Locally Adaptive Learning Loss for Semantic Image Segmentation

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Abstract

We propose a novel locally adaptive learning estimator for enhancing the inter- and intra-discriminative capabilities of Deep Neural Networks, which can be used as improved loss layer for semantic image segmentation tasks. Most loss layers compute pixel-wise cost between feature maps and ground truths, ignoring spatial layouts and interactions between neighboring pixels with same object category, and thus networks cannot be effectively sensitive to intra-class connections. Stride by stride, our method firstly conducts adaptive pooling filter operating over predicted feature maps, aiming to merge predicted distributions over a small group of neighboring pixels with same category, and then it computes cost between the merged distribution vector and their category label. Such design can make groups of neighboring predictions from same category involved into estimations on predicting correctness with respect to their category, and hence train networks to be more sensitive to regional connections between adjacent pixels based on their categories. In the experiments on Pascal VOC 2012 segmentation datasets, the consistently improved results show that our proposed approach achieves better segmentation masks against previous counterparts.

1 Introduction

Convolutional Neural Networks are rapidly driving advances in semantic image segmentation [13, 19, 40], which aims to predict accurate and effective masks on different classes of targets. To fulfill this challenge, previous study focused on designing or finetuning different network architectures [27, 20, 16, 30, 6]. To our knowledge, all these frameworks adapt the estimator (i.e. loss) proposed in [27], which averages pixel-wise cross-entropy over prediction maps and ground truths of input batches. However, this kind of estimator only measures pixel-wise distances between predictions and ground truths, neglecting the interactions between pixels of same category within their neighborhoods. Whereas, such interactions are crucial especially when the appearances of targets change due to the deformation, illumination variations, occlusion and so forth [14, 17].

Previous loss functions for enhancing the intra-class features were designed for image classification [19, 24, 33, 36], which usually measures batch costs between predicted classes and labels over batches of images, such like contrastive loss [18, 8], triplet loss [32] and center loss [37]. However, as mentioned in [37], the approaches, like contrastive loss and triplet loss, require image pairs or triplets.
for each training iteration, which result in the dramatic growth of training samples, and thus significantly increase the computational complexity. Center loss overcomes such problem by introducing k-nearest neighbor (k-NN) [15] algorithms into softmax cross-entropy. At each training iteration, it computes the distances between deep features and every class centers of the features over a mini-batch of images, and updates the centers after each iteration. Center loss can effectively minimize the intra-class variations while keeping the features of different classes separable. Even though, such kind of estimation is still computational expensive, let alone, for image segmentation tasks, each pixel is considered as a training sample. Moreover, most semantic segmentation datasets exhibit long tail distributions with few object categories, which means inter- and intra-classes are imbalanced, and consequently biasing networks training towards major classes [4]. To address class imbalance problem, in the realm of object detection, Lin et al. [26] modified standard cross entropy loss to down-weight the losses assigned to well-classified examples, and proposed focal loss.

In this paper, we introduce a novel locally adaptive loss for semantic image segmentation by estimating selectively filtered predictions based on their categories. Figure 1 illustrates the training framework of our proposed method at a glance. The selective pooling filter slides over output feature maps and ground truths simultaneously, meanwhile at each striding step, it selectively pools predicted vectors into a merged one, then computes cost between the merged vector and center pixel’s category label inside file. Such operation is conducted on each valid pixel over input batches, and finally it computes a global loss for each input batch (see Figure 2). During training, such loss layer emphasizes on the interactions from same category over neighborhood, which intuitively indicates that stochastic gradients descent(SGD) solver should optimize entire predictions on same category in a scale rather than per pixel. Such loss can effectively supervise networks to summarize features of the same category, meanwhile, indirectly enlarge the differences of inter-class features. Thus, the discriminative capabilities of learned models are significantly improved with higher robustness and object sensitivity. Via this loss, we trained deep neural networks (DNN), and demonstrate that our learned models outperforms against previous state-of-arts.

In summary, we make the following contributions:

- We propose a novel locally adaptive loss layer for semantic image segmentation. During learning procedure, it helps networks to improve the capabilities of discriminating targets from both inter- and intra-classes. In our experiments We also verified that the learned models trained with our loss outperform against their counterparts.

- We explore a simple method for rebalancing losses from image segmentation datasets, which often exhibit long-tail distribution. Our correction mechanisms can prevent networks from biasing towards majority classes.

- We implement other well-known losses (i.e., center loss and focal loss) for image semantic segmentation tasks as our additional contribution. With these losses, the learned models can also predict decent masks, and thus we use them as our counterparts.

The remainder of this paper has the following structure: Section 2 briefly summarizes related work. Section 3 constructs the locally adaptive loss. Section 4 illustrates and evaluates our locally adaptive loss via several numerical experiments using different training frameworks. Section 5 draws conclusions and proposes direction for future work.

