Neutrino Counting, NuTeV Measurements, Higgs Mass and $V_{us}$ as Probes of Vectorlike Families in ESSM/SO(10)

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Abstract

The Extended Supersymmetric Standard Model (ESSM), motivated on several grounds, introduces two vector-like families $[16 + \overline{16}$ of SO(10)] with masses of order one TeV. In an earlier work, a successful pattern for fermion masses and mixings (to be called pattern I) has been proposed within a unified SO(10)-framework, based on MSSM, which makes seven predictions, in good accord with observations, including $V_{cb} \approx 0.04$, and $\sin^2 2\theta_{\nu_{\mu} \nu_{\tau}} \approx 1$. Extension of this framework to ESSM, preserving the successes of pattern I, has been recently proposed in an accompanying paper, where it is noted that ESSM can provide a simple explanation of the indicated anomaly in $(g - 2)_{\mu}$. To exhibit new phenomenological possibilities which may arise within ESSM, we present here a variant pattern (to be called pattern II) for fermion masses and mixings, within the SO(10)/ESSM framework, which possesses the same degree of success as pattern I as regards the masses and mixings of all fermions including neutrinos. The main point of this paper is to first note that either one of these two patterns, embedded in ESSM, would lead to a reduction in the LEP neutrino-counting from $N_{\nu} = 3$ (in good agreement with the data) and also provide a simple explanation of the $(g - 2)_{\mu}$-anomaly, as pointed out in the accompanying paper. They can, however, be distinguished from each other by (a) a sharpening of our understanding of the true magnitude of the anomaly in $\nu_{\mu}$-nucleon scattering, recently reported by the NuTeV group, (b) improved measurements of $m_t$, $m_H$ and $m_W$, (c) improved tests of $\mu$ lepton-universality in charged current processes, and (d) improvements in the measurements of $V_{ud}$ and $V_{us}$. Pattern II (extended to ESSM) would predict departures from the standard model in the right direction with regard to (a) and (b), though not as regards (c) and (d) (as judged by the current data). Pattern I on the other hand would practically coincide with the standard model as regards its predictions for all four features: (a)-(d). Both patterns would predict some departure from the SM as regards tau lifetime. The probes listed above, and, of course, direct searches for the vectorlike families at the LHC and a future NLC can clearly test ESSM, and even distinguish between certain variants.
1 Introduction

The recently reported NuTeV result on $\nu$-nucleon scattering suggests that quite possibly there is an anomaly in $(\sigma_{NC}/\sigma_{CC})$-ratios ($R_\nu$ and $R_\bar{\nu}$) compared to expectations of the Standard Model. The results on $R_\nu$ and $R_\bar{\nu}$ have been interpreted in Ref. [1] to reflect either (a) a higher on-shell value of $\sin^2\theta_W$ which is at 3$\sigma$ above the value extracted from other experiments within the Standard Model (SM), or (b) a reduced coupling of the left-handed quarks to $Z^0$($g_{eff}^L$), compared to the SM value for the same. A third possibility has also been mentioned in the context of a two-parameter fit corresponding to a reduced overall strength ($\rho_0$) of the neutral current four fermion coupling together with a possible non-standard value of $\sin^2\theta_W$. If the result persists against (even) more precise data, and improved theoretical scrutiny involving Standard Model effects, it would clearly have some profound implications.

In addition to the NuTeV anomaly [counted as (a)], there also exist a few other possible discrepancies in the predictions of the Standard Model (SM). These in particular include: (b) the LEP neutrino-counting which is about 2$\sigma$ below the SM value of 3, (c) the empirical Higgs mass limit $m_H > 115$ GeV (while typically the SM would suggest $m_H < 100$ GeV, see later), (d) the measured value of $m_W = 80.446 \pm 0.040$ GeV (LEP data), which is typically higher than the SM value (see discussions below), and (e) the possible 1.6 to 3$\sigma$ discrepancy in $(g - 2)_\mu$ (for experimental and theoretical analysis, see respectively Ref. [4] and [5]) [6]. The purpose of this note is to seek for correlated explanations of the two indicated discrepancies (a) and (b), together with desired shifts in $m_H$ and $m_W$, in the context of an old idea motivated on several grounds (see below) [7, 8]. A simple explanation of the indicated anomaly in $(g - 2)_\mu$, together with associated tests through radiative processes ($\tau \to \mu \gamma$ and $\mu \to e \gamma$), in the context of the same idea, has been noted in an accompanying paper [9].

The idea in question pertains to the so-called “Extended SuperSymmetric Standard Model” (ESSM), which introduces two complete vectorlike families of quarks and leptons – denoted by $Q_{L,R} = (U,D,N,E)_{L,R}$ and $Q'_{L,R} = (U',D',N',E')_{L,R}$ – with masses of order few hundred GeV to one TeV. Both $Q_L$ and $Q_R$ transform as (2,1,4), while $Q'_{L}$ and $Q'_{R}$ transform as (1,2,4) of the quark-lepton unifying symmetry $G(224)=SU(2)_L \times SU(2)_R \times SU(4)_C$. Thus, together, they transform as a pair $16 + \overline{16}$ of SO(10), to be denoted by $16_V = (Q_L|Q'_R)$ and $\overline{16}_V = (\overline{Q}_R|Q'_L)$. The subscript “V” signifies two features: (a) $16_V$ combines primarily with $16_V$, so that the pair gets an SO(10)-invariant (thus SU(2)$_L \times U(1)$-invariant) mass-term of the form $M_V 16_V \cdot \overline{16}_V + h.c.$, at the GUT scale, utilizing for example the VEV of an SO(10)-singlet, where $M_V \sim$ few hundred GeV to one TeV [10], (b) since $Q_L$ and $Q_R$ are doublets of SU(2)$_L$, the massive four-component object $(Q_L \oplus Q'_R)$ couples vectorially to $W_L$’s; likewise $(Q'_L \oplus Q'_R)$ couples vectorially to $W_R$’s. Hence the name “vectorlike” families.

It has been observed in earlier works [11] that addition of complete vectorlike families...
[16+\overline{16} \text{ of } \text{SO}(10)], with masses \(\gtrsim 200 \text{ GeV}\) to one TeV (say), to the Standard Model naturally satisfies all the phenomenological constraints so far. These include: (a) neutrino-counting at LEP (because \(M_{N,N'} > m_Z/2\)), (b) measurement of the \(\rho\)-parameter [because the \(\text{SO}(10)\)-invariant mass for the vectorlike families ensure up-down degeneracy – i.e., \(M_U = M_D\), etc. – to a good accuracy], and (c) those of the oblique electroweak parameters [12] (for the same reasons as indicated above) [13]. We will comment in just a moment on the theoretical motivations for ESSM. First let us note why ESSM is expected to be relevant to the NuTeV anomaly and why it would simultaneously have implications for the LEP neutrino-counting. As a central feature, ESSM assumes that the three chiral families \((e, \mu\) and \(\tau)\) receive their masses primarily (barring corrections \(\lesssim \) a few MeV) through their mixings with the two vectorlike families \([7, 8]\). As we will explain in Sec. 2, this feature has the advantage that it automatically renders the electron family massless (barring corrections as mentioned above); and at the same time it naturally assigns a large hierarchy between the muon and the tau family masses, \textit{without} putting in such a hierarchy in the respective Yukawa couplings \([7, 8, 14]\). In short, ESSM provides a simple reason for the otherwise mysterious interfamily mass hierarchy, i.e., \((m_{u,d,e} \ll m_{c,s,\mu} \ll m_{t,b,\tau})\).

Now, since the chiral families get masses by mixing with the vectorlike families, the observed neutrinos \(\nu_i\) naturally mix with the heavy neutrinos \(N_L\) and \(N'_L\) belonging to the families \(Q_L\) and \(Q'_L\), respectively. The mixing parameters get determined in terms of fermion masses and mixings. As we will explain, it is the mixing of \(\nu_\mu\) and likewise of \(\nu_\tau\) with the \(\text{SU}(2)_L\)-singlet heavy lepton \(N'\) belonging to the family \(Q'_L\), that reduces the overall strengths of the couplings \(i) \ Z^0 \rightarrow \nu_\mu \bar{\nu}_\mu\), \(ii) \ Z^0 \rightarrow \nu_\tau \bar{\nu}_\tau\), as well as of \(iii) \ W^+ \rightarrow \mu^+ \nu_\mu\), and \(iv) \ W^+ \rightarrow \tau^+ \nu_\tau\), compared to those of the Standard Model, all in a predictably correlated manner. The forms of the couplings remain, however, the same as in the Standard Model.

In accord with the interfamily hierarchy of fermion masses and mixings, the reduction in the couplings as above is found to be family-dependent, being maximum in the \(\nu_\tau\), intermediate in \(\nu_\mu\) and small \(< 1\text{ part in } 10^4\) in the \(\nu_e\)-channel.

These effects would manifest themselves as (a) a deficit in the LEP neutrino-counting from the Standard Model value of \(N_\nu \equiv N_{\nu_e} + N_{\nu_\mu} + N_{\nu_\tau} = 3\), (b) as a correlated reduction in the strength of \(\nu_\mu N \rightarrow \nu X\) interaction (which is relevant to the NuTeV anomaly), as (c) small departures from lepton universality in the charged current processes, and as (d) a decrease in \(V_{ud}\) (depending upon \(u-U'\) mixing) compared to its SM value. Qualitative aspects of these effects arising from \(\nu_\tau-N'\)-mixing (without a quantitative hold on the reduction in the \(\nu_\mu \bar{\nu}_\mu\) and \(\nu_\tau \bar{\nu}_\tau\)-couplings to \(Z^0\)) were in fact noted in an earlier work [11] almost ten years ago. In that work, motivated by an (overly) simplified version of understanding the inter-family hierarchy, the effect of the \(\nu_\mu \bar{\nu}_\mu\)-channel was considered to be too small.

Two interesting developments have, however, taken place in the meanwhile. First, SuperK discovered atmospheric neutrino oscillations, showing that \(\nu_\mu\) oscillates very likely into \(\nu_\tau\)
with a surprisingly large oscillation angle: \( \sin^2 2\theta^{\text{osc}}_{\nu_\mu,\nu_\tau} \gtrsim 0.92 \) [13]. Second, motivated in part by the SuperK result, an economical SO(10)-framework, based on MSSM, has been proposed in the context of a minimal Higgs system \((10_H, 16_H, 16_H\overline{H} \text{ and } 45_H)\) to address the problem of fermion masses and mixings [16]. Within this framework, a few variant patterns of fermion mass-matrices are possible, each of which is extremely successful in describing the masses and mixings of all fermions including neutrinos. For example, the pattern exhibited in [16] (to be called pattern I) makes seven predictions, including \( V_{cb} \approx 0.042 \) and \( \sin^2 2\theta^{\text{osc}}_{\nu_\mu,\nu_\tau} \approx 0.85-0.99 \), all in accord with the data to within 10\% [17]. Extension of this SO(10)-framework to ESSM, preserving the successes of pattern I, has recently been proposed in an accompanying paper [9], where it is noted that ESSM can provide a simple explanation of the indicated anomaly in \((g - 2)\mu\) without requiring the presence of light sleptons.

To exhibit new phenomenological possibilities, which may arise in ESSM, we present here a variant pattern\(^\text{II}\) for fermion masses and mixings, within the SO(10)-framework, incorporating ESSM. This variant possesses the same degree of success as pattern I as regards the masses and mixings of all fermions including neutrinos. One of the main points of this paper is to note that both patterns (I or II), embedded in ESSM, would lead to a reduction in LEP neutrino-counting from \( N_\nu = 3 \), owing to a mixing of \( \nu_\mu \) and \( \nu_\tau \) with the singlet heavy neutrino \( N' \) (in good agreement with the data), and would also provide a simple explanation of the indicated anomaly in \((g - 2)\mu\), as pointed out in the accompanying paper [9].

Interestingly enough, we observe that the two variants (I and II) can clearly be distinguished from each other, however, by other phenomena. In particular, owing to a relative enhancement of \( \nu_\mu - N' \) mixing, pattern II, to be presented in sec. 3, can provide (a) a partial explanation of the anomaly in \( \nu_\mu \)-nucleon scattering reported recently by the NuTeV group, and simultaneously (b) an increase in the predictions for \( M_H \) and \( m_W \) compared to those of the standard model (in good accord with experiments), (c) small departures from \( e-\mu \) universality (up to 1 to 1.8\( \sigma \) deviation from experiments), and (d) small decreases in the effective values of \( V_{ud} \) and \( V_{us} \) compared to those in the standard model, which are currently disfavored by the data on grounds of unitarity of the \( 3 \times 3 \) CKM-matrix (this latter turns out to be well respected even for ESSM, especially for the sum of the absolute squares of the entries in the first row). Pattern I, proposed in [13], embedded in ESSM [1], on the other hand, practically coincides with the standard model as regards its predictions for all four features: (a)-(d). Both patterns (I and II) would lead to some departures from the standard model (though of differing magnitudes) as regards their predictions for the tau lifetime. Improved experimental and theoretical studies involving these five features can thus clearly distinguish between the two variants (I and II) and thereby shed light on GUT/string-scale physics.

We stress that the two variants I and II, embedded in ESSM, while differing as regards
(a)-(d), share the common feature that they both depart from the standard model as regards their predictions for $N_\nu$ and $(g - 2)_\mu$, in good accord with the present data.

Before discussing the relevance of ESSM to the phenomena mentioned above, a few words about motivations for ESSM might be in order. Note that it, of course, preserves all the merits of MSSM as regards gauge coupling unification and protection of the Higgs masses against large quantum corrections. Theoretical motivations for the case of ESSM arise on several grounds: (a) It provides a better chance for stabilizing the dilaton by having a semi-perturbative value for $\alpha_{\text{unif}} \approx 0.25$ to 0.3 \cite{8}, in contrast to a very weak value of 0.04 for MSSM; (b) It raises the unification scale $M_X$ \cite{5, 13} compared to that for MSSM and thereby reduces substantially the mismatch between MSSM and string unification scales \cite{13}; (c) It lowers the GUT-prediction for $\alpha_3(m_Z)$ compared to that for MSSM \cite{8}, as needed by the data; (d) Because of (b) and (c), it naturally enhances the GUT-prediction for proton lifetime compared to that for MSSM embedded in a GUT \cite{16, 20}, also as needed by the data \cite{21}; and finally, (e) as mentioned above, it provides a simple reason for interfamily mass-hierarchy. In this sense, ESSM, though less economical than MSSM, offers some distinct advantages.

In an accompanying paper \cite{9} we have noted that ESSM provides a simple explanation of the indicated anomaly in $(g - 2)_\mu$ and that such an explanation can be clearly tested by improved searches for $\tau \to \mu \gamma$ and $\mu \to e \gamma$-decays. The main purpose of this paper is two-fold: (1) First, we present in sec. 3 a variant pattern (called pattern II) of fermion masses and mixings within SO(10)-framework, which is interesting in its own right, in that it has the same degree of success as pattern I \cite{9, 10} in describing fermion masses and mixings, regardless of whether it is embedded in ESSM or not. (2) Second, we study (in secs. 4, 5 and 6) the phenomenological consequences of pattern II, embedded in ESSM and those for pattern I in sec. 7. We observe that pattern II (thus embedded) not only leads to a decrease in LEP neutrino counting from $N_\nu = 3$ (like pattern I), but also leads to departures from standard model predictions as regards (i) NuTeV-type measurements, (ii) masses of the Higgs and W-bosons, (iii) lepton-universality and (iv) effective values of $V_{ud}$ and $V_{us}$ (unlike pattern I). These issues are discussed in secs. 4, 5 and 6 and a comparison between the predictions of pattern I and pattern II is made in sec. 7. A summary is presented in sec. 8.

