Tracking photon jumps with repeated quantum non-demolition parity measurements

L. Sun1†, A. Petrenko1, Z. Leghtas1, B. Vlastakis1, G. Kirchmair1†, K. M. Sliwa1, A. Narla1, M. Hatridge1, S. Shankar1, J. Blumoff1, L. Frunzio1, M. Mirrahimi1,2, M. H. Devoret1 & R. J. Schoelkopf1

Quantum error correction is required for a practical quantum computer because of the fragile nature of quantum information. In quantum error correction, information is redundantly stored in a large quantum state space and one or more observables must be monitored to reveal the occurrence of an error, without disturbing the information encoded in an unknown quantum state. Such observables, typically multi-quantum-bit parities, must correspond to a special symmetry property inherent in the encoding scheme. Measurements of these observables, or error syndromes, must also be performed in a quantum non-demolition way (projecting without further perturbing the state) and more quickly than errors occur. Previously, quantum non-demolition measurements of quantum jumps between states of well-defined energy have been performed in systems such as trapped ions7–9, electrons7, cavity quantum electrodynamics8–10, nitrogen–vacancy centres7–9 and superconducting quantum bits10,11. So far, however, no fast and repeated monitoring of an error syndrome has been achieved. Here we track the quantum jumps of a possible error syndrome, namely the photon number parity of a microwave cavity, by mapping this property onto an ancilla quantum bit, whose only role is to facilitate quantum state manipulation and measurement. This quantity is just the error syndrome required in a recently proposed scheme for a hardware-efficient protected quantum memory using Schrödinger cat states (quantum superpositions of different coherent states of light) in a harmonic oscillator12. We demonstrate the projective nature of this measurement onto a region of state space with well-defined parity by observing the collapse of a coherent state onto even or odd cat states. The measurement is fast compared with the cavity lifetime, has a high single-shot fidelity and has a 99.8 per cent probability per single measurement of leaving the parity unchanged. In combination with the deterministic encoding of quantum information in cat states realized earlier13,14, the quantum non-demolition parity tracking that we demonstrate represents an important step towards implementing an active system that extends the lifetime of a quantum bit.

As well as being necessary in quantum error correction (QEC) and quantum information, quantum non-demolition (QND) measurements have a central role in quantum mechanics. The application of an ideal projective QND measurement yields a result corresponding to an eigenvalue of the measured operator, and projects the system onto the eigenstate associated with that eigenvalue. Moreover, the measurement must leave the system in that state, so that subsequent measurements always return the same result. The hallmark of a continuously repeated high-fidelity QND measurement is that it demonstrates a canonical thought experiment: individual quantum jumps between eigenstates are resolved in time on a single quantum system. This ideal measurement capability has been experimentally realized only in the past few decades. The jumps of a two-level system (quantum bit, or qubit) between energy eigenstates with different numbers of excitations (Fock states), were first observed for the motion of an electron in a Penning trap1. More recently, the observation of quantum jumps of light in cavity quantum electrodynamics10–13 (QED), where the number of microwave photons in a cavity is probed with Rydberg atoms, has enabled a range of new experiments in quantum feedback and control15,16.

An analogous system to cavity QED is the combination of microwave photons in a superconducting resonator with superconducting qubits, known as circuit QED17. The strong dispersive interaction of a qubit and a photon, as in Rydberg-atom cavity QED, allows either the qubit or the cavity to act as a QND probe of the other component. With the advent of quantum-limited parametric amplifiers18,19, measurement techniques for superconducting devices have rapidly advanced. For instance, the frequency shift of a cavity has recently been used to observe the quantum jumps of a qubit between energy eigenstates20,21. So far, however, there have been no observations of jumps for the cavity field in circuit QED. A recent paper measured a different quantity, the parity of two qubits, in a step towards the conventional approach of QEC20. However, that work did not present real-time tracking of the jumps due to the natural error rate of that quantity.

In this work, we use the dispersive qubit–cavity interaction of circuit QED to observe the jumps of photon number parity. Importantly, these jumps reveal the loss of individual photons without projecting the system onto a state of definite number or energy, but rather into an eigenspace of even or odd photon number. This characteristic is a crucial requirement for future applications in quantum information, where the parity measurement serves as the error syndrome for correcting a quantum memory. Even in the presence of rapidly repeated measurements, the smooth decay of the ensemble-averaged parity is largely unperturbed. However, when individual time records of the measurement are examined, the parity is observed to take on only the extremal values, ±1, indicating the projective nature of each individual measurement. On examining the statistics of the jumps recorded over many trajectories, we find excellent agreement with a numerical simulation, suggesting that 85% of the jumps for states with an average photon number $n = 4$ are faithfully detected (see Methods section on photon jump statistics). When selecting on the outcome of a single parity measurement, we observe, by Wigner tomography22, the creation of cat states with $n$ up to 4.

In our experiment, we use a three-dimensional circuit QED architecture27 with a single ‘vertical’ superconducting transmon qubit (the qualifier ‘vertical’ indicates that the dominant electric field is perpendicular to the film plane) coupled to two waveguide cavities14,22, as shown in Fig. 1a. Our qubit has a transition frequency of $\omega_{0}2\pi = 5.938$ GHz, an energy relaxation time of $T_{1} = 8$ µs and a Ramsey time of $T_{2} = 5$ µs. The high-frequency cavity, with a resonant frequency of $\omega_{0}2\pi = 8.174$ GHz and a lifetime of 30 ns, serves only as a fast readout of the qubit state. To perform a high-fidelity single-shot dispersive readout of the qubit, we use a Josephson bifurcation amplifier (JBA) operating in a double-pumped

1Departments of Applied Physics and Physics, Yale University, New Haven, Connecticut 06511, USA. 2INRIA Paris-Rocquencourt, Domaine de Voluceau, BP 105, 78153 Le Chesnay Cedex, France. 3Present address: Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing 100084, China (L.S.); Institut für Experimentelle Physik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria and Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Otto-Hitlmaer-Platz 1, A-6020 Innsbruck, Austria (G.K.).
mode\textsuperscript{4,25} as the first stage of amplification. The low-frequency cavity, with a resonant frequency of $\omega_{cl}/2\pi = 7.216 \text {GHz}$ and a lifetime of $\tau_0 = 55 \mu\text{s}$, stores the photon states which are measured and manipulated. Exploiting the nonlinearities induced in both resonators, we use the transmon qubit to track the parity of the storage cavity state. For simplicity, we will refer to the storage cavity as the ‘cavity’ henceforth.

