CP Violation in the Width Difference of the Scalar Electron Decay due to the Complex Gaugino Masses

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Abstract

We calculate the width difference of the scalar electron decay, $\Gamma(\tilde{e}_L \to e_L \omega) - \Gamma(\tilde{e}_L \to \bar{e}_L \omega)$ in the softly broken supersymmetrized standard model. The CP asymmetry is assumed to arise from the complex gaugino masses. Even in limit that the electron mass $m_e$ is vanishing, the difference can be of the order of $\alpha$.

PACS: 11.30.Er, 12.60.Fr, 14.80.Cp
Introduction

Supersymmetric extensions of the Standard Model (SM) has been widely considered to be the most likely new physics beyond SM. In particular, such extension can contains various new mechanisms of CP violation. It is the purpose of this note to explore the consequence one of such mechanism.

In the softly broken supersymmetrized standard model, the dimension-3 gaugino masses can be complex, i.e. $-\frac{1}{2}m_{\psi} e^{-i2\phi} \bar{\psi}_L \psi_L + $ h.c., which can be transformed into the canonical Majorana form $-\frac{1}{2}m_{\psi} \bar{\psi} \psi$ with the Majorana field $\psi$ defined to be $e^{-i\phi} \psi_L + e^{i\phi} (\psi_L)^c$. However, such rearrangement only shift the absorbed phase into interaction terms such as gauge interactions. For example, since the electron has the quantum numbers

\begin{align*}
y/2 & T_3 \\
\epsilon_L & -\frac{1}{2} -\frac{1}{2} \\
\epsilon_R & -1 0
\end{align*}

its supersymmetric couplings to the scalar electron and the gauginos [the bino $\beta$ and the (neutral) wino $\omega$] become complex, and can be tabulated as:

\begin{align*}
\text{Bino } \beta & \quad \text{Wino } \omega \\
\bar{\epsilon}_L & \frac{\sqrt{2}e}{2 \cos \theta_W} e^{i\phi_\beta} \bar{\epsilon}_L e_L \beta + \frac{\sqrt{2}e}{2 \sin \theta_W} e^{i\phi_\omega} \bar{\epsilon}_L e_L \omega \\
\bar{\epsilon}_R & \frac{\sqrt{2}e}{\cos \theta_W} e^{i\phi_\beta} \bar{\epsilon}_R e_R \beta
\end{align*}

The supersymmetric part of gauge interaction Lagrangian is

\begin{align*}
\mathcal{L} = \frac{\sqrt{2}e}{2 \cos \theta_W} e^{i\phi_\beta} \bar{\epsilon}_L e_L \beta + \frac{\sqrt{2}e}{2 \sin \theta_W} e^{i\phi_\omega} \bar{\epsilon}_L e_L \omega + \frac{\sqrt{2}e}{\cos \theta_W} e^{i\phi_\beta} \bar{\epsilon}_R e_R \beta + \text{h.c.} + \cdots
\end{align*}

The CP phase dependence of the CP-even parameters in the cross section of $e_L e_L \rightarrow \bar{\epsilon}_L \bar{\epsilon}_L$ has been recently studied in Ref.\,\[. However, it is important also to establish direct CP violation by searching for CP-odd effects, such as decay width differences. In this article, we calculate the asymmetry between conjugated channels, like $\text{\Gamma}(\bar{\epsilon}_L \rightarrow e_L \omega) - \text{\Gamma}(\bar{\epsilon}_L \rightarrow \bar{\epsilon}_L \omega)$. It is clear that in the limit when the electron mass is vanishing, only the phase difference $\phi_\omega - \phi_\beta$ is physical in the above Lagrangian.
**Formalism**

