Numerical investigations of internal stresses on carbon steel based on ultrasonic LCR waves

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Abstract. Internal stresses or residual stresses in the structural elements are very crucial in carrying out in-service evaluations and fitness-for-purpose assessments. The generation of these internal stresses can occur as result of the fabrication of the steel members, installation sequence or other ad-hoc events such as accidents or impact. The accurate prediction of the internal stresses will contribute towards estimating the integrity state of the structural elements, with respect to their material allowable stresses. This paper investigates the explicit FE based numerical modelling of the ultrasonic based non-destructive technique, utilising the measurable longitudinal critical refracted wave (LCR) and relating these to the internal stresses within the structural elements by the evaluation of the material dependent acoustoelastic factors. The subsurface travel path of the LCR wave inside the structural elements makes it a subsurface stress measurement technique and the linearised relationship with corresponding internal stresses can be systematically applied repeatedly. The numerical results are compared against laboratory tests data to correlate the findings and to establish modelling feasibility for future proof-of-concepts. It can be concluded from this numerical investigation, that the subsurface ultrasonic LCR wave has great potential to be implemented for in-situ structural residual stress measurements, as compared to other available surface measurements such as strain gauges or x-ray diffraction.

1. Introduction

1.1. Background
Residual stresses or internal stresses are major contributing factors in mechanical structures, especially when carrying out integrity assessments and subsequent rejuvenation plans. These residual stresses are possibly induced at various stages, from fabrication to installation sequences, and in some cases, are imparted from ad-hoc scenarios such as accidents or impacts. When stress analyses are carried out, almost always, only the operational design loads are considered in assessing the in-place integrity state, whilst these residual stresses are ignored completely in some cases, or analytical and numerical evaluations are carried out to provide some approximations. In most situations, these approximations are over-conservative, and results in unnecessary and costly repairs on structures with adequate resistance, whilst on the other extreme may result in underestimated stresses which may result in catastrophic failures without any warning.
1.2. Available Measurement Method

Currently, there are several prominent methods being used in measurement of residual stresses. One of the most elegant non-destructive method is the x-ray diffraction method [1][2] which involves the use of x-ray to measure the strain in the material microstructural crystal lattice arrangements, and calculation of the residual stress that causes this pre-existing strain assuming a linear elastic distortion of the lattice. This technique utilises Bragg’s law in determining the diffraction angle, and any shift in the lattice spacing due to residual strain results in shift in the diffraction angle. The problem associated with this method is the depth of penetration of the monochromatic x-rays are limited to about 5μm beneath the surface [1]. This can be unfeasible as the bulk of the aged structural steel may be covered with rust or coatings and therefore thorough chemical based surface preparation is required, such that this does not induce any unnecessary stresses on the surface (which may be the case when sanding or grinding is involved). The presence of radioactive material within this system is also a problem with the in-place guidelines for explosive environments such as ATEX [3] and overall bulky configuration of the system makes it less portable for field operations. Another method for residual stress measurement is the hole drilling method as specified in ASTM [4], which involves the precision drilling of a hole through a residual strain gauge. The residual strain gauge is in fact a strain rosette forming the foundation for the acoustoelastic wave. The incident wave at an angle of \( \theta_1 \) with the vertical axis entering the medium will be refracted at angle \( \theta_2 \) into the shear component, longitudinal component and the surface ripple or Rayleigh wave. The particles riding the shear wave will oscillate perpendicular to the propagation path, whereas the particles riding the longitudinal wave will be oscillating in parallel to the propagation path. The Rayleigh component, however, oscillates the particles in an elliptic motion near the surface of the medium causing the weak ripple moving away from the source. In the situation where residual stress measurement is present in the solid medium, the wave propagation path should ideally be parallel to the principal stress axis, i.e. the wave components travel along the stress direction. In this work, the longitudinal beam is utilised to measure the residual stress in structures due to the more significant changes in measurable propagation speed, as compared to the other components. Furthermore, longitudinal wave which travels subsurface along the principal stress axis will vary much more significantly in the presence of stress in the medium, in addition to the material properties. This

