Analytical method for the out-of-plane buckling of the telescopic boom with guy cables

Lixia Meng¹,², Zhaojian Gui¹, Ke Zhang¹, Junru Liu³ and Shiming Liu¹
¹School of Mechanical Engineering, Shenyang Jianzhu University, Shenyang, China
²Shenxi Machinery Corporation Ltd., Jiangsu, China
³South East Fujian Motor Corporation Ltd., Fujian, China

Abstract
The guy cables can effectively improve the bearing capacity for the telescopic boom of the crane, while the introduction of tensioned cables makes the buckling analysis complicated. The primary objective of this study was to propose an analytical method for the out-of-plane buckling of the telescopic boom with the spatial symmetric guy cables. To analyze the influence of the guy cables on the out-of-plane buckling property of the telescopic boom, the deflection differential equation of the multi-stepped telescopic boom with guy cables was established based on the theory of elastic beam, then the buckling characteristic equation to determine the critical load of the telescopic boom was derived. Comparison of results with that given by the finite element method showed the high accuracy of the proposed method. In the end, with this equation, the influences of the structural geometric parameters on the critical load were investigated. The results indicated that, in the engineering application, the critical load of the telescopic boom can be increased by decreasing the length ratio a/L or increasing the angle φ between the two cables. The influence of angle θ on the out-of-plane buckling analysis of the telescopic boom can be neglected. With the proposed method, the buckling behavior of the telescopic boom with guy cables can be solved accurately. The present work is significant to structural design and safety analysis of the telescopic boom, and it can be utilized to provide technical support for the structural design of telescopic boom.

Keywords
Telescopic boom, out-of-plane buckling, guy cables, differential equation method, critical buckling load

Corresponding author:
Shiming Liu, School of Mechanical Engineering, Shenyang Jianzhu University, No. 25, Hunnan Middle Road, Hunnan District, Shenyang 110168, China.
Email: liushiming_1983@163.com

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Introduction

With the advantages of high bearing capacity and high percentage of material usage, stepped columns are widely used in the fields of lifting machinery, steel structure of bridges, and buildings et al. The buckling problem is one of the hot topics for such structure that draw many scholars’ attention.\(^1\)–\(^4\) As the key bearing component of the truck crane, the telescopic boom usually works in the form of the stepped column, the buckling analysis of the telescopic boom as stepped column has been proved to be correct and reasonable in the Chinese national crane design standard.\(^5\)\(^,\)\(^6\) As the increasing of the rated load and the lifting height, guy cables are equipped to improve the stability of the telescopic boom for large truck cranes. Determination of the critical load of the telescopic boom with guy cables is a critical issue for structure design of the truck cranes.

Previously, the approximate method represented by the energy method is usually adopted to determinate the critical load of beam structure.\(^7\)\(^,\)\(^8\) The accuracy of the energy method depends on the correctness of the assumed deformation configuration, and the error is difficult to be estimated. To determinate the critical load of beam structure, the differential equilibrium equation method is employed to deduce the characteristic equation of instability. Based on the differential equilibrium equation, the buckling of a column under axial compressive force has been thoroughly investigated.\(^8\)\(^,\)\(^9\) Finite element method is another effective method for buckling analysis, the characteristic equation to solve the buckling problem is the determinate of the stiffness matrix of the structure. When the determinate of the stiffness matrix becomes zero, the buckling of the structure occurs.\(^6\)\(^,\)\(^10\)–\(^12\) Compare to the differential equilibrium equation method, the finite element method is more convenient for complex structure. While the differential equation method is more appropriate for engineering application in the beginning of the structure design. Currently, the buckling of the telescopic boom or stepped columns under different boundary conditions has already been analyzed by different methods.\(^13\)–\(^17\)

