SU(3) symmetry breaking in charmed baryon decays

C.Q. Geng\textsuperscript{1,2}, Y.K. Hsiao\textsuperscript{1,2}, Chia-Wei Liu\textsuperscript{2} and Tien-Hsueh Tsai\textsuperscript{2}

\textsuperscript{1}School of Physics and Information Engineering, Shanxi Normal University, Linfen 041004, China

\textsuperscript{2}Department of Physics, National Tsing Hua University, Hsinchu, Taiwan 300

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Abstract

We explore the breaking effects of the SU(3) flavor symmetry in the singly Cabibbo-suppressed anti-triplet charmed baryon decays of $B_c \rightarrow B_n M$, with $B_c = (\Xi_c^0, \Xi_c^+, \Lambda_c^+)$ and $B_n(M)$ the baryon (pseudo-scalar) octets. We find that these breaking effects can be used to account for the experimental data on the decay branching ratios of $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 K^+, \Lambda^0 K^+)$ and $R'_K/\pi = \mathcal{B}(\Xi_c^0 \rightarrow \Xi^- K^+)/\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+)$. In addition, we obtain that $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- K^+, \Sigma^- \pi^+) = (4.6 \pm 1.7, 12.8 \pm 3.1) \times 10^{-4}$, $\mathcal{B}(\Xi_c^0 \rightarrow p K^-, \Sigma^+ \pi^-) = (3.0 \pm 1.0, 5.2 \pm 1.6) \times 10^{-4}$ and $\mathcal{B}(\Xi_c^+ \rightarrow \Sigma^0(+) \pi^+(0)) = (10.3 \pm 1.7) \times 10^{-4}$, which all receive significant contributions from the breaking effects, and can be tested by the BESIII and LHCb experiments.
I. INTRODUCTION

It is known that the theoretical approach based on the factorization and quantum chromodynamics (QCD) barely explains the charmed hadron decays \[1\]. This is due to the fact that the mass of the charm quark, \(m_c \simeq 1.5\) \(\text{GeV}\), is not as heavy as that of the bottom one, \(m_b \simeq 4.8\) \(\text{GeV}\), resulting in an undetermined correction to the heavy quark expansion, such that the alternative models have to take place for this correction \[2, 7\]. On the other hand, the \(SU(3)\) flavor (\(SU(3)_f\)) symmetry that works in the \(b\)-hadron decays \[8–12\] can be well applied to \(D \to MM\) and \(B_c \to B_n M\) \[13–24\], where \(B_c = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+\) are the lowest-lying anti-triplet charmed baryon states, while \(B_n\) and \(M\) represent baryon and pseudoscalar meson states, respectively. Particularly, the \(SU(3)_f\) symmetry has been extended to investigate the singly charmed baryon sextet states as well as the doubly and triply charmed baryon ones \[22, 23\]. For \(D \to MM\) decays, the measurements produce \[25\]

\[
\begin{align*}
\mathcal{R}_{D^0(K/\pi)} & \equiv \frac{\mathcal{B}(D^0 \to K^+K^-)}{\mathcal{B}(D^0 \to \pi^+\pi^-)} = 2.82 \pm 0.07, \\
\mathcal{B}_{D^0(2K^0)} & \equiv \mathcal{B}(D^0 \to K_s^0K_s^0) = (1.70 \pm 0.12) \times 10^{-4},
\end{align*}
\]

in comparison with \((\mathcal{R}_{D^0(K/\pi)}, \mathcal{B}_{D^0(2K^0)}) \simeq (1, 0)\) given by the theoretical calculations based on the \(SU(3)_f\) symmetry. The disagreements between the theory and experiment imply that the breaking effects of the \(SU(3)_f\) symmetry cannot be ignored in the singly Cabibbo-suppressed (SCS) processes. We note that, in the literature, the \(SU(3)_f\) breaking effects were used to relate \(\mathcal{R}_{K/\pi}\) to the possible large difference of the \(CP\) violating asymmetries of \(\Delta A_{CP} \equiv A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-)\) \[14, 26, 27\], which is recently measured to be \((-0.10 \pm 0.08 \pm 0.03)\%\) by LHCb \[28\].

