Flavor Changing Neutral Current Processes in a SO(10) SUSY GUT with Family Symmetry

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Abstract. We report on a detailed analysis of a SO(10) SUSY GUT model of Dermišek and Raby (DR) with a $D_3$ family symmetry. The model is completely specified in terms of only 24 parameters and is able to successfully describe both quark and lepton masses and mixings, except for $|V_{ub}|$ that turns out to be too low. However, a global fit shows that flavor changing (FC) processes like $B_s \to \mu^+ \mu^-$, $B_s$-mixing, $B^+ \to \tau^+ \nu$, $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$ pose a serious problem to the DR model. The simultaneous description of these FC processes forces squarks to have masses well above 1 TeV, not appealing on grounds of naturalness and probably beyond the reach of the LHC.

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1 Introduction

Many extensions of the Standard Model (SM), like the general Minimal Supersymmetric Standard Model (MSSM), typically introduce a large set of additional parameters to the SM ones and therefore largely lose their predictivity. On the other hand, in Supersymmetric Grand Unified Theories (SUSY GUTs) with additional family symmetries, the number of parameters can even be smaller than in the SM. In a top down approach, these models then allow to predict observables at the low scale in terms of a manageable number of GUT scale parameters.

One of such highly predictive models is the SO(10) SUSY GUT introduced by Dermišek and Raby in [1]. In [2] lepton flavor violating processes and electric dipole moments were studied extensively within this model. Here we report on a detailed analysis [3] of the same model in the light of the best measured FC processes in the quark sector.

2 The Model

The DR model is a supersymmetric SO(10) Grand Unified Theory that is supplemented by a $D_3 \times [U(1) \times Z_2 \times Z_3]$ family symmetry.

The three generations of quarks and leptons are each unified in a 16. Furthermore, the model has one additional 10 that contains the two Higgs doublets of the MSSM. The family symmetry ensures that only a universal third generation Yukawa coupling is allowed. Yukawa couplings for the first and second generation are then generated by a Froggatt-Nielsen mechanism [4]. It turns out that the resulting Yukawa matrices can be parameterized in terms of only 11 parameters. All parameters of the model are summarized in Table 1. Among them, the most important role in the numerical analysis of the FC processes is played by the universal fermion mass $m_{16}$, the universal trilinear coupling $A_0$ and the Higgsino mass parameter $\mu$. Moreover, $\tan \beta$ is forced to be around 50, because of third generation Yukawa unification.

The total number of model parameters is 24 and once they are fixed, the complete MSSM Lagrangian at the electro-weak scale is specified.

3 Basic Procedure of the Analysis

Starting with the model parameters at the GUT scale, the Yukawa matrices, the right-handed (RH) neutrino mass matrix, the gauge couplings and the soft SUSY breaking parameters are run down using renormalization group equations. The RH neutrinos are integrated out at their respective scale and the remaining parameters are further run down to the electro-weak scale,
4 Interplay of Flavor Changing Processes

Although the DR model is minimal flavor violating, one expects interesting effects in various FC processes due to the large value of \(\tan \beta\). In this section we discuss the general pattern of these effects.

4.1 \(B_s \to \mu^+ \mu^-\) and \(B_s\) mixing

Combining data from CDF and DØ results in the following upper bound on the branching ratio of the rare decay \(B_s \to \mu^+ \mu^-\) at 95% C.L.:

\[
\text{BR}(B_s \to \mu^+ \mu^-)^{\text{exp}} < 5.8 \times 10^{-8},
\]

that is still much larger than the SM prediction [8].

\[
\text{BR}(B_s \to \mu^+ \mu^-)^{\text{SM}} = (3.37 \pm 0.31) \times 10^{-9}.
\]

The helicity suppression of the SM result can be lifted in the MSSM with large \(\tan \beta\) by neutral Higgs penguins [9] that lead to contributions to the branching ratio that are strongly enhanced by \(\tan \beta\)

\[
\text{BR}(B_s \to \mu^+ \mu^-) \propto \frac{\tan^6 \beta}{M_A^4}.
\]

On the experimental side, this quantity is known very precisely [10]

\[
(\Delta M_s)^{\text{exp}} = (17.77 \pm 0.10 \pm 0.07) \text{ps}^{-1}.
\]

On the other hand, the theory prediction in the SM suffers from large hadronic uncertainties [6]

\[
(\Delta M_s)^{\text{SM}} = (18.6 \pm 2.3) \text{ps}^{-1},
\]

leaving still some room for new physics contributions. But as \(\tan \beta\) is forced to be around 50 by third generation Yukawa unification, both observables constrain the pseudoscalar Higgs mass \(M_A\). In fact, in the DR model we find a lower bound on \(M_A > 450 \text{ GeV}\), that then approximately also holds for the other heavy Higgs particles.

