Flavoring Monochromatic Neutrino Flux from Dark Matter Annihilation

Yasaman Farzan

School of physics, Institute for research in fundamental sciences (IPM)
P.O. Box 19395-5531, Tehran, Iran

Abstract

As is well-known, if the scattering cross section of the Dark Matter (DM) particles off the nuclei inside the Sun is large enough, the Sun can trap DM particles. In principle, the annihilation of DM pair inside the Sun can then lead to the detection of a relatively large flux of neutrinos in the neutrino telescopes. If the annihilation directly produces a neutrino pair, the flux of neutrinos on Earth will be monochromatic. In this case, the oscillatory terms in the oscillation probability lead to a novel seasonal variation of detected events which is sensitive to the initial flavor composition. In this paper, we propose two models that predict such a detectable monochromatic neutrino flux from the DM annihilation. Model I, which is based on augmenting the type II seesaw mechanism, predicts a flavor composition for the monochromatic flux determined by \((m_\nu)_{\alpha\beta}\). In model II, the DM pair first annihilates to a pair of sterile neutrinos which oscillate into active neutrinos with a flavor composition determined by the flavor structure of the active-sterile neutrino mixing.

1 Introduction

Although reasonably large amount of evidence has been accumulating in favor of Dark Matter (DM) as the explanation of the missing mass problem of the universe, the nature of DM particles is still unknown. Under the assumption that the DM particles are produced thermally in the early universe and there is only one kind of DM, the measured DM abundance in the universe
determines the self-annihilation cross section to be 1 pb. However, the DM mass or its annihilation products are still unknown.

Arguably one of the most plausible classes of DM is Weekly Interacting Massive Particles (WIMP) with a mass, \( m_{DM} \), in the range 100 GeV – few TeV. This range of DM mass is interesting both for direct and indirect techniques of DM particle detection. Direct detection techniques are based on measuring the recoil energy in the scattering of DM particles off nuclei in a target. Indirect detection techniques are based on detecting the particles produced by DM annihilation in regions such as the Sun or the galaxy center where the DM concentration is relatively high.

Within certain popular models such as MSSM, the DM pairs first annihilate to pairs such as \( b\bar{b}, \tau\bar{\tau}, ZZ \) and \( W^+W^- \). Neutrinos are then produced as secondaries through their decays. However, as discussed in [1], there are various ways to build a model in which DM particles dominantly annihilate to neutrinos.

In this paper, we are interested in detecting the neutrino flux from the annihilation of DM particles that have been accumulated inside the Sun. These DM particles are non-relativistic. As a result, if neutrinos are directly produced by the DM pair annihilation, the spectrum will be monochromatic with \( E_\nu = m_{DM} \). However, if neutrinos are secondary products of the DM annihilation, their spectrum will be continuous with \( E_\nu < m_{DM} \). In either case, if the DM mass, \( m_{DM} \), is larger than the detection threshold of the neutrino telescopes, a high energy neutrino flux pointing towards the Sun is expected at ICECUBE. Remember that the energy of ordinary neutrinos produced by proton fusion in the Sun center is too small to be detectable by neutrino telescopes with detection energy threshold of \( E_{th} \gg a \text{ few } 10 \text{ MeV} \).

The purpose of the present paper is to build models within which the method proposed in [2, 3] is effective. Such a model should have the following properties: (1) The Dark matter mass is larger than the detection
energy threshold of the neutrino telescope: $O(50 \text{ GeV})$-$O(100 \text{ GeV})$. (2) The DM pair dominantly annihilates to a neutrino pair with a non-trivial flavor composition and a total annihilation cross section of 1 pb. (3) As it will be discussed in the next section, to obtain enough statistics, the scattering cross section of DM particles off nucleons should be greater than $10^{-9}$ pb. On the other hand, it should be smaller than $10^{-8}$ pb to evade the bounds from direct detection \cite{4}. Other examples of such models can be found in Ref. \cite{5}.

In sect. 2, we briefly review the method proposed in \cite{2, 3} and formulate the conditions a model has to satisfy to predict a sizable seasonal variation in the number of events from the DM annihilation in the Sun center. In sect. 3, we propose Model I which embeds type II seesaw mechanism. Within this model, the flavor structure of $\sigma(DM + DM \rightarrow \nu_\alpha \nu_\beta)$ is determined by $(m_\nu)_{\alpha\beta}$. In sect. 4, we propose Model II within which the DM pair first annihilates to a pair of sterile neutrinos and then the sterile neutrinos oscillate into active neutrinos on the way to Earth. In sect. 5, we review our conclusions.

2 A novel method to extract information about dark matter particles

The DM particles propagating in the solar system have velocities about a few hundred km/sec. When these particles enter the Sun, they can lose their kinetic energy by scattering off the nuclei. They will then fall in the gravitational well of the Sun. As a result, during the Sun lifetime, the density of DM in the Sun has increased. The number density increases with the scattering cross section of these particles off nuclei. The trapped DM particles virilize and come to thermal equilibrium with the nuclei in the Sun center. Equating $|E_{\text{kinetic}}| = 3k_B T_\odot/2$ with $|V_{\text{gravity}}| = 4\pi G_N \rho_{\text{DM}} r_{\text{DM}}^2/3$, we find that the virilized DM particles are centered in the Sun within a volume of
radius $r_{DM} \sim (9k_B T_\odot/8\pi G N_\odot \rho m_{DM})^{1/2}$. The DM particles inside the Sun are non-relativistic so the annihilation of a DM pair into a pair of on-shell particles will result in a monochromatic spectrum. In case that a neutrino pair is directly produced by the annihilation of the DM pair, the energy of each neutrino will be equal to the DM mass, $m_{DM}$. However, if the neutrinos are the decay products of unstable particles produced by the DM pair annihilation, their spectrum will be continuous. In the latter case because of the very large distance between the Sun and Earth, $L_{Sun-Earth}$, the oscillatory terms in neutrino oscillation probability given by $\sin(\Delta m^2 L_{Sun-Earth}/2E_\nu)$ will average to zero. However, as discussed in [2], in the former case where the spectrum is monochromatic, the oscillatory effects in the oscillation probability are not averaged out and therefore lead to a seasonal variation in the number of events at ICECUBE as the Sun-Earth distance varies during a year because of the eccentricity of the Earth orbit. In [3], it was shown that studying this seasonal variation provides information on the flavor structure of $\nu_\alpha \rightarrow \nu_\alpha + (\nu_\beta)$ as well as on the value of $m_{DM}$ (through the combination $\Delta m^2_{21}/m_{DM}$). In particular, observing oscillatory behavior on top of the trivial variation of inverse of the square of the Earth Sun distance ($i.e., 1/L^2_{Earth-Sun}$) means the initial flux is monochromatic with a non-trivial non-democratic flavor composition ($i.e., F_{\nu_e}:F_{\nu_\mu}:F_{\nu_\tau} \neq 1:1:1$).

