Comparison of DoA Estimation Algorithms in SDMA System

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With Space Division Multiple Access (SDMA) schemes employing digital signal processing algorithms in its smart antenna systems, it is possible to improve the system capacity of modern wireless communication systems by enabling user’s angular separation. Angular separation ability enhances reception in Signal-of-Interest direction and minimizes interference in Signal-of-Not-Interest direction. Direction of Arrival (DoA) algorithms are used for estimation of a number of incident plane waves on the antenna array and their incidence angles. This paper compares performance of three DoA algorithms: MUSIC, root-MUSIC and Capon applied on the uniform linear array in the presence of uncorrelated white noise. The simulation results show that MUSIC outperformed root-MUSIC and Capon in both required number of snapshots and number of array elements as well as in signal-to-noise ratio requirements.

Key words: Capon, DoA estimation, MUSIC, root-MUSIC, MVDR

1 INTRODUCTION

In our day to day life, overwhelming amount of devices such as personal digital assistants (PDA), TV remote controls, cellular phones, satellite TV receivers and mobile computers are based on wireless communications technology. Many more new technologies are emerging, which demand more spectrum and bandwidth for faster growth. But since the electromagnetic spectrum is a limited resource, it is not possible to get new spectrum allocation without the international coordination on the global level. Therefore efficient use of existing spectrum is of prime interest as a research objective. Efficient source and channel coding as well as reduction in transmission power, transmission bandwidth or both are significantly contributing to this challenging issue. With the advances in digital techniques, the frequency efficiency can be improved by Multiple Access Technique (MAT), which improves mobile users’ access to the scarce resources of base station and hence improves the system’s capacity [1]. By adding a new parameter of ‘space’ or ‘angle’ to the existing family of Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA), Code Division Multiple Access (CDMA) and Orthogonal Frequency Division Multiple Access (OFDMA) schemes, a new MAT known as ‘Space Division Multiple Access’ (SDMA) is established [2]. Generally, at the receiver’s side, the signal received is a superposition of multipath components combined with interferers’ signals, and with present noise. Thus, detection of the Signal-of-interest (SoI) is a tough task. The Smart Antenna System (SAS) embeds the antenna elements and the digital signal processing unit which enables it to form a beam to a desired direction taking into account the multipath signal components. Hence, Signal-to-Interference-and-Noise Ratio (SINR) can be improved
due to producing nulls towards the interferers in the direction of Signal-of-not-Interest (SonI) [3-5] and overall spectrum efficiency can be increased. In order to form beam in SoI direction, estimate of the number of plane waves arriving at the antenna array and the angle at which the waves are incident on antenna array is essential. The angle of wave incidence on antenna array is calculated using Direction of Arrival (DoA) estimation algorithms [3-5]. Thus the performance of SAS greatly depends on the performance of its DoA estimation algorithm.

A previous paper [4] investigated performance of the DoA algorithms MUSIC, ESPRIT and root-MUSIC on the uniform linear array in the presence of white noise. The simulation results showed to which extent the resolution of DoA techniques improves as number of snapshots, number of array elements and signal-to-noise ratio increase. In [4] mean square estimated error (MSE) was used for describing quality of DoA estimation algorithms. The simulation results showed that MUSIC algorithm was superior to the other two.

In this paper, we have investigated the performance of three very popular algorithms: Capon, MUSIC, and root-MUSIC. In our investigation, following parameters were considered: number of array elements, user spatial distribution (wide/narrow/combined angular separation), number of snapshots and Signal-to-Noise Ratio (SNR). Further, limitations and sensitivity of all algorithms to the DoA angles around direction along the antenna array are analysed.

The research and comparison of performance was limited to the special case of continuous wave (unmodulated signal), 2-dimensional model is used and the antennas used were with omnidirectional pattern (dipole like). Also we did not take mutual coupling into account.

The organisation of paper is as follows: Section 2 describes the background of what is SDMA scheme with SAS. Section 3 describes the framework for DoA estimation algorithms comparison, Section 4 describes the simulation results and is followed by conclusions in Section 5.

