Collective transition densities in neutron-rich nuclei

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Abstract

Quadrupole transition densities in neutron-rich nuclei in the vicinity of the neutron drip-line are calculated in the framework of the Random Phase Approximation. The continuum is treated by expansion in oscillator functions. We focus on the states which contribute to the usual Giant Quadrupole Resonance, and not on the low-lying strength which is also expected in such nuclei and whose collective character is still under debate. We find that, due to the large neutron skin in these nuclei, the isoscalar and isovector modes are in general strongly mixed. We further show that the transition densities corresponding to the GQR states can be reasonably well described by the collective model in terms of in phase and out of phase oscillations of neutron and proton densities which have different radii.

During recent years, in view of the prospects of new dedicated radioactive beam facilities, considerable interest has been focussed to the study of the effect of neutron skin on the collective properties of neutron-rich nuclei \cite{1}. Largely the interest has been directed towards the possible occurrence of sufficient multipole strength at low excitation energies in such nuclei. It is now excepted that this feature is primarily a consequence of the extremely weak binding of the neutrons close to the Fermi surface, whereas the collective or single-particle nature of these excitations is still unclear \cite{2}. It is also of interest to study neutron-rich nuclei which are still far from the neutron drip-line, so that the last nucleons are not so weakly bound, and ask what the effect of the neutron skin is on the traditional collective oscillations of these systems. In view of the presence of the neutron skin one would expect an enhanced mixing of the isoscalar and isovector modes.

To investigate this point, we have performed microscopic calculations for nuclei \textsuperscript{28}O and \textsuperscript{60}Ca, based on spherical Hartree-Fock model with Skyrme SGII interaction. These interaction predicts the last neutron to be bound by 3.25
MeV in $^{28}\text{O}$ and 5.1 MeV in $^{60}\text{Ca}$. The proton and neutron densities for both $^{28}\text{O}$ and $^{60}\text{Ca}$ are shown in fig. 1. These nuclei display a neutron skin (the difference in the root-mean-square radii amounts to about 0.6 fm and 0.4 fm, in the two cases, respectively), without any significant neutron halo, since the last neutrons in these nuclei are not too weakly bound. As a further consequence of the relatively large binding energy, we do not expect any significant multipole strength at very low excitation energies in these nuclei. In view of this, we believe that the continuum states can be adequately treated by expanding them in oscillator functions of different principal quantum numbers, a procedure which is cumbersome for a system just on the drip line, which would require an extremely large number of oscillator functions. The adequacy of the oscillator expansion was however tested by varying the number of oscillator states and verifying that the multipole strength distribution is not strongly affected for the collective part of the response. The strength distributions were calculated for multipole operators of the form \( \hat{O}_{\lambda \mu} = r^{\lambda} Y_{\lambda \mu}(\hat{r}) \) for both neutrons and protons.

The collective excitations of these nuclei were determined in the RPA, using the full residual interaction, i.e. both the isoscalar and isovector components of the Skyrme force were included simultaneously. The RPA states are there-

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1. Different choices of the Skyrme interaction may lead to lower binding energies of the last neutron and may result in significant multipole strength at low excitation energy [2,3]
for in general connected to the ground state by both isoscalar and isovector operators.

We already know that even for $\beta$-stable nuclei with $N \geq Z$ the collective vibrations are mixtures of isoscalar and isovector vibrations. In the absence of neutron skin one expects the ratio of isovector and isoscalar amplitude for the isoscalar giant resonance to be approximately equal to $(N-Z)/A$. As an example we show in fig. 2 the isoscalar and isovector quadrupole response for $^{208}$Pb. In correspondence with the concentration of the isoscalar strength around 11 MeV one also observes some isovector strength of the order of 3 % (namely close to ratio ($(N-Z)/A)^2$, which is 4 %) while the main concentration of the isovector strength occurs around 22 MeV.

In view of the interest in the effect of the neutron excess on the isoscalar and isovector response functions the RPA results for $^{28}$O and $^{60}$Ca are shown in fig. 3, compared with the analogous quantities for $^{16}$O and $^{40}$Ca, respectively. In the case of the $N = Z$ nuclei $^{16}$O and $^{40}$Ca the isoscalar and isovector distributions of strength are rather well separated, corresponding to pure isoscalar
Fig. 3. Comparison of the isoscalar and isovector strength distributions for the RPA quadrupole states for $^{16}$O and $^{28}$O (left panel) and for $^{40}$Ca and $^{60}$Ca (right panel).

and isovector excitations. In contrast to this, one observes in the case of $^{28}$O and $^{60}$Ca firstly a large fragmentation of the isoscalar strength and secondly a very large fragmentation of the isovector strength, down to the region where there is a large isoscalar strength. In tables 1 and 2 the $B(E2)$’s for certain representative RPA states for $^{28}$O and $^{60}$Ca are shown, together with the percentage of the isoscalar and isovector sum rules that each state exhausts. Note that the total EWSR for isoscalar and isovector are different due to the exchange term in the latter [4]. Note also that at variance with the situation for systems around the $\beta$-stability, the proton and neutron EWSR’s do not simply scale according to $Z$ and $N$, due to the presence of the neutron skin which leads to different rms radii for protons and neutrons.

