Antenna Array-Aided Real-Time Kinematic Using Moving Base Stations

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ABSTRACT A global navigation satellite systems (GNSS) real-time kinematic (RTK) using multiple base stations is referred as an antenna array-aided RTK. This positioning method more reliably resolves carrier phase integer ambiguities by processing additional measurements provided from multiple GNSS reference stations whose antenna locations are accurately surveyed. In this study, a conventional antenna array-aided RTK is extended to improve RTK performance with multiple moving base stations. Because accurately surveyed antenna coordinates are not available, we propose an antenna array-aided Kalman filter approach to first check the sanity of the fixed integer ambiguities and baselines estimated by moving reference stations and use them as a-priori information in RTK measurement formulations. Our flight test results with three unmanned aerial vehicles confirmed that the proposed Kalman filter formulation outperformed the conventional RTK and a priori antenna array-aided RTK approaches in the aspects of positioning accuracy, time to first fix, and success probability of integer ambiguity fixes.

INDEX TERMS Global navigation satellite system (GNSS), real-time kinematic (RTK), relative position, multi agent operation.

I. INTRODUCTION Real-time kinematic (RTK) positioning has been one of the common methods for precise relative positioning using global positioning satellite systems (GNSSs) [1], [2], [3]. An RTK system usually consists of a rover and reference receivers, and can provide centimeter-level relative positioning accuracy. Since an RTK process uses double difference code and carrier phase measurements as ranging sources, it must solve for integer ambiguities in carrier phase measurements, such as using least-squares (LS) ambiguity decorrelation adjustment (LAMBDA) algorithms [4], [5]. The performance of LAMBDA is typically determined by the success probability of integer ambiguity resolution (or fix) and time to first fix (TTFF). It has been reported that the RTK performance in terms of these criteria improves as the number of GNSS constellations and measurement frequencies increase [6], [7], [8], [9]. For example, the performance of a single-frequency RTK integer ambiguity resolution and positioning accuracy was evaluated for Global Positioning System (GPS) and Galileo constellations [6]. The comparison of the RTK performance for single and dual frequency GPS/BeiDou navigation satellite system with survey-grade GNSS receivers was reported in [7], [8]. The RTK performance of some commercial low-cost single-frequency GNSS receivers was evaluated in [9]. To further improve the probability of integer ambiguity resolution or fix in RTK, a known baseline information between GNSS receivers has been incorporated in the integer ambiguity resolution process. This approach is often applied to a GNSS-based attitude determination problem of a vehicle by using the known baseline length between receivers as a constraint in integer ambiguity search strategies. References [10], [11] introduced so-called GNSS compass algorithms that

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modified the LAMBDA method of searching for integer ambiguity solutions through quadratically constrained least-squares. Reference [12] proposed search and shrink strategies for the baseline-constrained LAMBDA, and reference [13] used the known baseline length information to narrow the integer ambiguity search space that had been first increased due to an inaccurate float solution. References [14], [15] further proposed a multivariate constrained LAMBDA method that can be used as a general framework for solving various GNSS attitude-determination problems with multiple antennas by determining integer ambiguities and attitudes in an integral manner. The performance of some commercial GNSS attitude-determination products have also been reported in various literature [16], [17], [18].

The idea of using known baseline information among multiple GNSS receivers and antennas has further evolved to antenna array-aided GNSS precise positioning, ionospheric spatial gradient monitoring, and integrity monitoring. In these approaches, it is assumed that the known baseline coordinates between multiple GNSS antennas and double difference (DD) integer ambiguities associated with the baselines are also available. This a-priori information is used to produce more accurate DD integer ambiguity float solutions, which in turn lead to higher probabilities of integer ambiguity resolutions or fixes [19]. It is found to be particularly helpful in resolving integer ambiguities when there is a small number of satellites in view.

Authors in [20] introduced an antenna array-aided precise point positioning (A-PPP) or instantaneous positioning and attitude solutions, wherein integer ambiguities were resolved by an orthonormality-constrained multivariate mixed-integer least-squares method. The principles of A-PPP have been further enhanced to speed up the resolution of integer ambiguities of Continuously Operating Reference Stations [21]. References [22], [23] used multiple static GNSS receivers and antennas to perform LS based array-aided RTK (A-RTK) positioning with soft and hard constraints. Also, an integer ambiguity dilution of precision was derived for A-RTK. Other researchers applied an antenna array-aided GNSS positioning concept to kinematic positioning problems. Authors in [24] used three GNSS receivers and antennas on a ship carrying a gravimeter to obtain centimeter-level kinematic positioning solutions without solving for integer ambiguities. Luo et al. also introduced centimeter-level relative positioning of moving platforms [25], [26], wherein the integer ambiguities of three moving GNSS receivers were concurrently found based on the constraint that the sum of the DD integer ambiguities of the three GNSS receivers must be equal to zero. Authors in [27] proposed tensor and antenna array-aided GNSS positioning to reduce severe multipath effects for the operation of autonomous cars.

