Enhanced Black-Hole Mergers in Binary-Binary Interactions

Bin Liu\textsuperscript{1,2}, Dong Lai\textsuperscript{1,2,3}

\begin{itemize}
\item \textsuperscript{1} Cornell Center for Astrophysics and Planetary Science, Cornell University, Ithaca, NY 14853, USA
\item \textsuperscript{2} Shanghai Astronomical Observatory, Chinese Academy of Sciences, 80Nandan Road, Shanghai 200030, China
\item \textsuperscript{3} Tsung-Dao Lee Institute, Shanghai 200240, China
\end{itemize}

24 September 2018

ABSTRACT

We study the orbital evolution of black hole (BH) binaries in quadruple systems, where the tertiary binary excites large eccentricity in the BH binary through Lidov-Kozai (LK) oscillations, causing the binary BHs to merge via gravitational radiation. We show that binary-binary interactions can significantly increase the LK window for mergers (the range of companion inclinations that allows the BH binary to merge within 10 Gyrs). This increase arises from a secular resonance between the LK oscillation of the BH binary and the nodal precession of the outer (binary-binary) orbit driven by the tertiary binary. Therefore, in the presence of tertiary binary, the BH merger fraction is increased to $10^{-30\%}$, an order of magnitude larger than the merger fraction found in similar triple systems. Since the occurrence rate of stellar quadruples in the galactic fields is not much smaller than that of stellar triples, our result suggests that dynamically induced BH mergers in quadruple systems may be an important channel of producing BH mergers observed by LIGO/VIRGO.

Key words: binaries: general - black hole physics - gravitational waves - stars: black holes - stars: kinematics and dynamics

1 INTRODUCTION

Since 2015, a number of black hole (BH) binary and neutron star (NS) binary mergers have been observed in gravitational waves by aLIGO/VIRGO (e.g., Abbott et al. 2016a,b, 2017a,b,c,d). To bring two BHs into sufficiently close orbits and allow gravitational-radiation driven binary coalescence, several different formation scenarios have been proposed. These include isolated binary evolution, either through common-envelop phases (e.g., Lipunov et al. 1997, 2017; Podsiadlowski et al. 2003; Belczynski et al. 2010, 2016; Dominik et al. 2012, 2013, 2015) or through chemically homogeneous evolution associated with rapid stellar rotations (e.g., Mandel & de Mink 2016; Marchant et al. 2016), three-body encounters and/or secular interactions in dense star clusters such as globular cluster (e.g., Portegies Zwart & McMillan 2000; Miller & Hamilton 2002; Wen 2003; Miller & Lauburg 2009; O'Leary et al. 2006; Banerjee et al. 2010; Downing et al. 2010; Thompson 2011; Rodriguez et al. 2015; Chatterjee et al. 2017; Samsing et al. 2018) or galactic nuclei (e.g., O’Leary et al. 2009; Antonini & Perets 2012; Antonini & Rasio 2016; VanLandingham et al. 2016; Petrovich & Antonini 2017; Hoang et al. 2018; Leigh et al. 2018), and secular/nonsecular Lidov-Kozai oscillations (e.g., Lidov 1962; Kozai 1962; Naoz 2016) in isolated triples in the galactic fields (e.g., Silsbee & Tremaine 2017; Antonini et al. 2017; Liu & Lai 2018).

The BH binary merger rate inferred from the LIGO detections ($10-200$ Gpc$^{-3}$yr$^{-1}$) is higher than expected and challenges existing models. Additional mechanisms/effects may be required to produce a greater BH merger rate to match observations. Lidov-Kozai (LK) oscillations driven by tertiary companions (either another star/BH in the galactic triple scenario, or a supermassive BH for binaries near galactic nuclei) provide a natural, purely dynamical mechanism to induce binary BH merger (e.g., Miller & Hamilton 2002; Wen 2003; Thompson 2011; Antonini & Perets 2012; Antonini et al. 2014; Hoang et al. 2018). In a recent paper (Liu & Lai 2018), we systematically study the merger window (the range of companion inclinations that allows the inner binary to merge within $\sim$10 Gyrs) and merger fraction for BH binaries in triples for a wide range of param-
eters, taking account of both (octupole-level) secular and non-secular effects. We find that for a “typical” inner binary system (with masses $m_1 = 30M_\odot$, $m_2 = 20M_\odot$, initial separation $a_\text{in} = 100$ AU) and a random orientation of the tertiary binary orbits, the merger fraction ranges from $\sim 1\%$ at $e_\text{out} = 0$ (quadrupole LK effect) to $\sim 10-20\%$ at $e_\text{out} = 0.9$ (octupole LK effect).

The merger fraction of BH binaries in triples can increase when the tertiary companion is a binary by itself (see Figure 1). Such binary-binary systems may allow Lidov-Kozai (LK) eccentricity excitation to operate over a wide range of inclinations (Hamers & Lai 2017). The qualitative reason is as follows: the second binary induces nodal precession of the outer binary (at the characteristic rate $\Omega_\text{out}$); when $\Omega_\text{out}$ matches the LK rate of the first (inner) binary, a secular resonance occurs; this can generate large mutual inclinations (between the first binary and the outer binary), and therefore induce eccentricity excitation of the first (inner) binary. Fang et al. (2018) and Hamers (2018a) studied this “enhanced LK effect” in the context of white dwarf (WD) binaries, with emphasis on WD-WD mergers relevant to Type Ia supernovae. Petrovich & Antonini (2017) considered a similar effect where stellar-mass BH binaries merging around a supermassive BH are embedded in a non-spherical galactic potential. They found that extreme eccentricity excitation is possible if the LK timescale driven by the central massive BH is comparable to the nodal precession timescale of the binary centre of mass driven by the non-spherical potential. An enhanced merger rate may also be achieved due to the effect of vector resonant relaxation of BH binaries in galactic nuclei (Hamers et al. 2018b).

