Perturbative world-volume dynamics of the bosonic membrane and string

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Abstract

We study the world-volume theory of a bosonic membrane perturbatively and discuss if one can obtain any conditions on the number of space-time dimensions from the consistency of the theory. We construct an action which is suitable for such a study. In order to study the theory perturbatively we should specify a classical background around which perturbative expansion is defined. We will discuss the conditions which such a background should satisfy to deduce the critical dimension. Unfortunately we do not know any background satisfying such conditions. In order to get indirect evidences for the critical dimension of the membrane, we next consider two string models obtained via double dimensional reduction of the membrane. The first one reduces to the Polyakov string theory in the conformal gauge. The second one is described by the Schild action. We show that the critical dimension is 26 for these string theories, which implies that the critical dimension is 27 for the membrane theory.

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1 Introduction

Finding a microscopic definition of M-theory [1] is one of the most important problems in string theory. The low energy effective theory of M-theory is the 11 dimensional supergravity, but since it is nonrenormalizable, we can do only classical analysis by using it. There exist extended objects, membranes and fivebranes in M-theory. Thus it is tempting to consider the extended objects in M-theory as the fundamental degrees of freedom in a microscopic setting of M-theory, as was examined in Ref. [2]. It is widely known that the supermembrane has an unstable ground state [3]. In this respect, the M(atrix) theory conjecture [4] gives a fascinating explanation which accommodates the idea of matrix regularization of the membrane [5]: a membrane is a solitonic state which can decay into its elementary dynamical constituents.

Membranes may not be fundamental objects, but it will be possible to get some information about M-theory by studying their world-volume quantum dynamics. Although the world-volume theory is nonrenormalizable, perturbative analysis as a cut-off theory will make it possible for us to discuss anomalies in the membrane world-volume gauge symmetries in the low energy approximation. The attitude is similar to that of the effective string theory [6]. We will restrict ourselves to bosonic membranes in this paper. Such membranes may be related to the bosonic string theory via dimensional reduction (See Ref. [7] for some observation on this point.). Then the critical dimension $D = 27$ is expected to emerge as a special number $D$ of target space-time coordinates from the requirement of the absence of anomalies on the world-volume so that its double dimensional reduction naturally yields the critical dimension 26 of the bosonic string theory. This subject has been pursued before in a series of papers [8, 9] and appearance of some specific features at $D = 27$ has been demonstrated. On the other hand, in [10] it was concluded that critical dimension cannot be obtained for bosonic membrane theory. See also Ref. [11] for discussion on this topic, and Ref. [12, 13] for supermembranes.

In the first part of this paper, we would like to reexamine this issue by using perturbative analysis based on Lagrangian and path integral approach. In order to start perturbative expansions, we should specify a classical background of the world-volume theory, around which the fields fluctuate. We can get conditions on the number of space-time dimensions, if the reparametrization symmetry on the world-volume becomes anomalous in general. We will argue that, in perturbation theory, anomaly can occur only when there exist no variables which can be used as a
metric on the world-volume being nondegenerate in the classical background. The induced metric $\partial_a X^\mu \partial_b X_\mu$ is such an example. Therefore if the classical value of this is nondegenerate, any potential anomaly for the reparametrization symmetry can be cancelled. In order to deduce the critical dimension of the theory, we should find a classical background in which such metrics are singular, and simultaneously the perturbative expansion around which is possible. Finding such a background is a difficult problem and unfortunately, at present, we do not know any examples of classical backgrounds that satisfy these conditions.

However it is possible to find a classical background satisfying such conditions after dimensional reduction of the model. The dimensional reduction we will consider is actually the double dimensional reduction and we get a string model as a result. There are two inequivalent ways of double dimensional reduction in the world-volume action of membranes we use. One gives the usual Polyakov string theory and the other gives a string theory with the Schild action [14]. We will demonstrate that the critical dimension for such string theories is 26. This result implies that the critical dimension for bosonic membrane theory is 27.

The string theory described by the Schild action suffers from a similar difficulty as the one encountered in the case of the membrane. In order to analyze the world-sheet theory perturbatively, one should specify a classical background and usually a good background in this respect yields a nondegenerate induced metric. Therefore, it is difficult to deduce the critical dimension of such a string theory, although we expect that it is 26. Our result shows how to do so, and may give a clue as to how to deduce the critical dimension in the membrane theory.

The organization of this paper is as follows. In Sec. 2, we discuss bosonic membrane theory. We consider how one can deduce the critical dimension of the theory, if it is possible. We construct an action suitable for perturbative analysis and discuss its symmetry. In Sec. 3 and Sec. 4, we discuss two inequivalent ways of dimensional reduction of the theory in Sec. 2 and deduce the critical dimension. Sec. 5 is devoted to discussion.

2 Perturbative membrane dynamics

2.1 Membrane world-volume action

In this section, we would like to study the world-volume theory of bosonic membranes perturbatively and pursue if there is any critical dimension. Before doing
so, let us recapitulate the standard procedure of quantizing the membrane theory. The action to start from is the Nambu-Goto action:

\[ S_{NG} = -\int d^3\sigma \sqrt{-\gamma}. \]  \hspace{1cm} (1)

Here \( \sigma^a (a = 0, 1, 2) \) are the coordinates on the world-volume, \( X^\mu (\mu = 0, 1, \cdots, D - 1) \) represents the embedding of the world-volume in the \( d \)-dimensional space-time, \( \gamma_{ab} = \partial_a X^\mu \partial_b X_\mu \) is the induced metric and \( \gamma = \text{det}(\gamma_{ab}) \). Canonical quantization of the action goes in the usual way. The momentum variable \( P_\mu \) conjugate to \( X^\mu \) can be obtained as

\[ P_\mu = -\sqrt{-\gamma} \gamma^{0b} \partial_b X_\mu. \]  \hspace{1cm} (2)

One can find the following constraints in the system:

\[ \phi_0(\sigma) = \frac{1}{2} (P_\mu(\sigma) P^\mu(\sigma) + h(\sigma)), \]
\[ \phi_r(\sigma) = P_\mu(\sigma) \partial_r X^\mu(\sigma), \]  \hspace{1cm} (3)

with \( r = 1, 2 \). Here \( h(\sigma) \) is the determinant of the spatial part of the metric \( h_{rs}(\sigma) \) induced from \( X^\mu(\sigma) \),

\[ h_{rs}(\sigma) \equiv \partial_r X^\mu(\sigma) \partial_s X_\mu(\sigma), \]
\[ h(\sigma) \equiv \text{det}(h_{rs}(\sigma)). \]  \hspace{1cm} (4)

The inverse of \( h_{rs}(\sigma) \) will be denoted as \( h^{rs}(\sigma) \). \( \phi_0(\sigma) \) is the Hamiltonian constraint and \( \phi_r(\sigma) \) are the momentum constraints. They can be regarded as the generators of reparametrization on the world-volume. The Hamiltonian made from the action in eq. (1) vanishes as is usual for a theory with reparametrization invariance.

Using the Poisson bracket

\[ \{X^\mu(\sigma^0, \vec{\sigma}), P_\nu(\sigma^0, \vec{\sigma}')\}_P = \delta^\mu_\nu \delta^2(\vec{\sigma} - \vec{\sigma}'). \]  \hspace{1cm} (5)

one readily finds that all the constraints \( \phi_0(\sigma), \phi_r(\sigma) \) are of the first class, and they satisfy the algebra of

\[ \{\phi_0(\vec{\sigma}), \phi_0(\vec{\sigma}')\}_P = (\phi_r(\vec{\sigma}) h(\vec{\sigma}) h^{rs}(\vec{\sigma}) + \phi_r(\vec{\sigma}') h(\vec{\sigma}') h^{rs}(\vec{\sigma}')) \partial_s \delta^2(\vec{\sigma} - \vec{\sigma}'), \]
\[ \{\phi_0(\vec{\sigma}), \phi_r(\vec{\sigma}')\}_P = (\phi_0(\vec{\sigma}) + \phi_0(\vec{\sigma}')) \partial_r \delta^2(\vec{\sigma} - \vec{\sigma}'), \]
\[ \{\phi_r(\vec{\sigma}), \phi_s(\vec{\sigma}')\}_P = \phi_r(\vec{\sigma}') \partial_s \delta^2(\vec{\sigma} - \vec{\sigma}') + \phi_s(\vec{\sigma}) \partial_r \delta^2(\vec{\sigma} - \vec{\sigma}'). \]  \hspace{1cm} (6)
where \( \bar{\sigma} \equiv (\sigma^1, \sigma^2) \), and \( \sigma^0 \)-dependence is not made explicit. For a later use, we arrange the algebraic structure (3) in the following form;

\[
\{ \phi_a(\bar{\sigma}), \phi_b(\bar{\sigma}') \}_P = \int d^2 \bar{\sigma}'' C_{ab} \, c(\bar{\sigma}, \bar{\sigma}'; \bar{\sigma}'') \phi_c(\bar{\sigma}''),
\]

