CALCULATION OF THE ELECTRIC FIELD DISTRIBUTION IN THE VICINITY OF THE CONDUCTIVE ROD

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ABSTRACT The article reviews the methods of mathematical modeling of electric fields in the vicinity of conducting rods and presents a method developed for calculating the distribution of the electric field strength and potential in systems with conducting rods. This method allows to use a computational spatial grid with a step proportional not to the radius of the rod, but to its length, which is relevant when the ratio of the rod length to its radius is large. The method is applied to the calculation of rods EF, for which this ratio is of the order of \( 10^2 - 10^3 \). The proposed method is based on the finite integration method. At the same time, the non-linear decrease in the levels of strength and potential when moving away from the rod in directions perpendicular to its axis is taken into account. The difference coefficients at the nodes surrounding the rod were obtained by integrating over the computational grid cell surfaces of expressions describing the strength and potential of the electric field for an elongated conducting ellipsoid under potential. With this representation of the conducting rod, it was possible to achieve the greatest accordance of calculations with the analytical solution. In practice, the application of the presented method allows for a more accurate calculation of the electric field in the vicinity of a conducting rod, which is either under potential, or in a homogeneous electric field, using a computational grid with a step proportional not to the radius of the rod, but to its length. The non-linear character of the decrease in the strength and potential of the electric field near the rod is taken into account using analytical expressions for a conductive ellipsoid under potential. In the area surrounding the rod and above its top, when using a spatial grid step proportionate with the length of the rod, and not with its radius, the relative errors in calculating the strength decreased from 27 % to 3 %. The results of calculating the electric field of a lightning rod are presented in order to analyze the conditions for the occurrence of upward leaders.

Keywords: electric field potential; electric field strength; conducting rods; finite integration method; lightning rods; mathematical modeling; upward leaders

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АНОТАЦІЯ У статті проведений огляд методів математичного моделювання електричних полів в околі електропровідних стрижнів і представлений метод, що розробленний для розрахунку розподілу напруженості і потенціалу електричного поля в системах з електропровідними стрижнями. Цей метод дозволяє використовувати розрахункову просторову сітку, що має крок, пропорційний не радіусу стрижня, а його довжині, що є актуальним при великому співвідношенні довжини стрижня до його радіусу. Метод заснований для розрахунку стрижнів, у яких таке співвідношення стяга порядок \( 10^2 \) – \( 10^3 \). Запропонований метод побудований на базі методу скінченного інтегрування. При цьому враховує нелінійне спадання рівня напруженості і потенціалу при віддаленні від стрижня в напрямках, перпендикулярних його осі. Різниці коефіцієнти у вузлах, що оточують стрижень, були отримані шляхом інтегрування по поверхнях комірок розрахункової сітки таких виразів, що описують напруженість і потенціал електричного поля для видовженого еліпсоїда, який перебуває під потенціалом. При такому поділі електропровідного стрижня вдавалося досягти найбільшого збігу розрахунків з аналітичними рішеннями. Практичне застосування представленого методу дозволяє здійснювати більш точний розрахунок електричного поля в околі електропровідного стрижня, що знаходиться або під потенціалом, або в однорідному ЕП, з використанням розрахункової сітки з кроком, співрозмірним не з радіусом стрижня, а з його довжиною. Врахування нелінійного характеру спадання напруженості і потенціалу електричного поля поблизу стрижня проводиться за допомогою аналітичних виразів для електропровідного еліпсоїда, що перебуває під потенціалом. У випадку, що оточує стрижень, і над його вершиною при використанні кроку просторової сітки, що віддалені від стрижня в напрямках, перпендикулярних його осі, результати розрахунку електричного поля стрижневого блискавкоприймача з метою дослідження умов виникнення зустрічних лідерів.

