A Solution for the Full-Load Collection Vehicle Routing Problem With Multiple Trips and Demands: An Application in Beijing

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ABSTRACT Municipal solid waste (MSW) collection has become a major challenge for clean city management and social sustainable development in developing economies. A new variant of the collection vehicle routing problem (CVRP) is addressed with the characteristics of full loads and multiple trips of the collection vehicles, and multiple demands of the garbage facilities, which is called the collection vehicle routing problem of the garbage facilities (CVRPGF) in this study. Dummy customers are introduced to equivalently transform the CVRPGF problem to the vehicle routing problem with simultaneous pickup-delivery and time windows (VRPSPDTW). A parallel simulated annealing algorithm (Par-SAA) is developed to solve the VRPSPDTW problem. When applied to an international benchmark dataset, the computational results prove the superiority of the proposed algorithm, in which the number of collection vehicles (NV) in four instances is reduced by one. Finally, when the model and algorithm are applied to a real CVRPGF problem in the Xuanwu District of Beijing, the NV needed is reduced by 30%, and the total travel time is decreased by 12%. Thus, the effectiveness of the Par-SAA is demonstrated, and the proposed solution has practical value in China.

INDEX TERMS Municipal solid waste collection, vehicle routing, full load, multiple trips, multiple demands, dummy customer, parallel simulated annealing.

I. INTRODUCTION

Due to the substantial increase in solid waste, municipal solid waste (MSW) management has increasingly attracted social attention [1]. It is important to maintain a healthy and clean environment and to reduce the social problems of collecting MSW in a timely manner [2]. Furthermore, high-efficiency collection can reduce the management costs for the government because the operating cost of solid waste collection accounts for 60%-80% of the total cost of solid waste management [3], [4]. Moreover, MSW collection is also the key process for the development of the circular economy [5].

In the last two decades, researchers have paid attention to solving the collection vehicle routing problem (CVRP) of MSW in developed economies [6]. The successful and effective practices have proved that the optimization of MSW collection vehicle routing can sharply decrease the costs of a collection system [7]. Some scholars have analyzed the efficiency of different waste collection services to identify suitable services for many cities [8]. Moreover, several comprehensive collection systems combining renewable waste recycling have been formed in some developed economies [9].

In developing economies, MSW collection has become an increasingly urgent issue in urban cities and areas due to the lack of strategic MSW plans [10]. China is no exception and is experiencing problems regarding the lack of timely garbage collection in most cities. Meanwhile, the collection plan is scheduled according to an empirical approach based on the operational characteristics of the facility [11]. Although efficient models and algorithms have been proposed in previous
studies, most of them are based on the cities in developed economies, which are not applicable to China [12], [13]. MSW collection in developing economies has not been sufficiently studied.

To fill this gap, this research focuses on a new variation of the classical CVRP in developing economies. In this case, garbage facilities are the collection points located in the city. Each garbage facility requires multiple collection services by large-container-level vehicles per day. The garbage collection vehicle makes multiple trips between garbage facility and the transfer stations (or disposal sites) each day. This case is referred to as the collection vehicle routing problem of the garbage facilities (CVRPGF). The purpose of this study is to develop an efficient model and algorithm to address the CVRPGF and to investigate a real CVRPGF case in China.

The rest of this paper is organized as follows. Section 2 presents a literature review on various CVRP variants and solution methods. Section 3 develops a theoretical model that transforms the CVRPGF problem into a vehicle routing problem with simultaneous pickup and delivery time windows (VRPSDPTW). Section 4 proposes a parallel simulated annealing algorithm (Par-SAA) to solve the VRPSDPTW problem. Section 5 presents a real CVRPGF case in Beijing and provides the solution by the proposed algorithm. Section 6 draws the conclusions.

II. LITERATURE REVIEW
Due to various forms of MSW collection, several CVRP variants have been developed. In this section, we briefly summarize the CVRP variants and the algorithms to address this problem.

A. VARIOUS KINDS OF CVRP
The CVRP problem is a famous application of the classical vehicle routing problem (VRP) in the MSW field [14]. Various kinds of VRP variants and solution methods can be found in the CVRP of different cities [15]. TABLE 1 summarizes various kinds of CVRP according to their characteristics. Only two of the papers in TABLE 1 are related to developing economies [13], [16], and the remainder focus on developed economies.

According to the collection time of the collection point, there are two approaches to waste collection.

1) PERIODIC COLLECTION
The first study on the CVRP problem was the periodic CVRP [17]. As the population density of most areas of the USA is relatively sparse, each collection point needs to be visited only once every few days, and the vehicle schedule is usually organized in a periodic (mostly weekly) manner. To solve the periodic CVRP, we first determine which collection points will be visited each day; second, we optimize the schedule of the vehicles every day [18].

2) COLLECTION WITH TIME WINDOWS
Some collection points have service time requirements, so the vehicle needs to provide waste collection service within the time window, which is called the CVRP with time windows [19], [20].

According to the characteristics of collecting areas, whether residential, commercial or industrial, there are three types of MSW collection [21].

3) COLLECTION BY THE STREET
The collection vehicles servicing residential areas move along the streets to collect the garbage accumulated by each house [22]. The corresponding VRP is the arc routing problem [23].

