The response of drought in Beiluo River Basin of China based on the comprehensive method of Pa, SPI and fuzzy

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Abstract. The meteorological drought is threatening the agricultural economic development with the change of the climate. In order to analyze the characteristics of drought spatio-temporal change, the precipitation data of eight meteorological stations in the Beiluo River Basin of Shaanxi Province of China have been collected, and the drought index of Pa, SPI and FSE have been selected to analyze the drought in Shaanxi Province for the last 55 years. The results of Pa, SPI and FSE test show that the droughts happened in the Beiluo River Basin are 149, 215 and 203 times in the past 55 years, respectively. Overall, the Beiluo River has a tendency to dry out. The main type of drought is low-grade drought, followed by the medium-grade drought, and the specially-grade drought happened least. The average rainfall decreases in the Beiluo River Basin from the southeast to the northwest, and the change of the number of drought is just opposite to that of precipitation trend, which increases from southeast to northwest. The results will provide the scientific basis for the monitoring, evaluation, early warning and drought relief.

1. Introduction

Drought is one of the most common natural disasters in the world and is regarded as one of the most serious types of natural disasters in the world [1]. In the context of global warming, the frequency and intensity of extreme weather events, such as severe drought and high temperatures, are increasing [2, 3]. China is one of the countries with the largest number of natural disasters, the most frequent disasters and the most serious disasters. More than 70% of the annual economic losses of national average are caused by natural disasters because of the meteorological disasters. In the context of climate warming, drought disasters have been intensified. In the 20th century, there were many serious droughts in the northwest, the interval of drought occurred was shorter and shorter, the drought duration was longer and longer, and the drought intensity was bigger and bigger. The drought assessment, monitoring and forecasting research of the government and academia attract great importance to the hot issues, which has great practical significance. For a long time, a large number of research works have been carried out in the related industries all over the world, and many quantitative evaluation indexes have been established from climate, hydrology and agriculture [4]. Drought indicators are the basis for droughts study and the key to measuring the degree of drought. Over the years, many scholars have studied drought changes in different research areas based on different drought indicators [5-7], but it is rare to use a variety of indicators to compare and study at the same area.
Climate is the basis cause of drought, but also the main cause of drought heterogeneity. Among these climatic factors, precipitation is an important basis for the quantitative study of regional dry and wet succession. The standardized precipitation index (SPI) and the percentage of precipitation anomalies (Pa) are common indicators for the assessment of drought trends based on precipitation changes [8, 9]. SPI is recommended by the World Meteorological Organization (WMO) as a useful indicator of drought and is widely used around the world. Pa is more extensive used in China. Huang et al used the SPI and Copula functions to calculate the joint recurrence period in two typical drought scenarios of the Weihe River Basin [2, 10]. Wei et al used the Pa to analyze the spatial distribution of drought in Shaanxi Province [3]. Zhao used the Pa, SPI and SPEI to compare and analyze the eastern Qinghai-Tibet Plateau alpine grassland drought [4].

The Weihe River Basin is located in the semi-humid and semi-arid areas in the northwest of China. The ecological environment is fragile and the natural disasters are frequent. The drought is the most frequent, the heaviest and the most harmful disaster. It has great influence on the regional economic structure adjustment and rational exploitation and utilization of water resources [11]. The Beiluo River Basin is located in the Weihe River Basin. The climate and environment are complex and the drought disaster threatens the development of agriculture. The identification of the drought index is a powerful tool for monitoring drought and is the basis for studying the drought. Therefore, this paper chooses the comprehensive method of SPI, Pa and fuzzy to analyze the temporal and spatial characteristics of arid in the Beiluo River Basin, aiming at adjusting the agricultural planting structure, disaster prevention and mitigation, and comparing the applicability of SPI and Pa in drought monitoring in Shaanxi Province, providing scientific advices for water resources assessment and drought monitoring in the area.

In recent years, great progress has been made in the development and application of drought indexes. Drought index, as quantitative indicators of drought degree, plays an important role in drought monitoring, forecasting and management of water resources, especially drought index provides method and basis for the application of regional resource environment and interdisciplinary fields. At the same time, drought index is also an effective tool for hydrology and water resources research. It can provide a basis for slowing down and preventing the adverse effects of drought on agriculture and making scientific policy.

2. Data source

In this study, the daily precipitation, daily temperature, sunshine hours and other meteorological data from eight meteorological stations (figure 1) in the Beiluo River Basin (1960-2015) were collected from the Chinese Meteorological Science Data Sharing Service Network. The distribution of each meteorological station is uniform, which can be used to reflect the spatial changes of hydrological meteorology in the basin. For the missing meteorological data, linear interpolation was applied in the station. The precipitation and surface evaporation are calculated from the precipitation of each station. The evaporation data are calculated by Tyson polygon method in ArcGIS.