with a set of 0-th order structure functions \( C_{ab} \, c(\bar{\sigma}, \bar{\sigma}'; \bar{\sigma}'') \). Their explicit expressions read

\[
C_{00}^{00}(\bar{\sigma}, \bar{\sigma}'; \bar{\sigma}'') = 0,
\]

\[
C_{00}^{rs}(\bar{\sigma}, \bar{\sigma}'; \bar{\sigma}'') = h(\bar{\sigma}'') h^{rs}(\bar{\sigma}'') \partial_s \delta^2(\bar{\sigma} - \bar{\sigma}') \left( \delta^2(\bar{\sigma} - \bar{\sigma}'') + \delta^2(\bar{\sigma}' - \bar{\sigma}'') \right),
\]

\[
C_{0r}^{0}(\bar{\sigma}, \bar{\sigma}'; \bar{\sigma}'') = \delta_r \delta^2(\bar{\sigma} - \bar{\sigma}') \left( \delta^2(\bar{\sigma} - \bar{\sigma}'') + \delta^2(\bar{\sigma}' - \bar{\sigma}'') \right) = C_{r0}^{0}(\bar{\sigma}, \bar{\sigma}'; \bar{\sigma}''),
\]

\[
C_{0r}^{s}(\bar{\sigma}, \bar{\sigma}'; \bar{\sigma}'') = 0 = C_{r0}^{s}(\bar{\sigma}, \bar{\sigma}'; \bar{\sigma}''),
\]

\[
C_{rs}^{0}(\bar{\sigma}, \bar{\sigma}'; \bar{\sigma}'') = 0,
\]

\[
C_{rs}^{u}(\bar{\sigma}, \bar{\sigma}'; \bar{\sigma}'') = \left( \delta_r \delta^2(\bar{\sigma} - \bar{\sigma}'') + \delta_r \delta_s \delta^2(\bar{\sigma}' - \bar{\sigma}'') \right) \partial_t \delta^2(\bar{\sigma} - \bar{\sigma}') .
\]  

If there exists any critical dimension for the membrane theory at all, Schwinger terms should appear on the right hand side of the above algebra when quantized, and the condition that they vanish will determine the number of the space-time dimensions. There are many ways to quantize the system but the critical dimension essentially originates from the Schwinger terms in the algebra. The existence of such Schwinger terms implies that the diffeomorphism symmetry is anomalous. In the following we would like to discuss under what conditions such an anomaly can occur.

Here we will quantize the system perturbatively. As a three-dimensional field theory, the world-volume theory of membranes is nonrenormalizable. Hence we consider the theory as a theory with a cut-off. The perturbative expansion gives the low energy approximation, and we examine if the theory is consistent in the low energy regime. The action most convenient for the perturbative analysis of this system can be obtained as follows. Since there are constraints, we introduce the Lagrangian multiplier fields \( \lambda^0(\sigma), \lambda^r(\sigma) \) \((r = 1, 2)\) to respective constraints and we have the Hamiltonian:

\[
H = \int d^2 \bar{\sigma} \left( \lambda^0(\sigma) \phi_0(\sigma) + \lambda^r(\sigma) \phi_r(\sigma) \right) = \int d^2 \bar{\sigma} \left[ \lambda^0(\sigma) \frac{1}{2} (P_\mu(\sigma) P^\mu(\sigma) + h(\sigma)) + \lambda^r(\sigma) P_\mu(\sigma) \partial_r X^\mu(\sigma) \right] .
\]

By Legendre transformation, we obtain the action

\[
S_0 = \int d^3 \sigma \, P_\mu(\sigma) \partial_0 X^\mu(\sigma) - \int d\sigma^0 \, H
\]

\[
= \int d^2 \bar{\sigma} \left[ \lambda^0(\sigma) \frac{1}{2} (P_\mu(\sigma) P^\mu(\sigma) + h(\sigma)) + \lambda^r(\sigma) P_\mu(\sigma) \partial_r X^\mu(\sigma) \right] .
\]  

5
\[
= \int d^3\sigma \left( P_\mu(\sigma) \partial_0 X^\mu(\sigma) \right. \\
- \lambda^0(\sigma) \frac{1}{2} \left( P_\mu(\sigma) P^\mu(\sigma) + h(\sigma) \right) - \lambda^r(\sigma) P_\mu(\sigma) \partial_r X^\mu(\sigma) \Bigg) . \quad (10)
\]

Gaussian integration of (10) over momenta \( P_\mu(\sigma) \) gives
\[
S_0 = \int d^3\sigma \left( \frac{1}{2\lambda^0} \left( \partial_0 X^\mu - \lambda^r \partial_r X^\mu \right)^2 - \frac{1}{2} \lambda^0 h \right) \\
= \int d^3\sigma \left( \frac{1}{2\lambda^0} \left( \partial_0 X^\mu - \lambda^r \partial_r X^\mu \right)^2 - \frac{1}{4} \lambda^0 \left\{ X^\mu, X^\nu \right\}^2 \right) , \quad (11)
\]
where
\[
\left\{ A, B \right\} \equiv \epsilon^{rs} \partial_r A \partial_s B . \quad (12)
\]
This is the action we start from. This action was obtained in [15]. Since it is in the form of polynomials of \( \partial_a X^\mu \), after an appropriate gauge fixing and expansion around an appropriate background, we will be able to get an action in which perturbative analysis is possible.

### 2.2 BRS transformation

Since we would like to study if the symmetry corresponding to the constraints \( \phi_0, \phi_r \) is anomalous or not by using the action (11), we need to know how such a symmetry is realized in the action (11). In this subsection, we will discuss the local symmetry of the action and treat it by using the BRS formalism.

The symmetry generated by \( \phi_0, \phi_r \) is realized in eq. (11) as
\[
\delta_\epsilon X^\mu = \epsilon^0 \frac{1}{\lambda^0} \left( \partial_0 X^\mu - \lambda^r \partial_r X^\mu \right) + \epsilon^r \partial_r X^\mu , \\
\delta_\epsilon \lambda^0 = \partial_0 \epsilon^0 + \epsilon^0 \partial_r \lambda^r - \partial_r \epsilon^0 \lambda^r + \epsilon^s \partial_s \epsilon^0 \lambda^0 - \partial_s \epsilon^0 \lambda^0 , \\
\delta_\epsilon \lambda^r = \partial_0 \epsilon^r + \epsilon^0 \partial_r \lambda^0 - \partial_r \epsilon^0 \lambda^r - \partial_s \epsilon^r \lambda^s + \epsilon^s \partial_s \lambda^r , \quad (13)
\]
with three parameters, \( \epsilon^0, \epsilon^r \) (\( r = 1, 2 \)). In eq. (13), the transformation law for \( X^\mu \) has been inferred from that in the Hamiltonian description
\[
\delta_\epsilon X^\mu(\vec{\sigma}) = \int d^2\vec{\sigma}' \left\{ X^\mu(\vec{\sigma}), \phi_a(\vec{\sigma}')\epsilon^a(\vec{\sigma}') \right\}_P \\
= \epsilon^0(\vec{\sigma}) P^\mu(\vec{\sigma}) + \epsilon^r(\vec{\sigma}) \partial_r X^\mu(\vec{\sigma}) , \quad (14)
\]
with the use of the value of \( P_\mu \) obtained from the equations of motion of the action (11)
\[
P_\mu = \frac{1}{\lambda^0} \left( \partial_0 X^\mu - \lambda^r \partial_r X^\mu \right) . \quad (15)
\]
In general, the action becomes invariant if the Lagrange multipliers transform in
the manner
\[
\delta \epsilon^a (\vec{\sigma}) = \partial_b \epsilon^b (\vec{\sigma}) - \int d^2 \vec{\sigma}' \int d^2 \vec{\sigma}'' \epsilon^c (\vec{\sigma}'') \lambda^b (\vec{\sigma}') C_{bc}^a (\vec{\sigma}', \vec{\sigma}'', \vec{\sigma}),
\]
using the structure functions \( C_{ab}^c (\vec{\sigma}, \vec{\sigma}' ; \vec{\sigma}'') \) of the constraint algebra \((7)\). The explicit form of \( C_{ab}^c (\vec{\sigma}, \vec{\sigma}' ; \vec{\sigma}'') \) in eq. \((8)\) led to the last two equations in eq. \((13)\). Defining \( \hat{\epsilon}^0, \hat{\epsilon}^r \) as
\[
\hat{\epsilon}^0 \equiv \epsilon^0 \frac{1}{\lambda^0}, \quad \hat{\epsilon}^r \equiv \epsilon^r - \hat{\epsilon}^0 \lambda^r,
\]
the gauge transformation \((13)\) becomes
\[
\delta \epsilon X^\mu = \hat{\epsilon}^a \partial_a X^\mu,
\]
\[
\delta \lambda^\alpha = \hat{\epsilon}^a \partial_a \lambda^\alpha + \lambda^\alpha \left( \partial_0 \hat{\epsilon}^0 - \partial_r \hat{\epsilon}^r - 2 \lambda^r \partial_r \hat{\epsilon}^0 \right),
\]
\[
\delta \lambda^r = \hat{\epsilon}^a \partial_a \lambda^r + (\partial_0 - \lambda^s \partial_s) \hat{\epsilon}^r + \lambda^r \partial_0 \hat{\epsilon}^0 - I^{rs} \partial_s \hat{\epsilon}^0,
\]
where
\[
I^{rs} \equiv (\lambda^0)^2 h h^{rs} + \lambda^r \lambda^s.
\]
The transformation law for \( X^\mu \) in eq. \((18)\) shows that the gauge symmetry corresponds to the reparametrization on the world-volume.

