Fractal express methods evaluation of a breaking stress of concrete

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Abstract. We have offered the methods of the operative evaluation of a breaking stress in testing strength on compression of heavy concrete of grade 400 with the use of fractal formalism. The methods are based on the setting of the relations of the values of a breaking stress and fractal measurement of concrete: 1. Areas with crushed stone prevailing \( R^2 = 0.7224 \); 2. Areas with sand prevailing \( R^2 = 0.6102 \); 3. Pin holes \( R^2 = 0.6874 \). During fractal experimenting the indexes of breaking were increased from 391.63 to 515.13 kН in reducing fractal dimension of areas 1 from 1.866 to 1.588; areas 3 from 1.826 to 1.684 and internal quality (boundary of its elements) from 1.617 to 1.353. Increase of fractal dimension was fixed only for areas containing sand (areas 2) from 1.755 to 1.944. These results make possible to apply fractal dimension as the indicator of structural changes of concrete in the prediction of its properties. In fractal modelling the accuracy of the results depends on the option of the task way for space metric that is proved by the obtained linear model \( R^2 = 0.9254 \), which describes the connection between the elements of macrostructure and strength criterion of concrete.

Such methods provide a satisfactory in practical purposes operative prediction for the values of breaking stress of concrete of grade 400 with significant reduction of time and money expenditures on full-scale testing and the application of microscopy.

1. Introduction

A great number of parameters influence the quality criteria of concretes including the conditions of its obtaining, mineralogical and phase compositions [1, 2]. Real structure of concrete on various scale levels is characterised by a great number of elements that reflect its properties in a varying degree [3]. Approximation of the elements of concrete structure by Euclid’s integral figures (appraisal of points, square etc.) not always allows to use these results in the models of the prediction of its quality. One of the reasons of such a phenomenon is insufficiency of formal axiomatics in the identification of a real structure of material with complex geometric configuration of its elements.

For partial compensation of the existing insufficiency of formal axiomatics to identify materials structure, you can use the methods of mathematical modelling [4, 5], system analysis [6, 7], fractal theory [8, 9]. One of the advantages of using the fractal approach in order to describe qualitative transformations of materials is an arbitrary way of metrics definition. Fractal dimension as a quantitative feature of fractal set gives more differential evaluation for similar objects at first view. That is why fractal formalism is used in quality ranking of multiparameter technology [10, 11].
Basing on the results of the work for the application of fractal theory to model the structure and qualitative features of concretes, for example [12-14], the work dealt with the evaluation of breaking stress on the compression of heavy concrete on the base of fractal characteristics of the elements of its macrostructure. The application of such an approach allows to model strength criterion according to the photos of structure without any additional expenditures on full-scale testing and microscopy.

2. Materials and methods
Concrete of grade 400 produced from Portland Cement was investigated. Five samples from one pilot run were produced (Fig. 1). There are three main constituents on the surface of concrete:
1. Coarse aggregate (crushed stone) of 10… 20 mm fraction. Percentage 54…67 %. On the expanding fragment of Sample 5 Fig.1, these are the darker areas 1 of the structure of concrete containing crushed stone coming to the surface.
2. Fine aggregate (sand) with fractions not less 1.5 mm. Lighter areas 2 of the structure of concrete in Fig. 1 with sand prevailing. Percentage 30…40 %.
3. Pores sized 0.2…2 mm. Area of pores 3…6%.

![Sample 1](image1)
![Sample 2](image2)
![Sample 3](image3)
![Sample 4](image4)
![Expanding fragment of Sample 4](image5)

**Figure 1.** Structure 5 cubes of concrete (100×100×100 mm), where 1 – areas with gravel prevailing; 2 – areas with sand prevailing; 3 – pores.

To test strength the samples were imposed to destruction. At the same time the values of a breaking stress were changed within the limits from 391.63 to 515.13 kN (Table 1).
Table 1. Values of breaking stress of samples of concrete.

| Sample № | Breaking stress $P$, kN |
|----------|------------------------|
| 1        | 406.25                 |
| 2        | 391.63                 |
| 3        | 483.50                 |
| 4        | 438.75                 |
| 5        | 515.13                 |

To increase the reliability of the obtained values of fractal dimension we used patented methods described in detail in [15]. The main point of the methods lies in the determination of fractal dimension of tested object on the base of precision of cell and point-like dimensions.

Cell dimension $D$ was calculated by Fejerváry Hausdorff equation (1) [16]. At the core of this method, there is an idea of covering an object with cells by the number $N$ with linear size $l$:

$$D = -\lim_{\delta \to 0} \frac{\ln N(l)}{\ln l},$$  \hspace{1cm} (1)

Fractal dimension is based on a point-like method [16], it was calculated by the method of determining probability $P(m,L)$. This probability describes the number of points $m$ of a fractal object in the cell sized $L$. At the same time the average number of cells $\tilde{N}(L)$, which covers the object, proportionally to the value of its fractal dimension (2).

$$\tilde{N}(L) = \sum_{m=1}^{K} (1/m)P(m,L) \sim L^{-D},$$  \hspace{1cm} (2)

Fig. 2 demonstrates the results of calculation of dimension of areas of concrete structure with rushed stone and sand prevailing for Sample 5. To calculate fractal dimension, figures are presented in 256-coloured format with shades of grey. Analysis of the range of 256 colours for the areas of the structure allowed to set the range of colour changes in which the fractal dimension was calculated: for darker Areas 1 with crushed stone prevailing from 0 to 141; for lighter Areas 2 with sand prevailing – from 142 to 255. Fractal dimension of areas of structure 3 was calculated separately, the range of colour change fluctuated from 0 to 118.

