PHOTONS, DILEPTONS
AND HARD THERMAL LOOPS * †

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Abstract
The production rate of soft real photons or soft lepton pairs by a hot QCD plasma is dominated
by strong collinear divergences. As a consequence, it appears that the effective theory based on the
resummation of hard thermal loops fails to handle properly these light-cone sensitive processes since
some formally higher order diagrams are in reality the dominant ones.

ENSLAPP-A-646/97
hep-ph/9705360

*Work done in collaboration with P. Aurenche, R. Kobes and E. Petitgirard.
†Talk given at the XXXIInd Rencontres de Moriond, Les Arcs, 22-29 March 1997.
1 Introduction

1.1 Plasma observables

There are several quantities that could be interesting probes for a quark-gluon plasma. Let us briefly describe a few such possibilities\(^\ddagger\), recalling the theoretical tools used for the calculation of each of them. The first possibility would be to measure the thermodynamic properties of such a plasma, by measuring for instance its pressure as a function of temperature and volume. From a theoretical point of view, this quantity requires the calculation of the free energy, which has been achieved up to the order \(g^5\), where \(g\) is the strong coupling constant, in thermal QCD. Another interesting experiment would be consist in studying the scattering of an external hard particle by a plasma. Since this situation involves the interaction of the plasma with some non thermalized object, it cannot be dealt with by the standard methods of thermal field theory. So far, it has been studied by semi classical methods. The last observable I have selected is the production rate of some weakly interacting particle, like photons or lepton pairs. This quantity has been calculated both by semi-classical methods\(^1\) and by thermal field theory\(^2\)\(^-\)\(^4\). The main part of this talk is devoted to the latter, while a comparison with the results of semi classical methods is presented briefly at the end.

1.2 Thermal field theory tools for the photon rate

The real photon production rate, per unit time and per unit volume of the plasma, is related simply to the thermal photon polarization tensor by:

\[
\frac{dN}{q_0^2 \, d^3 \mathbf{q}} = \frac{1}{(2\pi)^3} n_B(q_0) \text{Im} \Pi_{\mu \mu}(q_0, \mathbf{q}),
\]

where \(n_B(q_0) \equiv 1/(\exp(q_0/T) - 1)\) is the Bose-Einstein function. The production rate of lepton pairs (or, equivalently of virtual photons), is also related to \(\text{Im} \Pi_{\mu \mu}(q_0, \mathbf{q})\), with a somewhat different phase space. It is worth saying that this formula is valid to all orders in the strong coupling constant, but only to first order in \(\alpha\), since effects like the re-interactions of the emitted photons in the medium are neglected. This is justified by the smallness of the electromagnetic coupling constant.

1.3 Infrared problems and hard thermal loops summation

Nevertheless, the game consisting in the calculation of \(\text{Im} \Pi_{\mu \mu}\) in thermal QCD is not as simple as it seems, because thermal field theories are plagued by infrared singularities. One may easily understand why these singularities can be stronger than at \(T = 0\), by noticing that a Bose-Einstein statistical factor can be very large for soft energies: \(n_B(l_0) \sim T/l_0 \gg 1\) for \(l_0 \ll T\). In the following, it is useful to distinguish between two energy scales: the hard scale, of order \(T\), corresponds to the typical energy of partons in the plasma, and the soft scale, of order \(gT\), which is the typical scale of the quanta exchanged by interacting partons\(^5\). Noticing that certain loops carrying hard momentum can be as large as their bare counterparts if all their external legs are soft, and need therefore to be taken into account already to calculate consistently the first order of perturbation theory, Braaten and Pisarski set up an effective theory resumming these “Hard Thermal Loops” (HTL in the following), while preserving the gauge symmetry of the bare theory\(^5\). Physically, this resummation is closely related to the Debye screening in a plasma: through the resummation of HTLs, the gauge bosons can acquire a thermal mass \(m_g \sim gT\), which is nothing but the inverse of the range of the screened interaction.

