In the solidification processes of transparent organic and metallic alloys, it is known that fluid flow induced by natural or forced convection affects dendrite growth and resultant dendritic grain structures. Through decreasing the thickness of boundary layer around dendritic grain and prompting the mass transfer in upstream direction, the fluid flow causes the dendrites to grow inclined toward the upstream direction and consequently modifies the preferred growth direction. It is found in unidirectional solidification experiments of Cyclohexanol that dendrites perfectly aligned with thermal gradient in the absence of fluid flow are deflected a few degrees by forced flow perpendicular to the growth direction, and the deflection angle of primary dendrite arms is dependent on the growth rate. Also in casting experiments of steel, this mechanism is responsible for the tendency of columnar grains growing toward upstream direction, and the deflection angle is deduced as a function of flow velocity and solidification rate.

To understand the formation of dendritic grain structure in solidification processes, Cellular Automaton (CA) model has been developed in parallel to experimental techniques. CA model includes the algorithms which describe heterogeneous nucleation procedure, dendrite tip growth kinetics and competitive growth procedure. Especially, the decentred quadrilateral growth algorithm, which is developed from decentred square growth algorithm with proper dendrite tip growth kinetics, can describe the evolution of unsymmetrical envelope of grain in the presence of fluid flow. With the help of the decentred quadrilateral growth algorithm and the dendrite tip growth kinetics in consideration of the influences of flow intensity and flow direction, the formation of dendritic grain structure of Pb–Sn alloy is studied.

In this paper, focusing on the deflective growth behaviors of dendrites in the flow field, the dendrite tip growth kinetics (the Gandin, Guillemot, Appolaire and Niane (GGAN) model) and the decentred quadrilateral growth algorithm are combined in CA model to predict the deflective growth of dendrites inclined toward upstream direction. The influences of flow intensity, cooling rate (or solidification rate), nucleation density at ingot surface on the deflective growth of dendrites are discussed. The dendrite tip growth kinetics in the flow field and the decentred quadrilateral growth algorithm for describing the evolution of grain growth are combined in Cellular Automaton model to predict the deflective growth of dendrites inclined toward upstream direction. The increases of flow intensity and cooling rate suppress slightly this deflective growth. The relations predicted among deflection angle, flow intensity and solidification rate for Al–Si alloy and Al–Cu alloy show the same tendency as that in Okano et al.’s empirical expression deduced from experiments on steel. The deflection angle predicted for Al–Cu alloy fits well with previous experimental results.

KEY WORDS: dendrite tip growth kinetics; decentred quadrilateral growth algorithm; Cellular Automaton; deflective growth of dendrites; flow intensity; cooling rate; nucleation density.
quadrilateral growth algorithm are combined in two dimensional (2D) CA model to describe the deflective growth behaviors of dendrites in a fluid field.

2.1. Dendrite Tip Growth Kinetics in the Presence of Fluid Flow (the GGAN Model)

The presence of fluid flow remarkably influences the dendrite tip growth kinetics by prompting the mass transfer in the boundary layer and consequently increasing the constitutional undercooling in the upstream direction ahead of the dendrite tip. Here, the dendrite tip growth kinetics (the GGAN model)\(^7\) based on Stokes flow approximation is adopted to describe the steady state growth of a paraboloidal dendrite in an undercooled melt in the presence of fluid flow. The dimensionless supersaturation, \(\Omega\), is written as a function of the growth Peclet number, \(P_v\), the Reynolds number, \(Re_p\), and the Schmidt number, \(Sc\):

\[
\Omega = P_v \exp(P) \left\{ E_v(P_v) - E_r \left( \frac{1}{1 + \frac{4}{ARe^2_vSc^2 \sin(\theta/2)}} \right) \right\}
\]

............................(1)

\[P_v = \frac{rv_{tip}}{2D_v} \]..........................(1a)

\[Re_p = \frac{2ru}{v} = \frac{4P_v}{Sc} \]..........................(1b)

\[P_v = \frac{ru}{2D_v} \]..........................(1c)

\[Sc = \frac{v}{D_v} \]..........................(1d)

where \(P_v\) is the flow Peclet number, \(v_{tip}\) is the dendrite tip growth rate, \(u\) is the relative velocity of liquid with respect to solid dendrite tip, \(v\) is the kinematic viscosity, \(D_v\) is the diffusion coefficient of solute in liquid, \(\theta\) is the angle in radians between dendrite tip growth direction and flow direction. The constants in Eq. (1) are set as \(A=0.5773\), \(B=0.6596\) and \(C=0.5249\). The exponential integral function, \(E_v(P_v) = \int_{P_v}^{\infty} (\exp(-\tau)/\tau) d\tau\), is estimated by polynomial interpolations.\(^7\)

