Investigation of plasma parameters of a single-gas ion engine using a single langmuir probe

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Abstract. This paper presents the results of an experimental study of the parameters of nitrogen, argon and helium plasma obtained in high-frequency capacitive ring discharge at a pressure of 3.5 Pa. The parameters were determined by a single Langmuir probe. Work is being carried out as part of a project to develop a backlash ion microwave plasma generator. The object of study is ionized gas formed in the ring capacitance gap of a microwave solid-state generator emitter on the MOSFET transistor MRF284. The power consumed by the microwave generator in the study is no more than 5 W. The work resulted in getting the volt-ampere characteristics of a high-frequency discharge in low-pressure gas and determining the main plasma parameters.

1. Introduction
The development of high-frequency and microwave technology, in particular, the creation of solid-state microwave cells has opened up opportunities to create low-power plasma generators. This fact has revived interest in ion and plasma propulsion systems for small spacecraft with low excess on-board electrical power. The application of classical gas jet propulsion systems (cold or hot gas) on small micro- and nano-class spacecraft (1 to 10 kg) is not always feasible due to the high level of thrust generated (which has a negative impact on the controllability of the spacecraft) and low specific momentum.

Currently, all countries claiming a sovereign place in space are actively seeking technical solutions in the field of miniaturization (both in terms of mass and size, as well as energy consumption) of devices providing controlled motion by unmanned spacecraft. Advances in microelectronics opened the way to miniature installations, where jet propulsion is provided by Lorenz's power. To date, researchers in Japan and the USA have achieved the greatest results in terms performance. For example, an engine with a microwave plasma generator [1] with a 15 W power consumption and a xenon as a working body. The thrust declared by the authors is 0.297 mN at a specific pulse rate of 1100 seconds. The plasma is generated in the annular gap of the transmitter at 4.2 GHz (0.5-3 W). The construction has ring permanent magnets whose magnetic field directs the ionized substance to a two-electrode ion-optical system. A similar design has been presented by developers from the USA and is called MMIT [2]. According to the authors' assurances, this ion engine provides a thrust of 0.217 mN on argon and 0.392 mN on xenon with a power consumption of 8 W. The specific pulse is 10700 seconds and 7200 seconds, respectively. The potential difference on the ion-optical system was 2280 V for argon and 3350 V for xenon.

OmSTU's research laboratory "The propulsion system of micro-thrust small spacecraft" ("PSMT SSC") is working on the creation of an ion engine with microwave plasma generation of low power consumption. The main differences from existing designs are the application of elements of acceleration technology (single and
multiple clearances) and the absence of a guiding and focusing magnetic field of permanent and electric magnets.

The relevance of work in the field of ion-plasma thrusters for small spacecraft’s is determined by the worldwide search for an optimal solution to ensure controlled movement of unmanned devices in interplanetary space. Research of microwave plasma of working bodies of various chemical compositions generated in the discharge chamber of the engine provides the basis for choosing the best experimental working substance in terms of "price/efficiency" ratio.

This paper presents a probe-based study of the parameters of three working bodies of undisturbed microwave plasma: nitrogen, argon and helium. The plasma was generated in the ring gap of the transmitter, the design of which was originally adopted by the authors as the main (further revised) for implementation in a micro-thruster.

2. Problem statement

The main object of research is a single-gap ion engine of low power [3]. In the cavity of a single-gap ion engine, plasma is a gas with a high degree of ionization. The random separation of charges that can be generated inside it is small compared to the microscopic density of the charges. Consequently, macroscopic plasma is relatively neutral, although the main components are ions and electrons. The challenge, therefore, is to develop a methodology for calculating and implementing a physical model for determining the plasma parameters of the ring radiator, which forms the plasma in the engine cavity, with a single Langmuir probe.

For practical implementation of the methodology, the material and technical base of research laboratory "PSMT SSC" must be sufficient without additional resources.

The tasks to be solved in the course of research and reflected in this work:
1. Develop a schematic diagram for determining instantaneous values of the probe current;
2. Develop the required configuration of a single Langmuir probe suitable for studying plasma created by the transmitter;
3. Develop a method for calculating plasma parameters with a single Langmuir transducer;
4. Carry out experimental studies on the bench and obtain instantaneous values of voltage and current of the probe for the construction and approximation of volt-ampere characteristics.

