Experimental Identification of Electric Dipoles Induced by Magnetic Monopoles in Tb\(_{2}\)Ti\(_{2}\)O\(_{7}\)

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Experimental identification of electric dipoles induced by magnetic monopoles in Tb$_2$Ti$_2$O$_7$

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The fundamental principles of electrodynamics allow an electron carrying both electric monopole (charge) and magnetic dipole (spin) but prohibit its magnetic counterpart. Recently, it was predicted that the magnetic “monopoles” carrying emergent magnetic charges in spin ice systems can induce electric dipoles. The inspiring prediction offers a novel way to study magnetic monopole excitations and magnetoelectric coupling. However, no clear example has been identified up to now. Here, we report the experimental evidence for electric dipoles induced by magnetic monopoles in spin frustrated Tb$_2$Ti$_2$O$_7$. The magnetic field applied to pyrochlore Tb$_2$Ti$_2$O$_7$ along [111] direction, brings out a “3-in-1-out” magnetic monopole configuration, and then induces a subtle structural phase transition at $H_c \sim 2.3$ T. The transition is evidenced by the non-linear phonon splitting under magnetic fields and the anomalous crystal-field excitations of Tb$^{3+}$ ions. The observations consistently point to the displacement of the oxygen O" anions along [111] axis which gives rise to the formation of electric dipoles. The finding demonstrates that the scenario of magnetic monopole having both magnetic charge and electric dipole is realized in Tb$_2$Ti$_2$O$_7$ and sheds light into the coupling between electricity and magnetism of magnetic monopoles in spin frustrated systems.

The interplay of electricity and magnetism is always a central topic in fundamental physics. In recent decades, the topic has received renewed attention in many exciting fields such as multiferroics, magnetoelectrics and spintronics [1, 2]. The coexistence of various degrees of freedom (lattice, charge and spin) and their mutual entanglement, output a large variety of unusual effects and responses [3]. These systems represent excellent platforms for quantum control and engineering in both fundamental research and practical applications.

Recently, Khomskii et al. proposed a fascinating scenario that in spin ice compounds the magnetic monopoles should be accompanied with electric dipoles [4, 5]. The attachment of electric dipoles on magnetic monopoles enables us to study and control such exotic magnetic monopoles by means of electric fields [6], and offers a novel way to develop potential applications in quantum computation. In spin ice systems, magnetic monopoles are topological defects of spin ice textures where within one tetrahedra the 2-in-2-out ice rule is violated [7, 8]. Spin ice realistic systems can be realized in rare-earth pyrochlore and spinel compounds. Besides, many of these materials exhibit large magnetoelectric coupling that are also regarded as one of the essential ingredients to realize multiferroicity [9, 10].

The effort of looking for the ideal candidate demonstrating both magnetic monopole and electric dipole, is thus focused on the pyrochlore family with strong magnetoelectric coupling. Recently, it was theoretically pointed out that one of the family members, Tb$_2$Ti$_2$O$_7$, is perhaps a good candidate to realize such scenario in its magnetic monopole structures [11]. Despite the complications of low-lying crystal field levels [12–14] in this material, it shows unfreezing behaviors down to 50 mK [15] and the pinch point correlations at low temperatures [16–19], suggesting that this material may be in a quantum spin ice state. Moreover, this material hosts giant spin-lattice coupling [20–24]. Thus, it is believed that Tb$_2$Ti$_2$O$_7$ represents a promising candidate to realize Khomskii’s proposal. While the field-induced magnetic monopole/anti-monopole structure has been reported by previous neutron experiments [25, 26], to the best of our knowledge, the induced electric dipoles and their coupling with magnetic monopoles have not been clearly identified on the experimental side.

In this Letter, we employed magneto-Raman and magnetodielectric technique to search for electric dipoles induced by magnetic monopoles in the pyrochlore Tb$_2$Ti$_2$O$_7$. The alternating magnetic monopole/anti-monopole structure is stabilized by magnetic fields along [111] axis [25]. Meanwhile, the field-induced electric dipoles are manifested by the pronounced rise of dielectric permittivity and the subtle structural changes captured by our Raman measurements. The application of
magnetic fields results in the unusual non-linear phonon splitting, the anomalous splitting of crystal field excitations (CFEs) and the emergence of new CFEs. The observations can be consistently and well explained in term of the shift of the oxygen (O’') along [111] axis that gives rise to electric dipoles. The findings demonstrate that electric dipoles induced by magnetic monopoles and the strong coupling between them are unambiguously identified in Tb$_2$Ti$_2$O$_7$. This opens new possibilities to control magnetic monopoles with electric fields.

