Specific heat of high temperature superconductors: Role of $|\psi|^4$ term in the Ginzburg-Landau free energy

B. C. Gupta† and K. K. Nanda‡∗

Institute of Physics, Bhubaneswar-751005, India

Abstract

We have derived the expression for the specific heat by using Ginzburg-Landau (GL) theory by taking $|\psi|^4$ into account in the Hartree approximation. Without this term, the specific heat diverges at $T = T_c(B)$. It is also shown that width and shape of the transition depends on the value of $\alpha$ and $\beta$, the coefficients in the GL free energy.

Keywords: Ginzburg-Landau theory, specific heat in magnetic fields

† e-mail: bikash@iopb.ernet.in

‡ e-mail: nanda@iopb.ernet.in

* Corresponding author
1 Introduction

The most interesting feature of high temperature superconductors (HTSCs) is that the superconducting transition is broadened in presence of magnetic field which has been understood in terms of thermodynamic fluctuations near the superconducting phase transition and has drawn a great interest and many theories have been developed [1, 2, 3, 4, 5]. It is expected that fluctuations of the normal phase into the superconducting phase above the transition temperature $T_c$ can give rise to additional contributions to both thermodynamic and transport quantities. These fluctuations can be viewed as superconducting droplets spontaneously appear and disappear on time scales of $\hbar/k_B | T - T_c |$ [6]. Our discussion is based on the lowest landau level approximation. According to this approximation, the applied field forces the superconducting electrons to move in the lowest landau orbitals perpendicular to the field, thereby, reducing the effective dimensionality of the system. Within this approximation, it is shown [3] that physical properties exhibit scaling with scaling variable $x = [\frac{T - T_c(B)}{(T_B)^n}]$, where $n=2/3$ for 3-dimensional (3D) systems and 1/2 for 2D systems that has been observed experimentally [7].

If the quartic term is dropped in the GL free energy, an analytic expression for the specific heat is obtained which diverges at $T = T_c(B)$ [8]. Close to $T_c(B)$, this term plays an important role and cannot be ignored. The effect of including the quartic term for layered superconductors is studied implicitly by Quader and Abrahams [9]. But, if the variation of order parameter along the direction of the applied field is smooth enough, then the problem is reduced to ordinary 3D GL equation [10]. Sufficiently near $T_c$, the coherence length along the $c-$axis $\xi_c$ will be so large that $\xi_c > s$, the interplanar distance and
the 3D description of GL theory is justified.

An exact evaluation of the partition function \(Z\) is not possible in the presence of the quartic term. However, the partition function can be evaluated within a Hartree-type approximation where \(|\psi|^4\) is replaced by \(2<|\psi|^2>|\psi|^2\). In this paper, we take the quartic term into account and derive the specific heat in the Hartree approximation.

## 2 Hartree Approximation

The free energy functional for a superconductor in a uniform flux density \(B\) near the \(T_c\) has the form

\[
F_s = F_n + \int dV \left[ \frac{1}{2m} |(-i\hbar \nabla - eB \times r)\psi|^2 + \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \ldots \right]
\]  

(1)

where \(\alpha = \alpha_0 t\) and \(\beta = \beta_0\).

To calculate the fluctuations above \(T_c(B)\), we can use the free energy functional of Eq. (1). The terms quadratic in \(\psi\) is considered which gives the specific heat to be divergent at \(T = T_c(B)\). Close to \(T_c(B)\), the terms quartic in the fluctuations cannot be ignored. Within the Hartree approximation,

\[
\Delta F[\psi] = F_s - F_n = \int dV \left[ \alpha |\psi|^2 + \beta <|\psi|^2|\psi|^2 + \frac{1}{2m} |(-i\hbar \nabla - eB \times r)\psi|^2 \right]
\]  

(2)

In the free energy expression, we have dropped the term \(B^2/2\mu_0\) because \(B\) is taken to be the mean value of the induction and neglects the fluctuations. The free energy can be diagonalized in terms of the solutions of the Eq.

\[
\frac{1}{2m} (-i\hbar \nabla - eB \times r)^2 \psi + \alpha \psi + \beta <|\psi|^2 = E\psi
\]  

(3)

Writing \(\psi = \sum_q C_q \psi_q(r)\), we get the eigen values are

\[
E_q = \alpha + (2n + 1) \frac{eB}{m} + \frac{\hbar^2 k_q^2}{2m} + \beta <|\psi|^2 = \alpha_B + 2n \frac{eB}{m} + \frac{\hbar^2 k_q^2}{2m} + \beta <|\psi|^2
\]  

(4)
with degeneracy $eB/\pi \hbar$ per unit area and $\alpha_B = \alpha + eB/m$.

