The Evolution toward Electron Capture Supernovae: The Flame Propagation and the Pre-bounce Electron–Neutrino Radiation

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Abstract

A critical-mass ONe core with a high ignition density is considered to end in gravitational collapse leading to neutron star formation. Being distinct from an Fe core collapse, the final evolution involves combustion flame propagation, in which complex phase transition from ONe elements into the nuclear statistical equilibrium (NSE) state takes place. We simulate the core evolution from the O+Ne ignition until the bounce shock penetrates the whole core, using a state-of-the-art 1D Lagrangian neutrino radiation hydrodynamic code, in which important nuclear burning, electron capture, and neutrino reactions are taken into account. Special care is also taken in making a stable initial condition by importing the stellar equation of state, which is used for the progenitor evolution calculation, and by improving the remapping process. We find that the central ignition leads to intense $\nu_e$ radiation with $L_{\nu_e} \gtrsim 10^{51}$ erg s$^{-1}$ powered by fast electron captures onto NSE isotopes. This pre-bounce $\nu_e$ radiation heats the surroundings by the neutrino–electron scattering, which acts as a new driving mechanism of the flame propagation together with the adiabatic contraction. The resulting flame velocity of $\sim 10^4$ cm s$^{-1}$ will be more than one order of magnitude faster than that of a laminar flame driven by heat conduction. We also find that the duration of the pre-bounce $\nu_e$ radiation phase depends on the degree of the core hydrostatic/dynamical stability. Therefore, the future detection of the pre-bounce neutrino is important not only to discriminate the ONe core collapse from the Fe core collapse but also to constrain the progenitor hydrodynamical stability.

Key words: neutrinos–nuclear reactions – nucleosynthesis – abundances–stars: evolution–supernovae: general

1. Introduction

In a standard theory of stellar evolution, two types of stellar cores have been known to collapse to form a neutron star (NS; Janka 2012; Langer 2012, for recent review papers). One is a core made of iron-group elements, and the other is a core mainly made of oxygen and neon. A star massive enough to form an Fe core is called a massive star, and the lowest initial mass of the massive star is often indicated by $M_{\text{max}}$. Because of several theoretical uncertainties, including the uncertain efficiency of the convective overshoot, it is difficult to precisely determine the value of $M_{\text{max}}$ while the current estimates are around $9$–$11 M_\odot$ for solar-metallicity stars. The ONe core is formed in a super-AGB star, which is less massive than $M_{\text{max}}$ but massive enough to ignite core carbon burning.

Evolution of a collapsing Fe core is relatively well understood (e.g., Arnett 1977; Weaver et al. 1978). An Fe core contracts owing to neutrino cooling, electron capture, and continuous core mass growth. Core collapse takes place when the instability due to the photodisintegration sets in at the central part of the Fe core. The collapse lasts until a nascent NS supported by nucleon degeneracy and nuclear repulsive force forms at the center, at which time the bounce shock is created at the core surface and stalls on the way. Although extensive investigations have not yet fully revealed what mechanism(s) accounts for the revival of the stalled shock and how properties of Fe core-collapse supernovae (FeCCSNe) such as the explosion energy are determined, it is widely believed that the core collapse of an Fe core triggers a variety of observed SNe of Type II, Ib, and Ic.

Meanwhile, the evolution of a super-asymptotic-redgiant-branch (SAGB) star until central oxygen+neon ignition has been investigated in detail by several authors (Garcia-Berro & Iben 1994; Ritossa et al. 1996; Ventura & D’Antona 2005; Siess 2006; Doherty et al. 2010; Lau et al. 2012; Jones et al. 2013; Takahashi et al. 2013; Schwab et al. 2015; and recent review by Doherty et al. 2017). The explosion of a so-called electron capture SN (ECSN) as a result of collapse of a critical-mass ONe core has been intensively investigated as well (Hillebrandt et al. 1984; Mayle & Wilson 1988; Dessart et al. 2006; Kitaura et al. 2006; Janka et al. 2008; Fischer et al. 2010; Miyaji et al. 1980; Nomoto 1987; Takahashi et al. 2013). Accordingly, the late evolution of collapsing ONe cores has not been fully understood.

The distinctive composition of the ONe core complicates the investigations of the last phase of the ONe core evolution: the major components of the ONe core are still combustible. While the Fe core is mainly made of iron-group nuclei so that no additional heating due to nuclear reactions is expected, $\sim 0.7$ MeV per baryon on average is released through the $^{56}$Ni synthesis in the ONe core. As the highly degenerate ONe core typically has a total energy (the sum of the gravitational energy and the internal energy) of $\sim 5 \times 10^{50}$ erg with a core mass of $1.37 M_\odot$ and a core radius of $1.4 R_\odot$, the available nuclear

$^{6}$ The degenerate pressure is already relativistic and has a nearly 4/3 adiabatic index. Consequently, the virial theorem $E_{\text{g}} = -3(\gamma - 1)E_{\nu}$ does not agree with $E_{\text{g}} = -1/2E_{\nu}$, the limit obtained for the nonrelativistic monoatomic ideal gas.
energy of \( \sim 1.8 \times 10^{51} \) erg is enough to explode the entire core, if they burn out instantaneously.

Of course, this simple energy estimate is not enough to determine the fate of the critical-mass ONe core because several important nuclear processes take place in the collapsing ONe core. On one hand, the nuclear burning liberates nuclear binding energies, increasing the internal energy and the pressure to expand the core. On the other hand, immediately after the nuclear burning, rapid electron capture reactions proceed, reducing the internal energy and the electron fraction to accelerate core contraction. Moreover, after the central oxygen-\( n \)-neon ignition, the flame front starts to propagate outward, accompanying those important nuclear reactions. Therefore, not only the reactions but also the flame propagation and their interplay must be taken into consideration to understand the hydrodynamic evolution of the critical-mass ONe core.

Flame propagation is a successive process, in which nuclear burning recursively takes place at a region just above the front. Therefore, flame propagation can be driven by efficient heat transfer that transports the energy from the hot ash region into the cold fuel region to trigger the subsequent nuclear reactions. And if the front propagation is predominantly powered by a certain mechanism of heat transfer, it is referred to as a deflagration.

So far, heat conduction at the flame front has been considered as a main driving mechanism of the propagation of the deflagration in an ONe core. In this case, heat transfer results from countless energy exchanges by high-energy relativistic electrons that travel between the hot and cold regions at a microscopic scale. However, it is difficult to resolve the flame structure in a global simulation because the length scale of the flame structure of \( \sim 10^{-4} \) cm (Timmes & Woosley 1992) is far smaller than the system scale of \( \sim 10^{8} \) cm. Besides, the deflagration velocity in reality may be faster than the 1D laminar flame velocity, when a multidimensional corrugation effect on the flame front is considered. Therefore, one needs to apply a certain degree of approximation for the flame propagation in a global simulation of an ONe core, if the flame propagation is driven by the heat conduction.

In the pioneering work by Miyaji et al. (1980), who have investigated the collapse of a critical-mass ONe core using a 1D core model composed of oxygen, neon, and magnesium, the propagation has been modeled by setting a simple propagation speed by Nomoto et al. (1976). In work by Nomoto (1984, 1987) that used a more realistic helium star model and work by Takahashi et al. (2013) with a full stellar model, the efficiency of the heat transportation by turbulent mixing is estimated using the time-dependent mixing-length theory developed by Únno (1967), which reduces the intrinsically multidimensional and highly nonlinear properties of turbulent mixing into two simple time-differential equations.

While these previous works have found that the ONe core with a high ignition density of \( \rho_{\text{ig}} \gtrsim 2.4 \times 10^{10} \) g cm\(^{-3}\) finally collapses as a result of efficient energy reduction due to the neutrino radiation and electron reduction due to the electron capture, only the early core collapse before the central densities reach \( \rho_c \lesssim 1 \times 10^{11} \) g cm\(^{-3}\) has been calculated. In order to consistently describe the core collapse and the succeeding explosion, one needs to follow the evolution up to further high densities ideally until the formation of a nascent NS at \( \rho_c \gtrsim 1 \times 10^{14} \) g cm\(^{-3}\). The requirements are to incorporate the nuclear equation of state (EOS) and to handle complicated interactions between neutrinos and matter.

The purpose of this work is thus to investigate the late evolution of a collapsing ONe core by conducting a hydrodynamical simulation as consistent as possible. The hydrodynamic code used in this work incorporates effects of nuclear burning, electron capture reactions in the nuclear statistical equilibrium (NSE) region, and complicated neutrino transfer as well. We utilize two different progenitor models for the initial condition. In order not to break the hydrostatic structure that the ONe core should initially have, we newly calculate stellar evolution of a \( 9.0 \, M_\odot \) super-AGB star as in Takahashi et al. (2013), which has an ignition density of \( 1.76 \times 10^{10} \) g cm\(^{-3}\), using the same EOS as the hydrodynamical calculation. Special care has been taken for the remapping process as well. The other progenitor is the model calculated by Nomoto (1984, 1987), which has an ignition density of \( 2.4 \times 10^{10} \) g cm\(^{-3}\) and has been widely used as the only progenitor model for ECSNe in the community.

Three limitations exist in this work. The first is omission of electron captures by intermediate-mass elements such as \( ^{24}\text{Mg} \) and \( ^{20}\text{Ne} \). In order to minimize the effect, we use the stellar structure just before the ignition of oxygen and neon as the initial condition for the new progenitor model. The second and third limitations are omissions of heat transfer by heat conduction and of the multidimensional effects of turbulence at the flame front. Discussions on these limitations are made in the text.

Apart from the physical limitations, there is a debatable uncertainty in the progenitor evolution. That is, if no effective matter mixing takes place after the initiation of the electron capture on \( ^{20}\text{Ne} \), the central temperature suddenly increases and leads to the ignition of the central O+Ne with a low ignition density of \( \sim 9 \times 10^{9} \) g cm\(^{-3}\) (Miyaji & Nomoto 1987; Canal et al. 1992; Gutiérrez et al. 1996; Schwab et al. 2015). Recent multidimensional simulations indicate that explosive mass ejection similar to the thermonuclear explosion can take place with such a low ignition density (Jones et al. 2016; Leung & Nomoto 2017; see also Nomoto & Kondo 1991; Isern et al. 1991). In this context, our progenitor model having an ignition density of \( 1.76 \times 10^{10} \) g cm\(^{-3}\) would be destined for a core collapse.

