Guided waves modes in a slab waveguide formed from the isotropic dielectric layer embedded by hyperbolic materials are investigated. Optical axis is normal to the slab plane. The dispersion relations for TE and TM waves are found. The differences between hyperbolic waveguide and conventional one are demonstrated. In particular, for each TM mode of hyperbolic waveguide there are two cut-off frequencies and the number of modes is limited. For the TE and TM modes Poynting vector component along the wave’s propagation axis could be equal to zero.

I. INTRODUCTION

Metamaterials and their optical properties attract great attention over the past decade. Metamaterials are artificial materials which are composed of unit cells far below the size of the wavelength. These materials can exhibit exotic electromagnetic properties. The negative refraction is the famous property of these media [1–6]. As usually such materials are materials with simultaneously negative real parts of the dielectric permittivity and the magnetic permeability in some frequency region [7, 8].

It has been known, that the negative refraction can take place in the anisotropic media [9–13]. It should be pointed, that uniaxial anisotropy is typical property of the metamaterials. It is important to remark, that an anisotropic metamaterials can demonstrate negative refraction in one direction and positive refractions in the orthogonal directions.

Let us assume, that in the uniaxial anisotropic medium coordinate axes $OX$, $OY$ and $OZ$ are chosen to be equal to the principal axes of the dielectric permittivity tensor with the principal values of dielectric permittivity, which satisfy the conditions $\varepsilon_{xx} = \varepsilon_e$ and $\varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_o$. The dispersion relation for extraordinary wave connecting frequency $\omega$ with Cartesian components of wave vector $k$, takes the following form:

$$k_z^2 + k_y^2 \frac{\varepsilon_e(\omega)}{\varepsilon_o(\omega)} + k_x^2 \frac{\varepsilon_o(\omega)}{\varepsilon_e(\omega)} = \frac{\omega^2}{c^2},$$

(1)

This relation shows, that in the case of either $\varepsilon_e$ or $\varepsilon_o$ is negative, iso-frequency dispersion surface (1) represents the hyperboloid. The hyperboloid of one sheet is realized if $\varepsilon_e > 0$, $\varepsilon_o < 0$ and hyperboloid of two sheets is obtained if $\varepsilon_e < 0$, $\varepsilon_o > 0$ [14][18].

$$\frac{k_z^2 + k_y^2}{\varepsilon_e(\omega)} - \frac{k_x^2}{\varepsilon_o(\omega)} = \frac{\omega^2}{c^2}, \quad \frac{k_z^2 + k_y^2}{\varepsilon_o(\omega)} - \frac{k_x^2}{\varepsilon_e(\omega)} = \frac{\omega^2}{c^2},$$

(2)

These anisotropic materials are referred to as hyperbolic materials.

The hyperbolic materials can be fabricated as a multilayer structure consisting of alternating metallic and dielectric layers [14][19][20], or as a nanowire structure consisting of metallic nanorods embedded in a dielectric host [15][21][22].

Iso-frequency dispersion surfaces (2) allow the infinitely large wave vectors. It results in the different effects, among which are the Purcell enhancement of the spontaneous emission rate in hyperbolic metamaterials [23][26] and the subwavelength resolution effect [14][27].

The optical phenomena on interface between conventional dielectric and hyperbolic metamaterial has attracted attention. The surface waves were studied in [28]. The extremely large Goos-Hänchen shift has been studied in some details in [29]. Plasmonic planar waveguide cladded by hyperbolic metamaterials (Fig.1) was proposed and investigated in [30][31].
The purpose of this paper is to investigate the dispersion properties of the linear guided waves in a planar waveguide previously studied in [30]. Unlike [30], the guided waves localized in the dielectric core will be considered here instead of surface waves. The anisotropy axes of substrate and cladding layers are directed along \( OX \) axis that is normal to the interface (Fig.1). In the planar geometry, as is known [32], the Maxwell equations can be separated into two uncoupled systems of equations, which describe the propagation of the waves having a different polarization. These waves are referred to as TE and TM waves. Analysis of the guided TE and TM waves will be done independently. In both cases the expression for dispersion relation connecting effective waveguide index with frequency will be obtained. The symmetrical waveguide will considered in details.

