Photon thermal Hall effect

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A near-field thermal Hall effect (i.e., Righi-Leduc effect) in networks of magneto-optical particles placed in a constant magnetic field is predicted. This many-body effect is related to a symmetry breaking in the system induced by the magnetic field which gives rise to preferential channels for the heat-transport by near-field interaction thanks to the particles anisotropy tuning.

The Righi-Leduc effect [1] is the thermal analog of classical Hall effect [2]. It consists in the appearance of a heat flux transversally to a heat current induced by a temperature gradient inside a solid under the presence of a magnetic field. Like the Hall effect, it is due to the curvature of carriers trajectories through the magnetic field. At macroscopic scale this effect is related to a symmetry breaking in the transport equations due to the presence of an external magnetic field. At microscopic numerous mechanisms can be responsible for this effect. In semiconductors, metals or high-Tc superconductors it is the Lorentz force acting on the free electrons which is responsible for a transversal heat current. In ferromagnetic materials, magnons (spin waves) [3, 4] currents have been shown to be the source of thermal Hall effect. Recently, a phonon mediated thermal Hall effect [5, 6] has been highlighted in neutral objects of zero electrical charge. But, so far, no magnetotransverse effect have been predicted for photonic materials which are arranged in a four-terminal junction made with magneto-optical nanoparticles used to demonstrate the existence of a photon Hall effect under the action of an external magnetic field H when the left and right particles are held at two different temperatures. (a) If H = 0 the particles are optically isotropes so that the system is thermally symmetric (i.e., T_3 = T_4) and the Hall flux ϕ_H ≡ ϕ_y is null. (b) If a magnetic field is applied in the z direction, the particles become biaxial breaking the system symmetry (the optical axis are two complex valued vectors V_1 = (i, 1) and V_2 = (−i, 1), the eigenvectors of permittivity tensor) a temperature gradient is generated in the y direction giving rise to a non null Hall flux. The black arrows illustrate the particles anisotropy.

Figure 1: Sketch of the four terminal junction made with magneto-optical nanoparticles used to demonstrate the existence of a photon Hall effect under the action of an external magnetic field H when the left and right particles are held at two different temperatures. (a) If H = 0 the particles are optically isotropes so that the system is thermally symmetric (i.e., T_3 = T_4) and the Hall flux ϕ_H ≡ ϕ_y is null. (b) If a magnetic field is applied in the z direction, the particles become biaxial breaking the system symmetry (the optical axis are two complex valued vectors V_1 = (i, 1) and V_2 = (−i, 1), the eigenvectors of permittivity tensor) a temperature gradient is generated in the y direction giving rise to a non null Hall flux. The black arrows illustrate the particles anisotropy.

To start, we consider the system sketched in Fig. 1. It consists in four identical spherical particles made with a magneto-optical material which are arranged in a four-terminal junction. Those particles can exchange electromagnetic energy between them and with the surrounding medium which can be assimilated to a bosonic field at ambient temperature T_a. By connecting the two particles along the x-axis to two heat baths at two different temperatures, a heat flux flows through the system between these two particles. Without external magnetic field all particles are isotropic, so that the two others unthermostated particles have, for symmetry reasons, the same equilibrium temperatures and therefore they do not exchange heat flux through the network. On the contrary, when a magnetic field is applied orthogonally to the particles network, the particles become anisotropic so that the symmetry of system is broken (Fig. 1). As we will see hereafter, when the steady state regime is reached, the two unthermostated particles display two different temperatures. Therefore a heat flux propagates transversally to the primary applied temperature gradient.

Using the Landauer formalism for N-body systems [8–13] the heat flux exchanged between the i-th and the j-th particle in the network reads

\[ ϕ_{ij} = \int_0^\infty \frac{dω}{2π} [Θ(ω, T_i) − Θ(ω, T_j)] T_{i,j}(ω), \]

where Θ(ω, T) = ℏω/(e^(-ℏω/kT) − 1) is the mean energy of a harmonic oscillator in thermal equilibrium at temperature T and T_{i,j}(ω) denotes the transmission coefficient, at the frequency ω, between the two particles. When the
where particles are small enough compared with their thermal wavelength $\lambda_T = \hbar c/(k_B T_c)$ ($c$ is the vacuum light velocity, $2\pi \hbar$ is Planck’s constant, and $k_B$ is Boltzmann’s constant) they can be modeled by simple radiating electrical dipoles. In this case the transmission coefficient is defined as [11]

$$T_{ij}(\omega) = 2\text{Im} \text{Tr}[\tilde{A}_{ij} \text{Im} \tilde{C}_{ij}^\dagger],$$