High quality of Tb$_2$Ti$_2$O$_7$ single crystal was grown by the floating-zone technique [27] and has been characterized before measurements [28]. Confocal micro-Raman measurements were performed with a backscattering configuration using a Jobin Yvon T64000 system and a 532-nm diode-pumped solid-state laser. The laser power was kept at a level of 500 µW to avoid overheating. Magnetic fields were generated up to 9 T using a superconducting magnet, and the direction of magnetic field was along the [111] axis with an accuracy of ± 2°. The dielectric measurements were carried out in a Cryogen-free Superconducting Magnet System (Oxford Instruments, TeslatronPT). An Agilent 4980A LCR meter was used to measure the dielectric permittivity with the frequency $f = 1$ MHz.

The Raman spectra taken at 10 K and 0 T are shown in Fig. 1a, in which three strong phonon modes appear at ∼ 289 (F$_{2g}$), 320 (E$_{g}$) and 511 (A$_{1g}$) cm$^{-1}$. According to previous reports [29, 30], the F$_{2g}$ mode is assigned as the combined vibration of O’ (48f) and O’’ (8a) anions while the A$_{1g}$ and E$_{g}$ modes are solely due to the vibration of O’ anions. With the application of magnetic field along [111] axis, the A$_{1g}$ mode is nearly unchanged but the F$_{2g}$ mode shows a clear splitting and eventually evolves into two well-resolved modes (P1 and P2) at 9 T (Fig. 1b).

The photoluminescence origin of P1 or P2 can be easily excluded since they remain unchanged under different excitation sources (Fig. 1a). The magnetic origin is also unlikely because of the very small magnetic exchange energy in Tb$_2$Ti$_2$O$_7$ ($\Theta_{CW} \approx -19$ K) [15]. And the CFE origin is incompatible with the following characteristics of the two modes [28]: (1) the P1 and P2 intensities are almost one order of magnitude larger than that of a typical CFE at ∼ 100 cm$^{-1}$; (2) the P1 and P2 energies are well below that of the CFE (∼ 339 cm$^{-1}$) revealed by neutron experiments [14]; and (3) the P2 energy, which goes to saturation with increasing fields, exhibits a field dependence distinguished from that of a typical CFE, which normally manifests a pronounced linear field dependence.

By ruling out the above origins, we attribute the P1 and P2 modes to the splitting of the F$_{2g}$ phonon mode, which is strongly supported by the field dependence of their energies (Fig. 1c) and the polarization dependence of their intensities (Fig. 1d). At $H = 9$ T, the P2 mode locates at ∼ 16 cm$^{-1}$ above the P1 mode. The energy difference of the two modes decreases with decreasing fields and the two modes eventually merge into a single mode at ∼ 0 T. Although it is not easy to precisely extract the P1 and P2 positions at low magnetic fields through fitting process, the polarized Raman spectra (inset of Fig. 1c) clearly show that the energy difference between the two modes approaches zero with decreasing fields, suggesting that the P1 and P2 modes stem from the split of the F$_{2g}$ phonon. Moreover, the polarization dependence of the P1 and P2 intensities exhibits a clear anti-phase correlation (Fig. 1d), indicating they share the same origin, i.e., the F$_{2g}$ splitting. Then we conclude that the F$_{2g}$ mode splits into two modes with the application of magnetic field along [111] axis.

The splitting of the F$_{2g}$ mode suggests the breaking of cubic symmetries, which can be either due to a lattice modulation or a redistribution of electrons, or both. To clarify this issue, let’s turn to the CFEs which directly
probe the local environments around Tb$^{3+}$ ions. Generally, magnetic fields split the CFEs and their energies are expected to linearly depend on fields due to Zeeman effects. However, if there exists a strong field-induced lattice modulation that substantially affects the crystal field environments, the CFEs will behave anomalously, such as the nonlinear field dependence of the CFE energies, further splitting of CFEs and the emergence of new CFEs at non-zero field.

