Light-induced swirling and locomotion

Ameneh Maghsoodi and Kaushik Bhattacharya

Division of Engineering and Applied Science, California Institute of Technology, Pasadena, CA 91125, USA

The design of remotely activated, untethered devices without onboard power is a continuing challenge in soft robotics. This work describes a method of generating a whirling motion in pre-stressed photomechanical liquid crystal elastomer fibres using steady illumination that can be exploited for propulsion and mixing. Photomechanical liquid crystal elastomers (LCEs) can convert light directly into mechanical deformation, making them attractive candidates for soft actuators capable of remote and multi-mode actuation. We propose a three-dimensional multi-scale model of the nonlinear and non-local dynamics of fibres of photomechanical LCEs under illumination. We use the model to show that pre-stressed helix-like fibres immersed in a fluid can undergo a periodic whirling motion under steady illumination. We analyse the photo-driven spatio-temporal pattern and stability of the whirling deformation, and provide a parametric study. Unlike previous work on photo-driven periodic motion, this whirling motion does not exploit instabilities in the form of snap-through phenomena, or unilateral constraints as in rolling. More broadly, our work provides an unusual example of a physical system capable of periodic motion under steady stimulus that does not exploit instabilities. We finally show that such motion can be exploited in developing remote controlled bioinspired microswimmers and novel micromixers.

1. Introduction

Liquid crystal elastomers (LCEs) are rubbery networks composed of cross-linked polymer chains that contain liquid-crystalline mesogens in their main or in side chains. External stimuli like heat changes the ordering of the liquid-crystalline (LC) mesogens leading to a change in shape or stimuli-induced deformation.
This phenomenon is reversible. LCEs containing light-sensitive molecules, such as azobenzene (azo) photochromes, exhibit a reversible photomechanical behaviour: they absorb light energy and convert it into mechanical energy by changing their shape. This fascinating photomechanical effect arises from the trans-cis isomerization of azobenzene dyes, a process in which the azo molecules absorb light energy and change their conformation from a linear trans isomer to a bent cis isomer. The steric interaction between the azo and LC molecules consequently changes the LC ordering leading to a photo-induced shape-change. Upon removing the light, the cis isomers thermally relax to the trans state leading to a reversal of the photo-induced shape-change.

Recent works demonstrate the potential of photomechanical materials for soft robotics because of their remarkable features: (1) properly synthesized photomechanical materials can undergo large, reversible deformation upon light irradiation; (2) these materials are lightweight and soft, and hence, appropriate to generate flexible motion in soft robotics; (3) light is a clean power source that can actuate a photomechanical object from distance, thereby eliminating the need for on-board power or tethers; and (4) light-induced deformations can be controlled by changing light intensity, wavelength or polarization [1–3], thereby providing a platform for multiplexing and control.

In particular, recent attention has focused on locomotion in soft-robotics. A long-standing problem in the use of active materials is the need to reset the material after each actuation stroke. The ability to rapidly turn on and turn off light enables such a reset. This has been exploited to build legged microwalkers [4], leg-free caterpillar-like crawling [5] and swimming flagellum [6]. Even more promising, the coupling between photo-isomerization, light propagation and nonlinear mechanics can be exploited to generate cyclic motion even under steady illumination. Yamada et al. [7] demonstrated a ring can be made to roll by simultaneously illuminating it with a combination of UV and natural light at two different but carefully chosen locations. Wie et al. [8] demonstrated that a flat monolithic polymer film made of azo-containing liquid crystalline polymer networks changes its conformation to a spiral ribbon under illumination. The ribbon also translates a large distance with continuous irradiation. Gelebart et al. [9] created a wave-like motion in a doubly clamped azo-LCE strip using steady illumination and exploited this for a framed walker under steady illumination. In these examples, the motion is generated by gravity and the reaction from the surface. In this work, we explore cyclic motion under steady illumination in an immersive environment, and propose potential applications as bioinspired microswimmers and micromixers, as shown in figure 1.

The generation of cyclic deformation has also been a subject of interest in chemo-mechanical systems since the demonstration of oscillatory swelling and deswelling in hydrogels [10,11]. In these systems, cyclic chemistry like the Belousov–Zhabotinsky reactions or Landolt pH oscillators are combined with hydrogels to obtain chemo-mechanical oscillations. These have recently been used to demonstrate spatio-temporal oscillatory motion that can be exploited for peristaltic
motion and walking [12], as well as homeostatic feedback loops [13]. A theoretical model of these systems shows that the system is statically bistable and the dissipative kinetics leads to these chemo-mechanical oscillations. In this way, these systems are conceptually similar to the wave-like motion [9]. In this work, we explore the emergence of periodic behaviour in a stable system.

