ON THE SUPER FIELD REALIZATION OF SUPER CASIMIR $\mathcal{W}_{A_n}$ ALGEBRAS

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We give an explicit quantum super field construction of the $\mathcal{N}=2$ super- Casimir $\mathcal{W}_{A_n}$ algebras, which is obtained from supersymmetric Miura transformation for the Lie superalgebra $\mathcal{A}_{n,n-1}$. And also we give an extension of this algebras including a super vertex operator which depends on simple root system of $\mathcal{A}_{n,n-1}$.

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1. Introduction

Superconformal symmetry has played an important role in the developments of the many branches of theoretical physics and mathematics. Extended superconformal symmetry is an extension of superconformal symmetry containing higher conformal spin generators in addition to the usual superconformal algebra generators. Many superconformal algebras are known (for review see Ref.1 and references therein). It is well known that a usual and a fermionic \((N=1)\) Casimir algebra corresponding to a simple Lie algebra \(L_n\) can be denoted by \(\mathcal{W}L_n\). Analogous, a \((N=2)\) super Casimir algebra associated with the Lie superalgebra \(A_{n,n-1}\) can be also shown by super-\(\mathcal{W}L_n\). Therefore super-Casimir \(\mathcal{W}A_n\) algebras based on the Lie superalgebras \(A_{n,n-1}\). With the above identification, one says that \(\mathcal{W}A_n\) algebra is the simplest \((N=2)\) superconformal algebra. This superconformal algebra coincides with Neveu-Schwarz algebra \((N=1)\) super Virasoro algebra. We must emphasize here that the \((N=1)\) super-Casimir algebra is a subalgebra of the \((N=2)\) super-Casimir algebra. L.J.Romans have proven that super-\(\mathcal{W}_3\) algebras \(^{11}\) are associative for all values of central charge \(c\). Some realizations of this algebra in terms of the \((N=2)\) superfields has been constructed with \((n=2)\) two scalar fields by using the supersymmetric Miura transformation \(^{10,15,16}\).

In this letter, we study a super Casimir algebra for a Lie superalgebras \(L_n = A_{n,n-1}\), which are called super-Casimir \(A_n\) algebras, for \(n = 3\). We must emphasize here that all pure conformal fields depend on \(n\), so this is the first pure spin field definitions for this algebra in the literature. We reserve a notation of our spin content as module-1: \(\{1, +\frac{1}{2}, -\frac{1}{2}, 2\}\) and module-2: \(\{2, +\frac{1}{2}, -\frac{1}{2}, 3\}\).

Let us now give outline of this paper. In Sec. 2, we give a basis for super-Casimir \(A_n\) algebra by using the well-known supersymmetric Miura transformation with \(2n\) free massless bosonic fields and real fermion fields\(^{12-16}\). Then, we constructed a free field realization of the super-Casimir \(A_n\) algebra by calculating all primary fields explicitly in general \(n\), but we can not give all nontrivial OPEs among primary fields, since all OPEs exist in the literature with some normalizations. In Sec.3, we constructed, in the component language, a super-vertex operator extension of the super-Casimir \(A_n\) algebras by calculating explicitly nontrivial OPEs between primary fields and super-vertex operators which are of two forms. The calculations of OPEs have been done with the help of Mathematica Package OPEDefs.m of Thielemans \(^{17}\) under the Mathematica\(^{TM}\) in Ref.18.

2. Super- Miura Basis For Super Casimir \(\mathcal{W}A_n\) Algebras In The Component Formalism

In this section, we begin with the review of the supersymmetric Miura transformation based on a Lie superalgebra \(A_{n,n-1}\). We use odd simple root system \(\{\alpha_1, \cdots, \alpha_{2n}\}\) satisfying Cartan matrix \(C_{ij} = \alpha_i \cdot \alpha_j = (-1)^{i+j} \delta_{i+j}\) and \(\{\lambda_1, \cdots, \lambda_{2n}\}\) are the fundamental weights of \(A_{n,n-1}\), satisfying \(\alpha_i \cdot \lambda_j = \delta_{ij}\). We show \((N=1)\) super coordinate \(Z = (z, \theta)\) and a super derivative \(D = \frac{\partial}{\partial z} + \theta \frac{\partial}{\partial \theta}\). Then we introduce a primary basis for the super-Casimir \(\mathcal{W}A_n\) algebras from \(2n\) free massless bosonic fields \(\varphi_i(z) (i = 1, \cdots, 2n)\) and real fermion fields \(\psi_i(z) (i = 1, \cdots, 2n)\) realization point of view, which are single-valued functions on the complex plane and its mode expansion are given by