For the two-body \(B_c \to B_n M\) decays, both Cabibbo flavored (CF) and SCS decays are not well explained. In particular, the experimental measurements show that

\[
\begin{align*}
\mathcal{B}_{p\pi^0} & \equiv \mathcal{B}(\Lambda_c^+ \to p\pi^0) < 3 \times 10^{-4} \ (90\% \ \text{C.L.}) \quad [29, 30], \\
\mathcal{R}_{K/\pi}' & \equiv \frac{\mathcal{B}(\Xi_c^0 \to \Xi^-K^+)}{\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)} = 0.028 \pm 0.006 \simeq (0.6 \pm 0.2)s_c^2 \quad [25], \\
\mathcal{B}(\Lambda_c^+ \to \Lambda^0K^+) & = (6.1 \pm 1.2) \times 10^{-4} \quad [25], \\
\mathcal{B}(\Lambda_c^+ \to \Sigma^0K^+) & = (5.2 \pm 0.8) \times 10^{-4} \quad [25],
\end{align*}
\]

where \(s_c \equiv \sin \theta_c = 0.2248 \quad [25]\) with \(\theta_c\) the well-known Cabbibo angle. However, theoretical evaluations based on the \(SU(3)_f\) symmetry lead to \(\mathcal{B}_{p\pi^0} = (5.7 \pm 1.5) \times 10^{-4}\) and \(\mathcal{R}_{K/\pi}' \simeq\)
1.0s^2_c [20], and those in the factorization approach give \( B_{\pi^0} = f^2_\pi/(2f_K^2)s^2_c B(\Lambda^+_c \to pK^0) = (5.5 \pm 0.3) \times 10^{-4} \) and \( R^\prime_{K/\pi} = (f_K/f_\pi)^2 s^2_c \simeq 1.4s^2_c \), where we have used the data of \( B(\Lambda^+_c \to pK^0) = (3.16 \pm 0.16) \times 10^{-2} \) [23]. In addition, the fitted results of \( B(\Lambda^+_c \to \Lambda^0 K^+, \Sigma^0 K^+) = (4.6 \pm 0.9, 4.0 \pm 0.8) \times 10^{-4} \) [21] are (1.3 – 1.6)\( \sigma \) away from the data in Eq. (2). In this study, we will consider the breaking effects of the \( SU(3)_f \) symmetry due to the fact of \( m_s \gg m_{u,d} \) in the \( B_c \to B_n M \) decays, particularly, the SCS processes, in accordance with the \( D \to M M \) ones. Our goal is to find out whether the data in Eq. (2) can be understood by introducing the breaking effects.

The paper is organized as follows. In Sec. II, we provide the formalism, in which the amplitudes of the \( B_c \to B_n M \) decays with and without the breaking effects of \( SU(3)_f \) symmetry are presented. The numerical analysis is performed in Sec. III. In Sec. IV, we discuss our results and give the conclusions.

II. FORMALISM

The two-body charmed baryon weak decays, such as \( \Xi^0_c \to \Xi^- \pi^+ (\Xi^- K^+) \) and \( \Lambda^+_c \to p\pi^0 \), proceed through the quark-level transitions of \( c \to u \bar{d}, c \to u d \bar{d} \) and \( c \to u s \bar{s} \), with the effective Hamiltonian given by [31]

\[
\mathcal{H}_{\text{eff}} = \sum_{i=+,-} \frac{G_F}{\sqrt{2}} c_i \left( V_{cs} V_{ud} O_i + V_{cd} V_{ud} O^d_i + V_{cs} V_{us} O^s_i \right),
\]

where \( G_F \) is the Fermi constant, \( c_{\pm} \) are the scale-dependent Wilson coefficients, and the CKM matrix elements \( V_{cs} V_{ud} \simeq 1 \) and \( V_{cs} V_{us} \simeq -V_{cd} V_{ud} \simeq s_c \) correspond to the Cabibbo-favored (CF) and singly Cabibbo-suppressed (SCS) charmed hadron decays, respectively, while \( O^{(d,s)}_{\pm} \) are the four-quark operators, written as

\[
O_{\pm} = \frac{1}{2} \left[ (\bar{u}d)(\bar{s}c) \pm (\bar{s}d)(\bar{u}c) \right],
\]

\[
O^{d}_{\pm} = \frac{1}{2} \left[ (\bar{u}q)(\bar{q}c) \pm (\bar{q}q)(\bar{u}c) \right],
\]