We begin by developing a three-dimensional theory of photomechanical rods. The general theory of photomechanical coupling was developed by Corbett & Warner [14], and used by Warner & Mahadevan [15] to study the development of light-induced spontaneous curvature in beams and plates under the assumption of shallow penetration. Corbett et al. [16] studied deep penetration, and the deformation of beams under stress-free conditions (i.e. without loads). Korner et al. [17] developed the theory of photomechanical beams under loads and used it to explain the rolling of rings [7] and wave-like motion of doubly clamped beams [9].

All of this work was limited to the two-dimensional setting of beams. In this paper, we extend these ideas to the three-dimensional setting of rods by combining the photomechanical actuation with nonlinear Kirchhoff rod theory [18–20]. We do so in a dynamical setting with inertia.

We then use the theory to exploit photomechanical coupling and pre-stressed structural configuration to generate periodic whirling motion under steady illumination; a fascinating photomechanical response that, to our knowledge, has not been reported in the literature. Importantly, we show that such whirling motion of a fibre immersed in a viscous fluid generates propulsion along its rotational axis. This can be exploited in developing remote controlled bioinspired microswimmers and micromixers (figure 1). We also show that this phenomenon does not rely on instabilities of flapping, or on gravity. Finally, we show that we can control the photoresponse using illumination conditions (light intensity and direction), pre-stressed configuration of fibre (twist and bending) and cross-sectional shape (circular and rectangular). These provide opportunities to design microactuators for different conditions, and for control. Together, the model and the results open a new avenue for the development of remote actuated and controlled soft actuators.

2. Model

Consider a long, thin azo-LCE fibre subjected to a steady illumination $\Lambda$ that makes angles $\psi_1$ and $\psi_2$ with fixed frame $XYZ$, as illustrated in figure 2a. We describe the centreline of fibre as a time-dependent space curve $R(s,t)$, where $s$ is the arc-length along the fibre and $t$ is time. Let $\{\hat{a}_i(s,t)\}$ be a body-fixed orthonormal frame at point $s$ and time $t$ with unit vectors $\hat{a}_1$ and $\hat{a}_2$ coinciding with the principal axes of the cross section and $\hat{a}_3$ normal to the cross section [18]. We assume that the fibre is inextensible and unshearable so that $\hat{a}_3$ is always tangent to the space curve. We define the curvature vector $\kappa = [\dot{a}_3 \cdot \hat{a}_1, \dot{a}_3 \cdot \hat{a}_2, \dot{a}_1 \cdot \dot{a}_2]$ in the $\{\hat{a}_i\}$ frame. Note that the rod may have intrinsic (stress-free, illumination-free) curvature.

We now specify the photomechanical constitutive behaviour of the rod building on the ideas of [14,17]. Consider a short section of the filament at position $s$ and time $t$. Suppose for the moment that it is subjected to an illumination along axis $\hat{a}_1$ with intensity $\Lambda_1$, as shown in figure 2b. As the light diffracts into the cross section, it is absorbed by the photochrome molecules and attenuates. In this work, we assume shallow penetration governed by Beer’s Law [15]. Recall that, according to Beer’s Law, when a light impinges on a flat surface with intensity $\Lambda_0$, the intensity $\Lambda(d)$ at a depth $d$ is given by $\Lambda(d) = \Lambda_0 \exp(-d/d_0)$, where $d_0$ is the penetration depth [15]. Generalizing this to our general cross section, the intensity at any point $y$ on the cross section is given by $\Lambda(y) = \Lambda_1 f_1(y)$ for some function $f_1 : A \rightarrow [0, 1]$, which quickly decays to zero away from the surface, as illustrated in figure 2b. Examples of this function for circular and rectangular cross sections are provided in the electronic supplementary material.

The photochromes in the trans state absorb light and transform into the cis state, which can thermally relax back to the trans state. Therefore, the number density $n_c(y)$ of cis molecules at
Figure 2. Dynamic model of a photomechanical fibre under steady illumination. (a) Nonlinear dynamics of the fibre is described using Kirchhoff rod theory. Frame XYZ represents the fixed inertial frame and \{\hat{a}_i\} represents the body-fixed frame at each cross section of the fibre. The steady illumination is defined by the intensity \(\Lambda\) and angles \(\psi_1\) and \(\psi_2\) with respect to the X-axis and plane XY, respectively. (b) Profile of light intensity for an arbitrary (e.g. elliptical) cross section under illumination along \(\hat{a}_1\) in the limit of shallow penetration. (Online version in colour.)

