MPC-enabled Privacy-Preserving Neural Network Training against Malicious Attack

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1 INTRODUCTION

During this last decade, with the development of machine learning, especially deep neural network (DNN), the scenario where different parties, e.g., data owners or cloud service providers, jointly solve a machine learning problem while preserving their data privacy has attracted tremendous attention from both academia and industry. Federated learning scheme [45] seems to offer the possibility for distributed privacy-preserving machine learning, focusing on cross-device and cross-silo setting [39] where multiple clients train local models using their raw data, then aggregate their models under the coordination of a central server. However, there is still no formal privacy guarantee to this baseline learning model. Therefore, secure multiparty computation (MPC) [31] as a practical mature privacy-preserving technique aiming to enable multiple parties jointly evaluating a function, is natural to be applied to address such privacy issues of training neural networks in a distributed manner.

Previous Results: There have been several schemes proposed to perform distributed neural network. This research direction is arguably pioneered by the design of SecureML by Mohassel and Zhang [47] where they propose several MPC-friendly activation functions to enable neural network training on secret shared data. The study of neural network on shared data can be classified to several classes based on their goals, number of parties and threat model. First, some models focus on the neural network prediction. A scheme designed for neural network prediction called MiniONN [44] was constructed based on the design of SecureML [47]. With the extensive use of packing techniques and additive homomorphic encryption (AHE) cryptosystem along with garbled circuit, Gazelle [38] provides a neural network prediction protocol with a more efficient linear computation.

Secondly, some other models provides secure neural network training for two active parties providing security against one corrupted party. As discussed above, SecureML [47] provides us with a neural network training protocol for two parties which is secure against semi-honest adversary controlling one party. Chameleon [54] proposes a different neural network training protocol for two parties. Instead of using the relatively more costly oblivious transfer (OT) protocols, the design relies on the use of external parties that aid the computation without having their own private inputs. In this case, the protocol is proven to be secure against semi-honest adversary under honest majority setting. ABY [30] presents a framework of efficient conversion between various two-party...
computation schemes to support different machine learning algorithms. It is proven to be secure against semi-honest adversary controlling one party. SecureNN[59] relies on more sophisticated protocols and up to two non-colluding external parties to provide neural network training protocols for two active parties. Its security is guaranteed against semi-honest adversary controlling up to 1 party. QUOTIENT [4] deeply integrates OT protocols with advanced neural network techniques such as ternary weight neural network [43] in 2-server setting against semi-honest adversary controlling 1 of the two servers. Note that all the designs mentioned here are designed for two parties with only semi-honest security guarantee with number of corrupted parties being at most half of the total number of participating parties. Despite some efforts to extend these designs to a larger number of active parties[46], the improvement has been rather limited.

Lastly, there have also been some designs dedicated for system with more than two parties with active security guarantee in dishonest majority setting. Most of these works are based on the SPDZ scheme[28]. These works[20, 24, 56] utilizes SPDZ to provide several accurate and efficient machine learning algorithms. Nevertheless, these works only rely on existing libraries, such as SCALE-MAMBA [7] and FRESCO [5], which do not offer primitives and optimizations for neural network training. Therefore, in this work, we present dedicated MPC protocols based on SPDZ for convolutional neural networks (CNN) training and demonstrate that our protocols obtain active security with affordable overheads compared to the existing secure neural network training in semi-honest setting.

For SPDZ protocol, the main efficiency improvement and active security come from the extensive use of pre-computed Beaver triples [9] with Message authentication code (MAC) (see Section 3.2) to accelerate arithmetic operations. During this last decade, following the initial scheme proposed in BDOZ [10], many researchers have been working on protocols of efficient Beaver triple generation in malicious setting. Both HE based schemes such as [27, 28, 41], and OT based schemes such as [23, 40, 48] offer reasonable efficiency and security. Relying on these schemes, all parties are able to jointly generate Beaver triples over a finite field $\mathbb{F}_Q$ or a ring $\mathbb{Z}_N$, which can be directly used for MAC checking in the online phase of SPDZ and its variants. Specifically, for HE based schemes, we have to choose a proper crypto-system to be used in the offline phase, e.g., leveled BGV [15] is used in [27, 41, 52] for its high performance due to the extensive use of packing technique, e.g., single-instruction multiple data (SIMD) trick [57]. Unfortunately, Beaver triples generated based on AHE crypto-systems over $\mathbb{Z}_N$, e.g., Paillier [53] or DGK [26] as used in SecureML [47], cannot be directly used for the verification of standard SPDZ which is based on a finite field. Therefore, an instance of SPDZ that is based on Paillier or DGK requires a secure scheme to transform the triples generated modulo $\mathbb{Z}_N$ to the underlying field of the SPDZ, $\mathbb{Z}_Q$.

Our contributions: In this work, we propose construction of efficient $n$-party protocols for secure CNN training in malicious majority setting, including linear and convolutional layer, Rectified Linear Unit (ReLU) layer, Maxpool layer, normalization layer, dropout layer, and their derivatives. In addition, we present a secure conversion scheme for shares defined over an integer ring $\mathbb{Z}_N$ to shares over a prime field $\mathbb{Z}_Q$ which can also be used to correctly convert shared Beaver triples. We believe that this result may be of independent interest. Our experimental results show that our protocols for secure neural network training provide affordable overheads compared with existing schemes in semi-honest setting.

Organisation of the paper: The rest of the paper is organized as follows. In Section 2, we provide notations and threat model used in this paper, as well as a brief discussion of secure computation and neural network. In Section 3, we introduce several supporting protocols including Distributed Paillier crypto-system, SPDZ protocol, and protocols of secure computation of fixed-point numbers. Section 4 contains our MPC protocols which can be used to construct an efficient secure neural network protocol as illustrated in Section 5. Then we analyze the performance of our protocol in Section 6. Finally, we present our experimental evaluation in Section 7 and give the conclusion in Section 8.

2 PRELIMINARIES

2.1 Neural Network Training

Neural network consists of many layers with nodes defined by a set of linear operations, such as addition and multiplication, and non-linear operations such as ReLU, Maxpool, and dropout. At a very high level, we represent a neural network as a function $w = f(x)$ where $x$ represents the set of input data with their respective labels, $w$ represents the set of weights of neural network, and function $f$ can be represented with linear operations and non-linear operations as mentioned above. The target of training neural network is to obtain these weights $w$ which can be used to map a new unlabeled data $x^*$ to its predicted label $\ell^*$, i.e., prediction.

2.2 Secure multiparty computation

Privacy-preserving technology provides privacy guarantee for data in various purposes such as for computation or for publishing. This technology broadly encompasses all schemes for privacy preserving function evaluations, including but not limited to differential privacy (DP), secure multiparty computation (MPC), and homomorphic encryption (HE). It is well known that DP provides a trade-off between accuracy and privacy that can be mathematically analyzed while MPC and HE offer cryptographic privacy but with high communication or computation overheads. In this work, we focus on MPC to construct efficient neural network training protocols. Since its general definition in [60], thanks to both theoretical and engineering breakthroughs, MPC has moved from pure theoretical interests to practical implementations, e.g., Danish sugar beets auction [13] and Estonian students study [12].

