Electronic transport through a correlated quantum wire connected to a superconducting lead

P P Aseev, S N Artemenko

Kotel’nikov Institute of Radio-engineering and Electronics of Russian Academy of Sciences, Moscow 125009, Russia and Moscow Institute of Physics and Technology, Dolgoprudny 141700, Moscow region, Russia

Abstract

We study theoretically the electron transport in a 1D conductor adiabatically connected to a superconducting and normal metal leads. In the case of non-interacting electrons in the wire, we obtain full I-V curve. We show that ac voltage applied along with dc voltage modifies I-V curve resembling a regime of photon-assisted tunneling. We obtain in the limit of low voltages, that the electron-electron interaction affects electronic transport resulting in a decrease of conductance and features in the frequency dependence of the impedance.
I. INTRODUCTION

It is well-known for a long time that a nonlinear conductivity arises in superconductor-normal metal contacts with multimode constriction, in case of both dirty constrictions\textsuperscript{1} and constrictions without scattering\textsuperscript{2}. This non-linearity is related to Andreev reflection. One can also expect nonlinear I-V curves in a junction of a superconductor with a quantum wire.

In contrast to narrow ballistic quantum wires connected to wide normal metal reservoirs where the conductance is quantized in units of $2e^2/h$\textsuperscript{3,4}, it was shown that in a wire with non-interacting electrons, which is connected to superconducting and normal leads Andreev reflection results in a double conductance quantum at zero temperature\textsuperscript{5}. For the wire with normal leads electron-electron interaction does not modify I-V curves\textsuperscript{6–8}, although the simple Fermi liquid description with single-electron quasiparticles breaks down, and the Luttinger liquid (LL) picture that describes 1D system of interacting electrons becomes an alternative to the Fermi liquid (for a review see Ref.\textsuperscript{9} and \textsuperscript{10}). However, in a carbon atomic wire connected to a superconducting Al lead and to a normal Al lead density functional theory calculations\textsuperscript{11} predict reduction of the conductance due to electron-electron interaction.

In this paper we study electronic transport in a quantum wire adiabatically connected to superconducting and normal leads. We derive boundary conditions for an adiabatic contact of a quantum wire with a superconducting lead. Then we calculate full I-V curves without taking into account electron-electron interaction in the quantum wire. We also consider static I-V curve in the case when the applied voltage contains dc and ac components simultaneously. The result resembles the regime of photon-assisted tunneling\textsuperscript{12} like in the systems with multimode constrictions. Finally, we study how the interaction affects conductance and obtain analytically contribution to the conductance due to electron-electron interaction.

Below we set $e$, $\hbar$ and $k_B$ to unity, restoring dimensional units in final expressions when necessary.

II. BOUNDARY CONDITIONS

We consider a long (in comparison with the Fermi length) 1D conductor connected adiabatically to superconducting (S) and normal (N) massive leads, see Fig.\textsuperscript{11} As the width of the quantum wire is of the order of the Fermi wavelength and small compared to super-
FIG. 1. System in study. S stands for superconducting lead, N denotes normal metal lead

conducting correlation length, we can neglect suppression of the superconducting gap in the
superconductor near the quantum wire due to the proximity effect. Using the quasi-classical
approach one can justify the replacement of the adiabatically widening high-dimensional
superconducting lead with an effective 1D lead with a non-zero order parameter $\Delta \neq 0$ at
$x < 0$, as it was done in Ref.6,8. Note that in case of normal metal leads the approach
based on the replacement of the massive lead by an effective 1D lead13 and more detailed
treatment of the boundary with a massive lead12,14 yield similar results.

At $x > 0$ we describe interacting electrons by Tomonaga-Luttinger model (for a review
see9,10). The field operator at $x > 0$ can be represented as

$$\hat{\Psi}_s(x, t) = \hat{\Psi}_{s,+}(x, t)e^{iqFx} + \hat{\Psi}_{s,-}(x, t)e^{-iqFx}, \tag{1}$$

where $s$ relates to spin, $q_F$ is the Fermi wave-number, and $\hat{\Psi}_\pm(x)$ are field operators of
right(left) moving electrons, and operators $\hat{\Psi}_\pm(x)$ slightly vary at distances of order $q_F^{-1}$.

Boundary conditions for the contact of a quantum wire with a normal lead were derived
in15. In the energy representation for an adiabatic contact these boundary conditions read

$$\hat{\Psi}_{s,-}(x = L, \varepsilon) = \hat{R}_s^N(\varepsilon), \tag{2}$$

$$\hat{R}_s^N(\varepsilon) = \sum_{k<0} \hat{c}_{s,k} 2\pi \delta(\varepsilon - \varepsilon_k), \tag{3}$$

where $\hat{c}_{s,k}$ are annihilation operators of the incident electrons with spin $s$ and momentum $k$
in the normal lead, $\varepsilon_k$ is a spectrum of electrons in the normal lead.