where \( q = (d, s) \) and \( (\bar{q}_1 q_2) = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2 \). With \( q_i = (u, d, s) \) as the triplet of 3, the operator of \( (\bar{q}^i q_k \bar{q}^j)c \) can be decomposed as the irreducible forms, that is, \( (\bar{3} \times 3 \times \bar{3})c = (\bar{3} + 3' + 6 + 15)c \). Accordingly, the operators \( O^{(d,s)}_{\pm} \) and \( O^{(q)}_{\pm} \) belong to 6 and \( \bar{15} \), respectively,
given by \[16\]

\[
O_+ \simeq O_{6(15)} = \frac{1}{2}(\bar{u}d\bar{s} + \bar{s}d\bar{u})c, \\
O_0 \simeq O_{6(15)}^d = \frac{1}{2}(\bar{u}q\bar{q} + \bar{q}q\bar{u})c,
\]

such that the effective Hamiltonian in Eq. (3) can be transformed into the tensor form of the Hamiltonian

\[
\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[ c_{ij} \frac{e^{ij}}{2} H(6)_{ik} + c_+ H(15)^{ij}_k \right],
\]

with the non-zero entries:

\[
H_{22}(6) = 2, H_{21}^{13}(15) = H_{2}^{31}(15) = 1,
H_{23}(6) = H_{32}(6) = -2s_c, H_{12}^{13}(15) = H_{21}^{31}(15) = s_c,
\]

where the notations of \((i, j, k)\) are quark indices, to be connected to the initial and final states in the amplitudes. Note that \(H_{23}(6)\) and \(H_{32}(6)\) are derived from \(O_6^s\) and \(O_6^d\), respectively.

The lowest-lying charmed baryon states \(B_c\) are an anti-triplet of 3 to consist of \((ds - sd)c\), \((us - su)c\) and \((ud - du)c\), presented as

\[
B_c = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+),
\]

together with the baryon and meson octets, given by

\[
B_n = \begin{pmatrix}
\frac{1}{\sqrt{6}}\Lambda^0 + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\
\Sigma^- & \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0 & n \\
\Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0
\end{pmatrix},
\]

\[
M = \begin{pmatrix}
\frac{1}{\sqrt{2}}\pi^0 & \pi^- & K^- \\
\pi^+ & -\frac{1}{\sqrt{2}}\pi^0 & \bar{K}^0 \\
K^+ & \bar{K}^0 & 0
\end{pmatrix},
\]

where we have removed the octet \(\eta_8\) and singlet \(\eta_1\) meson states to simplify our discussions.

Subsequently, the amplitudes of \(B_c \rightarrow B_n M\) can be derived as

\[
\mathcal{A}(B_c \rightarrow B_n M) = \langle B_n M | \mathcal{H}_{eff} | B_c \rangle = \frac{G_F}{\sqrt{2}} T(B_c \rightarrow B_n M),
\]

with \(T(B_c \rightarrow B_n M) = T(\mathcal{H}_6) + T(\mathcal{H}_{15})\), where \(T(\mathcal{H}_{6,15})\) are decomposed as \[20–22\]

\[
T(\mathcal{H}_6) = a_1 H_{ij}(6) T^{ik} (B_n)_i^j (M)_k^j + a_2 H_{ij}(6) T^{ik} (M)_j^i (B_n)_i^j + a_3 H_{ij}(6) (B_n)_i^j (M)_k^j T^{kl},
\]

\[
T(\mathcal{H}_{15}) = a_4 H_{ij}^{13}(15) (B_n)_i^j (M)_k^j (B_c)_i^j + a_5 (B_n)_i^j (M)_j^i (B_c)_k + a_6 (B_n)_i^j (M)_j^i (B_c)_k + a_7 (B_n)_i^j (M)_j^i (B_c)_k,
\]

\[
T(\mathcal{H}_{15}) = a_4 H_{ij}^{13}(15) (B_n)_i^j (M)_k^j (B_c)_i^j + a_5 (B_n)_i^j (M)_j^i (B_c)_k + a_6 (B_n)_i^j (M)_j^i (B_c)_k + a_7 (B_n)_i^j (M)_j^i (B_c)_k,
\]

\[
T(\mathcal{H}_{15}) = a_4 H_{ij}^{13}(15) (B_n)_i^j (M)_k^j (B_c)_i^j + a_5 (B_n)_i^j (M)_j^i (B_c)_k + a_6 (B_n)_i^j (M)_j^i (B_c)_k + a_7 (B_n)_i^j (M)_j^i (B_c)_k,
\]
with $T^{ij} \equiv (B_c)_k \epsilon^{ijk}$. In Eq. (11), $a_{1,2,3}$ and $a_{4,5,6,7}$ are the $SU(3)$ parameters from $H(6)$ and $H(15)$, respectively, in which $c_{\mp}$ have been absorbed. We hence obtain

