Dynamical Properties in the Bilayer Quantum Hall Ferromagnet

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(February 23, 2001)

The spectral functions of the pseudospin correlation functions in the bilayer quantum Hall system at \( \nu = 1 \) are investigated numerically, where the pseudospin describes the layer degrees of freedom. In the pseudospin-ferromagnetic phase, the lowest-energy excitation branch is closely connected with the ground state through the fluctuations of pseudospin \( S_y \) and \( S_z \), and it plays a significant role on the tunneling properties in this system. For the system with very small tunneling amplitude and layer separation smaller than the critical one, the system-size dependence of calculated spectral function \( A_{\nu z} \) suggests the superfluidity on the tunneling current in the absence of impurities.

Recently the bilayer quantum Hall (QH) system [1] has often been investigated from the viewpoint of quantum ferromagnetism. In fact, the pseudospin can be introduced to this system by a simple definition of assigning the upper/lower layers to the pseudospin \( \uparrow / \downarrow \) [23], and the pseudospin ferromagnetism can be realized for the total Landau level filling \( \nu = 1/m \) (\( m \): odd integers). In particular, the \( \nu = 1 \) bilayer QH system has extensively been studied both experimentally and theoretically [1].

The pseudospin rotational symmetry in this system is degraded because of the difference between the intralayer and interlayer Coulomb interaction for a finite layer separation \( d \) (and of the interlayer tunneling). Since the two layers tend to have equal numbers of electrons to reduce the interaction energy, the \( z \) component of the total pseudospin (half the difference in the numbers) tends to vanish, \( \langle S_z \rangle = 0 \), in the ground state. Thus the pseudospin will lie in the \( xy \) plane in the pseudospin space, where the system maintains an invariance under rotations about the \( z \) axis, i.e., the SU(2) symmetry is reduced to U(1) [4].

The electron correlation alone pushes the ground state for \( \nu = 1/m \) towards a ferromagnetic one [3], so the bilayer \( \nu = 1/m \) QH system behaves like an easy-plane itinerant-electron ferromagnet. The U(1) symmetry is further degraded in the presence of the interlayer tunneling, since the tunneling amplitude behaves like a magnetic field acting on the pseudospin, as seen in the tunneling Hamiltonian, \( H_T = -\Delta_{\text{SAS}} S_z \). Here \( \Delta_{\text{SAS}} \) is the energy difference between the symmetric and antisymmetric combinations of isolated layer states and is proportional to the interlayer-tunneling amplitude [4].

So a bilayer QH system is characterized by two dimensionless parameters, \( d/l \) and \( \Delta_{\text{SAS}}/(e^2/dl) \) (\( l \): the magnetic length, \( c \): dielectric constant of the host material). For \( \nu = 1 \), the phase diagram against these two parameters is obtained experimentally [3], and the diagram shows that the QH state disappears for \( d > d_c \) and that the critical separation \( d_c \) increases as \( \Delta_{\text{SAS}} \) increases.

Within this QH effect region, the ground state for \( \nu = 1 \) evolves continuously from tunneling-dominated (single-particle like) to correlation-dominated (many-body like) as \( \Delta_{\text{SAS}} \) is decreased, and there is no intervening non-ferromagnetic region between the two regimes [3]. Moreover, within this pseudospin-ferromagnetic region, the ground-state wavefunction is analytically given as a spin-squeezed state [3], where the fluctuation \( \Delta S_z \) is suppressed by the electron correlation.

By using a sample with very weak interlayer tunneling \( (\Delta_{\text{SAS}}/(e^2/dl) \sim 10^{-6}) \), Spielman et al. have recently showed the huge resonant enhancement of the zero bias tunneling conductance [3]. The possibility of the excitonic superfluidity was also discussed because of the tunneling spectroscopy reminiscent of the Josephson effect, of which controversial predictions have been done [5]. Very recently, further tunneling-spectroscopy measurements were performed with the in-plane magnetic field applied to the system, and the observation of a linearly-dispersing Goldstone mode in a bilayer QH ferromagnet was claimed [6]. This in-plane field effect has been discussed in some theoretical papers [7 13].

