Quasi-Degradation Probability of Two-User NOMA over Rician Fading Channels

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Abstract

Non-orthogonal multiple access (NOMA) has a great potential to offer a higher spectral efficiency of multi-user wireless networks than orthogonal multiples access (OMA). Given two users, previous work has established the condition, referred to quasi-degradation (QD) probability, under which NOMA has no performance loss compared to the capacity-achieving dirty paper coding. Existing results assume Rayleigh fading channels without line-of-sight (LOS). In many practical scenarios, the channel LOS component is critical to the link quality where the channel gain follows a Rician distribution instead of a Rayleigh distribution. In this work, we analyze the QD probability over multi-input and single-output (MISO) channels subject to Rician fading. The QD probability heavily depends on the angle between two user channels, which involves a matrix quadratic form in random vectors and a stochastic matrix. With the deterministic LOS component, the distribution of the matrix quadratic form is non-central that dramatically complicates the derivation of the QD probability. To remedy this difficulty, a series of approximations is proposed that yields a closed-form expression for the QD probability over MISO Rician channels. Numerical results are presented to assess the analysis accuracy and get insights into the optimality of NOMA over Rician fading channels.

Keywords

Gamma distribution, multi-input single-output (MISO), non-orthogonal multiple access (NOMA), quasi-degradation, Rician fading.

I. INTRODUCTION

Due to the scarcity of wireless spectrum, high spectral efficiency has been the primal design goal of contemporary wireless communication systems that impose multiple users to share the
same spectrum. Traditionally, spectrum sharing is performed through orthogonal multiple access (OMA). Recently, non-orthogonal multiples access (NOMA) has received tremendous attentions from both academia and industry for its improved spectral efficiency over OMA [1]. The great potential of NOMA has stimulated the interest of the standardization body to include NOMA in the standard of the 5th generation (5G) mobile network [2].

While NOMA has been extensively studied in the literature, there remain numerous challenges to the successful application of NOMA in practice. One major concern is the implementation complexity. For power-domain NOMA, the superimposed user signals need to be decoded using advanced receiving techniques such as successive interference cancellation that increases receiver complexity. Also, the power allocated to different users is crucial to the achievable rate of NOMA. To determine the optimal power allocation, channel state information (CSI) is required at the transmitter side that is generally not accurate. Besides, the power allocation for sum rate maximization is a non-convex optimization problem that does not have closed-form solutions even for the simplest two-user case [3]. Furthermore, there is no guarantee that the sum rate achieved by NOMA through optimal power allocation can approach the capacity region of multi-input and multi-output (MIMO) broadcast channels. For example, when different users’ channels are nearly orthogonal or have similar channel gains, severe performance loss is incurred using NOMA.

Most existing work focuses on maximizing the sum rate for NOMA users by optimizing the power allocation for given user channel realizations that determine the decoding order for SIC. There is little discussion on whether the given user channels are attainable to achieving the capacity region. In [4], the authors characterize the relationships among the capacity region of MIMO broadcast channels and rate regions achieved by NOMA and time-division multiple access (TDMA), which is a typical member of OMA family. For a given single-antenna user pair served by a single-antenna transmitter, the probability that NOMA outperforms TDMA in terms of sum rate and individual user rates is derived. In [5], the notion of quasi-degraded channels is introduced to evaluate the condition of users channels that achieve the capacity region of broadcast channels using NOMA. Since the exact capacity region of broadcast channels is not known, the sum rate of capacity achievable dirty paper coding (DPC) is considered as the benchmark [6]. For a pair of users, their channels are quasi-degraded if applying NOMA incurs no performance loss compared to that of using sophisticated DPC, which employs the
encoding order same as the decoding order in NOMA. The notion of quasi-degradation (QD) is also exploited in [7] to address the user pairing and optimal precoding design for multi-user NOMA systems, where the precoders are designed for two users that are paired if their channels are quasi-degraded. Besides, the authors derive the QD probability when a multi-antenna base station (BS) serves two single-antenna users, i.e., a multi-input and single-output (MISO) setup. All the existing work [4], [5], [7] assumes Rayleigh fading with no line-of-sight (LOS) in the propagation channel. For some scenarios suitable for NOMA, the quality of a communication link is dominated by the LOS component in which case the fading statistics follow Rician distribution instead of Rayleigh distribution. For example, measurement results confirm that the 28 GHz millimeter wave outdoor channels follow a Rician distribution with the Rician-factor ranging from 5-8 dB [8].

In this paper, we extend the work [7] and intend to characterize the optimality of NOMA over Rician fading channels. Given two fixed users, we theoretically analyze the probability that the two user channels are quasi-degraded, namely, using NOMA to serve these two users can achieve the identical performance as non-linear DPC. In our work, a general model for the MISO Rician fading channel is considered, including a deterministic component that captures the azimuthal angle of the LOS signal and a non-deterministic component due to the randomness of NLOS. Unlike the Rayleigh fading case, the presence of LOS component raises numerous challenges to the characterization of QD probability. Firstly, the deterministic LOS component results in non-zero and distinct means for each element in the channel vector, i.e, the channel vectors are not isotropic. Consequently, the channel power of the MISO Rician channel has a non-central distribution whose statistical characteristics are difficult to be obtained. Secondly, the QD probability heavily depends on the squared cosine of the angle between two user channels, which can be represented in the matrix quadratic form [7] as given by $X^HAX$ where $X$ is a complex random vector and $A$ is a symmetric matrix. When $X$ is the channel vector of an isotropic channel, the distribution of the matrix quadratic form follows Chi-squared distribution [7], but this is not the case for non-isotropic channels. Moreover, the matrix $A$ in our work is stochastic but existing results on the distribution of the quadratic form are limited to the constant matrix $A$. It is worth to mention that there have been fruitful results on the distribution of the matrix quadratic form in random vector $X$ when $X$ is real [9], [10]. For complex random vectors, [11] considers the quadratic form in a zero-mean complex random vector and the case of a non-
zero mean complex random vector is studied in [12], [13]. However, the matrix $A$ considered in [11]–[13] is constant. To the best of our knowledge, the statistical properties of the matrix quadratic form with a stochastic matrix has not been explored in the literature.

In view of the aforementioned difficulties, it is not plausible to find the exact QD probability over Rician fading channels. In this work, we propose an analytical framework that derives the QD probability based on some celebrated approximation techniques. Our contributions are two-fold.

