Optimal sensor placements using modified Fisher information matrix and effective information algorithm

Eun-Taik Lee¹ and Hee-Chang Eun²

Abstract
This article presents an optimal sensor placement algorithm for modifying the Fisher information matrix and effective information. The modified Fisher information matrix and effective information are expressed using a dynamic equation constrained by the condensed relationship of the incomplete mode shape matrix. The mode shape matrix row corresponding to the master degree of freedom of the lowest-contribution Fisher information matrix and effective information indices is moved to the slave degree of freedom during each iteration to obtain an updated shape matrix, which is then used in subsequent calculations. The iteration is repeated until the target sensors attain the targeted number of modes. The numerical simulations are then applied to compare the optimal sensor placement results obtained using the number of installed sensors, and the contribution matrices using the Fisher information matrix and effective information approaches are compared based on the proposed parameter matrix. The mode-shape-based optimal sensor placement approach selects the optimal sensor layout at the positions to uniformly allocate the entire degree of freedom. The numerical results reveal that the proposed F-based and effective information–based approaches lead to slightly different results, depending on the number of parameter matrix modes; however, the resulting final optimal sensor placement is included in a group of common candidate sensor locations. However, the resulting final optimal sensor placement is included in a group of common candidate sensor locations.

Keywords
Optimal sensor placement, effective independence, sensor, structural health monitoring, Fisher information matrix

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Introduction
It is necessary to determine the effects of aging and degradation, owing to natural disasters, such as earthquakes and hurricanes, on structural performance. Structural health monitoring (SHM) is an essential technique used for maintaining existing structures. An SHM system is used to monitor structural integrity and detect and assess structural damage using measured responses. It collects and analyzes the response data of the sensors attached to the structure. Its necessity has considerably increased with the construction and maintenance of high-rise building structures, long-span structures, and bridges. Innovative and sensitive sensors and real-time measurement systems that can collect more accurate information and evaluate structural performance more explicitly have been developed and remain a research hotspot.

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In SHM, the selection, number, and placement of measurement sensors are critical. It is impractical and uneconomical to place sensors at all degrees of freedom (DOFs) of an entire finite element model. Optimal sensor placement (OSP) techniques are applied to select the target sensor locations or coordinates and to form the dynamic characteristics of the system from the sensor measurements. Different OSP techniques have been developed to optimize sensor layout. The choice of sensor placement is necessary for system identification, damage detection, control, and health monitoring.

The deviation of predicted responses from the actual path is attributed to incomplete mode shape results. The predicted responses satisfy the dynamic constraints of the relationship between the generalized and incomplete modal coordinates. This inconsistency arises because high modal data are ignored. In this study, we derived the Fisher information matrix (FIM) and effective information (EI) algorithms expressed using the weighting coefficient matrix, which minimized the differences between the observed and estimated responses and constructed a novel OSP algorithm. During each iteration, the weighting matrix that satisfied the constraints was updated. The OSP iteration was repeated until the target sensors finally coincided with the prescribed mode number. The validity of the approaches developed in this study is verified using two numerical examples, in which the OSPs of a cantilevered beam and a truss structure were established. The numerical experiments were compared with the OSP results obtained using the number of installed sensors and the algorithms of the EI approach based on the mode shape of the dynamic system, and contribution matrix F and EI approaches based on the parameter matrix were proposed in this study. It is shown that the proposed OSP algorithm designates sensor locations that are more reasonable than the mode-shape-based EI approach. The numerical results revealed that although the proposed F-based and EI-based approaches showed slightly different results, the final OSPs gradually converged to close values within a narrower range for optimal measurements.

**State of the art**

The optimal sensor locations were assessed using various concepts, such as Fisher matrix determination, the modal assurance criterion error, and the singular value decomposition ratio. The EI approach is a widely used OSP technique. The sensors should be located at the common nodes for the key parts of the structure associated with a high EI index. The fundamental principle of the EI sensor placement method was introduced by Kammer. The method maximizes both the spatial independence and signal strength of the targeted mode shapes. Optimal measurement locations based on the Guyan reduction method were established by eliminating the DOF at each iteration. The effect of removing the mode shape at each step was distributed between the other DOFs in the subsequent process. The target sensor DOFs corresponded to where the inertia is high and the stiffness is low. This indicated that the EI index at the final stage approaches 1.0, which represents the rigid-body mode.

