The chiral and deconfinement phase transitions

Fukun Xu 1, Mei Huang 1,2

1 Institute of High Energy Physics, Chinese Academy of Sciences,
Yuquan Road 19B, 100049, Beijing, China
2 Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences,
Yuquan Road 19B, 100049, Beijing, China

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Abstract: By introducing the dressed Polayakov loop or dual chiral condensate as a candidate order parameter to describe the deconfinement phase transition for light flavors, we discuss the interplay between the chiral and deconfinement phase transitions, and propose the possible QCD phase diagram at finite temperature and density. We also introduce a dynamical gluodynamic model with dimension-2 gluon condensate, which can describe the color electric deconfinement as well as the color magnetic confinement.

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1. Introduction

The interplay between chiral and deconfinement phase transitions at finite temperature and density continue to be of interest in the study of the QCD phase diagram. The chiral restoration is characterized by the restoration of chiral symmetry and the deconfinement phase transition is characterized by the breaking of center symmetry, which are only well defined in two extreme quark mass limits, respectively. In the chiral limit when the current quark mass is zero \( m = 0 \), the chiral condensate \( \langle \bar{q}q \rangle \) is the order parameter for the chiral phase transition. When the current quark mass goes to infinity \( m \to \infty \), QCD becomes pure gauge \( SU(3) \) theory, which is center symmetric in the vacuum, and the usually used order parameter is the Polyakov loop expectation value \( \langle P \rangle \) [1], which is related to the heavy quark free energy.

At zero density and chiral limit, lattice QCD results show that the chiral and deconfinement phase transitions occur at the same critical temperature [2]. It has been widely believed for a long time that chiral symmetry restoration always coincides with deconfinement phase transition in the whole \((T,\mu)\) plane. It has been conjectured in Ref. [3] that in the large \( N_c \) limit a confined but chiral symmetric phase, which is called the quarkyonic phase, can exist in the high baryon density region. It is interesting to question whether or not this quarkyonic phase can survive in a practical QCD phase diagram.

For the case of finite physical quark mass, neither the chiral condensate nor the Polyakov loop are suitable order parameters. The Wuppertal-Budapest group [4, 5] found that for the case of \( N_f = 2 + 1 \), there are three pseudo-critical temperatures: the transition temperature for chiral restoration of \( u, d \) quarks

\[ T_c^{ud}(N_f = 2 + 1) = 151(3) \text{ MeV} \]

the
transition temperature for s quark number susceptibility $T_s^c = 175(2)(4)\text{ MeV}$; and the deconfinement transition temperature $T_s^d = 176(3)(4)\text{ MeV}$ from the Polyakov loop. These results are supported by the HotQCD collaboration [6, 7] which uses an improved HISQ action. (The critical temperature for deconfinement phase transition recently extracted from RHIC data is $T_s^d = 175.3\text{ MeV}$ [8].)

This paper discusses two topics relating to deconfinement phase transition. In Sec. 2, we will discuss the possible order parameter candidate for describing the deconfinement phase transition for light flavors, and investigate the interplay between the chiral and deconfinement phase transitions. Based on our results, we will propose the possible QCD phase diagram at finite temperature and density. In Sec. 3, we will try to introduce a gluodynamic model to describe the deconfinement phase transition.

# 2. Dressed Polyakov loop and deconfinement phase transition for light flavors

Recent investigation has revealed that quark propagator heat kernels can also act as order parameters as they transform non trivially under the center transformation related to deconfinement transition [9–14]. The exciting result is the behavior of the spectral sum of the Dirac operator under this center transformation. A new order parameter, called the dressed Polyakov loop, has been defined which can be represented as a spectral sum of the Dirac operator. The infrared part of the spectrum has been found to play a leading role in confinement. This result is encouraging because it suggests that it may be possible to relate the chiral phase transition to the confinement-deconfinement phase transition. The order parameter for chiral phase transition is related to the spectral density of the Dirac operator through the Banks-Casher relation [15]. Therefore, both the dressed Polyakov loop and the chiral condensate are related to the spectral sum of the Dirac operator.

Consider a $U(1)$ valued boundary condition for the fermionic fields in the temporal direction $\psi(x, \beta) = e^{-i\beta}\psi(x, 0)$, where $0 \leq \beta < 2\pi$ is the phase angle and $\beta$ is the inverse temperature. The dual quark condensate, or the dressed Polyakov loop $\Sigma_1$, is then defined as $\Sigma_1 = -\int_0^{2\pi} d\beta \frac{e^{-i\beta}}{2\pi} \langle \bar{\psi} \psi \rangle$. It transforms in the same way as the conventional thin Polyakov loop under center symmetry and hence is an order parameter for the deconfinement transition. It reduces to the thin Polyakov loop and to the dual of the conventional chiral condensate in infinite and zero quark mass limits respectively: in the chiral limit $m \to 0$ we get the dual of the conventional chiral condensate and in the $m \to \infty$ limit we have the thin Polyakov loop. Therefore, we extend the dressed Polyakov loop as a candidate order parameter to describe the deconfinement phase transition of a quark with any mass.

