IMPLICATIONS OF PLASMA BEAM INSTABILITIES FOR THE STATISTICS OF THE FERMI HARD GAMMA-RAY BLAZARS AND THE ORIGIN OF THE EXTRAGALACTIC GAMMA-RAY BACKGROUND

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\textbf{ABSTRACT}

Fermi has been instrumental in constraining the luminosity function and redshift evolution of gamma-ray bright BL Lac objects, a subpopulation of blazars with almost featureless optical spectra. This includes limits on the spectrum and anisotropy of the extragalactic gamma-ray background (EGRB), redshift distribution of nearby Fermi active galactic nuclei (AGNs), and the construction of a $\log N$--$\log S$ relation. Based on these, it has been argued that the evolution of the gamma-ray bright BL Lac population must be much less dramatic than that of other AGNs. However, critical to such claims is the assumption that inverse Compton cascades reprocess emission above a TeV into the Fermi energy range, substantially enhancing the strength of the observed limits. Here we demonstrate that in the absence of such a process, due, e.g., to the presence of virulent plasma beam instabilities that preempt the cascade, a population of TeV-bright BL Lac objects that evolve similarly to quasars is consistent with the population of hard gamma-ray BL Lac objects observed by Fermi. Specifically, we show that a simple model for the properties and luminosity function is simultaneously able to reproduce their $\log N$--$\log S$ relation, local redshift distribution, and contribution to the EGRB and its anisotropy without any free parameters. Insofar as the naturalness of a picture in which the hard gamma-ray BL Lac population exhibits the strong redshift evolution observed in other tracers of the cosmological history of accretion onto halos is desirable, this lends support for the absence of the inverse Compton cascades and the existence of the plasma beam instabilities.

\textbf{Key words:} BL Lacertae objects: general – gamma rays: diffuse background – gamma rays: general – infrared: diffuse background – plasmas – radiation mechanisms: non-thermal

\textbf{Online-only material:} color figures

1. INTRODUCTION

1.1. Blazars in the Fermi Era

The Large Area Telescope (LAT) onboard the Fermi gamma-ray space telescope has become a powerful tool for studying gamma-ray bright active galactic nuclei (AGNs), placing the most stringent constraints to date upon their numbers and evolution. In practice, this is performed via a variety of methods, including the flux and redshift distributions of nearby sources and the extragalactic gamma-ray background (EGRB) due to unresolved sources at high redshift. Each of these effectively probes different projections of the evolving luminosity function of the gamma-ray bright objects, and thus taken together they provide considerable traction on their populations at low and high redshifts.

The Fermi AGN sample is overwhelmingly dominated by blazars, with a handful of radio and starburst galaxies comprising the remainder (see, e.g., Table 5 of Ackermann et al. 2011). The population of blazars is itself often subdivided into a number of subcategories, depending primarily upon their optical properties. The most populous are the flat-spectrum radio sources (FSRQs; which show strong broad emission lines in their optical spectra) and BL Lac objects, which are instead characterized by almost featureless optical spectra that at most show weak emission lines or absorption features. The multi-frequency spectrum of blazars is dominated by a nonthermal, jet-related component that is composed of two broad humps: one at low energies that is attributed to synchrotron emission in the jet and spans from radio to optical respectively X-ray wavelengths (depending on the blazar type), and one at high energies that is usually thought to be due to inverse Compton radiation and extends from the X-rays to gamma rays.

The BL Lac objects are further segregated into low-, intermediate-, and high-synchrotron peaked sources (LSP, ISP, HSP, respectively). While the latter categories are based on the peak position of the synchrotron component, they roughly correspond to the hardness or softness of the gamma-ray spectrum, $E^2 dN/dE$, at energies relevant for Fermi, with HSPs being harder than ISPs, which are harder than LSPs. Typically, the FSRQs are considerably softer than the BL Lac objects and thus appear primarily at low energies ($\lesssim 10$ GeV). In contrast, a number of BL Lac objects exhibit rising spectra between 1 GeV and 100 GeV, which we define as “hard gamma-ray blazars.”

The rising Fermi spectra of the hard gamma-ray blazars suggest a natural identification with the observed set of TeV blazars, detected and characterized by imaging atmospheric Cerenkov telescopes such as H.E.S.S., VERITAS, and MAGIC.\textsuperscript{5}

As for Fermi, the extragalactic TeV universe is dominated by blazars: of the 28 objects with well-defined spectral energy distributions (SEDs) listed in Broderick et al. (2012), 24 are blazars, which we refer to as the “TeV blazars. Thus, any limitation on the evolution of the hard gamma-ray blazar

\textsuperscript{5} High Energy Stereoscopic System, Very Energetic Radiation Imaging Telescope Array, Major Atmospheric Gamma Imaging Cerenkov Telescope.

\textsuperscript{6} For an up-to-date list, see http://www.mppmu.mpg.de/~rwagner/sources.
population implies a corresponding constraint on the TeV blazars, and vice versa.

The recent detection of a gamma-ray horizon provides strong evidence that the TeV gamma rays are generated at cosmological distances (Ackermann et al. 2012b; Domínguez et al. 2013), either in the vicinity of the source AGN or as a result of the hadronically induced cascade (e.g., Essey & Kusenko 2010). This is consistent with the fact that all of the observed TeV blazars are relatively nearby, with $z \sim 0.1$ typically. The depletion of gamma rays is assumed to be due to the annihilation of TeV gamma rays upon the extragalactic background light (EBL) and the subsequent generation of a relativistic population of TeV gamma rays upon the extragalactic background light depletion of gamma rays is assumed to be due to the annihilation of TeV gamma rays upon the extragalactic background light (EBL) and the subsequent generation of a relativistic population of gamma rays. These are referred to as the pure luminosity evolution and luminosity-dependent density evolution models by Narumoto & Totani (2006), respectively.

Historically, it has been assumed that these cool primarily by Comptonizing the cosmic microwave background, resulting in an inverse Compton cascade that effectively reprocesses the original TeV emission to energies below 100 GeV. The assumption that this reprocessing occurs has a dramatic impact on the implications Fermi has for the TeV blazar population. Early attempts to reconstruct the gamma-ray blazar luminosity function ignored absorption on the EBL and found that consistency with the EGRET EGRB required fixed or negatively evolving comoving number densities (e.g., Narumoto & Totani 2006). While absorption on the EBL provides a natural explanation for the absence of direct contributions from high-redshift gamma-ray blazars above 100 GeV, the reprocessing of this emission to GeV energies by putative inverse Compton cascades is more than sufficient to offset the absorption losses within the Fermi band (e.g., see Figure 2 of Inoue & Ioka 2012). As a result, stringent constraints on the number of hard gamma-ray blazars at high redshift can be derived from the EGRB (e.g., Venters 2010; Murase et al. 2012; Inoue & Ioka 2012), for which Fermi has now provided the most precise estimate. Based upon these, it is now well established that in the presence of inverse Compton cascades, the comoving number density of gamma-ray blazars, and by extension the TeV blazars, must be either fixed or decreasing with redshift (Kneiske & Mannheim 2008; Venters 2010; Abazajian et al. 2011; Inoue & Ioka 2012). That is, the TeV blazar population cannot exhibit the dramatic evolution that characterizes other AGNs specifically and other tracers of the cosmological history of accretion onto galactic halos more generally (e.g., star formation).

This represents a substantial obstacle to unifying the hard gamma-ray blazar population with that of other AGNs, is at odds with the underlying physical picture of accreting black hole systems, and suggests an unlikely conspiracy between accretion physics and the formation of structure. In analogy with Galactic X-ray binaries, AGNs are believed to undergo transitions in their accretion states, cycling between the high/soft state (i.e., quasars) and the low/hard state (i.e., radio-loud AGNs) on timescales set by the dynamics of the inner accretion flow instead of those associated with the supply of available gas (Maccarone et al. 2003; McHardy et al. 2006). In this case, we would anticipate that AGN types associated with the various accretion states have contemporaneous populations. This receives indirect support from the apparent rapid redshift evolution of radio-loud AGNs, the presumed parent population of blazars, which peak near $z \sim 1.85$ (Willott et al. 2001; Wall et al. 2005). Direct support comes from a recent demonstration that X-ray and gamma-ray selected BL Lac objects and FSRQs are consistent with a similar evolution (Giommi et al. 2012, 2013); the apparent decrease in number density of BL Lac objects with redshift found in previous studies (e.g., Padovani et al. 2007) arising due to selection effects (including possible misclassifications) associated with the relative dominance of the various emission components.8

In a series of papers (Broderick et al. 2012; Chang et al. 2012; Pfrommer et al. 2012, hereafter Paper I, Paper II, and Paper III, respectively) and Puchwein et al. (2012), we have explored the possible impact of beam-plasma instabilities on the gamma-ray emission of bright TeV sources and their subsequent cosmological consequences. We found that in this second scenario a variety of cosmological puzzles, most importantly the statistics of the high-redshift Lyα forest, were naturally resolved if the VH EGR emission from TeV blazars was dumped into heat in the intergalactic medium, as anticipated by such plasma instabilities.9 However, to do so requires a much more rapidly rising TeV blazar comoving number density than implied by previous analyses of the EGRB, namely one similar to that of quasars and qualitatively consistent with these other examples that depend on the cosmological history of accretion (e.g., star formation, radio galaxies, AGNs, and galactic merger rates). As shown in Paper I, the apparent tension with the above-discussed constraints from the EGRB is reconciled by the lack of significant inverse Compton cascades, preempted by the plasma instabilities responsible for depositing the VHEGR luminosity into the intergalactic medium. Without the inverse Compton cascades, it is possible to quantitatively reproduce both the redshift-dependent number of hard gamma-ray blazars listed in the First Fermi LAT AGN Catalog (1LAC, Abdo et al. 2010b) and the EGRB spectrum above 10 GeV.

The recent release of the 2 Year Fermi-LAT AGN Catalog (2LAC, Ackermann et al. 2011) and the First Fermi-LAT Catalog of > 10 GeV sources (1FHL; Ackermann et al. 2013), as well as the forthcoming releases of the third Fermi-LAT point source and AGN Catalogs (3LAC and 3FGL, respectively), motivates a reevaluation of the Fermi constraints on the evolution of the number and luminosity distribution of the hard gamma-ray blazars within the context of a considerably more complete set of resolved Fermi sources. With the luminosity function posited in Paper I, here we explicitly construct the expected flux and redshift distributions and the Fermi EGRB and directly assess the viability of a quasar-like evolution in the hard gamma-ray blazar population. Generally, we find excellent agreement where expected, implying that in the absence of inverse Compton

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8 These are referred to as the pure luminosity evolution and luminosity-dependent density evolution models by Narumoto & Totani (2006), respectively.