- We show that the distribution of the quadratic form with random matrix $A$ and complex vector $X$ can be approximated to the gamma distribution. This is achieved with the aid of the second-order moment matching technique. To this end, we obtain the mean and the variance of the quadratic form with random matrix $A$ and complex vector $X$ by extending the existing results for constant $A$ and real vector $X$.

- We show that the channel power of the MISO Rician fading channel, which follows the non-central chi-square distribution, can be also well approximated to the gamma distribution. This approximation greatly facilitates the analysis of the QD probability that involves the ratio between the channel powers of two independent Rician channels.

- Using the approximated distributions aforementioned, we obtain the QD probability over MISO Rician fading channels. Numerical results are presented to validate the accuracy of the approximated QD probability and provide insights to the optimality of NOMA subject to Rician fading. It is shown that the strength of the LOS component relative to the NLOS component affects the QD probability in a different way, depending on the angular difference between the user channels.

The remainder of this paper is organized as follows. The MISO Rician fading channel is first defined in Sec. II. Some useful distributions and important results on the distributions of the matrix quadratic form are established in Sec. III. Theoretical analysis for the QD probability over Rician fading channels is conducted in Sec. IV. Numerical results are presented and discussed in Sec. VI. Finally, concluding remarks are drawn in Sec. VII.

Notations: In our notations, italic letters are used for scalars. Vectors and matrices are noted by bold-face letters. For a square matrix $A$, $A^{-1}$, $\text{tr}(A)$, $A^T$ and $A^H$ denote its inverse, trace, transpose and conjugate transpose, respectively. $I$ and $0$ denote an identity matrix and an all-zero matrix, respectively. For a complex-valued vector $x$, $\|x\|$ denotes its Euclidean norm.
Fig. 1. Illustration of the two-user NOMA system.

The distribution of a circularly symmetric complex Gaussian random vector with mean $\mu$ and covariance matrix $\Sigma$ is denoted by $\mathcal{CN}(\mu, \Sigma)$, and ‘$\sim$’ stands for ‘distributed as’. $\mathbb{E}[\cdot]$ and $\mathbb{V}[\cdot]$ denote the statistical expectation and variance, respectively. $\mathfrak{R}(\cdot)$ and $\mathfrak{I}(\cdot)$ denote the real and the imaginary part of a complex number. Finally, $\mathbb{C}^{m\times n}$ denotes the space of $m \times n$ complex-valued matrices.

II. SYSTEM MODEL AND REVIEW OF RELEVANT SCHEMES

A. System Model

Consider a downlink wireless network consisting of one BS and multiple users. The BS has $N$ ($N \geq 2$) antennas and each user has a single antenna. The channel vector between a user and the BS follows the Rician fading channel model given by

$$
\mathbf{g} = \sqrt{\beta} \left( \sqrt{\frac{1}{K+1}} \mathbf{h} + \sqrt{\frac{K}{K+1}} \mathbf{a} \right)
$$

where $\beta$ accounts for the large-scale fading due to pathloss, $\mathbf{h} \in \mathbb{C}^{N} \sim \mathcal{CN}(0, \mathbf{I}_N)$ models the normalized NLOS component and $\mathbf{a} \in \mathbb{C}^{N}$ is a deterministic vector that captures the LOS component. The power ratio between the LOS component and NLOS component is determined by the Rician factor $K$. Assuming the $N$ BS antennas form a uniform linear array (ULA) with half-wavelength antenna spacing, the LOS component is modeled as
\[ \mathbf{a} = [1, e^{-i\pi \sin(\theta)}, \ldots, e^{-i\pi (N-1) \sin(\theta)}]^T \] where \( \theta \) is the azimuth angle of the user. While the one-dimensional ULA is considered for its popularity, our work can be extended to other antenna array patterns such as uniform rectangular arrays (URAs) and uniform circular arrays (UCAs) with minor modifications. It is also worth to mention that our work generalizes the existing work that considers Rayleigh fading channels with the Rician factor \( K \to 0 \).

In this work, we focus on two arbitrary users and characterize the QD probability, namely, the users channels permit the same performance using NOMA as that by DPC. The definition of QD probability will be given in Sec. IV. Fig. 1 illustrates the considered scenario.

### III. Preliminaries

In this section, we establish the distribution of the matrix quadratic form that serves as the core of the analysis for the QD probability in Sec. IV. Besides, some useful distributions and the second-order moment matching technique relevant to our work will be reviewed.

**Definition 1.** Given a multivariate random vector \( \mathbf{X} = (x_1, \ldots, x_N)^T \) and a symmetric matrix \( \mathbf{A} = (a_{ij}) \), the quadratic form of \( \mathbf{X} \) is defined as

\[
Q_\mathbf{A}(\mathbf{X}) = \mathbf{X}^T \mathbf{A} \mathbf{X} = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} x_i x_j. \tag{2}
\]

From (2), \( Q_\mathbf{A}(\mathbf{X}) \) is a scalar function of \( \mathbf{X} \). When \( \mathbf{X} \) is a real random vector and \( \mathbf{A} \) is a constant matrix, the mean and variance of \( Q_\mathbf{A}(\mathbf{X}) \) are well known as given below.

**Lemma 1.** For a real random vector \( \mathbf{X} \) with mean \( \mathbf{\mu} \) and covariance matrix \( \mathbf{\Sigma} \), the mean and variance of \( Q_\mathbf{A}(\mathbf{X}) \) is given by [9],

\[
\mathbb{E}[Q_\mathbf{A}(\mathbf{X})] = \text{tr}(\mathbf{A} \mathbf{\Sigma}) + Q_\mathbf{A}(\mathbf{\mu}) \tag{3}
\]

\[
\mathbb{V}[Q_\mathbf{A}(\mathbf{X})] = 2\text{tr}((\mathbf{A} \mathbf{\Sigma})^2) + 4Q_\mathbf{A} \mathbf{\Sigma} \mathbf{A}(\mathbf{\mu}^T). \tag{4}
\]

The quadratic form \( Q_\mathbf{A}(\mathbf{X}) \) is useful for defining sums of squares.

