Optical force acting on strongly driven atoms in free space or modified reservoirs

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Received 29 November 2012, in final form 14 January 2013
Published 8 February 2013
Online at stacks.iop.org/JPhysB/46/045502

Abstract
We investigate the quantum dynamics of a moderately driven two-level particle in free space or modified electromagnetic field reservoir. Particularly, we calculate the optical force acting on the radiator in such an environment. We found that the modified environmental reservoir influences significantly the optical force. Very intense driving in free space also modifies the maximal force.

1. Introduction

The force acting on a two-level atom in a resonance light field can be estimated as follows: in the field of a strong running wave, the atom absorbs a photon from the light beam and acquires the momentum $\hbar k$ of the photon. Respectively, a force $\hbar k\gamma$ acts on the atom, where $2\gamma$ is the spontaneous-decay rate of the upper level [1–3]. This force can be even stronger in a field of a standing wave. If the atom is accelerated in such a field only to a distance of half the wavelength, it acquires an energy greater than in the thermal case. The acceleration effect can be substantially enhanced if the frequency of one of the opposing waves varies with time. Acceleration of neutral particles was achieved in [4]. Furthermore, the scattering rate from a coherent stimulated process can be made arbitrarily large by detuning the optical field far from resonance and increasing the intensity. When detuned from resonance, very large accelerations in the $10^{11}$ g range have been realized and this process has been termed optical Stark deceleration or acceleration [5]. Thus, the mechanical effect of a resonant or non-resonant field on atoms can be considerable. In another context, laser acceleration of charged particles up to GeV energies was obtained as well [6, 7]. Previous force studies include dielectrics [8, 9] and plasmas [10, 11].

Another important related issue is the cooling and trapping in laser fields [12]. The experimental feasibility of Bose–Einstein condensation is already history [13]. Laser-cooled atoms are used, for example, as frequency standards [14], in quantum information processing [15] or in atomic clocks [16]. Therefore, it is not surprising that the optical force received a lot of attention. In particular, cooling of atoms with stimulated emission was observed in [17]. Laser cooling of atoms in squeezed vacuum was investigated in [18, 19], respectively. An overview on cold atoms and quantum control was given in [20] while laser cooling of atoms, ions or molecules by coherent scattering was studied in [21]. Adiabatic cooling of atoms by an intense standing wave was experimentally achieved in [22]. Further, in [23] trapping and cooling of atoms in a vacuum perturbed in a frequency-dependent manner was investigated, while three-dimensional cavity Doppler cooling and cavity sideband cooling via coherent scattering was studied in [24]. Light-pressure cooling of crystals was analysed as well [25]. Stopping atoms with laser light was achieved in [26] while collective-emission-induced cooling of atoms in an optical cavity was observed in [27]. In the radiation field of an optical waveguide, the Rayleigh scattering of photons was shown to result in a strongly velocity-dependent force on atoms [28]. Cavity-induced atom cooling in the strong coupling regime was discussed in [29]. Furthermore, a large velocity capture range and low temperatures with cavities was obtained in [30]. Finally, these techniques were exported to other systems such as mesoscopic systems. For instance, the resolved sideband laser cooling was used to cool a mesoscopic mechanical resonator to near its quantum ground state [31].

In this paper we investigate the optical force acting on two-level atoms in moderately intense running laser fields in free space or modified electromagnetic field (EMF) reservoirs like low-quality optical cavities or photonic crystals. We show that modified reservoirs lead to a significant enhancement of optical forces. In contrast, very intense driving in free space contributes to a maximal force, which is slightly smaller than for moderate pumping. This tendency occurs in the presence of
counter-rotating terms too. This is somehow surprising as one may expect the force to be larger for bigger intensities. Note also that modified environmental reservoirs are responsible for the recovery of the interference pattern [32], enhanced squeezing [33], population inversion [34] or thresholdless lasing [35].

The paper is organized as follows. In section 2 we describe the system of interest and obtain an expression for the optical force acting on strongly driven two-level atoms in free space or modified reservoirs. Section 3 deals with discussions of the obtained results. The summary is given in section 4.

