The Strange Attractor Model of Bipedal Locomotion and its Consequences on Motor Control

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Abstract

\textbf{Background:} Despite decades of study, many unknowns exist about the mechanisms governing human locomotion. Current models and motor control theories can only partially capture the phenomenon. This may be a major cause of the reduced efficacy of lower limb rehabilitation therapies. Recently, it has been proposed that human locomotion can be planned in the task-space by taking advantage of the gravitational pull acting on the Centre of Mass (CoM) by modelling the attractor dynamics. The model proposed represents the CoM transversal trajectory as a harmonic oscillator propagating on the attractor manifold. However, the vertical trajectory of the CoM, controlled through ankle strategies, has not been accurately captured yet. \textbf{Research Questions:} Is it possible to improve the model accuracy by introducing a mathematical model of the ankle strategies by coordinating the heel-strike and toe-off strategies with the CoM movement? \textbf{Method:} Our solution

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consists of closed-form equations that plan human-like trajectories for the CoM, the foot swing, and the ankle strategies. We have tested our model by extracting the biomechanics data and postural during locomotion from the motion capture trajectories of 12 healthy subjects at 3 self-selected speeds to generate a virtual subject using our model. Our virtual subject has been based on the average of the collected data. **Results:** The model output shows our virtual subject has walking trajectories that have their features consistent with our motion capture data. Additionally, it emerged from the data analysis that our model regulates the stance phase of the foot as humans do. **Significance:** The model proves that locomotion can be modelled as an attractor dynamics, proving the existence of a nonlinear map that our nervous system learns. It can support a deeper investigation of locomotion motor control, potentially improving locomotion rehabilitation and assistive technologies.

**Keywords:** Bipedal Locomotion, Motor Control, Locomotion Stability

1. **Introduction**

Human locomotion has been studied for decades due to its relevance to the medical field. The description of gait as a set of "six determinants" as proposed in the classic paper from Inman and Eberhart in 1953 [1] led the clinical evaluation of gait to focus on pelvic tilt and rotation, knee and hip flexion, knee and ankle interaction, and lateral pelvic displacement. Moreover, roboticists have also become interested in understanding biological locomotion to develop models that reproduce its robustness and efficiency. Nowadays, there are different and, sometimes, confronted theories about the mechanisms that animals employ to generate locomotor strategies [2]. This is made even more fascinating when accounting for both mechanical and computational performances of the human body [3, 4, 5, 6, 7, 8, 9]. Multiple proposed models aim to capture biological locomotion and its motor control, but they can only partially capture the locomotion dynamics [4, 9].

The limited understanding of locomotion dynamics also affects our knowledge about the control of gait. Among these, the motor primitives theory is based on the hypothesis that human actions are composed of a superposition of basic movements called primitives[8, 10, 5, 11, 12]. They have been observed in multiple movement strategies involving both upper and lower limbs. Furthermore, this theory also justifies how the human body can deal
both with redundancy and information delay due to neural transmission. The Passive Motion Paradigm (PMP) is a computational method that can capture the motor primitives by minimising the energetic cost of the movement [10, 13, 5]. Such optimisation is performed on an energetic function that is defined to account for both the desire to perform a specific action and the energetic cost associated with the mechanical impedance of the human body [10, 13, 5]. The main limitation of this approach is that its formulation is based on the hypothesis that the human body’s mechanical impedance mainly determines the task dynamics. Thus, they only apply to tasks where the environmental dynamics are negligible compared to the body impedance, which is not the case when there is contact with the ground.

The dynamic motion primitives theory has been introduced to solve this issue by accounting for the environmental dynamics by characterising the task attractor generated at the interface with the human [5, 6]. It is also compatible with the hypothesis that humans have a hierarchical semi-autonomous controller. Under this theory, every module can autonomously manage the task assigned by a higher level controller [14, 6]. This can be visualised as a sequence of connecting dots puzzles where the lower-level controller is tasked to connect the dots (called via points) provided by the higher controller, which also supervises the outcome [15]. This decentralised architecture enables the Central Nervous System (CNS) to deal with the neuronal transmission delay and allows the coordination of the body [14, 15, 5].