The paper is organized as follows. Description of the radiation hydrodynamic code is given in Section 2. We explain the initial conditions in Section 3. Results of hydrodynamic calculations are reported in Section 4 for the new progenitor model and in Section 5 for Nomoto’s progenitor. In Section 6, possible contributions of other neutrino reactions to the flame propagation are examined, and then the effect of turbulent corrugation on the conductive flame propagation in the ONe core is discussed. Conclusions are drawn in Section 7.

### 2. Radiation Hydrodynamic Code

The explosion is simulated by a 1D time-implicit Lagrangian general relativistic radiation hydrodynamic code (Yamada 1997; Yamada et al. 1999). This code has been utilized to study the FeCCSNe from core massive stars (Sumiyoshi et al. 2005, 2007, 2008; Nakazato et al. 2007, 2013). The code comprises the approximate Riemann solver by the Roe method. Four flavors of neutrino, electron-, anti-electron-, \( \mu \)-/\( \tau \)-, and anti-\( \mu \)/\( \tau \)-neutrino, are considered in this work. The neutrino transport is formulated based on the Boltzmann equation, in
which the evolution of the neutrino distribution function in the 3D phase space (mass coordinate × neutrino energy × azimuth angle from the radial direction) is solved. The mass coordinate is in common with both hydrodynamic and neutrino transfer equations. Resolutions are 511 grid points for the mass coordinate, 14 grid points for the neutrino energy, and 6 grid points for the azimuth angle. Below we describe extensions of the code for the current study.

2.1. Treatment of Non-NSE Compositions

In the collapsing phase of the ONe core, nuclear burnings modify the original non-NSE chemical composition (e.g., oxygen–neon and carbon–oxygen) to achieve the NSE. As a result of the phase transition, the matter entropy increases and a rapid electron capture reaction initiates. In order to properly deal with these phenomena, the evolution of chemical composition is carefully treated in this work.

Two types of EOSs are included in the hydrodynamic code. One is a nuclear EOS based on a relativistic mean field theory (the STOS EOS; Shen et al. 1998). Since reaction equilibrium among baryons is assumed in the nuclear EOS, it is applicable for a region where NSE is realized. For non-NSE regions, a composition-dependent EOS is imported from the stellar code (Takahashi et al. 2016). The stellar EOS consists of ideal gases of radiation, electron and positron, proton, neutron, alpha-particle, and heavy nuclei. A consistency check between the two EOSs has been done for a wide parameter range, which is provided in Appendix A.

The EOS comparison shows that the two EOSs can be smoothly connected if the switching temperature is adequately determined. The switching over the two EOSs in the current work is carried out in a simple temperature–density-dependent manner. A transition region is defined in a temperature–density plane by \( \Lambda_{\text{trans}} = \Lambda_2 - \Lambda_1 \), where \( \Lambda_i \) is \( \{(T, \rho)|T < T_i, \rho < \rho_i\} \), with \( T_1 = 5.80 \times 10^9 \text{K} (=0.5 \text{MeV}) \) and \( T_2 = 9.28 \times 10^9 \text{K} (=0.8 \text{MeV}) \) and \( \rho_1 = 3 \times 10^{12} \text{g cm}^{-3} \) and \( \rho_2 = 7 \times 10^{13} \text{g cm}^{-3} \). A weight function \( W(T, \rho) \) is defined as

\[
W(T, \rho) = \begin{cases} 
\max \left( \frac{\log T - \log T_1}{\log T_2 - \log T_1}, \frac{\log \rho - \log \rho_1}{\log \rho_2 - \log \rho_1} \right) & (T, \rho) \in \Lambda_{\text{trans}} \\
0 & (T, \rho) \in \Lambda_1 \\
1 & \text{otherwise}
\end{cases}
\]

so as to connect zero and unity in the transition region. Using \( W_t \), a thermodynamic quantity \( f \) is calculated as \( f = (1 - W_t)f_{\text{stellar}} + W_t f_{\text{STOS}} \), where \( f_{\text{stellar}} \) or \( f_{\text{STOS}} \) are quantities derived by each EOS.

Chemical composition in a non-NSE region is described by 49 species of isotopes (Table 1). For a low-temperature region of \( T < 10^{9.7} \text{K} \), evolution of chemical composition is calculated by solving a reaction network,

\[
\frac{dY_i}{dt} = Y_i(T, \rho, Y_j)
\]

\[
= -\lambda_{j \rightarrow i} Y_i + \lambda_{j \rightarrow i} Y_j - \sum_{j \neq k} \lambda_{j \rightarrow k} Y_i Y_j \\
+ \sum_{j \neq k} \lambda_{k \rightarrow i} Y_k \cdots ,
\]

where \( Y_i \) is the mole fraction of the \( i \)th isotope and \( \lambda \) are the reaction rates. Otherwise, chemical-potential-based NSE equations,

\[
\mu(A, Z) = Z \mu_p + (A - Z) \mu_n, \tag{4}
\]

are solved. Here \( A \) and \( Z \) are the mass and the proton numbers, and \( \mu(A, Z), \mu_p \), and \( \mu_n \) are chemical potentials of \( (A, Z) \) isotope, proton, and neutron, respectively. These equations are simultaneously and iteratively solved with other hydrodynamic equations. For this purpose, mole fractions \( Y_i \) are added to a set of independent variables in our code.

Note that nuclear weak reactions such as electron captures and beta decays are not treated in the nuclear reaction network but included in the Boltzmann equation. This causes an inconsistency between \( Y_e \) calculated by the reaction network, which becomes constant for each mass grid, and that calculated by the Boltzmann equation. We use \( Y_e \) determined by the Boltzmann equation for the hydrodynamical calculation. Consistent treatment between the nuclear reaction network and the Boltzmann equation will be done in the future.

2.2. Reaction Kernel of the Electron-type Neutrino Absorption on Nuclei

The collision term in the Boltzmann equation in this work is composed of six nuclear weak reactions and three thermal pair emissions (Friman & Maxwell 1979; Maxwell 1987; Brueenn 1985; Braaten & Segel 1993; Mezzacappa & Brueenn 1993; Yamada et al. 1999; Sumiyoshi et al. 2005). Nuclear weak reactions are as follows: electron-type neutrino absorption on neutron (and its inverse reaction), electron-type anti-neutrino absorption on proton and its inverse reaction, electron-type neutrino absorption on nuclei (and its inverse reaction), neutrino–nucleon scattering, neutrino–electron scattering, and neutrino–nuclei coherent scattering. Thermal pair emissions are as follows: electron–positron pair processes, plasmon processes, and bremsstrahlung. Among them, treatment of the electron-type neutrino absorption on nuclei, or electron capture on nuclei in other words, is improved in the present study.

Emission and absorption kernels, \( R^e \) and \( R^n \), appear in the collision term as

\[
\left( e^{-\varepsilon_{\nu}} \frac{\partial f_{\nu}}{\partial t} \right)_{\text{coll}} = R^e (1 - f_{\nu}) - R^n f_{\nu}, \tag{5}
\]

where \( c \) is the speed of light, \( \varepsilon_{\nu0} \equiv e^{2\varepsilon_0} \) is the (0,0) component of the metric, and \( f_{\nu} \) is the neutrino distribution function. The absorption kernel is related to the emission kernel as

Table 1

| Element | 49 Isotopes Included in the Hydrodynamic Code |
|---------|---------------------------------------------|
| n       | Ne 20                                      |
| H       | He 23                                      |
| He      | Mg 24                                      |
| Li      | Al 27                                      |
| Be      | Si 28                                      |
| B       | P 31                                       |
| C       | S 32                                       |
| N       | Cl 35                                      |
| O       | Ar 36                                      |
| F       | K 39                                       |
The Rankin–Hugoniot relation as
\[ e^{-\phi} \frac{\partial \epsilon}{\partial t} = \left[ e^{-\phi} \frac{\partial \epsilon}{\partial t} \right]_{\text{shock}} - \tau Q \]  
(11)

\[ \left[ e^{-\phi} \frac{\partial \epsilon}{\partial t} \right]_{\text{shock}} = -\frac{1}{\Gamma} \frac{\partial}{\partial \eta} (4\pi^2pU) - \frac{h}{\Gamma^2} e^{-\phi} \frac{\partial}{\partial \eta} \left( \frac{1}{2} U^2 \right) \]

\[ + \frac{h}{\Gamma^2} 2\pi e^{-\phi} \frac{\partial^2}{\partial \eta^2} (p + p_e), \]

\[ - \frac{1}{\Gamma} \tau Uq + \frac{p}{\Gamma} A_4 \pi \tau F_e, \]

(12)

where \( \Gamma, U \equiv e^{-\phi} \frac{\partial \epsilon}{\partial t}, h \equiv 1 + \epsilon + p \tau \), and \( \bar{m} \) are the general relativistic gamma factor, the radial fluid velocity, the specific enthalpy, and the gravitational mass, respectively, and \( p, q \), and \( F_e \) are quantities related to neutrinos (see Yamada 1997; Yamada et al. 1999, for detailed definitions).