$$
T(\Lambda_c^+ \to p\pi^0) = -\sqrt{2}(a_2 + a_3 - \frac{a_6 - a_7}{2})s_c,
$$

$$
T(\Xi^0_c \to \Xi^-\pi^+) = 2(a_1 + \frac{a_5 + a_6}{2}),
$$

$$
T(\Xi^0_c \to \Xi^-K^+) = -2(a_1 + \frac{a_5 + a_6}{2})s_c,
$$

(12)

based on the exact $SU(3)_f$ symmetry.

According to Refs. [20, 21], the numerical analysis with the minimum $\chi^2$ fit has well explained the ten observed $B_c \to B_n M$ decays by neglecting the terms associated with $a_{4,5,6,7}$ to reduce the parameters [19–22, 24]. This reduction is due to the fact that the contributions to the branching rates from $H(15)$ and $H(6)$ lead to a small ratio of $R(15/6) = c_+^2/c_-^2 \simeq 17\%$ with $(c_+, c_-) = (0.76, 1.78)$ calculated at the scale $\mu = 1$ GeV in the NDR scheme [32, 33]. There remain two measurements to be explained. In Eq. (2), the prediction for $B_{n\pi^0}$ has the 2σ gap to reach the edge of the experimental upper bound. However, with $R(15/6)$ to be small, it is nearly impossible that, by restoring $a_{4,5,6,7}$ that have been ignored in the literature [19–22, 24], one can accommodate the data of $B_{n\pi^0}$ but without having impacts on the other decay modes, which are correlated with the same sets of parameters. Moreover, as seen from Eq. (12), there is no room for $R'_{K/\pi}$ as it is fixed to be $(1.0)s_c^2$. On the other hand, the results for $D \to MM$ decays in Eq. (11) suggest some possible corrections from the breaking effects of the $SU(3)_f$ symmetry in the SCS processes. In the charm baryon decays, we consider the similar effects. Due to $m_s \gg m_{u,d}$, we present the matrix of $M_s = \epsilon(\lambda_s)_{ij}$ [13] to break $SU(3)_f$, where $\epsilon \sim 0.2 - 0.3$ and $\lambda_s$ is given by [13, 14, 17]

$$
\lambda_s = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -2
\end{pmatrix},
$$

(13)

which transforms as an octet of 8, such that its coupling to $H(6)$ is in the form of $8 \times 6 = \bar{3} + 6 + 15 + 24$, and $\bar{3}$ is for the simplest break effects to be confined in the SCS processes [17, 34]. Note that from $\frac{1}{17} \epsilon_{ijk} \epsilon^{imn} (\lambda_s)_m^{(m) n} H(6)^n_{mo}$ and the nonzero entry of $H(3)^1 = s_c$ from the coupling of $H(6)_{23}$ and $H(6)_{32}$ [13], one can trace back to the break effect between SCS $c \to us\bar{s}$ and $c \to udd\bar{d}$ transitions. As a result, the $SU(3)_f$ symmetry breaking gives rise to
TABLE I: Amplitudes of $T(B_c \to B_n M)$, where $T$-amps refers to $T(B_c \to B_n M)$ and CF (SCS) represents Cabbibo favored (singly Cabbibo-suppressed).

