The dynamical solar atmosphere

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Abstract. High resolution space and ground-based telescopes made it clear that one of the most fundamental characteristics of solar atmosphere is that it is dynamic, showing stationary and non-stationary motion on all time and space scales. Dynamical changes in the solar atmosphere are rather difficult to interpret given the presence of the magnetic field and the high-degree of ionization of the plasma. After all, the magnetic field is the quantity which controls the dynamics, stability, and thermal state of the plasma. The present contribution aims to review some fundamental knowledge on plasma dynamics in the solar atmosphere with special emphasis on coronal seismology, i.e. the science which is using the observable dynamical changes in the solar corona to infer quantities that cannot be measured (structure and magnitude of the coronal magnetic field, scale-heights, density filling factors, etc.).

1. Introduction

High resolution observations made in the last decade have changed dramatically our perception about the dynamical phenomena occurring in different layers of the solar atmosphere. Waves, which until mid 90s were sporadically observed (mainly in radio frequency), have become now a reality, being observed in all sort of wavelengths. The new achievement also meant that new areas of solar physics can have now an observational background.

The dynamical response of the plasma in the solar atmosphere to rapid changes in the solar interior can be manifested through waves propagating within the solar atmosphere. The dynamics of a large class of waves with wavelengths and periods large compared with the ion Larmor radius (10⁻⁴ m in the solar photosphere and about 1 m for almost all combinations of coronal parameters) and the gyroperiod (10⁻⁷ s in the solar photosphere and of the order of 10⁻³ s in the solar corona), respectively, can be described within the framework of collisional magnetohydrodynamics (MHD). Waves perturb macro-parameters of the plasma, such as density, temperature, bulk velocity and magnetic field. The period range of waves and oscillations observed in the solar atmosphere (from a few seconds to a few hundreds of minutes) is well covered by temporal resolution of presently available ground-based and spaceborne observational tools. However, it is very important to understand that in normal circumstances both spatial and temporal resolution are necessary ingredients of successful detection of the waves, e.g. the pixel size must be much smaller than the wavelength and the temporal cadence time must be shorter than the period. Only in certain exceptional cases, e.g. when the wave passes through a bright object with the geometrical size smaller than the wavelength, the wave can be detected with poorer spatial resolution.
The importance of studying waves and oscillations in the solar atmosphere resides on at least three aspects. First of all, MHD waves are able to transport energy and momentum from the solar deeper regions up to the chromosphere and corona, therefore contributing to the thermalization of the upper solar atmosphere and/or acceleration of the solar wind. Secondly, waves and oscillations are able to transport information about the medium in which they propagate, therefore seismological techniques are used to obtain information about the solar interior, solar atmosphere and the coupling between different regions in the solar atmosphere. Finally, waves and oscillations are important as they are able to trigger some other secondary effects (very important though) in space plasmas, such as magnetic reconnection, instabilities, turbulences, etc.

Given the abundance of results, a full review of waves and oscillations including observation, theory and modelling could be a very challenging and time consuming task, different aspects were summarized previously in great detail (see, e.g. Banerjee et al 2007 for observations and Nakariakov and V erwichte 2005 for theory and modelling).

2. MHD waves in solar plasma – an observational overview

Ordinary sound waves in air owe their existence to the restoring force created by a gas pressure gradient. If we introduce a magnetic field to this system we create two more restoring forces, the magnetic tension and magnetic pressure gradient. If we assume that the plasma is homogeneous (every element of the plasma is identical to any other element) and infinite, the ideal MHD equations predict the existence of three different types of waves called the slow magnetoacoustic, Alfvén and fast magnetoacoustic waves (if gravitational and non-inertial forces are neglected).

The slow magnetoacoustic (or just simply slow) waves are driven by the magnetic and plasma pressure interacting destructively, whereas the fast magnetoacoustic (FMA) waves are produced when the magnetic and gas pressure interact constructively. The Alfvén wave has an intermediate wave speed, and is entirely driven by the magnetic tension. A polar diagram of the phase speeds of the three types of waves is given in Fig 1. It is obvious that slow waves can only travel at very small angles off parallel to the magnetic field lines and fast waves travel in any direction, with a larger speed when they propagate perpendicular to the ambient magnetic field lines. Alfvén waves have a preference to transport energy along field lines and never across them (analogous to vibrations on a guitar string).

The presence of waves and oscillations in the solar atmosphere is strongly connected to the magnetic field; one cannot fully understand the possible spectrum of waves without a detailed knowledge of the magnitude and structure of the magnetic field. Waves are generated, in general, by buoyoncy forces (gravity, magnetic field, pressure gradients, Coriolis forces, etc.) but also due to the convection motion in the sub-surface region and transient and energetic phenomena occurring at different heights in the solar atmosphere. Waves and oscillations can be categorized in many ways, however for our purposes we can distinguish between local and global waves depending whether the wave is connected or not to a certain magnetic structure. Although they may seem separate phenomena, in fact they are very much related in the sense that often global waves are able to generate local waves and oscillations.

The solar atmosphere is a highly structured medium, with large gradients created by the magnetic field. The magnetic field is not dispersed uniformly, instead it tends to accumulate in entities known as flux tubes and this imposes a strict ordering of solar magnetism. The process of emergence of flux tubes is believed to be caused by the massive convective motions below the photosphere.

