Spectroscopy, decay properties and Regge trajectories
of the B and B_s mesons*

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Abstract: A Gaussian wave function is used for detailed study of the mass spectra of the B and B_s mesons using a Cornell potential incorporated with a $O(1/m)$ correction in the potential energy term and expansion of the kinetic energy term up to $O(p^{10})$ for relativistic correction of the Hamiltonian. The predicted excited states for the B and B_s mesons are in very good agreement with results obtained by experiment. We assign $B_2(5747)$ and $B_{s2}(5840)$ as the $1^3P_2$ state, $B_1(5721)$ and $B_{s1}(5830)$ as the $1P_1$ state, $B_0(5732)$ as the $1^3P_0$ state, $B_{s1}(5850)$ as the $1P_1'$ state and $B(5970)$ as the $2^3S_1$ state. We investigate the Regge trajectories in the $(J,M^2)$ and $(n_+,M^2)$ planes with their corresponding parameters. The branching ratios for leptonic and radiative-leptonic decays are estimated for the B and B_s mesons. Our results are in good agreement with experimental observations as well as outcomes of other theoretical models.

Keywords: potential model, mass spectrum, decay constant, Regge trajectories

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1 Introduction

The last decade has been marked by noteworthy experimental progress in understanding the spectroscopy of mesons containing heavy-light quarks. In the case of the B and B_s mesons, the ground state as well as first few orbitally excited states have been well established experimentally. Other radially and orbitally excited states, however, require further experimental investigation [1–5].

In 2013, the CDF collaboration investigated the $B^0\pi^+$ and $B^+\pi^-$ invariant mass distributions using data at $\sqrt{s} = 1.96$ TeV pp collisions corresponding to an integrated luminosity of 9.6 fb$^{-1}$. They found evidence for a new resonance, the B(5970). The reported mass and width of the neutral state are $5978 \pm 5 \pm 12$ MeV and $70^{+30}_{-26} \pm 30$ MeV respectively. More recently, the LHCb collaboration also studied the $B^0\pi^+$ and $B^+\pi^-$ invariant mass distribution using $\sqrt{s} = 7$ and $\sqrt{s} = 8$ TeV corresponding to an integrated luminosity of 3.0 fb$^{-1}$. Precise measurements were made of the mass and width of the $B_1(5721)$ and $B_2^*(5747)$, as well as observing two excited states, $B_3(5840)^{0,+}$ and $B_3(5960)^{0,+}$ [6, 7].

As well as the experimental observations, the states of the B and B_s mesons have already been predicted by various theoretical models. These predictions employ various relativistic or relativized quark models, potential models, the Bethe-Salpeter equation as well as constituent quark models based on the Dirac equation [2–4, 8–13]. Experimental exploration of newly observed and unconfirmed states of the B and B_s mesons motivates us to carry out a comprehensive theoretical study.

In this article, we employ a potential model, incorporating corrections to the potential energy part besides the kinematic relativistic correction of the Hamiltonian, to understand the B and B_s mesons. Using our predicted masses for the B and B_s mesons, we plot the Regge trajectories in both the $(M^2 \to J)$ and $(M^2 \to n)$ planes (where J is the spin and n is the principal quantum number). These play a vital role in identifying any new (experimentally) excited states as well as for information about the quantum numbers of the particular state [14].

In the present work, the leptonic and radiative leptonic decay widths are estimated. The radiative leptonic

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decay of mesons can be equal to or larger than the pure-leptonic decay, which can allow study of the effect of the strong interaction in the decay [15].

The article is organized as follows. In Section 2.1, we present the theoretical framework for the mass spectra. Section 2.2 introduces the theoretical framework for leptonic, dileptonic and radiative leptonic decay widths. The results of mass spectra, leptonic and radiative-leptonic decays are discussed in Section 3. In Section 3.1, we investigate Regge trajectories in the ($J, M^2$) and ($n_c, M^2$) planes. Finally, we present the conclusion in Section 4.

2 Methodology

2.1 Cornell potential with $O\left(\frac{1}{m}\right)$ corrections

For spectroscopic study of the B and B* mesons, we employ the Hamiltonian [16–18]

$$H = \sqrt{p^2 + m^2} + \sqrt{p^2 + m^2} + V(r),$$  

where $p$ stands for the relative momentum of the meson, $m_Q$ for the mass of the heavy quark, $m_\ell$ for the mass of the light anti-quark, and $V(r)$ is the meson potential, which can be written as [19],

$$V(r) = V^{(0)}(r) + \left(\frac{1}{m_Q} + \frac{1}{m_\ell}\right)V^{(1)}(r) + O\left(\frac{1}{m^2}\right).$$  

Here, $V^{(0)}$ is a Cornell-like potential [20],

$$V^{(0)}(r) = -\frac{\alpha_c}{r} + A r + V_0$$  

and $V^{(1)}$ in leading order perturbation theory yields

$$V^{(1)}(r) = -C_F C_A \alpha_s^2/4r^2$$  

where $\alpha_c = (4/3)\alpha_s(M^2)$, $\alpha_s(M^2)$ is the strong running coupling constant, $A$ is a potential parameter, $V_0$ is a constant, and $C_F = 4/3$ and $C_A = 3$ are the Casimir charges [19].

