M \), given by \( \eta M S_p \), where the \( \eta \) is the disease transmission coefficient from \( M \) to \( S_p \). The
parameters $\phi_p$ and $\sigma_p$ of the infected symptomatic and asymptotically infected respectively contributing the virus into the seafood market $M$. 

The removing rate of the virus from the seafood market $M$ is given by the rate $\xi$.  

So we can write this model in the fractional differential differentiation sense as follows:

\[
\text{FFP}_0 &\text{D}^{\alpha} S_p = \lambda_p - \mu_p S_p - \frac{\eta_p \Sigma_p (I_p + A_p)}{N_p} + \eta_p S_p M,
\]

\[
\text{FFP}_0 &\text{D}^{\alpha} I_p = \frac{\eta_p \Sigma_p (I_p + A_p)}{N_p} - \eta_p S_p M - (1 - \theta_p)\omega_p I_p - \theta_p\rho_p I_p - \mu_p I_p,
\]

\[
\text{FFP}_0 &\text{D}^{\alpha} A_p = \tau_p I_p - (\tau_p + \mu_p) A_p,
\]

\[
\text{FFP}_0 &\text{D}^{\alpha} R_p = \tau_p I_p - (\tau_p + \mu_p) R_p,
\]

\[
\text{FFP}_0 &\text{D}^{\alpha} M = \psi_p I_p + \sigma_p A_p - \pi M,
\]

(2.5)

with the initial conditions

\[
S_p(0) = S_0(0) \geq 0, E_p(0) = E_0(0) \geq 0, I_p(0) = I_0(0) \geq 0, A_p(0) = A_0(0) \geq 0, R_p(0) = R_0(0) \geq 0, M(0) = M(0) \geq 0.
\]

(2.6)

Positiveness of systems solutions for three cases

we now present that for each case of $S_p(0)$, $E_p(0)$, $I_p(0)$, $A_p(0)$, $R_p(0)$ and $M(0)$ are positive then the solution are also positives. We shall start with power law case: To do this, we define the following norm,

\[
\|f\|_\infty = \sup_{t \in [0,T]} |f(t)|
\]

(2.7)

we suppose that the all the classes constituting function of time have the same sign, therefore any product of the classes is positive.

\[
\text{FFP}_0 &\text{D}^{\alpha} E_p = \frac{\eta_p \Sigma_p (I_p + A_p)}{N_p} + \eta_p S_p M - (1 - \theta_p)\omega_p E_p - \theta_p\rho_p E_p - \mu_p E_p,
\]

(3.1)

from the hypothesis

\[
\frac{\eta_p \Sigma_p (I_p + A_p)}{N_p} > 0
\]

and $\eta_p S_p M > 0$ then

\[
\text{FFP}_0 &\text{D}^{\alpha} E_p(t) \geq - (1 - \theta_p)\omega_p + \theta_p\rho_p + \mu_p) E_p(t)
\]

(3.2)

Thus

\[
\text{FFP}_0 &\text{D}^{\alpha} E_p(t) \geq - (1 - \theta_p)\omega_p + \theta_p\rho_p + \mu_p E_p(t)
\]

(3.3)

we consider the case with exponential decay law

\[
\text{FFP}_0 &\text{D}^{\alpha} E_p(t) \geq (1 - \theta_p)\omega_p + \theta_p\rho_p + \mu_p) E_p(t)
\]

(3.10)

Also

\[
\text{FFP}_0 &\text{D}^{\alpha} E_p(t) \geq - (1 - \theta_p)\omega_p + \theta_p\rho_p + \mu_p) E_p(t)
\]

(3.11)

and finally

\[
\text{FFP}_0 &\text{D}^{\alpha} E_p(t) \geq - \pi (t)\omega_p + \theta_p\rho_p + \mu_p) E_p(t)
\]

(3.13)

Thus

\[
\text{FFP}_0 &\text{D}^{\alpha} E_p(t) \geq - \pi (t)\omega_p + \theta_p\rho_p + \mu_p) E_p(t)
\]

(3.14)
with same hypothesis, we have

$$\begin{align*}
E_s(t) &\geq E_s(0) \exp \left( -\frac{\theta \Lambda_s t}{M(\theta) - (1 - \theta)\Lambda_s} \right) \\
F^\theta E_s(t) &\geq -\Lambda_S S_p(t) \\
\text{FFE}_{\theta} D^\theta_t S_p(t) &\geq -\Lambda_S S_p(t)
\end{align*}$$