2. Quantum dynamics in moderately strong laser fields and modified EMF reservoirs

We proceed by briefly introducing the main steps of the analytical formalism involved and then rigorously describing the obtained results. The Hamiltonian characterizing the interaction of a two-level particle possessing the frequency \( \omega_0 \) with a coherent source of frequency \( \omega_L \), in a frame rotating at \( \omega_L \) is [36]

\[
H = H_0 + H_L + H_F,
\]

where

\[
\begin{align*}
H_0 &= \sum_k \hbar (\omega_k - \omega_L) a_k^\dagger a_k + \hbar (\omega_0 - \omega_L) S_z, \\
H_L &= \hbar \Omega(S^+ e^{i \vec{k} \cdot \vec{r}} + S^- e^{-i \vec{k} \cdot \vec{r}}), \\
H_F &= i \sum_k (\vec{g}_k \cdot \vec{d})(a_k^\dagger S^- e^{-i \vec{k} \cdot \vec{r}} - a_k S^+ e^{i \vec{k} \cdot \vec{r}}).
\end{align*}
\]

Here \( H_0 \) describes the free Hamiltonians of the EMF and the atomic subsystems, respectively. The interaction between the laser of field strength \( E_0 \), i.e. Rabi frequency \( \Omega = (d \cdot E_0)/(2\hbar) \), and wave vector \( \vec{k}_L \), and the two-level radiator is given by \( H_L \). \( H_F \) characterizes the interaction of the free EMF reservoir and in free space \( \vec{g}_k = \sqrt{2 \pi l m_{0k}/V \vec{e}_\lambda} \) with \( \vec{e}_\lambda \) being the photon polarization vector with \( \lambda = 1, 2 \) and \( V \) is the EMF quantization volume. Further, \( S^+ = \ket{2}\bra{1} + \ket{1}\bra{2} \) describes the excitation (deexcitation) of the two-level particle at position \( \vec{r} \) and obeys the commutation relations for \( \text{su}(2) \) algebra: \( \left[ S^+, S^- \right] = 2S_z \), and \( \left[ S_z, S^\pm \right] = \pm S^\pm \). Here \( S_z = (\ket{2}\bra{2} - \ket{1}\bra{1})/2 \) is the bare-state inversion operator. \( a_k^\dagger \) and \( a_k \) are the creation and annihilation operators of the EMF, respectively, and satisfy the standard bosonic commutation relations, i.e. \( [a_k, a_{k'}] = \delta_{kk'} \), and \( [a_k, a_{k'}^\dagger] = [a_k^\dagger, a_{k'}] = 0 \).

The optical force acting on two-level particles in a travelling laser wave is given by the following expression:

\[
\vec{F} = -\nabla H_L(\vec{r}) = -i\vec{k}_L \hbar \Omega (S^+ - S^-),
\]

where we considered that the particle is located at the origin, i.e. \( e^{i \vec{k} \cdot \vec{r}} \to 1 \).

In what follows we shall describe our system using the dressed-state formalism (see figure 1):

\[
\begin{align*}
\ket{1} &= \cos \theta \ket{\bar{1}} + \sin \theta \ket{\bar{2}}, \\
\ket{2} &= \cos \theta \ket{\bar{2}} - \sin \theta \ket{\bar{1}},
\end{align*}
\]

with \( \cot 2\theta = (\Delta/2)/\Omega \) and \( \Delta = \omega_0 - \omega_L \). In this picture the force is given by the relation:

\[
\vec{F} = -i\vec{k}_L \hbar \Omega (R^+ - R^-).
\]

Figure 1. The schematic shows the involved energy levels of a two-level atom. Here \( \Omega \) is the Rabi frequency while \( \gamma \) is the bare state spontaneous decay rate. The dressed-state decay rates \( \gamma_{\pm} \) are different for a very intense laser field in free space or for a modified environmental electromagnetic field reservoir.

