GAMMA-RAY EMISSION FROM ROTATION-POWERED PULSARS

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ABSTRACT

Using a simplified model of cascade pair creation over pulsar polar caps, presented in two previous papers, we investigate the expected γ-ray output from pulsars' low-altitude particle acceleration and pair creation regions. We divide pulsars into several categories, based on which mechanism truncates the particle acceleration off the polar cap, and give estimates for the expected luminosity of each category. We find that inverse Compton scattering above the pulsar polar cap provides the primary γ-rays that initiate the pair cascades in most pulsars. This reduces the expected γ-ray luminosity below previous estimates, which assumed that curvature γ-ray emission was the dominant initiator of pair creation in all pulsars. Even for the brightest pulsars, where curvature radiation sets the height of the pair formation front, we find predicted luminosities too low to explain the EGRET pulsars, suggesting that the source of that emission is an outer magnetosphere accelerator. The predicted polar cap luminosities are high enough, however, to be observable by upcoming γ-ray instruments, providing a firm test for this theory.

Subject headings: acceleration of particles — gamma rays: theory — pulsars: general

1. INTRODUCTION

Although only a few pulsars have so far been detected in γ-rays, mostly by the EGRET telescope on CGRO (Ulmer 1994; Thompson et al. 1997), this number should steadily grow as a new generation of γ-ray instruments, such as INTEGRAL and GLAST, is brought on-line. This should greatly assist theoretical understanding of pulsars, since the highest-energy photons provide a direct window into the underlying mechanisms thought to lead to pulsar emission of all types.

The primary photon emission mechanisms important in radio pulsars are inverse Compton scattering (ICS), in both the resonant (RICS) and Klein-Nishina nonresonant (NRICS) modes, and curvature emission. The relative importance of these mechanisms is still unclear. Most γ-ray pulsar papers have concentrated on curvature emission (Zhang & Harding 2000; Romani & Yadgaroglu 1995; Daugherty & Harding 1982), while an increasing number of polar cap physics papers have emphasized the importance of ICS (Dermer 1990; Sturner 1995; Sturner, Dermer, & Michel 1995; Luo 1996; Harding & Muslimov 1998).

In the polar cap acceleration model, particles are extracted from the polar cap and accelerated by large rotation-induced electric fields, forming the primary beam. These particles then emit primary γ-ray photons via ICS and curvature emission, and these photons interact with the pulsar magnetic field to create electron-positron pairs. The density of these secondary pairs increases with height as more and more photons pair produce, until the pair density is sufficient to short out the accelerating field.

Historically, this shorting of the accelerating field was thought to occur in a thin layer after the creation of the first pair; hence, the region of no pair creation, \( E_{\|} \neq 0 \), was thought to be separated from the region of copious pair creation, \( E_{\|} \approx 0 \), by a thin "pair formation front" (PFF). ICS photons, however, create small numbers of pairs at low altitudes, breaking the connection between the altitude of the first created pair and the altitude at which the electric field disappears. Since the altitude of the first created pair has no inherent dynamical significance, the term PFF in this paper always applies to the altitude at which the accelerating field is shorted. Other effects, such as the polarization of the generated pairs, will also begin at the point of first pair creation, but these effects must likewise reach some threshold before they affect the dynamics of the beam.

Once the accelerating field has been shorted out, the primary beam coasts, continuing to emit γ-rays. The total γ-ray output of a polar cap is then a combination of the synchrotron γ-rays produced by the created secondary particles, the ICS radiation emitted by the secondary particles, and the primary γ-rays emitted by the primary beam. If the primary beam is radiation reaction limited, this emission efficiently converts the beam energy into γ-rays; otherwise, only a small fraction is extracted.

We find that the γ-ray output of pulsars falls into two categories. For the majority of pulsars, nonresonant ICS stops the beam acceleration at low Lorentz factors, where γ-ray emission is inefficient, leading to low luminosities. For the remaining pulsars, the beam is accelerated to high Lorentz factors, resulting in efficient γ-ray emission and a high luminosity. Using the results of Hibschman & Arons (2001a, 2001b, henceforth Papers I and II), we examine the boundary between these categories of pulsars and predict the luminosities and spectral characteristics of these objects.

2. MODEL

Inverse Compton scattering depends strongly on the temperature of the neutron star polar cap, making the thermal cooling model chosen for the neutron star an important part of the theory. For simplicity, we will assume that the temperature of the polar cap is entirely due to the polar cap heating discussed in Paper I. We use the acceleration model of Muslimov & Tsygan (1992) and Muslimov & Harding (1997), in the simplified form described in Paper I, and neglect spatial variations across the polar cap. This accelerating potential, expressed in units of \( mc^2/e \) so as to yield the expected particle Lorentz factor, is

\[
\Phi_{\text{low}}(t) = \Phi_1 t^{2} \frac{mc^2e^{-1}}{\gamma}, \quad t < 1
\]

\[
\Phi_{\text{high}}(t) = \Phi_1 t \frac{mc^2e^{-1}}{\gamma}, \quad t > 1
\]

(1)
where $\Phi_d = 5.14 \times 10^4 B_{12} P^{-5/2}$, $t = s / s_1$, and $s_1 = 8.87 \times 10^{-6} P^{-1/2} R_\ast$, while $s$ is the altitude above the stellar surface.

