The use of multivariate page test in dealing the violations of principal assumptions in completely randomized block design

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Abstract. There are various variance analysis tests, one of which is the Friedman test, it is used as an alternative to two way variance analysis technique. If there are unmet assumptions in the Friedman test, that is, they are not normally distributed and do not have a homogeneous variant, then the multivariate Page test can be used to determine whether there is a median difference between k (k ≥ 3), a median population. The Page test will be applied in the case of the growth rate of soybean plants that obtain 5 types of treatment that are conditioned, namely seedling, watering, giving 100% KL, 75%, and 50% KL. Based on statistical test using multivariate page indicated that there is influence on 5 types treatment of seed conditioning i.e dry seed (control), distilled water, 100 g PEG L⁻¹ of water, 200 g PEG L⁻¹ of water, and 300 g PEG L⁻¹ of water against 3 response measurements i.e germination, growth rate and growing Randomness Of Soybean.

1. Introduction

The objective of research is to get the effect of 5 types treatment of seed conditioning were dry seed (control), distilled water, 100 g PEG L⁻¹ of water, 200 g PEG L⁻¹ of water, and 300 g PEG L⁻¹ of water against 3 measurement responses namely germination, growth rate and growing Randomness of soybean, by using multivariate analysis of variance (MANOVA) testing. Based on data characteristics, the nonparametric multivariate statistics is appropriate to use, i.e multivariate page as it more common in obtaining a comparison of treatment effects on completely randomized block design [1].

Pre-testing of multivariate analysis of variance (MANOVA) is first tested the principal assumption by using formal test [2]. If there is a violation of assumption, then it required another test method on multivariate data, i.e non-parametric statistics. Nonparametric statistics are statistics that require no assumption making of distribution or free-distribution form, so it does not require assumptions about the population to be tested [3]. Several nonparametric testing alternatives in experimental design have been developed. showed nonparametric multivariate testing in a completely randomized block design, i.e Multivariate Page. Multivariate Page test is an alternative test of nonparametric statistics which is often used to get the treatment effect of comparison on a basic completely randomized block design, which is limited to a fixed model [4]. As an application on the treatment type of seed conditioning that is soybean stress data by observing 3 (three) response variables i.e germination, growing rate, and growing Randomness, it consisting of 3 levels and 3 responses to the treatment observed in the block, where there is a violation of the principal assumption
of multivariate normality and the homogeneity of variance-covariance matrixes that causes both assumptions to be violated [5].

2. Literature Review
2.1 Completely Multivariate Completely Randomized Block Design

In the experimental design of response to the treatment observed in the block is often not single, but as much $p$ ($p \geq 2$), so it requires multivariate [6]. Data consists of $N = nk$ vector $p$-dimension. $N = n$ subject in $n$ block and in each block set $k$ random treatment. Observation of $p$-variation can then be presented in table $n \times k$ as follows:

| Block | Replicate | Treatment | Total |
|-------|-----------|-----------|-------|
|       | i         | j         | K     |
| 1     | 1         | $y_{11} = \begin{pmatrix} y_{111} \\ y_{112} \\ \vdots \\ y_{11p} \end{pmatrix}$ | $y_{12} = \begin{pmatrix} y_{121} \\ y_{122} \\ \vdots \\ y_{12p} \end{pmatrix}$ | $\vdots$ | $y_{1k} = \begin{pmatrix} y_{1k1} \\ y_{1k2} \\ \vdots \\ y_{1kp} \end{pmatrix}$ | $y_{1.} = \begin{pmatrix} y_{101} \\ y_{102} \\ \vdots \\ y_{10p} \end{pmatrix}$ |
|       | 2         | $y_{21} = \begin{pmatrix} y_{211} \\ y_{212} \\ \vdots \\ y_{21p} \end{pmatrix}$ | $y_{22} = \begin{pmatrix} y_{221} \\ y_{222} \\ \vdots \\ y_{22p} \end{pmatrix}$ | $\vdots$ | $y_{2k} = \begin{pmatrix} y_{2k1} \\ y_{2k2} \\ \vdots \\ y_{2kp} \end{pmatrix}$ | $y_{2.} = \begin{pmatrix} y_{201} \\ y_{202} \\ \vdots \\ y_{20p} \end{pmatrix}$ |
|       | i         | $y_{ni} = \begin{pmatrix} y_{n11} \\ y_{n12} \\ \vdots \\ y_{n1p} \end{pmatrix}$ | $y_{n2} = \begin{pmatrix} y_{n21} \\ y_{n22} \\ \vdots \\ y_{n2p} \end{pmatrix}$ | $\vdots$ | $y_{nk} = \begin{pmatrix} y_{nk1} \\ y_{nk2} \\ \vdots \\ y_{nkp} \end{pmatrix}$ | $y_{n.} = \begin{pmatrix} y_{n01} \\ y_{n02} \\ \vdots \\ y_{n0p} \end{pmatrix}$ |
| Total | $y_{1} = \begin{pmatrix} y_{o11} \\ y_{o12} \\ \vdots \\ y_{o1p} \end{pmatrix}$ | $y_{2} = \begin{pmatrix} y_{o21} \\ y_{o22} \\ \vdots \\ y_{o2p} \end{pmatrix}$ | $\vdots$ | $y_{k} = \begin{pmatrix} y_{ok1} \\ y_{ok2} \\ \vdots \\ y_{okp} \end{pmatrix}$ | $y_{.} = \begin{pmatrix} y_{o01} \\ y_{o02} \\ \vdots \\ y_{o0p} \end{pmatrix}$ |

