Quantum phase transitions in rotating nuclei

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We extend the classical Landau theory for rotating nuclei and show that the backbending in $^{162}\text{Yb}$, that comes about as a result of the two-quasiparticle alignment, is identified with the second order phase transition. We found that the backbending in $^{156}\text{Dy}$, caused by the instability of $\gamma$-vibrations in the rotating frame, corresponds to the first order phase transition.

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Quantum phase transitions (QPTs), that occur at zero temperature as a function of some nonthermal control parameter, are studied extensively in diverse areas of physics including, for example, low-dimensional systems in condensed matter physics [1], atomic nuclei and molecules [2]. In nuclear structure, QPTs associated with transitions between different shapes or geometric configurations of a chain of nuclei in the ground state are being studied intensively during the past years within the interacting boson model (cf [2]). Thanks to novel experimental detectors, a new frontier of discrete-line $\gamma$-spectroscopy at very high spins has been opened in the rare-earth nuclei. These nuclei can accommodate the highest values of the angular momentum, providing one with various nuclear structure phenomena. Evidently, from analysis of the rotational states one expects to obtain further insight into the nature of QPTs in finite quantum systems.

One of the well known phenomena at high spins is a backbending. A sudden increase of a nuclear kinematical moment of inertia $\mathcal{I}_1 = J/\Omega$ of the lowest (yrast) level sequence as a function of a rotational frequency $\Omega$ (see experimental results for $^{156}\text{Dy}$ and $^{162}\text{Yb}$ in Fig.1) is a paradigm of structural changes in a nucleus under rotation. While one observes a similar picture for the level sequence as a function of a rotational frequency $\Omega$ (see experimental results for $^{156}\text{Dy}$) from perspective of the Landau theory of phase transitions within a microscopic approach [3] based on the cranked Nilsson model plus random phase approximation (CRPA). We will show that the interplay between the alignment (single-particle degrees of freedom) and collective quantum fluctuations determines the type of the shape-phase transition at the backbending.

The phase transition is usually detected by means of an order parameter as a function of a control parameter. In rotating nuclei one can suggest a few order parameters like deformation parameters of a nuclear effective potential, $\beta$ and $\gamma$, that characterize the geometrical configuration (cf [2]), as a function of the rotational frequency, i.e., the control parameter. To analyze the experimental data of above nuclei, we use a cranked Hamiltonian

$$ H_\Omega = H - \sum_{\tau=N,P} \lambda_{\tau} \hat{N}_\tau - \Omega \hat{J}_x + H_{\text{int}}. \quad (1) $$

The term $H = H_{\text{Nil}} + H_{\text{add}}$ contains the Nilsson Hamiltonian $H_{\text{Nil}}$ and the additional term that restores the local Galilean invariance of the Nilsson potential in the rotating frame. The Nilsson potential naturally incorporates the quadrupole deformation parameters ($\beta$ and $\gamma$) of a nuclear shape. The interaction includes separable monopole pairing, double stretched quadrupole-quadrupole (QQ) and monopole-monopole terms. The details about the model Hamiltonian $H_{\text{Nil}}$ can be found in [4,5]. In our approach mean field parameters are determined from the energy-minimization procedure (see [4]). The consistency between the mean field and the residual interactions of the Hamiltonian $H_{\text{Nil}}$ was achieved by varying the strength constants of the pairing and QQ interactions in the CRPA. It results in the separation collective excitations from those that are related to the symmetries broken by the mean field. Among them are the conservation of particle numbers and space symmetries (see details in [4,5]).

We found that the triaxiality of the mean field sets in at the rotational frequency $\hbar \omega_\tau = 0.250, 0.301$ MeV for $^{162}\text{Yb}, ^{156}\text{Dy}$, respectively, which triggers a backbending in the considered nuclei (see Fig.1 in [4]). To elucidate the different character of the shape transition from axially symmetric to the triaxial shape and its relation to a phase transition, we consider potential landscape sections in the vicinity of the shape transition. Since we
analyze a shape transition from the axially symmetric shape \((\gamma = 0)\) to the triaxial one \((\gamma \neq 0)\), we choose only the deformation parameter \(\gamma\) as the order parameter that reflects the broken axial symmetry. Such a choice is well justified, since the deformation parameter \(\beta\) preserves its value before and after the shape transition in both nuclei: \(\beta_t \approx 0.2\) for \(^{162}\text{Yb}\) and \(\beta_t \approx 0.31\) for \(^{156}\text{Dy}\). Thus, we can consider a mean field value of the cranking Hamiltonian, \(E_\Omega(\gamma; \beta_t) \equiv \langle H_{\Omega}\rangle\), for different values of \(\Omega\) (our state variable) and \(\gamma\) (order parameter) at fixed value of \(\beta_t\).