In order to treat this theory with local symmetries perturbatively, we will use the BRS formalism. Since the structure constants of the symmetry are field-dependent, we should resort to the Batalin-Vilkoviski formalism\([16, 17]\). In order to do so, let us explore the algebraic structure of the gauge transformation \((13)\). The gauge transformation is summarized abstractly by
\[
\delta \phi^i = \epsilon^\alpha R_{\alpha}^i,
\]
where the index \( i \) distinguishes the dynamical variables while \( \alpha \) labels the gauge degrees of freedom. We adopt the convention that both \( i \) and \( \alpha \) include the dependence on the world-volume coordinates. The algebraic structure possessed by the gauge symmetry \((20)\) is summarized as
\[
R_{\alpha}^i \frac{\delta R_{\beta}^j}{\delta \phi^j} - R_{\beta}^j \frac{\delta R_{\alpha}^i}{\delta \phi^j} = D_{\alpha \beta}^\gamma R_{\gamma}^i + M_{\alpha \beta}^{ij} \delta S_0^{\phi^j} \frac{\delta \phi^j}{\delta \phi^j}.
\]
Here \( D_{\alpha \beta}^\gamma \) are the structure functions which can be read off as
\[
D_{ab}^c (\sigma', \sigma'' ; \sigma) = \delta_{ab}^{c} \partial_3 (\sigma - \sigma') \delta_3 (\sigma - \sigma'') - \delta_{ab}^{c} \partial_3 (\sigma - \sigma') \partial_3 (\sigma - \sigma'').
\]
\(^1\)We will follow the notation used in \([17]\) in the following.
The second term in eq. (21) corresponds to the trivial symmetry

$$\delta_\mu \phi^j = \mu^{jk} \frac{\delta S_0}{\delta \phi^k},$$

with the parameters $\mu^{jk} = -\mu^{kj}$. A straightforward calculation shows that

$$M_{ab}^{ru} (\sigma', \sigma''; \sigma, \sigma'') = 3[\lambda^0(\sigma)]^3 \delta_s^0 \delta_t^0 \epsilon^{st} \partial_t \delta^3(\sigma - \sigma') \partial_s \delta^3(\sigma - \sigma'') \epsilon^{ru} \delta^3(\sigma - \sigma'').$$

With these ingredients we can apply the Batalin-Vilkoviski procedure to the membrane quantum mechanics and obtain a solution of the classical master equation as well as the BRS transformation.

A solution of the classical master equation is found as

$$(25) \quad S = (0) S + (1) S + (2) S,
\begin{align*}
(0) S &= S_0 = \int d^3 \sigma \left( \frac{1}{\lambda^0} \left( \frac{1}{2} \lambda^0 h \right) - 1 \right),
(1) S &= \int d^3 \sigma \left( X^*_\mu \partial_\mu X^a \right.
\left. - \lambda^0 C^0 + 2 \lambda^0 X^r \partial_r C^0 + \lambda^*_a \left( \lambda^0 \partial_t + (\partial_t \lambda^0) \right) C^r - \lambda^a X^0 - \lambda^*_a I^{rs} \partial_s C^0 + \lambda^*_a (\partial_0 - \lambda^s \partial_s) C^r + \lambda^r (\partial_s \lambda^r) C^s \right),
(2) S &= \int d^3 \sigma \left( -C^a_\mu \partial_\mu C^a + \frac{3}{4} (\lambda^0)^3 \epsilon^{ru} \lambda^*_u \left\{ C^0, C^0 \right\} \right),
\end{align*}$$

Here, $X^*_\mu$, $\lambda^a$ and $C^a_\mu$ are the antifields of $X^\mu$, $\lambda^a$ and $C^a$ respectively. The number $i$ in the parenthesis of $(i) S$ denotes the antighost number. The BRS transformation is generated by $S$ in terms of the antibracket,

$$s \phi^i = (\phi^i, S) = \frac{\delta S}{\delta \phi^i},
\quad s \phi^*_i = (\phi^*_i, S) = -\frac{\delta S}{\delta \phi^i},$$

where the functional derivatives are understood to act from the left. The explicit expression for the BRS transformation law becomes, for instance,

$$s X^\mu = \partial_\mu X^\mu C^a,
\quad s \lambda^0 = \partial_0 (\lambda^0 C^0) - 2 \lambda^0 \lambda^r \partial_r C^0 - \left( \lambda^0 \partial_t + (\partial_t \lambda^0) \right) C^r,
\quad s \lambda^r = C^a_\mu \partial_\mu X^a + \lambda^r \partial_0 C^0 - I^{rs} \partial_s C^0 + (\partial_0 - \lambda^s \partial_s) C^r + \frac{3}{2} (\lambda^0)^3 \epsilon^{ru} \lambda^*_u \left\{ C^0, C^0 \right\},
\quad s \lambda^*_r = \frac{1}{\lambda^0} (\partial_0 X^\mu \partial_r X^\mu - \lambda^s h_{sr})$$
+2\lambda^0\lambda^0_0\partial_0C^0 + \partial_0\lambda^0_0C^0 + \lambda^s_0\lambda^s\partial_sC^0 + \lambda^s_0\lambda^s\partial_0C^0 \\
+\lambda^s\lambda^s_0\partial_0C^s + \partial_s(\lambda^s_0C^s),

sC^a = C^b\partial_bC^a. \quad (27)

2.3 BRS covariant metric and perturbative anomaly

Now let us consider the question under what conditions the reparametrization symmetry or the BRS symmetry of the action \(S\) in eq. (24) can become anomalous. In order to do so, it is instructive to recall how the reparametrization symmetry becomes anomalous in the case of string theory. In the string case, starting from the Nambu-Goto action, we can follow the same procedure as that in Sec. 2.1 and obtain the action

\[
S_0 = \int d^2\sigma \left( \frac{1}{2\lambda^0} \left( \partial_0 X^\mu - \lambda^1 \partial_1 X^\mu \right)^2 - \frac{\lambda^0}{2} \left( \partial_1 X^\mu \right)^2 \right) . \quad (28)
\]

This action is actually the Polyakov string action \[18\]

\[
\int d^2\sigma \frac{1}{2} \sqrt{-g} g^{ab} \partial_a X^\mu \partial_b X_\mu , \quad (29)
\]

with \(\sqrt{-g} g^{ab}\) \((a, b = 0, 1)\) expressed in terms of \(\lambda^0, \lambda^1\). Since \(\sqrt{-g} g^{ab}\) do not include the conformal mode of the metric, \(\lambda^0\) and \(\lambda^1\) express the modes of the metric other than the conformal mode.

Starting from the action in eq. (28) one can quantize the theory using the BRS formalism in the usual way and find that the BRS symmetry is anomalous if \(D \neq 26\). The reason for the existence of the anomaly is obvious. In order to define a quantum theory, we need a metric to define the regularization procedure, etc. \[19\]. If a metric \(g_{ab}\) which is transformed as a tensor under the reparametrization is available, we can regularize the action by using the Laplacian made from \(g_{ab}\) for example. Then the theory can be defined preserving the reparametrization symmetry. In order to regularize the action in the BRS invariant manner, we need a metric \(g_{ab}\) which is transformed under the BRS symmetry as

\[
s g_{ab} = g_{ac}\partial_b C^c + \partial_a C^c g_{cb} + C^c \partial_c g_{ab}, \quad (30)
\]

where \(C^a\) \((a = 0, 1)\) are the reparametrization ghost fields. Let us call such a metric a BRS covariant metric. If a metric which behaves properly under the reparametrization symmetry or BRS symmetry is not available, it can be a source of an anomaly. In the present case, we have the variables \(\lambda^0, \lambda^1\) from which
we can construct $\sqrt{-gg^{ab}}$ but not $g_{ab}$ itself. Therefore we cannot have a metric which behaves properly under the symmetry, and the symmetry becomes anomalous. On the other hand, in the case of Polyakov string theory, we consider $g_{ab}$ as the fundamental degrees of freedom and the reparametrization invariance can be made nonanomalous. Then what matters is the Weyl symmetry, or the conformal symmetry in the conformal gauge.