II. ELECTRIC AND MAGNETIC FIELD DISTRIBUTIONS FOR GUIDED WAVES

Let us consider a slab waveguide. We assume, that the material of the waveguide core is nonmagnetic \( \mu_i = 1 \) and has an isotropic permittivity \( \varepsilon_i \). The core thickness is \( h \). The dielectric core is cladded by the uniaxial hyperbolic metamaterials which are characterized by symmetric dielectric tensor with the principal dielectric constants \( \varepsilon^{(1)}_o \), \( \varepsilon^{(1)}_e \), \( \varepsilon^{(3)}_o \), \( \varepsilon^{(3)}_e \) and the magnetic permeabilities \( \mu_1 \) and \( \mu_3 \). All permeabilities are assuming to be positive. The anisotropy axes are aligned with a unit normal vector to the interface, i.e., along the \( OX \) direction. (Fig.1). Axes \( OY \) and \( OZ \) are parallel to interface. Axis \( OZ \) is directed along the wave propagation. In this case the Maxwell equations are invariant under the shifting along \( OY \) axis. Thus, the strengths of the electric and magnetic fields of the guided wave are independent of variable \( y \). From it follows, that the Maxwell equations are splitting into two independent systems of equations describing the TE and TM waves [32].

![FIG. 1: A schematic illustration of the hyperbolic waveguide](image)

The TE wave is defined by the tangent component of the electric field vector \( E_y \) and by two components of magnetic field: \( H_x \) and \( H_z \). These values are assumed to be harmonic functions of the time: \( \exp(i\omega t) \). The wave equation for complex amplitude \( E = E_y(x, z, \omega) \) looks like:

\[
\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2} + k_0^2 \varepsilon_o(x) \mu(x) E = 0,
\]

where \( k_0 = \omega/c \), \( \omega \) is frequency of radiation. The principal dielectric constants and magnetic permeabilities are piecewise functions (Fig.1):

\[
\varepsilon_o(x) = \begin{cases} 
\varepsilon^{(1)}_o & x < 0, \\
\varepsilon_i & 0 \leq x \leq h, \\
\varepsilon^{(3)}_o & x > h,
\end{cases} \quad \varepsilon_e(x) = \begin{cases} 
\varepsilon^{(1)}_e & x < 0, \\
\varepsilon_i & 0 \leq x \leq h, \\
\varepsilon^{(3)}_e & x > h,
\end{cases} \quad \mu(x) = \begin{cases} 
\mu_1 & x < 0, \\
\mu_i & 0 \leq x \leq h, \\
\mu_3 & x > h,
\end{cases}
\]

The components of magnetic field can be found from the following relations:

\[
H_x = -\frac{i}{\kappa_0 \mu(x)} \frac{\partial E}{\partial z}, \quad H_z = \frac{i}{\kappa_0 \mu(x)} \frac{\partial E}{\partial x},
\]

(3)

The TE wave is ordinary one according to choosing of the direction of anisotropy axis. In the case of \( \varepsilon_o > 0 \) the problem is reduced to a familiar case. However, one can expect the interesting result for hyperbolic material with \( \varepsilon_o < 0 \).
The electric field can be represented in the form \( E(x, z) = \tilde{E}(x) \exp(i\beta z) \), because the waveguide is homogeneous along \( OZ \), where parameter \( \beta \) is the propagation constant. This parameter is similar to the wave number in the case of homogeneous medium. Solutions of the wave equation should be found with taking into account the boundary conditions \( \mathbf{E} \to 0, \mathbf{H} \to 0 \) at \(|x| \to \infty \). The solution can be obtained by standard procedure [22]. Distribution of the electric field is given by the following equations

\[
 x < 0 : \quad E^{(1)} = A \exp(px + i\beta z) + \text{c.c.}, \\
 0 \leq x \leq h : \quad E^{(2)} = A \left[ \cos(\kappa x) + \xi_p \sin(\kappa x) \right] e^{i\beta z} + \text{c.c.}, \quad (4) \\
 x > h : \quad E^{(3)} = A \left[ \cos(\kappa h) + \xi_p \sin(\kappa h) \right] e^{-q(x-h)} e^{i\beta z} + \text{c.c.}
\]

where following parameters are used

\[
p_2^2 = \beta^2 + k_0^2\mu_1 |\varepsilon_o^{(1)}|, \quad q^2 = \beta^2 + k_0^2\mu_3 |\varepsilon_o^{(3)}|, \quad \kappa^2 = k_0^2\mu_1 \varepsilon_i - \beta^2.
\]