In the photosphere one of the most obvious magnetic feature is the sunspot typified by a cool dark region, with strong (3kG) magnetic field, often with a $10^4$ km diameter with temperatures of about 4000 K, about 2000 K cooler that sunspots’ surrounding. Sunspots are not homogeneous, but generally exhibit an irregular pattern of bright points, called umbral dots, some 150 km in
Phase velocity parallel $v_{ph,z}/v_A$

Phase velocity perpendicular $v_{ph,x}/v_A$

$\overline{c_s}$ slow mode

Alfven wave

$\overline{v_A}$ fast mode

Figure 1. Polar plot of the phase speeds of the three types of MHD waves. The magnetic field lines lie parallel to the horizontal axis. Here the sound speed ($c_S$) is 70% of the Alfvén speed ($v_A$). The Alfvén speed and sound speed are given by the dashed and dotted line, respectively.

diameter with brightness similar to that of the surrounding 6000 K photosphere and with an average lifetime of 1500 s. Magnetic field in these dots are expected to be somehow less then in their surroundings.

It is well-know that several oscillations are connected to the sunspots, most important being the 180 s (chromospheric oscillation) and the 300 s (photospheric oscillation). Observations of sunspots oscillations have been summarised (see, e.g. Lites et al. 1982; Brynildsen et al. 2000) as comprising vertical oscillations within the umbrae (with periods of 130-190 s), and horizontal penumbral waves running outwards at speeds of 10-20 km s$^{-1}$ and periods of 210-300 s. Apart from sunspots, the rest of the photosphere is far from being a homogeneous medium. It is now generally accepted that the magnetic field over most of the solar surface is concentrated into regions of magnetic flux ranging from small ($\sim 100$ km, 1 kG) flux tubes, through knots (500 km, 1.5 kG) and pores ($\sim 10^2$ km, 2 kG) to sunspots.

It now widely agreed that the magnetic flux concentrations of the photosphere (usually at the supergranules boundaries) appear to be co-spatial with magnetic features higher in the solar atmosphere, the chromosphere. Briefly, the principal features of the chromosphere may be classified as horizontal ($\leq 300$ km above the limb) fibrils which appear to outline flux tubes joining areas of opposite polarity and spicules which are radial jets of gas which shoot upwards with average velocities of 20 km s$^{-1}$ from the chromospheric network along magnetic field lines. Recently several attempts have been made to explain spicules as the effect of p-mode buffeting on photospheric network of magnetic field lines or reconnection of opposite polarity magnetic field lines (see, e.g. De Pontieu et al. 2004, Pasachoff et al. 2009) while the multitude of waves and oscillations in spicules has been reviewed recently by Zaqarashvili and Erdélyi (2009).

X-ray and (E)UV observations by high resolutions space satellites (SOHO, TRACE, HINODE) have changed our view of the dynamical state of the corona. Since the Skylab mission we know that the corona consists of multitude of high emissivity coronal loops with different lengths, temperature, density, etc. at very high temperature (majority have temperature in excess of 1 MK). Since the plasma is controlled by magnetic forces (in corona the plasma-beta is much smaller than 1), coronal loops are tracers of coronal magnetic field lines. Loops may vary in their characteristics over a wide range. The small X-ray bright points which have time scales of the order of 8 hours are thought to be loop structures while coronal loops believed to form large
prominences, some extending 100 Mm above the limb, have an average lifetime of a few days to a few months. Some of the loops are "closed" being anchored by magnetically opposite footpoints in the denser chromosphere and photosphere, extending as high as 300 Mm whereas others are "open" fading high in the corona and coinciding with the source of the high-speed solar wind. Values for magnetic field inside coronal loops were obtained using extrapolation techniques from the photosphere but also with seismological methods revealing that intensities vary between a few G up to 100 G resulting in Alfvén speeds of the order of 1000-2000 km s$^{-1}$. Emission line widths also revealed that coronal loops are denser structures than their environment, the filling factor (density ratio between inside and outside the loop) varying between 5 and 10. These values usually refer to loops in active regions, being the easier to observe, especially those appearing shortly after a solar flare.

Given the complex structure of coronal loops it is expected that these magnetic entities will support a rich variety of waves and oscillations. One of the most studied type of waves is the so-called kink wave which propagates along a magnetic flux tube so that the symmetry axis of the tube is distorted (see, e.g. Aschwanden et al. 1999, Nakariakov et al. 1999, Douglas et al. 2010). It is believed that kink waves and oscillations are the result of the interaction between an external driver (possibly a global wave or nearby sudden energy release) and the coronal loop (Selwa et al. 2006, Ogrodowczyk and Murawski 2007, Hindman and Jain 2008, Ballai et al. 2009). Kink oscillations in coronal loops and their very rapid damping allowed the estimations of magnetic fields, density scale-heights, sub-resolution structure, heating functions, etc. (see, e.g. Nakariakov et al. 1999, Andries et al. 2005; Verth et al. 2008, Douglas et al. 2009). On the other hand, if the oscillations occur such that the symmetry axis of loops is not dislocated we are dealing with sausage modes and they were observed already in coronal loops by, e.g. Aschwanden et al. (2003), Erdélyi and Taroyan (2008). Both of these waves are belonging to the family of magnetoacoustic waves which arise due to the combined effect of forces due to the magnetic tension and pressure gradients.

