Non-diagonal disorder enhanced topological properties of graphene with laser irradiation

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Laser irradiation, as a versatile tool to tune topological properties of electronic systems, is under intensive studies. Experimentally, laser irradiation induced anomalous Hall effect in graphene has been observed (McIver et al., Nat. Phys. 16, 38 (2020)). Disorder is ubiquitous in real materials, and it has been shown that diagonal disorders, i.e., onsite disorder, can enhance topological properties of time-periodically driven quantum materials (Titum et al., Phys. Rev. Lett. 114, 056801 (2015)). Here, we investigate circularly polarized laser irradiated graphene with non-diagonal disorders, i.e., disordered tunneling, and find that disorder can induce nontrivial topological properties, characterized by Bott index and the real-space Chern number. Moreover, we show that one can turn on the laser irradiation non-adiabatically to drive the disordered graphene into non-trivial topological phase. It is a scheme which is especially interesting for experimental implementations.

I. INTRODUCTION

Quantum materials with non-trivial topology have attracted great attention in last decades. Laser irradiation, as an effective way of Floquet engineering to introduce time-periodic driving to a static system, is a powerful way to tune the topological properties of solid state systems. Aoki and Oka [1] have pioneered the prediction that a circularly polarized laser can open a gap in the Dirac cone of graphene. This work sparked series of work in the direction of tuning topological properties with polarized laser. Kitagawa et al. [2] pointed out that it was, in fact, a realization of the Haldane model [3] in the regime of nonequilibrium physics. Lindner el al.[4], also proposed that using irradiation at microwave frequencies, a non-trivial topological phased can be induced in a semiconductor quantum well. Torres et al. carried out detailed work [5–10] about transport properties of multiterminal graphene irradiated by laser, and proposed possible experimental schemes. Furthermore, in the context of quantum simulation [11–13] and quantum materials [14], for both non-interacting and interacting cases, lots of interesting proposals on Floquet systems have been made. The theoretical insight is experimentally carried out firstly in the platform of ultracold atoms in hexagonal optical lattices [13, 15, 16]. Remarkably, using an ultrafast circularly polarized laser pulse [17], McIver et al. observed the anomalous Hall effect in graphene. Nevertheless, the anomalous Hall conductance is not fully quantized.

Disorder, ubiquitous in real materials, plays an important role to enhance topological properties. Topological Anderson insulator (TAI) is predicted in Ref.[18] and experimentally realized using one-dimensional disordered ultracold atoms [19]. By introducing diagonal disorders, i.e., onsite disorder, to circularly polarized laser irradiated graphene, Titum el al., [20] identified a non-trivial topological phase induced by disorder, for which the bulk state is totally localized [21], dubbed as Floquet topological Anderson insulator (FTAI).

These progresses motivate us to ask whether non-diagonal disorder, i.e., disordered tunneling, can enhance topological properties of a circularly laser irradiated graphene, and more importantly, how to implement the laser irradiation experimentally. Here, we theoretically investigated a circularly polarized laser irradiated graphene in the presence of non-diagonal disorders in the hopping terms. A non-trivial topological phase induced by disorder has been identified. We characterized its topological properties with Bott index [22, 23] and the real space Chern number [24, 25], and demonstrated that it could be realized at a moderate frequency. For the possible experimental implementation, we showed that the system could be driven into a non-trivial topological phase by non-adiabatically ramping up of the driving, in sharp contrast to the adiabatically turning on of the driving for the pristine case [26]. We stress that it is a scenario which is especially appealing experimentally.

The rest of the manuscript is organized as follows. In Sec. II, we present the model and topological invariants we have investigated. In Sec. III, results on ramping up of the laser irradiation are shown and discussed. A conclusion and outlook is given in Sec. IV.

II. THE MODEL AND METHOD

In this section, we firstly present the model for the non-interacting fermions in the circularly polarized laser irradiated graphene as,

\[
H = -J \sum_{\langle i, j \rangle} (1 + W_{ij}) e^{i A_{ij}} c_{i \alpha}^\dagger c_{j \beta} + M \sum_{i, \alpha} \sigma^z_{\alpha \alpha} c_{i \alpha}^\dagger c_{i \alpha}
\]  

(1)

where \( J \) is the hopping amplitude, \( A_{ij} = A(t) \cdot (\mathbf{r}_i - \mathbf{r}_j) \) for the bond between site \( i \) and \( j \), and \( A = \)
\( A_0 \) \((\cos(\Omega t), \sin(\Omega t))\). \( \Omega \) is the driving frequency. \( c_{i\alpha} \) (\( c_{i\alpha}^\dagger \)) is the fermion annihilation (creation) operator at atom \( \alpha \) in unit cell \( i \). \( \alpha, \beta = A, B \) label sub-lattices. \( M \) is the staggered potential. Effectively, in the high-frequency limit the Hamiltonian (1) without disorder is the Haldane model [15, 16]. Non-zero disorder strength \( W \) introduces non-diagonal disorder, respectively. \( \eta_{ij} \) is the random number uniformly distributed in the range \([-0.5, 0.5]\).

