Gravity waves on the surface of topological superfluid $^3$He-B

V.B. Eltsov, P.J. Heikkinen, and V.V. Zavjalov
O.V. Lounasmaa Laboratory, Aalto University, P.O. Box 15100, FI-00076 AALTO, Finland
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We have observed waves on the free surface of $^3$He-B sample at temperatures below 0.2 mK. The waves are excited by vibrations of the cryostat and detected by coupling the surface to the Bose-Einstein condensate of magnon quasiparticles in the superfluid. The two lowest gravity-wave modes in our cylindrical container are identified. Damping of the waves increases with temperature linearly with the density of thermal quasiparticles, as expected. Additionally finite damping of the waves in the zero-temperature limit and enhancement of magnetic relaxation of magnon condensates by the surface waves are observed. We discuss whether the latter effects may be related to Majorana fermions bound to the surface of the topological superfluid.

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Waves on the surface of a fluid in a gravitational field present a universal phenomenon in a wide range of systems, from a glass of drink to hot astrophysical objects and cold superfluids. Properties of the waves provide an important information about the fluid itself which in turn can result in useful practical applications. For example using the well-known phenomenon that oil film stills the water waves one can find oil pollutions in the ocean by observing its calm regions from a satellite.

Recently the surface properties of the fermionic $^3$He in its superfluid B phase have attracted a lot of attention owing to non-trivial topology of this superfluid: It is expected that fermionic bound states with Majorana character emerge at the surface. While in solid-state systems complex engineering efforts are required to obtain Majorana fermions, the free surface of $^3$He-B should be naturally covered by a thin layer of such states. Could this ‘film’ in a sense ‘still’ the surface waves of $^3$He-B and is it possible to observe this damping in the experiment? While this question awaits a proper theoretical consideration we report here the first, to our knowledge, observation of gravity waves on the surface of $^3$He-B.

For such observation it is not enough to cool $^3$He below its critical temperature $T_c \approx 10^{-3}$ K. At temperatures close to $T_c$, viscosity of the normal component of $^3$He is high, oil-like, and the surface waves are overdamped. Only the third sound waves in a thin film, where the normal component is clamped, have been previously observed in $^3$He-B at $(0.3 \div 0.8) T_c$. We have performed measurements at temperatures below $0.2 T_c$ where the normal component becomes a rarefied gas of ballistic quasiparticles and its contribution to the damping of the surface waves rapidly decreases.

Experiment. The $^3$He-B sample is contained in a vertical quartz cylinder with internal diameter of $2R = 6$ mm and length of $150$ mm, Fig. 1. The free surface of the superfluid is placed about $8$ mm below the upper wall of the cylinder within the pick-up coil of the nuclear magnetic resonance (NMR) spectrometer. To detect surface oscillations we use their influence on the frequency of the coherently-precessing NMR mode known as the trapped Bose-Einstein condensate of magnon quasiparticles.

The order parameter of $^3$He-B in the magnetic field is anisotropic. The orientation of the orbital anisotropy axis $l$ slowly varies in the sample (forms a texture) which creates a trapping potential for magnon quasiparticles in the radial direction owing to the spin-orbit interaction energy:

$$F_{so} = \frac{4}{5} \frac{\Omega_B^2}{\omega_L} \sin^2 \frac{\beta_l}{2} |\Psi|^2. \tag{1}$$

Here $\omega_L = \gamma H$ is the Larmor frequency, $\Omega_B$ is the Leggett frequency in the B phase, $\beta_l$ is the deflection angle of the orbital anisotropy axis $l$ from the vertical direction (growing from $\beta_l = 0$ at...
the cylinder axis to $\beta_l = \pi/2$ at the cylindrical wall) and $\Psi$ is the wave function of the magnon condensate. Density of magnons $|\Psi|^2$ is related to the tipping angle of magnetization $\beta_M$ as $|\Psi|^2 = \chi H (1 - \cos \beta_M) / \gamma h$.

