Dynamic Response of Piles under Wave Loading Based on Comsol Multiphysics

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Abstract. In order to simulate the action of waves on Submarine Structures, COMSOL Multiphysics Finite Element Analysis Software was used. By solving Helmholtz equation in frequency domain of given frequency and introducing artificially applied background acoustics field, considering incidence and penetration of acoustics pressure at the junction of seabed and water surface, a total acoustics field is established. Porous media was characterized by Biot’s consolidation theory to compute the displacement field and acoustics pressure fluctuation in porous media propagating porous elastic waves, considering the viscous loss of the model, so the deformation of Solid Matrix can be described. Finally, a numerical model of wave-pile-seabed was established by coupling the pressure acoustic physical field with the physical field of porous media. The influence of different wave frequencies and wave incidence angles on the displacement of piles and the influence of different wave incidence angles on the stress of piles were analyzed. Finally, the liquefaction of soil around piles was studied.

1. Introduction

With the development of science and technology and the promotion of national policies, more and more marine structures have been built[1]. For example: oil platforms, transportation and port facilities. As the core component of this kind of structure, it is necessary to study the displacement, pile stress and pore pressure of surrounding soil under wave load. These piles are constructed on porous seabed. Waves can cause significant changes in surface pressure of sediments, resulting in deformation and vibration of marine structures. If the magnitude of deformation or vibration is large enough, these marine structures will be damaged, seabed or even liquefied, which will cause huge property losses and may endanger the safety of people working around them.

Since the last century, scholars have carried out a series of studies on wave and seabed response. The discussion can be divided into three categories: laboratory test, analytical calculation and numerical simulation. Zen and Yamazaki[2-4] conducted a one-dimensional cylinder test in the 1990s to simulate the change of pore water pressure under wave loads and to discuss seabed liquefaction. Based on the improved cylindrical test equipment, Lv[5] controlled the wave pressure above the model soil by a loading device to simulate the response test of the soil around the pile under wave load. In the field of analytical calculation, most of the researchers' research on the simulation of seabed dynamic response under wave action is based on the mathematical model proposed by Biot[6] in 1941 to describe the interaction between solid particles and pore water in porous elastic media. In numerical simulation, Guo [7] takes Navier-stokes equation as wave governing equation and establishes a
numerical model of wave-seabed-pile dynamic problem to simulate the wave action on pile foundation and seabed. Duan Lunliang \[8\] established the finite element numerical model of dynamic response of the compact seabed under box girder under extreme wave action by solving RANS equation and Biot equation. Researchers have given different discriminants for seabed liquefaction. Okusa \[9\] first gave the formula of seabed liquefaction:

\[-(\gamma_s - \gamma_w)z - \sigma_{zd} \leq 0\]

Tsui \[10\] put forward the discriminant formula of average stress:

\[
\frac{1}{3} \left[-(\gamma_s - \gamma_w) (1 + 2K)z - (\sigma_{zd} - \sigma_{yd} - \sigma_{zd}) \right] \leq 0.
\]

Although some achievements have been made in the study of wave-seabed, the analysis of wave-structure interaction is less, and the research on piles is even more limited. The influence of wave frequency and incident angle on seabed and structure is also very limited.

In this paper, two physical fields of pressure acoustics and porous media are established and coupled by COMSOL Multiphysics finite element analysis software. The background sound field is applied by pressure acoustics to describe the propagation of mechanical waves in homogeneous media and control the size and direction of waves. In the seabed, i.e. porous media region, based on Biot theory, the viscous loss of the model is considered to describe the deformation of solid matrix in saturated porous media. In conclusion, the coupling of multi-physical fields can be realized, and the numerical model of wave-pile-seabed can be established.

2. Model introduction

2.1 Model overview

In order to accurately simulate the dynamic response of pile and seabed under wave load, the technical route of this paper is as follows: The background sound field is applied and the total sound field is further solved. Then the total sound field is applied to the pile and seabed. Finally, the relevant results are solved. The computational model in this paper is shown in Figure 1.

![Fig.1 Wave-Pile-Seabed Model](image)

2.2 Mesh generation

In order to reduce the amount of calculation without losing the accuracy of calculation, the calculation model follows the following three principles: (1) The grid scale should be fine enough to approximate the spatial variation of the physical layer; (2) In order to accurately simulate the characteristics of sinusoidal wave, the principle of one fifth wavelength is followed. (3) Considering the periodic boundary conditions, the left and right boundary grids must correspond one by one. The gridding diagram of the computational model is shown in Figure 2.

