

\textbf{\textit{F(R) nonlinear massive gravity and cosmological implications}}

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We propose a nonlinear massive gravitational theory which includes \textit{F(R)} modifications. This construction inherits the benefits of the dRGT model and is free of the Boulware-Deser ghost due to the existence of a Hamiltonian constraint accompanied by a nontrivial secondary one. However, the advantage is that the scalar perturbations in a cosmological background can be stabilized at linear level in an FRW universe by tuning the \textit{F(R)} form. Finally, due to the combined contribution of the \textit{F(R)} and graviton-mass sectors, the proposed theory allows for a huge class of cosmological evolutions, such as the simultaneous and unified description of inflation and late-time acceleration.

\textbf{Introduction:} The search for a consistent theory of massive gravity has been open for decades. Its motivations arise from both theoretical considerations, namely to understand the construction procedure of a massive spin-2 theory, as well as (lately) from observational requirements, that is to explain the universe acceleration through such an Infra-Red (IR) modification of General Relativity. However, since the first, linear approach \textsuperscript{[1]}, the whole subject remains a theoretically intriguing problem.

In the instructive idea of Fierz and Pauli \textsuperscript{[1]} General Relativity is extended by introducing a linear mass term and thus the theory involves at least 5 degrees of freedom (dof), representing a massive spin-2 field in a Poincaré invariant background. However, it turns out that the graviton’s longitudinal degree of freedom remains coupled to the trace of the energy-momentum tensor regardless the smallness of the graviton mass. This leads to the famous van Dam-Veltman-Zakharov (vDVZ) discontinuity \textsuperscript{[2]} and thus to a severe challenge by experiments and observations. This discontinuity can be alleviated at the nonlinear level through the Vainshtein mechanism \textsuperscript{[3]}, however, due to the constraints on the dynamical variables, the same nonlinearities give rise to a ghost instability, called Boulware-Deser (BD) ghost \textsuperscript{[4]}. Using the effective field theory approach one can show that the BD ghost is related to the Goldstone boson associated with the broken general covariance \textsuperscript{[5]}.

The above inconsistencies puzzled physicists for years. Recently de Rham, Gabadadze and Tolley (dRGT) showed that the BD ghost can be removed in a suitable nonlinear massive gravitational theory \textsuperscript{[6]}. In particular, due to a delicate construction of the graviton potential, the Hamiltonian constraint and the associated secondary one are restored, and thus this IR modified theory becomes free from BD ghosts \textsuperscript{[7]}. Unfortunately, although safe at the fundamental level, when applied in a homogeneous and isotropic cosmological background, dRGT massive gravity exhibits ghost instabilities as it is shown in detailed analyses of cosmological perturbations \textsuperscript{[8]}.

On the other hand, after the 60’s physicists realized that although General Relativity is not renormalizable, possible high-energy corrections could improve renormalizability and thus quantization \textsuperscript{[8,10]}. Although these Ultra-violet (UV) corrections are expected to be of quantum origin or arise from an underlying fundamental theory such as string theory (for example see \textsuperscript{[11,12]}), one can describe them effectively, by investigating a classical, modified, gravitational action. The simplest model of such an UV modified gravity, that can sufficiently encapsulate the basic properties of higher-order gravitational theories, is the \textit{F(R)} paradigm, in which the gravitational Lagrangian is extended to an arbitrary function of the Ricci scalar (see \textsuperscript{[13]} for a review). The corresponding \textit{F(R)} cosmology is able to describe the inflationary epoch, and in particular the well-known Starobinsky’s \textit{R\textsuperscript{2}}-inflation scenario \textsuperscript{[14]} proves to be the best-fitted scenario with the recently-released Planck data \textsuperscript{[15]}.

Inspired by the above discussion, in this Letter we propose a modification of General Relativity both in the UV and IR regimes, that is the \textit{F(R)} nonlinear massive gravity. In this theory, the extra scalar dof of the \textit{F(R)} sector, clearly seen through a conformal transformation, has a positive-defined kinetic term as usual, and its interaction with the massive sector can stabilize metric perturbation of scalar type at linear order (this is a novel feature, not present in \textsuperscript{[16]}). In summary, the total theory is not only free of BD ghosts at the fundamental level, but it is also free of linear cosmological perturbative instabilities for the largest part of its parameter space, even in homogeneous and isotropic geometries. Finally, the increased freedom of both \textit{F(R)} and massive-graviton sectors can lead to a huge class of interesting cosmological behaviors at early and late times, in agreement with observations.

