The Istanbul BFT Consensus Algorithm

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1 Introduction

In this paper, we present Istanbul BFT (IBFT), a Byzantine fault-tolerant (BFT) consensus algorithm that is used for implementing state-machine replication in the Quorum blockchain\footnote{https://www.goquorum.com}. Quorum is an open source permissioned blockchain platform. It is based on Ethereum and designed for enterprise applications.

IBFT was initially proposed informally in EIP-650 \cite{EIP650}. The first proposal had safety issues where two correct processes could decide on different values. A revision addressed these safety issues but introduced liveness issues where some executions could lead to a deadlock \cite{IBFT2}. This paper offers a precise and correct description of the algorithm along with correctness proofs.

IBFT belongs to a class of BFT algorithms that assume a partially synchronous communication model \cite{BFT1}. Under this model, although messages can be arbitrarily delayed, the system is characterized by also having periods of good communication in which messages are timely delivered. The IBFT algorithm is deterministic, leader-based, and optimally resilient - tolerating $f$ faulty processes out of $n$, where $n \geq 3f + 1$. During periods of good communication, IBFT achieves termination in three message delays. The communication complexity is $O(n^2)$ for normal operation and $O(n)$ for round (or view) changes.

We also introduce a variant of IBFT, dubbed LinearIBFT, which achieves $O(n)$ communication complexity during both normal operation and view changes.
As we can see, our two protocols occupy an interesting point in the design space. IBFT minimizes worst-case latency (i.e., the number of message delays to termination without optimistic assumptions about the environment). It achieves termination in three message delays - matching PBFT, Zyzzyva, and Spinning - while having a quadratic complexity during normal operation and linear complexity during view changes. LinearIBFT achieves linear complexity during normal operation, which is beneficial for scalability.

Table 1: Performance of related algorithms when communication is timely. Message delays for dual-mode protocols are shown as \( x/y \) where \( x \) is for the optimistic environment and \( y \) otherwise.

* Zyzzyva requires, for the slow path, waiting for the maximum network delay \( \Delta \) in addition to the 3 message delays.

† \( O(fn^2) \) if applying the recommended heuristic. See Section 2 for context.
communication complexity during both normal operation and view changes, matching HotStuff, while improving on the latency from 8 to 5 message delays.

The remainder of the paper is organized as follows. Section 2 discusses the related work. Section 3 defines the system model under which the algorithm is designed. Section 4 describes the IBFT algorithm in detail. Section 5 proves the correctness of the new algorithm. Section 6 presents LinearIBFT, a variant of IBFT with linear communication complexity. Finally, Section 7 concludes the paper.

2 Related Work

The problems of consensus and state machine replication (SMR) are closely associated with one another. Consensus requires processes in a distributed system to reach agreement on some value [23]. SMR requires agreement on a total order of commands [17, 22]. When consensus is solvable, so is SMR. As such, consensus is often used as a building block to implement SMR.

The problem of a distributed system reaching agreement in the presence of Byzantine process failures was first devised by Pease et al. [18, 20]. They also propose solutions for synchronous systems, where there is a known bound on the message transmission delays and the relative speeds of processes.

Of more practical utility are solutions to the problem of consensus in asynchronous systems, where there are no timing assumptions. Fischer et al., however, proved there is no deterministic solution to consensus in asynchronous system where a single process is allowed to fail [14]. There is an abundant body of research dedicated to circumventing this impossibility result using different techniques. The most notable examples being partial synchrony [11, 12], failure detectors [1, 7], and randomization [2, 8, 21].

In this paper, we are concerned with implementing Byzantine fault-tolerant state machine replication in a partially synchronous system. We thus restrict our comparison to protocols that, like IBFT, assume a partially synchronous model, tolerate Byzantine process failures, and are optimally resilient (i.e., \( n \geq 3f + 1 \)).

The partially synchronous model was introduced by Dwork et al. [12]. Along with it, they also proposed a Byzantine fault-tolerant consensus algorithm (DLS), which, although inefficient, proved that the problem has a solution in partially synchronous systems.
The PBFT algorithm was the first to provide a complete solution for state machine replication with Byzantine faults in a partially synchronous system where safety does not depend on timing assumptions [6]. This work was seminal in that it inspired a long line of algorithms that explore the design space within the same partially synchronous model.

The Zyzzyva algorithm introduced the idea of using speculative execution to improve performance [16]. Replicas optimistically adopt the order proposed by the primary and delegate to the clients the detection of inconsistencies, which help replicas resolve their state to one that is consistent with a total ordering of requests. Clement et al. later showed that this approach suffers from significant performance problems if even a single malicious client is present in the system [10]. As a solution, they propose a new algorithm - Aardvark - that makes more robust design decisions at a cost of best-case performance.

Spinning was the first algorithm to use a rotating leader replica [24], a concept later applied by many blockchain-motivated BFT algorithms. The leader replica is changed after every request execution instead of only when it is suspected to have failed.