## 2 Fermion Masses and Mixings in ESSM

Following the discussion in the introduction (see Ref. \cite{8} for details and notation), the $5 \times 5$ mass-matrix involving the three chiral ($q_{L,R}$) and the two vectorlike families ($Q_{L,R}$ and $Q'_{L,R}$)
is assumed to have the see-saw form:

\[
M^{(0)}_{f,c} = \begin{pmatrix}
\bar{q}^i_R & Q_L \\
0_{3 \times 3} & X_f \langle H_f \rangle \\
Y'_f \langle H_f \rangle & z_c \langle H_V \rangle & 0 \\
X'_f \langle H_f \rangle & 0 & z'_f \langle H_V \rangle
\end{pmatrix}.
\]

(1)

Here the symbols \(q, Q\) and \(Q'\) stand for quarks as well as leptons; \(i=1, 2, 3\) corresponds to the three chiral families. The subscript \(f\) for the Yukawa-coupling column matrices \(X_f\) and \(X'_f\) denotes \(u, d, l\) or \(\nu\), while \(c=q\) or \(l\) denotes quark or lepton color. The fields \(H_f\) with \(f = u\) or \(d\) denote the familiar two Higgs doublets, while \(H_s\) and \(H_V\) are Higgs Standard Model singlets \([23]\), whose VEVs are as follows: \(\langle H_V \rangle \equiv v_0 \sim 1\ \text{TeV} \gtrsim \langle H_s \rangle \equiv v_s \gtrsim \langle H_u \rangle \equiv v_u \sim 200\ \text{GeV} \gg \langle H_d \rangle \equiv v_d\). The zeros in Eq. (1), especially the direct coupling terms appearing in the upper \(3 \times 3\) block, are expected to be corrected so as to lead to masses \(\lesssim\) a few MeV.

The parametrization in Eq. (1) anticipates that differences between \(z_c\) and \(z'_f\), between \(X_f\) and \(X'_f\), and between \(Y_c\) and \(Y'_f\) may arise at the electroweak scale in part because renormalization effects distinguish between \(Q_{L,R}\), which are SU(2)\(_L\)-doublets, and \(Q'_{L,R}\), which are SU(2)\(_L\)-singlets (see Eq. (10) of Ref. [8]), and in part because \((B-L)\)-dependent and \(L-R\) as well as family-antisymmetric contributions may arise effectively by utilizing the VEV of a 45-plet, sometimes in conjunction with that of a 10\(_H\) (see Refs [10] and [11] for details), which can introduce differences between \(X_f\) and \(X'_f\), etc.

Denoting \(X^T_f = (x_1, x_2, x_3)_f\) and \(Y^T_c = (y_1, y_2, y_3)_c\), it is easy to see [8] that regardless of the values of these Yukawa couplings, one can always transform the basis vectors \(\bar{q}^i_R\) and \(q^i_L\) so that \(Y^T_c\) transforms into \(\hat{Y}^T_c = (0, 0, 1)y_c\), \(X^T_f\) simultaneously into \(\hat{X}^T_f = (0, p_f, 1)x_f\), \(X'^T_i\) into \(\hat{X}'^T_i = (0, p'_f, 1)x'_f\) and \(Y'^T_i\) into \(\hat{Y}'^T_i = (0, 0, 1)y'_c\). It is thus apparent why one family remains massless (barring corrections of \(\lesssim\) a few MeV), despite lack of any hierarchy in the Yukawa couplings \((x_i)_f\) and \((y_i)_c\), etc. This one is naturally identified with the electron family. To a good approximation, one also obtains the relations [8]: \(m^0_{c,s,\mu} \approx m^0_{b,\tau}(p_f p'_f/4)\). Even if \(p_f, p'_f\) are not so small (e.g. suppose \(p_f, p'_f \sim 1/2\) to 1/4), their product divided by four can still be pretty small. One can thus naturally get a large hierarchy between the masses of the muon and the tau families as well. We note that while the transformation of the \(X, X', Y\) and \(Y'\) matrices into their hat-forms as above demonstrates the masslessness of the electron family, such transformations would in general induce some mixing among the three chiral families, which could be important for the first family. Ordinarily we may, however, expect the entries of the \(X\) and \(X'\) matrices in the gauge basis to be hierarchical \((x_1 \ll x_2 \ll x_3, \text{etc.})\) owing to flavor symmetries. In this case, such mixings would be unimportant. We will assume this to be the case in the text, but allow for possible departures from this assumption in the appendix.

As shown in Ref. [4], the SO(10) group-structure of the (2,3)-sector of the effective 3\(\times\)3
mass matrix for the three chiral families, proposed in Ref. [16], can be preserved (to a good approximation) for the case of ESSM, simply by imposing an SO(10)-structure on the off-diagonal Yukawa couplings of Eq. (1), that is analogous to that of Ref. [16] (see [23]), while small entries involving the first family can be inserted, as in Ref. [16], through higher dimensional operators. (We refer the reader to Ref. [9] and to a forthcoming paper [24] devoted entirely to “fermion masses in ESSM” for more details.)

It is the Dirac mass-matrices of the neutrinos and of the charged leptons that are relevant to the present paper. In the hat-basis mentioned above, where the first family is (almost) decoupled from the two vectorlike families, the Dirac mass-matrix of the neutrinos (following notations of Ref. [16] and [9]) is given by [25]:

\[
M^D_\nu = \begin{pmatrix}
\nu^c_L & \nu^\mu_L & \nu^\tau_L \\
0_{3\times3} & N_L & N'_L \\
(0, 0, 1) & \kappa^\nu_u & M_N \\
M'_N & 0 & \kappa^\nu_s
\end{pmatrix}
\]

Here, \(\kappa^\nu_u \equiv x_{\nu}(H_u), \kappa^\nu_l \equiv x_l(H_u), \kappa^l_s \equiv y_l(H_s), \kappa^\nu_u \equiv y^\nu_l(H_s), \) \(M_N \approx M_E = z_l(H_V), \) \(M'_N \approx M'_E = z'_l(H_V).\) The mass matrix for the charged leptons is obtained by replacing the suffix \(\nu\) by \(l\) and \(u\) by \(d\), so that \(H_u \to H_d, \kappa^\nu_u \to \kappa^l_d, \) but \(\kappa^l_s \to \kappa^l_s, \) etc. Analogous substitution give the mass matrices for the up and down quarks. We stress that the parameters of the mass-matrices of the four sectors \(u, d, l, \) and \(\nu, \) and also those entering into \(X\) versus \(X'\) or \(Y\) versus \(Y'\) in a given sector, are of course not all independent, because a large number of them are related to each other at the GUT-scale by the group theory of SO(10) and the representation(s) of the relevant Higgs multiplets [26]. For convenience of writing, we drop the superscript \(\nu\) on kappas, from now on.

We now proceed to determine some of these parameters in the context of a promising SO(10)-model, which would turn out to be especially relevant to the NuTeV-anomaly and the LEP neutrino counting.

3 A Variant Pattern For Fermion Masses Within the SO(10)/ESSM Framework

Following the approach of [16] we now present a concrete example wherein the effective mass-matrices for ESSM, exhibited in Eq. (1) and (2), emerge from a unified SO(10) framework, and which turns out to be relevant to the NuTeV anomaly. The pattern of the mass-matrices for the three light families in the \((u, d, l, \nu)\)-sectors, which result from this example
upon integrating out the heavy families \((Q\text{ and }Q')\), turns out to be a *simple variant* of the corresponding pattern presented in Ref. [16]. The ESSM-extension [9] of the pattern of Ref. [16] will be called Model I, while that of the variant presented here will be called Model II. The variant preserves the economy (in parameters) and the successes of Ref. [16] as regards predictions of the masses and mixings of quarks as well as leptons including neutrinos; these include \(V_{cb} \simeq 0.04\) and \(\sin^2 2\theta_{\nu\mu\nu\tau} \approx 1\). At the same time, the variant, extended to ESSM (Model II), turns out to be relevant to partially account for the NuTeV-anomaly and simultaneously for the LEP data on neutrino counting, while Model I [9, 16] can account for the LEP neutrino counting, but not for the NuTeV anomaly.

Let us first assume for the sake of simplicity that the electron family is (almost) decoupled from the heavy families \((Q\text{ and }Q')\) in the gauge-basis – that is to say, the gauge and the hat–basis (defined earlier) are essentially the same, so that Eq. (2) holds to a good approximation, already in the gauge basis. (In the appendix we will consider possible departures from this assumption.) Consider then the following superpotential, which involves the \(\mu\) and the \(\tau\) families \((16_2\text{ and }16_3)\) and the two vector-like families \((16^V\text{ and }\overline{16}^V)\):

\[
W_{\text{Yuk}} = h V_{16^V \overline{16}^V} H V + h_{3V} 16_3 16_6 10_H + h'_{3V} 16_3 16_6 16_H 16^d_H / M \\
+ h_{3V} 16_5 16^V H_s + h_{2V} 16_2 16_6 10_H 45_H / M + h'_{2V} 16_2 16_6 16_H 16^d_H 16_H / M .
\]  

(3)

Contrast this from the superpotential of Model I given in Ref. [4]. Here, \(\langle 16_H \rangle \sim \langle 45_H \rangle \sim \langle X \rangle \sim M_{\text{GUT}},\) and \(M \sim M_{\text{string}},\) with \(X\) being an SO(10) singlet and \(\langle 45_H \rangle\) being proportional to \(B - L\). As mentioned before, \(\langle H_V \rangle \sim 1 \text{ TeV} > \langle H_s \rangle \sim \langle H_u \rangle \sim 200 \text{ GeV} \gg \langle H_d \rangle\). \(10_H\) contains the Higgs doublets \(H_u\) and \(H_d\) of MSSM. While \(H_u = 10^u_H\), the down type Higgs is contained partly in \(10^d_H\) and partly in \(16^d_H\) (see Ref. [16]) – that is \(H_d = \cos \gamma 10^d_H + \sin \gamma 16^d_H\). If \(\sin \gamma = 0\), one would have \(\tan \beta = \frac{m_t}{m_b}\), but with \(\cos \gamma \ll 1\), \(\tan \beta\) can have small to intermediate values of \(3 - 20\). \((\tan \beta / \cos \gamma = \frac{m_t}{m_b}\) is fixed.) The entries in Eq. (3) with a factor \(1/M\) are suppressed by \(M_{\text{GUT}}/M \sim 1/10\). Note however that the contribution from \(h_{3V}\) and \(h'_{3V}/M\) terms to the down quark and charged lepton mass matrices could be comparable, \(\cos \gamma \sim 1/10\), which is what we adopt.

One can verify that Eq. (3) will induce mass–matrices of the type shown in Eqs. (1) and (2) with definite correlations among \(X_f, X'_f, Y_c\) and \(Y'_c\) sectors. To see these correlations, it is useful to block–diagonalize the \(5 \times 5\) mass matrix given by Eq. (2) and its analogs, so that the light families \((e, \mu\text{ and }\tau)\) get decoupled from the heavy ones. From now on, we denote the gauge basis (in which Eqs. (1) and (2) are written) by \(\psi_{L,R}^0\) and the transformed basis which yields the block–diagonal form by \(\psi_{L,R}'\). Given the SO(10) group structure of Eq. (3), it is easy to see that the effective Dirac mass matrices of the muon and the tau families in the up, down, charged lepton and neutrino sectors, resulting from block-diagonalization,
would have the following form at the GUT scale \[27\], which is referred to as pattern II:

\[
\begin{align*}
M_u &= \begin{pmatrix} 0 & -\epsilon \\ \epsilon & 1 \end{pmatrix} M^0_u, \\
M_d &= \begin{pmatrix} 0 & \eta - \epsilon \\ \eta + \epsilon & 1 + \xi \end{pmatrix} M^0_d, \\
M^D_\nu &= \begin{pmatrix} 0 & 3\epsilon \\ -3\epsilon & 1 \end{pmatrix} M^0_\nu, \\
M_l &= \begin{pmatrix} 0 & \eta + 3\epsilon \\ \eta - 3\epsilon & 1 + \xi \end{pmatrix} M^0_l.
\end{align*}
\]

(4)

Here the matrices are written in the primed basis (see above), so that the Lagrangian is given by \[\mathcal{L} = \overline{\psi}_R M \psi'_L + h.c.\]. (These should be compared with the transpose of the corresponding matrices in Ref. \[16\].) It is easy to verify that the entries \(1\) in ESSM compared to MSSM, the predicted value of \(m\) are in good agreement with observations (to within 10%). Owing to the larger QCD effect would have the following form at the GUT scale \[27\], which is referred to as pattern II:

As mentioned earlier, there are only three independent parameters \((\eta, \epsilon, \xi)\), leading to non-trivial correlations between observables.

The eight \(p\)-parameters of Eq. \[2\] and its analogs can be readily obtained from Eqs. \(3\) and \(4\). They are (for the present case of Model II):

\[
\begin{align*}
p_u &= -2\epsilon, \quad p'_u = 2\epsilon, \quad p_d = \frac{2(\eta - \epsilon)}{1 + \xi}, \quad p'_d = \frac{2(\eta + \epsilon)}{1 + \xi}, \\
p_\nu &= 6\epsilon, \quad p'_\nu = -6\epsilon, \quad p_l = \frac{2(\eta + 3\epsilon)}{1 + \xi}, \quad p'_l = \frac{2(\eta - 3\epsilon)}{1 + \xi}.
\end{align*}
\]

(5)

As mentioned earlier, there are only three independent parameters \((\eta, \epsilon, \xi)\), leading to non-trivial correlations between observables.

The matrices of Eq. \[1\] can be diagonalized in the approximation \(\epsilon, \eta \ll \xi, 1\). One obtains (for pattern II):

\[
\begin{align*}
\frac{m_b^0}{m_t^0} &\approx 1 - \frac{8|\epsilon|^2}{|1 + \xi|^2}, \\
\frac{m_\mu^0}{m_\tau^0} &\approx |\epsilon|^2, \\
\frac{m_\mu^0}{m_b^0} &\approx \frac{|\eta^2 - \epsilon^2|}{|1 + \xi|^2}, \\
|V_{cb}^0| &\approx \frac{|\epsilon\xi - \eta|}{|1 + \xi|}.
\end{align*}
\]

(6)

Here the superscript “\(0\)” denotes that these relations hold at the unification scale. A reasonably good fit to all observables can be obtained (details of this discussion will be given in a separate paper \[24\]) by choosing

\[
\begin{align*}
\epsilon &= -0.05, \quad \eta = 0.0886, \quad \xi = -1.45, \quad (\text{Model II})
\end{align*}
\]

(7)

which leads to \[28\] \(m_\mu^0/m_\tau^0 \approx 1/17.5, \quad m_\mu^0/m_b^0 \approx 1/44.5, \quad m_\mu^0/m_t^0 \approx 1/400, \quad |V_{cb}^0| \approx 0.031\). After renormalization group extrapolation is used (using \(\tan \beta = 10\) for definiteness) these values lead to \(m_e(m_e) = 1.27\text{ GeV}, \quad |V_{cb}| = 0.036, \quad m_s(1\text{ GeV}) = 160\text{ MeV}, \) all of which are in good agreement with observations (to within 10%). Owing to the larger QCD effect in ESSM compared to MSSM, the predicted value of \(m_b(m_b)\) is about 20% larger than the experimentally preferred value. Allowance for either larger values of \(\tan \beta \approx 35 - 40\) \[29\],
or gluino threshold corrections, and/or a 20% $B - L$ dependent correction to the vector family mass at the GUT scale (see Ref. [9]) could account for such a discrepancy.

