SOFT X-RAY EXCESS FROM SHOCKED ACCRETING PLASMA IN ACTIVE GALACTIC NUCLEI

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ABSTRACT
We propose a novel theoretical model to describe the physical identity of the soft X-ray excess that is ubiquitously detected in many Seyfert galaxies, by considering a steady-state, axisymmetric plasma accretion within the innermost stable circular orbit around a black hole (BH) accretion disk. We extend our earlier theoretical investigations on general relativistic magnetohydrodynamic accretion, which implied that the accreting plasma can develop into a standing shock under suitable physical conditions, causing the downstream flow to be sufficiently hot due to shock compression. We perform numerical calculations to examine, for sets of fiducial plasma parameters, the physical nature of fast magnetohydrodynamic shocks under strong gravity for different BH spins. We show that thermal seed photons from the standard accretion disk can be effectively Compton up-scattered by the energized sub-relativistic electrons in the hot downstream plasma to produce the soft excess feature in X-rays. As a case study, we construct a three-parameter Comptonization model of inclination angle \( \theta_{\text{obs}} \), disk photon temperature \( kT_{\text{in}} \), and downstream electron energy \( kT_e \) to calculate the predicted spectra in comparison with a 60 ks XMM-Newton/EPIC-pn spectrum of a typical radio-quiet Seyfert 1 active galactic nucleus, Ark 120. Our \( \chi^2 \)-analyses demonstrate that the model is plausible for successfully describing data for both non-spinning and spinning BHs with derived ranges of \( 61.3 \text{ keV} \lesssim kT_e \lesssim 144.3 \text{ keV} \), \( 21.6 \text{ eV} \lesssim kT_{\text{in}} \lesssim 34.0 \text{ eV} \), and \( 17^\circ \lesssim \theta_{\text{obs}} \lesssim 42^\circ \), indicating a compact Comptonizing region of three to four gravitational radii that resembles the putative X-ray corona.

Key words: accretion, accretion disks – black hole physics – galaxies: individual (Ark 120) – galaxies: Seyfert – magnetohydrodynamics (MHD) – methods: numerical

1. INTRODUCTION

The broad-band synergistic study of active galactic nuclei (AGNs) in recent years has revealed a number of underlying spectroscopic components that are critical for understanding the fundamental physical process around the central engines of AGNs. In particular, in close proximity to the nucleus, well within the sphere of influence of the supermassive black hole (BH), state-of-the-art X-ray spectroscopy has clearly demonstrated the complexity of AGN physics, which is associated with inflows as well as outflows. Among other features the "soft X-ray excess", which is the excessive amount of spectral features below \( \sim 2 \text{ keV} \) above a baseline continuum extrapolated from a hard X-ray range (typically from a \( 2-10 \text{ keV} \) power-law component of photon index \( \Gamma \sim 2 \)). It is known to be present among Seyfert galaxies, particularly in the so-called narrow-line Seyfert 1 (NLS1) AGNs—a sub-class of the type 1 Seyfert galaxies with certain spectroscopic properties. Since the first extensive analysis, as a part of a multi-wavelength campaign (extreme-UV (EUV) and soft/hard X-rays with ROSAT), of NLS1s about two decades ago (e.g., Osterbrock & Pogge 1985; Goodrich 1989; Walter & Fink 1993; Boller et al. 1996; Leighly 1999a, 1999b), only small fragments of observational detail have been obtained (even from more recent observations seeking the origin of soft excess) as follows. (1) Soft excess is almost ubiquitous in NLS1 galaxies of both smaller and larger BH masses. (2) Its equivalent blackbody temperature appears to be almost universally \( \sim 0.1-0.2 \text{ keV} \) regardless of the object. (3) Its output power occupies a good fraction of the total AGN luminosity (Boller et al. 1996). In the context of the standard accretion disk (Shakura & Sunyaev 1973) its maximum effective temperature is well-defined to be only \( kT \sim 10 (\dot{m}/M_{\odot})^{1/4} \text{ eV} \) where \( \dot{m} \equiv M/M_{\text{Edd}} \) is the AGN mass-accretion rate normalized by the Eddington mass-accretion rate for a BH mass \( M \approx M_{\odot}/(10^8 M_{\odot}) \). Not only is it challenging to account for the observed nearly constant "temperature" of the soft excess using the standard disk model (e.g., Shakura & Sunyaev 1973), even with an extreme assumption of high accretion rate and a low-mass BH the model can barely bring the peak of the disk spectrum up to \( kT \lesssim 0.1 \text{ keV} \). This means that the standard disk emission should make virtually no significant contribution to the observed soft excess (e.g., Laor et al. 1997) thus the model fails to explain its physical origin. Although long-sought, the nature of this spectral component has been quite elusive so far.

The number of plausible scenarios to explain the physics of the soft excess includes: (1) a continuum component strongly absorbed by a series of ionized absorbers in a relativistic outflow whose spectral curvature could then be interpreted as a falsified “excess” feature (Gierliński & Done 2004; Schurch & Done 2006, 2008; Middleton et al. 2007); (2) ionized atomic processes from an inner part of the disk illuminated via light bending (e.g., Fabian et al. 2004; Miniutti & Fabian 2004; Kara et al. 2015) to produce a series of relativistically blurred emission lines to mimic an apparently smooth "excess" spectral shape (e.g., Ross & Fabian 2005; Crummy et al. 2006; Fabian).
et al. 2009; Ponti et al. 2010; Nardini et al. 2011; De Marco et al. 2013) with a predicted correlation between the hard and soft X-rays (e.g. Vasudevan et al. 2014, Boissay et al. 2016); and (3) a Comptonization of the disk photons by some means such as the corona ($kT_e = 1$ keV) or upper layer of the disk (e.g., Petrucci et al. 2004; Mehdipour et al. 2011; Done et al. 2012; Noda et al. 2013; Zhong & Wang 2013; Di Gesu et al. 2014) with an expectation of a correlation between UV and soft X-ray flux (Mehdipour et al. 2011). Petrucci et al. (2013), for example, performed a synergistic spectral analysis (from UV to hard X-ray) based on the multi-wavelength campaign on a bright Seyfert galaxy, Mrk 509, focusing in part on the observed soft excess. Motivated by the implied correlation between UV and soft X-ray flux from XMM-Newton and INTEGRAL observations, the authors proposed a thermal Comptonization model to describe the physical origin of both the soft excess and power-law components.

Accretion physics has been extensively studied for decades, particularly in terms of the theoretical aspects including semi-analytic investigations as well as global numerical simulations, in an effort to further understand its physical nature and observational consequences. Many of the works on BH accretion have, in general, revealed an important generic feature of accretion, i.e., the formation of shocks as an accreting plasma is subject to outward forces via a number of decelerating mechanisms (e.g., Abramowicz & Prasanna 1990) and develops a shock front at $r = r_{sh}$ within the radius of the inner edge of a magnetized accretion disk, perhaps equivalent to an innermost stable circular orbit (ISCO) for a pure HD Keplerian disk, before crossing an event horizon at $r = r_{H}$. Previous studies include hydrodynamic shocks (e.g., Nobuta & Hanawa 1994; Lu et al. 1997; Chakrabarti 1990; Fukumura & Tsuruta 2004) and magnetohydrodynamic (MHD) shocks (e.g., Koide et al. 1998, 2000; Das & Chakrabarti 2007; Takahashi et al. 2002, 2006, hereafter T02, T06, respectively; Fukumura & Kazanas 2007b; Fukumura et al. 2007, hereafter F07; Takahashi & Takahashi 2010). In particular, extensive theoretical studies of various types of shocks have been conducted to date in an attempt to understand their dynamical behavior, e.g., shock oscillation in the context of quasi-periodic oscillations and its spectroscopic signatures (e.g., Chakrabarti & Titarchuk 1995; Molteni et al. 1996, 1999; Acharya et al. 2002; Okuda et al. 2004, 2007; Nagakura & Yamada 2008) that may be relevant for X-ray Binaries (XRBs), for example. Independent general relativistic (GR) MHD (GRMHD) simulations of the tilted accretion disk clearly show that the compression of the plunging plasma in the inner region ($r \lesssim 10r_g$) leads to the formation of standing shocks (e.g., Fragile et al. 2007; Fragile & Blaes 2008; Generozov et al. 2014) depending on the characteristics of the disk geometry and the BH spin (e.g., Morales et al. 2014). The expected highly magnetized shocked region may perhaps correspond to the magnetically arrested plasma seen in other large-scale simulations (e.g., Tchekhovskoy et al. 2011).

As a generic feature of accretion shocks unambiguously revealed in earlier theoretical works by many authors, the downstream flow across the shock front is compressed and heated up efficiently to generate additional entropy all the way down to the horizon unless a cooling process is sufficiently efficient (e.g., Chakrabarti 1995; Chakrabarti & Titarchuk 1995). While the detailed formalisms and numerical methodologies in these works are different, the presence of shocks in accretion is strongly favored in these calculations. As a result, the post-shock region at small radii will provide an ideal site where the accelerated electrons could Compton up-scatter the thermal photons from an accretion disk. This process can produce a characteristic excess component in the soft X-ray band below $\sim 1–2$ keV as a substantially modified disk blackbody radiation, and its spectral shape depends on a number of variables related to MHD accretion processes. Our current work is thus motivated by this long-standing implication of shock formation in accretion.

