$B \to K$ Transition Form Factor up to $\mathcal{O}(1/m_b^2)$ within the $k_T$ Factorization Approach

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Abstract

In the paper, we apply the $k_T$ factorization approach to deal with the $B \to K$ transition form factor $F_{+0}^{B \to K}(q^2)$ in the large recoil regions. The B-meson wave functions $\Psi_B$ and $\bar{\Psi}_B$ that include the three-particle Fock states’ contributions are adopted to give a consistent PQCD analysis of the form factor up to $\mathcal{O}(1/m_b^2)$. It has been found that both the wave functions $\Psi_B$ and $\bar{\Psi}_B$ can give sizable contributions to the form factor and should be kept for a better understanding of the $B$ meson decays. Then the contributions from different twist structures of the kaon wavefunction are discussed, including the $SU_f(3)$-breaking effects. A sizable contribution from the twist-3 wave function $\Psi_p$ is found, whose model dependence is discussed by taking two group of parameters that are determined by different distribution amplitude moments obtained in the literature. It is also shown that $F_{+0}^{B \to K}(0) = 0.30 \pm 0.04$ and $[F_{+0}^{B \to K}(0)/F_{+0}^{B \to \pi}(0)] = 1.13 \pm 0.02$, which are more reasonable and consistent with the light-cone sum rule results in the large recoil regions.

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I. INTRODUCTION

A study on the heavy-to-light exclusive processes plays a complementary role in determination of the fundamental parameters of the standard model and in developing the QCD theory. And there is an increasing demand for more reliable QCD calculations of the heavy-to-light form factors. We have done a consistent analysis of the $B \to \pi$ transition form factor in Refs. [1, 2], which shows that the results from the PQCD approach, the lattice QCD approach and the QCD light cone sum rules (LCSRs) are complementary to each other and by combining the results of those three approaches, one can obtain an understanding of the $B \to \pi$ transition form factor in the whole physical regions. It is argued that by applying the $k_T$ factorization approach [3, 4, 5], where the transverse momentum dependence for both the hard scattering part and the non-perturbative wavefunction, the Sudakov effects and the threshold effects are included, one can regulate the endpoint singularity from the hard scattering part effectively and derive a more reliable PQCD result of the B-meson decays. Furthermore, by applying the B-meson wave functions up to next-to-leading Fock state, Ref. [2] calculated the $B \to \pi$ transition form factor up to $\mathcal{O}(1/m_b^2)$ and also made some discussions on the reasonable regions for the two phenomenological B-meson wave function parameters $\Lambda$ and $\delta$, where $\Lambda$ is the effective mass of B meson that determines the B-meson’s leading Fock state behavior and $\delta$ is a typical parameter that determines the broadness of the B-meson transverse distribution. Since both pion and kaon are pseudo-scalar mesons, it will be interesting to give a consistent PQCD analysis of the $B \to K$ transition form factor up to order $\mathcal{O}(1/m_b^2)$ based on the results of $B \to \pi$ transition form factor.

In the literature, the $B \to K$ transition form factor has been studied under several approaches [6, 7, 8, 9, 10, 11]. A PQCD calculation has been done in Ref. [7], which can be roughly treated as a leading-order estimation $\mathcal{O}(1/m_b)$ since some of the power suppressed terms both in the hard scattering amplitude and the B-meson wave function have been neglected. The $B \to K$ transition form factor has also been analyzed by several groups under the QCD LCSR approach [8, 9, 10, 11], where some extra treatments to the correlation function either from the B-meson side or from the kaonic side are adopted to improve their LCSR estimations. New sum rule for the $B \to K$ form factor is derived by expanding the correlation function near the light cone in terms of B-meson distributions [8], in which the contributions of the quark-antiquark and quark-antiquark-gluon components in the B-meson
are taken into account. While in Ref. [9] an improved LCSR approach that had been raised in Ref. [12] was adopted to eliminate the contributions from the most uncertain kaonic twist-3 wave functions and to enhance the reliability of sum rule calculations of the $B_s \to K$ form factor. A systematic QCD LCSR calculation has been done in Ref. [10] by including the one-loop radiative corrections to the kaonic twist-2 and twist-3 contributions, and the kaonic leading-order twist-4 corrections. It can be found that the main uncertainties in estimation of the $B \to K$ transition form factor come from the B-meson wave function and the kaonic twist-2 and twist-3 wave functions.

In doing the PQCD calculations on the B-meson decays, an important issue is whether we need to take both the two wavefunctions $\Psi_B$ and $\bar{\Psi}_B$ into consideration or simply $\Psi_B$ is enough? By taking the frequently used first type definition of $\Psi_B = \frac{\Psi_B^+ + \Psi_B^-}{2}$ and $\bar{\Psi}_B = \frac{\Psi_B^+ - \Psi_B^-}{2}$, where $\Psi_B^\pm$ are defined in Ref. [13], it can be found that [14, 15] their distribution amplitudes have quite different endpoint behaviors even under the Wandzura-Wilczek (WW) approximation [16], such difference may be strongly enhanced by the hard scattering kernel. For example, the ratio between the contributions of $\bar{\Psi}_B$ and $\Psi_B$ is about ($-70\%$) [1, 17] for the $B \to \pi$ from factor in the large recoil regions. So the contribution from $\bar{\Psi}_B$ under the above definition can not be neglected and it is needed to suppress the big contribution coming from $\Psi_B$ so as to obtain a reasonable total contributions. To derive more accurate estimation, Ref. [7] raised the second type definition of $\Psi_B = \Psi_B^+$ and $\bar{\Psi}_B = (\Psi_B^+ - \Psi_B^-)$, under which the contribution from $\bar{\Psi}_B$ is of order $O(1/m_b^2)$ to that of $\Psi_B$ [18]. For convenience, in the following, we shall adopt the second type definition of $\Psi_B$ and $\bar{\Psi}_B$ to do our calculation. Then one may ask is it enough to give a $O(1/m_b^2)$ estimation with $\Psi_B$ and $\bar{\Psi}_B$ under the WW approximation? As has been pointed out in Ref. [19], the 3-particle Fock states' contributions to the B-meson wave function can be estimated by attaching an extra gluon to the internal off-shell quark line, and then $(1/m_b)$ power suppression is induced in comparison to that of the WW-part B-meson wave functions. Recently, the B-meson light-cone wave functions have been investigated up to next-to-leading order in Fock state expansion in the heavy quark limit [2]. It was shown that by using the relations between 2- and 3- particle wave functions derived from the QCD equations of motion and the heavy-quark symmetry, one can give a constraint on the transverse momentum dependence of the B-meson wave function, whose distribution tends to be a hyperbola-like curve other than a simple delta function that is derived under the WW approximation. These results provide us a chance
to give a consistent PQCD analysis of the $B \to K$ form factor up to order $O(1/m_b^2)$.