Finally we should mention that apart from magnetoacoustic waves, magnetic structures can support pure magnetic waves (Alfvén waves) which always propagate along magnetic field lines, are transversal and very little influenced by non-ideal effects. Since Alfvén waves are incompressible they will not show any variation of intensity in EUV wavelengths. The only way to observe Alfvén waves is by Doppler line broadening. Although the propagation speed and characteristics of Alfvén waves are very much similar to the kink waves, the observation of Alfvén waves is very sporadic (e.g. Harrison et al. 2002, Jess et al. 2009), however future observational facilities will provide the required resolution for observing Alfvén waves.

Apart from waves propagating in various magnetic structures, the solar atmosphere permits the propagation of waves which are able to cover a very large extent of the solar surface, i.e. global waves. One of the most known global waves are the so-called $p$-modes propagating in the solar photosphere and can be considered as acoustic modes able to penetrate the solar interior. These oscillations were first discovered by Leighton (1960). $p$-modes are generated by the convection motion inside the Sun where convective cells are continuously buffeting the convectively stable photosphere (Stix 1970). It is believed that the solar photosphere is covered by thousands of $p$-modes at all times. The power spectrum of these standing oscillations shows a maximum peak at about 300 s. Higher frequency $p$-modes are observed in the chromosphere. Their velocity amplitude lies between 0.1 and 1.6 km s$^{-1}$ in the photosphere but it is increasing with height. The horizontal wavelength of these oscillations is observed to be between 50 to 10,000 km, the higher limit being larger than a typical size of a granule. Vertical and horizontal speeds lie in the interval 30 to 100 km s$^{-1}$. The great advantage of $p$-modes is that they can carry information about the structure (density, temperature, chemical composition, etc.) and dynamical state (shear meridional and poloidal flows, stability, etc.) of the solar interior. Helioseismology is the science which studies the properties of $p$-modes for solar interior
The other family of global waves consists of large-extent waves that are generated by sudden and powerful energy releases (flares/CMEs). Impulsively driven global waves have been known since the early 1960s. Although it is still not known how the release of energy and energized particles will transform into waves, today it is widely accepted that these disturbances are similar to the circularly expanding bubble-like shocks after atomic bomb explosion or shock waves which follow the explosion of a supernova. Thanks to the available observational facilities, global waves were observed in a range of wavelengths in different layers of the solar atmosphere. A pressure pulse can generate seismic waves in the solar photosphere propagating with speeds of 200–300 km s\(^{-1}\) (Kosovichev and Zharkova 1998; Donea et al. 2006). Higher up, a flare generates very fast super-
\Alfvenic shock waves known as Moreton waves (Moreton and Ramsey, 1960), best seen in the wings of \(H\alpha\) images, propagating with speeds of 1000–2000 km s\(^{-1}\). In the corona, a flare or CME can generate an EIT wave (Thompson et al. 1999) first seen by the SOHO/EIT instrument or an X-ray wave seen in SXT (Narukage et al. 2002). There is still a vigorous debate how this variety of global waves are connected (if they are, at all). Co-spatial and co-temporal investigations of various global waves have been carried out but without a final widely accepted result being reached.

Unambiguous evidence for large-scale coronal impulses initiated during the early stage of a flare and/or CME has been provided by the Extreme-ultraviolet Imaging Telescope (EIT) observations onboard SOHO and by TRACE/EUV. EIT waves propagate in the quiet Sun with speeds of 250–400 km s\(^{-1}\) at an almost constant altitude. At a later stage in their propagation EIT waves can be considered a freely propagating wavefront which is observed to interact with coronal loops (see, e.g. Wills-Davey and Thompson 1999). Using TRACE/EUV 195 Å observations, Ballai et al. (2005) have shown that EIT waves (seen in this wavelength) are waves with average periods of the order of 400 seconds. Since at this height, the magnetic field can be considered vertical, EIT waves were interpreted as fast MHD waves. This conclusion is further supported by other observations (see, e.g. Long et al. 2008, Patsourakos et al. 2009).

3. MHD waves in solar plasma – a theoretical overview
All waves in which we are interested in the present review can be described within the framework of MHD, which is a set of highly nonlinear equations combining the Navier-Stokes equation with Maxwell system of equations. The general solution of the MHD system of equation is complicated but various realistic assumptions may be made to assist the analysis. First of all, we assume that all waves in which we are interested are linear, so perturbations of physical quantities are much smaller than their equilibrium value. Secondly, since here we are interested in the basic characteristic of waves, we will assume that the plasma is ideal, so free from any non-ideal effects. Further simplifications can be made by considering the location of waves’ appearance. For example, in the magnetically dominated corona the so-called zero-beta (or cold plasma) approximation involving the neglect of sound speed compared with the \Alfven speed, is an acceptable simplification. In addition when wavelengths are smaller than the density scale-height, gravitational effects (i.e. inhomogeneity along the direction of propagation) can be also neglected. This latter assumption is valid for those coronal waves which are observed to propagate for a rather limited length, but not possible for oscillations involving the whole coronal loop. Since the density scale-height decreases from the corona to photosphere in the lower regions of the solar atmosphere gravitational effects will be more important. One property common to the variety of solar phenomena mentioned above which can be often exploited mathematically is that the transversal dimensions of magnetic structures are much smaller than longitudinal ones and as a result the so-called slender flux tube approximation can be used.