Actually in the formulation using the action in eq. (28), things are more subtle than it appears. It is not possible to construct a metric from $\lambda^0$, $\lambda^1$ alone, but it is possible to do so using $X^\mu$. Indeed, the induced metric $\partial_a X^\mu \partial_b X_\mu$ is a BRST covariant metric. The reason why such a metric is not considered to make the symmetry nonanomalous is because we always consider the world-sheet theory of strings around the classical background $X^\mu = 0$. Therefore, at least perturbatively, the induced metric is a singular metric and cannot be used.

In Ref. [6], Polchinski and Strominger studied Nambu-Goto string theory perturbatively around a background. For a perturbation theory to be well-defined, the induced metric, necessarily becomes nondegenerate. Hence, an anomaly in the reparametrization symmetry can be cancelled in this setting. Indeed they showed that it is possible to construct the counterterms which cancel the potential anomaly for the reparametrization invariance in such a background.

Now let us turn to the membrane theory. The action in eq. (11) can be considered as a generalization of the action in eq. (28), but there are several differences. Firstly, this action does not coincide with the Polyakov-type action with an intrinsic metric $g_{ab}$, but rather can be obtained by classically integrating out the spatial metric in the Polyakov-type action after taking a gauge considered in [20]. Since the Polyakov-type action does not possess a symmetry like the conformal symmetry, we cannot attribute the problem to such a symmetry being anomalous. Another difference is that we cannot obtain a well-defined perturbation theory from the action in eq. (11) by expanding around the classical background $X^\mu = 0$. Therefore we should specify a good classical background to start perturbative expansions.

The problem is if a metric which behaves properly under the symmetry is available or not in the classical background. Also in this case, one of such metrics is the three-dimensional induced metric

$$\gamma_{ab} = \partial_a X^\mu \partial_b X_\mu \quad (a, b = 0, 1, 2).$$ (31)
We can construct another BRS covariant metric \( g_{ab} \) defined as

\[
g_{ab} = \begin{bmatrix}
\lambda^t G_{tu} \lambda^u - (\lambda^0)^2 G & G_{st} \lambda^t \\
G_{rt} \lambda^t & G_{rs}
\end{bmatrix},
\]

\[
g^{ab} = \begin{bmatrix}
-(\lambda^0)^2 G & \lambda^s G \\
\lambda^r G (\lambda^0)^2 G & G^{rs} - \lambda^r \lambda^s (\lambda^0)^2 G
\end{bmatrix}.
\]

Here

\[
G_{rs} \equiv \partial_r X^\mu \partial_s X_\mu + \lambda^0 \left( \lambda^r \partial_s C^0 + \lambda^s \partial_r C^0 \right).
\]

The metric in eq. (32) is in the form where \( \sqrt{G} \lambda^0 \) is the lapse, \( \lambda^r \) are the shift and \( G_{rs} \) is the space metric in the ADM decomposition of the metric [21]. \( G_{rs} \) is exactly the combination appeared in [20]. A similar kind of metric can also be constructed in the same fashion for higher dimensional branes and strings.

It is of course probable that there exist other BRS covariant metrics. The potential anomaly of the BRS symmetry can be cancelled by adding counterterms, if any one of such metrics is nondegenerate in the classical background. The classical background we should consider is a solution of the classical equations of motion around which the fields corresponding to quantum fluctuations obtain regular propagators in all directions after a proper gauge fixing.

One typical solution to realize this last requirement is an infinitely extended static membrane configuration;

\[
X^0_{(0)}(\sigma) = \sigma^0, \quad X^1_{(0)}(\sigma) = \sigma^1, \quad X^2_{(0)}(\sigma) = \sigma^2,
\]

\[
X^I_{(0)}(\sigma) = 0 \quad (I = 3, \cdots, D - 1),
\]

with \( \lambda^0_{(0)}(\sigma) = 1 \) and \( \lambda^r_{(0)}(\sigma) = 0 \). We introduce a coupling constant \( g \) characterizing the order of perturbation. Accordingly, the left hand side of eq. (25) is regarded as \( g^2 S \) and all the fields \( \phi \) except for \( X^\mu \) should be replaced by \( g\phi \). For the choice of the gauge fixing condition

\[
\lambda^0(\sigma) = 1, \quad \lambda^r(\sigma) = g \epsilon^s A_s(\sigma),
\]

the fluctuations around the background (34),

\[
X^r(\sigma) = \sigma^r + g \epsilon^r A_s(\sigma) \quad (r = 1, 2),
\]

\[
X^A(\sigma) = g Y^A(\sigma) \quad (A = 0, 3, \cdots, D - 1),
\]

\[
(36)
\]
are governed by the action similar to the Yang-Mills action

\[ S_0 = \int d^3\sigma \left( \frac{1}{g^2} - \frac{1}{4} F_{ab} F^{ab} - \frac{1}{2} D_a Y^A D^a Y^A - \frac{g^2}{4} \left( \{ Y^A, Y^B \} \right)^2 \right), \]  

(37)

where

\[ F_{ab} \equiv \partial_a A_b - \partial_b A_a + g \{ A_a, A_b \}, \]

\[ D_a Y^A \equiv \partial_a Y^A + g \{ A_a, Y^A \}. \]  

(38)

After setting a gauge fixing condition as in the usual Yang-Mills theory, all the fields have free propagators. However, the metrics (31) and (32) are both regular in the background (34). Thus, a path integral measure invariant under the BRST symmetry can be constructed using either of these metrics so that the perturbative quantum theory around (34) does not have anomalies. Although we do not dictate its detail, it is tedious but straightforward to perform the explicit one-loop analysis similar to that to be performed in Sec. 4.2 and show the absence of an anomaly in the perturbation theory around the static membrane solution (34).

Obviously the discussion here applies to any backgrounds. An anomaly can arise only on the background which gives no regular BRST covariant metrics. Unfortunately, at present we do not know any example of such a background around which all the fields have propagator suitable for perturbation theory. However it is possible to find a background around which the kinetic terms for some fields lack derivatives in some directions on the world-volume. In such a case, we encounter severe divergences in perturbative expansions, because propagators in the position space include delta function of coordinates in some directions. However, even in such a background, if one dimensionally reduces the theory, one can get a well-defined perturbation theory. Then we can get a little indirect evidence for the critical dimension of membrane theory. We will consider two examples in Sec. 3 and Sec. 4.

3 Dimensional reduction of one spatial direction

Let us consider the following configuration of the membrane world-volume

\[ X^2_{(0)}(\sigma) = \sigma^2, \]

\[ X^M_{(0)}(\sigma) = 0 \quad (M = 0, 1, 3, \cdots, D - 1). \]  

(39)
It is easy to check that this is a solution of the equation of motion of the action (23). The fluctuation $\hat{Y}^\mu$ defined by

$$X^2 = \sigma^2 + g\hat{Y}^2,$$

$$X^M = g\hat{Y}^M \ (M = 0, 1, 3, \cdots, D - 1),$$  \hspace{1cm} (40)

is governed by the action

$$S_0 = \int d^3\sigma \left[ \frac{1}{2\lambda^0} \left( -\frac{\lambda^2}{g} + \partial_0\hat{Y}^2 - \lambda^r\partial_r\hat{Y}^2 \right)^2 + \frac{1}{2\lambda^0} \left( \partial_0\hat{Y}^M - \lambda^r\partial_r\hat{Y}^M \right)^2 \right.$$

$$-\frac{\lambda^0}{2}(\partial_1\hat{Y}^M)^2 - g^2\frac{\lambda^0}{4} \left\{ \hat{Y}^\mu, \hat{Y}^\nu \right\}^2 \bigg].$$  \hspace{1cm} (41)

Taking the gauge $\lambda^0 = 1, \lambda^r = 0$, the kinetic terms for $\hat{Y}^M$ lack the derivatives in $\sigma^2$ direction. Thus, we will encounter severe divergences in the perturbative expansion.

In order to avoid this problem, let us compactify the $X^2$-direction to a circle and reduce its radius $R$ to zero. The background (39) implies that we are considering the membrane wrapped around $X^2$-direction. Therefore, what we are doing is the so-called double dimensional reduction and as a result we get a string theory. Since the action we started from was a generalization of the Polyakov string action, we expect that we get the Polyakov string action after the dimensional reduction. We will demonstrate that this is indeed the case.

