Problem formulation

During the operation of roller forming units designed for forming reinforced concrete products, considerable dynamic loads appear both in drive and forming trolley elements [1–14]. Despite a rather extensive research into the technological process of forming reinforced concrete products by the non-vibration roller forming method [1–4], the dynamics of a forming trolley movement and its influence on the formation process have not been investigated yet. Little attention has been paid to the modes of a forming trolley movement and the forces arising in drive elements.

Analysis of recent research and publications

In the existing theoretical and experimental studies of roller forming units designed for forming reinforced concrete products, the design parameters and productivity of the units are substantiated [1–4]. At the same time, insufficient attention is paid to the study of the existing dynamic loads and movement modes, which greatly affect both the operation of the units and the quality of the finished products. During continuous start-stop modes, considerable dynamic loads appear both in the drive and forming trolley elements, which may lead to the premature failure of the units [1–6]. Therefore, the task of studying dynamic loads in the elements of the units is actual. In [15–19], loads in the roll forming unit elements were determined, however, the rigidity and dissipation coefficients of the drive were not taken into account.

Purpose of the paper

The purpose of this paper is to determine the loads in the elements of a roll forming unit with an energy-balanced drive, taking into account the drive rigidity and dissipation coefficient.

Statement of the research problem

In order to reduce energy consumption in roller forming machines, a design of a roller forming unit [20, 21] was proposed to provide the compaction of reinforced concrete products on a single technological line. It consists of three forming trolleys, located parallel to each other on one side of the drive shaft, which are set in reciprocating movement from one drive. It is composed of three slider-crank mechanisms, whose cranks are tightly fixed on one drive shaft and shifted to each other at the angle $\Delta \phi = 120^0$. Fig. 1 shows a roller forming unit with an energy-balanced drive. Each of the forming trolleys 11, 8 and 15 (Fig. 1, a) is mounted on the gantry 14 and performs reciprocating movement in the guide rails 9 over the cavity of the form 10. The forming trolley 11 consists of the feeding hopper 12 and coaxial sections of the compaction rollers 13. The other two trolleys have the same design. The trolleys 11, 8, and 15 with distribution hoppers are set into reciprocating movement by a drive made in the form of three slider-crank mechanisms, whose cranks 5, 3, and 2 are rigidly fixed on one drive shaft 4 and shifted to each other at the angle $\Delta \phi = 120^0$. The connecting rods 7, 6 and 1 are hinged to the forming trolleys 11, 8, and 15 while their other ends are connected to the connecting rod 17. During the reciprocating movement of the trolleys, the elements of the units are actual. In [15–19], loads in the roll forming unit elements were determined, however, the rigidity and dissipation coefficients of the drive were not taken into account.
connected to the cranks 5, 3, and 2. Such a design of a roller forming unit makes it possible to reduce the dynamic loads in the drive elements, extra devastating loads on the frame structure and, accordingly, increase the unit durability as a whole. Fig. 1, b shows a kinematic scheme of a roller forming unit with an energy-balanced drive for compacting reinforced concrete products on a single technological line. In this kinematic scheme, \( r \) is the radius of the cranks 5, 3, and 2; \( l \) is the length of the connecting rods 7, 6, and 1; \( \varphi \) is the angular coordinate of the first trolley crank position; \( \Delta \varphi \) is the displacement angle of the cranks 5 and 3, 3 and 2, and 2 and 5 with respect to each other; \( x_{B_1}, x_{B_2}, \) and \( x_{B_3} \) are the coordinates of the mass centers of the trolleys 11, 8, and 15 respectively; \( \beta_1, \beta_2, \) and \( \beta_3 \) are the angular coordinates determining the positions of the connecting rods 7, 6 and 1 relative to the horizontal.

**Fig. 1. Roller forming unit with an energy-balanced drive:**
a – general view; b – kinematic scheme

During the operation of a roller forming unit with an energy-balanced drive, there arise significant dynamic loads in the elements of the transfer mechanism from the electric motor to the cranks, leading to the premature destruction of the drive structural elements. To study these loads, we use a two-mass dynamic model of a roller forming unit (Fig. 2). In this model, the following symbols are used: \( M_{dr1} \) is the driving moment on the driving electric motor shaft, reduced to the crank rotation axis; \( M_{dr2} \) is the moment from the movement resistant forces of the forming trolleys with compaction rollers, reduced to the crank rotation axis; \( J_{dr1} \) is the inertia moment of the electric motor rotor and drive elements, reduced to the crank rotation axis; \( J_{dr2} \) is the inertia moment of the forming trolleys and crank-and-rod mechanisms, reduced to the crank rotation axis; \( c \) is the drive mechanism rigidity, reduced to the crank rotation axis; \( \varphi_1 \) and \( \varphi_2 \) are the generalized coordinates of the reduced masses \( J_{dr1} \) and \( J_{dr2} \), respectively.

The drive mechanism reduced moment of inertia can be determined by the following dependence:

\[
J_{dr1} = (J_{ret} + J_{eth}) \cdot \delta \cdot u^2, \tag{1}
\]

where \( J_{ret} \) and \( J_{eth} \) are the moments of inertia of the engine and clutch, connecting the motor shaft and the reducer input shaft, respectively; \( \delta \) is the coefficient taking into account the moments of inertia of the reducer elements, reduced to the rotor shaft; \( u \) is the reducer transmission ratio.

