Quantum discord in a spin-1/2 transverse $XY$ chain following a quench

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Abstract. We report a study on the zero-temperature quantum discord as a measure of the two-spin correlation of a transverse $XY$ spin chain following a quench across a quantum critical point and investigate the behavior of the mutual information, the classical correlations and, hence, of the discord in the final state as a function of the rate of quenching. We show that although the discord vanishes in the limit of very slow as well as very fast quenching, it exhibits a peak for an intermediate value of the quenching rate. We show that although the discord and also the mutual information exhibit a behavior similar, as regards the quenching rate, to that of the concurrence or negativity following an identical quenching, there are quantitative differences. Our studies indicate that like the concurrence, the discord also exhibits a universal power law scaling with the rate of quenching in the limit of slow quenching although it may not be expressible in a closed power law form. We also explore the behavior of the discord on quenching linearly across a quantum multicritical point and observe a scaling similar to that of the defect density.

Keywords: spin chains, ladders and planes (theory), quantum phase transitions (theory), defects (theory), entanglement in extended quantum systems (theory)

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1. Introduction

Quantum phase transitions (QPT) [1]–[4] driven by quantum fluctuations arising due to the change of a parameter in the Hamiltonian at absolute zero temperature have been studied extensively. A QPT is characterized by a fundamental change in the symmetry of the ground state of a quantum many-body system and is also associated with diverging correlation length as well as a diverging relaxation time at the quantum critical point (QCP). Over the past few years, numerous efforts have been directed to understanding the connection between quantum information and QPTs [5]–[8]. In recent years, QPTs have been observed experimentally in a large number of systems, for example in optical lattices where a Mott insulator to superfluid transition is observed [9]–[11].

The entanglement between two spins is a measure of the correlations between them [6] and is usually quantified in terms of quantities like concurrence and negativity [12,13]. For a transverse field Ising model, concurrence has been found to maximize close to the QCP and its derivatives show scaling behavior characteristic of that QCP [5]. However, a different and significant measure other than the entanglement, namely the ‘quantum discord’, was introduced by Olliver and Zurek [14] which exploits the fact that different quantum analogs of equivalent classical expressions can be obtained because of the fact that a measurement perturbs a quantum system. This property enables us to probe the ‘quantumness’ of a system. Quantum discord which ideally is a subject of interest in quantum information theory [15]–[20] has been studied for spin systems and also close to QCPs [14], [21]–[27] and thereby establishes a natural connection between these two fields. Very recently, an experimental study aiming to measure quantum discord using an NMR setup has been reported [28].

In this paper, we study the quantum correlations present in the final state of a one-dimensional transverse $XY$ model after quenching the system through an Ising critical point between two spins separated by a lattice spacing $n$ and quantify it in terms of quantum discord. In the process, we also investigate the classical correlations and the mutual information of two spins in the final state and study their behavior as a function of the rate of quenching. We compare our observations with the behavior of two-spin entanglement in the final quantum state following a quench similar to one reported in a

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recent study [29]. We also investigate quantum discord following a quantum quench across a multicritical point (MCP) along a linear path. We note that similar quenching studies have been carried out to establish the universal scaling relation, namely the Kibble–Zurek scaling [30,31], of the defect density generated following critical [32]–[37] and multicritical quenches [38]–[40]. In [29], it was established that concurrence also follows the same Kibble–Zurek scaling relation as the defect density; this prediction was found to hold good also for quenching through a MCP in a later study [41]. We attempt to address the same question related to the scaling of discord; our studies indicate a similar scaling.

The organization of the rest of the paper is as follows. In section 2, we quantify discord in terms of classical correlations and quantum mutual information. In section 3, we compute quantum discord of the transverse XY model after quenching the system through critical and multicritical points. This is followed by a discussion on our main results in section 4. We present some concluding remarks in section 5.

