Estimating the EOS from the measurement of NS radii with 5% accuracy

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ABSTRACT

Context. Observations of heavy (\(\gtrsim 2 \, M_\odot\)) neutron stars, such as PSR J1614-2230 and PSR J0348+0432, in addition to the recent measurement of tidal deformability from the binary neutron-star merger GW170817, place interesting constraints on theories of dense matter. Currently-operating and future observatories, such as the Neutron star Interior Composition Explorer (NICER) and the Advanced Telescope for High E\textsc{nergy} Astrophysics (ATHENA) are expected to collect information on the global parameters of neutron stars, namely masses and radii, with the accuracy of a few percent. Such accuracy will allow for precise comparisons of measurements to models of compact objects and significantly improve our understanding of the physics of dense matter.

Aims. The dense-matter equation of state is still largely unknown. Here we investigate how the accuracy of the measurements expected from the NICER and ATHENA missions will improve our understanding of the dense-matter interior of neutron stars.

Methods. We compare global parameters of stellar configurations obtained using three different equations of state: a reference (SLy4 EOS) and two piecewise polytropes manufactured to produce mass-radius relations indistinguishable from the observational point of view i.e. within the predicted error of the radius measurement. We assume observational errors on the radius determination corresponding to the accuracies expected for the NICER and ATHENA missions. The effect of rotation is examined using high-precision numerical relativity computations. Due to the fact that masses and rotational frequencies might be determined very precisely in the most optimistic scenario, only the influence of observational errors on the radius measurements is investigated.

Results. We show that \(\pm 5\%\) errors in radius measurement lead to \(\sim 10\%\) and \(\sim 40\%\) accuracy in central parameter estimation, for low-mass and high-mass neutron stars, respectively. Global parameters, such as oblateness and surface area, can be established with \(8\%–10\%\) accuracy, even if only compactness (instead of mass and radius) is measured. We also report on the range of tidal deformabilities corresponding to the estimated masses of GW170817, for the assumed uncertainty in radius.

Key words. stars: neutron – pulsars – equation of state

1. Introduction

Neutron stars (NS) are the most extreme material objects in the Universe. Formed in the aftermath of core-collapse supernovae, some few thousands are currently known. Most of them are seen as pulsars. These rotating compact objects allow astronomers to perform direct measurements of global parameters of NSs and, indirectly, to investigate the properties of dense matter in their interiors, which is in a state impossible to reproduce in terrestrial conditions. High-precision observations compared with theoretical models for the equation of state (EOS) of dense matter in NSs are currently the only way to study physics in such extreme conditions.

Precise determinations of the masses of PSR J1614-2230 (Demorest et al.\textsuperscript{[2010]}, Fonseca et al.\textsuperscript{[2016]}, Arzoumanian et al.\textsuperscript{[2017]} and PSR J0348+0432 (Antoniadis et al.\textsuperscript{[2013]}),\textsuperscript{[2016]} have set an observational bound on the maximum mass of a NS not lower than about \(2 \, M_\odot\), and constrain theoretical models of the EOS. Present determinations of NS radii \(R\) are roughly between 10-15 km (for a review see, e.g. Özel & Freire\textsuperscript{[2016]}, Fortin et al.\textsuperscript{[2016]}), but due to systematic errors, uncertainties on the values are still large (see, e.g. Heinke et al.\textsuperscript{[2014]}, Fortin et al.\textsuperscript{[2015]), Eshamouty et al.\textsuperscript{[2016]}, Miller & Lamb\textsuperscript{[2016]}, Haensel et al.\textsuperscript{[2016]}). Additionally, GW170817, the recent first direct detection of gravitational waves from the last orbits of a relativistic binary NS system (Abbott et al.\textsuperscript{[2017]}), yielded tidal deformability parameter measurements that disfavour radius larger than 14 km at 1.4 \(M_\odot\).