Utilizing the models of T02 and F07, we thus make a preliminary attempt in this paper to calculate the expected soft X-ray excess spectrum in the context of the well-explored GRMHD shocked accretion models in the previous works (T02, F07). As depicted in Figure 1, we assume magnetized equatorial accretion with the inner edge of $r_{in}$ (similar to the standard Shakura–Sunyaev disk, Shakura & Sunyaev 1973) in this model as a reservoir for incoming EUV photons. Plasma near $r = r_{in}$ (i.e., an ISCO-like boundary radius as discussed in Armitage et al. 2001) eventually begins to plunge in and subsequently develops an adiabatic standing shock at a very small radius determined primarily by the plasma conditions such as energy, angular momentum, and mass-accretion rate, for example. We recall, in the case of GRMHD accretion, that there exist multiple magnetosonic points between $r_{in}$ and $r_{H}$ (see T02 and T06 for a more detailed discussion). The downstream region is therefore heated by adiabatic shock compression creating a centrally concentrated, compact hot region similar to the putative X-ray coronae (e.g., Fabian et al. 2015; Wilkins et al. 2015). The incoming blueshifted disk photons toward the downstream plasma are then Comptonized by hot electrons in the post-shock flow, producing the soft excess. Note, however, that the proposed Comptonization in our model is attributed exclusively to accelerated electrons in the rest-frame of the downstream plasma accretion, independent of the bulk motion of the flow (e.g., see, Titarchuk et al. 1996, for a fundamental difference from the standard bulk motion Comptonization model). Our study focuses on explicitly constraining the defining parameters of the GRMHD accretion models by clearly identifying the physical origin of the observed soft excess feature as a shock-heating innermost downstream flow. Our ultimate goal is to systematically understand the physics of the observed soft excess component in a coherent scenario by applying the model to a large sample of relevant Seyfert AGNs. As an example in this paper, we have analyzed a stereotypical radio-quiet Seyfert AGN, Ark 120, as

![Diagram](https://example.com/diagram.png)

**Figure 1.** A schematic diagram illustrating the nonthermal Comptonizing process of thermal disk photons of energy $kT_{m}$ (labeled $\gamma_{bb}$) by downstream energetic electrons of energy $kT_{e}$ to produce the Comptonized photons (labeled $\gamma_{\text{Comp}}$). See the text for the radius labels.
an application of the model and to demonstrate its viability via $\chi^2$-statistics.

We will briefly review the characteristics of the GRMHD accretion in Section 2 by presenting fiducial shocked accretion solutions. Then, our methodology for calculating the Comptonized spectrum for a series of solutions in Kerr geometry is discussed in Section 3 with GR effects being fully implemented. In Section 4 we show our preliminary results for a 60 ks XMM-Newton/EPIC-pn spectrum of Ark 120 as a case study successfully constraining primary variables in the model. We summarize and discuss the implications of the model in Section 5.

2. GRMHD MODELS WITH SHOCKED ACCRETING PLASMA

2.1. Formalism

We adopt the well-defined model for GRMHD shock formation in accreting plasma, discussed in a series of papers (T02, T06, F07) in a formalism closely aligned with other simulations, for example, by Pu et al. (2015). We consider stationary ($\partial_t = 0$) and axisymmetric ($\partial_\phi = 0$) ideal MHD accretion in Kerr geometry whose spacetime metric component $g_{\mu\nu}$ is described by the Boyer–Lindquist coordinates $(t, r, \theta, \phi)$

$$ds^2 = \left(1 - \frac{2Mr}{\Sigma}\right)dt^2 + \frac{4Mar}{\Sigma} \sin^2 \theta \, dtd\phi - \frac{A \sin^2 \theta}{\Sigma} d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2,$$

with the conventional (+−−−) metric signature where $M$ is BH mass and $a$ is its angular momentum per BH mass (i.e., spin parameter) with $\Delta \equiv r^2 - 2Mr + a^2$, $\Sigma \equiv r^2 + a^2 \cos^2 \theta$, and $A \equiv (r^2 + a^2)^2 - a^4 \sin^2 \theta$. The length scale in this paper is normalized to the gravitational radius $r_g$ where $r_g \equiv GM/c^2$ with $G$ and $c$ being the gravitational constant and speed of light, respectively.

In the context of ideal GRMHD, the properties of the accreting plasma are governed by (1) the particle number conservation law, $(n u^\alpha)_{\alpha}$, where $n$ is the proper particle number density and $u^\alpha$ is the plasma four-velocity; (2) the equation of motion, $T^{\alpha\beta} = 0$, where $T^{\alpha\beta}$ is the energy–momentum tensor for magnetized plasmas; and (3) the ideal MHD condition, $u^\alpha F_{\alpha\beta} = 0$, where $F_{\alpha\beta}$ is the electromagnetic field tensor. The poloidal plasma four-velocity is given as $u^\alpha_p \equiv -u^\alpha_n$. The energy–momentum tensor $T^{\alpha\beta}$ is given by

$$T^{\alpha\beta} = n \mu u^\alpha u^\beta - P g^{\alpha\beta} + \frac{1}{4\pi} \left( F^{\gamma\alpha} F^\beta_{\gamma} + \frac{1}{4} g^{\alpha\beta} F^2 \right),$$

where $F^2 \equiv F_{\mu\nu} F^{\mu\nu}$ and $\mu = (\rho + P)/n$ is the relativistic enthalpy, $P$ is the thermal gas pressure, $\rho$ is the total energy density, and $n$ is the plasma number density. Note that we assume the polytropic relation as $P = K_p \rho^\Gamma_p$ where $K_p$ is related to the entropy of the plasma by the polytropic index $\Gamma_p$, and $\rho_0 = n m_e$ is the rest-mass density with the particle rest-mass $m_e$.

Assuming a steady-state, axisymmetric plasma, one can describe a topology of magnetic field lines using the magnetic stream function $\Psi(r, \theta)$ which is constant along a given field line. The plasma is frozen-in and flows along the field lines with five constants of motion in this formalism: the angular velocity of field lines $\Omega_f(\Psi)$; the plasma flux to magnetic flux ratio $\eta(\Psi)$; the total energy of the accreting plasma $E(\Psi)$; the total angular momentum of the plasma $L(\Psi)$; and the entropy $S(\Psi)$. The total energy and angular momentum of the adiabatic plasma are given by

$$E \equiv \mu u_t - \frac{\Omega_f B_\phi}{4\pi \eta},$$

$$L \equiv -\mu u_\phi - \frac{B_\phi}{4\pi \eta},$$

where $B_\phi = (\Delta/\Sigma)F_\phi \sin \theta$ is the poloidal component of the magnetic field seen by a distant observer. From the poloidal components of the equation of motion with the above five constants, one can derive the GR Bernoulli equation (also called the poloidal/wind equation) as

$$\mu^2 (1 + u_\phi^2) \equiv E^2 [(\alpha - 2M^2) f^2 - \delta],$$

where $u_\phi^2 \equiv -(u_t u^\phi) = -(u_t u^\phi + u_\phi d^\phi)$ is the poloidal plasma velocity with $f$, $\delta$, and $\alpha$ being the functions of the metric components and conserved quantities (see T06 and F07). Here, $M_A$ is the relativistic Alfvén Mach number defined as

$$M_A^2 \equiv \frac{4\pi \mu_0 u_p^2}{B_p^2},$$

where $B_p$ is the poloidal magnetic field in the distant observer’s frame (e.g., T06; F07). Technically speaking, a field geometry should be self-consistently calculated by the force-balance equation in a direction parallel to the streamline described by the Grad–Shafarmon (GS) equation in the GR regime. However, we will adopt a simplistic approach (e.g., T02; F07) and specify a purely conical field line geometry such that the poloidal magnetic field is given by $|B_\phi| \propto (\Delta/\Sigma)^{1/2}$ following the split-monopole approximation (e.g., Michel 1973; Wald 1974; Blandford & Znajek 1977; see also Section 5). In this formalism, the field topology is thus parameterized to be conical. Plasma is assumed to be adiabatic of single temperature and we ignore its self-gravity and viscous nature.