Here \( R^+ = \ket{\bar{2}}\bra{\bar{1}} \) and \( R^- = \ket{\bar{1}}\bra{\bar{2}} \) are new quasiprinciples operators operating in the dressed-state picture. They obey the same commutation relations as the old ones. To obtain the explicit expression for the force we need the equations of motion for the new dressed-state operators. Therefore, we write the Hamiltonian in the dressed-state representation:

\[
H = \sum_k \hbar (\omega_k - \omega_L) a_k^\dagger a_k + \hbar \Omega R_c
\]

\[
+i \sum_k (\vec{g}_k \cdot \vec{d})\{[\sin 2\theta R_2, 2 + \cos^2 \theta R^- - \sin^2 \theta R^+] a_k^\dagger
\]

\[- \text{H.c.}\}.
\]

Here \( R_c = \ket{\bar{2}}\bra{\bar{2}} - \ket{\bar{1}}\bra{\bar{1}} \) is the dressed-state inversion operator while \( \Omega = \sqrt{\Omega^2 + (\Delta/2)^2} \) is the generalized Rabi frequency. The Heisenberg equation for an arbitrary dressed-state atomic operator \( \hat{Q} \) is

\[
\frac{d}{dt} \hat{Q}(t) = \frac{i}{\hbar} [\hat{H}, \hat{Q}(t)].
\]

Here the notation \( \{ \cdots \} \) indicates averaging over the initial state of both the atoms and the EMF environmental reservoir.

Introducing the Hamiltonian (5) in equation (6) one arrives at

\[
\frac{d}{dt} a_k^\dagger(t) = i\Delta_k a_k^\dagger + \frac{(\vec{g}_k \cdot \vec{d})}{\hbar} \{[\sin 2\theta R_2(t)/2 + \cos^2 \theta R^- - \sin^2 \theta R^+] a_k^\dagger
\]

\[+ \text{H.c.}\}.
\]

where, in general, for the non-Hermitian atomic operators \( Q \), the H.c. terms should be evaluated without conjugating \( Q \), i.e. by replacing \( Q^+ \) with \( Q \) in the Hermitian conjugate parts. Assuming that the atomic subsystem couples weakly to the surrounding EMF, i.e. in the bad-cavity limit, the EMF operators can be eliminated from the above equation of motion, equation (7). Therefore, we represent the Heisenberg equation for the field operators:

\[
\frac{d}{dt} a_k(t) = i\Delta_k a_k + \frac{(\vec{g}_k \cdot \vec{d})}{\hbar} \{[\sin 2\theta R_2(t)/2 + \cos^2 \theta R^+(t)
\]

\[+ \text{H.c.}\}.
\]

where \( \Delta_k = \omega_k - \omega_L \) and \( a_k = [a_k^\dagger]^\dagger \).
On integrating formally the above Heisenberg equation for the EMF field operators one obtains

\[ a_k(t) = a_k(0)e^{i\delta t} + \frac{(\gamma \bar{g}_k - \bar{d})}{\hbar} \int_0^t dt'[\sin 2\theta R_c(t')/2 + \cos^2 \theta R^+(t') - \sin^2 \theta R^-(t')]. \]  

(9)

Now, ignoring the memory effects in the Born–Markov approximations [36] one can show that \( R_i(t) \approx R_i(t') \) and \( R^+(t') \approx R^+(t) e^{2i\Omega(t-t')} \). Substituting these expressions in equation (9) and using the relation \( \lim_{t \to \infty} \int_0^t e^{i\omega t} dt = \pi \delta(x) + i\pi \bar{P}_{\pm} \) (here \( P_c \) denotes the Cauchy principal part) one arrives at

\[ a_k(t) = a_k(0)e^{i\delta t} + \pi \frac{(\gamma \bar{g}_k - \bar{d})}{\hbar} \left[ \sin 2\theta R_c(t)\delta(\omega_k - \omega_L)/2 + \cos^2 \theta R^+(t)\delta(\omega_k - \omega_L - 2\Omega) - \sin^2 \theta R^-(t)\delta(\omega_k - \omega_L + 2\Omega) \right]. \]  

(10)