Here, we use the Ritz variational strategy for the study of the B and B* mesons. In the heavy-light mesons, the confining interaction plays an important role. We employ a Gaussian wave function to predict the expectation values of the Hamiltonian [18, 21, 22]. The Gaussian wave function in position space has the form

$$R_{nl}(\mu, r) = \mu^{3/2} \left(\frac{2(n-1)!}{F(n+1/2)}\right)^{1/2} (\mu r)^l e^{-\mu^2 r^2} L_{n-1}^{1/2}(\mu^2 r^2)$$  

and in momentum space has the form

$$R_{nl}(\mu, p) = \frac{(-1)^n}{\mu^{3/2}} \left(\frac{2(n-1)!}{F(n+1/2)}\right)^{1/2} \left(\frac{p}{\mu}\right)^l e^{-\mu^2 p^2} L_{n-1}^{1/2}(\mu^2 p^2).$$

Here, $L$ and $\mu$ represent the Laguerre polynomial and the variational parameter respectively. Using the virial theorem [17], we found the value of variational parameter $\mu$ for each state, for the chosen value of potential parameter $A$,

$$\langle K.E. \rangle = \frac{1}{2} \left(\frac{dV}{dr}\right).$$  

To justify the relativistic approach for quarks within the heavy-light mesons, we expand the kinetic energy of the quarks, retaining powers up to $O(p^3)$, from the Hamiltonian Eq. (1) [18]. In the virial theorem, we use a momentum space wave function to determine the expectation value of the kinetic energy part, whereas a position space Gaussian wave-function is used to determine the expectation value of the potential energy part.

Here, the center of weight mass is the expectation value of the Hamiltonian. By fixing the potential constant $V_0$, $\alpha_s$, and $A$, we fitted the ground state center of weight mass and matched it with the PDG value. The fitted potential parameters are listed in Table 1. Using the following equation, we fitted the ground state center of weight mass [23, 24]:

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P),$$  

where $M_V$ is a vector and $M_P$ is a pseudoscalar meson ground state mass. Using the potential parameter listed in Table 1, we predicted the $S$, $P$, and $D$ state wave center of weight masses of the mesons, which are tabulated in Table 3. For the $nJ$ state comparison, we computed the center of weight mass from the respective theoretical values as [23]:

$$M_{CW,n} = \frac{\sum_j (2J+1) M_{nJ}}{\sum_j (2J+1)},$$  

where $M_{CW,n}$ is the center of weight mass of the $n$ state and $M_{nJ}$ is the meson mass in the $nJ$ state. The hyperfine and spin-orbit shifting of the low-lying $S$, $P$ and $D$ states have been estimated by the spin-dependent part of the conventional one gluon exchange potential between the quark and anti-quark [18, 25–27]:

$$V_{SD}(r) = \left(\frac{L \cdot S_Q + L \cdot S_\ell}{2m_Q^2 m_\ell^2} + \frac{1}{2m_Q m_\ell} \frac{dV^{(0)}(r)}{dr} + \frac{8}{3} \alpha_s \frac{1}{r^2} \right)$$

$$+ \frac{4}{3} \alpha_s \frac{1}{m_Q m_\ell} \frac{L \cdot S}{r^3} + \frac{4}{3} \alpha_s \frac{2}{3 m_Q m_\ell} S_Q \cdot S_\ell 4\pi \delta(r)$$

and

$$+ \left(3S_Q \cdot n\right) \left(3S_\ell \cdot n\right) \frac{1}{r^2}, \quad n = \frac{r}{r}.$$  

where $V^{(0)}(r)$ stands for the phenomenological potential. In the spin-dependent part, the first part stands for the
conjugating mixing matrix obtained using Eq. (10) [27]. Charge the light degrees of freedom phase convention depends on the order of coupling

\[ j \]

of the light quark spin and \( L \) is the orbital angular momentum of the light quark. The quantum numbers of the excited \( L = 1 \) states are formed by combining \( S_\text{Q} \) and \( j_\text{q} \). For \( L = 1 \) we have \( j_\text{q} = 1/2 \) (\( j = 0, 1 \)) and \( j_\text{q} = 3/2 \) (\( j = 1, 2 \)) states. These states are denoted as \( ^3P_0, ^1P_1 \) \( (j_\text{q} = 1/2) \), \( ^1P_1 \) \( (j_\text{q} = 3/2) \) and \( ^3P_2 \) in the case of the B and B\( _s \) meson. [18, 27]

\[
\text{Mass eigenstates for heavy-light meson are constructed by } jj \text{ coupling. The quantum numbers } S_\text{Q} \text{ and the light degrees of freedom } j_\text{q} = s_\text{q} + L \text{ are individually conserved. Here, } S_\text{Q} \text{ is the heavy quark spin, } s_\text{q} \text{ is the light quark spin and } L \text{ is the orbital angular momentum of the light quark. The quantum numbers of the excited } L = 1 \text{ states are formed by combining } S_\text{Q} \text{ and } j_\text{q} \text{. For } L = 1 \text{ we have } j_\text{q} = 1/2 \text{ (} j = 0, 1 \text{)} \text{ and } j_\text{q} = 3/2 \text{ (} j = 1, 2 \text{)} \text{ states. These states are denoted as } ^3P_0, ^1P_1 \text{ (} j_\text{q} = 1/2 \text{), } ^1P_1 \text{ (} j_\text{q} = 3/2 \text{) and } ^3P_2 \text{ in the case of the B and B}_s \text{ meson. [18, 27]}
\]

