Research Article

Effect of Vugs on Hydraulic Fracture Propagation with Phase Field Method

Qingdong Zeng,1,2,3 Wenzheng Liu,3,4 and Jun Yao3

1Department of Mechanics, College of Energy and Mining Engineering, Shandong University of Science and Technology, Qingdao 266590, China
2State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu 610500, China
3Research Center of Multiphase Flow in Porous Media, China University of Petroleum (East China), Qingdao 266580, China
4Bohai Oilfield Research Institute, Tianjin Branch, CNOOC China Limited, Tianjin 300459, China

Correspondence should be addressed to Qingdong Zeng; upc.zengqd@163.com

Received 17 September 2021; Accepted 13 October 2021; Published 1 November 2021

Copyright © 2021 Qingdong Zeng et al. Exclusive Licensee GeoScienceWorld. Distributed under a Creative Commons Attribution License (CC BY 4.0).

The interaction between hydraulic fracture and vugs affects the formed fracture pattern, which is of great significance for developing carbonate reservoirs efficiently. To investigate the effect of vugs on hydraulic fracture propagation, a hydromechanical coupling model is established based on the phase field method. Fluid flow in the porous media with vug is described by using phase field indicator functions. The pressure, displacement, and phase fields are solved by finite element approximation and iterative method, and the proposed model is validated against analytical solution. Some influencing factors including vug geometry and property are analyzed, and results show that the vug has important impact on hydraulic fracture propagation. The fracture diverts to the vug, and it could propagate across a small vug but be arrested by a large vug. As the distance between fracture and vug increases, the effect becomes weaker until it disappears. Besides, the modulus of vug fillings determines the fracture propagation pattern. The increase of vug filling modulus makes the fracture direct crossing difficult. The study could provide some guidance for hydraulic fracturing design of carbonate reservoirs.

1. Introduction

Carbonate rock is characterized by low porosity, low permeability, and complicated pore throat structure. However, natural fractures and vugs are abundant and dispersed inside the rock. Through hydraulic fracturing, these natural fractures and vugs can be connected together to form a favorable channel for oil and gas flow [1]. There are lots of researches that have been done to investigate the interaction between hydraulic fracture and preexisting natural fractures [2–4]. However, few studies are focusing on the interaction between hydraulic fracture and vugs. It still remains unclear how hydraulic fracture propagates when it encounters the vugs.

Experimental and numerical studies have been done on hydraulic fracture propagation with vugs. Liu et al. [5] conducted experiments to investigate the effect of vugs on hydraulic fracture propagation by using true triaxial apparatus and identified three main interaction modes; that is, the hydraulic fracture may cross, bypass, or be arrested by the vug. Guo et al. [6] carried out a numerical simulation to examine the effect of dissolved cavern on hydraulic fracture by using the seepage stress damage coupling model, and some influencing factors including cavern property, reservoir parameter, and treatment parameters were analyzed. Luo et al. [7, 8] presented numerical simulation to investigate the effect of caverns on hydraulic fracture propagation direction under triaxial stress condition based on the extended finite element method and concluded that fractures tend to deflect from the maximum horizontal principal stress direction due to caverns. The effect of holes on crack propagation is also studied in other research areas [9, 10].

Regarding hydraulic fracturing modeling, there are many different kinds of numerical methods that have been proposed. The commonly used methods include the finite element method, boundary element method, and discrete element method [11–15]. The finite element method is
usually combined with the cohesive zone model for the hydraulic fracturing problem [16]. Fractures propagate along element boundaries, and thus, mesh refinement around fractures is needed for accuracy due to stress singularity of fracture tips. Instead, the extended finite element method is a good alternative, which allows fracture propagation across elements without remeshing [17, 18]. However, it becomes cumbersome when dealing with intersection of multiple fractures [19]. The boundary method only discretizes the fracture surface boundary, which results in much less elements and lower computational cost. The displacement discontinuity method is a kind of indirect boundary element method, which has been extensively employed for simulation of complex fracture propagation in unconventional reservoirs [20, 21]. Although the boundary element method has the advantage of fast computational velocity, it is difficult to consider the rock anisotropy and heterogeneity. The discrete element method treats rock as an assembly of discrete particles. The movement of particles obeys Newton’s Second Law. It is easy to simulate complex fracture propagation by using the discrete element method, but the computational cost is high [22, 23]. Moreover, much progress has been made in considering multiphysics coupling effect and rock plastic strain on fracture propagation [24–26].

The above methods represent fractures explicitly, and there is strong displacement discontinuity between fracture top and bottom surfaces. In recent years, the phase field method has been introduced to simulate hydraulic fracture propagation, which was initially applied for microstructure evolution, grain growth, and two-phase flow problems [27–30]. Without intention to review all models proposed so far, only some representative ones are mentioned. Wheeler et al. proposed an augmented-Lagrangian method for pressurized fracture problem with the phase field approach [31] and extended it to fluid-filled fractures in poroelastic media [32]. Miehe and Mauthé [33] presented a modular model to link the crack phase field evolution with the hydro-poroelastic response of the porous media for hydraulic fracturing and put forward a robust finite element implementation. Zhou et al. also conducted phase field modeling of fracture propagation in poroelastic media [34] and investigated the dynamic cracking phenomenon [35]. It has been well proved that it becomes very convenient to handle fracture initiation, branching, and joining with the phase field method in contrast with other methods for these crack behaviors which are automatically captured without other additional criterions. Considering the complex interaction modes between hydraulic fracture and vugs, the phase field method is an excellent approach for the problem. Thus, a phase field model is established to simulate hydraulic fracture propagation with preset vugs in this study for the purpose of getting a more comprehensive understanding of vug effects in hydraulic fracturing.

