Signatures of Cosmic Strings on the Microwave Background

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Abstract

We review recent progress on testing the hypothesis of the existence of cosmic string perturbations in microwave background maps. Using an analytical model for the string network we show that the predicted amplitude and spectrum of MBR fluctuations are consistent with the COBE data for a reasonable value of the single free parameter of the string model (the mass per unit length of the string \(\mu\)). To distinguish the predictions of cosmic strings from those of Gaussian models it is necessary to apply specific statistical tests which are sensitive to non-random phases of the MBR temperature maps. We discuss two such statistical tests: First, the probability distribution and the moments of fluctuations \(\delta T\) and second the corresponding statistical quantities for the gradient of fluctuations along a fixed direction. We show that the second statistic can detect cosmic string specific signatures on MBR temperature maps with resolution of a few arcminutes or smaller.

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1 Introduction

The quest for an understanding of the origin of Microwave Background Radiation (MBR) anisotropies has become a central open issue since their recent discovery. There are three main candidate types of primordial fluctuations that may have caused these anisotropies. First, Gaussian perturbations produced by quantum fluctuations of a scalar field during inflation. Second, seed perturbations which are naturally generated by topological defects produced during a phase transition in the early universe. Third, other types of non-Gaussian perturbations produced for example by special inflationary potentials or late time phase transitions. Here we review recent progress on testing for seed-like perturbations. In particular we focus on the case of cosmic strings (Kibble 1976; Vilenkin 1985; Brandenberger 1992) and address the following questions:

- Are the cosmic string predictions consistent with the COBE data?
- What statistical tests can identify cosmic string signatures on MBR temperature maps?

In order to address these questions we use a simple analytic model (Vachaspati 1992) for the effects of a distribution of cosmic strings which was first applied to the study of MBR fluctuations by Perivolaropoulos (1993a) (see also Hara & Miyoshi 1993). Some of the main results obtained using this model include the predicted amplitude and spectrum of MBR fluctuations on COBE angular scales and the predicted probability distribution and moments of the temperature fluctuations $\frac{dT}{T}$ and their gradient. It turns out that the statistical study of the gradient of temperature fluctuations offers a powerful probe of the signatures of cosmic strings on scales of a few arcminutes or smaller (Moessner et al. 1994). Other interesting statistical tests of non-Gaussianity have also been proposed recently (see e.g. Luo 1993; Graham et al. 1993; Coulson et al. 1993; Perivolaropoulos 1993c).

The main assumptions on which the analytical model is based are the following: First, long strings, present after the time of recombination are assumed to dominate perturbations (this is also suggested in the string evolution simulations of Bouchet et al. (1988)). Second, strings are approximately straight on horizon scales (Bennett & Bouchet 1990, Allen & Shellard 1990). Third, string positions and velocities are uncorrelated on Hubble space- and time- scales (by causality).

The method used is based on a discretization of the photon path from the time of recombination to today into a set of “Hubble” time-steps (Fig. 1).
The first step begins at recombination when the horizon scale is of the order $t_{\text{rec}}$ and each subsequent time step occurs when the horizon has doubled in size. Each individual string present in a given Hubble slice produces a Kaiser-Stebbins temperature kick (Kaiser & Stebbins 1984; Stebbins 1988) equal to $\pm 4\pi G \mu \beta$ which is effective only within a horizon distance from the string (Traschen et al. 1986; Veeraraghavan & Stebbins 1990, Magueijo 1992). Here $\beta = \hat{k} \cdot (\vec{v}_s \gamma_s \times \hat{s})$ where $v_s$ is the magnitude of the average string velocity, $\hat{s}$ is a unit vector along the string and $\hat{k}$ is the unit photon wave-vector. It is then straightforward to add up the contributions of all strings (Perivolaropoulos 1993a) using the assumption that there is no correlation among strings in different Hubble time-steps and Hubble volumes.

The model offers a fairly powerful analytical tool to compute several observable predictions of seed-based models. For example it is straightforward to obtain analytical expressions for the temperature correlation function $C(\alpha)$ (Perivolaropoulos 1993a), the probability distribution of temperature fluctuations (Perivolaropoulos 1993b) or their gradient (Moessner et al. 1994), properties of peculiar velocities (Vachaspati 1992; Perivolaropoulos & Vachaspati 1993) etc.