Ongoing and future missions like the Neutron star Interior Composition Explorer (NICER, Arzoumanian et al.\textsuperscript{[2014]} and Advanced Telescope for High E\textsc{nergy} Astrophysics (ATHENA, Motch et al.\textsuperscript{[2013]}),\textsuperscript{[2017]} are or will be able to measure NS gravitational masses and radii, with accuracies of a few percent. Generally, the radius \(R\) and the gravitational mass \(M\) measurement methods consist of

- Pulse profile modelling related to brightness variations from the non-uniform surface of the NS. Such fluctuations may be caused by hot and cold spots, originated by magnetic field in rotation-powered pulsars or by non-uniform thermonuclear burning on the surface of an X-ray burster (see, e.g., Özel et al.\textsuperscript{[2016]}, Sotani\textsuperscript{[2017]}). According to Psaltis et al.\textsuperscript{[2014]} and Psaltis \& Özel\textsuperscript{[2014]}), an uncertainty of \(\lesssim 5\%\) in the NS radius measurements is feasible, assuming that the rotational frequency of the object is in the 300 – 800 Hz range,
and sufficiently long observations are possible (10⁶ counts in the pulse profile). However, other authors pointed out that realistic observational errors might be larger, even up to 10% (see e.g. Lo et al. 2013; Miller & Lamb 2016) and may depend strongly on the system geometry. Nevertheless, in our work we decided to assume 5% accuracy, as an optimistic scenario.

To break the degeneracy, four quantities have to be measured - the amplitude of the bolometric flux oscillation, the amplitude of its second harmonic, the amplitude of the spectral colour oscillation and the phase difference between the bolometric flux and colour oscillation. These requirements can be fulfilled by the NICER mission’s long exposure time and/or by combining pulse profile modelling methods with other measurements (for example X-ray modelling from ATHENA and mass determination from radio timing). For lower spins (≤ 300 Hz), the amplitude of the second harmonic is too low to perform a full analysis. In this case only measurements of the compactness M/R are possible. For much higher spins (≥ 800 Hz), higher order multipoles become important in the modelling and solutions of the field equations are EOS-dependent.

- Observations of Eddington-limited X-ray bursts from accreting NSs, which may put constraints on the maximum radius of the compact object (see, e.g. Galloway et al. 2008).
- Fitting spectra with an appropriate atmosphere model to observations of the quiescent emission from Low-Mass X-ray Binaries. This method was proposed by van Paradijs 1979 and improved since that time by using realistic NS atmospheres, as well as relativistic NS models and ray tracing (e.g. Vincent et al. 2017). A recent analysis by Steiner et al. 2017 suggests NS radii between 10 to 14 km.
- Detection of gravitational waves from NS binary systems, allowing for measurements of NS masses and radii from both observations (e.g. Neijssel et al. 2005; Damour et al. 2012; Abbott et al. 2017a) and post-merger (e.g. Bauswein et al. 2015; Abbott et al. 2017b; Annala et al. 2017; Margalit & Metzger 2017; Bauswein et al. 2017; Rezzolla et al. 2018).

All these methods are necessarily limited by their intrinsic measurement errors. The current state of the art is such that in several cases mass can be measured with much smaller errors than radii. Here we focus on future measurements of radii, and in particular on how the planned radius measurement accuracy reflects on the ability to discriminate similar M(R) relations obtained using different NS interior descriptions (that is, based on different EOSs). Specifically, we manufacture stable M(R) sequences for parametric (piecewise-polytropic) EOSs and compare them to a reference sequence of configurations based on the SLy4 EOS (Douchin & Haensel 2001). In this test case we investigate ways of telling apart the M(R) relations that are indistinguishable because of observational errors in the radius measurement, which we assume to be equal to 5% of the radius of the reference configuration based on the SLy4 EOS (the M(R) sequences of the piecewise-polytropes trace the ΔR = ±5% outline of the SLy4 EOS non-rotating M(R) sequence). We also study different spin frequency cases: from non-rotating objects to extremely rapidly spinning ones, in order to check if rotation aids the discrimination between various functionals of the EOS (mass, radius, quadrupole moment, moment of inertia). In addition, we study cases in which the rotation rate is not known, and quantify the magnitude of the errors on the EOS parameters related to the radius measurement error and/or the lack of spin frequency measurement.

The paper is composed as follows. In Sect. 2 we present the EOS models, the numerical methods used to obtain the rigidly-rotating sequences of configurations, and list the global NS parameters of interest. Section 3 contains the results. Section 4 presents the discussion and conclusions.

2. Methods and equations of state

The state of matter is relatively well known below the nuclear saturation density \( \rho_s = 2.7 \cdot 10^{14} \) g cm⁻³. Above this density several competing theories describing the EOS of dense matter exist (for a textbook review, see e.g., Haensel et al. 2007). In order to compare the effects of the uncertainty in the radius measurement on the EOS, we choose as the reference the SLy4 EOS (Douchin & Haensel 2001), consistent with recent radius and mass constraints, described in the introduction. Furthermore, we select two parametric EOSs, named Model1 and Model2, manufactured to be barely consistent with the expected radius uncertainty measurement with respect to the non-rotating reference model.