More recently, we have seen a new wave of protocols that are motivated by their application to blockchain systems. Within our model, we highlight Tendermint, SBFT, and HotStuff.

Tendermint is a protocol whose main novelty is that it does not have a separate round (i.e., view) change algorithm [3]. Replicas change to a new round \( r + 1 \) as part of the normal operation by reaching a decision on round \( r \). Like Spinning, this allows for leader rotation as part of the normal operation and not just when a leader is suspected to be faulty. The main drawback of Tendermint is that even with timely communication and a honest leader it does not guarantee that a decision will be reached. This is because if a correct process is locked on a block \( b \) that is not the one being proposed, then the algorithm needs to keep advancing the view until it reaches one where the leader proposes \( b \). This results in a total communication complexity of \( O(fn^2) \) and message delays of \( O(f) \).

SBFT is a dual-mode protocol, employing a faster optimistic protocol - inspired by Zyzzyva - when there are no faulty replicas and the system is synchronous, and a slower fallback protocol - similar to PBFT - otherwise [15]. SBFT makes use of the concept of a collector. During a communication round, each replica, instead of broadcasting its message, sends it to a designated replica that aggregates the messages from all replicas into a single
message and broadcasts it. Messages are signed using threshold signatures, which allow for the aggregated message to have a constant size. Since a single slow or failed collector would be sufficient to make the system switch to the fallback, slower protocol, SBFT allows the optimistic protocol to tolerate a parameterized number $c$ of slow or failed replicas out of $n = 3f + 2c + 1$. Thus, for any $c > 0$, the algorithm fails to achieve optimal resiliency. SBFT has $O(cn)$ communication complexity during the normal case and $O(n^2)$ complexity during view changes. If $c$ is a constant, then this results in linear complexity during the normal case. It is unlikely, however, for $c$ to remain a constant value as a system scales. The authors recommend $c \leq f/8$ as a good heuristic, in which case the communication complexity would be $O(fn)$.

HotStuff is another protocol that employs the concept of a collector combined with threshold signatures to reduce communication complexity [25]. Unlike SBT, however, it only uses the primary replica as the collector. Like Tendermint, HotStuff does not employ a separate view change protocol. Instead, the view is advanced as part of the normal execution. This allows it to achieve $O(n)$ communication. The trade-off is a higher number of message delays to reach a decision.

3 System Model

We assume of system of $n$ processes with a Byzantine fault model, where $f$ processes can be faulty such that $n \geq 3f + 1$. A faulty process can take any arbitrary actions, including sending purposely wrong messages with the intent to obviate the correct execution of the algorithm. A process that is not faulty is said to be correct.

We assume a partially synchronous system, where there is an unknown bound $\Delta$ on execution and communication delays that holds after an unknown global stabilization time (or GST, for short).

Processes communicate by broadcasting messages. Before GST, messages can be arbitrarily delayed or lost. After GST, we assume the following two properties:

**GST-1** A message broadcast by a correct process at some time $t$, such that $t \geq GST$, is guaranteed to be delivered by all correct processes by time $t + \Delta$. 

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GST-2 If a correct process receives a message $m$ at some time $t$, then every correct process receives $m$ by time $\max(t, GST) + \Delta$.

We should note that GST-2 holds regardless of whether the broadcaster is correct or faulty, and it can be achieved, for instance, by point-to-point channels with some message retry mechanism or by using an epidemic dissemination algorithm [13]. This property also reflects the nature of some systems that assume some form of overlay network as in the case of the Ethereum network.

4 The IBFT Algorithm

We present IBFT in the pseudocode of algorithms 1, 2 and 3. The pseudocode is presented from the perspective of a correct process $p_i$. Algorithm 1 has the constants, state variables, and ancillary procedures. Algorithm 2 describes the normal operation of the algorithm, which happens during periods where the leader is correct and the messages are delivered in a timely manner. Finally, Algorithm 3 details how round changes are performed, for when a leader is suspected to be faulty. The logic in the algorithms is expressed as a set of event-driven upon rules that are triggered when some condition is met.

4.1 Preliminaries

The IBFT algorithm proposes a solution for the consensus problem. Particularly, each execution instance of IBFT guarantees the following properties:

**Agreement.** If a correct process decides some value $v$, then no correct process decides a value $v'$ such that $v' \neq v$.

**Validity.** There is an externally provided predicate $\beta$ such that if a correct process decides some value $v$, then $\beta(v)$ is true.

