The $^{1}S_{0}$ pairing in neutron matter has been investigated in presence of realistic two- and three-nucleon interactions. We have adopted the Argonne $v_{8}$ NN and the Urbana IX 3N potentials. Quantum Monte Carlo theory, specifically the Auxiliary Field Diffusion Monte Carlo method, and Correlated Basis Function theory are employed in order to get quantitative and reliable estimates of the gap. They both fully take into account the medium modifications due to the interaction induced correlations. The two methods are in good agreement up to the maximum gap density and both point to a slight reduction with respect to the standard BCS value. In fact, the maximum gap is about 2.5 MeV at $k_{F} \sim 0.8 \text{ fm}^{-1}$ in BCS and 2.3-2.4 MeV at $k_{F} \sim 0.6 \text{ fm}^{-1}$ in correlated matter. At higher densities the Quantum Monte Carlo gap becomes close to BCS. In general, the computed medium polarization effects are much smaller than those previously estimated within all theories. Truncations of Argonne $v_{8}$ to simpler forms give the same gaps in BCS, provided the truncated potentials have been refitted to the same NN data set. Differences among the models appear in the correlated theories, most of the reduction being attributable to the tensor force. The three–nucleon interaction provides an additional increase of the gap of about 0.35 MeV.

PACS numbers: 21.30.-x, 21.60.-n, 21.60.Ka, 26.60.+c

Keywords: nuclear forces, neutron matter, nuclear pairing, superfluidity, Quantum Monte Carlo methods

Superfluidity in highly asymmetrical nuclear matter, with a low concentration of protons, can lead to major consequences in modern astrophysics. $^{1}S_{0}$ pairing may occur in the low density neutron gas in the inner crust of neutron stars and in the proton phase in the interior of the star. Anisotropic $^{3}P_{2}^{3}F_{2}$ neutron pairing may also appear at higher interior densities. Understanding microscopically such phenomena as the cooling rate and the post–glitch relaxation times requires as accurate as possible knowledge of the properties of the phases (normal and superfluid) of nuclear matter.

The role that tensor force and $N–N$ correlations have on the superfluid properties of neutron matter of $T = 0$ is not yet fully understood. In this paper we address this problem by performing ab initio calculations of the $^{1}S_{0}$ BCS–type pairing in neutron matter, using (i) a realistic Hamiltonian and (ii) a trustworthy many–body theories. As far as the Hamiltonian is concerned, a number of nucleon–nucleon interactions have been derived in the last decade, all of them fitting the large body ($\sim 3000$) of the available $pp$ and $np$ scattering data with a $\chi^{2}$/datum $\sim 1$. The Argonne $v_{14}$, Nijmegen, and CD Bonn interactions are members of this family of phase–equivalent potentials. The situation is much less established for the three–nucleon interaction (TNI) because of the much smaller number of experimental data to be fitted as well as the larger uncertainty of the theoretical framework on which its construction is based.

There has been a rapid advance in dealing with strongly interacting systems in their normal phase within modern many–body theories. Parallel to the more traditional BHF and CBF theories there has been significant development of the Quantum Monte Carlo methods (QMC) in dealing with spin-dependent Hamiltonian. In the different versions of QMC the normal phase many–body Schrödinger equation is solved via stochastic sampling. For homogeneous systems, a finite number of particle is put inside a simulation box with periodic boundary conditions to simulate the infinite gas. Then either the spatial configurations only (Green’s Function Monte Carlo, GFMC) or both the spatial and spin–isospin ones, by introducing auxiliary fields (Auxiliary
Field Diffusion Monte Carlo, AFDMC \[11\]), are sampled. GFMC calculations are limited to a low number of particles in the box (N=14 in neutron matter) since it sums over all the \(2^N\) spin states of the N interacting
neutrons, whereas AFDMC does not have this limitation. As a consequence, GFMC has typically large finite size corrections. GFMC \[12\] and AFDMC \[13\] have been so far used to study pure neutron matter with comparable results. Actual calculations restrict the sampling within the nodal surface of some trial wave function, where the sampling guiding function has a constant sign (Path Constraint approximation \[13\]). An accurate choice of the nodal surface is essential in obtaining good quality results \[14\]. This is more difficult for AFDMC since the method samples the spin degrees of freedom, and has less inherent cancellation.