Model1 and Model2 EOSs were constructed using the SLy4 prescription of the crust for densities lower than the nuclear saturation density, and with three piecewise relativistic polytropes

\[
P(n) = \kappa_i n^{\gamma_i}, \quad \epsilon(n) = \rho c^2 = \frac{P}{\gamma_i - 1} + \frac{nm_b c^2}{\gamma_i - 1},
\]

for higher densities, where \( P(n) \) and \( \epsilon(n) \) denote the pressure and mass-energy density as function of the baryon density \( n \), \( \kappa_i \), \( \gamma_i \) and \( m_b \) are the pressure coefficient, the polytropic index (characterizing the stiffness of the EOS at given density) and the baryon mass for a given polytropic \( i \)-th segment (\( i = 1, \ldots, 3 \)). Index \( \gamma_i \) is a parameter of choice, and \( \kappa_i \) and \( m_b \) are fixed for a polytropic segment by the mechanical and chemical equilibrium.

In this study we limit ourselves to stationary, axisymmetric, rigidly-rotating NS configurations. Non-rotating static NS solutions are obtained by solving the TOV equations ( Tolman 1939; Oppenheimer & Volkoff 1939). Sequences of rotating stars
parametrized by the spin frequency \( f \) and the EOS parameter at the stellar center (e.g., the central pressure \( P_c \)) are obtained by solving coupled partial differential equations using a multidomain spectral methods library LORENE\(^1\) (Langage Objet pour la RELativité NumériquE, Gourgoulhon et al. 2016) nrotstar code (Bonazzola et al. 1993). The accuracy of the solutions is inspected by checking the validity of the 2D general-relativistic virial theorem (Bonazzola & Gourgoulhon 1994). Global parameters describing the NS are gravitational and baryon masses \( M \) and \( M_b \), angular momentum \( J \) and quadrupole moment \( Q \); their definition can be found in Bonazzola et al. 1993, Gourgoulhon 2010. From electromagnetic observations one obtains some estimation of the flux from the stellar surface, \( F_\infty \sim T^4R^2 \), proportional to its effective temperature \( T \) and the size \( R \) of the star. For a rotating star its visible size and radius are not uniquely defined (depends on many factors: viewing angle, compactness, rotation rate, physical parameters of the atmosphere, see e.g. Vincent et al. 2017 for details); in order to compare configurations rotating with different rates we adopt the mean radius \( R_{\text{mean}} = \sqrt{S/4\pi} \), where \( S \) is the surface area of the star, as a sufficiently good approximation.

We also compare the tidal deformabilities related to different models. The tidal deformability \( \Lambda \) which represent the reaction of the star on the external tidal field (such as that in a tight binary system) were obtained in the lowest-order approximation by integrating the TOV equations supplemented by an additional equation for the second tidal Love number \( k_2 \) (Flanagan & Hinderer 2008; Van Oeveren & Friedman 2017), \( \Lambda = 2R^3k_2/3 \), where \( k_2 \) is the quadrupole Love number (Love 1911) and \( R \) is the non-rotating star radius. We use the normalized value of the \( \Lambda \) parameter, \( \Lambda = G\Lambda \left( GM/c^2 \right)^{-5} \).

### 3. Results

Parametric models, denoted Model1 and Model2, are chosen in such a way to produce non-rotating \( M(R) \) relations tracing the \( \Delta R = \pm 5\% \) outline of the SLy4 EOS non-rotating \( M(R) \) sequence; they are plotted in the left panel in Fig. 1. At the present moment the data-analysis methods allow to treat objects with rotational frequency \( \lesssim 800 \) Hz, as was mentioned in the Sect. 1. Nevertheless, one can expect that future theoretical and observational progress will enable studies of NSs with larger spins. We therefore research a broader range of spin frequencies, \( 0 - 1400 \) Hz, as shown in the right panel in Fig. 1. The EOSs parameters are collected in Table 1. In addition to the required difference in radius, we select the parameters so that (i) the maximum mass \( M_{\text{max}} > 2M_\odot \), (ii) the speed of sound \( \sqrt{\partial P/\partial \rho} \), is always smaller than the speed of light \( c \) for stable configurations.

#### 3.1. Non-rotating neutron stars

The pressure-density \( P(\rho) \) relations for non-rotating configurations are presented in Fig. 2. The two bottom panels show the differences between the reference SLy4 model, Model1 (bottom panel) and Model2 (middle panel). Note that Model1 is initially stiffer than the SLy4 EOS, which provides a larger radius. For higher densities Model2 becomes stiffer, which is necessary to reach the desired maximum mass.