\[
i \partial \varphi(z) = \sum_{n \in Z} a_n z^{-n-1}, \quad \psi(z) = \sum_{n \in Z} \psi_n z^{-n-\frac{1}{2}}.
\]

respectively. A scalar superfield can be defined by \(\Phi(Z) = \varphi(z) + i \theta \psi(z)\). Canonical quantization gives the commutator and anti-commutator relations

\[
[a_m, a_n] = m \delta_{m+n,0}, \quad [\psi_m, \psi_n] = \delta_{mn},
\]

and these relations are equivalent to the contractions, as \(z_{12} = z_1 - z_2\)

\[
\partial \varphi(z_1) \partial \varphi(z_2) = \frac{1}{z_{12}}, \quad \psi(z_1) \psi(z_2) = \frac{1}{z_{12}}
\]

respectively.
Let us consider the supersymmetric Miura transformation $^{12-16}$, which is defined as

$$\mathcal{R}_n(\mathcal{Z}) = \mathcal{D}^{2n+1} + \sum_{j=2}^{2n+1} W_j(\mathcal{Z}) (\alpha_0 \mathcal{D})^{2n+1-i}$$

$$= \prod_{j=1}^{2n+1} (\alpha_0 \mathcal{D} - \Theta_j(\mathcal{Z})) ;,$$

(2.4)

where $\Theta_j(\mathcal{Z}) = (-1)^{j-1} (\lambda_i - \lambda_{i-1}) \cdot \mathcal{D} \Phi(\mathcal{Z})$ ($\lambda_0 = \lambda_{2n+1} = 0$) the symbol : $*$ : shows the normal ordering, and $\alpha_0$ is a free parameter.

The super- Casimir $\mathcal{W} \mathcal{A}_n$ algebra generated by a set of chiral currents $\mathcal{W}_\frac{i}{2}(\mathcal{Z})$, of conformal dimension $\frac{i}{2}$ ($i = 2, \cdots, 2n + 1$) We show first few examples of currents, under $\mathcal{H}_i = \lambda_i \cdot \mathcal{D} \Phi(\mathcal{Z})$ definition :

$$\mathcal{W}_i(\mathcal{Z}) = - \sum_{i=1}^{2n-1} (\mathcal{H}_i, \mathcal{H}_{i+1})(\mathcal{Z}) - a_0 \sum_{i=1}^{2n} (-1)^i (\mathcal{H}_i)(\mathcal{Z})$$

(2.5)

$$\mathcal{W}_\frac{i}{2}(\mathcal{Z}) = - \sum_{i=1}^{n} (\mathcal{D} \mathcal{H}_{2i-1}, \mathcal{H}_{2i})(\mathcal{Z}) + \sum_{i=1}^{n-1} (\mathcal{H}_{2i}, \mathcal{D} \mathcal{H}_{2i+1})(\mathcal{Z}) - a_0 \sum_{i=1}^{n} (\mathcal{D}^2 \mathcal{H}_i)(\mathcal{Z})$$

(2.6)

In the component formalism all bosonic and fermionic currents are expressed as in module-(k-1) :

$$\mathcal{W}_{k-i}(\mathcal{Z}) = \mathcal{J}_{k-i}(z) + i \theta \left[ \mathcal{G}_{k-i}^+(z) + \mathcal{G}_{k-i}^-(z) \right]$$

$$\mathcal{W}_{k-\frac{i}{2}}(\mathcal{Z}) = \alpha_0 \left[ i \mathcal{G}_{k-\frac{i}{2}}^+(z) + \theta \mathcal{T}_{k}(z) \right]$$

(2.7)

For the module-2

$$\mathcal{J}_i(z) = \sum_{i=1}^{2n-1} (\psi_i \psi_{i+1})(z) - a_0 \sum_{i=1}^{2n} (-1)^i \psi_i(z)$$

$$\mathcal{G}_{\frac{i}{2}}^-(z) = \sum_{i=1}^{2n-2} (1 - t_i)(\psi_i h_{i+1})(z) - \sum_{i=1}^{2n} t_i(h_i \psi_{i+1})(z) - a_0 \sum_{i=1}^{2n} (-1)^i (1 - t_i) \psi_i(z)$$