Another computational theory for motor control is based on the Optimal Feedback Theory (OFT). The concept behind this approach is that the evolution of system dynamics can be associated with energetic costs, and the optimal strategy can be obtained by minimising a cost function [16, 10, 5]. This approach has two major drawbacks: the impossibility of accurately measuring a muscle cost function in humans and the extremely high computational cost, which increases with task complexity, especially in the presence of redundancies [16, 10, 5, 17].

From an experimental perspective, several authors have studied gait as a periodic process with stride-to-stride fluctuations that, while resembling random behaviour, possess long-time correlations, as it occurs in fractal systems [18, 19]. This idea has led to the analysis of gait as a Limit Cycle attractor. This approach has also been used to analyse experimentally gait data from multiple strides or to assess gait perturbations [20]. In this respect, the strange attractor has fractal dynamics, and it has been shown experimentally [19].
Recent works suggest that there is an alternative formulation for locomotion dynamics by using the potential energy of the bipedal system in the task space and removing the switching dynamics in the swing-stance transitions [21, 22, 15, 23, 24]. Tiseo et al derived closed-form equations for both CoM and the swing foot trajectory from the bipedal potential energy surface, which is a saddle [22]. The system dynamics can be obtained by applying the gradient operator to the potential energy manifold, which shows the direction of the energy flow within the system and, consequently, identifies the autonomous trajectory of the system. A reference frame centred in the saddle point and aligned to the surface’s principal directions was defined to derive the equations, using energy conservation to identify the optimal trajectory for moving along the gravitational manifold [23]. The saddle frame is then used as an observer to describe the COM trajectory, which is modelled as a harmonic oscillation between the two maxima of potential energy. Then a coordinate transformation is used to express the trajectories in the task space. The model accurately tracks the Swing foot and CoM traversal plane trajectories, but it cannot track as well the vertical CoM trajectory [22].

This work presents a new model to estimate and predict the trajectory of the CoM during gait. It includes an accurate model of heel-strike and toe-off to remove the tracking error of the CoM trajectories. Afterwards, the trajectories generated from the model simulation were compared with experimental data of healthy subjects in order to check its accuracy. In this way, it is possible to use this model to predict CoM trajectories in the context of gait stability controllers for biped robots and exoskeletons.

2. Method

The methods explain the experimental gait data recordings, the model description and its validation.

2.1. Experimental gait data

To validate the model, the gait data from 12 healthy subjects (7 males and 5 females) were recorded. All the subjects have given written informed consent to the experiment that was approved by the corresponding Institutional Review Board (IRB-2016-09-015).

The subjects’ age, mass, and height averages are $28.3 \pm 3.42$ years old, $71.14 \pm 12.6$ kg and $171 \pm 10.7$ cm, respectively. The subjects were instructed to walk ten times in a straight line for about 6 metres at three self-selected
speeds: slow, normal and fast. Their movements were recorded with ten markers placed on the body according to [25] using an 8-Cameras Vicon Motion Capture System (Vicon Motion System Inc, UK). However, technical issues with the equipment resulted in losing five trajectories at the normal speed of one subject. The four markers at the pelvis (left and right anterior and posterior superior iliac spines) were used to define the centre of mass (COM) by taking the centre of these coordinates. The foot trajectories \((CoR_{R,L})\) were obtained by computing the barycentre of the triangle formed by the heel, first and fifth metatarsal markers and projecting it to the foot sole. The feet inclination relative to the ground is the slope of the segment between the heel marker and the mid-point of the segment between the first and fifth metatarsal markers. The 3-D trajectories of the COM and both feet were retained for further analyses (Figure 1).

