CP Violation in the Higgs Sector and a Phase Transition in the MSSM

Koichi Funakubo,1,∗ Shuichiro Tao2,∗∗ and Fumihiko Toyoda3,∗∗∗

1Department of Physics, Saga University, Saga 840-8502, Japan
2Department of Physics, Kyushu University, Fukuoka 812-8581, Japan
3Kyushu School of Engineering, Kinki University, Iizuka 820-8555, Japan

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We investigate the electroweak phase transition in the presence of a large CP violation in the squark sector of the MSSM. When the CP violation is large, scalar-pseudoscalar mixing of the Higgs bosons occurs, and a large CP violation in the Higgs sector is induced. It, however, weakens the first-order phase transition before the mixing reaches its maximal level. Even when the CP violation in the squark sector is not so large that the phase transition is strongly first order, the phase difference between the broken and symmetric phase regions grows to $O(1)$, which leads to successful baryogenesis when the charged Higgs bosons are light.

§1. Introduction

The baryon asymmetry of the universe (BAU) is one of the most obvious facts regarding the makeup of the universe, and its explanation has been a longstanding problem in astrophysics.1) To explain the light-element abundances within the framework of the standard big-bang nucleosynthesis, it is required that 2)

$$\frac{n_B}{s} = (0.21 - 0.90) \times 10^{-10}. \tag{1.1}$$

It is well known that in order to obtain this asymmetry starting from a baryon-symmetric state, three requirements must be satisfied: baryon number violation, $C$ and CP violation, and departure from equilibrium.3) In general, electroweak theories satisfy baryon number violation through the chiral anomaly and may be capable of describing the BAU.4) In the minimal standard model (MSM), the main source of CP violation comes from the phase $\delta_{KM}$ in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Although this phase is able to account for the experimentally observed CP violation in neutral $K$-mesons and, as recently measured, in the $B_d$ system, it has been shown that it is not possible to generate sufficient BAU through $\delta_{KM}$. Furthermore, the strength of the phase transition is so weak in the MSM in the case that the Higgs scalar is heavier than 115 GeV5),6) that the universe is approximately in equilibrium when the process of baryon number change is in effective.

In the context of the supersymmetric (SUSY) extension of the MSM, it has been

∗) E-mail: funakubo@cc.saga-u.ac.jp
∗∗) E-mail: tao@higgs.phys.kyushu-u.ac.jp
∗∗∗) E-mail: ftoyoda@fuk.kindai.ac.jp
pointed out that in the presence of a light stop, the electroweak phase transition (EWPT) can be strong enough for baryogenesis to take place.\textsuperscript{7} Moreover, SUSY models contain many complex parameters as new \( CP \)-violating sources in addition to \( \delta_{\text{KM}} \): the Higgs bilinear term, \( \mu \), and soft SUSY breaking terms (gaugino masses and scalar trilinear couplings).\textsuperscript{8} In addition to these complex parameters, a non zero relative phase of the expectation values of the two Higgs doublets \( \theta \) might be induced by radiative and finite-temperature effects, although it vanishes at the tree level. Without any complex parameter, a phase \( \theta \) could be induced by loop effects of SUSY particles. This idea of spontaneous \( CP \) violation was studied at zero temperature in the minimal supersymmetric standard model (MSSM), and it was pointed out that there inevitably appears a pseudoscalar boson as light as \( \sim O(1 \text{ GeV}) \).\textsuperscript{9} The same mechanism at finite temperatures was suggested in Ref. 10) and extensively studied by several people, including two of the present authors.\textsuperscript{11} They found that \( \theta \) could be large only in the transient region between the symmetric and broken phase regions, with a pseudoscalar Higgs whose mass satisfies \( m_A \lesssim 85 \text{ GeV} \). This mechanism was considered appealing because such a large \( \theta \) can result in a sufficiently large BAU, and it does not induce a large \( \theta \) in the vacuum, which is consistent with the bound obtained from the neutron electric dipole moment (nEDM). This scenario, however, is now excluded, because the mass of the pseudoscalar must satisfy \( m_A \geq 90.1 \text{ GeV} \).\textsuperscript{2}

A nonzero \( \theta \) is also induced by complex parameters in the MSSM. However, the magnitude of explicit \( CP \) violation is constrained by nEDM measurements. For example, the physical \( CP \) phase relevant to the EDM must be as small as \( O(10^{-3}) \) when the masses of the SUSY particles are on the order of the weak scale, while if these phases are \( O(1) \), these masses will be heavier than 1 TeV.\textsuperscript{8,12} Recently it was observed that the spectrum and interactions of the neutral Higgs bosons are affected by a large explicit \( CP \) violation in the third generation of the squark sector, which is not restricted by the nEDM constraints.\textsuperscript{13} The authors of Ref. 13) found that when the imaginary part of the product of \( \mu \) and the Higgs-squark trilinear coupling is large, the lightest Higgs boson, \( H_1 \), which is composed of the scalar and pseudoscalar Higgs fields, becomes much lighter than the present bound, 115 GeV, but it is difficult to observe, because its couplings to the gauge boson and to the bottom quarks are very small.

One may think that this \( CP \) violation can generate sufficient BAU in the MSSM with parameter sets consistent with experiments. Our purpose is to investigate the effects of explicit \( CP \) violation on the EWPT in the MSSM and to evaluate the magnitude of \( CP \) violation relevant to electroweak baryogenesis, which should be measured at the transition temperature. The organization of the paper is as follows. In §2, we formulate the effective potential of the Higgs fields, including the one-loop corrections from the gauge bosons and the third generation of quarks and squarks, both at zero and finite temperatures. The masses of the three neutral Higgs bosons and the charged Higgs boson are defined as the derivatives of the effective potential at the zero-temperature vacuum. The mass formulas are almost the same as those given in Ref. 13), with the most significant difference being the inclusion of the gauge boson contributions. In §3, we study the EWPT for parameter sets that are consistent with the mass bounds on the neutral lightest and charged Higgs bosons,
in the absence of explicit \( CP \) violation. Next, we introduce the phase of the trilinear coupling, examine how the strength of the phase transition changes, and evaluate the magnitude of the \( CP \) violation relevant to electroweak baryogenesis. Section 4 is devoted to concluding remarks. We summarize the formulas for the Higgs masses in the Appendices.