2 Amplitude, Spectrum

Numerical simulations (Bennett & Bouchet 1990, Allen & Shellard 1990) have shown clear evidence that the evolving cosmic string network rapidly approaches a scale invariant configuration called the scaling solution. According to the scaling solution there are a fixed number of long strings per horizon volume per Hubble time moving with relativistic velocities. There is also a distribution of loops smaller than the Hubble radius whose statistical properties do not depend on time if all lengths are scaled to the horizon. This scale invariant string configuration produces density fluctuations and induced MBR anisotropies which are also scale invariant and may therefore be approximately described by a Harrison-Zeldovich ($n = 1$) spectrum. Thus, to a first approximation the shape of the spectrum is the same as that predicted in the simplest inflationary Universe models.

A more quantitative study of predicted temperature fluctuations may be obtained using the analytic model described in the previous section. In this way, it is straightforward to obtain an expression for the angular autocorrelation function $C(\alpha)$ of temperature fluctuations, valid on angular scales
larger than about a degree. The result is (Perivolaropoulos 1993a)

\[ C(\alpha) = \frac{\xi^2}{3} \cos \alpha \log_2 \left( \frac{t_0}{t_{\text{rec}}} \right) - 3 \log_2 (1 + \frac{\alpha}{\alpha_{\text{rec}}}) \]  

(1)

where \( \xi \equiv 4\pi G \mu v_s \gamma_s \sqrt{M} \), \( v_s \) is the magnitude of the string velocity and \( M \) is the number of strings per Hubble volume. Fig. 2 shows a plot of \( C(\alpha) \) smoothed on angular scales of 10\(^{{\circ}}\) (as in Smoot et. al. (1992)) with \( v_s \gamma_s = 0.15 \) and \( M = 10 \) as suggested by string network numerical simulations (Allen & Shellard (1990)).

The units for the curve that corresponds to strings are \( (\mu_6 \mu K)^2 \) where \( \mu_6 \equiv \frac{\mu}{10^{-6}} \) is the single free parameter of the cosmic string model. A value of \( \mu_6 \approx 1 \) is required in order to obtain reasonable agreement of cosmic string predictions with large scale structure (Stebbins et. al. 1987; Perivolaropoulos et. al. 1990), galaxy observations (Turok & Brandenberger 1986) and the temperature autocorrelation function obtained by the DMR of COBE (Smoot et. al. (1992)). In addition, such a value of \( \mu_6 \) is consistent with constraints coming from particle physics if cosmic strings were produced during a grand unification phase transition.

The best fit to the COBE data can be obtained for \( \mu_6 \approx 1.7 \pm 0.7 \) (Perivolaropoulos 1993a) while the effective power spectrum index \( (n = 1.1 \pm 0.4) \) is consistent with a scale invariant Zeldovich spectrum. As mentioned above, such a spectrum could have been expected since the perturbations were constructed in a scale invariant way by the string scaling solution. These results are in good agreement with other recent studies based on numerical simulations of string evolution (Bennett et. al. (1992)).

3 Statistical Properties of Fluctuations

The predictions for the correlation function can test the cosmic string model by comparing its predictions of amplitude and spectrum of perturbations with the COBE data but can not distinguish it from other theories which are also consistent with COBE. This distinction can be made by identifying the non-Gaussian properties of string induced perturbations. The simplest test of non-Gaussianity is provided by calculating the probability distribution and the moments of \( \delta T \).

It is straightforward to use the analytical model described above to find expressions for the probability distributions and the moments of string induced perturbations (Perivolaropoulos 1993b). The large number of strings...
per horizon volume implies (by the central limit theorem) that the resulting perturbations are approximately Gaussian. This is shown in Fig. 3a where we plot the probability distribution \( P(\delta T) \) for strings vs a variable proportional to \( \delta T \). The maximum relative difference from the corresponding Gaussian probability distribution is about 1% even though the existence of long non-Gaussian tails is clear (Fig. 3b). The predicted value of the skewness is 0 due to the symmetry between positive and negative perturbations in the mechanism that creates them. The relative difference of the kurtosis from the Gaussian varies inversely proportional to the number of strings \( M \) per Hubble volume and is less than 1%. These results show that the predicted difference from the Gaussian for this statistic is negligible. It may be shown by using the central limit theorem that the effects of smoothing (not taken into account in the above analysis) always tend to increase further the Gaussian character of fluctuations.