2.2. Derivation of the Model Parameters

To plan the strategies, the model requires identifying some gait strategies and measuring some biomechanical parameters. First, based on the biomechanical data of our sample we define:

- \(h_{body} = 1.71\) m is the body height of the model, which is the mean height of our dataset;
- \(L_{P} = \sqrt{0.765^2 + 0.0955^2}\) \(h_{body} = 0.5943\) \(h_{body}\) m is the maximum length of the pendulum [26, 27].
- \(d = 0.0921\) cm is the average distance of the CoRs from both the metatarsus and the heel in our dataset.
- \(d_{xAH} = 0.0253\) \(h_{body}\) m is the distance of the ankle joint from the heel in the anteroposterior direction [26].
- \(d_{x} = 0.039\) \(h_{body}\) m is the distance of the ankle joint from the CoR in the vertical direction [26].
- \(d_{xA} = d - (d_{AH} h_{body})\) m is the distance of the ankle joint from the CoR in the anteroposterior direction.
- \(d_{xAMT} = 2d - d_{AH}\) m is the distance of the ankle joint from the metatarsus in the anteroposterior direction.
The gait-dependent parameters are the relationships between step length $d_{SL}$, step width $d_{SW}$ and the maximum heel-strike angle $\theta_{MaxHS}$ as functions of the walking velocity $v_W$. These were identified by providing as input the mean gait parameters of the 12 subjects to the Matlab Curve Fitting App (Matworks Inc, USA) and selecting a first-order polynomial regression. The expressions obtained are:

$$\begin{align*}
d_{SW} &= -0.009149 \ v_W + 0.1072 \ m \quad R^2 = 0.04 \\
d_{SL} &= 2 \ (0.09399 \ v_W + 0.1624) \ m \quad R^2 = 0.89 \\
\theta_{MaxHS} &= -1.162 \ v_W + 9.198 \ deg \quad R^2 = 0.02
\end{align*}$$

(1)

The low $R^2$ values for the heel-strike angle and step width indicate that the data variability cannot be captured using the velocities, indicating that other factors may influence the choice of these parameters during locomotion. On the other hand, the high value for the step length indicates that the walking speed mainly determines this gait parameter. These observations align with what is reported in literature [22, 28], which indicates that step width is highly influenced by lateral perturbations [28].

The above-mentioned relationships make it possible to define the following parameters that are required to characterise the bipedal mechanism shown in Figure 1:

$$\begin{align*}
L_{leg} &= \sqrt{L^2_{P0} - \left(\frac{d_{SW}}{2}\right)^2 - d^2_{zA} - d_{zA}} \ m \\
D_{AH} &= \sqrt{d^2_{zA} + d^2_{zAH}} \ m \\
\theta_{DAH} &= tan^{-1} \left(\frac{d_{zA}}{d_{zAH}}\right) \ deg \\
D_{AMT} &= \sqrt{d^2_{zA} + d^2_{zAMT}} \ m \\
\theta_{DMT} &= tan^{-1} \left(\frac{d_{zA}}{d_{zAMT}}\right) \ deg
\end{align*}$$

(2)