Although we use general relativity though the Muslimov-Tsygan accelerating potential, we neglect other relativistic effects, such as the changes to the magnetic field near the surface and the gravitational redshift of the emitted photons. These effects are of order 10%–15%, so including them would pretend to greater accuracy than justified. The accelerating potential is unique, because it is only the charge difference created by the relativistic contribution that creates the starvation electric field.

### 2.1. Emission Rates

First, we consider the emission of a single primary beam particle, moving along the field lines above the pulsar cap. We neglect, for now, discussion of the secondary particles created by pair production, because the total energy emitted by these particles is clearly limited by that emitted by the primaries. For this section, we effectively assume that only a negligible fraction of the energy emitted in primary $\gamma$-rays remains in the generated electron-positron plasma. From the results of Paper II, this naturally follows in low-B pulsars ($B_{12} < 1$) and results in other pulsars because of the RICS of the secondary-pair plasma.

Since NRICS is only logarithmically dependent on the Lorentz factor of the beam, the NRICS power emitted is limited primarily by the attenuation of the background thermal photons as the beam particles move away from the star. If we assume a hot polar cap of angular radius $\theta_c = (R_{\odot}/c)^{1/2}$, where $\Omega$ is the angular velocity of the pulsar, $\Omega = 2\pi/P$, and $R_{\odot}$ is the radius of the neutron star, assumed to be 10 km, the total energy radiated by one particle is

$$E_{NR} = (\theta_c R_{\odot} c) P_{NR} = 6.1 \times 10^{-4} P^{-1/2} T_6^2 \text{ ergs}.$$  \hspace{1cm} (2)

The approximate form used for the NRICS power, $P_{NR}$, is that given in Paper I, and $T_6$ is the temperature of the polar cap in units of $10^6$ K.

The power emitted via RICS, while only logarithmically sensitive to the decline in the thermal photon flux, decreases quickly with increasing Lorentz factor. Because of this dependence, most of the RICS power is emitted at low altitudes, where the Lorentz factor is still low. The expected total output per particle is

$$E_R = (s_{\text{min}}/c) P_{\gamma}(s_{\text{min}}) = 4.5 \times 10^{-5} P^{3/4} B_{12} T_6^{-1/2} \text{ ergs},$$  \hspace{1cm} (3)

where $s_{\text{min}} = \gamma_{\text{m}} / kT$ is the minimum Lorentz factor at which thermal photons are upscattered into resonance with the beam, $\gamma_{\text{m}}$ is the altitude at which the beam particles reach $s_{\text{min}}$, and $P_{\gamma}$ is the power emitted by RICS emission, as given in Paper I. In the acceleration model from Paper I, $s_{\text{min}} = 450 P^{1/4} T_6^{-1/2}$ m, assuming a star of radius 10 km.

The power emitted by curvature radiation is strongly dependent on the Lorentz factor of the beam, varying as $\gamma^4$, so most of the energy emitted by curvature radiation is emitted as the beam coasts above the PFF. Once particle acceleration stops, the Lorentz factor of the primary beam declines according to

$$\gamma(s > s_{\text{PFF}}) = \frac{\gamma_{\text{PFF}}}{c} \left[ 1 + 3 \frac{P_{\gamma}(s_{\text{PFF}})}{P_{\gamma}(s_{\text{PFF}})} \right]^{-1/3} R_\ast \ln \frac{1 + s_{\text{PFF}}}{1 + s_{\text{PFF}}},$$  \hspace{1cm} (4)

assuming a dipolar magnetic field. The upper limit may be estimated by finding the altitude at which the high-energy primary beam decouples from the magnetic field. Equating the energy density in the beam to the energy density in the magnetic field yields a decoupling height of $r_{\text{max}} = 4123 B_{12}^{1/2} P^{1/2} R_\ast$, where $\gamma_{\text{m}}$ is the Lorentz factor of the beam in units of 10. The PFF is close to the surface, so the logarithm in equation (4) is approximately 8.3. The total curvature energy emitted is then

$$E_C = \frac{[\gamma_{\text{PFF}} - \gamma(s_{\text{max}})]mc^2}{(s_{\text{max}})} = 8.2[\gamma_{\text{PFF}} - \gamma(s_{\text{max}})] \text{ ergs}.$$  \hspace{1cm} (5)

Comparing the emitted energies, equations (2), (3), and (5), we find that, for typical Lorentz factors of $\gamma_{\text{PFF}} > 10^4$, only curvature emission may radiate any appreciable fraction of the beam particle energy. The minimum Lorentz factor at which radiation reaction is important is then

$$\gamma_{\text{RR}} = 2.33 \times 10^4 P^{1/3},$$  \hspace{1cm} (6)

which is the Lorentz factor at which the primary beam loses half their energy to curvature radiation. Above this Lorentz factor, roughly all the energy in the beam is lost to $\gamma$-rays; below it, the beam propagates without significant radiation losses.

