UNEXPECTED GOINGS-ON IN THE STRUCTURE OF A NEUTRON STAR CRUST

Aurel Bulgac*, Paul-Henri Heenen†, Piotr Magierski**, Andreas Wirzba‡ and Yongle Yu*

*Department of Physics, University of Washington, Seattle WA 98195-1560, USA
†Service de Physique Nucléaire Théorique, Université Libre de Bruxelles, B 1050, Brussels, Belgium
**Faculty of Physics, Warsaw University of Technology, ul. Koszykowa 75, 00-662, Warsaw, Poland
‡Helmholtz-Institut für Strahlen- und Kernphysik, Universität Bonn, D-53115 Bonn, Germany

Abstract.
We present a brief account of two phenomena taking place in a neutron star crust: the Fermionic Casimir effect and the major density depletion of the cores of the superfluid neutron vortices.

FERMIONIC CASIMIR EFFECT AND NEUTRON STAR CRUST

At a depth of about 500 m or so below the surface of a neutron crust the nuclear matter (which consists mostly of neutrons plus a small percentage of protons and electrons in β-equilibrium) organize themselves in some exotic inhomogeneous solid phase [1]. As a matter of fact, neutron star crusts seem to be just about the only other places in the entire Universe, apart from planets, where one can find condensed matter, in particular a solid phase [2]. Moving from the neutron star surface inward, one finds at first a Coulomb crystal lattice of nuclei immersed in a very low density neutron gas and even lower density electron gas. With increasing depth, the density and pressure increase, the nuclei get closer to each other and start evolving into some unusual elongated nuclei, which eventually become rods. These nuclear rods evolve gradually into plates, their place being taken later by tubes and bubbles (dubbed “inside out” nuclei) just before the average density becomes almost equal to the nuclear saturation density and the entire mixture of neutrons, protons and electrons become an homogeneous phase. The properties of this part of the neutron star have been the subject of a lot of studies, see Refs. [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] and other references therein. Most of these approaches however have missed a rather subtle and apparently important physical phenomenon, the fermionic counterpart of the Casimir interaction in such a medium [10, 11, 12, 13, 14].

In order to quickly explain the main physics ideas behind this new phenomenon, let us consider an over-simplified model of the neutron star crust. One can ask the rather innocuous question: “What is the ground state energy of an infinite homogeneous Fermi sea of noninteracting neutral particles with two hard spheres of radii a, separated by a distance r?” The naive and somewhat startling answer that perhaps one can place the two hard spheres almost anywhere with respect to each other and that the energy
of the system will not be affected if one were to move the hard spheres around. The “theoretical argument” which can lead to such a conclusion is based on the same type of argumentation, which was used in Refs. [1, 5, 6, 7] and allowed these authors to establish that by going deeper and deeper into the interior of the neutron star one finds a well defined sequence of “exotic” nuclear shapes. This traditional argumentation is based essentially on liquid drop model, which includes the volume, surface, Coulomb contributions to the ground state energy only. This is basically “classical thinking.” For a person using “quantum reasoning” instead, the fact that the ground state of such a system in infinitely degenerate (corresponding to an arbitrary relative arrangement of the two hard spheres) will find such an answer most likely wrong. An indeed, a careful analysis of the problem reveals the fact that indeed a system of two hard spheres, immersed in an infinite Fermi sea of non-interacting particles at zero temperature has a well defined ground state. The correct answer, namely that the “interaction energy” of the two hard spheres of radius $R$, at distance $r$ from each other, is somewhat even more surprising. One finds that

$$E_C \approx -\frac{\hbar^2 k_F^2}{m} \frac{R^2}{2\pi r(2R)} j_1[2k_F(r-2R)],$$

where $j_1(x)$ is the spherical Bessel function, $k_F$ is the Fermi momentum and $m$ is the fermion mass. “Why would this “interaction energy” be a non-monotonic function of the hard sphere separation $r$?” and, moreover, “How does interaction really emerges here, where one starts with such a simple system of non-interacting particles?” As one soon “discovers” the “culprit” is the wave character of the Quantum Mechanics really. Fermions even at zero temperature do not stop moving and the space is really “filled” with an infinite number of de Broglie’s waves. These waves reflect from the two hard spheres and as in the case of any wind musical instrument, for some frequencies one would have a favorable wave interference while for other frequencies there will not such a favorable interference. In an infinite Fermi sea there is an infinite number of waves with all frequencies ranging from zero to the Fermi frequency. If one carefully adds up the effects of all these waves one readily arrives at the result above [10, 11]. Things get a little bit more complicated when one adds more hard spheres, as then one naturally discovers that besides the “natural” two-body interactions there are genuine three- and four- and many-body interactions among these spheres. Moreover, there is absolutely no reason why not consider other type of objects, which could be immersed in this Fermi sea, like “logs” and “boards” and in principle almost anything else. Surprisingly all these combinations of various objects in various arrangements can be analyzed rather easily. What is surprising however is the fact that the characteristic interaction energy between such objects is of the same order as the energy differences between various phases in a neutron star crust [10, 12, 13, 14] and when taken into account this fermionic Casimir energy can in “ruin perfect crystalline structures” found in all previous studies. These conclusions have been backed by more sophisticated fully microscopic calculations of the nuclear matter in a neutron star crust [8, 9].

