Conditions for Viable Affleck–Dine
Baryogenesis–
Implications for String Theories *

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Abstract
We examine the conditions for a viable Affleck–Dine baryogenesis in supergravity (SUGRA) scenarios, finding surprisingly strong constraints on the type of SUGRA theory. These constraints are beautifully fulfilled by string-based SUGRA models provided that inflation is driven by a modulus ($T$) field.

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1 Introduction

The fundamental problem of how a net baryon number has been generated in the universe is still far from being solved. The hopes that a sufficient baryogenesis could be produced at the electroweak transition of the Standard Model have been proven not to be viable \[1, 2\], which clearly points out to the need of new physics. Weak scale baryogenesis has also been studied in the context of supersymmetric models \[3\]. The results are somewhat better than in the Standard Model, but a working scenario would require a substantial amount of artificial arrangement of the supersymmetric parameters \[4\].

Given this situation it is important to study efficient alternative mechanisms for baryogenesis, the most promising of which is probably the one of Affleck and Dine (AD) \[5\]. This is based on the possible production in the early universe of large vacuum expectation values (VEV’s) of fields (or combinations of fields) carrying baryon or lepton number \((B \text{ or } L)\). If, in addition, \(B \text{ or } L\) is explicitly broken by some of these condensates, it is possible to excite a net baryon number in the universe. This mechanism is remarkably efficient; as a matter of fact, it tends to generate too much \(B\), which needs to be subsequently diluted. The AD mechanism requires scalars carrying \(B \text{ or } L\), which is happily the case of supersymmetric (SUSY) theories. The aim of this paper is to establish which conditions are to be fulfilled by a SUSY theory (more precisely, by a supergravity (SUGRA) theory) to be consistent with AD baryogenesis, and apply the results to relevant models (particularly string models).

Let us briefly review the basics of the AD mechanism. As mentioned above, the AD mechanism requires large VEV’s of scalar fields (say AD fields) in the early universe. These have been commonly associated in the literature with the existence of (approximately) flat directions involving the AD fields. Then quantum fluctuations during inflation may yield initial large VEV’s, which are eventually driven to zero by e.g. low-energy SUSY breaking mass terms. This was the implementation of the AD mechanism originally considered by Affleck and Dine. As has been stressed in Ref. \[6\], this picture ignores the following basic fact. During inflation SUSY is necessarily spontaneously broken since the scalar potential \(V\) gets a VEV, \(\langle V \rangle = 3H^2M_P^2\), and effective SUSY soft breaking terms, in particular effective soft masses of the order of the Hubble constant \(H\) are generated \((M_P = M_{Planck}/\sqrt{8\pi} \text{ is the usual SUGRA Planck scale})\). These spoil the flat directions (although we will maintain this denomination), since \(H\) is expected to be \(O(10^{13-14} \text{ GeV})\) in order to obtain the observed magnitude of the microwave background radiation anisotropy in most models. However, large VEV’s are still possible if these effective masses squared are negative. In Ref. \[3\] it was pointed out that this is perfectly possible in a generic SUGRA theory. Then, the sequence of events is schematically as follows.

- During inflation an effective potential for the AD field(s), say \(\phi\), is produced by the \(O(H)\) soft terms. These must include negative soft mass and, possibly, generalized A-type terms. For large values of \(\phi\) the flat direction of the potential must be lifted, which is likely done by F–terms coming from \(\sim \phi^n\) terms in the
superpotential $W$. The $\phi$ field evolves rapidly towards its minimum during this period.

- After inflation the inflaton starts oscillating coherently about its minimum. The energy density in these oscillations, which dominates the universe, red-shifts as matter so that $H = (2/3t)$ and the position of the minimum of $V$ in the $\phi$ direction becomes time-dependent (through $H$). The $\phi$ field follows the instantaneous minimum provided that the operators that lift the flat direction of the potential are non-renormalizable (i.e. $n \geq 4$ in the previous paragraph), which is perfectly possible.

- At $H = O(\text{TeV})$ the low-energy SUSY soft breaking terms become important. They produce a positive mass term for $\phi$ and $B(L)$–violating A–type terms, which become comparable to the other terms in the potential. For arbitrary phases of the $\phi$ fields the A–terms can also generate CP violation. In this period a large (maximal) $B$ or $L$ number is generated. Subsequently the decay of the inflaton partially dilutes the asymmetry yielding the final value of $B$. It turns out that for $n = 4$ the latter can be very naturally the observed one, though other values of $n$ (preferably $n \geq 4$) are also possible.

From the above picture it is clear that a successful AD baryogenesis requires that the $\phi$ effective potential during inflation contains

1. negative effective mass terms, $m_{\phi}^2 \leq 0$,
2. non-renormalizable terms to lift flat directions of the potential.

