Anisotropic effect on two-dimensional cellular automaton traffic flow with periodic and open boundaries

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Abstract

By the use of computer simulations we investigate, in the cellular automaton of two-dimensional traffic flow, the anisotropic effect of the probabilities of the change of the move directions of cars, from up to right ($p_{ur}$) and from right to up ($p_{ru}$), on the dynamical jamming transition and velocities under the periodic boundary conditions in one hand and the phase diagram under the open boundary conditions in the other hand. However, in the former case, the first order jamming transition disappears when the cars alter their directions of move ($p_{ur} \neq 0$ and/or $p_{ru} \neq 0$). In the open boundary conditions, it is found that the first order line transition between jamming and moving phases is curved. Hence, by increasing the anisotropy, the moving phase region expand as well as the contraction of the jamming phase one. Moreover, in the isotropic case, and when each car changes its direction of move every time steps ($p_{ru} = p_{ur} = 1$), the transition from the jamming phase (or moving phase) to the maximal current one is of first order. Furthermore, the density profile decays, in the maximal current phase, with an exponent $\gamma \approx \frac{1}{4}$.

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1 Introduction

Transport phenomena in complex systems, in particular models of highway traffic flow, attracted much attention in recent years. Much of the effort was concentrated on discrete stochastic models of traffic flow, first proposed by Nagel and Schreckenberg [1], and subsequently studied by many other authors using a variety of techniques [2]-[5]. Since the introduction of the Nagel Shrekenberg (NaSch) model [1], cellular automata became a well established method of traffic flow modeling. Comparatively low computational cost of cellular automata models made it possible to conduct large-scale real-time simulations of urban traffic in the city of Duisburg [6] and Dallas/Forth Worth [7]. Compared with the fluid dynamical approaches to traffic flow problems, the CA models are conceptually simpler, and can be readily implemented on computers. These models have the advantages that they can be easily modified to deal with the effects of different kinds of realistic conditions, such as road blocks and hindrances, traffic accident [8], highway junctions [9], vehicle acceleration [10], stochastic delay due to drivers reactions [5], anisotropy of car distributions in different driving directions [11], faulty traffic lights [12]. Traffic flow is a kind of many body systems of strongly interacting cars. Recent studies reveal physical phenomena such as the dynamical phase transitions and nonlinear waves [13],[14]. When the car density increases, the jamming transition occurs and traffic jams appear. The jamming transitions from the freely moving traffic to the jammed traffic have been studied by microscopic and macroscopic models. The two-dimensional traffic flow is more complex than the one-dimensional case. The two-dimensional traffic has been investigated only by the cellular automaton models [15]-[18]. It has been shown that the jamming transition occurs in the two-dimensional case and is similar to the one-dimensional case.

The NaSch model [1] is a probabilistic CA model for one-dimensional highway traffic. It considered the effects of acceleration and stochastic delay of vehicles with high speed. A vehicle can move at most \( v_{\text{max}} \) sites in a time step, where \( v_{\text{max}} \) is the maximal velocity. The speed at a time step depends on the gap (number of empty sites) between successive vehicles. If the speed in the present time step is less than \( v_{\text{max}} \) and the gap ahead allows, the speed increases by one unit in the next time step. If the spacing ahead is less than the speed in the present time step, then the speed is reduced to the value allowed by the spacing. The speed of a car is reduced by one unit in the next time step with a probability \( p \), exhibiting a randomization in realistic traffic flows. We introduced in a recent article [19] the boundary effects on the NaSch model, a vehicle can enter without constraint, with a probability \( \alpha \), in the first site being to the left side of the road if this site is empty. While, a vehicle being on the right in the last site can leave the road with a probability \( \beta \). There is a free flow and jamming phase separated by a line of first order transitions, this transition occurs at \( \alpha < \beta \) for \( p \neq 0 \) and \( \alpha = \beta \) for \( p = 0 \), and the maximal current
phase is obtained only for $v_{\text{max}} = 1$ (this case coincides with the asymmetric exclusion process model), otherwise it vanishes. Cheybani et al. [20] introduced a constraint during the entry of a vehicle, where at site $i = 0$, that means out of the system a vehicle with the probability $\alpha$ and with the velocity $v = v_{\text{max}}$ is created. This car immediately moves according to the NaSch rules. If the velocity of the injected vehicle on $i = 0$ is $v = 0$, then the injected vehicle is deleted. For $p$ greater than a critical value, the maximal current phase separated by second order transitions occurs like for the asymmetric simple exclusion process (ASEP).

