Pressure drop and heat transfer characteristics for single-phase developing flow of water in rectangular microchannels

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Abstract. Experiments were conducted to investigate the pressure drop and heat transfer characteristics of single-phase flow of de-ionized water in single copper microchannels of hydraulic diameters 0.438 mm, 0.561 mm and 0.635 mm. The channel length was 62 mm. The experimental conditions covered a range of mass flux from 500 to 5000 kg/m$^2$s in the laminar, transitional and low Reynolds number turbulent regimes. Pressure drop was measured for adiabatic flows with fluid inlet temperatures of 30°C, 60°C and 90°C. In the heat transfer tests, the heat flux ranged from 256 kW/m$^2$ to 519 kW/m$^2$. Friction factors and Nusselt numbers determined from the measurements were higher than for fully-developed conditions, but in reasonable agreement with predictions made using published solutions for hydrodynamically and thermally developing flow. When entrance effects, experimental uncertainties, heat losses, inlet and exit losses, thermal boundary conditions and departure from laminar flow were considered, the results indicate that equations developed for flow and heat transfer in conventional size channels are applicable for water flows in microchannels of these sizes.

1. Introduction

In recent years, microchannels have been studied extensively due to the rapid growth of applications which require the transfer of high heat fluxes [1]. Applications range from compact heat exchangers in the process and refrigeration industries to small scale devices for the cooling of electronic equipment such as computer processors, see [2] for example. The advantages of these microscale devices, in addition to compactness, include reduced material requirement, low weight, smaller amount of coolant and high overall system efficiency. Microscale devices therefore offer long term benefits of resource conservation and environmental protection.

The applicability of thermo-fluid results obtained with conventional sized channels to microchannels is still questioned and there are many contradictory published results. Early transition from laminar to turbulent flow is also often reported in the literature, which itself may lead to the conclusion that the flow behaviour in microchannels deviates from that conventionally expected. In some cases discrepancies between results can be attributed to scaling effects, e.g. entrance effects, surface roughness effects, temperature dependence, and conjugate heat transfer, as presented in Rosa et al. [3]. Entrance effects due to hydrodynamic and thermal development are very important in short microchannels and influence the pressure drop and heat transfer coefficient. Webb [4] states that for a short channel, the portion of pressure drop due to the entrance and exit losses may reach 30% of the...
The total pressure drop. Entrance effects have also been investigated by Wilding et al. [5] and Costaschuk et al. [6]. They explained that the deviation of the experimental Poiseuille number from the expected macroscale value was due to the entrance length, minor losses and inlet geometry rather than microscale effects. Surface roughness may also have a significant influence on flow in microchannels and could be a reason for the deviations from conventional theory as reported by Wu and Cheng [7], Kandlikar et al. [8] and Jiang et al. [9]. They all noted that higher friction factors and Nusselt numbers could be obtained in microchannels with rough walls. Qu et al. [10] studied heat transfer to water flowing through smooth trapezoidal silicon microchannels with hydraulic diameters ranging from 0.062 to 0.169 mm. They found that the experimental Nusselt number was lower than the Nusselt number calculated numerically. The effect of temperature on the friction factor has been explored experimentally by Shen et al. [11] and Urbanek et al. [12]. They concluded that as the temperature increased, the friction factor also increased. In contrast, other published results indicated simply that there was no deviation of flow behaviour in microchannels from that in channels of conventional size, Harms et al. [13], Judy et al. [14] and Qu and Mudawar [15].

The transition from laminar to turbulent flow became an issue in microchannel studies because several reports indicated that the transitional regime started at a lower Reynolds number than the conventionally expected value, e.g. Harms et al. [13], Jiang et al. [9], Jiang et al. [16]. They found that the transition Reynolds number was between 600 and 1500. On the contrary, Costaschuk et al. [6] found that the average critical Reynolds number was 2370, which is in close agreement with the value normally expected.

