Enhanced superconducting proximity effect in clean ferromagnetic domain structures

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We investigate the superconducting proximity effect in a clean magnetic structure consisting of two ferromagnetic layered domains with antiparallel magnetizations in contact with a superconductor. Within the quasiclassical Green’s function approach we find that the penetration of the superconducting correlations into the magnetic domains can be enhanced as compared to the corresponding single domain structure. This enhancement depends on an effective exchange field which is determined by the thicknesses and the exchange fields of the two domains. The pair amplitude function oscillates spatially inside each domain with a period inversely proportional to the local exchange field. While the oscillations have a decreasing amplitude with distance inside the domain which is attached to the superconductor, they are enhancing in the other domain and can reach the corresponding normal metal value for a zero effective exchange field. We also find that the corresponding oscillations in the Fermi level proximity density of states as a function of the second domain’s thickness has an growing amplitude over a range which depends on the effective exchange field. Our findings can be explained as the result of cancellation of the exchange fields induced phases gained by an electron inside the two domains with antiparallel magnetizations.

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I. INTRODUCTION

In recent years mesoscopic ferromagnet-superconductor (FS) hybrid structures have been studied very extensively. These structures provide the possibility for the controlled studies of the coexistence of the magnetic and superconducting orderings in the presence of the phase coherent effects. The motivation also has come from the interesting potential applications, such as the proposal of the so called π-SFS Josephson junctions as the solid state qubits. The properties of a normal metal (N) in close contact with an S, find superconducting characteristics due to the penetration of the superconducting correlations (induced order parameter) inside the normal metal. This proximity effect is rather specific in an F metal, in which the interplay between the spin splitting exchange interaction and the singlet superconducting correlations makes the induced superconducting order parameter to have a spatial damped oscillatory behaviour. One of the manifestations of this effect is that the density of states (DOS) of an F layer in contact to S oscillates with varying the thickness and amplitude of the exchange field of the F layer. The oscillations of the proximity DOS in thin F layers with different thicknesses have been observed in experiments.

The proximity effect in F is rather short range as compared to a normal metal case in which the superconducting correlations can penetrate over the normal coherence length \( \xi_N = v_F / T \). Here \( v_F \) is the Fermi velocity and we use the units in which \( h = k_B = 1 \). When the exchange field \( h \) is homogeneous, the induced order parameter contains the singlet component and also a triplet component with the zero projection \( S_z = 0 \) of the total spin. These components can survive in a clean F only over a short distance \( \xi_F = v_F / h \) from the FS interface. In spite of this observation there have been experiments in which the superconducting proximity effect in F are reported which have an amplitude and range much stronger and longer than what the above described theory predicts.

In a pioneering work Bergeret et al. studied the possibility of a long range proximity in ferromagnets with a local inhomogeneous magnetization. They showed that in the presence of a nonhomogeneous spiral magnetization vector close to the FS interface a triplet component of the superconducting order parameter with projections \( S_z = \pm 1 \) can be generated in addition to the singlet and the triplet \( S_z = 0 \) components. These triplet components are not affected by the exchange splitting and can spread over a long distance of the normal coherence length \( \xi_N \). Very recently new experimental evidences of the long range superconducting correlations in ferromagnets have been reported. Sosnin et al. observed the superconducting phase-periodic conductance oscillations of a ferromagnetic Ho wire at a magnetic phase with a conical magnetization vector, in contact with the Al superconducting ring. They found that the phase coherent oscillations sustain for the Ho wire lengths up to 150nm at \( T = 0.27 \text{K} \), which is much longer than \( \xi_F \sim 6 \text{nm} \).

There has been also reports of observing Josephson supercurrent in SFS contacts with stronger CrO ferromagnetic contacts at even longer distances \( 0.3 - 1 \mu \text{m} \), by group of Klapwijk in Delft. In order to explain such long range effects, they refer to the generation of the Bergeret’s triplet component with \( S_z = \pm 1 \), due to the inhomogeneity of the magnetization.