#### 3.2. Slowly-spinning neutron stars

As was mentioned in the Sect. 1 a full pulse profile analysis cannot be performed for NSs with rotational frequencies \( \lesssim 300 \) Hz, because the signal cannot be distinguished from a sinusoid and its second harmonic is too weak to perform a Fourier decomposition. As a consequence, only compactness can be measured (here defined as \( C = 2GM/R_{\text{mean}}c^2 \) ratio, where \( G \) is the gravitational constant, and \( c \) is the speed of light). Here we analyse the influence of the measured compactness on global and central NS parameters.

In Fig. 3 we show the compactness as a function of the NS surface area. Selected values of constant \( \rho_s \) are also marked. Configurations with \( C \gtrsim 0.6 \) are possible only for Model2. Ad-

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1. http://www.lorene.obspm.fr

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**Table 1.** Parameters of the three polytropes employed for the EOSs of Model1 and Model2 (indices correspond to the number of the polytrope). \( \gamma \) is the index of polytrope, \( n_b \) is baryon density, change of the polytrope occurred, \( \kappa \) is the pressure coefficient and \( m_0 \) the baryon mass at the point where the crust joins with the first polytrope (\( m_{b1} \)) or where two polytropes join (\( m_{b2} \) and \( m_{b3} \)). Other parameters were calculated from the conditions of mechanical and chemical equilibration.

| \( \gamma \) | Model1 | Model2 |
| --- | --- | --- |
| \( \gamma_1 \) | 3.20 | 2.50 |
| \( \gamma_2 \) | 2.83 | 3.22 |
| \( \gamma_3 \) | 2.50 | 3.00 |
| \( n_{b12} \) | 0.21 | 0.24 |
| \( n_{b23} \) | 0.70 | 0.50 |
| \( m_{b1} \) | 1.017982 | 1.016573 |
| \( m_{b2} \) | 1.014858 | 1.021916 |
| \( m_{b3} \) | 0.977851 | 1.015670 |
| \( \kappa_1 \) | 0.006646 | 0.006646 |
| \( \kappa_2 \) | 0.008745 | 0.003538 |
| \( \kappa_3 \) | 0.016621 | 0.005042 |

**Fig. 2.** (Color online) Upper panel: \( P(\rho) \) profiles corresponding to the \( M(R) \) relations from the left panel of Fig. 1 with the SLy4 model denoted by a red solid line, Model1 by a green dashed line and Model2 by a blue dash-dotted line. Thick semi-transparent lines denote \( P(\rho) \) ranges for the two components of the GW170817 binary neutron star system (Abbott et al. 2017a, with mass estimates using the low-spin priors). We also plot the pressure difference between the SLy4 model and Model1 (dashed green line, middle panel) and Model2 (dash-dotted blue line, lower panel).

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ditionally, if in addition to \( C \) the rotational frequency \( f \) is also known, the NS surface area can be estimated with \( \sim 10\% \) accuracy, for all values of \( C \) and \( f \). Unfortunately, obtaining the central parameters, such as the central density \( \rho_c \), is difficult when only the compactness, instead of \( M \) and \( R \), is known. For example, a central density \( \rho_c = 0.7 \cdot 10^{15} \text{g} \cdot \text{cm}^{-3} \) spans \( C \) between 0.2 and 0.3.

\[ 0.1 \leq C \leq 0.7 \]

\[ M \approx 2.05 M_\odot, \text{ red stars to } R_{\text{mean}} \approx 11.9 \text{ km and } M \approx 1.76 M_\odot, \text{ green diamonds to } R_{\text{mean}} \approx 12.5 \text{ km and } M = 1.06 M_\odot, \text{ blue triangles to } R_{\text{mean}} \approx 11.9 \text{ km and } M = 0.91 M_\odot. \]

Models with a fixed mass and radius may thus belong to configurations with different EOS and \( f \), but similar masses and radii can cross each other. Such cases are denoted by markers: black dots correspond to \( R_{\text{mean}} \approx 10.6 \text{ km and } M \approx 2.05 M_\odot \) (denoted by a black dot), three different NSs are possible: a non-rotating configuration described by Model1, the reference model SLy4 rotating at \( f = 700 \text{ Hz} \), and Model2 at \( f = 1200 \text{ Hz} \). Such a setup corresponds to a situation in which the masses and radii are determined, but the spin frequency is unknown, e.g., for a NS in a binary system for which bursts are observed, but which is not observed as a pulsar. We compare these configurations in the \( P(n_b) \) plane in Fig. 5.