The structure of the paper is organized as follows. Section 2 presents the mathematical model, which phase field representation of crack propagation and fluid flow equation considering crack and vug with phase field. Section 3 describes the numerical solution, including calculation of driving force of phase field, and finite element approximation as well as iterative scheme. Section 4 validates the proposed model and analyzes the effect of vug property on fracture propagation. Section 5 gives some conclusions.

2. Mathematical Model

2.1. Crack Representation by Phase Field Method. Following Miehe et al. [36], the sharp crack topology can be approximated by a diffusive one, which is described by an auxiliary field variable as shown in Figure 1. The field variable is characterized for $d = 0$ the intact state and for $d = 1$ the fully broken state in rock, denoted as phase field. The crack surface can be represented by a functional as follows:

$$\Gamma_c(d) = \int_{\Omega} Y(d,\nabla d) dV,$$

(1)

where $Y(d,\nabla d)$ is the crack surface density function defined as

$$Y(d,\nabla d) = \frac{1}{2l_0^2} d^2 + \frac{l_0}{2} |\nabla d|^2,$$

(2)

where $l_0$ is the length parameter which controls the spread of damage. As the length $l_0$ approaches 0, the sharp crack topology is recovered.

In the variational theory, the total potential energy is the sum of elastic strain energy and crack surface energy as follows:

$$\Psi(u, \Gamma) = \int_{\Omega} \psi(\varepsilon_e) d\Omega + \int_{\Gamma} G_c d\Gamma,$$

(3)

where $\psi(\varepsilon_e)$ is the elastic strain energy density and $G_c$ is the critical energy release rate of the rock.

Furthermore, it needs to consider the effect of fluid pressure on the total potential energy in the porous media. There are several ways to add the pressure term [37, 38]. The expression given by Lee et al. [38] is adopted as follows:

$$\Psi(u, \Gamma) = \int_{\Omega} \psi(\varepsilon_e) d\Omega + \int_{\Gamma} G_c d\Gamma - \int_{\Omega} ap \cdot (\nabla \cdot u) d\Omega,$$

(4)

where $p$ is the fluid pressure and $a$ is Biot’s coefficient.

Considering the diffusive crack topology, the crack surface energy can be rewritten in the form of domain integral with crack phase field:

$$\int_{\Gamma} G_c d\Gamma = \int_{\Omega} \frac{G_c}{2} \left( \frac{1}{l_0} d^2 + l_0 |\nabla d \cdot \nabla d| \right) d\Omega.$$

(5)

The involution of the crack phase field represents the damage of rock, which gives rise to stress degradation. Hence, the elastic strain energy density can be written as

$$\psi(\varepsilon_e) = \frac{1}{2} \sigma : \varepsilon_e = \frac{1}{2} \varepsilon_e : \left[ (1 - d)^2 + k_0 \right] C : \varepsilon_e.$$

(6)
where $\sigma$ is the total stress tensor and $\varepsilon_e$ is the elastic strain tensor. $C$ is the stiffness tensor of the rock. $k_0$ is a parameter taken as small as possible while keeping the system of equations well conditioned.

Substituting equations (5) and (6) into equation (4) gives the final expression of total potential energy:

$$
\Psi(u, d) = \int_\Omega \frac{1}{2} \varepsilon : \left[(1 - d)^2 + k_0\right] C \varepsilon \, d\Omega + \int_{\partial\Omega} G_c \frac{d}{l_0} \delta\varepsilon \cdot \nabla \phi \, d\Omega + \int_{\partial\Omega} \frac{1}{4} \varepsilon_0^2 \, d\Omega - \int_{\partial\Omega} a \varepsilon_0 \cdot \nabla u \, d\Omega.
$$

The variation of internal energy increment is written as follows:

$$
\delta W_{\text{int}} = \delta\Psi = \frac{\partial\Psi}{\partial\varepsilon_e} \delta\varepsilon_e + \frac{\partial\Psi}{\partial\delta d} \delta d + \frac{\partial\Psi}{\partial\delta v} \cdot \delta\nabla d.
$$

After some mathematical manipulations on equation (7), the variation of internal energy can be expressed as follows:

$$
\delta W_{\text{int}} = \int_\Omega \sigma' \cdot \delta\varepsilon_e \, d\Omega + \int_{\partial\Omega} \left[-2(1 - d)\psi_0(\varepsilon_e) + G_c \frac{d}{l_0} \right] \delta\varepsilon_e \cdot \nabla \phi \, d\Omega + \int_{\partial\Omega} G_c l_0 \delta\varepsilon_e \cdot \nabla \phi \, d\Omega.
$$