Position \(y\) evolves according to

\[
\tau \dot{n}_t(y, t) = -n_t(y, t) + \alpha \Lambda(y) = -n_t(y, t) + \alpha \Lambda f_1(y),
\]

where the dot represents the material time derivative, \(\tau\) is thermal relaxation time (or cis lifetime) and \(\alpha\) is a constant depending on material properties and forward–backward reaction rates. The trans-cis isomerization changes the LC ordering that consequently introduces a spontaneous axial strain \(\varepsilon_c = -\lambda n_c\), where \(\lambda\) is a constant of proportionality with \(\lambda > 0\) corresponds to a contraction \([21]\). It follows that the axial normal stress \(\sigma(y) = E(\kappa_2 - \kappa_2^\text{in})\hat{a}_2\) and \(\kappa_2^\text{in}\) is the intrinsic (stress-free, illumination-free) curvature in the 2-direction, and \(E\) is Young’s modulus. The bending moment along \(\hat{a}_2\) is

\[
q_2 = \int_A (y \cdot \hat{a}_1)\sigma(y) \, dA = B_2(\kappa_2 - \kappa_2^0) \quad \text{and} \quad \kappa_2^0 = \kappa_2^\text{in} - \int_A \frac{n_c(y)}{I_2} (y \cdot \hat{a}_1) \, dA,
\]

where \(B_2 = EI_2\) is the bending rigidity along \(\hat{a}_2\) and \(I_2\) is the second moment of inertia along \(\hat{a}_2\). Now, differentiating \(\kappa_2^0\) with respect to time and using (2.1), we conclude that

\[
\tau \dot{\kappa}_2^0 + (\kappa_2^0 - \kappa_2^\text{in}) = \gamma_1 \Lambda_1 \quad \text{and} \quad \gamma_1 = -\frac{\alpha \lambda}{I_2} \int_A f_1(y)(y \cdot \hat{a}_1) \, dA.
\]

In the case of general illumination, we split it into components along \(\hat{a}_1\) and \(\hat{a}_2\) to conclude that the photomechanical constitutive behaviour of the fibre is given by

\[
q = B(\kappa - \kappa^0)
\]

and

\[
\tau \kappa^0 - (\kappa^0 - \kappa^\text{in}) = \Gamma \Lambda,
\]

where \(q\) is the vector of bending moments and torque, \(B\) is the tensorial rigidity (diag\([EI_1, EI_2, GJ]\)) in the \(\{\hat{a}_i\}\) basis with bending and torsional rigidities and \(\Gamma = \gamma_1 \hat{a}_1 \otimes \hat{a}_1 + \gamma_2 \hat{a}_2 \otimes \hat{a}_2\) is the photomechanical rate tensor (diag\([\gamma_1, \gamma_2, 0]\) in the \(\{\hat{a}_i\}\) basis).
For details on the numerical method, and its verification, see [24–26].

Finally, we complete the system of equations by considering the balance of linear and angular momenta [18,22]

\[
\left( \frac{\partial f}{\partial s} + \kappa \times f \right) + f_{ext} = m \left( \frac{\partial v}{\partial t} + \omega \times v \right)
\]

and

\[
\left( \frac{\partial q}{\partial s} + \kappa \times q \right) + \hat{a}_3 \times f + q_{ext} = \left( I_m \frac{\partial \omega}{\partial t} + \omega \times I_m \omega \right),
\]

where \( v \) is the particle velocity, \( \partial / \partial t \) is the spatial time derivative, \( \omega \) is the angular velocity of the frame \( \{ \hat{a}_i \} \), and \( f_{ext} \) and \( q_{ext} \) are the external or applied force and moment per unit length, respectively; the inextensibility

\[
\frac{\partial v}{\partial s} + \kappa \times v = \omega \times \hat{a}_3;
\]

the compatibility between curvature and angular velocity

\[
\frac{\partial \omega}{\partial s} + \kappa \times \omega = \frac{\partial \kappa}{\partial t};
\]

and appropriate initial and boundary conditions.

If the fibre is immersed in a fluid, it experiences a hydrodynamic drag force. For the range of values of the parameters in this paper, the dimensionless Reynolds number \( Re = \rho u D / \mu \) is less than 10 and the fluid–structure interaction remains in the laminar (in fact creeping flow) regime. Above, \( \rho \) is the fluid density, \( u \) is the velocity of fluid with respect to fibre, \( \mu \) is the dynamic viscosity of fluid and \( D \) is the diameter of fibre. Thus, using Stokes Law in the limit of low Reynolds number [23], the hydrodynamic drag force per unit length that acts on the fibre is given by

\[
f_{ext} = f_{\text{drag}} = -\text{diag}(C_1, C_2, C_3) v;
\]

where the normal, bi-normal and tangential drag coefficients are estimated to be \( C_1 = C_2 = 4\pi \mu / (\ln(L/D) + 0.84) \) and \( C_3 = 2\pi \mu / (\ln(L/D) - 0.2) \) for a circular cross section, and \( L \) is the length of fibre. The drag force is incorporated into the dynamic model through the term \( f_{ext} \) in (2.6).