| CF mode | $T$-amp | SCS mode | $T$-amp |
|---------|---------|----------|---------|
| $\Xi^0_c \to \Sigma^+ K^0$ | $2(a_2 + \frac{a_3 + a_4}{2})$ | $\Xi^0_c \to \Sigma^+ K^0$ | $2(a_2 + v_1 + v_3 + \frac{a_4 + a_5}{2})s_c$ |
| $\Xi^0_c \to \Sigma^0 K^0$ | $-\sqrt{2}(a_2 + a_3 - \frac{a_4 + a_5}{2})$ | $\Xi^0_c \to \Sigma^+ K^0$ | $2(a_1 + v_1 + v_2 + \frac{a_4 + a_5}{2})s_c$ |
| $\Xi^0_c \to \Xi^- \pi^+$ | $\sqrt{2}(a_1 - a_2 - \frac{a_4 + a_5}{2})$ | $\Xi^0_c \to \Sigma^0 K^0$ | $(a_2 + a_3 - 2v_1 - v_2 - v_3 - \frac{a_3 + a_5 - a_6 - a_7}{2})s_c$ |
| $\Xi^0_c \to \Lambda^0 K^0$ | $2(a_1 + \frac{a_3 + a_4}{2})$ | $\Xi^0_c \to \Xi^- K^0$ | $-2(a_1 - v_1 - v_2 + \frac{a_4 + a_5}{2})s_c$ |
| $\Xi^0_c \to \Lambda^0 K^0$ | $-\sqrt{2}(2a_1 - a_2 - a_3 + \frac{2a_1 - a_3 + a_5}{2})$ | $\Xi^0_c \to \Xi^0 K^0$ | $-2(a_2 - v_1 - v_2 + \frac{a_4 + a_5}{2})s_c$ |
| $\Xi^+ \to \Sigma^0 K^0$ | $2(a_3 - \frac{a_4 + a_5}{2})$ | $\Xi^0_c \to \Xi^0 K^0$ | $-2(a_1 - a_2 - a_3 + \frac{a_4 + a_5}{2})s_c$ |
| $\Xi^+ \to \Xi^0 \pi^+$ | $2(a_3 + \frac{a_4 + a_5}{2})$ | $\Xi^0_c \to \Xi^0 K^0$ | $2(a_1 - a_2 - a_3 + v_1 + \frac{a_4 + a_5}{2})s_c$ |
| $\Lambda^+_c \to \Sigma^0 K^+$ | $-\sqrt{2}(a_1 - a_2 - a_3 - \frac{a_4 + a_5}{2})$ | $\Xi^0_c \to \Xi^0 K^0$ | $2(a_1 - a_2 - a_3 - v_1 + \frac{a_4 + a_5}{2})s_c$ |
| $\Lambda^+_c \to \Sigma^+ K^0$ | $\sqrt{2}(a_1 - a_2 - a_3 - \frac{a_4 + a_5}{2})$ | $\Xi^0_c \to \Xi^0 K^0$ | $\sqrt{2}(-a_1 - a_2 + 2a_3 + v_2 + v_3 + \frac{a_4 + a_5 - a_6 - a_7}{2})s_c$ |
| $\Lambda^+_c \to \Xi^0 K^+$ | $-2a_2 + \frac{a_4 + a_5}{2}$ | $\Xi^0_c \to \Xi^0 K^0$ | $\sqrt{2}(-a_1 - a_2 + 2a_3 - v_2 + v_3 - \frac{a_4 + a_5 + a_6 - a_7}{2})s_c$ |
| $\Lambda^+_c \to \Xi^0 K^+$ | $-2a_2 + \frac{a_4 + a_5}{2}$ | $\Xi^0_c \to \Xi^0 K^0$ | $\sqrt{2}(-a_1 - a_2 + 2a_3 - v_2 + v_3 - \frac{a_4 + a_5 + a_6 - a_7}{2})s_c$ |
| $\Lambda^+_c \to \Xi^0 K^+$ | $-2a_2 + \frac{a_4 + a_5}{2}$ | $\Xi^0_c \to \Xi^0 K^0$ | $\sqrt{2}(-a_1 - a_2 + 2a_3 - v_2 + v_3 - \frac{a_4 + a_5 + a_6 - a_7}{2})s_c$ |
| $\Lambda^+_c \to \Xi^0 K^+$ | $-2a_2 + \frac{a_4 + a_5}{2}$ | $\Xi^0_c \to \Xi^0 K^0$ | $\sqrt{2}(-a_1 - a_2 + 2a_3 - v_2 + v_3 - \frac{a_4 + a_5 + a_6 - a_7}{2})s_c$ |
| $\Lambda^+_c \to \Lambda^0 K^+$ | $-2a_2 + \frac{a_4 + a_5}{2}$ | $\Xi^0_c \to \Xi^0 K^0$ | $\sqrt{2}(-a_1 - a_2 + 2a_3 - v_2 + v_3 - \frac{a_4 + a_5 + a_6 - a_7}{2})s_c$ |
| $\Lambda^+_c \to \Lambda^0 K^+$ | $-2a_2 + \frac{a_4 + a_5}{2}$ | $\Xi^0_c \to \Xi^0 K^0$ | $\sqrt{2}(-a_1 - a_2 + 2a_3 - v_2 + v_3 - \frac{a_4 + a_5 + a_6 - a_7}{2})s_c$ |

