Application of a $\sqrt{\text{area}}$-Approach for Fatigue Assessment of Cast Aluminum Alloys at Elevated Temperature

Roman Aigner 1,*, Christian Garb 2, Martin Leitner 1, Michael Stoschka 1 and Florian Grün 2

1 Christian Doppler Laboratory for Manufacturing Process based Component Design, Chair of Mechanical Engineering, Montanuniversität Leoben, 8700 Leoben, Austria; martin.leitner@unileoben.ac.at (M.L.); michael.stoschka@unileoben.ac.at (M.S.)
2 Chair of Mechanical Engineering, Montanuniversität Leoben, 8700 Leoben, Austria; christian.garb@alumni.unileoben.ac.at (C.G.); florian.gruen@unileoben.ac.at (F.G.)
* Correspondence: roman.aigner@unileoben.ac.at; Tel.: +43-3842-402-1454

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Abstract: This paper contributes to the effect of elevated temperature on the fatigue strength of common aluminum cast alloys EN AC-46200 and EN AC-45500. The examination covers both static as well as cyclic fatigue investigations to study the damage mechanism of the as-cast and post-heat-treated alloys. The investigated fracture surfaces suggest a change in crack origin at elevated temperature of 150 °C. At room temperature, most fatigue tests reveal shrinkage-based micro pores as their crack initiation, whereas large slipping areas occur at elevated temperature. Finally, a modified $\sqrt{\text{area}}$-based fatigue strength model for elevated temperatures is proposed. The original $\sqrt{\text{area}}$ model was developed by Murakami and uses the square root of the projected area of fatigue fracture-initiating defects to correlate with the fatigue strength at room temperature. The adopted concept reveals a proper fit for the fatigue assessment of cast Al-Si materials at elevated temperatures; in detail, the slope of the original model according to Murakami should be decreased at higher temperatures as the spatial extent of casting imperfections becomes less dominant at elevated temperatures. This goes along with the increased long crack threshold at higher operating temperature conditions.

Keywords: aluminum cast; fatigue strength; defects; hardness; tensile tests; elevated temperature

1. Introduction

Aluminum cast parts enable the manufacturing of quite complex geometries, due to their sound castability [1]. Furthermore, the investigated alloys also feature a proper fatigue strength; hence the achievement of lightweight design goals is additionally supported [2]. Therefore, Al-Si cast alloys are commonly used materials for automotive engine components, as reported in [3–6]. However, the manufacturing process, as well as the subsequent heat treatments, must be considered as they possess distinctive impacts on the resulting mechanical properties [7–9]. As such cast aluminum components are also exposed to elevated temperatures during service, the characterization of the material properties under these conditions is inevitable. Hence, the material properties of Al-Si cast alloys are investigated under elevated temperatures in a preliminary study [10]. This work extends the investigations from Garb et al. [10] by means of sampling positions, the statistical evaluation of fatigue fracture-initiating inhomogeneities and a defect-based fatigue strength model at elevated temperature of 150 °C, which was chosen in this study to reflect typical service conditions [11]. Another study [12] proposed the estimation of fatigue life under enhanced temperatures from tensile...
Yet, increased temperatures significantly influence the microstructure as well as the fatigue lifetime [13,14]. The corresponding investigated sampling position parts can inherit different local microstructures due to the varying local cooling conditions. Therefore, the inhomogeneities and their statistically distribution regarding shape and spatial extent differ from each sample position to another. Tiryakioğlu proposed in [15] that fatigue-initiating defect sizes in aluminum cast material can be properly described by Gumbel [16], see Equation (1) or the Generalized Extreme Value distribution [17], see Equation (2).

\[
G_{(d_{eq})} = \exp \left[ -\exp \left( -\frac{d_{eq} - \lambda}{\delta} \right) \right]
\]

(1)

With \( G_{(d_{eq})} \) being the cumulative distribution function of the Gumbel distribution, its course is characterized by the distribution parameters \( \lambda \) and \( \delta \), also referred to as location and scale parameter.

\[
P_{(d_{eq})} = \exp \left\{ -\left[ 1 + \xi \left( \frac{d_{eq} - \mu}{\sigma} \right)^{-\frac{1}{\xi}} \right] \right\}
\]

(2)