3. Theory

3.1. Kinetic theory of plasma in the removal of voltamper characteristics

The concept of the ionized gas velocity distribution function is central to equilibrium kinetic theory. The particles in the gas have different velocities and the velocity of each particle changes due to collision with other particles. In theory, a particle in an ionized gas can have any velocity. The distribution function is based on the assumption that at any given time there is a statistical probability that a certain number of particles will have a certain velocity. According to existing concepts, the electron does not have an internal structure, so to characterize electrons it is sufficient to specify a distribution function for their velocities.

\[ f(v) = f(v_x, v_y, v_z). \]  

By which we mean the number of particles in a volume unit having velocity projections in some Cartesian coordinate system in intervals.

\[ v_x + dv_x \quad v_y + dv_y \quad v_z + dv_z. \]  

If plasma is isotropic, then \( f(v) \) depends only on the speed module, and the function for distributing electrons by energy can be entered \( F(\varepsilon) \) indicating the number of particles per unit volume that have kinetic energy \( \varepsilon = \frac{mv^2}{2} \) in the interval \( \varepsilon + \varepsilon \) \( dc \). It is known that in thermodynamic equilibrium, the Maxwell energy distribution of particles is established

\[ F(\varepsilon) = \frac{2}{\sqrt{\pi}} \frac{N \varepsilon}{(kT)^{3/2}} \exp(-\varepsilon / kT), \]  

and \( N = \int_0^\infty F(\varepsilon) \) \( dc \) – full particle concentration, \( T \) – temperature, \( k \) – Boltzmann's constant.

The function of electron distribution by energy is important in plasma research and modelling.
Similar to the speed distribution, the energy distribution function \( F(\varepsilon) \) determines the total number of ionised particles with energy. It is known from the FDEE that the concentration of charged plasma particles and their temperature can be determined. The concentration is equal to the integration of FDEE and the temperature is \( 2/3 \) of the average energy. This definition is true for any generalised distribution function. \( F(\varepsilon) \).

\[
n = \int_{0}^{\infty} F(\varepsilon) d\varepsilon, \quad (4)
\]

\[
T_e = \frac{2}{3} \left( \frac{\langle \varepsilon \rangle}{2} \right) = \frac{1}{2} \int_{0}^{\infty} \varepsilon F(\varepsilon) d\varepsilon. \quad (5)
\]

It was said that the distribution of velocities for gas in equilibrium is set by the Maxwell distribution. This expression is the product of three distribution functions for velocity in each of the three main directions, and is also isotropic in the velocity space.

\[
f(v) = \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left[ -\frac{m}{2kT} (v_1^2 + v_2^2 + v_3^2) \right]. \quad (6)
\]

Equation (6) is a normalized form of recording the Maxwell distribution function. This means that the integration of a function is one. The energy distribution is related to the velocity starting from the known ratio for kinetic energy of a particle with mass \( m \)

\[
\varepsilon = \frac{1}{2} m v^2. \quad (7)
\]

Since energy is related to the speed value, it is useful to work in terms of the speed distribution function.

\[
\chi(v) = \int_0^\infty f(v) dv = \left( \frac{m}{2\pi kT} \right)^{3/2} \varepsilon^2 \exp \left( -\frac{m}{2kT} \varepsilon^2 \right). \quad (8)
\]

Particles with a speed \( v \) have energy accordingly \( \varepsilon = \varepsilon(v) d\varepsilon \). Consequently

\[
F(\varepsilon) d\varepsilon = \chi(v) dv. \quad (9)
\]

The normalized energy distribution function is equal to the normalized speed distribution function multiplied by the derived speed in relation to energy.

\[
F(\varepsilon) = \chi(v) \frac{dv}{d\varepsilon}, \quad (10)
\]

\[
and \quad \frac{dv}{d\varepsilon} = \left( 2m \right)^{-1/2}. \quad (11)
\]

The final result is a normalized Maxwell energy distribution function.

\[
\chi(v) = \frac{2}{(kT)^{3/2} \sqrt{\pi}} \varepsilon \exp\left( -\frac{\varepsilon^2}{kT} \right). \quad (11)
\]

The energy distribution of gas is not always described in the Maxwell equation. One example of a not Maxwell or disequilibrium distribution is the distribution of Druyvesteyn. The Druyvesteyn distribution was obtained specifically for FDEE in gas discharges where the electron temperature is significantly higher than the ion temperature. This is the main difference between the Druyvesteyn and Maxwell distributions. Let us briefly summarize the conclusion presented in the paper [4]. The Druyvesteyn FDEE is based on the collision of solid shells between energy electrons and stationary ions in the electric field \( E \). The amount of kinetic energy lost in one collision is equal to twice the mass ratio multiplied by the initial energy of the electron.
\[ \Delta \varepsilon = \frac{2m_e \varepsilon}{m_i}. \quad (12) \]

