A chiral $\bar qqqq$ nonet?

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We point out that meson spectrum indicates the existence of a degenerate chiral nonet in the energy region around 1.4 GeV with a slightly inverted spectrum with respect to a $\eta$ nonet. Based on the observation and the approximately linear rising of the mass of the quark with the number of constituent quarks we conjecture the existence of a tetraquark chiral nonet in this energy region with chiral symmetry implemented directly. We realize this idea in a chiral model and take into account the mixing of the tetraquark chiral nonet with a conventional $\eta$ nonet. We find that the mass spectrum of mesons below 1.5 GeV is consistent with this picture. In general, pseudoscalar states arise as mainly $\eta$ states but scalar states turn out to be strong admixtures of $\eta$ and tetraquark states.

The understanding of the puzzling scalar mesons is of primary interest since these mesons have the same quantum numbers as the QCD vacuum and their properties are expected to be strongly influenced by the latter. The hope is that a better understanding of the properties of scalar mesons yield some information on the structure of the QCD vacuum.

The quark structure of mesons is usually inferred from the accuracy of quark model predictions for their properties or the interpretation of their decay products according to a flavor structure. This structure is mapped onto the transformation properties of quark composites under the $SU(3)_F$ group. In the case of the scalars, the quasi-degeneracy of the firstly discovered $f_0(980)$ and $a_0(980)$ mesons suggesting an $\omega - \rho$ like system was in contradiction with the strong coupling of the former to $K\bar K$ system. It was also soon realized that naive quark model calculations fail to describe the small two photon decay widths of the $f_0(980)$ and $a_0(980)$ mesons, which triggered the interest in other possible structures for these mesons.

The classic study of tetraquark structures by Jaffe \cite{1} in the framework of a quark-bag model predicted a light scalar nonet with an inverted spectrum which nowadays can be identified with a nonet composed by the $f_0(980)$, $a_0(980)$, $f_0(600)$ (or $\sigma$) and the more controversial $\kappa(900)$. In this framework, the inverted mass spectrum and couplings are related to the flavor structure. A strong attractive gluon-magnetic interaction is essential here in order to pull the natural scale $M \sim 4m_q$ down to $M \approx 900$ MeV. Similar results for the members of the scalar nonet are obtained in a variety of formalisms. In particular, a recent analysis reach this conclusion analyzing three different independent aspects of the problem \cite{2}. The extensive literature to the scalar mesons problem can be traced back from this work.

Recently, chiral Lagrangians have been formulated trying to understand the spectrum and interactions of scalar mesons. Clearly, the effective Lagrangian approach would be able to describe the long distance physics of these mesons in contrast with quark model descriptions which are based on the internal degrees of freedom therefore taking into account effects at distances of the order of meson radius and smaller. Although chiral Lagrangians deal with effective degrees of freedom (d.o.f.), in principle, transformation properties of these d.o.f. can be mapped directly to a quark content. This mapping must be taken carefully as discussed in \cite{3}.

An alternative explanation to the inverted spectrum of light scalars was formulated based on the properties of the QCD vacuum \cite{4, 5}. This explanation relies on the long distance description of these mesons embraced in effective Lagrangians which take into account the main properties of the QCD vacuum, the spontaneous breaking of chiral symmetry and the breaking of the $U_A(1)$ symmetry by non-perturbative effects. The role of the effective six-quark interaction due to instantons \cite{6} in the inversion of the light scalar spectrum was emphasized in \cite{3}.

The existence of two scalar nonets, one below 1 GeV and another one around 1.4 GeV, lead to the exploration of these two nonet models. A first step in this direction was given in \cite{7} where a light four-quark scalar nonet and a heavy $\eta$-like nonet are mixed up to yield physical scalar mesons above and below 1 GeV. In the same spirit, in Ref. \cite{8} a linear sigma model is coupled to a nonet field with well defined transformation properties under chiral rotations and trivial dynamics, except for a $U_A(1)$ (non-determinantal) violating interaction. The possibility for the pseudoscalar light mesons to have a small four-quark content was speculated on the light of this “toy” model. An alternative approach was formulated in \cite{8} where two linear sigma models were coupled using the same interaction as in \cite{7}. The novelty here is a Higgs mechanism at the hadron level in such a way that the members of a pseudoscalar nonet are “eaten up” by axial vector mesons and one ends up with only one nonet of pseudoscalars and two nonets of scalars. The model is used as a specific realization of the conjectured “energy-dependent” composition of scalar mesons based on a detailed analysis of the whole experimental situation \cite{8}.

