Plasma profile tomography for EAST based on integrated data analysis

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Abstract
In this study, a plasma profile reconstruction algorithm based on integrated data analysis (IDA) is proposed, which incorporates various diagnostics and can provide two-dimensional distributions of plasma current and electron density. The IDA algorithm based on Bayesian inference combines limited data from multiple diagnostics and builds models in a probabilistic manner, overcoming the limitations of models based on just external magnetic diagnostics and providing more accurate results. To reduce the probability of unreasonable solutions, two Gaussian priors are established: conditional autoregressive prior and squared exponential kernel function prior, which constrain the plasma current and electron density, respectively. Compared to the models based on only magnetic diagnostics, the IDA model improves the current distribution in the core and increases the accuracy of plasma profile reconstruction.

Keywords: tomography, integrated data analysis, Bayesian inference, plasma

(Some figures may appear in colour only in the online journal)

1. Introduction
The distribution of plasma current plays an important role in the study of magnetic confinement fusion as it determines the magnetic field geometry of the plasma. The magnetic field geometry can provide the locations of plasma boundary, magnetic axis, X-point, etc, which can help in realizing an accurate plasma control. In addition, it provides the canonical coordinate system and magnetic surface coordinates [1]. The reconstructions of some profile parameters are based on the magnetic surface coordinates, for example electron density [2]. The assumption of constancy on flux surfaces of certain physics parameters can reduce the dimensionality of models from two-/three-dimensional (2D/3D) to one-dimensional (1D), thereby decreasing the number of measurements necessary for the spatial inference of these quantities [3].

In the experimental advanced superconducting tokamak (EAST), the plasma current reconstruction is usually realized by equilibrium fitting (EFIT) [4] code, which interleaves the equilibrium and fitting iterations with a Picard linearization scheme to find the optimum solution by allowing a distributed plasma current source constrained by magneto-hydrodynamic (MHD) equilibrium [5]. EFIT is used to compute the distributions in the R, Z plane of the poloidal flux and the toroidal current density, that provide a least squares best fit to the diagnostic data and that simultaneously satisfy the model given by the MHD equilibrium equation. However, it has been proved that the plasma current profile cannot be accurately inverted by only using external magnetic
diagnostics [6]. Therefore, based on EFIT, kinetic equilibrium fitting (K-EFIT) code has been developed [7]. K-EFIT code is not only constrained by the magnetic diagnostics but also by the internal pressure profile and edge current profile. The pressure and edge current profiles are determined using the diagnostics and theoretical bootstrap current model. The pressure profile can be obtained by Thomson scattering, electron cyclotron emission, charge exchange recombination spectroscopy, x-ray imaging crystal spectrometry, fast-ion D-alpha, NUBEAM calculation, etc and the edge current profile is obtained by motional Stark effect, lithium beam emission spectroscopy, soft x-ray measurements, etc [8]. The process of separately analyzing individual physics measurements and then combining them into kinetic constraints and applying them to K-EFIT [9] contrasts with integrated data analysis (IDA) [10], where the same problem is addressed by instead integrating analysis of raw data from different diagnostics into a single, consistent process. IDA can make full use of limited data, the idea of IDA can be traced back to 1985 [11]. IDA is generally based on a Bayesian framework, integrates the diagnostic systems through a joint probability density, and the probability distribution is restricted by constructing a suitable prior probability. The optimal plasma profile is obtained when the joint probability density function becomes maximum. In addition, Minerva in JET device is an IDA platform based on Bayesian inference [12]. Compared with K-EFIT, IDA based on Bayesian inference is more flexible, and easier to integrate diagnostics by means of joint probability density. When integrating multiple diagnostics, the error of diagnostics can be considered uniformly. IDA based on Bayesian inference can not only give reconstruction results, but also quantitatively give uncertainties. In this study, IDA based on Bayesian inference can rapidly give the distribution of toroidal current density and poloidal flux in a probabilistic manner without iteratively solving MHD equilibrium equation to find an optimal fit for the external magnetic diagnostic measurements.

In a previous report, a tomographic technique for plasma current reconstruction based on magnetic diagnostics has been established for EAST [13]. Since the plasma current in the core lacks the diagnostic constraint, the reconstruction error of the plasma current is relatively large. In this study, polarimetric interferometry (POINT) technique is introduced into plasma current tomography to provide the constraints of the core and improve the reconstruction accuracy. Since the POINT signal is determined by the plasma current and electron density, the distribution of electron density is first required for obtaining a signal of plasma current constraint. A hydrogen cyanide (HCN) laser interferometer is also employed for a more accurate reconstruction of electron density profile.

The rest of the paper is organized as follows. Firstly, the diagnostics required in the tomography process are introduced in section 2. The tomographic method for plasma profile reconstruction (including the electron density and plasma current profiles) is established in section 3. The plasma profile tomography method based on IDA is validated in section 4. Finally, the study is concluded in section 5.