Convert the expression of energy loss (12) by supplementing it with FDEE to represent the total energy loss for all electrons in a unit volume, in a unit time. Based on the average length of free travel \( \lambda \) and electron speed \( v_e \) get

\[ F(\varepsilon) = F(\varepsilon) \left( \frac{2m_e \varepsilon}{m_i} \right) \left( \frac{2\varepsilon}{m_e} \right). \quad (13) \]

Druyvesteyn equates equation (12) with energy taken from an electric field.

\[ J(\varepsilon)E = F(\varepsilon) \left( \frac{2m_e \varepsilon}{m_i} \right) \left( \frac{2\varepsilon}{m_e} \right). \quad (14) \]

The final expression of the Electron Distribution Function for Druyvesteyn energy is as follows

\[ F(\varepsilon) = \frac{4}{e^2 \cdot A_p} \sqrt{\frac{m_e}{e}} \sqrt{\frac{d^2 E(V)}{dV^2}}. \quad (15) \]

The most significant difference between Maxwell's energy distribution and Druyvesteyn's distribution is the different power of the exponential argument.

Based on the above, the raw data for calculating plasma parameters are instantaneous voltage and current, the probe surface area and the basic physical constants.

Volt-ampere characteristic approximation was performed according to the empirical equation drawn up, which is presented in [5]. This equation is suitable for most Langmuir probes. The equation includes four free parameters related to the basic properties and parameters of plasma. These properties include ion and electron saturation currents and the temperature of plasma electrons. The equation is easy to differentiate in order to obtain FDEE by the Druyvesteyn expression.

In this paper [5], the author uses a hyperbolic tangential function. This function is symmetrical and well suited to the double probe. However, for a single probe, this function must have an offset to the electronic saturation current region. The Volt-ampere characteristic of the Langmuir single probe is therefore asymmetrical. This problem is solved by adding an exhibitor to the symmetrical function, and the resulting expression for the probe current is as follows:

\[ I = \exp \left[ a_1 \tanh \left( \frac{V + a_2}{a_3} \right) \right] + a_4. \quad (16) \]

Four free parameters \( a_1, a_2, a_3, a_4 \) are related to the properties of plasma as follows. Coefficients \( a_2 \) and \( a_3 \), are independent, and because tensions \( V \) becomes very negative, and the hyperbolic function takes the limit value of -1.

The solution to equation (16) is the plasma potential.

\[ V_p = a_3 \tanh \left[ \ln \left( \frac{-a_4}{a_1} \right) \right] - a_2. \quad (17) \]

Ionic and electronic saturation currents are presented as follows, respectively

\[ I_{\text{ion\_sat}} = e^{-a_1} + a_4, \quad (18) \]

\[ I_{\text{e\_sat}} = e^{a_1} + a_4. \quad (19) \]

The concentration of electrons was determined by expression.

\[ N_e = \int f(\varepsilon) d\varepsilon. \quad (20) \]
Ion concentration by expression

$$N_{ion} = \frac{-I_{ion,sat}}{0.6eA_p} \sqrt{\frac{m_i}{eT_e}}$$  \hspace{1cm} (21)

The temperature of electrons is determined by expression (5) using the Electron Distribution Function for Druyvesteyn energy (15). A study on this topic is given in [6].

The methodology developed allows for the calculation of the temperature of electrons, ion concentration, plasma potential and floating potential by volt-ampere characteristic of a single Langmuir probe.

3.2. Development of a circuit diagram for the current sensor of the Langmuir probe

When designing the sensor circuitry, special attention was paid to low-noise amplifiers capable of amplifying small amplitude signals. After reviewing and analysing the amplifiers, the AD620 chip was selected. The low noise level, low input bias current and low power consumption of the AD620 make it well suited to the design of the Langmuir current sensor.

