Non Standard $\eta - \eta'$ mixing and the Nonleptonic $B$ and $\Lambda_b$ Decays to $\eta$ and $\eta'$

Alakabha Datta\textsuperscript{1}, Harry J. Lipkin\textsuperscript{2} and Patrick. J. O'Donnell\textsuperscript{3}

\textsuperscript{1}Laboratoire René J.-A. Lévesque, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7
\textsuperscript{2}Department of Particle Physics, Weizmann Institute, Rehovot 76100, Israel and School of Physics and Astronomy, Tel-Aviv University, Tel-Aviv 69978, Israel
\textsuperscript{3}Department of Physics and Astronomy, University of Toronto, Toronto, Canada.

Abstract

Radial mixing in the pseudoscalar $\eta - \eta'$ systems can be generated from hyperfine interactions and annihilation terms. For the $\eta - \eta'$ system we find the effects of radial mixing are appreciable and seriously affect the decay branching ratios for $B \to \eta(\eta')K(K^*)$, mainly by modifying the $B \to \eta(\eta')$ form factors. In particular, the effect of radial mixing in conjunction with the interference effects among penguin

\textsuperscript{1}email: datta@lps.umontreal.ca
\textsuperscript{2}email: harry.lipkin@weizmann.ac.il
\textsuperscript{3}email: pat@medb.physics.utoronto.ca
amplitudes can resolve puzzles in the $B \to \eta(\eta')K$ decays. The decay $\Lambda_b \to \Lambda\eta(\eta')$ on the other hand is dominated by a single amplitude so that the significant interference effects of $B$ decays are absent here. Moreover, since no $\Lambda_b \to \eta(\eta')$ form factors are involved here, the effect of radial mixing is essentially negligible. Hence, unlike the $B$ system, we do not predict a large enhancement of $\Lambda_b \to \Lambda\eta'$ relative to $\Lambda_b \to \Lambda\eta$.

Nonleptonic $B$ decays play a very important role in the study of CP violation. It is expected that these will test the standard model (SM) picture of CP violation or provide hints for new physics. Some clues to possible new physics may be given by recent experimental data for $B$ and $D$ decays into final states containing the $\eta$ and $\eta'$ pseudoscalar mesons. So far these have remained unexplained in the standard treatments. One possible source of this disagreement comes from the fact that most models describe the $\eta$ and $\eta'$ mesons as node-less ground-state s-wave $q\bar{q}$ systems. In a previous paper [1], we have shown that in a factorization approximation, suitable for the treatment of two body decays, radial excitations are favored. We will follow this treatment for the pseudoscalar mesons and consider the $\eta$ and $\eta'$ wave functions to be mixtures of the ground state and radially excited $q\bar{q}$ states. This will alter the high momentum behavior of the $\eta$ and $\eta'$ wave functions. So, for example, for $B \to \eta(\eta')K(K^*)$ decays, as shown in Fig. 1(a), factorization results in the kaon leaving the weak vertex with its full momentum; the remaining quark carries the full momentum of the final $\eta(\eta')$ meson. A large internal momentum transfer is needed to hadronize this quark with the spectator anti-quark to form an $\eta$ or $\eta'$ final state. This will favor radial excitations since they have a much higher mean internal momentum. On the other hand, there is no such diagram in the decay $\Lambda_b \to \Lambda\eta(\eta')$ and we should not expect any similar radial enhancement of the form factors.

The most general description of $\eta-\eta'$ system involves four different radial wave functions and cannot be described by diagonalizing a simple $2 \times 2$ matrix with a single mixing angle [2]. The normalized $\eta-\eta'$ wave functions may be written in terms of two angles as

$$
|\eta\rangle = \cos \phi \, |N\rangle - \sin \phi \, |S\rangle,
$$

$$
|\eta'\rangle = \sin \phi' \, |N'\rangle + \cos \phi' \, |S'\rangle,
$$

where $|N\rangle$, $|N'\rangle$, $|S\rangle$ and $|S'\rangle$ are arbitrary isoscalar non-strange and strange quark-antiquark wave functions, respectively. In the traditional picture where the $\eta-\eta'$ mixing is described by a single mixing angle,

$$
|N\rangle = |N'\rangle,
$$
Figure 1: The diagrams contributing to $B \to \eta(\eta')K(K^*)$ decays. The dashed line represents a gluon or a $\gamma$ or a $Z$ boson. Tree diagrams are not shown. For the decays $\Lambda_b \to \Lambda\eta(\eta')$ diagram (a) is absent.

$$|S\rangle = |S'\rangle, \quad \phi = \phi'.$$

Some theoretical input [3] is needed to obtain the mixing angle $\phi$ from experimental data; tests of the standard mixing case involving $B$ decays have been discussed in Ref. [2].