The nature of the different states and the effect of the neutron skin are best evidenced by looking at the transition densities. To this end transition densities to the selected states in $^{28}$O and $^{60}$Ca listed in the tables 1 and 2 are shown in Fig. 4. In each one of the frames, the isoscalar and isovector transition densities along with the separate neutron and proton transition densities are shown. The figure confirms that the isoscalar and isovector modes are mixed and contribute to each state.

We can compare the corresponding transition densities with two different prescriptions for the collective model. In one of the prescriptions (Bohr and Mottelson model [5]), we assume that the change in the neutron and proton den-
Table 1
Properties of some selected RPA quadrupole states in $^{28}$O. For each state we quote the excitation energy, fraction (in percentage) of isoscalar and isovector EWSR exhausted by the state, $B(E2)$ values for the proton and neutron components, and corresponding isoscalar and isovector values.

| Energy (MeV) | EWSR (%) | EWSR (%) | $B(E2)_p$ ($e^2 fm^4$) | $B(E2)_n$ ($e^2 fm^4$) | $B(E2)_{is}$ ($e^2 fm^4$) | $B(E2)_{iv}$ ($e^2 fm^4$) |
|-------------|----------|----------|-------------------------|-------------------------|---------------------------|---------------------------|
| 17.417      | 48.9     | 14.4     | 28.39                   | 399.32                  | 640.66                    | 214.76                    |
| 20.315      | 14.2     | 0.5      | 25.55                   | 58.08                   | 160.67                    | 6.59                      |
| 30.968      | 0.0      | 12.7     | 24.18                   | 30.47                   | 0.36                      | 108.94                    |

Table 2
As in Table 1, for the case of $^{60}$Ca.

| Energy (MeV) | EWSR (%) | EWSR (%) | $B(E2)_p$ ($e^2 fm^4$) | $B(E2)_n$ ($e^2 fm^4$) | $B(E2)_{is}$ ($e^2 fm^4$) | $B(E2)_{iv}$ ($e^2 fm^4$) |
|-------------|----------|----------|-------------------------|-------------------------|---------------------------|---------------------------|
| 15.315      | 44.0     | 4.0      | 241.11                  | 912.03                  | 2091.00                   | 215.28                    |
| 16.373      | 15.5     | 0.0      | 154.88                  | 188.45                  | 685.01                    | 1.64                      |
| 27.542      | 0.1      | 8.0      | 52.73                   | 82.37                   | 3.29                      | 266.91                    |

Densities are proportional to their derivatives, according to

$$
\delta \rho_n = \beta_n^2 R_n \frac{d\rho_n}{dr}
$$

$$
\delta \rho_p = \beta_p^2 R_p \frac{d\rho_p}{dr}
$$

where $\rho_n$ and $\rho_p$ are the Hartree-Fock densities. In the second prescription we use the Tassie model [6], where $\beta_2 R$ is replaced by $\beta_2 r$. By imposing that the collective transition densities lead to the same $B(E2)$'s as the RPA ones and choosing $R_{n,p} = \sqrt{3/5} < r_{n,p}^2 >$ we have obtained the $\beta$ values reported in Table 3. It is worthwhile to notice that the $\beta$ values corresponding to the two collective model prescriptions are very close to each other.

As quoted in Table 1, the states at 17.4 MeV in $^{28}$O and at 15.3 MeV in $^{60}$Ca exhaust approximately half of the isoscalar EWSR, and therefore can be associated with the usual collective Isoscalar Giant Quadrupole Resonance (GQR). This interpretation is confirmed by the fact that the $\beta$ values for neutrons and protons have the same sign and practically the same magnitude, which in the collective picture means that the proton and neutron densities $\rho_n$ and $\rho_p$ oscillate in phase and with the same amplitude. The predictions of the two models in the case of $^{28}$O for the neutron and proton transition densities are shown in Fig. 5a,b along with the RPA results, while the corresponding isoscalar and isovector transition densities are compared in Fig. 5c,d.
Fig. 4. RPA transition densities for the selected states in $^{28}$O listed in Table 1 are shown in the left panel. The proton and neutron transition densities are shown together with the isoscalar and isovector transition densities. The right panel shows the corresponding quantities for the states listed in Table 2 for $^{60}$Ca.

