On Quadratic Divergences and the Higgs Mass

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Abstract

The quadratic divergences in the scalar sector of the standard model are con-
sidered. Since the divergences are present also in the unbroken theory, a natural
scale for the divergence formula is proposed to be at the scale of new physics.
The implications of top quark mass on the Higgs mass are investigated by means
of the renormalization group equations. The Coleman-Weinberg mechanism for
spontaneous symmetry breaking is also considered.

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Supersymmetric theories have attracted a great deal of attention mainly due to
the fact that they have the property of being free of quadratic divergences. It is
therefore natural to ask whether the cancellation of such divergences may occur in
nonsupersymmetric theories as well and, in particular, in the standard model.

According to the discussion given by Veltman [1], suggestive of an underlying the-
ory with a symmetry protecting the mass, the quadratic divergences in the standard
model should cancel as happens with the electron and gauge boson masses. For the
masses in the broken phase this implies the relation

$$\frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{3}{4} m_H^2 = \sum_f m_f^2,$$

where $f$ stands for the fermions. Indeed this relation follows from the relation among
the coupling constants:

$$\frac{3}{2} g_1^2 + \frac{9}{2} g_2^2 + 6\lambda = 4 \sum_f h_f^2,$$

with $g_1$ and $g_2$ being the $U(1)$ and $SU(2)$ gauge couplings respectively, $\lambda$ the Higgs
self-coupling and $h_f$ the Yukawa couplings.

Much work on the mass formulas has been done in details and in different aspects
in refs. [2]-[8]. In [5, 6], using the dimensional regularization the quadratic divergences
have been put to zero in two-loop order to determine unambiguously the top and Higgs
masses. In this case, however, the equations from the higher loop corrections are not
compatible with the formulas from the lower loop orders [2]. Equating, in addition to
quadratic divergences, one of the logarithmic divergences to zero has also been used
to find a second equation [4, 7, 8]. For this purpose, logarithmic divergences of either
the Higgs self-energy, or electron self-energy, or $eeH$ coupling have been considered.
For instance, in the papers by Osland and Wu [7], by imposing the cancellation of
quadratic divergences and the logarithmic divergences in the $eeH$ vertex, the Higgs
and top masses were determined to be $m_H \sim 190$ GeV and $m_t \sim 120$ GeV.

The physical motivation of these approaches seems not to be quite satisfactory:
one problem with the mass formulas found from the vanishing of the divergences is
that the formulas are not invariant under the renormalization group transformations.
Thus the formula (1) is defined at some scale, which is not determined. In [3], it was
shown that the formula (1) can be required to be scale independent provided that
strong interactions are ignored. Taking the strong interactions (which give important
contribution) into account, one does not find a scale independent solution.

In this letter we study the formula (1) as a function of the scale. We shall not
consider the contributions from leptons and lighter quarks which are negligible in
Eq.(1). Since the quadratic divergences exist already before the symmetry breaking,
it is natural to require that the equation should be valid at a large scale \( \Lambda \). The scale is not a priori determined, since we do not know when new physics enters into play. The corrections due to (1) should not however exceed the physical scalar mass. We consider the consequences of these requirements on the predictions for the Higgs mass at the electroweak scale. Furthermore, we shall consider the possibility of combining the Coleman-Weinberg idea [9] for the spontaneous symmetry breaking with Veltman’s idea of cancellation of quadratic divergences in the standard model.

1 RGE and Quadratic Divergences

Since Eq. (I) is not invariant under the renormalization group transformations, one should take into account the running of the couplings according to the renormalization group equations (RGE) in finding the physical masses at the electroweak scale.

The RGE up to two loops for the gauge couplings \( g_i, i = 1, 2, 3 \) \((g_3 \text{ is the strong coupling})\), the top Yukawa coupling \( h_t \) and the scalar coupling \( \lambda \) are given by [10]-[12]

\[
\frac{dg_i}{dt} = \frac{1}{16\pi^2} \beta_i, \quad \frac{dh_t}{dt} = \frac{1}{16\pi^2} \beta_t, \quad \frac{d\lambda}{dt} = \frac{1}{16\pi^2} \beta_\lambda, \quad (2)
\]

where \( t = \ln(\mu/\mu_0) \) and

\[
\beta_1 = \frac{41}{6} g_1^3 + \frac{1}{16\pi^2} \left\{ \frac{199}{18} g_1^2 + \frac{9}{2} g_2^2 + \frac{44}{3} g_3^2 - \frac{17}{6} h_t^2 \right\} g_1^3,
\]

