Statistical properties of driven Magnetohydrodynamic turbulence in three dimensions: Novel universality

Abhik Basu (∗)
Abteilung Theorie, Hahn-Meitner-Institut, Glienicker Strasse 100, D-14109 Berlin, Germany, and Poornaprajna Institute of Scientific Research, Bangalore, India.

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Abstract. – We analyse the universal properties of nonequilibrium steady states of driven Magnetohydrodynamic (MHD) turbulence in three dimensions (3d). We elucidate the dependence of various phenomenologically important dimensionless constants on the symmetries of the two-point correlation functions. We, for the first time, also suggest the intriguing possibility of multiscaling universality class varying continuously with certain dimensionless parameters. The experimental and theoretical implications of our results are discussed.

In the vicinity of a critical point, equilibrium systems show universal scaling properties for thermodynamic functions and correlation functions: These are characterised by universal scaling exponents that depend on the spatial dimension $d$ and the symmetry of the order parameter (e.g., Ising, XY etc.) [1], but not on the parameters that specify the details of the Hamiltonian. A notable exception is the class of two-dimensional models, such as the XY model in $d = 2$, in which continuously varying scaling exponents are found [2]. Dynamic scaling exponents, that characterise the behaviour of time-dependent correlation functions also show universality [3]. A different situation arises in driven, dissipative, nonequilibrium systems with nonequilibrium statistical steady states (NESS). We show here that driven homogeneous and isotropic MHD turbulence in three dimensions (3d) is a good natural candidate for a system with an NESS whose universal properties vary continuously with the degree of cross-correlations between the velocity and magnetic fields. Our results illustrate the sensitivities of the statistical properties of nonequilibrium steady states on the parameters of the model, a situation which does not arise in equilibrium systems.

Turbulence in plasmas is described by the equations of MHD [4] in 3d for the coupled evolution of the velocity field $\mathbf{v}$ and the magnetic field $\mathbf{b}$. Solar-wind data [5] and numerical simulations of the 3dMHD equations as well as shell models [6] show multiscaling corrections to the simple Kolmogorov (K41) scaling [7] for the structure functions of $\mathbf{v}$ and $\mathbf{b}$: $S^a_{\mathbf{v}}(r) \equiv \langle |a_i(x + r) - a_i(x)|^a \rangle \sim r^{\zeta a}$, $a = v, b$ for $r$ in the inertial range between the forcing scale $L$ and the dissipation scale $\eta_d$ (i.e., $L \gg r \gg \eta_d$). In observations and experiments magnetic Prandtl number $P_m (=\text{the ratio of the magnetic to kinetic viscosity})$ and $\epsilon = \frac{E_m}{E_v}$ (the ratio

(*) email:basu@hmi.de
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of the magnetic- to the kinetic- energy) are known to have a wide range of values: \(e \ll 1 \Rightarrow \) kinetic regime of 3dMHD (as on the solar surface), \(e \sim O(1) \Rightarrow \) equipartition regime (e.g., solar wind in its rest frame) and \(e \gg 1 \Rightarrow \) magnetic regime (e.g., fusion plasmas) [8]. Similarly \(P_m\) can be high (> 1), e.g., in galactic and proto galactic plasmas [9]. Low values of \(P_m\) are found in liquid metals [10]. In numerical simulations, the structure functions of \(v\) and \(b\) in 3d exhibit multiscaling with exponents different from their fluid analogues [6, 11]. In particular Ref. [8] elucidated different multiscaling exponents for \(v\) and \(b\) (i.e., \(\zeta_v \neq \zeta_p\)).