The reduced moment of inertia \( J_{dr2} \) can be determined from the second part of the mechanism (Fig. 3), which includes crank-and-rod mechanisms with forming trolleys. In this case, we divide the mass of the connecting rods 7, 6, and 1 \( m_{conrod} \) in equal parts in the points \( A_1 \) and \( B_1 \), \( A_2 \) and \( B_2 \), and \( A_3 \) and \( B_3 \). Then the moment of inertia of the cranks can be determined by the dependence

\[
J_{crank} = 3 \cdot J'_{crank} + 3 \cdot \frac{m_{conrod}}{2} \cdot r^2 = 3 \left( J'_{crank} + \frac{m_{conrod}}{2} \cdot r^2 \right), \tag{2}
\]

and the trolley masses will have the look
\[ m_B = m'_B + \frac{m_{\text{controd}}}{2}; \quad m_B = m'_B + \frac{m_{\text{controd}}}{2}; \quad m_B = m'_B + \frac{m_{\text{controd}}}{2}. \] (3)

In the expressions (3) \( m_{\text{controd}} \) is the mass of each of the connecting rods; \( m'_B, m'_B, m'_B \) are the masses of the forming trolleys 11, 8 and 15, respectively (Fig. 1, b); \( J'_{\text{crank}} \) is the moment of inertia of each of the cranks relative to their own rotation axes; \( r \) is the radius of each of the cranks; \( J_{\text{crank}} \) is the moment of inertia of each of the cranks with half the mass of the connecting rod relative to their own rotation axes; \( m_B, m_B, m_B \) are the masses of the trolleys 11, 8, and 15 with half the mass of the connecting rod.

The reduced moment of inertia \( J_{dr/2} \) can be determined from the condition of equality of the kinetic energies of the crank-and-rod mechanisms with the trolleys \( T_r \) (Fig. 3) and the second disk of the dynamic model (Fig. 2) \( T_m \), i.e., \( T_r = T_m \).

![Fig. 3. Calculation scheme of the loads on the elements of a roller forming unit with an energy-balanced drive: a – load on the forming trolley; b – load on the drive mechanism](image)

We find the kinetic energy of crank-and-rod mechanisms with trolleys

\[ T_r = \frac{1}{2} \cdot J_{\text{crank}} \cdot \dot{\varphi}_2^2 + \frac{1}{2} \cdot m_B \cdot \dot{x}_S^2 + \frac{1}{2} \cdot m_B \cdot \dot{x}_S^2 + \frac{1}{2} \cdot m_B \cdot \dot{x}_S^2, \] (4)

where \( \dot{x}_S, \dot{x}_S, \) and \( \dot{x}_S \) are the velocities of the mass centers of the first, second and third forming trolleys, respectively.

Since the three trolleys move progressively, all their points have the same velocity. Therefore, we can accept that \( \dot{x}_S = \dot{x}_B, \dot{x}_S = \dot{x}_B, \) and \( \dot{x}_S = \dot{x}_B \). We express the velocities of the points \( B_1, B_2 \) and \( B_3 \) through the coordinates of the cranks and their derivatives in time. In order to do this, we use the dependencies

\[ \dot{x}_B = \frac{\partial x_B}{\partial \varphi_2}; \quad \dot{x}_B = \frac{\partial x_B}{\partial \varphi_2}; \quad \dot{x}_B = \frac{\partial x_B}{\partial \varphi_2}. \] (5)

Then, taking into account (5), the dependence (4) will have the form

\[ T_r = \frac{1}{2} \cdot \dot{\varphi}_2^2 \left[ J_{\text{crank}} + m_B \cdot \left( \frac{\partial x_B}{\partial \varphi_2} \right)^2 + m_B \cdot \left( \frac{\partial x_B}{\partial \varphi_2} \right)^2 + m_B \cdot \left( \frac{\partial x_B}{\partial \varphi_2} \right)^2 \right]. \] (6)

The kinetic energy of the second disk in figure 2 is expressed by the dependence

\[ T_m = \frac{1}{2} \cdot J_{dr/2} \cdot \dot{\varphi}_2^2. \] (7)

Equating the dependences (6) and (7), we have

\[ \frac{1}{2} \cdot \dot{\varphi}_2^2 \left[ J_{\text{crank}} + m_B \cdot \left( \frac{\partial x_B}{\partial \varphi_2} \right)^2 + m_B \cdot \left( \frac{\partial x_B}{\partial \varphi_2} \right)^2 + m_B \cdot \left( \frac{\partial x_B}{\partial \varphi_2} \right)^2 \right] = \frac{1}{2} \cdot J_{dr/2} \cdot \dot{\varphi}_2^2. \] (8)
From the equation (8) we obtain

$$J_{dr} = J_{crank} + m_b \left( \frac{\partial x_{b}}{\partial \varphi_2} \right)^2 + m_{b_2} \left( \frac{\partial x_{b_2}}{\partial \varphi_2} \right)^2 + m_{b_3} \left( \frac{\partial x_{b_3}}{\partial \varphi_2} \right)^2,$$

(9)