To describe the plasma kinematics from an intuitive perspective, we calculate and express the three-velocity components of the plasma in two different locally flat inertial frames (e.g., Mannoto 2000), i.e., the radial component $v_{\phi_{\text{CRF}}}$ defined in a co-rotating reference frame (CRF) where a local observer is co-rotating with the plasma such that

$$v_{\phi_{\text{CRF}}} = \left(\frac{-u_t u^\phi}{1 - u_t u^\phi}\right)^{1/2},$$

and toroidal component $v_{\phi_{\text{LNRF}}}$ is defined in a locally non-rotating reference frame (LNRF) where a zero-angular-momentum-observer (ZAMO) is co-rotating with a BH such that

$$v_{\phi_{\text{LNRF}}} = \frac{A}{r^2 \Delta^{1/2}} (\Omega - \omega),$$

$^8$ The Mach number $M_A$ can decrease while plasma speeds up if the magnetic field strength increases faster.
where $\Omega \equiv u^0/u^t$ is the angular velocity of the plasma and the frame-dragging $\omega \equiv -g_{00}/g_{00}$ has been subtracted from the LNRF so one sees the intrinsic plasma rotation locally. Note that in these reference frames it is guaranteed to have $v^0_{\text{CRF}} / v^0_{\text{LNRF}} \to 1$ and $v^\phi_{\text{CRF}} / v^\phi_{\text{LNRF}} \to 0$ as $r \to r_H$ by definition (see Section 2.2 for results). To characterize the magnetized nature of the plasma, we examine the magnetization parameter $\sigma$ defined in LNRF as the ratio of the outward Poynting flux to the inward net mass-energy flux of the accreting plasma, such that

$$\sigma \equiv \frac{B_0 g_{00} (\Omega_p - \omega)}{4 \pi n_0 u^t \rho_w^2},$$

where $\rho_w^2 \equiv g_{10}^2 - g_{tt} g_{00}^2$. Thus, the sign of $\sigma$ in general can change as $B_0$ may switch its direction due to the global field geometry (see T02 and F07 for details).

Assuming that the accreting plasma is initially injected from a plasma source (most likely near the inner edge of a magnetized accretion disk; see Figure 1) with its toroidal velocity being predominant (i.e., $v^0_{\text{CRF}} < v^\phi_{\text{LNRF}}$) onto a BH, the physical plasma accretion must be trans-magnetosonic before reaching the event horizon, going through two magnetosonic points (i.e., slow and fast-magnetosonic points) and the Alfvén point. During the course of accretion, furthermore, the accreting plasma is subject to various “obstacles” that slow it down inwards, e.g., gas pressure, radiation pressure, magnetic force, and centrifugal barriers, for example. Via nonlinear processes, the flow can develop into a shock front at some radius determined by a certain physical condition. Considering a proper jump condition across the shock front, one can determine a physically valid shock location $r_{sh}$. Note that both upstream and downstream plasma must be trans-magnetosonic on its own, i.e., the former (latter) must pass through the outer (inner) magnetosonic points and the Alfvén point. To simplify the problem we set the surface of the shock front to be normal to the magnetic field lines as in the previous calculations (e.g., T02, F07). Of the different types of shocks, we consider here adiabatic (i.e., Rankine–Hugoniot) perpendicular shock conditions where cooling processes are so inefficient at the shock front that no energy (or angular momentum) is dissipated away. The condition can be analytically simplified as

$$\frac{1}{\Delta \Sigma} \left( \frac{M_1}{4 \pi \eta} \right)^2 + \frac{\mu_1 - 1}{1 + N} \frac{\mu_1 - 1}{M_1^2} + \left( \frac{E_i}{2} \right)^2 = \frac{1}{\Delta \Sigma} \left( \frac{M_2}{4 \pi \eta} \right)^2 + \frac{\mu_2 - 1}{1 + N} \frac{\mu_2 - 1}{M_2^2} + \left( \frac{E_f}{2} \right)^2,$$

where the roots ($r = r_{sh}$) to this equation are “shock locations” and the subscripts “1” and “2”, respectively, denote upstream and downstream quantities. We have used $1 + N \equiv \Gamma_p (\Gamma_p - 1)$. We will numerically calculate this radius, across which particle number, energy, angular momentum, and magnetic flux are all simultaneously conserved (while density, temperature, and velocity are discontinuous). Note that the enthalpy $\mu$ remains conserved due to adiabatic assumptions but increases across the shock because of entropy generation. The local shock compression in the Newtonian view is then given by the velocity ratio as

$$\frac{\rho_2}{\rho_1} = \frac{u^t_2}{u^t_1} = \frac{\mu_2}{\mu_1} \frac{M^2_1}{M^2_2}.$$  \tag{11}$$

As in the past works (e.g., T02; F07), we simply treat the shock front as a mathematical discontinuity rather than considering its actual finite internal structure (see, e.g., Le & Becker 2005; Becker et al. 2011, for considering particle transport at a diffusive shock front). We recall that the post-shock downstream flow is heated across the shock front in the absence of efficient cooling mechanisms and remains hot under the adiabatic flow assumption in this work.

Most importantly in the present work, we also introduce the normalized electron thermal energy $\Theta_e$ to assess the energetics of accelerated electrons in the heated downstream plasma due to shock compression as a function of radius as

$$\Theta_e(r) \equiv \frac{k T_e}{m_e c^2} = \frac{E_i}{4 \pi \eta} = \frac{1}{1 + N} \left( \frac{\mu}{m_e c^2} - 1 \right).$$ \tag{12}$$

where $k$ is the Boltzmann constant, $m_e c^2 = 511$ keV is the electron’s rest-mass energy and $T_e$ is electron’s equivalent thermal temperature in a single-fluid approximation. Hence, the temperature is uniquely determined by the plasma density and entropy, which is closely related to plasma pressure as well from the polytropic assumption. Unlike the hot downstream flow, the upstream (preshock) plasma is assumed to have negligible thermal energy (i.e., $K \sim 0$) compared to its rest-mass energy in the cold flow limit (i.e., $\mu_1 \sim m_e c^2$ thus $\Theta_e \sim 0$ in the upstream region). In this limit, the slow magnetosonic point for the upstream plasma vanishes, leaving only the Alfvén point and a fast-magnetosonic point.

2.2. Numerical Solutions for Shocked Plasma Accretion

Following previous works (T02, F07), we calculate a global property of physically valid plasma accretion for a given set of conservative quantities described in Section 2.1. Our calculations throughout this paper are restricted to the equatorial flows for simplicity (i.e., $\theta = \pi/2$ and $u^t = B^t = 0$) and we set the polytropic index $\Gamma_p = 4/3$ in the presence of the conical magnetic field. To exploit the parameter space as systematically as possible, a fiducial value of the plasma energy is chosen as $E = 6.1$ in all cases discussed here (see F07 for its relevance) along with the other conserved quantities and radii as listed in Table 1. Note that these characteristic radii are not free parameters but are determined by the shock conditions. We then vary the rest of the primary parameters $L/E$ and $\Omega_p$ for a given BH spin $a$ and $\eta$ in order to find the valid solutions (see T02 and F07 for a detailed numerical methodology).

As a representative solution for a given BH spin, normalized radial profiles of major characteristics of each shocked accretion are shown in Figures 2–4 for (a) $a/M = 0$, (b) 0.5, and (c) $-0.5$ along with its corresponding outer/inner fast-magnetosonic radius and the Alfvén radius (vertical dotted lines). A standing shock is denoted as a solid gray vertical line and a diffusive shock as a dotted gray vertical line.
Alfvén point and the outer fast point. The upstream flow then forms a shock at $r = r_{sh}$ developing a hot downstream region, after which it passes through the inner fast point before entering the event horizon in each BH spin case (a)–(c). Note that the upstream plasma starts slowing down due to a number of outward forces (e.g., the Lorentz force and centrifugal barrier) just before forming a shock. In the case of a Schwarzschild BH in (a), the plasma motion becomes more and more radial toward the horizon (i.e., $u^r > u^\phi$). For a prograde BH in (b), the plasma starts to plunge in from the ISCO at a smaller radius compared to (a) because the ISCO radius shifts more inward. Therefore, the plunging plasma acquires a larger toroidal velocity $u^\phi$ due to a faster Keplerian motion of the disk in the beginning. At a small radius, the frame-dragging effect is prominent, forcing the downstream flow to co-rotate with the BH and $u^\phi$ to dominate over $u^r$. On the other hand, around a retrograde BH in (c), the rotational sense of the upstream plasma ($u^\phi > 0$ at large radii) is eventually switched the other way around ($u^\phi < 0$ near the horizon) due to the frame-dragging, as expected. In other words, a distant observer in a flat spacetime sees the accreting plasma momentarily turn around at some point (i.e., $u^\phi = 0$) between the outer fast point and the shock location, changing the direction of its toroidal motion.