In this letter we develop a one-loop, self-consistent theory for the randomly forced 3dMHD equations and use it to calculate several interesting results concerning its NESS. We show, in particular, that (i) the variation of \(e\) and renormalised (effective) \(P_m\) are connected to each other (ii) \(P_m, e,\) Kolmogorov’s constants \(K_v\) and \(K_b\) (see below) vary with the degree of crosscorrelations between \(v\) and \(b\). We calculate the intermittency exponents (these in a log-normal model provide approximate estimates of the deviations from the K41 scaling) which also vary continuously with the degree of crosscorrelations between \(v\) and \(b\). This suggests an interesting possibility of continuously varying multiscaling universality classes for 3dMHD. We show that these variations are linked with the ratios of the different two-point correlation functions. Our observations on multiscaling of 3dMHD based on our self-consistent calculations on a log-normal model allows for different multiscaling exponents for the velocity and magnetic fields as reported in Ref. [6].

The 3dMHD equations consist of the Navier-Stokes equation for the velocity field \(v\) supplemented by the Lorentz force due to the magnetic field \(b\):

\[
\frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla p + \frac{(\nabla \times b) \times b}{4\pi\rho} + \nu_o \nabla^2 v + f
\]

with \(\nabla \cdot v = 0\) (incompressibility) and Ampère’s law for a conducting fluid

\[
\frac{\partial b}{\partial t} + v \cdot \nabla b = b \cdot \nabla v + \mu_o \nabla^2 b + g.
\]

In [11] and [2], \(\rho\) is the fluid density, \(p\) is the pressure, \(\mu_o\) is the “magnetic viscosity”, arising from the nonzero resistivity of the plasma, \(\nu_o\) is the kinematic viscosity and \(f\) and \(g\) are forcing functions. Under rescaling of space and time \(v(x, t)\) and \(b(x, t)\) scale as \(v_i(\ell x, \ell^2 t) \rightarrow \ell v_i(x, t), b_i(\ell x, \ell^2 t) \rightarrow \ell^2 b_i(x, t)\). Since \(v\) and \(b\) are polar and axial vectors respectively, crosscorrelation tensor \(\langle v_i(k, t) b_j(-k, 0) \rangle\) is odd and imaginary in wavevector \(k\) [12].

We employ a self-consistent mode coupling scheme (SCMC). The response and the correlation functions of the fields \(v\) and \(b\) are defined by \(v_i \equiv G_v f_i, b_i \equiv G_b g_i\), and correlators \(C_v^{ij} \equiv \langle v_i(k, t) v_j(-k, 0) \rangle, C_b^{ij} \equiv \langle b_i(k, t) b_j(-k, 0) \rangle\) with the crosscorrelator \(C_c^{ij} \equiv \langle v_i(k, t) b_j(-k, 0) \rangle\) in the scaling limit, in terms of the dynamic exponent \(z\) and the roughness exponents \(\chi_1\) and \(\chi_2\), \(G_v^{-1} = i\omega - \Sigma_v(k, \omega), G_b^{-1} = i\omega - \Sigma_b(k, \omega)\) where \(\Sigma_v = k^{-\chi_1} \eta_v(\omega/k^2)\), \(\Sigma_b = k^{-\chi_2} \eta_b(\omega/k^2)\) are the self-energies whereas the correlators are given by \(C_v^{ij} = P_{ij} k^{-d-2\chi_1} \eta_v(\omega/k^2), C_b^{ij} = P_{ij} k^{-d-2\chi_2} \eta_b(\omega/k^2)\). \(\eta_v(\omega/k^2), \eta_b(\omega/k^2), \eta(\omega/k^2)\) are scaling functions and \(P_{ij}\) is the transverse projection operator. In systems out of equilibrium, there is no particular relation between the noise variance and the dissipation coefficient, unlike their equilibrium counterparts where such a relation exists due to the Fluctuation-Dissipation-Theorem. It is also well-known that the statistical properties of the nonequilibrium systems depend strongly on the noise variances. We, here in particular, take the forcing terms \(f, g\) to be Gaussian noises with zero mean and covariances proportional to \(k^{-d}\) in \(d\)-dimension [13]. This ensures that, in absence of a mean magnetic field, the energy spectra are K41-like [5, 7]. In this model, the advective nonlinearities and the noise variances do not renormalize. This means under
simultaneous rescaling of space and time, the nonlinear vertices and the noise variances are affected only by their naive scaling dimensions. At the fixed point, these conditions leads to strong dynamical scaling, i.e., same dynamic exponent for both the fields and immediately yield \( z = 2/3 \), \( \chi_1 = \chi_2 = \chi = 1/3 \) corresponding to K41 scaling \cite{12,13}. Hence the ratios of various correlations are dimensionless numbers. We assume Lorentzian line shapes for the self-energies and the correlation functions. The fact that the values of the scaling exponents \( z \) and \( \chi \) are dimension independent is because the noise correlations change as a function of dimensionality in such a way as to render the exponents dimension independent.

