Deep-learning-based Signal Detection for Faster-than-Nyquist Transmission

Peiyang Song, Fengkui Gong, Qiang Li, Shenghua Zhai and Haiyang Ding

Abstract

Faster-than-Nyquist (FTN) is a promising paradigm to improve bandwidth utilization at the expense of additional intersymbol interference (ISI). In this letter, we develop a signal detection architecture based on deep learning (DL) for FTN system, which employs the sliding window and works without any iteration. To the best of our knowledge, this is the first attempt to apply a six-layer deep neural network (DNN) for FTN signal detection. As demonstrated by simulation results, our proposed DL-based detection can achieve a near-optimal bit error rate (BER) performance and shows the potential in high order modulations. In addition, the proposed DL-based detection has a robustness to signal to noise ratio (SNR). In a nutshell, DL has been proved to be a powerful tool for FTN signal detection.

Index Terms

Faster-than-Nyquist, signal detection, deep learning, intersymbol interference

I. INTRODUCTION

The last couple of decades have seen the exponential growth of wireless devices and data traffic. Nowadays, spectral efficiency has become extremely valuable. With increasingly demanding requirements for spectral resources, a promising technology named FTN is rediscovered and has attracted a lot of attention in both industrial and academic communities [1]–[8].
As known, in conventional Nyquist-criterion transmission, when available bandwidth is $W$ Hz, the symbol interval $T$ is always set as $T \geq T_N = 1/(2W)$. The strict orthogonality between transmitted symbols guarantees the signal recovery in the receiver. In contrast, the symbol interval reduces to $T < T_N$ in FTN signaling to achieve a higher transmission rate, which, at the same time, destroys the orthogonality and introduces unavoidable ISI. Although the additional interference increases the complexity to recovery original signals in the receiver, the Mazo limit [1] proves that without the expansion of bandwidth and loss of BER performance, the FTN signaling can achieve an up to 25% higher transmission rate than conventional Nyquist-criterion design in additive white Gaussian noise (AWGN) channel.

To eliminate the ISI caused by the smaller symbol interval, a lot of detection algorithms for FTN signaling have been reported, which can be roughly classified as time-domain [2]–[5] and frequency-domain [6]–[8] methods. Among time-domain approaches, [2] and its simplified version [3] formulate the FTN signal as convolutionally encoded symbols and applies the Viterbi algorithm for detection. [4] employs symbol-by-symbol signal detection, which achieves the near-optimal performance with very low complexity. More recently, [5] proposes a reduced-complexity equalization based on Ungerboeck observation model. Meanwhile, a few effective frequency-domain detection algorithms for FTN can also be found in the literature. [6] designs a minimum mean square error (MMSE) frequency-domain equalization (FDE) for FTN detection. An H-ARQ technique for FTN signaling is proposed in [7]. [8] focuses on iterative FDE architectures to joint estimate the channel and eliminate the introduced ISI.

Signal detection for FTN based on sequence estimation can be regarded as a classification problem which aims to divide a multiple dimension space into several parts. For example, when we try to recover $M$ transmitted symbols from $M$ received symbols in BPSK modulation, we practically divide an $M$-dimension space into $2^M$ parts. Motivated by the power of DL in solving such classification problems, which has been proved by its successful application in image and voice recognition, we propose a DL-based detection for FTN signaling.
Although various DL-based algorithms have been proposed until now to improve the performance of conventional communication technologies (e.g. channel estimation and signal detection in orthogonal frequency-division multiplexing (OFDM) systems [9], channel state information (CSI) sensing and recovery [10], channel coding [11], modulation classification [12], etc.), DL-based detection for FTN signaling in QAM modulations, as far as we know, has not been studied yet in the literature.

The contribution of this letter can be summarized as follows.

- We propose a DL-based detection for FTN signaling, which, to the best of our knowledge, is the first attempt to apply a six-layer DNN for FTN signal detection. The proposed detection, as proved by the near-optimal BER performance, can intelligently and effectively recover transmitted symbols interfered by both ISI and colored noise.

- We investigate the robustness of the proposed DL-based detection to SNR. And results show that after training by the FTN data set under a specific SNR value, the DL-based detection can fit the scenarios with different SNRs and achieve a near-optimal performance in the offline recovery.

- We carry out comprehensive evaluations, especially with small $\beta$ and $\tau$ values, to verify and analyze the proposed DL-based detection.

Herein, we give the definition of notations which we will encounter throughout the rest of the letter. Bold-face lower case letters (e.g. $x$) are applied to denote column vectors. Light-face italic letters (e.g. $x$) denote scalers. $x_i$ is the $i$th element of vector $x$. And $x(t) \ast y(t)$ denotes the convolution operation between $x(t)$ and $y(t)$. $\lfloor x \rfloor$ is the maximum integer less than or equal to $x$.