\textbf{The setup:} Imposing both the UV (\textit{F(R)} sector) and IR (graviton-mass sector) modifications, the total action becomes

\begin{equation}
S = \frac{M_p^2}{2} \int d^4x \sqrt{|g|} \left[ F(R) + 2m_g^2 \mathcal{U}_M \right],
\end{equation}

where \textit{M}_p the Planck mass, \textit{g} the physical metric and \textit{m}_g the graviton mass. As usual in dRGT construction, to build the dimensionless graviton potential \text{U}_M one needs to define the matrix \textit{X} \equiv 1 - \text{K}, where \text{K} \equiv \sqrt{g^{-1}f}.
involves a non-dynamical (fiducial) metric $f$. Then, the regular antisymmetrization in 4D space-time yields the following polynomials

$$U_2 = K_{[\mu}^{\nu} K_{\nu]}^{\rho}, U_3 = K_{[\mu}^{\nu} K_{\nu}^{\rho} K_{\rho]}^{\sigma}, U_4 = K_{[\mu}^{\nu} K_{\nu}^{\rho} K_{\rho}^{\sigma} K_{\sigma]}^{\rho};$$

and the graviton potential is given by $U_M = U_2 + \alpha_3 U_2 + \alpha_4 U_2$, containing two dimensionless parameters ($\alpha_3, \alpha_4$).

The UV sector inherits the remarkable properties of the $F(R)$ term. In particular, by performing the conformal transformation $g_{\mu\nu} \rightarrow g_{\mu\nu} = \Omega^2 \delta_{\mu\nu}$, the $F(R)$ part can be reformulated as the standard General Relativity minimally coupled to a canonical scalar field $\phi$, with effective potential

$$U(\phi) = M_p^2 [R F_{,R} - F]/2F_{,R}^2,$$  

where $F_{,R} \equiv \partial F/\partial R$. Additionally, the conformal transformation acts on the IR sector too, with the graviton potentials transforming as

$$\tilde{U}_M = \sum_{i=0}^{4} \Omega^{-4} \beta_i \tilde{E}_i,$$

where $\beta_i = (-1)^i [(4 - i)(3 - i)/2 + (4 - i)\alpha_3 + \alpha_4]$. In this expression, based on the transformed matrix $\tilde{X} = \sqrt{\gamma}^{-1} J$, we have introduced the elementary symmetric polynomial $\tilde{E}_i$ as:

$$\tilde{E}_0 = 1, \quad \tilde{E}_1 = \tilde{X}_\mu^{\nu}, \quad \tilde{E}_2 = \tilde{X}_\mu^{\nu} \tilde{X}_\nu^{\sigma}, \quad \tilde{E}_3 = \tilde{X}_\mu^{\nu} \tilde{X}_\nu^{\rho} \tilde{X}_\rho^{\sigma}, \quad \tilde{E}_4 = \tilde{X}_\mu^{\nu} \tilde{X}_\nu^{\rho} \tilde{X}_\rho^{\sigma} \tilde{X}_\sigma^{\mu}.$$  

Then the resulting Lagrangian in the Einstein frame can be written as

$$\mathcal{L} = \sqrt{-\gamma} \left[ \frac{M_p^2}{2} (\tilde{R} + 2m_y^2 \tilde{U}_M) - \frac{1}{2} \partial \varphi \partial^\mu \varphi - U(\varphi) \right].$$

Note that through this reformulation the present model can be also seen as an interesting phenomenological extension of the quasi-dilaton massive gravity [11, 14], since a potential term for the “dilaton” is introduced due to the UV modification.