\section{Effective potential of the MSSM}

We consider the MSSM that has the superpotential

\[ W = \epsilon_{ij} \left( f_{AB}^{(d)} H_d^i Q_j^A D_B - f_{AB}^{(u)} H_u^i Q_j^A U_B - \mu H_d^i H_u^i \right). \]

In addition to the supersymmetric Lagrangian, the low-energy MSSM contains the soft-SUSY-breaking terms

\[ \mathcal{L}_{\text{soft}} = - \tilde{m}_d^{2} \Phi^d \Phi_d^\dagger - \tilde{m}_u^{2} \Phi^u \Phi_u^\dagger + \epsilon_{ij} \left( \tilde{m}_{ij} \Phi^d_i \Phi^d_j + \text{h.c.} \right) - m_{\tilde{g}_{AB}^{A}} \tilde{g}_{AB}^{A} \tilde{g}_{AB}^{A} - m_{\tilde{A}_{AB}^{A}} \tilde{A}_{AB}^{A} \tilde{A}_{AB}^{A} - \epsilon_{ij} \left[ \left( f^{(d)} A^{(d)} \right)_{AB} \tilde{f}^{(d)}_{ij} \tilde{d}_{AB}^{ij} - \left( f^{(u)} A^{(u)} \right)_{AB} \tilde{f}^{(u)}_{ij} \tilde{u}_{AB}^{ij} + \text{h.c.} \right]. \]

We calculate the effective potential of the Higgs fields by taking into account the one-loop contributions from the gauge bosons and the third generation of quarks and squarks. We consider the gauginos to be heavy enough to decouple, so that the most dangerous contribution to nEDM from the gluino is negligible. The correction from the leptons, the other quark and squarks can be ignored, because of their small Yukawa couplings to the Higgs fields. Now, the effective potential at zero temperature is given by

\[ V_{\text{eff}}(\Phi_d, \Phi_u; T = 0) = V_0(\Phi_d, \Phi_u) + \Delta V(\Phi_d, \Phi_u), \]

where \( V_0(\Phi_d, \Phi_u) \) is the tree-level potential,

\[ V_0 = \frac{1}{2} \sum_{q=t,b} \sum_{j=1,2} \left( \tilde{m}_{ij}^{2} \epsilon_{ij} \Phi^d_i \Phi^d_j + \text{h.c.} \right) + \frac{g_2^2 + g_1^2}{8} \left( \Phi^d_i \Phi^d_d - \Phi^d_i \Phi^u_u \right)^2 + \frac{g_2^2}{2} \left| \Phi^d_i \Phi^u_u \right|^2, \]

and \( \Delta V(\Phi_d, \Phi_u) \) is the one-loop correction written as

\[ \Delta V = \frac{N_C}{32 \pi^2} \sum_{q=t,b} \sum_{j=1,2} \left( \tilde{m}_{ij}^{2} \epsilon_{ij} \right)^2 \left( \log \frac{\tilde{m}_{ij}^{2}}{M^2} - \frac{3}{2} \right) - 2 \left( \tilde{m}_{ij}^{2} \right)^2 \left( \log \frac{\tilde{m}_{ij}^{2}}{M^2} - \frac{3}{2} \right) \]

\[ + \frac{3}{64 \pi^2} \left[ \left( \tilde{m}_{Z}^{2} \right)^2 \left( \log \frac{\tilde{m}_{Z}^{2}}{M^2} - \frac{3}{2} \right) + 2 \left( \tilde{m}_{W}^{2} \right)^2 \left( \log \frac{\tilde{m}_{W}^{2}}{M^2} - \frac{3}{2} \right) \right]. \]

Here \( \tilde{m}_{ij}^{2}, \tilde{m}_{ij}^{2}, \tilde{m}_{ij}^{2}, \tilde{m}_{ij}^{2}, \tilde{m}_{ij}^{2} \) and \( \tilde{m}_{ij}^{2} \) are field-dependent masses defined by (A-1), (A-2), (A-3), (A-5) and (A-6), respectively, and \( M \) denotes the renormalization scale, which we choose such that the loop corrections vanish at the vacuum. The expression (2.5)
is the same as the one-loop correction in Ref. 13), except for our inclusion of the gauge boson contributions, which strengthen the first-order EWPT.

It is well known that the masses of the Higgs bosons receive large corrections from the loops of the top quark and squarks. Here, the masses of the Higgs bosons are defined by the second derivative of the effective potential at the vacuum. To evaluate them, we parameterize the Higgs fields by the vacuum \((v_d, v_u, \theta)\) and fluctuations around it as

\[
\Phi_d = \left( \frac{1}{\sqrt{2}} (v_d + h_d + i a_d) \right), \quad \Phi_u = e^{i\theta} \left( \frac{1}{\sqrt{2}} (v_u + h_u + i a_u) \right).
\]

(2.6)

In the following, we represent the quantities evaluated at the vacuum by \(\langle \cdot \rangle\), that is, evaluated with all the fluctuations being set to zero. Requiring that the first derivatives of the effective potential with respect to the neutral Higgs fields evaluated at the vacuum vanish, that is,

\[
\langle \frac{\partial V_{\text{eff}}}{\partial h_d} \rangle = 0, \quad \langle \frac{\partial V_{\text{eff}}}{\partial a_d} \rangle = 0,
\]

(2.7)

we can express \(m_d^2\) and \(m_u^2\) in (2.4) in terms of \(\text{Re}(m_3^2 e^{i\theta})\), \(\tan \beta = v_u/v_d\), and the particle masses from the first equation, and we can express \(\text{Im}(m_3^2 e^{i\theta})\) in terms of the particle masses from the second equation. Now the mass-squared matrix of the neutral Higgs scalars is expressed as

\[
\begin{pmatrix}
M_S^2 & M_{SP}^2 \\
(M_{SP}^2)^T & M_P^2
\end{pmatrix},
\]

(2.8)

where

\[
(M_S^2)_{11} = \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d^2} \rangle, \quad (M_S^2)_{22} = \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_u^2} \rangle, \quad (M_S^2)_{12} = (M_S^2)_{21} = \langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d \partial h_u} \rangle,
\]

(2.9)

and

\[
(M_P^2)_{11} = \langle \frac{\partial^2 V_{\text{eff}}}{\partial a_d^2} \rangle, \quad (M_P^2)_{22} = \langle \frac{\partial^2 V_{\text{eff}}}{\partial a_u^2} \rangle, \quad (M_P^2)_{12} = (M_P^2)_{21} = \langle \frac{\partial^2 V_{\text{eff}}}{\partial a_d \partial a_u} \rangle.
\]

where each component is given in Appendix B. It can be shown that the pseudoscalar elements are factorized as

\[
M_P^2 = (M_P^2)_{12} \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix},
\]

(2.10)

so that, the unphysical Goldstone mode \(G^0\) can be extracted as

\[
\begin{pmatrix} a_d \\ a_u \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^0 \\ a \end{pmatrix}.
\]

(2.11)
Hence, the mass-squared eigenvalues of the neutral Higgs bosons are the eigenvalues of the matrix

\[
\mathcal{M}_0^2 \equiv \begin{pmatrix} (\mathcal{M}^2_S)_{11} & (\mathcal{M}^2_S)_{12} & \frac{1}{\cos \beta} (\mathcal{M}^2_{SP})_{12} \\ \frac{1}{\cos \beta} (\mathcal{M}^2_{SP})_{11} & (\mathcal{M}^2_S)_{22} & \frac{1}{\sin \beta} (\mathcal{M}^2_{SP})_{21} \\ \frac{1}{\sin \beta} (\mathcal{M}^2_{SP})_{12} & \frac{1}{\sin \beta} (\mathcal{M}^2_{SP})_{21} & (\mathcal{M}^2_p)_{12} \end{pmatrix}.
\] (2.12)