**Termination.** Every correct process decides.

Our validation condition deserves further explanation. It uses the notion of external validity, originally proposed by Cachin et al. [5]. The application calling the algorithm provides an arbitrary predicate $\beta$ whose purpose is to
Algorithm 1 IBFT pseudocode for process $p_i$: constants, state variables, and ancillary procedures

1: constants:
2: $p_i$ \hfill $\triangleright$ The identifier of the process

3: state variables:
4: $h_i$ \hfill $\triangleright$ The identifier of the consensus instance
5: $r_i$ \hfill $\triangleright$ The current round
6: $\text{preparedRound}_i$ \hfill $\triangleright$ The round at which the process has prepared
7: $\text{preparedValue}_i$ \hfill $\triangleright$ The value for which the process has prepared

8: procedure Start($height$, $inputValue$)
9: $h_i \leftarrow height$
10: UpdateRound(1)
11: $\text{preparedRound}_i \leftarrow \perp$
12: $\text{preparedValue}_i \leftarrow \perp$
13: if Leader($h_i$, $r_i$) = $p_i$ then
14: broadcast $\langle$PRE-PREPARE, $h_i$, $r_i$, $inputValue$\rangle$ message

15: procedure UpdateRound($round$)
16: if $round > r_i \lor r_i$ is not set then
17: $r_i \leftarrow r$
18: reset round timer
Algorithm 2 IBFT pseudocode for process $p_i$: normal operation

1: **upon** receiving a $\langle$PRE-PREPARE, $h_i$, round, value$\rangle$ message $m$ from $\text{LEADER}(h_i, \text{round})$ s.t. $\text{round} \geq r_i \land \text{VALIDATEPrePrepare}(m)$ do
2: \hspace{1em} $\text{UPDATERound}(\text{round})$
3: \hspace{1em} broadcast $\langle\text{PREPARE}, h_i, r_i, \text{value}\rangle$
4: **upon** receiving a quorum of valid $\langle$PREPARE, $h_i$, round, value$\rangle$ messages s.t. $\text{round} \geq r_i$ do
5: \hspace{1em} $\text{UPDATERound}(\text{round})$
6: \hspace{1em} $\text{preparedRound}_i \leftarrow r_i$
7: \hspace{1em} $\text{preparedValue}_i \leftarrow \text{value}$
8: \hspace{1em} broadcast $\langle\text{COMMIT}, h_i, \text{value}\rangle$
9: **upon** receiving a quorum of valid $\langle$COMMIT, $h_i$, value$\rangle$ messages do
10: \hspace{1em} output $\text{value}$ and stop execution

ensure that the decided value is acceptable. For instance, a blockchain implementation might want to check that the decided value is a block containing legitimate transactions.

The algorithm proceeds in rounds. During each round, one of the processes acts as a leader that tries to drive the execution to a decision by proposing a value. During a *good* round, where communication is timely and the leader is not faulty (i.e., after GST), the algorithm guarantees that all correct processes will reach a decision. There is a function $\text{LEADER}(\text{height}, \text{round})$ that identifies the leader. This function can be any deterministic mapping from $\text{height}$ and $\text{round}$ to the identifier of the leader as long as it allows $f + 1$ processes to eventually assume the leader role.

**Messages.** Messages are represented as tuples enclosed in angle brackets. There are four types of messages: PRE-PREPARE, PREPARE, COMMIT, and ROUND-CHANGE. The first three types are of the form $\langle$message-type, height, round, value$\rangle$ and comprise the normal operation of the algorithm. The ROUND-CHANGE message is of the form $\langle$ROUND-CHANGE, height, round, preparedRound, preparedValue$\rangle$ and is used ensure progress of the algorithm when the current leader is suspected to have failed or communication is not timely.
Algorithm 3 IBFT pseudocode for process $p_i$: round changes and message validation

1: upon round timer expiring do
2:   UpdateRound($r_i + 1$)
3:     broadcast $\langle \text{ROUND-CHANGE}, h_i, r_i, \text{preparedRound}_i, \text{preparedValue}_i \rangle$

4: upon receiving a quorum $Q_{rc}$ of valid $\langle \text{ROUND-CHANGE}, h_i, \text{round}, pr_j, pv_j \rangle$ messages such that $\text{round} \geq r_i \land \text{LEADER}(h_i, \text{round}) = p_i \land \text{ValidateRoundChange}(Q_{rc})$ do
5:   UpdateRound($\text{round}$)
6:   $(\ldots, v) \leftarrow \text{HighestPrepared}(Q_{rc})$
7:   broadcast $\langle \text{PRE-PREPARE}, h_i, r_i, v \rangle$

8: predicate $\text{ValidateRoundChange}(Q_{rc})$
9:   return true if received a quorum of valid $\langle \text{PREPARE}, h_i, pr, pv \rangle$ messages s.t. $(pr, pv) = \text{HighestPrepared}(Q_{rc})$

10: predicate $\text{ValidatePrePrepare}(\langle \text{PREPARE}, h_i, \text{round}, value \rangle)$
11:   return true if $\text{round} = 1$ or received:
12:     (1) a quorum of valid $\langle \text{ROUND-CHANGE}, h_i, r, \perp, \perp \rangle$ messages or
13:     (2a) a quorum $Q_{rc}$ of valid $\langle \text{ROUND-CHANGE}, h_i, r, pr, pv \rangle$ messages s.t. $(pr, pv) = \text{HighestPreparedValue}(Q_{rc})$
14:     (2b) and a quorum of valid $\langle \text{PREPARE}, h_i, pr, v \rangle$ messages