2 Related Work

2.1 Image Segmentation

Semantic image segmentation using convolutional neural networks or deep neural networks (DNN) has achieved several breakthroughs in recent years [2, 6, 20, 27, 9, 11]. Inspired by the work [27], researchers commonly remove last fully connected (FC) layers of neural networks, and then utilize the in-network upsampled or deconvolved predictions of convolutional layers as predicted feature maps. The estimating procedure for training generally computes pixel-wise losses between the maps and ground truths over each batch, and then pools them into a global value for back propagation (BP).

The pixel-wise losses in [27, 6] are based on softmax and multinomial cross-entropy between predicted vectors of neurons and labels. However, this computation collapses the spatial dimensions of both predicted maps and labeled images into vectors. The methods like [10, 28, 29] resort to FC layers to establish the prediction masks, which requires more complex hyper-parameters. Recently, He et al. [20] proposed a regional loss computation, using aligned Region of Interest (ROI) [16] to maintain each object’s spatial layout. On each aligned ROI, it conducts a pixel-wise sigmoid and binary loss between predictions and targets labels, eliminating inter-class competition.

Very recently, Loss Max-Pooling [4] was proposed for handling the imbalanced inter-class datasets of semantic segmentation. It selectively assigns weights to each pixel based on their losses, and rebalances datasets by up-weighting losses contributed by minority classes.
2.2 Weighted Ensemble Entropy Estimator

Density functions, like cross entropy, are widely used as estimators for training CNN and DNN frameworks (e.g., AlexNet [24], the VGG net [33], ResNet [19], DenseNet [23], etc.). As discussed in [34], the ensemble of weak estimators can improve the performance of learned models, similar to the methods (e.g., boosting [31], etc.) proposed in the context of classification. Meanwhile, a weighted ensemble entropy estimator was introduced by optimally combining multiple weak entropy-like estimators (e.g., k-NN entropy functional estimators [21], intrinsic dimension estimators [35], etc.). The weighted ensemble entropy estimator is defined as:

$$\hat{E}_w = \sum_{i=1}^{n} w(i) \hat{E}_i$$  \hspace{1cm} (1)

where $\hat{E}_i$ stands for an individual entropy-like estimator, \(n\) means the number of estimators, and $w(i)$ is the weight to be optimized, which subject to $\sum_{i\in\omega} w(i) = 1$. It was also verified that such weighted estimator can provide better prediction accuracy and stronger discrimination ability with higher convergence rate. Note that each weighted weak estimator $\hat{E}_i$ operates on the same set of input variables $X$. Similarly, center loss [37] also estimates on the same set of features.

In contrast, we explored a new training loss for image segmentation task, which estimates on the merged intra-correlated predictions for neighboring pixels with same category. We demonstrate that our new loss can help networks to better learn the interactions of neighbor pixels with same category, and thus improve the discriminative abilities on both intra- and inter- classes. Our learned models outperform their counterparts trained via plain pixel-wise estimators.

3 The Locally Adaptive Loss

In this section, we elaborate the estimating procedure of our proposed method, and demonstrate that our locally adaptive loss can improve the discriminative power of the learned models, followed by some discussions.

3.1 Selective Pooling Estimator in Scale

In image semantic segmentation, the main task of an effective loss function is to improve the discriminative capability of learned model. However, in contrast to the image detection and classification where each training batch contains independent samples (e.g., labeled images, bounded objects, etc.), each batch for image segmentation contains all the labeled pixels from different objects, which means several groups of input samples are partially correlated to each other once they belong to same object category. Intuitively, estimating the predictions within a small scale of pixels with same category and minimizing the loss over that scale give a way.

To this end, at each spatial point (i.e., pixel) $p_{ij}$ with location $(i,j)$ on feature maps, we firstly obtain its normalized predicted distribution vector $\mathbf{x}_{ij} \in \mathbb{R}^{c \times 1}$. Then we conduct our selective pooling kernel with size $(w, h, c)$, operating on predicted vectors of neighboring points (where $w$, $h$, and $c$ represent: width and height of the kernel window, number of classes respectively). The filtered vector $\mathbf{f}(i,j) \in \mathbb{R}^{c \times 1}$ is then computed as follows:

$$f(i,j) = \frac{1}{\xi} \sum_{u} \sum_{v} \mu(i+u, j+v) \omega_d(u,v) \mathbf{x}_{i+u,j+v}$$  \hspace{1cm} (2)

$$\mu(i+u, j+v) = \begin{cases} 1, & \text{if } y_{i+u,j+v} = y_{ij} \\ 0, & \text{otherwise} \end{cases}$$