The purpose of the present paper is precisely to show what kinds of SUGRA theories are consistent with these conditions. We will find surprisingly strong constraints, which select a class of SUGRA theories. As an additional surprise, the possibility $m_{\phi} \simeq 0$, which had been considered implausible after the arguments of Ref.[6], turns out to be perfectly possible, allowing for scenarios in which the AD mechanism can be implemented in the original, “old-fashioned”, way.

In Sect. 2 we examine the viability of AD baryogenesis in SUGRA models when inflation (and the corresponding SUSY breaking) is triggered by a non-vanishing D–term (D–inflation). Sect. 3 is devoted to the other possible case, i.e. F–inflation. In Sect. 4 we apply our results to string scenarios, finding very significant and encouraging results. Finally, in Sect. 5 we summarize the conclusions.

## 2 D–Inflation

The possibility of D–inflation in SUGRA scenarios is very attractive since, as has been often claimed in the literature, F–inflation seems to lead naturally to too large inflaton
mass terms that disable the inflationary process (however, see footnote 4). Models of D–inflation were first proposed in Ref.[7] and have been recently revived in Ref.[8].

In order to be concrete, it is convenient to suppose that inflation is mainly triggered by a single D–term (this does not reduce the generality of the present analysis). Then, a suitable choice is to suppose that the relevant D–term is associated to one “anomalous” $U(1)$, which takes the form:

$$V_D = \frac{1}{2} D^2 = \frac{1}{2} g^2 \left( \xi + \sum_j q_j |z_j|^2 K_{jj} \right)^2,$$

where $g$ is the corresponding gauge coupling, $q_j$ are the charges of all the chiral fields, $z_j$, under the anomalous $U(1)$ and $K_{jj}$ is the Kähler metric, i.e. the second derivative of the Kähler potential $K = \partial K/\partial z_j \partial \bar{z}_j$ (we are assuming here a basis for the $z_j$ fields where the Kähler metric is diagonal). Finally, the constant $\xi$ is related to the apparent anomaly, $\xi = g^2 M_P^2 (\sum_j q_j / 192 \pi^2)$. This was precisely the scenario considered in Refs. [7, 8]. At low energy the D–term is cancelled by the VEV’s of some of the scalars entering Eq.(2), but initially $\langle D \rangle$ may be different from zero, thus triggering inflation.

Let us also notice that the scenario is quite insensitive to the details of the Kähler potential $K$ (note in particular that $(K_{jj})^{1/2} z_j$ are simply the canonically normalized chiral fields). In consequence we will ignore the Kähler factors, taking $K_{jj} = 1$, in the rest of this section.

The AD mechanism requires the existence of a field $\phi$ (or several ones) carrying $B$ or $L$ that develops a non-vanishing VEV during the inflationary process, say $\langle \phi \rangle_{in} \neq 0$ (the subindex in denotes either inflation or initial). Experimental evidence requires the final (low-energy) VEV to be vanishing, i.e. $\langle \phi \rangle_f = 0$. There are two main possibilities to consider, depending on $q_\phi \neq 0$ or $q_\phi = 0$.

Let us start with the first case. Taking$^1$ $(1/2)|\langle D \rangle|^2 \propto g^2 |\xi|^2$, it is clear that the D–term induces an effective mass term for $\phi$ whose sign depends on the relative sign of $\xi$ and $q_\phi$. If

$$\text{sign}(\xi) \text{ sign}(q_\phi) = -1,$$

then the effective mass squared is negative and we expect $\langle \phi \rangle_{in} \neq 0$.

However, this cannot be the whole story, since in the absence of additional $\phi$–dependent terms in $V$, $\langle \phi \rangle_{in}$ would adjust itself to cancel the D–term, thus disabling the inflationary process and breaking $B$ or $L$ at low energy. Thus, we need extra contributions yielding $\langle D \rangle_{in} \neq 0$, $\langle \phi \rangle_f = 0$. These may come from

a) low-energy soft breaking terms,

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$^1$These “anomalous” $U(1)$ symmetries appear frequently in string theories. The apparent anomaly is actually cancelled by the transformation of the dilatonic axion. The form of the corresponding D–term is that of a Fayet–Iliopoulos D–term.

$^2$E.g. in the model of the first article of Ref. [8] the combination of the D–term with appropriate F–terms provides nearly flat directions for a slow rollover transition, in a sort of hybrid-like inflation. Then, during the inflationary epoch, $(1/2)|\langle D \rangle|^2 = (1/2) g^2 |\xi|^2$, i.e. $H^2 \sim (1/2) g^2 |\xi|^2 / M_P^2$.