To improve the bearing capacity, the guy cables system is equipped in truck cranes, which makes the buckling analysis complicated. It should be noted that the out-of-plane buckling is easier to occur than the in-plane buckling for the telescopic boom. Meanwhile, the out-of-plane buckling will occur suddenly once the lifting load reaches the critical value without any symptoms, which makes the out-of-plane buckling is more harmful than the in-plane buckling. Hence, many scholars have focused on the out-of-plane buckling. An et al.\(^18\) and Liu et al.\(^19\) analyzed the buckling of the boom tensioned by a single cable. Wang presented the buckling characteristic equation of the jib system with middle strut.\(^20\) However, the research on the out-of-plane buckling analysis of the telescopic boom with double cables spatially equipped to the boom is rarely reported. The finite element method is the main option for analysis of that problem currently.\(^21\)\(^,\)\(^22\) The characteristic equation of the telescopic boom with the spatial double cables is established in Lu et al.,\(^23\) the telescopic boom is simply considered as a uniform beam, which is inconsistent with the actual model.
Hence, the main purpose of the present paper is to propose an analytical method for the out-of-plane buckling of the telescopic boom with the spatial symmetric guy cables. The mechanics model of the out-of-plane buckling analysis considering the tension of the cables is built. Then, the deflection differential equations of the multi-stepped telescopic boom under the buckling critical state are established, and the analytical expression of the buckling characteristic equation is derived. Additionally, a range of influential parameters are investigated to study their effects on the buckling behavior of the telescopic boom.

**Buckling analysis of telescopic boom with guy cables**

The buckling of the multi-stepped telescopic boom with guy cables are analyzed by using differential equilibrium equation method. The flowchart of the scheme is shown in Figure 1. The equivalent model of the telescopic boom with guy-cables is built. The tensions of the cables are calculated in the deformed configuration. The mechanics model when the out-of-plane buckling occur is then built. Based on the mechanics model, differential equilibrium equation of the boom is deduced and its general solution is given. By introducing the boundary conditions to the general solution, the characteristic equation, which is a transcendental equation, is obtained. The transcendental equation can be solved by using dichotomy method, then the critical load is obtained.

**The establishment of out-of-plane buckling model**

Due to the existence of spatial symmetric double cables, the configuration of the telescopic boom is shown in Figure 2. A coordinate system is built at the root end

| Building mechanics model |
|--------------------------|
| a) Building equivalent model of the telescopic boom with guy cables. |
| b) Calculating tensions of the cables $T_1$, $T_2$. |

| Deducing the characteristic equation |
|-------------------------------------|
| a) Building differential equilibrium equation and getting its general solution. |
| b) Introducing boundary conditions to deduce the characteristic equation. |

| Solving the buckling load |
|---------------------------|
| a) Solving characteristic equation by using dichotomy method. |

**Figure 1.** The flowchart of the scheme.
point \( o \) of the boom. The lifting plane is set as the \( xz \) plane with the boom axis as the \( x \)-axis. And the \( xy \) plane is the out of lifting plane, in which the model of out-of-plane buckling analysis is built.

According to the connection between the root of the telescopic boom and the slewing platform, the telescopic boom can be regarded as a clamped-free stepped column out of the lifting plane. As the buckling of the telescopic boom occurs, the out-of-buckling mode of the telescopic boom with the spatial symmetric double cables is shown in Figure 3. It should be noted that, as shown in Figure 2, the luffing cylinder is mounted with two hinge joints in its both ends, that the luffing cylinder can afford only axial force. The influence of the luffing cylinders to the in-plane (\( xz \) plane) deformation is significant while neglectable to the deformation in the out-of-plane (\( xy \) plane). Hence, the luffing cylinder is neglected in the buckling analysis model.

In the buckling mode shown in Figure 3, \( \delta \) is the lateral deformation of the free end \( D \), \( l_s \) is the length of the guy cable before deformation, \( c \) is the distance between

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**Figure 2.** The configuration of telescopic boom with spatial symmetric double cables.

![Figure 2](image)

**Figure 3.** The out-of-plane buckling mode of telescopic boom tensioned by guy cables.

![Figure 3](image)
point A and point G and \( d \) is the distance between point G and point D. According to the geometric relationship shown in Figure 3, the length of the tensioned guy cables can be expressed as

\[
\begin{align*}
ls_1 &= \sqrt{(\delta + c)^2 + d^2} = \sqrt{\delta^2 + 2\delta c + l_2^2} \\
ls_2 &= \sqrt{(\delta - c)^2 + d^2} = \sqrt{\delta^2 - 2\delta c + l_2^2}
\end{align*}
\]  

(1)

According to Hooke law, the tensions of the two cables \( T_1 \) and \( T_2 \) caused by deformation can be written as

\[
\begin{align*}
T_1 &= (l_2 - l_1) \frac{E_s A_s}{l_s 1} \\
T_2 &= (l_2 - l_1) \frac{E_s A_s}{l_s 2}
\end{align*}
\]

where \( E_s \) is the Young’s modulus of the cable, \( A_s \) is the cross-section area of the cable.