Ключові слова: потенціал електричного поля; напруженість електричного поля; електропровідні стрижні; метод скінченного інтегрування; стрижніві блискавкоприймачі; математичне моделювання; зустрічні лідери

Introduction

Solving the problems of modeling rod lightning rods, incomplete insulation breakdown channels, channels of lightning leaders [1-6] may be performed by
The aim of the work

The aim of this work is to develop a refined method for the numerical calculation of the electric field in the vicinity of conducting rods with a large ratio of length to radius. The difference coefficients in the nodes of the computational grid surrounding the rod are determined in a special way. As a result, the used step of the computational grid can be proportional to the length of the rod rather than its radius. This approach increases the accuracy of calculating the electric field strength in systems containing conducting rods.

Basic research materials

The calculation used a system (Fig. 1) with a grounded rod with potential \( U_0 \).

The computational equations used in the finite volume method [21,22] were obtained by integrating Maxwell's equations over the volumes \( V \) of the unit cells that make up the computational domain. The nodes of the computational grid \((i, j, k)\), in which the electric field potentials are determined, are located at the interface between the media, and therefore on the axis of the conducting rod (Fig. 2). With such an arrangement of the computational grid nodes, the conditions at the interfaces between the media are satisfied automatically.

To obtain the difference coefficients, Maxwell's equation was used [20]:

\[
\text{rot} \mathbf{H} = \gamma \mathbf{E} + \frac{\partial \mathbf{D}}{\partial t},
\]

where \( D = \varepsilon_0 \varepsilon E = \varepsilon_0 \varepsilon \varphi \) - electric induction; \( \varepsilon_0 = 8.85 \times 10^{-12} \) F/m; \( E \) - electric field strength; \( \varphi \) - electrical potential; \( \gamma \) - conductivity; \( H \) - magnetic field strength.
After applying the divergence operation to the equation, the resulting expression must be integrated over the volumes of the unit cells \( V \) that make up the computational domain using the Ostrogradsky-Gauss theorem. In the absence of space charges, the second term on the right-hand side of the last equation can be neglected. As a result, an equation will be obtained for determining the electric potential \( \phi \):

\[
\int_V \gamma E dV = \frac{\gamma E_s}{s} dS = \int_S -\gamma \frac{\partial \phi}{\partial n} dS = 0,
\]

where \( n \) – normal to the surface \( S \) enclosing the volume \( V \), \( E_s \) – the projection of the vector strength \( E \) normal to the surface \( S \).

For each node \((i, j, k)\) of the computational grid, an equation of the form (1) is drawn up. In this case, each cell of the computational domain is characterized by its specific conductivity \( \gamma_{i,j,k} \). Since the length of the rod is several orders of magnitude larger than its radius, the rod in the design model is replaced by a set of nodes located on the axis with indices \( r \) (Fig. 2).

The non-linear nature of the decrease in the strength and potential of the electric field in the direction perpendicular to the rod axis is taken into account using the conductivity tensor:

\[
\underline{\tau}_{ijkl} = \begin{bmatrix}
\gamma & k_x & 0 & 0 \\
0 & \gamma & k_y & 0 \\
0 & 0 & \gamma & k_z \\
0 & 0 & 0 & \gamma
\end{bmatrix},
\]

where \( k_x, k_y, k_z \) – coefficients equal to 1 for all nodes except \((i, j, k), (i-1, j, k), (i, j, k-1)\).

The presence of conductivity between the nodes \((i, j, k), (i, j, k-1)\) and the coefficient \( k_r \) of the node at the top of the rod are determined from analytical expressions for the coefficients \( k_r = 10^6 \) for these nodes.

The coefficients \( k_r \) and \( k_z \) of the nodes surrounding the rod and lying on its axis, and the coefficient \( k_z \) of the node at the top of the rod are determined from analytical expressions for the potential level and the electric field strength depending on the voltage applied to the rod. The rod is represented as an elongated ellipsoid [20]. In this case, the potential is determined by the expression:

\[
\phi = U_0 \cdot f_{\phi}\;,
\]

where

\[
f_{\phi} = \frac{1}{2 \ln(2L/R)} \ln\frac{\sqrt{L^2 + L^2} + \sqrt{L^2 - R^2}}{\sqrt{L^2 + L^2} - \sqrt{L^2 - R^2}};
\]

components of the electric field strength vector:

\[
E_x = -U_0 \cdot f_{E_x};
\]