Eqs. (5) and (7) lead to (8):

\[
\begin{align*}
p_\nu &= -0.30, \quad p'_\nu = +0.30, \quad p_l = 0.282, \quad p'_l = -1.05 \\
p_u &= 0.10, \quad p'_u = -0.10, \quad p_d = -0.607, \quad p'_d = -0.163.
\end{align*}
\]

(Model II)

Now, the light neutrino masses are induced by the seesaw mechanism. In addition to Eq. (3), there are terms in the superpotential that induce heavy Majorana masses for the right handed neutrinos of the three chiral families: $W' \supset \nu'_R M_R \nu'_R$. Let us assume that the matrix $M'_R$ for the dominant $\nu_\mu - \nu_\tau$ sector has the simple form

\[
M'_R = M_R \begin{pmatrix} 0 & y \\ y & 1 \end{pmatrix}
\]

as in Ref. [10]. (We are ignoring here the masses and mixings of the first family. Their inclusion will modify the present discussion only slightly.) The effective light neutrino mass matrix for the $\nu_\mu - \nu_\tau$ sector is then

\[
M^\text{light}_\nu = \frac{1}{y^2 M_R} \begin{pmatrix} 0 & ye^2 \\ ye^2 & \epsilon^2 + 2\epsilon y \end{pmatrix}.
\]

The $\nu_\mu - \nu_\tau$ oscillation angle is then

\[
\theta_{\nu_\mu,\nu_\tau} \simeq \left| \frac{ye^2}{\epsilon^2 + 2\epsilon y} - \frac{\eta - 3\epsilon}{1 + \xi} \right|.
\]

The second term in Eq. (10), $(\eta - 3\epsilon)/(1 + \xi) \simeq -0.53$, arises from the charged lepton sector, while the first term, arising from the neutrino sector is approximately equal to $\sqrt{m_{\nu_2}/m_{\nu_3}}$. Varying $m_{\nu_2}/m_{\nu_3}$ in the range $(1/25 - 1/8)$, so as to be compatible with the solar and the atmospheric neutrino oscillation data, we find, for $y$ being positive, that $y = (1/42.5 \text{ to } 1/44.8)$, and for this range of $y$, $\sin^2 2\theta^\text{osc}_{\nu_\mu,\nu_\tau} \approx (0.95 - 1.0)$. (Such a hierarchical value of $y$ goes well with the flavor symmetries that were assumed in the Dirac mass matrices.) We remark that the inclusion of the first family can be carried out in the context of ESSM as in Ref. [10], which would lead to eight predictions for the masses and mixings of quarks and leptons.

4 LEP Neutrino Counting and NuTeV Anomaly in the ESSM Framework: Applications To Model II

Having described the general framework, we now proceed to show how ESSM can modify expectations for neutral current interactions at NuTeV as well as neutrino counting at LEP.
Although the analytical formulae to be derived here in terms of the parameters $p_l, p'_l, p_\nu$ and $p'_\nu$ [see Eq. (2)] are general, and thus apply to both Model I and Model II, we will apply them only to Model II in this section and in the next two. The consequences for Model I will be discussed in sec. 7. Since the system is quite constrained by its structure and symmetries, in particular by its description of fermion masses and mixings, we will see that the consequences for NuTeV and LEP, and also for charged current interactions are fully determined within the model in terms of a single parameter $\eta_\nu = \kappa'_\nu / M_N$. To see these, we have to go from the gauge basis, in which the mass matrices of Eqs. (1) and (2) are written, to the mass eigenbasis for the charged and the neutral leptons. The same transformation should then be applied to the neutral currents and the charged currents.

The diagonalization of the mass matrices can be carried out in two steps. Consider the Lagrangian term $\bar{\psi}_R M \psi^0_L$, where $\psi^0_{L,R}$ may stand for fermions in any of the four sectors of ESSM in the gauge basis. The mass matrix $M$ has a form as shown in Eq. (2). Let us write it as

$$M = \begin{pmatrix} 0 & A \\ B & Z \end{pmatrix},$$

where $0$ is a $3 \times 3$ block matrix with all its entries equal to zero, $A$ is a $3 \times 2$ matrix, $B$ is a $2 \times 3$ matrix and $Z$ is a $2 \times 2$ matrix. The transformation $\psi'_{L,R} = U_{L,R} \psi^0_{L,R}$, where

$$U_{L,R} = \left( \begin{array}{cc} 1 - \frac{1}{2} \rho_{L,R} \rho_{L,R}^+ & \rho_{L,R}^+ - \frac{1}{2} \rho_{L,R} \rho_{L,R} \\ -\rho_{L,R} & 1 - \frac{1}{2} \rho_{L,R}^+ \rho_{L,R} \end{array} \right) + O(\rho_{L,R}^3)$$

and $\rho_{L} = Z^{-1} B, \rho_{R}^+ = A Z^{-1}$ will bring $M$ to a block-diagonal form, in which the three light chiral families get decoupled from the two heavy ones. The effective mass matrix from the light sector is given as $M_{\text{light}} = -AZ^{-1}B$. Such a block diagonalization may, of course, conveniently be achieved by a two-step process involving: (a) the transformation of the $(X, Y, X', Y')$-matrices from the gauge to the hat-basis which decouples the electron-family from the rest [see Eq. (3)], and (b) the subsequent block-diagonalization of the $4 \times 4$-matrix which decouples the $\mu$ and the $\tau$-families from the vectorlike families.

For simplicity of writing, we will ignore here the mixings of the fermions in the first (i.e., the electron) family with those of others, which could be introduced in step (a). Ordinarily, we would expect these mixings to be negligible if the $X$ and $X'$ matrices are sufficiently hierarchical (i.e., $x_1 \ll x_2 \ll x_3$). We will discuss the possible importance of such mixings in Appendix B. Let us apply the procedure just described to the charged lepton sector. The effective $\mu' - \tau'$ mixing matrix is found to be

$$M_{\mu' - \tau'} = \begin{bmatrix} 0 & p_l \kappa'_d \kappa'_s / M'_{E} \\ p_l \kappa'_d \kappa'_s / M_{E} & \kappa'_d \kappa'_s / M_{E} + \kappa'_d \kappa'_s / M'_{E} \end{bmatrix}$$

(12)
where \( M_E = M_N \) [see Eq. (1)] by \( SU(2)_L \) symmetry. The physical \( \mu \) and \( \tau \) leptons, denoted by \( \mu_{L,R} \) and \( \tau_{L,R} \) are then

\[
\begin{align*}
\mu'_{L,R} &= c_{L,R} \mu_{L,R} + s_{L,R} \tau_{L,R} \\
\tau'_{L,R} &= -s_{L,R} \mu_{L,R} + c_{L,R} \tau_{L,R}
\end{align*}
\]

(13)

with \( c_L = \cos \theta_L, s_L = \sin \theta_L, \) etc. From Eq. (12) we have \( \theta_R \approx \frac{p_1}{2} \) and \( \tan \theta_L \approx \frac{p_1}{2} \) (where we have set \( \kappa_s = \kappa'_s, \kappa_d = \kappa'_d \)). Note that \( \mu_L - \tau_L \) mixing can be quite large in our framework [see Eq. (8)], while \( \theta_R \) is small (so that the correct \( \mu - \tau \) mass hierarchy is reproduced), hence the use of \( \theta_L \), rather than \( \theta_R \).

Applying the same transformation to the relevant neutral current of the charged lepton: 
“\( J_{Z^0}^{NC} = \overline{\psi}^0_{L,R} diag(1,1,1,1,a_L) \psi^0_L + \overline{\psi}^0_{L,R} diag(1,1,1,1,a_R) \psi^0_R \)”, where \( \psi^0_{L,R} = (e^0, \mu^0, \tau^0, E^0, E^{0'})_{L,R} \), will lead to the following new couplings of the \( Z^0 \) boson to the leptons (i.e., in addition to their Standard Model couplings):

\[
\Delta \mathcal{L}_{NC}^{\text{leptons}} = -\frac{gZ^0}{2 \cos \theta_W} (a_L - 1) \eta_d^2 \left[ (s_Lp'_L - s_L)^2 \overline{\tau}_L \mu_L + (s_Lp'_L + c_L)^2 \overline{r}_L \tau_L \\
+ (c_Lp'_L - s_L)(s_Lp'_L + c_L) \overline{\eta}_L \tau_L + \overline{\tau}_L \mu_L \right].
\]

(14)

Here we have defined \( \eta_d = \kappa'_d/M_E \). We shall also use related quantities \( \eta'_u = \kappa'_u/M_{E'} \), \( \eta'_d = \kappa'_d/M_N \) and \( \eta'_u = \kappa'_u/M_{N'} \). The new interactions of \( Z^0 \) with \( \mu_R \) and \( \tau_R \) can be obtained from Eq. (14) by the replacement \( L \to R, p'_L \to p_l, \eta'_d \to \eta_d \). Here, \( a_{L,R} \) are defined as \( a_{L,R} = T_3 - Q \sin^2 \theta_W \).

To obtain numerical estimates of violations flavor and of universality, we note that to a very good approximation, \( \eta'_u = \eta_u, \eta'_d = \eta_d \), and \( \kappa'_u = \kappa_u \) (in all four sectors). We then have \( \eta'_d/\eta'_u \approx m_b/m_t \approx 1/60 \). Since violations of universality and flavor-changing effects in the up and neutrino sectors can at most be about 1-2%, we expect \( \eta_u \approx 1/8-1/10 \). Such a magnitude for \( \eta_u \) is quite plausible \[31\]. \( \eta_d \) is then \( \approx 1/500 \), leading to extremely tiny effects in the charged lepton sectors. For example, the ratio \( \Gamma(Z \to \mu^+\mu^-)/\Gamma(Z \to \tau^+\tau^-) \) deviates from the Standard Model value only by about 1 part in \( 10^5 \). The decay \( Z^0 \to \mu^+\mu^- \) has a rate proportional to \( \eta_d^4 \approx 10^{-10} \).

Violations of flavor and universality, analogous to those in Eq. (14) exist in the quark sector as well. For charm and top quarks, such effects are larger, by a factor of \( (m_t/m_b)^2 \) in the amplitude, compared to the charged lepton sector. The \( Z^0 \to c\bar{c} \) coupling deviates from the Standard Model value by an amount given by \( (p'_u/\eta_u)^2 \) or \( (p_u/\eta_u)^2 \) \[32\]. Owing to the smallness of \( |p_u| \) and \( |p'_u| \) (\( \approx \pm 0.1 \) in the example given in the previous section), the deviation of the rate for \( Z^0 \to c\bar{c} \) from the Standard Model value is only about \( 10^{-4} \).

The interesting feature having its origin in the SO(10) group theorectic structure of Eq. (3) is that while non-universality in neutral current interactions involving quarks and the charged leptons is extremely tiny, it is not so in the neutral lepton sector. This difference affects both NuTeV neutral current cross section and LEP neutrino counting. There are
two reasons for the difference. First, LEP neutrino counting is sensitive to the $Z^0 \rightarrow \nu_\mu \bar{\nu}_\tau$ coupling (while $Z^0 \rightarrow t\bar{t}$ is kinematically forbidden). Second, since the $\nu_\mu - \nu_\tau$ oscillation angle is large, as required by SuperKamiokande, and also as predicted by our framework, the effective $p'_t$ parameter is large ($\approx -1.05$), unlike the case for charm ($p'_u \approx -0.10$). To see the effects more concretely we need to diagonalize the neutral lepton mass matrix of Eq. (4), to which we now turn.

In addition to Eq. (2), the three $\nu'_R$ fields have superheavy Majorana masses parametrized by a matrix $M'_{R}$. Once the $\nu'_R$ are integrated out, small masses for $N_L$ and $N'_L$ fields will emerge. (There is no direct $\nu'_L \nu'_L$ mass term after seesaw diagonalization because of the structure of Eq. (2).) Let us write these effective mass terms (of order eV or less) as

$$L_{\text{mass}}^{\text{eff}} = m_{11}N^0_LN^0_L + m_{22}N^0_LN'^0_L + 2m_{12}N^0_LN'^0_L.$$  

If we denote $(M'_{R})^{-1} = a_{ij}$, the mass terms are $m_{11} = \kappa_u^2(a_{33} + 2a_{23}p_\nu + a_{22}p'^2_\nu)$, $m_{22} = \kappa_s^2a_{33}$, $m_{12} = \kappa_u\kappa_s(a_{33} + p_\nu a_{23})$. The $N^0_L$ and $N'^0_L$ fields have Dirac masses (by combining with $N^0_R$ and $N'^0_R$ respectively) of order few hundred GeV; they also possess non-diagonal Dirac mass mixing terms involving the light neutrinos. Upon identifying the light components, Eq. (15) will generate small Majorana masses of the standard left-handed neutrinos.

We can block diagonalize the Dirac mass matrix of the neutral leptons which is obtained from Eq. (2) after integrating out the superheavy $\nu'_R$ fields. This can be done by applying a unitary transformation on $(\nu'^0_R, \nu'^0_L, N'^0_L)$ fields. Note that the $(N_R, N'_R)$ fields do not mix with the light neutrinos, since $\nu'_R$ are superheavy. [We remind the reader that (for the sake of simplicity) we ignore for the present the mixing of the fermions of the first family with those of the vectorlike families. Such mixings will have a very small effect on LEP neutrino-counting, which we will incorporate, and likewise on NuTeV measurements. Possible consequences of such mixing are discussed in Appendix B.] Define

$$a = p'_\nu \eta_\nu, \quad b = \eta_\nu, \quad c = \kappa'_s/M_N = \eta_\kappa, \quad N_2 = \sqrt{1 + a^2 + b^2}, \quad N_3 = \sqrt{1 + b^2 + c^2}, \quad N_4 = \sqrt{(1 + a^2)(1 + b^2) + c^2}.$$  

The transformation $(\nu'_2, \nu'_3, \nu'_4, \nu'_5)^T = U^\nu(\nu'_0, \nu'_R, N, N')^T_L$, where

$$U^\nu = \begin{pmatrix}
N_3 & -ab & -abc & a(1+c^2) \\
N_2 & a & 0 & -abc \\
0 & N_3 & N_2 & N_4 \\
-abc & c(1+a^2) & N_2 & N_4 \\
\frac{a}{N_2} & \frac{b}{N_2} & 0 & \frac{1}{N_2}
\end{pmatrix}$$  

(17)

block diagonalizes the Dirac mass entries, so that there is no mixing between the massless states $(\nu'_2, \nu'_3)$ and the massive states $(\nu'_4, \nu'_5)$. From Eq. (17), one can read off the light mass eigenstate components in the original fields defined in the gauge basis. For example, $\nu^0_\mu = (N_3/N_4)\nu'_2 + ..., \quad N_L = [abc/(N_3N_4)]\nu'_2 - (c/N_3)\nu'_4 + ...$, etc, where the dots denote the
heavy components, which we drop since they are not kinematically accessible to NuTeV and LEP. Once these heavy components are dropped, the resulting states are not normalized to unity and it is this feature that is relevant to the NuTeV anomaly and LEP neutrino counting.