It has been experimentally determined that the current flowing through the probe varies between 1 and 300 µA. Input data for the design of the circuit diagram for the current sensor of the probe is given in Table 1. The schematic diagram of the Langmuir current transducer is shown in (fig. 1)

| Table 1. Input data for design |
|-----------------------------|
| Supply voltage $V_{in}$ | Minimum current flowing through shunt resistance $I_{min}$ | Maximum current flowing through shunt resistance $I_{max}$ | Sensor output voltage limit range $V_{out}$ |
| -15 V | ±1 µA | ±300 µA | ±12 V |

Figure 1. Circuit diagram of the Langmuir Probe Current Meter

In the diagram, R5 and R6 resistors represent the shunt resistance $R_{sh}$, through which the current of the probe flows. In the study of high-frequency plasma, noises and parasitic harmonics are produced that distort the useful signal. The circuit is therefore equipped with a second-order passive filter, which is organized by R3, R4 resistors and C2, C3, C4 capacitors. The connectors XW1 and XW2 are connected in series to the measuring circuit. Accordingly, XW2 is connected to a bipolar voltage shaper amplifier and XW1 to the Langmuir probe. Connector XW3 is connected to the input of one of the oscilloscope channels. In this way, a useful current signal is generated on the oscilloscope screen. The scheme developed has an independent battery power supply which is connected to XP1. The MC78L15 and L79L15 chips are equipped with a linear voltage stabiliser, which allows a stable supply voltage of $V_{in} = ±15B$. 

5
Based on the initial data in Table 1, we will present the expressions of the main parameters of the current sensor circuit diagram. As a result of the experiments, it has been established that the most optimal resistance is to accept shunt resistance $R_{sh} = 50$ Ohms. As the effect of bias voltage mistake for this resistance is minimal. Therefore, the gain is equal:

$$G = \frac{49.4 \times 10^7 (R_1 + R_2)}{R_1 R_2} + 1.$$  \hspace{1cm} (22)

The gain is set by resistors $R_1$ and $R_2$ connected between leads 1 and 8. The minimum and maximum output voltage is calculated as:

$$V_{out_{\min}} = I_{\min} R_{sh} G + x,$$

$$V_{out_{\max}} = I_{\max} R_{sh} G + x,$$  \hspace{1cm} (23) \hspace{1cm} (24)

and $x$ – zero drift correction factor.

The conversion of the sensor output voltage into output current is determined by the expression

$$I_{out} = \frac{V_{out} - x}{R_{sh} G}.$$  \hspace{1cm} (25)

3.3. Development of a single Langmuir probe

A single Langmuir probe is relatively easy to construct because all it takes is one open electrode, which is held in place by an insulating support. Depending on the area in which the plasma is studied, there may be significant or slight heating of the probe. A photograph of the design of a single Langmuir Probe is shown in (fig. 3). A metal tube is used to support the probe, which additionally serves as an electrostatic and electromagnetic shield connected to the ground bus. A ceramic tube was used as an insulator between the tungsten measuring wire and the shield. The design of the probe is recessed 0.2 cm into the end of the cylindrical Teflon tube. The kapton tape and heat shrink fixes the copper signal conductors in the fluoroplastic insulation.

4. Results experiments

A developed calculation methodology and a physical model have been used to investigate the plasma parameters created by the radiator. A photograph of the experiment to obtain a volt-ampere characteristic is
shown on (fig. 4). The experiment was conducted in a vacuum chamber at a pressure of 35 Pa. The working body was chosen to be nitrogen gas, argon and helium which was ionized by a microwave generator.

![Figure 4. Photo of an experiment to generate plasma in the radiator ring gap. Left to right gas a- nitrogen, b - argon, c - helium](image)

The instantaneous voltage and current values of the Langmuir probe are based on the volt-ampere characteristics given in Figure

![Figure 5. Volt-ampere characteristics. Left to right Gas a – nitrogen, b – argon, c – helium](image)

Functions for the distribution of electrons by energy have been defined and constructed using an approximated voltage-ampere characteristic function, knowing the function of filling in energy levels (or the function of energy distribution) makes it possible to calculate the average of any function from energy (fig.6).

![Figure 6. Electron distribution functions for nitrogen gas energies (a), argon (b), helium (c)](image)

Based on the results of the experiment, Table 2 shows the power consumption of a microwave generator and calculations of plasma parameters of a single-gap ion engine created using a ring radiator.
Table 2. Results of plasma parameters calculation with a single Langmuir probe

