Sudakov Logarithms in Four-Fermion Electroweak Processes at High Energy

S. Moch\textsuperscript{a}

\textsuperscript{a} Deutsches Elektronen-Synchrotron, DESY, Platanenallee 6, 15738 Zeuthen, Germany

We discuss recent results for the asymptotic behavior of fermion scattering amplitudes in the Sudakov limit. The results include next-to-next-to-leading logarithmic electroweak corrections and are used for the analysis of fermion-antifermion pair production in $e^+e^-$ annihilation at high energy. The importance of the subleading logarithmic contributions is emphasized and the fermionic contributions to the Abelian form factor at two loops are discussed.

1. Introduction

Four-fermion processes are generally considered as benchmark processes at high energy colliders, with electron positron annihilation into muon or quark pairs at LEP and the Drell Yan process at hadron colliders as characteristic examples. At energies presently accessible, typically up to 200 GeV, radiative corrections are dominated by the shift in the $W$ and $Z$ masses as parametrized by the $\rho$ parameter and by the running of the coupling constant. Vertex corrections and box diagrams involving gauge bosons are generally of minor importance. In the TeV region, which will be probed at future colliders like the LHC or TESLA, this picture changes drastically. There, double logarithmic corrections become relevant and rapidly dominant. They were first observed by Sudakov \cite{1} in the context of Quantum Electrodynamics for reactions with cuts on the radiated energy of the photons. For electroweak interactions, the large negative corrections arise from the exchange of gauge bosons which remain uncompensated if one restricts the analysis to exclusive final states, consisting for instance of a fermion antifermion pair only.

Complete one-loop corrections to the four-fermion process are available since long (see e.g. \cite{2}). In recent years, the systematic study of double logarithmic electroweak corrections has received a lot of attention [3–9], (see \cite{10} for a review). While these studies are fairly straightforward for a theory with massive gauge bosons only, an important complication arises in the Standard Model due to the presence of massless photons in the final state. Events with soft and hard photon radiation are normally included in the sample, thus a “semi-inclusive” definition of the cross section is closest to the experimental analysis.

At present, all large contributions up to the next-to-next-to-leading logarithms have been evaluated \cite{9} for the form factor, for four-fermion scattering in a spontaneously broken $SU(2)$ gauge theory and, last not least, for the Standard Model with $W$, $Z$-bosons and the massless photon. In addition, other reactions have been investigated in the high energy limit, such as gauge boson pair production. There, the next-to-leading logarithmic terms have been calculated \cite{11,12} for the exclusive production of transversly and longitudinally polarized $W$-bosons. But even in inclusive boson fusion processes, enhanced electroweak corrections are present \cite{13}, a consequence of the lack of compensation between virtual and real emission due to the non-Abelian charges of the incoming particles.

2. The Abelian form factor

Let us begin with a discussion of the form factor of an Abelian vector current in the Sudakov limit. In Born approximation, one writes

\[ F_B = \bar{\psi}(p_2)\gamma_\mu \psi(p_1), \]

and we study the limit $s = (p_1 - p_2)^2 \to -\infty$ with on-shell massless fermions, $p_1^2 = p_2^2 = 0$, and
massive gauge bosons, $M^2 \ll -s$. For convenience we choose $p_{1,2} = (Q/2, 0, 0, +Q/2)$ so that $2p_1 \cdot p_2 = Q^2 = -s$.

The large logarithmic corrections in the Sudakov limit can be resummed to all orders of perturbation theory [14,15], such that the asymptotic behavior of the form factor is obtained from

$$F = F_0(\alpha(M^2)) \exp\left\{ \int_{M^2}^{Q^2} \frac{dx}{x} \right\} \times \left[ \int_{M^2}^{Q^2} \frac{dx'}{x'} \gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2)) \right] \} F_B. \tag{2}$$

The resummation exploits the factorization of the scattering amplitude in the high energy limit into products of functions $h$, $J$ and $S$, which organize the large corrections corresponding to a particular momentum region. The jet function $J$ contains the full dynamics of collinear momentum regions. It includes all leading logarithms. The soft function $S$ summarizes the dynamics of soft momenta, while the function $h$ describes the short distance dynamics of the hard scattering process. It contains no large logarithms. The predictive power of the factorized amplitude then follows from the properties of the individual functions. In particular, $J$ and $S$ can be defined in terms of operators, which obey renormalization group equations with calculable anomalous dimensions [14–18].