As a digression, we may mention that when the light neutrino mass matrix resulting from Eq. (13) is written in terms of \((\nu_2', \nu_3')\) fields, it is given approximately (by setting \(a_{33} = 0\), and \(a_{23} \ll a_{22}\), both of which follow from the form of \(M_R^\nu\) given before) by,

\[
M_\nu^{\text{light}} \simeq \begin{pmatrix}
0 & a_{23}p_{\nu} \\
\kappa_2^2 & a_{22}p_{\nu}^2 + 4a_{23}p_{\nu}
\end{pmatrix} \frac{\kappa_1^2\kappa_2^2}{M_N^2},
\]

Eq. (18) is of course completely equivalent to Eq. (9), except for having a reparametrization, and thus preserves the prediction of large \(\nu_\mu-\nu_\tau\) oscillation angle [see discussion below Eq. (9)].

Having identified the light neutrino states through Eq. (17), we can calculate the correction to neutrino counting at LEP. In the gauge basis, the \(Z^0\) coupling is given by \([g/(2\cos \theta_W)]Z^0[\bar{\nu}_e\nu_\mu + \bar{\nu}_\mu\nu_\mu + \bar{\nu}_\tau\nu_\tau + \bar{N}_L^0N_L^0 + \bar{N}_R^0N_R^0]\). The last term does not affect \(N_\nu\) (the number of light neutrinos counted at LEP), since \(N_R\) is heavier than \(Z\), and since it has no mixing with the light neutrinos. Applying the transformation of Eq. (17) we find that owing to mixing of the current eigenstates \(\nu_\mu\) and \(\nu_\tau\) with \(N'\), the couplings of \(Z^0 \rightarrow \bar{\nu}_e\nu_\mu\) and \(Z^0 \rightarrow \bar{\nu}_\tau\nu_\tau\) are reduced compared to their standard model values by the factors \((1 - \delta_\mu)\) and \((1 - \delta_\tau)\) respectively, where

\[
\delta_\mu \equiv \eta_\mu^2(c_Lp_{\nu}^\prime - s_L)^2; \quad \delta_\tau \equiv \eta_\mu^2(c_L + s_Lp_{\nu}^\prime)^2.
\]

In this description, we have ignored corrections of order \(\eta_\mu^4 \lesssim 10^{-4}\) [see Eqs. (21)-(23) for an accurate description]. The corresponding angles for mixing of \(\nu_\mu\) with \(\nu_\tau\) with \(N'\) are given by \(\theta_{\nu_\mu,N'} \approx \sqrt{\delta_\mu}\) and \(\theta_{\nu_\tau,N'} \approx \sqrt{\delta_\tau}\). Now, in general, \(\nu_\tau\) also mixes with \(N'\) owing to (a) the transformation of the \(X'\) matrix to the hat-basis discussed in sec. 2 and in Appendix B, which introduces \(\nu_\tau-\nu_\mu\) mixing given by \(\theta'_{\nu_\tau,\nu_\mu}\) (see Eq. (17)), and (b) \(\nu_\mu-\nu'\) mixing as mentioned above. Thus we have: \(\theta_{\nu_\mu,N} \approx (\theta'_{\nu_\tau,\nu_\mu})/(\theta_{\nu_\tau,N'}) \approx (\theta'_{\nu_\tau,\nu_\mu})\sqrt{\delta_\mu}.\) This mixing would reduce the \(Z^0 \rightarrow \bar{\nu}_\tau\nu_\mu\) coupling by the factor \((1 - \delta_\epsilon)\), where \(\delta_\epsilon \equiv (\theta'_{\nu_\tau,\nu_\mu})^2\delta_\mu \lesssim \delta_\mu/16\), if \(\theta'_{\nu_\tau,\nu_\mu}\) is set to be \(\lesssim 1/4\) (see Appendix B). Thus ordinarily \(\delta_\epsilon\) is expected to be extremely small. we will still exhibit it in the equations for the sake of generality. The net neutrino counting observed at LEP involving \(Z^0\)-decays into \(\bar{\nu}_\tau\nu_\tau\)-pairs is then given by:

\[
N_\nu(\text{LEP}) = 3 - 2(\delta_\mu + \delta_\tau + \delta_\epsilon) = 3 - 2\eta_\mu^2(1 + p_{\nu}^2) - 2\delta_\epsilon.
\]

This expression for \(N_\nu\) holds for both Models I and II. For numerical purposes, we will apply it in this section only to Model II. The experimental value from LEP is \(N_\nu = 2.9841 \pm 0.0083\) [2]. We see that ESSM leads to a reduction in \(N_\nu\), which is in good agreement with the LEP
data. Setting $p'_\nu = 0.3$ for Model II [see Eq. (8)], and $\eta_u = 1/10 - 1/15$, we have $N_\nu = (2.9782$ to $2.9903) - 2\delta_e$, where the $(2\delta_e)$-term makes a negligible contribution: $(2\delta_e \lesssim 0.0005$ for $\eta_u$ as stated above). The suggested two sigma deviation in $N_\nu$ measured at LEP compared to Standard Model may thus be taken as a hint for $\nu_\tau$-$N'$ and $\nu_\mu$-$N'$ mixings. The LEP value for $N_\nu$ would imply a magnitude for $\eta_u \approx (1/10 - 1/15)$, which can then be used to predict deviations in the other experiments, such as NuTeV. Theoretically, values of $\eta_u \approx 1/5 - 1/20$ are perfectly plausible (see Ref. [31, 32]).

There are modifications in the charged current interactions as well, which is straightforward to compute:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}W^+[(1 - \delta_e/2)\bar{\nu}_e\nu_L + (1 - \delta_\mu/2)\bar{\nu}_\mu\mu_L + (1 - \delta_\tau/2)\bar{\nu}_\tau\tau_L] + h.c. \quad (21)$$

Here we define $\nu_\mu$ as the normalized state that couples to $\mu_L W^+$ and similarly $\nu_\tau$ as the normalized state that couples to $\tau_L W^+$. In terms of $\nu'_2$ and $\nu'_3$, $\nu_\mu$ is given as

$$\nu_\mu = \cos \phi \nu'_2 + \sin \phi \nu'_3 \quad (22)$$

where $\sin \phi = -s_L(1 - b/2) - (s_L/2)(b s_L - a c_L)^2$. The state orthogonal to $\nu_\mu$, viz., $\tilde{\nu}_\tau = -\sin \phi \nu'_2 + \cos \phi \nu'_3$, is not exactly $\nu_\tau$. (Thus, $\nu_\mu$ produced in $\tau$ decays can produce $\tau$ leptons, but numerically this cross section is very small, being proportional to $\eta_u^4 \sim 10^{-5}$.)

In order to see how the model accounts for the NuTeV neutral current anomaly, it is useful to rewrite the $Z^0\bar{\nu}_i\nu'_j$ interaction in terms of the current eigenstate $\nu_\mu$ and the state orthogonal to it ($\tilde{\nu}_\tau$). Including the first family, it is given as

$$\mathcal{L}_{NC}^Z = \frac{g}{2\cos \theta_W}Z^0[\bar{\nu}_e\nu_\mu(1 - \delta_\mu) + (\bar{\nu}_\mu \tilde{\nu}_\tau + \tilde{\nu}_\tau \nu_\mu)\{(b^2 - a^2)c_L s_L - (c_L^2 - s_L^2)ab\} + \tilde{\nu}_\tau \nu_\mu(1 - \delta_\tau) + \tilde{\nu}_e \nu_\mu(1 - \delta_e)] \quad (23)$$

Using Eq. (21), and including radiative corrections, the Fermi coupling $G_\mu$ is given in our model (including radiative corrections) by

$$\frac{G_\mu^{ESSM}}{\sqrt{2}} = \frac{g^2}{8m_W^2}(1 + \Delta r_W^{(\nu_\mu,i)})(1 - \delta/2), \quad (24)$$

where $\delta \equiv \delta_\mu + \delta_e \approx \delta_\mu$ and $\Delta r_W^{(\nu_\mu,i)}$ denotes electroweak radiative corrections to $\mu$-decay, which is usually denoted by $\Delta r$ in the literature [33, 34, 35]. Note that by definition, it is the right side of Eq. (24) which is determined by the observed muon decay rate: thus $G_\mu^{ESSM} = G_\mu^{obs}$. The amplitude for the CC process ($\nu_\mu N \rightarrow \mu X$) is then given by $(G_\mu^{obs}/\sqrt{2})(1 + \Delta r'_W)$ in both the SM and ESSM. Here, $\Delta r'_W$ denotes the difference [36] between the radiative correction $(\Delta r_W^{(\nu_\mu,h)})$ to the amplitude for ($\nu_\mu N \rightarrow \mu X$) and that for $\mu$-decay; thus $\Delta r'_W = \Delta r_W^{(\nu_\mu,h)} - \Delta r_W^{(\nu_\mu,l)}$. Since $\Delta r'_W$ is the same for both ESSM and the SM, the CC cross sections are the same in the two cases: $[\sigma_{CC}]^{ESSM} = [\sigma_{CC}]^{SM}$. 

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On the other hand, the NC cross section $\sigma(\nu_{i}N \rightarrow \nu X)$ will be modified. To see this, we use the fact that in ESSM, the amplitude for the process $\nu_{i}q_{i} \rightarrow \nu_{j}q_{j}$ (including radiative corrections) is proportional to:

$$A(\nu_{i}q_{i} \rightarrow \nu_{j}q_{j})^{\text{ESSM}} \propto \frac{g^2}{8m_{Z}^2 \cos^2 \theta_{W}}(1 + \Delta r_{Z}^{i})a_{i}(1 - \delta_{\mu})$$

$$= \left(\frac{G_{\mu}^{\text{obs}}}{\sqrt{2}}\right)a_{i}(1 + \Delta r_{Z}^{i} - \Delta r_{W}^{(\nu_{\mu,l})})(1 - \delta'/2),$$

(25)

where $\delta' \equiv \delta_{\mu} - \delta_{e} \approx \delta_{\mu}$. Here $a_{i} = (I_{3L} - Q \sin^2 \theta_{W})_{i}$ and $\Delta r_{Z}^{i}$ denotes the radiative correction to the amplitude for $\nu_{i}q_{i} \rightarrow \nu_{j}q_{j}$. In getting the second step of Eq. (25), we have used Eq. (24) and the on-shell relation $m_{W}^2/(m_{Z}^2 \cos^2 \theta_{W}) = 1$, where $m_{W}$ and $m_{Z}$ denote the physical masses of $W$ and $Z$, respectively. Since the radiative corrections to the NC processes, discussed in detail in Ref. [37], are the same for the SM and ESSM (barring small differences owing to differences between predictions for the Higgs mass in the two cases [38]), we obtain (using Eq. (25)):

$$\sigma(\nu_{i}N \rightarrow \nu X)^{\text{ESSM}} = \sigma(\nu_{i}N \rightarrow \nu X)^{\text{SM}}(1 - \delta_{\mu}),$$

(26)

where we have set $\delta' = \delta_{\mu}$ Notice that the neutral current cross section is reduced compared to its SM value. Using the values corresponding to Model II – that is $p'_{\nu} = 0.3$, $s_{L}/c_{L} = -0.526$ [fixed by our considerations of fermion masses and mixings, see Eqs. (4)-(5)], and $\eta_{u} = 1/11.6-1/13.3$, we get $\delta_{\mu} = 0.003$ to 0.004. While this range for $\eta_{u}$ and equivalently for $\delta_{\mu}$ is quite reasonable [31], we stress that we can not choose $\delta_{\mu}$ to be much higher owing to constraints from charged-current universality, discussed below. We thus obtain a deviation:

$$\sigma(\nu_{i}N \rightarrow \nu X)^{\text{ESSM}}/\sigma(\nu_{i}N \rightarrow \nu X)^{\text{SM}} \approx 1 - (0.3 \text{ to } 0.4\%)$$

in Model II. Considering that NuTeV measurements may be interpreted as a reduction of the NC cross section by about $(1.2 \pm 0.4\%)$, compared to the SM [39], keeping the CC cross section and $\sin^2 \theta_{W}$ unchanged, we see that ESSM would reduce the deficit in the NC cross section to about $0.8 \pm 0.4\%$. This can partially account for the NuTeV anomaly by reducing it from a $3\sigma$ to a $2\sigma$-effect. This is of course statistically quite significant. It remains to be seen whether a portion or even a bulk of the NuTeV anomaly can be attributed to a Standard Model-based QCD and other effects, considered by several authors, including the experimenters [40].

We stress the intimate quantitative link between the reduction in LEP neutrino-counting $N_{\nu}$ and that in the NuTeV cross section [see Eqs. (20) and (21)], which emerges because all the relevant parameters are fixed owing to our considerations of fermion masses and mixings. It is worth noting that owing to hierarchical masses of the three families, and thus nonuniversal mixings of $(\nu_{e}, \nu_{\mu}$ and $\nu_{\tau})$ with $N'$, the reduction in $N_{\nu}$ from 3 is not simply three times the relative reduction in $\sigma(\nu_{i}N \rightarrow \nu X)$ [compare Eqs. (20) and (26)].
5 Changes in $m_H$ and $m_W$ in Model II due to modification in $G_\mu$

It is worth noting that the tree-level modification of the expression for $G_\mu$ containing the $\eta_\mu^2$-term [see Eq. (23)] can have a testable consequence as follows. The familiar Standard Model expression, in the on-shell scheme, for $G_\mu$, including electroweak radiative corrections \cite{37,38,39,40}, is now modified to

$$G_\mu = \frac{\pi \alpha}{\sqrt{2} m_W^2 (1 - m_W^2/m_Z^2)(1 - \Delta r + \delta/2)}, \quad (27)$$

where $\delta/2 = (\delta_\mu + \delta_\epsilon) \approx \delta_\mu/2 = \eta_\mu^2 (c_{LP'} - s_L)^2/2 \approx 0.0015 - 0.0020$ (for Model II, see above), and we have used $\sin^2 \theta_W \approx 1 - m_W^2/m_Z^2$ \cite{37,39}. Now, $\Delta r$ depends on $m_t$ and $m_H$. Using the CDF/D0 value of $m_t = 174.3 \pm 5.1$ GeV, and $\alpha^{-1}(m_Z) = 128.933(21)$, one obtains \cite{34} $\Delta r = (0.03402, 0.03497, 0.03575, 0.03646)$ for $m_H = (75, 100, 125$ and $150$) GeV with an uncertainty $\delta(\Delta r) = \pm 0.0020 \pm 0.0002$, where the first uncertainty is primarily due to that in $m_t$ by $\pm 5.1$ GeV. Note that $\Delta r$ increases with $m_H$, for a fixed $m_t$. Defining $(\Delta r)_{\text{eff}} \equiv \Delta r - \delta/2$, we see that for any given reference point of $(m_t, m_H)$, and thus for $\Delta r$, $(\Delta r)_{\text{eff}}$ is necessarily lower than $\Delta r$. Eq. (27) then tells us that, with a lower $(\Delta r)_{\text{eff}}$, compared to $\Delta r$, ESSM will predict a higher value of $m_W$, compared to that of the SM, so that $G_\mu$ and $m_Z$ are held fixed, which are measured most accurately. To be specific, for any given choice of $(m_t$ and $m_H)$, in order that $G_\mu$ and $m_Z$ may be held fixed, Eq. (27) imposes the condition

$$\hat{m}_W^2 (1 - \hat{m}_W^2/m_Z^2)(1 - \Delta r) = m_W^2 (1 - m_W^2/m_Z^2)(1 - \Delta r_{\text{eff}}), \quad (28)$$

where $\hat{m}_W$ and $m_W$ denote the mass of the $W$-boson for the SM and ESSM, respectively. Defining $m_W = (m_W)_{\text{ESSM}} \equiv \hat{m}_W + \delta m_W$, it is easy to verify that Eq. (28) yields (to a very high accuracy):

$$(\delta m_W)_{\text{fixed } (m_t, m_H)} = \left( \frac{m_Z^2 - m_W^2}{2 m_W^2} \right) \left[ 1 - \frac{m_Z^2 - m_W^2}{m_W^2} \right]^{-1} m_W (\delta/2) \approx (0.20) m_W (\delta/2). \quad (29)$$

Thus, the shift $\delta m_W$ (for fixed $m_t$ and $m_H$) is positive. Taking $\delta/2 = 0.0015 - 0.0020$ (for Model II), we obtain: $m_W = (m_W)_{\text{SM}} + (24-32)$ MeV, for any fixed value of $(m_t$ and $m_H)$ \cite{43}.