To solve the nonlinear initial-boundary-value problem above, we employ a finite difference method in both space and time to discretize the equations. Starting with the initial condition of fibre at \( t = 0 \), the discretized equations are integrated over space at each successive time step. During spatial integration, the boundary conditions are satisfied using the shooting method for boundary-value problems. For details on the numerical method, and its verification, see [24–26].

### 3. Photo-driven motion of a buckled and twisted fibre

We now use the model to study the deformation of initially straight (\( \kappa^\text{in} = 0 \)) fibres with circular (\( \gamma_1 = \gamma_2, I_1 = I_2 \)) cross section that are buckled and possibly twisted, clamped at the two ends and illuminated. We use the parameters listed in table 1 unless otherwise specified.

| parameter (symbol) | value |
|--------------------|-------|
| Young's modulus (\( E \)) | 4 GPa [27] |
| fibre diameter (\( D \)) | 15 \( \mu \)m [27] |
| fibre length (\( L \)) | 15 mm [27] |
| penetration depth (\( a \)) | 0.56 \( \mu \)m [17] |
| relaxation time (\( \tau \)) | 0.1 s |
| cis-strain proportionality (\( \lambda \)) | 0.05 |

Table 1. Simulation parameters.
Figure 3. Light-induced periodic motion of a twisted-buckled fibre under steady illumination. (a) Schematic of fibre under illumination. (b) The buckled-twisted fibre undergoes a periodic whirling; dark blue refers to start of the cycle and light blue indicates the end of the cycle. (c) Radial distance of fibre relative to the X axis during one cycle. (d–f) Three orthogonal views of the photoactivated fibre. Red dots refer to maximum radius of fibre and black dots refer to mid-point. (g–i) Evolution of the light-induced spontaneous curvature $\kappa_0$ of the fibre in XYZ frame. (j–l) Evolution of velocity of the fibre in cylindrical coordinate $r\theta$X shown in (a). In this simulation, $L_b = 0.9L$, $\phi = 60^\circ$, $\Lambda = 300 \text{ W m}^{-2}$, $\psi_1 = 30^\circ$ and $\psi_2 = 20^\circ$. (Online version in colour.)

(a) Initialization

We begin by defining a pre-stressed doubly clamped conformation as the initial state of the fibre. Assume that the fibre is initially straight ($\kappa_{\text{in}} = 0$) with length $L$ and the nematic director is along the axis of the fibre (so $\lambda > 0$ in equation (2.2)). We compress and twist the fibre quasi-statically by applying a compressive force along and a torque about the axis of fibre till the fibre buckles (the final end to end distance is $L_b < L$ and angle of twist is $\phi$). We then clamp the two ends by constraining their position and rotation till the fibre is in equilibrium. This is the initial state of fibre, which we then illuminate as shown in figure 3a.

(b) Typical result

A typical result ($L_b = 0.9L$, $\phi = 60^\circ$, $\Lambda = 300 \text{ W m}^{-2}$, $\psi_1 = 30^\circ$, $\psi_2 = 20^\circ$) is shown in figure 3b–l. After an initial transient, the fibre undergoes a steady cyclic motion, snapshots of which over
one cycle are displayed in figure 3b (in perspective view) and d–f (three normal projections). Note from figure 3d that the rotation about the X axis (line joining the endpoints) is uniform and from figure 3e,f that the shape is slightly asymmetric about the X-axis and about the centre. Snapshots of the radial distance (distance to the line joining the endpoints) during the cycle is shown in figure 3c; note that this is almost invariant. In short, the buckled and twisted fibre rotates about the axis quite uniformly and with an almost rigid shape. However, the rotation is not rigid since the ends are clamped. Figure 3g–i plots the spatio-temporal evolution of the spontaneous curvature of the fibre (components of \( \kappa_0 \) with respect to the laboratory frame) over first five cycles, while figure 3j–l plots the velocity. These confirm that the motion is indeed periodic after an initial transient. We also note that after an initial transient, the radial and axial components of velocity remain very small confirming the almost fixed shape we observed earlier. The angular velocity shows small periodic fluctuations showing that the rotational motion is almost, but perfectly, uniform.