Table 3
Deformation parameters $\beta_2^p$ and $\beta_2^n$ obtained within the collective Tassie and Bohr-Mottelson models, for each quadrupole state considered in Tables 1 and 2. The values have been obtained from the condition of yielding, within the collective models, the microscopic RPA values, separately for neutrons and protons.

| Nuclei | Energy (MeV) | $\beta_2^p$ Tassie | $\beta_2^n$ Tassie | $\beta_2^p$ B. & M. | $\beta_2^n$ B. & M. |
|--------|--------------|---------------------|---------------------|---------------------|---------------------|
| $^{28}$O | 17.417       | 0.096               | 0.099               | 0.098               | 0.103               |
|        | 20.315       | 0.091               | 0.038               | 0.093               | 0.039               |
|        | 30.968       | 0.088               | -0.027              | 0.091               | -0.029              |
| $^{60}$Ca | 15.315       | 0.068               | 0.053               | 0.069               | 0.054               |
|        | 16.373       | 0.054               | 0.024               | 0.055               | 0.025               |
|        | 27.542       | 0.032               | -0.016              | 0.032               | -0.016              |
Fig. 5. Neutron and proton RPA transition densities for the quadrupole state at 17.4 MeV in $^{28}$O are compared with the predictions of the Tassie model in Fig. 5a and with the predictions of the Bohr-Mottelson model in Fig. 5b. The corresponding isoscalar and isovector transition densities are compared with the Tassie and Bohr-Mottelson models in Figs. 5c and 5d, respectively.

with the RPA results. Both collective model predictions and the RPA transition densities seem to agree qualitatively with each other. Similar agreement between the collective model predictions and the RPA transition densities was observed for the isoscalar collective state at 15.3 MeV in $^{60}$Ca (see fig. 6).

In fig.4c and 4f we have shown the transition densities for two representative states which are dominantly of isovector character. Again in this case one can compare the RPA results with the collective model. In this case the $\beta$ values for neutrons and protons have opposite sign, corresponding to the picture in which the neutron and proton densities oscillate out of phase. As an example we show in Fig. 7 the comparison of the RPA transition densities with those predicted by both Tassie and Bohr and Mottelson versions of the collective model for the “isovector” state at 30.9 MeV in $^{28}$O, with the value of $\beta$ reported in table 3. Unlike the case of the isoscalar resonances (see Fig. 5), these isovector states exhaust very small fraction of the EWSR. It is not clear if their description in terms of a collective model is appropriate. It can be seen from the table that the corresponding $\beta$ parameters for the neutrons and protons are quite different. So they can only be qualitatively interpreted in terms of the collective
Fig. 6. Same as in Fig. 5 for the state at 15.3 MeV in $^{60}$Ca.

The shape of the transition densities displayed in Fig. 4 can put in evidence further aspects which are a direct consequence of the displacement of proton and neutron radii and which do not necessarily show up in the integrated $B(E2)$ values. For the states of isoscalar character the proton and neutron components to the transition densities peak in different positions, thus leading to a surface-peaked isoscalar transition density and conversely to an isovector density which has a node on the surface. As a consequence, the isovector electromagnetic matrix elements are quenched because of cancellation, and the isoscalar matrix elements dominate. Yet, beyond the nuclear surface, the neutrons give practically the only contribution, and therefore isoscalar and isovector transition densities are of comparable magnitude. We therefore expect that these states, although of isoscalar character, will respond in an equivalent way to isoscalar and isovector heavy-ion probes that are only sensitive to the surface. There may still be some effects of the nodes of the transition density due to the finite range of the effective nucleon-nucleon interaction. A similar statement may be valid for the isovector modes (cf. for example Fig. 4f). In this case it is the isoscalar transition density which displays a node on the surface.

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$^2$ Similar findings were reported by Halbert and Satchler [7] in their analysis of proton inelastic scattering by $^{208}$Pb.
and this leads to a small isoscalar matrix element. Again, however, in the tail of the distribution, isoscalar and isovector transition densities are comparable, and therefore we may expect such a state to be also appreciably excited by a nuclear isoscalar field. One should, in general, expect to see the effect of the neutron skin in nuclear excitation which is sensitive to the behaviour of the transition densities near the nuclear surface.

To summarize, quadrupole transition densities as well as the quadrupole response of neutron rich nuclei were investigated in the framework of the RPA. The transition densities clearly exhibit the effect of the neutron skin in the case of neutron excess nuclei beyond the region of $\beta$-stability. The effect of the neutron skin is less apparent in the case of the $B(E2)$'s since these involve an integral of the transition densities and the occurrence of nodes in the latter leads to cancellation effects. The results clearly indicate a strong mixing of isoscalar and isovector strengths in the RPA states. In the case of the nuclei $^{28}$O and $^{60}$Ca, states were observed which exhaust around 40-50 % of the EWSR. In these cases, the transition densities were seen to behave like collective in-phase oscillations of the neutrons and protons with the same amplitude. The isovector strength, on the other hand, was observed to be strongly fragmented with no single state carrying more than 10-12 % of the EWSR. No realistic candidate for a collective isovector out-of-phase oscillation of the densities was observed.
The effect of the neutron skin is expected to become apparent in nuclear excitation where the details of the transition densities near the nuclear surface will be effective. The detailed study of Coulomb and nuclear inelastic excitation of the neutron-rich nuclei is therefore of interest and should be pursued in the future.

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