In this paper, we study binary BH mergers in quadruple systems (Figure 1). We show that binary-binary interactions increase the LK window for extreme eccentricity excitations, and therefore significantly increase the BH binary merger fraction. We quantify the parameter space (e.g., the orbital properties of the tertiary binary) where this increase occurs. Our result suggests that although the quadruple stellar systems may not be as common as triples (e.g., Sana 2017), they could be the dominant sources for dynamically enhanced BH mergers in the galactic field.

Our paper is organized as follows. In Section 2, we summarize the secular equations of motion in the octupole order to evolve the quadruple systems with gravitational reaction. These equations are based on the double-averaged approximation (averaging over both inner and outer orbits) for the orbital evolution of hierarchical quadruple systems. In Section 3, we present the basic properties of LK oscillations for general stellar quadruples. In Section 4, we perform a suite of numerical integrations to determine the merger windows for LK-induced binary mergers, assuming isotropic distribution of the orientations of tertiary binaries. The associated merger fractions of BH binaries are then obtained. We summarize our main results in Section 5.

![Figure 1. Illustration of the binary-binary system. The first (inner) binary is comprised of two BHs ($m_1$ and $m_2$); the second binary consists of another two bodies ($m_3$ and $m_4$) and orbits the center mass of the first inner binary, constituting the outer orbit. Here, $a_{1,2,\text{out}}$ are the semi-major axes, $e_{1,2,\text{out}}$ are the eccentricities of each binary. The total angular momentum $J_\text{tot} \equiv L_1 + L_2 + L_\text{out}$ is along the $z$-axis, where $L_1$, $L_2$ and $L_\text{out}$ (not to scale) denote the angular momenta of the first, second (inner) binaries and outer orbit, respectively. “c.m.” indicates the center of mass of each system. $I_1$ and $I_2$ are the mutual inclinations between $L_1$ and $L_\text{out}$, $L_2$ and $L_\text{out}$, respectively.](image-url)

## 2 OCTUPOLE-LEVEL EQUATIONS OF MOTION FOR BINARY-BINARY SYSTEMS

We consider a hierarchical quadruple system, composed of two binaries orbiting each other, as depicted in Figure 1. The first (inner) BH binary has the masses $m_1$, $m_2$ and the distant second (inner) binary has the masses $m_3$ and $m_4$. The reduced mass for the first binary is $\mu_1 \equiv m_1m_2/m_{12}$, with $m_{12} \equiv m_1 + m_2$ and the second binary has $\mu_2 \equiv m_3m_4/m_{34}$, with $m_{34} \equiv m_3 + m_4$. The outer binary ($m_{12}$ orbits around $m_{34}$) has $\mu_\text{out} \equiv (m_1m_2m_3m_4)/m_\text{tot}$ with $m_\text{tot} \equiv m_{12} + m_{34}$. The semi-major axes and eccentricities are denoted by $a_1$, $a_2$, $a_\text{out}$ and $e_1$, $e_2$, $e_\text{out}$, respectively. The orbital angular momenta of three orbits are

\begin{align}
L_1 &= L_1 \dot{L}_1 = \mu_1 \sqrt{Gm_2a_1(1 - e_1^2)} \dot{L}_1, \\
L_2 &= L_2 \dot{L}_2 = \mu_2 \sqrt{Gm_3a_2(1 - e_2^2)} \dot{L}_2, \\
L_\text{out} &= L_\text{out} \dot{L}_\text{out} = \mu_\text{out} \sqrt{Gm_\text{out}a_\text{out}(1 - e_\text{out}^2)} \dot{L}_\text{out},
\end{align}

where $\dot{L}_1$, $\dot{L}_2$ and $\dot{L}_\text{out}$ are unit vectors. We also define the eccentricity vectors as $\mathbf{e}_1 = e_1 \mathbf{e}_1$, $\mathbf{e}_2 = e_2 \mathbf{e}_2$ and $\mathbf{e}_\text{out} = e_\text{out} \mathbf{e}_\text{out}$. For simplicity, we only study the LK-induced orbital decay in the first inner binary, considering the second one as an external perturber. Thus, for convenience of notation, we will frequently omit the subscript “1” for the first inner binary.