In [3], the details of this novel method to extract information on the properties of DM annihilation modes have been discussed. Of course, to study the time variation of the neutrino flux, sufficient statistics is also required. The statistics is determined by the rate at which the Sun traps DM particles and this rate in turn depends on the scattering cross section of the DM particles off the nuclei in the Sun. Scattering of DM particles off nucleons inside a nucleus is coherent so in the case that scattering is spin independent, the cross section of DM particles off a nucleus of $Z$ protons and $A-Z$ neutrons is proportional to $|Z\mathcal{M}_p + (A-Z)\mathcal{M}_n|^2$ where $\mathcal{M}_p$ and $\mathcal{M}_n$ are respectively
the scattering amplitude of the DM particles off a single proton and neutron. (Notice that if the scattering was not coherent, the cross section would be given by $Z|\mathcal{M}_p|^2 + (A - Z)|\mathcal{M}_n|^2$.) Because of the non-linear dependence on $Z$ and $A - Z$, heavier nuclei can trap the DM more effectively than the same number of separate nucleons. Thus, although the majority of the mass of the Sun is composed of protons, in evaluating the capture rate via spin-independent scattering, the presence of the heavier nuclei inside the Sun has to be taken into account. It can be shown that for $m_{DM} > 100$ GeV, if the spin-independent cross section is larger than $10^{-9}$ pb and each DM pair produces a neutrino pair, a few hundred events can be registered each year in a detector such as ICECUBE \cite{3, 6}. In this mass range, the direct bounds on the cross-section of the spin-independent scattering of DM particles off nucleons is $\sim 10^{-8}$ pb.

In \cite{2}, various possible sources of widening of the monochromatic spectrum from $\text{DM} + \text{DM} \rightarrow \nu_\alpha + \nu_\beta$ have been studied. The main source of widening turns out to be the thermal distribution of the velocities of the initial DM particles. This thermal distribution widens the monochromatic line to a narrow Gaussian with a width $\Delta E/E \sim 10^{-4}(100 \text{ GeV}/m_{DM})^{1/2}$ \cite{2}. At these energies a fraction of active neutrinos undergo interaction with the nuclei inside the Sun while they cross the Sun. The charged current interactions of $(\nu_e)$ and $(\nu_\mu)$ produce charged leptons which are absorbed. The charged current interaction of $(\nu_\tau)$ produces a charged tau which in turn decays producing $(\nu_\tau)$ with a lower energy. The neutral current interaction of the neutrinos produces a neutrino of a lower energy. The outcome is that the scattering will reduce the height of the monochromatic line but will add a tail to the spectrum. Since the cross section of the neutral current interaction is finite in the forward scattering, the sharp line will remain sharp. At the sun surface the spectrum will be composed of a sharp line superimposed on the end of a continuous spectrum consisting of the scattered and regenerated
neutrinos. For $m_{DM} < 500 \text{ GeV}$, the energy and therefore cross section of the produced neutrinos are small which means a good fraction of the neutrino will remain unscattered resulting in a significant seasonal variation due to the oscillatory terms in the oscillation probability. However, for larger $m_{DM}$, the majority of the neutrinos are supposed to be scattered before leaving the Sun. This means in general for $m_{DM} > 500 \text{ GeV}$, the method proposed in [3] is ineffective. In building Model I, we take this consideration into account. However, in the case of Model II, DM pair first annihilates into sterile neutrinos that leave the Sun unscattered. These neutrinos then oscillate to active neutrinos. This means in the case of Model II, even if $m_{DM} > 500 \text{ GeV}$, the method might be effective.

On the other hand, if $m_{DM} < \text{ a few } \times 10 \text{ GeV}$, the energy of neutrinos will be below the detection threshold of ICECUBE. Moreover, for lower values of the DM mass, the correlation between the direction of incoming neutrinos and the produced charged lepton will be lost. This in turn means using the directionality to reduce the background from the atmospheric neutrinos will become less efficient. Considering these facts, in model building we set $m_{DM} > 100 \text{ GeV}$.

3 MODEL I

In this section, we build a model for neutrino mass and dark matter by slightly augmenting the type II seesaw mechanism. Through the standard type II seesaw mechanism, this model will lead to a Majorana mass for neutrinos. The DM candidate in this model can be either a complex scalar or a Dirac fermion that we add to the model. A $Z_2$ symmetry guarantees its stability.

As is well-known, within the type II seesaw mechanism, a $SU(2)$ triplet
scalar exists with a nonzero hypercharge as follows

$$\Delta = \begin{bmatrix} \Delta^+ \sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+ \sqrt{2} \end{bmatrix}.$$  \hspace{1cm} (1)

The left-handed leptons couple to this triplet as follows

$$\mathcal{L} = \frac{f_{\alpha \beta}}{2} \epsilon_{ik} \bar{L}_i \Delta_{kj} L_j = \frac{f_{\alpha \beta}}{2} \left( \Delta^0 \nu_{\alpha \beta}^T c_{\nu} + \frac{\Delta^+}{\sqrt{2}} (\nu_{\alpha \beta}^T c_{\nu} + i_{\alpha \beta}^T c_{\nu}) + \Delta^{++} i_{\alpha \beta}^T c_{\nu} \right)$$ \hspace{1cm} (2)

in which $\alpha$ and $\beta$ are flavor indices, $i$, $j$ and $k$ are $SU(2)$ indices and $C$ is the $2 \times 2$ charge conjugation matrix: $C_{11} = C_{22} = 0$ and $C_{12} = -C_{21} = 1$. By assigning lepton number equal to $-2$, the lepton number will be preserved by the coupling in Eq. (2). The pure scalar part of the potential involving only $H$ and $\Delta$ is given by

$$V = m^2 H^\dagger H + M_3^2 Tr[\Delta^\dagger \Delta] + \frac{\lambda_1}{4} (H^\dagger H)^2 + \lambda_2 H^\dagger \Delta^\dagger \Delta H + \lambda_3 Tr[\Delta^\dagger \Delta]H^\dagger H$$

$$+ \frac{\lambda_{\Delta 1}}{4} (Tr[\Delta^\dagger \Delta])^2 + \frac{\lambda_{\Delta 2}}{4} |Tr[\Delta \Delta]|^2$$

$$+ \mu \left( (\Delta^0)^\dagger (H^0)^2 + \sqrt{2} \Delta^--H^+H^0 + \Delta^{--}H^+H^+ \right).$$  \hspace{1cm} (3)

Notice that other forms of quartic coupling of $H$ and $\Delta$ which preserves lepton number and electroweak symmetry can be rewritten as the combinations of the above terms. For example, $Tr(\Delta \Delta^\dagger [\Delta^\dagger, \Delta]) = |Tr(\Delta \Delta)|^2 - [Tr(\Delta^\dagger \Delta)]^2$, $Tr(\Delta^\dagger \Delta \Delta^\dagger \Delta) = (Tr(\Delta^\dagger \Delta))^2 - |Tr(\Delta \Delta)|^2/2$ or $H^\dagger \Delta \Delta^\dagger H = Tr(H^\dagger H)Tr(\Delta^\dagger \Delta) - H^\dagger \Delta \Delta^\dagger H$. If the quartic couplings are all positive the potential will be stable; i.e., as $\Delta$ and/or $H \to \infty$, $V$ remains positive. In fact, a weaker condition guarantees stability: For example, as long as $\lambda_{\Delta 1} > 0$, the condition $\lambda_{\Delta 1} + \lambda_{\Delta 2} > 0$ guarantees stability even if $\lambda_{\Delta 2}$ is negative.