2. Quantum discord

Let us consider a classical bipartite system comprising two subsystems $A$ and $B$. The information associated with the system is quantified in terms of the Shannon entropy $H(p)$ where $p$ is the probability distribution of the system. The classical mutual information is defined as

$$I(p) = H(p^A) + H(p^B) - H(p),$$

(1)

where the $H(p^i), i = A, B$, stand for the entropy associated with the subsystem $i$; this can alternatively be expressed as

$$J(p) = H(p^A) - H(p|p^B),$$

(2)

where $H(p|p^B) = H(p) - H(p^B)$ is the conditional entropy. In the quantum context, the classical Shannon entropy functional gets replaced by the quantum von Neumann entropy expressed in terms of the density matrix $\rho$. The natural quantum extension of equation (1) is given by

$$I(\rho) = s(\rho^A) + s(\rho^B) - s(\rho),$$

(3)

The conditional entropy based on local measurement, however, alters the system. The measurement are of von Neumann type, having a set of one-dimensional projectors $\{\hat{B}_k\}$ that sum up to the identity. Following a local measurement only on the subsystem $B$, the final state $\rho_k$ of the composite system, which is the generalization of the classical conditional probability, is given by

$$\rho_k = \frac{1}{p_k}(\hat{I} \otimes \hat{B}_k)\rho(\hat{I} \otimes \hat{B}_k),$$

(4)

with the probability $p_k = \text{tr}(\hat{I} \otimes \hat{B}_k)\rho(\hat{I} \otimes \hat{B}_k)$ where $\hat{I}$ is the identity operator for the subsystem $A$. The quantum conditional entropy can be defined as $s(\rho|\{\hat{B}_k\}) = \sum_k p_k s(\rho_k)$, such that the measurement based quantum mutual information takes the form $J(\rho|\{\hat{B}_k\}) = s(\rho^A) - s(\rho|\{\hat{B}_k\})$. This expression maximized on the basis of the local measurement gives the classical correlation [42]. Hence we have

$$C(\rho) = \max_{\{\hat{B}_k\}} J(\rho|\{\hat{B}_k\}).$$

(5)

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This line of argument provides us with two quantum analogs of the classical mutual
information: the original quantum mutual information \( I(\rho) \) (equation (3)) and the
measurement induced classical correlation (equation (5)). As introduced by Olliver and
Zurek [14], the difference between these two, i.e.
\[
Q(\rho) = I(\rho) - C(\rho),
\]
(6)
is the quantum discord which measures the amount of quantumness in the state. It
is noteworthy that \( I \) represents the total information (correlation) whereas
\( C \) is the information gained about \( A \) as a result of a measurement on \( B \). If \( Q = 0 \), we conclude
that the measurement has extracted all the information about the correlation between
\( A \) and \( B \); on the other hand, a non-zero \( Q \) implies that the information cannot be extracted
by local measurement and the subsystem \( A \) gets disturbed in the process, a phenomenon
not usually expected in classical information theory.

3. The model and pairwise correlations

We study pairwise correlations in a one-dimensional spin-1/2 XY model in a transverse
field with nearest neighbor ferromagnetic interactions described by the Hamiltonian [43]–
[47]
\[
H = -\frac{1}{2} \sum_i \left[ (1 + \gamma)\sigma_i^1\sigma_i^{1+1} + (1 - \gamma)\sigma_i^2\sigma_i^{2+1} + h\sigma_i^3 \right],
\]
(7)
where the matrices \( \sigma \) are the Pauli spin matrices and the subscripts stand for the
spin direction and the superscripts for the lattice index. The parameter \( h \) is the
magnetic field applied in the transverse direction and \( \gamma \) measures the anisotropy in
the in-plane interactions; \( \gamma = 1 \) refers to the transverse Ising model [2]. The model
can be exactly solved by mapping the spins to spinless fermions via a Jordan–Wigner
transformation [43,44,46]; the phase diagram for the model is shown in figure 1. Using
Jordan–Wigner transformation the Hamiltonian (7) can be reduced to decoupled 2 \( \times \) 2 matrices
in the two-dimensional subspace spanned by \( |0\rangle \) (empty state) and \( |k, -k\rangle \) (a
two-fermion state with wavevectors \( k \) and \( -k \)). The general ground state for the reduced
2 \( \times \) 2 Hamiltonian for wavevector \( k \) can be written as \( |\psi_k\rangle = C_{1k}|0\rangle + C_{2k}|k, -k\rangle \), with
\[
|C_{1k}|^2 + |C_{2k}|^2 = 1.
\]
We study the behavior of quantum discord in the final state after quenching the
system across an Ising critical point following the quench scheme \( h(t) = t/\tau \) with \( t \) going
from \( -\infty \) to \( \infty \) [34]. The diverging relaxation time close to the QCPs at \( h = \pm 1 \)
lead to defects in the final state. At \( t \to -\infty \), the system is in the ground state, \( |0\rangle \), for all
\( k \) where all the spins are aligned in the \( -z \) direction (\( C_{2k}(-\infty) = 0 \)). At \( t \to \infty \), the
probability of excitation in the final state for the mode \( k \) is given by
\[
p_k = |C_{1k}(\infty)|^2 = \exp(-\pi \tau \gamma^2 \sin^2 k).
\]
(8)
In the limit \( \tau \to \infty \) (i.e., slow quenching), only the modes close to the critical modes
\( (k = 0 \) or \( \pi \)) contribute and one gets \( p_k = \exp(-\pi \gamma^2 k^2 \tau) \). The defect density \( n \)
in the final state is obtained by integrating over all momentum modes and scales as \( n \sim 1/\sqrt{\tau} \)
in the limit of slow quenching [33,34]. This is in agreement with the Kibble–Zurek scaling
\( n \sim 1/\tau^{d/(\nu z + 1)} \) [30,31] where \( d \) is the spatial dimension and \( \nu, z \) are the correlation length

