Turbulent natural convection combined with entropy generation in a nanofluid cavity with non-uniformly heated side walls

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Abstract. The two-dimensional turbulent natural convection and entropy generation within an alumina-water nanoliquid chamber in the presence of sinusoidal temperature profiles at vertical walls is calculated. The considered process has been modelled using the RANS (Reynolds-averaged Navier–Stokes) approach for turbulent regime and Boussinesq approximation for the buoyancy force. The control equations written using the non-dimensional stream function, vorticity and temperature variables combined with parameters of standard $k$-$\varepsilon$ turbulence model have been solved by the finite difference method with non-uniform computational mesh. The special algebraic transformation has been used for an introduction of the non-uniform mesh in physical domain. The influence of the nano-sized particles concentration and Rayleigh number on the nanofluid circulation and energy transport patterns has been examined.

1. Introduction

Nowadays, a development of modern engineering devices and technological systems demands to understand the physical processes within these systems. Taking into account the intensive physical phenomena and very often the huge sizes of the analyzed regions these considered processes have the turbulent nature [1–3].

During last two decades, special “smart” liquids are used for an intensification of heat transfer in various engineering applications. These liquids are a solution of the conventional heat transfer fluids (water, oil, ethylene glycol mixture) and solid nano-size particles (metal, metal oxide and others). The name of these liquids is nanofluids. Experimental and numerical investigations show that an application of nanofluids in various engineering devices allows enhancing the energy transport [4, 5]. Thus, Choudhary and Subudhi [6] studied experimentally the turbulent thermogravitational convection of alumina/water nanofluids within the closed cavity under the bottom heating and top cooling. Authors found the heat transfer enhancement in the case of the lower nanoparticles concentration and high Rayleigh numbers. Kumar et al. [7] analyzed numerically and experimentally the turbulent nanofluid flow and heat transfer in a square passage with protruded rib shape for high Reynolds numbers. They revealed that alumina nanoparticles allow obtaining the highest average Nusselt number in comparison with CuO and ZnO nanoparticles. Moreover, the energy transport enhancement occurs with nanoparticles concentration. Bianco et al. [8] numerically investigated turbulent convection of alumina/water nanoliquid in a circular tube under the constant wall temperature.
influence. It was ascertained that a rise of the nanoparticles concentration leads to the energy transport and entropy generation enhancement. Khairat Dawood et al. [9] experimentally and numerically investigated turbulent natural convection of delafossite nanofluid within a rectangular cavity. The heat transfer enhancement was found for high Rayleigh number and nanoparticles concentration of 5%.

The objective of this study is a numerical simulation of turbulent thermogravitational energy transport and entropy generation within a square alumina/water nanofluid chamber under the effect of non-uniformly heated vertical walls.

2. Physical statement and mathematical model

The domain of interest is a square cavity with thermally insulated horizontal walls and active vertical ones (see figure 1). The vertical borders are kept at sinusoidal temperature distributions. The considered chamber is filled with alumina-water nanofluid. The nanofluid is supposed to be incompressible and radiatively non-participating. It is assumed that the flow is turbulent. The nanoliquid flow and heat transfer are described using two-dimensional RANS equations with $k$-$\varepsilon$ turbulence model. The special algebraic coordinate transformation was used for introducing the non-uniform mesh in physical domain, while in the computational domain we used the uniform mesh [10, 11]. Such technique allows reducing the computational load.

Equations of the nanoliquid flow and heat transfer using the dimensionless stream function, vorticity, temperature, turbulent kinetic energy and its dissipation rate have the following form:

\[
\frac{\partial^2 \xi}{\partial \xi^2} \frac{\partial \psi}{\partial \xi} + \left( \frac{\partial \xi}{\partial x} \right)^2 \frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial \eta}{\partial y} \frac{\partial^2 \psi}{\partial \eta^2} + \left( \frac{\partial \eta}{\partial y} \right)^2 \frac{\partial^2 \psi}{\partial \eta^2} = -\omega
\]  

