Assessment of thermodynamic parameters of plasma shock wave

O V Vasileva¹, Yu N Isaev¹, A A Budko¹ and A I Filkov²
¹National Research Tomsk Polytechnic University, 30 Lenina avenue, Tomsk, 634050, Russia
²National Research Tomsk State University, 36 Lenina avenue, Tomsk, 634050, Russia
E-mail: vasileva.o.v@mail.ru

Abstract. The work is devoted to the solution of the one-dimensional equation of hydraulic gas dynamics for the coaxial magneto plasma accelerator by means of Lax-Wendroff modified algorithm with optimum choice of the regularization parameter artificial viscosity. Replacement of the differential equations containing private derivatives is made by finite difference method. Optimum parameter of regularization artificial viscosity is added using the exact known decision of Soda problem. The developed algorithm of thermodynamic parameter calculation in a braking point is proved. Thermodynamic parameters of a shock wave in front of the plasma piston of the coaxial magneto plasma accelerator are calculated on the basis of the offered algorithm. Unstable high-frequency fluctuations are smoothed using modeling and that allows narrowing the ambiguity area. Results of calculation of gas dynamic parameters in a point of braking coincide with literary data. The chart 3 shows the dynamics of change of speed and thermodynamic parameters of a shock wave such as pressure, density and temperature just before the plasma piston.

1. Introduction
The precursor shock wave is formed before head part of plasma substance of the coaxial magneto plasma accelerator [1]. The assessment of thermodynamic parameters in a wake of the shock wave is presented in the article. Some simplifications were introduced – substance is conditionally called the blunted body or the piston, calculation was made for one dimension. Not perturbed gas-air will move on the piston with a piston speed if to pass to system of coordinates connected with the piston. This model is presented when modeling, as movement of two identical waves on a meeting each other [2-4].

2. Technique
The equations of conservation of mass, momentum and energy for the solution of one-dimensional nonstationary gas-dynamic tasks is written in divergent form were used:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial (u \rho)}{\partial x} &= 0, \\
\frac{\partial (u \rho)}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial x} &= 0, \\
\frac{\partial \left( \rho (e + pu^2 / 2) \right)}{\partial t} + \frac{\partial \left( u (\rho e + pu^2 / 2 + p) \right)}{\partial x} &= 0,
\end{align*}
\]

(1)

here \( \rho \) – gas density; \( p \) – gas pressure; \( u \) – velocity of gas; \( e \) – internal energy of the gas; \( t, x \) – time and coordinate.
The same equations (1) which have been written down in a vector form, convenient for numerical realization:

\[
\begin{aligned}
\mathbf{s} &= \begin{pmatrix} \rho \\ u \rho \\ \rho (\varepsilon + \rho u^2 / 2) \end{pmatrix}, \\
\mathbf{f}(\mathbf{s}) &= \begin{pmatrix} u \rho \\ pu^2 + p \\ u (\rho \varepsilon + \rho u^2 / 2 + p) \end{pmatrix}
\end{aligned}
\]

\[
\frac{\partial \mathbf{s}}{\partial t} + \frac{\partial \mathbf{f}(\mathbf{s})}{\partial x} = 0,
\]

where \(\mathbf{s}\) – vector of conservative variables; \(\mathbf{f}(\mathbf{s})\) – flux vector.

Algorithm description:
1. Replacement of the differential equations containing private derivatives by finite differences (2).
2. Addition of optimum parameter of regularization artificial viscosity in the MathCAD wrapper.
3. Choice of the optimum artificial viscosity using the exact known decision (Soda problem).
4. Approbation of the developed algorithm of thermodynamic parameter calculation in a braking point (analytically).
5. Calculation of thermodynamic parameter change dynamic in front of plasma piston of the accelerator.

The modified of Lax-Wendroff algorithm used for the numerical decision of equations system which is that the equations containing private derivatives are replaced by finite differences [5]. Instability appears in finite differences, i.e. influence of high-frequency noise because of strong shock waves existence. The artificial viscosity was added. The optimum parameter of regularization (artificial viscosity) was picked up for ensuring regularization of the decision and suppression of its noise in the on-line mode.

