Simple experimental setup for producing polarization-entangled photons

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Abstract. Quantum entanglement is an important action in quantum mechanics. Basically, quantum entanglement of correlated photon pairs can be produced by spontaneous down-conversion process inside the two birefringence crystals. This work aims to create a pair of photons, i.e. signal and idler, that are entangled and to assay the relation between these photons by using polarization-entangled photon pairs to demonstrate quantum non-locality by comparing with the Malus’s law. From the experiments, the coincidence counts as a function of relative angle (α - β) between transmission axis of the polarizer and analyzer were obtained and the polarization entanglement curve was demonstrated. This result corresponding with polarization entanglement prediction term of ½ cos² (α - β) which confirmed the entanglement of photons.

1. Introduction
The non-local effects in quantum mechanics involve the measurement of the polarization state of entangled particles separated at a long distance called “action at a distance” [1-4]. The non-locality of the measuring process is a measurement of one particle that instantly collapses the states of both particles, even if they are not near each other [2]. This is a non-classical feature of quantum theory. When the down-conversion process produces photon pairs, they will share non-classical correlations. Bell [2, 3] considered alternative theories without non-local effects and observed an inequality showing that the local theories disagreed with quantum mechanics concerning the degree of the polarization correlation. Experimental tests are in good agreement with quantum mechanics and in disagreement with local theories [2].

In this work, the polarization-entangled photon pairs were produced by the Type-I spontaneous parametric down-conversion process in two beta-barium-borate (BBO) crystals. The researcher used the pairs to demonstrate quantum non-locality by comparing with Malus’s law represented as locality in classical mechanics by placing the polarizer and analyzer in the same path. The photon pairs were set to demonstrate the entanglement at different polarization angles to confirm Malus’s law. The results show that the results from this experimental setup are in good agreement with Malus’s law.

2. Theory

2.1. Type-I spontaneous parametric down-conversion process
The spontaneous parametric down conversion process is a studied non-linear optics effect that produces two photons called signal and idler photons from one pump photon. The birefringent crystal is used to
demonstrate the non-linear optics that convert an incident pump photon into two photons. This process requires energy conservation: 

\[ E_p = E_s + E_i \]

with 

\[ E = \hbar c k \]

so that 

\[ k_p = k_s + k_i \]  

(1)

where \( k \) is the wave number in the vacuum and \( c \) is the speed of light. The subscripts, \( p \), \( i \) and \( s \), are referred to as the pump, idler and signal photons, respectively. In this study, we treat only a degenerate down conversion where \( k_s = k_i = k_p/2 \). In addition to energy conservation, parametric down-conversion requires that the photon momentum \( \mathbf{p} \) is conserved inside the crystal. The momentum is related to the wave vector by the relation \( \mathbf{p} = h \mathbf{k} \). The wave number inside the crystal is \( |\mathbf{k}| = n k = 2 \pi n / \lambda \), where \( n \) is the index of refraction of the crystal [5]. This momentum conservation condition can be expressed in terms of the wave vectors as

\[ n_p k_p = 2 n_s k_s \cos \theta_c. \]  

(2)

Although the direction taken by down-conversion photons is of specific wavelength, other complementary wavelengths are determined by the phase-matching angle \( \theta_m \) formed by the optic axis of the crystal (OA) and the propagation direction of the pump beam.

![Diagram](image)

**Figure 1.** The signal and idler beams form a laboratory angle \( \theta_c \) with the pump beam outside the crystal.

Consider the case where the down-conversion photons leave the crystal as shown in figure 1. If \( \theta_c \) is the angle the signal and idler photons form with the direction of propagation of the pump beam inside the crystal, then equation (2) becomes

\[ n_p k_p = 2 n_s k_s \cos \theta_c. \]  

(3)

Next, combine \( k_s = k_p/2 \) with equation (3) and obtain

\[ n_p = n_s \cos \theta_c. \]  

(4)

It is not possible to satisfy the equation (4) in an isotropic medium because the normal dispersion in the index of refraction decreases with increasing wavelength as in \( n_p > n_s \). This problem can be overcome
with a birefringent crystal. The top red dashed line in figure 2 represents the ordinary index of refraction where the polarization of the light is perpendicular to the optic axis (OA) of the crystal [5].

![Index of refraction curves for a beta-barium-borate crystal with its optic axis aligned perpendicular (top/red dash curve) and parallel (bottom/blue solid curve) to the input polarization. The blue dash-dot middle curve corresponds to the phase-matching condition, $n_p = n_e^*(\theta_m) = 1.6603$.](image)

**Figure 2.** Index of refraction curves for a beta-barium-borate crystal with its optic axis aligned perpendicular (top/red dash curve) and parallel (bottom/blue solid curve) to the input polarization. The blue dash-dot middle curve corresponds to the phase-matching condition, $n_p = n_e^*(\theta_m) = 1.6603$.