Almost all waves and oscillations presented earlier are strictly connected to the magnetic structure in which they propagate, therefore the magnetic flux tube acts as a waveguide (that is
why these waves can be labelled as local waves). The fact that these solar features have lateral boundaries means that the usual waves of an infinite medium – presented earlier – are modified by the structuring.

3.1. MHD waves in structured media

In order to investigate the effect of structuring in plasmas we assume that the structure has infinite extent in the $z$-direction of a Cartesian coordinate system and the effect of gravitational stratification is neglected. In the equilibrium state we suppose that the plasma pressure, $p$, density, $\rho$, and magnetic field induction $B\hat{z}$ are all $x$ dependent. In equilibrium the total pressure, made up of gas pressure, $p(x)$, and magnetic pressure, $B^2(x)/2\mu$, must be constant, i.e.

$$
\frac{d}{dx} \left( p + \frac{B^2}{2\mu} \right) = 0,
$$

where $\mu$ is the permeability of free space.

Linear and isentropic perturbations for velocity (with components $\mathbf{v} = (v_x, v_y, v_z)$) and magnetic field ($\mathbf{b} = (b_x, b_y, b_z)$) about the equilibrium state (1) may then be Fourier-decomposed in space and time by writing the variables in the form

$$
v_x = \hat{v}_x(x)e^{i(\omega t + ky + kz)}, \text{etc.,}
$$

where the $x$-dependent amplitude of the velocity component, $\hat{v}_x$ satisfies

$$
\frac{d}{dx} \left[ \rho(x) \frac{d}{dx} \left( \frac{k^2 v_A^2(x) - \omega^2}{m^2(x) + l^2} \right) \hat{v}_x \right] - \rho(x) \left[ k^2 v_A^2(x) - \omega^2 \right] \hat{v}_x = 0,
$$

where $m^2(x)$ is the magnetoacoustic parameter defined as

$$
m^2(x) = \frac{(k^2 c_S^2(x) - \omega^2)(k^2 v_A^2(x) - \omega^2)}{(c_S^2(x) + v_A^2(x))(k^2 c_T^2(x) - \omega^2)}.
$$

In the above equations the sound, $c_S$, Alfvén, $v_A$, and cusp (tube), $c_T$, speeds are defined as

$$
c_S(x) = \left[ \frac{\gamma p(x)}{\rho(x)} \right]^{1/2}, \quad v_A(x) = \frac{B(x)}{[\mu \rho(x)]^{1/2}}, \quad c_T(x) = \frac{c_S(x)v_A(x)}{[c_S^2(x) + v_A^2(x)]^{1/2}},
$$

with $\gamma$ being the adiabatic index. Eq. (3) is the Cartesian equivalent of the Hain-Lüst equation arising in cylindrical geometry and has been obtained earlier by Goedbloed (1971), Chen and Hasegawa (1974), Wentzel (1979), Roberts (1981).

A general solution of Eq. (3) is a rather difficult task due to the singularities of the equation occurring when $\omega^2 = k^2 v_A^2$ (Alfvén singularity) and $\omega^2 = k^2 c_T^2$ (slow singularity). Due to the transversal inhomogeneity, the nature of modes changes dramatically. While homogenous plasmas have a spectrum of linear eigenmodes which can be divided into slow, fast and Alfvén subspectra, with the slow and fast subspectra having discrete eigenmodes and the subspectrum of Alfvén waves being infinitely degenerated, in inhomogeneous plasmas the three subspectra are changed. The infinite degeneracy of the Alfvén point spectrum is lifted and replaced by the Alfvén continuum along with the possibility of discrete Alfvén modes occurring, the accumulation point of the slow magnetoacoustic eigenvalues is spread out into the slow continuum and a number of discrete slow modes may occur, and the fast magnetoacoustic point spectrum accumulates at infinity. Eigenfunctions that correspond to frequencies in the continuum spectra are improper as they contain a non-square integrable singularity at the
two singular positions. The continuous spectra appearing in these plasma allows the resonant interaction between plasmas and external waves if the frequency of the external wave matches any frequency in the slow and/or Alfvén continuum.

The resonant coupling of system is always accompanied by a bi-directional exchange of energy; so that if the energy of the external wave is transferred to the plasma, we speak about resonant absorption while if the energy flow is directed from the plasma to the wave, we are dealing with instabilities. The process of resonant interaction of waves and the possibility of energy transfer between global and local waves has been studied intensively for the past few decades. First resonant absorption was studied as a means of supplementary heating fusion plasmas, but was later rejected due to technical difficulties (see, e.g. Tataronis and Grossman 1973, Chen and Hasegawa 1974, Poedts et al. 1989, van Eester et al. 1991).