The expected curvature energy loss in these two regimes is

$$E_C \approx \frac{\gamma_{\text{PFF}} mc^2}{(s_{\text{min}})} = 8.2 \times 10^{-7} \frac{\gamma_{\text{PFF}}}{c} \text{ ergs},$$  \hspace{1cm} (7)

if radiation reaction is important and

$$E_C \approx (8.3 R_{\odot}/c) P_{\gamma}(\gamma_{\text{PFF}}) = 1.51 \times 10^{-28} P^{-1} \gamma_{\text{PFF}}^4 \text{ ergs},$$  \hspace{1cm} (8)

if not.

### 2.2. Luminosity

Using the polar cap model from Paper I, we can classify pulsars according to which mechanism sets the PFF and whether the beam is radiation reaction limited ($\gamma_{\text{PFF}} > \gamma_{\text{RR}}$). The pair formation model then yields the altitude of the PFF, $s_{\text{PFF}}$, and the Lorentz factor at that altitude, $\gamma_{\text{PFF}}$.

Given $\gamma_{\text{PFF}}$, we can compute the total expected luminosity by multiplying the total energy emitted by a single beam particle by the number of particles emitted by the polar cap, $N = n_{GJ} \theta_c R_{\odot} = 1.37 \times 10^{30} B_{12} P^{-2} s^{-1}$, where $n_{GJ}$ is the expected Goldreich-Julian number density, $n_{GJ} = \Omega B/2\pi c$. Using a slight modification to the model in Paper I, we find that, in comparison with the numerical results of Paper II, the PFF from curvature emission is more accurately found by finding the altitude at which the first pair is formed. This is because of the steadily increasing intensity of curvature emission with increasing Lorentz factor.

A curvature photon emitted at altitude $s$ will pair produce at

$$s_{\text{c}} = s + \frac{1}{4} \frac{\epsilon_{\gamma c} L(s)}{c} R_\ast,$$  \hspace{1cm} (9)

where $\epsilon_{\gamma c}$ is the typical curvature photon energy, $\epsilon_{\gamma c} = 5.8 \times 10^{-19} P^{1/2} R_\ast m c^2$, and $c$ is the scaling energy for pair production from Paper I, $\epsilon_{\gamma} = 2166B_{12}^{1/2} P^{1/2} \text{ ergs}$. Here $f_{\text{c}}$ is the ratio of the actual field line radius of curvature to the radius of curvature of the dipole field line that intersects the stellar surface at $\theta_c$; for the remainder of the paper, this is taken to be 1. The minimum value of this is at

$$s_{\text{FF}} = 1.91B_{12}^{-1} P^{1/4} f_{\text{c}}^{1/2} R_\ast,$$  \hspace{1cm} (10)

if the PFF takes place in the linear regime of the accelerating potential at $s \geq \theta_c R_\ast$, or

$$s_{\text{FF}} = 0.211B_{12}^{-4/7} P^{11/4} f_{\text{c}}^{2/7} R_\ast,$$  \hspace{1cm} (11)

if in the quadratic regime at $s \leq \theta_c R_\ast$. 

1184 HIBSCHMAN Vol. 565
Using the seminumerical model of Paper II, we then classify pulsars by which the emission mechanism produced the PFF, by whether the PFF occurred at low or high altitude (in comparison with the polar cap width), and by whether the beam was radiation reaction limited. In principle, this yields 12 categories; in practice, there are only five important divisions.

Of the 540 pulsars in the Princeton pulsar catalog with positive $P$, the majority, 315, had a PFF set by NRICS at high altitude, with $\gamma_{\text{PFF}} < \gamma_{\text{RR}}$. For all these objects, curvature radiation is the primary energy-loss mechanism in the beam drift region above the PFF. Although most of the radiated energy is curvature emission, it is the relatively sparse radiation that sets the PFF. In principle, this is much lower than previous estimates of pulsar radiative energy; the 23 of these where this is not true are among those set by both curvature and NRICS emission, but at low altitudes rather than high. These are the 21 brightest, highest-potential pulsars. Using equations (11) and (7) yields a Lorentz factor and luminosity of

$$\gamma_{\text{PFF,C}}^{\text{low}} = 2.90 \times 10^7 B_{12}^{-1/7} P_{1/4}^{1/14},$$
$$L_{12}^{\text{low}} = 3.3 \times 10^{31} B_{12}^{12/7} P_{-27/14}^{27/14} \text{ergs s}^{-1},$$

(corresponding to regime I of Zhang & Harding (2000).

However, the correlation between the NRICS–dominated and curvature-dominated modes is at $B_{12} = 0.061 P^{1/4}$. Pulsars with weaker magnetic fields are dominated by curvature, and equation (19) is the appropriate luminosity, while pulsars with fields stronger than this are dominated by NRICS, and equation (14) is appropriate. However, this ignores the weakening of NRICS with increasing field discussed in Paper II and so should be taken as a statement that the millisecond pulsars are certainly controlled by curvature, while the higher field, longer period pulsars are favored by NRICS but must be examined carefully, using either the full algebraic results from Paper I or the seminumerical model of Paper II.