In Figure 3, the tensions \( T_1 \) and \( T_2 \) can be decomposed to \( F_x \), \( F_y \), and \( F_z \) at the tip of the telescopic boom. Employing the geometric relationship, then

\[
\begin{align*}
F_x &= a \left( \frac{l_1}{l_1 - l_2} + \frac{l_2}{l_1 - l_2} \right) = E_s A_s \cos \varphi \cos \theta \left( 2 - \frac{l_1}{l_1 - l_2} - \frac{l_2}{l_1 - l_2} \right) \\
F_y &= T_1 \frac{\delta + c}{l_1} + T_2 \frac{\delta - c}{l_2} = \frac{F_x \delta}{a} + c \left( \frac{l_1}{l_1} - \frac{l_2}{l_2} \right) \\
F_z &= h \left( \frac{\delta}{l_1} + \frac{\delta}{l_2} \right) = F_x \tan \theta
\end{align*}
\]

(3)

where \( a \) and \( h \) are the length of the cable projection onto \( x \) and \( z \) axis respectively, \( \varphi \) is a half of the angle between the two cables (i.e. the angle between the cable and the line GD), \( \theta \) is the angle between the telescopic boom and the cables’ plane.

**The buckling characteristic equation of telescopic boom**

Decomposing the tensions \( T_1 \) and \( T_2 \) at the tip of the telescopic boom, the calculation model for the out-of-plane buckling analysis can be established, as shown in Figure 4, the moment of inertia of the \( i \)-th section of the telescopic boom is \( I_i \), \( l_i \) denotes the length from the top of the \( i \)-th section to the fixed root, \( L \) is the total length of the telescopic boom(\( L = l_i \)), and \( E \) is the Young’s modulus of the telescopic boom.

As shown in Figure 4, based on the small deformation assumption of elastic beam, the deflection differential equations of the \( n \)-section telescopic boom with guy cables can be established as

\[
\begin{align*}
EIy''_1 &= F_x (\delta - y_1) - F_y (L - x) (0 \leq x \leq x_1) \\
& \quad \cdots \\
EIy''_i &= F_x (\delta - y_i) - F_y (L - x) (l_{i-1} \leq x \leq l_i)
\end{align*}
\]

(4)
Replacing the axial force $F_x$ with the symbol $P$, and substituting equation (3) into equation (4), yields

$$EI_{i}y''_{i} + Py_{i} = P\delta \left(1 - \frac{L-x}{a}\right) - cE_{s}A_{s}\left(\frac{1}{l_{s2}} - \frac{1}{l_{s1}}\right)(L-x)$$

(5)

Equation (5) can be expressed in a general form by introducing $k_{i} = \sqrt{P/(EI_{i})}$.

$$y''_{i} + k_{i}^{2}y_{i} = k_{i}^{2}\delta \left(1 - \frac{L-x}{a}\right) - \frac{cE_{s}A_{s}}{EI_{i}}\left(\frac{1}{l_{s2}} - \frac{1}{l_{s1}}\right)(L-x)$$

(6)

Where $i = 1, 2, 3\ldots n$.

The general solution of the deflection differential equation (6) can be presented as

$$y_{i} = A_{i}\sin (k_{i}x) + B_{i}\cos (k_{i}x) + \delta - (L-x)\left(\frac{\delta}{a} + \frac{cE_{s}A_{s}}{P}\left(\frac{1}{l_{s2}} - \frac{1}{l_{s1}}\right)\right)$$

$$y'_{i} = A_{i}k_{i}\cos (k_{i}x) - B_{i}k_{i}\sin (k_{i}x) + \frac{\delta}{a} + \frac{cE_{s}A_{s}}{P}\left(\frac{1}{l_{s2}} - \frac{1}{l_{s1}}\right)$$

(7)