In the energy region around 1.4 GeV the Particle Data Group lists the following scalar states: $f_0(1370)$, $K_0^*(1430)$, $a_0(1450)$, and $f_0(1500)$ \cite{7}. In addition we have the $f_0(1710)$ at a slightly higher mass \cite{8}. We note also that if we consider the former states as the members of a nonet then it presents a slightly inverted mass spectrum. Furthermore,
a look onto the pseudoscalar side at the same energy yields the following states: \( \eta(1295), \eta(1440), K(1460), \pi(1300) \). Thus data seems to indicate the existence of a quasi-degenerate chiral nonet around 1.4 GeV whose scalar component has a slightly inverted mass spectrum. On the other hand, the linear rising of the mass of a hadron with the number of constituent quarks indicates that four-quark states should lie slightly below 1.5 GeV. This lead us to conjecture that this chiral nonet comes from tetraquark states mixed with conventional \( \overline{q}q \) mesons to form physical mesons. The quasi-degeneracy of this chiral nonet suggests that chiral symmetry is realized in a direct way for tetraquark states.

In this letter we report results on the implementation of this idea in the framework of an effective chiral Lagrangian. We start with two chiral nonets, one around 1.4 GeV with chiral symmetry realized directly and another one at low energy with chiral symmetry spontaneously broken. In contrast to previous studies, mesons in the “heavy” nonet are considered as four-quark states. The nature of these states is distinguished from conventional \( \overline{q}q \) mesons by terms breaking the \( U_A(1) \) symmetry. We introduce also mass terms appropriate to four-quark states which yield an inverted spectrum for the “pure” four-quark structured fields. These states mix with conventional \( \overline{q}q \) states to yield physical mesons. As a result we obtain weak mixing in the isospinor and isovector pseudoscalar sectors hence the \( \eta(1380) \) and \( K(940) \) states arise as mainly \( \overline{q}q \) states whereas the \( \pi(1400) \) and \( K(1460) \) turn out to be mainly tetraquarks. In contrast, scalars in these isotopic sectors are strongly mixed thus the \( a_0(980) \) and \( \kappa(900) \) states are strong admixtures of \( \overline{q}q \) and tetraquark states. Concerning isoscalar pseudoscalars we also find \( \eta(1440) \) and \( \eta(958) \) as strong admixtures of \( \overline{q}q \) and tetraquark whereas \( \eta(547) \) and \( \eta(1295) \) turn out to be mainly \( \overline{q}q \) and tetraquark states respectively. As for isoscalar scalar mesons we find the \( f_0(600) \) and \( f_0(1370) \) as mainly \( \overline{q}q \) and tetraquark states whereas the \( f_0(980) \) and \( f_0(1500) \) arise as strong admixtures of \( \overline{q}q \) and tetraquark states. We remark that results for isosinglet scalars are expected to get modified by the mixing of pure \( \overline{q}q \) and four-quark-structured mesons with the lowest lying scalar glueball field.

The idea of a chiral tetraquark nonet is implemented in the framework of an effective model written in terms of “standard” (\( \overline{q}q \)-like ground states) meson fields and “non-standard” (four-quark like states) fields denoted as \( \hat{\Phi} = S + iP \) and \( \hat{\Phi} = S + i\hat{P} \) respectively. Here, \( S \), \( \hat{S} \) scalar and \( P \), \( \hat{P} \) pseudoscalar nonets are given by

\[
S = \begin{pmatrix}
\frac{S_+ + S_0}{\sqrt{2}} & S^+ & Y^+ \\
S^- & \frac{S_0 - S^0}{\sqrt{2}} & Y^0 \\
Y^- & \bar{Y}^0 & S_s
\end{pmatrix};
\]

\[
P = \begin{pmatrix}
\frac{H_{+} + P^0}{\sqrt{2}} & P^+ & X^+ \\
\frac{H_{-} + P^0}{\sqrt{2}} & X^0 & H_s
\end{pmatrix},
\]

\[
\hat{S} = \begin{pmatrix}
\frac{\bar{S}_+ + S_0}{\sqrt{2}} & \bar{S}^+ & \bar{Y}^+ \\
\bar{S}^- & \frac{S_0 - S^0}{\sqrt{2}} & \bar{Y}^0 \\
\bar{Y}^- & \bar{Y}^0 & \bar{S}_s
\end{pmatrix};
\]

\[
\hat{P} = \begin{pmatrix}
\frac{\bar{H}_{+} + \bar{P}^0}{\sqrt{2}} & \bar{P}^+ & \bar{X}^+ \\
\frac{\bar{H}_{-} + \bar{P}^0}{\sqrt{2}} & \bar{X}^0 & \bar{H}_s
\end{pmatrix}.
\]