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Figure 1. The phase diagram of the one-dimensional $XY$ model in a transverse field. The vertical bold line at $h = 1$ denotes the Ising transition from a ferromagnetic phase to a paramagnetic phase. The horizontal bold line stands for the anisotropic phase transition between the ferromagnetic phase ordering in the $x$ and $y$ directions. A linear path to approach the MCP ‘A’ is shown.

and dynamical exponent, respectively, associated with the QCP across which the system is quenched because for the Ising transition, $\nu = z = 1$.

We further extend our study by quenching the system across a quantum multicritical point (MCP) by approaching along a linear path [40]

$$h(\gamma) = 1 + |\gamma(t)| \text{sgn}(t); \quad \gamma(t) = -\frac{t}{\tau},$$

and investigate the dependence of the discord. The probability of defect formation $p_k$ is given by [38]–[40]

$$p_k = \exp(-\pi \tau (1 + \cos k)^2 \sin^2 k).$$

We emphasize that we have used the expressions (8) and (10) which are valid for all $\tau$ in deriving the concurrence and quantum discord.

We shall now calculate various elements of the two-spin density matrix in the final paramagnetic phase of the Hamiltonian (7) for spins at the sites $i$ and $j = i + n$ using the generic form of the density matrix given by [6, 23, 48]

$$\rho^n = \frac{1}{4}(I_i \otimes I_j + c_1 \sigma_1^i \otimes \sigma_1^j + c_2 \sigma_2^i \otimes \sigma_2^j + c_3 \sigma_3^i \otimes \sigma_3^j + c_4 I_i \otimes \sigma_3^j + c_5 \sigma_3^i \otimes I_j),$$

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where \( c_1 = \langle \sigma_1^x \sigma_1^x \rangle, c_2 = \langle \sigma_2^x \sigma_2^x \rangle, c_3 = \langle \sigma_3^x \sigma_3^x \rangle, c_4 = c_5 = \langle \sigma_3^z \rangle \). It is to be noted that the averages are calculated over the final state of the system following the quench which is a linear combination of the states \(|0\rangle\) and \(|k, -k\rangle\) whose probabilities are given by \( p_k \) and \((1-p_k)\), respectively. For the Hamiltonian (7), the X and Y directions are equivalent and hence \( c_1 = c_2 \). The density matrix can also be expressed in the form

\[
\rho^n = \begin{pmatrix}
a^n_+ & 0 & b^n_1 \\
0 & a^n_0 & b^n_2 \\
b^n_1* & 0 & a^n_-
\end{pmatrix},
\]

(12)

where the matrix elements are given in terms of the two-spin correlation functions in the following manner:

\[
a^n_\pm = \frac{1}{4}\langle(1 \pm \sigma_3^z)(1 \pm \sigma_3^{i+n})\rangle = 1 + c_3 \pm 2c_4,
\]

\[
a^n_0 = \frac{1}{4}\langle(1 \pm \sigma_3^z)(1 \mp \sigma_3^{i+n})\rangle = 1 - c_3,
\]

\[
b^n_{1(2)} = \langle \sigma_3^\pm \sigma_3^{i+n} \rangle.
\]

(13)

We note that the up–down symmetry of the Hamiltonian simplifies the density matrix and some of the elements vanish [48]. Defining a quantity \([29, 34]\)

\[
\beta_n = \int_0^\pi \frac{dk}{\pi} p_k \cos(nk),
\]