The term $\sigma'$ is the effective stress, and it is related to the total stress as follows:

$$
\sigma' = \sigma - a\varepsilon_0 I.
$$

The term $\psi_0(\varepsilon_e)$ represents the undamaged elastic strain energy density, which is given as

$$
\psi_0(\varepsilon_e) = \frac{1}{2} \varepsilon_0 : C : \varepsilon_e.
$$

By using the divergence theorem, the equation of energy variation is deduced to

$$
\delta W_{\text{int}} = \int_\Omega \left[- \left(\nabla \cdot \sigma'\right) \right] \cdot \delta\varepsilon_e \, d\Omega + \int_{\partial\Omega} \left(\sigma \cdot n\right) \cdot \delta\varepsilon_e \cdot \nabla \phi \, d\Omega
$$

$$
+ \int_{\partial\Omega} \left[-2(1 - d)\psi_0(\varepsilon_e) + G_c \frac{d}{l_0} \right] \delta\varepsilon_e \cdot \nabla \phi \, d\Omega
$$

$$
- \int_{\partial\Omega} G_c l_0 \delta\varepsilon_e \cdot \nabla \phi \, d\Omega.
$$

On the other hand, the variation of external work is expressed as

$$
\delta W_{\text{ext}} = \int_\Omega b \cdot \delta\varepsilon_e \, d\Omega + \int_{\partial\Omega_h} h \cdot \delta\varepsilon_e \cdot \nabla \phi \, d\Omega.
$$

According to the variational theory, it is required that $\delta W_{\text{int}} = \delta W_{\text{int}}$ holds for arbitrary values of $\delta\varepsilon_e$ and $\delta d$, which results in the strong form of governing equation for rock deformation and phase field evolution:

$$
\nabla \cdot (\sigma - apI) + b = 0,
$$

$$
G_c \frac{d}{l_0} - G_c l_0 (\nabla \cdot \delta\varepsilon_e) = 2(1 - d)\psi(\varepsilon_e).
$$

And the corresponding boundary conditions are given as

$$
\sigma \cdot n = h \quad \text{on} \quad \partial\Omega_h,
$$

$$
\frac{d}{l_0} = u_0 \quad \text{on} \quad \partial\Omega_u,
$$

$$
\nabla \cdot n = 0 \quad \text{on} \quad \partial\Omega,
$$

where $\bar{u}$ is the prescribed displacement on boundary $\partial\Omega_u$.

2.2. Fluid Flow in Porous Media with Vug. Fracturing fluid is injected from the wellbore and used to initiate and extend fractures. Due to high pressure in the fracture, the fluid leaks into surrounding porous media. As the fracture propagates, the permeability of media changes. However, the fracture is not explicitly represented but by using phase field, which gives rise to the difficulty in capturing the effect of fracture on fluid flow. Moreover, the vug has also significant effect on fluid flow. Lee et al. [38] proposed a way to separate the domain into three parts including the unbroken reservoir domain, the fractured domain, and the transition domain by using phase field. Considering the existence of vug, the unbroken reservoir domain is further divided into the matrix domain and vug domain with different mechanical and seepage properties.

Two thresholds parameters are given to separate the domain, denoted as $c_1$ and $c_2$. If the phase field $d$ is equal to or less than $c_1$, the domain is considered the reservoir domain. If the phase field $d$ is equal to or greater than $c_2$, the domain is considered to be in the transition region.
the domain is considered the fractured domain. The rest domain is the transition domain.

In the reservoir domain, the equation of mass conservation is written as

\[ \rho S_r \frac{\partial p}{\partial t} + \nabla \cdot (\rho \nu_r) = Q_r - \rho c \frac{\partial \varepsilon_{vol}}{\partial t}, \]  

where \( \rho \) is the fluid density, \( S_r \) and \( \alpha_r \) are the composite compressibility coefficient and Biot’s coefficient of the reservoir domain, and \( \nu_r \) and \( Q_r \) are the fluid velocity and source term in the reservoir domain.

The composite compressibility coefficient \( S_r \) is given by

\[ S_r = \phi c + \frac{(\alpha_r - \phi)(1 - \alpha_r)}{K_r}, \]

where \( \phi \) is the porosity of the reservoir domain, \( c \) is the fluid compressibility, and \( K_r \) is the bulk modulus of the reservoir domain.

The fluid velocity \( \nu_r \) is related to the pressure gradient as follows:

\[ \nu_r = -\frac{k_r}{\mu} \nabla p, \]

where \( \mu \) is the fluid viscosity and \( k_r \) is the permeability of the reservoir domain.

To capture the fluid flow in the vug, the reservoir domain is further divided into two domains, which are the matrix domain and vug domain with different mechanical and seepage properties. The vug domain is viewed as a domain with very large permeability \( k_v \) (several orders of magnitude larger than matrix permeability \( k_m \)) and small modulus \( E_v \) (several orders of magnitude smaller than matrix modulus).

In the fractured domain, the equation of mass conservation is written as

\[ \rho S_f \frac{\partial p}{\partial t} + \nabla \cdot (\rho \nu_f) = Q_f, \]

where \( S_f \) is the equivalent compressibility coefficient of the fracture domain, which is equal to the fluid compressibility. \( \nu_f \) and \( Q_f \) are the fluid velocity and source term in the fractured domain.

The fluid velocity \( \nu_f \) in the fractured domain is given by Darcy’s law as follows:

\[ \nu_f = -\frac{k_f}{\mu} \nabla p, \]

where \( k_f \) is the permeability of the fractured domain.

To describe fluid flow in the transition domain, two linear indicator functions are defined as presented by Lee et al. [38], which have shown good effectiveness and accuracy.

