THERMAL EVOLUTION AND LIFETIME OF INTRINSIC MAGNETIC FIELDS OF SUPER-EARTHS IN HABITABLE ZONES

C. Tachinami, H. Senshu, and S. Ida

1 Department of Earth and Planetary Sciences, Tokyo Institute of Technology, Meguro, Tokyo 1528551, Japan; ctchnm@geo.titech.ac.jp
2 Planetary Exploration Research Center, Chiba Institute of Technology, 2-17-1 Tsudanuma, Chiba 2750016, Japan

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ABSTRACT

We have numerically studied the thermal evolution of different-mass terrestrial planets in habitable zones, focusing on the duration of dynamo activity to generate their intrinsic magnetic fields, which may be one of the key factors in habitability of the planets. In particular, we are concerned with super-Earths, observations of which are rapidly developing. We calculated the evolution of temperature distributions in the planetary interior using Vinet equations of state, the Arrhenius-type formula for mantle viscosity, and the astrophysical mixing-length theory for convective heat transfer modified for mantle convection. After calibrating the model with terrestrial planets in the solar system, we apply it for 0.1–10 $M_\oplus$ rocky planets with a surface temperature of 300 K (in habitable zones) and Earth-like compositions. With the criterion of heat flux at the core–mantle boundary (CMB), the lifetime of the magnetic fields is evaluated from the calculated thermal evolution. We found that the lifetime slowly increases with planetary mass ($M_p$), independent of the initial temperature gap at the CMB ($\Delta T_{CMB}$), but beyond the critical value $M_{c,p}$ ($\sim O(1) M_\oplus$) it abruptly declines from the mantle viscosity enhancement due to the pressure effect. We derived $M_{c,p}$ as a function of $\Delta T_{CMB}$ and a rheological parameter (activation volume, $V^*$). Thus, the magnetic field lifetime of super-Earths with $M_p > M_{c,p}$ sensitively depends on $\Delta T_{CMB}$, which reflects planetary accretion, and $V^*$, which has uncertainty at very high pressure. More advanced high-pressure experiments and first-principle simulation, as well as planetary accretion simulation, are needed to discuss the habitability of super-Earths.

Key words: planets and satellites: interiors – planets and satellites: magnetic fields

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1. INTRODUCTION

Many of the exoplanets so far detected may be gas giants with masses $\gtrsim 100 M_\oplus$, because massive planets are more easily detected. However, recently several super-Earths with masses of a few to 10 $M_\oplus$ have been discovered by improved radial velocity measurements (e.g., Udry et al. 2007) or microlensing observations (e.g., Beaulieu et al. 2006). Ongoing radial velocity (Mayor et al. 2009) and microlensing (Gould et al. 2010) surveys and theoretical studies (e.g., Ida & Lin 2004, 2008, 2010) strongly suggest the ubiquity of super-Earths in extrasolar planetary systems.

Space transit surveys such as CoRoT and Kepler will also detect many super-Earths. In fact, CoRoT has detected a minimum mass transiting planet (CoRoT-7b; Léger et al. 2009; Queloz et al. 2009). With an assumed composition, a mass–radius relationship of the super-Earths yields planetary masses from transit observational data. On the other hand, if the planetary masses are obtained by follow-up radial velocity observations, the mass–radius relationship can be used to estimate planetary composition, although there is some ambiguity depending on the amount of $H_2O$ (Sotin et al. 2007). Valencia et al. (2006) used the Birch–Murnaghan equation of state (EOS) for rocks and metals to obtain a mass–radius relationship of different-mass terrestrial planets under some conditions (core ratio, surface temperature, etc.). Sotin et al. (2007) also considered ocean planets that contain 50% $H_2O$. They used the EOS including thermal pressure to describe P–V–T relationships for ice under extremely high pressure and obtained a mass–radius relationship for both terrestrial and ocean planets.

Because super-Earths should also exist in habitable zones, the aspects related to the habitability of super-Earths are being discussed. Planetary habitability is often discussed in terms of the stability of liquid water on the planetary surface (Kasting et al. 1993). Assuming planets that are massive enough to maintain a dense atmosphere, a range of orbital radius in which liquid water is stable is called a “habitable zone.”

In addition to the existence of liquid water, the evolution of the amount and composition of planetary atmosphere may also be an important factor for habitability. It is believed that a large fraction of the present atmosphere of the Earth was formed by impact degassing (e.g., Abe & Matsui 1985), and it consisted of CO$_2$ and H$_2$O with more than 100 bars. The plate tectonics on the Earth have removed huge amounts of CO$_2$ from Earth’s atmosphere on Gyr timescales (Tajika & Matsui 1992). Valencia et al. (2007a) and O’Neill & Lenardic (2007) investigated the possibility of plate tectonics on the surface of super-Earths. The plate tectonics would significantly affect the amount and composition of planetary atmosphere through a carbonate–silicate cycle with degassing and weathering. They would also have a stabilizing effect on planetary surface temperature, because the temperature’s dependence on the weathering rate of carbonate works as a negative feedback mechanism for surface temperature change (Tajika & Matsui 1992). Valencia et al. (2007a) argued that super-Earths could invoke plate tectonics, because terrestrial planets larger than Earth may have a thinner surface thermal-boundary layer and increased yield stress. On the other hand, O’Neill & Lenardic (2007) showed that super-Earths would have a stagnant-lid-style mantle convection without plate tectonics. It is noted that these studies a priori assumed that super-Earths have a thermal structure, as did the studies on a mass–radius relationship.

Although the thermal effect on the mass–radius relationship may be negligible, plate tectonics should depend on planetary
thermal-evolution history. Dynamo activity to generate a magnetic field also depends on thermal evolution. A planetary magnetic field prevents stellar winds from splitting the planet’s atmosphere. The magnetic field also prevents cosmic rays from penetrating the planetary surface. Thus, it may be one of the most important factors for land-based life to be maintained. It is widely accepted that Earth’s magnetic field is attributed to a dynamo effect (e.g., Glatzmaier & Roberts 1995; Kuang & Bloxham 1997; Kageyama & Sato 1997) in the metallic core. The intrinsic magnetic field may be sustained if convective fluid motions in the core are vigorous enough, i.e., the heat flux through the core surface is large enough. Thus, a detailed study on the core’s thermal evolution is needed to evaluate magnetic field generation.

Using the box model (see below), Papuc & Davies (2008) modeled the thermal evolution of the different-mass terrestrial planets on geological timescales to discuss the evolution of planetary surface activity, i.e., the plate velocity and the degassing rate. Their results showed that the super-Earths may have dense atmospheres in their early histories, because larger planets have higher degassing rates. Because they were concerned with planetary surface activity, they focused on the treatment of heat generation of radio activity in the mantle and surface heat flow, conducting only a simple treatment of cores. We will show that a careful treatment of the core, considering factors such as the effects of inner core nucleation and increases in heat capacity due to high compression and gravitational energy stored in the core, which Papuc & Davies (2008) neglected, is important for the study of dynamo activity (note that the evaluation of surface activity is hardly affected by the careful treatment of the core).

In a series of papers, Schubert, Stevenson, and their colleagues developed a “box” model for the thermal evolution of terrestrial planets in our solar system (e.g., Schubert et al. 1979; Stevenson et al. 1983). In their model, the thermal structure is described by two boxes that correspond to the mantle and the core. The temperature variation in each box is neglected, and the temperature distributions in the Earth are represented by three distinct temperatures of the core, the mantle, and the planetary surface. The heat flux is evaluated by thermal conduction through the thermal-boundary layers at the core–mantle boundary (CMB) and the planetary surface using the thermal boundary layer theory (BLT; Stevenson et al. 1983). In the BLT, the heat flux is determined by the thickness of the thermal-boundary layer, which is given by the local Rayleigh number. Because this model is easily treated, it provides a powerful tool to explore general trends of thermal evolution of terrestrial planets.

For Earth’s thermal evolution, we can use observational constraints such as surface heat flow and inner core size. With the calibrated model, unknown parameters such as initial temperature distribution of Earth’s interior (Yukutake 2000; see also Section 2.7) or potassium abundance in the core (Nimmo et al. 2004) can be constrained. Rheological parameters and impurity abundance in the core are also estimated (see Section 3.3). The existence of the magnetic field for Mercury and early decay of the magnetic field for Venus and Mars are consistent with calculations using a reasonable choice of initial temperature or impurity abundance in the cores via the BLT (Stevenson et al. 1983) and mixing-length theory (MLT; Appendix B).

Gaidos et al. (2010) simulated thermal evolution of various-sized super-Earths via the BLT to evaluate the magnetic activity of super-Earths. They concluded that massive rocky planets (>2.5 \( M_{\oplus} \)) cannot sustain a magnetic field, because they found an inversion of the gradients of the melting and adiabatic curves in the core under such high pressure. Since the outer core is solid in the inverse state, the cooling of the inner liquid core, in which the dynamo operates, is inhibited. Although the possibility of the inversion raised by the paper is very important, the conclusion depends on high-pressure material properties, such as melting and adiabatic curves, that need to be confirmed. In the present paper, we point out another important factor to inhibit dynamo activity in super-Earths: a drastic increase in the mantle viscosity due to the pressure effect. Even if the inversion in the core does not occur, the enhanced mantle viscosity quickly terminates dynamo activity in the core.

In the present paper, we are concerned with the lifetime of magnetic fields of super-Earths. There is no observational constraint for super-Earths to calibrate parameters for the initial temperature distribution of the planetary interior. Here, clarifying key quantities for generation of the magnetic field, we derive planetary mass dependence on the lifetime of magnetic activity and how it depends on the model parameters and initial/boundary conditions. To reduce unknown parameters, we consider super-Earths in nearly circular orbits in habitable zones. The analysis on the key rheological parameters will provide new motivations for high-pressure experiments and first-principle simulations, because state-of-the-art high-pressure experiments have already reached the pressure at the bottom of Earth’s mantle. We will point out that initial temperature distribution sensitively affects the lifetime of the magnetic field, which gives new motivations for theories of planet accretion from planetesimals and core formation.