Next we implement the idea of a chiral nonet around 1.4 GeV using an effective linear Lagrangian in terms of the four-quark structured fields \( \hat{\Phi} \) with chiral symmetry realized linearly and directly (\( \mu^2 > 0 \))

\[
\mathcal{L}(\hat{\Phi}) = \mathcal{L}_{sym}(\hat{\Phi}) + \mathcal{L}_{SB}^{(1)}(\hat{\Phi})
\]

\[
\mathcal{L}_{sym}(\hat{\Phi}) = \frac{1}{2} \left( \partial_\mu \hat{\Phi} \partial^\mu \hat{\Phi}^\dagger \right) - \frac{\mu^2}{2} \left( \hat{\Phi} \hat{\Phi}^\dagger \right) - \frac{\tilde{\lambda}}{4} \left( (\hat{\Phi} \hat{\Phi}^\dagger)^2 \right)
\]

\[
\mathcal{L}_{SB}^{(1)}(\hat{\Phi}) = -\frac{\tilde{\lambda}}{4} \left( \mathcal{M}(\hat{\Phi} \hat{\Phi}^\dagger + \hat{\Phi}^\dagger \hat{\Phi}) \right)
\]

Here, \( \tilde{\mu} \) sets the scale at which four-quark states lie, it is expected to be slightly below 1.4 GeV. Although some of the fields in \( \hat{\Phi} \) have the same quantum numbers as the vacuum they do not acquire vacuum expectation values (vev’s) since we require a direct realization of chiral symmetry. The symmetry breaking term in Eq. 4 requires some explanation. This is an explicit breaking term quadratic in the fields and proportional to a quadratic quark mass matrix. The point is that in a quiral expansion the quark mass matrix has a non-trivial flavor structure and enters as
an external scalar field. In the case of a four-quark nonet there must be breaking terms with the appropriate flavor structure. This matrix is constructed according to the flavor structure of the quark mass matrix as
\[(M_Q)_{ab} = \frac{1}{2} \epsilon_{cdef} (M_q^d)^c (M_q^e)^f,
\]
where \(M_q = \text{Diag}(m, m, m_s)\) stands for the conventional quark mass matrix in the good isospin limit which we will consider henceforth. Explicitly we obtain \(M_Q = \text{Diag}(m m_s, m m_s, m_s^2)\). This structure yields to pure 4q-structured fields an inverted spectrum with respect to conventional states. A word of caution is necessary concerning the notation in Eq. (3). The matrix field for four-quark states has a schematic quark content
\[\hat{\Phi} \sim \begin{pmatrix}
\overline{qq} \overline{ss} \\
\overline{qq} \overline{ss} \\
\overline{qq} \overline{qs}
\end{pmatrix},\]
where \(q\) denote a \(u\) or \(d\) quark. The subindex \(s\) and \(ns\) in these fields refer to the notation for \(SU(3)\) matrices in Eq. (1) but do not correspond with the hidden quark-antiquark content, e.g. \(\hat{S}_s \sim \overline{qq} \overline{ss}\) and \(\hat{S}_s \sim \overline{qq} \overline{qq}\).

Conventional \(\overline{q}q\)-structured fields are introduced in a chirally symmetric way with chiral symmetry spontaneously broken. We introduce also an instanton inspired breaking for the \(U_A(1)\) symmetry. Notice that the determinantal interaction is appropriate for \(\overline{q}q\)-structured fields but not for four-quark fields since this is a six-quark interaction
\[L(\Phi) = L_{\text{sym}}(\Phi) + L_A + L_{SB}^{(2)}(\Phi)\]
\[L_{\text{sym}}(\Phi) = \frac{1}{2} \langle \partial_\mu \Phi \partial^\mu \Phi^\dagger \rangle - \frac{\mu^2}{2} \langle \Phi \Phi^\dagger \rangle - \frac{\lambda}{4} \langle (\Phi \Phi^\dagger)^2 \rangle \]
\[L_A = -B \langle \text{det} \Phi + \text{det} \Phi^\dagger \rangle, \quad L_{SB}^{(2)}(\Phi) = \frac{b_0}{\sqrt{2}} \langle M_q (\Phi + \Phi^\dagger) \rangle\]

