Possible retardation effects of quark confinement on the meson spectrum II

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Abstract

We present the results of a study of heavy-light-quark bound states in the context of the reduced Bethe-Salpeter equation with relativistic vector and scalar interactions. We find that satisfactory fits may also be obtained when the retarded effect of the quark-antiquark interaction is concerned.

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Because of the limited understanding for confinement at present, more theoretical efforts to be made related to this issue are worthwhile. In an earlier paper [1], we presented results of a relativistic analysis of the spectrum of light- and heavy-quark-antiquark bound states based on the reduced Bethe-Salpeter(BS) equation, while retardation effects in the quark-antiquark interaction kernel were taking into consideration. The results are stimulating and appears having clarified the problem pointed out by Durand et al. [2] for the static scalar confinement in reduced Salpeter equation. The ”intrinsic flaw” of the Salpeter equation with static scalar confinement could be remedied to some extent by taking the retardation effect into confinement. In the on-shell approximation for the retardation term of linear confinement, the notorious trend of narrow level spacings for quarkonium states especially for light quarkonium states is found to be removed. A good fit for mass spectrum of S-wave heavy and light quarkonium states (except for the light pseudoscalar mesons) is obtained using one-gluon exchange potential and the scalar linear confinement potential with retardation taken into account.

In this paper we extend our previous study to the heavy-light-quark systems (Q̄q or q̄Q) in order to get a fully understanding of the retardation effects on meson spectra. Based on the same procedure taken in Ref. [1], by solving the reduced Bathe-Salpeter equation numerically, the previous conclusion is further substantiated through this study.

We assume the confinement kernel in momentum space taking the form

\[ G(p) \propto \frac{1}{(-p^2)^2} = \frac{1}{(p^2 - p_0^2)^2}, \]  

(1)

as suggested by some authors as the dressed gluon propagator to implement quark confinement [3]. Here \( p \) is the 4-momentum exchanged between the quark and antiquark in a meson. If the system is not highly relativistic we may make the approximation

\[ G(p) \propto \frac{1}{(p^2 - p_0^2)^2} \approx \frac{1}{(p^2)^2} \left(1 + \frac{2p_0^2}{p^2}\right), \]  

(2)

and may further express \( p_0 \) in terms of its on-shell values which are obtained by assuming that quarks are on their mass shells. This should be a good approximation for \( c\bar{c} \) and \( b\bar{b} \) systems. However, to get a qualitative feeling about the retardation effect considered here, we will also use (2) for heavy-light-quark mesons, though the approximations are not as good as for heavy-heavy-quark mesons. With the above procedure, the scalar confinement kernel becomes instantaneous again but with some retardation effects have been taken into the kernel. In the static limit, the retardation term vanishes and the kernel returns to \( G(q) \propto \frac{1}{(p^2)^2} \), which is just the Fourier transformation of the linear confining potential. In this paper, we will use this modified scalar confining potential in which the retardation effect is incorporated and the one-gluon-exchange potential in the framework of the reduced
Salpeter equation to study the mass spectrum of \( q \bar{Q} \) mesons, and the structure of the hyperfine splittings of the heavy-light-quark systems will also be investigated in this paper.

In quantum field theory, one of the basic descriptions for the bound states is the Bethe-Salpeter equation \([4]\). We can define the BS wave function of the bound state \( | P \rangle \) of a quark \( \psi(x_1) \) and an antiquark \( \bar{\psi}(x_2) \) as

\[
\chi(x_1, x_2) = \langle 0 | T\psi(x_1)\bar{\psi}(x_2) | P \rangle.
\] (3)