The third category consists of those pulsars where either curvature or NRICS sets the PFF at high altitudes and where the beam is radiation reaction limited, for a total of 57 objects. In practice, we find that these pulsars are all well modeled by the expected PFF for curvature emission, equation (10), and by equation (7) for the energy loss. This yields a final Lorentz factor and total luminosity of

$$\gamma_{\text{PFF,C}}^{\text{high}} = 1.10 \times 10^3 P^{-1/4},$$
$$L_{12}^{\text{high}} = 1.2 \times 10^{31} B_{12}^{-9/4} \text{ergs s}^{-1},$$

essentially the same as the Zhang & Harding (2000) values for their regime II. As they mentioned, this preserves the empirical $L_{12} \propto L_{45}^{1/2}$ relation. These are among the brightest $\gamma$-ray pulsars, behind only the fourth category in total luminosity.

The fourth category is similar to the third, in that it includes radiation reaction–limited beams where the PFF is set by both curvature and NRICS emission, but at low altitudes rather than high. These are the 21 brightest, highest-potential pulsars. Using equations (11) and (7) yields a Lorentz factor and luminosity of

$$\gamma_{\text{PFF,R,pc}}^{\text{low}} = 4.03 B_{12}^{-2} P^{3/8} R_*,$$
$$\gamma_{\text{PFF,R,pc}}^{\text{low}} = 1.06 \times 10^9 B_{12}^{-3} P_{-3/4}^{3/4},$$

(24)

As in Paper I, the emission mechanism with the smallest PFF height is the dominant mechanism. Because of the multiplicity of categories, generalizations are difficult, but we can derive a few formulae for the boundaries between regimes by considering only the high-altitude limits.

In this simplest approximation, the boundary between the NRICS–dominated and curvature-dominated pulsars is at $B_{12} = 0.061 P^{1/4}$. Pulsars with weaker magnetic fields are dominated by curvature, and equation (19) is the appropriate luminosity, while pulsars with fields stronger than this are dominated by NRICS, and equation (14) is appropriate. However, this ignores the weakening of NRICS with increasing magnetic field discussed in Paper II and so should be taken as a statement that the millisecond pulsars are certainly controlled by curvature, while the higher field, longer period pulsars are favored by NRICS but must be examined carefully, using either the full algebraic results from Paper I or the seminumerical model of Paper II.

The boundary between NRICS and RICS lies at $P_{\text{NR}} = 6.81 B_{12}^{-5/7}$, with higher periods favoring RICS, while that between RICS and curvature is at $P_{\text{R,C}} = 3.66 B_{12}^{-30/35}$, again with RICS dominating at longer periods. For pulsars with magnetic fields stronger than approximately $4 \times 10^{12}$ gauss, NRICS is ineffective in setting the PFF, and the only active mechanisms are RICS and curvature; at midrange fields between $3 \times 10^{12}$ and $4 \times 10^{12}$ gauss, NRICS sets the PFF, while at lower fields, curvature sets it.

2.3. Flux

Since in this model the observed $\gamma$-rays originate from particles moving along the field lines above the pulsar polar cap, the predicted luminosities translate directly into a predicted flux. Since the polar cap is the source of the $\gamma$-rays, they are beamed into a cone of opening angle $\theta \approx (3/2)\theta_c$. In general, this opening angle will vary with the altitude of the emission, but for most pulsars the altitude of the PFF is small compared to the stellar radius, so simply evaluating the opening angle at the surface is sufficient. If the lumi-
where a peak flux of

$$\phi_{\text{peak}} = L/(\pi\theta^2 d^2),$$

where $d$ is the distance to the pulsar. This is the flux seen while looking “down the barrel of the gun.” Only if the spin axis of the pulsar is nearly aligned with the line of sight will the observed flux be on this order. The average flux is reduced by $\theta/\pi$, if the spin axis is perpendicular to the line of sight, and by approximately $\theta/\pi \sin \alpha$ in the general case, where $\alpha$ is the angle between the magnetic moment and the rotation axis and we have effectively assumed that the observer’s line of sight passes through the center of the emitting cone ($\beta = 0$). This represents the fraction of the pulse period where the emitting cone is directed toward the observer,

$$\phi_{\text{ave}} = L/(2\pi^2 \theta \sin \alpha d^2).$$

This average flux is the observable, not the total, luminosity. In terms of the luminosities from § 2.2, this flux is

$$\phi_{\text{ave}} = 3.7 \times 10^{-43} P^{1/2}(\sin \alpha)^{-1} d_{\text{kpc}}^{-2} L \text{ ergs cm}^{-2} \text{ s}^{-1}.$$  

Since the $\sin \alpha$ term is a factor of order unity, and these approximations are only good to approximately a factor of 2, we simply set it to 1 for the remainder of the paper. We plot the expected observable flux as a function of the pulsar period, using a fiducial pulsar distance of 1 kpc in Figure 1 to show the inherent dependencies on pulsar parameters and using the estimated distances of each individual object in Figure 2 to show the predicted observable flux.

