GRIBOV CONFINEMENT AND
CHIRAL PERTURBATION THEORY

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ABSTRACT

I discuss the chiral dynamics of Gribov’s theory of confinement. At a critical coupling \( \alpha_s^c \) the light quark vacuum undergoes a series of phase transitions which should be taken into account in the application of chiral perturbation theory. The Gribov theory offers a simple explanation of the value of the pion nucleon sigma term without need to invoke a large strange quark component in the nucleon.

1 Introduction

The pion nucleon sigma term \( \sigma_{\pi N} \), which is measured in pion nucleon scattering, is a sensitive probe of chiral dynamics. The sigma term is a measure of chiral symmetry breaking in the nucleon. It is commonly thought that \( \sigma_{\pi N} \) can be calculated using the quark model and the mass splitting in the baryon octet. However, there is a long outstanding discrepancy between the measured value of \( \sigma_{\pi N} \) and this theoretical prediction (see Ref. [1] and references therein).

In this paper I discuss the chiral dynamics associated with Gribov’s theory of confinement [2, 3], which offers a dynamical explanation of spontaneous chiral symmetry breaking. At a critical coupling \( \alpha_s^c \approx 0.6 \), the light quark vacuum undergoes a rich series of phase transitions. The energy level of a quark in a background colour field falls below the Fermi surface of the perturbative vacuum and the quark becomes a resonance — it is not seen as a free particle. As I explain below, these phase transitions mean that hadronic physics does not change continuously as we vary the light quark mass to zero. This result has important consequences for chiral perturbation theory. Indeed, the Gribov theory anticipates the “discrepancy” between the two determinations of \( \sigma_{\pi N} \) [4].

2 The pion nucleon sigma term

We first review the theory of \( \sigma_{\pi N} \) in the usual theory of pions and chiral symmetry [5]. The QCD Lagrangian exhibits exact chiral symmetry for massless quarks. Since there are no parity doublets in the hadron spectrum we know that the chiral symmetry must be spontaneously broken, whence Goldstone’s theorem tells us to expect a zero mass boson. In the real world the light quarks have a small mass, which breaks the exact chiral symmetry. We use \( H_m = \hat{m}(\bar{u}u + \bar{d}d) \) to denote the chiral symmetry breaking term in the QCD Lagrangian where \( \hat{m} \) is the mean light “current-quark” mass. If we assume that the chiral dynamics does not change as we vary the light quark mass to zero, then we must assign a value to \( \sigma_{\pi N} \) for massless quarks and then use this value to determine \( \hat{m} \).
quark mass from zero to $\tilde{m}$ then the chiral Goldstone state acquires a small mass. It is identified with the physical pion. The theory of chiral symmetry gives a relation between the value of $\tilde{m}$ and the pion mass, viz.

$$m_\pi^2 = \tilde{m} \left( -\frac{<\text{vac}|\bar{u}u + \bar{d}d|\text{vac}>}{f_\pi^2} \right)$$ (1)

which we shall need later in our discussion. The sigma term is formally defined as

$$\sigma_{\pi N} = \frac{1}{3} \sum_{i=1}^{3} <N|[Q_5^i, [Q_5^i, H_m]]|N>$$ (2)

where $Q_5^i$ is the axial charge. After we use the QCD equations of motion to evaluate the commutators in equ.(2) $\sigma_{\pi N}$ becomes

$$\sigma_{\pi N} = \int d^3x \tilde{m} <N|\bar{u}u + \bar{d}d|N>$$ (3)

Hence, $\sigma_{\pi N}$ provides a measure of chiral symmetry breaking in the nucleon. The value of $\sigma_{\pi N}$ is measured in $\pi N$ scattering to be $\sigma_{\pi N} = 45 \pm 8$ MeV. In renormalised QCD $[m(\bar{u}u + \bar{d}d)]$ is scale invariant. Using the QCD sum-rule determination of $-\frac{<\text{vac}|\bar{u}u + \bar{d}d|\text{vac}>}{f_\pi^2}$ one finds that $\tilde{m} = 6$ MeV at a scale $\mu^2 = 1$ GeV$^2$.

The determination of $\sigma_{\pi N}$ from hadron spectroscopy goes as follows. The mass degeneracy in the baryon octet is broken by the finite quark masses. If we make a leading order analytic (linear) expansion in the quark mass and assume that there is a negligible strange quark component $<N|\bar{s}s|N> = 0$ in the nucleon, then it is easy to show that

$$\sigma_{\pi N}^{th} = \frac{3(M_\Xi - M_\Lambda)}{(m_q/m_s - 1)}$$ (4)

where $m_q$ and $m_s$ are the running light quark and strange quark masses respectively. If we identify $m_q = \tilde{m}$ (whence $m_s/m_q = 25$), then we find the familiar prediction of hadron spectroscopy $\sigma_{\pi N}^{th} = 25$ MeV.

