Confirmed Width–$E_{\text{iso}}$ and Width–$L_{\text{iso}}$ Relations in Gamma-Ray Bursts: Comparison with the Amati and Yonetoku Relations

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Abstract

The well-known Amati and Yonetoku relations in gamma-ray bursts show strong correlations between the rest-frame $v_{fi}$ spectrum peak energy, $E_{p,i}$, and the isotropic energy, $E_{\text{iso}}$, as well as isotropic peak luminosity, $L_{\text{iso}}$. Recently, Peng et al. showed that the cosmological rest-frame spectral widths are also correlated with $E_{\text{iso}}$ and with $L_{\text{iso}}$. In this paper, we select a sample including 141 BEST time-integrated F spectra and 145 BEST peak flux P spectra observed by Konus–Wind with known redshift to recheck the connection between the spectral width and $E_{\text{iso}}$ as well as $L_{\text{iso}}$. We define six types of absolute spectral widths as the differences between the upper ($E_{2}$) and lower energy bounds ($E_{1}$) of the full width at 50%, 75%, 85%, 90%, 95%, and 99% of maximum of the $EF_{i}$ versus E spectra. It is found that all of the rest-frame absolute spectral widths are strongly positively correlated with $E_{\text{iso}}$ as well as $L_{\text{iso}}$ for the long burst for both the F and P spectra. All of the short bursts are outliers for the width–$E_{\text{iso}}$ relation, and most of the short bursts are consistent with the long bursts for the width–$L_{\text{iso}}$ relation for both F and P spectra. Moreover, all of the location energies, $E_{2}$ and $E_{1}$, corresponding to various spectral widths, are also positively correlated with $E_{\text{iso}}$ as well as $L_{\text{iso}}$. We compare all of the relations with the Amati and Yonetoku relations and find that the width–$E_{\text{iso}}$ and width–$L_{\text{iso}}$ relations, when the widths are at about 90% maximum of the $EF_{i}$ spectra, almost overlap with the Amati relation and the Yonetoku relation, respectively. The correlations of $E_{2} - E_{\text{iso}}$, $E_{1} - E_{\text{iso}}$ and $E_{2} - L_{\text{iso}}$, $E_{1} - L_{\text{iso}}$ when the location energies are at 99% of maximum of the $EF_{i}$ spectra are very close to the Amati and Yonetoku relations, respectively. Therefore, we confirm the existence of tight width–$E_{\text{iso}}$ and width–$L_{\text{iso}}$ relations for long bursts. We further show that the spectral shape is indeed related to $E_{\text{iso}}$ and $L_{\text{iso}}$. The Amati and Yonetoku relations are not necessarily the best relationships for relating the energy to the $E_{\text{iso}}$ and $L_{\text{iso}}$. They may be special cases of the width–$E_{\text{iso}}$ and width–$L_{\text{iso}}$ relations or the energy–$E_{\text{iso}}$ and energy–$L_{\text{iso}}$ relations.

Unified Astronomy Thesaurus concepts: Gamma-ray bursts (629)

1. Introduction

Thanks to their extreme brightness, gamma-ray bursts (GRBs) are thought to be the most energetic explosions in the universe. The emission mechanisms of the GRB prompt emission are still unresolved. One of the methods is to analyze the GRB prompt emission, especially for the spectroscopy, which can provide valuable clues to the underlying processes giving rise to the phenomenon. Until now, no physical model has been demonstrated to systematically fit the observed spectra, though some attempts to fit physical models to the observational data have been made (e.g., Titarchuk et al. 2012; Ahlgren et al. 2019a, 2019b; Oganesyan et al. 2019; Burgess et al. 2020). Instead, a single simple empirical Band model (Band et al. 1993) or the cutoff power-law (CPL) model can well represent these GRB spectra because the overall spectral shape is similar. By performing the time-resolved spectral analysis, it was found that the prompt emission undergoes significant spectral evolution (e.g., Preece et al. 2000; Kaneko et al. 2006; Peng et al. 2009).

Most researchers still used the empirical Band or CPL model to describe the time-integrated or time-resolved spectrum. They also found some interesting correlations (spectrum–energy correlations) between the model parameters and the energy-related physical parameters of the overall GRB. For example, the well-known Amati relation shows that the intrinsic peak energy $E_{p,i}$ in the $v_{fi}$ spectra correlates with the isotropic energy ($E_{\text{iso}}$; Amati et al. 2002). Moreover, $E_{p,i}$ is also correlated with the isotropic peak luminosity ($L_{\text{iso}}$; Yonetoku relation; Yonetoku et al. 2004), as well as the collimation-corrected energy ($E_{\gamma}$; Ghirlanda relation; Ghirlanda et al. 2004).

Traditionally, GRBs can be roughly divided into short- and long-duration classes based on $T_{90}$, where all of the bursts are likely to be separated at about 2 s (Kouveliotou et al. 1993). In fact, $T_{90}$ depends on the energy range used for the calculation, and it is highly affected by the number of selection effects (e.g., Minaev & Pozanenko 2020). Therefore, some authors derived burst types using I (merger origin), II (collapsar origin), or I/II (the type is uncertain; Horváth et al. 2010) categories. A method similar to Horváth et al. (2010) is put forward by Svinkin et al. (2016). They divide the burst types as I > 0.9-Type I, 0.1 < I < 0.9-Type II, or I < 0.1-Type II based on the short/hard burst indicator function $I(T_{50}, HR_{32})$ (see Equation (5) in Horváth et al. 2010). For different burst classes, the spectrum–energy correlation properties may be different. It is shown that short burst classes do not follow the Amati relation (e.g., Qin & Chen 2013). Until now, the nature of the Amati correlation is still unclear. Some authors think it may be connected with selection effects (e.g., Band & Preece 2005). More authors claimed that the $E_{p,i}$–$E_{\text{iso}}$ or $E_{p,i}$–$E_{\gamma}$ correlations were not significantly affected by the selection effects.
Recently, Axelsson & Borgonovo (2015) used the relative spectral widths (the ratio of two energies) of the observed $\nu F_\nu$ spectrum to compare with those of known emission mechanisms. Peng et al. (2019, hereafter Paper I) further studied the properties of spectral width fitted by the BEST model with Fermi/GBM data. They have found there are correlations between the relative spectral width and the isotropic energy ($E_{iso}$) as well as the isotropic peak luminosity ($L_{iso}$) in the Fermi/GBM data. Moreover, the short bursts extend the correlations to the long ones, which seems to show that the long and short bursts share the same physics origins.