To determine the reduced moment of the resistance forces $M_{dr}$ we use figure 3, on which the following symbols are used: $F_1$, $F_2$, and $F_3$ are the forces in the connecting rods necessary for overcoming the resistance forces acting on the trolleys; $\beta_1$, $\beta_2$, and $\beta_3$ are the angular coordinates determining the positions of the connecting rods of the first, second and third trolleys relative to the horizontal; $F_{011}$ and $F_{012}$ are the horizontal forces of interaction of the compaction rollers with a concrete mixture for the first forming trolley; $R_{011}$ and $R_{012}$ are the vertical forces of interaction of the compaction rollers with a concrete mixture; $N_{11}$ and $N_{12}$ are the normal reactions of the forming trolley guide rails to the guide rollers; $F_{f11} = N_{11} \cdot f_{red}$, $F_{f12} = N_{12} \cdot f_{red}$ are the friction forces of the guide rollers by the forming trolley guide rails; $f_{red}$ is the reduced friction coefficient of the guide rollers by the forming trolley guide rails; $G$ is the forming trolley gravity force; $a$, $b$, $p$, $e$ are the geometric dimensions of the forming trolley; $D$ is the pressure roller diameter; $d$ is the guide roller diameter; $l$ is the connecting rod length. For the second and third forming trolleys, the force parameters $F_{021}$, $F_{022}$, $F_{031}$, $F_{032}$, $R_{021}$, $R_{022}$, $R_{031}$, $R_{032}$, $N_{21}$, $N_{22}$, $N_{31}$, $N_{32}$, $F_{21}$, $F_{22}$, $F_{31}$, $F_{32}$, $f_{red}$, $G$ and the geometric characteristics $a$, $b$, $p$, $e$, $D$, $d$, $l$ will be identical.

To determine the reactions of the guide rollers $N_{11}$, $N_{12}$, $N_{21}$, $N_{22}$, $N_{31}$, and $N_{32}$ as well as the forces in the connecting rods $F_1$, $F_2$, and $F_3$, let us consider the static equilibrium of the first, second and third forming trolleys. We design all the forces acting on each of the trolleys on the coordinate axes $x$ and $y$, and compute the sum of the moments of these forces relative to the points, $B_1$, $B_2$, and $B_3$ (Fig. 3). As a result, for the first forming trolley we obtain

$$\sum X = -F_1 \cdot \cos \beta_1 + N_{11} \cdot f_{red} + N_{12} \cdot f_{red} + F_{011} + F_{012} = 0;$$

$$\sum Y = F_1 \cdot \sin \beta_1 - N_{11} - N_{12} - G + R_{011} + R_{012} = 0;$$

$$\sum M_{b_1} = -N_{11} \cdot (p - b) - N_{12} \cdot (p + b) - G \cdot p - (N_{11} + N_{12}) \cdot f_{red} \cdot d +$$

$$+ (F_{011} + F_{012}) \cdot e + R_{011} \cdot (p + a) + R_{012} \cdot (p + a) = 0;$$

(10)

for the second forming trolley we obtain

$$\sum X = -F_2 \cdot \cos \beta_2 + N_{21} \cdot f_{red} + N_{22} \cdot f_{red} + F_{021} + F_{022} = 0;$$

$$\sum Y = F_2 \cdot \sin \beta_2 - N_{21} - N_{22} - G + R_{021} + R_{022} = 0;$$

$$\sum M_{b_2} = -N_{21} \cdot (p - b) - N_{22} \cdot (p + b) - G \cdot p - (N_{21} + N_{22}) \cdot f_{red} \cdot d +$$

$$+ (F_{021} + F_{022}) \cdot e + R_{021} \cdot (p + a) + R_{022} \cdot (p + a) = 0;$$

(11)

for the third forming trolley we obtain

$$\sum X = -F_3 \cdot \cos \beta_3 + N_{31} \cdot f_{red} + N_{32} \cdot f_{red} + F_{031} + F_{032} = 0;$$

$$\sum Y = F_3 \cdot \sin \beta_3 - N_{31} - N_{32} - G + R_{031} + R_{032} = 0;$$

$$\sum M_{b_3} = -N_{31} \cdot (p - b) - N_{32} \cdot (p + b) - G \cdot p - (N_{31} + N_{32}) \cdot f_{red} \cdot d +$$

$$+ (F_{031} + F_{032}) \cdot e + R_{031} \cdot (p + a) + R_{032} \cdot (p + a) = 0.$$

(12)

Having solved the systems of equations (10) – (12), we find
\[ F_1 = \frac{1}{\cos \beta_1 - f_{\text{red}} \sin \beta_1} \left[ (R_{011} + R_{012} - G) \cdot f_{\text{red}} + F_{011} + F_{012} \right]; \]  
(13)

\[ F_2 = \frac{1}{\cos \beta_2 - f_{\text{red}} \sin \beta_2} \left[ (R_{021} + R_{022} - G) \cdot f_{\text{red}} + F_{021} + F_{022} \right]; \]  
(14)

\[ F_3 = \frac{1}{\cos \beta_3 - f_{\text{red}} \sin \beta_3} \left[ (R_{031} + R_{032} - G) \cdot f_{\text{red}} + F_{031} + F_{032} \right]; \]  
(15)

\[ N_{12} = \frac{1}{2 \cdot b} \left[ R_{011} \cdot \left( b - a - f_{\text{red}} \cdot \frac{d}{2} \right) + R_{012} \cdot \left( b + a - f_{\text{red}} \cdot \frac{d}{2} \right) \right] - G \left( b - f_{\text{red}} \cdot \frac{d}{2} \right) + F_1 \cdot \sin \beta_1 \left( b - p - f_{\text{red}} \cdot \frac{d}{2} \right) \right]; \]  
(16)