$\triangleright$ Helper function that returns a tuple $(pr, pv)$ where $pr$ and $pv$ are, respectively, the prepared round and the prepared value of the $\text{ROUND-CHANGE}$ message in $Q_{rc}$ with the highest prepared round

12: function $\text{HighestPrepared}(Q_{rc})$
13:   return $(pr, pv)$ such that
14:     $\exists \langle \text{ROUND-CHANGE}, h_i, \text{round}, pr, pv \rangle \in Q_{rc} :$
15:     $\forall \langle \text{ROUND-CHANGE}, h_i, \text{round}, pr_j, pv_j \rangle \in Q_{rc} : pr \geq pr_j$
State. The algorithm state is composed of four variables: the height \( h_i \), the round \( r_i \), \( \text{preparedRound}_i \), and \( \text{preparedValue}_i \).

The variable \( h_i \) is the height. It is set upon a call to the START procedure and it never changes throughout the execution of the algorithm. It serves two purposes. First, it is an identifier for each instance of the algorithm. Second, when the algorithm is used to implement state machine replication, it determines the order in which to execute the commands. For example, in a blockchain system, it determines the position in the blockchain where the block resulting from the algorithm execution (i.e., the decision value) will be inserted in.

The variable \( r_i \) identifies the round in which process \( p_i \) is currently on and it starts at 1.

The \( \text{preparedRound}_i \) and \( \text{preparedValue}_i \) variables are, respectively, the highest round and the corresponding value for which \( p_i \) has prepared. During the execution of an algorithm instance with height \( h_i \), we say that a process \( p_i \) has prepared for a round \( r \) and a value \( v \) if it receives a quorum of valid messages \( \langle \text{PREPARE, } h, r, v \rangle \). These variables are initialized with a default value \( \perp \), which means that \( p_i \) has not prepared yet. This mechanism is essential for the safety of the algorithm and we explain how it works further below.

Upon rules. Most upon rules are triggered when receiving a quorum of valid messages that match a certain pattern. We say that a process has received a quorum of valid messages if it has received valid messages from \( \lceil \frac{n+f}{2} \rceil + 1 \) different processes. For instance, the first upon rule in Algorithm 2 is triggered when process \( p_i \) receives \( \lceil \frac{n+f}{2} \rceil + 1 \) valid messages from different processes that have type \( \text{PRE-PREPARE} \), height \( h_i \), round \( r_i \), and the same value.

While not explicit in the pseudocode, we impose the restriction that within an instance of the algorithm, for any round value carried by the messages, each upon rule is triggered at most once.

One of the upon rules in Algorithm 3 does not depend on receiving messages, but on a timer expiring. This timer is reset at the beginning of each round by the \texttt{UpdateRound} procedure. Naturally, the above restriction does not apply to this upon rule because it does not depend on receiving a quorum of messages.

\[ \text{We explain in the paragraph below what this entails.} \]
Validation. A correct process only accepts a message if it considers it to be valid. To be valid, a message must carry some proof of integrity and authentication of its sender. For example, a digital signature. The external validity predicate $\beta$ must also hold for the value carried by the message.

Furthermore, PRE-PREPARE messages (for any round higher than 1) and ROUND-CHANGE messages require additional validation, which is expressed, respectively, by the predicates $\text{VALIDATEPREPARE}$ and $\text{VALIDATEROUNDCHANGE}$ in Algorithm 3. This is to ensure the correctness of round changes. We defer their explanation to Section 4.3 where we describe round changes in detail.

4.2 Normal case operation.

We now explain how the algorithm works during its normal case operation, i.e., when communication is timely and the leader is correct.

For any correct process $p_i$, an execution of an instance $h_i$ of the algorithm begins with a call to the Start procedure, which takes as input parameters the height, which sets the value of $h_i$, and a value $\text{inputValue}$ to be proposed.

The procedure then initializes the state variables and if $p_i$ is the leader for the current round, it broadcasts a PRE-PREPARE message.

Upon receiving a valid PRE-PREPARE message from the leader for height $h_i$ and current round $r_i$, a process $p_i$ broadcasts a PREPARE message.

Upon receiving a quorum of valid PREPARE messages for height $h_i$ and current round $r_i$ with the same value, a process $p_i$ updates its preparedRound$_i$ and preparedValue$_i$ variables to match the values of the received messages. We now say that $p_i$ has prepared for round $r_i$ and value $\text{value}$. It then broadcasts a COMMIT message carrying $\text{value}$.