**Lemma 2.** The quadratic form \( Q_\mathbf{A}(\mathbf{X}) \) reduces to the squared norm of \( \mathbf{X} \) when \( \mathbf{A} = \mathbf{I}_N \), e., \( \mathbf{X}^H \mathbf{X} = x_1^2 + \cdots + x_N^2 = \|\mathbf{X}\|^2 \). If the entries of \( \mathbf{X} \) are independent but not necessarily identical (i.n.i.d.) Gaussian distributed with nonzero means and unit variance, then \( \|\mathbf{X}\|^2 \) follows the non-central chi-squared \((\chi^2)\) distributed with \( 2N \) degrees of freedom.
**Remark 1.** The exact distribution of $Q_A(X)$ is only known when $A$ is deterministic. For a stochastic, matrix $A$, approximations are required to obtain the statistics of $Q_A(X)$.

Although the distribution of the non-central $\chi^2$ distribution is well known, the PDF involves the modified Bessel function of the first kind that yields no closed-form expressions for the QD probability. For analytical tractability, we approximate the squared Gaussian random variables to be gamma distributed, which is defined as follows.

**Lemma 3.** If $X$ is Gamma distributed with the shape parameter $k$ and the scale parameter $\theta$, the PDF is given by

$$f_X(x) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}},$$

where $\Gamma(\cdot)$ denotes the Gamma function. Moreover, $X \sim \Gamma(k, \theta)$ has the mean and variance given as

$$\mathbb{E}[X] = k\theta,$$
$$\mathbb{V}[X] = k\theta^2.$$

The approximation of the squared Gaussian random variable to gamma distribution is established based on the following lemma.

**Lemma 4.** If the random variable $X \sim \mathcal{N}(\mu, \sigma^2)$, then $X^2$ has the same first and second order statistics as $\Gamma(k, \theta)$ where

$$k = \frac{(\sigma^2 + \mu^2)^2}{2\sigma^2(1 + \mu^2)}, \quad \text{and} \quad \theta = \frac{2\sigma^2(1 + \mu^2)}{\sigma^2 + \mu^2}. \tag{5}$$

*Proof:* See Appendix A. 

We will need to work on the sum of i.n.i.d. gamma random variables. The exact distribution of the sum of i.n.i.d. gamma random variables can be obtained using numerical methods such as inverting the characteristic function or the saddlepoint approximation [14]. However, the distribution obtained from numerical computations does not permit a closed-form expression that is necessary to the derivation of the QD probability of interest. In this work, we resort to the second-moment matching technique to obtain the approximated distribution for the sum of i.n.i.d. gamma random variables.
Lemma 5 (Second-order moment matching). Let \( \{X_n\}_{n=1}^N \) be a set of \( N \) i.n.i.d. gamma random variables where \( X_n \sim \Gamma(k_n, \theta_n) \). Then the sum \( Z = \sum_{n=1}^N X_n \) has the same first and second order statistics as a gamma random variable with the shape and scale parameters given as

\[
k = \left( \frac{\sum_n k_n \theta_n}{\sum_n k_n \theta_n^2} \right)^2, \quad \theta = \frac{\sum_n k_n \theta_n^2}{\sum_n k_n \theta_n}.
\]  

(6)

Our analysis for the QD probability also relies on the inverse gamma distribution given as follows.

Lemma 6. If a random variable \( Z \sim \Gamma(k, \theta) \), then \( Z^{-1} \) follows the inverse gamma distribution with the PDF given as

\[
f_{Z^{-1}}(z) = \frac{1}{\Gamma(k) \theta^k} z^{-(k+1)} e^{-\frac{1}{\theta z}}.
\]  

(7)

The mean and the variance of \( Z^{-1} \) are

\[
\mathbb{E}[Z^{-1}] = \frac{1}{(k-1)\theta}, \quad \mathbb{V}[Z^{-1}] = \frac{1}{(k-1)^2(k-2)\theta^2}
\]  

(8)

for \( k > 2 \).

The subsequent analysis involves the quotient of two independent gamma random variables.

Lemma 7. Given \( V \sim \Gamma(k_v, \theta_v) \) and \( W \sim \Gamma(k_w, \theta_w) \), \( \frac{V}{W} \) follows the Beta prime distribution with the PDF given by

\[
f_{V/W}(x) = \frac{x^{\alpha_v-1}(1/\theta_v)^{\alpha_v}(1/\theta_w)^{\alpha_w}}{(x/\theta_w + 1/\theta_v)^{\alpha_v+\alpha_w} B(\alpha_v, \alpha_w)}
\]  

(9)

where \( B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} \, dt \) is the Beta function.

Finally, the random vector \( X \) in the quadratic form encountered in our analysis is complex. The following lemma adopted from [13] establishes the connection between a complex-valued random vector and its real-valued counterpart.

Lemma 8. A complex random vector \( Z \in \mathbb{C}^N \) is constructed from a pair \( X = (X_1^T, X_2^T)^T \) of real random vectors as

\[
Z = X_1 + jX_2
\]  

(10)
where \( X_i \in \mathbb{R}^N \) for \( i = 1, 2 \). Equivalently, \( Z \) can be represented as a pair \( Z = (Z^T, Z^H)^T \). The connection between \( Z \) and \( X \) is

\[
X = MZ, \ Z = M^{-1}X
\]

(11)

where \( M \) is a \( 2N \times 2N \) matrix given by

\[
M = \frac{1}{2} \begin{bmatrix}
I_N & I_N \\
-jI_N & jI_N
\end{bmatrix}
\]

(12)

Since \( M^{-1} = 2M \), we have \( Z = 2M^H X \).

IV. Quasi-degradation Probability

Consider the two-user NOMA where the BS transmits the combined signals of the two users \( i \) and \( j \) using superposition coding. The precoding vectors are designed to minimize the transmission power subject to the target rates constraints \( r_i \) and \( r_j \). The same objective can be achieved by using DPC at the BS, where the encoding order \( (i, j) \) is assumed to be fixed and identical to the decoding order of NOMA. For both NOMA and DPC, the design of the precoding vectors heavily depend on the users’ channels. If the two users’ channels denoted as \( g_i \) and \( g_j \), respectively, permit the same minimum transmission power of NOMA as that of DPC, their channels are quasi-degraded with respect to \( r_i \) and \( r_j \) [5]. It is shown in [7] that the condition of quasi-degraded channel can be expressed as

\[
Q(\Theta) \leq \frac{\|g_i\|^2}{\|g_j\|^2},
\]

(13)

where

\[
Q(\Theta) = \frac{1 + r_i}{\Theta} - \frac{r_i \Theta}{[(1 + r_j)(1 - \Theta)]^2}, \ \Theta = \cos^2(u)
\]

