Observation of the hindered electromagnetic Dalitz decay $\psi(3686) \to e^+e^-$
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The electromagnetic (EM) Dalitz decays $V \to \ell^+\ell^-P$ arise from an internal conversion of the virtual photon ($\gamma^*$) in the corresponding decays $V \to \gamma^*P$. These decays can be used to probe the dynamic EM structure of the transition $V \to P$, and investigate the fundamental mechanisms for the interactions between photons and hadrons [1]. Here, $V$ and $P$ are vector and pseudoscalar mesons, and $\ell$ denotes leptons ($\ell = e, \mu$), respectively. Normalized to the width of the corresponding radiative decay $V \to \gamma P$, the differential decay width of $V \to \ell^+\ell^-P$ is described by the formula:

$$\frac{d\Gamma(V \to \ell^+\ell^-P)}{dq^2\Gamma(V \to \gamma P)} = \left| \frac{f_{VP} (q^2)}{f_{VP}(0)} \right|^2 \times \text{QED}(q^2) \quad (1)$$

Using a data sample of $(448.1 \pm 2.9) \times 10^6 \psi(3686)$ decays collected at an $e^+e^-$ center-of-mass energy of 3.686 GeV by the BESIII detector at BEPCII, we report an observation of the hindered electromagnetic Dalitz decay $\psi(3686) \to e^+e^-\eta_c$ with a significance of 7.9$\sigma$. The branching fraction is determined to be $B(\psi(3686) \to e^+e^-\eta_c) = (3.77 \pm 0.40_{\text{stat}} \pm 0.18_{\text{syst}}) \times 10^{-5}$, agreeing well with the prediction of the vector meson dominance model. This is the first measurement of the electromagnetic Dalitz transition between the $\psi(3686)$ and the $\eta_c$, which provides new insight into the electromagnetic properties of this decay, and offers new opportunities to measure the absolute branching fractions of $\eta_c$ decays.

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I. INTRODUCTION
where $q^2$ is the squared four-momentum transfer that equals the square of the invariant mass of the lepton pair $(M^2_{e^+e^-})$, $F_{VP}(q^2)$ is the transition form factor (TFF) that characterizes the EM structure of the region in which $V$ converts into $P$, $F_{VP}(q^2) \equiv f_{VP}(q^2)/f_{VP}(0)$ is the normalized TFF with a normalization of $F_{VP}(0) = 1$, and QED($q^2$) represents the quantum electrodynamics (QED) calculations of the differential decay width of $V \rightarrow \ell^+\ell^-P$ assuming $V$ and $P$ to be point-like particles [2].

According to Eq. (1), the $q^2$-dependent $f_{VP}(q^2)$, or $F_{VP}(q^2)$ can be extracted from the experimental data after the QED factors have been taken into account. It can also be calculated theoretically using nonperturbative QCD models [3–7]. For example, the vector mechanism is appropriate in the time-like kinematic region, $q^2 > (m_V - m_P)^2$, where the resonant behavior of $\gamma^*$ arises near $q^2 = m_V^2$ since $m_V$ is approaching or even reaching the mass shell [1]. Under a single pole approximation, the TFF can be parametrized with the formula,

