The Color-Flavor Transformation and Lattice QCD*

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We present the color-flavor transformation for gauge group SU($N_c$) and discuss its application to lattice QCD.

1. Introduction

Motivated by the study of disordered systems in condensed matter physics, the color-flavor transformation was first derived by Zirnbauer in 1996 for an integral over U($N_c$)\textsuperscript{1}.

\[
\int_{U(N_c)} dU \exp \left( \bar{\psi}^i_{x+\mu,a} U^{ij}_{\mu,a} \psi^j_{x,a} + \bar{\psi}^i_{x,b} U^{ij}_{\mu,b} \psi^j_{x+\mu,b} \right) = \int D\rho_{N_c}(Z, \tilde{Z}) \times \exp \left( \bar{\psi}^i_{x+\mu,a} Z_{ab} \psi^i_{x+\mu,b} + \bar{\psi}^i_{x,b} \tilde{Z}_{ba} \psi^i_{x,a} \right). \tag{1}
\]

In the above equation, $\psi$ and $\bar{\psi}$ are $\mathbb{Z}_2$-graded tensors, $i, j = 1, \ldots, N_c$ are color indices, $a = 1, \ldots, n_+$ and $b = 1, \ldots, n_-$ are flavor indices, and the supermatrices $Z$ and $\tilde{Z}$ parameterize the coset space $U(n_+ + n_-) / U(n_+) \times U(n_-)$. Their boson-boson parts satisfy $\tilde{Z}_{BB} = Z_{BB}^t$ and for the fermion-fermion parts, $\tilde{Z}_{FF} = -Z_{FF}^t$. The measure is given by $D\rho_{N_c}(Z, \tilde{Z}) = d(Z, \tilde{Z})$ Sdet$(1 - \tilde{Z})^{N_c}$. For further progress, see \textsuperscript{2}.

Notice that before the transformation, the color indices of $\psi$ and $\bar{\psi}$ are coupled by the matrix $U$, while the flavor indices are diagonal. In the transformed integral, the flavor indices are coupled by the matrix $Z$, and the color indices are diagonal — thus the name of the transformation.

The integral over the gauge field closely resembles the one for a single link in the partition function of lattice QCD at infinite coupling. We could therefore apply the transformation to all links of a lattice and then integrate out the fermions. In this regard, the most important property of the transformation (\textsuperscript{1}) is that in terms of $\psi$ and $\bar{\psi}$, the right-hand side is local in coordinate space. Therefore, the fermion matrix, which for the usual lattice gauge action is an irreducible matrix of dimension $\sim N_c V$, decomposes into blocks of size $\sim N_f$ (number of flavors). This means that the fermion determinant of the transformed action is the product of determinants of fairly small matrices, which bears promise for a new approach to fermion algorithms.

In order to apply this strategy to lattice QCD, we need to derive a color-flavor transformation for gauge group SU($N_c$). This is done in Sec. \textsuperscript{3}. In Sec. \textsuperscript{3} we address the issue of how to go beyond infinite coupling (i.e. how to include the plaquette action), discuss baryon loops that arise in SU($N_c$), and comment on simulation algorithms.

2. Color-flavor transformation for SU($N_c$)

We consider two types of tensors $\psi$ and $\varphi$ with only fermionic degrees of freedom. This simplifies the calculations and is more appropriate for the application to lattice QCD that we have in mind. The reader may think of $\bar{\psi}$ as corresponding to $\bar{\psi}_{x+\mu}$, $\psi$ to $\bar{\psi}_x$, $\varphi$ to $\bar{\psi}_x$, and $\varphi$ to $\bar{\psi}_{x+\mu}$. The result for the transformation for SU($N_c$) is

