Modelling for the integer optimization of the production resources of the organization in the conditions of warranty support of goods

V N Ivanov
Omsk State Technical University, 11 Mira ave., Omsk, 644050, Russia
E-mail: vitaly-ivanov@yandex.ru

Abstract. The article describes the economic and mathematical model to optimize the distribution of production resources of the organization in conditions of warranty support of goods. The model allows the integer assignment of such resources as, for example, equipment. The mathematical apparatus allows in a single system to consider the production of all goods from the nomenclature of the organization for each technological operation for each year of the project together with the warranty support of the goods. The system of limitations of the model, in addition to taking into account all items of possible costs, assumes the possibility of taking into account the impact of additional efforts aimed at ensuring quality in production on the subsequent volumes of work associated with the warranty.

1. Introduction

Currently, mathematical models and decision-making methods based on them are increasingly used to solve the problems of stability and development of various spheres of society in conditions of limited resources [1]. In this process, a significant place is occupied by the creation of a mathematical apparatus designed to improve the work of organizations by optimizing its use of production resources. The main effects can be obtained through optimal synchronization of the use of different types of resources interacting with each other in complex production networks [2]. At the same time, both the amount of such resources and their properties experience constant fluctuations in the production system.

Only a small number of mathematical models describing the work of organizations include competitiveness. This is due to the fact that the simultaneous presence in the model of production and marketing indicators makes it, as a rule, cumbersome and difficult to apply [3]. Such models are difficult to analyze even with the help of existing computer applications. On the one hand, the desire of scientists who create such models to make them as knowledge-intensive, complex and accurate as possible, and on the other hand, the unwillingness of managers of organizations to use such models in their work contribute to this. Nevertheless, the concept of competitiveness is widely accepted by the business community and is actively implemented in the activities of the organization. There are also scientific works devoted to this topic, such as the source [4], which presents various approaches to the competitiveness of modern organizations, a comprehensive model of the competitiveness of the company, checking the relationship between its elements.

The relevance of mathematical modeling of the use of production resources of the organization is based on the confirmation that with the great importance of the interaction of resources to ensure competitive advantages, individually they do not matter, and different configurations of the interaction...
of resources give different manifestations and degree of competitive advantage in specific conditions for the organization and the economic market [5].

Despite the large amount of data that determine the competitive advantage of the company [6], there are three main factors of competitiveness of any of its products. These factors include the price, quality and level of after-sales service, including warranty support of the goods. Under the product the author, as is customary in management, refers to products, services or work.

Organizations are regularly faced with the need to organize their production processes in conditions of the established contractual terms, and hence the rate of release of goods, compliance with the requirements for their quality and at the same time minimizing the use of resources [7].

To date, the rules of business planning require an assessment of the parameters of design and organization of production of goods by analyzing the cost of its life cycle [8], it is necessary to synchronize the distribution of resources across the range of products.

Performing cost estimation of organizations they often transfer a certain group of quality costs, which are intangible costs, in conditions where there is no reliable method of their measurement [9]. However, organizations that account for all quality costs are in a competitive advantage over others. The relationship between the costs of production and warranty support of goods of the organization is obvious and requires its mathematical description together with other laws that form the economic efficiency of business processes. In itself, warranty support does not bring profit to the organization, as a whole increases its profit [10]. Managers need an instrument of decision support based on available statistical information in the organization, which helps manufacturers automatically identify the priority in improving the quality of products that have a large impact on the costs of guarantees, through redeployment of resources after the optimization procedure.

The above requirements make it difficult to describe the processes with a fairly simple model, an attempt of which the author made in the materials of this article, allowing the simplification of the perception of some production characteristics.

2. Problem statement
The purpose of this article is to acquaint the reader with the economic and mathematical model that allows to optimize the distribution and use of production resources of the organization for the production and warranty support of goods. The model allows to find values on distribution of such integer resources as means of production of the organization, for example, appointing separate machines, machines, and other equipment on each technological operation of production and guarantee maintenance on each type of goods. The model implements the ability to redistribute costs between the production of goods and its warranty support in the process of optimization. In addition to the cost of means of production, the model takes into account the cost of materials and components, wages, overhead, other costs, as well as the results obtained from the sale of goods.

The objective function of the created model is the economic effect of the release of goods for the period of production of goods, taking into account the warranty period. This effect is an annual effect based on the net present value. Maximization of net present value is often used as a target function of mathematical models describing production systems, as it was done, for example, in the mathematical modeling of the production process of mining [11] or in road construction [12]. Released high-quality goods in one period reduces the cost of its warranty support in others, and therefore all-period design optimization for the selected target function will be justified.