In the absence of a model independent extraction of $\phi$ we will use as our standard mixing the Isgur mixing [4]

$$|\eta\rangle_{std} = \frac{1}{\sqrt{2}} [N_0 - S_0]$$

$$|\eta'\rangle_{std} = \frac{1}{\sqrt{2}} [N_0 + S_0]$$

which corresponds to the mixing angle $\phi = 45^0$. We will treat the mixing given here as “standard” in the sense that we will compare results calculated with non-standard mixing
coming from radially excited components in the $\eta(\eta')$ wave functions with those given in Eq. (3).

To obtain the eigenstates and eigenvalues in the $\eta-\eta'$ system, including radial excitations, we diagonalize the mass matrix

$$<q_a'q_b', n'|M|q_aq_b, n> = \delta_{aa'}\delta_{bb'}\delta_{nn'}(m_a + m_b + E_n) + \delta_{aa'}\delta_{bb'} \frac{B}{m_a m_b} \vec{s}_a \cdot \vec{s}_b \psi_n(0)\psi_{n'}(0)$$

$$+ \delta_{ab}\delta_{a'b'} A \frac{1}{m_a m_b} \psi_n(0)\psi_{n'}(0).$$

(4)

where $\vec{s}_{a,b}$ and $m_{a,b}$ are the quark spin operators and masses. Here $n = 0, 1, 2$ and the basis states for the isoscalar mesons are chosen as $|N> = |u\bar{u} + d\bar{d}>/\sqrt{2}$ and $|S> = |s\bar{s}>$ for the non–strange and strange wavefunctions. The excitation energy of the $n^{th}$ radially excited state is $E_n$.

This has the same structure as that of the vector system involving the $\omega - \phi$ mesons. The only differences between the pseudoscalar and vector mass matrices are in the values of the parameters, which are determined by fitting the experimental masses.

The mixing of the strange and non–strange components in the $\eta$ and $\eta'$ wave functions is assumed usually to come from a short-range flavor-singlet interaction which is absent for the pion and kaon. This was shown to have a negligible effect on the vector mesons. This flavor mixing contribution was called the “annihilation” contribution in earlier papers on $\eta-\eta'$ mixing. This phenomenological contribution, $A$, which we shall continue to refer to as an annihilation contribution, has no direct relation to the annihilation diagrams of QCD. The QCD origin of flavor mixing in the $\eta-\eta'$ system is not fully understood and probably has to do with gluon effects and the non–trivial structure of the QCD vacuum. We will assume that these effects are included in an effective parameter $A$. When the parameter $A=0$, we see that in the vector case the $\rho$ and $\omega$ masses are equal while in the pseudoscalar case the $\pi$ and $\eta$ masses are equal. Thus, fitting the experimentally small $\rho-\omega$ mass difference and the experimentally large $\pi-\eta$ mass difference immediately requires a small value for $A$ and small flavor-mixing for the vectors and a large value for $A$ and large flavor-mixing for the pseudoscalars.

In addition to this flavor–mixing interaction we also include a short–range hyperfine interaction with a strength $B$. As in we have allowed the annihilation term to have a flavor dependence from mass factors and the wave functions, modeling it on the hyperfine interaction. Evidence for flavor dependence of the annihilation term in $\eta-\eta'$ mixing has been known for some time. Even without knowledge of the QCD origin of this term we
can see that it may be considered as a short-range repulsion which acts only on a singlet state, with the matrix element vanishing for the octet state in the $SU(3)$ limit. In a simple quark potential model this implies that the singlet and octet states have different radial wave functions.

These effects break nonet symmetry and necessarily correct all results which assume that members of the pseudoscalar nonet all have the same radial wave function. It is this non–standard $\eta(\eta')$ mixing that has important implications for the nonleptonic decays $B \to \eta(\eta')K(K^*)$.

In a quark potential model, flavor-$SU(3)$ breaking by quark masses makes the kinetic energies of the strange and non–strange components of the pseudoscalars different. Thus the $\eta - \eta'$ system has now four different radial wave functions and is no longer described by a simple $2 \times 2$ matrix with one mixing angle. Our approach here is to follow the example of Refs. [6, 8] and choose a basis for our radial wave functions. In this basis the difference in radial wave functions of the strange and non–strange parts of the $\eta - \eta'$ system is described by admixtures of radial excitations. This allows us to write the normalized $\eta - \eta'$ wave functions as