and then our proposed local estimator is formulated as:

$$\mathcal{L}^S(i,j) = \mathcal{E}(f(i,j))$$  \hspace{1cm} (3)

where $\mathcal{E}(\cdot)$ represents a local cost function (e.g., softmax cross-entropy, etc.), one-hot label vectors $y_{ij}, y_{i+u,j+v} \in \mathbb{R}^{c \times 1}$ denotes the $y$th category of $p_{ij}$ and its neighboring points $p_{i+u,j+v}$. $\mathbf{x}_{i+u,j+v} \in \mathbb{R}^{c \times 1}$ means the normalized predicted vector at each neighbor point $p_{i+u,j+v}$. $\omega_d(u,v)$ is the Gaussian weighting function based on spatial distance to center $p_{ij}$, and $\mu(i+u, j+v)$ is a indicator function.
for eliminating the predictions of neighboring points with different classes from center \( p_{ij} \), \( \xi \) is the number of points which have the same labels to \( p_{ij} \). Figure 2 illustrates the computation details of our selective pooling filter.

The intuition behind Eqn. 3 is that the local estimation computes the entire loss over a group of neighboring points with same category, which indicates the optimization should emphasize on minimizing the overall loss towards a certain category in a scale rather than per point. Note that, for each point, such operation only modifies the distribution of its predicted vector, and does not increase any predicted vector.

For the normalization of predicted vector \( x_{ij} \), we use standard score normalization, which is defined as: \( x_{ij} = \frac{(x_{ij} - \mu)}{\sigma} \), where \( x_{ij} \) means the raw predicted vector, \( \mu \) and \( \sigma \) stand for mean and standard deviation of \( x_{ij} \). In practice, we adapt softmax cross-entropy as our local cost function \( \mathcal{L}(\cdot) \). Thus, our local estimator can be rewritten as:

\[
\mathcal{L}^S(i, j) = -\sum_l y_l \log \left( \frac{e^{f_l(i, j)}}{\sum_k e^{f_k(i, j)}} \right) \tag{4}
\]

where \( y_l \) represents each element value of center point’s label vector \( y_{ij} \). \( f_l \) means each element value of the merged vector \( f(i, j) \) from the filter.

### 3.2 Striding and Batch Pooling Strategy

The selective estimator above constructs an ensemble cost based on the category label of center point inside filter. Then, we propose our Locally ADaptive Loss \( \mathcal{L}^{AD} \) by sliding such filter (with striding step \( s \)) over all the input batches \( B \) (i.e., images), and averaging all the local estimator values with Minkowski pooling. Here we take one image as an input batch, and drop indices \( b \) values with \( [7, 38] \). However, as mentioned before, our method does not increase any losses number after local cost estimation on each point, therefore the computational time maintains the same at each iteration.

In practice, we set the striding step \( s \) to 1, which means the filter slides pixel by pixel over input feature maps and ground truths. We use Gaussian-like weighting as a neat allocation of \( \omega_d \) to each neighbor pixel. Specifically, we allocate down-weights \( \omega_d \) to prediction vectors of neighbor pixels, according to their chessboard distances \( D8 \) to center pixel (i.e., \( \omega_d = 2^1 \) to center pixel, \( \omega_d = 2^0 \) to the pixels with \( D8 = 1 \), \( \omega_d = 2^1 \) to the pixels with \( D8 = 2 \), \( \omega_d = 2^2 \) to the pixels with \( D8 = 3 \), and so forth).

In order to determine \( k \), we tried different \( k \) values (see Table 1), and adapted \( k = 3 \) accordingly. However, in comparison with the learned model trained by plain softmax cross-entropy (last column), no matter which \( k \) values adapted, the models via our locally adaptive loss provide consistently higher Mean IoU values on testing dataset. It means the local selective estimator primarily contributes to the effectiveness of locally adaptive loss, rather than batch pooling strategy.

The derivative of \( \mathcal{L}^{AD} \) w.r.t. the input vector \( x_{ij} \), written in an element-wise is as follows (\( m, n \) stand for each neighbor point’s indices inside filter):

\[
\frac{\partial \mathcal{L}^{AD}}{\partial x_{ij}} = \frac{\partial \mathcal{L}^{AD}}{\partial \mathcal{L}^S} \cdot \frac{\partial \mathcal{L}^S}{\partial x_{ij}} \tag{6}
\]

\[
\frac{\partial \mathcal{L}^{AD}}{\partial \mathcal{L}^S} = \frac{1}{M_p} \sum (\mathcal{L}^S)^{k-\frac{1}{2}} \tag{7}
\]

\[
\frac{\partial \mathcal{L}^S}{\partial x_{ij}} = \frac{\partial \mathcal{L}^S}{\partial f(i, j)} \cdot \frac{\partial f(i, j)}{\partial x_{ij}} \tag{8}
\]

### Table 1. Influences of batch pooling strategies with different \( k \) values.

| k value | 1    | 2    | 3    | 5    | SoftMax CE |
|---------|------|------|------|------|------------|
| Mean IoU| 70.1 | 70.9 | 71.2 | 70.6 | 66.8       |

Both locally adaptive loss and loss max-pooling methods are designed for image semantic segmentation. Locally adaptive loss directly focuses on connections of adjacent pixels with same category, while loss max-pooling...
aims to rebalance the datasets between majority and minority classes.