2 BACKGROUND ON SDMA USING SAS

As shown in Fig. 1, SDMA can be realised using SAS that generates multiple beam patterns; each beam would be assigned to one user (SoI) while producing nulls towards the interferers in the direction of Interferer (SonI), improving frequency reuse capability and increase in channel capacity. After the system downconverts the received signals to the baseband and digitizes them, it locates the SoI (user) using DoA algorithm and it continuously tracks the SoI and SonIs by dynamically changing the complex weights \( w_k \) (amplitudes and phases of the signals) using adaptive beamforming [6-8]. Each antenna system performs DoA estimation of all the signals to find SoI by calculating time delays between antenna elements. This is done in Parameter Estimator block as shown in Fig. 1. The output of Parameter Estimator block is fed to adaptive beamforming where a digital signal processor (DSP) uses cost (error) function for calculating the optimum filter weights using adaptive algorithm (Least Mean Square (LMS)) that generates an array factor for an optimal Signal-to-Interference Ratio (SIR). Specifically, this results in an array pattern, where ideally the maximum of the pattern is placed towards the intended user (SoI) while nulling or attenuating the interfering users (SonI) [1],[3].

3 FRAMEWORK FOR DOA ESTIMATION ALGORITHMS COMPARISON

In practice we are interested in estimation of signal parameters such as code, time, frequency and direction of arrival. There are two main categories of parameter estimation techniques: spectral-based and parametric approaches. Spectral-based approaches form some spectrum-like function of the parameters of interest, e.g., DoA. Locations of distinct separated highest peaks of the function are recorded as DoA estimates. The parametric approaches require simultaneous search for all parameters of interest and therefore often results in more accurate estimates at the expense of increased computational complexity, which may limit the capability of estimating parameters in the real-time [4-5].

3.1 System Model

An Uniform Linear Array (ULA) is considered, with \( J \) number of signals of frequency \( f_0 \) arriving at \( K \) number of array elements which are equally spaced at distance \( d \) between the elements. The channel noise for all channels is modeled as mutually non-coherent narrowband noise at \( f_0 \).

The steering vector of dimensions \( K \times 1 \) corresponding to DoA at some angle \( \theta \) is given by a column vector:

\[
v(\theta) = \left[ e^{-j(m-1)2\pi\sin(\theta)/\lambda} \right]^T, \quad m = 1, 2, \ldots K, \quad (1)
\]

where \( \lambda = c/f_0 \) is the wavelength, \( c \) being the velocity of light and \( d \) is the spacing between antenna elements, set for this research to 0.5\( \lambda \). The columnwise combination of all \( J \) steering vectors is called array manifold matrix \( V \) of dimensions \( K \times J \) given by \( V(\theta) = [v(\theta_1) : v(\theta_2) : \ldots : v(\theta_J)] \).

The spatial correlation (covariance) matrix for the \( N \) number of snapshots is given by:

\[
S_x = \frac{1}{N} \sum_{t=1}^{N} x(t) x(t)^H, \quad (2)
\]
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Antenna Elements

Fig. 1. SDMA employing Smart Antenna System (θ₁ to θₗ are DoAs of transmitted signals s₁ to sₗ; x₁ to xₖ are antenna-specific received signals; d = 0.5λ)

where H denotes the Hermitian operator and x denotes a vector of dimensions K × 1 consisting of received signals xₖ. Substitution of (1) into (2) results in

\[ S_x = \frac{1}{N} \sum_{t=1}^{N} V(\theta) s(t) s(t)^H V(\theta)^H + n(t) n(t)^H, \]  

(3)

\[ S_x = V(\theta) S_n V(\theta)^H + \sigma_w^2 I, \]  

(4)

where \( \sigma_w^2 \) is noise variance, I is an identity matrix of size \( K \times K \) and \( S_n \) is received signal power matrix.

3.2 Beamforming Techniques

The principle behind beamforming technique is to "steer" the array in one direction at a time and measure output power. The steering locations that give maximum power yield DoA estimates. A number of sources will correspond to a number of peaks. The array response is steered by forming a linear combination of the sensor outputs [5],[9].

The array output is:

\[ y = \sum_{i=1}^{K} w_i^* x_i = w^H x(t), \]  

(5)

where \( w = [w_1, w_2, \ldots, w_K]^T \) is a complex weighting vector, which determines the radiation pattern. The array output samples \( y(1), y(2), \ldots, y(N) \) give output power as:

\[ P_o/p(w) = \frac{1}{N} \sum_{t=1}^{N} |y(t)|^2 = \frac{1}{N} \sum_{t=1}^{N} w^H x(t) x(t)^H w \]

\[ = w^H S_x w. \]  

(6)

Bartlett’s [5] and Capon’s methods are based on Beamforming techniques.