Trapping in the axial direction is provided by an additional pinch coil which creates a minimum in the static NMR field $H$ and thus minimum in the Zeeman energy $E_z = h\omega_t |\Psi|^2$. The lowest magnon levels in this trapping potential typically closely follow harmonic-trap relation [12] and can be enumerated by the radial and axial quantum numbers $m$ and $n$, respectively:

$$f_{mn} = f_L + \nu_r (m + 1) + \nu_z (n + 1/2).$$

Here $f_L = \omega_t / 2\pi \approx 0.826$ MHz and $\nu_r \approx 220$ Hz and $\nu_z \approx 40$ Hz are the radial and axial trapping frequencies, respectively. When magnons are pumped to the trap using NMR they relax in sub-second time to the ground level [12], where spontaneous coherence appears and a Bose-Einstein condensate is formed. The magnetization of the condensate precesses around the magnetic field at $f_{00} = f_L + \nu_r + \nu_z / 2$ frequency. The precession induces signal in the NMR pick-up coil from which the frequency of the precession can be determined. The free surface changes $\nu_z$ by limiting the trap in the axial direction and also modifies $\nu_r$ owing to orientation of $\hat{I}$ perpendicular to the surface. When the waves modify the geometry of the surface, the frequency of the precession $f_{00}$ changes as a result.

In the measurements we keep a small cw pumping usually at $m = 2$ level to compensate for the loss of the magnons from the ground level. Such pumping is applied at the frequency $f_{00} > f_{00}$ and thus it does not interfere with the measurements of the precession of the ground-level condensate. To the signal recorded from the NMR pick-up coil we apply the band-pass filter to keep only the contribution from the ground-state condensate including all the side bands, resulting from the frequency modulation. The frequency $f_{00}$ of the precession of the condensate is found from the time intervals between zero crossings in the filtered signal.

**Surface resonances.** An example of the measured $f_{00}(t)$ record and its Fourier transform are shown in Fig. 2. Peaks in the frequency modulation spectrum can be attributed to the lowest-frequency gravity-wave modes in a vertical cylinder. The height profile $h(r, \theta)$ of such surface oscillations is

$$h(r, \theta) = J_i(k_{ij} r) e^{i \theta}, \quad i = 0, 1, \ldots, j = 1, 2, \ldots$$

Here $(r, \theta)$ are the polar coordinates of the point on the surface, $J_i$ are the Bessel functions, and wave numbers $k_{ij}$ satisfy the equation $J'_i(k_{ij} R) = 0$. The spectrum of these modes follows simple relation for the gravity waves on deep water $\omega^2 = g k_{ij}$, where $g$ is the free-fall acceleration. This applies since the length of the sample cylinder significantly exceeds its diameter and the surface tension of $^3$He is small and can be neglected here [13].

The primary mode is the non-axisymmetric $(1,1)$ mode with $k_{11} = 1.8412 / R$ and its frequency in our cylinder is $\omega_{11} / 2\pi = 12.4$ Hz. The lowest next-frequency mode has $k_{21} = 3.0542 / R$ and $\omega_{21} / 2\pi = 17.8$ Hz. These two modes are clearly seen in Fig. 2 with the second harmonic of the $(1,1)$ mode being the most prominent peak. The frequency doubling occurs since the frequency of the magnon precession is the same for the two trap configurations symmetric relative to the vertical plane passing through the nodal line of the surface mode.

The amplitude of the surface waves $h_0$ can be connected to the change of the frequency of the magnon precession $\Delta f_{00}$ with Eq. (11) as $h \cdot 2\pi \Delta f_{00} \sim h(\Omega_B / \omega_{21}) (h_0 / R)^2$. With $\Delta f_{00} = 50$ Hz and $\Omega_B / 2\pi \approx 10^3$ Hz we get for the amplitude of the wave $h_0 \sim 0.2$ mm. At such amplitudes the flow velocity along the surface $v \sim (kR) h_0 / \pi \lesssim 1$ cm/s is substantially below the critical and we have not observed formation of vortices by surface oscillations.