Here \( T \) represents the time-order product, and the wave function can be transformed into the momentum space,

\[
\chi_P(q) = e^{-iP \cdot X} \int d^4 x e^{-iq \cdot x} \chi(x_1, x_2),
\] (4)

where \( P \) is the four-momentum of the meson and \( q \) is the relative momentum between quark and antiquark. As the standard measure in solving the BS equation, we choose center-mass and relative coordinates as variables,

\[
X = \eta_1 x_1 + \eta_2 x_2, \quad x = x_1 - x_2,
\] (5)

where \( \eta_i = \frac{m_i}{m_1 + m_2} \) (\( i = 1, 2 \)). After taking the Fourier transformation, in the momentum space the BS equation reads as

\[
(\not{p}_1 - m_1)\chi_P(q)(\not{p}_2 + m_2) = \frac{i}{2\pi} \int d^4 k G(P, q - k) \chi_P(k),
\] (6)

where \( p_1 \) and \( p_2 \) represent the momenta of quark and antiquark, respectively,

\[
p_1 = \eta_1 P + q, \quad p_2 = \eta_2 P - q,
\] (7)

and \( G(P, q - k) \) is the interaction kernel which acts on \( \chi \) and is determined by the interquark dynamics. Note that in Eq.(6) \( m_1 \) and \( m_2 \) represent the effective constituent quark masses so that we could use the effective free propagators of quarks instead of the full propagators. This is an important approximation and simplification for light quarks. Furthermore, because of the lack of a fundamental description for the nonperturbative QCD dynamics, we have to make some approximations for the interaction kernel of quarks. In solving the Eq.(6), the kernel is taken to be instantaneous, but with some retardation effect being taken into it; the negative energy projectors in the quark propagators are neglected, because in general the negative energy projectors only contribute in higher orders due to \( M - E_1 - E_2 \ll M + E_1 + E_2 \), where \( M, E_1, \) and \( E_2 \) are the meson mass, the quark kinetic energy, and the antiquark kinetic energy respectively. Based on the above assumptions the BS equation can be reduced to a three-dimensional equation, i.e. the reduced Salpeter equation,
\[(P^0 - E_1 - E_2)\Phi_{\vec{P}}(\vec{q}) = \Lambda_+^1 \gamma^0 \int d^3kG(\vec{P}, \vec{q}, \vec{k})\Phi_{\vec{k}}(\vec{k})\gamma^0\Lambda_-^2.\] (8)

Here
\[\Phi_{\vec{P}}(\vec{q}) = \int dq^0 \chi_{P}(q^0, \vec{q}),\] (9)

is the three dimensional BS wave function, and
\[\Lambda_+^1 = \frac{1}{2E_1}(E_1 + \gamma^0 \vec{\gamma} \cdot \vec{p}_1 + m_1\gamma^0),\] (10)
\[\Lambda_-^2 = \frac{1}{2E_2}(E_2 - \gamma^0 \vec{\gamma} \cdot \vec{p}_2 - m_2\gamma^0),\] (11)

are the remaining positive energy projectors of the quark and antiquark respectively, with \(E_1 = \sqrt{m_1^2 + p_1^2}, \ E_2 = \sqrt{m_2^2 + p_2^2}.\) The formal products of \(G\Phi\) in Eq.(8) take the form
\[G\Phi = \sum_i G_i O_i \Phi O_i = G_s\Phi + \gamma_\mu \otimes \gamma^\mu G_v\Phi,\] (12)

where \(O_i = \gamma_\mu\) corresponding to the perturbative one-gluon-exchange interaction and \(O_i = 1\) to the scalar confinement potential.

From Eq.(8) it is easy to see that
\[\Lambda_+^1 \Phi_{\vec{P}}(\vec{q}) = \Phi_{\vec{P}}(\vec{q}), \ \Phi_{\vec{P}}(\vec{q})\Lambda_-^2 = \Phi_{\vec{P}}(\vec{q}).\] (13)