### Hamiltonian constraint analysis

In order to see whether a given massive-gravity construction suffers from BD ghosts, one must perform the Hamiltonian constraint analysis following the powerful method developed in [7]. For simplicity we work within the Einstein frame and expand the metrics using the famous Arnowitt-Deser-Misner (ADM) formalism:

$$\tilde{g}_{\mu\nu} dx^\mu dx^\nu = -(N_0^g)^2 dt^2 + \gamma_{ij} (dx^i + N_{ij}^g dt)(dx^j + N_{ij}^g dt),$$

$$f_{\sigma\mu} dx^\mu dx^\sigma = -(N_0^f)^2 dt^2 + \omega_{ab} (dx^a + N_{ab}^f dt)(dx^b + N_{ab}^f dt).$$

The lapse $N_0^g$ and shift $\tilde{N}_g$ (the three $N_i^g$ are expressed as vector) of the physical metric, as well as the corresponding ones for the fiducial metric, $N_0^f$ and $\tilde{N}_f$ respectively, are all non-dynamical.

In massive gravity $\gamma_{ij}$ allows for at most six propagating modes, one of them being the origin of the BD ghost. Thus, a potentially healthy theory must maintain a single constraint on $\gamma$ (from now on a bar denotes the matrix form) and the corresponding conjugate momenta, along with an associated secondary constraint, which will lead to the elimination of the ghost dof. In the following we briefly show the existence of these constraints in $F(R)$ massive gravity, referring to [13] for the details.

In order to introduce the Lagrange multiplier explicitly, we define a new shift $\tilde{\eta}$ through $\tilde{N}_g - \tilde{N}_f = (N_0^g \bar{I} + N_0^f \bar{D}) \tilde{\eta}$, where $I$ is the $3 \times 3$ unit matrix and the $3 \times 3$ matrix $D$ satisfies $\lambda D = [\gamma^{-1} - (D \eta)(D \eta)^T] \tilde{\omega}$ with $\lambda = 1 - \tilde{\eta}^T \tilde{\omega} \tilde{\eta}$. In addition, the conjugate momenta are defined as $\pi = \frac{\delta \tilde{L}}{\delta \dot{\eta}}$ and $\bar{P} = \frac{\delta \tilde{L}}{\delta \dot{\bar{\eta}}}$. After some algebra we can derive the Hamiltonian of the model as

$$\mathcal{H} = \int d^3 x \left[ \mathcal{H}_N \mathcal{G}(\varphi, \pi, \gamma, \bar{\eta}, \bar{\bar{\eta}}) \right],$$

where

$$\mathcal{H} = -(N_0^g \bar{\eta} + \bar{N}_f) \bar{R} - N_0^g \sqrt{\gamma} \tilde{S}_H m_y^2 \gamma^{2} \gamma^{2},$$

$$C = \mathcal{R} + \tilde{R} D \bar{\eta} + \sqrt{\gamma} \tilde{S}_C m_y^2 \gamma^{2} \gamma^{2}.$$  

In the above expressions we have introduced the coefficients:

$$\mathbf{R} = \frac{\sqrt{|\gamma|}}{2} \left[ M_p^2 R_{i\bar{j}} - \varphi_{,i} \varphi_{,\bar{j}} + \mathcal{U}(\varphi) + \frac{(1 \varphi - \varphi^2 - \varphi^3)}{\gamma^{1/2}} / \sqrt{|\gamma|} \right]$$

while the coefficients appearing in the mass terms are

$$\tilde{S}_H = \frac{\beta_1 \lambda^2}{\Omega^2} - \frac{\beta_2 \lambda^2}{\Omega^2} \left[ \lambda \bar{\eta} D^T \bar{\eta} + \tilde{\omega} T D \bar{\eta} \right]$$

$$+ \frac{\beta_3}{\Omega} \left[ 2 \lambda^2 \varphi^2 + \varphi_{,i} D^T \bar{\eta} \varphi_{,i} + \lambda^2 \varphi_{,i} D^T \bar{\eta} \varphi_{,j} + \beta_4 \lambda^2 \varphi_{,i} D^T \bar{\eta} \varphi_{,j} \right] / |\gamma|^{1/2}.$$  

$$\tilde{S}_C = \frac{\beta_0}{\Omega^2} + \frac{\beta_1 \lambda^2 \varphi_{,i} D^T \bar{\eta} \varphi_{,i}}{\Omega^2} + \frac{\beta_2 \lambda^2 \varphi_{,i} D^T \bar{\eta} \varphi_{,j}}{\Omega^2} + \frac{\beta_3 \lambda^2 \varphi_{,i} D^T \bar{\eta} \varphi_{,j}}{\Omega^2}.$$  

