Ablation driven by hot electrons in shock ignition

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Abstract. An analytical model for the ablation driven by hot electrons is developed. The hot electrons are assumed to carry on the totality of the absorbed laser energy. Efficient energy coupling requires to keep the critical surface sufficiently close to the ablation front. To achieve this goal for high laser intensities a short enough laser wavelength is required. Scaling laws for the ablation pressure and the other relevant magnitudes of the ablation cloud are found in terms of the laser and target parameters.

1. Introduction

The shock ignition (SI) approach to inertial confinement fusion (ICF) critically depends on the possibility to generate a very strong shock launched at the stagnation phase of the implosion by a pressure of the order of 1 Gbar [1, 2]. In directly driven laser fusion the generation of this shock needs to rise the laser intensity \( I \) up to 10 PW/cm\(^2\) or more, for a laser wave length \( \lambda = 0.35 \mu m \). At such intensities the laser energy is absorbed in the subcritical region by collisionless processes that produce copious amounts of hot electrons (HE) [3], and it has been argued that they could be beneficial for the generation of the ignitor shock [1, 2]. The main difficulty to address this problem is that a detailed treatment of the HE generation requires to deal with kinetic mechanisms that are difficult to couple with the hydro-codes used for describing the target implosion and the later propagation of the ignitor shock. Therefore, it is necessary to resort to approximations in order to get insight into the role of the HE in SI [2, 4]. In this regard, analytical models can be a valuable complement to the other approaches to the problem.

Here, we present a model based in the evidence available at present from experiments and numerical simulations that show that the effects of the collisionless absorption depend on the irradiance \( I\lambda^2 \) [5]. Therefore, much of the relevant physics overlaps with early work performed with nanosecond Nd and CO\(_2\) lasers of wavelengths \( \lambda = 1.06 \mu m \) and \( \lambda = 10.6 \mu m \), respectively. That work is in agreement with recent PIC simulation results with \( \lambda = 0.35 \mu m \) [3]. Then, consistently with the available evidence we construct our model assuming that the absorbed laser energy is totally transferred to a Maxwellian population of HE with mean temperature \( \theta_H \) and particle density \( n_H \) that are produced in the neighborhood of the surface with mass density...
\( \rho_c \) which we will taken as the critical density. Thus, the HE energy flux \( q_H \) can be written:

\[
q_H = \frac{1}{2} \Phi_H \theta_H; \quad \Phi_H = n_H v_H; \quad v_H = \left( \frac{8 \theta_H}{\pi m_e} \right)^{1/2}.
\] (1)

As the electrons penetrate into the supercritical plasma the energy flux becomes attenuated by energy loss \((d\Phi_H/dr)\) and by backscattering \((d\Phi_H/dr)\). Since both processes have approximately the same mean free path \( \lambda_H \), we have:

\[
\frac{dq_H}{dr} = \frac{2q_H}{\lambda_H}; \quad \frac{q_H}{\theta_H^2} \approx \text{const}; \quad \lambda_H \approx \frac{Am_p}{Z} \frac{\theta_H^3}{3 \pi \rho c \ln \Lambda_H} \equiv \frac{\alpha \theta_H^3}{\rho},
\] (2)

where \( \rho \) is the mass density, \( r \) represents the radial coordinate in spherical geometry and \( \ln \Lambda_H \approx 10 \) is the Coulombian logarithm of the HE.

Then, for studying the ablation driven by HE we will introduce the previous energy deposition equation into the hydrodynamic equation for the energy conservation.

2. Ablation model and scaling laws

In spherical geometry we can consider quasi-stationary conditions provided that the transit time of a fluid element for traveling from the ablation surface up to the critical surface is shorter than the characteristic time of variation of the ignitor pulse. Then, the conservation equations for mass, momentum and energy are:

\[
\dot{m} = 4 \pi \rho v r^2 = \text{const}; \quad \rho v \frac{dv}{dr} + \frac{dp}{dr} = 0; \quad \frac{\dot{m}}{4 \pi r^2} \frac{d}{dr} \left( \gamma \epsilon + \frac{v^2}{2} \right) = \frac{dq_H}{dr},
\] (3)

where \( v \) is the fluid velocity, \( \epsilon = (Z+1)kT/Am_p \) is the specific internal energy \((k\) is the Boltzmann constant, \( Z \) and \( A \) are the charge and mass numbers, respectively, \( m_p \) is the proton mass, \( T \) is the electrons and ions temperature), \( p = (\gamma - 1)\rho \epsilon \) is the pressure, and \( \gamma = 5/3 \) is the enthalpy coefficient of the ablated plasma.

