Robust correlations between electromagnetic moments of low-lying $2^+$ states from realistic nuclear systems and random-interaction ensembles

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We observe and discuss proportional correlations between $Q$ and $\mu$ values of the first two $I^\pi = 2^+$ states in both two-body random-interaction and experimental ensembles. $Q(2^+_1) = -Q(2^+_2)$, $Q(2^+_2) = Q(2^+_1)$ and $\mu(2^+_2) = \mu(2^+_1)$ correlations robustly exist in both pseudo and realistic nuclear systems. An experimentally unconfirmed $Q(2^+_1) = -\frac{1}{4}Q(2^+_2)$ correlation is also reported in the IBM1 with random interactions. These observation is attributed to the interacting two-body nature of finite many-body systems, and demonstrates the underlying robustness of nuclear collective structure.

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I. INTERACTION

Finite many-body systems (e.g., nuclei, small metallic grains, metallic clusters) robustly maintains similar regularities, despite they are bound by very different interactions. For example, they all present the odd-even staggering on their binding energy, which is, however, attributed to various mechanisms. Particularly for the nuclear system, the nucleon-nucleon interaction numerically exhibits a “random” pattern with no trace of symmetry groups, and yet the nuclear spectrum follows robust dynamical features: the nuclear spectral fluctuation is universally observed; low-lying spectra of even-even nuclei are orderly and systematically characterized by seniority, vibrator and rotor, beyond $I^\pi = 0^+$ ground states without exception.

To demonstrate these robust regularities insensitive to interaction detail, and reveal underlying origins of them, the random interaction is employed to simulate (or even introduce) the variety and chaos into finite many-body systems. Predominante behaviors in random-interaction ensembles are analyzed, correspondingly to dynamical features of realistic systems. Many efforts have devoted into this field. Typically, predominances of the $I = 0$ ground state and collective band structures have been observed in the presence of random two-body interactions, and related to the spectral properties of realistic even-even nuclei mentioned above. However, the robustness of nuclear electromagnetic properties was rarely discussed.

Recently, a complete experimental survey of nuclear electric quadrupole moments ($Q$ values) of the first two $I^\pi = 2^+$ states presented a general $Q(2^+_1) = -Q(2^+_2)$ correlation (see Fig. 1) across a wide range of masses, deformations and collective patterns. As inspired by this robust correlation, this work would like to make use of random-interaction calculations to provide an interacting-particle vision to this correlation and other potential correlations between $Q$ values or magnetic moments ($\mu$ values) in both random-interaction ensemble and experiments. Thus, theoretical roots of these correlations will be discussed.

II. CALCULATION FRAMEWORK

In our microscope calculations, two-body interactions are randomized without the single-nucleon/boson-energy degree of freedom. Any two-body interaction matrix el-
ment is symmetric to insure spectra with random interactions is laboratorially observable. Here, it’s denoted by $V_{\alpha\beta}$, where $\alpha$ or $\beta$ represents an arbitrary two-body configuration with total angular momentum $J$ as a good quantum number. To maintains the angular-momentum conservation, $\alpha$ and $\beta$ have the same $J$ value. The isospin is omitted, because only like nucleons or bosons are introduced to simplify our analysis. We also require our random interactions statistically invariant under any arbitrary orthogonal transformation. According to the random-matrix theory \cite{21}, this orthogonal invariance can be achieved, if interaction-matrix elements follow the statistical property as

$$
\langle V_{\alpha\beta} \rangle = \langle V_{\gamma\kappa} \rangle = 0,
$$

$$
\langle V_{\alpha\beta} V_{\gamma\kappa} \rangle = (1 + \delta_{\alpha\beta}) \delta_{\alpha\gamma} \delta_{\beta\kappa}.
$$

(1)

In practice, these elements are normally independent Gaussian random numbers constrained by Eq. (1). All the possibilities of random interactions and corresponding microscope-calculation outputs construct the “two-body random ensemble” (TBRE) \cite{21,22}.