In the rest of this section we focus on the contact with a superconductor. In the super-
conducting region field operators read
\[ \hat{\Psi}_\uparrow(x, t) = \sum_k \hat{\gamma}_k \uparrow u_k(x) e^{-i\varepsilon_k t} - \hat{\gamma}_k \uparrow v_k(x) e^{i\varepsilon_k t}, \]  
\[ \hat{\Psi}_\downarrow(x, t) = \sum_k \hat{\gamma}_k \downarrow u_k^*(x) e^{i\varepsilon_k t} + \hat{\gamma}_k \downarrow v_k(x) e^{-i\varepsilon_k t}, \]  

where \( \hat{\gamma}_{k,s} \) are annihilation operators of quasiparticles with momentum \( k \) and spin \( s \), and \( u_k(x), v_k(x), \varepsilon_k \) are corresponding solutions and an eigen energy of Bogolyubov equations

\[ \left( \frac{\partial^2}{\partial x^2} - \frac{k_F^2}{2m} \right) u(x) + \tilde{\Delta} v(x) = \varepsilon u(x), \]  
\[ \tilde{\Delta}^* u(x) - \left( \frac{\partial^2}{\partial x^2} - \frac{k_F^2}{2m} \right) v(x) = \varepsilon v(x). \]  

Solving Bogolyubov equations (6)–(7) on a half-line \( x < 0 \), one can find a set of standard solutions

\[ u_k(x) = \left(1/2 + \frac{\xi_k}{(2\varepsilon)}\right)^{1/2} e^{i\chi/2} e^{ikx}, \]  
\[ v_k(x) = \left(1/2 - \frac{\xi_k}{(2\varepsilon)}\right)^{1/2} e^{-i\chi/2} e^{ikx}, \]  

with an eigen-energy above the gap \( \varepsilon_k = \sqrt{\xi_k^2 + \Delta^2} \), where \( \xi_k = k^2/(2m) - k_F^2/2m \), \( \chi \) is a phase of the complex order parameter \( \tilde{\Delta} \). The solutions with \( k > k_F \) and \( -k_F < k < 0 \) correspond to right-moving quasiparticles, while the other solutions correspond to left-moving quasiparticles. Beside of these solutions, there are two set of solutions with an eigen-energy below the gap, which decay into the depth of the superconductor. We parametrize these solutions by a decay parameter \( \kappa \)

\[ u_{\pm,i\kappa} = \frac{1}{\sqrt{2}} e^{\mp i\alpha/2 + i\chi/2} e^{\pm ik_F x + i\kappa x}, \]  
\[ v_{\pm,i\kappa} = \frac{1}{\sqrt{2}} e^{\pm i\alpha/2 - i\chi/2} e^{\pm ik_F x + i\kappa x}, \]  
\[ \varepsilon_\kappa = \sqrt{\Delta^2 - |\xi_\kappa^2|}, \]

where \( \xi_{\pm,k} = \mp i v_F \kappa, e^{i\alpha} = (iv_F \kappa + \varepsilon)/\Delta. \)

For the purpose of simplicity, below we take \( \chi = 0 \).

Since the field operators \( \hat{\Psi}_s(x) \) and their spatial derivatives continuously depend on \( x \), we equate the operators in (1) and their spatial derivatives at \( x = 0 \) with the corresponding operators in (4)–(5). For the purpose of convenience we transform operators to the energy
representation $\hat{\Psi}(\varepsilon) = \int_{-\infty}^{+\infty} \hat{\Psi}(t)e^{i\varepsilon t}dt$. Finally, after some algebra we relate fields $\hat{\Psi}_{s,r}$ at the boundary of 1D conductor with creation-annihilation operators of right-moving quasiparticles in the superconductor:

$$\hat{\Psi}_{\uparrow,+}(\varepsilon) - \frac{\varepsilon - \varepsilon^R}{\Delta} \left[ \hat{\Psi}_{\downarrow,-}(\varepsilon) \right]^\dagger = \hat{R}^S_{\uparrow}(\varepsilon),$$

$$\left[ \hat{\Psi}_{\downarrow,+}(\varepsilon) \right]^\dagger - \frac{\varepsilon - \varepsilon^R}{\Delta} \hat{\Psi}_{\uparrow,-}(\varepsilon) = \left[ \hat{R}^S_{\downarrow}(\varepsilon) \right]^\dagger,$$

where $\varepsilon^R(\varepsilon) = \sqrt{(\varepsilon + i0)^2 - \Delta^2}$ is analytic on the complex plane with a cut: $(-\Delta, \Delta)$, and the right hand side is given by relations

$$\hat{R}^S_{\uparrow}(\varepsilon) = \frac{2\pi}{\varepsilon + \varepsilon^R} \left[ \sum_{k > k_F} \gamma_{\uparrow,k}\delta(\varepsilon - \varepsilon_k) - \theta(-\varepsilon - \Delta) \sum_{k = -k_F}^{0} \gamma_{\downarrow,k}^\dagger\delta(\varepsilon + \varepsilon_k) \right],$$

$$\left[ \hat{R}^S_{\downarrow}(\varepsilon) \right]^\dagger = \frac{2\pi}{\varepsilon + \varepsilon^R} \left[ \theta(\varepsilon - \Delta) \sum_{k = -k_F}^{0} \gamma_{\uparrow,k}\delta(\varepsilon - \varepsilon_k) + \theta(-\varepsilon - \Delta) \sum_{k > k_F} \gamma_{\downarrow,k}^\dagger\delta(\varepsilon + \varepsilon_k) \right].$$