Another issue we need to be more careful is about the kaonic wave functions. The distribution amplitude (DA) for the twist-2 wave function $\Psi_K$ has been deeply studied, e.g. by the light-front quark model [20], the LCSR approach [21, 22, 23, 24] and the lattice calculation [25, 26] and etc. In Ref. [21], the QCD sum rule for the diagonal correlation function of local and nonlocal axial-vector currents is used, in which the contributions of condensates up to dimension six and the $O(\alpha_s)$-corrections to the quark-condensate term are taken into account. The first Gegenbauer moment $a^K_1(1\text{GeV})$ of the twist-2 DA derived there, i.e. $a^K_1(1\text{GeV}) = 0.05 \pm 0.02$, is consistent with that of the lattice calculations [25, 26], so we shall constrain $a^K_1(1\text{GeV})$ within this range when constructing a model for $\Psi_K$. As for the twist-3 wave function $\Psi_p$, the calculations of it has more uncertainty than that for the leading twist, e.g. its DA moments in Refs. [24, 27, 28] are quite different from each other, where the DA moments in Refs. [24, 27] are derived by using the QCD light-cone sum rules and the moments in Ref. [28] are derived based on the effective chiral action from the instanton. Under the PQCD approach, according to our experience on the $B \to \pi$ transition form factor [1] and the pion electro-magnetic form factor [29], it can be found that for a twist-3 wave function with a better endpoint behavior other than the asymptotic one, the twist-3’s contributions are indeed power suppressed to the leading twist’s contribution that favor the conventional power counting rules. In the present paper, we shall adopt two groups of DA moments [24, 28] together with the Brodsky-Huang-Lepage (BHL) prescription [30] to construct a model for $\Psi_p$, and then make a discussion on its uncertainty to the $B \to K$ transition form factor. The $SU_f(3)$-breaking effects shall also be included for constructing the kaonic wave functions.

The purpose of the paper is to reexamine the $B \to K$ transition form factor in the PQCD $k_T$ factorization approach up to $O(1/m_b^2)$. Under the $k_T$ factorization approach, the full transverse momentum dependence ($k_T$-dependence) for both the hard scattering part and the non-perturbative wave function, the Sudakov effects and the threshold effects are included to cure the endpoint singularity. Furthermore, we shall analyze the power suppressed contributions from both the wave functions and the hard scattering amplitude and then give a consistent analysis of the form factor up to $O(1/m_b^2)$, which have not been considered in the literature. In section II, we give the calculated technology for the form factor in the large recoil regions. Also we present the model wave functions of the kaon.
II. CALCULATION TECHNOLOGY FOR THE $B \rightarrow K$ TRANSITION FORM FACTOR

The $B \rightarrow K$ transition form factors $F_{+}^{B\rightarrow K}(q^2)$ and $F_{0}^{B\rightarrow K}(q^2)$ are defined as follows:

$$
\langle K(P_K)|\bar{s}\gamma_{\mu}b|B(P_B)\rangle = \left[ (P_B + P_K)_{\mu} - \frac{M_B^2 - M_K^2}{q^2}q_{\mu} \right] F_{+}^{B\rightarrow K}(q^2) + \frac{M_B^2 - M_K^2}{q^2}q_{\mu}F_{0}^{B\rightarrow K}(q^2)
$$

where $F_{+}^{B\rightarrow K}(0)$ should be equal to $F_{0}^{B\rightarrow K}(0)$ so as to cancel the poles at $q^2 = 0$. The amplitude for the $B \rightarrow K$ transition form factor can be factorized into the convolution of the wave functions for the respective hadrons with the hard-scattering amplitude. In the large recoil regions, the $B \rightarrow K$ transition form factor is dominated by a single gluon exchange in the lowest order. In Ref.[1], we have done a consistent analysis of the $B \rightarrow \pi$ transition form factor within the $k_T$ factorization approach, where the power suppressed terms up to $O(1/m_b^2)$ have been kept explicitly in the hard scattering amplitude. The interesting reader may refer to Ref.[1] for more details ¹. More specifically, for the present case, one needs to know the momentum projection for the matrix element of the kaon and B meson in deriving the hard scattering amplitude. By keeping the transverse momentum dependence in the wave function, the momentum projection for the matrix element of the kaon has the following form,

$$
M_{\alpha\beta}^{K} = \frac{if_{K}}{4}\left\{ \phi_{5}\gamma_{5}\Psi_{K}(x, k_{\perp}) - \mu_{K}\gamma_{5}\left( \Psi_{p}(x, k_{\perp}) - i\sigma_{\mu\nu}\left( n^{\mu}\bar{n}^{\nu}\frac{\Psi'_{\sigma}(x, k_{\perp})}{6} - p^{\mu}\frac{\Psi_{\sigma}(x, k_{\perp})}{6}\frac{\partial}{\partial k_{\perp}^{\nu}} \right) \right) \right\}_{\alpha\beta},
$$

where $f_{K}$ is the kaon decay constant and $\mu_{K}$ is the phenomenological parameter $\mu_{K} = M_{K}^2/(m_s + m_u)$, which is a scale characterized by the chiral perturbation theory. $\Psi_{K}(x, k_{\perp})$ is the twist-2 wave function, $\Psi_{p}(x, k_{\perp})$ and $\Psi_{\sigma}(x, k_{\perp})$ are twist-3 wave functions, respectively. $\Psi'_{\sigma}(x, k_{\perp}) = \partial\Psi_{\sigma}(x, k_{\perp})/\partial x$, $n = (\sqrt{2}, 0, 0_{\perp})$ and $\bar{n} = (0, \sqrt{2}, 0_{\perp})$ are two null vectors that

¹ Three typo errors are found in Ref.[1], i.e. in Eq.(3) $P_{0}M_{B}$ should be changed to $P_{0}+M_{B}$, in Eq.(5) the factor $[3 - \eta - x\eta]$ should be changed to $[3 - \eta + x\eta]$ and in Eq.(7) $y$ should be changed to $\eta$. 

with better endpoint behavior in the same section, which are constructed based on the BHL prescription [30] and the DA moments obtained in Ref.[24, 28]. In section III, we give our numerical results. Conclusion and a brief summary are presented in the final section.
point to the plus and the minus directions, respectively. And the momentum projection for the matrix element of the B meson can be written as \[17, 31\]:

\[
M^B_{\alpha\beta} = \left\{ \frac{i f_B + M_B}{4} \left[ \not\! p_B^\perp (\xi, \mathbf{l}_\perp) + \not\! n_B^\perp (\xi, \mathbf{l}_\perp) - \Delta(\xi, \mathbf{l}_\perp) \gamma^\mu \frac{\partial}{\partial l^\mu_\perp} \right] \gamma_5 \right\}_{\alpha\beta},
\]

where \( \xi = \frac{l^+}{M_B} \) is the momentum fraction for the light spectator quark in the B meson and \( \Delta(\xi, \mathbf{l}_\perp) = M_B \int_0^\xi d\xi' (\Psi_{B'}(\xi', 1_\perp) - \Psi_{B'}^+(\xi', 1_\perp)) \). The four-component \( l^\mu_\perp \) in Eq. (3) is defined through, \( l^\mu_\perp = l^\mu - \frac{(l^+ n^\mu + l^- \bar{n}^\mu)}{2} \) with \( l = (\frac{l^+}{\sqrt{2}}, \frac{l^-}{\sqrt{2}}, \mathbf{l}_\perp) \). By including the Sudakov form factors and the threshold resummation effects, one can obtain the formulae for the \( B \to K \) transition form factors \( F_{B \to K}^+(q^2) \) and \( F_{B \to K}^0(q^2) \) in the transverse configuration \( b \)-space, which can be simply obtained from Ref. [1] by changing the pion wave functions to the present case of kaon and by changing \( \Psi_B \) and \( \bar{\Psi}_B \) to the second type definition as described in the INTRODUCTION.

