The Kauffman Constraint Coefficients $K_\phi(n_0, n_1, \ldots, n_{\beta-1})$

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I. Introduction

In Reference [1], Louis H. Kauffman presents an *Algebra of Constraints*. Its purpose is to explicitly define the algebraic conditions that best align non-commutative operators with their corresponding classical variables in the usual context of continuum calculus.

In creating this *classical variable* (CV) to *non-commuting operator* (NCO) correspondence, two assumptions are made:

1. there exists an exact mapping between CVs and NCOs of the form

(I-1) 

\[
\text{classical variable} \leftrightarrow \text{Non–Commutating Operator}
\]

\[
x_1^n \leftrightarrow X_1^n
\]

\[
x_2^n \leftrightarrow X_2^n
\]

\[
\vdots
\]

\[
x_\beta^n \leftrightarrow X_\beta^n
\]

where \( n \) is a positive integer, and

2. there exists an exact mapping between linear combinations of CVs and linear combinations of NCOs of the form

(I-2) 

\[
\text{classical variables} \leftrightarrow \text{Non–Commutating Operators}
\]

\[
\left( \sum_{\alpha=1}^{\beta} x_\alpha \right)^n \leftrightarrow \left( \sum_{\alpha=1}^{\beta} X_\alpha \right)^n
\]

These two requirements establish constraint conditions on the NCOs that take the form of *Symmetrizers*; namely,

(I-3) 

\[
\{ X_1 X_2 \cdots X_{\beta} \} \leftrightarrow \left( \frac{1}{n!} \right) \sum_{\sigma \in S_n} (X_{\sigma_1} X_{\sigma_2} \cdots X_{\sigma_n})
\]

where we are summing over all \( n \)-permutations of the permutation group \( S_n \).

\[^a \text{Kauffman uses \{\} brackets to designate Symmetrizers as opposed to the [] which designate Commutators. We will only use his Symmetrization convention in equation (I-3). In all cases that follow, curly brackets \{\} designate simple parenthesis.} \]
These mappings enable Kauffman to align classical variable differential formulas with their corresponding non-commutative operator differential forms. In fact, by defining the first temporal derivative of a variable \( \theta \) with respect to time

\[
\theta^{(1)} = h \theta = h^{(0)} \theta
\]

Kauffman provides a general algorithm for obtaining a \( \beta \)-order time-derivative \( \theta^{(\beta)} \) in a recursive fashion. Namely, the next derivative is obtained from the previous by applying the product rule for differentiation and using the \( (I - 4) \) identity. By programming this algorithm into Mathematica™, Kauffman is able to calculate the first nine levels of derivatives. However, Kauffman notes that

"The structure of the coefficients in this recursion is unknown territory...To penetrate the full algebra of constraints we need to understand the structure of these derivatives and their corresponding non-commutative symmetrizations."

While Kauffman employs computational differentiation using Mathematica™, we present below a closed-form expression for the Kauffman Constraint Coefficient \( K_\omega(n_0, n_1, n_2, \cdots, n_{\beta-1}) \) both as a solution to his challenge and to foster a greater understanding of the full Algebra of Constraints.

II. Solution

The Algebra of Constraints can be described combinatorically in terms of its structures. We begin by defining a \( \beta \)-order time-derivative \( \theta^{(\beta)} \) in terms of three structures: the first is the original variable \( \theta \); the second is a set of \( \beta \)-derivatives \( \{ H^{(0)}, H^{(1)}, \cdots, H^{(\alpha)}, \cdots, H^{(\beta-2)}, H^{(\beta-1)} \} \); and the third is a set defining the multiplicity of those \( \beta \)-derivatives \( \{ n_0, n_1, \cdots, n_\alpha, \cdots, n_{\beta-2}, n_{\beta-1} \} \).

From these structures, two generalized functions are defined: firstly, there is the \( \omega \)-Elemental \( E_\omega \), which is a product of the \( \beta \)-derivatives with their \( n_\alpha \) multiplicities; secondly, there is the Kauffman Constraint Coefficient \( K_\omega \), which is a count of the \( \omega \)-Elementals.