However, the correlation between the relative spectral width and $E_{iso}$ as well $L_{iso}$ was not very significant for time-integrated spectra, and it was not significant for the peak flux spectra. When they used the intrinsic absolute spectral width defined by the difference of two energies, the correlations of width–energy and width–luminosity improved noticeably. Moreover, the two correlated relationships were also established for the peak energy spectra. Both the Amati and Yonetoku relations are based on the peak energy of GRB spectra, and the peak energy and the spectral widths are related to the spectral shape of the GRB. Therefore, they further deduced that $E_{iso}$ and $L_{iso}$ might be connected with the GRB spectral shape combined with the Amati and Yonetoku relations. They also suspected that the other shape parameters $E_2$ and $E_1$ (where $E_2$ and $E_1$ are the upper and lower energy bounds of the FWHM of the $E_F$ versus $E$ spectra) also correlate with luminosity and energy. Which parameter is the relatively better indicator of energy and luminosity? We shall investigate these issues in detail in our work. Besides these, some other issues are not resolved. First, are the short burst outliers of the width–energy relation or width–luminosity relation? Second, are there connections between the width–energy relation, width–luminosity, and the Amati and Yonetoku relations? These issues motivate our investigations here. In Section 2, we present a sample description and data analysis. The results of the analysis are given in Section 3. Discussion and conclusions are presented in the last section.

### 2. Sample Selection and Data Analysis

In this paper, we would like to investigate the correlation properties between the intrinsic absolute spectral width and the isotropic-equivalent radiated energy $E_{iso}$ as well as the isotropic-equivalent peak luminosity $L_{iso}$. Therefore, we need spectral data that can well represent the observed spectral model data. Tsvetkova et al. (2017) provide just such a large spectral sample. They have presented the results of a systematic study of GRBs with reliable redshift estimates detected in the triggered mode of the Konus–Wind (KW) experiment during the period from 1997 February to 2016 June. The sample consists of 150 GRBs (including 12 short/hard bursts) and represents the largest set of cosmological GRBs studied to date over a broad energy band (13 keV–10 MeV). The wide energy range facilitates deriving the complete spectral shape directly from the KW data, and the GRB energetics can also be estimated using fewer extrapolations coupled with reliable redshift estimates, which is very important for obtaining a complete spectral width for our work. These can provide an excellent testing ground for widely discussed correlations between rest-frame spectral hardness and energetics (Tsvetkova et al. 2017). The KW catalog provides two types of spectra: time-integrated spectral fits (F spectra) and spectral fits at the brightest time bin (P spectra). They employed two different spectral models, the Band GRB function (BAND) and the exponential cutoff power law (COMP), to fit the data, respectively. They also compare the two models and give the BEST models for these bursts.

In order to investigate in detail the connections between the width–energy and width–luminosity relations and compare the Amati and Yonetoku relations, we want to define six types of absolute spectral widths, $W_{ab,50}$, $W_{ab,75}$, $W_{ab,85}$, $W_{ab,90}$, $W_{ab,95}$, and $W_{ab,99}$, based on Equation (1):

$$W_{ab} = \log (E_2 - E_1)$$

where $E_1$ and $E_2$ are the lower and upper energy bounds of the full width at 50%, 75%, 85%, 90%, 95%, and 99% of maximum of the $E_F$ versus $E$ spectra, respectively.

Similar to Axelsson & Borgonovo (2015), we also minimize selection effects due to parameter limits by requiring $\alpha > -1.90$ and $\beta < -2.10$ to obtain better observed spectral widths. Finally, there are 141 F burst spectral widths (including 12 short bursts) and 145 P spectral widths (including 12 short bursts) in our sample under these restrictions. All of the spectra are fitted by the BEST models with curvature shapes (Band function and Compton). The numbers of the two models for the subset are listed in Table 1. The uncertainty for each burst spectral width is estimated by using Monte Carlo methods.

### 3. Analysis Results

#### 3.1. Distribution of the Burst Absolute Spectral Width

In this section, we first check the distributions of the different absolute spectral widths defined in the previous section. Then we compare the distributions between the F spectra and the P spectra as well as the two types of bursts, long and short bursts. The various $W_{ab}$ distributions for the long and short bursts are demonstrated in Figure 1, and the characteristics are summarized in Table 2. We only analyze in detail the popular half-width $W_{ab,50}$ distributions below.

We find from Figure 1 and Table 2 for the long burst of two types of spectra that (1) the widths range from 2.16 ± 0.03 to 4.00 ± 0.17 for the F spectra and from 2.16 ± 0.03 to 4.00 ± 0.21 for the P spectra; (2) for both the P spectra and the F spectra, with the distribution peak at $<3$, there is a very small fraction of bursts extending toward larger widths; and (3) the corresponding median values of $W_{ab,50}$ are 2.90 ± 0.11 and 2.93 ± 0.19 for the P spectra and F spectra, respectively.

For the short bursts of two types of spectra, (1) the widths range from 2.88 ± 0.02 to 3.86 ± 0.05 for the F spectra and from 2.88 ± 0.02 to 3.85 ± 0.02 for the P spectra; (2) the distribution peaks at $>3$ for both spectra; (3) the corresponding median values of $W$ are 3.16 ± 0.05 and 3.22 ± 0.08 for the F spectra and P spectra, respectively. It is found that the widths of

| Sample Sizes of the Two Models |
|--------------------------------|
| F Spectra | P Spectra |
| Long GRBs | Short GRBs | Entire Bursts | Long GRBs | Short GRBs | Entire Bursts |
| BAND | 48 | 0 | 48 | 44 | 0 | 44 |
| COMP | 81 | 12 | 93 | 89 | 12 | 64 |
| All | 129 | 12 | 141 | 133 | 12 | 145 |
When comparing the long and short bursts for two types of spectra from Figure 1 and Table 2, we found that the peak value and the median of the short burst widths are evidently greater than those of the long burst for both spectra. These absolute width distributions are contrary to those of the relative spectral width of Paper I. Whether or not only the median value is different for the two types of bursts is the question. We do a Kolmogorov–Smirnov (K-S) test to check this and find that the significance probabilities and $D$ values are $5.39 \times 10^{-3}$, 0.50 and $8.55 \times 10^{-3}$, 0.48 for the F and the P spectra, respectively. Therefore, there are significant differences between the two types of GRBs for both the F spectra and the P spectra. However, there is substantial overlap between the long burst and the short burst (see Figure 1 and Table 2). Therefore, the distributions of the two types of bursts are perfectly compatible when taking into account the variances of the distributions.

We also compare the width distributions between the F spectra and the P spectra. For the short burst, the probability (0.99) and $D$ value (0.17) reveal that the distribution of the short burst between the F and the P spectra is the same. The K-S test gives a significance probability of 0.58 and $D$ value of 0.10, which also shows that the distribution of the long burst between the F spectra and the P spectra is also the same.

The distributions of $W_{ab,75}$, $W_{ab,85}$, $W_{ab,90}$, $W_{ab,95}$, and $W_{ab,99}$ are also shown in Table 2 and Figures 2–6. Naturally the median and peak width decrease. The K-S test also shows that there are significant differences between the two types of GRBs for both the F spectra and the P spectra, and the significance increases as the width decreases. However, the distributions between the F spectra and the P spectra are the same for both long and short bursts.