2. Ultrasonic Stress Measurement Method

2.1. The Concept

Ultrasonic wave propagation in a solid structure is affected by the internal stress field [5][6][7], and this makes the non-destructive internal stress measurement to be feasible. This fundamental behaviour allows the measurement of time of flight (TOF) of the wave from the transmitting probe to the receiving probe. The change in measured speed depends on the internal stress magnitude along the propagation direction, in a linear relationship hence forming the foundation for the acoustoelastic theory. When an ultrasonic transmitter probe emits the wave into the solid medium, there are three components of wave generated at different speeds and angles in the medium, as shown in Figure 1. The incident wave at an angle of \( \theta_1 \) with the vertical axis entering the medium will be refracted at angle \( \theta_2 \) into the shear component, longitudinal component and the surface ripple or Rayleigh wave. The particles riding the shear wave will oscillate perpendicular to the propagation path, whereas the particles riding the longitudinal wave will be oscillating in parallel to the propagation path. The Rayleigh component, however, oscillates the particles in an elliptic motion near the surface of the medium causing the weak ripple moving away from the source. In the situation where residual stress measurement is present in the solid medium, the wave propagation path should ideally be parallel to the principal stress axis, i.e. the wave components travel along the stress direction. In this work, the longitudinal beam is utilised to measure the residual stress in structures due to the more significant changes in measurable propagation speed, as compared to the other components. Furthermore, the longitudinal wave which travels subsurface along the principal stress axis will vary much more significantly in the presence of stress in the medium, in addition to the material properties. This
longitudinal component which is refracted at $\theta_2 = 90^\circ$ is termed the longitudinal critical refracted (LCR) wave, and will be the focus of this work.

![Figure 1. Schematics of Wave Components.](image)

2.2. Theoretical Overview

For an unstressed elastic solid, the propagation speed terms of the ultrasonic wave components can be written as follows [8][9]:

$$V_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$  \hspace{1cm} (1)

$$V_S = \sqrt{\frac{\mu}{\rho}}$$  \hspace{1cm} (2)

$$V_R \approx \frac{0.862 + 1.14\nu}{1 + \nu} V_S$$  \hspace{1cm} (3)

where the terms $V_L$, $V_S$ and $V_R$ are the longitudinal, shear and Rayleigh wave speeds respectively, $\nu$ and $\rho$ are the Poisson’s ratio and density of the medium. The terms $\lambda$ and $\mu$ are the second order elastic constants for the materials. Snell’s law is used to orientate the signal source such that to produce the LCR wave, by introducing an intermediate medium with the critical incident angle of $90^\circ$.

Polymethylmethacrylate (PMMA), or more commonly known as plexiglass or acrylic is found to be ideal for this purpose, and a machined wedge with the correct incident angle can create the LCR waves close to the surface to facilitate the residual stress measurements. The relationship between the LCR wave speed ($V$) and stress ($\sigma$) is as shown in Equation (4) [10], and reduces to Equation (1) in the absence of any residual stresses.

$$V^2 = \lambda + 2\mu + \frac{\sigma}{3\lambda + 2\mu} \left[ \frac{\lambda + \mu}{\mu} (4\lambda + 10\mu + 4m) + \lambda + 2l \right]$$  \hspace{1cm} (4)

Here, the parameters $m$ and $l$ are the part of the third order elastic constants of the material. Further manipulations and rearranging will lead to the more useful form in Equation (5)

$$\sigma = \frac{1}{k} \left( \frac{V^2}{V_0^2} - 1 \right)$$  \hspace{1cm} (5)

where

$$k = \frac{4\lambda + 10\mu + 4m + 2l - 3\lambda - 10\mu - 4m}{3\lambda + 2\mu} \frac{\mu}{\lambda + 2\mu}$$  \hspace{1cm} (6)

The relative change in LCR wave is now directly proportional to the stress along the same direction with by a factor $k$. Experimental procedures can therefore be setup to calibrate the relative LCR speed to the stress.
2.3. Materials
The specimen selected for this investigation is Grade-B carbon steel (with yield strength of 240MPa), which is commonly used in the constructions of various structural systems decades ago ranging from offshore platforms to infrastructural buildings, and now can be found on majority aged structures worldwide where the residual stress measurements are ever more crucial. PMMA wedges cut to the correct angle are used to generate the LCR waves. The material specifications for the specimen and wedge are presented in Table 1, followed by the evaluation of the second order elastic constants and the respective wave speeds in Table 2.