\[
E_y = -U_0 \cdot f_{E_y};
\]

\[
E_z = -U_0 \cdot f_{E_z},
\]

where

\[
f_{E_x} = \frac{1}{2 \ln(2L/R)} \frac{\sqrt{L^2 - R^2}}{\sqrt{L^2 + L^2} \cdot (\xi + R^2)} d_{E_x} \cdot x;
\]

\[
f_{E_y} = \frac{1}{2 \ln(2L/R)} \frac{\sqrt{L^2 - R^2}}{\sqrt{L^2 + L^2} \cdot (\xi + R^2)} d_{E_y} \cdot y;
\]

\[
f_{E_z} = \frac{1}{2 \ln(2L/R)} \frac{\sqrt{L^2 - R^2}}{\sqrt{L^2 + L^2} \cdot (\xi + R^2)} d_{E_z} \cdot z;
\]

\[
d_{E_x} = \left[ 1 - \frac{p - L^2}{\sqrt{p^2 + q}} \right] d_{E_y} = \left[ 1 - \frac{p - R^2}{\sqrt{p^2 + q}} \right] d_{E_z};
\]

\[
\xi = -p - \sqrt{p^2 + q}; \quad \xi > -b \quad \text{[9]}; \quad p = \frac{L^2 + R^2 - (x^2 + y^2 + z^2)}{2};
\]

\[
q = L^2 - L^2 \cdot (x^2 + y^2) - R^2 y^2; \quad x, y, z - \text{Cartesian coordinates (} i, j, k \text{)-th computational grid node}.
\]

The coefficients \( k_{x}, k_{y}, k_{z} \) of the nodes surrounding the bar and lying on its axis \((i, j, k), (i, j, k-1), (i, j, k))\), and the coefficient \( k_{z} \) of the node at the top of the rod are determined using in (3) instead of \( U_0 \) the potential difference at the nodes on the rod and spaced one step of the computational grid from it and using this expression in the formulas for the components of the EF strength (4)–(6). So, for the component \( E_x \) of the electric field strength, we express \( \partial \phi / \partial x \) through \( U_0 \) in the form of the potential difference at a node on the rod axis \( \phi(x_0, y_0, z_0) = U_0 \) and at a node spaced from the axis at the distance of the computational grid step \( \phi(x_{0-1}, y_0, z_0) \), corresponding to the derivative step back:

\[
\frac{\partial \phi}{\partial x} \bigg|_{x=x_{0-1}} \approx \Delta \phi / \Delta x = \frac{\phi(x_0, y_0, z_0) - \phi(x_{0-1}, y_0, z_0)}{\Delta x} = U_0 / \Delta x.
\]

Let us write the expression for \( U_0 \) in terms of \( \partial \phi / \partial x \) considering that the sign of the derivative is taken into account automatically when solving the system of equations:

\[
U_0 = \frac{\Delta \phi}{\Delta x} \cdot D_x,
\]

where

\[
D_x = \frac{\Delta x}{1 - f_{\phi}(x_{0-1}, y_0, z_0)}.
\]

For derivatives \( \partial \phi / \partial x \) and \( \partial \phi / \partial y \) \text{[}y=\text{y}_{\text{min}}\text{]} expressions are similar:

\[
\frac{\partial \phi}{\partial x} \approx \Delta \phi / \Delta x = \frac{\phi(x_0, y_{0-1}, z_0) - \phi(x_0, y_{0-1}, z_0)}{\Delta x} = U_0 / \Delta x.
\]

\[
\frac{\partial \phi}{\partial x} \approx \Delta \phi / \Delta x = \frac{\phi(x_0, y_{0-1}, z_0) - \phi(x_0, y_{0-1}, z_0)}{\Delta x} = U_0 / \Delta x.
\]