2.3. CoM Trajectory on the Transversal Plane

The CoM trajectory is modelled as a harmonic oscillator moving forward at a constant speed ($v_W$) in the transversal plane while oscillating at a frequency $\omega_{Step}/2 = (2t_{Step})^{-1}$, which is based on the model presented
Figure 1: (a) The mechanical bipedal structure used to formulate the model is composed of two identical legs connected to the CoM via spherical joints. Each leg comprises two rigid bodies (\(L_{\text{Leg}}\) and the foot) connected via the revolute joint at the ankle. The feet are modelled as two triangles, with one vertex in the ankle joint, the second congruent to the middle point of the segment connecting the metatarsal joints and the last vertex in the heel. \(\text{CoR}_R\) and \(\text{CoR}_L\) were obtained by computing the barycentre of the triangle formed by the heel, first and fifth metatarsal markers. \(\text{vCoR}_R\) and \(\text{vCoR}_L\) are the projections of the CoRs that are used for the planning, and they coincide with the CoRs when the feet are in contact with the ground. The lengths of the two pendula (\(L_{P_R}\) and \(L_{P_L}\)) are defined as the Cartesian distances between the CoM and the \(\text{vCoR}s\). (b) The Heel-Strike is modelled as a rotation about the \(\text{CIR}_{HS}\) of the foot. (c) The Toe-Off is modelled as a rotation about the \(\text{CIR}_{TO}\). (d) The planner uses two reference systems. The Task-Space frame (TS) is aligned with the anatomical planes, and it is used to describe walking trajectories. The Saddle-Space frame (Saddle) is used to describe posture-dependent quantities, which are later projected in the TS.

in [22]. The oscillation amplitude is determined by imposing the alignment of the harmonic oscillator asymptote with the ordinate of the Saddle Space (\(y_{\text{Saddle}}\) in Figure 1.(d)) during the step-to-step transition. The Saddle Space is defined by a posture-dependent frame as proposed previously [21, 15, 29] to model the potential energy of the system. Hence, the amplitude \(A_y = d_{SW}/(2\pi \omega_{\text{Step}} d_{SL})\) also depends from the step width (\(d_{SW}\)) and
the step length \(d_{SL}\), which leads to the following equation:

\[
\begin{aligned}
    x_{CoM}(t) &= v_W t \\
    y_{CoM}(t) &= A_y \cos(\pi \omega_{\text{Step}} t)
\end{aligned}
\]  

(3)

The vertical trajectory is generated by imposing the constraint imposed by the mechanical system, which requires having at least one of the two feet in contact with the ground, and it depends on the legs’ postures, and it is presented at the end of the ankle strategies model.

2.4. vCoRs Anteroposterior Trajectory

The foot swing trajectory is modelled by synchronising the movement with the biped potential energy, identified by the Saddle Space [21]. This implies that if the CoM maintained \(y_{\text{Saddle}}\), the gravitational pull is always directed toward the opposite foot [15]. The virtual Centres of Rotation (vCoRs) are defined by projecting to the ground the segment between the CoM and the CoR, based on the recently reported findings that human motor control relies on a static reference in the foot [30, 15]. Therefore, the vCoRs (Figure 1) anteroposterior trajectories are described with a cycle starting when both feet are aligned in the frontal plane. This is a periodic state that occurs every \(t_{\text{Step}}\) when the CoM reaches its maximum amplitude in the mediolateral oscillation in the opposite direction (Equation 3).

\[
\begin{aligned}
    x_{vCoR-i}(t) &= \begin{cases} 
    x_{vCoR_i}(t_0), & \text{from } t_0 \text{ until } TO \\
    -d_{SW}/m_{\text{saddle}}(t) + x_{vCoR_i}(t_0), & \text{swing} \\
    x_{vCoR_i}(t_0) + d_{SL}, & \text{from } HS \text{ to } GC
    \end{cases}
\end{aligned}
\]

(4)

where \(i = R, L\) indicates the right or left foot, \(t_0\) is the time at the beginning of the cycle, and \(m_{\text{saddle}}(t)\) is the trajectory of the slope of \(y_{\text{Saddle}}\) determined from the position of the support foot and the CoM. TO describes the end of the toe-off, HS is the beginning of the heel strike, and GC stands for the end of the gait cycle.