(1)

\[
\frac{\partial \omega}{\partial \tau} + \left( \frac{\partial \xi}{\partial y} \frac{\partial \omega}{\partial \eta} - \frac{\partial \xi}{\partial \xi} \frac{\partial \omega}{\partial \xi} \right) \frac{\partial \xi}{\partial y} + \left( \frac{\partial \eta}{\partial y} \frac{\partial \omega}{\partial \eta} \right) \frac{\partial \eta}{\partial y} = \frac{\partial \xi}{\partial x} \left[ \sqrt{\frac{Pr_{nf} \cdot V_t}{Ra_{nf}}} + \frac{Pr_{nf} \cdot V_t}{Ra_{nf}} \right] \frac{\partial \xi}{\partial y} \frac{\partial \eta}{\partial \eta} + \frac{Pr_{nf} \cdot V_t}{Ra_{nf}} \frac{\partial \xi}{\partial y} \frac{\partial \eta}{\partial \eta} \]  

(2)

\[
\frac{\partial \theta}{\partial \tau} + \frac{\partial \eta}{\partial \eta} \left[ \frac{1}{\sqrt{Ra_{nf} \cdot Pr_{nf}}} + \frac{V_t}{Pr} \right] \frac{\partial \eta}{\partial \eta} \frac{\partial \theta}{\partial \eta} + \frac{\partial \xi}{\partial x} \left[ \frac{1}{\sqrt{Ra_{nf} \cdot Pr_{nf}}} + \frac{V_t}{Pr} \right] \frac{\partial \xi}{\partial \eta} \frac{\partial \theta}{\partial \eta} + \frac{\partial \xi}{\partial \xi} \frac{\partial \eta}{\partial \eta} \left[ \frac{Pr_{nf} \cdot V_t}{Ra_{nf}} \right] \frac{\partial \xi}{\partial \eta} \frac{\partial \theta}{\partial \eta} \]  

(3)

\[
\frac{\partial k}{\partial \tau} + \frac{\partial \eta}{\partial y} \frac{\partial \xi}{\partial \xi} \frac{\partial \xi}{\partial \eta} - \frac{\partial \xi}{\partial \xi} \frac{\partial \eta}{\partial \eta} \frac{\partial \xi}{\partial \eta} = \frac{\partial \xi}{\partial \xi} \left[ \frac{Pr_{nf} \cdot V_t}{Ra_{nf}} \right] \frac{\partial \xi}{\partial \eta} \frac{\partial k}{\partial \eta} + \frac{\partial \eta}{\partial \eta} \left[ \frac{Pr_{nf} \cdot V_t}{Ra_{nf}} \right] \frac{\partial \eta}{\partial \eta} \frac{\partial k}{\partial \eta} \]  

(4)
\[
\frac{\partial \varepsilon}{\partial \tau} + \frac{\partial \eta}{\partial y} \frac{\partial \varepsilon}{\partial \eta} - \frac{\partial \varepsilon}{\partial \xi} \frac{\partial \eta}{\partial \xi} = \frac{\partial \varepsilon}{\partial \xi} \left( \frac{Pr_{nf}}{Ra_{nf}} + \frac{\nu_{i}}{\sigma_{\varepsilon}} \right) \frac{\partial \xi}{\partial \xi} + \frac{\partial \eta}{\partial \eta} \left( \frac{Pr_{nf}}{Ra_{nf}} + \frac{\nu_{i}}{\sigma_{\varepsilon}} \right) \frac{\partial \varepsilon}{\partial \varepsilon} + c_{\nu} \left( \frac{\bar{\rho}}{\rho_{nf}} \right) \left( \frac{\partial \eta}{\partial y} \right)^{2} - c_{\mu} \left( \frac{\partial \varepsilon}{\partial \varepsilon} \right)^{2} + c_{\mu} \left( \frac{\partial \varepsilon}{\partial \varepsilon} \right)^{2} 
\]

Here
\[
\bar{\rho}_{i} = \nu_{i} \left[ 4 \left( \frac{\partial^{2} \eta}{\partial x \partial \eta} \right)^{2} + \left( \frac{\partial^{2} \varepsilon}{\partial y \partial \eta} \right)^{2} + \left( \frac{\partial^{2} \varepsilon}{\partial \xi \partial \eta} \right)^{2} - \left( \frac{\partial^{2} \varepsilon}{\partial \xi \partial \xi} \right)^{2} - \left( \frac{\partial^{2} \varepsilon}{\partial \eta \partial \eta} \right)^{2} \right] \quad \bar{G}_{i} = -\nu_{i} \frac{\partial \eta}{\partial y} \frac{\partial \theta}{\partial \eta}. 
\]

Figure 1. Analyzed physical region.

