About the preliminary design of the suspension spring and shock absorber

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Abstract. The aim of this paper is to give some recommendation for the design of main-spring and shock absorber of motor vehicle suspensions. Starting from a 2DoF model, the suspension parameters are transferred on the real vehicle on the base of planar schemes for the linkage. For the coil spring, the equations that must be fulfilled simultaneously permit to calculate three geometrical parameters. The indications presented for the shock absorber permit to obtain the damping coefficients in the compression and rebound strokes and to calculate the power dissipated during the vehicle oscillatory movement.

1. Introduction

The aim of this work is to give the main directions to be followed during the earliest design stages of passive-suspension’s spring and shock absorber, or, in other words, to indicate the main steps needed to be done when the design starts from scratch.

The design of a vehicle suspension system starts with very few input parameters. Simple models are used during initial simulations in order to ensure the desired compromise between comfort and dynamic performance qualities, at different vehicle speeds and loads. That stage leads to the setup of the needed suspension parameters on the model and vehicle.

2. The suspension model and its correspondence with the real vehicle

To design the vehicle suspension, a set of input data are necessary. To obtain this information, the automotive engineers imagine and use models to simulate the ride behaviour of the suspension. Corresponding to the needs, the models can have different complexities.

For basic studies of vehicle’s ride behaviour and quality, the most used model is the so-called “quarter-vehicle model” or “vehicle-corner model” [17], [2], [4], [19]. As presented in the left side of figure 1, this includes only two elements with concentrated inertial properties: one for a wheel and the other for the part of the body supported by that wheel. Because normally the up-and-down movement is of most interest, only the mass of those inertial elements will be considered (neglecting the moments of inertia involved in the oscillation). That means the model will have only two degrees of freedom, the vertical translation of the masses \( m_W \) of the wheel (unsprung mass) and \( m_B \) of the vehicle “quarter” (sprung mass). In the model, the vehicle body is linked to the wheel through massless spring and damper, while the connection between the wheel and the ground is realised by the tyre (represented here as a spring-damper combination). Suited for frequencies up to 30 – 50 Hz (bigger of 2 to 4 times as the natural frequency of the unsprung mass), this model has only six parameters: the masses, the stiffness \( k_S \) of the suspension spring, the damping coefficient \( c_D \) of the shock absorber, and the
stiffness and damping coefficient of the tyre, \(k_T\) and \(c_T\). Due to the mechanical properties of the tyre rubber and of the low frequencies of interest (0…25 Hz) in the case of the ride study, the tyre damping may be neglected (\(c_T \approx 0\)).

Accordingly with the destination of the vehicle, the design engineer will set-up the best values for the model parameters \(k_T\), \(k_S\) and \(c_D\). In the literature there are indications about how to do that by simulation in time or frequency domain in the case of deterministic of stochastic excitation. A possible way to do that it is presented in [9].

The next step after the tuning of the “quarter-car” model is to “transfer” or “translate” the values of the design parameters from the model to the real vehicle, i.e. the values \(k_S\) of the suspension main spring stiffness, \(c_D\) of the shock absorber damping and \(k_T\) of the tyre stiffness.

To realize the correspondence between all the parameters of the model and the real vehicle it is necessary to obtain almost similar dynamic behaviours for the both systems: the mechanical energy (both, kinetic and potential components) and theirs rates of change must be equal at any moment for the dynamic model and the real vehicle.

The results of parameters translation from the model to the vehicle depend on the type, layout and dimensions of the suspension linkage. How the values of the coefficients \(k_S\) and \(c_D\) are changed during this “transfer” (while the tyre stiffness \(k_T\) remains the same) it is presented in [8]. In that paper, the demonstration is based on the generally adopted (even though it is not entirely true) hypothesis of planar mechanisms approximating some of the most spread independent suspension linkages. Such a scheme of calculus it is presented in the right side of figure 1.

![Figure 1. Calculus scheme of the short-long-arm suspension linkage and its 2DoF model.](image)

Starting from the values of the real vehicle’s tyre stiffness and shock absorber damping, some recommendations regarding the design of the coil spring and shock absorber will be made further.