In the limit $R \to 0$, we will keep the coupling constant $g_2 \equiv g/\sqrt{2\pi R}$ fixed. The fluctuation in the world-sheet theory corresponds to $Y^\mu \equiv \sqrt{2\pi R}\hat{Y}^\mu$. Redefining $g_2$ as new $g$ and dropping all the derivatives in $\sigma^2$-direction, eq. (41) reduces to a two-dimensional theory

$$S_0 = \int d^2\sigma \left[ \frac{1}{2\lambda^0} \left( -\frac{\lambda^2}{g} + \partial_0Y^2 - \lambda^1\partial_1Y^2 \right)^2 + \frac{1}{2\lambda^0} \left( \partial_0Y^M - \lambda^1\partial_1Y^M \right)^2 \right.$$

$$-\frac{\lambda^0}{2}(\partial_1Y^M)^2 \bigg],$$  \hspace{1cm} (42)

where $d^2\sigma \equiv d\sigma^0d\sigma^1$. The symmetry of the action $S_0$ is given by the dimensional reduction of (18), except for the transformation law for $Y^2$,

$$\delta_\epsilon Y^2 = \frac{1}{g}\epsilon^2 + \hat{\epsilon}^j\partial_jY^2,$$  \hspace{1cm} (43)

where $j = 0, 1$. We can fix the symmetry corresponding to the parameter $\hat{\epsilon}^2$ by taking $Y^2 = 0$. Such a gauge fixing does not invoke any dynamical ghost field.
After a Gaussian integration over $\lambda^2$, we end up with the action in eq. (28). In this form, it is a familiar procedure to construct the BRS charge and examine if it is nilpotent or not and find that the BRS symmetry is anomalous unless the space-time dimension is 26. This fact indirectly implies that the critical dimension of the bosonic membrane is 27. From the membrane theory point of view, the anomaly can appear because the metrics (31) and (32) are singular in the background (39).

Trying to study the membrane theory more directly, let us recover the radius $R$ of the compactified circle and discretize it with a finite cutoff length $a$. This cutoff regularizes the divergences in the perturbation theory. The membrane can then be regarded as a collection of Polyakov-like strings placed on this circle, each of which interacts with its neighbors. For each Polyakov string to be consistent, we may be able to conclude that the critical dimension of bosonic membrane theory is 27. It is beyond the scope of this paper to investigate in detail whether this model gives a nonperturbative definition of the world-volume theory of membranes by taking an appropriate nontrivial continuum scaling limit.

## 4 Dimensional reduction of time-like direction

### 4.1 Schild string theory

Since the coordinates $\sigma^0$ and $\sigma^r$ are not on the equal footing in the action in eq. (11), there exists a way of dimensional reduction inequivalent to the one in the previous section. Namely we can wrap the membrane along the coordinate $\sigma^0$. Let us describe the string theory we can get as the result of such a dimensional reduction.

In this section, we consider the world-volume with the Euclidean signature. Since $\lambda^0$ is like an einbein field, this is achieved by replacing $\lambda^0$ and its antifield $\lambda^*_0$ as

$$\lambda^0 \rightarrow -i\lambda^0, \quad \lambda^*_0 \rightarrow i\lambda^*_0.$$  

The Euclidean action is then obtained by multiplying the Minkowskian action by $-i$. We will use the name $\sigma^0$, $\lambda^0$ and $\lambda^*_0$ even after Euclideanization.

Let us wrap the $\sigma^0$-direction around a circularly compactified space-time direction $X^0$,

$$X^0_{(0)} = \sigma^0, \quad X^I_{(0)} = 0 \quad (I = 1, \cdots, D - 1),$$  

and consider the string model obtained by taking the radius of the circle $R \rightarrow 0$. In this process, the coupling constant $g_2^2 = g^2/(2\pi R)$ in the two-dimensional theory

14
is kept fixed, which is redefined hereafter as new $g^2$. The above configuration itself is not a classical solution of the original action $S_0$ in eq. (11), but later we will turn on further backgrounds so that the final configuration satisfies the equations of motion.

Now, in terms of the fluctuation $Y^\mu$ defined by

$$X^0 = \sigma^0 + gY^0,$$
$$X^I = gY^I \quad (I = 1, \ldots, D - 1),$$

the action becomes

$$S_0 = \int d^2\sigma \left[ \frac{1}{2\lambda^0} \left( \frac{1}{g} - \lambda^0 \omega^r \partial_r Y^0 \right)^2 + \frac{\lambda^0}{2} \left( \omega^r \partial_r Y^I \right)^2 + \frac{\lambda^0}{4} g^2 \left( \{Y^\mu, Y^\nu\} \right)^2 \right],$$

where $\omega^r \equiv \lambda^r / \lambda^0$ and $\mu, \nu = 0, \ldots, D - 1$. The transformation properties are expressed in terms of $\omega^r$ as

$$\begin{align*}
\delta_\epsilon Y^0 &= \frac{1}{g} \bar{\epsilon}^0 + \bar{\epsilon}^s \partial_s Y^0, \\
\delta_\epsilon Y^I &= \bar{\epsilon}^s \partial_s Y^I, \\
\delta_\epsilon \lambda^0 &= -\lambda^0 \partial_s \bar{\epsilon}^s - 2(\lambda^0)^2 \omega^s \partial_s \bar{\epsilon}^0 + \bar{\epsilon}^s \partial_s \lambda^0, \\
\delta_\epsilon \omega^r &= \partial_s (\omega^r \bar{\epsilon}^s) + \lambda^0 J^{rs} \partial_s \bar{\epsilon}^0 - \omega^s \partial_s \bar{\epsilon}^r,
\end{align*}$$

where

$$J^{rs} = \eta^{rs} + \omega^r \omega^s.$$ 

If one fixes the symmetry corresponding to $\bar{\epsilon}^0$ by taking $Y^0 = 0$, the action becomes

$$S_0 = \int d^2\sigma \left[ \frac{1}{2\lambda^0 g^2} + \frac{\lambda^0}{2} \left( \omega^r \partial_r Y^I \right)^2 + \frac{\lambda^0 g^2}{4} \left( \{Y^I, Y^J\} \right)^2 \right].$$

Here $I, J = 1, \ldots, D - 1$. Now, since $\omega^r$ are just auxiliary fields, we can integrate them out and we obtain

$$S_0 = \int d^2\sigma \left[ \frac{1}{2\lambda^0 g^2} \left( \lambda^0 + \frac{\lambda^0 g^2}{4} \left( \{Y^I, Y^J\} \right)^2 \right) \right],$$

which is nothing but the Schild action in a $D - 1$ dimensional space-time [14]. Therefore we can obtain Schild string by dimensionally reducing our membrane action.
In the rest of this section, we would like to demonstrate the existence of the critical dimension starting from the action in eq. (47). Since this action is equivalent to the Schild string action, what we will show gives a way to get the critical dimension of the Schild string theory. The purpose of studying Schild string theory is twofold. For one thing, we want to check if the critical dimension of the bosonic membrane is 27 or not, by trying another way of dimensional reduction. For another thing, by studying Schild strings, we may be able to get some clue as to how we can treat the membrane action perturbatively. Like the membrane action in eq. (11), we should find out an appropriate classical background in the Schild action (51) to start perturbative expansions. Usually, a BRS covariant non-degenerate metric is available in such a background. However, starting from the action (17), which is a simple modification of eq. (51), one can deduce the critical dimension. Therefore we can expect that there exists a modification of eq. (11) from which one can deduce the critical dimension.

For our purpose, it is convenient to fix the gauge symmetry corresponding to \( \hat{\epsilon}^0 \) by a gauge fixing condition \( \partial_r \omega^r = 0. \) Then, \( \omega^r \) should take the form

\[
\omega^r = g \epsilon^r s \partial_s Y, \tag{52}
\]

and the action becomes

\[
S_0 = \int d^2 \sigma \left[ \frac{1}{2 \lambda^0} \left( \frac{1}{g} + g \lambda^0 \{ Y, Y^0 \} \right)^2 + \frac{\lambda^0}{2} \left( g \{ Y, Y^I \} \right)^2 + \lambda^0 \frac{g^2}{4} \left( \{ Y^\mu, Y^\nu \} \right)^2 \right]. \tag{53}
\]

We will consider a classical background

\[
g Y^0_0 = \sigma^2, \quad g Y^0_0 = \sigma^1. \tag{54}
\]

The kinetic terms for all the fields around this background are not pathological. In this background, the induced metric \( \partial_r Y^I \partial_s Y^I \) is singular. It may appear that one can construct a metric using \( Y \) and \( Y^0 \) but such a metric does not transform properly under the symmetry corresponding to \( \hat{\epsilon}^0. \) This action also looks like a Schild action but with two more coordinates \( Y, Y^0 \) compared to eq. (51). Supersymmetrization of this action may be relevant to F-theory [22]. We examine the perturbative quantum dynamics starting from this action, in particular focusing on the quantum-mechanical consistency of the gauge symmetries (18).
4.2 Anomaly and critical dimension

After further fixing the symmetry by a condition $\lambda^0 = 1$, the dynamics around the background in eq. (54) with respect to the fluctuation $A_r$ defined by

$$Y = \frac{1}{g} \sigma^2 - A_1,$$
$$Y^0 = \frac{1}{g} \sigma^1 + A_2,$$

is described by the action,

$$S_{\text{matter}} = \int d^2 \sigma \left[ \frac{1}{4} (F_{rs})^2 + \frac{1}{2} (D_r Y^I)^2 + \frac{g^2}{4} \left\{ \{Y^I, Y^J\} \right\}^2 \right],$$

where

$$F_{rs} \equiv \partial_r A_s - \partial_s A_r + g \left\{ A_r, A_s \right\},$$
$$D_r Y^I \equiv \partial_r Y^I + g \left\{ A_r, Y^I \right\}.$$ (57)

This action looks like the Yang-Mills action and it is invariant under the area-preserving diffeomorphism. To fix the area-preserving diffeomorphism, we will take a covariant gauge fixing term mimicking the Yang-Mills case to respect the covariance of the free parts of $A_r$ in eq. (56).