Most OSP algorithms utilize incomplete modal data, such as the natural frequency and the corresponding mode shape of the dynamic system. The Guyan condensation approach has the advantage of using the reduced DOFs of the transformation matrix condensed from the eigenfunction. Therefore, it is necessary to develop a more explicit and simpler process and formulation for the reduced system. Typical approaches, such as reducing the model DOFs until the measurement locations coincide with the master coordinates based on the Guyan reduction technique and the FIM, are performed to maximize some metrics of the FIM. Penny et al. established an optimum set of measurement locations. Lu et al. proposed an OSP method using the Guyan reduction method and a genetic algorithm. Kammer and Peck introduced a sensor placement method using an iterative Guyan expansion for mass weighting of target modes and EI sensor set expansion. Despite its merits, the Guyan reduction method focuses on only the lower modes and requires a complicated derivation and computation of the transformation matrix.

Bakir compared six different OSP techniques using the determinant, trace, and condition number of the FIM and concluded that the sensor set expansion technique is the best in computational effort, engineering aspects, and sensor distribution. Liu et al. considered sensor placement for parameter estimation by minimizing the inverse trace of the Bayesian FIM. They derived a closed form of the FIM with respect to the selected variables of the sensor. Chen et al. developed a hybrid method for OSP using modal assurance criterion matrices and EI vectors. Jiang et al. investigated the effect of different weighting coefficients on the maximization of the FIM using a mathematical property of the product of the target mode and its transpose and an alternative EI formula. Blachowski et al. established an OSP method based on the FIM matrix and the structural topology optimization concept to convert a discrete optimization problem into a continuous problem. He et al. introduced the generalized equivalent stiffness and importance coefficient of the component and used statistical data to obtain a sensor placement structure.

The frequency response function (FRF) can include responses within a broader frequency range rather than a narrower frequency range based on the mode shapes of the OSP. Ulriksen et al. utilized the FRF as an observable variable and developed a sensor placement
approach based on the maximization of the minimum FIM of the frequency responses within the selected modal parameter subset.

The OSP algorithm searches for more independent DOFs. Nontraditional optimization methods have been developed using genetic algorithms and neural networks. Gomes et al.\textsuperscript{12} developed an OSP method that adopted multiobjective genetic algorithms using the FIM and mode shape interpolation. Sun and Buyukozturk\textsuperscript{13} proposed a discrete optimization scheme based on an artificial bee colony algorithm for a reduced model. Tongco and Meldrum\textsuperscript{14} derived D-optimality using an optimal input design and optimal sensor design. The D-optimality criterion is defined as the maximum determinant of the FIM.

The OSP can also be selected using static response data. Xiao et al.\textsuperscript{15} identified optimal strain sensor placement based on an assumed set of applied static forces. Sanaye and Javdekar\textsuperscript{16} introduced sensor placement for parameter estimation and structural model updating using static nondestructive data. Song et al.\textsuperscript{17} constructed an OSP algorithm using reduced strain sensors based on the axial strain and nodal displacements in a truss structure. Castro-Triguero et al.\textsuperscript{18} investigated the influence of parametric uncertainties on four sensor configurations and optimization methodologies. Shi et al.\textsuperscript{19} proposed an OSP method based on a weighted standard deviation norm to obtain the Hadamard product of the standard deviation and damage estimation weight. Papadopoulos and Garcia\textsuperscript{20} established a structural sensor placement method using the Gram–Schmidt orthogonalization procedure and principal component analysis. Tan and Zhang\textsuperscript{21} reviewed computational methodologies for OSP and formulated evaluation criteria for sensor configurations and optimization methodologies.

**Methodology**

*Constrained dynamic equation by incomplete mode shape matrix*

The dynamic behavior of a structure, which is assumed linear and approximately discretized for $n$ DOFs without damping and external excitations, can be described by the following equation of motion in the time domain

\[
M\ddot{q} + Kq = 0
\]  

(1)

where $M$ and $K$ denote the $n \times n$ mass and stiffness matrices, respectively, $q$ and $\dot{q}$ denote the $n \times 1$ displacement and acceleration vectors, respectively.

