Determining the geometric parameters of a blade runner that has a geometry obtained through the photogrammetry technique

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Abstract. For a blade runner of a Kaplan turbine, with its 3D geometry obtained through the Photogrammetry technique, the paper aims to determine the following geometric parameters of the blade profile: the stagger angle, the chord length, the relative thickness (maximum thickness / chord length) and relative thickness position, the relative maximum camber and the maximum position of the profile camber, using the following software packages: SolidWorks and Visual Basic for Applications included in Microsoft Excel. The blade profiles are generated through multiple sections of the axial blade runner with coaxial cylinders with the runner axis and plane unfolding. These parameters will be compared with the corresponding design parameters in order to investigate the precision of the Photogrammetry technique.

1. Introduction

The geometric parameters of the profiles that have an effect on the hydrodynamic performances are:

- the chord line L, defined as the straight line connecting the leading and trailing edges;
- the stagger angle \( \beta_S \);
- the maximum mean camber line \( f/L = (Y_S)_{\text{max}} \); the camber is the distance between the mean camber line and the chord line, measured perpendicular to the chord line;
- the location of the maximum mean camber line \( X_f/L \);
- the maximum thickness \( d/L = (Y_G)_{\text{max}} \); the thickness is the distance between the suction (upper) side and the pressure (lower) side;
- the location of the maximum thickness \( X_d/L \).

2. The hydrodynamic profile

We will consider that the ordinate \( Y \) of the hydrodynamic profile (Figure 1) is expressed through equation (1) and the profile can be described by superimposing two functions: the profile mean camber line function \( Y_S \) (Figure 2) and the thickness function \( Y_G \) (Figure 3). As a consequence, the suction (upper) side \( Y^E \) and pressure (lower) side \( Y^I \) ordinates can be calculated using equation (2) and (3) respectively.
\[
Y = Y_s \pm Y_g \\
Y^E = Y_s + Y_g \\
Y^I = Y_s - Y_g
\] (1) (2) (3)

A number of six 3D profiles were generated through reconstructed blade intersection [1] with 6 cylinders (Figure 4) and each profile was divided in 100 points using SolidWorks (Figure 5). Using a macro, these points (X, Y, Z coordinates) were exported from SolidWorks to Microsoft Excel.

The correspondence between the 3D profile generated through the blade intersection with cylinders and the same profile projected in the YZ plane is presented in (Figure 6), where:
X – the rotation axis of the blade;
Z – the runner axis;
Y – direction perpendicular to the XZ plane;
R – the radius of the intersection cylinder;
P – point on the 3D profile;
O – the origin point of the blade, placed on the runner axis Z;
θ – the angle of the point P, measured between the X axis and R radius;
\(Y_{desf}\) – the curvilinear coordinate of point P;
P₁ – the projection of the point P in the YZ plane;
BA, BF – the leading and trailing edges of the profile;
L₁ – the profile chord projected in the YZ plane, which join BA and BF;
O₁ – the point corresponding to the X axis of the blade;
χₛ – the angle of the profile projected in the YZ plane.

Figure 7 show the profile unfolded, where the Z ordinate is preserved, and the Y abscissa is substituted by the \( Y_{\text{desf}} \) abscissa, calculated using the following equation:

\[
Y_{\text{desf}} = R \cdot \theta^{[\text{rad}]} = R \cdot \arcsin(Y/R)
\]  

(4)

The unfolded profile is expressed in the \( Y_{\text{desf}}Z \) coordinate system, which originates at point O₁ and has a positive direction to the right for the \( Y_{\text{desf}} \) axis and to up for the Z axis.

Figure 7. The unfolded profile in \( Y_{\text{desf}}Z \) plane.

For every profile, based on (Figure 7), the following parameters can be calculated:

- L – the profile chord;

\[
L = \sqrt{[Y_{\text{desf}}(BF) - Y_{\text{desf}}(BA)]^2 + [Z(BF) - Z(BA)]^2}
\]

(5)

- \( \beta_S \) – the stagger angle;

\[
\beta_S = \arctg \left( \frac{Z(BA) - Z(BF)}{Y_{\text{desf}}(BF) - Y_{\text{desf}}(BA)} \right)
\]

(6)

where:
Yₐₑₜₕₜ(BA) – the leading edge abscissa;
Z(BA) – the leading edge ordinate;
Yₐₑₜₕₜ(BF) – the trailing edge abscissa;
Z(BF) – the trailing edge ordinate.