3. Theory
The objective function in accordance with the task has the form presented in the formula (1):

$$N = \sum_{s=0}^{s_{\text{max}}} N_s \rightarrow \max, \quad i = 1, I,$$  

(1)
where $N_s$ – the effect of the release and warranty support of the goods for the year $s$; $S$ – the period of production of goods under the project; $R_i$ – the warranty period for the $i$-th type of manufactured goods; $l$ – the range of manufactured goods.

Each annual effect is described by the formula (2):

$$N_l = \left\{ \sum_{l=1}^{J} \sum_{s=1}^{S} \left( V_{iljs}^{(b)} \cdot \rho_{iljs}^{(b)} - \sum_{j=1}^{J} Z_{iljs}^{(b)} - \sum_{l=1}^{L} Z_{ils}^{(e)} \right) \right\} \cdot \frac{1}{(1+E)^s}, \quad s=0, \{S + \max(R_i)\},$$

where $(b)$ – the index to denote indices related to the production of goods; $(e)$ – the index to indicate indicators associated with the warranty support of products; $j$ – the number of technological operations in the manufacture of the goods; $J$ – the quantity of technological operations in the manufacture of the goods; $l$ – the number of technological operations for warranty support of products; $L$ – quantity of technological operations for warranty support of products; $V_{iljs}^{(b)}$ – the estimated annual production for the $j$-th technological operation of the $i$-th product; $\rho_{iljs}^{(b)}$ – the price of the $i$-th commodity in the $s$-th year, monetary unit/unit of production; $Z_{iljs}^{(b)}$ – annual costs of the $j$-th technological operation in the production of $i$-th type of goods in the $s$-th year; $Z_{ils}^{(e)}$ – annual costs of the $l$-th technological operation in the warranty support of the $i$-th type of goods in the $s$-th year; $E$ – discount rate.

Approaches to determining costs may differ depending on the characteristics of national legislation, as well as the chosen methods [13]. When specifying in the model the determinants of costs should be taken into account in business transfer pricing for certain products [14], when sales of products and components within the process or supply chain related counterparties is carried out by intra-firm, not market prices.

The annual costs for each technological operation in the production and warranty support of the $i$-th type of goods in the $s$-th year can be described respectively by formulas (3) and (4):

$$Z_{iljs}^{(b)} = \begin{cases} W_{iljs}^{(b)} + M_{iljs}^{(b)} + A_{iljs}^{(b)} + H_{iljs}^{(b)} + Q_{iljs}^{(b)} + Z_{ils}^{(D)}, & i=1, \ldots, I, \quad j=1, \ldots, J, \quad W_{iljs}^{(b)}, M_{iljs}^{(b)}, A_{iljs}^{(b)}, H_{iljs}^{(b)}, Q_{iljs}^{(b)}, Z_{ils}^{(D)} \geq 0, \quad \text{если} \quad s \leq S, \\ 0, & \text{если} \quad s > S; \end{cases}$$

$$Z_{ils}^{(e)} = W_{ils}^{(e)} + M_{ils}^{(e)} + A_{ils}^{(e)} + H_{ils}^{(e)} + Q_{ils}^{(e)}, \quad i=1, \ldots, I, \quad l=1, \ldots, L, \quad W_{ils}^{(e)}, M_{ils}^{(e)}, A_{ils}^{(e)}, H_{ils}^{(e)}, Q_{ils}^{(e)} \geq 0,$$

where $W_{iljs}^{(b)}, M_{iljs}^{(b)}, A_{iljs}^{(b)}, H_{iljs}^{(b)}, Q_{iljs}^{(b)}$ – respectively costs of operation of the fleet of equipment, materials and accessories, the salary of workers (except for workers working on equipment that makes up a fleet), overheads and other costs of $j$-th technological operation at production of $i$-th type of goods in $s$-th year; $W_{ils}^{(e)}, M_{ils}^{(e)}, A_{ils}^{(e)}, H_{ils}^{(e)}, Q_{ils}^{(e)}$ – respectively costs of operation of the fleet of equipment, materials and accessories, the salary of workers (except for workers working on equipment that makes up a fleet), overheads and other costs of $l$-th technological operation at guarantee maintenance of $i$-th type of goods in $s$-th year; $Z_{ils}^{(D)}$ – additional costs for the $j$-th technological operation in the production of the $i$-th type of goods in the $s$-th year, providing a reduction in warranty claims in the future, described by the formula (5):