Considering the constraint of Eq.(13) and the requirements of space reflection of bound states, in the meson rest frame \((\vec{P} = 0)\) the wave function \(\Phi_{\vec{P}}(\vec{q})\) for the \(0^-\) and \(1^-\) mesons can be expressed as
\[\Phi_{P}^{0^-}(\vec{q}) = \Lambda_+^1 \gamma^0(1 + \gamma^0)\gamma_5\gamma^0\Lambda_-^2 \varphi(\vec{q}),\] (14)
\[\Phi_{P}^{1^-}(\vec{q}) = \Lambda_+^1 \gamma^0(1 + \gamma^0) \varphi'\gamma^0\Lambda_-^2 f(\vec{q}),\] (15)

where \(\varphi = \gamma_\mu e^\mu\) is the polarization vector of \(1^-\) meson; \(\varphi(\vec{q})\) and \(f(\vec{q})\) are scalar functions of \(\vec{q}^2\). It is easy to show that Eqs.(14) and (15) are the most general forms of the S-wave wave functions for \(0^-\) and \(1^-\) mesons in the rest frame.

Substituting Eqs.(12), (14), and (15) into Eq.(8), one can derive the equations out for \(\varphi(\vec{q})\) and \(f(\vec{q})\) in the meson rest frame \[3\].
\[ M \varphi_1(\vec{q}) = (E_1 + E_2) \varphi_1(\vec{q}) - \frac{E_1 E_2 + m_1 m_2 + \vec{q}^2}{4E_1 E_2} \int d^3 k (G_S(\vec{q}, \vec{k}) - 4G_V(\vec{q}, \vec{k})) \varphi_1(\vec{k}) \]

\[ - \frac{E_1 m_2 + E_2 m_1}{4E_1 E_2} \int d^3 k (G_S(\vec{q}, \vec{k}) + 2G_V(\vec{q}, \vec{k})) \frac{m_1 + m_2}{E_1 + E_2} \varphi_1(\vec{k}) \]

\[ + \frac{E_1 + E_2}{4E_1 E_2} \int d^3 k G_S(\vec{q}, \vec{k})(\vec{q} \cdot \vec{k}) \frac{m_1 + m_2}{E_1 m_2 + E_2 m_1} \varphi_1(\vec{k}) \]

\[ + \frac{m_1 - m_2}{4E_1 E_2} \int d^3 k (G_S(\vec{q}, \vec{k}) + 2G_V(\vec{q}, \vec{k}))(\vec{q} \cdot \vec{k}) \frac{E_1 - E_2}{E_1 m_2 + E_2 m_1} \varphi_1(\vec{k}), \tag{16} \]

with

\[ \varphi_1(\vec{q}) = \frac{(m_1 + m_2 + E_1 + E_2)(E_1 m_2 + E_2 m_1)}{4E_1 E_2 (m_1 + m_2)} \varphi(\vec{q}), \tag{17} \]

and

\[ M f_1(\vec{q}) = (E_1 + E_2) f_1(\vec{q}) - \frac{1}{4E_1 E_2} \int d^3 k (G_S(\vec{q}, \vec{k}) - 2G_V(\vec{q}, \vec{k}))(E_1 m_2 + E_2 m_1) f_1(\vec{k}) \]

\[ - \frac{E_1 + E_2}{4E_1 E_2} \int d^3 k G_S(\vec{q}, \vec{k}) \frac{E_1 m_2 + E_2 m_1}{E_1 + E_2} f_1(\vec{k}) \]

\[ + \frac{E_1 m_2 - E_2 m_1}{4E_1 E_2 \vec{q}^2} \int d^3 k (G_S(\vec{q}, \vec{k}) + 4G_V(\vec{q}, \vec{k}))(\vec{q} \cdot \vec{k}) f_1(\vec{k}) \]

\[ - \frac{E_1 + E_2 - m_2 - m_1}{2E_1 E_2 \vec{q}^2} \int d^3 k G_S(\vec{q}, \vec{k})(\vec{q} \cdot \vec{k})^2 \frac{1}{E_1 + E_2 + m_1 + m_2} f_1(\vec{k}) \]

\[ - \frac{m_2 + m_1}{E_1 E_2 \vec{q}^2} \int d^3 k G_V(\vec{q}, \vec{k})(\vec{q} \cdot \vec{k})^2 \frac{1}{E_1 + E_2 + m_1 + m_2} f_1(\vec{k}), \tag{18} \]

with

\[ f_1(\vec{q}) = - \frac{m_1 + m_2 + E_1 + E_2}{4E_1 E_2} f(\vec{q}). \tag{19} \]