As the fibre is illuminated, the photomechanical coupling causes a change in spontaneous curvature which leads to a change of shape. The overall length of the fibre endows it with a very small twisting stiffness, and this enables the fibre to accommodate this change of shape by a rotary motion. This changes the illumination conditions (the angle between the direction of light propagation and the tangent to the fibre) leading to a further change of shape and this cascades into a cyclic motion that we observe. We emphasize that the fibre is clamped at both ends, so it is not free to rotate about the X-axis. This is consistent with the periodic fluctuations in the angular velocity. Still, the small twisting stiffness makes this a low energy manifold of deformation, thereby enabling the motion. Importantly, there is no instability, which we discuss further in the case with zero twist.

(c) The effect of illumination intensity and direction

Figure 4 compares the frequency \( f \) and pattern of oscillations over a range of values of illumination intensity \( \Lambda \) and angles \( \psi_1 \) and \( \psi_2 \). One needs a critical intensity before one has periodic motion (figure 4a); below this critical intensity, the fibre deforms and attains a photo-stationary state. Beyond this critical intensity, the frequency increases with the light intensity: this is consistent with the fact that greater intensity leads to a higher rate of increase in the photo-induced strain and spontaneous curvature.

The angle \( \psi_1 \) between the direction of light propagation and the X-axis (the line joining the endpoints) has a significant effect on the motion. When the illumination is perpendicular to the X-axis, then the fibre reaches a photo-stationary state (figure 4b). The frequency increases as \( \psi_1 \) decreases and reaches a peak when the illumination is parallel to the X-axis. Note that the whirling motion is symmetric with respect to the YZ plane perpendicular to the helical axis of fibre (i.e. \( \psi_1 = 90^{\circ} \)) such that the amplitude of frequency is the same but the direction is the opposite with respect to the YZ plane.

Figure 4c shows that angle \( \psi_1 \) also affects the shape of the rod, and the rotary motion becomes less steady as the illumination is perpendicular and the more steady as it is parallel to the X-axis. The angle \( \psi_2 \) which describes orientation in the YZ plane has no impact, consistent with almost rotary motion.

(d) The effect of pre-stressed conformation of fibre

Figure 5 examines the frequency as a function of the pre-stressed conformation, both the buckled length \( L_b \) and the angle of twist \( \phi \) (with \( \Lambda = 500 \text{ W m}^{-2} \) with \( \psi_1 = 30^{\circ} \) and \( \psi_2 = 0^{\circ} \)). The figure shows that the case of no twist, \( \phi = 0 \), is distinct and so we defer its discussion. Once twisted, figure 5a shows that the frequency increases almost linearly but only slightly with increasing twist. Increasing the buckling (i.e. reducing \( L_b \)) also increases the frequency. Figure 5b shows that the shape changes with a larger bulge with reduced \( L_b \).
Figure 4. The effect of illumination on the frequency and pattern of oscillations. (a) Frequency of oscillations as a function of light intensity, where $\psi_1 = 30^\circ$ and $\psi_2 = 0^\circ$. (b) Frequency of oscillations as a function of illumination direction for various illumination angles $\psi_1$ and $\psi_2$, where $L_b = 0.95L$ and $\phi = 120^\circ$, $\Lambda = 200 \text{ W m}^{-2}$. (c) Comparison of conformational change of fibre during periodic oscillations at three different illumination directions. Red dots refer to maximum radius of fibre and black dots refer to mid-point. (Online version in colour.)

(e) The effect of cross-sectional geometry

We now briefly examine whether the cross section of fibre affects the photo-driven motion. In the photomechanical model of fibre, the function $f_1(y)$ depends on both the geometrical and material properties of fibre. Examples of function $f_1(y)$ are presented and compared for circular and rectangular cross section in the electronic supplementary material. Figure 6 compares the photomechanical response of the fibres with square and rectangular cross sections where the illumination is along the X-axis from left to right. From figure 6a–c for a square cross-sectional fibre, the motion is periodic. The fibre rotates uniformly about its helical axis with an almost
constant shape; this is similar to that of circular cross-sectional fibre explained in previous sections.

Figure 6d–f shows the photomechanical response of a rectangular cross-sectional fibre. The photo-driven motion is periodic; however, the pattern of deformation is significantly different from circular cross-sectional fibre. The fibre rotates about its axis; however, the rotation velocity is largely non-uniform during the cycle, as observed in figure 6d. From figure 6e,f, the radial distance of fibre relative to rotation axis significantly changes. Also the maximum radius (red dots) translates along the X-direction while rotating about the X-axis. These results conclude
that the fibre undergoes a coupled flapping-whirling motion about its axis. Future analysis and parametric study on rectangular cross section can reveal the contribution of bending and torsional elasticity and aspect ratio on the coupled flapping-whirling motion.