We start with the zero-frequency forms for the self-energies (or the relaxation rates) \( \Sigma_{s}(k, \omega = 0) = \nu k^d \), \( \Sigma_{b}(k, \omega = 0) = \mu k^d \), and the correlations \( C_{ij}^{s}(k, \omega = 0) = \frac{2 \nu}{\mu^2} P_{ij} k^{-2\chi - d - 1} \), \( C_{ij}^{b}(k, \omega = 0) = \frac{2 \nu}{\mu^2} P_{ij} k^{-2\chi - d - 1} \), in \( d \)-dimension. The crosscorrelation tensor \( C_{ij}^{\nu} \) in general has both symmetric \( C_{ij}^{s} \) and antisymmetric \( C_{ij}^{a} \) parts: \( C_{ij}^{s} = C_{ij}^{s} + C_{ij}^{a} \), \( C_{ij}^{b} = C_{ij}^{b} \) for \( k, \omega = 0 \) = \( \frac{2 \nu D_{ij}^{\nu}}{\mu} k^{-2\chi - d - 4} \), this form is for \( 3d \) only. The mean crosshelicity \( \sum_k \langle \mathbf{v}(k) \cdot \mathbf{b}(-k) \rangle \) and mean electromotive force \( \sum_k \langle \mathbf{v}(k) \times \mathbf{b}(-k) \rangle \) are proportional to \( \hat{D} \) and \( \bar{D} \) respectively. Here \( \epsilon_{ijp} \) is the totally antisymmetric tensor in \( 3d \). Previous renormalisation group/self-consistent approach to \( 3d \) MHD did not take any crosscorrelation function \cite{14} fully into consideration. The noises in stochastically driven models, such as Eqs. \( \mathbf{1} \) and \( \mathbf{2} \), represent coarse-grained effects of the degrees of freedom whose time-scales of decay are faster than the coarse-grained variables \( (\mathbf{v} \text{ and } \mathbf{b}) \) used. Since both the noises in Eqs. \( \mathbf{1} \) and \( \mathbf{2} \) have same microscopic origin there is no a priori reason for their crosscorrelations to vanish always. In SCMC approach vertex corrections are neglected. Lack of vertex renormalisations in the zero wavevector limit in \( 3d \) MHD allows SCMC to yield exact relations between the scaling exponents \( z \) and \( \chi \), as in the the noisy Burgers/Kardar-Parisi-Zhang equation \cite{15}. In the context of the noisy Burgers equation in 1+1 dimensions, Frey et al have shown, by using nonrenormalisation of the advective nonlinearities and a second-order perturbation theory, that the effects of the vertex corrections on the correlation functions are very small \cite{16}. Similar conclusions for \( 3d \) MHD will presumably follow from the vertex renormalisations in \( 3d \) MHD, though a rigorous calculation is still lacking.

One may note that the symmetries of the symmetric and the antisymmetric parts of the crosscorrelations are different under the exchange of the cartesian indices. Hence, the presence of one does not lead to the generation of the other by non-linear interactions. Thus it is possible to treat them separately without any loss of generality. It is straight forward to extend the various expressions derived in the paper for the case when both the symmetric and the anti-symmetric parts are present.