In the presence of CP violation, which induces scalar-pseudoscalar mixing, the pseudoscalar is no longer a mass eigenstate. In what follows, we use the mass of the charged Higgs boson as an input parameter, instead of the pseudoscalar. The mass matrix of the charged Higgs scalar has the form

\[
\mathcal{M}^2 = \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial \phi_d^+ \partial \phi_u} \right\rangle \begin{pmatrix} \tan \beta & 1 \\ 1 & \cot \beta \end{pmatrix}.
\]

Similarly, we can extract the Goldstone mode so that the mass of the charged scalar is given by

\[
m^2_{H^\pm} = \frac{1}{\sin \beta \cos \beta} (\mathcal{M}^2_\pm)_{12}.
\] (2.13)

The detailed form of the mass is given in Appendix C, with which we can express Re\(m^2_3 e^{i\theta}\) in terms of \(m^2_{H^\pm}\).

The effective potential at finite temperature contains the temperature-dependent corrections

\[
V_{\text{eff}}(\Phi_d, \Phi_u; T) = V_{\text{eff}}(\Phi_d, \Phi_u; T = 0) + \Delta_T V(\Phi_d, \Phi_u; T),
\] (2.14)

where

\[
\Delta_T V = \frac{T^4}{2\pi^2} \left[ 6I_B \left( \frac{\bar{m}_W^2(\Phi)}{T^2} \right) + 3I_B \left( \frac{\bar{m}_Z^2(\Phi)}{T^2} \right) \right]
+ 6 \cdot \frac{T^4}{2\pi^2} \sum_{q=t,b} \left[ -2I_F \left( \frac{\bar{m}_q^2(\Phi)}{T^2} \right) + \sum_{j=1,2} I_B \left( \frac{\bar{m}_{tq}^2(\Phi)}{T^2} \right) \right].
\] (2.15)

Here, the functions \(I_B(a^2)\) and \(I_F(a^2)\) are defined by

\[
I_{B,F}(a^2) = \int_0^\infty dx \, x^2 \log \left( 1 \mp e^{-\sqrt{x^2 + a^2}} \right).
\] (2.16)

The function \(I(a^2)\) yields a \(a^3\)-term with a negative coefficient when expanded for \(a^2 \ll 1\). This qualitatively explains why the EWPT becomes first order with bosons whose field-dependent mass-squared values behave as \(\bar{m}^2(v) \sim v^2\) for small \(v^2\). Because of its large Yukawa coupling, a stop with a vanishing soft mass causes the first-order EWPT to be stronger. In the following, we do not use the high-temperature expansion \((m^2/T^2 \ll 1)\) but, rather, give the results of a numerical calculation of the integrals in order to study the EWPT quantitatively.

An important constraint on the finite-temperature effective potential comes from the requirement that the sphaleron process decouples immediately after the EWPT,
in order for the baryon asymmetry produced at the EWPT not to be washed out. If we denote the minimum of the effective potential as \((v_d, v_u, \theta)\), the first-order EWPT is characterized by the degenerate minima at the transition temperature \(T_C\) \((v_C \cos \beta_C, v_C \sin \beta_C, \theta_C)\) in the broken phase region and \((0, 0, \theta_0)\) in the symmetric phase region. The sphaleron decoupling condition can then be written as\(^{16}\)

\[
\frac{v_C}{T_C} > 1. \tag{2.17}
\]

The difference between \(\theta_C\) and \(\theta_0\) is crucial to determine the total amount of baryon asymmetry produced at the EWPT. The profile of the bubble wall created at the first-order EWPT is derived from the equation of motion for the gauge-Higgs system with the effective potential at \(T_C\).\(^{17}\) Then, the boundary conditions for the Higgs fields are provided by \((v_C \cos \beta_C, v_C \sin \beta_C, \theta_C)\) and \((0, 0, \theta_0)\). Because the bubble wall profile smoothly interpolates between the degenerate minima, the phase \(\theta\) varies from \(\theta_0\) to \(\theta_C\) at the phase boundary. If there is no local minimum of the effective potential at \(T_C\) that leads to transitional \(CP\) violation,\(^{11}\) we expect the phase to monotonically vary, so that the baryon number generated would be characterized by \(\theta_C - \theta_0\). Once we choose the phase convention such that \(\theta = 0\) at the zero-temperature vacuum, both \(\theta_C\) and \(\theta_0\) can be unambiguously calculated from the effective potential in the presence of the explicit \(CP\) violation in the squark sector. As seen from (2.7), \(\text{Im}(m_3^2)\) becomes nonzero, and \(\theta_0 = -\delta\) with \(\delta \equiv \text{Arg}(m_3^2)^{10}\). Then \(\theta_C + \delta\) is another important quantity that we must evaluate. As shown in various works, when \(\theta_C + \delta\) is \(\mathcal{O}(1)\), sufficient BAU is generated by the charge transport mechanism.\(^{18}\)

### §3. Numerical results

There are many parameters in the model, some of which can be fixed by requiring the vacuum at zero temperature to be that prescribed by \(v_0 = 246\ \text{GeV}, \tan \beta\) and \(\theta\). In practice, we determine \(m_1^2\) and \(m_2^2\) by use of the tadpole condition (2.7). \(\text{Re}(m_3^2 e^{i\theta})\) can also be determined once the charged Higgs mass \(m_{H^\pm}\) is given by (C.1). In order to evaluate the right-hand sides of these equations, we must prepare \(\mu\) and the parameters in the squark sector: soft masses \(m_{\tilde{q}}, m_{\tilde{t}_R}\) and \(m_{\tilde{b}_R}\), and trilinear couplings \(A_t\) and \(A_b\). Since the \(CP\) symmetry is violated by nonzero \(\text{Im}(\mu A_t)\) and \(\text{Im}(\mu A_b)\), we take \(\mu\) to be real and regard the phases of the \(A\) parameters as inputs. The EWPT is strongly first order when the soft mass of squark is very small, as noted in the last section. One usually chooses \(m_{\tilde{t}} \simeq 0\) and \(m_{\tilde{q}} = \mathcal{O}(100)\ \text{GeV}\) to avoid too large corrections to the \(\rho\) parameter. For definiteness, we take \(m_{\tilde{t}_R} = 10\ \text{GeV}\), \(m_{\tilde{b}_R} = 100\ \text{GeV}\), and several values of \(m_{\tilde{q}}\) larger than 1 TeV. As seen from the scalar-pseudoscalar mixing elements of the mass-squared matrix (2.12), the \(CP\) violation induced in the Higgs sector is enhanced for larger \(\text{Im}(\mu A_t)\). In the following, we choose large values for \(\mu\) and \(|A_t|\), but to avoid a color-and-charge-breaking vacuum,

*\(^{10}\) In the second paper listed in Ref. 17), the relation \(\theta_0 = -\delta\) is proved by use of the kink ansatz for the wall profile. One can show that this relation holds generally by use of the asymptotic expansion of the wall profile in the symmetric phase.
we maintain the relation $\mu \cot \beta = A \equiv |A_t| = |A_b|$.$^\ast$ Thus, the full set of input parameters is $\tan \beta$, $\mu$, $m_{H^\pm}$, $m_{\tilde{q}}$ and $\delta_A = \text{Arg} A_t = \text{Arg} A_b$.