$^3$If some fields $\eta_j$ with $q_j \neq 0$ get $\langle \eta_j \rangle_{in} \neq 0$, we simply replace $\xi \rightarrow \xi + \sum_j q_j |\langle \eta_j \rangle_{in}|^2$. 

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b) F–terms,
c) D–terms.

The first possibility, a), cannot work in practice. Certainly, all the scalar fields get soft-breaking masses, \( m = O(\text{TeV}) \), but they are too small to be useful here. Schematically, the \( \phi \)–dependent potential reads

\[
V = \frac{1}{2} g^2 \left[ \xi + q_\phi \phi \right]^2 + m^2 |\phi|^2 ,
\]

which leads to \( |\langle \phi \rangle_{in}|^2 \simeq (-\xi/q_\phi) - (m^2/g^2 q_\phi^2) \) and \( \langle V \rangle_{in} \simeq (-m^2/q_\phi)[\xi + (m^2/2g^2 q_\phi)] \), which is too small to produce inflation. Even if, in order to preserve inflation, \( m \) were abnormally large, we notice from (3) that the potential in this direction would be eventually lifted (for large \( \phi \)) by the quartic term \( \sim g^2 q_\phi^2 |\phi|^4 \). As mentioned in the previous section, such a term would imply that the \( \phi \)–field does not follow the instantaneous minimum of \( V \) after inflation. This may spoil the AD mechanism, that depends crucially on the initial value of the field when it begins to oscillate freely, at later times.

The possibility b) is more plausible but still difficult to implement. In principle F–terms may contain masses much larger than \( O(\text{TeV}) \), thus yielding a sizeable \( \langle V \rangle_{in} \), suitable for inflation. However, this possibility is still unattractive since the problem of the field being driven away from the instantaneous minimum remains. Nevertheless, it may well occur that the relevant F–terms that lift the flat direction of the potential are not mass terms, but non-renormalizable terms, coming e.g. from a term \( \sim \lambda M^{-(n-3)} \phi^n \) \((n \geq 4)\) in the superpotential \( W \), where \( M \) is some large mass scale and we take for convenience \( \lambda = O(1) \). Then, schematically, the potential reads

\[
V = \frac{1}{2} g^2 \left[ \xi + q_\phi \phi \right]^2 + \frac{|\lambda|^2}{M^{2n-6}} |\phi|^{2n-2} .
\]

It is easy to check that in order to ensure that the non-renormalizable term (and not the quartic one) be responsible for the lifting of the flat direction of the potential, it is necessary to have \( \xi/M^2 > 1 \) (preferably much larger than 1). This is precisely the opposite to what one expects in these models, since non-renormalizable terms in SUGRA are likely to be suppressed by inverse powers of \( M_P \), and thus \( \xi/M^2 \ll 1 \). Besides, it is straightforward to check that this would yield \( |\langle \phi \rangle_{in}|^2 \simeq (-\xi/q_\phi) \) and a too small \( \langle V(\phi) \rangle_{in} \) for inflation.

The possibility c) arises if \( \phi \) is charged under extra gauge groups apart from the inflationary one, which is quite plausible. The extra D–terms would induce in general quadratic and quartic terms in \( \phi \). For instance, a quartic term \( \frac{1}{2} b |\phi|^4 \), with \( b = (g^2 q_\phi^2) \), together with the anomalous D–term (3), i.e.

\[
V = \frac{1}{2} g^2 \left[ \xi + q_\phi \phi \right]^2 + \frac{1}{2} b |\phi|^4
\]
would give $|\langle \phi \rangle_{\text{in}}|^2 \simeq -g^2 q_\phi \xi / (g^2 q_\phi^2 + b) < (-\xi / q_\phi)$, $\langle V \rangle_{\text{in}} = O(g^2 \xi^2)$, which is perfectly consistent with inflation. The potential problems here are that the flat direction of the potential is lifted by quartic terms, which is inescapable in this scenario, and the absence of A–terms at high energy scales. $B$ and $CP$ violating A–terms are required by the AD mechanism to drive the generation of $B$. These terms are absent in the scenario outlined above, but they can appear at $H \leq O(\text{TeV})$, when the low-energy soft breaking terms become relevant.

It is clear, therefore, that in D–inflationary scenarios, it is enough to have the AD field $\phi$ carry a charge of the appropriate sign under the $U(1)$ responsible for the inflation, in order to obtain an effective negative mass squared at $\phi=0$ during inflation (and, consequently, $\langle \phi \rangle_{\text{in}} \neq 0$). However, the remaining details for a successful AD mechanism (and D–inflation) are not so easy to arrange.

