Mathematical Modelling of Silica Scaling Deposition in Geothermal Wells

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Abstract. Silica scaling is widely encountered in geothermal wells in which produce two-phase geothermal fluid. Silica scaling could be formed due to chemical reacting by mixing a geothermal fluid with other geothermal fluid in different compositions, or also can be caused by changes in fluid properties due to changes pressure and temperature. One of method to overcome silica scaling which is occurred around geothermal well is by workover operation. Modelling of silica deposition in porous medium has been modeled in previously. However, the growth of silica scaling deposition in geothermal wells has never been modeled. Modelling of silica deposition through geothermal is important aspects to determine depth of silica scaling growth and best placing for workover device to clean silica scaling. This study is attempted to develop mathematical models for predicting silica scaling through geothermal wells. The mathematical model is developed by integrating the solubility-temperature correlation and two-phase pressure drop coupled wellbore fluid temperature correlation in a production well. The coupled model of two-phase pressure drop and wellbore fluid temperature correlation which is used in this paper is Hasan-Kabir correlation. This modelling is divided into two categories: single and two phase fluid model. Modelling of silica deposition is constrained in temperature distribution effect through geothermal wells by solubility correlation for silica. The results of this study are visualizing the growth of silica scaling thickness through geothermal wells in each segment of depth. Sensitivity analysis is applied in several parameters, such as: bottom-hole pressure, temperature, and silica concentrations. Temperature is most impact factor for silica scaling through geothermal wellbore and depth of flash point. In flash point, silica scaling thickness has reached maximum because reducing of mole in liquid portion.

Keywords: mathematical modelling, silica scaling, silica solubility, well-bore modelling

1. Introduction
Silica scaling is widely encountered in geothermal wells which produce two-phase geothermal fluid. Silica scaling could be occurred in production wells, reinjection wells, turbines and surface pipeline. This phenomenon is caused by transport of chemical in hot water from geothermal reservoir which allows deposition of chemical during exploitation process. In exploitation processes, heat loss and pressure drop through geothermal wells are occurred and can causes changes in enthalpy, the vapor fraction, and drop temperature. These phenomena will cause changes solubility of chemical content dissolved in the geothermal fluid. Changing solubility will cause precipitation of solid content through
the geothermal wells. Deposition of silica will be deposited on the inside of the pipe radially [1]. This precipitation will change the size of the pipe diameter becomes smaller as a function of time and will also affect the ability of brine flow.

The primary objective of this study is to develop mathematical model to determine silica scaling deposition through geothermal wells. This mathematical model is developed by integrating the solubility-temperature correlation and two phase pressure drop coupled wellbore fluid temperature correlation in geothermal production well.

Solubility-temperature correlation is used because temperature is most impact factor compared to other factor such as enthalpy, salinity and pressure. Two-phase pressure drop coupled heat loss calculation which is used for model development is Hasan-Kabir correlation [2]. This modelling is divided into two main categories: single and two phase fluid model.

Silica deposition can be split into two sections: thermodynamics and kinetics of silica deposition. The thermodynamics of silica solutions allows the prediction of what must eventually happen when equilibrium is reached. The kinetics try to explain how fast equilibrium is achieved.

2. Fluid Flow and Heat Transfer Through Geothermal Wells

Mechanistic Model of Multiphase Flow
This mechanistic model consisting to gas-liquid fraction and pressure gradient. In predicting of pressure in the geothermal well as function of depth, the mechanistic model which were developed by Hasan-Kabir are used.