The difference between \( e^{-\phi} \frac{\partial \epsilon}{\partial t} \) and \( \left[ e^{-\phi} \frac{\partial \epsilon}{\partial t} \right]_{\text{shock}} \) expresses the effect of shock heating. Hence, we define
\[ \tau Q_{\text{shock}} = \left[ e^{-\phi} \frac{\partial \epsilon}{\partial t} \right]_{\text{shock}} - e^{-\phi} \frac{\partial \epsilon}{\partial t} \]
(13)
as the shock heating rate. Accordingly, we reformulate the entropy equation as
\[ e^{-\phi} T \frac{\partial s}{\partial t} = -e^{-\phi} \mu \frac{\partial Y_e}{\partial t} - e^{-\phi} \sum_{\text{ion}} \mu_i \frac{\partial Y_i}{\partial t} + \tau Q \]

\[ + \tau Q_{\text{shock}} \times i_{\text{shock}}, \]

(14)

where \( i_{\text{shock}} \) is a switching function defined as
\[ i_{\text{shock}} = \begin{cases} 1 & (s_t > 1) \\ 0 & \text{(otherwise)} \end{cases} \]

(15)

### 3. Initial Conditions

#### 3.1. Model T9.0

We have calculated a new progenitor model, which hereafter is referred to as model T9.0. Using a stellar evolution code described in Takahashi et al. (2014), the evolution of a solar-metallicity, 9.0\( M_\odot \) model is calculated from the pre-main-sequence phase until just before the oxygen ignition at the center of the ONe core. A reaction network of 62 isotopes is solved in the evolution code, in which electron capture reactions by\(^{20}\)Ne, \(^{20}\)F, \(^{24}\)Mg, \(^{24}\)Na, \(^{23}\)Na, \(^{25}\)Mg, and \(^{27}\)Al are taken into account. The baryon core mass becomes 1.365\( M_\odot \) at the end of the evolution calculation, but it is slightly reduced to 1.361\( M_\odot \) when the data are mapped onto the hydrodynamic code. The ignition density is 1.76\( \times 10^{10}\) g cm\(^{-3}\). The effect of stellar rotation has not been taken into account in this model. The initial structure used in the hydrodynamical calculation is referred to as model T9.0.
The description of convective matter mixing has been modified. We solve a diffusion equation to consider the mixing of chemical species. We apply the Ledoux criterion for the convective criterion, i.e., dynamically unstable regions are defined according to the condition

$$\nabla_{\text{rad}} > \nabla_{\text{ad}} + \frac{\varphi}{\delta} \nabla_{\mu},$$

where \(\varphi \equiv (\partial \ln \rho / \partial \ln \mu)_{p,T}\) and \(\delta \equiv -(\partial \ln \rho / \partial \ln T)_{p,\mu}\) are thermodynamic functions, \(\nabla_{\mu} \equiv d \log \mu / d \log p\) is the \(\mu\)-gradient, and \(\nabla_{\text{rad}} \equiv (\kappa L / 16\pi c GM)(3P/aT^4)\) and \(\nabla_{\text{ad}} \equiv (\partial \ln T / \partial \ln P)_{p,\mu}\) are the radiative and adiabatic temperature gradients, respectively. For this region, the diffusion coefficient is determined as

$$D_{\text{conv}} = \frac{1}{3} \nu_{\text{mix}} l_{\text{mix}},$$

where \(\nu_{\text{mix}}\) and \(l_{\text{mix}} \equiv \alpha_{\text{mix}} H_p\) are the velocity and the mixing length of convective blobs determined by the mixing-length theory, respectively (Böhm-Vitense 1958). Also, the vibrational instability is assumed to grow in a region of

$$\nabla_{\text{ad}} + \frac{\varphi}{\delta} \nabla_{\mu} \geq \nabla_{\text{rad}} > \nabla_{\text{ad}},$$

and a semi-convective diffusion coefficient of Spruit (1992),

$$D_{\text{sc}} = f_{\text{sc}} \frac{\nabla_{\text{rad}} - \nabla_{\text{ad}}}{(\varphi / \delta) \nabla_{\mu}} D_{\text{therm}},$$

where \(D_{\text{therm}} = (1/c_p \rho)(4acT^3 / 3k(\rho))\) is the thermal diffusivity, is used for the region. The free parameter \(f_{\text{sc}}\) is set to be \(f_{\text{sc}} = 0.3\), which results in semi-convective mixing of intermediate strength (Umeda et al. 1999; Umeda & Nomoto 2008). In addition, the effect of convective overshooting is taken into account for core hydrogen and core helium burning stages. An exponentially decaying function (Herwig 2000) is used to determine an additional diffusion coefficient from the edge of the convective regions as

$$D_{\text{conv,ov}} = D_{\text{conv,0}} \exp \left(-2 \frac{\Delta r}{f_{\text{ov}} H_{p,0}}\right),$$

where \(f_{\text{ov}}\) is an adjustable parameter, \(D_{\text{conv,0}}\) and \(H_{p,0}\) are the convective mixing coefficient and the pressure scale height at the edge of the convective region, respectively, and \(\Delta r\) is the distance from the edge. Parameters are calibrated to explain the position of the red giants \((\alpha_{\text{mix}} = 1.5)\) and the main-sequence width of stars in open clusters \((f_{\text{ov}} = 0.015)\) observed in our Galaxy in the H-R diagram. A star forms a more massive core as a result of inclusion of the overshooting mixing. This is why

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**Figure 1.** Distributions of density (top row), temperature (second row), electron mole fraction (third row), and entropy per baryon (bottom row) of the initial model T9.0. As the horizontal axis, mass coordinates (left) or radii (right) are taken. Plus signs indicate quantities every 10 grids to show the resolution of the calculation.
the initial mass of the current model has been reduced from 10.4–10.8 $M_\odot$ in Takahashi et al. (2013) to 9.0 $M_\odot$.

The evolution calculation has once been halted soon after convective regions in the helium layer and the hydrogen-rich envelope have merged (the dredge-out episode; Iben et al. 1997). The further evolution of the degenerate core is calculated by removing the outer region and by setting new boundary conditions. By assuming that fitting with the highly inflated condensed-type envelope (Chandrasekhar 1939) is always achieved at the surface of the core, two relations of

$$0 = \nabla_{\text{rad}} - 1/4$$  \hspace{1cm} (21) \\
$$0 = 2U + V - 4$$  \hspace{1cm} (22)

are imposed (e.g., Sugimoto & Fujimoto 2000), where $U \equiv 4\pi r^3 \rho / M$ and $V \equiv GM_\rho / PR$ are the homologous parameters. This not only gives a more physically consistent surface structure of the core but also improves the stability of the calculation. The mass of the core is increased with a constant rate of $1.0 \times 10^{-6} M_\odot \text{yr}^{-1}$. The value is fairly consistent with recent estimates of a mass accretion rate of thermal pulses in an SAGB star (Poelarends et al. 2008; Siess 2010). Note that although the rest time until collapse significantly depends on the mass accretion rate, the core structure does not much depend on the rate (Takahashi et al. 2013).

New rates for electron capture reactions by isotopes of $^{20}\text{F}$, $^{20}\text{Ne}$, $^{23}\text{Na}$, $^{24}\text{Na}$, $^{24}\text{Mg}$, $^{25}\text{Mg}$, and $^{27}\text{Al}$ calculated by Suzuki et al. (2016) are applied. Because the data table by Oda et al. (1994) is more sparse especially for the density grid, the steep rise in the electron capture rate around the critical density has not been well resolved. Accordingly, the reaction rates have been underestimated in our previous calculation. As a result of using the new rates, for example, electron capture by $^{25}\text{Mg}$ initiates when the central density reaches $4.5 \times 10^6 \text{g cm}^{-3}$ in the current calculation, which is earlier than the previous result of $7.6 \times 10^6$; usage of those new rates does not significantly alter the result, since both rates agree for much higher density than the critical density.

After the evolution calculation, an SAGB envelope having a physically consistent structure has been recovered on the highly degenerate ONe core. First, the surface structure of the ONe core is reconstructed by integrating four stellar equations (e.g., Kippenhahn & Weigert 1990). The integration starts from a point at which the entropy per baryon becomes $s_b = 2.0k_B$. Note that time derivative terms of $Tds/\text{d}t$ and $dv/\text{d}t$ are set to be zero, as we do not have information of the previous time step. Constant composition is taken from the point (mostly being composed of carbon and oxygen; $X(C) = 0.370$ and $X(O) = 0.581$) and is applied for the core surface layer during the integration. At a point where the boundary condition of $\rho = 2U + V - 4 = 0$ (Sugimoto & Fujimoto 2000) is fulfilled, the luminosity and composition are artificially changed. A hydrogen-rich composition is applied, $X(\text{H}) = 0.70$ and $X(\text{He}) = 0.21$. The luminosity of the envelope is tuned so that a reasonable amount of mass ($\sim 1 M_\odot$) is enclosed inside a reasonable radius ($\sim 100 R_\odot$).

The initial structure has a steep gradient of density and temperature at the boundary between the core and the envelope. In order to resolve the steep gradient, we apply an improved grid reconstruction method, which is described in Appendix B.

3.2. Model NB8: Nomoto’s Progenitor

The other progenitor model we use is a $2.2 M_\odot$ He star model calculated by Nomoto (1984, 1987).\footnote{This model has been often referred to as an “8.8 $M_\odot$ ECSN progenitor” in the SN community (Janka et al. 2008; Fischer et al. 2010; Radice et al. 2017).} We refer to this model as model NB8. This progenitor has formed a cool $1.3769 M_\odot$ ONe core. Prior to collapse, the electron mole fraction of the central $\sim 0.7 M_\odot$ region is slightly reduced to 0.488, while the outer region in the core has a uniform $Y_e$ of 0.50. The central $\sim 0.1 M_\odot$ region has already experienced a passage of the ONe deflagration and has a small electron mole fraction and high temperature and entropy. The ONe core is surrounded by a diffuse hydrogen-rich envelope, which has been attached from a point where the density is $3.54 \times 10^9$ cm$^{-3}$. The envelope extends to $1.09 \times 10^9$ cm and has $Y_e$ of 0.60.

The new grid reconstruction method is also applied for this progenitor to yield well-defined grid points for the hydrodynamic calculation. The result is shown by Figure 2. Due to the remapping process, the core mass of this initial condition is slightly reduced from the original value of $1.3769 M_\odot$ to $1.3703 M_\odot$. The initial condition is not in a hydrostatic equilibrium, probably due to the low $Y_e$ at its center.

4. Result of Model T9.0

Hydrodynamic evolution of model T9.0 is calculated for $0.3942 \text{s}$. Core bounce takes place $0.2830 \text{s}$ after the calculation starts. Hence, hereafter we use the post-bounce time, $t_{pb} = (\text{calculation time}) - 0.2830 \text{s}$, for a time indicator in addition to the calculation time. Trajectories, as well as the evolution of the flame and the shock fronts, are shown in Figure 3 using the post-bounce time.

4.1. Until Core Bounce

4.1.1. Oxygen+Neon Ignition

At $0.1507 \text{s}$ after the calculation begins ($t_{pb} = -0.1323 \text{s}$), oxygen and neon at the center of the star are burned out. Because of the high electron degeneracy, oxygen+neon burning in the ONe core becomes a runaway reaction. The nuclear reaction increases the temperature, but the degenerate pressure only slightly rises at the same time. The rise of the temperature significantly enhances the reaction rate, and the rate of the temperature rise is recursively enhanced. As a result, the reaction proceeds much faster than the hydrodynamical response time. Because of the high temperature, the reaction finally reaches a reaction equilibrium as a steady state, and NSE is established for the chemical composition.