### 3. The coil spring

The suspension spring design can be realized relying on machine elements publications such as [18], [6], or automotive design publications, such as [17], [2], [1]. Accordingly to the literature, the next three equations must be simultaneously fulfilled:

\[
D_m = R_{dia} \cdot d; \quad \tau = F \cdot [8 \cdot c_{cor}(R_{dia}) \cdot D_m]/\pi \cdot d^2; \quad k = (G \cdot d^4)/(8 \cdot N_c \cdot D_m^3) \quad (1)
\]

The notations in this equation system represent: \(d\) – wire diameter; \(D_m\) – mean coil (winding) diameter; \(E\) – modulus of elasticity (Young’s modulus); \(\nu\) – Poisson’s ratio; \(G=E/[2 \cdot (1+\nu)]\) – shear modulus; \(\tau\) – torsion stress; \(R_{dia}\) – ratio of coil and wire diameters; \(c_{cor}\) – stress correction factor (as
function of \( R_{\text{dia}} \) considering the uneven distribution of torsion stress in the case of variable spring load (which is the case of vehicle suspension); \( N_c \) – number of active coils; \( F \) – compression force; \( k \) – spring stiffness. The spring dimensions \((d, D_m)\) and the force \( F \) are illustrated also in figure 2.

![Figure 2. Scheme with the calculus elements of the coil spring](image)

The stress correction factor \( c_{\text{cor}} \) (also named What’s factor) depends on \( R_{\text{dia}} = D_m/d \) and can be calculated with the theoretical derived equation \([18]\)

\[
c_{\text{cor}} = \frac{(4R_{\text{dia}} - 1)}{(4R_{\text{dia}} - 4)} + 0.615/R_{\text{dia}}
\] (2)

or can be taken from tables with experimentally determined data \([1]\).

From the equations (1), three unknowns from the previous list of magnitudes can be determined, while the others must be adopted. For example, the three unknowns may be: \( d, D_m \) and \( N_c \). In that case, the material properties must be adopted (the shear modulus \( G \) and the admissible torsion stress \( \tau_{\text{adm}} \)), while the stiffness coefficient \( k \) (the spring rate) is already known (as presented previously).

The usual maximum dynamic load on the tyre is indicated in the literature: about 2 times \([17]\) up to 2.5 times \([11]\) bigger than the static load. That means the maximal compression force \( F_{\text{max}} \) acting on the coil spring will not exceed 2 times the static load, because at the extremities of the linkage stroke, the main spring is “helped” (protected) by rubber or polyurethane spring buffers. For example, in the case of the MacPherson suspension presented in \([5]\), the maximum force acting on the spring is approximately \( 1.7 \cdot F_w \).

For other spring types, as leaf and torsion bar springs, the equation system (1) will be replaced by other specific equations, but involving also dimensional and material (strength) parameters.

### 4. The shock absorber

At low frequencies, the shock absorber is the suspension main element reducing the energy of oscillations. It serves to dissipate the kinetic and potential energy existing in the suspension during relative wheel-body movements. The shock absorber is an essential element to realize a good compromise between a low level of the vibrations transmitted from the ground to the vehicle body (comfortability) and an adequate control of the unsprung and sprung masses, in order to keep good dynamic performances (active safety and handling).

The working principle of the modern shock absorbers consists in generating friction forces opposing to the relative movements of the damper ends by forcing a quantity of oil to pass through calibrated orifices, as illustrated in figure 3. Many of the automotive specialists recommend, and motor-vehicle manufacturers adopt, designs with asymmetric characteristics that offer, for the same relative speed, smaller damping force on the compression stroke as in the rebound (extension) stroke. That behaviour will reduce the summative force transmitted together by the spring and shock absorber to the body when the wheel escalates a bump, while at the rebound the damper will consume most of the potential energy accumulated in the spring. If \( c_c \) and \( c_r \) are the damping coefficients of the shock absorber in the compression and rebound strokes, theirs ratio

\[
R_f = c_r/c_c
\] (3)
may have values in the interval 2…5 [17], [7], [19], with a tendency at the modern vehicles to limit that value to 2…3, in order to avoid the body “sinking” when the vehicle starts passing over obstacles [4] and not to impede the tyre to maintain the contact with the ground.

![Figure 3](image)

**Figure 3.** Simple principle scheme presenting the oil circulation into a shock absorber

When the oil passes through orifices with constant flowing section, the damping force is practically proportional with the square of the relative speed,

\[ F = \text{sgn}(v) \cdot c \cdot v^2. \]  \hspace{1cm} (4)

But, with modern shock absorber designs, that parabolic force-speed characteristic can be transformed due to supplementary flowing sections opened by complex control valves when the oil pressure difference in the working chambers attains certain values. Therefore, using valves, the actual characteristic shape \( F = F(v) \) may be very different as the natural parabolic one from equation (4). Today, the characteristics are preferred almost linear (\( F = c \cdot v \)), figure 4, or even regressive, in order to reduce the possibility of excessive forces apparition.