It is clear from the above brief review that there are still discrepancies between published results for the fluid flow and heat transfer characteristics of microchannels. One of the main objectives of the first part of the work presented in this paper was to perform careful and accurate experiments measuring pressure drop and heat transfer rates in single rectangular microchannels with water and contribute to the debate. The second part of the research programme, not reported here, looked at flow boiling characteristics in these channels.

2. Experimental facility and procedure

A schematic diagram of the test facility is depicted in figure 1. The working fluid is deionized water, which was degassed before and during the tests by continuous boiling and venting in the main reservoir. The fluid temperature in the reservoir was maintained at 102°C for all tests using a PID - controlled immersion heater. Water was drawn from the main reservoir through a subcooler and circulated around the closed loop by a magnetically coupled gear pump (Micropump GA-T23, PFSB) equipped with a programmable variable speed drive (Ismatec Reglo ZS-Digital). The mass flow rate of water was measured by a Coriolis flowmeter (Micromotion Elite CMF010). Two filters were fitted in the flow loop to remove any particles suspended in the fluid. Electric preheaters with PID controllers were installed upstream of the microchannel test section to heat the fluid to the desired inlet temperature. After exiting the test section, the water passed through a cooler before returning to the main reservoir. The water-glycol solution used as the coolant in the subcooler, reflux condenser and cooler was supplied from an external chiller unit, not shown in this schematic.

The central component is the microchannel test section detailed in figure 2, which was made from an oxygen-free copper block of overall dimensions 12 mm wide x 25 mm high x 72 mm long. A single rectangular microchannel was cut in the top surface of the block between the 2 mm diameter inlet and outlet plenums using a Kern HSPC 2216 high-speed micro-milling machine. The microchannel length between the plenums was measured as 62 mm. Three test sections were manufactured, all with the same channel length (L = 62 mm) and channel depth (H = 0.39 mm), but with three different channel widths, see table 1. The measurements were accurate to ± 2µm giving a mean uncertainty of ±0.34% for the hydraulic diameters. The average surface roughness Ra of the channel base was measured using a Zygo NewView 5000 surface profiler with an accuracy of ±1 nm.
Heat input to the microchannel test section was provided by a cartridge heater inserted into a 7 mm diameter hole located parallel to and below the microchannel. The AC electrical supply to the cartridge heater was controlled by a variable transformer. Six holes, 0.6 mm diameter x 6 mm deep, were drilled in the side of the copper test section, 1.5 mm below the top surface, to accommodate thermocouples for measuring the temperature distribution along the microchannel. The thermocouple holes are located 1.1 mm below the microchannel and equi-spaced 12.4 mm apart in the flow direction. The sides of the copper block were insulated with nitrile foam rubber to minimise heat loss.

**Table 1.** Dimensions and surface roughness of the microchannels.

| Test section   | Width $W$ (mm) | Height $H$ (mm) | Hydraulic diameter $D_h$ (mm) | Aspect ratio $\beta$ | Length $L$ (mm) | Surface roughness $Ra$ ($\mu$m) |
|---------------|---------------|----------------|-------------------------------|---------------------|----------------|-------------------------------|
| Test section #1 | 0.50          | 0.39           | 0.438                         | 0.78                | 62.0           | 1.012                         |
| Test section #2 | 1.00          | 0.39           | 0.561                         | 0.39                | 62.0           | 1.048                         |
| Test section #3 | 1.71          | 0.39           | 0.635                         | 0.23                | 62.0           | 1.190                         |

A transparent polycarbonate cover was clamped to the top of the copper test section and sealed with an O-ring. The cover forms the upper surface of the microchannel and incorporates flow connections and the 2 mm diameter plenums leading to and from the microchannel. Thermocouples were inserted into the plenums to measure the inlet and outlet temperatures of the water. Holes drilled through the cover provide static pressure tappings at six positions: inlet plenum, outlet plenum and four equi-spaced locations along the microchannel. Differential pressure sensors (Honeywell 26PCC type) were
connected between the tapping points and atmosphere to measure the gauge pressure distribution along the microchannel test section.