Eqs. (16) and (18) can also be formally expressed as more compact forms

\[ (M - E_1 - E_2) \varphi_1(\vec{q}) = \int d^3 k \sum_{i=S,V} F_i^0 (\vec{q}, \vec{k}) G_i (\vec{q}, \vec{k}) \varphi_1(\vec{k}), \tag{20} \]

\[ (M - E_1 - E_2) f_1(\vec{q}) = \int d^3 k \sum_{i=S,V} F_i^1 (\vec{q}, \vec{k}) G_i (\vec{q}, \vec{k}) f_1(\vec{k}). \tag{21} \]
In case of taking the nonrelativistic limit for both quark and antiquark, expanding in terms of $\vec{q}^2/m_1^2$ and $\vec{q}^2/m_2^2$, it can be approved that Eqs.(16) and (18) are identical with the Schrödinger equation to the zeroth order, and with the Breit equation to the first order.

To solve Eq.(6) one must have a good command of the potential between two quarks. At present, the reliable information about the potential only comes from the lattice QCD result, which shows that the potential for a heavy quark-antiquark pair $Q\bar{Q}$ in the static limit is well described by a long-ranged linear confining potential (Lorentz scalar $V_S$) and a short-ranged one gluon exchange potential (Lorentz vector $V_V$), i.e. \[ V(r) = V_S(r) + \gamma_\mu \otimes \gamma_\mu V_V(r), \tag{22} \]

with
\[
V_S(r) = \lambda r \frac{(1 - e^{-\alpha r})}{\alpha r}, \tag{23}
\]
\[
V_V(r) = -\frac{4}{3} \alpha_s(r) \frac{e^{-\alpha r}}{r}, \tag{24}
\]
where the introduction of the factor $e^{-\alpha r}$ is not only for the sake of avoiding the infrared(IR) divergence but also incorporating the color screening effects of the dynamical light quark pairs on the "quenched" $Q\bar{Q}$ potential \[8\]. Although the lattice QCD result for the $Q\bar{Q}$ potential is supported by the heavy quarkonium spectroscopy including both spin-independent and spin-dependent effects \[9\] \[11\]. we will employ this static potential below to the heavy-light-quark systems as an assumption. The interaction potentials in Eq.(22) can be transformed straightforwardly into the momentum space, where the strong coupling constant
\[
\alpha_s(\vec{p}^2) = \frac{12\pi}{27} \frac{1}{\ln(a + \frac{\vec{p}^2}{\Lambda_{QCD}^2})}. \tag{25}
\]
is assumed to be a constant of $O(1)$ as $\vec{p}^2 \to 0$. The constants $\lambda$, $\alpha$, $a$, and $\Lambda_{QCD}$ are the parameters that characterize the potential.

In taking the retardation effect of scalar confinement into consideration, as discussed in Ref. \[1\], the confinement will be approximately introduced by adding a retardation term $\frac{2\vec{p}_0^2}{\rho^6}$ to the instantaneous part $\frac{1}{(\vec{p}_0^2)^2}$ as given in Eq.(4), and $\vec{p}_0^2$ will be treated to take its on-shell values which are obtained by assuming that the quarks are on their mass shells, which means that the retardation term will become instantaneous rather than convoluted. By this procedure the modified scalar confinement potential will include retardation effect and become
\[ G_S(\vec{p}) \rightarrow G_S(\vec{p}, \vec{k}) = -\frac{\lambda}{\alpha} \delta^3(\vec{p}) + \frac{\lambda}{\pi^2 (\vec{p}^2 + \alpha^2)^2} \frac{1}{(\vec{p}^2 + \alpha^2)^2} + \frac{2\lambda}{\pi^2 (\vec{p}^2 + \alpha^2)^3} (\sqrt{(\vec{p} + \vec{k})^2 + m^2} - \sqrt{\vec{k}^2 + m^2})^2, \]  

which is related not only to the interquark momentum exchange \( \vec{p} \) but also the quark momentum \( \vec{k} \) itself.