Then

$$\begin{align*}
S_p(t) &\geq S_p(0) \exp \left( -\frac{\theta \Lambda_s t}{M(\theta) - (1 - \theta)\Lambda_s} \right) \\
I_p(t) &\geq I_p(0) \exp \left( -\frac{\theta \Lambda_s t}{M(\theta) - (1 - \theta)\Lambda_s} \right) \\
A_s(t) &\geq A_s(0) E_f \left[ -\Lambda_S \theta^\rho \frac{AB(\theta) - (1 - \theta)\Lambda_s}{AB(\theta) - (1 - \theta)\Lambda_s} \right] \\
R_s(t) &\geq R_s(0) E_f \left[ -\Lambda_S \theta^\rho \frac{AB(\theta) - (1 - \theta)\Lambda_s}{AB(\theta) - (1 - \theta)\Lambda_s} \right] \\
M &\geq M(0) E_f \left[ -\Lambda_S \theta^\rho \frac{AB(\theta) - (1 - \theta)\Lambda_s}{AB(\theta) - (1 - \theta)\Lambda_s} \right]
\end{align*}$$

Numerical approximation

Recently Atangana and Shafiq [4] suggested an alternative numerical scheme for solving fractal fractional differential equation. So in this section we can solve this method by using that numerical scheme as follows:

Let us consider the following Cauchy problem

$$y(t) = \frac{1}{\Gamma(\theta)} \int_0^t (t - \tau)^{\theta - 1} f(t, y(t)) \tau^\theta \left( \xi(t) \ln(r) + \frac{\xi(t)}{r} \right) d\tau,$$

at the given point \( t_m - 1 \), here we chose \( m = 0, 1, 2, \ldots \), then the above equation can be written as

$$y(t) = \frac{1}{\Gamma(\theta)} \sum_{j=0}^{m-1} (t_m - 1 - \tau)^{\theta - 1} f(t, y(t)) \tau^\theta \left( \xi(t) \ln(r) + \frac{\xi(t)}{r} \right) d\tau,$$

For simplicity we consider

$$F(t, y(t)) = f(t, y(t)) \tau^\theta \left( \xi(t) \ln(r) + \frac{\xi(t)}{r} \right),$$

$$y^{n+1} = \frac{1}{\Gamma(\theta)} \sum_{j=0}^{m-1} \int_{t_j - 1}^{t_{n+1} - 1 - \tau} (t_{n+1} - \tau)^{\theta - 1} F(t, y(t)) d\tau,$$

within \([t_j, t_{j+1}]\), we can apply the Lagrange polynomial interpolation. Thus within this interval, we approximate

$$F(t, y(t)) = \frac{t - t_{j+1}}{t_{j+1} - t_j} F(t_j, y(t_j)) - \frac{t - t_j}{t_{j+1} - t_j} F(t_{j-1}, y(t_{j-1})).$$

Then replacing above in original equation, we obtain

$$y^{n+1} = \frac{1}{\Gamma(\theta)} \sum_{j=0}^{m-1} \int_{t_j}^{t_{n+1} - 1 - \tau} (t_{n+1} - \tau)^{\theta - 1} \left[ \frac{t - t_{j+1}}{t_{j+1} - t_j} F(t_j, y(t_j)) - \frac{t - t_j}{t_{j+1} - t_j} F(t_{j-1}, y(t_{j-1})) \right] d\tau.$$

As given in the Atangana and Toufik [20], the above equation can be reformulated as after integration

$$y^{n+1} = \sum_{j=0}^{m-1} \left[ \frac{(\Delta t)^{\theta}}{\Gamma(\theta + 2)} F(t_j, y_j) \left\{ (m + 1 - j)^\theta (m - j + 2 + \theta) - (m - j)^\theta (m - j + 1 + \theta) \right\} \right],$$

replacing \( F(t_j, y_j) \) and \( F(t_{j-1}, y_{j-1}) \) by their respective values, we obtain
We next consider the corresponding Cauchy with exponential decay kernel.

\[
\text{Re}_\phi \mathbf{K}^{(\phi)}(y(t)) = f(t, y(t))
\]

\[
y(0) = y_0
\]

The corresponding Volterra type is given by

\[
y(t) = \frac{1 - \theta}{M(\theta)} \int_0^t f(x, y(x)) \varphi(t - x) dx + \frac{\varphi(t)}{\tau} dt,
\]

\[
\theta = \frac{\theta}{M(\theta)} \int_0^\infty f(x, y) \varphi(x) dx + \frac{\varphi(t)}{\tau} dt.
\]

