Hot thermal universe endowed with massive dark vector fields and the Hubble tension

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The value of the Hubble constant inferred from Planck measurements of anisotropies in the cosmic microwave background is at 4.4σ tension with direct astronomical measurements at low redshifts. Very recently, it has been conjectured that this discrepancy may be reconciled if a small fraction of the dark matter is described by three mutually orthogonal vector fields of the same mass. We study the thermal description of this model and use the observationally-inferred primordial fractions of baryonic mass in $^4$He to constrain its phase space. We show that while the sterile vector fields may help to alleviate a little bit the existing tension in the measurements of the Hubble parameter, they cannot eliminate the discrepancy between low- and high-redshift observations.

Over the past decade or so, cosmology has witnessed an avalanche of data which has signaled the inception of “precision cosmology.” During this period and through many experiments, ΛCDM has become established as a well-tested cosmological model. Within this set up, the expansion of the universe today is dominated by the cosmological constant $\Lambda$ and cold dark matter (CDM). Nevertheless, various discrepancies have persisted. Most strikingly, the emerging tension in the inferred values of the Hubble constant $H_0 = 100\ h\ km\ s^{-1}\ Mpc^{-1}$ \cite{1}. $H_0$ parametrizes the expansion rate and thus provides clues about the cosmological energy content of the universe.

Numerous independent measurements of $h$ at low-redshift, including those from Cepheids and type-Ia supernovae, seem to indicate that $h = 0.735 \pm 0.016$ \cite{2–6}. All these observations are local and consequently nearly independent of the cosmological expansion history. On the other hand, when the all-sky map from the temperature fluctuations on the cosmic microwave background (CMB) is combined with data from Baryon Acoustic Oscillations (BAO) to calibrate the sound horizon, the inferred $H_0$ value is $h = 0.6775 \pm 0.0075$ \cite{7,10}. The disagreement is significant at 4.4σ level \cite{10,11}, and systematic effects do not seem to be responsible for this discrepancy \cite{12–16}. This then substantiates a possible physics-beyond-ΛCDM origin of the $H_0$ tension. For example, free-streaming relativistic particles from a dark sector may alter the epoch of matter-radiation equality, boosting the expansion rate.

To accommodate new physics in the form of extra relativistic degrees of freedom it is convenient to account for the extra contribution to the standard model (SM) energy density, by normalizing it to that of an “equivalent” neutrino species. The number of “equivalent” light neutrino species,

$$N_{\text{eff}} \equiv \frac{\rho_R - \rho_{\nu}}{\rho_{\nu}}, \quad \text{(1)}$$

quantifies the total “dark” relativistic energy density (including the three left-handed SM neutrinos) in units of the density of a single Weyl neutrino:

$$\rho_{\nu} = \frac{7 \pi^2}{120} \left( \frac{4}{11} \right)^{1/3} T_\gamma^4, \quad \text{(2)}$$

where $\rho_{\nu}$ is the energy density of photons (with temperature $T_\gamma$) and $\rho_R$ is the total energy density in relativistic particles \cite{17}. The Hubble tension hints at the presence of an excess $\Delta N_{\text{eff}}$ above the SM expectation. Note that the normalization of $N_{\text{eff}}$ is such that it gives $\Delta N_{\text{eff}} = 3$ for three families of massless left-handed neutrinos. For most practical purposes, it is accurate enough to consider that SM neutrinos freeze-out completely at about 1 MeV. However, as the temperature dropped below this value, these neutrinos were still interacting with the electromagnetic plasma and hence received a tiny portion of the entropy from pair annihilations. The non-instantaneous neutrino decoupling gives a minor correction to the SM contribution $\Delta N_{\text{eff}} = 0.046$ \cite{18}.

The impact of the $h$ determination is particularly complex in the investigation of $N_{\text{eff}}$. For example, combining CMB observations with BAO data the Planck Collaboration reported $N_{\text{eff}} = 2.99 \pm 0.17$ \cite{9}. However, a combination of the space telescope measurement $h = 0.738 \pm 0.024$ with the Planck CMB data gives $N_{\text{eff}} = 3.62 \pm 0.25$, which suggests new neutrino-like physics (at around the 2.3σ level) \cite{8}. Finally, the simultaneous fit to: (i) CMB observations, (ii) lensing and BAO data, and (iii) local H$_0$ measurements leads to $N_{\text{eff}} = 3.27 \pm 0.15$ and $h = 0.6932 \pm 0.0097$ \cite{9}. In addition, light-element abundances probing big-bang nucleosynthesis (BBN) have also hinted at the presence of extra relativistic degrees of freedom. More concretely, the observationally-inferred primordial fractions of baryonic mass in $^4$He favor $N_{\text{eff}} = 3.80^{+0.80}_{-0.70}$ (with 2σ errors) \cite{19}.

