Heat and Mass Transfer Effects Past a Vertical Cone with Electric Conductivity

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Abstract

Numerical investigation of the transient natural convective double diffusion in a Walter-B viscoelastic fluid from a vertical cone with electric conductivity is introduced. Numerical analysis for non dimensional concentration, velocity and temperature fields by utilizing implicit finite difference scheme have been studied for the effect of electric conductivity parameter, Prandtl number (Pr), viscoelasticity parameter. The present discussion fixates on the local Nusselt number skin-friction and Sherwood number. It is observed that the electric conductivity has critical impact on the effect on the viscoelastic fluid flow past vertical cone proximate to the cone surface. Our numerical conclusions are compared with anterior work and a good acquiescent is descried.

Keywords: Cone, Finite Difference Method, Non-Darcian Porous Medium, Viscoelasticity

1. Introduction

The impacts of non-uniform heat source/sink and variable electric conductivity on unsteady, free convective viscoelastic fluid, higher order chemical reaction flow over a moving vertical cone and a caliber plate soused with porous medium was dissected in ⁵.

The above reviews did not consider consolidated viscoelastic double diffusion past a vertical cone within the visual perception of electric conductivity and heat radiation. Attributable to the noteworthiness of this issue in substance mechanical preparing the temperamental mundane convective double diffusion from a vertical cone with electric conductivity is considered in this paper.

2. Mathematical Formulation

An axi-symmetric transient natural convective double diffusion in a viscoelastic fluid within the sight of electric conductivity is considered. The Boussinesq’s approximation is considered for the lightness impacts incited by heat and mass dissemination. The co-ordinate framework picked (as appeared in Figure 1) is the neighbourhood span of the cone.

Figure 1. Physical Model.
We execute the accompanying non-dimensional amounts:

\[
X = \frac{X}{L}, \quad Y = \frac{Y}{L}, \quad r = \frac{x \sin \phi}{L},
\]

where \( r = \sqrt{\frac{r}{L}} \) for all \( X, Y, t \), and it tends to zero as 

\[ \begin{align*}
\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} &= \frac{\partial}{\partial y} \left( \frac{\partial}{\partial y} \right) + \frac{1}{\alpha} \frac{\partial}{\partial y} \left( \frac{\partial}{\partial y} \right) \left( \beta C' - C'' \right) \\
\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} &= \frac{\partial}{\partial y} \left( \frac{\partial}{\partial y} \right) + \frac{1}{\alpha} \frac{\partial}{\partial y} \left( \frac{\partial}{\partial y} \right) \left( \beta C' - C'' \right) \\
C &= - \frac{C''}{C'} \end{align*}
\]

Administering Equations in the non-dimensional form are

\[
\frac{\partial(U\tau)}{\partial X} + \frac{\partial(V\tau)}{\partial Y} = 0
\]

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) + \frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) \left( \beta C' - C'' \right)
\]

\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) + \frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) \left( \beta C' - C'' \right)
\]

The comparing beginning and limit conditions are

\[
t \leq 0: U = 0, V = 0, T = 0, C = 0 \quad \text{for all} \quad X, Y,
\]

\[
t > 0: U = 0, V = 0, T = 1, C = 1 \text{at} \quad Y = 0,
\]

\[
U = 0, \quad T = 0, \quad C = 0 \quad \text{at} \quad X = 0,
\]

\[
U \rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0 \quad \text{as} \quad Y \rightarrow \infty .
\]

The dimensionless local values of the skin friction, the Nusselt and Sherwood numbers are given by:

\[
t, U = \left( \frac{\partial U}{\partial Y} \right)_{x=0}
\]

\[
N_{U} = -X \left( \frac{\partial T}{\partial Y} \right)_{x=0}
\]

\[
S_{h} = -X \left( \frac{\partial C}{\partial Y} \right)_{x=0}
\]

3. Numerical Solution

The transient, non-linear equation (2)–(5) under the conditions (6) are solved by a numerical scheme of numerical scheme type which is discussed in \([1,2,4,6,7]\). The local truncation error is \( O(\Delta t^2 + \Delta X^2 + \Delta Y^2) \) and it tends to zero as \( \Delta t, \Delta X \) and \( \Delta Y \) tend to zero.

\[
\frac{U_{i,j}^{k+1} - U_{i,j}^{k}}{\Delta t} + \frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) + \frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) \left( \beta C' - C'' \right)
\]

\[
\frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) + \frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) \left( \beta C' - C'' \right)
\]

\[
\frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) + \frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) \left( \beta C' - C'' \right)
\]

\[
\frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) + \frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) \left( \beta C' - C'' \right)
\]

\[
\frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) + \frac{1}{\alpha} \frac{\partial}{\partial Y} \left( \frac{\partial}{\partial Y} \right) \left( \beta C' - C'' \right)
\]
### 3.4 Species (Concentration) Conservation

\[
\frac{C_{i,j+1}^{n+1} - C_{i,j}^{n}}{\Delta t} + U_{i,j}^{n} \left( \frac{C_{i,j+1}^{n+1} - C_{i,j}^{n+1}}{2\Delta x} + \frac{C_{i,j}^{n+1} - C_{i,j+1}^{n+1}}{2\Delta x} \right) + \frac{V_{j}^{n} - V_{j+1}^{n}}{4\Delta y} = \frac{1}{Sc} \left( \frac{C_{i,j}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j+1}^{n+1} - 2C_{i,j+1}^{n+1} + C_{i,j+1}^{n+1} - C_{i,j+1}^{n+1}}{2(\Delta Y)^{2}} \right) \tag{13}
\]

