Metastable gauged
O’Raifeartaigh

Borut Bajc

J. Stefan Institute, Ljubljana, Slovenia

B. Bajc, A. Melfo, 08
In SUSY GUTs almost all predictions (proton decay, fermion masses, etc.) depend on soft terms. Soft terms usually derived from new (GUT unrelated) ad-hoc assumed sectors which break susy and mediate it. To minimize inputs: is it possible to break SUSY in the same sector that breaks GUTs? This would make SUSY GUTs more predictive, especially for the low energy sector (fermion and sfermion masses and mixings)
Purpose of this talk

Consider a global supersymmetric theory

- perturbative regime
- no singlets

We will look for ways of spontaneously break both supersymmetry and gauge symmetry
So far:

- either perturbative regime with singlets
  
  Witten, 81; Dine, Fischler, 82; Banks, Kaplunovsky, 83;
  
  Dimopoulos, Raby, 83; Kaplunovsky 84

- or non-perturbative regime without singlets
  
  Murayama, 97; Dimopoulos, Dvali, Rattazzi, Giudice, 97;
  
  Luty, 97; Agashe, 98
• perturbative regime: it is the simplest way of having a theory under control, possible to use minimal and realistic gauge groups

• no singlets:
  1. we know no gauge singlets in nature (even $\nu_R$ is usually only a SM singlet, but usually lives in a nontrivial representation of a GUT)
  2. it is just a cheap trick to avoid constraints from gauge symmetries
  3. can destabilize hierarchies in a supergravity framework
Standard O’Raifeartaigh

Why we can not work without singlets in usual O’Raifeartaigh superpotential?

\[ W = \mu S_1 \phi + \lambda S_2 (\phi^2 - M^2) \]

\( \mu S_1 \phi \) could come from a gauge invariant \( Tr(\ldots) \)

\( \lambda S_2 \phi^2 \) could come from a gauge invariant \( Tr(\ldots) \)

But \(-\lambda M^2 S_2 \rightarrow S_2\) is a singlet.

But the form of the above \( W \) needed only if we want that SUSY broken in a global (stable) minimum
But susy breaking vacuum could well be metastable

If local (metastable) minima allowed then

- linear term can be dropped
- no need for two fields $S_1, S_2$, just one ($S$) enough
\[ W = S \left( \mu \phi + \lambda \phi^2 \right) \]

Equations of motion:

\[ \frac{\partial W}{\partial \phi} = S (\mu + 2\lambda \phi) = 0 \]
\[ \frac{\partial W}{\partial S} = \mu \phi + \lambda \phi^2 = F \]
Susy preserving minima:

\[ \langle \phi \rangle = 0 \text{ or } -\frac{\mu}{\lambda} \ ; \ \langle S \rangle = 0 \ ; \ F = 0 \]

Susy breaking minimum

\[ \langle \phi \rangle = -\frac{\mu}{2\lambda} \ ; \ \langle S \rangle \text{ arbitrary} \ ; \ F = \frac{\mu^2}{4\lambda} \]

No tachyonic states if

\[ |\langle S \rangle| \geq \frac{|\langle \phi \rangle|}{\sqrt{2}} \]

Tree order metastable minimum without linear term in \( W \)!
Example: SU(5) with two adjoints

\[ W = \mu Tr (\Sigma_1 \Sigma_2) + \lambda Tr (\Sigma_1^2 \Sigma_2) \]

U(1)\(_R\) symmetry with \( R[\Sigma_1] = 0, R[\Sigma_2] = 2 \)

\[ \Sigma_i = \frac{v_i + s_i}{\sqrt{30}} \begin{pmatrix} 2_{3\times3} & 0_{3\times2} \\ 0_{2\times3} & -3_{2\times2} \end{pmatrix} + \begin{pmatrix} O_i & X_i \\ \bar{X}_i & T_i \end{pmatrix} \]
Vevs from

\[ W = v_2 \left( \mu v_1 - \frac{\lambda}{\sqrt{30}} v_1^2 \right) \]

Susy preserving (global) minimum if

\[ v_1 = 0 \text{ or } \frac{\sqrt{30} \mu}{\lambda} \; ; \; v_2 = 0 \; ; \; F = 0 \]

Susy breaking (local-metastable) minimum if

\[ v_1 = \frac{\sqrt{30} \mu}{2 \lambda} \; ; \; v_2 \text{ undetermined} \; ; \; F = \frac{\lambda}{\sqrt{30}} v_1^2 \]
Two things to check

- there is no tachyonic state provided

\[ v_2 \geq \frac{v_1}{\sqrt{2}} \]

- 1-loop corrections stabilize the minimum: Coleman-Weinberg formula

\[
\Delta V = \frac{1}{64\pi^2} \sum_i (-1)^{2s_i+1}(2s_i + 1)m_i^4(\phi) \left( \ln \frac{m_i^2(\phi)}{\mu^2} - \frac{3}{2} \right)
\]

Interested just in leading non-zero order

\[
(F/M_{GUT}^2) \approx (v_1/v_2)^2 \ll 1
\]
easier to solve the RGE \((\tau = \ln (\mu/M_{GUT})/(8\pi^2))\)

\[
\frac{dg^{-2}}{d\tau} = -2
\]

\[
\frac{d\ln \lambda^2}{d\tau} = 21\lambda^2 - 30g^2
\]

\[
\frac{d\ln Z_2}{d\tau} = 10g^2 - \frac{21}{5}\lambda^2
\]

The leading order corrected potential

\[
V(\sigma_2) = \frac{F^2}{Z_2(\sigma_2)}
\]