3. SPECTRAL SHAPE

In § 2.3, we discussed the total energy output of the pulsar; here, we turn to the expected shape of the spectrum itself. In Paper II, we found that a cascade of pair creation from a single absorbed photon produces a response with a power-law index of $-3/2$, because of the reprocessing of synchrotron photons. In order to be absorbed, however, the photon must have an energy greater than $\epsilon_{\text{min}} = 5134B_{\text{E}}^{-1} P^{1/2} mc^2$, presuming that the photon was emitted parallel to the magnetic field at the edge of the polar cap at the surface of the star.

Since most of the power emitted in $\gamma$-rays from pulsar polar caps is due to curvature emission, except for the few extreme high-field objects where RICS dominates, the energetics of the curvature photons determine the shape of the spectrum. The minimum Lorentz factor for curvature pair production may be found by equating $\epsilon_c(\gamma) = \epsilon_{\text{min}}$, yielding

$$\gamma_C = 2.01 \times 10^7 B_{\text{E}}^{-1/3} P^{1/3},$$

which is the Lorentz factor at which the critical curvature energy, $\epsilon_c(\gamma) = 5.8 \times 10^{-19} P^1 m c^2$, equals the minimum energy to pair produce, $\epsilon_{\text{min}}$, assuming a dipole field radius of curvature.

If $\gamma_{\text{PFF}} > \gamma_C$, then the copious curvature photons will pair produce, and the observed radiation will be the synchrotron emission of the generated pairs. At low energies, $\epsilon < \epsilon_a/(1 + a^2)$ In A in the notation of Paper II, the spectral index is the $-2/3$ of unprocessed synchrotron radiation, while at higher energies, the spectral index is the characteristic $-3/2$.

If $\gamma_{\text{PFF}} < \gamma_C$, then the curvature photons will be observed directly. The beam energy loss in this case, for all pulsars examined, is small enough that the characteristic curvature energy remains effectively unchanged. Therefore, the unmodified $-2/3$ spectrum of curvature radiation will be seen, extending from low energies up to $\epsilon_c(\gamma)$. This energy will clearly be lower than the traditional estimate of the cutoff energy of $\epsilon_a$ and dependent on the mechanism that sets the PFF.

Figure 3 shows the results of dividing the pulsars into two categories, based on this division. Of the 20 brightest
expected polar cap \( \gamma \)-ray pulsars, only two are expected to have the \(-2/3\) power law, J0953+0755 and J1932+1059. To illustrate the difference, we plot in Figures 4 and 5 the numerically calculated \( \gamma \)-ray spectra of J1952+3252, which is expected to have a \(-3/2\) spectrum, and J1932+1059, respectively. The numerical method used was that described in Paper II, and the results confirm the expectations.

The maximum energies of these spectra depend on whether the beam is radiation reaction limited not on the minimum energy for pair production, \( \epsilon_\gamma \). All photon energies higher than \( \epsilon_\gamma \) are converted by pair production to photons of lower energy, but \( \epsilon_\gamma \) steadily increases with altitude as \( r^3 \), eventually surpassing the maximum energy of the raw photon spectrum emitted by the beam particles. At that point, and beyond, the maximum energy of the spectrum is set by the energy of the beam itself.

Since curvature radiation is the strongest emission mechanism for all the brightest pulsars and all but a small minority of the others, the maximum energy will be the characteristic curvature energy, evaluated at the high-altitude coasting Lorentz factor of the beam. However, the relevant Lorentz factor is slightly different from that calculated in § 2.3, equation (6). Since the radius of curvature of the field steadily increases with altitude, the curvature energy of a coasting beam will steadily decrease, as \( r^{-1/2} \). The maximum energy observed will then arise from emission from an intermediate regime: high enough that the magnetic field no longer absorbs photons, but low enough that the radius of curvature is still small. In this case, the appropriate limiting Lorentz factor is where the scale height for energy loss is equal to the stellar radius, \( \gamma_{\text{RR}} = 3.56 \times 10^7 P^{1/3} \).

If the beam is radiation reaction limited in this sense, i.e., if \( \gamma_{\text{PF}} > \gamma_{\text{RR}} \), then the maximum energy is the curvature energy evaluated at \( \gamma_{\text{RR}} \), or

\[
\epsilon_{\text{max}} = \epsilon_c(\gamma_{\text{RR}}) = 10.2P^{1/2} \text{ GeV},
\]

where we have evaluated the curvature energy at a radius of \( 2R_\star \). If the beam is not radiation reaction limited, then the maximum energy depends on the Lorentz factor of the
expected observable fluxes and maximum energies for
lower than the theoretical results, although the brightest
on the tails of the distribution. Over the 25 brightest

\[ \epsilon_{\text{max}}^{\text{coast}} = \epsilon_{\gamma}^{\text{PFF}} = 3.2 \times 10^{-2} P^{-1/2} \gamma_{\text{PFF}} \text{ GeV}. \]  

The expected observable fluxes and maximum energies for the 25 brightest pulsars are shown in Table 1.

4. NUMERICAL MODEL

Using the full numerical system described in § 4 of Paper II, we have run several simulations of these objects. These results confirm our conclusions about the different regimes of pair production discussed above, with the numerically calculated luminosity remaining within approximately a factor of 3 of the simple analytic model, with the numerical model always lower, because of the effects of pair creation on the tails of the distribution. Over the 25 brightest pulsars, the numerical results are on average a factor of 2.1 lower than the theoretical results, although the brightest pulsars show more variance.