Alternatively, noting that $m_H$ is fixed by $\Delta r$ in both the SM and the ESSM for a given $m_t$, if one wishes to fix $m_W$ [say within about one sigma of its measured value $m_W^{\exp} = 80.446 \pm 0.040$ GeV (LEP data)], which in turn fixes $(\Delta r)_{\text{eff}}$ via Eq. (27), one would be led to predict (using $\Delta r = (\Delta r)_{\text{eff}} + \delta/2$) a higher value of $m_H$ in ESSM compared to that in the SM for any fixed value of $m_t$.

To be concrete, using $m_t = 174.3$ GeV, and choosing $m_H = 125$ GeV (in accord with LEP search that shows $m_H \geq 115$ GeV \cite{3}), one gets $(m_W)_{\text{SM}} = 80.372$ GeV \cite{34}, which is about
1.8 sigma below the measured value, whereas one obtains \((m_W)_{ESSM} = 80.396 - 80.404\) GeV, for \(\delta/2 = 0.0015 - 0.002\) (in Model II), which is in better agreement with the observed value. Alternatively, with \(m_t = 174.3\) GeV, if one chooses to fix \(m_W = 80.401\) GeV (1.1 sigma below the measured value), one would predict (as a central value) \((m_H)_{SM} \approx 75\) GeV \[^{34, \ 44}\], whereas \((m_H)_{ESSM} \approx 125\) GeV (for \(\delta/2 = 0.0017\)), in Model II.

In general, for any fixed \(m_t\), the replacement of \(\Delta r\) by \(\Delta r_{eff} = \Delta r - \delta/2\) would, of course, correspond to changes in both \(m_H\) and \(m_W\). A few sample cases, exhibiting the SM and ESSM predictions for \((m_H\) and \(m_W)\), based on Eq. (27) are exhibited in Table I, for the case of Model II. \((m_t = 174.3\) GeV is held fixed \[^{45}\]).

| case | (a) | (b) | (c) | (d) |
|------|-----|-----|-----|-----|
|      | Fix \(m_W\) | Fix \(m_H\) | Vary \((m_H, m_W)\) | Vary \((m_H, m_W)\) |
| SM   | \(m_H = 75\) | \(m_W = 80.401\) | \(m_H = 125\) | \(m_W = 80.372\) | \(m_H = 115\) | \(m_W = 80.377\) | \(m_H = 100\) | \(m_W = 80.385\) |
| ESSM | 116.5-135 | 80.401 | 125 | 80.396-80.404 | 123-140 | 80.397 | 108-125 | 80.405 |

**Table I.** Predictions for \(m_H\) and \(m_W\), based on Eq. (27), with \(m_t = 174.3\) GeV and \(\delta/2 = 0.0015-0.0020\). The masses are in units of GeV. The ESSM-predictions given above are for Model II.

We thus see that, ESSM (for Model II) would generically predict higher values of \(m_H\) and/or of \(m_W\) compared to that in the SM \[^{40}\]. While this feature seems to be in better accord with observations at present, its validity can be ascertained once (a) \(m_t\) is measured to within 1 – 2 GeV, (b) \(m_W\) is measured to better than 10 – 20 MeV accuracy, and (c) if the light Higgs particle (say with a mass \(\lesssim 150\) GeV) is discovered in future searches at the Tevatron, the LHC, and the NLC.

### 6 Charged Current Processes In Model II

We now turn to the question of universality in leptonic charged current processes. The flavor dependence of the charged current couplings predicted by our framework, Eq. (21), will lead to nonuniversality in leptonic decays, correlated with the \(\nu_\mu\)-nucleon neutral current cross section measured at NuTeV, as well as neutrino counting at LEP.

The leptonic decays of \(\pi^+\) mesons provide a sensitive probe of \(e-\mu\) universality. In the Standard Model, the branching ratio \(R_{e/\mu}^{SM} = \frac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)}\) has been computed quite accurately, including radiative corrections to be \[^{17}\] \(R_{e/\mu}^{SM} = (1.2352 \pm 0.0004) \times 10^{-4}\). In our framework, this prediction is modified to [see Eq. (21)]

\[
R_{e/\mu}^{ESSM} = R_{e/\mu}^{SM}(1 + \delta'),
\]

(30)
where $\delta' = \delta_\mu - \delta_\tau \approx \delta_\mu$. The PSI experiment \[18\] measures this ratio to be $R_{e/\mu}^{\exp - \text{PSI}} = (1.2346 \pm 0.0050) \times 10^{-4}$, whereas the TRIUMF experiment \[19\] finds it to be $R_{e/\mu}^{\exp - \text{TRIUMF}} = (1.2285 \pm 0.0056) \times 10^{-4}$. If we choose $\delta_\mu = \eta_u^2 (c_L p'_\mu - s_L)^2 = 0.3$ to $0.4\%$ (for Model II), so that the deviation from the Standard Model in $\nu_\mu$–nucleon neutral current cross section at NuTeV is $0.3$ to $0.4\%$, we have $R_{e/\mu}^{\text{ESSM}} = (1.2389$ to $1.2401) \times 10^{-4}$. This value is about $0.86$ to $1.1$ sigma above the PSI measurement, and about $1.8$ to $2.1$ sigma above the TRIUMF measurement. We consider these deviations arising within Model II, although not insignificant for the TRIUMF experiment, to be within acceptable range. Modest improvements in these measurements can either confirm or entirely exclude our explanation of the indicated NuTeV anomaly.

It should also be mentioned that $e - \mu$ universality is well tested in $\tau^+ \rightarrow e^+ \nu_e \bar{\nu}_\tau$ versus $\tau^+ \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau$ decays as well. The effective Fermi coupling strength $G_{\tau e}$ and $G_{\tau \mu}$ characterizing these decays are in the ratio \[50\] $G_{\tau e}/G_{\tau \mu} = 0.9989 \pm 0.0028$. Our framework predicts it to be $(G_{\tau e}/G_{\tau \mu})_{\text{ESSM}} = (G_{\tau e}/G_{\tau \mu})_{\text{SM}}[1 + \delta_\mu/2]$. Using the correction factor in this ratio to be $0.15$ to $0.2\%$ (so that deviation at NuTeV is $0.3$ to $0.4\%$), we find the deviation from experiment to be at the level of $0.9$ to $1.1$ sigma, which is quite acceptable. In Appendix A, we discuss the modifications in ESSM for $V_{ud}$ and $V_{us}$ and the resulting implications for the unitarity of the CKM matrix. The results in this regard for Model I and Model II are compared in the next section.

7 Results for Model I: Comparison with Model II

As noted in the introduction, pattern II for fermion masses and mixings, presented in sec. 3, is an interesting variant of pattern I \[4, 16\]. Both of these arise within the SO(10)-framework and possess the same degree of success as regards their predictions for fermion masses and mixings. They can, however, be clearly distinguished from each other if they are extended to ESSM. The extensions of patterns I and II to ESSM are referred to as Models I and II, respectively. So far, in secs. 4, 5 and 6, we have considered the consequences of Model II only. We now turn attention to Model I.

Following Ref. \[16\] we recall that pattern I for the fermion masses for the $\mu$-$\tau$ sector is given by:

$$M_u = \begin{pmatrix} c_L & t_L \\ 0 & \sigma - \epsilon \\ \sigma + \epsilon & 1 \end{pmatrix} \mathcal{M}_U^0; \quad M_d = \begin{pmatrix} s_L & b_L \\ 0 & \eta - \epsilon \\ \eta + \epsilon & 1 \end{pmatrix} \mathcal{M}_D^0;$$

$$M^D = \begin{pmatrix} \nu_{L\mu}^\mu & \nu_{L\tau}^\tau \\ 0 & \sigma + 3\epsilon \\ \sigma - 3\epsilon & 1 \end{pmatrix} \mathcal{M}_U^0; \quad M_l = \begin{pmatrix} \mu_L & \tau_L \\ 0 & \eta + 3\epsilon \\ \eta - 3\epsilon & 1 \end{pmatrix} \mathcal{M}_D^0. \quad (31)$$
Compare these with pattern II given in Eq. (I). These, together with their extension to include the first family, yield seven predictions, all in good accord with observations [16]. The parameters $\eta$, $\epsilon$, $\sigma$, $M_U^0$ and $M_D^0$ are determined to be:

$$\begin{align*}
\eta &\approx -0.151, & \epsilon &\approx 0.095, & \sigma &\approx -0.110, \\
M_U^0 &\approx 110 \text{ GeV}, & M_D^0 &\approx 1.5 \text{ GeV}.
\end{align*}$$

(32)

Embedding of pattern I in ESSM, which we call Model I, is discussed in Ref. [9]. Using Eq. (32), the corresponding $p_f$ and $p_f'$ parameters defined in sec. 2 (see e.g., Eq. (2)) are given by:

$$\begin{align*}
p_f &= 2(\sigma - \epsilon) = -0.41; & p_f' &= 2(\sigma + \epsilon) = -0.03; \\
p_d &= 2(\eta - \epsilon) = -0.492; & p_d' &= 2(\eta + \epsilon) = -0.112; \\
p_\nu &= 2(\sigma + 3\epsilon) = 0.350; & p_\nu' &= 2(\sigma - 3\epsilon) = -0.79; \\
p_l &= 2(\eta + 3\epsilon) = 0.268; & p_l' &= 2(\eta - 3\epsilon) = -0.872.
\end{align*}$$

(33)

Compare these with the corresponding values for Model II given in Eqs. (2) and (5). Note the crucial difference in magnitude and sign between the values of the parameter $p_\nu'$ in the two models: $p_\nu' = -0.79$ (for Model I), while $p_\nu' = +0.30$ (for Model II). The values of the parameter $p_\nu'$ (which determines the mixing angle $\tan \theta_L \approx p_\nu'/2$ (see Eq. (13)) on the other hand are quite similar in the two models: $p_\nu' = -0.87$ (for Model I), while $p_\nu' = -1.05$ (for Model II). These two parameters together (whose magnitudes and relative signs are completely fixed within each model) lead to a host of differences between the predictions of the two models.

For comparison purposes, the values of certain relevant quantities are noted below for the Model I and II [we recall the definitions $\delta_\mu = \eta_u^2(c_Lp_\nu' - s_L)^2$, $\delta_\tau = \eta_u^2(c_L + s_Lp_\nu')^2$ given in Eq. (19), and the relations $\tan \theta_L \approx p_\nu'/2$ (see Eq. (13)) and $N_\nu = 3 - 2(\delta_\mu + \delta_\tau) = 3 - 2\eta_u^2(1 + p_\nu'^2)$ (see Eq. (20)), where we have neglected $\delta_\nu$]:

| Model I | Model II |
|---------|----------|
| $p_\nu' = -0.87$, $p_\nu' = -0.79$ | $p_\nu' = -1.05$, $p_\nu' = 0.30$ |
| $\theta_L \approx \tan^{-1}(p_\nu'/2) \approx 156.49^0$ | $\theta_L \approx 152.3^0$ |
| $\delta_\mu \approx \eta_u^2(0.1060)$ | $\delta_\mu \approx \eta_u^2(0.5335)$ |
| $\delta_\tau \approx \eta_u^2(1.5182)$ | $\delta_\mu \approx \eta_u^2(0.5564)$ |
| $N_\nu = 3 - \eta_u^2(3.248)$ | $N_\nu = 3 - \eta_u^2(2.18)$ |

(34)

The following features are worth noting:

1. For any choice of $\eta_u$, $\delta_\mu$ is suppressed in Model I relative to that in Model II by about a factor of 5. This is the main reason for the differences between the predictions of the two models (as we elaborate below).

2. $\delta_\tau$ on the other hand is enhanced in Model I relative to Model II by about a factor of 2.7. This would lead to differences between the two models in their predictions for the tau lifetime.
Despite the differences between the two models as regards the values of $\delta_\mu$ and $\delta_\tau$, in particular despite the smallness of $\delta_\mu$ in Model I, it is rather interesting that both models lead to a substantial decrease in $N_\nu$ from 3 (see elaborations below), in good accord with the LEP data [2]. This is because the decrease in $N_\nu$ depends on the sum $(\delta_\mu + \delta_\tau)$ and the smallness of $\delta_\mu$ in Model I is fully compensated by the largeness of $\delta_\tau$. Note that these features are fixed even qualitatively by the nature of the SO(10)-based patterns [Eqs. (4) and (31)] and our consideration of the masses and mixings of quarks and leptons.