The new $T$-amplitudes, given by

$$T(\mathcal{H}_3) = v_1(B_c)H(3)^j(B_n)^k(M)^k_j + v_2(B_c)H(3)^j(B_n)^k(M)^k_j + v_3(B_c)H(3)^j(B_n)^k(M)^k_j,$$  \hspace{1cm} (14)

where $v_{1,2,3}$ are the parameters related to the $SU(3)_f$ breaking. It is interesting to note that the $v_1$ terms associated with $(B_c)_iH(3)^j$ in Eq. (14) occur in some of the $\Xi^0_c, \Lambda^+_c$ decays, but disappear in all $\Lambda^+_c$ modes. By adding $T(\mathcal{H}_3)$ into $T(B_c \to B_n M)$ in Eq. (10), the full expansions of $T(B_c \to B_n M)$ are given in Table I to be used to calculate the decay widths, given by [25]

$$\Gamma(B_c \to B_n M) = \frac{|\vec{p}_{cm}|}{8\pi m^2_{B_c}} |A(B_c \to B_n M)|^2,$$  \hspace{1cm} (15)

where $|\vec{p}_{cm}| = \sqrt{[(m_{B_c}^2 - (m_{B_n} + m_M)^2)](m_{B_c}^2 - (m_{B_n} - m_M)^2)/(2m_{B_c})}$. 


III. NUMERICAL ANALYSIS

In the numerical analysis, we examine \( B(\Lambda_c^+ \to \Lambda^0 K^+, \Sigma^0 K^+, p\pi^0) \) and \( R'_{K/π} \) by including the breaking effects of the \( SU(3)_f \) symmetry to see if one can explain their data in Eq. (2). The theoretical inputs for the CKM matrix elements are given by [25]

\[
(V_{cs}, V_{ud}, V_{us}, V_{cd}) = (1 - \lambda^2/2, 1 - \lambda^2/2, \lambda, -\lambda),
\]

with \( \lambda = 0.2248 \) in the Wolfenstein parameterization. We perform the minimum \( \chi^2 \) fit, in terms of the equation of [21]

\[
\chi^2 = \sum_i \left( \frac{B_{th}^i - B_{ex}^i}{\sigma_{ex}^i} \right)^2 + \sum_j \left( \frac{R_{th}^j - R_{ex}^j}{\sigma_{ex}^j} \right)^2,
\]

with \( B = B(\Lambda_c^+ \to B_n M) \) and \( R = B(\Xi_c^0 \to B_n M)/B(\Xi_c^0 \to \Xi^- π^+) \), where the subscripts \( th \) and \( ex \) are denoted as the theoretical inputs from the amplitudes in Table II and the experimental data points in Table II respectively, while \( \sigma_{i,j} \) correspond to the 1σ errors. By following Refs. [20–22], we extract the parameters, which are in fact complex numbers, given by

\[
a_1, a_2 e^{iδ_{a2}}, a_3 e^{iδ_{a3}}, v_1 e^{iδ_{v1}}, v_2 e^{iδ_{v2}}, v_3 e^{iδ_{v3}},
\]
where $a_{4,5,...,7}$ have been ignored as discussed in Sec. III. Since only the relative phases contribute to the branching ratios, $a_1$ is set to be real without losing generality. However, we take $v_i$ to be real numbers in order to fit 8 parameters with the 9 data points in Table III. Subsequently, the fit with the breaking effects in the $SU(3)_f$ symmetry yields

$$(a_1, a_2, a_3) = (0.252 \pm 0.005, 0.127 \pm 0.009, 0.091 \pm 0.015) \text{GeV}^3,$$

$$(\delta_{a_2}, \delta_{a_3}) = (73.0 \pm 27.3, 40.2 \pm 4.7)^\circ,$$

$$(v_1, v_2, v_3) = (0.090 \pm 0.032, -0.037 \pm 0.013, 0.025 \pm 0.012) \text{GeV}^3,$$ (19)

with $\chi^2/d.o.f = 3.0/1$, where $d.o.f$ represents the degree of freedom. Note that $a_{1,2,3}$ and their phases are nearly the same as those without the breaking of $SU(3)_f$ [21]. With the parameters in Eq. (19), we obtain the branching ratios of the CF and SCS $B_c \to B_n \Lambda$ decays, shown in Table III.