(14)

one gets

\[
c_4 = c_5 = \langle \sigma_3^i \rangle = 1 - 2\beta_0, \quad c_3 = \langle \sigma_3^i \sigma_3^{i+n} \rangle = \langle \sigma_3^i \rangle^2 - 4\beta_0^2.
\]

(15)

The expressions for \( c_1 \) and \( c_2 \) differ for different values of \( n \); these are presented below for \( n \leq 6 \):

\[
c_1 = c_2 = \begin{cases}
\frac{\beta_2}{2}(1 - 2\beta_0), & n = 2, \\
(1 - 2\beta_0)^2 \beta_2^2 - 4\beta_2^4 + \frac{\beta_4}{2}(1 - 2\beta_0)^3 - 2\beta_2^2\beta_4(1 - 2\beta_0), & n = 4, \\
\frac{1}{2} \{\beta_0 \{1 - 2\beta_0\}^2 - 4\beta_2^2\} + 4\beta_2 \{\beta_2^2 + \beta_4^2 - \beta_4(1 - 2\beta_0)\} \\
\times |16\beta_2^2\beta_4 + (1 - 2\beta_0) \{1 - 2\beta_0\}^2 - 8\beta_2^2 - 4\beta_2^2|, & n = 6.
\end{cases}
\]

The functional forms of \( c_1 \) and \( c_2 \) become complicated with increasing \( n \) because the correlation function matrices grow as \( n \times n \) [43].

The eigenvalues of the density matrix are obtained in terms of the correlators \( c_i \) [22, 23] as

\[
\lambda_0 = \frac{1}{4} \left[ (1 + c_3) + \sqrt{4c_1^2 + (c_1 - c_2)^2} \right], \quad \lambda_1 = \frac{1}{4} \left[ (1 + c_3) - \sqrt{4c_1^2 + (c_1 - c_2)^2} \right],
\]

\[
\lambda_2 = \frac{1}{4} \left[ (1 - c_3) + (c_1 + c_2) \right], \quad \text{and} \quad \lambda_3 = \frac{1}{4} \left[ (1 - c_3) - (c_1 + c_2) \right],
\]

(16)

which can be expressed entirely in terms of the parameters \( \beta \) using equations (13)–(15).

The expression for the concurrence [7, 29] is given by \( C_{nc} = \max\{0, \sqrt{\lambda_0 - \sqrt{\lambda_1} - \sqrt{\lambda_2}} - \sqrt{\lambda_3} \}\), where the \( \lambda_i \) are the eigenvalues of \( \rho^n \) (12) in decreasing order. One can show that the \( \sqrt{\lambda_i} \) are given by \( \sqrt{a^n_1 a^n_2} \) (appearing twice) and \( a^n_0 \pm |b^n_2| \). Thus the spin chain has a non-zero concurrence if \( |b^n_2| > \sqrt{a^n_1 a^n_2} \) and it is given by

\[
C_{nc} = \max\left\{0, 2\left( |b^n_2| - \sqrt{a^n_1 a^n_2} \right) \right\}.
\]

(17)

This expression is to be used later for comparison with the quantum discord.

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4. Results

In this section, we present results for the pairwise correlations in the final state of the spin chain as a function of the quenching rate $\tau^{-1}$. Using equation (14) we note that $\beta_n = 0$ for odd $n$ as $p_k$ is invariant under $k \rightarrow \pi - k$. At the same time, $\langle \sigma^+_i \sigma^+_n \rangle = b^n_1 = 0$ for all $n$ since the expectation values of a pair of fermionic annihilation or creation operators do always vanish. Moreover, $\langle \sigma^+_i \sigma^+_n \rangle = b^n_2 = 0$ for odd $n$ since the quantities $b^n_2$ are odd under the $\mathbb{Z}_2$ transformation [29,34]. On the other hand, $b^n_2 = c_1 + c_2$ for even $n$.

