Negative reflections of electromagnetic waves in chiral media

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We investigate the reflection properties of electromagnetic/optical waves in isotropic chiral media. When the chiral parameter is strong enough, we show that an unusual negative reflection occurs at the interface of the chiral medium and a perfectly conducting plane, where the incident wave and one of reflected eigenwaves lie in the same side of the boundary normal. Using such a property, we further demonstrate that such a conducting plane can be used for focusing in the strong chiral medium. The related equations under paraxial optics approximation are deduced. In a special case of chiral medium, the chiral nihility, one of the bi-reflections disappears and only single reflected eigenwave exists, which goes exactly opposite to the incident wave. Hence the incident and reflected electric fields will cancel each other to yield a zero total electric field. In another word, any electromagnetic waves entering the chiral nihility with perfectly conducting plane will disappear.

PACS numbers. 78.20.Ci, 41.20.Jb, 42.25.Bs, 42.25.Gy

The chiral medium was first explored in the beginning of 19th century for its optical rotation phenomenon. Then it is proved that right and left-hand circularly polarized waves have different velocities and hence have different refraction indexes in the chiral medium. Different polarized rotations correspond to different modes. Therefore, bi-refraction happens at the boundary of chiral media even if they are isotropic, due to the coexistence of two different modes caused by chirality.

In 1968, Veselago introduced the concept of negative refraction when both permittivity and permeability are simultaneously negative.\textsuperscript{1} In the currently hot research of left-handed metamaterials, the chirality may be used to split the degenerated transverse wave modes. If the chiral parameter is strong enough or the chirality is combined with electric plasma,\textsuperscript{2–10} one eigenwave becomes backward wave, and a negative refraction is generated naturally in one of the circularly polarized waves.

The earlier research on chiral media is concentrated in the negative refraction and the relevant physical properties like the subwavelength focusing. The task of this paper is to discuss the extraordinary reflection properties of electromagnetic/optical waves in isotropic chiral media. We will prove that bi-reflection exists at the boundary of isotropic media. When the chiral parameter is strong enough, there will be a negative reflection for one of the reflected eigenwaves. Based on such a property, we show that a plane mirror instead of a lens can be used for focusing in the strong chiral medium and get a real image for paraxial rays. Finally, we discuss the behavior of electromagnetic waves in a chiral nihility with perfectly conducting plane. It is proved that only single reflected eigenwave exists, which goes exactly opposite to the incident wave. The incident and reflected electric fields will cancel each other to yield a zero total electric field. Hence we discover an exotic phenomenon that any electromagnetic waves entering the chiral nihility with perfectly conducting plane will disappear.

In this paper, we define the right-hand polarized wave is the one whose electric vector rotates clockwise when looking along the energy stream. Consider a half-infinite space problem, where the left region is an isotropic chiral medium and the right region is a perfectly electric conductor (PEC). An incident right-polarized wave propagates toward the boundary at an oblique angle $\theta_i$ in the $yoz$ plane, as illustrated in Fig. 1(a). Here, $k_x = 0$ has been assumed under the shown coordinate system.

Assume that the electric field of incident wave is expressed using the $e^{-i\omega t}$ system as

$$\vec{E}_i = E_0 e^{ik_y y + ik_z z}, \quad (1)$$

where $\vec{k} = \hat{y} k_y + \hat{z} k_z$ indicates the wavevector. According to Maxwell equations and the constitutive relation for
isotropic chiral media\textsuperscript{11}

\[
\begin{align*}
\vec{D} &= \varepsilon \vec{E} + i\kappa \vec{H}, \\
\vec{B} &= \mu \vec{H} - i\kappa \vec{E},
\end{align*}
\] (2, 3)

we obtain the following dispersion relation for the wavenumber \(k\):

\[
k_{\pm} = \omega (\sqrt{\mu/\varepsilon} \pm \kappa),
\] (4)

where “+” and “-” represent two different eigenwaves. In above expressions, \(\kappa\) indicates the chirality, which is assumed to be positive in this paper. Similar dual conclusions can be easily expanded to the negative chirality.