According to the boundary conditions of the root of the telescopic boom, that is $y_{1} = y'_{1} = 0$ at $x = 0$, the integral constants $A_{1}$ and $B_{1}$ can be obtained as

$$\begin{cases} 
A_{1} = -\frac{\delta}{ak_{1}} - \frac{cE_{s}A_{s}}{Pk_{1}}\left(\frac{1}{l_{s2}} - \frac{1}{l_{s1}}\right) \\
B_{1} = \frac{\delta L}{a} + \frac{cE_{s}A_{s}L}{P}\left(\frac{1}{l_{s2}} - \frac{1}{l_{s1}}\right) - \delta 
\end{cases}$$

(8)

On the basis of the displacement compatibility when $x = l_{i}$, $y_{i} = y'_{i+1}$, and $y'_{i} = y'_{i+1}$, the relationship of the integral constants are presented as

$$\begin{cases} 
A_{i}\sin (k_{i}l_{i}) + B_{i}\cos (k_{i}l_{i}) = A_{i+1}\sin (k_{i+1}l_{i}) + B_{i+1}\cos (k_{i+1}l_{i}) \\
\frac{k_{i}}{k_{i+1}}A_{i}\cos (k_{i}l_{i}) - \frac{k_{i}}{k_{i+1}}B_{i}\sin (k_{i}l_{i}) = A_{i+1}\cos (k_{i+1}l_{i}) - B_{i+1}\sin (k_{i+1}l_{i}) 
\end{cases}$$

(9)
Equation (9) can be expressed in matrix form as

$$\begin{bmatrix} A_{i+1} \\ B_{i+1} \end{bmatrix} = T_i \begin{bmatrix} A_i \\ B_i \end{bmatrix}$$  \hspace{1cm} (10)

Where $T_i$ is

$$T_i = \begin{bmatrix} \sin(k_i + 1l_i) & \cos(k_i + 1l_i) \\ \cos(k_i + 1l_i) & -\sin(k_i + 1l_i) \end{bmatrix} \cdot \begin{bmatrix} \frac{k_i}{k_{i+1}} \cos(k_i l_i) - \frac{k_i}{k_{i+1}} \sin(k_i l_i) \\ \frac{k_i}{k_{i+1}} \sin(k_i l_i) + \frac{k_i}{k_{i+1}} \cos(k_i l_i) \end{bmatrix}$$  \hspace{1cm} (11)

The recursion expression of the integral constants $A_n$ and $B_n$ are obtained from equation (10).

$$\begin{bmatrix} A_n \\ B_n \end{bmatrix} = T_{n-1} T_{n-2} \ldots T_1 \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \prod_{i=n-1}^1 T_i \begin{bmatrix} A_1 \\ B_1 \end{bmatrix}$$  \hspace{1cm} (12)

According to the boundary conditions of the free end of the telescopic boom, where $x = l_n = L$, $y_n = \delta$, we can have

$$A_n \sin(k_n L) + B_n \cos(k_n L) = 0$$  \hspace{1cm} (13)

By substituting equation (8) and equation (12) into equation (13), the buckling characteristic equation of the telescopic boom can be written as follows

$$\begin{bmatrix} \sin(k_n L) & \cos(k_n L) \end{bmatrix} \cdot \prod_{i=n-1}^1 T_i \begin{bmatrix} -\frac{\delta}{ak_1} - \frac{cE_A l_i}{P_k} \left( \frac{1}{l_{s1}} - \frac{1}{l_{s2}} \right) \delta l_2 \cos \left( \frac{1}{l_{s1}} - \frac{1}{l_{s2}} \right) - \delta \right) = 0$$  \hspace{1cm} (14)

In equation (14), the lateral displacement $\delta$ is an unknown quantity and the length $l_{s1}$ and $l_{s2}$ are also functions of the lateral displacement. Therefore, further simplification is necessary to solve the characteristic equation. We can define a function $f(\delta)$ as

$$f(\delta) = \frac{1}{l_{s2}} - \frac{1}{l_{s1}} = \frac{\sqrt{\delta^2 + 2c\delta + l_2^2} - \sqrt{\delta^2 - 2c\delta + l_2^2}}{\sqrt{\delta^2 + 2c\delta + l_2^2} \sqrt{\delta^2 - 2c\delta + l_2^2}}$$  \hspace{1cm} (15)