\textit{Symmetric Cross-correlations.-} We obtain the one-loop self-consistent integral equations for the self-energies and correlation functions. The one-loop diagrammatic corrections to the crosscorrelation function vanish as it is odd in momentum (see, e.g., \cite{17}). We first consider the case when \( \hat{D} = 0 \), i.e., no antisymmetric part of the crosscorrelations. By matching at \( \omega = 0 \), for the self-energies we obtain (by using \( \chi = 1/3 \), \( z = 2/3 \)) in any dimension \( d \)

\begin{equation}
\nu = \frac{3D_1}{4\nu^2} \frac{S_d}{(2\pi)^d} \frac{d - 1}{d + 2} + \frac{3D_2}{4\mu^2} \frac{S_d}{(2\pi)^d} \frac{d^2 + d - 4}{d(d + 2)},
\end{equation}

\begin{equation}
\mu = \frac{3D_1}{2\nu(\nu + \mu)} \frac{S_d}{(2\pi)^d} (1 - \frac{1}{d}) + \frac{3D_2}{2\mu(\nu + \mu)} \frac{S_d}{(2\pi)^d} (1 - \frac{3}{d}).
\end{equation}
Similarly, by demanding consistency in the amplitudes of the correlations we get

\[
\frac{D_1}{D_2} = \frac{D_1^2 + D_2^2}{\mu^2 - \frac{4D_1^2}{\nu(\nu + \mu)}} \left[ \frac{2}{3} \frac{2}{d(d+2)} \right].
\] (5)

In terms of the renormalised magnetic Prandtl number \( P_m = \mu/\nu \) and \( \Gamma = D_2/D_1 \) we can write Eqs. (3), (4) and (5) as

\[
P_m^{-1} = \frac{\frac{d-1}{2d+2} + \frac{1}{2} \frac{d^2 + d - 4}{d(d+2)}}{1 + P_m(1 + P_m)} \left[ \frac{2}{3} \frac{2}{d(d+2)} \right],
\] (6)

and \( \Gamma = \frac{\int_{\infty}^\infty \frac{1}{P_m^{(1 + P_m)^{1/2}}} \left[ \frac{2}{3} \frac{2}{d(d+2)} \right]}{\int_{\infty}^\infty \frac{1}{P_m^{(1 + P_m)^{1/2}}} \left[ \frac{2}{3} \frac{2}{d(d+2)} \right]} \). (7)

where \( \tilde{\beta} = (\frac{D_1}{D_2})^2 \). Equations (6) and (7) explicitly demonstrate that \( P_m \) and the magnetic-to kinetic-energy ratio \( e = \frac{1}{P_m^2} \) depend on each other. When \( \tilde{\beta} = 0 \), i.e., zero crosshelicity, Eqs. (6) and (7) yield \( P_m = 0.67 \) and \( \Gamma = 0.63 \) and hence energy-ratio \( e = \Gamma/P_m \approx 0.94 \) in 3d. Thus for zero crosshelicity \( P_m \) and \( e \) are fixed numbers. But with finite crosshelicity, i.e., a finite \( \tilde{\beta} \) in Eqs. (6) and (7), \( P_m \) and \( \Gamma \) and hence \( e \) depend on \( \tilde{\beta} \) in any dimension \( d \). For \( \tilde{\beta} > 0 \) few representative values of \( P_m \) and \( \Gamma \) in 3d are given in Table 1.

Anti-Symmetric Cross-Correlations. For the effects of the antisymmetric part (under the exchange of \( i \) and \( j \), \( i, j \) are Cartesian indices) of the crosscorrelation function in 3d on the long wavelength properties we again use self-consistent methods. Equations (3) and (4) remain unaltered. However, Eq. (5) is changed to

\[
\frac{D_1}{D_2} = \frac{D_1^2 + D_2^2}{\mu^2 - \frac{4D_1^2}{\nu(\nu + \mu)}} \left[ \frac{2}{3} \frac{2}{d(d+2)} \right].
\] (8)

Again by defining \( \tilde{\beta} = (\frac{D_1}{D_2})^2 \), we obtain self-consistently

\[
\Gamma = \frac{1 + \frac{\Gamma^2}{P_m(1 + P_m)}}{\frac{16\Gamma^2}{3(1 + P_m)^{1/2}}} \left[ \frac{2}{3} \frac{2}{d(d+2)} \right].
\] (9)

Thus Eqs. (6) and (7) together provide self-consistent relations between \( P_m \) and \( \Gamma \) and hence between \( P_m \) and \( \epsilon \) when there is a finite antisymmetric crosscorrelation. A few representative values of \( P_m \) and \( \Gamma \) for \( \tilde{\beta} > 0 \) are given in Table 1.