3. Experimental part

The value of artificial viscosity was determined from the misfits-mismatch (figure 1, a). The exact known solution of Soda problem was compared to our algorithm if mismatches were not more than 10%, the regularization coefficient (artificial viscosity) was considered to be optimal, figure 1, b.

Values of pressure, density and temperature in environments were identical before collision of waves, and speeds of waves identical in size, but different in the direction (sign).

An inspection of work of algorithm was carried out on calculation of critical parameters – pressure, density and temperature braking \(p_T, \rho_T, T_T\) for given initial data of the undisturbed gas \(p_0=1/\gamma, \rho_0=1, T_0=1\) (in relative units) and given Mach number \(M_0\). Parameters of inhibition were calculated on known ratios [6, 7]. The Hugoniot-Rankine’s adiabatic (shock wave) used to calculate the pressure, density and temperature of the undisturbed environment through the leap seal:

\[
\begin{aligned}
p_1 &= p_0 \left( \frac{2\gamma}{\gamma + 1} M_0^2 - \frac{\gamma - 1}{\gamma + 1} \right), \\
\rho_1 &= \rho_0 \left( \frac{\gamma + 1}{(\gamma - 1) + 2 / M_0^2} \right), \\
T_1 &= T_0 \left( \frac{\gamma - 1}{\gamma + 1} \right)^2 \left( \frac{2\gamma}{\gamma + 1} - \frac{1}{M_0^2} \right) \left( \frac{2}{\gamma - 1} + M_0^2 \right).
\end{aligned}
\]

(3)
Figure 1. The restored thermodynamic parameters of a shock wave and exhaustion wave: a) with non-optimal viscosity $\mu=10^{-7}$; b) with optimal viscosity $\mu=10^{-6}$.

Formulas for engineering calculations use (3, 4):

$$\rho_i = \frac{1}{4} (5M_0^2 - 1) \rho_0, \quad \rho_T = 4 \left( 1 + \frac{3}{M_0^2} \right)^{-1} \rho_0, \quad T_T = \frac{5}{16} \left( 1 - \frac{1}{M_0^2} \right) (3 + M_0^2) T_0,$$

(5)

The Poisson's adiabatic (rarefaction wave) used to calculate the parameters of inhibition:

$$p_T = \left( 1 + M_1^2 \frac{\gamma - 1}{2} \right)^{-\frac{1}{\gamma - 1}} p_1, \quad \rho_T = \left( 1 + M_1^2 \frac{\gamma - 1}{2} \right)^{\frac{1}{\gamma - 1}} \rho_1, \quad T_T = \left( 1 + M_1^2 \frac{\gamma - 1}{2} \right) T_1,$$

(6)

where $M_1$ – Mach numbers after shock wave, are defined through Mach number in not indignant environment by expression: $M_1 = \left( \frac{2 + (\gamma - 1)M_0^2}{2\gamma M_0^2 - (\gamma - 1)} \right)^{1/2}$.

For engineering calculations use (5, 6):

$$M_1 = \left( \frac{3 + M_0^2}{5M_0^2 - 1} \right)^{1/2}, \quad p_T = \left( 1 + \frac{M_1^2}{3} \right)^{5/3} p_1, \quad \rho_T = \left( 1 + \frac{M_1^2}{3} \right)^{3/2} \rho_1, \quad T_T = \left( 1 + \frac{M_1^2}{3} \right) T_1.$$

(7)

Initial data in relative units: $p_0 = 1/\gamma = 0.6$, $\rho_0 = 1$, $T_0 = 1$.

Results of calculation of methods are given in the table.
The distance between a shock wave and the end of the piston $S$ is brought, using Lunev’s formula and expression (7):

$$\frac{\rho_v}{\rho_0} = \frac{1}{K_\rho} = \frac{4}{1 + \frac{3}{M_0^2}}, \quad K_\rho = \frac{1}{4} \left( 1 + \frac{3}{M_0^2} \right), \quad S = \sqrt{K_\rho \left( 1 + 0.6K_\rho \right)}.$$  \hspace{1cm} (8)

The program gives good results according to table. By means of Lax-Wendroff numerical scheme and the entered artificial viscosity are calculated pressure, density and environment temperature just before the piston on values of speed of the shock wave, received of experiment (8). Experimental values of coordinate of a shock wave $L(t)$ (substance) and speed $v(t)$ are given in figure 2, a. Smoothed values $L(t)$ and $v(t)$ are given in figure 2, b. Values of coordinate of a shock wave approximated by splines and smoothed by "sliding average" filter, values of speed obtained by taking the derivative of a spline curve.

