A Possibility of Determining the WIMP Mass by Using the Angular Recoil–Energy Spectra from Directional Direct Dark Matter Detection Experiments

CHUNG-LIN SHAN

Preparatory Office of the Supporting Center for Taiwan Independent Researchers
P.O.BOX 21 National Yang Ming Chiao Tung University, Hsinchu City 30099, Taiwan, R.O.C.

E-mail: clshan@tir.tw

Abstract

In this article, as an extension of our study on the angular distribution of the recoil flux of WIMP–scattered target nuclei, we demonstrate a possibility of determining the mass of incident halo WIMPs by using or combining “ridge–crater” structures of the angular recoil–energy spectra with different target nuclei observed in directional direct Dark Matter detection experiments. Our simulation results show that, for a WIMP mass of only a few tens GeV, the stereoscopic angular recoil–flux distributions of both of light and heavy target nuclei would have a (longitudinally) “ridge–like” structure. However, once the WIMP mass is as heavy as a few hundreds GeV, the angular recoil–flux distributions of heavy target nuclei would in contrast show a (latitudinally) “crater–like” structure.
1 Introduction

Direct Dark Matter (DM) detection experiments aiming to observe scattering signals of Weakly Interacting Massive Particles (WIMPs) off target nuclei would still be the most reliable experimental strategy for identifying Galactic DM particles and determining their properties [1, 2, 3, 4]. While most direct DM detection experiments measure only recoil energies deposited in underground detectors, the “directional” direct detection experiments could provide additional 3-dimensional information (recoil tracks and/or head–tail senses) of (elastic) WIMP–nucleus scattering events for discriminating WIMP signals from isotropic backgrounds and/or some incoming–direction–known astronomical events [5, 6, 7].

In Refs. [8, 9], we have developed the double Monte Carlo scattering–by–scattering simulation package for the 3-dimensional elastic WIMP–nucleus scattering process and studied the angular distribution of the recoil flux of WIMP–scattered target nuclei in different celestial coordinate systems. Then, in Ref. [10], we have shown the angular recoil–flux distributions of different target nuclei in several narrow recoil energy windows in the Equatorial and (geocentric) Galactic coordinate systems and observed the WIMP–mass and target–nucleus dependence of such angular recoil–energy spectra. In this article, we discuss therefore a possibility of determining the mass of incident halo WIMPs by using or combining the angular recoil–energy spectra observed (hopefully in the future) in directional direct DM detection experiments (with different target nuclei).

In the next section, we will consider two WIMP masses and present the corresponding angular recoil–energy spectra of different target nuclei whose masses could basically cover the mass range of almost all nuclei used in direct DM detection experiments. Then we conclude in Sec. 3.

2 Angular recoil–energy spectra

This work is based on our double Monte Carlo (MC) simulations for 3-D elastic WIMP–nucleus scattering described in detail in Refs. [8, 9]: we MC generate first a 3-D velocity of an incident WIMP in the Galactic coordinate system according to the theoretical isotropic Maxwellian velocity distribution, transform it to the laboratory coordinate system, and, in the laboratory (more precisely, the incoming–WIMP) coordinate system, we generate an equivalent recoil angle of a scattered target nucleus and validate this candidate scattering event according to the criterion [8]:

$$f_{N_R}(v_{\chi,\text{Lab}}, \theta_{N_R,\chi_{\text{in}}}) = \frac{v_{\chi,\text{Lab}}}{v_{\chi,\text{cutoff}}} \left[ \sigma_0^{\text{SI}} F_{\text{SI}}^2(Q) + \sigma_0^{\text{SD}} F_{\text{SD}}^2(Q) \right] \sin(2\theta_{N_R,\chi_{\text{in}}}).$$

(1)

Here $v_{\chi,\text{Lab}}$ and $\theta_{N_R,\chi_{\text{in}}}$ are the transformed WIMP incident velocity in the laboratory coordinate system and the generated equivalent recoil angle of the scattered target nucleus, $v_{\chi,\text{cutoff}}$ is a cut–off velocity of incident halo WIMPs in the Equatorial/laboratory coordinate systems, which is set as 800 km/s in our simulations, $\sigma_0^{(\text{SI,SD})}$ are the spin–independent/dependent (SI/SD) total cross sections ignoring the nuclear form factor suppressions, and $F_{(\text{SI,SD})}(Q)$ indicate the elastic nuclear form factors corresponding to the SI/SD WIMP interactions, respectively. Note that the recoil energy of the scattered target nucleus appearing in Eq. [1] is now estimated by the equivalent recoil angle $\theta_{N_R,\chi_{\text{in}}}$ [8]:

$$Q(\theta_{N_R,\chi_{\text{in}}}) = \left( \frac{2m_{\text{N}}^2}{m_N} \right) v_{\chi,\text{Lab}}^2 \sin^2(\theta_{N_R,\chi_{\text{in}}}).$$