3.3 Capon’s method

Capon’s method is also called Minimum Variance Distortionless Response algorithm (MVDR). The aim is to
minimize power contributed by noise and any signals coming from other direction than desired [9-11].

\[
\min_w \left( w^H S_x w \right) \quad \text{Subject to} \quad \left| w^H v(\theta) \right| = 1. \tag{7}
\]

The Capon’s weight vector is found to be:

\[
w_{\text{capon}} = \frac{S_x^{-1} v(\theta)}{v(\theta)^H S_x^{-1} v(\theta)}. \tag{8}
\]

Thus Capon’s output spectrum is:

\[
P_{\text{capon}}(\theta) = \frac{1}{v(\theta)^H S_x^{-1} v(\theta)}. \tag{9}
\]

### 3.4 Subspace Based Methods

In Subspace based method, the observed covariance matrix is decomposed into two orthogonal spaces: signal subspace and noise subspace. The DoA estimation is calculated from any one of the subspaces. The subspace based DoA estimation algorithms MUSIC and ESPRIT provide high resolution, they are more accurate and not limited to physical size of array aperture [4][9][12].

### 3.5 MUSIC algorithm

MUSIC stands for MUltiple SIgnal Classification, one of the high resolution subspace DoA algorithms, which gives an estimate of a number of arrived signals, hence their direction of arrival [5][9][12-16]. Estimation of DoA is performed from one of subspaces either signal or noise, assuming that noise in each channel is highly uncorrelated. This makes the covariance matrix diagonal.

Writing the spatial covariance matrix in terms of eigenvalues and eigenvectors[7-9] gives

\[
S_x = \sum_{i=1}^{K} T_i \varphi_i \varphi_i^H, \tag{10}
\]

\[
S_x = \varphi_i \beta \varphi_i^H, \tag{11}
\]

\[
\beta = \text{diag} \left[ T_1, T_2, \ldots, T_K \right]. \tag{12}
\]

The noise subspace eigenvalues and eigenvectors are:

\[
T_i, \quad i = J+1, J+2, \ldots, K, \tag{13}
\]

\[
\varphi_i, \quad i = J+1, J+2, \ldots, K. \tag{14}
\]

The noise subspaces can be written in the form of \( K \times (K - J) \) matrix:

\[
\mathbf{\vartheta}_N = [\varphi_{J+1}, \varphi_{J+2}, \ldots, \varphi_K]. \tag{15}
\]

Equation (15) indicates that the desired value DoA of \( \theta_1, \theta_2, \ldots, \theta_J \) can be found out by finding a set of vectors that span \( \mathbf{\vartheta}_N \) and projecting \( v(\theta) \) onto \( \mathbf{\vartheta}_N \) for all values of \( \theta \) and evaluating the \( J \) values of \( \theta \), where the projection is zero:

\[
\| v_i^H \mathbf{\vartheta}_N \|^2 = 0, \quad i = 1, 2, \ldots, J. \tag{16}
\]

Thus, MUSIC Pseudospectrum is given as:

\[
P_{\text{music}}(\theta) = \frac{1}{\left| v(\theta)^H \mathbf{\vartheta}_N \mathbf{\vartheta}_N^H v(\theta) \right|}. \tag{17}
\]

### 3.6 Root-MUSIC algorithm

Root-MUSIC is the polynomial version of MUSIC. The array manifold matrix is expressed in polynomial form by evaluating at \( z = e^{i\theta} \). If the eigendecomposition corresponds to the true spectral matrix, then MUSIC spectrum \( P_{\text{music}}(\theta) \) becomes equivalent to the polynomial on the unit circle and peaks in the MUSIC spectrum exist as roots of polynomial that lie close to the unit circle [5, 17-19]. That is \( P_{\text{music}}(z) \big|_{z = e^{i\theta}} = P_{\text{music}}(\theta) \). Ideally in absence of noise, the poles will lie exactly on the unit circle at the locations determined by DoA. Ultimately, we calculate the polynomial and select the \( J \) roots that are inside the unit circle. A pole of polynomial, \( D(z) \big|_{z = z_q} = |z_q| = |\exp[i q \arg(z_q)]| \) will result in a peak in the MUSIC spectrum at:

\[
\theta = \sin^{-1} \left\{ \lambda/2\pi d \right\} \arg [z_q], \quad q = 1, 2, \ldots, J. \tag{18}
\]

### 3.7 ESPRIT

The ESPRIT algorithm is described here for completeness. Its acronym stands for Estimation of Signal Parameter via Rotational Invariance Technique. This algorithm is more robust with respect to array imperfections than MUSIC [4]. Computation complexity and storage requirements are lower than for MUSIC algorithm, as it does not involve extensive search throughout all possible steering vectors, but rather it explores the rotational invariance property in the signal subspace created by two subarrays derived from original array with a translation invariance structure.