The terms in the third line of Eq. (3) can be rewritten as $\epsilon_{ik} H_i \Delta_{kj}^\dagger H_j$. Since we have assigned lepton number of $-2$ to $\Delta$, lepton number will be broken only softly by $\mu$. Along with $H$, $\Delta^0$ also receives a tiny VEV proportional...
to the lepton number violating $\mu$ parameter

$$
\langle H \rangle = \frac{v}{\sqrt{2}} \quad \langle \Delta^0 \rangle = \frac{-\mu v^2}{2M^2_\Delta + \lambda_3 v^2}.
$$

(4)

Since $\mu v/m^2_\Delta \ll 1$, we expect $\langle \Delta^0 \rangle \ll \langle H \rangle$. The first term in Eq. (2) then gives a Majorana mass to neutrinos

$$(m_\nu)_{\alpha\beta} = -f_{\alpha\beta} \langle \Delta^0 \rangle = f_{\alpha\beta} \frac{\mu v^2}{2M^2_\Delta + \lambda_3 v^2}.$$  

Although the coupling in Eq. (2) preserves the total lepton number, it violates lepton flavor and can therefore give a significant contribution to Lepton Flavor Violating (LFV) processes. For a comprehensive review see [7]. In particular, the bounds on $\mu \rightarrow eee$ sets the bound $f_{ee}f_{\mu e}/m^2_\Delta < 1.2 \times 10^{-5}$ TeV$^{-2}$. Taking $m_\Delta \sim 1$ TeV and $f_{\alpha\beta} \sim 0.01 - 0.001$, all the bounds will be satisfied.

For $m_\Delta \sim$ TeV and $f_{\alpha\beta} \sim$ few $\times 10^{-3}$, the values of $\mu$ smaller than 10 keV result in small enough neutrino mass. The smallness of the $\mu$ parameter can be justified by 'tHooft criterion: In the limit that $\mu$ vanishes, the lepton number is preserved.

Let us now discuss the DM sector. In the following, we discuss two kinds of candidates: (1) complex scalar; (2) Dirac Fermions. In both cases, we assign a lepton number equal to 1 to the DM candidate. We introduce a $Z_2$ symmetry under which only the DM candidate is odd. The $Z_2$ symmetry stabilize the DM candidate. In order for the DM pair to annihilate to neutrinos, we need to introduce another scalar, $\eta$, which mixes with $\Delta^0$. A similar model in the context of extra dimensions has been studied in [8]. Before discussing the DM particles, let us focus on $\eta$. We take $\eta$ to be a singlet complex scalar with lepton number opposite to that of $\Delta$ with the following interaction term

$$V = \lambda_4 \eta \epsilon_{jk} H^*_i \Delta_{ik} H^*_j$$
which after electroweak symmetry breaking leads to a mixing term:

\[ \lambda_4 \frac{v^2}{2} \eta \Delta^0. \]

Taking \( m_\Delta^2 \gg m_\eta^2 \sim \lambda_4 v^2 / 2 \), we shall have two mass eigenstates with masses approximately equal to \( m_\eta^2 \) and \( m_\Delta^2 \) and mixing of \( \lambda_4 v^2 / (2 m_\Delta^2) \).

A term such as \( \eta^2 H^\dagger \cdot H \) is a lepton number breaking term with a dimensionless coupling so we do not include it. We should protect \( \eta \) from getting a large VEV; otherwise it will lead to a large \( \langle \Delta^0 \rangle \) and therefore large neutrino mass. A term of form

\[ \mu' \eta H^\dagger \cdot H \]

leads to a mixing between \( \eta \) and \( h \) given by \( \mu' v / (m_\eta^2 - m_\eta^2) \) and induces \( \langle \eta \rangle = -\mu' v^2 / m_\eta^2 \). Notice that \( \mu' \) breaks lepton number softly, so it should be also suppressed: \( \mu' \ll m_\eta \). The subsequent shift of \( \Delta^0 \) will be given by \( \lambda_4 (\mu')^2 v^2 / m_\eta^2 \). This small shift in \( \langle \Delta^0 \rangle \) does not change the situation. A small lepton number violating mass term of form \( \eta^2 \) can be also added to the Lagrangian but it has no serious impact on the discussion.

As mentioned above, the DM can be either a complex scalar or a Dirac fermion. In both cases, the DM pair annihilates to a neutrino pair via a s-channel exchange of a scalar mass eigenstates that are mixtures of \( \eta \) and \( \Delta^0 \). Let us now discuss both possibilities one by one.

- **Complex scalar, \( \Phi \), as DM**

The general \( Z_2 \) invariant and lepton number conserving Lagrangian involving \( \Phi \) can be written as

\[
m_\Phi^2 \Phi^\dagger \cdot \Phi + \frac{m_{\eta \Phi \Phi}}{2} \eta \Phi \Phi + H.c.) + \\
\frac{\lambda_\Phi}{4} (\Phi^\dagger \cdot \Phi)^2 + \lambda_{H \Phi} H^\dagger \cdot H \Phi^\dagger \Phi + \lambda_{\eta \Phi \eta} \eta \Phi^\dagger \cdot \Phi + \lambda_{\Delta \Phi} \text{Tr}(\Delta^\dagger \Delta) \Phi^\dagger \cdot \Phi.
\]
The coupling in the last line mixes $\eta$ and $\Delta$ so leads to

$$\sigma(DM + DM \rightarrow \nu_\alpha \nu_\beta) = \frac{m_{\eta \Phi \Phi}^2}{32\pi} \left( \frac{\lambda_4 v^2 f_{\alpha \beta}}{m_{\eta}^2 - (2m_{DM})^2} \right)^2,$$

where $m_{DM}^2 = m_\Phi^2 + \lambda_H v^2 / 2$. To obtain annihilation rate indicated by DM abundance in the standard thermal DM scenario, $\sigma_{\text{tot}} \sim 10^{-36} \text{ cm}^2$, the following relation should hold

$$|m_{\eta}^2 - (2m_{DM})^2| \sim 300 \text{ GeV}^2 \frac{\lambda_4 m_{\eta \Phi \Phi} f_{\alpha \beta}}{500 \text{ GeV} \times 0.01} \left( \frac{1 \text{ TeV}}{m_\Delta} \right)^2.$$

As mentioned earlier, a $Z_2$ symmetry stabilizes $\Phi$ against decay. If $\langle \Phi \rangle$ is nonzero, the $Z_2$ symmetry will be broken and various decay modes will become open for $\Phi$. Vanishing $\langle \Phi \rangle$ sets bounds on the parameters of the model. A necessary condition for vanishing $\langle \Phi \rangle$ is

$$m_{\eta \Phi \Phi}^2 < 4(4\lambda_\eta + \lambda_\Phi + \lambda_\eta)(m_{DM}^2 + m_{\eta}^2),$$

where $\lambda_\eta$ is the quartic coupling of $\eta$. If this bound is not satisfied, the minimum of the potential will lie at

$$\Phi = \eta = \frac{-3m_{\eta \Phi \Phi} \pm \sqrt{9m_{\eta \Phi \Phi}^2 - 32(m_\Phi^2 + m_{\eta}^2)(\lambda_\Phi + \lambda_\eta + 4\lambda_{\eta \Phi})}}{4(\lambda_\Phi + \lambda_\eta + 4\lambda_{\eta \Phi})},$$

where $+$ is for negative $m_{\eta \Phi \Phi}$ and $-$ is for positive $m_{\eta \Phi \Phi}$. Eq. (7) combined with Eq. (8) imply

$$|m_{\eta} - 2m_{DM}| < \lambda_4 \sqrt{\lambda_\Phi + \lambda_\eta + 4\lambda_{\eta \Phi}} \text{ GeV}.$$

This means a mild fine tuning between $m_{\eta}$ and $2m_{DM}$ is required to obtain $\sigma_{\text{tot}} \sim 10^{-36} \text{ cm}^2$.