The secular equations of motion for the two inner binaries...
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...angular momentum vectors as
\[ \mathbf{dL} = \frac{dL}{dt} \mathbf{L}_{\text{LK}} + \frac{dL}{dt} \mathbf{L}_{\text{GR}}, \]
\[ \frac{de}{dt} = \frac{de}{dt} \mathbf{L}_{\text{LK}} + \frac{de}{dt} \mathbf{L}_{\text{GR}}, \]
\[ \frac{dL_2}{dt} = \frac{dL_2}{dt} \mathbf{L}_{\text{LK}}, \]
\[ \frac{de_2}{dt} = \frac{de_2}{dt} \mathbf{L}_{\text{LK}} + \frac{de_2}{dt} \mathbf{L}_{\text{GR}}, \]
and the outer orbit follows
\[ \frac{dL_{\text{out}}}{dt} = \frac{dL_{\text{out}}}{dt} \mathbf{L}_{\text{L1st}} + \frac{dL_{\text{out}}}{dt} \mathbf{L}_{\text{2nd}}, \]
\[ \frac{de_{\text{out}}}{dt} = \frac{de_{\text{out}}}{dt} \mathbf{L}_{\text{L1st}} + \frac{de_{\text{out}}}{dt} \mathbf{L}_{\text{2nd}}, \]

In the first binary, we include the contributions from the outer binary (with perturber mass \( m_{34} \)) that generate \( \text{LK} \) oscillations (subscripted by “LK”), the general relativistic (GR) post-Newtonian correction, and the dissipation due to gravitational wave (GW) emission. The outer binary’s angular momentum and eccentricity are affected by both the first and second inner binaries (subscripted by “1st” and “2nd”).

To describe the \( \text{LK} \) oscillations, we introduce the reduced angular momentum vectors as
\[ \mathbf{j} \equiv \mathbf{jL} = \sqrt{1 - e^2} \mathbf{L}, \]
\[ \mathbf{j}_{\text{out}} \equiv \mathbf{j}_{\text{outLout}} = \sqrt{1 - e^2_{\text{out}} \mathbf{L}_{\text{out}}} \]

Therefore, for the first binary, we have, to the octupole order (Liu et al. 2015; Petrovich 2015)
\[ \frac{dj}{dt} \mathbf{L}_{\text{LK}} = \frac{3}{4} \frac{dL_{\text{L12}}}{dt} \left( \mathbf{j} \cdot \mathbf{L}_{\text{out}} \right) \mathbf{j} \times \mathbf{L}_{\text{out}} - 5 \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) \mathbf{e} \times \mathbf{L}_{\text{out}} \]
\[ -\frac{75 \varepsilon_{\text{oct,12}}}{64 \frac{m_{12}}{m_{34}}} \left( \mathbf{j} \cdot \mathbf{L}_{\text{out}} \right) \mathbf{j} \times \mathbf{L}_{\text{out}} \]
\[ + \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) \left( \mathbf{j} \cdot \mathbf{L}_{\text{out}} \right) \]
\[ + 7 \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) \]
\[ - \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) ^2 \left( \mathbf{j} \cdot \mathbf{L}_{\text{out}} \right) ^2 \]

and
\[ \frac{dL}{dt} \mathbf{L}_{\text{GR}} = \frac{3}{4} \frac{m_{12}}{m_{34}} \left( \mathbf{j} \cdot \mathbf{L}_{\text{out}} \right) \mathbf{e} \times \mathbf{L}_{\text{out}} + 2 \mathbf{e} \times \mathbf{L}_{\text{out}} \]
\[ -5 \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) \left( \mathbf{j} \cdot \mathbf{L}_{\text{out}} \right) \]
\[ - \frac{75 \varepsilon_{\text{oct,12}}}{64 \frac{m_{12}}{m_{34}}} \left( \mathbf{j} \cdot \mathbf{L}_{\text{out}} \right) \mathbf{j} \times \mathbf{L}_{\text{out}} \]
\[ + \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) \left( \mathbf{j} \cdot \mathbf{L}_{\text{out}} \right) \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) \]
\[ + 2 \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) \left( \mathbf{j} \cdot \mathbf{L}_{\text{out}} \right) \]
\[ + \frac{16}{5} \left( \mathbf{e} \cdot \mathbf{L}_{\text{out}} \right) \left( \mathbf{j} \cdot \mathbf{e} \right), \]

where
\[ \varepsilon_{\text{oct,12}} \equiv \frac{m_{12} - m_{34}}{m_{12}} \left( \frac{a_{\text{out}}}{a_{\text{out}}} \right)^3 \]

measures the relative strength of the octupole potential compared to the quadrupole one. The quadrupole term induces the oscillations in the eccentricity and mutual orbital inclination on the timescale of
\[ t_{\text{L12}} \equiv \frac{m_{12}}{m_{34}} \frac{a_{\text{out}}}{a_{\text{out}}} \sqrt{1 - e_{\text{out}}}, \]

where \( n = (G m_{12}/a^3)^{1/2} \) is the mean motion of the first inner binary and the effective outer binary separation is defined as
\[ a_{\text{out},\text{eff}} \equiv a_{\text{out}} \sqrt{1 - e_{\text{out}}}. \]

General Relativity (1-PN correction) introduces peri-center precession as
\[ \frac{de}{dt} \mathbf{L}_{\text{GR}} = \Omega_{\text{GR}} \mathbf{L} \times \mathbf{e}, \]
with the precession rate given by
\[ \Omega_{\text{GR}} = \frac{3 G_{\text{mn}} m_{12}}{e^2 a (1 - e^2)^5}. \]

Gravitational radiation draws energy and angular momentum from the BH orbit. The rates of change of \( \mathbf{L} \) and \( \mathbf{e} \) are (Peters 1964)
\[ \frac{dL}{dt} \mathbf{L}_{\text{GR}} = - \frac{32}{5} \frac{G}{c^3} \frac{m_{5/2}}{a_{12}^{5/2}} \left( \frac{1}{1 - e^2} \right)^2 \mathbf{L}, \]
\[ \frac{de}{dt} \mathbf{L}_{\text{GR}} = - \frac{304}{15} \frac{G^3}{c^5} \frac{m_{7/2}}{a_{12}^{7/2}} \left( \frac{1}{1 - e^2} \right)^5 \mathbf{e}. \]

