Dark state localization of quantum emitters in a cavity

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We study a disordered ensemble of quantum emitters collectively coupled to a lossless cavity mode. The latter is found to modify the localization properties of the “dark” eigenstates, which exhibit a novel character of being localized on multiple, non-contiguous sites. We denote such states as semi-localized and characterize them by means of standard localization measures. We show that those states can very efficiently contribute to coherent energy transport. Our work underlines the important role of dark states in systems with strong light-matter coupling.

When quantum emitters and a cavity mode coherently exchange energy at a rate faster than their decay, hybrid light-matter states play an important role [1–3]. Such polaritonic states are superpositions composed of “bright” emitter modes and cavity photons, while the numerous remaining emitter modes have no photon contribution, i.e. they remain “dark”. Collective strong light-matter coupling has been intensively pursued in atomic [4–6] and condensed matter physics [7–12]. For example, polaritonic quasi-particles have been demonstrated to undergo Bose-Einstein condensation [13–18] and superfluidity [19–22] in laser-driven experiments. In contrast, strong coupling has been recently explored as a tool to engineer fundamental properties of matter, e.g. the critical temperature of superconductors [23, 24] or chemical reaction rates [25–32], without any external drive. Much interest is currently raised by the possibility of modifying energy [33–40] and charge [40–43] transport.

For transport, disorder plays a crucial role. It is well studied that coherent transport can be inhibited due to Anderson localization (AL) [44]. Here, an arbitrarily small disorder can lead to a localization of eigenstates in 1D and 2D [44, 45], while in 3D a metal–insulator transition driven by the disorder strength occurs [44, 46]. In this work, we study the fate of this coherent phenomenon in a cavity. While it is known that polaritonic states are largely unaffected by disorder [47] and can lead to a considerable enhancement of energy transmission [34–38, 40], the localization and transport properties of the dark states have remained largely unexplored. Moreover, disorder leads to a mixing of the bright with the dark states [48], which largely upends the usual description of light-matter coupling. Addressing these issues is expected to have important applications for the control of radiative energy transmission in mesoscopic systems.

In this work, we investigate a simple model for Anderson localization and coherent energy transport with N emitters collectively coupled to a cavity mode [Fig. 1(a)]. We focus on the impact of the cavity coupling on localized eigenstates, i.e. for a disorder strength much larger than the excitation hopping rate. We focus on the dark states of the system and find that they exhibit several surprising features: for any strength of light-matter interactions, they acquire a squared amplitude ∼ 1/N, on average, for arbitrary distances while remaining localized according to standard localization measures, such as the inverse participation ratio. However, we find that localization is distributed over two or more sites, which can be arbitrarily distant from one another. For large enough cavity coupling, they can be considered as hybridizations of a few localized states of the uncoupled system, and their energy lies in between those of the bare states. This results in semi-Poissonian statistics of the energy level spacings, which neither corresponds to that of a fully lo-
calized nor extended phase. We find that the dark states are responsible for a diffusive behavior, which is at odds with their localized nature. On average, the exponential decay of the excitation current with the system size for the uncoupled system is turned into an algebraic decay \( \sim 1/N \), and can thus dominate over the unknown component expected from polaritonic states. Our results are based on both numerical calculations and analytical results for vanishing excitation hopping, and should be directly relevant to coherent transport experiments with semiconductors interacting with confined electromagnetic vacuum fields [49–51].

We consider a 3D cubic lattice of \( N \) two-level systems embedded in a cavity. The Hamiltonian (\( \hbar = 1 \)) is

\[
\hat{H}_1 = -J \sum_{\langle i,j \rangle} \sigma_i^{\dagger} \sigma_j + \sum_i w_i \sigma_i^{\dagger} \sigma_i + g \sum_i (\hat{a}^{\dagger} \sigma_i + \hat{a} \sigma_i^{\dagger}),
\]