The mass flow rate was measured using a Coriolis flowmeter with an uncertainty of ±0.6 g/min. All copper block temperatures were measured with 0.5 mm diameter K-type sheathed thermocouples. The same K-type thermocouples were used to record the fluid temperatures. All the thermocouples were calibrated in a constant temperature circulating bath against a precision thermometer (ASL, F250 MK II) with an accuracy of ±0.025 K and their uncertainty was ±0.2 K. The electrical power dissipated by the test section cartridge heater was determined from voltage and current measurements obtained using calibrated digital multimeters (Black Star 3225) with uncertainties of ±0.3 V and ±0.01 A respectively. Differential pressure sensors (Honeywell 26PCC type) were used to measure the local pressures with an uncertainty of ±0.2 kPa.

\[ \Delta p_{ch} = \Delta p_{meas} - \Delta p_{loss} \]  
\[ \Delta p_{loss} = \frac{1}{2} \rho \bar{V}^2 (K_c + K_e) \]  

1. Cover plate, polycarbonate; 2. Channel cover, polycarbonate; 3. O-ring seal; 4. Cartridge heater; 5. Copper block; 6. Nitrile foam rubber insulation; 7. Bottom plate, polycarbonate.

**Figure 2.** Test section construction showing the main parts (all dimensions in mm).

The pressure drop along the microchannel due to friction and the developing flow is obtained by subtracting the inlet and outlet pressure losses from the total measured pressure drop. That is

\[ \Delta p_{ch} = \Delta p_{meas} - \Delta p_{loss} \]

where

\[ \Delta p_{loss} = \frac{1}{2} \rho \bar{V}^2 (K_c + K_e) \]

\[ K_{90} \] is the loss coefficient associated with each of the 90° turns at the channel inlet and outlet and is approximately 1.2 according to Phillips [17]. \( K_c \) and \( K_e \) are the inlet and exit loss coefficients for the sudden contraction and the sudden enlargement and can be estimated from Kays and London [18] based on the ratio of the channel to plenum flow areas and the flow regime. The Fanning friction factor based on the channel pressure drop is given by
The rate of heat loss from the test section to the ambient was determined by energy balance tests conducted with and without water flowing (see also [19]) and found to be on average approximately 6.8% of the input electrical power to the cartridge heater. Thus the rate of heat removal, \( q_{\text{rem}} \), by the working fluid is determined as follows:

\[
q_{\text{rem}} = \dot{m}c_p(T_o - T_i) = P - q_{\text{loss}}
\]  
(4)

where \( P \) is equal to the product of the voltage \( V \) and current \( I \) supplied to the cartridge heater. The average heat flux at the heated walls of the channel is defined as

\[
q^* = \frac{q_{\text{rem}}}{A_{ht}}
\]  
(5)

where \( A_{ht} = (2H + W)l \) since the polycarbonate channel cover is assumed to be adiabatic. The channel wall temperature, \( T_w \), is assumed to be uniform and equal to the average of the readings from the six thermocouples located in the copper test section. The average heat transfer coefficient is calculated as

\[
\bar{h} = \frac{q_{\text{rem}}}{A_{ht} \Delta T_{lm}} = \frac{q^*}{\Delta T_{lm}}
\]  
(6)

where \( \Delta T_{lm} \) is the logarithmic-mean temperature difference:

\[
\Delta T_{lm} = \frac{(T_w - T_f) - (T_w - T_i)}{\ln \left(\frac{T_w - T_f}{T_w - T_i}\right)}
\]  
(7)

The corresponding average Nusselt number is defined as

\[
\overline{N_u} = \frac{\bar{h} D_h}{k}
\]  
(8)

The fluid bulk temperature variation with distance \( z \) along the channel can be evaluated as follows, (see [20]):

\[
T_f(z) = T_w - (T_w - T_i) \exp\left(-\frac{(2H + W)z}{mc_p \bar{h}}\right)
\]  
(9)