This illustrates the importance of the spin information for the inference of the EOS from observations. The most extreme example is shown in the left panel of Fig. 5, where the accuracy of the estimation of the central density between Model1 with \( f = 0 \text{ Hz} \) and Model2 with \( f = 1200 \text{ Hz} \) is approximately 40%. Additionally, the pressure difference between the Model1 \( (f = 0 \text{ Hz}) \) and SLy4 \( (f = 700 \text{ Hz}) \) configurations is almost 20%. Other properties of the NSs from this case are shown in Table 2: the surface area \( S \), oblateness \( O \), \( T/W \) ratio and the angular momentum \( J \). Interestingly, the configuration described by Model1 \( (f = 0 \text{ Hz}) \) has almost the same surface area \( S \) as the fast-spinning \( (f = 1200 \text{ Hz}) \) Model2 configuration. In the other panels of Fig. 5 less extreme cases are presented; their differences between central pressures are around 10%.

The influence of rotation on the global parameters, like the oblateness \( O \) and the surface area \( S \), differs between low-mass and massive NSs, as is shown in Figs. 6 and 7. Grey vertical lines correspond to PSR J1748-2446ad. As expected, from these two figures one can notice that NSs with masses \( \approx 1 M_\odot \) reach the mass-shedding frequency much faster (\( \approx 750 \text{ Hz} \) for Model1)

3.3. Unknown rotational frequency

We limit our analysis of rotating stars to axisymmetric, rigidly-rotating, stable cases (frequencies below the Keplerian frequency). We have studied a wide range of possible spin frequencies and central parameters, as shown on the right panel on the Fig. 4. The fastest-known pulsar, PSR J1748-2446ad (Hessels et al., 2006) rotates at 716 Hz. However, to fully cover the possible parameter space, we survey spin frequencies much higher than this current limit, up to 1200 Hz, which is just above the spin frequency of 1122 Hz, suggested for XTE J1739-285 (reported by Kaaret et al., 2007) but not confirmed burst oscillation frequency. The typical accuracy of the results, monitored by the GRV2 virial error, is of the order of \( 10^{-6} \). Figure 4 shows stable configurations for the three models: no rotation (left panel), 700 Hz (middle panel) and 1200 Hz (right panel). For the 700 Hz spin frequency, the \( M(R) \) relations for Model1 and Model2 go beyond the shaded region representing the tentative observational errors. The deviation is stronger for less-massive stars, \( \lesssim 1.4 M_\odot \). As expected, for extremely rapidly-rotating NSs (1200 Hz) only objects with large masses, \( \approx 2 M_\odot \), are able to counterbalance the centrifugal force; all stable Model1 and Model2 configurations are outside the \( \Delta R = \pm 5\% \) region. In the following, we will discuss the scenario in which, when the measurements of spin are uncertain, configurations of different EOS and spin may have the same mass and radius. Such ambiguous configurations are marked with symbols on Fig. 4.
and ≈ 850 Hz for Model2) than in the case of massive NSs. If PSR J1748-2446ad is a light NS, its oblateness is between ≈ 0.73–0.83. In other words, a ΔR = ±5% accuracy in the radius measurement leads to ±8% accuracy in $O$ and to ±10% accuracy in $S$ estimation. For stars with $M = 2M_⊙$, the dependence of oblateness on EOSs is much weaker: at the spin frequency of PSR J1748-2446ad the oblateness is around 0.95 for all three models (±1% accuracy). For the faster rotation, $f ≈ 1200$ Hz, the error of the $O$ increases up to ±11%.

The NS surface area for masses $M ≈ 1M_⊙$ is almost constant with increasing rotational frequency. A significant increase of $S$ appears for $f ≥ 650$ Hz, close to the mass-shedding limit. Massive NSs are also not very susceptible to centrifugal force deformations. Significant deformations occur at ≈ 1000 Hz. For the whole range of $f$ (for NSs with $M ≈ 1M_⊙$ and $M ≈ 2M_⊙$), $S$ changes about ±10%. For PSR J1748-2446ad, for which the mass is unknown, the error in the surface area estimation is large: $S$ for Model2 $2M_⊙$ NS is twice as small as for Model1 $1M_⊙$ NS. As was mentioned in Sect. 3.4, a precise measurement of compactness $C$ leads to ±10% accuracy in the surface area estimation, if $f$ is known. This is comparable with the result that we get for low-mass NSs with unknown spins.