(f) Zero twist

We now consider the case of zero twist ($\phi = 0$). The pre-stressed shape is planar and we assume that it belongs to the $XY$ plane, as shown schematically in figure 7a. There are two cases. The first is the planar case where the illumination is also in the $XY$ plane ($\psi_2 = 0^\circ$). This is exactly the situation studied by Gelebart et al. [9] experimentally (although they used a strip) and through modelling by Korner et al. [17] (although they used a quasi-static model with no inertia). When illuminated, we observe that the fibre goes into a flapping motion, as shown in figure 7b, coinciding with the results of Korner et al. [17]. The second case is the non-planar case where the illumination is not in the $XY$ plane ($\psi_2 \neq 0^\circ$). In this case, we see in figure 7c that the fibre goes into a whirling motion similar to that of the case with twist.

There is a crucial difference between the flapping motion in the planar case and the whirling motion in the non-planar case. Korner et al. [17] showed that the flapping motion in the planar case is enabled by snap-through buckling instability between two distinct buckled states. This is also shown in figure 7b where the arrows trace the position of the peak during one cycle. There are two segments where the evolution is slow as the peak/trough moves to the right (dashed horizontal arrows), and two segments where the evolution is fast as the fibre snaps between the up and down buckled states (diagonal arrows). The snap-through is confirmed by analysing the stability by examining the smallest eigenvalue of the stiffness matrix in figure 7d. We note that this becomes negative indicating instability. By contrast, the motion in whirling is always smooth and there is no instability. This is confirmed by analysing the stability by examining the smallest eigenvalue of the stiffness matrix in figure 7e. This is also in the case of an initially buckled and twisted fibre ($\phi \neq 0^\circ$) (see the electronic supplementary material). Figure 7f shows the frequency of flapping is smaller than that of whirling.
4. Photo-driven propulsion

We now examine whether the photo-driven motion discussed in the previous section can be used to generate propulsion. We consider that the fibre is immersed in a fluid and therefore subject to a hydrodynamic drag force. The total drag force $F_{\text{pro}}$ on the filament in laboratory frame is given by integrating the drag force per unit length over the length of the filament

$$F_{\text{pro}} = \int_0^L f_{\text{drag}} \, ds.$$  \hspace{1cm} (4.1)

Further, the average propulsive force over the cycle is

$$\bar{F}_{\text{pro}} = \frac{1}{T} \int_0^T F_{\text{pro}} \, dt;$$  \hspace{1cm} (4.2)

where $T$ is the period.

We assume that the fibre is attached to a bead, and therefore this force propels the bead through the fluid with the velocity $V = -C^{-1}F_{\text{pro}}$ where $C$ is the drag coefficient for the bead and $V$ is the velocity of bead in laboratory frame. We solve the equations of the fibre to obtain the total force, and then use this force to study the motion of the bead. We now describe results where the immersing fluid is water with viscosity of $\mu = 0.001 \, \text{Pa.s}$, and for simplicity, we assume $C$ is identity.

Figure 8 shows the results for a planar fibre ($\phi = \psi_2 = 0^\circ$) with a flapping motion and a spatial fibre ($\phi \neq 0^\circ$) with a whirling motion. We first consider the planar case. Figure 8a shows that the propulsive force is periodic after an initial transient; this is expected since the motion and the velocity $v$ of fibre are periodic, as noted earlier. There is a large propulsive force in the $Y$-direction that is normal to the axis of the fibre; however, the average of this force is zero. The propulsive force in the $Z$-direction normal to the plane is zero as expected. Finally, we have spikes in the propulsive force in the $X$-direction during the snap-through phase of the cycle; importantly, this is always in the same direction, and thus the net propulsive force (equation (4.2)) in the $X$-direction...
Figure 9. The effect of illumination and pre-stressed conformation of fibre on propulsion. (a) Propulsive force as a function of light intensity, where $\psi_1 = 30^\circ$ and $\psi_2 = 0^\circ$. (b) Propulsive force as a function of illumination direction for various illumination angles $\psi_1$ and $\psi_2$, where $L_b = 0.95L$ and $\phi = 120^\circ$, $\Lambda = 200$ W m$^{-2}$. (c) Propulsive force as a function of the initial twist $\phi = \{0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 180^\circ\}$ and buckling $L_b = \{0.9, 0.925, 0.95, 0.975\}L$. In these simulations $\Lambda = 500$ W m$^{-2}$, $\psi_1 = 30^\circ$ and $\psi_2 = 0^\circ$. (d) Propulsive force versus frequency at different illumination intensities $\Lambda = [50 - 500]$ W m$^{-2}$. (Online version in colour.)

is non-zero. All of this leads to the motion of the bead shown in figure 8b. Note that it follows a sinusoidal path of cyclic excursions in the Y-direction that produce no net translation, but a quasi-steady motion in the X-direction. This sinusoidal trail is reminiscent of the crawling of the nematode and Caenorhabditis elegans.