The variation of the mutual information $I$, the classical correlation $C$ and the quantum discord $Q = I - C$ with $\tau$ are therefore studied for both critical and multicritical quenches (9) for even $n$. Let us rename the spins $i$ as subsystem $A$ and the spins $j$ as subsystem $B$. The reduced density matrix for the subsystems $A$ and $B$ can be expressed as

$$\rho_A = \frac{1}{2}(I^i \otimes I^j + c_4 I^i \otimes \sigma_3^j), \quad \text{and} \quad \rho_B = \frac{1}{2}(I^i \otimes I^j + c_4 \sigma_3^i \otimes I^j)$$

with eigenvalues

$$\lambda_4 = \frac{1}{2}(1 + c_4), \quad \text{and} \quad \lambda_5 = \frac{1}{2}(1 - c_4).$$

The total mutual information $I(\rho)$ is expressed in terms of von Neumann entropies, which when substituted in equation (1) give

$$I(\rho) = s(\rho_A) + s(\rho_B) - \sum_{\alpha=0}^{3} \lambda_\alpha \log_2 \lambda_\alpha,$$

where $s(\rho_A) = s(\rho_B) = -\lambda_4 \log_2 \lambda_4 - \lambda_5 \log_2 \lambda_5$. To calculate the classical correlation, we introduce a set of projectors for local measurement on the subsystem $B$ given by

$$B_k = \Pi_k V^\dagger$$

where $\Pi_k = |k\rangle\langle k|, k = +, -$, is the set of projectors on the computational basis $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \langle -| = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ and $V \in U(2)$ where $V$ is parametrized over a Bloch sphere given by

$$
\begin{pmatrix}
\cos \frac{\theta}{2} & \sin \frac{\theta}{2} e^{-i\phi} \\
\sin \frac{\theta}{2} e^{i\phi} & -\cos \frac{\theta}{2}
\end{pmatrix},
$$

where the polar angle $\theta$ lies between 0 and $\pi$ and the azimuthal angle $\phi$ is from 0 to $2\pi$. Following a technique used in [23], we can obtain the classical correlation by maximizing

$$C(\rho) = s(\rho_A) - s(\rho_+),$$

where $\rho_+$ is the density matrix for the outcome $|k\rangle = |+\rangle$. Below we summarize the final results; e.g., for $n = 2$, we get

$$c_1 = c_2 = \frac{\beta_2}{2}(1 - 2\beta_0), \quad c_3 = (1 - 2\beta_0)^2 - 4\beta_2^2, \quad c_4 = 1 - 2\beta_0.$$

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The exact expressions for the mutual information and classical correlation for \( n = 2 \) are given below.

\[
I = -2(1 - \beta_0) \log_2(1 - \beta_0) + ((1 - \beta_0)^2 - \beta_2^0) \log_2((1 - \beta_0)^2 - \beta_2^0) - 2\beta_0 \log_2(\beta_0) \\
+ \left( \beta_0^2 - \beta_2^0 \right) \log_2(\beta_0^2 - \beta_2^0) + \frac{1}{4} \{4\beta_0(1 - \beta_0) + 4\beta_2^0 + \beta_2(1 - 2\beta_0)\} \\
\times \log_2 \left[ \frac{1}{4} \{4\beta_0(1 - \beta_0) + 4\beta_2^0 - \beta_2(1 - 2\beta_0)\} \right] \\
+ \frac{1}{4} \{4\beta_0(1 - \beta_0) + 4\beta_2^0 - \beta_2(1 - 2\beta_0)\} \\
\times \log_2 \left[ \frac{1}{4} \{4\beta_0(1 - \beta_0) + 4\beta_2^0 - \beta_2(1 - 2\beta_0)\} \right],
\]
and

\[
C = -(1 - \beta_0) \log_2(1 - \beta_0) - \beta_0 \log_2(\beta_0) + \frac{1}{2} \left( 1 - (1 - 2\beta_0) \sqrt{1 + \beta_2^0} \right) \\
\times \log_2 \left[ \frac{1}{2} \left\{ 1 - (1 - 2\beta_0) \sqrt{1 + \beta_2^0} \right\} + \frac{1}{2} \left( 1 + (1 - 2\beta_0) \sqrt{1 + \beta_2^0} \right) \right] \\
\times \log_2 \left[ \frac{1}{2} \left\{ 1 + (1 - 2\beta_0) \sqrt{1 + \beta_2^0} \right\} \right].
\]

Similarly one can obtain the expressions for \( n = 4 \) and 6 using the appropriate equations. We note that \( I, C \), and hence, \( Q = I - C \) depend entirely on the values of \( \beta \) which are in turn dependent on the quench rate \( \tau^{-1} \) through the defect density \( p_0 \). In deriving the above expressions, we have used \( \gamma = 1 \); however, qualitatively the mathematical forms of the eigenvalues presented in equation (16) remain unaltered, though the values of \( \beta \) are modified for \( \gamma \neq 1 \).