Our TBRE calculations are based on the Shell Model \cite{24} and the interacting sd-boson model (IBM1) \cite{23}. For shell-model calculations, four model spaces are constructed with 4 or 6 valence protons in $sd$ or $pf$ shell, correspondingly to four nuclei: $^{24}$Si, $^{26}$S, $^{44}$Cr and $^{46}$Fe. IBM calculations are performed for nuclei with valence boson numbers $N_b = \{12, 13, 14 \text{ and } 15\}$. These nuclei in the TBRE do not match, and do not mean to match, realistic nuclei. They are “pseudo” nuclei in the computational laboratory. Only their statistical properties can be related to the robustness of dynamical features in realistic nuclear systems. To insure the statistical validity of our conclusions, 1 000 000 sets of random interactions are generated for each pseudo nucleus, and inputted into the shell-model or IBM1 code. If a single code running produces $J = 0$ ground state, $Q$ and $\mu$ matrix elements of $2^+_1$ and $2^+_2$ states are further calculated and recorded for following statistic analysis.

The obvious $Q(2^+_1) = -Q(2^+_2)$ correlation in Fig. 1 is a proportional relation. Thus, this work still focus on potential proportional correlations of $Q$ or $\mu$ values in the TBRE. Such correlation can be conveniently characterized by the polar angle ($\theta$ coordinate) of the polar coordinate system in $[Q(2^+_1), Q(2^+_2)]$ or $[\mu(2^+_1), \mu(2^+_2)]$ planes. Therefore, all the statistics, analysis, and discussion in this work are based on $\theta_Q$ and $\theta_\mu$ coordinates as defined by

$$
\theta_Q = \arctan \left( \frac{Q(2^+_1)}{Q(2^+_2)} \right),
\theta_\mu = \arctan \left( \frac{\mu(2^+_1)}{\mu(2^+_2)} \right).
$$

(2)

We visualize the polar coordinate system of the $[Q(2^+_1), Q(2^+_2)]$ plane in Fig. 1. The experimental $Q(2^+_1) = -Q(2^+_2)$ correlation \cite{10} corresponds to $\theta_Q = -45^\circ$, and thus the experimental concentration along the $\theta_Q = -45^\circ$ direction is obvious in Fig. 1.

![FIG. 2. (Color online) $\theta_Q$ distributions in the experiments (Exp) \cite{10} and the shell-model TBRE for four pseudo nuclei: $^{24}$Si, $^{26}$S, $^{44}$Cr and $^{46}$Fe. $\theta_Q = -45^\circ$ and $\theta_Q = 45^\circ$ peaks are highlighted correspondingly to $Q(2^+_1) = -Q(2^+_2)$ and $Q(2^+_1) = Q(2^+_2)$ correlations, respectively. Error bars are from the statistics counting.]

III. RESULTS AND ANALYSIS

A. Q statistics in the TBRE

In Fig. 2 we present the $\theta_Q$ distributions of four pseudo nuclei in the shell-model TBRE compared with the experimental distribution from Ref. \cite{10}. A rough consistency is achieved between experiments and TBRE calculations. The main peak around $\theta_Q = -45^\circ$ confirms the $Q(2^+_1) = -Q(2^+_2)$ correlation in both experiments and the TBRE. On the other hand, slight $\theta_Q = 45^\circ$ concentration in both experiments and pseudo nuclei are also observed correspondingly to a potential $Q(2^+_1) = Q(2^+_2)$ correlation.