II. System Model

We consider the communication system with the $M$-point complex-valued quadrature amplitude modulation (QAM) scheme and AWGN channel. As known, after constellation mapping, the baseband signal should pass through a shaping filter $h(t)$. Hence, the transmitted signal $s(t)$ can be written as
\[ s(t) = \sqrt{E_s} \sum_{k=-\infty}^{+\infty} x_k h(t - k\tau T_N), \quad (1) \]

where \( E_s \) is the average energy of constellation symbols, \( x_k \) (\( k = 0, \pm 1, \pm 2, \cdots \)) is the \( k \)th transmitted symbol and \( \tau \) is the time acceleration factor which satisfies \( 0 < \tau \leq 1 \). Due to the existence of \( \tau \), practical symbol interval \( T \) is smaller than Nyquist limit \( T_N \), which helps the system achieve a higher transmission rate.

Actually, when \( \tau = 1 \), benefiting from the orthogonality between any two symbols, each sample of the symbols will not be influenced by the others. However, when \( 0 < \tau < 1 \), each sample becomes a weighted sum of different symbols, which will make it difficult for the receiver to recover the transmitted signal.

Fig. 1 shows the block diagram of a communication system with FTN signaling, where \( \tau T_N \) is not only the practical symbol interval of the symbols passing through the shaping filter but also the sampling interval for signal passed through the matched filter. The two DL-based detection components for real and imaginary parts of the received symbols share the same structure and parameters. The received signal after passing through the matched filter can be written as
\[ y(t) = (s(t) + n(t)) \ast h(t) \]
\[ = \sqrt{E_s} \sum_{k=\infty}^{\infty} x_k g(t - k\tau T_N) + \tilde{n}(t), \tag{2} \]

where \( g(t) = \int h(x) h(t - x) dx \), \( \tilde{n}(t) = \int n(x) h(t - x) dx \), and \( n(t) \) is a zero mean complex-valued Gaussian random process with variance \( \sigma^2 \). Throughout this letter, time synchronization error is not taken into consideration. Hence, the \( k \)th sample of received signal \( y(t) \) can be obtained as

\[
y_k = \sqrt{E_s} \sum_{n=-\infty}^{n=\infty} x_n g(k\tau T_N - n\tau T_N) + \tilde{n}(k\tau T_N) \\
= \sqrt{E_s} \sum_{n=-\infty}^{k-1} x_n g((k - n) \tau T_N) + \sqrt{E_s} x_k g(0) \\
+ \sqrt{E_s} \sum_{n=k+1}^{+\infty} x_n g((k - n) \tau T_N) + \tilde{n}(k\tau T_N). \tag{3} \]

As seen, each sample of the received waveform contains not only the expected symbol but also the weighted sum of both its previous and upcoming symbols. A key problem for receivers is eliminating the ISI from both directions and recover the transmitted sequence \( x \) from the received symbols \( y \). It will certainly increase the complexity of signal detection, which can be regarded as the price of higher transmission rate.

Actually, the degree of ISI mainly depends on \( \beta \) and \( \tau \). \( \beta \) is the roll-off factor of the square root raised cosine (SRRC) filter, which affects the attenuation speed of the shaping function \( h(t) \). A smaller \( \beta \) means quicker attenuation of \( h(t) \) and stronger ISI. \( \tau \) directly determines the transmission rate. With \( \tau \) increasing, ISI can be alleviated.
III. DL-BASED FTN SIGNAL DETECTION

A. Architecture

The proposed DL-based detection is essentially a deep neural network (DNN) which includes six layers (an input layer, an output layer, and four hidden layers). As shown in Fig. 2, each hidden layer is designed to be composed of a fully connected (FC) sublayer, a batch normalization (BN) and a rectifier linear unit (ReLU) function while the output layer is simply an FC sublayer. FC sublayer can be regarded as a vector of independent neurons. Each neuron has its own parameters $w$ and $b$ which denote weight vector and bias respectively [13]. With the backpropagation (BP) algorithm, $w$ and $b$ of each neuron can be updated after a complete forward and backward propagation process.

B. Dataset

The data sets we used for training and testing share the same structure. The input data set is composed of the received symbols which have passed through the matched filter. And the label set contains the corresponding transmitted symbols. These two original symbol sequences are sliced by the sliding window parameters $L$ and $m$ which will be introduced in the following part.

C. Workflow

A notable feature of our proposed DL-based detection is the sliding window, as shown in Fig. 3. The input in each recovery is an $L$-length sliding window on received symbols $y$. During each recovery, the DL-based detection tries to recover the middle $m$ symbols of the input window. Then, the input...
window slides forward over \( m \) symbols to start the next recovery. This special architecture results from the uncertainty of edge symbols of the input window. They always suffer from severe ISI from either the previous or subsequent symbols while these symbols are unknown since they are not contained in the input.

