A Mathematical Model of Laser Drilling with Laser Absorption in the Plasma

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Abstract
Despite being an accurate process with high processing speed, laser drilling suffers from several drawbacks including wall taper, recast layer and spatter. Hence, in order to improve the process efficiency and achieve the good hole quality, it is crucial to have a precise model which can be used as a tool to relate the process parameters to the process output. In this paper, an analytical model of laser drilling is developed. The governing equations are established from the energy equations at the solid-liquid and liquid-vapor interfaces. Absorption of laser energy in the plasma and an additional energy generated from the exothermic reaction are also included in the model. Aiming at simplifying the model, a constant value of the plasma absorption coefficient of the laser beam is employed. Validation of the model is done by comparing with the available experimental data. It is concluded that the recommended value of the plasma absorption coefficient for low carbon steel drilling is 0.2.

Keywords: laser drilling, mathematical model, plasma absorption

1. Introduction
Laser drilling of metal can be performed by means of trepanning drilling, single pulse drilling or percussion drilling. In trepanning drilling, the process begins by drilling a central hole followed by contour cutting to enlarge the hole diameter. It typically generates the hole with diameter larger than 25 mm in the material with thickness ranging from 0.25 to 20 mm [1]. For single pulse drilling, a single shot of high energy laser pulse is used to drill a hole on a thin substrate. This technique is recommended when the production time is more significant than the hole quality. In percussion drilling, multiple laser pulses impinge on the same spot to produce a hole. Between each successive pulse is the pulse off period which solidification may take place. This technique has recently become an attractive candidate for a wide range of industrial applications due to its ability to produce a large number of small diameter holes with tight tolerance in a short time period.

Despite continuous improvement over the decades, laser drilling has not yet achieved its maximum performance. This is due to the fact that there are numerous complex phenomena and parameters involving in the process. During laser drilling of the metals, the irradiated laser beam heats and melts the solid substrate. At high laser beam intensity, vaporization occurs almost immediately. Vaporization of the melt not only remove material in the form of vapour, it also generates the recoil force which acts on the melt surface and pushes the melt up along the cavity wall. The melt expelled from the cavity may
adhere around the hole entrance or may be ejected as a group of small droplets. Once the cavity depth reaches the exit of the workpiece material, the remaining melt is flushed down through the hole bottom. In laser drilling, oxygen is generally supplied coaxially with the laser beam. In addition to protecting the optical lens from the debris, it plays three roles in the process [2]. Firstly, the molten metal may oxidize with oxygen gas and the exothermic energy produced will enhance the drilling rate. Secondly, the oxygen gas convects heat away from the interaction zone, hence lowering the material temperature. Lastly, the assist gas pressure blows the melt away from the cavity resulting in increasing of the material removal rate.

In some cases, further absorption of the laser energy by metal vapour may lead to formation of laser induced plasma (LIP) which reflects the laser beam and traps a large amount of laser energy in it. Sankaranarayanan et al. [3] examined the effects of plasma plume on laser attenuation above the workpiece surface. They found that at low laser intensity, the etch rate per pulse increased as the laser energy increased. However, there was a critical laser intensity in which the etch rate per pulse decreased. At incident laser intensity higher than $4.4 \times 10^6 \text{ W/cm}^2$, less than 20% of the laser energy was delivered to the workpiece surface. Pandey et al. [4] proposed a numerical model of percussion laser drilling. The numerical results obtained were compared with those obtained from their experiments. They found that for laser drilling of SUPERNI 163A, laser induced plasma was observed in all experiments conducted. Therefore, only 5% of the laser energy was assumed to reach the workpiece surface in their model. Hoffman et al. [5] studied effects of the laser wavelength on ablated carbon plume. Experiments were conducted using the first, the second and the third harmonic of Nd:YAG laser. They found that the plasma temperature was highest for the case of 1064 nm laser wavelength compared to those of 532 and 355 nm wavelengths. This implied that the largest amount of laser energy was trapped in the plasma when 1064 nm laser beam is employed. Tokarev et al. [6] pointed out that up to 80% of the laser energy was transformed into plasma energy, hence slowed down the drilling speed. Moreover, the plasma formed can be treated as a secondary drilling tool which enlarged the hole diameter.