Source: Processed, 2018

An additive linear model was formed for multivariate variance analysis of randomized block design, by substituting each $y_{ij}$ into an observation vector $y_{ij}$ as follows:

$$
Y_{ij} = \left( \begin{array}{c} Y_{ij1} \\ Y_{ij2} \\ \vdots \\ Y_{ijp} \end{array} \right) = \left( \begin{array}{c} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{array} \right) + \left( \begin{array}{c} \alpha_{j1} \\ \alpha_{j2} \\ \vdots \\ \alpha_{jp} \end{array} \right) + \left( \begin{array}{c} \beta_{i1} \\ \beta_{i2} \\ \vdots \\ \beta_{ip} \end{array} \right) + \left( \begin{array}{c} \epsilon_{ij1} \\ \epsilon_{ij2} \\ \vdots \\ \epsilon_{ijp} \end{array} \right); i = 1, ..., n; j = 1, ..., k
$$

(2.1)

Where $y_{ij}$ was a vector in observation $p \times 1$ for $i$-th block in $j$-th treatment. The observation vector is written as follows:

- $\mu$: overall mean vector $\alpha$
- $\alpha_{ij}$: vector $j$-th treatment effect
- $\beta_{ij}$: vector $i$-th bllock effect
- $\epsilon_{ij}$: vector error of $j$-th treatment in $i$-th group
- $n, k$: level number of factor A and B

Fixed model assumption for multivariate randomized block design is [8].

$$
\sum_{j=1}^{k} \alpha_j = 0, \sum_{i=1}^{n} \beta_i = 0, \text{and} \epsilon_{ij} \sim N(0, \Sigma)
$$

(2.2)
Based on equation (2.2), treatment effect ($\alpha_j$) and the effect of $\beta_i$ group is fixed and the experimental error ($\varepsilon_{ij}$) is independent, normal distributed with median equal to zero and the covariance variance matrix equals ($\Sigma$).

The general form of hypothesis to be tested is the effect of treatment as follows:
- $H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_k = 0$ There is no effect of block effect
- $H_1: \text{there is } \alpha_j \neq 0 \text{ for } j = 1, 2, \ldots, k$ There is effect of block effect.

### 2.2 Principal Assumptions in Multivariate Completely Randomized Block Design

Consider the equation (2.1) of linear model for multivariate completely randomized block design as follows:

$$Y_{ij} = \mu + \alpha_j + \beta_i + \varepsilon_{ij} ; i = 1, 2, \ldots, n; j = 1, 2, \ldots, k.$$  
where $\varepsilon_{ij} \sim (0, \Sigma)$

In practice, the implied meaning of model is:

a) In observation of each treatment group came from the normal population
b) All treatment groups has homogeneous variances
c) Experiment unit is determined and placed randomly in each treatment group so that $\varepsilon_{ij}$ are independent each other
d) The effect of treatment factor ($\alpha_j$) and group ($\beta_i$) and error ($\varepsilon_{ij}$) is additive, hence the assumptions that must be fulfilled in multivariate variance analysis are multivariate normality, homogeneity of variance-covariance matrices, independence (error freedom), and additive [9].

### 2.3 Assumption Test of Multivariate Normality

Statistical method of multivariate complete randomized block design requires the assumption of normality distribution with hypothesis:
- $H_0$: Data normally-distributed multivariate
- $H_1$: Data not normally-distributed multivariate

Normality assumption of error vectors ($\varepsilon$) using $q$-$q$ plots or scatter-plot quartile is approximated by chi-square quartile. The steps as follows:

a) Determine the distance value mahalanobis or general square of each observation point with mean vector using equation:

$$MD^2 = (\varepsilon - E(\varepsilon))S^{-1}(\varepsilon - E(\varepsilon))$$  
(2.3)

Where:
- $\varepsilon$: error vector
- $S$: covariance matrix of error

b) Order the distance value of $\text{mahalanobis}$ (MD2) in equation (2.3) from smallest to argest such as:

$$MD_1^2 < MD_2^2 < \cdots < MD_n^2$$

where $n$ is number of data

c) Find the chi-square value of $(i-0.5)/n$ with freedom degrees $p$ and $i = 1, 2, \ldots, n$, it denoted by $\chi^2_{p(i-0.5)n}$ with $n$ = number of data and $p$ = number of treatment response observed in block (number replicates).

d) Create $q$-$q$ plot $MD^2$ with $\chi^2_{p(i-0.5)n}$.

e) If this $q$-$q$ plot tends to follow a straight line pattern and more than 50% of $MD^2$ $\leq \chi^2_{p(0.50)}$, then $H_0$ is accepted and multivariate normally-distributed data.