The next-to-next-to-leading logarithmic corrections include all the terms of the form $\alpha^n \log^{2n-m}(Q^2/M^2)$ with $m = 0, 1, 2$. To this accuracy, one needs for the anomalous dimensions $\gamma$, $\zeta$ and $\xi$ in Eq. (2) the one-loop results

$$\gamma^{(1)} = -2CF, \quad \zeta^{(1)} = 3CF, \quad \xi^{(1)} = 0,$$

$$F_0^{(1)} = -CF \left( \frac{7}{2} + \frac{2}{3} \pi^2 \right), \tag{3}$$

as well as the two-loop result [19] for the anomalous dimension $\gamma^{(2)}$, where we define $\gamma = \left( \frac{\beta_0}{\pi^2} \right) \gamma^{(1)} + \left( \frac{\beta_0}{\pi^2} \right)^2 \gamma^{(2)} + \ldots$ and similarly for $\zeta$, $\xi$ and $F_0$.

In the $\overline{MS}$-scheme, including $n_f$ light fermions and $n_s$ light scalars in the fundamental representation, $\gamma^{(2)}$ reads

$$\gamma^{(2)} = -2CF \left[ \left( \frac{67}{9} \frac{\pi^2}{3} \right) C_A - \frac{20}{9} T_F n_f - \frac{8}{9} T_F n_s \right], \tag{4}$$

It is instructive, to expand Eq. (2) for the form factor at two loops up to next-to-next-to-next-to-leading logarithmic ($N^3LL$) accuracy,

$$F_{LL}^{(2)} = \frac{1}{8} (\gamma^{(1)})^2 \ln^4(z) F_B, \tag{5}$$

$$F_{NLL}^{(2)} = \frac{1}{2} \left( \frac{1}{3} \beta_0 - \zeta^{(1)} \right) \gamma^{(1)} \ln^3(z) F_B, \tag{6}$$

$$F_{N^2LL}^{(2)} = \frac{1}{2} \left( \gamma^{(2)} + \left( \zeta^{(1)} - \beta_0 \right) \zeta^{(1)} + F_0^{(1)} \gamma^{(1)} \right) \ln^2(z) F_B, \tag{7}$$

$$F_{N^3LL}^{(2)} = - \left( \gamma^{(2)} + \zeta^{(2)} + F_0^{(1)} \zeta^{(1)} \right) \ln(z) F_B, \tag{8}$$

where $z = M^2/Q^2$.

Employing the results of Eqs. (3) and (4), we see a particular pattern of growing coefficients of the logarithms, which reflects the general structure of logarithmically enhanced electroweak corrections. The relatively small coefficient of the leading logarithm and the large coefficient of the $N^2LL$ term in the form factor are clearly indicative of the importance of subleading logarithmic corrections. At $N^3LL$ accuracy, the still unknown quantities $\zeta^{(2)}$ and $\xi^{(2)}$ enter.

Finally, we mention an equivalent approach to calculate the anomalous dimensions in Eq. (2). This is the so-called strategy of regions [20–22], which applies expansions in various kinematical regions directly to Feynman diagrams contributing to the form factor. From an evaluation of the Feynman diagrams to logarithmic accuracy, one may read off the results of Eqs. (3) and (4).

3. Electroweak four-fermion processes

Let us now investigate the four-fermion scattering at fixed angles in the limit when all the invariant energy and momentum transfers of the process are far larger than the gauge boson mass, $|s| \sim |t| \sim |u| \gg M^2$.

