Numerical studies of tunneling in a nonharmonic time-dependent potential

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Abstract

Tunneling through time-dependent potentials is of relevance to a number of physical problems. Using a WKB analysis Azbel' has recently studied the effects of a nonharmonic time-dependent perturbation embedded in an opaque potential barrier. He suggests the existence of three different regimes for transmission in such systems: direct tunneling, activation assisted tunneling, and elevator resonant activation. We address the same problem with a numerical technique based on wave-packet simulations. Our numerical results are in qualitative agreement with Azbel’s picture. The outcome depends on the characteristic time $T$ of the nonharmonic potential. There is a transition from direct tunneling to the activation regime around a ”crossover” value $T_c$ which is determined by the Büttiker-Landauer time, the distance between the initial energy and the lowest resonant level, and the amplitude of the time-dependent perturbation. The total transmission probability is strongly enhanced when entering the activation regime.

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Tunneling through time-modulated potential barriers has attracted considerable interest over the last ten years [1]-[9]. As argued in Refs. [10, 11] it may be important in a variety of physical problems, such as tunneling chemical reactions, charge exchange between impurity centers and resonant tunneling in semiconductors. Until recently mostly harmonic time-dependent potentials have been studied theoretically. In that case the tunneling particle with initial energy $E_i$ may absorb or emit energy quanta corresponding to the modulation frequency $\omega$, and the transmitted and reflected beams develop ”sidebands” at energy $E_i \pm \hbar \omega$, $E_i \pm 2\hbar \omega$, . . . . Expressions for the sideband amplitudes have been found analytically for simple rectangular barriers [2], and formal generalizations to barriers of arbitrary shape were derived in Ref. [6]. In addition, numerical calculations involving wave packets have confirmed the analytic results and completed our understanding of the physics involved [3, 4, 8, 9].

In two recent papers [10, 11] Azbel’ showed that tunneling and activation in a nonharmonic time-dependent potential can reveal new and interesting physics. Azbel’ considered an opaque static barrier $V(x)$ with an adiabatic time-dependent component $V_T(t)$ localized to a narrow region in the interior of the static part [12], see Fig. 1. The potential $V_T(t)$ varies on a time scale $T$ which limits the range of activation energies to roughly $\hbar/T$. If the potential varies very slowly, there is virtually no activation, and the transmission probability is essentially unchanged from its stationary value. However, by decreasing $T$ there may, at a given instant of time $t_0$, be sufficient energy available to activate the particle to the lowest instantaneous resonant level $E_r(t_0)$ in the well created by the time-dependent potential. Since the resonant-state lifetime $\tau_d$ is long for opaque barriers, the activated particle will be trapped in the resonant level and only escape after it has been lifted above the static-barrier edge by the time-dependent potential. The resonant level acts like an elevator for the tunneling particle; hence the name ”elevator resonant activation” (ERA) [10]. Ac-
cording to Azbel’ the transition from ordinary stationary tunneling to ERA happens when

\[ T \sim \tau_{BL}, \]

where \( \tau_{BL} \) is the Büttiker-Landauer time [1] for tunneling out of the well with

energy \( E_i \). With an opaque barrier, as in Fig. 1, one has \( \tau_{BL} \approx \int_{x_0}^{x} dx \sqrt{\frac{m^*}{2(V(x) - E_i)}}. \)

Here \( V(x) \) is the static potential barrier, \( x_0 \) is the classical turning point of the potential, i.e., \( V(x_0) = E_i \), and \( m^* \) is the effective mass of the tunneling particle. For a rectangular barrier with \( V(x) = V_0 \) and \( x_0 = d \) one has \( \tau_{BL} \approx d \sqrt{\frac{m^*}{2(V_0 - E_i)}}. \) Azbel’ also predicts

the existence of an intermediate regime, ”activation assisted tunneling” (AAT), where the

particle is activated to an energy \( \hat{E} < E_r(t_0) \) before it tunnels out. Since all transmission

processes require tunneling into the well at energy \( E_i \), the transmission probability is al-

ways exponentially small. However, the total transmission probability is predicted to be

greatly enhanced in the transition from direct tunneling to AAT and ERA.