\(^\ddagger\)In this theoretical description, we present only “ideal” experiments, in the sense that we assume having at our disposal a sufficient amount of plasma at thermal equilibrium. Unfortunately, they will remain thought experiments for a long time, and it is therefore important to develop also theoretical tools to study the real world experiments. This requires to deal with out-of-equilibrium plasmas, a subject beyond the scope of the present talk.
1.4 Further problems

Despite its nice features, the effective theory one gets after the resummation of HTLs has still a few problems that are not solved by this resummation. The first one is related to the absence of thermal mass for the static transverse gauge bosons. In QED, this is known to be true to all orders, and is physically related to the fact that static magnetic fields are not screened in a plasma. In QCD, the status of this “magnetic mass” is not so clear since the self interaction of gauge bosons can generate such a mass. Nevertheless, if such a mass exists, it is expected to be at most of order $g^2 T$ and beyond the abilities of perturbative calculations.

Another problem that is not solved by the HTL resummation is related to collinear singularities. Technically, this is due to the fact that the building blocks of the HTLs are bare massless propagators. As a consequence, the HTLs, which are the building blocks of the effective theory, become singular when some of their external legs are put on the light–cone, due to angular integrals like $\int d\cos\theta /E(1 - \cos\theta)$. A solution to overcome these singularities consists in taking into account an asymptotic thermal mass $m_F$ for the hard fermions that run inside the HTLs. Naïvely, the previous integral now becomes $\int d\cos\theta / (E - p \cos\theta)$, with $E = \sqrt{p^2 + m_F^2})$. The important result lies in the fact that such a procedure preserves gauge invariance while regularizing the collinear divergences.

2 Photon production rate at 1 loop in the effective theory

2.1 Results of previous calculations

At one loop in the effective theory, there are 3 diagrams contributing to the photon production rate. For each of them, we give some of the relevant physical amplitudes, as well as the result of previous calculations for real photons (for dileptons, see 2):

\[ a \] : 
\[ b \] : 
\[ c \] :

\[ \text{Im} \Pi_{\mu\mu}(Q) \mid_{[a]} \sim e^2 g^4 T^3 q_0 \]
\[ \text{Im} \Pi_{\mu\mu}(Q) \mid_{[b]} \approx 0 \]
\[ \text{Im} \Pi_{\mu\mu}(Q) \mid_{[c]} \approx 0 \]

A few comments are useful concerning these results. First of all, the only non vanishing term $[a]$ corresponds to a very complicated physical amplitude, which is not intuitive at all. Secondly, the result found for $[a]$ is smaller than what could be expected for soft photons by simple power counting. This fact seems to be due to a cancelation particular to the trace $\cdots \mu \mu$, since it does not occur in $\text{Im} \Pi^{00}$ for instance. Therefore, one should expect that this result is not complete since the approximations made in the calculation were designed to deal with dominant terms, whereas the first non vanishing order for $\text{Im} \Pi_{\mu\mu}$ is subdominant because of this cancelation.

As a consequence, one should perform the calculation of the same diagrams beyond the HTL ap-
proximation\textsuperscript{4}, including $[b]$ and $[c]$, since the vanishing result obtained so far is only a consequence of the HTL approximation associated to the trace $\cdots^\mu_{\mu}$, that does not tell us anything about subdominant terms. In reality, by noticing that $[b]$ involves a Fermi-Dirac weight evaluated at a soft energy whereas $[c]$ involves a Bose-Einstein weight at the same energy, one can avoid the calculation of $[b]$, which is negligible in front of $[c]$. Moreover, it turns out that the result for $[c]$ is much greater than the partial result already obtained for $[a]$, which means that only $[c]$ is needed.