\[
\beta_2 = -\frac{19}{6} g_2^3 + \frac{1}{16\pi^2} \left\{ \frac{3}{2} g_1^2 + \frac{35}{6} g_2^2 + 12 g_3^2 - \frac{3}{2} h_t^2 \right\} g_2^3,
\]

\[
\beta_3 = -7 g_3^3 + \frac{1}{16\pi^2} \left\{ \frac{11}{6} g_1^2 + \frac{9}{2} g_2^2 - 26 g_3^2 - 2 h_t^2 \right\} g_3^3,
\]

\[
\beta_t = \left( \frac{9}{2} h_t^2 - \frac{17}{12} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2 \right) h_t
+ \frac{1}{16\pi^2} \left\{ -12 h_t^4 + h_t^2 \left( \frac{131}{16} g_1^2 + \frac{225}{16} g_2^2 + 36 g_3^2 - 6 \lambda \right) + \frac{3}{2} \lambda^2
+ \frac{1187}{216} g_1^4 - \frac{3}{4} g_1^2 g_2^2 + \frac{19}{9} g_1^2 g_3^2 - \frac{23}{4} g_2^4 + 9 g_2^2 g_3^2 - 108 g_3^4 \right\} h_t,
\]

\[
\beta_\lambda = 12 \lambda^2 - (3 g_1^2 + 9 g_2^2 - 12 h_t^2) \lambda + \frac{3}{4} g_1^4 + \frac{3}{2} g_1^2 g_2^2 + \frac{9}{4} g_2^4 - 12 h_t^4
+ \frac{1}{16\pi^2} \left\{ -78 \lambda^3 - 72 \lambda^2 h_t^2 + 18 \lambda^2 (g_1^2 + 3 g_2^2) - 3 \lambda h_t^4 + 80 \lambda g_3^2 h_t^2
+ \frac{45}{2} \lambda g_2 h_t^2 + \frac{85}{6} \lambda g_1^2 h_t^2 - \frac{73}{8} \lambda g_1^4 + \frac{39}{4} \lambda g_1^2 g_2^2 + \frac{629}{24} \lambda g_1^4
+ 60 h_t^6 - 64 h_t^4 g_3^2 - \frac{16}{3} h_t^4 g_1^2 - \frac{9}{2} h_t^2 g_2^2 + 21 h_t^2 g_1^2 g_2^2
+ \frac{19}{2} h_t^2 g_1^4 + \frac{305}{8} g_1^6 - \frac{289}{24} g_1^2 g_2^4 - \frac{559}{24} g_1^4 g_2^2 - \frac{379}{24} g_1^6 \right\}. \quad (3)
\]
Neglecting the two-loop contributions, which from our further analysis turn out to be of the order of a few percent, the one-loop equations (2) for gauge couplings can be solved analytically and their solutions read as

\begin{align*}
g_1^2(\mu) &= \frac{g_1^2(\mu_0)}{1 - \frac{11}{48\pi^2} g_1^2(\mu_0) \ln(\mu/\mu_0)} , \\
g_2^2(\mu) &= \frac{g_2^2(\mu_0)}{1 + \frac{19}{48\pi^2} g_2^2(\mu_0) \ln(\mu/\mu_0)} , \\
g_3^2(\mu) &= \frac{g_3^2(\mu_0)}{1 + \frac{7}{8\pi} g_3^2(\mu_0) \ln(\mu/\mu_0)}. \tag{4}
\end{align*}

We have solved the equations for the top Yukawa coupling $h_t$ and Higgs self-coupling $\lambda$ numerically using the experimental values of $g_1^2 = 0.13$, $g_2^2 = 0.42$, $g_3^2 = 1.46$ \[^{13}\] and $h_t = 1.01$ (for $m_t = 176$ GeV from CDF), $h_t = 1.14$ (for $m_t = 199$ GeV from D0) \[^{14}\] at the electroweak scale. After symmetry breaking the masses of the particles are related to the couplings as $m_W^2 = \frac{1}{2} g_2^2 v^2$, $m_Z^2 = \frac{1}{2} (g_1^2 + g_3^2) v^2$, $m_H^2 = 2 \lambda v^2$ and $m_t = h_t v$ with $v \simeq 174$ GeV being the vacuum expectation value of the Higgs field.