Independently of the total spin \( J \) projection, one has

\[
\begin{align*}
|^{2L+1}L_{L+1}\rangle &= |J=L+1, S=1\rangle, \\
|^{2L+1}L_L\rangle &= \sqrt{\frac{L}{L+1}} |J=L, S=1\rangle + \sqrt{\frac{L+1}{2L+1}} |J=L, S=0\rangle, \\
|^{2L-1}L_L\rangle &= \sqrt{\frac{L+1}{2L+1}} |J=L, S=1\rangle - \sqrt{\frac{L}{2L+1}} |J=L, S=0\rangle,
\end{align*}
\]

where \( |J,S\rangle \) are the state vectors with the given values of the total quark spin \( S=s_\text{q}+S_\text{Q} \), hence the potential terms of the order of \( 1/m_\text{q}^2 m_\text{Q} \), \( 1/m_\text{Q}^2 \) lead to the mixing of the levels with the different \( j_\text{q} \) values at the given \( J \) values. The tensor forces (fourth term in Equation (10)) are zero at \( L=0 \) or \( S=0 \).

The heavy-heavy flavored meson states with \( J = L \) are mixtures of spin-triplet \( |^3L_L\rangle \) and spin-singlet \( |^1L_L\rangle \) states:

\[
\begin{align*}
|\psi_J\rangle &= |^1L_L\rangle \cos \phi + |^3L_L\rangle \sin \phi, \\
|\psi'_J\rangle &= -|^1L_L\rangle \sin \phi + |^3L_L\rangle \cos \phi,
\end{align*}
\]

where \( \phi \) is the mixing angle and the primed state has the heavier mass. Such mixing occurs due to the nondiagonal spin-orbit and tensor terms in Eq. (10). The masses of the physical states were obtained by diagonalizing the mixing matrix obtained using Eq. (10) [27]. Charge conjugating \( q\bar{q} \) into \( \bar{q}q \) flips the sign of the angle and the phase convention depends on the order of coupling \( L, S_\text{Q} \) and \( s_\text{q} \). Radiative transitions are particularly sensitive to the \( ^3L_L-^1L_L \) mixing angle, with predictions giving radically different results in some cases of different models [28, 29]. The values of mixing angles for the \( P \) and \( D \) states are tabulated in Table 2.

### Table 1. Potential parameters.

| meson | \( \alpha \) | \( A/\text{GeV}^2 \) | \( V_0/\text{GeV} \) |
|-------|---|---|---|
| B     | 0.6675 | 0.118 | -0.00742 |
| B\( _s \) | 0.59025 | 0.140 | -0.0108 |

### Table 2. Mixing angles \( \theta \) for B and B\( _s \) mesons.

| meson | \( \theta^1_P \) | \( \theta^3_P \) | \( \theta^1_D \) | \( \theta^3_D \) | \( \theta^1_{1D} \) | \( \theta^3_{1D} \) |
|-------|---|---|---|---|---|---|
| B     | -14.66 | -15.71 | -16.04 | 71.99 | 71.89 | 72.86 |
| B\( _s \) | -9.97 | -12.41 | -13.01 | 72.17 | 72.04 | 71.99 |

In the present study, the quark masses are \( m_{u/d} = 0.46 \text{ GeV} \), \( m_u = 4.530 \text{ GeV} \) and \( m_s = 0.586 \text{ GeV} \), to reproduce the ground state masses of the B and B\( _s \) mesons.

#### 2.2 Leptonic, radiative leptonic and dileptonic branching fractions

To predict the leptonic branching fractions for the \((1^1S_0) \) B mesons, we employed the formula

\[
BR = \Gamma \times \tau,
\]

where \( \Gamma \) (leptonic decay width) is given by [30]

\[
\Gamma(B^+ \rightarrow \ell \nu) = \frac{G_F^2 |V_{ub}|^2 m_B^3}{8\pi^3} \left[ 1 - \frac{2 m_{\ell}^2}{M_B^2} \right] M_B.
\]

For the calculation of the radiative leptonic decay \( B^- \rightarrow \gamma \nu(l=e, \mu) \) width, we employ the formula [31]

\[
\Gamma(B^- \rightarrow \gamma \nu) = \frac{\alpha G_F^2 |V_{ub}|^2}{2592\pi^2} f_B^2 m_{B^-}^3 \left[ x_u + x_\mu \right]
\]

where

\[
x_u = \left( 3 - \frac{2 m_B}{m_u} \right)^2,
\]

and

\[
x_\mu = \left( 3 - \frac{2 m_B}{m_\mu} \right)^2
\]