In the present work we carry out numerical simulations for tunneling systems with

parameter values appropriate to semiconductor heterostructures. Such systems, with

their large flexibility and tunability, seem to be a promising candidate for observing the

new effects predicted by Azbel’. We study the tunneling dynamics by solving the time-

dependent Schrödinger equation numerically [13], and we restrict ourselves to one spatial

dimension. The initial state \( \psi(x; t = 0) \) is a quasi-monoenergetic minimum uncertainty

Gaussian wave packet with mean energy \( E_i \), energy width \( \Delta E \), and mean velocity \( v_i \).

Since we have semiconductor heterostructures in mind, suitable units of measure will be

Ångström [Å] for length, millielectronvolts [meV] for energy, and femtoseconds [fs] for

time. To be specific, we shall assume GaAs contacts and AlGaAs barriers with height

230 meV. We choose the spatial width of the wave packet to be \( \Delta x = 1000 \) Å. Then

\[ \Delta E \approx (\partial E/\partial k)\Delta k = \sqrt{2E_i/m^*\hbar/\Delta x} \approx 5 \text{ meV} \]

if \( E_i \) equals half the barrier height, and \( m^* = 0.067m_0 \) (the effective mass in GaAs). Thus the initial wave packet has a well-defined

energy in all the numerical examples below, in the sense that \( \Delta E(\partial P_{tr}/\partial E)/P_{tr} < 1 \) [14].
Here $P_{tr}$ is the instantaneous transmission probability of the potential barrier at energy $E_i$. The initial state has its center of mass $<x(0)>$ more than 4000 Å away from the barrier structure. Thus the overlap between $\psi(x; t = 0)$ and the barriers is negligible.

Our choice of initial state is not of purely academic interest. By using modulation-doped semiconductor heterostructures it is possible to fabricate tunneling barriers which transmit a fairly monoenergetic beam of electrons. These electrons can move ballistically over tens of nanometers \cite{13} and should be well described by wave packets of the kind used here.

The static part of the potential $V(x)$ can be made by standard epitaxial-growth techniques. Alternatively, the transport could take place in a two-dimensional electron gas. In that case the potential-barrier structure is created by applying external gate potentials. Possible ways of producing the perturbation $V_T(t)$ could be by means of ultra-short laser pulses or time-dependent gate potentials \cite{16}. It may be difficult to control $V_T(t)$ on the length and time scales which are typical for the systems that we have in mind (of the order of 100 Å and 100 fs, respectively), although laser pulses with a duration less than 100 fs have already been demonstrated in several experiments.

We will present numerical results for two different kinds of potential-barrier structures, see Fig. 2. Structure A is a single rectangular barrier of width $W_0$ and height $V_0$. The time-dependent part of the potential acts on a segment of width $W_1$ located at the center of the static barrier. In this region the total potential is $V_0$ when $|t| \to \infty$ and $V_0 - V_1$ for $t = t_0$. We have used $V_1 = V_0$ in all the examples below. Within the adiabatic picture the lowest instantaneous resonant level starts at $E_\infty \equiv E_r(|t| \to \infty)$ (i.e., the ”virtual” state above the barrier), falls down to $E_0 \equiv E_r(t_0)$ at $t = t_0$, and rises back to $E_\infty$ as $t \to \infty$. Structure A will be used to investigate ERA. Structure B is similar to A, but has in addition a wide and low barrier of height $\tilde{V}$ on each side, leading to a total width $\tilde{W}$.
for the structure. There is still a resonance in the time-dependent well, moving from $E_\infty$ to $E_0$ and back again. The positions of $E_0$ and $E_\infty$ can be tuned by changing the well width $W_1$ (cf. Figs. 2B1 and 2B2). In addition there are one or more “quasi-resonant” levels $\tilde{E}_j > \tilde{V}$ ($j = 1, 2, \ldots$) corresponding to resonances above a single rectangular barrier of width $\tilde{W}$ and height $\tilde{V}$. The positions of these levels are quite insensitive to both the instantaneous value and the width of the time-dependent potential. With structure B we will demonstrate AAT and ERA, where the outcome will depend on the relative positions of $E_0$ and the various $\tilde{E}_j$. In order to observe a transition from direct tunneling to the activation regime, the initial-state energy $E_i$ must be chosen to lie below all the resonant levels. In the opposite case, with $E_0 \leq E_i$, the elevator effect will be present for any finite $T$ since no activation energy is required to trap the particle in the resonance [11]. In the tables in Fig. 2 we have collected numerical values for the most important parameters in the examples described below. We have also plotted the transmission probability of the instantaneous potential, both at $t = t_0$ and $t \to \infty$. Notice the peaks in $P_{tr}$ for $E = \tilde{E}_j$ in Fig. 2B. Although the amplitudes of these peaks depend strongly on the value of $V_T(t)$, their positions remain more or less unchanged. The parameter $t_0$ is chosen such that the center of mass at $t = t_0$, $< x(t_0) > = < x(0) > + v_i t_0$, of a freely moving wave packet with group velocity $v_i$ would coincide with the center position of the time-dependent part of the potential. In this way our simulations are as close as possible to the stationary case ($\Delta x \to \infty$) studied in Refs. [10, 11].