To be more convincing that its correlation is linear then it can be done by calculating the Pearson’s correlation $\chi^2_{p(0.50)n}$ with $MD^2$. If correlation coefficient value $MD^2 \leq \chi^2_{p(i-0.5)n}$ on the percent point of the normal probability plot correlation coefficient table, then $H_0$ is accepted, it fulfill multivariate normality assumption, and vice versa. Creating $q$-$q$ plot of the value of $MD^2 = (\varepsilon - E(\varepsilon))S^{-1}(\varepsilon - E(\varepsilon))$ can use PROC program IML in SAS’s software [10].
2.4 Assumption Test of Matrix Homogeneity of Variance-Covariance

Test statistics are needed to test the matrix homogeneity of variance-covariance, with hypothesis:

- $H_0: \sum_{i=1}^n \Sigma_i = \sum_{n} \Sigma$; Matrix of homogeneous variance-covariance
- $H_1$: at least one $\Sigma_i \neq \Sigma$; for $i = 1, 2, ..., n$

Matrix homogeneity test of variance-covariance can be done by Box’s M test. If sig. value $> \alpha$, then $H_0$ is accepted so that it can be concluded that the matrix of variance-covariance of $l$-population is equal or homogeneous [11].

2.5 Multivariate Analysis of Variance (MANOVA) of Completely Randomized Block Design

When in an experiment studies the effect of multiple treatments on more than one response, the appropriate method of analysis was multivariate analysis of variance (MANOVA) [12]. In MANOVA testing, the first step is to create the core values observed each block:

$$y_{ij} \rightarrow \hat{y}_{ij} = y_{ij} - \bar{y}_{i0}, \quad i = 1, ..., n, \quad j = 1, ..., k. \tag{2.4}$$

where $\bar{y}_{i0} = \frac{1}{n} \sum_{j=1}^k y_{ij}$.

Then to calculate the number of each treatment can formulated as follows:

$$\hat{y}_{o j} = \sum_{i=1}^n \hat{y}_{ij}, \quad j = 1, ..., k \tag{2.5}$$

and $C = \frac{1}{nk} \sum_{i=1}^n \sum_{j=1}^k y_{ij} \hat{y}_{ij}^t \tag{2.6}$

where $C$ is the estimator of variance-covariance matrix in block.

A table of multivariate analysis of variance (MANOVA) can be presented as follow:

| Source | Df | Sum of Matrix Squares |
|--------|----|-----------------------|
| Treatment | $k-1$ | $H_1$ |
| Block | $n-1$ | $H_2$ |
| Error | $(n-1)(k-1)$ | $E$ |
| Total | $nk-1$ | $T$ |

Table 2

Multivariate analysis of variance in the Completely Randomized Block Design

Source: Processed, 2018

Where:

- $H_1$: Sum of squares and cross-multiplying result of matrix treatment
- $H_2$: Sum of squares and cross-multiplying result of matrix block
- $E$: Sum of squares and cross-multiplying result of matrix error
- $T$: Total sum of squares and cross-multiplying result

The formula for calculating the sum of squares and cross-multiplying result of matrix in multivariate data of completely randomized block design is formulated as follows:

Total sum of squares and cross-multiplying result (T) for $(n, k)$ combination matrix can be defined as follows:

$$T = \sum_{i=1}^n \sum_{j=1}^k (y_{ij} - \bar{y}_{i0})^2 \tag{2.7}$$

Sum of squares and cross-multiplying result of treatment ($H_1$) for $(n, k)$ combination matrix can be defined as follows:

$$H_1 = \sum_{i=1}^n \sum_{j=1}^k (\hat{y}_{o j} - \bar{y}_{i0})^2 \tag{2.8}$$

Sum of squares and cross-multiplying result of block ($H_2$) for $(n, k)$ combination matrix can be defined as follows:
\[
H_2 = \sum_{i=1}^{n} \sum_{j=1}^{k} (\hat{y}_{io} - \hat{y}_{oo})^2
\]  
(2.9)

Sum of squares and cross-multiplying result of error (E) for (n, k) combination matrix can be defined as follows
\[
E = \sum_{i=1}^{n} \sum_{j=1}^{k} (Y_{ij} - \hat{y}_{io} - \hat{y}_{oj} - \hat{y}_{oo})^2
\]  
(2.10)

Test statistics of MANOVA to test \(H_0\) are
\[
Q^2 = \frac{k-1}{nk} \sum_{j=1}^{k} \hat{y}_{oj}^t - C^{-1} \hat{y}_{oj}
\]  
(2.11)

If it is written in the equation, it can use PillaiTrace’s test statistic as follows:
\[
Q^2 = \frac{k-1}{k} \text{tr}(H_1 C^{-1})
\]  
(2.12)

Criteria of hypothesis testing as follows:
If \(Q^2 < \chi^2_{(d (k-1))}\) then \(H_0\) is accepted and applies otherwise if \(Q^2 > \chi^2_{(d (k-1))}\) then \(H_0\) is rejected.