\[
\int_{SU(N_c)} dU \exp \left( \bar{\psi}^i_a U^{ij}_{\mu,a} \psi^j_a + \bar{\varphi}^i_a U^{ij}_{\mu,a} \varphi^j_a \right) = C \cdot C_0 \int_{\mathbb{C}^{N_f} \times \mathbb{C}^f} \frac{dZ d\tilde{Z}}{\det(1 + ZZ^t)^{2N_f}} \times \exp \left( \bar{\psi}^i_a Z_{ab} \psi^i_b - \bar{\varphi}^i_a Z_{ab} \varphi^i_b \right) \sum Q=0 \chi_Q, \tag{2}
\]
where \(a, b = 1, \ldots, N_f\), \(\chi_0 = 1\) and
\[
\chi_{Q > 0} = C_Q \left[ \det(\mathcal{M})^Q + \det(\mathcal{N})^Q \right].
\]
The matrices in the determinants in Eq. (3) carry color indices. They are \(\mathcal{M}^{ij} = \bar{\psi}_a^i (I + Z Z^\dagger)_{ab} \psi_b^j\) and \(\mathcal{N}^{ij} = \bar{\varphi}_b^i (I + Z^\dagger Z)_{ab} \varphi_a^j\). Finally, the constants are given by
\[
C = (1/\pi^{N_f^2}) \prod_{n=0}^{N_f-1} (N_f + n)!/n!
C_0 = \prod_{n=0}^{N_f-1} 1! (N_c + N_f + n)!/(N_c + n)!(N_f + n)!
C_Q = \frac{1}{(Q!)^{N_f}(N_f!)^Q} \prod_{n=0}^{Q-1} (N_c + n)!(N_f + n)!/n!(N_c + N_f + n)!.
\]

The basic idea behind the proof of the color-flavor transformation is the following. One can define an auxiliary Fock space, with fermion creation and annihilation operators acting in this space. The integrand of the SU\((N_c)\) integral can be regarded as a complex-valued overlap of two states in this Fock space. Integration over SU\((N_c)\) can be identified with a projection onto the color-neutral sector of the Fock space. The idea is to develop another implementation of this very projector, which then leads to the color-flavor transformed integral. The details of the proof are rather complicated, and we shall only sketch some of the steps involved. For the full proof, we refer to Ref. [3] (see also Ref. [4]).

As mentioned above, we define two sets of fermionic creation and annihilation operators \(\bar{c}_a^i, c_b^j\) and \(\bar{d}_a^i, d_b^j\) which carry color indices \(i = 1, \ldots, N_c\), and flavor indices \(a = 1, \ldots, N_f\). These operators obey canonical anti-commutation relations. The Fock space is obtained by acting on the Fock vacuum \(|0\rangle\) with linear combinations of the creation operators.

It is useful to define a certain “half-filled” state in this Fock space as
\[
|\psi_0\rangle = (\bar{c}_1^1 \cdots \bar{c}_N^1)(\bar{c}_1^N \cdots \bar{c}_N^N)|0\rangle.
\]
The outline of the proof is as follows,
\[
\int_{\text{SU}(N_c)} dU \exp \left( \bar{\psi}_a^i U^{1ij} \psi_b^j + \bar{\varphi}_b^i U^{1ij} \varphi_a^j \right)
= \int_{\text{SU}(N_c)} dU \left\langle \psi_0 \right| \exp \left( \bar{\psi}_a^i U^{1ij} \psi_b^j + \bar{\varphi}_b^i U^{1ij} \varphi_a^j \right) \exp \left( \bar{d}_a^i U^{1ij} \psi_b^j + \bar{\delta}_b^i U^{1ij} \varphi_a^j \right)|\psi_0\rangle
= \left(\psi_0\right| \exp \left( \bar{\psi}_a^i d_a^i - \bar{\varphi}_b^i \varphi_b^i \right)
\times P \exp \left( \bar{\varphi}_b^i \varphi_b^i + \bar{\delta}_b^i \delta_b^i \right)|\psi_0\rangle
= \sum_{Q = -N_f}^{N_f} \left(\psi_0\right| \exp \left( \bar{\psi}_a^i d_a^i - \bar{\varphi}_b^i \varphi_b^i \right)
\times P \exp \left( \bar{\varphi}_b^i \varphi_b^i + \bar{\delta}_b^i \delta_b^i \right)|\psi_0\rangle
= C_Q \int \frac{D(Z, Z^\dagger)}{\det(1 + ZZ^\dagger)^{N_c}} \sum_{Q = -N_f}^{N_f} \chi_Q.
\]

The interpretation of integration over SU\((N_c)\) as a projection onto the color-neutral sector of Fock space can be understood by analogy with the case of projecting a vector in \(\mathbb{R}^3\) onto the \(z\)-axis. By averaging over all SO\((2)\) rotations around the \(z\)-axis, what remains is the \(z\)-component of the initial vector, which is precisely the component that is invariant under such rotations.

