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On the Method of N-Body Hyperspherical Basis Symmetrization

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Abstract

Parentage Scheme of Summarization (PSS) to the N-body symmetrized basis construction necessary for the description of the structural characteristics and decay reactions of the hypernuclear and nuclear systems with arbitrary amount of particles is introduced. Proposed method allows to construct N-particle symmetrized hyperspherical functions on the bases of N-particle hyperspherical functions symmetrized with respect to N−1-particles by the use of the Kinematic Rotation Coefficients (KRC) related with the (N-1)-th and N-th particle permutations. The main problem that arises when investigating dynamics of few-body systems in physics is the problem of kinematic rotations under particle permutations. When number of particles increases, kinematic rotations include not only particle permutation but also transitions between different possible configurations, and mathematical calculations using complex general formula become impossible. Moreover, no general formula exists for systems with more than four particles. In order to solve kinematic rotation problem for N-particle systems, the recurrence method of determination of the KRC is applied. According to this method, the initial coefficients with the lowest quantum numbers are calculated by solving the overlap integral analytically, wave functions with arbitrary quantum numbers are expanded in terms of the basic hyperspherical functions, and the kinematic rotations of the obtained expansion are performed with the use of already known coefficients with the lowest quantum numbers. Significant advantage of the recurrence method is that no principal difficulties arise when increasing number of particles. Furthermore, recurrence relations contain numerical coefficients that are easy to evaluate by substituting appropriate quantum numbers.

The problem of symmetrization of N-Body hyperspherical functions is solved by the use of the PSS. A construction scheme is given for the symmetrized hyperspherical basis in (4+1) and (5+1) configurations. The relations between parentage coefficients and the KRC of the basic hyperspherical functions are obtained. Parentage coefficients for N-Body systems are introduced. It is demonstrated that no principal difficulties arise when increasing number of particles.

1. Introduction

While investigating few-body systems with the use Hyperspherical Function Method (HFM) the problem of the hyperspherical basis transformation between different sets of Jacobi coordinates arises (the kinematic rotation problem) [1-6]. Coefficients of unitary transformations of three-particle hyperspherical functions under particle permutations were introduced in [1] (Raynal-Revai coefficients). For four- and more particle systems kinematic rotations include both particle permutations and transitions from one configuration to another. Four-particle transformations were discussed in [2-3,7-10, 17]. When number of particles increases, mathematical calculations using complex general formula become impossible. Moreover, no general formula exists for systems with more than four particles.

In order to solve kinematic rotation problem for N>3 particle systems, the recurrence method of determination of the kinematic rotation coefficients (KRC) was introduced [2, 11, 18]. The symmetrized basis may be constructed easily if the KRC are available [2, 3, 8, 9, 10, 16, 17, and 18]. However, the transformations of Hyperspherical Functions (HF) become sufficiently complex when the number of particles increases. The recurrence method allows avoiding
difficulties related with the direct calculations of the KRC. A construction scheme for recurrence relations, proposed in [11], can be easily generalized and applied to the determination of the transformation coefficients for hyperspherical basis with any number of particles. According to this method the coefficients with $K_N = 2$ and $L_N = 0$ with arbitrary hyperspherical function are expanded in terms of hyperspherical basis and the kinematical rotation of the expansion is performed with the use of the known coefficients with $K_N = 2$ and $L_N = 0$. This procedure allows to obtain recurrence relations for the transformation coefficients with any $K_N$ and $L_N$. A general formula for the coefficients is not needed at all. Transformation coefficients for N=3 and N=4 were obtained in [10].

This paper discusses the problem of symmetrization of N-body hyperspherical functions. In the next section, parentage scheme of symmetrization is applied to the construction of the symmetrized five- and six- particle hyperspherical basis in the (4+1) and (5+1) configurations. The relations between parentage coefficients and the KRC of the corresponding hyperspherical functions under particle permutations are obtained. It is demonstrated that the KRC contain only permutations related with the last two particles.

In the third section parentage scheme of symmetrization is generalized to construct N-particle symmetrized hyperspherical functions.