Finally, upon receiving a quorum of valid COMMIT messages for height $h_i$ with the same value, a process $p_i$ decides by (1) outputting $\text{value}$ and (2) stopping the execution of the algorithm for height $h_i$. Note that the COMMIT message does not carry a round number. This is because, at this point, there are no more algorithm steps to take that depend on the round.

4.3 Round changes

The previous section explains how the algorithm works under good conditions. The algorithm, however, must be able to tolerate arbitrary periods where communication is untimely or the leader is faulty.
Liveness is ensured by having a timer set that at beginning of each round (within the \textsc{UpdateRound} procedure). If the algorithm has not made sufficient progress for a process $p_i$ to decide, then that timer will eventually expire. When this happens, $p_i$ advances to the next round and send a \textsc{RoundChange} message to the leader of the new round. This message carries the values of the $\text{preparedRound}_i$ and $\text{preparedValue}_i$ variables, which will be used by the leader to select a value to propose in a \textsc{Pre-Prepare} message for the new round, as explained in the paragraph below.

In order to broadcast a \textsc{Pre-Prepare} message for a new round $r$, the leader for $r$ waits to receive a quorum of valid \textsc{RoundChange} messages for round $r$ and selects the $\text{preparedValue}$ within these messages with the highest corresponding $\text{preparedRound}$. It then broadcasts a \textsc{Pre-Prepare} message proposing this selected $\text{preparedValue}$. If none of the received \textsc{RoundChange} messages carries a $\text{preparedRound}$ (and $\text{preparedValue}$) different than $\bot$, then the new leader is free to propose any value value in the \textsc{Pre-Prepare} message.

4.3.1 Message Validation

To guarantee correctness during round changes, both \textsc{RoundChange} and \textsc{Pre-Prepare} messages need additional validation. The goal with validating these messages is to ensure that a \textsc{Pre-Prepare} message cannot carry a proposal value different than one that could have potentially already been decided by some correct process.

A process validates a message of either type \textsc{RoundChange} or \textsc{Pre-Prepare} by observing other messages that legitimize its content. The principle is to ensure that the message content is congruent with the execution of the protocol.

For example, a message $\langle \text{RoundChange}, h, r, pr, pv \rangle$ is only congruent with the execution of the protocol if it was ever broadcasted a quorum of $\langle \text{Prepare}, h, pr, pv \rangle$ messages because that is the only way a correct process could have prepared for round $pr$ and value $pv$.

There are to ways for a correct process to observe the necessary messages that validate a \textsc{RoundChange} or \textsc{Pre-Prepare} message. One is \textit{implicit}, in which the process waits to receive these messages from the network. The other is \textit{explicit}, in which the validating message are piggybacked in the message that they are validating. We further discuss these these two validation methods below.
ROUND-CHANGE validation. Within a quorum of ROUND-CHANGE messages, the message with the highest preparedRound needs additional validation by the leader $p_i$ of the new round. A correct leader process $p_i$ considers a \langle ROUND-CHANGE, h, r, preparedRound, preparedValue \rangle message to be valid, where preparedRound is the highest among a quorum of ROUND-CHANGE messages, only if $p_i$ observes a quorum of valid \langle PREPARE, h, preparedRound, preparedValue \rangle messages. This is expressed by the predicate VALIDATEROUNDCHANGES in Algorithm 3.

PRE-PREPARE validation. For PRE-PREPARE messages, we have to guarantee that the value proposed after a round change is safe. We deem a proposed value to be safe if no correct process could have decided on a different value before the new round. More precisely, a valid PRE-PREPARE message for round $r$ proposing value $v$ guarantees that no correct process has decided on a value $v' \neq v$ before broadcasting its ROUND-CHANGE message for round $r$. Such a message is thus safe for a correct process to accept and make progress on.

A correct process $p_i$ considers a \langle PRE-PREPARE, h_i, r, v \rangle message for $r > 1$ to be valid when one of the following two conditions holds:

1. $p_i$ observes a quorum of valid \langle ROUND-CHANGE, h_i, r, ⊥, ⊥ \rangle messages.

2. $p_i$ observes both:
   
   (a) a quorum of valid \langle ROUND-CHANGE, h_i, r, preparedRound, preparedValue \rangle messages and the message with the highest preparedRound$_j$ has preparedValue$_j = v$; and
   
   (b) a quorum of valid \langle PREPARE, h_i, preparedRound, v \rangle messages.

This is expressed by the predicate VALIDATEPRE-PREPARE in Algorithm 3. Note that property GST-2 of our system model guarantees that these validating messages are received within a bounded delay after GST.

Validation Methods: Implicit and Explicit. There are two main methods of implementing the validation of ROUND-CHANGE and PRE-PREPARE messages.

The first one is what we call implicit validation. Here, a correct process simply waits to receive the validating messages from the network. This form
is validation is depending on property GST-2 from our model as this is what guarantees that a correct process eventually receives these messages.