4.2 \(B^+ \to \tau^+ \nu\)

Using the most recent experimental results one obtains the following average for the branching ratio of the tree level decay \(B^+ \to \tau^+ \nu\) (see [3] and references therein)

\[
\text{BR}(B^+ \to \tau^+ \nu)^{\text{exp}} = (1.41 \pm 0.43) \times 10^{-4}.
\]

\footnote{In the numerical analysis, we resum large \(\tan \beta\) corrections following [4].}
The SM branching ratio is proportional to $|V_{ub}|^2$. Using the exclusive and the inclusive value for $|V_{ub}|$ from eq. (2) yields the following SM predictions \[ 3 \]

\[
\begin{align*}
\text{BR}(B^+ \to \tau^+ \nu)^{\text{SM}} &= (0.50 \pm 0.20) \times 10^{-4}, \\
\text{BR}(B^+ \to \tau^+ \nu)^{\text{incl}} &= (1.31 \pm 0.23) \times 10^{-4}. \quad (10)
\end{align*}
\]

In the MSSM there is an additional contribution to this decay coming from the exchange of a charged Higgs boson. It interferes destructively with the SM contribution and one finds \[ 11 \]

\[
R_{Br_{\tau\nu}} = \frac{\text{BR}(B^+ \to \tau^+ \nu)^{\text{DR}}}{\text{BR}(B^+ \to \tau^+ \nu)^{\text{SM}}} = \left( 1 - \frac{\tan^2 \beta}{M_{H^+}^2 + \epsilon_0 \tan^2 \beta} \right)^{\frac{1}{2}} \frac{|V_{ub}^{\text{DR}}|}{|V_{ub}^{\text{SM}}|^2}. \quad (11)
\]

As confirmed in \[ 3 \], the value for $|V_{ub}|$ in the DR model is always lower than in the SM, which leads to a further suppression of the branching ratio with respect to the SM value. In the DR model, we typically find

\[
\text{BR}(B^+ \to \tau^+ \nu)^{\text{DR}} < 0.6 \times 10^{-4}, \quad (12)
\]

which is however not yet excluded, given the large experimental error in eq. (9).

4.3 $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$

The experimental value for the branching ratio of the inclusive decay $B \to X_s \gamma$ reads \[ 13 \]

\[
\text{BR}(B \to X_s \gamma)^{\text{exp}} = (3.55 \pm 0.27) \times 10^{-4}, \quad (13)
\]

which is slightly above the NNLO SM prediction \[ 14 \]

\[
\text{BR}(B \to X_s \gamma)^{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}. \quad (14)
\]

As discussed in Sec. 4.1, Higgs masses are forced to be quite large, implying that new physics contributions to $C_7$, the Wilson coefficient governing $B \to X_s \gamma$, are dominated by chargino-stop loops. For large values of $\tan \beta$ these chargino contributions obey the following approximate relation \[ 15 \]

\[
C_7^{\chi^+} \propto \mu A_t \tan \beta \times \text{sign}(C_7^{\text{SM}}). \quad (15)
\]

For $\mu > 0$ and $A_t < 0$ the sign of the chargino contribution is opposite to the SM one. Without invoking further constraints, the model favors very large chargino contributions $C_7^{\chi^+} \approx -2C_7^{\text{SM}}$ that lead to $C_7 \approx -C_7^{\text{SM}}$ which accommodates the data on $B \to X_s \gamma$.

A further important point to consider is then $B \to X_s \ell^+ \ell^-$. Both the forward backward asymmetry and the branching ratio of this decay are sensitive to the sign of $C_7$. In case the sign of $C_7$ is opposite to its SM value, the forward backward asymmetry has no zero, which is however not yet excluded experimentally. On the other hand, the experimental data on the branching ratio in the low $s$ region $1 \text{ GeV}^2 < s < 6 \text{ GeV}^2 \ [16]$

\[
\text{BR}(B \to X_s \ell^+ \ell^-)^{\text{exp}} = (1.60 \pm 0.51) \times 10^{-6}. \quad (16)
\]

is in very good agreement with the SM prediction \[ 17 \]

\[
\text{BR}(B \to X_s \ell^+ \ell^-)^{\text{SM}} = (1.59 \pm 0.11) \times 10^{-6}. \quad (17)
\]

It has been shown \[ 18, 19 \] that the experimental result \[ 16 \] excludes the “wrong sign” solution for $C_7$ if the Wilson coefficients $C_9$ and $C_{10}$ are SM-like. This is especially the case in a minimal flavor violating MSSM \[ 19 \] and also in the DR model. Thus chargino contributions to $C_7$ have to be suppressed, which can only be done by raising the stop masses.

5 Results of the Numerical Analysis

The main features of the interplay of the FC processes described in Sec. 4 are then also reflected in the performed numerical fits. The adopted strategy in \[ 3 \] was to roughly set the scale for the fermion masses by fixing $m_{16}$, while all the other model parameters where left free in the fits.

5.1 Fits with $\mu > 0$

For positive values of $\mu$, the fit strongly prefers values for $A_0$ that obey the following approximate relation at the GUT scale

\[
A_0 \approx -2m_{16}, \quad (18)
\]

which helps to obtain third generation Yukawa unification \[ 20 \] and leads to an inverted mass hierarchy for squarks. These large values for $A_0$ also result in large negative values for $A_t$ at the electro-weak scale, that in turn lead to the large chargino contributions to $B \to X_s \gamma$ discussed in Sec. 4.3. The only possibility to tame these corrections is then to decouple stops, which can be done by choosing a very large $m_{16}$.
the CKM matrix element $V_{ub}$ that is even smaller than the central exclusive value.

Given such a small value of $|V_{ub}|$, we then find a very low upper bound \[^{12}\] on the branching ratio of $B^+ \to \tau^+ \nu$ in the DR model. Consequently, this decay will turn out to be quite problematic for the model, if the central experimental value for the branching ratio stays above $1.0 \times 10^{-4}$.

Furthermore, we find that the model is not able to simultaneously fit the branching ratios of the decays $B_s \to \mu^+ \mu^-$, $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$, unless the squark spectrum is made very heavy ($m_{t_1} > 1.8$ TeV). Such large squark masses may be problematic from the point of view of naturalness and squarks may be even beyond the reach of the LHC.

As the example of the DR model shows, it is essential to check simultaneously many flavor changing processes to test the validity of models for fermion masses and mixings.

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