In addition to pure homomorphic encryption based MPC, there are two schemes which can be used to construct MPC protocols, i.e., circuit garbling and secret sharing. Circuit garbling, as used in SecureML [47], involves encrypting and decrypting keys in a specific order [8], while the latter emulates a function evaluation more efficiently based on the "secretly-shared" inputs between all parties. Our work leverages on an additive secret sharing MPC protocol called SPDZ (see Section 3.2), such that for each data $x$, it will be randomly split up into $n$ pieces and then be distributed among $n$ parties (see Algorithm 1 in Section 3.2). In the rest of the paper, we write $[x]^T = [(x_1)^T, ..., (x_n)^T]$ to denote $x \in \mathbb{Z}_N$ being secretly shared between all parties such that each $P_i$ holds $(x_i)^T$. In this paper, we present dedicated MPC protocols based on SPDZ for convolutional neural networks in malicious setting. Both BE based schemes such as [27, 28, 41], and OT based schemes such as [23, 40, 48] offer reasonable efficiency and security. Relying on these schemes, all parties are able to jointly generate Beaver triples over a finite field $\mathbb{F}_Q$ or a ring $\mathbb{Z}_N$, which can be directly used for MAC checking in the online phase of SPDZ and its variants. Specifically, for HE based schemes, we have to choose a proper crypto-system to be used in the offline phase, e.g., leveled BGV [15] is used in [27, 41, 52] for its high performance due to the extensive use of packing technique, e.g., single-instruction multiple data (SIMD) trick [57]. Unfortunately, Beaver triples generated based on AHE crypto-systems over $\mathbb{Z}_N$, e.g., Paillier [53] or DGK [26] as used in SecureML [47], cannot be directly used for the verification of standard SPDZ which is based on a finite field. Therefore, an instance of SPDZ that is based on Paillier or DGK requires a secure scheme to transform the triples generated modulo $\mathbb{Z}_N$ to the underlying field of the SPDZ, $\mathbb{Z}_Q$.

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and $x = \sum_{i=1}^{n} (x_i)^4$. For simplicity of notation, when the context is clear, we abuse the notation and use $x_i$ instead of $(x_i)^4$, the share of $x$ owned by party $P_i$. Similarly, when the underlying space $\mathbb{Z}_4$ is clear from the context, we write $[x]$ instead of $[x]^4$.

### 2.3 Threat model and security

In many real life neural network applications, the training data are distributed across multiple parties which are independent business entities and are required to comply with the applicable data privacy regulations. Therefore, due to the competitive nature of business organizations, in this work, we consider the scenario where a majority of parties may collude to obtain the data from the other parties by sending fraudulent messages to them. Such threat model is the same as the one used in SPDZ, i.e., a security against malicious adversary controlling up to $n-1$ parties. This means that in an $n$-party setting, such MPC protocol is secure even if $n-1$ parties are corrupted by a malicious adversary. Such threat model is different from that of MPC protocols in SecureML [47] and SecureNN [59], which are against semi-honest adversary. As demonstrated in [22], semi-honest protocols can be elevated into the malicious model, which may incur infeasible cost overhead. However, thanks to the online-offline architecture of SPDZ, such overhead can be moved from online phase to offline phase and thus the amortized efficiency of function evaluation can be improved. Our security definition is based on Universal Composability (UC) framework and we refer interested readers to [16] for the details.

The correctness and security of our proposed protocol depends on the supporting building blocks, i.e., distributed Paillier cryptosystem, SPDZ, and secure computation of fixed-point numbers. Data representation follows the format in [18] and we use standard SPDZ scheme over a finite field $\mathbb{Z}_Q$. The subprotocol of SPDZ involved in this work includes protocol for data resharing Resharing($\cdot$), multiplication MulTri($\cdot$), Paillier based Beaver triple generation TriGen($\cdot$), and MAC checking. The details of all the above mentioned supporting protocols can be found in the Section 3.

### 3 SUPPORTING PROTOCOLS

#### 3.1 Distributed Paillier cryptosystem

Paillier [33] is a public key encryption scheme that possesses partial homomorphic property [3]. The public key is $N = p \cdot q$ and the secret key is $(p, q)$ pair where $p$ and $q$ are large primes. First, we fix $g$ to be a random invertible integer modulo $N^2$. The encryption of a message $m$ is defined as $c = E(m) = g^{m \cdot r} \mod N^2$ for a randomly chosen invertible $r \in \mathbb{Z}_N$. The decryption function is $D(c) = \frac{L(c^{\lambda} \mod N^2)}{L(g^{\lambda} \mod N^2)} \mod N$ where function $L$ is defined as $L(x) = \frac{1}{x^{L}}$ and $\lambda = \varphi(N)$ where $\varphi$ is Euler’s totient function. Paillier supports homomorphic addition between two ciphertexts and homomorphic multiplication between a plaintext and a ciphertext, in particular, $E(m_1) \cdot E(m_2) \mod N^2 = E(m_1 \cdot m_2 \mod N)$ and $E(m_1)^{m_2} \mod N^2 = E(m_1 \cdot m_2 \mod N)$. For simplicity, we denote the following two functions: $E(m_1 + m_2 \mod N) = PAdd(E(m_1), E(m_2))$ and $E(m_1 \cdot m_2 \mod N) = PMult(E(m_1), m_2)$. We can easily generalize the two notations such that $E(m_1 + m_2 + \cdots + m_r \mod N) = PAdd(E(m_1), E(m_2), \ldots, E(m_r))$ and $E(m_1 \cdot m_2 \cdot \cdots \cdot m_r \mod N) = PMult(E(m_1), m_2, \ldots, m_r)$. Due to the inverting property of $g$ and $r$ modulo $N^2$, it is easy to see that a Paillier ciphertext is invertible modulo $N^2$. Hence, if $c = E(m)$, we will also have $c^{-1} \mod N^2$ to be a valid encryption of $-m$. We denote $Plinv(E(m)) = E(\bar{m} \mod N) \equiv E(m)^{-1} \mod N^2$. Using $Plinv$, Paillier can support homomorphic subtraction, $E(m_1 - m_2 \mod N) = PAdd(E(m_1), Plinv(E(m_2)))$. These homomorphic properties enable several protocols proposed in Section 3.2 and Section 4. However, in terms of MPC scenario, the secret key pair is not allowed to be owned by any party. To keep the hardness of composite residuosity [37] used for the security of Paillier, the value of $p$ and $q$ cannot be known by anyone. Hence we need to generate the public key in distributed manner and keep its factors secret while still enabling joint decryption to be done without revealing the private key. Fortunately, such distributed Paillier does exist. Distributed Paillier key generation includes two sub-protocols, i.e., (i) distributed RSA modulus generation, and (ii) distributed biprimality test to verify the validity of generated RSA modulus in (i). Inspired by [14] which proposed the first RSA modulus generation protocol in multiparty setting, several works [32, 34, 49] provides solutions in different threat models. Note that to ensure our protocol is secure against malicious adversaries, we have to guarantee that all the sub-protocols are also secure against same threat model in $n$-party setting. Thus we rely on the scheme proposed in [35]. We denote the Paillier cryptosystem with plaintext space $\mathbb{Z}_N$ as $\text{Pailler}_N$, as well as its encryption and distributed decryption, i.e., $Enc_N(\cdot)$, $Dec_N(\cdot)$.

#### 3.2 SPDZ

SPDZ is a well known secret sharing based MPC protocol against malicious majority proposed in [28]. Following this initial somewhat homomorphic encryption (SHE) based work, several variants are proposed including its improved version [27], OT based version called MASCOT [40], improved SHE based version called Offline [41], and the versions over $\mathbb{Z}_{2k}$ such as SPDZ2k [23] and Overdrive2k [52]. We will refer to all of these variants in SPDZ family as SPDZ.

SPDZ consists of a pre-processing or offline phase, which is independent from both the input data and the very efficient online phase for function evaluation. In the offline phase, all parties jointly generate some "raw materials", typically the Beaver triples [9]. In the online phase, the parties only need to exchange some shares and perform some efficient verification. The active security is guaranteed by the MAC which enables the validation of parties’ behavior during computation. In the rest of this section, several important techniques in SHE based SPDZ are introduced in order to construct some higher level protocols proposed in Section 4.