2.1.1 Gas Volume Fraction. Gas-volume fraction is calculated from the known flow rates in gas or liquid phase. Normally, flow regime of fluid flow is modelled because gas-volume fraction depends on the flow pattern; that is, bubbly, slug, churn, and annular. However, all of flow regimes show that gas phase moves faster than the liquid phase because of its buoyancy and its tendency to flow close to the channel center. Therefore, the generalized form of gas-volume fraction relationship with measured superficial velocity variable of gas and liquid phases can be written as [2]:

\[ f_g = \frac{v_{sg}}{C_0v_m \pm v_{\infty}}, \quad (1) \]

where \(v_{sg}\) is superficial velocity of gas, \(v_{\infty}\) is average rise of velocity and \(C_0v_m\) is the channel-center mixture velocity. In this equation, the values of flow parameter \(C_0\) and rise velocity \(v_{\infty}\) dependent on the type of flow and flow pattern. Parameters for flow type and flow pattern are presented in Table 1.

| Flow Pattern | Flow Parameter \(C_0\) | Rise Velocity \(v_{\infty}\) |
|--------------|------------------------|--------------------------|
| Bubbly       | 1.2                    | \(v_{\infty b}\)         |
| Slug         | 1.2                    | \(v_{\infty}\)          |
| Churn        | 1.15                   | \(v_{\infty}\)          |
| Dispersed Bubbly | 1.15                | \(v_{\infty b}\)         |
| Annular      | 1                      | 0                        |
In addition, we can define gas mass fraction, \( x \), (called quality in steam/water flow) and can be expressed by the following equation:

\[
x = \frac{h - h_L}{h_g - h_L},
\]

(2)

where \( h \) is enthalpy of fluid, \( h_L \) is enthalpy of saturated liquid and \( h_G \) is enthalpy of saturated vapor. Moreover, superficial velocity of liquid and gas can be calculated by the following equation:

\[
v_{sg} = \frac{m_{fluid}x}{\rho_gA}; \quad v_{SL} = \frac{m_{fluid}(1-x)}{\rho_LA}
\]

(3)

**Flow Pattern (Transition Criteria)**

An alternative and more flexible approach to empirical flow-pattern maps is to examine each pattern transition and develop criteria. Later, will be explained how to determine transition criteria such as bubbly-slug flow transition, slug churn, flow transition and transition to annular flow by using several equations which were developed by some researcher.

\( v_m \) is mixture velocity of two phase flow and can be obtained by the following equation:

\[
v_m = v_{sg} + v_{SL}
\]

(4)

\( v_\infty \) is terminal rise-velocity of single bubble which can be calculated by using Harmathy equation:

\[
v_\infty = 1.53 \left[ g \left( \rho_L - \rho_g \right) \frac{\sigma}{\rho_L \rho_g} \right]^{\frac{1}{2}}
\]

(5)

where surface tension (\( \sigma \)) is calculated by the following equation: [3]

\[
\sigma = B \left( \frac{T_c - T}{T_c} \right)^{1.256} \left[ 1 + b \left( \frac{T_c - T}{T_c} \right) \right]
\]

(6)

Where \( B = 235.8 \times 10^{-3} \), \( b = -0.625 \), \( T_c = 647.15 \)

And \( v_{oT} \) is terminal rise velocity of small bubble of Taylor bubble which strongly dependent on the pipe diameter, proposed by Nicklin *et al*

\[
v_{oT} = 0.35 \sqrt{g d (\rho_L - \rho_g) / \rho_L}
\]

(7)

where \( v_{oT} \) is greater than \( v_\infty \) the flow would be transition from bubbly to slug flow.

Flow regime would be bubbly flow if \( v_{sg} < v_t \). \( v_t \) value can be obtained by the following equation:

\[
v_t = 0.429 v_{SL} + 0.357 v_\infty
\]

(8)

At high gas flow rates, transition from churn or slug flow to annular flow takes place. The transition between churn (or slug) and annular flow can be understood by visualizing a long vertical tube with porous section at the middle. If the liquid is fed through the porous section at the middle and gas from the bottom, then at low gas flow rates, the liquid will flow down the tube as a film. However, at high gas flow rates, the shear force of the gas on the liquid will pull it upward, just as in the case of annular

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flow. The minimum of gas velocity need for reversal in the direction of the liquid flow can be expressed by the following equation

\[ v_{sg} > 3.1 \left( g \left( \rho_l - \rho_g \right) \sigma \rho_g^2 \right)^{\frac{1}{8}} \]  

(9)

**Bubbly Flow**

In bubbly flow, gas bubbles tend to flow mostly through the center of the channel. At the channel center, the mixture velocity is significantly higher than the cross-sectional average velocity, which means that gas bubbles are moving faster than the average velocity of the mixture.