The dynamic equation of equation (1) is decoupled by modal analysis. By substituting $q = \phi e^{j\Omega t}$, where $j = \sqrt{-1}$, $\Omega$ is the excitation frequency, and $\phi$ is the analytical mode shape vector, into equation (1), the eigenvalue equation is obtained and can be used to obtain the natural frequency and the corresponding mode shape vector. It can be derived as follows

\[
(K - \Omega^2M)\phi_i = 0, \quad i = 1, 2, \ldots, n
\]

(2)

It is impractical to obtain a full set of modal data. We predict the optimal number of sensors and their layout using an incomplete mode shape matrix. In this study, the target sensor number $s (s \leq n)$ was designed to be greater than the number of incomplete modes. The observation acceleration vector is considered the measurement dataset. In addition, the estimator acceleration vector at all DOFs is expanded and predicted using constraint conditions expressed by the relationship between generalized coordinates and modal coordinates.

Assuming the $n \times s$ measured modal matrix $\phi$, the condensation process at modal analysis begins with the division of the mode shape matrix. It is divided into the $r \times s$ mode shape matrix corresponding to the slave DOFs, $\phi_s$, and the $(n-r) \times s$ mode shape matrix corresponding to the master DOFs, $\phi_m$, where $r (r \leq n)$ denotes the number of candidate sensor locations. The generalized displacement vector can be written as a linear combination of the product of the mode vector matrix and modal coordinate vector

\[
Q = \phi y
\]

(3)

where $q = [q_a^T \ q_b^T]^T$, $\phi = [\phi_a^T \ \phi_b^T]^T$, the subscripts $a$ and $b$ denote the slave and master DOFs, respectively, and $y$ is the $s \times 1$ modal displacement vector.

Solving the second equation of equation (3) with respect to the modal displacement and inserting the result into the first equation of equation (3) yields

\[
q_a = \phi_a \phi_a^T q_b
\]

(4a)

or

\[
[I \ -\phi_a \phi_a^T] [q_a \ q_b] = 0
\]

(4b)

where $I$ is an $r \times r$ identity matrix, and “ $+$ ” denotes the Moore–Penrose inverse. Equation (4) is the constraint condition expressed by the reduced DOFs. The dynamic responses cannot be explicitly calculated by the solution of equation (4) with respect to the generalized DOFs because the coefficient matrix is not a square matrix.

The coefficient matrix on the left-hand side of equation (4b) represents the $r \times s$ constraint matrix that relates the slave DOFs to the master DOFs. Udawadia and Kalaba\textsuperscript{22} provided a mathematical form to describe the constrained motion of a dynamic system based on Gauss’s principle of least constraint. It states that the
constrained acceleration set is as close as possible to the unconstrained acceleration set while satisfying the constraints. Substituting the relationship between equation (4b) and equation (1) into the constrained dynamic equation proposed by Udwadia and Kalaba, the constrained dynamic equation can be approximately derived as

$$\ddot{q} = a + M^{-1/2} \left( A M^{-1/2} \right)^{+} (b - Aa)$$

where $\ddot{q}$ denotes the observation acceleration vector needed to satisfy the constraint of equation (4), the coefficient matrix $A = [I - \phi_a \phi_b^T]$, $a = -M^{-1}Kq$, and $b = 0$. The right-hand side of equation (5) represents the estimated acceleration vector $\ddot{q}$ needed to satisfy the constrained conditions of equation (4b). It includes the measurement errors and truncated modal effect between $\ddot{q}$ and $\ddot{q}$, which can be obviated by utilizing the complete mode shape matrix. The OSP algorithm in this study searches for the sensor layout to minimize the second term on the right-hand side of equation (5) based on Gauss’s principle.

By multiplying both sides of equations (4b) and (5) by the coefficient matrix $R$, we can obtain the acceleration responses at the candidate sensor locations. $R$ is the $r \times n$ Boolean matrix used to define the active sensor locations. The OSP algorithm is derived by establishing the sensor layout to minimize the errors included in the constrained dynamic equation of equation (5).