Based on the formalism and discussions above, we can now embark on the numerical calculations, in which we take the following values for input parameters

\[ \lambda = 0.21 \text{ GeV}^2, \quad \alpha = 0.06 \text{ GeV}, \quad a = e = 2.7183, \quad \Lambda_{QCD} = 0.19 \text{ GeV}, \quad C = -0.05 \]  

and

\[ m_u = m_d = 0.35 \text{ GeV}, \quad m_s = 0.5 \text{ GeV}, \quad m_c = 1.68 \text{ GeV}, \quad m_b = 4.925 \text{ GeV}, \]  

which fall in the scopes of customarily usage. The numerical results with retardation are listed in Table I. For the convenience of comparisons, results obtained without retardation are also presented.

From Table I one can immediately see that the calculated masses with retardation effect are generally well fitted compared with those without retardation considered, and the spin splittings, \( 1^3S_1 - 1^1S_0 \) are significantly improved by adding the retardation term to the scalar confinement potential. These conclusions obviously shed light on the usefulness of the Bethe-Salpeter equation in describing systems containing light quarks, however, in the mean time they also ask for further investigations on the interaction kernels.

As noticed in Ref. [2], the smallness of the hyperfine splitting obtained for \( q\bar{Q} \) mesons is due to the weakness of the binding potential in these systems. However, similarly as showed in Ref. [1] this situation may also be changed after including the retardation effect in the interaction kernel. For demonstration, in the equal-quark-mass special case the coefficients for the scalar potential \( G_s \) which plays the main role in setting the spin splittings in Eqs.(20) and (21) will reduce to

\[ F_{S}^{0^-}(\vec{q}, \vec{k}) = -\frac{1}{2} + \frac{\vec{q} \cdot \vec{k} - m^2}{2E_qE_k}, \]  

\[ F_{S}^{1^-}(\vec{q}, \vec{k}) = -\frac{m(E_q + E_k) - \vec{q} \cdot \vec{k}}{2E_q^2} - \frac{(\vec{q} \cdot \vec{k})^2}{2E_q^2q^2} (E_q + m)(E_k + m), \]  

coresponding to \( 0^- \) and \( 1^- \) mesons, respectively. It is clear that these coefficients will limit to \( m/E \) or it higher order while \( \vec{q} \rightarrow \vec{k} \). On the other hand, however, the static
linear confining potential in momentum space, which behaves as $G_s(\vec{q} - \vec{k}) \propto (\vec{q} - \vec{k})^{-4}$, is strongly weighted as $\vec{q} \to \vec{k}$ in Eq. (21) and (22). Because the coefficients will diminish in the relativistic limit as $\vec{q} \to \vec{k}$, the strength of the confining potential would be reduced in turn. This is the reason which leads to the small spin splittings obtained for heavy-light-quark mesons. However, this depressing situation would be changed if the retardation is taken into account. In fact, the covariant form of confinement interaction may take the form $G_S(q, k) \propto [(\vec{q} - \vec{k})^2 - (q_0 - k_0)^2]^{-2}$, and in the on-shell approximation, $q_0^2 = m^2 + q^2$, $k_0^2 = m^2 + k^2$, it becomes

