Deep Learning for Optimal Deployment of UAVs with Visible Light Communications

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Abstract

In this paper, the problem of optimizing the deployment of unmanned aerial vehicles (UAVs) equipped with visible light communication (VLC) capabilities is studied. In the studied model, the UAVs can predict the illumination distribution of a given service area and determine the user association with the UAVs to simultaneously provide communications and illumination. However, ambient illumination increases the interference over VLC links while reducing the illumination threshold of the UAVs. Therefore, it is necessary to consider the illumination distribution of the target area for UAV deployment optimization. This problem is formulated as an optimization problem, which jointly optimizes UAV deployment, user association, and power efficiency while meeting the illumination and communication requirements of users. To solve this problem, an algorithm that combines the machine learning framework of gated recurrent units (GRUs) with convolutional neural networks (CNNs) is proposed. Using GRUs and CNNs, the UAVs can model the long-term historical illumination distribution and predict the future illumination distribution. Based on the prediction of illumination distribution, the optimization problem becomes nonconvex and is then solved using a low-complexity, iterative physical relaxation algorithm. The proposed algorithm can find the optimal UAV deployment and user association to minimize the total transmit power. Simulation results using real data from the Earth observations group (EOG) at NOAA/NCEI show that the proposed approach can achieve up to 64.6% reduction in total transmit power compared to a conventional optimal UAV deployment that does not consider the illumination.

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distribution and user association. The results also show that UAVs must hover at areas having strong illumination, thus providing useful guidelines on the deployment of VLC-enabled UAVs.

Index Terms

Visible light communication, unmanned aerial vehicles, drones, machine learning, gated recurrent units, convolutional neural networks, energy efficiency.

I. INTRODUCTION

Deploying unmanned aerial vehicles (UAVs) as flying base stations (BSs) for wireless networking is a flexible and cost-effective approach to providing on-demand communications [2]–[6]. However, for tomorrow’s ultra dense wireless networks that encompass a large number of ground BSs, UAVs may not have enough radio frequency (RF) resources to service ground users. Moreover, UAVs deployed as aerial BSs using RF will interfere with ground devices, hence significantly affecting the performance of the ground network. In addition, the limited energy will restrict the applicability of UAVs using RF resource to provide high-speed communication services for ground users. These challenges can be addressed by equipping UAVs with visible light communication (VLC) capabilities [7]. Indeed, VLC has recently attracted attention due to its large license-free bandwidth and high energy efficiency. For instance, a VLC system that uses light-emitting diodes (LEDs) to transmit wireless signals can provide both illumination and communication services. Moreover, the altitude of the UAVs ensures the line of sight channel for VLC. Therefore, using VLC can be a promising approach to provide energy-efficient UAV communications with sufficiently available bandwidth. However, deploying VLC-enabled UAVs also faces many challenges that include illumination interference detection and prediction, UAV deployment optimization, and energy efficiency.

The existing literature such as in [3]–[6] and [8]–[12] has studied a number of problems related to UAV deployment. The work in [3] proposed to deploy UAVs using the notion of truncated octahedron shapes in cellular networks so as to minimize latency of ground users. In [4], the authors studied the optimal UAVs’ locations based on the prediction of human behavior so as to optimize the quality-of-experience of wireless devices. The authors in [5] derived the average

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coverage probability and the system sum-rate as a function of the UAV altitude and the number of users. In [6], the authors designed a shortest-path-routing algorithm to minimize the outage probability and the bit error rate of UAVs in a UAV-assisted emergency network. However, the works in [3]–[6] ignored the energy efficiency of UAVs in optimizing the deployment of UAVs. In [8], the authors considered the efficient deployment and mobility of multiple UAVs to enable reliable uplink communications for the Internet of Things (IoT) devices with a minimum total transmit power. The authors in [9] jointly optimized the transmit power and trajectory of UAVs to improve the energy harvesting efficiency while guaranteeing the secrecy rate in the presence of multiple eavesdroppers. However, the works in [8] and [9] only optimized the locations of UAVs under fixed user association. The authors in [10] optimized resource allocation and user association in an integrated satellite-drone network. In [11], the authors analyzed user association, power control, and computational resource allocation to find the optimal position of each UAV. The work in [12] maximized the minimum throughput over all ground users by optimizing the multiuser communication scheduling, user association, as well as the UAV’s trajectory and transmit power. However, all of the existing works such as in [3]–[6] and [8]–[12] are over limited capacity radio frequency bands which may not allow the UAVs to meet the high data rate demands of ground users. Instead, VLC-enabled UAVs can be considered to provide high speed communications [13]. In [14], the authors developed a novel integrated VLC and UAV framework that simultaneously provide communication and illumination and optimized the locations of UAVs to minimize the total power consumption. However, this work does not consider the impact of night-time illumination such as vehicle lights, street lights, and building lights, which will cause strong interference to VLC links [15]. Therefore, it is necessary to analyze the illumination distribution of the service areas so as to optimize the deployment of VLC-enabled UAV. Naturally, machine learning (ML) [16] can be used to predict future illumination distribution due to its strong ability on the analysis of historical illumination distribution.

More recently, there has been significant interest in applying ML techniques to optimize UAV deployment such as in [17]–[20]. The authors in [17] used a Q-learning method for dynamically designing placement and movement of UAVs in a non-orthogonal multiple access (NOMA) based wireless network. In [18], the authors used deep reinforcement learning for UAV control to maximize energy efficiency with joint consideration of communication coverage, fairness, and
connectivity. The works in [17] and [18] that used reinforcement learning algorithms to optimize network performance did not consider the use of the data related to the wireless environment to analyze wireless network states which can also improve network performance. The work in [19] studied the use of artificial neural networks (ANNs) to predict future network states, thus adaptively optimizing UAV energy efficiency. The authors in [20] analyzed the instantaneous traffic demands of the users for predictive deployment of UAVs. Based on the analysis of wireless network states, UAVs can be optimally deployed in the service area in advance thus providing on-demand and power-efficient wireless service to ground users. Despite these promising results, existing works such as [17]–[20] do not analyze the potential of using ML for the predictions of illumination distribution. In particular, the existing works in [19] and [20] only consider the temporal correlation of the wireless network states, which can be easily captured by a simple model such as Gaussian mixture model (GMM) in [20]. However, such works cannot deal with the prediction of illumination distribution which needs a comprehensive analysis of joint spatial and temporal features of illumination distribution. Nighttime illumination causes interference over the VLC link while reducing the illuminance requirements of users, hence affecting the data rate of each user that is serviced by VLC links and the deployment of VLC-enabled UAVs. The distribution and intensity of nighttime illumination caused by human activities can vary in real time. For example, during evenings, the illumination of factories will decrease while the illumination of residential or commercial areas will increase. Meanwhile, the illumination of each road changes as the density of vehicles in the road changes. In consequence, it is necessary to develop a novel ML framework for the analysis and prediction of illumination distribution over an hourly scale. Based on the predictions, the network can optimally deploy UAVs to the service area in advance thus providing a power-efficient and on-demand wireless service to ground users.

The main contribution of this work is a novel framework for dynamically optimizing the locations of VLC-enabled UAVs based on accurate predictions of the illumination distribution of a given area. Our key contributions include:

- We consider a VLC-enabled UAV network, in which the UAVs must find their optimal locations and user association by predicting the distribution of ambient lighting so as to provide illumination as well as communication services to ground users. This problem is formulated as an optimization problem whose goal is to minimize the total transmit power of UAVs under illumination, communication, and user association constraints.
• To solve this optimization problem, we propose a deep learning-based prediction model approach by marrying gated recurrent units (GRUs) with convolutional neural networks (CNNs). The proposed approach can analyze the temporal and spatial characteristics of the long-term historical illumination distribution thus enabling the UAVs to predict future illumination distributions.

• Given the predicted illumination distribution, we transform the original, nonconvex problem into a convex equivalent by using a physical relaxation for the user association constraints. Then, we develop a feasible, efficient, and low-overhead iterative algorithm via dual decomposition, which can be implemented in VLC-enabled UAV networks.