The two-dimensional models have been presented by Biham, Middeleton and Levine (BML) to mimic the traffic flow in the whole city. The BML model [15] is a simple two-dimensional (square lattice) CA model. Each cell of the lattice represents an intersection of an east-bound and a north-bound street. The spatial extension of the streets between two intersections is completely neglected. The cells (intersections) can either be empty or occupied by a vehicle moving to the east or to the north. In order to enable movement in two different directions, east-bound vehicles are updated at every odd discrete time-step whereas north-bound vehicles are updated at every even time-step. The velocity update of the cars is realized following the ASEP rules, a vehicle moves forward by one cell if the cell in front of is empty, otherwise the vehicle does not move. The alternating movement to a traffic lights cycle of one time-step. The traffic-flow model is given by a three-state CA on the square lattice. Biham, Middeleton and Levine have studied the traffic-flow problem only in the case $\rho_x = \rho_y = \frac{\rho}{2}$ where $\rho_x$ and $\rho_y$ are respectively the density of cars moving to the right and the density of cars moving upwards, and $\rho$ is the total density of cars. They have found that a dynamical jamming transition occurs at a critical density $\rho = \rho_c$ with increasing the density of cars. The dynamical jamming transition separates between the low-density moving phase and the high-density jamming phase. Nagatani [16] has investigated the anisotropic effect of density of cars on BML models, it was shown that the traffic-jam transition occurs at higher density of cars with increasing the difference between the density of right moving cars and the density of up moving cars. The difference of the densities of cars has an important effect on the dynamical jamming transition. Cuesta et al. [21] introduced the randomness parameter $\gamma$ which allows to control the trend of the motion of every car. Half of cars move horizontally (vertically) with the probability $\gamma (1 - \gamma)$ (accordingly, $1 - \gamma (\gamma)$ is the probability to move vertically (horizontally) ). The system in this case exhibits the phase diagram of a first order phase transition from a freely moving to a jammed phase. The curves presenting the variation of the mean velocity versus the density ($\rho$) undergo a discontinuous transition of magnitude $\Delta v (\gamma)$ at the transition $\rho_c$. As $\gamma$ increases, $\rho_c$ increases and $\Delta v (\gamma)$ decreases, and eventually vanishing for some randomness $\gamma_c$. Our aim in this paper is to study, using numerical simulations, the effect of the anisotropy of the probabilities of the change.
of the move directions of cars, from up to right \((p_{ur})\) and from right to up \((p_{ru})\), in the cellular automaton of two-dimensional traffic flow, on the dynamical jamming transition and velocities under the periodic boundary conditions in one hand and the phase diagram and density profile behaviour under the open boundary conditions in the other hand.

The paper is organized as follows; in the following section we define the model, the section 3 is reserved for results and discussions, the conclusion is given in section 4.

2 Model

We exhibit a simpler cellular automaton model that describes traffic flow in two dimensions (BML model), for periodic and open boundaries. The traffic flow model is given by a three-state CA on a square lattice. The CA model is defined on a square lattice of \(L \times L\) sites. Each site \((i, j)\), with \(1 \leq i \leq L\) and \(1 \leq j \leq L\), contains either a car moving upwards, a car moving to the right, or empty (Fig. 1). In order to take into account the anisotropic effect on BML model, we introduce in this paper two parameters, \(p_{ur}\) and \(p_{ru}\), which exhibit, respectively, the probability that an up-moving car change its direction to become a right-moving car, and the probability that a right-moving car change its direction to become an up-moving car. At the initial configuration, cars are randomly distributed at the sites on the square lattice in such manner that a uniform random number \(\rho\) \((0 \leq \rho \leq 1)\) is generated independently at each site and if \(0 \leq \rho \leq \rho_x\) its site is occupied by a right car, if \(\rho_x \leq \rho \leq \rho_x + \rho_y\) its site is occupied by a upwards car and if otherwise its site is empty.