Figure 2. Determination of precision of values of fractal dimension of the elements of structure calculated with cell and point-like ways.
Graph from Fig. 2 shows that the best precision of cell and point-like dimensions for darker areas with gravel prevailing corresponds to $D_l=(D_1+D_2)/2=1.588$ and is observed on step 5 of iterations.

On this stage of calculations the cell dimension $D_1$ corresponds to 1.593, but point dimension $D_2=1.583$. For lighter areas with sand prevailing the best precision $D_f=(D_3+D_4)/2=1.944$ was fixed on step 11 of iterations with cell dimension $D_3=1.949$ and point dimension $D_4=1.938$.

Boundaries in a multicomponent system i.e. concrete, contributes in its strength properties [1, 2]. In order to study the influence of boundaries between the elements of the structure of concrete on a breaking stress, the values of their fractal dimensions between areas of the structure were calculated separately.

### 3. Results and discussion

The values of the calculated values of fractal dimensions of the areas of the structure and their boundaries are shown in Table 2.

**Table 2.** Values of fractal dimensions of concrete macrostructure.

| Sample № | Dimensions of structure 1 with breakstone, $D_{breakstone}$ | Dimensions of structure 2 with sand prevailing, $D_{sand}$ | Dimensions of areas of structure 3 (Pores), $D_{pores}$ | Dimension of boundary of elements of structure, $D_{bound}$ |
|----------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| 1        | 1.866                                           | 1.755                                           | 1.826                                           | 1.617                                           |
| 2        | 1.787                                           | 1.854                                           | 1.798                                           | 1.560                                           |
| 3        | 1.749                                           | 1.897                                           | 1.776                                           | 1.490                                           |
| 4        | 1.790                                           | 1.800                                           | 1.750                                           | 1.477                                           |
| 5        | 1.588                                           | 1.944                                           | 1.684                                           | 1.353                                           |

Fig. 3 demonstrates the ratio between fractal dimensions of the areas of the structure and values of a breaking stress of concrete on compression that are described by linear models.
On account of the analysis of the obtained ratio, fractal dimension of a coarse aggregate (crushed stone $R^2 = 0.7224$ in Fig. 3 a) and boundary dimension ($R^2 = 0.7851$ in Fig. 3 d) have the significant influence on the strength of concrete. Sensitivity of indicators of a breaking stress to fractal characteristics of crushed stone forming matrix of concrete and to interphase boundaries as failures reducing strength indicators [1, 2], is proved by the processes of their physical and chemical influence on the properties of concrete. The obtained results correspond to the data from [13] for ceramsite concrete where in all cases there is the reduction of strength indicators on compression $fck.cube$ in increasing the values of fractal dimensions of the elements of macrostructure (expanded clay gravel, cement-sand matrix, large feldspar grains in the sand, large pores, large fractions of quartz).

Particularly, in [13] indicators $fck.cube$ are reduced from 31.3 to 42.6 MPa in the reduction of fractal dimension of a coarse aggregate (gravel) from 1.998 to 1.910. Such an effect shows the influence of space metrics in evaluation of concrete structure. Lower sensitivity is determined between indicators of a breaking stress and fractal dimensions of areas with sand prevailing (light areas of the structure) in Fig. 3 b, and fractal dimensions of pores (Fig. 3 c). There are many publications devoted to research of fractal dimensions, e.g. works [17, 18] confirm their fractal nature. In [18] fractal dimension of pores of aerated concrete is calculated at the range of 1.775…1.805, it is increased linearly with porosity, area, size and an average diameter of pores. Results from [18] gave the opportunity to use fractal dimension of pores as an integral indicator of pore structure of aerated concrete. In our case the values of a breaking stress of concrete correlate with all the elements of the structure of concrete.

As all the elements under research of macrostructure influence on physical and chemical properties of concrete, in particular, its strength, there was obtained a multiparameter fractal model (3) of their complex influence.

$$P = -1384.15 \, D_{\text{breakstone}} - 1382.56 \, D_{\text{sand}} + 5591.77 \, D_{\text{pores}} - 2963.94 \, D_{\text{bound}} \quad R^2 = 0.9254 \quad (3)$$

The model is adequate according to the statistics Durbin-Watson = 2.21.

From the practical point of view, the application of the model (3) allows to predict promptly the indicators of a breaking stress of heavy concrete on the compression without methods of microscopy. Coefficient of pair correlation of the fractal model (3) corresponds to the value of 0.9254, it confirms that metric factor is taken into account in the models of prediction of the structure and properties of concretes of various grades.
4. Summary
The application of fractal approach in the prediction of concrete quality criteria is due to its multicomponent structure, in which real elements of the structure are described with the help of Euclid’s integral geometry, however, they have a complex geometric configuration of shape. Thus, the use of traditional results of the evaluation of the structure in present models of prediction structure-properties is limited.

There was considered the approach for the evaluation of a breaking stress of heavy concrete on compression on the base of fractal analysis for determined elements of macrostructure (areas with crushed stone, sand, pores, their boundaries). The advantage of the given methods over traditional ones is the use of fractal dimension as an indicator of structural changes that is proved by a number of publications. To calculate fractal dimension of the elements of the structure of concrete, photos of 256-coloured format were used. Obtained results show the efficiency of the chosen methods for prediction of strength indicator on the base of obtained model with correlation coefficient $R^2 = 0.9254$.

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