In PQCD approach, the parton transverse momenta \( \mathbf{k}_\perp \) are not negligible around the endpoint region. The relevant Sudakov factors from both \( \mathbf{k}_\perp \) and the threshold resummation \[32\] can cure the endpoint singularity which makes the calculation of the hard amplitudes infrared safe, and then the main contribution comes from the perturbative region. Also it is necessary to keep the transverse momentum dependence in the wave functions to derive a more reliable estimation in PQCD. In principle, the Bethe-Salpeter formalism \[33\] and the discretized light cone quantization approach \[34\] could determine the hadronic wave functions, but in practice there are many difficulties in getting the exact wave functions at present. The BHL prescription \[30\], which connects the equal-time wave function in the rest frame and the wave function in the infinite momentum frame, provides a useful way to use the approximate bound state solution of a hadron in terms of the quark model as the starting point for modeling the hadronic wave function. So in the present paper, we will adopt the BHL prescription for constructing the kaonic wave functions. While for the \( B \)-meson wave function, they have been investigated up to next-to-leading order in Fock state expansion in the heavy quark limit in Ref. [2], which shall be adopted to do our discussions.

A simple model has been raised in Ref. [2] for the \( B \)-meson wave functions \( \Psi_B^+ \) and \( \Psi_B^- \), which keep the main features caused by the 3-particle Fock states and whose transverse momentum dependence are still the like-function of the off-shell energy of the valence quarks but shall broaden the transverse momentum dependence under the WW approximation to
a certain degree. And in the compact parameter $b_B$-space, it reads

$$
\Psi^+_{B}(\xi, b_B) = (16\pi^3)\frac{M_B^2\xi}{\omega_0} \exp\left(-\frac{M_B\xi}{\omega_0}\right) \left(\Gamma[\delta]J_{\delta-1}[\kappa] + (1 - \delta)\Gamma[2 - \delta]J_{1-\delta}[\kappa]\right) \left(\frac{\kappa}{2}\right)^{1-\delta}
$$

(4)

and

$$
\Psi^-_{B}(\xi, b_B) = (16\pi^3)\frac{M_B^2\xi}{\omega_0} \exp\left(-\frac{M_B\xi}{\omega_0}\right) \left(\Gamma[\delta]J_{\delta-1}[\kappa] + (1 - \delta)\Gamma[2 - \delta]J_{1-\delta}[\kappa]\right) \left(\frac{\kappa}{2}\right)^{1-\delta},
$$

(5)

where $\omega_0 = 2\bar{\Lambda}/3$, $\xi = \bar{\Lambda}/M_B$ and $\kappa = \theta(2\bar{\xi} - \xi)\sqrt{\xi(2\bar{\xi} - \xi)M_Bb_B}$. The factor $(16\pi^3)$ is introduced to ensure their Fourier transformation, i.e. $\Psi^\pm_{B}(\xi, k_{\perp})$, satisfy the normalization,

$$
\int d\xi d^{2}k_{\perp} 16\pi^3 \Psi^\pm_{B}(\xi, k_{\perp}) = 1.
$$

It can be found that both $\Psi^+_B$ and $\Psi^-_B$ have the same transverse momentum dependence and only two phenomenological parameters $\bar{\Lambda}$ and $\delta$ are introduced. $\bar{\Lambda}$ is the effective mass of $B$ meson, $\bar{\Lambda} = M_B - m_b$, which determines the $B$ meson’s leading Fock state behavior. $\delta$ is a typical parameter that determines the broadness of the $B$-meson transverse distribution in comparison to the WW-like one. The WW-like $B$-meson wave functions in the compact parameter $b_B$-space can be found in Ref.[1]. And a direct comparison shows that when $\delta \rightarrow 1$, the transverse momentum dependence of the $B$-meson wave function in Eqs.(4,5) returns to a simple $\delta$-function, which is the same as that of the $B$-meson wave function under the Wandzura-Wilczek approximation [14, 35]. According to the definitions, we have $\Psi_{B}(\xi, b_B) = \Psi^+_{B}(\xi, b_B)$, $\Psi_{B}(\xi, b_B) = \Psi^-_{B}(\xi, b_B) - \Psi^+_{B}(\xi, b_B)$ and $\Delta(\xi, b_B) = -M_B \int_0^{\xi} d\xi' \Psi_{B}(\xi', b_B)$.

Next, we construct the kaonic twist-2 wave function based on its first Gegenbauer moment $a^K_1$ and on the BHL prescription [30]. The first Gegenbauer moment $a^K_1$ has been studied by the light-front quark model [20], the LCSR approach [21, 22, 24] and the lattice calculation [25, 26] and etc. In Ref.[21], the QCD sum rule for the diagonal correlation function of local and nonlocal axial-vector currents is used, in which the contributions of condensates up to dimension six and the $O(\alpha_s)$-corrections to the quark-condensate term are taken into account. The moments derived there are close to that of the lattice calculation [25, 26], so we shall take $a^K_1(1\text{GeV}) = 0.05 \pm 0.02$ to determine the model wave function $\Psi_{K}$. Based on the BHL prescription, we take the twist-2 wave function of kaon as

$$
\Psi_{K}(x, k_{\perp}) = [1 + B_{K}C_{1}^{3/2}(2x - 1)] \times \frac{A_{K}}{x(1 - x)} \exp\left[ -\beta_{K}^2 \left( \frac{k_{\perp}^2 + m_q^2}{x} + \frac{k_{\perp}^2 + m_s^2}{1 - x} \right) \right],
$$

(6)

where $q = u, d$, $C_{1}^{3/2}(1 - 2x)$ is the Gegenbauer polynomial. In comparison to the pion wave function (e.g. [30]), it can be found that the $SU_f(3)$ symmetry is broken by a non-zero $B_{K}$
and by the mass difference between the s quark and u (or d) quark in the exponential factor. The $SU_f(3)$ symmetry breaking in the lepton decays of heavy pseudoscalar mesons and in the semileptonic decays of mesons have been studied in Ref.[37]. For definiteness, we take the conventional values for the constitute quark masses: $m_q = 0.30 \text{GeV}$ and $m_s = 0.45 \text{GeV}$. The parameters $A_K, B_K$ and $\beta_K$ can be determined by the value of $a_1^K$ together with the normalization condition:

$$
\int^1_0 dx \int^{k^2_\perp < \mu^2_0} \frac{d^2k^\perp}{16\pi^3} \Psi_K(x, k^\perp) = 1
$$

(7)

and the constraint $(k^2_\perp/K)^{1/2} \approx (k^2_\perp/\pi)^{1/2} = 0.350 \text{GeV}$ [38], where the average value of the transverse momentum square is defined as

$$
\langle k^2_\perp/K \rangle = \frac{\int dx d^2 k^\perp |k^2_\perp| |\Psi_K(x, k^\perp)|^2}{\int dx d^2 k^\perp |\Psi_K(x, k^\perp)|^2}.
$$

The parameter $\mu_0$ in the model wave function stands for some hadronic scale that is of order $O(1 \text{ GeV})$. For clarity, we set $\mu_0 = 1 \text{ GeV}$. The DA $\phi_K(x, \mu_0)$ is defined as $\phi_K(x, \mu_0) = \int^{k^2_\perp < \mu^2_0} \frac{d^2k^\perp}{16\pi^3} \Psi_K(x, k^\perp)$. The first Gegenbauer moment $a_1^K(\mu_0)$ of Refs.[21, 22, 24] can be defined as

$$
a_1^K(\mu_0) = \frac{\int^1_0 dx \phi_K(1-x, \mu_0) C^{3/2}_1(2x-1)}{\int^1_0 dx 6x(1-x)[C^{3/2}_1(2x-1)]^2},
$$

(8)

where $\phi_K(1-x, \mu_0)$ other than $\phi_K(x, \mu_0)$ should be adopted, since in Refs.[21, 22, 24] $x$ stands for the momentum fraction of s-quark in the kaon ($K$), while in the present paper we take $x$ as the momentum fraction of the light $q$-(anti)quark in the kaon ($K$)². Based on the above discussions, we can obtain the values for $A_K, B_K$ and $\beta_K$:

$$
A_K \approx 2.71 \times 10^2 \text{GeV}^{-1}, \quad B_K \approx [0.116 - 0.9a_1^K(\mu_0)], \quad \beta_K \approx 0.877 \text{GeV}^{-1},
$$