In order to induce the long range triplet components \( S_z = \pm 1 \) in F from a singlet superconducting condensate in S, the nonlocal noncollinear orientation of the magnetization vectors is essential. The triplet correlations will not be induced for a collinear configuration of the magnetization vector. In this paper we study the pos-
sibility of a long range superconducting proximity effect in a ferromagnetic structure with collinear magnetization vectors. We consider a clean FS proximity system in which F consists of two layered domains $F_1$ and $F_2$ whose magnetization vectors are pointed antiparallel to each other and are separated by a sharp domain wall of negligible width (see Fig. 1). Using the quasiclassical Eilenberger equation\textsuperscript{18} we calculate the superconducting pair amplitude function (order parameter) and the proximity DOS in the structure. The pair amplitude function (PAF) spatially oscillates within each domain with a period which is determined by the amplitude of the local exchange field.

We find that while in $F_1$ the amplitude of the PAF oscillations decreases with the distance from S, in $F_2$ it can increase over a distance which depends on an effective exchange field determined by the thicknesses and the exchange fields of $F_1$ and $F_2$. When $F_1$ and $F_2$ have the same thicknesses and exchange fields, the PAF oscillations are amplified over whole of $F_2$ and reaches to the value of the PAF for the corresponding NS structure. We can understand this effect by noting that a quasiparticle travelling through F with a given spin state will gain additional phases in $F_1$ and $F_2$ due to the exchange field splittings. Since the exchange fields are oriented antiparallel to each other the phases gained in $F_1$ and $F_2$ will have opposite signs, leading to a phase cancellation and suppression of the exchange fields effect. The similar effect was found before in Josephson SFS junctions with F having a domain structure\textsuperscript{19,20} We further study the effect of this phase cancellation in the local proximity DOS at the Fermi level. We show that it has an oscillatory behaviour with respect to the exchange field and the thickness of $F_2$. The oscillations are amplified up to the values of the thickness and exchange field which cancels the phase effect of $F_1$ completely. At this point the DOS goes to zero corresponding to the normal metal case. This kind of long range penetration differs from the one introduced by Bergeret et al., as it has an oscillatory behaviour and is superposition of the singlet and triplet components with $S_z = 0$.

The rest of this paper is organized as follows. In the next section we introduce our model of the ferromagnetic domain structure (FDS) in contact with an S and present solutions of the Eilenberger equation for the quasiclassical Green’s functions. In Sec. \textsuperscript{III} we calculate the local subgap DOS and PAF. Sec. \textsuperscript{IV} is devoted to the analysis of the PAF and the proximity DOS in terms of the involved parameters. Finally in Sec. \textsuperscript{V} we present the conclusion.

II. THE MODEL AND BASIC EQUATIONS

The system we study is sketched in Fig. 1. An FDS consisting of two layered domains $F_1$ and $F_2$ is connected to a superconductor (S) in one side and is bounded on the other side by an insulator or vacuum. $F_{1,2}$ have thicknesses $d_1$ and $d_2$, respectively, with magnetization vectors which are oriented antiparallel to each other. We characterize $F_1$ and $F_2$ by mean field exchange splittings $h_1$ and $h_2$, respectively, which are included in the Hamiltonian with different signs for antiparallel orientation of the magnetization vectors. The thicknesses $d_{1,2}$ are larger than the Fermi wave length $\lambda_F$ and smaller than the elastic mean free path $\ell_{imp}$, which allows for a quasiclassical description in the clean limit. We apply the collisionless Eilenberger equation\textsuperscript{18} which in the absence of the spin-flip scatterings, is reduced to the following equation for each of $\sigma = \pm 1$ spin directions:

$$
\nu_F \nabla \hat{G}_\sigma(\omega_n, \nu_F, \mathbf{r}) + \left[ (\omega_n - i\sigma h(\mathbf{r})) \hat{\tau}_3 + \hat{\Delta}(\mathbf{r}), \hat{G}_\sigma(\omega_n, \nu_F, \mathbf{r}) \right] = 0.
$$

