Finite temperature gluon self-energy in a class of temporal gauges

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The approach which relates thermal Green functions to forward scattering amplitudes of on-shell thermal particles is applied to the calculation of the gluon self-energy, in a class of temporal gauges. We show to all orders that, unlike the case of covariant gauges, the exact self-energy of the gluon is transverse at finite temperature. The leading $T^2$ and the sub-leading $\ln(T)$ contributions are obtained for temperatures $T$ high compared with the external momentum. The logarithmic contributions have the same structure as the ultraviolet pole terms which occur at zero temperature.

I. INTRODUCTION

Thermal gauge field theories in the temporal gauge have been the subject of many investigations both in the imaginary and in the real time formalisms [1–3]. One of the main advantages of the non-covariant temporal gauge is that it is physical and effectively ghost-free. At finite temperature, it may be considered a more natural choice, since the Lorentz invariance is already broken by the presence of the heat bath. It is also convenient for calculating the response of the QCD plasma to a chromo-electric field, since in this case only the gluon self-energy is needed (in this gauge, the chromo-electric field depends only linearly on the gauge field $A_\mu^a$) [1–6]. Despite these advantages, explicit calculations of loop corrections to Green functions are known to be more complicated than in covariant gauges. One of the difficulty is associated with the more involved tensorial structure of the propagator (see Eq. (6)). The other more fundamental problem is how to deal with the extra poles at $q \cdot n = 0$ in the gluon propagator, where $q$ is the loop momentum and $n = (n_0, \vec{0})$ ($n_0^2 > 0$) is the temporal axial four-vector.

In the imaginary time formalism, the standard method of calculation of thermal Green functions consists in replacing the Matsubara frequency sum by a contour integral in the complex plane $\xi$. Using this approach, all Green functions, at one-loop order, can be written as the sum of the vacuum part plus the a thermal part which involves the Bose-Einstein (boson loop) or Fermi-Dirac (fermion loop) statistical distributions.

In the case of the temporal gauge, the extra poles at $q \cdot n = 0$ in the gluon propagator demand more care and make the standard contour integral method of calculation very involved. In order to deal with this problem, Leibbrandt and Staley [6] have developed a technique for performing perturbative calculations at finite temperature in the temporal gauge (see also reference [3] for a different approach to this problem). Employing a special version of the Mandelstam-Leibbrandt prescription [3,11] combined with the $\zeta$-function method [12], they have obtained the complete 1/$T$-expansion for the one-loop self-energy component $\Pi_{00}^H(k_0 = 0, \vec{k})$. Their results show that the leading and sub-leading contributions to the self-energy can be unambiguously determined in the temporal gauge, and are in agreement with the standard contour integral method employed in reference [6].

The temperature dependent part of Green functions can be described in terms of forward scattering amplitudes of on-shell thermal particles of the thermal medium. This idea was described in reference [12] and has been further elaborated in the Feynman gauge and shown to be very useful for determining the partition function in QCD at high temperature [13]. More recently this result has been generalized to the case of general covariant gauges [12]. This method has been derived using both the imaginary time formalism and the real time formalism up to two-loop order [13,14]. There is also an interesting relation with the path integral approach [17].

One of the purposes of this work is to extend the forward scattering method to a class of temporal gauges. The main advantage of this method is that in the high-temperature limit, it is straightforward to obtain the full tensor structure of both the leading $T^2$ and the sub-leading logarithmic contributions for temperatures $T$ such that $T \gg k$, where $k$ denotes the external four-momentum. For the leading $T^2$ part we obtain the well known gauge invariant hard thermal loop result. The gauge dependent sub-leading logarithmic part shares with the previous calculations in general covariant gauges [14] the interesting property of having the same structure as the ultraviolet pole contributions which occur at zero temperature. Another purpose of this work is to study the transversality property of the thermal gluon self-energy. As is well known in general covariant gauges, the exact self-energy of the gluon is not transverse at finite temperature.

In section II we derive, in a class of temporal gauges, the full tensor structure of the forward scattering amplitude associated with the gluon self-energy. In section III we show that, in contrast to the behavior in general covariant gauges, the exact thermal self-energy of the gluon is transverse to all orders. In section IV we compute the leading
gauge-invariant $T^2$ and the sub-leading $\ln T$ contributions of the non-vanishing transverse structures. The later is shown to be simply related to the ultraviolet pole terms which occur at zero temperature. Finally, we present a brief conclusion in section V.