Table 1. Material Specifications.

| Material | \( \rho \) (kg/m\(^3\)) | \( E \) (GPa) | \( \nu \) (-) |
|----------|-----------------|-------------|-------------|
| Steel    | 7950            | 207         | 0.3         |
| PMMA     | 1178            | 4.3         | 0.4         |

Table 2. Material Elastic & Acoustic Parameters.

| Material | \( \lambda \) (GPa) | \( \mu \) (GPa) | \( V_L \) (m/s) | \( V_S \) (m/s) | \( V_R \) (m/s) |
|----------|--------------------|----------------|----------------|----------------|----------------|
| Steel    | 119.42             | 79.62          | 5920.37        | 3164.57        | 2930.88        |
| PMMA     | 6.15               | 1.54           | 2798.00        | 1142.28        | 1077.98        |

To generate the LCR which refracts into the steel from PMMA medium at 0\(^\circ\) from vertical, the wedges need to be cut at about 28.2\(^\circ\) based on Equation (5). To account for some small variations in material properties for repeatability of the tests, the base of the wedge is slightly curved to enable adjustments of up to \( \pm 15^\circ \) on the beam incident angle such that it produces the maximum LCR wave amplitudes during tests as well as usable on most types of metal specimens, as shown in Figure 2. The ultrasonic transducer is placed tightly into the hole on the wedge, with water-based ultrasound gel applied into the hole to create an acoustical contact to propagate the longitudinal waves through into the specimen. The resulting LCR wave at right angle from the vertical axis will be accompanied by a shear wave component at about 32\(^\circ\) as calculated from Snell’s Law. Placing a similar arrangement of receiving transducer at a specified distance away from the transmitting transducer along the specimen will permit the acquisition of the LCR signals during the test.

Figure 2. PMMA Wedge Design for LCR Wave on Steel.

The steel test specimens selected for the experimental work consist of hollow sections of various shapes and sizes cut at 500mm length. It should be noted that the objective of the study is to test the LCR’s feasibility on stress measurement on the steel material, and the shapes/sizes are merely introduced to verify that the derived properties are independent of shapes/sizes and exclusively
material dependant. Their limiting load limits are also calculated to ensure loadings remain within elastic limits and to ensure overall safety.

3. Numerical Analysis

3.1. Model Description

The numerical model, utilising the finite element (FE) method is carried out in the general purpose nonlinear code ABAQUS [13] to correlate to the theoretical procedures described in the preceding sections of this paper. The model is setup to analyse the case as outlined in Figure 3, highlighting the specimen and wedges modelled in a two-dimensional plane strain assembly. The specimen is modelled at 10mm depth and 100mm long, while the wedge is about 5mm depth and 20mm long. The wedges are modelled with straight base, as compared to the curved ones used in the tests since the specific material properties are used and the distinctive critical incident angle can be modelled directly to generate the LCR waves. The specimen is restrained along the vertical by boundary conditions at its base, and compressive residual stresses at the specimen edges, applied as shown over a range of 10MPa to 1000MPa.

![Figure 3. Outline of the FE Model for LCR Analysis.](image)

The ultrasonic transducer signals are modelled by force amplitude $F(t)$, and depends on the frequency of the transducer $(f)$ used, and can be expressed over a range of time $(t)$ in a cosine function as follows [7]:

$$F(t) = \left[1 - \cos\left(\frac{2\pi ft}{3}\right)\right]\cos(2\pi ft) \quad (7)$$

The model is constructed in a plane strain condition, using the quadrilateral plane strain elements with reduced integration or CPE4R [13]. The interface between the wedge and specimen is kinematically coupled for all degrees of freedom, creating a continuity to enable the ultrasonic perturbation to travel across these boundaries.