Zaminpardaz et al. investigated the precision and integrity of the ionospheric spatial gradient estimation based on an array-aided GNSS DD observation model. More over, they developed closed-form expressions for the variance matrix of the ionospheric spatial gradient and the corresponding minimal detectable biases [28]. Datta-Barua et al. developed an estimation method for the ionospheric irregularity layer height and thickness with an array of GNSS scintillation receivers spaced within a kilometer [29], [30], [31]. Tsikin et al. developed an antenna array processing for the integrity and anti-spoofing monitoring of GNSS signals [32], [33]. Yang et al. proposed a blind anti-interference algorithm based on a novel antenna array design to mitigate pseudorange and carrier phase biases that may occur during interference suppression [34].

During the operation of a swarm of unmanned vehicles (UV), the precise relative position of the UVs should be
ensured in a reliable and fast manner. An A-RTK would be an effective solution for the determination of the relative positioning among the UVs in a short baseline, and could be implemented without requiring additional hardware because an UV is typically equipped with GNSS receiver and communication modules. A circumstance that we consider in operating UVs is shown in Fig. 1. The baseline and DD integer ambiguity between two moving bases have been estimated by a conventional RTK. Then, a rover UV approaches to the base UVs and tries to obtain precise relative position to the base UVs. The prior arts in [22], [23] [25], and [26] are not quite suitable solutions to this problem. First, the LS A-RTK formulation proposed in [22] and [23] is based on static GNSS reference receivers and would perform poorer with moving reference receivers. The approaches in [25] and [26] considered moving reference receivers but they are rather inefficient because the a-priori information of fixed integer ambiguities and baselines was not used. In this study, we propose an extended Kalman filter based A-RTK that fully uses the a-priori information and is better suited for dynamic baseline changes. We assumed that the a-priori baseline was determined by reference UVs and sent to a rover for A-RTK. However, the a-priori information may have a large bias due to computation or communication faults. Therefore, we also propose a novel fault monitoring scheme to check the sanity of the a-priori information.

The rest of the paper is organized as follows. Section II gives an overview of an LS based A-RTK, followed by a Kalman filter formulation of A-RTK in Section III. Section IV introduces fault monitoring of a-priori information, and Section V presents flight test results of the Kalman filter-based A-RTK.

II. OVERVIEW OF LEAST-SQUARES BASED ANTENNA ARRAY-AIDED RTK FORMULATION

This section discusses the previously proposed least-squared based A-RTK introduced in [22]. For the sake of simplified mathematical expressions, we assume that there are three reference receivers.

Fig. 2 shows three reference antennas, denoted $a_1$, $a_2$, and $a_3$, and one rover UV, denoted $a_b$. Here, it is assumed that the baselines, $b_{31}$ and $b_{21}$, and DD integer ambiguities, $n_{31}$ and $n_{31}$, were computed from a conventional RTK, referred as one-to-one RTK in this paper. The one-to-one RTK DD code and carrier phase measurements between the rover $a_b$ and the reference receivers $a_1$ through $a_3$ can be modeled as follows

$$
E(\phi_1) = Gb_1 + \Lambda n_1, \quad E(\rho_1) = Gb_1
$$
$$
E(\phi_2) = Gb_2 + \Lambda n_2, \quad E(\rho_2) = Gb_2
$$
$$
E(\phi_3) = Gb_3 + \Lambda n_3, \quad E(\rho_3) = Gb_3
$$