In an undercooled melt, the local dendrite tip undercooling, \(\Delta T\), can be written as follows by neglecting the contributions of thermal undercooling and kinetics undercooling:

\[
\Delta T = \Delta T_c + \Delta T_v \]..........................(2)

\[
\Delta T_c = mC^0 \left( 1 - \frac{1}{1 - (1 - k^{3/2}L)/\Omega} \right) \]..........................(2a)

\[
\Delta T_v = 2\Gamma/r \]..........................(2b)

where \(\Delta T_c\) is the constitutional undercooling, \(\Delta T_v\) is the curvature undercooling, \(m\) is the slope of liquidus line, \(C^0\) is the initial solute concentration, \(k^{3/2}\) is the partition coefficient, \(\Gamma\) is the Gibbs–Thomson coefficient, \(r\) is the dendrite tip radius. Supersaturation \(\Omega\) is defined as

\[
\Omega = \frac{(C^* - C^0)(1 - k^{3/2}L)}{mC^*(k^{3/2}L - 1)} \]..........................(3)

where \(C^*\) is the local concentration in liquid at solid/liquid (S/L) interface.

The dendrite tip radius is dependent on the dendrite tip growth rate following the marginal stability criterion\(^9\) by ignoring the effect of the thermal gradients on each side of the solid–liquid interface,

\[
r_{\text{tip}}^2v_{\text{tip}} = \frac{D_v}{\sigma^* mC^*(k^{3/2}L - 1)} \]..........................(4)

where the stability constant, \(\sigma^*\), is approximately taken as \((4\pi)^{-1}\).

The dendrite tip growth rate in the presence of fluid flow is resolved by the following sequences, as shown in Fig. 1:

1. Give values of local undercooling \(\Delta T\), fluid flow velocity \(u\) and physical properties of alloy.
2. Assuming the dendrite tip radius \(r\) is infinite (i.e. curvature undercooling is zero, local undercooling equals constitutional undercooling), calculate supersaturation \(\Omega\) from Eq. (2).
3. Calculate interface concentration \(C^*\) from definition of supersaturation \(\Omega\), Eq. (3).
4. Rewire dendrite tip radius \(r\) as a function of growth Peclet number \(P_v\) by combining Eq. (4) and Eq. (1a),

\[
r = \frac{2\sigma^* mC^*(k^{3/2}L - 1)P_v}{\Gamma} \]..........................(5)

5. Express supersaturation \(\Omega\) as a function of growth Peclet number \(P_v\) (i.e. \(\Omega = f(P_v)\)) according to Eq. (1).
6. Calculate \(P_v\) from \(\Omega = f(P_v)\) by Van Wijngaarden–Dekker–Bren method.\(^1\)
7. Calculate dendrite tip radius \(r\) from Eq. (5).
8. Obtain dendrite tip growth rate \(v_{\text{tip}}\) from Eq. (1a).

\[\text{Fig. 1. Flow chart for calculation of dendrite tip growth rate in the presence of fluid flow.}\]
(9) Calculate curvature undercooling $\Delta T_c$ from Eq. (2b).
(10) Obtain new constitutional undercooling $\Delta T_u$ from Eq. (2).
(11) Calculate new dendrite tip radius $r$ following the step from (3) to (8). Once the error of the dendrite tip radius $r$ between sequent iterations is small enough (i.e. $|r_n - r_{n+1}| < e = 10^{-10}$ m), the calculation is finished and the exact dendrite tip radius $r$ and the dendrite tip growth rate $v_{tip}$ are obtained.