In order to derive an alternative expression for the projector \(P\), we consider bilinear products of the creation and annihilation operators of the form \(\bar{c}_A^i c_B^j\). Here, \(A, B = 1, \ldots, 2N_f\) are composite indices introduced so that we don’t have to distinguish between the \(c_b^i\)’s and \(d_b^i\)’s. These bilinears generate a gl\((2N_f N_c)\) algebra, which has two commuting subalgebras that are of interest to us: the sl\((N_c)\) color algebra and the gl\((2N_f)\) flavor algebra. The former is generated by operators \(E^{ij} = \sum_A \bar{c}_A^i c_A^j\), \(i \neq j\), and \(H^i = \sum_A (\bar{c}_A^i c_A^j - \bar{c}_A^j c_A^i)\), \(i = 1, \ldots, N_c\), \(N_c - 1\), and the latter is generated by \(E_{AB} = \sum_i (\bar{c}_A^i c_B^i - \bar{c}_B^i c_A^i)\).

States in the color-neutral sector satisfy \(T|\mathcal{N}\rangle = 0\) for \(T \in \text{sl}(N_c)\), or more explicitly,
\[
\sum_{A=1}^{2N_f} \bar{c}_A^i c_A^j |\mathcal{N}\rangle = (N_f + Q) \delta^i_j |\mathcal{N}\rangle.
\]
The possible values for \(Q\) are \(Q = -N_f, \ldots, N_f\). Thus, the color-neutral sector splits into subsectors labelled by \(Q\). Under the action of U\((2N_f)\) on Fock space, given by
\[
g \in \text{U}(2N_f) \mapsto T_g = \exp(c_A^i (\log g)_{AB} c_B^j),
\]
each sector \(Q\) subsumes an irreducible representation of the flavor group U\((2N_f)\) with a rectangular Young diagram that has \(N_f + Q\) rows and \(N_c\) columns.
Using the method of generalized coherent states, we can express the projector onto the color-neutral sector as a sum of projectors onto the different subspaces $Q$, i.e. $P = \sum_{Q = -N_f}^{N_f} \mathbb{1}_Q$.

To derive the projectors $\mathbb{1}_Q$, we start out with the highest weight vector of the representation $\mathcal{Q}$, which is given by $|\psi_Q \rangle = \prod_{i=1}^{N_c} \prod_{A=1}^{N_f+Q} c_A^i |0\rangle$. An overcomplete set of states is obtained by acting on this state with all group elements $T_g$, $g \in U(2N_f)$. In the sector $Q = 0$, the highest-weight vector $|\psi_0 \rangle$ corresponds to "half-filling." Clearly, $T_h |\psi_0 \rangle \propto |\psi_0 \rangle$ for $h = \text{diag}(h_+, h_-)$ with $h_{\pm} \in U(N_f)$. The subgroup $H = U(N_f) \times U(N_f)$ of $U(2N_f)$ is called the isotropy subgroup of $|\psi_0 \rangle$.

In the sector $Q = 0$, coherent states may thus be parameterized without overcounting by the elements of the coset space $U(2N_f)/H$. We label the elements of this coset space by picking a representative of each equivalence class $gH$, $s(\pi(g))$. Each element $g \in U(2N_f)$ can be decomposed into a product $g = s(\pi(g))h(g)$. As an explicit expression we choose $s(\pi(g)) = (1 + Z_1^1Z)^{-1/2} - Z_1^1(1 + ZZ_1^1)^{-1/2}$, where $Z$ is an arbitrary $N_f \times N_f$ complex matrix.

The resolution of the identity in coherent states in the $Q$-sector is given by

$$\mathbb{1}_Q = C_Q \int_{U(2N_f)} d\mu(g) T_g |\psi_Q \rangle \langle \psi_Q | T_g^{-1}. \quad (11)$$

In fact, this is a projector onto the $Q$-sector since it is easily seen to annihilate all other states.

An integral over the whole group $U(2N_f)$ can be decomposed into two integrals, one over $H = U(N_f) \times U(N_f)$, and one over the coset space $U(2N_f)/H$, where the measure of integration over the coset space is given by $D(Z, Z^\dagger) = C_d DdZdZ^\dagger/\text{det}(1 + ZZ^\dagger)^{2N_f}$. For $Q = 0$, the integrand is indeed independent of $h$, and with the normalization $\text{vol}(H) = 1$, the projector in this sector is simply

$$\mathbb{1}_0 = C_0 \int D(Z, Z^\dagger) |Z\rangle \langle Z|. \quad (12)$$

The case of $Q \neq 0$ is more complicated, since the integration over $H$ is non-trivial. However, this integration can be done using standard group theory results, which finally leads to the color-flavor transformed integral of Eq. (13).