The matrix Green’s function for spin $\sigma$ has the form

$$
\hat{G}_\sigma = \left( \begin{array}{cc} g_\sigma & f_\sigma \\ f^*_\sigma & -g_\sigma \end{array} \right),
$$

where $g_\sigma$ and $f_\sigma$ are the normal and anomalous Green’s functions, respectively, which depend on Matsubara’s frequency $\omega_n = (2n + 1)\pi T$ ($T$ is the temperature), the direction of the Fermi velocity $\nu_F$, and the coordinate $\mathbf{r}$. Here $\hat{\Delta}(\mathbf{r}) = \Delta(\mathbf{r}) \hat{\tau}_3$ is the superconducting pair potential matrix (taken to be real) and $\hat{\tau}_i$, ($i = 1, 2, 3$) denote the Pauli matrices. The matrix Green’s function obey the normalization condition $\hat{G}^2_\sigma = 1$. We assume that the exchange field is homogeneous in $F_{1,2}$ with opposite signs, and vanishes inside S. Inside $F_{1,2}$ we take $\Delta(\mathbf{r}) = 0$. We also neglect the selfconsistent variation of the pair potential close to the $F_1S$ interface, thus $\Delta(\mathbf{r})$ is constant inside S.

In the ballistic limit we can solve Eq. (2.1) along a classical electronic trajectory. A typical trajectory is shown in Fig. 1. We parameterize the trajectory by a variable $-\infty < \tau < \infty$. An electron coming from bulk of the S, $\tau = -\infty$, enters FDS at $F_1S$ interface, $\tau = 0$, and subsequently $F_2$ at $F_1F_2$ interface, $\tau = l_{11}/\nu_F$. The electron undergoes a reflection at the insulator and travels back along the returned segment of the trajectory toward the S. In the returned path the variable $\tau$ at $F_1F_2$ and $F_1S$ interfaces is given by $\tau = (l_{11} + l_2)/\nu_F$ and...
\( \tau = \tau_0 = (l_1 + l_2)/v_F \), respectively. After passing through \( F_1 S \) interface, the electron returns to bulk of the S where \( \tau = +\infty \). Here \( l_{11} (l_{12}) \) is the length of the trajectory segment inside \( F_1 \) at the coming (returned) part. The total length of the trajectory inside \( F_{1,2} \) are \( l_1 = l_{11} + l_{12} \) and \( l_2 \), respectively. To take into account the roughnesses at the insulator surface we consider the reflections to be diffusive. Thus the directions of the coming and returned parts of the trajectory are completely uncorrelated.

The solutions of Eq. (2.4) are restricted by the boundary conditions. These conditions determine the behaviour of the matrix Green’s function upon crossing different interfaces in the structure, and also its limiting equilibrium values at bulk of the S. The matrix Green’s function approaches to the bulk value \( G_{\sigma}(\text{bulk}) = (\omega_n \tau_3 + \Delta \tau_1)/\Omega_{\sigma} \), where \( \Omega_{\sigma} = \sqrt{\omega_n^2 + \Delta^2} \), at the beginning and the end of a trajectory \( (\tau = \pm \infty) \), and \( \Delta(T) \) is the superconducting gap at the temperature \( T \). For simplicity we assume that the \( F_1 S \) and \( F_1 F_2 \) interfaces are ideally transparent. For \( F_1 F_2 \) interface our assumption means that the corresponding domain wall is sharp with a negligible width. In this case the boundary conditions are reduced to the continuity of the matrix Green’s function at these interfaces.

By imposing the above mentioned boundary conditions on the solutions of Eq. (2.1), we found that along a trajectory the normal Green’s function \( g_\sigma \) inside FDS \( (0 \leq \tau \leq \tau_0) \) does not depend on \( \tau \). It depends only on the lengths \( l_1 \) and \( l_2 \), and has the form

\[
g_\sigma(l_{11}, l_{12}) = \tanh(\beta_{n\sigma} + \alpha_n),
\]

where \( \beta_{n\sigma} = (\omega_n^{(1)} l_1 + \omega_n^{(2)} l_2)/\nu_F \) and \( \alpha_n = \sinh^{-1}(\omega_n/\Delta), \) with \( \omega_n^{(j)} = \omega_n - i\sigma h_j \). Inside S, the normal Green’s function depends on \( \tau \). For the coming segment of the trajectory \( \tau \leq 0 \), we obtain the following result

\[
g_\sigma(\omega_n, \tau, l_{11}, l_{12}) = \tanh \alpha_n + \frac{\sinh(\beta_{n\sigma})}{\cosh \alpha_n \cdot \cosh(\beta_{n\sigma} + \alpha_n)} \cdot e^{2\Omega_n \tau}.
\]

For the returned segment of the trajectory, \( \tau \geq \tau_0 \), the normal Green’s function is related to the one for incoming segment via \( g_\sigma(\omega_n, \tau_0 - \tau, l_{11}, l_{12}) \).