Correspondingly, the eigenwave vectors are given in terms of the free variable \(E_x\) as

\[
\begin{align*}
E_y &= \pm i E_x k_z/k_{\pm}, \\
E_z &= \mp i E_x k_y/k_{\pm}, \quad (5) \\
H_x &= \mp i E_y/\eta, \\
H_y &= E_z k_x/k_{\pm} \eta, \quad (6) \\
H_z &= -E_z k_y/k_{\pm} \eta,
\end{align*}
\] (7, 8, 9)

in which \(\eta = \sqrt{\mu/\varepsilon}\) is the wave impedance of chiral media. It is self-evident that it is left circularly polarized wave if \(k_{\pm}^2 + k_{\pm}^2 = k^2_+\), and right circularly polarized wave if \(k_{\pm}^2 + k_{\pm}^2 = k^2_-\). Hence the incident right polarized wave must match with one of the eigenwaves and be written as

\[
\vec{E}_i = E_0 (\hat{x} - \hat{y}ik_{\pm}/k_+ + \hat{z}ik_{\pm}/k_-) e^{ik_{\pm}y + ik_{\pm}z},
\] (10)
in which \(E_0\) is a free variable indicating the amplitude.

First we consider the case of weak chirality, where |\(\kappa| < \sqrt{\mu/\varepsilon}\). Traditionally, it is regarded as a natural limit to all chiral media for positive energy requirement. However, in the recent research on left-handed materials, we know that the energy calculation in dispersive media is not so simple, and negative wavenumber do not result in negative energy at all.\textsuperscript{12} Hence, it is fairly possible that strong chiral medium with \(\kappa > \sqrt{\mu/\varepsilon}\) also exists at some frequency,\textsuperscript{2} which will be discussed later in this paper.

Under weak chirality, both \(k_+\) and \(k_-\) are positive.

We need point out that reflected waves with different eigenmodes have different \(k\) and \(k_z\), but the same \(k_y\) due to the phase matching on the boundary. So we may draw

\[
k_{y+} = k_{y-} = k_y.
\] (11)

We assume that the projections of reflected energies and phase vectors on the \(z\) axis are both negative as the common sense. Though the incident wave is a right-polarized wave, we cannot ensure whether the reflected wave is right or left polarized. Hence, we suppose that both exist, and then use the boundary condition to calculate their coefficients.

It is clear that the projections of right- and left-polarized reflected wavenumbers on the \(z\) axis are \(-k_-\) and \(-k_+\). Here, both \(k_-\) and \(k_+\) are positive for propagating waves \((k_y < k_z)\). From the boundary condition on the PEC boundary, we have

\[
1 + A + B = 0, \quad (12)
\]

\[
-ik_-/k_- + A(ik_-)/k_- - B(ik_+)/k_+ = 0, \quad (13)
\]

where \(A\) and \(B\) are reflected coefficients of the right- and left-polarized waves separately. After simple derivation, we obtain

\[
A = (k_- k_- - k_- k_-)/k_- k_- + k_- k_- k_-, \quad (14)
\]
\[
B = -2k_- k_- / (k_- k_- + k_- k_- k_-). \quad (15)
\]

Hence, if there is any chirality, we have \(k_+ \neq k_-\), leading to \(A \neq 0\) and \(B \neq 0\). That is to say, both circularly polarized reflected waves exist. If there is no chirality, we have \(A = 0\). Hence the whole reflected wave is left polarized for the right-polarized incident wave. And this degeneration is just the common case of circularly polarized wave reflection in non-chiral medium.

For the left- and right-polarized reflected waves, their \(k\) vectors are: \(k_\pm = \hat{y}k_y - \hat{z}k_z\), and the corresponding Poynting vectors are written as

\[
\begin{align*}
\vec{S}_{r+} &= \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*) = \frac{|E_x B|^2}{k_+} \sqrt{\frac{\mu}{\varepsilon}} (\hat{y}k_y - \hat{z}k_z), \quad (16) \\
\vec{S}_{r-} &= \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*) = \frac{|E_x A|^2}{k_-} \sqrt{\frac{\mu}{\varepsilon}} (\hat{y}k_y - \hat{z}k_z). \quad (17)
\end{align*}
\]

We can see that neither \(k\) nor \(S\) vectors of the two polarized reflected waves are the same in chiral media.