Kolmogorov’s constants. According to the Kolmogorov’s hypothesis for fluid turbulence [7] in the inertial range energy spectrum \( E(k) = K_0 k^{-5/3} \), where \( K_0 \), a universal constant, is the Kolmogorov’s constant and \( \epsilon \) is the energy dissipation rate per unit mass. Various calculations, based on different techniques by different groups [15, 19, 13, 20] show that \( K_0 \sim 1.5 \) in three dimensions. For MHD, by using Novikov’s theorem [21], we connect the dissipation of the total energy with the sum of the velocity and magnetic field correlation-amplitudes. We can then relate the dissipation of the reduced energy \( E_R = E_v - E_b \) with the correlation-amplitudes. Thus, in the lowest order of the perturbation theory we calculate the coefficients of the velocity
and magnetic field correlations in terms of dissipation rates: \( E_v(k) = K_v \epsilon_v^{2/3} k^{-5/3} \); \( E_b(k) = K_b \epsilon_b^{2/3} k^{-5/3} \). \( K_v \) and \( K_b \) are the Kolmogorov’s constants for MHD, and \( \epsilon_v \) and \( \epsilon_b \) are the mean dissipation rates of the kinetic energy and magnetic energy respectively. In absence of a mean magnetic field energy spectra scale as \( k^{-5/3} \) in the inertial range [6]. In Ref. 22 Kolmogorov’s constants for MHD turbulence (for the Elsässer variables) have been calculated incorrectly for various energy-ratio \( \epsilon \) due to the absence of crosscorrelation functions of appropriate structure. We work out the Kolmogorov’s constants with finite crosscorrelation functions. From Eq. (6) in dimension \( d = 3 \), we have \( \nu = \left[ \frac{0.6 D_1 \rho_s}{(2\pi)^{3/2}} \right]^{3/2} (1 + 3 \frac{\Gamma}{\pi m})^{3/4} \). In our notations, \( \langle u_i (k, \omega) u_j (-k, -\omega) \rangle = \frac{2 D_1 \rho_s k^{-3}}{\omega^2 + \nu^2 k^2} \) which in 3d gives (in the inertial range) kinetic energy spectrum \( E_v(k) = 1.186 [2 D_1 (1 + 0.62 \{ 1 + \frac{\Gamma^2}{\pi^2 m} - \frac{4 a}{\pi m (1 + \pi m)} \})^{3/2} \frac{S_m}{(2\pi)^3} (1 + 3 \frac{\Gamma}{\pi m})^{3/4}] \). To connect dissipation rate \( \epsilon_v \) with noise correlation amplitude \( D_1 \) we use Novikov’s theorem [21] which gives \( 2 D_1 \frac{S_m}{(2\pi)^3} = \epsilon_v \). Substituting this in \( E_v(k) \) above, we obtain Kolmogorov’s constant

\[
K_v = 1.186 \left[ 1 + 0.62 \{ 1 + \frac{\Gamma^2}{\pi^2 m} - \frac{4 a}{\pi m (1 + \pi m)} \} \right]^{2/3},
\]

where \( a = \frac{\Gamma}{\pi m} \), \( -\frac{\Gamma}{\pi m} \) for finite symmetric and antisymmetric crosscorrelations respectively. A similar calculation yields \( K_b = K_v \). Our results show, unsurprisingly, that \( K_v \) and \( K_b \) are not fixed numbers, rather they depend upon other dimensionless numbers like magnetic Prandtl number \( P_m \) and energy-ratio \( \epsilon \). For \( \Gamma = 0 \), i.e., for no magnetic fields we get the result \( K_v = 1.63 \) for pure fluid turbulence which is well-within the range of accepted values [13, 23].