9 In practice, the ability of plasma instabilities to efficiently thermalize the pairs’ kinetic energy depends upon their nonlinear evolution. This is currently highly uncertain (see, e.g., Paper I; Schlickeiser et al. 2012; Miniati & Elyiv 2013). Here we assume only that the inverse Compton cascades are preempted, presumably by such plasma instabilities, and explore the consequences for the Fermi hard gamma-ray blazar population and EGRB.
cascades (preempted, e.g., by plasma instabilities) it is possible to unify the hard gamma-ray blazars with AGNs generally.

1.2. Methodology and Outline

Our primary goal is to observationally probe the redshift-dependent luminosity function of the TeV blazars. We do this by first making the ansatz that the intrinsic TeV blazar population evolves in a manner similar to most other AGNs and specifically exhibits the rapid increase in comoving number density exhibited by quasars and then assessing the implications for the gamma-ray blazar population. This ansatz is motivated both by a natural theoretical desire to unify AGNs, the analogy between AGN types and X-ray binary states, and the success that such a model has in reproducing the properties of the high-redshift Lyα forest when intergalactic plasma instabilities operate efficiently.

In practice, this is further complicated by the small number and limited redshift range of the known TeV blazars. The sample of observed TeV blazars is strongly biased in favor of nearby, X-ray selected BL Lac objects. To address these selection effects, we use the Fermi hard gamma-ray blazars (defined by an intrinsic photon spectral index \( \leq 2 \)) as proxies.

At low redshift \((z \lesssim 0.2)\), the cosmological redshift and absorption on the EBL are negligible below 100 GeV, and the two blazar populations are directly comparable. We exploit this to empirically define the distribution of intrinsic spectra relevant for the hard gamma-ray blazars, extending the luminosity function described in Paper I and removing a key degeneracy therein.

However, even at moderate redshifts \((z \gtrsim 0.2)\) the absorption on the EBL substantially softens the spectra below 100 GeV (Ackermann et al. 2012b), and this must be taken into account in the source identification. Where direct comparisons to the Fermi blazar sample are made, we make the conservative choice of considering only objects with observed photon spectral indexes \( \leq 2 \), which necessarily implies that the intrinsic photon spectral indexes are also \( \leq 2 \) (Sections 3.1 and 3.3). For concreteness, we define the “hard Fermi” blazars to be those objects that exhibit a rising spectrum, with index \( \Gamma_F \lesssim 2 \) in the energy band 1–100 GeV, where \( E^2dN/dE \propto E^{2-\Gamma_F} \). For consistency it is necessary to restrict the expected source population as well, and therefore we construct an approximate relationship between the observed and intrinsic photon spectral indexes. Where comparison with the Fermi blazar sample is not required, e.g., for modeling of the EGRB, we consider the full hard gamma-ray population, restricting only the intrinsic spectra (Section 3.4).

In Section 2, we define the TeV blazar luminosity function, describe its regime of validity, and relate it to the luminosity function of the Fermi hard gamma-ray blazars generally. In Section 3, we review the definitions of the various Fermi constraints and compare the expectations from our TeV blazar luminosity functions. Finally, discussion and conclusions are contained in Section 4.

2. THE HARD GAMMA-RAY BLAZAR LUMINOSITY FUNCTION

Here we construct a luminosity function for the hard gamma-ray blazars, beginning with a review of the luminosity function for the TeV blazars constructed in Paper I. Critical to producing an analogous luminosity function for the Fermi hard gamma-ray blazars is the relationship between the Fermi band (here 100 MeV–100 GeV) and the intrinsic isotropic equivalent TeV luminosity (100 GeV–10 TeV).\(^{10}\)

2.1. The TeV Blazar Luminosity Function

The vast majority of extragalactic TeV sources have also been identified by Fermi, and thus there is a close relationship between the TeV blazars and the Fermi hard gamma-ray blazars (defined explicitly below).

The TeV blazars typically have falling SEDs above a TeV, with the brightest sources having a photon spectral index of \( \Gamma_{\text{TeV}} \approx 3 \) (where \( \Gamma_{\text{TeV}} \) is defined by \( dN/dE \propto E^{-\Gamma_{\text{TeV}}} \) from 100 GeV to 10 TeV), implying a peak in the SED at energies \( \lesssim 1 \) TeV. Below 100 GeV these sources are among the hardest in the Fermi AGN sample with rising SEDs, implying a peak above 100 GeV.

In principle, we define the TeV-band luminosity function of TeV blazars by

\[
\phi_B(z, L_{\text{TeV}}) = \frac{dN}{d\log_{10} L_{\text{TeV}} d\chi},
\]

where \( N^\prime \) is the number of TeV blazars with isotropic equivalent TeV luminosities above \( L_{\text{TeV}} \) within a physical volume of \( d\chi \), and in keeping with the notation in Papers I–III we denote quantities defined in terms of physical volumes by tildes (as opposed to comoving volumes).

Measuring \( \phi_B \) in practice is complicated by the large optical depth to annihilation on the EBL for gamma rays with energies above 100 GeV. The pair-production mean free path, \( D_{\text{pp}} \), is both energy and redshift dependent. We use the approximate form for \( D_{\text{pp}} \) obtained in Paper I, based upon Neronov & Semikoz (2009) and extended to higher redshifts as in Kneiske et al. (2004),

\[
D_{\text{pp}}(E, z) = 35 \left( \frac{E}{1 \text{ TeV}} \right)^{-1} \left( \frac{1 + z}{2} \right)^{-\xi} \text{Mpc},
\]

where \( \xi = 4.5 \) for \( z < 1 \) and \( \xi = 0 \) for \( z \geq 1 \). While more recent determinations of the EBL spectrum and thus the corresponding mean free path exist (e.g., Domínguez et al. 2011; Gilmore et al. 2012; Inoue et al. 2013), over the energy range of interest the resulting optical depths are similar, and we expect the updated EBL estimates to make little difference for our purposes here. The redshift evolution is due to the EBL and is sensitive primarily to the star formation history. The associated optical depth for a gamma ray emitted at redshift \( z \) and observed at an energy \( E_{\text{obs}} \) is then

\[
\tau(E_{\text{obs}}, z) = \int_0^z \frac{dD_{\text{pp}}}{dz'} d\chi' \frac{d\chi'}{D_{\text{pp}}(E_{\text{obs}}(1+z'), z')},
\]

\(^{10}\)In practice, blazars are highly beamed, and thus the true intrinsic luminosity is reduced by the appropriate beaming factor. However, this beaming factor is degenerate with the overall normalization of the blazar number, with smaller beams offset by correspondingly larger intrinsic numbers. More precisely, a structured jet viewed from different angles will have a range of beaming factors, and thus the emission will be boosted to different degrees. Thus given an intrinsic luminosity function, the corresponding apparent luminosity will not simply be boosted by the on-axis beaming factor (or some such characteristic beaming), but also broadened by the range in angles. It turns out that the luminosity function is dominated by the highest beaming factors, which generate a disproportionately large boost. Thus, in the interest of simplicity, here we consider only the isotropic equivalent luminosities for our empirical approach.
where \( D_P \equiv \int c dt = \int c dz/[H(z)(1+z)] \) is the proper distance.\(^\text{11}\) At 1 TeV this is unity at a redshift of \( z \approx 0.14 \), and TeV blazars are visible at only low redshifts, preventing a direct measurement of the evolution of \( \tilde{\Phi}_B \).

The existing collection of TeV blazars is the result of targeted observations, motivated by features in other wavebands, and is therefore subject to a number of ill-defined selection effects. Nevertheless, in Paper I we constructed an approximate luminosity function for these objects at \( z \approx 0.1 \). It was found that at low redshifts this was in excellent agreement (\( \chi^2/\text{dof} = 0.38/3 \)) with the quasar luminosity function, \( \Phi_Q \), given by Hopkins et al. (2007) and summarized in Appendix A, upon rescaling the bolometric luminosity and overall normalization:

\[
\tilde{\Phi}_B(0.1, L_{\text{TeV}}) \simeq 3.8 \times 10^{-3} \Phi_Q(0.1, 1.8 L_{\text{TeV}}). \quad (4)
\]

This is despite the fact that the shape of the quasar luminosity function evolves rapidly at low redshifts (\( \lesssim 1 \)), and thus even at moderately higher redshifts (\( z \sim 0.5 \)) the two would differ notably.\(^\text{12}\)

Included in this are a variety of uncertain corrections for various selection effects. Previously, we have attempted to estimate these by identifying the TeV blazars with the Fermi hard gamma-ray blazars. Within the context of the 2LAC we reconsider these, focusing upon the duty cycle (\( \eta_{\text{duty}} \)) and source selection (\( \eta_{\text{sel}} \)) corrections. There remains considerable uncertainty in the relevant source populations to compare, however.

The TeV blazars are necessarily at very low redshift, suggesting that we should compare them only to the nearby Fermi hard gamma-ray blazar population. Less clear is what redshift cut to impose. At \( z = 0.1, 0.15, \) and 0.2, there are 9, 14, and 17 TeV blazars and 16, 37, and 49 Fermi hard gamma-ray blazars with measured redshifts in the 2LAC, respectively (i.e., above the catalog’s flux limit). Noting that nearly all of the TeV blazars have now been detected by Fermi, this implies that the selection bias induced by the incomplete sky and time coverage of TeV observations requires a correction factor of 1.8–2.9. Furthermore, of the 277 Fermi hard gamma-ray blazars in the 2LAC, only 110 have measured redshifts. Assumed these are drawn from the same underlying population, this provides an additional correction of 2.5.\(^\text{13}\) In combination, the associated selection correction ranges from 4.5 to 9.8.

In Paper I and implicitly employed in Equation (4), we assumed \( \eta_{\text{sel}} \times \eta_{\text{duty}} \simeq 6.4 \), intermediate to those inferred from the above. However, the origin of this factor is rather different: the decrease in \( \eta_{\text{duty}} \) to unity has been nearly exactly offset by the increase in the overall number of Fermi sources and thus in \( \eta_{\text{sel}} \). Hence, the numerical factors in Equation (4), as derived in Paper I, remain unchanged.