The detailed comparison between MUSIC, root-MUSIC and ESPRIT was made in [4].

### 4 RESULTS

For reliable comparison between algorithms, 50 trials were run for each case and their results were averaged before the comparison. Also, standard deviation (in degrees)
of DoA estimation (also in degrees) in these 50 trials is used for presenting accuracy and deviation of DoA estimation results: the higher the deviation – the higher the unreliability of the algorithm for given conditions. The MUSIC, Capon and root-MUSIC techniques for DoA estimations were simulated using MATLAB.

The simulations were run for three different sets of environment: one with wide angular separation $E_W = \{0^\circ, 25^\circ, 55^\circ\}$, one with narrow angular separation $E_N = \{-5^\circ, 10^\circ, 20^\circ\}$ and the last one with combination of wide and narrow separations $E_C = \{0^\circ, 10^\circ, 15^\circ, 40^\circ\}$. Furthermore, accuracy in the case of DoA close to $90^\circ$ (alongside the ULA) is considered. For analysing the performance of these algorithms, regarding impact of number of array elements, number of snapshots and SNR, simulation parameters were set as follows:

1. **Impact of number of array elements:** at SNR of 10 dB 200 snapshots for environment $E_W$ and $E_N$ were considered.

2. **Impact of number of snapshots:** array of 10 elements at SNR of 10 dB for environment $E_N$ was considered.

3. **Impact of SNR:** 16 element arrays with 200 snapshots for environment $E_C$ were considered.

4. **Performance with DoAs around $90^\circ$:** 16 element arrays with 200 snapshots and SNR of 10 dB were considered, with environments containing one DoA around $90^\circ$.

### 4.1 Impact of number of array elements

Table 1 and Table 2 show the performance of algorithms for different number of array elements. Figure 2 and Fig. 3 show standard deviation of DoAs for 50 runs as a measure of error of algorithms. Root-MUSIC showed significant down performance compared to MUSIC and Capon in case of wide angular separation.

While MUSIC and Capon errors are less than $1^\circ$ already for 5 element array and above, root-MUSIC error is still above $40^\circ$ coming down to $17^\circ$ for 6 element array and reaching below $1^\circ$ only for 7 element array and above.

Further the spectrum for wide angular separation and narrow angular separation has been plotted for Capon and MUSIC. As shown in Fig. 4 for wide angular separation, both Capon and MUSIC algorithms accurately detect DoA at $\{0^\circ, 25^\circ, 55^\circ\}$ and sharp peaks in the spectrum for estimated DoAs are visible.

Figure 3 shows that for narrow angular separation case with 5 element array, all three algorithms still had significant DoA estimation errors (around $33^\circ$, $32^\circ$ and $42^\circ$ for MUSIC, Capon and root-MUSIC algorithms respectively). MUSIC algorithm proved again to be more robust than the other two, having its error reduced to below $1^\circ$ already for 6 element array case.

Capon algorithm turned out to be the most sensitive to reduction in angular separation having an error of less than $1^\circ$ only at 9 elements array and above, root-MUSIC sensitivity to angular separation is higher than for MUSIC algorithm, but better than for Capon algorithm.

For case of narrow angular separation, Fig. 5 illustrates how for 16 element arrays both MUSIC and Capon spectra easily enable detection of all three DoAs.

### 4.2 Impact of number of snapshots

Figure 6 shows the performance of algorithms as a function of snapshots for narrower angular separation. It gives standard deviation of DoAs for 50 runs as a measure of error of algorithms. Below 40 snapshots all three

