Y(4626) as a $P$-wave $[cs][ar{c}s]$ tetraquark state

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Motivated by the Belle Collaboration’s new observation of $Y(4626)$, we investigate the possibility of its configuration as a $P$-wave $cs$-scalar-diquark $car{s}$-scalar-antidiquark state from QCD sum rules. Eventually, the extracted mass $4.60^{+0.14}_{-0.20}$ GeV agrees well with the experimental data of $Y(4626)$, which could support its interpretation as a $P$-wave $[cs][ar{c}s]$ tetraquark state.

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I. INTRODUCTION

Very newly, Belle Collaboration reported the first observation of a vector charmoniumlike state $Y(4626)$ decaying to a charmed-antistrange and anticharmed-strange meson pair $D_s^+ D_{s1}(2536)^-$ with a significance of $5.9\sigma$ [1]. Its mass and width were measured to be $4625.9^{+6.2}_{-6.0} \pm 0.4$ MeV and $49.8^{+13.9}_{-11.5} \pm 4.0$ MeV, respectively. This state is near the $Y(4660)$ observed in the hidden-charm process $e^+e^- \to \psi(2S)\pi^+\pi^-$ [2, 3] and also consistent with the $Y(4630)$ searched in the $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ [4, 5]. Considering their close masses and widths, $Y(4660)$ and $Y(4630)$ were suggested to be the same resonance [6–8], and there have been various theoretical explanations for them, such as a conventional charmonium [9–11], a baryonium state [15–17], a hadro-charmonium state [18], a tetraquark state [19–25], and so on.

The new observation of $Y(4626)$ by Belle immediately aroused one’s great interest [26–32]. With an eye to the multiquark viewpoint, an assignment of $Y(4626)$ was proposed as a $D_s^+\bar{D}_{s1}(2536)$ molecular state in a quasipotential Bethe-Salpeter equation approach with the one-boson-exchange model [27]. Later, the mass spectrum of a $D_s^+\bar{D}_{s1}(2536)$ system was calculated within the framework of Bethe-Salpeter equations [28], and in the end the authors may not think $Y(4626)$ to be a $D_s^+\bar{D}_{s1}(2536)$ bound state, but something else. Otherwise, some authors employed a multiquark color flux-tube model with a multibody confinement potential and one-glue-exchange interaction to make an exhaustive investigation on the diquark-antidiquark state [29], and they concluded that $Y(4626)$ can be well interpreted as a $P$-wave $[cs][\bar{c}s]$ state.

Under the circumstance, it is interesting and of significant to study that whether $Y(4626)$ could be a candidate of $P$-wave $[cs][\bar{c}s]$ tetraquark state by different means. It is known that one has to face the complicated nonperturbative problem in QCD while handling a hadronic state. Established on the QCD basic theory, the QCD sum rule acts as one authentic way for evaluating nonperturbative effects, which has been successfully applied to plenty of hadronic systems (for reviews see [34–37] and references therein). Therefore, in this work we devote to investigate that whether $Y(4626)$ could be a $P$-wave $[cs][\bar{c}s]$ tetraquark state with the QCD sum rule method.

This paper is organized as follows. The QCD sum rule for the $P$-wave tetraquark state is derived in Sec. II, followed by the numerical analysis in Sec. III. The last part is a brief summary.

II. THE $P$-WAVE $[cs][\bar{c}s]$ STATE QCD SUM RULE

According to our previous analysis [22, 38], a $P$-wave $[cs][\bar{c}s]$ state having the flavor content $[cs][\bar{c}s]$ with the spin momentum numbers $S_{[cs]} = 0, S_{[\bar{c}s]} = 0, S_{[cs][\bar{c}s]} = 0$, and the orbital momentum number $L_{[cs][\bar{c}s]} = 1$. To characterize the studied state, the following current could be constructed from the $cs$-
scalar-diquark $\bar{s}s$-scalar-antidiquark configuration and a derivative could be included to generate $L = 1$, 

$$j_\mu = \epsilon_{def} \epsilon_{d'e'} (\bar{s}d^T C \gamma_5 \bar{c}e) D_\mu (\bar{s}d^T C \gamma_5 \bar{c}e').$$  

(1)

Here the index $T$ means matrix transposition, $C$ denotes the charge conjugation matrix, $D^\mu$ is the covariant derivative, and $d$, $e$, $f$, $d'$, and $e'$ are color indices.