**Data Resharing**: Given $Enc_{ij}(x)$, all parties can follow the protocol given in Algorithm 1 to obtain $[x]^4$ as the shares of $x$. We note that this resharing of an encrypted value is only done during pre-processing phase to help in the generation of auxiliary values and it is not used in the sharing protocol of private inputs of the function $f$. For the sharing of private inputs during the online phase of the computation, it follows the protocol given in Algorithm 2.
Algorithm 1 Data Resharing: \([x]_t^j \leftarrow \text{Resharing}(\text{Enc}_t(x))\)

1. Each party \(P_j\) publishes \(\text{Enc}_t(r_j)\), where \(r_j\) is uniformly selected from \(\mathbb{Z}_N\).
2. All parties calculate \(\text{Enc}_t(r + x) = \text{PAdd}(\text{Enc}_t(r_1), \text{Enc}_t(r_2), \ldots, \text{Enc}_t(r_n), \text{Enc}_t(x))\) using homomorphic addition.
3. All parties jointly decrypt \(\text{Enc}_t(r + x)\) to obtain \(r + x\).
4. \(P_i\) sets its share \(x_1 = r + x - r_i\), \(P_j\) sets its share \(x_j = -r_j\) for \(j \neq 1\).
5. Return \([x]_t^j\)

Algorithm 2 Data Sharing: \([x]_t^j \leftarrow \text{Share}(x)\) where \(x\) is a private value owned by \(P_i\)

Require: A shared random value \([r]_t^j\)

1. Each party \(P_j\) sends his share of \([r]_t^j, (r_j)\) to \(P_i\) enabling \(P_i\) to recover the value of \(r\).
2. \(P_i\) sets \((x_j) = x + r - (r_j)^t\) and for \(j \neq i\), \(P_j\) sets \((x_j) = -(r_j)^t\).
3. Return \([x]_t^j = ((x_1)^t, \ldots, (x_n)^t)\)

Arithmetic operation: SPDZ is based on secret sharing, thus there is no communication cost for addition and scaling by a public constant. Multiplication between two secretly shared values is more complex but we can use the well known Beaver triple trick to accelerate this operation. In the following discussion, all values are secretly shared using the additive secret sharing scheme over the same space. Assume that we have generated three secret shared values \([a], [b], [c]\), called Beaver triples, such that \(c = a \cdot b\). Given \([x]_t^i\) and \([y]_t^i\), all parties can follow the protocol in Algorithm 3 to calculate \([x \cdot y]_t^i\). Correctness and security are proven in [9]. Note that all the protocols in SPDZ can be applied to matrices.

Algorithm 3 Multiplication based on Beaver triple: \([x \cdot y] \leftarrow \text{MulTri}(x), (y), (a), (b), (c)\)

1. Each party \(P_i\) publishes \(x_i - a_i\) and \(y_i - b_i\).
2. Each party \(P_i\) computes \(x - a\) and \(y - b\).
3. \(P_i\) sets its share \(x_i = c_i + (x - a) \cdot b_i + (y - b) \cdot a_i + (x - a) \cdot (y - b)\), \(P_j\) sets its share \(x_j = c_j + (x - a) \cdot b_j + (y - b) \cdot a_j\) for \(j \neq 1\).
4. Return \([z]_t^i\)

Considering that 1 triple cannot be used to perform 2 multiplications for privacy reason [10], the number of triples we need to generate depends on the number of multiplications we want to complete. Furthermore, these Beaver triples do not depend on inputs data as well as the function to be evaluated, which means they can be generated at any point prior to evaluating the function, i.e., the offline phase in SPDZ, thus enabling highly efficient online phase.

Beaver triple generation: Algorithm 4 describes the \(n\)-party protocol for Beaver triple generation based on Paillier such that \([a]_N^N, [b]_N^N, [c]_N^N \leftarrow \text{TriGen}()\). For simplicity of discussion, we only discuss the protocols under semi-honest setting. As discussed before, such protocols can be made secure against malicious adversary by the combination of zero knowledge proof and standard technique of sacrificing an auxiliary value to check the correctness of another. A more detailed discussion of this technique can be found in [55]. As mentioned in Section 1, these Beaver triples generated using Algorithm 4 are over \(\mathbb{Z}_N\), thus cannot be directly in the online phase of SPDZ which is over a finite field.

Algorithm 4 Beaver triple generation based on Paillier: \([a]_N^N, [b]_N^N, [c]_N^N \leftarrow \text{TriGen}()\)

1. Each party \(P_i\) publishes \(\text{Enc}_N(a_i)\) and \(\text{Enc}_N(b_i)\), where \(a_i\) and \(b_i\) are uniformly selected from \(\mathbb{Z}_N\).
2. Each party \(P_j\) computes \(\text{Enc}_N(a) = \text{PAdd}(\text{Enc}_N(a_1), \ldots, \text{Enc}_N(a_n))\).
3. Each party \(P_i\) computes and publishes \(\text{Enc}_N(a \cdot b_i) = \text{PMult}(\text{Enc}_N(a), b_i) = \text{Enc}_N(a)^{b_i}\) using homomorphic multiplication in Paillier.
4. Each party \(P_i\) computes \(\text{Enc}_N(c) = \text{PAdd}(\text{Enc}_N(a \cdot b_1), \ldots, \text{Enc}_N(a, b_n))\).
5. All parties call Resharing(\(\text{Enc}_N(c)\)) to get \([c]_N\).
6. Return \([a]_N^N, [b]_N^N, [c]_N^N\)

Note that in step 3 of Algorithm 4, only \(P_i\) knows the value of \(b_i\), which enables the homomorphic multiplication between a plaintext and ciphertext (refer Section 3.1), i.e., \(\text{Enc}_N(a)\) and \(b_i\), thus no information leaks.

MAC checking: To obtain active security over \(\mathbb{Z}_Q\), the main idea of SPDZ is to use unconditional MAC which enables verification of computation correctness. This authentication scheme prevents parties from cheating on their interactive computation with high probability. In SPDZ, to enable authentication, each private value, including the Beaver triples that are generated, come with their respective tags. To obtain this, first the parties agree on a random MAC key \(\alpha \in \mathbb{Z}_Q\) which is secretly shared among all parties. To compute the tag of a secretly shared value \([x]_Q^i\), the parties compute \([\alpha \cdot x]_Q^i\) and store it along with their shares of \([x]_Q^i\). We can observe that if some adversaries cheat such that the secretly shared value is changed from \(x\) to \(x’\), they can do so undetected only if they can modify the corresponding tag \([m]\) to \([m']\) such that \(m' = \alpha \cdot x’\). This means that the probability of cheating without being detected is equal to the probability of guessing \(\alpha\) correctly, which is inversely proportional to the finite field size \(Q\). When considering a similar scheme over a ring \(\mathbb{Z}_N\), the security is no longer as strong. This is due to the fact that, contrary to \(\mathbb{Z}_Q\), not all non-zero value in \(\mathbb{Z}_N\) is invertible. Because of this, the probability that \(m' = \alpha \cdot x'\) becomes larger. For example, if \(N = 2^k\) and \(\alpha = 2^{k-1}\), \(m'\) can only be either 0 or 1 making the probability 1/2. As illustrated above, we use Paillier to generate Beaver triples with MAC, which means all the secret shared values are in \(\mathbb{Z}_N\), hence we have to convert all these shares from \(\mathbb{Z}_N\) to \(\mathbb{Z}_Q\) while preserving the relationship between them. Note that for any sub-protocols, all the inputs and outputs should always be secretly shared and not be in clear. In addition to the shares of the outputs \([y]_Q^i\), the parties should also hold a secret share of their respective tags \([a \cdot y]_Q^i\).

3.3 Secure computation of fixed-point numbers

For typical neural networks, data and weights are represented by floating point numbers. However, in terms of combining neural
network with cryptographic techniques such as HE and MPC, we have to use a very large finite field to preserve full accuracy [33]. This method supports for only limited number of multiplications to avoid the overflow, which is prohibitive for neural network where a large number of multiplications are involved. Fortunately, the authors in [47] illustrate that rational numbers can be treated as fixed point number relying on a truncation technique while preserving appropriate accuracy of neural network prediction. Other prior works regarding neural network with MPC, such as [59] and [4] also show their accuracy in terms of truncation. In our work, we follow the same methodology and extend its applications to secure n-party truncation, comparison, and arithmetic operations based on the protocols given in [17, 18, 25, 58]. Note that the correctness of these protocols are given in above mentioned works and security is guaranteed by the MAC-checking method in SPDZ.