The propagated experimental uncertainties were calculated based on the method described in Coleman and Steele [21]. The maximum uncertainty for the friction factor was ±37.7% at the lowest Reynolds number but was much lower, down to ±3.6%, at the highest Reynolds number. The uncertainty in the Nusselt number ranged from ±5.2% to 10%.
3. Experimental results and discussion

3.1. Friction factor

The results presented in this section were obtained from tests performed without heating applied to the test sections in order to eliminate fluid property variations. Local pressure measurements are plotted in figure 3 at equi-spaced locations along the \( D_h = 0.438 \) mm test section from the inlet plenum \((z/L = 0)\) to the exit plenum \((z/L = 1)\) for several Reynolds numbers and a fluid temperature \( T = 30^\circ C \). The marked decrease in pressure evident between the inlet plenum and \( z/L = 0.2 \) includes contributions due to the flow area change and the losses associated with the 90° turn and sudden contraction at channel inlet, in addition to the pressure drop due to wall shear stress and flow development. Similarly, the pressure change between \( z/L = 0.8 \) and the outlet plenum includes contributions due to the flow area change and the losses at channel exit. As mentioned in section 2, the channel pressure drop was determined using equation (1), by subtracting the inlet and exit losses from the total test section pressure drop measured between the plenums, and the corresponding friction factor was calculated using equation (3).

![Figure 3](image_url)

**Figure 3.** Measured axial pressure distributions (adjusted to zero outlet pressure) along the \( D_h = 0.438 \) mm microchannel between the inlet and outlet plenums at several Reynolds numbers.

For each test section, the channel pressure drop increased with increasing flow rate, but decreased as the fluid temperature was increased due to the reduction of the water viscosity. Experimental friction factors are plotted versus Reynolds number in figure 4 for the \( D_h = 0.438 \) mm microchannel and three fluid temperatures, \( T = 30^\circ C, 60^\circ C \) and \( 90^\circ C \). The data points for the different temperatures fall in close agreement with each other and certainly within the experimental uncertainty, suggesting that the \( f\text{-}Re \) relationship is independent of the fluid temperature for the range of conditions considered. This finding appears to contradict the results of Urbanek et al. [12] who reported that the product \( f\text{-}Re \) (or Poiseuille number) increased with increasing fluid temperature.

In this work, hydrodynamic development occurred over part or all of the microchannel flow length. The length of this entry region is more significant at higher laminar Reynolds numbers and for larger \( D_h \) channels. Accordingly, the laminar flow results are compared with an equation proposed by Shah [22] to predict apparent friction factors for developing flow in circular and noncircular ducts, given by
where \( L^* = L/(D_h \text{Re}) \) is the dimensionless channel length and \( K(\infty) \) is the fully developed Hagenbach factor. For rectangular channels, \( K(\infty) \) is presented in graphical form by Shah and London [23] and can be described by the equation

\[
K(\infty) = 0.6611 + 1.1182\beta + 2.1758\beta^2 - 5.8322\beta^3 + 4.4683\beta^4 - 1.1553\beta^5
\]

where \( \beta \) is the channel aspect ratio, defined as the ratio of the short side to the long side, or \( H/W \) for the microchannels tested in this study. For the \( D_h = 0.438 \text{ mm microchannel} \) considered in figure 4 the aspect ratio \( \beta = 0.78 \) (see Table 1) and the corresponding value of \( K(\infty) \) given by equation (11) is 1.41.

The constant \( C \) in equation (10) also depends on \( \beta \) and it was estimated that \( C \approx 0.00025 \) for \( \beta = 0.78 \) using values tabulated by Shah [22]. The friction factor for fully developed laminar flow in a rectangular channel can be calculated using the following equation given by Shah and London [23]:

\[
f_{FD} = \frac{24}{\text{Re}} \left( 1 - 1.3553\beta + 1.9467\beta^2 - 1.7012\beta^3 + 0.9564\beta^4 - 0.2537\beta^5 \right)
\]

When evaluated for \( \beta = 0.78 \), as shown in figure 4, equation (12) becomes

\[
f_{FD} = 14.42 \text{ Re}^{-1}.
\]

Figure 4. Experimental friction factors for the \( D_h = 0.438 \text{ mm microchannel} \) at three different fluid temperatures.