The merger time due to GW radiation of an isolated binary with the initial semi-major axis \( a_0 \) and eccentricity \( e_0 = 0 \) is given by
\[ T_{\text{m,0}} = \frac{5 c^3 v_0^3}{256 G^3 m_{12}^2 \mu^3} \]
\[ \simeq 10^{10} \left( \frac{60 M_{\odot}}{m_{12}} \right)^2 \left( \frac{15 M_{\odot}}{\mu} \right) \left( \frac{a_0}{0.202 \text{AU}} \right)^4 \text{yrs.} \]

In our calculations, we also evolve the second binary,
except that we do not include the GW terms for the sake of clarity. By switching the indices \( j \rightarrow j_2, e \rightarrow e_2, e_{\text{oct},34} \) and \( t_{\text{LK},34} \), Equations (12)-(13) can be applied to the second binary (with \( m_1 \rightarrow m_3, m_2 \rightarrow m_4, m_{12} \rightarrow m_{34}, a \rightarrow a_2 \) and \( n \rightarrow (G m_3 / a_2^3)^{1/2} \) in Equations 14-15, 18). The outer orbit is influenced by both first and second binary. The first piece of Equation (8) is given by

\[
\frac{d\dot{L}_{\text{out}}}{dt}\bigg|_{1st} = \frac{3}{4t_{\text{LK},12} \Lambda_{\text{out}}} \left[ (j \cdot \dot{\dot{L}}_{\text{out}}) \dot{\dot{L}}_{\text{out}} \times j \right] - 5(e \cdot \dot{\dot{L}}_{\text{out}}) \dot{\dot{L}}_{\text{out}} \times e + \frac{75 e_{\text{oct},12} \Lambda}{64t_{\text{LK},12} \Lambda_{\text{out}}} \left[ 2(e \cdot \dot{\dot{L}}_{\text{out}})(j \cdot \dot{\dot{L}}_{\text{out}}) \dot{\dot{L}}_{\text{out}} + (e \cdot \dot{\dot{L}}_{\text{out}})(j \cdot \dot{\dot{L}}_{\text{out}}) \dot{\dot{L}}_{\text{out}} \times j \right] + \left[ 2(e \cdot \dot{\dot{L}}_{\text{out}})(j \cdot \dot{\dot{L}}_{\text{out}}) \dot{\dot{L}}_{\text{out}} - 14(e \cdot \dot{\dot{L}}_{\text{out}})(e \cdot \dot{\dot{L}}_{\text{out}}) \dot{\dot{L}}_{\text{out}} \times j \right] + \left[ \frac{5}{2} \frac{e^2}{\Lambda_{\text{out}}} \left( \frac{5}{2} - 7(e \cdot \dot{\dot{L}}_{\text{out}})^2 + (j \cdot \dot{\dot{L}}_{\text{out}})^2 \right) \dot{\dot{L}}_{\text{out}} \times j \right].
\]

The evolution equation of \( \dot{L}_{\text{out}} \) is \( \langle d\dot{L}_{\text{out}} / dt \rangle_{1st} = \mu_{\text{out}} \sqrt{Gm_{\text{tot}} a_{\text{out}}} \langle d\dot{L}_{\text{out}} / dt \rangle_{1st} \). Also

\[
\frac{d\dot{e}_{\text{out}}}{dt} \bigg|_{1st} = \frac{3}{4t_{\text{LK},12} \sqrt{1 - e_{\text{out}}^2}} \frac{\Lambda}{\Lambda_{\text{out}}} \left[ (j \cdot \dot{\dot{L}}_{\text{out}}) \dot{\dot{L}}_{\text{out}} \times j \right] - 5(e \cdot \dot{\dot{L}}_{\text{out}}) \dot{\dot{L}}_{\text{out}} \times e - \frac{1}{2} - 3e^2 + \frac{25}{2} (e \cdot \dot{\dot{L}}_{\text{out}})^2 - \frac{5}{2} (j \cdot \dot{\dot{L}}_{\text{out}})^2 \dot{\dot{L}}_{\text{out}} \times e - \frac{75}{64t_{\text{LK},12} \sqrt{1 - e_{\text{out}}^2}} \frac{\Lambda}{\Lambda_{\text{out}}} \left[ e^2 \left( \frac{5}{2} - 5(e \cdot \dot{\dot{L}}_{\text{out}})^2 + (j \cdot \dot{\dot{L}}_{\text{out}})^2 \right) \dot{\dot{L}}_{\text{out}} \times j \right] + \left[ 2(e \cdot \dot{\dot{L}}_{\text{out}})(j \cdot \dot{\dot{L}}_{\text{out}}) \dot{\dot{L}}_{\text{out}} - 14(e \cdot \dot{\dot{L}}_{\text{out}})(e \cdot \dot{\dot{L}}_{\text{out}}) \dot{\dot{L}}_{\text{out}} \times j \right] + \left[ \frac{5}{2} \frac{e^2}{\Lambda_{\text{out}}} \left( \frac{5}{2} - 7(e \cdot \dot{\dot{L}}_{\text{out}})^2 + (j \cdot \dot{\dot{L}}_{\text{out}})^2 \right) \dot{\dot{L}}_{\text{out}} \times j \right].
\]

3 EXCITATION OF ECCENTRICITY IN BINARY-BINARY SYSTEMS

Before considering the population of binary mergers in quadruple systems (Section 4), we first examine how binary-binary interaction influences the excitation of eccentricity in the inner binary.