Here, the contributions due to Cauchy principal part \( P_c \) leading to a small Lamb shift compared to Rabi frequency \( \Omega \) were ignored. Next, substituting equation (10) in equation (7) and replacing the summation over the discrete wave vectors \( k \) by an integral over the continuum modes [36] one arrives at the following master equation:

\[ \frac{d}{dt} \langle \Omega \rangle - i\Omega \langle R_c, Q \rangle = -\langle \gamma_0 \sin 2\theta R_c/2 + \gamma_+ \cos^2 \theta R^+ - \gamma_- \sin^2 \theta R^- \rangle \times \sin 2\theta R_c/2 + \cos^2 \theta R^+ - \sin^2 \theta R^- \rangle + \text{H.c.}. \]  

(11)

Here, in free space, \( \gamma(\omega) = 2\omega^2\omega^3/(3\hbar^2) \) and for \( \gamma_0 \) we have \( \omega = \omega_L \) while for \( \gamma_{\pm} \) we have \( \omega = \omega_L \pm 2\Omega \). Note that for usual vacuum modes and moderate driving, i.e. \( \Omega/\omega_L \to 0 \), we have \( \gamma_0 = \gamma_+ = \gamma_- \). We anticipate that, for modified environmental reservoirs such as low-quality optical cavities or photonic crystals, or for very intense driving in free space with \( \Omega/\omega_L \ll 1 \) but not zero, this is not the case, namely, \( \gamma_0 \neq \gamma_+ \neq \gamma_- \) [37].

We emphasize here that the master equation (11) describes also, under the Born–Markov conditions, the case of a driven two-level particle that is damped by a modified reservoir, i.e. when the density of EMF modes is different at various dressed-state transitions. In this case \( \gamma_{\pm} \propto g(\omega_0 \pm 2\Omega) \) while \( \gamma_0 \propto g(\omega_L) \), where \( g(\omega) \) characterizes the atom–environment coupling strength [32–37]. In particular, for a low-quality optical cavity one obtains (see also [38])

\[ \gamma_0 = \frac{\kappa g^2}{k^2 + \delta^2}, \quad \gamma_{\pm} = \frac{\kappa g^2}{k^2 + (\delta \mp 2\Omega)^2}. \]  

(12)

Here \( \kappa \) is the cavity decay rate and \( \delta = \omega_L - \omega_0 \), where \( \omega_0 \) is the cavity frequency. Note that the spontaneous emission modification via optical cavities in the visible frequency domain was experimentally demonstrated in [39]. Finally, for photonic crystals we have \( \gamma_{\pm} = \pi\sum_k (\bar{g}_k d^2) (\delta(\omega_k - \omega_L \pm 2\Omega)) \) while \( \gamma_0 = \pi\sum_k (\bar{g}_k d^2) (\delta(\omega_k - \omega_L)) \) and these decay rates depend on concrete atom–environment coupling [34, 40].

In the next subsections, we shall obtain the equations of motion as well as the expression for the optical force acting on a strongly driven two-level particle in various environments.

2.1. Equations of motion

Using equation (11) one can obtain the equations of motion for the operators of interest:

\[ \frac{d}{dt}(R_c) = -2\gamma_0 R_c + \gamma_0 \sin 4\theta (R^+ + R^-)/2 - 2\gamma_m. \]

(13)

\[ \frac{d}{dt}(R^+) = (R^+)[2i\Omega - (\gamma_0 \sin^2 \theta + \gamma_p)] - (\gamma_+ + \gamma_-) \sin^2 \theta (R^-)/4 + \gamma_f + \sin 2\theta (R_c)(\gamma_+ \cos^2 \theta - \gamma_- \sin^2 \theta)/2. \]

(14)

\[ \frac{d}{dt}(R^-) = -(R^-)[2i\Omega + (\gamma_0 \sin^2 \theta + \gamma_p)] - (\gamma_+ + \gamma_-) \sin^2 \theta (R^+)/4 + \gamma_f + \sin 2\theta (R_c)(\gamma_+ \cos^2 \theta - \gamma_- \sin^2 \theta)/2. \]

(15)

Here \( \gamma_p = \gamma_+ \cos^3 \theta + \gamma_- \sin^4 \theta, \gamma_m = \gamma_+ \cos^3 \theta - \gamma_- \sin^4 \theta \) while \( \gamma_f = \sin 2\theta (\gamma_0 + \gamma_+ \cos^2 \theta + \gamma_- \sin^2 \theta)/2 \). Further, \( \cos^2 \theta = (1 + (\Delta/2)/\Omega)/2, \sin^2 \theta = (1 - (\Delta/2)/\Omega)/2 \) and \( \sin 2\theta = \Omega/\Omega \).

