Helical Properties of Sheared and Rotating Turbulence

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Abstract. Helical properties of two homogeneous turbulent flows are compared by using velocity fields obtained by means of direct numerical simulation (DNS). Probability distribution functions (PDFs) of relative helicity show a preference for two-dimensionalization in the case of growing shear-driven turbulence, while an approximately equidistribution is obtained for the case of rotating turbulence. PDFs of super-helicity show a higher probability for alignment or anti-alignment of vorticity with dissipation of velocity in both cases. Isovorticity surfaces colored with relative helicity and relative super-helicity show that the signs of the two quantities tend to be strongly correlated again in both cases. This sign correlation is further quantified by their joint PDF and it suggests that super-helicity dissipates helicity.

1. Introduction

Turbulent flows typically exhibit a self-organization into coherent vorticity structures. Depending on the flow configuration the vortical structures show different topologies, such as columnar structures in rotating turbulence (Liechtenstein et al., 2005), inclined structures in shear-driven turbulence (Jacobitz et al., 2008), or sheet-like structures in stratified turbulence (not considered in this study). Helicity allows to characterize the flow topology statistically. Therewith swirling structures can be distinguished from non-swirling motion. Helicity is of importance in meteorology for the formation and prediction of hurricanes and tornadoes. Historically helicity can be traced back to Betchov (1961), Moreau (1961), and Moffatt (1969). A review on helicity in laminar and turbulent flow can be found in Moffatt & Tsinober (1992) and Sagaut & Cambon (2008). Recent high resolution simulations of helical rotating turbulence have been presented by Mininni & Pouquet (2010a,b).

In the current work we consider two homogeneous turbulent flows without mean helicity, one with pure rotation and one with pure shear from a uniform mean velocity gradient. Both flows are computed by means of direct numerical simulation (DNS). Details on the flows can be found in Liechtenstein et al. (2005) and Jacobitz et al. (2008), respectively.

The aim of the performed flow analyzes is two-fold. First, we consider statistics of the helicity and super-helicity, the latter defined as the scalar product between vorticity and its curl. We
Table 1. Properties of the turbulent flows considered in this study.

| Flow                  | Reference              | $Re_{\lambda}$ | $K$  | $Z$   | fate   |
|----------------------|------------------------|----------------|------|-------|--------|
| Sheared Turbulence   | Jacobitz et al. (2008) | 72             | 1.157| 176.0 | growth |
| Rotating Turbulence  | Liechtenstein et al. (2005) | 50            | 0.062| 2.3   | decay  |

also analyze the probability distribution of the angle between velocity and the dissipation term, defined by the negative of the Laplacian of velocity. Second, we focus on the helicity dynamics by considering its transport equation. We also address the question if super-helicity is responsible for dissipating helicity.

The remainder of the manuscript is organized as follows. After a short description of the DNS data we present the helicity statistics. Then we study the helicity transport equation and finally some conclusions are drawn.

2. Helicity Statistics

The scalar-valued quantity helicity $H_u = u \cdot \omega$ is defined as the scalar product of velocity $u$ and vorticity $\omega = \nabla \times u$ and it is an important topological quantity to characterize the local structure of turbulence. Relative helicity $h_u = H_u/(|u||\omega|)$ describes the cosine of the angle between velocity and vorticity vectors. For helical structures (swirling motion), $h_u$ has values of $\pm 1$, which correspond to alignment or anti-alignment of vorticity and velocity. For non-helical structures, local two-dimensionalization of the flow occurs and the helicity assumes a value $h_u = 0$ as velocity is perpendicular to vorticity. In this case, the depletion of non-linearity due to local helicity fluctuations in minimal. Corresponding quantities for super-helicity $H_\omega = \omega \cdot (\nabla \times \omega)$ and relative super-helicity $h_\omega = H_\omega/(|\omega||\nabla \times \omega|)$ are defined in a similar way.

In this work, the helical properties of growing sheared turbulence (Jacobitz et al., 2008, 2010) and decaying rotating turbulence (Liechtenstein et al., 2005) are compared. The flows were studied using DNS based on a Fourier collocation method on grids with $256^3$ grid points. Some flow parameters are given in table 1 and further details can be found in the respective publications. These flows initially do not contain mean helicity and they remain free from it.

Figure 1. PDFs of relative helicity $h_u$ for shear-driven and rotating turbulence. The PDFs are estimated using histograms with 100 equidistant bins.
However, they possess local helicity and regions with strong helicity can exist in a flow free from mean helicity.