In the vicinity of the upper critical field, the dominant contribution comes from the lowest Landau level ($n=0$). The order parameter averages are given by \[11\]

$$<|\psi|^2| = k_B T \sum_q \frac{1}{E_q}$$

and the specific heat can be calculated as \[8\],

$$C = k_B T^2 \sum_q \frac{1}{E_q} \left( \frac{dE_q}{dT} \right)^2$$

2.1 Specific Heat in 2D Systems

In case of 2D, for the applied field perpendicular to the film, the fluctuations are zero dimensional and the $k_z$ component is suppressed. The average of the $|\psi|^2$ for the film of thickness $d$ can be evaluated as \[12\],

$$<|\psi|^2| = \frac{-\alpha_B + \sqrt{\alpha_B^2 + \left(\frac{eB}{\pi \hbar}\right) \beta k_B T}}{2\beta}$$

This along with Eq.(6) gives

$$C_{2D} = \frac{\left(\frac{eB}{\pi \hbar}\right) k T^2}{\frac{\alpha_B}{2T_c} + \frac{1}{4} \sqrt{\alpha_B^2 + \beta k T \left(\frac{eB}{\pi \hbar}\right)}} \left( \frac{2\alpha_B}{T_c} + \beta k \left(\frac{eB}{\pi \hbar}\right) \right)$$

$$\frac{1}{4} \left( \alpha_B + \sqrt{\alpha_B^2 + \beta k T \left(\frac{eB}{\pi \hbar}\right)} \right)^2$$

As $\beta$ is independent of temperature and $\alpha$ decreases and as $T \to T_c(B)$, $\alpha_B \to 0$

$$<|\psi|^2| = \left(\frac{eB}{\pi \hbar}\right)^{1/2} \beta^{-1/2}$$

which is non-zero and give rise to a non divergent specific heat.
2.2 Specific Heat in 3D Systems

Replacing the sum by integration, the average of the $|\psi|^2$ is obtained as

$$<|\psi|^2> = \frac{(\frac{eB}{4\pi\hbar})k_B T \sqrt{\frac{m}{2}}}{\sqrt{\alpha_B + \beta} <|\psi|^2>}$$

which gives,

$$<|\psi|^2> = \left(\frac{m}{2}\right)^{2/3} (\frac{eB}{\pi^2})^{2/3} \beta^{-2/3}$$

as $T \to T_c(B)$. Hence the specific heat is non-divergent and is given by

$$C_{3D} = \frac{(\frac{eB}{\pi\hbar})kT^2}{\left(\frac{\alpha_0}{T_c} + \frac{2}{3} \left(\sqrt{\frac{m}{2}} \frac{eB}{\pi\hbar} \beta k\right)^{\frac{2}{3}} \frac{1}{\pi^2}\right)}$$

\[\alpha_B + \left(\sqrt{\frac{m}{2}} \frac{eB}{\pi\hbar} kT\right)^{\frac{2}{3}}\]

\[\frac{4}{3}\]

3 Discussion

The fluctuation in specific heat above $T_c(B)$ in case of 3D differs by a factor $\sqrt{2}$ at a corresponding distance below $T_c(B)$ because the coherence length for $\psi(r)$ is smaller by a factor $\sqrt{2}$ below $T_c(B)$ in comparison with the region above $T_c(B)$ \[4\]. Whereas, the specific heat in case of 2D will be the same and that has been observed experimentally.