Here, we develop a thermal evolution model to discuss the existence of the intrinsic magnetic field in terrestrial planets with various masses. Because we are concerned with heat flux across the CMB, we calculate detailed radial temperature distribution in both core and mantle. We use the MLT to calculate the heat flow. The MLT is commonly used to study stellar interiors. We use the modified version for low-Reynolds-number flow in solid planets that have radial discontinuities in their interiors (Sasaki & Nakazawa 1986; Abe 1995; Senshu et al. 2002; Kimura et al. 2009, and references therein). The modified MLT is useful for calculations of super-Earths that may have additional higher-pressure phase transitions, such as a post–post-perovskite transition (although we do not consider this in the present paper), and early-stage planets that may have convection barriers at the upper/lower mantle boundary (e.g., Honda et al. 1993) or density crossover at the melt/solid boundary (Labrosse et al. 2007). We will also show the results for the two-layer convection case in which convective flow does not penetrate the spinel–perovskite transition at the upper/lower mantle boundary, although most calculations involve one-layer convection. We compare the MLT with the conventional BLT in detail and show that they are in good agreement with each other in the case of one-layer convection (Appendix A).

We will show that the lifetime is shorter for super-Earths than for Earth-mass planets for nominal solid-state parameters. The mechanism to suppress dynamo activity in super-Earths found in this paper is independent of that in Gaidos et al. (2010), so that it may be likely that super-Earths are magnetically inactive. We also point out that the choice of initial conditions and rheological parameters highly affects the thermal evolution of the planets. In Section 2, we explain our numerical model. The numerical results will be shown in Section 3. Finally, we discuss the habitability of terrestrial planets in view of the intrinsic magnetic field.
2. NUMERICAL MODEL

We follow planetary thermal evolution, defined as a cooling process from a hot early stage due to accretion from planetesimals and core–mantle differentiation, for various-mass terrestrial planets by using a one-dimensional spherically symmetric model.

As described below, there are unknown parameters for rheological properties and initial conditions of planets. Furthermore, even in our solar system, the terrestrial planets have different compositions (a water–rock–iron ratio) and surface temperatures that are regulated by orbital radius. In extrasolar planetary systems, more variations in the water–rock–iron ratio and surface temperature should exist due to different metal abundance of host molecular clouds, as well as different pressure/temperature states of disks, planetary atmosphere, and planetary formation processes. These unknown variations could make the study of mass dependence meaningless.

In order to reduce the uncertainty, in a “nominal” case (see Section 2.8), we consider planets with a surface temperature ($T_{\text{surf}}$) of 300 K and a mantle/core mass ratio ($\zeta_{\text{m/c}}$) of 7:3, which is the same as that for the Earth. The planets may correspond to extrasolar terrestrial planets in nearly circular orbits in habitable zones. In the nominal case, the other unknown parameters in mantle rheology, core impurity, and initial temperature are calibrated with data from the Earth.

The restriction to the nominal case enables us to derive clear planetary mass dependences. We also discuss how these dependences change with different parameters. In Appendix B, we also carry out the runs with different $T_{\text{surf}}$ and $\zeta_{\text{m/c}}$ to calculate thermal evolution of Mercury, Venus, and Mars. The results show that our model can be applied for these planets, too, if we use reasonable non-nominal parameters. A systematic survey of thermal evolution of super-Earths in non-nominal cases is left for future research.

The thermal evolution is calculated by the following methods.

1. The radial density distribution is calculated by using VINET EOS, taking into account pressure dependence (Section 2.1). Because this distribution is almost independent of the evolution of temperature distribution and the inner core growth, as explained below, we use the distribution calculated in the initial state ($t = 0$) throughout the entire thermal evolution.

2. Given a temperature distribution in the interior at time $t$, the radius of the inner solid core is calculated by the melting temperature with pressure and composition dependences (Section 2.2). Sulfur is considered an impurity in the core, and its concentration in the outer liquid core is self-consistently calculated with the condensation of the inner core (Section 2.2).

3. The heat transfer throughout the mantle is calculated by the astrophysical MLT modified for solid planets (we discuss its validity and usefulness in Section 2.3). The mantle viscosity in the heat transfer equation is estimated by using the Arrhenius-type formulation (Section 2.5). By subtracting the energy loss during the time step ($\Delta t$), we obtain a new temperature distribution at $t + \Delta t$ and go back to step 2. Time evolution is calculated by iterations of steps 2 and 3.

In the following subsections, we will explain each step in more detail.

2.1. Density Profile

The hydrostatic stratification is calculated by

$$\frac{dP}{dr} = \rho(r)g(r); \quad \frac{dr}{dm} = \frac{1}{4\pi r^2 \rho(r)},$$

where $P, r, \rho, g,$ and $m$ are pressure, radius, density, gravitational acceleration, and mass inside the radius $r$, respectively. Vinet EOS (Vinet et al. 1987) is given by

$$P = 3K_0 \frac{1-x}{x^3} \exp(\phi(1-x)),$$

where

$$\phi = \frac{3}{2}(K_0' - 1),$$

where $x = \rho_0/\rho$, where $\rho_0$, $K_0$, and $K_0'$ are density, bulk modulus, and its pressure differentiation at zero pressure, respectively.

The Birch–Murnaghan EOS is a “finite strain” EOS, in which pressure is expressed by Taylor-series expansions of finite strain, and it is often used for the calculation of interiors of solid planets. However, the finite strain EOS does not accurately represent volume variation under very high compression (if pressure exceeds the bulk modulus of zero pressure, the expansion never converges). The Vinet EOS is derived from a general interatomic potential energy function. For simple solids, the Vinet EOS provides more accurate representations of volume variations with pressure under very high pressure. Because we are concerned with interiors of super-Earths under very high pressure, we adopt the Vinet EOS rather than the Birch–Murnaghan EOS, following Valencia et al. (2007b).

Because thermal contraction is small enough (it is less than $1\%$ in physical size for a $100$ K change in the average temperature of Earth’s interior), we neglect the temperature dependence in the Vinet EOS. The density profile of the planet is calculated by numerically solving Equations (1) and (2).

The compositions are assumed to be olivine and $\gamma$-spinel for the upper mantle, perovskite and post-perovskite for the lower mantle, Fe and FeS for the outer core, and Fe for the inner core; properties are given in Table 1. As temperature decreases, the inner solid core grows and sulfur moves from the inner core to the outer core (see Section 2.2). In general, the inner core growth changes the volume of the whole core, because the parameters $\rho_0$, $K_0$, and $K_0'$ are different between Fe and FeS. However, because we numerically found that the volume change of the whole core is very small, we neglect it. Figure 1 shows the result of numerical calculations of the density profile for 0.1 to 10 Earth-mass planets. This result is different in the radii of the core and the mantle from the results found in Valencia et al. (2006) and Sotin et al. (2007) by a few percentage points, which may be due to different choices of parameter values in the EOS. However, this difference does not affect thermal evolution.

2.2. Thermal Evolution of the Core

The temperature distribution of the core is determined as follows.

1. The inner solid core. We assume that each part of the inner core memorizes the temperature at which it solidified because of inefficient heat transfer due to conduction in the solid core.

2. The outer liquid core. We assume that the liquid core has an adiabatic temperature distribution by vigorous convection.
3. Time evolution. The radius of the inner core and the temperature at the CMB is determined by the total energy of the inner and outer cores as described below, and the total energy is given as a function of time by integrating heat flux at the CMB.

The adiabatic temperature gradient in the outer core is given by (Sohl & Spohn 1997; Yukutake 2000; Valencia et al. 2006)

\[
\frac{\partial T}{\partial r} = \frac{\rho g \gamma_G}{K_s} T, \tag{4}
\]

where \( \gamma_G \) and \( K_s \) are the Grüneisen parameter and bulk modulus of the liquid core, respectively. Depth variation of \( \gamma_G \) is calculated as \( \gamma_G = \gamma_G(0)(\rho_0/\rho)^q \) (the parameter values used are summarized in Table 1). The density at 0 pressure \( \rho_{0\text{OC}} \), bulk modulus \( K_{0\text{OC}} \), and its pressure derivation \( K'_{0\text{OC}} \) of the outer core are given by impurity concentration \( x_S \) as

\[
x_{\text{FeS}} = x_S \frac{Z_{\text{Fe}} + Z_S}{Z_S} \tag{5}
\]

\[
\rho_{0\text{OC}} = \left( \frac{1 - x_{\text{FeS}}}{\rho_{\text{Fe}}} + \frac{x_{\text{FeS}}}{\rho_{\text{FeS}}} \right)^{-1} \tag{6}
\]

\[
K_{0\text{OC}} = \frac{1}{\rho_{0\text{OC}}} \left[ \frac{1}{\rho_{\text{Fe}}} \frac{1}{K_{\text{Fe}}} + \frac{1}{\rho_{\text{FeS}}} \frac{1}{K_{\text{FeS}}} \right] \tag{7}
\]

\[
K'_{0\text{OC}} = -1 + \rho_{0\text{OC}} K_{0\text{OC}} \left( \frac{1 - x_{\text{FeS}}}{\rho_{\text{Fe}}} + \frac{1}{\rho_{\text{FeS}}} \right)^2 \left( \frac{1}{\rho_{\text{Fe}}} \frac{1}{K_{\text{Fe}}} + \frac{1}{\rho_{\text{FeS}}} \frac{1}{K_{\text{FeS}}} \right), \tag{8}
\]

where \( x_{\text{Fe}}, x_{\text{FeS}}, Z_{\text{Fe}}, \) and \( Z_S \) are mass fractions of Fe and FeS and molar weights of Fe and S, respectively.