### 2. Construction of a Symmetrized Hyperspherical Basis for (4+1) and (5+1) Configurations

Four body Hyperspherical Functions symmetrized with respect to three identical particles (nuclei) were introduced in [10]:

$$\phi_{\mu l_{1}}^{(f)} \phi_{\nu l_{2}}^{(l)} \phi_{\lambda l_{3}}^{(l)} \phi_{\omega l_{4}}^{(k)} \left( \rho \right) = \sum_{l_{i}} C_{\phi l_{i}}^{(f)} \phi_{\nu l_{2}}^{(l)} \phi_{\lambda l_{3}}^{(l)} \phi_{\omega l_{4}}^{(k)} \left( \rho \right)$$ (1)

where $C_{\phi l_{i}}^{(f)} \phi_{\nu l_{2}}^{(l)} \phi_{\lambda l_{3}}^{(l)} \phi_{\omega l_{4}}^{(k)}$ are three body symmetrization coefficients, $l_{1}, l_{2}, l_{3}$ are angular momenta corresponding to x, y, z four- body systems’ Jacobi coordinates for either (2+2) or (3+1) configurations [10], $\mu$ and $\kappa$ are four and three particle hypermomenta, L is a total four-particle angular momentum. $\rho \equiv \rho(x, y, z, \alpha, \beta)$ is an element of volume, $\rho_{f}$ and $\rho_{k}$ are Young diagrams for the four and three particle systems correspondingly ($\rho_{f}$ is obtained from $\rho_{k}$ by means of the removal of a cell, corresponding to the fourth particle), $\lambda_{\{j\}}$ denotes the rows of the $\rho_{f}$ representation and $\nu_{\{j\}}$ is the number of $\rho_{f}$ representation with given $\kappa$ and $\mu$.

Transformation coefficients of the functions under particle permutations were introduced in [10]

$$P_{\mu l_{1}}^{\phi_{\mu l_{1}}^{(f)} \phi_{\nu l_{2}}^{(l)} \phi_{\lambda l_{3}}^{(l)} \phi_{\omega l_{4}}^{(k)}} \left( \rho \right) = \sum_{\rho_{f}^{(f)}} \sum_{\rho_{k}^{(k)}} \phi_{\mu l_{1}}^{(f)} \phi_{\nu l_{2}}^{(l)} \phi_{\lambda l_{3}}^{(l)} \phi_{\omega l_{4}}^{(k)}$$ (2)

Transformations of four-body hyperspherical functions under particle permutations were obtained with the use of four-body coefficients [3], similar to three body coefficients introduced by Raynal and Revai [1]
where \( i = 1, 2, 3 \) and \( P_{i4} \) includes only permutations involving the fourth particle. From Equations (1) and (2) we obtain KRC for four-particle hyperspherical functions symmetrized with respect to three identical particles [10]

\[
\langle l_1 l_2 l_3 K \rangle \langle l_1 l_2 l_3 K \rangle \langle l_1 l_2 l_3 K \rangle \langle l_1 l_2 l_3 K \rangle \rangle_{P_{i4}} \Phi_{m}^{\mu \nu} (\omega)
\]

(4)

where

\[
\langle l_1 l_2 l_3 K \rangle \langle l_1 l_2 l_3 K \rangle \langle l_1 l_2 l_3 K \rangle \rangle_{P_{i4}} = \int \Phi_{m}^{\mu \nu} (\omega) \psi_{K_{4}K_{4}}^{m} (\omega) d\omega
\]

Four-body symmetrized hyperspherical basis in (3+1) configuration was constructed by the use of the parentage scheme of symmetrization [10]. Complete set of Young operators acting on the four-body functions symmetrized with respect to three particles were obtained and all relations between parentage coefficients and the coefficients (4) were found [10].

Parentage Scheme of Symmetrization can be easily generalized for N-body systems by introducing Young operators acting on N-particle functions corresponding to the irreducible representation of the N-1 particle permutation group \( S_{N-1} \)

\[
\omega_{[f]}^{[j]} \psi^{[j]} = \sum_{\mu} \Gamma^{[f]}_{[\mu]} (P_{i\mu}) P_{iN} \psi^{[j]}
\]