We use \( U_0 \) in the last two equations, expressed through the change in potential in directions along the axis \( x \) (\( \Delta \phi / \Delta x \)) and \( z \) (\( \Delta \phi / \Delta z \)):

\[
U_0 = \frac{\Delta \phi}{\Delta x} \cdot D_x; \quad U_0 = \frac{\Delta \phi}{\Delta y} \cdot D_y,
\]

\[
D_x = \frac{\Delta x}{1 - f_{\phi}(x_{0-1}, y_0, z_0)};
\]

\[
D_y = \frac{\Delta y}{1 - f_{\phi}(x_0, y_{0-1}, z_0)}.
\]
where
\[ D_x = \frac{\Delta x}{|1 - f_{EE}(x, y, z, z_{ir})|}; \quad D_y = \frac{\Delta y}{|f_{EE}(x, y, z, z_{ir}) - 1|}. \]

The coefficient \( k_{ss} \) characterizes the non-linear potential drop between the nodes located on the rod axis and at a step away from it. To determine it, we write Eq. (4) for the cell surface \( S_{ir} \) at a distance of half a step from the rod axis \((x = x_0 + 1/2)\) and substitute \( U_0 \) into it in the form (7):
\[ E_s(x_{ir+1/2}, y_{ir}, z_{ir}) = \frac{\Delta \varphi}{\Delta x} \cdot D_y \cdot f_{EE}(x_{ir+1/2}, y_{ir}, z_{ir}). \]

Then
\[ E_s = \frac{\Delta \varphi}{\Delta x} k_{ss}, \]
where \( k_{ss} = D_y \cdot f_{EE}(x_{ir+1/2}, y_{ir}, z_{ir}). \)

Similar transformations made it possible to obtain expressions for the coefficients \( k_{s_1}, k_{s_2}, k_{s_3} \) (see (2)) are obtained by integrating \( k_{ss}, k_{s_1}, k_{s_2} \) over the corresponding surfaces \( S_{ir'}, S_{ir}, S_{ir'} \) (see fig. 2) using any standard subprogram.

Because the computational domain refers to open areas, then in order to reduce its dimensions when calculating the EF, uniaxially perfectly matched layers (UPML) were placed on its boundaries [17].

The unknown potentials of the nodes of the computational scheme are determined by solving a system of difference equations composed of equations of the form (1) for each node, using the iterative method of alternating directions and the sweep method [16,17]. The boundary conditions used in this calculation are shown in Fig. 1. The case is considered when the space charge is still absent in the zone where the development of the negative streamer stage begins [4]. The step of the spatial grid is chosen equal to 1 m, the absorbing layers located at the boundaries of the computational domain have the number of steps \( N=10, M=3, k_{max}=300 \) [18,19].

In the first calculated case, the downward lightning leader is at potential \( U_1 \) and is located at a height of several hundred meters above a grounded lightning rod of length \( L \) and radius \( R \). The leader's EF does not affect the EF of the lightning rod top due to the significant distance between them. EF strength in the vicinity of the lightning rod under thunderstorm conditions is \( E_0 = U_0/H_0 \), where \( H_0 \) is the thunderstorm cloud height. The calculation was carried out for the potential of the downward leader \( U_1 = 100 \text{ MV} \), launched from a thundercloud from a height of \( H_0 = 2000 \text{ m} \). The strength around the top of the lightning rod with a height of \( L = 60 \text{ m} \) and a diameter of \( 2R = 0.1 \text{ m} \) (see Fig. 1) was \( E_0 = 50 \text{ kV/m} \), the diameter of the lightning leader channel is \( 2R_L = 0.01 \text{ m} \). The calculation showed up that the obtained strength above the top of the lightning rod (no more than \( 100 \text{ kV/m} \)) is not enough to fulfill the condition for the appearance of an upward leader. The case was also calculated when the top of the downward leader approached the top of the lightning rod at a distance of 20 m. The distance between the axes of the leader channel and the lightning rod was \( D_{LR} = 5 \text{ m} \). The strength obtained in this case \( (300 \text{ kV/m}) \) is also insufficient to fulfill the condition of occurrence an upward leader. The calculation under the condition of the location of the downward leader streamer top at a height of \( 10 \text{ m} \) from the top of the lightning rod top showed up the level of the obtained tension exceeding \( 500 \text{ kV/m} \), thus only in this case the occurrence of an upward leader from the lightning rod is possible.

