Interferometric measurement of the quadrature coherence scale using two replicas of a quantum optical state

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Multi-copy observables for the detection of optically nonclassical states

Distinguishing quantum states that admit a classical counterpart from those that classify has been a central issue in quantum optics. Finding a way to do so in a reasonable manner has been one of the major challenges in the field of quantum information and quantum computing.

A new approach to this problem is based on the use of multi-copy observables. These observables are defined as the expectation values of observables that are taken in different copies of the quantum state. The key advantage of multi-copy observables is that they can provide information about the non-classical nature of the state, which is not available from single-copy observables.

Interferometric measurement of the quadrature coherence scale using two replicas of a quantum optical state

Interferometric methods are a powerful tool for measuring the coherence properties of quantum states. One of the key quantities that describe the coherence of a quantum state is the quadrature coherence scale. This quantity is defined as the ratio of the quadrature variance to the mean-square displacement of the quantum state.

In this paper, we propose a new method for measuring the quadrature coherence scale using two replicas of a quantum optical state. The method is based on interferometric techniques and allows for a precise and accurate measurement of the coherence properties of the state.

Measuring the quadrature coherence scale on a cloud quantum computer

Quantum computers are a promising technology for solving a wide range of problems that are intractable for classical computers. One of the key challenges in using quantum computers is the measurement of the coherence properties of quantum states. In this paper, we propose a new method for measuring the quadrature coherence scale on a cloud quantum computer.

The method is based on a novel interferometric technique that allows for a precise and accurate measurement of the coherence properties of the quantum state. The method is implemented on a cloud quantum computer, which allows for a scalable and efficient measurement of the coherence properties of the state.
Measuring polynomial functions of states

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Abstract

In this paper I show that any $m$th-degree polynomial function of the elements of the density matrix $\rho$ can be determined by finding the expectation value of an observable on $m$ copies of $\rho$, without performing state tomography. Since a circuit exists which can approximate the measurement of any observable, in principle one can find a circuit which will estimate any such polynomial function by averaging over many runs. I construct some simple examples and compare these results to existing procedures.
The multicopy technique [1]

Goal: Evaluate the following polynomial function

\[ f(\rho) = \sum_{i_1, j_1, \ldots, i_m, j_m} c_{i_1 j_1 \ldots i_m j_m} \rho_{i_1 j_1} \rho_{i_2 j_2} \cdots \rho_{i_m j_m} \quad \hat{\rho} = \sum_{i, j=0}^{d-1} \rho_{ij} |i\rangle \langle j| \]

Then there always exists a multicopy operator whose mean value on multiple copies of the state evaluate the function:

\[ f(\rho) = \langle \langle \hat{A}_f \rangle \rangle_{\hat{\rho}} = Tr \left( \hat{A}_f \hat{\rho}^{\otimes m} \right) \]

where

\[ \hat{\rho}^{\otimes m} = \sum_{i_1, j_1, \ldots, i_m, j_m} \rho_{i_1 j_1} \cdots \rho_{i_m j_m} |i_1\rangle \langle j_1| \otimes \cdots \otimes |i_m\rangle \langle j_m| \]  

\[ \hat{A}_f = \sum_{i_1, j_1, \ldots, i_m, j_m} c_{i_1 j_1 \ldots i_m j_m} \hat{A}_{i_1 j_1 \ldots i_m j_m} \]

[1] T.A. Brun, Measuring polynomial functions of states, Quant. Inf. Comp. 4, 401 (2004)
Optical nonclassicality

Glauber-Sudarshan P-function [2]: \[ \hat{\rho} = \int P(\alpha) |\alpha\rangle \langle \alpha| d^2 \alpha \]
where \(|\alpha\rangle\) coherent state

A state is said to be \textit{classical} if its Glauber-Sudarshan P-function is a probability distribution:

\[ P(\alpha) = P_{cl}(\alpha) \]

If the Glauber-Sudarshan P-function of a state \( \hat{\rho} \) fails to be interpreted as a probability distribution, then the state is \textit{nonclassical}.

\[ P(\alpha) \neq P_{cl}(\alpha) \]

Known witnesses: Mandel parameter, squeezing parameter, Shchukin et al. hierarchy of nonclassicality witnesses, quadrature coherence scale

[2] R. J. Glauber, Phys. Rev. \textbf{131}, 2766 (1963) & E. C. G. Sudarshan, Phys. Rev. Lett. \textbf{10}, 277 (1963)
Multicopy observables for the detection of optically nonclassical states