$$\mathcal{G}_{\frac{i}{2}}^+(z) = - \sum_{i=1}^{2n-2} (1 - t_i)(h_i \psi_{i+1})(z) + \sum_{i=1}^{2n} t_i(\psi_i h_{i+1})(z) + a_0 \sum_{i=1}^{2n} (-1)^i t_i \psi_i(z)$$

and

$$\mathcal{T}_2(z) = \sum_{i=1}^{2n-1} (-1)^i (h_i h_{i+1})(z) + \sum_{i=1}^{2n-1} (1 - t_i)(h_i h_{i+1})(z) + \sum_{i=1}^{2n-1} t_i(h_i h_{i+1})(z) - a_0 \sum_{i=1}^{2n} (-1)^i (1 - t_i) h_i(z)$$

(2.8)

where $h_i(z) = \lambda_i \partial \varphi(z)$, $t_i = \begin{cases} 1 & \text{odd } i \\ 0 & \text{even } i \end{cases}$ and also module-3 components including higher order $\mathcal{J}_{3}(z)$, $\mathcal{G}_{\frac{i}{2}}^-(z)$, $\mathcal{G}_{\frac{i}{2}}^+(z)$ and $\mathcal{T}_3(z)$ currents are not given here since their formal complexity $h_i(z)$’s and $\psi_i(z)$’s satisfy $h_i(z_1) h_j(z_2) = [C^{-1}]_{i j} z_{12}^{[i-1]_{j} - 1}$ and $\psi_i(z_1) \psi_j(z_2) = [C^{-1}]_{i j} z_{12}^{[i-1]_{j} - 1}$ OPE’s respectively, under (2.3) contractions , where $[C^{-1}]_{i j}$ is inverse matrix of Cartan matrix $C_{i j}$ of Lie superalgebra $\mathcal{A}_{n,n-1}$.

A definition $\mathcal{T}(z) = \mathcal{T}_2(z) - \frac{1}{2} \mathcal{J}_{2}^{'}(z)$ denotes the energy-momentum tensor of $\mathcal{N}=2$ model with the central charge $c^{(\mathcal{N}=2)} = 3 n (1 - (n + 1) a_0^2)$. The OPE with itself is

$$\mathcal{T}(z_1) \mathcal{T}(z_2) = \frac{c^{(\mathcal{N}=2)}}{z_{12}^{2}} + \frac{2 \mathcal{T}(z_2)}{z_{12}^{2}} + \frac{\partial \mathcal{T}(z_2)}{z_{12}^{2}} + \cdots$$

(2.9)
Note that in the component definitions (2.7), all bosonic and fermionic currents are not primary fields with respect to the energy-momentum tensor \( \mathcal{T}(z) \), except \( \mathcal{J}_i(z) \), for bosonic currents, i.e.

\[
\mathcal{T}(z_1)\mathcal{J}_i(z_2) = \frac{1}{2} \sum_{s=2}^{k} \frac{(n-k+s)!}{(n-k)!} z_1^{n-2} a_o^{n-s-2} \left(1 + (n+1+2(k-s))a_o^2\right) \frac{\mathcal{J}_i(z_2)}{z_1^{n+2}} + \frac{(n-k+1)!}{(n-k)!} z_1^{n} a_o^{n} \frac{\mathcal{J}_i(z_2)}{z_1^{n+2}} + \ldots \tag{2.10}
\]

and also fermionic currents, i.e.

\[
\mathcal{T}(z_1)\mathcal{G}^+(z_2) = (2n-5)a_o^2 \left(1 + (n+3)a_o^2\right) \frac{G^+(z_2)}{z_1^{n+2}} + \frac{(n-1)(3 + (2 + 3n)a_o^2)}{3n - 1 + 3n(n+1)a_o^2} \mathcal{T}(z)
\]

In order to construct all pure bosonic and fermionic currents for the \( \mathcal{N}=2 \) super-Casimir \( \mathcal{W}_\mathcal{A}_n \) algebras, one may calculate suitable coefficients for quasi-primary bosonic and fermionic fields, which form a \( \mathcal{N}=2 \) super multiplet \( \kappa \{ \mathcal{J}_2(z) , \sqrt{2} \mathcal{G}^+(z) , \mathcal{T}_3(z) \} \)

\[
\kappa \mathcal{J}_2(z) = \mathcal{J}_2(w) - \frac{1}{2} (n-1)a_o^2 \partial \mathcal{J}_1(z) - \frac{(n-1)(3 + (2 + 3n)a_o^2)}{6n - 2 + 6n(n+1)a_o^2} \mathcal{J}_1(z) \mathcal{J}_1(z)
\]