• We perform fundamental analysis on the lower bound of the minimum transmit power of each UAV for satisfying illuminance and data rate requirements of its associated users. Our result shows that, when the illumination requirement is smaller than the data rate requirement, the transmit power achieves the lower bound if the illuminance is 0. In contrast, when the illumination requirement is larger than the data rate requirement, the unique optimal illuminance can be derived.

Simulation results show that the proposed approach can achieve up to 64.6% reduction in terms of transmit power compared to a conventional optimal UAV deployment without considering illumination distribution. Furthermore, our results also show that UAVs should hover over areas with strong illumination. To the best of our knowledge, this is the first work that *studies the use of the predictions of the illumination distribution to provide a power-efficient deployment of VLC-enabled UAVs.*

The rest of this paper is organized as follows. The system model and the problem formulation are described in Section [II](#). The integrated GRU and CNN deep learning model to predict the future illumination distribution is proposed in Section [III](#). The proposed iterative UAV deployment, user association, and power efficiency algorithm is presented in Section [IV](#). In Section [V](#), the numerical results are presented and discussed. Finally, conclusions are drawn in Section [VI](#).

### II. System Model and Problem Formulation

Consider a wireless network composed of a set $\mathcal{D}$ of $D$ VLC-enabled UAVs that serve a set $\mathcal{U}$ of $U$ ground users distributed over a geographical area $\mathcal{A}$. The UAVs provide downlink
transmission and illumination simultaneously, as shown in Fig. 1. Hereinafter, we use aerial cell to refer to the service area of each UAV.

A. Transmission Model

Given a UAV $i \in D$ located at $(x_i, y_i, H)$ and a ground user $j \in U$ located at $(x_j, y_j) \in A$, the channel gain of the VLC link between UAV $i$ and user $j$ can be given by [21]:

$$h_j(x_i, y_i) = \begin{cases} \frac{(m+1)\rho}{2\pi d_{ij}^2} g(\psi) \cos^m(\phi) \cos(\psi), & 0 \leq \psi \leq \Psi_c, \\ 0, & \psi > \Psi_c, \end{cases}$$

(1)

where $\rho$ is the detector area and $d_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + H^2}$ is the distance between UAV $i$ and ground user $j$. $m = -\ln 2 / \ln(\cos \Phi_{1/2})$ is the Lambert order with $\Phi_{1/2}$ being the transmitter semiangle (at half power); $\psi$ and $\phi$ represent the angle of incidence and irradiance, respectively. As such, $\cos \phi = \cos \psi = \frac{H}{d_{ij}}$. Let $\Psi_c$ be the receiver field of vision (FOV) semi-angle. The gain of the optical concentrator $g(\psi)$ is defined as:

$$g(\psi) = \begin{cases} \frac{n_e^2}{\sin^2 \Psi_c}, & 0 \leq \psi \leq \Psi_c, \\ 0, & \psi > \Psi_c, \end{cases}$$

(2)

where $n_e$ represents a refractive index.

Let $u_{ij,t}$ be the association for UAV $i$ and user $j$ at time $t$, i.e., $u_{ij,t} = 1$ indicates that user $j$ is associated with UAV $i$ at time $t$; otherwise, we have $u_{ij,t} = 0$. Assuming that each user is associated with only one UAV, we have:

$$\sum_{i \in D} u_{ij,t} = 1, \forall j \in U.$$
For static user $j$ located at $(x_j, y_j)$ associated with UAV $i$, the channel capacity at time $t$ can be given by:

$$C_{ij,t} = \frac{1}{2} \log_2 \left( 1 + \frac{e}{2\pi} \left( \frac{\xi P_{ij,t} h_j(x_i, y_i)}{n_w + I_t(x_j, y_j)} \right)^2 \right) ,$$

(4)

where $\xi$ is the illumination target, $P_{ij,t}$ is the transmit power of UAV $i$ serving user $j$ at time $t$, and $n_w$ represents the standard deviation of the additive white Gaussian noise. In (4), $I_t(x_j, y_j)$ is the ambient illumination at $(x_j, y_j)$, which also indicates the interference over the VLC link between the UAV and the user $j$. To obtain the illumination for each location, we define the illumination distribution of the service area as $I_t$ that will be specified in Section III.

Due to the limited energy of UAVs, their deployment must be optimized to minimize the transmit power while satisfying the data rate and illumination requirements of users. Since the area of aerial cells is small and ground users served by UAVs are static, as done in [22]–[24], we do not consider the mobility energy consumption of the UAVs.

B. Problem Formulation

To formulate the deployment problem, we must first determine the minimum transmit power that each UAV $i$ uses to meet the data rate and illumination requirements of its associated users. To satisfy the data rate constraint $R_j$ of each user $j$ located at $(x_j, y_j)$, the power required from UAV $i$ at time $t$ is:

$$P_{ij,t} = \frac{u_{ij,t}(n_w + I_t(x_j, y_j)) \sqrt{2\pi (2^{2R_j} - 1)}}{\xi h_j(x_i, y_i)} .$$

(5)

A UAV can successfully satisfy all the users requirements once the user that has the maximum power requirement is satisfied. Therefore, the minimum transmit power of UAV $i$ satisfying the data rate requirements of its associated users is given by:

$$P_{i,t}^{\text{min}} = \max\{P_{ij,t}\}, \forall j \in \mathcal{U} .$$

(6)

Given this system model, our goal is to find an effective deployment of UAVs that meets the data rate and illumination requirements of each user while minimizing the transmit power of the UAVs. This problem involves predicting the illumination and adjusting the user association, the locations as well as the transmit powers of UAVs. The optimization problem is formulated as
follows:

$$\min_{x_i, u_i, u_{i,t}} \sum_{i \in D} P_{i,t},$$

$s.t.$

$$\xi P_{i,t} h_j(x_i, y_i) \geq u_{ij,t}(\eta_r - I_t(x_j, y_j)), \forall i \in D, \forall j \in U,$$

(7a)

$$P_{i,t} \geq P_{i,t}^{\min}, \forall i \in D,$$

(7b)

$$\sum_{i \in D} u_{ij,t} = 1, \forall j \in U,$$

(7c)

$$u_{ij,t} \in \{0, 1\}, \forall i \in D, \forall j \in U,$$

(7d)

where $u_{i,t} = [u_{i1,t}, u_{i2,t}, \ldots, u_{iU,t}]$ is the user association vector of UAV $i$, $\eta_r$ denotes the illumination demand, and $\xi P_{i,t} h_i(x_i, y_i)$ is the illumination of UAV $i$ at time $t$ [25]. (7a) indicates that each UAV needs to provide illumination to meet the illumination threshold of each user $j$. (7b) indicates that the transmit power of UAV $i$ should satisfy the data rate requirements of its associated users from (6). (7c) and (7d) imply that each user can only associate with one UAV at each time slot. Here, we ignore the interference caused by other UAVs, since the service area of each UAV does not overlap with the service areas of other UAVs. Note that ambient illumination causes interference over the VLC link while reducing the illuminance requirements of users. The distribution of ambient illumination at night that consists of vehicle, street, and building lights varies in real time. For example, during nights, the illumination of the factories will decrease while the illumination of residential or commercial areas will increase. In addition, the illumination of each road changes as the vehicle density changes. Therefore, it is necessary to predict the illumination distribution of the target area to deploy the UAVs at the beginning of each time interval. Hence, we will next introduce a machine learning algorithm to predict the illumination distribution of the service area (at an hourly scale). Based on the prediction, UAVs are deployed according to the solution of (7) which remain unchanged during each prediction period.