A corresponding three-velocity of the plasma is calculated in a local reference frame of flat spacetime as shown in Figure 3. In all the cases the plasma is seen in the CRF to radially approach the speed of light ($v_{\text{CRF}} \to 1$) in the course of accretion while the toroidal motion seen in the LNRF eventually approaches the spacetime rotational speed ($\Omega \to \omega$ thus $v_{\text{LNRF}} \to 0$) as expected. Note in (c) that the same “turn-around” behavior of the plasma toroidal motion is clearly seen in the LNRF between the outer fast point and the shock location (i.e., $v_{\text{LNRF}} > 0 \to v_{\text{LNRF}} < 0$) due to the frame-dragging and, in fact, the plasma “overshoots” the ZAMO in the LNRF until the shock occurs (thus $v_{\text{LNRF}}$ continues to increase in the same sense as the BH rotation). In the downstream flow, the shocked plasma stops “overshooting” the ZAMO, eventually converging to the frame-dragging at the horizon as expected. It is counterintuitive to see in (c) that the plasma seemingly appears to “speed up” in the toroidal direction across the shock (i.e., $|v_{\text{LNRF}}(r)|$ increases across the shock) in the LNRF, although in radial direction the plasma indeed slows down (i.e., $v_{\text{LNRF}}$ decreases across the shock, although this is difficult to see in the plot) in CRF. This is explained as follows; we see $\Omega > 0$ and $\omega < 0$ at large radii in the upstream flow (i.e., $v_{\text{LNRF}} > 0$). Plasma is inevitably forced to slow down as it accretes due to frame-dragging and at some point it appears to come to a stop momentarily (i.e., $\Omega = 0$) with respect to the observer (i.e., still $v_{\text{LNRF}} > 0$). The plasma then turns around in the same sense as the BH rotation (i.e., still $\omega < 0$), thus $v_{\text{LNRF}} > 0$ again and later converges to the frame-dragging (i.e., $\Omega = \omega < 0$, thus $v_{\text{LNRF}} = 0$). Because of the initial angular momentum, the plasma subsequently “overshoots” in the toroidal direction allowing for $\Omega < 0$, $\omega < 0$, thus $v_{\text{LNRF}} < 0$. Past this phase, the plasma starts to converge to the BH rotation at small radii and eventually acquires $\Omega = \omega < 0$, thus $v_{\text{LNRF}} = 0$.

In addition to plasma kinematics, magnetization $\sigma(r)$ and plasma number density $n(r)$ are shown in Figure 4. Across the shock the upstream flow becomes compressed in all cases by definition causing the downstream flow to be heated where particles (primarily electrons) can be efficiently accelerated to later participate in Comptonization (see Section 3). The magnetization parameter is initially negative at large radii because the Poynting flux in the upstream flow is directed radially outward while the accretion energy flux always points inward. The Poynting flux is then shifted inward because $B_\phi$ switches its direction due to the curvature of the field line (see T02). In terms of the energy budget of the plasma flow across fast MHD shocks, a fraction of the upstream accretion energy is redistributed to both the magnetic field and thermal energies and hence the magnetization always increases (via increasing $B_\phi$) and entropy generation (via $K$) occurs due to shock compression. The downstream region is therefore always more magnetized (i.e., $|\sigma_f| > |\sigma_i|$).

Finally, we show in Figure 5 the downstream thermal energy $\Theta_e(r)$ of the trans-magnetosonic plasma as a function of normalized radius $x \equiv (r - r_{sh})/(r_{sh} - r_{H})$ where $r_{sh}$ is the shock location. As seen, the shock heating can raise the plasma thermal energy up to $\lesssim40\%$ of the rest-mass energy of the plasma. It is found that $\Theta_e(r)$ only slowly varies with radius in almost all cases because (i) it is closely related to the density $n(r)$ which is almost constant in the post-shock plasma (except for the retrograde case in (c)), (ii) the shock heating is not dissipated due to adiabatic assumption but is advected, and (iii) due to the small radial size of the downstream flow. We will discuss later, in Section 3, that the value of $\Theta_e(r)$ plays a fundamental role in determining the degree of Comptonization of incoming thermal disk photons.

| Table 1 |
| Characteristics of Fiducial GRMHD Plasma Accretion |
| Parameter | Description | BH Spin $a/M$ |
| $E$ | Energy | -0.5 | 0 | 0.5 |
| $L/E$ | Specific angular momentum | 6.1 | 6.1 | 6.1 |
| $\Omega_f$ | Angular velocity of field line | 0.02725 | 0.08334 | 0.233 |
| $4\pi f$ | Scaled accretion energy | 0.0082 | 0.006 | 0.005 |
| $v_{\text{out}}^r/r_f$ | Outer Alfvén radius | 7.01 | 3.28 | 2.82 |
| $v_{\text{out}}^r/r_f$ | Outer fast radius | 3.52 | 3.01 | 2.49 |
| $r_{sh}/r_f$ | Shock location | 2.23 | 2.70 | 1.99 |
| $v^r/r_f$ | Inner fast radius | 2.09 | 2.41 | 1.90 |
| $r_{H}/r_f$ | Event horizon | 1.86 | 2.0 | 1.86 |
| $\Theta_e(r = r_{sh})$ | Electron energy | 0.358 | 0.199 | 0.285 |

Note. Superscripts “in” and “out”, respectively, denote those radii for the “downstream” and “upstream” plasma.
efficiently transported into the particle (i.e., electron) acceleration in the downstream flow with the energy of \((\gamma - 1)m_e c^2\), one can express the corresponding electron velocity ratio \(\beta(\Theta_e)\) via the usual Lorentz factor \(\gamma \equiv (1 - \beta^2)^{-1/2}\). We consider that \(\Theta_e(r)\) primarily characterizes the upper cut-off energy of the accelerated electron number distribution in the downstream flow, i.e., \(\beta_2(r) = \beta(\Theta_e)\), whereas the lower cut-off is arbitrarily assumed to be \(\beta \equiv \beta_1 = 0.01\) in this work. Hence, shock heating can produce a nonthermal electron distribution in the downstream flow in the form of the power-law, i.e., the...
Figure 5. Same as Figure 2 but showing the downstream electron energy \( \Theta_e \equiv kT_e/(m_ec^2) \) for an accreting plasma with \( a/M = 0 \) (solid), 0.5 (dashed), and -0.5 (dotted) where \( x \equiv (r - r_H)/(r_H - r_{in}) \) is a normalized radius between the horizon \( r_H \) and the shock location \( r_{sh} \) such that \( x = 1 \) at the horizon \( r_H \) while \( x = 0 \) at the shock location \( r_{sh} \) as defined in the text. The letter “F” denotes the corresponding inner fast point in each case.

electron spectrum distribution is assumed to obey \( \epsilon^q \) (e.g., Droege & Schlickeiser 1986) as speculated for solar flares. In our work we assume a conservative slope of \( q = -2 \) (e.g., Zhong & Wang 2013) while the spectrum is only weakly sensitive to the exact value of \( q \).

In the conventional view of the standard accretion disk scenario, the disk surface radiates like a blackbody of different temperature at a given point on the disk. The local intensity of the disk blackbody between energy \( \epsilon \) and \( \epsilon + d\epsilon \) that is liberated at a point \((r, \phi)\) on the disk surface is given by the Planck distribution

\[
B_D(\epsilon, kT_{in}) d\epsilon = \frac{8\pi\epsilon^2}{h^2c^2} \left( \frac{1}{\epsilon/kT_{in}} - 1 \right) d\epsilon,
\]

where \( \epsilon \) is the thermal photon energy in a local disk frame and \( kT_{in} \) denotes the maximum disk temperature in the Shakura–Sunyaev model with GR correction as

\[
kT_{in} \approx 10 \left( \frac{\dot{m}}{0.5} \right)^{1/4} \left( \frac{1}{m_s} \right)^{1/4} \left( \frac{r}{r_g} \right)^{-3/4} \left\{ 1 - \left( \frac{r_{in}}{r} \right)^{1/2} \right\}^{1/4} \text{eV},
\]

emitted at the characteristic radius \( r = r_D \equiv (49/6)r_g \approx 8r_g \) for \( a/M = 0 \) (e.g., Frank et al. 1992; Kato et al. 2008). While in reality the disk radiation is known to be a multi-color spectrum (Mitsuda et al. 1984), in this work we assume that the hottest part of the disk predominantly contributes to a subsequent Comptonization. In other words, seed disk photons for Comptonization are assumed to originate primarily from \( r = r_D \) where the disk temperature is maximum. Note, however, that both \( kT_{in} \) and \( r_D \) depend on the BH spin \( a \).

The disk continuum is then reprocessed by Compton up-scattering via energetic electrons in the hot downstream region with a shock front at \( r = r_{sh} < r_D \) to produce the soft excess.