The inner core nucleation decelerates cooling of the core by release of gravitational energy due to the change in the density distribution and by release of latent heat (Stevenson et al. 1983; Gubbins et al. 2004). The light elements are kicked into the outer core, resulting in depression of the melting point of the outer core (Stevenson et al. 1983; Yukutake 2000). The boundary between inner and outer cores is located at the intersection between adiabatic and melting curves in the core. We use Linderman’s equation for the melting curve of pure iron,

\[
\Gamma(\rho) = \Gamma_0 \left( \frac{\rho_0}{\rho} \right)^{2/3} \exp \left\{ \frac{2\gamma_0}{q} \left[ 1 - \left( \frac{\rho_0}{\rho} \right)^q \right] \right\}. \tag{9}
\]

We also consider the depression of the melting point by concentration of light elements. We define the melting point of Fe–FeS alloy as

\[
T_{\text{melt}} = (1 - 2x_S)\Gamma(\rho), \tag{10}
\]

and the factor \((1 - 2x_S)\) expresses the depression of the melting point due to dissolution of light elements (Usselman 1975; Stevenson et al. 1983). Assuming that the outer core is well mixed by convection

\[
x_S = x_{\text{OS}} \frac{M_c}{M_c - M_{ic}}, \tag{11}
\]

where \( M_c \) and \( M_{ic} \) are the inner core mass and total mass of the inner and outer cores, respectively, and \( x_{\text{OS}} \) is the initial impurity concentration. In the nominal case, we adopt \( x_{\text{OS}} = 0.1 \).

Given the inner core radius, we can calculate the total energy of the core \( E_{\text{core}} \), which is the sum of the gravitational energy \( E_G \), latent heat \( E_L \), and thermal energy \( E_{th} \). As described above, the temperature at the CMB is given as a function of the radius of the inner core. As a result, we can obtain \( E_{\text{core}} \) as a function of the temperature at the CMB. Conversely, the radius of the inner core and the temperature at the CMB are given as a function of \( E_{\text{core}} \).

The energies are given by

\[
E_G = - \int_{r_e}^{r_i} 4\pi r^3 \rho_{\text{ic}}(r) g_{\text{ic}}(r) dr - \int_{r_e}^{r_i} 4\pi r^2 \rho_{\text{oc}}(r) g_{\text{oc}}(r) dr, \tag{12}
\]

\[
E_L = L M_{ic}, \tag{13}
\]

\[
E_{th} = \int_{r_e}^{r_i} 4\pi r^2 \rho(r) C_p(r) T(r) dr, \tag{14}
\]

where \( L \) is the latent heat released by solidification of the unit mass of iron, which is assumed to be constant at \( 1.2 \times 10^6 \text{ J kg}^{-1} \) (Anderson & Duba 1997) and not to depend on the impurity concentration in the outer core; and \( C_p \) is specific heat with constant pressure. Both the gravitational energy and the latent heat are released after the inner core starts to solidify. Gravitational energy is also released by thermal contraction, which will be discussed in Section 2.6. The total energy \( E_{\text{core}} \) decreases at a rate that is equal to the heat flux at the bottom of the mantle (see Section 2.3). Detailed calculations of the energies are given in Appendix C.
2.3. Heat Transfer Throughout the Mantle

The mantle is cooled by irradiation from the planetary surface and heated by heat flow from the core and internal radioactivity (see below). The heat transfer equation is

\[ \rho C_p \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 k_c \frac{\partial T}{\partial r} \right) + r^2 k_v \left[ \left( \frac{\partial T}{\partial r} \right)_s - \left( \frac{\partial T}{\partial r} \right) \right] + \rho Q, \]

(13)

where \( k_c \) is the thermal diffusion coefficient, \( Q \) is the radioactive heat production rate, \( (\partial T/\partial r)_s \) is the adiabatic temperature gradient, and the first and second terms on the right-hand side represent conductive and convective fluxes.

To evaluate the convective flux in the mantle, we use the astrophysical MLT modified for solid planets (Sasaki & Nakazawa 1986; Abe 1995; Senshu et al. 2002; Kimura et al. 2009, and references therein) rather than the conventional parameterized convection model (PCM; e.g., Sharpe & Peltier 1979) or the commonly used BLT (e.g., Stevenson et al. 1983).

The PCM is very simple (Appendix A). However, it uses the values of \( k_c \) and the global averaged Rayleigh number \( Ra \), which represent the whole mantle and are difficult to evaluate for the real mantle because of huge spatial variation in mantle viscosity. As a result, although the PCM can be applied to study the overall trend of thermal evolution, it may not be accurate enough for the evaluation of heat flux across the CMB \( F_{\text{CMB}} \), which we are concerned with in this paper. In the BLT, because the heat flux is expressed only by quantities in the thermal-boundary layer (Appendix A), the BLT has better resolution for the evaluation of heat production rate at the CMB, \( H_{\text{CMB}} \).

In Section 2.3.1, we use the modified MLT, which is easily applied to low-Reynolds-number flow in the mantle. To apply the model to the upper/lower mantle boundary, we will consider the effects of other barriers.

Table 2

| Element | \( U/\text{ppb} \) | \( H/\mu\text{W kg}^{-1} \) | \( \lambda/\text{yr}^{-1} \) |
|---------|----------------|----------------|----------------|
| K\(^{40}\) | 28.0 | 29.17 | 5.54 \times 10^{-10} |
| Th\(^{232}\) | 76.4 | 26.38 | 4.95 \times 10^{-11} |
| U\(^{235}\) | 0.14 | 568.7 | 9.85 \times 10^{-10} |
| U\(^{238}\) | 20.1 | 94.65 | 1.55 \times 10^{-10} |

In future papers, we will consider the effects of other barriers. In the MLT, the coefficient for convective heat transfer is given by

\[
\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 k_c \frac{\partial T}{\partial r} \right) + r^2 k_v \left[ \left( \frac{\partial T}{\partial r} \right)_s - \left( \frac{\partial T}{\partial r} \right) \right] + \rho Q,
\]

where \( \eta \) and \( \ell \) are viscosity and the mixing length, respectively. Here, the velocity of fluid blobs is evaluated by Stokes velocity rather than free-fall velocity, as used in the original MLT, in order to apply the model to low-Reynolds-number flow in the mantle. In an astrophysical context such as a stellar interior, the density scale height is usually adopted as \( \ell \). For calculation of thermal evolution of the Earth, it is proposed that a distance \( (D) \) from the closest barrier, such as the CMB or the top of the mantle layer, is appropriate for \( \ell \). (Sasaki & Nakazawa 1986; Abe 1995; Senshu et al. 2002; Kimura et al. 2009, and references therein). The detailed comparison with the PCM and BLT in Appendix A shows that \( \ell = 0.82 \) D is the best choice. We adopt \( \ell = 0.82 \) D for all the runs in the present paper.

With this choice, when approaching a barrier, \( k_c \) rapidly decreases in proportion to \( \ell^4 \), and the conductive term dominates in Equation (13). As a result, thermal-boundary layers, in which the conductive heat transfer dominates, are automatically represented. Thereby, the modified MLT is easily applied to calculation for thermal evolution of the proto-Earth or super-Earths. Thermal conductivity, specific heat, and thermal expansion of each layer are listed in Table 3.
values of these parameters for upper and lower mantles. The activation volume of the mantle, respectively. We use different strain rate, creep index, Barger coefficient, activation energy, and model (Ranalli 2001):

\[ \eta(T, P) = \frac{1}{2} \left( \frac{1}{B^{1/n}} \exp \left( \frac{E^* + PV^*}{nRT} \right) \right)^{(1 - n)/n}, \]  \tag{15} 

where \( R, \dot{\varepsilon}, n, B, E^*, \) and \( V^* \) are the universal gas constant, strain rate, creep index, Barger coefficient, activation energy, and activation volume of the mantle, respectively. We use different values of these parameters for upper and lower mantles. The mineral properties we use are listed in Table 4. Note that the prescription for the mantle viscosity may include uncertainty. The formula is based on the theoretical rate equation for the creep law of rocks. In this formula, the most important parameter to study thermal evolution of super-Earths is the activation volume \( (V^*) \), because \( V^* \) determines the dependence of the viscosity on pressure and the pressure in the mantle can be increased by orders of magnitude as the planetary mass increases. The activation volume is related to atomic volume, but the exact values under extremely high pressure are not well determined. In the nominal case, we use \( V^* = 10 \times 10^{-6} \text{ m}^3\text{ mol}^{-1} \), but we also test a smaller value of \( V^* = 3 \times 10^{-6} \text{ m}^3\text{ mol}^{-1} \). In Section 3.4, we will discuss how the conclusion in the present paper depends on a formula for the viscosity.

2.6. Release of Gravitational Energy by Thermal Contraction

Although the thermal contraction is negligible for the physical radius, the gravitational energy released by the thermal contraction cannot be neglected (a little about 50 kJ kg\(^{-1}\) for a 100 K change of the core in an Earth-Mass planet). In our model, the released energy is regarded as an increase in the specific heat of the core (Yukutake 2000):

\[ \Delta C_p = \frac{\alpha P}{\rho}. \]  \tag{16} 

The gravitational energy released by thermal contraction is more effective in deeper regions. For an Earth-Mass planet, \( \Delta C_p/C_p \) is as large as 50% at the CMB in our calculation.