III. MACHINE LEARNING FOR ILLUMINATION PREDICTION

Since predicting the illumination distribution requires both spatial and temporal sequence information, we propose a deep learning approach that integrates GRUs with CNNs. The proposed approach enables the UAVs to analyze the relationship among historical illumination distributions
Fig. 2: Overall architecture of the proposed learning model.

and to predict the future illumination distribution. Specifically, we first apply an CNN to extract spatial features of the illumination distribution at each time slot $t$. Then, the time-varying spatial features are fed to GRUs for predicting the features of illumination distribution at time $t + 1$ based on the learned temporal dependencies. Finally, a deconvolution network (DeCNN) is used to transform the multidimensional features, which are predicted by GRUs, to the illumination distribution. The architecture of the integrated GRU and CNN predictive model is shown in Fig. 2. Next, we introduce the proposed model that consists of three components: a) CNNs, b) GRUs, and c) DeCNNs.

A. CNN for Encoding Illumination Distribution

Since illumination is caused by human activities such as business and industrial operation, the illumination at a given position always has very strong spatial correlations with the illumination distribution of nearby regions. Therefore, we use CNNs to capture spatial correlations between the illumination of a given location and the illumination of its nearby regions, and then build the feature representations that preserve the changes in local illumination.

Given an illumination distribution $I_t$ at time $t$, a CNN encoder is used to extract the feature vector $x_t$, which represents the spatial features extracted from $I_t$. The proposed CNN algorithm consists of $L$ convolutional layers, $L$ max-pooling layers, and a flatten layer. In particular, each convolutional layer is followed by a max-pooling layer and the last layer of the CNN is a flatten layer. Next, we introduce each layer of the proposed CNN.
• **Convolutional layer**: In a CNN, a convolutional layer is used to extract spatial features which are useful in the next illumination distribution predicting stage. Mathematically, the input of each convolutional layer $l$ is $H^{l-1,m}$, where $H^{l-1,m}$, $l = 1, \ldots, L$ is the feature map $m$ in convolutional layer $l - 1$ and the input $H^{0,1}_t$ of convolutional layer 1 is an illumination distribution at time $t$ (e.i., $I_t = H^{0,1}_t$). The output of each convolutional layer $l$ is given by

$$H^{l,m}_t = f\left(\sum_{k=1}^{K^{l-1}_c} H^{l-1,k}_t \otimes W^{l,m}_{c,t} + b^{l,m}_{c,t}\right),$$

(8)

where $f(\cdot) = \max(0, \cdot)$ is rectifier activation function, $K^{l-1}_c$ is the number of feature maps in convolutional layer $l - 1$, $\otimes$ denotes the convolution operation, and $W^{l,m}_{c,t} \in \mathbb{R}^{S \times S}$ and $b^{l,m}_{c,t}$ are convolution kernels and bias of feature map $m$ in convolutional layer $l$, respectively, with $S$ being a constant that controls the spatial granularity. Note that $H^{0,0}_t \in \mathbb{R}^{\lambda_0 \times \lambda_0}$ and the size of feature maps $H^{l,m}_t \in \mathbb{R}^{\lambda_l \times \lambda_l}$ in convolutional layer $l$ satisfy $\lambda_l = \lambda_{l-1} - S + 1$.

• **Max-pooling layer**: The input of each max-pooling layer $l$ is the feature map $H^{l,m}_t$. Max-pooling layers compress the input feature map, which allows a CNN encoder to extract robust spatial features while reducing the computation complexity. The position of max-pooled features in feature maps are recorded in switch variables (switches), which will be used to decode the predicted features of the future illumination distribution in the DeCNN.

• **Flatten layer**: A flatten layer is used at the end of the CNN encoder, whose input is the combination of feature maps extracted by max-pooling layer $L$. The flatten layer generate a spatial feature vector $x_t \in \mathbb{R}^N$, where $N = \lambda_L^2 K_L$ is the number of the features extracted by the CNN encoder.

**B. Illumination Distribution Prediction**

Next, we introduce the use of GRUs [27] for the prediction of the illumination distribution. GRUs are extensions of conventional recurrent neural networks (RNNs) [28]. GRUs can effectively solve the gradient vanishing and the gradient exploding problem in long-term memory RNNs. Due to interconnected neurons at hidden layers and their internal gating mechanisms, GRUs can model the temporal characteristics of the long-term illumination distribution. In addition, GRUs can dynamically update the model based on the current illumination distribution due to the variable-length recurrent structure, hence, GRUs enable the UAVs to predict future illumination distribution.
A GRU-based prediction algorithm consists of three components: a) input, b) output, and c) GRU model. The key components of our GRU-based prediction approach are:

- **Input**: The input of the GRU-based prediction algorithm is the output of the CNN encoder which is represented as \( X = (x_1, x_2, \cdots, x_t, \cdots, x_T) \).

- **Output**: The output of the GRU-based prediction algorithm is a vector \( x_{T+1} \), that represents the spatial features of illumination distribution at time slot \( T + 1 \).

- **GRU model**: A GRU model is used to approximate the function between the input \( X \) and output \( x_{T+1} \), thus building a relationship between historical illumination distribution and future illumination distribution. A GRU model is essentially a dynamic neural network that consists of an input layer, a hidden layer, and an output layer. The hidden states \( h_t \) of the units of the in hidden layer at time \( t \) are used to store information related to the illumination distribution from time slot 1 to \( t \). For each time \( t \), the hidden states \( h_t \) of the GRU are updated based on the input \( x_t \) and \( h_{t-1} \). Next, we introduce how to update the hidden state \( h_t^j \) of hidden unit \( j \) given a new illumination distribution \( x_t \).

At each time slot \( t \), the hidden state \( h_t^j \) is determined by two gates: reset gate \( r_t^j \) and update gate \( z_t^j \). First, the reset gate \( r_t^j \) is used to determine the historical illumination distribution information retained in the candidate hidden state \( \tilde{h}_t^j \), which can be given by:

\[
r_t^j = \sigma ([W_r x_t]_j + [U_r h_{t-1}]_j),
\]

where \( \sigma(\cdot) = \frac{1}{1+e^{-\cdot}} \) is the logistic sigmoid function and \([\cdot]_j \) is element \( j \) of a vector. \( W_r \in \mathbb{R}^{N \times D_h} \) and \( U_r \in \mathbb{R}^{D_h \times D_h} \) represent the weight matrices of reset gate, where \( N \) is the length of the input \( x_t \) and \( D_h \) is the number of the units in hidden layer. Based on the value of the reset gate \( r_t^j \), the candidate hidden state \( \tilde{h}_t^j \) that is used to combine the input illumination distribution \( x_t \) with the previous memory \( h_{t-1} \) is given by:

\[
\tilde{h}_t^j = \text{tanh} \left( [W_h x_t]_j + [U_h (r_t \odot h_{t-1})]_j \right),
\]

where \( r_t \in \mathbb{R}^{D_h} \) is a reset gate vector at time \( t \) and \( \odot \) is an element-wise multiplication. For example, given two vectors \( p = (a, b) \) and \( q = (c, d) \), \( p \odot q = (ac, bd) \). \( W_h \in \mathbb{R}^{N \times D_h} \) and \( U_h \in \mathbb{R}^{D_h \times D_h} \) represent the hidden state weight matrices.

Similarly, the update gate \( z_t^j \) is used to decide the size of the information stored in the candidate hidden state to update the hidden state \( h_t^j \), which can be given by:

\[
z_t^j = \sigma ([W_z x_t]_j + [U_z h_{t-1}]_j),
\]
where $W_z \in \mathbb{R}^{N \times D_h}$ and $U_z \in \mathbb{R}^{D_h \times D_h}$ represent the weight matrices of the update gate. The actual hidden state $h^j_t$ of hidden unit $j$ is updated by:

$$h^j_t = z^j_t h^j_{t-1} + (1 - z^j_t)\tilde{h}^j_t. \quad (12)$$

The proposed GRU model iteratively updates the hidden states to store the input $X$ until the hidden state of the current time $T$ is computed. The output layer of the GRU model will predict the illumination distribution at time $T+1$ based on the hidden state $h_T$:

$$x_{T+1} = W_o h_T, \quad (13)$$

where $W_o \in \mathbb{R}^{D_h \times N}$ is the output weight matrix. Based on (13), we get output $x_{T+1}$ from the hidden state $h_T$ that stores the information of input $X$.