Beiluo River is first tributary of the Weihe River, the second tributary of the Yellow River, a total length of 680.3 km, and is the longest river in Shaanxi Province. The average runoff of the Beiluo River Basin is 943 million m³, and in Shaanxi Province is 873 million m³, and the inter-annual variation is large [2, 3]. Annual average sediment transmission is 0.833 × 10⁸ t. The basin is semi-arid continental monsoon climate, the annual average precipitation is 548mm, the distribution of precipitation in the Beiluo River is uneven during the year, and the precipitation in June to September accounts for about 70% of the total annual precipitation, during which rainstorm happens frequently, the 3 months contain 60% of the annual runoff, and more than 90% of the flood occurred from July to August. Climate is semi-arid and semi-humid to the dry climate of the transition zone, with obvious continental monsoon climate characteristics, cold and dry in winter, in spring dry and windy, cool and humid in autumn. The average annual temperature of 7.8°C, the extreme maximum temperature is 37.4°C, and the extreme minimum temperature is -28.0°C.
3. Methodology

3.1. Pa

The percentage of precipitation anomalies can be used to reflect the degree of deviation from the average state of precipitation over a period of time and to directly reflect the drought caused by precipitation anomalies [4, 9]. The formula is:

\[ Pa = \frac{P - \bar{P}}{P} \times 100\% \]

where, \( P \) is the precipitation in a certain period of time, \( \bar{P} \) is the average monthly precipitation in the same period. In this study, \( P \) is the monthly precipitation, which has been the average precipitation for the past 55 years. The advantage of the Pa is that the calculation method is simple and intuitive, but the disadvantage is that the response is relatively slow. In China, Pa index is used in the animal husbandry drought test, and the China Meteorological Administration has determined the semi-arid, semi-humid areas of drought criteria (table 1).

| Drought magnitude | Non drought | Low-grade drought | Medium-grade drought | Heavy-grade drought | Specially-grade drought |
|-------------------|-------------|-------------------|----------------------|---------------------|------------------------|
| Pa                | 40<Pa       | 60<Pa<40          | 80<Pa<60             | 95<Pa<80            | Pa<95                  |
| SPI               | -0.5<SPI    | -1<SPI<0.5        | -1.5<SPI<1           | -2<SPI<1.5          | SPI<2                  |
| FSE               | FSE<0.7     | 0.7<FSE<1.5       | 1.5<FSE<2            | 2<FSE<2.6           | 2.6<FSE<4              |

3.2. SPI

The SPI index uses mathematical methods to convert the cumulative frequency distribution of precipitation into a standard normal distribution, which has dimensionless and standardized...
characteristics and is applicable in different regions [4, 8, 10]. Suppose the precipitation in some time
is \( x \), assuming that the precipitation sequence obeys the Gamma distribution, then the probability
density function is:

\[
P(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, x > 0
\]

\[
\alpha = \frac{1+\sqrt{1+4A/3}}{4A}
\]

\[
\beta = \frac{x}{\alpha}
\]

\[
A = \ln(\bar{x}) - \frac{\sum_{i=1}^{n} \ln x_i}{m}
\]

where, \( \beta \) and \( \alpha \) are the scale parameters and the shape parameters respectively, greater than zero, are
obtained by the maximum likelihood estimation method, \( n \) is the length of the precipitation sequence,
where the number of items for zero is \( m \), \( \bar{x} \) is the average of non-zero precipitation sequence.

Let \( q = m/n \), the cumulative probability of a certain time scale is:

\[
H(x) = q + (1-q)G(x)
\]

\[
G(x) = \int_0^x p(x)dx
\]

The cumulative probability distribution is converted to the standard normal distribution, that is, the
corresponding SPI value.

When \( 0 < H(x) \leq 0.5 \), and let \( k = \sqrt{\ln \frac{1}{H(x)}} \), then

\[
SPI = -\left( k - \frac{c_0 + c_1k + c_2k^2}{1 + d_1k + d_2k^2 + d_3k^3} \right)
\]

When \( 0.5 < H(x) < 1 \), and let \( k = \sqrt{\ln \left( \frac{1}{1-H(x)} \right)} \), then

\[
SPI = k - \frac{c_0 + c_1k + c_2k^2}{1 + d_1k + d_2k^2 + d_3k^3}
\]

where, \( c_0 = 2.515517, c_1 = 0.802853, c_2 = 0.010328, d_1 = 1.432788, d_2 = 0.189269 \)
\( d_3 = 0.001308 \)

The advantage of SPI index is that the calculation stability is great, the time scale is flexible, and the
indication of spatiotemporal change is effective and can reflect the drought and flood conditions
effectively. This paper chose the time scale of 6 months SPI as the research object, and the drought
magnitude standard is shown in table 1.