Eventually, we can now vary Hamiltonian with respect to the new shift $\tilde{\eta}$ and lapse $N_0^g$ functions, and obtain the corresponding momentum and Hamiltonian constraints. In particular, the latter one leads to

$$C(\varphi, \pi, \gamma, \bar{\eta}, \bar{\bar{\eta}}) = 0.$$  

Next, similarly to [7], a secondary constraint is obtained by demanding that the primary constraint $C$ is time independent on the constraint surface and thus $\{C, H\} = 0$.

As long as the relation $\{C(x), C(y)\} = 0$ is satisfied at nonlinear order, this condition becomes a nontrivial constraint on $\varphi$, $\gamma$, and their conjugate momenta:

$$\{C, \int d^3 x \mathcal{H}(x)\} = 0.$$
Finally, since the Poisson bracket of the above constraint vanishes automatically, only the BD ghost is removed and the model will not be over-constrained.

Cosmology: When applied in cosmological frameworks, the scenario of $F(R)$ massive gravity exhibits a huge class of phenomenological behaviors due to the combination of the $F(R)$ and graviton-mass sectors. Let us start with a Minkowski fiducial metric $f_{μν} = η_{μν}$. The model allows only for open Friedmann-Robertson-Walker universe, and thus we consider the physical metric in Jordan frame as

$$ds^2 = -N^2dt^2 + a^2(t)\gamma_{ij}^K dx^i dx^j,$$

with $\gamma_{ij}^K dx^i dx^j = δ_{ij} dx^i dx^j - \frac{a_0^2(δ_{ij} x^i x^j)}{1-a_0^2 t^2 x^2}$ and $a_0 = \sqrt{|K|}$ is associated with the spatial curvature. The St¨ uckelberg scalars are: $\varphi^0 = b(t) \sqrt{1 + a_0^2 t^2 x^2}$, $\varphi^i = a_0 b(t) x^i$. Then the polynomials defined in (2) take the forms of

$$U_2 = 3a(a - a_0)b(2Na - b'a - Na_0b),$$
$$U_3 = (a - a_0)b^2(4Na - 3a'b' - Na_0b),$$
$$U_4 = (a - a_0)b^3(N - b'),$$

where primes denote derivatives with respect to $t$. Finally, for simplicity we assume that the gravitational sector couples minimally to the regular matter component.

Variation of the action with respect to $b$, $N$ and $a$ gives respectively the constraint and the two Friedmann equations, namely

$$\dot{(a - a_0)}Y_1 = 0,$$
$$3M_p^2F_{\dot{R}} H^2 - \frac{α_2}{a^2} = ρ_m + ρ_{IR} + ρ_{UV},$$
$$M_p^2F_{\dot{R}} \left(-2\ddot{H} - 3H^2 + \frac{α_2}{a^2}\right) = p_m + p_{IR} + p_{UV},$$

with $\dot{a} = \frac{\dot{r}}{r}$ and $H = \frac{\dot{r}}{r}$. In the above expressions we have defined the IR (massive gravity) effective contribution

$$ρ_{IR} = m_g^2 M_p^2 (B - 1) (Y_1 + Y_2),$$
$$p_{IR} = -m_g^2 M_p^2 (B - 1) Y_2 - m_g^2 M_p^2 (\dot{b} - 1) Y_1,$$

as well as the UV ($F(R)$ sector) effective contribution

$$ρ_{UV} = M_p^2 \left[ \frac{RF_{FR} - F}{2} - 3HF_{FFR} \right],$$
$$p_{UV} = M_p^2 \left[ \dot{R}F_{RRR} + 2H\dot{R}F_{FR} + \dot{R}F_{RR} + \frac{F - RF_R}{2} \right],$$

where the polynomials $Y_{1,2}$ are given by $Y_1 = (3 - 2B) + \alpha_3(3 - B)(1 - B) + \alpha_4(1 - B)^2$ and $Y_2 = (3 - B) + \alpha_3(1 - B)$, with $B = \frac{a_0b}{a}$.