Furthermore, \( P_{(d_{eq})} \) equals the cumulative distribution function of the Generalized Extreme Value distribution depending on the equivalent circle diameter \( d_{eq} \), the location \( \mu \), the scale \( \sigma \) and the shape parameter \( \xi \). Furthermore, it was shown that the fatigue strength in the finite life region can be assessed by linking the cumulative probability density of fracture-initiating defect size distribution with the Paris-Erdogan equation for stable crack growth [18]. Due to the fact that the focus of this work is the fatigue strength assessment of Al-Si cast materials in the long-life fatigue region, Murakami’s \( \sqrt{\text{area}} \) concept is used, see [19]. Hereby, the author in [19] stated that in the presence of extrinsic heterogeneities, such as flaws, the fatigue strength correlates well with the size of those defects. The spatial extent of the fatigue fracture-initiating defects is characterized by the square root of defect projection area \( \sqrt{\text{area}} \), whereby the area is projected onto the plane perpendicular to the direction of maximum stress. A summary of this procedure is additionally given in [20]. The \( \sqrt{\text{area}} \) is used as the size parameter of fatigue fracture-initiating flaws, due to the sound correlation with the maximal stress intensity factor at the crack tip \( K_{\text{max}} \), see [20]. Murakami’s approach is based on the material and defect location dependent coefficient \( C_1 \), the material constants \( C_2 \) and \( m \), as well as on the Vickers hardness HV, see Equation (3).

\[
\sigma_{f,\text{FE}} = C_1 \cdot \frac{(HV + C_2)}{\sqrt{\text{area}}^{1/(2m)}}
\]

(3)

Murakami proposed the exponent \( m \) of the defect size to possess a constant value of 3, which revealed sound results for preliminary studies [21]. Furthermore, Murakami estimated the material and defect location coefficient \( C_1 \) as 1.43 for surface and 1.56 for subsurface cracks and the material dependent parameter \( C_2 \) to possess a constant value of 120 by applying the least squares method [19].

To assess the fatigue strength for operating conditions, this paper focuses on the influence of elevated temperature on the mechanical properties of two Al-Si cast materials, namely EN AC-46200 and EN AC-45500. Both tensile tests to assess the quasi-static properties as well as fatigue tests are performed at room and at elevated temperature to assess an advanced fatigue strength model based on the \( \sqrt{\text{area}} \)-approach by Murakami [19,20]. In summary, this paper scientifically contributes with the following working packages:

- Investigation of the fatigue strength of Al-Si cast materials at an elevated temperature of 150 °C
- Assessment of statistically defect distribution regarding spatial extent and shape
- Investigation of damage mechanisms at enhanced operating temperatures
- Evaluation of the material constants \( C_1 \), \( C_2 \) and \( m \) of Murakami’s \( \sqrt{\text{area}} \) concept for elevated temperatures
2. Materials and Methods

2.1. Materials

The examined materials are two Al-Si cast alloys with Strontium (Sr) as eutectic modifier and post-heat treatments to obtain T5 and T6 condition, see previously performed studies [10,22,23]. The T5 heat treatment consists of a quenching process and subsequent artificial aging whereas the T6 heat treatment usually involves three stages; the solution treatment at high temperatures, the quenching, and the age hardening process [24]. A summary of the T6 heat-treatment procedure and its impact on the resulting mechanical properties, depending on the chemical composition of the alloy and the exposure temperature and time, is given in [25]. The specimens are taken from gravity casted crankcases (CC) and cylinder-heads (CH) from varying positions (denoted as Pos #1 to Pos #3) exhibiting different local cooling conditions and therefore a variation in microstructure such as secondary dendrite arm spacing (sDAS) and micro pore size distribution. The nominal chemical composition of the investigated alloys is given in Table 1. The eutectic modifier Sr acts as micro alloying element and is measured in the ppm-range in the final cast material condition.

| Table 1. Nominal chemical composition of the investigated cast alloys in weight percent. |
|---|---|---|---|---|---|---|---|
| Alloy | Si [%] | Cu [%] | Fe [%] | Mn [%] | Mg [%] | Ti [%] | Al [-] |
| EN AC-46200 | 7.5–8.5 | 2.0–3.5 | 0.8 | 0.15–0.65 | 0.05–0.55 | 0.25 | balance |
| EN AC-45500 | 6.5–7.5 | 0.2–0.7 | 0.25 | 0.15 | 0.45 | 0.20 | balance |

In addition, an overview of the component’s material specifications is listed in Table 2.