(1)

where $\phi_i = \left[ \phi_{i,1}^{1,2} \ldots \phi_{i,1}^{1,m+1} \ldots \phi_{i,f}^{1,2} \ldots \phi_{i,f}^{1,m+1} \right]^T$ is the DD carrier phase measurement vector and $\rho_i = \left[ \rho_{i,1}^{1,2} \ldots \rho_{i,1}^{1,m+1} \ldots \rho_{i,f}^{1,2} \ldots \rho_{i,f}^{1,m+1} \right]^T$ is the DD code measurement. $E[\cdot]$ is the expectation operator. $i$ is a reference receiver index and $f$ indicates a GNSS carrier frequency. The superscript indicates a pair of $m + 1$ satellite indices having 1 as a pivot satellite, $b_i = [b_{i,1} \ b_{i,2} \ b_{i,3}]^T$ is the baseline vector between $a_i$ and rover, and $n_i = [n_{i,1}^{1,2} \ldots n_{i,1}^{1,m+1} \ldots n_{i,f}^{1,2} \ldots n_{i,f}^{1,m+1}]^T$ is the corresponding DD integer ambiguity vector; $\Lambda = \lambda \otimes I_m$ is a wavelength matrix with $\lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_f)$, and $\otimes$ denotes the Kronecker product; $I_m$ is an identity matrix having a dimension of $m \times m$; $G = e_f \otimes g$ is a DD user-to-satellite line of sight matrix for all frequencies; $e_f$ is a column vector of 1’s corresponding to the number of $f$ used in the measurements; $g$ is a DD user-to-satellite line of sight matrix for a $f$. In a short baseline RTK, the DD user-to-satellite geometry would be very close for all reference receivers. Also, the ephemeris error, satellite clock error, ionospheric delay error, and tropospheric delay error can all be assumed to be cancelled out.

Using the relationships of the baseline and integer ambiguities vectors between the reference receivers and rover, (1) can be rewritten as follows

$$
E(\phi_1) = Gb_1 + \Lambda n_1
$$
$$
E(\phi_2) = G(b_{21} + b_1) + \Lambda(n_{21} + n_1)
$$
$$
E(\phi_3) = G(b_{31} + b_1) + \Lambda(n_{31} + n_1)
$$
$$
E(\rho_1) = Gb_1
$$
$$
E(\rho_2) = G(b_{21} + b_1)
$$
$$
E(\rho_3) = G(b_{31} + b_1)
$$

(2)

Rearranging (2) for the unknown $b_1$ and $n_1$ results in

$$
E(\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}) = G \begin{bmatrix} 0 & b_{21} & b_{31} \end{bmatrix} + G \begin{bmatrix} b_1 \\ b_1 \\ b_1 \end{bmatrix}
$$
$$
+ \Lambda \begin{bmatrix} 0 & n_{21} & n_{31} \end{bmatrix} + \Lambda \begin{bmatrix} n_1 \\ n_1 \\ n_1 \end{bmatrix}
$$
\[ E[\begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{bmatrix}] = G \begin{bmatrix} 0 & b_{21} & b_{31} \end{bmatrix} + G \begin{bmatrix} b_1 & b_1 & b_1 \end{bmatrix} \] (3)

Then, (3) can be further simplified by subtracting the known baselines and DD integer ambiguities from measurements as follows

\[ E(\Phi - GB_1 - \Lambda N_1) = G \begin{bmatrix} b_1 & b_1 & b_1 \end{bmatrix} + \Lambda \begin{bmatrix} n_1 & n_1 & n_1 \end{bmatrix} \]

\[ E(Pr - GB_1) = G \begin{bmatrix} b_1 & b_1 & b_1 \end{bmatrix} \] (4)

where \( \Phi = [\phi_1 \, \phi_2 \, \phi_3] \) and \( Pr = [\rho_1 \, \rho_2 \, \rho_3] \) are the matrices consisting of DD carrier and code phase measurements, respectively; \( B_1 = \begin{bmatrix} 0 & b_{21} & b_{31} \end{bmatrix} \) is the matrix with baseline vectors between reference receivers; \( N_1 = \begin{bmatrix} 0 & n_{21} & n_{31} \end{bmatrix} \) is the matrix with the DD integer ambiguity vector of the baselines.

Equation (4) can be written in vectorized form as follows

\[ E(y_\Phi) = \hat{\Lambda} n_1 + \hat{G} b_1 \]

\[ E(y_{Pr}) = \hat{G} b_1 \] (5)

where \( y_\Phi = \text{vec} (\Phi - AN_1 - GB_1) \) and \( y_{Pr} = \text{vec} (Pr - GB_1) \) with a vectorizing operator vec, \( e_r \) is a column vector of 1’s and its rows and numbers is the same as the number of reference receivers, \( r \). Then, \( \hat{\Lambda} = e_r \otimes \Lambda \) and \( \hat{G} = e_r \otimes G \) with the Kronecker product \( \otimes \) and the \( e_r \).