Taking Eqs.(2)-(4) into account we have found the running mass of the Higgs as a function of the large scale $\Lambda$, where the formula (4) is assumed to be valid. The results for the top masses 176 GeV and 199 GeV are given in Fig. 1. It is seen that for the scale of new physics being at $\Lambda = 10^{15} - 10^{19}$ GeV and with $m_t = 176$ GeV (199 GeV) at the electroweak scale, one obtains for the Higgs mass $m_H \sim 170$ GeV (210 GeV). With lower than $10^{15} - 10^{19}$ GeV values for $\Lambda$, however, the Higgs mass $m_H$ increases. If in addition one imposes that the quadratic corrections to the Higgs mass, i.e.

\[ \Delta m_H^2 = \frac{3}{2} g_1^2 + \frac{9}{2} g_2^2 + 6 \lambda - 12 h_t^2 \left( \frac{\Lambda^2}{16\pi^2} \right), \tag{5} \]

can become at most equal to the physical mass value \[^{1}\] (“naturalness”), then for $m_t = 176$ GeV (199 GeV) one obtains $\Lambda \simeq 1.8$ TeV and, correspondingly, a Higgs mass around 260 GeV (300 GeV) at the electroweak scale (see Fig. 1). Notice also that if one would impose the relation (1) to be valid at the electroweak scale one would obtain higher values for the Higgs mass, namely $m_H = 320$ GeV (370 GeV).

Thus we see that under the assumption that quadratic divergences in the standard model are cancelled at GUT-Planck mass scale the Higgs mass is expected to be in the range 170 - 210 GeV for $m_t \sim 176 - 199$ GeV. If one insists however on the naturalness of the theory one obtains an upper bound on the scale, $\Lambda \lesssim 1.8$ TeV, and correspondingly, a lower bound on the Higgs mass $m_H \gtrsim 260$ GeV for $m_t \gtrsim 176$ GeV.
2 Coleman-Weinberg mechanism for spontaneous symmetry breaking

As first pointed out by Coleman and Weinberg \[9\] the spontaneous symmetry breaking may be driven by radiative corrections in theories which at the tree level do not exhibit such breaking. The advantage of this dynamical mechanism is that the symmetry breaking does not have to be put in by hand. In the framework of the latter mechanism it has been recently argued \[15\] that top quark loops may trigger the symmetry breaking in the standard electroweak model and as a consequence, the Higgs boson mass is expected to be \( m_H \leq 400 \text{ GeV} \) depending on the value of the top quark mass and the physical cutoff \( \Lambda \).

Here we shall study the implications of the Coleman-Weinberg mechanism for the spontaneous symmetry breaking in combination with the cancellation of quadratic divergences in the standard model. The starting point is the one-loop effective potential which includes the one-loop top quark and gauge boson contributions. The latter is easily calculated and the result is well-known \[9, 15\]. We have

\[
V = m^2 \phi^2 + \frac{1}{32\pi^2} \int_0^{\Lambda^2} dq^2 q^2 \left\{ 6 \ln \left( 1 + \frac{g_2^2 \phi^2}{2q^2} \right) + 3 \ln \left( 1 + \frac{(g_1^2 + g_2^2)\phi^2}{2q^2} \right) - 12 \ln \left( 1 + \frac{h_t^2 \phi^2}{q^2} \right) \right\}. \tag{6}
\]

Note that we have not included quartic scalar self-interactions, i.e. we start with a simple Lagrangian of a massive scalar \( \phi (m^2 > 0) \) interacting with a massless fermion (top quark). Thus the scalar self-interactions will be induced by quantum corrections. We postpone comments on this point to the end of this section.

After performing the integrals in (6) and neglecting terms that vanish as \( \Lambda \to \infty \) we obtain finally

\[
V = m^2 \phi^2 + \left( \frac{3}{2} g_1^2 + \frac{9}{2} g_2^2 - 12 h_t^2 \right) \frac{\Lambda^2 \phi^2}{32\pi^2} + \frac{1}{64\pi^2} \left\{ \frac{3}{2} g_2^4 \phi^4 \left( \ln \frac{g_2^2 \phi^2}{2\Lambda^2} - \frac{1}{2} \right) \right. \\
+ \frac{3}{4} (g_1^2 + g_2^2)^2 \phi^4 \left( \ln \frac{(g_1^2 + g_2^2)\phi^2}{2\Lambda^2} - \frac{1}{2} \right) - 12 h_t^4 \phi^4 \left( \ln \frac{h_t^2 \phi^2}{\Lambda^2} - \frac{1}{2} \right) \left\} \right. \tag{7}
\]

To extend the region of validity of the one-loop effective potential we can use its RG improved version \[16, 12\] and with this aim we shall run the couplings in Eq.(7) according to their RGEs given by Eqs.(2)-(4).