$$
|\eta\rangle = a_1 |N\rangle + a_2 |S\rangle,
$$

$$
|\eta'\rangle = a'_1 |N'\rangle + a'_2 |S'\rangle,
$$

where the normalized states $|N\rangle$, $|N'\rangle$, $|S\rangle$ and $|S'\rangle$ have the general structure

$$
|N\rangle = \frac{|N_0\rangle + b_1 |N_1\rangle + b_2 |N_2\rangle}{\sqrt{1 + b_1^2 + b_2^2}},
$$

$$
|N'\rangle = \frac{|N_0'\rangle + b_1' |N_1'\rangle + b_2' |N_2\rangle}{\sqrt{1 + b_1'^2 + b_2'^2}},
$$

$$
|S\rangle = \frac{|S_0\rangle + c_1 |S_1\rangle + c_2 |S_2\rangle}{\sqrt{1 + c_1^2 + c_2^2}},
$$

$$
|S'\rangle = \frac{|S_0'\rangle + c_1' |S_1'\rangle + c_2' |S_2\rangle}{\sqrt{1 + c_1'^2 + c_2'^2}},
$$

with $|N >_{0,1,2} = |u\bar{u} + d\bar{d} >_{0,1,2} / \sqrt{2}$ and $|S >_{0,1,2} = |s\bar{s} >_{0,1,2}$ representing the various non–strange and strange radial excitations. The normalization of the $\eta - \eta'$ wave functions in Eq. (5) then leads to $a_1^2 + a_2^2 = a_1'^2 + a_2'^2 = 1$.

The values of $A$ and $B$ in the mass matrix, Eq. (4), are fitted from the measured masses. We use the phase convention of Ref. [8] where the wave functions at the origin in configuration...
space, which enter in the hyperfine and annihilation terms in the mass matrix, are positive (negative) for the even (odd) radial excitations. The mass matrix is a $6 \times 6$ matrix which is diagonalized to give the six masses and mixings. However, for our purposes we will only need the predictions for the $\eta$ and $\eta'$ masses and wave functions. Several solutions that give acceptable values of the masses can be obtained. We choose those solutions for the linear, quadratic and quartic confining potentials that are similar in predictions for the $\eta(\eta')$ masses.

With $B = 0.065m_u^2$ we obtain the eigenvalues and eigenstates in Table 1 for the linear potential, in Table 2 for the harmonic potential and in Table 3 for the quartic potential. Our results for the harmonic potential are similar to those of Ref. [8] where a slightly different mass mixing matrix has been used to obtain the $\eta - \eta'$ mixing.

Table 1: Eigenvalues and eigenstates for the $\eta - \eta'$ system with $A = 0.045m_u^2$

| Linear | $N_0$ | $N_1$ | $N_2$ | $S_0$ | $S_1$ | $S_2$ |
|--------|--------|--------|--------|--------|--------|--------|
| $\eta$(0.544) | 0.961 | -0.198 | 0.108 | -0.150 | 0.050 | -0.032 |
| $\eta'$(0.924) | 0.170 | 0.039 | -0.016 | 0.974 | -0.126 | 0.049 |

Table 2: Eigenvalues and eigenstates for the $\eta - \eta'$ system with $A = 0.065m_u^2$

| Harmonic | $N_0$ | $N_1$ | $N_2$ | $S_0$ | $S_1$ | $S_2$ |
|----------|--------|--------|--------|--------|--------|--------|
| $\eta$(0.547) | 0.913 | -0.252 | 0.154 | -0.249 | 0.107 | -0.076 |
| $\eta'$(0.931) | 0.316 | 0.109 | -0.049 | 0.925 | -0.148 | 0.088 |

Table 3: Eigenvalues and eigenstates for the $\eta - \eta'$ system with $A = 0.11m_u^2$

| Quartic | $N_0$ | $N_1$ | $N_2$ | $S_0$ | $S_1$ | $S_2$ |
|---------|--------|--------|--------|--------|--------|--------|
| $\eta$(0.547) | 0.764 | -0.287 | 0.198 | -0.441 | 0.248 | -0.196 |
| $\eta'$(0.940) | 0.623 | 0.350 | -0.177 | 0.658 | -0.134 | 0.087 |

The entries for $\eta(\eta')$ mixing in Tables 1, 2 and 3 are the coefficients for the eigenstates in Eqs. 5, 6. These are sensitive to the confining potential; there can be substantial radial mixing, affecting the predictions for $B \to \eta(\eta')K(K^*)$ decays. In going from linear
to quadratic to quartic potential the $\eta(\eta')$ mixing deviates more from the ideal mixing case. This corresponds to the increase in value of $A$.