Already inherent in the duty-cycle correction is the distinction between the observing cadences of Fermi and atmospheric Cerenkov telescopes. The Fermi flux measurements reported in the 2LAC represent 24 month averages, substantially smoothing the considerable intraday variability characteristic of blazars. In comparison, the TeV observations by atmospheric Cerenkov telescopes typically occur over many hours, probing the instantaneous source state. As a consequence, it is possible that the relationship between the two is complicated by variability. Nevertheless, we note first that the rising Fermi-band SEDs of the hard gamma-ray blazars provide strong evidence for the presence of TeV emission and second that the TeV luminosity for the putative plasma instabilities that are critical here are precisely the time-averaged values that we might infer from Fermi. For these reasons, we will assume that up to the corrections discussed above, we may directly compare the Fermi hard gamma-ray blazar and the observed TeV blazar populations.

Motivated by the strong similarities with the local quasar luminosity function, we posited the ansatz that this relationship held at large \( z \) as well. That is, we extrapolate the luminosity function obtained for \( z < 0.2 \) out to \( z \sim 5 \) and explore the potential consequences for Fermi. This extrapolation has already received indirect circumstantial support via the observational consequences of the plasma-instability-induced heating of the intergalactic medium described in Papers II, III, and Puchwein et al. (2012). Of particular note is the great success in the quantitative reproduction of the high-z Ly\( \alpha \) forest.

2.2. Relationship to the Fermi Blazars

While relating the TeV blazars and the Fermi hard gamma-ray blazars is natural in principle, some care must be taken in practice. Difficulties arise from the uncertain relationship between the observed Fermi-band fluxes and \( L_{\text{TeV}} \), the annihilation of the high-energy gamma rays, and the distribution of source properties. Here we assume a specific family of SEDs for the TeV blazars, use these to define the associated Fermi observables, and discuss the inherent restrictions upon the Fermi blazar population implied by these choices.

2.2.1. Intrinsic Fermi Hard Gamma-Ray Blazar SED

Relating the fluxes above a TeV and at energies relevant for Fermi (\( \lesssim 100 \text{ GeV} \)) requires some knowledge about the intrinsic SED of the TeV blazars. As already mentioned, the SED above a TeV is slowly falling, with a photon spectral index of 3 typical (i.e., \( E^2dN/dE \propto E^{-1} \)). However, for the two brightest TeV blazars in the sky, Mkn 421 and 1ES 1959+650, the Fermi photon spectral indexes, defined from 1 GeV to 100 GeV, are \( \Gamma_F = 1.77 \) and 1.94, respectively (i.e., \( E^2dN/dE \propto E^{-1.9} \)). This spectral shape is generic; TeV sources with spectra well characterized by Fermi below 100 GeV show a median shift between their photon spectral indexes below 100 GeV and above 1 TeV of 1.2 (see Figure 44 of Ackermann et al. 2011), and the typical \( \Gamma_F \) for the hard gamma-ray blazars is \( \sim 1.8 \) (see below).

Thus, it is clear empirically that a single power law is a poor model for the intrinsic SED (see, e.g., Figures 13 and 21 of Abdo et al. 2010a).

A more complicated SED is further motivated theoretically by the identification of the high-energy gamma-ray emission with the Comptonized synchrotron bump. Models of the blazar spectrum exhibit a peak near the TeV for the TeV-bright objects, suggesting that a similarly peaked SED must be considered in practice. Here, we model the intrinsic SED as a family of broken power laws:

\[
\frac{dN}{dE} = f \left[ \left( \frac{E}{E_b} \right)^{\Gamma_1} + \left( \frac{E}{E_b} \right)^{\Gamma_2} \right]^{-1}, \quad (5)
\]

\(^{11}\) In the definition of the Hubble function, we adopt the WMAP7 parameters, \( H_0 = 70.4 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \Omega_m = 0.272 \), and \( \Omega_{\Lambda} = 0.728 \), in terms of which, \( H(z)^2 = H_0^2 [\Omega_m (1+z)^3 + \Omega_{\Lambda} (1+z)^{3\Lambda}] \).

\(^{12}\) At \( z = 0.5 \) the resulting \( \chi^2 \) is nearly twice that at \( z = 0.1 \), though interpreting this difference in the presence of so few degrees of freedom (3) is difficult. More important, however, is that the fits at higher redshifts exhibit clear features in the residuals that are not present at \( z = 0.1 \).

\(^{13}\) This estimate should be taken with some caution, however. There seems to be growing evidence that most of the BL Lac objects without redshifts lie on average at larger redshift (e.g., Ajello et al. 2014). Hence, this would decrease the normalization somewhat.
for some normalization $f$ (with units of photons GeV$^{-1}$ s$^{-1}$), break energy $E_b$, and low- and high-energy photon spectral indexes, $\Gamma_h$ and $\Gamma_l$, respectively. We assume that the high-energy photon spectral index is synonymous with the photon spectral index above a TeV, and by definition the break energy lies within 100 GeV for the hard gamma-ray blazars. While in principle it is necessary to consider a distribution of $E_b$ and $\Gamma_h$, as we have done for $\Gamma_l$, in practice the statistics of interest are comparatively insensitive to reasonable variations in these (see, e.g., Figure 7 of Paper I). Therefore, we fix them to characteristic values based upon the properties of nearby, bright objects: after correcting for annihilation on the EBL, for the two objects that dominate the TeV flux at Earth, this is consistent with a value of roughly $\Gamma_h \simeq \Gamma_{\text{TeV}} = 3$ and a break energy of $E_b = 1$ TeV.

The choice of $\Gamma_l$ is complicated by the fact that some of the observable tests described in Section 3 are sensitive to its value. However, we have some observational guidance in the form of the Fermi photon spectral indexes for the TeV blazars themselves. Figure 1 shows the $F_{\Gamma}$ distribution of the nearby TeV blazars (blue solid) and the Fermi hard gamma-ray blazars (red dashed). The former is well fit by a Gaussian, with mean $\bar{\Gamma}_l = 1.78$ and standard deviation $\sigma_l = 0.18$. A Kolmogorov–Smirnov (K-S) test comparing the TeV blazar $\Gamma_l$ distribution to that implied by the fit produces a K-S probability of 0.68. Hence we conclude that the observed and fit $\Gamma_l$ are currently indistinguishable given the current TeV blazar sample. Based upon this we adopt the expanded luminosity function:

$$\tilde{\Phi}_B(z, L_{\text{TeV}}, \Gamma_l) = \frac{dN}{d\log_{10}L_{\text{TeV}} d\Gamma_l} e^{-((\Gamma_l - \Gamma_l)^2/2\sigma_l^2)} \sqrt{2\pi\sigma_l}. \tag{6}$$

Note that the $F_{\Gamma}$ distribution of the TeV blazars is somewhat harder than that of the hard gamma-ray blazars. This is likely due to the dramatic drop in TeV luminosity when the location of the Compton peak falls well below 100 GeV. We discuss this point and the limitation it implies in more detail in Section 2.2.3.

### 2.2.2. Relating the TeV Blazars and the Hard Gamma-Ray Blazars

The luminosity function in Equation (6) is still presented in terms of intrinsic quantities (e.g., $\Gamma_l$, $L_{\text{TeV}}$). However, frequently it will be necessary to relate these to quantities that are directly measurable by Fermi. These will be impacted both by the redshifting of the intrinsic spectrum and the gamma-ray annihilation on the EBL.

It is straightforward to show that the flux and fluence observed by Fermi between energies $E_m$ and $E_M$ (e.g., 100 MeV and 100 GeV) from a source at redshift $z$ are

$$F_F = \frac{1}{4\pi D_L^2} \int_{\Gamma_l(z,Em)}^{(1+z)E_M} dE \frac{dN}{dE} e^{-\tau(E/(1+z),z)}, \tag{7}$$

and

$$F_F = \frac{1+z}{4\pi D_L^2} \int_{\Gamma_l(z,Em)}^{(1+z)E_M} dE \frac{dN}{dE} e^{-\tau(E/(1+z),z)}, \tag{8}$$

respectively, where $D_L$ is the luminosity distance. While the particular units of flux and fluence depend upon those employed to define $D_L$, $E$, and $dN/dE$, here we adopt erg cm$^{-2}$ s$^{-1}$ and photons cm$^{-2}$ s$^{-1}$, respectively. Similarly, the intrinsic TeV luminosity is

$$L_{\text{TeV}} = \int_{0.1\text{TeV}}^{10\text{TeV}} dE \frac{dN}{dE}. \tag{9}$$

Thus, we have a redshift and SED-dependent relationship between $F_F$ and $L_{\text{TeV}}$:

$$\frac{L_{\text{TeV}}(z, \Gamma_l)}{4\pi D_L^2 F_F} = \frac{\int_{0.1\text{TeV}}^{10\text{TeV}} dE \frac{dN}{dE} (dN/dE)}{\int_{\Gamma_l(z,Em)}^{(1+z)E_M} dE \frac{dN}{dE} e^{-\tau(E/(1+z),z)}}, \tag{10}$$

where the denominator is simply the isotropic equivalent Fermi-band luminosity. This is shown for a handful of redshifts as a function of $\Gamma_l$ in Figure 2. Note that this neglects any inverse Compton cascade component, which would otherwise increase $F_F$ beyond the intrinsic emission.

Similarly important for the definition of the Fermi sources is the Fermi-band photon spectral index, $\Gamma_F$. Again this is modified by the redshift (different portions of the intrinsic spectrum are
being observed) and by absorption on the EBL. Assessing the impact these have upon the measured $\Gamma_F$ depends on how it is defined. Here we estimate $\Gamma_F$ via a least-squares fit to the redshifted and absorbed intrinsic spectrum\footnote{In practice this is done via a linear fit in $\log dN/dE$ vs. $\log E$.} between 1 GeV and 100 GeV, the energy range over which it is defined in the 2LAC. The impact upon $\Gamma_F$ is shown in Figure 3. At high redshift, even intrinsically hard spectra appear soft due to absorption. For example, a source at $z = 0.667$ with $\Gamma_i = \Gamma_l = 1.78$ will have a $\Gamma_F \simeq 2$. Thus, even moderate redshifts are sufficient to move objects out of the hard gamma-ray blazar class.

Also shown in Figure 3 is the evolution of the distribution of $\Gamma_F$ for the Fermi blazars. This may occur for a variety of reasons, including a correlation between $\Gamma_F$ and bolometric luminosity (see, e.g., Ghisellini 2011). Recently, it has been shown explicitly that this cannot account for the entirety of the spectral evolution, with absorption necessarily playing a role (Ackermann et al. 2012b). Here, we note that there is excellent agreement between the lower envelope of the Fermi sources and the evolution of the $\Gamma_F$ associated with the $2\sigma$ lower limit on $\Gamma_l$ from the TeV blazars alone (shown by the short-dash line).