We then turn to the spatial case. Figure 8d shows that the propulsive force is periodic after an initial transient as expected. Once again we see large forces in the Y- and Z-directions, but they average to zero. However, there is a small but steady force in the X-direction. This leads to the helical or spiral motion as shown in figure 8e that is reminiscent of swimming of sperm and flagellated bacteria.

Figure 9 presents a parameter study. Figure 9a shows that the average propulsive force over the cycle increases with illumination. This is consistent with our finding that the frequency increases with illumination (cf. figure 4). Figure 9b shows the role of illumination direction, and this reflects the dependance of the frequency since the twist and the buckled length remain the same. Note that the propulsive force is symmetric with respect to the YZ plane perpendicular to the helical axis of fibre (i.e. $\psi_1 = 90^\circ$) such that the amplitude of force is the same but the direction is the opposite with respect to the YZ plane. Figure 9c shows that propulsive force increases with increasing twist, and that this increase is greater than that can be expected from the increase of frequency alone (cf. figure 4). Also, the propulsive force increases with decreasing $L_b$, and that this dependance is greater than that of the frequency.
**Figure 9d** shows that the propulsive force and the frequency are proportionally correlated. This is consistent with the resistive force theory and experimental data [28,29] for a rotating rigid cylindrical helix at low Reynolds number that the propulsive thrust increases linearly with respect to rotation frequency. We note that the slight deviation from a perfect linear function can correspond to the slight deformation of fibre during rotation while its overall shape remains almost constant (cf. figure 3).

**Data accessibility.** All algorithms and data are included in the paper and the electronic supplementary material [30].

**Authors’ contributions.** A.M.: conceptualization, data curation, formal analysis, investigation, methodology, writing—original draft, writing—review and editing; K.B.: conceptualization, formal analysis, funding acquisition, investigation, methodology, project administration, supervision, writing—original draft, writing—review and editing.

Both authors gave final approval for publication and agreed to be held accountable for the work performed therein.

**Conflict of interest declaration.** We declare we have no competing interests.

**Funding.** This study was supported by the US Office of Naval Research through Multi-investigator University Research Initiative (grant no. ONR N00014-18-1-2624).

**Acknowledgements.** We gratefully acknowledge the support of the US Office of Naval Research through Multi-investigator University Research Initiative grant no. ONR N00014-18-1-2624.

**References**

1. Kumar K, Knie C, Bléger D, Peletier MA, Friedrich H, Hecht S, Broer DJ, Debije MG, Schenning APHJ. 2016 A chaotic self-oscillating sunlight-driven polymer actuator. *Nat. Commun.* 7, 11975. (doi:10.1038/ncomms11975)

2. Zeng H, Wasylczyk P, Wiersma DS, Priimagi A. 2018 Light robots: bridging the gap between microrobotics and photomechanics in soft materials. *Adv. Mater.* 30, 1703554. (doi:10.1002/adma.201703554)

3. Tabiryan N, Serak S, Dai X-M, Bunning T. 2005 Polymer film with optically controlled form and actuation. *Opt. Express* 13, 7442–7448. (doi:10.1364/OPEX.13.007442)

4. Zeng H, Wasylczyk P, Parmeggiani C, Martella D, Burresi M, Wiersma DS. 2015 Light-fueled microscopic walkers. *Adv. Mater.* 27, 3883–3887. (doi:10.1002/adma.201501446)

5. Zeng H, Wani OM, Wasylczyk A, Priimagi P. 2018 Light-driven, caterpillar-inspired miniature inching robot. *Macromol. Rapid Commun.* 391, 1700224. (doi:10.1002/marc.201700224)

6. Huang C, Lv J, Tian X, Wang Y, Yu Y, Liu J. 2015 Miniaturized swimming soft robot with complex movement actuated and controlled by remote light signals. *Sci. Rep.* 5, 17414. (doi:10.1038/srep17414)

7. Yamada M, Kondo M, Mamiya J, Yu Y, Kinoshita M, Barrett CJ, Ikeda T. 2008 Photomobile polymer materials: towards light-driven plastic motors. *Angew. Chem.* 120, 5064–5066. (doi:10.1002/ange.200800760)

8. Wie JJ, Shankar MR, White TJ. 2016 Photomotility of polymers. *Nat. Commun.* 7, 13260. (doi:10.1038/ncomms13260)

9. Gelebart AH, Jan Mulder D, Varga M, Konya A, Vantomme G, Meijer EW, Selinger RLB, Broer DJ. 2017 Making waves in a photoactive polymer film. *Nature* 546, 632–636. (doi:10.1038/nature22987)