Let $|\eta\rangle_g$ and $|\eta'\rangle_g$ represent the (unnormalized) portions of the physical wave functions that are in the ground state configuration in Tables (4-6). These states may then be written in terms of the “standard” $\eta-\eta'$ states defined in Eq. (3). For the linear potential we find,

\begin{align}
|\eta\rangle_g &= 0.81 |\eta\rangle_{std} - 0.57 |\eta\rangle_{std}, \\
|\eta\rangle_g &= 0.79 |\eta\rangle_{std} + 0.57 |\eta\rangle_{std}. 
\end{align}

(7)

For the harmonic potential we find,

\begin{align}
|\eta\rangle_g &= 0.88 |\eta\rangle_{std} - 0.43 |\eta\rangle_{std}, \\
|\eta\rangle_g &= 0.82 |\eta\rangle_{std} + 0.47 |\eta\rangle_{std}. 
\end{align}

(8)

For the quartic potential we find,

\begin{align}
|\eta\rangle_g &= 0.91 |\eta\rangle_{std} - 0.025 |\eta\rangle_{std}, \\
|\eta\rangle_g &= 0.85 |\eta\rangle_{std} + 0.23 |\eta\rangle_{std}. 
\end{align}

(9)

Thus all three confining potentials give mixings for the $\eta - \eta'$ that have substantial overlap with the “standard mixing”. The mixing from the quartic potential is closest to the standard mixing in the sense that here one has the smallest component of the $|\eta\rangle_{std} (|\eta\rangle_{std}$ in $|\eta\rangle_g (|\eta\rangle_g)$. Several tests of non–standard mixing have been discussed in Ref[2]. Of particular interest, for non lepton B decays, are the ratios defined in [2] as

\begin{align}
rd &\equiv \frac{p_{\eta}^{2} \Gamma(B^{0} \rightarrow J/\psi\eta)}{p_{\eta}^{2} \Gamma(B^{0} \rightarrow J/\psi\eta')}, \\
rs &\equiv \frac{p_{\eta}^{2} \Gamma(B^{0}_{s} \rightarrow J/\psi\eta)}{p_{\eta}^{2} \Gamma(B^{0}_{s} \rightarrow J/\psi\eta')}. 
\end{align}

(10)

(11)

We then have the prediction, for the standard mixing,

\begin{align}
r &= \sqrt{rdrs} = \sqrt{\cot^{2}\phi \tan^{2}\phi} = 1. 
\end{align}

(12)

On the other hand, with the non–standard mixing, the ratio $r$ can be quite different from unity. In particular, with the mixing in Table [3], we found that $r$ could be as low as $\sim 0.2$ [4].
Having now defined how we will construct the $\eta(\eta')$ wave functions we now consider $B \to \eta(\eta')K(K^*)$ decays. These are dominated by the penguin diagrams shown in Fig. 1 since the tree term is color and CKM suppressed.

In Fig. 1(a) factorization results in the kaon leaving the weak vertex with its full momentum; the remaining quark combines with the spectator quark to form the final $\eta(\eta')$ meson. Fig. 1(b) shows the $\bar{s}$ quark in the QCD penguin combining with the $s$ quark from the $b \to s$ transition to form the $\eta(\eta')$. Another possibility is shown in Fig. 1(c) in which a $q\bar{q}$ pair (where $q = u, d, s$) appearing in the same current in the effective Hamiltonian, hadronizes to the $\eta(\eta')$. This term is often referred to as being OZI suppressed with respect to the other terms in the decay amplitude. However, this may not be the case for $\eta'$ in the final state because the OZI suppressed terms add constructively while, for the $\eta$ in the final state the OZI suppressed terms tend to cancel among themselves.

We expect the OZI suppressed terms to be more important in $B \to KP$ than in $B \to KV$ decays where $P$ is a pseudoscalar and $V$ is a vector state. This follows from the fact that in $J/\psi$ and $\Upsilon$ decays we know that the OZI-forbidden process requires three gluons for coupling to a vector meson and two gluons for coupling to a pseudoscalar. Thus the contribution of the OZI suppressed term should be much smaller in the $B \to K\rho^0(\omega)$ and $B \to K\phi$ decays than in $B \to K\eta$ and $B \to K\eta'$ decays.