Due to the conservation of charge, single charged lepton decays as well as decays to two muons are forbidden at the primary transition, but such types of decay occur in higher-order transitions. Due to Cabibbo-Kobayashi-Maskawa and helicity suppression, there is an expectation of very small branching fraction for the \( B^0 \rightarrow \mu^+ \mu^- \) and \( B^0 \rightarrow \mu^+ \mu^- \) compared to the dominant \( b \rightarrow \ell \) transitions. Hence, one can consider dileptonic decays as rare decays. The decay width for the \( B^0 \) and \( B^0 \) mesons is given by [3, 32, 33]

\[
\Gamma(B^0 \rightarrow \ell^+ \ell^-) = \frac{G_F^2}{\pi} \left[ \frac{\alpha}{4\pi \sin^2 \Theta_W} \right]^2 f_B^2 m_{B^0}^3 m_{B^0} \times \left[ 1 - \frac{4 m_{\ell}^2}{m_{B^0}^2} \right] |V_{us} V_{ub}|^2 |C_{10}|^2
\]
The branching ratio for $B_0^\pm \rightarrow l^+l^-$ is

$$BR \rightarrow \Gamma_{B_0^\pm \rightarrow l^+l^-} \times \tau_{B_0^\pm},$$

(22)

$G_F$ is the Fermi coupling constant, $f_{B_0}$ is the corresponding decay constant, and $C_{10}$ is the Wilson coefficient given by [3, 34, 35].

$$C_{10} = \frac{x_t}{8} \left[ \frac{x_t + 2}{x_t - 1} + \frac{3x_t - 6}{(x_t - 1)^2 \ln x_t} \right],$$

(23)

where $\eta_1 = 1.026$ is the next-to-leading-order correction [3, 35], $\Theta_W \approx 28^\circ$ is the weak mixing angle (Weinberg angle) [36], and $x_t = (m_t/m_w)^2$.

The decay constants were obtained from the Van-Royen-Weisskopf formula [37]. Incorporating the first order QCD correction factor,

$$f_{P/S}^2 = \frac{12 |\psi_{P/S}(0)|^2}{M_{P/S}^2} \tilde{C}^2(\alpha_s),$$

(24)

where $\tilde{C}^2(\alpha_s)$ is the QCD correction factor given by [38]

$$\tilde{C}^2(\alpha_s) = 1 - \frac{\alpha_s}{\pi} \left[ 2 - \frac{m_q - m_{\bar{q}}}{m_q + m_{\bar{q}}} \ln \frac{m_q}{m_{\bar{q}}} \right].$$

(25)

We calculate the leptonic branching fractions using Eq. (17), the radiative leptonic branching ratio using Eq. (18) for the B meson and the dileptonic branching ratio with corresponding decay width using Eq. (21) for the B and $B_s$ mesons. We have taken $\tau_B = 1.638$ ps, $\tau_{B_0} = 1.510$ ps [1] and the calculated values of the pseudoscalar decay constant $f_{B} = 0.146(0.150)$ GeV and $f_{B_0} = 0.187(0.203)$ GeV with/without QCD correction using the masses obtained from Tables 4 and 5.

3 Results and discussion

The center of weight masses for the S, P and D states are estimated using Eq. (8) and Eq. (9), and the results are tabulated in Table 3. In the case of the B meson, the center of weight masses for the 1S, 2S, 3S, 1P, 2P, 1D and 2D states are in good agreement with the outcomes of other theoretical models, whereas masses for 4S, 5S, 3P and 3D are somewhat underestimated. In the case of the $B_s$ meson, the center of weight masses are in good agreement with the outcomes of other theoretical models.

The estimated mass spectra for the B and $B_s$ mesons are tabulated in Tables 4 and 5 with the spectroscopic notation $m_{2S+1}L_J$. The mass spectra are also depicted graphically in Figs. 1 and 2. The predicted masses of the B and $B_s$ mesons are in close agreement with experimental observations. The difference between the predicted and experimentally observed values of the mass of the B meson is 7 MeV for $1^1S_0$, 2 MeV for $1^3S_1$, 20 MeV for $1^3P_0$, 7 MeV for $1^3P_1$ and matches for the $1^3P_2$ state. Similarly, the difference between the predicted and experimentally observed values of the mass of the $B_s$ meson is 1 MeV for $1^1S_0$, 2 MeV for $1^3S_1$, 1 MeV for $1^3P_1$, 11 MeV for $1^3P_1'$ and matches for the $1^3P_2$ state.