### 3.2. Confirmed Correlated Relationships of Width–$E_{\text{iso}}$ and Width–$L_{\text{iso}}$

In this section, we first reanalyze the correlations between the absolute spectral width in the rest frame $W_{ab,r} = W_{ab} + \log(1+z)$ and the $E_{\text{iso}}$ as well as $L_{\text{iso}}$. The case of the most popular spectral width at half (50%) maximum $W_{ab,50}$ is first considered. We calculate the Spearman rank-order correlation coefficients ($\rho$) and the associated null-hypothesis
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(chance) probabilities or $P$ values to check the existence of correlations. In fact, astronomical data are often uncertain with errors that are heteroscedastic (Robotham & Obreschkow 2015). The correlation is often characterized by the “intrinsic” scatter for the two subsamples. A Hyper-fit package can derive the general likelihood function to be maximized to recover the best-fitting model if a set of $D$-dimensional data points can be described by a $(D-1)$-dimensional plane with intrinsic scatter (Robotham & Obreschkow 2015). We can use the Hyper-fit package to perform the linear model fitting (Equation (2)) with both statistical errors in $X$ and $Y$ values. The correlation coefficients ($\rho$), $P$ values, slope ($a$), intercept ($b$), and dispersion of the points around the best-fit relations ($\sigma$) are summarized in Table 3, where

$$y = a \times x + b. \quad (2)$$

Figure 7 and Table 3 demonstrate the relationships between $W_{\text{ab,50,i}}$ versus $E_{\text{iso}}$ and $W_{\text{ab,50,i}}$ versus $L_{\text{iso}}$. The short GRBs are shown as red filled circles, while long GRBs are denoted with black filled circles. We first check the relationships between $W_{\text{ab,50,i}}$ and $E_{\text{iso}}$ for the F spectra. For the F spectra, a strong
correlation exists between them for the entire sample (including long and short bursts) with the Spearman rank-order correlation coefficient $\rho = 0.72$ ($P = 2.25 \times 10^{-23}$). The short bursts clearly deviate from the correlations of the long bursts in Figure 7, which can also be seen from the fact that the correlation of the long burst is $\rho = 0.80$ ($P = 8.75 \times 10^{-30}$). This correlation of the long burst is much more significant than for the entire bursts. The short bursts are the outliers for the $W_{ab,50,i} - E_{iso}$ relation. This property is similar to the Amati relation.

Then we examine the relationships between $W_{ab,50,i}$ and $L_{iso}$ for the P spectra. The corresponding Spearman rank-order correlation coefficients are $\rho = 0.69$ ($P < 0.0001$) and $\rho = 0.70$ ($P < 0.0001$) for the entire bursts and long bursts, which also reveal the very strong correlations for $W_{ab,50,i} - L_{iso}$. In addition, this shows that the short and long bursts are not well separated, and the short bursts almost follow the same correlated relationships as the long bursts for the case of $W_{ab,50,i} - L_{iso}$. This property is consistent with the Yonetoku relation.

Therefore, we try to investigate the possible reason for the larger scatter. We suspect that the wider spectra cause larger scatter by inspection of Figure 7. In fact, the upper energy bounds of the FWHM $E_i$ defined in Equation (1) are less reliable because the detector effective area and the flux fall rapidly with increasing energy, which can be seen from the larger errors of the wide spectral bursts.

Therefore, we check the correlations between other much smaller absolute spectral widths defined in the previous section in the rest frame and the $E_{iso}$ as well as $L_{iso}$. The diagrams and correlated properties constructed for different spectral widths $W_{ab,75,i}$, $W_{ab,85,i}$, $W_{ab,90,i}$, and $W_{ab,95,i}$ with $E_{iso}$ and $L_{iso}$ are presented in Figures 8–12 and Table 4. It is found that the smallest correlation coefficient among the correlation parameter pairs arrives at 0.64 ($p < 0.0001$). These show that all of the absolute spectral widths in the rest frame are strongly correlated with $E_{iso}$ as well as $L_{iso}$. Short bursts are evident outliers for width–$E_{iso}$ for the F spectra. For the width–$L_{iso}$, short and long GRBs are not well separated.

| Correlation | Number | $\rho_E$ | $P_E$ | $\rho_L$ | $P_L$ | $\alpha_L$ | $b_L$ | $\sigma_{\text{int},L}$ |
|-------------|--------|---------|-------|---------|-------|------------|-------|-----------------|
| $W_{ab,50,i} - E_{iso}$ | 141     | 0.72    | <0.0001 | 0.80    | <0.0001 | 0.57 ± 0.04 | 2.07 ± 0.09 | 0.30 ± 0.022  |
| $W_{ab,50,i} - L_{iso}$ | 145     | 0.68    | <0.0001 | 0.71    | <0.0001 | 0.55 ± 0.05 | 2.44 ± 0.08 | 0.346 ± 0.027 |

Note. Subscripts $F$, $P$, $E$, and $L$ denote the F spectra, P spectra, the entire burst set, and the long burst set, respectively.

| Correlation | Number | $\rho$ | $P$ | $\alpha$ | $b$ | $\sigma$ |
|-------------|--------|-------|-----|----------|-----|--------|
| $W_{ab,50,i} - E_{iso}$ | 129     | 0.80  | <0.0001 | 0.57 ± 0.04 | 2.07 ± 0.09 | 0.30 ± 0.02 |
| $W_{ab,75,i} - E_{iso}$ | 129     | 0.75  | <0.0001 | 0.42 ± 0.03 | 2.08 ± 0.07 | 0.240 ± 0.02 |
| $W_{ab,85,i} - E_{iso}$ | 129     | 0.72  | <0.0001 | 0.38 ± 0.03 | 2.02 ± 0.07 | 0.23 ± 0.02 |
| $W_{ab,90,i} - E_{iso}$ | 129     | 0.71  | <0.0001 | 0.37 ± 0.03 | 1.94 ± 0.07 | 0.23 ± 0.02 |
| $W_{ab,95,i} - E_{iso}$ | 129     | 0.69  | <0.0001 | 0.35 ± 0.03 | 1.81 ± 0.06 | 0.23 ± 0.02 |
| $W_{ab,99,i} - E_{iso}$ | 129     | 0.66  | <0.0001 | 0.33 ± 0.03 | 1.48 ± 0.06 | 0.23 ± 0.02 |
| $W_{ab,50,i} - L_{iso}$ | 133     | 0.71  | <0.0001 | 0.55 ± 0.05 | 2.44 ± 0.08 | 0.35 ± 0.03 |
| $W_{ab,75,i} - L_{iso}$ | 133     | 0.69  | <0.0001 | 0.41 ± 0.04 | 2.35 ± 0.06 | 0.26 ± 0.02 |
| $W_{ab,85,i} - L_{iso}$ | 133     | 0.68  | <0.0001 | 0.39 ± 0.03 | 2.24 ± 0.06 | 0.25 ± 0.02 |
| $W_{ab,90,i} - L_{iso}$ | 133     | 0.67  | <0.0001 | 0.37 ± 0.03 | 2.16 ± 0.06 | 0.25 ± 0.02 |
| $W_{ab,95,i} - L_{iso}$ | 133     | 0.65  | <0.0001 | 0.35 ± 0.03 | 2.01 ± 0.06 | 0.25 ± 0.02 |
| $W_{ab,99,i} - L_{iso}$ | 133     | 0.64  | <0.0001 | 0.34 ± 0.03 | 1.67 ± 0.05 | 0.25 ± 0.02 |
| $E_{F,i} - L_{iso}$ | 133     | 0.69  | <0.0001 | 0.38 ± 0.03 | 2.17 ± 0.05 | 0.24 ± 0.02 |