Notice the crucial role played by the gauge contribution \((g \neq 0)\)
New extremum at $dZ_2/d\tau = 0$:

$$\lambda^2 = \frac{50}{21} g^2$$

It is a minimum because

$$\frac{1}{Z_2} \frac{d^2 Z_2}{d\tau^2} = -180g^4 < 0$$

The flat direction got stabilized by radiative corrections!
\[ \frac{V}{F^2} \]

\[ |\sigma_2|/M_{GUT} \]
Notice that

- the superpotential considered the most general renormalizable one with given symmetry and charges
- \( v_2 = M_{GUT} \) breaks \( SU(5) \) gauge symmetry
- \( v_1 = \sqrt{\sqrt{30} F/\lambda} \) breaks susy
- no \( \sigma_1 \) tachyon providing \( v_2 \geq v_1/\sqrt{2} \)
- no fine-tuning involved
Unfortunately light states \((O, T)\) present

\[
\mathcal{M} = \lambda \begin{pmatrix} c_2 v_2 & c_1 v_1 \\ c_1 v_1 & 0 \end{pmatrix} ; \quad c_i \text{ CG coeff.}
\]

In the limit \((v_1/v_2) \approx \sqrt{F}/M_{GUT} \ll 1\) eigenvalues

\[
\lambda c_2 v_2 = \mathcal{O}(M_{GUT}) ; \quad \frac{\lambda c_1^2 v_1^2}{c_2 v_2} = \mathcal{O}(F/M_{GUT}) \ll M_{GUT}
\]

A light colour octet and weak triplet in the spectrum!

Unification still possible? Yes, but \(M_{GUT} > M_{Planck}\): need to change the model
Different solutions possible

- Add non-renormalizable interactions
  
  Idea: make $v_1$ large possible

\[
W = \mu Tr (\Sigma_1 \Sigma_2) + \lambda Tr (\Sigma_1^2 \Sigma_2) \\
+ \frac{\alpha_1}{M} Tr (\Sigma_1^3 \Sigma_2) + \frac{\alpha_2}{M} Tr (\Sigma_1^2) Tr (\Sigma_1 \Sigma_2)
\]

Still $U(1)_R$ symmetric

Relation between $F$ and $v_1$ now changed:

\[
F = v_1^2 \left[ \frac{\lambda}{\sqrt{30}} - \frac{2}{M} \left( \frac{7}{30} \alpha_1 + \alpha_2 \right) \right]
\]

$F \ll v_1^2$ possible provided $\lambda$ and $\alpha_i$ terms cancel: fine tuning needed
• Add more adjoint fields (but still renormalizable)
  
  Idea: the previous example can be obtained by integrating out new heavy fields

\[
W = - M Tr(\Omega_1 \Omega_2) + Tr[\Omega_2 (\mu_1 \Sigma_1 + \lambda_1 \Sigma_1^2)] \\
+ Tr[\Omega_1 (\mu_2 \Sigma_2 + \lambda_2 \Sigma_1 \Sigma_2)]
\]

Again $U(1)_R$ symmetric with $R[\Sigma_2] = R[\Omega_2] = 2$ and $R[\Sigma_1] = R[\Omega_1] = 0$

Same fine-tuning as in the previous example obviously needed ($F \ll M^2_{GUT}$)
• Add new representations

Idea: Clebsches different

Renormalizable example with two adjoints:

\[ F \propto F(m_i, \lambda_i, v_i) \Rightarrow \text{e.o.m. } f(m_i, \lambda_i, v_i) \]
\[ \det(\mathcal{M}) \propto G(m_i, \lambda_i, v_i) \Rightarrow \text{e.o.m. } f(m_i, \lambda_i, v_i) \]

Same combination!

This could be avoided if a different representation (than 24) is used:

for example 75(= Φ)

\[ W = \mu Tr(\Sigma_1 \Sigma_2) + \lambda Tr(\Sigma^2_1 \Sigma_2) \]
\[ + \eta_1 Tr(\Phi \Sigma_1 \Sigma_2) + \eta_2 Tr(\Phi^2 \Sigma_2) \]
Metastability

All such models necessarily have susy breaking metastable.

Is it safe?

One can show that

\[ S \approx \frac{M_{GUT}^4}{F^2} \gg 10^3 \]

Lifetime much longer than the age of the universe
Inflation

Byproduct of this exercise: potentials of this type are automatically flat enough to allow for slow-roll inflation

CMB anisotropy $\rightarrow V \approx M_{GUT}^4$ (\(\neq F^2\) as before!)

This possible since \(v_1\) can be chosen freely \((V \approx v_1^4)\)

In the region \(10M_{GUT} \lesssim \sigma_2 \lesssim 100M_{GUT}\)

\[\epsilon, \eta \ll 1\]

Very similar to F-term hybrid inflation

\textit{Dvali, Shafi, Schaefer, 94}

but here for SU(5) without gauge singlets (and no fine-tuning)!
Conclusions

- It is possible to construct models without gauge singlets that break supersymmetry spontaneously in the perturbative regime
- A generic problem of these models is the appearance of light states that spoil realistic GUT models
- To solve this unwanted behaviour one can add
  1. nonrenormalizable interactions (1/$M$ terms)
  2. more fields of the same type (four $24_H$ at least)
  3. different type fields (two $24_H$ and one $75_H$)
- Still work to be done to make such models realistic and predictive (mediation, other gauge groups, doublet-triplet splitting, sugra corrections, ...)

Susy 08 - Seoul