II. THE FORWARD SCATTERING AMPLITUDE

The diagrams which contribute to the gluon self-energy are shown in the Figure 1. We employ the Feynman rules for the three- and four gluon vertices given in reference [19]. In the class of temporal gauges characterized by a gauge parameter $\alpha$, the gluon propagator can be written as

$$\frac{1}{q^2} \left\{ -i\delta^{ab} \left[ g^{\mu\nu} - \frac{1}{q \cdot u} (q_{\mu} u_{\nu} + q_{\nu} u_{\mu}) + \frac{q_{\mu} q_{\nu}}{(q \cdot u)^2} \left( \frac{\alpha}{n_0 q^2} + 1 \right) \right] \right\},$$

where the usual axial vector has been written as $n = n_0 (1, 0) \equiv n_0 u$.

Before considering the more involved diagrams in Figures (1a) and (1b), let us show that the finite temperature contribution to the diagram in Figure (1c) vanishes. In the imaginary time formalism this contribution is proportional to the following integral [7]

$$\int \frac{d^3 \vec{q}}{\pi^3} \int_{-\infty + \epsilon}^{+\infty + \epsilon} dq_0 N(q_0) \left[ \frac{t_{\mu\nu}}{n \cdot q n \cdot (q + k)} + q_0 \leftrightarrow -q_0 \right],$$

where

$$N(q_0) = \frac{1}{e^{\frac{q_0}{T}} - 1}$$

is the Bose-Einstein statistical distribution and $t_{\mu\nu}$ is a momentum independent tensor which is obtained from the explicit expression for gluon-ghost vertex (note that the gluon-ghost vertex is not equal to zero in the class of gauges considered here). Using the partial fraction decomposition (for $n \cdot k$ different from zero)

$$\frac{1}{n \cdot q n \cdot (q + k)} = \frac{1}{n \cdot k} \left[ \frac{1}{n \cdot q} - \frac{1}{n \cdot (q + k)} \right]$$

and making the change of variable $q \rightarrow q - k$ in the second term on the right hand side, we can see that Eq. (2) vanishes when one takes into account the property

$$N(q_0 \pm k_0) = N(q_0 \pm 2\pi iT) = N(q_0); \quad n = 0, \pm 1, \pm 2 \cdots.$$  

This result holds independently of any explicit prescription for the poles at $q_0 = 0$.

Let us now consider the diagrams in Figures (1a) and (1b). We are specifically interested in those contributions which can be expressed in terms of forward scattering amplitudes of on-shell thermal particles. Following the steps described in the appendix, such an expression can be obtained closing the contour of the $q_0$-integration in the right hand side of the complex plane and considering only the poles from the propagator at $q_0 = |\vec{q}|, |\vec{q} + \vec{k}| - k_0$ and at $q_0 = |\vec{q}|$, respectively for the diagrams in the Figures (1a) and (1b). After shifting the momentum in the contribution

\[\text{FIG. 1. One-loop diagrams which contribute to the gluon self-energy. Wavy and dashed lines denotes respectively gluons and ghosts. All external momenta are inwards.}\]
from the second pole in the diagram of Figure (1a) and using the property \[19\], we obtain the following expression for the forward scattering (FS) part of the gluon self-energy

\[
\Pi_{\mu\nu}^{ab}|_{FS} = -\frac{1}{(2\pi)^3} \int \frac{d^3q N(|q|)}{2|q|} \frac{1}{2} \left\{ \begin{array}{c}
q \quad q+\bar{q} \\
\quad q \quad q+k \\
\quad q \quad q-k \\
\quad \quad q \quad q \quad + q \leftrightarrow -q
\end{array} \right\} , \tag{6}
\]

where the factor 1/2 in front of the curly bracket takes into account the symmetry of the graphs in figures (1a) and (1b). It is understood that the Lorentz and color indices of the thermal particles (horizontal lines) are contracted using two Lorentz indices, the gluon four momentum \( k \) is a set of four independent tensors which can be constructed using two Lorentz indices, the gluon four momentum \( k \) and a vector \( u \) which describes the four velocity of the heat bath. The tensors \( P_{\mu\nu}^T, P_{\mu\nu}^L, P_{\mu\nu}^C, P_{\mu\nu}^D \) are transverse with respect to \( k \) and satisfy \( k^i P_{\nu i}^T = 0 \) and \( k^i P_{\nu i}^L \neq 0 \) \((i = 1, 2, 3)\), whereas the tensors \( P_{\mu\nu}^C \) and \( P_{\mu\nu}^D \) are non-transverse.