Note. $F$ and $P$ correspond to the $F$ spectra and $P$ spectra, respectively.
When comparing the correlated properties between all of the absolute spectral widths in the rest frame and $E_{\text{iso}}$ as well as $L_{\text{iso}}$ from Table 4 and Figures 7–12, we find the following: (1) the correlation coefficient and significance of $W_{\text{ab,50},i} - E_{\text{iso}}$ are the largest and most significant among the six correlated parameter pairs; (2) the correlation coefficients, significances, and scatters of correlations decrease with the decrease in width. However, the variability of the Spearman rank-order correlation coefficients for $W_{\text{ab},i} - E_{\text{iso}}$ and $W_{\text{ab},i} - L_{\text{iso}}$ is very different. Note that the variability here is the amount between the smallest and the largest item in the data set. The correlations for $W_{\text{ab},i} - E_{\text{iso}}$ have a large variability ($\Delta \rho \sim 0.14$), but the amplitude of

### Table 6

| Correlation                | Number | $\rho$   | $P$     | $a$     | $b$     | $\sigma$ |
|----------------------------|--------|----------|---------|---------|---------|----------|
| $E_{2,50,\text{F}} - E_{\text{iso}}$ | 129    | 0.80     | <0.0001 | 0.59 ± 0.04 | 2.08 ± 0.09 | 0.33 ± 0.02 |
| $E_{2,75,\text{F}} - E_{\text{iso}}$ | 129    | 0.76     | <0.0001 | 0.48 ± 0.04 | 2.08 ± 0.09 | 0.30 ± 0.02 |
| $E_{2,50,\text{L}} - E_{\text{iso}}$ | 129    | 0.73     | <0.0001 | 0.39 ± 0.03 | 2.15 ± 0.07 | 0.24 ± 0.02 |
| $E_{2,25,\text{L}} - E_{\text{iso}}$ | 129    | 0.72     | <0.0001 | 0.38 ± 0.03 | 2.13 ± 0.07 | 0.23 ± 0.02 |
| $E_{2,50,\text{L}} - L_{\text{iso}}$ | 129    | 0.70     | <0.0001 | 0.36 ± 0.03 | 2.09 ± 0.06 | 0.23 ± 0.02 |
| $E_{2,25,\text{L}} - L_{\text{iso}}$ | 129    | 0.68     | <0.0001 | 0.35 ± 0.03 | 2.02 ± 0.06 | 0.23 ± 0.02 |
| $E_{p,\text{F}} - E_{\text{iso}}$ | 129    | 0.69     | <0.0001 | 0.36 ± 0.03 | 1.95 ± 0.07 | 0.23 ± 0.02 |
| $E_{p,\text{L}} - E_{\text{iso}}$ | 133    | 0.72     | <0.0001 | 0.56 ± 0.05 | 2.49 ± 0.08 | 0.36 ± 0.03 |
| $E_{p,\text{L}} - L_{\text{iso}}$ | 133    | 0.70     | <0.0001 | 0.45 ± 0.04 | 2.43 ± 0.07 | 0.32 ± 0.02 |
| $E_{2,50,\text{F}} - L_{\text{iso}}$ | 133    | 0.70     | <0.0001 | 0.40 ± 0.03 | 2.40 ± 0.06 | 0.26 ± 0.02 |
| $E_{2,25,\text{F}} - L_{\text{iso}}$ | 133    | 0.69     | <0.0001 | 0.39 ± 0.03 | 2.36 ± 0.06 | 0.25 ± 0.02 |
| $E_{2,50,\text{L}} - L_{\text{iso}}$ | 133    | 0.68     | <0.0001 | 0.38 ± 0.03 | 2.31 ± 0.06 | 0.25 ± 0.02 |
| $E_{2,25,\text{L}} - L_{\text{iso}}$ | 133    | 0.68     | <0.0001 | 0.37 ± 0.03 | 2.24 ± 0.05 | 0.24 ± 0.02 |

Note. F and P correspond to the F spectra and P spectra, respectively.
variability $\Delta \rho \sim 0.07$ for $W_{ab,i} - L_{iso}$ for the P spectra is very small; see Table 4. These seem to show that the width–$L_{iso}$ relations for the peak flux spectra are more stable than that of the time-integrated spectra.

3.3. Comparison of Width–$E_{iso}$ and Width–$L_{iso}$ with the Amati and Yonetoku Relations

Since the correlated properties of the $W_{ab,i} - E_{iso}$ and $W_{ab,i} - L_{iso}$ pairs are very similar to the well-known Amati relation (Amati et al. 2002) and Yonetoku relation (Yonetoku et al. 2004), we would like to compare the correlation properties of the rest-frame width–$E_{iso}$ and rest-frame width–$L_{iso}$ with the Amati and Yonetoku relations. For the sake of comparison, Figures 7–12 and Table 4 also give the correlated properties of $E_{p,i} - E_{iso}$ and $E_{p,i} - L_{iso}$ for both the F and P spectra.

It can be seen from Figure 7 and Table 4, for the case of the F spectra, that the correlation of $W_{ab,50,i} - E_{iso}$ ($\rho = 0.80$, $P < 0.0001$) is much stronger than the Amati relation ($\rho = 0.69$, $P < 0.0001$). However, the dispersion of $W_{ab,50,i} - E_{iso}$ (0.300 ± 0.022) is also slightly greater than the Amati relation (0.227 ± 0.016) obtained with the same spectral data. Nevertheless, it is found that the two correlated properties ($\rho = 0.71, P < 0.0001$ for $W_{ab,50,i} - L_{iso}$) are almost consistent with the Yonetoku relation ($\rho = 0.69, P < 0.0001$) when comparing the P spectra. But the dispersions of $W_{ab,50,i} - L_{iso}$ (0.35 ± 0.03) for the F spectra and P spectra are also slightly greater than the Yonetoku relation (0.24 ± 0.02) obtained with the same spectral data.