Lepton number violating terms such as $\eta \Phi^{\dagger} \Phi$ and $\eta^{\dagger} \Phi \Phi$ can be added to the Lagrangian but since their couplings should be much smaller.
than $m_{\eta \Phi \Phi}$, they cannot change the discussion. Moreover, once the lepton number is broken, a small mass term of form $\tilde{m}_\Phi^2 \Phi \Phi / 2$ can be also added. This means that there can be a splitting between the imaginary and real components of $\Phi$. Notice that a real $m_{\eta \Phi \Phi}$ coupling leads to the annihilation of a pair of real components together and a pair of imaginary components together. This is unlike the annihilation through a neutral gauge boson that takes place between the imaginary and real components of $\Phi$. As a result, a small splitting between the real and imaginary components will not change the annihilation processes. However, the heavier component can decay into the lighter one and a pair of neutrinos via the mixing of $\Delta^0$ and $\eta$ and via the $f_{\alpha \beta}$ coupling. If the decay takes place when the DM particles have become non-relativistic, the energy of neutrinos will be given by the mass difference between lighter and heavier components of $\Phi$ which is given by $|\tilde{m}_\Phi^2|/(2m_{DM})$. If this splitting is much smaller than 1 MeV (which is natural with taking $\tilde{m}_\Phi^2/(2m_{DM}) \sim \mu \ll 1$ MeV), the energy of these neutrinos will be too small to destroy the products of nucleosynthesis even if the decay takes place at or after nucleosynthesis era.

• **Dirac fermion, $\psi$, as DM**

The general $Z_2$ invariant and lepton number conserving Lagrangian involving $\psi$ can be written as

$$
\lambda_5 \eta \psi_L^T C \psi_L \frac{1}{2} + \lambda_6 \eta \psi_R^T C \psi_R \frac{1}{2} + m_{DM} \bar{\psi}_R \psi_L + \text{H.c.}
$$

Again through a $s$-channel diagram, this Lagrangian leads to

$$
\sigma(\text{DM}+\text{DM} \rightarrow \nu_\alpha \nu_\beta) = \frac{(\lambda_5^2 + \lambda_6^2)}{64\pi} \left( \frac{\lambda_{f_{\alpha \beta} m_{DM} v^2}}{[m_\Delta^2 - (2m_{DM})^2][m_\eta^2 - (2m_{DM})^2]} \right)^2.
$$
To obtain $\sigma_{\text{tot}} \sim 10^{-36}$ cm$^2$,

$$|m_\eta^2 - (2m_{DM})^2| \sim 300 \text{ GeV}^2 \frac{\lambda_5 m_{DM} f_{\alpha\beta}}{500 \text{ GeV}} \left( \frac{1 \text{ TeV}}{m_\Delta} \right)^2 \left( \frac{\lambda_5^2 + \lambda_6^2}{2} \right)^{1/2}. \quad (9)$$

Similarly to the case with complex scalar DM, to obtain the required value of $\sigma_{\text{tot}}$ a fine tuning between $m_\eta$ and $2m_{DM}$ is required.

Lepton number violating mass terms of form $m_R \psi^T \bar{C} \psi_R/2$ and $m_L \psi^T \bar{C} \psi_L$ can be added to the Lagrangian. These terms lead to a mass splitting. Remember that we discussed the effects of decay and annihilation in the presence of mass splitting for scalar DM. The same discussion applies here, too.

Notice that in this model the flavor structure of DM + DM $\rightarrow \nu_\alpha \nu_\beta$ is given by $|\langle m_\nu \rangle_{\alpha\beta}|^2$. It is also possible to have a two component DM scenario with both $\psi$ and $\Phi$. A $Z_2 \times Z_2$ symmetry can stabilize both of them.

Through the coupling of the DM with $\eta$ and the mixing of $\eta$ with $h$, the DM can interact with nuclei but the scattering cross section will be suppressed by the lepton number violating parameter $\mu'$; i.e., by the mixing of $h$ and $\eta$. To obtain significant scattering cross section off nuclei, we add a real scalar $\xi$ as the portal to quark sector through mixing with the SM Higgs. The term that mixes $H$ and $\xi$ is

$$m_{\xi HH} \xi H^\dagger H$$

which leads to a mixing of

$$\tan 2\alpha_{h\xi} = \frac{2m_{\xi HH} v}{m_\xi^2 - m_h^2}.$$

The DM candidates also couple to $\xi$. The coupling to the scalar DM, $\Phi$,

$$m_{\xi \Phi} \xi \Phi^\dagger \Phi.$$

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leads to

\[ \sigma(\text{DM} + N \to \text{DM} + N) = \sigma(\Phi N \to \Phi N) = \frac{f_N^2}{\pi} \left( \frac{m_{\xi \phi} m_{\xi HH}}{m_{\xi}^2} \right)^2 \frac{\mu_{\text{DMN}}^2 m_N^2}{m_{\text{DM}}^2 m_h^2} \]

\[ \sim 10^{-8} \text{ pb} \left( \frac{m_{\xi \phi} m_{\xi HH}/m_{\xi}^2}{0.1} \right)^2 \left( \frac{200 \text{ GeV}}{m_{\text{DM}}} \right)^2 \left( \frac{120 \text{ GeV}}{m_h} \right)^4 \left( \frac{f_N}{0.3} \right)^2 , \]

where \( N \) collectively denotes nucleons (i.e., \( n \) and \( p \)) and \( \mu_{\text{DMN}} \approx 1 \text{ GeV} \) is the reduced mass of the \( \Phi-N \) system [9].

The same coupling and mixing lead to \( \Phi + \Phi^\dagger \to \xi^*, h^* \to f + \bar{f}, W^+ + W^-, Z + Z \) with \( \langle \sigma v \rangle \) equal to

\[ (2m_{\text{DM}} \Gamma(h \to \text{final states}))|_{m_h \to 2m_{\text{DM}}} \frac{m_{\xi \phi}^2 m_{\xi HH}^2 v^2}{4m_{\text{DM}}^2 (4m_{\text{DM}}^2 - m_{\xi}^2)^2 (4m_{\text{DM}}^2 - m_h^2)^2}. \]

\[ \Gamma(h \to \text{final states}) \text{ versus } m_h \text{ can be found in [10]. Taking } m_{\xi \phi} m_{\xi HH}/|4m_{\text{DM}}^2 - m_{\xi}^2| \lesssim 0.1 \text{ and } m_{\text{DM}} < 300 \text{ GeV, } \langle \sigma_{\text{tot}} v \rangle \text{ will be smaller than } 10^{-36} \text{ cm}^2. \]

However, for \( m_{\text{DM}} \sim 200 - 400 \text{ GeV} \), the annihilation mode via \( s \)-channel \( h \) and \( \xi \) exchange can be significant along with \( \Phi + \Phi \to \nu + \nu \). As discussed in [3], in this case still the seasonal variation of muon track events at neutrino telescopes can be significant.

