Vortex Quantum Nucleation and Tunneling in Superconducting Thin Films: Role of Dissipation and Periodic Pinning

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Abstract

We investigate the phenomenon of decay of a supercurrent in a superconducting thin film *in the absence* of an applied magnetic field. The resulting zero-temperature resistance derives from two equally possible mechanisms: 1) quantum tunneling of vortices from the edges of the sample; and 2) homogeneous quantum nucleation of vortex-antivortex pairs in the bulk of the sample, arising from the instability of the Magnus field’s “vacuum”. We study both situations in the case where quantum dissipation dominates over the inertia of the vortices. We find that the vortex tunneling and nucleation rates have a very rapid dependence on the current density driven through the sample. Accordingly, whilst normally the superconductor is essentially resistance-free, for the high current densities that can be reached in high-$T_c$ films a measurable resistance might develop. We show that edge-tunneling appears favoured, but the presence of pinning centres and of thermal fluctuations leads to an enhancement of the nucleation rates. In the case where a periodic pinning potential is artificially introduced in the sample, we show that current-oscillations will develop indicating an effect specific to the nucleation mechanism where the vortex pair-production rate, thus the resistance, becomes sensitive to the corrugation of the pinning substrate. In all situations, we give estimates for the observability of the studied phenomena.

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1 Introduction

There has been a strong revival of interest, recently, in the physics of magnetic vortices in type II and high-temperature superconductors [1]. The motion of these vortices subsequent to the applied supercurrent gives rise to a non-zero resistance, especially important in the case of the high-$T_c$ materials. Most research efforts have been devoted to phenomena relating to the nature of the mixed phase of a superconductor in some externally applied magnetic field and supercurrent. Issues connected with the pinning of the flux lines by defects have been widely studied.

We [2], as well as Ao and Thouless [3] and Stephen [4], have addressed the problem of the quantum dynamics of vortices in the absence of an external field but in the presence of an externally driven supercurrent, quantum dissipation and pinning. This leads to the decay of a supercurrent, or a residual low-temperature resistance in the superconductor. Whilst most of the dissipation seems to be ascribed to vortices tunneling in the sample from the edge [2], another interesting and novel possibility also explored by us in depth is that of a residual resistance arising from spontaneous vortex-antivortex pair creation in the bulk of, for example, a superconducting thin film. We have set up a powerful theory to study both bulk nucleation and tunneling from the edge of vortices spontaneously created either by the quantum fluctuations of the Magnus force field or by the current’s gradient at the border of the sample. We find that, due to the high values of the critical currents in thin films, these dissipative effects can become particularly important in the two-dimensional (2D) geometry, where lateral fluctuations of the flux lines can be neglected.

In this report we discuss our findings, comparing the rates of edge tunneling and bulk nucleation, to find that both effects can become important and observable under suitable conditions. Our theory is set up to study vortex tunneling and nucleation in the 2D geometry in the presence of quantum dissipation and of pinning potentials. The central result is that the tunneling/nucleation rate $\Gamma$ has a strong exponential dependence on the number current density $J$

$$\Gamma \propto \eta_{\text{eff}}^{1/2} J^{-1} \exp\left\{-\eta_{\text{eff}} \mathcal{E}_R^2 / 4 \pi J^2 \right\}$$

(1.1)

Here $\eta_{\text{eff}}$ is an effective viscosity coefficient as renormalised by the magnetic-like part of the Magnus force, and $\mathcal{E}_R$ is the barrier or nucleation energy for a single vortex as renormalized by screened Coulomb interactions and (fake) Landau-level corrections. This exponential dependence would make the tunneling/nucleation (folded, e.g., into the sample’s resistance) observable in a rather narrow range of $J$-values. Thus, normally the superconductor is practically resistance-free. However, the high values of $J$ that can be reached in the high-$T_c$ materials make the possibility of observing both tunneling and pair creation within reach for thin films. One particular feature that would uniquely relate the residual resistance to the phenomenon of spontaneous vortex-pair creation is the presence of oscillations in the $J$-dependence of $\Gamma(J)$ in case a periodic pinning potential is artificially created in the film. These oscillations are in fact strictly connected to the pinning-lattice spacing $d = 2\pi/k$ of the periodic potential (we assume a square lattice), e.g.