$$F_{VP}(q^2) = \frac{1}{1 - q^2/\Lambda^2}, \quad (2)$$

where $\Lambda$ serves as an effective pole mass, subsuming effects from all possible resonance poles and scattering terms in the time-like kinematic region. The VMD assumption has been phenomenologically very successful in describing the experimental TFF behaviors for many analogous EM Dalitz decays, such as $\eta(1) \rightarrow \ell^+\ell^-\gamma/\omega$ [8–13], $\phi \rightarrow e^+e^-\pi^0/\eta$ [14, 15], $J/\psi(3686) \rightarrow e^+e^-\pi^0/\eta(1)$ [16, 17], $\psi(3686) \rightarrow e^+e^-\chi_{cJ}$ [18], and $D^{\ast0} \rightarrow e^+e^-D^0$ [19]. On the other hand, one case where VMD dramatically fails is in the TFF of the $\omega \rightarrow \mu^+\mu^-\pi^0$ decay [8, 9, 20]. Neither the VMD model nor other models [21–25] can explain the steep rise of the experimental $\omega$ TFF in a larger $q^2$ region that shows a relative increase close to the kinematic cut-off by a factor of $\sim 10$ [9]. This discrepancy of the $\omega$ TFF between theoretical calculations and experimental measurements motivates our experimental investigation of other $V \rightarrow \ell^+\ell^-P$ decays, such as the hindered electromagnetic Dalitz decay $\psi(3686) \rightarrow e^+e^-\eta_c$, where ‘hindered’ means the transition matrix element between the $\psi(3686)$ and the $\eta_c$ would vanish in the limit of zero virtual photon energy because of the orthogonality of the wave functions with different principal quantum number [26, 27].

Based on the VMD model, the branching fraction (BF) is predicted to be $B(\psi(3686) \rightarrow e^+e^-\eta_c) = (3.04 \pm 0.45) \times 10^{-5}$ [28, 29], after considering the effects of the polarization of the $\psi(3686)$ produced in $e^+e^-$ collisions. Comparing the theoretical and experimental BFs and $q^2$-dependent TFFs can enrich our understanding of the nature of the $\psi(3686)$ meson, for example by distinguishing between a pure $S$-wave state and an $S$- and $D$-wave mixture [30]. In addition, the $\psi(3686) \rightarrow e^+e^-\eta_c$ decay makes it possible for absolute measurements of $\eta_c$ decays.

In this Letter, we present the first experimental study of the EM Dalitz decay $\psi(3686) \rightarrow e^+e^-\eta_c$. We analyze (448.1 $\pm$ 2.9) $\times 10^6$ $\psi(3686)$ decays [31] taken at a center-of-mass energy of $\sqrt{s} = 3.686$ GeV with the BESIII detector.

## II. DETECTOR AND MONTE CARLO SIMULATIONS

The BESIII detector is a magnetic spectrometer [32] located at the Beijing Electron Positron Collider (BEPCII). The cylindrical core of the BESIII detector consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field.

The simulated data samples are produced using a GEANT4-based [33] Monte Carlo (MC) package that incorporates the geometric description of the BESIII detector and the detector response. An inclusive MC sample containing $5.06 \times 10^8$ inclusive $\psi(3686)$ decays is used to investigate the potential backgrounds. The detection efficiency is determined by a signal MC sample of $\psi(3686) \rightarrow e^+e^-\eta_c$ events with inclusive $\eta_c$ decays, and using a TFF parameterized by Eq. (2) with $\Lambda = 3.773$ GeV/$c^2$ [28]. The production of the $\psi(3686)$ resonance is simulated by the MC event generator KKM [34], where the beam energy spread and initial-state radiation (ISR) in the $e^+e^-$ annihilation have been taken into account. The known decay modes are generated by EVTGEN [35, 36] utilizing BFs taken from the Particle Data Group [37], and the remaining unknown decays are modelled with LUNDCHARM [38, 39].

## III. EVENT SELECTION

To select the signal process, we first save all $e^+e^-$ combinations in each event, and then search for the $\eta_c$ signal in the recoiling system of each combination. The BF of $\psi(3686) \rightarrow e^+e^-\eta_c$ is determined by

$$B_{\text{sig}} = \frac{N_{\text{sig}}}{N_{\psi(3686)}}, \quad (3)$$

where $N_{\text{sig}}$ is the signal yield obtained through a fit to the recoil mass distribution of $e^+e^-\eta_c$. $N_{\psi(3686)}$ is the total number of $\psi(3686)$ decays, and $\varepsilon_{\text{sig}}$ is the detection efficiency.