First, we turn off $CP$ violation and calculate the masses of the neutral Higgs bosons for various values of $\tan \beta$, $m_{H^\pm}$, $\mu$ and $m_{\tilde{q}}$. Among the parameter sets consistent with the present bounds on the lightest Higgs boson mass $m_{H_1}$, we pick several for which the EWPT is investigated. We numerically calculated the effective potential (2.14) and searched for the minimum by use of the downhill simplex algorithm. We define $T_C$ as the temperature at which this minimum becomes degenerate with the symmetric phase, and evaluate $v_C$, $\tan \beta_C$ and $\theta_C$. Next, we gradually increase $\delta_A$ and examine $T_C$, $v_C$ and $\theta_C + \delta$.

Before presenting the numerical results, we roughly describe how the spectrum, $CP$ violation, and the strength of the EWPT depend on the parameters. If we increase $m_{H^\pm}$, which implies larger $\text{Re}(m^2_3)$, the masses of the neutral Higgs boson increase, and scalar-pseudoscalar mixing decreases, since the diagonal elements of (2-12) increase. Then, as is well known, the EWPT becomes stronger for larger $m_{H^\pm}$. If we decrease $\tan \beta$, which implies a larger top Yukawa coupling $y_t$ and a larger $A$ for fixed $\mu$, the EWPT becomes stronger, and Higgs mixing is enhanced. For the effects of $\delta_A$, we expect that the first-order EWPT will be weakened. This is because a nonzero $\delta_A$ modifies the $v$ dependence of the mass-squared of the lighter stop, which is roughly proportional to $v^2$ when $m_{\tilde{t}_R} = 0$ and $\delta_A = 0$, as seen from (A-8).

Now we present our numerical results. In the absence of $CP$ violation, the bounds on the mass of the lightest neutral Higgs exclude the largest portion of the theoretically allowed region.$^2$) For small $\tan \beta$, the lower bound is the same as that in the MSM, i.e. $m_h \geq 115$ GeV. For $\tan \beta$ between 8 and 40, the lightest Higgs boson can be as light as 92 GeV. A very large $\tan \beta$ region is excluded by the fact that CDF at Fermilab have not observed a $b\bar{b}$ pair from the lightest Higgs boson. For the charged scalar, the bounds are satisfied with $m_{H^\pm} \geq 90$ GeV for $\tan \beta = 1 - 50$. First, we turned off the $CP$ violation and calculated the masses of the Higgs bosons for $\tan \beta = 5, 10, 20$ and 30. For each $\tan \beta$, we studied the EWPT at several points $(\mu, m_{\tilde{q}})$ that are allowed by the Higgs mass bounds. An example of the contour plots of the lightest Higgs mass in the $(m_{H^\pm}, m_{\tilde{q}})$-plane is shown in Fig. 1. There is no parameter set with $\tan \beta = 5$ for which the EWPT is strongly first order, satisfying (2-17). For $\tan \beta = 10$ and 20, we found several parameter sets with small $m_{\tilde{q}}$ for which the EWPT is strongly first order. The lightest Higgs boson mass is smaller than about 110 GeV for such parameter sets.

Next, introducing the $CP$ violating phase $\delta_A$, we studied the strength of the EWPT and the $CP$ violation relevant to electroweak baryogenesis with several parameter sets for which the Higgs mass bounds are satisfied and the EWPT is strongly first order in the absence of the $CP$ violation. In our convention, $\delta_A \simeq \pi$ is disfavored by the $b \rightarrow s\gamma$ constraint.$^{19)}$ For example, the results are shown in Fig. 2 for the point in Fig. 1 with $m_{H^\pm} = 150$ GeV and $m_{\tilde{q}} = 1$ TeV. Although we do not show

$^\ast$ In practice, we also studied the case with $\mu \cot \beta \neq A$, but the results are not altered significantly as long as the squarks do not have nonzero expectation values.
Fig. 1. The lightest Higgs mass for $\tan \beta = 10$ and $\mu = 1500$ GeV. Dots indicate parameter sets for which the EWPT is strongly first order, and crosses indicate those not satisfying (2.17).

Fig. 2. In the left-hand figure, the $\delta_A$-dependences of $v_C$ (solid curve), $T_C$ (dashed curve) and the lightest Higgs mass $m_{H_1}$ (dotted curve) are plotted for $\tan \beta = 10$, $\mu = 1500$ GeV, $m_{H^\pm} = 150$ GeV and $m_{\tilde{q}} = 1$ TeV. In the right-hand figure, the dashed curve represents $\delta \equiv \text{Arg}(m_3^2)$, the dotted curve $\theta_C$ and the solid curve $\theta_C + \delta$ for the same parameter set.

The mass of the second-lightest Higgs boson, the mixing of the Higgs bosons becomes maximal for $\delta_A > 90^\circ$. The strength of the EWPT becomes too weak to satisfy the sphaleron decoupling condition (2.17) for $\delta_A \gtrsim 40^\circ$. The magnitude of the $CP$ phase relevant to baryogenesis is sufficiently large for $\delta_A \gtrsim 10^\circ$, in spite of the small Higgs
mixing. For a larger charged Higgs mass, the EWPT becomes stronger, while the Higgs mixing becomes smaller, as expected. The results for $m_{H^\pm} = 200$ GeV are shown in Fig. 3. The strongly first-order EWPT persists for $\delta_A \lesssim 55^\circ$, while the magnitude of $\theta_C + \delta$ decreases. We also examined the EWPT and the CP phase for larger values of $m_{H^\pm}$ and found that the strongly first-order EWPT persists for larger $\delta_A$, but $|\theta_C + \delta|$ becomes very small. For example, the maximal value of $|\theta_C + \delta|$ is about 0.02 for $m_{H^\pm} = 1$ TeV and 0.005 for $m_{H^\pm} = 2$ TeV.

For larger $\mu$, the effect of $\delta_A$ is expected to become stronger; that is, the EWPT is weakened as $\delta_A$ decreases. The results for $\mu = 2500$ GeV and $m_{\tilde{q}} = 1100$ GeV are depicted in Fig. 4. The strongly first-order EWPT persists for $\delta_A \lesssim 30^\circ$, while $|\theta_C + \delta|$ is $\mathcal{O}(1)$ for $\delta_A \simeq 20^\circ$. If we increase $m_{H^\pm}$, the EWPT becomes stronger, so that (2.17) is satisfied up to larger values of $\delta_A$, but the magnitude of the CP violation $|\theta_C + \delta|$ decreases, as in the case with smaller $\mu$. As an example of a larger $\tan \beta$, we show the results for $\tan \beta = 20$, $\mu = 2500$ GeV and $m_{\tilde{q}} = 1220$ GeV in
Fig. 5. The same as Fig. 3, but with \( \tan \beta = 20, \mu = 2500 \text{ GeV} \) and \( m_{\tilde{q}} = 1220 \text{ GeV} \).