In the Earth’s magnetosphere resonant MHD wave coupling is believed to generate low frequency pulsation or energize ULF waves (see, e.g. Southwood and Hughes 1983, Taroyan and Erdélyi 2003). Within the context of solar physics Ionson (1978) was the first who proposed resonant absorption as a possible mechanism for coronal heating. His idea was further developed in the last few decades by many authors (see, e.g. Kuperus et al. 1981, Hollweg 1991, Goossens 1991, Sakurai et al. 1991, Goossens and Ruderman 1995, Ofman and Davila 1995, Ballai et al. 1998ab, Ballai and Erdélyi 1998, Ballai et al. 2000, Ruderman and Roberts 2002, etc.), however it became clear soon that resonant absorption alone cannot explain the very high temperature of the corona. Resonant absorption was also used to explain the loss of power in p-modes when interacting with sunspots in the solar photosphere (see, e.g. Goossens and Poedts 1992, Spruit and Bogdan 1992, Tirry et al. 1998).

Recently resonant absorption acquired a new application in coronal seismology where the rapid damping (with damping times of the order of a few periods) of kink oscillations in coronal loops is explained in terms of resonant interaction of global kink modes and the local Alfvén waves (Ruderman and Roberts 2002, Goossens et al. 2006, Terradas et al. 2010). Coronal seismology is dependent on theoretical relations (dispersion relations) which link plasma parameters, such as the plasma density, to wave parameters, such as the wave frequency, in a precise way. Generally, plasma parameters are determined from wave parameters, which themselves are determined observationally. The dispersion relations for many complicated plasma structures under the assumptions of ideal MHD are well known; they were derived long before accurate EUV observations were available using simplified models within the framework of ideal and linear MHD. Although the realistic interpretation of many observations are made difficult by the spatial and temporal resolution of present satellites not being adequate, considerable amount of information about the state of the plasma and the structure and magnitude of the coronal magnetic field have been already obtained.

When the medium is discretely structured, comprising slabs and cylinders of piecewise uniform magnetic field and density, Eq. (3) can be solved in a straightforward way. Let us consider first a slab of magnetic field, having a width $x_0$, with equilibrium described by quantities with an index ”0” for the plasma inside the slab ($|x| < x_0$) and by quantities having an index ”e” describing the plasma outside the slab ($|x| > x_0$). The structure is taken as infinite in the $y$-direction, and for simplicity we restrict our analysis to two-dimensional analysis, i.e. $v_y=0$ and $l=0$. To model waves and oscillations in magnetic structures, let us consider waves that are confined to the inhomogeneity $|x| < x_0$, thus the inhomogeneity acts as a wave guide with disturbances outside the slab ($|x| > x_0$) being laterally evanescent, i.e. $\hat{v}_x \rightarrow 0$ as $|x| \rightarrow \infty$. In this case the solutions of Eq. (3) are given by

$$\hat{v}_x(x) = \begin{cases} 
\alpha_0 e^{-m_0(x-x_0)}, & x > x_0, \\
\alpha_0 \cosh m_0 x + \beta_0 \sinh m_0 x, & |x| < x_0, \\
\alpha_0 e^{-m_0(x+x_0)}, & x < -x_0, 
\end{cases}$$

(6)
where $\alpha_0$, $\beta_0$, $\alpha_e$ and $\beta_e$ are arbitrary constants, while $m_0$ and $m_e$ are values of the magnetoacoustic parameter, $m$, in the two regions. In order to satisfy the condition of evanescence of waves outside the slab we also require that $m_e > 0$. Solutions of Eq. (3) are obtained by applying conservation laws at the boundaries of the slab, i.e. at $|x| = x_0$ we impose the continuity of the normal component of the velocity and total pressure (magnetic plus kinetic).

For an incompressible plasma, corresponding to $\gamma \to \infty$, both $m_0$ and $m_e$ tend to $|k|$ so the explicit solution of Eq. (3) can be written simply as

$$\frac{\omega}{k} = \pm \left[ \frac{\rho_0 v_A^2 + \rho_e v_{Ae}^2 \tanh |k|x_0}{\rho_0 + \rho_e \tanh |k|x_0} \right]^{1/2},$$

(7)

for symmetric modes (also called sausage modes) and

$$\frac{\omega}{k} = \pm \left[ \frac{\rho_0 v_A^2 + \rho_e v_{Ae}^2 \coth |k|x_0}{\rho_0 + \rho_e \coth |k|x_0} \right]^{1/2},$$

(8)

for asymmetric or kink modes. The solutions of Eqs. (7) and (8) are Alfvénic surface waves which owe their existence to the presence of boundaries and their amplitude attain the maximum value at the boundaries and decline on either side of $x = \pm x_0$.

In the case of waves with their wavelength much longer than the width of the slab (i.e. $kx_0 \ll 1$) Eqs. (7) and (8) reduce to

$$\omega = \pm kv_A \left(1 + \frac{\rho_e}{\rho_0} \left(\frac{v_{Ae}^2}{v_A^2} - 1\right) |k|x_0\right)^{1/2},$$

(9)

for sausage waves and to

$$\omega = \pm kv_{Ae} \left(1 + \frac{\rho_e}{\rho_0} \left(\frac{v_A^2}{v_{Ae}^2} - 1\right) |k|x_0\right)^{1/2},$$

(10)

for the kink wave. In a wide slab, i.e. $|k|x_0 \gg 1$, both modes have phase speed given approximately by $\omega^2 = k^2 c_K^2$, where

$$c_K^2 = \frac{\rho_e v_{Ae}^2 + \rho_0 v_A^2}{\rho_e + \rho_0},$$

(11)

is known as the kink speed and it can be regarded as a density weighted Alfvén speed.