The results for Model II pertaining to $N_\nu$, the NuTeV-measurements and other entities have been discussed in detail in secs. 4, 5 and 6. We now present the corresponding results for Model I in table II, where the results for Model II are also listed for the sake of a comparison. Following our discussion in Appendix A, we have included, in table II, the predictions for the effective values of $|V_{ud}|$ and $|V_{us}|_{\text{unitarity}}$ in ESSM.
Table II. Results for Models I and II of ESSM. The deviations from SM predictions for the NuTeV measurements involving the ratio of NC to CC cross sections are simply given by deviations of the ratio \((\sigma^{\text{ESSM}}_{\text{NC}})/\sigma^{\text{SM}}_{\text{NC}})\) from unity, because \(\sigma^{\text{ESSM}}_{\text{CC}} = \sigma^{\text{SM}}_{\text{CC}}\), see discussions following Eq. (24). The quantity \(\Delta u_s\) in the last row stands for 0.0021, see Eq. (40).

| \(\eta_u\) | \(1/12\) | \(1/15\) | \(1/18\) | \(1/20\) |
|---|---|---|---|---|
| Models | I | II | I | II | I | II | I | II |
| \(\delta_\mu\) | 0.00074 | 0.00370 | 0.00047 | 0.00237 | 0.00033 | 0.00165 | 0.00027 | 0.00133 |
| \(\delta_\tau\) | 0.0105 | 0.00386 | 0.00675 | 0.00247 | 0.00469 | 0.00171 | 0.00380 | 0.00139 |
| \(N_\nu = 3 - 2(\delta_\mu + \delta_\tau)\) | 2.9775 | 2.9850 | 2.9855 | 2.9903 | 2.9900 | 2.9933 | 2.9919 | 2.9946 |
| \((\sigma^{\text{ESSM}}_{\text{NC}})/\sigma^{\text{SM}}_{\text{NC}}) - 1 = -\delta_\mu\) | -0.074% | -0.37% | -0.05% | -0.24% | -0.03% | -0.17% | -0.027% | -0.13% |
| \((R^{\text{ESSM}}_{\ell\mu}/R^{\text{SM}}_{\ell\mu}) - 1 = +\delta_\mu\) | 0.074% | 0.37% | 0.05% | 0.24% | 0.03% | 0.17% | 0.027% | 0.13% |
| \((r^{\text{ESSM}}_{\tau}/r^{\text{SM}}_{\tau}) - 1 \approx -\delta_\mu/5.7\) | 1.06% | 0.45% | 0.68% | 0.29% | 0.47% | 0.20% | 0.38% | 0.16% |
| \(|V^{\text{ESSM}}_{ud}/(V_{ud})_{\text{SM}}| - 1 \approx -\delta_\mu/2\) | 0.00037 | 0.00185 | 0.00024 | 0.00119 | 0.00016 | 0.00082 | 0.000135 | 0.00066 |
| \(|V^{\text{ESSM}}_{us}|_{\text{unitarity}}\) | 0.22850 | 0.23470 | 0.2279 | 0.2319 | 0.2276 | 0.2304 | 0.2274 | 0.2297 |
| \(\pm \Delta u_s\) | \(\pm \Delta u_s\) | \(\pm \Delta u_s\) | \(\pm \Delta u_s\) | \(\pm \Delta u_s\) | \(\pm \Delta u_s\) | \(\pm \Delta u_s\) | \(\pm \Delta u_s\) | \(\pm \Delta u_s\) |
A glance at table II reveals the following features:

(1) As mentioned above, both Model I and Model II lead to a significant reduction in \( N_\nu \) from 3 (for plausible values of \( \eta_u \)), in accord with the data. Considering that \((N_\nu)_{\text{LEP}} = 2.9841 \pm 0.0083\) [2], we see that Models I and II would yield nearly the central value of \((N_\nu)_{\text{LEP}}\) for \( \eta_u \approx 1/14.5 \) and 1/11.8 respectively, and \((N_\nu)_{\text{ESSM}}\) would be within \( \pm 1\sigma \) from the central value for

\[ \eta_u \approx 1/11.5 \text{ to } 1/20 \text{ (in Model I), and } \eta_u \approx 1/10 \text{ to } 1/16 \text{ (in Model II).} \] (35)

(2) As discussed in secs. 4, 5 and 6, and as can also be inferred from table 2, Model II would lead to a reduction in \((\sigma_{\text{NC}}/\sigma_{\text{CC}})\) by 0.4 to 0.3% for \( \eta_u = 1/11.6 \) to 1/13.3, corresponding to \( \delta_\mu = 0.0040 \) to 0.0030. This would (a) reduce the NuTeV anomaly from a reported 3\( \sigma \) [10] to a 2\( \sigma \)-effect, and simultaneously (b) lead to an increase in the predictions for \( m_H \) and \( m_W \) compared to those in the SM (for a fixed \( m_{\text{top}} \)). Both of these changes appear to be in the right direction, as judged by the current data, in conjunction with SM theory. By the same token, however, (with \( \delta_\mu \approx 0.4 \) to 0.3 %), Model II would predict (c) a 1 to 1.8\( \sigma \) deviation from the present experimental value of the ration \( R_{e/\mu} \) (see sec. 6), and (d) a decrease in the effective values of \( V_{ud} \) and \( V_{us} \) (see Appendix A for definitions) by a factor of \( (1 - \delta_\mu/2) \) which is in conflict with the currently measured values and the constraints of CKM-unitarity. (See table II and Appendix A for the theoretical and observed values of these parameters.) As expressed in the Appendix, better judgement in this regard should wait till improved measurements of \( V_{us} \) (expected from BNL-E865 and KLOE experiments), together with consistency checks of different measurements of \( V_{ud} \) and reduction of theoretical and systematic uncertainties in these parameters are in hand [21].

Meanwhile, Model I leads to extremely small values of \( \delta_\mu \) (for \( \eta_u \) being in the parameter range that is relevant to measurements of \( N_\nu \) (see eq. (35))). As a result, although it leads to significant reduction in \( N_\nu \) (because \( \delta_\tau \) is large), it practically coincides with the predictions of the SM (see table II and discussion in sec. 5) as regards (a) NuTeV-measurements, (b) \( m_H \) and \( m_W \), (c) \( R_{e/\mu} \), and most important (d) \( V_{ud} \) and prediction for \( V_{us} \) based on CKM unitarity. For example, with \( \eta_u = 1/18 \) (thus \( \delta_\mu = 0.00033 \)), \((N_\nu)_{\text{Model I}} = 2.9900\) (in good accord with the LEP data), but \((\sigma_{\text{NC}}^{\text{ESSM}}/\sigma_{\text{NC}}^{\text{SM}}) = 1 - 0.03\%\), and \(|V_{us}^{\text{ESSM}}|_{\text{unitarity}} = 0.2276 \pm \Delta us\), to be compared with \(|V_{us}^{\text{SM}}|_{\text{unitarity}} = 0.2269 \pm \Delta us\).

Thus Model I, while accounting for the LEP data, would be essentially on par with the SM (especially for \( \eta_u \approx 1/28 \) to 1/20 (say)), and thereby free from a large conflict as regards predictions for \( V_{ud} \) and \( V_{us} \), that Model II currently seems to have. Model I of ESSM would thus gain prominence, even over the SM, if (a) the present 2\( \sigma \) discrepancy in \( N_\nu \) becomes even a 3 or a 4\( \sigma \) effect, but at the same time, (b) NuTeV anomaly is explained away as a standard model-based QCD and other effects [10], and if (c) improved measurements of \( V_{ud} \) and \( V_{us} \) remove the current 2.2\( \sigma \) discrepancy of the SM in the prediction for \( V_{us} \) based on CKM-unitarity. Model II on the other hand could gain prominence under the following (less
likely) set of circumstances: (a) the current discrepancy in $N\nu$ stays, (b) NuTeV anomaly survives though at a reduced level (of 0.3 to 0.4% (say), see sec. 4), (c) $R_{e/\mu}$ measurement deviates somewhat from the prediction of the SM (see sec. 6), and most important (d) $V_{ud}$ and/or $V_{us}$ increase significantly (by say $2\sigma$ each or by $2.5\sigma$ in one and $2\sigma$ in the other) so as to remove the current discrepancy of Model II in the prediction for $V_{us}$ based on CKM-unitarity.

(3) Both models predict an increase in the tau lifetime compared to the prediction of the SM, by about 1.0 to 0.38% in Model I, and 0.45 to 0.16% in Model II. These effects are, however, difficult to disentangle at present from uncertainties in (a) non-perturbative effects for $\tau \rightarrow \nu_\tau + \text{hadrons}$, and (b) in $\alpha_s$. With a reduction of these uncertainties, tau lifetime can be an important probe of ESSM.

(4) One general comment is in order. It should be clear from the discussion above that in a model based on ESSM, if $\nu_\mu$-$N'$ mixing is important enough to cause noticeable departure from the SM for NuTeV-type measurements (as in Model II), it would also be important in causing significant departures from the SM-predictions for (a) $N_\nu$, (b) $m_H$ and $m_W$, (c) $R_{e/\mu}$, and (d) $|V_{us}|$ unitarity, those for the first two being favored and last two being disfavored by current data. Generically, one would of course expect $\nu_\mu$-$N'$ mixing in an ESSM-type model to be important, especially if vectorlike leptons have masses $\lesssim 1$ TeV (say) (see [31]), and thus one would expect to see departures from the SM in $N_\nu$ as well as in all the other features listed above. Model I presents a very interesting and notable exception in this regard in that it does lead to significant decrease in $N_\nu$, but hardly affects any of the other features listed above relative to the standard model.

(5) Last but not least, it is worth recalling here the result of Ref. [9] that both Models I and II of ESSM can provide a simple explanation of the indicated anomaly in $(g-2)_\mu$, by utilizing heavy vectorlike leptons in the loop, and (thus) without requiring sleptons to be rather light.

8 A Summary

Motivations for the ESSM framework have been noted in our earlier papers [7, 8] and are summarized here in the introduction. We have argued that the mixing of $\nu_\mu$ and $\nu_\tau$ with the singlet $N'$ which is naturally expected to arise within this framework, would generically modify (i) $\nu_\mu$ neutral current interaction (relevant to the NuTeV-type measurements), (ii) LEP neutrino counting, (iii) $e-\mu$ and $\mu-\tau$ universalities in charged current processes, (iv) predictions for $m_H$ and $m_W$, as well as (v) effective values of $V_{ud}$ and $V_{us}$. The degree of modification of these features depends on the nature of the fermion mass-matrix.

In an earlier work [10], a successful pattern of fermion masses and mixings (herein called pattern I) has been proposed within a MSSM-based SO(10)-framework that makes seven
predictions all in good accord with observations, including $V_{cb} \approx 0.04$ and $\sin^2 2\theta_{\nu_{\mu}N'} \approx 1$. An extension of this framework to ESSM, preserving the successes of pattern I, has been proposed in a recent paper, where it is noted that ESSM can provide a simple explanation of the indicated anomaly in $(g - 2)_\mu$ by utilizing contributions from the vectorlike heavy leptons ($M_{E,E'} \sim (300-700)$ GeV) in the loop.

The main purpose of this paper has been twofold:

(1) First, to bring out new phenomenological possibilities which may arise within ESSM, in particular to see whether it can be relevant to NuTeV-type experiments, we have presented here a variant pattern (called pattern II) of fermion masses and mixings within the SO(10)-framework, and have shown that the extension of this pattern to ESSM can account partially for the NuTeV anomaly. The variant pattern is, of course, interesting in its own right, in that it has the same degree of success as pattern I in describing fermion masses and mixings, regardless of whether it is embedded in ESSM or not.

(2) Second, we have studied here the phenomenological consequences of both patterns I and II, extended to ESSM (herein called Models I and II). In this regard, we have argued that both Models I and II lead to a sizable decrease in LEP neutrino-counting $N_\nu$ (in good accord with the LEP data). Following the results of Ref. [9], one can see that both models also provide a simple explanation of the indicated anomaly in $(g - 2)_\mu$. We have further noted that the two models, interestingly enough, can be clearly distinguished from each other, however, by other phenomena. In particular, owing to a relative enhancement of $\nu_\mu$-$N'$ mixing, Model II can provide (a) a partial explanation of the NuTeV anomaly (as mentioned above), and simultaneously (b) an increase in the predictions for $m_H$ and $m_W$ compared to the standard model (in accord with the data), (c) small departures in $e$-$\mu$ universality in charged current processes (up to 1 to 1.8$\sigma$ deviation from experiments), and (d) small decreases in the effective values of $V_{ud}$ and $V_{us}$ compared to those in the SM (which are currently disfavored by the data on grounds of unitarity of the CKM-matrix). Model I ([11, 18]) on the other hand nearly coincides with the SM as regards its predictions for these four features: (a)-(d) (see Table II). (e) Both Models I and II lead to some departures from the standard model (those for Model I being more prominent) as regards their predictions for tau lifetime (see table II). Improved experimental and theoretical studies involving LEP neutrino counting $N_\nu$, $(g - 2)_\mu$ and the five features (a)-(e) listed above can thus not only distinguish between ESSM versus the standard model, but also between the Models I and II, and thereby shed light on GUT/string-scale physics.

We stress that the two models I and II, while differing as regards (a)-(d), share the common feature that both depart from the standard model as regards their predictions for $N_\nu$ and $(g - 2)_\mu$, in good accord with the present data.

The hallmark of ESSM (irrespective of the NuTeV and LEP neutrino-counting results) is, of course, the existence of the two complete vectorlike families $(U,D,N,E)_{L,R}$ and
$(U', D', N', E')_{L,R}$ with masses in the range of 200 GeV to 2 TeV (quarks being heavier than the leptons owing to QCD effects), which can certainly be tested at the LHC and a future linear collider.

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Appendix A: $V_{ud}$, $V_{us}$ and Unitarity of the CKM Matrix

First, let us recall the status of $V_{ud}$ and $V_{us}$ and of unitarity of the $3 \times 3$ CKM-matrix in the context of the standard model. Traditionally, $|V_{ud}|$ is determined by measuring the ratio of $\beta$-decay and $\mu$-decay rates and using the SM relation: $(G^V_A/G^\mu)_\text{SM} = K^{V,A}(V^{SM}_{ud})$, where $K^{V,A}$ denote the ratio of the relevant matrix elements, including radiative corrections ($V$ and $A$ stand for vector and axial vector couplings). A very recent analysis leads to a world average value of $|V_{ud}|$ given by $|V^{\text{SM}}_{ud}| = 0.9739 \pm 0.0005$ [54]. This value includes the averages of those based on measurements of superallowed nuclear Fermi transitions as well as highly polarized neutron $\beta$-decay [55]. Using this value for $|V_{ud}|$ and unitarity of the CKM-matrix, one is led to predict $|V^{\text{SM}}_{us}|_{\text{unitarity}} = \sqrt{1 - |V_{ud}|^2 - |V_{ub}|^2} = 0.2269 \pm 0.0021$ (where we have used $|V_{ub}|^{\text{exp}} = 0.0038 \pm 0.0008$; $|V_{ub}|$ has, of course, a negligible effect on this prediction). The direct determination of $V_{us}$, obtained from $K_{l3}$ decay rates, however, yields: $|V_{us}|^{\text{exp}} = 0.2196 \pm 0.0026$ [56, 57] which is about $2.2\sigma$ below the prediction based on CKM-unitarity. It has been expressed in Ref. [54] that such a discrepancy may have its origin in an underestimate of theoretical and systematic errors. An improved determination of $|V_{us}|$ is expected from high-statistics measurements of $K_{l3}$ decay rates at the BNL-E865 [57] and KLOE experiments [58]. It is worth mentioning here that preliminary results of the BNL-E865 experiment indicate that $|V_{us}| = 0.2278 \pm 0.0029$ [57, 54]. Such a value, if it holds up, would of course be in good accord with the constraints of unitarity of the CKM-matrix. It thus seems that a better assessment of the extent to which CKM-unitarity is respected, in the context of even the standard model, should wait until improved values of $|V_{us}|$ and $|V_{ud}|$, together with reduction of theoretical and systematic uncertainties are available.

Turning to ESSM, one would expect, in principle, two modifications here pertaining to
quark-mixings: (i) Even in the absence of SU(2)$_L$-singlet quarks ($U', D'$), the 3 × 3 CKM-matrix would extend to a 4 × 4-matrix, to be denoted by $\tilde{V}_{ij}$, which describes mixings among the four SU(2)$_L$-doublets ($q^c_L$, and $Q_L = (U_L, D_L)$). The CKM-unitarity constraint is then extended to satisfy conditions such as $\tilde{V}_{ud}^2 + \tilde{V}_{us}^2 + \tilde{V}_{ub}^2 + \tilde{V}_{uD}^2 = 1$. (ii) Allowing for the $u-U'$-mixing (one can safely neglect $d-D'$-mixing, because $\theta_{dD'/\theta_{uU'}} \sim (\kappa_d/\kappa_u) \sim m_b/m_t \ll 1$), each of the tilde-entries involving the up-quark, which together satisfy the unitarity-constraint as above, would be reduced by $(1 - \delta_u/2)$, so that $V_{ud}^{ESSM} = \tilde{V}_{ud}(1 - \delta_u/2)$, where $j = d, s, b, D$ and $\delta_u = \theta_{uU'}^2$.

