MODELING THE TIME VARIABILITY OF ACCRETING COMPACT SOURCES

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ABSTRACT

We present model light curves for accreting black hole candidates (BHCs) based on a recently proposed model for their spectro-temporal properties. According to this model, the observed light curves and aperiodic variability of BHCs are due to a series of soft photon injections at random (Poisson) intervals near the compact object and their reprocessing into hard radiation in an extended but nonuniform hot plasma corona surrounding the compact object. We argue that the majority of the timing characteristics of these light curves are due to the stochastic nature of the Comptonization process in the extended corona, whose properties, most notably its radial density dependence, are imprinted in them. We compute the corresponding power spectral densities (PSD), autocorrelation functions, time skewness of the light curves, and time lags between the light curves of the sources at different photon energies and compare our results to observation. Our model light curves compare well with observations, providing good fits to their overall morphology, as manifest by the autocorrelation and skewness functions. The lags and PSDs of the model light curves are also in good agreement with those observed (the model can even accommodate the presence of quasi-periodic oscillations). Finally, while most of the variability power resides at timescales approximately a few seconds, at the same time, the model allows also for shots of a few milliseconds in duration, in accordance with observation. We suggest that refinements of this type of model along with spectral and phase lag information can be used to probe the structure of this class of high-energy sources.

Subject headings: black hole physics — radiation mechanisms: nonthermal — stars: neutron — X-rays: stars

1. INTRODUCTION

The study of the physics of accretion onto compact objects (neutron stars and black holes) whether in galactic (X-ray binaries) or extragalactic systems (active galactic nuclei) involves length scales much too small to be resolved by current technology or that of the foreseeable future. As such, this study is conducted mainly through the theoretical interpretation of spectral and temporal observations of these systems, much in the way that the study of spectroscopic binaries has been used to deduce the properties of the binary system members and the elements of their orbit. In this endeavor, the first line of attack in unfolding their physical properties is the analysis of their spectra. For this class of objects and in particular the black hole candidate (BHC) sources, a multitude of observations have indicated that their energy spectra can be fitted very well by Comptonization of soft photons by hot electrons; the latter are naturally expected to be present in the sources at different photon energies and can provide clues to their overall variability as manifest by the autocorrelation and skewness functions. The lags and PSDs of the model light curves are also in good agreement with those observed (the model can even accommodate the presence of quasi-periodic oscillations). Finally, while most of the variability power resides at timescales approximately a few seconds, at the same time, the model allows also for shots of a few milliseconds in duration, in accordance with observation. We suggest that refinements of this type of model along with spectral and phase lag information can be used to probe the structure of this class of high-energy sources.

Recent RXTE (W. Focke 1998, private communication), as well as older HEAO 1 observations (Meekins et al. 1984) of the BHC Cyg X-1, which resolved X-ray flares of duration approximately a few milliseconds, appear to provide a validation of our simplest expectations. At the same time, however, the X-ray fluctuation power spectral densities (PSD) of accreting compact sources generally contain most of their power at frequencies \( \omega \lesssim 1 \) Hz, far removed from the kHz frequencies expected on the basis of the arguments given above. Flares of a few milliseconds in duration, while present in the X-ray light curves of Cyg X-1, contribute but a very small fraction to its overall variability as manifest in its PSD, which exhibits very little power at frequencies

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photon emission has several direct implications concerning the spectral properties of these sources due to the high energy of the escaping photons, which increases with their residence time in the scattering medium. Thus, while there may not be any coherence in the absolute phases of the various Fourier components of the light curves of this class of sources, the relative phases between photons in different energy bands may not be random, provided that the observed radiation is produced by a single hot electron cloud rather than a large number of individual, disjoint sites. As a result, the hard photon light curves should lag with respect to those of softer photons by amounts that depend on the photon scattering time in the plasma. If the sources are of rather limited spatial extent and the scattering takes place only in the vicinity of the compact object, then the corresponding time lags should be roughly constant (independent of the Fourier frequency) and of order of the scattering time in this medium, i.e., $\sim 10^{-3}$ s for galactic sources and correspondingly larger for AGNs. On the other hand, if the scattering medium is extended, the lags should decrease with the square of the separation distance, leading to an overall decrease in the observed time lags.

Searches for these lags in the light curves of the BHC sources Cyg X-1 and GX 339-4 in the Ginga (Miyamoto et al. 1988, 1991) and the more recent RXTE data (Cui et al. 1997), have detected their presence and, more importantly, discovered that the hard time lags $\Delta t$ depend on the Fourier period $P$, increasing roughly linearly with $P$ from $\Delta t \approx 0.001$ s at $P \approx 0.05$ s to $\Delta t \lesssim 0.1$ s for $P \approx 10$ s. These long lags and in particular their dependence on the Fourier frequency $P$ are very difficult to interpret in the context of a model in which the X-ray emission is due to soft photon Comptonization in the vicinity of the compact object; in such a model, the hard lags should simply reflect the photon scattering time in the specific region (i.e., $\sim 10^{-3}$ s), and, moreover, they should be independent of the Fourier frequency. While this type of time lag was found originally in the light curve of the BHC Cyg X-1, similar lag behavior has been recorded also for the transient J0422+32 (Grove et al. 1998) and also the source GR 1758−238 (Smith et al. 1997). Finally, to indicate that these results may not be universal, the data of Wilms et al. (1997) indicate that the lags associated with Cyg X-1 during their observation increase much more gradually.