If it is assumed that the OZI terms are forbidden then definite predictions about the branching ratios $B \to \eta K/B \to \eta'K$ and $B \to \eta K^*/B \to \eta'K^*$ are possible. Here we will first derive these predictions using the factorization assumption and the standard $\eta - \eta'$ mixing in Eq. (3). We will then study how the predictions change with non–standard mixing for $\eta - \eta'$ and with the inclusion of the OZI terms. For $B$ decays, we define the two ratios

$$R_K = \frac{BR(B^- \to K^-\eta)}{BR(B^- \to K^-\eta')}$$

and

$$R_{K^*} = \frac{BR(B^- \to K^{*-}\eta)}{BR(B^- \to K^{*-}\eta')}$$

and, for $\Lambda_b \to \Lambda(\eta')$, define the ratio

$$R_\Lambda = \frac{BR(\Lambda_b \to \Lambda\eta)}{BR(\Lambda_b \to \Lambda\eta')}$$

It has been shown that there is a parity selection rule for the decays $B \to \eta(\eta')K(K^*)$. This fixes the relative phase between the penguin amplitudes, Fig. 1(a) and Fig. 1(b), for
the strange and non–strange contributions to the $\eta$ and $\eta'$ final states. In particular, the parity selection rule predicts that there is a relative sign difference between the strange and non–strange penguin amplitudes in $B \to K\eta(\eta')$ and $B \to K^*\eta(\eta')$.

With the use of factorization and this parity selection rule, we find

$$R_K \approx \left| \frac{f_K F_{\eta}^+ + f_{\eta}^s F_{\eta}^+}{f_K F_{\eta'}^+ + f_{\eta'}^s F_{\eta'}^+} \right|^2,$$

$$R_{K^*} \approx \left| \frac{f_{K^*} F_{\eta}^+ - f_{\eta}^s F_{\eta}^+}{f_{K^*} F_{\eta'}^+ - f_{\eta'}^s F_{\eta'}^+} \right|^2,$$

$$R_\Lambda \approx \left| \frac{f_{\eta}^s F_\Lambda + f_{\eta'}^s F_\Lambda}{f_{\eta'}^s F_{\eta}^+ + f_{\eta'}^s F_{\eta'}^+} \right|^2,$$

where we have set the masses of the pseudoscalars in the final states to zero and introduced the form factor $F^+$ [16]. For $\Lambda_b$ decays the form factors $F_\Lambda$ as well as the heavy quark relations between them can be found in Ref. [17]; we have also neglected terms of $O(m_\Lambda/m_{\Lambda_b})$.

In the above equation $f_K$ is the kaon decay constant and the decay constants $f_{\eta}^q$ and $f_{\eta'}^q$ are defined by

$$i f_{\eta(\eta')}^q P^\mu_{\eta(\eta')} = \langle \eta(\eta') | \bar{q} \gamma^\mu (1 - \gamma_5) q | 0 \rangle.$$

For standard mixing, Eq. (3), and assuming $SU(3)$ flavor symmetry, we obtain the following relations between the various form factors and decay constants appearing in Eq. (16).

$$f_{\eta}^{u,d} \approx \frac{f_K}{2}; f_{\eta}^s \approx \frac{-f_K}{\sqrt{2}}; f_{\eta'}^{u,d} \approx \frac{f_K}{2}; f_{\eta'}^s \approx \frac{f_K}{\sqrt{2}},$$

$$F_{\eta}^+ \approx \frac{F_K^+}{2}; F_{\eta'}^+ \approx \frac{F_K^+}{2}.$$

From Eqs. (16, 18) we find

$$R_K \approx \left| \frac{\frac{1}{2} - \frac{1}{\sqrt{2}}}{\frac{1}{2} + \frac{1}{\sqrt{2}}} \right|^2 \approx 0.03.$$

Hence there is substantial interference between the two penguin amplitudes. This gives a small value for $R_K$, consistent with the current experimental limits. Neglecting differences in form factors between $B \to P$ to $B \to V$ transitions, we find

$$R_{K^*} \approx \left| \frac{\frac{1}{2} + \frac{1}{\sqrt{2}}}{\frac{1}{2} - \frac{1}{\sqrt{2}}} \right|^2 \approx \frac{1}{R_K} = 33.$$
For the $\Lambda$ ratio, only Fig. Ib contributes so that

$$R_\Lambda \sim 1$$  \hspace{1cm} (21)

We now calculate the ratios $R_{K(K^*)}$ and $R_\Lambda$ with a nonstandard $\eta - \eta'$ mixing. This is a more complicated calculation and so we will use the mixing for a quartic potential, given in Table (3), as an example, since the ground states of this mixing are those most similar to the standard mixing, Eq. (3).

A key ingredient in the parity selection rule that predicts the relative phases of the strange and non–strange contributions to $\eta$ and $\eta'$ final states is approximate flavor symmetry. In the flavor symmetry limit the radial wave functions of $\pi$, $K$, $\eta$ and $\eta'$ (up to mixing factors in $\eta(\eta')$) are all the same. Since all are ground state wave functions with no nodes and a constant phase over the entire radial domain, flavor symmetry breaking may change the radial shape and size of the wave function but will not change the relative phases. However, when radially excited wave functions, with nodes that differ from the ground state wave functions, are also considered, there may be complicated changes in the relative phases.