For a grain growing in the flow field, Sherwood number $(Sh_2 = 2 + ARe_2^b Sc^c \sin(\theta/2))$ approaches maximum ($=2 + ARe_2^b Sc^c$) in its upstream direction ($\theta = \pi$), and minimum ($=2$) in downstream direction ($\theta = 0$), respectively. It indicates that the boundary layer thickness related to the mass transfer $(\delta = 2r/(Sh_2 - 2))$ is the thinnest in the upstream direction and the thickest in the downstream direction. The faster mass transfer near the thinner boundary layer contributes to a larger constitutional undercooling for grain growth. So the dendrite tip growth rate in upstream direction is larger than that in downstream direction. Figure 2 shows variations of dendrite tip growth rate $v_{tip}$ of Al–7mass%Si alloy as a function of the angle $\theta \in [0-\pi]$ between dendrite tip growth direction and fluid flow direction by using the GGAN model. In a purely diffusive growth regime $(u=0 \text{ m/s, i.e. } P_v = 0)$, and $\Omega = P_v \exp(P_v)E_1(P_v)$), the dendrite tip growth rate $v_{tip}$ is same in all directions. While in the presence of fluid flow, the dendrite tip growth rate $v_{tip}$ approaches maximum in upstream direction $(\theta = \pi)$, and approaches minimum in downstream direction $(\theta = 0)$, i.e. $\Omega = P_v \exp(P_v)E_1(P_v)$).

Figure 3 shows the comparison of the dendrite tip growth rate $v_{tip}$ of Al–7mass%Si alloy from the GGAN model with that from the simplified KGT model. The ones by the GGAN model are in the case of flow direction opposite to the dendrite tip growth direction, and the one by the simplified KGT model is valid in purely diffusive growth regime. The dashed line represents the result from the simplified KGT model as a function of undercooling: $v_{tip} = 2.9 \times 10^{-6} \times (\Delta T)^2 \text{ m/s}$(12) As shown in Fig. 3, the dendrite tip growth rate from the GGAN model in a purely diffusive growth regime $(u=0 \text{ m/s, i.e. } P_v = 0)$ fits perfectly with that from the simplified KGT model. From Fig. 2 and Fig. 3, it is clear that the GGAN model is efficient to calculate the dendrite tip growth rate in the presence of fluid flow.

2.2. Decentred Quadrilateral Growth Algorithm in Consideration of the GGAN Model

Schematic two dimensional (2D) decentred quadrilateral growth algorithm in consideration of dendrite tip growth kinetics in the presence of fluid flow (the GGAN model).
crystallographic direction, with respect to the x-axis), the virtual growth center (intersection point of diagonals of a quadrilateral envelope) and the growth length for each dendrite arm (length of four half diagonals of the quadrilateral). Each of the four dendrite arms of a grain evolves with an individual rate calculated by the dendrite tip growth kinetics. In the presence of fluid flow, the growth length for each dendrite arm is unequal due to the different angle between the dendrite tip growth direction and the fluid flow direction. Thus the consequent quadrilateral is not a regular square shape as that in the purely diffusive growth regime.

The two open circle marks (O) labeled m and n correspond to the centers of two neighbor cells in CA network. At a certain time, cell m becomes solid, and its growth envelope, centered at virtual growth center \( G_m \), propagates from the quadrilateral with dotted lines to the one with solid lines ABCDA. When this envelope engulfs the centre of cell n, the entrapment of cell n is a random procedure. Once cell n is entrapped, its state is changed into solid and it inherits the crystallographic orientation from its parent cell. Its initial growth envelope is firstly inherited as the quadrilateral ABCDA, then truncated according to the length of side AB which is the closest to cell n. The length IA and IE are the truncations from IA and IB to keep them not greater than \( \sqrt{2} t_\text{CA} \) (\( t_\text{CA} \) is CA cell spacing, I is a pedal, i.e. a foot from n to side AB). Then the new quadrilateral AEFHA is obtained by similarity analysis with the parent quadrilateral ABCDA. And the virtual growth center \( G_n \) corresponding to cell n is settled as intersection point of the diagonals in quadrilateral AEFHA.