The Taylor series expansion of $f(\delta)$ is performed, the quadratic and higher order terms on $\delta$ are omitted based on the small deformation assumption, then equation (15) can be approximately expressed as follows.

$$f(\delta) = \frac{1}{l_{s2}} - \frac{1}{l_{s1}} \approx \frac{2c\delta}{l_2^3}$$  \hspace{1cm} (16)

Substituting equation (16) into equation (14) yields
As shown in Figure 3, the following relationships are established between the length $a$, $c$, and $l_s$.

\[
\begin{align*}
    a &= l_s \cos \varphi \cos \theta \\
    c &= l_s \sin \varphi
\end{align*}
\]  

Substituting equation (18) into equation (17), the buckling characteristic equation of the telescopic boom with spatial symmetric guy cables is deduced.

\[
\begin{align*}
    \{ \sin(k_n L) \cos(k_n L) \} \cdot \prod_{i = n-1}^{1} T_i \cdot \\
    \left\{ - \frac{1}{ak_1} - \frac{2E_s A_s}{Pl_l^2} \frac{L}{a} + \frac{2E_s A_s}{Pl_l^2} - 1 \right\}^T = 0
\end{align*}
\]  

Equation (19) is a transcendental equation containing only one unknown quantity $P$. Among the solutions of the equation, we are interested in the minimum solution, which is the first critical load of the telescopic boom. By solving the nonlinear characteristic equation, the critical load $P_{cr}$ of the telescopic boom with guy cables can be obtained.

When the angle $\varphi = 0$, the spatial symmetric double cables becomes a single cable action in the lifting plane. Let $c = 0$ in equation (18), the characteristic equation (17) degenerates to the same expression as presented in [19]:

\[
\begin{align*}
    \{ \sin(k_n L) \cos(k_n L) \} \cdot \prod_{i = n-1}^{1} T_i \cdot \\
    \left\{ \frac{1}{k_1 \cos \varphi \cos \theta} - \frac{2\sin \varphi^2 E_s A_s}{Pl_l} \frac{L}{l_s \cos \varphi \cos \theta} + \frac{2E_s A_s}{Pl_l} - 1 \right\}^T = 0
\end{align*}
\]  

In addition, when $I_1 = I_2 = \cdots = I_n = I$, equation (19) agrees well with the expression in the case of uniform boom buckling:

\[
\tan(kL) = kL \left( 1 - \frac{a}{Lk^2EI + 2E_sA_s \sin \varphi^2 \cos \varphi \cos \theta} \right)
\]  

Buckling analysis of the telescopic boom with finite element method

The buckling of the boom with guy cables can also be solved with finite element method by using commercial software ANSYS. The finite element model of the multi-stepped boom with guy cables are shown in Figure 5.
The telescopic boom is meshed with BEAM44 element, each step of the boom is discretized with 10 elements. The boom is made by steel material with Young’s modulus $E = 206$ GPa, and the Poisson ratio $v = 0.3$. Each cable is meshed with 1 element of the type LINK180, the Young’s modulus of the cable material is $E_s = 100$ GPa, and the Poisson ratio $v = 0.3$. The root node of the boom is constrained, only the rotational degree of freedom around $y$ axis is allowed. The degrees of freedoms of the root nodes of the cables are all constrained. A node force $F_z = 1$ N is applied to the free tip node of the boom in $z$ direction. The eigenvalue buckling solver is selected to obtained the critical load of the boom. In ANSYS, the characteristic equation for solving the buckling problem can be written as

$$\left| K_0 + \lambda K_g \right| = 0$$  \hspace{1cm} (22)

where $K_0$ is the elastic stiffness matrix which is constant, $K_g$ is the geometrical stiffness matrix which is function of axial force of the beam element, and $\lambda$ is the eigenvalue which is equal to the critical load of the structure.

The solution from the finite element method can be used to verify the correctness of the analytical solution which is obtained from section “Buckling analysis of telescopic boom with guy cables.”