The indicator functions are given as

\[ \chi_r(d) = \frac{c_2 - d}{c_2 - c_1}, \]

\[ \chi_f(d) = \frac{d - c_1}{c_2 - c_1}, \]

By using the indicator functions, the equation of mass conservation in the transition domain can be written as

\[ \rho S_t \frac{\partial p}{\partial t} + \nabla \cdot (\rho \nu_t) = Q_t - \rho c \frac{\partial \varepsilon_{vol}}{\partial t}, \]

where \( S_t \) and \( \alpha_t \) are the equivalent compressibility coefficient and Biot’s coefficient of the transition domain and \( \nu_t \) and \( Q_t \) are the fluid velocity and source term in the transition domain.

Biot’s coefficient \( \alpha_t \) is expressed as

\[ \alpha_t = \alpha_r \chi_r + \chi_f. \]

The equivalent compressibility coefficient \( S_t \) in the transition domain is rewritten as

\[ S_t = \phi c + \frac{(\alpha_t - \phi)(1 - \alpha_t)}{K_r}, \]

where \( \phi \) denotes the porosity of the transition domain, which is given as

\[ \phi_t = \phi \chi_r. \]

The fluid velocity \( \nu_t \) is written as

\[ \nu_t = -\frac{k_t}{\mu} \nabla p, \]

where \( k_t \) is the effective permeability of the transition domain, which is expressed by

\[ k_t = k_r \chi_r + k_f \chi_f. \]

So far, the governing equations for all domain parts can be generalized into the unified form (equations (25) and (29)) by using the phase field indicator functions. Through solving the unified governing equations, the fluid pressure can be obtained in all domains including the matrix domain, vug domain, fractured domain, and transition domain.

**3. Numerical Solution**

3.1. Driving Force of Phase Field. In the governing equation of crack phase field, the driving force of the damage is the elastic strain energy density. To avoid crack initiation in compression, the elastic strain energy density is divided into the tensile and compressive parts [39].
Firstly, the elastic strain is decomposed into the tensile and compressive parts as follows:

$$\varepsilon^e = \varepsilon^t + \varepsilon^c,$$

(31)

where $\varepsilon^t$ and $\varepsilon^c$ are the tensile and compressive modes of the elastic strain. And they can be calculated as

$$\varepsilon_i^t = \sum_{j=1}^{m} \langle \varepsilon_i^t \rangle ^+ n_i \otimes n_j,$$

(32)

where $m$ is the dimension number of the problem. $\varepsilon_i^t$ and $n_i$ are the eigenvalue and eigenvector of the elastic strain.

The Macaulay bracket is defined as

$$\langle \varepsilon_i^t \rangle ^+ = \frac{\varepsilon_i^t + \varepsilon_i^t}{2}. $$

(33)

The decomposition of the elastic strain energy density can be written as

$$\psi^e(\varepsilon_i) = \frac{\lambda}{2} (\text{tr}(\varepsilon_i))^2 + \omega \text{tr}\left( (\varepsilon_i^t)^2 \right),$$

(34)

where $\lambda$ and $\omega$ are the Lame constants of rock. The operator $\text{tr}$ is to access the trace of the tensor.

Finally, the local history field of the maximum tensile reference energy density is adopted as the driving force of the crack phase field as follows:

$$\mathcal{H}(x, t) = \max_{s \in [0, t]} \psi^e(\varepsilon(x, s)).$$

(35)

3.2. Finite Element Approximation. There are three primary fields needed to be solved: displacement, fluid pressure, and phase field. The finite element method is adopted to discretize the governing equations of these fields. And then, the problem is coupled solved by a staggered iterative scheme.

The weak form of the displacement field can be derived by the principle of virtual work as follows:

$$\int_{\Omega} (\sigma \cdot \delta u) \, d\Omega = \int_{\Omega} b \cdot \delta u \, d\Omega + \int_{\Gamma_{h}} h \cdot \delta u \, d\Gamma_{h},$$

(36)

where $\delta u$ is the virtual displacement.

The weak form of fluid pressure is given as

$$f_i^p = 2 \int_{\Gamma_{p}} (N_i)^T \mathcal{H} \, d\Omega.$$  

(37)

The temporary discretization of the transient term in the above equation is approximated by using the implicit backward method. The discretized form is

$$\frac{\partial p}{\partial t} = \frac{P^{n+1} - P^{n}}{\Delta t},$$

(38)

where $n$ and $n+1$ represent the current step and next step and $\Delta t$ is the time interval between them. The superscript $n+1$ is omitted for simplicity in the following derivation.