Data representation: Rational numbers can be treated as a sequence of digits including integer and fractional parts split by a radix point. More specifically, for any real number \( \tilde{x} \) sequence of digits including integer and fractional parts split by a radix point. More specifically, for any real number \( \tilde{x} \) sequence of digits including integer and fractional parts split by a radix point. More specifically, for any real number \( \tilde{x} \) sequence of digits including integer and fractional parts split by a radix point. More specifically, for any real number \( \tilde{x} \) sequence of digits including integer and fractional parts split by a radix point. More specifically, for any real number \( \tilde{x} \) sequence of digits including integer and fractional parts split by a radix point.

\( \tilde{x} \) is greater than 0 or not, which can be obtained by simulating multiplication computation, deterministic truncation is needed. Protocol of Truncation and comparison: In order to maintain the same resolution of secretly shared values and enable comparison, two truncation protocols are used in our work, i.e., probabilistic truncation TruncPr and deterministic truncation Trunc, as given in [18]. Probabilistic truncation supports efficient truncation as no bit-wise operation is involved and some “raw materials” needed in the protocol can be prepared during the pre-processing phase. However, it introduces error with probability depending on the size of the Least Significant Bit (LSB) after truncation. In terms of data representation of \( \tilde{x} \), the Most Significant Bit (MSB) of \( \tilde{x} \) determines whether \( \tilde{x} \) is greater than 0 or not, which can be obtained by simply truncating the last \( k-1 \) least significant bits. Due to the fact that the LSB after truncation is now large, using TruncPr for this purpose will yield a non-negligible error. Hence, an alternative truncation protocol is required. Deterministic truncation is less efficient but enables truncation with zero error probability. Therefore, although probabilistic truncation may be used to avoid overflow during multiplication computation, deterministic truncation is needed for comparison computation. We denote by GEZ, an adapted version of comparison protocol LTZ following the notation given in [17] such that GEZ(\( x, k \)) = 1 if \( x \geq 0 \), and 0 otherwise, in order to keep the consistency with ReLU function (see Section 2.1).

Arithmetic operations: Addition and public scaling on additive shares can be done without interaction, while maintaining the same resolution. Multiplication can be done using Beaver triple method (see Section 3.2) with its resolution changing from \( 2^{-f} \) to \( 2^{-2f} \), which means probabilistic truncation is needed. Protocols of division with public divisor and secretly shared divisor are also given in [18] that offers reasonable accuracy and efficiency.

4 PROPOSED PROTOCOLS

In this section, we describe protocols to support Beaver triple conversion and neural network training. We assume that there are two distributed Paillier crypto-systems with different plaintext space \( \mathbb{Z}_p \) and \( \mathbb{Z}_q \). Based on the definition of Paillier and the notation introduced in Section 3.1, let \( N = p \cdot q \) where \( 2^k < p, q < 2^{k+1} \) and \( N' = p' \cdot q' \) where \( 2^k < p', q' < 2^{k+1} \) such that \( N > N' \). All the multiplications involved in these protocols can be done following the similar multiplication protocol in Paillier based Beaver triple generation given in Section 3.2. In the rest of the paper, we write \( [x]^f = (\langle x_1 \rangle^f, ..., \langle x_t \rangle^f) \) to denote \( x \in \mathbb{Z}_P \) being secretly shared between all parties such that each \( P_i \) holds \( \langle x_i \rangle^f \) and \( x = \sum_{i=1}^n \langle x_i \rangle^f \). For simplicity of notation, when the context is clear, we abuse the notation and use \( x_i \) instead of \( \langle x_i \rangle^f \), the share of \( x \) owned by party \( P_i \). Similarly, when the underlying space \( \mathbb{Z}_q \) is clear, we write \([x] \) instead of \([x]^f \).

4.1 Comparison over \( \mathbb{Z}_N \)

The first supporting protocol that we want to introduce is the secure comparison protocol GEZ\( \mathbb{Z}_N \) over \( \mathbb{Z}_N \). More specifically, this function receives a secretly shared value \([x]^N \) and the bit length \( k \) of \( x \). It then outputs \([s]^N \) where \( s = 1 \) if \( x \geq 0 \) and 0 otherwise. Note that this function can be used to compare two secretly shared values \([x]^N \), \([y]^N \) by computing \( \text{GEZ}(x - y)^N \). This algorithm is an adapted version combining LTZ protocol in [17] and the deterministic comparison protocol in [18], which is based on the following remark: for \( x \) with \( k \)-bit length (refer to data representation in Section 3.3), if \( x < 0 \) then \( [x/2^{k-1}] = 1 \), and if \( x \geq 0 \) then \( [x/2^{k-1}] = 0 \). This protocol, which depends on a subprotocol BitLTc, cannot be directly applied in our case due to the difference in the underlying space. BitLTc([x_{k-1}, ..., x_0], [y_{k-1}, ..., y_0]) is a protocol that returns 1 if \( x < y \) and 0 otherwise where \( x = \sum_{i=0}^{k-1} x_i 2^i \) and \( y = \sum_{i=0}^{k-1} y_i 2^i \) are secretly shared in their binary representations. In the scheme proposed in [1], BitLTc involves protocols to generate random bits and random invertible integers over the underlying finite field. In order to have a similar protocol to BitLTc over \( \mathbb{Z}_N \), we first describe some subprotocols that are required, namely random bit generation and random invertible integer generation modulo \( \mathbb{Z}_N \).

Algorithm 5 describes our \( n \)-party protocol for random bit generation over \( \mathbb{Z}_N \). The standard protocol in \( \mathbb{Z}_Q \) setting, such as RAN2() in [25], is to take an inverse of root from squaring modulo \( Q \) by dividing the initial value, i.e., \( x/\sqrt{x} \in (-1, 1) \). However, this may no longer works when working modulo \( N \). Fortunately, we can rely on existing Paillier crypto-system and Resharing protocol in SPDZ (see Section 3.2). Correctness is easily proved and security depends on the Resharing protocol in SPDZ and Indistinguishability under chosen-plaintext attack (IND-CPA) security of Paillier crypto-system which guarantees the security when \( n < N \). Furthermore, \( \text{Enc}(a_1 \cdot a_{i+1}) \) in Step 2 must be calculated either by \( P_i \) or \( P_{i+1} \) but not anyone else.

Algorithm 6 describes our \( n \)-party protocol for random integer with inverse generated over \( \mathbb{Z}_N \), which is an adapted version of PRandInv in [1]. Algorithm 6 is built on another secure protocol Rdmlnt(\( \mathbb{Z}_N \)). This protocol generates a random share \([r]^N \) where \( r \) is unknown to any of the parties. This can be done by letting
Algorithm 5 Random bit generation over $\mathbb{Z}_N$: $[a]^N \leftarrow \text{RndBit}_N(\cdot)$

1. Each party $P_i$ generates a uniformly random bit $a_{i,j,k} \in \{0, 1\}$ and computes $c_i = \text{Enc}_N(a_{i})$. Let $b_1 = a_1$ and $d_1 = c_1$
2. for $i = 1, \ldots, n = 1$ do
3. $P_i$ sends $d_i = \text{Enc}(b_i)$ to $P_{i+1}$
4. $P_{i+1}$ calculates $d_{i+1} = \text{Enc}(b_{i+1}) = \text{Enc}(b_{i} @ a_{i+1}) = \text{Enc}(b_{i} + a_{i+1} - 2b_{i+1}) = PAdd(a_i, c_{i+1}, PMult(PInc(d_i), 2a_{i+1}))$
5. end for
6. Return $[a]^N \leftarrow \text{Resharing}(\text{Enc}(b_n))$.