![Fig.2 Mesh generation graph](image)
3. Model establishment

3.1 governing equation

3.1.1 Sound Waves in the Ocean

According to the Fourier transform theorem, waves can be regarded as combinations of harmonic excitations of different frequencies. In this case, the waves can be considered as sound waves controlled by Helmholtz equation, which is as follows:

\[
\nabla \cdot \left( -\frac{\nabla p_t}{\rho_c} - \frac{\kappa_{eq} \rho_t}{\rho_c} \right) = Q_m
\]

\[
K_{eq} = \frac{\omega^2 M}{\rho_c} - K_2
\]

In formula, \(K_{eq}\) is wave number; \(Q_m\) is an external sound source, 0 in this calculation model; \(p_t\) is the sound pressure and \(\rho_c\) is the density of the medium.

3.1.2 Porous Elastic Waves in Seabed

The elastic wave module of the numerical simulation software COMSOL multiphysics can be used to calculate the elastic displacement and sound propagation in the elastic material of porous media. The constitutive equation of porous elastic materials can define the properties of solid matrix and fluid respectively, and consider the isotropy and anisotropy of drainage matrix. The governing equations of porous media are as follows:

\[
\nabla \cdot \left( -\frac{\rho_{av} - \rho_f^2}{\rho_c} \omega^2 u \right) = \nabla \cdot S + F_V e^{i\phi} + \frac{\rho_f}{\rho_c} \nabla p
\]

\[
\nabla \cdot \left( -\frac{1}{\rho_c} \left( \nabla p - \omega^2 \rho_f u \right) \right) = \frac{\kappa_{eq} \rho}{\rho_c} = \omega^2 \alpha_B \nabla \cdot u
\]

\[
\frac{1}{M} = \frac{\epsilon_p \chi_f}{1 - \epsilon p} + \frac{\alpha_B - \epsilon_p}{\epsilon_p} (1 - \alpha_B)
\]

In formula, \(\rho_{av}\) is average density; \(\rho_f\) is fluid density; \(\rho_c\) is the density of drainage matrix; \(\omega = 2\pi f\) is angular frequency; \(u\) is the displacement field of material. \(S\) is the internal stress in porous media. \(F_V\) represents volume force; \(P\) is the pressure of elastic wave; \(K_d\) is the volume modulus of drainage; \(\alpha_B\) is the coefficient of Biot-Wills and is defined as:

\[
\alpha_B = 1 - \frac{K_d}{K_S};
\]

In formula, \(K_S\) is the bulk modulus of the whole solid porous material block; The definition of \(\alpha_B\) shows that the volume modulus (compressibility) of drainage porous matrix and the bulk modulus of solid materials play a controlling role in \(\alpha_B\).

Fluid density, dynamic viscosity and compressibility play a decisive role in fluid properties in porous media. The compressibility of fluids is mainly affected by the volume modulus \(K_f\) and sound velocity \(C\) of fluids. The expression is as follows:

\[
\chi = \frac{1}{K_f} = \frac{1}{c^2 \rho_f}
\]

The modulus \(m\) is determined by the compressibility \(\chi\). The expression is as follows:

\[
\frac{1}{M} = \frac{K_s}{1 - \epsilon p} + \frac{\alpha_B - \epsilon_p}{\epsilon_p} \chi
\]

3.1.3 Linear Elastic Waves in Marine Structures

On the basis of elastic theory, the deformation of ocean structures under wave pressure and elastic pressure is numerically simulated by using linear elastic theory. The expression is as follows:

\[
-\rho \omega^2 u = \nabla \cdot S + F_V e^{i\phi}
\]

In formula, \(\rho\) is the density of the structure.

3.2 boundary condition

3.2.1 Free boundary

Porous free boundary conditions are the default conditions without constraints and loads. Detailed expressions can be written as follows:
3.2.2 Periodic boundary conditions
When the structure and physical field are periodic distribution, the periodic condition is an effective method to reduce the amount of calculation. In this model, the Floquet condition is used to simulate the abnormal sea wave pressure on the infinite periodic seabed sediments. The equation of Floquet condition can be expressed as:

\[
P_t, z_{st} = p_t, ss e^{-ik_F(r_{dst} - r_{src})} \]

(11)

\[
n_t, z_{st} \cdot \left( -\frac{p_t}{p_c} \right) = p_t, ss e^{-ik_F(r_{dst} - r_{src})} \]

(12)

\[u_{dst} = u_{src} e^{-ik_F(r_{dst} - r_{src})} \]

(13)

3.3 Coupling of Multiple Physical Fields

3.3.1 Porous Media-Structure Boundary
Porous structure boundary assumes that the adjacent boundary between ocean structure and seabed sediment has the same displacement field, and its boundary constraints are as follows:

\[u_{porous} = u_{solid} \]

(14)

3.3.2 Acoustic-Structural Boundary
The formulas of sound pressure control and elastic wave equation of porous media are given respectively.