For \(^{156}\text{Dy}\) we observe the emergence of the order parameter \(\gamma\) above the critical value \(h\Omega_c = 0.301\) MeV of the control parameter \(\Omega\) (see a top panel in Fig.2). Below and above the transition point there is a unique phase whose properties are continuously connected to one of the coexistent phases at the transition point. The order parameter changes discontinuously as the nucleus passes through the critical point from axially symmetric shape to the triaxial one. The polynomial fit of the potential landscape section at \(h\Omega_c = 0.301\) MeV yields the following expression

\[
F(\Omega;\gamma) = F_0(\Omega) + F_2(\Omega) \gamma^2 - F_3(\Omega) \gamma^3 + F_4(\Omega) \gamma^4, \tag{2}
\]

where the coefficients \(F_0(\Omega) = 0.3169\) MeV, \(\gamma\) in degrees and \(F_2(\Omega) = 0.12239, F_3(\Omega) = 0.009199, F_4(\Omega) = 1.7 \times 10^{-4}\) are defined in corresponding units. We can transform this polynomial to the form

\[
\bar{F} = \frac{F(\Omega;\gamma) - F_0(\Omega)}{F_0} \approx \alpha \frac{\eta^2}{2} - \frac{\eta^3}{3} + \frac{\eta^4}{4}, \tag{3}
\]

where \(\bar{F}_0 = (3F_3)^4/(4F_4)^3, \alpha = 8F_2F_4/(9F_3^2), \eta = 4F_4/(3F_3)\gamma\). The expression \(\bar{F}\) represents the generic form of the anharmonic model of the structural first order phase transitions in condensed matter physics. The condition \(\partial \bar{F}/\partial \eta = 0\) determines the following solutions for the order parameter \(\eta : \eta_1 = 0, \eta_{2,3} = (1 \pm \sqrt{1 - 4\alpha})/2\). If \(\alpha > 1/4\), the functional \(\bar{F}\) has a single minimum at \(\eta = 0\). Depending on values of \(\alpha\), defined in the interval \(0 < \alpha < 1/4\), the functional \(\bar{F}\) manifests the transition from one stable minimum at zero order parameter via one minimum+ metastable state to the other stable minimum with the nonzero order parameter. In particular, at the universal value of \(\alpha = 2/9\) the functional \(\bar{F}\) has two minimum values with \(\bar{F} = 0\) at \(\eta = 0 \rightarrow \gamma \approx 0^0\) and \(\eta = 2/3 \rightarrow \gamma \approx 27^0\) and a maximum at \(\eta = 1/3 \rightarrow \gamma \approx 13.5^0\). The correspondence between the actual value \(\gamma \approx 20^0\) and the one obtained from the generic model is quite good.

In general, the alignment of angular momenta of a nucleon pair occupying a high-\(j\) intruder orbital near the Fermi surface is considered as a main driving force that leads to the backbending. Although two-quasiparticle states align their angular momenta along the axis \(x\) (collective rotation), the axial symmetry persists till the transition point (see also discussion in [4]). The RPA analysis of \(\gamma\)-vibrational (lowest) excitations of the positive signature in the vicinity of the shape transition demonstrates a collective nature of these excitations. The mode blocks a transition to the triaxial shape. However, at the transition point, this mode is anomalously low in the rotating frame. It appears that soft positive signature \(\gamma\)-vibrations (fluctuations of the order parameter \(\gamma\)) coupled to the other modes are responsible for the shape-phase transition of the first order. A drastic change of the mean field configuration leads to large fluctuations of

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**FIG. 1:** Rotational behavior of the experimental, kinematical \(\Im^{(1)} = I/\Omega\) and dynamical \(\Im^{(2)} \approx 4/\Delta E_\gamma\) moments of inertia. Here, \(h\Omega = E_\gamma/2, E_\gamma\) is the \(\gamma\)-transition energy between two neighboring states that differ on two units of the angular momentum \(I\) and \(\Delta E_\gamma\) is the difference between two consecutive \(\gamma\)-transitions. The experimental data denoted by black squares are taken from [3]. The experimental rotational frequency at the transition point is \(h\Omega_c \approx 0.27, 0.32\) MeV for \(^{162}\text{Yb}\) and \(^{156}\text{Dy}\), respectively. The results of calculations for \(\Im^{(1)}\) are connected by a solid line.
the dynamical moment of inertia at the transition point, since $\Sigma^{(2)} = -d^2E_\Omega/d^2\Omega$, which is reproduced successfully in the CRPA (see details in [4]). It seems reasonable to say that the backbending in $^{156}Dy$ possesses typical features of the first order phase transition.