(9)

where the values of $A_K$ and $\beta_K$ are almost constant, i.e. their changes ($\delta A_K/A_K$) and ($\delta \beta_K/\beta_K$) are less than 0.001 by varying $a_1^K(\mu_0)$ within the range of $[0.03, 0.07]$. More specifically, for the case of $a_1^K(\mu_0) = 0.05$, we have

$$
A_K = 2.71 \times 10^2 \text{GeV}^{-1}, \quad B_K = 0.071, \quad \beta_K = 0.877 \text{GeV}^{-1}.
$$

As will be seen that the contributions from twist-3 wave function $\Psi_{\sigma}(x, k^\perp)$ is less important in comparison to that of $\Psi_K(x, k^\perp)$ and $\Psi_{\rho}(x, k^\perp)$, which is similar to the case of

² In the literature, there are some ambiguities in use of $\phi_{K,\rho}(x, \mu_0)$ or $\phi_{K,\rho}(1-x, \mu_0)$ in connection to the hard scattering part. This will cause errors when the $SU_f(3)$-symmetry is broken.
FIG. 1: Kaon $\phi_p(x, \mu_0)$ with its parameters determined by the two groups of DA moments \[24, 28\].

The solid line and the dashed line are for $\phi^1_p(x, \mu_0)$ and $\phi^2_p(x, \mu_0)$ respectively. For comparison, the big dotted line and the dash-dot line are for $\phi^{sr}_p(x, \mu_0)$ \[24\] and $\phi^{in}_p(x, \mu_0)$ \[28\] respectively. The dotted line is the asymptotic behavior of $\phi^{as}_p(x, \infty) = 1$.

$B \rightarrow \pi$ transition from factor \[1\]. So basing on the BHL prescription, we directly take the twist-3 wave function $\Psi_\sigma$ of kaon as

$$
\Psi_\sigma(x, \mathbf{k}_\perp) = A_\sigma \exp \left[ -\beta_\sigma^2 \left( \frac{k^2_\perp + m^2_q}{x} + \frac{k^2_\perp + m^2_s}{1 - x} \right) \right],
$$

where $A_\sigma$ can be determined by its normalization condition, i.e. $A_\sigma = 1.36 \times 10^3 \text{GeV}^{-1}$.

As for the twist-3 wave function $\Psi_p(x, \mathbf{k}_\perp)$, its DA’s asymptotic behavior, $\phi^{as}_p(x, \infty) = 1$, so its endpoint singularity is much more serious. Then the transverse momentum dependence of $\Psi_p(x, \mathbf{k}_\perp)$ is much more important than that of $\Psi_K(x, \mathbf{k}_\perp)$ and $\Psi_\sigma(x, \mathbf{k}_\perp)$ in order to cure the endpoint singularity. One can construct $\Psi_p(x, \mathbf{k}_\perp)$ in the following form,

$$
\Psi_p(x, \mathbf{k}_\perp) = \left[ 1 + B_p C_1^{1/2}(2x - 1) + C_p C_2^{1/2}(2x - 1) \right] \times \frac{A_p}{x(1 - x)} \exp \left[ -\beta_K^2 \left( \frac{k^2_\perp + m^2_q}{x} + \frac{k^2_\perp + m^2_s}{1 - x} \right) \right],
$$

where $x$ stands for the light quark $q$’s momentum fraction, $C_1^{1/2}(2x - 1)$ and $C_2^{1/2}(2x - 1)$ are Gegenbauer polynomials and the coefficients $A_p$, $B_p$ and $C_p$ can be determined by its DA moments. The DA $\phi_p(x, \mu_0)$ is defined as $\phi_p(x, \mu_0) = \int_{k^2_\perp < \mu_0^2} \frac{d^2k_\perp}{16\pi^2} \Psi_p(x, \mathbf{k}_\perp)$.

To discuss the uncertainty caused by $\Psi_p$, we take two groups of DA moments that have been obtained in Refs.\[24, 28\] to determine the coefficients $A_p$, $B_p$ and $C_p$, where the mo-
ments in Ref. [24] are derived by using the QCD light-cone sum rules and the moments in Ref. [28] are derived based on the effective chiral action from the instanton:

\[
\begin{align*}
\text{Group 1} \ [23]: & \quad \langle x^0 \rangle^K_p = 1, \quad \langle x^1 \rangle^K_p = 0.06124, \quad \langle x^2 \rangle^K_p = 0.36757, \\
\text{Group 2} \ [27]: & \quad \langle x^0 \rangle^K_p = 1, \quad \langle x^1 \rangle^K_p = 0.00678, \quad \langle x^2 \rangle^K_p = 0.35162.
\end{align*}
\] (12)

Here the moments are defined as \(\langle x^i \rangle^K_p = \int_0^1 dx (2x - 1)^i \phi_p(1 - x, \mu_0)\) with \(i = (0, 1, 2)\). It should be noted that the moments defined in Ref. [24, 28] are for \(\phi_p(1 - x, \mu_0)\) other than \(\phi_p(x, \mu_0)\), since in these references \(x\) stands for the momentum fraction of \(s\)-quark in the kaon (\(\bar{K}\)), while in the present paper \(x\) stands for the momentum fraction of the light quark \(q\) in the kaon (\(K\)). Taking the above two groups of DA moments for \(\phi_p\), the parameters of \(\Psi_p(x, \vec{k}_\perp)\) can be determined as,

\[
\begin{align*}
\text{Group 1}: & \quad A^1_p = 382.\text{GeV}^{-1}, \quad B^1_p = 0.311, \quad C^1_p = 1.61, \\
\text{Group 2}: & \quad A^2_p = 422.\text{GeV}^{-1}, \quad B^2_p = 0.257, \quad C^2_p = 1.52.
\end{align*}
\] (14) (15)

The distribution amplitudes for these two groups of parameters are shown in Fig. (1), where \(\phi^1_p(x, \mu_0)\) is determined by Group 1 parameters and \(\phi^2_p(x, \mu_0)\) is determined by Group 2 parameters respectively. For comparison, we also draw the distributions derived in Ref. [24, 28] in Fig. (1), i.e. \(\phi^{sr}_p(x, \mu_0)\) stands for the DA obtained in Ref. [24] and \(\phi^{in}_p(x, \mu_0)\) stands for that of Ref. [28]. One may observe that different from \(\phi^{sr}_p(x, \mu_0)\) and \(\phi^{in}_p(x, \mu_0)\), both \(\phi^1_p(x, \mu_0)\) and \(\phi^2_p(x, \mu_0)\) are double humped curves and are highly suppressed in the endpoint region. Such feature is necessary to suppress the endpoint singularity coming from the hard-scattering kernel and then to derive a more reasonable results for the twist-3 contributions to the \(B \to K\) form factor.