Three causes can contribute to the central temperature increase in the ONe core to trigger the runaway nuclear burning. The first one is adiabatic compression, the second is electron capture reactions onto $^{20}\text{Ne}$ and $^{24}\text{Mg}$, and the third is nuclear reactions of oxygen+neon burning itself. Until the nuclear burning significantly changes the chemical composition, a temperature rise can be separated into two terms:

\[
\frac{\Delta T}{T} = \frac{\Delta T_{\text{comp}}}{T} + \frac{\Delta T_{\text{reac}}}{T} \hspace{1cm} (23)
\]
The density rise increases the temperature through the first term on the right-hand side, and the entropy change due to reactions affects through the second term. For the central grid, time evolution of these two terms until ignition is shown in Figure 4. The assumption that the chemical composition only slightly changes during the temperature rise can be verified since the summation of the two terms, \( \Delta T_{\text{sum}} \equiv \Delta T_{\text{comp}} + \Delta T_{\text{reac}} \), well explains the evolution of the total temperature difference. This figure shows that the central temperature steadily increases not by heating but by compression for the first \(~0.1\) s. Then, after the temperature increases by \(~0.02\) MeV, the runaway heating by the nuclear reaction initiates. The initial central temperature is 0.12 MeV. Therefore, the heating by oxygen+neon burning is estimated to be dominant after the temperature rises to \(~0.14\) MeV. The heating rate exceeds \(~1 \times 10^{16}\) erg g\(^{-1}\) s\(^{-1}\) at this moment and keeps increasing.

Thus, the adiabatic compression importantly increases the central temperature of model T9.0. This is because the initial structure of model T9.0 is nearly but not completely in the hydrostatic equilibrium, so that the core slowly contracts from the start of the calculation. Despite the effort of remapping the structure as consistently as possible, the loss of the hydrostatic equilibrium is likely due to the remapping process from the stellar evolution calculation to the hydrodynamic calculation, because a much longer contraction timescale has been obtained in the evolution calculation. This suggests that the ONe core in reality can be hydrostatic even at the moment of the central ignition. In order to investigate how the different degree of the initial hydrostatic/dynamical stability affects the late core evolution, we have conducted a similar hydrodynamical calculation using an ONe core model in which the central Ye distribution is artificially changed to ensure the initial hydrostatic stability. The result is reported in Section 4.3.

Our hydrodynamical calculation does not include heating by electron capture reactions onto \(^{20}\)Ne and \(^{24}\)Mg. However, this can be well justified because the progenitor structure just before the central ignition is taken from the evolution calculation for the initial condition. Here we give an estimate for the heating effect of the electron capture reaction by \(^{20}\)Ne. The reaction accompanies the other electron capture by \(^{20}\)F, and one sequential electron capture heats the surroundings by \(2\mu_e + \mu_{\nu_{\mu}} - \mu_{\nu_e} - 2E_\nu\), where \(2E_\nu\) shows energy emitted by neutrinos, which are assumed to escape from the system at this stage. Stellar evolution calculation provides values of \(\mu_e \sim 10\) MeV, \(\mu_{\nu_{\mu}} - \mu_{\nu_e} \sim \Delta m_{\nu_{\mu}-\nu_e} c^2 - \Delta m_{\nu_e} c^2 = -10.8\) MeV, and \(2E_\nu \sim 0.8 + 2.7\) MeV with the electron capture rate of \(dY_{\nu_e}/dt \sim 9.0 \times 10^{-5}\) s\(^{-1}\) baryon\(^{-1}\). The equivalent heating rate is \(\sim 4.9 \times 10^{14}\) erg g\(^{-1}\) s\(^{-1}\). Thus, an entropy change during a short period \(\Delta t\) becomes \(\Delta s_k = (\epsilon_{\nu_e} \Delta t)/T \sim 5.1 \times 10^{-4} \times (\Delta t/0.1\) s) baryon\(^{-1}\) with the temperature of \(T \sim 0.1\) MeV.

As the initial entropy of the core is \(s_k \sim 0.5\), this gives \(\Delta T_{\text{heat}}/T \sim 3.2 \times 10^{-3}\) for \(\Delta t = 0.1\) s, which is much smaller than the effect of compression.

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**Figure 2.** Same as Figure 1, but for model N8.8. The distribution of model T9.0 is overlaid as thin dashed lines for the sake of comparison.
4.1.2. Flame Propagation and Neutrino Radiation

As a result of the oxygen-neon burning, the NSE region has an entropy per baryon of $s_k \sim 1.5$, as well as the high temperature of $T \sim 1.1 \text{ MeV} (\sim 1.3 \times 10^{10} \text{ K})$. The NSE region is surrounded by a still cold ($s_k \sim 0.5$ and $T \sim 0.1 \text{ MeV} \sim 1.2 \times 10^9 \text{ K}$) ONe region. In this simulation, the boundary layer that connects the hot ash and cold fuel is resolved by a 1D zoning with a radial resolution of $\gtrsim 10^6 \text{ cm}$. With this resolution, the boundary layer looks like a discontinuity surface in terms of temperature and chemical composition, which is hereafter referred to as a flame front.

As time passes, the flame front moves outward and finally reaches $\sim 1.0 \, M_\odot$ by core bounce. The propagation speed of the flame front is shown in Figure 5. The propagation velocity in the observer frame, $V_{\text{obs}}$, is calculated as the time derivative of the radius of the flame front, $r_f$. To do so, the original sawtooth-shape data of $r_f$ (shown by the dashed line in the top panel) are numerically smoothed to make continuous data (solid line). The contraction speed at the flame front, $V_{\text{cont}}$, is calculated every 0.01 s before the core bounce. Finally, the flame propagation velocity in the comoving frame, or the local propagation velocity $V_{\text{prop}}$, is calculated as $V_{\text{prop}} = V_{\text{obs}} - V_{\text{cont}}$.

Until $t_{\text{pb}} \sim -0.04 \text{ s}$, or until the flame front passes the inner $0.3 \, M_\odot$, $V_{\text{prop}}$ roughly keeps a constant value of $\sim 2 \times 10^8 \text{ cm s}^{-1}$ except for the first ignition phase. Later, the local propagation velocity and the contraction velocity are accelerated. $V_{\text{prop}}$ reaches $9.1 \times 10^8 \text{ cm s}^{-1}$ at core bounce; however, the local propagation Delta T [10^2 MeV]
0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16
calculation time [sec]

Delta T [10^2 MeV]
0 1 2 3 4 5

Figure 3. Trajectories of Lagrangian grids are shown for model T9.0. Grids of 0.100, 1.000, and 1.360 $M_\odot$ are shown by black thick lines, and the boundary between CO and He compositions (1.36145 $M_\odot$) is shown by the red thick line. Radii of the flame and shock fronts are shown by thick orange and thick green lines, respectively.

Figure 4. Details of the prior temperature rise at the center of the star, calculated for model T9.0. See the text for definitions of each temperature difference.

Figure 5. Time evolution of the radius of the flame front (top) and the propagation velocities (bottom). For the flame radius, the original data are shown by the dashed line, while the smoothed data are shown by the solid line. Gray lines are trajectories of arbitrarily selected Lagrangian grids (thin) or 0.1 and 0.3 $M_\odot$ (thick). For the flame velocities, lines are the propagation velocity in the observer frame (red solid), the contraction velocity at the flame radius (green dashed), and the local propagation velocity (blue dotted). The contraction velocity is calculated every 0.01 s before the core bounce, which is indicated by points in the figure.
velocity never exceeds the sound velocity of the core, \( \sim 1 \times 10^9 \text{cm s}^{-1} \). Thus, a supersonic mode of flame propagation (detonation) does not take place here. Note that a negative bump of \( V_{\text{det}} \) can be seen at \( t_{\text{pb}} \sim -0.017 \) to \(-0.016 \text{ s} \). This is because the flame front is locally trapped at \( \sim 0.7M_\odot \). There is a temperature discontinuity, which is made by a convection powered by \(^{20}\text{Ne}\) electron capture during a previous evolutionary stage. Although the existence of this discontinuity itself is debatable (e.g., Schwab et al. 2015), the stagnation of the flame front will have a minor effect for core collapse, since the collapse has already begun at this moment.

Evolution of distributions of density, temperature, electron mole fraction, and entropy per baryon until core bounce is shown in Figure 6. Inside the NSE region, fast electron capture on free protons rapidly reduces \( Y_e \). The characteristic timescale depends on the mass fraction of the free protons. In a region where the electron mole fraction is still larger than \( \sim 0.4 \), typically a mass fraction of \( 10^{-2} \) to \( 10^{-3} \) exists as free protons. This gives the electron reduction timescale of 0.01–0.1 s for a typical density of \( \sim 1 \times 10^{10} \text{g cm}^{-3} \). The proton mass fraction decreases with decreasing \( Y_e \). As a result, electron capture by heavy nuclei becomes important in a region with \( Y_e \lesssim 0.36 \). The electron capture reaction plays an important role for the core contraction. First, it reduces the degenerate pressure, by which the core is supported. Second, a significant amount of energy is radiated away by neutrino emission when electron capture takes place.

In Figure 7, evolution of neutrino luminosities and mean neutrino energies recorded at the grid having an initial radius of \( 3 \times 10^4 \text{cm} \) (\( \sim c \times 0.01 \text{ s} \)) is shown for three types of neutrinos: electron-type (red), anti-electron-type (green), and mu- and tau-type neutrinos (blue). The ONe core starts to radiate \( \nu_e \) just after the central ignition of oxygen and neon. The \( \nu_e \) radiation is mainly due to the rapid electron capture by free protons and lasts for \( \sim 0.1 \text{ s} \). Even before the core bounce takes place, the luminosity exceeds \( L_{\nu_e} > 10^{51} \text{erg s}^{-1} \).

Similar to \( \nu_e \), other flavors of \( \bar{\nu}_e \) and \( \nu_{\mu,\tau} \) (as well as \( \bar{\nu}_{\mu,\tau} \)) are emitted from the central NSE region before the core bounce. In our simulation, these emissions are largely due to thermal pair emissions of \( e^+e^- \) pair annihilation and plasmon decay, so that their luminosities of \( L_{\nu} < 10^{46} \text{erg s}^{-1} \) are much smaller than that of \( \nu_e \). Recent works have revealed that \( \beta^+ \) decay of NSE isotopes enhances pre-core-bounce emission of \( \nu_e \) for both FeCCSNe and ECSNe (Kato et al. 2017; Patton et al. 2017). The result for an ECSN in Kato et al. (2017), however, shows that the \( \nu_e \) luminosity resulting from the \( \beta^+ \) decay is only one order of magnitude larger than that of the thermal processes. Considering the far more energetic \( \nu_e \) emission by the electron capture, the \( \beta^+ \) decay does not affect the pre-collapse hydrodynamic evolution of the ONe core.