Hence two different eigenwaves will be generated from the same incident wave in a boundary shown in Fig. 1(a), resulting in bi-reflections. It seems not to satisfy geometrical optics principles at the first glance. However, the chiral medium is a special material with such unique characters: the refraction indexes \(n\) for a pair of right- and left-polarized waves do not equal. In other words, the chiral medium is one material to the right-polarized wave and another to the left one. Considering the difference between \(n_+\) and \(n_-\), the direction of each polarized reflected wave satisfies Fermat principle.

For the right-polarized reflected wave, its reflected angle equals the incident angle, i.e., \(\theta_{r-} = \theta_i\), because the refraction index of reflected wave equals the incident one. For the left-polarized one, the reflected angle \(\theta_{r+}\) satisfies

\[
\frac{\sin \theta_{r+}}{\sin \theta_{r-}} = \frac{k_+}{k_-}, \quad \frac{n_+}{n_-}, \quad (18)
\]

which is similar to the Snell’s law, since the refraction index of reflected wave is different from the incident one. Here, \(n_\pm = \sqrt{\mu/\varepsilon} \pm \kappa\) represent refraction indexes of the two eigenwaves in chiral media.
We make another explanation of the bi-reflections in chiral media. If we consider the PEC boundary as a perfect mirror, then we may get an image of incident wave as an effective problem shown in Fig. 1(b), in which the mirrored chiral medium has an opposite \( \kappa \). The chirality is generated by spatial asymmetry, hence it should be reversed if the material structure is mirrored. In the mirrored chiral medium, the right-polarized wave corresponds to \( k_+ \) and the left-polarized wave to \( k_- \). Then we may turn the reflection problem into a refraction problem. From the boundary condition, we get the same result as Eqs. (14) and (15), indicating that bi-refraction happens on the boundary between the two dual chiral media. Hence we may also put \( k_{r\pm} \) as the transmission of \( k_i' \). In other words, bi-reflection shares the common essence with bi-refraction.

Next, we consider the case of strong chiral media, where more interesting characters will appear. When \( \kappa > \sqrt{\mu \varepsilon} \), we have \( k_- = \sqrt{\mu \varepsilon} - \kappa < 0 \). Hence the right-polarized wave turns into a backward wave. That is to say, \( E, H \) and \( k \) form a left-handed triad and the Poynting vector \( S \) is antiparallel to \( k \). However, the left-polarized wave remains right-handed as in common media. When the incident wave is left circularly polarized, it is a forward wave, as shown in Fig. 2(a). As the reflection happens, the left-polarized reflected wave goes normally while the right-polarized one is a backward wave. Here, we will illustrate that a negative reflection happens for the backward eigenwave.

Fig. 2. Strong chirality makes negative reflections. (a) Left-polarized incidence. (b) Right-polarized incidence.

Based on the phase matching on the boundary, the \( k_y \) components of both reflected waves should be \( +\hat{y} \) directed. Based on the causality principle, the \( S_z \) components of both reflected waves should be \(-\hat{z}\) directed. Hence the left-polarized reflected wave goes normally as that in the weak-chirality case with \( \theta_{r-} = \theta_i \). For the right-polarized reflected wave, \( k_z \) is antiparallel to \( S_z \) and hence a negative reflection occurs, where the incident and reflected wavevectors lie in the same side of the boundary normal, as shown in Fig. 2(a). The reflected angle \( \theta_{r-} \) satisfies the Snell-like law

\[
\frac{\sin \theta_{r-}}{\sin \theta_i} = -\left| \frac{k_+}{k_-} \right|, \tag{19}
\]

which yields a negative reflected angle.