In each time steps, the cars are randomly selected within sequential dynamics, if the selected car is in right-moving (up-moving) state, it moves to the right (up) unless the adjacent site on its right (upwards) hand side is occupied by another car, which can be either an up or right driver. If it is blocked by another car it does not move. After that, if the selected car is in the right-moving (up-moving) state, its state is altered into up-moving (right-moving) state with the probability \(p_{ru}\) \((p_{ur})\). Then, we perform computer simulations of the CA model starting with a set of random initial conditions for the system size \(L = 10 - 500\), the density \(\rho = 0.0 - 1.0\) of cars. Each run is obtained after 10000 - 50000 time steps. After a transient period that depends on the system size, on the random initial configuration and on the density of car, the system reaches its asymptotic state. In order to compute the average of any parameter \(u\) \((u)\), the values of \(u(t)\) obtained in the asymptotic state are averaged.

If we denote \(\tau(i, j)\) the state of the site \((i, j)\). The periodic boundaries case is defined by the conditions:

for \(1 \leq i \leq L\) : \(\tau(i, 0) = \tau(i, L)\) and \(\tau(i, L + 1) = \tau(i, 1)\)

for \(1 \leq j \leq L\) : \(\tau(0, j) = \tau(L, j)\) and \(\tau(L + 1, j) = \tau(1, j)\)

While for the open boundaries case, in each time steps, we visit the sites \((i, 1)\) \((1 \leq i \leq L)\),
each time that a site is empty then a right-moving car is injected with the probability $\alpha$. While the up-moving cars are injected in the sites $(1, j)$ $(1 \leq j \leq L)$ with the probability $\alpha$ when the site is empty. If an up-moving (right-moving) car reaches one of the sites located in the upper (right) of the lattice, i.e. the sites $(L, j)$ $(i, L)$ with $1 \leq j \leq L$ $(1 \leq i \leq L)$, it leaves the lattice with the probability $\beta$.

In the following section we have used the parameters $v_u$, $v_r$ and $v_g$ defined, respectively, as the mean up-velocity, the mean right-velocity and the mean velocity. In the periodic boundaries condition, $v_u$ ($v_r$) is the number of move performed by the up-moving (right-moving) cars calculated in each time steps averaged over the up-moving (right-moving) cars. The same procedure is carried out in order to compute $v_g$ except that we average over all cars. In the open boundaries condition, the parameters are calculated inside the square of length $2l + 1$ ($l = 6$) centered in the middle of the lattice (i.e. $(l + 1, l + 1)$). We define the parameters $d_u$, $d_r$ and $d$ ($j_u$, $j_r$ and $j$), as the density (current) at the middle of the lattice of, respectively, the up-moving cars, right-moving cars and all types of cars. The currents $j_u$ and $j_r$ at the site $(i, j)$ are defined, respectively, by $\langle u(i, j) (1 - g(i + 1, j)) \rangle_l$ and $\langle r(i, j) (1 - g(i, j + 1)) \rangle_l$, where $\langle \rangle_l$ is the average over the square $(2l + 1)^2$, while the global current is the summation of the both currents, $j = j_u + j_r$. The parameters $u(i, j)$, $r(i, j)$ and $g(i, j)$ are defined as the probability to find the site $(i, j)$ occupied, respectively, by the up-moving car, right-moving car and any type of cars. Hereafter, we use the following parameters, $\rho_u = \langle d_u \rangle$, $\rho_r = \langle d_r \rangle$ and $\rho = \langle d \rangle$.

3 Simulations and results

3.1 Periodic boundaries

We exhibit in Fig.2 the variation of the global mean velocity as a function of the density for $p_{ur} = p_{ru} = 0$, for different system sizes. The system presents two different asymptotic state, which are separate by a sharp dynamical transition. Before the transition, all cars move freely and the average velocity is $\langle v_g \rangle = 1$, while when the transition occurs, they are all stuck and $\langle v_g \rangle = 0$ separate rows of right and up cars along the diagonals from the upper-left to the lower-right corners, this situation prevents the cars to move. As the system size increases, the critical density $\rho_c$ tends to decrease giving rise to sharper transition, and stabilize for high system sizes ($L \geq 300$).