2.2 Diagram (c) beyond the HTL approximation

In order to perform the calculation, we come back to the diagrammatic expansion of the effective $\gamma - \gamma - g - g$ vertex, which amounts to the following two topologies:

![Diagram](image)

The first step is now to perform the Dirac’s algebra associated to the fermion loop. Na"ively, for a loop with four hard fermions, the Dirac’s traces should be of order $T^4$. Nevertheless, due to the $\cdots^\mu_{\mu}$ cancelation, these traces are suppressed and their order of magnitude is in reality $g^2 T^4$. More precisely, these traces are $r^2 L^2$ in the case of the vertex correction topology, and $r^2 Q^2$ for the self energy correction. Therefore, if the produced photon is close enough to the light-cone, we can neglect the self-energy diagram with respect to the vertex correction.

Another feature of the vertex diagram lies in the strong collinear singularities that can potentially appear when the photon is emitted collinearly to the quark. Indeed, the imaginary part of this diagram contains the following factors, relevant for these considerations: \(\delta(P^2)\delta((R + L)^2)/(P + L)^2(P + Q)^2\), where we omitted for the sake of simplicity the fermion thermal mass. We easily see that the dangerous denominators, both of them leading to a simple pole in $\cos \theta$ where $\theta$ is the angle between the quark and the photon, can vanish almost simultaneously since $p$ and $r + l$ are collinear to $q$ almost at the same time ($p$, $r$ are hard, whereas $q$ and $l$ are soft). As a consequence, these factors behave very much like a double pole, which means that the regulator (a thermal mass $m_F \sim gT$) will appear as $T^2/m_F^2$ instead of inside a logarithm as it would be for two separate simple poles, and this fact can change the order of magnitude of the whole result. More precisely, taking the fermion thermal mass into account, and being very rough with algebra, such a double pole will give\textsuperscript{5}: \(\int_{-1}^{1} \ d\cos \theta / (1 - \cos \theta + m_g^2/2r^2)^2 \sim r^2/m_F^2 \sim 1/g^2 \gg 1\), whereas such a dimensionless angular integral would have been of order 1 in the absence of collinear singularities, and at most of order $\ln(r^2/m_F^2)$ in the case of separate simple poles.

Let us now summarize the features of the result we get for this diagram:

(i) \(\Im \Pi_{\mu}^a(Q)_{\mid_{|c|}} \approx - \sum_{T,L} e^2 g^2 J_{T,L} \frac{T^2}{m_F^2} \sim e^2 g^2 T^2/\eta_0\), where $J_{T,L}$ is a numerical factor depending on the two dimensionless ratios $m_g^2/m_F^2$ and $Q^2 T^2/\eta_0^2 m_g^2$, where $m_g$ is the gluon thermal mass. Moreover, the transverse gluon contribution is of the same importance as the longitudinal one. The functions $J_{T,L}$ are plotted on the following left figure, for $m_g^2/m_F^2 = 1.5$ and $m_g^2/T^2 = 0.1$. We see that the result is considerably enhanced in the region of very small photon invariant mass.

(ii) The result is free of any infrared divergence, even for the transverse gluon exchange. As a consequence, there is no need for a magnetic mass. In fact, including such a mass has no effect on the result provided that $m_{\text{mag}} \ll m_F$. The effect of $m_{\text{mag}}$ has been evaluated numerically on the following right figure, at $Q^2 = 0$, for $m_{\text{mag}}^2/m_F^2 = 1$ (solid line) and $m_{\text{mag}}^2/m_F^2 = 10$ (dotted line).

\textsuperscript{5}This is the basic structure of the angular integral when the emitted photon is massless ($Q^2 = 0$). In the case $Q^2 > 0$, a more precise study of the kinematics shows that it is sufficient to replace in this integral the fermion thermal mass by an effective mass taking into account the regularizing effect of a positive $Q^2$: $m_{\text{eff}}^2 = m_F^2 + Q^2 r^2/\eta_0^2$.
(iii) If $Q^2$ increases, the scattering becomes sensitive to the energy scale of $m_{\text{eff}}$, intermediate between the hard and soft scales. This is a qualitative difference with the situations where the hard thermal loops framework holds.