Verifying that the BL Lac $\Gamma_l$ distribution is independent of redshift and described by Equation (6) is complicated by the fact that there are no BL Lac objects with $\Gamma_F > 2.5$. This is likely due to the aforementioned correlation between $\Gamma_l$ and luminosity, implying that high-redshift soft BL Lac objects preferentially fall below the Fermi detection threshold (see, e.g., Figure 14 of Ackermann et al. 2011). Nevertheless, we may invert the relationship between $\Gamma_l$, $\Gamma_F$, and $z$ to obtain the distribution of $\Gamma_l$ directly, shown in the bottom panel of Figure 3. Apart from the threshold on $\Gamma_F$, there does not appear to be a substantial evolution of the lower envelope with redshift.

We have verified this explicitly at $z < 1$ by performing a K-S test on the populations projected in redshift. To avoid issues with the detection threshold, we restricted ourselves to $\Gamma_l \leq 2$. The resulting K-S probabilities (i.e., the likelihood of random fluctuations resulting in a larger disparity from our Gaussian fit of Figure 1 than observed) are listed in Figure 3. All are quite large, implying that the lower envelope of the intrinsic photon spectral index distribution is indistinguishable from that at low redshifts.

### 2.2.3. Limitations on the Hard Gamma-Ray Blazar Luminosity Function

Our empirical TeV blazar luminosity function necessarily was constructed only for TeV-bright objects and thus only describes the TeV-bright blazar population. As a consequence, some care must be taken in extending this to the entire Fermi blazar population. In particular, the TeV blazar luminosity function poorly constrains the population of soft gamma-ray blazars. To address this, here we restrict ourselves to the class of Fermi blazars with flat or rising spectra and thus to the objects with intrinsic gamma-ray spectra likely to peak well above 100 GeV. That is, we consider only objects for which $\Gamma_l \leq 2$, for which the intrinsic SED in Equation (5) peaks around $E_0 = 1$ TeV. Empirically, this is evident from the lack of TeV blazars with $\Gamma_F > 2.1$.

Due to the spectral softening arising from absorption on the EBL and redshift, this condition on the intrinsic SED does not translate into a unique condition on the observed SED. That is, $\Gamma_l \leq 2$ does not generally imply that $\Gamma_F \leq 2$. The converse is, however, true: $\Gamma_F \leq 2$ does imply $\Gamma_l \leq 2$ generally, as may be seen immediately in Figure 3. Thus, where we wish to construct populations of TeV-bright objects from the Fermi blazar sample for comparison with the hard gamma-ray blazar luminosity function described in the previous section, we will consider only the Fermi hard gamma-ray blazars. This includes the log $N$–log $S$ relation and redshift distributions described in Sections 3.1 and 3.3, respectively.

When we consider the TeV blazar contribution to the Fermi EGRB, we do not require a corresponding population of Fermi blazars and thus retain only the more conservative condition on $\Gamma_l$. Concerns regarding the generality of the associated high-energy EGRB are discussed in Section 3.4; here we simply note that the neglected population of soft gamma-ray blazars is significant only below a few GeV.
Superficially, it remains unclear if this is consistent with recent efforts to empirically construct the luminosity function of Fermi BL Lac objects. In particular, the apparent negative evolution in the comoving number density of HSPs found by Ajello et al. (2014) suggests that we may not be justified in extrapolating the TeV blazar luminosity function to high redshift using the strongly positive evolution found in quasars. However, we note a number of reasons for caution here. First, we identify both the HSPs and nearly half of the ISPs with TeV blazars, where the latter are found to have a positive evolution in Ajello et al. (2014), ameliorating the situation marginally. Second, our luminosity function is consistent with various other attempts at constructing the luminosity functions of the gamma-ray blazars and associated populations (e.g., Giommi et al. 2012, 2013; Willott et al. 2001; Wall et al. 2005). Third, and perhaps most importantly, in the following section we demonstrate that just such a rapidly evolving luminosity function is able to simultaneously reproduce the statistics of the hard Fermi blazars and the EGRB.

3. COMPARISONS WITH THE FERMI HARD GAMMA-RAY BLAZARS

We now turn our attention to comparing the implications of the luminosity function obtained in the previous section with various measures of the Fermi hard gamma-ray population. We note that the specific properties of the luminosity function and the intrinsic spectra are now completely defined, and thus in the following comparisons to the Fermi blazar sample there are no degrees of freedom to adjust.

Both the \( \log N - \log S \) and redshift distribution probe the recent evolution of the hard gamma-ray blazar luminosity function. Since they are both one-dimensional, they are both necessarily projections of \( \phi_B \). They differ in the form of the projection, measuring in different degrees the redshift evolution and the luminosity distribution of the hard gamma-ray blazars. In contrast, the Fermi isotropic EGRB is most sensitive to the unresolved sources at high redshifts and thus probes the peak of the luminosity function in both redshift and luminosity. While all are important, the isotropic EGRB is likely to provide the most significant constraint upon the viability of rapidly evolving hard gamma-ray blazar luminosity functions.

3.1. 2LAC \( \log N - \log S \) Relation

The \( \log N - \log S \) relation describes the flux distribution of a particular source class. In it, \( N(S) \) is simply the number of sources with fluxes \( > S \), making it straightforward to define empirically. Complications arise in selecting the particular source class of interest, the definition of “flux” to be employed, and the treatment of observational selection effects. All of these are relevant for Fermi, and thus here we describe how we constructed the Fermi \( \log N - \log S \) relation for the hard gamma-ray blazars and its relation to the hard gamma-ray blazar luminosity function discussed in the previous section.

3.1.1. Observational Definition

Depending on the application, the fluence from 100 MeV to 100 GeV (\( F_{25} \)), the fluence from 1 GeV to 100 GeV (\( F_{35} \)), and the flux from 100 MeV to 100 GeV (\( F_{25} \)) have all been used as “flux” measures for Fermi sources. Primarily, \( F_{25} \) and \( F_{25} \) have been used to assess statistical properties of Fermi sources (see, e.g., Abdo et al. 2010b; Ackermann et al. 2011). This includes an empirical reconstruction of the \( \log N - \log S \) relation for the Fermi blazars in the 1LAC in terms of \( F_{25} \) (Abdo et al. 2010c; Singal et al. 2012). However, within the 2LAC itself, \( F_{35} \) is the flux measure reported.

We relate these here by assuming the spectrum across the Fermi LAT band (100 MeV to 100 GeV) is well approximated by a single power law, \( dN/dE \propto E^{−Γ_F} \), and therefore the fluence and flux are

\[
F = f_F \int_{E_{\text{min}}}^{E_{\text{max}}} dE E^{−1+Γ_F},
\]

and

\[
F = f_F \int_{E_{\text{min}}}^{E_{\text{max}}} dE E^{−1+Γ_F} / \Gamma_F,
\]

respectively.\(^{15} \) The normalization, \( f_F \), is set by the reported \( F_{35} \), from which \( F_{25} \) and \( F_{25} \) may then be readily computed (see the Appendix B for explicit expressions).

Figure 4 shows the \( \log N - \log S \) relation for all of the Fermi blazars in the 2LAC with a signal-to-noise ratio (S/N) of \( \geq 7 \), defined in terms of \( F_{25} \) and \( F_{25} \). The precision with which the \( \log N - \log S \) relation can be reconstructed empirically is limited by both the intrinsic measurement uncertainty \( (\delta_x) \) and the limited number of AGNs. We attempt to assess this uncertainty via a Monte Carlo simulation of the Fermi catalog, using the reported measurement uncertainties (assuming normal and log-normal error distributions for \( \Gamma \) and \( F_{35} \), respectively) and constructing bootstrap samples of the 2LAC. The 2σ regions are shown by the gray-shaded regions in Figures 4 and 5. However, we note that the errors at various fluxes are strongly correlated due to the cumulative definition of \( N(S) \) and thus must be interpreted cautiously.

The \( F_{25} \) \( \log N - \log S \) relation is in good agreement with the empirically constructed \( \log N - \log S \) relation from Singal et al. (2012), providing some confidence in our reconstructed \( F_{25} \) itself. Clearly evident in both forms of the \( \log N - \log S \) relation is a flattening at small fluxes. Singal et al. (2012) identify this with a systematic bias induced by a correlation between the \( F_{25} \) flux limit and the source spectral index, resulting in fewer soft sources being detected below a few \( \times 10^{-8} \) photons cm\(^{-2} \) s\(^{-1} \) (see Figure 14 of Ackermann et al. 2011). This results in a break in the \( \log N - \log S \) relation roughly at the value for \( F_{25} \) at which the sample becomes incomplete. Unlike \( F_{25} \), the flux limit in \( F_{25} \) is only weakly dependent upon \( \Gamma_F \) (see Figures 14 and 15 in Ackermann et al. 2011), and the break is correspondingly weaker.

Despite the known bias, Singal et al. (2012) have argued based upon the 1LAC that the intrinsic \( \log N - \log S \) relation does indeed have a break near \( F_{25} \approx 6 \times 10^{-8} \) photons cm\(^{-2} \) s\(^{-1} \). We believe this is suspect for three reasons. First, its location is very near the bias-induced break in the 1LAC. Second, the location of the break in the \( \log N - \log S \) relation constructed from the 2LAC blazars appears to have moved toward marginally lower fluxes. Third, the break is considerably less prominent when a less-biased flux is employed, namely \( F_{25} \). This points to an as-yet- unidentified source of bias for \( F_{25} \lesssim 10^{-11} \) erg cm\(^{-2} \) s\(^{-1} \), and thus we will restrict ourselves to fluxes above this cutoff.

The \( F_{25} \) \( \log N - \log S \) relation for the Fermi hard gamma-ray blazars specifically is shown in Figure 5. Aside from the

\(^{15} \) Here we assumed \( \Gamma_F \neq 1 \) and \( \Gamma_F \neq 2 \), respectively. While the first condition is empirically true, in the case of \( \Gamma_F = 2 \) we have \( F = f_F \log (E_M/E_{\text{min}}) \).
restriction to blazars with $\Gamma < 2$, this is constructed in a fashion identical to those described above. Apart from the reduced number of sources, it shares many of the qualitative features found for the log $N$–log $S$ relation from Singal et al. (2012) is shown by the red line. For the latter, the Singal et al. (2012) log $N$–log $S$ relation has been inferred using the average spectral index, 2.13.

(A color version of this figure is available in the online journal.)