Using the orthogonality of the tensors given in Eq. (8) as well as the normalization \( \frac{1}{2} P_{\mu \nu}^T P_{\mu \nu}^T = P_{\mu \nu}^L P_{\mu \nu}^L = -\frac{1}{4} P_{\mu \nu}^C P_{\mu \nu}^C = P_{\mu \nu}^D P_{\mu \nu}^D = 1 \), it is straightforward to obtain the explicit expressions for \( \Pi^L, \Pi^T, \Pi^C, \Pi^D \) from Eqs. \(19 \) and \(20 \) (the tensor algebra was performed using the MapleVR3 version of the symbolic computer package HIP 20). The resulting expressions for \( \Pi^C \) and \( \Pi^D \) turn out to be equal to zero. The vanishing of the one-loop correction to \( \Pi^D \) is a general consequence of the finite temperature Slavnov-Taylor identities \[18 \] while the vanishing of \( \Pi^C \) is a property which is verified by the forward scattering part of the gluon self-energy. From the transversality property of \( P_{\mu \nu}^T \) and \( P_{\mu \nu}^L \) it follows that

\[
k^\mu \Pi_{\mu \nu}^{ab}|_{FS} = 0 \tag{9}
\]

### III. Transversality of the Exact Gluon Self-Energy

In order to ascertain whether the transversality property of the thermal gluon self-energy holds beyond the approximation of forward amplitudes with on-shell particles, it is useful to study the consequences of the Becchi-Rouet-Stora identities \[21 \] on the structure of \( \Pi_{\mu \nu}^{ab} \) at finite temperature. To derive these, we start from the effective action:

\[
I = \int d^4x d^4y J^{\mu a}(x) \tilde{G}^{ab}_\mu(x-y) \eta^b(y) + \frac{1}{2} \int d^4x d^4y A^{\mu a}(x) \tilde{\Gamma}^{ab}_\mu(x-y) A^{ab}(y) + \cdots \tag{10}
\]

where the ellipsis denote ghost terms which are unimportant for our purpose \[19 \]. \( J^{\mu a} \) is the source of the BRS transformation, and \( \eta^b \) is a ghost field (Although in our case the closed ghost loops do not contribute to Feynman diagrams \[22 \], the ghost fields need not necessarily decouple from the BRS identity, which involves open ghost lines). \( \tilde{G}^{ab}_\mu \) is a quantity related to the gauge transformation of the gluon field \( A^{\mu a} \), being given to lowest order by the covariant derivative \( \delta^{ab}_\mu \eta^a - g^{abc} A^{bc} \) \[19 \]. The quadratic part \( \tilde{\Gamma}^{ab}_\mu \) is the sum of free kinetic energy, without the gauge fixing term and the self-energy:
The relevant BRS invariance of the gluon self-energy can now be written as

\[ \int d^4x \frac{\delta I}{\delta A_\mu^0(x)} \frac{\delta I}{\delta J\mu\nu(x)} = 0. \]  

(12)

Differentiating this equation with respect to \( A_\nu^b(y) \) and \( \eta^a(z) \), and setting the sources and fields equal to zero leads in momentum space to the identity:

\[ \tilde{\Gamma}_{\mu,ca}(k_0, \vec{k}) \tilde{\Gamma}^{cb}_{\mu}(k_0, \vec{k}) = 0 \]  

(13)

Since, to lowest order, \( \tilde{\Gamma}_{\mu,ca} \) is proportional to \( k_\mu \), it follows from Eq. (11) that the above identity implies the relation:

\[ k_\mu \Pi^{ab}_{\mu\nu}(k_0, \vec{k}) = (k^2 g_{\mu\nu} - k_\mu k_\nu) G_{\mu,ca}^{\nu,ab} - G_{\mu,ca}^{\nu,cb} \]  

(14)

where \( G_{\mu} \) denotes the contributions of Feynman loops to \( \tilde{\Gamma}_{\mu} \).

Let us first consider the one loop order function \( G_{(1)}^{\mu,ab} \), which may be represented as shown in figure 2a.