Following the same formalism by Zhong & Wang (2013), the differential Compton spectrum (i.e., Compton flux generated per solid angle per energy) in the local reference frame of the downstream flow can be described as a photon spectrum with \( B_{pl}(\epsilon, kT_{in}) \) being the blackbody intensity seen in the plasma frame; i.e., photon counts per time per area per energy is expressed as

\[
I_{\text{Comp}}(\epsilon', kT_{in}) \propto \frac{1}{H_1(\beta_1, \beta_2)} \int_{e_0}^{\epsilon'} B_{pl}(\epsilon_{pl}, kT_{in}) H\left( \frac{\epsilon'}{\epsilon_{pl}} \right) d\epsilon_{pl},
\]

where \( \epsilon_{pl} \) and \( \epsilon' \) are, respectively, the incoming disk photon energy and the outgoing Comptonized photon energy measured in the rest-frame of the downstream plasma and

\[
H\left( \frac{\epsilon'}{\epsilon} \right) = \begin{cases} \int_{\beta_1}^{\beta_2} (\gamma - 1)^{-q} \gamma^{-1} \beta^{-3} \zeta \left( \frac{\epsilon'}{\epsilon}, \beta \right) d\beta, & \text{if } 1 \ll \epsilon' < 1 + \beta_1, \frac{1 + \beta_2}{1 + \beta_1}, \\ \int_{\beta_1}^{\beta_2} (\gamma - 1)^{-q} \gamma^{-1} \beta^{-3} \zeta \left( \frac{\epsilon'}{\epsilon}, \beta \right) d\beta, & \text{if } 1 + \beta_1 \ll \epsilon' \ll 1 + \beta_2, \frac{1 + \beta_2}{1 + \beta_1}, \\ 0, & \text{if } \frac{1 + \beta_2}{1 + \beta_1} < \epsilon' \ll \epsilon, \\ \end{cases}
\]

\[
\zeta \left( \frac{\epsilon'}{\epsilon}, \beta \right) = \frac{\epsilon'}{\epsilon} (\beta + 1)(\beta^2 + 1)^2 \gamma - \frac{\epsilon'}{\epsilon} \frac{1}{(\beta + 1)^2} + 2 \ln \left( \frac{\epsilon'}{\epsilon} \frac{1}{(\beta + 1)^2} \right) + \frac{\beta}{\beta + 1},
\]

with a cut-off plasma velocity \( \beta_c \) defined as

\[
\beta_c \equiv \frac{\epsilon'}{\epsilon} - 1, \frac{\epsilon'}{\epsilon} + 1
\]

and the lower integration limit \( \epsilon_0 \) is given by

\[
\epsilon_0(\epsilon', \beta_2) = \epsilon' \left( \frac{1 - \beta_2}{1 + \beta_2} \right).
\]

Finally, the above coefficient \( H_1(\beta_1, \beta_2) \) in Equation (16) has been defined as

\[
H_1(\beta_1, \beta_2) \equiv \int_{\beta_1}^{\beta_2} \frac{\gamma^3 \beta (1 + \frac{\beta^2}{3})(\gamma - 1)^{-q} d\beta}{\beta + 1}.
\]

In addition, in terms of the photon energy, it is crucial to take into account the redshift factor of photons as they propagate from one point to the other under a curved spacetime because the Comptonizing region—the hot downstream flow—is in close proximity to the BH. Realizing that the post-shock downstream region in this scenario typically lies within the ISCO (for both spinning and non-spinning BH cases), as depicted in Figure 1, disk photons are strongly subject to relativistic effects in several ways while they propagate from the disk surface at \( r = r_D \) to the plasma downstream region at \( (r, \phi) \), where \( r_H < r < r_{sh} < r_D \) and \( 0 \ll \phi < 2\pi \); i.e., classical
Doppler motion of the downstream plasma, special relativistic time dilation, and gravitational redshift (e.g., Bardeen et al. 1972; Cunningham 1975; Kojima 1991; Hollywood & Melia 1997). The photon frequency is then shifted between the disk frame of the Keplerian motion and accreting plasma by factor of

$$g_1(r, \phi) = \frac{(p_\phi u^\gamma)_{pl}}{(p_\phi u^\gamma)_{pl}}_{\text{Disk} \rightarrow \text{Plasma}},$$

where $p_\phi = (-E_{ph}, \pm E_{ph}(R(r)1/2/\Delta, \pm(\Theta(\theta))1/2, \xi E_{ph})$ denotes the four-momentum of photons defined, respectively, in the plasma rest-frame (“pl”) and disk rest-frame (“D”). In Equation (22) above, note that we numerically solve null geodesic equations by GR ray-tracing (see Section 3.2) in radial $R(r)$ and angular $\Theta(\theta)$ directions with the photon energy $E_{ph}$ in the emitted frame and its angular momentum component $\xi$ (Chandrasekhar 1983, p. 347). The disk blackbody intensity in Equation (14) in the plasma downstream frame can then be expressed as

$$B_{pl}(\epsilon_{pl}, kT_{pl}) = g_1^3 B_D(\epsilon_D, kT_{in}),$$

where $\epsilon_D = \epsilon_{pl}/g_1$ is the seed photon energy in the disk frame using Lorentz invariance of photon intensity. Note that $g_1$ depends on $(r, \phi)$ because of the relative motion between the Keplerian disk and accreting plasma. These photons are then locally Comptonized in all parts of the post-shock flow as expressed in Equations (16)–(21).

Assuming that the Comptonization takes place in a spatially uniform manner within the downstream region ($r_H \leq r \leq r_{sh}$), the observed Comptonized intensity is given by

$$I_{obs}(\epsilon_{obs}, kT_{in}) \propto g_2(r, \phi)^3 I_{\text{Comp}}(\epsilon', kT_{in}) = g_2^3 I_{\text{Comp}}(\epsilon_{obs}, \frac{kT_{in}}{g_2}),$$

where

$$g_2(r, \phi) = \frac{(p_\phi u^\gamma)_{obs}}{(p_\phi u^\gamma)_{pl}}_{\text{Plasma} \rightarrow \text{Observer}}.$$  

This provides the additional redshift factor of the reprocessed photon energy due to secondary relativistic effects with respect to a distant observer, while those photons propagate under curved spacetime from the downstream plasma to the observer, as illustrated in Figure 1, such that $\epsilon' \equiv \epsilon_{obs}/g_2$. Therefore, the observed Comptonized intensity per radius is affected by the coupling between the two redshift effects, $g_1$ and $g_2$, and is found by

$$I_{obs}(\epsilon_{obs}, kT_{in}) \propto \frac{g_1(r, \phi)^3 g_2(r, \phi)^3}{H_1(\beta_1, \beta_2)} \frac{B_D(\epsilon_{pl}/g_1, kT_{in})}{\epsilon_{pl}} d\epsilon_{pl},$$

Note in Equations (22) and (25) that different photon trajectories are considered and therefore $g_1$ is totally independent of $g_2$. Thus, the corresponding differential flux via Comptonization is given by integrating over a solid angle subtended by the downstream region as

$$\frac{dF_{obs}}{dr} \propto \int \int_{\text{downstream}} I_{obs}(\epsilon_{obs}, kT_{in}) d\Omega_{obs}.$$  

By further integrating over the downstream region in radius (i.e., $r_H \leq r \leq r_{sh}$) one can calculate the observed Comptonized spectrum

$$F_{obs} = \int_{r_H}^{r_{sh}} \left( \frac{dF_{obs}}{dr} \right) dr,$$

where $r_H \equiv [1 + (1 - \alpha^2/M^2)^{1/2}] r_s$ is the radius of the event horizon normalized by the gravitational radius $r_g$, and $r_{sh}$ is the radius of the shock front in the accreting plasma. Finally, the normalization of the model spectrum is currently treated as an independent parameter to be constrained by data (see Section 5).

### 3.2. Transfer Functions with GR Ray-tracing

Given a kinematic field of MHD accretion coupled with the conventional thermal disk radiation, one can calculate two redshift factors expressed in Equations (22) and (25). We call that these factors are functions of the radius and toroidal positions of the plasma, and the observed photon flux also depends on both $g_1$ and $g_2$ due to relativistic beaming. While incoming disk photons traveling toward the downstream plasma are always blueshifted ($g_1 > 1$), Comptonized photons from the plasma region are always redshifted ($g_2 < 1$) due to strong gravitational redshift being dominant over the longitudinal Doppler effect (at least for the small angle $\theta \leq 45^\circ$ considered here). The transfer function (i.e., effective redshift factor), $g_{eff}$ is then the product of the two, $g_{eff} \equiv g_1 g_2$.

Employing the standard GR ray-tracing approach we calculate and store the transfer functions for different sets of inclination angle $\theta$, BH spin $a$, and the fiducial accretion solutions. As an example, Figure 6 shows $g_1$, $g_2$, and $g_{eff}$ as a function of azimuthal angle $0 \leq \phi \leq 2 \pi$ at the shock location $r = r_{sh}$ for (a) $a/M = 0$ and (b) $a/M = 0.5$ assuming $\theta = 30^\circ$ with fiducial shocked plasma solutions similar to those in Table 1. With respect to the initial disk photon energy, the observed photons are found, as expected, to be always redshifted despite the blueshift from $g_1$. This is because the Comptonization region (i.e., the downstream plasma) is sufficiently close to the horizon, allowing the factor $g_2$ to always dominate over the factor $g_1$ in redshift such that $0.5 \lesssim g_{eff} \lesssim 1.0$, as calculated. Note that the ranges of the effective redshift factor for both $a/M = 0$ and 0.5 appear to be very similar despite the spatial difference in the downstream plasma region, i.e., closer in toward the horizon for $a/M = 0.5$ case. This is due to the competition between $g_1$ and $g_2$ for a given BH spin. In other words, $g_1$ may be comparatively very large for the $a/M = 0.5$ case because the downstream region is closer in, while $g_2$ should then be correspondingly very small as Comptonized photons must climb up farther out toward the observer. As a result, a modest value of the effective redshift $g_{eff}$ is always achieved, almost regardless of the BH spin. These profiles are also found to be almost independent of radius $r_H \leq r \leq r_{sh}$ since the downstream region is very compact in radius. In Section 3.3 the computed transfer function is used to calculate the Comptonized spectrum in combination with the MHD plasma solutions.
3.3. Modeling Comptonized Spectra