Proposed modifications of ΛCDM to accommodate the $H_0$ tension reshape either the local expansion rate or else the early universe pre-CMB emission. Among the plethora of proposals that have been put forward are those based on sterile neutrinos \cite{20–24}, Goldstone bosons \cite{25}, axions \cite{26–27}, scalar fields \cite{28–31}, and
decaying dark matter \cite{32,37}. In a similar fashion, it has been recently conjectured that the $H_0$ tension may be reconciled if a small fraction of the dark matter is described by three mutually orthogonal dark vector fields of the same mass \cite{38}. In this paper we study the thermal description of this model and use tests of BBN to constrain its phase space. We show that while the sterile vector fields may help to alleviate a little bit the existing tension in the measurements of the Hubble parameter, they cannot eliminate the discrepancy between low- and high-redshift observations.

The $Λ$CDM extension under investigation evolves in a Friedmann-Lemaitre-Robertson-Walker geometry, with generic metric

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2dΩ^2),$$  \hspace{1cm} (3)

where the scale factor $a(t)$ is normalized such that at the present time we have $a = 1$. The renormalizable Lagrangian density of the model can be expressed in terms of the visible (SM) and dark sectors

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{dark}},$$  \hspace{1cm} (4)

with

$$\mathcal{L}_{\text{dark}} \supset \sqrt{-g} \left( -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{m^2}{2} A_\mu A^\mu \right),$$  \hspace{1cm} (5)

where $A_\mu = (A_\mu^0, A_\mu^\alpha, A_\mu^i)$ represents a set of three massive vector fields and $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The equation of motion that follows from the Lagrangian of the dark vector fields \cite{5} is found to be

$$\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} F^{\mu \nu} \right) - m^2 A^\nu = 0.$$  \hspace{1cm} (6)

Using the decomposition $A_\mu = (-\phi, \vec{A})$, the electric and magnetic fields can be defined as

$$\vec{E} = -\dot{\vec{A}} - \vec{\nabla} \phi \quad \text{and} \quad \vec{B} = \vec{\nabla} \times \vec{A},$$  \hspace{1cm} (7)

where a dot represents the derivative with respect to time. Now, restricting to homogeneous solutions, it is straightforward to see that $\phi = 0$ and $\vec{A}$ obeys

$$\ddot{\vec{A}} + \frac{a}{a} \dot{\vec{A}} + m^2 \vec{A} = 0;$$  \hspace{1cm} (8)

see \cite{38} for details. The solutions of (8) yield, through (7), a time-dependent electric field with a null magnetic field.

The stress-energy tensor that follows from the Lagrangian density \cite{5} is

$$T_{\mu \nu} = F_{\mu \rho} F^{\mu \rho} - \frac{1}{2} g_{\mu \nu} \left( \frac{1}{2} F_{\rho \sigma} F^{\rho \sigma} + m^2 A_\alpha A^\alpha \right) + m^2 A_\mu A_\nu.$$  \hspace{1cm} (9)

This tensor must be diagonal to comply with the symmetries of the metric. It can be taken to a diagonal form by averaging over the three fields, when the they are chosen to be mutually orthogonal, and with the same mass and norm $|A_\mu A^\mu|$. In this case,

$$\bar{T}_{ij} = \frac{1}{6} \delta_{ij} (E^2 - m^2 A^2),$$  \hspace{1cm} (10)

where the bar denotes the average over “the triplet.” Combining (9) and (10) we obtain an expression for the energy density

$$\rho_A = \frac{1}{2 \pi^2} (E^2 + m^2 A^2),$$  \hspace{1cm} (11)

and another one for the pressure

$$p_A = \frac{1}{6 \pi^2} (E^2 - m^2 A^2).$$  \hspace{1cm} (12)

Hence, the equation of state is

$$w \equiv \frac{p_A}{\rho_A} = \frac{1}{3} \frac{E^2 - m^2 A^2}{E^2 + m^2 A^2},$$  \hspace{1cm} (13)

with $-1/3 < w < 1/3$.