The simple relation of gas-volume fraction can be expressed by the following equation:

\[ f_g = \frac{v_{sg}}{C_0v_m + \frac{v_d}{f_g}} \]  

(10)

where \( v_d \) is the expression of drift flux which were proposed by Findlay and Wallis. The expression is semi theoretical in terms of the volume fraction and terminal rise velocity.

\[ v_d = v_\infty f_g (1 - f_g)^n \]  

(11)

For gas/liquid flow in vertical pipes, exponent n has a zero value. Therefore, the expression of drift flux could be simplified to:

\[ v_d = v_\infty f_g \]  

(12)

And the expression of simple relation of gas-volume fraction is expressed by the following equation:

\[ f_g = \frac{v_{sg}}{C_0v_m + v_\infty} \]  

(13)

**Slug Flow**

Fluid configuration in slug flow is quite different from that in bubbly flow. During slug flow, there are two distinct zones; one is dominated by the large Taylor bubble and the other consisting of small bubble dispersed in the liquid slug.

The average rise velocity is expressed in terms of the Taylor bubble-rise velocity, small bubble rise velocity, and bubbly slug-transition velocity as

\[ \bar{v}_\infty = v_\infty \left( 1 - e^{-\frac{v_l}{v_{sg}}} \right) + v_\infty T e^{-\frac{v_l}{v_{sg}}} \]  

(14)

And \( f_g \) can be calculated by the following equation:

\[ f_g = \frac{v_{sg}}{C_0v_m + v_\infty} \]  

(15)

**Annular Flow**

In annular flow, gas flow through the central of the pipeline, while liquid flows along the wall as a film. Therefore, the system can be viewed as single phase flow of gas through a pipe, in which its wall is formed by the liquid film. For such an ideal annular flow, it is not necessary to estimate holdup, as we are modeling a true single phase flow, except that diameter must be reduced somewhat. Unfortunately, annular flow is rarely ideal. Usually, a certain fraction of the liquid is carried as droplets in the gas
stream. Often, the amount of entrained liquid is a substantial portion of the total liquid flow. In annular flow, average rise velocity should be neglected.

The simple relation of gas-volume fraction can be expressed by the following equation:

\[ f_g = \frac{v_g}{c_{gm}} \]  

(16)

2.1.2 Pressure Gradient. Total pressure gradient during fluid flow is the sum of static, friction, and kinetic gradient should be written as

\[ -\frac{dp}{dz} = f_m v_m^2 \rho_m + \frac{g \sin \alpha}{g_c} \rho_m + \frac{\rho_m v_m dv_m}{g_c} \]  

(17)

\( \rho_m \) is in-situ density of the two-phase mixture which can be expressed by:

\[ \rho_m = f_L \rho_L + (1 - f_L) \rho_g \]  

(18)

\( \mu_m \) is mixture viscosity of two-phase mixture which can be expressed by [4]:

\[ \mu_m = x \mu_g + (1 - x) \mu_L \]  

(19)

Friction factor is usually expressed as a function of Reynolds number (Re). Friction factor can be estimated reliably from the Blassius equation by the following condition: smooth pipes such as plastic pipes and tubulars coated with PVC lining.

\[ f_m = 0.32 Re^{-0.25} \]  

(20)

where:

\[ Re = \frac{\rho_m v_m D}{\mu_m} \]  

(21)

Energy Balance for Wellbore Fluid

Temperature difference between the wellbore fluid and the surrounding formation results in energy exchange. An energy balance for the fluid may be performed following any standard text on thermodynamics. Wellbore heat transfer to surrounding formation gives impact for fluid properties and, in turn, the dynamics of fluid flow. Consequently, pressure drop coupled heat loss correlation require simultaneous solutions for both process. A general energy balance for the wellbore fluid which have developed by Hasan and Kabir are

\[ \frac{dH}{dz} + \frac{g \sin \alpha}{g_c} + \frac{v dv}{g_c} = \frac{Q}{w} \]  