The exact form of the time-dependent part of the potential is not important. However, in order to observe ERA, it is crucial to have a nonharmonic variation in time. A nonharmonic time dependence implies, as we shall see below, a continuous activation probability. Under these conditions a particle which has been activated from $E_i$ to the lowest instantaneous resonant level $E_0$, can continuously absorb infinitesimal amounts of energy and
follow the resonance as it rises from $E_0$ to its maximum value $E_\infty$. In contrast, in a purely harmonic potential with frequency $\omega$ a particle can only absorb energy in multiples of $\hbar \omega$, and hence cannot be trapped in the continuously moving resonant level. In the numerical calculations below we will use the time-dependent potential

$$V_T(t) = -\frac{V_1}{\cosh((t - t_0)/T)},$$

which was also used in an example in Ref. [10].

Let us try to gain some insight into when the transition from direct tunneling to activation takes place. This must happen when the characteristic time $T$ of the nonharmonic potential is such that the probability $P_{act}(E_i \to E_0; T)$ for activation from $E_i$ to the lowest resonant level $E_0$ is approximately equal to the probability $P_{dir}(E_i)$ for direct tunneling out of the time-dependent region with the initial energy $E_i$. An estimate of the activation probability can be made by using standard time-dependent perturbation theory [17]. We find

$$P_{act}(E_i \to E_0; T) = \left| -\frac{i}{\hbar} \int_{-\infty}^{\infty} dt < 0|V_T(t)|i > e^{i(E_0 - E_i)t/\hbar} \right|^2 \approx \left[ \frac{\pi V_1 T}{\hbar \cosh[\frac{1}{2} \pi T(E_0 - E_i)/\hbar]} \right]^2,$$

where $|i>$ and $|0>$ denote the initial state and the lowest resonant state, respectively, and we have used $V_T(t)$ from Eq. (1). We have also taken into account that $P_{act}$ is the activation probability conditional on the particle having tunneled into the time-dependent region, and we assume that the lowest resonant state can be approximated by a symmetric and normalized wave function. A "crossover" time $T_c$ can now be defined via

$$P_{act}(E_i \to E_0; T_c) \simeq P_{dir}(E_i).$$

Since the barrier is opaque when seen from the time-dependent region, we have
We have found that the argument of the cosh-function in Eq. (4) is typically much larger than unity. In that case we may rearrange Eq. (4) and obtain a simpler equation for the crossover time,

\[ T_c \simeq \frac{4}{\pi} \tau_{BL} V_0 - E_i + \frac{2\hbar}{\pi(E_0 - E_i)} \ln \frac{2\pi V_1 T_c}{\hbar}. \]  

(5)

From Eq. (5) it is evident that the Büttiker-Landauer time \( \tau_{BL} \) sets a scale for the crossover from direct tunneling to activated processes. As mentioned earlier, this was pointed out very clearly in Ref. [10]. However, equally important parameters in this connection are the distance from the initial energy to the lowest resonance, \( E_0 - E_i \), and the amplitude of the potential modulation, \( V_1 \).

In order to analyze the tunneling and activation process in energy space, we perform a spatial Fourier transform of the wave packet in the asymptotic limit \( t \to \infty \), thus obtaining the momentum (or energy) distribution of the transmitted (\( p > 0 \)) and reflected (\( p < 0 \)) parts of the wave packet [18].