Figure 2 shows the maximum excited eccentricity achieved in the first binary \( (e_{\text{max}}; \text{in the absence of GW emission}) \) and merger window (including GW emission; to be discussed in Section 4) as a function of the initial mutual inclination angle \( I_0 \) (the initial value of \( I_1 \)) for a system with \( m_1 = 30M_\odot, m_2 = 20M_\odot, a_0 = 100 \text{AU} \) (the initial semimajor axis of the first binary), \( m_3 = m_4 = 15M_\odot \). We fix the initial inclination of the second binary to be \( I_{2,0} = 30^\circ \), so that no LK oscillations occur in the second binary, and we concentrate on the eccentricity excitation of the first binary. As in Liu & Lai (2018), we introduce the effective outer binary semimajor axis as \( a_{\text{out,eff}} = a_{\text{out}} \sqrt{1 - e_{\text{out}}^2} \) and define

\[
\tilde{a}_{\text{out,eff}} = \left(a_{\text{out,eff}} \sqrt{10000 \text{AU}} \right)^{1/3} \left(\frac{m_{34}}{30M_\odot}\right)^{-1/3}
\]

This quantity characterizes the “quadrupole strength” of the outer perturber \( m_3 + m_4 \). In the examples depicted in Figure 2, we adopt \( a_{\text{out}} = 4400 \text{AU} \) for \( e_{\text{out}} = 0 \) and \( a_{\text{out}} = 5500 \text{AU} \) for \( e_{\text{out}} = 0.6 \), so that in all cases \( a_{\text{out,eff}} = 4.4 \).

The top two panels of Figure 2 show the results when \( a_2 \ll a_{\text{out}} \). In these cases, the binary-binary system effectively reduces to a triple system, with the first binary perturbed by \( m_{34} \). When \( e_{\text{out}} = 0 \) (the top left panel of Figure 2), the octupole effect vanishes, and the maximum eccentricity \( e_{\text{max}} \) achieved by the first binary (starting from \( e_0 \approx 0 \)) can be evaluated analytically (Liu et al. 2015; Anderson et al. 2017):

\[
\frac{3j_{\text{max}}^2 - 1}{8} j_{\text{max}}^2 + \eta \left( \frac{\cos I_0 + \eta}{2} \right)^2 - \left( 3 + 4\eta \cos I_0 + \frac{9}{4} \eta^2 \right) j_{\text{min}}^2 + \eta^2 j_{\text{min}}^4 + \varepsilon GR \left( 1 - j_{\text{min}}^{-1} \right) = 0,
\]

where \( j_{\text{min}} = \sqrt{1 - e_{\text{max}}^2} \), \( \eta \equiv (L/L_{\text{out}}) e_0 = 0 \), and

\[
\varepsilon GR = \frac{3 G m_{12} m_{34} a_{\text{out,eff}}}{c^2 a^4 m_{34}} \left( \frac{m_{34}}{30M_\odot} \right)^{-1} \left( \frac{a_{\text{out,eff}}}{10^5 \text{AU}} \right)^3 \left( \frac{10^5 \text{AU}}{c^2 a} \right)^{-4},
\]

which measures the strength of the GR precession (relative...
Figure 2. Eccentricity excitation and merger window in binary-binary systems for different values of $e_{\text{out}}$ and $a_2$. All four panels have the same $a_{\text{out,eff}} = 4.4$ (Equation (27)). In each panel, the upper and lower plots show the maximum eccentricity $e_{\text{max}}$ (assuming no GW emission), and the first (inner) binary merger time $T_m$ (with GW emission) as a function of $I_0$ (the initial value of $I_1$). The system parameters are: $m_1 = 30M_\odot$, $m_2 = 20M_\odot$, $a_0 = 1000$AU (initial value of $a$), $m_3 = m_4 = 15M_\odot$, $a_{\text{out}} = 4400$AU (for $e_{\text{out}} = 0$) and $a_{\text{out}} = 5500$AU (for $e_{\text{out}} = 0.6$). The numerical results (blue and black dots) are from the double-averaged secular equations (each black dot represents a successful merger event within $10^{10}$ yrs). The dashed horizontal line ($e_{\text{lim}}$) is given by Equation (30).

to the LK oscillations. Note that in the limit of $\eta \to 0$ and $\varepsilon_{\text{GR}} \to 0$, Equation (28) yields the well-known relation $e_{\text{max}} = \sqrt{1 - (5/3)\cos^2 I_0}$. The maximum possible $e_{\text{max}}$ for all values of $I_0$, called $e_{\text{lim}}$, is given by

$$
\frac{3}{8} J_{\text{lim}}^2 - 1 \times \frac{9}{4} \left[ -3 + \frac{\eta^2}{4} \left( \frac{3}{5} J_{\text{lim}}^2 - \frac{1}{3} \right) \right] \times \varepsilon_{\text{GR}} \left( 1 - J_{\text{lim}}^{-2} \right) = 0.
$$

From the top panels of Figure 2 (with $a_2 = 1$ AU), we see that for $e_{\text{out}} = 0$, the limiting eccentricity can be achieved only in a very narrow inclination window around $I_0 = 92.2^\circ$. For $e_{\text{out}} = 0.6$ (corresponding to $e_{\text{out,12}} \approx 0.003$), the same limiting eccentricity applies (see Liu et al. 2015), but it can be achieved over a wide range of $I_0 \in [92^\circ, 94.5^\circ]$.