![Diagram](a)

![Diagram](b)

**FIG. 2.** Examples of source-ghost diagrams. The traced/wave-line represents the external source and the crossed vertex is proportional to \( g_{\mu\nu} \).

Using Eq. (1) for gluon propagator and the fact that the gluon-ghost vertex is proportional to \( n^\sigma \), one finds that \( G_{(1)}^{\mu,ab} \) is given at finite temperature by

\[ G_{(1)}^{\mu,ab} = \alpha g^2 N \delta^{ab}(iT) \sum_{l=-\infty}^{+\infty} \int \frac{d^3-q}{(2\pi)^3} \frac{q^\mu}{(q \cdot n)(q + k \cdot n)} \]  

(15)

where \( N \) is the number of colors, the space-time dimension is \( 4 - \epsilon \) and \( q_0 = 2\pi i T \). One can now employ the identity (11) and make the change of variable \( q \rightarrow q - k \) in the second term, to get

\[ G_{(1)}^{\mu,ab} = \alpha g^2 N \delta^{ab}(iT) \frac{k_\mu}{k \cdot n} \sum_{l=-\infty}^{+\infty} \int \frac{d^3-q}{(2\pi)^3} \frac{1}{(q \cdot n)} \]  

(16)

Since this expression is orthogonal to \( (k^2 g_{\mu\nu} - k_\mu k_\nu) \), it follows from Eq. (14), that the exact gluon self-energy is transverse to one-loop order.

In order to study the behavior of \( G_{(1)}^{\mu,ab} \) in higher orders, let us consider, for example, the two-loops diagram shown in Figure 2b. This yields at finite temperature a contribution given by:

\[ G_{(2)}^{\mu,ab} = (\alpha g^2 N)^2 \delta^{ab}(iT)^2 \sum_{j=-\infty}^{+\infty} \int \frac{d^3-p}{(2\pi)^3} \frac{1}{(p \cdot k + k \cdot n)} \left[ g^{\mu\sigma} - \frac{p^{\mu} n^\sigma}{p \cdot n} \right] \]  

(17)

\[ \times \sum_{l=-\infty}^{+\infty} \int \frac{d^3-q}{(2\pi)^3} \frac{q^\sigma}{(q \cdot n)((q + k) \cdot n)(p - q \cdot n)} \]

where \( p_0 = (2\pi i T) j \) and \( q_0 = (2\pi i T) l \).
Using partial fraction decompositions and making appropriate changes of variables, one finds that Eq. \(17\) can be written in the form:

\[
G_{(2')}^{\mu,ab} = (\alpha g^2 N)^2 \delta^{ab}(iT)^2 \sum_{j=\infty}^{+\infty} \int \frac{d^3\epsilon}{(2\pi)^3} \frac{1}{(p \cdot n)^2} \left[ \frac{k^\mu}{k \cdot n} - \frac{p^\mu}{p \cdot n} \right] \times \sum_{l=\infty}^{+\infty} \int \frac{d^3\epsilon}{(2\pi)^3} \frac{1}{(q \cdot n)}
\]

which is rather typical of the contributions associated with two-loop diagrams.

Comparing the relation \(18\) with the one-loop result given by Eq. \(16\), one can see that both expressions have the same \(q\)-dependence. But such contributions vanish, in consequence of the antisymmetry of \((q \cdot n)^{-1}\). Note that, thus far, we have made no assumptions about the specific form of \(n\), so that this conclusion is also true in a class of axial gauges \(22\).

In the specific case of temporal gauges where \(q \cdot n = 0\), it is worth noticing that the integrals decouple, in fact, from the sums. Then, we can set, in the sense of dimensional regularisation:

\[
\int d^{3-\epsilon}q |\tilde{\xi}|^\omega = 0, \quad \omega \geq 0
\]

Furthermore, using the Mandelstam-Leibbrandt prescription to remove the singular terms at \(p_0 = 0\) and \(q_0 = 0\), the sum over \(l\) also gives zero because of the antisymmetry of \(q_0 = (2\pi i T)l\).