Algorithm 6 Random integer with inverse generation over $\mathbb{Z}_N$: $([r]^N, [r^{-1}]^N) \leftarrow \text{RndInt}_N(N)$

1. All parties call RndInt($\mathbb{Z}_N$) to generate $[x]^N \leftarrow \text{RndInt}(\mathbb{Z}_N)$, $[y]^N \leftarrow \text{RndInt}(\mathbb{Z}_N)$, and then compute $u \leftarrow \text{Output}([x] \cdot [y])$.
2. Repeat step 1 until $u$ is invertible.
3. Return $([x]^N, u^{-1}[y]^N)$.

Now we are ready to present our $n$-party protocol GEZ for comparison over $\mathbb{Z}_N$. This algorithm is an adapted version combining LTZ protocol in [17] and the deterministic protocol in [18], which is based on the following remark: for $x$ with $k$-bit length (refer to data representation in Section 3.3), if $x < 0$ then $x/2^k - 1 = 1$, and if $x \geq 0$ then $x/2^k - 1 = 0$.

However this protocol which depends on a subprotocol BitLTC, cannot be directly applied in our case due to the following condition in the underlying space. $[b] \leftarrow \text{BitLTC}([x_{-1}], \ldots, [x], ([y_{-1}], \ldots, [y]))$ is a protocol that returns 1 if $x < y$ and 0 otherwise where $x = \sum_{i=0}^{k-1} x_2^i$ and $y = \sum_{i=0}^{k-1} y_2^i$ are secretly shared in their binary representations. In the proposed protocol in [1], BitLTC involves protocols to generate random bits and random invertible integers over the underlying finite field. In order to have a similar protocol to BitLTC over $\mathbb{Z}_N$, we propose the protocol over $\mathbb{Z}_N$ for generating random bit in Algorithm 5.

For random integer generation over $\mathbb{Z}_N$, we can simply follow the protocol PRandInt given in [51]. Correctness and security are proven in [17].

4.2 Wrap, modulo reduction, share conversion

In this section, we discuss the secure conversion protocol that will help us in converting the values we generated during offline phase (modulo $N$ for some RSA modulus $N > n$) to equivalent value that is compatible with the online phase (modulo $Q$ for a prime $Q$). More specifically, given $[a]^N$, the additive share of a secret value $a$ modulo $N$, we want to calculate $[a]^Q$, the additive share of the same secret value modulo $Q$ for some prime $Q$. First, for simplicity, we discuss about the transformation of the secret sharing values. Note that initially, we want our secret value and its shares to be an element in $S_1 = \{\frac{-a_0}{2^n}, \ldots, \frac{-a_n}{2^n}\}$. For simplicity of our argument in this section, we transform all these values to be non-negative value in $S_2 = \{0, \ldots, n-1\}$ via congruence operation. Note that this does not change the correctness of any sharing and transformation between the two formats can be done trivially.

Note that if $[x]^N = (a_0, \ldots, a_n)^N$, there exists $\delta \in \{0, \ldots, n-2, n-1\}$ such that $x = x_1^N + \cdots + x_n^N - \delta N$. (1)

Hence we can rewrite the value of $\delta$ as $\bar{\delta} = \frac{\sum_{i=0}^{n} a_i}{N}$. (2)

In order to calculate $[x]^Q$ from $[x]^N$, we need to calculate the value of $\bar{\delta}$ which can be rewritten as $\bar{\delta} = \frac{\delta}{N}$. (3)

Now we discuss how we can calculate the value of $\delta$. Note that Equation (1) will not yield the value of $\delta$ if it is computed modulo $N$. Intuitively, if we consider the equation modulo $N'$ for some $N'$ such that $N' > N$, the equation does not become equivalence and hence we can use it to calculate $\delta$. Once we have the equation modulo $N'$, we can find the maximum value of $j$ such that $\sum_{i=1}^{n} x_i^N = jN' \geq 0$. It is easy to see that $\delta = \sum_{i=1}^{n} x_i^N$ when $jN' - jn \geq 0$. Now since the equation is modulo $N'$, the calculation will give us $[\delta]^N'$. We let this procedure to be called $\text{LiftWrap}([x]^N, N')$ which is only applicable if $N' > N^2 > n^2$. Algorithm 8 provides the complete LiftWrap protocol. Note that the security of the protocol is guaranteed due to the security of GEZ protocol and the fact that no intermediate values is revealed.

Algorithm 7 Comparison over $\mathbb{Z}_N$: $[s]^N \leftarrow \text{GEZ}_N([x], k)$

1. For each $i \in [0, \ldots, k-1]$, all parties calculate $[r_i] = \text{RndBit}_N(\cdot)$ in parallel, and thus obtain $[r'] = \sum_{i=0}^{k-2} x_i \cdot [r_i]$
2. All parties calculate $[r'''] = \text{RndInt}_N(\mathbb{Z}_N + 1)$.
3. All parties publish $c \leftarrow \text{Output}(2^{k-1}[r'] + [r'''] + 2^{k-1} + [x])$, and then calculate $c' = c \mod 2^k$.
4. All parties call $[u] = \text{BitLTC}(c', (\lceil r_{k-2} \rceil, \ldots, \lceil r_0 \rceil))$.
5. All parties compute $[x'] = c' + [r'''] + 2^{k-1}[u]$
6. All parties compute $[s] = 1 + ([x] - [x']) (2^{-k-1} \mod N)$.

The next step is to convert $[\delta]^N$ to $[\delta]^N$. In other words, we need a secure conversion protocol DropMod to convert a secretly shared value $[x]^N$ back to $[x]^N$ where $N' > N^2 > n^2$. In order to prove the correctness, first, we observe that given $[\delta]^N = (\delta_1, \ldots, \delta_n)$, setting $y_i = x_i - \delta_i N \mod N'$, we have $\sum_{i=1}^{n} y_i = x \mod N'$.
In other words, for any $[x]^N$, we can calculate $[x]^{N'}$. Let this procedure be called $[x]^{N'} \leftarrow \text{Li/f}_t\text{Mod}([x]^N, N')$ which is only applicable if $N' > N^2 > n^2$. Algorithm 9 provides the complete LiftMod protocol. The security is guaranteed based on the security guarantee of LiftWrap protocol.

**Algorithm 9** Lift shares in $Z_N$ to $Z_{N'}$: $[x]^{N'} \leftarrow \text{LiftMod}_r([x]^N, N')$

1. Parties jointly compute $\{\delta\}^{N'} = (\delta'_1, \ldots, \delta'_n) = \text{LiftWrap}_r([x]^N, N')$.
2. For each $i \in \{1, \ldots, n\}$, having $x_i$ and $\delta'_i$ (the shares of $[x]^N$ and $[\delta]^{N'}$ respectively), $P_i$ calculates $x'_i \equiv x_i - \delta'_iN \text{ (mod } N')$.
3. Return $[x]^{N'} = (x'_1, \ldots, x'_n)$.

Now we are ready to discuss the last subprotocol needed for the Wrap function, $[x]^N \leftarrow \text{DropMod}([x]^N', N)$. Intuitively, assuming the existence of the protocols RndInt and LiftMod over $Z_N$, we mask $x$ by a random string $r$ such that $r$ is taken from a space that is $2^n$ larger than $N$. In order to do this, we further set a third RSA modulus $N''$ such that $N'' > 2^nN$ while $N'$ is said so that $N' = \Omega(N''^2)$, our aim is to have a random value $r \in Z_{N''}$ that is secretly shared three times, over $Z_N, Z_{N'}$ and $Z_{N''}$. Having the secret sharings of $r$, we use the secret sharing over $Z_{N''}$ to reveal $y \equiv x + r \text{ (mod } N')$. We can then use $z \equiv y \text{ (mod } N)$ and $[r]^N$ to get $[x]^N = z - [r]^N$. Note that the only value being revealed is a calculation over integer which is masked by another integer that is at least $2^nN$ larger than it. So, along with the security guarantees of the other subprotocols, the security of DropMod is guaranteed with security parameter $\kappa$ for $x < N$. Algorithm 10 provides a complete DropMod protocol.