After pion corrections are included, this number increases to $\sigma_{\pi N}^{th} = 35 \pm 5$ MeV. The difference between this theoretical prediction and the value of $\sigma_{\pi N}$ measured in $\pi N$ scattering is the sigma term “discrepancy”. Taken at face value, it implies that

$$y = \frac{2 <N|\bar{s}s|N>}{<N|\bar{u}u + \bar{d}d|N>} = 0.2 \pm 0.2$$ (5)

Modulo the large error, there appears to be a large strange quark component $<N|\bar{s}s|N> = 0$ in the nucleon. If this were true then a significant fraction of the nucleon’s mass would be due to strange quarks – in contradiction with the quark model.

The derivation of equs.(1-5) assumed that hadronic physics changes continuously (with no phase transition) as we take $m_q \to 0$. At this point we have to be careful. Even in QED we know that the theory of the electron differs from the theory with a zero mass gap. The Born level cross section for $e^+e^-$ production when a hard (large $Q^2$) transverse photon scatters from a soft (small $-p^2$) longitudinal photon is non-vanishing when we let $-p^2 \to 0$ in QED with a zero mass gap. This is in contrast to the familiar physical situation where the electron has a finite mass and this cross-section vanishes as $-p^2 \to 0$. As we take $m_e \to 0$ the vacuum in QED becomes strongly polarised: the perturbation theory expression for the vacuum polarisation $\Pi(q^2)$ diverges logarithmically and one cannot renormalise the theory using on-mass-shell subtraction. This suggests that the vacuum state for QED with a zero mass gap is not perturbative: it exists in a different phase of the theory.

In QCD there is every reason to expect vacuum polarisation to play an important role in the physics of light quarks since the QCD dynamics at strong coupling must spontaneously break the chiral symmetry of the classical theory.
3 Gribov confinement

I now present a simple explanation of the sigma term “discrepancy” in terms of Gribov’s theory of confinement\[2,3\]. The Gribov theory is obtained via the simultaneous solution of the Schwinger-Dyson equation for the quark propagator and the Bethe-Salpeter equation for meson bound states. At a critical coupling $\alpha_s \sim 0.6$ the theory exhibits a rich sequence of phase transitions. There appear multiple solutions to the quark propagator equation which correspond to new states in the light quark vacuum. These new vacuum states correspond to quasi-particle excitations with both positive and negative total energies\[2\].

Each phase transition leading to new states in the vacuum is characterised by a critical mass $m_P$. Provided that the running quark mass $m_P$ at the critical scale $\lambda$ (where $\alpha_s = \alpha_s^c$) is less than the critical mass $m_P^c \ll \lambda$ one finds that the solution of the Schwinger-Dyson equation in the new vacuum states matches onto the solution of the perturbative renormalisation group equation, which describes the physics at small coupling. If this condition is satisfied then the new vacuum states yield physical excitations in the theory. (For technical details and a derivation of these results see Ref.[2].)

Each transition is characterised by a pseudo-scalar Goldstone bound state. (The appearance of the new vacuum states spontaneously breaks the chiral symmetry.) The wavefunctions of the physical Goldstone states are found by perturbing the solution of the Bethe-Salpeter equation about the critical mass $m_P^c$. The mass of the Goldstone state associated with any given phase transition is determined by $m_P$ and the value of $m_P^c(\lambda)$ for that transition. One finds that the Goldstone meson mass is

$$\mu^2 = \kappa^2 (m_P^c - m_P)$$

where $\kappa^2$ is a positive constant. The fact that we see only one Goldstone pion state in the physical spectrum tells us that the light quark mass lies somewhere between the critical mass for the first and the second vacuum transitions.

We compare equs. (6) and (1), whence it follows that the “current-quark” mass in pion physics (and in particular equ.(3)) is

$$\hat{m} = m_P^c - m_P$$

rather than $m_P$ and

$$\kappa^2 = \left[ -\langle \text{vac} | \overline{q}_q | \text{vac} \rangle \right]_{\mu^2 = \lambda} \frac{f_\pi^2}{}\mu^2 = \lambda$$

The QCD dynamics which lead to the chiral phase transition tell us that chiral perturbation theory (in pion physics) is really an expansion about $(m_P^c - m_P) = 0$ rather than about $m_P = 0$. (The usual formulation of chiral symmetry assumes that $m_P^c = 0$, which need not be the case.) As we decrease $m_P \to 0$ the mass of the physical pion increases in the Gribov theory. When $m_P$ coincides with the critical mass for the $n$th transition ($n \geq 2$) a new Goldstone state appears in the hadron spectrum with zero mass. The mass of this state increases as we decrease $m_P$ further. At $m_P = 0$ one finds an infinite number of pseudo-scalar Goldstone states with small masses $\mu_{\pi,n}^2$, where $\mu_{\pi,n}^2 \geq \mu_{\pi,n+1}^2 \to 0$ as $n \to \infty$.