| Part | Position | Alloy | Modifier | Heat Treatment | sDAS [µm] |
|---|---|---|---|---|---|
| CH | Pos #1 | EN AC-46200 | Sr | T5 | 24 ± 4.8 |
| CC | Pos #2 | EN AC-46200 | Sr | T6 | 30 ± 7.3 |
| CC | Pos #3 | EN AC-46200 | Sr | T6 | 72 ± 24.9 |
| CH | Pos #1 | EN AC-45500 | Sr | T6 | 27 ± 6.6 |

2.2. sDAS and Metallographic Analysis

The sDAS is evaluated by means of a procedure proposed by [26], referred to as measurement method D. Hereby, metallographic specimens are taken out at the very sample positions and prepared by polishing. Afterwards, the specimens are investigated by digital optical microscopy. The secondary dendrite arm spacing is then calculated invoking the number of secondary arms along one side of a primary arm, such that dendrite asymmetry does not affect this method. This procedure is used, as it provides the most accurate estimation of secondary dendrite arm spacing, according to [26]. The investigation of metallographic sections in the testing region proposes a similar secondary dendrite arm spacing of Pos #1 for both alloys. The investigation of metallographic parameters in Pos #2 proposes a slightly enhanced local sDAS of 30 µm, regarding the values of Pos #1. The evaluated sDAS in sampling position Pos #3 is significantly higher though, due to lower cooling conditions compared to Pos #1 and #2, see [27]. The hardness measurements are conducted by means of a system of the type Zwick ZHU 2.5 TS1S (Ulm, Germany) and the metallographic samples are prepared by means of a Buehler BETA and a Struers CitPress system.

2.3. Quasi-Static and Fatigue Testing

The evaluation of the quasi-static material properties is carried out for room and elevated temperature, see [28] and [29] respectively. Each test series contains a minimum number of three specimens to statistically assess quasi-static properties. All tensile specimens and are conducted
strain-controlled at a strain rate of $3.6 \times 10^{-3}$ 1/s with a gauge length of $l_0 = 25$ mm, using an extensometer. Both the tensile tests and the fatigue tests are conducted at a hydraulic Instron Schenk testing system. To additionally evaluate both the fatigue strength and yield strength of the investigated materials not only at room but also at the ET of 150 °C, an Instron SFL 3119-400 Series temperature controlled chamber (Darmstadt, Germany) is used. The tensile test specimen geometry is displayed in Figure 1.

![Figure 1. Tensile test specimen geometry in units of millimetre.](image-url)

2.4. X-ray Computed Tomography

The investigated samples possess a low density and a small specimen diameter. Thus, the non-destructive investigation of selected specimens is carried out by a Phoenix/X-ray Nanotom 180, enabling a resolution of just 5 µm voxel-size, such that extrinsic flaws with a spatial extent of about 15 µm can be evaluated properly.

3. Results

3.1. Tensile Tests

Preliminary studies [30] stated that the fatigue resistance correlates linearly with the quasi-static material properties as the ultimate tensile strength (UTS) or the yield stress (YS) at corresponding temperatures. Furthermore, it is well known that exposure of Al-Si cast material at enhanced temperature leads to a significant decrease of the fatigue resistance [14]. Therefore, quasi-static tensile tests are not only conducted at room temperature, but also at an elevated temperature of 150 °C, using a heat chamber. An overview of the quasi-static test results is provided in Table 3.

| Abbreviation | Temperature | UTS [MPa] | YS [MPa] | A [%] |
|--------------|-------------|-----------|----------|-------|
| EN AC-46200 Pos #2 | RT | 326 | 277 | 1.58 |
| EN AC-46200 Pos #2 | ET | 265 | 245 | 2.96 |
| EN AC-46200 Pos #3 | RT | 208 | 207 | 0.18 |
| EN AC-46200 Pos #3 | ET | 187 | 187 | 0.19 |
| EN AC-46200 Pos #1 | RT | 287 | 198 | 2.31 |
| EN AC-46200 Pos #1 | ET | 234 | 184 | 5.25 |
| EN AC-45500 Pos #1 | RT | 334 | 273 | 6.78 |
| EN AC-45500 Pos #1 | ET | 259 | 233 | 9.55 |

EN AC-45500 Pos #1 reveals the highest UTS with 334 MPa at room temperature. In addition, EN AC-45500 Pos #1 also possesses the highest fracture elongation values $A$ at room and elevated.
temperature. As listed in Table 3, the fracture elongation increases if tested at elevated compared to room temperature.