| meson | state | $\mu$ | $M_{CW}$ | Expt [1] | [2] | [3] | [4] | [39] | [40] | [14] | [41] |
|-------|-------|-------|----------|----------|-----|-----|-----|------|------|------|------|
| B     | 1P    | 0.309 | 5.740    | 5.748    | 5.737 | 5.759 | 5.785 | 5.774 | 5.745 | 5.696 |
|       | 2P    | 0.249 | 6.301    | 6.224    | 6.127 | 6.188 | 6.213 | 6.250 | 6.249 | 6.030 |
|       | 3P    | 0.216 | 6.828    | 6.482    |       |       |       | 6.669 |       | 6.265 |
|       | 1D    | 0.278 | 6.057    | 6.048    | 6.065 | 6.108 | 6.079 | 6.106 | 6.924 |       |
|       | 2D    | 0.232 | 6.596    | 6.467    | 6.429 | 6.377 | 6.466 | 6.495 | 6.540 | 6.183 |
|       | 3D    | 0.205 | 7.110    | 6.769    |       |       |       |       |       |       |
| $B_s$ | 1P    | 0.376 | 5.835    | 5.827    | 5.838 | 5.838 | 5.858 | 5.851 | 5.844 | 5.805 |
|       | 2P    | 0.303 | 6.380    | 6.280    | 6.233 | 6.254 | 6.290 | 6.310 | 6.343 | 6.161 |
|       | 3P    | 0.264 | 6.889    | 6.603    |       |       |       | 6.768 |       | 6.413 |
|       | 1D    | 0.337 | 6.150    | 6.116    | 6.181 | 6.117 | 6.181 | 6.147 | 6.200 | 6.047 |
|       | 2D    | 0.283 | 6.668    | 6.513    | 6.626 | 6.450 | 6.539 | 6.546 | 6.635 | 6.323 |
|       | 3D    | 0.251 | 7.162    | 6.912    |       |       |       |       |       |       |
Table 4. Predicted masses (in GeV) for the B meson.

| state  | JPT | present work | Expt. [1] | [2] | [3] | [4] | [39] | [40] | [14] | [41] |
|--------|-----|--------------|-----------|-----|-----|-----|------|------|------|------|
| 1S0    | 0−  | 5.287        | 5.280 (B0) | 5.280 | 5.279 | 5.273 | 5.309 | 5.266 | 5.280 | 5.277 |
| 1S1    | 1−  | 5.323        | 5.325 (B⁺) | 5.329 | 5.325 | 5.331 | 5.369 | 5.330 | 5.326 | 5.325 |
| 2S0    | 0−  | 5.926        | 5.910      | 5.804 | 5.893 | 5.904 | 5.930 | 5.890 | 5.822 |
| 2S1    | 1−  | 5.947        | 5.961 B(5970) [42] | 5.939 | 5.824 | 5.932 | 5.934 | 5.946 | 5.906 | 5.848 |
| 3S0    | 0−  | 6.492        |           | 6.369 | 6.242 | 6.334 | 6.387 | 6.379 | 6.117 |
| 3S1    | 1−  | 6.508        |           | 6.391 | 6.254 | 6.355 | 6.396 | 6.387 | 6.136 |
| 4S0    | 0−  | 7.027        |           | 6.641 |       | 6.773 | 6.781 | 6.335 |
| 4S1    | 1−  | 7.039        |           | 6.649 |       | 6.779 | 6.786 | 6.351 |
| 5S0    | 0−  | 7.538        |           |       |       |       |       |       |
| 5S1    | 1−  | 7.549        |           |       |       |       |       |       |

Fig. 1. (color online) Mass spectrum of the B meson.

Fig. 2. (color online) Mass spectrum of the Bₙ meson.

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The predicted leptonic branching fractions for the B meson are tabulated in Table 6. The predicted BR$_\mu$ and BR$_e$ indicates that the predictions are in good agreement with the experimental outcomes, while BR$_\tau$ is slightly underestimated with respect to experimental observations. In the literature, various methods are used to calculate the radiative leptonic decay rate and branching ratio. In Ref. [51], the calculated branching ratio $B \rightarrow l\nu\gamma$ is of the order of $10^{-6}$ in a non-relativistic quark model. In Ref. [48] with the perturbative QCD approach, it is found that the branching ratio $B^+ \rightarrow e^+\nu\gamma$ is of the order of $10^{-6}$. In the factorization approach, it is found to be of order $10^{-6}$ for the B meson [11, 49, 50, 52]. In the form factors parameterizing approach, it is found to be the order of $10^{-7}$ [48]. We also found a branching ratio of the order of $10^{-7}$ for the B meson.

Our predicted decay widths and branching ratios for the rare leptonic decays $B \rightarrow l^+l^-$ and $B \rightarrow l^+l^-,(l=\mu, \tau, e)$ are shown in Tables 7 and 8. The predicted branching ratios $B \rightarrow \mu^+\mu^- = 2.529 \times 10^{-9}$ and $B \rightarrow \mu^+\mu^- = 1.002 \times 10^{-10}$ are in excellent agreement with the experimental results published by CMS and LHCb [44, 46, 53]. The predicted branching ratios $B_{s}^0 \rightarrow e^+e^- = 5.921 \times 10^{-14}$ and $B^0 \rightarrow e^+e^- = 2.345 \times 10^{-15}$ are in good agreement with Ref. [3] (Dirac formalism), Ref. [32] (Standard Model with reduced theoretical uncertainty) and Ref. [47] (lattice results). We have also predicted the branching ratio with corresponding decay width of other rare leptonic $(l=\tau,e)$ decays for the B and Bs mesons. Due to the large uncertainty in experimental observations in rare leptonic $(l=\tau,e)$ decays for the B and Bs mesons, it is difficult to come to a reasonable conclusion, but our results are in relatively good agreement with the predictions by other theoretical models.