These parameters can be used to define the Goos-Hänchen phase shifts \( \phi_q \) and \( \phi_p \):

\[
 \xi_q = - \tan(\phi_q / 2) = \frac{q\mu_i}{\kappa\mu_3}, \quad \xi_p = - \tan(\phi_p / 2) = \frac{p\mu_i}{\kappa\mu_1}.
\]

Normalized amplitude \( A \) of the electrical field at \( x = 0 \) is arbitrary.

TM wave is determined by the component of a magnetic field \( H_y \) and an electric field components \( E_x, E_z \). The wave equation for magnetic field \( H = H_y(x, z, \omega) \) is

\[
 \frac{1}{\varepsilon(x)} \frac{\partial^2 H}{\partial z^2} + \frac{\partial}{\partial x} \left( \frac{1}{\varepsilon_o(x)} \frac{\partial H}{\partial x} \right) + k_0^2 \mu(x) H = 0.
\]

The electric field components can be found from the following expressions:

\[
 E_x = - \frac{i}{k_0 \varepsilon_o(x)} \frac{\partial H}{\partial z}, \quad E_z = \frac{i}{k_0 \varepsilon_o(x)} \frac{\partial H}{\partial x}.
\]

The principal dielectric constants and permeability are piecewise functions previously defined in the case of TE waves.

A consideration of the equation (5) shows, that if \( \varepsilon_e^{(a)} < 0 \) and \( \varepsilon_o^{(a)} > 0 \) (\( a = 1, 3 \)), then there is no solution of this equation decreasing at \(|x| \) tends to infinity. Hence, there are no wave localized in waveguide. Alternatively, if \( \varepsilon_e^{(a)} > 0 \) and \( \varepsilon_o^{(a)} < 0 \) then under conditions:

\[
k_0^2 \mu_1 \varepsilon_e^{(1)} > \beta^2, \quad k_0^2 \mu_3 \varepsilon_e^{(3)} > \beta^2
\]

equation (5) admits a solution describing wave confinement in this waveguide. The magnetic field distribution is written as

\[
 x < 0 : \quad H^{(1)} = A \exp(px + i\beta z) + \text{c.c.}, \\
 0 \leq x \leq h : \quad H^{(2)} = A \left[ \cos(\kappa x) - \xi_p \sin(\kappa x) \right] e^{i\beta z} + \text{c.c.}, \quad (8) \\
 x > h : \quad H^{(3)} = A \left[ \cos(\kappa h) - \xi_p \sin(\kappa h) \right] e^{-q(x-h)} e^{i\beta z} + \text{c.c.}
\]

In these expressions following parameters

\[
p_2^2 = k_0^2 \mu_1 |\varepsilon_o^{(1)}| - \frac{|\varepsilon_o^{(1)}|}{\varepsilon_e^{(1)}} \beta^2, \quad q^2 = k_0^2 \mu_3 |\varepsilon_o^{(3)}| - \frac{|\varepsilon_o^{(3)}|}{\varepsilon_e^{(3)}} \beta^2, \quad \kappa^2 = k_0^2 \mu_1 \varepsilon_i - \beta^2,
\]

are used. The GoosHänchen phase shifts \( \phi_q \) and \( \phi_p \) are defined by the following expressions:

\[
 \xi_q = \tan(\phi_q / 2) = \frac{q\varepsilon_i}{\kappa|\varepsilon_o^{(3)}|}, \quad \xi_p = \tan(\phi_p / 2) = \frac{p\varepsilon_i}{\kappa|\varepsilon_o^{(1)}|}.
\]
III. DISPERSION RELATIONS

Taking into account decreasing of the magnetic and electric fields in the limit $|x| \to \infty$, the Maxwell equations solutions describe the waves which are confined by the waveguide. It is necessary to distinguish the coupled surface waves and the waveguide modes. In a linear waveguide the amplitude of surface wave takes the maximum at interface. In the case of the dielectric waveguide cladded by metal this coupled surface wave is said to be the plasmon-polariton wave. Guided plasmon-polariton wave in the dielectric waveguide arranged from a hyperbolic metamaterial has been proposed and studied in [30] [31]. The dispersion relation of this plasmon-polariton wave can be obtained by changing parameter $k^2$ to $\beta^2 = k_0^2 \mu_i \varepsilon_i$. However, other than these, there are a number of waves localized in the dielectric core that are designated as guided modes or waveguide modes [32].