The LS float solution \( b_1, n_1 \) of (5), and its variance-covariance (VC) matrices are given as

\[ \hat{b}_1 = R_{b}^{-1} G_{\text{Pr}} y_{Pr} \]

\[ \hat{n}_1 = R_{n}^{-1} \hat{R}_{b}^{-1} \left( y_\Phi - \hat{G} b_1 \right) \]

\[ R_{b}^{-1} = \left( G_{b}^{T} R_{Pr}^{-1} G_{b} \right)^{-1} \]

\[ R_{n}^{-1} = \left( G_{n}^{T} R_{Pr}^{-1} \hat{G} \right)^{-1} \hat{\Lambda}^{-1} \] (6)

where \( R_b \) and \( R_{Pr} \) are the VC matrix of carrier and code measurements, respectively, and have the following expressions

\[ R_{\Phi} = \left( D_b D_b^{T} \right) \otimes \left( \sigma_{\rho}^2 \otimes D_t W^{-1} D_t^{T} \right) \]

\[ R_{Pr} = \left( D_b D_b^{T} \right) \otimes \left( \sigma_{\rho}^2 \otimes D_t W^{-1} D_t^{T} \right) \] (7)

where \( D_b = [e_m - I_m] \) is the between-satellite single-differencing matrix; \( D_t = [e_3 - I_{3 \times 3}] \) is the between-antenna differencing matrix of the three reference receivers; \( \sigma_{\rho}^2 = \text{diag} \left( \sigma_{\phi_1}^2, \sigma_{\phi_2}^2, \cdots, \sigma_{\phi_3}^2 \right) \) and \( \sigma_{\rho}^2 = \text{diag} \left( \sigma_{\rho_1}, \sigma_{\rho_2}, \cdots, \sigma_{\rho_3} \right) \) are the variance matrices of phase and code measurement errors for each frequencies, respectively; \( W = \text{diag} \left( w_1, w_2, \cdots, w_m \right) \) is the weighting matrix taking into account the signal quality. After the integer ambiguities are resolved using an algorithm such as the LAMBDA, the fixed baseline solution and its covariance matrix are obtained as

\[ b_1 = R_{b_1} \left( G_{b}^{T} R_{Pr}^{-1} y_{Pr} \right) \]

\[ b_{b_1} = \left( G_{b}^{T} R_{Pr}^{-1} + G_{b}^{T} \right)^{-1} \hat{G} \] (8)

### III. KALMAN FILTER BASED ANTENNA ARRAY-AIDED RTK FORMULATION

For the formulation of an extended Kalman filter (EKF) based A-RTK, a state vector is defined as follows

\[ x_k = \begin{bmatrix} b_{11}^T & v_1^T & n_1^T \end{bmatrix}^T \] (9)

where \( v_1 \) is the relative velocity vector between \( a_1 \) and rover. The time update of EKF from \( t_{k-1} \) to \( t_k \) is given as

\[ x_{k-1} = F_k^{-1} x_{k-1} \]

\[ P_{k-1} = F_k^{-1} P_{k-1} F_k^{-1} T + Q_{k-1} \] (10)

where \( \hat{x}_k \) and \( P_k \) are the estimated states and its VC matrix at \( t_k \) epoch, respectively. The superscripts \(-\) and \(+\) respectively denote before and after measurement update of the EKF; \( Q_{k-1} \) and \( F_k \) are the VC matrix of the system noise and the state transition matrix from epoch \( t_{k-1} \) to \( t_k \), respectively.

The movement of the relative position vector, \( b_1 \), is modeled to have a constant velocity in the two consecutive epochs. Therefore, the state transition matrix and system noise VC matrix in (10) are as follows

\[ F_k = \begin{bmatrix} I_{3 \times 3} & I_{3 \times 3} & 0_{(mf) \times (mf)} \end{bmatrix} \]

\[ Q_k = \begin{bmatrix} Q_b & Q_v \end{bmatrix} \] (11)

where \( dt = t_k - t_{k-1} \) is GNSS receiver sampling interval. \( Q_b, Q_v, \) and \( Q_n \) are the covariance matrix of the baseline, baseline velocity, and DD integer ambiguity noise, respectively.