Numerous laser drilling models have been proposed to date [7-15]. During the initial period, most of the work emphasized on analytical approach of the one-dimensional heat conduction problem [7-11]. The predicted drilling rates obtained from these models were in reasonable agreement with the experimental data. With the recent availability of the high performance computer, the current trends of the laser drilling models have been shifted to the numerical solutions [12-15]. Drilling speed, hole shape and temperature distribution in the substrate were predicted and effects of the relevant parameters were evaluated. Although the accuracy of the laser drilling models have been greatly improved, there are plenty of gaps to be completed. This is due to the fact that a number of assumptions was typically made in order to simplify the model. For examples, effects of the laser-induced plasma, multiple reflections inside the cavity and the exothermic reaction, which in fact have considerable influence on the laser drilling mechanisms, were often disregarded in the model.

In this paper, a mathematical model of single pulse laser drilling is developed. Laser absorption in the plasma plume is also included. Values of the laser absorption in the plasma are assumed and the appropriate value is determined from comparison with the experimental data. Effects of the oxygen assist gas are also taken into consideration.

2. Mathematical Formulation
Irradiation of the high intensity laser beam on solid substrate, which is initially at temperature $T_0$, leads to formation of the solid-liquid and liquid-vapour interfaces at $z_m(t)$ and $z_v(t)$ as shown in Figure 1. Because the laser intensity used for laser drilling is typically high, it can be assumed that melting and vaporization occur instantly. Moreover, the hole diameter is normally much larger than the melt layer thickness, hence the one-dimensional heat transfer is applicable.

In addition to the above, followings are assumptions made for the present model.
1) The laser beam has uniform spatial profile and rectangular temporal distribution. This assumption is reasonable especially for the Nd:YAG laser, which is widely used for metal drilling.

2) The changes in laser absorptivity, melting and boiling temperatures of the substrate due to oxidation are ignored.

3) Not all of the metal oxidises with the oxygen assist gas. The oxidation efficiency is introduced in the model to account for the portion of the metal that actually oxidises.

4) Thermophysical properties of the materials and oxygen assist gas are constant i.e. these properties are independent of the temperature.

5) Multiple reflections inside the cavity is ignored.

![Figure 1. Schematic diagram of the model](image)

At the liquid-vapour interface, the energy equation can be expressed as:

\[
I_{abs} + \rho_l H_{ox} \eta_{ox} \frac{\partial z_m}{\partial t} - h_g (T_{l0} - T_a) + k_j \frac{\partial T_l}{\partial z} = \rho_v L_v \frac{\partial z_v}{\partial t}
\] (1)

At the solid-liquid interface, the energy balance can be written as:

\[
k_s \frac{\partial T_s}{\partial z} - k_l \frac{\partial T_l}{\partial z} = \rho_s L_m \frac{\partial z_m}{\partial t}
\] (2)

where

- \(I_{abs}\) is the laser energy absorbed by the liquid surface,
- \(k_1, k_s\) are the thermal conductivities of liquid and solid metal,
- \(\rho_l, \rho_s\) are the densities of liquid and solid,
- \(L_m, L_v\) are the latent heat of melting and the latent heat of vaporization,
- \(H_{ox}\) is the enthalpy of oxidation,
- \(\eta_{ox}\) is the oxidation efficiency,
- \(z_{m}, z_v\) are the locations of the solid-liquid and liquid-vapour interfaces,
- \(h_g\) is the convection heat transfer coefficient at the liquid surface,
- \(T_l, T_s, T, T_{l0}\) are the temperatures in the liquid and solid, temperature of the surrounded gas and the temperature of the liquid surface which can be estimated from:

\[
T_{l0} = I_{abs} \frac{4t}{\pi \rho_l c_{eff} k_j}
\] (3)
Note that \( \tau \) is the laser heating time and \( c_{\text{eff}} \) is the effective specific heat defined by \( c_{\text{eff}} = c_p s + \frac{L_m}{T_m} \), where \( c_p s \) is the specific heat of the solid and \( T_m \) is the melting point of material.

The second term on the left hand side of equation (1) is the energy added due to exothermic reaction. Because not all of the metal oxidises, the oxidation efficiency \( \eta_{\text{ox}} \) is included in the model. Ng et al.\[2\] laser drilled the mild steel plate and deduced the oxidation percentage from the cross-sectional area of the melt droplets. The oxidation percentage was found to be in the range of 26±18%. Hence, the oxidation efficiency \( \eta_{\text{ox}} \) is assumed to be 0.26 in this present model.

As the laser beam passes through the plasma, a fraction of the laser energy is absorbed in the plasma. If the laser intensity entering the plasma is \( I_i \), the laser intensity exiting the plasma \( I_{po} \) can be approximated by:

\[
I_{po} = (1 - A_{h\beta}) I_i
\]

where \( A_{h\beta} \) is the absorption coefficient.