Correlated Basis Functions \[15\] (CBF) theory provides an alternative way of addressing interacting nuclear systems. The strong nuclear interaction modifies the short range structure of the wave function (short range correlations, SRC) and introduces many–body contributions. Within CBF the SRC are introduced through a many–body correlation operator acting on a model function (Fermi gas for normal phase infinite systems or shell model wave function for finite nuclei). Since the NN potentials have important spin– and isospin–dependent components, the nuclear correlation operator is highly state dependent. An effective correlation operator has been shown to be:

\[
F_6(1,2,..N) = S \left[ \prod_{i<j=1,N} f_6(ij) \right] , \tag{1}
\]

where

\[
f_6(ij) = \sum_{p=1,6} f^{(p)}(r_{ij})O^{(p)}(ij) , \tag{2}
\]

with \(O^{(p=1,2,3)}(ij) = 1, \sigma(i) \cdot \sigma(j), S(ij) = (3\hat{r}_a(i)\hat{r}_b(j) - \delta_{a\beta})\sigma_a(i)\sigma_b(j)\), and \(O^{(p'=p+3)}(ij) = O^{(p)}(ij) \times [\tau(i) \cdot \tau(j)], S\) being a symmetrizer operator since the \(f_6\) operators do not commute among each other. In pure \(T = 1\) neutron matter, the isospin components may be actually embodied into the spin dependent ones. This choice of the correlation is consistent with the use of a momentum independent \(v_6\) potential:

\[
v_6(ij) = \sum_{p=1,6} v^{(p)}(r_{ij})O^{(p)}(ij) . \tag{3}
\]

Modern potentials also contain momentum dependent (spin–orbit and so on) components. Accordingly, spin–orbit correlations have been used in CBF nuclear matter studies \[16\]. The correlation is variationally fixed by minimizing the ground state energy as computed via cluster expansion.

The normal phase of neutron matter has been studied in details by both QMC and CBF. Very recently we have extended the two methods to deal with the \(1S_0\) superfluid pairing. A detailed account of the theories is in preparation and will be given in subsequent publications \[15\]. We present here results obtained within the AFDMC and the CBF methods for the gap energy computed starting from realistic potentials (Argonne\(\upsilon_{18}\) and its reductions plus the Urbana IX TNI) and compare with the pure BCS results. A brief outline of the methods will also be given.

A correlated wave function for the neutron matter superfluid phase is constructed as

\[
|\Psi_s\rangle = \hat{F} |\text{BCS}\rangle , \tag{4}
\]

where the model BCS–state vector is

\[
|\text{BCS}\rangle = \prod_k (u_k + v_k a^{\dagger}_k a^{-}_k)|0\rangle . \tag{5}
\]

\(u_k\) and \(v_k\) are real, variational BCS amplitudes, satisfying the relation \(u_k^2 + v_k^2 = 1, |0\rangle\) is the vacuum state and \(a^{\dagger}_k\) is the fermion creation operator in the single–particle state, whose wave function is

\[
\phi_{m\equiv k,\sigma}(x) = \frac{1}{\sqrt{\Omega}} \eta_\sigma(s)\exp(ik \cdot r) . \tag{6}
\]