To simplify the engulfing judgment and the truncation, a coordinate transformation from xoy system to AG mB system is done for each growing cell as m. In AG mB system, \( G_m \) is the origin; [01] direction of cell m, \( G_{mA} \), is the x-axis; [01] direction of cell m, \( G_{mB} \), is the y-axis. The coordinates of the liquid neighbor of cell m, for example, cell n, are transformed into AG mB system. Cell n is engulfed when its new coordinates are inside the quadrilateral ABCDA with four half diagonals (\( G_{mA}, G_{mB}, G_{mC}, G_{mD} \), i.e. the line segments on the axes of AG mB system. The inherited growth envelope of cell n is truncated according to the length of side AE by triangular similarity analysis, for example, based on the similarity of triangle \( AG_mA \) and \( AEG_e \). With the growth length \( G_{mA} \) and \( G_{mB} \) and the hypotenuse length AB and AE, the length of \( G_{mA}, G_{mE} \) can be obtained. They represent the two lengths of the initial growth of cell n. Other growth lengths, \( G_{mF} \) and \( G_{mH} \), can be deduced similarly. At the same time, the coordinates of virtual growth center \( G_n \) for cell n can easily be determined in AG mB system. They are then transformed to the original xoy system.

2.3. Two Dimensional (2D) CA Model

In two dimensional (2D) CA model, it is assumed that the ingot is cooled down in a uniform temperature field or in a gradient temperature field under a uniform flow field. The flow field and the temperature field do not influence each other, i.e. the flow field and the temperature field are not coupled. Consequently, fluid flow only influences the dendrite tip growth kinetics through the change of undercooling. The local undercooling (here only the contributions of constitutional and curvature undercooling are considered) is regarded as the difference between liquidus temperature at initial concentration and local temperature at dendrite tip. The local temperature is obtained by the calculation of temperature decrease. When the melt is undercooled, it is permitted to nucleate. For a nucleus in the undercooled melt, its dendrite tip growth rates in the four [10] directions (i.e. directions of the four dendrite arms, [10], [01], [10], [01] are calculated by the GGAN model as a function of local undercooling, flow intensity and angle between dendrite tip growth direction and flow direction. During a time step each dendrite arm grows by the product of dendrite tip growth rate and time step. In two dimensional (2D) case, the four dendrite arms, which are the four half diagonals, compose the skeletons of the dendritic grain envelope. The envelope evolves through the growth of the dendrite arm and the entrapment of the neighbor cells in a CA network.

3. Evolution of Single Grain Envelope in the Presence of Fluid Flow

In Al–7mass%Si alloy ingot with cross section size 0.01 m x 0.01 m, the evolution of a single grain envelope in the presence of fluid flow is examined using the above CA model, assuming that a nucleus with a random orientation is located at the bottom center of ingot (for convenience, an angle of \( \pi/4 \) is chosen with respect to x-axis). Figure 5 shows the typical evolution of the envelope of a single grain in the uniform temperature field with \( T=1.0 \) K/s and in the gradient temperature field with \( T=1.0 \) K/s and \( G=250 \) K/m under different flow conditions for Al–7mass%Si alloy.

In the case of non-fluid flow (Fig. 5(a)), the envelope of grain keeps the regular square shape due to the same growth rate at the four dendrite tips in the uniform temperature field. The envelope of grain in the gradient temperature field evolves slower than that in the uniform temperature field. The reason behind this phenomenon is that the undercooling at the upper part of the calculation region in the gradient temperature field is less than that in the uniform temperature field. Thus, the dendrite tip growth rate in the gradient temperature field is smaller and consequently the evolution of the envelope is slower. In the case of transverse flow, \( u=0.1 \) m/s (Fig. 5(b)), the angle between [01] and [10] direction of dendrite arm and flow direction is \( 3/4 \pi \) and \( 1/4 \pi \), respectively. The dendrite in [01] direction grows faster than that in [10] direction, as illustrated in Fig. 2. In the case of vertical flow, \( u=0.1 \) m/s (Fig. 5(c)), the dendrites in [01] and [10] direction grow in the same growth rate because both angles are \( 1/4 \pi \). In addition, in both cases of transverse flow (Fig. 5(b)) and vertical flow (Fig. 5(c)), the envelopes of grains in the gradient temperature field evolve slower than those in the uniform temperature field due to the same reason as that in the case of non-fluid flow (Fig. 5(a)).