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**Table 1. Effect of number of array elements on performance of algorithms [SNR=10 dB, snapshots=200] in case of wide angular separation $E_W = \{0^\circ, 25^\circ, 55^\circ\}$**

| Array Size | Mean DoA estimation (in deg.) out of 50 runs |
|------------|--------------------------------------------|
|            | MUSIC | Capon | Root-MUSIC |
| 5          | -0.5  | 25.8  | 56          |
| 6          | -0.1  | 24.9  | 54.9        |
| 7          | 0.1   | 24.9  | 54.9        |
| 8          | 0.1   | 25    | 54.8        |
| 9          | 0.1   | 25.1  | 55.1        |
| 10         | -0.1  | 25    | 54.9        |
| 11         | -0.1  | 25    | 54.9        |
| 12         | 0.1   | 25    | 55          |
| 13         | 0     | 25    | 55          |
| 14         | 0     | 25    | 55          |
| 15         | -0.1  | 25    | 55.1        |
| 16         | 0     | 25    | 55          |
### Table 2. Effect of number of array elements on performance of algorithms [SNR=10 dB, snapshots=200] in case of narrow angular separation $E_N = \{-5^\circ, 10^\circ, 20^\circ\}$

| Array Size | Mean DoA estimation (in deg.) out of 50 runs |
|------------|-------------------------------------------|
|            | MUSIC | Capon | Root-MUSIC |
| 5          | -58.3 | -53.9 | -34.47     |
|            | -3.9  | -1.9  | 3.78       |
| 10.7       |       | 12.9  | 69.07      |
| 6          | -3.9  | -38.3 | -29.91     |
| 9.9        | -2.3  | 13.2  | 46.05      |
| 19.9       | 12.1  | 15.22 |
| 7          | -5    | -4.1  | -5.15      |
| 9.7        | -5.08 | 10.04 |
| 20         | 50    | 39.56 |
| 8          | -4.8  | -1.9  | 5.15       |
| 10.2       | -5    | 11    |
| 20.3       |       | 20.20 |
| 9          | -4.9  | -5.2  | -4.98      |
| 9.9        | -5.08 | 10.04 |
| 20         | 19.1  | 20.16 |
| 10         | -4.9  | -5.1  | -5.02      |
| 9.9        | -5.08 | 10.04 |
| 19.9       | 19.6  | 19.90 |
| 11         | -5    | -5    | -4.9       |
| 10         | -10.2 | 10.07 |
| 20         | 19.8  | 20.13 |
| 12         | -4.9  | -4.9  | -5.02      |
| 10         | -5    | 10    |
| 19.9       | 19.8  | 20.09 |
| 13         | -5    | -5    | -4.99      |
| 10         | -10.2 | 10.04 |
| 20         | 19.9  | 19.92 |
| 14         | -5    | -5    | -5.02      |
| 10         | -5.1  | 9.98  |
| 19.9       | 19.9  | 19.85 |
| 15         | -5    | -5    | -5.08      |
| 10         | -5.1  | 9.98  |
| 19.9       | 19.9  | 19.85 |
| 16         | -5    | -5    | -4.99      |
| 10         | -5    | 10.02 |
| 19.9       | 19.9  | 19.91 |

algorithms performed poorly. For snapshots of 10, Capon gives error as 25°, root-MUSIC nearly 23° and MUSIC 11°. This error for all three algorithms starts decreasing gradually to 10° for snapshots of 30 and to 1° at 50 snapshots.

From 50 snapshots onwards all three algorithms converge to the correct value having an error of less than 1°. In this region MUSIC algorithm performed slightly better than the other two.

As shown in Table 3, up to 40 snapshots all algorithms down-perform and from 50 snapshots onwards all three algorithms perform well, having an error around 1°. In this region MUSIC algorithm performed slightly better than the other two.

### 4.3 Impact of SNR

The comparison of Capon and MUSIC algorithms’ performance as a function of SNR value for $E_C$ environment is given by plotting their spectra (figures 7 and 8). The analysis was performed using environment $E_C$ where one separation was only 5° (other separations being 10° and 25°). In good SNR conditions like 10 dB and 0 dB, Capon has good resolution capability to estimate DoA at $\{0^\circ, 10^\circ, 15^\circ, 40^\circ\}$. However, as SNR value decreases, peaks in the spectrum start to disappear and hence decreases resolution capability of Capon for closely spaced signals like DoAs at 10° and 15°. From SNR of -6 dB onwards the peak related to DoA at 15° starts disappearing.

The performance of MUSIC algorithm as a function of SNR value is shown in Fig. 8 by plotting its spectrum using also environment $E_C$. In good SNR conditions like 10 dB
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Fig. 4. Capon and MUSIC spectrum [SNR = 10 dB, snapshots = 200, 16 element arrays and in case of wide angular separation $E_W = \{0^\circ, 25^\circ, 55^\circ\}$]

Fig. 5. Capon and MUSIC spectrum [SNR = 10 dB, snapshots = 200, 16 element arrays in case of narrow angular separation $E_N = \{-5^\circ, 10^\circ, 20^\circ\}$]

and 0 dB, MUSIC has good resolution capability to estimate DoA at $\{0^\circ, 10^\circ, 15^\circ, 40^\circ\}$. As SNR value decreases, peaks in spectrum start to disappear and hence decreases resolution capability of MUSIC for closely spaced signals like $10^\circ$ and $15^\circ$. But this effect of vanishing peak related to DoA at $15^\circ$ is much less severe compared to Capon’s even for SNR of -6 dB.