**Numerical examples**

Numerical results are presented to demonstrate the versatility and accuracy of the developed approach in solving buckling problems of the telescopic boom in this section. The buckling of two telescopic booms are analyzed by the proposed method and the finite element method performed by the commercial FE software ANSYS.

The moment of inertia of the first section $I_1$ is used to describe the critical buckling load of the telescopic boom, which is written as follows.

![Figure 5. Finite element model of the multi-stepped telescopic boom with guy cables.](image-url)
The effective length factors $\mu$ of the four-section telescopic boom obtained by the proposed method and FE software ANSYS are shown in Table 1.

**Buckling analysis of eight-section telescopic boom**

For further verification of the correctness and accuracy of the proposed method, setting $n = 8$, a eight-section telescopic boom shown in Figure 6 is analyzed. The parameters are listed as follows: $E = 2.06 \times 10^{11}$ Pa, $I_1 = 43.26 \times 10^{-3}$ m$^4$, the ratios of the moment of inertia $\beta_2 = I_1/I_2$, $\beta_3 = I_2/I_3$, $\beta_4 = I_3/I_4$, $\beta_5 = I_4/I_5$, $\beta_6 = I_5/I_6$, $\beta_7 = I_6/I_7$, $\beta_8 = I_7/I_8$, and the certain values of $\beta_i (i = 2,3,4...8)$ for different cases are shown in Table 2. $L = 96.875$ m, $l_1 = 0.16L$, $l_2 = 0.28L$, $l_3 = 0.40L$, $l_4 = 0.52L$, $L_a = 40$m, $l_1 = 0.34L$, $l_2 = 0.56L$, $l_3 = 0.78L$, $l_4 = L$. The projection length of the cable $a = 36$m. The material and cross-sectional properties of the cables are $A_s = 1.0 \times 10^{-3}$ m$^2$, $E_s = 1.0 \times 10^{11}$ Pa. The angle $\phi = 10^\circ$, $\theta = 20^\circ$.

The geometric model of generalized $n$-section telescopic boom with guy cables is shown in Figure 6.
In the same way, the results of the eight-section telescopic boom are shown in Table 3.

As shown in Tables 1 and 3, the maximum errors between the solutions of the proposed method and ANSYS simulation are 0.06% and 0.22%, respectively. The comparisons in these two cases indicate that the proposed method can give
buckling solutions with high accuracy, and it can be applied to the out-of-plane buckling analysis of the telescopic boom with guy cables in practical engineering.

### Parametric study

According to the characteristic equation (19), the critical load of the telescopic boom is related to the projection length \( a \) of the cable (or the length of the cable \( l_s \)), the angle \( \phi \) of the cable, and the angle \( \theta \) between the telescopic boom and the cables’ plane. In order to analyze the influence of these geometric parameters on the stability of the telescopic boom, the buckling property of the eight-section telescopic boom in section “Buckling analysis of eight-section telescopic boom” is analyzed with varying parameters, including the length ratio \( a/L \), angle \( \phi \), and angle \( \theta \).

#### Influence of \( a/L \) on critical buckling load

For the sake of analyzing the influence of the length ratio \( a/L \) on the critical load, the buckling property of the boom is analyzed with a series of projection length \( a \) which leads to different values of \( a/L \). The ratios of the moment of inertia are \( b_2 = b_3 = b_4 = b_5 = b_6 = b_7 = b_8 = 1.2 \), the successive solutions of \( \mu \) are shown in Table 4.

The results in Table 4 imply that, the maximum error of the proposed method is 0.2864\%, which further verifies that the presented characteristic equation of the telescopic boom is correct for different range of \( a/L \).

On the purpose of revealing the influence of the length ratio \( a/L \) on the critical load clearly, curves of the dimensionless length factors \( \mu \) according to the length ratio \( a/L \) are shown in Figure 7. In order to better describe the variation trend of length factors \( \mu \), the abscissa is represented by logarithmic coordinate in Figure 7.

