Model tests and theoretical analysis of the vertical vibration of a single pile in saturated sandy soil

X Y Li¹, *, Q Li¹, L Hu¹, L Q Huang¹

¹Department of Civil Engineering, Zhejiang Ocean University, Zhoushan 316022, China

*Corresponding author’s e-mail address: lixinyi9510@163.com

Abstract. The study of vibration dynamic response of pile foundation has critical applications for seismic structure design. In this paper, firstly, the vibration characteristics of single pile with different length to diameter ratios under different vibration frequency loads were studied by the foundation pile model test method. Moreover, the amplitude-frequency curves of single pile vibration are plotted by using Fourier transform. Then the theoretical model of the pile-soil-cap system under sinusoidal periodic vibration load is established by using saturated porous medium theory, and the amplitude and frequency curves are obtained. Numerical results show that the theoretical solution can be in good agreement with the experimental results, which verifies the validation of the theoretical model. Furthermore, the resonant frequency is proven to be negatively related to the length to diameter ratio of the pile under the same pile diameter.

1. Introduction

Pile foundation engineering is widely used, especially in some soft ground area, the existence of pile foundation can effectively ensure the stability of the structure. Pile foundations not only have high bearing capacity, high strength and good durability. For the theoretical study of pile-soil interaction, the three-dimensional continuum model is widely used in theoretical studies [1-4]. In pile foundation tests, Manna and Baidya [5] performed vertical vibration tests under different eccentricity conditions to determine the frequency amplitude response of the piles. Both theoretical and experimental studies have shown [6-8] that the dynamic response of a pile is very sensitive to the properties of the soil in the vicinity of the pile. Zhu et al. [9] studied the dynamic response of dry loose sand foundations, medium dense sand foundations and saturated medium dense sand foundations under different frequencies of sinusoidal vibratory loads.

Under the action of dynamic load, vibrations of different frequencies will be generated between the pile and soil, and the experimental research results provide strong support for pile foundation vibration analysis. In this paper, similar constants are formulated according to π theorem, and model tests are used to study the vertical vibration characteristics of single piles in saturated sandy soils, and validated with theoretical analysis to provide the necessary basis for the analysis of single pile characteristics under vertical dynamic loads.

2. Single pile dynamic model test

2.1. Model material

The sand used in the model test is taken from the coast of Zhoushan. The model box (length*width*height =2000*1500*3000mm) is constructed by steel, as shown in Figure 1(a). The test
system consists of a vibration loading section and a signal acquisition section. The vibration loading section consists of an electrodynamic shaker, a DF1405 digital synthesized function signal generator and a HEAS-50 power amplifier. The signal acquisition part mainly consists of acceleration sensors, dynamic signal test analyser and server.

![image](a)  
(a)  

![image](b)  
(b)  

![image](c)  
(c)  

Figure 1. Schematic diagram of the experimental system for the model experimental study.

The model pile was made of PVC pipe, as shown in Figure 1(b). The elastic modulus of PVC pipe pile is 3000MPa, the density is 1400kg/m³, and the longitudinal wave propagation wave speed is about 1464m/s. In the model design, the PVC pile diameter is 32mm and the pile lengths are 1m, 1.25m, 1.75m and 2m respectively.

2.2. Test procedure
(1) A back filter layer is laid on the bottom of the test chamber.
(2) The model piles were buried in the model box as in the literature [10].
(3) The shaker is installed in alignment with the model pile as shown in Figure 1(c).
(4) A sinusoidal cyclic load was applied to the top of four groups of PVC pipe piles with different L/D ratios to test single piles with caps in the form of a frequency sweep.
(5) Monitor specific excitation frequencies and record data to obtain point frequency curves of PVC pipe piles when vibrating in sandy soils with water saturation.

2.3. Similarity ratios of model tests
The relationship between the physical parameters of a pile-soil-cap structure under dynamic load is expressed in general form as

\[ f \left( \rho_s, L, D, E_p, \rho_p, m, g, a, F, \omega_F \right) = 0 \]  

(1)

where: soil density \( (\rho_s) \), pile length \( (L) \), pile diameter \( (D) \), pile modulus of elasticity \( (E_p) \), pile density \( (\rho_p) \), mass of the cap \( (m) \), the acceleration of gravity \( (g) \), the acceleration of the dynamic response \( (a) \), the magnitude of the excitation force \( (F) \), and the frequency of the excitation load \( (\omega_F) \);

The similarity ratio is calculated by derivation according to Bockingham \( \pi \) theorem. All these 7 similarity ratios \( \left( \frac{C_{\rho_s}}{C_{\rho_p}}, \frac{C_D}{C_p}, \frac{C_m}{C_p}, \frac{C_L}{C_p}, \frac{C_{\rho_p}}{C_p}, \frac{C_a}{C_p}, \frac{C_F}{C_p}, \frac{C_{\omega_F}}{C_p}, \frac{C_L C_{\rho_p}}{C_p}, \frac{C_{\rho_p}}{C_p}, \frac{C_L}{C_p} \right) \) should theoretically be equal to 1.