Possibilities of variable multifractality — Experiments and numerical simulations [6, 11] find nonlinear multiscaling corrections to the K41 prediction of \( \zeta^a = p/3 \) for the structure functions in the inertial range. Until the date, no controlled perturbative calculation for \( \zeta^a \) is available. To account for multiscaling in fluid turbulence, however, Obukhov [24] and Kolmogorov [25] assumed a log-normal distribution for dissipation \( \epsilon \) to arrive at \( S^\epsilon(r) = \langle |\Delta u|^p \rangle = C_p \tau^{p/3} \rho^{p/3} \left( \frac{L}{r} \right)^S (p-3) \), where \( \tau \) is the mean value of \( \epsilon \) and \( \langle \epsilon(x+r) \epsilon(x) \rangle \propto (\langle \Delta u \rangle^6 / r^6) \sim (L/r)^{5\delta} \). A standard calculation on the randomly stirred model yields intermittency exponent \( \delta = 0.2 \) [23] whereas the best possible estimates from experiments is 0.23 [26]. In MHD dissipations of kinetic- \( \left( \epsilon_v \right) \) and magnetic- \( \left( \epsilon_b \right) \) energies fluctuate in space and time. Consequently we define two intermittency exponents \( \delta_v, \delta_b \) for the kinetic \( \left[ \epsilon_v = \frac{1}{2} \nu \sum_{\alpha \beta} \left( \frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} \right)^2 \right] \), and magnetic energy dissipations \( \left[ \epsilon_b = \frac{1}{2} \nu \sum_{\alpha \beta} \left( \frac{\partial b_\alpha}{\partial x_\beta} + \frac{\partial b_\beta}{\partial x_\alpha} \right)^2 \right] \) to explain the deviations from the K41 scaling. We calculate the exponents below by using the Eqs. 11 and 2 following closely Ref. 20. We work with the self-consistent forms for the self-energies and correlation functions given above along with the consistency relations for the amplitude-ratios \( \Gamma, \beta, \tilde{\beta} \) and \( P_m \).

Following Ref. 20, we find the dissipation correlation functions in 3d to be \( \langle \epsilon_v (x+r) \epsilon_v (x) \rangle \approx 12.4 c_\alpha c_\beta K_v^2 \ln \frac{L}{r}, \langle \epsilon_b (x+r) \epsilon_b (x) \rangle \approx 12.4 c_\alpha c_\beta K_v^2 \ln \frac{L}{r} \). \( \alpha_v \) is defined by the relation \( \nu = \alpha_v \epsilon_v^{1/3} \). By using Eqs. 3, 4, 5, 7 and 9 we find, in 3d,

\[
\alpha_v = 0.4 \left[ 1.62 + 0.62 \left( \frac{\Gamma^2}{P_m^3} - \frac{4 a}{P_m (1 + P_m)} \right) \right]^{1/3} \left[ 1 + 3 \frac{\Gamma}{P_m^3} \right]^{1/3},
\]

where \( a = \frac{\Gamma}{\pi m} \) and \( -\frac{\Gamma}{\pi m} \) for finite symmetric and antisymmetric crosscorrelations respectively. For the pure fluid case \( \alpha_v \approx 0.5 \) [13] and is universal. For MHD, however we see that \( \alpha_v \)
varies with $\tilde{\beta}$ and $\hat{\beta}$, similar to $K_v$. We then obtain fluid intermittency exponent

$$\delta_v = 0.2 \left[ 1 + 0.4 \left( \frac{\Gamma^2}{P_m^{3/2}} - \frac{4a}{P_m(1+P_m)} \right) \right]^{4/3}, \tag{12}$$