Motivated by the discrepancies between our expectations of X-ray variability based on simple dynamical models and the observed form of their PSDs and their frequency dependent hard lags, we have revisited the issue of time variability of BHC sources (Kazanas, Hua, & Titarchuk 1997, hereafter KHT; Hua, Kazanas, & Titarchuk 1997, hereafter HKT; Hua, Kazanas, & Cui 1999, hereafter HKC). The central point of our considerations has been that, contrary to the prevailing notions about the timing behavior of accreting sources, features in the observed PSD correspond not to variations in the accretion rate but rather to properties of the electron density distribution of an extended ($\gtrsim 10^3 R_\odot$) rather than a compact $\lesssim 10 R_\odot$ hot electron corona. With this assumption in place, features in the PSDs translate to well-defined properties of the scattering corona that can in principle be checked for consistency with observations. Thus we have proposed that the low-frequency ($\nu \lesssim 1$ Hz) break of the PSD to white noise is associated with the outer edge of the Comptonizing corona, while its inner edge is very close to the compact source. We further indicated that
the power-law–like PSDs depend on both the density profile and the total Thomson depth of the corona, implying correlations between the energy and the variability power spectra, which can be sought for in the data.

Using this extended corona model, we were able to show explicitly that the observed energy spectra convey rather limited information about the structure of the scattering medium (KHT, HKC) and that very different hot electron configurations can indeed yield the same energy spectrum as long as the \textit{total} probability of scattering remains the same across the different configurations.

We have also shown (HKC) that the fact that the lags between different energy bands depend on the \textit{differential} probability of scattering at a given range of radii (contrary to the spectrum that depends on the total probability) affords a means of \textit{probing the density structure of the corona} through the study of the lag dependence on the Fourier period $P$. Finally, we were able to show that because of the linearity of Compton scattering, the coherence function (as defined by Vaughan & Nowak 1997) of the extended hot corona configuration is very close to one, in agreement with observations (Vaughan & Nowak 1997), suggesting, in addition, that the properties of the scattering hot electron cloud remain constant over the observation timescales (Hua et al. 1997).

Thus the timing observations indicate, on one hand, that the absolute phase of the light curves of accreting compact objects are random, implying an incoherent process, while on the other hand that the relative phases between different energy bands are indeed extremely well correlated, indicating a coherent underlying process. The purpose of the present paper is to produce model light curves of accreting compact sources compatible with these apparently contradictory aspects of their coherence, as well as the observed PSDs. We also examine the structure of the latter (including the presence of quasi-periodic oscillations [QPOs]) and their dependence on the sources’ luminosity in the general context of these models. Having provided a prescription for producing model light curves, we subsequently examine their properties in the time rather than frequency domain and indicate tests that may confirm or disprove the fundamentals of our model.

In § 2 we outline the extended corona configuration under consideration and we present models of the Comptonization response function and the corresponding power spectra associated with it. We also elaborate on the fundamental tenet of our models, namely, that of association of PSD features with corresponding features in the density structure of the extended corona by providing a generic QPO model within this framework. In § 3 we provide a prescription for generating model light curves using the coronal response function and we do generate a number of such curves for different values of the model parameters. With the model light curves at hand, we further elaborate our analysis by computing and comparing their attributes to observation. Thus in § 4 we compute phase lags as a function of the Fourier frequency and in § 5 the associated autocorrelation and skewness functions. Finally, in § 6 the results are summarized and discussed.

2. THE EXTENDED CORONA AND ITS RESPONSE

Kazanas et al. (1997) proposed that the timing properties of BHCs discussed above, i.e., the PSD and the form and frequency dependence of the time or phase lags, can be easily accounted for with the assumption that all associated variability is due to the Compton scattering of soft photons in an extended, nonuniform corona surrounding the compact object and spanning several decades in radius. Specifically, they proposed the following density profile for the Comptonizing medium:

$$\eta(r) = \begin{cases} \eta_1 & \text{for } r \leq r_1, \\ \eta_1(r_1/r)^p & \text{for } r_2 > r > r_1, \end{cases}$$

where the power index $p > 0$ is a free parameter, $r$ is the radial distance from the center of the spherical corona, and $r_1$ and $r_2$ are its inner and outer radii, respectively.

KHT considered $p$ to be an arbitrary parameter; however, as indicated in that reference, the value of $p = 1$ allows for scattering of the photons with equal probability over the entire extent of the corona introducing time lags at every available frequency with equal probability, a fact that is in agreement with the observations; hence they considered the value $p = 1$ as the fiducial value of this parameter. However, they also considered different values of $p$, in particular the value $p = 3/2$, as it corresponds to the density profiles of the currently popular advection-dominated accretion flows (ADAFs; Narayan & Yi 1994).