\(\Omega\) is the normalization volume and \(\eta_{\sigma=1,1}(s)\) is the spin wave function with spin projection \(\sigma\). The second–quantized correlation operator \(\hat{F}\) is written in terms of the N–particle correlation operators, \(\hat{F}_N\), as

\[
\hat{F} = \sum_{N,m_N} \hat{F}_N |\Phi_{m_N}^n\rangle \langle \Phi_{m_N}^n| , \tag{7}
\]

where \(m_N\) specifies a set of single–particle states. In coordinate representation and for a \(f_6\)–type correlation, we have:

\[
\langle x_1,x_2,..x_N|\hat{F}_N|\Phi_{m_N}^n\rangle = F_6(1,2,..N) \{\phi_{m_1}(x_1)\phi_{m_2}(x_2)\cdots \phi_{m_N}(x_N)\}_A . \tag{8}
\]

The suffix \(A\) stands for an antisymmetrized product of single–particle wave functions.

The cluster expansion of the two–body distribution function, \(g(r_{12})\), and of the one–body density matrix, \(n(r_{11})\), for a simple Jastrow correlations \((f_j(i,j) = f_j^{(1)}(r_{ij}))\) was developed in Ref. \[15\]. It is possible to sum, in a Fermi Hypernetted Chain (FHNC) theory, all the cluster diagrams contributing to the two quantities and constructed by the dynamical correlation lines \((h_j = f_j^2 - 1)\) and the by BCS statistical correlations,

\[
l_v(r) = \frac{\nu}{\rho_0} \int \frac{d^3k}{(2\pi)^3} \exp(ik \cdot r) v^2(k) , \tag{9}
\]

\[
l_u(r) = \frac{\nu}{\rho_0} \int \frac{d^3q}{(2\pi)^3} \exp(ik \cdot r) u(r)v(r) , \tag{10}
\]
where \( \rho_0 \) is the average density of the uncorrelated BCS model and \( \nu = 2 \) is the neutron matter spin degeneracy. For more complicated correlations, like the \( f_6 \) model, complete FHNC expansion cannot be derived. So, we have computed the expectation values at low orders of their cluster expansions. In particular, the average density,

\[
\rho = \frac{\langle \hat{N} \rangle}{\Omega} = \frac{\sum_m \langle a_m^\dagger a_m \rangle}{\Omega},
\]

is computed at the first order of the expansion in a series of powers of the dynamical correlations. This expansion provides at each order the correct density normalization in the normal phase, since it fully takes into account cancellations between same order diagrams but with different numbers and types of statistical correlations. Similar, if not complete cancellations, hold in the superfluid phase. Consistently, the matrix elements of the Hamiltonian on the correlated BCS state are evaluated at the two-body cluster level with vertex corrections at the interacting pair.

In AFDMC for a BCS-like phase the neutrons are paired in a pfaffian constructed by a trial BCS amplitude. To this aim, we have modified the normal phase AFDMC code, substituting the Slater determinant with the pfaffian. We have taken the BCS amplitudes from the CBF calculations. The AFDMC simulation has been performed for \( N=12-18 \) neutrons in a periodic box and interacting through the Argonne \( v_{18}' \) potential. Preliminary results with larger \( N \)-values confirm the findings of this paper. Table I shows the results obtained by using a time step of \( \Delta \tau = 5 \times 10^{-5} \) MeV \(^{-1} \). The energy per particle for the normal phase at \( k_F = 0.6 \) fm \(^{-1} \) for the closed shell case, \( N=14 \), is \( E_{N_{D}}/N = 2.548(3) \) MeV. For even \( N \)-values all neutrons are paired in the pfaffian, whereas for odd \( N \)-values the configuration of the unpaired neutron providing the best energy must be found. The gap energy for odd \( N \)-values is calculated according to:

\[
\Delta(N) = E(N) - \frac{1}{2}(E(N+1) + E(N-1)).
\]

The present calculations show that, at shell closure for \( N=14 \), the normal phase is energetically slightly preferred to the superfluid one. Two effects must be considered: improvements to the pairing functions (e.g. a better choice of the nodal surface) and that the 14 particles box dimensions are too small for the correlation length of the pairing function. Again, we have indications that the superfluid phase is actually preferred from preliminary calculations with larger number of particles.