| state  | $J^P$ | present work | Expt. [1] | 2 | 3 | 4 | 39 | 40 | 14 | 41 |
|--------|-------|--------------|-----------|---|---|---|----|----|----|----|
| $1^+S_0$ | 0$^-$ | 5.367 | 5.366 $B_{s}^0$ | 5.362 | 5.366 | 5.355 | 5.390 | 5.355 | 5.372 | 5.366 |
| $1^3S_1$ | 1$^-$ | 5.413 | 5.415 $B_{s}^+$ | 5.413 | 5.415 | 5.416 | 5.447 | 5.417 | 5.414 | 5.417 |
| $2^1S_0$ | 0$^-$ | 6.003 | 5.977 | 5.939 | 5.962 | 5.985 | 5.998 | 5.976 | 5.939 |
| $2^3S_1$ | 1$^-$ | 6.029 | 6.003 | 5.956 | 5.999 | 6.013 | 6.016 | 5.999 | 5.966 |
| $3^1S_0$ | 0$^-$ | 6.556 | 6.415 | 6.419 | 6.409 | 6.441 | 6.467 | 6.254 |
| $3^3S_1$ | 1$^-$ | 6.575 | 6.435 | 6.427 | 6.429 | 6.449 | 6.475 | 6.274 |
| $4^1S_0$ | 0$^-$ | 7.071 | 6.859 | 6.812 | 6.874 | 6.487 |
| $4^3S_1$ | 1$^-$ | 7.087 | 6.864 | 6.818 | 6.879 | 6.504 |
| $5^1S_0$ | 0$^-$ | 7.565 | 6.029 | 6.003 | 5.956 | 5.999 | 6.013 | 6.016 |
| $5^3S_1$ | 1$^-$ | 7.797 | 6.157 | 6.159 | 6.209 | 6.172 | 6.185 | 6.110 | 6.189 | 6.043 |

Table 5. Predicted masses (in GeV) for the $B_s$ meson.
Table 6. Leptonic branching fractions of the B meson.

| process          | $\Gamma(B^0_{\theta}\rightarrow l^+l^-)/$keV | $BR_{\ell}$ | $BR_{\mu}$ | $BR_{e}$ |
|------------------|---------------------------------------------|-------------|-------------|-----------|
| this work        |                                              | 0.822×10^{-4} | 0.37×10^{-7} | 8.64×10^{-12} |
| PDG [1]          |                                              | (1.14±0.27)×10^{-4} | <1.0×10^{-6} | <9.8×10^{-7} |

Table 7. Branching ratio with corresponding rare leptonic decay width of the $B^0$ meson.

| process          | $\Gamma(B^0_{\theta}\rightarrow l^+l^-)/$keV | $BR$ | others          |
|------------------|---------------------------------------------|------|-----------------|
| $B^0\rightarrow\mu^+\mu^-$ | 4.341×10^{-17}          | 1.002×10^{-10} | $(3.9^{+1.6}_{-1.4})×10^{-10}$ [1] |
|                  | 4.406×10^{-17}          |      | <1.1×10^{-9} [44] |
|                  |                            |      | <9.4×10^{-10} [45] |
|                  |                            |      | <7.4×10^{-10} [46] |
|                  |                            |      | 1.20×10^{-10} [47] |
|                  |                            |      | (1.06±0.09)×10^{-10} [32] |
|                  |                            |      | 1.018×10^{-10} [3] |
| $B^0\rightarrow\tau^+\tau^-$ | 9.097×10^{-15}          | 2.099×10^{-8} | <4.1×10^{-3} [1] |
|                  | 9.232×10^{-15}          |      | 2.55×10^{-8} [47] |
|                  |                            |      | (2.22±0.19)×10^{-8} [32] |
|                  |                            |      | 2.133×10^{-8} [3] |
| $B^0\rightarrow e^+e^-$  | 1.016×10^{-21}          | 2.345×10^{-15} | <8.3×10^{-8} [1] |
|                  | 1.028×10^{-21}          |      | 2.82×10^{-15} [47] |
|                  |                            |      | (2.48±0.21)×10^{-15} [32] |
|                  |                            |      | 2.376×10^{-15} [3] |

Table 8. Branching ratio with corresponding rare leptonic decay width of the $B^0_{\theta}$ meson.