![Figure 4](image-url)

Figure 4. The log $N$–log $S$ relation defined in terms of $F_{25}$ (top) and $F_{25}$ (bottom) for the Fermi blazars in the 2LAC uncorrected for selection effects. The shaded region provides an estimate of the 2σ uncertainty in the Fermi log $N$–log $S$ relation due to the measurement uncertainty on $F_{25}$ and the Poisson fluctuations in the sample itself. The empirically reconstructed log $N$–log $S$ relation from Singal et al. (2012) is shown by the red line. For the latter, the Singal et al. (2012) log $N$–log $S$ relation has been inferred using the average spectral index, 2.13.

(A color version of this figure is available in the online journal.)

not be well approximated by a single power law. Above $F_{25} \simeq 10^{-11}$ erg cm$^{-2}$ s$^{-1}$, Singal et al. (2012) found $N \propto F_{25}^{-1.37 \pm 0.13}$, which were it to continue indefinitely to small fluxes would imply that $F_{25,\text{tot}}$ diverges at the faint end.

### 3.1.2. Relationship to $\phi_B$

The primary difficulty in producing an log $N$–log $S$ relation to compare with that constructed using the Fermi 2LAC blazars is the treatment of the particular selection effects relevant for the population of interest. Specifically, it is necessary to produce cuts on $\Gamma_F$ and $F_{25}$. Thus, we define

$$N = \eta_F \int_0^{\Gamma_F} d\Gamma_F \int_0^{\Gamma_F} d\Gamma \int_{\log_{10} L_{\text{TeV}}(F_{25}, \Gamma)}^{\infty} d\log_{10} L_{\text{TeV}}$$

$$\times 4\pi D_A^2 \frac{dD}{dz} \Theta[2 - \Gamma F(\Gamma, z)] \phi_B(z, L_{\text{TeV}}, \Gamma),$$

where $L_{\text{TeV}}(F_{25}, \Gamma)$ is given by Equation (10), $\Gamma F(\Gamma, z)$ is obtained as described in Section 2.2.2, $\Theta(x)$ is the Heaviside function (vanishing for $x < 0$ and unity otherwise), and the cuts on $\Gamma_F$ and $\Gamma_F$ are motivated by Section 2.2.3 (note that the cut on $\Gamma_F$ is redundant). The coefficient $\eta_F \approx 0.826$ is the correction due to the sky coverage of the Fermi clean sample ($b > 10^\circ$, where $b$ is the Galactic latitude). This is compared to the observed Fermi log $N$–log $S$ relation for the hard gamma-ray blazars in Figure 5.

![Figure 5](image-url)

Figure 5. The log $N$–log $S$ relation in terms of the flux from 100 MeV to 100 GeV associated with the expanded hard gamma-ray blazar luminosity function presented in Equation (6) in comparison with that from the 2LAC Fermi hard gamma-ray blazar sample. The shaded region provides an estimate of the 2σ uncertainty in the Fermi log $N$–log $S$ relation due to the measurement uncertainty on $F_{25}$ and the Poisson fluctuations in the sample itself. The flattening in the Fermi log $N$–log $S$ relation at low fluxes is probably an artifact of the Fermi flux limit. For reference, the contributions to the log $N$–log $S$ relation from hard gamma-ray blazars with $z \lesssim 0.1$ (green dotted) and $z \lesssim 0.3$ (red dashed) are shown.

(A color version of this figure is available in the online journal.)

This provides an independent constraint upon the overall normalization once resolved point sources have been removed. The associated Fermi limit on $F_{25,\text{tot}}$ is $18 \pm 2.4 \times 10^{-5}$ photons cm$^{-2}$ s$^{-1}$, while that implied by the log $N$–log $S$ relation in Singal et al. (2012) is $11 \times 10^{-5}$ photons cm$^{-2}$ s$^{-1}$. Note the condition that $F_{25,\text{tot}}$ be finite implies that $N$ can-
is part of the justification for using the extended luminosity function in Equation (6). The location of the resulting break after integrating over the TeV blazar $\Gamma_1$ distribution is near $F_{25} = 1.6 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$, roughly a factor of three below Fermi’s stated flux limit for the 2LAC and a factor of six below the point at which unknown systematic effects appear to produce an artificial flattening of the Fermi log $N$–log $S$ relation.

Above and below the break we obtain $N \propto F_{25}^{1.42}$ and $N \propto F_{25}^{-0.75}$, respectively. Notably, despite the difference in the location of the cutoff, both of the power laws are consistent with those reported in Singal et al. (2012). In the case of the latter, however, we suspect the agreement is incidental.

Above a flux of $F_{25} = 10^{-11}$ erg cm$^{-2}$ s$^{-1}$, the minimum flux at which we trust the Fermi log $N$–log $S$ relation, Equation (14) reproduces the observed relation qualitatively. This is especially true for $F_{25} \lesssim 10^{-10}$ erg cm$^{-2}$ s$^{-1}$. At higher fluxes the paucity of sources induces large Poisson errors, and thus the excess bump at and above this flux is not significant.

Nevertheless, the predicted and observed distributions do differ at statistically significant levels. In principle, we must specify the flux range over which the comparison is made, which we characterize by a lower flux limit corresponding to the minimum flux at which the detection efficiency is close to unity. In practice, this conclusion is insensitive to the range in flux over which the comparison is made. For all lower limits on $F_{25}$ above $10^{-11}$ erg cm$^{-2}$ s$^{-1}$ the K-S probabilities were $\lesssim 10^{-4}$, implying that the theoretical and observed log $N$–log $S$ relations are different.

Because the hard gamma-ray blazar contributions to the EGRB in detail in Section 3.4, here we simply note that the anticipated contribution to the Fermi EGRB is $F_{25} \text{tot} \simeq 1.19 \times 10^{-5}$ photons cm$^{-2}$ s$^{-1}$. This corresponds to roughly 6.6% of the total Fermi EGRB from 100 MeV to 100 GeV and 11% of that implied by the empirical reconstruction from the 1LAC by Singal et al. (2012). That the hard gamma-ray blazars are responsible for such a small fraction of the EGRB is not surprising; below 10 GeV the EGRB is dominated by soft blazars, i.e., FSRQs and soft BL Lac objects. Nevertheless, even at 100 MeV we expect the hard gamma-ray blazars to account for roughly 10% of the EGRB.

### 3.2. The 1FHL log $N$–log $S$ Relation

Because the hard sources necessarily dominate at high energies, the recently published 1FHL, a catalog of Fermi sources detected above 10 GeV, provides a means to probe the hard-source population directly. Already it is clear that for hard sources the flattening at low fluxes is almost entirely, if not entirely, an artifact of the LAT detection efficiency near the flux threshold (see Appendix C.1 and Figures 31–33 of Ackermann et al. 2013). Thus, there is currently no evidence for a break in the log $N$–log $S$ relation for the high-energy Fermi population.

As with the 2LAC sources, we may compare the log $N$–log $S$ relation of 1FHL sources with that anticipated by the extended luminosity function in Equation (6). Because the 1FHL reports the fluence between 10 GeV and 500 GeV explicitly ($F_{45.7}$), eliminating the need to perform a spectral correction to this energy band, constructing the observed log $N$–log $S$ relation within this energy band is somewhat simplified. It is, however, complicated by the fact that the 1FHL also includes a Galactic component that must be removed. We do this by considering only high-latitude ($|b| > 20^\circ$) sources that are identified as BL Lac objects. The resulting log $N$–log $S$ relation is shown in Figure 6 and is comparable to Figure 33 of Ackermann et al. (2013).

The anticipated log $N$–log $S$ relation is constructed in a manner similar to that in the previous section, replacing the relevant flux measure with $F_{45.7}$ and adjusting the correction to account for the differing sky coverage of the high-latitude 1FHL sample adopted. In addition, since the 1FHL detection efficiency is provided in Ackermann et al. (2013), we make an effort to correct the log $N$–log $S$ relation near the detection threshold, as described in Section C.1. This is compared to the measured high-energy BL Lac log $N$–log $S$ relation in Figure 6, providing an excellent fit over more than an order of magnitude in fluence. As with the 2LAC log $N$–log $S$ relation shown in Figure 5, there is an excess of sources at high fluxes in the 1FHL, though this is not significant. We do predict a weak break near $F_{45.7} \simeq 2\sim 3 \times 10^{-11}$ photons cm$^{-2}$ s$^{-1}$, or roughly 50% of the fluence of the dimmest 1FHL source and hence potentially accessible in the future.

This nearby excess notwithstanding, for the 1FHL the agreement is also quantitatively excellent. Performing a Kolmogorov–Smirnov test in this case requires applying the 1FHL detection efficiency to the theoretically obtained log $N$–log $S$ relation (as opposed to removing it from the observed source distribution, as done in Figure 6). Unlike for the 2LAC comparison, with the detection efficiency known, we can perform this comparison with some confidence. Upon doing this, we find a K-S probability of 73%, implying that the predicted and observed 1FHL flux distributions are indistinguishable.

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16. Note that in Singal et al. (2012), the power law indexes are for $dN/dF_{25}$.

17. These comprise roughly half of the 1FHL sample and are the dominant extragalactic component.
3.3. Hard Gamma-Ray Blazar Redshift Distribution

In principle, the evolution in the number density of the nearby Fermi hard gamma-ray blazars is directly probed by their observed redshift evolution. As with the log \( N \)–log \( S \) relation, this is straightforward to define observationally. However, in practice, it is complicated by the flux-limited nature of the 2LAC, significantly impacting even moderate redshifts, and the limited number of sources with known redshifts (roughly 39%). Nevertheless, it represents a different projection of the hard gamma-ray blazar luminosity function and provides a powerful additional test of the viability of a rapidly evolving blazar population.

Within the context of the 1LAC, we demonstrated in Paper I that the relatively large flux limit was capable of generating a precipitously declining observed number of blazars, \( N_B \), with redshift. However, there we made a number of assumptions and approximations regarding the intrinsic hard gamma-ray blazar spectra and their relationship to \( L_{\text{TeV}} \). Here we revisit this within the more complete 2LAC and in terms of the more fully self-consistent TeV blazar model described in Section 2. Particular improvements over the computation in Paper I are the self-consistent relationship between \( L_{\text{TeV}} \) and \( F_{25} \), the distribution of \( \Gamma_l \), and the ability to now dispense with the upper limit on the TeV luminosity, to which our results are insensitive.