Thermophysical properties of the considered nanomaterial were determined as follows [12]:

\[
(\rho \beta)_{nf} = (1 - \phi)(\rho \beta)_{f} + \phi(\rho \beta)_{p}, \quad (\rho c)_{nf} = (1 - \phi)(\rho c)_{f} + \phi(\rho c)_{p}, \quad \rho_{nf} = (1 - \phi)\rho_{f} + \phi\rho_{p}
\]

The dynamic viscosity and thermal conductivity for nanofluid are [12, 13]:

\[
\mu_{nf} = \mu_{f} \left( 1 + 4.93\phi + 222.4\phi^{2} \right) \\
\lambda_{nf} = \lambda_{f} \left( 1 + 2.944\phi + 19.672\phi^{2} \right)
\]

Initial and boundary conditions:

\[
\begin{align*}
\psi &= 0, \quad \omega = 0, \quad \theta = 0, \quad \kappa = 0, \quad \varepsilon = 0 \quad \text{at} \quad \tau = 0 \\
\psi &= 0, \quad \frac{\partial \psi}{\partial \xi} = 0, \quad \theta = \sin(2\pi y), \quad \kappa = 0, \quad \frac{\partial \varepsilon}{\partial \xi} = 0 \quad \text{on} \quad \xi = 0 \\
\psi &= 0, \quad \frac{\partial \psi}{\partial \xi} = 0, \quad \theta = \gamma \sin(2\pi y), \quad \kappa = 0, \quad \frac{\partial \varepsilon}{\partial \xi} = 0 \quad \text{on} \quad \xi = 1 \\
\psi &= 0, \quad \frac{\partial \psi}{\partial \eta} = 0, \quad \frac{\partial \theta}{\partial \eta} = 0, \quad \kappa = 0, \quad \frac{\partial \varepsilon}{\partial \eta} = 0 \quad \text{on} \quad \eta = 0 \text{ and } \eta = 1
\end{align*}
\]

Here \(\xi, \eta\) are the computational independent Cartesian coordinates; \(x, y\) are the non-dimensional physical Cartesian coordinates; \(\tau\) is the non-dimensional time; \(\psi\) is the non-dimensional stream function; \(\omega\) is the non-dimensional vorticity; \(\theta\) is the non-dimensional temperature; \(\mu\) is the dynamic viscosity; \(\beta\) is the coefficient of thermal expansion; \(\rho\) is the density; \(k\) is the non-dimensional turbulent...
4

kinetic energy; \( \varepsilon \) is the non-dimensional dissipation rate of turbulent kinetic energy; \( c \) is the heat capacity; \( \phi \) is the volume fraction of nanoparticles; \( \lambda \) is the thermal conductivity; and the subscripts: \( p \) is the nanoparticle, \( f \) is the base fluid; \( nf \) is the nanofluid.

Partial differential equations (1)–(5) combined with boundary conditions (6) were solved using the finite difference method [10–12]. The developed numerical code was verified using the experimental and theoretical results of other researchers. Detailed verification of the created code can be found in [10–12].

The non-dimensional local entropy generation \( S_{gen} \) is [14]:

\[
S_{gen} = \frac{\lambda_{nf}}{\lambda_f} \left[ \left( \frac{\partial^2 \xi}{\partial x \partial \xi} \right)^2 + \left( \frac{\partial \xi}{\partial y} \right)^2 + 4 \left( \frac{\partial \xi}{\partial y} \right)^2 \left( \frac{\partial \eta}{\partial y} \right)^2 \right] + \chi \frac{\mu_{nf}}{\mu_f} \left[ \frac{\partial \xi}{\partial x} \frac{\partial \eta}{\partial \xi} \frac{\partial^2 \psi}{\partial y^2} \right]^2 + \left( \frac{\partial^2 \eta}{\partial y^2} \right)^2 + \left( \frac{\partial \eta}{\partial y} \right)^2 \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial \eta} \frac{\partial \psi}{\partial \eta} \frac{\partial \psi}{\partial \eta} \frac{\partial^2 \psi}{\partial \eta^2} \right]^2 \right] = S_{gen,lt} + S_{gen,ff}
\]  

(7)

Here \( \chi \) is the irreversibility factor \( \chi = \frac{\mu_f T_0}{\lambda_f} \left( \frac{g \beta_f L}{T_u - T_l} \right) \).