Similarly, the coupling to the fermionic DM, \( \psi \),

\[ Y_{\xi \psi} \bar{\psi} \psi H.c. \]

results in

\[ \sigma(\text{DM} + N \to \text{DM} + N) = \sigma(\psi N \to \psi N) = \frac{f_N^2}{\pi} \left( \frac{Y_{\xi \psi} m_{\xi HH}}{m_{\xi}^2} \right)^2 \frac{\mu_{\text{DMN}}^2 m_N^2}{m_h^2} \sim \]

\[ 10^{-8} \text{ pb} \left( \frac{Y_{\xi \psi} m_{\xi HH} \times 3 \text{ TeV}}{m_{\xi}^2} \right)^2 \left( \frac{120 \text{ GeV}}{m_h} \right)^4 \left( \frac{f_N}{0.3} \right)^2 \]

\[ \leq 10^{-8} \text{ pb} \left( \frac{120 \text{ GeV}}{m_h} \right)^4 \left( \frac{f_N}{0.3} \right)^2 . \]
where $\mu_{\text{DMN}} \simeq 1 \text{ GeV}$ is the reduced mass of the $\psi-N$ system [9]. In both cases, $0.14 < f_n, f_p < 0.66$. Thus, within the favored range of parameters ($m_{\text{DM}} \sim 200 \text{ GeV}$), the scattering cross section will be of order of $10^{-8} \text{ pb}$ which is high enough to lead to a few hundred neutrino events (or even more for larger $m_{\text{DM}}$) at ICECUBE each year. We can have a detectably large monochromatic neutrino flux from DM annihilation inside the Sun with a non-democratic flavor composition determined by $(m_\nu)_{\alpha\beta}$. The $Y_{\xi\psi}$ coupling also leads to $\psi + \bar{\psi} \to f + \bar{f}, W^+ + W^-, Z + Z$ with $\langle \sigma v \rangle$ equal to

\begin{equation}
(2m_{\text{DM}} \Gamma(h \to \text{final states}))|_{m_h \to 2m_{\text{DM}}} \frac{Y^2_{\xi\psi} m^2_{\xi HH} v^2 / 2}{(4m_{\text{DM}}^2 - m^2_{\xi})(4m^2_{\text{DM}} - m^2_h)} v^2_{\text{rel}}
\end{equation}

where $v_{\text{rel}}$ is the relative velocity of the DM pair. Notice that in Eq. (10) for the scalar DM case, such a factor of $v^2_{\text{rel}}$ does not appear. At the decoupling era, $v_{\text{rel}} \sim 1/\sqrt{20}$ so for $\sqrt{2}m_{\text{DM}} m_{\xi HH} Y_{\xi\psi}/|4m^2_{\text{DM}} - m^2_{\xi}| \lesssim 0.1$, the annihilation to the Higgs decay products in the early universe can be only subdominant: $\langle \sigma(\psi + \bar{\psi} \to \xi^*, h^* \to \text{anything}) v \rangle / \langle \sigma_{\text{tot}} v \rangle < 0.01$. For the DM particles trapped inside the sun,

$$v^2_{\text{rel}} \sim \frac{3k_B T}{m_{\text{DM}}} \sim 10^{-9} \frac{300 \text{ GeV}}{m_{\text{DM}}} \frac{k_B T}{100 \text{ eV}}$$

so the annihilation to the Higgs decay products can be safely neglected. Thus, for the purpose of this paper, the $v^2_{\text{rel}}$ dependence is favored. Had we taken the interaction of $\xi$ with $\psi$ to be of form $i \xi \bar{\psi} \gamma^5 \psi$ instead of $\xi \bar{\psi} \psi$, such factor of $v^2_{\text{rel}}$ would not have appeared in Eq. (12).

Let us now discuss the flavor structure of the neutrinos produced by the DM annihilation. The amplitude of $\text{DM} + \text{DM} \to \nu_\alpha + \nu_\beta$, $\mathcal{M}_{\alpha\beta}$, is proportional to $f_{\alpha\beta}$ which is in turn proportional to $(m_\nu)_{\alpha\beta}$. This means in the mass basis where $(m_\nu)$ is diagonal, the amplitude $\mathcal{M}$ is also diagonal. Let us denote the neutrino mass eigenstates in vacuum by $|i\rangle = |1\rangle, |2\rangle$ and $|3\rangle$. The neutrino production is in the form $|1\bar{1}\rangle, |22\rangle$ and $|33\rangle$ (but not for example of form
The production rate of $|i\bar{i}\rangle$ is given by $|f_{ii}|^2 \propto |m_i|^2$. That is the density matrix in the mass basis is proportional to $\text{Diag}(m_1^2, m_2^2, m_3^2)$. As a result, for the quasi-degenerate neutrino mass scheme with $|m_1|^2 \simeq |m_2|^2 \simeq |m_3|^2$, the neutrino production will be democratic and as discussed in detail in Ref. [3], the seasonal variation will vanish. Let us now evaluate the seasonal variation for the general neutrino mass scheme. Due to the matter effects inside the Sun, a pure mass eigenstate while crossing the Sun converts to a combination of mass eigenstates as follows: $|i;\text{surface}\rangle = \sum_j a_{ij} |j\rangle$ and $|\bar{i};\text{surface}\rangle = \sum_j \bar{a}_{ij} |\bar{j}\rangle$ where due to the matter effects $a_{ij}$ may differ from $\bar{a}_{ij}$. $a_{ij}$ and $\bar{a}_{ij}$ are unitary matrices: $\sum_j a_{ij}^* \bar{a}_{kj} = \delta_{ik}$ and $\sum_j a_{ij} \bar{a}_{kj}^* = \delta_{ik}$. After traversing the distance between the Sun and Earth, $L$, $|j\rangle$ and $|\bar{j}\rangle$ will pick up a phase of $e^{-im^2_{jL}/2E}$. Neglecting the average of the phase $e^{i\Delta m^2_{31}L/2E}$, we can write $\langle P(\nu_i \rightarrow \nu_\mu) \rangle = \sum_j |U_{\mu j}|^2 |a_{ij}|^2 + 2 \Re[a_{i1}^* a_{i2} U_{\mu 1} U_{\mu 2}^* e^{i\Delta m^2_{31}L/2E}]$ (see Ref. [3]). Considering the fact that the production of $|i\rangle$ is given by $m_i^2$, the seasonal variation due to $\Delta m^2_{12}L/2E \sim 1$ can be evaluated as

$$\text{variation} = \frac{2 \sum_i m_i^2 \Re[a_{i1}^* a_{i2} U_{\mu 1} U_{\mu 2}^*]}{\sum_{ij} m_i^2 |a_{ij}|^2 |U_{\mu j}|^2}.$$  

Since $\Delta m^2_{21} \ll \Delta m^2_{31}$, we can take $m_1^2 \simeq m_2^2$ and can therefore write

$$\text{variation} = \frac{2 \sum_i \Delta m^2_{31} \Re[a_{31}^* a_{32} U_{\mu 1} U_{\mu 2}^*]}{m_1^2 + \Delta m^2_{31} \sum_{ij} |a_{ij}|^2 |U_{\mu j}|^2}$$  \hspace{1cm} (13)$$

where we have used the unitarity of $a_{ij}$. If $\theta_{13}$ is exactly zero, despite the matter effects, $|3\rangle$ will not change which means $a_{31} = a_{32} = 0$, so the variation will vanish. Remembering that the matter density of the Sun falls as $e^{-r/(0.1R_{\text{sun}})}$ we find that for $\sin \theta_{13} \gtrsim [(0.1R_{\text{sun}})V_{\text{e}}|_{\text{center}}]^{-1} \sim 0.001$, the values of $a_{31}$ and $a_{32}$ can in general be of order of 1 which means the variation can be sizeable (larger than 10 %) and can be measured by a few hundred events. Eq. (13) also confirms that for quasi-degenerate mass scheme with $m_1^2 \gg |\Delta m^2_{31}|$, the variation is suppressed even for $a_{31}^* a_{32} \sim 1/2$. 