4) COLLECTION FROM DESIGNATED POINTS
The collection vehicles servicing commercial districts visit the customers, such as hotels, shopping malls, government departments and stations, one by one to collect garbage. This method most closely resembles the classic VRP [1].

5) COLLECTION WITH CONTAINERS
The main characteristic of this problem is that substantial amounts of garbage need to be collected in industrial regions, involving construction sites, hospitals and downtown areas, which requires large-container-level services (Roll-on-roll-off VRP, RR-VRP) [24]. In this case, a collection vehicle only has the capacity to serve one customer each time. Hence, this problem is called the full-load VRP with container [25].

According to the collection vehicle types and collection times, and the number of disposals, there are three ways to perform waste collection.

6) COLLECTION WITH MULTICOMPARTMENT VEHICLES
In some developed economies, MSW is collected separately for recycling. In that situation, MSW can be transported by various vehicles or by one vehicle with multiple compartments [13].

7) COLLECTION WITH MULTIPLE TRIPS
One vehicle may visit the collection points and the transfer stations (or disposals) during the working time making several trips [16].

8) COLLECTION TO MULTIPLE DISPOSALS
In some cities, there are several transfer stations (or disposal sites) in one region, and the collection vehicle can unload the waste at any of these sites according to the optimized schedule [4].

Because of the complicated situation in each region, the CVRP usually has several characteristics and many different CVRPs have been studied. However, the CVRPGF problem has not been explored, which is currently a suitable collection method in China.
TABLE 1. Variants and characteristics of various CVRP.

| CVRP variants                       | Characteristics                                      | Literature |
|-------------------------------------|------------------------------------------------------|------------|
| Periodic collection                 | Periodic VRP: frequency of customer service is less than one day | [17][18]  |
| Collection with time windows        | VRPTW: collecting point should be serviced in time     | [19][20][21] |
| Collection by the street            | Arc routing problem: vehicles move along the streets to collect the garbage | [22] |
| Collection from designated points   | Classic VRP: vehicles visit customers one by one      | [1]        |
| Collection with containers          | Full-load VRP with container: customers require large-container level services | [24], [25] |
| Collection with multiple trips      | VRP with multiple trips: times of vehicle to the disposal | [16] |
| Collection with multi-compartment   | VRP with multi-compartment: types of vehicle compartments | [13] |
| Collection with multiple disposals  | VRP with multiple disposals: garbage delivered to different disposals | [4] |

B. METHODS

1) MODELS

The models are important for solving the problem effectively because the characteristics of various MSW collection types increase the difficulty of solving the CVRP variants. For example, the subtour elimination constraints are needed to avoid the formation of vehicle routes that are disconnected from a depot [26]. Furthermore, in the multi-trip vehicle routing problem, multiple constraints are required to distinguish between a round trip of the vehicle (between the collection point and the transfer station) and the total travel of the vehicle during working hours (including multiple round trips) [19].

To simplify the problem complexity, many studies transformed the complicated problem into a VRP variant. For example, some scholars have transformed the arc routing problem into a node routing problem, which simplifies the complexity of modeling and the algorithm [27]. In the CVRP with containers, Song et al. equivalently transformed the container drayage problem under a separation mode into a variant of the asymmetric VRP with time windows (a-VRPTW) based on a determined-activities-on-vertex (DAOV) graph [28]. Desrosiers J et al. considered the situation of multiple trips and transformed the problem into an asymmetric traveling salesman problem by assuming a constraint in which the vehicles have to return to the parking area after completing their tasks [26].

Although many modeling methods have been explored for various kinds of CVRP variants, there have been few research attempts to model the CVRPGF problem.

2) ALGORITHMS

The classical VRP is an NP-hard problem [29]. As a special case of VRP, CVRP is also an NP-hard problem. Therefore, the main algorithms applied to solve the CVRP are exact algorithms, heuristic algorithms, and metaheuristic algorithms. Exact algorithms have not been widely explored to solve the CVRP problem because the computational complexity grows factorially with the number of customers. The heuristics and metaheuristics are popular methods that have been used by previous studies [30].

The CVRP with the full container load has been addressed by Imai et al. with the subgradient heuristic based on Lagrangian relaxation [25]. A two-stage heuristic has been adopted in the periodic CVRP [18]. Large neighborhood search-based iterative heuristic approaches consisting of several algorithms have been proposed to solve the RR-CVRP [24]. With the increase of complexity in the CVRP variant, it becomes more difficult to solve by heuristics.

It is indisputable that metaheuristics can produce higher-quality solutions than conventional heuristics [31]. Thus, metaheuristic approaches have been the most popular algorithms for the CVRP in the past decade. Numerous researchers have developed metaheuristics for the CVRP. To solve the multicompartiment capacitated arc routing problem with intermediate facilities for MSW collection, an adaptive large neighborhood search algorithm (ALNS) and a hybrid ALNS with the whale optimization algorithm have been proposed [13]. To solve the RR-VRP problem, a LS/LNS-based metaheuristic was proposed [32].

Among them, the simulated annealing algorithm (SAA) is a metaheuristic with the characteristics of robustness and efficiency, which is applicable to many problems. Moreover, approaches have been proposed to solve the CVRP with multiple trips [16]. A hybrid metaheuristic algorithm is developed based on an SAA and a heuristic algorithm to solve the capacitated arc routing problem in the urban waste collection [33].