Eqs.(2) and (3) can be combined to get the following system [6]:

\[
\frac{dv}{dr} = \frac{2\gamma(\gamma - 1)\epsilon v}{\gamma(\gamma - 1) - \epsilon - v^2} \left( \frac{q_H}{1 - \alpha \theta_H^2} \frac{1 - \frac{1}{r}}{\gamma \epsilon v} \right); \quad \frac{de}{dr} = \frac{2q_H}{\alpha \theta_H^2 \gamma v} - \frac{v dv}{\gamma dr}; \quad \frac{dq_H}{dr} = \frac{2q_{Hc}}{\alpha \theta_{Hc}^2 \rho}.
\] (4)

Eqs.(4) must be solved with the boundary conditions on the ablation surface at \( r = r_0 \): \( q_H(r_0) = \theta_H(r_0) = v(r_0) = \epsilon(r_0) = 0 \), and on the absorption surface at \( r = r_c \): \( \rho(r_c) = \rho_c \), \( \theta_H(r_c) = \theta_{Hc} \), and \( q_H(r_c) = q_{Hc} = \beta I \), where \( \beta \) is the fraction of absorbed laser energy. Since we consider a free expansion, we have to look for the unique transonic solution of the previous equations. Such a solution can be easily obtained by numerical calculation, but for the purpose of finding relationships with the character of scaling laws for the ablation driven by HE it is more convenient to obtain an approximated analytical solution. To that end we first find the asymptotic solution of Eqs.(4) \( (r >> r_0) \) and then, we extend it up to the target surface:

\[
v \approx \sqrt[3]{\frac{30(\gamma - 1) q_H r_0}{7\gamma - 5}} \left( \xi - 1 \right)^{1/3}; \quad \epsilon \approx \left( \frac{1}{M^2(\gamma - 1)} \right) \frac{30(\gamma - 1) q_H r_0}{7\gamma - 5} \left( \xi - 1 \right)^{2/3}.
\] (5)

\[
\rho \approx \frac{\dot{m}}{4 \pi r_0^2} \left[ \frac{7\gamma - 5}{30(\gamma - 1)} \frac{\alpha \theta_H^2}{q_H} \right]^{1/3} \frac{1}{\xi^2(\xi - 1)^{1/3}}; \quad p \approx \frac{\dot{m}}{4 \pi M^2 r_0^2} \left[ \frac{30(\gamma - 1) q_H r_0}{7\gamma - 5} \right]^{1/3} \frac{1}{\xi^2}.
\] (6)
I regime the critical surface must remain attached to the sonic surface so that below the above
\[ q_H \approx \frac{m}{4\pi r_0^2} \left( \frac{2(7\gamma - 5)}{30(\gamma - 1)} \right)^{2/3} \left( \frac{q_H}{\alpha \theta_H^2} \right)^{1/3} F(\xi), \]
\[ F(\xi) = \left( \frac{\xi - 1}{\xi} \right)^{2/3} + \frac{1}{\sqrt{3}} \xi^{-1} \left[ \frac{2(\xi - 1)^{1/3} - 1}{\sqrt{3}} \right] + \frac{1}{6} \log \left( \frac{1}{(\xi - 1)^{1/3} + 1} \right)^{2/3} \] + \frac{\pi}{6\sqrt{3}}. \]
where \( \xi = r/r_0 \). These equations are not applicable for \( \xi < 1.06 \) but we will use them for radii larger than the isothermal sonic radius \( r_s = 1.2r_0 \ (\xi \geq 1.2) \).

