The astrophysics of rotational energy extraction from a black hole

Recent work has questioned whether nature can extract the rotational energy of a black hole via electromagnetic fields. Although we show that the Blandford–Znajek effect is sound, the deeper physics of the electric nature of black holes remains unresolved.

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Recently, King and Pringle have challenged the Blandford–Znajek mechanism for the extraction of black hole rotational energy by suggesting that the electric field near the horizon gets shorted out by the black hole's ability to absorb charge. That highly curved spacetime near black hole horizons leaves an indelible imprint on the accreting plasma is certain, but we show that it is the plasma astrophysics that holds the key to understanding the relevant electrical properties of the Blandford–Znajek process. Much of the confusion probably stems from the 3+1 form of the magnetohydrodynamics (MHD) equations in curved spacetime, which avoid both a full covariant treatment and the familiar vector formulation. As a result, those approaching the subject from an astrophysical perspective often acquire the false impression that black holes behave dramatically differently from other regions of empty spacetime. Here we try to amend this by illustrating the process needed to determine the electrical structure of the region near a black hole, with emphasis on the increased complexity in more realistic astrophysical environments (see also ref. 1). Ultimately, this leads us to the recognition that to determine the astrophysical behaviour of electric fields and currents near black holes, we must implement the covariant and causal general relativistic Ohm's law.

Although the black hole horizon cannot affect the physics of the magnetosphere, it has long been recognized that the near-horizon region must influence the dynamics of the inflowing plasma. This requires evaluating the equations of the accretion flow on the so-called stretched horizon to produce a 'regularity' condition. While the highly curved spacetime near the horizon alters the form of the equations, it cannot invalidate the physical content of the equations. In other words, black holes cannot determine or change the causal nature of the physics of the inflowing plasma, which, instead, is determined by the plasma itself. Accordingly, the nature of the electrical properties of the accreting flow near a black hole is determined by the astrophysical parameters of the flow. As the plasma astrophysics becomes complicated for realistic flows, various simplifications are adopted. In this work, we explore the strategy for determining the electrical properties near black holes for increasingly realistic flows. What we show is that while much of this remains unresolved currently, the criticisms of King and Pringle amount to ignoring the strategy that is needed to determine the behaviour of the flow near black holes.

Physics of the accreting fluid

Below, we first explore the strategy for determining the electric field near black holes in the ideal MHD limit. Next, we extend that to a more realistic astrophysical flow by implementing a finite conductivity. This is a highly idealized, and ultimately problematic, set of equations. In addition to the infinite conductivity and the absence of a constraint on current in these equations (it is obtained via Ampère's law), the fluid does not distinguish between particle species. If there is any place in the Universe where such an idealization is poorly motivated, it is in the extreme environment near a black hole. From a circuit perspective, the induced voltage or electromotive force near the black hole requires integrating the induced electric field, which from Eq. (3) and up to a minus sign is

\[ \mathbf{E} \cdot \mathbf{v} = 0 \quad (2) \]

\[ \mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \quad (3) \]

where \( \mathbf{E} \), \( \mathbf{v} \) and \( \mathbf{B} \) are the electric field, the fluid velocity and the magnetic field, respectively. Of course, Eq. (2) is implied by Eq. (3) and so is

\[ \mathbf{E} \cdot \mathbf{B} = 0. \quad (4) \]

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where we have implemented the relation between the Faraday tensor and the vector potential
\[ F_{cb} = A_{b,c} - A_{c,b}. \] (8)

But the invariant magnetic flux on the black hole is
\[ \varphi_{BH} = \int A_{\varphi} \, d\phi = \int A_{\phi} \, d\varphi = 2\pi A_{\phi} \] (9)

Hence,
\[ F_{\phi \theta} U^\theta = -A_{\phi,\theta} U^\theta = -\left( \partial \varphi_{BH} / \partial \theta \right) \left( U^\theta / 2\pi \right) \] (10)

which leads to
\[ F_{\phi \theta} = -\left( \partial \varphi_{BH} / \partial \theta \right) U^\theta / (2\pi U^\phi). \] (11)

Now, \( U^\phi / U^\theta = d\varphi / dt \) (\( \varphi \) is the azimuthal Boyer–Lindquist coordinate) is the angular velocity of the fluid with respect to the Boyer–Lindquist frame in which the electric field components are determined. For the second Boyer–Lindquist electric field component, we have
\[ F_{\phi \theta} U^\theta = -F_{\phi \theta} U^\theta - F_{\phi \phi} U^\phi = -A_{\phi,\theta} U^\theta = -\left( \partial \varphi_{BH} / \partial \theta \right) U^\theta / 2\pi \] (12)

which gives
\[ F_{\phi \theta} = -\left( 1/2\pi \right) \left( \partial \varphi_{BH} / \partial \theta \right) \, d\varphi / dt. \] (13)

By the same analysis, one obtains that the azimuthal electric field component vanishes.