Returning to the compressible case, the four constants in Eq. (6) may be eliminated by applying the continuity conditions at the two interfaces, obtaining

$$\rho_e (k^2 v_{Ae}^2 - \omega^2)m_0 \left\{ \frac{\tanh}{\coth} \right\} m_0x_0 + \rho_0(k^2 v_A^2 - \omega^2)m_e = 0,$$

(12)

where the two cases correspond to sausage and kink modes, respectively. Since Eq. (12) is a transcendental equation, it possesses a rich spectrum of solutions and within the context of solar plasma physics it has been first derived by Roberts (1981). Although a full solution of this equation might look complicated, specific applications to, e.g. photospheric or coronal structures allow an easier interpretation. According to the standard nomenclature, solutions corresponding to $m_0^2 > 0$ are labelled as surfaces waves, while waves with $m_0^2 < 0$ are body
waves. This difference in solutions refers to the spatial behaviour inside the structure; surface waves have their maximum amplitude on the boundary are are not oscillatory inside the slab, while body waves show an oscillatory pattern inside the slab.

In the unstructured plasma one expects three groups of waves, the slow and fast sets of magnetoacoustic waves and the set of transversal Alfvén waves. Our analysis has uncoupled the latter, so these pure magnetic waves will not be consider further but we must note that the slow and fast waves met in infinite plasmas are altered in a complicated way depending on the relative magnitudes of sound and Alfvén speed inside and outside the slab. In some cases, for example, that of a cold plasma, in which \( v_{Ae} > v_A \) (often applied for coronal structures) only two sets of body waves occur whereas in photospheric applications where \( v_{Ae} < v_A \), the slow mode can always propagate (either as a surface or body mode) but the fast mode may only propagate in a structure that is cooler than its surrounding (i.e. \( c_e > c_0 \)).

4. Waves in coronal structures

In the solar corona the dynamics is controlled by the magnetic field, here the plasma-beta parameter is much less than 1, therefore it is right to use the so-called low-beta approximation (\( \beta = 2c_S^2/\gamma v_A^2 \ll 1 \)) or cold-plasma approximation (\( \beta = 0 \)). In the cold plasma approximation only fast waves will survive. Writing \( n_0^2 = -m_e^2 \) Eq. (12) for fast sausage body and fast kink body mode reduces to

\[
\tan(n_0 x_0) = -\frac{n_0}{m_e}, \quad \tan(n_0 x_0) = \frac{m_e}{n_0}.
\]  

(13)

In the cold plasma limit, \( n_0 \) and \( m_e \) are given by

\[
\frac{n_0}{m_e} = \frac{\omega^2 - k^2 v_A^2}{v_A^2}, \quad m_e^2 = \frac{k^2 v_{Ae}^2 - \omega^2}{v_{Ae}^2}.
\]  

(14)

It is interesting to note that the equations given by Eq. (13) have arisen in a number of other, physically dissimilar, situations. The first equation of Eq. (13) arises in seismological and oceanographical studies (see, e.g. Pekeris 1948) while the second equation has been earlier obtained within the context of elastic layers of the Earth’s crust (also known as Love waves, Love 1911).

It is also interesting to consider the case of a cylindrical inhomogeneity. For simplicity we will assume that \( c_e = c_0 = 0 \) and the plasma is described within the low-beta plasma would give qualitatively similar results. For a cylinder of radius \( a \) and plasma density \( \rho_0 \) surrounded by a plasma of density \( \rho_e \), all embedded in a uniform field \( \mathbf{B} \), waves of the form

\[
v_r = \hat{v}_r(r)e^{i(\omega t + n\theta + kz)},
\]

(15)

satisfy the dispersion relation

\[
\frac{J_n'(n_0 a) K_n(m_e a)}{J_n(n_0 a) K_n'(m_e a)} = -\frac{n_0}{m_e}
\]  

(16)

and \( J_n \) and \( K_n \) are Bessel functions of order \( n \) with their derivatives \( J_n' \) and \( K_n' \) (Edwin and Roberts 1983).

The behaviour of phase speed \( \omega/k \) as a function of the dimensionless wavenumber \( ka \) is shown in Figure 2, allowing for the effects of non-zero sound speeds (but keeping the low-\( \beta \) approximation). These modes are guided waves of a region of low Alfvén speed (i.e. \( v_A < v_{Ae} \)), corresponding to a region of high density (\( \rho_0 > \rho_e \)). If the Alfvén speeds inside the loop would be higher than outside, that would give rise to radial propagation in the outside cylinder, i.e. waves become leaky. In coronal loops there are no surface waves, i.e no solutions are found for \( n_0^2 < 0 \), however, as Figure 2 shows, there are two sets of body modes widely separated.
The phase speed $\omega/k$ for magnetoacoustic modes in terms of the dimensionless wavenumber $ka$. The above dispersion curves are obtained for typical coronal conditions ($v_{\text{Ae}} > v_A > c_S > c_T$). Sausage modes and kink modes are indicated by solid and dashed lines, respectively (adopted from Edwin and Roberts 1983).