The results for the gap function at the Fermi momentum are collected in Figure 10. The curves labeled \( \Delta_{v_x}^0 \) refer to the pure BCS estimates using operatorial reductions of the Argonne \( v_{18} \) potential, down to the \( v_4 \) one.

Most of the reduced potentials have been refitted to reproduce the \( S- \) and \( P- \) wave experimental phase shifts \( (v_{4} - v_{8}') \). It is remarkable that all these potentials give the same BCS gap, provided they fit the same data set. Presumably this is a consequence of getting the effective interaction near the Fermi surface correct. A version of Argonne \( v_{18} \) simply cut to \( v_{6} \), without refitting, provides the higher BCS gaps of the \( \Delta_{v_6}^0 \) curve. In BCS, \( \Delta_{v_6}^0 = \Delta_{v_{18}}^0 \). The \( \Delta_{v_{6} - v_{6}'} \) curves show the CBF gaps for the corresponding refitted potentials. The points with error bars are the AFDMC gap estimates with Argonne \( v_{8}' \). The AFDMC results are the average values of the gaps computed around \( N=13, 15 \) and 17.

The Figure shows that the low density (up to \( k_F \sim 0.6 \) fm \(^{-1} \)) CBF and AFDMC gaps, for the same \( A_{8}' \) potential model, are in good agreement. The low order cluster expansion provides a CBF gap slightly smaller than the quantum MC one. The highest Fermi momentum AFDMC result, at \( k_F = 0.8 \) fm \(^{-1} \), is much higher of the corresponding CBF gap, stressing the need of pushing the cluster expansion to higher orders at increasing densities. It is clear, by looking at the \( \Delta_{v_{6} - v_{6}'} \) curves, that most of the reduction of the gap with respect to BCS is due to the tensor force, only partially compensated by the spin–orbit potential. The CBF \( \Delta_{v_6} \) is the lowest curve, pointing once more to the importance of tensor force, in the sense that experimental data must be fitted in order to obtain meaningful results.

The reduction of the gap due to medium effects is much smaller than previous estimates, which provided suppressions of a factor of two and more. The CBF theory shows an early disappearance of the gap, around \( \rho \sim 0.15 \\rho_{N_{M}} \), \( \rho_{N_{M}} = .16 \) fm \(^{-3} \) being the empirical nuclear matter saturation density. However, the QMC calculations do not seem to support this finding, and, surprisingly enough, give a gap energy that is still close to the standard BCS one. A similar result was found in a preliminary study employing GFMC and simple model potentials.

The square in Figure gives the AFDMC gap, at \( k_F = 0.6 \) fm \(^{-1} \), for the \( A_{8}' \) model implemented by the Ur-
bana IX three–nucleon interaction \[21\]. The gap with the three–body force results to be \(\Delta_{v_{\nu}+U_{IX}} = 2.810(146)\) MeV, slightly increased with respect to \(\Delta_{v_{\nu}} = 2.457(76)\) MeV. A qualitatively different result was found in Ref. \[24\], where the authors have obtained a small decrease of the gap in a Bruckner \(G\)–matrix based approach.

In this letter we have used Quantum Monte Carlo and Correlated Basis Functions theories to microscopically evaluate the \(1S_0\) superfluid gap in pure neutron matter with modern interactions. These methods allow for taking into account medium modification effects in a consistent and realistic way. In particular, QMC is expected to give a solution of the many–body Schrödinger equation very close to the true one. Both theories are in good agreement up to the maximum gap density, and show a slight reduction of the maximum gap with respect to standard BCS. At higher densities QMC gives a larger gap than CBF, probably because the CBF gap is computed at low order of the cluster expansion and the missing diagrams become more and more relevant with the density. The gap reduction is essentially due to the tensor interaction. A novel and important effect is the small influence of the medium polarization, contrary to all the previous estimates. The three–nucleon interaction provides a small increase of the gap. It will be most interesting to extend these analysis to other types of pairing, as the \(^3P_2-^3F_2\) neutron pairing and the \(^1S_0\) proton pairing in highly asymmetrical nuclear matter.