2.5. Vertical Trajectory of the CoM

The vertical trajectory has been modelled as a cycloid curve based on recent findings reported by Carpentier et al [30]. To derive the trajectory equation the amplitude of the movement \(A_z\) and the time of heel strike
\( (t_{HS0}) \) are calculated from Equation 3 and Equation 4.

\[
\begin{align*}
    &d_{SL} - d \cos(\theta_{MaxHS}) - \frac{d_{SW}x_{CoM}(t_{HS0})}{y_{CoM}(t_{HS0}) - 0.5d_{SW}} = 0 \\
    &d(CoM, CoR_{HS})_{t_{HS}} = \sqrt{(L_{P0} - \Delta X_{HS})^2 - \Delta Y_{HS}^2} + d sin(\theta_{MaxHS} + \theta_{DAH}) \\
    &A_z = d(CoM, CoR_{Support})_{t_0} - d(CoM, CoR_{HS})_{t_{HS}}
\end{align*}
\]

(5)

where \( d(\cdot, \cdot)_t \) indicates the Euclidean distance between two points at the instant \( t \), \( CoR_{HS} \) is the CoR of the landing leg, \( CoR_{Support} \) the CoR of the legs passing over support of the CoM. \( \Delta X_{HS} \) and \( \Delta Y_{HS} \) are the distances between CoM and \( CoR_{HS} \) along the anteroposterior and mediolateral directions, respectively. Lastly, \( t_0 \) is the instant when the moving leg has surpassed the leg providing support (\( x_{COR-Support} = x_{COR-Swing} \)). Once these parameters are determined, the CoM vertical trajectory can be expressed as follows:

\[
z_{CoM}(t) = \begin{cases} 
(z_{MAX} - A_z) + A_z \cos(\pi \frac{t + d_{xAnk}/v_{W}}{t_{HS0} + d_{xAnk}/v_{W}}) & \text{if } t \leq t_{HS0} \\
(z_{MAX} - A_z) + A_z \cos(\pi \frac{t - t_{HS0}}{t_{Step} - t_{HS0} - d_{xAnk}/v_{W}}) & \text{if } t_{HS0} \leq t \leq t_{Step} - \frac{d_{xAnk}}{v_{W}} \\
(z_{MAX} - A_z) + A_z \cos(\pi \frac{t - t_{Step} + d_{xAnk}/v_{W}}{t_{HS0} + d_{xAnk}/v_{W}}) & \text{Otherwise}
\end{cases}
\]

(6)

where \( z_{MAX} = \sqrt{L_{P0}^2 - (0.5d_{SW})^2 - d_{xAH}} \) is the maximum height of the CoM that is reached when the CoM is aligned with the ankle joint of the foot providing support.

2.6 Validation Method

The results are validated by comparing the mediolateral amplitude of the CoM and the vertical amplitudes generated from the proposed model with the CoM trajectories recorded during our experiment. The regressions were computed with a 95%.

\[
\begin{align*}
    A_{y-Des} &= (0.011 \pm 0.007)v_{W}^2 - (0.041 \pm 0.022)v_{W} + (0.051 \pm 0.015) \\
    A_{z-Des} &= (0.012 \pm 0.013)v_{W}^2 - (0.003 \pm 0.040)v_{W} + (0.021 \pm 0.028)
\end{align*}
\]

(7)
Furthermore, we analysed the double support phase duration to observe if there is an intrinsic regulation that can justify the variation of double support observed in humans when walking at different speeds [31, 4].

3. Results

The results show that the proposed model can generate human-like behaviour for the 3-D trajectory of the centre of mass and the anteroposterior trajectory of the foot swing (Figure 2, Figure 3 and Figure 4). Figure 4 shows how the model trajectory is tracking human behaviour, which is always close or superimposed to the mean behaviour of the collected data. The data also show that ankle strategies are responsible for the intrinsic regulation of the double support duration. The analysis of the double support duration shows a value of 14% of the gait cycle at 2.2 m/s, 16% at 1.8 m/s, 18% at 1.4 m/s, 24% at 1 m/s, and 46% at .6 m/s. This implies that the foot stance duration at the same speeds is 57%, 58%, 59%, 62%, and 73%, respectively. This behaviour is consistent with the experimental studies on regulating the double support and foot stance at different speeds [32, 11], and it is an emergent behaviour in the proposed model identified during the data analysis.