Instead of describing in more detail results which have been published already, we shall instead draw the attention of our readers here to another element which was overlooked in studies of the neutron star crust, and which is apparently going to influence a great deal of properties. In order to analyze the thermal and electric conductivities of
the crust, which are important for understanding of the thermal evolution of neutron stars one has to go beyond the static approximation. The “nuclei” which are immersed in the neutron fluid, which indeed is a superfluid, can and do move. As with boats on a lake, when they start moving they make waves and one has to include the dynamics of the surrounding superfluid in any analysis. We shall limit ourselves here to quoting a single result, namely the kinetic energy of two penetrable spheres located at the distance \( r \), immersed in a superfluid at velocities below the critical velocity for the loss of superfluidity. One then finds [18] that the kinetic energy of two such spheres becomes:

\[
T = \frac{1}{2} (M_1^{\text{ren}}u_1^2 + M_2^{\text{ren}}u_2^2) + 4\pi m\rho_{\text{out}} \left( \frac{1 - \gamma}{2\gamma + 1} \right)^2 \left( \frac{R_1 R_2}{r} \right)^3 \left[ \vec{u}_1 \cdot \vec{u}_2 - \frac{3}{r^2} (\vec{u}_1 \cdot \vec{r}) (\vec{u}_2 \cdot \vec{r}) \right]
\]

where the renormalized masses of nuclei have the form:

\[
M_i^{\text{ren}} = \frac{4}{3} m\rho_{\text{in}} \pi R_i^3 \left( \frac{1 - \gamma}{2\gamma + 1} \right)^2 = M_i \left( \frac{1 - \gamma}{2\gamma + 1} \right)^2,
\]

where \( \vec{u}_i \) are the velocities of the two nuclei, \( i = 1, 2 \) and \( M_i \) and \( R_i \) denote the nuclear bare mass and radii of the \( i \)-th nucleus, \( \gamma = \rho_{\text{out}}/\rho_{\text{in}} \) and \( \rho_{\text{in, out}} \) are the densities inside and outside the two nuclei. The somewhat unexpected cross term appearing above shows that the existence of mere motion of the two objects in a perfect fluid can lead to a velocity-dependent interaction, which decays with the separation as slows as the static Casimir Fermionic energy, namely as \( 1/r^3 \). Further analysis shows that this velocity dependent-interaction is important as well when considering dynamical properties of neutron star crust [19].

THE SPATIAL STRUCTURE OF A VORTEX IN LOW-DENSITY SUPERFLUID NEUTRON MATTER

There is a long held belief that vortices in Fermi systems do not show any appreciable normal density variations and that only the anomalous density vanishes along the vortex axis, similarly to the behavior of the density (which is the order parameter) in Bose systems [20, 21, 22]. Thus it came as somewhat of a surprise the fact that in Fermi systems one can have a spatial structure of a vortex with a significant normal density depletion along the vortex axis [23, 24, 25]. What happens in low density superfluid neutron matter for example is the following. The magnitude of the pairing gap becomes comparable with the Fermi energy.