(22)

where \( Q \) is rate of heat transfer between the fluid and the surrounding formation per unit depth of the wellbore; the negative sign applies to fluid production and the positive sign applies to fluid injection. Heat loss to the formation can be expressed by the following equation [2]:

\[ Q \equiv -L_R w c_p (T_f - T_{ei}) \]  

(23)

where \( L_R \) is the relaxation length parameter defined as
\[ L_R = \frac{2\pi}{c_p w} \left[ \frac{r_{to} U_{to} k_e}{k_e + (r_{to} U_{to} T_D)} \right] \tag{24} \]

### 2.3 Wellbore Fluid Temperature

By substituting the expression for heat loss to the formation, \( Q \), into the energy balance equation, fluid temperature to well depth can be obtained by the following equation:

\[ \frac{dT_f}{dz} = \pm (T_f - T_{ei}) L_R - \frac{g \sin \alpha}{c_p \rho_G} \frac{d}{dz} \left( c_j \frac{dp}{dz} \right) \tag{25} \]

where the + sign applies to production, and the – sign applies to injection. We assume that the undisturbed formation temperature, \( T_{ei} \), varies linearly with depth and can be expressed by the following equation:

\[ T_{ei} = T_{eibh} - (L - z) g_G \sin \alpha \tag{26} \]

where \( g_G \) is the geothermal gradient in terms of vertical depth and \( T_{eibh} \) is the static earth temperature at the bottom hole. Note that \( z \) is positive in the downward direction.

#### 2.3.1 Produced Fluid Temperature (Single Phase Liquid)

Liquids, being essentially incompressible and allow a number of simplifications of wellbore fluid temperature equation. In addition, fluid velocity is changing with depth become negligible and liquid density variation with pressure is usually very small. For single phase liquid, wellbore fluid temperature equation become a simple first-order linear differential equation, which can be solved by the integrating factor method. The solution is

\[ T_f = T_{eibh} - g_G \sin \alpha \left[ (L - z) - \frac{1 - e^{(z-1)l_R}}{l_R} \right] \tag{27} \]

Equation above shows that the fluid temperature decreases exponentially from the bottom hole (\( z = L \)) to the wellhead.

#### 2.3.2 Produced Fluid Temperature (Two-Phase Flow)

For two-phase fluid, the equation is also solved by the integrating factor methods and resulting first order linear differential equation. The solution is

\[ T_f = T_{ei} + \frac{1 - e^{(z-1)l_R}}{l_R} \left( g_G \sin \alpha + \frac{\phi - g_G \sin \alpha}{c_p \rho_G} \right) \tag{28} \]

where \( \phi \) is empirical expression which depend on many variables, such as flow rate, gas/liquid ratio, and wellbore pressure. Hasan and Kabir used the empirical expression for \( \phi \) which were developed by Sagar et al, which is valid for flow rates less than 5 lbm/sec:

\[ \phi = -0.002978 + 1.006 \times 10^{-6} P_{wh} + 1.906 \times 10^{-4} w - 1.047 \times 10^{-6} GLR + 3.229 \times 10^{-5} API + 0.004009 \gamma g - 0.3551 g_G \tag{29} \]

For flow rates higher than 5 lbm/sec, Sagar et all. set \( \phi = 0 \). An alternative to this empirical approach is the use of Joule-Thompson coefficient for two-phase flow, which is derived from first principles and is given by
\[ C_j \equiv \frac{1}{c_p} \left[ \frac{x}{\rho_g} \left( -\frac{T}{z} \frac{\partial z}{\partial T} \right)_p + \frac{1-x}{\rho_L} \right] \]  

(30)

where \( x \) is the mass fraction of the vapor phase.