(14)

with \( u \in [0, \pi] \) being the angle between \( g_i \) and \( g_j \). Hence,

\[
\Theta = \frac{\|g_j^H g_i\|^2}{\|g_i\|^2 \|g_j\|^2}
\]

(15)

For convenience, denote the power ratio of the users’ channels as

\[
\frac{\|g_i\|^2}{\|g_j\|^2} \triangleq \Xi.
\]
Then the QD can be expressed as

\[ P_{QD} = P[Q(\Theta) \leq \Xi] \]
\[ = \mathbb{E}_{\Theta}[P[\Xi \geq Q(\Theta)]] \]
\[ = \int_{0}^{1} \int_{Q(\vartheta)}^{\infty} f_{\Xi}(\xi) d\xi f_{\Theta}(\vartheta) d\vartheta. \]  

(16)

In the sequel, we derive the PDFs of \( \Xi \) and \( \Theta \) that are essential to the evaluation of \( P_{QD} \). To begin with, we first analyze the distribution of the channel power for the considered Rician MIMO channel (1). It is worth to mention that when the channel vector is subject to different pathloss and LOS components as considered in this work, the distribution of channel power is non-isotropic.

A. Gamma Approximation for Channel Power

We first establish the approximated distribution of the channel power \( \|g\|^2 \) for the MIMO Rician channel in (1), which serves as the root of our work. Without loss of generality, the user index is dropped from the channel vector. Recall that each entry in \( g \) is a complex Gaussian random variable and thus the \( n \)th entry can be expressed as

\[ g_n = X_n + iY_n, \quad n = 0, 1, \cdots, N - 1. \]  

(17)

Since \( e^{-ix} = \cos(x) - i\sin(x) \), we have

\[ X_n = \sqrt{\frac{K}{K+1}} \cos(\varphi_n) + \sqrt{\frac{1}{K+1}} a_n \]
\[ Y_n = -\sqrt{\frac{K}{K+1}} \sin(\varphi_n) + \sqrt{\frac{1}{K+1}} b_n \]  

(18)

where \( \varphi_n = (n-1)\pi \sin(\theta) \), \( a_n \) and \( b_n \) are independent and follow the zero mean Gaussian distribution with variance 1/2. Together with the fact that \( \varphi_n \) is deterministic, both \( X_n \) and \( Y_n \) are Gaussian distributed as given by

\[ X_n \sim \mathcal{N} \left( \sqrt{\frac{K}{K+1}} \cos(\varphi_n), \frac{1}{2(K+1)} \right), \]  

(19)

\[ Y_n \sim \mathcal{N} \left( -\sqrt{\frac{K}{K+1}} \sin(\varphi_n), \frac{1}{2(K+1)} \right). \]  

(20)
With each entry in $g$ characterized above, the channel power $\|g\|^2$ is the sum of squared complex Gaussian random variables with distinct means and the same variance. When $X_n$ and $Y_n$ have unit variances, $\|g\|^2$ follows the non-central $\chi^2$ distribution with $2N$ degrees of freedom according to Lemma 2. Unfortunately, the unit variance condition is valid only when the Rician factor $K = -1/2$, which can not be true. Motivated by the gamma approximation for the non-isotropic fading channels in [15], we propose to approximate $\|g\|^2$ as follows. Firstly, $X_n^2$ and $Y_n^2$ are approximated as two independent gamma random variables that are fully characterized by the first two moments with closed form expressions using Lemma 4. Then the channel power is approximately gamma distributed according to Lemma 5.

**Remark 2.** The exact distribution of $\|g\|^2$ can be obtained using the result in [16], which represents $\|g\|^2$ in the Euler form and gives the distribution of the amplitude and the phase components. In their results, the Fourier series representation is used to numerically evaluate an improper integral appeared in the density function of $\|g\|^2$. Hence, their results are not useful to our work.

**B. Distribution of $\Xi$**

Given that the user’s channel power can be approximated to the gamma distribution, $\Xi$ as the ratio of $\|g_i\|^2$ and $\|g_j\|^2$ follows the Beta prime distribution according to Lemma 7.

**C. Distribution of $\Theta$**

First we examine the structure of $\Theta$. From (15), the numerator of $\Theta$ is the squared norm of the inner product between user $i$’s and user $j$’s channel vectors. The channel vector contains the complex entries with non-central means, in which case the PDF of the numerator of $\Theta$ involves complicated expressions. On the other hand, the denominator is the product of the user $i$’s channel power and user $j$’s channel power. As shown in Sec. IV-A, the channel power is non-central $\chi^2$ distributed. The PDF of the product of two independent non-central $\chi^2$ random variables is given in [17], which involves the infinite sum of the modified Bessel function of the second kind.

From the above discussion, the original form of $\Theta$ in (16) is analytically non-tractable.
\[ P_{QD} = \int_0^1 \int_{Q(\theta)}^\infty f_{\xi}(\xi) d\xi f_\Theta(\theta) d\theta \]

(i) \[ \int_0^1 \frac{(1/\theta_S)^{\alpha_S}(1/\theta_W)^{\alpha_W}}{B(\alpha_W, \alpha_S)} \int_{Q(\theta)}^\infty \frac{\xi^{\alpha_{\omega}-1}}{(\frac{1}{\theta_S} + \frac{\xi}{\theta_W})^{\alpha_W+\alpha_S}} d\xi f_\Theta(\theta) d\theta \]

(ii) \[ \int_0^1 (Q(\theta))^{-\alpha_S \theta_W^{\alpha_S}} \frac{2F_1(\alpha_W + \alpha_S, \alpha_S + 1; -\frac{\theta_W}{\theta_S Q(\theta)})}{\alpha_S B(\alpha_W, \alpha_S) B(\alpha_V, \alpha_W)} d\theta \]

(iii) \[ \left( \frac{\theta_W}{\theta_S} \right)^{\alpha_S} \left( \frac{\theta_W}{\theta_V} \right)^{\alpha_V} \frac{1}{\alpha_S B(\alpha_W, \alpha_S) B(\alpha_V, \alpha_W)} \int_0^1 \frac{2F_1(\alpha_W + \alpha_S, \alpha_S + 1; -\frac{\theta_W}{\theta_S Q(\theta)})}{(Q(\theta))^{\alpha_S} \left( 1 + \frac{\theta_W}{\theta_V} \right)^{\alpha_V+\alpha_W}} d\theta \]