To analyze this system perturbatively, we provide the gauge fixing and ghost terms à la Batalin-Vilkoviski procedure. The action satisfying the classical master equation can be obtained by either dimensionally reducing the action (25) or starting from the dimensionally reduced action (47). Both give the same result. In order to impose the gauge fixing conditions described above, it is convenient to introduce a ghost field $C^{-1}$ and its antifield $C^{-1}_{*1}$, an antighost field $\overline{C}_{-1}$ and its antifield $\overline{C}^{* -1}_{-1}$, auxiliary fields $B, \overline{\lambda}_0, \overline{\omega}_r$ and their antifields $B^*, \overline{\lambda}^*, \overline{\omega}^r$, and add the following term to the action:

$$S_{-1}^E = \int d^2 \sigma \left( -i Y^* C^{-1} + \overline{C}^{* -1} B + \overline{C}^{* 0} \overline{\lambda}_0 + \overline{C}^{* r} \overline{\omega}_r \right).$$ (58)

By taking the gauge fermion $\psi$,

$$\psi = \int d^2 \sigma \left[ \overline{C}_{-1} \left( \partial_1 Y - \frac{1}{g} \partial_2 X^0 - \frac{\alpha}{2} B \right) + \overline{C}_0 (\lambda^0 - 1) + \overline{C}_r \left( \frac{1}{g} \omega^r - \epsilon^s \partial_s Y \right) \right]$$
$$= \int d^2 \sigma \left[ \overline{C}_{-1} \left( -\partial_r A_r - \frac{\alpha}{2} B \right) + \overline{C}_0 (\lambda^0 - 1) + \overline{C}_r \left( \frac{1}{g} \omega^r - \epsilon^s \partial_s Y \right) \right],$$ (59)
the antifields are fixed as follows:

\[ C^{-1} = - \left( \partial_r A_r + \frac{\alpha}{2} B \right), \]
\[ Y^* = - \partial_1 C_{-1} + \epsilon^{rs} \partial_s C_r, \]
\[ X^* = \frac{1}{g} \partial_2 C_{-1}, \]
\[ \omega^*_r = C_r, \]
\[ \lambda^*_0 = C_0, \]
\[ C^{*r} = \frac{1}{g} \omega^* - \epsilon^{rs} \partial_s Y, \]
\[ C^{*0} = \lambda^0 - 1. \] (60)

Hence the action \( S_{-1}^E \) becomes

\[ S_{-1}^E \psi \equiv S_{-1}^E \bigg|_{\phi^*_i = \partial_\psi / \partial \phi^i} \]
\[ = \int d^2 \sigma \left[ i \partial_1 C_{-1} C^{-1} + i \left( \epsilon^{rs} \partial_r C_s \right) C^{-1} \right. \]
\[ + \left( - \partial_r A_r - \frac{\alpha}{2} B \right) B \]
\[ + \bar{\lambda}_0 \left( \lambda^0 - 1 \right) + \bar{\omega}_r \left( \frac{1}{g} \omega^* - \epsilon^{rs} \partial_s Y \right) \]. (61)

Integrating over the auxiliary fields \( B, \bar{\lambda}_0, \bar{\omega}_r \), we get the gauge fixing conditions \( \lambda^0 = 1, \omega^r = g \epsilon^{rs} \partial_s Y \), and a covariant gauge fixing term

\[ S_{GF} = \int d^2 \sigma \frac{1}{2\alpha} (\partial_r A_r)^2. \] (62)

Finally we get the action

\[ S_{\text{matter}} + S_{GF} + S_{\text{ghost}}, \] (63)

where

\[ S_{\text{ghost}} = \int d^2 \sigma \left[ \right. \left. \right. \left. \right. \right.
\[ + \left( - \partial_r A_r - \frac{\alpha}{2} B \right) B \]
\[ + \bar{\lambda}_0 \left( \lambda^0 - 1 \right) + \bar{\omega}_r \left( \frac{1}{g} \omega^* - \epsilon^{rs} \partial_s Y \right) \].
\[ +ig \mathcal{C}_r \{A_1, C^r\} - ig (\partial_s \mathcal{C}_r) \epsilon^{rt} (\partial_t A_1) C^s \\
+ig^2 \mathcal{C}_r \epsilon^{rs} (\partial_s A_1) \{A_l, C^0\} + ig^2 \mathcal{C}_r \epsilon^{rs} \partial_s Y^{r} \{Y^l, C^0\} \\
+ \frac{3}{4} g^2 \epsilon^{tu} \mathcal{C}_r \mathcal{C}_u \{C^0, C^0\} \]  \hspace{1cm} (64)

Now that we have the gauge-fixed action which is BRS invariant at least classically, the most honest way to examine if the BRS symmetry is anomalous or not is to check if the BRS charge is nilpotent quantum mechanically. Here we take a by-pass which is usually taken in this kind of situation. We will define the quantities \( j_0 \) and \( j_r \) which we will call currents as the variations of \( C_0 \) and \( C_r \) under the BRS transformation as

\[
\begin{align*}
j_0 &\equiv sC_0, \\
j_r &\equiv sC_r \quad (r = 1, 2).
\end{align*}
\]  \hspace{1cm} (65)

Essentially \( j_0 \) and \( j_r \) are the BRS invariant version of the constraints \( \phi_0 \) and \( \phi_r \) in the original action. Therefore, by checking if there is any Schwinger terms in the algebra satisfied by \( j_0 \) and \( j_r \), we can see if the BRS symmetry is anomalous or not.

There is another way to look at these currents. Since \( C_0 \) and \( C_r \) are the antifields of \( \lambda^0 \) and \( \omega^r \), \( j_0 \) and \( j_r \) can be written as

\[
\begin{align*}
j_0 &= \left. \frac{\delta S}{\delta \lambda^0} \right|_{\phi^i = \partial \phi/ \partial \phi^i}, \\
j_r &= \left. \frac{\delta S}{\delta \omega^r} \right|_{\phi^i = \partial \phi/ \partial \phi^i},
\end{align*}
\]  \hspace{1cm} (66)

where \( S \) is the action before the gauge fixing. Therefore the currents can be considered as representing the response of the action to variations of the gauge fixing conditions for \( \lambda^0 \) and \( \omega^r \). These currents play a role similar to the one played by the energy-momentum tensor in the Polyakov string theory. If the reparametrization symmetry is not anomalous, the correlation functions of these currents should vanish up to local terms proportional to derivatives of delta functions in the position space.

The explicit form of these currents is found to be \( j_a = j_a \mid_{\text{matter}} + j_a \mid_{\text{ghost}}, \) where the matter contributions \( j_a \mid_{\text{matter}} \) are

\[
\begin{align*}
j_0 \mid_{\text{matter}} &= -\frac{1}{2g^2} + \frac{g^2}{2} \left( \{Y, Y^0\} \right)^2 + \frac{g^2}{2} \left( \{Y, Y^I\} \right)^2 + \frac{g^2}{4} \left( \{Y^\mu, Y^\nu\} \right)^2
\end{align*}
\]
\[
\frac{1}{g} \epsilon^{rs} \partial_r A_s \\
+ \frac{1}{4} (\partial_r A_s - \partial_s A_r)^2 + \frac{1}{2} \epsilon^{rs} \{A_r, A_s\} + \frac{1}{2} (\partial_r Y^I)^2 + \mathcal{O}(g),
\]
\[
j_r \big|_{\text{matter}} = -\frac{1}{g} \partial_r Y^0 - g \partial_r Y^\mu \{Y, Y_\mu\}
\]
\[
= \delta_{r1} \frac{1}{g} \epsilon^{st} \partial_s A_t \\
+ (\partial_r A_2) \epsilon^{st} \partial_s A_t + \delta_{r1} \frac{1}{2} \epsilon^{st} \{A_s, A_t\} + \partial_r Y^I \partial_1 Y^I + \mathcal{O}(g),
\]
while the ghost contributions \( j_a \big|_{\text{ghost}} \) are
\[
\begin{align*}
\bar{j}_0 \big|_{\text{ghost}} &= 4i \overline{C}_0 \partial_1 C^0 + 2i \overline{C}_0 \partial_s C^s + i \partial_s \overline{C}_0 C^s - i \overline{C}_r \partial_r C^0 \\
&\quad + 4ig \overline{C}_s \{A_1, C^0\} - ig \overline{C}_r \{A_r, C^0\} + ig \overline{C}_r \epsilon^{rs} (\partial_s A_t) \partial_t C^0 \\
&\quad + ig^2 \overline{C}_r \epsilon^{rs} (\partial_s A_t) \{A_t, C^0\} + ig^2 \overline{C}_r \epsilon^{rs} (\partial_s Y^I) \{Y^I, C^0\} \\
&\quad + \frac{3}{4} g^2 \epsilon^{tu} \overline{C}_t \overline{C}_u \{C^0, C^0\}, \\
\bar{j}_r \big|_{\text{ghost}} &= 2i \overline{C}_0 \partial_r C^0 - i \overline{C}_r \partial_1 C^0 - i \overline{C}_1 \partial_r C^0 + i \overline{C}_s \partial_r C^s + i (\partial_s \overline{C}_r) C^s \\
&\quad - ig \overline{C}_r \{A_1, C^0\} + ig \overline{C}_s \epsilon^{st} (\partial_t A_1) \partial_r C^0.
\end{align*}
\]

We will calculate the two-point functions of these currents to see if there is any anomaly. The local terms in such two-point functions proportional to derivatives of the delta function in position space are irrelevant, because such terms can be cancelled by adding appropriate local counterterms. If the nonlocal terms remain even after the addition of any possible counterterms to the bare action and the currents, the symmetry is anomalous.