2.1. The Shot Profiles

The time response of the given corona, i.e., the flux of photons escaping at a given energy range as a function of time, following the injection of a $\delta$ function of soft photons at its center at $t = t_0$ has been computed using the Monte Carlo code of Hua (1997). As discussed in KHT and in HKC, its form $g(t)$, ignoring its rising part, can be approximated by a gamma distribution function, i.e., a function of the form

$$g(t) = \begin{cases} t^{\alpha-1}e^{-t/\beta} & \text{if } t \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $t$ is the time and $\alpha > 0$ and $\beta > 0$ are parameters determining the shape of the light curves that depend on the scattering depth $\tau_0$ and photon escape energy (see Figs. 1a and 1b in KHT). As indicated in KHT, for small values of the total depth of the corona $\tau_0 \ll 1$, $\alpha = 1 = -p$, while as $\tau_0$ increases the power-law part of the curve becomes progressively flatter, i.e., $|\alpha - 1| < p$. For the cloud with $p = 1$ in equation (1), $\alpha$ is small compared to 1 and $\beta$, determined by the outer edge of the scattering cloud, is taken to be of the order of 1 s so that the light curves have exponential cutoff at $\sim 1$ s. For uniform cloud, $p = 0$, $\alpha = 1$, and the light curve is a pure exponential (leading to a PSD proportional to $\omega^{-2}$; one should note though that the precise value of $\alpha$ depends also on the photon energy). The simplified form of the response function given by equation (1) allows one to compute analytically its Fourier transform (HKC)

$$G(\omega) = \frac{\Gamma(\alpha)\beta^2}{\sqrt{2\pi}} \left(1 + \beta^2\omega^2\right)^{-\alpha/2} e^{i\theta},$$

where $\Gamma(\alpha)$ is the Gamma function, $\omega$ is the Fourier frequency, $\theta$ is the phase angle, and $\tan \theta = 1/\beta$.

However, the form of the realistic response functions is more complicated. Figure 1 shows the shape of the response function as computed using the Monte Carlo code (Hua 1997). The parameters used in the Monte Carlo calculation...
were as follows: \( r_1 = 6.35 \times 10^{-3} \) lt-s \( s = 1.9 \times 10^8 \) cm, \( r_2 = 6.24 \) lt-s \( s = 1.87 \times 10^{11} \) cm, \( T_e = 100 \) keV, \( \tau_0 = 1 \), and \( n_1 = 10^{15} \) cm\(^{-3} \) (long-dashed curve) along with its fit by eq. (4) with \( \alpha = 0.4, \beta = 10 \) s, \( t_0 = 0.02 \) s, \( b = 1 \) (solid curve), and the corresponding power spectrum (short-dashed curve).

To simulate the precise form of the shots in the coronae we consider we use a more complex function

\[
g(t) = \begin{cases} 
A_1(1 - B_1 x^{b})x^\gamma & \text{if } x \equiv t/t_0 \leq 1, \\
A_2(1 + B_2 x^{-b})x^\gamma e^{-(xt0/\beta)^{3}} & \text{if } x \equiv t/t_0 > 1,
\end{cases}
\]

where \( b, \gamma > 0 \). The parameters \( A_2, B_1, B_2 \) are given in terms of the arbitrary normalization \( A_1 \), and the form parameters \( \alpha, \beta, \gamma, t_0 \), and \( b \) by the requirement that the function \( g(t) \) and its first derivative be continuous at \( x = 1 \), with that point being also a local maximum. The parameters \( \alpha, \beta \) have the same meaning as those used in equation (2) above, while \( t_0 \) and \( \gamma \) indicate the time at which the response function achieves its maximum value (of order \( r_i/c \)) and the rate at which this maximum is achieved. The cutoff form of \( g(t) \) is steeper than exponential to mimic the detailed curve produced by the Monte Carlo simulation. One should note the additional parameter \( b \), which regulates the sharpness of the transition from the rising to the falling part of the model response function. The values of the parameters used in fitting the response function were \( \alpha = 0.4, \beta = 10 \) s, \( t_0 = 0.02 \) s, \( b = 1 \). In the same figure we also present the PSD that corresponds to a single shot with this specific form of the response function. As will be argued in the next section, under certain assumptions, this is also the PSD of the entire light curve.

The particular light curve and the associated values of the corresponding fits should be considered only as fiducial values. Monte Carlo runs with smaller values of \( r_1 \) gave shots achieving their peak flux at timescales proportionally shorter. Thus fits of corona response functions with \( r_1 = 4.77 \times 10^{-4} \) lt-s \( s = 1.43 \times 10^{7} \) cm and \( n_1 = 10^{16} \) cm\(^{-3} \) gave rise times of order \( t_0 = 0.001 \) s.

### 2.2. The Power Spectra

As will be discussed in the next section, the PSDs of the model X-ray light curves of accreting neutron stars and BHC sources that we present, to a large extent, reflect the properties of the response functions of the corresponding coronae; we therefore feel that a short discussion of their form is, at this point, necessary. To begin with, we note that in the limit of infinitely sharp turn-on of the shots under consideration, the Fourier transform of \( g(t) \) is given by \( G(\omega) \) of equation (2), and therefore the PSD, under these conditions, can be computed analytically (HKC)

\[
|G(\omega)|^2 = \frac{\Gamma(\alpha^2\beta^2\omega^2)}{2\pi} (1 + \beta^2\omega^2)^{-\frac{3}{2}}.
\]

As discussed in KHT, these PSDs consist of a power-law section with slope that is related to the corresponding slope of the power-law segment of the response function while flattening to white noise for \( \omega \ll 1/\beta \). Since we have adopted the value \( \alpha \approx 0.5 \) in the form of the response function, the PSD consists mainly of a power-law segment of slope \( 2\alpha - 2 \approx -1 \), i.e., flicker noise, representative of the power-law segment of the time response function. However, for the case of more realistic shots that have a finite rise time, such as those shown in Figure 1, the computation of the PSD can no longer be done analytically. Moreover, the finite size of the shot rise time introduces additional structure in the PSD that manifests as a break in its high-frequency portion. In Figure 1 we present, in addition to the profiles of the response functions, the corresponding PSDs (short-dashed lines).