4. Discussion

Our results describe how the proposed deterministic model, based on a harmonic oscillator centred in the saddle point, produces human-like gait trajectories for both the CoM and the foot swing at different gait speeds. Furthermore, it identifies the ankle strategies as the mechanism that allows to control of the vertical movement of the CoM. Therefore, it simultaneously regulates walking speed and stability. Moreover, the slower gait speed also showed an increase in the double stance phase in agreement with well-established experimental results [11, 32]. The results presented here confirm and extend previous results obtained with different data sets [21, 15, 33]. The data also indicate that humans control step length, step frequency and mediolateral amplitudes according to an a priori optimised behaviour, which is similar to what has been observed by Collins et al [34]. In contrast, they seem to optimize the step width ($d_{SW}$) and the vertical amplitude of the CoM (via the ankle strategies) depending on local conditions. This result is also supported by multiple studies that observed how these two parameters are modulated based on the environmental conditions [4, 14, 28]. Moreover, the
role of ankle strategies in the energetic efficiency of gait is widely reported in the literature [14, 35, 34, 36, 37, 38, 39, 9]. Furthermore, the proposed model intrinsically captures the regulation of the CoM vertical trajectory and the double support observed in human data [31, 32, 11]. As reported by Orendurff et al human trajectories are not symmetric at low speeds due to the anticipation of the CoM minimum height in the gait cycle. The model proposed here is based on the Saddle Space proposed by Tiseo et al [21, 15, 23, 29, 40], it also provides a complete characterisation of the potential energy which can be calculated from the derivative of the CoM trajectory. Hence, it models comprehensively the attractor produced by the interaction between CoM and
Figure 3: The anteroposterior trajectories of the vCoRs for different walking speed shows the intrinsic regulation of the support and swing phases. A lower speeds, the support phase dominates the gait cycle, but this dominance gets mitigated at higher speeds.

4.1. Unpredictable local conditions transform the Harmonic Oscillator in a Strange Attractor

Human behaviour is characterised by significant non-random variability. It has multiple sources, e.g. neural and effector noise or external pertur-
Figure 4: The model can generate foot swing, mediolateral and vertical trajectories that are within the 95% confidence level of the human data. Furthermore, the curves are very close to the average human behaviour.

bations, making it very unlikely that the two steps are exactly identical. Therefore, the chaotic nature of the process transforms the deterministic behaviour of the proposed model into a strange attractor centred in the saddle point, as observed in human data. This can be seen from the comparison between the data from the model (Figure 2) and the data from a typical subject (Figure 5). It shows that human trajectories are consistent with the behaviour predicted by the model. Moreover, they are affected by local conditions, thus showing the intrinsic variability in tracking the ideal behaviour. Some experimental observations of the strange attractor dynamics were made by Kang et al[41] and Orendurff et al[31], but they have never connected them to the attractor.
Figure 5: The data at the slow, normal and fast speed for one of the subjects’ trajectories on the 3 planes. The trajectories show confirms that the model describes idealised locomotion strategies, which are then affected by local conditions (e.g., body state and environmental interaction). The One Step Frontal Plane highlights this, showing how, when a single step is isolated, we observed a similar trajectory adaptation to the velocity change as in the trajectories generated in the model shown in Figure 2.