With these two functions, Kauffman’s derivative series \( \theta^{(\beta)} \) takes the general form

\[
\theta^{(\beta)} = \sum_{\omega=1}^{N_\beta} K_\omega(n_0, n_1, \cdots, n_{\beta-1}) E_\omega(n_0, H^{(0)}, n_1, H^{(1)}, \cdots, n_{\beta-1}, H^{(\beta-1)})
\]

where every Elemental \( E_\omega \) of the derivative series is explicitly given by

\[\text{Ibid., p. 31.}\]
\[\text{Because the Coefficient } K_\omega \text{ is independent of the existence of the time functional } \theta=\theta^{(\beta)}=T \text{ (where T is used in Kauffman’s paper) for every Elemental of the derivative } \theta^{(\beta)} \text{ expansion, it is not displayed in the } E_\omega \text{ Elementals.}\]
The Kauffman Constraint Coefficients $K_\omega$

(II-2)

$$E_\omega(n_0, H^{(0)}, n_1, H^{(1)}, \ldots, n_{\beta-1}, H^{(\beta-1)}) = \left\{ \prod_{\alpha=0}^{\beta-1} (H^{(\alpha)})^{n_\alpha} \right\}_{\omega}$$

$$= \left\{ (H^{(0)})^{n_0} (H^{(1)})^{n_1} \ldots (H^{(\alpha)})^{n_\alpha} \ldots (H^{(\beta-2)})^{n_{\beta-2}} (H^{(\beta-1)})^{n_{\beta-1}} \right\}_{\omega}$$

with

$$\beta \equiv \text{the number of derivatives of } \theta \text{ with respect to the Hamiltonian } T, \theta^{(\beta)};$$

$$N_\beta = \text{the total number of distinct Elementals } E_\omega, \text{ with } 1 \leq \omega \leq N_\beta, \text{ of the derivative series;}$$

$$n_\alpha = \text{the exponent and number of } \alpha \text{-derivatives, } H^{(\alpha)n_\alpha};$$

$$K_\omega(n_0, n_1, \ldots, n_{\beta-1}) = \text{the } \omega\text{-Kauffman Constraint Coefficient for the } \omega\text{-Elemental of } \theta^{(\beta)}$$

The key to this construction is that all $\alpha$-derivatives $H^{(\alpha)}$, where $0 \leq \alpha \leq \beta - 1$, are present for every Elemental $E_\omega$ of the series $\theta^{(\beta)}$. As such, the form of the Elemental is encoded in the set of exponents $n_\alpha$. Determining the correct $\omega$-sets $\{n_0, n_1, \ldots, n_{\beta-1}\}_\omega$ of exponents generates the correct $E_\omega$ elementals and their corresponding Kauffman Constraint Coefficients $K_\omega$.\(^d\)

Because of this, the Kauffman Constraint Coefficient for each $E_\omega$ Elemental is purely a function of the $n_\alpha$ exponents instead of the $\alpha$-derivatives $H^{(\alpha)}$. As such, the Kauffman Constraint Coefficient is defined as

(II-3)

$$K_\omega(n_0, n_1, \ldots, n_{\beta-1}) = \frac{\beta!}{\prod_{\alpha=0}^{\beta-1} n_\alpha! [(\alpha + 1)!]^{n_\alpha}}$$

III. Redefining the Kauffman Recursion Relation for $\theta^{(\beta)}$

As explained in the Introduction, Kauffman generates each time derivative $\theta^{(\beta)}$ via a commutative recursion relation with all smaller derivatives

(III-1)

$$\{ \theta^{(0)}, \theta^{(1)}, \ldots, \theta^{(\beta-1)} \}$$

We have derived a new recursion relation that does not depend on using the product rule for differentiation but instead involves only summations and substitutions.\(^e\) Namely,

(III-2)

$$\theta^{(\beta)} = (\beta - 1)! \sum_{\alpha=1}^{\beta} \left\{ \frac{H^{(\alpha-1)} \theta^{(\beta-\alpha)}}{[(\alpha - 1)! (\beta - \alpha)!]} \right\}$$

\(^d\) Calculating the $E_\omega$ Elementals can be performed as an iterative optimization problem through recursive differentiation and application of the product rule of differentiation or alternatively by Combinatoric methods which will be explored in an upcoming paper.