Correspondingly, the reflection coefficients are given by

\[
A_L = (k_- k_{z+} + k_+ k_{z-})/(k_- k_{z+} - k_+ k_{z-}), \tag{20}
\]

\[
B_L = -2k_- k_{z+}/(k_- k_{z+} - k_+ k_{z-}), \tag{21}
\]

in which \( A_L \) corresponds to the left-polarized wave, and \( B_L \) corresponds to the right-polarized wave. For the right-polarized reflected wave,

\[
\bar{E}_{r-} = B_L E_0 (\hat{x} - \frac{ik_z}{k_-} \hat{y} + \frac{ik_y}{k_-} \hat{z}) e^{ik_y y + ik_z z}, \tag{22}
\]

\[
S_{r-} = \frac{|E_r B_L|^2}{k_+} \sqrt{\mu} (k_y \hat{y} + k_z \hat{z}). \tag{23}
\]

We remark that \( k_{z-} \) has been assigned as positive in this paper. Considering \( k_- < 0 \), it is clear that \( S_{r-} \) is antiparallel to \( \bar{E}_{r-} \) for the right-polarized reflected wave. Negative reflection really happens.

In case of right circularly polarized incident wave, it is a backward wave, as shown in Fig. 2(b). Now the wave vector for incident wave is \(-k_y \hat{y} - k_z \hat{z}\). Similarly, there are a normal reflection with \( \theta_{r-} = \theta_i \), and a negative reflection for the left-polarized reflected wave with

\[
\frac{\sin \theta_{r+}}{\sin \theta_i} = -\left| \frac{k_-}{k_+} \right|. \tag{24}
\]

Correspondingly, the reflection coefficients are given by

\[
A_R = (k_- k_{z-} + k_+ k_{z+})/(k_- k_{z-} - k_+ k_{z+}), \tag{25}
\]

\[
B_R = -2k_- k_{z-}/(k_- k_{z-} - k_+ k_{z+}), \tag{26}
\]

in which \( A_R \) corresponds to the right-polarized wave, and \( B_R \) corresponds to the left-polarized wave. For the left-polarized reflected wave,

\[
\bar{E}_{r+} = B_R E_0 (\hat{x} - \frac{ik_z}{k_+} \hat{y} + \frac{ik_y}{k_+} \hat{z}) e^{-ik_y y - ik_z z}, \tag{27}
\]

\[
S_{r+} = \frac{|E_r B_R|^2}{k_+} \sqrt{\mu} (-k_y \hat{y} - k_z \hat{z}). \tag{28}
\]

We see that the left-polarized reflected wave and the incident wave lie in the same side of normal. Negative reflection happens again.

Using such unusual reflection properties, we may realize partial focusing of a source using a simple PEC mirror. Actually the field generated by a source can be decomposed as left- and right-polarized waves. For both polarized-wave incidences, the reflected waves will be partially focused, as shown in Figs. 3(a) and 3(b), respectively.

Taking paraxial approximation in Gaussian optics, we may get a reasonably good image for this partial focusing. Assume that the distance from the source to
PEC mirror is $s$. Then the position of image point is 
$s_L' = -sk_+/k_+$ for the left-polarized incident wave, and 
$s_R' = -sk_-/k_-$ for the right-polarized incident wave.

It is true that the positively reflected wave will diverge, 
not participating in the partial focusing but forming an 
imaginary image. However, in the paraxial case, we have 
k_+ \cong k_{++} and $k_- \cong -k_{--}$ for the strong chiral medium.
Hence the amplitudes of positively-reflected waves are 
close to zero, which may be neglected. That is to say, 
most paraxial rays reflected negatively for partial focusing.

Considering the negative reflections in the strong chiral 
medium, some conclusions in the conventional Gaussian optics need to be improved. For one thing, the real 
images we get are not upsidedown as real images always do. On the other hand, we may generalize our analysis into spherical reflection surface. In strong chiral medium, the reflection relationship between object and image distances can be written as:

$$\frac{k_+}{s} + \frac{k_-}{s_L'} = -\frac{k_+ + k_-}{R},$$
$$\frac{k_-}{s} + \frac{k_+}{s_R'} = -\frac{k_+ + k_-}{R},$$

where $R$ is the radius of the spherical surface, which is 
positive if convex and negative if concave, and $s' < 0$ 
for the imaginary image. In the weak chirality case, we may draw the same results as those in Eqs. (29) and (30) 
though there is no negative reflection. These are the 
general reflection relationships of all chiral media in 
paraxial Gaussian optics.