![Figure 4](image)

**Figure 4.** Schematic typical work diagram (left) and characteristic diagram (right) of a modern bi-linear shock absorber

There are many specialized publications presenting models for calculating different types of shock absorbers: [17], [7], [10], [14], [15], [3], [13], [16], that may be used for mono- or bi-tubular designs; without or with separation piston; with normal- or magnetorheological oil. These theoretical models are then verified in practice. At some designs with expansion reservoir, the influence of the pressured gas may be considered in a more complex model of the shock absorber or as a supplementary spring added to the main one.

To experimentally obtain the characteristics, the damper is excited with sinusoidal displacement \( u \) and speed \( v \) [12]:

\[ u = a \sin(\omega t) = a \sin(2\pi ft); \quad v = \frac{du}{dt} = a \omega \cos(\omega t) = a \cdot 2\pi f \cos(2\pi ft) \]  \hspace{1cm} (5)

where \( a \) is a constant displacement amplitude and \( f \) and \( \omega \) are constant frequency and pulsation. By varying the frequency in several steps, different force-displacement curves \( F = F(u) \), named *work diagrams*, are obtained, as shown in figure 4. Then, by taking the peak values of the damper force at the displacement \( u = \pm a \), which corresponds with the velocity \( v = \pm 2\pi f a \), the curve \( F = F(v) \), named
characteristic diagram, is generated. Normally, the half-cycle representing the rebound stroke is associated with negative damper velocities and forces.

In [4] it is demonstrated that an individual characteristic diagram of any shock absorber can be replaced for simulation purpose with an equivalent bilinear one:

\[ F_c = c_c \cdot v \quad \text{if} \quad v \geq 0; \quad F_i = c_i \cdot v \quad \text{if} \quad v < 0; \quad R = c_i / c_c = \text{constant} \quad (6) \]

At that point, knowing the mass \( m_B \) of the vehicle body, the spring stiffness \( k_S \) and adopting the damping ratio

\[ \zeta_D = \frac{c_D}{2 \sqrt{k_S m_B}} \quad (7) \]

it can be calculated the equivalent damping coefficient \( c_D \), considered constant for the both strokes of the shock absorber.

The instantaneous power dissipated by the shock absorber is

\[ P_D(t) = F(t) \cdot v(t) = c \cdot v^2(t). \quad (8) \]

that, for sinusoidal displacement, will become

\[ P_{Ds}(t) = c \cdot a^2 \cdot \omega^2 \cdot \cos^2(\omega t). \quad (9) \]

Further, integrating this power \( P_{Ds}(t) \) over a half cycle it obtains the dissipated energy

\[ L_{s1/2} = \int P_{Ds}(t) = c \cdot a^2 \cdot \omega \cdot \pi / 4. \quad (10) \]

The problem is now to realize the equivalence of the constant coefficient \( c_D \) with the coefficients \( c_c \) and \( c_r \) in the equation (6). The logical possibility to make this approximation must take into account the equality of the energy consumed in an oscillation cycle by the equivalent and by the real shock absorbers:

\[ 2 \ c_D \cdot a^2 \cdot \omega \cdot \pi / 4 = (c_c + c_i) \cdot a^2 \cdot \omega \cdot \pi / 4 \quad \text{which means} \quad c_D = (c_c + c_i) / 2, \quad (11) \]

i.e. the equivalence equation indicated in the literature.

Now, with the equations (6) and (11), the true compression and rebound damping coefficients \( (c_c \) and \( c_i) \) of the shock absorber can be calculated.

For a sinusoidal excitation with the frequency \( f \) [cycles/s], the mean power dissipated by the suspension (transformed in heat) is

\[ P_{Dmean} = f \cdot (c_c + c_i) \cdot a^2 \cdot \omega \cdot \pi / 4 = f \cdot c_D \cdot a^2 \cdot \omega \cdot \pi / 8 \quad (12) \]

and may be used for the calculus of the thermal balance.

Because the damping coefficients \( c_c \) and \( c_i \) in the compression and rebound strokes are known now, the design of the shock absorber may continue with the dimensioning of the piston orifices (as presented in [17] and [7]) and also with the strength calculus of the parts, because the mean and maximal forces produced by the shock absorber (\( F_c \) and \( F_i \)) can be estimated.

5. Conclusions

For the study of the ground vehicles independent suspensions, simple dynamic and mathematic models are used, mainly in the first stages of the design process. After the tuning of the suspension parameters realized on the model, the suspension stiffness and the damping coefficient are “transferred” (with bigger values) to the real vehicle’s main-spring and shock absorber. Based on these necessary values, in the paper are presented some recommendations for the design process of the suspension spring and shock absorber. These may be useful to establish both the spring’s wire and mean coil winding diameters and the shock absorber’s damping coefficients and mean power dissipated at different working regimes.
6. References

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