Nevertheless, there is a beautiful alternative to get AD baryogenesis. Namely, if $q_\phi = 0$ (i.e. the AD field(s) is (are) not charged under the inflationary $U(1)$), then $m_{\phi}^2 = 0$ during inflation. In this way there is a truly flat direction along $\phi$ and the AD mechanism can be implemented as originally considered by Affleck and Dine. This argument is only exact at tree level. Strictly speaking, there are small contributions to $m_\phi$ coming from higher loop corrections. These arise at two–loop or three–loop levels, depending on whether $\phi$ does or does not carry any charge in common with the $U(1)$ charged fields \([11]\). In addition, there are the expected $O(\text{TeV})$ low-energy supersymmetry breaking contributions. In any case, $\phi$ will acquire a large VEV during inflation due to quantum fluctuations, if the correlation length for de Sitter fluctuations $l_{\text{coh}} \simeq H^{-1} \exp(3H^2/2m_\phi^2)$ \([12]\), is large compared to the horizon size. In fact, the present length corresponding to $l_{\text{coh}}$, namely $l_{\text{coh}}(a_0/a_{\text{infl}})$ (where $(a_0/a_{\text{infl}})$ is the ratio of scale factors), should be larger than the horizon size at present, $c t_0$. Using $H = 10^{13}\text{GeV}$, $t_0 = 18$ sec and assuming a radiation dominated universe ($a \sim t^{1/2}$), one obtains the condition $H^2/m_\phi^2 \gtrsim 40$ \([11]\) ($\gtrsim 30$, if matter domination is assumed, since $a \sim t^{2/3}$), which is easily fulfilled in this context.

In conclusion, the AD mechanism can be implemented in a D–inflationary model and the preferred case is that of $q_\phi = 0$, i.e. when the AD field(s) is (are) not charged under the relevant D–symmetry. Then $m_\phi^2 \ll H^2$ and a large $\langle \phi \rangle_{\text{in}}$ is produced by quantum fluctuations. The AD mechanism can thus be implemented in the original, “old-fashioned”, way. This scenario illustrates the fact that scalar fields do not necessarily get effective masses squared (of either sign) of $O(H)$ during inflation. Essentially vanishing masses are also perfectly possible. Let us also emphasize that this scenario is quite insensitive to the SUGRA details, in particular to the form of the Kähler potential.
3 F–Inflation

In this section we will consider the case when inflation (and the corresponding SUSY breaking) is driven by a non-vanishing F–term of the appropriate size\(^4\). Concerning the implementation of the AD mechanism in F–inflationary scenarios, the main question (see Sect. 1) is whether it is possible to get an effective mass squared \(m_\phi^2 < 0\) or \(m_\phi^2 \simeq 0\) for the AD field, \(\phi\), during inflation. This will automatically yield \(\langle \phi \rangle_m \neq 0\), setting the onset for subsequent baryogenesis, as explained in the Sect. 1. To answer this question, already addressed in Ref.\(^{[6]}\), we need to examine the effective potential, \(V\), in a SUGRA theory. Neglecting the contribution of the D–terms (analyzed in the previous section) this is given by

\[
V = e^G \left( G_j K^{j\bar{j}} G_l - 3 \right) = F^{j\bar{k}} K_{j\bar{k}} F^j - 3e^G .
\] (6)

Here \(G = K + \log |W|^2\) where \(W\) is the superpotential, \(K^{j\bar{j}}\) is the inverse of the Kähler metric \(K_{j\bar{l}} \equiv \partial K/\partial z_j \partial \bar{z}_l\), \(z_j\) are the (scalar components) of the chiral superfields and \(F^j = e^{G/2} K^{j\bar{k}} G_{\bar{k}}\) are the corresponding auxiliary fields. During inflation

\[
\langle V \rangle_m = V_0 \simeq H^2 M_p^2,
\] (7)

which implies that some \(F\) fields are different form zero, thus breaking SUSY. The effective gravitino mass squared during the inflationary epoch is given by \(m_{3/2}^2 = e^G = e^K |W|^2\) in \(M_p\) units. Notice that \(V_0 = F^{j\bar{k}} K_{j\bar{k}} F^j - 3m_{3/2}^2\), so, unless there is some fine–tuning, \(m_{3/2}^2\) is at most of \(O(V_0)\), \(m_{3/2}^2 \leq O(V_0)\). The SUSY breakdown induces soft terms for all the scalars, in particular for \(\phi\). More precisely, the value of the effective mass squared, \(m_{\phi}^2\), is intimately related to the form of \(K\). (As we will see shortly, the form of \(W\) is relevant for higher order terms, but not for \(m_{\phi}^2\).) It is convenient to parametrize \(K\) as

\[
K = K_0(I) + K_{\phi\bar{\phi}}|\phi|^2 + \cdots ,
\] (8)

where \(I\) represents generically the inflaton or inflatons\(^5\).