The qubit and cavity are in the regime of strong dispersive coupling, which can be described by the Hamiltonian

$$H = \hbar \omega_{cl} |e\rangle \langle e| + \left( \omega_a - \chi_{qp} |e\rangle \langle e| \right) a^\dagger a$$

where $a$ and $a^\dagger$ are the annihilation and creation operators, respectively, $|e\rangle$ is the excited state of the qubit and $\chi_{qp}/2\pi = 1.789 \text{MHz}$ is the qubit-state-dependent frequency shift of the cavity. The readout cavity has been neglected because it remains in the ground state while the system evolves. The interaction between the qubit and the cavity entangles qubit and photon. In the rotating frame of the cavity, Fock states associated with the qubit in the excited state acquire a phase $\Phi = a^\dagger \chi_{qp} t$ proportional to their photon number\textsuperscript{24}. By waiting for $t = \pi/\chi_{qp}$, one can realize a controlled-phase gate $C_{\pi} = |0\rangle \langle 0| + |e\rangle \langle e| \otimes \mathbb{1}$, where $|e\rangle$ is the ground state of the qubit, adding a phase shift of $\pi$ per photon to the cavity state conditioned on the cavity state\textsuperscript{4,27,28}. Therefore, $C_{\pi}$ can be inserted between two $\pi/2$-pulses on the qubit in a Ramsey-type measurement to map the photon parity of any cavity state onto the qubit (black enclosure labelled ‘P’ in Fig. 1b). The result of a qubit measurement after the second $\pi/2$-pulse, together with prior knowledge of the initial qubit state, indicates whether the number of photons in the cavity is even or odd, but reveals nothing about the actual value of the photon number.

The creation of cat states is a natural consequence of a parity measurement on a coherent state $|x\rangle$ (x is a complex amplitude) because the phase cat states defined by

$$|\chi\rangle = \frac{1}{\sqrt{2(1 + e^{-i\pi|a|^2})}} \left( |+\rangle + (-1)^a |\rangle \right)$$

are eigenstates of the parity operator $e^{i\pi a^\dagger a}$ (refs 28, 29). After applying a microwave pulse at frequency $\omega_a$ to the cavity, initially in vacuum, to create a coherent state $|a\rangle$ with the qubit initially in $|g\rangle$, we use the parity protocol to take $|a\rangle \langle g| + |e\rangle \langle e| \otimes \mathbb{1}$ after the first $\pi/2$-pulse to $|\chi\rangle = (|+\rangle + |\rangle)/\sqrt{2}$ after the second pulse, at which point the parity of the cavity state is entangled with the state of the qubit. Detection of the qubit state using the readout cavity then projects the storage cavity onto one of the two cat states.

Figure 1 | Experimental device and parity measurement protocol of a photon state. a, Bottom half of the device containing a ‘vertical’ transmon qubit located in a trench and coupled to two waveguide cavities. The low-frequency cavity, with $\omega_{cl}/2\pi = 7.216 \text{GHz}$ and a lifetime of $\tau_0 = 55 \mu\text{s}$, is used to store and manipulate quantum states. The high-frequency cavity, with $\omega_{cl}/2\pi = 8.174 \text{GHz}$ and a lifetime of 30 ns, allows for fast readout of the qubit. b, Protocol (P) for measuring the parity of the storage cavity field. After an initial displacement of cavity vacuum $D(x)(0) = |x\rangle$ to create a coherent state with a complex amplitude $x$, a Ramsey-type measurement is performed. It consists of two $\pi/2$-pulses separated by $t = \pi/\chi_{qp}$ (during which a controlled-phase gate $C_{\pi} = |0\rangle \langle 0| + |e\rangle \langle e| \otimes \mathbb{1}$ is realized), followed by a projective measurement of the qubit, where $\chi_{qp}$ is the dispersive interaction between the cavity and the qubit states. In this schematic, with the qubit initially in the ground state, $|g\rangle$, the Ramsey-type measurement maps the even and odd photon states onto the $|e\rangle$ and $|g\rangle$ states of the qubit, respectively. A subsequent projective measurement indicates the cavity state parity. The second $\pi/2$-pulse can be either $R_{z,\pi/2}$ or $R_{x,\pi/2}$, simply swapping the interpretation of the result of the qubit measurement.

The weak visibility of the fringes comes from the slightly lower fidelity of the even cat state, wherein the qubit ends up in the $|e\rangle$ state, which is more susceptible to qubit relaxation. Figure 2d shows the normalized difference between the two cat states to emphasize the interference fringes. The high contrast between even and odd cat states is a central requirement in implementing a recently proposed QEC scheme\textsuperscript{26,27}, where these form the code and error spaces, respectively.

Because the loss of a single photon changes the parity of a cat state, monitoring parity repeatedly in real time allows us to track photon jumps of our cavity. Here we note that to interpret the result of a single parity measurement we must know the state of the qubit before the first $\pi/2$-pulse. In other words, it is the correlation of the qubit states before and after the parity measurement (a pattern of oscillation between $|g\rangle$ and $|e\rangle$ versus a constant pattern remaining in either $|g\rangle$ or $|e\rangle$) that reveals the photon state parity. For the following data we have chosen $R_{z,-\pi/2}$ as the second qubit pulse, instead of $R_{x,\pi/2}$, to maintain a constant pattern when the cavity is in the even parity state. Apart from reversing which pattern we assign to be even and which we assign to be odd, this change makes no difference. Figure 3a shows the measurement protocol and Fig. 3b–e shows typical 400 μs single-shot traces. The initial displacement is $|\alpha\rangle = 1.0$ and the repetition interval of the parity measurement is 1 μs, which is much smaller than the average photon lifetime, $\tau_0 = 55 \mu\text{s}$.

Figure 2 | Ensemble-averaged Wigner functions of cat states in the cavity created by single-shot parity measurements of an initial coherent state in the cavity. The Wigner functions are mapped out with varying displacements $\beta$ and a measurement of the mean photon parity (P) (ref. 21). Here we follow the protocol depicted in Fig. 1b, using $R_{z,\pi/2}$ as the second pulse. The qubit is always initialized to the $|g\rangle$ state through post-selection on an initial measurement.

a, Odd cat state by post-selection on the $|g\rangle$ state as the result of the parity measurement. b, Even cat state by post-selection on the $|e\rangle$ state. c, No post-selection of the parity measurement, thus tracing over the qubit state. Fringes almost disappear, indicating a mixture of two coherent states. d, The normalized difference (data in a minus data in b, all divided by two), or the expectation of the parity weighted by $\langle \sigma_x \rangle$ of the ancilla, emphasizing the interference fringes.
over an entire trajectory (see Methods section on photon jump statistics). We note that parity, we have applied a quantum filter that best estimates the photon jump. a. In this protocol we switch the sign of the second pulse, using \( R_y = \pi/2 \) instead of \( R_y = \pi/2 \). The repetition time of the parity measurement is 1 μs, and the traces in b–e all have an initial displacement of |x\rangle = 1. b. For the most part, the correlation between neighbouring measurements is positive, indicating an even-parity state for the whole 400 μs. The changes in the qubit state between 120 μs and 320 μs are probably due to qubit decoherence during the parity measurement. c. One parity jump is observed by the change in the measurement pattern (oscillating versus constant) at about 130 μs. d. Two parity jumps are recorded at about 10 μs and then again at 260 μs. The change of pattern at about 200 μs is a result of the qubit leaving the computational space for higher excited states, a feature that disables the parity measurement until the qubit returns to either |g\rangle or |e\rangle. e. A trace with all features described above included. In this particular trajectory, the filter can clearly resolve five photon jump events.