Without loss of generality, we assume the bino $\beta$ and the wino $\omega$ do not have significant mixing with the Higgsinos. To simplify our discussions, we also assume that $\tilde{e}_L$ and $\tilde{e}_R$ do not mix. If they do mix, it will constitute another mechanism of CP violation which should be treated separately. In this limit, electric dipole measurements are not directly sensitive to the phase difference $\phi_\omega - \phi_\beta$, which can be of order unity. It is very likely that both these weak gauginos, $\beta$ and $\omega$, are lighter than the scalar left handed electron $\tilde{e}_L$. In this case the asymmetry mentioned above exists through the final state interaction between the two decay channels. We denote physical masses of $\tilde{e}_L$, $\beta$, and $\omega$ as $m$, $m_1$, and $m_2$, respectively. Note that $\tilde{e}_R$ is never involved in our process. The amplitude at the tree level for the process $\tilde{e}_L(P) \to e_L(p)\omega(p')$ is

$$M^0 = \left(\frac{\sqrt{2} e^{i\phi_\omega}}{2 \sin \theta_W}\right) \bar{u}(e_L,p) \frac{1}{2} + \frac{\gamma_5}{2} v(\omega,p') .$$

(4)

The one-loop amplitude occurs with the intermediate state $e_L\beta$. The Feynman diagram, as shown in Fig. 1, for the process, $\tilde{e}_L(P) \to \beta(k) e_L(k') \to e_L(p)\omega(p')$ gives

$$M^1 = \left(\frac{\sqrt{2} e^{-i\phi_\omega}}{2 \sin \theta_W}\right) \left(i \frac{\sqrt{2} e^{i\phi_\beta}}{2 \cos \theta_W}\right)^2 \frac{i d^4 q/(2\pi)^4}{q^2 - m^2} \times \bar{u}(e_L,p) \frac{1}{2} + \frac{\gamma_5}{2} i(k + m_1) \frac{1}{2} + \frac{\gamma_5}{2} (-i k') \frac{1}{2} - \frac{\gamma_5}{2} v(\omega,p') .$$

(5)

The amplitude involves the integration of the virtual momentum $q$ which runs through the loop. We use the Feynman rules adopted in Refs.[3] to deal with couplings of Majorana fermions. The rules were summarized in Ref.[4]. Since the dispersive part of the amplitude will not produce CP violation in the decay width difference at the 1-loop level, in the following we keep only the absorptive part which is easily obtained[5] from the above expression by requiring the on-shell condition $k^2 = m^2_1$ and $k'^2 = 0$, removing the corresponding denominators, and integrating over the solid angle $\Omega$ of $k$ in the rest frame of $\tilde{e}_L$. This method of calculation is very similar to those of CP violation in baryogensis[6] or in Higgs decays[7].

$$M^1 = \frac{i}{16\pi} \frac{\sqrt{2} e^{-i\phi_\omega}}{2 \sin \theta_W} \frac{e^{2i\phi_\beta}}{2 \cos^2 \theta_W} \bar{u}(e_L,p) \frac{1}{2} + \frac{\gamma_5}{2} m_1 (\not{q} + \not{q'} + \not{p'}) v(\omega,p') \frac{1 - m^2_2/s}{q^2 - m^2} d\Omega .$$

(6)
Although the incoming momentum squared $s$ and the scalar electron mass squared $m^2$ in the $t$-channel propagator are the same in our case, we use different symbols for them above for possible extension of the formulas. First, we need to establish the following covariant expression,

$$\int \frac{q^\mu}{q^2 - m^2} \frac{d\Omega}{4\pi} = Ap^\mu + Bp'^\mu.$$  

(7)

Only the $B$ term is relevant to our calculation. It is obtained by the dot products with $p$ on both sides of the above equation.

$$B = \frac{1}{s - m^2} \left( 1 + \frac{(m^2 - m_1^2)s}{(s - m_2^2)(s - m_1^2)} \ln \frac{m^2 - m_1^2 m_2^2/s}{s + m^2 - m_1^2 - m_2^2} \right).$$  

(8)

Finally we obtain,

$$\mathcal{M}^1 = - \left( \frac{\sqrt{2} e^{-i\phi_\omega}}{2 \sin \theta_W} \right) \left( \frac{i\alpha e^{2i\phi_\beta}}{8 \cos^2 \theta_W} \right) \bar{u}(e_L, p) \frac{1 + \gamma_5}{2} v(\omega, p') \frac{m_1 m_2 s - m_1^2}{s} \times \mathcal{I},$$  