The lower panels of Figure 2 show $e_{\text{max}}$ versus $I_0$ when the second binary has a semimajor axis $a_2 = 81$AU. We see that regardless of the value of $e_{\text{out}}$ (i.e., the strength of the octupole potential), extreme eccentricity excitation can be achieved over a much wider range of inclinations, roughly from $90^\circ$ to $130^\circ$.

The enhanced inclination range for LK oscillations in binary-binary systems can be understood as a resonance phenomenon (Hamers & Lai 2017). Considering the simple case where the second binary does not experience LK oscillation and stays circular ($e_2 = 0$) and the outer binary is also circular ($e_{\text{out}} = 0$), the angular momentum axis of the
outer binary is affected by the second binary via
\[
\frac{dL_{\text{out}}}{dt}_{\text{2out}} = \frac{3}{4} \frac{L_2}{L_{\text{out}}} \cos I_2 \ L_{\text{out}} \times \dot{L}_2,
\]
where \( t_{\text{LK,34}} \) is the LK timescale in the second binary, given by
\[
t_{\text{LK,34}} = \frac{1}{n_2} \left( \frac{a_{\text{out,eff}}}{a_2} \right)^3,
\]
where \( n_2 = (Gm_{34}/a_2^3)^{1/2} \). Thus, \( L_{\text{out}} \) is driven into precession around the \( L_{\text{24,34}} \equiv L_2 + L_{\text{out}} \) axis at the rate
\[
\Omega_{\text{out}} = \frac{3}{4} \frac{L_2 + L_{\text{out}}}{L_{\text{out}}} \cos I_2 \simeq \frac{3}{4} t_{\text{LK,34}} \cos I_2.
\]
On the other hand, the outer binary drives LK oscillations of the (first) inner binary on timescale \( t_{\text{LK,12}} \). Thus, we define the dimensionless parameter
\[
\beta \equiv \Omega_{\text{out}} t_{\text{LK,12}} = \frac{3}{4} \cos I_2 \left( \frac{a_2}{a_1} \right)^{3/2} \left( \frac{m_1 + m_2}{m_3 + m_4} \right)^{3/2},
\]
The value of \( \beta \) measures the ratio between the LK timescale in the first binary and the precession timescale of the outer orbit. When \( \beta \ll 1 \), the second binary essentially acts like a single mass \((m_3 + m_4)\), and “normal” LK oscillations apply. When \( \beta \gg 1 \), \( L_{\text{out}} \) precesses rapidly around the \( L_{\text{24,34}} \), the problem again reduces to that of “normal” LK oscillations, with \( L_{\text{24,34}} \) serving as the effective \( L_{\text{out}} \). When \( \beta \sim 1 \), a secular resonance occurs that generates large \( I \) even for initially low-inclination systems, and this resonantly excited inclination then leads to LK oscillations of the inner binary.

In the lower panels of Figure 2, the parameters of the system (with \( a_2 = 81\text{AU} \)) gives \( \beta \approx 1 \). So we indeed see that the width of LK window for extreme eccentricity excitation is significantly enhanced due to the presence of the tertiary binary. Note that the eccentricity of the inner binary can undergo excursions to more extreme values than the analytical prediction of \( \varepsilon_{\text{lim}} \). Also, when the outer binary is eccentric (\( \varepsilon_{\text{out}} = 0.6 \)), the octupole effect comes into play, and the LK window is further extended (although slightly). Overall, Figure 2 shows that the orbital properties of the second binary play a more important role compared to the octupole terms in exciting eccentricity of the first inner binary and largely determine the LK window.

4 MERGER WINDOW AND MERGER FRACTION

In this section, we study the LK oscillations including gravitational radiation for binary-binary systems. We focus on the merger window of the first inner binary (i.e., the initial inclination \( I_0 \) that gives mergers in less than \( 10^{10} \) yrs).

First consider the examples shown in Figure 2. The upper two panels (with \( a_2 = 1\text{AU} \), so that the second binary behaves like a single mass) correspond to the result already found in Liu & Lai (2018): the inner binary can merge within \( 10^{10} \) yrs only if its eccentricity is excited to sufficiently large value, and the merger window increases as the octupole effect (measured by \( \varepsilon_{\text{oct}} \)) becomes stronger. Note that for \( \varepsilon_{\text{oct}} = 0 \) and \((1 - \varepsilon_{\text{max}}) \ll 1 \), the merger window can be determined analytically: the merger time is given by
\[
T_m \simeq T_{m,0}(1 - \varepsilon_{\text{max}}^2)^3
\]
to a good approximation (see Equation 48 of Liu & Lai (2018) and regime of validity of this equation), with \( T_{m,0} \) given by Equation (21), Combining Equations (28) and (35) and setting \( T_m = 10^{10} \) yrs, the upper and lower boundaries of the merger window, \( I_{\text{lim}} ^{\pm} \), can be obtained.

The lower panels of Figure 2 show that for \( \beta \approx 1 \), as a direct consequence of the widened LK eccentricity excitation window, the binary merger window also significantly widens compared to the case with small \( a_2 \) (or \( \beta \ll 1 \)).