In the isotropic case, i.e. $p = p_{ur} = p_{ru}$ ($p \neq 0$), Fig.3 presents the variation of $\langle v_g \rangle = \langle v_r \rangle = \langle v_u \rangle$ as a function of the density for different values of $p$. The sharp dynamical transition vanishes and the mean velocity decreases monotonically with the density. In the case $p = 1$, for low density ($\rho \leq 0.34$) the mean velocity is equal to 1, which means that each car moves without undergoing interactions with the other cars, so the cars take routes that never overlapping. For $\rho \geq 0.34$, the mean velocity decreases
almost linearly with increasing the density. For \( p \neq 1 \), the mean velocity decreases with increasing the density, in this case the cars interact with each other since a car leaves its oblique route and block other cars. By fixing the density, the mean velocity increases with \( p \). So, the growing of \( p \) has for effect to avoid the formation of the traffic jam since the cars leave the tails very fast, this liberates the route for other cars which come from perpendicular direction or moving behind it. In the anisotropic case (i.e., \( p_{ur} \neq p_{ru} \)), we exhibit in the Figs.4a, 4b and 4c, respectively, the global, up and right mean velocities versus the density for different values of \( p_{ur} \) and taking \( p_{ru} = 1 \). The mean velocities (i.e., \( \langle v_r \rangle, \langle v_u \rangle \) and \( \langle v_g \rangle \)) decrease with increasing the density, both the right and up mean velocities vary with \( p_{ur} \). Indeed \( \langle v_r \rangle \) increases with \( p_{ur} \), while \( \langle v_u \rangle \) doesn’t vary monotonically with varying \( p_{ur} \).

In order to understand the variation of the velocities versus the anisotropy for fixed density, one have to investigate the behavior of the mean velocities versus \( p_{ur} \) and \( p_{ru} \). Since the symmetry of the system, it suffice to study the variation \( \langle v_r \rangle \) and \( \langle v_u \rangle \) as a function of \( p_{ur} \) instead of studying them versus both parameters. So, we exhibit the variation of \( \langle v_r \rangle \) and \( \langle v_u \rangle \) as a function of \( p_{ur} \) for different values of \( p_{ru} \) in the Figs. 5 and 6 for, respectively, \( \rho = 0.3 \) (low density) and \( \rho = 0.7 \) (high density). In the case \( p_{ur} = 0 \), \( \langle v_u \rangle = 1 \) (Fig.5a), there is only the up-moving cars in the lattice, and since the system is in the moving phase, the up-moving cars move freely. This is not the case at the jamming phase (Fig. 6a), in which the traffic jam tails of up-moving cars form, then limiting the mean velocity of up-moving cars (\( \langle v_u \rangle \neq 1 \), for \( p_{ur} = 0 \)). By increasing \( p_{ur} \) from \( p_{ur} = 0 \), the mean up-velocity decreases at low values of \( p_{ur} \), so, with the augmentation of \( p_{ur} \) there is the formation of right-moving cars in the lattice provoking the blockage of the up-moving cars. The decrease is more important for low \( p_{ru} \), in fact, for low values of \( p_{ru} \), the right-moving cars preserve their direction for long time yielding the formation of more and more long traffic jam tails. At low density, the up-velocity starts increasing from a value of \( p_{ur} \) which grows with \( p_{ru} \). This augmentation of the up-moving velocity is the consequence of the fact that the up-moving cars change their direction frequently avoiding the grow of the local small tails in the upwards direction. Since at low values of \( p_{ru} \), the formation of the traffic jam tails is more important, the augmentation of the up-moving velocity is manifested from low values of \( p_{ur} \). At high density, for \( p_{ru} \leq 0.5 \), the mean up-velocity preserve the same behavior noted at low density, except the fall occurred at low \( p_{ur} \) is more important for \( p_{ru} = 0.1 \), while for \( p_{ru} > 0.5 \) the mean up-velocity decreases with increasing \( p_{ur} \) for any value of \( p_{ur} \). In fact, at high density, the number of the empty sites is reduced, so the cars could not always leave the traffic jam tails. This explain the fall of \( \langle v_u \rangle \) for very low values of \( p_{ur} \) for \( p_{ru} = 0.1 \), indeed, by increasing \( p_{ur} \) under these conditions the number of right-moving cars increases without allowing to up-moving cars to leave the tails. For \( p_{ru} \geq 0.5 \), even for high values of \( p_{ur} \),
\( \langle v_u \rangle \) decreases with increasing \( p_{ur} \). The up-moving cars in this case could not always leave the tails when they change their state to become right-moving vehicles, for high values of \( p_{ru} \), they don’t stay in the same state for long time, they change their state to upward without leaving the tails. At high values of \( p_{ur} \), the variation of \( \langle v_u \rangle \) is more and more weak with increasing \( p_{ur} \) until becoming constant. In fact, for high values of \( p_{ur} \), even if \( p_{ur} \) increases the situation of the traffic jam does not change since there is the balance between the formation of the obstacles (right-moving cars), and the rate of unblocking the local clusters of traffic jam. The mean velocity of right-moving cars increases with \( p_{ur} \) for any value of \( p_{ru} \), except, at low density, the decrease occurred at lower \( p_{ur} \) for \( p_{ru} < 0.5 \) (Fig.5b) and at hight density when \( p_{ru} \leq 0.5 \) (Fig.6b). In fact, by increasing \( p_{ur} \) the number of up-moving cars decreases, yielding the diminish of the obstacles for the right-moving cars. For low values of \( p_{ru} \), \( \langle v_u \rangle \) diminish for very low values of \( p_{ur} \) before undergoing an increase. In fact, under these conditions, as we have quoted above, there is the formation of small number of right-moving cars, which block the up-moving cars during their move, but \( p_{ur} \) is not so higher to unblock the tails of the up-moving cars (the decrease of \( \langle v_u \rangle \) for low \( p_{ur} \)), this affects the right-moving cars during their move which are slowed down by the tails of up-moving cars. In the Fig. 7, we present the variation of the global, right and up velocities as a function of \( p_{ur} \) for \( p_{ru} = 0.5 \) and \( \rho = 0.7 \). We note that for \( p_{ur} < p_{ru} \) (\( p_{ur} > p_{ru} \)), the number of up-moving (right-moving) cars is more important than the number of right-moving (up-moving) cars, therefore the value of the mean global velocity tends to the right (up) velocity. For \( p_{ur} = p_{ru} \), the velocities are equal.