$$G_S(q, k) \propto (-2m^2_q + qk - \vec{q} \cdot \vec{k})^{-2}$$

in the high energy limit, i.e., $p, k \gg m$. We can see that with the retardation effect the scalar interaction $G_S(q, k)$ is heavily weighted as $\vec{q}$ and $\vec{k}$ are co-linear ($\vec{q} \parallel \vec{k}$), whereas the static linear potential only peaks at $\vec{q} = \vec{k}$. This indicates that the former is weighted in a much wider kinematic region than the latter, especially for systems including light components. As a consequence, the wave functions in coordinate space would tend to be short ranged and hence the magnitudes of the wave function at the origin, $\psi(0)$, would increased. Because to leading order in $1/m^2$ expansion the hyperfine splitting is proportional to $|\psi(0)|^2$, the Hermitan square of the wave function of meson at the origin, the splittings would enlarge as well. The above analysis indicates that in the equal-quark-mass situation the modified effective scalar interaction will not be weakened too much as $\vec{q}$ and $\vec{k}$ approach to be parallel, and this is just due to the retardation effect. In our opinion, the difficulty in the reduced Salpeter equation with the static scalar confinement is probably due to the improper treatment that the confining interaction is purely instantaneous. In practice, for the constituent quark model, which is essentially used in the present work, the equal mass and the on-shell approximation maybe not a good simplification for the $q\bar{Q}$ systems, but in any case the analysis given above is qualitatively correct, which is supported by the numerical results listed in Table I. And, it is obvious that the new procedure gives a much better fit to data than the previous one, in especially the spin splittings. The results obtained about the mass spectrum, particularly the $s\bar{d}$ system, seems still not fully satisfactory, but it is noted that our results listed are just a schematic ones. A fine tuning may lead an improvement on them.

In conclusion, we have extended our previous study in clarifying the problems pointed out by Durand et al. for the static scalar confinement in reduced Salpeter equation to the heavy-light-quark systems in this paper. The conclusion remains the same that the ”intrinsic flaw” of the Salpeter equation with static scalar confinement could be remedied to some extent by taking the retardation effect of the confinement into consideration. A fit for mass spectrum of S-wave heavy-light-quark systems is obtained by using the scalar
linear confinement potential with retardation and the one-gluon exchange potential. Result shows that a great improvement is achieved on fitting the data with retardation taken into account. Although the on-shell approximation may not be a rigorous treatment here, the qualitative feature of the retardation effect is still manifest. Nevertheless, it is still premature to assess whether or not the quark confinement is really represented by the scalar exchange of the form of \((\vec{p}^2 - p_0^2)^{-2}\), as suggested by some authors and being used here, as the dressed gluon propagator to implement quark confinement. Therefore, further investigations on this subject are necessary.

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Table I

Calculated mass spectrum of $qQ$ states using reduced Salpeter equation with retardation for scalar confinement. The Experiment data are taken from Ref. [13]

| State   | Quarks | Data (MeV) | Fit I$^a$ (MeV) | Error (MeV) | Fit II$^b$ (MeV) | Error (MeV) |
|---------|--------|------------|-----------------|-------------|-----------------|-------------|
| $\bar{B}^0$ | $bd$   | 5279       | 5381            | +102        | 5258            | -21         |
| $D_+^s$  | $c\bar{s}$ | 1969       | 2097            | +128        | 1946            | -23         |
| $D_s^{++}$ | $c\bar{s}$ | 2112       | 2148            | +35         | 2094            | -19         |
| $D^+$    | $cd$   | 1869       | 1983            | +114        | 1862            | -7          |
| $D^{**}$ | $cd$   | 2010       | 2010$^*$        | 0           | 2003            | -7          |
| $\bar{K}^0$ | $sd$   | 498        | 743             | +245        | 652             | +154        |
| $\bar{K}^{*0}$ | $sd$   | 892        | 870             | -22         | 898             | +6          |
| $D^{*+} - D^{++}$ |        | 144        | 51              | -93         | 148             | +4          |
| $D^{*+} - D^+$   |        | 141        | 27              | -114        | 141             | 0           |
| $K^{*0} - K^0$   |        | 394        | 127             | -267        | 244             | -150        |

$^a$Results without retardation. $^b$Results with retardation. $^*$Used to fix the parameters.