The above arguments, which show that \(G_{(1)}^{\mu,ab}\) and \(G_{(2')}^{\mu,ab}\) are equal to zero, may be extended to higher order to establish that \(G_{(2')}^{\mu,ab}\) must generally vanish. It then follows from Eq. \(14\) that the exact self-energy of the gluon is transverse at all temperatures. This important property implies that, to all orders, the structure functions \(\Pi\) establish that \(\eta - 4\eta_0 = 0\), the exact self-energy of the gluon is transverse at all temperatures. This important property implies that, to all orders, the structure functions \(\Pi\) should be equal to zero. (We show in the appendix \(3\) that these results are confirmed by explicit calculations to one-loop order.) In consequence of this behavior, the non-linear relation among the structure functions \(14\)

\[
\Pi_D = \frac{\Pi_2}{k^2 - \Pi_L}
\]

which must hold at finite temperature in any linear gauge, is manifestly satisfied in the class of temporal gauges.

**IV. HIGH TEMPERATURE CONTRIBUTIONS**

The non-vanishing structures \(\Pi_T\) and \(\Pi_L\), can be expanded as a series of high-temperature terms which can come from the hard thermal loop region of large \(q\), such that \(q \sim T >> k\). (There are also other contributions, such as those proportional to \(T\), which originate from the region of small \(q\).) This can be done in a straightforward way expressing the integrands in terms of the dimensionless four-vector \(Q \equiv (\tilde{\xi}/|q|)\) and performing the expansion in powers of \(1/|q|\). The results up to sub-leading contributions are given by

\[
\Pi_T^{hl} = -\frac{\alpha^2 N}{4(2\pi)^3|k|^2} \int d\Omega \left\{ \left[ \frac{4k^4}{(Q \cdot k)^2} + \frac{8k^2 k_0}{Q \cdot k} + 4k^2 - 8k_0^2 \right] \int d|q| |\tilde{\xi}| N(|\tilde{\xi}|) \right. \\
+ \left. \frac{2}{Q \cdot k} \frac{k^4}{(Q \cdot k)^2} + 2k^2 k_0^2 + 4k^2 k_0 - 8k^2 k_0^2 + 2k^4 + \frac{5k^6}{(Q \cdot k)^2} \right) \int d|q| |\tilde{\xi}| N(|\tilde{\xi}|) \right\}
\]

and

\[
\Pi_L^{hl} = -\frac{\alpha^2 N}{4(2\pi)^3|k|^2} \int d\Omega \left\{ \left[ \frac{8k^4}{(Q \cdot k)^2} - \frac{16k^2 k_0}{Q \cdot k} + 8k^2 \right] \int d|q| |\tilde{\xi}| N(|\tilde{\xi}|) \right. \\
+ \left. \frac{2k^4}{(Q \cdot k)^2} + \frac{4k^2 k_0}{Q \cdot k} + 4k^4 + \frac{2k^6}{(Q \cdot k)^2} + \frac{4k^6}{(Q \cdot k)^2} \right) \int d|q| |\tilde{\xi}| N(|\tilde{\xi}|) \right\},
\]

}[17][22][3]
where \( \int d\Omega \) denotes the integration over the directions of \( \vec{q}/|\vec{q}| \). Using that \( \int d|\vec{q}| N(|\vec{q}|) = \pi^2 T^2 / 6 \) and performing the angular integrations, we obtain

\[
\Pi_T^{ht} = -g^2 N \left\{ \frac{T^2}{12|k|^2} \left[ \frac{k^2 k_0}{|k|} \ln \left( \frac{k_0 + |\vec{k}|}{k_0 - |\vec{k}|} \right) - 2k_0^2 \right] \right. \\
- \frac{k^2}{12\pi^2} \left( 11 - 4 \frac{\alpha}{n_0^2} \right) \int \frac{d|\vec{q}|}{|\vec{q}|} N(|\vec{q}|) \right\} \quad (23)
\]

\[
\Pi_L^{ht} = g^2 N \left\{ \frac{T^2}{6|k|^2} \left[ \frac{k_0 k^2}{|k|} \ln \left( \frac{k_0 + |\vec{k}|}{k_0 - |\vec{k}|} \right) - 2k_0^2 \right] \right. \\
+ \frac{k^2}{12\pi^2} \left( 11 - 4 \frac{\alpha}{n_0^2} \right) \int \frac{d|\vec{q}|}{|\vec{q}|} N(|\vec{q}|) \right\}. \quad (24)
\]

The above contributions proportional to \( T^2 \) constitute the well-known gauge independent hard thermal loop result which has been previously obtained in the Feynman gauge as well as in general covariant gauges \[23,14\]. It also agrees with the results obtained in the static and long-wavelength limits using the temporal gauge with \( \alpha = 0 \) \[13\].