Next we impose that the quadratic divergences (proportional to \( \Lambda^2 \)) in the RG improved effective potential (7) are cancelled. This implies the relation
\[
\frac{3}{2}g_1^2 + \frac{9}{2}g_2^2 - 12h_t^2 = 0 \tag{8}
\]

Since the latter equation is scale-dependent, we should require it to be valid at some fixed scale. Notice that if one would assume relation (8) to be valid at the electroweak scale one would obtain a light top quark \(m_t^2 = (m_Z^2 + 2m_W^2)/4\), i.e. \(m_t \simeq 75\) GeV), which is experimentally excluded. As in our previous analysis, a natural choice for the scale will be the cutoff scale \(\Lambda\) where new physics enters into play.

At low energies the effective potential (7) develops a new minimum \(\langle \phi \rangle = v \neq 0\) and thus the symmetry is spontaneously broken due to quantum corrections. The Higgs mass at the one-loop level will be given by

\[
m_H^2 = \left. \frac{1}{2} \frac{\partial^2 V}{\partial \phi^2} \right|_{\phi = v} = \frac{3}{32\pi^2 v^2} \left\{ 4m_t^4 \left( 2 \ln \frac{\Lambda^2}{m_t^2} - 1 \right) - m_Z^4 \left( 2 \ln \frac{\Lambda^2}{m_Z^2} - 1 \right) - 2m_W^4 \left( 2 \ln \frac{\Lambda^2}{m_W^2} - 1 \right) \right\}. \tag{9}
\]

The numerical procedure for evaluating the top quark and the Higgs mass at the electroweak scale is then as follows: we require Eq.(8) to be valid at some scale \(\Lambda\) and solve the RGE for the top Yukawa coupling (given in Eqs.(2)-(4)) to find the top mass at the electroweak scale. Finally, we evaluate the Higgs mass by using Eq.(9). The results for \(m_t\) and \(m_H\) as a function of the scale are given in Fig. 2 assuming \(\alpha_s(m_Z) = 0.116\) \cite{13}. We notice that in order to have a top quark mass in agreement with the recent experimental results \cite{14} the scale \(\Lambda\) at which relation (8) is valid should be sufficiently high and at most of the order of the Planck scale, i.e. \(\Lambda \sim 10^{19}\) GeV. In the latter case the Higgs mass will be \(m_H \lesssim 300\) GeV, while for the top quark mass we obtain \(m_t \lesssim 150\) GeV. The above results are of course sensitive to the initial value of the strong coupling constant \(\alpha_s\). For instance, if we use the value \(\alpha_s(m_Z) = 0.123 \pm 0.006\) from LEP event shapes \cite{17}, we obtain the upper bounds \(m_H \lesssim 330\) GeV and \(m_t \lesssim 155\) GeV.

It is worth mentioning that we have used in our calculations the running masses for the Higgs scalar and top quark. The physical pole masses can be computed from the running ones through the corresponding corrections which are typically of the order of a few percent. In the case of the top quark the latter corrections increase the value of \(m_t\) by about 7 %, thus implying \(m_t \lesssim 160 - 170\) GeV depending on the value of \(\alpha_s\).

\*It is interesting that a preliminary top mass result from the dilepton channels reported by D0 collaboration is \(m_t = 145 \pm 25 \pm 20\) GeV \cite{18}.
It is obvious that in the Coleman-Weinberg case the naturalness (i.e. the requirement that the quadratic corrections to the mass are smaller than the physical Higgs mass in (9)) cannot be required since this would lead to a cutoff scale in the TeV range and thus \( m_t \) would be too low (about 90 GeV, cf. Fig. 2), which is excluded by the recent experimental limits on the top quark mass.