From (5), the EKF measurement vector, \( z_k \), is

\[ z_k = \begin{bmatrix} y_\Phi \\ y_{Pr} \end{bmatrix} = \begin{bmatrix} \text{vec} (\Phi - AN_1 - GB_1) \\ \text{vec} (Pr - GB_1) \end{bmatrix} \]

where \( z_k \) is a nonlinear function of \( x_k \) such that \( z_k = h(x_k) \). The measurement matrix, \( H \), is obtained from linearizing \( z_k \) at \( \hat{x}_k \)

\[ H = \begin{bmatrix} e_r \otimes H_{\Phi} \\ e_r \otimes H_{Pr} \end{bmatrix} \]

\[ H_{\Phi} = \begin{bmatrix} G & 0_{(mf) \times (3)} & \Lambda \end{bmatrix} \]

\[ H_{Pr} = \begin{bmatrix} G & 0_{(mf) \times (3)} & 0_{(mf) \times (mf)} \end{bmatrix} \] (14)

where \( H_{\Phi} \) and \( H_{Pr} \) are the measurement matrices for the DD carrier phase measurements and the DD code measurements, respectively; \( 0 \) is the zero matrix having the dimension of the subscripts.

The EKF gain matrix is given as

\[ K_k = R_k^{-1} H \left( H H^{T} + R_k \right)^{-1} \]

where \( R_k \) is the VC matrix of the measurement errors and has the following expressions

\[ R_k = \begin{bmatrix} R_{y_\Phi} & R_{y_{Pr}} \end{bmatrix} \]
In (16), the a-priori DD integer ambiguities associated with reference UVs are treated as constants. Then, the measurement updated state vector is

$$\hat{x}_k = \hat{x}_k + K_k (z_k - \hat{z}_k)$$  \hspace{1cm} (17)

where

$$\hat{z}_k = \begin{bmatrix} \tilde{\Delta} h_1^- + \tilde{\Delta} b_1^- \\ \dot{G}_1 \hat{b}_1^- \end{bmatrix}$$  \hspace{1cm} (18)

The measurement updated state VC matrix is

$$P_k^+ = (I - K_k H (\hat{x}_k^-)) P_k^-$$  \hspace{1cm} (19)

Moreover, $P_k^+$ can be expressed as the following block matrix form

$$P_k^+ = \begin{bmatrix} P_{k,ib} & P_{k,bv} & P_{k,bn} \\ P_{k,bv}^T & P_{k,vv} & P_{k,vn} \\ P_{k,bn}^T & P_{k,vn}^T & P_{k,nn} \end{bmatrix}$$  \hspace{1cm} (20)

The integer ambiguities are found using $P_k^+$ and the LAMBDA algorithms. After, an integer ambiguity is obtained from the LAMBDA process, $\hat{n}_{1,k}$, the fixed baseline is computed as follows

$$\hat{b}_{1,k} = \hat{b}_{1,k}^- + P_{k,bn}^T P_{k,nn}^{-1} (\hat{n}_{1,k}^+ - \hat{n}_{1,k})$$  \hspace{1cm} (21)

**IV. FAULT MONITORING OF A-PRIORI INFORMATION**

For the implementation of an A-RTK, reference UVs must send the fixed integer ambiguities and estimated baselines to a rover UV. However, there are chances that reference UVs resolve wrong integer ambiguities and the corresponding baselines have large errors, which would lead to corrupted measurements for the A-RTK process. Therefore, an appropriate monitoring scheme should be implemented to detect faults in the received a-priori information.

Using the carrier phase measurements compensated by the a-priori information, $y_\Phi$, a test vector can be formed as follows

$$T_y = D_y y_\Phi = \begin{bmatrix} y_{\Phi_1} - y_{\Phi_2} \\ y_{\Phi_1} - y_{\Phi_3} \end{bmatrix}$$  \hspace{1cm} (22)

where $D_y$ is the single difference matrix of $y_\Phi$ between UVs such that

$$D_y = \begin{bmatrix} I_{(mf)\times(mf)} & -I_{(mf)\times(mf)} & 0_{(mf)\times(mf)} \\ I_{(mf)\times(mf)} & 0_{(mf)\times(mf)} & -I_{(mf)\times(mf)} \end{bmatrix}$$  \hspace{1cm} (23)