In case of very weak interlayer tunneling as realized in these experiments [3 6], the U(1) symmetry of the system is expected to recover almost completely. Some simple model systems with this symmetry have been investigated and the tunneling properties have been discussed [7 13]. The Chern-Simons Landau-Ginzburg approaches for bilayer QH systems [4 14] were also performed for the case of vanishing interlayer tunneling.

On the other hand, some discussions based a BCS-like wavefunction with a phase variable \( \phi \), which determines the orientation of the pseudospin magnetic moment, have also been done [3]. This wavefunction, however, is compatible with the SU(2) symmetry, and does not reflect the U(1) symmetry of the system as well as the Hartree-Fock approximation does not. Thus an approach that fully reflects the system’s symmetry and takes small (but non-zero) tunneling amplitudes into consideration has yet to come for the study of the tunneling-current properties in this system. That is exactly the purpose of the present paper.

By the exact diagonalization method for small-size systems, we obtain the spectral functions of pseudospin correlation functions numerically. We show that in the pseudospin-ferromagnetic phase the lowest-energy excitation branch is closely connected with the ground state...
through the fluctuations of pseudospin $S_y$ and $S_z$, and that it plays a significant role for the tunneling properties in this system. For the system with very small tunneling amplitude and layer separation smaller than the critical one, the system-size dependence of calculated spectral function $A_{yz}$ suggests the superfluidity on the tunneling current in the absence of impurities.

Our numerical calculations were done for bilayer spherical systems [8], where each surface of the spheres is passed through by $2S$ flux quanta. For the case of $\nu = 1$ we consider in this paper, $2S = N - 1$ for $N$-electron systems. In terms of annihilation and creation operators for the $m$-th spatial orbit in the lowest Landau level with pseudospin $\sigma$, the total pseudospin operators are given as $S_z = \sum_m(a_{m\uparrow}a_{m\downarrow} - a_{m\downarrow}a_{m\uparrow})/2$, $S_x = \sum_m(a_{m\uparrow}a_{m\downarrow} + a_{m\downarrow}a_{m\uparrow})/2i$, $S_x = \sum_m(a_{m\uparrow}a_{m\downarrow} + a_{m\downarrow}a_{m\uparrow})/2i$. Pseudospin $S_z$ and $S_x$ are related with the interlayer-tunneling and interlayer-voltage part of the Hamiltonian, respectively, as $H_T = -\Delta_{SAS}S_x$ and $H_V = eV S_y$. Our numerical calculations were done for bilayer spher-

Table II indicates that the U(1) symmetry in the easy-plane ferromagnet is recovered almost completely in case of very small $\Delta_{SAS}$. In fact, for $t \lesssim 10^{-3}$, intra- and inter-layer correlations between electrons in the ground state are found to be little dependent on the value of $t$.

In Fig. 1(a), the excitation spectra are shown by open circles in Fig. 1(a). In Fig. 1(b) and (c), the strength for each excitation in the spectral functions $A_{yy}(L, \omega)$ and $A_{zz}(L, \omega)$ is shown against the angular momentum $L$ and excitation energy $\omega$ by the area of the corresponding circle. We note that the energy is measured from the ground-state energy and that the unit is $e^2/\ell$.

The lowest-energy excitation branch can be seen for $0 < L \leq 3$ in Fig. 1(a). We made sure that the energy values in this branch (in particular, the one for $L = 0$) are quite dependent on the system size $N$ and that these ones decrease as $N$ increases. This branch is expected to be seen as a linearly-dispersing excitation mode [19] for sufficiently large $N$ and very small $\Delta_{SAS}$. This pseudospin-wave excitation branch is closely connected with the ground state through the fluctuations of pseudospin $S_y$ and $S_z$.