**Algorithm 10** Convert shares in $Z_{N'}$ to $Z_N$: $[x]^N \leftarrow \text{DropMod}_{\text{L}}([x]^{N'}, N)$

1. Parties jointly compute a random bit $[b_i]^N = \text{RndBit}(Z_N)$ for $i = 0, \ldots, \lfloor \log(N) \rfloor + \kappa - 1$.
2. Parties jointly compute $[b_i]^{N'} = \text{LiftMod}([b_i]^N, N')$ for $i = 0, \ldots, \lfloor \log(N) \rfloor + \kappa - 1$.
3. Parties locally compute $[r]^N = \sum_{i=0}^{\lfloor \log(N) \rfloor + \kappa - 1} 2^i[b_i]^{N'}$
4. Parties locally compute $[r \text{ (mod } N)]^N = \sum_{i=0}^{\lfloor \log(N) \rfloor + \kappa - 1} 2^i \text{ (mod } N)[b_i]^{N'}$
5. Parties locally compute $[y]^{N'} = [x]^{N'} + [r]^N$, publish their shares of $[y]^{N'}$ and recover $y$.
6. Return $[x]^N = y \text{ (mod } N) - [r]^N$.

Note that we can apply DropMod to obtain $[\delta]^N$ from $[\delta]^{N'}$. We note that in this use, DropMod is secure since $\delta < n < N$. So this also guarantees the security of the protocol $[\delta]^N \leftarrow \text{Wrap}_{\text{L}}([x]^N, N')$. Algorithm 12 provides the complete Wrap$_\text{L}$ protocol.

Note that the conversion protocols LiftMod and DropMod are only securely applicable in very restrictive case. More specifically, LiftMod can only convert from $Z_S$ to $Z_{S'}$ where $S' > S^2 > n^2$ and $\text{GEZ}$ must be well defined over $Z_S$. So in other words, $S$ must be either an RSA modulus or a prime. On the other hand, DropMod can only convert from $Z_{S'}$ to $Z_S$ with $S$ and $S'$ having the same requirements as the ones in LiftMod. Furthermore, DropMod$_\text{L}([x]^S, S)$ is only secure when $x < S$.

Recall that our main objective of this part is to have a secure conversion protocol to convert a secret sharing $[x]^Q$ to $[x]^{Q'}$ where $N$ is an RSA modulus while $Q$ is a prime. Since this needs to be used to convert secret sharing of random values or Beaver Triple generated modulo $N$, in order to have all possible random values modulo $Q$, we need to have $N > Q$. Note that if we use DropMod for this purpose, the value of $N$ needs to be much bigger than $Q$, more specifically, $N > Q^2$. In the following, we propose another conversion protocol ShaConv which can accomplish this goal securely as long as $N > Q$.

Following Equation (2), having $[x]^N = (x_1, \ldots, x_n)$, the first $n$ terms can be calculated locally by each party $P_i$. Now in order for the conversion to be completed, we need the last term, $(\delta \text{ (mod } Q)) \cdot (N \text{ (mod } Q))$. Recall that the only information we have about $\delta$ is its secret sharing modulo $N, [\delta]^N$. Note that to be able to calculate $[\delta]^Q \cdot (N \text{ (mod } Q))$, we need to first convert $[\delta]^N$ to $[\delta]^Q$. Note that we can use a variant of DropMod to achieve this. However, this can only be achieved securely if $\delta < Q$. So in order to guarantee this, in our discussion, we will assume $n < Q < N < \sqrt{N^2}$. Now suppose that we have $[\delta]^N$ and we would like to calculate $[\delta]^Q$. Since we do not have the guarantee that $\delta < n^2$, we cannot apply DropMod directly. Instead, we will again use the space $Z_{N^2}$ for this purpose. More specifically, after the calculation of LiftWrap to obtain $[\delta]^N$, we can directly call DropMod to obtain $[\delta]^Q$ instead of $[\delta]^N$. Now once $[\delta]^Q$ is obtained, we can obtain $[\delta]^Q \cdot (N \text{ (mod } Q))$ completing the calculation of Equation (2). It is easy to see that since all the subprotocols being used here are secure, the protocol that calculates Equation (2) we have just discussed is secure. The complete protocol of ShaConv can be found in Algorithm 13.

Note that Algorithm 13 can be used for any $Q$ and $N$ securely as long as they satisfy the following requirements: (i) $n < Q < N$, (ii) we have a secure GEZ protocol modulo $N$, and (iii) we have secure RndInt and GEZ protocols modulo $Q$. Due to this observation, we
Algorithm 13 Share conversion: \([x]^Q \leftarrow \text{ShaConv}([x]^N, Q, N')\)

1. Parties jointly compute \(\delta^N = \text{LiftWrap}([x]^N, N')\)
2. Parties jointly compute \([\delta]^Q = (\delta_1, \ldots, \delta_n) = \text{DropMod}_Q(\delta^N, Q)\)
3. for \(i = 1, \ldots, n\) do
4. \(P_i\) possesses \(x_i\) and \(\delta_i\), the \(i\)-th share of \([x]^N\) and \([\delta]^Q\) respectively
5. \(P_i\) locally computes \(x'_i \equiv x_i - \delta_i \cdot (N \mod Q)\) \((\mod Q)\).
6. end for
7. Return \([x]^Q = (x'_1, \ldots, x'_n)\).

Algorithm 14 Beaver triple conversion: \(([a']^Q, [b']^Q, [c']^Q) \leftarrow \text{TripConv}([[a]^N, [b]^N, [c]^N], Q)\)

1. Parties agree on \(N'\) such that \(N' > Q^2\) and \(N'\) is either an RSA modulus or a prime. Parties also agree on \(N''\) such that \(N'' > (N')^2\).
2. Parties jointly compute \(([a]^Q, [b]^Q, [c]^Q) = \text{ShaConv}([[a]^N, [b]^N, [c]^N], Q, N', N'')\).
3. Parties jointly compute \(([a]^{N''}, [b]^{N''}, [c]^{N''}) = \text{LiftMod}([a]^N, [b]^N, [c]^N, N')\).
4. Parties jointly compute \([\sigma N]^{N''} = [a]^{N''} \cdot [b]^{N''} - [c]^{N''}\)
5. Parties jointly compute \([\sigma N]^Q = \text{ShaConv}([[\sigma N]^N, Q, N'')\)
6. Set \([c']^Q = [c]^Q + [\delta N]^Q\).
7. Return \(([a]^Q, [b]^Q, [c]'^Q)\).

Algorithm 15 Probabilistic bit generation over \(Z_Q\): \([b]^Q \leftarrow \text{PrRndBit}_Q(p)\)

1. All parties deal a random sharing \([a]^Q\), where \(a \in \mathbb{Z}_Q\).
2. All parties calculate \([b]^Q = \text{GEZ}_Q([a]^Q - [p \cdot Q], l, \mathbb{Z}_Q\).
3. Return \([b]^Q\).

4.3 Beaver triple conversion

Share conversion is not trivial in terms of MPC as proved in [11], let alone Beaver triple conversion. Inspired by [21], several share conversion protocols have been developed. To convert a sharing over \(Z_Q\) to a sharing over \(Z_{Q'}\), we can rely on the method of [6] and the mixed-protocol framework given in ABY [30] which generalizes the conversion protocols between different sharing schemes including Arithmetic sharing, Boolean sharing, and Yao’s garbled circuit. However, for additive sharing of integer over \(Z_Q\), the conversion protocol follows the scheme \(Z_Q \rightarrow Z_{Q'} \rightarrow Z_Q\), which involves bit decomposition and bit sharing conversion over a field to another field [29]. These approaches are complicated and expensive. In our work, we do not use these protocols to construct share conversion, instead, we rely on two Paillier crypto-systems which enables us to convert Beaver triples from \(Z_N\) to \(Z_Q\).