Fig. 5. As noted above, a larger \( \tan \beta \) implies a smaller top Yukawa coupling, so that the effect of \( CP \) violation in the stop sector decreases. In fact, Fig. 5 shows that the strongly first-order EWPT persists for a larger \( \delta_A \), and \( |\theta_C + \delta| \) is smaller than in the case with \( \tan \beta = 10 \) (Fig. 4). We also explored other parameter sets with the same \( \tan \beta \) and observed the same tendency as in the case with \( \tan \beta = 10 \).

§4. Conclusion

We have studied the EWPT and \( CP \) violation in the Higgs sector of the MSSM. For parameter sets consistent with the present experimental bounds on the masses of neutral and charged Higgs bosons, we found strongly first-order EWPT when the lighter stop mass is less than the mass of the top quark and the lightest Higgs mass is less than about 110 GeV for \( 8 \lesssim \tan \beta \lesssim 30 \), in the absence of \( CP \) violation. These results without \( CP \) violation are not new. Because transitional \( CP \) violation cannot occur for parameter sets consistent with the updated Higgs boson mass bounds, viable \( CP \) violations for electroweak baryogenesis are among those in the complex parameters in the model. The relative phase between \( \mu \) and the gaugino soft mass is essential for the scenario in which the charginos and neutralinos play the role of charge carriers, while the phase is strongly constrained by the neutron EDM bound. If the masses of charginos and neutralinos are found to be as heavy as 1 TeV, they cannot participate the baryogenesis, since in this case they would be excited only very little at the EWPT temperature. Another source of \( CP \) violation is the relative phase between \( \mu \) and the squark-Higgs trilinear coupling \( A \). In particular, that phase in the third generation is free from the nEDM constraint, and it is expected to play an important role in baryogenesis. Further, it was pointed out that the explicit \( CP \) violation in the stop sector can cause the scalar-pseudoscalar mixing in the Higgs bosons to become very large, so that a Higgs boson lighter than the present bound is allowed, because of their couplings to the gauge bosons and \( b \)-quarks.\footnote{13} We investigated how this phase affects the EWPT, and we found that as the phase increases,
the scalar-pseudoscalar mixing increases, but the first-order EWPT becomes weaker. The EWPT cannot continue to be strong enough for successful electroweak baryogenesis before the mixing becomes maximal. Hence, a Higgs scalar lighter than the present bound that is allowed with a large explicit \( CP \) violation is not consistent with the strongly first-order EWPT, which is essential for electroweak baryogenesis. We, however, found that the phase relevant for baryogenesis can be sufficiently large for parameter sets consistent with the present mass bounds on the Higgs bosons. Baryon asymmetry produced by electroweak baryogenesis depends on the phase difference in the Higgs sector between the broken and symmetric phase regions, which are separated by the bubble wall created at the first-order EWPT. In our phase convention, the phase is \( \theta_C + \delta \). This phase can be \( O(1) \) for \( m_{H_1} \lesssim 110 \text{ GeV} \), \( m_{H^\pm} \lesssim 200 \text{ GeV} \) and \( 8 \lesssim \tan \beta \lesssim 20 \), and decreases for a larger \( m_{H^\pm} \).

The calculations presented here were done at the one-loop level. At finite temperatures, the naive loop expansion is not always reliable, and the resummed perturbation with temperature-corrected masses will improve the approximation. In particular, the infrared behavior of the Higgs scalar loop must be treated carefully. This is the reason that the results based on perturbation in the MSM are modified by an improved perturbation or nonperturbative lattice calculation. For the MSSM, the EWPT is mainly controlled by the stop loops, and the one-loop results given in Ref. 20) are consistent with the results obtained using the improved perturbation\(^{21)}\) and nonperturbative lattice calculations\(^{22),23)}\) for parameters with small \( m_{\tilde{t}_R}, \mu \) and \( A \). We expect that the results in this work will not be altered by such improvements.

The parameter region of the MSSM allowed by the experimental Higgs mass bounds is now much narrower than that allowed theoretically. Within that region, strongly first-order EWPT is possible only for \( m_{\tilde{t}_1} \lesssim m_t \) and \( m_{H_1} \lesssim 110 \text{ GeV} \), which corresponds to \( 8 \lesssim \tan \beta \lesssim 30 \). Explicit \( CP \) violation in the stop sector can induce a large \( CP \) phase in the Higgs sector, which is relevant to electroweak baryogenesis. Whether or not this scenario provides a realistic model of baryogenesis depends on the spectrum of the lightest Higgs boson, the charged Higgs boson and the lighter stop. The masses of these particles should be clarified in the near future by LHC. If the lightest Higgs boson is found to be heavier than 110 GeV, it will be shown that the EWPT is too weak for the sphaleron process to take place out of equilibrium. Even if the EWPT is strongly first order, the model with heavy charged Higgs bosons and large \( \mu \) requires a source of \( CP \) violation in addition to that in the Higgs sector for realistic electroweak baryogenesis. This source might be the relative phases of \( \mu \) and the gaugino masses, which are constrained by nEDM experiments. For these phases to provide the necessary source of \( CP \) violation, the masses of the charginos and neutralinos should be as light as the weak scale. This will also be checked in the near future.

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Appendix A

Field-Dependent Masses

In this appendix, we summarize the field-dependent masses of the quarks and squarks of the third generation and those of the gauge bosons, which appear in the definition of the effective potential (2.3). These are functions of the neutral components of the Higgs fields, while the charged components are needed to calculate the mass of the charged Higgs boson. The masses of the quark of the third generation are given by

\[ m^2_{t,b} = \frac{1}{2} \left[ |y_t|^2 \Phi_u^\dagger \Phi_u + |y_b|^2 \Phi_d^\dagger \Phi_d \mp \sqrt{(|y_t|^2 \Phi_u^\dagger \Phi_u - |y_b|^2 \Phi_d^\dagger \Phi_d)^2 + 4 \Phi_d^\dagger \Phi_d} \right] . \]  

Similarly, the top and bottom squark masses are

\[ m^2_{\tilde{t}_{1,2}} = \frac{1}{2} \left\{ m^2_{\tilde{q}} + m^2_{\tilde{t}_R} + 2 |y_t|^2 \Phi_u^\dagger \Phi_u + \frac{g_2^2 + g_1^2}{4} \left( \Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u \right) \right\} \]

\[ \pm \left[ m^2_{\tilde{q}} - m^2_{\tilde{t}_s} + x_t \left( \Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u \right) \right]^2 \]

\[ + 2 |y_t|^2 \left( \mu (v_d + h_d + i a_d) - A_t^* e^{-i\theta} (v_u + h_u - i a_u) \right) \frac{1}{2} \right\} , \]