It is more convenient to transform the kaon wave functions into the compact parameter \(b_K\)-space, which can be done with the help of the Fourier transformation

\[
\Psi(x, b_K) = \int_{|\vec{k}_\perp| < 1/b_K} d^2\vec{k}_\perp \exp (-i\vec{k}_\perp \cdot \vec{b}_K) \Psi(x, \vec{k}_\perp),
\]

where \(\Psi\) stands for \(\Psi_K, \Psi_p\) and \(\Psi_\sigma\), respectively. The upper edge of the integration \(|\vec{k}_\perp| < 1/b_K\) is necessary to ensure that the wave function is soft enough [39]. After doing the Fourier transformation, we obtain the kaonic wave functions in the compact parameter \(b_K\)-space:

\[
\Psi_K(x, b_K) = \frac{2\pi A_K}{x(1 - x)} [1 + B_K C_1^{3/2} (2x - 1)] \exp \left[ -\beta^2_K \left( \frac{m_s^2}{1 - x} + \frac{m_q^2}{x} \right) \right]
\]
\[
\times \int_0^{1/b_K} \exp \left( -\beta_k^2 \frac{k_\perp^2}{x(1-x)} \right) J_0(b_K k_\perp) k_\perp dk_\perp,
\]

\[
\Psi_\sigma(x, b_K) = 2\pi A_\sigma \exp \left[ -\beta_k^2 \left( \frac{m_s^2}{1-x} + \frac{m_u^2}{x} \right) \right] \int_0^{1/b_K} \exp \left( -\beta_k^2 \frac{k_\perp^2}{x(1-x)} \right) J_0(b_K k_\perp) k_\perp dk_\perp,
\]

and

\[
\Psi_p(x, b_K) = \frac{2\pi A_p}{x(1-x)} \left[ 1 + B_p C_1^{1/2}(2x-1) + C_p C_2^{1/2}(2x-1) \right] \exp \left[ -\beta_k^2 \left( \frac{m_s^2}{1-x} + \frac{m_u^2}{x} \right) \right] \times \int_0^{1/b_K} \exp \left( -\beta_k^2 \frac{k_\perp^2}{x(1-x)} \right) J_0(b_K k_\perp) k_\perp dk_\perp.
\]

### III. NUMERICAL CALCULATIONS

In the numerical calculations, we adopt

\[
\Lambda^{(n_f=4)} = 250\text{MeV}, \quad f_B = 190\text{MeV}, \quad M_B = 5.279\text{GeV}, \quad f_K = 160\text{MeV}, \quad M_K = 494\text{MeV}.
\]

As for the phenomenological parameter \(\mu_K = M_K^2/(m_s + m_u)\), which is a scale characterized by the chiral perturbation theory, we take its value to be \(\mu_K \simeq 1.70\ \text{GeV}\).

In the following, we first discuss the properties of \(F_B^{B\rightarrow K}(q^2)\) and \(F_0^{B\rightarrow K}(q^2)\) that are calculated up to \(O(1/m_b^2)\), i.e. to show how \(F_B^{B\rightarrow K}(q^2)\) and \(F_0^{B\rightarrow K}(q^2)\) are affected by the B-meson wave function and the kaonic wave functions. The B-meson wave functions \(\Psi_B\) and \(\bar{\Psi}_B\) up to next-to-leading order Fock state expansion depend on two phenomenological parameters \(\bar{\Lambda}\) and \(\delta\). An estimate of \(\bar{\Lambda}\) using QCD sum rule approach gives \(\bar{\Lambda} = 0.57 \pm 0.07\text{GeV}\) [40]. By comparing the PQCD results of the \(B \rightarrow \pi\) form factor with the QCD LCSR results and the lattice QCD calculations, Ref. [1] shows that \(\bar{\Lambda} = 0.55 \pm 0.05\text{GeV}\). As for the value of \(\delta\), it has been pointed out that if the contribution from the B-meson three-particle wave function is limited to be within \(\pm 20\%\) of that of the WW-like wave function within the energy region of \(Q^2 \in [0, \sim 10\text{GeV}^2]\), then the value of \(\delta\) should be restricted within the region of \([0.25, 0.30]\) [2]. For clarity, we take the same regions as obtained from the \(B \rightarrow \pi\) case [1, 2] for both \(\bar{\Lambda}\) and \(\delta\), i.e. \(\bar{\Lambda} \in [0.50, 0.60]\text{GeV}\) and \(\delta \in [0.25, 0.30]\), to study the form factors \(F_B^{B\rightarrow K}(q^2)\) and \(F_0^{B\rightarrow K}(q^2)\) in the large and intermediate energy regions. Furthermore, according to the discussion in the last section, the remaining uncertainty of the kaonic twist-2 wave function \(\Psi_K\) is caused by the value of \(a_1^K(1\text{GeV})\), cf. Eq. (9). There we take \(a_1^K(1\text{GeV}) = 0.05 \pm 0.02\) [21] to do our discussion. As for the twist-3 wave function \(\Psi_p\), we take two groups of parameters as shown in Eqs. (14,15) to do the calculation.
Next, we compare the $\mathcal{O}(1/m_b^2)$ result of the form factor with the leading order one that is of order $\mathcal{O}(1/m_b)$ and is calculated by using the WW-like B-meson wave function, and also make a comparison with the LCSR results of Ref. [8, 10] in the large and intermediate energy regions. Through comparison, preferable values for the undetermined parameters can be found. The $B \to K$ transition form factors $F_{+0}^{B\to K}(q^2)$ and $F_{00}^{B\to K}(q^2)$ have been studied within the framework of QCD LCSR [10], especially at $q^2 = 0$, it shows

$$F_{+0}^{B\to K}(0) = 0.331 \pm 0.041 + 0.25[a_1^K(1\text{GeV}) - 0.17],$$  \hspace{1cm} (16)$$
eq \text{e.g. when } a_1^K(1\text{GeV}) = 0.05, F_{+0}^{B\to K}(0) = 0.301 \pm 0.041. \text{ More generally, } F_{+0}^{B\to K}(q^2) \text{ and } F_{00}^{B\to K}(q^2) \text{ can be parameterized in the following form [10]:}$$

$$F_{+0}^{B\to K}(q^2) = f^{as}(q^2) + a_1^K(\mu_0)f_{a}^{aK}(q^2) + a_2^K(\mu_0)f_{a}^{aK}(q^2) + a_4^K(\mu_0)f_{a}^{aK}(q^2),$$  \hspace{1cm} (17)$$

where $f^{as}$ contains the contributions to the form factor from the asymptotic DA and all higher-twist effects from three-particle quark-quark-gluon matrix elements, $f_{a}^{aK,a}K_{a}^K$ contains the contribution from the higher Gegenbauer term of DA that is proportional to $a_1^K$, $a_2^K$ and $a_4^K$ respectively. Here the factorization scale $\mu_0$ should be taken as $2.2\text{GeV}$, since the functions $f^{as,a}K_{a}^K,a_2^K,a_4^K$ are determined with $\mu_0 = 2.2\text{GeV}$ [10]. The explicit expressions of $f^{as,a}K_{a}^K,a_2^K,a_4^K$ can be found in Table V and Table IX of Ref. [10]. For the Gegenbauer moments $a_2^K(2.2\text{GeV})$ and $a_4^K(2.2\text{GeV})$, we take their preferred values: $a_2^K(2.2\text{GeV}) = 0.080$ and $a_4^K(2.2\text{GeV}) = -0.0089$ [10]. While for $a_1^K(2.2\text{GeV})$, it equals to $0.793a_1^K(1\text{GeV})$ with the help of QCD evolution.