2.4 Calculation Procedure

To obtain pressure, temperature, dryness and flow regime at each section of pipeline, there are several steps have to be undertaken based on the modelling of fluid flow and heat transfer through geothermal wells. The flow chart of calculation is presented in Figure 1:

![Figure 1: Pressure Drop and Heat Transfer Modelling Flow Chart](image-url)
3.1 Silica Solubility
Silica is one of the chemical components found in many earth faults. At the regional manifestations, silica is commonly found in soil and water output surface of the earth. Silica can be formed in a variety of forms called polymorphs such as quartz, chalcedony, cristobalite, and amorphous [1]. All of these various polymorphs are known to occur in the nature. The surrounding rocks of most geothermal reservoirs contain quartz, and this will dissolve in hot water. In study of hydrothermal system, quartz and amorphous silica are the two polymorphs most commonly encountered.

Temperature is most impact factor that affects mineral solubility of geothermal fluid and cause scaling deposition not only in geothermal wells, but also in surface pipeline, turbines and geothermal wellbore. Other factor such as enthalpy, salinity, and pressure also give impact in changing of solubility of geothermal fluid, but not dominant.

In a geothermal system, there are 3 properties of the effect of temperature on solubility [1]:
1. Solubility increases with increasing temperature
2. Solubility increases with decreasing temperature
3. Solubility increases with increasing temperature but increased solubility only up to its maximum value and then decreases with increasing temperature

Silica has the third properties because it has a maximum value. There are two aspects which influence to silica scaling deposition: thermodynamic aspect and kinetic aspect

3.2 Thermodynamic Aspect
In geothermal reservoir (T > 230°C), the equilibrium of quartz between the solid and dissolved species can be expressed by the following reaction [4,5]:

\[ SiO_2(s) + 2H_2O(l) \rightleftharpoons Si(OH)_4(aq) \]

or

\[ SiO_2(s) + 2H_2O(l) \rightleftharpoons H_4SiO_4(aq) \]

The reaction is dependent to temperature, pH and salinity. However, temperature is most dependent parameter for silica scaling deposition. In this study, only temperature dependent is used to obtain silica scaling deposition.

Normally, hot water is in equilibrium with quartz in subsurface. But in the surface, form of silica will be precipitated is amorphous silica. Amorphous silica has no crystalline structure and is more soluble than quartz. The solubility of amorphous silica can be obtained by the following equation at saturated vapor condition [1,5]:

\[ \log C = -\frac{731}{T} + 4.52 \]  \hspace{1cm} (31)

where \( C \) = silica concentration in mg/L and \( T \) = absolute temperature (K)

The solubility equations for quartz and amorphous silica have been calculated at the saturated vapor pressure of pure water. As the concentration of other dissolved species is increased (e.g. in NaCl solutions) the solubility of both quartz and amorphous silica is decreased.

The important parameter which is related to silica scaling deposition is silica saturation index (SSI). SSI is ratio between silica concentration in solution between silica solution at same condition. SSI parameter could be used to estimate possibility of silica scaling occurring, with this criteria [6]:
1. If SSI > 1, fluid is in supersaturated condition and possible to deposit silica
2. If SSI = 1, fluid is in saturated condition
3. If SSI < 1, fluid is in under saturated condition and impossible to deposit silica
3.3 Kinetic Aspect

Kinetic aspect in silica deposition is related to mechanism and silica deposition reaction rate. Silica deposition kinetic are influenced by several factor: Supersaturation level, pH, temperature, flow rate, aeration, and other ion in solution. Kinetic aspect is hardly estimated. Two possible mechanism to deposit silica are [6]:
1. Initial deposition of colloid is followed by in phase deposition of colloid
2. Direct deposition at solid surface

3.4 Mass Silica Deposition

Normally, heat loss which is occurred through geothermal well causes decreasing in temperature. Changing temperature can cause changing of silica solubility. Mass rate of silica deposition can be obtained by calculating the difference $\Delta C$ silica concentrations due to changes in temperature.