We then compare other width–$E_{iso}$ and width–$L_{iso}$ relations with the Amati relation and Yonetoku relation, respectively. It is very interesting that, for both F spectra and P spectra, we can also find from Figures 8–12 that the rest-frame absolute spectral width approaches $E_{p,i}$ with the decrease in the widths until $W_{ab,90,i}$ almost overlaps with $E_{p,i}$. Meanwhile, the best regression lines of $W_{ab,90,i} - E_{iso}$ and $W_{ab,90,i} - L_{iso}$ also almost

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**Figure 8.** $W_{ab,75,i}$ vs. $E_{iso}$ for the F spectra (a) and $W_{ab,75,i}$ vs. $L_{iso}$ for the P spectra (b), along with the Amati and Yonetoku relations. All symbols are the same as in Figure 7.

**Figure 9.** $W_{ab,85,i}$ vs. $E_{iso}$ for the F spectra (a) and $W_{ab,85,i}$ vs. $L_{iso}$ for the P spectra (b), along with the Amati and Yonetoku relations. All symbols are the same as in Figure 7.
overlap with those of the Amati and Yonetoku relations, respectively. Moreover, it can be observed from Table 4 that the derived slopes, intercept, and intrinsic scatter of \(W_{90,\text{iso}}^{90,\text{iso}}\) and \(W_{90,\text{iso}}^{90,\text{iso}}\) are also very close to those of the Amati and Yonetoku relations, respectively. It is also can be seen that \(W_{95,\text{iso}}^{90,\text{iso}}\) and \(W_{95,\text{iso}}^{90,\text{iso}}\) deviate from the Amati and Yonetoku relations, and the correlation coefficients and the significances also decrease when the absolute spectral width decreases.

These striking similarities between \(W_{90,\text{iso}}^{90,\text{iso}}\) and the Amati relation as well as \(W_{90,\text{iso}}^{90,\text{iso}}\) and the Yonetoku relation seem to show that the Amati and Yonetoku relations are the special cases of width–Eiso and width–Liso. In addition, it also can be seen that the correlation of the Amati relation is not the strongest among the width–Eiso relations when the scatter is very close, while the Yonetoku relation also does not show the most significant correlation properties compared with width–Liso.

3.4. Is \(E_{\text{p},i}\) a Best Energy Correlated with Eiso and Liso?

As we know, \(E_{2}, E_{1}\), and \(E_{p}\) are the location energies of the \(EF_{E}\) spectra. The absolute spectral width is the difference between location energies \(E_{2}\) and \(E_{1}\), and it must be related to \(E_{2}\) and \(E_{1}\). For the wide spectra, the absolute width should be strongly correlated with \(E_{2}\), while the absolute width should also be correlated with \(E_{1}\) for the narrower absolute spectral width. The corresponding Spearman correlation coefficients \(\rho\) and \(P\) are shown in Table 5, and the absolute spectral widths indeed correlate with both \(E_{2}\) and \(E_{1}\).

As a consequence, we suspect there must be correlations between rest-frame \(E_{\text{rest},i}\) and \(E_{\text{iso}}\) as well as \(L_{\text{iso}}\), so we check the correlations between the two location energies \(E_{\text{rest},i}\) and \(E_{\text{iso}}\) in various spectral widths and \(L_{\text{iso}}\). The correlations between them are shown in Table 6 and Figures 13–24. It is found that both \(E_{\text{rest},i}\) and \(E_{\text{iso}}\) in various spectral widths are also strongly correlated with \(E_{\text{iso}}\) as well as \(L_{\text{iso}}\). The short bursts for \(E_{\text{rest},i} - E_{\text{iso}}\) and \(E_{\text{iso}} - E_{\text{iso}}\) deviate...
from the long bursts for both F spectra and P spectra, while a few short bursts for \( E_{2,i} \) vs. \( E_{\text{iso}} \) and \( E_{1,i} \) vs. \( L_{\text{iso}} \) deviate from the long bursts for both F spectra and P spectra. The correlated properties are very similar to the correlations between various absolute spectral widths and \( E_{\text{iso}} \) and \( L_{\text{iso}} \) above. As the widths decrease, both \( E_{2,i} \) and \( E_{1,i} \) approach \( E_{\text{iso}} \) (see Figures 13–24 and Tables 6 and 7). The correlation coefficients, significances, and intrinsic scatters also decrease, and the best regression lines of \( E_{2,99,i} \) vs. \( E_{\text{iso}} \), \( E_{2,99,i} \) vs. \( L_{\text{iso}} \), \( E_{1,99,i} \) vs. \( E_{\text{iso}} \), and \( E_{1,99,i} \) vs. \( L_{\text{iso}} \) almost overlap with those of the Amati and Yonetoku relations, respectively.

In addition, we find there are several correlations between location energy \( (E_{2,i}, E_{1,i}) \) and \( -E_{\text{iso}} \), such as \( E_{2,95,i} \) vs. \( E_{\text{iso}} \) and \( E_{2,90,i,F} \) vs. \( E_{\text{iso}} \), that are stronger than that of the Amati relation. Likewise, we also find there are several correlations of \( E_{2,i} \) vs. \( L_{\text{iso}} \) and \( E_{1,i} \) vs. \( L_{\text{iso}} \), such as \( E_{2,85,i,P} \) vs. \( L_{\text{iso}} \) and \( E_{1,95,i,P} \) vs. \( L_{\text{iso}} \), that are stronger than that of the Yonetoku relation. Therefore, the Amati and Yonetoku relations are not the most significant correlated relationships among the relations \( E_{2,i} \) vs. \( E_{\text{iso}} \), \( E_{1,i} \) vs. \( E_{\text{iso}} \), \( E_{2,i} \) vs. \( L_{\text{iso}} \), and \( E_{1,i} \) vs. \( L_{\text{iso}} \). These seem to also show that the Amati and Yonetoku relations are only the special correlations between location energy in the GRB spectra and \( E_{\text{iso}} \) as well as \( L_{\text{iso}} \).