(A.1)

\[ m^2_{\tilde{b}_{1,2}} = \frac{1}{2} \left\{ m^2_{\tilde{q}} + m^2_{\tilde{b}_R} + 2 |y_b|^2 \Phi_d^\dagger \Phi_d + \frac{g_2^2 + g_1^2}{4} \left( \Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u \right) \right\} \]

\[ \pm \left[ m^2_{\tilde{q}} - m^2_{\tilde{b}_s} + x_b \left( \Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u \right) \right]^2 \]

\[ + 2 |y_b|^2 \left( \mu e^{i\theta} (v_u + h_u + i a_u) - A_b^* (v_d + h_d - i a_d) \right) \frac{1}{2} \right\} , \]

(A.2)

where

\[ x_t \equiv \frac{1}{4} \left( g_2^2 - \frac{5}{3} g_1^2 \right) , \quad x_b \equiv - \frac{1}{4} \left( g_2^2 - \frac{1}{3} g_1^2 \right) . \]

(A.3)

The gauge bosons masses are

\[ m^2_Z = \frac{1}{4} \left( g_2^2 + g_1^2 \right) \left[ (v_d + h_d)^2 + a_d^2 + (v_u + h_u)^2 + a_u^2 \right] , \]

(A.5)

\[ m^2_W = \frac{1}{4} g_2^2 \left[ (v_d + h_d)^2 + a_d^2 + (v_u + h_u)^2 \right] . \]

(A.6)

The masses evaluated at the zero-temperature vacuum are given as follows:

\[ m^2_t = \langle \bar{m}^2_t \rangle = \frac{1}{2} |y_t|^2 v_u^2 , \quad m^2_b = \langle \bar{m}^2_b \rangle = \frac{1}{2} |y_b|^2 v_d^2 , \]

(A.7)
\[ m_{t_{1,2}}^2 = \langle \bar{m}_{t_{1,2}}^2 \rangle \]
\[ = \frac{m_q^2 + m_{t_R}^2}{2} + m_t^2 + \frac{1}{4} m_Z^2 \cos(2\beta) \]
\[ \pm \frac{1}{2} \sqrt{\left[ m_q^2 - m_{t_R}^2 + \frac{x_t}{2} v_0^2 \cos(2\beta) \right]^2 + 4 m_t^2 \mu \cot \beta - A_t^* e^{-i\theta}} \] (A.8)

\[ m_{b_{1,2}}^2 = \langle \bar{m}_{b_{1,2}}^2 \rangle \]
\[ = \frac{m_q^2 + m_{b_R}^2}{2} + m_b^2 - \frac{1}{4} m_Z^2 \cos(2\beta) \]
\[ \pm \frac{1}{2} \sqrt{\left[ m_q^2 - m_{b_R}^2 + \frac{x_b}{2} v_0^2 \cos(2\beta) \right]^2 + 4 m_b^2 \mu e^{i\theta} \tan \beta - A_b^* \beta} \] (A.9)

\[ m_Z^2 = \frac{1}{4} (g_2^2 + g_1^2) v_0^2, \quad m_W^2 = \frac{1}{4} g_2^2 v_0^2, \] (A.10)

where \( v_0^2 \equiv v_d^2 + v_u^2 \), \( \tan \beta = v_u/v_d \).

**Appendix B**

**Mass Matrix of the Neutral Higgs Boson**

The calculation of the elements of the mass matrix is presented in Ref. 13. We added the gauge-boson contributions to that calculation, and here we give the resulting explicit forms. For later convenience, we introduce the following quantities:

\[ M_t^2 = m_q^2 - m_{t_R}^2 + \frac{x_t}{2} v_0^2 \cos(2\beta), \quad M_b^2 = m_q^2 - m_{b_R}^2 + \frac{x_b}{2} v_0^2 \cos(2\beta), \] (B.1)

\[ \Delta m_q^2 = m_{q_1}^2 - m_{q_2}^2, \quad \Delta m_{b_R}^2 \] (q = t, b) (B.2)

\[ R_q = \text{Re} \left( \mu A_q e^{i\theta} \right), \quad I_q = \text{Im} \left( \mu A_q e^{i\theta} \right), \quad (q = t, b) \] (B.3)

\[ P_t = |\mu|^2 - R_t \tan \beta, \quad Q_t = |A_t|^2 - R_t \cot \beta, \] (B.4)

\[ P_b = |\mu|^2 - R_b \cot \beta, \quad Q_b = |A_b|^2 - R_b \tan \beta. \] (B.5)

The elements of the mass matrix in the scalar sector including the gauge-boson contributions are given by

\[ (\mathcal{M}_S^2)_{11} \]
\[ = \text{Re}(m_Z^2 e^{i\theta}) \tan \beta + m_Z^2 \cos^2 \beta \]
\[ + \frac{N_C}{16\pi^2} \left\{ \frac{1}{2} m_Z^2 \cos^2 \beta \left( \frac{x_t M_t^2 + 2 |y_t|^2 P_t}{\Delta m_{t_R}^2} \log \frac{m_{t_1}^2}{m_{t_2}^2} - \frac{x_b M_b^2 + 2 |y_b|^2 Q_b}{\Delta m_{b_R}^2} \log \frac{m_{b_1}^2}{m_{b_2}^2} \right) \right\} \]
\[ + 2 m_b^2 \left( \frac{x_b M_b^2 + 2 |y_b|^2 Q_b}{\Delta m_{b_R}^2} \log \frac{m_{b_1}^2}{m_{b_2}^2} - 2 |y_b|^2 \log \frac{m_{b_1}^2}{M^2} \right) \]
\[ + v_d^2 \left( \frac{g_2^2 + g_1^2}{8} \right)^2 \left( \log \frac{m_{t_1}^2}{M^2} + \log \frac{m_{t_2}^2}{M^2} \right) \]
\[
+ v_d^2 \left( |y_0|^2 - \frac{g_2^2 + g_1^2}{8} \right)^2 \left( \log \frac{m^2_{b_1}}{M^2} + \log \frac{m^2_{b_2}}{M^2} \right) \\
+ x_t^2 v_d^2 + 2 |y_t|^2 R_t \tan \beta \left( m^2_{t_1} \log \frac{m^2_{t_1}}{M^2} - m^2_{t_2} \log \frac{m^2_{t_2}}{M^2} - \Delta m^2_t \right) \\
+ x_b^2 v_d^2 + 2 |y_b|^2 R_b \tan \beta \left( m^2_{b_1} \log \frac{m^2_{b_1}}{M^2} - m^2_{b_2} \log \frac{m^2_{b_2}}{M^2} - \Delta m^2_b \right) \\
+ v_d^2 (x_t M_t^2 + 2 |y_t|^2 P_t)^2 \left( 1 - \frac{m^2_{t_1} + m^2_{t_2}}{2 \Delta m^2_t} \log \frac{m^2_{t_1}}{m^2_{t_2}} \right) \\
+ v_d^2 (x_b M_b^2 + 2 |y_b|^2 Q_b)^2 \left( 1 - \frac{m^2_{b_1} + m^2_{b_2}}{2 \Delta m^2_b} \log \frac{m^2_{b_1}}{m^2_{b_2}} \right) \right\} \\
+ \frac{3}{128 \pi^2} v_d^2 \left\{ (g_2^2 + g_1^2) \log \frac{m^2_{b_1}}{M^2} + 2 g_2^2 \log \frac{m^2_{W}}{M^2} \right\}, \quad (B.6)
\]