From the second lines in Eqs. (23) and (24) one can extract the part proportional to \( \ln T \) which comes from the region of high internal momenta. Such contributions can be readily obtained noticing that \( \int \frac{d|\vec{q}|}{|\vec{q}|} N(|\vec{q}|) = -\frac{1}{2} \ln(T) + \cdots \). In this way, inserting the logarithmic part of Eqs. (23) and (24) into Eq. (4), we obtain the following result for the logarithmic contribution to the gluon self-energy in the temporal gauge

\[
\Pi_{\mu\nu}(\ln T) \bigg|_{FS} = -\frac{g^2 N \delta^{\mu\nu}}{8\pi^2} \left\{ \left[ \frac{11}{3} + \frac{4\alpha k^2}{3n^2} \right] (k_\mu k_\nu - k^2 g_{\mu\nu}) \right. \\
+ \frac{4\alpha}{3n^2} \left[ (n \cdot k) k_\mu - k^2 n_\mu \right] \left( n \cdot k k_\nu - k^2 n_\nu \right) \right\} \ln T, \quad (25)
\]

where we have substituted \( u = n/n_0 \) and \( n_0^2 = n^2 \).

Comparing the above logarithmic contribution with the dimensionally regularized zero-temperature gluon self-energy in the general axial gauge \[24\], we find that the coefficients of \( \ln(T) \) and of \( 1/\epsilon \) are identical up to a relative sign.

V. CONCLUSION

We have shown that, in a class of temporal gauges, the exact thermal self-energy of the gluon is transverse to all orders. (Except for the Feynman gauge, where the transversality has been verified only to one loop order, this property is not valid in the class of covariant gauges \[18\].) The forward scattering amplitude associated with the thermal gluon self-energy has been obtained, to one-loop order, using the approximation which consists in neglecting the contributions from the prescription poles. Our explicit calculations of the \( T^2 \) and \( \ln(T) \) terms, together with the arguments in \[8\], show that such corrections affect only the contributions proportional to powers of \( 1/T \), which become negligible at high temperature. Furthermore, our approach preserves the transversality property of the exact thermal gluon self-energy. The leading \( T^2 \) and the sub-leading \( \ln(T) \) contributions are consistently obtained from the forward scattering amplitude of on-shell thermal particles. The relation between the dimensionally regularized zero temperature gluon self-energy and the \( \ln T \) contribution, given in Eq. (23), is relevant to explain the cancellation of the \( \ln(-k^2) \) terms between the zero temperature and the temperature dependent parts of the gluon self-energy (such a cancellation has also been verified by explicit calculations in covariant gauges \[24\]).

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APPENDIX A:

In this appendix we illustrate the forward scattering amplitude technique by considering the following type of integral which arises in the calculation of the two-point function in the imaginary time formalism. 

\[ I = \int d^3\bar{q} \int^{+i\infty+\delta}_{-i\infty+\delta} \frac{dz}{2\pi i} N(z) \left[ \frac{t(q; p)}{(q \cdot q)^s (p \cdot p)^t} + (z \leftrightarrow -z) \right]. \]  

\[ (A1) \]

where \( p = q + k, k_0 = 2m\pi i, m = 0, \pm 1, \pm 2, \ldots, q = (q, \bar{q}) \), and we have factorized the denominators \( 1/(q \cdot q)^s \) from the propagators. In covariant gauges \( t(q; p) \) has no poles in the complex \( z \)-plane, while for temporal gauges Eq. \( (1) \) yields extra prescription poles. For the ghost loop diagram in Fig 1c we have (in the temporal gauge) the trivial situation \( i = j = 0 \). The two special cases \( i = 1, j = 1 \) and \( i = 1, j = 0 \) corresponds to Fig 1a and Figs 1b, respectively. In what follows we will consider the case \( j \neq 0 \) so that there will be contributions from the poles at \( q \cdot q = 0 \) and \( p \cdot p = 0 \). The case \( j = 0 \) can be easily derived using the same basic steps described below.