The possibility that the value of the superfluid gap can attain large values was raised more than two decades ago in connection with the BCS \( \rightarrow \) BEC crossover [29, 30]. One can imagine that one can increase the strength of the two–particle interaction in such a manner that at some point a real two–bound state forms, and in that case \( a \rightarrow -\infty \). By continuing to increase the strength of the two–particle interaction, the scattering length becomes positive and starts decreasing. A dilute system of fermions, when \( \rho r_0^3 \ll 1 \) (here \( r_0 \) is the interaction radius), will thus undergo a transition from a weakly coupled BCS system, when \( a < 0 \) and \( a = \Theta(r_0) \), to a BEC system of tightly bound Fermion
pairs, when \( a > 0 \) and \( a \Rightarrow \mathcal{O}(r_0) \) again. In the weakly coupled BCS limit the size of the Cooper pair is given by the so-called coherence length \( \xi \approx \frac{\hbar^2 |k_F|}{m} \), which is much larger than the inter-particle separation \( \approx \lambda_F = 2\pi / k_F \). In the opposite limit, when \( k_F a \ll 1 \) and \( a > 0 \), and when tightly bound pairs/dimers of size \( a \) are formed, the dimers are widely separated from one another. Surprisingly, these dimers also repel each other with an estimated scattering length \( \approx 0...2a \) [27, 28] and thus the BEC phase is also (meta)stable. The bulk of the theoretical analysis in the intermediate region where \( k_F |a| > 1 \) was based on the BCS formalism [27, 29, 30, 31] and thus is highly questionable. Even the simplest polarization corrections have not been included into this type of analysis so far. In particular, it is well known that in the low density region, where \( a < 0 \) and \( k_F |a| \ll 1 \) the polarization corrections to the BCS equations lead to a noticeable reduction of the gap [26]. Only a truly \textit{ab initio} calculation could really describe the structure of a many Fermion system with \( k_F |a| \gg 1 \). In the limit \( a = \pm \infty \), when the two-body bound state has exactly zero energy, and if \( k_F r_0 \ll 1 \), one can expect that the energy per particle of the system is proportional to \( \varepsilon_F = \frac{\hbar^2 k_F^2}{2m} \), as it was recently confirmed by the variational calculations of Refs. [32, 33]. The normal density at the vortex core is lowered, while the pairing field vanishes at the vortex axis as expected. In hindsight this result could have been expected. Large values of the pairing field correspond to the formation of atom pairs/dimers of relatively small sizes. When these dimers are relatively strongly bound and when they are also widely separated from one another, they undergo a Bose–Einstein condensation. For a vortex state in a 100% BEC system the density at the vortex axis vanishes identically. Therefore, by increasing the strength of the two–particle interaction, the Fermion system simply approaches more and more an ideal BEC system, for which a density depletion of the vortex core is expected.

Almost thirty years ago Anderson and Itoh [2] put forward the idea that vortices should appear in neutron stars and that they can also get pinned to the solid crust. They argued that the “star–quakes,” observable on Earth as pulsar “glitches,” apparently are caused by the vortex de–pinning in neutron star crust. This idea and its various implications have been examined by numerous authors, see Refs. [21, 24] and further references therein, but a general consensus does not seem to have emerged so far.

The profile of a vortex in neutron matter is typically determined using a Ginzburg–Landau equation, which is expected to give mostly a qualitative picture and its accuracy is difficult to estimate. Surprisingly, prior to Ref. [23] there exists only one microscopic calculation of a vortex in low density neutron matter [22]. The existence of a strong density depletion in the vortex core is going to affect appreciably the energetics of a neutron star crust. One can obtain a gross estimate of the pinning energy of a vortex on a nucleus as \( E_{\text{pin}}^V = [\varepsilon(\rho_{out})\rho_{out} - \varepsilon(\rho_{in})\rho_{in}]V \), where \( \varepsilon(\rho) \) is the energy per particle at density \( \rho \), \( \rho_{in} \) and \( \rho_{out} \) are the densities inside and outside the vortex core and \( V \) is the volume of the nucleus. Naturally, this simple formula does not take into account a number of factors, in particular surface effects and the changes in the velocity profile and the pairing field. These last contributions were accounted for (with some variations) in the past [2, 21]. However, if the density inside the vortex core and outside differ significantly one expects \( E_{\text{pin}}^V \) to be the dominant contribution. In the low density region, where \( \varepsilon(\rho_{out})\rho_{out}/\varepsilon(\rho_{in})\rho_{in} \) is largest, one expects a particularly large
anti–pinning effect \( (E_{\text{pin}}^V > 0) \). The energy per unit length of a simple vortex is expected to be significantly lowered when compared with previous estimates \[\approx [\varepsilon(\rho_{\text{out}})\rho_{\text{out}} - \varepsilon(\rho_{\text{in}})\rho_{\text{in}}]\pi R^2, \]
where \( R \) is an approximate core radius.