To calculate the geometric parameters of the profile, the maximum mean camber line f/L and its locations, the maximum thickness d/L and its location, it’s imperative to translate the coordinates of the profile from the \( Y_{\text{desf}}Z \) coordinate system, (Figure 7), to the \( X_P, Y_P \) coordinate system, (Figure 8), by rotating the profile around the O₁ point, so that the L chord is placed in the \( X_P \) direction, oriented positively to the right, and the \( Y_P \) ordinate is perpendicular to the chord; the rotation is based on the following equations, where the “e” and “i” indices refer to the suction (upper) side and pressure (lower) side respectively:
\[ X_e = X_l = Y_{desf} + Y_{desf}(BA) \] (7)

\[ Y_e = Z_e + Z(BF) \] (8)

\[ Y_{Pe} = \frac{[L \cdot \sin(\beta_S) - Y_e] \cdot \cos(\beta_S) - X_e \cdot \sin(\beta_S)}{L} \] (9)

\[ X_{Pe} = \frac{X_e + Y_{Pe} \cdot L \cdot \sin(\beta_S)}{L \cdot \cos(\beta_S)} \] (10)

\[ Y_{Pi} = \frac{[L \cdot \sin(\beta_S) - Y_i] \cdot \cos(\beta_S) - X_i \cdot \sin(\beta_S)}{L} \] (11)

\[ X_{Pi} = \frac{X_i + Y_{Pi} \cdot L \cdot \sin(\beta_S)}{L \cdot \cos(\beta_S)} \] (12)

Thus, by rotating the profile, the current point M from the Y_{desf} Z coordinate system, (Figure 7), is transposed into the profile’s own coordinate system at the M_P point, which has its X_P, Y_P coordinates (Figure 8).

From the equations (7) ÷ (12) one can notice that the X_Pe, X_Pi abscises and the Y_Pe and Y_Pi ordinates are divided by the profile chord L, so they are without units. In this way, we can associate the profile with a parametric representation \( Y_P = Y_P(\phi) \), where the \( \phi \) angle is the argument of the parametric representation, (Figure 9), (Figure 10), with the following particular values: \( \phi = 0^\circ = 360^\circ \) – correspond to the trailing edge BF, \( \phi = 180^\circ \) – correspond to the leading edge BA, \( \phi \in (0^\circ \div 180^\circ) \) – correspond to the suction (upper) side and \( \phi \in (180^\circ \div 360^\circ) \) – correspond to the pressure (lower) side.

Through this parametric representation, every point on the suction and pressure side of the profile is associated with an \( \phi \) angle, which can be calculated using the following equations:

\[ \phi^e = \arccos(2 \cdot X_{Pe} - 1) \] (13)

\[ \phi^i = 360^\circ - \arccos(2 \cdot X_{Pi} - 1) \] (14)

**Figure 9.** The correspondence between the profile points and the \( \phi \) angle.

**Figure 10.** The values of the ordinates profile \( Y_{Pe,i} \) as a function of the \( \phi \) angle.

For each section (Figure 11) – (Figure 16) the profiles projected in the YZ plane are shown, over which the unfolded profiles were placed.
Table 1 and (Figure 17) – (Figure 18) show the values of the chord $L$ and the stagger angle $\beta_S$ of the profiles, resulted from the project, compared to those calculated by equations (5) and (6) for the reconstructed blade using Photogrammetry technique (3D scanning). The maximum percentage deviation for the stagger angle $\beta_S$ is 3.89% and 1.07% for the chord.
Table 1. The comparison of the stagger angle and the chord.

| The profile section | The stagger angle $\beta_s$ [°] | From project | From 3D scanning | Deviance $\Delta\beta_s$ [%] | The chord $L$ [mm] | From project | From 3D scanning | Deviance $\Delta L$ [%] |
|---------------------|--------------------------------|--------------|-----------------|-----------------------------|------------------|--------------|-----------------|-----------------------------|
| 1                   | 29.48443                      | 29.339       | 0.49            | 2092.646                    | 2110.2191        | -0.84        |
| 2                   | 24.5521                       | 25.137       | -2.38           | 2443.661                    | 2449.491         | -0.24        |
| 3                   | 20.85922                      | 20.940       | -0.39           | 2808.686                    | 2828.7963        | -0.72        |
| 4                   | 18.0322                       | 18.313       | -1.55           | 3182.622                    | 3216.748         | -1.07        |
| 5                   | 15.80784                      | 16.259       | -2.86           | 3561.574                    | 3573.2842        | -0.33        |
| 6                   | 14.00279                      | 14.547       | -3.89           | 3945.68                      | 3984.6823        | -0.99        |

Figure 17. The stagger angle comparison of the profiles.