2. Diagnostics on EAST

2.1. Polarimetric interferometry

A multichannel far-infrared laser-based POINT system based on the three-wave technique is under development at EAST [14, 15]. As shown by the pink dotted line in figure 1, the POINT system consists of 11 horizontal double-pass chords equally spaced from $Z = -0.425$ to $Z = +0.425$ m. POINT system launches electromagnetic waves into the plasma and measures the phase difference $\phi$ and polarization angle $\psi$ between these injected waves and the reference wave. The phase difference is related to the electron density $n_e$ along the path of the electromagnetic waves, and the polarization angle is related to the magnetic field strength $B$ and electron density $n_e$ along the path of the electromagnetic waves. They are defined in equations (1) and (2), respectively.

$$\phi = 2.82 \times 10^{-15} \lambda \int n_e dl \quad (1)$$
$$\psi = 2.62 \times 10^{-13} \lambda^2 \int n_e B_\parallel dl \quad (2)$$

where $\lambda$ is the wavelength of the injected electromagnetic waves and $B_\parallel$ represents the magnetic field strength parallel to the incident direction of the electromagnetic waves. $l$ indicates the path of electromagnetic waves. Since the electromagnetic waves are injected into the plasma along the horizontal direction, $B_\parallel$ is equal to the component of the magnetic field strength in the direction $R$. According to equations (1) and (2), the phase difference and polarization angle are proportional to the line integrals $\int n_e dl$ and $\int n_e B_\parallel dl$, respectively, along the path of the electromagnetic waves.

2.2. HCN laser interferometer

A three-channel far-infrared HCN laser interferometer is routinely employed for the precise measurement of electron density profiles [16–18]. HCN provides the integrated electron density signal, as shown by the black dashed lines in figure 1. It is an integral in the $Z$ direction, as shown in equation (3), which is different from the POINT signal.

$$\phi = \frac{\pi}{\lambda n_e} \int_{z_1}^{z_2} n_e(z) dz \quad (3)$$

where $\lambda$ is the wavelength of injected electromagnetic waves. $n_e$ is the cutoff density, which can be calculated by equation (4). $\omega$ is the frequency of injected electromagnetic waves. $m_e$ and $e$ are electron mass and electron charge, respectively. $\varepsilon_0$ is the permittivity, which is a constant.

$$n_e = \frac{\omega^2 \varepsilon_0 m_e}{e^2} \quad (4)$$

2.3. Magnetic diagnostics

The magnetic diagnostic systems include the pickup coils, flux loops, and Rogowski loops. As shown in figure 1, there are 38
pickup coils (red dot) for local magnetic field measurement and 35 flux loops (blue asterisk) for magnetic flux measurement. They are all functions of the position \((R, Z)\), as shown in equations (5) and (6).

\[
D_{\text{pickup}} = B_R(R, Z) \cos \theta + B_Z(R, Z) \sin \theta \tag{5}
\]

\[
D_{\text{fluxloop}} = \psi(R, Z) \tag{6}
\]

where \(B_R\) and \(B_Z\) are the \(R\) and \(Z\) directions of the magnetic field at the position with angle \(\theta\). \(\psi\) is the magnetic flux at \((R, Z)\). Rogowski loop is used to measure the total plasma current, as shown in equation (7). \(I_{R,Z}\) represents the plasma current at\((R, Z)\).

\[
D_{\text{Rogowski}} = \sum I_{R,Z} \tag{7}
\]

3. Integrated data analysis model

3.1. Bayesian theorem

The tomographic reconstruction based on Bayesian inference [12] can be realized by defining a joint probability of unknown parameters and observations \(P(H, D)\). The joint probability includes the predictive probability \(P(D|H)\) and prior probability \(P(H)\), which can be expressed as follows:

\[
P(H, D) = P(D|H) \cdot P(H). \tag{8}
\]

The prior probability \(P(H)\) encodes the prior knowledge of the unknown parameters such as physical/empirical assumptions, that assumed that the prior expected value of a given plasma current is approximately equal to the mean value of its four neighbors [13]. The predictive probability \(P(D|H)\) is
also called the likelihood probability in Bayesian inference, which reflects the relationship between the observations $D$ and unknown parameters $H$. In general, when an unknown parameter is defined, the observation value is calculated through a physical formula (or forward model). In actual applications, it is generally desirable to obtain unknown parameters based on observation, namely $P(H \mid D)$. Generally, the optimal solution of the model is achieved when the posterior probability $P(H \mid D)$ becomes maximum. The posterior probability $P(H \mid D)$ is obtained through the Bayes formula as follows:

$$P(H \mid D) = \frac{P(D,H)}{P(D)} = \frac{P(D \mid H)P(H)}{P(D)}$$

(9)

where $P(D) = \int P(D \mid H)P(H) dH$ which is a marginal probability of the observation, also known as the model evidence. $P(D)$ is the normalization constant, which does not affect the peak value of posterior probability. Therefore, it is usually ignored. In many cases, the number of unknown parameters is always more than the number of observations, which is an ill-posed problem in mathematics. Bayesian inference congregates the prior knowledge (prior probability) and measured information (likelihood probability) and objectively provides the occurrence probability of the target knowledge (posterior probability).

3.2. Boundary constraints

The EAST device is shown in figure 2. The region containing the plasma is within the first wall (the green portion). The plasma is represented as a grid of beams, and the beams fill the entire volume available to the plasma. It is assumed that the plasma current or the electron density of each beam is uniform. Considering axisymmetric equilibrium, the 3D problem can be converted to a 2D problem. The plasma current or electron density on each grid point is an unknown parameter that needs to be determined. The number of diagnostics is much lower than the number of unknown parameters. To improve the reconstruction accuracy, it is necessary to eliminate the grid points without plasma current and electron density. To reduce the number of grid points, von Nessi et al proposed to invert the current density inside the last closed flux surface [19]. The same strategy can also be used for electron density region. The initial grid boundary is the first wall, and the plasma boundary is obtained after inverting the plasma current. The plasma boundary is used as the new grid boundary for reconstruction until the convergence condition is met.