It is easy to check that the tree level contributions to the current-current correlation functions are local. Thus, we are interested in the one-loop correction. However, as seen from the bilinear part of (61) and (64), the various ghosts and antighosts mix in a complicated way, making all the calculations rather cumbersome. Thus, we pose here to make the ghost sector as simple as possible. We note that the ghost \( C^{-1} \) appears only in the first line of eq. (61) and does not appear in the currents to the relevant order of \( g \). Writing
\[
\overline{C}_1' \equiv \overline{C}_1, \quad \overline{C}_2' \equiv \overline{C}_2 + \overline{C}_{-1},
\]
integration over \( C^{-1} \) gives
\[
\epsilon^{rs} \partial_r \overline{C}_s' = 0.
\]
If we consider only world-sheets with trivial topology, this condition is solved by introducing a new variable $b$ as
\[ C'_r = i\partial_r b. \] (71)

Hence, the original variables $C_r$ are expressed in terms of $b$ and $C_{-1}$ as
\[ C_r = i\partial_r b - \delta_{r2} C_{-1}. \] (72)

The ghost sector now consists of three pairs of ghost and antighost. Further change of the variables from $C_r$ to $E_r$
\[ C_r = E_r - 2\delta^{r1} C^0 + 2g(e^r s \partial_s A_1) C^0, \] (73)

simplifies the structure of interactions. Now the free propagators of the ghost variables are found as
\[
\begin{pmatrix}
\langle C^0(\sigma) b(\sigma') \rangle & \langle C^0(\sigma) C_0(\sigma') \rangle & \langle C^0(\sigma) C_{-1}(\sigma') \rangle \\
\langle E^1(\sigma) b(\sigma') \rangle & \langle E^1(\sigma) C_0(\sigma') \rangle & \langle E^1(\sigma) C_{-1}(\sigma') \rangle \\
\langle E^2(\sigma) b(\sigma') \rangle & \langle E^2(\sigma) C_0(\sigma') \rangle & \langle E^2(\sigma) C_{-1}(\sigma') \rangle 
\end{pmatrix}
= \int \frac{d^2q}{(2\pi)^2} e^{iq(\sigma-\sigma')} \frac{1}{q^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -q_1 & -q_2 \\ 0 & -q_2 & q_1 \end{pmatrix}. \] (74)

In the present example, the one-loop quantum correction to the two-point current correlation functions consists of three types of Feynman diagrams shown in Fig. 1. We demonstrate the existence of an anomaly except for $D = 27$ at the one-loop level by evaluating the nonlocal part of the simplest two-point function
\[
\Pi_{22}(p) \equiv \int d^2\sigma e^{-ip\cdot\sigma} \langle j_2(\sigma) j_2(0) \rangle \bigg|_{\text{nonlocal}}. \] (75)

Since $j_2$ involves no terms linear in fields, the two-particle irreducible diagram shown in Fig. 1(c) gives the lowest order contribution. In perturbation theory, the effect of adding counterterms is a higher order effect. Thus, if this lowest order contribution is nonzero, the theory is anomalous. From eq. (77), we find the nonlocal contributions to $\Pi_{22}(p)$ from the scalar $Y^I$ and the gauge field $A_r$ as
\[
\Pi_{22}(p)|_{Y^I} = \frac{1}{4\pi} \left[ \frac{D - 1}{3} \right] \frac{1}{p^2},
\Pi_{22}(p)|_{A_r} = \frac{1}{4\pi} \left[ \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right) + \frac{1}{6} \frac{1}{p^2} \right], \] (76)

\footnote{We use the Feynman gauge, $\alpha = 1$ in the calculation.}
where $\mu$ is an arbitrary scale to be chosen as the renormalization point. The ghost contribution to $\Pi_{22}(p)$ becomes

$$
\Pi_{22}(p)|_G = \frac{1}{4\pi} \left[ -\frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right) - \frac{53}{6} \frac{(p_1)^2(p_2)^2}{p^2} \right].
$$

(77)

The logarithmic term in this ghost contribution cancels that in the gauge boson contribution. Now we find that the sum of (76) and (77) becomes

$$
\Pi_{22}(p) = \frac{1}{4\pi} \frac{D - 27}{3} \frac{(p_1)^2(p_2)^2}{p^2}.
$$

(78)

The nonlocal terms arising from Fig. (c) cannot be removed by an addition of any local counterterms to the action and the currents, showing that the gauge
symmetry is anomalous except for $D = 27$ in this order of approximation. We need to check that the other two-point functions vanish if and only if $D = 27$. This fact is demonstrated in Appendix B. The result we get implies that the critical dimension of the bosonic membrane is 27 and the critical dimension of the Schild string is 26, at least to this order of approximation.

5 Discussion

In the first part of this paper, we have explored a perturbative regime of the membrane world-volume theory from Lagrangian and path integral point of view, in order to examine the existence of a potential anomaly in gauge symmetries. The perturbation theory is defined only around an appropriate membrane background which guarantees the existence of non-pathological propagators for all dynamical degrees of freedom. On the other hand, the gauge anomaly can appear only in the background where BRS covariant metrics are degenerate so that neither BRS invariant measures nor local counterterms cancelling anomalies are allowed. As far as we checked, all the classical solutions with nondegenerate metrics do not induce perturbative membranes with an anomaly. Even if we can check that some of the metrics are degenerate, there is always a possibility that there exists another BRS covariant metric we do not know which becomes nondegenerate. Therefore a careful study is necessary to argue that the theory is anomalous around a background.

The analysis here does not contradict with the results given in Ref. [8, 9], which is based on the operator description. The operator methods do not refer to any backgrounds. One of the motivation to develop the perturbative method in this paper is to avoid being annoyed about the ambiguity in the operator ordering and regularization procedure. Unfortunately we do not know any classical solutions of the membrane world-volume action examined here, with singular metrics which make the reparametrization symmetry anomalous.

In the second part, we examined two string models obtained by dimensional reduction of membranes. Each of these string models accommodates an anomaly and critical dimension. We may say that this aspect indicates that the original membrane theory also has the critical dimension, although it is of course much better to be derived in a three-dimensional theory. Another possibility is that the critical dimension appears as the dimension where the regularized membrane model proposed in Sec. 3 possesses a nontrivial continuum limit. This limiting procedure
may be related rather to the nonperturbative aspects of the membrane world-volume theory, which cannot be pursued by the perturbative analysis developed here.

In particular, we have found that the string theory with the Schild action possess the critical dimension 26. We expect that we can get the usual bosonic string theory also from the Schild action. In order to confirm this, we should check if this string theory yields the same space-time equations of motion. \( D - 1 = 26 \) is a part of the space-time equations of motion, and it is an intriguing problem to generalize this to get the whole equations of motion in the low energy approximation. The Schild action is interesting because it enables one to take the tensionless limit. Thus, by studying this action, we may be able to reveal the huge gauge symmetries of string theory.

Also the experience of dealing with the Schild action gives some hint as to how to deal with the membrane theory. The reason why we were able to deduce the critical dimension for the Schild string is that we started from an action with some auxiliary fields and extra gauge symmetries. These features originate from the fact that we started from a higher dimensional theory. Therefore, an obvious strategy which should be tried to deal with the membrane theory is to consider an action obtained from reduction of higher dimensional branes. Dimensional reduction of one spatial or time-like direction of a 3-brane is the simplest possibility. Another possibility is to use the Born-Infeld action. We hope to come back to this problem in the future.

The matrix regularization of Schild string model is also interesting, recalling the role played by the Schild action in [23]. The matrices can be considered as regularizing the area-preserving diffeomorphism. Obviously, the matrix regularization is possible only when the action obtained after fixing the other gauge redundancy is written in terms of the Poisson bracket. The matrix model should have the associated ghost sector so as to reproduce the critical dimension \( D - 1 = 26 \) in the naive continuum limit.