For the anomalous Green’s function we find the following expressions in different segments of the trajectory

\[
f_\sigma(\omega_n, \tau, l_{11}, l_{12}) = \begin{cases} 
(1 - e^{2\Omega_n \tau})/\cosh \alpha_n + \exp(\beta_{n\sigma} + 2\Omega_n \tau)/\cosh(\beta_{n\sigma} + \alpha_n) & \tau \leq 0 \\
\exp(\beta_{n\sigma} - 2\omega_n^{(1)} \tau)/\cosh(\beta_{n\sigma} + \alpha_n) & 0 \leq \tau \leq l_{11}/\nu_F \\
\exp[\beta_{n\sigma} + 2(\omega_n^{(2)} - \omega_n^{(1)}) l_1/\nu_F - 2\omega_n^{(2)} \tau]/\cosh(\beta_{n\sigma} + \alpha_n) & l_{11}/\nu_F \leq \tau \leq (l_{11} + l_2)/\nu_F \quad (2.5) \\
\exp[-\beta_{n\sigma} + 2\omega_n^{(1)} (\tau_0 - \tau)]/\cosh(\beta_{n\sigma} + \alpha_n) & (l_{11} + l_2)/\nu_F \leq \tau \leq \tau_0 \\
(1 - e^{2\Omega_n (\tau_0 - \tau)})/\cosh \alpha_n + \exp[-\beta_{n\sigma} + 2\Omega_n (\tau_0 - \tau)]/\cosh(\beta_{n\sigma} + \alpha_n) & \tau \geq \tau_0.
\end{cases}
\]

Eqs. (2.4, 2.5) specify the matrix Green’s function Eq. (2.2) in all points of the structure. Using these results we can extract all the interested physical quantities. In the next section we use these results to calculate the local electronic DOS and the profile of the superconducting PAF.

III. PROXIMITY DOS AND THE SUPERCONDUCTING PAIR AMPLITUDE FUNCTION

The DOS is expressed in terms of the normal Green’s function. To find the DOS per trajectory, we have to calculate

\[
N(E, \tau, l_{11}, l_{12}) = \frac{N_0}{2} \sum_{\sigma = \pm 1} \Re g_\sigma(\omega_n = -iE + 0), \quad (3.1)
\]

where \( N_0 \) is the DOS at the Fermi level in the normal state. In the following we will calculate the proximity DOS for energies below the gap \( (|E| \leq \Delta) \). Replacing Eq. (2.4) into Eq. (3.1), for the DOS inside the FDS \( (0 \leq \tau \leq \tau_0) \) we find

\[
N(E, l_{11}, l_{12}) = \frac{N_0}{2} \sum_{\sigma = \pm 1} \sum_{n = -\infty}^{+\infty} \pi \delta(k^{(i)}_\sigma l_1 + k^{(2)}_\sigma l_2 - \Phi - n\pi),
\]

where \( k^{(i)}_\sigma = (E + \sigma h_j)/\nu_F \) and \( \Phi = \arccos(E/\Delta) \) is the Andreev phase. From equation (3.2) we can see the fact that the subgap DOS is a sum of \( \delta \) functions resulting from Andreev bound states of electrons of \( E \geq 0 \) (positive \( n \)’s) and holes of \( E < 0 \) (negative \( n \)’s). The energies also follow from the quasiclassical quantization condition that sets the argument of the \( \delta \)-function equal to zero. For a given spin direction \( \sigma \), the phase shift of the An-
dreeve states caused by the exchange field has two contributions $\sigma h_1 l_1 / v_F$ and $\sigma h_2 l_2 / v_F$, resulting from the phase gained by a quasiparticle inside $F_1$ and $F_2$, respectively. For antiparallel $h_1$ and $h_2$ these phases have opposite signs which can lead to a cancellation of the exchange fields phase effects. As we will see in the following this cancellation is responsible for a long range penetration of the superconducting correlations inside the FDS.