**Surface damping.** To determine the damping of the surface waves we have measured the resonance width $\Delta f_{\text{surf}}$ of the forced oscillations in the primary mode, Fig. 3. Oscillations are excited by periodically tilting the
cryostat at a frequency \( f_{\text{exc}} \) using the active air-spring dampers on which the cryostat is suspended. With the vertical distance from the suspension point to the sample surface of about 1.5 m this transforms essentially to horizontal oscillations of the sample container. We measure \( f_{\text{iso}}(t) \) dependence like in Fig. 2(top) and extract the amplitude of the response \( A \) at 2\( f_{\text{exc}} \) frequency using a lock-in-like detection.

As seen in Fig. 3 the damping increases with increasing temperature as the density of the normal component grows. We are not aware of the rigorous calculation of the damping applicable at these temperatures, where the normal component of \(^3\)He-B presents a gas of ballistic quasiparticles, which would include also the effects of Andreev reflection from the surface \(^{14}\) and from the flow along it. A simple model of thermal damping of quartz tuning forks in the ballistic regime \(^{15}\) predicts that the width of the resonance is proportional to \( (d/M) \exp(-\Delta/T) \), where \( d \) is the size of the object perpendicular to the direction of the oscillations, \( M \) is the effective mass and the exponential factor reflects temperature-dependent density of thermal quasiparticles. Using this expression we can roughly scale the thermal effect from the thermometer fork, which has \( d/M \approx 270 \text{ cm/g} \[^{16}\] \), to the oscillations of the surface. For the primary mode of the oscillating surface we estimate \( d \sim R \) and \( M \sim \rho_{\text{He}} R^2 / k \) which gives \( d/M \sim 200 \text{ cm/g} \). Thus we may expect that the thermal contribution to the width of the resonant response will be of the same order for our quartz tuning forks and for the oscillating surface. This is indeed demonstrated by the data in Fig. 3, where the slope of the fit line is close to 1. We note, however, that this slope has not been exactly reproducible in the runs which differ by the azimuthal orientation of the cryostat (and thus different directions of the forcing with respect to the residual misalignment of the sample axis and the vertical direction), which is probably related to the sensitivity of the surface wave pattern in a vibrating cylinder to the exact conditions of the forcing \[^{17}\].

Another feature seen by Fig. 3 is the finite value of the surface resonance width (0.3 Hz) when extrapolated to zero temperature. This zero-temperature damping has been reproducible in all measurement runs. Among possible explanations for this damping is the surface friction at the cylindrical wall of the sample or non-linear interactions with other surface wave modes and possible creation of wave turbulence \[^{18, 19}\]. An intriguing possibility is contribution to the damping from the surface-bound Majorana fermions. No calculations of such contribution exist up to date, though, and even the physical mechanism of possible damping is not entirely clear. It can be similar to the recently proposed temperature-independent but frequency-dependent dissipation mechanism in the motion of quantized vortices originating from the vortex-core-bound fermions \[^{20}\]. Alternatively surface-bound fermions can directly mediate energy and momentum transfer to the container walls without transferring them to the bulk quasiparticles first. Such contribution to damping should have power-law dependence on temperature which in our experimental temperature range would mimic a finite zero-temperature damping.

Relaxation of the magnon BEC. Additional information about the surface waves is provided by the measurements of the relaxation of the magnon condensates after switching the NMR pumping off. For the magnon condensate in a time-independent trap which is completely in bulk the relaxation is determined by the bulk thermal quasiparticles \[^{21}\] and by the interaction with a NMR pick-up circuit, which will be described elsewhere. For our condensates in a time-dependent potential modulated by the surface waves we observe an additional relaxation channel so that the life time of magnon condensates decreases with increasing amplitude of the surface oscillations, Fig. 3.

In principle, for a quantum-mechanical system in a time-dependent potential one can expect transitions to other states. However, for our trapped condensates in the ground state we expect the highest probability of excitation at modulation frequencies corresponding to tran-
The thermometer fork has tines with dimensions $108 \times 10^{-6}$, and $12 \times 25$. We thank G.E. Volovik, M. Krusius, M.A. Silaev and I.A. Todoshchenko for stimulating discussions. The work is supported by the Academy of Finland (Center of Excellence 2012-2017) and the EU 7th Framework Programme (grant 228464 Microkelvin). P.J.H. acknowledges financial support from the Väisälä Foundation.

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