\[ N_{22} = \frac{1}{2 \cdot b} \left[ R_{021} \cdot \left( b - a - f_{\text{red}} \cdot \frac{d}{2} \right) + R_{022} \cdot \left( b + a - f_{\text{red}} \cdot \frac{d}{2} \right) \right] - G \left( b - f_{\text{red}} \cdot \frac{d}{2} \right) + F_2 \cdot \sin \beta_2 \left( b - p - f_{\text{red}} \cdot \frac{d}{2} \right) \right]; \]  
(17)

\[ N_{32} = \frac{1}{2 \cdot b} \left[ R_{031} \cdot \left( b - a - f_{\text{red}} \cdot \frac{d}{2} \right) + R_{032} \cdot \left( b + a - f_{\text{red}} \cdot \frac{d}{2} \right) \right] - G \left( b - f_{\text{red}} \cdot \frac{d}{2} \right) + F_3 \cdot \sin \beta_3 \left( b - p - f_{\text{red}} \cdot \frac{d}{2} \right) \right]; \]  
(18)

\[ N_{11} = R_{011} + R_{012} + F_1 \cdot \sin \beta_1 - N_{12} - G; \]  
(19)

\[ N_{21} = R_{021} + R_{022} + F_2 \cdot \sin \beta_2 - N_{22} - G; \]  
(20)

\[ N_{31} = R_{031} + R_{032} + F_3 \cdot \sin \beta_3 - N_{32} - G. \]  
(21)

On the basis of the dependences (13)–(15) we find the moments of the resistance forces \( M_{\text{red1}}, M_{\text{red2}} \) and \( M_{\text{red3}} \) from each of the forming trolleys and the total moment of the resistance forces \( M_{\text{drag}} \), reduced to the crank rotation axis

\[ M_{\text{red1}} = F_1 \cdot r \cdot \cos \left( \frac{\pi}{2} - \phi_2 - \beta_1 \right) = F_1 \cdot r \cdot \sin \left( \phi_2 + \beta_1 \right); \]  
(22)

\[ M_{\text{red2}} = F_2 \cdot r \cdot \cos \left( \frac{\pi}{2} - (\phi_2 + \Delta \phi) - \beta_2 \right) = F_2 \cdot r \cdot \sin \left( \phi_2 + \Delta \phi + \beta_2 \right); \]  
(23)

\[ M_{\text{red3}} = F_3 \cdot r \cdot \cos \left( \frac{\pi}{2} - (\phi_2 + 2\Delta \phi) - \beta_3 \right) = F_3 \cdot r \cdot \sin \left( \phi_2 + 2\Delta \phi + \beta_3 \right); \]  
(24)

\[ M_{\text{drag}} = M_{\text{red1}} + M_{\text{red2}} + M_{\text{red3}} = F_1 \cdot r \cdot \sin \left( \phi_2 + \beta_1 \right) + F_2 \cdot r \cdot \sin \left( \phi_2 + \Delta \phi + \beta_2 \right) + F_3 \cdot r \cdot \sin \left( \phi_2 + 2\Delta \phi + \beta_3 \right). \]  
(25)

The magnitude of the angles \( \beta_1, \beta_2, \) and \( \beta_3 \) can be determined from the ratios

\[ r \cdot \sin \phi_2 = 1 \cdot \sin \beta_1; \quad r \cdot \sin (\phi_2 + \Delta \phi) = 1 \cdot \sin \beta_2; \quad r \cdot \sin (\phi_2 + 2\Delta \phi) = 1 \cdot \sin \beta_3. \]  
(26)

From here

\[ \beta_1 = \arcsin \left( \frac{r}{l} \cdot \sin \phi_2 \right); \quad \beta_2 = \arcsin \left( \frac{r}{l} \cdot \sin (\phi_2 + \Delta \phi) \right); \quad \beta_3 = \arcsin \left( \frac{r}{l} \cdot \sin (\phi_2 + 2\Delta \phi) \right). \]  
(27)

Using the Lagrange equation of the second kind, we make differential equations of the motion of a roller forming unit with an energy-balanced drive, represented by a two-mass dynamic model.
\[ \frac{d}{dt} \frac{\partial T}{\partial \phi_1} - \frac{\partial T}{\partial \phi_1} = Q_{\phi_1} \frac{\partial \Pi}{\partial \phi_1}; \]
\[ \frac{d}{dt} \frac{\partial T}{\partial \phi_2} - \frac{\partial T}{\partial \phi_2} = Q_{\phi_2} \frac{\partial \Pi}{\partial \phi_2}, \]
where \( t \) is the time; \( T \) is the system kinetic energy; \( Q_{\phi_1}, Q_{\phi_2} \) are the generalized forces corresponding to the coordinates \( \phi_1 \) and \( \phi_2 \); \( \Pi \) is the system potential energy that has the form
\[ \Pi = \frac{1}{2} c \cdot (\phi_1 - \phi_2)^2. \] (29)

The system kinetic energy, taking into account the expression (9), is expressed by the dependence
\[ T = \frac{1}{2} J_{\text{dr}} \cdot \dot{\phi}_1^2 + \frac{1}{2} J_{\text{crank}} \cdot \left( m_B \cdot \left( \frac{\partial x_{B_1}}{\partial \phi_2} \right)^2 + m_{B_2} \cdot \left( \frac{\partial x_{B_2}}{\partial \phi_2} \right)^2 + m_{B_3} \cdot \left( \frac{\partial x_{B_3}}{\partial \phi_2} \right)^2 \right) \cdot \dot{\phi}_2^2. \] (30)