In the turbulent regime, the experimental friction factor results are compared with equation (13) due to Phillips [17]. This equation can be applied for both developing and fully developed turbulent flow.
and is also given by Kandlikar [24]. The laminar-equivalent Reynolds number, Re*, appearing in equation (13) was proposed by Jones [25] for rectangular channels and is defined by equation (14).

\[ f_{app} = \left( 0.0929 + \frac{1.01612}{L/D_h} \right) Re^*^{\left( -0.268 - \frac{0.3293}{L/D_h} \right)} \]

\[ Re^* = Re \left( \frac{2}{3} + \frac{11}{24} \beta (2 - \beta) \right) \]  

(13)  
(14)

The turbulent flow results are also compared with the well known Blasius equation:

\[ f = 0.079 Re^{-0.25} \]  

(15)

As shown in figure 4, the experimental friction factors for the \( D_h = 0.438 \text{ mm} \) (\( \beta = 0.78 \)) microchannel are in reasonable agreement with the developing flow line calculated using equation (10) at low laminar Reynolds numbers. The data are observed to depart from the predicted line at Reynolds numbers between 1000 and 2000 suggesting transition from laminar to turbulent flow. However, the 90° change in flow direction at the channel inlet may be responsible for this apparent early transition and it is not necessarily indicative of differences with conventional results. In the turbulent regime, the experimental friction factor results are consistently higher than predicted by equations (13) and (15), the latter relationship providing slightly better agreement.

**Figure 5.** Comparison of experimental friction factors for three microchannels (\( D_h = 0.438, 0.561 \) and 0.635 mm) at a fluid temperature of 60°C.

Figure 5 compares friction factor results obtained for the three microchannels (\( D_h = 0.438 \text{ mm}, 0.561 \text{ mm and} 0.635 \text{ mm} \)) tested in this study at a fluid temperature \( T = 60^\circ C \). The channel height was the same (\( H = 0.39 \text{ mm} \)) for each channel and the corresponding channel aspect ratios were \( \beta = 0.78, 0.39 \) and 0.23 respectively. The results for the different channel sizes shown in figure 5 are closely aligned and no systematic effect of aspect ratio, or hydraulic diameter, can be detected. In contrast,
equations (10) and (13) predict that friction factor increases with decreasing aspect ratio in the laminar and turbulent regimes respectively. To avoid overcrowding figure 5, these two equations are shown for only one value of aspect ratio (β = 0.23).

3.2. Nusselt number

Results are presented in this section for the average Nusselt number, determined using equation (7), based on the average wall heat flux, \( q'' \), at the heated walls of the channel and the logarithmic-mean wall-to-fluid bulk temperature difference, \( \Delta \text{T}_{lm} \). The wall temperature was treated as uniform in \( \Delta \text{T}_{lm} \) and equal to the average of the wall temperature measurements along the channel. The validity of the assumed thermal boundary condition is evidenced in figure 6, which shows measured wall temperature distributions obtained for the \( D_h = 0.438 \) mm microchannel at two different Reynolds numbers with the same average heat flux of 286 kW/m². At Re = 443, the wall temperature varies very slightly, whereas at Re = 2291, the wall temperature distribution is virtually uniform. Corresponding fluid bulk temperature distributions, determined using equation (9), are also shown in figure 6. The wall temperature uniformity achieved along the flow direction is attributed to axial heat conduction in the copper test section due to its high thermal conductivity and large cross sectional area compared to the microchannel dimensions.