By using the local history field, the weak form of phase field is written as

$$\int_{\Omega} \left( \frac{G_{ij}}{\kappa} + 2 \mathcal{H} \right) d\delta d\Omega + \int_{\Omega} G_{ij} \nabla d \cdot \nabla (\delta d) d\Omega = \int_{\Omega} \mathcal{H} d\delta d\Omega.$$  

(39)

Using the Voigt notation, the primary fields can be discretized at the element level as follows:

$$u = \sum_{i=1}^{n} N_i^u u_i,$$

(40)

$$p = \sum_{i=1}^{n} N_i p_i,$$

(41)

$$d = \sum_{i=1}^{n} N_i d_i,$$

(42)

where $n_i$ is the number of the element and $N_i^p$ for the displacement vector is written as

$$N_i^u = \begin{bmatrix} N_i & 0 \\ 0 & N_i \end{bmatrix}.$$  

(43)

Substituting the above expressions for displacement approximation into the weak form (equation (36)) and invoking the arbitrariness of the test functions, the element stiffness matrix and the external force of an element at node $i$ for the displacement field can be obtained as follows:

$$K_i^u = \int_{\Gamma_{p}} \left[ (1 - d)^2 + k \right] (B^u_i)^T C B^u_i \, d\Omega,$$

(44)

$$f_i^u = \int_{\Gamma_{p}} (B^u_i)^T ap \, d\Omega + \int_{\Gamma_{p}} (N_i)^T b \, d\Omega + \int_{\Gamma_{h}} (N_i)^T h \, d\Gamma_{h}.$$  

(45)

Correspondingly, the element stiffness matrix and the external contribution at node $i$ for the fluid pressure field can be obtained as follows:

$$K_i^p = \int_{\Gamma_{p}} \left[ \frac{\rho S}{\Delta t} (N_i)^T N_i d\Omega + \int_{\Gamma_{p}} \frac{p}{\mu} (B_i)^T B_i \, d\Omega,$$

(46)

$$f_i^p = \int_{\Gamma_{p}} \left[ \frac{\rho S}{\Delta t} (N_i)^T p^n \, d\Omega + \int_{\Gamma_{p}} (N_i)^T (Q - \rho \alpha x \frac{\Delta \varepsilon}{\Delta t}) \, d\Omega.$$

(47)
The element stiffness matrix and the driving force at node $i$ for phase field can be obtained as follows:

$$K^{d}_{ij} = \int_{\Omega} \left( \frac{G_c}{l_0} + 2\mathcal{R} \right) (N_i)^T N_j d\Omega + \int_{\Omega} G_c I_0 (B_i)^T B_j d\Omega,$$

$$f^d_i = 2 \int_{\Omega} (N_i)^T \mathcal{R} d\Omega.$$  

### Table 1: The input parameters for numerical simulation.

| Parameter                  | Magnitude/unit | Parameter                  | Magnitude/unit |
|----------------------------|----------------|----------------------------|----------------|
| Domain length $L$          | 1 m            | Length parameter $l_0$     | 0.01 m         |
| Domain width $H$           | 1 m            | Threshold constants $c_1$, $c_2$ | 0.4, 1.0     |
| Young’s modulus $E$        | 20 GPa         | Matrix porosity $\varphi$  | 0.05           |
| Poisson’s ratio            | 0.25           | Fluid viscosity $\mu$      | $1 \times 10^{-3}$ Pa·s |
| Critical energy release rate $G_c$ | 50 N/m      | Fluid density $\rho$       | $1 \times 10^3$ kg/m$^3$ |
| Matrix permeability $k_m$  | $1 \times 10^{-15}$ m$^2$ | Biot’s modulus of matrix $\alpha_r$ | 0.8           |
| Fracture permeability $k_f$ | $1 \times 10^{-8}$ m$^2$ | Fluid compressibility $c$  | $1 \times 10^{-8}$ 1/Pa |

3.3. **Iterative Scheme.** Miehe et al. [39] proposed a robust operator split scheme that successively updated the local history field, the phase field, and the displacement field for modeling crack propagation. When the fluid flow is considered, the scheme is extended by updating the pressure field in the time step successively. The iterative scheme is illustrated in detail as follows. To obtain the fluid pressure, displacement, and phase field for next time step $n+1$, a staggered iterative scheme is presented.
4. Result and Analysis

The proposed model to simulate hydraulic fracture propagation is validated against the classic KGD model. And then, the effect of vug on fracture propagation is analyzed in detail in terms of vug radius, distance from fracture to vug, and vug modulus. The effect of the injection rate on its propagation is also investigated.

4.1. Validation of Proposed Model. To validate the proposed model for hydraulic fracturing, the classic KGD fracture model is solved numerically, and then, the numerical solution is compared with the analytical solution [40]. The input parameters are given in Table 1. Based on numerical solution, the phase field evolutions at different times are shown in Figure 2. The phase field evolution represents fracture propagation, and it shows that the fracture propagates along the horizontal direction from two sides. Besides, the comparison between numerical solution and analytical solution is shown in Figure 3. It can be seen that the numerical solution is generally consistent with the analytical solution. Some inconsistency may take its rise from separating the domain into different parts for fluid flow in the porous media. However, the relative error is still in an allowable range. This proves that the proposed model is capable of capturing hydraulic fracture propagation in porous media.

4.2. Effect of Vug Radius. Vugs are of different shapes and sizes. Here, the vugs are assumed to be circles with different radii, and the physical model of hydraulic fracture propagation with preset vug is illustrated in Figure 4. To investigate the effect of vug radius on fracture propagation, three cases are considered by setting the radius to 2.5 mm, 3.5 mm, and 4.5 mm. The results of phase field distribution are shown in Figure 5, in which the white circle represents the vug. It shows that the fracture firstly propagates along the horizontal direction from both ends and then diverts to the vug. This indicates that fracture propagation direction changes due to the vug, and it shows an attraction effect of vug on fracture propagation. If the vug radius is 2.5 mm, fracture reinitiates from the vug after its approaching. However, when the vug radius increases to 3.5 mm and 4.5 mm, fracture propagation mainly occurs on the other side, and it is difficult to reinitiate from the vug surface. It indicates that a large vug would hinder fracture propagation.