2.7. Initial Conditions

Initial temperature distribution in the mantle is determined by the procedure following Yukutake (2000), as illustrated in Figure 2: (1) an adiabat is drawn from the bottom of the surface-boundary layer with 1500 K down to the top of the boundary layer at the CMB (the obtained tentative temperature is denoted by \( T_2 \)), assuming efficient thermal convection;

![Figure 2. Schematic diagram of the procedure to obtain initial temperature distributions. For more details, see the text.](image-url)

(2) the initial temperature at the bottom of the CMB is assumed to be \( T_{\text{CMB}} = T_2 + \Delta T_{\text{CMB}} \), where \( \Delta T_{\text{CMB}} \) is determined by step 6; (3) the adiabat is drawn from the CMB with temperature \( T_{\text{CMB}} \) to the surface; (4) the initial temperature distribution in the mantle is given by the average of the two adiabats obtained by steps 1 and 3; (5) core temperature is determined by the procedure given in Section 2.2 with \( T_{\text{CMB}} \), and (6) the amount of \( \Delta T_{\text{CMB}} \) (\( \sim 1000 \text{ K} \)) is determined by the requirement that the predicted surface heat flux and inner core radius for an Earth-Mass planet are comparable to the observed values for the present Earth. We use this value for all the cases with various planetary masses. Note that the temperature distribution is quickly relaxed to an equilibrium distribution, as long as we use the initial conditions created by the above procedures.

2.8. Simulation Parameters

We summarize parameters for the “nominal” case:

1. Boundary conditions
   (a) surface temperature: \( T_{\text{surf}} = 300 \text{ K} \),
   (b) a mass ratio between mantle and core: \( \zeta_{\text{m/c}} = 7:3 \), and
   (c) the CMB is a barrier for convection, while convection penetrates the upper/lower mantle boundary.

2. Initial conditions
   (a) impurity fraction: \( x_{\text{0S}} = 0.1 \).

3. Rheological conditions
   (a) activation volume: \( V^* = 10 \times 10^{-6} \text{ m}^3\text{ mol}^{-1} \).

We first investigate planetary mass dependence for planets with the above nominal parameters. For Earth-mass planets, we adopt \( \Delta T_{\text{CMB}} = 1000 \text{ K} \) in most runs, because the nominal case with \( \Delta T_{\text{CMB}} = 1000 \text{ K} \) reproduces the present Earth. We also systematically study dependences of the results on \( \Delta T_{\text{CMB}} \), because \( \Delta T_{\text{CMB}} \) is not well determined. In some runs, the upper/lower mantle boundary is treated as a barrier to convection. We also carry out calculations with different values of \( T_{\text{surf}}, \zeta_{\text{m/c}}, \) and \( x_{\text{0S}} \) to reproduce the results that are consistent with the current magnetic fields of Mercury, Venus, and Mars in Appendix B (we do not systematically survey the dependences on these parameters).

2.9. Definition of the Lifetime of a Planetary Intrinsic Magnetic Field

To drive dynamo action, a liquid metallic core must be in an active convection state. Following Stevenson et al. (1983), we adopt the threshold heat flux in the core for generation of
A dynamo action (conducted heat flux along the core adiabatic thermal structure) as

\[ F_{\text{crit}} = k_c \left( \frac{\partial T_{\text{CMB}}}{\partial r} \right)_s = k_c \frac{\rho g Y_G}{K_s} T_{\text{CMB}}. \]  

We define the lifetime of a magnetic field as a period during which the core heat flux exceeds the threshold value.

3. NUMERICAL RESULTS

3.1. Thermal Evolution of the Earth

We now show the evolution of temperature distribution calculated by the procedures in Section 2. Figure 3(a) shows the evolution of internal temperature distribution in the case of \( \Delta T_{\text{CMB}} = 1000 \) K for an Earth-mass planet with \( \Delta T_{\text{CMB}} = 1000 \) K with \( 1 M_\oplus \). The planet radius is 6385 km and the core radius is 3375 km. The surface heat flux declines to \(-0.08 \) W/m\(^2\) and the inner core grows to 1200 km after 4.5 Gyr, which is nearly equal to the present observed value of the inner core radius of the Earth. The core heat flux monotonically decreases with time, but its time derivative discontinuously changes at the initiation of the inner core at around 2.2 Gyr. The solid curve represents the threshold flux \( (F_{\text{cond}}) \) to maintain dynamo activity in the outer core (Equation (17)).

(A color version of this figure is available in the online journal.)

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(A color version of this figure is available in the online journal.)

Some paleomagnetism data suggest that the magnetic field of the Earth has been enhanced to its present level of \(-2 \) Gyr (e.g., Hale 1987). It might be due to formation of the inner core, because nucleation of the inner core provides an additional heat source. Paleomagnetism data may include large uncertainty. If more detailed data are provided, they will constrain the condition of generation of the intrinsic magnetic field.

Figures 4 and 5 show the results of two-layer convection. In this calculation, we set the upper/lower-mantle boundary as a barrier to convection. The other boundary conditions and the model parameters are the same as those in the nominal case. The mixing length is shorter in the entire region of the mantle, and cooling is slower than in the one-layer case. As is shown in Figure 4, if we adopt initial \( \Delta T_{\text{CMB}} = 1000 \) K, as in the case of one-layer convection, the inner core cannot grow to 1200 km because of the low heat transfer efficiency of layered convection, and core temperature is somewhat higher than that obtained by one-layer convection. We also carried out a calculation with initial \( \Delta T_{\text{CMB}} = 800 \) K. Figure 5(a) shows the evolution of the
thermal profile. A thermal-boundary layer at the upper/lower-mantle boundary is clearly established. Because the lower initial $\Delta T_{\text{CMB}}$ is compensated with the inefficient two-layer convection, the evolution of the heat flux through CMB ($F_{\text{CMB}}$) and the lifetime of the magnetic field (11 Gyr in this case) are similar to those in the one-layer convection case.

If two-layer convection is assumed only in the Archean and Hadean ($t < 2$ Gyr), with the same initial $\Delta T_{\text{CMB}}$ (1000 K) the surface heat flux at 4.5 Gyr is $F_{\text{surf}} \sim 0.12$ Wm$^{-2}$, which is somewhat higher than the observed value ($\sim 0.09$ Wm$^{-2}$) because the thermal energy beneath the upper/lower-mantle boundary was stored until 2 Gyr and then

Figure 4. Same figure as Figure 3 but using a two-layer mantle convection model with $\Delta T_{\text{CMB}} = 1000$ K. (A color version of this figure is available in the online journal.)

Figure 5. Same as Figure 3, except that a two-layer mantle convection model with $\Delta T_{\text{CMB}} = 800$ K is considered instead of a one-layer model with $\Delta T_{\text{CMB}} = 1000$ K. (A color version of this figure is available in the online journal.)
supplied to the upper mantle after 2 Gyr. However, the evolution of $F_{\text{CMB}}$ is not so different between one- and two-layer convection. The lifetime of the magnetic field is about 12 Gyr.

Thus, if we tune the initial $\Delta T_{\text{CMB}}$ with the present observed values of $F_{\text{surf}}$ and $R_c$, the expected lifetime of the magnetic field is not affected by the mode (one-layer or two-layer) of mantle convection.

### 3.2. Thermal Evolution of Mercury, Venus, and Mars

The existence of the magnetic field for Mercury and the early decay of the magnetic field for Venus and Mars were addressed by Stevenson et al. (1983), using the box model with parameter values such as surface temperature and a mantle-core mass ratio different from those in the nominal case. The validity of these parameter values is discussed in Appendix B. Adopting the same parameter values as Stevenson et al. (1983), we have performed simulations for Mercury, Venus, and Mars with our model. As discussed in Appendix B, our model produces results that are not inconsistent with the magnetic activity of Mercury, Venus, and Mars.

### 3.3. Thermal Evolution of Super-Earths

For super-Earths, we use the nominal parameters (the surface temperature is 300 K and the mantle/core mass ratio is 7:3), assuming that their orbits are nearly circular and in habitable zones. We also assume one-layer convection throughout the mantle. A detailed study on the effects of phase transitions is left for future research. Figure 6(a) shows the evolution of temperature distribution for a planet with mass $M_p = 5 M_\oplus$.

Compared with the case of $M_p = 1 M_\oplus$ in Figure 3(a), a thicker thermal boundary layer is established on the CMB within the first few Gyr, because the viscosity of the bottom of the mantle is higher. The increase in the viscosity due to higher pressure dominates the decrease due to higher temperature (Equation (15)). Thus, $F_{\text{CMB}}$ is lower than the case of $M_p = 1 M_\oplus$ (Figures 6(b) and 3(b)). On the other hand, effective heat capacity rapidly increases with $M_p$. Figure 15 in Appendix C shows that for fixed $T_{\text{CMB}}$, thermal energy is $E_{\text{th}} \propto M_p^2$, while the core surface area $S_{\text{core}}$ increases with $M_p$ only weakly ($S_{\text{core}} \propto M_p$). Therefore, the core for higher $M_p$ cools much more slowly. It is shown that $R_c$ does not grow at all for 10 Gyr.

On the other hand, $F_{\text{surf}}$ is not so different from that in the case of $M_p = 1 M_\oplus$. The heat bath of surface heat flow comprises radiogenic elements in the mantle and is proportional to $M_p$. To balance heat generation and cooling, $F_{\text{surf}}$ should be proportional to $M_p^{2/3}$, provided $R_p \propto M_p^{1/3}$. Thereby, $F_{\text{surf}}$ changes by a factor of only $5^{1/3} \approx 1.7$.

As discussed above, thermal evolution of super-Earths differs from that of Earth-mass planets in many aspects. Here, we focus on the evolution of the heat flux through the CMB, $F_{\text{CMB}}$, and the inner core radius, $R_{ic}$, in order to study magnetic activity of super-Earths. Figure 7 shows the evolution of $F_{\text{CMB}}$ (left column) and $R_{ic}$ (right column) with various initial $\Delta T_{\text{CMB}}$ for the case of (a) $M_p = 1 M_\oplus$, (b) $2 M_\oplus$, (c) $5 M_\oplus$, and (d) $10 M_\oplus$.