In the second method, called explicit validation, the validating messages are explicitly sent along with the message that they validate. This second method does not require for the system to have property GST-2 and it allows for a small change in the protocol in the case of validating PRE-PREPARE messages: for a round $r$, instead of broadcasting a ROUND-CHANGE message, a correct process can send it only to the leader of round $r$.

While both validation methods are correct, they have different performance characteristics. Table 2 shows the communication complexity of each validation method. We can see that using implicit validation for ROUND-CHANGE messages and explicit validation for PRE-PREPARE messages results in the best communication complexity.

## 5 Correctness

In this section, we demonstrate the correctness of our algorithm by presenting theorems and their respective proofs for each of the properties of agreement, validity, and termination as expressed in Section 4.1.

### 5.1 Agreement

To prove the agreement property we first need to construct supporting Lemma 1 and Corollary 2. The agreement property is proved in Theorem 3.

**Lemma 1** If $\lfloor \frac{n-f}{2} \rfloor + 1$ correct processes prepare for some value $v$ and round $r$, then no correct process can prepare for a value $v'$ and round $r'$ such that $v' \neq v$ and $r' \geq r$.

1. We assume that
   1. $\lfloor \frac{n-f}{2} \rfloor + 1$ correct processes prepared for value $v$ and round $r$
2. \( v' \neq v \)
3. \( p \) is a correct process
and we prove by induction on \( r' \) that \( p \) does not broadcast a \((\text{PREPARE}, h, r', v')\) message.

2. Case: (base step) \( r' = r \)

2.1. \( \left\lceil \frac{n-f}{2} \right\rceil \) + 1 correct processes received a quorum of valid \((\text{PREPARE}, h, r', v)\) messages.

Proof: By the case assumption \( r' = r \) and the assumptions in 1, since to prepare for a value \( v \) and a round \( r' \), a correct process must receive a quorum of valid \((\text{PREPARE}, h, r, v)\) messages.

2.2. \( \left\lceil \frac{n-f}{2} \right\rceil \) + 1 correct processes broadcasted a valid \((\text{PREPARE}, h, r', v)\) message.

Proof: From 2.1 it follows that a quorum of such messages must have been broadcasted by at least \( \left\lceil \frac{n-f}{2} \right\rceil \) + 1 correct processes.

2.3. At most \( \left\lceil \frac{n+f}{2} \right\rceil \) valid \((\text{PREPARE}, h, r', v)\) messages were broadcasted.

Proof: By 2.2. Otherwise, we would have \( \left\lceil \frac{n+f}{2} \right\rceil + 1 + \left\lceil \frac{n-f}{2} \right\rceil + 1 > n \) processes in the system, which is a contradiction.

2.4. Q.E.D.

Proof: By 2.3, since to prepare for a value \( v' \) and round \( r' \), a correct process requires \( \left\lceil \frac{n+f}{2} \right\rceil \) + 1 valid \((\text{PREPARE}, h, r', b)\) messages.

3. Case: (inductive step)

1. \( r' > r \)
2. no correct process broadcasts a valid \((\text{PREPARE}, h, r'', v')\) message such that \( r \leq r'' < r' \)

3.1. No correct process broadcasts a valid \((\text{PREPARE}, h, r', v')\) message.

3.1.1. We assume that some correct process broadcasts a valid \((\text{PREPARE}, h, r', v')\) message and demonstrate that this leads to a false statement.

Proof: By contradiction.

3.1.2. There is a valid \((\text{PREPARE}, h, r', v')\) message.
Proof: It follows from 3.1.1. A correct process can only broadcast a \langle PREPARE, h, r', v' \rangle message in response to receiving a valid \langle PRE-PREPARE, h, r', v' \rangle message.

3.1.3. There must be

- either
  - (a) \( \lceil \frac{n+1}{2} \rceil + 1 \) valid \langle ROUND-CHANGE, h, r', \bot, \bot \rangle messages
  - or
  - (b) \( \lceil \frac{n+1}{2} \rceil + 1 \) valid \langle ROUND-CHANGE, h, r', preparedRound_j, preparedValue_j \rangle messages where some message has \( preparedRound_j \neq \bot \) and \( preparedValue_j \neq \bot \)
  - (c) \( n - f \) valid \langle PREPARE, h, r_h, v' \rangle messages such that \( r_h \) is the highest \( preparedRound_j \) among the messages from (b) and \( r_h < r' \)

Proof: By 3.1.2 and the validation definition of PRE-PREPARE messages.

3.1.4. There are at most \( \lceil \frac{n+1}{2} \rceil \) valid \langle ROUND-CHANGE, h, r', \bot, \bot \rangle messages.

3.1.4.1. For any correct process that prepared at a round \( r \) and value \( v \), for any round \( r' \) such that \( r' > r \), any valid \langle ROUND-CHANGE, h, r', \bot, \bot \rangle message that it broadcasts has \( preparedRound \geq r \).