$$Z_{ils}^{(D)} = W_{iljs}^{(D)} + M_{iljs}^{(D)} + A_{iljs}^{(D)} + H_{iljs}^{(D)} + Q_{iljs}^{(D)}, \quad i=1, \ldots, I, \quad l=1, \ldots, L, \quad W_{iljs}^{(D)}, M_{iljs}^{(D)}, A_{iljs}^{(D)}, H_{iljs}^{(D)}, Q_{iljs}^{(D)} \geq 0,$$

where $W_{iljs}^{(D)}, M_{iljs}^{(D)}, A_{iljs}^{(D)}, H_{iljs}^{(D)}, Q_{iljs}^{(D)}$ – accordingly, the additional costs of the $j$-th technological operation in the production of the $i$-th type of goods in the $s$-th year, providing a reduction in warranty claims in the future for the operation of the fleet of equipment, materials and components, wages of workers (except for workers working on equipment that makes up a fleet), overhead and other costs.
The cost of operation of the fleet of equipment for the j-th technological operation in the production of i-th type of goods in the s-th year can be described by the equation (6):

$$W_{ijs}^{(b)} = \sum_{s=1}^{X} C_{x_{ijs}} \cdot n_{x_{ijs}}^{(b)} \cdot T_{CM} \cdot K_{CM}^{(b)} \cdot \left[ t_{y_{ijs}}^{(b)} - t_{x_{ijs}}^{(b)} \right], \quad i = \overline{1,I}, \quad j = \overline{1,J}, \quad s = \overline{1,S};$$

where $x$ – the standard size of the machine; $X$ – the quantity of standard sizes of equipment on the j-th technological operation; $C_{x_{ijs}}$ – the cost of machine hours equipment x-standard size on the j-th technological operation in the production of i-th type of goods in the s-th year; $n_{x_{ijs}}^{(b)}$ – the quantity of units of equipment x-standard size, performing j-th technological operation in the production of i-th type of goods in the s-th year; $T_{CM}$ – shift duration; $K_{CM}^{(b)}$ – coefficient indicating the accepted number of work shifts; $t_{y_{ijs}}^{(b)}$ – permissible for the s-th year duration of production work on the j-th technological operation of the i-th product, in days; $t_{x_{ijs}}^{(b)}$ – duration for the s-th year of activities for maintenance and repair of equipment of the x-th standard size working on the j-th technological operation for the production of the i-th goods, in days.

Similar look, as shown in formulas (7) and (8) dependences for the definition and, the only difference is the replacement of the indices by the corresponding:

$$W_{ils}^{(e)} = \sum_{s=1}^{X} C_{ils} \cdot n_{ils}^{(e)} \cdot T_{CM} \cdot K_{CM}^{(e)} \cdot \left[ t_{ils}^{(e)} - t_{x_{ils}}^{(e)} \right], \quad i = \overline{1,I}, \quad l = \overline{1,L}, \quad s = \overline{1,S};$$

$$W_{ijs}^{(D)} = \sum_{s=1}^{X} C_{ijs}^{(D)} \cdot n_{ijs}^{(D)} \cdot T_{CM} \cdot K_{CM}^{(D)} \cdot \left[ t_{ijs}^{(D)} - t_{x_{ijs}}^{(D)} \right], \quad i = \overline{1,I}, \quad j = \overline{1,J}, \quad s = \overline{1,S};$$

In order to be able to optimize the quantity of pieces of equipment for each j-th process step in the production of i-type products in the s-th year in the model introduced by the formula (9):

$$\sum_{s=1}^{X} \left[ x_{ijs}^{(b)} \cdot n_{ijs}^{(b)} \right] \cdot T_{CM} \cdot K_{CM}^{(b)} \geq \frac{V_{ijs}^{(b)}}{T_{ijs}^{(b)} - \sum_{s=1}^{X} n_{ijs}^{(b)}}, \quad i = \overline{1,I}, \quad j = \overline{1,J}; \quad s = \overline{1,S},$$

where $x_{ijs}^{(b)}$ – average hourly operational productivity of the equipment of the x-th standard size performing j-th technological operation at production of i-th type of goods.