Finally, let us comment on the quartic scalar self-coupling \( \lambda \). We have assumed it to be zero at \( \Lambda \) scale (cf. Eq.(8)). At the electroweak scale it is defined as

\[
\lambda = \frac{1}{12} \left. \frac{\partial^4 V}{\partial \phi^4} \right|_{\phi=v},
\]

where \( V \) is the effective potential given in Eq.(7). (We have chosen the coefficient in the definition of \( \lambda \) so that the quartic term in the effective potential is \( \lambda \phi^4/2 \)). Then using Eqs.(9) and (10) it is straightforward to show that in the leading ln \( \Lambda \) approximation \( m^2_H = 2\lambda v^2 \), which is the same expression as in the usual mechanism for spontaneous symmetry breaking.

To conclude, our suggestion in this letter is that if the standard model could be rendered free of quadratic divergences, then the cancellation of such divergences should occur at the scale of new physics and not at the electroweak scale. As a consequence of this approach the mass relations between \( m_H \) and \( m_t \) are drastically changed at the electroweak scale in the direction of lowering the Higgs mass.

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References

[1] M. Veltman, Acta Phys. Pol. B12 (1981) 437.

[2] I. Jack, D.R.T. Jones, Phys. Lett. 234B (1990) 321; Nucl. Phys. B342 (1990) 127.

[3] M. S. Al-sarhi, I. Jack, D.R.T. Jones, Nucl. Phys. B345 (1990) 431; Z. Phys. C55 (1992) 283.

[4] R. Decker, J. Pestieau, Mod. Phys. Lett. A4 (1989) 2733;
Lett. Nuovo Cimento 29 (1980) 560; preprint UCL-IPT-79-19 (1979).

[5] M. Ruiz-Altaba, B. González, M. Vargas, preprint CERN-TH.5558/89.

[6] M. Capdequi Peyranère, J.C. Montero, G. Moultaka, Phys. Lett. B260 (1991) 138.
[7] P. Osland, T.T. Wu, Z. Phys. C55 (1992) 569; Z. Phys. C55 (1992) 585; Z. Phys. C55 (1992) 593; Phys. Lett. 291B (1992) 315; Bergen preprint 1992-02.

[8] G. Lopez Castro and J. Pestieau, Mod. Phys. Lett. A10 (1995) 1155.

[9] S. Coleman and E. Weinberg, Phys. Rev. D 7 (1973) 1888; see also S. Weinberg, Phys. Rev. D 7 (1973) 2887.

[10] H. Georgi, S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438; H. Georgi, H.R. Quinn, S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.

[11] M.E. Machacek, M.T. Vaughn, Nucl. Phys. B222 (1983) 83; Nucl. Phys. B236 (1984) 221; Nucl. Phys. B249 (1985) 70.

[12] C. Ford, D.R.T. Jones, P.W. Stephenson and M.B. Einhorn, Nucl. Phys. B395 (1993) 17.

[13] Review of Particle Properties, Phys. Rev. D 50 (1994) 1173.

[14] F. Abe et al. (CDF collaboration), Phys. Rev. Lett. 74 (1995) 2626; S. Abachi et al. (D0 collaboration), Phys. Rev. Lett. 74 (1995) 2632.

[15] J.P. Fatelo, J.-M. Gérard, T. Hambye and J. Weyers, Phys. Rev. Lett. 74 (1995) 492; see also for the minimal supersymmetric standard model case: M. Chaichian, P. Chiappetta, J.-M. Gérard, R. Gonzalez Felipe and J. Weyers, Helsinki University preprint HU-SEFT R 1995-08/UCL-IPT-95-07/CPT-95-PE.3185 (submitted to Phys. Lett. B).

[16] M. Sher, Phys. Rep. 179 (1989) 273.

[17] Review on Experimental Results on Precision Tests of Electroweak Theories, talk given by P.B. Renton at the XVII International Symposium on Lepton-Photon Interactions, Beijing, China, 10-15 August 1995.

[18] Top Physics at D0, talk given by B. Klima at the XVII International Symposium on Lepton-Photon Interactions, Beijing, China, 10-15 August 1995.
Figure 1: Higgs mass $m_H$ as a function of the scale $\Lambda$ where cancellation of quadratic divergences is assumed. The bullets denote the intersection points at which the quadratic corrections $\Delta m_H$ (cf. Eq.(5)) equal the physical mass $m_H$. 
Figure 2: Higgs mass $m_H$ and top quark mass $m_t$ as functions of the scale $\Lambda$ in the case of spontaneous symmetry breaking via Coleman-Weinberg mechanism.