The anomalous evolutions of several CFEs under $H \parallel [111]$ are illustrated in Fig. 2a. At zero field, three CFEs are observed at $\sim 2117$ (CFE1), 2072 (CFE2) and 2060 (CFE3) cm$^{-1}$. According to the crystal field calculations [31], the three CFEs involve transitions from the $^7F_g$ manifolds to $^7F_5$ manifolds, as schematically shown in Fig. 2b. The CFE1 splits into three strong peaks with increasing magnetic fields. The peak energies (Fig. 2c) exhibit nearly a linear field dependence below $\sim 2.5$ T due to the Zeeman effect, and clearly deviate from the linear behaviors at $H > 2.5$ T. The non-linear field behaviors at $H > 2.5$ T are also witnessed by the CFE3 (Fig. 2e) and many other CFEs [28], systematically suggesting a change of crystal field environments of Tb$^{3+}$ ions.

The change of crystal field environments is more clearly evidenced by the splitting of the CFE2 and the emergence of a new CFE at $\sim 2.5$ T (Fig. 2d and 2e), which point to a distortion of the local geometry around Tb$^{3+}$ ions. Figure 2d shows the field dependence of the CFE2 energies, which is linear at low fields and then splits into two peaks between 2-3 T. The CFE2 splitting starting at $\sim 2.5$ T but not 0 T (inset of Fig. 2d), is quite unusual and needs to be understood with the distortion of the local geometry around Tb$^{3+}$ ions beyond the simple Zeeman effect. Meanwhile, a new CFE accompanying the CFE2 splitting appears between 2060 and 2070 cm$^{-1}$ (Fig. 2a). The normalized integrated intensities (I.I.) of the new CFE are shown in the inset of Fig. 2e and a transition-like upturn is seen above a critical field $H_c = 2.3$ T. This demonstrates that the new CFE does not originate from the Zeeman splitting of any CFE but is related to the distortion of the local geometry around Tb$^{3+}$ ions.

The above findings, including the $F_{2g}$ splitting and the anomalous behaviors of CFEs under magnetic fields, allow us to conclude that a field-induced subtle structural transition occurs at $H_c$ in Tb$_2$Ti$_2$O$_7$. Now the question is how to understand the structural transition. The low-lying first excited crystal field state [32] and the magnetoelastic mode [23, 24] seems unlikely to be the origin of the observed transition because of the small energy shift of the first excited level for $H < H_c$ ($< 1$ cm$^{-1}$ [28]). The fact that the observed anomalous behaviors appear only in the magnetic correlated state [28], suggests that the field-induced structural transition must be related with magnetism. Having in mind that the lattice structure of frustrated spin systems strongly depends on their magnetic structure [9, 33, 34] and that a field-induced magnetic transition occurs in Tb$_2$Ti$_2$O$_7$ from the zero field spin ice/liquid state to the magnetic monopole structure (3-in-1-out/3-out-1-in, Fig. 3a) [25], we propose that the field-induced structural transition observed in Tb$_2$Ti$_2$O$_7$ stems from the field-induced magnetic monopole structure which will be further explained below.

Unlike 3d ions, rare-earth ions possess a very strong spin-orbit coupling ($\sim 1$eV), which tightly locks the spin and the orbital angular momentum together, where the orbital angular momentum results from the spatially anisotropic $4f$ wave functions that can be simplicity envisioned as oblate electron charge cloud (equatorially expanded, upper of Fig. 3b) [35]. Due to the strong spin-orbit coupling, the orientation of the anisotropic shaped electron charge cloud is rigidly attached to the direction of the spin moment. At zero field, the system stays in the spin ice/liquid phase with magnetic moments randomly pointing into or out of tetrahedra. Meanwhile,
FIG. 3. Schematic illustration of magnetoelastic mechanism. (a), Electric dipoles and magnetic monopoles induced by [111]-field. Note that the displacement of \(O''\) ions is along the [111] direction and away from the Kagomé plane (shaded). \(\alpha\) is the angle between the Tb1 magnetic moments and the applied field. (b), Schematic of the oblate \(4f\) charge cloud of Tb1\(^{3+}\) ions (Upper) and its rotation under magnetic fields (Lower). (c), Field induced magnetic monopole \((H_0 < H < H_c)\) and monopole plus electric dipole phase \((H > H_c)\).