The results confirm preliminary studies [31], which proposed a function of the ascending elongation with increasing testing temperature. Yet, the authors in [31] do not register a significant increase of \( A \) below 270 °C. Another study [32] shows a strong increase of the elongation at fracture only for testing temperatures above 300 °C testing temperature. However, the test results observed in this study propose an increased fracture elongation already at 150 °C for the examined alloys, see Figure 2. Figure 2 displays representative tensile test results for the varying sampling positions at room and elevated temperature. Furthermore, representative microstructures at the very specimen locations are illustrated in detail in Figure 3.

On the other hand, the yield strength in general decreases at elevated temperature, as listed in Table 3. The mean reduction of the UTS from room to elevated temperature is approximately 17%. This goes along with preliminary studies in [31–33].
3.2. Hardness Measurements

As stated in [34,35] the quasi-static material properties such as the yield strength exhibit a basic coherence with the macrohardness. Also, the investigation of the position depending hardness is from significance to set up Murakami’s $\sqrt{\text{area}}$ model [19] in a uniform way for the investigated specimen series. Therefore, Vickers hardness measurements are executed at room temperature in line with [36] applying a testing force of 3 kp. A preliminary study [37] describes a linear relationship of Al-Zn-Mg alloys between the Vickers hardness HV and the flow stress. In [38] it is also stated that the Brinell hardness BHN corresponds well to the YS for both A356 and A357 alloys. In [39] an evaluation of models for hardness-yield strength relationships is presented. Drouzy and Richard [40] presented a quite simple linear ratio between the YS and the Brinell hardness BHN of underaged Al-Si alloys. However, it is also shown that the YS-BHN relationship of Al-Si alloys can be rather described by a hysteresis, which shows a significantly higher slope in the underaged region than the proposed relationship from [40]. Murakami’s $\sqrt{\text{area}}$ model takes the local Vickers hardness HV into account, as it relates well to the fatigue strength [20]. Therefore, the local hardness for the sample positions must be investigated not only at room but also at elevated temperatures.

To estimate the hardness at an elevated temperature, a linear fit, using the least square method, between the measured hardness and calculated yield strength, is carried out, see Figure 4. The evaluated slope of the fit is almost two times the proposed value of approximately 2.85 from [40]. Each test series contains a minimum number of three tensile specimens and three hardness tests at the corresponding position to tone down outlier values. Subsequently, the yield strength of tensile tests at elevated temperatures is re-inserted in the estimated YS-HV fit, to estimate the Vickers hardness at elevated temperatures.

![Figure 4](image_url)

**Figure 4.** Evaluated correlation between Vickers hardness and Yield strength for Al-Si castings.

3.3. Fatigue Tests

The fatigue tests are conducted under alternating tension-compression testing. This deduces a load stress ratio, lower load level to upper load level, of $R = -1$. Therefore, the mean stress is equal to zero. All fatigue tests are executed at a hydraulic testing machine with a testing frequency of 30 Hz until either specimen burst failure or run out at $1 \times 10^7$ load cycles. In line with the quasi-static tests, the fatigue tests at elevated temperature are performed using a heat chamber. The fatigue resistance in the finite life region is statistically evaluated by means of the ASTM E 739 standard [41]. This methodology assumes a constant variance in the finite life region. Furthermore, the fatigue data in the run out region is statistically investigated based on the $\arcsin \sqrt{p}$ methodology, see [42]. This procedure approximates the probabilities of survival in the long-life region by a $\arcsin \sqrt{p}$ function.
and is a proven methodology to statistically estimate the mean value and standard deviation of the fatigue strength and likewise minimum life, see [43]. As proposed in [44], the slope $k_2$ of the run out region is assumed to be five times the slope $k_1$ in the finite life region. This assumption is also verified in [45–47]. All fatigue data has been normalized by the UTS at room temperature of EN AC-46200 -Pos #2, evaluated to 326 MPa, see Table 3.

The tests are performed until a total number of $1 \times 10^7$ load cycles, because preliminary studies showed defect correlated specimen failure in the HCF region between $1 \times 10^6$ and $1 \times 10^7$ load cycles, see [3,48,49]. All evaluated S/N-curves are displayed including the 10 and 90 % probability of survival scatter band. The statistical evaluation of scatter bands in the run out region is conducted by means of the $\arcsin \sqrt{p}$ methodology [42,43]. The evaluated fatigue strength data for each sample position as well as the scatter band in the run out region is listed in Table 4. In Figure 5 the S/N-curve of EN AC-45500 with T6 heat treatment at Pos #1 at room and at elevated temperature is displayed. While the slope $k_1$ is almost identical for RT and ET, the number of cycles $N_D$ for the transition region rises with increased testing temperature from approximately $1 \times 10^6$ to about $4.27 \times 10^6$ load cycles.