Mostly, locally adaptive loss is a metric approach in the feature space (i.e., activations of last upsampled DNN layer), using the selective pooling filter to increase network attentions on ensemble predicting correctness of neighboring pixels. It applies the filtering operations before local cost computations with ground truths. For loss max-pooling, it only re-weights losses after local cost computations, aiming to increase the contributions of under-represented object classes.

Loss max-pooling is in some way similar to our batch cost pooling strategy, as we use simple Minkowski pooling for handling the imbalanced class datasets. Loss max-pooling can be also embedded into our loss as a replacement for batch pooling strategy.

Loss max-pooling is essentially a weighting and sampling method on outputs of cost function, which can be considered as hard sample mining \cite{7,38}, whereas locally adaptive loss operates on both inputs and outputs.

4 Experiments

We have evaluated our novel locally adaptive training loss ($\mathcal{L}^{AD}$) on the extended Pascal VOC \cite{12} semantic image segmentation datasets. We adapt Intersection-over-Union (IoU) as the evaluating metric on over all classes of datasets.
classification, we manually adjusted their hyper parameters since they were originally designed for object detection and recognition. For center loss and focal loss, we used kernel sizes of $\alpha = 0.5, \lambda = 3 \times 10^{-4}$ for center loss, and $\alpha = 0.25, \gamma = 2$ for focal loss. We report the mean IoU values in Table 2 after 20,000 iterations. As shown in Table 2, the learned models trained via locally adaptive loss predict consistently improved results, in particular, using $L_{AD}$ with kernel size: $\alpha = 0.5, \lambda = 3 \times 10^{-4}$ can give 1.5% more in predicting accuracy, compared with plain softmax cross-entropy. In contrast, applying center loss leads to mean IoU values similar to plain softmax cross-entropy, while we obtain 4.2% lower values by using focal loss.

Additionally, in Figure 3, we exhibit several segmented examples on testing set to visually demonstrate improvements via our training framework against others. The first two columns show the original images (randomly cropped) and ground truths. From the 3rd to 6th columns, we observe the masks predicted by learned models trained via plain softmax cross-entropy, center loss, focal loss and our locally adaptive loss (with kernel size: $\alpha = 0.5, \lambda = 3 \times 10^{-4}$, $\gamma = 2$). And we can see that the models via our training framework predict more accurate and effective masks with higher robustness and object sensitivity, compared to its counterparts.

| Training Framework   | Hyper Para. | M IoU  |
|----------------------|-------------|--------|
| RN-101 + SoftMax CE  | -           | 74.6   |
| RN-101 + Center Loss | $\alpha = 0.5, \lambda = 3 \times 10^{-4}$ | 74.5   |
| RN-101 + Focal Loss  | $\alpha = 0.25, \gamma = 2$ | 70.4   |
| RN-101 + $L_{AD}$ (ours) | $\alpha = 0.5, \lambda = 3 \times 10^{-4}$, $\gamma = 2$ | 75.7   |
| RN-101 + $L_{AD}$ (ours) | $\alpha = 0.25, \gamma = 2$ | 76.1   |
| RN-101 + $L_{AD}$ (ours) | $\alpha = 0.5, \lambda = 3 \times 10^{-4}$, $\gamma = 2$ | 75.1   |

Table 2. Experimental results (in %) on Pascal VOC 2012 segmentation validation data. The training framework is based on ResNet-101 [6].

5 Conclusions and Future Work

In this work, we introduced a novel approach to increase networks discriminative capabilities of inter- and intra-class for semantic image segmentations. At each pixel’s position our method firstly conducts adaptive pooling filter operating over predicted feature maps, aiming to merge predicted distributions over a small group of neighboring pixels with same category, and then computes cost between the merged distribution vector and their category label. Our locally adaptive loss does not increase any loss numbers, thus the time complexity maintains the same at each iteration. In the experiments on Pascal VOC 2012 segmentation datasets, the consistently improved results show that our proposed approach achieves more accurate and effec-
tive segmentation masks against its counterparts. More extensive experiments will be launched on Cityscapes dataset \[9\] and COCO dataset \[25\] to further verify our training framework.

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