The calculated pulsar spectra matched expectations, although in several cases a low-intensity, high-energy tail due to NRICS was observed, extending up to \( \gamma_{\text{PFF}} \), with a power law exponent of roughly \(-2\). In general, this tail contains a negligible portion of the energy, namely, the energy predicted by equation (2).

We also ran the numerical model using alternative heating models. We found that, since in the brightest objects curvature emission sets the PFF, the precise temperature model mattered little for the observed objects. Using different models of the stellar temperature only changed the expected luminosity by on the order of 20%. However, for the lower luminosity ICS–dominated pulsars, the temperature is far more important, because the ICS process relies on the thermal photon bath, potentially allowing future \( \gamma \)-ray observations to discriminate between thermal models.

5. DISCUSSION

In Table 2, we compare the observed flux, the flux from the model of Zhang & Harding (2000), the flux predicted by the seminumerical model of this paper, and the flux computed by running the full numerical cascade model for each of the observed \( \gamma \)-ray pulsars. The flux predicted for both

### TABLE 1

#### BRIGHTEST PULSARS

| Pulsar         | \( P \) (s) | \( B \) (gauss) | PFF Mechanism | RR Limited? | Curvature Pairs? | \( \epsilon_{\text{max}} \) (GeV) | Predicted Flux (ergs cm\(^{-2}\) s\(^{-1}\)) |
|---------------|-------------|----------------|---------------|-------------|------------------|-----------------|----------------------------------|
| J0835 - 4510  | 0.089       | 6.8 \times 10^{12} | Curvature     | Yes         | Yes              | 3.07            | 1.6 \times 10^{-8}               |
| J0633 + 1746  | 0.237       | 3.3 \times 10^{12} | NRICS         | Yes         | Yes              | 3.73            | 9.5 \times 10^{-9}               |
| J0437 - 4715  | 0.006       | 1.2 \times 10^{9}  | Curvature     | Yes         | Yes              | 0.78            | 5.7 \times 10^{-9}               |
| J0534 + 2200  | 0.033       | 7.6 \times 10^{12} | Curvature     | Yes         | Yes              | 1.88            | 4.8 \times 10^{-9}               |
| J1932 + 1059  | 0.227       | 1.0 \times 10^{12} | NRICS         | No          | No               | 0.81            | 1.1 \times 10^{-9}               |
| J1709 - 4428  | 0.102       | 6.3 \times 10^{12} | Curvature     | Yes         | Yes              | 3.29            | 9.2 \times 10^{-10}              |
| J0953 + 0755  | 0.253       | 4.9 \times 10^{11} | NRICS         | No          | No               | 0.55            | 6.2 \times 10^{-10}              |
| J1300 + 1240  | 0.006       | 1.7 \times 10^{9}  | Curvature     | Yes         | Yes              | 0.81            | 3.7 \times 10^{-10}              |
| J1952 + 2252  | 0.040       | 9.7 \times 10^{11} | NRICS         | Yes         | Yes              | 2.04            | 3.3 \times 10^{-10}              |
| J1048 - 5832  | 0.124       | 7.0 \times 10^{12} | Curvature     | Yes         | Yes              | 3.61            | 2.9 \times 10^{-10}              |
| J1102 + 5307  | 0.005       | 5.6 \times 10^{8}  | Curvature     | Yes         | Yes              | 0.74            | 2.3 \times 10^{-10}              |
| J0403 + 2740  | 0.096       | 7.0 \times 10^{11} | NRICS         | Yes         | Yes              | 3.18            | 2.0 \times 10^{-10}              |
| J0742 - 2822  | 0.167       | 3.4 \times 10^{12} | Curvature     | Yes         | Yes              | 4.19            | 1.9 \times 10^{-10}              |
| J0034 - 0534  | 0.002       | 2.3 \times 10^{8}  | Curvature     | Yes         | Yes              | 0.44            | 1.9 \times 10^{-10}              |
| J1826 - 1334  | 0.101       | 5.6 \times 10^{12} | Curvature     | Yes         | Yes              | 3.27            | 1.7 \times 10^{-10}              |
| J1803 - 2137  | 0.134       | 8.6 \times 10^{12} | Curvature     | Yes         | Yes              | 3.75            | 1.6 \times 10^{-10}              |
| J1730 - 2304  | 0.008       | 7.9 \times 10^{8}  | Curvature     | Yes         | Yes              | 0.93            | 1.6 \times 10^{-10}              |
| J0117 + 5914  | 0.101       | 1.6 \times 10^{12} | NRICS         | Yes         | Yes              | 3.27            | 1.4 \times 10^{-10}              |
| J1959 + 2048  | 0.002       | 3.3 \times 10^{8}  | Curvature     | Yes         | Yes              | 0.41            | 1.4 \times 10^{-10}              |
| J1801 - 2451  | 0.125       | 8.1 \times 10^{12} | Curvature     | Yes         | Yes              | 3.63            | 1.3 \times 10^{-10}              |