A. Basic properties of the form factor up to $\mathcal{O}(1/m_b^2)$

First, we discuss the properties of $F_{+0}^{B\to K}(q^2)$ and $F_{00}^{B\to K}(q^2)$ caused by the B-meson wave function. For such purpose, we fix the kaonic wave functions by setting $a_1^K(1\text{GeV}) = 0.05$ and by using the Group 1 parameters for $\Psi_p$. We show the $B \to K$ transition form factors $F_{+0}^{B\to K}(q^2)$ and $F_{00}^{B\to K}(q^2)$ with $\delta = \delta_c = 0.275$ in Fig. [2], where $\bar{\Lambda}$ varies within the region of $[0.5\text{GeV}, 0.6\text{GeV}]$. For comparison, we show the QCD LCSR result with $a_1^K(1\text{GeV}) = 0.05$ and its theoretical error ($\sim \pm 10\%$) [10] by a fuscous shaded band in Fig. [2]. The results show that the $B \to K$ transition form factors will decrease with the increment of $\bar{\Lambda}$. And the best fit of the QCD LCSR result at $q^2 = 0$ shows that $\bar{\Lambda} \cong \bar{\Lambda}_c = 0.525\text{GeV}$. Moreover, we show
FIG. 2: PQCD results for the $B \to K$ transition form factors $F^{B\to K}_{\pm}(q^2)$ (Left) and $F^{B\to K}_{0}(q^2)$ (Right) with $\delta = 0.275$ and $a_{1}^{K}(1\text{GeV}) = 0.05$. The dash-dot line, the dashed line and the dotted line stand for $\bar{\Lambda} = 0.50$ GeV, 0.55 GeV and 0.60 GeV respectively. For comparison, the solid line comes from the QCD LCSR result as shown in Eq.(17) and the fuscous shaded band shows its theoretical error $\pm 10\%$.

FIG. 3: PQCD results for the $B \to K$ transition form factors $F^{B\to K}_{\pm}(q^2)$ (Left) and $F^{B\to K}_{0}(q^2)$ (Right) with $\bar{\Lambda} = 0.525$ GeV and $a_{1}^{K}(1\text{GeV}) = 0.05$. The dotted line, the dashed line and the dash-dot line stand for $\delta = 0.25, 0.275$ and 0.30 respectively. For comparison, the solid line comes from the QCD LCSR as shown in Eq.(17) and the fuscous shaded band shows its theoretical error $\pm 10\%$. 
FIG. 4: PQCD results for the $B \rightarrow K$ transition form factors $F_{+}^{B \rightarrow K}(q^2)$ (Left) and $F_{0}^{B \rightarrow K}(q^2)$ (Right) with $\bar{\Lambda} = 0.525$ GeV and $\delta = 0.275$. The dotted line, the dashed line and the dash-dot line stand for $a_{1}^{K}(1\text{GeV}) = 0.03$, 0.05 and 0.07 respectively.

$F_{+}^{B \rightarrow K}(q^2)$ and $F_{0}^{B \rightarrow K}(q^2)$ with $\bar{\Lambda} = \bar{\Lambda}_c = 0.525$ GeV in Fig.3, where $\delta$ varies within the region of [0.25, 0.30]. The results show that the $B \rightarrow K$ transition form factors will increase with the increment of $\delta$. It can be found that when setting $a_{1}^{K}(1\text{GeV}) = 0.05$, and by varying $\delta$ within the region of [0.25, 0.30] and $\bar{\Lambda}$ within the region of [0.5GeV, 0.6GeV], $F_{+}^{B \rightarrow K}(0)$ runs within the region of [0.23,0.34]. since the best agreement between the PQCD result and the QCD LCSR result at $q^2 = 0$ is obtained around $\bar{\Lambda}_c = 0.525$GeV and $\delta_c = 0.275$, we shall always take $\bar{\Lambda} = \bar{\Lambda}_c$ and $\delta = \delta_c$ to do our following calculations if not specially stated.

Second, we discuss the properties of $F_{+}^{B \rightarrow K}(q^2)$ and $F_{0}^{B \rightarrow K}(q^2)$ caused by the twist-2 wave function $\Psi_K$, i.e. by the value of $a_{1}^{K}(1\text{GeV})$. For such purpose, we fix the B-meson wave functions by setting $\delta = \delta_c$ and $\bar{\Lambda} = \bar{\Lambda}_c$ and by using the Group 1 parameters for $\Psi_p$. We show the $B \rightarrow K$ transition form factors $F_{+}^{B \rightarrow K}(q^2)$ and $F_{0}^{B \rightarrow K}(q^2)$ in Fig.4 with $a_{1}^{K}(1\text{GeV}) = 0.03, 0.05$ and 0.07 respectively. It can be found that the form factors shall be increased with the increment of $a_{1}^{K}(1\text{GeV})$, which agree with the observation of Ref.[10].

Furthermore, since the contribution from $\Psi_p$ is sizable to that of $\Psi_K$, it is necessary to make a discussion on its uncertainty to the $B \rightarrow K$ transition form factor. Fig.5 shows $F_{+}^{B \rightarrow K}(q^2)$ and $F_{0}^{B \rightarrow K}(q^2)$ with two groups of parameters for $\Psi_p$. The results are very close to each other due to the close shape of their $\phi_p$ as shown in Fig.1, e.g. around the region of $q^2 \sim 0$ the difference between them is less than 6%. So by taking proper transverse momentum
FIG. 5: PQCD results for the $B \rightarrow K$ transition form factors $F^{B\rightarrow K}_{+}(q^2)$ and $F^{B\rightarrow K}_{0}(q^2)$ with $\Lambda = 0.525\, GeV$, $\delta = 0.275$ and $a^{K}_{1}(1\, GeV) = 0.05$. The dash-dot and the dashed lines are for $\Psi_p$ with Group 1 parameters Eq. (14), Group 2 parameters Eq. (15) respectively. For comparison, the solid lines come from the QCD LCSR with $a^{K}_{1}(1\, GeV) = 0.05$ [10].

FIG. 6: PQCD results for the $B \rightarrow K$ transition form factor $F^{B\rightarrow K}_{+}(q^2)$ with fixed $\lambda = 0.55\, GeV$, $\delta = 0.275$ and $a^{K}_{1}(1\, GeV) = 0.05$. The left diagram is for the different kaon twist structures, $\Psi_K$, $\Psi_p$ and $\Psi_\sigma$. The right diagram is for the different B meson structures, $\Psi_B$, $\bar{\Psi}_B$ and $\Delta$.

dependence for the wave function $\Psi_p$, where we have taken the BHL prescription for its transverse momentum dependence, the uncertainties from its distribution amplitude $\phi_p$ can be reduced.