the \(O''\) ions reside at the center of Tb4 tetrahedra because of the equivalence of the four \(O''\)-Tb bonds. Upon increasing field, the direction of Tb2 moment stays parallel to the field while that of Tb1 moment continuously tilts from its original direction towards the field direction (Fig. 3a) [25]. Meanwhile, the strong spin-orbit coupling will drive the charge cloud (\(4f\) orbit) of Tb1 ions to rotate accordingly (Fig. 3b), which breaks the equivalence of the four \(O''\)-Tb bonds with the \(O''\)-Tb1 bonds having much more charge clouds overlap than that of the \(O''\)-Tb2 bonds (Fig. 3b, Tb1: on the Kagomé planes; Tb2: out of Kagomé planes). This will increase the Coulomb repulsion between the Tb1 and \(O''\) ions. To minimize the overall energy, the \(O''\) ions tend to displace away from the Kagomé planes along the [111] direction with the distorted phase having \(R3m\) space group (No. 166, see ref. [28] for the distorted structure). As a result, the induced electric dipoles emerge and the system should in principle develop an antiferroelectric order (Fig. 3c).

The scenario well explains our experimental observations. For \(H < H_c\), the Coulomb repulsion is too small to drive the displacement of \(O''\) ions. Therefore, the field dependence of the CFEs is governed by the Zeeman effect. On the other hand, the triple \(F_2g\) mode–vibration of \(O''\) ions which is surrounded by Tb4 tetrahedra (inset of Fig. 1a)–is very sensitive to the bonding environment of \(O''\)-Tb1 and \(O''\)-Tb2 bonds. Accompanied with the rotation of the Tb1 charge cloud, the equivalence of \(O''\)-Tb1 and \(O''\)-Tb2 breaks (Fig. 3b), which results in the splitting of the \(F_{2g}\) mode even at \(H < H_c\). For \(H > H_c\), the \(O''\)-Tb1 and \(O''\)-Tb2 bonds are not further broken by the displacement of \(O''\) ions. It explains why the P1 and P2 modes show only a kink in energy rather than a jump when crossing the transition. However, the real displacement of \(O''\) ions changes the local geometry of Tb3\(^{3+}\) ions, resulting in the observed anomalous behaviors of CFEs. The peak bifurcation of CFE2 above \(H_c\) is a natural consequence of the inequivalence of crystal field environments between Tb1 (\(C_1\) site symmetry) and Tb2 (\(D_{3d}\) site symmetry) [28]. And the variations of CF wave functions caused by the displacement in principle relax the CF transition rules and render some transitions visible in Raman channel, i.e., the emergence of the new CFE peak.

To confirm the formation of electric dipoles, we turn to the magnetodielectric response of this material. As shown in Fig. 4a and 4b, the low-temperature dielectric permittivity \(\varepsilon\) exhibits a pronounced rise with increasing field and its slope reaches maximum value at around 2-3 T, consistent with our Raman observations. The enhancement of \(\varepsilon\) strongly suggests the emergence of extra electric dipoles with increasing magnetic field. In contrast, \(\varepsilon\) is nearly constant with magnetic field at high temperatures. These results further support our proposal that the induced electric dipoles are related to the magnetic monopole structures at low temperatures, thus, they should strongly couple with each other. Based on above, we conclude that the concurrence of monopoles and coupled electric dipoles is realized in Tb2Ti2O7.
and analysis similar to Fig. 2e for different temperatures, we tracked the temperature dependence of $H_s$ (see Fig. 4c). From 10 to 100 K, $H_s$ linearly increases from $\sim$2.3 T to 3.5 T (see Fig. 4c). Interestingly, the extrapolated field at zero temperature ($\sim$2.1 T) agrees well with the field $H_2$ given by susceptibility [36] and $H_s$ by thermal conductivity [37]. It suggests that the transition at $H_2/H_s$ reported by thermodynamic measurements, is related to the structural transition observed here. The finding may be a key to understand the anomalous thermal conductivity observed in Tb$_2$Ti$_2$O$_7$ [37].

Let’s conclude the paper by discussing the key consequence of the field-induced structural transition in Tb$_2$Ti$_2$O$_7$, the emergence of electric dipole on its magnetic monopole. Tb$_2$Ti$_2$O$_7$ is thus a promising candidate in which the magnetic “monopoles” have both magnetic charges and coupled electric dipoles. The study makes the close analogy of electricity and magnetism go even further than usually assumed, i.e., the counterpart of a point charge (electron) not allowed in the fundamental level, can be realized as an emergent particle in condensed matter systems. This may bring many new and intriguing possibilities and greatly extend the study of pyrochlore spin systems. For example, the coupling between magnetic monopoles and electric dipoles allow to study and control the monopoles by external electric fields, i.e., creation, elimination and separation of monopoles and anti-monopoles.

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