### 3.5 Stability of the Scheme

The stability of finite difference has been discussed in the well-known Von-Neumann technique in this section. This method introduces an initial error represented by a finite difference scheme of Crank-Nicolson type. It is observed that the stability of the general terms of the Fourier expansion for the solution. The general terms of the Fourier expansion for the solutions (15) - (17) can be written as

\[
\begin{align*}
\frac{F' - F}{\Delta t} &= \frac{2A}{2\Delta X} \sin(\beta \Delta Y) + \frac{V}{2} \left( F' + F \right) \sin(\beta \Delta Y) + \frac{V}{2} \left( F' + F \right) \cos(\beta \Delta Y - 1) \Delta Y \tag{15} \\
\frac{G' - G}{\Delta t} &= \frac{2A}{2\Delta X} \sin(\beta \Delta Y) + \frac{V}{2} \left( G' + G \right) \cos(\beta \Delta Y - 1) \Delta Y \tag{16} \\
\frac{H' - H}{\Delta t} &= \frac{2A}{2\Delta X} \sin(\beta \Delta Y) + \frac{V}{2} \left( H' + H \right) \cos(\beta \Delta Y - 1) \Delta Y \tag{17}
\end{align*}
\]

Equations (15) - (17) can be written as

\[
\begin{align*}
(1 + A)F' &= (1 - A)F + \frac{Gr(1 + G')}{2} \Delta t + \frac{1}{2} \frac{Gm(H + H')}{2} \Delta t \tag{18} \\
(1 + B)G' &= (1 - B)G \tag{19} \\
(1 + E)H' &= (1 - E)H \tag{20}
\end{align*}
\]

where

\[
\begin{align*}
A &= \frac{V}{2} \frac{\Delta t}{\Delta X} \sin(\beta \Delta Y) - \frac{Gr(1 + G')}{2} \frac{\Delta t}{(\Delta Y)^2} \\
B &= \frac{V}{2} \frac{\Delta t}{\Delta Y} \sin(\beta \Delta Y) - \frac{Gm(H + H')}{2} \frac{\Delta t}{Pr(\Delta Y)^2} \\
E &= \frac{V}{2} \frac{\Delta t}{\Delta Y} \sin(\beta \Delta Y) - \frac{Gm(H + H')}{2} \frac{\Delta t}{Sc(\Delta Y)^2}
\end{align*}
\]

After eliminating $G'$ and $H'$ in Equation (18) using Equation (19) and (20), the resultant equation and equations (19) and (20) can be written in the following matrix form:

\[
\begin{bmatrix}
F' \\
G' \\
H'
\end{bmatrix} =
\begin{bmatrix}
1 + A & 1 & A \\
-1 & 1 & 0 \\
0 & 0 & 1 + E
\end{bmatrix}
\frac{\Delta t}{(1 + A)(1 + E)} \begin{bmatrix}
F \\
G \\
H
\end{bmatrix}
\]

\[
d_1 = \frac{\Delta t Gr}{(1 + A)(1 + E)} \quad \text{and} \quad d_2 = \frac{\Delta t Gm}{(1 + A)(1 + E)}
\]

where

\[
a = \frac{U \Delta t}{2\Delta X}, \quad b = \frac{V |\Delta t|}{2\Delta Y} \quad \text{and} \quad c = \frac{\Delta t}{(\Delta Y)^2}
\]

### 4. Results and Discussion

For brevity, selective figures are reproduced here. In order to prove the accuracy of the computations in steady state at $X = 1.0$, $Pr = 0.7$, $\eta = 4$ and considering $Gr_i = Gr_{max} \frac{x}{(T_1 - T_2)(J)}$ are compared with available similarity solutions in the literature. Figures 1(a) and 1(b) demonstrate the impact of radiation parameter $F_i$ on steady state velocity ($U$) and temperature ($T$) distributions. An expansion in $F_i$ from 0 through 0.5, 1.0, 3.0, 5.0, 10.0 to 100.0, causes a significant reduction in velocity.

### 5. Conclusions

The unsteady natural convective heat and mass transfer in a viscoelastic fluid past a vertical cone with electric conductivity is considered. The dimensionless conservation equations for the boundary layer regime are solved by the implicit finite difference scheme of Crank-Nicolson type. It is observed that the electric conductivity has a significant effect on the viscoelastic fluid flow past vertical cone close to the cone surface. An increase in cone semi-apex angle ($\phi$) is observed to decelerate the flow near the cone surface but to increase temperatures and concentration in the boundary layer regime.
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