Development of an automatic birefringence measuring device of mirror substrates for gravitational wave detectors

M Tokunari, H Hayakawa*, K Yamamoto, T Uchiyama, S Miyoki, M Ohashi, K Kuroda
Institute for Cosmic Ray Research, The University of Tokyo, Kashiwa, Chiba 277-8582, Japan
E-mail: tokunari@icrr.u-tokyo.ac.jp

Abstract. We developed an automatic measuring device of birefringence inhomogeneity in synthetic sapphire substrates to evaluate their crystal quality suitable for laser interferometric gravitational wave (GW) detectors. The orientation of the projection of the c-axis on a plane orthogonal to the beam was measured with an accuracy of $2 \times 10^{-2}$ rad. The phase retardation was measured with an accuracy of $7 \times 10^{-4}$ rad, which was equivalent to $4 \times 10^{-10}$ in terms of the fluctuation of the refractive index for samples of 150 mm thickness. The reproducibility was 1% for both the orientation and phase retardation. The automatic measuring device meets the requirements of the LCGT project, which is a next-generation laser interferometer project for GW detection.

1. Introduction
Interferometers for GW detection, such as the LCGT (Large-scale Cryogenic Gravitational-wave Telescope) project[1], require very high optical quality in their mirror substrates. Advanced LIGO[2] has selected to use fused silica primarily both because the fabrication technology of sapphire to date is not mature enough to consistently produce substrates of adequate quality and because fused silica is optically appropriate substrate at room temperature. However, fused silica is not eligible for substrates at cryogenic temperature from the perspective of low thermal conductivity. On the other hand, since sapphire has a high mechanical quality factor, a high thermal conductivity and a small thermal-lensing effect at cryogenic temperature[3, 4], cryogenic sapphire mirrors are being introduced for the CLIO (Cryogenic Laser Interferometer Observatory) project[5], and are planned to be used in the LCGT project.

However, sapphire exhibits a higher optical absorption than fused silica. Large absorption may create heat production inside the mirrors, which would prevent the reduction of thermal noise. Although some measurements of the optical absorption shows that sapphire meets the requirement of LCGT thanks to continuous progress on improving quality in the manufacturing process and a further reduction of the optical absorption is achieved by annealing[6], there might be piece-to-piece variations. Therefore, it is necessary to establish the evaluation method of optical quality of sapphire substrates. Measuring the fluctuation of the birefringence gives information about the crystal quality of the sample at the position where the beam passes

* Present address: Graduate School of Science, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan

© 2006 IOP Publishing Ltd
inside the sample. For this reason, we have developed an automatic measuring device of the birefringence of a sapphire crystal.

2. Phase retardation of light

Sapphire is a uniaxial crystal, which means there is a light axis, along which a linearly polarized light beam propagates with the same phase speed regardless of its polarization direction. This axis is called c-axis. Although the cylindrical axis of the mirror substrate is normally taken to coincide with the c-axis of the crystal in applications to laser interferometers, there can be small local differences that fluctuate due to a possible intrinsic irregularity or impurity of a practical crystal sample, and also due to extrinsic mechanical and thermal stress.

Consider two linearly polarized beams that propagate along the z-axis from which the c-axis is inclined by an angle $\theta$, as shown in Fig. 1. If the polarization plane of the electric field vector $\mathbf{E}$ of the light is parallel to the projection of the c-axis onto the xy-plane, which inclines by an angle $\phi$ from the x-axis, as shown in Fig. 1, the light propagates with a speed of $c/n'_e$, where $c$ and $n'_e$ are the speed of light in vacuum and the extraordinary refractive index, respectively. This ray is called an extraordinary ray. If the polarization plane of the electric field vector is perpendicular to the projection of the c-axis onto the xy-plane, the phase speed of light is $c/n_o$, where $n_o$ is the ordinary refractive index. This ray is called an ordinary ray. The extraordinary refractive index ($n'_e$) is obtained by the following equation of an index ellipsoid:

$$
\frac{1}{n^2_e} = \frac{\cos^2 \theta}{n^2_o} + \frac{\sin^2 \theta}{n'^2_e},
$$