\[
\kappa \mathcal{G}^+(z) = \mathcal{G}^+(z) + \frac{(n-1)(1 - na_o^2)(1 + (n-1)a_o^2)}{3n - 1 + 3n(n+1)a_o^2} \partial \mathcal{G}^+(z) + \frac{(n-1)(3 + (2 + 3n)a_o^2)}{3n - 1 + 3n(n+1)a_o^2} \mathcal{J}_2(z) \mathcal{G}^+(z)
\]

\[
\kappa \mathcal{G}^-(z) = \mathcal{G}^-(z) - \frac{(n-1)(1 + 3na_o^2 + 2n(n+1)a_o^4)}{3n - 1 + 3n(n+1)a_o^2} \partial \mathcal{G}^-(z) - \frac{(n-1)(3 + (2 + 3n)a_o^2)}{3n - 1 + 3n(n+1)a_o^2} \mathcal{J}_2(z) \mathcal{G}^-(z)
\]

\[
\kappa \mathcal{W}_3(z) = \mathcal{T}_3(z) - \frac{1}{2} \mathcal{J}_3(z) + \frac{(n-1)(3 + (2 + 3n)a_o^2)}{3n - 1 + 3n(n+1)a_o^2} \left[(\mathcal{G}^+ \mathcal{G}^-)(z) - (\mathcal{J}_2 \mathcal{W}_2)(z) + \frac{1}{4} \partial(\mathcal{J}_3 \mathcal{J}_1)(z)\right] - \frac{(n-1)(1 + na_o^2)(2 + (n+1)a_o^2)}{6n - 2 + 6n(n+1)a_o^2} \partial \mathcal{J}_1(z) \mathcal{J}_1(z)
\]

where

\[
\kappa = \sqrt{\frac{3n - 1 + 3n(n+1)a_o^2}{(1 - na_o^2)(2 + (n+1)a_o^2)(1 + (n+2)a_o^2)}} \quad \tag{2.12}
\]

From (2.12), we may compute the operator product expansions for the \( \mathcal{N}=2 \) super multiplet \( \kappa \{ \mathcal{J}_2(z) , \sqrt{2} \mathcal{G}^+(z) , \mathcal{T}_3(z) \} \). It is easy to see that these OPE's coincide with the results of the OPE method \(^1\), under (2.12) normalization.

3. (OPEs) for Chiral Super-Vertex Operators

In this section we define a super-vertex operator \( \mathcal{V}_\beta(Z) \) which corresponds to the root system \( \{ \beta \} \) of the Lie superalgebra \( \mathcal{A}_{n,n-1} \) and a super field \( \Phi(Z) = \varphi(z) + i \theta \psi(z) \),

\[
\mathcal{V}_\beta(Z) = :e^{i\beta \Phi}(Z): \quad \tag{3.1}
\]
In the component formalism, one gets

\[ \mathcal{V}_\beta(Z) = \mathcal{V}_\beta(z) + \theta \mathcal{V}_\beta(z) \]  

(3.2)

where the bosonic \( \mathcal{V}_\beta(z) \) and fermionic \( \mathcal{V}_\beta(z) \) components of \( \mathcal{V}_\beta(Z) \) are

\[ \mathcal{V}_\beta(z) = : e^{i \beta \varphi(z)} : \]  

(3.3)

and

\[ \mathcal{V}_\beta(z) = \beta \Psi(z) : e^{i \beta \varphi(z)} : \]  

(3.4)

respectively. We must emphasize here that we will concentrate over the bosonic vertex operator \( \mathcal{V}_\beta(z) \), in our advanced calculations. By using conformal spin-0 contraction \( \varphi(z_1) \varphi(z_2) = -\ln |z_{12}| \). The standard OPEs are of two forms:

\[ \mathcal{V}_\beta(z_1) \mathcal{V}_\beta(z_2) = (z_{12})^{\beta \cdot \beta} : \mathcal{V}_\beta(z_1) \mathcal{V}_\beta(z_2) : \]  

(3.5)

and

\[ \mathcal{V}_\beta(z_1) \mathcal{V}_\beta(z_2) = \frac{\beta^2}{z_{12}} \mathcal{V}_\beta(z_1) \mathcal{V}_\beta(z_2) \]  

(3.6)