| process          | $\Gamma(B^0_{\theta}\rightarrow l^+l^-)/$keV | $BR$ | others          |
|------------------|---------------------------------------------|------|-----------------|
| $B^0_{\theta}\rightarrow\mu^+\mu^-$ | 1.101×10^{-15}          | 2.529×10^{-9} | $(2.9^{+0.7}_{-0.6})×10^{-9}$ [1] |
|                  | 1.583×10^{-15}          |      | 3.04×10^{-9} [44] |
|                  |                            |      | 3.24×10^{-9} [45] |
|                  |                            |      | 2.9×10^{-9} [46] |
|                  |                            |      | 3.40×10^{-9} [47] |
|                  |                            |      | (3.65±0.23)×10^{-9} [32] |
|                  |                            |      | 3.602×10^{-9} [3] |
| $B^0_{\theta}\rightarrow\tau^+\tau^-$ | 2.335×10^{-13}          | 5.364×10^{-7} | 7.22×10^{-7} [47] |
|                  | 3.361×10^{-13}          |      | (7.73±0.23)×10^{-7} [32] |
|                  |                            |      | 7.674×10^{-7} [3] |
| $B^0_{\theta}\rightarrow e^+e^-$  | 2.577×10^{-20}          | 5.921×10^{-14} | <2.8×10^{-7} [1] |
|                  | 3.695×10^{-20}          |      | 7.97×10^{-14} [47] |
|                  |                            |      | (8.54±0.55)×10^{-14} [32] |
|                  |                            |      | 8.408×10^{-14} [3] |

Table 9. Branching ratio with corresponding radiative leptonic decay width for the B meson.

| decay constant | $\Gamma$/GeV | this work | [48] | [49] | [50] |
|---------------|--------------|-----------|------|------|------|
| $fp$          | 1.51×10^{-19} | 0.38×10^{-6} | 0.23×10^{-6} | 1.66×10^{-6} | 5.21×10^{-6} |
| $fp_{cor}$    | 1.36×10^{-19} | 0.34×10^{-6} |      |      |      |
3.1 Regge trajectories

The Regge trajectories play a vital role in identifying any new (experimentally) excited state as well as in providing information about the quantum numbers of particular states. We use our predicted ground, radial and orbital excited state masses for the B and Bs mesons to constitute the Regge trajectories for the \((n,M^2)\) and \((J,M^2)\) planes. Here, \(M\) stands for the mass of the B and Bs mesons, \(n\) stands for the principal quantum number and \(J\) is the total spin.

The Regge trajectories with natural \((P=(−1)^J; J^P=1^−,2^+,3^−)\) and unnatural \((P=(−1)^J; J^P=0^+,1^+,2^−)\) parity in the \((J,M^2)\) plane for the B and Bs mesons are depicted in Figs. 3–6. The masses predicted by our potential model are shown by solid triangles and the experimentally available values by hollow squares with the corresponding meson name. Straight \(\chi^2\) fit lines were obtained for the predicted mass values. We have used the following definition

\[
J=\alpha M^2+\alpha_0
\]

(26)
to find the slope \((\alpha)\) and the intercept \((\alpha_0)\). The slopes and intercepts for the \(\chi^2\) fitted \((J,M^2)\) Regge trajectories are tabulated in Table 10.

Figures 7 and 8 depict the Regge trajectories using the pseudoscalar \((J^P=0^-)\) and vector \((J^P=1^-)\) \(S\) state, excited \(P\) \((J^P=2^+)\), \(D\) \((J^P=1^-)\) and \(D\) \((J^P=3^-)\) state masses of the B and Bs mesons for \(n_r=n-1\) principal quantum number in the \((n_r,M^2)\) plane. Available experimental values are given by solid dots with the corresponding meson name. Figures 9 and 10 depict the Regge trajectories using the \(S\), \(P\) and \(D\) state center of weight masses of the B and Bs mesons for \(n_r=(n-1)\) principal quantum number in the \((n_r,M^2)\) plane. We have used the following definition

\[
n_r=\beta M^2+\beta_0
\]

(27)
to find the slope \((\beta)\) and the intercept \((\beta_0)\). The slopes and intercepts for the \(\chi^2\) fitted \((n,M^2)\) Regge trajectories are tabulated in Tables 11 and 12.
Fig. 7. (color online) Regge trajectory \((M^2 \rightarrow n_r)\) for the pseudoscalar and vector \(S\) state and excited \(P\) and \(D\) state masses of the \(B\) meson.

Fig. 8. (color online) Regge trajectory \((M^2 \rightarrow n_r)\) for the pseudoscalar and vector \(S\) state and excited \(P\) and \(D\) state masses of the \(B_s\) meson.

Fig. 9. (color online) Regge trajectory \((M^2 \rightarrow n_r)\) for the \(S\)-\(P\)-\(D\) states center of weight mass of the \(B\) meson.

Fig. 10. (color online) Regge trajectory \((M^2 \rightarrow n_r)\) for the \(S\)-\(P\)-\(D\) states center of weight mass of the \(B_s\) meson.