In the radiation-dominated epoch, the scale factor can be written as

$$a(t) = \left( \frac{2 \sqrt{Ω_0} H_0 t}{\Omega_3} \right)^{1/2},$$  \hspace{1cm} (14)

where $Ω_3$ is the radiation density normalized to the critical density and the subindex zero indicates it is evaluated today. Using this expression, a particular solution of (8) can be written as

$$A(t) = A_0 \left[ mt \right]^{1/4} \left[ c_1 J_{1/4}(mt) + c_2 Y_{1/4}(mt) \right],$$  \hspace{1cm} (15)

where $J(x)$ and $Y(x)$ are Bessel functions, $A_0$ is a constant vector, and $c_1$ and $c_2$ are constants. With the expression for $A(t)$, we can calculate the energy density, which has the following asymptotic limits:

$$\rho_A \approx |A_0|^2 \begin{cases} \frac{\sqrt{2} n(c_1 + c_2)}{\Gamma^2(1/4)} \frac{1}{a^2 t}, & \text{for } mt \ll 1 \\ 2m^2(c_1^2 + c_2^2) \frac{1}{a^2 \sqrt{mt}}, & \text{for } mt \gg 1 \end{cases}.$$  \hspace{1cm} (16)

Substituting (14) into (16) it is easily seen that $\rho_A \propto a^{-4}$ for $t \ll m^{-1}$, and $\rho_A \propto a^{-3}$ for $t \gg m^{-1}$, so that the energy density of the set of vector fields, when conveniently averaged, behaves as radiation first and then as matter, always during the radiation phase.

In the light of this behaviour, a modification to the functional form of the $Λ$CDM Hubble parameter,

$$E(a) = \left( \frac{Ω_{R0}}{a^4} + \frac{Ω_{m0} - Ω_{A0}}{a^3} + \frac{Ω_R}{a^2 \Gamma(a+1)} + Ω_Λ \right)^{1/2},$$  \hspace{1cm} (17)

was proposed in \cite{38}, with $Ω_{R0} = Ω_{r0} + Ω_{ ν0}, \ Ω_{ν0} \approx 5.37 \times 10^{-5}, Ω_{r0} \approx 3.66 \times 10^{-5}, Ω_{m0} = Ω_{k0} + Ω_{CDM0}, Ω_{k0} \approx 0.048,
\( \Omega_{\text{CDM}} \approx 0.258, \Omega_{\Lambda} \approx 0.692 \) [39]. Here, \( \Omega_{A0} \) is a free parameter of the model with \( w = 1/3 \) for \( mt < 1 \), and \( w = 0 \) for \( mt > 1 \). The extra contribution to the radiation density at early times leads to an increment of the Hubble parameter

\[
H \equiv H_0 \, E(a),
\]

with respect to its value in the SM.

Next, in line with our stated plan, we use experimental data to constrain the model. BBN is the earliest observationally verified landmark at a temperature \( T_{\text{BBN}} = (1 + z)T_{\nu0} \approx 2.348 \times 10^{-4} \) GeV, where \( T_{\nu0} = 2.348 \times 10^{-4} \) eV and the redshift is \( z \approx 10^9 \). Using the number of relativistic degrees of freedom at temperature \( T \),

\[
N_R(T) = \sum_B g_B \left( \frac{T_B}{T} \right)^4 + \frac{7}{8} \sum_F g_F \left( \frac{T_F}{T} \right)^4,
\]

we can write the total energy density of all types of relativistic particles in thermal equilibrium as

\[
\rho_R = \frac{\pi^2}{30} N_R(T) T^4,
\]

where \( g_{B(F)} \) is the total number of boson (fermion) degrees of freedom and the sum runs over all boson (fermion) states with \( m \ll T \) [40]. The factor of \( 7/8 \) is due to the difference between the Fermi and Bose integrals. \( \Omega_{A0} \) defines the effective number of degrees of freedom, \( N_R(T) \), by taking into account new particle degrees of freedom as the temperature is raised. Comparing the Friedmann equation,