We also investigate central EOS parameters (pressure $P_c$, density $ρ_c$ and the baryon density $n_b$) for various masses, spin frequencies and EOS models. Their behaviour as a function of $f$ strongly depends on the $M$: for low-mass stars central EOS parameters are almost the same for a broad range of $f$ (Fig. 8). For $M = 1M_⊙$, for all EOSs and $f$, $P_c ≈ 7 \times 10^{35}$ dyne cm$^{-2}$ with negligible errors, $ρ_c ≈ 0.75 \times 10^{15}$ g cm$^{-3}$ ±10% and $n_b ≈ 4.1 \times 0.1$ fm$^{-3}$ ±15%. For NSs with masses $≈ 2M_⊙$, above few hundreds Hz, the decrease in the values of central parameters, with increasing rotational frequency, is rather fast especially for Model2. For example, the difference in $P_c$ between 0 Hz and 1400 Hz configurations is almost five times smaller (Fig. 8). This case, once again, shows how important the knowledge of $f$ is. Large uncertainties exist also between the interiors of various models: the difference between SLy4 and Model2 is ≈ 40% in $P_c$ and ≈ 35% in $ρ_c$ and $n_b$. The difference between SLy4 and Model1 is slightly less: ≈ 27% in $P_c$ and ≈ 20% in $ρ_c$ and $n_b$. Compared to global parameters $O$ and $S$, estimates of $P_c$, $ρ_c$ and $n_b$ result in larger differences. For larger spin frequencies, the central parameters converge to similar values.

### 3.4. Known rotational frequency

In Fig. 9 the relation between central pressure $P_c$ and mass $M$ is shown for different spin frequencies. $P_c$ slowly increases up
to the mass $\approx 1.9 M_\odot$, whereas for higher $M$ the growth of $P_c(M)$ is rapid. This is reflected in the uncertainty of the central parameters: for $M \leq 1.9 M_\odot$, differences between models for $P_c$ and $\rho_c$ are $\leq 10\%$ (this value increases up to 30\% when $f$ is unknown). For objects with higher masses ($\geq 1.9 M_\odot$), uncertainties in $P_c$ are around 50\%, if $f$ is known, and increase to 85\% if $f$ is unknown. Increasing uncertainty in central parameters at higher masses is related to the softening of the $P_c(M)$ curve due to general-relativistic effects near the maximum mass. Although massive stars are much more interesting from the point of view of the dense-matter EOS, they are more challenging to study than low-mass NSs.

In Fig.~9, the global angular momentum $J$ for NSs with $M = 1 M_\odot$ and $2 M_\odot$ is shown. For a specified mass, results for all three models are very similar for frequencies $\lesssim 1000$ Hz. Above this point one can observe a fast increase in $J$ until the object reaches the Keplerian frequency. Differences between models for low-mass and slowly-rotating massive NS are much smaller than for rapidly-rotating massive stars. The largest differences, about $\pm 20\%$, are present for sub-millisecond rotation rates.

According to Yagi & Yunes (2013a) there exists a universal (indepenent on the EOS) relation between the quadrupole moment $Q$ and the moment of inertia $I$. It might be used, for example, to determine rotational frequencies of NSs or distinguish between "normal" NSs and strange stars (see e.g., Urbanec, Miller & Stuchlík 2013; Yagi & Yunes 2013a), or employed in the description of binary NS inspiral waveforms. We use the following normalization: $I = I/M^6$ and $Q = Q/(M^3\chi^2)$, where $M$ is gravitational mass, $\chi = J/M^2$ and $J$ is angular momentum of the object. Our results presented in Fig.~11 show the $I-Q$ relation for the three models, for two different spin frequencies: 700 and 1200 Hz. The results are consistent with the $I-Q$ relation. This is especially true for massive stars (occupying the lower right corner of Fig.~11), which are less deformed by the centrifugal force.

3.5. ±10\% uncertainty in radius measurements

As mentioned in Sect. 1, ±5\% errors in $R$ may be considered too optimistic and a ±10\% uncertainty is a more realistic value (Lo et al. 2013; Miller & Lamb 2016). In order to study this case two additional piecewise polytropic EOSs with the SLy4 EOS crust were constructed. The parameters of the Model3 and Model4 EOSs are collected in Table 3.