Figure 18. The chord comparison of the profiles.

Through equations (7) – (12), every point of the profiles from (Figure 11) – (Figure 16) was transposed into a profile with its own coordinate system $X_p, Y_p$ using the Excel application and the Visual Basic programming language embedded in Excel, by following the next steps:

- a number of 51 values were generated for the $X_{spline}$ abscissa in the range 0÷1 with a 0.02 step;
- for every $X_{spline}$ abscissa, the ordinates values $Y_{E_{spline}}$ and $Y_{I_{spline}}$ where calculated using spline interpolation;
- for every $X_{spline}$ abscissa, the relative thickness $(d/L_{spline})_X$ was calculated using equation (15); the maximum relative thickness $d/L$ is the maximum of the $(d/L_{spline})_X$ values and its position $X_d/L$ is the abscissa value $X_{spline}$ where the maximum was recorded;
- for every $X_{spline}$ abscissa, the relative camber $(f/L_{spline})_X$ was calculated using equation (16); the maximum mean camber line $f/L$ is the maximum of the $(f/L_{spline})_X$ values and its position $X_f/L$ is the abscissa value $X_{spline}$ where the maximum was recorded.

$$\frac{d}{L_{spline}}_X = Y_{E_{spline}} - Y_{I_{spline}}$$

$$Y_{S_{spline}} = (f/L_{spline})_X = Y_{I_{spline}} + \frac{1}{2} \cdot X(\frac{d}{L_{spline}})_X$$

The values of $d/L$, $X_d/L$, $f/L$, $X_f/L$ were calculated for equidistant and discreet values of the $X_{spline}$ abscissa, so they are obtained with approximation, but the precision can improve by increasing the number of the $X_{spline}$ values.

For each section (Figure 19) – (Figure 24) and Table 2, the profiles transposed into their own profile coordinate system $X_p, Y_p$ are presented. The maximum relative thickness $d/L$ and its position $X_d/L$ is marked with a dash line and the maximum mean camber line $f/L$ values and its position $X_f/L$ are marked with diamond point.
Figure 19. The profiles of section 1.

Figure 20. The profiles of section 2.

Figure 21. The profiles of section 3.

Figure 22. The profiles of section 4.

Figure 23. The profiles of section 5.

Figure 24. The profiles of section 6.
3. Smoothing the profiles

The geometry of any object cannot coincide perfectly with its theoretical one, imposed by the drawing, because it is affected by deviations that should fall within a range of tolerances imposed. Additionally, the Photogrammetry technique can generate errors specific to any measurement process. As a consequence, the intersection of the scanned geometry of the blade with the cylinders can generate slightly irregular profiles. This section proposes an algorithm for smoothing the boundary of any profile, in order to generate a profile as close to the original as possible and with a continuous boundary, but with the removal of existing irregularities. In order to achieve this goal, the following algorithm is proposed, based on the mathematical expression of the boundary of the profile through a Fourier series:

\[
x_k = \frac{X_k}{L} = \frac{1}{2} \cdot (1 + \cos(\varphi))
\]

\[
y_k = \frac{Y_k}{L} = \frac{a_0}{2} + \sum_{n=1}^{N_0} \left[ a_n \cdot \cos(n \cdot \varphi) + b_n \cdot \sin(n \cdot \varphi) \right]
\]

\[
y_S(\varphi) = \frac{a_0}{2} + \sum_{n=1}^{N_0} a_n \cdot \cos(n \cdot \varphi)
\]

\[
y_G(\varphi) = \sum_{n=1}^{N_0} b_n \cdot \sin(n \cdot \varphi)
\]

where:
- L – the profile chord;
- \((X_k, Y_k)\) – the boundary profile coordinates;
- \((x_k, y_k)\) – the boundary profile coordinates with no units;
- \(a_n, b_n\) – the trigonometric polynomial coefficients of the Fourier interpolation;
- \(y_S(\varphi)\) – the profile mean camber line function, expressed only with \(a_n\) coefficients;
- \(y_G(\varphi)\) – the thickness function, expressed only with \(b_n\) coefficients;
- \(\varphi\) - the angular argument of the parametric representation.