The generalized forces have the form
\[ Q_{\phi_1} = M_{\text{dr}1} \cdot \dot{\phi}_1; \quad Q_{\phi_2} = M_{\text{dr}2}, \] (31)
where \( M_{\text{dr}1} \) is the driving moment on the electric motor drive shaft, reduced to the crank rotation axis, which is determined by the Kloss formula
\[ M_{\text{dr}1} = \frac{2 \cdot M_{\text{crit}} \cdot u \cdot \eta_{\text{dr}}}{1 - \frac{\dot{\phi}_1}{\omega_0} + \frac{u}{\omega_0}}. \] (32)

Here \( M_{\text{crit}} \) is the critical (maximum) moment on the electric motor shaft; \( u \) is the drive mechanism transmission ratio; \( \eta_{\text{dr}} \) is the drive mechanism efficiency coefficient; \( \omega_0 \) is the synchronous angular speed of the electric motor rotor; \( s_{\text{crit}} \) is the electric motor critical slip determined by the dependence
\[ s_{\text{crit}} = s_{\text{nom}} \cdot \left( \lambda + \sqrt{\lambda^2 - 1} \right), \]
where \( \lambda \) is the electric motor maximum ratio (electric motor overload capacity); \( s_{\text{nom}} \) is the electric motor nominal slip determined as follows:
\[ s_{\text{nom}} = 1 - \frac{\omega_{\text{nom}}}{\omega_0}. \]

Here \( \omega_{\text{nom}} \) is the nominal angular speed of the driving electric motor rotor. Taking the derivatives of the system kinetic energy expression (30), we obtain
\[ \frac{\partial T}{\partial \phi_1} = \left[ J_{\text{crank}} + m_B \cdot \left( \frac{\partial x_{B_1}}{\partial \phi_2} \right)^2 + m_{B_2} \cdot \left( \frac{\partial x_{B_2}}{\partial \phi_2} \right)^2 + m_{B_3} \cdot \left( \frac{\partial x_{B_3}}{\partial \phi_2} \right)^2 \right] \cdot \dot{\phi}_1; \]
\[ \frac{\partial T}{\partial \phi_2} = \left[ J_{\text{crank}} + m_B \cdot \left( \frac{\partial x_{B_1}}{\partial \phi_2} \right)^2 + m_{B_2} \cdot \left( \frac{\partial x_{B_2}}{\partial \phi_2} \right)^2 + m_{B_3} \cdot \left( \frac{\partial x_{B_3}}{\partial \phi_2} \right)^2 \right] \cdot \dot{\phi}_2. \] (33)
After substituting the expressions (29), (31)–(33), (13)–(15) and (25) into the system of equations (28) we have

\[
\begin{align*}
J_{d1} \ddot{\varphi}_1 &= \frac{2 \cdot M_{\text{cr}} \cdot u \cdot \eta_{\text{dr}}}{1 - \dot{\varphi}_1 \cdot \omega_0} + \frac{s_{\text{crit}}}{1 - \dot{\varphi}_1} \cdot \omega_0 \\
J_{\text{crank}} + m_B \dot{\varphi}_2 \left[ \frac{\partial x_B}{\partial \varphi_2} \right]^2 + m_{B_2} \dot{x}_B_2 \frac{\partial^2 x_B}{\partial \varphi_2^2} + m_{B_3} \dot{x}_B_3 \frac{\partial^2 x_B}{\partial \varphi_2^2} + m_{B_4} \dot{x}_B_4 \frac{\partial^2 x_B}{\partial \varphi_2^2} \right] \\
+ \dot{\varphi}_2^2 \left[ m_{B_1} \frac{\partial x_B}{\partial \varphi_2} \right]^2 + m_{B_2} \frac{\partial^2 x_B}{\partial \varphi_2^2} + m_{B_3} \frac{\partial^2 x_B}{\partial \varphi_2^2} + m_{B_4} \frac{\partial^2 x_B}{\partial \varphi_2^2} \right] = 0 \tag{34}
\end{align*}
\]

Results of the solution

For a roller forming unit with the parameters [4] to be given below, the functions are determined and the graphs are built, expressing the changes in the reactions of the guide rollers \(N_11, N_12, N_21, N_22, N_31, \) and \(N_32\) (Fig. 4), forces in the connecting rods \(F_1, F_2, \) and \(F_3\) (Fig. 5), and moments of the resistance forces \(M_{\text{res1}}, M_{\text{res2}}, M_{\text{res3}} \) and \(M_{d2}\) (Fig. 6), depending on the rotation angle of the cranks. These parameters are the following: \(r = 0.2 \text{ m}; l = 0.8 \text{ m};\) horizontal interaction forces \(F_{011} = F_{012} = F_{021} = F_{022} = F_{031} = F_{032} = 7962 \text{ N}\) between the compaction rollers and a concrete mixture during the process of its being compacted by a roller forming trolley with two \(R = 0.11 \text{ m}\) compaction rollers, the mixture having the following characteristics: product height \(h_0 = 0.22 \text{ m};\) product width \(B = 1.164 \text{ m};\) type of compacted mixture is fine-grained; concrete mixture humidity \(W = 10\%;\) required product compactness \(k_{\text{comp}} = 0.98;\) value of the maximum contact pressure providing \(k_{\text{comp}} = 0.98 \) at \(W = 10\%,\) according to experimental data \(p = 625 \text{ kPa};\) \(R_{011} = R_{012} = R_{021} = R_{022} = R_{031} = R_{032} = 9740 \text{ N};\) \(m_{\text{conrod}} = 64 \text{ kg};\) \(m'_B = m''_B = m'''_B = 1000 \text{ kg};\) \(f_{\text{red}} = 0.008;\) \(D = 0.22 \text{ m};\) \(d = 0.046 \text{ m};\) \(a = 0.27 \text{ m};\) \(b = 0.37 \text{ m};\) \(p = 0.52 \text{ m};\) \(e = 0.21 \text{ m};\) trolley mass with half the mass of the connecting rod \(m_{B_1} = m_{B_2} = m_{B_3} = 1032 \text{ kg};\) \(G = 10124.9 \text{ N}\) by the expressions (13)-(25), taking into account (27).