For example, with factorization, $B$ decay into the non–strange part of the $\eta$ and $\eta'$ involves a point–like form factor for the kaon and a hadronic overlap integral for the $\eta(\eta')$. On the other hand, $B$ decay into the strange part of the $\eta$ and $\eta'$ involves a point–like form factor for the $\eta(\eta')$ and a hadronic overlap integral for the kaon. A phase ambiguity can therefore arise between the two penguin amplitudes coming from the hadronic overlap integral for the $\eta$ and $\eta'$ since it involves a mixing among different parts of wave functions.

One way to simulate flavor symmetry breaking would be to define, for the $\eta(\eta')$, an effective wave function

$$\Psi_{eff} = a_0 \Psi_{N_0} + a_1 r_1 \Psi_{N_1} + a_2 r_2 \Psi_{N_2}$$  \hspace{1cm} (22)

where the $a_i$'s are from Table 3 and $r_1$ and $r_2$, representing flavor breaking effects, can have either sign; form factors can then be obtained from this effective wave function. We will consider two choices of $r_1$ and $r_2$. In the first, and simplest, case we set $r_1 = r_2 = 1$. Otherwise, we choose $r_1 = -r_2$ so that the contributions from the various radial excitations and the ground state add constructively.

The problem of calculating form factors for transitions to radially excited states warrants a separate investigation of its own and is beyond the scope of this work. Therefore for a decay to a radially excited state $M'$ we use for the form factor $B \to M'$

$$B \to M' = (B \to M'/B \to M) \times B \to M.$$
The first factor in the right hand side of the above, \( B \to M'/B \to M \), is obtained from a constituent model \[18\]. While this is a simplified quark model that is not expected to correctly predict the absolute values of the form factors, it does correctly exhibit the scaling properties of the form factors expected from heavy quark symmetry. We use this model to predict the ratio \( B \to M'/B \to M \) assuming that corrections to the form factors, calculated in a more complete model, cancel in the ratio. The second factor \( B \to M \), representing the form factors for transition from \( B \) to the ground state \( M \), is taken from Ref. \[16\]. The decay constants in Eq. (17) are calculated from the wave functions for the quartic potential using the formula in Ref \[19\].

For the \( F_+ \) form factor we find

\[
\frac{F^{\eta'}_{+\text{nonstandard}}}{F^{\eta'}_{+\text{standard}}} \approx 1.5, 1.7 \quad ; \quad \frac{F^{\eta}_{+\text{nonstandard}}}{F^{\eta}_{+\text{standard}}} \approx 0.5, 2.1, \quad (23)
\]

where \( F_{+\text{standard}} \) and \( F_{+\text{nonstandard}} \) are the form factors calculated in the standard mixing in Eq. (3), and for the non–standard mixing in Table (3). The two numbers in the equation above correspond to the two choices for \( r_{1,2} \) mentioned above. The form factor with non–standard mixing does not change much for the \( \eta' \) but changes significantly for the \( \eta \) for the two choices of \( r_{1,2} \). For the decay constants we find

\[
\frac{f_{\text{ud}}^{\eta}_{\text{nonstandard}}}{f_{\text{ud}}^{\eta}_{\text{standard}}} \approx 1 \quad ; \quad \frac{f_{\text{ud}}^{\eta'}_{\text{nonstandard}}}{f_{\text{ud}}^{\eta'}_{\text{standard}}} \approx 1.1 \quad ; \quad \frac{f_{s}^{\eta}_{\text{nonstandard}}}{f_{s}^{\eta}_{\text{standard}}} \approx 0.8 \quad ; \quad \frac{f_{s}^{\eta'}_{\text{nonstandard}}}{f_{s}^{\eta'}_{\text{standard}}} \approx 1.2. \quad (24)
\]

Using the second entry in Eq. (23) we then have, approximately,

\[
\frac{F^{\eta'}_{+\text{nonstandard}}}{F^{\eta'}_{+\text{standard}}} \approx 2.0 \approx \frac{F^{\eta}_{+\text{nonstandard}}}{F^{\eta}_{+\text{standard}}} \quad ; \quad f_{\text{ud},s}^{\eta}_{\text{nonstandard}} \approx f_{\text{ud},s}^{\eta}_{\text{standard}} \quad ; \quad f_{\text{ud},s}^{\eta'}_{\text{nonstandard}} \approx f_{\text{ud},s}^{\eta'}_{\text{standard}}. \quad (25)
\]