A probable fault scenario assumed in our study is that at least one cycle offset exists in one or more number of the DD integer ambiguities and a corresponding baseline has a position bias. Therefore, some elements of $T_y$ would have a large value for the cases of a faulty a-priori information. Because $T_y$ is highly correlated mainly due to the DD process, it should be de-correlated to be properly used for a monitoring. The VC matrix of $T_y$, $S$, is

$$S = D_y R_{y_\Phi} D_y^T$$  \hspace{1cm} (24)

Therefore, a de-correlated test metric can be formulated as

$$v_1 = T_{y_{21}}^T S_{21}^{-1} T_{y_{21}}$$
$$v_2 = T_{y_{31}}^T S_{31}^{-1} T_{y_{31}}$$  \hspace{1cm} (25)

where $T_{y_{21}}$ and $T_{y_{31}}$ are the partial vectors of $T_y$ associated with baselines $b_{21}$ and $b_{31}$, respectively. $S_{21}^{-1}$ and $S_{31}^{-1}$ are the partial matrix of $S^{-1}$ associated with $T_{y_{21}}$ and $T_{y_{31}}$, respectively. $v_1$ and $v_2$ follow a Chi-squared distribution with the degree of freedom equal to the number of measurements [35], [36].
Fig. 3 and Fig. 4 show the distributions of $\nu$ obtained from flight tests using hexacoptors and ground static GNSS receivers in normal and fault cases, respectively. A more detailed test setup will be discussed in the following section. There are a total of 3700 samples in each of Fig. 3 and Fig. 4 and 22 measurements were used in each epoch to compute $\nu$. The distribution of $\nu$ in Fig. 3 also shows a theoretical central chi-square distribution with a degree of freedom of 22. The experimental distribution appears to have a higher probability density at large test metric values, and the theoretical distribution is similar in tendency. The fault measurements in Fig. 4 were generated by injecting one cycle offset to one of DD integer ambiguities and computing resultant baselines. A clear separation between the normal and fault $\nu$ distributions can be observed in Fig. 3 and Fig. 4. This seems to support our reasoning that a threshold of $\nu$ can be determined from a desirable false alarm rate with a central chi-square distribution and the clear separation of the $\nu$ distributions in normal and fault cases would allow limited missed detection. However, more test samples need to be obtained.

Notably, the monitor aims to detect outliers in the a-priori information sent from base stations. Inaccurate measurements from a rover itself should be checked through a cycle-slip detector and Kalman filter innovation and residual tests.

V. EXPERIMENTAL RESULT

A. EXPERIMENT SETUP

To see the performance of the proposed EKF filter-based A-RTK, three hexacopters and four static GNSS receivers on ground were used as shown in Fig. 5. The hexacopters in Fig. 6 were equipped with Ublox ZED-F9P receivers and Trimble AV16 antenna. Novatel OEM7700 receivers with 3GNSSA-XT antenna were used on ground. During the tests, the hexacopters flew at the speed of about 1 to 2 m/s following the trajectories shown in Fig. 7, and the total flight time was approximately 90 minutes. The visible GNSS satellites during the test are shown in Fig. 8, with a measurement interval of 1 second. The true or reference values of static and dynamic GNSS receiver position as well as DD integer ambiguities were estimated from using one-to-one RTK with open source RTK algorithms and nearby national GNSS reference stations at Suwon, South Korea [37]. The one-to-one RTK and A-RTK were implemented on a
laptop in a post-processing mode using collected GNSS measurements.

**B. IMPLEMENTATION PROCESS OF EKF A-RTK**

Fig. 9 shows the overall procedure of the EKF A-RTK process, wherein most computation blocks and flows are similar to one-to-one EKF RTK except for the a-priori information from other UVs and a fault monitor. Slightly different ranging measurement time among a rover and UV reference receivers were synchronized using doppler and carrier phase measurements, which was a critical part in a moving baseline RTK. There are a total of three monitors used in the process, namely cycle slip detector, a-priori information fault monitor, and integer ambiguity-monitoring ratio test. If a monitor detects a fault, the corresponding measurements are excluded from further processing.