C. Illumination Distribution Deconvolution Network

We now study the decoding of the predicted feature vector $x_{T+1}$ into the illumination distribution $I_{T+1}$. Since GRU-based predictions $x_{T+1}$ only contain the spatial features of illumination distribution $I_{T+1}$ rather than a complete illumination distribution, we use a DeCNN to decode the predicted features. The proposed DeCNN decoder is a mirrored version of the CNN encoder introduced before, which consists of $L$ unpooling layers and $L$ deconvolutional layers. Next we introduce each layer of the proposed DeCNN.

- **Unpooling layer**: The input of the first unpooling layer is $x_{T+1}$ predicted by GRUs and the input of unpooling layer $l$ ($l > 1$) is the feature maps output from the deconvolutional layer $l-1$. The unpooling layers are used to reconstruct the illumination distribution of service area to the original size. Therefore, the output of an unpooling layer is an enlarged, yet sparse feature map.

- **Deconvolutional layer**: The input of each deconvolutional layer $l$ is the enlarged feature maps output from the unpooling layer $l-1$. The deconvolutional layers effectively reconstruct the detailed structure of illumination distribution based on the learned weights, which is defined as:

$$\tilde{H}^{l,m}_{T+1} = f\left(\sum_{k=1}^{K_d^{l-1}} \tilde{H}^{l-1,k}_{T+1} \otimes W_{d,T+1}^{l,m} + b_{d,T+1}^{l,m}\right), \quad (14)$$

where $\tilde{H}^{l,m}_{T+1}$ is reconstructed feature map $m$ in deconvolutional layer $l$, $K_d^{l-1}$ is the number of feature maps in deconvolutional layer $l-1$, and $W_{d,T+1}^{l,m}$ and $b_{d,T+1}^{l,m}$ are convolution kernels and bias of feature map $m$ in deconvolutional layer $l$, respectively. The output of
Algorithm 1 Integrated GRU and CNN Predictive Model for Illumination Distribution Prediction.

1: **Input:** The time series illumination distribution of service area, $I_1, I_2, \cdots, I_t, \cdots, I_T$.
2: **Initialize:** $W_{c,1}, \cdots, W_{c,T}, W_{d,T+1}, W_c, U_r, W_o, U_z, W_{\tilde{h}}, U_{\tilde{h}}$, and $W_o$ are initially generated randomly via a uniform distribution. The number of iterations $e$.
3: for $i = 1 \rightarrow e$ do
4: for each time $t$ do
5: Input $I_t$ and encode $I_t$ into a feature vector $x_t$ based on (8).
6: Predict the spatial feature vector $x_{t+1}$ based on (13).
7: Decode the predicted $x_{t+1}$ into the illumination distribution $I_{t+1}$.
8: end for
9: Calculate the loss $E$ based on (16).
10: Update the weight matrices based on (15).
11: end for
12: **Output:** Prediction $I_{T+1}$.

Finally, the trained integrated GRU and CNN predictive model can output the illumination distribution prediction based on the input historical illumination distributions.

D. Integrated GRU and CNN Predictive Model Training

The proposed integrated GRU and CNN predictive model build the relationship between output $\tilde{I}_{T+1}$ and the input time series historical illumination distribution $I_1, I_2, \cdots, I_t, \cdots, I_T$ using the weight parameters. To build this relationship, a batch gradient descent approach is used to train the weight matrices which are initially generated randomly via a uniform distribution [29].

The update rule of the gradient descent approach is given by:

$$W^{i+1}_n = W^i_n - \alpha \nabla E(W_n),$$
$$U^{i+1}_m = U^i_m - \alpha \nabla E(U_m),$$  \hspace{1cm} (15)

where $\alpha$ is the learning rate, $n \in \{c, d, r, z, \tilde{h}, o\}$, and $m \in \{r, z, \tilde{h}\}$. $\nabla E(W_n) = \frac{\partial E}{\partial W_n}$ and $\nabla E(U_m) = \frac{\partial E}{\partial U_m}$ are the gradients of the loss function $E$ which is defined as:

$$E = \frac{1}{2\lambda_0^2} \sum_{x=1}^{\lambda_0} \sum_{y=1}^{\lambda_0} ||I_{T+1}(x, y) - \tilde{I}_{T+1}(x, y)||^2.$$  \hspace{1cm} (16)

$I_{t+1}$ and $\tilde{I}_{t+1}$ represent the actual illumination and the predicted illumination at location $n$ at time $t+1$, respectively. The specific process of using the proposed deep learning model to predict the illumination distribution for each UAV $i$ is summarized in Algorithm 1.
IV. OPTIMIZATION OF UAV DEPLOYMENT, USER ASSOCIATION, AND POWER EFFICIENCY

Once the illumination distribution is predicted, the UAVs can determine their optimal deployment at the beginning of each time interval by solving the optimization problem defined in (7). As analyzed in Section II, a UAV only needs to consider the users with the maximum power requirement since, by doing so, the requirements of all other users will be automatically satisfied. Therefore, substituting (1), (5), and (6) into (7), we have:

\[
\begin{align*}
\min_{x_i,y_i,P_{i,T+1}} & \quad \sum_{i \in \mathcal{D}} P_{i,T+1}, \\
\text{s.t.} & \quad P_{i,T+1} \geq lM_j d_{ij}^{m+3} u_{ij,T+1}, \quad \forall i \in \mathcal{D}, \forall j \in \mathcal{U}, \\
& \quad P_{i,T+1} \geq lN_j d_{ij}^{m+3} u_{ij,T+1}, \quad \forall i \in \mathcal{D}, \forall j \in \mathcal{U}, \\
& \quad \sum_{i \in \mathcal{D}} u_{ij,T+1} = 1, \quad \forall j \in \mathcal{U}, \\
& \quad u_{ij,T+1} \in \{0, 1\}, \quad \forall i \in \mathcal{D}, \forall j \in \mathcal{U},
\end{align*}
\]

where \( l = \frac{2\pi}{\xi (m+1) \rho g H^{m+1}} \), \( M_j = \eta_e - I_{T+1}(x_j, y_j) \), and \( N_j = (n_w + I_{T+1}(x_j, y_j)) \sqrt{\frac{2\pi}{e} (2^{2R_j} - 1)} \). Note that problem (17) is nonconvex. We present an iterative algorithm for solving the nonconvex problem. In particular, we first optimize the UAV deployment and power allocation with fixed user association. Then, given the UAV deployment, we find the optimal user association.

A. UAV Deployment and Power Efficiency with Fixed User Association

Since constraints (17c) and (17d) are only determined by user association \( u_{T+1} \), the UAV deployment and power efficiency problem (17) with fixed user association \( u_{T+1} \) is expressed as:

\[
\begin{align*}
\min_{x_i,y_i,P_{i,T+1},u_{T+1}} & \quad \sum_{i \in \mathcal{D}} P_{i,T+1}, \\
\text{s.t.} & \quad P_{i,T+1} \geq lM_j d_{ij}^{m+3} u_{ij,T+1}, \quad \forall i \in \mathcal{D}, \forall j \in \mathcal{U}, \\
& \quad P_{i,T+1} \geq lN_j d_{ij}^{m+3} u_{ij,T+1}, \quad \forall i \in \mathcal{D}, \forall j \in \mathcal{U}, \\
& \quad \sum_{i \in \mathcal{D}} u_{ij,T+1} = 1, \quad \forall j \in \mathcal{U}, \\
& \quad u_{ij,T+1} \in \{0, 1\}, \quad \forall i \in \mathcal{D}, \forall j \in \mathcal{U},
\end{align*}
\]

where \( \mathcal{U}_i = \{ j \in \mathcal{U} | u_{ij,T+1} = 1 \} \). Since optimizing the location of each UAV \( i \) is independent, problem (18) can be decoupled into multiple subproblems. For each UAV \( i \), the location optimization subproblem can be formulated as follows:

\[
\begin{align*}
\min_{x_i,y_i,P_{i,T+1}} & \quad P_{i,T+1}, \\
\text{s.t.} & \quad P_{i,T+1} \geq lM_j d_{ij}^{m+3}, \quad \forall i \in \mathcal{D}, \forall j \in \mathcal{U}_i, \\
& \quad P_{i,T+1} \geq lN_j d_{ij}^{m+3}, \quad \forall i \in \mathcal{D}, \forall j \in \mathcal{U}_i,
\end{align*}
\]
where \( a_j = (\max \{lM_j, lN_j\})^{\frac{2}{m+3}}. \)

Given the user association, problem (19) is a convex problem due to its convex objective functions and constraints, which can be optimally solved by using the dual method [30]. The Lagrange function of problem (19) will be:

\[ L = P_{i,T+1} + \sum_{j \in U_i} \lambda_j \left( ((x_i - x_j)^2 + (y_i - y_j)^2 + H^2)a_j - P_{i,T+1}^{\frac{2}{m+3}} \right), \]

where \( \lambda_j \) is the dual variable associated with constraint \( j \) in (19a).