The nominal rated power of the electric motor is determined by the mean value of the reduced moment of resistance for one crank rotation cycle [22, 23]. According to these data, a 4A series 4A160M6Y3 basic-version asynchronous electric motor with a short-circuit rotor was chosen, having the following parameters: engine rotor synchronous speed \(\omega_o = 104.72 \text{ rad/s};\) engine rotor nominal speed
\( \omega_{\text{nom}} = 102.1 \, \text{rad} / \text{s} \); engine rotor critical speed \( \omega_{\text{crit}} = 94.95 \, \text{rad} / \text{s} \); engine rotor moment of inertia \( J_{\text{tr}} = 0.183 \, \text{kg} \cdot \text{m}^2 \); starting moment \( M_{\text{start}} = 176.3 \, \text{N} \cdot \text{m} \); nominal moment \( M_{\text{nom}} = 146.915 \, \text{N} \cdot \text{m} \); critical moment \( M_{\text{crit}} = 293.83 \, \text{N} \cdot \text{m} \); overload capacity \( \lambda = \frac{M_{\text{crit}}}{M_{\text{nom}}} = 2.0 \); nominal slip \( s_{\text{nom}} = 0.025 \); critical slip \( s_{\text{crit}} = 0.0933 \). In addition, the PF (pin flexible) claw clutch [25] having the nominal transfer torque \( M_{\text{nom}} = 500 \, \text{N} \cdot \text{m} \) and moment of inertia \( J_{\text{cth}} = 0.32 \, \text{kg} \cdot \text{m}^2 \) as well as the C2-400 reducer [25] with the transmission ratio \( u = 9.8 \) and moment of inertia \( J_{\text{red}} = 0.046 \, \text{kg} \cdot \text{m}^2 \) were selected.

The values of the first and second transfer functions of the trolleys are determined from the expressions of the functions of changing the coordinates of the first, second and third forming trolleys in accordance with (Fig. 1, b) [5, 15, 22, 23]

\[
x_{B_1} = r \cdot \cos \phi_2 + l \cdot \cos \beta_1; \quad (35)
\]

\[
x_{B_2} = r \cdot \cos (\phi_2 + \Delta \phi) + l \cdot \cos \beta_2; \quad (36)
\]

\[
x_{B_3} = r \cdot \cos (\phi_2 + 2\Delta \phi) + l \cdot \cos \beta_3. \quad (37)
\]

From the equations (26) we can obtain
\[
\sin \beta_1 = \frac{r}{l} \cdot \sin \varphi_2 \quad \iff \quad \cos \beta_1 = \left[ 1 - \left( \frac{r}{l} \cdot \sin \varphi_2 \right)^2 \right]^{1/2}; \tag{38}
\]

\[
\sin \beta_2 = \frac{r}{l} \cdot \sin (\varphi_2 + \Delta \varphi) \quad \iff \quad \cos \beta_2 = \left[ 1 - \left( \frac{r}{l} \cdot \sin (\varphi_2 + \Delta \varphi) \right)^2 \right]^{1/2}; \tag{39}
\]

\[
\sin \beta_3 = \frac{r}{l} \cdot \sin (\varphi_2 + 2\Delta \varphi) \quad \iff \quad \cos \beta_3 = \left[ 1 - \left( \frac{r}{l} \cdot \sin (\varphi_2 + 2\Delta \varphi) \right)^2 \right]^{1/2}. \tag{40}
\]

The expressions \( \cos \beta_1 \), \( \cos \beta_2 \), and \( \cos \beta_3 \) in the expressions (38)–(40), can be expanded into series by Newton's binomial formula

\[
\left[ 1 - \left( \frac{r}{l} \cdot \sin \varphi_2 \right)^2 \right]^{1/2} = 1 - \frac{1}{2} \left( \frac{r}{l} \cdot \sin \varphi_2 \right)^2 - \frac{1}{8} \left( \frac{r}{l} \cdot \sin \varphi_2 \right)^4 - \ldots; \tag{41}
\]

\[
\left[ 1 - \left( \frac{r}{l} \cdot \sin (\varphi_2 + \Delta \varphi) \right)^2 \right]^{1/2} = 1 - \frac{1}{2} \left( \frac{r}{l} \cdot \sin (\varphi_2 + \Delta \varphi) \right)^2 - \frac{1}{8} \left( \frac{r}{l} \cdot \sin (\varphi_2 + \Delta \varphi) \right)^4 - \ldots; \tag{42}
\]

\[
\left[ 1 - \left( \frac{r}{l} \cdot \sin (\varphi_2 + 2\Delta \varphi) \right)^2 \right]^{1/2} = 1 - \frac{1}{2} \left( \frac{r}{l} \cdot \sin (\varphi_2 + 2\Delta \varphi) \right)^2 - \frac{1}{8} \left( \frac{r}{l} \cdot \sin (\varphi_2 + 2\Delta \varphi) \right)^4 - \ldots. \tag{43}
\]