3.3. Fuzzy synthetic evaluation based on entropy method

Entropy weigh method is an objective empowerment method, and the weight of each evaluation index
can be determined according to the difference degree of each index value in drought evaluation. The fuzzy comprehensive evaluation method is to quantify the fuzzy indexes of the object by constructing the hierarchical fuzzy subsets (ie, determine the membership), and then use the fuzzy transformation principle to evaluate the indexes [12, 13]. When the fuzzy comprehensive evaluation method is used to evaluate some results, the evaluation results are intuitionistic and can be solved satisfactorily. The fuzzy evaluation can objectively solve the problems of uncertainty existed in the evaluated things, and can deal with the fuzzy evaluation object exactly by digital means, and give a comprehensive, accurate, and realistic quantification results. The generally follow the steps below:

Step 1: Standardize raw data matrix and Define entropy, count and record the Entropy Weight.
Step 2: divide the study area \( D \) into \( k \) small drought evaluation areas, that is, \( D = [D_1, D_2,..., D_k] \). Determine \( U \), the set of factors for comprehensive drought assessment, and determine the \( m \) drought evaluation indexes, \( U = [u_1, u_2,..., u_m] \). Determine the set of possible assessment results in the drought evaluation, the set of drought magnitude set \( B = [B_1, B_2,..., B_n] \).

Step 3: Determine the membership matrix, starting from a single index, determine the evaluation of the degree of membership of the drought magnitude. Membership can be obtained by membership function:

\[
r_{ij} = \begin{cases} 
1, & x < x_1 \\
(x_2 - x)/(x_2 - x_1), & x_1 < x < x_2 \\
0, & x > x_2 
\end{cases}
\]

where, \( 1 < k < 5 \), \( x_1, x_2, x_3, x_4, x_5 \) are the critical values of drought index at the different drought magnitude, mild drought, moderate drought, severe drought and extreme drought, respectively.

Step 4: Determine the membership weight of the evaluation index. Use the entropy method to establish the mathematical model to calculate the weight of the index. \( A = (\alpha_1, \alpha_2,..., \alpha_m) \), and \( \alpha_i \) is the drought evaluation index membership weight coefficient.

Step 5: The membership matrix \( R \) is multiplied by the evaluation index membership weight \( A \), and the matrix \( W \) is obtained. \( W \) shows all the effects on different drought magnitude, and calculates the fuzzy comprehensive drought evaluation index \( DI \).

\[
w_i = \sum_{j=1}^{m} (r_{ij} \alpha_i)
\]

\[
DI = \sum_{i=1}^{m} (w_i B_i)
\]

where, \( B \) is the weight coefficient for different drought magnitude. Drought is divided into five magnitudes: drought, mild drought, moderate drought, severe drought and extreme drought. The standard of classification is shown in table 1, using the classification method of "meteorological drought magnitude" (GB/T 20481-2006).

4. Results and discussion

4.1. Time scale analysis of drought

The Pa and SPI16 indexes of each station were calculated by the meteorological data from 1961 to 2015. Pa and SPI of each month in recent 55 years in the Beiluo River Basin were shown in the following figures 2(a) and 2(b), and at the same time according to the drought magnitude (table 1) the number of Pa and SPI in the Beiluo River was tallied (table 2). It can be seen from figure 2(a) that the overall Pa of the Beiluo River Basin shows a decreasing trend, and the Mann-Kendall test has a statistical value
of -0.812, and its absolute value is less than 1.96, failing to pass the 95% confidence test, which indicates that the Beiluo River Basin has no obvious dry-out trend. It can be seen from table 2 that the results of Pa test show that the number of drought in the Beiluo River Basin was 149, and 72 (10.91%) and 41 (6.21%) occurred in Low-grade drought and Medium-grade drought, while the number of Heavy-grade and Specially-grade drought was relatively small, which occurred for 18 (2.73%) times. It can be seen from figure 2(b) that the overall SPI6 exhibits a decreasing trend, which is statistically -3.165 and an absolute value larger than 1.96, which pass the Mann-Kendall test. The 95% confidence test shows that the Beiluo River Basin has a dry-out trend. From table 2, the results of SPI test showed that the number of drought in the Beiluo River Basin was 215 times in the past 55 years, among which 102 (15.45%) and 74 (11.21%) occurred in Low-grade drought and Medium-grade drought, while the number of Heavy-grade drought and Specially-grade drought was relatively small, which occurred for 25 (3.79%) and 14 (2.12%) times, respectively.