Figure 1 shows the PDFs of relative helicity $h_u$. The sheared turbulence case has a maximum at $h_u = 0$, corresponding to a preference for velocity and vorticity to be perpendicular, i.e., a tendency to two-dimensionalization of the flow. The decaying rotating turbulence case shows an approximately uniform distribution of relative helicity. The corresponding PDFs of relative super-helicity $h_\omega$ are given in figure 2. In contrast to the PDFs of $h_u$, the PDFs of $h_\omega$ for both flows exhibit pronounced maxima at $h_\omega = \pm 1$, corresponding to alignment or anti-alignment of vorticity $\omega$ with the negative of velocity dissipation $-\Delta u$.

3. Helicity Transport Equation

Just as energy, the mean helicity $\langle H_u \rangle = \int H_u d^3x$ satisfies a transport equation:

$$\frac{d}{dt} \langle H_u \rangle = -2\nu \langle H_\omega \rangle + \langle F \rangle \tag{1}$$

Here, $F = 2f \cdot \omega$ accounts for the forcing term $f$ in the momentum equation, and $\nu$ is the kinematic viscosity of the fluid. Contrary to energy, neither helicity $H_u$ nor super-helicity $H_\omega$ are positive definite quantities. Therefore, super-helicity can only be interpreted as helicity dissipation, if both have the same sign. Considering isotropic turbulence, Sanada (1993)
conjectured that helicity and super-helicity indeed have the same sign and thus the super-helicity acts as a dissipative mechanism for helicity. Further evidence supporting Sanada’s conjecture was given more recently by Galanti & Tsinober (2006) for isotropic turbulence with helical or non-helical forcing.

As shown in figure 3, the PDF of the cosine of the angle between velocity $u$ and $-\nabla^2 u = \nabla \times \nabla \times u$ indicates a much larger probability that the two vectors are aligned, which holds for both flows. Using integration by parts, it follows that the mean value is positive $-\langle u \cdot \nabla^2 u \rangle = \langle \omega \cdot \omega \rangle > 0$. This implies thus the tendency that $H_u = u \cdot \omega$ and $H_\omega = -\nabla^2 u \cdot \omega$ have the same sign (Sanada, 1993) - a tendency that also holds for the relative helicities (Galanti & Tsinober, 2006).

Figures 4 and 5 show isovorticity surfaces of shear-driven turbulence and rotating turbulence, respectively. Inclined structures are observed for sheared turbulence, while vertical columns are present in rotating turbulence. The isovorticity surfaces are colored with relative helicity $h_u$ (left) and relative super-helicity $h_\omega$ (right). For a given flow structure, it is likely that the same sign is found. This visual observation thus indicates a high likelihood that the signs of $h_u$ and $h_\omega$ are the same.
$h_\omega$ are correlated. This suggests that super-helicity acts as a dissipative process for helicity also in anisotropic turbulence. Further confirmation is obtained by consideration of the joint PDFs of relative helicity and relative super-helicity, shown in figure 6. The surface plots show indeed a strong correlation of the signs of the two helicities for both flows. For rotating turbulence the PDF is almost symmetric, while for shear-driven turbulence the symmetry with regard to the diagonal is broken and a bias is observed.

4. Conclusion

DNS data of rotating and shear-driven homogeneous turbulent flows without mean helicity have been analyzed with respect to their helical properties. Flow visualizations of isovorticity surfaces colored either with relative helicity or relative super-helicity illustrated that indeed both quantities exhibit a much larger likelihood to have the same sign. Hence we gave evidence that super-helicity is responsible for dissipating helicity not only for isotropic turbulence as proposed in Sanada’s conjecture (Sanada, 1993), but also for rotating and sheared turbulence.

For more detailed studies on helical properties for a large variety of homogeneous flows, such as isotropic turbulence, sheared and rotating turbulence, we refer to Jacobitz et al. (2011a). Therein scale-dependent analyses based on orthogonal wavelet decomposition can be found. A helical multiscale characterization of the different flows has been performed and we showed that small scales of motion are helical, while large scales have a higher probability that velocity and vorticity are perpendicular, corresponding to a trend to two-dimensionalization. The influence of initial mean helicity on different homogeneous shear flows has been studied in Jacobitz et al. (2011b). Therein we have shown that the decay of the initial turbulent kinetic energy is weakened in the presence of strong mean helicity. The direct numerical results have been theoretically underpinned by considering the spectral tensor, i.e., the Fourier transform of the two-point correlation of the velocity. The evolution equations of the symmetric and anti-symmetric part, the latter corresponding to helicity, nicely showed that both equations are only coupled through the nonlinear interaction terms.

Figure 6. Joint PDFs of relative helicity $h_u$ and $h_\omega$ for shear-driven turbulence (left) and rotating turbulence (right).
Acknowledgments

We thank Claude Cambon, Fabien Godeferd, and Lukas Liechtenstein for many discussions and access to their DNS data of rotating turbulence. FGJ acknowledges support from an International Opportunity Grant from the University of San Diego. KS thanks the Université de Provence, Relations Internationales for travel support. MF and KS acknowledge financial support from the PEPS program of INSMI–CNRS.

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