The pressure in the domain changes due to fluid injection and fracture propagation. And the pressure evolutions at two observation points including the injection point and the bottom point of vug circle are shown in Figure 6.
the injection point, the fluid pressure rises quickly and then descends slightly, rises, and finally tends to stabilize. At the bottom point of the vug, the fluid pressure rises heavily by the time the fracture approaches the vug and then rises slowly and finally tends to stabilize. In terms of vug, the final stabilized pressure increases as the vug radius increases.

**Figure 6:** Evolution of fluid pressure at different points: (a) injection point; (b) the bottom point of vug.

**Figure 7:** Distribution of phase field with different vertical distances from fracture to vug center: (a) 0 mm; (b) 3.75 mm; (c) 5 mm; (d) 6.25 mm.
4.3. Effect of Distance to Vug. The vug shows an attraction effect on the hydraulic fracture as stated above. However, when the vug is far away from the fracture, the effect disappears. To investigate the effect of distances between fracture and vug, four cases are simulated by setting the y coordinate of vug circle to 0 mm, 3.75 mm, 5 mm, and 6.25 mm. The distributions of phase field are obtained as shown in Figure 7. It shows that when the vertical distance from fracture and vug center is less than and equal to 5 mm, the fracture diverts to vug. Otherwise, when the vertical distance between them increases to 6.25 mm, the fracture propagation direction keeps unchanged, which indicates the vug has little impact on fracture propagation.

The evolutions of fluid pressure at the two observation points are shown in Figure 8. At the injection point, when the fracture diverts to the vug, the fluid pressure evolution becomes complex at a local time interval, and the fluid pressure could rise above that pressure when the vug has no effect on fracture propagation and finally descends below that pressure. However, at the bottom point of the vug, when the fracture diverts to the vug, the fluid pressure is greater than that when the diversion of fracture does not occur.

4.4. Effect of Vug Modulus. The vug is usually filled with some minerals, so it is of different equivalent Young’s modulus. To investigate the effect of vug equivalent modulus on fracture propagation, three cases are conducted by setting the vug modulus to $10^{-6}$, $10^{-3}$, and 10 times matrix modulus. The distributions of phase field are shown in Figure 9. It shows that new fracture reinitiates from the vug without offset when the vug modulus is less than that of the matrix, while the fracture propagates around the vug circumference and reinitiates from a favorable point. The intrinsic mechanism is that the vug modulus determines the deformation and failure in the vug. If the vug modulus is much smaller than the matrix modulus, the vug is very vulnerable to failure, and hydraulic fracture is prone to crossing the vug. And as the vug modulus increases, the vug develops to be an obstruction and it becomes difficult for the hydraulic fracture crossing.
Figure 11: Distribution of phase field with different injection rates: (a) $q_0 = 0.24$ mm$^2$/s; (b) $q_0 = 0.28$ mm$^2$/s; (c) $q_0 = 0.32$ mm$^2$/s.

Figure 12: Evolution of fluid pressure at different points: (a) injection point; (b) the bottom point of vug.
Similarly, the evolutions of fluid pressure at the injection point and top point of the vug are shown in Figure 10. As the vug modulus increases, the final stabilized fluid pressure rises at the injection point but drops at the top point of the vug.

4.5. Effect of Injection Rate. At last, the effect of the injection rate on fracture propagation is analyzed by setting it to 0.24 mm$^2$/s, 0.28 mm$^2$/s, and 0.32 mm$^2$/s. The obtained phase field is shown in Figure 11. It can be seen that as the injection rate increases, the newly initiated fracture from the vug has longer length, which indicates that the injection rate could affect the propagation velocity and length of the fracture.

The evolutions of fluid pressure at the two observation points are shown in Figure 12. It shows that as the injection rate increases, the final stabilized pressure rises slightly at both the injection point and the bottom point of the vug. The final distributions of fluid pressure are also presented as shown in Figure 13.

5. Conclusion

A hydromechanical coupling model based on the phase field method is presented to simulate hydraulic fracture propagation with preset vugs. The numerical model is validated by comparison with analytical solution to the classic KGD fracture model. A series of numerical cases are done to study the effect of vug on fracture propagation. Based on the simulation results, some conclusions are summarized as follows.

(1) Due to the existence of vugs, the fracture deflects from its initial propagation direction and tends to divert to the vug. When the vug radius is small, the fracture can propagate across the vug. Otherwise, the fracture may be arrested by it. As the distance between fracture and vug increases, the effect becomes weaker until it disappears, and the fracture proceeds along its initial propagation direction.

(2) The modulus of vug fillings determines the fracture propagation pattern. When the vug filling modulus is small, the fracture crosses the vug without offset.

However, when the vug filling modulus is large, the fracture propagates around the vug circumference and reinitiates from the offset point. The increase of the injection rate is conducive to fracture crossing the vug, which results in longer fracture length.