In order to systematically explore how the merger window and merger fraction vary for different binary-binary parameters, we carry out calculations for different values of \( \beta \) by changing \( a_2 \). Note that for a given \( a_{\text{out}} \) and \( \varepsilon_{\text{out}} \), the semimajor axis of the second binary must satisfy the stability criterion of Markle & Aarseth (2001):
\[
a_{\text{out}} > 2.8 \left( \frac{1 + m_2}{m_3} \right)^{2/5} \left( \frac{1 + \varepsilon_{\text{out}}}{1 - \varepsilon_{\text{out}}} \right)^{2/5} \left( 1 - \frac{0.3I_20}{180} \right)^{12/5}.
\]

Figure 3 shows the results for systems with \( m_1 = 30M_\odot \), \( m_2 = 20M_\odot \), \( a_0 = 100\text{AU} \), and \( m_3 = m_4 = 15M_\odot \). The semimajor axis of the second binary is \( a_2 = 56000\text{AU} \) (\( \varepsilon_{\text{out}} = 0 \)), \( a_2 = 58700\text{AU} \) (\( \varepsilon_{\text{out}} = 0.3 \)), \( a_2 = 70000\text{AU} \) (\( \varepsilon_{\text{out}} = 0.6 \)) and \( a_2 = 12847\text{AU} \) (\( \varepsilon_{\text{out}} = 0.9 \)), all given \( a_{\text{out,eff}} = 5.6 \) (Equation 27). We see that the merger window indeed is much wider for \( \beta \approx 0.3 - 3 \). This range is somewhat larger when the octupole effect (\( \varepsilon_{\text{oct}} \)) increases. Note that the initial mutual inclinations for successful mergers inside the merger window are not uniformly distributed. This is because the overlap of resonances from both binary-binary interactions (e.g., Hamers & Lai 2017) and octupole terms (e.g., Lithwick et al. 2011; Li et al. 2015) together induces chaos of the systems with intermediate \( \beta \).

To calculate the merger fraction, we assume that the initial inclination of the outer binary is uniformly distributed in \( \cos I_0 \in [-1,1] \). As shown in Figure 3, \( f_{\text{merger}} \) exhibits a clear dependence on \( \beta \). The secular resonance around \( \beta \approx 1 \) gives the the maximum \( f_{\text{merger}} \sim 30\% \), which is \( 6 - 30 \) times larger than the cases with \( \beta \ll 1 \) (equivalent to a “pure” triple). We also see that compared to the octupole contribution, the resonance plays an more significant role in determining the merger fraction.

Equation (34) indicates that \( \beta \) has a dependence on \( I_2 \). In the calculations shown above (Figures 2-3), the angular momentum vector of the second binary \( L_2 \) is always placed initially at \( 30^\circ \) with respect to \( L_{\text{out}} \). In Figure 4, we set the initial \( I_{2,0} \) to \( 15^\circ \) and \( 45^\circ \), and all other parameters are sampled identically to the case of \( \varepsilon_{\text{out}} = 0.6 \) depicted in Figure 3. The different results for \( I_{2,0} = 15^\circ \) and \( 45^\circ \) arise from the fact that \( I_2 \) varies in time in the case of \( I_{2,0} = 45^\circ \), giving rise to time-dependent \( \beta \). Also, the amplitude of nodal precession of the outer binary (i.e., the angle between \( L_{\text{out}} \)
and $L_{\text{tot}}$) for the two cases are different, and this difference can affect the LK oscillations of the first inner binary (see Hamers & Lai 2017).

To illustrate how the merger window and merger fraction depend on the properties of the outer binary, Figure 5 shows our results as a function of $a_{\text{out, eff}}$ for several values of $\beta$. When $\beta \ll 1$, for a given $e_{\text{out}}$, the merger window shows a general trend of widening as $a_{\text{out, eff}}$ decreases. Note that for $e_{\text{out}} \simeq 0$, the merger window (the dashed curve in each panel) and merger fraction can be obtained analytically using Equations (28) and (35) (see Equations 51, 53 and 54 of Liu & Lai 2018). For the same value of $a_{\text{out, eff}}$ (thus the same quadrupole effect), the merger window and merger fraction can be different for different $e_{\text{out}}$. In general, the larger the eccentricity $e_{\text{out}}$, the stronger the octupole effect, and the wider the window. The merger fraction ranges from $\sim 1\%$ (for $e_{\text{out}} \simeq 0$) to a few $\%$ (for $e_{\text{out}} = 0.9$). Note that for some values of $a_{\text{out, eff}}$, the irregular distribution of merger events inside the merger window is evident; this results from the chaotic behaviors of the octupole-level LK oscillations (see also the examples in Figure 2, particularly the $e_{\text{out}} = 0.6$ case).

For $0.3 \lesssim \beta \lesssim 3$, the merger window and merger fraction are significantly larger for all values of $e_{\text{out}}$. At $\beta \simeq 1$, different values of $e_{\text{out}}$ give the similar $f_{\text{merger}}$ for each $a_{\text{out, eff}}$. The secular resonance enhances $f_{\text{merger}}$ to ten of percent.