2.6 Multivariate Page in Completely Randomized Block Design

As with univariate, data analysis of a multivariate experimental design is based on principal assumptions, sometimes there is a violation or not fully of assumption is met. If there is an assumption violation, then the testing in nonparametric multivariate statistics method becomes very useful. Nonparametric multivariate statistics of multivariate page test were more general in obtaining a comparison of treatment effects in a complete randomized block design.

Multivariate page can be derived in equal way as MANOVA test. If in MANOVA testing, block wise multivariate is centered on \(y_{ij}\) response, then in multivariate page test is only replaced by a block wise multivariate is centered on rank vector \(R_{ij}\). Vector \(R_{ij}\) is central rank of observation \(y_{ij}\) between all observations on \(i\)-th block, i.e, between \(y_{i1}, ..., y_{ik}\) \(j = 1, ..., n\). The rank can be presented in a following table.

**Table 3**

| Block | Treatment | \(1\) | \(2\) | \(\ldots\) | \(K\) |
|-------|-----------|------|------|----------|------|
| 1     | \(R_{11}\) | \(R_{12}\) | \(\ldots\) | \(R_{1k}\) |
| 2     | \(R_{12}\) | \(R_{22}\) | \(\ldots\) | \(R_{2k}\) |
| \vdots| \vdots    | \vdots| \vdots| \vdots  |
| \(N\) | \(R_{n1}\) | \(R_{n2}\) | \(\ldots\) | \(R_{nk}\) |
| \(\sum\) | \(R_{1}\) | \(R_{2}\) | \(\ldots\) | \(R_{k}\) |

Source: Processed, 2018

Let \(R_i = (R_{i1}^T, \ldots, R_{i(k-1)}^T)^T\) denote the relationship of the rank vector in \(i\)-th block \((R_{ik} = -\sum_{j=1}^{k-1} R_{ij})\) and let \(R = (R_{11}^T, \ldots, R_{(k-1)}^T)^T = \sum_i R_i\) be a vector of rank number of blocks.

Vector calculation of the number of rank upon treatment is formulated as follows:
\[
R_{oij} = \sum_{i=1}^{n} R_{ij}, \ j = 1, \ldots, k
\]  
(2.13)

With estimator formula for variance-covariance matrices given as follows:
\[
C_r = \frac{1}{nk} \sum_{i=1}^{n} \sum_{j=1}^{k} R_{ij} R_{ij}^T
\]  
(2.14)

In multivariate page, a test statistic used is formulated as follows:
\[
P = -\sum_{i=1}^{n} \sum_{j=1}^{k} L_{ij} C_r^{-1} L_i
\]  
(2.15)

Where \(L_{\text{rend}}\) value is traced first by using the following formula:
The test criteria on the Multivariate page test are:

\[ H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_k = 0 \]

\[ H_1: \text{there is } \alpha_j \neq 0 \text{ for } j = 1, 2, \ldots, k \]

If \( P > \chi^2_{(d(k-1))} \) then \( H_0 \) is rejected and applies otherwise, if \( P < \chi^2_{(d(k-1))} \) then \( H_0 \) is accepted.

### 3. Method of Research

#### 3.1 Data Source

The data used are secondary data i.e multivariate data of completely randomized block design comes from a research report of Agriculture Department named [10] titled “The Role of Seed Conditioning in Increasing Adaptation Rate of Soybean against Drought Stress.” The results of research such as germination (% per etmal), growth rate (% per etmal) and growing randomness (% per etmal) of conditioning treatment of dry seed, distilled water, 100 g PEG L\(^{-1}\) of water, 200 g PEG L\(^{-1}\) of water, and 300 g of PEG L\(^{-1}\) of water with a watering factor of 50% KL, 75% KL, and 100% KL as a block for observation because theoretically different watering is considered to affect the drought stress of soybean.