10. Yoshida R, Takahashi T, Yamaguchi T, Ichijo H. 1996 Self-oscillating gel. *J. Am. Chem. Soc.* 118, 5134–5135. (doi:10.1021/ja9602511)

11. Crook CJ, Smith A, Jones RA, Ryan AJ. 2002 Chemically induced oscillations in a ph-responsive hydrogel. *Phys. Chem. Chem. Phys.* 4, 1367–1369. (doi:10.1039/b109977a)

12. Kim R, an Tamate YS, Akimoto AM, Yoshida R. 2017 Recent developments in self-oscillating polymeric systems as smart materials: from polymers to bulk hydrogels. *Materials Horizons* 4, 38–54. (doi:10.1039/C6MH00435K)

13. He X, Aizenberg M, Kuksenok O, Zarzar LD, Shastri A, Balazs AC, Aizenberg J. 2012 Synthetic homeostatic materials with chemo-mechano-chemical self-regulation. *Nature* 487, 214–218. (doi:10.1038/nature11223)

14. Corbett D, Warner M. 2006 Nonlinear photoresponse of disordered elastomers. *Phys. Rev. Lett.* 96, 237802. (doi:10.1103/PhysRevLett.96.237802)
15. Warner M, Mahadevan L. 2004 Photoinduced deformations of beams, plates, and films. Phys. Rev. Lett. 92, 134302. (doi:10.1103/PhysRevLett.92.134302)
16. Corbett D, Xuan C, Warner M. 2015 Deep optical penetration dynamics in photobending. Phys. Rev. E 92, 013206. (doi:10.1103/PhysRevE.92.013206)
17. Korner K, Kuenstler AS, Hayward RC, Audoly B, Bhattacharya K. 2020 A nonlinear beam model of photomotile structures. Proc. Natl Acad. Sci. USA 117, 9762–9770. (doi:10.1073/pnas.1915374117)
18. Dill EH. 1992 Kirchhoff’s theory of rods. Arch. Hist. Exact Sci. 44, 1–23. (doi:10.1007/BF00379680)
19. Antman SS. 2005 Nonlinear problems of elasticity. Berlin, Germany: Springer.
20. Audoly B, Pomeau Y. 2010 Elasticity and geometry: from hair curls to the nonlinear response of shells. Oxford, UK: Oxford University Press.
21. Corbett D, Warner M. 2007 Linear and nonlinear photoinduced deformations of cantilevers. Phys. Rev. Lett. 99, 174302. (doi:10.1103/PhysRevLett.99.174302)
22. Goyal S, Perkins NC, Lee CL. 2005 Nonlinear dynamics and loop formation in kirchhoff rods with implications to the mechanics of dna and cables. J. Comput. Phys. 209, 371–389. (doi:10.1016/j.jcp.2005.03.027)
23. Howard J. 2001 Mechanics of motor proteins and the cytoskeleton. Sunderland, MA: Sinauer associates.
24. Gatti-Bono C, Perkins NC. 2002 Physical and numerical modelling of the dynamic behavior of a fly line. J. Sound Vib. 255, 555–577. (doi:10.1006/jsvi.2001.4180)
25. Maghsoodi A, Chatterjee A, Andricioaei I, Perkins NC. 2016 A first model of the dynamics of the bacteriophage T4 injection machinery. J. Comput. Nonlinear Dyn. 11, 041026. (doi:10.1115/1.4033554)
26. Gobat JL, Grosenbaugh MA, Triantofyllou MS. 2002 Generalized-alpha time integration solutions for hanging chain dynamics. Technical report, Woods Hole Oceanographic Institution.
27. Smith ML, Shankar MR, Backman R, Tondiglia VP, Lee KM, McConney ME, Wang DH, Tan L-S, White TJ. 2014 Designing light responsive bistable arches for rapid, remotely triggered actuation. In Behavior and mechanics of multifunctional materials and composites 2014, vol. 9058 (eds NC Goulbourne, HE Naguib), pp. 121–130. Bellingham, WA: SPIE.
28. Rodenborn B, Chen C-H, Swinney HL, Liu B, Zhang HP. 2013 Propulsion of microorganisms by a helical flagellum. Proc. Natl Acad. Sci. USA 110, E338–E347. (doi:10.1073/pnas.1219831110)
29. Zhong X, Moored KW, Pinedo V, Garcia-Gonzalez J, Smits AJ. 2013 The flow field and axial thrust generated by a rotating rigid helix at low Reynolds numbers. Exp. Therm Fluid Sci. 46, 1–7. (doi:10.1016/j.expthermflusci.2012.10.019)
30. Maghsoodi A, Bhattacharya K. 2022 Light-induced swirling and locomotion. Figshare. (doi:10.6084/m9.figshare.c.6260162)