Solid, dotted, dashed, and long-dashed lines represent the results with initial $\Delta T_{\text{CMB}} = 1000$ K, 2000 K, 5000 K, and 10,000 K, respectively. In all cases, $V^* = 10^9 \times 10^{-9}$ m$^3$ mol$^{-1}$.

The results in the left column show that $F_{\text{CMB}}$ is generally higher for higher initial $\Delta T_{\text{CMB}}$. The dependence is more pronounced for relatively large $M_p$ cases. For $M_p = 1 M_\oplus$, the dependence is very weak. We found the dependence is also very weak for $M_p < 1 M_\oplus$. For $M_p \lesssim 1 M_\oplus$, the temperature dependence of the mantle viscosity (Equation (15)) dominates the pressure dependence. Then, $F_{\text{CMB}}$ is high when the core temperature is high, and $F_{\text{CMB}}$ declines as the core cools. Thus, the heat flux is self-regulated to be quickly relaxed independent of the initial values. On the other hand, as will be shown later, when $\Delta T_{\text{CMB}} \lesssim 1000$ (for $M_p/M_\oplus$), the pressure dependence is more effective. Then, the self-regulation does not work and the dependence of $F_{\text{CMB}}$ on initial $\Delta T_{\text{CMB}}$ is retained for more than 20 Gyr. The threshold flux for driving dynamo action is marked by a black line in each case. The decline of the threshold value is due to a decrease in core surface temperature (Equation (17)). The duration for $F_{\text{CMB}} > F_{\text{cM}}$ determines the lifetime of magnetic field generation.

Papuc & Davies (2008) obtained $F_{\text{CMB}} \propto M_p^{2/3}$, whereas our results show $F_{\text{CMB}} \propto M_p$, provided that $\Delta T_{\text{CMB}}$ is sufficiently high. The difference may come from the assignment of specific heat from the core. Papuc & Davies (2008) assumed constant specific heat from the core. $C_p = 1000$ J kg$^{-1}$ K$^{-1}$, for all sizes of planets. As we discussed in Section 2.6, however, thermal contraction results in an increase in the effective $C_p$, and the effect is more pronounced for larger $M_p$. In our calculations that include this effect, the core tends to cool less efficiently, and the dependence of $F_{\text{CMB}}$ on $M_p$ is stronger than that obtained by Papuc & Davies (2008).

The right column of Figure 7 shows the growth of inner solid cores for $\Delta T_{\text{CMB}} = 1000, 2000, 5000$, and 10,000 K. In the case of $M_p = 1 M_\oplus$, an inner core is nucleated at 2–3 Gyr, almost independent of initial $\Delta T_{\text{CMB}}$, because core cooling is self-regulated. For $M_p = 2 M_\oplus$, inner core growth depends on $\Delta T_{\text{CMB}}$ for $\Delta T_{\text{CMB}} \gtrsim 2000$ K. For such high $\Delta T_{\text{CMB}}$, because the core has larger thermal energy initially and the heat flux is not self-regulated, it takes more time for the core temperature to fall below the nucleation temperature. For $M_p \gtrsim 5 M_\oplus$, the core hardly cools for 20 Gyr, and the inner core does not grow from the initial state. In these cases, the inner core size is determined by a relationship between the adiabatic curve and the melting curve of iron. The inner core of super-Earths ($M_p > M_\oplus$) has never nucleated for $\Delta T_{\text{CMB}} = 10,000$ K. For massive planets, the increase in viscosity due to higher pressure is overcome only by a very high initial temperature. The high $\Delta T_{\text{CMB}}$ also delays nucleation of the inner solid core. As a result, there is a trade-off between the heat flux and inner core growth through the relationship between the melting point of the iron core and the temperature and pressure dependence of mantle viscosity.

Figure 8 shows the dependence of the evolution of $F_{\text{CMB}}$ on $M_p$ for fixed values of $\Delta T_{\text{CMB}}$. For $\Delta T_{\text{CMB}} = 1000$ K, we have already mentioned that $F_{\text{CMB}}$ was lower for $M_p = 5 M_\oplus$ than for $M_p = M_\oplus$ (Figures 6(b) and 3(b)), because the increase in viscosity due to higher pressure dominates the decrease due to higher temperature for $M_p = 5 M_\oplus$. This trend is clearly shown in Figure 8(a).

However, this is not always the case. If the core temperature is high enough (i.e., $\Delta T_{\text{CMB}}$ is high enough), or if pressure is low enough ($M_p$ is small enough), the viscosity should decrease with an increase in $M_p$ due to the temperature effect. For $\Delta T_{\text{CMB}} = 10,000$ K (Figure 8(d)), $F_{\text{CMB}}$ is approximately proportional to $M_p$. Even for $\Delta T_{\text{CMB}} = 1000$ K, $F_{\text{CMB}}$ increases with $M_p$ for a low-mass regime ($M_p < 1 M_\oplus$). Thus, $F_{\text{CMB}}$ has a peak at some value of $M_p$ for a given value of $\Delta T_{\text{CMB}}$. Figures 8(b) and (c) show that the critical planet mass ($M_{pc}$) at which $F_{\text{CMB}}$ achieves the maximum value is $2 M_\oplus$ for
\[ \Delta T_{\text{CMB}} = 2000 \text{ K and } 5 M_\oplus \text{ for } 5000 \text{ K. We empirically found that} 
\]
\[ M_{p,c} \simeq \frac{\Delta T_{\text{CMB}}}{1000} M_\oplus. \quad (18) \]

Because the mantle viscosity depends on the activation volume, \( V^* \) (Equation (15)), and the values of \( V^* \) may have uncertainty at high pressure, we also performed calculations with a smaller value of \( V^* \). Figure 9 shows the results with \( V^* = 3 \times 10^{-6} \text{ m}^3\text{ mol}^{-1} \). Due to the weakened pressure effect, \( M_{p,c} \) is increased by a factor of several Earth mass. In Figure 9, the viscosity is artificially increased (Equation (15)) by a factor of 6000 in order to compensate for the smaller value of \( V^* \) and reproduce Earth’s observed values. Note that the artificial increase does not affect \( M_{p,c} \).

The critical planetary mass \( M_{p,c} \) is approximately derived by the \( M_p \) dependence of the mantle viscosity at the CMB. Because we empirically found that \( P_{\text{CMB}} \sim (M_p/M_\oplus)P_{\text{CMB}} \) and \( T_{\text{CMB}} \sim 5\Delta T_{\text{CMB}} \), Equation (15) is reduced to

\[ \eta_{\text{CMB}}(M_p, T_{\text{CMB}}) \propto \exp \left( \frac{E^* + (M_p/M_\oplus)P_{\text{CMB}} V^*}{nRT_{\text{CMB}}} \right). \quad (19) \]

When the argument of the exponential is larger than unity, the viscosity rapidly increases with \( M_p \) to depress \( F_{\text{CMB}} \), because \( F_{\text{CMB}} \propto \eta^{-1/3} \). For \( M_p < M_{p,c} \), we found that \( F_{\text{CMB}} \propto M_p \). When the argument exceeds some critical value (\( C > 1 \)), the viscosity enhancement eventually overwhelms the factors for the positive \( M_p \) dependence of \( F_{\text{CMB}} \). Thus, \( M_{p,c} \) is given by the value of \( M_p \), with which the argument of the exponential is \( \simeq C \),

\[ M_{p,c} \simeq \frac{5nCR\Delta T_{\text{CMB}} - E^*}{V^*} \frac{M_\oplus}{P_{\text{CMB}}} \simeq \frac{5nCR\Delta T_{\text{CMB}}}{V^*} \frac{M_\oplus}{P_{\text{CMB}}} \]

\[ \sim \frac{C\Delta T_{\text{CMB}}}{10000 \text{ K}} \left( \frac{V^*}{10 \times 10^{-6} \text{ m}^3\text{ mol}^{-1}} \right)^{-1} M_\oplus, \quad (20) \]

which explains the dependences on \( \Delta T_{\text{CMB}} \) and \( V^* \) that we found numerically. (If we adopt \( C \sim 10 \), the numerical factor is also explained.)

### 3.4. The Lifetime of Intrinsic Magnetic Fields

The lifetime of the intrinsic magnetic fields is calculated for \( \Delta T_{\text{CMB}} = 1000, 2000, 5000, \text{ and } 10,000 \text{ K with a fixed value of } V^* = 10 \times 10^{-6} \text{ m}^3\text{ mol}^{-1} \). The results are summarized in Figure 10. It is clearly shown that the lifetime declines for \( M_p \gtrsim M_{p,c} \) by the increase in mantle viscosity due to the pressure effect that we discussed in detail in the previous subsection. The results for \( V^* = 3 \times 10^{-6} \text{ m}^3\text{ mol}^{-1} \) show a similar property.

In Figure 10, the lifetime weakly increases with \( M_p \) for \( M_p \lesssim M_{p,c} \). The dependence is explained as follows. The lifetime is approximately given by \( \tau_{\text{life}} \sim E_{\text{th}}/(S_{\text{CMB}}F_{\text{CMB}}) \), where \( E_{\text{th}} \) is the thermal energy of the core and \( S_{\text{CMB}} \) is the surface area of the core. According to our calculation, \( S_{\text{CMB}} \propto M_p^{5/2} \) rather than \( M_p^{2/3} \) due to self-compression. Figures 7 and 13 show that \( F_{\text{CMB}} \propto M_p \) and \( E_{\text{th}} \propto M_p^{3/2} \) for a fixed \( T_{\text{CMB}} \). As we mentioned in Section 3.3, \( T_{\text{CMB}} \sim 5\Delta T_{\text{CMB}} \). Thus, for a fixed \( \Delta T_{\text{CMB}} \), it is predicted that \( \tau_{\text{life}} \sim E_{\text{th}}/(S_{\text{CMB}}F_{\text{CMB}}) \propto M_p^{-1/2-1} = M_p^{1/2} \), which is consistent with the numerical results in Figure 10.