The definition of \( N_B(z) \) differs from \( N \) only by the limits of integration:

\[
N_B(z) = \int_0^2 d\Gamma_l \int_0^z dz' \int_{\log_{10} L_{\text{TeV}}(F_{25,\min}, z, \Gamma_l)}^{\infty} d\log_{10} L_{\text{TeV}} \\
\times 4\pi d_A^2 \frac{dD_P}{dz'} \mathcal{Q}(2 - \Gamma_F(\Gamma_l, z')) \tilde{\phi}_B(z', L_{\text{TeV}}, \Gamma_l),
\]

(15)

where \( F_{25,\min} \approx 5 \times 10^{-12} \) erg cm\(^{-2}\) s\(^{-1}\) is the flux limit of Fermi (see Figures 15 and 36 of Ackermann et al. 2011). As with the log \( N \)–log \( S \) relation, the spectral cut on \( \Gamma_F \) induces a \( \Gamma_l \)-dependent redshift cutoff, limiting the potential contributions from very bright objects at high \( z \). This mimics the luminosity upper limit we applied in Paper I, removing its necessity.\(^{18}\) In practice, we compare \( d\log N_B/dz \), both to avoid correlations in the errors at subsequent redshifts and because the overall normalization has already been compared in the context of the log \( N \)–log \( S \) relation.

Figure 7 shows the \( d\log N_B/dz \) for Fermi hard gamma-ray blazars with \( S/N \geq 7 \) in comparison to that implied by Equation (15). As in Paper I, the agreement is quite good: for \( z < 2 \), the resulting \( \chi^2/\text{dof} = 10.6/10 \) and the K-S probability is 14%, both comfortably implying that the source redshift distribution is reproduced quantitatively. Despite this, we miss what appears to be a small population of high-redshift objects. This may suggest an issue with our estimation of \( \Gamma_F(z, \Gamma_l) \), our distribution in \( \Gamma_l \), or with the source identification in the 2LAC at high \( z \). Alternatively, it may suggest a possibly faster evolution for blazars in comparison with quasars, as is the case for jet sources (i.e., radio-loud quasars, see, e.g., Singal et al. 2011, 2013). In any case, it is clear that a rapidly evolving TeV blazar population is explicitly consistent with the observed

\[^{18}\text{That such a limit exists, however, is strongly supported by the lack of a significant number of HSPs in the 2LAC with } z > 1.\text{ Note that unlike the hard gamma-ray blazars, the HSPs are defined by the location of the synchrotron peak, and thus their definition is unaffected by the annihilation on the EBL suffered by the gamma rays.}\]

\[^{19}\text{We note that we only limit our analysis to sources with reported redshifts in the comparison of the redshift distribution and the validation of the spectral softening shown in Figure 3 while we include the sources without redshifts when we compare our model log } N \text{–log } S \text{ relations and the spectrum and the anisotropy of the EGRB to the data.}\]
Figure 8. Fermi isotropic EGRB anticipated by the hard gamma-ray blazars. The dotted, dashed, and solid lines correspond to the unabsorbed spectrum, the spectrum corrected for absorption on the EBL, and the spectrum corrected for resolved point sources (assuming all hard gamma-ray blazars with $z < 0.291$ are resolved; see text). These are compared with the measured Fermi EGRB reported in Abdo et al. (2010d, red squares) and Ackermann (2012, blue circles). Note that below ~10 GeV the EGRB is dominated by soft sources, specifically, the Fermi FSRQs.

(A color version of this figure is available in the online journal.)

Figure 9. Contribution to the Fermi EGRB from hard gamma-ray blazars below various redshifts, specifically, for objects with $z < 0.5$ (blue dot), 0.75 (green short dash), 1.0 (yellow long dash), 1.5 (orange short dash dot), 2.0 (red long dash dot), 2.5 (dark red long dash–short dash), and all redshifts (black solid). In all cases it was assumed that all sources with $z < 0.291$ are resolved. These are compared with the measured Fermi EGRB reported in Abdo et al. (2010d, red squares) and Ackermann (2012, blue circles). Note that below ~10 GeV the EGRB is dominated by soft sources, specifically, the Fermi FSRQs. The inset shows the cumulative flux fraction as a function of redshift for 1 GeV (red solid), 10 GeV (green long dash), and 100 GeV (blue short dash).

(A color version of this figure is available in the online journal.)

The EGRB must be substantially suppressed above a TeV. That is, the power-law behavior implied by the Abdo et al. (2010d) measurement of the Fermi EGRB cannot extend significantly beyond the 100 GeV upper limit for which it was reported. This is seen explicitly in Figure 8, where for the distribution of $\Gamma_{\ast}$ adopted in Section 2.2.1 the anticipated contribution to the EGRB peaks near 10 GeV is followed by a rapid decline at larger photon energies. This provides a remarkable agreement with the recent estimate of Fermi EGRB spectrum by Ackermann (2012), shown by the blue circles in Figure 8; above 3 GeV, $\chi^2$/dof = 10.4/14. Moreover, the spectrum of the EGRB above 6 GeV appears to show a high-energy bump on a monotonically decreasing spectrum, which we identify with the specific population of hard gamma-ray blazars.

Even in the presence of substantial absorption, the bulk of the EGRB is produced at high redshifts, seen explicitly in Figure 9. At 10 GeV, roughly half of the observed EGRB is due to objects with $z > 1$. The typical redshifts that contribute are necessarily energy-dependent, with the higher energy EGRB arising from more nearby sources. Nevertheless, it is clear that even above a few GeV the Fermi EGRB is probing the high-redshift blazar population.

The contribution to the EGRB spectrum from the hard gamma-ray blazars below 10 GeV is nearly flat and consequently dominates their contribution to $F_{\text{25S}}$, consisting of roughly 10% and responsible for the value obtained in Section 3.1. However, this number is quite uncertain, depending upon the behavior of the extension of the gamma-ray blazar luminosity function to $\Gamma_{\ast} > 2$.

Below a few GeV the Fermi EGRB is dominated by soft gamma-ray sources, potentially including FSRQs (see, e.g., Cavolini et al. 2011; Stecker & Venters 2011; but also see Ajello et al. 2012), radio galaxies (Inoue 2011), and millisecond pulsars (Faucher-Giguère & Loeb 2010). The spectral shape, distribution on the sky, and total magnitude of the contribution...
from pulsars are highly uncertain, depending critically on the details of pulsar evolution. The intrinsically soft spectra of the potential extragalactic sources combined with their typically larger luminosities (and thus higher redshifts) confine their contribution to below ~3 GeV. Thus, our neglect of these is unlikely to significantly change the Fermi EGRB above 10 GeV, where the hard gamma-ray blazars successfully reproduce the observed background.

We note that the overall normalization of the TeV blazar luminosity density is subject to an uncertain correction factor that depends primarily on the incomplete census of the observed TeV blazar population and enters linearly into the overall normalization of the EGRB. We estimated this correction factor using the source counts of hard Fermi blazars and confirm its value by comparison to the redshift and cumulative flux distributions of these objects. Nevertheless, there are remaining uncertainties associated with the contribution of sources without measured redshifts and with the extrapolation of that population of the Fermi band to TeV energies. If another plausible source population such as starburst galaxies contributes a non-negligible, but subdominant, signal to the Fermi-GeV blazar redshift evolution. Despite this uncertainty, the impressive match between the EGRB shapes at energies above ∼3 GeV strongly suggests that it is dominated by a rapidly evolving hard gamma-ray blazar population.

3.5. Anisotropy of the Extragalactic Gamma-Ray Background

In principle, the anisotropy of the EGRB limits the potential contribution from discrete sources, providing a second direct constraint on the fraction of the EGRB associated with blazars. The angular power in the EGRB on small angular scales is the constraint on the fraction of the EGRB associated with blazars. It is possible, however, to unambiguously compare the anticipated hard gamma-ray blazar contribution to the EGRB anisotropy spectrum with that anticipated by the hard gamma-ray blazar luminosity function in Equation (6). The estimates associated with the power-law extension of the Fermi LAT 2FGL source population (Broderick et al. 2013) are shown by the green, orange, and magenta triangles (left to right). The corresponding expectations from the hard gamma-ray blazars are shown by the dark green, red, and purple circles (left to right). In all cases, bars denote the 1σ cosmic variance uncertainty. For reference, the gray bars show the energy bins employed and values reported in Ackermann et al. (2012a), though see Broderick et al. (2013) regarding a discussion of their normalization. Points are horizontally offset within each bin for clarity.

Figure 10. Various estimates of the EGRB anisotropy spectrum compared with that anticipated by the hard gamma-ray blazar luminosity function in Equation (6). The estimates associated with the power-law extension of the Fermi LAT 2FGL source population (Broderick et al. 2013) are shown by the green, orange, and magenta triangles (left to right). The corresponding expectations from the hard gamma-ray blazars are shown by the dark green, red, and purple circles (left to right). In all cases, bars denote the 1σ cosmic variance uncertainty. For reference, the gray bars show the energy bins employed and values reported in Ackermann et al. (2012a), though see Broderick et al. (2013) regarding a discussion of their normalization. Points are horizontally offset within each bin for clarity.

(A color version of this figure is available in the online journal.)

where \( F_{\text{MM}} \) is the fluence in the specified energy band, specified in Equation (8), with the energy range explicitly identified, and \( u(F_{35}) \) is a weighting that describes the detection efficiency for the sample under consideration (see below).

In practice, the normalization of the EGRB anisotropy spectrum reported in Ackermann et al. (2012a) is inconsistent with the contributions arising from sources already resolved in the 2 Year Fermi LAT Source Catalog (2FGL). Detected sources in the 1FHL alone, without correcting for the 1FHL detection efficiency, are sufficient to account for the entirety of the reported EGRB anisotropy signal above 10 GeV (Broderick et al. 2013). Thus, consistency with the reported EGRB anisotropy would require a dramatic, and implausible, suppression in the log \( N_{\text{ps}} \) relation, shown in Figure 6, immediately below the 1FHL detection threshold.

It is possible, however, to unambiguously compare the anticipated hard gamma-ray blazar contribution to the EGRB anisotropy spectrum with that either from the 2FGL sources alone (for which an unambiguous estimate does exist) or simple extrapolations of the 2FGL source population (providing a reasonable upper limit). In the former, we consider the contribution to the EGRB arising from blazars that lie between the 2FGL and 1FGL detection thresholds. In the latter we consider all sources below the 1FHL flux limit but compare the result to the EGRB anisotropy due to the power-law extrapolation of the 2FGL fluence distribution described in Broderick et al. (2013).

These comparisons are distinguished by the form of the weighting function, \( u(F_{35}) \), appearing in Equation (18), which in both cases may be constructed from the detection efficiencies of the 1FGL and 2FGL, \( u_{1\text{FGL}}(F_{35}) \) and \( u_{2\text{FGL}}(F_{35}) \), respectively (explicit expressions for these are provided in Appendix C.2).