Model3 and Model4 were chosen to reproduce $\Delta R = \pm 10\%$ errors of the radius measurements of the non-rotating reference SLy4 model. Results, as before, depend on the NS mass. Trends are similar to the ±5\% case (NSs with masses $\approx 1 M_\odot$ are more favourable for the estimations of the central parameters like $P_c, \rho_c, r_c$, whereas massive stars produce small errors in the determination of global parameters like $O$ and $S$). As expected, most of the uncertainties increase with increasing $\Delta R$.

For low-mass NSs, the accuracy of the $O$ determination is comparable with the result for the $\Delta R = \pm 5\%$ assumption (the oblateness for NSs with masses $\approx 1 M_\odot$ depends weakly on the accuracy of the $R$ measurements). For massive NSs, the $O$ uncertainty is $\approx 3\%$ for the frequency of the fastest-known pulsar.
and ≈ 18% for 1200 Hz (for the ±5% case the values were 1% and 11%, respectively). The errors in the estimation of the surface area $S$ do not depend on frequency: for the whole spin range they are similar and around 22% (twice what they are in the case of $\Delta R = \pm 5\%$).

As expected, uncertainties in the central parameters are smaller for low-mass stars. For NSs with $M \approx 1M_\odot$, $P_c \approx 7 \cdot 10^{15}$ dyn cm$^{-2}$ with negligible errors (similarly as for $\Delta R = \pm 5\%$), $\rho_c \approx 0.75 \cdot 10^{15}$ g cm$^{-3}$ ±30% (10% in case $\Delta R = \pm 5\%$) and $n_c \approx 4.1 \cdot 0.1$ fm$^{-3}$ ±30% (15% in case $\Delta R = \pm 5\%$). For massive stars again one can observe a very fast decrease in the central values with frequency, especially for Model4. An example of the central density $\rho_c$ as a function of the rotational frequency, for all five models $2M_\odot$, NS is shown on Fig. 12. Differences between SLy4 and Model4 are as follows: 64% in $P_c$, 38% in $\rho_c$, and 32% in $n_c$, and between SLy4 and Model3 are 33% in $P_c$, 19% in $\rho_c$, and 15% in $n_c$.

### 3.6. Tidal deformability

We also investigate the recent gravitational-wave estimate of the tidal deformabilities in the GW170817 binary NS system. Figure 13 reproduces the $\Lambda_1 - \Lambda_2$ relation for the deformabilities in the range of masses corresponding to the measured chirp mass $M = (M_1 M_2)^{1/3} / (M_1 + M_2)^{1/3} = 1.188_{-0.004}^{+0.004} M_\odot$, and the range of component masses in the case of low-spin priors, $M_1 = 1.36 - 1.60 M_\odot$ and $M_2 = 1.17 - 1.36 M_\odot$ (the definition of the tidal deformability $\Lambda$ was introduced in Sect. 2). For the three EOS models discussed here, the difference in radius $\Delta R = \pm 5\%$ is consistently reflected in the $\Delta \Lambda$ values: between 250 for 1.17 $M_\odot$, and 50 for 1.6 $M_\odot$. Increasing $\Delta R$ by a factor of two gives a two times larger $\Delta \Lambda$. This trend can be understood by analysing Fig. 2 where the $P(\rho)$ ranges for GW170817 component masses are displayed, as well as Fig. 13 where the relation between $R$, $M$, $\Lambda$ and $k_2$ is plotted. The EOSs yield, to first approximation, a constant difference in the stellar radius, and have similar stiffness in the relevant mass ranges, which is reflected in the $k_2$ values, while the difference in $\Lambda$ between models is dominated by the $M^{-3}$ term. Regular behaviour of $\Lambda$ with $R$ and $M$ may be exploited to approximately reproduce $R$, given $\Lambda$ and $M$ values. A linear relation $R \approx a(M)\Lambda + b(M)$ [km], where $a(M) = \sum_{i=0}^3 a_i M^i$, $b(M) = \sum_{i=0}^3 b_i M^i$ ($a_4 = 0.1332$, $a_3 = -0.6426$, $a_2 = 1.1886$, $a_1 = -0.9855$, $a_0 = 0.3076$, $b_1 = -0.1133$, $b_0 = 9.9216$, $M$ is the $M_\odot$ units) recovers the values of radii $R$ for the models considered here with an error of typically 0.1 km.

### 4. Discussion

In this work we have estimated how much information about the EOS parameters can be drawn from current and future measurements of neutron star radii, assuming a target accuracy of about 5%. To address this question, we have compared the widely accepted SLy4 EOS (Douchin & Haensel 2001), treated here as a reference EOS, with two parametric EOSs designed to yield TOV $M(R)$ sequences with ±5% of the SLy4 radius.