When \( M_p > M_{p,c} \), higher mantle viscosity due to the effect of higher pressure significantly depresses heat transfer at the bottom of the mantle. The suppressed heat flux cannot maintain the vigorous core convection. As a result, the magnetic field lifetime is shorter for \( M_p > M_{p,c} \).

Figure 10 also shows that the lifetime for \( M_p \lesssim M_{p,c} \) does not depend on \( \Delta T_{\text{CMB}} \) at all. The initial temperature is high enough to overcome the pressure dependence, even for the case of \( \Delta T_{\text{CMB}} = 1000 \text{ K} \), resulting in the effective self-regulation of \( F_{\text{CMB}} \). As a result, the lifetime does not depend on the initial value of \( \Delta T_{\text{CMB}} \).
3.5. The Strength of Magnetic Fields

The strength of magnetic fields is as important as their lifetime when discussing habitability of planets. Here, we evaluate the strength of the magnetic fields using the scaling law derived by Christensen et al. (2009). For planets with sufficiently rapid spins, Christensen et al. (2009) derived the magnetic strength at the core surface as

$$B_c \sim 0.5 \mu_0 \frac{1}{12} \frac{1}{\rho_c^{1/3}} F_{\text{conv}}^{1/3},$$

where $\rho_c$ is the average density of the core and $\mu_0$ is permeability. If the magnetic moment is dipole dominant and the dipole moment is $\propto 1/r^3$ (Gaidos et al. 2010), the strength of the magnetic dipole at the planetary surface is

$$B_{\text{surf}} = B_c \left( \frac{r_c}{r_p} \right)^3.$$

With $F_{\text{conv}} = F_{\text{CMB}} - F_{\text{cond}}$, we calculated $B_{\text{surf}}$ from our simulation results. Figure 11 shows the calculated $B_{\text{surf}}$ at $t = 5$ Gyr for various $M_p$ and $\Delta T_{\text{CMB}}$. The strength monotonically increases if the initial $\Delta T_{\text{CMB}}$ is sufficiently high. The relatively weak dependence ($B \propto M_p^{1/3}$) comes from the adopted scaling law, $B \propto F_{\text{conv}}$, and the numerically obtained relationship, $F_{\text{CMB}} \propto M_p$. If $\Delta T_{\text{CMB}}$ is not high enough, the pressure effect is dominant and the strength is significantly suppressed for $M_p \gtrsim M_{p,c}$. 

(A color version of this figure is available in the online journal.)
4. CONCLUSION AND DISCUSSION

We have developed a numerical model to simulate thermal evolution of various-mass terrestrial planets in habitable zones. The density distribution of the planetary interior is calculated by the Vinet EOS, taking into account pressure dependence. Using the interior structure model, we calculate heat transfer through the mantle, using the astrophysical MLT modified for mantle convection. The modified MLT is easily applied to multilayer convection, which may be the dominant convection mode in super-Earths. We have calibrated the modified MLT with the conventional parameterized convection model and the BLT in simple one-layer convection cases.

With nominal parameters of surface temperature \( T_{\text{surf}} = 300 \text{ K} \), a mantle-core mass ratio of \( \zeta_{m/c} = 7:3 \), initial core impurity \( x_{\text{CS}} \) of 10 wt%, and an initial temperature gap at CMB \( \Delta T_{\text{CMB}} = 1000 \text{ K} \), our model for \( M = 1M_\oplus \) reproduces the surface heat flow and inner core radius of the present Earth. With different parameter values suitable for Mercury, Venus, and Mars, our model also reproduces results that are consistent with present magnetic activity on these planets.

With this model, we calculated thermal evolution of terrestrial planets with mass \( M_p = 0.1-10 M_\oplus \) in habitable zones, using the nominal parameters, to study the lifetime of the intrinsic magnetic field, which is one of the most important factors for the planets to be habitable. We found from the numerical calculations that the lifetime is maximized at

\[
M_{p,c} \sim \frac{\Delta T_{\text{CMB}}}{1000 \text{ K}} \left( \frac{V^*}{10 \times 10^{-6} \text{ m}^3 \text{ mol}^{-1}} \right)^{-1} M_\oplus, \tag{22}
\]

where \( V^* \) is the activation volume of mantle material. Planets with smaller masses cool more rapidly, so they cannot maintain core heat flux to generate a dynamo long enough. For \( M_p > M_{p,c} \), the rapid increase in mantle viscosity caused by high pressure significantly depresses heat transfer throughout the mantle and in the core. As a result, the dynamo cannot last long. Although the temperature effect tends to decrease the mantle viscosity as planetary mass becomes large, the pressure effect that increases viscosity overwhelms the temperature effect for \( M_p > M_{p,c} \). With the numerically obtained empirical relationship \( T_{\text{CMB}} \sim 5 \Delta T_{\text{CMB}} \), we can analytically derive Equation (22) from the Arrhenius-type formula for mantle viscosity that we adopt (Equation (15)).

We found that while the lifetime of magnetic fields does not depend on \( \Delta T_{\text{CMB}} \) for \( M_p < M_{p,c} \), it sensitively depends on \( \Delta T_{\text{CMB}} \) for \( M_p > M_{p,c} \) because \( M_{p,c} \propto \Delta T_{\text{CMB}} \) (Equation (22)). The initial \( \Delta T_{\text{CMB}} \), that is, the initial temperature profile of the planetary interior, is one of the most uncertain parameters, because it highly depends on the processes of planetary formation and differentiation of the planetary interior. As is shown by smoothed particle hydrodynamics (SPH) simulations, if a planet undergoes giant impacts, its metallic core is heated as high as several tens of thousands K for \( M_p \sim 1 M_\oplus \) (Canup 2004). On the other hand, if a planet accretes from small planetesimals without giant impacts, the initial temperature profile is determined by the balance between gravitational energy buried by planetesimals and thermal transfer efficiency through the rocky mantle. The process includes crystallization of a magma ocean and depends on the mechanical property of the molten mantle (Abe & Matsui 1986; Zahnle et al. 1988; Senshu et al. 2002). Thus, to evaluate the lifetime of magnetic fields, in particular for super-Earths that are likely to satisfy \( M_p > M_{p,c} \), detailed analyses of accretion and early thermal evolution of terrestrial planets are needed.

It is also found that a higher initial temperature profile delays inner core nucleation. For super-Earths, in order to maintain a
Figure 9. Same as Figure 7, except $V^* = 3 \times 10^{-6}$ m$^3$ mol$^{-1}$.
(A color version of this figure is available in the online journal.)

Figure 10. Lifetime of magnetic fields as a function of planetary mass ($M_p$) with various $\Delta T_{CMB}$ for (a) $V^* = 10 \times 10^{-6}$ m$^3$ mol$^{-1}$ and (b) $V^* = 3 \times 10^{-6}$ m$^3$ mol$^{-1}$. (A color version of this figure is available in the online journal.)
magnetic field more than 10 Gyr, the initial temperature has to be high enough to overwhelm the pressure dependence. However, in that case, the temperature of the core center never reaches its condensation temperature and the inner core cannot grow. Some geo-dynamo simulations suggest that the presence of the inner core stabilizes the dipole moment of the geomagnetic field (Sakuraba & Kono 1999). It is also suggested that because thermally driven convection is not sufficient to drive dynamo action against the ohmic dissipation within the core of Earth (Gubbins et al. 2003), the compositional convection induced by light elements released to the outer core by solidification of the inner core plays an essential role in dynamo generation (Stevenson et al. 1983; Gubbins et al. 2004).

Because our planet does not have an intrinsic magnetic field, its atmosphere could be directly detected in the future by the polarization observation of the photon from transiting planets or observation of H\textsubscript{2} trapped by the magnetic fields. Another possibility for detecting planetary magnetic fields is, although it is indirect, observation of the composition of planetary atmosphere or atmospheric tail. If the planet has an intrinsic magnetic field, its atmosphere could keep H\textsubscript{2}O molecules for a long period. Venus may have lost H\textsubscript{2}O molecules over a short timescale (Bullock & Grinspoon 2001). Thus, if water-series molecules, such as H\textsubscript{2}O, H\textsubscript{3}O, and HO, are detected in the planetary atmosphere, it would indicate the existence of an intrinsic magnetic field, although super-Earths might be able to sustain the H\textsubscript{2}O molecules in the atmosphere with their high gravity even without the protection of magnetic fields.

The existence of magnetic fields on extrasolar planets could be directly detected in the future by the polarization observation of the photon from transiting planets or observation of H\textsubscript{2} trapped by the magnetic fields. Another possibility for detecting planetary magnetic fields is, although it is indirect, observation of the composition of planetary atmosphere or atmospheric tail. If the planet has an intrinsic magnetic field, its atmosphere could keep H\textsubscript{2}O molecules for a long period. Venus may have lost H\textsubscript{2}O molecules over a short timescale (Bullock & Grinspoon 2001). Thus, if water-series molecules, such as H\textsubscript{2}O, H\textsubscript{3}O, and HO, are detected in the planetary atmosphere, it would indicate the existence of an intrinsic magnetic field, although super-Earths might be able to sustain the H\textsubscript{2}O molecules in the atmosphere with their high gravity even without the protection of magnetic fields.