First we observe that given a triple \(([a]^N, [b]^N, [c]^N)\), we have \(c \equiv ab \pmod N\) or equivalently, \(ab \equiv c + \sigma N\) for some integer \(\sigma\) such that \(|\sigma| \leq n - 1\). Similar to the discussion of \(\delta\) in the previous section, in order to get the value of \(\sigma\), we need to lift the equation modulo \(N'\) for some \(N' > N^2 > n^2\). Using the algorithm LiftMod described above, we can obtain \(([a]^{N'}, [b]^{N'}, [c]^{N'})\). Then \([\sigma N]^{N'} = [ab - c]^{N'}\). Note that it is not secure to use \text{DropMod} to obtain \([\sigma N]^Q\) from \([\sigma N]^{N'}\) even if \(N'\) is an RSA modulus or a prime. This is because it is impossible that \(\sigma N < Q\). Hence we will need to use \text{ShaConv} to achieve this. In order to make this possible, we require that the \(N'\) we choose to be either an RSA modulus or a prime. Once we have \([\sigma]^Q\), it is easy to see that \([ab]^Q \equiv [c]^Q + [\delta N]^Q \pmod Q\). This protocol, denoted by \text{TripConv} is secure due to the security of all the subprotocols involved. The complete protocol of \text{TripConv} can be found in Algorithm 14.

4.4 Probabilistic bit generation

Algorithm 15 describes our \(n\)-party protocol for probabilistic random bit generation over \(Z_Q\) such that \([b]^Q = \text{PrRndBit}_Q(p)\), where \(b = 0\) with probability \(p\), and \(b = 1\) with probability \(1 - p\). The generated bit share can be used for computation in dropout layer in neural network. Note that \(a \in \mathbb{Z}_Q\), thus \(a - [p \cdot Q] < 0\) with probability approximately \(p\), and \(a - [p \cdot Q] \geq 0\) with probability approximately \(1 - p\). Therefore, \(\text{GEZ}_Q([a - [p \cdot Q]], l) = 0\) with approximate probability \(p\) and \(\text{GEZ}_Q([a - [p \cdot Q]], l) = 1\) with approximate probability \(1 - p\). Note that we can have a more accurate probability by using a larger \(Q\).

5 MPC FOR NEURAL NETWORK

In this section, we describe various protocols to support efficient secure neural network training based on protocols given in Section 3.2, Section 3.3, and Section 4. Our protocols focus on \(n\)-party setting where correctness and security are guaranteed by our supporting protocols. Compared with MPC based neural network protocols in SecureML [47] and SecureNN [59], our protocols are applicable to a larger number of parties. Furthermore, compared to SecureNN[59], our protocols do not require external parties to assist the computation. Note that all the secret shares in this section are over finite field \(Z_Q\).

5.1 Linear and convolutional layer

Since operations in linear layer and convolutional layer are exactly multiply-and-accumulates on matrix [19], all parties can jointly call addition and multiplication protocols in SPDZ to make an efficient evaluation. Note that multiplications on matrix rely on matrix Beaver triples such that \(a_{rs} \cdot b_{xt} = c_{rt}, \quad a_{rs}, b_{xt}, c_{rt}\) are matrix. Indeed, matrix Beaver triple generation involves extra multiply-and-accumulates compared with that of single Beaver triple, hence takes more time. However, this can be done during offline phase, thus greatly improves the efficiency of evaluating multiply-and-accumulates in online phase.

5.2 ReLU with derivative

In neural network, ReLU function is a function that depends on the non-negativity of the input such that
ReLU(x) = \begin{cases} 
    x & x \geq 0 \\
    0 & x < 0 
\end{cases}

Therefore, evaluating ReLU function boils down to comparison between x and 0.

In addition, it is easy to see that the derivative of ReLU, denoted by DReLU, can be formulated as follows.

DReLU(x) = \begin{cases} 
    1 & x \geq 0 \\
    0 & x < 0 
\end{cases}

Therefore we can conclude that for any matrix x of any size, DReLU(x) = s = (x \geq 0) where the comparison is done entry-wise and ReLU(x) = x \times s where \times is an entry-wise matrix multiplication. Following this argument, the parties can then consecutively calculate ReLU and DReLU given a secretly shared input [x]\textsuperscript{⊗} following the protocol described in Algorithm 16. We note that the security of this protocol is guaranteed by the security of all the subprotocols being used during its calculation.

Algorithm 16 ReLU: ([y],[s]) \leftarrow \text{ReLU}([x])

1. All parties call GEZ([x]) to obtain [s] = [x \geq 0].
2. All parties call MulTrip([x],[s]) to obtain [y] = [x] \times [s] where \times represents entry-wise matrix multiplication.
3. Return [y] and [s].

5.3 Maxpool with derivative

Maxpool is a layer of neural network that outputs the maximum values of various submatrices of the input matrix determined by several parameters, namely filter size and stride. Since each submatrix can be handled independently in parallel, we simply focus on finding the maximum value of a submatrix, which can be represented as a list of s elements. To find such maximum value, we use the divide and conquer strategy where the comparison can be done in log s rounds. To simplify the description of the protocol, we first assume that s = 2\textsuperscript{p} for some positive integer p. In each round, we can pair up the elements and keep the larger element for the next round of comparison. This way, the number of elements to be compared in each round is reduced by half from the previous round. This can be done until we are left with one element, which is the largest element required as the output of Maxpool.

In order to enable backward propagation, we will need the derivative of Maxpool, which we denote by DMaxpool. Suppose that given an input list x = (x_1, \ldots, x_n) with Maxpool(x) = x^* where x^* is the i\textsuperscript{th}-th entry of x. Then DMaxpool(x) = v = (v_1, \ldots, v_n) where v_i = 1 and v_j = 0 for all other j. It is easy to see that the intermediate comparison results from the protocol Maxpool can be used to provide “path” from the maximum value to the x_i which is the maximum value. So multiplying all the intermediate comparison results in the path from x_i to the maximum value.

Algorithm 17 Maxpool: ([y],[comp]) \leftarrow \text{Maxpool}([x]) = ([x_1], \ldots, [x_n]), s = 2^p for some positive integer p

1. for i = j, \ldots, s do
2. Parties set [y(i,j)] = [x_i].
3. end for
4. for i = 1, \ldots, p do
5. for j = 1, \ldots, 2\textsuperscript{i-1} do
6. All parties compute [c_{i,j}] = \text{GEZ}([y(i-1,2\textsuperscript{j-1})] - [y(i-1,2\textsuperscript{j})])
7. Parties set [y(i,j)] = [y(i-1,2\textsuperscript{j-1})] + [c_{i,j}] \cdot ([y(i-1,2\textsuperscript{j-1})] - [y(i-1,2\textsuperscript{j})])
8. for k = 1, \ldots, 2\textsuperscript{i-1} do
9. Parties set [comp(j-1,2\textsuperscript{i-1} + k,i)] = [c_{i,j}] + [0] and [comp(j-1,2\textsuperscript{i-1} + k,i)] = [1 - c_{i,j}] + [0]
10. end for
11. end for
12. end for
13. Return ([y(p,1)], [comp]).

Algorithm 18 DMaxpool: [ind] \leftarrow \text{DMaxpool}([x],[comp])

1. for i = 1, \ldots, s do
2. Parties compute [ind_i] = \prod_{j=1}^{p} [\text{comp}(i,j)]
3. end for
4. Return [ind] = ([ind_1], \ldots, [ind_s]).