The optimal first-order conditions of (19) will be:

\[ \frac{\partial L}{\partial P_{i,T+1}} = 1 - \frac{2}{m+3} \sum_{j \in U_i} \lambda_j P_{i,T+1}^{\frac{m+1}{m+3}} = 0, \]

(21)

\[ \frac{\partial L}{\partial x_i} = 2 \sum_{j \in U_i} \lambda_j a_j (x_i - x_j) = 0, \]

(22)

\[ \frac{\partial L}{\partial y_i} = 2 \sum_{j \in U_i} \lambda_j a_j (y_i - y_j) = 0. \]

(23)

Solving (21) to (23) yields

\[ P_{i,T+1} = \left( \frac{2}{m+3} \sum_{j \in U_i} \lambda_j \right)^{\frac{m+1}{m+3}}, \]

(24)

\[ x_i = \frac{\sum_{j \in U_i} \lambda_j a_j x_j}{\sum_{j \in U_i} \lambda_j a_j}, \]

\[ y_i = \frac{\sum_{j \in U_i} \lambda_j a_j y_j}{\sum_{j \in U_i} \lambda_j a_j}. \]

(25)

Given \( x_i, y_i, \) and \( P_{i,T+1} \), the value of \( \lambda_j \) can be determined by the gradient method [31]. The updating procedure is:

\[ \lambda_j = \lambda_j + \gamma \left( ((x_j - x_i)^2 + (y_j - y_i)^2 + H^2)a_j - P_{i,T+1}^{\frac{2}{m+3}} \right), \]

(26)

where \( \gamma \) is a dynamic step size. With regards to the optimality, each subproblem (19) is a convex problem which can always converge to the optimal solution according to [30]. Therefore, the solution in (25) of each subproblem (19) is the optimal solution of the original problem (18).

To find the lower bound of the minimum transmit power, \( \inf P_{i,T+1}^{\min} \), we state the following result:

**Proposition 1:** If the illumination at the location of user \( j \) satisfies the following conditions:

\[ I^*_{T+1}(x_j, y_j) = \begin{cases} \frac{\eta_r + n_w}{1 + \sqrt{\frac{2\pi}{e} (2^2Rj - 1)}} - n_w, & \eta_r \geq n_w \sqrt{\frac{2\pi}{e} (2^2Rj - 1)}, \\ 0, & \eta_r < n_w \sqrt{\frac{2\pi}{e} (2^2Rj - 1)}, \end{cases} \]

(27)

...
then the transmit power of each UAV $i$ achieves the lower bound, which is given by:

$$
\inf P_{i,T+1}^{\min} = \max_{j \in \mathcal{U}} \left\{ \left( n_w + I_t^* (x_j, y_j) \right) \sqrt{\frac{2\pi}{e}} \left( 2^2 R_j - 1 \right) l d_{ij}^{m+3} u_{ij,T+1} \right\}.
$$

(28)

**Proof:** See Appendix A.

Proposition 1 captures the relationship between the illumination distribution of service area and the minimum transmit power of each UAV. From Proposition 1 we can see that, given the illuminance requirement $\eta_r$ and data rate constraint $R_j$ of each user $j$, the minimum transmit power of each UAV depends on the illuminance at $(x_j, y_j)$. Based on Proposition 1 we can compute the optimal illuminance that allows the transmit power of each UAV $i$ to achieve the lower bound.

**B. User Association and Power Efficiency with Fixed UAV Deployment**

The original optimal problem in (17) is combinatorial due to the binary variable $u_{ij,T+1}$. Due to the complexity of solving combinatorial problems, the computation is essentially impossible even for a modest-sized wireless network [32]. To overcome this, we temporarily adopt the fractional user association relaxation, where association variable $u_{ij,T+1}$ can take on any real value in $[0, 1]$. We will later show that the optimal solution to $u_{ij,T+1}$ must be either 1 or 0 even though the feasible region of $u_{ij,T+1}$ is relaxed to be continuous. Therefore, the relaxation fortunately does not cause any loss of optimality to the final solution to the original problem in (17). Given the optimal UAV deployment in (18), the relaxed problem (17) can be formulated as:

$$
\min_{P_{i,T+1}, u_{T+1}} \sum_{i \in \mathcal{D}} P_{i,T+1},
$$

(29)

s.t. $P_{i,T+1} \geq l a_j d_{ij}^{m+3} u_{ij,T+1}, \forall i \in \mathcal{D}, \forall j \in \mathcal{U},$

(29a)

$$
\sum_{i \in \mathcal{D}} u_{ij,T+1} = 1, \forall j \in \mathcal{U},
$$

(29b)

$$
u_{ij,T+1} \geq 0, \forall i \in \mathcal{D}, \forall j \in \mathcal{U}.\tag{29c}
$$

To obtain the optimal solution of problem (29), we can state the following theorem:

**Theorem 1:** For problem (29), the optimal user association $u_{ij,T+1}$ and transmit power $P_{i,T+1}$
can be respectively expressed as:

$$u_{ij,T+1}^* = \begin{cases} 1, & \text{if } i = \arg\min_{k \in D} \beta_{kj} d_{kj}^{m+3} \\ 0, & \text{otherwise} \end{cases} \quad (30)$$

and

$$P_{i,T+1}^* = \max_{j \in U} l a_j d_{ij}^{m+3} u_{ij,T+1}^*, \quad (31)$$

where $\beta_{ij}$ is the Lagrange multiplier associated with constraint (29a), and $\sum_{j \in U} \beta_{ij} \leq 1$. If there are multiple minimal points in $\arg\min_{k \in D} \beta_{kj} d_{kj}^{m+3}$, we will choose any one of them.

**Proof:** See Appendix B. \qed

From Theorem 1, we can see that, even though the feasible region of $u_{ij,T+1}$ is relaxed to be continuous, the optimal solution to problem (29) can be effectively solved via its dual problem, while satisfying the discrete constraints $u_{ij,T+1} \in \{0, 1\}, \forall i \in D, \forall j \in U$.

The values of $\beta_{ij}$ can be determined by the gradient method $[31]$. The updating procedure is given by:

$$\beta_{ij} = [\beta_{ij} + \delta (l a_j d_{ij}^{m+3} u_{ij,T+1} - P_{i,T+1})]^+, \quad (32)$$

where $\delta > 0$ is a dynamically chosen step-size sequence. By iteratively optimizing primal variable and dual variable, the optimal user association and transmit power are obtained. Notice that the optimal $u_{ij,T+1}$ is either 0 or 1 according to (30).

The proposed algorithm used to solve problem in (6) is summarized in Algorithm 2, which includes predicting illumination distribution in service area and iteratively optimizing UAV deployment, user association, and energy efficiency.