Similar to all massive gravity scenarios, Eq. (17) constrains the dynamics significantly. As in self-accelerating backgrounds of dRGT [10], the nontrivial solutions correspond to the case of $Y_1 = 0$ and yield

$$B_{±} = 1 + 2α_3 - 2α_4 ± \sqrt{1 + α_3 + α_3^2 - α_4} \over \overline{α_3 + α_4}.$$

This relation can be always fulfilled by choosing $b(t) \propto a(t)$, and therefore it yields $ρ_{IR} = -p_{IR}$ to be constant, as it is expected similarly to standard nonlinear massive gravity [21]. However, the crucial issue is that in the present model the remaining $F(R)$ sector can be taken at will, leading to a huge class of cosmologies. Amongst them, an interesting class is when the $F(R)$ sector is important at early times and thus responsible for inflation, while the massive graviton is dominant at late times and can drive the universe acceleration as observed today.

In order to provide a representative example we consider the well-known Starobinski model with $F(R) = R + \frac{α}{M^2} R^2$ in numerical estimates. In the left panel of Fig. 1 we present the early-time inflationary solutions for three parameter choices, while in the right panel we depict the late-time self-accelerating solutions. In this particular choice, the Ricci scalar becomes very small at late times and thus the $F(R)$’s contribution is dramatically suppressed by the Planck scale. Therefore, only the massive-gravity part contributes to the late-time acceleration. However, note that in the general case the total effective dark energy constitutes of both the massive gravity as well as the $F(R)$-modification sectors, that is $ρ_{DE} \equiv ρ_{IR} + ρ_{UV}$. For instance, for a class of $F(R)$ models involving $1/R^α$ terms the contribution of the graviton-mass and of the $F(R)$ sectors may get mixed in late-time universe. Therefore, our model is expected to be very interesting phenomenologically.

Perturbation analysis: The scenario at hand is free of the BD ghost and its cosmological applications allow for

FIG. 1: The left panel presents three inflationary solutions corresponding to a) $m_g = 10^{-10}$, $α_3 = 2$, $α_4 = -1$, $a_0 = 0.5$, $ξ = 10^{-10}$ (black-solid), b) $m_g = 10^{-10}$, $α_3 = 10$, $α_4 = 10$, $a_0 = 0.1$, $ξ = 10^{-9}$ (red-dashed), c) $m_g = 10^{-10}$, $α_3 = 1$, $α_4 = 1$, $a_0 = 1$, $ξ = 10^{-10}$ (blue-dash-dotted). The two horizontal lines mark the $N = 50$ and $N = 60$ e-folding regimes. The right panel depicts three late-time accelerating solutions corresponding to a) $m_g = 10^{-6}$, $α_3 = 3$, $α_4 = -5$, $a_0 = 0.05$, $ξ = 10^{-24}$ (black-solid), b) $m_g = 10^{-5}$, $α_3 = 1$, $α_4 = -2$, $a_0 = 0.01$, $ξ = 10^{-21}$ (red-dashed), c) $m_g = 10^{-5}$, $α_3 = 10$, $α_4 = 1$, $a_0 = 0.05$, $ξ = 10^{-20}$ (blue-dash-dotted). We have imposed $a = 1$, $Ω_m ≈ 0.3$, $Ω_{DE} ≈ 0.7$, $Ω_k ≈ 0.01$ and $m_g \approx H_0$. All parameters are in Planck units.
a huge class of behaviors. However, the last and necessary step is to examine whether such cosmological applications remain free of instabilities at the perturbative level, which is exactly the weak and disastrous point of standard nonlinear massive gravity pointed out in [5] (see also [21]). In the rest of the Letter we briefly show that the scalar perturbations in our model can be stable at the linear perturbative level under certain parameter choices, referring to [18] for the details.