In general, the two-point correlator

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T [j_\mu(x) j^\nu_\mu(0)] | 0 \rangle.$$  

(2)

can be parameterized as

$$\Pi_{\mu\nu}(q^2) = \frac{g_{\mu\nu} q^2}{q^2} \Pi^{(0)}(q^2) + \left( \frac{g_{\mu\nu} q^2}{q^2} - g_{\mu\nu} \right) \Pi^{(1)}(q^2).$$  

(3)

Furthermore, the part $\Pi^{(1)}(q^2)$ of the correlator proportional to $g_{\mu\nu}$ is employed to attain the sum rule, which can be evaluated in two different ways: at the hadronic level and at the quark level. Phenomenologically, $\Pi^{(1)}(q^2)$ can be written as

$$\Pi^{(1)}(q^2) = \frac{\lambda^2}{M_H^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im} \Pi^{(1)}(s)}{s - q^2},$$  

(4)

where $M_H$ denotes the hadron’s mass. In the OPE side, $\Pi^{(1)}(q^2)$ can be expressed as

$$\Pi^{(1)}(q^2) = \int_{(2m_c + 2m_s)^2}^\infty ds \frac{\rho(s)}{s - q^2},$$  

(5)

for which the spectral density $\rho(s) = \frac{4}{\pi} \text{Im} \Pi^{(1)}(s)$.

To derive $\rho(s)$, one works at leading order in $\alpha_s$. The $s$ quark is treated as a light one and the diagrams are considered up to the order $m_s$. Keeping the heavy-quark mass finite, one uses the heavy-quark propagator in momentum space. The correlator’s light-quark part is calculated in the coordinate space and Fourier-transformed to the momentum space in $D$ dimension, which is combined with the heavy-quark part and then dimensionally regularized at $D = 4 - \epsilon$. Lastly, the spectral density is concretely given by

$$\rho^{\text{pert}}(s) = \rho^{(\bar{s}s)} + \rho^{(g^2 G^2)} + \rho^{(g \bar{s}s Gs)} + \rho^{(\bar{s}s + \rho^2 G^2)},$$

with

$$\rho^{(\bar{s}s)} = \frac{\langle \bar{s}s \rangle}{3} \left\{ \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^4} \left[ (2 - \alpha - \beta)m_c + (1 - \alpha - \beta)m_s \right] r^2 - m_s \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \left[ m_c^2 - (1 - \alpha) s \right] \right\},$$

$$\rho^{(g^2 G^2)} = -\frac{m_c}{3} \left( \frac{g^2 G^2}{2} \right)^2 \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^2} \left[ (1 - \alpha - \beta)(\alpha^2 + \beta^3) \pi \{ (m_c - 3m_s) - 2m_s m_c^2 (\alpha + \beta) \} \right],$$

$$\rho^{(g \bar{s}s Gs)} = \frac{1}{\alpha_{\text{min}}} \left\{ \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^2} \left[ -3m_c \alpha (\alpha + \beta) - 4m_s^2 \alpha (1 - \alpha) \right] \right\},$$

$$\rho^{(\bar{s}s)^2} = \frac{m_c^2 \langle \bar{s}s \rangle^2}{3} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \left\{ -2m_c [m_c^2 - (1 - \alpha) s] + m_s [m_c^2 - 2(1 - \alpha) s] \right\},$$