(3) The vugs have great influence on hydraulic fracture propagation. Thus, to obtain clear characterization of vug properties and geometrical location is very important, which underlies the optimization design. Besides, there are also many natural fractures in carbonate reservoirs; the combined effects of natural fracture and vugs on hydraulic fracturing are important and will be investigated in the future work.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This study is jointly supported by the National Natural Science Foundation of China (51904321, 52034010, and 42174143) and Open Fund (PLN2020-5) of State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation (Southwest Petroleum University).

References

[1] P. Liu, J. Li, S. Sun, J. Yao, and K. Zhang, “Numerical investigation of carbonate acidizing with gelled acid using a coupled thermal-hydrologic-chemical model,” *International Journal of Thermal Sciences*, vol. 160, pp. 106700–106718, 2021.

[2] N. R. Warpinski and L. W. Teufel, “Influence of geologic discontinuities on hydraulic fracture propagation (includes associated papers 17011 and 17074),” *Journal of Petroleum Technology*, vol. 39, no. 2, pp. 209–220, 1987.
[3] C. E. Renshaw and D. D. Pollard, “An experimentally verified criterion for propagation across unbounded frictional interfaces in brittle, linear elastic materials,” *International Journal of Rock Mechanics & Mining Sciences & Geomechanics Abstracts*, vol. 32, no. 3, pp. 237–249, 1995.

[4] H. R. Gu, X. Weng, J. Lund, M. Mack, U. Ganguly, and R. S. Rivera, “Hydraulic fracture crossing natural fracture at non-orthogonal angles: a criterion and its validation,” *SPE Production Operation*, vol. 27, no. 1, pp. 20–26, 2012.

[5] B. Liu, Y. Jin, and M. Chen, “Influence of vugs in fractured-vuggy carbonate reservoirs on hydraulic fracture propagation based on laboratory experiments,” *Journal of Structural Geology*, vol. 124, pp. 143–150, 2019.

[6] T. Guo, S. Tang, Y. Li et al., “The effect of dissolved cavern on the fracture propagation in vuggy carbonate reservoir,” *Journal of Petroleum Science and Engineering*, vol. 143, no. 5, pp. 1–14, 2021.

[7] Z. Luo, N. Zhang, L. Zhao, J. Zeng, P. Liu, and N. Li, “Interaction of a hydraulic fracture with a hole in poroelasticity medium based on extended finite element method,” *Engineering Analysis with Boundary Elements*, vol. 115, pp. 108–119, 2020.

[8] Z. Luo, N. Zhang, L. Zhao, N. Li, D. Ren, and F. Liu, “An extended finite element method for the prediction of acid-etched fracture propagation behavior in fractured-vuggy carbonate reservoirs,” *Journal of Petroleum Science and Engineering*, vol. 191, article 107170, 2020.

[9] Z. Li, Z. Wen, S. Gu, H. Pei, H. Gao, and Q. Mao, “In-situ observation of crack initiation and propagation in Ni-based superalloy with film cooling holes during tensile test,” *Journal of Alloys and Compounds*, vol. 793, pp. 65–76, 2019.

[10] R. Ghifiri, H. Shi, R. Guo, and G. Mesmacque, “Effects of expanded and non-expanded hole on the delay of arresting crack propagation for aluminum alloys,” *Materials Science & Engineering A*, vol. 286, no. 2, pp. 244–249, 2000.

[11] Z. Chen, “Finite element modelling of viscosity-dominated hydraulic fractures,” *Journal of Petroleum Science and Engineering*, vol. 88-89, no. 2, pp. 136–144, 2012.

[12] E. Gordeliy and A. Peirce, “Coupling schemes for modeling hydraulic fracture propagation using the XFEM,” *Computer Methods in Applied Mechanics and Engineering*, vol. 253, pp. 305–322, 2013.

[13] K. Wu and J. E. Olson, “Simultaneous multifracture treatments: fully coupled fluid flow and fracture mechanics for horizontal wells,” *SPE Journal*, vol. 20, no. 2, pp. 337–346, 2015.

[14] Y. Zou, X. Ma, Z. Tong et al., “Hydraulic fracture growth in a layered formation based on fracturing experiments and discrete element modeling,” *Rock Mechanics and Rock Engineering*, vol. 50, no. 2-3, pp. 1–15, 2017.

[15] X. Cai and W. Lin, “Hydromechanical-coupled cohesive interface simulation of complex fracture network induced by hydrofracturing with low-viscosity supercritical CO2,” *Lithosphere*, vol. 2021, pp. 1–8, 2021.

[16] B. Carrier and S. Granet, “Numerical modeling of hydraulic fracture problem in permeable medium using cohesive zone model,” *Engineering Fracture Mechanics*, vol. 79, pp. 312–328, 2012.

[17] Q. Zeng, J. Yao, and J. Shao, “Numerical study of hydraulic fracture propagation accounting for rock anisotropy,” *Journal of Petroleum Science and Engineering*, vol. 160, pp. 422–432, 2018.

[18] T. Mohammadnejad and A. R. Khoei, “An extended finite element method for hydraulic fracture propagation in deformable porous media with the cohesive crack model,” *Finite Elements in Analysis and Design*, vol. 73, pp. 77–95, 2013.

[19] X. L. Wang, F. Shi, C. Liu, D. Lu, H. Liu, and H. Wu, “Extended finite element simulation of fracture network propagation in formation containing frictional and cemented natural fractures,” *Journal of Natural Gas Science and Engineering*, vol. 50, pp. 309–324, 2018.