So at \( t = t_{n+1} \), we have

\[
y(t_{n+1}) = \frac{1 - \theta}{M(\theta)} \int_0^{t_{n+1}} f(x, y(x)) \varphi(t - x) dx + \frac{\varphi(t)}{\tau} dt,
\]

\[
\theta = \frac{\theta}{M(\theta)} \int_0^{t_{n+1}} f(x, y) \varphi(x) dx + \frac{\varphi(t)}{\tau} dt.
\]

Therefore
so we can solve the (2.5) equation numerically with constant fractional order and variable fractal dimension (see [14]) by using above solution and get

\[ S_{p}(t_{n+1}) = \frac{(1 - \theta)}{M(\theta)} f(t_{n}, y(t_{n})) e^{[t_{n+1}]} \left[ \frac{\xi(\tau_{n+1}) - \xi(\tau_{n})}{\Delta \tau} \ln_{\tau_{n}} + \frac{\xi(\tau_{n})}{\tau_{n}} \right] \\
- \frac{3}{2} \Delta \tau S_{p}(t_{n}) e^{[t_{n+1}]} \left[ \frac{\xi(\tau_{n+1}) - \xi(\tau_{n})}{\Delta \tau} \ln_{\tau_{n}} + \frac{\xi(\tau_{n})}{\tau_{n}} \right] \\
- \frac{\Delta \tau}{2} S_{p}(t_{n}) e^{[t_{n+1}]} \left[ \frac{\xi(\tau_{n+1}) - \xi(\tau_{n})}{\Delta \tau} \ln_{\tau_{n}} + \frac{\xi(\tau_{n})}{\tau_{n}} \right] \\
\]

(4.25)

so our model can be written in exponential law form as

\[ FFM_{D_{0+}^{\alpha}} S_{p} = S_{p}(t, S_{p}, E_{p}, I_{p}, A_{p}, R_{p}, M) \]

\[ FFM_{D_{0+}^{\alpha}} E_{p} = E_{p}(t, S_{p}, E_{p}, I_{p}, A_{p}, R_{p}, M) \]

\[ FFM_{D_{0+}^{\alpha}} A_{p} = A_{p}(t, S_{p}, E_{p}, I_{p}, A_{p}, R_{p}, M) \]

\[ FFM_{D_{0+}^{\alpha}} R_{p} = R_{p}(t, S_{p}, E_{p}, I_{p}, A_{p}, R_{p}, M) \]

\[ FFM_{D_{0+}^{\alpha}} M = M(t, S_{p}, E_{p}, I_{p}, A_{p}, R_{p}, M) \]

(4.24)

so we can solve the (2.5) equation numerically with constant fractional order and variable fractal dimension (see [14]) by using above solution and get

\[ L_{p}(t_{n+1}) = \frac{(1 - \theta)}{M(\theta)} \left[ S_{p}(t_{n+1}) e^{[t_{n+1}]} \left[ \frac{\xi(\tau_{n+1}) - \xi(\tau_{n})}{\Delta \tau} \ln_{\tau_{n}} + \frac{\xi(\tau_{n})}{\tau_{n}} \right] \\
- \frac{3}{2} \Delta \tau S_{p}(t_{n+1}) e^{[t_{n+1}]} \left[ \frac{\xi(\tau_{n+1}) - \xi(\tau_{n})}{\Delta \tau} \ln_{\tau_{n}} + \frac{\xi(\tau_{n})}{\tau_{n}} \right] \\
- \frac{\Delta \tau}{2} S_{p}(t_{n+1}) e^{[t_{n+1}]} \left[ \frac{\xi(\tau_{n+1}) - \xi(\tau_{n})}{\Delta \tau} \ln_{\tau_{n}} + \frac{\xi(\tau_{n})}{\tau_{n}} \right] \\
\]

(4.26)

The existence and uniqueness of this solution is given in the paper Atangana and Shafiq.

Now we consider the case of Mittag Leffler Kernel.

\[ FFM_{D_{0+}^{\alpha}} y(t) = f(t, y(t)) \]

(4.31)

\[ y(0) = y_{0} \]

Using the corresponding integral operator we convert the above equation as
\[ y(t) = \frac{1 - \theta}{AB(\theta)} f(t, y(t))e^{\xi(t)} \left( \xi(t) \ln(t) + \frac{\xi(t)}{t} \right) + \frac{\theta}{\Gamma(\theta)AB(\theta)} \int_0^t (t - \tau)^{\theta-1} e^{\xi(t)} \left( \xi(t) \ln(t) + \frac{\xi(t)}{t} \right) f(\tau, y(\tau)) d\tau, \]

(4.32)