3.2. Results

The LCR model will now be analysed and results extracted to investigate the acoustoelastic constant from a numerical perspective. The pressure contours are extracted and are presented for the wave propagation leaving the wedge in Figure 4, to identify the various wave components. The LCR wave components, being the more prominent and travelling at higher speed can be seen travelling parallel to the specimen surface, with its wave front propagating across the specimen depth, whereas the shear wave is propagating at an angle of approximately 30° from the vertical. The Rayleigh surface ripple is propagation outward away from the source and lagging before the much faster LCR wave. The head wave component which is observed as adjoining the LCR and shear components, i.e. interactions between the surface and inclined planes of the propagation path. The LCR wave can be expressed also as a net effect of the longitudinal, shear, Rayleigh and the head wave [14].
The series of LCR wave propagation sequence can also be extracted and are presented in Figure 5 (i) to (vi) showing the progression of the LCR wave from the transmitting wedge until it reaches the receiving wedge. The longitudinal wave front is also observed to be reflected by the specimen backwall, and must be identified and ignored when post-processing the signal acquisition data. A quick check on the depth of the LCR wave on the FE output also shows an approximately 2mm depth for a 5MHz transducer which is being used, and is within acceptable order of magnitude from the measurements made earlier in [10][5] of same order of magnitude.

A quick comparison is made to check the signals received at the receiving probing location on the numerical model and the test data, as shown in Figure 6. The plot shows an acceptable correlation with the test data [15], in terms of the LCR TOF arriving at the receiving probe point, and the approximate size of its amplitude.

The signals at the receiver probing point is further extracted and plotted in Figure 7, with the exaggerated view on total TOF varying against residual stress. This is subsequently used to generate the plot for relative speed against stress shown in Figure 8, resulting in the acoustoelastic constant
value of $9.13 \times 10^{-4}$ MPa-$1$, and lies within 5% limit in comparison of the experimentally measured average value due to the numerical idealisations.

Figure 6. Comparison of FE and Test Signals.

Figure 7. Numerical TOF Measurement at Receiver Wedge Location.

Figure 8. Comparison of Experimentally and Numerically Obtained LCR Relative Speed and Stress Relationship.

4. Conclusion
The primary objective of this study is to investigate the feasibility of using explicit FE based numerical simulations to effectively predict the internal stresses on carbon steel structural components based on the ultrasonic LCR wave propagations. LCR wave is generated by use of approximately 28° PMMA wedge which produces critically refracted wave components from a piezoelectric ultrasonic transducer. Compressive stresses were introduced into the specimen at controlled increments whilst the relative speed change is evaluated from the measured TOF of the LCR wave. The numerical model constructed was used to validate the experimental procedures carried out in a similar work, and to study the wave behaviours and extending the loadings beyond material allowable which are limited during the tests and found to correlate well with the experimentally measured values within 5% deviation.

In a field inspection of internal stresses, the sample material is required to evaluate the acoustoelastic constant prior to site measurements. This accounts for material properties and compositions used in the actual fabrication and construction. Once the constant is determined, it can be directly input into a customised computer routine to automatically produce internal stress magnitude along the ultrasonic wave propagations or transducer orientations during in-situ measurement activities. The transmitting and receiving wedges can also be integrated to be more easily handled, and the ultrasonic box interfaced with a tablet for mobility. The numerical model
presented in this study is also deemed suitable for quick estimate of the acoustoelastic constant of idealised materials prior to setup of costly experiments to assess the feasibility and practicality of the ultrasonic LCR based residual stress measurements and the acquisition system resolutions and sampling rates required to successfully undertake any specific field measurements. Overall the LCR method is regarded as very portable and safe to operate within any conditions and adheres to stringent requirements as compared to other available techniques and is deemed to have great potential as a robust and reliable method in determining internal stresses on structures.

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