The previous literature has shown that the study of the CVRPGF is rare compared with that of other CVRP variations. This study fills this gap and (1) we propose a CVRPGF problem, (2) we develop a Par-SAA to solve this problem; and (3) we apply the proposed method to a practical CVRPGF case in Beijing.

III. PROBLEM DESCRIPTION AND MATHEMATICS FORMULATION

This section first describes the CVRPGF problem, and then transforms this problem to VRPSPDTW problem. Finally, a mixed integer programming formulation (MIP) is presented.

A. CVRPGF PROBLEM

In the CVRPGF problem, a garbage facility serves as the collection point and there are sufficient empty containers at the garbage facility. The waste collection process for the garbage facilities is shown in FIGURE 1. First, the community
sanitation workers collect the MSW from the neighborhoods and send it to the corresponding garbage facility, where the staff loads the waste into garbage containers. The collection vehicle, which has a single-container capacity, parks in the yard and visits the garbage facility to collect MSW. Then, the collection vehicles transport the fully loaded garbage containers to the transfer stations (or disposal sites) near the city and return empty containers to the garbage facility. All collection vehicles must return after transporting all the garbage containers gathered by every garbage facility to the transfer facility. This can be regarded as the switch-out (S/O) problem of the RR-VRP [34], which means that a full container is exchanged for an empty one [24]. Finally, the collection vehicles will come back to the yard when they have finished their task.

Most of the garbage facilities deal with large amounts of garbage and generate many full garbage containers daily. Therefore, the collection vehicles need to transport the containers for each garbage facility several times. Practically, the intercity transport of the collected MSW is short-distance transport. Therefore, one vehicle makes multiple trips between the garbage facility and transfer station.

The optimization objective of this problem is to find a collection schedule with the minimization of the objective, which determines the collection vehicle and the collection sequence of each collection task of the garbage facilities. The primary objective is to minimize the number of collection vehicles (NV) and the secondary objective is to minimize the total travel time (TT).

B. PROBLEM TRANSFORMATION

In previous studies, the concept of dummy customers is introduced based on the characteristics of real customers to simplify the problem. It is a popular way to address the dynamic VRP variants [6]. To solve the school bus routing problem, Bektash and Elikximastas transformed the open VRP into the classical VRP though the introduction of dummy customers [35]. Additionally, the dummy customer is used in a new proactive real-time control approach for dynamic vehicle routing problems in which the urgent delivery of goods is important [36]. To solve the proposed CVRPGF problem, this study follows this approach and develops an equivalent transformation approach in which dummy customers are introduced to the CVRP model to transform the CVRPGF into the VRPSPDTW.

Each dummy customer proposed in this study has a pickup task or a delivery task. The dummy customers can convert multiple round trips of a collection vehicle into a circular path starting from the yard and eventually back to the yard. Specifically, two kinds of dummy customers are introduced. The first kind of dummy customers are duplicated from the garbage facility with different time windows and pickup tasks, and the number of them is determined by the total collection times of the garbage facility. They are called dummy customers.
garbage facilities. The second kind of dummy customers are duplicated from the transfer station (or MSW disposal facility) with delivery tasks, and the number of them is determined by the total unloading times of the collection vehicles at the transfer station (or MSW disposal facility). They are called the dummy transfer stations.

According to the characteristics of this problem, the pickup amount and the delivery amount of each dummy customer is equal to the capacity of the garbage container. In addition, both kinds of dummy customers have the same location as the garbage facility or the transfer station from which they are duplicated.

FIGURE 2 presents the idea of assigning collection tasks of four garbage facilities to dummy customers. Assume that the number of daily collection times of the four garbage facilities \((A_1, A_2, A_3, A_4)\) are 2, 2, 1, and 3, respectively, and three collection vehicles are required to achieve the tasks. Then, each vehicle’s collection sequence is \([A_1, A_2, A_4]\), \([A_1, A_3, A_4]\) and \([A_2, A_4]\). FIGURE 2 (a) shows the actual collection routes of the three collection vehicles, which are represented by different lines. FIGURE 2 (b) displays the transformed collection routes after the dummy customer points are introduced. For example, \(A_{ic}\) represents the No. \(c\) dummy garbage facility of \(A_i\), \(A_{11}\) and \(A_{12}\) represent the No. 1 and No. 2 dummy garbage facilities of garbage facility 1, and the dashed circle represents the dummy transfer station. Through the above steps, the multiple trips of one collection vehicle have been transformed into a single route of one collection vehicle, departing from and ending at the yard.

C. MATHEMATICS FORMULATION

The transformation problem in this study can be formally defined on a graph, \(G = (V_0, E)\), where \(V_0 = \{O \cup V_l \cup V_z\}\) and \(E = \{<i, j> | i, j \in V_0, i \neq j\}\) are the sets of vertices and edges, respectively, according to the study of [37]. Every point has delivery account \(D_i\), pickup account \(P_i\), time window \([a_i, b_i]\) and service time \(T_i\). The vehicles are similar and with capacity \(Q\). \(K\) is the set of vehicles, and \(K = \{1, 2, 3, \ldots, M\}\). Every vehicle starts from and ends at the yard, archiving the services of all customers in working time \(T\). We introduce some basic notations in TABLE 2.