4. Discussion and Conclusions

Spectrum–energy correlations of GRBs are empirical connections between the measurable properties of the prompt gamma-ray emission spectra and the energy or luminosity of GRBs. These correlations are very important because they were proposed to measure the universe and classify GRBs as useful probes (e.g., Dai et al. 2004; Wang et al. 2017, Minaev & Pozanenko 2020). In this paper, we have analyzed in detail the correlations between the absolute spectral width of GRB spectra with BEST model parameters from KW and the equivalent isotropic energy \( (E_{\text{iso}}) \) as well as the isotropic peak luminosity \( (L_{\text{iso}}) \) for time-integrated F and peak flux P spectra, respectively. Different from Paper I, we mainly consider the
absolute spectral width in the rest frame and the isotropic-equivalent radiated energy rather than the relative spectral width. The sample includes 141 and 145 bursts for the F spectra and P spectra, respectively, which contain 12 and 12 short bursts. The majority of the BEST spectral data are fitted by a Compton model (93/141 and 101/145 for the F and P spectra, respectively). The sample size with known redshift we used in this paper is much larger than that of Paper I (86 and 75 for the F and P spectra). Using the best estimate of the observed spectral parameters of GRBs, we compute six different absolute spectral widths for the F and P spectra based on the $E_{F,50,i}$ spectra peak at 0.5, 0.75, 0.85, 0.90, 0.95, and 0.99 maximum to investigate the relationships of the rest-frame width–$E_{iso}$ and width–$L_{iso}$. Of course, the six different widths we defined are arbitrary. Our purpose is to check if the different spectral widths are also correlated with $E_{iso}$ and $L_{iso}$ and compare them with the well-known Amati and Yonetoku relations.

We first consider the popular rest-frame half-maximum width $W_{ab,50,i}$ for the F spectra and P spectra. The $W_{ab,50,i}$ is strongly correlated with $E_{iso}$ as well as $L_{iso}$ for both F spectra and P spectra. Similar to the Amati relation, it is found that the short bursts are also outliers of the long ones for the $W_{ab,50,i} - E_{iso}$ relation (see Table 3 and Figure 7), while for the $W_{ab,50,i} - L_{iso}$ relation, a few short bursts deviate from the long bursts even if most of the short bursts are consistent with long bursts. This property seems to show that the long bursts have a different origin from the short bursts. Recently, Zhang et al. (2018) found that the correlations for short GRBs are also established, and they had power-law indices consistent with long GRBs. Minaev & Pozanenko (2020) further confirmed the correlation with a much larger short-burst sample. Due to the huge difference of the number of long and short bursts, we only check the correlations by comparing the $W_{ab,50,i}$ and $E_{iso}$ as well as $L_{iso}$, with the Amati relation and Yonetoku relation for the case of short bursts. The correlation properties are demonstrated in Table 8. It is surprising that there is almost no difference for the correlated properties, the slopes, and the scatters, except for some slightly different intercepts for all of the spectral widths. Take the case of

Figure 14. $E_{F,50,i}$ vs. $E_{iso}$ for the F spectra (a) and P spectra (b) and $E_{F,50,i}$ vs. $L_{iso}$ for the F spectra (c) and P spectra (d), along with the Amati and Yonetoku relations for the F and P spectra. All symbols are the same as in Figure 7.

Figure 15. $E_{F,75,i}$ vs. $E_{iso}$ for the F spectra (a) and P spectra (b) and $E_{F,75,i}$ vs. $L_{iso}$ for the F spectra (c) and P spectra (d), along with the Amati and Yonetoku relations for the F and P spectra. All symbols are the same as in Figure 7.
for example: the relationships are shown in Figure 25, and the slopes of $W_{ab,50} - E_{\text{iso}}$ and $E_{\text{p,3i}} - E_{\text{iso}}$ are $0.278 \pm 0.069$ and $0.279 \pm 0.065$, respectively, while the slopes of $W_{ab,50} - L_{\text{iso}}$ and $E_{\text{p,3i}} - L_{\text{iso}}$ are $0.246 \pm 0.052$ and $0.240 \pm 0.048$. These show that the slopes of width-$E_{\text{iso}}$ and width-$L_{\text{iso}}$ are consistent with that of the Amati relation and the Yonetoku relation, respectively. Therefore, we deduce that the short burst slopes of $W_{ab,1} - E_{\text{iso}}$ and $W_{ab,1} - L_{\text{iso}}$ are also consistent with that of the long bursts since the short burst and the long burst of the Amati relation and Yonetoku relation have consistent slopes (Zhang et al. 2018). Of course, their validity needs further identification with a much larger short-burst sample.

We then consider if the correlations of width-$E_{\text{iso}}$ and width-$L_{\text{iso}}$ for other narrower widths defined above, such as $W_{ab,75}$, $W_{ab,85}$, $W_{ab,90}$, $W_{ab,95}$, and $W_{ab,99}$, exist and then compare the width-$E_{\text{iso}}$ and width-$L_{\text{iso}}$ with the Amati and Yonetoku relations. We find all of the correlations of width-$E_{\text{iso}}$ and width-$L_{\text{iso}}$ are established regardless of the F spectra and P spectra. In addition, the variabilities of the correlation of width-$E_{\text{iso}}$ and width-$L_{\text{iso}}$ for the F spectra are much larger than those of the P spectra. All of the scatters of width-$E_{\text{iso}}$ and width-$L_{\text{iso}}$ decrease with the decrease in the width regardless of the F and P spectra. The more stable correlations of width-$E_{\text{iso}}$ and width-$L_{\text{iso}}$ for the P spectra seem to reveal that the correlations of peak flux spectra can better represent the connections between width and $E_{\text{iso}}$ as well as $L_{\text{iso}}$. The F spectra are the average spectra, and the GRB spectra evolve with time (e.g., Crider et al. 1997; Kaneko et al. 2006; Peng et al. 2009). Hence, the peak flux spectra can better describe the observed spectra.

In fact, the spectral width may be a good physics quantity for describing the spectral shape. The observed spectral width was used to compare with the widths of spectra from fundamental emission processes (Axelsson & Borgonovo 2015; Vurm & Beloborodov 2016). As Paper I and Axelsson &
Borgonovo (2015) pointed out, most of the observed spectra are much narrower than the emission from an electron distribution and are also significantly wider than a blackbody spectrum. The discrepancy between the observed spectral width and the theory width can provide some evidence for the GRB radiation mechanism. Recently, Bharali et al. (2017) put forward the idea that if the system undergoes a rapid temperature evolution, the observed spectral shape can be broadened. Particularly, if invoking thermal radiation, a way must be found to broaden the spectrum (Vurm & Beloborodov 2016). Beloborodov (2013) pointed out that several factors inevitably broaden the synchrotron peak in a more realistic model. Moreover, they thought that the observed spectrum is hardly consistent with the synchrotron model, even if uncertainties in the observed spectrum due to the detector response and limited photon statistics are taken into account. Therefore, they thought that the narrow megaelectronvolt spectral peak provided strong evidence for thermalization of radiation at early, opaque stages of the GRB explosion.

Figure 18. $E_{1,85}$ vs. $E_{iso}$ for the F spectra (a) and P spectra (b) and $E_{1,85}$ vs. $L_{iso}$ for the F spectra (c) and P spectra (d), along with the Amati and Yonetoku relations for the F and P spectra. All symbols are the same as in Figure 7.

Figure 19. $E_{2,90}$ vs. $E_{iso}$ for the F spectra (a) and P spectra (b) and $E_{2,90}$ vs. $L_{iso}$ for the F spectra (c) and P spectra (d), along with the Amati and Yonetoku relations for the F and P spectra. All symbols are the same as in Figure 7.