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Distinguishing quantum states that admit a classical counterpart from those that exhibit nonclassicality has long been a central issue in quantum optics. Finding an implementable criterion certifying optical nonclassicality (i.e., the incompatibility with a statistical mixture of coherent states) is of major importance as it often is a prerequisite to quantum information processes. A hierarchy of conditions for detecting whether a quantum state exhibits optical nonclassicality can be written based on matrices of moments of the optical field [E. V. Shchukin and W. Vogel, Phys. Rev. A 72, 043808 (2005)]. Here, we design optical nonclassicality observables that act on several replicas of a quantum state and whose expectation value coincides with the determinant of these matrices, hence providing witnesses of optical nonclassicality that overcome the need for state tomography. These multicopy observables are used to construct a family of physically implementable schemes involving linear optical operations and photon number detectors.

Physical Review A, 106: 043705, October 2022.
Nonclassicality and matrix of moments \cite{3}

Shchukin, Richter and Vogel hierarchy of nonclassicality criteria based on normally ordered moments

$$d_N = \begin{pmatrix}
1 & \langle \hat{a} \rangle & \langle \hat{a}^\dagger \rangle & \langle \hat{a}^2 \rangle & \langle \hat{a}^3 \rangle & \langle \hat{a}^4 \rangle & \langle \hat{a}^5 \rangle & \langle \hat{a}^6 \rangle & \langle \hat{a}^7 \rangle & \langle \hat{a}^8 \rangle & \langle \hat{a}^9 \rangle & \langle \hat{a}^{10} \rangle & \langle \hat{a}^{11} \rangle & \langle \hat{a}^{12} \rangle & \langle \hat{a}^{13} \rangle & \langle \hat{a}^{14} \rangle & \langle \hat{a}^{15} \rangle & \langle \hat{a}^{16} \rangle & \langle \hat{a}^{17} \rangle & \langle \hat{a}^{18} \rangle \\
\langle \hat{a}^\dagger \rangle & \langle \hat{a}^2 \rangle & \langle \hat{a}^3 \rangle & \langle \hat{a}^4 \rangle & \langle \hat{a}^5 \rangle & \langle \hat{a}^6 \rangle & \langle \hat{a}^7 \rangle & \langle \hat{a}^8 \rangle & \langle \hat{a}^9 \rangle & \langle \hat{a}^{10} \rangle & \langle \hat{a}^{11} \rangle & \langle \hat{a}^{12} \rangle & \langle \hat{a}^{13} \rangle & \langle \hat{a}^{14} \rangle & \langle \hat{a}^{15} \rangle & \langle \hat{a}^{16} \rangle & \langle \hat{a}^{17} \rangle & \langle \hat{a}^{18} \rangle \\
\langle \hat{a}^2 \rangle & \langle \hat{a}^3 \rangle & \langle \hat{a}^4 \rangle & \langle \hat{a}^5 \rangle & \langle \hat{a}^6 \rangle & \langle \hat{a}^7 \rangle & \langle \hat{a}^8 \rangle & \langle \hat{a}^9 \rangle & \langle \hat{a}^{10} \rangle & \langle \hat{a}^{11} \rangle & \langle \hat{a}^{12} \rangle & \langle \hat{a}^{13} \rangle & \langle \hat{a}^{14} \rangle & \langle \hat{a}^{15} \rangle & \langle \hat{a}^{16} \rangle & \langle \hat{a}^{17} \rangle & \langle \hat{a}^{18} \rangle \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{pmatrix}$$

If there exists a $N$ s.t. $d_N < 0$ then the state is optically nonclassical

\bibitem{3} E. Shchukin et al., Phys. Rev. A 71, 011802(R) (2005)
## 2-copy nonclassicality observables: $d_{23}$ and $d_{15}$

\[ d_{23} = \begin{bmatrix} \langle \hat{a}^\dagger \hat{a} \rangle & \langle \hat{a}^\dagger \hat{a}^2 \rangle \\ \langle \hat{a}^2 \rangle & \langle \hat{a}^\dagger \hat{a} \rangle \end{bmatrix} \quad \text{and} \quad d_{15} = \begin{bmatrix} 1 & \langle \hat{a}^\dagger \hat{a} \rangle \\ \langle \hat{a}^\dagger \hat{a} \rangle & \langle \hat{a}^\dagger \hat{a}^2 \hat{a}^2 \rangle \end{bmatrix} \]

| Squeezed and even cat states | Fock and odd cat states |
|------------------------------|-------------------------|
| $\hat{D}_{23}' = \frac{1}{2} (\langle n_1' - n_2' \rangle^2 - \langle n_1' + n_2' \rangle)$ | $\hat{D}_{15} = \frac{1}{2} (\langle n_1 - n_2 \rangle^2 - \langle n_1 + n_2 \rangle)$ |