The following assumptions should be satisfied:

- The waste collection speed of each garbage facility follows a uniform distribution and there are enough empty containers in one garbage facilities;
- Each vehicle can serve only one route, and each vehicle starts from and ends at the yard;
- Each dummy garbage facility must be collected within the time window (hard time window) and can be collected by only one vehicle for one collection sever;
- Each dummy garbage facility has a service time, the vehicles travel at a uniform speed, and the time for loading (unloading) is negligible;
- The total travel time of each vehicle cannot exceed the fixed working time;

The first customer of each route is a dummy garbage facility, as the freight volume should be less than the capacity of

![FIGURE 2. Transformation approach.](image)
the vehicle. The vehicle travels between the dummy transfer station and one dummy garbage facility.

An MIP mathematical formulation of the VRPSDPTW problem is considered below. For this problem, $x_{ijk}$ is the decision variable. When collection vehicle $k$ continues to serve customer $j$ after serving customer $i$, this variable has a value of 1, and when the collection vehicle does not serve customer $i$ and $j$ continuously, it has a value of 0. $s_{ik}$ is the beginning time of service at customer $C_i$.

Min: $Z = \sigma \sum_{k \in K} \sum_{i \in V_0} c_v x_{0ik} + (1 - \sigma) \sum_{k \in K} \sum_{i \in V_0} c_t (s_{0k} x_{0ik} + T_0);$ \hspace{1cm} (1)

s.t: $\sum_{j \in V_0} x_{ij} = 1 \hspace{0.1cm} \forall i \in V_0;$ \hspace{1cm} (2)

$\sum_{i \in V_0} x_{ihk} = \sum_{j \in V_0} x_{hjk} \hspace{0.1cm} \forall k \in K;$ \hspace{1cm} (3)

$\sum_{i \in V_0} x_{0jk} = \sum_{i \in V_0} x_{0ik} \hspace{0.1cm} \forall k \in K;$ \hspace{1cm} (4)

$\sum_{k \in K} \sum_{i \in V_0} \sum_{j \in V_0} x_{ijk} = N;$ \hspace{1cm} (5)

$\sum_{i \in V_0} (s_{0k} x_{0ik} + T_0) \leq T, \hspace{0.1cm} \forall k \in K;$ \hspace{1cm} (6)

$l_{ij} > 0 \hspace{0.1cm} \forall i \in V_0, \forall j \in V_0;$ \hspace{1cm} (7)

$\sum_{i \in V_0} \sum_{j \in V_0} x_{ijk} - \left[ \sum_{i \in V_0} \sum_{j \in V_0} x_{ijk} / 2 \right] \times 2 = 0, \hspace{0.1cm} \forall k \in K;$ \hspace{1cm} (8)

$s_{ik} + T_i + T_{ij} - M \left( 1 - \sum_{k \in K} x_{ijk} \right) \leq s_{jk}, \hspace{0.1cm} \forall i \in V_0, \forall j \in V_0, M > 0;$ \hspace{1cm} (9)

$a_i \leq s_{ik} \leq b_i \hspace{0.1cm} \forall i \in V_0, \forall k \in K;$ \hspace{1cm} (10)

$s_{ik} + T_i \leq b_i \hspace{0.1cm} \forall i \in V_0, \forall k \in K;$ \hspace{1cm} (11)

$x_{ijk} \in \{0, 1\} \hspace{0.1cm} \forall i \in V_0, \forall j \in V_0, \forall k \in K;$ \hspace{1cm} (12)

The objective function (1) in this paper seeks to minimize the total cost. It is noted that $c_v$ is the dispatching cost of the collection vehicle, $c_t$ is the unit cost of travel cost, and $\sigma$ is a parameter that trades-off vehicle cost and travel cost. Hence, $\sigma \sum_{k \in K} \sum_{i \in V_0} c_v x_{0ik}$ denotes the dispatching cost of collection vehicles. It can be found that the total travel distance (TD) of collection vehicles in CVRPGF is constant when the yard is closed to the transfer station.
**TABLE 2.** The description of the symbols.

| Variables | Definition | Explanation |
|-----------|------------|-------------|
| $T$       | Working time | Equal to the capacity of container; |
| $v$       | Speed of vehicle | $K=\{1,2,3,\ldots,M\}$; |
| $Q$       | Capacity of vehicle | with the number of 0; |
| $K$       | Set for vehicle | $m=1,2,3,\ldots,n$, $m$ is the number of garbage facilities in the real world; |
| $O$       | Yard | |
| $A_m$     | Garbage facilities in the real world | |
| $N_m$     | Service time for $A_m$ | |
| $A_{mc}$  | No. $c$ dummy garbage facilities of $A_m$ | $c=1,2,\ldots,N_m$; |
| $N_i$     | Number of dummy garbage facilities points | $N_i=N_1+N_2+\ldots+N_n$; |
| $N_j$     | Number of dummy transfer station points | $N_j=N_i$; |
| $N$       | Number of dummy customers | $N=N_i+N_1$; |
| $C_i$     | Collection point (Customer) | $i=0,1,2,\ldots,N$; |
| $V_i$     | Set for dummy garbage facilities points | $V_i=\{V_1,V_2,\ldots,V_l\} = \{A_{11},A_{12},\ldots,A_{nm},A_{21},A_{22},\ldots,A_{nm},\ldots,A_{n1},A_{n2},\ldots,A_{nm}\}$; |
| $V_j$     | Set for dummy transfer stations | $V_j=\{V_{j1},V_{j2},\ldots,V_{jK}\} = N_i+N_{j1}+N_{j2}+\ldots+N_{jn}+N_n$; |
| $V_0$     | Set for points | $V_0=\{O\cup V_i\cup V_j\}$; |
| $V$       | Set for customer | $V=\{V_i\cup V_j\}$; |
| $D_i$     | Delivery amount of point $C_i$ | If $C_i\in V_i$, then $D_i=0$; if $V_i\in V_j$, then $D_i=Q$; |
| $P_i$     | Pick up amount of point $C_i$ | If $C_i\in V_i$, then $P_i=Q$; if $V_i\in V_j$, then $P_i=0$; |
| $[a_i,b_i]$ | Time window of the corresponding garbage facilities in real-world for dummy customers | $[a_i,b_i]=[a_{mc},b_{mc}]=[T/N_m\times(c-1),T]$; |
| $T_i$     | Service time for point $C_i$ | $i=0,1,2,\ldots,N$; |
| $I_{ij}$  | Distance between $C_i$ and $C_j$ | If $C_i,C_j\in V_i$ or $C_i,C_j\in V_j$, then $I_{ij}=0$, else $I_{ij}$ is the actual distance between $i$ and $j$; |
| $T_{ij}$  | Traveling time between $C_i$ and $C_j$ | $T_{ij}=I_{ij}/v$, $i,j\in V_0$; |