**C. RESULTS**

Table 1 compares the performance of LS and EKF based A-RTK with hexacopter 1 and static receivers on ground shown in the test set-up in Fig. 5. The number of ground static antennas increased from 1 through 4, and the number of visible satellites varied from 6 through 14. When the number of ground static antenna is one, this case corresponds to the one-to-one RTK. The integer ambiguities for a baseline were computed at every epoch, and the resolved integer ambiguities which passed a ratio test with a threshold of 3 were only used to form valid a-priori baselines. No faults in a-priori information occurred in the test data with the ratio test and threshold. The RTK performance evaluation criteria were integer ambiguity fix success rate, 3D RMS position error.
TABLE 1. KF and LS based A-RTK with hexa-copter 1 and four static receivers on ground.

| number of satellites | EKF number of antenna | LS number of antenna |
|----------------------|-----------------------|----------------------|
|                     | 1  | 2  | 3  | 4  | 1  | 2  | 3  | 4  |
| 6                    | 32.57 | 68.58 | 70.43 | 65.94 | 1.94 | 2.16 | 7.55 | 8.19 |
| 8                    | 82.33 | 90.64 | 93.7 | 96.44 | 16.51 | 30.33 | 48.37 | 46.8 |
| 10                   | 100 | 100 | 100 | 100 | 75.27 | 84.2 | 93 | 93.32 |
| 12                   | 100 | 100 | 100 | 100 | 99.76 | 100 | 100 | 100 |
| 14                   | 100 | 100 | 100 | 100 | 98.47 | 99.82 | 100 | 100 |

Table 2 shows the performance of the two types of A-RTK alongside the flying hexacopters. Hexacopter 1 was designated a rover, while the other two hexacopters were used as moving base stations. An antenna number of one represents the case of one-to-one RTK between hexacopter 1 and hexacopter 2. Similar to the results of Table 1, the EKF A-RTK, overall, performed better than the LS A-RTK. The difference in performance became significant when the number of visible satellites was small. However, there was an exception, with the TTFF of the LS A-RTK being slightly better than that of the EKF A-RTK. It is presumed that the longer TTFF of the EKF was caused by the dynamic movement of the hexacopters and relatively small number of satellites in view. As the number of antennas increased from 1 to 2 in the EKF A-RTK, the fix rate improved approximately from 2 to 14 percent, and the fixed 3D RMSE was reduced approximately up to 8 cm. The overall 3D RMSE were also reduced by approximately up to 88 cm, and TTFF became faster by about 30 epochs, based on the test dataset. Lastly, the test results showed that the performance of the EKF A-RTK with dynamic reference receivers was shown to be close to or slight better than that of the EKF A-RTK with static reference receivers.

The reason that A-RTK performed better than the one-to-one RTK can be inferred from the ratio test results shown in Fig. 10, 11, 12, and 13. These ratios were obtained from the case of 8 satellites in view with static and dynamic base stations. In general, a ratio over a threshold of 3 is
TABLE 2. EKF and LS based A-RTK results with three hexa-copters.

| number of satellites | 1 | 2 | 1 | 2 |
|----------------------|---|---|---|---|
| fixed rates (%)      | 6 | 85.18 | 99.08 | 17.91 | 25.51 |
|                      | 7 | 97.94 | 99.57 | 29.25 | 40.92 |
|                      | 8 | 100   | 100   | 64.86 | 78.33 |
|                      | 9 | 100   | 100   | 90.66 | 96.09 |
|                      | 10| 100   | 100   | 95.09 | 98.18 |

FIGURE 13. Ratio test results from LS A-RTK with dynamic base stations.

VI. CONCLUSION

This study introduced a novel EKF A-RTK approach with a fault monitoring scheme for a-priori information of fixed integer ambiguities and baselines determined by other UVs. The proposed EKF A-RTK technique was tested with flight tests using three hexacopters, two of which were used as moving reference stations, and three ground static receivers. The results showed that, overall, the EKF A-RTK overall outperformed the one-to-one RTK and LS A-RTK, and showed a significant performance improvement with a small number of satellites in view. With six and seven satellites in view, the EKF A-RTK provided 99.6% integer ambiguity fix rate and fixed 3D RMSE as large as 8 cm. In contrast, the prior art of LS A-RTK achieved approximately 41% fix rate and fixed 3D RMSE as large as 88 cm based on the test data. The proposed chi-square based fault monitoring metric was computed from using the flight test data as well as injected faulty integer ambiguity with one cycle offset on one satellite. The test metric was shown to be able to clearly differentiate normal and fault cases. We expect that the proposed EKF A-RTK would be a competent precise positioning solution for the operation of a swarm of UVs. Our future research focuses on the development of related protocols and standards to enable developers systematically apply the EKF A-RTK approach.

VII. CONFLICT OF INTEREST

Authors declare there is no conflict of interest.

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