In the IBM1 framework, the $Q$ operator is a linear combination of two independent rank-two operators as:

$$
Q = Q^1 + \chi Q^2, \quad Q^1 = d^\dagger \bar{s} + s^\dagger \bar{d}, \quad Q^2 = [d^\dagger \bar{d}]^2,
$$

(3)

where $\chi$ is a free parameter to fit data in calculations for realistic nuclei. A robust $[Q(2^+_1), Q(2^+_2)]$ correlation should be insensitive to the $\chi$ value, which requires $Q^1$ and $Q^2$ matrix elements of the first two $2^+$ states follow the same correlation. Thus, we define

$$
\theta_Q^1 = \arctan \left( \frac{Q(2^+_1)}{Q^1(2^+_1)} \right),
\theta_Q^2 = \arctan \left( \frac{Q(2^+_1)}{Q^2(2^+_1)} \right), \quad \theta_Q = \arctan(k)
$$

(4)

and a proportional correlation of $Q(2^+_1)/Q(2^+_2) = k$ can be observed with the concentration around $\theta_Q^1 = \theta_Q^2 = \arctan(k)$ in the two-dimensional [$\theta_Q^1, \theta_Q^2$] distribution.

It’s noteworthy that the IBM1 TBRE favors $R = E_{4^+_1}/E_{2^+_1}$ values around 2.0 and 3.33 \cite{12,14}. To avoid
of the bias for $R = 2.0, 3.33$ or any other potential $R$ predominances, and thus consider all the types of spectral patterns evenly in the IBM1 TBRE, we define the $R$-normalized $[\theta^1_Q, \theta^2_Q]$ distribution as

$$P(\theta_Q^1, \theta_Q^2) = \int_R P(R, \theta_Q^1, \theta_Q^2) \frac{1}{P(R)},$$

where $P(R, \theta_Q^1, \theta_Q^2)$ is the three-dimensional distribution of the $(R, \theta_Q^1, \theta_Q^2)$ vector, and $P(R)$ is the $R$ distribution in the IBM1 TBRE.

All the $P(\theta_Q^1, \theta_Q^2)$ distributions in Fig. 3 follow the same pattern: three sharp concentrations emerges. Similarly to $\theta_Q$ distributions in the shell-model TBRE and experiments (see Fig. 2), two concentrations are located around $\theta_Q^1 = \theta_Q^2 = -45^\circ$ and $\theta_Q^1 = \theta_Q^2 = 45^\circ$, correspondingly to $Q(2^+_g) = -Q(2^+_u)$ and $Q(2^+_g) = Q(2^+_u)$ correlations. Another concentration is around $\theta_Q^1 = \theta_Q^2 = -23^\circ$, which will be further discussed later.

We emphasize again that, concentrations of $\tilde{P}(\theta_Q^1, \theta_Q^2)$ distributions can not be attributed to the predominance of $R = 2.00$ and $3.33$ in the IBM1 TBRE due to the $R$ normalization by Eq. (3). One can also remove samples with $R \in [1.8, 2.2]$ and $R \in [3.1, 3.5]$ of the IBM1 TBRE, and replot the $[\theta_Q^1, \theta_Q^2]$ distributions, where three sharp concentrations can still be observed. Therefore, we conclude that these favorable $[Q(2^+_g), Q(2^+_u)]$ correlations are general many-body properties despitied of spectral patterns.

The $Q(2^+_g) = -Q(2^+_u)$ correlation has been analytically derived with the collective model, and numerically obtained with the consistent-$Q$ formalism of the IBM1 19. To search theoretical roots of other $[Q(2^+_g), Q(2^+_u)]$ correlations, we revisit symmetry limits of the IBM1 with $O(6)$, SU(3) and U(5) subgroup chains 23.

In the IBM calculation for $O(6)$-like nuclei, $\chi$ is empirically small, and thus the $Q$ selection rule requires $\langle \phi | Q | \phi \rangle = 0$ for any state $| \phi \rangle$ with $O(6)$-subgroup labels as good quantum numbers. Therefore, there is no proportional $[Q(2^+_g), Q(2^+_u)]$ correlation towards the $O(6)$ limit.