In our first example we use the structure from Fig. 2A. In Fig. 3 we have plotted on a logarithmic scale the energy distribution of both the transmitted (tr) and reflected (ref) parts of the wave packet for a fairly slow potential variation \( (T = 182 \text{ fs}) \). One can see that most of the wave packet is reflected at energy \( E_i \) without ever reaching the time-dependent region. In other words, we are in the opaque barrier regime, consistent with the assumptions in Refs. [10, 11]. Next, we see that for this slowly varying potential the energy distribution of the transmitted part of the wave packet is also centered at \( E_i \), indicating that hardly any activation has taken place. Notice, however, the small peak (or ”shoulder”) at \( E_\infty \), i.e., at the position of the resonant level as \( t \to \infty \). These energy components are
activated to $E_0$ by frequencies $\omega \sim (E_0 - E_i)/\hbar \gg 1/T$ in the exponential tail of the Fourier spectrum of $V_T(t)$ and subsequently follow the resonance adiabatically from $E_0$ to $E_\infty$. Finally, notice the symmetry of activated energy components in the reflected and transmitted parts of the wave packet. This is seen in all our numerical examples because activated components of the wave packet "see" a spatially symmetric potential. Hence they have equal probability of tunneling out in either direction and thus contribute with the same amount to reflection and transmission.

In the following we present numerical results which illustrate the transition from direct tunneling to activation. We will focus on the transmitted part of the wave packet, and in Fig. 4 we have plotted its energy distribution (now on a linear scale) for selected values of $T$, and for the three different structures presented in Fig. 2.

The first panel in Fig. 4A is a linear plot of the transmitted distribution in Fig. 3, and it is clear that direct tunneling dominates completely for such a slowly varying potential. Thus the total transmission probability for the wave packet $P_{wp}(T = 182 \text{ fs})$ is very little different from the stationary value $P_{wp}(T \to \infty)$ (see the rightmost panel in Fig. 4A; solid dot and dotted line, respectively) \[19\]. A faster potential, $T = 22 \text{ fs}$, yields a strikingly different picture. The energy spectrum of the transmitted packet is now dominated by components around $E_\infty$. In this case ERA dominates, and there is indeed a huge increase in the transmission probability. Finally, we have included the case with $T = 91 \text{ fs}$, which represents an intermediate situation with roughly equal amounts of direct tunneling and ERA. This is in good agreement with Eq. (5), which predicts a crossover between direct tunneling and ERA at $T_c \approx 80 \text{ fs}$.

As we decrease $T$ below 20 fs, the total transmission probability goes through a maximum value and then falls off rapidly. The reason is that the time-dependent potential, and hence the lowest resonance, is sufficiently low to permit "effective" activation only in
a period of time of the order of $T$. In other words, for decreasing $T$ a greater part of the wave packet is directly reflected \cite{20}. By varying the initial energy $E_i$ we have further verified that the onset of ERA scales as expected from Eq. (5). One should also bear in mind that when $T \lesssim 10$ fs, we are no longer in the adiabatic regime $\hbar/T \ll E_i, V_0 - E_i$, and thus outside the limits of the analytical work in Refs. \cite{10, 11}.

Activation assisted tunneling, AAT, implies activation to an energy below $E_0$ before tunneling out of the time-dependent region. In our numerical calculations this should show up as a significant weight between $E_i$ and $E_0$ in the energy spectrum of the transmitted wave packet. With a structure of type A (and with an activation probability as above) we have seen no indication of AAT. This is not surprising since both the (static) transmission coefficient and the activation probability behave monotonically as functions of energy between $E_i$ and $E_0$. Thus there is no energy between $E_i$ and $E_0$ at which the particle is preferably transmitted, and one has a transition directly from direct tunneling to ERA when $T$ is decreased. We should point out that this was predicted in Ref. \cite{10} for a similar time-dependent potential, but with an inverted parabola static part instead of the rectangular shape used here.