Based on the numerical approach described in Sections 3.1 and 3.2 we calculate the Comptonized spectra for various shocked plasma accretions with sets of fiducial disk blackbody temperatures. Although, in principle, the effective disk temperature $kT_{\text{in}}$ is strictly determined by the BH spin $a$, it is often speculated that the actual disk radiation is most likely subject to various scatterings and reflections due to the (presumably corona-related) atmospheric property above the disk, which could slightly alter (if not substantially) the effective temperature of emerging thermal radiation (i.e. the color temperature). This makes the degree of the color temperature index $f_k$, a sensitive quantity to assess the spectral property of the local thermal radiation. Following a number of previous works on disk color temperature (e.g., Ross et al. 1992; Shimura & Takahara 1995; Davis et al. 2005; Li et al. 2005; Done et al. 2012), we adopt the conventional value of $f_k = 1.7$ in this paper. Because of this uncertainty in estimating the exact disk temperature, in our calculations we introduce $kT_{\text{in}}$, replacing the conventional $kT_{\text{in}}$, and treated as a free parameter in the likelihood of the expected value (i.e., 10–40 eV) for a typical AGN disk environment. Among other independent model parameters, the primary ones to be intensively explored in this work include downstream electron energy $kT_e$, disk temperature $kT_{\text{in}}$, and inclination angle $\theta_{\text{obs}}$ as listed in Table 2. Hence, the entire model spectrum in the present approach is characterized by these three quantities. By seeking a best-fit model, one can constrain the three parameters followed by the other important plasma quantities.

Figure 7(a) presents the calculated normalized spectra (in $\nu F_\nu$) for different downstream electron energies $kT_e$ assuming $kT_{\text{in}} = 30$ eV and $\theta_{\text{obs}} = 30^\circ$ for different BH spins. For each spin value, three spectra are computed with the lowest (solid line), intermediate (dashed line), and highest (dotted line) shocked electron energy with $kT_{\text{in}} = 30$ eV and $\theta_{\text{obs}} = 30^\circ$; we select $kT_e = 33$ keV, 250 keV, and 378 keV for $a/M = -0.5$ (light gray); $kT_e = 75$ keV, 125 keV, and 256 keV for $a/M = 0$ (gray); and $kT_e = 126$ keV, 179 keV, and 296 keV for $a/M = 0.5$ (black). We stress here again that the energy $kT_e$ is a dependent quantity determined by the shock location in the model for a given BH spin, thus not arbitrarily selected a priori. It is noted that the spectral peak can exceed ~1 keV depending on how much the downstream plasma is heated by the shock and, as expected, they seem to be well correlated. The spectral shape is found to be more or less self-similar in a qualitative manner for different plasma flows and even BH spin values. This seems to be consistent with the observational fact that the detected soft excess can be almost uniquely accounted for in all AGNs with a single blackbody component of more or less the same temperature $kT \sim 0.1–0.2$ keV despite their potentially diverse circumnuclear conditions. Figure 7(b) shows a similar calculation but for different disk temperatures $kT_{\text{in}}$ with $a/M = 0.5$ and $\theta_{\text{obs}} = 30^\circ$: 10 V (solid line), 20 eV (dashed line), 30 eV (dotted line), and 40 eV (thick line) for the same photon emitting radius $r_D$. It is clear that the peak energy of the emergent spectrum has a strong dependence on the disk temperature $kT_{\text{in}}$ as expected. The expected Comptonized flux in this model is therefore strongly correlated with UV flux.

We also consider the effect of different inclination angle in Figure 8(a) for $\theta_{\text{obs}} = 15^\circ$ (solid line), $30^\circ$ (dashed line), and
45° (dotted line) assuming $a/M = 0$ and $kT_m = 30$ eV. We find that there is little change in the spectrum due to viewing angle. This is because the photon emitting region (i.e., the downstream plasma) is generically (i) very close to the BH where gravitational redshift is predominant and (ii) the size of the hot region is also very narrow in radius. Hence, in such proximity to the BH the longitudinal Doppler blueshift never becomes significant enough to overcome the degree of gravitational redshift in all three cases. For this reason, this weak angle-dependence is seemingly very different from what one typically finds in the disk emission line, for example, in broad iron fluorescence (e.g., Fabian et al. 1989; Kojima 1991). A spectral variation due to different BH spin $a$ is examined in Figure 8(b) where $a/M = 0$ (light gray), 0.5 (gray), and −0.5 (black) for $kT_m = 30$ eV and $\theta_{\text{obs}} = 30°$. The solid and dashed curves are obtained with the lowest and highest shocked electron energy in each BH spin, respectively.

As seen in this section, our calculations show that Comptonized disk photons in this scenario seem to be observationally plausible to account for the known soft excess feature in many Seyfert 1 X-ray spectra. To demonstrate its validity, we will apply this model in Section 4 to one of the well-known radio-quiet Seyfert 1 galaxies, Ark 120.

4. CASE STUDY: ARK 120

Based on the model spectra for the soft excess component as shown in Section 3, we now apply the model to Ark 120, one of the well-studied luminous Seyfert 1 AGNs ($z = 0.0323$ and $M \approx 2 \times 10^8 M_\odot$; e.g., Wandel et al. 1999; Peterson et al. 2004) in which few absorption features have been detected in...
both the UV and X-ray bands, despite the persistent presence of the soft excess (Turner & Pounds 1989; Brandt et al. 1993) which makes this particular AGN a “bare” nucleus (e.g., Vaughan et al. 2004).

As a case study to conclude the current work, we analyze a 60 ks XMM-Newton/EPIC-pn spectrum of Ark 120 observed in 2003 with the standard XSPEC 12.8.2 package (Arnaud 1996) to perform the $\chi^2$-statistics. All fit parameters are given in the source rest-frame ($z = 0.0323$) and errors are quoted at the 90% confidence level for one interesting parameter (i.e., $\Delta \chi^2 = 2.7$) unless otherwise stated. The Galactic column density, $N_{\text{HI}}$, toward Ark 120 is fixed at $1.31 \times 10^{20}$ cm$^{-2}$ (Dickey & Lockman 1990; Stark et al. 1992). Throughout this paper, $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ is assumed.

### 4.1. Best-fit compsh Model

By constructing a grid of Comptonized spectra spanned by the model parameters listed in Table 2, we develop a compsh model as an additive table model in the XSPEC tool, whose free parameters are (i) effective disk photon temperature $kT_{\text{in}}$ (eV), (ii) downstream electron energy $kT_e$ (keV), (iii) inclination angle $\theta_{\text{obs}}$ (deg), and (iv) corresponding normalization. Following the analysis by Vaughan et al. (2004) we freeze a single power-law of photon index $\Gamma = 2$ to account for the continuum to the energy up to 5 keV. As previously reported (Vaughan et al. 2004; Matt et al. 2014), there have been indications of an additional component at $E \gtrsim 7$–8 keV due, presumably, to disk (blurred) reflection as well as the well-defined iron emission line at ~6.4 keV. In this work we do not consider a putative reflection component since the proposed Comptonized model in this scenario is not directly (if at all) related to that of hard X-ray photon production.\(^\text{10}\)

After applying the compsh model, we note a residual bump at ~0.55 keV which can be attributed to the instrumental and Galactic oxygen edges (Vaughan et al. 2004). We independently treat this feature with a single Gaussian line (zga) as performed in Vaughan et al. (2004). Thus, our composite model is symbolically expressed as \texttt{phabs*(po+aatable\texttt{compsh}+\texttt{zga})}, where po is the power-law continuum and phabs denotes the Galactic absorption. As stated in Section 3.3, the compsh spectrum model is determined only by the above three primary parameters besides its normalization, i.e., $kT_{\text{in}}, kT_e$, and $\theta_{\text{obs}}$, and is independent of the other spectral components, at least explicitly.

We present our results for $a/M = 0$ (Figure 9), $a/M = 0.5$ (Figure 10), and $a/M = -0.5$ (Figure 11) where 60 ks XMM-Newton/EPIC-pn data are fitted with the compsh model in each case. First, the soft excess is found to be successfully accounted for by the proposed model yielding an excellent statistical significance in which the derived best-fit parameters are well constrained, as shown in the contour plots at 68%, 90%, and 99% levels.