Inside S or $\tau \leq 0$ the subgap DOS is obtained by replacing Eq. 2.4 in Eq. 3.1:

$$N(E, \tau, l_{11}, l_{12}) = \frac{N_0}{2} \pi e^{2\tau \sqrt{\Delta^2 - E^2}} \times \sum_{\sigma = \pm 1} \sum_{n = -\infty}^{+\infty} \delta(k^{(1)}_\sigma l_1 + k^{(2)}_\sigma l_2 - \Phi - n\pi),$$

where for returned part of the trajectory $\tau \geq \tau_0$, we have the relation $N(E, \tau_0 - \tau, l_{11}, l_{12})$.

The total DOS is obtained by averaging Eqs. 3.2, 3.3 over all different possible classical trajectories. This corresponds to an averaging over Fermi velocity directions. To take into account the weak bulk disorder, we include a factor $\exp(-\tau v_F \Lambda)$ in our averaging. Denoting the angles between the directions of the coming and returned segments with the normal to the insulator by $\theta_i$ and $\theta_r$, respectively, the relations $l_{11} = d_1 / \cos \theta_i$, $l_{12} = d_1 / \cos \theta_r$, hold. For diffusive reflections $\theta_i$ and $\theta_r$ are completely uncorrelated, and we make the averaging by an independent integration over these two angles. In this way from Eq. 3.2, inside FDS, we obtain the following result for the total subgap DOS

$$N(E) = \frac{N_0}{2} \sum_{\sigma = \pm 1} \sum_{n = -\infty}^{+\infty} P_{\sigma \sigma}(E) e^{2in \arccos(E/\Delta)} ,$$

where $P_{\sigma \sigma}(E) = E_2(b + 2imA_\sigma)/E_2(b)$ with $b = b_1 + b_2$, $b_1 = d_1 / \text{imp}$ and $E_2(z) = \int_0^\infty du \exp(-zu)/u^2$ is the exponential integral of the second order. Here

$$A_\sigma = k^{(1)}_\sigma d_1 + k^{(2)}_\sigma d_2 = k_\sigma d,$$

with $k_\sigma = (E + \sigma h_{\text{eff}})/v_F$, is an effective phase which is determined by the total thickness of FDS $d = d_1 + d_2$ and an effective exchange field defined as

$$h_{\text{eff}} = \frac{h_1 d_1 + h_2 d_2}{d_1 + d_2}.$$  

From the result given by Eqs. 3.3, 3.4 we conclude that the proximity DOS of the FDS is equivalent to an effective single domain ferromagnet with thickness $d$ and the exchange field $h_{\text{eff}}$. A similar result was obtain before for FDS Josephson junctions.

By a same averaging over Eq. 3.7 the total subgap DOS is obtained inside S

$$N(E, x) = \frac{N_0}{2} \sum_{\sigma = \pm 1} \sum_{n = -\infty}^{+\infty} Q_{\sigma \sigma}(E, x) e^{2in \arccos(E/\Delta)},$$

where we have introduced the coordinate $x$ along the normal to the interfaces with $x < 0$ and $0 \leq x \leq d$ correspond to the points at the S and FDS sides, respectively. Here $Q_{\sigma \sigma}(E, x) = E_2(b + 2imA_\sigma - 2\sqrt{\Delta^2 - E^2}/v_F)E_2(b + 2imA_\sigma)/E_2^2(b)$. While the subgap DOS at a given energy $E$ vanishes in bulk of S, it can have a finite value close to $F_1$S interface, due to the proximity effect. As we found above the proximity DOS in FDS, Eq. 3.7, is constant.

To calculate the PAF we have to do the same averaging over Fermi velocity direction $v_F$ as we did for the proximity DOS, but this time over the anomalous Green’s function given in Eq. 2.5. In fact for PAF normalized in the way that goes to unity at bulk of the S, we have the following relation:

$$F(x, T) = \frac{\lambda \pi T}{\Delta} \sum_{\sigma = \pm 1} \sum_{n = -\infty}^{+\infty} \langle f_\sigma(\sigma, \tau, l_{11}, l_{12}) \rangle,$$

in which $\langle \ldots \rangle$ denotes averaging over $v_F$ and the summation is taken over Matsubara frequencies. Here $\lambda$ is the electron-phonon interaction constant.