![Figure 6. Wall and fluid bulk temperature distributions along the \( D_h = 0.438 \) mm microchannel.](image-url)

Experimental average Nusselt numbers are presented in figure 7 for the \( D_h = 0.438 \) mm microchannel. The data cover three values of heat flux but there is no noticeable effect of \( q'' \) on Nusselt number. In the laminar flow regime, the Nusselt number increases with Reynolds number reflecting the increase of the combined entry length for simultaneous hydrodynamic and thermal development. Under fully developed conditions the local Nusselt number is constant in laminar flow. Shah and London [23] tabulated values of the laminar fully developed \( \text{Nu}_T \) against the aspect ratio for rectangular channels with three walls heated to uniform surface temperature and the fourth wall insulated. If one of the longer channel sides is adiabatic, as is the case in this work, \( \text{Nu}_T \) is closely approximated by equation (16).

\[
\text{Nu}_T = 4.861 \left( 1 - 3.656 \beta + 12.821 \beta^2 - 27.441 \beta^3 + 37.373 \beta^4 - 28.365 \beta^5 + 8.888 \beta^6 \right) \quad (16)
\]
For $\beta = 0.78$, equation (16) becomes $\text{Nu}_T = 2.78$, which is the fully developed value shown in figure 7.

For thermally and hydrodynamically developing laminar duct flow it is shown by Bejan [26] that the Nusselt number can be represented by a relationship of the following form:

$$\text{Nu} = C' \left( \frac{L / D_h}{\text{Re} \text{Pr}} \right)^{-0.5}$$

(17)

where $C'$ is a constant. When equation (17) was fitted to the average Nusselt number results obtained in this study, the constant $C'$ was estimated to be approximately 1.63.

*Figure 7.* Experimental Nusselt numbers for the $D_h = 0.438$ mm microchannel at three different values of heat flux. Correlations are plotted for $\text{Pr} = 4.3$. 
In the turbulent flow regime, the experimental Nusselt numbers are compared in figures 7 and 8 with two frequently referenced empirical equations, namely, the Dittus-Boelter correlation and the Gnielinski correlation, see for example [20]. Although the limited amount of turbulent data available falls below the normally accepted Re range of validity for the Dittus-Boelter correlation, this equation is in closer agreement with the experimental data than the Gnielinski equation.

Figures 7 and 8 also show heat transfer data obtained by Harms et al. [13] for forced convection of deionized water in deep rectangular microchannels ($H = 1$ mm, $W = 0.25$ mm, $D_h = 0.4$ mm) etched in a silicon substrate. The aspect ratio $\beta$ (= short side/long side) of their three-side heated channels was approximately 0.25, which is close to that of the $D_h = 0.635$ mm microchannel used in this work (see table 1). However, in the deep microchannels tested by Harms et al. [13], where one of the short channel sides was adiabatic, a much greater proportion of the channel wall area was available for heat transfer than in the channels investigated in this study, which were adiabatic on a long channel side. Nevertheless, the Nusselt numbers shown in figure 8 for the $D_h = 0.635$ mm microchannel are comparable with the data of Harms et al. [13].

Average Nusselt number results obtained for the three microchannel test sections at approximately the same average heat flux are presented in figure 8. These results and the comparison with published data suggest a possible effect of increasing Nusselt number with decreasing channel aspect ratio, rather than hydraulic diameter, for the range of parameters investigated. It is noted that for laminar fully developed conditions, $\text{Nu}_T$ does increase as $\beta$ decreases for rectangular channels heated on all four sides. However, for a three-side heated channel that is insulated on a long channel side, the fully developed $\text{Nu}_T$ given by equation (16) varies non-monotonically with $\beta$, and therefore does not support the observed dependency of average Nusselt number on aspect ratio for the simultaneously developing flow data plotted in figure 8.