For the analysis of the four-fermion amplitude, one has to account for the different “color” and Lorentz structures, which naturally induce the dependence on $s$, $t$ and $u$. A complete basis consists of four independent chiral amplitudes, each of them of two possible “color” structures.
Let us denote by $\tilde{A}$ the amplitude with the collinear logarithms factored out. It contains only the logarithms of soft nature and can be represented as a vector in the color/chiral basis. It satisfies an evolution equation which is known from QCD [16,18,23–25],

$$\frac{\partial}{\partial \ln Q^2} \tilde{A} = \chi(\alpha(Q^2)) \tilde{A}. \quad (9)$$

Here $\chi(\alpha)$ is the matrix of the soft anomalous dimensions, given for instance in [5,24,25], and the matrix elements of $\chi(\alpha)$ depend on the scattering angle through $t$ and $u$. Upon integration, eq. (9) resums all next-to-leading logarithms.

The solution of eq. (9) reads

$$\tilde{A} = \sum_i \tilde{A}_{0i}(\alpha(M^2)) \exp \left[ \int_{M^2}^{Q^2} \frac{dx}{x} \chi_i(\alpha(x)) \right], \quad (10)$$

where $\chi_i(\alpha)$ are eigenvalues of $\chi(\alpha)$, and $\tilde{A}_{0i}(\alpha)$ are $Q$-independent eigenvectors of $\chi(\alpha)$ which are determined by matching the initial conditions for the evolution equation at $Q = M$ to a full one-loop calculation to logarithmic accuracy including the constant terms.

These angular dependent terms as well as those in the starting values of $\tilde{A}$ discussed below are important for the calculation of the N$^2$LL terms in cross sections and asymmetries. To illustrate the significance of the subleading contributions, let us discuss a standard model inspired example such as the case of the $SU(2)_L$ group with $n_f = 6$ and one scalar boson. After integrating the differential cross section over all angles one obtains

$$\sigma^{(1)} = \left[ -3 \ln^2 \left( \frac{s}{M^2} \right) + \frac{80}{3} \ln \left( \frac{s}{M^2} \right) - \left( \frac{25}{9} + 3\pi^2 \right) \right] \sigma_B,$$

$$\sigma^{(2)} = \left[ \frac{9}{2} \ln^4 \left( \frac{s}{M^2} \right) - \frac{449}{6} \ln^3 \left( \frac{s}{M^2} \right) + \left( \frac{4855}{9} + \frac{37}{3} \pi^2 \right) \ln^2 \left( \frac{s}{M^2} \right) \right] \sigma_B, \quad (11)$$

if initial and final state fermions have the same isospin. Again we observe a relatively small coefficient of the leading logarithm and the large coefficient of the N$^2$LL term.

To analyze the electroweak correction to the process $f'f' \rightarrow f\bar{f}$, the approximation with $W$ and $Z$ bosons of the same mass $M$, a Higgs boson of the mass $M_H \sim M$ and massless quarks and leptons is employed. A fictitious photon mass $\lambda$ has to be introduced to regularize the infrared divergences. The photon is, however, massless and the corresponding infrared divergent contributions should be accompanied by the real soft photon radiation integrated to some resolution energy $\omega_{\text{res}}$ to get an infrared safe cross section independent of an auxiliary photon mass. To study the virtual corrections in the limit of the vanishing photon mass the general approach of the infrared evolution equations developed in [6] (see also references therein) is particularly useful.

Explicit expressions for the N$^2$LL results are given in [9]. Numerically, for the reaction $e^+e^- \rightarrow \mu^+\mu^-$ as a typical example, one obtains a correction factor

$$R_{\mu^+\mu^-} = 1 - 1.39 \ell(s) + 10.12 \ell(s) - 31.33 a$$

$$+ 1.42 L^2(s) - 18.43 L(s) - 99.89 \ell^2(s) \quad (12)$$

with $a = g^2/16\pi^2 = 2.69 \times 10^{-3}$, $\ell(s) = a \ln s/M^2$, $L(s) = a \ln^2 s/M^2$. In particular, we note the sizeable coefficient of the N$^2$LL term, thus demonstrating the importance of the subleading logarithmic corrections. To get the infrared safe result for the semi-inclusive cross sections one has to include the standard QED corrections due to soft photon emission and the pure QED virtual corrections.