It is of interest to compare the shape of the PSD to those associated with the real data of the BHCs Cyn X-1, GX 339–4, and Nova Muscae (Miyamoto et al. 1992, Fig. 1). One can see that, in addition to the flicker noise behavior, our model also produces the steepening observed at higher frequencies (see also Cui et al. 1997 for RXTE data of Cyg X-1 and Grove et al. 1998 for OSSE data of GRO J0422+32). The similarity of the model PSD shapes to the data notwithstanding, the most important feature of the present model is, in our view, the physical association between the specific PSD features and the physical characteristics of the hot electron corona responsible for the production of the high-energy radiation through Comptonization (most notably its size and radial density slope, although the latter is only determined unequivocally through the hard lags, HKC).

We would like to comment specifically on the effect of the parameter \( b \), the shot sharpness parameter, on the form of the PSD; as one might expect, for values \( b < 1 \), this parameter can affect significantly the form of the PSD, rendering it steeper than expected on the basis of the value of the parameter \( \alpha \). However, the realistic light curves produced by the Monte Carlo simulation of Hua (1997) fit quite well with values of \( b \approx 1 \), and we have therefore adopted this value for this parameter in the remaining of this work.

### 2.3. The QPOs

Given that the present model purports to provide a rather generic account of the variability of accreting sources, we feel that, even at this early stage of its development, it should also be able to provide a generic account of the features occasionally occurring in the PSDs of these sources, known as quasi-periodic oscillations (QPOs).
FIG. 2.—Response functions (a) and the corresponding power spectra (b) associated with the configuration consisting of a uniform sphere of $\tau_0 = 1$ and radius $r = 1$ lt-s, surrounded by a thin shell of the same Thomson depth $\tau_0 = 1$ and $r = 10$ lt-s.
These features have attracted the attention of both theorists and observers because it was considered that their quasi-periodic nature would lead to clues about the dynamics of accretion onto the compact object not available in their largely aperiodic light curves.

Since our present goal is not the study of the subject of QPOs, we forgo any discussion, references, or models associated with this subject matter with the exception of the reviews by van der Klis (1995, 1998), wherein the interested reader can find more about the QPO phenomenology, QPO systematics, and additional references. However, we would like to demonstrate that the tenet of our model—namely, that features of PSD correspond to features in the electron density of the hot corona—can indeed provide an account of the QPO phenomenon and some of its systematics.

In Figure 2a we present the response function of the following configuration: two concentric, uniform shells, each of \( q_0 = 1, kT = 100 \text{ keV} \), one extending from \( r = 0 \) to \( r = 1 \) lt-s, while the second extends from \( r = 10 \) lt-s to \( r + \Delta r \) with \( \Delta r/r_0 < 1 \). Soft photons of energy \( 0.1 \text{ keV} \) are released at the center, \( r = 0 \), of the configuration. Each panel of Figure 2a corresponds to the response function for photon in the energy range denoted in the figure. One can distinguish a cutoff at \( t \approx 1 \text{ s} \), indicative of the exponential drop-off of the photons escaping from the inner shell, and a second one at \( t \approx 10 \text{ s} \) due to escape through the outer thin shell. One should also note the presence of an additional peak at \( t \approx 20 \text{ s} \), corresponding to photons that were reflected by the outer shell (rather than transmitted through it), escaping at the opposite side of the configuration. It is our contention that it is the presence of these photons that is responsible for at least some of the aspects of the QPO phenomenon.

In Figure 2b we exhibit the power spectra corresponding to the shots given in Figure 2a. There is an apparent QPO peak at \( v \approx 1/20 \text{ s} \), as well as harmonically spaced peaks, indicative of the power associated with the timescale \( t_0 \approx 2r_0/c \), corresponding to the light crossing time of the outer shell. This additional power in the PSD is thus related to the spatial rather than timing properties of an otherwise stationary configuration with generally stochastic soft photon injection. As shown in Figure 2, the QPO contribution increases with photon energy, a fact also in agreement with observation. This model provides a straightforward account of this effect: the larger the photon energy, the greater the number of scatterings it has undergone, and therefore the greater the probability that it has crossed the outer shell, thereby increasing the QPO contribution to the PSD. We view this dependence of the QPO fractional power on the photon energy as an interesting interplay between the spectral and timing properties that the present model can easily accommodate within its general framework.

The presence of multiple QPOs in the PSD of accreting sources implies, therefore, within our model, the presence of multiple shells similar to that responsible for the PSD of Figure 2. We do not know at this point the dynamics that would lead to the presence of such features; the purpose of the present note is to simply indicate that their presence can lead to QPO-like features similar to those observed. Furthermore, while features in the spatial electron distribution can indeed produce QPOs, one should be cautioned that they are not necessarily the sole cause of all QPO features, and that more conventional models may be actually responsible for a number of them. We are in the process of studying these possibilities in greater depth (Kazanas & Hua 1999).