The decay chain of interest is $\psi(3686) \rightarrow e^+e^-\eta_c$, where the $\eta_c$ decays inclusively. Charged tracks detected in the MDC are required to have a polar angle ($\theta$) satisfying $|\cos\theta| < 0.93$ with respect to the positron beam, and a distance of closest approach to the interaction point within $\pm 10$ cm along the beam direction.
and 1 cm in the plane transverse to the beam direction. The number of good charged tracks is at least 2 with a net charge of 0. Positron (electron) PID uses the measured information in the MDC, TOF and EMC. The combined likelihoods ($L$) under the positron (electron), pion, and kaon hypotheses are obtained. Positron (electron) candidates are required to satisfy $L(e) > 0.001$ and $L(e)/L(\pi) > 0.8$. Within each event, at least one $e^+e^-$ pair is selected, and the momenta of $e^\pm$ should be less than 0.8 GeV/$c$. If there are multiple candidates, all of them are kept.

To reject events from the $\psi(3686) \rightarrow \pi^+\pi^- J/\psi$ decay, each charged track is treated as a pion and the recoil mass of each pair of oppositely charged tracks is required to be outside a range of $[3.090, 3.104]$ GeV/$c^2$. Background events from the process $\pi^+(\eta) \rightarrow e^+e^-$ are also possible since the recoil mass of the $e^+e^-$ pair may lie within the signal range of $\psi(3686) \rightarrow e^+e^-\eta_c$. Therefore, we combine each $e^+e^-$ pair with a soft photon to form the invariant mass $M_{\gamma e^+e^-}$. The soft photon candidates are identified using showers in the EMC. The deposited energy of each shower must be more than 25 MeV in the barrel region ($|\cos \theta| < 0.80$) and more than 50 MeV in the end cap region ($0.86 < |\cos \theta| < 0.92$). To suppress electronic noise and showers unrelated to the event, the difference between the EMC time and the event start time is required to be within [0, 700] ns. The resulting $M_{\gamma e^+e^-}$ for each $\gamma e^+e^-$ combination is required to be outside the ranges [0.115, 0.150] and [0.505, 0.570] GeV/$c^2$ to veto $\pi^0$ and $\eta$ backgrounds, respectively.

**IV. BACKGROUND AND SIGNAL YIELD**

The radiated photon in the $\psi(3686) \rightarrow \gamma \eta_c$ decay may convert into an $e^+e^-$ pair when it interacts with the beam pipe or the MDC inner wall, which may result in a fake signal. In order to suppress such background, we veto events in which the distance from the reconstructed vertex point of the $e^+e^-$ pair to the beam direction ($R_{xy}$) is larger than 2 cm and the angle between the $e^+$ and $e^-$ ($\theta_{e^+e^-}$) is larger than 40$^\circ$. Here, 40$^\circ$ is determined by maximizing the figure-of-merit, $S/\sqrt{S+B}$ with respect to $\theta_{e^+e^-}$, where $S$ and $B$ denote the expected signal yield and the background yield, respectively. MC-simulation studies show that the requirement of $\theta_{e^+e^-} < 40^\circ$ rejects about 15% of the signal, but suppresses about 60% of the background.

After applying the above event selection criteria, a clear $\eta_c$ signal peak appears in the $RM_{e^+e^-}$ distribution, as shown in Fig. 1. An unbinned maximum likelihood fit to the $RM_{e^+e^-}$ distribution is performed to obtain the signal yield. In the fit, the signal shape is described by a signal MC shape convolved with a Gaussian function, which is used to compensate for the discrepancy in the detection resolution between data and MC simulation. The shape of the non-peaking background is modelled with a second order Chebyshev polynomial function, and the size is allowed to float. The peaking background contains two contributions. One is from the two-photon process $\gamma^*\gamma^* \rightarrow \eta_c$, which is investigated using the $(2916.94 \pm 29.17) \text{ pb}^{-1} [40, 41]$ data sample taken at $\sqrt{s} = 3.773$ GeV by BESIII. Here the two virtual photons are radiated from the initial $e^+$ and $e^-$, respectively. With the same event selection criteria, the number of events for $\gamma^*\gamma^* \rightarrow \eta_c$ process $(N^{\gamma\gamma})$ is determined to be $N^{\gamma\gamma} = 0^{+19}_{-13}$.