### 3.2 Open boundaries

In this case, the system exhibits three phases; moving phase, jamming phase and maximal current phase \([22]-[24] \). These phases are governed by three factors; the flow of entering of cars, the flow of exiting the system and the velocity of cars inside the network. In the case of the moving phase, the current and the density don’t depend upon the variation of \( \beta \) with taking \( \alpha \) fixed, while the density and the current increase by increasing \( \alpha \) and taking \( \beta \) fixed (Fig. 8). At the jamming phase, the current and the density inside the lattice don’t depend upon \( \alpha \) for fixed \( \beta \) (Fig.8), while by increasing \( \beta \) for a given \( \alpha \), the current increases and the density decreases. At maximal current phase, the current reaches its maximal value, and by varying \( \alpha \) and \( \beta \), the flow remains unchanged. If the parameters of our model (i.e., \( p_{ur} \) and \( p_{ru} \)) favour the higher velocities (and eventually high maximal flow), we need high values of \( \alpha \) and \( \beta \) in order to maintain the flow in its high level, which implies the shrink of the maximal current phase zone. In the isotropic case \( (p = p_{ur} = p_{ru}) \), the velocities increase with \( p \), so the maximal current phase zone is contracted to high values of \( \alpha \) and \( \beta \) (Fig. 9). At the moving phase, the density \( \rho \) does
not depend upon \( p \) for low values of \( \alpha \), while at higher \( \alpha \), at vicinity of the transition, the density increases with decreasing \( p \) (Fig. 8). In fact, for a weak values of \( p \), there is the formation of small traffic jam clusters which block the cars inside the lattice, for very low density (i.e. very low \( \alpha \)), such clusters don’t form and the cars move freely even at very low values of \( p \). Like in the asymmetric exclusion model (ASEP) [22]-[24], at the moving phase close to the transition to jamming phase, the system is divided into two regions, namely, the higher density region at the exit of the system, in our model this region is located at vicinity of the upper-right exit (Fig. 10), and the lower density region elsewhere. For low values of \( p \), the system is blocked rapidly with the formation of local traffic jam clusters, hence the density inside the higher density region decreases. By increasing \( p \), the traffic jam clusters are unblocked on filling the empty sites located between these clusters, which increases the density inside the higher density region. Therefore, the area (density) of the higher density region increases (decreases) with decreasing \( p \). Like in the ASEP model, the transition between the moving and jamming phases occurs when the higher density region invade the lattice. Since at high values of \( p \), the higher density region is more compact, and the lower density is more empty, the cars arrive more rapidly to the higher density region. Increasing \( \alpha \) near the transition, the higher density region propagate inside the lattice and occupy the remaining exits, by increasing \( p \) above \( p = 0.5 \), there is the equilibrium between the large surface of the higher density region at low \( p \) (\( p \geq 0.5 \)) caused by the formation of clusters, and the great velocity of cars moving forward this region at high \( p \), which accelerate its propagation, so the transition point \( \alpha_c \) does not vary with the variation of \( p \) above \( p = 0.5 \). As we have quoted above, the density of the higher density region increases with \( p \), and since this region occupy the lattice in the jamming phase, the density increases with \( p \) in this phase. The transition between the high density and the moving phases is a first order transition, while the transition between the moving and jamming phases and the maximal current phase are a second order transition, except in the case \( p = 1 \), for which the transitions are a first order transition. The first order transition line between the jamming and moving phases exhibits a curvature which is due to the dimension effect. Such result is obtained in the NaSch model in one-dimension with open boundaries but in the case \( v_{\text{max}} > 1 \) [20]. In fact, in the both cases the car has more possibilities of move, in the NaSch model it has \( v_{\text{max}} \) opportunities of move, while in our case the car could move either right or upwards. In contrast with the transition from moving to jamming phases, the transition between the moving phase and the maximal current phase arises without the formation of the higher density region in the exit of the system. The density inside the lattice increases monotonically, except that the rate of the augmentation increases with moving from the entrance to the exit. In the case \( p = 1 \), with the absence of the clusters, the augmentation of the density is not important at vicinity of the transition, and since the density in the maximal current phase is larger because of
the absence of the great empty spaces, the intersection between these two situations at
the transition reflect an unstable equilibrium state which indicate a first order transition.
The first order transition occurs at $\alpha < \beta$, indeed, the system reaches its jamming state
at low densities because of the mutual blockage of cars moving in different directions.