For all guided waves the dispersion relation exists as expression connecting the propagation constant $\beta$ and frequency $\omega$. The dispersion relations of the guided waves under consideration can be determined by using the continuity conditions of an electric and magnetic field on the interferences. It is suitable to get the dispersion relations for TE and TM wave separately.

A. Case of TE wave

Distribution of the magnetic field in waveguide is derivable from (4) with taking into account the expressions (3). The continuity conditions for tangent components of both electric and magnetic field vectors result in following relation:

$$e^{2i \kappa h} \left( \frac{1 - i \xi_q}{1 + i \xi_q} \right) \left( \frac{1 - i \xi_p}{1 + i \xi_p} \right) = 1.$$  

Using the expression for the Goos-Hänchen phase shift one can write the dispersion relation in form:

$$2 \kappa h + \phi_p + \phi_q = 2 \pi m, \quad m = 0, 1, 2, \ldots \quad (9)$$

If the effective index of refraction $n_{ef}$ is defined according to formula $\beta = k_0 n_{ef}$, than equation (9) can be written as:

$$h k_0 \sqrt{n_i^2 - n_{ef}^2} = \arctan \left( \frac{\mu_i}{\mu_1} \sqrt{\frac{n_i^2 + n_{ef}^2}{n_i^2 - n_{ef}^2}} \right) + \arctan \left( \frac{\mu_i}{\mu_3} \sqrt{\frac{n_i^2 + n_{ef}^2}{n_i^2 - n_{ef}^2}} \right) + \pi m.$$  

Here the indexes of refraction $n_i^2 = \mu_1 \varepsilon_0^{(1)}$, $n_2^2 = \mu_3 \varepsilon_0^{(3)}$, $n_3^2 = \mu_i \varepsilon_i$ are introduced.

The dispersion relation shows that the effective index of refraction is limited by the condition $0 \leq n_{ef}^2 < n_i^2$. In the case of all dielectric waveguide the similar limitation appears as $\max(n_1^2, n_2^3) \leq n_{ef}^2 < n_i^2$. Difference between these inequalities is due to the fact that in dielectric waveguide embedded in a hyperbolic media the total internal reflection takes place at any incident angle. As in the case of dielectric waveguide cladded by metal.

Analysis of the dispersion relations will be performed for the case of symmetric waveguide, where $n_1^2 = n_3^2$. Then the dispersion relation takes the form:

$$k_0 h \sqrt{n_i^2 - n_{ef}^2} = 2 \arctan \left( \frac{\mu_i}{\mu_1} \sqrt{\frac{n_i^2 + n_{ef}^2}{n_i^2 - n_{ef}^2}} \right) + \pi m.$$  

Equation (10) can be rewritten in the normalized form. The parameter $b$ is introduced by the following equation $n_1^2 + n_{ef}^2 = b \Delta$, where $\Delta = n_1^2 + n_2^3$. The normalized waveguide thickness $V$ is introduced by formula $V = k_0 h \sqrt{n_i^2 + n_1^2}$. Then the relation (10) takes the form:

$$V \sqrt{1 - b} = 2 \arctan \left( \frac{b}{\mu_1 \sqrt{1 - b}} \right) + \pi m.$$  

This equation defines the function $b(V, m)$ that is implicit dependence of the normalized effective index of refraction $b$ on normalized waveguide thickness $V$. In the case under consideration $b$ is in the interval $[b_0, 1)$, where $b_0 = n_1^2/(n_1^2 + n_2^3)$. The plots of $b(V, m)$ vs $V$ are presented on Fig.2 at $\mu_1/\mu_2 = 1.2$ and $b_0 = 0.2$. The fact that $b_0 > 0$ means that zero-mode TE0 has non zero cut-off frequency $V_{c0}$. Substitution $b = b_0$ into equation (11) results in

$$V_{c0} = 2 \sqrt{1 + \frac{n_1^2}{n_1^2}} \arctan \left( \frac{\mu_i n_1}{\mu_1 n_i} \right).$$  