where $n_o$ is the refractive index under the condition that $\mathbf{E}$ is along the c-axis. Since $n_e$ is smaller than $n_o$ for negative crystals, such as sapphire, the direction of the electric field that gives $n'_e$ is called a fast axis. The phase difference (retardation) $R$ between the two ray beams is given by

$$
R = \frac{2\pi d(n_o - n'_e)}{\lambda},
$$

where $d$ and $\lambda$ are the optical path length and the wavelength of the light in vacuum, respectively; $R$ and $\phi$ are parameters that characterize the birefringence.

Figure 1. If the c-axis of the sapphire sample inclines by an angle $\theta$ from the z-axis, the phase speed of light propagating along the z-axis depends on the direction of its electric field vector $\mathbf{E}$. The phase speed of a beam whose electric field is parallel to the projection of the c-axis onto the xy-plane, which inclines by an angle $\phi$ from the x-axis, is $c/n'_e$ and the phase speed of a beam whose electric field is perpendicular to the direction is $c/n_o$. The phase retardation between these two rays is given by Eq. 2.
3. Setup of the measuring device

Figure 2 shows the experimental setup of the measuring device. A laser beam (Lightwave Electronics NPRO laser, model 124-PS) passes in succession through a polarizer oriented at 45 degrees from the horizontal axis, a compensator (OFR Soleil-Babinet Compensator, SB-10), a half-wave plate, the sapphire sample, a quarter-wave plate, an analyzer crossed with respect to the polarizer (135 degrees), and a photo-detector (PD). Firstly, the polarizer and the analyzer are set to extinguish the light at the PD in the initial setting without the sample. If the sample is inserted after that, a part of the light beam leaks due to its birefringence. The compensator can compensate the phase retardation and extinguish the light again while adjusting the orientation of the half-wave plate to align with the fast axis of the sample. This procedure was done by a computer-control system.

![Figure 2. Setup of the birefringence measuring device. The sapphire sample was scanned in two-dimensional directions in a plane orthogonal to the light beam using mechanical stages. Although the polarizer and the analyzer are arranged to extinguish the light passing to the PD without the sample, the inserted sample makes light leak due to its birefringence. The compensator can compensate the phase retardation and extinguish the light again while adjusting the orientation of the half-wave plate to align with the fast axis of the sample. This procedure was controlled by a personal computer (PC).](image)

We used a Soleil-Babinet compensator, which can make variable homogeneous birefringence over a broad area. It consists of three uniaxial quartz crystals. When one of the crystals is shifted by $\Delta x_c$ along the x-axis, the whole thickness of the compensator changes by $\Delta d_c \propto \Delta x_c$. Therefore, the phase retardation ($R$) varies linearly with the displacement ($x_c$) along the x-axis. Without the sample, the output voltage ($V$) of the PD changes according to the equation $V \propto \sin^2 \left( \frac{R}{2} \right)$. The relation between $R$ and $x_c$ was obtained by a fitted $V - x_c$ curve without the sample. This measurement was arranged to be repeatable at any time during the whole measurement run because the relation may change with a possible long-term change of the laser power and an electrical drift of the PD circuit. Figure 3 shows an example of the measured data of $V$ versus $x_c$ and its fitted curve.

The leaking light due to inserting the sample is extinguished again by adjusting the compensator and the orientation of the half-wave plate. The displacement of the compensator ($x_c$), which gives the extinction (minimum) point, results in the phase-retardation value. This point is obtained by fitting data acquired with scanning back and forth around the minimum light intensity with a quadratic function (Fig. 4). Since the standard error of $x_c$ was $3 \times 10^{-3}$ mm, the standard error of $R$ was $7 \times 10^{-4}$ rad, which is equivalent to $1 \times 10^{-9}$ in terms of the fluctuation of the refractive index for samples of 60 mm thickness. The orientation ($\phi$) of the sample was
calculated from the adjusted orientation of the half-wave plate. The orientation was determined with an accuracy of $2 \times 10^{-2}$ rad. The procedures were checked by measuring a quarter-wave plate as a sample with known birefringence. The average of measured $R$ data was $1.572 \ (\sim \pi/2)$. This is reasonable because the instrumental error of the quarter-wave plate is about 0.06 rad according to the manufacturer.