3. THE LIGHT CURVES

As discussed in the introduction, it is our opinion that one of the major obstacles in understanding the dynamics of accretion onto compact objects is the difficulty in providing concrete models of their observed time variability that could be easily compared to observation. In our view, this lack of models is largely due to the aperiodic character of their light curves, which offers few clues on the mechanism responsible for their formation. It is our contention that the light curves associated with the high-energy emission of accreting compact sources is due, to a large extent, to the stochastic nature of Compton scattering in an optically thin, hot, extended medium, which is responsible for the formation of their spectra, coupled with a (probably) stochastic injection of soft photons to be Comptonized. Our proposal, therefore, is that the observed light curves consist of the incoherent superposition of elementary events, each triggered by the injection of a soft photon pulse into the extended corona of hot electrons discussed above. To simplify matters, we assume that the soft photon injection takes place at the center of the corona and it is of vanishing duration; therefore the resulting high-energy light curve should be the incoherent sum of pulses with shapes given by the corresponding response function discussed in the previous section.

Following this prescription, one can easily produce model light curves of the resulting high-energy emission. These will have the form

\[
F(t) = \sum_{i=1}^{N} Q_i g(t - \tau_i)
\]

The variable \( \tau_i \) in the above equation is a random variable indicating the injection times of the individual shots, while \( Q_i \) is their normalization, a quantity that in our specific model depends on the number of soft photons injected at each particular shot event. Clearly, one could in principle arrange for any form of the PSD by choosing the values of the parameters \( Q_i, \tau_i \) from appropriately defined distributions. While the values of \( Q_i \) and \( \tau_i \) may in fact be associated with certain distributions, we have no a priori knowledge of such distributions and they are in no way restricted by any compelling dynamical arguments. Therefore, in order to avoid introducing extraneous information into our time series, we have chosen a constant value for all \( Q_i \)’s, while the values of \( \tau_i \)’s are chosen to be Poisson distributed with a given constant rate. As such, the \( \tau_i \)’s are chosen using the relation \( \tau_i = -\mu \log R_i \), where \( R_i \) is a random number uniformly distributed between 0 and 1 and \( f \) a real number, indicating the mean time between shots in terms of their rise time \( t_0 \).

In Figures 3a and 3b we present two such model light curves. The parameters of the shots used in constructing these curves were the same as those fitting the response function of Figure 1. The two figures are different in the value of the parameter \( f \), i.e., the parameter that indicates the mean arrival time between shots. The value of \( f \) was set to \( f = 3 \) in Figure 3a and \( f = 10 \) in Figure 3b. The two curves were produced with exactly the same sequence of
random numbers, a fact that can be discerned by identifying corresponding features in the two light curves. The random character of the parameter $f$ leads then to an apparently incoherent light curve in the sense discussed by Krolik et al. (1993), i.e., of random absolute phases as a function of the Fourier frequency.

In Figures 4a and 4b we present zoom-in sections of the light curves of Figures 3a and 3b, corresponding to the same ordinal in the sequence of random shots. The shape of the shots becomes progressively asymmetric as the value of $f$ increases, since that gives time to the contribution of an individual shot to the light curve to decrease substantially before the next one appears, thus manifesting the underlying asymmetry of the individual shots. At the same time, this also leads to an increase in the rms amplitude of the variability. One should note that the model light curves presented in these figures consist exclusively of the superposition of shots that die out like power laws rather than exponentially in time, without the presence of a DC component. The brief rising part of the light curve lasts for a time interval of $\approx \beta$ seconds, i.e., for the time it takes the earliest shot to die out, and it is a transient associated with the turn-on process. Beyond this point a steady state of rather well-defined mean is established, which, however, is not particularly smooth but characterized by large, aperiodic oscillations, the result of the random arrival and superposition of shots of shapes similar to those of Figure 1.

It is apparent from the above that an rms value for the light curve can be established only for timescales $t \gtrsim \beta$. We thus propose that observations in which the rms values of the corresponding light curves depend on the observation interval indicate that these light curves have not been sampled for sufficiently long intervals, i.e., $t \approx \beta$, even if the accretion rate has remained constant during this interval. Conversely, the sampling time over which a well-defined rms value of the light curve can be established could be used to estimate the timescale $\beta$ and thus the size of the high-energy emitting source. This size could then be compared to that obtained through study of the lags (see HKC) for consistency.

The present model allows for several different ways of implementing a change in the rms amplitude of the model light curves. These are, in principle, related to observables and could provide novel insights concerning the variability and dynamics of accreting sources. The simple prescription
for generating the light curves given above indicates clearly that increasing the value of $f$, i.e., the mean time between shots in units of their rise time, would result in larger rms fluctuations of the light curves, should the rest attributes of the response function remain the same. This particular case is discussed in more detail in §5.2 as it is expected to be correlated to the skewness of the light curves.

In this section we discuss the additional possibility of changes in the rms fluctuations of the light curves affected by changes in the value of the rise time of the shots $t_0$. For example, decreasing $t_0$ while keeping the other parameters (size, temperature, density) of the hot corona constant would lead to an increase in the source luminosity and a corresponding decrease in the rms fluctuation amplitude without any additional changes in the spectral or temporal properties of the source. However, if the timescales $t_0$ and $\beta$ are indeed related to a length scale associated with the size of the corona’s inner and outer edges, this would generally depend on the macroscopic parameters associated with the accretion flow, most notably the accretion rate. To illustrate with an example the insights that can be gained by comparing such simple models to observation, we assume that the inner and outer radii of the corona $r_1$, $r_2$ (and for that matter the entire flow within the corona), scale proportionally to $\nu_{\ell} t_{\text{cool}}$, where $\nu_{\ell}$ is the free-fall velocity and $t_{\text{cool}}$, the local cooling timescale. Assuming the latter to be inversely proportional to the local density, a prescription in accordance with, say, the models of ADAFs of Narayan & Yi (1994), then all the length scales of the corona, expressed in Schwarzschild radii, should be inversely proportional to the accretion rate of the flow, measured in units of its Eddington value. Therefore, an increase in $\dot{m}$ would lead not only to a decrease in $t_0$ and therefore in the rms amplitude of fluctuations, but also in a concomitant decrease of all other lengths of the system (e.g., its outer edge), resulting in an increase in the PSD frequencies corresponding to these features (e.g., the PSD low-frequency break).