$$U(q(t)) = U_0 \sum_{a=1}^{2} [1 - \cos(kq_a(t))]$$

(1.2)

acting on the nucleating vortex-pairs described by a coordinate $q$. The problem of quantum dissipation for a particle (the 2D vortex) moving in a periodic potential has some interesting
features in its own right \[5, 6, 7\]. It is characterised by a localization phase transition driven by dissipation; accordingly, two phases can occur depending on whether the dissipation coefficient \( \eta \) is greater (confined phase) or smaller (mobile phase) than a critical value \( \eta_c = k^2 / 2 \pi = 2 \pi / d^2 \). This localization transition is described by a Kosterlitz-type renormalization group (RG) approach. We have implemented the RG approach for the evaluation of the dependence of the spontaneous nucleation rate of vortex-antivortex pairs on the external parameters for our own quantum dynamical system. A remnant of the dissipation-driven phase transition is observed and the pair production rate \( \Gamma \) can be derived in both phases by means of a frequency-space RG procedure leading to observable current-oscillations if \( \eta > \eta_c \). These result from a correction factor \( K(J) \) in Eq. (1.1) which is sensitive to the periodicity \( d \) of the pinning substate. By measuring these oscillations one can confirm the spontaneous pair-creation related nature of the residual resistance in a superconducting thin film, as well as microscopically or even mesoscopically probe the lattice structure of the pinning substrate.

Below, we give a summary of our theoretical treatment and discuss the results obtained for the various cases relevant to supercurrent decay. Details of our work can be found in the literature [2].

2 Theory of Spontaneous Vortex-Antivortex Pair Nucleation in the Presence of Quantum Dissipation

One can show that a vortex moving in a superfluid experiences a force, the Magnus force, which is the equivalent of the electromagnetic Coulomb-Lorentz force. The classical equation of motion reads:

\[
\mathbf{m} \ddot{\mathbf{q}} = -\nabla U(\mathbf{q}) + e \mathbf{E} - e \dot{\mathbf{q}} \times \mathbf{B} - \eta \dot{\mathbf{q}}
\]  

(2.1)

with \( m \) the (negligible) inertial mass of the vortex, carrying topological charge \( e = \pm 2\pi \), treated as a single, point-like particle of 2D coordinate \( \mathbf{q}(t) \). \( U(\mathbf{q}) \) is the phenomenological potential acting on the vortex and \( \eta \) is a phenomenological friction coefficient taking dissipation into account. A supercurrent \( \mathbf{J} \) gives rise to an electric-like field \( \mathbf{E} = \times \mathbf{J} \) (a notation implying \( \mathbf{E} \cdot \mathbf{J} = 0 \)) superposed to a magnetic-like field \( \mathbf{B} = \hat{z} \rho_s^{(3)} \) for a thin film orthogonal to the vector \( \hat{z} \) and having thickness \( d \) with a superfluid component characterised by a 3D number density \( \rho_s^{(3)} \). The quantum-mechanical counterpart of Eq. (2.1) is constructed through the Feynman path-integral transposition in which the dissipation is treated quantistically through the formulation due to Caldeira and Leggett [8]. This approach views quantum dissipation as described by the linear coupling of the vortex coordinate to the coordinates of a bath of harmonic oscillators of prescribed dynamics.