In practice, however, each of these two modifications turns out to be insignificant, especially for mixing elements involving the up-quark. This is primarily because one can always transform from the gauge to the hat-basis (see discussions in sec. 2 and Appendix B) in which the first family gets decoupled from the vectorlike families. We estimate [59]: $\theta_{uU_L} \lesssim (1/3)(10^{-2})$ and $\theta_{uU_L} \lesssim 10^{-3}$. So, $\delta_u = \theta_{uU'}^2 \leq 10^{-5}$, and $|V_{ud}| = |\theta_{uU'} - \theta_{dD}| \leq 10^{-3}$. Thus, even in ESSM, we would expect that the unitarity-constraint of the 3 × 3 CKM-matrix, involving especially the up-quark – i.e., the condition $(V_{ud}^2 + V_{us}^2 + V_{ub})_{ESSM} = 1 – should be maintained, barring corrections given by $V_{uD}^2 \leq 10^{-6}$ and $\tilde{V}_{ud}^2 \lesssim 10^{-5}$. While these corrections might have been more important in a more general context, we can safely ignore them henceforth in the context of the model under discussion.

There is, however, a modification in the effective value of $V_{ud}$ in ESSM compared to its standard model-based experimental value, because the latter is extracted by utilizing the ratio of $\beta$-decay and $\mu$-decay rates, and there is a modification in the theoretical expression for $G_\mu$ in ESSM compared to that in the SM (see Eq. (24)). The same modification would apply also to $V_{us}$ which is determined from the ratio of $K_\beta$ and $\mu$-decay rates, and likewise for $V_{ub}$.

Writing $G_\mu^{ESSM} \propto (g^2/8m_W^2) V_{ud}^{ESSM}$, and $G_\mu^{ESSM} \propto (g^2/8m_W^2)(1 - \delta_\mu/2)$ (where we have suppressed radiative corrections which are partly common to $\beta$-decay and $\mu$-decay, see Eq. (24)), we obtain: $(G_\beta/G_\mu)^{ESSM} = V_{ud}^{ESSM}/(1 - \delta_\mu/2) = (V_{ud})_{SM}^{expt}$; and thus

$$|V_{ud}|_{ESSM}^{expt} = |V_{ud}|_{SM}^{expt} (1 - \delta_\mu/2),$$

and likewise $|V_{us}|_{ESSM}^{expt} = |V_{us}|_{SM}^{expt} (1 - \delta_\mu/2)$, and $|V_{ub}|_{ESSM}^{expt} = |V_{ub}|_{SM}^{expt} (1 - \delta_\mu/2)$. To see the effect of the reduction factor $(1 - \delta_\mu/2)$ on the unitarity constraint, we write:

$$V_{ud}^{ESSM} = V_{ud}^0(1 - \delta_\mu/2) \pm \Delta,$$

where $V_{ud}^0$ denotes the central value of $|V_{ud}|_{SM}^{expt}$, and $\pm \Delta$ denotes the error in it (with $\Delta > 0$). Using the unitarity condition for the first row of the CKM-matrix (which holds for ESSM, barring corrections $\lesssim 10^{-5}$), we would predict:

$$|V_{us}|_{ess}^{ESSM} = \sqrt{1 - (V_{ud}^2 + V_{ub})_{ESSM}} = |V_{ud}|_{ess} + \frac{(V_{ud}^0)^2(\delta_\mu/2)}{(V_{us})_{ess}^0} \pm \frac{V_{ud}^0}{(V_{us})_{ess}^0} \Delta,$$

27
where $|V^0_{us}|_{\text{unit}}$ is the central value for $V_{us}$ that is predicted by using the SM-based central value of $|V_{ud}|_{\text{SM}}^{\text{expt}}$, together with CKM-unitarity. Thus

$$|V^0_{us}|_{\text{unit}} \equiv \sqrt{1 - (V^0_{ud})^2 - (V^0_{ub})^2}. \quad (39)$$

In writing Eq. (38), we have ignored the error in $V_{ub}$ since even $V^0_{ub}$ is immaterial.

If one uses the world-average value of $|V_{ud}|_{\text{SM}}^{\text{expt}} = 0.9739 \pm 0.0005 \ [54]$, then $|V^0_{ud}| = 0.9739$ and $\Delta = 0.0005$, while $|V^0_{ub}| = 0.0038$. One then predicts, within the SM, $|V^0_{us}|_{\text{unit}} = 0.2269$, and thus (using Eq. (38)),

$$|V_{us}|_{\text{ESSM}}^{\text{unit}} = 0.2269 + 4.179(\delta_\mu/2) \pm \Delta us, \quad (40)$$

where $\Delta us \equiv 4.29(\Delta) = 0.0021$ (for $\Delta = 0.0005$). We will use Eq. (40) in computing the entries in table II of sec. 7.

**Appendix B: Inclusion of the First Family: Relevance To Solar Neutrino Oscillation?**

As remarked in sec. 2, the matrices ($X, X', Y$ and $Y'$), which denote the mixings of the three chiral with the vectorlike families in the gauge basis, can always be transformed to their hat-forms ($\hat{X}, \hat{X}', \hat{Y}$ and $\hat{Y}'$) so that one family is left exactly massless, which can then be identified with the electron family. Thus, the masses of the ($u, d, e, (\nu_e)_D$) must arise entirely through the direct mass terms of order few MeV (that enter into the $0_{3 \times 3}$ block of Eq. (2)). But the mixings of the electron family with the $\mu$ and the $\tau$-families can arise not only from the direct mass terms but also from the transformation from the gauge to the hat basis (i.e., $X \rightarrow \hat{X}$, $X' \rightarrow \hat{X}'$, etc.). As we now discuss, the mixings from this latter source can in principle be important especially for the first family, under the circumstances which we note below. To illustrate this possibility, we will consider only Model II. Qualitatively similar results will apply to Model I as well.

To include the first family ($16_1$), we start in the gauge basis and add the following effective terms to the superpotential for Model II, given in Eq. (3):

$$W_{\text{Yuk}} = h_{1V} 16_1 16_1 V 10_H + h'_{1V} 16_1 16_1 V 16_H 16_H 10_H + \bar{h}_{1V} 16_1 16_1 V 10_H 45_H 45_H. \quad (41)$$

The vectors $X_f$ and $X'_f$ of Eq. (11) will thus have the form:

$$X_f = (x_1, x_2, x_3)^T_f, \quad X'_f = (x'_1, x'_2, x'_3)_f. \quad (42)$$
At the GUT scale, the vectors $X'_f$ (defined in the gauge basis) are given by:

$$X'_u = [2(\sigma' + \epsilon'), 2\epsilon, 1]x'_u; \quad X'_d = \left[\frac{2(\eta' + \epsilon')}{1 + \xi}, \frac{2(\eta + \epsilon)}{1 + \xi}, 1\right] x'_d$$

$$X'_u = [2(\sigma' - 3\epsilon'), -6\epsilon, 1]x'_u; \quad X'_d = \left[\frac{2(\eta' - 3\epsilon')}{1 + \xi}, \frac{2(\eta - 3\epsilon)}{1 + \xi}, 1\right] x'_d$$

The vectors $X_f$ are obtained from Eq. (43) by the replacement $(\epsilon, \epsilon') \to - (\epsilon, \epsilon')$. Here $\sigma'$ and $\epsilon'$ are proportional to $h_{11V}$ and $h'_{11V}$ respectively, while $\eta' \equiv \hat{\eta}' + \sigma$, with $\hat{\eta}'$ being proportional to $h'_{11V}$. The parameters $\eta, \epsilon$ and $\xi$ have been defined before (see sec. 3).

As mentioned above, the mixing angle $V_{ij}$ involving the first family get contributions both from the direct mass terms, as well as from Eq. (III) through transformations of $X'_f$, etc. from the gauge to the hat basis. Let us denote the two contributions to $V_{ij}$ by $V_{ij}^{\text{dir}}$ and $V_{ij}'$, respectively; thus (for $i \neq j$, with small mixing angles $\leq 1/3$ (say)) $V_{ij} \approx V_{ij}^{\text{dir}} + V_{ij}'$. We can bring Eq. (13) into the hat basis, where the electron family decouples, by making a rotation in the 1-2 family-space. For example, a rotation in the $u_L - c_L$ quark sector by an angle $\tan \theta'_{uc} = (\sigma' + \epsilon')/\epsilon$ will bring $X'_u$ to the hat-form $[0, 2\sqrt{\epsilon^2 + (\epsilon' + \sigma')^2}, 1]x'_u$, etc. Now we can see that even with a very mild hierarchy between the entries for the first and the second families (in the gauge basis), the discussion of the 2-3 sector carried out in the previous section will remain essentially unchanged. For instance, even for a rather large value of the ratio $|\epsilon' + \sigma'|/|\epsilon| \approx 1/2$ (say), the correction terms relevant to the 2-3 sector are only of order $(1/2)(\epsilon' + \sigma')^2/\epsilon^2 \approx 12\%$.

At the same time, such a large ratio is fully compatible with the smallness of the up-quark mass because the $(\epsilon' + \sigma')$-entry contributes zero to $m_{up}$ by the rank argument. This is the new feature of ESSM. It provides an entirely new source for mixing of $(u, d, \epsilon, \nu_e)$ with the corresponding members of the second and the third families, which in principle could be relatively large without conflicting with the smallness of their masses. Of course, if flavor symmetries, distinguishing between the three families, dictate a large hierarchy already in the gauge basis, so that $x'_1 < x'_2 < x'_3$ etc., such mixings denoted by $V'_{ij}$ would be small. In this appendix, for purposes of illustration, however, we would allow for possible departures from such a hierarchical structure.

Including a rotation in the $d_L-s_L$ sector (analogous to the $u_L-c_L$ rotation), we obtain the following contributions (from the transformations of the $X'_{u,d}$ matrices) to the CKM mixings:

$$V'_{us} = \frac{\sigma' + \epsilon'}{\epsilon} - \frac{\eta' + \epsilon'}{\eta + \epsilon}, \quad V'_{ub} \approx V'_{us} \theta_{sb},$$

where $\theta_{sb}$ is the contribution to $V_{cb}$ arising from the diagonalization of the mass matrix ($M_d$) from the down sector (see Eq. (IV)). Thus, $\theta_{sb} = \eta + \epsilon = 0.0386$ (using values for $\eta$ and $\epsilon$ given in Eq. (IV)). It is clear from Eq. (44) that $V_{us}$ and $V_{ub}$ cannot be generated entirely or primarily through the mixings of the chiral with the vectorlike families. Because if $V_{us} \approx V'_{us}$
and $V_{ub} \approx V'_{ub}$, then Eq. (14) would yield $V_{ub} \approx (0.22)(0.0386) \approx 0.008$, which is a factor of two larger than the experimental value. The direct mass terms (entering into the $3 \times 3$ block of Eq. (2)), which should be of order $m_d \sim$ few MeV, are expected to contribute significantly to these mixings. We would expect $V_{ud}^{\text{dir}} \sim \mathcal{O}(m_d)/m_b \sim \mathcal{O}(0.0015)$ which is of the same order as $V_{ub}$, and similarly $V_{us}^{\text{dir}} \sim \mathcal{O}(m_d)/m_s \sim 0.06$.

Turning to the neutrino sector, the $\nu_e-\nu_\mu$ oscillation angle induced by the transformations of $X'_e$ and $X'_\mu$ to their hat-forms is given by:

$$\theta_{\nu e, \nu \mu}^{\text{osc}} = \frac{x'_{1e}}{x'_{2e}} \frac{x'_{1\mu}}{x'_{2\mu}} = \frac{\sigma' - 3\epsilon'}{-3\epsilon - \eta' - 3\epsilon}. \quad (45)$$

Combining with the expression for $V'_{us}$ in Eq. (14), we obtain:

$$\theta_{\nu e, \nu \mu}^{\text{osc}} \approx \frac{V'_{us}(1 + \eta/\epsilon)}{3 - \eta/\epsilon} \left[ \frac{\eta/\epsilon(1 - \sigma'/(3\epsilon')) + (\sigma/\epsilon' - \eta/\epsilon')}{(\eta/\epsilon)(1 + \sigma'/\epsilon') + (\sigma/\epsilon' - \eta/\epsilon')} \right]. \quad (46)$$

It is worth asking whether Eq. (46) can consistently lead to a large oscillation angle for solar neutrinos ($\sim 0.5, \text{as suggested by current data} [71]$). One way to obtain a large value for $\theta_{\nu e, \nu \mu}^{\text{osc}}$ is to maximize the numerator and minimize the denominator in the square bracket of Eq. (46). [Note $\eta$ and $\epsilon$ are fixed in the model (here we are considering Model II presented in sec. 3) from our considerations of fermion masses (see Eq. (7))]. To this end, parametrizing $\sigma'/\epsilon' \equiv -1 + r_1$ and $\eta'/\epsilon' \equiv -1 + r_2$ (with the presumption that $r_1, r_2 \ll 1$), we obtain:

$$\theta_{\nu e, \nu \mu}^{\text{osc}} \approx V'_{us} \left[ \frac{0.382}{0.772 r_1 + r_2} \right], \quad V'_{us} \approx \left( \frac{0.772 r_1 + r_2}{0.772} \right). \quad (47)$$

where we have inserted $\eta = 0.0886$ and $\epsilon = -0.05$ (see Eq. (7)). This in turn yields $\theta_{\nu e, \nu \mu}^{\text{osc}} \approx (0.382/0.772)(\epsilon'/\epsilon) \approx (0.50)(\epsilon'/\epsilon)$. Thus, keeping $|\epsilon'/\epsilon| \leq 1/2$ (this is to preserve the predictivity and the success of the 2-3 sector discussed in sec. 3), we see that the new source of mixing for the first family available in ESSM can provide a sizable value for the $\nu_e-\nu_\mu$ oscillation angle $\theta_{\nu e, \nu \mu}^{\text{osc}} \leq 0.25$, but not quite as big as the large angle ($\sim 0.5$) suggested by the data [64]. The $\nu_e-\nu_\mu$ mixing can of course receive contributions of the desired magnitude from other sources consistent with the SO(10)/G(224)-symmetry (see e.g., Ref. [20]). We note that given $\nu_e-\nu_\mu$ mixing from the source as above (Eq. (16)), and $\nu_\mu-N'$ mixing discussed in sec. 4, we will have $\nu_e-N'$ mixing given by

$$\theta_{\nu e, N'} \approx (\theta_{\nu e, \nu \mu}^{\text{osc}})(\theta_{\nu \mu, N'}), \quad (48)$$

where $\theta_{\nu e, N'} = \sqrt{\delta_\mu} = |\eta_\mu(c_{LP'} - s_L)|$ (see sec. 4). The effects of $\nu_e-N'$ mixing are reflected in the relevant expressions in sec. 4.