Let us first show that it is impossible to get the desired result, \(m_\phi^2 < 0\) or \(m_\phi^2 \simeq 0\), if there is no mixing between \(\phi\) and \(I\) in the quadratic term of \(K\), i.e. if \(K_{\phi\bar{\phi}} \neq K_{\phi\bar{\phi}}(I)\). To simplify the argument let us first assume that \(W\) does not contain either effective mass terms for \(\phi\) or couplings between \(\phi\) and \(I\). Then, it is clear from (3) that a mass term \(m_{\phi}^2 |\phi|^2\) has two effective contributions

\[
e^{K_0(I)} |W|^2 \left\{ K_{\phi\bar{\phi}} |\phi|^2 \right\} + e^{K_0(I)} |W|^2 K_{\phi\bar{\phi}} |\phi|^2 \left\{ G_j K^{j\bar{l}} G_l - 3 \right\} ,
\] (9)

\(^4\)F–inflation has been disputed (see e.g. Ref.\(^{[8]}\)) because one naturally expects \(O(H)\) effective masses during inflation for all the scalars, which when applied to the inflaton itself spoils the necessary slow rollover. This is not necessarily true. One of the most interesting conclusions of this section and the following one is that very tiny masses are also possible, which makes F–inflation as attractive, at least, as D–inflation.

\(^5\)If there are several AD fields which are mixed in the kinetic term, \(K_{\phi\bar{\phi}}|\phi|^2\), we are free to choose a basis of \(\phi\)–fields where it becomes diagonal.
which lead to an effective mass squared

\[ m_\phi^2 = m_{3/2}^2 + V_0/M_P^2, \]  

(10)

for the canonically normalized field \( (K_{\phi\phi})^{1/2}\phi \). Hence \( m_\phi^2 \) is of \( O(H^2) \) and positive and, therefore, \( \langle \phi \rangle_{in} = 0 \) and the AD mechanism cannot be implemented.

Things may get more complicated if \( W \) contains terms leading to effective masses. Such terms would generically read

\[ \lambda \eta_1 \cdots \eta_m \phi^m, \]  

(11)

where \( \eta_j \) are fields that develop non-vanishing VEV’s, at least during inflation (the inflaton \( I \) may be one of them), and \( m = 1, 2 \). Let us continue to assume that the quadratic term of \( K \) does not depend on \( \eta \), i.e. \( K_{\phi\phi} \neq K_{\phi\phi}(\eta) \). Then, calling \( \langle \lambda \eta_1 \cdots \eta_m \rangle_{in} = M^{3-m} \), besides the previously obtained positive mass terms (\( \sim H^2|\phi|^2 \)), we get positive F–terms of order \( M^{6-2m}|\phi|^{2m-2}, M^{6-2m}|\langle \eta \rangle_{in}|^{-2}|\phi|^{2m} \), which for \( m = 1, 2 \) are large positive mass terms. Furthermore, there appear soft terms of order \( HM^{3-m}\phi^m \) which are effectively negative for an appropriate choice of the phases of the fields. For \( m = 2 \) these terms provide an effective negative mass squared contribution. However, it is unlikely that this would dominate over the previous positive contributions of order \( H^2|\phi|^2 \) and \( M^2|\phi|^2 \), since we expect either \( MH \lesssim M^2 \) or \( MH \lesssim H^2 \). Finally, for \( m = 1 \) the effective linear term \( \sim HM^2\phi \) guarantees \( \langle \phi \rangle_{in} \neq 0 \) (more precisely, \( \langle \phi \rangle_{in} \lesssim H \)). We notice, nevertheless, that in this case again the flat direction of the potential is lifted by inappropriate (quadratic in this case) terms \( \sim |\phi|^2 \). As mentioned in the previous sections, according to Ref.\[6\] this implies that the \( \phi \)–field does not follow the instantaneous minimum of \( V \) after inflation, which may spoil the AD mechanism\[6\].