The repeated parity measurements shown above constitute just a single point, the origin, in the Wigner functions of the even and odd cat states (Fig. 2a, b). Thus, crucially, a parity measurement acquires no information about the phase of the cat states. Consequently, one could encode quantum information onto the computational bases \( |0\rangle_L = \mathcal{N}_+ (|x\rangle + |−x\rangle) \) and \( |1\rangle_L = \mathcal{N}_+ (|ix\rangle + |−ix\rangle) \), and any subsequent parity measurements would make no distinction between the two. The loss of a single photon will change the code space spanned by \( |0\rangle_L \) and \( |1\rangle_L \) into the error space spanned by \( |0\rangle_L = \mathcal{N}_− (|x\rangle − |−x\rangle) \) and \( |1\rangle_L = \mathcal{N}_− (|ix\rangle − |−ix\rangle) \) with a different parity. This error syndrome can thus be extracted by the parity measurement demonstrated here, but without gaining any knowledge of the information encoded in the cat states, as required by QEC.

The degree to which the measurements are QND can be determined by examining the decay rate for the parity of a coherent state with different measurement cadences. We extract the total decay rate of the parity \( (\tau_{\text{ref}}) \), from the ensemble-averaged parity dynamics obtained with the quantum filter (Fig. 4). This total decay rate is well modelled by the parallel combination of the free decay time \( (\tau_0 = 55\,\mu s) \) plus a constant demolition probability \( P_D = 0.002 \) per measurement interval \( \tau_m \) as shown by the fit in the inset of Fig. 4. In other words, a single parity measurement is 99.8% QND, leaving the parity of the cavity state largely unperturbed.

Several improvements and further investigations will be required to realize a truly robust error-corrected quantum memory. The probability of missing a photon jump, as a result of the finite measurement rate obtained from a free time evolution measurement of the parity of a coherent state (see Methods section on experimental set-up). We observe a range of photon jump statistics, from quiet traces that last for hundreds of micro-seconds with no apparent changes in parity, to those that have as many as five jumps. The clear dichotomy between the patterns in our traces indicates that, although the measurements are susceptible to qubit decoherence, as evidenced by intermittent, brief changes in measurement correlations and excitations to higher qubit states, they nonetheless exhibit a strong sensitivity to single-photon jump events.

When analysing these single-shot traces, to mitigate the effects due to qubit decoherence, excitation to qubit states higher than |e\rangle (denoted as |\tilde{f}\rangle) and other imperfections in the qubit readout in extracting the information encoded in the cat states, as required by QEC.
per cavity lifetime, would be greatly reduced if larger cavities were used\textsuperscript{30}. It is a necessary, but not sufficient, condition that the measurement itself, as shown here, is highly QND and unlikely to induce photon jumps. An additional requirement is that the measurement does not destroy the actual quantum information stored in the cat states. Dephasing of the cavity state will be non-negligible because the current realization is not yet robust against qubit decay or excitation (see Methods section on parity-tracking performance). Increasing qubit and cavity lifetimes, and further characterizing these types of error processes, could already allow extension of the average lifetime of an encoded qubit with an optimized measurement strategy, the current level of performance being about 90%, mainly limited by the short coherence times of the qubit. The quantum filter used as the best-parity estimator consists of two steps: a time evolution of the density matrix taking into account the cavity decoherence, and a modification of the density matrix based on the current measurement result. The effectiveness of this filter is confirmed by the good agreement between extracted numbers of jumps from the parity estimator during 500 μs repeated parity measurements and a numerical simulation (Extended Data Fig. 9). The Wigner tomography fidelity is limited by qubit $T_1$, $T_2^*$, photon jumps and detection accuracy. Among these factors, only qubit $T_1$ and missing of fast photon jumps can lead to the decay of the cat states. The high-QND nature of the parity measurement allows the incorrect parity measurement result due to qubit $T_1$ and detection inaccuracy to be removed by performing repeated parity measurements and taking a majority voting. Slow photon jumps can be tracked as demonstrated in the manuscript, and the resulting phase errors of the cat states can also be completely corrected. Repeated parity tracking can thus enhance the lifetime of the information encoded in cat states (Methods section on parity-tracking performance).

**Online Content** Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

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**METHODS SUMMARY**

Measurements are performed in a cryogen-free dilution refrigerator with a base temperature of about 10 mK. The ‘vertical’ transmon qubit is fabricated on a c-plane sapphire (Al$_2$O$_3$) substrate with a double-angle evaporation of aluminum after a single electron-beam lithography step. The state-dependent frequency shift between the qubit and the readout cavity is $\omega_{QR}/2\pi = 0.930$ MHz, which is not optimized for the best signal-to-noise ratio. The background photon number $n_{bg} = 0.02$ and the displacement $\alpha$ are calibrated on the basis of the Poisson distribution of photon numbers in the storage cavity (Extended Data Fig. 2). The qubit readout fidelity is about 90%, mainly limited by the short coherence times of the qubit. The quantum filter used as the best-parity estimator consists of two steps: a time evolution of the density matrix taking into account the cavity decoherence, and a modification of the density matrix based on the current measurement result. The effectiveness of this filter is confirmed by the good agreement between extracted numbers of jumps from the parity estimator during 500 μs repeated parity measurements and a numerical simulation (Extended Data Fig. 9). The Wigner tomography fidelity is limited by qubit $T_1$, $T_2^*$, photon jumps and detection accuracy. Among these factors, only qubit $T_1$ and missing of fast photon jumps can lead to the decay of the cat states. The high-QND nature of the parity measurement allows the incorrect parity measurement result due to qubit $T_1$ and detection inaccuracy to be removed by performing repeated parity measurements and taking a majority voting. Slow photon jumps can be tracked as demonstrated in the manuscript, and the resulting phase errors of the cat states can also be completely corrected. Repeated parity tracking can thus enhance the lifetime of the information encoded in cat states (Methods section on parity-tracking performance).