**IV. RESULTS AND DISCUSSIONS**

Eqs. 3.1, 3.7 and 3.8 express the proximity subgap DOS and the superconducting order parameter in different regions of the FDS proximity system. In the following we analyse the profile of these quantities as well as their dependence on the effective exchange field Eq. 3.8. We pay our attention to the weak ferromagnetic alloys like Pd$_{1-x}$Ni$_x$ with $x$ of the order of 10%, whose exchange field is estimated to be in the range $h = 5 - 20\text{meV}$. For a Nb superconductor with $\Delta_0 = \Delta(T = 0) = 1.4\text{meV}$ we take $h_{1/2}/\Lambda_0$ to be of order 10.

Let us start with analysing the PAF. In Fig. 2 the PAF is plotted versus the coordinate $x$, for different values of the exchange fields $h_{1,2}$ when the thicknesses $d_1 = d_2 = 0.3\xi_0$ and $T = 0.1T_c$. Here $\xi_0 = v_F/\Delta_0$ is the superconducting coherence length in the clean limit. At the S side close to the F$_1$S interface the PAF is suppressed with an amplitude which is almost independent of the values of $h_{1,2}$. Inside the FDS the PAF depends strongly on the values of $h_{1,2}$ as well as on their relative signs. While for a fully normal case of $h_1 = h_2 = 0$ (dashed curve) the PAF is exponentially decays with $x$ within the normal coherence length $\xi_N = v_F/T$, it has quite different behaviour for nonvanishing $h_{1,2}$. For $h_1 = h_2 = 20\Delta_0$ (dotted curve) the FDS is transformed to a single domain ferromagnet, for which PAF oscillates as a function of $x$ with a period $\xi_F = v_F/h$ and a damping amplitude. Here $\xi_F \ll \xi_N$, since $\Delta_0 \ll h$.

An interesting new effect is appeared when the ferromagnetic part is an FDS with the magnetization vectors oriented opposite to each other. In Fig. 2 the PAF is also plotted for the case $h_1 = -h_2 = 20\Delta_0$ (solid line), and in the inset for the cases $h_1 = 12\Delta_0$, $h_2 = -20\Delta_0$ (solid...
nonzero effective exchange fields \( h_1, h_2 \) and the ferromagnetic domain with \( h_{\text{eff}} = 0 \) at \( T = 0.1T_C \). The thicknesses of domains are taken to be \( d_1 = d_2 = 0.3\xi_0 \). The exchange fields \( h_{1,2} \) are written in the units of \( \Delta_0 \). Inset shows the curves for ferromagnetic domain structure with two nonzero effective exchange fields \( h_{\text{eff}} = -4\Delta_0 \) (solid curve) and \( h_{\text{eff}} = 4\Delta_0 \) (dotted curve). In the solid curve, the maximum of PAF happens at \( x_p = 0.48\xi_0 \).

In these cases the PAF is an oscillatory function in \( F_1 \) and \( F_2 \) with a period which is given by the local exchange fields \( h_1 \) and \( h_2 \). The PAF oscillations is always damping inside \( F_1 \). However the amplitude of the oscillations can enhance in \( F_2 \) over a distance from \( \text{SF}_1 \) interface, depending on the value of \( h_{\text{eff}} \). We obtain that the distance at which the PAF reaches its maximum amplitude is given by \( x_p = (1 - h_{\text{eff}}/h_2)d \). For the case \( h_1 = -h_2 = 20\Delta_0 \) the effective exchange field \( h_{\text{eff}} = 0 \) and hence \( x_p = d \), which implies the enhancement of the oscillations amplitude over whole the thickness of \( F_2 \). In this case the phase cancellation effect is complete and the PAF reaches the normal metal value at \( x = d \).