In the SU(3) limit, three low-lying rotational bands can be observed: the ground band with $[\lambda = 2N_b, \mu = 0, K = 0]$, the $\beta$ band with $[\lambda = 2N_b - 4, \mu = 2, K = 0]$, and the $\gamma$ band with $[\lambda = 2N_b - 4, \mu = 2, K = 2]$. $Q(2^)$ values in these three bands follow

$$Q(2^) = 4N_b - 3$$

where the superscript “g” represents the ground band. When $N_b \to \infty$, $Q(2^) = Q(2^g)$ and $Q(2^g) = -Q(2^u)$ are achieved. $\beta$ and $\gamma$ bands provide almost degenerated $2^+$ states. Namely, the $2^g$ state can belong to either the $\beta$ band or $\gamma$ band. Thus, $Q(2^g) = Q(2^g)$ and $Q(2^g) = -Q(2^u)$ correlation emerges with the SU(3) limit, correspondingly.

In the U(5) limit, the $Q(2^) / Q(2^u)$ value is a constant (i.e., $-3/7$ by the analytical calculation 25) independent of the total boson number $N_b$. The $\theta_Q^1 = \theta_Q^2 = -23^\circ$ concentration in Fig. 4 agrees with the $Q(2^) = -3/7Q(2^g)$ relation, considering $\arctan(-3/7) = -23^\circ$. Therefore, the $\theta_Q^1 = \theta_Q^2 = -23^\circ$ is labeled as “U(5)-like”.

B. $\mu$ statistics in the TBRE and experiments

Now, let’s turn to the $\mu$ statistics in the TBRE. In the IBM1, the $\mu$ operator is proportional to the angular-momentum operator, and $2^g$ and $2^u$ states shares the same angular momentum. Thus, we shall always obtain $\mu(2^g) = \mu(2^u)$ correlation in the IBM1. In shell-model calculations, the $\mu$ operator is defined as $\mu = g_{\pi} + g_{\sigma}\hat{\pi}$ for valence proton, where $g_{\pi} = 1$ and $g_{\sigma} = 5.586$. Corresponding shell-model TBRE distributions of $\theta_\mu$ defined by Eq. 2 are presented in Fig. 5, where the sharp peak around $\theta_\mu = 45^\circ$ also demonstrates the $\mu(2^g) = \mu(2^u)$ correlation.

We also perform an full survey on all experimentally available $\mu(2^g)$ and $\mu(2^u)$ of realistic nuclei in the ENSDF 24, which are plotted against each other in Fig. 5. One see most of experimental data scatter around the $\mu(2^g) = \mu(2^u)$ correlation. The experimental $\theta_\mu$ distribution is also compared with those in the shell-model TBRE in Fig. 4. One sees the perfect agreement between experiments and the TBRE, especially on the sharp peak.
with the $\mu(2^+_2) = \mu(2^+_1)$ correlation. Thus, we conclude the $\mu(2^+_2) = \mu(2^+_1)$ correlation universally exists in both random-interaction and experimental ensembles.

IV. SUMMARY

To summarize, we observe and discuss the proportional correlations between $Q$ and $\mu$ values of the first two $I^+ = 2^+$ states in both experiments and the TBRE. $Q(2^+_1) = -Q(2^+_2)$, $Q(2^+_1) = Q(2^+_2)$ and $\mu(2^+_2) = \mu(2^+_1)$ correlations robustly and universally exist despite of the low-lying spectral pattern and the randomness of interactions, which, at least in part, can be attributed to the interacting two-body nature of finite many-body systems. In even-even nuclei, the electromagnetic property of $2^+_1$ states represents the nuclear collective motive, and is more sensitive to the wave-function detail than the spectrum. Hence, the nuclear collective structure is maintained in far more deep level than the common realization based on the orderly spectral pattern.

In the IBM1 TBRE, the U(5)-like $Q(2^+_2) = -\frac{1}{3} Q(2^+_1)$ correlation is also observed, and yet unconfirmed by experiments. A serial of explorations to $Q(2^+_2)$ values is desired. Experimental evidences of the $\mu(2^+_2) = \mu(2^+_1)$ correlation are more uncertain and dispersed than those of $Q$ correlations, which requires more precise $\mu(2^+_2)$ measurement.

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