We need to elaborate our thermal evolution model by considering details of the mantle convection mode that is affected by phase transition between \(\gamma\)-spinel and perovskite (Christensen & Yuen 1985) at the upper/lower mantle. We also should take into account further mineral transitions suggested by ab initio calculations (Umemoto et al. 2006) that may appear in super-Earths, because they may affect internal density structure and the mantle convection mode.

The abrupt enhancement in mantle viscosity due to the pressure effect relies on the Arrhenius-type formula for mantle viscosity that we adopt here. The critical mass beyond which the pressure effect dominates is inversely proportional to the activation volume (Equation (22)). Thus, detailed rheological properties affect habitability of super-Earths. The values of the activation volume are not clear at such high pressure, as in the deep mantle in super-Earths. The mechanism to inhibit dynamo activity in super-Earths proposed by Gaidos et al. (2010) also depends on high-pressure material properties (melting and adiabatic curves), which also need to be confirmed. These provide new motivations for high-pressure experiments and first-principle simulations. Super-Earths provide valuable links between astronomy and high-pressure material science.

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APPENDIX A

COMPARISON AMONG THE Nu–Ra RELATIONSHIP, THERMAL-BOUNDARY LAYER, AND MIXING-LENGTH THEORY MODELS

In the evaluation of thermal transfer of mantle convection, we compare the modified mixing-length theory (MLT; Sasaki & Nakazawa 1986; Abe 1995) with the conventional parameterized convection model (PCM; e.g., Sharpe & Peltier 1979) and the commonly used thermal-boundary layer model (BLT; e.g., Stevenson et al. 1983). The original MLT (e.g., Vitense 1963; Spiegel 1963) is often used in the thermal transfer within the stellar interior to simulate the stellar evolution. Sasaki & Nakazawa (1986) modified the MLT for very low Reynolds-number convection in which the vertical flow is characterized by the Stokes velocity, which is determined by a balance between buoyant force and resident force of viscosity rather than by free-fall velocity. In the modified version, a distance (\(D\)) from the closest barrier, such as the CMB or the top of the mantle layer, is adopted for the mixing length \(\ell\), while in the original theory, the density scale height is usually adopted for \(\ell\).

The PCM uses the empirical Nu–Ra relationship,

\[
\text{Nu} = \zeta \left( \frac{Ra}{Ra_c} \right)^{1/3}, \tag{A1}
\]

where the Nusselt number represents the ratio between total heat flux and heat flux only due to conduction without convection,

\[
\text{Nu} = \frac{F_{\text{total}}}{F_{\text{cond}}}. \tag{A2}
\]
The thickness of the convective region, respectively, and $Ra$ is the critical Rayleigh number ($\sim 650$) for thermal convection. Because when $Ra \sim Ra_c$, $Nu$ must be $\sim 1$ and, $\zeta$ is $O(1)$. Sotin et al. (1999) derived $\zeta \sim 1.5–2.0$ through a three-dimensional fluid dynamical simulation, although the value of $\zeta$ is somewhat lower in the high-$Ra$ region. We adopt $\zeta = 1.7$ here. From Equations (A1) to (A3), the total heat flux through a fluid layer is represented by the Rayleigh number as

$$F_{\text{total}} = \zeta \left( \frac{Ra}{Ra_c} \right)^{1/3} F_{\text{cond}}. \quad (A5)$$

This model is very simple, but $Ra$ is a “mean” value of the whole mantle and is difficult to evaluate for a real mantle in which viscosity changes by orders of magnitude throughout the mantle. In particular, it may not have enough resolution to evaluate $F_{\text{CMB}}$, which we are concerned with in the present paper.

In the BLT, the heat flux is evaluated in the boundary layer. The thickness of the boundary layer is estimated with the assumption that the layer is marginally stable against thermal instability. Then, the local Rayleigh number of the thermal-boundary layer ($Ra_t$) is nearly equal to the critical Rayleigh number for thermal instability, that is,

$$Ra_t = \left( \frac{\rho g \alpha}{\kappa \eta} \right) \delta^3 \Delta T_{\text{TB}} \sim Ra_c, \quad (A6)$$

where $\delta$ is the thickness of the thermal-boundary layer and $\Delta T_{\text{TB}}$ is the temperature difference between the bottom and the top of the boundary layer, and subscript “$t$” denotes the values of the thermal-boundary layer. Thus, the heat flux through the layer is calculated as

$$F_{\text{total}} = k \frac{\Delta T_{\text{TB}}}{\delta}, \quad (A7)$$

where $\delta$ is calculated as

$$\delta = \zeta' \left[ \left( \frac{\kappa \eta}{\rho g \alpha} \right) \frac{Ra_c}{\Delta T_{\text{TB}}} \right]^{1/3}, \quad (A8)$$

and the factor $\zeta' \sim O(1)$ is determined as follows. If $\kappa, \eta, \rho, g$, and $\alpha$ are constant, $\Delta T_{\text{TB}} \sim \Delta T/2$, so that

$$F_{\text{total}} = \frac{1}{2^{4/3} \zeta'} \left( \frac{Ra}{Ra_c} \right)^{1/3} F_{\text{cond}}. \quad (A9)$$

To be consistent with the three-dimensional fluid dynamical simulation by Sotin et al. (1999), we set $2^{4/3} \zeta' = 1/1.7$, that is, $\zeta' = 0.23$.

Because the heat flux is expressed by quantities only in the thermal-boundary layer (Equations (A7) and (A8)), which is localized in the mantle, the BLT has better resolution than the PCM, in particular for the evaluation of $F_{\text{CMB}}$. However, because the values of viscosity change by orders of magnitude even in the thin thermal-boundary layer, it is not clear which value has to be chosen as a representative value of the viscosity in Equation (A8). For the terrestrial planets in our solar system, observational data can be used to constrain the uncertainty.

Because the modified MLT uses local values of physical quantities (Equation (13)), it quantitatively agrees with the BLT for a wide range of parameters, as shown below. There is no uncertainty for choice of a representative value of viscosity in the MLT, but the choice of mixing length has uncertainty. For calculation of thermal evolution of the Earth, it is proposed that a distance ($D$) from the closest barrier, such as the CMB or the top of the mantle layer, is appropriate for $\ell$ ($\ell$ Sasaki & Nakazawa 1986; Abe 1995; Senshu et al. 2002; Kimura et al. 2009, and references therein). Through comparison with the calibrated PCM and BLT, we adopt $\ell = 0.82 D$, as shown below.

To compare these models, we calculate the heat flux in the case of radially constant $\eta$ with the individual calibrated models. Internal heat generation due to radioactive elements is neglected. Figure 12 shows the heat flux at the base of the mantle as a function of $Ra$, obtained by each model. The values are normalized by $F_{\text{cond}}$, that is, the Nusselt number. Although the MLT does not assume the relationship of $Nu \propto Ra^{1/3}$, it produces the relationship. To match the absolute values, we set $\ell = 0.82 D$. The maximum value of $\ell$ is proportional to $d$, and heat flux is proportional to $\ell^4$ (Equation (14)). The sensitive dependence on $d$ is canceled, resulting in the rather weak dependence $Nu \propto Ra^{1/3} \propto d$, because we found that $[(\partial T/\partial r) - (\partial T/\partial r)_h]$ decreases with an increase in $\ell$ (Equation (14)). The analytical argument for this is found in Abe (1995).

We also examined a case in which the viscosity is strongly temperature dependent:

$$\eta(T) = \eta_0 \exp[\log(\eta_1/\eta_0)(1 - T)], \quad (A10)$$

where $\eta_0$ and $\eta_1$ are the viscosity at the top ($T = 0$) and the bottom ($T = 1$) of the convective region. Figure 13 shows the Nusselt number obtained by the PCM BLT, and MLT as a function of $Ra$. In the PCM, $Ra$ is a mean value for the whole mantle. The representative viscosity is evaluated using the average temperature of the mantle, that is, $T = 0.5$ if the mantle maintains thermal equilibrium because the PCM.

![Figure 12](image_url)
Figure 13. Temporal-averaged Nusselt numbers obtained from the PCM, BLT, and MLT models. The viscosity is changed from the bottom to the top with the ranges of (a) $\Delta \eta = 10$, (b) $10^2$, (c) $10^3$, and (d) $10^4$, respectively.

(A color version of this figure is available in the online journal.)

assumes a constant heat flux throughout the mantle. In the BLT and MLT, the heat flux is evaluated by local quantities. The BLT and MLT produce the same heat flux within 1% in all cases, while the results from the PCM deviate from those from the BLT and MLT for high Ra or high $\eta_0/\eta_1$. These results show that the MLT is as good as the BLT to calculate thermal evolution of terrestrial planets. Because the MLT is more easily applied to super-Earths that may have barriers to convection in their mantles, (Section 2.3), we adopt the MLT.

APPENDIX B

ON THE MAGNETISM OF PLANETS IN THE SOLAR SYSTEM

In order to confirm the validity of our model, we show that our model produces thermal evolution for individual terrestrial planets in the solar system that is consistent with their current magnetic activity, with appropriate non-nominal parameter values, in a similar way to Stevenson et al. (1983). Currently, Earth and Mercury have self-generating magnetic fields induced by dynamo action, while Venus and Mars do not (although some parts of the Martian crust have had a remnant magnetic field in the past (Acuña et al. 1999)).

To apply our model to Mercury, Venus, and Mars, we need to use non-nominal parameter values.

1. Mercury. $\zeta_{m/c} = 3.7$ (a significantly large metallic core) and $T_{\text{surf}} = 440$ K. These are observed values. We also tested smaller values of $x_{\text{OS}} = 0.01, 0.05$ according to Stevenson et al. (1983). We also tested higher mantle viscosity than that used in Equation (15) by multiplying the viscosity increase factor $\Delta \eta = 100$.