The investigated fatigue strength of EN AC-46200 with T5 heat treatment at Pos #1 states also a similar $k_1$ at room temperature, see Figure 6. On the other hand, the high-cycle fatigue strength at $1 \times 10^7$ load cycles $\sigma_{f,1 \times 10^7}$ significantly decreases by about 21 % at 150 °C. The investigation of the fatigue strength from EN AC-46200 with T6 heat treatment at Pos #2 assumes a slightly shallower S/N-curve at elevated temperature, represented by the slope in the finite life region $k_1$. Furthermore, the transition point $N_D$ rises to $1.75 \times 10^6$ load cycles, with a decrease of about 7% in fatigue strength $\sigma_{f,1 \times 10^7}$, as seen in Figure 7. Finally, EN AC-46200 with T6 heat treatment at Pos #3 shows a significant shallower slope in the finite life region $k_1$, while the evaluated fatigue resistance $\sigma_{f,1ET}$ decreases only by 2% at 150 °C testing temperature, see Figure 8. It must be pointed out that the depicted minor reduction of the fatigue strength in Pos #3 is within the scatter band of the S/N-curves.

![Figure 5. S/N-curves of EN AC-45500 T6 Pos #1 at RT and ET.](image-url)
Figure 6. S/N-curves of EN AC-46200 T5 Pos #1 at RT and ET.

Figure 7. S/N-curves of EN AC-46200 T5 Pos #2 at RT and ET.

Table 4. Fatigue test results at room and elevated temperature.

| Abbreviation | Temperature | $\sigma_1 \times 10^5 \text{,norm}$ | $N_D$ | $k_1$ | $\frac{1}{T}$ |
|--------------|-------------|----------------------------------|------|-------|-------------|
| EN AC-46200 Pos#2 | RT | 0.245 | $5.71 \times 10^5$ | 3.58 | 1.256 |
| EN AC-46200 Pos#2 | ET | 0.241 | $1.75 \times 10^6$ | 5.85 | 1.391 |
| EN AC-46200 Pos#3 | RT | 0.176 | $3.92 \times 10^5$ | 5.13 | 1.278 |
| EN AC-46200 Pos#3 | ET | 0.172 | $2.68 \times 10^6$ | 6.41 | 1.210 |
| EN AC-46200 Pos#1 | RT | 0.332 | $5.29 \times 10^5$ | 7.40 | 1.147 |
| EN AC-46200 Pos#1 | ET | 0.244 | $6.16 \times 10^6$ | 10.65 | 1.115 |
| EN AC-45500 Pos#1 | RT | 0.348 | $9.99 \times 10^5$ | 6.22 | 1.140 |
| EN AC-45500 Pos#1 | ET | 0.291 | $4.27 \times 10^6$ | 7.48 | 1.206 |
To investigate the fracture-initiating defects, it is from utmost importance to analyze all tested specimens either by means of digital microscope as well as by SEM. The detected crack-initiating flaws are characterized by means of geometrical parameters such as the square root of the effective defect area. In line with the procedure proposed in [20], the size of fatigue fracture-initiating defects is characterized by the square root of the projected area of the flaw, perpendicular to the load direction. This methodology is displayed in Figure 9 using the example of a fracture-initiating heterogeneity at Pos #3. Furthermore, to characterize not only the fracture-initiating defects by means of fractography, selected specimens are investigated non-destructively with X-ray computed tomography (XCT). This methodology supports the holistic characterization of the defect population respectively its spatial extent and is further described in [22,48].

Figures 9 and 10 show a fracture surface with a crack origin at a micro pore. At room temperature, all tested EN AC-46200 specimens initiate from such micro pores. On the other hand, Figure 11 displays a different cause of failure. At ET, the stress intensity is enhanced in the defect-near area while the activation energy of slip-planes decreases due to the increased operating temperature. Therefore, specimens tested at a higher temperature activate a different failure mechanism.
Some specimens would reveal a crack initiation right at the surface along with crack propagation at large slip bands. Nevertheless, some investigated specimens revealed mixed defect mechanisms of micro pores and large slipping areas. At the latter ones, the crack initiates at an intrinsic inhomogeneity and possesses a stable crack growth as with increasing crack length, the stress intensity rises near the crack tip. If the stress intensity factor and the activation energy based on the thermal energy reach a certain threshold, the crack starts to slip over a slip-band area during one single load-cycle and therefore significantly increases the crack growth. As a result, the remaining fatigue strength is reduced compared to an arbitrary defect with a similar $\sqrt{\text{area}}$. 