REFERENCES

1. G. Baym, H.A. Bethe, C.J. Pethick, Nucl. Phys. A175 225 (1971).
2. P.W. Anderson and N. Itoh, Nature, 256, 25 (1975).
3. J.W. Negele and D. Vautherin, Nucl. Phys. A207, 298 (1973).
4. P. Bonche and D. Vautherin, Nucl. Phys. A372 496 (1981); Astron. Astrophys. 112 268 (1982).
5. C.J. Pethick and D.G. Ravenhall, Annu. Rev. Nucl. Part. Sci. 45 429 (1995).
6. F. Douchin, P. Haensel, J. Meyer, Nucl. Phys. A 665 419 (2000).
7. F. Douchin, P. Haensel, Phys. Lett. B485 107 (2000).
8. P. Magierski, P.-H. Heenen, Phys. Rev. C65 045804 (2002).
9. P. Magierski, A. Bulgac, P.-H. Heenen, Int. J. Mod. Phys. A17 1059 (2002).
10. A. Bulgac, P. Magierski, Nucl. Phys. A683 695 (2001); Erratum: Nucl. Phys. A703 892 (2002).
11. A. Bulgac and A. Wirzba, Phys. Rev Lett. 87, 120404 (2001).
12. A. Bulgac, P. Magierski, Phys. Scripta T90 150 (2001).
13. A. Bulgac, P. Magierski, Acta Phys. Pol. B32 1099 (2001).
14. P. Magierski, A. Bulgac, Acta Phys. Pol. B32 2713 (2001).
15. G. Watanabe, K. Sato, K. Yasuoka, T. Ebisuzaki, Phys. Rev. C 66, 012801 (2002); Phys. Rev. C 68, 035806 (2003).
16. C. J. Pethick and A. Y. Potekhin, Phys. Lett. B 427, 7 (1998).
17. P.B. Jones, Phys. Rev. Lett. 83, 3589 (1999).
18. P. Magierski and A. Bulgac, preprint Nuclear Hydrodynamics in the Inner Crust of Neutron Stars, to appear in Acta Phys. Pol. B.
19. P. Magierski, preprint In-medium ion mass renormalization and lattice vibrations in the neutron star crust, to appear in Int. J. Mod. Phys. E.
20. P.G. de Gennes, Superconductivity of Metals and Alloys, Addison–Wesley, Reading MA, (1998); F. Gygi and M. Schlüter, Phys. Rev. B 43, 7609 (1991); P.I. Soininen et al., Phys. Rev. B 50, 13883 (1994); N. Hayashi, et al., Phys. Rev. Lett. 80, 2921 (1998); M. Franz and Z. Tešanović, Phys. Rev. Lett. 80, 4763 (1998); N. Nygaard, et al., Phys. Rev. Lett. 90, 210402 (2003).
21. P.W. Anderson, M.A. Alpar, D. Pines and J. Shaham, Phil. Mag. 45, 227 (1982); M.A. Alpar, P.W. Anderson, D. Pines and J. Shaham, Ap. J. 278, 791 (1984); M.A. Alpar, K.S. Cheng and D. Pines, Ap. J. 346, 823 (1989); R.I. Epstein and G. Baym, Ap. J. 328, 680 (1988); R.I. Epstein and G. Baym, Ap. J. 387, 276 (1992); B. Link, R.I. Epstein and G. Baym, Ap. J. 403, 285 (1993); P.B. Jones, Phys. Rev. Lett. 79, 792 (1997); ibid 81, 4560 (1998); ibid Mon. Not. R. Astron. Soc. 257, 501 (1992); ibid Mon. Not. R. Astron. Soc. 296, 217 (1998); P.M. Pizzochero, L. Viverit and R.A. Broglia, Phys. Rev. Lett. 79, 3347 (1997).
22. F.V. De Blasio and Ø. Elgarøy, Phys. Rev. Lett. 82, 1815 (1999); Ø. Elgarøy and F.V. De Blasio, A&A, 370, 939 (2001).
23. Y. Yu and A. Bulgac, Phys. Rev. Lett. 90, 161101 (2003).
24. P. Donati and P.M. Pizzochero, Phys. Rev. Lett. 90, 211101 (2003).
25. A. Bulgac and Y. Yu, cond-mat/0302325, Phys. Rev. Lett. 91, in press (2003).
26. L.P. Gorkov and T.K. Melik–Barkhudarov, Zh. Eksp. Teor. Fiz. 40, 1452 (1961) [Sov. Phys. JETP 13, 1018 (1961)]; H. Heiselberg, et al., Phys. Rev. Lett. 85, 2418 (2000).
27. M. Randeria, in Bose–Einstein Condensation, eds. A. Griffin, et al., Cambridge Univ. Press (1995), pp 355–392.
28. P. Pieri and G.C. Strinati, Phys. Rev. B 61, 15370 (2000) and cond-mat/0307421, D.S. Petrov, C. Salomon, G.V. Shlyapnikov, cond-mat/0309010.
29. A.J. Leggett, in Modern Trends in the Theory of Condensed Matter, eds. A. Pekalski and R. Przystawa, Springer–Verlag, Berlin, 1980; J. Phys. (Paris) Colloq. 41, C7–19 (1980).
30. P. Nozières and S. Schmitt–Rink, J. Low Temp. Phys. 59, 195 (1985).
31. C.A.R. Sá de Mello, et al., Phys. Rev. Lett. 71, 3202 (1993); J.R. Engelbrecht, et al., Phys. Rev. B 55, 15153 (1997).
32. J. Carlson, et al., [nucl-th/0302041](http://arxiv.org/abs/nucl-th/0302041).
33. J. Carlson, et al., Phys. Rev. Lett. 91, 050401 (2003).