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Let us now briefly discuss the impact of this model for the collider searches. Like the case of type II seesaw mechanism, we expect a triplet scalar which is produced by electroweak interactions. Like the type II seesaw mechanism, the $\Delta^+$ and $\Delta^{++}$ components decay into lepton pairs via the $f_{\alpha\beta}$ coupling. For a review of the possible decay modes of $\Delta$, see [11]. However, there will be a new decay mode for $\Delta^0$, which unlike $\Delta^0 \rightarrow \nu_\alpha \nu_\beta$, is visible. The new decay mode is $\Delta^0 \rightarrow \eta h$. If $\eta$ is heavier than $2m_{DM}$, it can decay into a DM pair which appears as missing energy. Since in this model $H$ mixes with $\xi$, the signature of Higgs can be in principle different. In fact, any process studied for the SM (e.g., $gg \rightarrow h$ and $h \rightarrow \tau\bar{\tau}, \gamma\gamma$) should be reconsidered for two SM-like Higgs fields (i.e., two mass eigenstates composed of $h$ and $\xi$) with rates suppressed by $\cos^2 \alpha_{\eta\xi}$ and $\sin^2 \alpha_{\eta\xi}$. The production of the second Higgs-like scalar will be suppressed by a factor of $\sin^2 \alpha_{\eta\xi}$. For $\sin \alpha_{h\xi} \simeq 0.1$, which is the favored range by bounds on $\sigma(\text{DM} + N \rightarrow \text{DM} + N)$, the production of the second Higgs can be neglected. In this case, the Higgs sector will be similar to what we had within the Standard Model.

4 MODEL II

In this section, we introduce another model within which DM is composed of Dirac fields, $\psi$, which are protected against decay again with a $Z_2$ symmetry. We introduce a new $U(1)'$ symmetry with gauge boson $Z'$ under which all the SM particles are invariant. In this model, $\nu_S$ with masses close to those of active neutrinos also exist such that oscillation can take place between the sterile and active neutrinos.

The fields that are added to this model are the following:

- A Dirac field, $\psi = (\psi_L \psi_R)$, which plays the role of the DM candidate;
- One (or more) left-handed sterile neutrino, $\nu_S$;
• Right-handed neutrinos $\nu_{R\beta}$ which are singlets under both electroweak symmetry and $U(1)'$. These neutrinos are included to give Dirac masses to the rest of neutrinos.

• $U(1)'$ gauge boson, $Z'$;

• A complex scalar field $H'$ which is an electroweak singlet but under $U(1)'$ has a charge equal to that of $\psi_L$. $H'$ receives a Vacuum Expectation Value (VEV) of $v'/\sqrt{2}$ which breaks the $U(1)'$ symmetry and gives a mass of $m_{Z'} = g'v'/\sqrt{2}$ to $Z'$. After electroweak and $U(1)'$ symmetry breaking, the $H'$ also mixes with $H$ and acts as a messenger to interact with nuclei.

Via a $s$-channel $Z'$ exchange, the DM particles annihilate into sterile neutrinos which in turn oscillate into active neutrinos with a non-trivial flavor composition given by the flavor structure of the mixing of the sterile neutrinos with the active neutrinos.

We assume that the right-handed $\psi$ is neutral under $U(1)'$ and only left-handed $\psi$ couples to $Z'$:

$$\mathcal{L} = g' \left( \bar{\nu}_S \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) \nu_S - \bar{\psi} \gamma^\mu \left( \frac{1 - \gamma^5}{2} \right) \psi \right) Z'_\mu.$$ 

With this charge assignment there is no $U(1)'$ anomaly. If we assigned the same $U(1)'$ charge to $\psi_R$, we must add additional chiral fermions to cancel the $U(1)'$ anomaly.

Through this coupling, the DM pair annihilates into a sterile neutrino pair with the annihilation cross section

$$\langle \sigma(\bar{\psi}\psi \to \bar{\nu}_S\nu_S) \rangle = \frac{g'^4}{8\pi} \frac{m_{DM}^2}{\left(2m_{DM}^2 - m_{Z'}^2\right)^2}$$

where the mass of $\nu_S$ which is taken to be of order of the active neutrino masses is neglected ($i.e.$, $m_{\nu S} \ll m_{DM}$). To have $\langle \sigma_{\text{tot}} v \rangle = 10^{-36}$ cm$^2$ ($i.e.$,
the value suggested by the DM abundance in thermal DM production scenario), we find

\[ m_{Z'}^2 = 4m_{DM}^2 - \left( \frac{g'}{0.16} \right)^2 m_{DM} \times (100 \text{ GeV}) . \]

For example, for \( g' = 0.16 \) and \( m_{DM} = 200 \text{ GeV} \), the desired abundance of DM can be achieved with \( m_{Z'} = 375 \text{ GeV} \). The \( Z' \) boson in this model does not couple to quarks or leptons and there is no mixing between \( Z' \) and the SM gauge bosons. As a result, none of the bounds from the new gauge boson searches at colliders applies in this case.

After breaking of \( U(1)' \) and the electroweak symmetry, the neutrinos and \( \psi \) receive Dirac mass which comes from the following Yukawa couplings:

\[ L = f_{\alpha\beta} \bar{\nu}_R^\beta L^\alpha_i H^j \epsilon_{ij} + f_{R\beta} \bar{\nu}_R^\beta H' \nu_S + Y_\psi \bar{\psi}_R \psi_L H' + \text{H.c.} . \]

We assumed that only \( \psi \) is odd under the \( Z_2 \) symmetry so the \( Z_2 \) symmetry prevents a coupling of form \( \bar{\nu}_R \psi_L (H')^\dagger \) or a mass term of form \( \nu^T_{R\beta} c \psi_R \). The masses of \( \psi_L \) and \( \psi_R \) are the same and equal to

\[ m_{DM} = Y_{\psi} \langle H' \rangle = Y_{\psi} \frac{v'}{\sqrt{2}} . \]

After \( H \) and \( H' \) obtain vacuum expectation values, \( \nu_S \) receives a Dirac mass and mixes with active neutrinos. For illustrative purposes, let us consider one sterile neutrino, one active flavor and two right-handed neutrinos. The masses will be given by

\[ \frac{1}{\sqrt{2}} [\bar{\nu}_{R1} \bar{\nu}_{R2}] \begin{bmatrix} f_{1\alpha} v \ f_{R1} v' \\ f_{2\alpha} v \ f_{R2} v' \end{bmatrix} \begin{bmatrix} \nu_{\alpha} \\ \nu_S \end{bmatrix} \]

(14)

The mixing between sterile and active neutrinos is given by the following formula

\[ \tan 2\theta = \frac{2(f_{1\alpha} f_{R1} + f_{2\alpha} f_{R2})v v'}{(f_{1\alpha}^2 + f_{2\alpha}^2)v^2 - (f_{R1}^2 + f_{R2}^2)v'^2} . \]
The Lagrangian of the model includes a term as follows
\[ Y'_{h'h} H'^\dagger \cdot H'^\dagger \cdot H. \]

In the unitary gauge \( H = (0 (v + h)/\sqrt{2}) \) and \( H' = (v' + h')/\sqrt{2}: \)
\[
\frac{1}{2} \left[ h' h \right] \begin{bmatrix}
    m_h^2 & Y_{h'h} v v' \\
    Y_{h'h} v v' & m_{h'}^2
\end{bmatrix} \left[ h \right].
\]

The mixing between \( h \) and \( h' \) is given by
\[ \tan 2\alpha_{hh'} = \frac{2 Y_{h'h} v v'}{m_h^2 - m_{h'}^2}. \]