(31)

Alternatively, we resort to an equivalent form as given by

\[ \Theta = \frac{g_i^H \Pi_{g_j} g_j^H g_i}{\|g_i\|^2 \|g_j\|^2} = \frac{g_i^H \Pi_{g_j} g_i}{\|g_i\|^2} \] (21)

where \( \Pi_{g_j} = g_j (g_j^H g_j)^{-1} g_j^H \). In (21), the denominator contains the channel power of one user only and thus is simpler than the original form. On the other hand, the numerator follows a matrix quadratic form of the random vector \( g_i \). Notice that \( \Pi_{g_j} \) is a function of \( g_j \) and thus it is a stochastic matrix of dimension \( N \times N \). Given that \( \|g_i\|^2 \) can be approximated by the gamma distribution, we show that the numerator of \( \Theta \) is also approximately gamma distributed in the following. The approximated distributions greatly simplify the analysis yet allow for numerical evaluation of the QD probability with reasonable accuracy. We note that when the user channel is subject to Rayleigh fading without the LOS component, the numerator of \( \Theta \) is \( \chi^2 \) distributed with degree of freedom 2 [7].

V. GAMMA APPROXIMATION FOR THE STOCHASTIC QUADRATIC FORM

The numerator of \( \Theta \) can be expressed in the quadratic form as \( Q_A(X) \) where \( A = \Pi_{g_j} = g_j (g_j^H g_j)^{-1} g_j^H \) and \( X = g_i \). Since \( g_i \) is a complex random vector and \( \Pi_{g_j} \) is stochastic, existing results shown in Lemma 1 for the real vector \( X \) and constant matrix \( A \) can not be used directly. In this section, we derive the mean and the variance of the complex quadratic form \( Q_S(Z) \), where \( S \) is a stochastic matrix and \( Z \) is a complex vector.
A. Mean of $Q_{\Pi_g}(g)$

We first establish the mean of the complex quadratic form $Q_A(Z)$ when $A$ is deterministic and $Z \in \mathbb{C}^N$.

**Lemma 9.** Consider a complex random vector $Z$ where the real and the imaginary parts have the same distribution. For a deterministic matrix $A$, the mean of the complex quadratic form $Q_A(Z)$ is given as

$$
E[Q_A(Z)] = \text{tr}(A\Sigma_Z) + Q_A(\mu_Z) \tag{32}
$$

where $\mu_Z = E[Z]$ and $\Sigma_Z = E[ZZ^H] - E[Z]E[Y^H]$.

**Proof:** The proof is deferred to Appendix B. ■

Now consider the complex quadratic form $Q_S(Z)$ with a stochastic matrix $S$ and a complex random vector $Z$. The following lemma gives the mean of $Q_S(Z)$.

**Lemma 10.** For a stochastic and hermitian matrix $S$, the mean of the quadratic form $Q_S(Z)$ where $Z$ is a complex random vector with mean $\mu_Z$ and covariance matrix $\Sigma_Z$ is given as

$$
E[Q_S(Z)] = \text{tr}(E[S])\Sigma_Z + Q_E[S](\mu_Z) \tag{33}
$$

**Proof:** The proof is given in Appendix C. ■

In using Lemma 10, $\Pi_{g_j}$ needs to be a Hermitian matrix. It is easy to verify that $\Pi_{g_j} = \Pi_{g_j}^H$. Moreover, the expectation of $\Pi_{g_j}$ is required. Denote the random variable $(g_j^H g_j)^{-1} \triangleq \Psi$ where the user index $j$ is dropped for brevity. The mean of $\Pi_g$ can be derived as

$$
E[\Pi_g] = E[g\Psi g^H] = E_{\Psi}[E_g[g\Psi g^H]] = \int_0^\infty E_g[\psi gg^H]f_\psi(\psi)d\psi. \tag{34}
$$

Because $\Psi$ is a function of $g$, the expectation in (34) requires the joint PDF of $\Phi$ and $g$, which is difficult to obtain.

For analytical tractability, we resort to an upper bound of $E[\Pi_g]$ by ignoring the correlation.
between $\Psi$ and $g$ such that (34) is simplified as

$$E[\Pi_g] = E[G] \cdot E[\Psi].$$  

(35)

where $G \triangleq gg^H \in \mathbb{C}^{N \times N}$. The two expectations in (35) are derived as follows. According to the structure of $g$, the $(m, n)$th entry of $G$ is given by

$$G(m, n) = \frac{\beta K}{K+1} \cos(\varphi_m - \varphi_n)$$

$$+ \frac{\beta \sqrt{K}}{K+1} \left( \cos(\varphi_m) \mathcal{R}(h_n) + \cos(\varphi_n) \mathcal{R}(h_m) - \sin(\varphi_m) \mathcal{I}(h_n) - \sin(\varphi_n) \mathcal{I}(h_m) \right)$$

$$+ \frac{\beta}{K+1} \left( \mathcal{R}(h_m) \mathcal{R}(h_n) + \mathcal{I}(h_n) \mathcal{I}(h_m) \right)$$

$$+ \left[ \frac{\beta K}{K+1} \sin(\varphi_n - \varphi_m) \right] - \frac{\beta \sqrt{K}}{K+1} \left( \cos(\varphi_m) \mathcal{I}(h_n) + \sin(\varphi_n) \mathcal{R}(h_m) - \mathcal{I}(h_m) \mathcal{R}(h_n) \right)$$

$$i$$

(36)

where $h_n$ denotes the $n$th entry of $h$. Given that $\mathcal{R}(h_m), \mathcal{I}(h_m), \mathcal{R}(h_n),$ and $\mathcal{I}(h_n)$ are independent zero-mean Gaussian random variable with variance $1/2$, $E[\mathcal{R}(h_m)\mathcal{R}(h_n)] = 0$ when $m \neq n$ and $E[\mathcal{R}(h_m)\mathcal{R}(h_n)] = \frac{1}{2}$ when $m = n$. After some arrangements, the expectation of $G(m, n)$ is equal to

$$E[G(m, n)]$$

$$= \begin{cases} 
\frac{K}{K+1} \left( \cos(\varphi_m - \varphi_n) - i \sin(\varphi_m - \varphi_n) \right), & \text{if } m \neq n \\
\frac{K}{K+1} \left( \cos(\varphi_m - \varphi_n) - i \sin(\varphi_m - \varphi_n) \right), & \text{if } m = n \\
+ \frac{1}{K+1}. 
\end{cases}$$  

(37)

Next, we derive the expectation of $\Psi$. As explained in Sec. IV-A, $g^H g$ can be well approximated to a gamma random variable. Thus, $\Psi$ follows the inverse gamma distribution as given by Lemma 6. Combining (8) and (37), (35) is obtained in closed form, which is not presented due to space limit.
B. Variance of $Q_{\Pi_g}(g)$

Similar to Sec. V-A, we first derive the variance of the complex quadratic form $Q_A(Z)$ for a deterministic matrix $A$.