A.Yu.I. acknowledges the support of INFN and of the Dipartimento di Fisica “Enrico Fermi” of the Pisa University.

* Electronic address: Adelchi.Fabrocini@df.unipi.it
† Electronic address: fantoni@sissa.it
‡ Electronic address: Alexei.Illarionov@pion.infn.it
§ Electronic address: Kevin.Schmidt@asu.edu

[1] S. Tsuruta, Phys. Rept. 292, 1 (1998).
[2] H. Heiselberg and M. Hjorth-Jensen, Phys. Rept. 328, 237 (2000). [nucl-th/9902033].
[3] J. A. Sauls, in H. Ogelman, E. P. J. van den Heuvel, and J. van Paradis, eds., "Cosme Lectures on "Timing Neutron Stars", NATO-ASI, Series C, April 1988 (Kluwer Academic Press, 1989), vol. 262, pp. 441–490.
[4] M. A. Alpar and D. Pines, in K. A. Van Riper, R. Epstein, and C. Ho, eds., "Proc. The Los Alamos Workshop "Isolated Pulsars"" (Cambridge University Press, 1993).
[5] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C51, 38 (1995).
[6] V. G. J. Stoks, R. A. Klomp, C. P. F. Terheggen, and J. J. DeSwart, Phys. Rev. C49, 2950 (1994).
[7] R. Macekith, F. Sammurrula, and Y. Song, Phys. Rev. C53, R1483 (1995).
[8] S. C. Pieper, V. R. Pandharipande, R. B. Wiringa, and J. Carlson, Phys. Rev. C64, 014001 (2001).
[9] K. E. Schmidt and M. H. Kalos, in K. Binder, ed., Monte Carlo methods in Statistical Physics (Springer–Verlag (Berlin), 2001), p. 125.
[10] R. B. Wiringa, S. C. Pieper, J. Carlson, and V. R. Pandharipande, Phys. Rev. C62, 044310 (2000).
[11] K. E. Schmidt and S. Fantoni, Phys. Lett. B446, 99 (1999).
[12] J. Carlson, J. J. Morales, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C68, 025802 (2003).
[13] A. Sarsa, S. Fantoni, K. E. Schmidt, and F. Pederiva, Phys. Rev. C68, 024308 (2003).
[14] L. Brualla, S. Fantoni, A. Sarsa, K. E. Schmidt, and S. A. Vitiello, Phys. Rev. C67, 065806 (2003).
[15] E. Feenberg, in Theory of Quantum Fluids (Academic Press, 1969).
[16] R. B. Wiringa, F. Ficks, and A. Fabrocini, Phys. Rev. C38, 1010 (1988).
[17] A. Fabrocini, S. Fantoni, A. Y. Illarionov, and K. E. Schmidt, in preparation (2005).
[18] A. Fabrocini, S. Fantoni, A. Y. Illarionov, and K. E. Schmidt, in preparation (2005).
[19] S. Fantoni, Nucl. Phys. A363, 381 (1981).
[20] J. Carlson, S.-Y. Chang, V. R. Pandharipande, and K. E. Schmidt, Phys. Rev. Lett. 91, 050401 (2003).
[21] B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, and R. B. Wiringa, Phys. Rev. C56, 1720 (1997).
[22] D. J. Dean and M. Hjorth-Jensen, Rev. Mod. Phys. 75, 607 (2003).
[23] S. Chang, J. Morales, V. Pandharipande, D. Ravenhall, J. Carlson, S. Pieper, R. Wiringa, and K. Schmidt, Nucl. Phys. A746, 215C (2004).
[24] W. Zuo, U. Lombardo, H. Schulze, and C. Shen, Phys. Rev. C66, 037303 (2002).