Shortly after the core bounce, the neutrino burst takes place. The peak luminosities reach \( 3.5 \times 10^{53} \text{erg s}^{-1} \) for electron-type neutrino and \( \sim 3 \times 10^{52} \text{erg s}^{-1} \) for other types as well. The \( \nu_e \) and \( \bar{\nu}_e \) mainly originate from electron capture and positron capture reactions, and thermal processes of \( e^+e^- \) pair process and bremsstrahlung are responsible for \( \nu_\mu \) and \( \nu_\tau \) emissions. \( \beta^+ \) decays and positron captures on NSE isotopes might be important for \( \bar{\nu}_e \) emission in the neutrino burst, though these have not been investigated in detail so far. Note that the increase in luminosities and mean energies at \( t_{\text{pb}} \sim 0.1 \text{ s} \) is caused by sudden acceleration of the referenced grid up to the speed of light.

**Figure 6.** Evolution of distributions of density (top), temperature (second top), electron mole fraction (third top), and entropy per baryon (bottom) in model T9.0 until core bounce takes place is shown. Thick gray lines are distributions at core bounce, while others are distributions every 0.01 s before the core bounce.

**Figure 7.** Time evolution of neutrino luminosities and mean neutrino energies for model T9.0. Red, green, and blue lines correspond to electron-type, anti-electron-type, and mu- and tau-type neutrinos, respectively. Arrows indicate the time of ignition (left, at \( t_{\text{pb}} = -0.1323 \text{ s} \)) and of core bounce (right).
from the ONe core collapse are expected to be more than one order of magnitude larger than Fe core collapse, while the number of $\nu_e$ events of ONe core collapse is much less, if they take place at the same distance of 200 pc from Earth. This work provides the theoretical understanding of differences between the two cores.

The other important consequence of the phase transition is that the adiabatic index $\gamma$ is significantly lowered when the region becomes NSE. In Figure 8, evolution of distributions of the density (top), the velocity (middle), and the adiabatic index (bottom) is shown. Times are selected when the ignition takes place at the center ($t_{\nu_b} = -0.1323$ s; light-yellow) and when the flame front reaches 0.1, 0.3, 0.5, and 0.7 $M_e$. This figure clearly shows that the hydrodynamical instability due to the photodisintegration, which is known to trigger the core collapse of an Fe core, also develops in the ONe core. Core contraction is accelerated by this instability, and runaway collapse takes place in the end.

### 4.1.3. Effect of Neutrino–Electron Scattering on the Flame Propagation

The flame front propagates as the temperature of the flame-above region first increases and successively a runaway nuclear reaction sets in, combusting the fuel into ash. Because of the runaway nature of the nuclear reaction, the overall timescale of the flame propagation is mainly determined by the timescale of the mechanism that is responsible for the prior temperature rise.

Heat conduction at the flame front has been considered as a main driving mechanism of the laminar flame in an ONe core. Applying Equation (44) in Timmes & Woosley (1992), the laminar flame driven by heat conduction in our calculation is estimated to have a slow velocity of $\lesssim 7 \times 10^3$ cm s$^{-1}$ until $t_{\nu_b} = -0.05$ s because of the small oxygen mass fraction of $X(\text{O}) \sim 0.48$. This is more than one order of magnitude less than the local flame propagation velocity of $V_{\nu e} \sim 10^6$ cm s$^{-1}$ obtained in this work. This will not only give a justification for omitting heat conduction from our calculation but also indicate the existence of other driving mechanisms of the flame front propagation in the ONe core. Note that the propagation velocity of the conductive flame can be enhanced owing to the burning front corrugation; therefore, the above estimate actually gives the lower limit of the propagation velocity of the conductive flame. We will briefly discuss this effect later in Section 6.2.

In order to find what kind of mechanisms are operating in this simulation, the detail of the local temperature rise is analyzed as shown in Figure 9. Differences between the initial values and the values when the local temperature exceeds a critical temperature of 0.16 MeV are used to calculate $\Delta T$, $\Delta T_{\text{comp}}$, and $\Delta T_{\text{react}}$ in this figure. Hereafter we refer to the temperature rise up to the 0.16 MeV as the prior temperature rise, since the nuclear heating rate exceeds $\sim 3 \times 10^{17}$ erg g$^{-1}$ s$^{-1}$ at this point and a time to reach NSE becomes less than $\sim 10^7$ s after that. This figure shows that the adiabatic compression is the main effect for the prior temperature rise for the inner $\sim 0.7 M_e$ region. In this meaning, this flame propagation is not a pure deflagration, in which the flame front propagation is predominantly powered by a certain mechanism of heat transfer. Meanwhile, not only compression but also heating by reactions plays an important role in our simulation as well, especially for the outer region of $\gtrsim 0.2 M_e$. The heating term even overcomes the other for the outer region of $> 1.00 M_e$.

We have found that neutrino–electron scattering is the most contributing reaction for the prior heating. Because of the fast electron capture reactions, the inner NSE region radiates high-energy electron-type neutrinos with a considerable luminosity (Figure 7). A part of these neutrinos hit the surroundings,
providing energy to heat up the material. Distributions of heating rates and the energy flux of the electron-type neutrino taken at $t_{pb} = -0.08$ and $-0.04$ s are shown in Figure 10. The heating rate by oxygen+neon burning shown as $\epsilon_{\nu,e}$ has a sharp peak in front of the flame front and a steep decline in the outer region. On the other hand, the electron scattering shown as $\epsilon_{\nu,\gamma}$ widely heats the flame-above region with a heating rate of $\sim 3 \times 10^{17}$ erg g$^{-1}$ s$^{-1}$. Hence, the local temperature of the flame-above region increases owing to the combination of the compression and the neutrino–electron scattering. The prior temperature rise leads to the runaway nuclear burning when the local temperature exceeds 0.16 MeV, with which the nuclear heating rate corresponds to $\sim 3 \times 10^{17}$ erg g$^{-1}$ s$^{-1}$.

One may have suspicions about the high efficiency of the neutrino–electron scattering. Indeed, the reaction has a small cross section of $\sigma_{\nu,e} \sim 0.06 \times \sigma_0 \left( \frac{E_{\nu}}{m_e c^2} \right)^2 \left( \frac{E_e}{m_e c^2} \right)$, where $E_{\nu}$ is the scattered neutrino energy and $\sigma_0 = \frac{1}{4} \left( \frac{m_e c^2}{\pi} \right)^2 \left( \frac{g_e}{2} \lambda_e \right)^2 = 1.76 \times 10^{-44}$ cm$^2$ (Shapiro & Teukolsky 1986). The mean neutrino energy during this phase is $E_{\nu} \sim m_e \sim 8$ MeV (see Figure 7). Thus, the cross section becomes $\sigma_{\nu,e} \sim 2.7 \times 10^{-43}$ cm$^2$, and the corresponding neutrino mean free path is $l_{\nu,e} = \frac{1}{n_e \sigma_{\nu,e}} \sim 1.23 \times 10^3$ cm for $\rho Y_e = 0.5 \times 10^{10}$ g cm$^{-3}$. This is 100 times larger than the radius of the flame propagation region of $\sim 10^2$ cm.

Nonetheless, the heating rate of $\sim 3 \times 10^{17}$ erg g$^{-1}$ s$^{-1}$ can be estimated as follows. First, the neutrino energy flux $F_\nu$, at the flame front of the radius $r_f$ is estimated as

$$4\pi r_f^2 F_\nu = 4\pi r_f^2 E_\nu D_{ec} \lambda_{ec},$$

where $D_{ec}$ is the thickness of an electron capture region and $\lambda_{ec}$ is the electron capture rate per unit volume. Since the electron capture is rapid, the thickness can be estimated as $D_{ec} = V_f \tau_{ec}$, with the flame propagation velocity $V_f$ and the timescale of the electron capture $\tau_{ec}$. As $\lambda_{ec} = \rho \frac{Y_e}{m_e} \tau_{ec}$, this yields

$$F_\nu = \frac{V_f \rho Y_e}{m_e \tau_{ec}}.$$ 

(26)

Supposing that 50% of the energy of the neutrino is passed to the electron by this scatter, an energy deposit rate of a neutrino that travels a short length $\Delta r$ relates to the energy flux as

$$4\pi r_f^2 \Delta r \rho \epsilon_{\nu,e} = 0.5 \times 4\pi r_f^2 F_\nu \Delta r / l_{\nu,e},$$

(27)

and the rate reduces to

$$\epsilon_{\nu,e} = 0.5 \times \frac{V_f Y_e}{m_e} \rho \sigma_{\nu,e}$$

(28)

at the flame front. Provided that $V_f = 2 \times 10^8$ cm s$^{-1}$, $E_\nu = 8$ MeV, $Y_e = 0.5$, and $\rho = 1 \times 10^{10}$ g cm$^{-3}$, this gives $\epsilon_{\nu,e} \sim 3.13 \times 10^{17}$ erg g$^{-1}$ s$^{-1}$ and well explains the simulation result. The radius dependence may be obtained by multiplying by a factor of $(r/r_f)^2$.

In order to confirm the importance of the contribution of the neutrino–electron scattering, another hydrodynamical calculation until core bounce is conducted using model T9.0 deactivating the neutrino–electron scattering in the surrounding ONe region. The radial resolution is reduced to 255 grid points in this additional calculation. The reduction effect is minimized, as the outermost grid is set to be at $3 \times 10^8$ cm, and we have confirmed that this resolution is enough to reproduce a very similar flame propagation speed for the case with the neutrino–electron scattering. As expected, the adiabatic compression now explains the vast majority of the prior temperature rise. The time until core bounce from the central ignition increases from the original 0.1322 to 0.1463 s, and furthermore, the extension rates in terms of both the radius and the enclosed mass of the flame front are reduced in the case without the neutrino–electron scattering (Figure 11).

The potential importance of neutrino–electron scattering has been discussed by Chechetkin et al. (1976, 1980) for a
degenerate ONe core. In this work, we show that this mechanism actually effectively works in a highly degenerate ONe core, in which higher efficiency than in a CO core is achieved by the higher density and the higher neutrino energy and luminosity. Note that electron capture on $^{20}$Ne only partly accounts for the post-shock material is significantly heated by the shock heating owing to coarse radial resolution in that region. This is why we have decided to focus on the early collapse phase of a highly degenerate ONe core in this work. Detailed results and discussions for the explosion properties will be reported in the near future.