To suppress electronic and $\eta_c$ background, the solid blue line is the peaking background. The statistical significance of the $\eta_c$ signal is computed to be $9.5\sigma$ with the formula of $S = \sqrt{-2\ln(L_0/L_{\text{max}})}$, where $L_{\text{max}}$ and $L_0$ are the likelihood values when $N^{\text{obs}}_{\text{sig}}$ is left free and fixed at 0 in the fit, respectively.

The other contribution is from $\gamma$ conversions within the $\psi(3686) \rightarrow \gamma \eta_c$ decay. An MC sample including $1 \times 10^7 \psi(3686) \rightarrow \gamma \eta_c$ events has been generated and is found to be consistent with data. The $\gamma$ conversion background is studied using this MC sample. Normalized with $B(\psi(3686) \rightarrow \gamma \eta_c)$ [37], the number of events $(N^{\gamma\eta_c})$ in the $\psi(3686)$ data is computed to be $N^{\gamma\eta_c} = 76 \pm 13$. Thus, the total number of events for the peaking background is $76^{+19}_{-13}$, where the uncertainty is the quadratic sum of the uncertainties for the two parts, and will be taken into account in the systematic uncertainty. The shape of the peaking background is extracted from the MC sample of $\psi(3686) \rightarrow \gamma \eta_c$, and the size is fixed to be 76. The fit result is depicted in Fig. 1. The signal yield determined from the fit is $N^{\text{obs}}_{\text{sig}} = 3078 \pm 329$. The detection efficiency is estimated to be $18.22\%$ using the signal MC simulation. The efficiency-corrected signal yields versus $M_{e^+e^-}$ for data and MC sample are shown in Fig. 2, which are consistent with each other.

![Fig. 1](image-url)  
**Fig. 1.** The $RM_{e^+e^-}$ distribution with the fit results overlaid. The dots with error bars are from data, the solid red line is the best fit, the long dashed green line is the non-peaking background, the solid blue line is the peaking background with a fixed size of 76, and the dashed red line is the signal.
Events/(5.0 MeV/c²)

0 0.1 0.2 0.3
Mₑₑ (GeV/c²)

10^3 10^2

FIG. 2. Efficiency-corrected signal yields versus $M_{e⁺e⁻}$. The solid red histogram with a bin width of 5 MeV/c² shown in y-axis is from the signal MC sample, and the four dots with error bars are from data due to the limited statistics, which have been averaged by a bin width of 5 MeV/c². Here the four bins are [2$m_{e,e}$, 0.01], [0.01, 0.03], [0.03, 0.06], [0.06, 0.30] GeV/c². The signal yields in data are obtained from fits to $RM_{e⁺e⁻}$ in each bin, and the signal yields in the signal MC sample are normalized by the BF of $\psi(3686) → e⁺e⁻\eta_c$ measured in this letter.

V. BRANCHING FRACTION

According to Eq. (3), the BF of $\psi(3686) → e⁺e⁻\eta_c$ is measured to be $B(\psi(3686) → e⁺e⁻\eta_c) = (3.77 ± 0.40_{stat.} ± 0.18_{syst.}) \times 10^{-5}$, where the first and second uncertainties are statistical and systematic uncertainties, respectively. The systematic uncertainties are elaborated upon below.

VI. SYSTEMATIC UNCERTAINTIES

The systematic uncertainty for the BF measurement comes from the $e⁺$ tracking and PID efficiencies, the requirements $R_{xy} < 2$ cm and $\theta_{e⁺} < 40^\circ$, vetos of the $\pi^0$, $\eta$ and $J/ψ$ backgrounds, the shape of the non-peaking background, the size of the peaking background, the signal model, and the total number of $\psi(3686)$ events.