Similar dependencies form the restrictions of being in the process of optimizing all $n_{x_{ils}}^{(e)}$ and $n_{ijs}^{(D)}$. At the same time, based on the statistical data on the organization, the model introduces a dependence (10), which establishes a functional relationship between the annual costs of the j-th technological operation with warranty support of the i-th type of goods in the s-th year and the estimated annual production volumes for the j-th technological operation of the i-th goods for the warranty period, as well as additional costs for the j-th technological operation in the production of the i-th type of goods for the warranty period, providing a reduction in warranty claims in the future:

$$V_{ijs}^{(e)} = \sum_{x=1}^{X} \left[ x_{ijs}^{(b)} \cdot n_{ijs}^{(b)} \right] \cdot T_{CM} \cdot K_{CM}^{(b)} \geq \frac{V_{ijs}^{(b)}}{T_{ijs}^{(b)} - \sum_{s=1}^{X} n_{ijs}^{(b)}}, \quad i = \overline{1,I}, \quad j = \overline{1,J}; \quad s = \overline{1,S}.$$

Boundary conditions (11) establish that the quantity of units of the equipment distributed on operations is nonnegative and integer:

$$n_{x_{ijs}}^{(e)}, n_{ijs}^{(D)} \geq 0, \quad n_{x_{ijs}}^{(e)} = \text{int}(n_{x_{ijs}}^{(e)}), \quad n_{ijs}^{(D)} = \text{int}(n_{ijs}^{(D)}), \quad s = \overline{1,S}, \quad x = \overline{1,X}, \quad i = \overline{1,I}, \quad j = \overline{1,J}, \quad l = \overline{1,L}. \quad (11)$$

The cost of materials and components for the j-th technological operation in the production of the i-th type of goods in the s-th year can be described in the model by the expression (12):

$$M_{ijs}^{(b)} = K_{F_{ijs}}^{(b)} \cdot Z_{M_{ij}}^{(b)} + \frac{Z_{P} + A_{R}}{S_{O} \cdot T_{M}} \cdot S_{F_{ijs}}^{(b)} + T_{F_{ijs}}^{(b)} + Z_{T_{R_{ijs}}}^{(b)} + K_{U_{ijs}}^{(b)} \cdot Z_{U_{ij}}^{(b)} \cdot O_{y_{ij}}^{(b)}, \quad i = \overline{1,I}, \quad j = \overline{1,J}; \quad s = \overline{1,S}. \quad (12)$$
где \( K_{ij}^{(b)}, Z_{ij}^{(b)}, S_{ij}^{(b)}, T_{ij}^{(b)}, Z_{ij}^{(b)}, K_{ij}^{(b)}, Z_{ij}^{(b)}, O_{ij}^{(b)} \) – для \( j \)-того технологического этапа в производстве \( i \)-того типа товаров в \( s \)-м году, соответственно, количество необходимых материалов и компонентов, их фактическая стоимость, занимаемое складское пространство материалами и компонентами, среднее количество времени для хранения материалов и компонентов, транспортные расходы на их доставку, количество устаревших товаров и материалов, которые должны быть утилизированы, например, при реализации программ замены с дополнительной стоимостью за устаревшие товары или при производстве нового продукта включают утилизацию предшествующего продукта, утилизационные затраты за единицу устаревшего материала, булевская переменная, которая указывает на то, что материал должен быть утилизирован.

\( P_Z \) – месячные затраты на использование складских площадей и помещений.

\( R_A \) – месячная заработная плата курьеров.

\( O_S \) – общее складское пространство.

\( M_T \) – количество рабочих часов в месяц.

4. *The discussion of the results*

The mathematical model described above allows for the best-in-terms of organizational efficiency for the period of the business project, as a result of optimization, to assign an integer number of different resources, such as equipment, as well as where it is required a non-integer assignment of resources. The mathematical apparatus allows in a single system to consider the production of all goods from the nomenclature of the organization for each technological operation for each year of the project together with the warranty support of the goods. The system of limitations of the model, in addition to taking into account all items of possible costs, assumes the possibility of taking into account the impact of additional efforts aimed at ensuring quality in production on the subsequent volumes of work associated with the warranty.

The created model involves the use of established technological sequences. In cases of multiple choice of technological or organizational solutions at a large number of stages of the production process, the problem under consideration is significantly complicated and can be considered as a traveling salesman problem with several details of the vertices [15]. However, the developed model has extensive adaptation capabilities to the needs of organizations. The proposed approach may become an additional potential opportunity when using artificial neural networks for production planning and management, developed to date, for example, as described in the source [16].

5. *Conclusions*

As a result of reading this article, the reader got acquainted with the economic and mathematical model that allows to optimize the distribution and use of production resources of the organization for the production and warranty support of goods. The objective function, constraints and boundary conditions fully allow to solve complex optimization problems of the organization in accordance with the topic of the article. An integrated approach to the allocation of resources and evaluation of the economic efficiency of the organization, while maintaining sufficient simplicity of the model allows it to be used in solving practical problems of organizations.

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