Hence, $(1-\sigma)\sum_{k\in K} x_{0k}\times x_{0kl} + T_0$ denotes the TT cost. Constraint (2) ensures that each point can be collected from by only one vehicle; constraint (3) restricts collection vehicle $k$ leaving after entering customer $h$; constraint (4) indicates that each vehicle departs from the yard and then returns to the yard; constraint (5) indicates that all customers’ needs must be met; constraint (6) ensures that the travel time of each collection vehicle is less than the total working time; constraint (7) ensures the first dummy garbage facility is a dummy garbage facility in each route, and then the collection vehicle travels between the dummy transfer station and dummy garbage facility; constraint (8) ensures the last served is a transfer station; constraints (9-11) are the time window, which ensures the feasibility of the vehicle routing; and constraint (12) is the decision variable.

**IV. PAR-SAA AND COMPUTATIONAL EXPERIMENTS**

The Par-SAA is a parallel construction procedure for the SAA route improvement approach in which an initial solution is the starting point for seeking improved solutions.

In this Par-SAA algorithm, we follow the procedure of the p-SA algorithm from [37]. However, we extend the p-SA by two ways. First, the optimal set $S$ of solutions is added to the SAA process; then, a route deletion stage is added to the local search to quickly reduce the NV.
A. SAA OPTIMAL SET S

The SAA is a local search metaheuristic. It accepts the suboptimal solution of the neighborhood in a certain probability, which is capable of escaping from local optima. To find the global optimum, the SAA needs a large initial temperature and slow cooling schedule, which results in a slow search process.

Therefore, this study adds an optimal set S to the SAA process, as shown in [20], which can save the first x(x > 0) optimal solutions and save the search time. The pseudocode of the modified SAA is shown in Table 3.

Table 3. Pseudocode of sequential SAA.

| No. | SAA with optimal solution set |
|-----|------------------------------|
| 1   | $s := s_0, T_0 := \gamma \times \text{cost}(s_0), \ S := \{s_0\}$; |
| 2   | $k = 0, \omega := 0$ |
| 3   | WHILE ($\omega < \omega_{\text{max}}$) DO |
| 4   | $l := 0, m := 0, \text{updateOptimalSolution} = \text{false}$ |
| 5   | WHILE ($l < L$) DO |
| 6   | BEGIN |
| 7   | $s_k = \text{local search}(s_k)$ |
| 8   | IF cost($s_k$) < cost($s_l$), THEN |
| 9   | ($s_i := s_k, \omega := 0, m := 0, \text{updateOptimalSolution} = \text{true}$ |
| 10  | IF number($S$) < $x$, THEN |
| 11  | $S := \{S \cup s_k\}$; ELSE |
| 12  | IF $s_{\text{min}} = \min(S) \& \& \text{cost}(s_k) < \text{cost}(s_{\text{min}})$ THEN |
| 13  | $s_{\text{min}} = \max(S)$, remove $s_{\text{min}}$ from $S, S := \{S \cup s_k\}$ |
| 14  | END IF) ELSE |
| 15  | $m := m + 1, s_k = s_k$ with the probability $p = \exp((\text{cost}(s_k) - \text{cost}(s_j))/T_k)$ |
| 16  | END IF |
| 17  | (select $s_k \in S$, let $s_l := s_k$) END IF |
| 18  | $l := l + 1$ |
| 19  | END WHILE |
| 20  | $k = k + 1, T_{k+1} = \beta \times T_k, \beta < 1$, |
| 21  | IF ! updateOptimalSolution |
| 22  | $\omega := \omega + 1$ |
| 23  | END WHILE |
| 24  | END | out put the solution to the VRPSDPTW |

B. PARALLELIZATION OF SAA

It is guaranteed that high-quality solutions in sequential SAA can be achieved if there is a sufficiently high initial temperature, slow annealing speed, large number of iterations, and holding time at the same temperature [38]. To speed up the search and improve the robustness and the quality of the best solutions, parallelization is necessary [39], [40].