Conclusions

The proposed method for calculating the EF of conducting rods, in which the non-linear decrease nature of the strength and potential near the rod is taken into account using analytical expressions describing the EF of an elongated conducting ellipsoid under potential, made it possible to reduce the relative error in calculating the EF strength to 3 % or less for rods with \( L/R > 10^2 \cdot 10^3 \). In this case, the space step is chosen proportional to the length of the rod, and not to its radius.
Список литературы

1. Cooray V. Lightning Protection. London: The Institution of Engineering and Technology, 2010. 1036 p.
2. Moore C. B., Rison W., Mathis J., Aulich G. Lightning rod improvement studies. Journal of applied meteorology. 2000. 39. P. 593-609. doi:10.1175/1520-0450-39.5.593.
3. Moore C. B, Aulich G., Rison W. Measurement of lightning rod responses to nearby strikes. Geophys. Res. Lett. 2000. 27. no. 10. P. 1487-1490. doi: 10.1029/1999GL011053.
4. Bazelyan E. M., Raizer Yu. P. Lightning Physics and Lightning Protection. Bristol: IOP Publishing, 2000. 325 p.
5. Petrov N. I., Waters R. T. Determination of the striking distance of lightning to earthed structures. Proc. R. Soc. 1995. 450. P. 589-601. doi:10.1098/rspa.1995.0102.
6. Akuyz M., Cooray V. The franklin lightning conductor: conditions necessary for the initiation of a connecting leader. Journal of Electrostatics. 2001. 51-52. P. 319-325. doi:10.1063/S0030-3886(01)00113-9.
7. Cole M. T., Teo K. B. K., Groening O., Gangloff L., Legagneux P., Milne W. I. Deterministic cold cathode electron emission from carbon nanofibre arrays. Scientific Reports. 2014. 4. P. 1-5. doi:10.1038/srep04840.
8. Park S., Gupta A. P., Yeo S. J., Jung J., Paik S. H., Mativenga M., Kim S. H., Shin J. H., Ahn J. S., Ryu J. Carbon nanotube field emitters synthesized on metal alloy substrate by PECVD for customized compact field emission devices to be used in X-ray source applications. Nanomaterials. 2018. 8. P. 378. doi: 10.3390/nano8060378.
9. Bocharov G. S., Eletskii A. V., Grigory S. Theory of carbon nanotube (CNT)-based electron field emitters. Nanomaterials. 2013. 3. P. 393-442. doi: 10.3390/nano8060378.
10. Collins C. M., Parme R. J., Milne W. I., Cole M. T. High performance field emitters. Advanced Science. 2016. 3. P. 1500318. doi:10.1002/advs.201500318.
11. Zhu N., Chen J., Cole M., Milne W. Anomalous improved electron field emission from hybridised graphene on Mo tip arrays. 19th International Conference on Solid-State Sensors, Actuators and Microsystems. 2017. P. 870-873. doi:10.1109/TRANSDUCERS.2017.7994187.
12. Papageorgiou L., Metaxas A. C., Georgiou G. E. Three-dimensional numerical modeling of gas discharges at atmospheric pressure incorporating photoionization phenomena. J. Phys. D: Appl. Phys. 2011. 44. P. 045203. doi:10.1088/0022-3727/44/4/045203.
13. Berenger J. Perfectly matched layer for the FD TD solution of wave–structure interaction problems, IEEE Trans. Antennas and Propag. 1996. 44. P. 110-117. doi:10.1109/87.477535.
14. Raitlon C. J., Schneider J. B. An analytical and numerical analysis of several locally conformal FD TD schemes, IEEE trans. Microwave Theory and Techn. 1999. 47. P. 56-66. doi:10.1109/MMW.1999.766317.
15. Dey S., Mittra R. A Conformal Finite-Difference Time-Domain Technique for Modeling Cylindrical Dielectric Resonators. IEEE Transactions on Microwave Theory and Techniques. 1999. 47. no. 9. P. 1737-1739. doi:10.1109/22.788616.
16. Ismail M. S., Al-Basyoni K. S., A Logarithmic Finite Difference Method for Troesch’s Problem. Applied Mathematics. 2018. 9. no. 5. P. 550-559. doi: 10.4236/am.2018.95039.
17. Taflove A., Hagness S. Computational electromagnetics: the finite difference time domain method. Boston – London: Artech House, 2000. 852 p.
18. Rezynkina M. M., Rezynkin O. L., Sosina O. V. Mathematical modeling of distribution of magnetic field in the vicinity of the magnetic rods. Tekhnikona Elektrodynamika. 2014. no. 6. P. 30-36.
19. Rezynkina M. M., Rezynkin O. L., Svetlichnay E. E. Electric field in the vicinity of long thin conducting rods. Technical Physics. 2015. 60. no. 9. P. 1277-1283. doi: 10.1134/S0040684X15090182.
20. Stratton J. A. Electromagnetic theory. NJ: IEEE Press, 2007. 614 p.
21. Clemens M., Weiland T. Discrete electromagnetism with the finite integration technique. Progress in Electromagnetics Research. 2001. 32. P. 65-87. doi:10.2528/PIER00081003.
22. Clemens M., Weiland T. Regularization of eddy current formulations using discrete grad–div operators. IEEE Transactions on Magnetics. 2002. 38. no 2. P. 569-572. doi:10.1109/20.996149.