**C. Complexity and Overhead of the Proposed Algorithms**

The complexity of the proposed algorithm lies in training an integrated GRU and CNN predictive model and iteratively updating UAV location $(x_i, y_i)$ and user association $u_{T+1}$. The complexity for training an integrated GRU and CNN predictive model is detailed in the following lemmas:

**Lemma 1:** For the CNN-based illumination distribution encoder and DeCNN-based decoder, the complexity are both $O(\sum_{i=1}^{L} \lambda_i^2 s^2 K_e^l K_{e-1}^l).$

**Proof:** See Appendix C. \qed

From Lemma 1, we can see that the complexity of CNN encoder and DeCNN decoder depends on the size and number of feature maps in each layer.
Algorithm 2 Proposed Algorithm for Deploying UAVs.

1: **Input**: A time series dataset of illumination distribution of service area $I$, the set of locations of users in $U$, height $H$ of UAVs, and the set of data rate requirement $R_i$ of users $U$.

2: **Initialize**: The user association $u_i$, Dual variables $\lambda_j, \beta$.

3: **Input** $I$ into Algorithm 1 to predict the illumination distribution $I_{T+1}$.

4: repeat
5: for $i = 1 \rightarrow D$ do
6: repeat
7: Update transmit power $P_{i,T+1}$ and UAV location $(x_i, y_i)$ according to (24)-(25).
8: Update dual variables $\lambda_j, j \in U_i$ based on (26).
9: until the objective function (19) converges.
10: end for
11: Update the user association $u_{ij,T+1}$ and power efficiency $P_{i,T+1}$ according to (30) and (31).
12: Update dual variable $\delta$ based on (32).
13: until the objective value (29) converges.
14: Calculate the transmit power $P_{i,T+1}$ based on the position of UAV $i$ being $(x_i, y_i, H)$ and the illumination distribution being $I_{T+1}(x,y)$.
15: **Output**: $P = \sum_{i \in D} P_{i,T+1}$.

**Lemma 2**: For the GRU-based illumination distribution predictor, the complexity is given as $O(TD_h(N + D_h))$.

**Proof**: See Appendix D.

From Lemma 2, we can see that the complexity of GRU predictor depends on the length of input time series and the size of weight matrices. Since the integrated GRU and CNN predictive model is trained by the BS which has enough computational ability for training, the overhead of training the predictive model can be ignored. Meanwhile, once the training process is completed, the trained integrated GRU and CNN model can used to predict the illumination distribution in a long term period.

Next, we investigate the complexity of solving the optimization problem, which lies in solving two subproblems: UAV deployment problem and user association problem. For the UAV deployment problem, the overhead of calculating $(x_i, y_i)$ of each UAV $i$ from (24) is $O(L_i |U_i|)$, where $L_i$ is the average number of iterations of UAV $i$ until (19) convergence and $|U_i|$ is the number of users covered by UAV $i$. Note that the UAV deployment optimization algorithm is distributed according to (18). For the user association problem, the overhead of obtaining $u_{ij,T+1}$ from (30) is $O(LDU)$, where $L$ is the average iteration number until (29) converges. Note that the user association optimization algorithm is centralized according to (29). The two subproblems are
TABLE I: System Parameters

| Parameters | Value | Parameters | Value |
|-----------|-------|------------|-------|
| $\Phi$    | $120^\circ$ | $\Psi_c$ | $120^\circ$ |
| $\rho$    | 1 cm$^2$ | $n_e$ | 1.5 |
| $H$       | 100 m | $\xi$ | 1 |
| $n_w$     | $1 \times 10^{-10}$ | $\eta_r$ | $5 \times 10^{-5}$ |
| $S$       | 3 | $\lambda_0$ | 256 |
| $D_h$     | 64 | $D_q$ | 16 |
| $\gamma$ | 0.01 | $\delta$ | 0.01 |
| $L$       | 4 | $N$ | 256 |
| $\epsilon$ | $10^4$ | $\epsilon$ | $10^{-4}$ |

Solved by dual method. According to [31], a sharp estimate of $L$ and each $L_i$ can be expressed as $\mathcal{O}\left(\frac{1}{\sqrt{\epsilon}}\right)$, where $\epsilon$ is the accuracy of the dual method. As a result, the complexity to solve the UAV deployment problem can be further simplified as $\mathcal{O}\left(\frac{|U|}{\sqrt{\epsilon}}\right)$, which can run independently on each UAV due to the linear algorithm complexity. The complexity to solve the user association problem can be simplified as $\mathcal{O}\left(\frac{DU}{\sqrt{\epsilon}}\right)$, which can run on a terrestrial BS.

V. SIMULATION RESULTS AND ANALYSIS

For our simulations, a $300 \times 300$ m square area is considered with $U = 40$ uniformly distributed users and $D = 4$ UAVs. The downlink rate requirement $R_j$ of each user $j$ is generated randomly and uniformly over $[0.5, 1.5]$ Mbps. Other parameters are listed in Table I. The time series illumination data used to train integrated GRU and CNN predictive model is a dataset of average radiances composite nighttime remote sensing images, obtained from the Earth observations group (EOG) at NOAA/NCEI [33].

Fig. 3 shows how the predicted illumination distributions change as the input time series change. We randomly select two areas for the predictions of illumination distribution. In Fig. 3, we can see that the prediction at the first time step is initialized to zero. Fig. 3 also shows that, as time elapses, the accuracy of illumination distribution prediction generated by the model increases. This is because the proposed model can build a relationship between the prediction and the historical illumination distribution. As the number of input historical illumination distribution increases, the proposed model can extract obtain more time-varying information about the illumination distribution.

In Fig. 4, we show how the integrated GRU and CNN model predicts the illumination distribution at next time slot. Here, we combine the representative features in each layer for
Fig. 3: Predicted illumination distribution of the target area.

effective visualization. Fig. 4(a) is an actual illumination distribution at time slot \( t \) and it is also an input of the proposed predictive model. Figs. 4(b) to 4(j) show the extracted feature maps in the CNN encoding components, which are extracted from 256×256 convolutional layer, 128×128 max-pooling layer, 128×128 convolutional layer, 64×64 max-pooling layer, 64×64 convolutional layer, 32×32 max-pooling layer, 32×32 convolutional layer, 16×16 max-pooling layer, and 16×16 flatten layer, respectively. Fig. 4(k) visualizes the predicted features of illumination distribution at time slot \( t + 1 \), \( x_{t+1} \), obtained by GRUs. Based on \( x_{t+1} \), Figs. 4(l) to 4(s) are the output maps in the DeCNN decoding components, which are reconstructed from 32×32 unpooling layer, 32×32 deconvolutional layer, 64×64 unpooling layer, 64×64 deconvolutional layer, 128×128 unpooling layer, 128×128 deconvolutional layer, 256×256 unpooling layer, and 256×256 deconvolutional layer, respectively. Fig. 4(t) shows the predicted illumination distribution at time slot \( t + 1 \) output from the integrated GRU and CNN model. From Figs. 4(b) to 4(j) we can see that the CNN encoder captures the boundary information and shading information of the illumination distribution. This is because the features that are closely related to the change of illumination distribution are amplified through forward-propagation while noisy features from background are suppressed. From Figs. 4(l) to 4(s) we can see that the coarse-to-fine structures of the illumination distribution are reconstructed after the predicted
features propagate through DeCNN decoder layers. This is due to the fact that, unpooling layers trace predicted features back to the original locations in service area and deconvolutional layers effectively reconstruct the detailed structure of illumination distribution based on the learned weights.

In Fig. 5, we show how the prediction accuracy of the illumination distribution on two test service areas changes as the size of input time series $t$ varies. In Fig. 5, for comparison, we include the results of an integrated GMM and GRU model [1] and an autoencoder in [34] trained on single time interval illumination distribution. 210 area samples are used to train the proposed model, with each area containing 78 illumination distributions in time series. We randomly choose 5% of each illumination series for validation and testing, and discard the chosen continuous segments from the training set. From Fig. 5, we can see that, as the length of input illumination series $t$ increases, the mean-square error (MSE) of the proposed model decreases, while the variation of the illumination distribution over each time slot is random. This is due

Fig. 4: Visualization of extracted features in the proposed predictive model.
to the fact that, as the input series $t$ increases, the proposed model can accumulate information on the change of illumination distribution. The average MSE of training data prediction and test data prediction are $6.01 \times 10^{-4}$ and $6.03 \times 10^{-4}$, respectively. Fig. 5 also shows that the proposed model can yield up to 46.5% and 53.6% reduction in terms of MSE compared with integrated GMM and GRU model and autoencoder model, respectively. These gains stem from the fact that, the proposed model can simultaneously extract the spatial and temporal features of historical illumination distributions so as to accurately predict future illumination distributions.

