Mathematical functional for thermal distribution calculating during the electron-beam welding process

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Abstract. In this paper, we researched the influence of the electron beam defocus on the quality of the weld. The carried-out research was based on the theory of the thermal field using mathematical models developed by the authors. As a criterion of optimality, the previously developed thermal functional was used. We obtained criteria that can be used to simulate processes in electron beam welding, and to obtain more accurate modes, which is necessary to bind to power equipment. The obtained results of the criteria can be used for electron beam welding of various materials and different thicknesses.

1. Introduction
Electron beam welding (EBW) is used in the aerospace industry to connect various structures and assemblies made from metals of alloys of high purity and hardness, therefore, special requirements are placed on the selection and observance of the technological regime. An important place is given to the selection of the optimal value of the welding speed, depending on the accelerating voltage.

Electron beam welding can also connect dissimilar metals. This process can also be applied to so-called black and white metals. Similar statements apply to various non-ferrous metals. Similar statements apply to various non-ferrous metals. However, the situation can be more complicated when completely different metals (for example, chemical composition, thermal conductivity, solidification properties, thermal expansion coefficient, etc.) must be connected to each other [1, 2].

If a vapor-gas channel is formed in a special shape, namely such that the lower part has a wide and rounded profile. This can be achieved by varying the distribution of the effective beam energy in the weld zone. [3-7].

Therefore, to the criteria obtained, researches were carried out with the aim of modernizing the technology of electron beam welding and improving the quality of welded joints. Welding was carried out on a plate with a thickness of 0.12 cm from VT-14 titanium alloy.

General objective of the researching was to estimation of the beam and the distribution of energy over the heating spot by electron beam welding [1, 8-10].

The previously obtained results were performed without taking into account the parameters of the heating source. To indicate the heating parameters of the electron beam welding source, along with the accelerating voltage, it is necessary to introduce the distribution of the electron beam. Therefore, the calculations in this work were carried out for an accelerating voltage of 20-60 kV for the distribution of the electron beam in the form of a normal distribution law [11-12].
2. Computation of the welding operation parameters

To calculate the parameters of electron beam welding, a mathematical functional is used. Using a mathematical functional implies the use of two formulas from the theory of a thermal field. These are formulas of the spatial state of the temperature field when exposed to a fast moving linear and point source.

The formula for the state of the temperature field when exposed to a point source [1, 4, 13]:

$$T_1(x, y, z, q, v, t) = T_i + \frac{2q}{c\rho\sqrt{4\pi \tau^2}} e^{-\frac{q^2}{2\lambda \tau}} \int_0^\tau \exp \left( -\frac{v^2 \tau}{4a} - \frac{q^2}{4ac} \right) d\tau,$$

where $T_i$ – initial temperature of the product;

$c * \rho$ – heat capacity of the product;

$q$ – effective beam power;

$\alpha$ – thermal diffusivity;

$v$ – welding speed;

$t$ – time counted from the moment the source passes through the section at which the point in question is located;

$R$ – distance from the direction of movement of the source to the point in question.

The formula for the state of the temperature field when exposed to a linear source [1, 4, 13]:

$$T_2(x, y, z, q, v, t) = T_i + \frac{q}{4\pi \lambda \delta} e^{-\frac{v^2}{2\lambda \tau}} \int_0^\tau \exp \left( -\frac{v^2 \tau}{4a} - \frac{2\lambda \tau}{c\rho} - \frac{(x^2+y^2)}{4\alpha t} \right) d\tau,$$

where $T_i$ – initial temperature of the product;

$\delta$ – thickness;

$\lambda$ – coefficient of thermal conductivity;

$b$ – plate heat transfer coefficient;

$v$ – welding speed;

$t$ – heat propagation time.

Point source equation (1) expresses the temperature increments in a semi-infinite body at the stage of heat saturation, i.e. when the temperatures of individual points are continuously increased. Equation (2) expresses the temperature increment in the plate at the stage of heat saturation.