In the case of the power-law extrapolation we need only to exclude sources that are detected in the 1FGL, i.e., \( u(F_{35}) = 1 - u_{1\text{FGL}}(F_{35}) \). When comparing to the 2FGL contribution, we...
must also consider the probability that sources are detected in the 2FGL, thus \( \psi(F_{35}) = (1 - \epsilon_{2FGL}(F_{35}))\epsilon_{1FGL}(F_{35}) \).

The EGRB anisotropy implied by the hard gamma-ray blazar luminosity function in Equation (6) is significantly larger than the values reported in Ackermann et al. (2012a) in all energy bands. This is unsurprising given that the reported values appear to be a substantial underestimate of the EGRB anisotropy spectrum itself (Broderick et al. 2013). Nevertheless, it is consistent with (i.e., lies below) the values anticipated from a smooth power-law extrapolation of the 2FGL, shown in Figure 10. More importantly, a similar conclusion follows from the comparison to the contribution to the EGRB anisotropy from the 2FGL sample alone. This holds for both the full 2FGL sample and the subsample of hard sources (i.e., \( \Gamma_F < 2 \)). Thus, despite being inconsistent with the reported values in Ackermann et al. (2012a), the hard gamma-ray blazar luminosity function in Equation (6) is able to reproduce the inferred anisotropy signal associated with the currently known and smoothly extrapolated point-source samples, respectively.

The \( C_{P,nM} \) are dominated by nearby sources, i.e., \( z \lesssim 1 \). This is clearly seen in Figure 11, in which the contribution from sources with \( z > 1 \) is below 33% above 1.99 GeV and rapidly decreasing with redshift cut. As a result, unlike the isotropic EGRB component, the EGRB anisotropy is a probe of the nearby blazar distribution (immediately below the 1FGL detection threshold). This is a direct result of the dominance of sources near the detection threshold in the definition of the \( C_{P,nM} \).

Hence, the agreement with the 2FGL contribution to the anisotropy is largely anticipated by the success at reproducing the statistics of the low-redshift hard gamma-ray blazar sample described in Sections 3.1 and 3.3.

The fractional contribution of intrinsically hard sources increases with energy, though even in the highest energy bin (10.4 GeV–50 GeV) sources with \( \Gamma_I < \Gamma_F \) contribute less than half of the anisotropy signal (see the blue squares in Figure 11). Therefore, even a moderate restriction on \( \Gamma_F \) produces a substantial reduction in the anisotropy at all energies, implying that our anisotropy estimates are sensitive to the high-\( \Gamma_F \) extension of the gamma-ray blazar luminosity function. Despite this uncertainty, the hard gamma-ray blazars contribute substantially, if not dominantly, to the anisotropy above roughly 3 GeV, consistent with their contribution to the isotropic EGRB. That is, it is possible to simultaneously match both the isotropic and anisotropic components of the EGRB with the single hard gamma-ray blazar population postulated here.

It is tempting to conclude that the absence of inverse Compton cascades, which could be preempted by the presence of virulent plasma beam instabilities, enables a notable consistency within the context of the simplest model conceivable. That is, the resolved source class of hard gamma-ray blazars, which dominates the extragalactic high-energy regime, also dominates the angular power as well as matches the detailed shape and normalization of the isotropic EGRB intensity above 3 GeV. However, as noted above, the current ambiguity in the normalization of the reported EGRB anisotropy currently precludes such a statement in general. The consistency obtained in Figure 10 is largely degenerate with the success in reproducing the low-redshift gamma-ray blazar population. Given the dominance of low-redshift source contribution to the anisotropy, this is likely to continue to be the case in the future. As a result, even with the dramatic evolution in the blazar population posited here, the EGRB anisotropy will predominantly probe the low-redshift blazar distribution generally.

4. CONCLUSIONS

In contrast to previous claims, a quasar-like evolution in the number density of TeV blazars is fully consistent with the properties of the currently observed Fermi population. Future enlargements of the statistical blazar samples through ongoing Fermi observations, including the forthcoming 3LAC and 3FLG, will provide a critical test for our proposed scenario and, in particular, will help to clarify whether the strong redshift evolution posited here continues to be consistent with the observed hard blazar population. The chief uncertainties in the analysis presented here remain fundamentally astrophysical: (1) how to relate the fluxes within the Fermi-relevant energy range and the intrinsic TeV luminosity, used to define the TeV blazar luminosity function; and (2) the efficiency of the inverse Compton cascades, if present at all.

A broken power-law model for the intrinsic TeV blazar spectrum, with a generic break energy and high-energy photon spectral index of 1 TeV and 3, respectively, is sufficient to reproduce many of the features of the Fermi hard gamma-ray blazar population. The TeV blazar luminosity function was constructed for blazars that were observed at TeV energies and hence is fundamentally limited to spectra that peak near \( \sim 1 \) TeV, and thus we necessarily impose an upper cutoff of 2 in the low-energy photon spectral index. This cutoff is empirically supported by the observed distribution of Fermi photon spectral indexes for the known TeV blazars.

Modeling systematic biases is crucial to relating the intrinsic blazar population and the Fermi blazar sample. Of these, the
most important is the softening of the Fermi-band spectra due to absorption on the EBL, which causes a strong redshift-dependent evolution in the observed photon spectral index from 1 GeV to 100 GeV. This, in turn, induces a significant sensitivity to the form of the intrinsic spectrum below 1 TeV, generally, and in our case the low-energy photon spectral index, specifically. For this reason, to obtain robust estimates of the anticipated log $N$–log $S$ relation and redshift distribution of nearby Fermi hard gamma-ray blazars, we found it necessary to expand the definition of the TeV blazar luminosity function to include the low-energy spectral index distribution. This is well approximated by a Gaussian peaked at a photon spectral index of 1.78 and standard deviation of 0.18. Due to the redshift-dependent spectral softening, the log $N$–log $S$ relation and hard gamma-ray blazar redshift distribution both probe primarily $z \lesssim 1$.

The 2LAC log $N$–log $S$ relation is well reproduced for 100 MeV–100 GeV fluxes above $10^{-11}$ erg cm$^{-2}$ s$^{-1}$. At smaller fluxes a catalog-dependent flattening of the log $N$–log $S$ relation suggests the presence of an unidentified systematic effect similar to that described by Singal et al. (2012). We predict the presence of a break in the hard gamma-ray blazar log $N$–log $S$ relation roughly at the current Fermi flux limit, $5 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$. However, the location of this break is determined primarily by objects near our low-energy photon spectral index cutoff ($\Gamma_I = 2$) and thus is potentially sensitive to the unmodeled soft end of the TeV blazar luminosity function.

Both the shape and magnitude of the 1FHL log $N$–log $S$ relation for 10 GeV–500 GeV fluxes above $4 \times 10^{-10}$ photons cm$^{-2}$ s$^{-1}$, presumably dominated by the hard, gamma-ray bright objects of interest here, is excellently reproduced after correcting for the 1FHL detection efficiency. Again, we predict a break in the log $N$–log $S$ relation at fluxes near the threshold, though the specific value is dependent upon the blazar luminosity function near the photon spectral index cutoff and is therefore somewhat uncertain.

Similarly, we are able to obtain a good fit to the Fermi 2LAC hard gamma-ray blazar redshift distribution. In contrast to similar calculations in Paper I, it is no longer necessary to specify an arbitrary $\Gamma_I$ relationship between the inferred Fermi-band and TeV luminosities, or maximum intrinsic TeV luminosity, substantially improving the robustness of the expected distribution. In comparison to that from the 2LAC, our $d \log N_B / dz$ falls marginally faster, either due to our assumption of a fixed-flux cutoff or suggesting an even more radical evolution of the TeV blazar luminosity function at low redshift.

In contrast to the log $N$–log $S$ relation and the hard gamma-ray blazar redshift distribution, the Fermi EGRB directly probes the high-$z$ evolution of the TeV blazar luminosity function. Below $\sim$3 GeV, the FSRQs and other soft sources dominate the EGRB. However, above $\sim$3 GeV, where soft sources contribute negligibly, the expected contribution from the hard gamma-ray blazars provide a remarkable fit to the most recently reported Fermi EGRB. Of particular importance is the now-observed strong suppression above 100 GeV; due to absorption on the EBL, this is a robust prediction of the TeV blazar luminosity function.

Simultaneously, the hard gamma-ray blazars match smooth extrapolations of the observed flux distributions at energies where they dominate the isotropic component. This is possible because the anisotropic and isotropic components of the EGRB are probing the hard gamma-ray blazar population at different redshifts (being dominated by nearby bright and distant dim objects, respectively), with the disparity being precisely that anticipated by the rapidly evolving TeV blazar luminosity function we have posited (note that this implies this success may be largely degenerate with the ability to reproduce the statistics of the hard gamma-ray blazars at low redshifts). Thus, above $\sim$3 GeV the Fermi EGRB may be fully explained within the context of the single resolved source class of hard gamma-ray blazars.

The comparisons described above are based on an a priori model for the TeV blazar population with no adjustable parameters. Thus, the success of the TeV blazar luminosity function is nontrivial; these are not “fits” in the normal sense. However, critical to these is the absence of the inverse Compton cascade emission that reprocesses the flux above $\sim$TeV into the Fermi-energy bands. If this occurs, the Fermi flux for a given TeV luminosity would increase substantially, moving the log $N$–log $S$ relations toward higher fluxes, $d \log N_B / dz$ toward higher $z$, the EGRB toward higher energy fluxes, and the EGRB anisotropy toward higher variances, thus in all cases badly violating the existing Fermi limits. Insofar as the evolution of TeV blazars may be expected to qualitatively reflect the cosmological history of accretion onto halos, this success may be seen as tentative support for the absence of the inverse Compton cascades and thus presumably circumstantial evidence in favor of the existence of the only known alternative, beam-plasma instabilities.

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APPENDIX A

AN EXPLICIT EXPRESSION FOR THE QUASAR LUMINOSITY FUNCTION

In the interests of completeness, here we reproduce the comoving quasar luminosity function, $\phi_Q(z, L)$ from Hopkins et al. (2007), corresponding to the “Full” case in that paper, that we employ. See Hopkins et al. (2007) for how this $\phi_Q(z, L)$ was obtained and caveats regarding its application.