2.2. Optical force

Substituting the steady-state solution of equation (13) in equation (4) we obtain the following mean expression for the optical force:

\[ \langle F \rangle = k_0 \hbar \gamma \frac{2\gamma \Omega^2 \sin 2\theta}{4\gamma_p \Omega^2 + 2\gamma_+ \gamma_- \gamma_f - \gamma_0 \gamma_f} \sin 2\theta / 4. \]

(16)

For strong fields, i.e. \( \Omega^2 \gg \Delta^2 + \gamma^2 \), one arrives at the maximal expression of the force:

\[ \langle F \rangle = k_0 \hbar \gamma \frac{\Omega^2}{\Delta^2 + \gamma^2 + 2\Omega^2}. \]

(17)

Notice, that the particle velocity \( v \) can be included in equation (15) via the modified detuning, i.e. \( \Delta \to \Delta - k_0 v \).

In the following we shall obtain the corresponding expression for the maximal force in the strong-field limit and in the modified reservoirs. In the intense-field limit, that is \( \Omega \gg |\Delta, \gamma_{\pm}, \gamma_0|, \) we have \( \cos^2 \theta \approx \sin^2 \theta \to 1/2 \) while \( \sin 2\theta \to 1 \). Therefore, the optical force acting on a strongly driven two-level atom in a modified reservoir is

\[ \langle F \rangle = k_0 \hbar \gamma (1/2 + \gamma_+ \gamma_- / \gamma_0 (\gamma_+ - \gamma_-)). \]  

(18)

3. Results and discussions

In the case when the driven atom is surrounded by the usual vacuum modes and \( \Omega/\omega_L \to 0 \), the spontaneous decay rates corresponding to different dressed-state transitions are equal, i.e. \( \gamma_0 = \gamma_+ = \gamma_- \equiv \gamma \). This will lead to the well-known expression for the optical force, that is

\[ \langle F \rangle = 2k_0 \hbar \gamma \frac{\Delta^2 + \gamma^2 + 2\Omega^2}{\Delta^2 + \gamma^2 + 2\Omega^2}. \]

(16)

For strong fields, i.e. \( \Omega^2 \gg \Delta^2 + \gamma^2 \), one arrives at the maximal expression of the force:

\[ \langle F \rangle = k_0 \hbar \gamma \frac{\Omega^2}{\Delta^2 + \gamma^2 + 2\Omega^2}. \]

(17)
One can observe that the optical force in the strong-field limit depends on the spontaneous decay rates at particular dressed-state transitions. By modifying these dressed-decay rates via a suitable environment reservoir one can influence the magnitude of the optical force. In particular, when \( \gamma_+ \gg \gamma_- \) we have for the optical force the following expression:

\[
\langle F \rangle = k_L h \gamma_0 \left( \frac{1}{2} + \frac{\gamma_-}{\gamma_0} \right).
\]  

(18)

Now if \( \gamma_- \gg \gamma_0 \) we obtain that the optical force is greater than \( k_L h \gamma_0/2 \). These conditions for the dressed-decay rates can be fulfilled in a photonic crystal environment \([34, 40]\) or in a two-coupled-cavity system where one cavity is in resonance with the lower dressed-state transition while another cavity being in resonance with the larger dressed-state transition (see also \([39]\)).

Conversely, if \( \gamma_- \gg \gamma_+ \) we get for the force in equation (17)

\[
\langle F \rangle = k_L h \gamma_0 \left( \frac{1}{2} + \frac{\gamma_+}{\gamma_0} \right),
\]  

(19)

which is again much larger than \( k_L h \gamma_0 \) when \( \gamma_+ \gg \gamma_0 \) or \( \langle F \rangle = k_L h \gamma_0/2 \) if \( \gamma_+ \ll \gamma_0 \). Finally, if \( \gamma_+ = \gamma_- \gg \gamma_0 \) we again have an increase in the optical force. These features are shown in Figure 2 where equation (17) is plotted.