3.3. Gaussian process prior

A suitable Gaussian process (GP) prior can improve the accuracy of the model because it constrains the plasma current distribution by the covariance (smoothness) function and exclude unreasonable solutions of model [12]. When constructing the prior probability, it is often necessary to optimize the hyperparameters, such as $\tau$ in conditional autoregressive (CAR) prior, $\sigma$ and $\ell$ in squared exponential (SE) prior. The choice of suitable hyperparameters is a key issue for the plasma profile tomography, as they determine the degree of smoothness of the reconstructed plasma current or electron density. In general, the hyperparameters are determined from the data by maximizing the evidence and the detailed procedure is presented in the literature [13]. To improve the reconstruction results, two different prior distributions are employed for plasma current and electron density, respectively.
3.3.1. Conditional autoregressive prior for plasma current. CAR prior is used for the reconstruction of plasma current. It is assumed that the prior expected value of a given current is approximately equal to the mean value of its four neighbors [13], i.e.

\[ f^j = 1/4 \left( f^{i-1,j} + f^{i,j-1} + f^{i,j+1} + f^{i+1,j} \right). \] (10)

For a particular grid point \( f^n \), the distribution is represented as follows:

\[ P(f^n | f^{-n}) \propto \exp \left( \frac{\| f^n - \sum_{m} \beta^{n,m} f^m \|^2}{2 \sigma^2} \right) \tag{11} \]

where \( \beta^{n,m} = 0 \), if \( n = m \)
\[ \beta^{n,m} = -\frac{\partial \psi}{\partial x}, \] if \( n \neq m. \] (12)

\( f^{-n} \) includes all the four neighbors of \( f^n \). All the variances \( \tau \) are set to be equal for the prior in our model. The CAR prior provides the covariance term, which expresses the current variation around the following mean:

\[ \bar{Q} = \frac{1}{\tau} \left( I - \frac{1}{4} \bar{W} \right) \tag{13} \]

where \( \bar{I} \) is the identity matrix, and \( \bar{W} \) is the adjacent matrix. If \( n \) and \( m \) are adjacent, then \( \bar{W}^{n,m} = 1 \); otherwise, it is 0. To reduce manual intervention, the non-informative prior mean is assigned to 0, thus the prior probability distribution can be written as follows:

\[ P(f) = \frac{1}{(2\pi)^2 |\bar{Q}|^{\frac{1}{2}}} \exp \left( -\frac{1}{2} f^T \bar{Q} f \right). \tag{14} \]

CAR prior enforces smoothness between the adjacent currents while minimizing the spatial long-range effects due to the manipulation of current in a selected grid point.

3.3.2. Squared exponential kernel prior for electron density. SE kernel prior is also known as the GP prior. Owing to its excellent features, it has become the default kernel of Gaussian distribution in machine learning algorithms such as support vector machine [20]. SE kernel is defined in equation (15). It determines the covariance matrix of the prior probability.

\[ K_{SE}(\vec{x}, \vec{x'}) = \sigma^2 \exp \left( -\frac{(\vec{x} - \vec{x'})^2}{2\ell^2} \right) \tag{15} \]

where \( \vec{x'} \) represents the location of present grid point and \( \vec{x} \) is any other location. \( \sigma \) is the standard deviation, determining the difference between the function and its average, which is a hyperparameter. \( \ell \) is characteristic length scale, which is also a hyperparameter. The core idea adopted for the magnetic flux surface geometry is that the correlation of the electron density between any two grid points depends on the distance between these grid points. Therefore, the position \( \vec{x} \) in the SE kernel function is converted to magnetic surface coordinates \( \vec{\psi} \). Based on this kernel function, \( \bar{\Sigma}_{\psi} \), is constructed.

\[ \bar{\Sigma}_{\psi} = \begin{pmatrix} K(\vec{\psi}, \vec{\psi}) & \cdots & K(\vec{\psi}, \vec{\psi}^n) \\ \vdots & \ddots & \vdots \\ K(\vec{\psi}, \vec{\psi}) & \cdots & K(\vec{\psi}, \vec{\psi}^n) \end{pmatrix}. \tag{16} \]

The mean of SE prior is set to \( \vec{0} \) like CAR prior. Accordingly, the prior probability of the electron density is obtained as follows:

\[ P(\bar{n}) = \frac{1}{(2\pi)^{\frac{n_c}{2}} |\bar{\Sigma}_{\psi}|^{\frac{1}{2}}} \exp \left( -\frac{1}{2} \bar{n}^T \bar{\Sigma}_{\psi}^{-1} \bar{n} \right). \tag{17} \]