\(^e\) This technique may provide a cleaner, more efficient algorithm for Mathematica™.
As a demonstration, we explicitly generate the first 4 such derivatives in the following Table III-1.

| $\vartheta^{(\beta)}$ | Recursive Calculation | Result |
|----------------------|-----------------------|--------|
| $\vartheta^{(1)}$   | $H^{(0)} \vartheta^{(0)} = H \vartheta$ |
| $\vartheta^{(2)}$   | $H^2 \vartheta + H^{(1)} \vartheta$ |
| $\vartheta^{(3)}$   | $H^3 \vartheta + 3HH^{(1)} \vartheta + H^{(2)} \vartheta$ |
| $\vartheta^{(4)}$   | $H^4 \vartheta + 6H^2 \vartheta^{(1)} \vartheta + 4HH^{(2)} \vartheta + 3H^{(1)} \vartheta + H^{(3)} \vartheta$ |

IV. Generating $K_\omega$ for $\vartheta^{(\theta)}$

As discussed in the Introduction, Kauffman calculates the first nine levels of derivatives via Mathematica™. We now present the calculation for those same coefficients using Equation (II-3). In so doing, we demonstrate how properly to use the Elementals $(E_1, E_2, E_3, \ldots, E_{30})$ corresponding to the $\vartheta^{(\theta)}$ derivative with the Kauffman Constraint Coefficient function to obtain the 30 coefficients $(K_1, K_2, K_3, \ldots, K_{30})$.

| $\omega$ | Mathematica™ | $E_\omega$ | $K_\omega(n_0, n_1, n_2, \ldots, n_k)$ | Calculation |
|----------|---------------|------------|----------------------------------|-------------|
| 1        | $1 \cdot H^{(0)} n^T$ | $H^{(0)} n^0$ | $K_1(9, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ | $\frac{\beta^1}{n_0^{(1)} n^{(0)}} = \frac{n_1^{(0)}}{g(1)^2} = 1$ |
| 2        | $36 \cdot H^{(0)} n T H^{(1)} h$ | $H^{(0)} n^1 H^{(1)} h$ | $K_2(7, 1, 0, 0, 0, 0, 0, 0, 0, 0)$ | $\frac{\beta^1}{n_0^{(1)} n^{(1)}} = \frac{n_1^{(2)}}{g(1)^2} = 36$ |
| 3        | $378 \cdot H^{(0)} n T H^{(1)} z$ | $H^{(0)} n^2 H^{(1)} z$ | $K_3(5, 2, 0, 0, 0, 0, 0, 0, 0, 0)$ | $\frac{\beta^1}{n_0^{(1)} n^{(2)}} = \frac{n_1^{(2)}}{g(1)^2} = 378$ |