The effect of the negative energy states is that the pole in the quark propagator occurs on an unphysical sheet — the quark becomes a resonance\[4\]. Furthermore, when we go beyond the quenched approximation we find pinch cuts, which run parallel to the real momentum axis and prevent us from making a Wick rotation at strong coupling. That is, the full, unquenched Gribov theory has a different solution in each of Euclidean and Minkowski times.

4 Chiral perturbation theory

Whilst the “current-quark” mass in pion physics is $\hat{m} = m_P^c - m_P$ (equ.(7)), the mass in the renormalised QCD Hamiltonian is (of course) the running quark mass $m_P$. The mass degeneracy of physical non-Goldstone states (like the baryon octet) is broken by the finite values of the running masses $m_P$. The
increases with \( m_P \). When \( m_P \) is less than the critical mass for the \( n^{th} \) transition the baryon mass also receives “pion” corrections associated with the \( n^{th} \) Goldstone state. The leading “pion” correction to \( M_B \) in improved chiral perturbation theory is proportional to \( \mu^2 \ln \mu^2 \) \[7\], where \( \mu^2 \) is proportional to \((m_P^c - m_P)\) in the Gribov theory. When \( m_P \) is increased above \( m_P^c \) these Goldstone states condense in the vacuum leading to a further increase in the mass of the constituent quark \[2\]. The quark mass which appears in the linear mass expansion (which does not include “pion” corrections) for the baryon octet is the running mass \( m_P \). This means that \( m_P \) is the light quark mass in equ.(4). The phase transitions in the light quark vacuum mean that the Gribov theory anticipates a “discrepancy” between the value of \( \sigma_{\pi N} \) which is measured in \( \pi N \) scattering and the value of \( \sigma_{\pi N}^{th} \) which is extracted from baryon spectroscopy. This “discrepancy” would remain even if the effect of “pion” corrections to \( M_B \) were included. There is no theoretical need to introduce a large strange quark matrix element in the nucleon in order to explain the \( \sigma_{\pi N} \) data.

We now estimate the the value of \( m_P^c \) at 1 GeV \(^2\). Since the “pion” corrections to equ.(4) are clearly different in the Gribov theory to those in the usual theory of chiral symmetry, we use the linear mass expansion to estimate \( m_P^c \). The old spectroscopy prediction of the sigma term used \( m_q = (m_P^c - m_P) \) instead of \( m_q = m_P \). We substitute the measured value \( \sigma_{\pi N} = 45 \text{MeV} \) into the linear mass formula

\[
\sigma_{\pi N}(\frac{m_s}{m_q} - 1) = \text{constant}
\]

and use the result that the strange “current-quark” mass \( m_s \simeq m_{s,P} \) to obtain \( m_q \simeq 10.5 \text{MeV} \) at a scale \( \mu^2 = 1 \text{GeV}^2 \). It follows that the value of the critical mass for the chiral phase transition is \( m_P^c \simeq 16.5 \text{MeV} \) after evolution to \( \mu^2 = 1 \text{GeV}^2 \).

Clearly, the strange quark mass is above \( m_P^c \). The strange quark part of the kaon and eta wavefunctions is not a Goldstone state. The kaon is the lowest mass pseudo-scalar which couples to the strange quark axial vector current. Chiral symmetry theory tells us that \( \mu_s^2 \) should be proportional to a strange quark mass. Since there is no critical mass for the strange quark, it follows that

\[
\mu_s^2 = \kappa^2 m_{s,P} \quad (m_{s,P} \geq m_P^c)
\]

If we decrease the strange quark mass from \( m_s \simeq 150 \text{MeV} \), then \( \mu_s \) decreases linearly from \( \simeq 485 \text{MeV} \) to \( \simeq 165 \text{MeV} \) at \( m_s = m_P^c \simeq 16.5 \text{MeV} \). At this point we reach the critical mass for the first chiral transition and \( \mu_s \) drops to zero. If we decrease \( m_s \) further, then \( \mu_s^2 \) rises according to equ.(6).

Let us summarise the state of chiral perturbation theory in the Gribov theory. If we wish to work with meson degrees of freedom (and not QCD), then the physical pseudo-scalar meson octet is described the effective chiral action that is used in the usual theory of chiral perturbation theory \[10\]. The SU(3)\(\otimes\)SU(3) symmetry of this action is broken by a mass term. If we wish to make the connection to QCD, then the pion mass squared is proportional to \( \hat{m} = (m_P^c - m_P) \) and the mass squared of the strange part of the kaon and eta is proportional to \( m_s \). The Gell-Mann, Okubo relation

\[
3m_{\eta}^2 + m_{\pi}^2 = 4m_K^2
\]

does not involve the quark masses and is not affected by the chiral phase transition. If we take the quark masses to zero, then we need to consider the effect of the chiral phase transitions. This is particularly important if we wish to include baryons in the same theoretical framework as the pseudo-scalar meson octet.
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