So we consider the following general nonlinear equation

\[ y(t) = \frac{1 - \theta}{AB(\theta)} e^{\xi(t)} \left( \xi(t) \ln(t) + \frac{\xi(t)}{t} \right) f(t, y(t)) + \frac{\theta}{\Gamma(\theta)AB(\theta)} \int_0^t (t - \tau)^{\theta-1} e^{\xi(t)} \left( \xi(t) \ln(t) + \frac{\xi(t)}{t} \right) f(\tau, y(\tau)) d\tau, \]

(4.33)

For simplicity, we put

\[ g(\tau, y(\tau)) = e^{\xi(t)} \left( \xi(t) \ln(t) + \frac{\xi(t)}{t} \right) f(t, y(t)), \]

(4.34)

then

\[ y(t_{n+1}) = \frac{1 - \theta}{AB(\theta)} g(t_{n+1}) \left( \xi(t_{n+1}) - \xi(t_n) \ln(t_{n+1}) - \xi(t_{n+1}) \right) + \frac{\theta}{\Gamma(\theta)AB(\theta)} \int_0^{t_{n+1} - \tau^{\theta-1} g(\tau, y(\tau)) d\tau, \]

(4.36)

\[ y(t_{n+1}) = \frac{1 - \theta}{AB(\theta)} g(t_{n+1}) \left( \xi(t_{n+1}) - \xi(t_n) \ln(t_{n+1}) - \xi(t_{n+1}) \right) + \frac{\theta}{\Gamma(\theta)AB(\theta)} \sum_{j=0}^{n+1} (t_{n+1} - t_j)^{\theta-1} g(t_j, y(t_j)) d\tau, \]

(4.37)

we approximate \( g(\tau, y(\tau)) \) within the interval \([t_j, t_{j+1}]\)

\[ q(\tau) = \frac{t_{j+1} - t_j}{t_{n+1} - t_j} g(t_{j+1}, y(t_{j+1})) - \frac{t - t_j}{t_{n+1} - t_j} g(t_j, y(t_j)) \]

(4.38)

\[ = \frac{g(t_j, y(t_j))}{h} (\tau - t_j) - \frac{g(t_{j+1}, y(t_{j+1})) - g(t_j, y(t_j))}{h} (t_{n+1} - \tau - t_j), \]

replacing \( g(\tau, y(\tau)) = q(\tau) \) by its value and integrating, we obtain

\[ y(t_{n+1}) = \frac{1 - \theta}{AB(\theta)} g(t_{n+1}) \left( \xi(t_{n+1}) - \xi(t_n) \ln(t_{n+1}) + \xi(t_{n+1}) \right) + \frac{\theta}{\Gamma(\theta)AB(\theta)} \sum_{j=0}^{n+1} (t_{n+1} - t_j)^{\theta-1} q(t_j, y(t_j)) \]

(4.39)

\[ \times \left\{ (m - j + 1)^\theta (m - j + 2 + \theta) - (n - j)^\theta (m - j + 1 + \theta) \right\} \]

\[ \left\{ (m - j + 1)^\theta - (n - j)^\theta (m - j + 1 + \theta) \right\}, \]

(4.40)

Now replacing

\[ g(t_j, y(t_j)) = \frac{1}{\Delta t} g(t_j, y(t_j)) \]

(4.41)

\[ y(t_{n+1}) = \frac{1 - \theta}{AB(\theta)} g(t_{n+1}) \left( \xi(t_{n+1}) - \xi(t_n) \ln(t_{n+1}) + \xi(t_{n+1}) \right) + \frac{\theta}{\Gamma(\theta)AB(\theta)} \sum_{j=0}^{n+1} (t_{n+1} - t_j)^{\theta-1} q(t_j, y(t_j)) \]

(4.42)

so our model can be written in exponential law form as

\[ S_{M}(t_{n+1}) = S_{M}(t_n) + \frac{\Delta t}{\Gamma(\theta)AB(\theta)} \left\{ (m - j + 1)^\theta - (m - j + 1 + \theta) \right\} \]

(4.43)

so we can solve the (2.5) equation numerically with constant fractional order and variable fractal dimension by using above solution and get

\[ S_{M}(t_{n+1}) = S_{M}(t_n) + \frac{\Delta t}{\Gamma(\theta)AB(\theta)} \left\{ (m - j + 1)^\theta - (m - j + 1 + \theta) \right\}, \]

(4.44)

\[ E_p(t_{n+1}) = E_p(t_n) + \frac{\Delta t}{\Gamma(\theta)AB(\theta)} \left\{ (m - j + 1)^\theta - (m - j + 1 + \theta) \right\} \]

(4.45)
Fig. 1. Numerical solution for $\theta = 0.4$.

Fig. 2. Numerical solution for $\theta = 0.5$. 
Fig. 3. Numerical solution for $\theta = 0.6$.