It is clear, therefore, that a successful implementation of the AD mechanism in SUGRA theories and F–inflation scenarios requires a mixing in the quadratic term of \( K \) of the inflaton \( I \) and AD fields \( \phi \), i.e. \( K_{\phi\phi} = K_{\phi\phi}(I) \) in Eq.(8). Our next point is to show that this mixing should be remarkably strong. Let us consider the following simple scenario

\[ K = K_0(I) + |\phi|^2 + a|I|^2|\phi|^2, \]  

(12)

where \( a \) is some unspecified coupling. This is the simplest possible modification of minimal SUGRA to include the required mixing between \( I \) and \( \phi \). Precisely a term \( |I|^2|\phi|^2 \) in \( K \) was invoked in Ref.\[6\] to get negative mass terms for \( \phi \). Assuming for simplicity that there is a single inflaton \( I \) with \( |F_I| \neq 0 \), we get from (12)

\[ \langle V \rangle_{in} = \left( F_I^T \right) K_{II} \left( F_I^T \right)^* - 3e^{K_0}|W|^2 \]

\[ = e^{K_0} \left\{ (W_I + K_I W)K_{II}(W_I + K_I W)^* - 3|W|^2 \right\}. \]  

(13)

\[ ^6 \text{This scenario, although rather artificial, would deserve further analysis anyway since, contrary to the scenarios considered in Ref.\[6\], \langle \phi \rangle \text{ is not due to a negative quadratic term in the potential, but to a linear one.} \]
where $W_I = \partial W / \partial I$, $K_I = \partial K / \partial I$ and $W$ simply denotes $\langle W \rangle$. Then, from the general expression for $V$, Eq. (3), we extract the various relevant contributions to the $\phi$ soft terms

$$
e^{K_0 + K_\phi |\phi|^2} \left\{ (W_I + K_I W) K^{II} (W_I + K_I W)^* - 3 |W|^2 \right\},$$

$$e^{K_0} \left\{ |W_\phi + K_\phi W|^2 K^{\phi\phi} - 3 (W^* W(\phi) + \text{h.c.}) \right\},$$

$$e^{K_0} \left\{ (W_\phi + K_\phi W) K^{I\phi} (W_I + K_I W)^* + \text{h.c.} \right\}. \quad (14)$$

The terms proportional to $W(\phi)$ give $A$-terms of $O(H)$ in the $\phi$ potential, while the terms proportional to $|\phi|^2$ give the effective mass squared $m^2_\phi$. For instance, if $I$ is a canonically normalized field up to $O(|\phi|^2)$ contributions, i.e. if $(K_0)_{II} = 1$, then

$$m^2_\phi = (1 + a |I|^2) \left[ |F|^2 \left( 1 - \frac{a}{(1 + a |I|^2)^2} \right) - 2m^2_{3/2} \right], \quad (15)$$

where the requirements of a positive kinetic energy for $\phi$ and of a positive cosmological constant $V_0$ yield, respectively, $1 + a |I|^2 > 0$ and $|F|^2 > 3m^2_{3/2}$. From (15) it is easy to determine the required value of $a$ in order to get $m^2_\phi \leq 0$. In particular, for $|I|^2 > 3/4$ (in Planck units) there is no value of $a$ for which $m^2_\phi \leq 0$. This can actually be frequently the case, since one expects the inflaton to get VEV’s of order $M_P$ during the inflationary process. For $|I|^2 \ll 1$, instead, we can obtain negative masses squared $m^2_\phi \leq 0$ if $a > 1/3$. This quite strong mixing between the inflaton and the AD field is not at all trivial to obtain in models. Still (as explained in Sect. 1), one has to assume the existence of F-contributions (preferably from non-renormalizable operators) that eventually lift the flat direction of $V$ for large values of $\phi$, which is not a strong requirement on models.

To summarize the results of this section, in order to obtain $m^2_\phi \leq 0$ during inflation, a strong mixing between the inflaton and $\phi$ in the quadratic term $(\propto |\phi|^2)$ of $K$ is required. This is quite a strong constraint for SUGRA scenarios, which excludes, in particular, minimal SUGRA.

On the other hand, the other desired possibility, that of a very small mass $m^2_\phi \sim 0$ (also welcome for a successful inflation, as mentioned in footnote 4) does not seem natural at first sight. However, the study of the SUGRA scenarios coming from strings provides beautiful surprises in this sense, as we are about to see.

### 4 String Scenarios

Undoubtedly, the best motivated SUGRA scenarios are those coming from string theories, which represent our best candidate for a fundamental theory. Here we examine the capability of string scenarios to implement AD baryogenesis.

As mentioned in Sect. 2 the D–inflation case is quite insensitive to the details of the particular SUGRA at hand. Correspondingly, all the results obtained there can
be translated integrally to string scenarios. Our only additional comment is that the existence of an anomalous $U(1)$, which seems to facilitate the possibility of D–inflation, is very common in string models, especially in the most realistic ones [10, 13].

Concerning F–inflation scenarios, string models do present special characteristics since the Kähler potential, $K$, which plays a crucial role, is greatly constrained. We have seen in the previous section that the implementation of the AD mechanism in this instance poses important restrictions on the form of $K$. So, we should first wonder whether the string Kähler potentials are able at all to accommodate AD baryogenesis. If the answer is positive, we should then ask what particular string models are favoured (or excluded) by this requirement.