#### 3.2 Identification of Variables

Multivariate pattern of completely randomized block design, namely:

1. Treatment factor consists of 5 levels are:
   \( p_1 = \text{treatment 1} \); \( p_2 = \text{treatment 2} \); \( p_3 = \text{treatment 3} \); \( p_4 = \text{treatment 4} \); \( p_5 = \text{treatment 5} \)

2. Block factor consists of 3 levels are:
   \( B_1 = \text{Group 1} \); \( B_2 = \text{Group 2} \); \( B_3 = \text{Group 3} \)

#### 3.3 Method of Analysis

Based on the objectives of research, the following steps are used:

1. Collect secondary data
2. Perform testing for principal assumption:
   a. Multivariate Normality Test, it is conducted on each population by creating \( q-q \) plot of a value of \( MD^2 = (e-E(e))^T S^{-1} (e-E(e)) \) \( i = 1, 2, \ldots, n \) using PROC IML program in SAS’s software. If these scatter-plot tend to form straight lines and more than 50% of the value of \( MD^2 \leq \chi^2_{p(0.50)} \), then \( H_0 \) is accepted, it means the data is multivariate normally-distributed.
   b. Homogeneity Test of Variance-Covariance Matrixes, by using SPSS ver. 23 program, homogeneity test of variance-covariance matrixes is done by Box’s M Test. If the value of \( \text{sig.} > \alpha \), then \( H_0 \) is accepted so that it can be concluded that the variance-covariance matrix of \( l \)-population is equal or homogeneous.
3. Perform Multivariate Analysis of Variance (MANOVA)
4. Perform Multivariate Page testing with the following steps:
   a. Gives ranks on each block
   b. Performs block wise multivariate calculation that centered on rank vector \( R_{ij} \)
   c. Calculate the number of vectors on each treatment \( R_{oj} = \sum_{i=1}^{n} R_{ij} \), \( j=1, \ldots, k \)
   d. Create and calculate the variance-covariance matrices

\[
C_r = \frac{1}{nk} \sum_{i=1}^{n} \sum_{j=1}^{k} R_{ij} R_{ij}^T
\]
e. Find $L_{Trend}$ value

$$L = R_{o1} + 2R_{o2} + \cdots + kR_{ok} = \sum_{j=1}^{k} jR_{oj}$$

f. Calculate Multivariate Page test

$$P = \frac{12}{nk^2(k+1)} L^T C^{-1} L$$

g. Hypothesis testing

$H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_k = 0$

$H_1: \text{there is } \alpha_i \neq 0 \text{ for } i = 1, 2, \ldots, k$

If $P > \chi^2_{(d(k-1))}$ then $H_0$ is accepted and applies otherwise.

5. Obtain conclusion of hypothesis as uses Multivariate Page test in observing treatment in a completely randomized block design.

4. Results and Discussion

Description of variable data can be seen in Table 3, wherein the secondary data consists of 3 levels on the block, 5 treatment levels and 2 response levels to the treatment observed in the block.

| Block | Replicate | Treatment | Total | Mean |
|-------|-----------|-----------|-------|------|
|       |           | 1 2 3 4 5 |       |      |
| 1     | 1         | 70 80 85 | 80   | 80   | 400 |
|       | 2         | 20.1 26.33 26.77 | 28.33 | 26.67 | 128.1 |
|       | 3         | 45 55 60 | 65   | 80   | 305 |
| Sub Total |       | 135.1 166.33 166.7 | 178.33 | 186.67 | 833.1 |
| 2     | 1         | 80 70 75 | 70   | 75   | 370 |
|       | 2         | 26 26.33 23.1 | 26.67 | 20   | 122.1 |
|       | 3         | 70 55 55 | 70   | 45   | 295 |
| Sub Total |       | 176 151.33 153.1 | 166.67 | 140 | 787.1 |
| 3     | 1         | 80 75 80 | 80   | 95   | 390 |
|       | 2         | 24.38 24.33 26 | 19.33 | 31 | 125.04 |
|       | 3         | 65 55 70 | 55   | 75   | 320 |
| Sub Total |       | 169.38 154.33 176 | 134.33 | 201 | 835.04 |
| Total |           | 230 230 235 215 250 | 1160 | 2372.24 |
|       | 70.48 76.99 75.77 74.33 77.67 | 920 |
|       | 180 165 185 190 200 | |
|       | 480.48 471.99 495.77 479.33 527.67 | 2455.24 163.68 |
| Mean  |           | 53.38 52.44 55.08 53.25 58.63 | |

Source: Processed, 2018

Based on Table 4 it appears that the block consists of 3 levels i.e watering 50% KL as block 1, 75% KL as block 2 and 100% KL as block 3. Given 5 levels treatment i.e dry seed (control) as treatment 1, distilled water as treatment 2, 100 g PEG L$^{-1}$ of water as treatment 3, 200 g PEG L$^{-1}$ of water as treatment 4, 300 g PEG L$^{-1}$ of water as treatment 5. Response observed in the block is said
as replicates consisting of 3 levels i.egermination as replicate 1, growing rate as replicate 2 and growing randomness as replicate 3.