The integration of Eq. (7) allows obtaining the non-dimensional average entropy generation, \( S_{gen,avg} \):

\[
S_{gen,avg} = \frac{1}{\delta} \int S_{gen} d\delta = S_{gen,lt,avg} + S_{gen,ff,avg}
\]  

(8)

For describing the heat transfer irreversibility in comparison with total entropy generation the Bejan number \( Be \) was defined as

\[
Be = \frac{S_{gen,lt}}{S_{gen,lt} + S_{gen,ff}}
\]  

(9)

3. Results

An analysis was performed for \( \phi = 0–0.04 \), \( Ra = 10^8–10^9 \), \( Pr = 6.82 \). It should be noted that

\[
Ra_{nf} = Ra \left( \frac{\rho \beta}{\rho c_f} \right)_{nf} \frac{\rho_f \mu_f}{\lambda_{nf}} \frac{\lambda_f}{\rho \beta c_f} \frac{\mu_{nf}}{\lambda_f} \left( \frac{\rho c_f}{pr} \right)_{nf} \frac{\rho_f \mu_f}{\lambda_{nf}} \frac{\lambda_f}{\rho \beta c_f} \frac{\mu_{nf}}{\lambda_f} \left( \frac{\rho c_f}{pr} \right)_{nf}
\]

Figure 2 shows the time profiles of average \( Nu \) at left vertical border and average total entropy generation for different \( Ra \) and \( \phi \). Such high magnitudes of Rayleigh number describe a formation of unsteady profiles where the steady state can not be reached due to intensive turbulent mixing of the nanoliquid. A growth of the Rayleigh number reflects the heat transfer enhancement, while an addition of nanoparticles illustrates the heat transfer rate reduction. It is worth noting that the diminution rate with \( \phi \) increases with the Rayleigh number. One can find that a growth of \( Ra \) leads to a rise of \( S_{gen,avg} \). The latter characterizes a formation of undesirable conditions for the process development. An inclusion of nano-sized particles to the host liquid (water) results in an augmentation of \( S_{gen,avg} \).
Figure 2. Profiles of average Nusselt number at left vertical wall (a) and average total entropy generation with dimensionless time, Rayleigh number and nanoparticles volume fraction at $\gamma = 0$.

Figure 3 shows the time dependences of the average Nusselt number at left wall and average total entropy generation within the cavity for different values of the temperature oscillation amplitude on the right border at $Ra = 10^9$ and $\phi = 0.04$. As it was mentioned above, the average Nusselt number has non-monotonic behavior with time and the effect of $\gamma$ illustrates also non-monotonic change of the heat transfer rate. $S_{\text{gen,avg}}$ rises with $\gamma$ due to more essential temperature differences not only at the left wall but also at the right one.

Figure 3. Time profiles of average $Nu$ at left vertical border (a) and $S_{\text{gen,avg}}$ with oscillation amplitude for $Ra = 10^9$ and $\phi = 0.04$. 
4. Conclusions
The effects of the buoyancy force magnitude, nanoparticles concentration and temperature oscillation amplitude on the right vertical wall on the heat transfer rate and average total entropy generation were studied in the case of turbulent natural convection and entropy generation within the square Al₂O₃/H₂O nanoliquid enclosure under the impact of sinusoidal temperature distributions at vertical walls. Numerical simulation was performed using the RANS approach with k-ε turbulence model and a single-phase nanofluid model with the experimentally-based correlations for the dynamic viscosity and thermal conductivity. It was revealed that the considered problem is unsteady taking into account the time dependences of the considered parameters. An addition of nanoparticles reflects a growth of the average total entropy generation and reduction of the average Nusselt number, while a raise of the temperature oscillation amplitude on the right border characterizes an increment of $S_{gen,avg}$ also.

Acknowledgments
The work of Mikhail A. Sheremet was supported by the Grants Council (under the President of the Russian Federation), Grant No. MD-821.2019.8. The work of Ioan Pop was supported from the grant PN-III-P4-ID-PCE-2016-0036, UEFISCDI, Romania.

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