(2)

1The elevation of the nuclear recoil direction in the incoming–WIMP coordinate system, namely, the complementary angle of the recoil angle [8].
where \( m_{rN} \equiv m_{\chi}m_{N}/(m_{\chi} + m_{N}) \) is the reduced mass of the WIMP mass \( m_{\chi} \) and that of the target nucleus \( m_{N} \).

As in our earlier works [9, 10], in our simulations presented in this article, the SI (scalar) WIMP–nucleon cross section has been fixed as \( \sigma_{\chi p}^{SI} = 10^{-9} \) pb, while the SD (axial–vector) WIMP–proton/neutron couplings have been tuned as \( a_{p} = 0.01 \) and \( a_{n} = 0.7a_{p} = 0.007 \), respectively. Considering the exponential–like decreased scattering event rate with increasing the recoil energy, the simulation energy range has been commonly limited between 0 and 20 keV for all considered target nuclei and sliced into four 5-keV energy windows [10]. 5,000 experiments with 500 accepted WIMP scattering events on average (Poisson–distributed) in one entire year in one experiment for one target nucleus have been simulated.

### 2.1 A light WIMP mass of 20 GeV

In this subsection, we consider at first the case of a light WIMP mass of \( m_{\chi} = 20 \) GeV.

In Figs. 1, we show the stereoscopic angular distributions of the WIMP–induced nuclear recoil flux (in unit of their all–sky average values) observed in the geocentric Galactic coordinate system in four 5-keV recoil energy windows, respectively. Four nuclei used frequently in (directional) direct detection experiments: \(^{19}\)F, \(^{40}\)Ar, \(^{73}\)Ge, and \(^{129}\)Xe have been considered as our targets. Note that 500 accepted WIMP scattering events on average in “each (5-keV)” energy window/plot have been simulated and binned into 12 × 12 bins for the azimuthal angle and the elevation, respectively. Note also that the distribution centers have been shifted to 90°W, so that the theoretical main direction of incident WIMPs (the opposite direction of the Solar Galactic movement toward the Cygnus constellation) [8]: 0.60°S, 98.78°W, is approximately at the center of our plots.

With these stereoscopic demonstrations, one can clearly observe a (longitudinally) “ridge–like” structure of the angular recoil–flux distributions in the energy range between 5 and 20 keV: the higher the recoil energy (window), the more concentrated the angular recoil–flux distribution and (relatively) the higher the crest–line/peak. Meanwhile, Figs. 1 show that, the heavier the target nuclei, the more obviously and rapidly the ridge–like structure of the angular recoil–flux distributions as well as the concentration and the increase of the angular recoil–energy spectrum would be.

### 2.2 A heavy WIMP mass of 200 GeV

As a comparison, in Figs. 2 the mass of incident WIMPs has been raised to \( m_{\chi} = 200 \) GeV.

Now the stereoscopic angular recoil–energy spectra become to a (latitudinally) “crater–like” structure: the lower the recoil energy, the more concentrated the angular recoil–flux distribution and (relatively) the higher the crest–line/peak. Meanwhile, Figs. 1 show that, the heavier the target nuclei, the more obviously and rapidly the ridge–like structure of the angular recoil–flux distributions as well as the concentration and the increase of the angular recoil–energy spectrum would be.

Interested readers can click each row in Figs. 1 and 2 to open the corresponding webpage of animated demonstrations (for more considered WIMP masses) [11].

---

---
Figure 1: The stereoscopic angular recoil–flux distributions induced by 20-GeV WIMPs (in unit of their all–sky average values) observed in the geocentric Galactic coordinate system in the energy range between 0 and 20 keV [10]. Four nuclei: $^{19}$F, $^{40}$Ar, $^{73}$Ge, and $^{129}$Xe have been considered. Note that the distribution centers have been shifted to 90°W, so that the theoretical main direction of incident WIMPs [8]: 0.60°S, 98.78°W, is approximately at the center of our plots. See the text for further details.

sharper). Additionally, the heavier the target nuclei, the higher the upper limit of the recoil energy range, in which one could observe this “reverse” inner–outer–asymmetry.