\[
(M^2_S)_{22} = \text{Re}(m^2_{S} e^{i\theta}) \cot \beta + m^2_{Z} \sin^2 \beta \\
+ \frac{N_C}{16 \pi^2} \left\{ \frac{1}{2} m^2_{Z} \sin^2 \beta \left( -x_t M_t^2 + 2 |y_t|^2 Q_t \log \frac{m^2_{t_1}}{m^2_{t_2}} \right) \\
+ \frac{2 m_t^2}{\Delta m^2_t} \left( -x_t M_t^2 + 2 |y_t|^2 Q_t \log \frac{m^2_{t_1}}{m^2_{t_2}} - 2 |y_t|^2 \log \frac{m^2_{t}}{M^2} \right) \log \frac{m^2_{t_1}}{m^2_{t_2}} \right\} \\
+ \frac{2 m_t^2}{\Delta m^2_t} \left( |y_t|^2 - \frac{g_2^2 + g_1^2}{8} \right)^2 \left( \log \frac{m^2_{t_1}}{M^2} + \log \frac{m^2_{t_2}}{M^2} \right) \\
+ \frac{x_t^2 v_u^2 + 2 |y_t|^2 R_t \cot \beta \left( m^2_{t_1} \log \frac{m^2_{t_1}}{M^2} - m^2_{t_2} \log \frac{m^2_{t_2}}{M^2} - \Delta m^2_t \right) \\
+ \frac{x_t^2 v_u^2 + 2 |y_t|^2 R_b \cot \beta \left( m^2_{b_1} \log \frac{m^2_{b_1}}{M^2} - m^2_{b_2} \log \frac{m^2_{b_2}}{M^2} - \Delta m^2_b \right) \\
+ \frac{x_u^2 (x_t M_t^2 + 2 |y_t|^2 P_t)^2 \left( 1 - \frac{m^2_{t_1} + m^2_{t_2}}{2 \Delta m^2_t} \log \frac{m^2_{t_1}}{m^2_{t_2}} \right) \\
+ \frac{x_u^2 (x_b M_b^2 + 2 |y_b|^2 Q_b)^2 \left( 1 - \frac{m^2_{b_1} + m^2_{b_2}}{2 \Delta m^2_b} \log \frac{m^2_{b_1}}{m^2_{b_2}} \right) \right\} 
\]
\begin{equation}
\begin{aligned}
(M_3^2)_{12} &= -\text{Re}(m_3^2 e^{i\theta}) - m_Z^2 \sin \beta \cos \beta \\
&+ \frac{N_C}{16\pi^2} \left\{ \frac{1}{2} m_Z^2 \sin \beta \cos \beta \left( \frac{-x_t M_t^2 + |y_t|^2 (Q_t - P_t)}{\Delta m_t^2} \log \frac{m^2_{t_1}}{m^2_{t_2}} + \frac{x_b M_b^2 + |y_b|^2 (Q_b - P_b)}{\Delta m_b^2} \log \frac{m^2_{b_1}}{m^2_{b_2}} \right) \\
&+ \frac{1}{2} m_Z^2 \sin \beta \left[ \left( |y_t|^2 - \frac{g_2^2 + g_1^2}{8} \right) \left( \log \frac{m^2_{t_1}}{M^2} + \log \frac{m^2_{t_2}}{M^2} \right) \\
&\quad + \left( |y_b|^2 - \frac{g_2^2 + g_1^2}{8} \right) \left( \log \frac{m^2_{b_1}}{M^2} + \log \frac{m^2_{b_2}}{M^2} \right) \right] \\
&- \frac{x_t^2 v_d v_u + 2 |y_t|^2 R_t}{2 \Delta m_t^2} \left( m_{t_1}^2 \log \frac{m_{t_1}^2}{M^2} - m_{t_2}^2 \log \frac{m_{t_2}^2}{M^2} - \Delta m_t^2 \right) \\
&- \frac{x_b^2 v_d v_u + 2 |y_b|^2 R_b}{2 \Delta m_b^2} \left( m_{b_1}^2 \log \frac{m_{b_1}^2}{M^2} - m_{b_2}^2 \log \frac{m_{b_2}^2}{M^2} - \Delta m_b^2 \right) \\
&+ v_d v_u \left( x_t M_t^2 + 2 |y_t|^2 P_t \right) (-x_t M_t^2 + 2 |y_t|^2 Q_t) \left( 1 - \frac{m_{t_1}^2 + m_{t_2}^2}{2 \Delta m_t^2} \log \frac{m_{t_1}^2}{m_{t_2}^2} \right) \\
&+ v_d v_u \left( x_b M_b^2 + 2 |y_b|^2 Q_b \right) (-x_b M_b^2 + 2 |y_b|^2 P_b) \left( 1 - \frac{m_{b_1}^2 + m_{b_2}^2}{2 \Delta m_b^2} \log \frac{m_{b_1}^2}{m_{b_2}^2} \right) \\
&+ \frac{3}{128\pi^2} v_d v_u \left\{ (g_2^2 + g_1^2)^2 \log \frac{m_W^2}{M^2} + 2 g_2^4 \log \frac{m_W^2}{M^2} \right\}.
\end{aligned}
\end{equation}

The matrix elements in the pseudoscalar sector are

\begin{equation}
(M_3^2)_{11} = (M_3^2)_{12} \tan \beta, \quad (B.9)
\end{equation}

\begin{equation}
(M_3^2)_{22} = (M_3^2)_{12} \cot \beta, \quad (B.10)
\end{equation}

\begin{equation}
(M_3^2)_{12} = \text{Re}(m_3^2 e^{i\theta}) + \frac{N_C}{16\pi^2} \left\{ \frac{|y_t|^2}{\Delta m_t^2} \left[ R_t \left( m_{t_1}^2 \log \frac{m_{t_1}^2}{M^2} - m_{t_2}^2 \log \frac{m_{t_2}^2}{M^2} - \Delta m_t^2 \right) \\
\quad + \frac{4 m_{t_1}^2 \cot \beta}{\Delta m_t^2} I_t \left( 1 - \frac{m_{t_1}^2 + m_{t_2}^2}{2 \Delta m_t^2} \log \frac{m_{t_1}^2}{m_{t_2}^2} \right) \right] \\
\quad + \frac{|y_b|^2}{\Delta m_b^2} \left[ R_b \left( m_{b_1}^2 \log \frac{m_{b_1}^2}{M^2} - m_{b_2}^2 \log \frac{m_{b_2}^2}{M^2} - \Delta m_b^2 \right) \right] \right\}.
\end{equation}
\[ + \frac{4m^2_t \tan \beta}{\Delta m_{\tilde{b}}^2} I^b_b \left( 1 - \frac{m_{b_1}^2 + m_{b_2}^2}{2m_b^2} \log \frac{m_{b_1}^2}{m_{b_2}^2} \right) \right\}. \tag{B.11} \]