As seen in Table 1, the U(1) symmetry is recovered almost completely for $t \lesssim 10^{-4}$. In fact, we made sure that the result for the spectral function $A_{xx}$ is almost the same as that for $A_{yy}$ shown in Fig. 1(b), and that each pair of excited states corresponding to $A_{xx}$ and $A_{yy}$, respectively, is nearly degenerate. On the other hand, as seen in Fig. 1(c), most of the spectral weights for $A_{zz}$ are occupied by the lowest-energy excitation branch. This is also the case for $A_{yz}$, because $A_{yy}$ and $A_{zz}$ include the same factor for each term in Eq. 1.

In Table 1, the strength of $i^{-1}A_{yz}(L, \omega)$ in the lowest-energy excitation branch in the $\nu = 1$ pseudospin ferromagnet is shown for $N = 10$, $d/l = 1.0$, and $t = 10^{-4}$, $10^{-5}$, $10^{-6}$. It indicates that these values of strength are proportional to $t$, i.e., to $\Delta_{SAS}$. Thus the tunneling conductance is considered to be proportional to the square of $\Delta_{SAS}$, because $g(L, \omega) \propto i^{-1}\Delta_{SAS}A_{yz}(L, \omega)$. It is also found that the strength increases as angular momentum $L$ decreases. That is, the long-wavelength part of the pseudospin-wave excitation mode plays a significant role on the interlayer-tunneling properties in this system.

In Table 11, the strength of $i^{-1}A_{yz}(L, \omega)/t$ in the lowest-energy excitation branch in the $\nu = 1$ pseudospin ferromagnet is shown for $d/l = 1.0$, $L = 0$, 1, 2, and $N = 6, 8, 10$. It has a system-size dependence varying approximately as $N^3$ ($\lambda \simeq 3$). This is also the case for $d/l = 0.5$. For larger layer separations such as $d/l = 1.5$ and 2.0, however, such a system-size dependence is not obtained. In fact, for $d/l = 1.5$ and 2.0, the strength increases or decreases as $N$ increases, and the system-size
dependence is much weaker than that for \( d/l = 0.5 \) and 1.0.

For sufficiently small \( \Delta_{\text{SAS}} \) (\( \lesssim 0.01 \epsilon^2/\epsilon_l \)), it is known that the pseudospin ferromagnetism is destroyed for layer separation \( d \) larger than the critical one \( d_c \simeq 1.3d \). Among the values of layer separation adopted by us, \( d/l = 0.5 \) and 1.0 belong to the case of \( d < d_c \), and \( d/l = 1.5 \) and 2.0 satisfy \( d > d_c \). As mentioned above, our results for \( d < d_c \) show a system-size dependence of the spectral function \( A_{yz} \) varying approximately as \( N^\lambda \) (\( \lambda \simeq 3 \)), which is much stronger than usual \( N \)-linear dependence. On the other hand, the possible superfluidity on the tunneling current in this system is considered to show an \( N \)-square dependence. It is not clear whether this discrepancy between our result and predicted ones results from small system sizes in our calculations or not. At least, however, the system-size dependence of \( A_{yz} \) in our results is much stronger than \( N \)-linear one and so suggests the superfluidity on the tunneling current in the absence of impurities. For clarifying interlayer tunneling properties in this system, further studies of the impurity effects and numerical calculations for larger systems are desired.

For \( d > d_c \), such a strong system-size dependence does not appear in our numerical results. Significant differences between the two cases of \( d > d_c \) and \( d < d_c \) are expected to become clearer in larger systems because of the differences in the system-size dependence. These qualitative differences between the two cases are consistent with the results in a recent experiment. Lastly we note that the spectral weights of \( \tilde{i}^{-1} A_{yz} \) occupied by the lowest-energy excitation branch decrease rapidly as the layer separation increases.