| \( a/L \) | The proposed method | ANSYS | Error (%) |
|---|---|---|---|
| 0.01 | 0.9764 | 0.9764 | 0.0 |
| 0.2 | 0.9910 | 0.9911 | 0.0101 |
| 0.5 | 1.0312 | 1.0318 | 0.0582 |
| 0.8 | 1.1888 | 1.1208 | 0.1784 |
| 1.0 | 1.2186 | 1.2221 | 0.2864 |
| 5.0 | 2.0953 | 2.0965 | 0.0572 |
| 10 | 2.2489 | 2.2494 | 0.0222 |
| 50 | 2.3795 | 2.3795 | 0.0000 |
| 100 | 2.3963 | 2.3962 | 0.0042 |
| 500 | 2.4099 | 2.4098 | 0.0041 |
| 1000 | 2.4116 | 2.4115 | 0.0041 |
| 5000 | 2.4130 | 2.4128 | 0.0083 |
| 10000 | 2.4131 | 2.4130 | 0.0041 |
It can be seen from Table 4 and Figure 7, as the length ratio $a/L$ increasing, the effective length factor $m$ is increased. In other words, the critical load of the telescopic boom decreased, the stability capacity of the structure becomes weaker with the increase of $a/L$.

When $a/L \to \infty$, the effective length factor $m$ approaches to a constant value. In Table 5, the results of the value $m$ are compared with those obtained by ANSYS.
and the method presented by Yao.\textsuperscript{13} The results show that, as the length ratio $a/L$ approaches to infinite, the effective length factor of the boom with guy cables approaches to a constant value which is coincident with the effective length factor of the stepped cantilever.

Influence of angle $\varphi$ on critical buckling load

The effective length factors of the boom are solved with successive value of angle $\varphi$. The curves of effective length factor $\mu$ with respect to angle $\varphi$ are plotted in Figure 8.

It can be seen from Figure 8, with constant values of $a/L$ and angle $\theta$, as the angle $\varphi$ increasing, the effective length factor $\mu$ goes downward firstly and then goes upward. As the angle $\varphi$ increasing from $0^\circ$ to $30^\circ$, the effective length factor $\mu$ is decreased significantly, and then it becomes steady when $\varphi$ is between $30^\circ$ and $70^\circ$, after that, it rises slightly when $\varphi$ changes from $70^\circ$ to $90^\circ$. In addition, according to Figure 8(a) and (b), the minimum value of $\mu$ occurs when $\varphi$ is about $55^\circ$, and the values of $\mu$ in Figure 8(c) and (d) conform to the same law.

Figure 8. The curves of the effective length factor with respect to the angle $\varphi$: (a) $a/L=10$, $\theta=45^\circ$; (b) $a/L=5$, $\theta=30^\circ$; (c) $a/L=1$, $\theta=15^\circ$; (d) $a/L=0.5$, $\theta=5^\circ$. 

The effective length factors of the boom are solved with successive value of angle $\varphi$. The curves of effective length factor $\mu$ with respect to angle $\varphi$ are plotted in Figure 8.
In this part, we focus on the influence of the angle \( \theta \) on the critical load. The successive solutions of the length factor \( m \) with respect to angle \( \theta \) is plotted in Figure 9. To avoid interference of other parameters, the value of the length ratio \( a/L \) and angle \( \theta \) are kept constant in the analysis.

In general, the length factor \( m \) increases gradually with the increase of the angle \( \theta \). Specifically, the change of \( m \) is obvious in the case of \( a/L = 5 \), \( \theta = 10^\circ \) as shown in Figure 9(a). By contrast, in the other three cases corresponding to Figure 9(b) to (d), \( m \) has a slight variation when \( \theta \) changes from 0\(^\circ\) to 60\(^\circ\) and increases slightly when \( \theta \) is bigger than 60\(^\circ\).

In the above analysis, the value of \( a/L \), angle \( \theta \) and \( \varphi \) are the theoretical values. In practice, the practical range of the parameters for a real telescopic boom system of an all-terrain crane with guy cables is limited. In general, the length ratio \( a/L < 1.0 \), the value of angle \( \varphi \) is \( 0^\circ < \varphi \leq 45^\circ \), and the value of angle \( \theta \) is \( 0^\circ < \theta \leq 45^\circ \). Therefore, in order to study the influence of geometric parameters in accordance with the practical structure on the critical load, different values of angle \( \theta \)
and \( \phi \) are adopted, the length factors \( \mu \) solved with varying \( \theta \) are replotted as shown in Figure 10.