4.1.1. Implications about the Control Architecture

The intrinsic unpredictability of long-term environmental conditions is a challenge for the human body, considering that a feedback loop involving the brain takes at least 200ms [7, 42, 43]. Therefore, it is impossible that the brain has pure feedback control of the step-to-step transition. Ahn et al[14] proposed a decentralised semi-autonomous architecture that can explain how the Nervous System (NS) can mitigate the hardware limitation of the human body. Their findings are confirmed by our model in which the ankle strategies are identified as the main actor in regulating the locomotion’s local stability and energy efficiency. Ahn et al[14] also reported that the ankle strategies behaviour seems to be synchronised by a non-linear oscillator, where the proposed model identified in the above-mentioned strange attractor.
We have recently proposed that the NS relies on two different models of the BoS to plan and supervise locomotion [21, 15, 23]. The Instantaneous-BoS (IBoS) has been theorised to be used as a region of finite time invariance by the Cerebellum to supervise the Central Pattern Generators (CPG) in the brain stem and the spinal cord. Where a region of finite time invariance is defined as a space surrounding the CoM where non-linear time-dependent dynamics can be considered invariant for a short time [15, 23]. This approach is usually used in control applications with highly variable environmental conditions, and, similarly to human behaviour, it does not guarantee that the system will always converge at the exact desired output, but it identifies the strategy to obtain a locally stable system. Moreover, we have also hypothesised a second BoS that describes the expected dynamic conditions in a given posture, called Ballistic-BoS (BBoS). Therefore, the BBoS describes a forward model implemented by the Cerebellum to predict future global stability conditions required by the motor cortex to plan stable movements.

4.2. An Integrated Motor Control Theory

Before proceeding into the explanation of how our findings can integrate all the most prominent motor control theories, we will summarise our deductions so far:

- Step length and lateral amplitude of the CoM trajectory are optimised based on a priori optimal strategies.
- Step width and vertical amplitude of the CoM trajectory depend on local conditions.
- Our findings support a hierarchy architecture of semi-autonomous controllers.
- The ankle strategies are the main actor in controlling the vertical movement of the CoM, and they seem to be controlled by CPG in the spinal cord under the supervision of the cerebellum.
- The motor cortex that is involved both in task-space and joint-space planning evaluates the stability conditions based on IBoS and BBoS.
- The task-space planner proposed by our model is based on the principal direction of the harmonic oscillator centred in the saddle point; therefore, it is compatible with the dynamic primitive theory.
The desired foot placement is determined by the combination of stereotyped optimal behaviour and stability conditions depending on both walking speed and environmental conditions.

4.2.1. How Dynamic Primitives becomes Motor Primitives

The dynamic motion primitives theory was developed as an extension of the motor primitives and, until now, it is unclear how they are connected[5]. Our theory is based on the integration of our task-space planner with the joint-space planner $\lambda_0 - PMP$ mentioned in the introduction. The current architecture of this planner considers that the task-space dynamics is determined only by a spring-based mechanism that pushes the user towards the desired posture[10]. However, if the forces generated by the attractor substitute the spring, the planning optimises an energy function which considers desired behaviour, body mechanical impedance and environmental dynamics[44]. Hence, the $\lambda_0 - PMP$ architecture will then produce a sequence of optimised postures that also account for the environmental dynamics. The CPG will then use this sequence of reference posture to generate the neural signals that control muscular activities. This latest part of the theory is also supported by animal experiments that show how it is possible to reproduce the gait-like behaviour in rats with an induced spinal injury, through an electrochemical stimulation of the spinal cord[45, 46]. Lastly, this architecture implies that when the environmental dynamics is negligible, then the task planning is based only on the "reward" or "discomfort" function of the $\lambda_0 - PMP$. Therefore, the motor primitives are dynamic primitives when the body’s intrinsic mechanical impedance dominates the task.

Lastly, the proposed architecture, as shown in Figure 6, is based on a local postural optimisation ($\lambda_0 - PMP$) of globally optimal strategies provided by Saddle Task-Space Planner; it implies that the output converges to an optimal strategy when the body can adequately compensate for the variability of the environmental conditions. Therefore, the output of the proposed architecture can converge to the one that would be predicted from an OFT at a significantly lower computational cost.

4.2.2. Anatomophysiological Parallels

The analysis of different studies about motor control revealed some parallels with current theories about NS anatomy physiology, and this paragraph is dedicated to describe such findings. However, the authors would like to
remark that the proposed organisation is not the structure of the NS but an architecture that can capture some of its characteristics.