In order to study the anisotropic case, i.e. $p_{ur} \neq p_{ru}$, we exhibit in the Figs. 11 and 12
the variation of $\rho$ as a function of $\alpha$ and $\beta = 0.4$ for, respectively, $p_{ru} = 0.1$ and $p_{ru} = 1$
and various values of $p_{ur}$. In the case $p_{ur} > p_{ru}$ ($p_{ur} < p_{ru}$), the up-moving (right-moving)
cars entering on the bottom (left) of the lattice change their direction at vicinity of that
entry to become the right-moving (up-moving) cars, more $p_{ur}$ ($p_{ru}$) is larger than $p_{ru}$
($p_{ur}$) more the move of cars is carried out at vicinity of bottom (left) entry in the right
(up) direction. This augmentation of the density in the entrance (Fig. 13) prevents other
cars to enter via this side, which implies the diminish of the density inside the lattice at
the moving phase. This behavior in the entrance caused by the difference between $p_{ur}$
and $p_{ru}$ delay the fill of the system, so as the difference is important as the transition
occurs at high $\alpha_c$. At the jamming phase, the difference between the rates $p_{ur}$ and $p_{ru}$,
cause an abundance of one type of cars in relation to other type, which limit the blockage
between different type of cars with decreasing the empty spaces, hence the augmentation
of the density. At higher values of $p_{ru}$ (i.e., $p_{ru} = 1$) and varying $p_{ur}$, the system vary
between the situation in which there is particularly the up-moving cars inside the lattice,
and the situation in which there is as many up-moving cars as right-moving ones without
the formation of traffic jam tails, the density between these both situations does not vary
considerably. While in the second case (i.e., $p_{ru} = 0.1$) and varying $p_{ur}$, the system vary
between the situation in which there is especially the right-moving cars inside the lattice,
and the situation in which there is as many right-moving cars as up-moving ones with the
formation of traffic jam tails yielding the diminish of the density provoked by the spaces
between the clusters. Indeed, the variation of the density undergone at high density with
the variation of $p_{ur}$ is more important in the second case (i.e., $p_{ru} = 0.1$). The zone of the
maximal current phase shrink with increasing $|p_{ru} - p_{ur}|$ (Figs. 14 and 15). In fact, as we
have quoted above, having a greater difference between $p_{ru}$ and $p_{ur}$ means the formation
of a condensate band in one entering side which prevent the cars to enter the lattice.
So, we have to increase $\alpha$ to overcome the traffic jam in the entrance, in order to obtain
the density corresponding to the maximal current, hence the transition line between the
moving phase and the maximal current phase increases. And as the density increases
with the augmentation of $|p_{ru} - p_{ur}|$ at the jamming phase, one must increase $\beta$ in order
to drain the higher number of cars, which provoke the augmentation of the transition
line between the jamming phase and the maximal current phase. This variation of the
transition line, with varying $\beta$ and fixed value of $\alpha$, is not important in the case $p_{ru} = 1$
since the density does not vary considerably at high density by varying $p_{ur}$. The density
profile in the middle line (i.e. $i = L/2 \ (j = L/2)$) and varying $j \ (i)$ or in the oblique line from the left-bottom corner to right-upper corner, decays in the maximal current phase with an exponent $\gamma \approx 0.20$ (Fig. 16), both in the anisotropic and isotropic cases. This means that the model belong to another universality class than the models studied in one dimension, such as the ASEP ($\gamma = \frac{1}{2}$) [22],[23] and the NaSch model for $v_{\text{max}} > 1 \ (\gamma \approx \frac{2}{3})$ [20].