IV. DISCUSSIONS AND CONCLUSIONS

As seen from Tables III and IV, the breaking effects associated with $v_2$ and $v_3$ on the branching ratios of the SCS $\Lambda^+_c \to B_n \Lambda$ decays are at most around 30%, which is close to the naive estimation of $(f_K/f_\pi)^2 \simeq 40\%$. In particular, we get $B(\Lambda^+_c \to \Lambda^0K^+, \Sigma^0K^+) = (6.1 \pm 0.9, 5.2 \pm 0.7) \times 10^{-4}$, which explain the data in Eq. (2) well and alleviate the (1.3–1.6)$\sigma$ deviations by the fit with the exact $SU(3)_f$ symmetry [21]. Meanwhile, the branching ratios for the CF modes are fitted to be the same as those without the breaking except $B(\Lambda^+_c \to \Xi^0K^+)$, which is slightly different in order to account for the recent observational value given by BESIII [35].

Moreover, the fitted value of $R'_{K_2/\pi} = (0.6 \pm 0.2)s_c^2 = 0.03 \pm 0.01$ explains the data very well for the first time. This leads to the prediction of $B(\Xi^0_c \to \Xi^-K^+) = (4.6 \pm 1.7) \times 10^{-4}$, with $v_1 + v_2$ as the destructive contribution to reduce the value of $(7.6 \pm 0.4) \times 10^{-4}$ under the exact $SU(3)_f$ symmetry, whereas $B(\Xi^0_c \to \Sigma^-\pi^+) = (12.8 \pm 3.1) \times 10^{-4}$ receives the constructive contribution from $v_1 + v_2$, with $T(\Xi^0 \to \Xi^-K^+, \Sigma^-\pi^+) = \mp[a_1 \mp (v_1 + v_2)]s_c$. Since there are other similar interferences between $a_i$ and $v_i$, which come from $T(\Xi^0 \to pK^-, \Sigma^+\pi^-) = \mp[a_2 \mp (v_1 + v_3)]s_c$ and $T(\Xi^+ \to \Sigma^0\pi^+, \Sigma^+\pi^0) = \mp\sqrt{2}[(a_1 - a_2) + (v_2 - v_3)]s_c$, it is predicted that $B(\Xi^0_c \to pK^-, \Sigma^+\pi^-) = (3.0 \pm 1.0, 5.2 \pm 1.6) \times 10^{-4}$ and $B(\Xi^+ \to \Sigma^0\pi^+(\Sigma^+\pi^0)) = (10.3 \pm 1.7) \times 10^{-4}$. It is interesting to note that the important roles of the terms associated
TABLE III: The branching ratios of the $B_c \rightarrow B_s M$ decays, where the numbers with the dagger ($\dagger$) correspond to the reproductions of the experimental data input, instead of the predictions.