Different coupling equations are applied to the interior and exterior of the boundary. The boundary interaction of acoustic structure can be expressed mathematically as follows:

\[-n \cdot \left( -\frac{p_t}{p_c} \right) = -n \cdot u_{tt} \]

(15)

\[F_A = p_t n \]

(16)

In formula, \(u_{tt} \) represents the acceleration of the structure; \( n \) is the normal vector of the boundary; \( F_A \) is the strength of each structural unit.

3.3.3 Acoustic-Porous Media Boundary
The acoustic porous boundary is applied to the adjacent boundary to couple the interaction between wave pressure and elastic wave pressure (biot model) in seabed sediments. Assuming the continuity of pressure on the boundary, the mathematical expression is as follows:

\[p_{pore} = p_t \]

(17)

4. Parameter analysis
In order to find out the relevant laws conveniently, the left boundary of the third pile from left to right is taken as the representative to study.

4.1 Effect of Wave Frequency on Pile Displacement

Figure 3 shows the variation of pile displacement under different wave frequencies at the same incident angle. It can be seen from the figure that under the action of low frequency waves, the displacement of piles decreases with the increasing of wave frequency. Under the action of high frequency waves, the displacement of piles increases with the increase of wave frequency. For the
general trend, the displacement of piles with low wave frequency is larger. It is worth noting that when the frequency is 2 Hz, the displacement of the pile is several times that of other frequencies, which indicates that the wave frequency is similar to the natural frequency of the pile.

4.2 Effect of Incident Angle of Wave on Pile Displacement

![Fig.4 Displacement of Piles at Different Incident Angles of Waves](image)

Figure 4 depicts the effect of different incident angles on pile displacement at the same wave frequency. It can be seen from the graph that when the wave frequency is 1 Hz, the displacement of pile decreases with the increase of incident angle, that is, the negative correlation. However, with the increase of frequency, the influence of incident angle on the displacement of pile gradually becomes positive correlation. When the wave frequency is 8 Hz, the influence of incident angle on pile displacement is completely positive correlation.

4.3 Effect of Incident Angle of Wave on Pile Stress

![Fig.5 Stresses of Piles at Different Incident Angles of Waves](image)

Fig. 5 shows the pile stress at different incident angles. Fig. A is the case when the frequency is 5 Hz and the incidence angle is 2 degrees; Fig. B is the case when the frequency is 5 Hz and the incidence angle is 14 degrees; Fig. C is the case when the frequency is 5 Hz and the incidence angle is 20 degrees. It can be seen from the figure that the stress in the pile body is much greater than that in the edge. When the incident angle decreases gradually, the stress of the whole pile body increases gradually. This is because the influence of wave on the seabed decreases with the decrease of incident angle. From figure c, it can be found that when the incident angle is very large, the stress of the pile body can almost be neglected. As can be seen from figure b, the stress of pile body decreases gradually from left to right with the spatial distribution of pile, which is caused by the wave propagation from left to right and the blocking and protecting effect of the pile in front of it.

5. Liquefaction analysis

Under the scouring action of wave cyclic load, pore water pressure in porous media decreases gradually, which leads to seabed instability and eventually liquefaction. In this paper, the seabed liquefaction region is studied by using Jeng\(^{[11]}\) liquefaction discriminant formula. The discriminant is as follows:

\[-\frac{1}{3}(\gamma_s - \gamma_w)(1 + 2K_0)z + [p_b - p(z)] \leq 0\ (18)\]
Fig. 6 is a schematic diagram of seabed liquefaction range at different wave frequencies. The dark red area in the figure indicates the liquefied area. By comparing figure a with figure b, it can be found that the liquefaction area of soil around pile decreases obviously with the increase of wave frequency. This is due to the fact that waves with higher frequencies exert less force, which makes it difficult to achieve substantial scouring effect on the seabed. By comparing Figure A and Figure c, it is found that there is no obvious correlation between liquefaction range and wave incidence angle, because the influence of incident angle on pore pressure is very weak.

6. Conclusion

In this paper, two physical fields of pressure acoustics and porous media are established and coupled by COMSOL Multiphysics finite element analysis software. In conclusion, the coupling of multi-physical fields can be realized, and the numerical model of wave-pile-seabed can be established. The main conclusions are as follows:

1. Under the action of low frequency waves, the displacement of piles decreases with the increase of wave frequency, while under the action of high frequency waves, the displacement of piles increases with the increase of wave frequency;

2. When the wave frequency is low, the pile displacement decreases with the increase of incident angle, i.e. negative correlation, but with the increase of frequency, the influence of incident angle on pile displacement becomes positive correlation.

3. The stress inside the pile body is far greater than that at the edge; when the incident angle becomes smaller, the stress of the whole pile body increases gradually.

4. With the increase of wave frequency, the liquefaction area of soil around pile decreases obviously. There is no obvious correlation between liquefaction range and incident angle of wave.

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