4 Conclusion

In this paper, we have established the anisotropy effect of directions of move on the cellular automaton of two-dimensional traffic flow (i.e. BML model), in the periodic and open boundaries conditions. We have shown in the periodic boundaries conditions that the first order jamming transition disappears when the cars could change their direction of move every time steps. The cars of different type play the role of obstacles for each other, by increasing the probability to change the direction from right (upward) to upward (right), $p_{ru \ (p_{ur})}$ by taking $p_{ur \ (p_{ru})}$ fixed, the mean velocity of up-moving (right-moving) cars increases, except the slight decrease at low $p_{ru \ (p_{ur})}$, while the mean velocity of right-moving (up-moving) cars decreases before undergoing an augmentation. In the open boundaries conditions, the system exhibits three phases, namely, moving phase, jamming phase and maximal current phase. The first order transition between the moving phase and the jamming one occurs at $\alpha < \beta$. The passage from first to second order transition occurs by decreasing the anisotropy. The zone of the high density (low density) phase contracts (expands) by the augmentation of the anisotropy. When the car change its direction of move every time steps, in the isotropic case, the transition from the jamming phase (or moving phase) to the maximal current phase is a first order transition instead of the second order one. The density profile in the middle line decays in the maximal current phase with an exponent $\gamma \approx \frac{1}{4}$, both in the anisotropic and isotropic cases.

References

[1] K. Nagel and M. Shreckenberg, J. Physique I 2, 2221 (1992)
[2] A. Schadschneider and M. Schreckenberg, J. Phys. A 26, L679 (1993)
[3] L. C. Q. Vilar and A.M.C. de Souza, Physica A 211, 84 (1994)
[4] K. Nagel and M. Paczuski, Phys. Rev. E 51, 2909 (1995)
[5] M. Schreckenberg, A. Schadschneider, K. Nagel and N. Ito, Phys. Rev. E 51, 2939 (1995)
[6] J. Esser and M. Schreckenberg, Int. J. Mod. Phys. C 8, 1025 (1997)
[7] P. M. Simon and K. Nagel, Phys. Rev. E **58**, 1286 (1998)
[8] T. Nagatani, J. Phys. A **26**, L1015 (1993)
[9] S. C. Benjamin, N. F. Johnson and P. M. Hui, J. Phys. A **29**, 3119 (1996)
[10] T. Nagatani, Physica A **233**, 137 (1996)
[11] T. Nagatani, J. Phys. Soc. Japan **62**, 2656 (1993)
[12] K. H. Chung, P. M. Hui and G. Q. Gu, Phys. Rev. E **51**, 772 (1995)
[13] T. Nagatani, Phys. Rev. E **58**, A271 (1998)
[14] T. Komatsu, S. Sasa, Phys. Rev. E **52**, 5574 (1995)
[15] O. Biham, A. A. Middelton and D. A. Levine, Phys. Rev. A **46**, R6124 (1992)
[16] T. Nagatani, J. Phys. Soc. Japan **62**, 2656 (1993)
[17] S. Tadaki and M. Kikuchi, Phys. Rev. E **50**, 4564 (1994)
[18] T. Horiguchi and T. Sakakibara, Physica A **252**, 388 (1998)
[19] A. Benyoussef, N. Boccara, H. Chakib and H. Ez-Zahraouy, Chinese J. Phys. **39**, 428 (2001)
[20] S. Cheybani, J. Kertész and M. Schreckenberg, cond-mat/0006223
[21] José A. Cuesta, Froilán C. Martínez, Juan M. Molera and Angel Sánchez, hep-lat/9305015
[22] B. Derrida, M.R. Evans and D. Mukamel, J. Phys. A **26**, 4911 (1993)
[23] B. Derrida, M.R. Evans, The asymmetric exclusion model: exact results through a matrix approach, in non equilibrium Statistical Mechanics in One Dimension, edited by V. Privman (Cambridge University Press, Cambridge, 1996)
[24] A. Benyoussef, H. Chakib and H. Ez-Zahraouy, Eur. Phys. J. B **8**, 275 (1999)
Figure captions:

Fig. 1: Schematic illustration of the traffic flow on the square lattice. There are two types of drivers: the right-moving car going to the right and the up-moving car going to the upwards. The right cars are presented by the symbol () and the upwards cars are presented by the symbol (△), the arrows indicate the cars which could move.

Fig. 2: The variation of the global mean velocity ⟨vg⟩ versus the density for different values of the lattice size L.

Fig. 3: The variation of the global mean velocity ⟨vg⟩ versus the density for different values of p = pur = pru (L = 100).

Fig. 4: The variation of a) the global mean velocity ⟨vg⟩, b) the up mean velocity ⟨vu⟩, c) the right mean velocity ⟨vr⟩, versus the density for different values of pur, and pru = 1 (L = 100).

Fig. 5: The variation of a) the up mean velocity ⟨vu⟩, b) the right mean velocity ⟨vr⟩, versus pur for different values of pru, and ρ = 0.3 (L = 100).

Fig. 6: The variation of a) the up mean velocity ⟨vu⟩, b) the right mean velocity ⟨vr⟩, versus pur for different values of pru, and ρ = 0.7 (L = 100).

Fig. 7: The variation of the mean velocities ⟨vg⟩, ⟨vu⟩ and ⟨vr⟩ versus pur for pru = 0.5 and ρ = 0.7 (L = 100).

Fig. 8: The variation of a) ρ and b) J, as a function of α for different values of p for β = 0.4 (L = 101).

Fig. 9: phase diagram (α, β) for different values of p, the continuous (dashed) line presents the first order (second order) transition.

Fig. 10: Schematic configuration of the system in the isotropic case for β = 0.4 and α = 0.148 and a) p = 0.5, b) p = 1. The up-moving cars are indicated by the vertical bar and the right-moving cars by the horizontal bar (L = 60).

Fig. 11: The variation of ρ as a function of α for different values of pur with β = 0.4 and pru = 0.1 (L = 101).

Fig. 12: The variation of ρ as a function of α for different values of pur with β = 0.4 and pru = 1 (L = 101).

Fig. 13: Schematic configuration of the system in the anisotropic case for β = 0.4, α = 0.1, pru = 0.1 and pur = 0.7. The up-moving cars are indicated by the vertical bar and the right-moving cars by the horizontal bar (L = 60).

Fig. 14: phase diagram (α, β) for different values of pur, pru = 0.1, the continuous (dashed) line presents the first order (second order) transition.

Fig. 15: phase diagram (α, β) for different values of pur, pru = 1, the continuous (dashed) line presents the first order (second order) transition.

Fig. 16: The variation of the density profile, (a) ρ(L, i) versus i (the horizontal middle line) and (b) ρ(i, i) versus i (the oblique line) at the maximal current phase for
$L = 500.$
\[ <V_g> \]

\[ \rho \]

\[ p_{ur} = 0.1 \]
\[ p_{ur} = 0.3 \]
\[ p_{ur} = 0.5 \]
\[ p_{ur} = 0.7 \]
\[ p_{ur} = 1 \]
\[ \langle V_u \rangle \]
\( \alpha \)

\( \rho \)

\( p_{ur} = 0.1 \)

\( p_{ur} = 0.3 \)

\( p_{ur} = 0.5 \)

\( p_{ur} = 0.7 \)

\( p_{ur} = 1 \)
Moving phase

Jamming phase

Max. current phase

\[ \beta \]

\[ \alpha \]

\[ p_{ur} = 0.1 \]

\[ p_{ur} = 0.3 \]

\[ p_{ur} = 0.5 \]

\[ p_{ur} = 0.7 \]

\[ p_{ur} = 1 \]
Moving phase

Jamming phase

Max. current phase

$\beta$

$\alpha$

$p_{ur} = 0.1$

$p_{ur} = 0.3$

$p_{ur} = 0.5$

$p_{ur} = 0.7$

$p_{ur} = 1$