If the orbital plane of the second inner binary has initially random orientation, LK oscillations in the second binary become possible, and the merger window and merger fraction can be changed. We show an example in Figure 6 for

![Figure 3](image_url)
the case of $a_2 = 81\text{AU}$, and the absolute value of $\beta \propto \cos I_2$ is in the range from 0 to 1.18. We see that the merger fraction for the random $\cos I_{2,0}$ case is similar to the $\beta \sim 1$ case depicted in Figure 5. However, unlike Figure 5, where the window lies in the retrograde regime ($\cos I_0 < 0$), in Figure 6 a large fraction of mergers occurs in the prograde regime ($\cos I_0 > 0$).

Note that the merger fractions presented above are based on the fiducial inner BH binary parameters ($m_1 = 30M_\odot$, $m_2 = 20M_\odot$, $a_0 = 100\text{AU}$). If we start with a closer binary or consider moderately hierarchical systems, where the double-averaged secular approximation may break down, the merger fraction can be even higher (see Liu & Lai 2018).

Having studied the role of binaries, we now summarize the distribution of the merger time for the merging systems studied in Figure 5, we consider systems with $a_{\text{out,eff}} \in [5.6, 8.8]$, and assume that the eccentricity of the tertiary companion has a uniform distribution in $e_{\text{out}}$ (i.e., $e_{\text{out}} = 0, 0.3, 0.6, 0.9$ are equally probable), and the initial mutual inclination is randomly distributed (uniform in $\cos I_0$). Figure 7 shows the result for four values of $\beta$. We see that most systems take long time to merge (with $T_m \sim 10^8 - 10^{10}$ yrs). In particular, a larger fraction of the systems with $\beta \sim 1$ merge with $T_m > 10^9$ yrs, compared to those with $\beta \ll 1$. This is because when $a_{\text{out,eff}} \gtrsim 5.6$, the merger window for systems with $\beta \lesssim 1$ is always larger than the other systems ($\beta = 0.0014, 0.29, 1.8$), providing more merger events even with the same quadrupole perturbation (same $a_{\text{out,eff}}$).

\section{Summary and Discussion}

In this paper, we have studied the mergers of binary BHs induced by the gravitational interaction with tertiary binaries. The binary-binary system is evolved in time using the octupole-level secular equations of motion, taking account of the post-Newtonian effect and gravitational radiation. We examine the dependence of the eccentricity excitation of the BH binary on the orbital properties of the tertiary binary. When the precession timescale of the outer orbit driven by the tertiary binary is comparable to the Lidov-Kozai oscillation time of the BH binary ($\beta \equiv 1$; Equation 34), the LK inclination window for $e$-excitation is enhanced drastically, leading to more BH mergers compared to the standard triple (“binary + perturber”) systems (see Figure 2).

By conducting a series of numerical integrations, we quantify the role of tertiary binaries in determining the BH merger windows and merger fractions. We find that the orbital properties of the external binary (especially the semimajor axis $a_2$) play a more important role in producing large merger fractions compared to the octupole effect (i.e., eccentric outer orbit). When $\beta \ll 1$ or $\beta \gg 1$, the merger windows are similar as in the standard triples, with the merger fraction less than a few \% (see Figure 3). This gives the lower limit of the merger fraction in the binary-binary interaction channel. However, for systems with $\beta \sim 0.3 - 3$, the merger fraction increases to $\gtrsim 10\%$, peaking at $\sim 30\%$, depending on the parameters of the outer orbits (see Figure 5). This places the upper limit to the BH merger fraction due to the presence of tertiary binaries.

In this paper we have focused on BH binaries in bound orbits around another binaries. To determine the global BH binary merger rate from such binary-binary channel, we would need to start from a population of main-sequence stellar quadruples, follow them through stellar evolution and BH formation, and eventually to eccentricity excitation and binary mergers. Such calculation is highly uncertain, and is beyond the scope of this paper. Recent population studies of BH binary mergers from field stellar triples gave a global merger rate of a few per Gpc$^3$ per year, which is within the low end of the observed BH merger rate determined by LIGO (Silsbee & Tremaine 2017; Antonini et al. 2017). The multiplicity fraction of high-mass main-sequence stars is quite high (as large as 90\%), with each star having more than 2 companions on average, suggesting that the stellar quadruple fraction is not much smaller than the stellar triple fraction (Sana 2017). With our finding that the merger fraction of quadruple systems is about 10 times larger than that of triple systems, we conclude that dynamically driven BH mergers in binary-binary systems may be more important than those produced in triple systems, and contribute appreciably to the BH merger events observed by LIGO/VIRGO.
Figure 5. Merger window and merger fraction as a function of the effective semi-major axis of outer binary $a_{\text{out,eff}}$ (Equation 27) for $\beta = 0, 0.29, 1, 1.8$ (corresponding to $a_2 = 1\,\text{AU}, 35\,\text{AU}, 81\,\text{AU}$ and $121\,\text{AU}$). In all examples, we assume a fixed $I_{2,0} = 30^\circ$. In each case, each dot (in the bottom four panels) represents a successful merger event within $10^{10}$ yrs. Note that merger events can have an irregular distribution as a function of $\cos I_0$, because of the chaotic behavior introduced by the octupole terms and binary-binary interactions. Also note that we only consider the range of $a_{\text{out,eff}}$ such that double-averaged secular equations are valid (see Equation 26).

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ACKNOWLEDGMENTS

This work is supported in part by the NSF grant AST-1715246 and NASA grant NNX14AP31G. BL is also supported in part by grants from NSFC (No. 11703068 and No. 11661161012). This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Shanghai Astronomical Observatory.

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