**Modified EI approach based on constraints**

The optimal sensor layout and number are designed based on the locations related to independent values to explain the entire modal data. The Cramer–Rao lower bound (CRLB) provides a lower bound on the variance of an unbiased estimator and is defined as the inverse of the FIM.

The OSP can be predicted by dynamic responses. Initially, we estimate the candidate sensor positions at a full set of DOFs of the finite element model. Evaluating the left-hand and right-hand sides of equation (5), the candidate sensor locations are reduced to the number of DOFs eliminated from the master DOFs and gradually approach the target locations. The acceleration-based observation equation including the estimation error can be written as

$$\ddot{u} = \ddot{u} + e(t)$$

where $\ddot{u} = Rq$ is the $r \times 1$ acceleration observation vector corresponding to the active sensors, and $\ddot{u} = R\ddot{q}$ is its unbiased acceleration-based estimation vector. $r$ is the number of candidate sensor DOFs. Furthermore, $e(t) \sim N(0, \psi_0^2)$ is assumed as an $r \times 1$ white noise vector to explain the measurement errors and truncated modal effect. The OSPs are located at the target coordinates, minimizing the estimation error.

The EI approach is an iterative method for selecting candidate sensor locations for optimizing the linear independence of mode shapes and the best target sensor locations. If $(\ddot{u} - \ddot{u})$ in equation (6) represents the difference between the observed and estimated acceleration vectors at candidate coordinates, the covariance matrix of the estimated acceleration error $P$ can be written as

$$P = E[(\ddot{u} - \ddot{u})(\ddot{u} - \ddot{u})^T] = \left[ \left( \frac{\partial \mu}{\partial \theta} \right)^T C^{-1} \left( \frac{\partial \mu}{\partial \theta} \right) \right]^{-1} = F^{-1}$$

where $\mu(\ddot{u})$ is the $r \times 1$ mean vector of deviated acceleration corresponding to the second term on the right-hand side of equation (5), and $C(\ddot{u}) = \psi_0^2 I$ represents the $r \times r$ stationary Gaussian white variance matrix. $E$ is the expected value, and $F$ is the $r \times r$ FIM. Owing to the constant variance of the stationary Gaussian measurement, equation (7) can be written as

$$E[(\ddot{u} - \ddot{u})(\ddot{u} - \ddot{u})^T] = \left[ \frac{1}{\psi_0^2} D^T D \right]^{-1} = F^{-1}$$

where the parameter weighting matrix $D$ is $M^{-1/2} (AM^{-1/2})^T A$ in the case of this proposed approach and $D$ for the case where the mode shape-based approach is $\phi$. The FIM is expressed by the product of the parameter weighting matrix and its transpose. Minimization of the covariance matrix of the estimated error suggests maximization of the FIM. Furthermore, the minimized FIM has little effect on reducing the size of the covariance matrix. Thus, the row of the mode shape corresponding to the lowest FIM is excluded at each iteration. The maximization is carried out by eigenvalue analysis of the FIM $F = D^T D$. It yields

$$[\phi^T \phi - \lambda I] \chi = 0$$

where $\lambda$ denotes the $r$ eigenvalues, and $\chi$ is the corresponding orthogonal eigenvector. By expressing the eigenvalues of the FIM by the absolute identification space, we can write the eigenvalue contribution matrix $G$ as

$$G = [\phi \chi] \otimes [\phi \chi]$$

where $\otimes$ denotes a term-by-term matrix multiplication. The $G$ index can be utilized to evaluate the degree of influence on the FIM. A low $G$ index is not significantly related to the sensor location.
Table 1. Test symbols.

| B: Beam test | D1 | 4 | F |
|--------------|----|---|---|
| T: Truss test| D2 = M^{-1/2}(AM^{-1/2})^+ A | 2: 2 sensors | EI: Effective information |
|              | 4: 4 sensors | F: FIM |

By multiplying G by the inverse of the matrix of eigenvalues, we can obtain the EI coefficients of the candidate sensor locations

\[ E_d = [\psi \chi] \otimes [\psi \chi] \lambda^{-1} \]  

(11)

where \( E_d \) is the EI index and collects the fractional contribution of the sensor locations to the eigenvalues. A large EI index plays a key role in sensor measurement, but a small EI index does not greatly affect the sensor measurement.