### 2.3 Comparison with semi-classical methods

To be complete, it is instructive to compare this result, obtained in the framework of thermal field theory, with the results obtained by means of semi-classical approaches\(^1\). To that purpose, we modified our expressions in order to recover the standard semi-classical factorization:

$$
\begin{align*}
\frac{dN}{d^4x} & \approx \frac{d^3q}{(2\pi)^32q_0} \int \frac{d^4P_1}{(2\pi)^4} \frac{d^4P_2}{(2\pi)^4} \frac{d^4P_1'}{(2\pi)^4} \frac{d^4P_2'}{(2\pi)^4} (2\pi)^4 \delta(P_1 + P_2 - P_1' - P_2' - Q) \\
& \times (2\pi)^4 n_\sigma(P_1)[1 - n_\sigma(P_1^0)] n_\sigma(P_2)[1 - n_\sigma(P_2^0)] \delta(P_1^2 - m_\sigma^2) \delta(P_2^2 - m_\sigma^2) \delta(P_1'^2 - m_\sigma^2) \\
& \times |M(P_1, P_1' + Q, P_2, P_2')|^2 e^2 \sum_{\text{pol.}\epsilon} \left( \frac{P_1 \cdot \epsilon}{P_1 \cdot Q} - \frac{P_1' \cdot \epsilon}{P_1' \cdot Q} \right)^2
\end{align*}
$$

where $M$ is the amplitude corresponding to the same scattering without photon emission, and the factor beginning by $e^2$ is the square of the electromagnetic current that couples the photon to the quark. We recognize in this formula the first term of the expansion performed in the semi-classical methods, which seems to indicate that the terms beyond the HTL approximation we have considered are actually relevant for the photo-emission process by a hot plasma. The complete expansion in the semi-classical approach gives the so-called Landau Pomeranchuk Migdal suppression\(^7\), but of course, we cannot recover this effect here with only one scattering. On the other hand, thermal QCD provides a much more rigorous framework to treat the transverse gluons, and it is not obvious that the static scattering center approximation at the basis of the semi-classical approach is a good one, since it implies that the transverse gluon exchange can be neglected which is not the case.

### 3 Conclusions

The main conclusion of this work is that the HTL expansion may breakdown in certain very specific circumstances, namely when effective vertices have light-like external legs. This has dramatic consequences for the calculation in thermal QCD of processes that are very sensitive to what happens in the vicinity of the light–cone, which is precisely the case of photon production. In this specific area, it appeared that the bremsstrahlung process is the dominant one, which is in agreement with the results of semi-classical methods.

Future work on thermal photon production should certainly consider higher order contributions, in order to determine whether the multiple scatterings are important or not, thereby allowing a field theoretical approach towards the LPM effect. On the other hand, from a more formal point of view, it would be important to determine precisely what kind of reorganization of the perturbative expansion would take care of the collinear enhancement encountered here.
Acknowledgments: It is a pleasure to thank the organizers of such an enjoyable conference and P. Aurenche for his very helpful comments in preparing this talk.

REFERENCES

1. J. Cleymans, V. Goloviznin, K. Redlich, Phys. Rev. D47, 989 (1993); Z. Phys. C59, 495 (1993).
2. E. Braaten, R. Pisarski, T. C. Yuan, Phys. Rev. Lett. 64, 2242 (1990).
3. R. Baier, S. Peigné, D. Schiff, Z. Phys. C62, 337 (1994).
4. P. Aurenche, F. Gelis, R. Kobes, E. Petitgirard, Phys. Rev. D 54, 5274 (1996); preprint hep-ph/9609256 (to appear in Z. Phys. C 74).
5. E. Braaten, R. Pisarski, Nucl. Phys. B337, 569 (1990); B339, 310 (1990).
6. F. Flechsig, A. Rebhan, Nucl. Phys. B464, 279 (1996).
7. L. Landau, I. Pomeranchuk, Dokl. Akad. Nauk 92, 535 (1953); 92, 735 (1953); A. Migdal, Phys. Rev. 103, 1811 (1956).