The systematic uncertainty from the $e⁺$ tracking and PID efficiency is investigated by analyzing a mixed control sample of radiative Blaheta events $e⁺e⁻ → γe⁺e⁻$ at $\sqrt{s} = 3.88$ GeV and $\psi(3686) → (\gamma_{ISR})e⁺e⁻$. According to the two-dimensional distributions of the polar angle and momentum, the $e⁺$ tracking and PID efficiencies in the control sample are weighted to match those in the signal decay. The data-MC simulation difference of $e⁺$ tracking efficiency is determined to be 0.5% per track and that of PID efficiency is 0.7% per track, both of which are assigned as the relevant systematic uncertainty.

The systematic uncertainty associated with the requirement $R_{xy} < 2$ cm for the suppression of the $γ$ conversion background is estimated to be 1.0% by using a highly pure sample of $J/ψ → π⁺π⁻π^0(→ γe⁺e⁻)$. The uncertainty due to the requirement $\theta_{e⁺} < 40^\circ$ is estimated using a control sample of $\psi(3686) → γ\chi_{cJ}, \chi_{cJ} → e⁺e⁻J/ψ(→ e⁺e⁻/µ⁺µ⁻)$ ($J = 1, 2$). The difference in the efficiency for the requirement $\theta_{e⁺} < 40^\circ$ between data and MC is determined to be 1.3%, which is assigned to be the corresponding systematic uncertainty. Utilizing the same control sample, the systematic uncertainty from vetoing $π^0(\eta) → γe⁺e⁻$ background is assigned to be 3.0% through analyzing the invariant mass of the radiative photon and the $e⁺e⁻$ pair directly from $\chi_{cJ}$ decays. The uncertainty caused by vetoing $J/ψ$ background is computed to be 0.4% by studying a control sample of $\psi(3686) → γ\chi_{cJ}, \chi_{cJ} → 2(π⁺π⁻)$ ($J = 0, 1, 2$).

The systematic uncertainty from the shape of the nonpeaking background is estimated by altering the order of the Chebyshev polynomial function from second to third or fourth, and the largest difference on the measured BF, 2.5%, is taken as the systematic uncertainty. The uncertainty due to the size of the peaking background is examined by varying its size within ±1σ, and the largest difference in the measured BF, 0.5%, is associated as the relevant systematic uncertainty.

In the nominal analysis, the signal MC sample is generated using a TFF modelled with a single pole, where $Λ$ is fixed at 3.773 GeV/c². Since the branching fraction is not sensitive to the pole mass, according to the theoretical calculation, alternative $Λ$ values of 1.5 and 6.0 GeV/c² are used in the signal MC sample samples, which is a conservative range for the estimation of the systematic uncertainty. The average difference in the measured BF is estimated to be 0.9%, which is taken as the systematic uncertainty from the signal model. The uncertainty from the total number of $\psi(3686)$ events is 0.6%.

The total systematic uncertainty on the measured BF of $\psi(3686) → e⁺e⁻\eta_c$ decay is computed to be 4.8% by adding the above systematic uncertainties in quadrature.

The signal significance is estimated again after considering the effects of the assumed background shape, the size of the peaking background and the signal model. Based on different variations, the lowest significance of the $\eta_c$ signal is calculated to be 7.9σ.

VII. SUMMARY

In summary, we observe the hindered EM Dalitz decay $\psi(3686) → e⁺e⁻\eta_c$ with a significance of 7.9σ using $(448.1 ± 2.9) \times 10^6$ $\psi(3686)$ decays. The measured BF is $B(\psi(3686) → e⁺e⁻\eta_c) = (3.77 ± 0.40_{stat.} ± 0.18_{syst.}) \times 10^{-5}$, which is consistent with the theoretical prediction from the VMD model [28, 29] within one standard deviation. The ratio over the corresponding radiative decay [37] is determined to be $(1.11 ± 0.21) \times 10^{-2}$, which is in accord with the QED predictions. In the future,
with the $3 \times 10^9 \psi(3686)$ decays recently collected by BESIII [45], the TFF for $\psi(3686) \rightarrow e^+e^+\eta_c$ can be extracted in the whole $M_{e^+e^-}$ kinematic range, and absolute BF measurements of $\eta_c$ decays can be performed.

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