Proof: It follows from a correct process setting its \( preparedRound_i \) variable to the highest round that it has prepared for.

3.1.4.2. There are \( \lceil \frac{n-f}{2} \rceil + 1 \) correct processes that if they broadcast a valid \langle ROUND-CHANGE, h, r', \bot, \bot \rangle message for round \( r' \), then it must be such that \( preparedRound \neq \bot \).

Proof: By 3.1.4.1, the inductive hypothesis that \( r' > r \), and the assumption that there are \( \lceil \frac{n-f}{2} \rceil + 1 \) correct processes prepared at a round \( r \) and a value \( v \).

3.1.4.3. Q.E.D.

Proof: By 3.1.4.2. Otherwise, we would have \( \lceil \frac{n-f}{2} \rceil + 1 + \lceil \frac{n-f}{2} \rceil + 1 > n \) processes in the system, which is a contradiction.
3.1.5. \( r_h \geq r \)

3.1.5.1. Any quorum of \( \langle \text{ROUND-CHANGE}, h, r', \text{preparedRound}, \text{preparedValue} \rangle \) messages has some message such that \( \text{preparedRound} \geq r \).

**Proof:** By the assumption that \( \lceil \frac{n-f}{2} \rceil + 1 \) correct processes are prepared at round \( r \) for \( v \). Any quorum of the aforementioned messages must contain a message from some such correct process. Otherwise, we would have \( \lceil \frac{n+f}{2} \rceil + 1 + \lceil \frac{n-f}{2} \rceil + 1 > n \) processes in the system, which is a contradiction.

3.1.5.2. Q.E.D.

**Proof:** By 3.1.5.1 and 3.1.3(c).

3.1.6. There must be \( \lceil \frac{n+f}{2} \rceil + 1 \) valid \( \langle \text{PREPARE}, h, r_h, v' \rangle \) messages such that \( r \leq r_h < r' \).

**Proof:** By 3.1.3, 3.1.4 and 3.1.5.

3.1.7. Q.E.D.

**Proof:** By 3.1.6, which contradicts the induction assumption.

3.2. No correct process receives a quorum of \( \langle \text{PREPARE}, h, r', v' \rangle \) messages.

**Proof:** By 3.1.

3.3. To prepare for a round \( r' \) and value \( v' \), a correct process has to receive a quorum of \( \langle \text{PREPARE}, h, r', v' \rangle \) messages.

**Proof:** This follows from a simple inspection of the algorithm.

3.4. Q.E.D.

**Proof:** By 3.2 and 3.3.

**Corollary 2** If \( \lceil \frac{n-f}{2} \rceil + 1 \) correct processes prepare for a value \( v \) and round \( r \), then no \( \lceil \frac{n-f}{2} \rceil + 1 \) correct processes can prepare for a value \( v' \) and round \( r' \) such that \( v' \neq v \) and \( r' \neq r \).

1. We assume
   1. \( \lceil \frac{n-f}{2} \rceil + 1 \) correct processes prepare for a value \( v \) and round \( r \).
   2. \( v' \neq v \)
   3. \( r' \neq r \)
4. without loss of generality, $r$ is the lowest round for which $\left\lfloor \frac{n-f}{2} \right\rfloor + 1$
correct processes prepare.

2. Q.E.D.

Proof: By 1 and Lemma [1].

Theorem 3 (Agreement) If a correct process $p_i$ decides some value $v$, then
no correct process $p_j$ decides a value $v'$ such that $v' \neq v$.

1. We assume
   1. $p_i$ is a correct process that has decided a value $v$
   2. $v' \neq v$
      and prove that no correct process $p_j$ can decide a value $v'$.

2. $p_i$ received a quorum of valid $\langle \text{COMMIT}, h, v \rangle$ messages.
   Proof: It follows from assumption 1.1. A correct process can only decide
   if it receives a quorum of valid COMMIT messages.

3. $\left\lfloor \frac{n-f}{2} \right\rfloor + 1$ correct processes broadcasted a $\langle \text{COMMIT}, h, v \rangle$ message.
   Proof: From 2 it follows that at least $\left\lfloor \frac{n-f}{2} \right\rfloor + 1$ correct processes must
   have broadcasted a $\langle \text{COMMIT}, h, v \rangle$ message.

4. $\left\lfloor \frac{n-f}{2} \right\rfloor + 1$ correct processes prepared for value $v$ and some round $r$.
   Proof: To broadcast a $\langle \text{COMMIT}, h, v \rangle$ message, a correct process must first
   prepare for value $v$ and some round $r$. From 3 it follows that $\left\lfloor \frac{n-f}{2} \right\rfloor + 1$
correct processes prepared for value $v$ and round $r$.