Thorough comparison of all three algorithms performance for different values of SNR in case of EC environment is given in Table 4.

Figure 9 gives the same comparison from the perspective of standard deviation of results. It visualises the fact that for environment $E_{EC}$ where one separation was only $5^\circ$, Capon performs poorly below 10 dB of SNR. Sensitivity of Capon towards narrower angular separation could be observed already by comparing Fig. 2 with Fig. 3. Root-MUSIC estimation’s standard deviation is higher than its MUSIC algorithm counterpart, but their performance aligns to satisfactory standard deviation of less than $1^\circ$.
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4.4 Performance with DoAs around 90°

Performance of all algorithms for DoAs around 90° is analysed for case of wide angular separation (Fig. 10, Fig. 11 and Fig. 12).

Figure 10 reveals that for DoA at 90° the algorithms fail to detect it due to appearance of false spectrum at negative 90° direction (180° flipping). Figure 11 reveals that for DoA at 86.5°, even though flipping is still observed in the spectrum, all the algorithms detect DOAs correctly. Detailed analysis at what extent algorithms can detect DoAs correctly near 90° is validated in Fig. 12 in the form of DoA detection percentage while varying DoA from 85.5° to 90°. For that purpose simulations were run for 100 times and the percentage of correct detection was calculated.

Figure 12 reveals that all algorithms detect the DoA correctly in 100% of cases for DoAs at 85.5° and 86°. For degrees closer to 90° there is degradation in correct DoA detection and likelihood for 180° flip in the result increases. For DoAs from 88.5° to 90° all the algorithms fail to detect correctly, since correct detection rate is around 50%, i.e. as random as flipping the coin.

| SNR(dB) | MUSIC | Capon | Root-MUSIC |
|---------|-------|-------|------------|
| 20      | 0     | 0     | 0.99       |
| 0       | 0     | -30.8 | -0.01      |
| 0.9     | 0     | 10.6  | 14.24      |
| 40      | 40.1  | 40.01 |
| -3      | -0.1  | -0.9  | 0.2        |
| 10      | 0.7   | 10.4  |
| 14.8    | 10.7  | 14.83 |
| 40.2    | 40.1  | 39.9  |
| -5      | -0.1  | -30.9 | -0.3       |
| 10.6    | -0.3  | 9.5   |
| 15.1    | 10.7  | 14.1  |
| 40.3    | 40.0  | 39.9  |
| -6      | -0.1  | -30.9 | -0.34      |
| 10.8    | 9.71  |
| 15.3    | 10.7  | 14.16 |
| 40.5    | 40.16 |
| -10     | -0.5  | -30.9 | -12.57     |
| 10.5    | -0.4  | -0.42 |
| 20      | 10.4  | 10.28 |
| 40.8    | 39.8  | 40.55 |
| -15     | -12   | -30.9 | -13.75     |
| -8.2    | -0.4  | -2.9  |
| -0.6    | 11.52 |
| 8.8     | 45    |

Table 4. Effect of varying SNR on algorithms [snapshots=200, array elements=16] in case of combination of wide and narrow angular separations $E_C = \{0^\circ, 10^\circ, 15^\circ, 40^\circ\}$
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Comparison included impact of number of snapshots, SNR, number of antenna array elements and performance comparison in the presence of wide, narrow and combined wide and narrow angular separation. Also, performance comparison for DoAs close to direction along the antenna array was given. As expected, performance improvement was observed with increasing number of array elements, increasing angular separation between the signals and with increasing SNR for all three algorithms. MUSIC algorithm showed superior performance by all criteria of comparison (SNR level, number of elements, number of snapshots) and in all environment conditions (narrow, wide and combined angular separation). When DoAs are close to the direction along the array, all three algorithms are prone to flip the result for 180° with similar likelihood. Incidence of this flipping was 0% at 86°, rising to incidence of around 50% for angles of 88.5° and above. The results presented offer both framework for DoA algorithms’ comparison and some hints about qualitative difference between Capon’s MUSIC and root-MUSIC algorithms.
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