Fig. 4. Numerical solution for $\theta = 0.7$. 
Fig. 5. Numerical solution for $\theta = 0.8$.

Fig. 6. Numerical solution for $\theta = 0.9$. 
\[ I_p(t_{n+1}) = 1 - \frac{\theta}{AB(\theta)} \rho^{(\alpha+1)} \left( \frac{\xi(t_{n+1}) - \xi(t_n)}{\Delta t} \ln(t_{n+1}) + \frac{\xi(t_{n+1})}{t_{n+1}} \right) S_1(t_n, I_p(t_n)) \]

\[ + \frac{\theta}{A(\theta)AB(\theta)} \sum_{j=1}^{n} \frac{\xi(t_j) - \xi(t_{j-1})}{\Delta t} \ln(t_j) + \frac{\xi(t_j)}{t_j} S_2(t_{j-1}, I_p(t_j)) \]

\times \{ \text{a function of } (m-j+1)^{\mu} (m-j+2+\theta) - (m-j)^{\mu} (m-j+1+\theta) \}.

(4.46)

\[ A_p(t_{n+1}) = 1 - \frac{\theta}{AB(\theta)} \rho^{(\alpha+1)} \left( \frac{\xi(t_{n+1}) - \xi(t_n)}{\Delta t} \ln(t_{n+1}) + \frac{\xi(t_{n+1})}{t_{n+1}} \right) S_1(t_n, A_p(t_n)) \]

\[ + \frac{\theta}{A(\theta)AB(\theta)} \sum_{j=1}^{n} \frac{\xi(t_j) - \xi(t_{j-1})}{\Delta t} \ln(t_j) + \frac{\xi(t_j)}{t_j} S_2(t_{j-1}, A_p(t_j)) \]

\times \{ \text{a function of } (m-j+1)^{\mu} (m-j+2+\theta) - (m-j)^{\mu} (m-j+1+\theta) \}.

(4.47)

\[ R_p(t_{n+1}) = 1 - \frac{\theta}{AB(\theta)} \rho^{(\alpha+1)} \left( \frac{\xi(t_{n+1}) - \xi(t_n)}{\Delta t} \ln(t_{n+1}) + \frac{\xi(t_{n+1})}{t_{n+1}} \right) S_1(t_n, R_p(t_n)) \]

\[ + \frac{\theta}{A(\theta)AB(\theta)} \sum_{j=1}^{n} \frac{\xi(t_j) - \xi(t_{j-1})}{\Delta t} \ln(t_j) + \frac{\xi(t_j)}{t_j} S_2(t_{j-1}, R_p(t_j)) \]

\times \{ \text{a function of } (m-j+1)^{\mu} (m-j+2+\theta) - (m-j)^{\mu} (m-j+1+\theta) \}.

(4.48)

\[ M(t_{n+1}) = 1 - \frac{\theta}{AB(\theta)} \rho^{(\alpha+1)} \left( \frac{\xi(t_{n+1}) - \xi(t_n)}{\Delta t} \ln(t_{n+1}) + \frac{\xi(t_{n+1})}{t_{n+1}} \right) S_1(t_n, M(t_n)) \]

\[ + \frac{\theta}{A(\theta)AB(\theta)} \sum_{j=1}^{n} \frac{\xi(t_j) - \xi(t_{j-1})}{\Delta t} \ln(t_j) + \frac{\xi(t_j)}{t_j} S_2(t_{j-1}, M(t_j)) \]

\times \{ \text{a function of } (m-j+1)^{\mu} (m-j+2+\theta) - (m-j)^{\mu} (m-j+1+\theta) \}.

(4.49)

Graphical simulation

In this section, using the obtained numerical solutions we present in this section numerical simulation for various fractional order and variable fractal order. The numerical simulations are depicted in Fig. 1–7.
Conclusion

Mathematical models with non local variable orders operators have been known to replicate sometime accurately as they are able to include into mathematical model real representation of complex patterns observed in nature. However, mathematical models depicted with such different and integral operators cannot be solved analytically due to highly non linearity of the operators. To have a solution of such model only numerical approximations are useful. In this paper a system of nonlinear ordinary differential equations with non local variable order operators were considered. The system describe the spread of COVID-19 in a given population. Numerical schemes based on Lagrange polynomial was used to derive numerical solutions for three cases, numerical simulations were depicted for different values of frac.

CRediT author statement

The first and second authors have both agreed to use the mathematical models and the methodology. Both authors did numerical schemes. The first author did numerical simulations. Both authors wrote the first original draft.

Compliance with ethics requirements

This article does not contain any studies with human or animal subjects.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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