**Interpretation**

\[ \langle \hat{D}_{23}' \rangle = \frac{1}{2} \left( \langle (n_1' - n_2')^2 \rangle - \langle n_1' + n_2' \rangle \right) = -\frac{1}{2} \langle n_1 + n_2 \rangle = -\sinh^2(r) = 0 \]
Interferometric measurement of the quadrature coherence scale using two replicas of a quantum optical state

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Assessing whether a quantum state $\hat{\rho}$ is nonclassical (i.e., incompatible with a mixture of coherent states) is a ubiquitous question in quantum optics, yet a nontrivial experimental task because many nonclassicality witnesses are nonlinear in $\hat{\rho}$. In particular, if we want to witness or measure the nonclassicality of a state by evaluating its quadrature coherence scale, this a priori requires full state tomography. Here, we provide an experimentally friendly procedure for directly accessing this quantity with a simple linear interferometer involving two replicas (independent and identical copies) of the state $\hat{\rho}$ supplemented with photon number measurements. This finding, that we interpret as an extension of the Hong-Ou-Mandel effect, illustrates the wide applicability of the multicopy interferometric technique in order to circumvent state tomography in quantum optics.

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**Quadrature coherence scale** [4]

Definition for a state $\hat{\rho}$ of $n$ bosonic modes:

$$
\mathcal{C}^2(\hat{\rho}) = \frac{1}{2n \mathcal{P}(\hat{\rho})} \left( \sum_{j=1}^{2n} \text{Tr} [\hat{\rho}, \hat{r}_j][\hat{r}_j, \hat{\rho}] \right)
$$

where

$$
\hat{\mathbf{r}} = (\hat{x}_1, \hat{p}_1, \cdots, \hat{x}_n, \hat{p}_n) \\
\mathcal{P}(\hat{\rho}) = \text{Tr}(\hat{\rho}^2) \quad \text{(purity)}
$$

**Properties:**

- $\mathcal{C}(\hat{\rho}) > 1 \quad \Rightarrow \quad \hat{\rho}$ is nonclassical
  - Classical states: QCS lower or equal to 1
- Nonclassicality measure: the larger the QCS, the further it is from $\mathcal{C}_{cl}$
- Average coherence scale of any pair of conjugated quadratures

[4] S. De Bièvre, D. B. Horoshko, G. Patera & M. I. Kolobov, Phys. Rev. Lett. **122**, 080402 (2019)
Measurement of the quadrature coherence scale \[4\]

QCS for one mode system:

\[ C^2(\hat{\rho}) = -\frac{1}{2\mathcal{P}(\hat{\rho})} \text{Tr} \left( [\hat{\rho}, \hat{x}]^2 + [\hat{\rho}, \hat{p}]^2 \right) \]

where \( \mathcal{P}(\hat{\rho}) = \text{Tr}(\hat{\rho}^2) \) (purity)

\[ C(\hat{\rho}) > 1 \implies \hat{\rho} \text{ is nonclassical} \]

How to measure the QCS?

\[ C^2(\hat{\rho}) = \frac{\mathcal{N}(\hat{\rho})}{\mathcal{P}(\hat{\rho})} \]

\[ \mathcal{N}(\hat{\rho}) = -\frac{1}{2} \text{Tr} \left( [\hat{\rho}, \hat{x}]^2 + [\hat{\rho}, \hat{p}]^2 \right) \]

Separated multicopy measurements

\[ \mathcal{P}(\hat{\rho}) = \text{Tr}(\hat{\rho}^2) \]

Need of 2 copies

[4] S. De Bièvre, D. B. Horoshko, G. Patera & M. I. Kolobov, Phys. Rev. Lett. 122, 080402 (2019)
Measurement of the purity $\mathcal{P}(\hat{\rho}) = \text{Tr}(\hat{\rho}^2)$

\[
\mathcal{P}(\hat{\rho}) = \text{Tr} \left( (\hat{\rho} \otimes \hat{\rho}) \hat{S} \right)
\]

where

\[
\hat{S} |\varphi\rangle |\psi\rangle = |\psi\rangle |\varphi\rangle, \forall |\varphi\rangle, |\psi\rangle
\]

\[
\hat{S} = e^{i \frac{\pi}{2} (\hat{a}^\dagger - \hat{b}^\dagger)(\hat{a} - \hat{b})}
\]

\[
\begin{align*}
\hat{c} &:= \hat{U}_{BS}^\dagger \hat{a} \hat{U}_{BS} = (\hat{a} + \hat{b}) / \sqrt{2} \\
\hat{d} &:= \hat{U}_{BS}^\dagger \hat{b} \hat{U}_{BS} = (-\hat{a} + \hat{b}) / \sqrt{2}
\end{align*}
\]

\[
\hat{S} = e^{i \pi \hat{d}^\dagger \hat{d}} = (-1)^{\hat{n}_d}
\]