3. Experimental results and discussion
This section discusses the analysis of vibration characteristics of monopile foundation with cap under high frequency vertical vibration load, and analyses the change of resonant frequency regularity under different L/D ratio.
3.1. Analysis of vertical vibration sweep and point frequency results of single pile foundation with cap

![Graphs showing vertical vibration sweep and point frequency results of single pile foundation with cap for different depths and frequencies.](image)

Figure 2. 2m, 1.75m, 1.25m and 1m sweep and spot frequency curves.
Figure 2 represents the sweep frequency curve and point frequency curve under different pile lengths. In the sweep frequency curve, it can be found that the absolute value of the acceleration magnitude increases from small to large, and then gradually decreases after reaching the resonance peak, basically showing a "spindle shape". For the phenomenon of irregular local variations on the sweep frequency curve, it may be caused by the noise of the environment or the instability of the pile-soil interface during the steady-state excitation.

The images of the point frequency curves at each point frequency vibration basically present harmonic load curves, and the acceleration amplitude is basically the same, indicating that the vibration load is accurately applied to the single pile foundation with cap. In the resonant frequency to higher frequency vibration interval, the point frequency curve then exhibits a less stable amplitude, probably in the case of high frequency excitation, the nature of the pile-soil interface is easily changed.

4. Analysis and discussion of test results
In this section, based on the experimental results, time-frequency conversion and filtering are performed to obtain the amplitude-frequency curves, and a theoretical model of the vibration of single pile with cap in saturated soil is established. The vibration characteristics of single pile with cap under different L/D ratios are discussed by comparing the theoretical and experimental analyses.

4.1. Theoretical model
The unified vector expression of porous media theory described in the form of displacement vector, which for saturated porous media, the control equation is:

\[ \ddot{\vec{u}} - \vec{m} \ddot{\vec{u}} = -\rho \frac{\partial^2 \vec{u}}{\partial t^2} + \frac{k}{\mu} \frac{\partial \vec{u}}{\partial t} = 0 \]  

(2)

The pile is simplified as a one-dimensional rod with the vibration control equation.

\[ E_b \pi a^2 \frac{d^2 \tilde{w}_b}{dx^2} - f(z) = \rho_b \pi a^2 \frac{d^2 \tilde{w}_b}{dt^2} \]  

(3)

where: \( f(z) = -2\pi a \tau_{zr} \) is the pile circumferential frictional resistance, \( \tau_{zr} \) represents the solid phase in the pile side shear stress.

Vibration control equation for the cap

\[ M \frac{d^2 \tilde{w}(z,t)}{dt^2} + c_{ms} \frac{d \tilde{w}(z,t)}{dt} + k_{ms} \tilde{w}(z,t) + p(t) \pi a^2 = F(t) \]  

(4)

where: \( w \) is the vibration displacement of the cap, \( M \) is the mass of the cap, \( c_{ms} \) and \( k_{ms} \) are the damping and stiffness of the cap respectively, \( p(t) \) is the reaction force at the top of the pile, and \( F(t) \) is the arbitrary excitation force at the top of the cap.

By the method of literature [11], the pile displacement is obtained by substituting the boundary conditions. The cap vibration solutions are obtained through the complex impedance transfer at the pile top as follows.

\[ \tilde{W}(z,s) = \frac{F(s)}{(\rho_m \delta^2 + c_{ms} \delta + k_{ms}) + Z_u(0) A^*} \]  

(5)

where: \( \rho_m^* = \frac{\rho_m}{\rho}, c_{ms}^* = \frac{c_{ms}}{\sqrt{\rho_m A_m}}, k_{ms}^* = \frac{k_{ms} A_m}{\mu A_m}, A^* = \frac{\pi a^2}{A_m}, F(s) = \frac{f(s)}{\mu A_m} \).