The dressed Polyakov loop may be considered in the framework of the three-flavor NJL model:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + G_1 \sum_a \left\{ (\bar{\psi} \tau_a \psi)^2 + (\bar{\psi} i\gamma_5 \tau_a \psi)^2 \right\} - K \left\{ \text{Det}(\bar{\psi}(1 + \gamma_5)\psi) + \text{Det}(\bar{\psi}(1 - \gamma_5)\psi) \right\}. \quad (1)$$

Where $\psi = (u, d, s)^T$ denotes the transpose of the quark field, and $m = \text{Diag}(m_u, m_d, m_s)$ is the corresponding mass matrix in the flavor space. $\tau_a$ with $a = 1, \ldots, N_f^2 - 1$ are the eight Gell-Mann matrices, and Det means determinant in flavor space. The last term is the standard form of the ’t Hooft interaction, which is invariant under $SU(3)_L \times SU(3)_R \times U(1)_B$ symmetry, but breaks down the $U(1)$ symmetry.

The chiral phase transition characterized by conventional chiral condensate, and the deconfinement phase transition characterized by the dressed Polyakov loop $\Sigma_1$, are investigated in Ref. [16, 17]. For the two flavor case, our results agree with those given by the Dyson-Schwinger Equations [18–20]. The three-flavor phase diagram in the $T - \mu$ plane for the case of $m_u = m_d = 5\text{ MeV}$ and $m_s = 140.7\text{ MeV}$ is shown in Fig. 1. The dash-dotted lines and the dashed lines are the critical lines for $\Sigma_1$ and the conventional chiral phase transition respectively. The solid lines indicate the 1st order phase transitions, and the solid circle indicates the critical end points for chiral phase transitions of $u, d$ quarks. It is noticed that in the region of $\mu < 270\text{ MeV}$, the strange quark experiences a smooth crossover, and there are no phase transition lines for the strange quark. It is found that the phase transitions are flavor dependent, and there is a phase transition range for each flavor. The transition range of s quark is located at higher temperature and higher baryon density than that of $u, d$ quarks. In the low baryon density region, it is found that the transition ranges of $u, d$ quarks are similar to that of the s quark, however, the separation of the transition ranges for $u, d$ quarks and s quark becomes wider with increasing chemical potential. Based on the above results, in Fig. 2, we show our conjectured 3 dimension (3D) QCD phase diagram for finite temperature $T$, quark chemical potential $\mu_q$ and isospin chemical potential $\mu_I$. 

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**The chiral and deconfinement phase transitions**
3. Gluodynamic model for color electric deconfinement and color magnetic confinement

In the framework of QCD effective models, there is still no dynamical model which describes the chiral symmetry breaking and confinement simultaneously. The main difficulty in establishing a QCD model which includes the confinement mechanism is calculation of the Polyakov loop statistically. Currently, the popular models used to investigate the chiral and deconfinement phase transitions are the Polyakov Nambu–Jona-Lasinio model (PNJL) and Polyakov linear sigma model (PLSM) [21–25], where the Polyakov loop is introduced to the framework statistically. A dynamical model which defines deconfinement phase transition is still missing. In [26], we introduce a pure gluodynamic model with dimension-2 gluon condensate, and investigate how well this model captures the main features of deconfinement phase transition. In the last decade, there has been growing interest in dimension-2 gluon condensates $< g^2 A^2 >$ in SU($N_c$) gauge theory [27–31], which is regarded to be closely related to confinement.

The pure gluon part of QCD Lagrangian is described by $\mathcal{L}_G = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$, with $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$. The gluon field can be decomposed into a condensate field $A^a_\mu$ and a fluctuating field $\delta A^a_\mu$ as, $A^a_\mu(x) = A^a_\mu + \delta A^a_\mu(x)$ [32]. Then the Lagrangian after this background expansion becomes $\langle \mathcal{L}_G \rangle = -\frac{1}{4} [g^2 + 2m_0^2 g^2 + 4b \phi_0^2]$, with $m_0^2 = \frac{g^2}{16\pi^2} \phi_0^2$ and $b = \frac{9}{16\pi^2} g^2$. The gluon has mass because of the existence of nonperturbative dimension-2 gluon condensate.

At finite temperature, the electric and magnetic screening masses as functions of temperature are shown in Fig. 3. It can be seen that the electric and magnetic components are degenerate at low temperatures and start to split at
higher temperatures, and that the electric screening mass rises rapidly with $T$. Correspondingly, the Polyakov loop expectation value as a function of $T/T_c$ in Fig. 4 is compared with lattice data in Ref.[33]. It is found that the Polyakov loop expectation value is zero in the vacuum and low temperature region, then rises sharply at high temperatures. However, the magnetic screening mass of the gluons remains almost the same as its vacuum value, which characterizes the color magnetic confinement feature of QCD.

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