The OSP algorithm based on the Guyan reduction method rearranges the array of the mode-shaped matrix during each iteration. The F and EI indices do not necessarily represent the lowest values for the same DOF. The EI index \( E_{d,i} \), corresponding to the \( i \)th sensor denotes the fractional contribution of the information matrix determinant if the \( i \)th sensor is removed from the candidate set. After the sensor location is removed from the master DOFs, the remaining coordinates become more independent. The eliminated row is moved to the slave DOFs, and its effect is distributed to the other DOFs in the next iteration process. The iteration is repeated until the number of rows of the master DOFs becomes the number of target sensors. During the final iteration, the EI index approaches 1.0, indicating independent target modes. The OSP is selected using the contribution index \( F \) in equation (8) and the EI index in equation (11) with \( D = M^{-1/2}(AM^{-1/2})^+ A \), respectively.

The measurement sensor layout must reduce the estimated uncertainties obtained using a limited number of sensors. The number of distinct sensor configurations in a finite element model is expressed as follows

\[ C_{N_d}^{N_s} = \frac{N_d!}{N_s!(N_d-N_s)!} \]  

(12)

where \( C_{N_d}^{N_s} \) represents the combinations, \( N_d \) is the number of candidate sensor locations, and \( N_s \) is the number of target sensors to be distributed over all candidate locations.

Numerical simulations were performed on the OSP of the beam–truss structure. The test variables included the number of sensors, evaluation index, and type of parameter matrix. The test symbols are listed in Table 1.

### Cantilevered beam

The cantilevered beam model shown in Figure 1 was utilized to determine the validity of the proposed OSP approach. In this example, the OSP results obtained using parameter matrix \( D \) in the FIM and the selected number of modes were compared. For this application, the beam length was 1m, the length of each beam element was 100mm, and the beam was modeled using 10 beam elements. The nodal points and elements are numbered in Figure 1. Each node had two DOFs of vertical displacement and slope, but the slope was ignored because it was difficult to measure the rotational responses of the actual structure. We considered two-parameter matrices of the mode shape matrix \( \phi \) in equation (3) and the weighting matrix \( M^{-1/2}(AM^{-1/2})^+ A \) in equation (5). Except for the boundary condition at the fixed end, the beam was a 10 DOF system and consecutively numbered from the left node (Figure 1).

The algorithm selected the optimal displacement sensor layout for the 10 candidate positions. Two incomplete mode shape matrices of the lowest four and two modes were considered to establish the OSP layout.
using four and two sensors, respectively. In this study, we assumed that the sensors had an equal number of modes. The initial candidate sensor locations coincided with the entire DOF of the beam model. Based on equation (2), the candidate sensor layout of the four and two sensors were 210 and 45 combinations, respectively, using equation (12). The beam had a modulus of elasticity of $2.1 \times 10^5$ MPa, and its gross cross-section was $b \times h = 50 \text{mm} \times 12\text{mm}$.

The test specimens of the B-D1 series use the mode shape matrix as the parameter matrix, $D_1 = \phi$, where $\phi$ can be the $10 \times 4$ and $10 \times 2$ mode shape matrices corresponding to the first four and two modes of the cantilevered beam model, respectively. The EI index was computed using equation (11), the mode shape matrix row corresponding to the lowest EI index $\min(\text{diag}(\text{EI}))$ was deleted during the first iteration, and the remaining mode shape matrix was retained. The second iteration was performed in the same manner. The deleted mode shape did not contribute to the remaining mode shapes. After six iterations, the final locations of the four sensors of B-D1-4 overlapped at the two sensors of B-D1-2. Tables 2 and 3 list the EI index and the OSP, respectively, during each iteration. The final OSPs that considered the lowest four and two modes were established at DOFs 2, 4, 7, and 10, and DOFs 4 and 10, respectively. The shaded numbers in the sixth column in Table 2 indicate the target sensor locations, and it is shown that the EI indices approach 1.0 at the sensor locations. This approach deletes the row of the mode shape matrix corresponding to the lowest EI index, and its influence on the other DOFs is disregarded. From Tables 2 and 3, it is shown that the optimal sensors are located at positions that ensure uniform allocation of the system DOFs, and the response information is provided at these positions.