Core bounce leaves a nascent NS at the center of the star (we refer to the inner high-density region with $p > 10^{11}$ g cm$^{-3}$ as the nascent NS hereafter). The nascent NS initially has a baryon mass of 0.4 $M_{\odot}$ and successively grows by continuous mass accretion. A strong bounce shock develops from the surface and propagates outward. The propagation speed is initially fast, and the shock passes through the inner $\sim 1.0 M_{\odot}$ region within 0.007 s. Then, it decelerates, passing the next $\sim 0.36 M_{\odot}$ in 0.063 s. The newly born proto-NS radiates a significant amount of energy by neutrino radiation. The strong neutrino irradiation heats the accreting matter, keeping the flame front at a radius of $\sim 2 \times 10^{7}$ cm (see Figure 3).

Mass accretion gradually ceases around $t_{pb} = 0.1$ s. With the decreasing ram pressure of the accretion flow, the shock rapidly accelerates to nearly the speed of light. At $t_{pb} = 0.1$ s, the shock completely passes through the core. At this moment, material of $\sim 0.02 M_{\odot}$ exists between the shock front and the surface of the nascent NS, and part of this is already unbound as enough energy has been provided by shock heating and neutrino reactions. The growing explosion energy, which is calculated as a sum of the thermal, kinetic, Newtonian gravitational, and nuclear binding energies of the unbound material, has already exceeded $\sim 4 \times 10^{50}$ erg and is already larger than the binding energy of the envelope of this progenitor model of $\sim 1.4 \times 10^{50}$ erg. Thus, our calculation confirms the successful explosion from the highly degenerate ONe core progenitor.

### 4.2. After Core Bounce

Here we give a short summary of the later result from the core bounce until the shock front passes the original core surface of $\sim 10^{6}$ cm at $t_{pb} \lesssim 0.1$ s. Although it is desired to conduct a longer simulation up to $t_{pb} \sim 1$ s to determine the explosion properties such as the explosion energy and the remnant mass (see Janka et al. 2008), our code has encountered a serious resolution problem after $t_{pb} \gtrsim 0.1$ s, in which the post-shock material is significantly heated by the shock heating owing to coarse radial resolution in that region. This is why we have decided to focus on the early collapse phase of a highly degenerate ONe core in this work. Detailed results and discussions for the explosion properties will be reported in the near future.

### 4.3. Core Collapse of a $Y_e$ Modified Progenitor

The adiabatic compression plays an important role for triggering oxygen-neon burning in model T9.0 even for the central ignition. This is because this initial condition is not in a complete hydrostatic equilibrium. The effect of the adiabatic compression will be minimized if the initial condition is hydrodynamically stable. In order to investigate how core collapse can be modified in this case, a $Y_e$ modified progenitor model is additionally set, and the core collapse is calculated until core bounce takes place.

This model is referred to as model T9.0ye in this work. Based on model T9.0, the inner $Y_e$ distribution is artificially increased from its original value 0.489 to 0.492. Model T9.0ye has an almost exact hydrostatic equilibrium: the structure is maintained more than $10^{3}$ s under a calculation without

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The conductive mean free path is estimated using the opacity $\kappa$ and the specific heat at constant pressure $C_p$ as $l_{cond} \sim 4aT^2/\rho C_p \kappa$. The following figure shows the time evolution of the central densities (top), the radii of the flame fronts (middle), and the mass coordinates of the flame front (bottom) estimated using the opacity $\kappa$, which higher efficiency than in a CO core is achieved by the higher density and the higher neutrino energy and luminosity. Note that electron capture on $^{20}$Ne only partly accounts for the post-shock material is significantly heated by the shock heating owing to coarse radial resolution in that region. This is why we have decided to focus on the early collapse phase of a highly degenerate ONe core in this work. Detailed results and discussions for the explosion properties will be reported in the near future.

![Figure 11](image-url) Time evolution of the central densities (top), the radii of the flame fronts (middle), and the mass coordinates of the flame front (bottom) estimated using the opacity $\kappa$, which higher efficiency than in a CO core is achieved by the higher density and the higher neutrino energy and luminosity. Note that electron capture on $^{20}$Ne only partly accounts for the post-shock material is significantly heated by the shock heating owing to coarse radial resolution in that region. This is why we have decided to focus on the early collapse phase of a highly degenerate ONe core in this work. Detailed results and discussions for the explosion properties will be reported in the near future.
reactions. When the nuclear reaction is switched on, the high initial central temperature of $1.60 \times 10^9$ K allows oxygen and neon to burn within $7.4 \times 10^{-7}$ s from the initiation of the calculation. The ignition density becomes $1.63 \times 10^{10}$ g cm$^{-3}$. Core bounce takes place 0.3758 s after the initiation of the calculation. Hence, the model has a longer pre-bounce neutrino radiation phase of $\sim 0.30$ s than model T9.0.

Details of the prior temperature rise are shown in Figure 12. The temperature rise for the inner $\leq 0.2 M_\odot$ is largely explained by the neutrino heating. The contribution from the adiabatic compression is only minor. This means that, even though the adiabatic compression is almost absent, the neutrino heating alone can drive the flame propagation in this earlier phase. On the other hand, both the adiabatic compression and heating by the neutrino scattering cause the temperature rise for the outer region of $\geq 0.3 M_\odot$ similar to model T9.0.

The evolution of the propagation velocity is shown in Figure 13. Because of the smaller compression rate, the early flame propagation takes place much slower than in model T9.0, and it takes about three times longer to propagate the inner $0.3 M_\odot$. Because the neutrino heating rate depends on the propagation velocity (Equation (28)), the slow velocity lowers the neutrino heating rate. For instance, $c_v \sim 1.3 \times 10^{17}$ erg g$^{-1}$ s$^{-1}$ when the flame front reaches 0.1 $M_\odot$. After the front passes $\sim 0.3 M_\odot$, the core becomes unstable and starts to collapse. In the collapsing core, the adiabatic compression effectively causes the prior temperature rise, accelerating the flame propagation.

Finally, evolutions of distributions of the density (top), the velocity (middle), and the adiabatic index (bottom) of models T9.0 and T9.0ye are compared in Figure 14. This figure clearly shows the similar dynamical evolutions of the two progenitors. A small difference in the velocity of $\sim 4 \times 10^7$ cm s$^{-1}$ can be seen for the initial distributions, which results from the different degree of the initial hydrostatic/dynamical stability. However, the two velocity distributions evolve almost identically through the collapse, since the contraction velocities are significantly accelerated.

5. Result of Model N8.8

We calculate the evolution of model N8.8 for 0.1310 s in total. Trajectories of this model are shown in Figure 15. The initial condition has a central NSE region of $\sim 0.1 M_\odot$, and the flame front is already located at $1.02 \times 10^7$ cm. Core bounce takes place after $5.07 \times 10^{-2}$ s from the start of the calculation. A successful explosion also takes place for this model in our work. Since we are focusing on the physics during the ONe core collapse, we leave detailed analysis of the explosion for future work. Here the result until core bounce is mainly discussed.

In Figure 16, time evolution of the central density (top), the radius of the flame front (middle), and the mass coordinate of the flame front (bottom) are compared for the three initial models T9.0 (green), T9.0ye (blue), and N8.8 (yellow). Most
of the time, the flame front in model N8.8 is located more inside than in the other two models in terms of both the radius and the mass coordinate. This more compact central NSE region originates from the initial structure. First of all, the initial model N8.8 is more compact than model T9.0, in which the central density has already increased to \(5.6 \times 10^{10} \text{ g cm}^{-3}\), though the flame front is still located at \(\sim 0.1 M_\odot\). This two times larger density having the same front position indicates that the early flame propagation velocity in the work by Nomoto (1987) might be much slower than in our model.

In Figure 17, the evolution of the neutrino luminosities and mean energies of model N8.8 is compared with the results of model T9.0. In spite of the more compact central NSE region, the results of model N8.8 agree well with that of \(t_{pb} > -0.05 \text{ s}\) of model T9.0. Having the smaller front radius with a comparable neutrino luminosity, the neutrino flux at the flame front in model N8.8 becomes about two times higher than in model T9.0. Distributions of heating rates and the energy flux of the electron-type neutrino are compared in Figure 18 for the two models. The about two times higher heating rate of neutrino–electron scattering in model N8.8 results from the two times higher neutrino flux. Because of the higher heating efficiency, the neutrino scattering dominates the prior temperature rise in model N8.8 (Figure 19). Therefore, a more compact initial structure of model N8.8 results in heating-dominated
propagation of the flame front, which is qualitatively different from the propagation mechanism observed in model T9.0.

However, in spite of the difference in the mechanism of the front propagation, the evolutions of the central densities of the three models shown in Figure 16 show striking resemblance for $t_{\text{fb}} > -0.05$ s. In Figure 20, the evolutions of distributions of density, velocity, and adiabatic index are compared for models N8.8 and T9.0. The figure shows that the velocity evolutions of the inner core coincide with each other, even though model N8.8 develops oscillation in the outer region. This result indicates that there is a particular dynamical evolution of core collapse for the critical-mass ONe core, which perhaps only slightly depends on how the deflagration propagates.

6. Discussion

6.1. Prior Heating by Other Neutrino Reactions

We have shown that the neutrino–electron scattering effectively heats the surroundings to drive the flame front propagation in the ONe core. Here we estimate whether other neutrino heating mechanisms contribute to the prior temperature rise.

The first important result is that the neutrino emitted during the collapsing phase is dominated by the electron-type neutrino. This is because the electron-type neutrino is emitted by the electron capture reactions, while other types of neutrinos are only weakly emitted from the low-temperature ONe core. Accordingly, neutrino reactions that require other types of neutrino such as antielectron-type neutrino absorption by nuclei ($\bar{\nu}_e + A \rightarrow e^+ + A$), or inverse processes of thermal neutrino emissions, such as bremsstrahlung, pair-annihilation, and plasmon decay, hardly take place in the surrounding ONe region. Thus, possible candidates will be electron-type neutrino absorption by free neutrons ($\nu_e + n \rightarrow e^- + p$), electron-type neutrino absorption by nuclei ($\nu_e + A \rightarrow e^- + A'$), neutrino–nucleon scattering ($\nu_e + p/n \rightarrow \nu_e + p/n$), and inelastic neutrino–nucleus scattering ($\nu_e + A \rightarrow \nu_e' + A'$). Among them, the only possible candidates are inelastic neutrino–nucleon scattering and neutrino absorption by nucleus, because almost no free nucleons exist in the outer cold ONe region.