The lower band corresponds to slow waves (nearly sound waves) constrained to propagate one dimensionally along the almost rigid field lines. These waves are weakly dispersive in a slender tube ($ka \ll 1$) and they propagate with the frequency

$$\omega \approx kc_T \approx kc_0 \quad \text{for} \quad c_0 \ll v_A.$$  \hspace{1cm} (17)

The upper band of Fig. 2 corresponds to fast oscillations of high density and are highly dispersive except the fundamental kink mode which has a phase speed close to $c_K$.

In a closed coronal loop with its footpoints anchored in the dense photosphere it is likely that standing modes occur. One of the mostly studied coronal oscillations are the kink oscillations mentioned earlier generated by the interaction of coronal loops with external drivers. The nature of possible standing modes in coronal loops was recently studied by Ballai et al. (2009) where the analysis was carried out for a wide range of drivers. The results show that the signal recovered in the loop is always a superposition of loops’ natural oscillations and the characteristics of the driver. For a loop of length $L$ and $k = j\pi/L$ the times scales involved in the analysis for slow/fast waves are

$$\tau_s \approx \frac{2L}{c_0}, \quad \tau_f \approx \frac{2L}{c_K}.$$  \hspace{1cm} (18)

For typical coronal sound speed of 200 km s$^{-1}$ and and kink speeds of 1000 km s$^{-1}$ for a loop length of 100 Mm we would obtain periods of about 1000 sec for slow waves and 200 sec for kink modes.
modes, such scales being already observed.

5. Waves in photospheric structures

Since temperatures in the solar photosphere are much lower than in the corona, gravitational effects are more important here, appearing as stratification in equilibrium quantities, i.e. all equilibrium quantities can depend on the vertical coordinate along the gravitational force. As specified earlier, the structuring manifests in the form of flux tubes from readily visible sunspots to the sub-resolution intense tubes. The lower solar atmosphere is divided into strong-field media (the intense flux tubes) or field-free media (the tubes’ surrounding) though within sunspots themselves there is fine structure in the form of umbral dots. Therefore some progress in describing possible waves in these structures can be made by using Eq. (12) with \( v_{Ae} = 0 \) or \( v_A = 0 \). In order to simplify the description but keep the essential physics we are going to suppose that we are dealing with slender tubes where we restrict our analysis to waves of long wavelength compared to the width of the structure. The effect of density and pressure decrease with height is that as we move upward the cross-section area of the tube is increasing however the flux within the tube is always conserved.

The equations describing the dynamics of longitudinal and isentropic motion (characterised by \( v(z, t) \)) of a gas of density \( \rho(z, t) \) and pressure \( p(z, t) \) confined within an elastic tube of cross-sectional area \( A(z, t) \) can be given as

\[
\frac{\partial \rho A}{\partial t} + \frac{\partial}{\partial z}(\rho v A) = 0, \tag{19}
\]

\[
\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} \right) + \frac{\partial p}{\partial z} + \rho g = 0, \tag{20}
\]

\[
\frac{\partial p}{\partial t} + v \frac{\partial p}{\partial z} = \frac{\gamma p}{\rho} \left( \frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial z} \right), \tag{21}
\]

\[
p + \frac{B^2}{2\mu} = p_e, \quad BA = \text{const.} \tag{22}
\]

The last two equations express that the tube will be stable if the total pressure inside the tube is balanced by the external pressure and that the magnetic flux is conserved at all times. Here the gravity is assumed to be aligned with the \( z \)-axis of the tube (\( \mathbf{g} = -g \hat{z} \)) and \( p_e \) is the gas pressure in the tube’s exterior.

The simplest way to deal with the above system of equation is to linearize the equations assuming small amplitude waves so we will assume that all quantities (except speed) can be written as the sum of an equilibrium value and its small perturbation. The equilibrium quantities can be written as

\[
p_0(z) = p_0(0)e^{-n_0(z)}, \quad p_0(z) = \rho_0(z)A_0(0)A_0(z)e^{-n_0(z)},
\]

\[
A_0(z) = A_0(0)e^{n_0(z)/2}, \quad B_0(z) = B_0(0)e^{n_0(z)/2}, \tag{23}
\]

where

\[
n_0(z) = \int_{z_0}^{z} \frac{dz'}{A_0(z')}, \quad A_0(z) = \frac{p_0(z)}{\rho_0(z)g},
\]

is the pressure scale-height (here we assumed that the temperatures inside and outside the tube are identical).