4. Evolution of Multi-grain Envelopes in the Presence of Fluid Flow

In Al–7mass%Si alloy ingot with cross section size 0.01 m x 0.01 m, the evolution of multi-grain envelopes in the presence of fluid flow is examined using the above CA
model, assuming that nuclei with random orientations are located uniformly at the bottom of ingot. It is to be noted that the number of CA cells used for the simulation was $1200$, thus resulting in a resolution of about 8.3 µm. This value is of the order of the secondary arm spacing and is adapted to the modelling of the extension of grains by branching mechanisms. With the nucleation density adopted in the present work, i.e. $n_0 = 12000$ to $30000$/m, there are 120 to 300 nuclei at the bottom of ingot, so the size of ingot cross section is large enough and well adapted to the prediction of grain growth in the presence of fluid flow. Figure 6 shows the typical evolution of the envelopes of multi-grains in the uniform temperature field for Al–7mass%Si alloy under different flow conditions for Al–7mass%Si alloy: (a) non-fluid flow, (b) transverse flow, $u=0.1$ m/s and (c) vertical flow, $u=0.1$ m/s.

Fig. 5. Typical evolution of the envelope of a single grain growing in the uniform temperature field with $\dot{T}=1.0$ K/s and in the gradient temperature field with $\dot{T}=1.0$ K/s and $G=250$ K/m under different flow conditions for Al–7mass%Si alloy: (a) non-fluid flow, (b) transverse flow, $u=0.1$ m/s and (c) vertical flow, $u=0.1$ m/s.

**Fig. 6.**


**4.1. Nucleation Density at Ingot Surface** $n^*_S$

The nucleation density at ingot surface $n^*_S$ is estimated using the relation between columnar grain density near ingot surface, $n^*_S_{\text{min}} (=1/\lambda_1)$, m$^{-1}$, and cooling rate $T$, K/s,

$$
\lambda_1 = 220 \times 10^{-6} \times (T')^{-0.55} \quad (7-1)
$$

where $\lambda_1$ is the primary arm spacing, m.

The nucleation density at ingot surface is adopted as $n^*_S = 12,000, 20,000$ and $30,000$/m under the condition of $n^*_S_{\text{min}} (=1/\lambda_1)$, since the values of $n^*_S_{\text{min}}$ are estimated as 3736, 7187 and 11016/m corresponding to cooling rate $T=0.7, 2.3$ and 5.0 K/s, respectively.

The influence of nucleation density at ingot surface $n^*_S$ on deflection angle $\phi$ is shown in Fig. 7. The horizontal solid line in Fig. 7(a) represents the maximum cooling rate corresponding to $n^*_S_{\text{min}} = 12,000$/m determined by Eq. (6). Values of deflection angles are labeled near each open circle mark (○). The increase of $n^*_S$ from 12,000 to 30,000/m results in only 2 degree decrease of deflection angle $\phi$. In addition, with the further increase of $n^*_S$, deflection angle $\phi$ changes very little. It is reasonable since the increase of $n^*_S$ promotes growth competition among grains and consequently suppresses grain growth inclined toward upstream direction, deflection angle $\phi$ decreases a little bit.

**4.2. Transverse Flow Intensity**

Figure 6 illustrates evolution of multi-grain envelopes in the case of flow intensities $u$ of 0.1, 0.2, 0.5 m/s which are encountered under common casting conditions. Obviously, fluid flow causes grain to grow inclined toward upstream direction. With flow intensity $u$ increasing from 0.1 to 0.5 m/s, the deflection angle $\phi$ is remarkably increased by 10 to 14 degrees as seen in Fig. 7. As explained in Fig. 2 and 3, the reason behind this phenomenon is the local increase of dendrite tip growth rate in upstream direction which is caused by the local decrease of the boundary layer thickness ahead of the dendrite tip due to fluid flow.

**4.3. Cooling Rate $T$ and Solidification Rate $v_L$**

The increase of cooling rate $T$ directly increases local undercooling $\Delta T$ and consequently increases the dendrite tip growth rate in all directions, as explained in Fig. 2. Thus, the grain growth inclined toward upstream direction is restricted by the growth of its neighbors in all directions. This results in a small decrease of deflection angle $\phi$ in Fig. 7. That is, the increase of $T$ from 0.7 to 5.0 K/s causes 2 to 6 degree decrease in deflection angle $\phi$.

Solidification rate $v_L$ (i.e. moving rate of solid–liquid interface) is mainly dependent on cooling rate $T$ but little dependent on nucleation density at ingot surface $n^*_S$ and flow intensity $u$, as shown in Fig. 8. Thus, $v_L$ can be adopted instead of $T$ from the approximately linear relation between $T$ and $v_L$. And the increase of $T$ from 0.7 to 5.0 K/s approximately corresponds to the increase of $v_L$ from 1 to 6 mm/s.