(2)

where $\tilde{\chi}_j$, $\tilde{A}_{ij}$ and $\tilde{C}_{ij}$ are the susceptibility tensor plus two matrices which read in terms of free space Green tensor $G_{ij}^{0}$

$$G_{ij}^{0} = \frac{\exp(ikr_{ij})}{4\pi r_{ij}} \left[ (1 + \frac{i kr_{ij}}{k^2 r_{ij}^3}) \mathbb{1} + \frac{3 - 3i k^2}{k^2 r_{ij}^3} \tilde{r}_{ij} \otimes \tilde{r}_{ij} \right]$$

($\tilde{r}_{ij} \equiv r_{ij}/r_{ij}$, $r_{ij}$ is the vector linking the center of dipoles i and j, while $r_{ij} = |r_{ij}|$ and $\mathbb{1}$ stands for the unit dyadic tensor) and of polarizabilities matrix $\alpha = \text{diag}(\alpha_1, ..., \alpha_N)$ ($\alpha_i$ being the polarizability tensor associated to the i-th object)

$$\tilde{\chi}_j = \tilde{\alpha}_j - \frac{k^3}{6\pi} \tilde{\alpha}_j \tilde{\alpha}_j^\dagger,$$

(3)

$$\tilde{A}_{ij} = [1 - k^2 \tilde{\alpha} B]^{-1},$$

(4)

$$\tilde{C}_{ij} = k^2 \tilde{G}_{ik}^0 \tilde{A}_{kj}.$$  

(5)

If the temperature difference between the two thermostated particles is small (i.e. $T_1 = T_{eq} + \Delta T_1$, $T_{eq}$ being the temperature of cold reservoir) then we can treat the system in linear regime. In this case, the heat flux received by each particle can be written as

$$\phi_i = \sum_{j \neq i} \varphi_{ij} = \sum_{j \neq i} G_{ij}(T_j - T_i),$$

(6)

where

$$G_{ij} = \frac{\partial \varphi_{ij}}{\partial T} \big|_{T=T_{eq}} = \int_{0}^{\infty} \frac{d\omega}{2\pi} \frac{\partial \Theta}{\partial T} T_{ij}(\omega)$$

(7)

is the thermal exchange conductance between the i-th and the j-th particle at temperature $T_{eq}$. In steady state, the net power received by each particle vanishes. By neglecting the far field interactions with the surrounding field (the power $\varphi_{i\text{env}} = C_{\text{abs}ij} \sigma_B (T_1^4 - T_i^4)$ exchanged in far field with the environment where $C_{\text{abs}ij}$ is the thermally averaged dressed absorption cross-section of the i-th particle and $\sigma_B$ is the Stefan-Boltzmann constant is negligible in front of near-field interactions [14]) the energy balance equation reads

$$P_{i,b_i} + \sum_{j \neq i} G_{ij}(T_j - T_i) = 0, \quad i = 1, 2$$

(8)

$$\sum_{j \neq i} G_{ij}(T_j - T_i) = 0, \quad i = 3, 4,$$

(9)

where $P_{i,b_i}$ is the power exchanged between the thermostated particle $i$ and the $i$-th heat bath. By solving the two last equations with respect to unknown temperatures $T_3$ and $T_4$ we get

$$T_3 = \frac{1}{\Upsilon} \left[ (G_{31} \sum_{j \neq 3} G_{4j} + G_{34} G_{41}) T_1 \right]$$

(10)

$$+ (G_{32} \sum_{j \neq 3} G_{4j} + G_{34} G_{42}) T_2],$$

$$T_4 = \frac{1}{\Upsilon} \left[ (G_{41} \sum_{j \neq 4} G_{3j} + G_{43} G_{31}) T_1 \right]$$

(11)

$$+ (G_{42} \sum_{j \neq 4} G_{3j} + G_{43} G_{32}) T_2]$$

$$\Upsilon = \sum_{j \neq 3} G_{3j} - G_{34} G_{43}.$$  

(12)

Thus, by breaking this symmetry condition into the system applying, for instance, an external magnetic field, a temperature difference must appear between the upper and lower particles giving rise to a thermal Hall flux.