**Appendix**
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Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to L.S. (luyansun@mail.tsinghua.edu.cn) or R.J.S. (robert.schoelkopf@yale.edu).
The qubit serves as an ancilla and provides the necessary nonlinearity for the manipulation of coherent states in the storage cavity. Both the storage and readout cavities are made of aluminum alloy 6061. The state-dependent frequency shifts between the qubit and the storage and readout cavities are $\Omega_{q}2\pi = 1.789$ MHz and $\Omega_{s}2\pi = 0.930$ MHz, respectively. For simplicity, we will refer to the storage cavity as the ‘cavity’ henceforth. The inset of Extended Data Fig. 2 shows the ‘number splitting peaks’ of the qubit due to different photon numbers in the cavity, which is displaced with a 10 ns square pulse right before the spectroscopy measurement. A second-order polynomial fit

\[ y = a_0 + a_1 x + a_2 x^2 \]

is the time of cavity 1, we post-select only the $|g\rangle$ state of the qubit. The system’s initial state before the first parity measurement $P_1$ (note that $R_{s,1/2}$ is the second $\pi/2$-pulse) is $|\psi\rangle = |g\rangle |\psi_s\rangle$. The rotating frame has been fixed to be the one rotating at the cavity frequency $\omega_0$. The qubit $T_2$ process between the two $\pi/2$-pulses in $P_2$ leads to $|\psi\rangle = (|g\rangle e^{i\phi_0} + |e\rangle) / \sqrt{2}$, where $t$ is the time of the $T_2$ jump happening. On average, this process gives 3% fidelity error. The qubit $T_2$ process between the two $\pi/2$-pulses in $P_1$ leads to $|\psi\rangle = (|g\rangle + |e\rangle + |\psi_s\rangle) + (|g\rangle + |e\rangle + |\psi_s\rangle)/\sqrt{2}$ right before measurement $M_5$, switching the entanglement relationship between the cat states and the qubit states and causing a full error. Here $N_{cat} = 1 / \sqrt{2(1 \pm 2^{-20})}$. Because in our experiment we mainly consider $|x\rangle = 2$, for reasons of simplicity, we use the approximation $N_{cat} = N_{cat} / \sqrt{2}$ in the large-$|x|$ limit in what follows. The photon jump process between the two $\pi/2$-pulses in $P_1$ leads to $|\psi\rangle = (|g\rangle + |e\rangle + |\psi_s\rangle + |\psi_s\rangle) / \sqrt{2}$, where $t$ is the time of the photon jump event happening. On average, in this case the fidelity to the ideal even/odd cat states is 50%. If the photon jump process happens during measurement $M_4$ and the following waiting time, it switches between the even and odd cat states, thus causing a full error. If the photon jump process happens between the two $\pi/2$-pulses during the Wigner tomography, the results in the two parity protocols cancel out on average, leading to a full error as well. Finally, the qubit measurement inaccuracy due to the full overlap between measurement histograms in $P_1$ plus the qubit transition up process during the waiting time right after the readout is about 1.3%. The sum of all sources of error limits the Wigner tomography fidelity $F = 84\%$ of the created cat states, consistent with the measurement. To create the even cat state, the measurement $M_5$ projects the qubit onto the $|e\rangle$ state. The extra qubit $T_2$ process after the projection lowers its creation fidelity by 280 ns/8 $\mu$s = 3.5%. This difference explains the imperfect cancellation of the fringes in Fig. 2c.

Similarly, the error budgets for the parity readout fidelity with $R_{s,1/2}$ as the second qubit pulse (Extended Data Fig. 6a) can also be estimated, as shown in Extended Data Fig. 6e. We again mainly consider qubit $T_1$, $T_3$ (f state) and photon jump processes. We examine the case with the system initially in the state $|\psi\rangle = (|g\rangle + |e\rangle) / \sqrt{2}$ (post-selected by the first five parity measurements), and consider the probability of not measuring the $|g\rangle$ state in the sixth parity measurement. In this case, the qubit $T_3$ process between the two $\pi/2$-pulses has a 50% chance of causing an error. The photon jump process between the two $\pi/2$-pulses leads to $|\psi\rangle = (|g\rangle - |e\rangle) / \sqrt{2}$ and $|\psi\rangle = (|g\rangle + |e\rangle) / \sqrt{2}$, where $t$ is the time of the photon jump event happening, on average also giving a 50% chance of a wrong answer. The qubit $T_3$ process between the two $\pi/2$-pulses flips the qubit mainly comes from qubit decoherence processes during the parity measurement (discussed later). Conditional probabilities $P(\pm 1 | \text{even})$, $P(\pm 1 | \text{odd})$, $P(\pm 1 | \text{even})$, $P(\pm 1 | \text{odd})$, $P(0 | \text{even})$ and $P(0 | \text{odd})$ are time-independent probabilities that have positive, negative and zero correlations (as indicated) between the digitized qubit readouts before and after a parity measurement for a given even or odd state. However, a pure even or odd state cannot be prepared easily in our system owing to the finite thermal population of the cavity, which is small but can still introduce systematic errors. We determine $P(\pm 1,0 | \text{even/odd})$ by post-selecting the cases with five consecutive identical parity results, which give the photon state parity with good fidelity, and then performing a histogram on the sixth parity measurement (Extended Data Fig. 6b).

Extended Data Fig. 6c shows the pulse sequence for producing the cat states and the Wigner tomography shown in Fig. 2. The protocol starts with a post-selection of the $|g\rangle$ state of the qubit after an initial qubit measurement $M_1$. A parity measurement is performed immediately after a storage cavity displacement $x$, followed by Wigner tomography with varying displacements $x$. A 280 ns waiting time after each measurement has been chosen to ensure that the readout cavity returns to the vacuum state. The qubit pulses have a Gaussian envelope truncated to 4$\tau$ = 8 ns, and the displacement pulses on the storage cavity are 10 ns square pulses. The dashed enclosures represent the pulse sequence for a parity measurement.

To remove the cross-Kerr effect between the readout cavity and the storage cavity which skews the readout signal for large storage cavity displacements, and also to convert the readout voltage to parity, we followed the procedure in the supplementary material of ref. 14. The idea is to perform two parity measurements of different protocols ($R_{s,2/3}$ or $R_{e,2/3}$) as the second $\pi/2$-pulse) for a vacuum state in the cavity. The difference between the two measurements corresponds to a parity value $P = 1$. We used this technique earlier in Extended Data Fig. 3 and here in the Wigner tomography in Extended Data Fig. 6c.