The derived values of the best-fit parameters of the proposed models are listed in Table 3 for each case, including the characteristic radii for plasma flows as well as the standing shock properties. Note that these radii derived in the table are not free parameters but dependent variables. We first note that all three cases are equally well constrained, yielding reasonable $\chi^2$ values for all three cases. With the seed disk photons of characteristic temperature ($\lesssim 20$–30 eV), the best-fit downstream energy $kT_e$ tends to increase with the BH spin, primarily because the shock location tends to slightly shift inward, bringing the downstream region inward with the spin. The Comptonized spectra for $a/M = 0.5$ are therefore subject to a more drastic gravitational redshift in the observer’s frame, requiring the downstream electron temperature $kT_e$ to be higher in the plasma rest-frame (before being redshifted) to balance. The derived viewing angle $\theta_{\text{obs}}$ seems to be systematically low to intermediate for three cases, consistent with the conventional classification scheme of the Seyfert galaxies such as Ark 120. Note also in Table 3 that the characteristic magnetosonic radii and the Alfven radius are all closer together in the case of $a/M = 0.5$. The shock compression (i.e., shock strength), $n_2/n_1$, is well correlated with the electron energy $kT_e$ as compression is the source of generating additional entropy in the downstream flow. Since we are focusing in this work on fast MHD shocks, the field line becomes more refracted away from the shock normal (i.e., away from the radial direction) across the shock front (i.e., $|B_{\phi,2}| > |B_{\phi,1}|$), and the dissipated plasma energy is partially transformed to the field energy consistent

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\(^{10}\) While reflection may be remotely related to disk photons via coronae, emitting regions in this scenario are physically very distant from each other and thus such a weak correlation, if any, would be smeared out.
Figure 10. (a) Same as Figure 9 but for $a/M = 0.5$ (prograde).

Figure 11. (a) Same as Figure 9 but for $a/M = -0.5$ (retrograde).

Table 3

| Parameter       | Description                  | $-0.5$  | 0      | 0.5     |
|-----------------|------------------------------|---------|--------|---------|
| $kT_{\text{e}}$ (keV) | Electron Energy              | $61.3^{+2.09}_{-2.08}$ | $83.5^{+4.3}_{-3.8}$ | $144.3^{+7.3}_{-6.6}$ |
| $kT_{\text{in}}$ (eV) | Disk Temperature             | $34.0^{+1.3}_{-1.1}$ | $21.7^{+0.4}_{-0.3}$ | $21.6^{+0.3}_{-0.2}$ |
| $\theta_{\text{obs}}$ (deg) | Inclination Angle           | $17.5^{+5.2}_{-1.4}$ | $42.6^{+7.3}_{-3.3}$ | $36.6^{+13}_{-5.5}$ |
| $r_{A}/r_{g}$ | Alfven Point                 | $8.27^{+0.02}_{-0.03}$ | $3.28^{+0.02}_{-0.01}$ | $2.48^{+0.02}_{-0.01}$ |
| $r_{\text{out}} / r_{g}$ | Outer Fast Point            | $3.53^{+0.0001}_{-0.0001}$ | $3.00^{+0.0001}_{-0.0001}$ | $2.35^{+0.0001}_{-0.0001}$ |
| $r_{\text{sh}} / r_{g}$ | Shock Location               | $2.16^{+0.0004}_{-0.0002}$ | $2.69^{+0.01}_{-0.001}$ | $1.99^{+0.04}_{-0.001}$ |
| $r_{\text{in}}^{\partial} / r_{g}$ | Inner Fast Point            | $2.11^{+0.0003}_{-0.0001}$ | $2.42^{+0.001}_{-0.001}$ | $1.90^{+0.01}_{-0.01}$ |
| $n_{2}/n_{1}$ | Compression Ratio           | $1.05^{+0.008}_{-0.008}$ | $1.10^{+0.002}_{-0.001}$ | $1.82^{+0.01}_{-0.01}$ |
| $\sigma_1$ | Upstream Magnetization      | $0.41^{+0.006}_{-0.004}$ | $0.037^{+0.001}_{-0.001}$ | $1.07^{+0.01}_{-0.05}$ |
| $\sigma_2$ | Downstream Magnetization    | $0.49^{+0.0005}_{-0.0004}$ | $0.21^{+0.004}_{-0.001}$ | $2.82^{+0.12}_{-0.11}$ |
| $B_{2/3}/B_{3}$ | Toroidal Field Enhancement  | $1.11^{+0.014}_{-0.014}$ | $5.01^{+0.25}_{-0.19}$ | $1.43^{+0.025}_{-0.021}$ |

$\chi^2$/dof

| 961.91/944  | 994.77/944  | 1014.64/944 |

Notes. All radii are normalized to the gravitational radius $r_{g}$.

* Assuming that $M_{8} = 1$, $q = -2$, $\beta_{1} = 0.01$, and $f_{c} = 1.7$.

* Lower parameter value reached.

* Upper parameter value reached.
with increasing magnetization parameter $\sigma_2 > \sigma_1$ as shown in Table 3. The rate of increase in magnetization $\sigma$ due to shock is also well correlated with the BH spin (see, e.g., T02). Among the three BH spins, for the $a/M = 0$ and 0.5 cases, the derived disk temperature is statistically consistent ($kT_{\text{in}} \approx 21-23$ eV) within the uncertainty. On the other hand, it appears that the retrograde BH spin of $a/M = -0.5$ is slightly more favored by observations indicating a relatively higher disk temperature $kT_{\text{in}} = 34.0$ eV and a slightly lower electron energy $kT_e \approx 61.3$ keV in comparison with the other two cases as seen in Table 3. One way to understand this result is the following; in the retrograde case, incoming disk photons are emitted primarily at a larger disk radius according to Equation (15), while the downstream region remains to be formed at small radii (i.e., $r_{sh}/r_g = 2.16$). Thus, the emitted photons from the disk are subject to more blueshift in the rest-frame of the downstream plasma. For such Comptonized photons with more blueshift, a high electron energy $kT_e$ would not be necessary to counteract against a subsequent gravitational redshift while propagating toward the observer. Whereas an even lower electron energy $kT_e (\lesssim 61$ keV) for $a/M = -0.5$ can be statistically viable as shown in Figure 11(b), we find that no plasma accretion in this case is allowed to develop a downstream region that is “colder” than $kT_e \approx 61$ keV, as seen in Figure 11(b), which is in contrast to the other two cases where the confidence level is unambiguously constrained, as shown in Figures 9(b) and 10(b).

5. SUMMARY AND CONCLUSION

In this work we proposed a novel scenario: the shock-heated downstream flow in GRMHD accretion, injected from near the ISCO, serves as an ideal heating site where accelerated electrons can Compton up-scatter thermal disk photons to imprint the observed soft excess component in the AGN X-ray spectrum. Extending our earlier work on GRMHD standing shock formation, we explored sets of fiducial solutions for an accreting plasma in Kerr geometry and studied their physical conditions in terms of liberated energy via shocks. Given a monochromatic blackbody radiation originating from the hottest part of the disk, we calculated Comptonized spectra by making use of the obtained plasma accretion. Our calculations are all fully relativistic, including photon redshift among the disk rest-frame, plasma rest-frame, and the distant observer’s rest-frame by employing a GR ray-tracing approach. As a simplistic three-parameter model (i.e., $\theta_{\text{obs}}$, $kT_e$, and $kT_{\text{in}}$), in addition to its normalization, we have constructed a grid of synthetic spectral models for the Comptonized component ($\text{compsph}$) and further demonstrated that the model can successfully explain the observed soft excess feature for a typical Seyfert 1 galaxy, Ark 120, as a case study.

We show that GRMHD plasma in this model begins to plunge in from radii very close to the ISCO within the standard disk paradigm. It is found that the downstream region, heated by fast MHD shocks, typically extends out to only a few gravitational radii. While we only consider equatorial accretion for simplicity, plasma in reality is expected to accrete along a closed-loop of poloidal field lines within the ISCO, developing shocks as shown in F07, where the vertical height of the shock is systematically found to be $h \lesssim 5r_g$ for various accretion parameters. Such a very compact and centrally concentrated region resembles the long-sought physical identity of the putative X-ray “coronal” region near the BH that is required to explain the basis of the major spectral components in AGN X-ray observations (e.g., Haardt & Maraschi 1991; Miniutti et al. 2003; Petrucci et al. 2004; Nardini et al. 2011; Fabian et al. 2015; Keck et al. 2015; Lohfink et al. 2015). For example, Kara et al. (2015) analyzed the broad-band X-ray spectrum of a narrow-line Seyfert galaxy, 1H0707-495, to estimate a very compact coronal source at the height of $h \sim 2r_g$ above a rapidly rotating BH in the context of the standard lamp-post model, although the physical identification of the X-ray source in that extreme proximity is yet to be confirmed theoretically. Our GRMHD calculations indicate that the formation of such a “corona” is ubiquitous and almost universal for a wide range of plasma parameter space, including BH spin, and the $\chi^2$ analysis successfully derived the range of $61.3$ keV $\lesssim kT_e \lesssim 144.3$ keV, $21.6$ eV $\lesssim kT_{\text{in}} \lesssim 34.0$ eV, and $17.5 \lesssim \theta_{\text{obs}} \lesssim 42.6^\circ$, although the model slightly favors the retrograde BH case ($a/M = -0.5$) as shown in Table 3. While the discussion of the physical arrangement of such a retrograde AGN system is beyond the scope of this paper, a counter-rotating plasma (with respect to BH rotation) allows for a larger Comptonizing area due to a larger ISCO radius (e.g., Bardeen et al. 1972; Cunningham 1975; Chandrasekhar 1983). Hence, the resulting composite spectrum from the downstream flow can afford to produce a more diverse spectral shape when integrating over the downstream region. This may be the reason why the soft excess in the data seems to be better accounted for by a retrograde BH case. The current model, however, does not provide a fundamental explanation of such a retrograde BH spin and it is only empirical in this framework. A retrograde BH, allowing for the disk to recede further out, may also be viable with the fact that Ark 120 seems to exhibit no strong signs of absorption (thus a “bare” nucleus), which would otherwise be expected to be present in the soft X-ray band. We note, however, that earlier studies of Ark 120 seem to imply a somewhat intermediate BH spin of $a/M \approx 0.5$–$0.6$ based on Ionized reflection models, although with some potential uncertainties (e.g., García et al. 2014; Matt et al. 2014).