3.4. Likelihood probability

3.4.1. POINT. As mentioned in section 2.1, POINT can provide two kinds of signals. According to these signals, two kinds of likelihood probability are established. For the phase difference \( \phi \), the likelihood probability is the integral signal of electron density \( \bar{n}_x \), which can be written in the following matrix form:

\[ \bar{D}_\phi = \bar{R}_\phi \cdot \bar{n}_x + \bar{\varepsilon}_\phi \tag{18} \]

where \( \bar{R}_\phi \) is the geometric matrix, whose element \( \bar{R}_{ij} \) represents the physical length of chord \( i \) through the grid point \( j \). \( \bar{\varepsilon}_\phi \) denotes an error term to account for the measurement uncertainty, which is usually limited to statistical errors only. According to the central limit theorem, it can be considered that \( \bar{\varepsilon}_\phi \) satisfies the Gaussian distribution. Thus, the likelihood probability can be written as

\[ P(D_\phi | \bar{n}_x) = \frac{1}{(2\pi)^{\frac{\frac{n_c}{2}}{2}} |\bar{\Sigma}_\phi|^{\frac{1}{2}}} \times \exp \left[ -\frac{1}{2} \left( \bar{R}_\phi \cdot \bar{n}_x - \bar{D}_\phi \right)^T \right. \bar{\Sigma}_\phi^{-1} \left. \left( \bar{R}_\phi \cdot \bar{n}_x - \bar{D}_\phi \right) \right]. \tag{19} \]

\( \bar{\Sigma}_\phi \) is the covariance matrix of POINT measurement. Since the diagnostic probabilities are independent of each other, the covariance matrix is a diagonal matrix, where the element value corresponds to the diagnostic noise level.

The polarization angle \( \psi \) contains the information of both magnetic field and electron density. Therefore, we can write

\[ \bar{D}_\psi = \bar{R}_\psi \cdot (\text{diag}(\bar{n}_c) \cdot \bar{B}_\parallel) + \bar{\varepsilon}_\psi \tag{20} \]

\( \bar{\varepsilon}_\psi \) is the error term and \( \bar{R}_\psi \) is the geometry matrix. \( \text{diag}(\bar{n}_c) \) represents a diagonal matrix with elements of vector \( \bar{n}_c \). The magnetic field \( \bar{B}_\parallel \) is induced by the poloidal field (PF) coil current and plasma current. The PF coil current can be directly obtained by measurement, while the plasma current \( \bar{I} \) is unknown. There is a linear relationship between the magnetic
field and plasma current, which can be described by the forward model as follows [21]:

$$\tilde{B}_\parallel = \tilde{R}I + \tilde{C}. \tag{21}$$

Here, $\tilde{R}$ is the coefficient matrix, which is equivalent to Green function of EFIT code [22]. $\tilde{C}$ is the contribution of PF coil current to the magnetic field. Combining equations (20) and (21), equation (22) can be obtained. If the electron density $\tilde{n}_e$ is known, the equation can be further simplified as follows:

$$\bar{D}_\psi = \bar{R}_\psi \cdot \text{diag}(\tilde{n}_e) \cdot (\tilde{R}I + \tilde{C}) + \tilde{\varepsilon}_\psi \tag{22}$$

where

$$\bar{R}_\psi = \bar{R}_\psi \cdot \text{diag}(\tilde{n}_e) \cdot \bar{R}$$

$$\bar{C}' = \bar{R}_\psi \cdot \text{diag}(\tilde{n}_e) \cdot \bar{C}. \tag{23}$$

It is considered that $\tilde{\varepsilon}_\psi$ satisfies the Gaussian distribution, and the likelihood probability can be written as:

$$P(\bar{D}_\psi \mid I) = \frac{1}{(2\pi)^{\frac{D}{2}} |\bar{\Sigma}_\psi|^{\frac{1}{2}}} \times \exp \left[-\frac{1}{2} \left(\bar{R}_\psi \cdot \bar{I} + \bar{C}' + \bar{D}_\psi \right)^T \bar{\Sigma}_\psi^{-1} \left(\bar{R}_\psi \cdot \bar{I} + \bar{C}' - \bar{D}_\psi \right) \right]. \tag{24}$$

3.4.2. HCN. As introduced in section 2.2, HCN laser interferometer also provides an integral signal of electron density. Using the same process employed for the phase difference of POINT, equation (25) can be obtained.

$$\bar{D}_{HCN} = \bar{R}_{HCN} \cdot \tilde{n}_e + \tilde{\varepsilon}_{HCN}. \tag{25}$$

$\bar{R}_{HCN}$ is the geometric matrix, and $\tilde{\varepsilon}_{HCN}$ is the measurement error of diagnostics. $\tilde{\varepsilon}_{HCN}$ also satisfies the Gaussian distribution, so the likelihood probability can be obtained as follows:

$$P(\bar{D}_{HCN} \mid \tilde{n}_e) = \frac{1}{(2\pi)^{\frac{D}{2}} |\bar{\Sigma}_{HCN}|^{\frac{1}{2}}} \times \exp \left[-\frac{1}{2} \left(\bar{R}_{HCN} \cdot \tilde{n}_e - \bar{D}_{HCN} \right)^T \bar{\Sigma}_{HCN}^{-1} \left(\bar{R}_{HCN} \cdot \tilde{n}_e - \bar{D}_{HCN} \right) \right]. \tag{26}$$

3.4.3. Magnetic diagnostics. The pickup coils, flux loops, and Rogowski loops can be expressed by the vector $\bar{D}_{Mag}$, and a linear relationship exists between the plasma current and these diagnostic values [13], which can be expressed in the following matrix form:

$$\bar{D}_{Mag} = \bar{R}_{Mag}I + \bar{C}_{Mag} + \tilde{\varepsilon}_{Mag}. \tag{27}$$