Together with the relations in Eq. (18) we obtain for the non–standard \( \eta - \eta' \) mixing in Eq. (23)

\[
\begin{align*}
  f_{\text{ud}}^{\eta}_{\text{nonstandard}} &\approx f_{\text{ud}}^{\eta}_{\text{standard}} \approx \frac{f_K}{2} \quad ; \quad f_{s}^{\eta}_{\text{nonstandard}} &\approx f_{s}^{\eta}_{\text{standard}} \approx \frac{-f_K}{\sqrt{2}}; \\
  f_{\text{ud}}^{\eta'}_{\text{nonstandard}} &\approx f_{\text{ud}}^{\eta'}_{\text{standard}} \approx \frac{f_K}{2} \quad ; \quad f_{s}^{\eta'}_{\text{nonstandard}} &\approx f_{s}^{\eta'}_{\text{standard}} \approx \frac{f_K}{\sqrt{2}}; \\
  F^{\eta}_{+\text{nonstandard}} &\approx 2F^{\eta}_{+\text{standard}} \approx F^{\eta}_{K}; \\
  F^{\eta'}_{+\text{nonstandard}} &\approx 2F^{\eta'}_{+\text{standard}} \approx F^{\eta'}_{K}. \quad (26)
\end{align*}
\]
From Eq. (26) it can easily be checked that the predictions $R_K$ and $R_{K^*}$ in Eqs. (19 and 20) remain unchanged under a scaling by a factor of two (last parts of Eq. (26)). Since $R_\Lambda$ does not depend on $B \to \eta(\eta')$ form factors and the decay constants change little with non-standard mixing the prediction for $R_\Lambda$ also remains essentially unchanged with non-standard $\eta(\eta')$ mixing.

It has been shown, by one of the authors of this paper, that flavor topology characteristics of charmless $B$ decays give rise to additional sum rules connecting $B$ decays to $K\eta(\eta')$ and $K\pi$ final states. One of the interesting sum rules \cite{13, 20, 21} is, neglecting phase space corrections,

$$R = \frac{\Gamma[B^\pm \to K^{\pm}\eta] + \Gamma[B^\pm \to K^{\pm}\eta']}{\Gamma[B^\pm \to K^{\pm}\pi^0]} \leq 3 \quad (27)$$

Recent experimental measurements \cite{22}, however, show the above sum rule to be invalid.

With the same assumptions that led to Eq. (16)

$$R \approx \frac{|f_KF^+_\eta + f^*_\eta F^+_K|^2 + |f_KF^+_{\eta'} + f^*_\eta' F^+_K|^2}{|F^+_{\pi^0} f_K|^2} \quad (28)$$

With standard mixing this gives $R \approx 3$. However, using the non-standard $\eta - \eta'$ mixing of Eq. (26), leads to $R \approx 6$, a value which is consistent with experiment. A similar result occurs for $K^*$ final states. In particular, we have

$$\frac{\Gamma[B^\pm \to K^{\pm}\eta]}{\Gamma[B^\pm \to K^{\pm}\pi^0]} \approx |\sqrt{2} + 1|^2 \approx 6,$$

$$\frac{\Gamma[B^\pm \to K^{\pm}\eta']}{\Gamma[B^\pm \to K^{\pm}\pi^0]} \approx |\sqrt{2} - 1|^2 \approx \frac{1}{6}. \quad (29)$$

Thus, with the new radial admixtures in the wave functions of the $\eta$ and $\eta'$, we can obtain an increase in the $B \to K\eta'$ decay without having the similar increase in the $B \to K^*\eta'$ decay as in the other OZI-violating models \cite{23}. In these models additional contributions depend only on the $\eta'$ wave function and are independent of the spin of the recoiling $K$ or $K^*$; interference effects do not play the key role of contributing with opposite signs to $\eta$ and $\eta'$ final states as in our scenario. Thus their predictions contrast sharply with our predicted suppression for $B \to K^*\eta'$ decay as indicated by Eq. (29) while also predicting enhancement for $B \to K^*\eta$ decay relative to $B \to K^*\pi$, similar to that of $B \to K\eta'$ decay relative to $B \to K\pi$. It is therefore crucial to obtain better experimental values for the $B \to K^*\eta'$, $B \to K^*\eta$ and $B \to K^*\pi$ decays since no other model gives these predictions.
We have suggested that the OZI suppressed terms Fig. (1.c) may play an important role in the decays with $\eta'$ in the final state. In fact, using Eqs. (3, 18) we find for the OZI suppressed contribution,

$$\frac{A_{\eta}^{OZI} K(K^*)}{A_{\eta}^{OZI} K(K^*)} \approx \frac{(1 + \frac{1}{\sqrt{2}})}{(1 - \frac{1}{\sqrt{2}})} \sim \frac{A_{\eta}^{OZI} \Lambda}{A_{\eta}^{OZI} \Lambda} \sim 6 \quad (30)$$

In particular, the OZI suppressed contribution may be important for case of $B \rightarrow K^*\eta'$ decay where there is large destructive interference between the two leading penguin contributions Figs.( 1.a and 1.b).