**Figure 2.** The monthly changes of Pa (a), SPI (b) and FSE (c) index.
Table 2. The quantitative statistics of drought occurrence on Beiluo River Basin.

| Drought index | Non drought | Low-grade drought | Medium-grade drought | Heavy-grade drought | Specially-grade drought |
|---------------|-------------|-------------------|----------------------|---------------------|------------------------|
| Pa            | 511         | 72                | 41                   | 18                  | 18                     |
| SPI           | 445         | 102               | 74                   | 25                  | 14                     |
| FSE           | 457         | 118               | 50                   | 18                  | 17                     |

In order to reflect the drought condition in the Beiluo River better, this paper uses the fuzzy synthetic evaluation method based on the entropy method to evaluate the wet and dry conditions of the basin (figure 2(c)) in case of avoiding the adverse effects of a single indicator. The results of FSE method showed that the number of month that drought happened was 203 times, and there were 118 (17.88%) and 50 (7.58%) times occurred in Low-grade drought and Medium-grade drought in the Beiluo River, respectively, and in Heavy-grade drought and Specially-grade drought was 18 (2.73%) and 17 (2.58%) times, respectively. Overall, the Beiluo River Basin has a dry trend, the main type of drought was low-grade drought, followed by Medium-grade drought, and the least was specially-grade drought.

The number of drought in different grades happened indifferent seasons was tallied according to the FSE by month. It can be seen from the figure 3 that the each grade drought happened most in winter, and the numbers are 41, 23, 7, and 6. The autumn was in the second place, and drought happened least in summer. Over all, there were large differences among the four seasons in which the drought happened in Beiluo River Basin, and the number that drought happened in the four seasons were 42, 32, 52, and 77, respectively, among which winter was the drought most likely happened. The Beiluo River Basin is a region that needs high attention and further study, autumn and winter are the most drought-prone seasons, high frequency of drought.

Figure 3. The frequency statistics of seasonal drought.

4.2. Spatial scale analysis of drought
The number of drought occurred years in each meteorological station in the Beiluo River Basin is shown in table 3, and the number of years of drought in each station is 177 – 251 times. As shown in figure 4(a), the average rainfall in the Beiluo River Basin gradually decreased from the southeast to the northwest, the precipitation of each station is between 430-810 mm, among which Huashan has the highest precipitation (810.33 mm), while the Huanxian station is the least (432.27 mm). The spatial...
distribution of drought situation on the Beiluo River Basin was shown in figure 4(b), and the times of drought occurred has the opposite trend of precipitation, increase from southeast to northwest, the most frequent occurrence in Low-grade drought is the Wuqi station, and in the Xifeng station the Medium-grade drought and Heavy-grade drought occurred most, and the most frequent occurrence in Specially-grade drought is Huanxian station.

Table 3. The quantitative statistics of drought occurrence with eight weather stations.

| Weather station | Non drought | Low-grade drought | Medium-grade drought | Heavy-grade drought | Specially-grade drought |
|-----------------|-------------|-------------------|----------------------|---------------------|------------------------|
| Huashan         | 483         | 79                | 61                   | 25                  | 12                     |
| Huanxian        | 412         | 85                | 75                   | 38                  | 50                     |
| Luochuan        | 444         | 80                | 64                   | 39                  | 33                     |
| Tongchuan       | 442         | 89                | 61                   | 42                  | 26                     |
| Wuqi            | 409         | 98                | 75                   | 31                  | 47                     |
| Xifeng          | 443         | 68                | 78                   | 43                  | 28                     |
| Yan’an          | 417         | 90                | 65                   | 42                  | 46                     |
| Changwu         | 455         | 70                | 68                   | 39                  | 28                     |

Figure 4. Spatial distribution of annual mean precipitation (a) and drought degree (b) in the Beiluo River Basin.

5. Conclusions
The spatial and temporal changes of the drought degree in the Beiluo River Basin were analyzed by Pa, SPI and FSE. The results are as follows:
- Pa and SPI test results showed that, numbers of years of drought occurred in the Beiluo River Basin in the past 55 years were 149 and 215, among which Low-grade drought occurred 72 times and 102 times, Medium-grade drought occurred, respectively, 41 times and 74 times. The occurrence of Heavy-grade drought and Specially-grade drought was relatively less, among which the number from Pa method test was 18 times. Heavy-grade drought and Specially-grade drought from SPI method was 25 times and 14 times, respectively.
• The result of FSE method showed that the number of drought happened in Beiluo River Basin was 203 times, among which the four grades drought happened for 118 (17.88%), 50 (7.58%), 18 (2.73%) and 17 (2.58%), respectively. Overall, the Beiluo River Basin has a dry-out trend, the main type of drought is low-grade drought, followed by the medium-grade drought, and the specially-grade drought happened least.

• The average annual precipitation in the Beiluo River Basin is decreasing from the southeast to the northwest, and the number of arid occurrences is opposite to that of the precipitation. And the most frequent occurrence of Low-grade drought is at the Wuqi station, Heavy-grade drought is the most at the Xifeng station, and the most frequent occurrence of Specially-grade drought is at Huanxian station.

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