4. Conclusions
Experimental data have been presented for the friction and heat transfer characteristics of single-phase flow of deionized water in single copper microchannels of rectangular cross-section. In the laminar
regime, the apparent friction factor is in reasonable agreement with the hydrodynamic entry region correlation of Shah [22]. The experimental Nusselt numbers for the simultaneously developing flow are higher than predicted by conventional theory for fully developed laminar flow but are well represented by a relationship of the form suggested in [26] with the leading constant set to 1.63. In the turbulent regime, the experimental friction factors are in reasonable agreement with a circular tube correlation modified by substituting a laminar-equivalent Reynolds number. The turbulent Nusselt numbers agree with the Dittus-Boelter correlation but are higher than those predicted using the Gnielinski equation. The effect of fluid temperature on friction factor is marginal. The results indicate early transition to turbulence but this could be due to disturbances at the channel inlet and probably does not indicate deviation from values predicted for larger channels. The effect of hydraulic diameter ($D_h = 0.438, 0.561, 0.635$ mm) was examined on both friction data and Nu number. The corresponding channel aspect ratios were 0.78, 0.39 and 0.23. It can be concluded that there is a negligible effect on friction characteristics. However, the Nusselt number appears to increase with decreasing aspect ratio. The effect of varying the hydraulic diameter is not clear in this range. Further investigation is needed to clarify these last two points.

| Nomenclature                  | Description                                                                 |
|-------------------------------|-----------------------------------------------------------------------------|
| $A_{ht}$                      | Heat transfer area ($m^2$)                                                  |
| $C$                           | Constant in equation (10)                                                  |
| $C'$                          | Constant in equation (17)                                                  |
| $c_p$                         | Specific heat ($J/kg K$)                                                    |
| $D_h$                         | Hydraulic diameter ($m$)                                                    |
| $f$                           | Fanning friction factor                                                    |
| $H$                           | Channel height ($m$)                                                        |
| $\bar{h}$                     | Average heat transfer coefficient ($kW/m^2 K$)                              |
| $I$                           | Current ($A$)                                                               |
| $k$                           | Thermal conductivity ($W/m K$)                                             |
| $K(\infty)$                  | Hagenbach factor                                                            |
| $K$                           | Loss coefficient                                                            |
| $L$                           | Channel length ($m$)                                                        |
| $L^*$                         | Dimensionless channel length                                                |
| $\dot{m}$                     | Mass flow rate ($kg/s$)                                                     |
| $\bar{Nu}$                    | Average Nusselt number ($= \bar{h}D_h/k$)                                  |
| $\bar{Nu}_T$                  | Nusselt number for uniform wall temperature.                                |
| $P$                           | Electrical power input ($W$)                                                |
| $Pr$                          | Prandtl number ($= c_p \mu / k$)                                           |
| $\Delta p_{meas}$             | Measured pressure drop between inlet plenum and outlet plenum ($kPa$)       |
| $\Delta p_{ch}$               | Pressure drop in the channel ($kPa$)                                        |
| $\Delta p_{loss}$             | Sum of pressure losses due to turns, sudden contraction and sudden enlargement ($kPa$) |
| $q^*$                         | Heat flux, based on heated area ($kW/m^2$)                                 |

| Greek symbols                  | Description                                                                 |
|-------------------------------|-----------------------------------------------------------------------------|
| $\beta$                       | Channel aspect ratio                                                        |
| $\rho$                        | Density ($kg/m^3$)                                                          |
| $\mu$                         | Dynamic viscosity ($kg/m s$)                                                |

| Subscripts                    | Description                                                                 |
|-------------------------------|-----------------------------------------------------------------------------|
| $app$                         | Apparent                                                                    |
| $ch$                          | Channel                                                                      |
| $c$                           | Contraction                                                                 |
| $e$                           | Enlargement                                                                 |
| $FD$                          | Fully developed flow                                                        |
| $i$                           | Inlet                                                                        |
| $o$                           | Outlet                                                                       |
| $w$                           | Wall                                                                         |

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L*

m

\bar{Nu}

\bar{Nu}_T

P

Pr

\Delta p_{meas}

\Delta p_{ch}

\Delta p_{loss}

q^*$
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