This mixing leads to DM scattering off the nuclei in the target of the direct DM search experiments with a cross section
\[ \sigma(DM + N \rightarrow DM + N) = \frac{f_N^2}{\pi} \left( \frac{Y_{h'h} v v'}{m_h^2, m_{h'}^2} \right)^2 \mu_{DMN}^2 m_N^2 = \]
\[ 10^{-8} \text{pb} \times \left( \frac{Y_{h'h} v v'/m_{h'}}{0.1} \right)^2 \left( \frac{200 \text{ GeV}}{m_{h'}} \right)^2 \left( \frac{120 \text{ GeV}}{m_h} \right)^4 \left( \frac{f_N}{0.3} \right)^2. \]

As expected, this cross section is the same as the one in Eq. (11) provided that we replace \( m_\xi \rightarrow m_{h'} \), \( m_{\xi HH} \rightarrow Y_{h'h} v v' \) and \( Y_{\xi\psi} \rightarrow Y_{\psi}/\sqrt{2} \). As mentioned before, \( 0.14 < f_n, f_p < 0.66 \). The cross section is large enough to lead to a significant capture rate and subsequently to a large neutrino flux detectable at ICECUBE. Again replacing \( m_\xi \rightarrow m_{h'} \), \( m_{\xi HH} \rightarrow Y_{h'h} v v' \) and \( Y_{\xi\psi} \rightarrow Y_{\psi}/\sqrt{2} \) in Eq. (12), the formula for \( \sigma(DM + DM \rightarrow \xi^*, h^* \rightarrow W^-W^+, ZZ, t\bar{t}) \) can be obtained. Similarly to the case of Eq. (12), we find that for \( Y_{hh'} v v'M_{DM}^2 v'/|4m_{DM}^2 - m_{h'}^2| < 0.1 \), this annihilation mode can be safely neglected.

Among the new particles that are added to this model, only \( h' \) has a significant coupling to the SM. As a result, only \( h' \) might appear in the collider searches through mixing with the Higgs field. The same discussion
in the case of Model I described in the previous section applies here, too: If
\( \sin \alpha_{hh'} < 0.1 \), the production of the new Higgs at collider will be suppressed
by \( \sin^2 \alpha_{hh'} < 0.01 \) relative to the ordinary SM Higgs.

Let us now address the oscillation of sterile neutrinos to active neutrinos
and the flavor composition of the neutrino flux from DM annihilation when
they reach the detector. This problem has recently been addressed in [12].
In Eq. (14), for simplicity we have assumed that \( \nu_S \) mixes only with one
active flavor. In general, \( \nu_S \) can mix with more than one flavor. In fact,
explaining the LSND and MiniBooNE results in the context of the present
scenario suggests that the sterile neutrino simultaneously mixes with \( \nu_e \) and
\( \nu_\mu \). Recent fits to the short baseline neutrino data can be found in [13].
Some hints for the oscillation of atmospheric \( \nu_\mu \) to \( \nu_S \) has also been found in
the ICECUBE data [14]. The scenario proposed in this section can include
two sterile neutrinos mixing with different flavors as required by the 3 + 2
scenario suitable for explaining the LSND and MiniBooNE results. Some
hints for sterile neutrinos have also been found by studying the solar neutrino
oscillation data [15]. Studying the general case is beyond the scope of the
present paper. We focus on the particular case that the mixing matrix in the
\( (\nu_e \ \nu_\mu \ \nu_\tau \ \nu_S) \) basis is of the following form

\[
U^{(4)} = O_{34} \cdot (U_{PMNS} \oplus 1_{1\times1})
\]

where

\[
O_{34} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \cos \theta_{34} & \sin \theta_{34} \\
0 & 0 & -\sin \theta_{34} & \cos \theta_{34}
\end{pmatrix}, \quad (16)
\]

\( (U_{PMNS} \oplus 1_{1\times1}) \) is the external sum of the standard three by three mixing
matrix of neutrinos \( U_{PMNS} \) with \( 1_{1\times1} \) which means its first \( 3 \times 3 \) block is
\( U_{PMNS} \), its 44 element is equal to one and the rest of its elements vanish.
Notice that, without matter effects, \( P(\bar{\nu}_e \rightarrow \nu_e) \), \( P(\bar{\nu}_\mu \rightarrow \nu_\mu) \) and \( P(\bar{\nu}_\mu \rightarrow \nu_e) \) as well as \( P(\bar{\nu}_e \rightarrow \nu_\mu) \) are the same as what we expected without the sterile neutrinos. As a result, the bounds from reactor searches as well as the measurement of \( P(\nu_\mu \rightarrow \nu_\mu) \) and \( P(\nu_\mu \rightarrow \nu_\mu) \) by atmospheric and long baseline experiment and various other experiments do not apply here. This mixing affects \( P(\bar{\nu}_\mu \rightarrow \nu_\tau) \) which is poorly constrained by observation. The strongest constraint comes from the measurement of the total neutral current interaction of the beam in the MINOS experiment [16]. If this constraint is saturated, \( \theta_{34} \) can be still relatively large leading to a significant probability of sterile to active oscillation.

A part of the \( \nu_S \) flux will oscillate into active neutrinos on the way to the Earth which can be detected at ICECUBE or its deepcore (provided that the detection threshold is lowered below \( m_{DM} \)). The number of muon track events in a time interval is given by \( \langle P(\nu_S \rightarrow \nu_\mu) \rangle \) which is the oscillation probability averaged over the time interval. As discussed in [3], due to the averaging effects the oscillatory terms given by \( \Delta m^2_{31} \) are subdominant however we should keep the oscillatory terms given by \( \Delta m^2_{21} \). A simple numerical calculation shows that the matter effects on the oscillation probability cannot be neglected. Adopting the formalism in [3] and generalizing it to a four-neutrino scheme, we can expand the evolved \( \nu_S \) at the Sun surface in terms of the mass eigenstates as follows

\[
|\nu_S; \text{surface}\rangle = a_{S1}|1\rangle + a_{S2}|2\rangle + a_{S3}|3\rangle + a_{S4}|4\rangle .
\]

By a numerical calculation taking into account matter effects, \( a_{Si} \) can be found in terms of the \( U^{(4)} \) elements and \( \Delta m^2_{34} \). Averaging out the oscillatory terms given by \( \Delta m^2_{31} \), we can then write

\[
\langle P(\nu_S \rightarrow \nu_\mu) \rangle = |U_{\mu 1}|^2|a_{S1}|^2 + |U_{\mu 2}|^2|a_{S2}|^2 + |U_{\mu 3}|^2|a_{S3}|^2 + \\
2\Re[U_{\mu 1}U^*_{\mu 2}a_{S1}a_{S2} \exp(i \frac{\Delta m^2_{21} L_{\text{Sun-Earth}}}{2E_\nu})] ,
\]

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where $E_\nu = m_{DM}$. As discussed in [2], by studying the periodicity of muon track events, the value of $\Delta m_{12}^2/m_{DM}$ can be derived. The results of a simple numerical calculation shows that if the bound from [16] on $\theta_{34}$ is saturated, $P(\nu_S \rightarrow \nu_\mu)$ can reach as large as 0.2. This means for $\sigma(DM + N \rightarrow DM + N) \sim 10^{-8}$ cm$^2$, up to a few hundred muon-track events can be registered each year at ICECUBE so the statistics will be enough to study a seasonal variation [3]. In ref. [3], we have found that for three-neutrino scheme in the case that $DM + DM \rightarrow \nu_\mu + \nu_\mu$ or $\nu_\tau + \nu_\tau$, the variation will be small. Obviously this consideration does not apply to the present model with four neutrinos and $DM + DM \rightarrow \nu_S + \nu_S$.