In the B-D2 series, $D$ in equation (8) was replaced with $D_2 = [M^{-1/2}(AM)^{-1/2}]A$ as the parameter matrix. It was observed that the coefficient matrix $A$ in parameter matrix $D_2$ is dependent on the sensor positions. This example also considers two and four final sensor locations. The mode shape at the master DOFs corresponding to the row of $\min(\text{diag}(\text{F}))$ and $\min(\text{diag}(\text{EI}))$ calculated using equations (8) and (11), respectively, was moved to the slave DOF mode during each iteration. Other F and EI indices were calculated, and the same process was repeated until the target sensor locations were determined. At each iteration step, the constraint conditions of equation (4b) were updated during the computations, and the mode shape was retained. The OSPs coincided with the sensor locations to minimize the second term on the right-hand side of equation (5).

The numerical results presented in Table 4 reveal that the OSPs obtained using the proposed method differed from those obtained in the first case. The optimal

Table 2. EI index at each iteration in the first case.

|    | First | Second | Third | Fourth | Fifth | Sixth |
|----|-------|--------|-------|--------|-------|-------|
| 1  | 0.3016| 0.6702 | 0.6894| 0.9426 | 0.9502| 0.9512|
| 2  | 0.4795| 0.3852 | 0.3865| 0.5638 | 0.5704| 0.5716|
| 3  | 0.3397| 0.4312 | 0.4350| 0.4455 | 0.5477| 0.5489|
| 4  | 0.4260| 0.4279 | 0.3994| 0.4008 | 0.4008| 0.4008|
| 5  | 0.4502| 0.4632 | 0.6164| 0.6245 | 0.8969| 0.9334|
| 6  | 0.3062| 0.3172 | 0.4199| 0.4199 | 0.4204| 0.4204|
| 7  | 0.5714| 0.5782 | 0.6019| 0.6028 | 0.6144| 0.9949|

DOF: degree of freedom. The shadowed cell indicates the deleted DOF from the system DOFs.
layouts of the four sensors on the F and EI indices of the B-D2-4 series were equally optimized at DOFs 2, 3, 4, and 10. It was observed that the OSPs focused on the free and fixed ends of the beam, similar to the B-D1 series. A slight difference in the OSPs caused by the parameter weighting matrix was attributed to the contribution of the deleted mode to the other DOFs, unlike the B-D1 series. The layouts of the two sensors on the B-D2-2 series specimens based on F and EI indices were optimized at the different locations of DOFs 3 and 10 using the EI index and 2 and 4 using the F index. The OSP difference was attributed to the consideration of very few modes of the lowest two modes. The initial difference started from the second iteration. However, the final OSPs were included in the target sensors selected by the four sensors. This indicated that the two sensor locations should be common candidate locations within a narrower range.

**Truss structure**

As another example, we considered the OSP of the truss structure (Figure 2). Figure 2 shows the structural composition, number of members, and number of joints of the structure. The structure was simply supported and consisted of 14 nodes and 30 members. Each node had two DOFs: horizontal and vertical displacements. The entire structure consisted of 25 DOFs, excluding the boundary conditions at Nodes 1 and 8. The 25 DOFs were arranged in order from Nodes 2 to 14 (Figure 2). For this numerical simulation, each chord and diagonal member had lengths of 4 and 5 m, respectively, and the height of the structure was 3 m. Each member had a modulus of elasticity of 210 GPa and a cross-sectional area of 250 mm². The first eight mode shapes were utilized to obtain the eight OSPs, and the condensation process was repeated similarly to the previous beam example until eight target sensors were obtained.