2. Venus. $T_{\text{surf}} = 737$ K, while the nominal values are used for $\zeta_{m/c}$ and $x_{\text{OS}}$. We also tested higher mantle viscosity, as in the case of Mercury. Note that the two-layer convection model is used for Venus, because the spinel-perovskite transition also could work as a barrier to the Venusian mantle.

3. Mars. $T_{\text{surf}} = 210$ K. $\zeta_{m/c}$ is the nominal value and $x_{\text{OS}} = 0.1, 0.15, 0.2$. The standard formula, Equation (15), is used for mantle viscosity.

$\Delta \eta$ is the viscosity increase factor due to lack of water in the case of Mercury and Venus. It is suggested by experiment that dry rock has higher viscosity than that of hydrated rocks by a factor of 100. Thereby, we multiply $\Delta \eta = 100$ in the case of Venus and Mercury.

The lifetime of magnetic fields calculated by our model is shown in Figure 14. In order to be consistent with the current conditions on Mercury, Venus, and Mars, the lifetime must be longer than 4.5 Ga for Mercury and shorter than 4.5 Ga for Venus and Mars. Because Martian crust aged $\sim 4$ Gyr retains a paleomagnetic field, the lifetime of the Martian magnetic field may be longer than 0.5 Gyr.

Figure 14 shows that for Mercury, the lifetime is longer than 4.5 Ga for relatively small values of $x_{\text{OS}}$ ($\sim 0.01-0.05$), except for extremely small $\Delta T_{\text{CMB}} (< 200-300 \text{ K})$. The relatively long lifetime results from nucleation of the inner core due to a lower solidification temperature corresponding to small values of $x_{\text{OS}}$. If the nominal value of $x_{\text{OS}}$ is used, the lifetime is short. The
vication suggests that Venus lacks H2O. That may be due to the runaway greenhouse effect of H2O itself and consequent dissipation by UV dissociation and heating of the molecules. Because the melting temperature of mantle viscosity is lowered by H2O, relatively high mantle viscosity is more likely, although we do not know the exact values of Venus’ mantle viscosity.

The predicted lifetime of the magnetic field on Mars is longer than 1 Gyr but shorter than 4.5 Gyr, if initial $\Delta T_{\text{CMB}}$ is $\sim$10–500 K. If Mars has never undergone giant impacts, which cause significant heating of the metallic core, such low initial $\Delta T_{\text{CMB}}$ is likely.

Thus, with non-nominal parameters that reflect the distance from the Sun and accretion history of individual planets, our model can produce results that are consistent with the current terrestrial planets in the solar system. However, in order to clarify the intrinsic physics of the generation of magnetic fields of extrasolar terrestrial planets, we focus on the results with the nominal parameters ($T_{\text{surf}} = 300$ K, $\zeta_{m/c} = 7.3$, and $x_{0S} = 0.1$), which correspond to the parameters of terrestrial planets with the same compositions as the Earth in habitable zones.

APPENDIX C
ENERGY IN THE CORE

Figure 15 shows the thermal and gravitational energy, released latent heat, and their sum as a function of $T_{\text{CMB}}$ for the nominal cases with planetary mass $M = 1, 2, 5,$ and $10 M_\oplus$, which are calculated by the procedures in Section 2.2. We set each value at zero at the temperature at the initiation of inner core growth. As is shown in this figure, the loss of thermal energy occupies about one third of the total energy loss of the core for the case of $M = 1 M_\oplus$. Released latent heat corresponds to about one fifth of the total energy loss, which depends on $T_{\text{CMB}}$ because of the nonlinear density dependence of the melting temperature of metal.

The gradient of the total energy in Figure 15 corresponds to an effective specific heat of the core. The total heat capacity is twice that of the specific heat of thermal energy solely after inner core initiation ($T_{\text{CMB}} \approx 4100$ K), while their values converge as temperature decreases. This is because the impurity concentration increases with a temperature decrease in the outer core. Inner core growth is moderated by the depression of the melting temperature of the outer core due to the concentration of impurities in the outer core. Released gravitational energy and latent heat become smaller than thermal energy as the temperature decreases. Note that the gravitational energy released by the thermal contraction of the core also works as resistance to cooling of the core (see Section 2.2).

The ratio of gravitational energy, latent heat, and thermal energy varies by planetary mass. Thermal energy is more dominant than other energies for more massive planets. This means that the gravitational energy and latent heat are not the main energy sources driving dynamo action within cores of massive super-Earths. This is mainly because of the change in slope of the adiabatic curve within the core. The higher gravity causes a steeper adiabatic thermal slope, and the core possesses large amounts of thermal energy inside it for the case of massive planets. This is also the reason why the effective specific heat of the core is increased as planetary mass increases.
Figure 15. Individual core energies as a function of $T_{CMB}$ for the nominal case with $M = 1, 2, 5,$ and $10 \, M_\oplus$, and $x_{\text{th}} = 0.1$. Dashed, dotted, and dot-dashed curves represent gravitational energy, latent heat, and thermal energy, respectively. The solid curve shows the total energy. The gradient of total energy corresponds to the specific heat of the core.

(A color version of this figure is available in the online journal.)

REFERENCES

Abe, Y. 1995, The Earth’s Central Part: Its Structure and Dynamics (Tokyo: Terra Scientific), 215
Abe, Y., & Matsui, T. 1985, in Proc. Lunar and Planetary Science Conference, Vol. 15, ed. G. Ryder & G. Schubert, 545
Abe, Y., & Matsui, T. 1986, J. Geophys. Res., 91, 291
Acuña, M. H., et al. 1999, Science, 284, 790
Anderson, O. L., & Duba, A. 1997, J. Geophys. Res., 102, 22659
Beaulieu, J., et al. 2006, Nature, 439, 457
Bullock, M. A., & Grinspoon, D. H. 2001, Icarus, 150, 19
Canup, R. M. 2004, Icarus, 168, 433
Christensen, U. R., & Yuen, D. A. 1985, J. Geophys. Res., 90, 10291
Christensen, U. R., Holzwarth, V., & Reiners, A. 2009, Nature, 457, 167
Glatzmaier, G. A., & Roberts, P. H. 1995, Nature, 377, 203
Gould, A., et al. 2010, ApJ, 720, 1073
Hale, C. J. 1987, Nature, 329, 233
Hale, C. J. 1987, Nature, 329, 233
Honda, S., Yuen, D. A., Balachandar, S., & Reuteler, D. 1993, Science, 259, 1308
Ida, S., & Lin, D. N. C. 2004, ApJ, 604, 388
Ida, S., & Lin, D. N. C. 2008, ApJ, 685, 584
Ida, S., & Lin, D. N. C. 2010, ApJ, 719, 810
Kagayama, A., & Sato, T. 1997, Phys. Rev. E, 55, 4617
Kasting, J. F., Whitmire, D. P., & Reynolds, R. T. 1993, Icarus, 101, 108
Kimura, J., Nakagawa, T., & Kurita, K. 2009, Icarus, 202, 216
Kuang, W., & Bloxham, J. 1997, Nature, 389, 371
Labrosse, S., Hernlund, J. W., & Colliere, N. 2007, Nature, 450, 866
Léger, A., et al. 2009, A&A, 506, 287
Mayor, M., et al. 2009, A&A, 493, 639
Nimmo, F., Price, G. D., Brodholt, J., & Gubbins, D. 2004, Geophys. J. Int., 156, 363
O’Neill, C., & Lenardic, A. 2007, Geophys. Res. Lett., 34, 19204
Papuc, A. M., & Davies, G. F. 2008, Icarus, 195, 447
Queloz, D., et al. 2009, A&A, 506, 303
Ranalli, G. 2001, J. Geodynam., 32, 425
Sakuraba, A., & Kono, M. 1999, Phys. Earth Planet. Inter., 111, 105
Sasaki, S., & Nakazawa, K. 1986, J. Geophys. Res., 91, 9231
Schubert, G., Cassen, P., & Young, R. E. 1979, Icarus, 38, 192
Senzhuu, H., Kuramoto, K., & Matsui, T. 2002, J. Geophys. Res. (Planets), 107, 5118
Sharpe, H. N., & Peltier, W. R. 1979, Geophys. J., 59, 171
Sohl, F., & Spohn, T. 1997, J. Geophys. Res., 102, 1613
Sotin, C., Grasset, O., & Moquet, A. 2007, Icarus, 191, 337
Sotin, C., et al. 1999, PEPL, 112, 171
Spiegel, E. A. 1963, ApJ, 138, 216
Stevenson, D. J., Spohn, T., & Schubert, G. 1983, Icarus, 54, 466
Stixrude, L., & Lithgow-Bertelloni, C. 2005, Geophys. J. Int., 162, 610
Tajika, E., & Matsui, T. 1992, Earth Planet. Sci. Lett., 113, 251
Tsuchiya, T., Tsuchiya, J., Umemoto, K., & Wentzcovitch, R. M. 2004, Earth Planet. Sci. Lett., 224, 241
Uchida, T., Wang, Y., Rivers, M., & Sutton, S. 2001, J. Geophys. Res., 106, 21799
Udry, S., et al. 2007, A&A, 469, L43
Umemoto, K., Wentzcovitch, R. M., & Allen, P. B. 2006, Science, 311, 983
Vine, T. J. 1963, Z. Astrophys., 32, 135
Williams, Q., & Knittle, E. 1997, Phys. Earth Planet. Inter., 100, 49
Wortel, M. J. R., & Spakman, W. 2000, Science, 290, 1910
Yukutake, T. 2000, Phys. Earth Planet. Inter., 121, 103
Zahnle, K. J., Kasting, J. F., & Pollack, J. B. 1988, Icarus, 74, 62