The SAA can be parallelized in many ways, including move acceleration, parallel moves, multiple Markov chains (MMC), and speculative computation, of which the MMC

Table 4. Configuration of parameters.

| $W$ | $\beta$ | $\Delta$ | $L$ | $\omega_{\text{max}}$ | $N$ | $n$ | $m$ |
|-----|---------|----------|-----|----------------------|-----|-----|-----|
| Value | 66 | 0.96 | 12 | $n^2$ | 40 | 0.3 | $L$ | 0.1 | $L$ | 4 |

with min and max costs in set $S$, respectively; $N$ is a constant; and $S$ is the optimal solution set that can save $x$ solutions.
method is the most common way of parallelization. We follow [37] to incorporate a master-slave structure, the framework of which is shown in FIGURE 3.

The master-slave thread generates a high-quality initial solution $x_0$. The $w$ slave threads receive the $x_0$, run a local search, and then exchange the best solutions with a periodic $l$. The slave threads find a high-quality solution $x_F$, which will be received by the master thread. In addition, the master thread then makes $x_F$, which is a new initial solution for the next iteration process. These steps will continue until the cease requirement is reached.

C. LOCAL SEARCH WITH ROUTE DELETION STAGE

The local search approach developed in this study is based on the research of [37], and the primary objective of the VRP is to minimize the NV. Therefore, the route deletion stage is added to the local search process. In contrast to the original p-SA algorithm, which selects each local search method with the same probability, we split the local search process into two stages, the revised route deletion stage and the classical local search stage, which is based on the research of [41]. FIGURE 4 shows the framework of the local search with the route deletion stage.

When the customer number of path $SR$ with the least number of customers is less than a fixed value $m$ or the failure time of route deletion is less than a fixed value $n$, the route deletion stage is started. All customers on the shortest route $SR$ are inserted into other routes in turn with the shift insertion method. If all the insertions are successful, the original shortest route $SR$ will be deleted from this solution, and the second stage of the general local search will start based on the new solution. If not all the customers in $SR$ are successfully inserted into other routes, the operation of the route deletion stage is repeated until the failure times of route deletions are greater than $n$; then, the original solution moves on to the second stage.

In the stage of the general local search, there are four types of improvement methods for the neighboring solutions: 2-opt*, $\lambda$-interchange, Or-opt and swap/shift. The selection probability of the methods is $1/4$.

D. COMPUTATIONAL EXPERIMENTS

In this subsection, we test our Par-SAA, using the well-known international benchmark data sets in [20] with 65 instances. Considering the characteristics of the CVRPGF, in which the collection customers are decentralized and fewer customers are disposed along one route, this paper tests the effectiveness of our algorithm by testing 12 instances in the R1 group with 100 customer sizes. It is noted that Wang et al. [37] and Wang and Chen [20] have a hierarchical objective of minimizing NV (primary objective) and TD (secondary objective). To maintain consistency, we modified objective function (1) and minimize the same objective as [37] and [20]. The Par-SAA algorithm works in a JDK7 environment, and all the computer results were executed on a 2.0 GHz desktop Intel Xeon CPU E5-2650 (2 processors) with 16 GB RAM.

Wang et al. tuned the algorithm parameters in order to ensure convergence and speed, and the parameters

| Problem/Customers | BKS NV | TD | p-SA NV | TD | ATD% | Par-SAA NV | ATD% |
|-------------------|-------|----|--------|----|------|------------|------|
| rdp101/100        | 19    | 1653.53 | 19    | 1659.76 | 0.38 | 19    | 1659.76 | 0.38 |
| rdp102/100        | 17    | 1488.04 | 17    | 1491.75 | 0.25 | 17    | 1490.25 | 0.15 |
| rdp103/100        | 14    | 1216.16 | 14    | 1226.77 | 0.87 | 13    | 1307.37 | 7.5  |
| rdp104/100        | 10    | 1000.65 | 10    | 1000.65 |     | 10    | 1000.65 |     |
| rdp105/100        | 14    | 1399.81 | 14    | 1399.81 |     | 14    | 1399.81 |     |
| rdp106/100        | 12    | 1275.69 | 12    | 1275.69 |     | 12    | 1271.95 | -0.29 |
| rdp107/100        | 11    | 1082.92 | 11    | 1082.92 |     | 10    | 1172.6 | 8.28 |
| rdp108/100        | 10    | 962.48  | 10    | 962.48  |     | 9     | 1003.78 | 4.29 |
| rdp109/100        | 12    | 1160    | 12    | 1181.92 | 1.89 | 11    | 1177.75 | 1.53 |
| rdp110/100        | 11    | 1106.52 | 11    | 1106.52 |     | 11    | 1106.52 |     |
| rdp111/100        | 11    | 1065.27 | 11    | 1073.62 | 0.78 | 11    | 1073.62 | 0.78 |
| rdp112/100        | 10    | 966.06  | 10    | 966.06  |     | 10    | 966.06  |     |

TABLE 5. Results comparison.
TABLE 6. Basic information of garbage facilities.