The correlation functions for these operators read:

$$\left\langle \hat{R}^S_{\uparrow}(\varepsilon')\hat{R}^S_{\uparrow}(\varepsilon) \right\rangle = \left\langle \hat{R}^S_{\downarrow}(\varepsilon')\hat{R}^S_{\downarrow}(\varepsilon) \right\rangle = \frac{2\xi^\theta(|\varepsilon| - \Delta)}{v_F(\varepsilon + \xi)} n_F(\varepsilon) 2\pi \delta(\varepsilon - \varepsilon'),$$

where $n_F(\varepsilon)$ is the Fermi distribution. Note that if we take $\varepsilon = 0$ the boundary conditions (8)–(9) are reduced to those obtained in[13].

**III. NON-INTERACTING CASE**

In order to obtain I-V curves for all voltages we assume first that electrons in quantum wire do not interact. We study the case when the voltage consisting of dc and ac components $V(t) = V_0 + V_1 \cos \omega_0 t$ is applied to the normal lead. The case of time-independent voltage can be obtained by taking $V_1 = 0$. In the presence of time-dependent potential Heisenberg operators $\hat{R}^N_s(t)$ in the right hand of boundary conditions (2) should be replaced by $\hat{R}^N_s(t) = \hat{R}^N_s(t)\exp(-i \int_0^t V(\tau)d\tau)$. Thus, a correlation function in the time representation reads

$$\left\langle \hat{R}^N_{\uparrow}(t')\hat{R}^N_{\uparrow}(t) \right\rangle = \left\langle \hat{R}^N_{\uparrow}(t')\hat{R}^N_{\uparrow}(t) \right\rangle \exp \left[ iV_0(t' - t) + i\frac{V_1}{\omega_0} (\sin \omega_0 t' - \sin \omega_0 t) \right].$$
After substituting (3) and using the expansion \( \exp(i \frac{V_1}{\omega_0} \sin \omega_0 t) = \sum_{k=-\infty}^{+\infty} J_k \left( \frac{V_1}{\omega_0} \right) \exp(-ik\omega_0 t) \) we obtain the correlation function in the energy representation

\[
\left\langle \hat{R}_s^N(\varepsilon') \hat{R}_s^N(\varepsilon) \right\rangle = \frac{1}{v_F} \sum_{m,k} n_F \left( \frac{\varepsilon + \varepsilon'}{2} - V_0 + \frac{k + m}{2} \omega_0 \right) \times J_k \left( \frac{V_1}{\omega_0} \right) J_m \left( \frac{V_1}{\omega_0} \right) 2\pi \delta(\varepsilon - \varepsilon' - k\omega_0 + m\omega_0),
\]

where \( n_F(\varepsilon) \) is the Fermi distribution. In the case of time-independent potential \( (V_1 = 0) \) the correlation function reduces to

\[
G^N(\varepsilon, \varepsilon') = \frac{1}{v_F} n_F(\varepsilon - V_0) 2\pi \delta(\varepsilon - \varepsilon').
\]

In the absence of interaction in the quantum wire operators \( \hat{\Psi}_{s,r}(x) \) in the general expression (11) read

\[
\hat{\Psi}_{s,r}(x, \varepsilon) = \hat{\Psi}_{s,r}(0, \varepsilon)e^{irqx}, \quad q(\varepsilon) = \frac{\varepsilon}{v_F}.
\]

Using boundary conditions (2), (8)–(9) and expressing pairings of fermionic operators \( \langle \hat{\Psi}_s^\dagger \hat{\Psi}_r \rangle \) at \( x = 0 \) in terms of correlators (10), (11) and then transforming into the time representation we obtain the current

\[
j(t) = v_F \sum_{s=\uparrow,\downarrow, r=\pm 1} r \left\langle \hat{\Psi}_{s,r}^\dagger(t) \hat{\Psi}_{s,r}(t) \right\rangle.
\]

In the case of stationary potential the current reads

\[
I_0(V) = G_0 \int_0^\infty \left[ \theta(\Delta - \varepsilon) + \theta(\varepsilon - \Delta) \frac{\varepsilon}{\varepsilon + \xi} \left( \tanh \frac{\varepsilon + V}{2T} - \tanh \frac{\varepsilon - V}{2T} \right) \right] d\varepsilon,
\]

where \( G_0 = 2e^2/h \) is the conductance quantum. In the limit of \( T = 0 \) integration yields

\[
I_0(V) = 2G_0 V \theta(\Delta - |V|) + \frac{2G_0}{3} \left[ \frac{V^3 - (V^2 - \Delta^2)^{3/2}}{\Delta^2} + 2\Delta \text{ sign } (V) \right] \theta(|V| - \Delta),
\]

The I-V curve is linear with double conductance quantum \( 2G_0 \) at the voltages below the gap \( |V| < \Delta \), and behaves sufficiently non-linearly at the voltages \( V \gtrsim \Delta \) and at high voltages \( V \gg \Delta \) it is almost linear with a normal conductance quantum \( G_0 \) and an excess current \( I_{exc} = \frac{4}{3} G_0 \Delta \).