4.1 Parameter Estimator

In estimating the parameters of fixed model assumptions that must be met are:

$$\sum_{i=1}^{\alpha_i} = 0, \sum_{i=1}^{\beta_i} = 0$$

To obtain estimator for parameters $\mu, \alpha_i$, and $\beta_i$ then equation (4.1) is converted into the following form:

$$\varepsilon_{ij} = Y_{ij} - \mu - \beta_i - \alpha_j$$

Equation (4.2) is squared and summed so as to obtain:

$$\sum_{i=1}^{n} \sum_{j=1}^{k} (Y_{ij} - \mu - \beta_i - \alpha_j)^2 = W$$

Estimator $\hat{\mu}, \hat{\alpha}_i$, and $\hat{\beta}_i$ for parameter $\mu, \alpha_i$, and $\beta_i$ are obtained by minimize the value of $W$ with Ordinary Least Square (OLS) by conducting first partial derivation $W$ against parameter $\mu, \alpha_i$, and $\beta_i$ and then it is equalized with zero so obtained:

Estimator parameter $\mu$

$$\frac{\partial W}{\partial \mu} |_{\mu = \hat{\mu}} = \frac{\partial (\sum_{i=1}^{n} \sum_{j=1}^{k} (Y_{ij} - \mu - \beta_i - \alpha_j)^2)}{\partial \hat{\mu}} = 0$$

$$2 \sum_{i=1}^{n} \sum_{j=1}^{k} (Y_{ij} - \hat{\mu} - \beta_i - \alpha_j)(-1) = 0$$

$$-2 \sum_{i=1}^{n} \sum_{j=1}^{k} (Y_{ij} - \hat{\mu} - \beta_i - \alpha_j) = 0$$

$$\sum_{i=1}^{n} \sum_{j=1}^{k} (Y_{ij} - \hat{\mu} - \beta_i - \alpha_j) = 0$$

$$\sum_{i=1}^{n} \sum_{j=1}^{k} Y_{ij} - \sum_{i=1}^{n} \sum_{j=1}^{k} \hat{\mu} - \sum_{i=1}^{n} \sum_{j=1}^{k} \beta_i - \sum_{i=1}^{n} \sum_{j=1}^{k} \alpha_j = 0$$

$$\sum_{i=1}^{n} \sum_{j=1}^{k} Y_{ij} - nk\hat{\mu} - k \sum_{i=1}^{n} \beta_i - n \sum_{j=1}^{k} \alpha_j = 0$$

By using fixed model assumption, so that

$$\sum_{i=1}^{n} \sum_{j=1}^{k} Y_{ij} - nk\hat{\mu} = 0$$

$$nk\hat{\mu} = \sum_{i=1}^{n} \sum_{j=1}^{k} Y_{ij}$$

$$\hat{\mu} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{k} Y_{ij}}{nk} = \frac{Y_{\alpha\alpha}}{nk} = \bar{Y}_{\alpha\alpha}$$

Estimator parameter $\beta_i$

$$\frac{\partial W}{\partial \beta_i} |_{\beta_i = \hat{\beta}_i} = \frac{\partial (\sum_{i=1}^{n} \sum_{j=1}^{k} (Y_{ij} - \mu - \beta_i - \alpha_j)^2)}{\partial \hat{\beta}_i}$$
By using fixed model assumption, so that

\[
\sum_{j=1}^{k} (Y_{ij} - \mu - \hat{\beta}_i - \alpha_j) = 0
\]

\[
-2 \sum_{j=1}^{k} (Y_{ij} - \mu - \hat{\beta}_i - \alpha_j) = 0
\]

\[
\sum_{j=1}^{k} (Y_{ij} - \mu - \hat{\beta}_i - \alpha_j) = 0
\]

\[
\sum_{j=1}^{k} Y_{ij} - \sum_{j=1}^{k} \mu - \sum_{j=1}^{k} \hat{\beta}_i - \sum_{j=1}^{k} \alpha_j = 0
\]

\[
\sum_{j=1}^{k} Y_{ij} - k\mu - k\hat{\beta}_i - \sum_{j=1}^{k} \alpha_j = 0
\]

Estimator parameter \( \alpha_j \)

\[
\frac{\partial W}{\partial \alpha_j} \bigg|_{\alpha_j = \hat{\alpha}_j} = \frac{\partial \left( \sum_{i=1}^{n} \sum_{j=1}^{k} (Y_{ij} - \mu - \beta_i - \alpha_j)^2 \right)}{\partial \hat{\alpha}_j}
\]

\[
2 \sum_{i=1}^{n} (Y_{ij} - \mu - \beta_i - \hat{\alpha}_j)(-1) = 0
\]

\[
-2 \sum_{i=1}^{n} (Y_{ij} - \mu - \beta_i - \hat{\alpha}_j) = 0
\]

\[
\sum_{i=1}^{n} (Y_{ij} - \mu - \beta_i - \hat{\alpha}_j) = 0
\]

\[
\sum_{i=1}^{n} Y_{ij} - n\mu - \sum_{i=1}^{n} \beta_i - n\hat{\alpha}_j = 0
\]

By using fixed model assumption, so that

\[
\sum_{i=1}^{n} Y_{ij} - n\mu - n\hat{\alpha}_j = 0
\]

\[
n\hat{\alpha}_j = \sum_{i=1}^{n} Y_{ij} - n\mu
\]

\[
\hat{\alpha}_j = \frac{\sum_{i=1}^{n} Y_{ij}}{n} = \mu
\]
4.2 Multivariate Page Testing against Completely Randomized Block Design

Data analysis of an experimental design is based on the principal assumptions, sometimes there is a violation or not fully the assumption is met. Based on data processing as performed, it seem that there is violation on the principal assumption of multivariate normality and homogeneity of variance-covariance matrices. If occur assumption violation, then test in non-parametric statistics method is more useful. Non-parametric statistic for data against multivariate form introduces more general multivariate page test in obtain comparison of effect treatment on a completely randomized block design.