and \( \hat{H}_0 = \omega_0 \hat{a}^{\dagger} \hat{a} + \omega_c \sum_i \sigma_i^{\dagger} \sigma_i \). We restrict our discussion to a Hilbert space with a single excitation, i.e. \( \sum_i \sigma_i^{\dagger} \sigma_i + \hat{a}^{\dagger} \hat{a} = 1 \). Then, there are \( N + 1 \) basis states, \( |i, 0\rangle \), \( |G, 1\rangle \), denoting states with an excitation on site \( \hat{a} \), or in the cavity, respectively. Considering also the state without excitation, \( |G, 0\rangle \), the spin lowering and photon annihilation operators are defined as \( \sigma_i^- = |G, 0\rangle \langle i | \) and \( \hat{a} = |G, 0\rangle \langle G, 0 | \). In all numerical calculations, we consider the cavity mode (frequency \( \omega_c \)) in resonance with the average emitter transition (\( \omega_0 \)), i.e. \( \delta = \omega_0 - \omega_c = 0 \). The first term in \( \hat{H}_1 \) governs hopping (with rate \( J \)) between nearest neighbor sites, indicated by the notation \( |i, j\rangle \). Assuming periodic boundaries, this term is diagonalized by introducing the operators \( b_q = \sum_i \exp(-iq \cdot i) \sigma_i^{\dagger} / \sqrt{N} \). The second term contains on-site disorder, with \( w_i \) random variables uniformly distributed in \([-W/2, W/2]\). The third term describes the Tavis-Cummings emitter-cavity coupling [1] with local strengths \( g \). This term can be written in the form \( g_c (\hat{a} \hat{b}^{\dagger}_0 + \hat{a}^{\dagger} \hat{b}_0) \) with the collective strength \( g_c = g \sqrt{N} \), and couples the symmetric bright mode \( \hat{b}_0 \) to cavity photons. Importantly, \( g \) decreases with the cavity-mode volume \( V \) as \( g \sim 1/\sqrt{V} \) [52] and \( g_c \) thus remains independent of \( N \) for a fixed density \( N/V \).

In the absence of disorder (\( W = 0 \)), \( \hat{H}_1 \) has two polariton eigenstates \( |\psi_{\pm}\rangle = (\hat{b}_{q=0} \pm \hat{a}^{\dagger}) / \sqrt{2} |G, 0\rangle \) with energies \( E_{\pm} = \pm g_c \) as well as \( N - 1 \) uncoupled dark states \( |\psi_{\alpha \neq \pm}\rangle = \hat{b}_{q \neq 0} |G, 0\rangle \) with vanishing photon weight, \( \langle G, 1 | \psi_{\alpha \neq \pm} \rangle = 0 \). Finite disorder (\( W \neq 0 \)) leads to a coupling between the bright and the dark states since the second term in \( \hat{H}_1 \) is non-diagonal in quasi-momentum space. The dark eigenstates therefore acquire a small photon weight \( \langle G, 1 | \psi_{\alpha \neq \pm} \rangle \sim 1/N \) [53], and can be thought of as “grey” states. In the following we are interested in the modification of the emitter part of the system, and define the normalized emitter amplitudes as

\[
a_{\alpha j} \equiv \langle j, 0 | \psi_{\alpha \neq \pm} \rangle / \sqrt{N_c} \quad \text{with} \quad N_c = 1 - |\langle G, 1 | \psi_{\alpha} \rangle|^2.
\]

For \( g_c = 0 \), \( \hat{H} \) corresponds to a usual Anderson model, displaying a \( W \)-dependent mobility edge that determines a metal-insulator transition at \( W_c \approx 16.5J \) (for energy states lying in the middle of the band) [54–57]. While for \( W < W_c \) the eigenstates \( |\psi_{\alpha} \rangle \) resemble extended Bloch states, they are localized around given sites for \( W > W_c \), e.g. \( |\psi_{\alpha}\rangle^2 \sim e^{-i-j\xi}/\xi \) for a state localized on site \( \hat{a} \), with \( \xi \propto W/J \)-dependent localization length. In the following, we investigate the case \( g_c, W \neq 0 \), and focus on spectral and transport properties of the Anderson insulator for strong collective light-matter couplings \( g_c > W > W_c \).