(5)

where \( \Gamma^{[f]}_{[\mu]} (P_{i\mu}) \) are the elements of \( P_{i\mu} \) permutation matrix related with the representation \( [f] \) of \( S_{N-1} \). These operators identify contributions of the \( S_{N-1} \) in N-particle symmetrized functions. For the five-particle systems we have the following representations [5], [11111], [2111], [311], and [221]. Each of these representations contain representations of the subgroup \( S_{4} \). For example, [2111] contains two representations [1111] and [211] of the subgroup \( S_{4} \). Below are presented some examples of the actions of the Young operators (5) on the five-body hyperspherical functions symmetrized with respect to four particles in the (4+1) configuration

\[
\omega_{[1]}^{[1]} \psi^{[1]} = \frac{1}{5} (1 + P_{15} + P_{25} + P_{35} + P_{45}) \psi^{[1]}
\]

\[
\omega_{[1]}^{[1]} \psi^{[1]} = \frac{1}{5} (1 - P_{15} - P_{25} - P_{35} - P_{45}) \psi^{[1]}
\]

\[
\omega_{[1]}^{[1]} \psi^{[1]} = \frac{2}{5} (P_{25} - P_{15}) \psi^{[1]}
\]

\[
\omega_{[1]}^{[1]} \psi^{[1]} = \frac{2}{15} (P_{15} + P_{25} - 2P_{35}) \psi^{[1]}
\]

\[
\omega_{[1]}^{[1]} \psi^{[1]} = \frac{1}{15} (3P_{45} - P_{15} - P_{25} - P_{35}) \psi^{[1]}
\]

(6)
\[ \psi^{[411]}_{111112} \psi^{[111]}_{1111} = \frac{1}{5} (4 + P_{15} + P_{23} + P_{35} + P_{45}) \psi^{[111]}_4 \]

As the functions under consideration are symmetrized relatively to four particles, the matrices \( P_{15} \), \( P_{23} \) and \( P_{35} \) may be in turn expressed by the element of the \( P_{34} \) matrix

\[ P_{15} = P_{14} \times P_{45} \times P_{14} \quad P_{23} = P_{24} \times P_{45} \times P_{24} \quad P_{15} = P_{34} \times P_{45} \times P_{34} \quad (7) \]

\[ P_{15}^{[111]}_{34} = \sum_{\alpha'} \left( \langle [11111] \alpha' | [1111] \alpha \rangle^{P_{45}}_{K_{34},L} \psi^{[111]}_{1111} - \langle [2111] \alpha' | [1111] \alpha \rangle^{P_{45}}_{K_{34},L} \right) \]

\[ = \pm \frac{\sqrt{6}}{3} \psi^{[211]}_{1111} - \frac{\sqrt{2}}{3} \psi^{[211]}_{1111} + \frac{1}{3} \psi^{[211]}_{1111} \]

\[ P_{35}^{[111]}_{34} = \sum_{\alpha'} \left( \langle [11111] \alpha' | [1111] \alpha \rangle^{P_{45}}_{K_{34},L} \psi^{[111]}_{1111} - \langle [2111] \alpha' | [1111] \alpha \rangle^{P_{45}}_{K_{34},L} \right) \]

\[ = \frac{2\sqrt{2}}{3} \psi^{[211]}_{1111} + \frac{1}{3} \psi^{[211]}_{1111} \quad (8) \]

where \( \alpha \) represents set of parameters defining 5-particle function \( \alpha \equiv \{ K_4, l_{123}, l_4, V \} \), \( V \) numbers one and the same representations \( [F] \) for given \( K_4, l_{123}, l_4 \); \( \langle [F'] | \alpha | [F] \rangle^{P_{45}}_{K_{34},L} \) are the KRC of five-particle hyperspherical functions symmetrized with respect to four identical particles.

Parentage coefficients for the symmetrized hyperspherical functions \( \psi^{[111]}_{K_{34},L} \) of five-body systems corresponding to the representation \( [F] \) of the permutation group \( S_5 \) can be determined by the equation

\[ \psi^{[111]}_{K_{34},L} = \sum_{\alpha} B^{[111]}_{[F]} (\tilde{F} | \alpha) \psi^{[111]}_{K_{34},L} \quad (9) \]

where \( \nu \) numbers one and the same representation \( [F] \) for a given \( K_4 \) and \( L \); \( \psi^{[111]}_{K_{34},L} \) are five-body hyperspherical functions symmetrized with respect to four identical particles corresponding to the subgroup \( S_4 \). These functions are obtained from the Young table corresponding to function \( \psi^{[111]}_{K_{34},L} \) by removing the square corresponding to the fifth particle. The method of obtaining relations between four-body parentage coefficients and the KRC \( (4) \) was described in [10]. The same method can be generalized for five-body systems and obtained the following relationships between parentage coefficients \( B^{[111]}_{[F]} (\tilde{F} | \alpha) \) and the KRC of five particle hyperspherical functions symmetrized with respect to four identical particles