Quite generally, we deal with the quantum decay of the “vacuum”, represented by a static e.m.-like Magnus field, through its quantum fluctuations: in this case the vortex-antivortex pairs. The decay rate \( \Gamma \) is given by the formula (\( \mathcal{N} \) is a normalization factor)

\[
\frac{\Gamma}{2L^D} = Im \int_0^\infty \frac{d\tau}{\tau} \mathcal{N}(\tau) \int_{q(0)=q(\tau)=q_0} Dq(\tau) e^{-\int_0^{\tau} ds \mathcal{L}_E}
\]  

(2.2)

where \( L^D \), with \( D = 2 \) here, is the volume of the sample, and the Euclidean single-particle Lagrangian for closed trajectories in space-time is
Here, the set \( \{x_k\} \) represents the Caldeira-Leggett bath of harmonic oscillators simulating quantum friction, with \( \frac{\pi}{2} \sum_k \frac{c_k^2}{m_k \omega_k^2} \delta(\omega - \omega_k) = \eta \omega \exp(-\omega/\Omega) \) (\( \Omega \) being some frequency cutoff) fixing the spectral distribution of frequencies (we restrict the present discussion to the ohmic case). Also, \( F_{\mu\nu} \) is the uniform Magnus field tensor and \( V(q) = 2E_0 U(q) \) is the relativistic counterpart of the classical potential, Eq. (1.2), thus having amplitude \( A_0 = 2E_0 U_0 \) (we have assumed \( A_0/E^2_0 \ll 1 \)). \( E_0 \) is the vortex nucleation energy. As one important remark, we assume here that the microscopic dissipation mechanism due to the normal material inside the vortex core, whether phonon- or electron-driven, can be always described phenomenologically by the Caldeira-Leggett type action.

The Feynman path integral in Eq. (2.2) can now be evaluated at the leading singularity of the \( \tau \)-integral; in the dissipation-dominated regime (\( m_1 = m_2 = m \to 0 \)) we obtain the exact analytic expression for the bulk nucleation rate:

\[
\frac{\Gamma_0}{L^2} = \frac{e \Omega \eta_{eff} \eta^{1/2}}{32\pi^2 J} e^{-\xi_0^2 \eta_{eff}/4\pi J^2}
\]  

where \( \eta_{eff} = \sqrt{\eta^2 + B^2/\eta} \) and \( \xi_0^2 = \xi_0^2 + \frac{\xi_0}{m} \sqrt{\eta^2 + B^2} \) are renormalizations due to the magnetic-like field. Further renormalizations of \( E_0 \) due to vortex-vortex interactions are understood. According to this formula, for small current densities \( J \) the nucleation rate, hence the resistance \([2]\), is negligible. However, depending on the material parameters, for higher \( J \) both the rate \( \Gamma_0 \) and the film resistance can become important. Before discussing realistic estimates, we consider the startling effect of a periodic pinning potential on bulk nucleation.

### 3 The Effect of a Periodic Pinning Potential: Current Oscillations

Suppose a regular lattice of pinning sites is introduced artificially in the sample, e.g. in a square-lattice configuration. This can be achieved by means of a variety of etching or laser ablation or other techniques combined with modern MBE-type sample growth processes. We are then confronted with the motion (and then the quantum-mechanical nucleation) of particles in a dissipative and periodic-potential situation. This type of quantum diffusion has been studied at length \([7]\). In the mobile phase, the RG analysis predicts a vanishing large-scale potential amplitude, thus no significant effect of the pinning potential can be expected on the nucleation rate, except for a further renormalization of \( E_0 \). We look therefore for a situation corresponding to the confined phase (\( \eta > \eta_c \)) where the effective pinning potential amplitude grows up to an upper limit as nucleation proceeds in the bulk of the film \([2]\).