Apparently such correlations between the source luminosity and the timing characteristics, while they may not be a universal phenomenon, have been observed in at least several sources. For example, van der Hooft et al. (1996) indicate that an increase in the luminosity of the black hole candidate GRO J1719 - 24 leads to an increase in the QPO frequency at 0.04 Hz by a factor of ~4, while at the same time the rms amplitude decreases by exactly the same amount preserving total variability power in the sense that the product $\omega |F(\omega)|^2$ remains constant. These authors indicate that scaling down all the frequencies of the PSD obtained when the source was in its high state leads to a PSD indistinguishable from that obtained when this source had a much lower luminosity. Similar general trends have also been observed in the transient source GRO J0422 + 32 (Grove et al. 1998) and are indicative that this behavior does not represent an isolated phenomenon associated with a particular source. At the same time, the simple example discussed above indicates how modeling the aperiodic variability of these sources could lead to new insights and probes of the dynamics of accretion onto compact objects.

Concerning the morphology of the light curves given in Figures 3a and 3b, eye inspection reveals shots with a variety of timescales, and indeed we believe that a distribution of such shots can be found, should one care to view and model them as such (see, e.g., Focke & Swank 1998). However, none of these additional timescales or shots have been used as input in this particular simulation. They result simply from the superposition of a large number of shots with power-law tails that extend to ~1 s. Zooming in to the highest time resolution, one can indeed discern shots with rise times of the order of $t_0 \sim 10^{-3}$ s in agreement with observations (Meekins et al. 1984). These shots are indeed the individual elementary events (with the power-law extended tails) that comprise our model light curves. However, given the rather limited time range over which one can distinguish the contribution of such individual shots over the flux produced by the incoherent sum of their ensemble (or the detector statistics), one would most likely attempt to fit their shape with an exponential rather than a power law because their long, extended tails cannot be discerned in the data. However, while fits in the time domain fail to uncover the true structure of these shots because of their crowding, the PSD can achieve this very easily and reliably.

The above analysis makes clear that our model for the variability of accreting sources stands in stark contrast to other models put forth to date to account for the flicker noise character of the observed variability of BHCs (Chen & Taam 1995; Takeuchi et al. 1995). Rather than attributing the observed PSD characteristics entirely to variations in the mass flux onto the compact object, it attributes them, for the most part, to the spatial distribution of electrons in the Comptonizing cloud and the stochastic nature of Compton scattering. The accretion rate is no doubt variable, especially at the shortest intervals–smallest radii (e.g., the soft photon injection events); however, its variability is consistent with a constant averaged-out accretion rate on longer timescales, as indicated by the coherence measurements and their interpretation (HKT). More importantly, this analysis provides a direct association between the physical properties of the scattering corona and specific features of the PSD, in particular its low- and high-frequency breaks. Since the scattering properties of the extended corona (i.e., $p$, $T_\alpha$, $n_s$, $t$, $r_2$) affect both the spectral and the temporal properties of the emitted radiation in a well-defined fashion, this model implies the presence of certain well-defined spectro-temporal correlations and provides the motivation to search for and model them in detail. Because such correlations will have to be related eventually to the dynamics of accretion, they could serve as a means for probing these dynamics along the lines of the simple example given above. Finally, the apparently random character of the observed light curves (Lochner et al. 1991) is attributed to the random (Poisson) injection of soft photons and the stochastic nature of the Comptonization process.

4. THE TIME AND PHASE LAGS

The fact that the variability associated with the light curves of Figure 3 is due in part to Compton scattering, rather than, for instance, the modulation of the accretion rate, affords an additional probe of the properties of scattering medium, namely, the study of time or phase lags in the light curves of two different energy bands as a function of the Fourier frequency. This particular issue has already been discussed in KHT and HKT, and in greater detail in HKC. It bears on the fact that time lags between photons of different energies depend on the scattering time of the plasma within which the Compton scattering takes place. In a corona with a density profile given by equation (1), there is
a linear relation between the scattering time and the crossing time of a given decade in radius, while in addition, the probability of scattering within a given (logarithmic) range in radius is constant (for \( p = 1 \)). Consider now the light curves in two different energy bands: the escaping photons in the higher of two energy bands suffer, on average, a larger number of scatterings, which, for \( p = 1 \), take place with equal probability at all radii; the information about the radii at which the additional scatterings took place is imprinted in the Fourier structure of these time lags. The linearity between the scattering and the crossing times then suggests that the lags grow proportionally to the Fourier period \( P \).

We have repeated the lag analysis outlined in the previous references, this time with model light curves appropriate for two different energies, generated artificially using the algorithm described in the previous section. These were generated by sums of shots as demanded by equation (6), with the function \( g(t) \) in each sum being the response function corresponding to a given photon energy. As noted in KHT, the very fact that higher energies require longer residence times of the photons in the scattering cloud leads to small but significant (from the point of view of the lags; see HKC) differences in their corresponding response functions. In particular, the power-law part of the response function is slightly flatter (larger \( z \)) and extends to slightly larger times (larger \( \beta \)). While eye inspection cannot discern any difference in the shape of the light curves corresponding to the two different energies, they are easily manifest in the Fourier decomposition of the time lags. As pointed out in HKC, these differences in the individual shot profiles suffice to produce lags in general agreement with observation.