\[ \sum_{\nu} B^{[5]}_{K_{34},L} (\tilde{F} | \alpha) B^{[5]}_{K_{34},L} (\tilde{F} | \alpha') = \frac{1}{5} \delta_{\nu \nu} + \frac{4}{5} \langle [4] \alpha | [4] \alpha \rangle^{P_{45}}_{K_{34},L} \]
Now it is not difficult to derive equations determining the number of the same representations 
\[ f \] with given \( K_s \) and \( L \)

\[
\begin{align*}
\sum_v B_{K_sL}^{[1111]}(\alpha \beta)B_{K_sL}^{[1111]}(\alpha') - \frac{4}{5}\langle \alpha \beta | \alpha' \beta \rangle_{K_sL} = \frac{1}{5}
\end{align*}
\]

\[
\begin{align*}
\sum_v B_{K_sL}^{[211]}(\alpha \beta)B_{K_sL}^{[211]}(\alpha') = \frac{4}{5}\delta_{\alpha\alpha'} + \frac{4}{5}\langle \alpha \beta | \alpha' \beta \rangle_{K_sL}
\end{align*}
\]

\[
\begin{align*}
\sum_v B_{K_sL}^{[111]}(\alpha \beta)B_{K_sL}^{[111]}(\alpha') = \frac{4}{\sqrt{15}}\langle \alpha \beta | \alpha' \beta \rangle_{K_sL}
\end{align*}
\]

\[
\begin{align*}
\sum_v B_{K_sL}^{[22]}(\alpha \beta)B_{K_sL}^{[22]}(\alpha') = \frac{1}{2}\delta_{\alpha\alpha'} - \langle \alpha \beta | \alpha' \beta \rangle_{K_sL}
\end{align*}
\]

\[
\begin{align*}
\sum_v B_{K_sL}^{[21]}(\alpha \beta)B_{K_sL}^{[21]}(\alpha') = \frac{2}{\sqrt{3}}\langle \alpha \beta | \alpha' \beta \rangle_{K_sL}
\end{align*}
\]

\[
\begin{align*}
\sum_v B_{K_sL}^{[31]}(\alpha \beta)B_{K_sL}^{[31]}(\alpha') = \frac{4}{\sqrt{15}}\langle \alpha \beta | \alpha' \beta \rangle_{K_sL}
\end{align*}
\]

\[
\begin{align*}
\sum_v B_{K_sL}^{[32]}(\alpha \beta)B_{K_sL}^{[32]}(\alpha') = \frac{2}{\sqrt{3}}\langle \alpha \beta | \alpha' \beta \rangle_{K_sL}
\end{align*}
\]

\[
\begin{align*}
\sum_v B_{K_sL}^{[31]}(\alpha \beta)B_{K_sL}^{[31]}(\alpha') = \frac{4}{\sqrt{15}}\langle \alpha \beta | \alpha' \beta \rangle_{K_sL}
\end{align*}
\]

Parentage coefficients with different quantum numbers can be easily found from equations (10). For example, when \( K_s = L = 1 \) and \( K_s = 2; \ L = 0 \) we obtain

\[
-B_{11}^{[4]}(\alpha_0) = B_{11}^{[3]}(\alpha_1) = 1;
\]

where \( \alpha_0 \equiv (K_4 = l_{121} = 0, l_4 = 1); \ \alpha_1 \equiv (K_4 = l_{121} = 1, l_4 = 1) \)
\[ B_{20}^{(41)}(4)\alpha_0 = 1 \quad B_{20}^{(41)}(31)\alpha_1 = -\sqrt{\frac{5}{6}} \quad B_{20}^{(41)}(31)\alpha_2 = -\sqrt{\frac{1}{6}} \quad B_{20}^{(32)}(31)\alpha_1 = 1 \quad B_{20}^{(32)}(31)\alpha_1 = \sqrt{\frac{1}{6}} \quad B_{20}^{(32)}(31)\alpha_2 = -\sqrt{\frac{5}{6}} \]  

(12)

where \( \alpha_0 \equiv (K_4 = l_1 = l_2 = 0); \quad \alpha_1 \equiv (K_4 = 2, l_1 = l_2 = 0); \quad \alpha_2 \equiv (K_4 = l_1 = l_2 = 1) \)

The symmetrized hyperspherical basis for five-particle systems can be easily constructed by substituting parentage coefficients (12) into formula (9).