[20] X. Weng, O. Kresse, C. Cohen, R. Wu, and H. Gu, “Modeling of hydraulic-fracture-network propagation in a naturally fractured formation,” *SPE Production & Operations*, vol. 26, no. 4, pp. 368–380, 2011.

[21] Q. Zeng and J. Yao, “Numerical simulation of fracture network generation in naturally fractured reservoirs,” *Journal of Natural Gas Science and Engineering*, vol. 30, pp. 430–443, 2016.

[22] Z. Chong, S. Karekal, X. Li, H. Peng, G. Yang, and S. Liang, “Numerical investigation of hydraulic fracturing in transversely isotropic shale reservoirs based on the discrete element method,” *Journal of Natural Gas Science and Engineering*, vol. 46, pp. 398–420, 2017.

[23] J. Zhou, L. Zhang, Z. Pan, and Z. Han, “Numerical studies of interactions between hydraulic and natural fractures by smooth joint model,” *Journal of Natural Gas Science and Engineering*, vol. 46, pp. 592–602, 2017.

[24] H. Wang, “Numerical modeling of non-planar hydraulic fracture propagation in brittle and ductile rocks using XFEM with cohesive zone method,” *Journal of Petroleum Science and Engineering*, vol. 135, pp. 127–140, 2015.

[25] M. Mcclure, M. Babazadeh, S. Shiozawa, and J. Huang, “Fully coupled hydromechanical simulation of hydraulic fracturing in 3D discrete-fracture networks,” *SPE Journal*, vol. 21, no. 4, pp. 1302–1320, 2016.

[26] Q. Zeng, J. Yao, and J. Shao, “An extended finite element solution for hydraulic fracturing with thermo-hydro-elastic-plastic coupling,” *Computer Methods in Applied Mechanics and Engineering*, vol. 364, article 112967, 2020.

[27] L. Chen, “Phase-field models for microstructure evolution,” *Annual Review of Material Research*, vol. 32, no. 1, pp. 113–140, 2002.

[28] N. Moelans, B. Blanpain, and P. Wollants, “An introduction to phase-field modeling of microstructure evolution,” *Calphad*, vol. 32, no. 2, pp. 268–294, 2008.

[29] G. Zhu, J. Kou, B. Yao, Y. Wu, J. Yao, and S. Sun, “Thermodynamically consistent modelling of two-phase flows with moving contact line and soluble surfactants,” *Journal of Fluid Mechanics*, vol. 879, pp. 327–359, 2019.

[30] Q. Zeng, W. Liu, J. Yao, and J. Liu, “A phase field based discrete fracture model (PFDFM) for fluid flow in fractured porous media,” *Journal of Petroleum Science and Engineering*, vol. 191, article 107191, 2020.

[31] M. F. Wheeler, T. Wick, and N. Wollner, “An augmented-Lagrangian method for the phase-field approach for pressurized fractures,” *Computer Methods in Applied Mechanics and Engineering*, vol. 271, pp. 69–85, 2014.

[32] A. Mikelic, M. F. Wheeler, and T. Wick, “Phase-field modeling of a fluid-driven fracture in a poroelastic medium,” *Computers and Geosciences*, vol. 19, no. 6, pp. 1171–1195, 2015.

[33] C. Miehe and S. Mauthke, “Phase field modeling of fracture in multi-physics problems. Part III. Crack driving forces in hydro-poro-elasticity and hydraulic fracturing of fluid-
saturated porous media,” *Computer Methods in Applied Mechanics and Engineering*, vol. 304, pp. 619–655, 2016.

[34] S. Zhou, X. Zhuang, and T. Rabczuk, “Phase-field modeling of fluid-driven dynamic cracking in porous media,” *Computer Methods in Applied Mechanics and Engineering*, vol. 350, pp. 169–198, 2019.

[35] S. Zhou, X. Zhuang, and T. Rabczuk, “A phase-field modeling approach of fracture propagation in poroelastic media,” *Engineering Geology*, vol. 240, pp. 189–203, 2018.

[36] C. Miehe, F. Welschinger, and M. Hofacker, “Thermodynamically consistent phase-field models of fracture: variational principles and multi-field FE implementations,” *International Journal for Numerical Methods in Engineering*, vol. 83, no. 10, pp. 1273–1311, 2010.

[37] A. Mikelic, M. F. Wheeler, and T. Wick, “A phase-field method for propagating fluid-filled fractures coupled to a surrounding porous medium,” *Multiscale Modeling and Simulation*, vol. 13, no. 1, pp. 367–398, 2015.

[38] S. Lee, M. F. Wheeler, and T. Wick, “Pressure and fluid-driven fracture propagation in porous media using an adaptive finite element phase field model,” *Computer Methods in Applied Mechanics and Engineering*, vol. 305, pp. 111–132, 2016.

[39] C. Miehe, M. Hofacker, and F. Welschinger, “A phase field model for rate-independent crack propagation: robust algorithmic implementation based on operator splits,” *Computer Methods in Applied Mechanics and Engineering*, vol. 199, no. 45-48, pp. 2765–2778, 2010.

[40] J. Geertsma and F. De Klerk, “A rapid method of predicting width and extent of hydraulically induced fractures,” *Journal of Petroleum Technology*, vol. 21, no. 12, pp. 1571–1581, 1969.