3.5 Changes of Inner Diameter of Pipelines

Deposition of silica scaling through the geothermal pipeline is assumed radially, then the radius of the well will be reduced at certain interval time. Silica scaling thickness $h_s$ can be obtained by the following equation: [1]

$$h_s = r_w - \sqrt{r_w^2 - \frac{m_{dep}}{\rho_{silica}\pi L}}$$

where $m_{dep}$ is a mass deposition rate of silica on the inner well, $t$ is interval time, $L$ is length of the pipeline and $\rho_{silica}$ is density of silica deposition. From precipitated silica thickness radially on the pipe wall, it can be obtained new inside diameter (ID) of pipeline at any given time

$$ID_{new} = ID_{old} - 2h_s$$

3.6 Calculation Procedure

To thickness and change of diameter of pipeline at each section of pipeline, there are several steps have to be undertaken based on the modelling of silica dissolution and precipitation. The step by step which also could say as the flow chart is presented in Figure 2.
4. Data
The proposed modelling of silica precipitation, fluid flow as well as heat transfer through geothermal wells has to be validated with valid data from published paper [7]. The data is presented in Table 2 and Table 3.

![Figure 2: Silica Solubility Modelling Flow Chart](image)

**Table 2: Completion of Well X**

| Description          | OD   | ID    | mVD  |
|----------------------|------|-------|------|
| Casing 13 3/8”       | 12.415”| 0-500 | mVD  |
| Liner 9 5/8”        | 8.618” | 500-2000 | mVD  |
Table 3: Production Data of Well X

|                                |          |          |
|--------------------------------|----------|----------|
| Static Pressure @ 2000 mVD     | 150      | Bara     |
| Static Temperature @ 2000 mVD  | 300      | Celsius  |
| Bottom Hole Pressure           | 140      | Bara     |
| Reservoir Fluid Enthalpy       | 1338     | kJ/kg    |
| Mass Flow Rate                 | 100      | kg/s     |
| Relaxation Length Parameter    | $9.51 \times 10^{-4}$ | m$^{-1}$ |

5. Result and Analysis

5.1 Pressure Drop Modelling

From the pressure drop by using Hasan-Kabir Modelling shows that wellhead pressure about 32 bar absolute with dryness at the wellhead is 11%, Flow regime occurred at well X is presented in Table 4. From the modelling result, the single-phase liquid (compressed liquid) flow at bottom-hole, flashing occurs at a depth of approximately 1030 meters from the surface, and annular flow condition occurred at near the surface.

Table 4: Flow Regime of Well X

| Depth      | Flow Regime        |
|------------|--------------------|
| 2000 – 1020| Single Phase Flow  |
| 1010 – 870 | Bubbly Flow        |
| 860 – 320  | Slug Flow          |
| 310 – 0    | Annular Flow       |

There are four flow regimes occurred in the well which are resulted by pressure drop modelling. Single phase flow occurs at depth 1040-2000 meter; bubbly flow occurs at depth of 870 - 1010 meter; slug flow occurs at depth of 320 - 860 meter; and annular flow occurs at depth 0 – 310 meter.

The profile of pressure, temperature and dryness versus depth is presented in Figure 3, 4 and 5, respectively:
5.2 Silica Solubility Modelling
Silica deposition is started at the flash point because there is fluid which release from the liquid and formed vapor so that liquid concentration increases. Silica solubility will decrease with decreasing temperature; give impact to deposition of silica through geothermal pipelines. Silica solubility versus temperature is presented in Figure 6, while silica mass rate deposition versus depth is presented Figure
7. Deposition of mass and thickness is lowest at a depth of 1030 meters which then gradually increase to the surface.

Thickness of silica deposition is presented in Figure 8. Thickness of silica deposition is gradually increasing with decreasing of depth. But in depth of 500 meters, thickness of silica deposition is suddenly decreasing due to increasing diameter of casing. Then, thickness of silica deposition is gradually increasing to the surface.