Figure 2 shows the variation of the quantum discord \( Q \) with \( \tau \) for \( n = 2, 4, 6 \) for a quench across the Ising critical point. As expected, \( Q \) vanishes in both the limits \( \tau \rightarrow 0 \) and \( \infty \); the final state is nearly a direct product state in either case. Discord initially increases with increasing \( \tau \), and starts decreasing monotonically after reaching a peak at \( \tau = \tau^m \). As \( n \) increases, \( \tau^m \) shifts toward the right. A similar behavior is observed for \( I \). Figure 3 shows that the classical correlations also exhibit a qualitatively identical variation with \( \tau \) though it is smaller in magnitude in comparison to that for the discord. The correlation between two spins decreases with increasing \( n \), which is reflected in the quantum discord and total mutual information, and as a result the peak value of \( Q \) decreases as \( n \) is increased. In the limit \( \tau \rightarrow \infty \), on the other hand, the discord tends to vanish because of the nearly adiabatic dynamics when defect generation is very much less; we therefore observe almost identical behavior for all \( n \) in this limit. These facts together lead to a faster decay of \( Q \) for smaller values of \( n \), as shown in figure 2. Surprisingly, the classical correlations show some fluctuating behavior for \( \tau \rightarrow 0 \) followed by a monotonic increase (see inset figure 3). The value of \( C \) is found to be one order of magnitude less than that of \( I \), implying that correlations present in the system are mainly quantum mechanical. Though \( C \) peaks at a larger \( \tau \) in comparison to \( I \) for a given \( n \), this does not substantially influence the behavior of \( Q \), as quantum correlations apparently dominate over classical correlations.

The variation of the concurrence (17) in the final state for a quench across an Ising critical point with \( \tau \) has been studied recently [29] and the comparison is shown.
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Figure 2. Quantum discord $Q$ as a function of $\tau$ for $n = 2$ (solid line), 4 (dotted line) and 6 (dashed line) in the final state following a linear quench across the Ising critical point with $\gamma = 1$. The inset shows the variation of the concurrence ($C_{nc}$) for the same parameter values as are reported in [29].

Figure 3. Variation of the classical correlation with $\tau$ for $n = 2$ (solid line), 4 (dotted line) and 6 (dashed line). The inset shows small peaks for $\tau \rightarrow 0$ and a monotonic increase.

in figure 2. Although the variations of the discord and concurrence are qualitatively similar, we emphasize the following differences. The magnitude of the discord is less than that of the concurrence for the same $n$, by one order, and it shows a peak at a value of $\tau$ which is very small in comparison to the corresponding $\tau$ for the concurrence. We conclude that the measurement based approach employed in calculating the discord

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The variations of the quantum discord $Q$ as a function of $\tau$ for $n = 2$ (solid line), 4 (dotted line) and 6 (dashed line) are shown on a log scale following a quench across the Ising critical point with $\gamma = 1$; the slope is $\approx 0.5$ in all cases, indicating that the discord satisfies the same scaling relation as the defect density. The inset shows the variation of the concurrence ($C_{nc}$) for $n = 2$ with the same parameter values as are reported in [29]; the slope is again $\approx 0.5$. This provides a quantitatively different result for quantum correlations. Moreover, the study on the concurrence [29] indicates the existence of a threshold value of $\tau$ above which the bipartite entanglement is generated. In contrast, the discord is non-zero for all $\tau$, as we observe a negligible shift close to $\tau = 0$ for different values of $n$ as shown in figure 2. We note that the classical correlation is given by $C(\rho) = s(\rho_A) - s(\rho_+)$. Our studies show that in the final state following the quench, $s(\rho_A)$ and $s(\rho_+)$ are of nearly the same magnitude though they are opposite in sign, leading to a small magnitude of the classical correlation.