In order to observe AAT we will in the next examples use a structure of type B (see Fig. 2B). As discussed earlier such a potential has quasi-resonances at $\tilde{E}_j > \tilde{V}$ to which the tunneling particle may be activated. Since the quasi-resonances $\tilde{E}_j$ do not move with the time-dependent potential, the activated particle will eventually tunnel out with energy $\tilde{E}_j$, and one has AAT rather than ERA. Again we use a symmetric potential, with parameters given in the tables of Fig. 2B. The time dependence is, as above, given by Eq. (1). In practice the low (and wide) barriers could be made from Al$_x$Ga$_{1-x}$As, with a lower Al content $x$ than in the high (and narrow) ones. By varying the width $W_1$ of the time-dependent region, the lowest time-dependent resonance $E_0$ is tuned to different positions.
relative to the quasi-resonances $\tilde{E}_j$. In Fig. 4B1 we use a narrow well ($W_1 = 18$ Å). As a result $E_0$ lies above $\tilde{E}_1$ and $\tilde{E}_2$, but below $\tilde{E}_3$. In Fig. 4B2 the well is somewhat wider ($W_1 = 35$ Å), and $E_0$ lies below all $\tilde{E}_j$. We have again plotted the energy distribution of the transmitted part of the wave packet for various values of $T$. In the case of the narrow well the direct tunneling component is significant for all $T$. This shows up as a peak at $E \simeq E_i$ in the transmitted energy distribution. Furthermore the selected examples in Fig. 4B1 illustrate AAT to $\tilde{E}_1$ when $T = 91$ fs, AAT to $\tilde{E}_1$ and $\tilde{E}_2$ (the latter only as a weak shoulder in the energy distribution) when $T = 46$ fs, and a combination of AAT to $\tilde{E}_1$ and $\tilde{E}_2$ and ERA from $E_0$ to $\tilde{E}_3$ and above when $T = 6$ fs. With the wider well we have observed substantial activation to $E_0$ followed by ERA already for $T \sim 350$ fs. When $T = 182$ fs (first panel, Fig. 4B2), most of the activated components follow the resonance to $\tilde{E}_1$ and tunnel out. Reducing $T$ to 46 fs results in ERA to $\tilde{E}_1$ and $\tilde{E}_2$. In that case the lifetime of the lowest quasi-resonance, $\tilde{\tau}_1 \simeq 70$ fs, is so long that many of the "elevating" wave-packet components are trapped in the moving resonance and do not tunnel out at $\tilde{E}_1$. However, the second quasi-resonance has a lifetime $\tilde{\tau}_2 \simeq 35$ fs which is shorter than $T$. Thus the elevating components that pass through the level at $\tilde{E}_1$ tend to tunnel out at $\tilde{E}_2$. Upon further reduction, to $T = 6$ fs, we observe ERA to $\tilde{E}_1$, $\tilde{E}_2$, and $\tilde{E}_3$, and some components even follow the resonance all the way up to $E_\infty$ before they tunnel out. In Figs. 4B1 and 4B2 (rightmost panel) we have also plotted the total transmission probability as function of $T$ for the two cases discussed above. As in Fig. 4A there is a large enhancement of $P_{wp}$ when $T$ is made short enough to allow for AAT or ERA, as predicted in Refs. [10, 11]. The crossover values calculated from Eq. (5) are $T_c = 137$ fs for Fig. 4B1 and $T_c = 285$ fs for Fig. 4B2, both in good agreement with what is observed. For sufficiently small values of $T$ the transmission probability goes through a maximum and falls off rapidly, for the same reason as discussed in connection with Fig. 4A.
We have also examined other time-dependent potentials, both harmonic and nonharmonic ones. As expected the qualitative features in the examples above stay unchanged as long as the activation probability is a continuous function of energy. On the other hand, with a purely harmonic potential, $V_1 \cos \omega t$, there is no elevator effect but simply activation and tunneling at energies $E_i + n\hbar \omega$ ($n = 0, \pm 1, \pm 2, \ldots$).

In conclusion, we have studied tunneling through a nonharmonic time-dependent potential by letting Gaussian wave packets scatter off the barrier. Analysis of the energy distribution of the transmitted wave packets for suitably chosen tunneling structures confirms Azbel’s prediction of a crossover from direct tunneling to elevator resonant activation and activation assisted tunneling. An estimate shows that activation dominates over direct tunneling when the characteristic time of the time-dependent potential is reduced below a crossover value which involves the Büttiker-Landauer time, the distance from initial energy to the lowest resonant level, and the amplitude of the time-dependent potential. The total transmission probability is strongly enhanced, in our numerical calculations by one or two orders of magnitude, when comparing activation with direct tunneling.

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[12] *Opaque* means that the transmission coefficient of the barrier is much smaller than one. *Adiabatic* means that the potential varies on a time scale $T$ such that $\hbar/T$ is small compared with other energy scales in the system.

[13] The time-dependent Schrödinger equation is solved with a standard numerical technique, see e.g. A.-P. Jauho in *Quantum Transport in Semiconductors*, eds. D. K. Ferry and C. Jacoboni (Plenum Press, New York, 1992), page 179, and references therein.

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[17] E. Merzbacher, Quantum Mechanics (Wiley, New York, 1970), chapter 18.

[18] In practice we must choose an ”observation time” $t_{obs} >> t_0$. However, $t_{obs}$ should be small enough to avoid reflections from the endpoints of the finite spatial line.

