Numeric Modeling of Phononic Crystal with Time-Dependent Properties

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Abstract. The research is devoted to numeric modeling of phononic crystals with time-dependent properties of periodic structure. There are many works describing phononic crystal with a tunable bandgap. The mechanism of restructuring can have a different physical nature. In this turn, we developed model of phonon crystal with fast-changing properties, where frequency of changing and propagating wave have the same order. The influence of time-dependent characteristics on propagation of wave packets, waves with different initial phase is shown in this work. Parameters of the crystal cell are given by time-dependent; signal propagation has significant differences from the static case. It was shown that under certain conditions a phononic crystal acquires the properties of a phase filter.

Keywords: modeling, wave propagation, material properties, phononic crystal, structure

Introduction

A phononic crystal is an artificial crystal of a finite-size periodic array composed of elements with different acoustic impedance. It should have full band-gaps, or complete band-gaps, where any sound wave is not allowed to propagate into the crystal but is reflected completely by the crystal [1, 2]. Several classes of phononic crystals materials differing by the physical nature of the inclusions and the matrix have been studied. By the nature of the materials they are solid/solid, fluid/fluid, and mixed solid/fluid composite systems. In the considered case, solid/solid material only longitudinal modes are allowed. There are opportunities of shear waves in suspension and viscoelastic materials.

Materials with tunable components are interesting for us. Rebuilding is possible by controlling the electronics, which can change material properties both independently of the transmitted signal and with feedback dependent on the signal [3, 4]. Properties changing of the material by influence to the medium by an external field were also investigated, for example, in work [5]. A number of studies have investigated the change in the speed of sound in materials located in magnetic and electric fields. For magnetic liquids in a magnetic field, the change can reach 50%. In our model, the change in sound velocity occurs due to a 10% change in the bulk modulus, which corresponds to a change in the sound velocity by approximately 3% [6].

Method

For simulation, we used finite-difference time-domain (FDTD) method. The FDTD method has recently become popular for acoustics simulation. FDTD solves acoustical wave field in medium, by employing finite differential equations to approximate the derivatives of pressure and the particle
velocity, both temporally and spatially. The use of the FDTD method allows solving problems with time-dependent parameters of the media [7].

Sound propagation in the fluid is described by two kinds of differential equations, Euler’s equation and the equation of continuity. For the loss media case [8]:

$$\frac{\partial u}{\partial t} + \frac{\partial P}{\partial x} + \alpha u u = 0,$$
$$\frac{\partial u}{\partial t} + \frac{\partial P}{\partial y} + \alpha v v = 0,$$
$$\frac{\partial u}{\partial t} + \frac{\partial P}{\partial z} + \alpha w w = 0,$$
$$\frac{\partial P_x}{\partial t} + K \frac{\partial u}{\partial x} + \rho \beta_x \rho P_x = 0,$$
$$\frac{\partial P_y}{\partial t} + K \frac{\partial v}{\partial y} + \rho \beta_y \rho P_y = 0,$$
$$\frac{\partial P_z}{\partial t} + K \frac{\partial w}{\partial z} + \rho \beta_z \rho P_z = 0,$$
$$P = P_x + P_y + P_z,$$

where $P_x, P_y, P_z$ - are components of pressure, $u, v, w$ - are particle velocity in $x, y, z$ direction, respectively, $K, \rho$ - volume elastic ratio and density and $\alpha, \beta$ - the factors responsible for sound absorption. The discretization of similar system is described in detail in [9].

**Result**

The model shown in the figure 1 is an acoustical waveguide of variable cross section. As can be seen in the figure, a port on the left is a source of acoustic waves, and on the right, respectively, is a receiver. The ends are absorbent and implemented as perfect matched layers. Other part of waveguide wall is rigid and velocity is equal zero in this wall.

![Diagram](image.png)

**Fig. 1.** General view of a numerical experiment

At the first stage, we calculated the dependence of pressure amplitude on frequency for the cases with a bulk modulus of $K = 2 \, \text{GPa}$ and $K = 2,2 \, \text{GPa}$, with the same density 1000 $\text{kg/m}^3$. The result of the calculation is shown in the figure. Different wave propagation is observed in the 22 kHz region.

We choose the frequency of 21750 Hz as the frequency at which the tuning is possible. Further the calculation was made for a crystal with time-dependent properties. Bulk modulus changes with a frequency four times the frequency of the signal from 2 GPa to 2,2 GPa for indicated on figure 2 region.
Fig. 2. Bandwidths for $K = 2.2$ (red) and $K = 2$ GPa (blue)

This variation leads to new properties of phononic crystal. Initial phase of signal have influence on signal transition. This dependence is shown in figure 3.

As you can see in figure 4 signal transition depends on phase. In this way the attenuation of acoustic wave with equal frequency but different phase can appear in time-dependent phononic crystal. This effect is provided by tuning of the properties at various period of signal transition causes changes in reflection coefficients on the elements of the crystal.

Fig. 3. Dependence of the pressure amplitude on the initial phase.
Frequency of signal is 21750 Hz, frequency of modulation is 87 kHz

Fig. 4. Comparison of pressure distribution in different phase $\phi$.
   a) $\phi = 1.25$ radians, b) $\phi = 0.25$ radians
Summary
In summary, we investigated the models of phononic crystal with time-dependent properties. In our numeric approach the changes of crystal properties occurs due to media properties variation. Supposed the turning mechanism may be changes of ferromagnetic liquid in magnetic field. It was shown that the modulation of media of a phononic crystal lead to dependence of signal amplitude on initial phase. It is possible to expand this model to 2-D and 3-D case.

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