In the limit \( T \gg \Delta \) the I-V curve differs from the normal-state Ohm’s law by the term that describes small correction to conductance at low voltages and saturates at high voltages giving the excess current

\[
I_0(V) = G_0 \left( V + \frac{4}{3} \Delta \tanh \frac{V}{2T} \right).
\]
FIG. 2. Dependence of differential conductance on the applied voltage. Solid line corresponds to the dc voltage. Dashed line corresponds to the voltage consisting of dc component $V$ and ac component $V_1 \cos \omega_0 t$ (the conductance was calculated for the case $eV_1 = \hbar \omega_0 = \Delta$)

If an ac voltage is also applied then the averaged over time current is given by a Tien-Gordon-like formula

$$
\bar{I}(V) = \sum_{k=-\infty}^{+\infty} I_0(V + k\hbar \omega_0) J_k^2 \left( \frac{eV_1}{\hbar \omega_0} \right).
$$

The dependence of differential conductance on the applied voltage at $T = 0$ is shown in Fig. 2.

Originally Tien-Gordon formula was derived for tunnel contacts, however, as we see it is applicable in case of superconductor and normal leads connected by a conducting constriction as well. This occurs because the current in the constriction does not affect the electronic states in the lead, and the applied voltage is taken into account by the factors $\exp i\left[ \int dt V(t) \right]$ added to Green’s functions in boundary conditions. As the current, in case of non-interacting electrons, linearly depends on the Green’s functions in the leads, this yields the results similar to photon-assisted transport. In case of interacting electrons, when the Hamiltonian in the quantum wire is not bi-linear in electron operators, the Tien-Gordon-like relation is not expected.

Up to now we considered the superconducting electrode as a natural superconductor, however, superconductivity can be induced in the two-dimensional electrode by the proximity effect. The proximity effect results in the superconducting pairing in the normal electrode described by the self-energy terms in the Green’s function of the electrode. In this case
the energy gap $\Delta$ induced in the 2D lead by the superconducting metal is smaller than the gap in the superconductor $\Delta_S$. In the limit of small tunneling rate $\Gamma \sim \frac{t^2 d k_F}{E_F} \ll \Delta_S$, where $t$ is tunneling matrix element, and $d$ is the thickness of the electrode, $\Delta \approx \Gamma$. It can be shown using the approach of Ref.\textsuperscript{17} that the equations (8-10) can be used with substitutions $\varepsilon \rightarrow \tilde{\varepsilon} = \varepsilon + \frac{i \Gamma \Delta_S}{\sqrt{(\varepsilon + i 0)^2 - \Delta_S^2}}$ and $\xi^R \rightarrow \sqrt{\tilde{\varepsilon}^2 - \Gamma^2}$. Therefore, our results are applicable to the case of the proximity-induced superconductivity if we substitute $\Delta$ in previous equations for $\Delta \approx \Gamma$. However, new features in the conductance are expected at voltage $V \approx \Delta_S \gg \Delta$ where the non-linear part of the I-V curves (13) is small. The conductance at this voltages reads

$$\frac{dI}{dV} \approx G_0 \left[ 1 + \Im \frac{\Gamma^2 (\Delta_S^2 - V^2)}{4V^2 \Gamma + \sqrt{\Delta_S^2 - (V + i 0)^2}} \right].$$

When the voltage approaches to $\Delta_S$ the non-linear correction to the conductance (the second term) has a dip of the approximate width $\Gamma^2/\Delta_S$ with a minimum value $-G_0 \Gamma^2/32V^2$ at $V \approx \Delta_S + \Gamma^2/6\Delta_S$.