Assuming that \( p_e \) remains equal to its equilibrium value, it may be shown that the re-scaled longitudinal component of the velocity

\[
u(z, t) = \frac{Q(z, t)}{[\rho_0(z)A_0(z)e^{n_0(z)}]^{1/2}}, \tag{24}\]
is governed by the Klein-Gordon equation (Webb and Roberts 1978, Rae and Roberts 1982, Ballai et al. 2006)

$$\frac{\partial^2 Q}{\partial t^2} - c_T^2(z) \frac{\partial^2 Q}{\partial z^2} + \omega_v^2 Q = 0.$$ \hfill (25)

The general expression of $\omega_v$ is rather cumbersome but in the case of an isothermal atmosphere ($\Lambda_0=\text{const.}$) it reduces to

$$\omega_v = \frac{c_T}{\Lambda_0} \left[ \frac{9}{16} - \frac{1}{2\gamma} + \left( \frac{\gamma-1}{\gamma} \right) \frac{c_0^2}{v_A^2} \right]^{1/2}.$$ \hfill (26)

Thus the geometry and elasticity of a magnetic flux tube give rise to a cut-off frequency, $\omega_v$, which is analogous to the cut-off frequency, $\omega_a = c_0 / 2 \Lambda_0$, for vertically propagating acoustic-gravity waves in a field-free medium. The presence of the cut-off frequency implies that only frequencies above the cut-off propagate under adiabatic conditions; frequencies below this value are evanescent. In the light of this statement, it is obvious that only those magnetoacoustic waves can propagate towards the chromosphere and corona whose frequency is above the cut-off value. The solution of the Klein-Gordon equation shows that under the effect of stratification a slow wave will propagate along field lines with speed $c_T$ followed by a wake oscillating with the frequency $\omega_v$.

In the chromosphere, due to the decreases density, many waves will have their amplitude increased and they can easily evolve into nonlinear waves (shocks or solitons), a subject which is not going to be discussed here. It is interesting to note that the Klein-Gordon equation presented earlier seems to have an universal character, the same equation was found to describe the propagation of externally driven kink oscillations in coronal loops (Ballai et al. 2009).

Strictly speaking the corona and solar wind are not fully collisional media, therefore the MHD system of equation may not be the best formalism to describe waves. The derivation of the isotropic MHD equations requires the assumption that electrons are tightly coupled to ions, so that plasma temperatures are isotropic. In this case the plasma density and temperature satisfy some equation of state. However, spacecraft observations often show anisotropic velocity and temperature distributions, implying that this model is not fully valid for many space plasmas. In this kind of plasma, different types of particle can have different temperatures parallel and perpendicular to the ambient magnetic field. Thus, more complex models based on more sophisticated closure relations may be necessary to model adequately the rich diversity of plasma dynamics.

Parker (1958) was the first to point out that the interplanetary magnetic field might lead to an anisotropic pressure tensor. When the cyclotron frequency of the ions is much higher than the collisional frequency, the particles gyrate many times around a line of magnetic force between two collisions. The magnetic field induces a splitting in the pressure, introducing a parallel and a perpendicular component. The two components of the pressure are not necessarily equal. However, the thermal anisotropy and electron conduction velocity relative to the ions generally excite plasma oscillations that scatter the particles, pulling the plasma toward isotropy, ($p_\parallel \sim p_\perp$).

The main complication arising from the presence of anisotropy relates to pitch angle scattering, a mostly kinetic effect. Such a description within fluid approximation remains a fundamental but unresolved theoretical problem. Technically speaking, the closure of the fluid equations requires two equations of state, one for the parallel and one for the perpendicular pressure. In spite of much effort, there is no generally accepted form for these two equations of state. In such a medium waves that are mostly generated by the free energy of the particle distribution functions are the means to relax this free energy. Waves, once amplified, can heat
particles, permit exchange of energy between different populations of particles and precipitate
particles.

An original approach to the problem of physical properties of rarefied plasmas has been the
theory developed by Chew, Goldberger and Low (1956) known as the Chew-Goldberger-Low
(CGL) approximation. The usual MHD equations are derived from the Boltzmann equation
using an expansion in powers of the collisional mean free path. In this case the plasma is collision-
dominated therefore the collisional term in the Boltzmann equation is the leading term, with
all other terms being treated as perturbations. When the density is so small that the plasma
can be considered as a (near) collisionless medium, a new form of governing equations can be
derived from the Boltzmann equation using an expansion in powers of the Larmor radius. This
approximation can be considered as an adiabatic approximation since it depends on the Larmor
frequency being large compared to other characteristic frequencies of the problem.

The properties of anisotropic plasmas have been studied intensively over the past few years
in the light of new observations (see, e.g. Kato, Tajiri and Taniuti 1966; Hau and Lin 1995;
Nakariakov and Oraevsky 1995; Ballai et al. 2002, Ballai and Zhugzhda 2002, Ballai and Marcu
2004). These studies revealed that the properties of the waves are different from the usual MHD
and these differences are crucial from the wave propagation and stability point of view. Since the
study of waves in anisotropic plasmas is in its infancy there is no general concept as to how these
waves are generated. However, it is very likely that the generation mechanism of these waves
in similar to the mechanisms that generate the well-known MHD (isotropic) waves: convection,
reconnection, resonances with other modes, even non-anisotropic, etc. For example, a wave that
propagates in the isotropic plasma can ‘decay’ into a two-component wave (i.e. parallel and
perpendicular sound speeds).

Thanks to the high resolution observations our knowledge about the dynamical state of the
solar atmosphere is ever changing, however every observation will help us understand how our
star is actually working.

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