**Figure 10.** Representative fracture-initiating micropore at Pos#2.

**Figure 11.** Representative fracture-initiating slipplane at Pos#1.
As the fractography results show, this failure mechanism occurs especially at EN AC-46200 T5 and EN AC-45500 T6, where a major part of the specimens at Pos #1 inherit a slip-band-induced failure. It must be noted that EN AC-45500 T6 Pos #1 even shows a slip-band-like failure mechanism also at room temperature. In Figure 12, the different damage mechanism fractions of the corresponding positions and alloys are displayed.

![Fractography Results](image)

**Figure 12.** Comparison of the crack-initiating failures at RT and ET.

As proposed by [50] either Gumbel or GEV distributions are applicable for fatigue-initiating defects. The distribution parameters are evaluated by means of a maximum likelihood method, as presented in [51].

\[
\phi = \frac{1}{n} \sum_{i=1}^{n} \frac{d_{eq,i}}{d_{max,i}}
\]  

(4)

The probability of occurrence of an arbitrary \(d_{eq}\), based on the cumulative density function of the GEV-fit for each sampling position is displayed in Figure 13. The probability of occurrence of a \(d_{eq}\) of 200 µm is less than 10% for sampling positions #1 and #2, whereas at Pos #3 the probability of occurrence for the same equivalent diameter possesses a value of just above 97%.

The cumulative density function of the distribution is computed by means of Equation 2, using the equivalent circle diameter \(d_{eq}\) from the most critical defects, whereas \(\xi\) is denoted as the shape \(\sigma\) the scale and \(\mu\) the location parameter. The equivalent circle diameter \(d_{eq}\) of one flaw can easily be derived by multiplying \(\sqrt{\text{area}}\) with the factor \(\frac{2}{\sqrt{\pi}}\). The ratio of the equivalent circle diameter \(d_{eq}\) to the maximum diameter \(d_{max}\) describes the shape \(\phi\) of the crack-initiating pores [52], see Equation (4). The shape factor \(\phi\) therefore ranges between zero and one, indicating the roundness of a crack-initiating pore. The lower \(\phi\), the more complexly shaped is the defect. Thus, a circle-shaped flaw would possess a shape factor \(\phi = 1\). The shape of representative defects with varying sphericity is presented in detail in [53].

Both the evaluated distribution parameters as well as the mean shape of the defects are listed in Table 5. The investigation of the different fracture-initiating defect sizes proposes a significant different micro pore size distribution at Pos #3, in line with the increased local sDAS. The mean shape \(\phi\) of the flaws in Pos #3 also reckon them to possess a more spherical shape.
3.4. Fatigue Assessment Model

To assess the fatigue strength of Al-Si alloys incorporating manufacturing process-based inhomogeneities, a defect size-based material model is set up. The main causes of failure, evaluated in both EN AC-46200 Pos #1 and EN AC-45500 Pos #1, are basically not based on micro pores. Therefore, the defect-based material model from Murakami [19] is mainly set up for EN AC-46200 Pos #2 and Pos #3 as herein the main failure cause can be assigned to micro porosity. However, specimens with defect correlated crack initiation from Pos #1 of both alloys are also displayed in the adopted material model, see Figure 14. A parameter set, containing the stress amplitude at $1 \times 10^7$ load cycles $\sigma_{f,1 \times 10^7}$ of run-outs, the stress amplitude $\sigma_a$ and number of load cycles $N$ of tests in the finite life region, as well as the corresponding defect size $\sqrt{\text{area}}$ are required. To increase the applicable data a power-function like projection method for specimens failed in the finite life region is executed. This method is suitable to increase the data points in the run out region of $N = 1 \times 10^7$ without significant falsification of the scatter band in the HCF region $1/T_s$, see [21].