IV. INTERACTING CASE

In this section we will study the effect of electron-electron interaction on the conductance when the applied voltage $|V| < \Delta$ and $V$ does not depend on time. We describe interacting electrons using Luttinger liquid (LL) model and bosonization technique\textsuperscript{9} in terms of boson displacement field $\hat{\Phi}_\nu(x, t)$ and its conjugated momentum $\hat{\Pi}_\nu(x, t) = \partial_x \hat{\Theta}_\nu(x, t)$. Here $\nu = \rho, \sigma$ stands for charge and spin sectors correspondingly, and $\hat{\Theta}_\nu$ is a field dual to $\hat{\Phi}_\nu$. The standard Tomonaga-Luttinger(TL) Hamiltonian reads

$$\hat{H} = \frac{1}{\pi v_F} \sum_{\nu = \rho, \sigma} \int dx \left[ \left( \pi \hat{\Pi} \right)^2 + \frac{v_F^2}{K_\nu^2} \left( \partial_x \hat{\Phi}_\nu \right)^2 \right].$$

Here the LL parameters $K_\nu$ playing the role of the stiffness coefficients of the elastic string described by Hamiltonian (13), are related to the electron-electron interaction potential, and measure the strength of interaction between electrons. We consider spin-rotation invariant case in our study and short-range electron-electron interaction. Under these assumptions $K_\sigma = 1$, and $K_\rho = 1/\sqrt{1 + \frac{g(q=0)}{\pi v_F}}$ where $g(q)$ is the Fourier-transformed interaction potential. Parameter $K_\rho = 1$ corresponds to non-interacting case, and $0 < K_\rho < 1$ for a repulsive interaction potential.
In the non-interacting case considered in the previous section only the states below the superconducting gap $\Delta$ contribute to a current if the applied voltage $|V| < \Delta$. In the interacting case as distribution function does not display a sharp jump at the Fermi level, a contribution from the states above the gap becomes significant. We study the case of weak enough interaction $1 - K_\rho \ll 1$ when this contribution can be considered as a perturbation.

In the zero-approximation we neglect the contribution of states with energy $|\varepsilon| > \Delta$. Therefore, the right-hand the of the boundary conditions (8)–(9) can be omitted, and since for $|\varepsilon| < \Delta$ the Bogolyubov amplitudes $|u(\varepsilon)| = |v(\varepsilon)|$, the boundary conditions (8)–(9) yield

\begin{align}
: \hat{\Psi}_{\uparrow,+}^\dagger(t) \hat{\Psi}_{\uparrow,+}(t) := & \hat{\Psi}_{\downarrow,-}(t) \hat{\Psi}_{\downarrow,-}^\dagger(t) :, \\
: \hat{\Psi}_{\downarrow,+}(t) \hat{\Psi}_{\downarrow,+}^\dagger(t) := & \hat{\Psi}_{\uparrow,-}(t) \hat{\Psi}_{\uparrow,-}^\dagger(t) :.
\end{align}

Here $: \hat{A} := \hat{A} - \langle \hat{A} \rangle_0$ denotes normal-ordering, $\langle \ldots \rangle_0$ stands for averaging over equilibrium state at $V = 0$. The boundary conditions (15)–(16) can be bosonized using the relations

\begin{align}
: \hat{\Psi}_{s,\nu}^\dagger(t) \hat{\Psi}_{s,\nu}(t) := & -\frac{\partial_x \hat{\Phi}_\rho}{2\sqrt{2\pi}} - s \frac{\partial_x \hat{\Phi}_\sigma}{2\sqrt{2\pi}} + r \frac{\partial_t \hat{\Phi}_\rho}{2\sqrt{2\pi v_F}} + rs \frac{\partial_t \hat{\Phi}_\sigma}{2\sqrt{2\pi v_F}},
\end{align}

resulting in the boundary conditions for displacement fields $\hat{\Phi}_\nu$ and dual fields $\hat{\Theta}_\nu$

\begin{align}
\partial_x \hat{\Phi}_\rho(x = 0) = 0, \\
\partial_t \hat{\Phi}_\sigma(x = 0) = 0, \\
\partial_t \hat{\Theta}_\rho(x = 0) = 0, \\
\partial_x \hat{\Theta}_\sigma(x = 0) = 0.
\end{align}

The boundary conditions for the normal contact were derived in (8–9) and earlier in (7). The operators $\hat{R}_{\nu}$ are defined in (3).

\begin{align}
- \frac{v_F}{K_\rho} \partial_x \hat{\Phi}_\rho - \partial_t \hat{\Phi}_\rho = \hat{P}_\rho, \\
- v_F \partial_x \hat{\Phi}_\sigma - \partial_t \hat{\Phi}_\sigma = \hat{P}_\sigma, \\
- v_F \partial_x \hat{\Theta}_\rho - \partial_t \hat{\Theta}_\rho = -\hat{P}_\rho, \\
- v_F \partial_x \hat{\Theta}_\sigma - \partial_t \hat{\Theta}_\sigma = -\hat{P}_\sigma,
\end{align}

where $\hat{P}_{\nu}(t) = \frac{v_F}{\sqrt{2}} \left[ \hat{R}_{\nu}^N(t) \hat{R}_{\nu}^N(t) \pm \hat{R}_{\nu}^N(t) \hat{R}_{\nu}^N(t) \right]$, $\langle \hat{P}_\rho \rangle = -\sqrt{2}V$ and operators $\hat{R}_\nu^N$ are defined in (3).
The TL Hamiltonian (14) yields equations of motion for the fields \( \hat{\Phi}_\nu, \hat{\Theta}_\nu \)

\[
\partial_t^2 \hat{\Phi}_\nu - \frac{v_F^2}{K_p^2} \partial_x^2 \hat{\Phi}_\nu = 0,
\]

\[
\partial_t^2 \hat{\Theta}_\nu - \frac{v_F^2}{K_p^2} \partial_x^2 \hat{\Theta}_\nu = 0.
\]