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Appendix

In this appendix, we collect the results for the one-loop correction to the current-current correlation functions, with the definition similar to (75)

\[ \Pi_{ab}(p) \equiv \int d^2 \sigma e^{-\psi_\sigma} \langle j_a(\sigma) j_b(0) \rangle \bigg|_{\text{nonlocal}}, \tag{79} \]

in the Schild-like string model. Eq. [77] shows that \( j_0(\sigma) \) and \( j_1(\sigma) \) have the common terms linear in fields. Thus, \( j_- \equiv j_0 - j_1 \) starts from the terms bilinear in fields, and we consider the two-point functions of \( j_0, j_2 \) and \( j_- \).

For \( \Pi_{--}(p) \)

\[
\begin{align*}
\Pi_{--}(p) |_{Y_I} &= \frac{1}{4\pi} \left( -\frac{D-1}{3} \right) \frac{(p_1)^2(p_2)^2}{p^2}, \\
\Pi_{--}(p) |_{A_r} &= \frac{1}{4\pi} \left[ -\frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right) - \frac{1}{6} \frac{(p_1)^2(p_2)^2}{p^2} \right], \\
\Pi_{--}(p) |_{G} &= \frac{1}{4\pi} \left[ \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right) + \frac{53}{6} \frac{(p_1)^2(p_2)^2}{p^2} \right], \tag{80}
\end{align*}
\]

which become in total

\[ \Pi_{--}(p) = \frac{1}{4\pi} \frac{27 - D}{3} \frac{(p_1)^2(p_2)^2}{p^2}. \tag{81} \]

The vacuum polarization contribution \( \Pi_{00}(p) |^{(a)} \) to \( \Pi_{00}(p) \) is found as

\[
\begin{align*}
\Pi_{00}(p) |^{(a)}_{Y_I} &= \frac{1}{4\pi} \times 2(D-1) \times \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right), \\
\Pi_{00}(p) |^{(a)}_{A_r} &= \frac{1}{4\pi} \left( -7 \right) \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right), \\
\Pi_{00}(p) |^{(a)}_{G} &= \frac{1}{4\pi} \left( -5 \right) \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right), \tag{82}
\end{align*}
\]

and thus,

\[ \Pi_{00}(p) |^{(a)} = \frac{1}{4\pi} \left( -12 + 2(D-1) \right) \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right). \tag{83} \]

The one propagator contribution \( \Pi_{00}(p) |^{(b)} \) to \( \Pi_{00}(p) \) is found as

\[
\begin{align*}
\Pi_{00}(p) |^{(b)}_{Y_I} &= \frac{1}{4\pi} \left( -4(D-1) \right) \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right), \\
\Pi_{00}(p) |^{(b)}_{A_r} &= \frac{1}{4\pi} \times 16 \times \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right), \\
\Pi_{00}(p) |^{(b)}_{G} &= \frac{1}{4\pi} \times 12 \times \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right), \tag{84}
\end{align*}
\]
the sum of which becomes

$$\Pi_{00}(p)|^{(b)} = \frac{1}{4\pi} \left( 28 - 4(D - 1) \right) \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right).$$  \hspace{1cm} (85)$$

The two-particle irreducible contribution $\Pi_{00}(p)|^{(c)}$ to $\Pi_{00}(p)$ is found as

$$\Pi_{00}(p)|^{(c)}_{Y_l} = \frac{1}{4\pi} \times 2(D - 1) \times \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right),$$
$$\Pi_{00}(p)|^{(c)}_{A_r} = \frac{1}{4\pi} (-8) \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right),$$
$$\Pi_{00}(p)|^{(c)}_{G} = \frac{1}{4\pi} (-8) \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right),$$  \hspace{1cm} (86)$$

and thus,

$$\Pi_{00}(p)|^{(c)} = \frac{1}{4\pi} (-16 + 2(D - 1)) \frac{p^2}{8} \ln \left( \frac{p^2}{\mu^2} \right).$$  \hspace{1cm} (87)$$

The total contribution to $\Pi_{00}(p)$, the sum of (83), (85) and (87), vanishes;

$$\Pi_{00}(p) = 0.$$  \hspace{1cm} (88)$$

There are potential contributions from Fig. 1(b) and (c) for $\Pi_{02}(p)$. The one propagator contribution $\Pi_{02}(p)|^{(b)}$ is found as

$$\Pi_{02}(p)|^{(b)}_{Y_l} = 0,$$
$$\Pi_{02}(p)|^{(b)}_{A_r} = \frac{1}{4\pi} \times 2p_1p_2 \times \frac{1}{8} \ln \left( \frac{p^2}{\mu^2} \right),$$
$$\Pi_{02}(p)|^{(b)}_{G} = \frac{1}{4\pi} \times 6p_1p_2 \times \frac{1}{8} \ln \left( \frac{p^2}{\mu^2} \right),$$  \hspace{1cm} (89)$$

the sum of which gives

$$\Pi_{02}(p)|^{(b)} = \frac{1}{4\pi} \times 8p_1p_2 \times \frac{1}{8} \ln \left( \frac{p^2}{\mu^2} \right).$$  \hspace{1cm} (90)$$

The two-particle irreducible contribution $\Pi_{02}(p)|^{(c)}$ is found as

$$\Pi_{02}(p)|^{(c)}_{Y_l} = 0,$$
$$\Pi_{02}(p)|^{(c)}_{A_r} = 0,$$
$$\Pi_{02}(p)|^{(c)}_{G} = \frac{1}{4\pi} (-8p_1p_2) \frac{1}{8} \ln \left( \frac{p^2}{\mu^2} \right),$$  \hspace{1cm} (91)$$

the sum of which becomes

$$\Pi_{02}(p)|^{(c)} = \frac{1}{4\pi} (-8p_1p_2) \frac{1}{8} \ln \left( \frac{p^2}{\mu^2} \right).$$  \hspace{1cm} (92)$$
Hence, eqs. (90) and (92) give

$$\Pi_{02}(p) = 0.$$  \(93\)

The two-point function \(\Pi_{2-}(p)\) receives corrections from the diagrams of topology in Fig. 1(c),

$$\Pi_{2-}(p) \big|_{Y,I} = \frac{1}{4\pi} \left( -\frac{D-1}{3} \right) \frac{(p_1)^3 p_2}{p^2},$$

$$\Pi_{2-}(p) \big|_{A_r} = \frac{1}{4\pi} \left( -\frac{1}{6} \right) \frac{(p_1)^3 p_2}{p^2},$$

$$\Pi_{2-}(p) \big|_{G} = \frac{1}{4\pi} \left( \frac{53}{6} \right) \frac{(p_1)^3 p_2}{p^2},$$ \(94\)

which give in total

$$\Pi_{2-}(p) = \frac{1}{4\pi} \frac{27 - D (p_1)^3 p_2}{3 p^2}.$$ \(95\)

Two types of diagrams shown in Fig. 1(b) and (c) contribute to \(\Pi_{0-}(p)\). The one propagator contribution \(\Pi_{0-}(p)\)\(^{(b)}\) is found as

$$\Pi_{0-}(p) \big|_{Y,I}^{(b)} = 0,$$

$$\Pi_{0-}(p) \big|_{A_r}^{(b)} = \frac{1}{4\pi} \left( -p^2 + 2(p_2)^2 \right) \frac{1}{8} \ln \left( \frac{p^2}{\mu^2} \right),$$

$$\Pi_{0-}(p) \big|_{G}^{(b)} = \frac{1}{4\pi} \left( -5p^2 + 6(p_2)^2 \right) \frac{1}{8} \ln \left( \frac{p^2}{\mu^2} \right),$$ \(96\)

which give

$$\Pi_{0-}(p)^{(b)} = \frac{1}{4\pi} \left( -6p^2 + 8(p_2)^2 \right) \frac{1}{8} \ln \left( \frac{p^2}{\mu^2} \right).$$ \(97\)

The two-particle irreducible contribution \(\Pi_{0-}(p)^{(c)}\) is found as

$$\Pi_{0-}(p) \big|_{Y,I}^{(c)} = 0,$$

$$\Pi_{0-}(p) \big|_{A_r}^{(c)} = 0,$$

$$\Pi_{0-}(p) \big|_{G}^{(c)} = \frac{1}{4\pi} \left( 6p^2 - 8(p_2)^2 \right) \frac{1}{8} \ln \left( \frac{p^2}{\mu^2} \right),$$ \(98\)

the sum of which becomes

$$\Pi_{0-}(p) \big|^{(c)} = \frac{1}{4\pi} \left( 6p^2 - 8(p_2)^2 \right) \frac{1}{8} \ln \left( \frac{p^2}{\mu^2} \right).$$ \(99\)

Hence, the sum of eqs. (97) and (99) vanishes;

$$\Pi_{0-}(p) = 0.$$ \(100\)
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