Figure 5 presents the phase lags of our model light curves as a function of the Fourier frequency. Its magnitude and Fourier frequency dependence are very similar to those discussed above, insights into the nature of variability of the BHC sources can also be obtained from moments of the light curves in the time rather than the frequency domain. These have been used in the analysis of the light curves of accreting compact objects several times in the past (Lochner et al. 1991). Since we are able to produce models of these light curves we feel that it is instructive to compute the corresponding statistics associated with them so that one could directly compare them to those associated with the light curves obtained from observation. At present we will pay particular attention to two such statistics, the autocorrelation function and the time skewness of the light curves.

5. THE AUTOCORRELATION AND TIME SKEWNESS FUNCTIONS

In addition to the information provided by the power spectral densities (PSD) and the phase or time lags discussed above, insights into the nature of variability of the BHC sources can also be obtained from moments of the light curves in the time rather than the frequency domain. These have been used in the analysis of the light curves of accreting compact objects several times in the past (Lochner et al. 1991). Since we are able to produce models of these light curves we feel that it is instructive to compute the corresponding statistics associated with them so that one could directly compare them to those associated with the light curves obtained from observation. At present we will pay particular attention to two such statistics, the autocorrelation function and the time skewness of the light curves.

5.1. The Autocorrelation Function

This statistic provides a measure of the dependence of the flux (counting rate) \( F(t) \) of the source at a given time \( t \) on its flux, \( F(t + \tau) \), at a prior time. Assuming that the source variability consists of flares of a particular timescale, the autocorrelation function (ACF) provides a measure of this timescale. While the information contained in the autocorrelation function is related to that of the PSD through the fluctuation-dissipation theorem, since this statistic is also used to gauge the variability of accreting sources, for purposes of comparison, it is instructive to provide its form for the model light curves we produce in conjunction with the PSD.

Given that our model light curves are the incoherent sum of a large number of shots of the form given by equation (4), in order to exhibit directly the effect of the random injection of shots used to produce the light curve, we have chosen to compute the autocorrelation function in two ways: (1) through the convolution of the response function \( g(t) \) with itself, i.e.,

\[
\text{ACF}(\tau) = \int_{0}^{\infty} g(t)g(t + \tau)\, dt, \tag{7}
\]

where \( \tau \) is the associated time lag, and (2) directly from the model light curves produced by the procedure described above. Considering that the light curve consists of measurements of the flux \( F \) at \( N \) points in time, denoted as \( t_i \), separated in time by an interval \( \Delta t \), the autocorrelation function at a given lag, \( \tau = u\Delta t \), is given by the sum (ignoring normalization factors)

\[
\text{ACF}(\tau) = \sum_{i}^{N-u} [F(t_i) - \bar{F}][F(t_i + u\Delta t) - \bar{F}], \tag{8}
\]
where $\bar{F}$ is the mean value of the flux over the interval we consider.

The results of these two procedures are shown in Figures 6a and 6b. As can be seen, the autocorrelation functions computed by these two procedures are consistent with each other, with the fluctuations at the largest lags of Figure 6b due to the statistical nature of our light curve. This latter one is also very similar to that computed from the Cyg X-1 light curve by Meekins et al. (1984) and Lochner et al. (1991). The similarity of the autocorrelation function of that obtained from the observations further corroborates the model we have just presented.

At this point we would like to note that, usually, the data associated with the autocorrelation function are presented (e.g., Meekins et al. 1984; Lochner et al. 1991) in linear time coordinate; we believe that this presentation masks most of the interesting physics that are contained in the interval near zero. It is our contention that in the study of systems whose variability apparently spans several decades in frequency, like the accreting compact objects considered in the present note, the use of logarithmic rather than linear coordinates is instrumental; it is only in terms of the former that one can capture the entire range of the physical processes involved. The forms of the autocorrelation functions corresponding to those of Figures 6a and 6b in logarithmic time coordinate are given in Figures 6c and 6d.

### 5.2. The Time Skewness Function

This statistic measures the time asymmetry of a given light curve, i.e., whether the latter is composed of pulses having sharper rise than decay or vice versa. Because the autocorrelation function is symmetric in the lag variable $\tau$, it cannot give any such information about the shape of the light curve. This property can be assessed by computing moments of the light curve higher than the second. In particular, the third moment, $Q(\tau) = \langle u \rangle (= u \Delta t)$, as defined in Priedhorsky et al. (1979), provides the proper statistic. For a light curve given as an array of the flux $F(t_i)$ as described in the previous subsection, the skewness is given by the sum (ignoring again the normalization factors)

$$Q(\tau) = \sum_{i}^{N-1} \left[ F(t_i) - \bar{F} \right] \left[ F(t_i + u \Delta t) - \bar{F} \right] \times \left[ F(t_i) - F(t_i + u \Delta t) \right].$$

Given that our model light curves are sums of shots with a unique time profile, one can infer a priori several properties...
of the resulting light curves. The form of the shot profile (eq. [4]) suggests that, since the shots are asymmetric in time, the corresponding light curves should be also asymmetric, at least in situations in which the contribution of individual shots can be perceived. However, the presence of the long power-law tails associated with the individual shots suggests that over sufficiently long timescales, which encompass a large number of shots, the light curves should be largely symmetric in time. This argument is born out by both inspection of our model light curves and computation of the skewness parameter.