![Diagram showing workflow of the proposed DL-based detection](image)

**Fig. 3.** Workflow of the proposed DL-based detection (when \( L = 16 \) and \( m = 4 \)).

Similar to most DL methods, the workflow of our proposed detection includes two stages named *offline training* and *online recovery.*

- **Offline training:** During this stage, the module is trained by received symbols that are generated with the given \( \beta \) and \( \tau \). The output estimated symbols are used for calculation of loss function and its derivative to each neural.

- **Online recovery:** During this stage, the DL-based detection can output the estimated symbols by both the received symbols and the well-trained DNN parameters without any backward calculation.

**IV. Simulation Results**

In this section, we assess the performance and robustness of our proposed DL-based FTN detection architecture. QAM modulation and SRRC filters with different roll-off factors are taken into consideration. A more detailed list of the parameters is provided in Table I.
| Item                        | Value                                                                 |
|-----------------------------|------------------------------------------------------------------------|
| Training Data Size          | $1.6 \times 10^9$ symbols                                             |
| Training $E_b/N_0$          | $E_b/N_0$@$\{BER=2 \times 10^{-4}\}$ without ISI in AWGN channel (e.g. 11.7dB for 16QAM) |
| Learning rate               | $0.001/5\lfloor\max\{k-120,0\}\rceil$ for the $k$-th $3.2 \times 10^6$ symbols |
| Loss function               | Mean square error (MSE)                                               |
| Optimizer                   | Adam                                                                  |
| Testing Data Size           | $3.2 \times 10^7$ for each $E_b/N_0$                                  |

A. Robustness to SNR Mismatching

It is very important for the proposed DL-based detection to be robust to the SNR without which the proposed DL-based detection will be trained and employed for different SNR independently and occupies large complexity resulting from SNR estimation and a great number of DL network parameters stored for different SNR values.

Throughout a lot of simulations, fortunately, we find an interesting point that the DL network trained with the $SNR@\{BER = 2 \times 10^{-4}\}$ always gives the near-optimal performance in the offline prediction stage. $SNR@\{BER = x\}$ here means the SNR under which the ideal QAM modulation with Nyquist-criteria can achieve a BER performance of $x$ in AWGN channel (e.g. 7.9dB in QPSK).

B. Performance with Different $\beta$ Values

As mentioned before, the ISI caused by FTN becomes severer with the smaller $\beta$. Here, QPSK with $\tau = 0.8$ and $\tau = 0.7$ is taken into consideration. And Fig. 4 illustrates the BER curves of our proposed DL-based detection with different $\beta$ values. From the figure, our proposed DL-based detection can achieve a near-optimal performance with even $\beta = 0.2$ or smaller.
C. Performance with Different $\tau$ Values

$\tau$ is an important metric for the acceleration in FTN signaling. For example, when $\tau = 0.6$, the FTN signaling can achieve a 67% higher rate than traditional Nyquist-criterion transmission. Fig. 5 illustrates the BER of our proposed DL-based detection with different $\tau$ values, where $\beta = 0.5$ and $\beta = 0.3$ are taken into consideration.

From the figure, our proposed DL-based detection can achieve an acceptable performance with different
\(\tau\) values. This result shows again that the feature of the ISI and colored noise introduced by FTN can be well learned by our proposed DL-based detection through the training stage.

Fig. 6. BER performance of our proposed DL-based detection versus some traditional algorithms.

Fig. 7. BER performance of the proposed DL-based detection in high order modulations with \(\tau = 0.8\) and \(\beta = 0.5\).
D. Performance versus Traditional Algorithms

To better evaluate the performance of our proposed DL-based detection, we choose two traditional algorithms for FTN detection MMSE FDE [6] and SSSgbKSE [4] as the baseline. The two algorithms are representative methods in time and frequency domain respectively with near-optimal performance and low complexity.

Fig.6 compares the BER performance of our proposed DL-based detection with the baselines. Results show that our DL-based detection can achieve performances which are the same as or beyond 2 baseline methods, especially in relatively severe ISI scenarios, confirming that the DL-based detection is applicable to FTN signal detection problems.

E. Performance in High Order Modulations

Fig. 7 illustrates the BER performance of the proposed DL-based detection in high order modulations with QPSK and $\tau = 0.8$. The result reveals the potential of DL-based detection for FTN signaling with high order modulations. In addition, the more practical scenarios (e.g. high order modulations with smaller $\beta$ and $\tau$ values) will be taken into consideration in our future works.

V. CONCLUSIONS

In this letter, we propose a DL-based signal detection for FTN signaling, which has shown a near-optimal performance. As far as we know, this is the first try to apply a six-layer DNN into FTN detection and has proved the feasibility of it. For the future work, we are going to consider the more general case where both the high order modulations and small $\beta$ values are taken into consideration. Another direction is to study new DL-based architecture with higher performance and/or lower complexity.

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