One can say that the operators \( \mathcal{V}_\beta(z) \) and \( \mathcal{V}_\beta(z) \) carry a root \( \beta \). From

\[ h_j(z_1) \mathcal{V}_\beta(z_2) = \frac{\theta_j}{z_{12}} \mathcal{V}_\beta(z_2) + \cdots \]  

(3.7)

and

\[ h_j(z_1) \mathcal{V}_\beta(z_2) = \frac{\theta_j}{z_{12}} \mathcal{V}_\beta(z_2) + \cdots \]  

(3.8)

where

\[ \theta_j = \theta_j(\beta) \equiv (\beta, \lambda_j) \]  

(3.9)

The OPEs with the stress-energy tensor \( \mathcal{T}(z) \) is

\[ \mathcal{T}(z_1) \mathcal{V}_\beta(z_2) = \frac{h^\beta(\beta)}{z_{12}^2} \mathcal{V}_\beta(z_2) + \frac{\eta^\beta \mathcal{V}_\beta(z_2)}{z_{12}} + \cdots \]  

(3.10)

where \( h^\beta(\beta) \) is given by

\[ h^\beta(\beta) = -\sum_{i=1}^{2n-1} (-1)^i \theta_i \theta_{i+1} + \frac{\theta^2}{2} \sum_{i=1}^{2n} \theta_i \]  

(3.11)

this means that the bosonic vertex operator \( \mathcal{V}_\beta(z) \) is a conformal field with spin \( h^\beta(\beta) \), and

\[ (\eta^\beta \mathcal{V}_\beta)(z) = -\sum_{i=1}^{2n-1} (-1)^i (\theta_i h_{i+1}(z) + h_i(z) \theta_{i+1}) \]  

(3.12)

similarly, for the fermionic vertex operator \( \mathcal{V}_\beta(z) \)

\[ \mathcal{T}(z_1) \mathcal{V}_\beta(z_2) = \frac{h_j^\beta \mathcal{V}_\beta(z_2)}{z_{12}^2} + \frac{\eta^\beta \mathcal{V}_\beta(z_2)}{z_{12}} + \cdots \]  

(3.13)

where \( (h_j^\beta \mathcal{V}_\beta)(z) \)

\[ (h_j^\beta \mathcal{V}_\beta)(z) = \frac{1}{2} \sum_{i=1}^{2n-1} (-1)^{i+1} (\theta_i \psi_{i+1}(z_2) + \psi_i(z_2) \theta_{i+1}) \mathcal{V}_\beta(z_2) \]
\[ + \sum_{i=1}^{2n-1} (-1)^i \theta_i \theta_{i+1} V_\beta^f(z_2) + \frac{a_0}{2} \sum_{i=1}^{2n} \theta_i V_\beta^f(z_2) \]  

(3.14)

and

\[ (\eta_\beta V_\beta^b)(z) = \sum_{i=1}^{2n-1} (-1)^{i+1} (\theta_i \partial \psi_{i+1}(z_2) + \partial \psi_i(z_2) \theta_{i+1}) V_\beta^b(z_2) \]  

\[ + \sum_{i=1}^{2n-1} (-1)^i (\theta_i h_{i+1}(z_2) + h_i(z_2) \theta_{i+1}) V_\beta^f(z_2) \]  

(3.15)

From (3.13), we can not define \( h^f(\beta) \) as the conformal spin of \( V_\beta^f(z) \), under (3.9) definition, since \( V_\beta^f(z) \) field does not seem in the r.h.s. of OPE (3.13).

Let us discuss the OPEs between the quasi-primary fields of the super-Casimir \( W_{\lambda_1} \) algebra and the bosonic vertex operator \( V_\beta^b(z) \), instead of primary fields since they are very complicated, which are given as in equation (2.12). First we write down the OPEs between module-1 components \( J_1(z) \) and \( G_2^\pm(z) \) are given by :

\[ J_1(z_1) V_\beta^b(z_2) = \sum_{i=1}^{2n} (-1)^{i-1} \theta_i V_\beta^b(z_2) - \frac{1}{z_2} + \cdots \]  

(3.16)

and, for fermionic currents :

\[ G_2^+(z_1) V_\beta^b(z_2) = \left( - \sum_{i=1}^{2n-2} (1-t_i) \theta_i \psi_{i+1}(z) + \sum_{i=1}^{2n} t_i \theta_{i+1} \psi_i(z) \right) \frac{V_\beta^b(z_2)}{z_2} + \cdots \]  