We proposed a structure of a semi-autonomous controller organised in a hierarchical architecture that has a parallelism in the biological motor control Figure 6. At the apex of the hierarchy is the TS planner, which acts on the input of walking towards a target. The TS planning appears to be based on stereotyped optimised strategies which depend on the environmental conditions Figure 6. Hence, this structure should perform the following tasks: receive the input of the voluntary decision, perform the assessment of both environmental and body conditions, select an adequate optimal strategy based on \textit{a priori} models, and generate the desired task-space trajectories and expected external dynamics. Based on these three characteristics, the brain regions involved in the process are the frontal and prefrontal cortices, which are tasked to produce the desired behaviour in the task-space. They are supported by the basal ganglia, which regulate the reward circuits of the brain and the cerebellum. In turn, it is considered that the cerebellum provides the internal models for planning and supervision, connected through the thalamus and the parietal cortex [42, 16, 47, 13]. This information is then forwarded to the task-space planner, which provides a bridge between the pre-motor and motor cortices. Afterwards, the latter is tasked with joint-space planning with the support of the parietal cortex and cerebellum [42, 16]. Its output is then passed to the spinal cord through the brain stem which implements the CPG and the spinal cord reflex, which is the final stage of the architecture allocated within the CNS [42, 16]. This stage of the CNS controller also provides the first centralised response to external perturbation, and it has a reaction time for the ankles of about 80ms [43, 14]. The last stage of the architecture is the intrinsic mechanical impedance of the musculoskeletal system that filters external perturbations and which controlled by the CNS to execute the planned movements.

4.2.3. Limitations

The main limitation of this work is that it is based on observations made for locomotion, and the current formulation can only reproduce straight walking. In other words, locomotion can be seen as the antipodes of reaching task, being dominated by the gravitational attractor dynamics rather than the body intrinsic mechanical impedance. Thus, the following questions need to be answered before the theory can be validated:
Figure 6: The parallel between the proposed architecture and the human nervous system based on literature information [17, 42, 8, 47, 16, 11, 13, 5, 10]. The full lines in the schematic represent information involved in the control, while the dotted lines indicate a supervision signal. Both types of connection are often mediated by other parts of the brain which are not included in this simplified architecture. Furthermore, the pre-motor cortex is drawn across the border of the task-space and joint-space planner because we hypothesise that it is acting as a bridge between the frontal cortex and the motor cortex.

- How does the NS balance between desired behaviour and environmental dynamics? (i.e., what happens in the middle ground between locomotion and upper-limb reaching task?)

- How does the NS regulate the change of control policy adopting the global strategies to changes in environmental conditions?

Lastly, it shall be pointed out that similar to the other theories, this theory does not imply that the human brain is effectively using this architecture. It only implies that the proposed controller is a good analogy to describe human motor control in the analysed tasks.

4.3. Insights on Locomotor Impairment Therapies

Our findings revealed that the locomotor strategies use gravitational forces to produce more energy-efficient locomotion, and the ankle strategies have a
leading role in the management of both stability and energy efficiency. This allows us to conclude that one of the possible reasons for the low effectiveness of available gait rehabilitation therapies is that they cannot provide a dynamic environment consistent with the activities of daily living. Therefore, we think the next generation of medical technologies should focus on transparent harness systems that provide minimal assistance to maintain an erect posture, with minimally invasive support of ankle strategies. There are already robots and mechanisms able to provide such support, but there is no integrated solution yet [36, 37, 38, 35, 48].

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6. Author contributions statement

C.T. formulated the model, conceived the experiment, conducted the data collection, performed the data analysis and wrote the paper, M.J.F. helped in the data collection, K.C.V. worked in the model formulation, experimental design and writing, A.F.C. helped in the writing of the manuscript, data analysis, and data presentation. A.W.T. supervised all the phases of the research and provided the funding. All authors reviewed the manuscript.

7. Conflict of Interest Statement

None of the authors has competing financial interests.

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