In fact, there are similar width–energy and width–luminosity relations in the astrophysics field. Many studies have shown that the temporal profile is also connected with the luminosity. For example, see the well-known lag–luminosity relation found by Norris et al. (2000) and the variability–luminosity relation found by Reichart et al. (2001). Norris et al. (2005) found that the pulse width is strongly correlated with the pulse lag. Hakkila et al. (2008) derived a new pulse width in the rest frame versus peak luminosity correlation in GRB pulses, which appear to be natural from the lag–luminosity and width–lag relations. Hashimoto et al. (2020) confirmed a positive time-integrated luminosity–duration relation of nonrepeating, fast radio bursts, but repeating, fast radio bursts do not indicate a clear correlation between luminosity and duration. These results reveal different physical origins of the two types of fast radio bursts. Tu & Wang (2018) also found a correlation between isotropic energy and intrinsic duration for the first time with a Swift GRB sample. In addition, the duration–energy correlation was also established for solar flares and stellar
superflares. It is very striking that the dependences of duration on the isotropic energy are very similar since they share a similar power-law index of duration on energy for the GRB, solar flares, and stellar superflares. Furthermore, many studies suggested there is a dependence of the temporal profiles of GRBs on energy (e.g., Fenimore et al. 1995; Norris et al. 2005; Peng et al. 2006). The connection between temporal profiles and energy seems to support the width–$E_{\text{iso}}$ and width–$L_{\text{iso}}$ relations.

When we compare the relations of the six width–$E_{\text{iso}}$ and width–$L_{\text{iso}}$ relations with the well-known Amati and Yonetoku relations, most of the correlations of width–$E_{\text{iso}}$ and width–$L_{\text{iso}}$ are much tighter than those of the Amati and Yonetoku relations for the F spectra, whereas for the P spectra their correlations are very consistent. Moreover, the width–$E_{\text{iso}}$ and width–$L_{\text{iso}}$ relations would approach the Amati and Yonetoku relations as the width decreases. The $W_{\text{ab},90,i} - E_{\text{iso}}$ and $W_{\text{ab},90,i} - L_{\text{iso}}$ relations almost overlap with the Amati and Yonetoku relations, and their correlation properties, slopes, and scatters are also well consistent. When the widths are less than $W_{\text{ab},90,i}$, the correlations of width–$E_{\text{iso}}$ and width–$L_{\text{iso}}$ deviate from the Amati and Yonetoku relations. As a consequence, the Amati and Yonetoku relations appear to be special cases of the width–$E_{\text{iso}}$ and width–$L_{\text{iso}}$ relations.

We further investigate the correlations between the upper ($E_2$) and lower energy ($E_1$) bounds of the $EFE$ spectrum from six different widths with $E_{\text{iso}}$ as well as $L_{\text{iso}}$. It is found that all of the $E_2$ and $E_1$ are also correlated with $E_{\text{iso}}$ as well as $L_{\text{iso}}$. The $E_2 - E_{\text{iso}}$, $E_1 - E_{\text{iso}}$ relations and $E_2 - L_{\text{iso}}$, $E_1 - L_{\text{iso}}$ relations are close to the Amati relation and Yonetoku relation when $E_2$ and $E_1$ approach the peak energy $E_p$. Once more the Amati and Yonetoku relations seem to be special cases of the corresponding $E_2 - E_{\text{iso}}$, $E_2 - L_{\text{iso}}$ or $E_2 - E_{\text{iso}}$, $E_2 - L_{\text{iso}}$ correlations. Moreover, the Amati and Yonetoku relations are not the best relationships by comparing the correlation parameters with almost the same scatters.

Both $E_{\text{iso}}$ and $L_{\text{iso}}$ are correlated with all of spectral widths as well as all of the location energies ($E_2$ and $E_1$), which further

![Figure 20](image_url)

**Figure 20.** $E_{90,i}$ vs. $E_{\text{iso}}$ for the F spectra (a) and P spectra (b) and $E_{90,i}$ vs. $L_{\text{iso}}$ for the F spectra (c) and P spectra (d), along with the Amati and Yonetoku relations for the F and P spectra. All symbols are the same as in Figure 7.

![Figure 21](image_url)

**Figure 21.** $E_{85,i}$ vs. $E_{\text{iso}}$ for the F spectra (a) and P spectra (b) and $E_{85,i}$ vs. $L_{\text{iso}}$ for the F spectra (c) and P spectra (d), along with the Amati and Yonetoku relations for the F and P spectra. All symbols are the same as in Figure 7.
suggests that \( E_{\text{iso}} \) and \( L_{\text{iso}} \) are related to the shape of the GRB spectra. At present, we cannot find the reasons behind the correlated relationships. These issues motivate us to suspect that \( E_{\text{iso}} \) and \( L_{\text{iso}} \) may be also related to the area of the EFE spectrum or the spectrum centroid energy, which deserve further investigations.

Paper I considered the correlations between the spectral widths and the photon indices \( \alpha \) and \( \beta \), and \( T_{90} \). We also inspect these relations with the KW sample for the popular half width \( W_{\text{ab},50} \). Because the sample consists of BAND and COMP, the absolute spectral widths also consist of the two models. For the BAND model, the widths are strongly correlated with the high-energy index \( \beta \) (\( \rho = 0.82, p < 10^{-4} \) for the F spectra and \( \rho = 0.87, p < 10^{-4} \) for the P spectra) and less correlated with \( \alpha - \beta \) (\( \rho = -0.57, p < 10^{-4} \) for the F spectra and \( \rho = -0.68, p < 10^{-4} \) for the P spectra). However, there are no correlations between the spectral width and the lower energy index \( \alpha \) (\( \rho = -0.02, p = 0.93 \) for the F spectra and \( \rho = -0.06, p = 0.71 \) for the P spectra). Likewise, we do not find that there are any correlations between the spectral width and \( \alpha \) (\( \rho = 0.04, p = 0.65 \) for the F spectra and \( \rho = 0.04, p = 0.68 \) for the P spectra).

We also check the relationships between the absolute spectral width \( W_{\text{ab},50} \) and the duration \( T_{90} \). We first examine the relationships for the case of the F spectra and the P spectra in the observed frame. The Spearman rank-order correlation coefficients and the \( P \) values (\( \rho = 0.24, p = 4.35 \times 10^{-3} \) for the F spectra and \( \rho = 0.23, p = 6.69 \times 10^{-3} \) for the P spectra) show that there are weak correlations between them. For the intrinsic case, the corresponding correlation coefficients and the \( P \) values (\( \rho = 0.21, p = 1.25 \times 10^{-2} \) for the F spectra and \( \rho = 0.22, p = 8.50 \times 10^{-3} \) for the P spectra) also show that the correlations between \( W_{\text{ab},50} \) and \( T_{90} \) are not strong for both the F spectra and the P spectra.