The modification of Anderson localization in a cavity can be understood by first considering the eigenstates of \( \hat{H} \) for \( J = 0 \), in which case the spatial dimensionality becomes irrelevant. In second-order perturbation theory, a trivially localized eigenstate on site \( \hat{a} \) (i.e. \( |i, 0\rangle \)), for \( g_c = 0 \) acquires an amplitude on site \( j \neq i \) via the cavity, \( b_{i \neq j} \equiv g^2/\delta (w_i - w_j) (w_i + \delta) \), for configurations with \( g^2 < \delta (w_j - w_i) (w_i + \delta) \). A lower bound for the squared amplitude of perturbed localized states is thus \( |b_{i \neq j}|^2 > 4g^4/(N^2 W^2) \), setting \( \delta = 0 \). We find that the averaged value over disorder realizations (keeping only the finite part of the averaging integral) is \( |b_{i \neq j}|^2 \sim 4g_c^4(4 - 2 \log(4))/NW^4 \) for large \( N \).

In Fig. 1(c) we show numerically that also for finite \( J \ll W \) the weights of an eigenstate localized in the center of a 3D cube, (logarithmically) averaged over disorder realizations, maintains an exponentially localized profile at short distances, followed by a constant tail rising with \( g_c \). The tails are consistent with our perturbative result for small \( g \) (dashed lines) and saturate for strong couplings \( g_c > W > J \). Note that a similar behavior was reported for dissipative couplings to a common reservoir [58, 59].

For strong coupling (\( g_c > W > J \)), two polaritonic states \( |\psi_{\pm}\rangle \) with \( |\langle G, 1 | \psi_{\pm}\rangle|^2 \approx 0.5 \) can be separated by a splitting \( \sim 2g_c \) (only slightly modified by disorder) emerge from the band of width \( W \). We find that the energies of the \( N - 1 \) dark states lie in between the \( N \) bare \( |g_c = 0\rangle \) levels, which can be seen as a simple consequence of the “arrowhead” matrix shape of the single-excitation Hamiltonian for \( J = 0 \) [60] [see Fig. 1(b)]. The strong cavity coupling leads to a hybridization between the bare levels that are close in energy, but not necessarily in real space. For a single disorder realization, the dark states appear strongly localized at multiple sites [see Fig. 1(d)]. We term this behavior as “semi-localization”.

Information about the spatial localization of the dark eigenstates with energy \( E_{\alpha} \) is given by the inverse participation ratio (IPR), \( IPR(E_{\alpha}) = \sum_{i=1}^{N} |a_{\alpha i}|^4 \). A finite, size-independent IPR indicates a localized eigenstate, while an IPR scaling as \( 1/N \rightarrow 0 \) indicates an extended one. Initializing the system in the state \( |i, 0\rangle \), the infinite-time-averaged probability to find an excitation at site \( j \) is

\[
\Pi_{ij} = \lim_{T \rightarrow \infty} \int_0^T dt P_{ij}(t) / T,
\]

with
For strong-couplings $g_c > W > W_c$, a plateau ($\Pi_{ii} \approx 0.4$) indicates a “semilocalized” regime. (b-c) Disorder-averaged inverse participation ratio $\text{IPR}(\epsilon_s)$ as a function of $W/J$ and the renormalized dark state energy $\epsilon_s$ (bins of widths 0.02, $\sim 100$ realizations, white dashed line: $W = W_c$). (b) $g_c = 0$ (no cavity); (c) $g_c = 30J$ (larger $W/J$-scale), showing an extended area with $\text{IPR}(\epsilon_s) \approx 0.4$. (d) Finite-size scaling of $\text{IPR}(\epsilon_s)$ for the parameters corresponding to the symbols in (c). Square ($W = 5J, \epsilon_s = 0.5$); circle ($W = 175J, \epsilon_s = 0.5$); triangle ($W = 175J, \epsilon_s = 0.9$).

$P_{ij}(t) \equiv |\langle j, 0|\phi(t)\rangle|^2$ and $\langle \phi(t)\rangle = e^{-i\epsilon_t} |j, 0\rangle$. The IPR is connected to the return probability $\Pi_{ii}$ by $\sum_i \Pi_{ii} = \sum_i \text{IPR}(E_s) N_s^2$. The IPR$(E_s)$ can thus be interpreted as the contribution of a given eigenstate to $\sum_i \Pi_{ii}$.