It has been known that coherent scattering of neutrinos on nuclei ($\nu_e + A \rightarrow \nu_e + A$), the effect of which is taken into account for the NSE region in our calculation, is the dominant neutrino interaction in a collapsing Fe core (e.g., Bruenn & Bruenn 1991). However, this process does not contribute to matter heating, since rest masses of nuclei are much larger than the neutrino energy, so that the scattering becomes almost elastic. Instead, inelastic neutrino–nucleus scattering is possible by exciting nuclei via neutral-current processes, so that $\nu_e + A \rightarrow \nu_e' + A'$. Bruenn & Haxton (1991) have shown that the cross section for $^{56}$Fe can be as high as one-third of the neutrino–electron scattering cross section in the high-temperature region of $T = 2 \times 10^{10}$ K. However, we expect that the heating effect in the ONe region will be minor. This is because, first, the ONe region is mainly composed of even–even nuclei, which requires a large neutrino energy for the excitation. This
in turn suggests the small cross section of the reaction (Langanke et al. 2008). Furthermore, since the temperature of the ONe region is merely $<0.1$ MeV, the effect of the thermal ensemble of the excited states (Sampaio et al. 2002; Juodagalvis et al. 2005; Dzhioev et al. 2011, 2014), which significantly enhances the reaction rate especially for neutrinos with small energies of $\lesssim 10$ MeV, will be negligible.

The heating rate of neutrino absorption by nuclei is estimated from the reaction rate of its inverse reaction of electron capture. Because of the detailed balance, the neutrino absorption kernel is related to the emission kernel as $R^a = \exp(\beta(E_\nu + \mu(c_{\nu(A,Z)} - \mu(\lambda\mu(A,Z+1))))R^e$. Similar to the discussion in Section 2.2, the emission kernel is estimated as $R^e = \left(\frac{\hbar c}{\mathcal{F}}\right)^3\lambda(A,Z+1)\psi(\nu(A,Z+1))\psi(\nu(A,Z+1))$, where $\lambda$ and $\psi_{\nu}$ are the reaction rate ($s^{-1}$) and the neutrino spectrum of the electron capture reaction by the $(A,Z+1)$ nucleus, respectively. Assuming that nuclei obey the Boltzmann statistic and $f_{\nu} \ll 1$, the collision term of the Boltzmann equation becomes

$$
\left\langle \exp\left(\frac{-\Phi}{e^\hbar}\right) \right\rangle_{\text{coll}} = R^e(1 - f_{\nu}) - R^af_{\nu},
$$

where $\Delta_A$ is a mass difference between $(A,Z)$ and $(A,Z+1)$ nuclei. The neutrino heating rate per unit mass can be equated with the rate of change of the specific neutrino energy density. Thus,

$$
\rho_{\nu} = -e^{-\Phi} \frac{\partial}{\partial t} \int d^3(p)e_{\nu}f_{\nu}
$$

$$
\sim \int E_{\nu}\lambda(A,Z+1)\psi_{\nu(A,Z+1)}
\times (e^\beta(E_\nu + \Delta_A - \mu_\nu)n_{\nu(A,Z+1)} - n_{\nu(A,Z+1)})dE_{\nu},
$$

is obtained. By approximating $\psi_{\nu(A,Z+1)} \sim \delta(E_\nu - E_{\nu(A,Z+1)})$, where $E_{\nu(A,Z+1)}$ is the mean energy of a neutrino emitted by the capture electron on the $(A,Z+1)$ nucleus, the energy integral can be done as

$$
\epsilon_{\nu} \sim E_{\nu(A,Z+1)} \frac{\lambda(A,Z+1)}{m_u}
\times (e^\beta(E_{\nu(A,Z+1)} + \Delta_A - \mu_\nu)Y_{\nu(A,Z+1)} - Y_{\nu(A,Z+1)}).
$$

The first and second terms on the right-hand side show the neutrino heating rate of the neutrino absorption by the $(A,Z)$ nucleus and the neutrino-loss rate of the electron capture on the $(A,Z+1)$ nucleus, respectively. Finally, considering the change of the number densities of electron and nuclei, the heating (or cooling) rates of the neutrino absorption and the electron capture are estimated as

$$
\epsilon_{\text{abs}} \sim (E_{\nu(A,Z+1)} + \Delta_A - \mu_\nu) \frac{\lambda(A,Z+1)}{m_u}
\times (e^\beta(E_{\nu(A,Z+1)} + \Delta_A - \mu_\nu)Y_{\nu(A,Z+1)})/f_{\nu}(E_{\nu(A,Z+1)}).
$$

The two equations show that the inverse process of the electron capture reaction should have a large reaction rate in order for the neutrino absorption to be efficient. Therefore, here we examine the absorption reactions by $^{20}\text{O}, ^{20}\text{F}, ^{24}\text{Ne},$ and $^{24}\text{Na}$, which are products of electron captures on $^{20}\text{Ne}$ and $^{24}\text{Mg}$. Moreover, the heat emitted per one reaction, $E_{\nu(A,Z+1)} + \Delta_A - \mu_\nu$, should be positive for heating for the neutrino absorption reaction; otherwise, it cools surroundings. Due to the large $\mu_\nu$, $E_{\nu(A,Z+1)} + \Delta_A - \mu_\nu$ tends to be negative in the whole region of the ONe core. As an exception in the considered reactions, the energy term can be positive in the outer region of $M_p > 0.4 M_{\odot}$. This is shown in the top panel of Figure 21, in which the distributions of relevant energies are shown.

Furthermore, in order to have a heating effect, the reaction rate of neutrino absorption should exceed that of the electron capture. Fractions of $e^\beta(E_{\nu(A,Z+1)} + \Delta_A - \mu_\nu, f_{\nu_a}, Y_{\nu_a}/Y_{\nu_e}$, and their product are shown in the middle panel of Figure 21. Thanks to the positive $E_{\nu_a}$, $E_{\nu(A,Z+1)} + \Delta_A - \mu_\nu$, the exponent exceeds unity in an outer region of $>0.4 M_{\odot}$. Because the neutrino distribution function $f_{\nu_a}$ gives a nearly energy-independent value of $\sim 1 \times 10^{-4}$ to $3 \times 10^{-3}$ for the concerning range of $E_{\nu} \lesssim 7$ MeV, $f_{\nu_a}(3.0 \text{ MeV})$ is shown as a representative case. During the evolutionary stage, a non-negligible amount of $^{20}\text{F}$ has been mixed up to the outer region by the convection powered by electron captures on $^{24}\text{Mg}$ and $^{56}\text{Ne}$. As a result,
the convective region has relatively high \( Y^{(20)F}/Y^{(20)Ne} \sim 10^{-2} \). In the end, the fraction \( e^{(E_e + \Delta \lambda - \nu_e)} \times f_e \times Y^{(20)F}/Y^{(20)Ne} \) exceeds unity in an outer region of \( 0.62-0.74 M_\odot \).

The heating or cooling rates of the reactions \( ^{20}F + e^- \rightarrow ^{20}Ne + e^- \) and \( ^{20}Ne + e^- \rightarrow ^{20}F + \nu_e \) are shown in the bottom panel of Figure 21. In the innermost region of \( M_b \leq 0.40 M_\odot \) with the high electron chemical potential \( \mu_e \), the electron capture reaction has a heating effect and thus is shown by the green solid line. The heating rate is much larger than the cooling rate of the neutrino absorption reaction, which is shown by the red dashed line. Meanwhile, in the middle region of \( 0.40 M_\odot < M_b < 0.72 M_\odot \), the electron capture reaction has a cooling effect and the neutrino absorption reaction has a heating effect. The cooling rate, shown by the red solid line, exceeds the cooling rate, shown by the green dashed line, in the region of \( 0.62 M_\odot < M_b < 0.72 M_\odot \). Besides, in the outermost region of \( 0.72 M_\odot < M_b < 0.74 M_\odot \), the heating rate of the beta decay of \( ^{20}F \rightarrow ^{20}Ne + e^- + \nu_e \), which is estimated as \( \epsilon_{\beta d} \sim (E_e + \Delta \lambda - \mu_e)(\lambda_{\beta d}/m_e)Y^{(20)F} \), becomes larger than the neutrino absorption reaction (blue solid line). In summary, although the neutrino absorption reaction has a heating effect only in the narrow region of \( 0.62 M_\odot < M_b < 0.72 M_\odot \), the weak reactions of the isotopes with the mass number of 20 in total have a net heating effect in the wide regions of \( M_b < 0.40 M_\odot \) and \( 0.62 M_\odot < M_b < 0.74 M_\odot \) in the ONe core.

However, none of them exceed the heating rate of neutrino–electron scattering, which reaches \( \epsilon_{\text{esc}} \sim 3 \times 10^{17} \text{erg g}^{-1} \text{s}^{-1} \) at the flame front. Similar to \( ^{20}F-^{20}O \), other nuclei also have heating effects mainly by the electron capture reactions in the innermost region, but these rates are merely \( \lesssim 1 \times 10^{15} \text{erg g}^{-1} \text{s}^{-1} \) and much weaker than the neutrino–electron scattering. Therefore, we conclude that neither neutrino absorption nor electron capture on nuclei effectively enhances the flame propagation in the ONe core.

6.2. Propagation Velocity of Conductive Flame with Corrugated Fronts

The conductive flame velocity may be enhanced owing to the corrugation effect by turbulence. In this subsection, we try to compare the flame propagation velocity obtained in this work to the velocity of the conductive flame with corrugated flame fronts.

As a result of the runaway oxygen+neon burning, the entropy and the temperature of the matter increase, and accordingly the density decreases to keep the pressure nearly constant. This makes a density inversion at the flame front, providing a satisfactory condition for the Rayleigh–Taylor (RT) instability. Under this instability, a large-scale convective flow may be developed. Small-scale turbulence is also possibly driven by the RT instability, or it appears as a result of the turbulent cascade, in which the Kelvin–Helmholtz instability plays an important role. The burning front can be corrugated by the turbulence, increasing the surface area of the fuel/ash boundary layer. As a result, the net consumption rate of the nuclear fuel, as well as the effective propagation velocity, is enhanced.