Since the integration in \( (A1) \) is over all values of \( \bar{q} \), it is convenient to make the change of variables \( \bar{q} \leftrightarrow -\bar{q} \) in all the terms \( (z \leftrightarrow -z) \) so that

\[ I = \int d^3\bar{q} \int^{+i\infty+\delta}_{-i\infty+\delta} \frac{dz}{2\pi i} N(z) \left[ \frac{t(q; p)}{(q \cdot q)^s (p \cdot p)^t} + q \leftrightarrow -q \right]. \]  

\[ (A2) \]

Factorizing the denominators in \( (A2) \) we can write

\[ I = \int q^3 d\bar{q} \int^{+i\infty+\delta}_{-i\infty+\delta} \frac{dz}{2\pi i} N(z) \left[ \frac{1}{(z + |\bar{q}|)^t} \left( \frac{1}{(z + k_0 + |\bar{p}|)^j} + \frac{t(q; p)}{(q \cdot q)^s (p \cdot p)^t} \right) + q \leftrightarrow -q \right]. \]  

\[ (A3) \]

The \( z \) integration can be readily performed using the Cauchy theorem and closing the contour in the right hand side plane where there are poles at \( z = |\bar{q}| \) and \( z = |\bar{p}| - k_0 \) \((k_0 \) is a pure imaginary quantity at this stage of the calculation). In this way we obtain

\[ I_{FS} = -\int q^3 d\bar{q} \left\{ \frac{1}{(i - 1)!} \lim_{q_0 \to |\bar{q}|} \frac{\partial^{i-1}}{\partial q_0^{i-1}} \left( \frac{N(q_0)}{(q_0 + |\bar{q}|)^t} \frac{t(q; p)}{(p \cdot p)^t} \right) + \frac{1}{(j - 1)!} \lim_{\bar{q}_0 \to |\bar{q}| - k_0} \frac{\partial^{j-1}}{\partial \bar{q}_0^{j-1}} \left( \frac{N(q_0)}{(q_0 + k_0 + |\bar{q}|)^j} \frac{t(q; p)}{(q \cdot q)^s} \right) + q \leftrightarrow -q \right\}, \]  

\[ (A4) \]

where the subscript \( FS \) is to remind us that we have only considered the mass shell poles. Performing the change of variables \( \bar{q} \to \bar{q} - \bar{k} \) in the second term of \( (A4) \) we can write

\[ I_{FS} = -\int q^3 d\bar{q} \left\{ \frac{1}{(i - 1)!} \lim_{q_0 \to |\bar{q}|} \frac{\partial^{i-1}}{\partial q_0^{i-1}} \left( \frac{N(q_0)}{(q_0 + |\bar{q}|)^t} \frac{t(q; p)}{(p \cdot p)^t} \right) + \frac{1}{(j - 1)!} \lim_{\bar{q}_0 \to |\bar{q}| - k_0} \frac{\partial^{j-1}}{\partial \bar{q}_0^{j-1}} \left( \frac{N(q_0)}{(q_0 + k_0 + |\bar{q}|)^j} \frac{t(q_0, \bar{q} - \bar{k}; q_0 + k_0, \bar{q})}{(q_0^2 - |\bar{q} - \bar{k}|^2)^s} \right) + q \leftrightarrow -q \right\}. \]  

\[ (A5) \]

Finally, using the property \( N(q_0 \pm k_0) = N(q_0) \) and taking into account the contributions \( q \leftrightarrow -q \) we obtain

\[ I_{FS} = -\int d^3q \left\{ \frac{1}{(i - 1)!} \lim_{q_0 \to |\bar{q}|} \frac{\partial^{i-1}}{\partial q_0^{i-1}} \left( \frac{N(q_0)}{(q_0 + |\bar{q}|)^t} \frac{t(q; p)}{(p \cdot p)^t} \right) + \frac{1}{(j - 1)!} \lim_{\bar{q}_0 \to |\bar{q}| - k_0} \frac{\partial^{j-1}}{\partial \bar{q}_0^{j-1}} \left( \frac{N(q_0)}{(q_0 + |\bar{q}|)^t} \frac{t(-p, q)}{(p \cdot p)^t} \right) + q \leftrightarrow -q \right\} \]  

\[ (A6) \]
Using $p = q + k$, the special case when $i = j = 1$ can be written as

$$I_{FS} = -\int \frac{d^3q}{2|q|} N(|\vec{q}|) \left[ \frac{t(q; p)}{k^2 + 2k \cdot q} + \frac{t(-p; -q)}{k^2 - 2k \cdot q} + q \leftrightarrow -q \right]_{q_0 = |\vec{q}|}.$$  \hspace{1cm} (A7)

The expression inside the bracket is a typical contribution to the on-shell forward scattering amplitude as represented, for example, by the first two terms in the Eq. (\ref{eq:fs}).