Definition of \( Z_u(s) \).

\[ Z_u(z) = A_1 \left[ e^{\kappa z} + \sum_{n=1}^{\infty} \frac{-2\eta_1 \cosh(\kappa n z)}{E_b^* h_n^* (e^{\kappa z} - \kappa^2)} \right] + B_1 \left[ e^{-\kappa z} + \sum_{n=1}^{\infty} \frac{-2\eta_1 \cosh(\kappa n z)}{E_b^* h_n^* (e^{-\kappa z} - \kappa^2)} \right] \]  

(6)

4.2. Vibration analysis of single pile foundation with cap
When a periodic sinusoidal load is applied on top of the cap, the amplitude-frequency curve can be obtained by preparing a numerical calculation program with MATLAB based on the theoretical calculation. The theoretical and experimental amplitude-frequency curves were compared, and the calculated parameters were: cap mass \( M = 0.258 \text{Kg} \); L/D ratios were taken as 31.25, 39, 54.7, 62.5. As shown in Figure 3.
Figure 3. Comparison of theoretical calculation and measured amplitude and frequency curves of floating pile vibration model.

From Figure 3, it can be seen that the magnitudes of resonant frequencies for four different aspect ratios calculated by the floating pile vibration model basically match with the experimental data, indicating that it is reasonable to adopt the floating pile vibration model proposed in this paper.

Figure 4. Resonant frequencies at different L/D ratios.

From the comparison results in Figure 4, it can be seen that the resonant frequency is basically negatively correlated with the L/D ratio of the pile for the same pile diameter. That is to say, as the pile L/D ratio increases, the resonant frequency of the pile basically tends to decrease. In the test, it was
found that the resonant frequency would not be as accurate as in the theory, but it basically matched. The experimental results were affected by factors such as the verticality of the shaker, the balance position of the assembly between the shaker and the cap, and the tightness of the device.

5. Conclusions

• The dynamic model test of the single pile by establishing similar ratios is feasible;
  • The frequency sweep of the single pile foundation with cap can better reflect the monopile vibration law, the amplitude of acceleration at resonance frequency is the maximum. However, the amplitude of acceleration presented in the certain frequency far from resonance frequency is obviously reduced;
  • In the case of the same pile diameter, the resonant frequency is negatively related to the L/D ratio of the pile;
  • The comparison with the pile dynamic test shows that the theoretical results of floating pile presented in this paper agree well with the experimental results, which illustrates the reasonablness of the model.

Acknowledgements
This work was supported by the Fundamental Research Funds for the Zhejiang provincial university under Grant No. 2019J00019 and Zhejiang Provincial Natural Science Foundation of China under Grant No. LY15E090008.

References
[1] Nogami T and Novak M 1977 Resistance of soil to a horizontally vibrating pile J. Int J Earthq Engng Struct Dynam.3(3):249-261
[2] Hesham M and El Naggar M H 2000 Vertical and torsional soil reactions for radially inhomogeneous soil layer J. Structural Engineering Mechanics. 10(4):299-312
[3] Li Q, Wang K H and Xie K H 2004 Vertical vibration of an end bearing pile emeddedin saturated soil J.Acta Mechanica Sinica.36(4):435-442
[4] Li Q, Wang K H and Xie K H 2004 Study on resistance factor of saturated soil caused by longitudinal vibration of pile J. Chinese Journal of Geotechnical Engineering.26(5):679-683
[5] Manna B and Baidya D K 2009 Vertical Vibration of Full-Scale Pile—Analytical and Experimental Study J. Journal of Geotechnical & Geoenvironmental Engineering. 135(10): 1452-1461
[6] Han Y and Novak M 1988 Dynamic behaviour of single piles under strong harmonic excitation J. Canadian Geotechnical Journal 25(3):523-534
[7] Han Y 1989 Coupled Vibration of Embedded Foundation J. Journal of Geotechnical Engineering
[8] El-Marsafawi H, Han Y C and Novak M 1992 Dynamic Experiments on Two Pile Groups J. Journal of Geotechnical Engineering.118(4):576-592
[9] Zhu W X ,Gu L L,Mei S,Nagasaki K,Chino N and Zhang F 2020 1g model tests of piled-raft foundation subjected to high-frequency vertical vibration loads -ScienceDirect J. Soil Dynamics and Earthquake Engineering
[10] Guo Y C,Liu H and Huang M S 2015 Model tests on nonlinear vibration characteristics of single piles with different length-diameter ratios under lateral loading in sand J. Chinese Journal of Geotechnical Engineering.37(S2): 57-60
[11] Wang G M, Li Q and Wang K H 2006 Simplified model for vertical vibration of pile single-layer saturated soil and its analyticalsolution J. Chinese Journal of Rock Mechanics and Engineering. S2:4233-4240