For simplicity we work in the Einstein frame, using the Lagrangian [6], and then consider perturbations around a homogeneous and isotropic background. In this regard, the perturbation analysis of our model is similar to the quasi-dilaton massive gravity, however in our case the dilatonic parameter is already fixed as $\omega = 6$. The scalar perturbations of our variables involve the metric part

$$\delta g_{00} = -2\ddot{N}\phi, \quad \delta g_{0i} = \dot{N}\dot{a}\partial_i B,$$

$$\delta g_{ij} = \ddot{a}^2[2\gamma_{ij}\dot{\psi} + (\nabla_i\nabla_j - \frac{1}{3}\gamma_{ij}\nabla^k\nabla^k)]E, \quad (24)$$

and the field fluctuation $\delta \varphi$. Using the Hamiltonian and momentum constraints, as well as the background equations of motion, we can integrate out the non-dynamical modes, namely $\phi, B, \dot{a}$, and $E$. Therefore, the would-be BD ghost is eliminated in our model. Furthermore, since the scalar dof of the graviton is non-dynamical at the linear level, we can apply momentum constraints, as well as the background equations of motion, we can integrate out the non-dynamical modes, namely $\phi, B, \dot{a}$, and $E$. Therefore, the would-be BD ghost is eliminated in our model. Consequently, the scalar dof of the graviton $\tilde{Q}$ in the Fourier space as

$$\ddot{Q} + 3H\dot{Q} + \frac{k^2}{a^2} + U_{,\varphi} - \frac{1}{M_p^2a^6}\left[2\gamma_{ij}\dot{\psi} + (\nabla_i\nabla_j - \frac{1}{3}\gamma_{ij}\nabla^k\nabla^k)\right]E = \frac{2m_\psi^2Y_Q}{304}Q_k - \frac{2K}{a^2H^2}\left(\frac{\dot{\varphi} - H\dot{\phi}}{H}\right)\psi_B, \quad (25)$$

where $Y_Q \equiv 4(1 - \tilde{B})\tilde{Y}_2$ is defined in the Einstein frame. Note that $Q$ is the only dynamical perturbation variable since $\psi_B$ can be determined by it as well.

The l.h.s. of the perturbation equation (25) is exactly the same as the usual one in GR plus a scalar field, but the r.h.s. involves a mass term due to the graviton potential. Its positivity depends on the coefficient $Y_Q$ and directly determines whether the model suffers from a tachyonic instability or not. Obviously, a healthy model of $F(R)$ massive gravity requires $Y_Q < 0$, which provides the corresponding allowed regime of the parameter space. Additionally, the last term of (25) appears due to the spatial curvature. Since this term would easily dilute out along the cosmic expansion, it is harmless to the model when applied into cosmology.

Conclusions: The study of massive gravity may be important in understanding the observed acceleration of present cosmic expansion, which is one of the greatest mysteries in modern physics. In this regard, the question on establishing a theoretically healthy and observationally viable model of nonlinear massive gravity has attracted the interest of the literature [22].

The theory of $F(R)$ nonlinear massive gravity, as a possible GR modification both at the IR and UV regimes, has significant advantages both at the theoretical as well as at the cosmological levels. Firstly, due to the common dRGT-like graviton potential it inherits its benefits and is free of BD ghosts. Furthermore, due to the freedom of the $F(R)$ sector combined with the graviton-mass, it allows for a huge class of cosmological evolutions. For instance a simple $R^2$ form is able to drive both early universe inflation and late-time acceleration, determining the whole cosmic evolution in a unified way.

We would like to end by highlighting the advantage of our model that the perturbations around a cosmological background can be stabilized due to the $F(R)$ term, which introduces a scalar dof at the linear level, and hence it constrains the scalar metric perturbations to be as in GR. In this respect, the possible instabilities that could appear at higher nonlinear regime becomes not important unless the perturbation theory itself break down. Additionally, we mention that once the perturbations evolve into nonlinear regime, higher curvature terms would become important accompanied by the high energy scale, and thus completely change the dynamics of the theory. The above features may reveal that the UV and IR behaviors of gravitation may not be independent.

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[23] We use small $f$ to denote the fiducial metric, and capital $F$ to denote the function $F(R)$.