The above procedures were automated with LabVIEW, which is a graphical programming tool. We used stepping motors to adjust $x_c$ and $\phi$, and also to translate the stages. A 12-bits-DAQ device was used to take data from the PD. The controller of these motors and the DAQ device were interfaced by GPIB and connected to a personal computer. It took about eight minutes to finish one automatic measurement for one line with LabVIEW, including the measurement, curve fitting and processing data.

4. Result and discussion

We used four high-purity Hemlite sapphire cylinder samples. They were 100 mm in diameter and 60 mm in thickness (samples A, B, C and D). These samples were made with the Heat Exchanger Method by Crystal Systems Inc.[7].

Figure 5 shows two-dimensional mappings of the phase retardations (the deviation from the mean) of the four samples. The X-Y plane corresponds to the circular plane of the sample. The reproducibility of a measurement to another, whose interval time was one week, was about 1% for both the phase retardation and the orientation. While the retardations of samples A and B change in a continuous fashion, those of samples C and D are checkered patterns. These patterns of samples C and D are attributed not to the error but to the real crystal structure\(^1\).

Table 1 gives a summary of the measured data: the average phase retardation, the average tilt angle of the c-axis against the cylindrical axis, standard deviation of the orientation angle, that of the phase retardation, that of the relative refractive index and that of the tilt angle. For a reference, we included the data of a Hemex-grade sample (sample E) whose properties were measured at the University of Western Australia (UWA)[8]. The average phase retardation represents a gap ($\bar{\theta}$) between the c-axis and the light axis. The standard deviation of the phase retardation, or the relative refractive index, represents a possible irregularity or inhomogeneity.

\(^1\) During the revision of this article, we made a close-up measurement for part of the sample C and found the small-scale structure.
Figure 5. Two-dimensional mappings of the phase retardation (the deviation from the mean): (a) sample A, (b) sample B, (c) sample C, (d) sample D. The measuring area of sample A was different from those of the others. The scales for all four plots are set to be the same.

of the crystal sample. The phase retardation of sample A, $4.2 \times 10^{-2}\text{rad}$ is equivalent to $11.8 \times 10^{-8}$ in terms of the fluctuation of the relative refractive index. It was confirmed that Hemex is of higher grade than Hemlite based on the results that the standard deviations of the phase retardations of four Hemlite samples were actually larger by several times than that of Hemex.

Due to a machining error, the cylindrical axis against the c-axis may differ on the order of $1 \times 10^{-2}\text{rad}$. The alignment error of the beam axis against the cylindrical axis was less than $1 \times 10^{-3}\text{rad}$. Therefore, the value of $\theta$ represents the tilt angle of the c-axis against the cylindrical axis. It was confirmed by our measurements that the machining error is around $1 \times 10^{-2}\text{rad}$.

In order to check the extrinsic effect of stress-induced birefringence, we rotated sample A by every 90 degrees, and made the same measurements for every 90 degrees. The mappings of those measurements rotated according to the sample rotations (Fig. 6). The gradient of differential phase retardation along the vertical axis ($\partial \Delta R / \partial Y$) was much less than $1 \times 10^{-3}\text{rad/mm}$. The phase retardation caused by mechanical stress-induced birefringence was reported to be about $0.03\text{rad}$ by F. Benabid et al. They measured a high-purity Hemex sapphire cylinder sample (sample E), which was 50 mm in diameter and 100 mm in thickness and weighed about 40% of our samples. They used a V-shaped holder of a sample, whereas we used a U-shaped holder. Even though sample A was 2.5-times heavier than sample E, it is conceivable that the phase
Table 1. Summary of the measured data with a reference sample: the average phase retardation, the average tilt angle of the c-axis against the cylindrical axis, standard deviation of the orientation angle, that of the phase retardation, that of the relative refractive index and that of the tilt angle. Samples A, B, C and D are of Hemlite grade. The reference sample E is of Hemex grade[8].