Here, $\bar{R}_{Mag}$ is the response matrix, and $\bar{C}_{Mag}$ is the contribution of PF coil current to the magnetic diagnostic signal. $\tilde{\varepsilon}_{Mag}$ is the error term of magnetic diagnostics, which satisfies the Gaussian distribution. The likelihood probability of magnetic diagnostics is shown in equation (28), where $\bar{\Sigma}_{Mag}$ is the covariance matrix of Gaussian distribution and is also a diagonal matrix.

$$P(\bar{D}_{Mag} \mid \bar{I}) = \frac{1}{(2\pi)^{\frac{D}{2}} |\bar{\Sigma}_{Mag}|^{\frac{1}{2}}} \times \exp \left(-\frac{1}{2} \left(\bar{R}_{Mag} \bar{I} + \bar{C}_{Mag} - \bar{D}_{Mag} \right)^T \bar{\Sigma}_{Mag}^{-1} \left(\bar{R}_{Mag} \bar{I} + \bar{C}_{Mag} - \bar{D}_{Mag} \right) \right). \tag{28}$$

3.5. Model

In this study, the obervation value is composed of signals from POINT, HCN interferometry, and magnetic diagnostics, and the unknown parameters are plasma current $I$ and electron density $\tilde{n}_e$. Since the polarization angle $\psi$ of POINT contains the information of both plasma current $I$ and electron density $\tilde{n}_e$, the two kinds of information need to be separated. The solution of the model can be obtained by iterations, as shown in figure 3. The main steps are as follows:

Step 1: The first wall is used as an initial boundary of grid, and the magnetic diagnostic signals are used as the input. Further, the plasma current $I$ of each grid point is calculated by the Bayesian model.

Step 2: The plasma boundary and magnetic flux are calculated by the plasma current and PF coil current through the Biot–Savart’s law (PF coil current is known).

Step 3: The plasma boundary in Step 2 is used as the new boundary of grid, and the prior probability of electron density is constructed according to the magnetic flux. Next, based on the HCN diagnostics and phase difference $\bar{\phi}$ of POINT, the electron density $\tilde{n}_e$ of each grid point is calculated.

Step 4: Based on the magnetic diagnostics, polarization angle $\bar{\psi}$ of POINT, and the obtained electron density in Step 3, the new plasma current distribution is determined.

Step 5: The difference between the two plasma currents is calculated. If the difference meets the convergence condition, the cycle is ended. Otherwise, the plasma current is used in the second step, and the cycle is repeated.

In Step 1, the Bayesian model includes the likelihood probability of magnetic diagnostics and CAR prior, which can be written as

$$P(I \mid \bar{D}_{Mag}) \propto P(\bar{D}_{Mag} \mid I)P(I) \times \exp \left(-\frac{1}{2} \left(\bar{R}_{Mag} \bar{I} + \bar{C}_{Mag} - \bar{D}_{Mag} \right)^T \bar{\Sigma}_{Mag}^{-1} \left(\bar{R}_{Mag} \bar{I} + \bar{C}_{Mag} - \bar{D}_{Mag} \right) - \frac{1}{2} \bar{Q}^T \bar{Q} \right). \tag{29}$$

Since the product of two GP functions is also a GP function, the posterior term can be written as a multivariate Gaussian distribution over $I$, i.e.
Figure 3. Flow chart of IDA.
Virtual diagnostic signals can be calculated and used as inputs.

\[ P(I \mid \hat{D}_{\text{Mag}}) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma_1|^2} \exp \left[ -\frac{1}{2}(I - \hat{m}_1)^T \Sigma_1^{-1}(I - \hat{m}_1) \right] \]  
\[(30)\]

where

\[ \hat{m}_1 = \left( \bar{R}^T_{\text{Mag}} \Sigma_{\text{Mag}}^{-1} R_{\text{Mag}} + \bar{Q} \right)^{-1} \bar{R}^T_{\text{Mag}} \Sigma_{\text{Mag}}^{-1} \left( D_{\text{Mag}} - C_{\text{Mag}} \right) \]
\[ \bar{\Sigma}_1 = \left( \bar{R}^T_{\text{Mag}} \Sigma_{\text{Mag}}^{-1} R_{\text{Mag}} + \bar{Q} \right)^{-1}. \]  
\[(31)\]

The maximum a posteriori (MAP) provides the most probable value of \( I \) equal to \( \hat{m}_1 \). In Step 3 and Step 4, the Bayesian model can be written as

\[ P(I \mid \hat{D}_\psi, D_{\text{Mag}}) \propto P(D_{\psi}, D_{\text{Mag}} \mid I) P(I) \]
\[ \propto P(D_{\psi} \mid I) P(D_{\text{Mag}} \mid I) P(I) \]
\[ P(\hat{n}_e \mid \hat{D}_\psi, D_{\text{HCN}}) \propto P(D_{\psi}, D_{\text{HCN}} \mid \hat{n}_e) P(\hat{n}_e) \]
\[ \propto P(D_{\psi} \mid \hat{n}_e) P(D_{\text{HCN}} \mid \hat{n}_e) P(\hat{n}_e). \]  
\[(32)\]