In multivariate data the use of multivariate page test is based on the rank vector of observation $y_{ij}$ between all observations on $i$-th block. Rank data is ordered from smallest to largest. If there are data that are double or similar then use mean rank that should be for that same set of data.

Rank result of secondary data observations in Table 5, is presented as follows:

| Block | Replicate | Treatment | Total | Mean |
|-------|-----------|-----------|-------|------|
| 1     | 1         | 1         |       |      |
|       | 2         | 2         |       |      |
|       | 3         | 3         |       |      |
|       | Sub Total |           |       |      |
| 2     | 1         | 5         |       |      |
|       | 2         | 3         |       |      |
|       | 3         | 4,5       |       |      |
|       | Subtotal  |           |       |      |
| 3     | 1         | 3,5       |       |      |
|       | 2         | 3         |       |      |
|       | 3         | 1,5       |       |      |
|       | Subtotal  |           |       |      |
| Total | 9,5       | 5,5       |       |      |
|       | 11        | 15.0      |       |      |
|       | 25.0      | 22.0      |       |      |
|       | 28.5      | 28.0      |       |      |
|       | 31,5      | 31.5      |       |      |
|       | 135.0     |           |       |      |

Based on Table 4 indicates the rank of observations replicate 1, 2, and 3 between all observations in each group (block).

In multivariate page testing in a completely randomized block design can be derived in equal way as MANOVA test first. If in MANOVA testing, blockwise multivariate is centered on the $y_{ij}$ response, then the multivariate page test is replaced only by the blockwisemultivariate is centered on

\[
\hat{a}_j = \frac{Y_{oj}}{n} - \mu = Y_{oj} - \bar{Y}_{oo}
\]
rank vector $R_{ij}$ first. Vector $R_{ij}$ is central rank of $y_{ij}$ observation between all observations on the $i$-th block, i.e., between $y_{it}$, ..., $y_{ik}$, $j = 1, ..., n$.

Given a formula for creating a central rank of observations between all observations on $i$-th block as follows:

$$
r_{ij} \rightarrow \bar{R}_{ij} = r_{ij} - \bar{r}_{io},
$$

where

$$
\bar{r}_{io} = \frac{1}{N} \sum_{j=1}^{5} r_{ij}, i = 1,2,3; j = 1,2,3,4,5
$$

Tabulation of observation rank data after establishing the center rank $y_{ij}$ between all observations on the block is given as follows:

| Block | Replicate | Treatment | Total |
|-------|-----------|-----------|-------|
| 1     | 1         | -2        | 1.5   | -0.5 | 1.5 | -0.5 | 0   |
|       | 2         | -2        | -1    | 0.5  | 2   | 0.5  | 0   |
|       | 3         | -2        | -1    | 0    | 1   | 2    | 0   |
| Sub Total |           | -6        | -0.5  | 0    | 4.5 | 2    | 0   |
| 2     | 1         | 1         | -1.5  | 0.5  | -1.5| 0.5  | 0   |
|       | 2         | 0         | 1     | -1   | 2   | -2   | 0   |
|       | 3         | 1.5       | -0.5  | -0.5 | 1.5 | -2   | 0   |
| Subtotal |           | 3.5       | -1.0  | -1.0 | 2.0 | -3.5 | 0   |
| 3     | 1         | 0.5       | -1    | 0.5  | -2  | 2    | 0   |
|       | 2         | 0         | -1    | 1.0  | -2  | 2    | 0   |
|       | 3         | 0         | -1.5  | 1.0  | -1.5| 2    | 0   |
| Subtotal |           | 0.5       | -3.5  | 2.5  | -5.5| 6    | 0   |
| Total |           | 0.5       | -1.0  | 0.5  | -2  | 2    | 0   |
|       |           | -2.0      | -1.0  | 0.5  | 2.0 | 0.5  | 0   |
|       |           | -0.5      | -3.0  | 0.5  | 1.0 | 2.0  | 0   |
|       |           | -2.0      | -5.0  | 1.5  | 1.0 | 4.5  | 0   |

Source: Processed, 2018

Based on Table 5 indicates the central rank of replicate 1, 2 and 3, between all observations on watering on each group (block).