We obtain quasi-energy spectrum of the Floquet Hamiltonian (1) by [27, 28],

\[
H_{\text{eff}} = \frac{i}{T} \log U(T, 0), \tag{2}
\]

where \( U(T, 0) = e^{-i \int_0^T H(t) dt} = \prod_{n=0}^{N-1} U(t_n + \Delta T, t_n) \), with \( U(t_n + \Delta T, t_n) = e^{-iH(t_n)\Delta T}; t_n = n\Delta T, \Delta T = \frac{T}{N} \) and \( T = \frac{2\pi}{\Omega} \). It is different from the usual method for the effective Hamiltonian [11], and one can obtain the exact quasi-energy spectrum in \([-\frac{\Omega}{2}, \frac{\Omega}{2}]\). We have a remark on the time-evolution operator \( U(t_n + \Delta T, t_n) = e^{-iH(t_n)\Delta T} \), which can be numerically implemented in a more efficiently way by introducing the Trotter–Suzuki decomposition [26] as

\[
U(t_n + \Delta T, t_n) = e^{-iH_0(t_n + \Delta T)\Delta T} e^{-iH_1(t_n + \Delta T)\Delta T} e^{-iH_0\Delta T},
\]

where

\[
H_0 = M \sum_{i,\alpha} \sigma_{\alpha}^{\dagger} c_{i\alpha} c_{i\alpha} \quad \text{and} \quad H_1 = -J \sum_{(i\alpha, j\beta)} (1 + W_{ij}) e^{iA_{ij}} c_{i\alpha} c_{j\beta}.
\]

We focus on topological properties of the bands between \(-\frac{\Omega}{2}\) and 0. Alternatively, one can obtain the quasi-energy spectrum by transforming the Hamiltonian (1) into the Floquet space [29, 30] by \( H^m_m = m\Omega \sum_{\alpha} \delta_{\alpha m} + \frac{T}{\Omega} \int_0^T dt e^{-i(m-m')\Omega t} H(t) \), where \( m \) and \( n \) are Floquet indices, and keeping the quasi-energy spectrum between \(-\frac{\Omega}{2}\) and 0. In this work, we mainly use the Hamiltonian defined in Eq. (2), and the Hamiltonian \( H^m_m \) in the Floquet space serves as a cross-check. For a non-interacting disordered system, we use the Bott index \( C_b \) \((-\frac{\Omega}{2}, 0)\) as a function of disorder strength to characterize different topological phases in the real space. In a time-independent Hamiltonian, it is shown that the Bott index is equivalent to the Chern number [31]. For a specific disorder configuration, the Bott index is defined by [22, 23],

\[
C_b \left( -\frac{\Omega}{2}, 0 \right) = \frac{1}{2\pi} \text{Im} \left[ \text{Tr} \left\{ \ln \left( \tilde{U}_Y \tilde{U}_X^\dagger \tilde{U}_X^\dagger \tilde{U}_Y \right) \right\} \right] \tag{3}
\]

where \( \tilde{U}_X(Y) = PU_X(Y)P \). The unitary matrices \( \tilde{U}_X = \exp (i2\pi X/L_y) \) and \( \tilde{U}_Y = \exp (i2\pi Y/L_y) \), where \( X(Y) \) is diagonal matrix of the \( x \) (\( y \)) coordinate of all the lattice sites. \( P \) is the projector operator to project the quasi-energy spectrum into the range between \(-\frac{\Omega}{2}\) and 0.

A second quantity can be used to characterize the topological properties of the disordered lattice system is the Chern number in the real space [24, 25], which is perfectly suited for a disordered case. Here we adopt in our calculations the formula for the real space Chern number presented in Ref.[25].