The author thanks Allan H. MacDonald, John Schliemann, and Yogesh Joglekar for fruitful discussion with them. He is also grateful to Hisatoshi Yokoyama for valuable conversations on spectral functions, and to Anju Sawada for valuable experimental information about the bilayer QH system. He was supported for the research in the USA by Japan Society for the Promotion of Science.

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\begin{table}[h]
\begin{tabular}{|c|c|c|c|}
\hline
 & \( t = 10^{-4} \) & \( 10^{-5} \) & \( 10^{-6} \) \\
\hline
\( \Delta S_\parallel \) & \( 6.845 \times 10^{-3} \) & \( 6.845 \times 10^{-4} \) & \( 6.845 \times 10^{-5} \) \\
\hline
\( \langle S_\parallel \rangle \) & \( 4.165 \times 10^{-5} \) & \( 4.165 \times 10^{-6} \) & \( 4.165 \times 10^{-7} \) \\
\hline
\end{tabular}
\caption{The fluctuations of pseudospin \( S_\parallel \) and expectation values of pseudospin \( S_\parallel \) in the \( \nu = 1 \) ground state are shown for \( N = 10, d/l = 1.0, \) and \( t \equiv \Delta_{\text{SAS}}/(\epsilon^2/\epsilon_l) = 10^{-4}, 10^{-5}, 10^{-6} \). For such small \( \Delta_{\text{SAS}} \), the values are proportional to \( \Delta_{\text{SAS}} \).}
\end{table}
TABLE II. The strength of the spectral function $i^{-1}A_{yz}(L, \omega)$ in the lowest-energy excitation branch in the $\nu = 1$ pseudospin ferromagnet is shown for $N = 10$, $d/l = 1.0$, and $t \equiv \Delta_{\text{SAS}}/(e^2/\ell) = 10^{-4}$, $10^{-5}$, $10^{-6}$. The strength increases as angular momentum $L$ decreases, and it is proportional to $t$. Most of the spectral weights are occupied by this pseudospin-wave excitation branch.

| $L$ | $t = 10^{-4}$ | $10^{-5}$ | $10^{-6}$ |
|-----|--------------|------------|------------|
| 0   | $16.02 \times 10^{-4}$ | $16.02 \times 10^{-5}$ | $16.02 \times 10^{-6}$ |
| 1   | $10.56 \times 10^{-4}$ | $10.56 \times 10^{-5}$ | $10.56 \times 10^{-6}$ |
| 2   | $8.328 \times 10^{-4}$  | $8.329 \times 10^{-5}$ | $8.329 \times 10^{-6}$ |
| 3   | $4.806 \times 10^{-4}$  | $4.806 \times 10^{-5}$ | $4.806 \times 10^{-6}$ |

TABLE III. The strength of $i^{-1}A_{yz}(L, \omega)/t$ in the lowest-energy excitation branch in the $\nu = 1$ pseudospin ferromagnet is shown for $d/l = 1.0$, $L = 0, 1, 2$, and $N = 6, 8, 10$, where $t \equiv \Delta_{\text{SAS}}/(e^2/\ell)$. It has a system-size dependence varying approximately as $N^\lambda$ ($\lambda \simeq 3$).

| $L$ | $N = 6$ | 8 | 10 |
|-----|--------|---|----|
| 0   | 4.389  | 8.670 | 16.02 |
| 1   | 2.144  | 5.225 | 10.56 |
| 2   | 1.617  | 4.150 | 8.329 |

FIG. 1. The excitation spectra and spectral functions in the $\nu = 1$ bilayer QH system are shown for $N = 10$, $d/l = 1.0$, and $t \equiv \Delta_{\text{SAS}}/(e^2/\ell) = 10^{-6}$. (a) The energy levels are shown against the total angular momentum by open circles. The strength for each excitation in the spectral functions (b) $A_{yy}(L, \omega)$ and (c) $A_{zz}(L, \omega)$ is shown against the angular momentum $L$ and excitation energy $\omega$ by the area of the corresponding circle. The energy is measured from the ground-state energy and the unit is $e^2/\ell$. 

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