(9)

where

$$\mathcal{I} = 1 + \frac{s + m^2 - m_1^2 - m_2^2}{(s - m_1^2)(s - m_2^2)} \ln \frac{m^2 - m_1^2 m_2^2/s}{s + m^2 - m_1^2 - m_2^2}.$$  

(10)

This finally gives

$$\mathcal{A}^\omega = \frac{\Gamma(\tilde{e}_L \to e_L \omega) - \Gamma(\tilde{e}_L \to \bar{e}_L \omega)}{\frac{1}{2}[\Gamma(\tilde{e}_L \to e_L \omega) + \Gamma(\tilde{e}_L \to \bar{e}_L \omega)]} = -\frac{\alpha}{2 \cos^2 \theta_W} \frac{\sin(2\phi) m_1 m_2 s - m_1^2}{s} \times \mathcal{I}.$$  

(11)

with $\phi = \phi_\omega - \phi_\beta$, i.e. the CP violation here depends only on the relative phase. Note that when either $m_1$ or $m_2$ goes to zero, the asymmetry disappears as expected because the phase $\phi$ loses its meaning in the limit $m_e = 0$. The asymmetry is of the order $\alpha$, and there is no $m_e$ suppression. Similarly, the complementary asymmetry is

$$\mathcal{A}^\beta = \frac{\Gamma(\bar{e}_L \to e_L \beta) - \Gamma(\bar{e}_L \to \bar{e}_L \beta)}{\frac{1}{2}[\Gamma(\bar{e}_L \to e_L \beta) + \Gamma(\bar{e}_L \to \bar{e}_L \beta)]} = \frac{\alpha}{2 \sin^2 \theta_W} \frac{\sin(2\phi) m_1 m_2 s - m_1^2}{s} \times \mathcal{I}.$$  

(12)

These expressions respect the CPT relations,

$$\Gamma(\tilde{e}_L \to e_L \omega) - \Gamma(\tilde{e}_L \to \bar{e}_L \omega) = -\Gamma(\bar{e}_L \to e_L \beta) + \Gamma(\bar{e}_L \to \bar{e}_L \beta).$$  

(13)

### Phenomenology

In $e^+e^-$ annihilation at high enough energy, scalar leptons can be copiously produced in pairs. If the bino is the lightest supersymmetric particle, the event profiles of the two
channels \( \tilde{e}_L \rightarrow e_L\beta \) and \( \tilde{e}_L \rightarrow e_L\omega \) are very different. In the \( e_L\beta \) mode, the bino \( \beta \) just carries away the missing momentum quietly. In the \( e_L\omega \) mode, the wino \( \omega \) continues to decay, \( \omega \rightarrow e_L\tilde{e}_L^* \rightarrow e_L\tilde{e}_L\beta \), producing \( e^+e^- \) pair and missing momentum. Distinguishing these two kinds of decay channels, we can test their CP asymmetry in the decay branching fractions. In this article, we have shown that such CP asymmetry can be of the order of \( \alpha \). It is not large, see Fig. 2, but \textit{not} suppressed by the tiny factor \( m_e^2/m^2 \).

**Acknowledgement**

F. B. and W.-Y. K. are supported by a grant from the Department of Energy, and D. C. by a grant from the National Science Council of R.O.C. W.-Y. K. thanks the hospitality and support of the KEK theory group, and useful discussions with V. Barger and T. Falk on gaugino phases.
Figure Caption

Fig. 1. (a) Tree level diagrams for $\bar{e}_L \to e_L \omega$; (b) One-loop diagram for $\bar{e}_L \to e_L \omega$.

Fig. 2. Plot of asymmetry $A^\beta / \sin(2\phi)$. 
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therein.
Figure 2: Plot of asymmetry $A^\beta / \sin(2\phi)$.
(a) Tree level diagram for $\tilde{e}_L \rightarrow e_L \omega$.

(b) One-loop diagram for $\tilde{e}_L \rightarrow e_L \omega$.

Figure 1