Ibid., p. 32.
| $\omega$ | Mathematica™ | $E_n$ | $K_\omega(n_0, n_1, n_2, n_3, \ldots, n_8)$ | Calculation |
|---|---|---|---|---|
| 4 | $1260 \cdot H^{(0)^3}TH^{(1)^3}$ | $H^{(0)^3}H^{(1)^3}$ | $K_4(3,3,0,0,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^0 \rangle \langle n_1(2)^3 \rangle} = \frac{91}{3 \cdot 288} = 1260$ |
| 5 | $945 \cdot H^{(0)^1}TH^{(1)^4}$ | $H^{(0)^1}H^{(1)^4}$ | $K_5(1,4,0,0,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^1 \rangle \langle n_1(2)^4 \rangle} = \frac{91}{4 \cdot 384} = 945$ |
| 6 | $84 \cdot H^{(0)^6}TH^{(2)^2}$ | $H^{(0)^6}H^{(2)^2}$ | $K_6(6,0,1,0,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^6 \rangle \langle n_2(3)^2 \rangle} = \frac{91}{6 \cdot 288} = 84$ |
| 7 | $1260 \cdot H^{(0)^4}TH^{(1)^1}H^{(2)^2}$ | $H^{(0)^4}H^{(1)^1}H^{(2)^2}$ | $K_7(4,1,1,0,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^4 \rangle \langle n_1(2)^1 \rangle \langle n_2(3)^2 \rangle} = \frac{91}{4 \cdot 21 \cdot 288} = 1260$ |
| 8 | $3780 \cdot H^{(0)^2}TH^{(1)^2}H^{(2)^2}$ | $H^{(0)^2}H^{(1)^2}H^{(2)^2}$ | $K_8(2,2,1,0,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^2 \rangle \langle n_1(2)^2 \rangle \langle n_2(3)^2 \rangle} = \frac{91}{2 \cdot 21 \cdot 3 \cdot 288} = 3780$ |
| 9 | $1260 \cdot TH^{(1)^3}H^{(2)^2}$ | $H^{(1)^3}H^{(2)^2}$ | $K_9(0,3,1,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^3 \rangle \langle n_2(3)^2 \rangle} = \frac{91}{3 \cdot 288} = 1260$ |
| 10 | $840 \cdot H^{(0)^3}TH^{(2)^2}$ | $H^{(0)^3}H^{(2)^2}$ | $K_{10}(3,0,2,0,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^3 \rangle \langle n_2(3)^2 \rangle} = \frac{91}{3 \cdot 21 \cdot 288} = 840$ |
| 11 | $2520 \cdot H^{(0)^2}TH^{(1)^2}H^{(2)^2}$ | $H^{(0)^2}H^{(1)^2}H^{(2)^2}$ | $K_{11}(1,1,2,0,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^2 \rangle \langle n_1(2)^2 \rangle \langle n_2(3)^2 \rangle} = \frac{91}{2 \cdot 21 \cdot 3 \cdot 144} = 2520$ |
| 12 | $280 \cdot TH^{(2)^3}$ | $H^{(2)^3}$ | $K_{12}(0,0,3,0,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_2(3)^3 \rangle} = \frac{91}{3 \cdot 288} = 280$ |
| 13 | $126 \cdot H^{(0)^5}TH^{(3)^1}$ | $H^{(0)^5}H^{(3)^1}$ | $K_{13}(5,0,0,1,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^5 \rangle \langle n_1(2)^1 \rangle \langle n_2(4)^3 \rangle} = \frac{91}{4 \cdot 2880} = 126$ |
| 14 | $1260 \cdot H^{(0)^4}TH^{(1)^1}H^{(3)^1}$ | $H^{(0)^4}H^{(1)^1}H^{(3)^1}$ | $K_{14}(3,1,0,1,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^4 \rangle \langle n_1(2)^1 \rangle \langle n_2(4)^3 \rangle} = \frac{91}{4 \cdot 3 \cdot 288} = 1260$ |
| 15 | $1890 \cdot H^{(0)^1}TH^{(1)^2}H^{(3)^1}$ | $H^{(0)^1}H^{(1)^2}H^{(3)^1}$ | $K_{15}(1,2,0,1,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^1 \rangle \langle n_1(2)^2 \rangle \langle n_2(4)^3 \rangle} = \frac{91}{4 \cdot 192} = 1890$ |
| 16 | $1260 \cdot H^{(0)^2}TH^{(2)^2}H^{(3)^1}$ | $H^{(0)^2}H^{(2)^2}H^{(3)^1}$ | $K_{16}(2,0,1,1,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_1(1)^2 \rangle \langle n_2(3)^2 \rangle \langle n_2(4)^3 \rangle} = \frac{91}{2 \cdot 3 \cdot 288} = 1260$ |
| 17 | $1260 \cdot TH^{(1)^1}H^{(2)^2}H^{(3)^1}$ | $H^{(1)^1}H^{(2)^2}H^{(3)^1}$ | $K_{17}(0,1,1,1,0,0,0,0,0)$ | $\frac{\beta^1}{\langle n_2(3)^3 \rangle \langle n_1(4)^3 \rangle} = \frac{91}{3 \cdot 288} = 1260$ |
### The Kauffman Constraint Coefficients $K_\omega$