*Figure 6: Silica Mass Rate Deposition vs Depth  Figure 7: Thickness of Silica Deposition vs Depth*

5.3 Sensitivity Analysis of Well X
Sensitivity analysis was performed on well X by varying bottom-hole pressure, bottom-hole temperature and silica concentration in solution to determine depth of flash point and also thickness of silica from the flash point into the surface. Depth of flash point is very important to estimate silica scaling deposition. In this case, silica scaling starts to precipitate in geothermal well when solubility of silica at flash point is equal to silica concentration in solution.

Sensitivity analysis of varying bottom-hole pressure with constant mass flow rate (100 kg/s) is presented at Table 5 and the result of calculation of thickness of silica with varying of bottom-hole is presented in Figure 9. From Table 5 could be concluded that increasing of bottom-hole pressure will cause depth of flash point become shallower. Moreover, decreasing of depth of flash point will cause silica scaling deposition depth will be decreasing (become shallower) too.
Table 5: Pressure variation to depth of flash point

| Pressure (bara) | Depth of flash point (m) |
|----------------|-------------------------|
| 130            | 1220                    |
| 140            | 1010                    |
| 150            | 780                     |

Sensitivity analysis of varying bottom-hole temperature with constant bottom-hole pressure (140 bara) is presented at Table 6. From Table 6 could be concluded that decreasing of bottom-hole temperature will cause depth of flash point become shallower. Moreover, decreasing of depth of flash point will cause silica scaling deposition depth will be decreasing (become shallower) too. Sensitivity analysis with varying of bottom hole temperature are undertaken to predict silica scaling thickness. Thickness of silica scaling is presented Figure 10.

Figure 9: Sensitivity Analysis of Silica Thickness (Bottom Hole Pressure)
Table 6: Bottom-hole temperature variation to depth of flash point

| Bottom hole temperature (°C) | Depth of flash point (m) |
|-----------------------------|-------------------------|
| 275                         | 630                     |
| 300                         | 1010                    |
| 325                         | 1620                    |

Moreover, we also undertake sensitivity analysis where silica concentration in solution is not equal to solubility of silica at flash point. We analyze in several parameters such as silica scaling deposited before reach flash point depth or after reach flash point depth. Depth of silica deposition with varying of silica concentration in solution is presented at Table 7. Thickness of silica at varying of silica concentration is presented at Figure 11.

Table 7: Silica concentration in solution variation to depth of deposition

| Silica Concentration in solution (mg/kg) | Depth of deposition point (m) |
|-----------------------------------------|------------------------------|
| 1600                                    | 490                          |
| 1650                                    | 810                          |
| 1700                                    | 1170                         |
6. Conclusion
A mathematical model has been successfully developed by integrating concept of pressure drop and heat transfer which were developed by Hasan-Kabir with solubility-temperature correlation for silica. This mathematical model has been also theoretically implemented in well X to estimate depth of flash point, flow regime, amount of silica mass deposition, thickness of silica deposition after a certain time interval. In addition, there are several conclusions were reached during the process of implementing the mathematical model and the sensitivity analysis by the following:
1. Decreasing of geothermal well temperature will cause decreasing of silica solubility in solution so that precipitation of silica scaling through the geothermal wells increase.
2. Increasing bottom hole pressure will cause decreasing of depth of flash point so that depth of silica deposition become shallower.
3. Mass flow rate has no effect on depth of flash point.

7. Recommendations
There are several recommendations for further studies:
1. It is necessary to test the validity of the mathematical model that has been developed on real geothermal field data where there are silica scaling problems.
2. The mathematical model needs to be developed with variation of bottom hole pressure and mass flow rate which is matched with well production history.
3. Prediction of silica scaling mass and thickness will be additional information for workover program to clean silica scaling in geothermal well.
4. Downhole sampling has to be undertaken to collect accurate fluid composition and chemistry and also determine silica concentration in geothermal fluid.
5. Real thickness of silica scaling is necessary to determine by using caliper log.
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