In Fig. 2(a), we compute numerically the disorder average of $\Pi_{ii}$, $\bar{\Pi}_{ii}$, for the central site of a cubic lattice ($N = 15^3$). For $g_c = 0$ (dashed line), $\bar{\Pi}_{ii}$ increases from 0 (extended phase) to 1 (localized phase) upon increasing the disorder strength $W/J$. Remarkably, we find that $\bar{\Pi}_{ii}$ exhibits a plateau $\approx 0.4$ for $g_c > W > J$, which persists up to large disorders strengths ($W \sim 100J$ for $g_c = 50J$).

The disorder-averaged IPR, $\text{IPR}(E_s)$, is shown in Figs. 2(b-c) as a function of $W/J$ for the Anderson model ($g_c = 0$) and with a cavity coupling $g_c = 30J$. As we only focus on the dark states (in the band of width $W$), we use a dimensionless, renormalized energy scale $\epsilon_s = (E_s - W/2)/W$ with $\epsilon_s \in [0, 1]$. For each disorder realization, we bin the different levels into groups with equal energy width and then average over realizations in each bin. Figure 2(b) shows the emergence of localized states upon increasing $W/J$, starting from the edges of the spectrum. A strong cavity coupling [Fig. 2(c)] leads to three distinct regimes: i) a delocalized region $\text{IPR}(\epsilon_s) \sim 0$ for $W \lesssim W_c$; ii) a fully localized region $\text{IPR}(\epsilon_s) \sim 1$ for $W > g_c$; and iii) an extended area with $\text{IPR}(\epsilon_s) \approx 0.4$ where the dark states feature semi-localized characteristics consistent with the return probability and the results shown in Fig. 1(c). The persistence of semi-localized states in the vicinity of $\epsilon_s = 0.5$ ($\delta = 0$) can be understood from the failure of perturbation theory, even for $W \gg g_c$. The energy separation between the two levels $(\epsilon_i, \epsilon_j)$ closest to $\delta = 0$ is $(w_{ij} - w_{jj}) \sim W/N$. For them, the perturbation condition $g_c^2 \ll W(w_{ij} - w_{jj} + \delta)$ is violated for all $W$ considered in Fig. 2(c), as they hybridize via the cavity.

Fig. 2(d), we analyze the finite size scaling of $\text{IPR}(\epsilon_s)$ in the three regions for parameters corresponding to the symbols in Fig. 2(c). We observe that the IPR of semi-localized states does not scale with the system size. These states exhibit the same behavior as in the fully localized region, only with a reduced value, which is consistent with states localized on multiple sites. In contrast, $\text{IPR}(\epsilon_s) \propto 1/N$ for extended states.

Localization properties of eigenstates are also characterized by their level statistics [57]. Here, we numerically analyze the probability distribution function $P(s_\alpha)$ for spacings between adjacent eigenenergies, $s_\alpha = \epsilon_{\alpha+1} - \epsilon_{\alpha}$. In Fig. 3(a), we plot $P(s_\alpha)$ for eigenstates corresponding to the symbols in Fig. 2(c). While in the delocalized region ($W \lesssim W_c$) $P(s_\alpha) = \frac{\pi}{8} s_\alpha \exp(-\frac{\pi}{4} s_\alpha^2)$ follows a Wigner-Dyson distribution, the fully localized phase is characterized by a Poissonian, $P(s_\alpha) = \exp(-s_\alpha)$ [61]. Interestingly, we observe that the semi-
localized region features semi-Poissonian [62] statistics, \( P(s_\alpha) = 4s_\alpha e^{-2s_\alpha} \). We have checked that this behavior appears in the entire semi-localized region and is independent of the system size. The semi-Poissonian form can be simply understood for \( J = 0 \). Then, bare \((g_c = 0)\) levels follow a Poisson distribution. Since for strong coupling \((g_c > W)\) dark states lie in between the bare levels, we can model the dark state distribution as
\[
P(s_\alpha) = \int \text{d}x \text{d}y \left( s_\alpha - \frac{x + y}{2} \right) e^{-x} e^{-y} = 4s_\alpha e^{-2s_\alpha},
\]
where we have assumed that the latter lie exactly at equal distance from the two closest bare levels. In order to check whether this property remains valid for \( J \neq 0 \), we also analyze numerically the disorder-averaged deviation \( \Sigma_\alpha = N \langle E_\alpha - [(w_i + w_{i+1})/2] \rangle \) in Fig. 3(b), with \( w_i \) and \( w_{i+1} \) the closest bare levels immediately below and above the energy \( E_\alpha \), respectively. While in the localized phase (triangle) the eigenenergies are found to be very close to the (fully localized) bare levels, they are much closer to \((w_i + w_{i+1})/2\) in the semi-localized region (square), thereby confirming our simple argument above.