For the cases shown in the inset of Fig. 2 the phase cancellation occurs only partially. For \( h_1 = 12\Delta_0, h_2 = -20\Delta_0 \), the effective field \( h_{\text{eff}} = -4\Delta_0 \) is negative and \( x_p = 4/5d < d \). In this case the amplitude of the PAF in \( F_2 \) is enhancing for \( x < x_p \) and decreasing for \( x > x_p \). For \( h_1 = 12\Delta_0, h_2 = -20\Delta_0 \), the effective field \( h_{\text{eff}} = 4\Delta_0 \) is positive and \( x_p = 6/5d > d \). For this case the PAF is enhancing in whole of \( F_2 \). Both the curves, in the inset, are reaching to the same value at \( x = d \), which is always smaller than the normal metal value.

We can understand the above mentioned results using the following simple picture. We attribute the proximity effect in FDS to the presence of a macroscopic number of the Cooper pairs which are escaped from the superconducting condensate of the attached S and enter into the ferromagnetic region. For an homogeneous exchange field in \( F \) region the Cooper pairs of electrons with opposite spins gain an additional phase by the exchange splitting induced momentum. The exchange field also tends to break the Cooper pairs by aligning the individual spins of the electrons. These effects, respectively, lead to oscillation and suppression of the Cooper pair wave function, or correspondingly the PAF. In the case of the FDS with antiparallel oriented magnetizations in \( F_1 \) and \( F_2 \), there is another alternative possibility. Two electrons of a Cooper pair with opposite spins can be in different domains. A fraction of these Cooper pairs with electrons whose spins are parallel to the local exchange fields \( h_{1,2} \), can survive from the above mentioned pair breaking. Such phenomena can lead to an enhancement of the Cooper pair wave function, compared to an homogeneous \( F \).

This effect manifests itself also in the dependence of the proximity DOS of the FDS on \( h_2d_2/\nu_F \). In Fig. 3 we plot the DOS at the Fermi level \( (E = 0) \) versus \( h_2d_2/\nu_F \) for \( d_1 = 0.3\xi_0 \) and \( h_1 = 20\Delta_0 \). The DOS oscillates around the normal state value \( N_0 \), with a period of \( \pi/2 \), similar to the DOS oscillations of an homogeneous \( F \). But now in contrast to the homogeneous \( F \) case for which the amplitude of the oscillations is damped with \( h_2d_2/\nu_F \) (see the inset), the amplitude can grow over a range of this quantity. In analogy with the PAF, this range is determined by the effective exchange field \( h_{\text{eff}} \). In this case the DOS amplitude enhances up to \( h_2d_2/\nu_F = -6 \), where it
drops to zero corresponding to the normal metal proximity DOS. At this point $h_{\text{eff}} = 0$, which implies a full cancellation of the exchange fields induced phases. We obtain the same dependence of the Fermi level DOS on $h_{2d}/v_F$, inside S. This is shown in Fig. 4 for different distances from $F_1S$ interface $x$, when $d_1 = 0.3\xi_0$ and $h_1 = 20\Delta_0$. The DOS oscillations with $h_{2d}/v_F$ is the same as the DOS of FDS, but now it is around a value which depends on $x$. Increasing $x$ from zero ($F_1S$ interface) this value decreases monotonically from $N_0$, and approaches zero at the bulk of the S, $x \gtrsim \xi_0$.

V. CONCLUSION

In conclusion we have studied the superconducting proximity effect in a ferromagnetic domain structure which consists of two clean layered domains $F_{1,2}$ separated by a sharp domain wall. Using the quasiclassical Green’s functions approach, we have calculated the pair amplitude function (PAF) and the proximity density of states (DOS). Our results show that in most cases the behaviour of the FDS proximity structure is equivalent to a single domain ferromagnet with thickness $d = d_1 + d_2$ and an effective exchange field $h_{\text{eff}} = (h_1d_1 + h_2d_2)/(d_1 + d_2)$.

We further have found that the exchange splitting compensation has an apparent effect in the oscillatory behaviour of the proximity DOS with the thicknesses of the ferromagnets. Inside the FDS, the Fermi level DOS oscillations around the normal state value has an enhancing amplitude with the thickness of $F_2$, and vanishes at a thickness for which $h_{\text{eff}} = 0$. Increasing further the thickness of $F_2$, the DOS increases back and start to have a damped oscillations around the normal state value. We have explained our results in a simple picture based on surviving of a penetrated Cooper pairs inside FDS, whose two electrons are in different domains with spin directions parallel to the local exchange fields.

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