Table 10. Fitted parameters of the \((J,M^2)\) Regge trajectories with unnatural and natural parity.

| parity | meson     | trajectory | \(\alpha/(\text{GeV}^{-2})\) | \(\alpha_0\) |
|--------|-----------|------------|-------------------------------|--------------|
|        | parent    |            | 0.234±0.022                   | -6.576±0.741 |
|        | \(B(b\bar{q})\) first daughter | 0.244±0.016 | -8.616±0.634 |
|        | second daughter | 0.235±0.005 | -9.813±0.261 |
| unnatural |            |            |                               |              |
|        | parent    |            | 0.226±0.023                   | -6.565±0.791 |
|        | \(b_s(bs)\) first daughter | 0.243±0.017 | -8.789±0.702 |
|        | second daughter | 0.246±0.013 | -10.605±0.611 |
| natural |            |            |                               |              |
|        | parent    |            | 0.229±0.008                   | -5.500±0.277 |
|        | \(b^*(c\bar{q})\) first daughter | 0.236±0.003 | -7.353±0.128 |
|        | second daughter | 0.235±0.0004 | -8.856±0.021 |
|        |            |            |                               |              |

Table 11. Fitted slope and intercept for the \((n_r,M^2)\) Regge trajectories.

| meson | state | \(J^P\) | \(\beta/(\text{GeV}^{-2})\) | \(\beta_0\) |
|-------|-------|----------|-------------------------------|--------------|
| \(S\) | 0\(^-\) | 0.139±0.0008 | -3.873±0.037 |
| \(S\) | 1\(^-\) | 0.139±0.001 | -3.946±0.045 |
| \(P\) | 2\(^+\) | 0.146±0.001 | -4.822±0.047 |
| \(D\) | 1\(^-\) | 0.143±0.001 | -5.182±0.057 |
| \(D\) | 3\(^-\) | 0.144±0.001 | -5.357±0.067 |
| \(B\) | 0\(^-\) | 0.141±0.0006 | -4.071±0.026 |
| \(S\) | 1\(^-\) | 0.142±0.0006 | -4.173±0.028 |
| \(B_s\) | 2\(^+\) | 0.149±0.0007 | -5.097±0.030 |
| \(D\) | 1\(^-\) | 0.147±0.001 | -5.526±0.051 |
| \(D\) | 3\(^-\) | 0.149±0.001 | -5.669±0.057 |
Table 12. Fitted parameters of Regge trajectory $(r_n, M^2)$ for the $S$-$P$-$D$ states center of weight mass.

| meson | trajectory | $\beta/ (\text{GeV}^{-2})$ | $\beta_0$ |
|-------|------------|----------------|----------|
| $B(q)$ | $S$ state | 0.139±0.001 | −3.922±0.042 |
|       | $P$ state | 0.145±0.001 | −4.810±0.042 |
|       | $D$ state | 0.143±0.001 | −5.283±0.061 |
| $B_s (qs)$ | $S$ state | 0.142±0.0006 | −4.147±0.027 |
|       | $P$ state | 0.149±0.0006 | −5.075±0.024 |
|       | $D$ state | 0.148±0.0012 | −5.609±0.054 |

With a comparison of the slopes, the slope values $\alpha$ are larger than the slope values $\beta$. The ratio of the mean of $\alpha$ and $\beta$ is 1.68 and 1.62 for the B and $B_s$ meson respectively. The estimated masses of the B and $B_s$ mesons fit well to the linear trajectories in the $(n, M^2)$ and $(J, M^2)$ planes and are almost parallel to and equidistant from each other. With the help of the Regge trajectories, we can identify any new (experimentally) excited states as well as ascertain information about quantum number of the state.

4 Conclusion

Our predicted masses for the $S$-$P$-$D$ states of the B and $B_s$ mesons are very close to the available experimental values. The excited state (4$S$, 5$S$, 3$P$ and 3$D$) masses are higher than other theoretical estimates. The estimated relativistic correction to the kinetic energy term is found to be less than 1%, while that of the potential energy term is found to be around 3%-5%. With limited experimental observations of excited states of the B and $B_s$ mesons, the LHCb collaboration’s recent measurements show the possibility of updating our knowledge of the excited states of these mesons. For the B meson, the 1P states $B_1(5721)$ and $B_2(5747)$ are in good agreement with our prediction [1, 6]. We have assigned newly observed states $B(5970)$ [42] to the $2^S_0$, state of the B meson. For the $B_s$ meson, the 1P experimental states $B_{s1}(5830)$ and $B_{s2}(5840)$ are in excellent agreement with our predicted values. These are only a few excited state measurements, but in the near future LHCb should provide new results in bottom meson spectroscopy.

The Regge trajectories for the B and $B_s$ mesons in both $(J, M^2)$ and $(n, M^2)$ planes are almost linear, equidistant and parallel. Regge trajectories provide information about the quantum numbers of a given state and are useful to identify any new excited state (experimentally). From Figs. 3–6, we found that the experimental states are sitting nicely on straight lines without deviation.

The pseudoscalar decay constants with QCD correction for both mesons are underestimated compared to other theoretical model estimates. Our calculated leptonic branching fractions of the B meson are fairly close to the PDG [1] values. Our calculated radiative leptonic branching ratio of the order of $10^{-6}$ is in good agreement with the branching ratio obtained with the form factor parameterizing approach [48]. The estimated rare leptonic decay widths as well as the branching ratios of the B and $B_s$ mesons have been compared with experimental observations and other theoretical estimates [3, 44, 46, 47], and the results are in good agreement (see Tables 7 and 8).

Finally, this study may help current experimental facilities to identify new states of the B and $B_s$ mesons as well as their $J^{PC}$ values.

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