| No. | Garbage facilities | Distance (km) | Collection times | Service time (min) |
|-----|-------------------|---------------|------------------|-------------------|
| 0   | Transfer station  | 0             | 0                | 20                |
| 1   | Dongli            | 10            | 2                | 18                |
| 2   | Chama             | 8.5           | 3                | 20                |
| 3   | Deyuan            | 7.5           | 3                | 18                |
| 4   | Wuyuan            | 8.5           | 1                | 18                |
| 5   | Meishijie         | 12            | 9                | 15                |
| 6   | Jiaozhi           | 9             | 3                | 12                |
| 7   | Ximen             | 8.5           | 4                | 20                |
| 8   | Jianxue           | 8.5           | 1                | 22                |
| 9   | Xianmengtan       | 9             | 1                | 20                |
| 10  | Houbaolou         | 9.5           | 5                | 18                |
| 11  | Lixuehu           | 11            | 4                | 16                |
| 12  | Nanwei            | 9.5           | 5                | 21                |
| 13  | Yaowu             | 9.5           | 1                | 15                |
| 14  | Shoupakou         | 10            | 7                | 16                |
| 15  | Daguanzi          | 6.5           | 6                | 18                |
| 16  | Xiaohongmao       | 8.5           | 4                | 16                |
| 17  | Luomashi          | 9.5           | 4                | 15                |
| 18  | Mialiandao        | 9.5           | 5                | 18                |
| 19  | Baicaowang        | 7.5           | 5                | 19                |
| 20  | Sanyi             | 9.5           | 1                | 20                |
| 21  | Honglian          | 9             | 5                | 16                |
| 22  | Beiweilu          | 9.5           | 3                | 18                |
| 23  | Ganzhu            | 9             | 3                | 22                |
| 24  | Chunfeng          | 8.5           | 3                | 18                |
| 25  | Huaihaibao        | 8             | 4                | 16                |
| 26  | Jingtie           | 8             | 3                | 21                |
| 27  | Yijian            | 7             | 3                | 20                |
| 28  | Kanglei           | 10            | 1                | 20                |
| 29  | Yinchaochang      | 6             | 1                | 20                |

$W$, $\beta$, $\Delta$, $L$, $\omega_{\text{max}}$ in this study are the same as in [37]. $W$ is the number of threads, $\beta$ is the cooling ratio, denotes the probability of accepting worse solutions, $L$ represents the maximum number of iterations, and $\omega_{\text{max}}$ is the maximum number of iterations for not improving the current solution. In addition, $N$, $n$ and $m$ are the new control parameters; all parameter values are listed in TABLE 4.

The effectiveness of Par-SAA is evaluated through comparison with others in TABLE 5, in which BKS denotes the best-known solution in the published literature and p-SA represents the optimal solution in the study of [37].

When the results of Par-SAA are compared with those of BKS and p-SA in TABLE 5, the superiority of our algorithm is revealed and can be summarized as follows. Four instances (33.3%) have reduced the NV compared with BKS. Of the remaining 8 instances with the same NV, there has one instance with better TD.

From a managerial perspective, it is worth using the proposed Par-SAA algorithm to acquire a better route plan, because less NV means a net decrease in overall fleet operating costs, due to the 100 percent elimination of the fixed cost and the slightly raise of the operating costs for the remaining fleet vehicles.

V. AN APPLICATION IN BEIJING
A. CASE DESCRIPTION
In the following, we study MSW management in Beijing, in which the garbage collection reached 9.25 million tons with an average of 25.3 million tons per day in 2017. Owing to the massive amount of solid garbage wastes, MSW collection with a garbage facility is very common in Beijing. Therefore, we apply the proposed model and the Par-SAA to the real CVRPGF case in Xuanwu District of Beijing. FIGURE 5 displays pictures of a garbage facility, collection vehicle and garbage container with dimensions of $3.5 \times 2.0 \times 1.4$ m.

The information, including the name and collection times of all garbage facilities and the origin of the collection route for each vehicle, was collected primarily by interviewing the collection manager of the Xuanwu District. We found that there are 29 garbage facilities in the district, and all the solid waste containers collected from them are transported to the
TABLE 7. Vehicle routing of the original plan.

| Vehicle Number | Sequence of garbage facilities | Traveling time |
|----------------|--------------------------------|----------------|
| 1              | 1-1-2-2-2                     | 378            |
| 2              | 3-3-3-4-5                     | 349            |
| 3              | 6-7-7-8-9                     | 368            |
| 4              | 10-12-12-12-12               | 370            |
| 5              | 11-12-12-12-12               | 384            |
| 6              | 5-5-26-15-16                 | 333            |
| 7              | 17-23-17-17-17               | 365            |
| 8              | 16-16-15-15-15               | 373            |
| 9              | 14-14-24-24-25               | 364            |
| 10             | 25-19-19-20-20               | 346            |
| 11             | 21-21-21-5-21                | 344            |
| 12             | 22-22-22-23-14               | 387            |
| 13             | 19-5-19-5-19                 | 348            |
| 14             | 15-15-18-18-28               | 336            |
| 15             | 26-26-27-27-27               | 340            |
| 16             | 18-23-6-6-7                  | 344            |
| 17             | 14-15-25-25-12               | 330            |
| 18             | 5-24-21-14-7                 | 367            |
| 19             | 14-18-14-5-18                | 375            |
| **Total Traveling Time** |                          | **7171**       |

TABLE 8. Vehicle routing of the PSO.