Considering the scale-invariant property in the turbulent front propagation, Pocheau (1994) has derived a general relation between the effective flame propagation velocity in a large scale, \( U_T \), the laminar flame velocity in a small scale, \( U_N \), and the turbulence intensity, \( U' \), as

\[
U_T^\alpha = U_N^\alpha + \beta U'^\alpha. \tag{36}
\]

For the two constants, \( \alpha = 2 \) is derived by imposing the energy conservation, and \( \beta = 4/3 \) is implied to be consistent with a numerical simulation (Peters 1999; Schmidt et al. 2006). For \( U_N, V_{\text{cond}} \) by Timmes & Woosley (1992),

\[
V_{\text{cond}} = 5.18 \times 10^{0}\left(\frac{\rho}{6 \times 10^9}\right)^{0.0688} \left(\frac{X^{(16)O}}{0.6}\right)^{0.16} \text{cm s}^{-1}. \tag{37}
\]

is applied. For \( U' \), we tentatively apply the Sharp–Wheeler relation (Davies & Taylor 1950; Sharp 1984)

\[
U' = V_{\text{RT}}(l) = 0.5 \sqrt{l_{\text{eff}}} \tag{38}
\]

where \( g_{\text{eff}} \equiv (\Delta \rho/\rho) g \) is the buoyancy force of the convective blob formed at the flame front and \( l = r_0 \), assuming that the turbulence is mostly driven by the RT instability. In the end, the effective flame velocity is estimated as \( V_{\text{eff}} = \sqrt{V_{\text{cond}}^2 + 4 V_{\text{RT}}^2/3} \).

The time evolution of the related velocities is shown in Figure 22 for models T9.0, T9.0ye, and N8.8 in the top, middle, and bottom panels, respectively. For all models, \( V_{\text{eff}} \approx \sqrt{V_{\text{cond}} V_{\text{RT}}} \) since \( V_{\text{cond}} \ll V_{\text{RT}} \). In model T9.0, the conductive energy transport will have a negligible contribution to
the flame propagation, as always $V_{\text{eff}} < V_{\text{prop}}$. The same can be found for model N8.8 and for the later propagation of $t_{\text{pb}} > \sim -0.15$ s in model T9.0ye. On the other hand, $V_{\text{eff}}$ is comparable to $V_{\text{prop}}$ in the early phase of $t_{\text{pb}} < \sim -0.15$ s in model T9.0ye, in which the flame propagation is mainly powered by the neutrino–electron scattering.

The conductive energy transport possibly helps the flame propagation in the early phase of an ONe core, if the core is initially hydrostatic. Therefore, it is important to determine how the ONe core is hydrostatically stable at the onset of the central O+Ne ignition. Model T9.0 seems to be destabilized owing to the remapping procedure from the stellar evolution code to the hydrodynamic code, and model T9.0ye is the other extreme case in which the hydrostatic stability is artificially posed. The real ONe core, if it exists, would have a gravitationally stable state in between the two models. Because only a very simple estimate has been done here, it will be interesting to investigate how the conductive burning front propagates through the hydrostatic ONe medium by a 3D simulation even for the case with the high ignition density.

One missing argument here is the effect of density increase caused by the electron capture reaction. The electron capture reaction by free protons that initiates immediately after the flame front passes the region has a short timescale of $\sim 0.01$ s. It decreases $Y_e$ from $\sim 0.5$ to $\lesssim 0.4$, so that causes the density increase of $\sim 25\%$. However, the timescale of the flame propagation, $t_{\text{flame}}$, is $\sim 0.1$ s, and the extent of the density inversion due to the oxygen+neon burning is merely $\Delta \rho / \rho \lesssim 0.1$. Therefore, the region just below the flame front will be rapidly stabilized by the electron capture reaction. In order to estimate the turbulent intensity more accurately, the effect of the rapid stabilization should be properly taken into consideration.

7. Conclusion

A critical-mass ONe core with a high ignition density of $\rho_{\text{ign}} \gtrsim 9 \times 10^{10}$ g cm$^{-3}$ is considered to be destined to gravitationally collapse to form an NS. Whereas a number of works have been performed to investigate phases of the super-AGB star evolution and the ECSN explosion, the final core evolution from the central ONe ignition toward core bounce has not been investigated in detail so far. This is because the ONe core consists of combustible elements, so that one has to follow a complex phase transition from the O+Ne composition into the NSE state. Thus, we have simulated the late core evolution using a neutrino radiation hydrodynamic code, which treats not only neutrino reactions by solving the elaborate Boltzmann equation but also the nuclear burning and electron capture reactions. Special care is also taken to remap the initial structure as consistent with the original evolution calculation as possible.

We have observed that the late core evolution is affected by the complex interplay among the nuclear reactions, the structure evolution, and the flame propagation. First, the oxygen+neon burning leads to (i) heating by the release of the rest-mass energy; (ii) $Y_e$ reduction due to accompanying electron capture reactions by free protons down to $Y_e \sim 0.36$, and even further by NSE heavy nuclei; and (iii) energy reduction due to $\nu_e$ emission by the electron capture. In addition, we have pointed out that (iv) reduction in $\gamma$ due to photodisintegration in the NSE region results from the oxygen +neon burning. As a consequence, the core becomes more and more unstable as the flame propagation extends the central NSE region. Moreover, the fast electron captures caused by the oxygen+neon burning result in $(\nu)$ the intense $\nu_e$ radiation with a luminosity of $L_{\nu_e} \gtrsim 10^{51}$ erg s$^{-1}$ even before the core bounce.

Second, owing to the destabilization described above and to the intrinsically unstable core structure due to the soft EOS with $\gamma \sim 4/3$, the ONe core starts to contract after the central ignition. Thus, the temperature rise due to the compression has a major contribution to trigger the succeeding nuclear burning ahead of the flame front. Furthermore, we have found that the intense pre-bounce $\nu_e$ radiation heats the broad cold region of the ONe core by neutrino–electron scattering, which acts as a new driving mechanism of the flame propagation in the collapsing ONe core. The resulting heating rate can be as high as $\sim 3 \times 10^{47}$ erg g$^{-1}$ s$^{-1}$ and much more efficient than any other neutrino reactions and electron capture reactions. In summary, the flame propagation in the collapsing ONe core is driven by both adiabatic compression and heating by neutrino–electron scattering.

Comparison of results of the progenitor model T9.0 and the artificially stabilized model T9.0ye shows that the different degree of the initial hydrostatic/dynamical stability affects the flame propagation velocity in the early phase of $t_{\text{pb}} \lesssim -0.05$ s. The early flame velocity is $\sim 1 \times 10^8$ cm s$^{-1}$ in the nearly hydrostatic core of model T9.0ye. On the other hand, faster velocity of $\sim 3 \times 10^8$ cm s$^{-1}$ is obtained for model T9.0, because the faster core contraction enhances the adiabatic compression, and besides the fast propagation velocity results in more efficient neutrino heating. Having different propagation velocities in the early phases, models with different degrees of the initial hydrostatic stability have different durations from the ignition until core bounce. The durations of the pre-bounce phases are 0.13 s for model T9.0 and 0.30 s for model T9.0ye, respectively. We note that the obtained flame velocity of $\sim 10^8$ cm s$^{-1}$ is more than one order of magnitude faster than the estimated laminar flame velocity driven by heat conduction, which has been considered as the main driving mechanism of the flame propagation in the ONe core.

Kato et al. (2017) have simulated the observability of neutrinos that are emitted during the pre-bounce phases for progenitors of an ECSN and FeCCSNe. They have found that a progenitor of an ECSN can be observationally distinguishable from a progenitor of an FeCCSNe based on the detection and nondetection of $\nu$ and $\bar{\nu}$, if the progenitor star is located close to Earth ($\lesssim 200$ pc). Besides, we predict that the duration of the pre-bounce neutrino emission phase can be determined by observing the pre-SN neutrinos since the luminosity of $\nu_e$ is large from the beginning. We have shown that this duration strongly depends on the different initial structures having different degrees of the core hydrostatic stability. Therefore, detection of pre-SN neutrinos has a potential importance to constrain the initial core hydrostatic stability state. In spite of the different propagation in the early phase, the core evolution in the late phase of $t_{\text{pb}} \gtrsim -0.05$ s becomes similar for models T9.0 and T9.0ye. Model N8.8 develops a more compact central NSE region during the whole collapsing phase. As a result, the flame front in model N8.8 is mostly driven by heating of neutrino–electron scattering, which qualitatively differs from models T9.0 and T9.0ye. However, important characteristics such as time evolutions of the central density and the neutrino luminosities still show striking resemblance to each other. In the end, successful explosions
Figure 23. Comparisons of pressure (left) and entropy per baryon (right) between the STOS and the stellar EOSs in density–temperature planes. Contours of the thermodynamic quantities are shown by lines. Attaching numbers show log values of pressure (in erg cm\(^{-3}\)) or entropy per baryon (in units of the Boltzmann constant \(k_B\)). Green lines are STOS EOS results, while orange lines are results of the stellar EOS.

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Appendix A

EOS Comparison

In Figure 23, pressure and entropy per baryon calculated by the two EOSs are shown for the case of \(Y_e = 0.5\). The NSE composition determined by the STOS EOS is used for the input of the stellar EOS in this case. Pressure well matches for \(T > 10^9\) K, if \(Y_e\) is less than 0.56. The STOS pressure will be unreliable below this temperature or above this \(Y_e\), since the original table has data points only at \(T = 0\) K and \(Y_e = 10^{-0.25}\). For entropy per baryon, \(s_k\), both EOSs provide almost the same values for a wide range of \(Y_e\), if \(s_k > 3k_B\) or \(T > 10^{9.7}\) K. A large part of the disagreement seen at the low-entropy region of \(s_k < 1k_B\) is explained by omission of the entropy of heavy nuclei in the STOS EOS. Since matters in such a region are always covered by the stellar EOS, this omission does not affect our calculation.

Appendix B

An Improved Grid Reconstruction Method

A frequency function \(f(r)\) and an integrated frequency function \(F(r) = \int_{r_{\text{center}}}^{r} f(r')d\ln r'\) are defined. Grid points are determined to be evenly spaced in terms of the integrated frequency function. Thus, a radius interval between two grid points becomes

\[
\Delta \ln r \approx \frac{d \ln r}{d F} \Delta F
\]

so that we named the function \(f(r)\) as the frequency function. As for the actual frequency function,

\[
f = \begin{cases} 
C 4\pi r^3 \rho & (P > 10^{20} \text{ erg cm}^{-3}) \\
\left| \frac{d \log P}{d \log r} \right| & \text{(otherwise)},
\end{cases}
\]

where \(C\) is a constant, is used.

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