Solving navigation-temporal tasks in different coordinate systems

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Abstract. Currently, lots of work is being done to improve the accuracy of navigation systems both from the hardware point of view [1-6] and from the software [7-11]. No less important issue is development of a single system within hardware and software complexes [12-14]. In this article we reveal the issues solving the navigation-temporal tasks in different coordinate systems. When creating multichannel GNSS receivers that implement interferometric methods for measuring spatial orientation on GLONASS and GPS signals, it is extremely important to solve navigation and time problems, especially in order to optimize the solution with missing satellite data. The article also presents proposals for the improvement of multichannel GNSS receivers in order to solve the problems of positioning in conditions of uncertainty and incompleteness of data.

1. Introduction
The hardware RTOS has a function of choosing a coordinate system that produces the results of navigational information. Switching control of the type of coordinate system is performed according to the Protocol or protocols of VIN VIN-M VIN-VIN-E at the exchange over Ethernet. For equipment with built-in remote control and indication, switching of coordinate systems is additionally provided.

With a fully deployed space grouping of navigation spacecraft, from six to ten spacecraft of each system are constantly visible. However, there are cases when the visible spacecraft are not enough for the standard solution of the navigation-temporal problem. Let us consider the cases where the GLONASS system is not fully deployed. Instead of 24 navigation spacecraft there may be only 20 or less ones in the orbit. And on the GLONASS system there are often the cases when only three or even two units are working. In addition, when the equipment is working on a mobile object, for example, on a car, when driving over rough terrain or in the forest, often there is a shading of the navigation spacecraft and signal monitoring failure of some ones. As a result, the navigation spacecraft number...
decreases in the calculation and may be less than the required number for the standard solution. In such cases, it is possible to solve the navigation problem for the minimum constellation of the spacecraft using a priori data.

2. The proposed solution method of navigation- temporal task in different coordinate system

As known, to solve the navigation problem in general case it is required to receive signals of four spacecraft, the number of unknown parameters are three spatial coordinates and time (tuning the equipment time scale from the system time scale). The required number of spacecraft can be reduced by using a priori data. It is advisable to use slowly changing parameters as a priori data. In practice, the height of an object over a quasigeoid or a general earth ellipsoid are often used as a priori data. This mode is often referred to as "2D". Height information can be obtained from the height sensors, for example, from barbastelle, or use previously obtained height values. It is very convenient to use this mode on sea objects, because the sea level corresponds to the zero level of height, and the height of the receiving antenna is known in advance.

In general, the equations for solving the navigation task have the following form:

\[
R_i = \frac{1}{2}(X_i - X)^2 + (Y_i - Y)^2 + (Z_i - Z)^2 + c\Delta T,
\]

where \(X_i, Y_i, Z_i\) are the coordinates of \(i\)-navigation spacecraft; \(X, Y, Z\) are the object coordinates to be found; \(\Delta T\) is tuning the equipment time scale from the system time scale, \(c\) is speed of light.

The navigation spacecraft coordinates and, accordingly, the object coordinates are usually specified in the selected geocentric system (GCS), in which the ephemeris is calculated. However, the task can be solved in any other rectangular coordinate system, for example, in the local topocentric system (TCS).

When solving a navigation problem with a known height, it is convenient to use the TCS, as the height there is expressed in an explicit form.

The topocentric coordinate system is associated with the location of the object. It is a rectangular coordinate system, the origin of which coincides with the location of the object, the X-axis is directed to the North (along the true Meridian), the Y-axis is directed vertically upwards, the Z-axis complements the system to the right coordinate system and is directed to the East horizontally. The transformation of the vector from TCS to GCS is carried out by the following linear transformation:

\[
\begin{bmatrix}
X_i \\
Y_i \\
Z_i
\end{bmatrix} = \begin{bmatrix}
\cos \varphi \cos \lambda & \sin \varphi & - \sin \lambda \\
- \sin \varphi & \cos \varphi \cos \lambda & \cos \varphi \sin \lambda \\
\cos \varphi \sin \lambda & - \cos \varphi \sin \lambda & \cos \varphi
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = C_{\varpi} \cdot \hat{X}_i,
\]

where \(\hat{X}_i = [X_i, Y_i, Z_i]^T\) is the vector in GCS, \(\hat{X}_i = [X_i, Y_i, Z_i]^T\) is the vector in TCS, \(\varphi\) is the latitude, \(\lambda\) is the longitude, \(C_{\varpi}\) is the matrix of a linear transformation of TCS in GCS.