3. Application to Lattice QCD

3.1. Induced QCD: How to generate the plaquette action

The color-flavor transformation can be applied to each lattice link for lattice actions at infinite coupling, i.e. without a plaquette interaction term. Any program aimed at reformulating full QCD using the color-flavor transformation must therefore address the issue of including an interaction term for the gauge fields. Here, we take advantage of the idea of induced QCD, which goes back to Kazakov and Migdal and was further developed in Ref. [3] leading to the result of interest to us. The idea is to couple a number of additional heavy fermions to the gauge field. This induces gauge interactions, which can be directly related to an effective (non-zero) gauge coupling.

To see how this works, we introduce $N_h$ heavy fermions, described by the Wilson-Dirac operator

$$D_{yx} = \delta_{yx} - \kappa \sum_{\mu=\pm 1} \delta_{y,x+\mu} (r + \gamma_\mu) U_\mu(x), \quad (13)$$

where we have not yet set $r = 1$. The hopping parameter $\kappa = 1/(2Ma + 8r)$ tends to zero in the limit $M \to \infty$. We now integrate out the heavy quark fields to obtain the determinant of the Wilson-Dirac operator, $D = \mathbb{1} - \kappa A$. The resulting expression can then be expanded in powers of the hopping parameter,

$$\det^{N_h} D = \exp(N_h \text{tr} \log(\mathbb{1} - \kappa A))
= \exp \left[ -N_h \text{tr} \left( \kappa A + \frac{\kappa^2}{2} A^2 + \frac{\kappa^3}{3} A^3 + \frac{\kappa^4}{4} A^4 + \ldots \right) \right]. \quad (14)$$

It is easily seen that terms containing odd powers of $A$ vanish when one takes the trace, since the matrix then has no entries which are diagonal in lattice site indices. The quadratic term is just a constant (equal to zero for $r = 1$), since $U_\mu(x)U_\mu^\dagger(x) = \mathbb{1}$. Similarly, $\text{tr} A^4 = \ldots$
The large-$N_c$ limit is unlikely to be a suitable method for studying this implies that an approach based on the color-flavor transformation is concerned. Thus, the dimension of the $Z$ matrices obtained after the transformation is given by $\dim(Z) = 4(N_f + N_b) \equiv 4N_q$.

For Wilson fermions, the action at infinite coupling is given by

$$S = -\frac{1}{2} \sum_{x,\mu>0} \left[ \bar{\psi}_{x+\mu,a}^\alpha (r + \gamma_\mu)^\alpha_\beta U_{x,\mu}^\beta \psi_{x,a}^\beta + \bar{\psi}_{x,a}^\alpha (r - \gamma_\mu)^\alpha_\beta U_{x,\mu}^\beta \psi_{x+\mu,a}^\beta \right] + \sum_{x,a} (m_a + 4r) \bar{\psi}_{x,a}^\alpha \psi_{x,a}^\alpha.$$  \hspace{1cm} (18)

We have included all the indices, namely Greek letters $\alpha, \beta = 1, \ldots, 4$, color indices $i, j = 1, \ldots, N_c$ and flavor indices $a = 1, \ldots, N_f$. Note that the $\gamma_\mu$ are traceless, idempotent matrices, which can be diagonalized by unitary matrices,

$$\gamma_\mu = \Gamma_\mu \text{diag}(1, 1, -1, -1) \Gamma_\mu^\dagger.$$  \hspace{1cm} (19)

If we now set $r = 1$, we have

$$\frac{1}{2}(r + \gamma_\mu) = \Gamma_\mu P_+ \Gamma_\mu^\dagger = \Gamma_\mu P_+ \Gamma_\mu^\dagger$$  \hspace{1cm} (20)

$$\frac{1}{2}(r - \gamma_\mu) = \Gamma_\mu P_- \Gamma_\mu^\dagger = \Gamma_\mu P_- \Gamma_\mu^\dagger.$$  \hspace{1cm} (21)