The magnitude of this Hall effect can be evaluated using the relative Hall temperature difference

$$R = \frac{T_3 - T_4}{T_1 - T_2}.$$  

(13)

In linear response regime, this expression reads from relations (10) and (11) and using the reciprocity of heat exchanges (i.e. $G_{ij} = G_{ji}$) we find the condition to fulfilled in order to get a null Hall flux (i.e. $T_3 = T_4$)

$$G_{31} G_{42} - G_{41} G_{32} = 0.$$  

(14)

As for the Hall conductance, it is defined from the ratio of Hall flux

$$\varphi_H = \sum_{i \neq 4} \varphi_{4i} - \sum_{i \neq 3} \varphi_{3i},$$  

(15)

over the primary temperature gradient $T_1 - T_2$ when $T_1 \to T_2$. After a straightforward calculation [15] this leads to

$$G_H = (G_{42} - G_{41}) T_{eq}$$

(16)

$$- R(G_{14} + G_{24} + 2G_{34}) T_{eq},$$

where the symbol ()$_T$ means that the conductances are calculated at temperature $T$. According to conditions [12], when $R = 0$ we verify that the Hall conductance vanishes.

Let us now consider a concrete situation by studying a four terminal junction made with four identical
InSb spherical particles of radius $r$ placed at the vertices of a square as sketched in Fig. 1. When a magnetic field is applied in the direction parallel to the $z$-axis, the permittivity tensor of InSb particles takes the following form [16, 17]

$$\tilde{\varepsilon} = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}$$

(17)

with

$$\varepsilon_{1}(H) = \varepsilon_{\infty}(1 + \frac{\omega_{L}^{2} - \omega_{T}^{2}}{\omega_{L}^{2} - \omega^{2} - i\Gamma}\frac{\omega^{2}(\omega + i\gamma)}{\omega(\omega + i\gamma)^{2} - \omega_{c}^{2}}),$$

(18)

$$\varepsilon_{2}(H) = \frac{\varepsilon_{\infty}\omega_{L}^{2}\omega_{c}}{\omega(\omega + i\gamma)^{2} - \omega_{c}^{2}},$$

(19)

$$\varepsilon_{3} = \varepsilon_{\infty}(1 + \frac{\omega_{L}^{2} - \omega_{T}^{2}}{\omega_{L}^{2} - \omega^{2} - i\Gamma}\frac{\omega^{2}(\omega + i\gamma)}{\omega(\omega + i\gamma)^{2} - \omega_{c}^{2}}).$$

(20)

Here, $\varepsilon_{\infty} = 15.7$ is the infinite-frequency dielectric constant, $\omega_{L} = 3.62 \times 10^{13}\text{rad.s}^{-1}$ is the longitudinal optical phonon frequency, $\omega_{T} = 3.39 \times 10^{13}\text{rad.s}^{-1}$ is the transverse optical phonon frequency, $\omega_{p} = (\frac{n e^{2}}{m^{*}\varepsilon_{\infty}})^{1/2}$ is the plasma frequency of free carriers of density $n = 1.07 \times 10^{17}\text{cm}^{-3}$ and effective mass $m^{*} = 1.99 \times 10^{-32}\text{kg}$, $\Gamma = 5.65 \times 10^{12}\text{rad.s}^{-1}$ is the phonon damping constant, $\gamma = 3.39 \times 10^{12}\text{rad.s}^{-1}$ is the free carrier damping constant and $\omega_{c} = eH/m^{*}$ is the cyclotron frequency. Thus, the polarizability tensor for a spherical particle can be described, including the radiative corrections, by the following anisotropic polarizability [18]

$$\bar{\alpha}_{i}^{0}(\omega) = (\bar{\varepsilon} - i\frac{k^{3}}{6\varepsilon}\bar{\alpha}_{0i})^{-1}\bar{\alpha}_{0i},$$

(21)

where $\bar{\alpha}_{0i}$ denotes the quasistatic polarizability of the $i$th particle which reads for spheres made with magneto-optical materials and which are embedded inside an isotropic host of permittivity $\varepsilon_{h}$

$$\bar{\alpha}_{0i}(\omega) = 4\pi r^{3}(\bar{\varepsilon} - \varepsilon_{h}\bar{1})(\bar{\varepsilon} + 2\varepsilon_{h}\bar{1})^{-1}.$$ (22)

As shown in the supplementary material [15] the particles polarizability becomes strongly anisotropic in presence of magnetic field. It is also shown, for particles smaller than the wavelength, that the contribution of magnetic moments can be neglected in front of electric contributions in the dissipation process.