In order to be concrete we will consider orbifold constructions, which are very well known string models and, besides, are extremely interesting for phenomenology. However, as it will become clear, most of the conclusions are completely general. The corresponding (tree–level) Kähler potential is given by [14]

$$K = - \log(S + \bar{S}) - 3 \log(T + \bar{T}) + \sum_j (T + \bar{T})^{n_j} |z_j|^2. \quad (16)$$

Here $S$ is the dilaton and $T$ denotes generically the moduli fields, $z_j$ are the chiral fields and $n_j$ the corresponding modular weights. The latter depend on the type of orbifold considered and the twisted sector to which the field belongs. The possible values of $n_j$ are $n_j = -1, -2, -3, -4, -5$. The discrete character of $n_j$ will play a relevant role later on.

Since a strong mixing between the inflaton and the AD field $\phi$ in the quadratic term ($\propto |\phi|^2$) of $K$ is required (see Sect. 3), our first conclusion is that $T$ is the natural inflaton candidate in string theories.

This is a strong conclusion, and a very satisfactory one, since $T$ and also $S$ are, in fact, very suitable candidates for inflatons for other reasons. Namely, $S$ and $T$ are present in all string constructions (at least in the perturbative approach) and their interactions with observable sector fields are gravitationally suppressed, as is expected for the inflaton. In addition, $S$ and $T$ have perturbatively flat potentials, which is appropriate for slow rollover. Finally, we expect $\langle S \rangle, \langle T \rangle \simeq O(M_P)$ at low energy, so some phase–transition-like process is expected for the $S$ and $T$ fields, which could well trigger inflation. It should be noted that all the comments in this paragraph are general for all string constructions.

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7Perturbative corrections to Eq.(16) are known at one-loop level [14] and are small, so they do not affect any of the results presented here. On the other hand, non-perturbative corrections are very poorly known (see e.g. Ref. [15] for an analysis of their possible phenomenological significance).

8Eq.(16) is written with the usual simplification of considering a single “overall modulus” $T$.

9This can also be the origin of a cosmological moduli (Polonyi) problem [16], but this is out of the scope of the present work.

10This comes from the fact that $\langle S \rangle$ and $\langle T \rangle$ have precise physical meanings. Namely, $\langle S \rangle$ is the value of the unified gauge coupling constant and $\langle T \rangle$ is the squared radius of the compactified space, both in Planck units.
We should recall, however, that a strong mixing in $K$ is a necessary but not sufficient condition for $m_\phi^2 \leq 0$. We must then examine the precise value of $m_\phi^2$ in the presence of a non-vanishing cosmological constant $\langle V \rangle_{in} = V_0 > 0$. If inflation is driven by the $T$ (and/or $S$) fields, we expect non-vanishing $T$ (and/or $S$) $F$–terms during inflation. Then, the corresponding soft terms, in particular mass terms, for $\phi$ are straightforwardly extracted from eqs.(6, 16). This was precisely the sort of scenario considered in Refs.[17, 18]. Although the motivation of these works was different, namely to study the form of the soft breaking terms at low–energy with $m_{3/2} = O(1 \text{ TeV})$, their results are applicable here. The only difference here is that the scale of the breaking is much higher and the non-vanishing cosmological constant, $V_0 > 0$, plays a major role. In particular, the effective $\phi$ mass squared, $m_\phi^2$, which is especially relevant in our context, is given by [18]

$$m_\phi^2 = m_{3/2}^2 \left\{ (3 + n_\phi \cos^2 \theta)C^2 - 2 \right\},$$

(17)

where $m_{3/2}^2 = e^K|W|^2$, $\tan^2 \theta = (K_{SS}/K_{TT}) \left| F_S/F_T \right|^2$, $C^2 = 1 + [V_0/(3M^2_P m_{3/2}^2)]$ and $n_\phi$ is the modular weight of the AD field $\phi$. Notice that $C^2 > 1$ and the condition for $m_\phi^2 \leq 0$ reads

$$n_\phi \leq \frac{1}{\cos^2 \theta} \left( \frac{2}{C^2 - 3} \right).$$

(18)

From (18) we conclude the following.

i) If $\cos^2 \theta = 0$ ($S$–driven inflation), then $m_\phi^2 = m_{3/2}^2 + V_0$, as expected. Thus, as we already concluded, “$S$–inflation” is excluded by AD baryogenesis.

ii) If $\cos^2 \theta = 1$ ($T$–driven inflation), then $m_\phi^2 \leq 0$ is perfectly possible. In particular, it certainly occurs if $n_\phi \leq -3$. In fact, states with $n_\phi \leq -3$ occur in all the orbifold constructions, so $m_\phi^2 \leq 0$ is perfectly natural in this context.