### III. RESULTS AND DISCUSSIONS

#### A. The Bott index

To characterize topological properties of laser-irradiated graphene, we firstly present the results for the Bott indices as a function of the disorder strength. In this work we choose the energy unit as \( J \tilde{\mathcal{J}}_0 \left( A_0 \right) \) with \( \tilde{\mathcal{J}}_0 \left( A_0 \right) \) as the zeroth order Bessel function of the first kind. In Fig. 1, we show that the disordered tunneling can induce non-trivial topological phase for the driving frequency \( \Omega = 0 \). The Bott index \( -1 \) shows the difference in the number of edge states in the gap \(-\frac{\Omega}{2}\) and 0. This phenomena exists for a relatively large range of staggered potentials. For different staggered potentials, the critical \( M \) is different. The introduction of the disordered tunneling effectively changes the staggered potential. For a certain staggered potential \( M = 0.8 \), we explore the properties of the Bott index for different driving frequencies. The phenomena that the disordered tunneling can induce non-trivial topological phase disappears for a relatively high driving frequency. We do not go to a very low driving frequency, which may be corresponding to the gap closing at the energy \(-\frac{\Omega}{2}\) and 0. We would like to point out that the Bott index as \(-1 \) or 1 is not essential, as can be tuned by \( A_{ij} \) in the Hamiltonian (1).

#### B. The real-space Chern number

To further confirm the non-trivial topological properties due to the non-diagonal disorder, we calculate the real-space Chern number, which is dubbed as the Chern marker. For a hexagonal lattice of \( 25 \times 24 \) sites with open boundary conditions, in Fig. 3, we show Chern markers versus the column indices for sites of the 12-th and 13-th rows in the bulk. For bulk sites in the 12-th and 13-th rows, the Chern markers are nearly uniform, meaning that they are well-defined topological index. More importantly, consistent with results for the Bott indices, in the non-trivial topological phase induced by non-diagonal disorder \( W = 0.5 \) in Fig. 3) Chern markers are about 1, in sharp contrast to the phase with trivial topology where it is largely deviated from 1.
It is clear in Figs.5 that for different disorder strengths, the system can turn into non-trivial topological phase. For disordered system, we investigate the time evolution from an initial state of a disordered static system with trivial topological properties as follows,

\[ |\psi(t)\rangle = \prod_{n_2=0}^{N_0} \prod_{n_1=0}^{N} U(t_n + \Delta T, t_n) |\psi(t=0)\rangle \tag{4} \]

with \(N_0\) the number total periods investigated and \(N\) the number of a single period divided. \(t_n = n_2T + n_1\Delta T\). As we have remarked before, we calculate the time evolution operator \(U(t_n + \Delta T, t_n)\) with the Trotter-Suzuki decomposition. We obtain the Bott index and level spacing ratio (LSR) [32] at \(t = nT\) in a stroboscopic way. The key result in this section is shown in Fig.4(a) that one needs to turn on the laser irradiation in a non-adiabatical way to enhance the topological properties, in stark contrast to a totally pristine driven system, where one needs to turn on the driving adiabatically[26]. Driving by laser irradiation tends to heat the system up. If it is turned on adiabatically, the system is not heated up before it enters into topological phase. We attribute it to the phenomena of coherent destruction[33] that driving contributes to localize the system. Once that the system enters into the phase with Bott index as \(-1\), the bulk state is totally localized, with LSR as about 0.39 in Fig.4(b) [21, 34], a necessary condition for FTAI. Therefore, disorder helps the system to become topologically non-trivial by localizing bulk states. The final resolution for the system is an interplay among heating, localization and topology.

To further clarify the issue of heating, localization and topology, we present in Figs.5 the same physical quantities but with a linear ramp-up of the laser irradiation by setting \(A = A_0 \frac{t}{\tau} ((\cos(\Omega t), \sin(\Omega t))\) with \(\tau = 200T\). It is clear in Figs.5 that for different disorder strengths,
the system ends up with Bott indices as 0 and LSR as about 0.6, meaning that the system is topologically trivial and delocalized\cite{21, 34}. It shows that the system can absorb energy from the driving effectively by gradually turning on the laser irradiation, leading to a fully delocalized phase. Therefore, localization is necessary for the system to enter into the non-trivial topological phase, and otherwise the system would be heated up.

The important experimental progress \cite{17} shows the power of laser irradiation. However, we would like to point out that the anomalous Hall conductance is not fully quantized. One possible way to enhance it is to introduce disorders, whether non-diagonal disorder in this work or diagonal disorder explored in literatures \cite{20, 21}, for both disorders are ubiquitous in a real materials. A second way to realize the Hamiltonian with non-diagonal disorder is to use electric circuits \cite{35}.

IV. CONCLUSION AND OUTLOOK

We have investigated a laser irradiated graphene system with disordered tunneling. By characterizing the topological properties with the Bott index and the local Chern marker, we find that topological properties of the system is enhanced by disordered tunneling. Moreover, we have shown an appealing experimental scenario that one needs to turn on the irradiation in a quenched way to drive the system into the non-trivial topological phase.

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