The product of the three GP functions is still a GP function, so their posterior probabilities can be written as follows:

\[ P(\hat{n}_e \mid \hat{D}_\psi, D_{\text{HCN}}) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma_2|^2} \exp \left[ -\frac{1}{2} (\hat{n}_e - \hat{m}_2)^T \Sigma_2^{-1} (\hat{n}_e - \hat{m}_2) \right] \]  
\[(33)\]

where

\[ m_2 = \left( \bar{R}^T_{\psi} \Sigma_{\psi}^{-1} R_{\psi} + \bar{R}^T_{\text{HCN}} \Sigma_{\text{HCN}}^{-1} R_{\text{HCN}} + \Sigma_{n_e}^{-1} \right)^{-1} \]
\[ \times \left( \bar{R}^T_{\psi} \Sigma_{\psi}^{-1} D_{\psi} + \bar{R}^T_{\text{HCN}} \Sigma_{\text{HCN}}^{-1} R_{\text{HCN}} D_{\text{HCN}} \right) \]
\[ \bar{\Sigma}_2 = \left( \bar{R}^T_{\psi} \Sigma_{\psi}^{-1} R_{\psi} + \bar{R}^T_{\text{HCN}} \Sigma_{\text{HCN}}^{-1} R_{\text{HCN}} \right)^{-1}. \]  
\[(34)\]

\[ P(I \mid \hat{D}_\psi, D_{\text{Mag}}) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma_3|^2} \exp \left[ -\frac{1}{2} (I - \hat{m}_3)^T \Sigma_3^{-1} (I - \hat{m}_3) \right]. \]  
\[(35)\]

Here,

\[ m_3 = \left( \bar{R}^T_{\psi} \Sigma_{\psi}^{-1} R_{\psi} + \bar{R}^T_{\text{Mag}} \Sigma_{\text{Mag}}^{-1} R_{\text{Mag}} + \bar{Q} \right)^{-1} \]
\[ \times \left( \bar{R}^T_{\psi} \Sigma_{\psi}^{-1} \left( D_{\psi} - C' \right) + \bar{R}^T_{\text{Mag}} \Sigma_{\text{Mag}}^{-1} \left( D_{\text{Mag}} - C_{\text{Mag}} \right) \right) \]
\[ \bar{\Sigma}_3 = \left( \bar{R}^T_{\psi} \Sigma_{\psi}^{-1} R_{\psi} + \bar{R}^T_{\text{Mag}} \Sigma_{\text{Mag}}^{-1} R_{\text{Mag}} + \bar{Q} \right)^{-1}. \]  
\[(36)\]

4. Results

The performance of this method is evaluated using phantom testing. Since the phantom is created for testing, its values may be known exactly. In this case, it consists of a plasma equilibrium with a corresponding density profile. Based on these, virtual diagnostic signals can be calculated and used as inputs to IDA. EFIT is used in a semi-fixed boundary calculation mode to design a self-consistent equilibrium, which is shown in figure 4. The electron density profile is then modeled as a modified tanh (MTANH) [23] near the pedestal and with a polynomial function in the core region:

\[ \hat{n}_e(\rho) = A^* \text{MTANH}(\alpha, z) + B \]  
\[(37)\]

where

\[ \text{MTANH}(\alpha, z) = [(1 + \alpha^* z) \exp(z) - \exp(-z)] / [\exp(z) + \exp(-z)] \]
\[ z = (\text{XSYM} - \rho) / \text{HWID}. \]  
\[(38)\]

Here, \( \rho \) is the normalized toroidal magnetic flux. XSYM is the location of the center of the barrier; HWID is the half-width of the barrier; \( \alpha \) allows a smooth transition to a linear fit near the core profile; A and B are used to adjust the magnitude and minimum value of the electron density. This function has continuous first derivatives everywhere. Since this function is based on the magnetic surface coordinates, it needs to be converted to obtain the electron density profile. The left side of figure 5 shows the electron density distribution along the normalized magnetic plane, and the right side shows the 2D electron density profile. When the electron density profile and current profile are determined, the virtual diagnostic signals of POINT and HCN interferometer can be calculated.

Before presenting the reconstruction results, two evaluation functions (RMSD, \( \xi \)) are introduced. RMSD represents the root-mean-square deviation, and \( \xi \) is the relative error. They are expressed as follows:

\[ \xi^i = \frac{|X^i_{\text{rec}} - \bar{X}^i|}{\text{max}[X]} \]
\[ \text{RMSD} = \sqrt{\frac{\sum_{i=1}^{n} (X^i_{\text{rec}} - \bar{X}^i)^2}{n}} \]  
\[(39)\]

where \( X_{\text{rec}} \) is the reconstruction parameter and \( \bar{X} \) is the true value obtained from phantom test data. \( i \) represents the \( i \)th grid point, and \( \text{max} \{ \} \) represents the maximum value. \( n \) is the...
number of grid points. According to the above expressions, the RMSD reflects the overall deviation between the calculated and true values, and the relative error reflects the reconstruction results for each grid point.

4.1. Phantom test

Based on the phantom test data, the reconstruction results of IDA are shown in table 1 and figure 6. Max $\xi$, Max $\xi$ and Max $\xi$ represent the maximum relative errors of plasma current, electron density and magnetic flux, respectively. $RMSD^\psi$, $RMSD^e$ and $RMSD^n$ represent the RMSD of plasma current, electron density and magnetic flux, respectively. Compared with the results based on only magnetic diagnostics in table 2, the plasma current and magnetic flux obtained using IDA are significantly improved. The maximum relative error of plasma current dropped from 22.56% to 9.39%, and $RMSD$ of plasma current dropped from 12.1007 to 5.5513 A. The maximum relative error of magnetic flux dropped from 8.59% to 1.51%, and $RMSD$ of magnetic flux dropped from 0.0177 to 0.0034 Wb. The plasma shape control system tries to control the positions of intersection of the plasma boundary with the control segments C1–C6 (shown in figure 6(f)) as well as the position of the X-point; therefore, these are natural places at which to check conformity of the reconstruction with the phantom. Maximum Error represents the maximum boundary error. The negative symbol in table 1 indicates the region within the boundary. It is evident from table 1 that the boundary errors of reconstruction are below 1 cm, which meet the requirements of control application.