For conventional dielectric waveguide cut-off frequency $V_{c0}$ is zero.
By the use of (8) and (6) the electric field strength can be found. The continuity conditions for tangent components of both electric and magnetic field vectors lead to the following dispersion relation for TM waves

\[ e^{2i\kappa h} \left( \frac{1 + i\xi_q}{1 - i\xi_q} \right) \left( \frac{1 + i\xi_p}{1 - i\xi_p} \right) = 1. \]

Using the Goos–Hänchen phase shifts the dispersion relation can be written as:

\[ 2\kappa h + \phi_p + \phi_q = 2\pi m, \quad m = 0, 1, 2, \ldots \] (13)

In terms of initial variables the relation (13) takes the form:

\[ h\sqrt{k_0^2(\mu_i\varepsilon_i - n_{ef}^2)} = -2 \arctan \left[ \frac{\varepsilon_i^2}{|\varepsilon_i^{(3)}|\varepsilon_e^{(3)}} \left( \frac{\varepsilon_e^{(3)} - n_{ef}^2}{\varepsilon_i - n_{ef}^2} \right) \right] - \arctan \left[ \frac{\varepsilon_i^2}{|\varepsilon_i^{(1)}|\varepsilon_e^{(1)}} \left( \frac{\varepsilon_e^{(1)} - n_{ef}^2}{\varepsilon_i - n_{ef}^2} \right) \right] + \pi m. \] (14)

Let us consider only symmetrical waveguide, where \( n_i^2 = n_e^3 \). The dispersion relation can be rewritten as

\[ h k_0 \sqrt{n_i^2 - n_{ef}^2} = -2 \arctan \left[ \frac{\varepsilon_i^2}{|\varepsilon_e^{(1)}|\varepsilon_e^{(1)}} \left( \frac{n_e^2 - n_{ef}^2}{n_i^2 - n_{ef}^2} \right) \right] + \pi m, \] (15)

where the effective indexes of refraction \( n_i \) and \( n_e \) are used. These parameters are defined by correlation \( n_i^2 = \mu_i \varepsilon_i \) for isotropic dielectric and \( n_e^2 = \mu_1 \varepsilon_e \) for extraordinary wave in the hyperbolic media.

The condition for effective index

\[ 0 \leq n_{ef}^2 < n_i^2, \quad 0 \leq n_{ef}^2 \leq n_e^2 \]

follows from the equation (14). In the case of a convenient dielectric medium this condition appears as \( n_e \leq n_{ef} < n_i \), where \( n_e \) is the refraction index of the substrate or cladding layer.

Equation (14) can be rewritten in terms of the uniform variables \((b, V)\) by using following correlations: \( n_e^2 - n_{ef}^2 = b\Delta > 0 \), where \( \Delta = n_i^2 - n_e^2 \), and normalized waveguide thickness \( V \) is introduced as \( V = k_0 h \sqrt{n_i^2 - n_e^2} \). \( b \) is normalized effective index of refraction of waveguide. It results in the dispersion relation in following form:

\[ V \sqrt{1 + b} = -2 \arctan \left[ \frac{\varepsilon_i^2}{|\varepsilon_e^{(1)}|\varepsilon_e^{(1)}} \left( \frac{b}{1 + b} \right) \right] + \pi m. \] (15)
In the case under consideration the parameter $b$ is in interval $[0, b_0]$, where $b_0 = n_i^2/(n_i^2 - n_e^2)$.

The dispersion curves corresponding to equation (15) are represented in Fig.3. The assumption $\varepsilon_i^2/|\varepsilon_0^{(1)}|\varepsilon_e^{(1)}| = 1.2$, $b_0 = 2$ is hold. For comparison the dispersion curves corresponding all dielectric waveguide are shown in Fig.4.

$$V\sqrt{1-b} = 2\arctan \left( \sqrt{\frac{b}{1-b}} \right) + \pi m, \quad m = 0, 1, 2, \ldots$$

where $u = \varepsilon_i^2/(\varepsilon_0\varepsilon_e)$. The curves in Fig. 4 were obtained at $u = 1.2$.