The above equations relate the radial and poloidal electric field components to the gradient of magnetic flux and the angular velocity of the fluid into which is anchored a magnetic field. As a result, a notion of ‘velocity of magnetic field lines’ is introduced. This is a poor choice of words, because it gives the impression of something physical, and is motivated by the fact that it is a constant along magnetic field lines. To recover physical significance, it should be replaced by ‘velocity of the fluid frame in which the electric field vanishes’. Alternatively, one could say that it can be interpreted as an electromagnetic angular velocity. This induced electric field produces the electromotive force that in this part of the circuit allows for the dissipation of energy (for example, ref. 1). More rigorously, in a force-free magnetosphere, the flow of energy from the black hole to infinity requires setting the covariant divergence of \( e_\phi G^{ab} \) to zero:
\[ \nabla_a e_\phi G^{ab} = 0 \] (14)

where \( G \) is the electromagnetic energy–momentum tensor, and \( e \) solves the Killing equation to produce the conserved quantity \( e_\phi G^{ab} \). The point is that the electric field components derived above appear in Eq. (14), which is then evaluated at the horizon. The point of Eq. (5) is to motivate the importance of those electric field components.

In the next section we show how these induced electric field components change when we implement a more realistic Ohm’s law.

**Resistive MHD**

A more realistic and familiar form of Ohm’s law in standard vector notation is
\[ J = \sigma (E + v \times B) \] (15)

where \( J \) is the current density vector and \( \sigma \) is the scalar conductivity. In covariant component form, this is
\[ J^a = \sigma F^{ab} U_b \] (16)

Using the same strategy as in the section on ideal MHD above, we obtain Boyer–Lindquist electric field components by evaluating
\[ F_{\phi \theta} U^\theta = J_\theta / \sigma, \quad F_{\phi \phi} U^\phi = J_\phi / \sigma. \] (17)

Clearly, the induced electric fields are related to additional terms. The usual procedure is to start with the fields and obtain the current. Our goal, instead, is to think of using Eq. (16) to constrain or determine the electric fields. But what determines the electric and magnetic fields? The charge and current distributions, of course. In a numerical simulation, one fixes the conductivity and assumes an initial electromagnetic field. Then the simulation proceeds to update the fields which remain finite over time. The finite fields and their relation to finite current densities are captured by Eq. (16) and can be evaluated near the black hole. But there is no physics that can be identified that will short out the electric fields. What you bring to the black hole will determine the conditions near there, despite the highly curved spacetime region. And the plasma that is approaching the horizon is characterized by electric fields that are determined by the accreting physics and not by the black hole.

Apart from the above considerations, we wish to point out that although Eq. (16) may be better than Eq. (1) in terms of being more physical, it suffers from the fact that it violates special relativity. In fact, Eq. (16) implies an instantaneous current in response to the fields. In the next section, we will further generalize Ohm’s law as we attempt to make our physics even more realistic.

The **generalized Ohm’s law**

Because of the absence of any time dependence in the current in \( J^a = \sigma P^{ab} U_b \), it must violate special relativistic causality. This has long been recognized. The time dependence of the current appears in the generalized Ohm’s law, which, for the reasons given above, we write in standard vector notation for a gas with electrons and ions as
\[ \left( \frac{m_i m_e c^2 / Z_i e^2}{\rho_i} \right) \partial J^a / \partial t + J^a / \sigma = E + v \times B \]
\[ + \left( \partial / \partial Z_p \right) \left[ m_e \nabla p_e - m_i \nabla p_i \right] \]
\[ - \left( m_i - Z m_e \right) \, \times B. \] (18)

Above, \( m_i \) is the mass of the ions, \( m_e \) the mass of the electrons, \( c \) the speed of light, \( Z \) the atomic number, \( \rho \) the mass density, \( e \) the electric charge, and \( p \) the pressure of each particle species.

Equation (18) is itself a simplification of a more general dynamical equation for the motion of charged particle species. Among the simplifications are (1) linearity, (2) a scalar pressure and (3) electrical neutrality. The last condition, in particular, would add terms to Eq. (18) by way of \( n_i Z - n_e \neq 0 \), where \( n_i \) and \( n_e \) are the number density of ions and electrons, respectively.

We emphasize that implementing the generalized Ohm’s law is not simply a matter of adding one equation but a complex network of equations that constrain the quantities in Eq. (18). In attempting to understand the nature of the electric fields, Meier points out that any number of phenomena may produce charge separation in the context of the generalized Ohm’s law that are both causal and covariant. It is worth emphasizing that the state-of-the-art in simulations of accretion around black holes is still wedged either to Eq. (1) or at most to Eq. (16). The reason for this is that turbulence allegedly makes much of the microphysics of Eq. (18) irrelevant. There are reasons to be sceptical about this, especially close to the black hole horizon where magnetic fields are strong.