For comparison, in the case of textures this test is more useful due to the small number of knots unwinding per Hubble volume. There are only 0.04 textures collapsing per Hubble volume per Hubble time while the corresponding number for strings is 10. It may be shown (Perivolaropoulos 1993b) that the maximum relative difference of the probability distribution from the Gaussian for textures is about 10% (without smoothing) while the relative deviation of the kurtosis from the Gaussian is about 30%. These deviations however become negligible when smoothing on COBE scales is taken into account.

Fortunately, there is a much more powerful test that can identify the non-Gaussian features due to strings. It involves the study of the statistics of temperature differences (gradients) rather than temperature fluctuations. The model described above can be applied to obtain the temperature differences between neighboring pixels in an MBR experiment. Since each long string produces a temperature discontinuity, the pattern of temperature differences will consist of a superposition of localized spikes. Such a pattern approaches much more slowly the Gaussian for a large number of spikes than a superposition of step-functions which would be the temperature fluctuation pattern. Fig. 4 shows the predicted probability distribution for an experiment with resolution 18” (e.g the VLA experiment (Fomalont et. al. 1993)). Superimposed are the corresponding Gaussian distribution with the same variance and the result of a Monte-Carlo simulation confirming the analytical result. The value of the kurtosis predicted for such an experiment is a factor of approximately 3 larger than the Gaussian value \( k_g = 3 \).
Since each experiment only makes a finite number of measurements, it can not measure the true value of moments. The probability distribution of the measured moments about their true values will be a Gaussian due to the central limit theorem. The variance $\sigma^2_{k_4}$ of the kurtosis $k_4$ depends on the number of measurements $n$ as $\sigma^2_{k_4} = \frac{1}{n}(k_8 - k^2_4)$ where $k_8$ and $k_4$ are the normalized 8th and 4th moments respectively (Moessner et. al. (1994)). Fig. 5 shows the predicted probability distribution of the kurtosis for the VLA experiment in the cases of underlying Gaussian and stringy perturbations. For a beam size of 18”, due to the large number of pixels ($n \simeq 1000$), the overlap of the Gaussian and stringy distributions is small enough to allow a clear distinction between the models. This is not so for a beam size of 80”. In this case, even though the predicted kurtosis is about 50% larger than the Gaussian, the overlap of the two distributions is too large to allow definite conclusions.

In conclusion, there are two main points supported by the work reviewed here: First, the cosmic string model is consistent with the COBE data and second there are well defined statistical tests that can probe non-Gaussian features induced by strings on scales of arcminutes or smaller.

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5 Figure Captions

Figure 1 : The discretization of the photon path from the time of recombination $t_{rec}$ to today. The dotted lines show two photon paths propagating from the last scattering surface to the observer at $O$.

Figure 2 : The predicted angular autocorrelation function $C(\alpha)$, in units of $\mu^2_0 (\mu K)^2$.

Figure 3: Probability distribution $P(\Delta T)$ as predicted by cosmic strings in the context of the analytic model (3a). The difference from the corresponding Gaussian distribution is shown in Fig. 3b. The variable $\Delta T$ is proportional but not equal to the predicted temperature fluctuations.
**Figure 4:** The analytically obtained probability distribution (tilted squares) for the gradient of $\delta T$ for an experiment with resolution 18”, superimposed with the corresponding Gaussian distribution (squares). The analytical result was also verified by a Monte-Carlo simulation (crosses).

**Figure 5:** The predicted probability distribution of the kurtosis for the VLA experiment (short dashed line) superimposed with the corresponding distribution for Gaussian underlying perturbations (continuous line). A similar superposition is also shown for 80” resolution in the same experiment (dotted vs long dashed lines).

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