We have also considered the influence of rigid axisymmetric rotation on the global properties of the NSs and their central EOS parameters. In some cases, certain configurations may mimic other ones with different rotational frequencies and EOSs. We show that even if $M$ is established precisely and errors are present only in the measurement of $R$, lack of the information
masses $\approx 2M_\odot$ and known rotational frequency $f$, the uncertainties are much higher, up to 40%.

For cases when only compactness can be measured (e.g., because of the slow rotation of the object), it is possible to estimate surface area with $\sim 10\%$ accuracy, for sources with known spin frequency.

We take as an example PSR J1748-2446ad, and show how limited our reconstruction of the properties of a compact object is, when only its rotational frequency is known. Assuming, for the sake of an example, that the SLy4 EOS is the true EOS, one can expect that the oblateness lies between 0.72 and 0.97 for masses between 1 and $2M_\odot$. If we assume $\pm 5\%$ observational uncertainty on radius, it leads to an $\approx 30\%$ accuracy in the determination of $\rho_c$, $\approx 20\%$ for $n_k$, and $\approx 50\%$ accuracy for $P_c$, if this object is massive. If its mass is only $\approx 1M_\odot$, central parameters for various models are very similar to each other.

We repeated our study for the $\Delta R = 10\%$ case. As expected, uncertainties on the central and global parameters increase. Oblateness $O$ can be estimated with $\leq 18\%$ and $S$ with $\approx 22\%$ error, which are approximately twice the measurement uncertainties of the $\pm 5\%$ case. The accuracy of the estimation of the central parameters depends on the NS mass. For low-mass stars $P_c$ can be estimate with negligible errors, and $\rho_c$ and $n_k$ with $\approx 30\%$ uncertainties. For massive NSs uncertainties are up to 64% in $P_c$, 38% in $\rho_c$, and 32% in $n_k$. These results suggest that increasing $AR$ by a factor of two decreases the accuracy of the estimation of the central NS parameters by a factor of $\sim 2$ to 3.

We also estimate the effect of the radius uncertainty on the tidal deformability $\Lambda$, using as an example the estimated masses of the components from the GW170817 event (Abbott et al. 2017a). The $\pm 5\%$ radius difference of the parametric models with respect to the SLy4 model results in $\Delta \Lambda$ between 250 at 1.17 $M_\odot$ and 50 at 1.6 $M_\odot$ (these values are two times higher for $\Delta R = \pm 10\%$), and since the EOSs possess similar stiffness in the relevant regimes and hence similar behavior of the tidal Love numbers $k_2$ (Figs. 2 and 13), the $\Delta \Lambda$ is mostly influenced by the dependence of $\Lambda$ on mass $M$.

Linking the NS global parameters with the properties of the dense matter EOS is an area of active effort in view of forthcoming observational results. An extensive study on the full currently allowed range of NS radii will be the subject of future work.

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Fig. 13. (Color online) Upper row, left panel: Comparison of tidal deformabilities $\Lambda$ for stellar component masses compatible with the GW170817 observation (assuming low-spin priors, Abbott et al. 2017a). The shaded area denotes the estimated 90% confidence region corresponding to the measurement. Right panel: Stellar mass as a function of tidal deformability $\Lambda$, for the mass range between 1.15 $M_\odot$ and the $M_{\text{max}}$. Lower row, left panel: tidal deformability $\Lambda$ as a function of stellar radius; points mark 1.2, 1.4 and 1.6 $M_\odot$ (from top to bottom). Approximate formula denoted in the text is represented by black dotted curves. Right panel: mass as a function of the $k_2$ tidal Love number.

about $f$ might lead to 40% error in central pressure estimation (see Figs. 4 and 5).

Rapid rotation may be a crucial factor in the distinction between NSs with different EOSs. For spins comparable with the rotational frequency of the fastest known pulsar (716 Hz for PSR J1748-2446ad), the radii deviate by more than 5% from the reference SLy4 radius for stars with masses $\leq 1.4M_\odot$, whereas for $f = 1200$ Hz the radius difference is larger than 5% for the whole available NS mass range. As expected, for low-mass stars, rotation manifests itself strongly by deforming the star, but the central parameters are almost unchanged. A 5% accuracy in the measurement of $R$ leads to 8–10% errors in the estimation of the stellar oblateness $O$ and the surface area $S$. Even if the rotational frequency of these stars is unknown, their central properties can be established with $\pm 10\%$ accuracy. For more massive stars, with

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