Finally, in order to get a deep understanding of the $B \rightarrow K$ transition form factor, we discuss the contributions from different parts of the B-meson wave function or the kaon
wave function, correspondingly. Here we take $F_{\pm B-K}(q^2)$ to do our discussions and the case of $F_{0 B-K}(q^2)$ can be done in a similar way. For convenience, we set $\bar{\Lambda} = \bar{\Lambda}_c$, $\delta = \delta_c$, $a_1^K(1\text{GeV}) = 0.05$ and by using the Group 1 parameters for $\Psi_p$. When discussing the contribution from one of the kaon wave function structures, the contribution from all the B-meson wave function structures are summed up, and vice versa. Fig. (6a) shows the contributions from the different twist structures of the kaon wave function, i.e. $\Psi_K$, $\Psi_p$ and $\Psi_{\sigma}$ (the contributions from the terms involving $\Psi'_{\sigma}$ are included in $\Psi_{\sigma}$), respectively. One may observe that the contribution from $\Psi_p$ is comparable to that of $\Psi_K$, e.g. its contribution is about 70% of that of $\Psi_K$ at $q^2 \simeq 0$, and the contribution from $\Psi_{\sigma}$ is small. Fig. (6b) presents the contributions from $\Psi_B$, $\bar{\Psi}_B$ and $\Delta$ respectively. It can be found that the contribution from $\bar{\Psi}_B$ is about 50% – 67% in comparison to that of $\Psi_B$ in the region of $q^2 \in [0, 10\text{GeV}^2]$, while the contribution from $\Delta$ is negligible in comparison to that of $\Psi_B$ and $\bar{\Psi}_B$. So the contribution from $\bar{\Psi}_B$ should be included for a consistent estimation to the next leading order. As a comparison, it can be found that under the leading order estimation the contribution from $\Psi_B$ is only about 20% in comparison to that of $\Psi_B$ at $q^2 = 0$, which agree with the rough order estimation that the contribution from $\bar{\Psi}_B$ is of order $O(1/m_b)$. So as to the leading order estimation $O(1/m_b)$, the contribution from $\bar{\Psi}_B$ is usually neglected in the literature. Such difference of $\bar{\Psi}_B$’s contribution between the leading order estimation and the next-to-leading order estimation is mainly due to the fact that the transverse momentum dependence of the B-meson wave functions are merely a delta function under the WW-approximation (the leading-order estimation), while it shall be broadened to a certain degree according to the value of $\delta$ by taking into account the 3-particle Fock states’ contributions (the next-to-leading order estimation), cf. fig.(2) of Ref.[2]. So qualitatively, the contributions from $\bar{\Psi}_B$ shall be raised to a certain degree for the next-to-leading order case, due to the less suppression of the end-point region ($\xi \to 0$) from the transverse momentum distributions than that of the leading order case. And then the naive order estimation for the contribution of $\bar{\Psi}_B$ is no longer correct, and the contributions from $\Psi_B$ and $\bar{\Psi}_B$ are both important in the next-to-leading order calculation.
The WW-like B-meson wave functions in the compact parameter $b_B$-space can be found in Ref. [1]. Taking the WW-like wave functions and cutting off the power suppressed terms in the hard scattering amplitude, we can obtain the leading order results ($O(1/m_b)$) for the form factors $F_{B+K}(q^2)$ and $F_{B+K}^+(q^2)$. Strictly, one should cut off the contribution from $\bar{\Psi}_B$ to obtain the leading order estimation, since $\bar{\Psi}_B$ is power suppressed in comparison to $\Psi_B$. However for easy comparison with the results in the literature, e.g. Ref. [7], we keep $\bar{\Psi}_B$ in the leading order estimation. For convenience, we take $\tilde{\Lambda} = \tilde{\Lambda}_c$, $\delta = \delta_c$, $a_1^K(1\text{GeV}) = 0.05$ and by using the Group 1 parameters for the wave function $\Psi_p$ to do a comparison of the leading order results with the total results that include the contributions up to order $O(1/m_b^2)$. It can be found that the leading order results are smaller than the total results by about 25% in the large recoil region, e.g. at $q^2 = 0$, the leading order $F_{B+K}^+(0) = 0.229$. One may observe that a larger leading order estimation has been obtained in Ref. [7], which shows $F_{B+K}^+(0) = 0.321 \pm 0.036$. We argue that the present leading order estimation is more reliable, and the larger value of $F_{B+K}^+(0)$ derived in Ref. [7] is mainly due to the following two reasons: 1) Even though the Sudakov and threshold resummation factors shall kill the endpoint singularity of the process [5, 7, 18, 19], the transverse momentum dependence of kaonic wave functions are still important to give a more reliable PQCD estimation, which is similar to the cases of $B \to \pi$ form factor [1] and the pion electromagnetic form factor [29]. In Ref. [7] the transverse momentum dependence of kaonic wave functions are lacking, i.e. the distribution amplitude other than the wave function is used. While in our present calculation, the BHL-prescription is adopted for the kaonic transverse momentum dependence. As for the wave function $\Psi_p(x, k_\perp)$, such transverse momentum dependence will results in a double humped DA $\phi_p$ as shown in Fig. and then it shall give more effective suppression in the end-point region than the one used in Ref. [7]. In fact, it can be found that the contributions from the end point region shall always be overestimated without taking the transverse momentum into the twist-3 wave function $\Psi_p(x, k_\perp)$ $^3$. Furthermore, by taking proper transverse momentum dependence for the wave function $\Psi_p$, the uncertainties from its distribution amplitude $\phi_p$ can be reduced as has been discussed in Sec.III.A; 2) the distribution amplitude of $\Psi_K$ with

\[ ^3 \text{For example, a detailed discussion on the model dependence of pionic twist-3 wave function } \Psi_p(x, k_\perp) \text{ can be found in Ref. [29].} \]
a much bigger value of $a^K_1(1\text{GeV})$, i.e. $a^K_1(1\text{GeV}) = 0.17$, is adopted by Ref.\[7\]. Since the form factors increases with the increment of $a^K_1(1\text{GeV})$, a larger value of $a^K_1(1\text{GeV})$ shall increase the form factors.

Furthermore, by varying $\bar{\Lambda}$ within the region of $[0.50, 0.55]$, the uncertainty caused by $\bar{\Lambda}$ is the biggest and is of order $(1/m_b)$. While by varying $\delta$ within the region of $[0.25, 0.30]$, the uncertainty caused by $\delta$ is smaller and are of order $(1/m_b^2)$. This can be qualitatively explained as that $\bar{\Lambda}$ is the characteristic parameter that determines the leading Fock-state behavior of the $B$-meson wave functions, while $\delta$ is the characteristic parameter that determines the higher Fock-state’s behavior of the $B$-meson wave functions. The uncertainties from $a^K_1$ and $\Psi_K$ are less than 10\% in the large recoil region.

C. Comparison with the LCSR results

The $B \to K$ transition form factor have been analyzed by several groups under the QCD LCSR approach $[8, 9, 10, 11]$. New sum rule for $B \to K$ is derived from the correlation functions expanded near the light cone in terms of B-meson distributions $[8]$, in which the contributions of the quark-antiquark and quark-antiquark-gluon components in the B-meson are taken into account. It has been found that the $B \to K$ transition form factor in the large recoil region does not receive contributions from the 3-particle B-meson DA’s. One may observe that if substituting the B-meson DAs, which are derived by doing the integration over $b_B$ in Eqs.(15), into the formulae of Ref.[8], then one can obtain the same results as that of Ref.[8], since our B-meson DAs are close to the exponential model wave functions adopted in Ref.[8]. Furthermore, one may observe that the result of $F_{+,0}^{B \to K}(0) = 0.31 \pm 0.04$ under the condition of $a^K_1(1\text{GeV}) = 0.05 \pm 0.03$ agrees well with our present PQCD estimation. Secondly, a systematic QCD LCSR calculation has been done in Ref.[10] by including the one-loop radiative corrections to the twist-2 and twist-3 contributions, and leading-order twist-4 corrections. Some comparison of their results with our present one can be found in Figs.(2,3,4), which also shows a good agreement within reasonable errors. For example, from Eq.(17) it can be found that the uncertainty of form factor caused by $a^K_1(1\text{GeV})$ within the region of $[0.03, 0.07]$ is less than 5\%, which is consistent with our present result as shown in Fig.(4).
IV. DISCUSSION AND SUMMARY

In this paper, we have examined the $B \rightarrow K$ transition form factor in the PQCD approach up to order $O(1/m_b^2)$, where the transverse momentum dependence for the wave function, the Sudakov effects and the threshold effects are included to regulate the endpoint singularity and to derive a more reasonable result. We have confirmed that the PQCD approach can be applied to calculate the $B \rightarrow K$ transition form factor in the large recoil regions. We emphasize that the transverse momentum dependence for both the $B$ meson and the kaon is important to give a better understanding of the $B \rightarrow K$ transition form factor. Fig. (6a) shows that the contribution from the pionic twist-3 wave function $\Psi_p$ is sizable in comparison to that of $\Psi_K$, and the contribution from $\Psi_\sigma$ is small. While Fig. (6b) shows that by using the $B$-meson wave functions up to next-to-leading order Fock state expansion, the contributions from $\Psi_B$ and $\bar{\Psi}_B$ are important.