The consideration of the heating source during the simulation is caused by need to evaluate the influence of the thickness of the product [13]:

$$T_l(x, y, z, q, v, t) = T_1(x, y, z, q, v, t) + T_2(x, y, z, q, v, t),$$

where $T_1(x, y, z, q, v, t)$ and $T_2(x, y, z, q, v, t)$ – mathematical models of movable, point (1) and linear (2) sources.

To find the parameters of the welding mode adopted next functional [4, 11]:

$$J = \left( \frac{1}{n-1} \sum_{i=1}^n \left( T_1(x, y, z, q, v, t) - \overline{T}(x, y, z, q, v, t) \right) \right)^2,$$

where $T_l(x, y, z, q, v, t) = \frac{T_1(x, y, z, q, v, t)}{T_{max}(x, y, z, q, v, t)} \cdot T_l(x, y, z, q, v, t)$ – temperature calculated by (1);

$\overline{T}(x, y, z, q, v, t)$ – average temperature;

$T_{max}(x, y, z, q, v, t)$ – maximum temperature value.

The criterion for choosing the scan path is the minimum value of the functional ($J_{min}$).

The calculation of the functional value is performed for a region whose dimensions are comparable with the dimensions of the penetration channel.
A complex source is selected from two sources equivalent to real sources that occur in the literature [11].

From the necessary optimality condition \( J \to min \) an optimality system of four equations was obtained [13]:

\[
\begin{bmatrix}
\frac{dj}{dt} \\
\frac{dj}{dv} \\
\frac{dj}{dT} \\
\frac{dj}{dq}
\end{bmatrix} = 0,
\]

\( \text{Equation (5)} \)

Next, write the welding current in expanded form, since the real parameters of the electron-beam equipment are taken into account, since the beam diameter, accelerating voltage, and beam coordinates:

\[
I = \frac{I_{eb}}{\sigma*\sqrt{2*\pi}} * \int_{-\infty}^{\infty} \varphi(x) * \exp \left( -\frac{(x-\epsilon)^2}{2*\sigma^2} \right) dx,
\]

\( \text{Equation (6)} \)

where \( I_{eb} \) – electron beam current;

\( x \) – abscissa coordinate;

\( \varphi(x) \) – effective secondary emission factor;

\( \sigma \) - standard deviation.

Table 1 shows the microsections of welded joints during welding of titanium alloy VT-14 with a thickness of 0.12 cm on the installation A306.13.

| Defocus | +3 mA | 0 mA | -0.5 mA | -3 mA |
|---------|------|-----|--------|------|
| ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) | ![Image](image4.png) |

The numerical values of the parameters are determined by the coordinates corresponding to the minimum of functional (4). Figure 1 shows a graph of the normal distribution law, which shows that the standard deviation is taken as the radius of the beam, and for the diameter of the beam is taken a value equal to two standard deviations.

The energy distribution in an electron beam can be described by a normal law, which is characterized by next formula [12, 14]:

\[
f(x) = \frac{1}{\sigma*\sqrt{2*\pi}} * \exp \left( -\frac{(x-m)^2}{2*\sigma^2} \right),
\]

\( \text{Equation (7)} \)

where \( x \) – abscissa coordinate,

\( m \) – expected value,

\( \sigma \) – standard deviation, describing the diameter of the electron beam. The smaller the diameter of the electron beam, the more high-quality beam the gun forms.
As a result of mathematical modeling, a functional change graph was plotted against the focal distance. Figure 3 shows the changing functional from the focal distance for an ideal source $Q = 299$ J and for an accelerating voltage of 20, 30, and 60 kV when the electron beam is distributed in the form of a normal distribution law.

3. Conclusions
The achieved means for calculating the focal distance of an electron beam is suitable for use in welding structures and products.

Numerical simulation of thermal processes to find the parameters of EBW can reduce the spent time on developing technologies for new nodes.

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