The $25 \times 8$ incomplete mode shape matrix was divided into mode shape matrices of the master and slave DOFs. The iteration was continued until there were eight master DOFs. Table 5 lists a comparison between the final OSPs obtained from the EI and F indices during each iteration using equations (8) and (11), respectively. The EI-based approach yielded the optimal sensor layout at DOFs 2, 3, 5, 6, 7, 9, 11, and 13, and the F-based approach at DOFs 2, 3, 4, 5, 15, 16, 23, and 25. It was shown that both approaches followed different paths during the iteration process. The difference started with the second iteration and was attributed to the contribution degree of a few considered modes, and the mode shape in the slave DOFs moved from the master DOFs. The F-based approach yielded the optimal sensor layout at DOFs 2, 3, 5, 6, 7, 9, 11, and 13, and the F-based approach at DOFs 2, 3, 4, 5, 15, 16, 23, and 25. It was shown that both approaches followed different paths during the iteration process. The difference started with the second iteration and was attributed to the contribution degree of a few considered modes, and the mode shape in the slave DOFs moved from the master DOFs. The F-based algorithm indicated the OSPs at the upper and lower chords, and the EI-based algorithm indicated the OSPs at the lower chords only. It was shown that the sensors obtained using both algorithms were evenly arranged along the span. A few target sensors determined by both

| First | Second | Third | Fourth | Fifth | Sixth | Seventh | Eighth |
|-------|--------|-------|--------|-------|-------|---------|--------|
| 1     |        |       |        |       |       |         |        |
| 2     |        |       |        |       |       |         |        |
| 3     |        |       |        |       |       |         |        |
| 4     |        |       |        |       |       |         |        |
| 5     |        |       |        |       |       |         |        |
| 6     |        |       |        |       |       |         |        |
| 7     |        |       |        |       |       |         |        |
| 8     |        |       |        |       |       |         |        |
| 9     |        |       |        |       |       |         |        |
| 10    |        |       |        |       |       |         |        |

DOF: degree of freedom. "O" indicates the final OSP. The shadowed cell indicates the deleted DOF from the system DOFs.

| Algorithm  | DOF |
|------------|-----|
| B-D2-4 series | EI index |
| B-D2-2 series | F index |
| B-D2-4 series | F index |
| B-D2-2 series | EI index |

DOF: degree of freedom. The shadowed cell indicates the final sensor locations.
algorithms can be common candidates within a narrower range for optimal measurements. It was inferred that the final OSPs provided sufficient information on structural performance.

The EI index based on the mode shape of the B-D1 series did not reflect the modal effect corresponding to the deleted DOF. However, the modified EI approaches of the B-D2 and T-D2 series proposed in this study distributed the effect of the deleted mode shape to the slave mode and led to reasonable results. The proposed method showed a slightly different sensor layout, owing to the contribution of the parameter matrix and the moved mode to the slave DOFs from the master DOFs. However, the final sensor layout contained common candidates within a narrower range for optimal measurements.

Conclusion

In this study, an OSP algorithm combining the FIM and EI methods was developed, and the dynamic equation subjected to constraints of the condensed relationship between generalized and modal coordinates was derived. A parameter weighting matrix was used to represent the truncated modes and measurement errors. Iterations were repeated until the target sensors coincided with the prescribed number of modes. The numerical simulations for the sensor layout obtained with the number of installed sensors and the EI approach based on the mode shape were compared, and the F and EI approaches based on the parameter matrix derived in this study were compared. The optimal sensors obtained using the mode shape-based EI approach were located at positions that uniformly allocated all the DOFs, unlike in the proposed approaches. In the proposed F-based and EI-based approaches, the mode shapes eliminated in the master DOFs influenced the mode shapes of the slave DOFs during each iteration. The numerical results revealed that the F-based and EI-based approaches yielded slightly different OSPs because they considered only a few modes from the first mode. Nevertheless, the final OSPs gradually converged common candidates within a narrower range for optimal measurements.

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Table 5. Final OSP of T-D2-8 series.

| Algorithm    | DOF |   |   |   |   |   |   |   |   |   |
|--------------|-----|---|---|---|---|---|---|---|---|---|
| T-D2-8 series| EI index |   |   |   |   |   |   |   |   |   |
|              | F index  |   |   |   |   |   |   |   |   |   |

DOF: degree of freedom. The shadowed cell indicates the final sensor locations.

Figure 2. A truss model.
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