(25) (26)

It is convenient to decompose the displacement field \( \hat{\Phi}_\rho \) and the dual field \( \hat{\Theta}_\rho \) in the sum of thermodynamically averaged \( c \)-number components \( \phi_\rho = \langle \hat{\Phi}_\rho \rangle, \theta_\rho = \langle \hat{\Theta}_\rho \rangle \) and fluctuating components \( \dot{\phi}_\rho = \hat{\Phi}_\rho - \phi_\rho, \dot{\theta}_\rho = \hat{\Theta}_\rho - \theta_\rho \). Solving wave equation (25) subject to boundary conditions (17)–(18), (21)–(22), we obtain a zero-approximation solution for \( \hat{\Phi}_\rho \):

\[
\phi_\rho = \sqrt{2} V t,
\]

\[
\dot{\phi}_\rho(x = 0, \omega) = \frac{K_\rho P_\rho(\omega)}{\omega (\sin(K_\rho \omega_0 L/v_F) + i K_\rho \cos K_\rho \omega_0 L/v_F)}.
\]

(27) (28)

The current can be calculated as \( j = \sqrt{2} \partial_t \phi_\rho / \pi = 2 G_0 V \). Thus, zero-approximation solution for interacting system at low and constant voltages yields the same conductance as in non-interacting case. However, if ac voltage with a frequency \( \omega_0 \) is applied \( V = V_1 \cos \omega_0 t \), then \( \langle \dot{P}_\rho(t) \rangle = -\sqrt{2} V_1 \cos \omega_0 t \), and the averaged solution for the wave equation (25) subject to boundary conditions (17)–(18), (21)–(22) reads

\[
\phi_\rho(x, t) = -\sqrt{2} K_\rho V_1 e^{i \omega_0 t} \omega_0 (\sin(K_\rho \omega_0 L/v_F) + i K_\rho \cos K_\rho \omega_0 L/v_F) \cos K_\rho \omega_0 x/v_F + c.c.
\]

Thus, even in the zero approximation which neglects contributions from energies \( |\varepsilon| > \Delta \), the electron-electron interaction affects an impedance \( Z(\omega) \) of the system

\[
Z(\omega) = \frac{\cos K_\rho \omega_0 L/v_F - i K_\rho^{-1} \sin K_\rho \omega_0 L/v_F}{2 G_0 \cos^2 \omega L/2 v_F}.
\]

It is worth noting that if an ac voltage is applied to the right electrode, since there is an average potential \( U_Q = V_\omega/2 \) the wire is being charged and discharged periodically, and the current flowing into and out of the wire is affected by the interaction

\[
I_Q = I_\omega(0) - I_\omega(L) = 2 G_0 U_Q \frac{2 K_\rho \sin^2 \omega L/2 v_F}{\sin \omega L/v_F + i K_\rho \cos \omega L/v_F}.
\]

(29)

Correlation functions and commutators for fluctuating parts \( \dot{\phi}_\rho, \dot{\dot{\phi}}_\sigma \) in case of \( T = 0 \) and a long wire (\( L \gg K_\rho^{-1} \)) read

\[
\langle \{ \dot{\phi}_\rho(0), \dot{\phi}_\rho(\tau) \} \rangle = 2 \langle \dot{\phi}_\rho(0), \dot{\phi}_\rho(0) \rangle - 2 K_\rho \ln \Lambda |\tau|,
\]

(30)

\[
[\dot{\phi}_\rho(0), \dot{\phi}_\rho(\tau)] = 2 i K_\rho \text{sign} \tau,
\]

(31)

\[
\langle \{ \dot{\dot{\phi}}_\sigma(0), \dot{\dot{\phi}}_\sigma(\tau) \} \rangle = 2 \langle \dot{\phi}_\sigma(0), \dot{\phi}_\sigma(0) \rangle - 2 \ln \Lambda |\tau|,
\]

(32)

\[
[\dot{\dot{\phi}}_\sigma(0), \dot{\dot{\phi}}_\sigma(\tau)] = 2 i \text{sign} \tau,
\]