Fig. 6 shows how the transmit power used to meet the users’ data rate and illumination requirements changes as the number of users varies. In Fig. 6, we can see that the proposed algorithm can reduce transmit power by up to 51.4% compared to a conventional optimal UAV deployment without considering the illumination distribution and user association. In Fig. 6, we can also see that the optimal UAV deployment only considering the illumination and the optimal UAV deployment only considering the user association can yield up to 30.1% and 23.7% of gain in terms of total transmit power, respectively. These gains are due to the fact that the power required by the users is related to the illumination of the service area and the deployment of the associated UAV. The proposed algorithm can iteratively optimize user association and UAV deployments, which will reduce the total transmit power of all the UAVs.

Fig. 5: Prediction accuracy of the illumination distribution as a function of the size of input series.
can also see that the proposed algorithm is closer to the UAV deployment optimization using actual illumination distribution and the gap between the two schemes is less than 2.8%. This is because the proposed prediction algorithm can accurately predict the illumination distribution so as to optimize UAV deployment. Fig. 6 also shows that, as the number of users increases, the performance gain of the proposed deployment becomes less significant. This is because when enough users are considered, the users will be uniformly distributed in the square and the optimal position of the UAV will be fixed.

Fig. 7 shows how the transmit power used to meet the users’ data rate and illumination requirements changes as the height of UAVs varies. In Fig. 7, we can see that, as the height of the UAVs increases, the total transmit power of all algorithms increases since the deployment of UAVs at a high altitude increases the distance from the user to the associated UAV. In Fig. 7, we can also see that the proposed algorithm achieves up to 64.6% gain in terms of transmit power reduction compared to a conventional optimal UAV deployment without considering the illumination distribution and user association. Fig. 7 also shows that the optimal UAV deployment only considering the illumination and the optimal UAV deployment only considering the user association can yield up to 23.7% and 45.1% of reduction in terms of total transmit power, respectively. This implies that, as the height of the UAVs increases, the transmit power gain achieved by considering the user association becomes more significant than the gain achieved
Fig. 7: The required sum power of UAVs as the height of UAVs varies.

by considering the illumination. This is because, when the UAVs are deployed at a very high altitude, the proposed algorithm prefers to associate all the users with as few UAVs as possible, while other UAVs are idle. Therefore, the optimal user association obtained by the proposed algorithm will significantly limit the increase in total transmit power of all the UAVs cause by the long distance between UAVs and users.

In Fig. 8, we show an example of how the proposed algorithm can optimize the deployment of UAVs. In the example, four UAVs are deployed at a height of 100 m to serve a 300 m × 300 m square area which is divided into four 150 m × 150 m subareas. Fig. 8 shows that the optimal location of each UAV without considering the illumination distribution and the user association is the center of the users located in the given subarea. From Fig. 8, we can see that the optimal locations of UAVs obtained by the proposed algorithm are shifted to the area with strong illumination. This is due to the fact that the illumination increases the interference for VLC link and, hence, the users located in a bright area need more transmit power compared to those located in a dark area. Under the collective effect of all the users in the service area, the optimal UAV locations move towards the area with strong illumination to minimize the total transmit power. Fig. 8 also shows that UAVs can serve users located at the boundaries of other subareas, resulting in the minimum total transmit power. This is because, the minimum transmit
power of each UAV depends on the maximum requirement of its associated users, which usually occurs at the boundary of the service subarea. Once the users located near boundaries of different subareas are simultaneously satisfied by one UAV, the minimum transmit power of other UAVs reduce significantly, thus achieving a minimum total transmit power.

VI. CONCLUSION

In this paper, we have developed a novel UAV deployment framework for dynamically optimizing the locations and user association of UAVs in a VLC-enabled UAV based network. We have formulated an optimization problem that seeks to minimize the transmit power while meeting the illumination and communication requirements of each user. To solve this problem, we have developed an integrated GRU and CNN prediction algorithm, which can model the long-term historical illumination distribution and predict the future illumination distribution. We have then transformed the nonconvex original problem into convex reformulation through physical relaxation of the user association. Therefore, the optimal solution of the optimization problem is obtained by an iterative algorithm. Simulation results have shown that the proposed approach yields significant power reduction compared to conventional approaches.
A. Proof of Proposition 1

Based on (5) and (19), the minimum transmit power of UAV $i$ to satisfy the requirements of user $j$ can be given by:

$$P_{ij,T+1}^{\min} = \max \{M_j, N_j\} \cdot l d_{ij}^{m+3}, \forall j \in U_i.$$  \hspace{1cm} (33)

where $M_j = \eta_r - I_T(x_j, y_j)$ and $N_j = (n_w + I_T(x_j, y_j)) \sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)}$. Given illuminance and data rate requirements of each user $j$, to obtain the lower bound of $P_{ij,T+1}^{\min}$, we derive the first derivative with respect $I_T(x_j, y_j)$ as:

$$\frac{\partial P_{ij,T+1}^{\min}}{\partial I_T(x_j, y_j)} = \begin{cases} 
-ld_{ij}^{m+3}, M_j > N_j, \\
\sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)l d_{ij}^{m+3}}, M_j < N_j.
\end{cases}$$  \hspace{1cm} (34)

Since $-ld_{ij}^{m+3} < 0$ and $\sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1) l d_{ij}^{m+3}} > 0$, there is a unique $I_T(x_j, y_j)$ that allows the minimum transmit power to reach the lower bound. Next, we analyze the optimal illumination, $I_{T+1}^*(x_j, y_j)$, that allows $P_{ij,T+1}^{\min}$ to reach the lower bound.

If $M_j < N_j$ for $\forall I_{T+1}(x_j, y_j) \geq 0$, that is $\eta_r < n_w \sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)}$, we have $P_{ij,T+1}^{\min} = N_j l d_{ij}^{m+3}$. Since

$$\frac{\partial P_{ij,T+1}^{\min}}{\partial I_T(x_j, y_j)} = \sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)l d_{ij}^{m+3}} > 0$$

and $I_{T+1}(x_j, y_j) \geq 0$, the optimal $I_{T+1}^*(x_j, y_j)$ that allows $P_{ij,T+1}^{\min}$ to reach the lower bound will be:

$$I_{T+1}^*(x_j, y_j) = 0,$$  \hspace{1cm} (35)

and the lower bound of the minimum transmit power of UAV $i$ to satisfy its associated user $j$ will be:

$$\inf P_{ij,T+1}^{\min} = n_w \sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)l d_{ij}^{m+3}}.$$  \hspace{1cm} (36)

Otherwise, we have $\eta_r \geq n_w \sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)}$. From (34), we can see that $P_{ij,T+1}^{\min}$ achieves the minimum value when $M_j = N_j$, that is $\eta_r - I_{T+1}^*(x_j, y_j) = (n_w + I_{T+1}^*(x_j, y_j)) \sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)}$. Then, we have $\eta_r - n_w \sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)} = I_{T+1}^*(x_j, y_j) \left(\sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)} + 1\right)$. Therefore, the optimal $I_{T+1}^*(x_j, y_j)$ will be:

$$I_{T+1}^*(x_j, y_j) = \frac{\eta_r - n_w \sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)}}{1 + \sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)}} = \frac{\eta_r + n_w}{1 + \sqrt{\frac{2 \pi}{e}(2^{2R_j} - 1)}} - n_w,$$  \hspace{1cm} (37)

and the lower bound of the minimum transmit power of UAV $i$ to satisfy its associated user $j$ will be:
\[ \inf P_{ij,T+1}^{\text{min}} = (n_w + I_{T+1}^*(x_j, y_j)) \sqrt{\frac{2\pi}{e} (2^{2R_j} - 1) ld_{ij}^{m+3}}. \]  