5.4 Dropout with derivative

Dropout layer is performed by dropping out some values with some fixed probability p such that

\text{Dropout}(x) = \begin{cases} 
    0 & \text{probability } p \\
    x/(1-p) & \text{probability } 1-p 
\end{cases}

Algorithm 19 describes our n-party protocol for Dropout which outputs the product of input matrix [x], matrix of probabilistic random bit [b], and public scaling factor c = (1-p)^{-1} which is encoded to \tilde{c} using the fixed point method discussed in Section 3.3. Since in Step 1, the matrix of probabilistic random bits [b] can be generated in the offline phase of SPDZ, only one multiplication is needed for the evaluation of the Dropout layer. In addition, according to the definition of Dropout and backward propagation, the derivative of Dropout is to propagate the gradients to the nodes except for the nodes that drop their values in the forward propagation. Therefore, DDropout can be simply obtained from the calculation of corresponding Dropout layer, i.e., DDropout([x]) = [b] \times \tilde{c}. Here [b] is the matrix with random bit entries used in the corresponding Dropout layer while \tilde{c} is the fixed point encoding of a public scaling factor c = (1-p)^{-1}.
Algorithm 19 Dropout: \((y), [b]) \leftarrow Dropout([x], p)

1. All parties call PrRndBit\((p)\) to generate a matrix which has the same size of \([x]\) of secret sharing probabilistic random bit \([b]\).
2. All parties compute \([y] = [x] \times [b] \times \bar{c}\) where \(\bar{c}\) is the fixed point encoding of \(c = (1 - p)^{-1}\) and the operator \(\times\) represents the entry-wise multiplication of involved matrices.
3. Return \((y), [b])\).

6 COMMUNICATION AND ROUNDS

We summarize the communication and round complexity of our neural network training protocols for 3PC compared with those of SecureNN [59] in Table 1.

We use the same \(\ell\) as the length of bits for data representation for the protocols both in SecureNN and ours. \(P\) and \(Q\) are the finite field size of SecureNN and our’s protocol respectively. Linear\(_{r,s,t}\) denotes multiplication between two matrix of dimension \(r \times s\) with \(s \times t\). Conv2d\(_{m,i,f,o}\) denotes the operations in convolutional layer with input matrix of dimension \(m \times m\), \(i\) input channels, \(o\) output channels, and a filter of dimension \(f \times f\). Maxpool\(_{i}\) and DMaxpool\(_{j}\) denote maxpool with its derivative over \(j\) elements. In addition, Dropout\(_{i}\) and DDropout\(_{j}\) denote dropout with its derivative over \(j\) elements, which are not available in SecureNN.

Note that (i) compared with SecureNN, we encode data in a larger finite field \(\mathbb{Z}_Q\) where \(Q\) is approximately \(\ell + \kappa\) bits, as our design relies on SPDZ which is more general than the specific design of SecureNN that enables protocols running over a small ring or field,

and (ii) we also do the same operation on MAC which increases the communication cost, although Beaver triples are generated offline in our protocol thus do not need a party as "assistant", i.e., \(P_2\) in [59], which saves the communication rounds. We can observe the round improvement of DMaxpool which is because we use a general constant-round comparison protocol instead of the protocol in [59] consisting of share conversion, reconstruction, and multiplication. In addition, the round improvement of DReLU and DMaxpool comes from the increase of storage of intermediate comparison results, while Dropout and DDropout save the rounds by moving some steps to the offline phase of SPDZ.

7 EXPERIMENTS

In this section, we present our experimental results for secure convolutional neural network training.

System setting. Our prototype is tested over three Linux workstations with an Intel Xeon Silver 4110 CPU (2.10GHz) and 128 GB of RAM, running CentOS7 in the same region. In this LAN setting, the average latency is 0.216 ms and the average bandwidth is 1.25 GB/s. In our experiment, data is represented in 64 bits including 12 bits (with sign bit) for integer part and 52 bits for fractional part. Our protocols are implemented using Gmpy2 [36] which is a Python version of GMP multiple-precision library [2] and several other standard libraries. The finite field size \(Q\) is set to be a prime which is greater than \(2^{145}\). Lastly, we set the security parameter \(\kappa\) to be 80. Note that to enable the comparison between SecureNN and our protocols, our experiments use the same bit length to represent the data \(\ell\) as the one used in the experiment conducted in SecureNN [59].

Neural network architecture. We implement two types of neural network: a deep neural network and a convolutional neural network. The former is the same model as used in [59] and [47], with architecture of fully connected layer (784, 128) - ReLU - fully connected layer (128, 10) - ReLU. The latter has the architecture of padding (32, 32) - convolutional layer (4, 28, 28) - ReLU - Maxpool (4, 14, 14) - convolutional layer (12, 10, 10) - ReLU - Maxpool (12, 5, 5) - flatten (1, 300) - fully connected layer (300, 120) - fully connected layer (120, 10) - ReLU. Both neural networks are implemented based on reproduced SecureNN protocols in [59] and our protocols, while plaintext implementation is based on Numpy [50]. We use MNIST dataset [42] which consists of 70,000 black-white hand-written digit images of size 28×28 in 10 classes. In our experiment, 60,000 images are used for training and 10,000 images are used for testing. Note that we only give the performance evaluation on neural network training, i.e., the online phase of SPDZ, as "raw materials" to be used can be prepared in the offline phase.

As shown in Table 2, with batch size of 64, our protocol offers a prediction accuracy of 97.75% after 10 epochs for DNN which takes 2.92 hours. For CNN, it takes 16.50 hours to complete 10 epochs training and achieves accuracy of 98.08%. Table 3 shows the training time of 1 epoch with different batch size for DNN and CNN. Table 4 summarizes the training comparison between our protocol, SecureNN, and plaintext in terms of training time and communication cost. We can observe that our protocol improves the threat model from semi-honest to dishonest majority with overheads of around 2X in LAN setting.

8 CONCLUSIONS

In this paper, we propose a new scheme with several primitives for secure neural network training in malicious majority setting leveraging on SPDZ. Our experimental results show that our protocols offer active security with affordable overheads of around 2X in LAN time compared with existing schemes in semi-honest setting. In addition, we propose a scheme for Beaver triple conversion from a ring \(\mathbb{Z}_N\) to a finite field \(\mathbb{Z}_Q\) to enable MAC checking in SPDZ, relying on two instances of Paillier crypto-systems.

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Table 2: Secure training for 3PC in LAN setting with batch size 64

| Type  | Epochs | Accuracy | Training time (hr) |
|-------|--------|----------|-------------------|
| DNN   | 1      | 96.99%   | 0.36              |
|       | 5      | 96.99%   | 1.45              |
|       | 10     | 97.75%   | 2.92              |
| CNN   | 1      | 97.00%   | 1.65              |
|       | 5      | 97.94%   | 8.27              |
|       | 10     | 98.08%   | 16.50             |

Table 3: Secure training for 3PC in LAN setting for 1 epoch

| Type  | Protocol | Accuracy | Training time (hr) | Comm (MB) |
|-------|----------|----------|--------------------|-----------|
| DNN   | SecureNN | 94.21%   | 0.13 hr            | 6.08      |
|       | Our’s    | 94.03%   | 0.29 hr            | 460.82    |
| CNN   | SecureNN | 97.01%   | 0.78 hr            | 56.92     |
|       | Our’s    | 97.00%   | 1.65 hr            | 2423.86   |

Table 4: Training comparison with SecureNN for 1 epoch

```
| Protocol | Accuracy | Training time | Comm (MB) |
|----------|----------|---------------|-----------|
| SecureNN | 94.21%   | 0.13 hr       | 6.08      |
| Our’s    | 94.03%   | 0.29 hr       | 460.82    |
| SecureNN | 97.01%   | 0.78 hr       | 56.92     |
| Our’s    | 97.00%   | 1.65 hr       | 2423.86   |
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