In conclusion, our analysis further confirms the tight width–energy and width–luminosity relations in GRBs, and the relations are also important correlation relationships in understanding the physics of GRBs. The Amati and Yonetoku

![Figure 22](image1.png)

**Figure 22.** \( E_{1.95} \) vs. \( E_{\text{iso}} \) for the F spectra (a) and P spectra (b) and \( E_{1.95} \) vs. \( L_{\text{iso}} \) for the F spectra (c) and P spectra (d), along with the Amati and Yonetoku relations for the F and P spectra. All symbols are the same as in Figure 7.

![Figure 23](image2.png)

**Figure 23.** \( E_{1.99} \) vs. \( E_{\text{iso}} \) for the F spectra (a) and P spectra (b) and \( E_{1.99} \) vs. \( L_{\text{iso}} \) for the F spectra (c) and P spectra (d), along with the Amati and Yonetoku relations for the F and P spectra. All symbols are the same as in Figure 7.
Correlation Analysis Results of Six Absolute Spectral Widths in the Rest Frame and $E_{iso}$ as well as $L_{iso}$ for the Short GRBs

| Correlation | Number | $\rho$ | $P$ | $a$  | $b$  | $\sigma$ |
|-------------|--------|--------|-----|------|------|----------|
| $W_{90,1,F} - E_{iso}$ | 12    | 0.60   | 3.86 x 10^{-2} | 0.28 ± 0.07 | 3.26 ± 0.07 | 0.22 ± 0.04 |
| $W_{90,1,F} - L_{iso}$ | 12    | 0.62   | 3.32 x 10^{-2} | 0.25 ± 0.05 | 3.34 ± 0.05 | 0.18 ± 0.04 |

Note. F and P correspond to the F spectra and P spectra, respectively.
relations are not necessarily the best-correlated relationships for relating the location energy to isotropic energy as well as isotropic peak luminosity. These results may provide some help in understanding the origins of the Amati and Yonetoku relations.

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References

Ahlgren, B., Larsson, J., Ahlberg, E., et al. 2019a, MNRAS, 485, 474
Ahlgren, B., Larsson, J., Valan, V., et al. 2019b, ApJ, 880, 76
Amati, L., Frontera, F., Tavani, M., et al. 2002, A&A, 390, 81
Axelsson, M., & Borgonovo, L. 2015, MNRAS, 447, 3150
Band, D. L., Matteson, J., Ford, L., et al. 1993, ApJ, 413, 281
Band, D. L., & Preece, R. D. 2005, ApJ, 627, 319
Beloborodov, A. 2013, ApJ, 764, 157
Bharali, P., Sahayathanath, S., Misra, R., & Boruah, K. 2017, NewA, 55, 22
Burgess, J., Bégué, D., Greiner, J., et al. 2020, NatAs, 4, 174
Crider, A., Liang, E. P., Smith, I. A., et al. 1997, ApJL, 478, L39
Dai, Z. G., Liang, E. W., & Xu, D. 2004, ApJL, 612, L101
Fenimore, E. E., in’t Zand, J. J. M., Norris, J. P., Bonnell, J. T., & Nemiroff, R. J. 1995, ApJL, 448, L101
Ghirlanda, G. W., Ghisellini, G., Lazzati, D., et al. 2004, ApJ, 616, 331
Guo-Jian Wang, G. J., Yu, H., Li, Z. X., Jun-Qing Xia, J. Q., & Zhu, Z. H. 2017, ApJ, 836, 303
Hakkila, J., Giblin, W., Norris, J., et al. 2008, ApJL, 677, L81
Hashimoto, T., Goto, T., Wang, T., et al. 2020, MNRAS, 494, 2886
Horváth, I., Bagoly, Z., Balázs, L. G., et al. 2010, ApJ, 713, 552
Kaneko, Y., Preece, R. D., Briggs, M. S., et al. 2006, ApJS, 166, 298
Kouveliotou, C., Meegan, C., Fishman, G., et al. 1993, ApJL, 413, L101
Minaev, P. Y., & Pozanenko, A. S. 2020, MNRAS, 492, 1919
Norris, J. P., Bonnell, J. T., Kazanas, D., et al. 2005, ApJ, 627, 324
Norris, J. P., Marani, G. F., & Bonnell, J. T. 2000, ApJ, 534, 248
Oganesyan, G., Nava, L., Ghirlanda, G., et al. 2019, A&A, 628, 59
Peng, Z. Y., Ma, L., Zhao, X. H., et al. 2009, ApJ, 698, 417
Peng, Z. Y., Qin, Y. P., Zhang, B. B., et al. 2006, MNRAS, 368, 1351
Peng, Z. Y., Zhao, X. H., Yin, Y., & Wang, D. Z. 2019, ApJ, 881, 51, (Paper I)
Preece, R. D., Briggs, M. S., Mallozzi, R. S., et al. 2000, ApJ, 126, 19
Qin, Y., & Chen, Z. 2013, MNRAS, 430, 163
Reichart, D. E., Lamb, D. Q., Fenimore, E. E., et al. 2001, ApJ, 552, 57
Robotham, A. S. G., & Obreschkow, D. 2015, PASA, 32, 33
Svinkin, D., Frederiks, D., Aptekar, R., et al. 2016, ApJ, 822, 10
Titarchuk, L., Farinelli, R., Frontera, F., et al. 2012, ApJ, 752, 116
Tsvetkova, A., Frederiks, D., Golenetskii, S., et al. 2017, ApJ, 850, 161
Tu, Z., & Wang, F. 2018, ApJL, 869, L23
Vurm, I., & Beloborodov, A. 2016, ApJ, 831, 175
Wang, G.-J., Yu, H., Li, Z.-X., Xia, J.-Q., & Zhu, Z.-H. 2017, ApJ, 836, 103
Yonetoku, D., Murakami, T., Nakamura, T., et al. 2004, ApJL, 612, L101
Zhang, Z. B., Zhang, C. T., Zhao, Y. X., et al. 2018, PASP, 130, 4202

Figure 25. Plots of $W_{ab,50,i} - E_{iso}$ (a) and $W_{ab,50,i} - L_{iso}$ (b) and $E_{pi,i} - E_{iso}$ and $E_{pi,i} - L_{iso}$ for the short burst sample. The black filled circles and black open circles represent the relations $W_{ab,50,i} - E_{iso}$, $W_{ab,50,i} - L_{iso}$, and $E_{pi,i} - E_{iso}$, $E_{pi,i} - L_{iso}$, respectively. The solid lines are the best-fitting lines obtained by fitting $W_{ab,50,i} - E_{iso}$, $W_{ab,50,i} - L_{iso}$, while the dashed lines are the fitting lines of $E_{pi,i} - E_{iso}$, $E_{pi,i} - L_{iso}$, respectively.