**5. Differences of the Defective Growth Behaviors of Dendrites among Three Alloys**

Based on Okano et al.’s empirical expression deduced from experiments on steel (C: 0.17 mass%, Si: 0.35 mass%, Mn: 1.35 mass%, Sol. Al: 0.031 mass%),

$$
\ln u = \frac{\phi + 9.73 \ln v_L + 33.7}{1.45 \ln v_L + 12.5} \quad 2 < u < 50 \text{ cm/s}
$$

$$
\ln u = \frac{\phi + 4.83 \ln v_L + 7.2}{0.1 \ln v_L + 5.4} \quad 50 \leq u < 100 \text{ cm/s}
$$

the relations among deflection angle $\phi$ (degree), flow intensity $u$ (cm/s) and solidification rate $v_L$ (cm/s) are derived for Al–7mass%Si alloy and Al–3mass%Cu alloy.

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5.1. Al–7mass%Si Alloy

Figure 9 shows relations among deflection angle $\phi$, flow intensity $u$ and solidification rate $v_L$ for Al–7mass%Si alloy and steel. Predicted values of deflection angle at various conditions are shown (the ones for Al–7mass%Si alloy are denoted by $\circ$, $\triangle$, and the ones for steel are denoted by thin solid line, thin dashed line and thin dotted line). It
seems that the relation among $\phi$, $u$ and $v_L$ for Al–7mass%Si alloy shows the same tendency as that of Okano et al. deduced empirically from experiments on steel. Best fit in the form of Eq. (7) for Al–7mass%Si alloy is derived by using Levenberg–Marquardt method,\textsuperscript{11} 

\[
\ln u = \frac{\phi + 2.008 \ln v_L + 0.322}{-0.374 \ln v_L + 2.008}, \quad 2 < u < 100 \text{ cm/s} \quad (8)
\]

where deflection angle $\phi$, degree, flow velocity, $u$, cm/s and solidification rate, $v_L$, cm/s. In Fig. 9, the fitting curves are denoted by bold solid line, bold dashed line and bold dotted line.

### 5.2. Al–3mass%Cu Alloy

Lee et al.\textsuperscript{15} measured the deflection angle $2\phi$ for the dendrites of Al–Cu alloy flowing down an inclined Cu chill plate (flow velocity, $u$: 0, 1 and 2 cm/s). Experimental values of $\phi$ at various conditions are shown in Fig. 10 (Al–4.5mass%Cu alloy denoted by $\square$, and Al–3mass%Cu alloy denoted by $\bigcirc$). For Al–3mass%Cu alloy, by using the present CA model with conditions and physical properties listed in Table 1, it is found that the predicted values of $\phi$ at $T=0.7$ K/s and $n_\text{S}^*=12000$ m$^{-1}$ (denoted by $\triangle$) fit well with Lee et al.'s experimental results (denoted by $\bigcirc$). This validates the predictions of the present CA model. Best fit in the form of Eq. (7) for Al–3mass%Cu alloy is derived by Levenberg–Marquardt method,\textsuperscript{11} 

\[
\ln u = \frac{\phi + 3.74 \ln v_L + 8.983}{-1.547 \ln v_L + 1.403}, \quad 0.001 < u < 2 \text{ cm/s} \quad (9)
\]

where deflection angle $\phi$, degree, flow velocity, $u$, cm/s and solidification rate, $v_L$, cm/s. In Fig. 11, the fitting curves are denoted by bold solid line, bold dashed line and bold dotted line.

The empirical relations between deflection angle $\phi$, flow intensity $u$ and solidification rate $v_L$ for Al–3mass%Cu alloy (bold solid line), Al–7mass%Si alloy (bold dashed line) and steel (bold dotted line) are compared in Fig. 12. Obviously, the tendency for deflective growth of dendrites in the presence of flow is in the order of Al–3mass%Cu alloy > Al–7mass%Si alloy > steel. The relations expressed in the same form can predict the tendencies of deflective growth of dendrites for these three alloys.