4. Fermionic contributions at two loops

In view of the large subleading logarithms, it seems desirable to attempt the evaluation of all two-loop terms linear in the logarithm or even of the two-loop constant terms.

As a first step, the corrections due to $n_f$ massless fermions and $n_s$ charged massless scalars in the fundamental representation have been calculated for the Abelian form factor [26]. Let us define the form factor in terms of scaling functions as

$$\frac{F}{F_B} = 1 + \frac{\alpha}{4\pi} F^{(1)} + \frac{\alpha^2}{16\pi^2} (n_f F^{(2)}_{n_f} + n_s F^{(2)}_{n_s}). \quad (13)$$
In the high energy limit, i.e. $z = M^2/Q^2 \to 0$, the large logarithms in the scaling functions are

$$\mathcal{F}^{(1)} = -\ln^2(z) - 3 \ln(z) - \frac{7}{2} - \frac{2}{3} \pi^2, \quad (14)$$

$$\mathcal{F}^{(2)}_{nf} = \frac{4}{9} \ln^3(z) + \frac{38}{9} \ln^2(z) + \frac{34}{3} \ln(z) + \frac{115}{9} + \frac{16}{27} \pi^2, \quad (15)$$

$$\mathcal{F}^{(2)}_{ns} = \frac{1}{9} \ln^3(z) - \frac{25}{18} \ln^2(z) - \frac{23}{6} \ln(z) - \frac{157}{36} + \frac{10}{27} \pi^2, \quad (16)$$

where the terms up to $\ln^2(z)$ in Eqs. (15) and (16) agree with the result from the evolution equation, i.e. Eqs. (5)–(7). The coefficients of the terms proportional to $\ln(z)$ in Eqs. (15) and (16) are a new result [26]. With the help of Eq. (8), one can determine the contributions of light fermions and light charged scalars to the sum of the anomalous dimensions $\zeta^{(2)}$ and $\xi^{(2)}$,

$$\zeta^{(2)} + \xi^{(2)} \bigg|_{n_f + n_s} = -C_F \left[ \frac{34}{3} T_F n_f + \frac{23}{6} T_F n_s \right]. \quad (17)$$

To study the numerical size of the logarithmic approximation of Eq. (15), we plot $\mathcal{F}^{(2)}_{nf}$ in Fig. 1 for a typical mass scale of $M = 100$ GeV. We show the complete result as calculated in Eq. (15) and the individual contributions of the large logarithms. We observe again the pattern of large cancellations between leading and subleading logarithms.

Finally, we note that the fermionic contribution to $\gamma^{(3)}$ at three loops has recently been calculated [27,28]. Together with the two-loop constant terms in Eq. (15), this result is a step towards the determination of the first five powers of logarithms of $n_f$-enhanced terms in the Abelian form factor to all orders in perturbation theory. Furthermore, with the known factorization properties of the full four-fermion amplitude [29–31], the result of Eq. (15) can even be used to get the complete fermionic contribution at two loops to logarithmic accuracy.

5. Conclusions

We have discussed the asymptotic behavior of the four-fermion scattering amplitudes in the Sudakov limit. We have emphasized the close connection between factorization and resummation and derived explicit expressions for the next-to-next-to-leading logarithmic electroweak corrections with the help of evolution equations.

The subleading logarithmic corrections were found to be sizeable and motivated the exact two-loop calculation of the fermionic contribution to the Abelian form factor.

The studies presented here are a first step towards refined precision analyses of the high energy asymptotic behavior of four-fermion amplitudes in non-Abelian gauge models and the Standard Model.