### TABLE 2

#### OBSERVED AND PREDICTED PULSAR FLUXES

| Pulsar         | \( B \) (gauss) | PFF Mechanism | \( \phi_{\text{obs}} \) (ergs cm\(^{-2}\) s\(^{-1}\)) | \( \phi_{\text{Zhang}} \) (ergs cm\(^{-2}\) s\(^{-1}\)) | \( \phi_{\text{pred}} \) (ergs cm\(^{-2}\) s\(^{-1}\)) | \( \phi_{\text{num}} \) (ergs cm\(^{-2}\) s\(^{-1}\)) |
|---------------|----------------|---------------|-----------------|-----------------|-----------------|-----------------|
| J0534 + 2200  | 7.6 \times 10^{12} | Curvature     | 1.3 \times 10^{-8} | 5.9 \times 10^{-9} | 4.8 \times 10^{-9} | 1.8 \times 10^{-9} |
| J0835 - 4510  | 6.8 \times 10^{12} | Curvature     | 9.9 \times 10^{-9} | 2.3 \times 10^{-8} | 1.6 \times 10^{-8} | 5.0 \times 10^{-9} |
| J0633 + 1746  | 3.3 \times 10^{12} | NRICS         | 3.9 \times 10^{-9} | 2.6 \times 10^{-8} | 9.5 \times 10^{-9} | 4.5 \times 10^{-9} |
| J1709 - 4428  | 6.3 \times 10^{12} | Curvature     | 1.3 \times 10^{-9} | 1.3 \times 10^{-9} | 9.2 \times 10^{-10} | 3.0 \times 10^{-10} |
| J1513 - 5908  | 3.1 \times 10^{13} | RICS          | 8.8 \times 10^{-10} | 5.3 \times 10^{-10} | 4.0 \times 10^{-13} | 2.0 \times 10^{-13} |
| J1952 + 2252  | 9.7 \times 10^{11} | NRICS         | 4.3 \times 10^{-10} | 5.1 \times 10^{-10} | 3.3 \times 10^{-10} | 1.8 \times 10^{-10} |
| J1057 - 5226  | 2.2 \times 10^{12} | NRICS         | 2.9 \times 10^{-10} | 2.5 \times 10^{-10} | 7.6 \times 10^{-11} | 5.1 \times 10^{-11} |
| J1048 - 5832  | 7.0 \times 10^{12} | Curvature     | 2.5 \times 10^{-10} | 4.2 \times 10^{-10} | 2.9 \times 10^{-10} | 7.5 \times 10^{-11} |

a Thompson et al. 1999.
b Zhang & Harding 2000.
models was derived from the total luminosity using equation (27).

The model of Zhang & Harding (2000) assumed that $\alpha = 30$, decreasing the expected flux, while invoking a proposed ICS instability to move the acceleration region off the surface of the star. These two effects roughly cancel, leaving the predictions of their model comparable to those of this model, which effectively uses $\cos \alpha = 1$ and acceleration near the surface. Because of the small numbers of reversed particles, we find no instability in the acceleration zone, especially in the cases where, in the language of this paper, curvature radiation sets the PFF. Hence, we find no reason to raise the altitude of the acceleration zone.

The numerically calculated result is substantially smaller than either the seminumerical result or that of Zhang & Harding (2000), because of two major effects. First, the numerically calculated PFF is lower than that predicted by the analytic model by approximately 25% on average, which, since these pulsars operate in the low-level quadratic portion of the accelerating potential, reduces the Lorentz factor of the beam to approximately 60% of its analytically calculated value. This occurs because photons on the exponential tail of the curvature spectrum pair produce and create a sufficiently dense plasma to short out the accelerating electric field at lower altitudes than expected in cruder calculations (Arons & Scharlemann 1979).

The second reason is that some of the primary photon energy remains in the generated pairs, rather than being reradiated. Both the analytic model of this paper and that of Zhang & Harding (2000) assume that all the energy emitted by the primary beam is eventually reemitted in $\gamma$-rays, because of the combination of synchrotron emission from the created pairs and RICS extracting any remaining energy. In the numerical model, however, we calculate the pair spectrum itself and can determine what fraction of the energy in the pairs is reradiated by RICS.

As a quick approximation of the effects of RICS on the pair spectrum, we assume that all particles with an energy-loss length scale equal to or less than the stellar radius reemit all their energy as lower energy $\gamma$-rays. The minimum Lorentz factor is the lowest at which thermal photons could be scattered into resonance with the field, $\gamma_{\text{min}} = \epsilon_{\gamma} / \Delta \mu kT = 134.5 B_{12}^{-1} T_6^{-1} \Delta \mu^{-1}$. The maximum Lorentz factor is the point where the particle’s energy-loss scale is a stellar radius, $\gamma_{\text{max}} = 4.05 \times 10^5 B_{12}^{-1/5} T_6^{1/2}$.

With these corrections, we see that the predictions of the polar cap model are low compared with the observations for most of the observed $\gamma$-ray pulsars. The difference is large for the Crab, for which the prediction is low by a factor of 10, and for the high-field object J1513−5908, for which the prediction is low by 3 orders of magnitude. Predictions for the other objects are typically low by a factor of 3.