When the RG analysis is folded into the evaluation of the leading contribution to the Feynman path integral in Eq. (2.2) for the periodic pinning potential (1.2), we end up with the following analytic result for the nucleation rate:
\[ \Gamma = \Gamma_0 K(J) \]
\[ K(J) = e(1 + \mu_0) \left( 1 + \frac{\mu_0 \Omega \eta}{8\pi J^2} \right) I_0 \left( \frac{A_0 \eta}{4\pi J^2} J_0(2k\ell_N) \right) \]

Here \( \Gamma_0 \) is given by Eq. (2.4), there is the further renormalization \( E_{0R}^2 \rightarrow E_{0R}^2 + A_0 \) for the nucleation energy, \( J_0(x) \) and \( I_0(x) \) are Bessel functions and the renormalised amplitude \( A_{0R} \) is a few times \( A_0 \) itself [2]. \( \ell_N \) is a nucleation length, given in the lowest approximation by \( \ell_N = E_{0R} / 2\pi J \). \( \mu_0 \) is a dimensionless parameter of order unity.

We then see that the competition between the nucleation length \( \ell_N \) and the pinning-lattice spacing \( d = 2\pi / k \) leads to oscillations, as the current \( J \) is changed, in the nucleation rate (hence in the resistance). This shows that in the nucleation process the dissociating vortex-antivortex pairs “feel” the periodicity of the pinning substrate. Now for the estimates of the expected values of \( \Gamma \). So far, the computation referred to \( T = 0 \). In [2] we proposed to take into account finite temperatures by adding incoherently, in the above formulas, a term proportional to \( T \) (note \( k_B = 1 \)): \( J^2 \rightarrow J^2 + \frac{\eta E_{0R}^2}{8\pi} T \), such that for \( J \rightarrow 0 \) the standard Boltzmann exponential form in the vortex density, \( \rho_v \propto \sqrt{\Gamma} \), is recovered in the absence of pinning. This might be a reasonable qualitative treatment of thermal effects for temperatures less than \( \approx 8\pi J^2 / \eta E_{0R}^2 \). Furthermore, we propose, following Minnhagen [9], to account for vortex-antivortex Coulomb-like interactions by means of a current-dependent (and temperature-independent) activation energy: \( E_R(J) = E_{0R} \ln(J_{max}/J) \), with \( E_{0R} \) including all other renormalization effects. This expression entails an \( E_R \) infinite for \( J = 0 \) (as is appropriate below the Kosterlitz-Thouless (KT) transition in the film) and changing sign, thus becoming unphysical, at \( J_{max} \). We take \( J_{max} \) an order of magnitude higher than the single-crystal YBCO critical current, [10] namely \( J_{max}^{em} = 10^8 \) A cm\(^{-2} \), and take \( E_R \approx 80 \) K for \( J = 10^7 \) A cm\(^{-2} \), of the order of magnitude of the KT transition temperature [1].

The important friction coefficient is taken to be \( \eta \approx 10^{-2} \) Å\(^{-2} \), from the Bardeen-Stephen formula [11] applied to YBCO films. A most sensitive parameter is related to the amplitude of the pinning potential, \( \epsilon = A_0 / E_{0R}^2 \), which is entirely unknown. We have taken a negative \( \epsilon = -0.5 \), borrowing from classical nucleation the point of view that vortex production is actually aided by the presence of pinning centers. Finally, \( d = 50 \) Å concludes our illustrative case which is clearly of the confined-phase type. As \( \eta > \eta_c \), we have taken \( A_{0R} \approx A_0 \), for simplicity’s sake, and find no significant dependence on the most uncertain of all parameters, \( \Omega \). Its dimensions being the same as \( E_{0R}^2 \), we have taken \( \Omega = E_{0R}^2 / \epsilon \). Also, we take the film thickness \( s = 10 \) Å as the typical interlayer spacing. Figure 1 then represents our estimate of the expected values of the nucleation rate in the above experimental conditions. Our claim is that both pair-production and current-oscillations should be observable with an appropriate technique (e.g. Tonomura’s electron holography [12]). We point out that, with no controlled pinning present, the decay of a supercurrent in the absence of a magnetic field has been measured experimentally in BiSCCO films, see e.g. [13], though in this case edge tunneling (which we discuss below) also plays a role.
4 Theory of Vortex Nucleation from the Edge of the Sample