The prescription for creating model light curves discussed in § 3 offers the possibility of testing the above arguments by producing light curves with the proper characteristics, through variation of one or more of their control parameters. The specific parameter in this case is the mean time between shots $f$. In Figure 7 we present the skewness function of light curves corresponding to two different values of this parameter, namely, $f = 2, 10$. As expected, for the small values of this parameter, the light curves are indeed symmetric as can be assessed both by inspection and the value of skewness. For $f = 10$ the light curves become distinctly asymmetric out to time lags roughly $10t_0$, beyond which they appear again symmetric due to the superposition of a large number of shots.

This specific property then offers itself to observational testing: one should note that large values of $f$ lead not only to nonzero short time skewness, but also to large rms fluctuations of the light curve as alluded in § 3; in fact, the larger the $f$ value, the larger the corresponding rms fluctuations and also the value of the lag $\tau$ for which the skewness $Q(\tau)$ of the light curve deviates from a nonzero value. To the best of our knowledge, such a correlation has never been proposed or sought in the data. It would be of interest to see to what extent it is born by observations.

6. Conclusions, Discussion

We have presented above a general prescription for producing model light curves of accreting compact objects. Within our model, the observed aperiodic variability of these light curves is due to the stochastic nature of the Comptonization process in conjunction with the soft photon injection near the compact object by a Poisson process. Our prescription thus accounts naturally for the apparent lack of coherence in the absolute phase of the observed light curves and the apparent high coherence in the relative phase of the light curves of two different energy bands, since both the hard and soft shots have the same origin in the impulsive injection of the soft photons. Concerning the most common test of variability, namely, the PSD, our model relates it to spatial rather than timing properties of these systems, in particular to the spatial distribution of electrons in the Comptonizing hot corona. Thus it produces PSDs in agreement with observation and provides a novel framework within which one can easily accommodate the existence of QPOs and some of their systematics and dependence on the sources' luminosity. Within the present framework for the variability of accreting sources, their timing and spectral properties are intimately related in a way that could allow a probe of the dynamics of accretion onto the compact object through the use a combined spectro-temporal analysis. Finally, the model light curves produced using the prescription indicated above have a morphology that is in general agreement with the observed aperiodic variability of this class of sources.

The morphology of the light curves has been examined by computation of two statistical properties in the time domain, namely, the autocorrelation function and the time skewness. These, as computed for our model light curves, appear to be in good agreement with their (albeit limited) published literature forms corresponding to the light curves of the BHC Cyg X-1. Considering the simplicity of our models (a single type of shot, Poisson distribution in injection times), we are quite surprised that they work as well as they do. Our investigation points, in addition, to a correlation between the rms variability and the skewness of the corresponding light curves, which we feel should be tested against the observations.

The models presented herein draw heavily on the ideas presented in KHT, HKT, and HKC, namely, of very extended ($\sim 10^3-10^4 R_g$) hot electron coronae with power-law dependence of the electron density in radius. In our view, the importance of these models lies in the implied direct association between features in the observed PSDs and time lags (i.e., features of their Fourier domain characteristics) with features in the spatial domain (i.e., size,
radial density structure). Such an association is not dictated in any of the, albeit very few, alternative models of BHC variability, and to the best of our knowledge, neither has been proposed before in the literature.

It is of interest that both the sources' sizes and density structures, as deduced from our models (see also KHT; HKT; HKC), are incommensurate with those predicted by the most popular models. These models generally require the sources' size to be only a few $R_s$ (Shapiro, Lightman, & Eardley 1976), leading to a very narrow range in density (which in the present framework can be considered uniform) and therefore to an equally narrow range in time lags, in disagreement with observation. The recent, popular ADAFs (Narayan & Yi 1994) are indeed extended in radius, as demanded by our models, but their density profiles are proportional to $r^{-3/2}$, rather than the $r^{-1}$ profiles preferred by our fits to most (but not all) of the time lag observations obtained to date (HKC), suggesting that a variant of these models may be more appropriate in describing the dynamics of accretion in these sources. It is nonetheless important to point out that both of these density profiles have been associated with data from the same source (Cyg X-1), at different viewing periods, indicating that, according to the present model, the structure of a given source can vary drastically as a function of time.

We believe that the present model is sufficiently well defined and makes concrete enough predictions to allow its falsification or confirmation by more detailed observations. As such, we expect observations in the time domain to be of vital importance. Because the timing and spectral properties are intimately related within our model, testing the particular paradigm would most likely involve a combination of spectral and temporal correlations. We have provided fits of our models to a small set of observations, and derived the corresponding physical parameters of the scattering coronae for these systems; these indicate a great departure from our previous notions as to what they should be. We do not know as yet how to justify the values of the parameters obtained by our models; however, this is not the purpose of the present paper. We simply hope that these models will stimulate additional observational scrutiny and reanalysis of the data within their framework, leading possibly to novel insights that will further our understanding of the dynamics of accretion onto the compact object.

Last but not least, these models will have to be modified to incorporate additional spectral features such as the reflection features and the Fe lines, which the more conventional models of thin, cold disks and their associated hot coronae have addressed so far with significant success.

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