| $\omega$ | Mathematica™ | $E_\omega$ | $K_\omega(n_0, n_1, n_2, n_3, \ldots, n_k)$ | Calculation |
|----------|--------------|-----------|------------------------------------------|-------------|
| 18       | $315 \cdot H^{(0)} TH^{(3)}$ | $H^{(0)} H^{(3)}$ | $K_{18}(1,0,0,2,0,0,0,0,0)$ | $\beta$ $\frac{1}{2^3(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^3}$ $\frac{9}{1152} = 315$ |
| 19       | $126 \cdot H^{(0)} TH^{(4)}$ | $H^{(0)} H^{(4)}$ | $K_{19}(4,0,0,0,1,0,0,0,0)$ | $\beta$ $\frac{1}{2^4(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{2880} = 126$ |
| 20       | $756 \cdot H^{(0)} TH^{(1)} H^{(4)}$ | $H^{(0)} H^{(1)} H^{(4)}$ | $K_{20}(2,1,0,1,0,0,0,0,0)$ | $\beta$ $\frac{1}{2^4(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{2880} = 756$ |
| 21       | $378 \cdot TH^{(1)} H^{(4)}$ | $H^{(1)} H^{(4)}$ | $K_{21}(0,2,0,0,1,0,0,0,0)$ | $\beta$ $\frac{1}{2^4(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{2880} = 378$ |
| 22       | $504 \cdot H^{(0)} TH^{(2)} H^{(4)}$ | $H^{(0)} H^{(2)} H^{(4)}$ | $K_{22}(1,0,1,0,1,0,0,0,0)$ | $\beta$ $\frac{1}{2^4(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{2880} = 504$ |
| 23       | $126 \cdot TH^{(3)} H^{(4)}$ | $H^{(3)} H^{(4)}$ | $K_{23}(0,0,0,1,1,0,0,0,0)$ | $\beta$ $\frac{1}{2^4(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{2880} = 126$ |
| 24       | $84 \cdot H^{(0)} TH^{(5)}$ | $H^{(0)} H^{(5)}$ | $K_{24}(3,0,0,0,0,1,0,0,0)$ | $\beta$ $\frac{1}{2^5(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{4320} = 84$ |
| 25       | $252 \cdot H^{(0)} TH^{(1)} H^{(6)}$ | $H^{(0)} H^{(1)} H^{(6)}$ | $K_{25}(1,1,0,0,1,0,0,0,0)$ | $\beta$ $\frac{1}{2^6(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{4320} = 252$ |
| 26       | $84 \cdot TH^{(2)} H^{(5)}$ | $H^{(2)} H^{(5)}$ | $K_{26}(0,0,1,0,0,1,0,0,0)$ | $\beta$ $\frac{1}{2^5(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{4320} = 84$ |
| 27       | $36 \cdot H^{(0)} TH^{(6)}$ | $H^{(0)} H^{(6)}$ | $K_{27}(2,0,0,0,0,0,1,0,0)$ | $\beta$ $\frac{1}{2^6(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{10080} = 36$ |
| 28       | $36 \cdot TH^{(1)} H^{(6)}$ | $H^{(1)} H^{(6)}$ | $K_{28}(0,1,0,0,0,0,1,0,0)$ | $\beta$ $\frac{1}{2^6(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{10080} = 36$ |
| 29       | $9 \cdot H^{(0)} TH^{(7)}$ | $H^{(0)} H^{(7)}$ | $K_{29}(1,0,0,0,0,0,1,0,0)$ | $\beta$ $\frac{1}{2^7(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{40320} = 9$ |
| 30       | $1 \cdot TH^{(8)}$ | $H^{(8)}$ | $K_{30}(0,0,0,0,0,0,0,0,1)$ | $\beta$ $\frac{1}{2^8(4)^2}$ $\frac{1}{2^1(4)^3}$ $\frac{9}{2^4}$ $\frac{9}{1} = 1$ |
The Kauffman Constraint Coefficients $K_o$