This is just the model used in [4, 5] except for an OZI-forbidden interaction whose corresponding coupling is consistent with zero when included in the present context. The Lagrangian
\[L_{\text{sym}}(\Phi, \hat{\Phi}) = L_{\text{sym}}(\Phi) + L_{\text{sym}}(\hat{\Phi})\]
is invariant under the independent chiral transformations
\[\Phi \rightarrow U_L(\alpha_L) \Phi U_R^\dagger(\alpha_R), \quad \hat{\Phi} \rightarrow \hat{\Phi}\]
\[\Phi \rightarrow \Phi, \quad \hat{\Phi} \rightarrow \hat{U}_L(\hat{\alpha}_L) \hat{\Phi} \hat{U}_R^\dagger(\hat{\alpha}_R).\]
i.e. it has \((U(3) \times U(3))^2\) symmetry. This symmetry is explicitly broken down to \(SU(3)_A \times SU(3)_V\) by the interaction
\[L_{c^2} = -\frac{c^2}{4} \langle \Phi \Phi^\dagger + \Phi \Phi^\dagger \rangle.\]

Further sources of breaking come from the anomaly term and quark mass terms in Eqs. (4, 6). Finally, since we are considering quark masses entering as external scalar fields we also consider the following terms
\[L_{SB}^{(3)} = \frac{b_0}{\sqrt{2}} \langle M_q (\hat{\Phi} + \hat{\Phi}^\dagger) \rangle + \frac{\hat{d}}{\sqrt{2}} \langle M_q (\hat{\Phi} + \hat{\Phi}^\dagger) \rangle.\]

A last term proportional to \(\langle M_q (\Phi + \Phi^\dagger) \rangle\) can also be added without altering the conclusions of this work. The linear terms in \(\Phi\) induce scalar-to-vacuum transitions which instabilizes the vacuum. We shift to the true vacuum, \(S \rightarrow S - V\) where \(V\) stands for the vacuum expectation values of \(S\) which we denote as \(V = \text{diag}(a, a, b)\). This mechanisms generates new meson mass terms and interactions. In particular, the shift generates linear terms in \(\hat{\Phi}\) which cancel against the linear terms in Eq. (10). Here, we present results for meson masses, details of the calculations and results for interactions will be published elsewhere [13].
For the isodoublets and isotriplets, the interaction term $\mathcal{L}_i$ mix up $\eta Q$ and four-quark states. We define the diagonal isotriplet pseudoscalar fields as

$$
\begin{pmatrix}
\pi \\
\eta
\end{pmatrix} = 
\begin{pmatrix}
\cos(\theta_1) & -\sin(\theta_1) \\
\sin(\theta_1) & \cos(\theta_1)
\end{pmatrix}
\begin{pmatrix}
\rho \\
\tilde{\rho}
\end{pmatrix}.
$$

For the isotriplet scalar sector we denote the physical fields as $\phi, A$ and the corresponding mixing angle is denoted by $\phi_1$. For the isodoublets we denote the physical fields as $K, \bar{K}$ ($\kappa, \bar{\kappa}$), and the mixing angle as $\theta_{1/2}$ ($\phi_{1/2}$) for pseudoscalars (scalars).

The isosinglet sectors are more involved due to the effects coming from the $U_A(1)$ anomaly which when combined with the spontaneous breaking of chiral symmetry produces mixing among four different fields. The mass Lagrangian for the isoscalar pseudoscalar sector reads

$$
\mathcal{L}_H = -\frac{1}{2} \langle H | M_H | H \rangle
$$

where

$$
|H\rangle = 
\begin{pmatrix}
H_{ns} \\
H_s \\
\bar{H}_{ns} \\
\bar{H}_s
\end{pmatrix},
M_H = 
\begin{pmatrix}
m_H^2 & 0 & 0 & 0 \\
0 & m_{H_s}^2 & 0 & 0 \\
0 & 0 & m_{H_{ns}}^2 & 0 \\
0 & 0 & 0 & m_{H_s}^2
\end{pmatrix},
$$

with

$$
m_{H_{ns}}^2 = \mu^2 - 2Bb + \lambda a^2,
\quad m_{H_s}^2 = \tilde{\mu}^2 + \tilde{c}nm_s,
\quad m_{H_{ns}}^2 = \mu^2 + \tilde{c}m,
\quad m_{H_s}^2 = \tilde{\mu}^2 + \tilde{c}m,
$$