Then, from Eqs.(5) to (8) we can find the scaling laws for the magnitudes at \( \xi_s = 1.2 \) in terms of the laser and target parameters, and of the HE temperature \( \theta_{Hc} \) at the critical surface:

\[ p_s \approx \frac{0.594}{F(\xi_c)} \frac{\alpha \theta_{Hc}^2}{r_0}; \quad v_s \approx 0.843 \left( \frac{r_0}{\lambda_{Hc}} \right)^{1/3} \left( \frac{q_{Hc}}{\rho_c} \right)^{1/3}; \quad \epsilon_s \approx 1.07 \left( \frac{r_0}{\lambda_{Hc}} \right)^{2/3} \left( \frac{q_{Hc}}{\rho_c} \right)^{2/3}, \]
\[ p_s \approx \frac{0.424}{F(\xi_c)} \left( \frac{\lambda_{Hc}}{r_0} \right)^{1/3} \left( \frac{\lambda_{Hc}}{r_0} \right)^{2/3} q_{Hc}; \quad \lambda_{Hc} \approx 2F(\xi_c)\xi_c^2(\xi_c - 1)^{1/3}, \]
where \( \lambda_{Hc} = \frac{\alpha \theta_{Hc}^2}{\rho_c} \) is the HE mean free path at the critical surface, \( \rho_c = 1.878 \times 10^{-3}(A/Z)/\lambda^2 \text{ g/cm}^3 \) (with \( \lambda \) in \( \mu \text{m} \)). \( \xi_c \geq \xi_s = 1.2 \), \( q_{Hc} = \beta I \). In spherical geometry the laser intensity \( I \) at the critical surface depends on the critical radius \( r_s \) so that \( I = I_0/\xi_s^2 \), where \( I_0 = W_0/4\pi r_0^2 \) is the nominal intensity (on the target surface) and \( W_0 \) is the laser power.

\subsection*{2.1. High intensity regime (\( r_c > r_s \))}

The second of the Eqs.(10) gives, in an implicit form, the critical distance \( \xi_c \) in terms of the ratio \( \lambda_{Hc}/r_0 \) and it shows that for \( \xi_c \geq \xi_s = 1.2 \) it is \( \lambda_{Hc}/r_0 \geq 0.75 \). For high laser irradiances \( 1\lambda^2 \geq 10^{15} \text{ W\textmu m}^2/\text{cm}^2 \), we can relate the HE temperature at the critical surface with the laser intensity by using the well known empirical law [7]: \( \theta_{Hc} = 7.5 \times 10^{-2}(I\lambda^2)^{1/3} \text{ eV} \), with \( I \) in \( \text{W/cm}^2 \), and \( \lambda \) in \( \mu \text{m} \). Then, we can get the HE mean free path at the critical surface and, in particular, the scaling law for the ablation pressure:

\[ \frac{\lambda_{Hc}}{r_0} \approx 2.56 \times 10^{-8} \left( \frac{I^{2/3}}{c} \right)^{10/3} \frac{\lambda^{10/3}}{r_0}; \quad p_s \approx 1.28 \times 10^{-7} \left( \frac{\beta^2 Z}{A} \right)^{1/3} \left( \frac{p_0^{8/15}}{\gamma_{0}^{8/15}} \right)^{1/5} \text{Mbar}, \]
where \( I \) is in \( \text{W/cm}^2 \), and \( \lambda \) and \( r_0 \) are in \( \mu \text{m} \).

As we can see from Eq.(11) the scaling of the ablation pressure with the laser intensity \( I_0 \) is weaker than the well known scaling \( p_s \sim I_0^{2/3} \) found at low intensities. From Eq.(10) one can see that this behavior arises from the combination of two opposite effects. From one side long mean free path of the HE increases the ablation pressure but, on the other side, the effective intensity at the critical surface is reduced as the critical radius \( r_c \) increases with the HE mean free path. The second effect dominates in this high intensity regime and the weaker scaling \( p_s \sim I_0^{8/15} \) results.