Here, $P_+ = \text{diag}(1, 1, 0, 0)$ and $P_- = \text{diag}(0, 0, 1, 1)$ are projection operators in Dirac space. On each link, we define

$$\phi_x^{\gamma,i} = \bar{\psi}_{x,a}^\alpha \Gamma_\mu^{\alpha_\beta} \psi_{x+\mu,a}^\beta,$$

$$\phi_x^{\gamma,i} = \bar{\psi}_{x,a}^\alpha \Gamma_\mu^{\alpha_\beta} \psi_{x+\mu,a}^\beta,$$

$$\phi_x^{\gamma,i} = \bar{\psi}_{x,a}^\alpha \Gamma_\mu^{\alpha_\beta} \psi_{x+\mu,a}^\beta.$$  \hspace{1cm} (20)

This definition is of course not the only possible choice for the inclusion of the Dirac matrices, and therefore the color-flavor transformed action will not be unique. This should, however, have no influence on the physics, since the transformation is in any case still exact. The $\phi$ and $\phi$ above are still Grassmann variables, and so we can now apply the color-flavor transformation to each link of the lattice to obtain a transformed partition function of the form

$$Z_{\text{QCD}} = \prod_{\mu > 0} \int \frac{dZ_\mu(x) dZ_\mu^\dagger(x)}{\det(1 + Z_\mu(x) Z_\mu^\dagger(x))^{2N_q + N_c}} \times \int D\bar{\psi} D\psi e^{-S_{\text{QCD}}(x)} \sum_{Q=0}^{4N_f} \chi_{Q\mu}(x).$$  \hspace{1cm} (22)
In the exponential, there is now a local term \( S_\mu(x) = \bar{\psi}_{x,a} B_\mu(x) \delta_{x,b} \psi_{x,b} \), where
\[
B_\mu(x) = [\Gamma_\mu^\alpha P^\beta Z_{x,\mu}^\gamma \delta_{ab} P^\delta \Gamma_\mu^\rho] - [\Gamma_\mu^\alpha P^\beta Z_{x-\mu,\mu}^\gamma \delta_{ab} P^\delta \Gamma_\mu^\rho] + (m_a + 4) \delta_{ab} \delta_{\alpha \rho}.
\]

Furthermore, we have \( \chi_{0\mu}(x) = 1 \) and
\[
\chi_{Q\mu}(x) = C_Q \left[ \det Q \mathcal{N}_\mu(x) + \det Q \mathcal{M}_\mu(x) \right].
\]

The matrices which appear here are defined as
\[
\mathcal{M}_\mu^{ij}(x) = \bar{\psi}_{x+\mu,a} \Gamma_\mu^\alpha P_+ (1 + Z_{x,\mu} Z_{x+\mu,\mu}^\dagger) \delta_{x,b} P_+ \Gamma_\mu^\rho \psi_{x,b},
\]
\[
\mathcal{N}_\mu^{ij}(x) = \bar{\psi}_{x,a} \Gamma_\mu^\alpha P_+ (1 + Z_{x,\mu} Z_{x+\mu,\mu}^\dagger) \delta_{x,b} P_+ \Gamma_\mu^\rho \psi_{x,b}.
\]

The next step is to integrate out the fermions. This is straightforward for \( Q = 0 \). With the definition of \( B_\mu(x) \) from above, we define \( B(x) = \sum_\mu B_\mu(x) \) and \( B = \bigotimes_x B(x) \). Then \( B \) has a block-diagonal structure, with different blocks corresponding to different lattice sites. After integration over the Grassmann variables, we thus obtain
\[
Z_{QCD}^Q = \det N_{\mu} B = \prod_x \det N_{\mu} B(x).
\]

For \( Q = 0 \), the situation becomes more complicated, since the \( \chi_{Q > 0} \) terms induce non-local contributions to the action. Notice that these terms correspond to baryon propagation, which explains why they were absent in the case of gauge group \( U(N_c) \).

The case of one static baryon (\( Q = 1 \)) was studied in saddle-point approximation (for large \( N_c \)) in 1+1 dimensions by Budczies et al. in Ref. [4]. However, our goal is an exact (numerical) approach to QCD, which takes into account all possible values of \( Q \).