For the isoscalar-scalar sector we obtain a similar structure. In principle, these matrices are diagonalized by a general rotation in $O(4)$ containing six independent parameters. However, modulo corrections of the order $m(m_s - m)/\tilde{\mu}^2$, it can be shown that they are actually diagonalized by a rotation in the subgroup $SO(2) \otimes SO(2) \otimes SO(2)$ and a straightforward analytic solution depending on three angles is obtained for the rotation matrix. Explicitly, under this approximation the physical pseudoscalar fields are given as

$$
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} =
\begin{pmatrix}
\alpha, \beta & \gamma, \delta & \gamma', \delta' \equiv s_{\alpha, \beta} \sin(\gamma, \delta) & \gamma, \delta \equiv s_{\alpha, \beta} & \gamma, \delta \equiv s_{\alpha, \beta} & \gamma, \delta \equiv s_{\alpha, \beta} \\
\alpha, \beta' & \gamma, \delta' & \gamma, \delta' & \gamma, \delta' & \gamma, \delta' & \gamma, \delta'
\end{pmatrix}
\begin{pmatrix}
H_{ns} \\
H_s \\
\bar{H}_{ns} \\
\bar{H}_s
\end{pmatrix},
$$

where $c_\alpha \equiv \cos \alpha$, $s_\alpha \equiv \sin \alpha$. Similar relations are also valid for the scalar sector. We denote as $\sigma$, $f_0$, $\sigma$, $f_0$ to the physical isosinglet scalar fields and $\gamma$, $\delta$ and $\gamma'$, $\delta'$ to the mixing angles analogous to $\alpha$, $\beta$, $\beta'$ respectively.

Finally, a calculation of the axial currents allow us to relate the vacuum expectation values of scalars to the weak decay constants of pseudoscalars as

$$
a = \frac{f_\pi}{\sqrt{2} \cos \theta_1},
\quad a + b = \frac{\sqrt{2} f_K}{\cos \theta_{1/2}},
$$

There are eight free parameters in the model which are relevant to meson masses: $\{\mu_1^2, \lambda, B, c^2, a, b, \tilde{\mu}_1^2, \tilde{\mu}_{1/2}^2\}$, where $\tilde{\mu}_{1/2}^2 \equiv \tilde{\mu}^2 + \tilde{c}m(m + m_s)$ and $\tilde{\mu}_1^2 \equiv \tilde{\mu}^2 + \tilde{c}nm_s$. Our input are the physical quantities listed in Table I, namely, the masses for $\pi(137)$, $a_0(980)$, $K(495)$, $\eta(547)$, $\eta'(958)$, $\eta(1295)$ in addition to the weak decay constants $f_\pi$ and $f_K$. Uncertainties listed in this table correspond to the measured values in the case of the isosinglets $K$. Since we are working in the good isospin limit we use the experimental deviations from this limit for the uncertainties in the masses of isotriplets and isodoublets. Using these values we fix the parameters to the values also listed in Table I. In Table II we show the predictions of the model for the remaining meson masses and mixing angles and the experimental values for these quantities when available. The quark content of mesons corresponding to the central values of these mixing angles are shown in Figs. 11, 12, 13. In Fig. 1 we show results for the light isodoublets and isotriplets. Heavy mesons
have the opposite quark content. We obtain pions and kaons as mainly \(q\bar{q}\) states whereas the heavy fields \(\pi(1300), K(1460)\) arise as mainly tetraquark states. In contrast, in the scalar sector isotriplets and isodoublets get strongly mixed and the physical mesons turn out to have almost identical amounts of \(q\bar{q}\) and four-quark content. In Fig. 2 we show results for isosinglet pseudoscalars. Here, in the case of \(\eta(547)\) we obtain also a small four-quark content, the \(\eta(1295)\) being almost a four-quark state. However, the \(\eta'(958)\) and \(\eta(1440)\), turn out to be strong admixtures of \(q\bar{q}\) and four-quark states with almost equal amounts of them. As to the isosinglet scalars Fig. 3 shows that the sigma meson (\(f_0(600)\)) arises as mainly \(q\bar{q}\) state and the opposite quark content is carried by the \(f_0(1370)\) which turns out to be mainly a four quark state. Finally, the \(f_0(980)\) turns out to be have roughly a 40% content of \(\pi\pi\) (more explicitly \(\pi\pi\) ) and 60% of \(\pi\pi\pi\pi\) whereas the \(f_0(1500)\) is composed 60% of \(\pi\pi\) (more explicitly \(\pi\pi\) ) and almost 40% of \(\pi\pi\pi\pi\). We stress again that quark content of isoscalar scalar mesons shown in Fig. 3 are expected to be modified by the inclusion of the scalar glueball in the present context although we do not expect strong modifications in the case of the \(f_0(600)\) which is far from the energy region around 1.6 GeV where this state is expected.