When \( K_5 = L = 1 \) we obtain the following expressions

\[\Psi^{(1)}_{K_1=1} = \Psi^{(1)}_{K_1=1} = \frac{64\sqrt{15}}{\pi} \cos \alpha \sin \beta \sin \gamma \]

\[\Psi^{(2)}_{K_1=1} = -\Psi^{(2)}_{K_1=1} = \frac{64\sqrt{15}}{\pi} \sin \alpha \sin \beta \sin \gamma \]

\[\Psi^{(3)}_{K_1=1} = -\Psi^{(3)}_{K_1=1} = \frac{64\sqrt{15}}{\pi} \cos \beta \sin \gamma \]

\[\Psi^{(4)}_{K_1=1} = -\Psi^{(4)}_{K_1=1} = \frac{64\sqrt{15}}{\pi} \cos \gamma \]

Formula (5) for Young operators can be applied for the summarization of the six-body hyperspherical functions in (5+1) configuration. For six-body systems \( \Gamma_{mn}^{f}(P_{16}) \) are the elements of \( P_{16} \) permutation matrix related with the representation \( [f] \) of the permutation group \( S_6 \). These operators identify contributions of the \( S_5 \) in six-particle symmetrized functions. When symmetrizing hyperspherical functions with \( K_6 = L = 1 \), we have two representations of \( S_5 \): \( [51] \) and \( [42] \). Representation \( [51] \) is five-dimensional and contains \( [41] \) and \( [5] \) representations of the subgroup \( S_5 \), whereas \( [42] \) is nine-dimensional and contains \( [41] \) and \( [32] \) representations of the subgroup \( S_5 \).

Parentage coefficients for symmetrized hyperspherical six-body functions \( \Psi^{(f)}_{K_5L}^{(m\nu)} \) corresponding to the representation \( [f] \) of the group \( S_6 \) can be determined by the equation

\[\Psi^{(f)}_{K_5L}^{(m\nu)} = \sum_{\alpha} B^{(f)}_{\alpha} \Psi^{(f)}_{K_5L}^{(m\nu)} \]

where \( \nu \) numbers one and the same representations of \( [f] \) for a given \( K_5 \) and \( L \), \( \Psi^{(f)}_{K_5L}^{(m\nu)} \) are six-body hyperspherical functions symmetrized with respect to five identical particles corresponding to the subgroup \( S_5 \). These functions are obtained from the Young table corresponding to function \( \Psi^{(f)}_{K_5L}^{(m\nu)} \) by removing the square corresponding to the sixth particle.
The relationship between six-body parentage coefficients and KRC of six particle hyperspherical functions symmetrized with respect to five identical particles for [42] and [51] representations is expressed by the following equations

\[ \sum_{\nu} B^{[42]\nu}_{K,L}[\{41]\alpha] B^{[42]\nu}_{K,L}[\{41]\alpha'] = \frac{3}{8} \delta_{\alpha\alpha'} - \frac{15}{32} \langle [41]\alpha | [41]\alpha' \rangle_{K,L} + \frac{15}{32} \langle [41]4\alpha | [41]4\alpha' \rangle_{K,L} \]

\[ \sum_{\nu} B^{[42]\nu}_{K,L}[\{41]\alpha] B^{[42]\nu}_{K,L}[\{32]\alpha'] = -\frac{3\sqrt{2}}{4} \langle [41]\alpha | [32]2\alpha' \rangle_{K,L} \]

\[ \sum_{\nu} B^{[51]\nu}_{K,L}[\{5\alpha] B^{[51]\nu}_{K,L}[\{5\alpha'] = \frac{5}{6} \delta_{\alpha\alpha'} - \frac{5}{6} \langle [5]\alpha | [5]\alpha' \rangle_{K,L} \]

\[ \sum_{\nu} B^{[51]\nu}_{K,L}[\{5\alpha] B^{[51]\nu}_{K,L}[\{41]\alpha'] = -\frac{5\sqrt{6}}{12} \langle [5]\alpha | [41]4\alpha' \rangle_{K,L} \]

Where \( \alpha \) represents set of parameters defining six-particle function \( \alpha \equiv \{ K_5, l_{1234}, l_5, \mathcal{F} \} \), \( \mathcal{F} \) numbers one and the same \( \{ j \} \) representations for given \( K_5, l_{1234}, l_5 \). For \( K_5 = 2; \ L = 0 \) we obtain the following results

\[ B^{[42]}_{20}[\{41]\alpha_1] = \frac{3}{\sqrt{10}}; \quad B^{[42]}_{20}[\{41]\alpha_2] = \frac{1}{\sqrt{10}}; \quad B^{[42]}_{20}[\{32]\alpha_2] = 1 \]

\[ B^{[51]}_{20}[\{5\alpha_0] = 1; \quad B^{[51]}_{20}[\{41]\alpha_1] = -\frac{1}{\sqrt{10}}; \quad B^{[42]}_{20}[\{41]\alpha_2] = \frac{3}{\sqrt{10}} \]