| CF mode       | Exact [21] | Broken | SCS mode       | Exact [21] | Broken |
|---------------|------------|--------|----------------|------------|--------|
| $10^3 B(\Xi_c^0 \rightarrow \Sigma^+ K^-)$ | 3.5 ± 0.9  | 3.8 ± 0.6 | $10^4 B(\Xi_c^0 \rightarrow \Sigma^+ \pi^-)$ | 2.0 ± 0.5  | 5.2 ± 1.6 |
| $10^3 B(\Xi_c^0 \rightarrow \Sigma^0 \bar{K}^0)$ | 4.7 ± 1.2  | 5.2 ± 0.8 | $10^4 B(\Xi_c^0 \rightarrow \Sigma^- \pi^+)$ | 9.0 ± 0.4  | 12.8 ± 3.1 |
| $10^3 B(\Xi_c^0 \rightarrow \Xi^0 \pi^0)$   | 4.3 ± 0.09 | 4.4 ± 0.4 | $10^4 B(\Xi_c^0 \rightarrow \Xi^0 \pi^-)$ | 3.2 ± 0.3  | 7.7 ± 2.2  |
| $10^3 B(\Xi_c^0 \rightarrow \Xi^- \pi^+)$   | 15.7 ± 0.7 | 15.2 ± 0.7| $10^4 B(\Xi_c^0 \rightarrow \Xi^- K^+)$  | 7.6 ± 0.4  | 4.6 ± 1.7  |
| $10^5 (\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0)$ | 8.3 ± 0.9  | 7.8 ± 0.5 | $10^4 B(\Xi_c^0 \rightarrow pK^-)$ | 2.1 ± 0.5  | 3.0 ± 1.0  |
| $10^3 B(\Xi_c^+ \rightarrow \Sigma^+ \bar{K}^0)$ | 8.0 ± 3.9  | 7.8 ± 2.7 | $10^4 B(\Xi_c^+ \rightarrow \Sigma^0 \pi^-)$ | 18.5 ± 2.2 | 10.3 ± 1.7 |
| $10^3 B(\Xi_c^+ \rightarrow \Xi^0 \pi^+)$   | 8.1 ± 4.0  | 7.9 ± 2.7 | $10^4 B(\Xi_c^+ \rightarrow \Sigma^+ \pi^0)$ | 18.5 ± 2.2 | 10.3 ± 1.7 |
| $10^2 B(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+)$ | (1.3 ± 0.2)$^\dagger$ | (1.3 ± 0.1)$^\dagger$ | $10^4 B(\Lambda_c^+ \rightarrow \Sigma^+ K^0)$ | 8.0 ± 1.6  | 10.4 ± 1.5 |
| $10^2 B(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0)$ | (1.3 ± 0.2)$^\dagger$ | (1.3 ± 0.1)$^\dagger$ | $10^4 B(\Lambda_c^+ \rightarrow \Sigma^0 K^+)$ | (4.0 ± 0.8)$^\dagger$ | (5.2 ± 0.7)$^\dagger$ |
| $10^2 B(\Lambda_c^+ \rightarrow \Xi^0 K^+)   $ | (0.5 ± 0.1)$^\dagger$ | (0.6 ± 0.1)$^\dagger$ | $10^4 B(\Lambda_c^+ \rightarrow p\pi^0)$ | 5.7 ± 1.5  | 5.4 ± 1.0  |
| $10^2 B(\Lambda_c^+ \rightarrow p\bar{K}^0)$ | (3.3 ± 0.2)$^\dagger$ | (3.2 ± 0.1)$^\dagger$ | $10^4 B(\Lambda_c^+ \rightarrow n\pi^+)$ | 11.3 ± 2.9 | 10.7 ± 1.9 |
| $10^2 B(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+)$ | (1.3 ± 0.2)$^\dagger$ | (1.3 ± 0.1)$^\dagger$ | $10^4 B(\Lambda_c^+ \rightarrow \Lambda^0 K^+)$ | (4.6 ± 0.9)$^\dagger$ | (6.1 ± 0.9)$^\dagger$ |

with $v_1$ in the $T$ amplitudes are also projected in the $\Xi_c^{0,+}$ modes, particularly, $\Xi_c^0 \rightarrow \Xi^- K^+$ and $\Xi_c^0 \rightarrow \Sigma^0 \pi^0$. Clearly, these SCS $\Xi_c$ decays all contain sizable $SU(3)_f$ breaking effects, and can be treated as golden modes to test the $SU(3)_f$ symmetry.

Finally, we remark that, even with the breaking effects, we are still unable to fit the data of $\Lambda_c^+ \rightarrow p\pi^0$ in Eq. (1) as our result for its branching ratio of $(5.4 \pm 1.0) \times 10^{-4}$, which is close to $(5.5 \pm 0.3) \times 10^{-4}$ from the factorization approach [24], is lower than the current experimental upper bound of $3 \times 10^{-4}$ [29, 30].
In sum, we have studied the singly Cabibbo-suppressed charmed baryon decays. We have shown that the breaking effect of the $SU(3)_f$ symmetry can be used to understand the experimental data of $B(\Lambda^+_c \rightarrow \Sigma^0 K^+, \Lambda^0 K^+)$ and $R_{K/\pi} = B(\Xi^0_c \rightarrow \Xi^- K^+)/B(\Xi^0_c \rightarrow \Xi^- \pi^+)$. With these effects, we have obtained that $B(\Xi^0_c \rightarrow \Xi^- K^+, \Sigma^+ \pi^-) = (3.0 \pm 1.0, 5.2 \pm 1.6) \times 10^{-4}$ and $B(\Xi^+_c \rightarrow \Sigma^0 \pi^+(\Sigma^0 \pi^0)) = (10.3 \pm 1.7) \times 10^{-4}$, which are quite different from those predicted by the approach with the exact $SU(3)_f$ symmetry. However, even with the breaking effects, our result for the branching ratio of $\Lambda^+_c \rightarrow p\pi^0$ is still higher than the current experimental upper bound, which clearly requires a close examination by a future dedicated experiment.

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