FIG. 3: Dispersion curves for TM modes of the hyperbolic waveguide.

FIG. 4: Dispersion curves for TM modes of the conventional dielectric waveguide.

Figures show, that in the case of TM wave the number of guided modes of the hyperbolic waveguide is limited. As the dielectric core thickness $h$ increases, one mode disappears but other mode appears. Furthermore, the zero-mode ($m = 0$) in this waveguide is absent. For an all dielectric waveguide (Fig.4) the number of guided modes increases with core thickness $h$.

Thus, for the each TM mode of the hyperbolic waveguide two cut-off frequencies exist: $b(V_{c_{m}}^{(2)}) = b_0$ and $b(V_{c_{m}}^{(1)}) = 0$. For each TM mode of conventional dielectric waveguide only single cut-off frequency exists $V_{c_{m}}^{(1)}$. 

C. Poynting vector of the guided waves in the hyperbolic waveguide

The Poynting vector defines density of the radiation energy flux and direction of wave’s energy propagation. It is instructive to consider an averaged projection of the Poynting vector along the $OZ$ axis. For TE wave it can be found from the equation

$$\langle S_z \rangle = -\frac{c}{16\pi} (E_y^* H_x + E_x^* H_y^*),$$

(16)

and for TM wave it follows from the relation

$$\langle S_z \rangle = \frac{c}{16\pi} (E_x^* H_y + E_y^* H_x^*).$$

(17)

The relations (16) and (17) can be rewritten with taking into account equation (3) for the TE wave and equation (6) for the TM wave as

$$\langle S_z \rangle_{TE} = \frac{c}{8\pi \mu_j} |E_y|^2, \quad \langle S_z \rangle_{TM} = \frac{c}{8\pi \varepsilon_j} |H_y|^2,$$

(18)

where $j = 1, i, 3$ for $\mu_j$ and $j = e^{(1)}, i, e^{(3)}$ for $\varepsilon_j$ depending on the layer under consideration.

As was obtained in the previous two subsections, the effective index of refraction $n_{ef}$ for TE and TM guided modes in the hyperbolic waveguide can achieve null value. In these cases the averaged energy flux along the guided wave propagation axis $OZ$ will be zero. Thus, the effect of slowing light in the waveguide takes place in these cases.

IV. CONCLUSION

The special kind of the hyperbolic slab waveguide is considered here. In the case of $\varepsilon_o < 0$ and $\varepsilon_e > 0$ the modes of directed waves were found. The TE wave is ordinary wave, but TM wave is extraordinary in the waveguide under consideration. If $\varepsilon_o > 0$ and $\varepsilon_e < 0$ TM wave is not confined, and TE guided waves are identical with waves in a conventional waveguide. It is the reason to study the case of hyperbolic waveguide with $\varepsilon_o < 0$ and $\varepsilon_e > 0$.

The effective index of refraction for TE wave obeys following inequality $0 \leq n_{ef} < n_i$, where $n_i$ is the index of refraction of the waveguide’s core. In the case of TM wave the effective index of refraction varies within the limits $0 \leq n_{ef} \leq n_e$, where $n_e$ is the index of refraction of the extraordinary wave in hyperbolic medium. (It is assumed that $n_e < n_i$.) Thus, both cases the effective index of refraction can be equal to zero, that leads to great phase velocity of the wave and zero Poynting vector component along the wave’s propagation axis in the hyperbolic waveguide discussed above. Hence the light wave can be slowed down in this waveguide. In the case of conventional (elliptic) anisotropic dielectric waveguide the effective index of refraction lies in the range $n_e \leq n_{ef} < n_i$ for TM wave, and in the range $n_o < n_{ef} \leq n_i$ for TE wave.

The dispersion relations for the case of TE and TM waves are derived. It was shown that for the TM wave the number of guided modes is limited. Each of these modes have two cut-off frequencies. One of them corresponds to mode appearance, another corresponds to mode disappearance. There is region of parameters in which the only single mode exists in this waveguide. It is worth noting that this phenomenon is unavailable in the case of a conventional waveguide. Usually the number of modes increases with core thickness, and only single cut-off frequency exists.

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