The form of $\phi_Q(z, L)$ is assumed to be a broken power law:

$$\phi_Q(z, L) = \frac{\phi_a}{[L/L_*(z)]^{\gamma_1} + [L_*/L_*(z)]^{\gamma_2(z)}}, \quad (A1)$$

where the location of the break ($L_*(z)$) and the power laws ($\gamma_1(z)$ and $\gamma_2(z)$) are functions of redshift. These are given by

$$\log_{10} L_*(z) = (\log_{10} L_*)_0 + k_{L,1} z + k_{L,2} z^2 + k_{L,3} z^3$$

$$\gamma_1(z) = \gamma_{1,0} 10^{k_{1,1} z}$$

$$\gamma_2(z) = 2\gamma_{2,0} (10^{k_{2,1} z} + 10^{k_{2,2} z})^{-1}. \quad (A2)$$
Finally, shown in Figure 12.

where we have assumed $\Gamma \neq F$ the corresponding value for $F$ is

$$f_F = \frac{\Gamma_F - 1}{1 - 100^{1-\Gamma_F}} F_{35} \text{GeV}^{-1} s^{-1},$$

the corresponding value for $F_{25}$ is

$$F_{25} = \frac{0.1 \log(10^F) - 100^{1-\Gamma_F}}{1 - 100^{1-\Gamma_F}} F_{35} \text{GeV}^{-1},$$

where the additional factor of a GeV sets the scale of the energy flux.

APPENDIX B

EXPLICIT FLUX DEFINITIONS

We employ three definitions of “flux” here: the fluences from 100 MeV to 100 GeV ($F_{25}$) and 1 GeV to 100 GeV ($F_{35}$) and the flux from 100 MeV to 100 GeV ($F_{25}$). These are related to the measured values $F_{35}$ via Equations (11) and (12). Explicitly, setting $F_{35}$ as

$$f_F = \frac{\Gamma_F - 1}{1 - 100^{1-\Gamma_F}} F_{35} \text{GeV}^{-1} s^{-1},$$

where $\Gamma_F \neq 1$. Similarly, $F_{25}$ is given by

$$F_{25} = \frac{0.1 \log(10^F) - 100^{1-\Gamma_F}}{1 - 100^{1-\Gamma_F}} F_{35} \text{GeV}^{-1},$$

where the additional factor of a GeV sets the scale of the energy flux.

APPENDIX C

DETECTION EFFICIENCIES OF HIGH-LATITUDE GAMMA-RAY POINT SOURCE SAMPLES

Here we summarize the detection efficiencies associated with various high-latitude point source samples employed in the text.

C.1. 1FHL

In the construction of the 1FHL log $N$–log $S$ relation we make an attempt to account for the detection efficiency using the values shown in Figure 30 of Ackermann et al. (2013). Specifically, we set

$$N(S) = \sum_j \frac{1}{\epsilon(F_{45.7})} \Theta(F_{45.7} - S),$$

where $\epsilon(F_{45.7})$ is the spline-interpolated detection efficiency shown in Figure 12.

Table 1

| Normalization | $\log_{10} L_*$ | $\gamma_1$ | $\gamma_2$ |
|---------------|-----------------|------------|------------|
| $\log_{10} \phi_4$ | $-4.825 \pm 0.060$ | $13.036 \pm 0.043$ | $0.417 \pm 0.055$ | $2.174 \pm 0.055$ |
| $\phi_{Q,L}$ | 0.632 ± 0.077 | $-0.623 \pm 0.132$ | $1.460 \pm 0.096$ |
| $\phi_{Q_2,L}$ | $-11.76 \pm 0.38$ | $-0.773 \pm 0.057$ |
| $\phi_{Q_3,L}$ | $-14.25 \pm 0.80$ | |

Notes.

$^a$ In units of comoving Mpc$^{-3}$

$^b$ In units of $L_\odot \equiv 3.9 \times 10^{33}$ erg s$^{-1}$

where

$$\xi \equiv \log_{10} \left( \frac{1 + z}{3} \right).$$

The values of the relevant parameters are given in Table 1. Finally, $\phi_Q$, defined in terms of physical volume, is related in the usual way:

$$\phi_Q(z, L) = (1 + z)^3 \phi_Q(z, L).$$

Figure 12. 1FHL Detection Efficiency for high-latitude sources ($|b| > 15^\circ$), taken from Figure 30 of Ackermann et al. (2013).

C.2. 1FGL and 2FGL

Unlike the 1FHL, the 1FGL and 2FGL point-source detection efficiency are not present in the literature. However, these are necessary to reconstruct the anticipated EGRB anisotropy spectra associated with populations either masked by or due to these populations. Here we approximately reconstruct these detection efficiencies, following the procedure employed in Broderick et al. (2013) (to which we direct the reader for a more complete discussion).

Figure 13 shows the distribution of all sources with $|b| > 15^\circ$ in the 1FGL and 2FGL catalogs in $F_{35}$. This flux measure was chosen because it is both reported in the 1FGL and 2FGL catalogs and is apparently uncorrelated with the photon spectral index. Above $F_{35} \approx 10^{-26}$ photons cm$^{-2}$ s$^{-1}$ the two populations are both consistent with a single power law, $\propto F_{35}^{-1.25}$. At lower fluences the number of 1FGL rapidly decreases. The extension of the 2FGL to even lower fluences, where it exhibits a rapid decline, implies that these are associated with the detection efficiency of the respective catalogs and not with some intrinsic feature of the underlying source population.

Assuming that the high-fluence power law provides an approximation of the true source population, the ratio of the observed fluence distribution to the power law provides an estimate of the desired detection efficiency, $\epsilon(F_{35})$. These are shown in the top two panels of Figure 13. We approximate the detection efficiencies by

$$\epsilon(F_{35}) = \begin{cases} 10^{-m[\log_{10}(F_{35}/F_{\max})]^2} & F_{35} < F_{\max} \\ 1 & \text{otherwise,} \end{cases}$$
where for the 1FGL we have $m_{1\text{FGL}} = 7$ and $F_{1\text{FGL}}^{\text{max}} = 1.12 \times 10^{-9}$ photons cm$^{-2}$ s$^{-1}$, and for the 2FGL we have $m_{2\text{FGL}} = 4$ and $F_{2\text{FGL}}^{\text{max}} = 0.89 \times 10^{-9}$ photons cm$^{-2}$ s$^{-1}$. These fits, shown in Figure 13, are most accurate in the immediate vicinity of the detection threshold, the region that dominates the contribution to the EGRB anisotropy measurements.

In these no attempt to correct for Eddington bias (Eddington 1913, 1940) has been made, despite being evident in the 1FHL and 1FGL (resulting in $\epsilon > 1$ near the fluence threshold, corresponding to lower-fluence sources being detected at higher fluences). Doing so would reduce the inferred EGRB anisotropies.

APPENDIX D

RELATIONSHIP BETWEEN REDSHIFT LIMITS, PRIORS, AND THE ESTIMATED REDSHIFT DISTRIBUTION

In Section 3.3, we restricted our comparison to sources with measured redshifts in the 2LAC sample, consisting of roughly 39% of the 2LAC hard blazar sample. That the majority of the hard blazars in the 2LAC lack redshift measurements is not unexpected, potentially resulting from confusion of optical emission lines from the galaxy due to the dominance of the nonthermal jet component in the blazar spectra (Giommi et al. 2012). Nevertheless, it is not immediately clear that the populations with and without measured redshifts are distributed similarly. Ajello et al. (2014) compiled a list of 211 Fermi BL Lac objects from the 1LAC with redshift information. Importantly, included within this sample are the redshift limits for objects without directly measured values are distributed similarly. However, these limits are not expected to be representative of the true source distribution. To study the relationship between redshift limits and the estimated redshift distribution, we can use the luminosity function in Equation (6) to calculate the expected redshift distribution for objects with only redshift limits.

As elsewhere, we restrict our attention to the hard blazars, for which the Ajello et al. (2014) sample contains 85 sources. Of these, 44 have spectroscopic redshift measurements. A further two have photometric redshifts, though both of these lie considerably above the typical spectroscopic values. The remaining 39 have limits, arising from intervening absorption, host galaxy spectroscopic modeling, and the absence of Ly$\alpha$ absorption features. Typically, the lower limits are comparable to the typical measured spectroscopic values, while the upper limits typically lie near $z \sim 1.2$ and $z \sim 2$ for photometric and spectroscopic limits, respectively.

The large values of the upper limits in comparison to the typical measured redshifts suggests that the actual values will be strongly biased toward low values. In practice, the resulting redshift distribution is dominated by the imposed prior, shown in Figure 14 for two choices. Following Ajello et al. (2014), for each source with redshift limits only, we set the probability...
distribution via
\[
\frac{dP}{dz} = \pi(z) \Pi(z, z_{\min}, z_{\max}) / \int_{z_{\min}}^{z_{\max}} dz \pi(z),
\tag{D1}
\]
where \(\pi(z)\) is some prior and \(\Pi(z, a, b)\) is the top-hat function, equal to unity for \(a \leq z \leq b\) and zero otherwise. The resulting redshift distribution for this subset of objects is then obtained by summing the corresponding probabilities.

Assuming that \(\pi(z)\) is constant produces the top panel of Figure 14, where the resulting redshift distribution of the sources with limits only is shown by the green squares. The large range between the upper and lower limits produces a nearly flat distribution extending to large redshifts, noticeably different from that associated with sources with redshift detections (red triangles), suggesting that it is unphysical. However, while this choice of \(\pi(z)\) clearly illustrates the importance of the prior, it is poorly motivated observationally.

Were the true redshift distribution known, the appropriate prior would be \(\pi(z) \propto dN_\theta/dz\). However, in lieu of the true redshift distribution, we adopt that implied by our ansatz luminosity function, presented in Section 2. This procedure is identical to that employed in Ajello et al. (2014, see Equation (21) therein), where the redshift distributions associated with the trial luminosity functions were used; we differ only in the particular choice of \(dN_\theta/dz\). Because of the overwhelming dominance of \(\pi(z)\) by sources at low redshift, the corresponding redshift distribution of the objects with limits is only weakly dependent upon the limits themselves. In practice, the independent constraining power arises almost solely from the values of \(z_{\min}\), and thus the redshift distribution is only marginally less dominated by low \(z\), shown explicitly in the lower panel of Figure 14.

When combined with the sources with redshift measurements, the resulting redshift distribution appears fully consistent with that anticipated in Section 3.3. Due to the choice of prior above, it is unclear how to interpret the fit quality. Ignoring the impact of the prior upon the statistical interpretation, we obtain a \(\chi^2/\text{dof} = 11/10\) for \(z < 2\), suggesting that the fit is quite good, though we strongly caution against over-interpreting this value. However, direct support for this conclusion comes from independent analyses of blazar evolution that carefully take selection effects in the X-ray and gamma-ray bands into account (Giommi et al. 2012, 2013) as well as the evolution of associated populations such as radio galaxies (e.g., Willott et al. 2001; Wall et al. 2005) all of which are consistent with a strongly evolving (unified) population of AGNs.

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