The original data from Murakami [19] proposes a coefficient of $m = 3$ for the exponent of the defect size. The parameters $C_1$ and $C_2$ are material dependent constants. This approach provides reasonable defect-based material models for Al-Si alloys at room temperature. However, the exponent of the defect size can vary at elevated temperatures, as it represents the slope of the material model. Therefore, the coefficient $m$ is not further considered to be a constant, see Equation (5).

$$
\sigma_{1E7} = C_{1,T} \cdot \frac{(HV + C_{2,T})}{\sqrt{\text{area}}^{1/(2 \cdot m_T)}}
$$

(5)

This model is supplied with all parameter sets depending on their alloy, position, and testing temperature. Furthermore, the hardness of the corresponding positions at elevated temperature is estimated based on the experimentally evaluated yield strength, as discussed in Section 3.2. Next, the parameters $C_{1,T}$ and $C_{2,T}$ as well as the slope $m_T$ are estimated applying a non-linear...
solver, using the least square method. The evaluated parameters maintain a slope of $m_T = 3.02$ for specimens tested at room temperature, which agrees with the proposed constant of $m = 3$ in the original model, see Equation (3). However, the data at elevated temperature leads to a change of the slope value. The least square method proposes a coefficient of $m_T = 4.05$ for specimens tested at an elevated temperature of 150 $^\circ$C. The defect-based material model for elevated temperatures with the 90 and 10% probability scatter bands is displayed in Figure 14. The defect correlated fatigue strength is restricted by two major limits. On the one hand, the upper boundary is set by the fatigue strength of near defect-free material where the area of the crack-initiating inhomogeneities tends to zero. On the other hand, the lower boundary is determined by huge defects, such that the stress intensity factor along the internal crack meets the long crack threshold $\Delta K_{th,lc}$, see Equation (6).

$$\Delta K_{th,lc} = Y \Delta \sigma_1 E \sqrt{\pi \sqrt{\text{area}}}$$

(6)

![Figure 14](image.png)

Figure 14. Defect correlated fatigue lifetime model with evaluated coefficient m at elevated temperature.

Preliminary studies [54–56] revealed that $\Delta K_{th,lc}$ rises in line with increasing testing temperature, until a critical temperature is met. This results from an increased plastic zone in front of the crack tip. The size of the monotonic plastic zone can be estimated by Irwin’s estimation, based on the YS of the material, see Equation (7) [57].

$$r_{pl} = \frac{K^2}{\pi Y S^2}$$

(7)

The relationship proposed by Irwin applies for a monotonic plastic zone with no crack closure occurring [58]. To evaluate the cyclic plastic zone, incorporating crack closure effects, $K$ is superimposed by $\Delta K_{eff}$, such that:

$$\Delta r_{pl} = \frac{\Delta K_{eff}^2}{\pi 4 Y S^2}$$

(8)

As the YS decreases with elevated testing temperature at an average of 10.7%, the size of the cyclic plastic zone $\Delta r_{pl}$ therefore rises by 25.4%, see Equation (8). Hence, the plastic-induced crack closure effects significantly increase, in line with the extended plastic zone [59–61], resulting in an elevated long crack threshold $\Delta K_{th,lc}$ at higher temperatures. As a result, defects with large spatial extent do not affect the fatigue strength at elevated temperatures as significantly as for room temperatures,
see Equation (6). The increased slope of the defect-based material model, represented by the coefficient \( m_T \), thus is deduced by increased \( \Delta K_{th,lc} \) at elevated temperatures.

### 4. Discussion

In this paper, fatigue strength, hardness and quasi-static tests are executed at both RT and an ET of 150 °C. The investigated materials are different Al-Si alloys with subsequent T5 or T6 heat treatment and varying local solidification times, leading to locally adjusted microstructural features, such as secondary dendrite arm spacing and micropore distribution. Furthermore, fractographic analysis is conducted to evaluate the fracture-initiating defects and their spatial extent. To consider the decreasing hardness at elevated temperature, a linear relation between the yield strength and Vickers hardness is evaluated to establish a link between room- and elevated temperature data. The hardness at elevated temperatures is subsequently estimated based on quasi-static test results at 150 °C testing temperature, using the evaluated YS-HV relationship displayed in Figure 4. The investigation of the fatigue strength at elevated temperature reveals a significant decrease of 21% for specimen Pos #1 of both observed alloys in respect to the fatigue strength at room temperature. The evaluation of fatigue strength at elevated temperature at specimen Pos #2 and Pos #3 revealed a minor decrease of 2%, though. The tensile test data at elevated temperature reveals an overall increase of elongation at fracture of about 65%, as well as a decrease of Young’s modulus of about 4%, due to elevated dislocation mobility.