If the polarization is in the same plane as OA, the index of refraction, also known as the extraordinary index of refraction, depends on the angle $\theta_m$ formed between the propagation direction $k_p$ and OA. The lower blue solid line in figure 2 corresponds to the case where $n_e = n_e^*(\theta_m = \pi/2)$ with

$$n_e^*(\theta_m) = \left(\sin^2 \theta_m/n_o^2 + \cos^2 \theta_m/n_e^2\right)^{1/2}$$

by means of $\theta_m$, the extraordinary index of refraction of $n_e^*$ between $n_o$ and $n_e$ is tuned. The graphs of the indices of refraction shown in figure 2 correspond to the negative uniaxial beta-barium-borate crystal with the index of refraction given by

$$n_e(\lambda) = [2.7359 + 0.01878/(\lambda^2 – 0.01822) – 0.01354 \lambda^2]^{1/2}$$

and

$$n_e(\lambda) = [2.3753 + 0.01224/(\lambda^2 – 0.01667) – 0.01516 \lambda^2]^{1/2}.$$  

For the beta-barium-borate, $n_o = 1.6603$ at 810 nm and $n_e = 1.5671$ at 405 nm are verified. Under the situation known as Type-I phase matching, the pump photon has an extraordinary index of refraction $n_e^*(\theta_m)$ and the down-conversion photons have an ordinary index of refraction. Suppose we want to have the two down-conversion photons collinear, $\theta_C = 0^\circ$ in the equation (4). By setting $\theta_m = 28.8^\circ$, the pump beam has an index of refraction represented by the blue dash-dot line in figure 2, or $n_p = n_e^*(\theta_m) = 1.6603$ at 405 nm. If the signal and idler beams form a laboratory angle of $\theta_L = 3^\circ$ with the pump beam outside the crystal as shown in figure 1, Snell’s law of $\sin \theta_l = n_e \sin \theta_C$, is used to obtain $\theta_C$. Then the phase matching angle $\theta_m$ is obtained to satisfy the equation (5). As a result, $\theta_C = 0.03154^\circ$ and the phase matching angle $(\theta_m)$ is 29.24° for 405 nm pump photon.
2.2. The polarization-entangled photon pairs (signal and idler)

It is now well-known that the photon pairs (signal and idler photons) produced via the down-conversion process share non-classical correlations. When a pump photon splits into two photons inside a non-linear crystal, the conservation of energy and momentum lead to entanglements between the pairs [3]. They are produced at nearly the same time. By appropriate angular placement of the detectors (corresponding with the momentum conservation), the signal and idler photons have the same wavelength corresponding with the energy conservation. The crystals are cut for Type-I down-conversion, in which the signal and idler photons have the same polarization, which is opposite to that of the pump photon. The polarization-entangled pair of photons are created by using two identical crystals with the polarization of one photon rotated at 90° from the other in the beam propagation direction, as shown in figure 3. In this arrangement, each crystal can support the down-conversion of one pump polarization.

A 45° polarized pump photon can down-convert in either crystal to produce a polarization-entangled pair of photons [1].

![Figure 3. The polarization-entangled photons are produced via the down-conversion process from two identical down-conversion crystals. The crystals oriented at 90° with respect to each other.](image-url)

The non-linear crystal in our experiment is a beta-barium-borate (BBO) crystal. The two BBO crystals are cut for Type-I phase matching, which means that the signal and idler photons emerge with the same polarization, which is orthogonal to that of the pump photon [5]. Each crystal can only support the down-conversion of one pump polarization. The other polarization passes through the crystal unchanged. In the present experiment, we used two crystals with one rotated at 90° from the other. By placing polarizers rotated to angles $\alpha$ and $\beta$ in the signal and idler paths, the polarization of the down-converted photon pairs are measured. The measurement of a pair in terms of the probability of coincidence detection is

$$P(\alpha - \beta) = \frac{1}{2} \cos^2 (\alpha - \beta)$$

which depends only on the difference between the angles, $\alpha - \beta$ [2]. However, the probability of coincidence detection is similar to Malus’s law at $I = I_{\text{max}} \cos^2 (\theta_1 - \theta_2)$ represented as locality in classical mechanics in which the polarizer and analyzer are placed in the same path where $\theta_1$ and $\theta_2$ are the transmission axis of the polarizer and analyzer, respectively.

3. Experimental setup

3.1. BBO crystals

In this work, the non-linear crystals were two beta-barium-borate (BBO) crystals, each with dimensions of 5 mm × 5 mm × 0.5 mm (width × length × thick). The crystals were cut with their crystal axes to produce signal and idler beams with half opening angles of 3° in the pump beam direction. The crystals
were mounted face-to-face with one crystal rotated at 90° from the normal to the large face (Newlight Photonics).