Extended Data Fig. 6d shows the error budgets for the Wigner tomography fidelity in Fig. 2. The fidelity is defined by the overlap between the measured Wigner state and that of an ideal cat state. We mainly consider qubit $T_1$, $T_3$ and photon jump processes. First of all, the qubit $T_1$ and $T_3$ processes and $|f\rangle$ states during Wigner tomography have been included in the parity calibration. After measurement $M_4$, we post-select only the $|g\rangle$ state of the qubit. Therefore, the system’s initial state before the first parity measurement $P_1$ (note that $R_{s,2/3}$ is the second $\pi/2$-pulse) is $|\psi\rangle = |g\rangle |\psi_s\rangle$. The rotating frame has been fixed to be the one rotating at the cavity frequency $\omega_0$. The qubit $T_3$ process between the two $\pi/2$-pulses in $P_2$ leads to $|\psi\rangle = (|g\rangle e^{i\phi_0} + |e\rangle) / \sqrt{2}$, where $t$ is the time of the $T_3$ jump happening. On average, this process gives 3% fidelity error. The qubit $T_2$ process between the two $\pi/2$-pulses in $P_1$ leads to $|\psi\rangle = (|g\rangle + |e\rangle + |\psi_s\rangle + |\psi_s\rangle) / \sqrt{2}$ right before measurement $M_5$, switching the entanglement relationship between the cat states and the qubit states and causing a full error. Here $N_{cat} = 1 / \sqrt{2(1 \pm 2^{-20})}$. Because in our experiment we mainly consider $|x\rangle = 2$, for reasons of simplicity, we use the approximation $N_{cat} = N_{cat} / \sqrt{2}$ in the large-$|x|$ limit in what follows. The photon jump process between the two $\pi/2$-pulses in $P_1$ leads to $|\psi\rangle = (|g\rangle + |e\rangle + |\psi_s\rangle + |\psi_s\rangle) / \sqrt{2}$, where $t$ is the time of the photon jump event happening. On average, in this case the fidelity to the ideal even/odd cat states is 50%. If the photon jump process happens during measurement $M_4$ and the following waiting time, it switches between the even and odd cat states, thus causing a full error. If the photon jump process happens between the two $\pi/2$-pulses during the Wigner tomography, the results in the two parity protocols cancel out on average, leading to a full error as well. Finally, the qubit measurement inaccuracy due to the full overlap between measurement histograms in $P_1$ plus the qubit transition up process during the waiting time right after the readout is about 1.3%. The sum of all sources of error limits the Wigner tomography fidelity $F = 84\%$ of the created cat states, consistent with the measurement. To create the even cat state, the measurement $M_5$ projects the qubit onto the $|e\rangle$ state. The extra qubit $T_1$ process after the projection lowers its creation fidelity by 280 ns/8 $\mu$s = 3.5%. This difference explains the imperfect cancellation of the fringes in Fig. 2c.

Similarly, the error budgets for the parity readout fidelity with $R_{s,1/2}$ as the second qubit pulse (Extended Data Fig. 6a) can also be estimated, as shown in Extended Data Fig. 6e. We again mainly consider qubit $T_1$, $T_3$ (f state) and photon jump processes. We examine the case with the system initially in the state $|\psi\rangle = (|g\rangle + |e\rangle) / \sqrt{2}$ (post-selected by the first five parity measurements), and consider the probability of not measuring the $|g\rangle$ state in the sixth parity measurement. In this case, the qubit $T_3$ process between the two $\pi/2$-pulses has a 50% chance of causing an error. The photon jump process between the two $\pi/2$-pulses leads to $|\psi\rangle = (|g\rangle - |e\rangle) / \sqrt{2}$ and $|\psi\rangle = (|g\rangle + |e\rangle) / \sqrt{2}$, where $t$ is the time of the photon jump event happening, on average also giving a 50% chance of a wrong answer. The qubit $T_3$ process between the two $\pi/2$-pulses flips the qubit
state on the equator of the Bloch sphere and leads to $|\psi_t\rangle = [(|x\rangle + |\bar{x}\rangle)/\sqrt{2}]$, giving a full error in the final readout. Finally, the qubit measurement inaccuracy in the fifth parity measurement plus the qubit transition up process during the waiting time is 1.3%, as in the case of the Wigner tomography. There is an extra error coming from the |0⟩ state between the two π/2-pulses in the sixth parity measurement, contributing about 0.5%. All the above sources of error add up to 7.7%, in good agreement with the 91.3% probability of faithfully measuring a positive correlation for an even cat state in Extended Data Fig. 6a. The lower fidelity for an odd cat state is because of the extra qubit $T_1$ process for the |e⟩ state due to the negative correlation under the same parity readout protocol.

**Quantum filter and correlated data.** To mitigate the effects due to qubit decoherence, $|f⟩$ states of the qubit (unobservable states that obscure the parity measurement) and other imperfections in the qubit readout in extracting the parity, we have applied a quantum filter\(^{1,3,4}\) that best estimates the photon state parity. We note that the quantum filter is an integration of the quantum stochastic master equation and depends on the measured trajectory. Extended Data Fig. 7 shows the schematic of the quantum filter. This quantum filter at each point in time is realized in two steps: first, a new density matrix $\rho(C_{t+\Delta t})$ is calculated from the best estimation $\rho(C_t)$ at the previous point, based only on the decoherence of the cavity; second, the density matrix $\tilde{\rho}(C_{t+\Delta t})$ gets updated as the best estimation $\rho(C_{t+\Delta t})$ according to Bayes’ law, based on the newly acquired knowledge from the current parity readout. This best estimated density matrix $\rho(C_{t+\Delta t})$ is then used as the input for the next iteration. We have truncated the dimension of the density matrix to $N = 5n$, which is large enough to cover all relevant number states. To initialize the density matrix after a displacement $D(z)$, we have set $\rho(t = 0) = 1 - 2n_0 D(z)(0)(0|D(z) + 2n_0 D(z)|1⟩⟨1|D(z)$, taking into account the background photon population in the limit $n_0 \ll 1$.