In Table 4 we present our full predictions for the decays $B \rightarrow \eta(\eta')K(K^*)$ and $\Lambda_b \rightarrow \Lambda\eta(\eta')$ decays. The expressions for the amplitudes for $B$ and $\Lambda_b$ decays can be found in Ref [24] and in Ref [25], respectively. These amplitudes include all terms in the effective Hamiltonian in the factorization assumption, including the OZI suppressed terms; also included are chirally enhanced terms which are formally suppressed by $m_b$ but which come with enhanced coefficients and so are important for these decays [26].

The values used for the decay constants are from Eq. (18) and Eq. (25) $f_{\eta}^{u,d} = f_{\eta'}^{u,d} \approx f_K/2$ and $f_{\eta}^s = -f_{\eta'}^s \approx f_K/\sqrt{2}$ with $f_K = 0.160$ GeV. The form factors for $B \rightarrow \eta(\eta')$ in the ground state configurations are taken from Ref [16] and $SU(3)$ has been used to relate the $\Lambda_b \rightarrow \Lambda$ form factors to $\Lambda_b \rightarrow p$ form factors calculated in Ref [27]. We see from Table 4

| Process          | Experimental BR | Theory BR |
|------------------|----------------|-----------|
| $B^- \rightarrow K^-\eta'$ | $(80.1_{-10}^{+6} \pm 7) \times 10^{-6}$ [28], $(70 \pm 8 \pm 5) \times 10^{-6}$ [29] | $62 \times 10^{-6}$ |
| $B^- \rightarrow K^-\eta$ | $< 6.9 \times 10^{-6}$ [28] | $2.2 \times 10^{-6}$ |
| $B^- \rightarrow K^-\eta'$ | $< 35 \times 10^{-6}$ [28] | $1.6 \times 10^{-6}$ |
| $B^- \rightarrow K^-\eta$ | $(26.4^{+9.6}_{-8.2} \pm 3.3) \times 10^{-6}$ [28] | $9 \times 10^{-6}$ |
| $\Lambda_b \rightarrow \Lambda\eta$ | - | $4.6 \times 10^{-6}$ |
| $\Lambda_b \rightarrow \Lambda\eta'$ | - | $12 \times 10^{-6}$ |

that our calculations are in reasonable agreement with experiment. From this table we find

$$R_K = 0.035 \quad R_{K^*} = 5.6 \quad R_\Lambda = 0.4 \quad (31)$$
Note that our prediction for $R_{K^*}$ in Eq. (31) is about a factor 6 smaller than in Eq. (24) because the latter result does not include the contribution of the OZI suppressed terms nor the chirally enhanced terms. The chirally enhanced terms contribute with opposite sign to $K$ and $K^*$ final states [24]. This slightly decreases $B \to K\eta'$ relative to $B \to K\eta$ but reduces substantially the enhancement of $B \to K^*\eta$ relative to $B \to K\eta'$. For $R_\Lambda$ the effect of the OZI and the chirally enhanced terms is to decrease the prediction of $R_\Lambda$ relative to the one in Eq. (21).

In summary we have considered the effect of radial mixing in the pseudoscalar systems generated from hyperfine interaction and the annihilation term. For the $\eta - \eta'$ system we found the effects of radial mixing can be appreciable and can seriously affect the branching ratios for the decays, $B \to \eta(\eta')K(K^*)$, mainly by modifying the $B \to \eta(\eta')$ form factors. We found that the effect of radial mixing, in conjunction with the interference effects between penguin amplitudes, can give a good description of the existing data on $B \to \eta(\eta')$ transitions. Moreover we have predictions for yet unobserved decays that are unique to our model. We also looked at the decay $\Lambda_b \to \Lambda\eta(\eta')$ which is dominated by a single amplitude so that interference effects significant for the $B$ decays are absent here. Moreover, since no $\Lambda_b \to \eta(\eta')$ form factors are involved here, the effect of radial mixing is negligible. We predict only a modest enhancement of $\Lambda_b \to \Lambda\eta'$ relative to $\Lambda_b \to \Lambda\eta$ unlike in the $B$ system; this which is also another unique prediction of our model.

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