| Working body | Power Consumption P, W | Temperature of electrons Te, eV | Ion concentration Ni, m⁻³ | Concentration of electrons Ne, m⁻³ | Potential of plasma Vp, V | Floating plasma potential Vf, V |
|--------------|-------------------------|---------------------------------|--------------------------|-------------------------------------|---------------------------|-------------------------------|
| Ar           | 2.34                    | 4.96                            | 3.76·10¹⁶                | 1.26·10¹⁵                          | 23.11                     | -2.41                         |
|              | 3.45                    | 4.59                            | 4.01·10¹⁶                | 1.38·10¹⁵                          | 20.59                     | -2.51                         |
|              | 4.43                    | 3.61                            | 3.86·10¹⁶                | 1.49·10¹⁵                          | 15.5                      | -1.71                         |
| He           | 2.68                    | 4.78                            | 8.88·10¹⁵                | 1.84·10¹⁶                          | 42.92                     | 1.09                          |
|              | 3.47                    | 4.08                            | 1.11·10¹⁶                | 2.49·10¹⁵                          | 33.92                     | 3.37                          |
| N₂           | 3.07                    | 5.69                            | 2.65·10¹⁶                | 6.89·10⁴                           | 15.77                     | -4.35                         |
|              | 3.81                    | 5.05                            | 2.80·10¹⁶                | 7.54·10⁴                           | 10.52                     | -5.36                         |
|              | 5.18                    | 4.88                            | 3.19·10¹⁶                | 9.36·10⁴                           | 12.29                     | -4.81                         |

The study of parameters of gases of various chemical compositions ionised in a ring capacitive high-frequency gap using the probe method is aimed at identifying the optimum working body used in the course of work on a multi-gap ion engine. Important in the practical part of the work are indicators of ion concentration in plasma with the lowest level of power consumption of a car generator. The plasma parameters obtained at different input capacities of a microwave generator for argon and helium gas do not differ significantly. As power consumption increases, the temperature of electrons decreases. This dependence is maintained for the types of gases listed in the table. The smallest concentration of electrons is nitrogen. It should be noted that a decrease in the concentration of electrons reduces the frequency of plasma.

5. Discussion of results
Determining the parameters of plasma generated by different gases is an important task for the design of a microwave ion source. It is known that the greatest efficiency of such a source is achieved by the appearance of resonant electron frequencies in plasma [7].

Here is a dispersion relation for electromagnetic waves in plasma.

\[ \omega^2 - \frac{N_e}{\varepsilon_0 m_e} c^2 k^2 = \omega_p^2 = \omega^2 - \omega_e^2, \]  
(26)

and \( k = \frac{2\pi}{\lambda} \) – wave number, \( c \) – light speed, \( m_e \) – electron mass.

In expression (26) the plasma frequency is equal to

\[ \omega_p^2 = \frac{N_e c^2}{\varepsilon_0 m_e}. \]  
(27)

The wavelengths created by a microwave generator and a ring emitter can be found as follows:

\[ \lambda = \frac{2\pi c}{\sqrt{\omega_p^2 - \omega_e^2}} = \frac{c}{\sqrt{f_p^2 - f_e^2}}, \]  
(28)

and \( f_p \) – plasma frequency, \( f_e \) – microwave generator frequency.

It follows from these expressions that if a microwave generator exceeds the frequency of electrons in plasma, the wavelength becomes infinitely long. In this case, the wave does not penetrate the plasma - it is reflected. By studying the parameters of plasma generated for different gases, it is possible to determine the frequency of the microwave generator, which allows for a more effective degree of ionization.

6. Conclusion
The article presents the intermediate results of the authors’ work in the field of measuring plasma parameters created by a ring emitter of a small thrust ion engine. According to the results of the study it was established that an alternative to electrostatic, emission and high-frequency induction plasma sources may be a microwave ring emitter. This solves the problem of constant voltage in the discharge chamber cavity and reduces the erosion of electrodes in the plasma. Electromagnetic waves propagate in plasma under certain conditions. If the plasma concentration is low enough or if the microwave generator has a high frequency, the microwave examination is reflected by the plasma surface. Conversely, if the conditions are such that microwaves penetrate the plasma, then microwave energy pumps the plasma by resonant heating the
electrons in the magnetic field. It is therefore necessary to ensure that the magnetic field of the ring emitter is sufficiently large to create resonance and pressure to begin discharging in the cavity of a single-gap ion engine. The effects given above affect the design and performance of the plasma generator. The methods and methodologies for probing the parameters of undisturbed plasma of three types of gas: nitrogen, argon and helium are presented. The design and technical implementation of a single Langmuir probe and a current meter from the probe are presented. The authors present the volt-ampere characteristics of discharges, which are used to determine the parameters of plasma components. The method of diagnosing plasma with a single Langmuir probe is of great practical interest in terms of finding the resonant frequency of plasma and the optimum consumption of the working body, which will ensure the greatest efficiency of ionization and increase the stability of the discharge.

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