From Figure 10, it can be seen that, the effective length factor \( \mu \) increases with the increase of the length ratio \( a/L \). By contrast, \( \mu \) drops along with the increase of angle \( \phi \). The length factor changes significantly when \( \theta \) is varying from 0\(^\circ\) to 20\(^\circ\), while its variation is slightly when the value of \( \theta \) is larger than 20\(^\circ\). The relative difference between the length factors given by \( \theta = 5^\circ \) and \( \theta = 45^\circ \) are all less than 3\%.

Hence, the influence of angle \( \theta \) on the buckling analysis of telescopic boom can be neglected in actual engineering calculation. It can be concluded that, by increasing the angle \( \phi \) or reducing the length ratio \( a/L \), the stability capacity of the telescopic boom can be improved.

**Conclusion**

On the purpose of studying the out-of-plane stability of telescopic boom with guy cables, the theoretical derivation and quantitative analysis are carried out. The out-
of-plane buckling of the telescopic boom with spatial symmetric guy cables is analyzed, and the characteristic equation of the telescopic boom is established. The accuracy of the proposed characteristic equation is verified by introducing a set of numerical examples. The comparisons between the proposed method and the simulation results from ANSYS show that the derived characteristic equation is accurate. Finally, the effects of the structural geometric parameters $a/L$, angle $\phi$, and angle $\theta$ on the critical buckling load of the telescopic boom are investigated. The results indicate that, in the engineering application, the critical load of the telescopic boom can be increased by decreasing the length ratio $a/L$ or increasing the angle $\phi$ between the two cables. The influence of angle $\theta$ on the out-of-plane buckling analysis of the telescopic boom can be neglected.

**Declaration of conflicting interests**

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**ORCID iD**

Shiming Liu https://orcid.org/0000-0002-3051-0947

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**Author biographies**

Lixia Meng is currently an Associate Professor of School of Mechanical Engineering at Shenyang Jianzhu University. She received her PhD from HarBin Institute of Technology in 2014. She has been an active academic and researcher who conducts research in the areas of structural stiffness, strength and reliability of construction machinery.

Zhaojian Gui is currently conducting his Master degree in School of Mechanical Engineering at Shenyang Jianzhu University. His research interests are structural stiffness, strength and reliability of construction machinery.
Ke Zhang is currently a Professor of Mechanical Engineering at ShenYang Jianzhu University. He has been participating directly and indirectly in many international research projects in the areas of construction machinery, precision machining, mechatronics technology, precision measurement and control technology.

Junru Liu has graduated from School of Mechanical Engineering at Shenyang Jianzhu University. His research interests are stability analysis and nonlinear analysis of mechanical structure.

Shiming Liu has been teaching in higher education and performing research for more than 6 years. He is currently an Associate Professor of School of Mechanical Engineering at ShenYang Jianzhu University. He received his PhD (2014) from HarBin Institute of Technology. His research work includes dynamics of mechanical system, stability and nonlinear analysis of construction machinery structure.

**Appendix**

**Notation**

- **δ**: lateral deformation of the free point D
- **l_s**: length of the guy cable before deformation
- **c**: distance between point A and point G
- **d**: distance between point G and point D
- **l_{s1}, l_{s2}**: length of the tensioned cables
- **T_1, T_2**: tensions of the two cables
- **E_s**: Young’s modulus of the cable
- **A_s**: cross-section area of the cable
- **a**: length of the cable projection onto x axis
- **h**: length of the cable projection onto z axis
- **φ**: half of the angle between the two cables
- **θ**: angle between the telescopic boom and the cables’ plane
- **I_i**: moment of inertia of the i-th section of the telescopic boom
- **l_i**: length from the top of the i-th section to the fixed root of the telescopic boom
- **L**: total length of the telescopic boom \((L = l_n)\)
- **E**: Young’s modulus of the telescopic boom
- **v**: Poisson ratio
- **K_0**: elastic stiffness matrix
- **K_g**: geometrical stiffness matrix
- **λ**: eigenvalue
- **P**: axial force of the telescopic boom \((P = F_x)\)
- **P_{cr}**: critical load of the telescopic boom
- **μ**: effective length factor
- **β_i**: ratios of the moment of inertia, \(β_i = I_{-i}/I_i\)