Further, we shall evaluate the maximal optical force when the two-level emitter is pumped with a very intense laser field in free space such that \( 2\Omega/\omega_L \ll 1 \) but not zero. In this case the dressed-decay rates can be determined as follows:

\[
\gamma_\pm = \gamma_0 (1 \pm 2\Omega/\omega_L)^3.
\]  

(20)

Substituting these expressions in equation (17) and to the second order in the small parameter \( \Omega/\omega_L \) one arrives at

\[
\langle F \rangle = k_L h \gamma_0 (1 - 3(2\Omega/\omega_L)^2).
\]  

(21)

One can observe here that the maximal force in free space is smaller for very intense driving than for moderate pumping (see equation (16)). The differences occur for non-vanishing values of \( \Omega/\omega_L \). However, here, the counter-rotating terms in the Hamiltonian (1) are not taken into account. Therefore, in free space, the expressions for the maximal force (17) and (21) should be corrected. The counter-rotating terms will contribute with the following term in the Hamiltonian (1) \([41]\):

\[
H_{\text{cr}} = i \frac{\Omega}{\omega_L} \sum_k (\vec{g}_k \cdot \vec{d})(a_k^+ - a_k)S_z.
\]  

(22)

This contribution is valid for \( \Omega \ll \omega_L \). The transition frequency \( \omega_0 \) modifies in this case due to a Bloch–Siegert shift \([42]\), i.e. \( \omega_0 \rightarrow \omega_0 + \Omega^2/\omega_L \). Inclusion of this Hamiltonian in our analytical formalism will result in the following expression for the maximal force:

\[
\langle F \rangle_{\text{cr}} = k_L h \gamma_0 (1 - (2\Omega/\omega_L)^2).
\]  

(23)

Now, taking into account the expressions (20) for the dressed-state decay rates in free space, and to the second order in the small parameter \( \Omega/\omega_L \) one arrives at the next formula for the maximal optical force:

\[
\langle F \rangle_{\text{cr}} = k_L h \gamma_0 (1 - (2\Omega/\omega_L)^2).
\]  

(24)

This expression is slightly different from the corresponding one, i.e. \( (21) \), obtained without the counter-rotating terms. However, the tendency is the same, i.e. the maximal force in free space is smaller for very intense driving than for moderate pumping (see equation (16)). This result is somehow counterintuitive as one may expect the force to be larger for higher field intensities, although the modification in free space is rather small. An intuitive explanation for the effect is as follows: in a weak field, the force would be dependent on the atoms on the excited state as they can spontaneously scatter photons. In a strong field, the force will depend on the distribution of atoms on the dressed states. Equation (17) shows a linear contribution to the force coming from the central band together with a nonlinear contribution due to photon scattering from the side-bands of the Mollow spectrum. The side-band free-space photon scattering rate is modified for stronger applied external fields and, therefore, the force modifies in comparison to lower pumping fields. Finally, the two-level approximation applies because \( \Omega/\omega_L \ll 1 \) and \( |\omega_0 - \omega_L|/\omega_L \ll 1 \). The two-level model can be obtained from a few-level atom where the frequency of the additional third level \( \omega_L \), which the laser may couple, must obey the relation \( |\omega_0 - \omega_L| \gg 2\Omega \).

4. Summary

In summary, we have investigated the optical force acting on a two-level atom in a running wave laser and in a modified surrounding EMF reservoir. For this, we obtained the master equation describing this process and, correspondingly, we obtained the equations of motion for the dressed-state operators of interest which helped to get the optical force. The obtained optical force shows a strong dependence on the modified electromagnetic reservoir via the dressed-decay rates at particular frequencies. In particular, it can be much larger than the corresponding force in the free space. Finally, we evaluated the maximal optical force acting on a two-level
emitter in very intense laser fields and in the free space. These effects can be useful for cooling various single-particle emitters via modified EMF environments.

Acknowledgment

We acknowledge valuable discussions with Christoph H Keitel.

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