Summarizing, in this work we point out the possibility that scalar and pseudoscalar mesons below 1.5 GeV be admixtures of conventional \(q\bar{q}\) and tetraquark states. This conjecture is risen by the key observation that beyond the light scalars lying below 1 GeV, there are nine states around 1.4GeV which, when considered as the members of a nonet, exhibit a slightly inverted mass spectrum as compared with a conventional \(q\bar{q}\) nonet. Furthermore this nonet lie at an energy scale compatible with that of tetraquarks according to the linear rising of the mass of a hadron with the number of constituents. There is a nonet of pseudoscalar states at the same mass which suggest these nonets form a chiral nonet. We implement this idea in the framework of a chiral model. As a result we obtain a meson spectrum consistent with the measured pseudoscalar and scalar meson spectrum below 1.5 GeV.

Acknowledgments

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Table I

| Parameter | Fit       |
|-----------|-----------|
| m_π       | 137.3 ± 2.3 MeV | 1257.5 ± 61.3 |
| m_η       | 984.3 ± 0.9 MeV  | 1206.2 ± 142.3 |
| f_π       | 92.42 ± 3.53 MeV | 1012.3 ± 75.9 |
| m_K       | 495.67 ± 2.00 MeV | 68.78 ± 4.47  |
| m_A       | 113.0 ± 1.3 MeV  | 104.8 ± 2.6   |
| m_κ       | 547.30 ± 0.12 MeV | -2.16 ± 0.28 |
| m_κ'      | 957.86 ± 0.14 MeV | 31.8 ± 5.9   |
| m_η'      | 1297.0 ± 2.8 MeV | 0.490 ± 0.107 |

Table I  Input used to fix the parameters entering the model and their values.

Table II

| Mass (MeV) | Prediction | Identification | Exp     |
|------------|------------|----------------|---------|
| m_π        | 1322.6 ± 32.4 | π(1300)      | 1300 ± 200[9] |
| m_K        | 1293.1 ± 3.5  | K(1460)      | 1400 − 1460[9] |
| m_A        | 1417.3 ± 51.0 | a_0(1450)    | 1474 ± 19[9] |
| m_κ        | 986.2 ± 19.1  | κ(900)       | 750 − 950[10, 11, 12] |
| m_κ'       | 1413.9 ± 76.4 | K_0*(1430)   | 1429 ± 4 ± 5[9] |
| m_σ        | 1394.0 ± 61.9 | η(1440)      | 1400 − 1470[9] |
| m_σ'       | 380.6 ± 91.0  | f_0(600) or σ| 478 ± 35[10] |
| m_f_0      | 1022.4 ± 25.6 | f_0(980)     | 980 ± 10[9] |
| m_f_0'     | 1284.7 ± 15.3 | f_0(1370)    | 1200 − 1500[9] |
| m_f_0''    | 1447.7 ± 84.6 | f_0(1500)    | 1500 ± 10[9] |
| θ_1        | 18.16° ± 4.34° |
| θ_1/2      | 22.96° ± 4.84° |
| φ_1        | 39.8° ± 4.5°   |
| φ_1/2      | 46.7° ± 9.5°   |
| α          | 53.4° ± 0.8°   |
| β          | 23.9° ± 5.1°   |
| β'         | 43.6° ± 7.2°   |
| γ          | -9.11° ± 0.49° |
| δ          | 21.45° ± 6.49° |
| δ'         | 51.36° ± 8.35° |

Table II  Predictions of the model for meson masses and mixing angles.
FIG. 1: Quark content of light isotriplet and isodoublets.

FIG. 2: Quark content of light isosinglet pseudoscalars.
FIG. 3: Quark content of light isosinglet scalars.