As \(C_{\varpi}\) describes the orthogonal transformation, the inverse transformation of the vector from GCS in TCS can be represented as

\[
\hat{X}_i = C_{\varpi} \cdot \hat{X}_i = [C_{\varpi}]^T \cdot \hat{X}_i = [C_{\varpi}]^T \cdot \hat{X}_i.
\]

Regarding to the task, we need to set the TCS. To set it we need to set the coordinate origin close to the true location of the object. As such, one can select the previous coordinate value of the object. The coordinates of the navigation spacecraft in the TCS are calculated using the following expression:

\[
x_{an} = (C_{\varpi})^T \cdot (X_a - X_0),
\]

where \(X_a\) are the initial point coordinates, calculated from the initial latitude and longitude.

The equation (5.10) is rewritten as:

\[
x_{an} = (C_{\varpi})^T \cdot (X_a - X_0),
\]
\[ R_i = \sqrt{(X_{i\alpha} - X)^2 + (Y_{i\alpha} - Y)^2 + (Z_{i\alpha} - Z)^2 + c \Delta T}, \]

where \( H \) is the known object height.

After linearization, the system of equations takes the form:

\[ k_{i\alpha} \Delta X + k_{i\alpha} \Delta Z + c \Delta T = \Delta R_i, \]

where \( k_{i\alpha} = \frac{X_{\alpha}}{R_0}, k_{i\alpha} = \frac{Y_{\alpha}}{R_0}, k_{i\alpha} = \frac{Z_{\alpha}}{R_0} \) are the guides of the cosines of the direction to the i-navigation spacecraft. \( R_0 = \sqrt{X_{\alpha}^2 + (Y_{\alpha} - H)^2 + Z_{\alpha}^2} \) is the estimated distance to the spacecraft, \( \Delta R_i = R_i - R_0 \) are the residuals, \( \Delta X, \Delta Z \) are the corrections.

Since the origin is close to the true coordinates of the object, a single iteration is required to solve the system of nonlinear equations, and the corrections \( \Delta X, \Delta Z \) can be replaced by the desired coordinates of the object in the selected TCS.

If the priori height of the object \( H \) is set with the error \( \Delta H \), it will cause the error of precision of the planned coordinates. The \( H \) value is included in the expression to determine the cosines guides of the directions for the spacecraft, as well as to determine the estimated range. In the first approach, the error of determining the cosines guide can be neglected, and the dominant error is the error of the calculated range \( \Delta r \), which is determined by the expression

\[ \Delta r = k, \Delta H. \]

The system of equations can be written in matrix form:

\[
\begin{bmatrix}
  k_{i1} & k_{i2} & 1 \\
  k_{i2} & k_{i3} & 1 \\
  k_{i3} & k_{i1} & 1
\end{bmatrix}
\begin{bmatrix}
  X \\
  Z \\
  c \Delta T
\end{bmatrix}
= \begin{bmatrix}
  \Delta R_1 \\
  \Delta R_2 \\
  \Delta R_3
\end{bmatrix}.
\]

Based on linearity of the system the error caused by inaccuracy of the height can be found from the equation:

\[
\begin{bmatrix}
  k_{i1} & k_{i2} & 1 \\
  k_{i2} & k_{i3} & 1 \\
  k_{i3} & k_{i1} & 1
\end{bmatrix}
\begin{bmatrix}
  \partial X \\
  \partial Z \\
  \partial c \Delta T
\end{bmatrix}
= \begin{bmatrix}
  k_{i1} \Delta H \\
  k_{i2} \Delta H \\
  k_{i3} \Delta H
\end{bmatrix}.
\]

The error depends on the configuration of the navigation constellation. Based on the simulation results, the coordinate and time error have the same order as the height error. The error decreases in all respects, if all the spacecraft are low on the horizon. If all spacecraft are high, then the height error almost completely passes into the time error. If there is one high spacecraft and two low, the error is distributed approximately evenly over all components.

The changing parameter is the discrepancy of time scales of the navigation equipment and of the system (parameter \( c \Delta T \)). Slow changes in this parameter are due to the use of highly stable reference generators in the navigation equipment with their frequency control by GLONASS signals. In case of loss of some spacecraft signals, the current value of \( c \Delta T \) can be used as a priori data, and thus reduce the number of spacecraft needed to solve the navigation task. Together with the known height, the required number of spacecraft for the calculation is reduced to two.

The value \( c \Delta T \) is clearly included in the system of equations in any coordinate system, and the implementation of the regime with a known time is by replacing the \( c \Delta T \) parameter by its priori value.