At time $t$, the density matrix of the photon state is $\rho(C_t)$, which depends on all previous correlations. At $t = t + \Delta t$, considering only the decoherence of the cavity, the expected density matrix from free evolution becomes $\rho(C_{t+\Delta t}) = \rho(C_t) + M_{nn} \rho(C_t) M_{nn}^\dagger + M_{nr} \rho(C_t) M_{nr}^\dagger + M_{rr} \rho(C_t) M_{rr}^\dagger$, where $M_{nn} = \sqrt{n_0} M_{nn} + M_{nr} = \sqrt{n_0} M_{nr} + M_{rr} = I - (\sqrt{n_0} M_{nn} + M_{nr} + M_{rr})/2$ are the Kraus operators for photon loss, absorption of thermal photons and no-jump events, respectively. We have $n_{down} = n_0 + 1/k_N$ and $n_{up} = n_0 - 1/k_N$, and $k_N$ is the energy decay rate in the cavity under repeated parity measurements. The additional information $C_{t+\Delta t}$ acquired from the parity measurement at $t + \Delta t$ changes the quantum state according to

$$\rho(C_{t+\Delta t}) = \begin{cases} \rho(C_{t+\Delta t}) & \text{if } C_{t+\Delta t} = 0 \\ \rho(C_{t+\Delta t}) & \text{if } C_{t+\Delta t} \neq 0 \end{cases}$$

which has been used to fit the curves in Fig. 4. **Statistics of photon jumps.** To test how faithfully our repeated parity measurement can track photon losses, we simply count the number of jumps extracted from the parity estimator during 500 μs repeated parity measurements. We have applied a Schmitt trigger to digitize the parity estimator to reject the unavoidable noise (spikes in the estimator) coming from qubit decoherence and erroneous parity readout. The two thresholds for the Schmitt trigger are chosen to be ±0.9 for a large discrimination. Then the number of parity jumps is inferred from the number of transitions in the digital data after the Schmitt trigger. Although our singles parity readout fidelity is about 80% (Extended Data Fig. 6a), owing to the averaging effect of the quantum filter we actually can achieve nearly unity detection sensitivity of single-photon jump events. However, because of the finite bandwidth of the filter, if two photon jumps occur within the response time of the filter $\tau_f$ (defined as the time to make a transition between the two thresholds for the Schmitt trigger), our Schmitt trigger will not catch both jumps. Extended Data Fig. 8c shows the time response of the quantum filter applied to typical photon jump events. Green and cyan curves are fits of the parity estimator at the transition based on a hyperbolic tangent function, giving a transition time constant of less than 1 μs. We also find the response time of the filter to make a transition between ±0.9 to be $\tau_f = 2\mu$s. The probability of having a second photon jump within $\tau_f$ after the first jump is simply $P_{jump} = \int_0^{\tau_f} e^{-(t-\tau_f)/\tau_f} dt = 1 - e^{-(\tau_f/\tau_f)}$. For $n = 1$ and $\tau_f = 49\mu$s, the above probability is $P_{jump} = 4\%$, and $P_{jump} = 15\%$ for $n = 4$, which is the probability of missing both jumps.
In reality, we have no way of knowing the true number of photon jumps for each parity measurement trajectory. The only way to test how faithfully our repeated parity measurement can track photon jumps is to see whether the distribution of jumps agrees with what we expect. Owing to the complication of background thermal excitation and the finite response time of the filter, to get an analytic solution is difficult. Instead, we perform a numerical Monte Carlo simulation to compare with the experiment. In the simulation, we use a coherent state as the initial state without distinguishing the parity. Each simulation trajectory is 300 μs long, and includes a transition probability of having a photon enter the cavity to change the photon number from n to n + 1 as a result of the background thermal excitation. In the simulation, we also neglect those who have neighbouring jumps within the response time τj of the quantum filter. Then for each trajectory we count the number of jumps and finally construct a histogram (black solid lines in Extended Data Fig. 9) of those numbers based on 100,000 trajectories. The good agreement between simulation and data demonstrates that the repeated parity measurement can track the error syndromes faithfully.

Quantifying parity-tracking performance. Our demonstrated parity-tracking protocol has several sources of infidelity that lead to a loss of the encoded information in the cat states, ultimately putting a bound on the improvement we would be able to achieve in an actual QEC protocol. This infidelity (Extended Data Fig. 6d) can be broken down into three categories: missed fast photon jumps (due to the limited bandwidth of the measurement), misinterpreted photon jumps (due to qubit Ts and readout inaccuracy), and dephasing of the cat states due to the relaxation of the ancilla qubit during a parity measurement protocol (qubit Tj process only, as explained later). Missing jumps certainly amounts to a complete loss of phase information. A distinction has to be made between the last two effects because misinterpreting photon jumps need not fully degrade our knowledge of the cavity state’s parity at a given point in its trajectory and can be minimized by using multiple quantum-measurement ‘packets’ (discussed more later). Recall that the Wigner tomogram in Fig. 2 aids in appreciating this point. Despite the 80% fidelity of a single parity measurement, after just three of them we can be very confident of the parity of our cavity state, because the probability of having three errant measurements is (0.2)3 = 0.8%. This is evident when inspecting the behaviour of the quantum filter in the single-shot traces; given three consecutive measurements that are the same, the filter converges to ± with nearly 100% confidence. In the Wigner tomography, this would amount to knowing the value at the origin very well, but, given the cat state dephasing due to qubit Tj, not knowing the full contrast of the fringes and coherent state populations.

To realize the cat states as a quantum memory, the entire state must be preserved in order that an eventual decoding procedure faithfully maps the information back onto some other component (for example a physical qubit). Given the long lifetime of our cavity, an 80% fidelity indicates that a large contribution to an incorrect parity measurement arises from two sources of qubit decoherence: Tj and Ts. The detrimental effects of Tj decay are apparent when recalling the entanglement between the cavity and the ancilla qubit, where the cat state begins to acquire a phase at a rate gq2q, that depends on the qubit state (g 0, |g| q = 1, |g| q = 2). Again, the rotating frame has been fixed to be the one rotating at the cavity frequency ω0 when the qubit is in the |g⟩ state. Concretely, if qubit Tj relaxation happens at t during the qubit-decay time τj, then the initial cat state becomes (|g⟩ 0, |g⟩ 1, |g⟩ 2) before the parity measurement will become ((|g⟩ 0, |g⟩ 1, |g⟩ 2))2/2. Again, for reasons of simplicity, we have used the approximation N = N = 1/2√2 in the large-|z| limit for the rest of Methods. Similarly, during the time between parity measurements, the ideal cat state (|x⟩ ± 2)√2 associated with different qubit states will rotate deterministically at a rate gq2q. Should the qubit state change at a random time without our knowledge, the cat state will change its rotation rate accordingly, and the phase of the cat state will thus become completely random. In the Wigner tomography, this would manifest itself as a washing out of the cat state’s features, where, unlike at the origin, successive measurements can only further reduce the fidelity. In the single-shot regime, the system evolves in a product state again; hence the excitation to higher qubit states imparts an arbitrary phase on the cat states that would be impossible to recover from without some auxiliary correction protocol.

The contribution of qubit dephasing Tj enters in a subtle, different way. Without loss of generality, let us assume the system is initially in state |x⟩ ± 2 |g⟩ 0, 1/2. The first π/2-pulse in the parity measurement brings the system to (|x⟩ + 2 |g⟩ 0, 2) |g⟩ 0, 1/2. The above state evolves to (|x⟩ + 2 |g⟩ 0, 2) |g⟩ 0, 1/2, where at time τ a random phase flip happens. Consequently, the system becomes (|x⟩ + 2 |g⟩ 0, 2) |g⟩ 0, 1/2 and then keeps evolving in the same way, regardless of the sign change of the term associated with |g⟩ 0, 2. Again, at the end of the π/2 pulse, the system is a product state again; hence the excitation to higher qubit states imparts an arbitrary phase on the cat states that would be impossible to recover from without some auxiliary correction protocol.