Figures 6(a) and (g) show the contour maps of plasma current and electron density obtained using IDA. Figure 6(d) displays the contour map of magnetic flux, which is calculated based on the PF coil current and plasma current of IDA. Figure 6(c) shows the plasma current profile along the minor radius, and the gray gradient color indicates the uncertainty from $\sigma$ to $3\sigma$. In GP distribution, the confidence interval is usually obtained as $[\sigma, \sigma]$, $[2\sigma, 2\sigma]$, and $[3\sigma, 3\sigma]$, which corresponds to the probability of 68.26%, 95.44%, and 99.74%, respectively. The closer the uncertainty distribution to the center, the larger the value of probability. It is reasonable that the uncertainty near the boundary is low, because most diagnostics are close to the boundary. The plasma current along the radius is in excellent agreement with the true value, even at the core position. In a previous work, only magnetic diagnostics were used, and the plasma current in the core was always below the true value [13] when the plasma current changed rapidly, which was due to the stationary hyperparameter adopted by the CAR prior. The IDA result suggests that the deficiency of CAR prior can be surmounted by adding the polarization angle of POINT. It also proves the idea suggested in the previous work that adding internal information can improve the reconstruction accuracy because the polarization angle includes the information of internal magnetic field [13]. Figures 6(b), (e) and (h) show the relative error maps of plasma current, magnetic flux and electron density, respectively. The black dots represent the locations of maximum relative error. It is clear that the error is larger near the boundary in figure 6(h). According to figure 5, the electron density varies greatly in this region. SE prior probability also adopts stationary hyperparameter and simply smooths the spatial distribution of electron density, so it is difficult to model the situation with large gradients. In addition, the diagnostic signals (POINT and HCN interferometer) are the

| Boundary | Error (cm) | Parameter | Error |
|----------|-----------|-----------|-------|
| C1       | 0.0568    | Max $\xi$ (%) | 9.3900 |
| C2       | 0.1100    | $RMSD^\psi$ (A) | 5.5513 |
| C3       | 0.1400    | Max $\xi$ (%) | 6.9900 |
| C4       | −0.0117   | $RMSD^e$ (10^17) | 15.0837 |
| C5       | −0.0384   | $RMSD^n$ (Wb) | 1.5100 |
| C6       | 0.0782    | Max $\xi$ (%) | 6.9900 |
| $X_{Point}$ | 0.0390   | $RMSD^\psi$ (Wb) | 0.0034 |

Maximum error $-0.6100$
integral signal of density without local information, and the number of unknown parameters (2D electron density profile) far exceeds the number of diagnoses. Therefore, it is difficult to accurately invert the distribution of electron density based on the diagnostics of POINT and HCN interferometer in the model. In previous studies, it was proposed to use a 1D model to constrain the electron density profile to the normalized flux surface, which can effectively reduce the number of unknown parameters. And adopting non-stationary hyperparameter can effectively satisfy the situation that the gradient of electron density is large near plasma boundary [24]. Figure 6(i) presents the deviation $\sigma$, which is the uncertainty. The maximal uncertainty is mainly observed near the X-point.

| Max $\xi^I$ (%) | RMSD$^I$ (A) | Max $\xi^\psi$ (%) | RMSD$^\psi$ (Wb) |
|-----------------|--------------|-------------------|-----------------|
| 22.5600         | 12.1007      | 8.5900            | 0.0177          |

The diagnostic chord (HCN interferometer and POINT) do not pass through this region, which is reasonable. This also demonstrates that the uncertainty of Bayesian model can be used to guide the design of the diagnostics, such as the location of the diagnostics. Figure 6(f) represents the boundary error and the green dot is the location of the maximum boundary error.
4.2. Diagnostics with noise

To simulate the actual situation, random noise is added to the virtual diagnostics. On EAST, the maximum diagnostic errors are considered to be less than 3%. Thus, the Gaussian noise \( \sigma \) is set to 1% of diagnostic measurement (1/3 of the maximum error). During the simulation, GP function is used to randomly generate 100 sets of diagnostic data with an error within 3%, which are used to evaluate the robustness of tomography against noise.

Based on the 100 sets of diagnostic data with errors, the reconstruction results are shown in figure 7. There are some boundary errors at location C1, and the maximum error exceeds 1 cm, but it is still less than 2.15 cm, which is the diagonal length of the rectangular grid; the size of the rectangular grid \( 1.08 \text{ cm} \times 1.86 \text{ cm} \) (\( \sqrt{1.08^2 + 1.86^2} = 2.15 \)).