[19] In fact, $P_{wp}(T \to \infty) > P_{wp}(T = 182 \text{ fs})$. This is as expected since $T \to \infty$ represents the stationary situation with $V = 0$ in the otherwise time-dependent region. As a result the effective tunneling barrier is lower for $T \to \infty$ than for any finite $T$, and $P_{wp}(\infty)$ exceeds $P_{wp}(T)$ for those $T$ where the amount of activation is negligible.

[20] This effect seems to be contained in Eq. (2) which indicates a maximum in the activation probability $P_{act}(T)$ for a finite $T_m$. However, Eq. (2) is based on perturbation theory and is only valid if $P_{act}(T) << 1$. For typical values of $V_1$ and $E_0 - E_i$, $T_m$ is well below the range of validity of Eq. (2) and cannot be used as a quantitative estimate of the maximum position in $P_{wp}(T)$.

**Figure Captions**

FIG.1. One-dimensional potential barrier $V(x)$ with a time-dependent component $V_T(t)$ embedded near the center. $x_0$ denotes the classical turning point for incoming energy $E_i$. 
Solid and dotted vertical lines in the well illustrate three possible processes, as described in the main text: direct tunneling at $E_i$ (DT), activation to $\tilde{E}$ and tunneling (AAT), activation to $E_r(t_0)$ and elevation to $E_r(\infty)$ (ERA).

FIG.2. Potential-barrier structures, static transmission probabilities versus energy $P_{tr}(E)$, and tables with potential-barrier parameters and resonant-level energies for the two types of structure used in our numerical calculations. (A) A rectangular static barrier of height $V_0$ and width $W_0$. The time-dependent part is located at the center of the static barrier and has width $W_1$ and maximum value $-V_1$. We have used $V_1 = V_0$ in all numerical calculations. The lowest instantaneous resonance moves between $E_\infty$ and $E_0$. (B) Similar to the structure in A, but here we have in addition wide and low barriers of height $\tilde{V}$ on each side. The total width is $\tilde{W}$. In addition to the moving resonance there are ”quasi-resonances” at $\tilde{E}_1, \tilde{E}_2, \ldots$. Figs. B1 and B2 represent results for structures which are identical apart from having different well width $W_1$. The levels in the well labeled with 1 and 2 denote the extremes $E_0$ and $E_\infty$ of the moving resonance (for the narrow and wide well, respectively). The transmission probability is in all three cases plotted for the maximum and minimum value of the time-dependent potential: Solid lines are for $t \to \infty$ whereas dashed lines are for $t = t_0$, i.e., for $V = V_0$ and $V = 0$, respectively, in the time-dependent region. The peaks in $P_{tr}$ are associated with the various resonant levels $E_0, E_\infty, \tilde{E}_1, \tilde{E}_2, \ldots$.

FIG.3. Logarithm of the energy distribution $|\Phi(E)|^2$ of the reflected (ref) and transmitted (tr) parts of the wave packet after scattering off the structure in Fig. 2A. (Note that the $E$-axis has positive values in both directions.) The characteristic time of the time-dependent potential is $T = 182$ fs. Both parts of the wave packet are dominated by energy
components around the initial energy $E_i \simeq 81$ meV. The "shoulders" with their maxima close to $E_\infty \simeq 248$ meV represent energy components that have been activated to the lowest instantaneous resonance $E_0 \simeq 127$ meV and elevated from $E_0$ to $E_\infty$ (ERA).

FIG.4. Numerical results corresponding to the structures presented in Fig. 2. Reading horizontally, the first three panels show the energy distribution (on a linear scale) of the transmitted part of the wave packet for selected values of $T$. Note that the scale along the vertical axis varies from curve to curve. The rightmost panel shows the total transmission probability as function of $T$. These curves result from integrating the transmitted energy distribution for each value of $T$. The solid dots represent the selected values of $T$. The dotted horizontal line (for the case B2 this line lies very close to $P_{wp} = 0$) represents the stationary value of the transmission probability, $P_{wp}(T \to \infty)$, for the particular structure.

(A) Results for the rectangular barrier in Fig. 2A. The first frame ($T = 182$ fs) is a linear plot of the transmitted part in Fig. 3. (B) Results for the structures of Fig. 2B, for the narrow well (B1) and the wide well (B2). In all cases the peaks in the transmitted energy distribution are connected to the initial energy $E_i$ or to the various resonant levels, which can be identified from the peaks in the static transmission probabilities in Fig. 2.