(3.17)

\[ G_2^-(z_1) V_\beta^b(z_2) = \left( \sum_{i=1}^{2n-2} (1-t_i) \theta_{i+1} \psi_i(z) - \sum_{i=1}^{2n} t_i \theta_i \psi_{i+1}(z) \right) \frac{V_\beta^b(z_2)}{z_2} + \cdots \]  

(3.18)

Second the OPEs between module-2 components \( J_2(z) , G_2^\pm(z) , T_a(z) \) and the bosonic vertex operator \( V_\beta^b(z) \) are given by :

\[ J_2(z_1) V_\beta^b(z_2) = \left( a_0^2 \sum_{i=1}^{2n-3} \sum_{j=i+2}^{2n} (-1)^{i+j} t_i \theta_j \theta_j + a_0^3 \sum_{i=1}^{2n-2} \sum_{j=i+1}^{2n} (-1)^{i} (1-t_i) \theta_i \theta_j \right) \frac{V_\beta^b(z_2)}{z_2} + \cdots \]  

(3.19)

and, for fermionic currents :

\[ G_2^+(z_1) V_\beta^b(z_2) = \left( a_0 \sum_{i=1}^{2n-3} \sum_{j=i+2}^{2n} (-1)^{i+j} t_i \psi_i(z_2) \theta_{j+1} \theta_j + a_0 \sum_{i=1}^{2n-3} \sum_{j=i+1}^{2n-1} (-1)^{i} (1-t_i) \theta_i \psi_{j+1}(z_2) \theta_j \right) \]  

\[ + a_0^2 \sum_{i=1}^{2n-3} \sum_{j=i+2}^{2n} (-1)^{i+1} t_i \psi_j(z_2) \theta_{j+1} \theta_j + a_0 \sum_{i=1}^{2n-4} \sum_{j=i+3}^{2n} (-1)^{j} (1-t_i) \theta_i \psi_{j+1}(z_2) \theta_j \]  

\[ + \frac{a_0^2}{2} \sum_{i=1}^{2n-2} i (-1)^{i} (1-t_i) \theta_i \psi_{i+1}(z_2) + \frac{a_0^2}{2} \sum_{i=1}^{2n-1} (i-1)(-1)^{i} t_i \psi_i(z_2) \theta_{i+1} \]  

\[ \cdot \frac{V_\beta^b(z_2)}{z_2} + \cdots \]  

(3.20)

\[ G_2^-(z_1) V_\beta^b(z_2) = \left( - a_0 \sum_{i=1}^{2n-2} \sum_{j=i+2}^{2n} (-1)^{i+j} (1-t_i) \psi_i(z_2) \theta_{j+1} \theta_j + a_0 \sum_{i=1}^{2n-3} \sum_{j=i+3}^{2n-1} (-1)^{j} t_i \theta_i \psi_{j+1}(z_2) \theta_j \right) \]  

\[ \cdot \frac{V_\beta^b(z_2)}{z_2} + \cdots \]  

(3.21)
\[-a_o \sum_{i=1}^{2n} \sum_{j=i+2}^{2n-2} (-1)^i (1 - t_i) \theta_i \psi_j (z_2) \theta_{j+1} + a_o \sum_{i=1}^{2n} \sum_{j=i+1}^{2n-1} (-1)^i t_i \theta_i \psi_j (z_2) \theta_{j+1} \]

\[-\frac{1}{2}a_o^2 \sum_{i=1}^{2n-1} (i - 1)(-1)^i t_i \theta_i \psi_{i+1} (z_2) - \frac{1}{2}a_0^2 \sum_{i=4}^{2n-2} (i - 2)(1 - t_i) \psi_i (z_2) \theta_{i+1} \right) \frac{V^b_\alpha (z_2)}{z_{12}^2} + \cdots \quad (3.21)\]

and also finally the last OPE is found to be:

\[\mathcal{T}_3(z_1) V^\beta_\alpha (z_2) = - \left( a_o \sum_{i=1}^{2n} \sum_{j=i+3}^{2n} (-1)^{i+j} \theta_i \theta_{i+1} \theta_j + a_o \sum_{i=1}^{2n} \sum_{j=i+2}^{2n-1} (-1)^{i+j} t_i \theta_i \theta_{j+1} \right) \frac{V^b_\alpha (z_2)}{z_{12}^2} + \cdots \quad (3.22)\]

Although the above advanced calculations was performed in the quasi-primary basis, this results can be carry on the primary basis.

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