and

$$\rho^{(g^3 G^3)} = -\frac{\langle g^3 G^3 \rangle}{3} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^2} \left[ (\alpha^3 + \beta^3) r + 4(\alpha^4 + \beta^4) m_c^2 \right],$$
which is coincident with our previous work \cite{22}. It is defined as \( r = (\alpha + \beta)m_c^2 - \alpha \beta s \) and \( \kappa = 1 + \alpha - 2\alpha^2 + \beta + 2\alpha \beta - 2\beta^2 \). The integration limits are \( \alpha_{\text{min}} = (1 - \sqrt{1 - 4m_c^2/s})/2, \alpha_{\text{max}} = (1 + \sqrt{1 - 4m_c^2/s})/2, \) and \( \beta_{\text{min}} = am_c^2/(sa - m_c^2) \). For the four-quark condensate \( \langle ss \rangle^2 \), a general factorization \( \langle ss\bar{s}s \rangle = \varrho \langle ss \rangle^2 \) \cite{35,12} has been used, where \( \varrho \) is a constant, which may be equal to 1 or 2.

After equating the two expressions (4) and (5) of the correlator, assuming quark-hadron duality, and making a Borel transform, the sum rule can be given by

\[
\lambda^2 e^{-M_H^2/M^2} = \int_{(2m_c+2m_s)^2}^{s_0} ds \rho e^{-s/M^2}. \tag{6}
\]

Eliminating the hadronic coupling constant \( \lambda \), one could yield

\[
M_H^2 = \int_{(2m_c+2m_s)^2}^{s_0} ds \rho e^{-s/M^2} / \int_{(2m_c+2m_s)^2}^{s_0} ds e^{-s/M^2}. \tag{7}
\]

### III. NUMERICAL ANALYSIS

Performing the numerical analysis of sum rule (7), the \( s \)-quark mass and the running charm quark mass are chosen as updated values \cite{43}: \( m_c = 93^{+11}_{-5} \) MeV and \( m_c = 1.27 \pm 0.02 \) GeV, respectively. Besides, other input parameters are taken as \cite{33,37}: \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01) \) GeV; \( m_\lambda^2 = 0.8 \pm 0.1 \) GeV; \( \langle \bar{s}s \rangle = m_0^2 \langle \bar{q}q \rangle \), \( \langle g\sigma \cdot Gs \rangle = m_0^2 \langle \bar{s}s \rangle \), \( \langle g^2 G^2 \rangle = 0.88 \pm 0.25 \) GeV; and \( \langle g^3 G^3 \rangle = 0.58 \pm 0.18 \) GeV.

Complying with the standard criterion of sum rule analysis, both the OPE convergence and the pole dominance would be considered to find appropriate work windows for the threshold \( \sqrt{s_0} \) and the Borel parameter \( M^2 \); the lower bound of \( M^2 \) is gained by analyzing the OPE convergence, and the upper one is obtained by viewing that the pole contribution should be larger than QCD continuum contribution. Meanwhile, the threshold parameter \( \sqrt{s_0} \) characterizes the beginning of the continuum state and is about \( 400 \sim 600 \) MeV above the lastly extracted value \( M_H \) in empirical.