References (transliterated)

1. Cooray V. Lightning Protection. London, The Institution of Engineering & Technology, 2010. 1036 p.
2. Moore C. B., Rison W., Mathis J., Aulich G. Lightning rod improvement studies. Journal of Applied Meteorology. 2000, 39, pp. 593-609, doi:10.1175/1520-0450-39.5.593.
3. Moore C. B, Aulich G., Rison W. Lightning rod responses to nearby strikes. Geophys. Res. Lett. 2000, 27, no. 10, pp. 1487-1490, doi:10.1029/1999GL011053.
4. Bazelyan E. M., Raizer Yu. P. Lightning Physics and Lightning Protection. Bristol: IOP Publishing, 2000. 325 p.
5. Petrov N. I., Waters R. T. Determination of the striking distance of lightning to earthed structures. Proc. R. Soc. 1995, 450, pp. 589-601, doi:10.1098/rspa.1995.0102.
6. Akuyz M., Cooray V. The franklin lightning conductor: conditions necessary for the initiation of a connecting leader. Journal of Electrostatics, 2001, 51-52, pp. 319-325, doi:10.1016/S0304-3886(01)00113-9.
7. Cole M. T., Teo K. B. K., Groening O., Gangloff L., Legagneux P., Milne W. I. Deterministic cold cathode electron emission from carbon nanofibre arrays. Scientific Reports, 2014, 4, pp. 1-5, doi:10.1038/srep04840.
8. Park S., Gupta A. P., Yeo S. J., Jung J., Paik S. H., Mativenga M., Kim S. H., Shin J. H., Ahn J. S., Ryu J. Carbon nanotube field emitters synthesized on metal alloy substrate by PECVD for customized compact field emission devices to be used in X-ray source applications. Nanomaterials, 2018, 8, p. 378, doi: 10.3390/nano8060378.
9. Bocharov G. S., Eletskii A. V., Grigory S. Theory of carbon nanotube (CNT)-based electron field emitters. Nanomaterials, 2013, 3, pp. 393-442, doi:10.3390/nano30303039.
10. Collins C. M., Parme R. J., Milne W. I., Cole M. T. High performance field emitters. Advanced Science, 2016, 3, pp. 1500318, doi:10.1002/advs.201500318.
11. Zhu N., Chen J., Cole M., Milne W. Anomalous improved electron field emission from hybridised graphene on Mo tip arrays. 19th International Conference on Solid-State Sensors, Actuators and Microsystems, 2017, pp. 870-873, doi:10.1109/TRANSDUCERS.2017.7994187.
12. Papageorgiou L., Metaxas A. C., Georgiou G. E. Three-dimensional numerical modeling of gas discharges at atmospheric pressure incorporating photoionization phenomena, J. Phys. D: Appl. Phys., 2011, 44, pp. 045203, doi:10.1088/0022-3727/44/4/045203.
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