**Lemma 11.** Consider a complex random vector $Z$ where the real and the imaginary parts have the same distribution. For a deterministic matrix $A$, the variance of the complex quadratic form $Q_A(Z)$ is given as

$$
\mathbb{V}[Q_A(Z)] = \text{tr}((A\Sigma_Z)^2) + 2Q_A\Sigma_Z(A\mu_Z^H)
$$

where $\mu_Z = \mathbb{E}[Z]$ and $\Sigma_Z = \mathbb{E}[ZZ^H] - \mathbb{E}[Z]\mathbb{E}[Y^H]$.

**Proof:** The proof is deferred to Appendix D.

Lemma 11 can be extended to the case when $A$ is a stochastic matrix.

**Lemma 12.** For a stochastic and hermitian matrix $S$, the variance of the quadratic form $Q_S(Z)$ where $Z$ is a complex random vector with mean $\mu_Z$ and covariance matrix $\Sigma_Z$ is given as

$$
\mathbb{V}[Q_S(Z)] = \text{tr}(\Sigma_Z^2(\mathbb{E}[S])^2) + 2Q_S(\mathbb{E}[S])^2(\mu_Z).
$$

**Proof:** The proof is deferred to Appendix E.

C. QD Probability

With the PDFs for $\Xi$ and $\Theta$ obtained through the gamma approximation, the QD probability in (16) can be derived as (31) on the top of this page where (i) is obtained by using (9); (ii) is reached with the help of [18, (3.194-2)] where $\text{$_2$F$_1$}(\cdot,\cdot;\cdot;\cdot)$ denotes the Gauss hypergeometric function. Finally, (iii) is obtained by approximating $\Theta$ as the ratio of two gamma random variables with the PDF given in (9). For readers’ convenience, the procedure for computing the QD probability is summarized in Algorithm 1.

VI. Numerical Results

Numerical results are presented in this section to evaluate the QD probability subject to different system parameters. The accuracy of the proposed analysis for the QD probability is also validated. Without loss of generality, consider two arbitrary users and they are assigned
Algorithm 1 Approximated QD probability

Require:
User channels $g_i$ and $g_j$; number of antennas $N$.

2: for $n \leftarrow 1$ to $N$ do
    Approximate $\Re(g_i[n])$ as $\Gamma(k_{\text{real},n}, \theta_{\text{real},n})$;
4:  Approximate $\Im(g_i[n])$ as $\Gamma(k_{\text{imag},n}, \theta_{\text{imag},n})$;
end for
6: Approximate $\|g_i\|^2$ as $W \sim \Gamma(\alpha_W, \theta_W)$ using (6);
   Approximate $\|g_j\|^2$ as $S \sim \Gamma(\alpha_S, \theta_S)$ using (6);
8: Approximate $g_i^H \Pi g_j g_i$ as $V \sim \Gamma(k_V, \theta_V)$;
    Compute $P_{QD}$ using (31);

with the index $i = 1$ and $j = 2$. Unless specified, we set the azimuth angle for user 1 and user 2 as $\theta_1 = 30^\circ$ and $\theta_2 = \theta_1 + \theta_\Delta$, respectively, where $\theta_\Delta = 5^\circ$. To reflect the difference of the channel strength, define the pathloss ratio of the two user channels as $\beta_\Delta = \beta_1 / \beta_2$. Thus a larger value of $\beta_\Delta$ mimics the scenario that the two NOMA users have very different channel gains. By choosing $\beta_\Delta \geq 1$, we can ensure that decoding user 1’s signal first satisfies the necessary condition of QD. Finally, the target rate constraint is set as $r_1 = r_2 = 1$.

Fig. 2 plots the QD probability versus Rician factor $K$ with $\beta_\Delta = 5$ and 25, respectively. One can see that when the user channels are more LOS dominant (i.e., larger $K$), the QD probability is higher and the increasing trend is more remarkable if the two user channels are more different in their strengths (i.e., larger $\beta_\Delta$). For example, the QD probability with $\beta_\Delta = 5$ is about half of that with $\beta_\Delta = 25$. This agrees with the known results that NOMA gain is more pronounced when the two NOMA users have more different channel strengths. Notice that the above discussions are obtained with a fixed angular difference between two user channels. The increasing trend of the QD probability with the Rician factor $K$ does not always hold, which will be illuminated later. In terms of the analysis accuracy, the analytical results mostly match to the simulated ones. The discrepancy revealed on the figure is the consequence of the approximated distributions used in the analysis. Since various approximations are employed, their impacts to the analysis accuracy shall be examined later.

The impact of the user’s angular difference to the QD probability is investigated in Fig. 3 where the QD probability is plotted as a function of Rician factor $K$ for $\theta_\Delta = 5$ and 10. Here, $\beta_\Delta$ is fixed to 100. A small angular difference implies that the two users are close in the angular domain and thus they are likely to be served by the same transmitting beam using the typical beam selection algorithm. It is interesting to see that when the angular difference is small, i.e.,
\( \theta_\Delta = 5 \), the QD probability decreases with \( K \), which is opposed to the case when \( \theta_\Delta = 10 \). This suggests that when the two users are close in their azimuth angles and their channels are LOS-dominated (namely, \( K \) is large), the probability for their channels to be quasi-degraded becomes small. This is true even the two user channels are very different in strength (e.g., \( \beta_\Delta = 100 \)). Consequently, NOMA is not preferable because the chance for NOMA to achieve the same performance as DPC is diminished. On the other hand, NOMA can be beneficial to serve the users with close azimuth angles if the LOS strengths of their channels are not significant (e.g., \( \theta_\Delta = 5 \) and \( K \) is small), yet the QD probability remains lower than the case with a larger angular difference (\( \theta_\Delta = 10 \)). Here, we observe a reasonable match between the analytical results and
the simulated ones except when $K$ and $\theta_{\Delta}$ are small. The cause will be discussed next.