The scalar-pseudoscalar mixing elements are given by

\[
\begin{align*}
(M_{SP}^2)_{11} &= (M_{SP}^2)_{12} \tan \beta, \\
(M_{SP}^2)_{22} &= (M_{SP}^2)_{21} \cot \beta, \\
(M_{SP}^2)_{12} &= \frac{N_C}{8\pi^2} \left\{ \frac{m_t^2 I_t \cot^2 \beta}{\Delta m_t^2} \left[ \frac{g_2^2 + g_1^2}{8} \log \frac{m_{t_2}^2}{m_{t_1}^2} \right] \\
&\quad + \frac{x_t M_t^2 + 2 |y_t|^2 P_t}{\Delta m_t^2} \left( 1 - \frac{m_{t_1}^2 + m_{t_2}^2}{2\Delta m_t^2} \log \frac{m_{t_1}^2}{m_{t_2}^2} \right) \right\}, \\
(M_{SP}^2)_{21} &= \frac{N_C}{8\pi^2} \left\{ \frac{m_b^2 I_b}{\Delta m_b^2} \left[ \left( |y_b|^2 - \frac{g_2^2 + g_1^2}{8} \right) \log \frac{m_{b_1}^2}{m_{b_2}^2} \right] \\
&\quad + \frac{-x_b M_b^2 + 2 |y_b|^2 Q_b}{\Delta m_b^2} \left( 1 - \frac{m_{b_1}^2 + m_{b_2}^2}{2\Delta m_b^2} \log \frac{m_{b_1}^2}{m_{b_2}^2} \right) \right\},
\end{align*}
\tag{B.14} \]

which are all composed of terms proportional to \( \text{Im}(\mu A_t)/\Delta m_t^2 \) or \( \text{Im}(\mu A_b)/\Delta m_b^2 \).

### Appendix C

**Charged Higgs Mass**

The calculation of the charged Higgs mass is a tedious task, and the method is described in the appendix of Ref. 13). Here we present the result with the contribution from the gauge bosons included:

\[
m_{H^\pm}^2 = \frac{1}{\sin \beta \cos \beta} \text{Re}(m_3^2 e^{i\theta}) + m_W^2
\]
\[ \frac{N_C}{16\pi^2 \sin\beta \cos\beta} \left\{ \frac{1}{\Delta m_t^2} \left[ \frac{f(m_{t_1}^2)}{(m_{t_1}^2 - m_{b_1}^2)(m_{t_1}^2 - m_{b_2}^2)} \right. \right. \\
+ |y_t|^2 R_t \left. \left. \right] m_{t_1}^2 \left( \log \frac{m_{t_1}^2}{M^2} - 1 \right) - (\tilde{t}_1 \to \tilde{t}_2) \right\} \\
+ \frac{1}{\Delta m_b^2} \left[ \frac{f(m_{b_1}^2)}{(m_{b_1}^2 - m_{b_2}^2)} + |y_b|^2 R_b \right] m_{b_1}^2 \left( \log \frac{m_{b_1}^2}{M^2} - 1 \right) \\
- (\tilde{b}_1 \to \tilde{b}_2) \right\} \\
- \frac{2 |y_t y_b| m_t m_b}{m_t^2 - m_b^2} \left[ m_t^2 \left( \log \frac{m_t^2}{M^2} - 1 \right) - m_b^2 \left( \log \frac{m_b^2}{M^2} - 1 \right) \right] \right\} \\
- \frac{3}{16\pi^2} (g_2^2 + g_1^2) \frac{g_2^2}{g_1^2} m_Z^2 \left( \log \frac{m_Z^2}{M^2} - 1 \right), \] (C.1)

where

\[ f(m_{q_k}^2) = \frac{1}{2} |y_t y_b|^2 v_u v_d \left[ 2m_{q_k}^4 - \text{Tr}(M_t^2 + M_b^2)m_{q_k}^2 + (M_t^2)_{11} \left( M_b^2 \right)_{11} + (M_t^2)_{22} \left( M_b^2 \right)_{22} \right] \\
- \frac{1}{4} g_2^2 v_u v_d \left( |y_t|^2 + |y_b|^2 - \frac{g_2^2}{2} \right) \left( m_{q_k}^2 - (M_t^2)_{22} \right) \left( m_{q_k}^2 - (M_b^2)_{22} \right) \\
+ \frac{1}{2} |\mu|^2 v_u v_d \left[ -(|y_t|^2 + |y_b|^2) \left( |y_t|^2 + |y_b|^2 - \frac{g_2^2}{2} \right) m_{q_k}^2 \right] \\
+ |y_t|^2 \left( |y_t|^2 - \frac{g_2^2}{2} \right) \left( M_b^2 \right)_{22} + |y_b|^2 \left( |y_b|^2 - \frac{g_2^2}{2} \right) \left( M_t^2 \right)_{22} \\
+ |y_t y_b|^2 \left( |\mu|^2 + 2m_t^2 + 2m_b^2 + 2m_t^2 - m_W^2 - \frac{g_2^2}{12} v_0^2 \cos(2\beta) \right) \right] \\
+ \frac{1}{2} |y_t A_t|^2 v_u v_d \left( |y_b|^2 - \frac{g_2^2}{2} \right) \left( m_{q_k}^2 - (M_b^2)_{22} \right) \\
+ \frac{1}{2} |y_b A_b|^2 v_u v_d \left( |y_t|^2 - \frac{g_2^2}{2} \right) \left( m_{q_k}^2 - (M_t^2)_{22} \right) \\
+ \frac{1}{2} |y_t y_b A_t A_b|^2 v_u v_d - |y_t y_b|^2 v_u v_d (R_t R_b + I_t I_b) \\
- 2m_t^2 m_b^2 \left[ \left( |y_b|^2 - \frac{g_2^2}{2} \right) R_t + \left( |y_t|^2 - \frac{g_2^2}{2} \right) R_b \right] \\
+ \frac{1}{2} |y_t y_b|^2 v_u v_d \text{Re}(A_t A_b^*) \left[ 2m_{q_k}^2 - 2m_{q_k}^2 m_W^2 + \frac{g_2^2}{12} v_0^2 \cos(2\beta) \right]. \] (C.2)
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