The specimen positions inherit major differences in fatigue fracture-initiating defects, as the statistical evaluation of flaw sizes states. This is mainly caused by the local significantly varying cooling rates between the sample positions and results in a broad spectrum of fatigue fracture-initiating material heterogeneities. Hence, the commonly used \( \sqrt{\text{area}} \) fatigue strength assessment model proposed by Murakami can be invoked as basic fatigue assessment strategy. The extended model, introduced in this paper, takes a modified and temperature dependent slope value of \( m_T \approx 4 \) into account to properly assess the fatigue strength in the long-life region at elevated temperatures, see Equation (5).

As shown in Figure 15, the proposed fatigue assessment model is compared with the normalized experimental results. The model for elevated temperatures properly meets the experimental fatigue lifetime data. The scatter band and the mean value are statistically estimated, approaching the methodology from [41]. The statistically evaluation of the fatigue assessment model reveals a comparably minor scatter band \( 1/T_m \) of 1.18.

![Figure 15. Validation of the \( \sqrt{\text{area}} \) model for elevated temperatures.](image-url)
5. Conclusions

Based on the conducted research work and assessed results, the following conclusions can be drawn:

- The statistically evaluated fatigue strength of all tested alloys drops when being exposed at elevated temperatures compared to fatigue lifetime at room temperature.
- A significant change in damage mechanism at elevated temperatures is observed. A major part of the specimens taken from both EN AC-46200 Pos #1, as well as EN AC-45500 Pos #1, maintain slipping areas as crack origins.
- While the original $\sqrt{\text{area}}$ model proposed by Murakami provides a sound fit for room temperature with a slope of $m = 3$, the slope changes at elevated temperature. The estimated slope $m_T \approx 4$ suggests an increased long crack threshold at elevated temperature, caused by more pronounced plasticity-induced crack closure effects. Therefore, the impact of increasing defect sizes on the anticipated fatigue resistance generally declines at elevated temperatures.
- Comparing the fatigue strength of the introduced extension of Murakami’s model for higher operating temperatures, the experiments reveal a proper relationship. The mean value of the suggested model turns out to be slightly conservative.

Subsequent work focuses on the validation of the proposed model for further elevated temperature values incorporating additional fatigue tests. Moreover, the yield strength versus hardness relationship may be investigated in more detail at higher temperatures by additional testing series. Finally, near defect-free material must be experimentally analyzed in terms of fatigue strength at elevated temperatures to define the very upper boundary of the defect-based material model most accurately.

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Abbreviations

The following abbreviations are used in this manuscript:

| Abbreviation | Description |
|--------------|-------------|
| RT           | Room temperature |
| ET           | Elevated temperature |
| $\lambda$    | Location parameter of the Gumbel distribution |
| $\delta$     | Scale parameter of the Gumbel distribution |
| GEV          | Generalized Extreme Value distribution |
| $\xi, \sigma, \mu$ | Shape, scale, and location parameter of the GEV |
$\sigma_{f,1 \times 10^7}$ Long-life fatigue strength amplitude at $1 \times 10^7$ load cycles
$\Delta \sigma_{1 \times 10^7}$ Long-life fatigue strength range at $1 \times 10^7$ load cycles
$\sigma_a$ Fatigue strength amplitude
HV Vickers hardness
$C_1, C_2, m$ Constants of the $\sqrt{\text{area}}$ approach by Murakami
sDAS Secondary dendrite arm spacing
YS Yield strength
UTS Ultimate tensile strength
HCF High-cycle fatigue
E Young’s modulus
A Fracture elongation
$N_D$ Number of load cycles at transition knee point of S/N-curve
$N$ Number of load cycles until failure
$k_1$ Slope in the finite life region of S/N-curve
$T_S$ Scatter band of S/N-curve in the HCF region
BHN Brinell hardness number
SEM Scanning-electron-microscopy
XCT X-ray computed tomography
d$_{eq}$ Equivalent circle diameter
d$_{max}$ Maximal spatial extent of an inhomogeneity
$\varphi$ Shape factor
$n$ Number of defects
$C_{1,T}, C_{2,T}, m_T$ Parameters of the extended $\sqrt{\text{area}}$ approach for elevated temperatures
$K$ Stress intensity factor
$K_{max}$ Maximal stress intensity factor
$\Delta K_{th,lc}$ Long crack threshold
$\Delta K_{eff}$ Effective crack threshold
$Y$ Geometry factor
$\Delta r_{pl}$ Cyclic plastic zone
$r_{pl}$ Monotonic plastic zone
$T_m$ Scatter band of the validation of the fatigue assessment model

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