---

### Table 1. Thermophysical properties of Al–7mass%Si alloy and Al–3mass%Cu alloy.

| Property                  | Al–7mass%Si | Al–3mass%Cu |
|---------------------------|------------|------------|
| Liquidus slope, m (K/°C)  | -650       | -337       |
| Partition coefficient, $k^{S/L}$ (-) | 0.13       | 0.17       |
| Initial solute concentration, $C^0$ (-) | 0.07       | 0.03       |
| Liquid density, $\rho$ (kg/m$^3$) | 2730       | 2448.5     |
| Gibbs-Thomson coefficient, $\Gamma$ (K m) | $1.96 \times 10^7$ | $2.41 \times 10^7$ |
| Stability constant, $\sigma^*$ (-) | 0.0253     | 0.0253     |
| Dynamic viscosity, $\mu$ (Pa s) | $1.38 \times 10^3$ | $1.2 \times 10^3$ |
| Liquidus temperature, $T_L$ (°C) | 618.       | 650.       |
| Diffusion coefficient of Si in liquid Al at $T_L$, $D_L$ (m$^2$/s) | $6.45 \times 10^8$ | $5 \times 10^8$ |
| Schmidt number, $S_c$ (-) | 90.6       | 98.        |

---

Fig. 10. Deflection angle $\phi$ for dendrites of Al–Cu alloy flowing down an inclined Cu chill plate (flow velocity, $u$: 0.1 cm/s, 1 cm/s and 2 cm/s). Experimental values of $\phi$: Al–4.5mass%Cu alloy denoted by $\square$, and Al–3mass%Cu alloy denoted by $\bigcirc$. Predicted values of $\phi$: Al–3mass%Cu alloy denoted by $\triangle$, $\blacksquare$, $\bigtriangleup$.

Fig. 11. Relations among deflection angle $\phi$, flow intensity $u$ and solidification rate $v_L$ for Al–3mass%Cu alloy: nucleation density at ingot surface $n_\text{S}^*$ = 12000 m$^{-1}$.

Fig. 12. Differences of the deflective growth behaviors of dendrites among three alloys: empirical relations among deflection angle $\phi$, flow intensity $u$ and solidification rate $v_L$ for Al–3mass%Cu alloy (bold solid line), Al–7mass%Si alloy (bold dashed line) and steel (bold dotted line).
6. Conclusions

The dendrite tip growth kinetics in the flow field and the decentred quadrilateral growth algorithm describing the evolution of grain growth are combined in CA model. With grain structures predicted by the CA model, some important features are found in the deflective growth behaviors of dendrites for Al-based alloys. The influencing factors on the deflection angle of dendrites are discussed. The results obtained are as follows:

1. From grain structures predicted for Al–7mass%Si alloy, flow intensity $u$, cooling rate $\dot{T}$ (or solidification rate $v_L$) and nucleation density at ingot surface $n^*_S$ are recognized as the main influencing factors on the deflective growth of dendrites in flow field. Their effects on deflection angle $\phi$ are in the order of $u/1.1022$, $\dot{T}/1.1022$, $v_L/1.1022$, $n^*_S$. That is, the increase of $u$ from 0.1 to 0.5 m/s causes 10 to 14 degree increase in $\phi$ and the increase of $\dot{T}$ from 0.7 to 5.0 K/s (in other words, the increase of $v_L$ from 1 to 6 mm/s) causes 2 to 6 degree decrease in $\phi$, while the increase of $n^*_S$ from 12 000 to 30 000/m causes only 2 degree decrease in $\phi$.

2. The empirical relations between $\phi$, $u$ and $v_L$ for Al–7mass%Si alloy and Al–3mass%Cu alloy show the same tendency as that in Okano et al.’s empirical expression deduced from experiments on steel:

$$
\ln u = \phi + 2.008 \ln v_L + 0.132 - 0.197 \ln v_L + 8.004,
$$

for Al–7mass%Si alloy, and

$$
\ln u = \phi + 3.741 \ln v_L + 8.983 - 1.547 \ln v_L + 1.403 - 0.001 < u < 2 \text{ cm/s}
$$

for Al–3mass%Cu alloy

The tendency for deflective growth of dendrites in the presence of flow is in the order of Al–3mass%Cu alloy > Al–7mass%Si alloy > steel. The relations expressed in the same form can predict the tendencies of deflective growth of dendrites for these three alloys.

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