Since the exact analysis for the QD probability is not tractable, several approximations are employed in this work. We first examine the gamma approximation for the channel powers because this is the root that leads us to a tractable analysis for the QD probability. Fig. 4 plots the theoretical mean and variance obtained by first computing the shape and scale parameters of a gamma random variable used to approximate $\|g_1\|^2$ according to Remark 5. Then Lemma 3 is used to compute the required theoretical mean and variance. The analytical results are compared with the simulated ones for $\beta_{\Delta} \in [1, 100]$ and $K = 10$ dB. As shown, the gamma approximation for $\|g_1\|^2$ is promising to capture the first two moments. Both the mean and the variance of $\|g_1\|^2$ increase with $\beta_{\Delta}$. The increasing trend can be explained by observing the mean and the
covariance matrix of the channel vector $\mathbf{g}$ given by

$$
\mu_{g} = \sqrt{\frac{\beta K}{K+1}}a
$$

(40)

$$
\Sigma_{g} = \frac{\beta}{K+1}I_{N}.
$$

(41)

Clearly, they are both proportional to $\beta$.

Next, we evaluate the approximation for the numerator of $\Theta$, which is the angle between two user channels. As explained in Sec. V, the numerator of $\Theta$ appears in a matrix quadratic form $Q_{\Pi_{g}}(g_{1})$. The projection matrix $\Pi_{g}$ is important in determining $\Theta$’s numerator and it can be expressed as $g_{j}\Psi g_{2}^{H}$ where $\Psi = (g_{2}^{H}g_{2})^{-1}$ being approximated by the inverse gamma
distribution. Due to the correlation between $\Psi$ and $g_2$, it is difficult to obtain the distribution of $\Pi_{g_2}$. By ignoring the dependence, $\mathbb{E}[\Pi_{g_2}]$ is approximately equal to the product of $\mathbb{E}[G_2]$ and $\mathbb{E}[\Psi]$, as indicated by (35) for $G_2 = g_2 g_2^H$. To validate (35), Fig. 5 plots the trace values for $\mathbb{E}[\Pi_{g_2}]$ and $\mathbb{E}[G_2]$, both being a square matrix, for $K \in [0, 10]$ dB. Notice that $\mathbb{E}[G_2]$ is given in (37) and $\Psi$ is approximated by the inverse gamma distribution using Lemma 6. It can be seen that the theoretical trace values of $\mathbb{E}[G_2]$ perfectly match with the simulated ones while the theoretical trace values of $\mathbb{E}[\Pi_{g_2}]$ slightly deviate from the simulated ones when $K$ is small. With a smaller Rician factor $K$, the user channel is more sensitive to the dynamics in the NLOS component and thus ignoring the dependence between $\Psi$ and $g_2$ introduces errors in evaluating
Finally, we validate the accuracy of the approximated mean and variance of the complex quadratic form $Q_{\Pi g_2}(g_1)$. In Fig. 6, the theoretical and simulated mean values are plotted as a function of $K$ for varied $\theta_\Delta$ and $\beta_\Delta$. One can see that the analytical mean values match to the simulated ones, confirming the effectiveness of the proposed approximations. We note that the change of $Q_{\Pi g_2}(g_1)$ with respect to the Rician factor $K$ in Fig. 6 follows that of the QD probability in Fig. 3. Meanwhile, $Q_{\Pi g_2}(g_1)$ is proportional to $\Theta$, according to (21). Consequently, the QD probability is proportional to $\Theta$. Although $\Theta$ is a function of the angle between channel
vectors (denoted as $u$ in (14)), the above coincidence should not be interpreted as a proportional relationship between $u$ and the QD probability because $\Theta$ is not linear to $u$. Therefore, the connection between the QD probability and the angle between channel vectors may not be as simple as it is. Our analysis can be used to conveniently assess the probability of quasi-degraded channels.

The variance of $Q_{\Pi_{g_2}}(g_1)$ is plotted in Fig. 7. Comparing with the mean values shown in Fig. 6, a larger difference between the theoretical variances and the simulated ones is revealed. This is because the variance computed from (33) involves the square of $E[\Pi_{g_2}]$ that pronounces the approximation error. Regardless the angle difference and the pathloss ratio, the variance decreases with $K$. This is because the variance is mainly caused by the NLOS component and thus it becomes smaller when the channel is more LOS dominant (i.e., larger $K$).

VII. CONCLUSION

For MISO Rician fading channels, an analytical framework is proposed to derive the QD probability that characterizes the optimality of NOMA in approaching the capacity region of the two-user broadcast channel. The QD probability of interest involves a matrix quadratic form whose exact distribution is not available. With the aid of a series of approximations based on the gamma distribution, we obtained the QD probability over MISO Rician fading channels in closed form. Our work is versatile in capturing important channel parameters including both the large-scale and the small-scale fading, the array factors, and the angular information of LOS paths. Numerical results indicate that the obtained expression is accuracy for a wide range of the Rician factor and the angle difference between two users. Our results also reveal the coupled impact of channel angles and LOS dominance. Unlike the Rayleigh fading channels that permit a high QD probability as long as two users have very different channel gains, the QD probability is diminished if the two user channels are LOS dominant and close in the angular domain. The results of our work may find some useful applications. For example, user grouping is essential to NOMA systems and the QD probability can be used to assess if the channels of potential NOMA users are likely to be quasi-degraded. Also, the matrix quadratic form commonly appears in the performance metric of multi-antenna wireless systems. One can approximate the matrix quadratic form in random vectors with non-central distributions to the gamma distribution with acceptance accuracy. Besides, it is possible to extend our work to the antenna configurations...
Fig. 7. Variances of $Q_{\Pi g_2}(g_1)$. Left: $\theta_\Delta = 5$; right: $\theta_\Delta = 10$.

other than ULA.