Where \( \alpha_0 \) corresponds to \( K_5 = 0, l_{1234} = 0, l_5 = 0; \ \alpha_1 \equiv \{ K_5 = l_{1234} = l_5 = 0 \} \) and \( \alpha_2 \) corresponds to \( K_5 = 2, l_{1234} = l_5 = 0 \)

Six particle symmetrized hyperspherical basis can be easily constructed by substituting parentage coefficients (15) into formula (13).

3. Construction of a Symmetrized Hyperspherical Basis for N-Body Systems

Construction scheme for the symmetrized hyperspherical basis for the (4+1) and (5+1) configurations presented in section 2 can be easily generalized and applied to the systems containing any number of particles. According to the PSS, N-body hyperspherical functions corresponding to the representation of the N-particle permutation group \( S_N \) can be obtained by finding parentage coefficients and constructing linear combinations of the N-particle functions corresponding to the irreducible representations of N-1 particle permutation group \( S_{N-1} \).

N-body hyperspherical functions and the complete sets of the recurrence relations for the KRC of these functions with arbitrary quantum numbers were obtained in [18]. According to the PSS we need to construct N-body functions symmetrized with respect to N-1 particles first and then calculate parentage coefficients acting on these functions. Formula (2) can be easily generalized for N-body systems.
\[
P_{N-1,N}^{[f],n,\alpha} = \sum_{[f'],n',\alpha'} \Psi_{K_{N-1},L}^{[f'],n',\alpha'}^{[f],n,\alpha}
\]

(16)

where \( \Psi_{[f],n,\alpha}^{[f'],n',\alpha'} \) is a linear combination of N-body hyperspherical functions corresponding to the irreducible representation \( [f] \) of the group \( S_{N-1} \); \( n \) - numbers functions of a given representation; \( \alpha \) denotes sets of parameters \( \alpha \equiv \{K_{N-1},l_{1},...,l_{N-1} (n = N-1),\bar{\nu} \} \), \( \bar{\nu} \) - numbers one and the same representations of \( [f] \) for a given \( K_{N-1},l_{1},...,l_{N-1} \). All other permutations \( P_{N-1} \) can be expressed through coefficients (16).

Parentage coefficients for N-body systems can be introduced with the following expression

\[
\Psi_{K_{N-1},L}^{[f],n,\alpha} = \sum_{\alpha} B^{[f],[\bar{\nu}]}_{[f],[\bar{\nu}]} \Psi_{K_{N-1},L}^{[f],n,\alpha}
\]

(17)

Where \( \nu \) numbers one and the same representations of \( [f] \) for a given \( K_{N} \) and \( L \). \( \Psi_{K_{N-1},L}^{[f],n,\alpha} \) are N-body hyperspherical functions symmetrized with respect to N-1 identical particles corresponding to the subgroup \( S_{N-1} \). These functions are obtained from the Young table corresponding to function \( \Psi_{K_{N-1},L}^{[f],n,\alpha} \) by removing the square corresponding to the N-th particle.

Relationships similar to (10) and (14) between N-body parentage coefficients \( B^{[f],[\bar{\nu}]}_{[f],[\bar{\nu}]} \) and KRC of N-body hyperspherical functions symmetrized with respect to N-1 identical particles can be easily obtained using the procedure introduced in the previous section.

4. Conclusion

The use of the parentage scheme of symmetrization allows developing rather simple construction scheme for N-body symmetrized hyperspherical basis in the \((N-1)+1\) configuration. This kind of basis may be used when considering different problems in molecular, atomic, nuclear and particle physics. The symmetrized N-body hyperspherical functions corresponding to the representations of the N particle permutation group \( S_{N} \) are constructed by the use of the parentage coefficients acting on the N-particle functions corresponding to the irreducible representation of the N-1 particle permutation group \( S_{N-1} \). Complete sets of relations between parentage coefficients and the KRC of the corresponding hyperspherical functions under (N-1)-th and N-th particle permutations are obtained.
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