Fig.1 shows the temperature dependence of the specific heat in case of 2D systems originated from fluctuations for different values of $\alpha_0$ and $\beta_0$. It is noted from fig.1 that the width and the shape of the transition depends on $\alpha_0$ and $\beta_0$. Fig.1(a) corresponds to $\alpha_0 = 10$, $\beta_0 = 5000$ and fig.1(b) corresponds to $\alpha_0 = 100$, $\beta_0 = 10000$. For both the figures the dashed and the solid curve corresponds to high (4T) and low (2T) external magnetic fields respectively. For our calculation we have taken $\frac{dB_{c2}}{dT} = -3.2$. It has been shown \[13\] that the field dependence of the specific heat below $T_c(B)$ can be accounted for in the mean field approximation by evaluating the interactions between vortices.
Here, the vortex structure has not been included in these calculations which influence the specific heat below $T_c(B)$. Therefore, below the transition temperature we do not expect the agreement of our result with the experimental data. However, we note from figure 1(a) and 1(b) that the width of the transition increases with magnetic field. This is in accordance with the experiments on both low $T_c$ \cite{14} as well as high-$T_c$ materials \cite{15,16}. Our result also gives similar behaviour above $T_c(B)$ as obtained by Bray \cite{17} using screening theory.

4 Conclusion

We have presented and discussed the results for the fluctuation specific heat, both with and without the quartic term in the free energy. An inspection of our results indicate that the interaction term plays an important role. The specific heat does not diverge in contrast to the GL theory without $|\psi|^4$ term. We have also shown that the decrease of the sharpness of transition of the fluctuation specific heat with the increase of magnetic field as is observed in experiment. Above $T_c(B)$, the nature of the fluctuation specific heat curve is also similar to that, obtained from screening theory, agreeing the experimental result.
References

[1] R. Ikeda, T. Ohmi, and T. Tsuento, J. Phys. Soc. Jap. 58, 1377 (1989).

[2] S. Ullah, and A. T. Dorsey, Phys. Rev. Lett. 65, 2066 (1990); Phys. Rev. B 44, 262 (1991).

[3] Z. Tēsanovic’, L. Xing, L. Bulaevskii, Q. Li, and M. Suenaga, Phys. Rev. Lett. 69, 3563 (1992).

[4] N. K. Wilkin and M. A. Moore, Phys. Rev. B 47, 957 (1993); Phys. Rev. B 48, 3464 (1993).

[5] I. D. Lawrie, Phys. Rev. B 50, 9456 (1994); N. Overend, M. A. Howson, and I. D. Lawrie, Phys. Rev. Lett. 72, 3238 (1994).

[6] M. Tinkham, An Introduction to the Superconductivity (McGraw-Hill, New York, 1975) chap. 7.

[7] U. Welp et al., Phys. Rev. Lett. 67 (1990) 3180.

[8] D. J. Thouless, Phys. Rev. Lett. 34, 946 (1975).

[9] Quader and Abraham, Phys. Rev. B 38, 11977 (1988)

[10] M. Tinkham, Physica C 235-240, 3 (1994).

[11] W. E. Masker, S. Marcelja and R. D. Parks, Phs. Rev. 188, 745 (1969).

[12] P. A. Lee and S. R. Shenoy, Phys. Rev. Lett. 28, 1025 (1972).

[13] K. K. Nanda, Physica C 245, 341 (1995).

[14] L. J. Barnes and R. R. Hake, Phys. Rev. 153, 435 (1967).
[15] S. E. Inderhees, M. B. Salmon, J. R. Rice and D. M. Ginsberg, Phys. Rev. Lett. 66, 232 (1991), Phys. Rev. B 47, 1053 (1993).

[16] E. Janod et al., Physica C 234 269 (1993).

[17] A. J. Bray, Phys Rev. B 9 4752 (1974).
FigureCaptions

Figure1:

(●)Thetemperaturedependenceofspecificheatin testcase of 2D systems that originatesfrom fluctuations in different magnetic fields and different values of $\alpha_0$ and $\beta_0$. a) $\alpha_0 = 10, \beta_0 = 5000$. b) $\alpha_0 = 100, \beta_0 = 10000$. 