Therefore, the optimal illumination at the location of user \( j \) is given by:

\[ I_{T+1}^*(x_j, y_j) = \begin{cases} \eta_r + n_w, & \eta_r \geq n_w \sqrt{\frac{2\pi}{e} (2^{2R_j} - 1)}, \\ 0, & \eta_r < n_w \sqrt{\frac{2\pi}{e} (2^{2R_j} - 1)}, \end{cases} \]  

Based on (36) and (38), the lower bound of the minimum transmit power of each UAV \( i \) at time \( T + 1 \) is given as:

\[ \inf P_{i,T+1}^{\text{min}} = \max_{j \in U} \left\{ (n_w + I_{T+1}^*(x_j, y_j)) \sqrt{\frac{2\pi}{e} (2^{2R_j} - 1) ld_{ij}^{m+3} u_{ij,T+1}} \right\}. \]

This completes the proof.

B. Proof of Theorem 1

The dual problem of problem (29) with relaxed constraints can be given by:

\[ \max_{\beta} D(\beta), \]  

where

\[ D(\beta) = \begin{cases} \min_{P_{i,T+1}, u_{T+1}} & \mathcal{L}(P_{i,T+1}, u_{T+1}, \beta) \\ \text{s.t.} & \sum_{i \in D} u_{ij,T+1} = 1, \quad \forall j \in U, \\ & u_{ij,T+1} \geq 0, \quad \forall i \in D, \forall j \in U, \end{cases} \]  

with

\[ \mathcal{L}(P_{i,T+1}, u_{T+1}, \beta) = \sum_{i \in D} P_{i,T+1} + \sum_{i \in D} \sum_{j \in U} \beta_{ij} (l_a d_{ij}^{m+3} u_{ij,T+1} - P_{i,T+1}) \]  

and \( \beta = \{\beta_{ij}\} \).

To minimize the objective function in (41), which is a linear combination of \( u_{ij,T+1} \), we should let the smallest association coefficient corresponding to the \( u_{ij,T+1} \) be 1 among all UAV \( i \) with given user \( j \). Therefore, the optimal \( u_{ij,T+1}^* \) is thus given as:

\[ u_{ij,T+1}^* = \begin{cases} 1, & \text{if } i = \arg \min_{k \in D} \beta_{kj} d_{kj}^{m+3} \\ 0, & \text{otherwise}. \end{cases} \]  

To obtain the optimal \( P_{i,T+1}^* \) from (42), we derive the first derivative with respect \( P_{i,T+1} \) as

\[ \frac{\partial \mathcal{L}(P_{i,T+1}, u_{T+1}, \beta)}{\partial P_{i,T+1}} = 1 - \sum_{j \in U} \beta_{ij}. \]

Note that the optimal \( P_{i,T+1}^* = +\infty \) if \( 1 - \sum_{j \in U} d_{ij} < 0 \) and dual value is \( -\infty \). To avoid this, we must have \( \sum_{j \in U} \beta_{ij} \leq 1 \). As a result, we can obtain the optimal solution \( P_{i,T+1}^* \) to problem (29) as (31). This completes the proof.
C. Proof of Lemma 1

The complexity of the CNN-based illumination distribution encoder and decoder depends on the calculations in convolutional (deconvolutional) layers, max-pooling (unpooling) layers, and a flatten layer.

For each convolutional layer, the calculations based on (8) is given as:

\[
\lambda_{i,j}^{l,m} = f\left(\sum_{k=1}^{K-l-1} h_{i,j}^{l-1,k} w_{1,1}^{l,m} + \cdots + h_{i,j+s}^{l-1,k} u_{1,1}^{l,m} + \cdots + h_{i+j+s}^{l-1,k} u_{s,s}^{l,m} + b_k^{l,m}\right),
\]

where \(h_{i,j}^{l,m}\) is the element of row \(i\) and column \(j\) in \(H_{i,j}^{l,m}\), \(h_{i,j}^{l-1,k}\) is the element of row \(i\) and column \(j\) in \(H_{i,j}^{l-1,k}\), \(w_{1,1}^{l,m}\) is the element of row 1 and column 1 in \(W_{1,1}^{l,m}\), and \(b_k^{l,m}\) is the element \(k\) of \(b_t^{l,m}\). For each \(h_{i,j}^{l,m}\), the complexity of calculation is \(O(K^{l-1}S^2)\). Note that, each convolutional layer \(l\) consists of \(K^{l}\) feature maps and each feature map \(H_{i,j}^{l,m} \in \mathbb{R}^{\lambda_t \times \lambda_t}\). Then, we have \(i = 1, \cdots, \lambda_t\), \(j = 1, \cdots, \lambda_t\) and \(m = 1, \cdots, K^{l}\). Therefore, the complexity of convolutional layer \(l\) is \(O(\lambda_t^2 K_t^l K^{l-1}S^2)\).

For each max-pooling layer \(l\), the max-pooling operation divides the input feature map \(H_t^{l-1,m}\) into \(\frac{\lambda_t^2}{S_m^2}\) square areas. In each \(S_m \times S_m\) square area, the max-pooling operation records the most robust feature, whose complexity is \(O(S_m^2)\). Hence, the complexity of max-pooling layer \(l\) is \(O(\frac{\lambda_t^2}{S_m^2} S_m^2) = O(\lambda_t^2)\).

For the flatten layer, the flatten operation rewrites input \(H_t^{l,m}\) to \(x_t \in \mathbb{R}^N\), where \(H_t^{l,m} \in \mathbb{R}^{\lambda_t \times \lambda_t}\), \(m = 1, \cdots, K^L\), and \(N = \lambda_t^L K^L\). Therefore, the complexity of the flatten layer is \(O(\lambda_t^2 K^L)\).

As a result, the complexity of the CNN-based illumination distribution encoder is:

\[
O\left(\sum_{l=1}^{L} \lambda_t^2 K_t^L K^{l-1}S^2 + \sum_{l=1}^{L} \lambda_t^2 + \lambda_t^2 K^L\right) = O\left(\sum_{l=1}^{L} \lambda_t^2 K_t^L K^{l-1}S^2\right).
\]

Due to the symmetry between the CNN-based illumination distribution encoder and the DeCNN-based decoder, the complexity of the decoder is also \(O(\sum_{l=1}^{L} \lambda_t^2 K_d^l K^{l-1}S^2)\). This completes the proof.

D. Proof of Lemma 2

Given representation \(x_t\) for illumination distribution at time slot \(t\), the GRU-based predictor extract the temporal characteristics based on (9)-(11). For each input \(x_t\), the complexity of reset gate operation in (9) is \(O(N D_h + D_h^2)\), which depends on the size of \(W_r \in \mathbb{R}^{N \times D_h}\) and \(U_r \in \mathbb{R}^{N \times D_h}\).
Similarly, the complexity of calculating candidate hidden state $\tilde{h}_t^j$ in (10) and the complexity of calculating update gate $z_t^j$ in (11) are both $O(ND_h + D_h^2)$. The proposed GRU model iteratively updates the hidden states based on (9)-(11). Therefore, the complexity of extracting temporal feature for all the input illumination distributions $X = (x_1, x_2, \cdots, x_t, \cdots, x_T)$ is given as $O(T \times 3(ND_h + D_h^2))$. Then, the complexity for the GRU model to output the illumination distribution prediction based on (13) is $O(ND_h)$, which depends on the size of $W_o \in \mathbb{R}^{N \times D_h}$.

Finally, the total complexity of the GRU-based predictor is given as:

$$O(T(3(ND_h + D_h^2)) + ND_h) = O(TD_h(N + D_h)).$$ (48)

This completes the proof.

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