5. No $\left\lfloor \frac{n-f}{2} \right\rfloor + 1$ correct processes can prepare for a value $v'$ and round $r'$
such that $v' \neq v$ and $r' \neq r$.
   Proof: By 4 and Corollary [2]

6. No $\left\lfloor \frac{n-f}{2} \right\rfloor + 1$ correct processes broadcast a $\langle \text{COMMIT}, h, v' \rangle$ message.
   Proof: It follows from 5 since to broadcast $\langle \text{COMMIT}, h, v' \rangle$ message, a
correct process has to prepare for value $v'$.

7. No process correct can decide $v'$.
   Proof: It follows from 6 and a trivial inspection of the algorithm - a
correct process can only decide $v'$ if there is a quorum of $\langle \text{COMMIT}, h, v' \rangle$
messages.
\section{Validity}

\textbf{Theorem 4} \textit{(Validity)} \textit{There is an externally provided predicate $\beta$ such that if a correct process decides some value $v$, then $\beta(v)$ is true.}

1. A process can only decide on a value $v$ if $v$ is broadcasted in a valid \texttt{PRE-PREPARE} message.
   \textbf{Proof:} It follows from a trivial inspection of the algorithm.

2. A \texttt{PRE-PREPARE} message carrying value $v$ is only considered valid if $\beta(v)$ is true.
   \textbf{Proof:} By definition.

3. Q.E.D.
   \textbf{Proof:} By 1 and 2.

\section{Termination}

\textbf{Theorem 5} \textit{(Termination)} \textit{Every correct process decides.}

We consider two cases. One, where some correct process $p_i$ has already decided. Two, where no correct process has decided yet. We show that in both cases every correct process must decide.

1. Case: some correct process $p_i$ has decided.
   1.1. $p_i$ has received a quorum of valid $\langle \text{COMMIT}, h, b \rangle$ messages.
   1.2. Every correct process receives a quorum of valid $\langle \text{COMMIT}, h, b \rangle$ messages.
      \textbf{Proof:} By property GST-2 and statement 1.1.
   1.3. Q.E.D.
      \textbf{Proof:} It follows from 1.2.

2. Case: no correct process has decided.
   2.1. We assume that no correct process decides and show that this leads to a false statement.
      \textbf{Proof:} By contradiction.
2.2. Every correct process $p_i$ must eventually reach some round $r$ (i.e., $r = r_i = r$) after GST with a correct leader $p_L$.

   **Proof**: By the assumption in 2.1 the round timer for $p_i$ must keep expiring indefinitely and its round variable $r_i$ increasing.

2.3. $p_L$ broadcasts a \langle PRE-PREPARE, h, r, b \rangle message.

   **Proof**: By 2.2 that $p_L$ is correct.

2.4. A valid \langle PRE-PREPARE, h, r, b \rangle message is delivered to every correct process.

   **Proof**: By 2.2, 2.3, and property GST-1.

2.5. Every correct process broadcasts a \langle PREPARE, h, r, b \rangle message.

   **Proof**: It follows from 2.4.

2.6. Every correct process receives a quorum of valid \langle PREPARE, h, r, b \rangle messages.

   **Proof**: By 2.2, 2.5, and property GST-1.

2.7. Every correct process broadcasts a \langle COMMIT, h, b \rangle message.

   **Proof**: It follows from 2.6.

2.8. Every correct process receives a quorum of valid \langle COMMIT, h, b \rangle messages.

   **Proof**: By 2.2, 2.7, and property GST-1.

2.9. Q.E.D.

   **Proof**: It follows from 2.8.

6 **Linear IBFT**

IBFT can be trivially adapted to produce an algorithm with $O(n)$ communication complexity and latency of 5 message delays (after GST).

To achieve this we need the following ingredients:

**Leader-relayed communication via threshold signatures.** During normal operation (i.e., PRE-PREPARE, PREPARE, and COMMIT messages), we
no longer use a broadcast primitive that sends \( n \) messages - one to each process. Instead, to broadcast a message, a correct process sends a message to the leader with a signature share. The leader waits to receive a quorum of messages for the same type, height, round and value, and produces a threshold signature on the tuple \((\text{type, height, round, value})\). It then sends a message containing the tuple and the threshold signature to all processes.

**Explicit validation.** The linear variant precludes implicit validation. As such, we always use explicit validation, which implies that processes do not broadcast \textsc{round-change} messages, but simply send them to the leader.

### 7 Conclusion

In this paper we proposed IBT, a flexible algorithm for consensus in Byzantine-fault tolerant systems that is used by the Quorum blockchain. IBFT assumes a partially synchronous model, with timely communication, has a worst-case \( O(n^2) \) communication complexity and terminates within three message delays. To ensure safety, IBFT relies on a validation mechanism of proposed values that can be flexibly implemented either implicitly or explicitly.

We also present LinearIBFT, a variant that has worst-case \( O(n) \) communication complexity and terminates within 5 message delays. LinearIBFT is the BFT consensus algorithm, within its class, with linear communication complexity that has lowest number of message delays.

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