In the case of parametric representation, only the first \(2 \cdot N_0 + 1\) coefficients of the trigonometric series are retained, as the coefficients of the Fourier series decrease rapidly as a value and, with them, the influence of higher order harmonics decreases.

Profiles can be generated analytically or numerically through points. Analytical expressions may exist in the case of taking profiles from catalogs or literature, for which a mathematical expression of the boundary is provided. However, in the case of numerical expressions, a mathematical form of expression of the profile border is required; this is the case of profiles which arise from the intersection of the rotating blade with the coaxial cylinders with the rotor axis as in the example of the present work, when the blade geometry arises from the Photogrammetry technique.

For a given number of \((X_k, Y_k)\) coordinates of the profile boundary, the calculation of the Fourier series coefficients, the harmonic analysis method (called the twelve-order algorithm \(2 \cdot N_0\)) is used [2] – vol. III, pp. 512 ÷ 517, where \(N_0 = 6\), by following the next steps:

- calculate the boundary profile coordinates \((x_k, y_k)\) with no units, by dividing the coordinates \((X_k, Y_k)\) with the profile chord L;
- for each point \((x_k, y_k)\), from equations (13), (14), the argument of the parametric representation \(\phi\), corresponding to the suction (upper) \(\phi_u\) and the pressure (lower) side \(\phi_l\) is calculated, thus obtaining the correspondence \(y_k = y_k(\phi)\) (Figure 25);
- by interpolating the correspondence \(y_k = y_k(\phi)\) by cubic spline functions [3], a number of \(2 \cdot N_0 = 12\) initial values of ordinates \(y_i\) corresponding to the equidistant values of the argument \(\phi\), can be calculated, (Figure 25);
- the calculation of the Fourier series coefficients can be obtained through the following equations:
\[ a_n = \frac{1}{N_o} \sum_{n=0}^{N_o-2} \left[ y_i \cdot \cos \left( i \cdot \frac{n}{N_o} \cdot \pi \right) \right] \text{ for } 0 \leq n < N_o \]  
\[ b_n = \frac{1}{N_o} \sum_{n=0}^{N_o-2} \left[ y_i \cdot \sin \left( i \cdot \frac{n}{N_o} \cdot \pi \right) \right] \text{ for } 0 \leq n < N_o \]  

- by generating 73 values of the \( \phi \) angle, in the range 0 ÷ 360\(^o\), with an equidistant step of 5\(^o\), through equations (17) ÷ (20), a number of 73 points are obtained for the smoothed boundary profile; (Figure 26) show the comparison of the initial profile of section 6 from (Figure 24), with the smoothed profile, that was calculated using the Fourier series; one can notice the border of the smooth profile overlaps the original border and is continuous and without irregularities; because the calculations are difficult to do manually, the Excel environment and Visual Basic were used as a programming language;
- considering the mathematical expression of the profile boundary through the Fourier series, the profile mean camber line and the thickness function, equations (19), (20), in Visual Basic one can calculate precisely, by using the bisection method, the geometric parameters of the profile: the relative thickness \( d/L \) and its position \( X_d/L \), the maximum mean camber line \( f/L \) and its position \( X_f/L \), (Figure 27) and Table 2.

**Figure 25.** The correspondence \( y_k = y_k(\phi) \).

**Figure 26.** The initial and the smoothed profile of section 6.

**Figure 27.** The geometrical parameters of the blade profiles comparison.
4. Conclusions
The paper calculates the geometric parameters of a blade’s profiles whose geometry is obtained through the Photogrammetry technique. By comparing with the design parameters, the deviation of the profile chord is 1.07%, while for the stagger angle $\beta$ is 3.89%. The relative thickness $d/L$ and its position $X_d/L$, the maximum mean camber line $f/L$ and its position $X_f/L$ were calculated utilizing two methods: through spline interpolation for equidistant and discreet values of the $X_{spline}$ abscissa and through the twelve-order algorithm combined with bisection method. The resulting values are quite close and inscribed in the general recommendation found in literature for turbine blade design [4], [5], [6]. A smoothing algorithm for the boundary of the profile based on a Fourier series is proposed and validated by the comparative calculated values. This algorithm can compensate the deviations that arise from the scanning process of a real blade, regardless of whether laser scanning or photo-scanning technology is used.

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