At first, the input parameters would be kept fixed at their central values. To obtain the lower bound of \( M^2 \), the OPE convergence is shown in FIG. 1 by comparing the relative contributions of different condensates from sum rule (6) for \( \sqrt{s_0} = 5.2 \) GeV. In numerical, the relative perturbative contribution begins to play a dominant role in the OPE side at \( M^2 = 3.0 \) GeV\(^2 \), which is increasing with the Borel parameter \( M^2 \). Thereby, the perturbative part could dominate comparing with other condensate contributions in OPE while taking \( M^2 \geq 3.0 \) GeV\(^2 \). On the other hand, the upper bound of \( M^2 \) is gained by considering the pole dominance phenologically. The comparison between pole and continuum contributions from sum rule (6) is shown in FIG. 2 for \( \sqrt{s_0} = 5.2 \) GeV. The relative pole contribution is approximate to \( 50\% \) at \( M^2 = 3.5 \) GeV\(^2 \) and descending with the \( M^2 \). Hence, the pole contribution dominance could be satisfied when \( M^2 \leq 3.5 \) GeV\(^2 \). Consequently, the Borel window of \( M^2 \) is fixed on \( 3.0 \sim 3.5 \) GeV\(^2 \) for \( \sqrt{s_0} = 5.2 \) GeV. In the similar analysis, the proper range of \( M^2 \) is gained as \( 3.0 \sim 3.4 \) GeV\(^2 \) for \( \sqrt{s_0} = 5.1 \) GeV, and \( 3.0 \sim 3.7 \) GeV\(^2 \) for \( \sqrt{s_0} = 5.3 \) GeV. In the chosen work windows, it is expected that two sides of QCD sum rules have a good overlap and information on the resonance can be safely extracted. The mass \( M_H \) of the \( P \)-wave \([cs][c\bar{s}]\) tetraquark state is shown in FIG. 3 as a function of \( M^2 \) from sum rule (7). In the work windows, the mass value is computed to be \( 4.60 \pm 0.11 \) GeV, for which the numerical error reflects the uncertainty due to variation of \( s_0 \) and \( M^2 \). By this time, the input QCD parameters have been kept at the central values.

Next, varying the quark masses and condensates and one could arrive at \( 4.60 \pm 0.11^{+0.03}_{-0.04} \) GeV (the first error resulted from the uncertainty due to variation of \( s_0 \) and \( M^2 \), and the second error reflects the variation of QCD parameters) or concisely \( 4.60^{+0.14}_{-0.15} \) GeV. At last, taking into account the variation of factorization factor \( \varrho \) in four-quark condensate \( \langle \bar{s}s \rangle^2 \) from 1 to 2, one could extract the final mass value \( 4.60^{+0.14}_{-0.20} \) GeV for the \( P \)-wave \([cs][c\bar{s}]\) tetraquark state, which is in good agreement with the experimental data for \( Y(4626) \) and could support its \( P \)-wave \([cs][c\bar{s}]\) explanation.
FIG. 1: The OPE convergence is shown by comparing the relative contributions of perturbative, two-quark condensate $\langle \bar{s}s \rangle$, two-gluon condensate $\langle g^2 G^2 \rangle$, mixed condensate $\langle g \bar{s} \sigma \cdot G s \rangle$, four-quark condensate $\langle \bar{s}s \rangle^2$, and three-gluon condensate $\langle g^3 G^3 \rangle$ from sum rule (6) for $\sqrt{s_0} = 5.2$ GeV.

FIG. 2: The phenomenological contribution in sum rule (6) for $\sqrt{s_0} = 5.2$ GeV. The solid line is the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) as a function of $M^2$ and the dashed line is the relative continuum contribution.

IV. SUMMARY

Stimulated by the Belle’s first observation of a vector charmoniumlike state $Y(4626)$, we have computed the mass of $P$-wave $[c\bar{s}][c\bar{s}]$ tetraquark state in QCD sum rules. The final result $4.60^{+0.14}_{-0.20}$ GeV for the $P$-wave $[c\bar{s}][c\bar{s}]$ tetraquark state is well compatible with the experimental data $4625.9^{+6.2}_{-6.0} \pm 0.4$ MeV of $Y(4626)$, which favors the explanation of $Y(4626)$ as a $P$-wave $[c\bar{s}][c\bar{s}]$ tetraquark state.

For the future, one can expect that further experimental observations and continually theoretical studies may shed more light on the nature of $Y(4626)$. 
FIG. 3: The dependence on $M^2$ for the mass $M_H$ of $[cs][\bar{cs}]$ from sum rule (7) is shown. The ranges of $M^2$ are $3.0 \sim 3.4$ GeV$^2$ for $\sqrt{s_0} = 5.1$ GeV, $3.0 \sim 3.5$ GeV$^2$ for $\sqrt{s_0} = 5.2$ GeV, and $3.0 \sim 3.7$ GeV$^2$ for $\sqrt{s_0} = 5.3$ GeV, respectively.

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