In Fig. 2-a we show the relative Hall temperature difference $R$ with respect to the magnitude $H$ of magnetic field both in near-field and far-field regimes when the particles are embedded in vacuum (i.e. $\varepsilon_{h} = 1$). For any separation distance, when the magnetic field is zero, all particles are isotropic so that the system is symmetric and, as expected, $R = 0$. On the contrary, for non null magnetic field the symmetry of system is broken and a Hall flux appears. The results plotted in Fig. 2-a show that, in near-field regime, $R$ keeps the same sign whatever the magnitude of magnetic field. On the contrary, in far-field regime we see that the sign of $R$ can changes with the magnitude of magnetic field. Since, the optical properties of particles are the same for both regimes of interaction, this difference of behavior comes from the spatial variation of electric field itself radiated by each
Under the action of a weak magnetic field, the spatial distribution of electric field radiated by the particles leads to a strongest dissipation of energy in the lower particle (particle 3) so that a Hall flux flows in the direction of positives $y$. On the other hand, for strong magnetic fields of magnitude larger than about $H = 5T$ it is the upper particle (particle 4) which is over excited and the Hall flux goes in the opposite direction. However, as shown in Fig. 2-a, in far-field regime, the difference of temperature between particles 3 and 4 is generally much smaller in far-field than in near-field regime for weak field. This comes from the efficiency of heat exchanges in near-field because of the presence of surface waves. Hence, as $d_{12} = 3r$, the Hall temperature difference is approximately equal to 28% of the primary temperature gradient when $H = 3T$ while it is equal to about 3% in far-field regime. Because of this, hereafter we focus exclusively our attention on the near-field regime.

To go further in the thermal Hall effect analysis, let us examine now the variation of thermal conductances with the magnitude of magnetic field. For low field, we observe that $G_{13}$ and $G_{14}$ are notably different. This difference gives rise to a preferential channel for heat exchanges through the network. Around $H = 6T$ the asymmetry inside the system becomes maximal as shown in Fig. 2-c so that $R$ becomes maximal as well. When $G_{13} > G_{14}$, a larger amount of energy is transmitted from the first (hot) particle to the third particle than from the first particle to the fourth. Therefore, the third particle becomes hotter than the fourth one and accordingly a Hall flux flows toward the latter (i.e. $R > 0$). To explain the near-field coupling between the particles, let us focus now our attention on the optical properties of particles. According to the Clausius-Mosotti-like relation (22), the particle resonances, which correspond to localized surface polaritons, are solutions of the following transcendental equations

$$\varepsilon_3(\omega) + 2\varepsilon_h = 0$$

and

$$(\varepsilon_1(\omega) + 2\varepsilon_h)^2 - \varepsilon_2^2(\omega) = 0.$$ (24)

These resonances are plotted in Fig. 3 with respect to the magnitude of magnetic field. We clearly see the presence of three different branches (bright areas). The vertical branch is independent on the magnetic field. This branch is related to the resonance which is solution of Eq. (23) and it corresponds to the presence of a surface phonon polariton (SPhP) at $\omega \sim 3.5 \times 10^{13}$ rad.s$^{-1}$. The two others branches are solutions of Eq. (24). Contrary to the first resonance, these resonances depend on the magnetic field and are of plasmonic nature. When the magnitude of magnetic field becomes sufficiently large, these plasmonic resonances get away from the Wien’s frequency so that they do not contribute anymore to heat exchanges. On the other hand, for weak magnetic fields, these resonances give rise to supplementary channels for heat exchanges which superimpose to the channel associated with SPhP. Moreover, the contribution of the high frequency plasmonic channel becomes more and more important as its frequency brings closer from the Wien’s frequency. The optimal transfer occurs for a magnetic field of magnitude $H = 6T$. This situation corresponds precisely to the condition where the Hall effect is maximal.

If an experimental observation of photon thermal Hall effect seems to be out of reach with nanoparticle networks, a direct measurement of the Hall temperature difference with measurements of electrical resistance variations in magneto-optical nanowires networks should be feasible. A similar experiment has been reported recently [19] to measure, with a very high accuracy, the near-field heat exchanges between two nanobeams. Besides the experimental observation of this effect, some potential applications of photon thermal Hall effect may be considered. As for the classical Hall sensor, a many body junction made with magneto-optical elements is a natural building block to make a purely thermal magnetic field detection. Indeed, in linear regime, the Hall flux $\varphi_H = G_H R \Delta T$ ($\Delta T$ being the primary temperature gradient) is directly proportional through the Hall conductance to the magnetic field. Another application is the use in nanoscale heat engines of thermal Hall effect in presence of an AC magnetic field to modulate the heat flow in multiple directions. Of course, the upper frequency for such a modulation is limited by the thermal relaxation of the Hall cell. But, with nanocomponents, this frequency is of the order of the inverse of phonon relaxation time $\tau_{ph} \sim 10^{-12}$s.

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