APPENDIX A

PROOF FOR LEMMA 4

Denote $Y = (\frac{X}{\sigma})^2$. For $X \sim N(\mu, \sigma^2)$, it follows that $Y$ is non-central $\chi^2$ distributed with one degree of freedom and the non-centrality parameter equal to $(\frac{\mu}{\sigma})^2$. Consequently, the mean and variance of $Y$ are given by $(1 + (\frac{\mu}{\sigma})^2)$ and $2(1 + 2(\frac{\mu}{\sigma})^2)$, respectively. Since $X^2 = \sigma^2 Y^2$, 
the mean and variance of \( X^2 \) can be found as

\[
\mathbb{E}[X^2] = \mathbb{E}[\sigma^2 Y] = \sigma^2 + \mu^2 \quad (A.1)
\]
\[
\mathbb{V}[X^2] = \mathbb{V}[\sigma^2 Y] = 2\sigma^2(1 + \mu^2). \quad (A.2)
\]

Using the first two moments and Lemma 3, (5) can be obtained that completes the proof.

**APPENDIX B**

**Proof for Lemma 9**

Following Lemma 8, the complex random vector \( Z \) can be constructed from a pair of real random vectors \( X = (X^T_1, X^T_2)^T \). When \( X_1 \) and \( X_2 \) are drawn from the same distribution, they have the same mean and variance, i.e.,

\[
\mathbb{E}[X_1] = \mathbb{E}[X_2] = \mu \\
\Sigma_{X_1} = \Sigma_{X_2} = \Sigma. \quad (A.3)
\]

By extending the result in Lemma 1 for the real random vector to the complex random vector, the mean of the complex quadratic form \( Q_A(Z) \) can be obtained as (32).

Since \( A \) is symmetric, it can be shown that the complex quadratic form \( Q_A(Z) \) is connected through the real quadratic form through the following equation.

\[
Q_A(Z) = (X^T_1 - jX^T_2)A(X_1 + jX_2) \\
= Q_A(X_1) + Q_A(X_2) \quad (A.4)
\]

If \( X_1 \) and \( X_2 \) are drawn from the same distribution, both their means and covariance matrices are identical. Therefore,

\[
\mathbb{E}[Q_A(Z)] = \mathbb{E}[Q_A(X_1)] + \mathbb{E}[Q_A(X_2)] \\
= 2\text{tr}(A\Sigma) + 2Q_A(\mu). \quad (A.5)
\]

which is obtained using Lemma 1 and the notations defined in (A.3). Let’s work on the covariance
matrix of $Z$, denoted by $\Sigma_Z$. Using (11), we have

$$\Sigma_Z = \mathbb{E}[ZZ^H] - \mathbb{E}[Z]\mathbb{E}[Z^H]$$

$$= 4M^H \Sigma M. \quad (A.6)$$

Therefore,

$$\text{tr}(\Sigma_Z) = 4\text{tr}(M^H \Sigma M)$$

$$= 4\text{tr}(\Sigma M^HM)$$

$$= 2\text{tr}(\Sigma) \quad (A.7)$$

where we have used the fact that $M^HM = \frac{1}{2}I_N$. By multiplying $\Sigma_Z$ with matrix $A$, we have

$$\text{tr}(A\Sigma_Z) = 2\text{tr}(A\Sigma). \quad (A.8)$$

On the other hand, we can establish the quadratic form $Q_A(\mu_Z)$ in terms of the real-numbered vectors $\mu$ as

$$Q_A(\mu_Z) = (\mu^T - j\mu^T)A(\mu + j\mu)$$

$$= 2\mu^TA\mu$$

$$= 2Q_A(\mu). \quad (A.9)$$

Based on (A.8) and (A.9), (A.5) can be rewritten as (32) and the proof is completed.

**APPENDIX C**

**PROOF FOR LEMMA 10**

For a given instant of the stochastic matrix $S$, (32) gives the conditional mean of the quadratic form $Q_S(Z)$. By taking the expectation over $S$, we have

$$\mathbb{E}_S[Q_S(Z)] = \mathbb{E}_S[\text{tr}(S\Sigma_Z) + Q_S(\mu_Z)]$$

$$= \mathbb{E}_S[\text{tr}(S\Sigma_Z)] + \mu_Z^H \mathbb{E}_S[S]\mu_Z$$

$$= \text{tr}(\mathbb{E}[S]\Sigma_Z) + \mu_Z^H \mathbb{E}_S[S]\mu_Z \quad (A.10)$$
where the last line is obtained because the trace is a linear operator.

APPENDIX D

PROOF FOR LEMMA 11

Using the same argument for obtaining (A.5), the variance of $Q_A(Z)$ can be given as

$$\mathbb{V}[Q_A(Z)] = \mathbb{V}[Q_A(X_1)] + \mathbb{V}[Q_A(X_2)]$$

$$= 4\text{tr}((A\Sigma)^2) + 8Q_{A\Sigma\Lambda}(\mu). \quad (A.11)$$

With the aid of (A.8), it can be established that

$$\text{tr}((A\Sigma Z)^2) = 4\text{tr}((A\Sigma)^2). \quad (A.12)$$

In addition, the complex quadratic form $Q_{A\Sigma Z\Lambda}(\mu_Z^H)$ can be expressed in terms of the real quadratic form as

$$Q_{A\Sigma Z\Lambda}(\mu_Z^H) = (\mu + j\mu)A(4M^H\Sigma M)(\mu^T - j\mu^T)$$

$$= 4\mu A\Sigma A\mu^T$$

$$= 4Q_{A\Sigma\Lambda}(\mu^T), \quad (A.13)$$

which is obtained because $M^H M = \frac{1}{2}I_N$. Using (A.12) and (A.13), (A.11) can be rewritten as (39) that completes the proof.

APPENDIX E

PROOF FOR LEMMA 12

The variance of $Q_S(Z)$ for stochastic $S$ can be found as

$$\mathbb{V}[Q_S(Z)] = \mathbb{E}_S[\mathbb{V}[Q_S(Z)]]$$

$$= \mathbb{E}_S[\text{tr}((S\Sigma Z)^2) + 2Q_{S\Sigma Z S}(\mu_Z)]$$

$$\overset{(i)}{=} \text{tr}(\mathbb{E}_S[(S\Sigma Z)^2]) + 2\mu_Z^H\mathbb{E}_S[S\Sigma Z S]\mu_Z$$

$$\overset{(ii)}{=} \text{tr}(\Sigma_Z^2(\mathbb{E}[S])^2) + 2\mu_Z^H\Sigma Z(\mathbb{E}[S])^2\mu. \quad (A.14)$$
where (i) is obtained by exchanging the expectation and the trace operations due to the linearity of the trace operator and (ii) is obtained because the covariance $\Sigma_Z$ is constant to the expectation over $\mathbf{A}$. Finally, (39) is obtained by writing the last term in the quadratic form.

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