It will be more interesting to discuss a special case of chiral medium: the chiral nihility with $\mu = 0$. In chiral nihility, we easily have $k_\pm = \pm \omega \kappa$. Hence, the corresponding physical features are quite similar to those in the strong-chirality medium, and the formulations (19)-(28) can be directly used. For propagating waves ($|k_y| < \omega \kappa$), we obtain $k_{z-} = k_{z+}$ under our definition in this paper.

If the incident wave is left polarized, one easily obtains 
$A_L = 0$ and $B_L = -1$. That is to say, the left-polarized 
reflected wave disappears, and a total reflection occurs to 
the right-polarized reflected wave, as shown in Fig. 4(a). 
Here, the wavevectors of incident and reflected waves are 
the same (directing to the up-right direction), while the 
Poynting vectors are opposite.

When the incident wave is right polarized, we then 
have $A_R = 0$ and $B_R = -1$. That is to say, the right-
polarized reflected wave disappears, and a total reflection 
occurs to the left-polarized reflected wave, as illustrated in 
Fig. 4(b). Again, the wavevectors of incident and reflected waves are the same (directing to the down-left direction), while the Poynting vectors are opposite.

Based on the above discussions, we can easily show 
that the totally reflected electric fields counteract the in-
cident electric fields exactly in both polarized incidences 
in the chiral nihility, which results in zero total electric fields. If $\epsilon = 0$ and $\mu \neq 0$, we can show that all magnetic fields must be zero from the dispersion equation. In such a case, all total electric and magnetic fields disappear 
in the chiral nihility. If $\epsilon = 0$ and $\mu = 0$, the magnetic 
fields may exist because the electric and magnetic fields 
are decoupled completely.

For evanescent waves ($|k_y| > \omega \kappa$), however, we have to 
set $k_{z-} = -k_{z+}$ to satisfy the causality under our defini-
tion in this paper. If the incident wave is left polarized, 
then $A_L \to \infty$ and $B_L \to \infty$, which is similar to the case 
of Pendry’s perfect lens. 

It is more interesting to consider a chiral nihility 
bounded by two PEC mirrors. When a wave is excited 
in the chiral nihility, the wave will be totally reflected 
forwardly and backwardly again and again between two 
mirrors based on the earlier discussions, as shown in Fig. 
5. All waves from the source will focus at the source 
point. Using the boundary conditions, we have shown 
exactly that the total reflected electric fields including 
left- and right-polarized components at any points sat-
sify

$$\vec{E}_r = \vec{E}_{r+} + \vec{E}_{r-} = -\vec{E}_i,$$

which is valid to both propagating-wave and evanescent-
wave incidences. Hence, the total electric fields at any 
points inside the chiral nihility are zero. If $\epsilon = 0$ and 
$\mu \neq 0$, the total magnetic fields are also zero. Hence,
a source could not radiate effectively inside the chiral nihility bounded by two PEC mirrors.

![Fig. 5. Chiral nihility bounded by two PEC mirrors.](image)

In conclusions, negative reflections occur at the boundary of strong-chiral medium and PEC mirror, which directly result in partial focusing using a simple plane mirror. Any propagating waves entering the chiral nihility ($\epsilon = 0$ and $\mu \neq 0$) with a PEC plane will disappear. Any sources could not radiate inside the chiral nihility bounded by two PEC mirrors.

This work was supported in part by the National Basic Research Program (973) of China under Grant No. 2004CB719802, in part by the National Science Foundation of China for Distinguished Young Scholars under Grant No. 60225001, and in part by the National Doctoral Foundation of China under Grant No. 20040286010. Email: tjcui@seu.edu.cn.

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