In order to evaluate terms from \( \chi_{Q > 0} \) contributions, we need to apply Wick’s theorem. This results in terms involving the inverse of the matrix \( B \). Note that \( B \) is diagonal in space and color, so that we have
\[
(B^{-1})^{ij}_{xy,pq} = \delta^{ij} \delta_{xy} B^{-1}(x)_{pq},
\]

\[
(1 + \cdot \cdot \cdot ) (1 + \cdot \cdot\cdot ) (1 + \cdot \cdot \cdot ) (1 + \cdot \cdot \cdot ) = 1 + \bigotimes_x
\]

Figure 1. Expansion of baryon contributions: Only closed loops survive the integration over the Grassmann variables.

where \( p, q = 1, \ldots, 4N_q \) combine flavor and Dirac indices. Let us denote
\[
\sum_{Q=1}^{4N_q} \chi_{Q\mu}(x) = \bigotimes_x B(x)
\]

Here, a solid link denotes the presence of 1, 2, \ldots, 4\( N_q \) baryons. Now consider
\[
\prod_{x,\mu} \left( 1 + \bigotimes_x \right)
\]

We can expand the product and note that because of \( (B^{-1})_{xy} \sim \delta_{xy} \), only those terms survive that correspond to closed loops (cf. for example Fig. [4]). Hence, in the partition function only closed baryon loops are allowed. These must not backtrack, so that each link occurs at most once in a graph. The paths may self intersect, and disconnected loops are also allowed. In addition, for each site the total number of baryons on the links connected to this site must not exceed \( 4N_q \).

Let us now consider observables, such as correlation functions. (In the color-flavor transformed formulation, gauge fields no longer appear, and it is therefore not surprising that non-gauge-invariant quantities are manifestly equal to zero.) As an example, take the baryon two-point function \( \langle \bar{B}_y B_z \rangle \), with \( B = \epsilon_{ijk} \psi^i \psi^j \psi^k \). Schematically, we have to compute
\[
\bar{B}_y B_z \prod_{x,\mu} \left( 1 + \bigotimes_x \right)
\]

Contributing baryon paths are those which originate at the site \( y \) and terminate at site \( z \). Of course, a background of closed baryon loops, such as those appearing for the partition function, is also allowed, in addition to the baryon propagating from \( y \) to \( z \).
3.3. Algorithmic considerations

While it is, in principle, clear how to perform a simulation for the $Q = 0$ case, the presence of baryon loops poses additional problems. Clearly, the number of possible loops grows prohibitively with the lattice volume so that a probabilistic approach to the generation of such loops is needed. A possibility is some form of random walk algorithm [7]. It should be noted, though, that the coefficients $C_Q$ decrease rapidly as functions of $Q$ so that multi-baryon paths are potentially strongly suppressed. For example, for $Q = 1$ and $N_c = 3$, we have $C_1 \propto 1/N_q^3$, which is already much smaller than in the $Q = 0$ case. In addition, since longer loops involve more factors of $C_Q$, only small loops, or shortest paths in the case of correlation functions, will significantly contribute. An expansion in baryonic loops may therefore be a far more systematic and better controlled approach than quenching, for example.

Unfortunately, the color-flavor transformed action is complex. This is already manifest for $Q = 0$, as can be seen from Eqs. (22) and (23), since $Z$ is a general complex matrix. We have tried to attack this problem by partially integrating out some of the degrees of freedom of $Z$ (cf. Ref. [8]), and also by finding different parameterizations of the coset space $U(2N_f)/[U(N_f) \times U(N_f)]$, but so far have not been able to obtain a satisfactory result. However, some progress has been made recently in attacking fermion sign and complex action problems [1], and this may also turn out to be helpful for the color-flavor transformation.

4. Summary and Outlook

We have derived the color-flavor transformation for gauge group $SU(N_c)$. This transformation converts a certain type of integral over color gauge fields to an integral over flavor matrices. The integral being transformed is of the same form as the one encountered on each link of the lattice QCD partition function at infinite coupling. Since (for zero baryons) the fermion determinant after the transformation is block-diagonal in the lattice site indices, this approach has the promise of leading to new ways of simulating dynamical fermions. However, in contrast to gauge group $U(N_c)$, the $SU(N_c)$ case gives rise to additional baryon loops, which lead to nearest-neighbor coupled baryon terms. Nevertheless, multi-baryon loops are potentially suppressed, and a systematic expansion of the partition function in terms of such loops is possible.

In order to go beyond the infinite coupling limit, a number $N_h$ of heavy flavors can be added to the theory to generate the plaquette action ($g^2 = 8N_h\kappa^4$ for Wilson fermions).

In the present formulation, the color-flavor transformed action is complex, which makes numerical simulations based on this approach impractical without further improvements. Work in this direction is in progress.

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