Despite the current limit in scope, recent particle-in-cell (PIC) simulations have begun to tease out the behaviour of...
charged particles near the horizon and find that the Blandford–Znajek conditions are valid\textsuperscript{14}. Komissarov\textsuperscript{5} also addresses King and Pringle’s critique of the Blandford–Znajek process in light of the work of ref. \textsuperscript{12}, concluding that although accumulation of electric charge onto the rotating black hole might affect dynamics in the magnetosphere, it cannot shut down the Blandford–Znajek process, but requires further investigation. He also emphasizes that the inner boundary conditions in PIC simulations (for example, refs. \textsuperscript{12,14}) are accurate. From ref. \textsuperscript{14}, one can estimate that the resolution needed just for nonideal (that is, resistive) MHD effects is 10 orders of magnitude smaller in length than the Schwarzschild radius, which, to use a specific source as example, is $10^{15}$ cm for M87. Similarly, characteristic resistive MHD timescales are 10 orders of magnitude smaller than ideal MHD. The transit time for M87 is of the order of $10^9$ seconds.

The development of PIC simulations is of fundamental importance, especially because a full Ohm’s law approach has not yet led to any tangible results. In short, we are quite removed from exploring the physics of the full Ohm’s law that is needed to rigorously determine the structure of the electric field near the black hole. Until then, any discussion that attempts to invalidate the Blandford–Znajek effect is moot.

As King and Pringle\textsuperscript{1} point out, however, jets are observed in sources that do not have black holes. Hence, the energy for jets need not come from black hole rotational energy as it does in general relativistic MHD simulations of black holes. This does not mean that the rotational energy of the object in question is not responsible. If the rotational energy of the rotating object is solely responsible for jets, one expects a correlation between the angular velocity of the object and the jet power. Additionally, the accretion disk carries gravitational potential energy and can be tapped via the Blandford–Payne mechanism\textsuperscript{14} by way of magnetic fields to produce a jet outflow. Evidence for an alternative source of energy is strongest in black hole X-ray binaries in ‘hard’ states, where no correlation is found between the black hole spin and the jet power\textsuperscript{15}.

**Summary and conclusions**

We have shown that to assess the true electrical properties near rotating black holes in both analytic and numerical work, one must implement more realistic physics. From reviews of the state-of-the-art, one acquires the impression that a physically well-motivated Ohm’s law is not necessary (for example, ref. \textsuperscript{10}), and the reason for this is the ubiquitous belief that turbulence tends to make discussion of microphysics irrelevant. It is also important to emphasize that resolving the scales on which plasma effects make their appearance is extremely computationally challenging. But although appealing to turbulent effects may be reasonable in much of a black hole magnetosphere, it is unlikely to be true in the region near the black hole where the intensity of gravity and the strength of the magnetic field are both large. If this is correct, we are left with an unresolved physical condition near black holes, and no first-principles approach can resolve this.

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**References**

1. King, A. R. & Pringle, J. E. *Astrophys. J.* 918, L22 (2021).
2. Komissarov, S. S. *Mon. Not. R. Astron. Soc.* https://doi.org/10.1093/mnras/stab2666 (2021).
3. MacDonald, D. & Thorne, K. *Mon. Not. R. Astron. Soc.* 198, 345–382 (1982).
4. Komissarov, S. S. *Mon. Not. R. Astron. Soc.* 367, L41–L44 (2001).
5. Znajek, R. L. *Mon. Not. R. Astron. Soc.* 179, 447–472 (1977).
6. McKinney, J. C. *Mon. Not. R. Astron. Soc.* 367, 1797–1807 (2006).
7. McKinney, J. C. *Mon. Not. R. Astron. Soc.* 368, 1561–1582 (2006).
8. Blandford, R. D. & Znajek, R. L. *Mon. Not. R. Astron. Soc.* 179, 433–456 (1977).
9. Meier, D. L. *Astrophys. J.* 605, 340–349 (2004).
10. Kandus, A. & Tiagas, C. G. *Mon. Not. R. Astron. Soc.* 385, 883–892 (2008).
11. Roide, S. *Astrophys. J.* 899, 95 (2020).
12. Crinquand, B., Cerutti, B., Philippov, A., Parfrey, K. & Dubus, G. *Phys. Rev. Lett.* 124, 141101 (2020).
13. Parfrey, K., Philippov, A. & Cerutti, B. *Phys. Rev. Lett.* 122, 035101 (2019).
14. Blandford, R. D. & Payne, D. G. *Mon. Not. R. Astron. Soc.* 199, 883–903 (1982).
15. Fender, R. P., Gallus, E. & Russell, D. *Mon. Not. R. Astron. Soc.* 406, 1425–1434 (2010).
16. Davis, S. W. & Tchekhovskoy, A. *Ann. Rev. Astron. Astrophys.* 58, 407–439 (2020).

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