| Vehicle Number | Sequence of garbage facilities | Traveling time |
|----------------|--------------------------------|----------------|
| 1              | 1-21-12-19-7-2                | 451            |
| 2              | 14-28-18-5-18-15              | 472            |
| 3              | 8-13-18-15-17-15              | 381            |
| 4              | 9-29-5-14-24-18               | 363            |
| 5              | 27-2-17-7-1-18                | 340            |
| 6              | 17-4-3-12-19-15               | 368            |
| 7              | 6-25-24-7-10-19               | 471            |
| 8              | 19-11-2-10-23-5               | 446            |
| 9              | 5-26-14-15-16-12              | 336            |
| 10             | 3-10-11-25-12                 | 450            |
| 11             | 16-19-13-16-23-25             | 458            |
| 12             | 11-20-24-7-16-26              | 394            |
| 13             | 23-24-13-21-3-14              | 440            |
| 14             | 22-10-22-14-17-22             | 461            |
| 15             | 18-5-27-5-21-21               | 417            |
| 16             | 10-14-26-14-15                | 427            |
| 17             | 12-5-6-25-6                   | 350            |
| **Total Traveling Time** |                          | **7025**       |

TABLE 9. Vehicle routing of the Par-SAA.

| Vehicle Number | Sequence of garbage facilities | Traveling time |
|----------------|--------------------------------|----------------|
| 1              | 21-3-18-25-7-15-18-27          | 435            |
| 2              | 28-1-5-17-15-5                 | 452            |
| 3              | 10-14-7-26-15-19-16-12        | 447            |
| 4              | 18-27-14-14-12-15-15-15       | 451            |
| 5              | 14-11-26-3-11-6-17-18         | 443            |
| 6              | 2-21-14-5                     | 459            |
| 7              | 8-21-15-16-24-7-23            | 465            |
| 8              | 5-19-5-11-17-18-16-3          | 447            |
| 9              | 20-29-12-22-14-5-21           | 456            |
| 10             | 4-10-12-2-10-12               | 447            |
| 11             | 12-6-11-21-5                  | 464            |
| 12             | 15-19-17-27-22-19-10          | 441            |
| 13             | 24-23-24-5-9-15-22-7          | 454            |
| 14             | 5-16-25-19-6-26-10-2          | 453            |
| **Total Traveling Time** |                          | **6314**       |

Majialou transfer station. FIGURE 6 shows the locations of the 29 garbage facilities in the Xuanwu District and the Majialou transfer station.

Detailed information about the garbage facility is tabulated in TABLE 6, including the names, the distance from each garbage facility to the transfer station, the number of collection times during the working time and the service time of each garbage facility. The total working time of collection for each vehicle is eight hours.

Due to the availability of information, the original collection vehicle routing plan of the Xuanwu District is shown in TABLE 7, which is made by the empirical approach based on the operational characteristics of the facility.

By conducting the interview and calculation, the value of the following parameters is obtained. The driving speed of the collection vehicle is 30 km/h, the cost of one collection vehicle is 3000 CNY and the cost of travel time is 50 CNY/min.

B. RESULTS COMPARISON

In this section, we compare the results obtained by different solutions to show the advantages of the introduction of dummy customers.

Our research group studied this CVRPGF case by particle swarm optimization (PSO) [42]. In this study, we introduce the dummy customers and transfer the CVRPGF problem to a VRPSPDTW problem. To show the advantages of this solution, we compare the results obtained by PSO and Par-SAA, which is shown in TABLE 8 and TABLE 9.

FIGURE 7 displays the results of the original plan, PSO and Par-SAA. The original plan denotes the original schedule made by works’ experiences. It is obvious that the proposed Par-SAA method obtains the best results, with the NV reduced by 30% and the total travel time reduced by 12%, compared with the original plan.

Through the comparison of three solutions, it can be found that by optimizing the vehicle routing, the working efficiency of each vehicle is improved. More importantly, the NV is reduced, which can lead to reductions in labor cost, vehicle maintenance costs and management costs, thus improving the
overall operational strength of the enterprise. Therefore, this study has an important practical value to MSW management in Beijing, which also provides an implication for MSW management and collection in the developing economies [43].

VI. CONCLUSION
MSW management has environmental and economic impacts, which are complicated due to different collection forms. Developed economies have extensively studied the CVRP to reduce collection costs. Moreover, they have built mature collection systems for recycling resources. However, the studies on this issue are still in their infancy in the developing economies. Therefore, it is an urgent concern to find solutions for the CVRP in developing countries.

This study provides a theoretical model and algorithm for the MSW collection vehicle routing problem, particularly the full-load CVRP with multiple trips and multiple demands. The model transforms the original CVRPGF problem into the VRPSPDTW through introducing two classes of dummy customers, which provides a new idea for solving the CVRPGF problem as well as the CVRP with multiple disposals.

When applied to the Wang and Chen’s benchmark instances, the computational results prove the superiority of the proposed Par-SAA algorithm, in which the NV in four instances is reduced by one. Finally, when the model and algorithm are applied to a real CVRPGF problem in the Xuanwu District of Beijing, the NV needed is reduced by 30%, and the total travel time is decreased by 12%, compared with the original plan.

The conventional CVRP always assumes the travel speed of the collection vehicle is constant. As we know, the travel speed cannot be constant all the time in municipal areas. To fill this limitation, the traffic congestion will be considered in our future study, which will be a time dependent CVRP problem. Moreover, we will study the time dependent CVRP problem with fuel consumption in Beijing.

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