The ratios \( \frac{r}{l} \) for roller forming units with crank-and-rod drive mechanisms do not exceed \( \frac{1}{3} \), and the series (41)–(43) converge fairly quickly, so, with satisfactory for the practice accuracy, the third and subsequent members of the series (41)–(43) can be thrown away. Then the dependencies (35)–(37) will look as

\[
x_{B_1} = r \cdot \cos \varphi_2 + l \cdot \left[ 1 - \frac{1}{2} \left( \frac{r}{l} \cdot \sin \varphi_2 \right)^2 \right]; \tag{44}
\]

\[
x_{B_2} = r \cdot \cos (\varphi_2 + \Delta \varphi) + l \cdot \left[ 1 - \frac{1}{2} \left( \frac{r}{l} \cdot \sin (\varphi_2 + \Delta \varphi) \right)^2 \right]; \tag{45}
\]

\[
x_{B_3} = r \cdot \cos (\varphi_2 + 2\Delta \varphi) + l \cdot \left[ 1 - \frac{1}{2} \left( \frac{r}{l} \cdot \sin (\varphi_2 + 2\Delta \varphi) \right)^2 \right]. \tag{46}
\]

The values of the first and second transfer functions of all three forming trolleys are determined from the expressions (44)–(46) by the following dependences:

\[
\frac{\partial x_{B_1}}{\partial \varphi_2} = -r \left( \sin \varphi_2 + \frac{r}{2 \cdot l} \cdot \sin 2 \varphi_2 \right); \quad \frac{\partial x_{B_2}}{\partial \varphi_2} = -r \left( \sin (\varphi_2 + \Delta \varphi) + \frac{r}{2 \cdot l} \cdot \sin 2 (\varphi_2 + \Delta \varphi) \right); \tag{47}
\]

\[
\frac{\partial x_{B_3}}{\partial \varphi_2} = -r \left( \sin (\varphi_2 + 2\Delta \varphi) + \frac{r}{2 \cdot l} \cdot \sin 2 (\varphi_2 + 2\Delta \varphi) \right).\]
\[
\frac{d^2 x_{B_1}}{d\phi_2^2} = -r \left( \cos \phi_2 + \frac{r}{l} \cdot \cos 2\phi_2 \right); \quad \frac{d^2 x_{B_2}}{d\phi_2^2} = -r \left( \cos (\phi_2 + \Delta \phi) + \frac{r}{l} \cdot \cos 2(\phi_2 + \Delta \phi) \right);
\]
\[
\frac{d^2 x_{B_3}}{d\phi_2^2} = -r \left( \cos (2\phi_2 + 2\Delta \phi) + \frac{r}{l} \cdot \cos 2(2\phi_2 + 2\Delta \phi) \right).
\]

(48)

As a result of the numerical experiment, it has been established that the optimal value of the rigidity, reduced to the crank rotation axis, of the drive of a roller forming unit with an energy-balanced drive with the above parameters equals \( c = 150000 \, \frac{N \cdot \text{sec}}{m} \). The determination of the optimal value of the drive rigidity was carried out according to the method described in papers [26, 27]. At this rigidity value, minimum loads are observed in the drive clutches. This rigidity value is used in the following calculations.

To study the movement dynamics of a roller forming unit with one taking into account the dissipation during the start-stop movement modes of forming trolleys, the system of equations (34) was supplemented by the drive dissipation value \( k \)
\[
J_{dr} \cdot \dot{\phi}_1 = \frac{2 \cdot M_{crit} \cdot u \cdot n_{dr}}{1 - \dot{\phi}_1 \cdot u} - c \cdot (\phi_1 - \phi_2) - k \cdot (\dot{\phi}_1 - \dot{\phi}_2);
\]
\[
= \frac{s_{crit} \cdot u}{\omega_0} + \frac{s_{crit}}{1 - \dot{\phi}_1 \cdot u}
\]
\[
= \left[ J_{crank} + m_{B_1} \left( \frac{\partial x_{B_1}}{\partial \phi_2} \right)^2 + m_{B_2} \left( \frac{\partial x_{B_1}}{\partial \phi_2} \right)^2 + m_{B_3} \left( \frac{\partial x_{B_2}}{\partial \phi_2} \right)^2 \right] \dot{\phi}_2 + \left[ m_{B_1} \frac{\partial^2 x_{B_1}}{\partial \phi_2^2} + m_{B_2} \frac{\partial^2 x_{B_1}}{\partial \phi_2^2} + m_{B_3} \frac{\partial^2 x_{B_2}}{\partial \phi_2^2} \right] \dot{\phi}_2^2
\]
\[
= c \cdot (\phi_1 - \phi_3) + k \cdot (\dot{\phi}_1 - \dot{\phi}_2) - \frac{1}{\cos \beta_1 - f_{red} \cdot \sin \beta_1} \left[ (R_{011} + R_{012} - G) \cdot f_{red} + F_{011} + F_{012} \right] \cdot r \cdot \sin (\phi_2 + \beta_1) - \frac{1}{\cos \beta_2 - f_{red} \cdot \sin \beta_2} \left[ (R_{021} + R_{022} - G) \cdot f_{red} + F_{021} + F_{022} \right] \cdot r \cdot \sin (\phi_2 + 2\Delta \phi + \beta_2) - \frac{1}{\cos \beta_2 - f_{red} \cdot \sin \beta_2} \left[ (R_{031} + R_{032} - G) \cdot f_{red} + F_{031} + F_{032} \right] \cdot r \cdot \sin (\phi_2 + 2\Delta \phi + \beta_3).
\]