The error below a grid can be considered acceptable. According to figures 7(a)–(c), the maximal relative errors of plasma current and magnetic flux are below 10%, but the maximal relative error of electron density is above 10%. The RMSD of electron density also proves that the reconstruction results are not very good. The addition of error to the diagnostics affects the plasma current reconstruction. The magnetic surface is calculated based on the reconstruction of the current. The prior probability of the electron density is based on the magnetic surface coordinates, and the constructed prior has a detrimental effect when the magnetic surface cannot be accurately inverted. In addition, the number of diagnostics used to invert the electron density profile is small, which leads to poor reconstruction results. The reconstruction of electron density
Figure 8. Distribution of pickup coils (blue dot) and unavailable pickup coils (blue dot in a red circle).

profile also affects the reconstruction of current profile. However, for the plasma current reconstruction based on magnetic diagnostic signal in addition to the POINT data, the reconstruction results are still good. Therefore, more diagnostic constraints are needed for the reconstruction of electron density, which will be further explored in future work.

4.3. Experimental validation

To experimentally validate the IDA model, the discharge 62 573 at 5.0 s is randomly selected. During this discharge, some pickup coils give fault signals. Therefore, they are carefully removed before retrieving the diagnostic data into IDA. As shown in figure 8, the faulty pickup coils are removed, which are represented by blue dot in a red circle. A phase difference of POINT at the position $z = 0$ is not used in this reconstruction. When the electron density profile is inverted, a virtual diagnostic value of the phase difference can be calculated based on the forward model. By comparing the calculated diagnostic value with the measured signal that is not used in model, the results of electron density reconstruction can be evaluated. The reconstruction results obtained using IDA are shown in figures 9 and 10.

Figures 9(a) and (b) show the plasma current distribution along the radius, where the red and blue lines denote the results of tomography and EFIT code, respectively. The difference is that the tomographic reconstruction in figure 9(b) is only based on magnetic diagnostics. From figure 9(b), it can be found that there is a large uncertainty on the reconstructed plasma current and it is mainly due to the prior probability. In previous studies [13], it can be found that when there is an error in the diagnostics, the value of the hyperparameter $\tau$ of the CAR prior becomes larger through the maximum evidence. Its increase leads to a larger uncertainty of the posterior probability in the reconstructed plasma current. In the Gaussian posterior probability, the larger uncertainty implies that the model is within the $2\sigma$ confidence interval, and the value of the plasma current may have a negative value. Actually, the prior probability is a Gaussian distribution with zero mean, and the probability of the plasma current taking a positive value is the same as the probability of a negative value. In addition, the total plasma current in magnetic diagnostics is a macroscopic quantity and can not constrain the currents of all grid points to remain positive. When sampling the posterior probability, the total current of the plasma is still within the error range of the diagnostics. The large uncertainty also implies that the information of diagnostics is insufficient when reconstructing the plasma current profile based on Bayesian model. Comparing figures 9(a) and (b), it can be seen that the uncertainty of the IDA model is significantly lower than that of tomography model using only magnetic diagnostics. Further, he plasma current distribution obtained using IDA is different from that obtained using magnetic diagnostics only. These indicate that the polarization angle provided by POINT diagnostics constrains the plasma current. Figure 9(c) shows the contour map of the magnetic flux, where the red and blue lines correspond to the results based on IDA and only magnetic diagnostics, respectively. It can be seen that the plasma boundary based on IDA is consistent with that based on magnetic diagnostics, but there are obvious differences in the magnetic surface structure inside the plasma. Therefore, it is difficult to accurately invert the magnetic structure inside the plasma by only using external magnetic diagnostics. This is also verified in an earlier study by Moret et al [6].

Figure 10(a) shows the inverted electron density profile, and figure 10(b) shows its deviation. According to the deviation distribution, the maximum error is still concentrated near the boundary, which is similar to the simulation results in
section 3.1. Next, a signal is used to verify the reconstruction of electron density. Based on the inverted electron density profile, virtual diagnostic values are calculated, as shown in figure 11. It is clear that the overall agreement between the calculated and measurement results is good. For the diagnostic not involved in the reconstruction (red pentagram), the calculated results are still similar to the measurement results.

5. Conclusion

In this study, a plasma profile tomography method based on IDA was established to invert the plasma current and electron density profiles for EAST. Compared to the previous models using only magnetic diagnostics, the plasma current reconstruction was greatly improved after adding the polarization angles of POINT (based on phantom data). The proposed model overcame the deficiency of CAR prior, i.e. when the plasma current in the core is large, the inverted plasma current is always lower than the true value. For electron density profile reconstruction, an effective method was proposed to combine SE prior with the magnetic surface coordinates. The reconstruction of electron density profile provided good results. Further, it was found that the reconstruction error of electron density mainly appeared near the pedestal region. The electron density had a large gradient in the pedestal region, and only a few diagnostics provided an integrated signal in our model, so it was difficult to accurately invert the profile. In addition, it was found that IDA is sensitive to noise during the reconstruction of electron density profile because it depends on the accurate magnetic surface coordinates.

Overall, the IDA model based on Bayesian inference can integrate multiple diagnostic systems. Compared with other integrated analysis models, it can not only calculate the parameters but also provide the parameter uncertainty. The uncertainty values can provide guidance to optimize and improve the diagnostics. In future work, we will consider adopting non-stationary hyperparameter priors to accommodate large electron density gradients, adopting a 1D model to reduce the number of unknown parameters, and integrate more diagnostics into the model to accurately invert the parameters. We believe that our results can serve as a useful reference for experiment data analysis on EAST and China Fusion Engineering Test Reactor (CFETR).
Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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