\[
H = 1.66 \sqrt{\Omega_{A0}(T)} \frac{T^2}{M_{\text{Pl}}},
\]

with \([17]\) and \([18]\), and using \( a = (1 + z)^{-1} \), we obtain a relation between \( \Omega_{A0} \) and \( N_R(T) \). The number of equivalent light neutrino species now follows from \([1]\) and is given by

\[
N_{\text{eff}}(T) = \frac{4}{7} \left[ N_R(T) - 2 \right] \left( \frac{11}{4} \right)^{1/3},
\]

assuming that all neutrinos flavors decouple at the same temperature. The 2\( \sigma \) upper limit \( \Delta N_{\text{eff}}(T_{\text{BBN}}) = 1.554 \) constrains \( \Omega_{A0} \) via \([17]\), \([18]\), \([21]\), and \([22]\). We note in passing that for \( h = 0.678 \) ΛCDM gives \( N_{\text{eff}}(T_{\text{BBN}}) \approx 2.99 \), whereas for \( h = 0.69 \) ΛCDM predicts \( N_{\text{eff}}(T_{\text{BBN}}) \approx 3.24 \); both these results are in agreement with those reported in [9]. For the model at hand, we find that the region of the parameter space that predicts \( \Omega_{A0} \gtrsim 5 \times 10^{-6} \) is excluded at the 2\( \sigma \) level.

There is a second theoretical constraint on the model, which is more restrictive. Since the dark vector fields must decouple before the electroweak phase transition there is a reheating of the left-handed neutrinos with respect to the dark vector fields. The reheating temperature can be computed comparing the 106.75 degrees of freedom of the SM with the 10.75 degrees of freedom of the primordial plasma before neutrino decoupling [40]. This leads to \( T_A/T_r = (10.75/106.75)^{1/3} \). By taking into account that each massive vector boson has 3 degrees of freedom and counting the 6 degrees of freedom of the left-handed neutrinos we obtain \( \Delta N_{\text{eff}}(T_{\text{BBN}}) = 0.24 \). Since it is not possible to reach \( \Delta N_{\text{eff}}(T_{\text{BBN}}) = 0.24 \) for a value \( h = 0.73 \), we take instead \( h = 0.69 \). The latter requires \( \Omega_{A0} \approx 2 \times 10^{-7} \). Finally, we use the constraint from the comoving sound horizon to show that the mass of the vector field satisfies \( 3.6 \times 10^{-28} < m/\text{eV} < 10^{-5} \) at the 1\( \sigma \) level; see Appendix.

In summary, we have examined the hot thermal description of the model proposed in [38] to resolve the Hubble tension. We have shown that while the addition of three dark massive vector fields may help to alleviate a little bit the existing tension in the measurements of the Hubble constant, it cannot eliminate the discrepancy between low- and high-redshift observations.

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Appendix

There is abundant evidence supporting the existence of acoustic waves that travelled through the tightly-coupled system of baryons and photons that filled the early universe [41]. These waves propagated until the decoupling of the photons at \( z = 1100 \), with a velocity given by

\[
c(a) = \sqrt{\frac{c}{3 \left( 1 + \frac{3a \Omega_\Lambda}{4 \Omega_\gamma} \right)}}.
\]

The sonic horizon, i.e. the distance that a sound wave could have travelled since since \( t = 0 \) up to the recombination time \( t \), is given by

\[
r_s = \int_0^t c_s \, dt,
\]
which can be written as
\[
\frac{r_s H_0}{c} = \int_0^\infty \frac{c_s(a)/c}{a^2E(a)} da = P^{-1}.
\]  
(25)

Observation leads to \( P = 29.63^{+0.48}_{-0.45} \) \[41\]. Since \( r_s \) can be determined for any given cosmological model, the value of \( P \) can be used to obtain \( H_0 \), and a decrement of the value of \( r_s \) leads to an increment of \( H_0 \). Numerical integration of \[25\] with \( \Omega_{m0} = 2 \times 10^{-7} \) and \( h = 0.69 \) requires \( 3.6 \times 10^{-5} < m/\text{eV} < 10^{-5} \) to satisfy the \( P \) constraint at \( 1\sigma \), and the lower limit from radiation-matter equivalence \( m > 3.6 \times 10^{-28} \, \text{eV} \) \[38\].

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