The influence of error \( c \Delta T \) is of a different nature than the uncertainty of the task height. If the height error affects the range calculation error, and this error is different for different spacecraft and depends on the angle of the spacecraft’s location, the time setting error is the same for all spacecraft.
If a mode with the known time is used without using height data, the time error has the greatest effect on the height error. Moreover, the height error increases with the decrease in the angle of all spacecraft location. Thus, in the presence of one high spacecraft the height error corresponds approximately to the error of the time setting, and the error in the plan is less than the error in height by 20-30%. At the angles of the spacecraft location from 10 to 20° the height error exceeds the error of the task $c \Delta T 3-4$ times, and the error in the plan is even reduced to $0.2-0.4 \ c \Delta T$.

If the mode with the known time is used together with the known height (measurement on two spacecraft), then at low spacecraft the error in the plan is approximately equal to the error of a priori data. In the presence of one high spacecraft the error increases by 3-4 times.

It should be noted that the mode with the known time can be used only for short time, since over time the scale of the equipment diverges relative to the system scale and the error of measurement of coordinates increases. The permissible operating time in this mode depends on the type of reference generator used and is from units of minutes in the case of low-cost thermocompensated generators up to a day or more in the case of quantum frequency standards.

If there is no a priori latitude and longitude data, the 2D mode should be implemented in the GCS. However, the height data is not explicitly contained in the equation system, so it is not possible to exclude one of the variables in this case. The solution can be obtained by introducing an additional equation that carries the height information of the object. The additional equation limits the possible object locations. If the height of the object is zero, the geometric location of the points of possible location is the surface of the earth ellipsoid. The equation of the ellipsoid is the following:

$$
\frac{X^2}{a^2} + \frac{Y^2}{b^2} + \frac{Z^2}{c^2} = 1,
$$

where a, b, c are the ellipsoid half-axes.

The general-earth ellipsoid is an ellipsoid of rotation in which the a and c half-axes are equal. If the object is at some height above the ellipsoid, then the area of possible locations in the first approximation can be described by an ellipsoid, in which the equatorial and polar half-axes are equal to the sum of the corresponding half-axes of the earth ellipsoid and height. If the height does not exceed 1000 m, the error of this approximation is negligible. The additional equation in this case has the form:

$$
\frac{X^2}{(a+H)^2} + \frac{Y^2}{(b+H)^2} + \frac{Z^2}{(a+H)^2} = 1.
$$

Currently, the following list of service (additional) geodetic tasks is implemented in the RTOS equipment. The forecast of GLONASS and GPS satellites visibility at any time according to the almanac. Calculation of flat rectangular x and y coordinates in the Gauss-Kruger projection by the geodesic coordinates B and L (Krasovsky ellipsoid). Recalculation of normal height in geodetic and geodetic in normal height. Conversion of angular values from the degree measure to small divisions of the goniometer and back. Determination of the directional angle and distance to the waypoint in real time. Recalculation of coordinates of points from one plane system to another according to the set parameters of connection. Entering the turning points of the route and drawing up routes. Notification about the exit to the area of a given waypoint with a given accuracy, as well as about the deviation from a given route.

3. Conclusion
In addition to the above tasks, the customer navigation device should solve the following service tasks. The solution of the direct geodesic problem on the plane for distances up to 600 km along the flat rectangular x, y coordinates, the directional angle of direction and the horizontal position between the points in the Gauss-Kruger projection (Krasovsky ellipsoid). The solution of the inverse geodesic problem on the plane for distances up to 600 km. The solution of the inverse geodesic problem, when points 1 and 2 are in different coordinate zones (adjacent, the difference between latitudes and
longitude less than 6°). The calculation of the convergence of meridians \( \gamma \) on the plane. The correction for the reduction of distances to projection Gauss-Kruger. The correction for the reduction direction. Correction in the directional angle for the transition from zone to zone \( \Delta \alpha \). Determination of coordinates by different types of serifs. Calculation of the directional angle by the magnetic azimuth and the magnetic azimuth by the directional angle. Calculation of communication parameters of two planar coordinate systems according to specified (introduced) coordinates. Calculation of orientation directional angles on the results of astronomical observations (with an error of less than 30 angular minutes). Calculation of coordinates and heights of the point based on the results of angular (horizontal and vertical) and linear measurements on the ground.

The problem of ensuring the minimum interval of the observational data update no more than 0.1 s (rate of 10 Hz output) is solved in the RTOS equipment for all defined parameters, including the values of spatial orientation. When working on manoeuvring objects, it was found that the rate of data output in 10 Hz for a number of applications is insufficient. In this regard, the rate of update of the observed samples was increased to 25 Hz, while maintaining all the accuracy characteristics. The tests have shown the effectiveness of using a higher rate of delivery of navigation information, especially for spatial orientation values.

For effective use of the possibility of further increasing the rate of data issuance, it is necessary to switch to using interfaces with greater bandwidth.

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