In the case of $^{162}Yb$ the energy $E_\Omega(\gamma; \beta_t)$ and the order parameter (Fig.2) are smooth functions in the vicinity of the transition point $\Omega_c$. This implies that two phases, $\gamma = 0$ and $\gamma \neq 0$, on either side of the transition point should coincide. Therefore, for $\Omega$ near the transition point $\Omega_c$ we can expand our functional $F(\Omega, \gamma) = E_\Omega(\gamma; \beta_t) - E_{\text{min}}$ in the form

$$F(\Omega; \gamma) = F_1(\Omega)\gamma + F_2(\Omega)\gamma^2 + F_3(\Omega)\gamma^3 + F_4(\Omega)\gamma^4 + \ldots$$

(4)

The energy surfaces are symmetric with regard of the sign of $\gamma$ and this also supports the idea that the effective energy $F$ can be expressed as an analytic function of the order parameter $\gamma$. With the aid of the conditions of the phase equilibrium, $\partial F/\partial \gamma = 0$ and $\partial^2 F/\partial \gamma^2 \geq 0$, one can show that $F_1(\Omega_c) = F_2(\Omega_c) = 0$. In virtue of these equations and of the fact that all phases at the transition point should coincide, we obtain from $\partial F/\partial \gamma = 0$ that $F_3(\Omega_c) = 0$. Assuming that $F_3 = 0$ for all $\Omega$, the minimum condition $\partial F/\partial \gamma = 0$ yields the following solution for the order parameter

$$\gamma_1 = 0, \quad \gamma_2,3 = -\frac{F_2(\Omega) F_4(\Omega)}{2 F_1(\Omega)} = \begin{cases} \neq 0 \text{ for } \Omega \neq \Omega_c \\ = 0 \text{ for } \Omega = \Omega_c \end{cases}$$

(5)

Since at the transition point $F_2(\Omega_c) = 0$, one can propose the following definition of the function $F_2(\Omega)$:

$$F_2(\Omega) \approx \frac{dF(\Omega; \gamma)}{d\Omega} (\Omega - \Omega_c)$$

(6)

Thus, we have $\gamma \sim (\Omega - \Omega_c)^\nu$ and the critical exponent $\nu = 1/2$, in accord with the classical Landau theory, where the temperature is replaced by the rotational frequency. Indeed, the numerical results (see Fig.2) are in an agreement with Eqs.(5),(6): $\gamma = 0$ for $h\Omega < h\Omega_c$, while $\gamma \neq 0$ for $h\Omega > h\Omega_c$. Thus, the backbending in $^{162}Yb$ can be classified as the phase transition of the second order. The smooth behavior of the function $F$ at the transition point implies a small amplitude of fluctuations of the dynamical moment of inertia. This result is nicely reproduced within the CRPA approach [4]. The RPA analysis of the lowest $\gamma$-vibrational mode in $^{162}Yb$ indicates on the breakdown of the quadrupole phonon. At the vicinity of the transition point one proton and one neutron two-quasiparticle components dominate ($\sim 95\%$) in the phonon structure and the backbending is caused by the alignment of the neutron two-quasiparticle configuration.

The identification of wobbling (negative signature) excitations near the yrast line provides sure evidence of the onset of the triaxiality. We found (see [2]) that a shape-phase transition produces relatively high-lying wobbling vibrational states in $^{156}Dy$. In contrast, a soft shape-phase transition from the axially deformed to nonaxial shapes produces the low-lying wobbling excitations in $^{162}Yb$. We have also established the relation between the sign of the $\gamma$-deformation and selection rules for the quadrupole electric transitions from the wobbling to the yrast states (see [3]). From our calculations it follows that at low angular momenta ($h\Omega \leq 0.28, 0.3$ MeV in $^{162}Yb$, $^{156}Dy$, respectively) the first negative signature one-phonon band populates with approximately equal probabilities the yrast states with $I' = I \pm 1$ ($I$ is the angular momentum of the excited state). At $h\Omega_c$ a shape-phase transition occurs, that leads to the triaxial shapes with the negative $\gamma$-deformation in the both nuclei. In turn, the negative signature phonon band decays stronger...
on the yrast states with angular momenta $I' = I - 1$, starting from $\hbar \Omega \geq \hbar \Omega_c$. We predict also the dominance of $\Delta I = 1\hbar$ magnetic transitions from the wobbling to the yrast states, independently from the sign of the $\gamma$-deformation (see [3]).

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