(33)
where \( \Lambda \) is cut-off parameter and is of order of \( \varepsilon_F \), and \( |\tau| \gg \Lambda^{-1} \). Note, that since the TL Hamiltonian (14) is quadratic in \( \hat{\Phi}_\nu \) and the boundary conditions (17)–(22) are linear, the fluctuations of the displacement fields \( \hat{\Phi}_\nu \) are Gaussian in zero approximation. Fermionic fields \( \hat{\Psi}_{s,r} \) are related to the field \( \hat{\Phi}_\nu, \hat{\Theta}_\nu \) by equation

\[
\hat{\Psi}_{s,r} = \hat{U}_{r,s} \sqrt{\frac{\Lambda}{2\pi v_F}} \exp \left[ -\frac{i}{\sqrt{2}} \left( r\hat{\Phi}_\rho - \hat{\Theta}_\rho + r s\hat{\Phi}_\sigma - s\hat{\Theta}_\sigma \right) \right],
\]

where \( \hat{U}_{r,s} \) are unitary Klein factors which decrease the number of fermions by one and commute with bosonic fields \( \hat{\Phi}_\nu \) and \( \hat{\Theta}_\nu \). Using (34), relations (30)–(33) and the identity \( e^{A_B} = e^{A+B} e^{[A,B]/2} \) we obtain fermionic correlation functions at the boundary \( x = 0 \) in time and energy representations

\[
\left\langle \hat{\Psi}_{+-}(t) \hat{\Psi}_{+-}^\dagger(t + \tau) \right\rangle = \left\langle \hat{\Psi}_{1+}^\dagger(t) \hat{\Psi}_{1+}(t + \tau) \right\rangle = \frac{\Lambda}{2\pi v_F} e^{-i\pi \tau F(\varepsilon)} \frac{\Lambda(1-K\rho)/2}{|\varepsilon - \varepsilon'|},
\]

\[
\left\langle \hat{\Psi}_{+-}(\varepsilon') \hat{\Psi}_{+-}^\dagger(\varepsilon) \right\rangle = \left\langle \hat{\Psi}_{1+}^\dagger(\varepsilon') \hat{\Psi}_{1+}(\varepsilon) \right\rangle = -2\pi \delta(\varepsilon - \varepsilon') \frac{\Lambda(1-K\rho)/2}{|\varepsilon - \varepsilon'|} F(\varepsilon),
\]

\[
F(\varepsilon) = \sin \pi \frac{1-K\rho}{4} \int_0^{+\infty} \frac{\sin x}{x^{1+K\rho}} dx - \cos \pi \frac{K\rho+1}{4} \int_0^{-\infty} \frac{\cos x}{x^{1+K\rho}} dx.
\]

In the rest of this section we focus on the case of dc voltage and take into account the contribution to the current due to the states above the superconducting gap considering this contribution as a perturbation. The fermionic field \( \hat{\Psi} \) at the boundary \( x = 0 \) can be represented as sum of the solution in zero approximation and a perturbation: \( \hat{\Psi} = \hat{\Psi}^{(0)} + \hat{\Psi}^{(1)} \).

We multiply fermionic boundary conditions (8)–(9) by the complex conjugate, and take into account that \( \left\langle \hat{\Psi}_{s,r}^\dagger \hat{R}_S \right\rangle = 0 \) since operators \( \hat{R}_S^N \) and \( \hat{R}_S^S \) are independent and the zero-approximation solution can be expressed in terms of \( \hat{R}_S^N \) using (28). Finally, we obtain boundary conditions for the first-order terms

\[
\left\langle :\hat{\Psi}_{1+}^\dagger(\varepsilon') \hat{\Psi}_{1+}(\varepsilon) : \right\rangle^{(1)} - \left\langle :\hat{\Psi}_{+-}(\varepsilon') \hat{\Psi}_{+-}(\varepsilon) : \right\rangle^{(1)} = \delta(|\varepsilon - \Delta)| \Xi \left( \left\langle :\hat{\Psi}_{+-}(\varepsilon') \hat{\Psi}_{+-}(\varepsilon) : \right\rangle^{(0)} + \left\langle :\hat{R}_S^S(\varepsilon') \hat{R}_S^S(\varepsilon) : \right\rangle \right),
\]

\[
\left\langle :\hat{\Psi}_{1+}^\dagger(\varepsilon') \hat{\Psi}_{1+}(\varepsilon) : \right\rangle^{(0)} + \left\langle :\hat{R}_S^S(\varepsilon') \hat{R}_S^S(\varepsilon) : \right\rangle,
\]

\[
\left\langle :\hat{\Psi}_{+-}^\dagger(\varepsilon') \hat{\Psi}_{+-}(\varepsilon) : \right\rangle^{(1)} - \left\langle :\hat{\Psi}_{+-}(\varepsilon') \hat{\Psi}_{+-}(\varepsilon) : \right\rangle^{(1)} = \delta(|\varepsilon - \Delta)| \Xi \left( \left\langle :\hat{\Psi}_{1+}^\dagger(\varepsilon') \hat{\Psi}_{1+}(\varepsilon) : \right\rangle^{(0)} + \left\langle :\hat{R}_S^S(\varepsilon') \hat{R}_S^S(\varepsilon) : \right\rangle \right),
\]