In the present work we have only considered moderate BH spin values ($-0.5 \leq a/M \leq 0.5$) because we found it challenging to systematically obtain the solutions for very high BH spin values ($a/M \geq 0.9$) as the size of the downstream region becomes even more compact in radius. A more thorough parameter search will form the basis of a future work.

In comparison with the recent thermal Comptonization scenario proposed by Petrucci et al. (2013) for MrK 509, the Comptonizing “corona” (i.e., $60$ keV $\lesssim kT_e \lesssim 150$ keV within the ISCO) in our model is very similar to the “hot corona” (e.g., $kT_{\text{hc}} \sim 100$ keV at the innermost region of accretion responsible for the power-law component) but is different to the “warm corona” ($kT_{wc} \sim 0.6$ keV exterior to the hot corona responsible for the soft excess) in their model in terms of its geometry and physical characteristics (see their Figure 10). A critical difference between the two models lies in the fact that the spectral distribution of Comptonizing particles (i.e., electrons) is assumed to be nonthermal (i.e., power-law) as described in Section 3.1, while Petrucci et al. (2013) consider thermal distribution. Although our current model focuses only on the production of soft excess through shocks, it is very likely that local magnetic field activity on the disk surface, such as reconnection, might play a major role in producing nonthermal continuum in the hard X-ray regime.
As has been widely debated to date in the literature, a relativistically blurred reflection model may also be able to explain the observed excess feature (e.g., Crummy et al. 2006; Ponti et al. 2010; Lohfink et al. 2012; De Marco et al. 2013; Vasudevan et al. 2014) while expecting a strong spectral correlation between the soft and hard X-rays due to their direct coupling in the production process. In the framework of our current model, on the other hand, the production site of the soft excess component is strictly confined to the innermost plasma accretion set by the shock formation within the ISCO radius and thus physically disentangled from the reflected hard X-rays, expecting no correlation between the two. In recent studies of broad-band spectroscopies from a number of Seyfert galaxies showing strong soft excess, however, we note that detailed spectral analyses seem to disfavor such an expected correlation, but indicate a correlation between UV and soft X-ray flux (e.g., Mehdipour et al. 2011; Petrucci et al. 2013; Boissay et al. 2014, 2015). Our model, by definition, is co-aligned with the latter findings. While not yet definitive, our scenario is consistent with those findings at least qualitatively. As discussed by Matt et al. (2014), on the other hand, the actual correlation could be rudimentary as the soft excess could be present without a pronounced relativistic reflection component, and it takes more effort to draw a conclusion.

It was found that Comptonization due to standing shock is very sensitive to the shock property, controlled primarily by the downstream energy $kTB$, while not so significantly dependent on BH spin $a$, in that the calculated spectra are almost generically self-similar regardless of the exact value of BH spin. This may thus be indicative of model degeneracy with BH spin. In the present calculations we arbitrarily normalize the Comptonized spectrum as a free parameter. That is, the flux level of the calculated excess component is not determined by the model but is only provided by the data. Technically speaking, the intensity of the excess component must be coupled with plasma properties (such as density, temperature, and magnetic field strength) and should be self-consistently calculated by the model. It is important, however, to examine how much energy is physically available in the post-shock region in comparison with the observed soft excess flux. To clearly address the modeled excess intensity, a more rigorous consideration of the $\text{comsh}$ normalization is necessary as a future work. While it is still open to debate how much Comptonized flux can be produced from the small downstream region, one might argue that relativistic beaming (via a light bending effect) of seed photons toward small radii could preferentially allow for a sufficient centrally concentrated illumination (e.g., Fukumura & Kazanas 2007a). This may yield enough Comptonized photons, possibly in a scenario analogous to the observed double-peaked Hα emission line from the AGN accretion disk (e.g., Chen et al. 1989; Eracleous & Halpern 1994; Strateva et al. 2008).

In this work, we assumed a split-monopole field configuration. Although not globally applicable, this is an approximate solution to the trans-field equation (i.e., the GR GS-equation) as originally discussed in Blandford & Znajek (1977). Interestingly, state-of-the-art numerical simulations in recent years from different groups indicate a characteristic field topology very similar to that described as the split-monopole (Hirose et al. 2004; Tchekhovskoy et al. 2009; Contopoulos et al. 2013). Although the detailed structure and strength of the actual magnetic field at the horizon scale still remains unclear, this approximation is a first step forward to address the problem. Nonetheless, the accreting plasma models listed in Table 1 yield a field strength on the order of $B \sim 10^{2-4}$ G, consistent with the known estimates to date (e.g., Krolik 1999; Wang et al. 2001; Fukumura et al. 2007).

While treated as fully relativistic under strong gravity, we note that the current model is time-independent based on axisymmetric plasma. This assumption makes it impossible for us to predict any temporal nature of the soft excess considered in this work, e.g., spectral time variabilities associated with shock compression and cooling effects. The downstream plasma properties are numerically solved by considering adiabatic (nonradiative) Rankine–Hugoniot jump conditions as a pure mathematical discontinuity with no energy/mass loss. Hence, most of the heat generated at the shock front is advected with the downstream plasma. A more realistic shock process, on the other hand, is most likely accompanied by radiative cooling to some degree in which the post-shock plasma temperature may stay comparatively as cool as that of the upstream one, as in the isothermal shocks (e.g., Lu & Yuan 1998; Das et al. 2003; Fukumura & Tsuruta 2004; Fukumura & Kazanas 2007b). Radiative dissipation at the shock front could therefore drastically change the subsequent downstream plasma condition, which in turn alters the Comptonization process. In reality, furthermore, accreting plasma may be characterized by a two-temperature gas between electrons and ions (e.g., Shapiro et al. 1976; Mahadevan 1998; Manmoto 2000) unless the Coulomb coupling between the two is very efficient, whereas in this work we prescribed a single-fluid approximation for simplicity. Becker et al. (2011) have considered a particle transport process (e.g., bulk advection, spatial diffusion, and particle escape) via the effects of the first-order Fermi acceleration across a standing shock. In a more self-consistent scenario such a calculation of diffusive shock acceleration should be incorporated to reflect the energetic outflows/jets from the shock front. Although all these micro-physics should be addressed and incorporated into more sophisticated calculations of GRMHD simulations for completeness, this is beyond the scope of this paper.

The other potentially important spectral components associated with magnetic fields include synchrotron radiation, and its Comptonization has also been extensively considered in the literature in the context of BH binaries such as Cygnus X-1, for example, via nonthermal power-law electrons (e.g., Wardziński & Zdziarski 2001; Chakrabarti & Mandal 2006). While this is particularly important in BH binaries, the characteristic synchrotron frequency in AGNs is estimated to be $\nu_{\text{syn}} \simeq 4 \times 10^{10} B_4 \gamma_e^2$ [Hz] $\sim 1.6 \times 10^{-4}$ MeV where $B_4$ is the field strength in units of $10^4$ G and we take $\gamma_e \sim 1$ in this model. Hence, those Comptonized photons are less likely to be significant for the composite spectrum from the innermost accretion region of AGNs that is considered in this work.

We have analyzed in this work a Seyfert galaxy, Ark 120, to demonstrate that the proposed model successfully describes the observed soft excess feature within the framework of a simplistic accretion model. As there are a lot of archival X-ray data available, mainly from typical narrow-line Seyfert AGNs and PG quasars that also exhibit strong excess components (e.g., Crummy et al. 2006, for 20 to 30 AGNs) in the XMM-Newton/EPIC-pn observations, we will extend the current study to those available for a more systematic analysis. Despite a simplistic prescription of the proposed scenario based
on GRMHD shock formation, our Comptonization model is successful in describing the observed soft excess feature in Ark 120. We thus find the current result to be an encouraging first step toward the next level, where additional relevant physics are employed to make the model more physically self-consistent and promising.

We anticipate that advanced new X-ray missions, such as *Athena*, will contribute significantly to this field of study, particularly the high-resolution X-ray microcalorimeter spectrometer, by simultaneously providing more detailed spectra on the soft and hard X-ray components. The expected data will thus help differentiate various (fundamentally) distinct models presented today and further clarify the current ambiguous views concerning the observed soft excess in the immediate circumnuclear region of AGNs.

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