Notice that $\langle P(\nu_S \rightarrow \nu_\mu) \rangle$ is not sensitive to $\Delta m_{34}^2$ because we have taken $U_{\mu 4} = 0$. The total oscillation probability to active neutrinos, $\langle 1 - P(\nu_S \rightarrow \nu_S) \rangle$, is given by

$$|U_{S1}|^2|a_{S1}|^2 + |U_{S2}|^2|a_{S2}|^2 + |U_{S3}|^2|a_{S3}|^2 + |U_{S4}|^2|a_{S4}|^2 + 2\Re[U_{S1}U_{S2}^*a_{S1}^*a_{S2} \exp(i \frac{\Delta m_{21}^2 L_{\text{Sun-Earth}}}{2E_\nu})] + 2\Re[U_{S3}U_{S4}^*a_{S3}^*a_{S4} \exp(i \frac{\Delta m_{34}^2 L_{\text{Sun-Earth}}}{2E_\nu})].$$

The number of the cascade-like events is given by $\sigma_{NC} \langle 1 - P(\nu_S \rightarrow \nu_S) \rangle + \sigma_{CC} \langle 1 - P(\nu_S \rightarrow \nu_S) - P(\nu_S \rightarrow \nu_\mu) \rangle$. Thus, by studying the variation of cascade-like events, it will be in principle possible to derive $\Delta m_{34}^2$. The possibility can be realized only if the following three conditions are fulfilled: (i) The energy threshold for cascade detection is below $m_{DM}$; (ii) The statistics is high enough; (iii) The oscillation length $L_{\text{osc}} = 4\pi m_{DM}/\Delta m_{34}^2$ is of the order of the seasonal variation of the Sun-earth distance: i.e., $L_{\text{osc}} \sim \Delta L_{\text{Sun-Earth}} \sim 5$ million km, which means

$$\Delta m_{34}^2 \sim 10^{-5} \text{ eV}^2 \left(\frac{m_{DM}}{200 \text{ GeV}}\right).$$
It is noteworthy that for $m_{DM} \sim 200$ GeV, this means $\Delta m^2_{12} \sim \Delta m^2_{34}$. Unlike the case of $\mu$-track events, the directionality cannot be used to reduce the background of shower-like events from atmospheric neutrinos. At ICECUBE, we expect a maximum of a few hundred cascade-like neutrino events from the DM annihilation inside the Sun per year which is within the statistical fluctuation of the background from the atmospheric neutrinos. This means that after 10 years of data taking by ICECUBE, the confidence level of the discovery of such cascade-like signal will be about 3-4 $\sigma$. To make a serious measurement, a detector ten times larger than ICECUBE with threshold below $m_{DM}$ is required. After about 2-3 years of data taking by such a detector, the statistics can be enough to claim discovery and after ten years, enough data can be collected to study variation and to extract information from the variation.

Let us now discuss the possibility of constraining this scenario by various other oscillation observations. As discussed above, $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ relevant for the reactor neutrino data is not affected by the presence of the sterile neutrino. However, because of the matter effects inside the Sun, the survival probability of the solar $\nu_e$ will be affected by the presence of the sterile neutrino. The effect in general is rather complicated. For our case with $|\Delta m^2_{41}| \simeq |\Delta m^2_{31}| \gg \Delta m^2_{21}$, the simplified formalism in [17] can be applied. Using the formalism in [17], we find that the deviation from SM neutrino oscillation without sterile neutrino is suppressed by $(N_n/2N_e) \cos 2\theta_{12} c^2_{23} s^2_{34}$ where $N_n$ and $N_e$ are respectively the neutron and electron number densities in the Sun. Inserting the numerical values, we find that the deviation is of order of $0.02 \sin^2 \theta_{34}$ and is therefore very small even for $\sin^2 \theta_{34} \sim 1$.

Measuring $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$ at long baseline experiment OPERA or by studying the atmospheric neutrino data at Super-Kamiokande constrains $\theta_{34}$ as in this scenario $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$ is suppressed by a factor of $\cos^2 \theta_{34}$ relative to the standard three-neutrino scheme. This scenario can be also probed by
improving the measurement of the rate of neutral current interaction events at long baseline experiments such as MINOS.

5 Summary

We have introduced two models within which the DM pair dominantly annihilates into a neutrino pair with a non-trivial flavor composition. As a result, the non-relativistic DM pair annihilation will lead to a sharp line in the neutrino energy spectrum with \( F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) \neq 1 : 1 : 1 \). As shown in [2, 3], despite the very large distance between the Sun and Earth, the oscillation probability does not average to zero, leading to a seasonal variation as the distance between the Sun and Earth varies during a year. The variation contains information on \( m_{DM} \) and the initial flavor composition.

In these models, the DM particles are stabilized by a \( Z_2 \) symmetry under which only the DM candidates are odd. In both models, the DM particles interact with nuclei via a new scalar which mixes with the SM Higgs so the interaction is spin-independent. The parameters are chosen to yield a scattering cross section in the range \( 10^{-9} - 10^{-8} \text{ pb} \). Thus, while the present bounds from direct searches are satisfied, still significant statistics (a few hundred events per year) are expected at ICECUBE for indirect DM detection. This means by a slight improvement in both direct and indirect DM searches, these models can be tested. The same coupling and mixing can also lead to \( \text{DM} + \text{DM} \rightarrow f + \bar{f}, Z + Z, W^+ + W^- \). However, these annihilation modes will be subdominant. When DM is composed of Dirac fermions, these modes, being P-wave effects, are further suppressed by \( v_{rel}^2 \) and for the case of the DM pair trapped inside the Sun can be safely neglected.

Model I embeds type II seesaw mechanism so neutrinos are Majorana particles. Within this model, the DM candidates can be either complex scalars or Dirac fermions with lepton number equal to 1. It is also possible
that both the complex scalars and the Dirac fermions contribute to the DM in the universe. The DM pair annihilates into $\nu_\alpha \nu_\beta$ with a flavor composition determined by $(m_\nu)_{\alpha\beta}$. The prediction of the model for LFV rare decays as well as the accelerator searches is similar to the predictions of type II seesaw mechanism, except that here the neutral component of the triplet, $\Delta^0$, can have a new decay mode, $\Delta^0 \to h + \text{missing energy}$.

In Model II, the DM is composed of Dirac fermions which via the exchange of a new $Z'$ gauge boson annihilate into a pair of sterile neutrinos. Since $Z'$ does not couple to quarks or ordinary leptons, it cannot be produced in the lepton or hadron colliders. In this model, neutrinos are Dirac particles so we expect a null result in searches for neutrinoless double beta decay.

Since the annihilation products in the Sun center are sterile, they do not scatter off the nuclei present inside the Sun so, to the first approximation, the height of the sharp line in the spectrum is not reduced by scattering. On their way to Earth, the sterile neutrinos oscillate into active neutrinos. The flavor composition of the flux on Earth is given by the mixing parameters of the sterile neutrinos with the active neutrinos. The number of the muon-track events is given by $P(\nu_S \to \nu_\mu)$ which in turn is given by the active-sterile mixing. As discussed in the text, the main constraint on the mixing comes from the MINOS measurement of the total neutral current interaction of the beam at the detector. Taking into account this bound, the statistics can be still high enough to employ the method introduced in [3]. By improving this bound, the model will be further constrained. While the variation of the muon-track events at neutrino telescope will be sensitive to $\Delta m^2_{12}/m_{DM}$, the variation of the cascade events at the neutrino telescopes will be sensitive to both $\Delta m^2_{34}/m_{DM}$ and $\Delta m^2_{12}/m_{DM}$. 
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