Finally, we briefly describe our theoretical treatment for evaluating the tunneling rate from the edge [2] (in the absence, however, of the periodic pinning potential). This is done by extending the treatment of Section 2 to the case where the vortices are already present in the edge strip of the sample. This requires that, in the formula for the rate, Eq. (2.2), the contribution from the antivortex paths is factored out and cancelled by the introduction of a suitable chemical potential $\mu^*$. Setting $B = 0$ in order to simplify the treatment (a finite $B$ results only in some renormalization), the formula for the tunneling rate becomes:

$$\Gamma \approx Im \int_0^\infty \frac{dT}{T} \Phi(T) \int_{q(T)=q(0)} Dq(t) e^{-S_{NR}}$$

(4.1)

where the non-relativistic (vortex-only) action reads

$$S_{NR} \approx \int_0^T dt \left\{ \frac{1}{2}m \left( \frac{dq}{dt} \right)^2 - E \cdot q + U(q) \right\}$$

(4.2)

and where the factor $\Phi(T)$ is fixed by a self-consistency procedure [2]. The potential $U(q)$ now contains the 2D Coulomb potential binding the vortices to the edge strip, a distance $y$ away, and has the form $U(q) = U_D(q) + K \ln(1 + y/a)$. $U_D$ is the usual Caldeira-Leggett potential and $K$ the Coulomb interaction strength. At this point the usual leading contribution to the $T$- and Feynman path-integral is extracted, however by means of a saddle-point approximation with inclusion of the Gaussian fluctuations [2], which yields the tunneling rate per unit length (for an edge strip having width $a$):

$$\Gamma = J a e^{-S_0}$$

(4.3)

where $\varphi$ is a slowly-varying dimensionless factor of order unity, $Q = \ln[\Omega T/(2\pi)]$, and $S_0$, $T$ and $\bar{y}$ are determined by the saddle-point conditions:

$$S_0 = \frac{\pi \eta}{2Q} \bar{y}^2 = \frac{\eta\{K \ln(1 + K/2\pi J a) + \cdots\}^2}{8\pi J^2 Q}$$

$$2\pi J \bar{y} + K \ln(1 + \bar{y}/a) = 0$$

(4.4)

$$\frac{\pi \eta}{Q} \bar{y} - 2\pi J T + K T \frac{1}{a + \bar{y}} = 0$$

This yields almost the same leading dependence (logarithmic corrections apart) $\Gamma \approx e^{-(J_0/J)^2}$ on the external current that was also obtained for the homogeneous bulk-nucleation phenomenon. Indeed, the effective Coulomb energy $K \ln(1 + K/2\pi J a)$ can be interpreted as playing the role of the activation energy $E_0$ renormalised by the vortex-antivortex interactions. As for the estimate of the edge tunneling rate, we use the same material parameters set up in Section 3, and in Figure 2 we report the tunneling rate $R = \Gamma/L$ for different possible values of the Coulomb strength $K$. The conclusion is that both edge-tunneling (normally dominant for small $J$) and bulk-nucleation can become important and concomitant mechanisms for supercurrent decay in a superconducting thin film.
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FIGURE CAPTIONS

Figure 1. Vortex-antivortex production rate (in $\mu m^{-2}s^{-1}$, log\textsubscript{10} scale) in the presence ($\Gamma$) and absence ($\Gamma_0$) of the periodic pinning potential, versus current density $J^m$ (in $A \ cm^{-2} \times 10^7$). $K(J)$ is the current-dependent correction factor due to the periodic substrate. Here, the temperature is $T = 2.5$ K.

Figure 2. Plot of the tunneling rate per unit length, $R$, as function of the supercurrent density, $J^m$, for the values $K = 30$ K and $K = 50$ K. Dashed lines, $T = 0$ K; full lines, $T = 2.5$ K.