**APPENDIX B:**

Here we show explicitly that the one-loop contributions to $\Pi_C$ and $\Pi_D$, in the class of temporal gauges characterized by the gauge parameter $\alpha$, are identical to zero. We start from the general expression

$$\Pi_{C,D} = g^2 N(iT) \sum_{l=-\infty}^{+\infty} \int \frac{d^{3-\epsilon}q}{(2\pi)^{3-\epsilon}} I_{C,D}(k, q)$$

$$= \frac{g^2 N}{2\pi} \int \frac{d^{3-\epsilon}q}{(2\pi)^{3-\epsilon}} \int_{-\infty+i\delta}^{i\infty+i\delta} dq_0 \frac{1}{2} \left[ I_{C,D}(k, q) + I_{C,D}(k, -q) \right] \coth \left( \frac{q_0}{2T} \right),$$  \hspace{1cm} (B1)

where the integrands $I_{C,D}(k, q)$ are obtained projecting the contributions from the diagrams in Figs. 1a and 1b onto the tensors $P_{\mu\nu}^{C,D}$. Using partial fraction decompositions, as in Eq. (\ref{eq:partial}), we perform shifts $q \rightarrow q - k$ in all the terms involving $(q + k) \cdot u$ in the denominator, which are justified in the dimensional regularization scheme. After using also the property $\coth((q_0 \pm k_0)/2T) = \coth(q_0/2T \pm i\pi n) = \coth(q_0/2T)$ as well as the invariance of the $\vec{q}$-integral under $\vec{q} \rightarrow -\vec{q}$, the rather involved expressions of $I_{C,D}(k, q)$ simplify to

$$I_C(k, q) = \frac{1}{|k||q|^2(q + k)^2} \left[ 8q \cdot uk \cdot q - k^2 k \cdot u - 6k \cdot qk \cdot u - 8(k \cdot q)^2 k \cdot u + 4k^2 q \cdot u \right]$$  \hspace{1cm} (B2)

and

$$I_D(k, q) = \frac{1}{q^2(q + k)^2} \left[ 2k \cdot q - 4q^2 + 8(k \cdot q)^2 k^2 - k^2 \right].$$  \hspace{1cm} (B3)

It is interesting to note that the terms with denominators involving factors like $q \cdot u$ or $(q + k) \cdot u$ cancel after performing shifts, so that we do not need to worry about prescription poles. Hence the only poles which may contribute are the ones at $q_0 = |\vec{q}|$ and $q_0 = |\vec{q} + \vec{k}| - k_0$. The corresponding residues of $I_{C,D}(k, q)$ are

$$\text{Res} \left[ I_C(k, q), q_0 = |\vec{q}| \right] = \frac{1}{2|\vec{k}| |\vec{q}|} \left. \left( 4q \cdot u - 4k \cdot u \frac{k \cdot q}{k^2} - k \cdot u \right) \right|_{q_0 = |\vec{q}|},$$  \hspace{1cm} (B4)

$$\text{Res} \left[ I_C(k, q), q_0 = |\vec{q} + \vec{k}| - k_0 \right] = -\frac{1}{2|\vec{k}| |\vec{q} + \vec{k}|} \left. \left( 4q \cdot u - 4k \cdot u \frac{k \cdot q}{k^2} - k \cdot u \right) \right|_{q_0 = |\vec{q} + \vec{k}| - k_0}$$  \hspace{1cm} (B5)

$$\text{Res} \left[ I_D(k, q), q_0 = |\vec{q}| \right] = \frac{1}{2|\vec{q}|} \left. \left( \frac{k \cdot q}{k^2} - 1 \right) \right|_{q_0 = |\vec{q}|}$$  \hspace{1cm} (B6)

and

$$\text{Res} \left[ I_D(k, q), q_0 = |\vec{q} + \vec{k}| - k_0 \right] = -\frac{1}{2|\vec{q} + \vec{k}|} \left. \left( \frac{k \cdot q}{k^2} + 3 \right) \right|_{q_0 = |\vec{q} + \vec{k}| - k_0}.$$  \hspace{1cm} (B7)

Performing the shift $\vec{q} \rightarrow \vec{q} - \vec{k}$ in the Eqs. (B5) and (B7) one can see that the contributions from the distinct poles in $I_{C,D}(k, q)$ exactly cancel each other.
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