The high-field object J1513−5908 clearly deserves further examination. Not only is the magnetic field of this pulsar so large that the applicability of this model is questionable, because of high-field effects as discussed by Harding, Baring, & Gonthier (1997), but preliminary studies of the spatial variation of $\gamma$-ray emission across the polar cap suggest that the core of this pulsar’s beam should be much brighter than the edge field line considered in this model.

The neglect of spatial variation across the polar cap limits the accuracy of these results; simple estimates suggest that including those effects would increase the expected flux by 50%, but more detailed study is required to be more concrete. Physically, the general relativistic acceleration is strongest at the center of the pulsar, while the field line radius of curvature is smallest at the edge. Because of the smaller radius of curvature and gentler acceleration, pair production due to ICS processes is far easier at the edges of the polar cap than at the center, while pair production via curvature radiation is more likely on the central field lines. Together, these effects combine to place the pair formation front at a higher altitude in the center of the polar cap and lower near the edges, so that the central field lines are comparatively brighter than the edges.

These effects should help raise the predicted luminosities closer to those observed, bringing them within the expected margins of error for this study. Further work on the variation of the $\gamma$-ray emission across the polar cap is certainly required before any firm conclusions can be drawn.

However, the polar cap model yields specific predictions for the beaming that are not consistent with the observations. First, the $\gamma$-ray peaks should be in phase with the radio emission, which is not typically the case. Second, for all the observed pulsars, the curvature emission energy loss, equation (4), predicts that 40%–60% of the emitted energy for these pulsars should be emitted within the first 10 km above the surface. This energy would go into a cone of angular width $\theta_{\text{emit}} = 2^{1/2} \theta_{\gamma}$, or $\theta_{\text{emit}} = 3.5 \theta_{\gamma}^{-1/2}$ degrees. This is far more focused than the observed pulsars, which typically have double-beam profiles spread over rotation phase 0.4. If the observed pulsars were all oriented with the magnetic axis, rotation axis, and line of sight all roughly parallel, such broad profiles could be generated by a polar cap model. Specific modeling of the polar cap model’s beaming properties require the angle between the rotation axis and the magnetic axis to be less than $45^\circ$ and the intrinsic opening angle of the beam to be as large as $30^\circ$ (Harding & Daugherty 1998). The restriction on obliquity is a priori improbable and is known to cause difficulty with the population statistics of pulsars (Romani 1996), while the large opening angle for the intrinsic emission beam requires adding new features to the basic dynamics of the polar cap model not supported by our calculations.

Given these beaming problems with the polar cap model, the outer gap models of Romani & Yadigaroglu (1995), Yadigaroglu & Romani (1995), and Zhang & Cheng (1997) remain strong contenders for the EGRET pulsars. These models easily produce beam profiles that resemble the data but are energetically more difficult. For example, recent work by Hirotani & Shibata (2001) addresses the over-abundance in TeV emission, but the emissivity in the GeV range remains an open question.

However, even if the observed $\gamma$-ray emission originates in the outer magnetosphere, the flux predicted to arise from the polar cap itself is not so much less than the observed fluxes as to be unobservable in the near future. With the advent of the new $\gamma$-ray instruments, the threshold of sensitivity should be low enough that the polar cap $\gamma$-rays can be observed. Detection of these $\gamma$-rays, seen when looking down the barrel of the radio gun, and comparison of their emission phase and profile with the radio counterpart would be a firm test of the long-established, but not directly tested, theory of polar cap pair creation, which underlies 30 years of theorizing about the origin of pulsar radio emission.
6. CONCLUSION

The primary conclusion of this paper is that the currently observed γ-ray pulsars are not representative of the bulk of pulsars. Currently, only the brightest γ-ray pulsars are observed; these pulsars are selected to be those where curvature radiation operates efficiently, which is not true of pulsars in general.

Most pulsars have a beam energy controlled by nonresonant inverse Compton scattering (NRICS), rather than by curvature emission, and will emit significantly fewer γ-rays. Locating examples of these objects will be a challenge to the new generation of γ-ray telescopes. The fainter the objects that can be seen, the stronger the differences should be between the NRICS cascades and the predictions of the curvature-based PFF models.

In these fainter objects, the separation of the mechanism producing the PFF (and thus shorting out the accelerating potential) from the mechanism producing most of the observable γ-rays should reveal itself through differing spectral indices. Because of the effects of inverse Compton scattering, there should exist two classes of observed γ-ray emission, one a raw curvature spectrum characterized by a spectral index of $-2/3$ and the other a saturated synchrotron response with a spectral index of $-3/2$.

For the currently observed γ-ray pulsars, the predicted fluxes are consistently low, although, because of the neglect of spatial variation across the polar cap, they are within the accuracy of this study. The beaming issues with the polar cap model remain; for this reason, the outer gap models are still strong contenders for the observed pulsars, despite the energetics problems of those models. Even if the currently observed emission is from an outer gap, improved γ-ray sensitivity should reveal the signal of the polar caps, which is at worst only a factor of 5 below the observed fluxes.

Further observations should also reveal the differences in the luminosity classes of polar cap γ-rays, according to which mechanism which creates the PFF, and the differences between the two spectral regimes, providing a straightforward test of this model.

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