Acknowledgments

We acknowledge gratefully pleasant collaborations with B. Feucht, J.H. Kühn, A.A. Penin and V.A. Smirnov.

REFERENCES

1. V. V. Sudakov, Sov. Phys. JETP 3, 65 (1956).
2. W. Beenakker, S. C. van der Marck, and
1. W. Hollik, Nucl. Phys. B365, 24 (1991).
2. P. Ciafaloni and D. Comelli, Phys. Lett. B446, 278 (1999), hep-ph/9809321.
3. M. Beccaria, G. Montagna, F. Piccinini, F. M. Renard, and C. Verzegnassi, Phys. Rev. D58, 093014 (1998), hep-ph/9805250.
4. J. H. Kühn, A. A. Penin, and V. A. Smirnov, Eur. Phys. J. C17, 97 (2000), hep-ph/9912503.
5. V. S. Fadin, L. N. Lipatov, A. D. Martin, and M. Melles, Phys. Rev. D61, 094002 (2000), hep-ph/9910338.
6. M. Beccaria, P. Ciafaloni, D. Comelli, F. M. Renard, and C. Verzegnassi, Phys. Rev. D61, 073005 (2000), hep-ph/9906319.
7. M. Hori, H. Kawamura, and J. Kodaira, Phys. Lett. B491, 275 (2000), hep-ph/0007329.
8. J. H. Kühn, S. Moch, A. A. Penin, and V. A. Smirnov, Nucl. Phys. B616, 286 (2001), hep-ph/0106298.
9. M. Melles, (2001), hep-ph/0104232.
10. M. Melles, Phys. Rev. D63, 034003 (2001), hep-ph/0004056; Phys. Rev. D64, 014011 (2001), hep-ph/0012157.
11. A. Denner and S. Pozzorini, Eur. Phys. J. C18, 461 (2001), hep-ph/0010201.
12. M. Ciafaloni, P. Ciafaloni, and D. Comelli, Phys. Rev. Lett. 84, 4810 (2000), hep-ph/0001142.
13. A. Sen, Phys. Rev. D24, 3281 (1981).
14. G. P. Korchemsky, Phys. Lett. B217, 330 (1989); Phys. Lett. B220, 629 (1989).
15. G. Sterman, Nucl. Phys. B281, 310 (1987).
16. L. Magnea and G. Sterman, Phys. Rev. D42, 4222 (1990).
17. H. Contopanagos, E. Laenen, and G. Sterman, Nucl. Phys. B484, 303 (1997), hep-ph/9604313.
18. J. Kodaira and L. Trentadue, Phys. Lett. 112B, 66 (1982).
19. M. Beneke and V. A. Smirnov, Nucl. Phys. B522, 321 (1998), hep-ph/9711391.
20. V. A. Smirnov and E. R. Rakhmetov, Theor. Math. Phys. 120, 870 (1999), hep-ph/9812529.
21. V. A. Smirnov, Phys. Lett. B465, 226 (1999), hep-ph/9907471.
22. A. Sen, Phys. Rev. D28, 860 (1983).
23. N. Kidonakis and G. Sterman, Phys. Lett. B387, 867 (1996).
24. N. Kidonakis, E. Laenen, S. Moch, and R. Vogt, Phys. Rev. D64, 114001 (2001), hep-ph/0105041.
25. B. Feucht, J. H. Kühn, and S. Moch, in preparation.
26. S. Moch, J. A. M. Vermaseren, and A. Vogt, (2002), hep-ph/0209100.
27. C. F. Berger, (2002), hep-ph/0209107.
28. S. Catani, Phys. Lett. B427, 161 (1998), hep-ph/9802439.
29. G. Sterman and M. E. Tejeda-Yeomans, (2002), hep-ph/0210130.
30. E. W. N. Glover, J. B. Tausk, and J. J. Van der Bij, Phys. Lett. B516, 33 (2001), hep-ph/0106052.