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\[ \hat{W}_{Yuk} = h_1 16_V \bar{16}_V 1_V + f_1 16_V \bar{16}_V (45_H/M) 1_V + h_{3V} 16_3 16_V 10_H \\
+ \tilde{h}_{3V} 16_3 16_V 10_H 45_H/M + h_{3V} 16_3 16_V H_S + h_{2V} 16_2 16_V 10_H \left( \frac{X}{M} \right) \\
+ a_{2V} 16_2 16_V 10_H 45_H/M + g_{2V} 16_2 16_V 16_H 16_H/M \]

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[25] The right-handed neutrinos of three chiral families ($\nu^c_R$, $\nu^c_R$ and $\nu^c_R$) acquire superheavy Majorana masses, which, following the familiar see-saw mechanism, leave the left-handed neutrinos light. Since RH neutrinos are superheavy, the mixing of LH neutrinos $\nu^c_L$ with $(N, N')_L$, obtained from Eq. (2), would not be affected by their presence.

[26] For example, if one drops the ($B$-$L$)-dependent contributions — that is with $(h_V16V16V1V + h_3V16316V10H + h_3V16316VH_S)$ being the leading terms of the effective superpotential (see Ref. 23) — one would get the following relations at the GUT-scale: $x_u = x_d = x_l = x_\nu = x_u' = x_l' = x_\nu'; y_q = y_l = y_\nu' = y_l'$, and $z_f = z'_c$. In this case, the entries denoted by “1” in Eq. (2) and its analogs in the quark and charged lepton sectors will be given by just three parameters ($\kappa_u, \kappa_d$ and $\kappa_s$) — rather than sixteen — at the GUT scale. Similar economy arises for the $p$ parameters (see text).

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[28] Since corrections to see-saw diagonalization for the $\mu$ – $\tau$ sector are not small, especially for $m^0_\mu/m^0_\tau$, the results quoted are based on a more accurate numerical diagonalization.

[29] See for e.g., J. Bagger, K. Matchev, D. Pierce and R. Zhang, Phys. Rev. Lett. 78, 1002 (1997).

[30] The corresponding values of the $p$-parameters for Model I of ESSM are given in sec. 7, where the predictions of Models I and II are compared.

[31] These may be estimated by making some plausible assumptions about the Yukawa couplings as follows. Assume that, in accord with generalized flavor symmetries, the Yukawa couplings of the vectorlike and the third families to each other involving the SO(10)-singlet Higgs fields $1_V$ and $1'_V$ (that is $h_V$ and $h_3V$, see Ref. 26 for definitions) are “maximal” ($\sim$ 1 to 2) at the GUT-scale; while the coupling involving the $10_H$ that mixes the vectorlike with the third family (i.e., the coupling $h_3V$) may be smaller than $h_V$ by a factor of 1/3 to 1/10 (say) at the GUT-scale. As discussed in Ref. 8, this would lead to fixed point values of the Yukawa couplings generated by $h_V$ and $h_3V$ at the electroweak scale, which are given by $z_l = 0.273$, $z'_l = 0.185$, $z'_\nu = 0.152$, $y_\nu' = 0.251$, $y_l = 0.184$ and $y_\nu = 0.20$ (see Eq. (10) of Ref. 8). For the choice indicated above, the couplings generated by $h_3V$ — that is $x_{l,\nu}$ and $x'_{l,\nu}$ — may, however, be smaller by factors of 1/2 to 1/4 (say) than the corresponding fixed point value of $\approx 0.3$ at the electroweak scale. We thus take $x_{l,\nu} \sim x'_{l,\nu} \approx 0.3 \xi_\nu$, where $\xi_\nu \approx 1$ to 1/4. These yield: $v_0 \equiv \langle H_V \rangle = M_{E'}/z'_l \approx M_{E'}/0.185 \approx (1.6 \, TeV)\rho_{E'}$, where $\rho_{E'} \equiv M_{E'}/(300 \, GeV)$. Thus, $v_u/v_0 \approx 175 \, GeV/v_0 \approx (1/9)(1/\rho_{E'})$. We also get: $\eta_u \equiv \kappa'_l/M_{N'} \approx x'_\nu v_u/M_{N'} \approx (0.3 \xi_\nu)(175 \, GeV)/(300 \, GeV)\rho_{E'} \approx (1/5.7$ to $1/20)(1/\rho_{E'})$ for $\xi_\nu \approx 1$ to 1/4.
Note, despite QCD effects, the parameters $\eta_u$ for the neutrinos and for up-quarks are essentially the same.

G. Degrassi, S. Fanchiotti and A. Sirlin, Nucl. Phys. B351, 49 (1991). For a current review and values of $\Delta r$, as well as references to other recent works, see Ref. [34] and Ref. [35].

G. Degrassi, P. Gambino, P. Passera and A. Sirlin, Phys. Lett. B418, 209 (1998); G. Degrassi, P. Gambino and A. Sirlin, Phys. Lett. B394, 188 (1997); W. Marciano, hep-ph/0003181.

J. Erler and P. Langacker, in Review of Particle Physics; K. Hagiwara et al., Particle Data Group, Phys. Rev. D66, 010001 (2002);

This difference $\Delta r'_W$ arises because of the difference between radiative corrections involving quark-lines (as in $\nu_\mu N \rightarrow \nu X$) and purely leptonic lines (as in $\mu$-decay), through vertex and box graphs. This difference is, however, the same for ESSM as for the SM.

W. Marciano and A. Sirlin, Phys. Rev. D22, 2695 (1980).

As shown in Ref. [37], the NC scattering amplitude is proportional to a universal renormalization factor $\rho_{NC}^{(\nu,h)}$ multiplying the overall amplitude, together with a correction factor $\kappa^{(\nu,h)}(q^2)$ multiplying $\sin^2 \theta_W$. Although these correction factors are the same in their analytical forms for ESSM and the SM, ESSM predicts a higher value for the Higgs mass compared to that in the SM (as discussed below). This in principle should introduce a correction factor on the right side of Eq. (26) depending upon the Higgs mass. In practice, there are, however, cancellations between the contributions to $\rho_{NC}^{(\nu,h)}$ and $\kappa^{(\nu,h)}(q^2)$ (see Appendix B of Ref. [37] for discussions), leaving only a weak dependence on $m_t$ as well as $m_H$. This may be inferred for example from the paper of K.S. McFarland et al. (hep-ex/0205080) where the corrections to $\rho_0$ through a shift in Higgs mass is noted to be $-0.00016 \ln(m_H/150 \text{ GeV})$. We have therefore not exhibited the Higgs-mass dependent correction factor on the right side of Eq. (26).

K. McFarland et al., NuTeV measurements, Ref. [1].

See e.g., S. Davidson, S. Forte, P. Gambino, N. Rius and A. Strumia, hep-ph/0112302; G.A. Miller and A.W. Thomas, hep-ex/0204007; S. Kovalenko, I. Schmidt and J.J. Yang, hep-ph/0207158, and also the discussion in the two papers in Ref. [1]. For a recent discussion on the interpretation of the NuTeV results, see P. Gambino, talk given at the ICHEP Conference, Amsterdam (July 24-31, 2002), hep-ph/0211003.

We are ignoring here the radiative corrections due to SUSY and due to ESSM, which seem to be rather small for our purposes. For example, the correction to the
The $\rho$ parameter from the vector-like quarks $U$ and $D$ of ESSM can be estimated to be 

$$\Delta \rho_{\text{ESSM}} \approx (\Delta \rho_{\text{top}}^\text{op} / 3)[(m_U^2 - m_D^2)]/(m_U^2 m_D^2) \approx (\eta_u^4 / 3)(m_U^2 / m_D^2) \Delta \rho_{\text{top}}^\text{op} \approx 3 \times 10^{-6},$$

where we used $m_U \approx m_D \sim 500$ GeV and $\eta_u \sim 1/10$ for the numerical estimate. The estimate is similar for $\Delta \rho$ correction from SUSY particles, if the masses of the SUSY particle are of order 500 GeV.

[42] M. Davier and A. Hocker, Phys. Lett. B439, 427 (1998).

[43] Constraint on increase in $m_W$ from measurements of $\sin^2 \theta_W$ is noted below (see [46]).

[44] Using Ref. [35], one would obtain a somewhat higher value of $m_H \approx 83$ GeV, which is still considerably lower than the ESSM value.

[45] Change in $m_t$ from its central value (174.3 GeV) by $\pm 5.1$ GeV would induce a shift in $m_W$ by $\pm 32$ MeV, as well as a shift in the prediction for $m_H$ (see Ref. [33] for details). For the sake of concreteness, columns (c) and (d) in Table I are obtained by keeping the difference in the predictions of $m_W$ for the two cases (SM versus ESSM) fixed at 20 MeV.

[46] As noted above, increase in $m_W$ by say 20-40 MeV (compared to the SM) would typically be in better accord with the data. This would, however, lead to a decrease in the prediction for the on-shell value of $\sin^2 \theta_W = 1 - m_W^2 / m_Z^2$ by $\delta(s_W^2) = -0.00039$ (1 to 2) for $\delta m_W = (20-40)$ MeV. A decrease by (say) 0.00039 is comparable to the experimental error in $\sin^2 \theta_W$ obtained from Standard Model fit by LEPEWWG which gives $\sin^2 \theta_W = 0.2227 \pm 0.00037$ [2], and thus quite acceptable. By itself, such a decrease would of course have negligible bearing on the anomaly in the NuTeV result, which yields $(\sin^2 \theta_W)_{\text{on shell}} = 0.2277 \pm 0.0013 \pm 0.0009$ (ignoring negligible corrections through $m_t$ and $m_H$), if the overall strength of the NC amplitude is kept at the SM value [1].

[47] W. Marciano and A. Sirlin, Phys. Rev. Lett. 71, 3629 (1993).

[48] C. Czapek et. al., Phys. Rev. Lett. 70, 17 (1993).

[49] D.I. Britton et. al., Phys. Rev. Lett. 68, 3000 (1992).

[50] W. Marciano, Phys. Rev. D60, 093006 (1999).

[51] In particular, if the true value of $|V_{us}|$ turns out to be higher than the present value of $0.2196 \pm 0.0026$ [34, 50], for instance, if the central value of $|V_{us}|$ is closer to that of the preliminary value of $0.2278 \pm 0.0029$ of the E865 experiment [54, 57], noted in Appendix A, the conflict in this regard with respect to Model II would be much milder (about 1.2$\sigma$ for $\eta_u = 1/15$).
Alternative explanations of the NuTeV-anomaly, based for example on $Z-Z'$ mixing have been proposed (see e.g. E. Ma and D. P. Roy [hep-ph/0111385]). In such an explanation, the correlation between the NuTeV-anomaly and the LEP neutrino-counting is, however, not obvious. For an interpretation based on $\nu_e - \nu_s$ oscillations with a large neutrino mass, see C. Giunti, and M. Laveder, [hep-ph/0202152].

After the completion of our work, which was presented by JCP at the WHEPP-7 Conference held at Allahabad, India (January 7, 2002), we came across a paper by S. Davidson, S. Forte, P. Gambino, N. Rius and A. Strumia, [hep-ph/0112302] which also notes the correlation between the NuTeV-anomaly and charged current universality in a general context, involving heavy fermions. We have recently come across another paper by W. Loinaz, N. Okamura, T. Takeuchi and L.C.R. Wijewardhana [hep-ph/0210193] which also discusses the effect of light-heavy neutrino mixing by assuming (in contrast to our case) lepton-universality in such a mixing.

V. Cirigliano, G. Colangelo, G. Isidori, G. Lopez-Castro, “Status of $V_{ud}$ and $V_{us}$”, Report to appear in the Proceedings of the First CKM Workshop, CERN, February 13-16, 2002; Private Communications from G. Isidori.

For completeness we note that superallowed nuclear Fermi transitions lead to a world average value for $|V_{ud}|_{\text{SM}}^{\text{expt.}} = 0.9740 \pm 0.0005$ [see J.C. Hardy and I.S. Towner, nucl-th/9907101, and also Ref. [54]]. Highly polarized $\beta$-decays on the other hand, lead to an average value of $|V_{ud}|_{\text{SM}}^{\text{expt.}} = 0.9731 \pm 0.0015$ [see Ref. [54] and references therein]. Combining these two averages, one obtains the value quoted in the text: $|V_{ud}|_{\text{SM}}^{\text{expt.}} = 0.9739 \pm 0.0005$ [54]. For comparison, it should be noted that the Particle Data Group, also using results from superallowed Fermi transitions and highly polarized neutron $\beta$-decays, quotes a world average of $|V_{ud}|_{\text{SM}}^{\text{expt.}} = 0.9734 \pm 0.0008$ [K. Hagiwara et al., PDG, Phys. Rev. D66, 010001 (2002)]. While these two values are certainly compatible, part of the reason for the differences between them is that PDG, taking a conservative approach, doubles the error given above for superallowed Fermi transitions.

K. Hagiwara et al., PDG, Phys. Rev. D66, 010001 (2002).

A. Sher (E865 Collaboration), talk given at the DPF 2002 Meeting, College of William and Mary, Williamsburg, May 2002; transparencies available at http://www.dpf2002.org.

B. Sciascia (KLOE Collaboration), talk given at the CKM Workshop, CERN, Geneva, February 2002; transparencies available at http://ckm-workshop.web.cern.ch.

As discussed in sec. 2 and in particular in Appendix B, as one transforms from the gauge to the hat basis in two steps: (a) the $Y^T$ and $Y'$ matrices are transformed into the forms
\( \propto (0, 0, 1) \), and next (b) the resulting \( X_f^T \) and \( X'_f \)-matrices are transformed into forms \( \propto (0, p_f, 1) \) and \( \propto (0, p'_f, 1) \) respectively; the first family gets decoupled from the vectorlike families. In this process, one introduces \( u_L - c_L \) mixing in step (b) given by \( \theta_{u_L c_L} \sim (x_1'/x_2')_{up} \lesssim 1/3 \) (say, see Appendix B). We thus estimate that \( \theta_{u_L U'_L} \approx (\theta_{u_L c_L})(\theta_{c_L U'_L}) \lesssim (1/3)(p'_u \eta_u) \lesssim (1/3)(10^{-2}) \) (where we have put \( \eta_u \leq 1/10; p'_u \approx 0.03 \) (for Model I), and \( |p'_u| \approx 0.1 \) (for Model II)). Furthermore, using a form for \( M_u \) analogous to Eq. (2), we estimate that \( \theta_{u_L U_L} \approx (\theta_{u_L c_L})(\theta_{c_L U'_L})(\theta_{U'_L U_L}) \approx (\lesssim 1/3)(p'_u \eta_u)(\sim 1/3 \text{ to } 1/2) \lesssim (10^{-3}), \) as mentioned in the text.

[60] Q. R. Ahmad et al. [SNO Collaboration], nucl-ex/0204008, nucl-ex/0204009; S. Fukuda et al. [SuperK Collaboration], Phys. Rev. Lett. 86, 5656 (2001); KamLand Collaboration, hep-ex/0212021. For a sample of pre-KamLand theoretical analyses see for example: V. Barger, K. Whisnant, D. Marfatia and B.P. Wood, hep-ph/0204253; J. Bahcall, C. Gonzalez-Garcia and C. Pena-Garay, hep-ph/0204314; A. Bandopadhyay, S. Choubey, S. Goswami and D.P. Roy, hep-ph/0204286; P. de Hollanda and A. Smirnov, hep-ph/0205241; M. Maltoni, T. Schwetz, M.A. Tortola and J.W.F. Valle, hep-ph/0207227.