We further verify this claim by investigating the scaling of the discord following a linear quenching across the MCP ‘A’ (see equation (9)) for which the defect density scales as $\tau^{-1/6}$ [38]–[40]. The mutual information and, hence, the quantum discord can be derived using equation (24) where the $\beta_n$ are obtained using equation (14) with excitation probability $p_k$ for a linear multicritical quench given in equation (10). In figure 5, we present the variation of the discord and also that of the concurrence (for $n = 2$) with respect to $\tau$ on a logarithmic scale; the slope in both cases is $\approx 1/6$ which again indicates that the scaling of the discord is likely to be the same as that of the defect density.
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Figure 5. The variation of the discord for $n = 2$ following a multicritical quench along a linear path with slope $\approx -0.19$, which matches well with the value of the exponent $-1/6$ obtained for the defect density [38]. In the inset, we show the similar variation of the concurrence with $\tau$ for $n = 2$ [41], and the slope is $\approx -0.13$ which is in close agreement with the exponent $-1/6$.

5. Conclusion

We have studied quantum discord and mutual information and their dependence on the quenching rate for two spins separated by $n$ lattice sites in the final state of a transverse $XY$ spin chain following a slow quench across a quantum critical and a multicritical point. Our studies show that the quantum discord and mutual information exhibit qualitatively similar behavior to the concurrence, as reported before [29]; all these quantities are in fact determined in terms of the eigenvalues of the two-spin density matrix. However, we do also highlight differences; discord is smaller in magnitude in comparison to concurrence and is non-zero even in the limit of $\tau \to 0$ for any $n$, unlike concurrence which sets in at a threshold value of $\tau$ especially for large $n$. The classical correlation is found to be smaller than the mutual information by one order of magnitude, suggesting a stronger quantum nature of the correlations. Finally, our studies indicate that for a linear quenching through a MCP, discord shows a power law scaling with the rate of quenching for large $\tau$ in a similar fashion to the defect density. This observation apparently suggests the existence of a universal scaling of discord in the final state, which is related to that of the defect density for both critical and multicritical quenches, though a closed power law form is not obtained.

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Quantum discord in a spin-1/2 transverse XY chain following a quench

References

[1] Sachdev S, 1999 Quantum Phase Transitions (Cambridge: Cambridge University Press)
[2] Chakrabarti B K, Dutta A and Sen P, 1996 Quantum Ising Phases and Transitions in Transverse Ising Models m41 (Heidelberg: Springer)
[3] Continentino M A, 2001 Quantum Scaling in Many-Body Systems (Singapore: World Scientific)
[4] Vojta M, 2003 Rep. Prog. Phys. 66 2069
[5] Osterloh A, Fazio R, Osterloh A and Vedral V, 2002 Nature 416 608
[6] Osborne T J and Nielsen M A, 2002 Phys. Rev. A 66 032110
[7] Amico L, Fazio R, Osterloh A and Vedral V, 2008 Rev. Mod. Phys. 80 517
[8] Gu S J, Tian G S and Lin H Q, 2005 New J. Phys. 71 052322
[9] Osborne T J and Nielsen M A, 2002 Phys. Rev. A 66 032110
[10] Makhlin Y, S ch en G and Sh ni rman A, 2001 Rev. Mod. Phys. 73 517
[11] Bennett C H, DiVincenzo D P, Smolin J A and Wootters W K, 1996 Phys. Rev. A 54 3824
[12] Horodecki M, 2001 Quantum Inf. Comput. 1 3
[13] Dakshinamurthy M, 2004 Quantum Inf. Comput. 1 27
[14] DiVincenzo D P, Smolin J A and Wootters W K, 1996 Phys. Rev. A 54 3824
[15] Peres A, 1996 Phys. Rev. Lett. 77 1413
[16] Olliver H and Zurek W H, 2001 Phys. Rev. Lett. 88 017901
[17] Zurek W H, 2003 Adv. Phys. 52 241109(R)
[18] Sengupta K and Sen D, 2009 Phys. Rev. A 80 032304
[19] Zurek W H, 2003 Rev. Mod. Phys. 75 715
[20] Bennett C H, DiVincenzo D P, Smolin J A and Wootters W K, 1996 Phys. Rev. A 54 3824
[21] Zurek W H, 2003 Adv. Phys. 52 241109(R)
[22] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 100 077204
[23] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[24] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[25] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[26] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[27] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[28] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[29] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[30] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[31] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[32] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[33] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[34] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[35] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[36] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[37] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[38] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[39] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[40] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[41] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[42] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[43] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[44] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[45] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[46] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[47] Sengupta K, Sen D and Mondal S, 2008 Phys. Rev. Lett. 101 016806
[48] Syljuasen O F, 2003 Phys. Rev. A 68 060301(R)

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