where, $a$ is the same as before. A similar calculation obtains magnetic intermittency exponent $\delta_b = \Gamma^2 \delta_v$. As expected, both $\delta_v$ and $\delta_b$ vary with $P_m$ and $\Gamma$ (or $\epsilon$). In general, the intermittency corrections to the simple K41 scaling of the $v$ and $b$ structure functions are unequal, as has been seen in the direct numerical simulations (DNS) and shell-model studies [6]. The presence of multiple intermittency exponents makes it difficult to directly extend the fluid log-normal intermittency model to 3dMHD. However, the continuous variations of the intermittency exponents with $\tilde{\beta}$ and $\hat{\beta}$ strongly suggest continuously varying multiscaling universality classes for 3dMHD.

In our calculations, $\tilde{\beta}$ and $\hat{\beta}$ parametrise the internal symmetries of the system. They represent the relative strengths of the symmetric and anti-symmetric parts of the cross correlation function. Therefore, $\tilde{\beta}$ and $\hat{\beta}$ characterise the symmetries of the two-point correlation functions under inversion of the parity and exchange of the cartesian indices. Results from our log-normal model-type calculations open up an intriguing possibility of multiscaling properties varying continuously with symmetries parametrised by the dimensionless numbers described above. We believe that our results (despite the limitations of mode coupling theories - see, e.g., [13, 27]) will provide a valuable insight to the understanding of the difference in the multiscaling properties of the velocity and magnetic fields [6] and may also explain the somewhat different numerical estimates of the multiscaling exponents by different groups [6,11]. We believe our results will stimulate further numerical as well as experimental studies for the measurements of the dimensionless parameters introduced above. To check our results we suggest MHD experiments on liquid sodium systems with active grids and random passage of current. Various $\tilde{\beta}$ and $\hat{\beta}$ can be achieved in experiments/simulations by controlling the appropriate Grashof numbers [28], constructed out of the various noise correlators in stochastically driven MHD and by measuring the cross helicity, the electromotive force and the kinetic energy in the steady state. Renormalised $P_m$ can be calculated by measuring time-dependent correlations. In closing we note that many natural systems, e.g., solar wind, plasmas in tokamac have strong mean magnetic fields making them anisotropic. Such effects can also be included in our scheme of calculations by appropriately modifying the form of the self-energies.

From a broader point of view, our results unveil qualitatively new properties in the steady-states of nonequilibrium driven, difusive systems. In contrast to the more well-known equilibrium systems, we illustrate the richness of nonequilibrium phenomenologies by using 3dMHD as an example. Recently, Drossel et al [29], in a set of coupled Langevin equations describing the interplay between phase ordering dynamics in the bulk and roughening dynamics of the interface of binary films, find a similar continuous variation of the dynamical exponent with the coupling strength of the bulk and surface fields. Recently, continuously varying universal properties in the context of a simple coupled Burgers-like model has been discussed in Ref. [30]. Here, for the first time, we illustrate a natural realisation of continuously varying multiscaling universality class in 3dMHD. We believe our studies will provide further stimuli to the studies of these new aspects of driven nonequilibrium systems.

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Table I – Representative values of $P_m$, $\Gamma$, $\epsilon$, $K_v$, $\alpha_v$ and $\delta_v$ as functions of $\tilde{\beta}$ and $\hat{\beta}$.

| $P_m$ | $\Gamma$ | $\epsilon$ | $\tilde{\beta}$ | $\hat{\beta}$ | $K_v$ | $\alpha_v$ | $\delta_v$ |
|-------|-----------|-------------|-----------------|---------------|-------|----------|----------|
| 0     | 0         | 0           | 0               | 0             | 1.6   | 0.47     | 0.2      |
| 1     | 1.76      | 1.76        | 3               | 0             | 0.6   | 0.6      | 0.04     |
| 0.1   | 0.06      | 0.6         | 0               | 0.15          | 3.5   | 1.6      | 10       |
| 0.67  | 0.63      | 0.04        | 0.0             | 1.52          | 0.76  | 0.76     | 0.6      |

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