In Refs. [1, 2], we have shown that the results from the PQCD approach, the lattice QCD approach and the QCD LCSRs are complementary to each other and by combining the results of those three approaches, one can obtain an understanding of the $B \rightarrow \pi$ transition form factor in the whole physical regions. And the best fit of the PQCD results with that of the QCD LCSR results in the large recoil region can be obtained by taking $\bar{\Lambda} \in [0.50, 0.60]$ and $\delta \in [0.25, 0.30]$ [2]. In the present paper, we show that within the regions of $\bar{\Lambda} \in [0.50, 0.55]$, $\delta \in [0.25, 0.30]$ and $a_1^K (1\text{GeV}) \in [0.03, 0.07]$, the PQCD results on the $B \rightarrow K$ form factor in the large recoil region also agree well with that of the QCD LCSR results [8, 10]. Our present PQCD results in some sense is more reliable than the LCSR calculations due to the fact that by taking the transverse momentum dependence properly for the wave functions the soft endpoint singularity have been effectively suppressed, e.g. as is shown in Sec.III.A the difference between the two models for $\Psi_p$ is less than 6% in the large recoil region, while for the LCSR approach large uncertainty comes from the kaonic twist-3 DA $\phi_p$ that is not too well-known. By running the parameters within the above regions, we obtain $F_{+,0}^{B \rightarrow K}(0) = 0.30 \pm 0.04$. Finally, to illustrate the $SU_f(3)$-breaking effects, we calculated the ratio with the help of the $B \rightarrow \pi$ results in Ref. [2]:

$[F_{+,0}^{B \rightarrow K}(0)/F_{+,0}^{B \rightarrow \pi}(0)] = 1.13 \pm 0.02$, which favors a small $SU_f(3)$-breaking effects.
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[1] T. Huang and X.G. Wu, Phys.Rev. D71, 034018(2005).
[2] T. Huang, C.F. Qiao and X.G. Wu, Phys.Rev. D73, 074004(2006).
[3] G.P. Lepage and S.J. Brodsky, Phys.Rev. D22, 2157(1980), ibid. D24, 1808(1981).
[4] G.P. Lepage, S.J. Brodsky, T. Huang and P.B. Mackenzie, in Particles and Fields-2, page 83, Invited talk presented at the Banff summer Institute on Particle Physics, Banff, Alberta, Canada, 1981.
[5] H.N. Li and G. Sterman, Nucl.Phys. B325, 129(1992); J. Botts and G. Sterman, Nucl.Phys. B225, 62(1989).
[6] C.D. Lu, W. Wang and Z.T. Wei, Phys.Rev. D76, 014013(2007).
[7] C.D. Lu and M.Z. Yang, Eur.Phys.J. C28, 515(2003).
[8] A. Khodjamirian, T. Mannel and N. Offen, Phys.Rev. D75, 054013(2007).
[9] Z.H. Li, F.Y. Liang, X.Y. Wu and T. Huang, Phys.Rev. D64, 057901(2001).
[10] P. Ball and R. Zwicky, Phys.Rev. D71, 014015(2005); hep-ph/0406232.
[11] P. Ball, J.High Energy Phys. 9809, 005(1998).
[12] T. Huang, Z.H. Li and X.Y. Wu, Phys.Rev. D63, 094001(2001); T. Huang and Z.H. Li, Phys.Rev. D57, 1993(1998).
[13] A.G. Grozin and M. Neubert, Phys.Rev. D55, 272(1997).
[14] T. Huang, X.G. Wu and M.Z. Zhou, Phys.Lett. B611, 260(2005).
[15] S.D. Genon and C.T. Sachrajda, Nucl.Phys. B625, 239(2002).
[16] S. Wandzura and F. Wilczek, Phys.Lett. B72, 195(1977).
[17] Z.T. Wei and M.Z. Yang, Nucl.Phys. B642, 263(2002).
[18] T. Kurimoto, H.N. Li and A.I. Sanda, Phys.Rev. D65, 014007(2002).
[19] Y.Y. Charng and H.N. Li, Phys.Rev. D72, 014003(2005).
[20] C.R. Ji, P.L. Chung and S.R. Cotanch, Phys.Rev. D45, 4214(1992); H.M. Choi and C.R. Ji, Phys.Rev. D75, 034019(2007).
[21] A. Khodjamirian, Th. Mannel and M. Melcher, Phys.Rev. D70, 094002(2004).
[22] P. Ball and M. Boglione, Phys.Rev. D68, 094006(2003).
[23] V.M. Braun and A. Lenz, Phys.Rev. D70, 074020(2004).
[24] P. Ball, V.M. Braun and A. Lenz, J.High Energy Phys. 05, 004(2006).
[25] V.M. Braun et al., Phys.Rev. D74, 074501(2006).
[26] P.A. Boyle et al., Phys.Lett. B641, 67(2006); hep-lat/0610025.
[27] T. Huang, M.Z. Zhou and X.H. Wu, Eur.Phys.J. C42, 271(2005).
[28] Seung-il Nam and Hyun-Chul Kim, Phys.Rev. D74, 096007(2006).
[29] T. Huang and X.G. Wu, Phys.Rev. D70, 093013(2004); X.G. Wu and T. Huang, Int.J.Mod.Phys. A21, 901(2006).
[30] S.J. Brodsky, T. Huang and G.P. Lepage, in Particles and Fields-2, Proceedings of the Banff Summer Institute, Banff, Alberta, 1981, edited by A.Z. Capri and A.N. Kamal (Plenum, New York, 1983), P143; G.P. Lepage, S.J. Brodskyk T.Huang, and P.B. Mackenize, ibid., p83; T. Huang, in Proceedings of XXth International Conference on High Energy Physics, Madison, Wisconsin, 1980, edited by L.Durand and L.G. Pondrom, AIP Conf.Proc.No. 69(AIP, New York, 1981), p1000.
[31] M. Beneke and T. Feldmann, Nucl.Phys. B592, 3(2001).
[32] G. Sterman, Phys.Lett. B179, 281(1986); Nucl.Phys. B281, 310(1987); S. Catani and L. Trentadue, Nucl.Phys. B327, 323(1989); Nucl.Phys. B353, 183(1991).
[33] E.E. Salpeter and H.A. Bethe, Phys.Rev.84, 1232(1951).
[34] H.C. Pauli and H.A. Bethe, Phys.Rev.D 32, 1993(1985); 32, 2001(1985); T. Eller, H.C. Pauli and S.J. Brodsky, ibid. 35, 1493(1987); S.J. Brodsky, H.C. Pauli and S.S. Pinsky, Phys.Rep.301, 299(1998).
[35] H. Kawanura, J. Kodaira, C.F. Qiao and K. Tanaka, Nucl.Phys. B(Proc.Suppl.)116, 269(2003); Mod.Phys.Lett. A18, 799(2003).
[36] T. Huang and X.G. Wu, Int.J.Mod.Phys.A22, 3065(2007).
[37] S.S. Gershtein and M.Yu. Khlopov, JETP Lett. 23, 338 (1976); M.Yu. Khlopov, Yad. Fiz. 18,
1134 (1978).

[38] X.H. Guo and T. Huang, Phys.Rev. D\textbf{43}, 2931(1991).

[39] J. Botts and G. Sterman, Nucl.Phys. B\textbf{325}, 62(1989); F.G. Cao and T. Huang, Mod.Phys.Lett. A\textbf{13}, 253(1998).

[40] M. Neubert, Phys.Rept. \textbf{245}, 259(1994).