|       | $R_{\text{rad}}$ | $\theta_{\text{rad}}$ | $\sigma(\phi)_{\text{rad}}$ | $\sigma(R)_{\text{rad}}$ | $\sigma(n_0 - n_e)$ | $\sigma(\theta)_{\text{rad}}$ |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| A(Hemlite) | $19.5 \times 10^{-2}$ | $8.8 \times 10^{-3}$ | $1.1 \times 10^{-4}$ | $4.2 \times 10^{-2}$ | $11.8 \times 10^{-8}$ | $9.6 \times 10^{-4}$ |
| B(Hemlite) | $4.9 \times 10^{-2}$ | $4.2 \times 10^{-3}$ | $4.5 \times 10^{-4}$ | $2.7 \times 10^{-2}$ | $7.7 \times 10^{-8}$ | $1.4 \times 10^{-4}$ |
| C(Hemlite) | $46.9 \times 10^{-2}$ | $13.7 \times 10^{-3}$ | $0.6 \times 10^{-1}$ | $6.3 \times 10^{-2}$ | $17.7 \times 10^{-8}$ | $9.3 \times 10^{-4}$ |
| D(Hemlite) | $30.4 \times 10^{-2}$ | $11.0 \times 10^{-3}$ | $0.9 \times 10^{-1}$ | $5.1 \times 10^{-2}$ | $14.5 \times 10^{-8}$ | $9.5 \times 10^{-4}$ |
| E(Hemex) | $2 \times 10^{-2}$ | $2 \times 10^{-3}$ | - | $1 \times 10^{-2}$ | $2 \times 10^{-8}$ | - |

The phase retardation was dominated by what caused by intrinsic birefringence, because the mean phase retardation of sample A was 0.195 rad.

5. Conclusion
We developed an automatic measuring device of the birefringence of a high-quality sapphire substrate. The phase retardation was measured with an accuracy of $7 \times 10^{-4}$ rad and the
orientation with an accuracy of $2 \times 10^{-2}$ rad. The measured standard deviations ranged from $0.7 \times 10^{-7}$ to $1.8 \times 10^{-7}$ in terms of the relative refractive index with a reproducibility of 1% along lines in parallel with the cylindrical axis of the sample. The fluctuation of the refractive index was obtained with an accuracy of $1 \times 10^{-9}$ for samples of 60 mm thickness. This corresponds to an accuracy of $4 \times 10^{-10}$ for LCGT substrates of 150 mm thickness. Since the requirement of LCGT on the inhomogeneity of the refractive index of the mirror substrate is $2 \times 10^{-7}$, these resolutions were sufficiently achieved by the developed automatic measuring device.

Acknowledgements
This study was supported by a Grant-in-Aid for Scientific Research on Priority Areas by the Ministry of Education, Culture, Sports, Science and Technology.

References
[1] Kuroda K et al 2003 Class. Quantum Grav. 20 S871-84
[2] Gustafson E, Shoemaker D, Strain K and Weiss R 1999 LIGO document T990080-00-D; http://www.ligo.caltech.edu/docs/T/T990080-00.pdf
[3] Uchiyama T et al 1999 Phys. Lett. A 261 5-11
[4] Tomaru T, Suzuki T, Miyoki S, Uchiyama T, Taylor C T, Yamamoto A, Shintomi T, Ohashi M and Kuroda K 2002 Class. Quantum Grav. 19 2045-49
[5] Ohashi M et al 2003 Class. Quantum Grav. 20 S599-607
[6] Benabid F, Notcutt M, Loriette V, Ju L and Blair D G 2000 J. Phys. D: Appl. Phys. 33 589-94
[7] Khattak C P, Schmid F and Smith M B 1997 Windows and Dome Technologies and Materials V ed R W Tustison, Proc. SPIE 3060 250-57
[8] Benabid F, Notcutt M, Ju L and Blair D G 1998 Phys. Lett. A 237 337-42