Measurement of the swap operator already implemented in a many-body Bose-Hubbard system [5][6]

[5] A. J. Daley, H. Pichler, J. Schachenmayer & P. Zoller, Phys. Rev. Lett. 109, 020505 (2012)
[6] R. Islam, R. Ma, P. M. Preiss, M. Eric Tai, A. Lukin, M. Rispoli & M. Greiner, Nature 528, 77–83 (2015)
Measurement of the purity

\[ \mathcal{P}(\hat{\rho}) = \text{Tr} \left( (\hat{\rho} \otimes \hat{\rho}) \hat{S} \right) \]
\[ \hat{S} = (-1)^{\hat{n}_d} \]

Fock state \( |1\rangle \)

\[ |1\rangle \]
\[ \hat{a} \quad \hat{d} \]
\[ \frac{1}{2} \]
\[ \hat{b} \quad \hat{c} \]
\[ \sqrt{2} \]

\[ |20\rangle - |02\rangle \]

\[ \mathcal{P}(\hat{\rho}) = 1 \]

Squeezed state

\[ |S_r\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{k=0}^{\infty} (-e^{i\phi} \tanh r)^k \frac{\sqrt{2k!}}{2^k k!} |2k\rangle \]

\[ \mathcal{P}(\hat{\rho}) = 1 \]
**Measurement of the QCS**

\[ \mathcal{N}(\hat{\rho}) = -\frac{1}{2} \text{Tr} \left( [\hat{\rho}, \hat{x}]^2 + [\hat{\rho}, \hat{p}]^2 \right) \]

**Objective**

\[ \mathcal{N}(\hat{\rho}) = \text{Tr} \left( (\hat{\rho} \otimes \hat{\rho}) \hat{N} \right) \]

\[ \mathcal{N}(\hat{\rho}) = \frac{1}{2} \left( \int (x - x')^2 |\langle x|\hat{\rho}|x'\rangle|^2 \, dx \, dx' + \int (p - p')^2 |\langle p|\hat{\rho}|p'\rangle|^2 \, dp \, dp' \right) \]

\[ = \frac{1}{2} \left( \int (x - x')^2 \langle x, x'|\hat{\rho} \otimes \hat{\rho}|x', x\rangle \, dx \, dx' + \int (p - p')^2 \langle p, p'|\hat{\rho} \otimes \hat{\rho}|p', p\rangle \, dp \, dp' \right) \]

\[ \hat{N} = \frac{1}{2} \left[ (\hat{x}_a - \hat{x}_b)^2 + (\hat{p}_a - \hat{p}_b)^2 \right] \hat{S} \]
Measurement of the QCS $\mathcal{N}(\hat{\rho}) = -\frac{1}{2} \text{Tr} \left( [\hat{\rho}, \hat{x}]^2 + [\hat{\rho}, \hat{p}]^2 \right)$

$\mathcal{N}(\hat{\rho}) = \text{Tr} \left( (\hat{\rho} \otimes \hat{\rho}) \hat{N} \right)$ where $\hat{N} = \frac{1}{2} \left[ (\hat{x}_a - \hat{x}_b)^2 + (\hat{p}_a - \hat{p}_b)^2 \right] \hat{S}$

$\hat{N} = (\hat{x}_d^2 + \hat{p}_d^2) \hat{S} = (1 + 2 \hat{n}_d) \hat{S}$
Other results and comment

QCS for n modes

Numerical simulations of the result

Experimental implementation already done [7]

[7] A. Z. Goldberg, G. S. Thekkadath & K. Heshami, Phys. Rev. A 107, 042610 (2023)
Conclusion and perspectives

• Multicopy technique: efficient way of measuring some expectation values experimentally.

• Perspectives:
  o Try to improve our results by using homodyne or heterodyne detection and/or active elements (e.g. implement $d_{1235}$)
  o Use multicopy technique to implement new criteria
  o Analyze the mathematics behind multicopy (link with algebra, Jordan-Schwinger map)

[7] A. Z. Goldberg, G. S. Thekkadath & K. Heshami, Phys. Rev. A 107, 042610 (2023)
Thank you for your attention!