The previous conclusions i) and ii) have been obtained in the context of orbifolds, but are, in fact, much more general. In particular, conclusion i) is completely general since the tree–level dependence of $K$ on $S$ is universal in all string constructions. Conclusion ii) is also very general since the coupling of the chiral fields to the moduli in $K$ is basically determined by modular invariance. We should mention here that generic Calabi–Yau compactifications with large radius ($T$) have Kähler potentials as in eq.(16), but with all the fields in the untwisted sector ($n = -1$), so $m_\phi^2 > 0$ from (17). Therefore, those constructions are not favorable for AD baryogenesis. If the Calabi–Yau compactification is not in the large radius limit, the corresponding expression for $K$ is more involved and depends on the type of Calabi–Yau compactification, so it would deserve a separate analysis.

From i) and ii) we conclude that $T$–inflation is required for AD baryogenesis (a mix of $S$ and $T$–inflation could also work). Hence, we will take in what follows

$$\cos^2 \theta = 1.$$  

(19)
In this context we examine next, two particularly interesting limits that could well be realized in practice. Namely, since \( V_0 = K_{TT} |F^T|^2 - 3m_{3/2}^2 = 3H^2 M_P^2 \), it may perfectly happen that \( K_{TT} |F^T|^2 \gg m_{3/2}^2 \), and thus \( C^2 \gg 1 \) (see definition of \( C^2 \) after eq.([17])). Another possibility is \( K_{TT} |F^T|^2 = O(m_{3/2}^2) \), and thus \( C^2 = O(1) \). Let us analyze the two limits separately.

- If \( C^2 \gg 1 \), then, from eq.([17])
  \[
m_{\phi}^2 \simeq H^2 (3 + n_{\phi}).
  \]  
  Hence for \( n_{\phi} = -3 \) we get \( m_{\phi}^2 \simeq 0 \). So, we see that the possibility of a very small mass \( m_{\phi}^2 \simeq 0 \) can occur in F–inflation, as it was the case in D–inflation. Notice that there is no fine-tuning here, since \( n_{\phi} \) is a discrete number which can only take the values \( n_{\phi} = -1, -2, -3, -4, -5 \).

  This is also good news for F–inflation itself. As has been pointed out in the literature (see footnote 4), F–inflation has the problem that if the inflaton mass is \( O(H) \), as expected at first sight during inflation, then the necessary slow rollover is disabled. We see here, however, that a hybrid–inflation scenario [19] in which \( T \) is the field responsible for the large \( V_0 \) and a second field (any one with \( n = -3 \)) is responsible for the slow rollover is perfectly viable.

- If \( C^2 = O(1) \), the probability of obtaining \( m_{\phi}^2 < 0 \) is larger. In particular, it is clear from ([17]) that if \( C^2 \leq 2 \), then \( m_{\phi}^2 \leq 0 \) whenever \( n_{\phi} \leq -2 \), which is a very common case.

## 5 Conclusions

Affleck–Dine baryogenesis requires the production of large expectation values for certain scalar fields (the AD fields) at early (inflationary) times. This is only possible if the effective soft masses squared coming from the inflationary breaking of SUSY are negative or very small. This poses strong constraints on the type of SUGRA theory. If inflation is driven by a D–term (D–inflation), the most favoured (and almost unique) scenario occurs if the AD fields, \( \phi \), are not charged under the gauge group of the relevant D–term. Then, \( m_{\phi}^2 \simeq 0 \) and the AD mechanism occurs as originally considered by Affleck and Dine. This possibility had been considered implausible in the recent literature. If inflation is driven by an F–term (F–inflation), then \( m_{\phi}^2 \leq 0 \) requires a strong mixing of \( \phi \) and the inflaton field in the quadratic term (\( \sim |\phi|^2 \)) of the Kähler potential. This (necessary but not sufficient) condition is not trivial and excludes in particular minimal SUGRA. Amazingly, string-based SUGRA theories satisfy the conditions for AD baryogenesis in a beautiful way, provided that inflation is driven by a modulus \( (T) \) field. Then \( m_{\phi}^2 \leq 0 \) for certain values of the AD field (in particular \( n_{\phi} \leq -3 \) always works). In addition, the possibility of \( m_{\phi}^2 \simeq 0 \) can also appear in a natural way, with no fine-tuning at all, thanks to the discrete character of \( n_{\phi} \). This is a nice
result which also shows, as explained after Eq. (20), that $T-$ hybrid inflation is viable in string-based SUGRA scenarios.

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