Recalling the 83% fidelity of our Wigner tomography to an ideal cat state, the k eff derived here indicates that this fidelity does not decrease as 0.83 k, with N the number of parity measurements performed. The latter would be the case only if we were to code a state, perform tomography on it with 83% fidelity, decode the state, whose fidelity to the initial one would then be reduced by 17%, and then repeat this procedure again and again. Of course this is not what we do. After the initial projection onto a cat state, we proceed with repeated parity measurements, and with each subsequent measurement we actually build up our confidence in the
state. At the same time, however, we pay the price of risking complete dephasing due to qubit decay. Thus, the number 0.83 reflects the amount of information we actually acquire when reconstructing the state through tomography rather than the degree to which the state has been corrupted. The actual corruption is related to how QND the measurement is regarding inducing extra photon loss and cavity state dephasing errors. In the main text, we show the former to be very high at 99.8% per measurement. The latter would be \( 1 - \epsilon_T = 94\% \).

Given our system’s parameters, we can quantify what level of improvement we can achieve with the demonstrated parity-tracking protocol over a photon jump rate \( n_{\text{sp}} \). As seen in Extended Data Fig. 6d, \( \epsilon_T = 6\% \), \( \epsilon_{\text{qs}} = 3\% \) and \( \epsilon_{\text{e}} = 1\% \) are all of the same order. Here we attribute a greater contribution of \( \epsilon_{\text{qs}} \), from that listed in Extended Data Fig. 6d because the sources of error there assumed that the final qubit state after each measurement was \( |g\rangle \). However, without any post-selection of trajectories, the qubit could just as well end up in \( |e\rangle \), enhancing the effect of qubit-induced dephasing. Therefore, the optimal \( N \) in our case is actually \( N = 1 \), notwithstanding the 80% fidelity of a single parity measurement. This can be understood by noting that for \( N > 1 \), the contribution of \( \epsilon_T \) quickly begins to outweigh any qubit dephasing and measurement errors, leading to a suboptimal choice of parameters. Given that the contributions of these terms together sum to about a 10% error, we now have

\[
\kappa_{\text{eff}} = \left[ \frac{W}{k} \right] \left( \tau_M + \tau_w \right)^2 + 0.1 - \frac{1}{\tau_M + \tau_w}
\]

The minimum \( \kappa_{\text{eff}} \) is achieved when the decay rates are equal:

\[
\left( \tau_M + \tau_w \right)^2 = \frac{2(0.1)}{\left( \frac{W}{k} \right)} \Rightarrow \kappa_{\text{eff}} = \frac{W}{k} \sqrt{0.2}
\]

The improvement over \( n_{\text{sp}} \) is thus of the order of \( \sqrt{0.2} \), which predicts an improvement in the effective cavity decay time by a factor of two over \( 1/k \). The corresponding \( \tau_w \) value is 4.6 \( \mu \)s. Given that \( \tau_M \) is dominated in large part by the parity protocol waiting time \( \pi/\kappa_{\text{eff}} \) a relevant benchmark for the overall performance becomes the product \( \tau_M \tau_T \). We emphasize that even for this system’s modest coherence properties, an improvement by a factor of two would be impressive.

Indeed, if \( \tau_M \) and \( \tau_T \) approach 20 \( \mu \)s, the protected lifetime of the information would exceed 50 \( \mu \)s, the lifetime of a single-photon Fock state in the storage cavity.

The highly QND nature of the parity measurements at 99.8%, expanded on in the main text, was omitted from the analysis above owing to its minor contribution relative to all other sources of error. We consider this number to be one of the two figures of merit for the success of the parity-tracking protocol. If the very act of measuring parity were to induce photon jumps without our knowledge, the parity-tracking protocol itself would be flawed. This success, however, belies the degree to which we perturb the information stored in the cat states. Referring once more the Wigner tomograms, although we can confidently claim that we are QND as far as the point at the origin is concerned, given low qubit \( \tau_T \) the same cannot be said of the rest of the information present in the Wigner tomogram at other points in \( \mathbb{J}-\mathbb{Q} \) space. In other words, the parity monitoring does not change photon number probabilities, but could change the relative phases between constituent Fock states. This second figure of merit, which can again be quantified as the contribution to cavity state dephasing due to qubit decay (1 - \( \epsilon_T = 94\% \) per measurement) still clearly leaves much room for improvement. Nonetheless, although certainly presenting challenges, shortcomings arising from qubit performance and other higher-order mechanisms of dephasing not discussed here (such as self-Kerr of the cavity and cross-Kerr due to readout) do not seem insurmountable.

Addressing the issue of cavity state dephasing due to measurement is an important next step in improving the performance of this QEC scheme. We are confident that we can address the issue of qubit \( \tau_T \) without substantially altering the parity-tracking protocol presented here, but we feel that this lies beyond the scope of this work.

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35. Mirrahimi, M. et al. Dynamically protected cat-qbets: a new paradigm for universal quantum computation. New J. Phys. 16, 045014 (2014).
Extended Data Figure 1 | Schematic of the measurement set-up. We use two separate lines to drive the readout and the storage cavity. Qubit state manipulations are realized through the readout cavity input line. The readout cavity output signal is first amplified by a JBA operating in a double-pumped mode, and the reflected signal then goes through three isolators in series before being further amplified by a HEMT at 4 K. The amplified signal is finally down-converted to 50 MHz and then digitized by a fast 1 GS data-acquisition card.
Extended Data Figure 2 | Poisson distribution of photon numbers in the cavity. Dotted colour lines are data for the first eight Fock states $n = 0, 1, 2, \ldots, 7$ as functions of displacement amplitude $|\alpha|$. The measurements are performed with a selective $\pi$-pulse on each number splitting peak, and the resulting signal amplitude should be proportional to the corresponding number population. These oscillation amplitudes have been normalized to probabilities such that the sum of the amplitudes corresponding to $n = 0$ and $n = 1$ equals unity. Dashed lines are theoretical curves with a Poisson distribution $P(n) = |\alpha|^n e^{-|\alpha|^2}/n!$, where the $x$ axis has had a single scale factor adjusted to fit all these probabilities. The excellent agreement indicates good control of the coherent state in the cavity and also gives a good calibration of the cavity displacement amplitude. On the basis of the probability of $n = 1$ at $|\alpha| = 0$, we find a background photon population of $n_{\text{bg}} = 0.02$ in the cavity.