3.2. Experimental procedure
Figure 4 shows a schematic of the experimental setup to produce the polarization-entangled photon pairs. A 50 mW, 405 nm, diode laser was used as the source of the pump beam. The laser beam was collimated by a convex lens attached to the front of the laser to produce the beam with a diameter of approximately 1 mm at 1 m. The pump beam passed through the iris diaphragm and a Glan-Thompson calcite polarizer mounted on a rotating angle. The Glan-Thompson calcite polarizer was used to rotate the laser polarization at 45° from the polarizing axis. Next, the pump beam reflected from a dielectric mirror attached to a mount that could adjust the plane mirror and passed through the second iris diaphragm, passing through two BBO crystals mounted on a crystal holder that could be aligned to make the face crystals perpendicular to the laser beam direction. Then the down-converted photons called signal and idler photons were produced by the BBO crystals. Both photons with wavelengths of 810 nm passed through the iris diaphragms to the near-infrared polarizers, A and B, mounted on the rotating holders. These were collimated by collimators connected with fiber collimators and passing to detector A and detector B. Along the signal and idler photon paths, the collimators, filters, polarizers and iris diaphragms were mounted on an aluminum rails and separated by 3°.

![Figure 4. Schematic of experimental setup to produce the polarization-entangled photons.](image)

3.3. Coincidence measurement
In the coincidence measurement, the researchers needed to detect only the photons with wavelengths of 810 nm. Therefore, the measurements of the 810 nm bandpass filters were placed in front of the fiber collimators to allow only 810 nm photons to pass through. The detectors were silicon avalanche photodiodes called single-photon counting modules (SPCMs). The signal and idler photons received by detectors A and B in the form of square pulses with a pulse width of 25 ns. The single counts of photons by detectors A and B in one second were recorded at the same time. Next, both single counts \(N_A\) and \(N_B\) were transmitted to the coincidence circuit, and the photon pairs were recorded only when the pairs arrived at detectors A and B at the same time, which is called a coincidence count. The coincidence...
counts from the coincidence detection between detectors $A$ and $B$ ($N_{AB}$) were recorded with the LABVIEW program in a personal computer.

3.4. Demonstration of the polarization entanglement

In figure 4, polarizers $A$ and $B$ were used as a polarizer and an analyzer, respectively. In the experiment, polarizer $A$ was set at different angles ($\alpha$) of 0°, 45°, and 90°. When polarizer $A$ was set at each angle ($\alpha$), the polarizing angle of analyzer $B$ ($\beta$) varied from 0° to 360°. Then the single and coincidence counts were recorded.

4. Results and discussion

Figure 5(a) shows the data demonstrating the polarization-entanglement and coincidence counts ($N_{AB}$) in one second as a function of analyzer $B$ angle in the idler path ($\beta$). The coincidence counts at different angles ($\alpha$) of polarizer $A$ in the signal path display the expected quantum mechanical correlations with the equation (8). The squares ( ■ ), circles ( ● ) and triangles ( ▲ ) are the coincident counts in one second of signal photons at angles $\alpha = 0^\circ$, 45°, and 90°, respectively, while Angle $\beta$ was varied by rotating polarizer $B$ (analyzer) in the idler path. Figure 5(b) shows the coincidence counts at $\alpha = 90^\circ$ and single counts ($N_B$) in one second of idler photons (diamonds (♦), right axis). The single counts were essentially constant, i.e. the individual idler photons were nearly unpolarized, while the coincidence counts of the signal photons with angles of $\alpha = 90^\circ$ (triangles (▲)), show the expected quantum mechanical correlations. The solid curves in figures 5(a) and (b) are the best fits with the equation (8). It should be pointed out, however, that the measurements were carried out only five times for each coincidence measurement, which might have led to the high standard deviation as shown by the large error bar in figure 5.

![Figure 5](image)

**Figure 5.** (a) the coincidence counts at different angles ($\alpha$) and (b) the coincidence count at $\alpha = 90^\circ$ and single count of the idler photon.

5. Conclusion

A simple experiment was conducted to demonstrate the entanglement of photon pairs. The photon pairs (signal and idler photons) were produced by two BBO crystals of Type-I by using a diode laser at a wavelength of 405 nm. In this work, the photon pairs were set to demonstrate the entanglement at different polarizing angles ($\alpha$ and $\beta$) to confirm Malus’s law. The results show that the results from this experimental setup are in good agreement with Malus’s law. Therefore, this work can be used in experimental laboratories to gain greater understanding about non-locality in quantum mechanics.
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References
[1] Dehlinger D and Mitchell M W 2002 Am. J. Phys. 70(9) 898–902
[2] Dehlinger D and Mitchell M W 2002 Am. J. Phys. 70(9) 903–10
[3] Kwiat P G, Waks E, White A G, Appelbaum I and Eberhard P H 1999 Phys. Rev. A 60(2) R773–6
[4] Kwiat P G, Mattle K, Weinfurter H, Zeilinger A, Sergienko A V and Shih Y 1995 Phys. Rev. Lett. 75(24) 4337–41
[5] Galvez E J, Holbrow C H, Pysher M J, Martin J W, Courtemanche N, Heilig L and Spencer J 2005 Am. J. Phys. 73(3) 127–40