V. Generating $K_7$ and $K_{33}$ for $\theta^{(12)}$

We extend our application of the Kauffman Constraint Coefficient $K_o$ to the slightly more difficult $\theta^{(12)}$ derivative. In so doing, we will only calculate the coefficients for two Elementals, namely,

\begin{equation}
E_7 = H^{(0)}H^{(2)}
\end{equation}

\begin{equation}
E_{33} = H^{(0)}H^{(4)}H^{(5)}
\end{equation}

The corresponding Kauffman Constraint Coefficients are

\begin{equation}
K_7(3,0,3,0,0,0,0,0,0,0,0,0,0,0,0) = \frac{\beta_7}{\nu_2(1)^3\nu_2(1)^2} = \frac{12}{7776} = 16,000
\end{equation}

\begin{equation}
K_{33}(1,0,0,1,1,0,0,0,0,0,0,0,0,0,0) = \frac{\beta_{33}}{\nu_2(1)^3\nu_2(1)^4\nu_2(6)^4} = \frac{12}{86,400} = 5,544
\end{equation}

The reader should find these calculations using Mathematica™, or many other software programs, computationally reasonable.

VI. The $\theta^{(40)}$ Challenge

To further demonstrate the usefulness of the Kauffman Constraint Coefficient, a quick calculation can be made that would otherwise push the computational limits of Mathematica™ on many non-supercomputers. Namely, the reader may find it challenging to find the coefficient of the Elemental

\begin{equation}
E_{40} = H^{(0)}H^{(1)}H^{(36)}
\end{equation}

of the fortieth time derivative $\theta^{(40)}$. The predicted Kauffman Constraint Coefficient is

\begin{equation}
K_{40}(1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0) = \frac{\beta_{40}}{\nu_2(1)^3\nu_2(1)^2\nu_3(3)^8} = \frac{40}{38,880} = 2,964
\end{equation}

VII. Conclusion

Because this paper is not designed to recount the full scope of Non-Commutative Worlds® as explicitly described by Louis H. Kauffman, we invite the reader to become more fully engaged in his revolutionary work. As such, we have presented only a summary of Kauffman’s mathematics and expressed motivations for creating an Algebra of Constraints. From this, we answered an outstanding question of the algebra: What is the structure of the coefficients in the Kauffman Recursion Relation for $\theta^{(\beta)}$?

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Footnote: 8 Ibid., pps. 1-34.
The Kauffman Constraint Coefficients $K_{\omega}$ and their corresponding Elementals $E_{\omega}$ are presented as solutions to the construction of the $\theta^{(d)}$ derivative. Additionally, a new recursion relation is provided that requires only operational substitutions and summations; this algorithmically simplifies Kauffman’s original technique. To demonstrate $K_{\omega}$, we generate the 30 $K_{\omega}$ Coefficients from the corresponding Elementals $E_{\omega}$ for $\theta^{(9)}$ and find that our results are in complete agreement with Kauffman’s Mathematica™ solutions. We further present a calculation of two coefficients for the $\theta^{(12)}$ derivative and invite readers to use Mathematica™ or any other means to calculate and verify our results. Finally, we present a challenging calculation for a coefficient of the $\theta^{(40)}$ derivative series; owing to the vast numbers of permutations involved, a Mathematica™ approach may require substantial computer resources to obtain the solution in a reasonable time.

These formula, calculations and techniques for the Kauffman Constraint Coefficients $K_{\omega}$ and their corresponding Elementals $E_{\omega}$ are presented to enable the further development and discussion of Kauffman’s emerging Algebra of Constraints.

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**VII. References**

1. Louis H. Kauffman [2011], Non-Commutative Worlds – Classical Constraints, Relativity and the Bianchi Identity, ArXiv:1109.1085v1 [math-ph], September 06, 2011, 23-26, 30-32.