3 Summary

In this article, as an extension of our study on the angular distribution of the recoil flux of WIMP-scattered target nuclei, we have demonstrated the possibility of determining the mass
of incident halo WIMPs by using or combining the “ridge–crater” structures of the angular recoil–energy spectra with different target nuclei observed in directional direct DM detection experiments.

Our simulation results show that, once the WIMP mass is as light as only a few tens GeV, the stereoscopic angular recoil–flux distributions of both of light and heavy target nuclei would have the (longitudinally) “ridge–like” structure: the higher the recoil energy, the more concentrated the angular recoil–flux distribution and (relatively) the higher the crest–line/peak. In addition, the heavier the target nuclei we use, the more obviously and rapidly the ridge–like structure of the angular recoil–flux distributions as well as the concentration and the increase of the angular recoil–energy spectrum would be.

In contrast, once the WIMP mass is as heavy as a few hundreds GeV, the angular recoil–flux distributions of heavy target nuclei would show the (latitudinally) “crater–like” structure:...
the lower the recoil energy, the wider the central plain of the angular recoil–flux distribution, whereas the distributions of light target nuclei could have such a crater–like structure only in (pretty) low energy range.

These WIMP–mass and target–nucleus dependent characteristics of the angular recoil–energy spectrum indicate the possibility of (analytic and perhaps model–independent) reconstruction(s) of the WIMP mass (and perhaps some other WIMP properties) by comparing and/or combining the angular recoil–energy spectra offered by directional direct detection experiments with one or several target nuclei.

Acknowledgments

This work was strongly encouraged by the “Researchers working on e.g. exploring the Universe or landing on the Moon should not stay here but go abroad.” speech.

References

[1] G. Jungman, M. Kamionkowski and K. Griest, “Supersymmetric Dark Matter”, Phys. Rept. 267, 195–373 (1996), arXiv:hep-ph/9506380.
[2] M. Schumann, “Direct Detection of WIMP Dark Matter: Concepts and Status”, J. Phys. G46, 103003 (2019), arXiv:1903.03026 [astro-ph.CO].
[3] L. Baudis and S. Profumo, contribution to “The Review of Particle Physics 2020”, Prog. Theor. Exp. Phys. 2020, 083C01 (2020), 27. Dark Matter.
[4] J. Cooley, “Dark Matter Direct Detection of Classical WIMPs”, SciPost Phys. Lect. Notes 55 (2022), arXiv:2110.02359 [hep-ph].
[5] S. Ahlen et al., “The Case for a Directional Dark Matter Detector and the Status of Current Experimental Efforts”, Int. J. Mod. Phys. A25, 1–51 (2010), arXiv:0911.0323 [astro-ph.CO].
[6] F. Mayet et al., “A Review of the Discovery Reach of Directional Dark Matter Detection”, Phys. Rept. 627, 1–49 (2016), arXiv:1602.03781 [astro-ph.CO].
[7] S. E. Vahsen, C. A. J. O’Hare and D. Loomba, “Directional Recoil Detection”, Ann. Rev. Nucl. Part. Sci. 71, 189–224 (2021), arXiv:2102.04596 [physics.ins-det].
[8] C.-L. Shan, “Monte Carlo Scattering–by–Scattering Simulation of 3-Dimensional Elastic WIMP–Nucleus Scattering Events”, arXiv:2103.06485 [hep-ph] (2021), in publication.
[9] C.-L. Shan, “Simulations of the Angular Recoil–Energy Distribution of WIMP–Scattered Target Nuclei for Directional Dark Matter Detection Experiments”, arXiv:2103.06486 [hep-ph] (2021), in publication.
[10] C.-L. Shan, “Incident Velocity–Recoil Angle Distribution and Angular Recoil–Energy Spectrum of 3-Dimensional WIMP–Nucleus Scattering Events”, arXiv:2203.05805 [hep-ph] (2022), in publication.
[11] C.-L. Shan, online interactive demonstrations of 3-dimensional elastic WIMP–nucleus scattering for (directional) direct Dark Matter detection experiments and phenomenology, http://www.tir.tw/phys/hep/dm/amidas-2d/ (2021).