Estimator of covariance matrix is obtained as follows:

$$
C_r = \frac{1}{nk} \sum_{i=1}^{n} \sum_{j=1}^{k} R_{ij} R_{ij}^T
$$

$$
= \frac{1}{3(5)} \left( \begin{array}{cccc}
R_{11} R_{11}^T + R_{21} R_{21}^T + \cdots + R_{35} R_{35}^T
\end{array} \right)
$$

$$
= \frac{1}{15} \left( \begin{array}{ccc}
4 & 4 & 4 \\
4 & 4 & 4 \\
4 & 4 & 4
\end{array} \right) + \left( \begin{array}{ccc}
0 & 0 & 3 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array} \right) + \left( \begin{array}{ccc}
0.25 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array} \right) + \cdots + \left( \begin{array}{ccc}
4 & 4 & 4 \\
4 & 4 & 4 \\
4 & 4 & 4
\end{array} \right)
$$

$$
= \frac{1}{15} \left( \begin{array}{ccc}
27.5 & 8.5 & 12.25 \\
8.5 & 29.5 & 24.5 \\
12.25 & 24.5 & 28.5
\end{array} \right)
$$
By using Matlab ver. 8.1 2013 obtained the estimator of covariance matrix as follows:
\[
\mathbf{C}_r^{-1} = \begin{pmatrix}
0.6898 & 0.1659 & -0.4392 \\
0.1659 & 1.8172 & -1.6334 \\
-0.4392 & -1.6334 & 2.1192
\end{pmatrix}
\]
While \(L_{\text{Trend}}\) value is obtained as follows:
\[
L = \sum_{j=1}^{k} n_j \mathbf{R}_{0j} = \sum_{j=1}^{5} \mathbf{R}_{0j}
\]
\[
= 1 \begin{pmatrix} 0.5 \\ -2 \\ -0.5 \end{pmatrix} + 2 \begin{pmatrix} -1 \\ 0.5 \\ 0.5 \end{pmatrix} + 3 \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix} + 4 \begin{pmatrix} 0.5 \\ -1 \\ -2 \end{pmatrix} + 5 \begin{pmatrix} 2 \\ 1 \\ 0.5 \end{pmatrix}
\]
\[
= 12 \begin{pmatrix} 2 \\ 8 \\ 9 \end{pmatrix}
\]

**4.3 Multivariate Page Test**

In statistical testing of multivariate page, the hypothesis to be tested is the effect on the completely randomized block design as follows:

- \(H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_5 = 0\); There is no effect for treatment
- \(H_1: \text{there is } \alpha_j \neq 0 \text{ for } j = 1, 2, \ldots, 5\); There is effect for treatment

Based on the hypothesis testing criteria is obtained the result of multivariate page test as follows

**Result of Multivariate Page Test**

\[
P = \frac{12}{n(k^2 - k + 1)} \mathbf{L}^T \mathbf{C}_r^{-1} \mathbf{L}
\]
\[
= \frac{12}{(3)(5^2)(5+1)} \begin{pmatrix} 2 & 8 & 9 \\ 0.6898 & 0.1659 & -0.4392 \\ 0.1659 & 1.8172 & -1.6334 \\ -0.4392 & -1.6334 & 2.1192 \end{pmatrix} \begin{pmatrix} 2 \\ 8 \\ 9 \end{pmatrix}
\]
\[
= 18.001
\]

Based on the results of testing obtained multivariate page 1.8001. as these results, when compared with the chi-square distribution table for the significance level \(\alpha = 0.05\) and degrees of freedom \((df = (p-1)(k-1) = (3-1)(5-1) = 8)\) are obtained table value 15.507, then the value of statistical count of multivariate page test is smaller than the value of chi-square table (18.001 > 15.507) so that the conclusion Reject \(H_0\), which means there is the influence of the 5 types of seed conditioning treatment namely dry (control), distilled water, 100 g PEG L⁻¹ water, 200 g PEG L⁻¹ water, and 300 g PEG L⁻¹ water against 3 measurement responses namely germination, growth rate and random growth of soybeans.

**5. Conclusions and Suggestions**

**5.1 Conclusion**

As a result of research and explanations given, it can be taken some conclusions as follows:

1. The result of a statistical calculation that contains a violation of principal assumptions by using a multivariate page test in completely randomized block design is obtained at 18.001. From these results, when compared with the Chi-square table obtained table value of 2.026, then the count value of the statistical test of multivariate page test is bigger than the Chi-square table value so it is significant.
Based on statistical test using multivariate page indicated that there is no effect influence on 5 types treatment of seed conditioning i.e dry seed (control), distilled water, 100 g PEG L\(^{-1}\) of water, 200 g PEG L\(^{-1}\) of water, and 300 g PEG L\(^{-1}\) of water against 3 response measurements i.e germination, growing rate and growing randomness of soybean.

5.2 Suggestions

For the next researcher can conduct more research on theoretical properties on the use of multivariate page test by using a random model or to study other tests of nonparametric multivariate completely randomized block design, and to study the nonparametric multivariate test of completely randomized block design with missing data.

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