If the potential is applied to the normal lead then \( \left\langle :\hat{R}_S^S(\varepsilon') \hat{R}_S^S(\varepsilon) : \right\rangle = 0 \). Using the values of fermionic correlators in zero approximation (36) we transform (38)–(39) to time
representation at low voltages $V < \Delta$

$$
\left\langle \hat{\Psi}_{\uparrow+}(t)\hat{\Psi}_{\uparrow+}(t) \right\rangle^{(1)} - \left\langle \hat{\Psi}_{\downarrow-}(t)\hat{\Psi}_{\downarrow-}(t) \right\rangle^{(1)} = \left\langle \hat{\Psi}_{\uparrow+}(t)\hat{\Psi}_{\uparrow+}(t) \right\rangle^{(1)} - \left\langle \hat{\Psi}_{\downarrow-}(t)\hat{\Psi}_{\downarrow+}(t) \right\rangle^{(1)} = -\frac{4VF(K_{\rho})A(K_{\rho}) \cdot (1 - K_{\rho})}{\pi v_F} \left( \frac{\Lambda}{\Delta} \right)^{(1-K_{\rho})/2} \quad (40)
$$

where $A(K_{\rho}) = \frac{\Lambda/\Delta}{\pi} \sqrt{\frac{x^2 - 1}{x(5-K_{\rho})/2}} dx$. Thus, the contribution from the states above the gap can be considered as small only if interaction is weak enough $(1 - K_{\rho}) \ln(\Lambda/\Delta) \ll 1$. In this case $F(K_{\rho}) \approx 1/4$ and $A(K_{\rho}) \approx \ln(\Lambda/\Delta)$. The boundary conditions for the first-order terms in the bosonized form for $K_{\rho} \approx 1$ read

$$
\partial_x \hat{\Phi}_{\rho}^{(1)} = \frac{\sqrt{2V(1-K_{\rho})}}{v_F} \ln \left( \frac{\Lambda}{\Delta} \right), \quad (41)
\partial_t \hat{\Phi}_{\sigma}^{(1)} = 0. \quad (42)
$$

Thus, we find an averaged solution $\phi_{\rho}^{(1)} = \sqrt{2V(1-K_{\rho})} \ln(\Lambda/\Delta) \cdot (x/v_F - t)$. Then the contribution to the conductance in the first order of perturbation theory reads

$$
\delta G = -2G_0(1 - K_{\rho}) \ln \left( \frac{\Lambda}{\Delta} \right). \quad (43)
$$

V. CONCLUSIONS

We have studied electronic transport in 1D conductor attached to a superconducting lead and to a normal metal lead. If electron-electron interaction in the 1D conductor is neglected, I-V curve is linear at low voltages $V < \Delta/e$ and is sufficiently non-linear at $V \gtrsim \Delta/e$. In the limit of high voltages $V \gg \Delta/e$ I-V curve is almost linear with an excess current $\frac{4}{3}G_0\Delta/e$. If the applied voltage contains ac component then the static I-V curve is modified resembling the regime of photon assisted tunneling\(^{12}\). We show analytically that in the case of a weak interaction in the quantum wire, the interaction results in features in the frequency dependence of the impedance and reduces the dc conductance.
ACKNOWLEDGMENTS

The work was supported by Russian Foundation for Basic Research (RFBR) and by Fund of non-profit programs "Dynasty".

1 S. N. Artemenko, A. F. Volkov, and A. V. Zaitsev, Solid State Comm. 30, 771 (1979).
2 A. V. Zaitsev, Sov. Phys. JETP 51, 111 (1980); 52, 1018 (1980).
3 B. J. van Wees, H. van Houten, C. W. J. Beenaker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. 60, 848 (1988).
4 D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, J. Phys. C 21, 209 (2000).
5 C. W. J. Beenakker, Phys. Rev. B 46, 12841 (1992).
6 D. L. Maslov and M. Stone, Phys. Rev. B 52, 5539 (1995).
7 I. Safi and H. J. Schulz Phys. Rev. B 52, 17040 (1995).
8 V. V. Ponomarenko Phys. Rev. B, 52, 8666 (1995).
9 T. Giamarchi, Quantum Physics in One Dimension (Clarendon Press, 2003).
10 J. Voit, Rep. Prog. Phys. 58, 977 (2007).
11 B. Wang, Y. Wei, and J. Wang Phys. Rev. B 86, 035414 (2012).
12 P. K. Tien, J. P. Gordon Phys. Rev. 129, 647 (1963).
13 D. Maslov, M. Stone, P. Goldbart, and D. Loss Phys. Rev. B 53, 1548 (1996).
14 R. Egger and H. Grabert Phys. Rev. Lett. 77, 538 (1996); 80, 2255 (1998).
15 S.N. Artemenko, P.P. Aseev, and D.S. Shapiro, JETP Letters 91, 589(2010).
16 A. F. Volkov, P. H. C. Magnée, B. J. van Wees, and T. M. Klapwijk, Physica C 242, 261 (1995).
17 N.B. Kopnin and A.S. Melnikov, Phys. Rev. B 129, 064524 (2011).