Inset bottom panel: spectroscopy (left axis) of the number splitting peaks of the qubit when populating different photon numbers in the cavity. Inset top panel: difference between peak positions and a linear fit. The curvature necessitates a second-order polynomial fit, resulting in a linear dispersive shift $\chi_{\text{disp}}/2\pi = 1.789 \pm 0.002$ MHz and a nonlinear dispersive shift $\chi_{\text{disp}}/2\pi = 1.9 \pm 0.1$ kHz.
Extended Data Figure 3 | Ensemble-averaged free parity evolution of a coherent state. The measurement protocol is shown in the inset. The single parity measurement gives a readout voltage that has been converted to parity through thresholding. All measured evolution curves saturate at the same value in the long time limit. This saturation level has been forced to 0.96 (because $n_{th} = 0.02$), represented by the dashed horizontal line. The solid lines are global fits, giving a time constant of $\tau_0 = 55 \mu s$. 
Extended Data Figure 4 | Effectiveness of the $R_{y,\pm\pi/2}$ pulse. Blue and red data (bottom axis) are ensemble-averaged qubit readouts after consecutively (with no wait time) applying ($R_{y,\pi/2}, R_{y,\pi/2}$) and ($R_{y,\pi/2}, R_{y,-\pi/2}$), respectively, as functions of different $n$ introduced into the cavity. The curvature for $n > 4$ comes from the finite bandwidth of the pulses in the frequency domain. Green curve (top axis) is a time Rabi trace for an amplitude comparison with no initial cavity displacement.
Extended Data Figure 5 | Qubit readout properties. a, Histogram of qubit readout for the parity protocol used in repeated single-shot traces in Fig. 3. The phase between the JBA readout and the pump has been adjusted such that \(|g\rangle\), \(|e\rangle\) and \(|f\rangle\) states can be distinguished with optimal spacings. Thresholds between \(|g\rangle\) and \(|e\rangle\), and between \(|e\rangle\) and \(|f\rangle\), have been chosen to digitize the readout signal to +1, −1 and 0 for \(|g\rangle\), \(|e\rangle\) and \(|f\rangle\), respectively. Note that we assign a zero to the \(|f\rangle\) states to indicate a ‘failed’ measurement with no useful information about the parity. b–d, Illustrations of pulse sequences (not to scale) producing the readout error matrix with the storage cavity left in vacuum. The \(|g\rangle\) state (b) is prepared through post-selection of an initial qubit measurement \(M_1\), whereas \(|e\rangle\) (c) and \(|f\rangle\) (d) are prepared by properly pulsing the selected \(|g\rangle\) state. A histogram of the second measurement, \(M_2\), gives the qubit readout properties. e, Qubit readout properties for qubit initially in \(|g\rangle\), \(|e\rangle\) and \(|f\rangle\), respectively.
Extended Data Figure 6 | Parity readout properties and Wigner tomography. a, Parity readout property for given even and odd parity states for the protocol ($R_y$, $p/2$ as the second qubit pulse) used in the single-shot traces in Fig. 3 (ii = 1). b, Protocol to measure parity readout fidelity. An initial qubit measurement allows a post-selection of the $|g\rangle$ state of the qubit, followed by six consecutive parity measurements. The pulse sequence of each parity measurement is shown in P1 in c. P1 (±1) (even/odd) are determined by post-selecting the cases with the first five consecutive identical parity results, which give the photon state parity with good confidence, and then constructing a histogram for the sixth parity measurement. c, Illustration of pulse sequence (not to scale) for producing the cat states and the Wigner tomography shown in Fig. 2. The protocol starts with a post-selection of the $|g\rangle$ state of the qubit through an initial qubit measurement $M_1$. A parity measurement is performed immediately after a storage cavity displacement $x$, followed by Wigner tomography with varying displacements $\beta$. A 280 ns waiting time after each measurement has been chosen to ensure that the readout cavity is in the vacuum state. The qubit pulses have a Gaussian envelope truncated to 48 ns, and the displacement pulses on the storage cavity are 10 ns square pulses. The dashed enclosures represent the pulse sequences for parity measurement. d, Error budgets for Wigner tomography fidelity. e, Error budgets for the parity readout fidelities with $R_y$, $p/2$ as the second qubit pulse.
Extended Data Figure 7 | Schematic of the quantum filter. At time $t$, the density matrix of the photon state is $\rho(C_t)$, which depends on all previous correlations. At $t + dt$, only considering the decoherence of the cavity, the expected density matrix from free evolution becomes $\tilde{\rho}(C_{t+dt})$. The additional information $C_{t+dt}$ acquired from the parity measurement at $t + dt$ changes the knowledge of the parity of the photon state according to equation (1).
Extended Data Figure 8 | Effectiveness and response time of the quantum filter. a. Ensemble-averaged parity dynamics obtained directly from the correlation of qubit states between neighbouring parity measurements. The data set is the same as that shown in Fig. 4. Solid lines are predictions based on equation (2), in excellent agreement with the measured data. The offset of the averaged parity at \( t = 0 \) comes from the asymmetry between the parity readout fidelities of the even and odd states. The fact that the saturated parity value in the long time limit is much lower than that in Fig. 4 is additional proof of the effectiveness of the quantum filter. b. Effectiveness of the quantum filter. Blue (raw) and red (filtered) curves are the same as those shown in Fig. 3e. The green curve is the direct correlation of qubit states between neighbouring parity measurements. The red curve is clearly much smoother and can reject the brief changes in the green curve. c. Response time of the quantum filter applied to typical photon jump events. The blue curve is the raw data from a repeated parity measurement. The red curve is the corresponding parity estimator based on the quantum filter. Green and cyan curves are fits to \( \tanh \) functions of the parity estimator at the transitions down and up, respectively, giving a transition time constant of less than 1 \( \mu \)s. However, the response time of the filter to make a transition between \(-0.9\) and \(+0.9\) is \( \tau \approx 2 \mu \)s.
Extended Data Figure 9 | Histograms of the number of jumps extracted from the parity estimator during 500 μs repeated parity measurements for an initial even or odd cat state by post-selection. a, b, |α| = 2.0; c, d, |α| = 1.4; e, f, |α| = 1.0. Solid lines are numerical simulations including the background thermal excitation and finite response time of the quantum filter. In the simulation, we use a coherent state as the initial state without distinguishing the parity. The good agreement between data and simulation demonstrates that the repeated parity measurement can track the error syndromes faithfully.
Extended Data Figure 10 | An optimized parity-tracking scheme would involve performing packets of \( N \) measurements, each lasting a time \( \tau_M \), followed by a waiting time of \( \tau_W \).