(49)

Having solved the system of equations (49), taking into account the expressions (2), (27), (47) and (48), a graphical dependency of the change of the maximum \( M_{ch} \max \) (Fig. 7) and the mean square \( M_{ch} \) (Fig. 8) torques in the clutch was determined and built, depending on the dissipation coefficient. The analysis of the graphs shows that at all the values of the dissipation coefficient in the range from \( k = 100 \, \frac{N \cdot \text{sec}}{m} \) up to \( k = 15000 \, \frac{N \cdot \text{sec}}{m} \) the maximum and the mean square torques decrease, however, a in the range from \( k = 100 \, \frac{N \cdot \text{sec}}{m} \) up to \( k = 8000 \, \frac{N \cdot \text{sec}}{m} \) they change sharply, and then stabilize smoothly, which almost does not affect the dynamics of the unit movement.

Proceeding from the system of equations (49), the graphs of the torque change in the clutch \( M_{ch} \) (Fig. 9) were calculated and built for the steady movement section, depending on the time at the values of the dissipation coefficient \( k = 2000 \, \frac{N \cdot \text{sec}}{m} \) (position 1) and dissipation coefficient \( k = 10000 \, \frac{N \cdot \text{sec}}{m} \) (position 2). The analy-
sis of these graphs shows that when the drive dissipation coefficient $k = 2000 \frac{N \cdot \text{sec}}{m}$, the torque value in the clutch in the steady movement mode changes in the range from $M_{\text{ch min}} = -1450 \ N \cdot m$ to $M_{\text{ch max}} = 1330 \ N \cdot m$. At the drive dissipation coefficient $k = 10000 \frac{N \cdot \text{sec}}{m}$, the torque value in the clutch in the steady movement mode changes in the range from $M_{\text{ch min}} = -600 \ N \cdot m$ to $M_{\text{ch max}} = 580 \ N \cdot m$.

Fig. 7. Chart of the maximum torque $M_{\text{ch max}}$ changes in the clutch, depending on the dissipation coefficient

Fig. 8. Chart of the mean square torque $M_{\text{ch}}$ change in the clutch, depending on the dissipation coefficient

Fig. 9. Chart of the torque in the clutch $M_{\text{ch}}$, depending on the time

Fig. 10. Graphic dependence $((\phi_1 - \phi_2), (\dot{\phi}_1 - \dot{\phi}_2))$

Fig. 11. Graphic dependence $((\phi_1 - \phi_2), (\dot{\phi}_1 - \dot{\phi}_2))$

Fig. 12. Graphic dependence $((\phi_1 - \phi_2), (\dot{\phi}_1 - \dot{\phi}_2))$
Fig. 10–12 show the graphic dependencies \( \left( \dot{\varphi}_1 - \dot{\varphi}_2, (\varphi_1 - \varphi_2) \right), \left( \ddot{\varphi}_1 - \ddot{\varphi}_2, (\dot{\varphi}_1 - \dot{\varphi}_2) \right) \) at the dissipation coefficient values \( k = 2000 \frac{N \cdot \text{sec}}{m} \) (position 1) and \( k = 10000 \frac{N \cdot \text{sec}}{m} \) (position 2), respectively. The analysis of these graphs shows the identical nature of their changes, but at the drive dissipation coefficient value \( k = 2000 \frac{N \cdot \text{sec}}{m} \) their amplitude along the abscissa and ordinate axes is more important than at \( k = 10000 \frac{N \cdot \text{sec}}{m} \).

Conclusions
1. As a result of the research carried out in order to increase the reliability and durability of a roller forming unit with an energy-balanced drive, the loads in both its elements and drive were calculated, and the function of changing the required torque on the crank drive shaft was determined to ensure the compaction of products from building mixtures, taking into account the rigidity and dissipation of the drive.
2. For a roller forming unit with an energy-balanced drive, the drive rigidity value, reduced to the crank rotation axis, is determined, at which the minimum loads are observed in the drive clutches.
3. The dependence of the torque in the drive clutch on the dissipation coefficient values is analyzed.
4. For a roller molding unit with an energy-balanced drive with the above parameters, the recommended value of the dissipation coefficient is in the range from \( k = 8000 \frac{N \cdot \text{sec}}{m} \) to \( k = 10000 \frac{N \cdot \text{sec}}{m} \). Increasing the dissipation coefficient value will not significantly affect the unit dynamics, but will require more precision in the manufacture of the drive.
5. The results of the work may in the future be useful to refine and improve the existing engineering methods for estimating the drives of roller forming units, both at design stages and in practical use.

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