Sankaranarayanan et al. \[3\] found that the fraction of laser energy that was absorbed by the plasma plume depends on the incoming laser intensity, \( I_i \). At \( I_i > 40 \, \text{MW/cm}^2 \), up to 80% of the laser energy might be trapped in the plasma. However, the results from their model which was based on this value of the absorption coefficient showed an overestimation of the laser absorption in the plasma. Mahdavi and Ghazizadeh \[18\], however, reported that for 1.064 \( \mu \text{m} \)-wavelength laser beam generated by a Nd:YAG laser, there was only 10% of the laser energy absorbed in the plasma. A large difference in the absorption coefficient values used in these studies indicated the uncertainty in approximating the plasma absorption coefficient. In this present model, values of the plasma absorption coefficient are assumed. An appropriate value is subsequently selected by comparing with the available experimental data.

After the laser beam leaving the plasma, it irradiates on the liquid substrate. The laser energy absorbed by the liquid layer can be determined from:

\[
I_{abs} = A_l I_{po} = A_l (1 - A_{h\beta}) I_i
\]

Oxygen assist gas plays several roles in laser drilling process. It protects the optical lens from debris, blows the melt away from the cavity, and adds exothermic energy to the system. Moreover, employing assist gas in the process increases the convection heat transfer from the interaction zone. The heat transfer coefficient \( h_g \) for this case can be determined from \[14\]:

\[
h_g = \frac{k_g}{2 \eta_b} \left( C_c \text{Re}^{n_c} \text{Pr}^{1/3} \right)
\]

where \( \eta_b \) is the beam radius, \( k_g \) is thermal conductivity of the assist gas, \( \text{Re} \) and \( \text{Pr} \) are Reynolds number and Prandtl number, respectively. The constants \( C_c \) and \( n_c \) for the forced convection heat transfer normal to the liquid surface are \( C_c = 0.228 \) and \( n_c = 0.731 \).

The Reynolds number of the gas flow is given by

\[
\text{Re} = \frac{\rho_g \nu_g 2 \eta_b}{\mu_g}
\]

where \( \rho_g \), \( \nu_g \) and \( \mu_g \) are the density, velocity and viscosity of the oxygen assist gas.
Thickness of the melt and the heat penetration depth in the solid layer are very thin compared with the material thickness. Therefore, it may be further assumed that the temperature profiles in the melt and penetration depth of the solid layers are linear. The energy equation at the two interfaces can now be rewritten as follows:

$$I_{abs} + \rho_l H_{ox} n_{ox} \dot{z}_m - h_g (T_{l0} - T_a) - k_l \frac{T_{l0} - T_m}{(\dot{z}_m - \dot{z}_v)\tau} = \rho_l L_v \dot{z}_v$$  \hspace{1cm} (8)

$$k_l \frac{T_{l0} - T_m}{(\dot{z}_m - \dot{z}_v)\tau} - k_s \frac{T_m - T_a}{2\sqrt{\alpha_s \tau}} = \rho_s L_m \dot{z}_m$$  \hspace{1cm} (9)

where $\alpha_s$ is the thermal diffusivity of the solid, $\dot{z}_m$ and $\dot{z}_v$ are the propagating velocity of the solid-liquid and liquid-vapour interfaces, $\tau$ is the pulse duration and $T_m$ is the melting temperature of the material.

Solving of equations (8) and (9) gives the velocity of the two interfaces $\dot{z}_m$ and $\dot{z}_v$.

3. Material properties

The thermophysical properties of low steel are listed in Table 1. Table 2 shows the laser absorptivity of 1.06 $\mu$m laser wavelength. Laser absorptivity of the liquid metal is approximated by $A_l \approx 1.05 A_s$ [16].

**Table 1. Thermophysical properties of the low carbon steel [17-19]**

| Property | Value |
|----------|-------|
| $\rho_s$ (kg/m$^3$) | 7800 |
| $\rho_l$ (kg/m$^3$) | 6980 |
| $c_{ps}$ (J/kg K) | 628 |
| $c_{pl}$ (J/kg K) | 748 |
| $\alpha_s$ (m$^2$/s) | $0.014 \times 10^{-3}$ |
| $\alpha_l$ (m$^2$/s) | $0.007 \times 10^{-3}$ |
| $T_m$ (K) | 1808 |
| $T_v$ (K) | 3100 |
| $L_m$ (J/kg) | $276 \times 10^3$ |
| $L_v$ (J/kg) | $6088 \times 10^3$ |

**Table 2. Laser absorptivity of low carbon steel at 1.06 $\mu$m laser wavelength [16-17].**

| Property | Value |
|----------|-------|
| $A_s$ | 0.37 |
| $A_l$ | 0.0389 |
4. Results and discussion