Finally, we investigate the role of semi-localized states on the transport and diffusion properties, in 1D for convenience. In Fig. 4(a), we analyze the excitation current flowing through the chain as a function of \( N \) (strong coupling, \( g_c = 30J, W = 10J, \gamma = 0.05J \), see text). Shown are the mean \((\bar{T}, \text{blue circles})\) and maximum/minimum currents \((I_{\text{max/min}}, \text{red lines})\) of 100 disorder realizations. Dashed lines are guides to the eye for \( 1/N \) and \( 1/N^2 \). Inset: \( T \propto e^{-N} \) for \( g_c = 0 \). (b) Disorder-averaged mean square displacement \( \sigma^2/N \) \((1D, g_c = 50J, W = 30J, 200 \text{ realizations, see text})\). While absence of diffusion is found for \( g_c = 0 \) \((\sigma^2/N \simeq 0, \text{grey line, expected for 1D Anderson localization})\), diffusive dynamics, \( \sigma^2 \propto t \), occurs for \( g_c \gg W \).

Localized region can be simply understood for \( J = 0 \). Here, \( \hat{L} = \hat{L}_1 \hat{L}_N + 2\hat{L}_\eta \hat{L}_h \), with \( \hat{L}_\eta = \sqrt{\gamma/2\delta} \) and \( \hat{L}_h = \sqrt{\gamma/2\delta} N \) (with \( \gamma \) a pump-}

**FIG. 4.** (a) Excitation currents through a 1D chain as function of \( N \) (strong coupling, \( g_c = 30J, W = 10J, \gamma = 0.05J \), see text). Shown are the mean \((\bar{T}, \text{blue circles})\) and maximum/minimum currents \((I_{\text{max/min}}, \text{red lines})\) of 100 disorder realizations. Dashed lines are guides to the eye for \( 1/N \) and \( 1/N^2 \). Inset: \( T \propto e^{-N} \) for \( g_c = 0 \). (b) Disorder-averaged mean square displacement \( \sigma^2/N \) \((1D, g_c = 50J, W = 30J, 200 \text{ realizations, see text})\). While absence of diffusion is found for \( g_c = 0 \) \((\sigma^2/N \simeq 0, \text{grey line, expected for 1D Anderson localization})\), diffusive dynamics, \( \sigma^2 \propto t \), occurs for \( g_c \gg W \).

**In conclusion,** we have shown that Anderson localization can be strongly modified by coupling the disordered ensemble to a cavity. This is manifested by the emergence of dark states localized on multiple sites with energy spacings following semi-Poissonian statistics. These states are responsible for a diffusive behavior and an algebraic decay of energy transmission for strong light-matter couplings. It is an interesting prospect to investigate how dephasing and dissipation [58] can affect our results.
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\[
\hat{H}' = \frac{1}{2} \sum_{i,j} \left( \frac{g^2}{w_i + \delta} + \frac{g^2}{w_j + \delta} \right) \hat{a}^+_i \hat{a}^-_j.
\]

It differs from other known models with long-range couplings, such as power-law hopping \( \sim J/|i-j|^\alpha \) with \( \alpha < d \) (\( d \) is the dimensionality). In this latter case, for random \( J \) all states are delocalized \([66, 67]\), while for constant \( J \) and \( \alpha = 0 \) essentially all states are localized \([68]\). Another model with random particle positions was studied in Refs. \([69, 70]\), where wave functions were found to be localized algebraically for all \( \alpha \).

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