\subsection*{2.2. Low intensity regime (\( r_c = r_s \))}

For the lower intensities the condition \( \lambda_{Hc}/r_0 \geq 0.75 \) breaks and Eqs.(9) to (11) are not valid any longer. This limit is achieved when the laser intensity becomes \( I \leq 1.6 \times 10^{11} \left( \frac{c}{\lambda} \right)^{3/2} \text{W} \) ( \( r_0 \) and \( \lambda \) in \( \mu \text{m} \), and \( I \) in \( \text{W/cm}^2 \)). Since beyond the absorption region density and temperature must be decreasing functions of \( r \), the critical surface cannot enters inside the surface with the isothermal sonic radius and it must always be \( r_c \geq r_s \) [8]. Therefore, in the lower intensity regime the critical surface must remain attached to the sonic surface so that below the above
mentioned limit it is \( r_c = r_s \leq 1.2 r_0 \). Then, \( \rho_s = \rho_c \) and from the first of Eqs.(6) we get the critical distance in this lower intensity regime:

\[
\xi_c - 1 \approx 0.27 \frac{\epsilon \theta H_c}{\rho_c r_0} \quad \text{or} \quad r_c - r_0 \approx 0.27 \lambda H_c \ .
\]

(12)

In a similar manner, from Eqs.(5) and (6) we re-obtain the scaling laws as follows:

\[
\epsilon_s = \frac{\nu_s^2}{\gamma - 1} \approx 1.3 \left( \frac{q H_c}{\rho_c} \right)^{2/3} \quad ; \quad p_s \approx 0.87 \left( \frac{1}{\epsilon_c} \right)^{2/3} \left( \frac{1}{\xi_c} \right)^{2/3} \ .
\]

(13)

These are the very well known scalings obtained in planar geometry ablation driven by thermal conduction. In fact, when \( r_c = r_s \) they become independent of the particular mechanism driving the ablation, and such a mechanism determines only the position of the critical surface with respect to the ablation front. Therefore, we have:

\[
p_s \approx 5 \times 10^{-9} \frac{A \beta^2}{Z} \left( \frac{I_0}{\lambda} \right)^{2/3} Mbar \ ; \ \left( \xi_c - 1 \right) \xi_c^{4/3} \approx \frac{6.9 \times 10^{-9}}{r_0} \frac{I_0^{2/3} \lambda^{10/3}}{r_0} ,
\]

(14)

\((I_0) \) in \( W/cm^2 \) and \( \lambda \) in \( \mu m \) where for obtaining the equation for \( \xi_c \) we have used again the empirical law \( \theta H_c = 7.5 \times 10^{-2} (I/\lambda^2)^{1/3} eV \) (for \( I \lambda^2 \geq 10^{15} \ W/\mu m^2/cm^2 \)). As an example, we calculate the ablation pressure \( p_s \) for the maximum intensity for which Eq.(14) can be applied \( (\xi_c = \xi_s = 1.2, \lambda H_c = 0.75 r_0) \), for \( \lambda = 0.351 \ \mu m \) \( (I_0 \approx 4.3 \times 10^{16} \ W/cm^2) \), \( A/Z = 2.5 \), and assuming \( \beta = 0.5 \). In such a case, we get \( p_s \approx 830 \ Mbar \). Therefore, it seems that a suitable ignitor shock could be generated by the ablation driven by HE within the regime of lower laser intensities for which we have the best laser/target coupling.

Finally, it is possible to verify the validity of the main model assumptions: a) We can see that the electric field required to maintain the return current is negligible. Thus, it cannot inhibit the inward hot electrons flux. b) The radiation pressure is also negligible in comparison with thermal pressure. Nevertheless, density jumps in the absorption region may still be present. It would place the laser absorption region near the critical surface. c) Quasi-steady state is reached in times \(( \sim 10 \ ps) \) much shorter than the foreseen spike duration \(( \geq 100 \ ps) \). d) The plasma preformed by the compression laser pulse can be neglected except when its characteristic scale-length \( l_T c \) is larger than the supra-thermal critical distance \( r_c - r_0 \sim \lambda H_c \). The matching condition \( l_T c \leq \lambda H_c \) imposes a relationship between the compression pulse intensity \( I_{cp} \) and the spike intensity \( I \): \( I \geq 2.2 \times 10^{-11} (\beta_p l_{cp} \lambda)^2 \), in the usual units. In the opposite case, \( I \) at the critical surface will be too small, \( \theta H_c \) too low, and the supra-thermal ablation cloud may not be assembled.

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