In order to validate the model, the calculated results are compared with the available experimental data. In Low et al. [11]’s experiment, a hole was drilled on low carbon steel plate using a single pulse laser generated from the Nd:YAG fiber laser. Figures 2 and 3 show the comparison of the predicted drilling velocity with the measured data at various absorbed laser intensity for the pulse duration of 0.5 and 1.0 ms, respectively. Values of the absorption coefficient are varied from 0 to 0.2 with the incremental step of 0.1 in the model. It is found that $A_{ib} = 0$, which is the case of no absorption in the plasma, gives the best prediction for both cases. For the case of 0.5 ms pulse width as shown in Figure 2, average errors are found to be 11.4 and 16.0% for $A_{ib} = 0$ and $A_{ib} = 0.1$. However, for the case of 1.0 ms pulse duration, average errors are quite close i.e. the average errors are 16.1 and 16.9% for $A_{ib} = 0$ and $A_{ib} = 0.1$, respectively. These errors may be due to the assumptions used in the model. For example, the beam multiple reflections which normally occur in the actual drilling process are not accounted in the model. If the beam multiple reflections are included, more laser energy will be absorbed by the cavity wall which will subsequently leads to the higher predicted drilling speed. Moreover, the present model assumes a constant value of the material absorptivity whereas the actual value varies with temperature and material properties.

Although the model with $A_{ib} = 0$ gives good prediction of the drilling velocity, it is important to note that to estimate the absorbed laser intensity from the impinging laser beam intensity, Low et al. [11] assumed the reflectivity of the low carbon steel to be 0.21. In other words, the material absorptivity used in their calculation was 0.79. However, in this present model, the absorptivity at the liquid surface is $A_l = 0.389$ which is much lower than that employed by Low et al. This results in less laser energy absorbed by the liquid. In addition to that, the present model also accounts for absorption of laser energy in the plasma which lessens the amount of laser energy reaching the liquid surface. Thus, the drilling velocity predicted by the present model tends to lower than that obtained experimentally.

![Figure 2](image url)

**Figure 2.** Comparison of the predicted drilling velocity with the experimental data of Low et al. [11]. Pulse duration is 0.5 ms.
Figure 3. Comparison of the predicted drilling velocity with the experimental data of Low et al. [11]. Pulse duration is 1.0 ms.

Figure 4. Comparison of the predicted hole depth with the experimental data of Maturose and Sheikh [20].
Figure 4 shows the predicted hole depth in comparison with the experimental data of Maturose and Sheikh [20]. The laser beam used in their experiment was Nd:YAG laser which produced a laser beam at 1.06 \( \mu \)m wavelength. The predicted results in this case show fair agreement with the measured hole depth. The smallest average error is 18.5\% which is obtained using \( A_{ib} = 0.5 \). It can also be seen from Figure 4 that during the initial time of laser drilling, the measured hole depth is much deeper than the calculated ones. This is probably because there is only small amount of plasma formed at the beginning of the actual drilling process. Thus, more laser energy is allowed to irradiate on the material surface and hence the deeper hole.

All in all, according to Figures 2 and 3, the proper value of the plasma absorption coefficient is \( A_{ib} = 0 \) while in Figure 4, \( A_{ib} = 0.5 \) is recommended. Therefore, the approximated value of plasma absorption coefficient \( A_{ib} = 0.2 \) is used throughout this present model.

Figure 5 illustrates number of pulses required to initiate breakthrough. It is obvious that including of the plasma absorption results in more pulses required. This is because less laser energy is delivered to the workpiece surface, therefore less material is melted, vaporized and removed from the interaction zone. This subsequently leads to a shallower hole depth. At 6 kW peak power, although 4 laser pulses are required for both cases, the hole depths obtained are different. By including the plasma absorption, the calculated hole depth is 2.53 mm whereas excluding the plasma absorption predicts 2.98 mm hole depth.

Figure 5 also shows that as the pulse peak power increases, less pulse is required to produce a through hole. This is not surprised because the higher the peak power, the more laser energy impinges on the surface. Consequently, more material is removed resulting in the formation of the deeper hole.
5. Conclusion
This paper presents a mathematical model of laser drilling with laser beam absorption in the plasma. A range of plasma absorption coefficients is assumed and the suitable value is selected from comparison with the available experimental data. It is found that the absorption coefficient of 0.2 gives the most promising prediction. However, it is crucial to note that this recommended value of the absorption coefficient is based on the laser drilling of low carbon steel using 1.06 µm wavelength Nd:YAG laser.

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