![Figure 2](image-url)

**Figure 2.** Values of coordinate $L(t)$ and speed of a shock wave $v(t)$ respectively: a) experimental values; b) smoothed values.

At calculation of thermodynamic parameters not indignant environment was considered as one-nuclear gas: constant of polytropic $\gamma = 5/3$, pressure $p_i = 105$ Pa, air-density $\rho_i = 1.2$ kg/m$^3$, the air temperature $T = 15°C$, $t = T_1 = 273.15 + 15 = 288.15$ $K$, the speed of sound in not indignant environment $c = 340$ m/s. Dynamics of change of thermodynamic parameters such as the shock wave velocity, pressure, density and temperature, directly in front of a plasma piston, shown in figure 3.

---

**Table.** Results of comparison engineering (E) and program (P) calculations.

| Mach number $M_0$ | $\rho_i$ | $\rho_T$ | $\rho_i$ | $\rho_T$ | $T_i$ | $T_T$ | Calculation |
|-------------------|---------|---------|---------|---------|------|------|------------|
| 1.5               | 1.536   | 2.280   | 1.714   | 2.172   | 1.495| 1.750| E          |
|                   | 1.542   | 2.287   | 1.717   | 2.175   | 1.497| 1.752| P          |
| 2                 | 2.850   | 3.807   | 2.286   | 2.719   | 2.078| 2.333| E          |
|                   | 2.914   | 3.893   | 2.306   | 2.744   | 2.106| 2.364| P          |
| 3                 | 6.600   | 8.204   | 3.000   | 3.418   | 3.667| 4.000| E          |
|                   | 6.557   | 8.151   | 2.995   | 3.413   | 3.649| 3.980| P          |
| 5                 | 1.600   | 22.300  | 3.571   | 3.982   | 8.680| 9.333| E          |
|                   | 18.522  | 22.206  | 3.570   | 3.980   | 8.647| 9.298| P          |
| 10                | 74.850  | 88.397  | 3.883   | 4.291   | 32.123| 34.333| E          |
|                   | 75.050  | 88.643  | 3.884   | 4.291   | 32.210| 34.426| P          |
When plasma flies from an electrode trunk with a supersonic speed at a temperature of 1300 … 3000 K it represents the gas extending in the form of a stream which is called as an underexpanded stream [8]. The theory shows that when gas flies from Laval's nozzle [5] with a supersonic speed originally gas is accelerated, and then slowed down. This process is repeated periodically, but the intensity is less. It is observed on graph (figure 2, figure 3, a).

4. Results
Modeling of parameters of the plasma piston in of front a shock wave of the coaxial magneto plasma device is carried out on the basis of Lax-Wendroff modified algorithm with introduction of artificial viscosity of plasma. Unstable high-frequency fluctuations are smoothed using modeling and that allows narrowing the ambiguity area and to allocate only smooth decisions. Results of calculation of gasdynamic parameters in a point of braking are consistent with literature data.

References
[1] Sivkov A A, Saygash A Ya, Pak A A and Evdokimov A A 2009 Nanotechnics 2 (18) 38-44
[2] Kolesnikov P M 1971 Electrodynamic acceleration of plasma (Moscow: Atom publishing) 388
[3] Landau L D and Lifshits E M 1992 Theoretical physics vol 8 (Moscow: Science) p 664
[4] Morozov A I 2008 Introduction in plasma dynamics (Moscow: Physics mathematics literature) p 613
[5] Zaliznyak V E 2008 Fundamentals of computing physics part 1 (Moscow: Techno sphere) p 224
[6] Zeldovich Ya B and Rayzer Yu P 2008 Physics of shock waves and high-temperature hydrodynamic phenomena (Moscow: Physics mathematics literature) p 656
[7] Chernyak V G and Suetin P E 2006 Mechanics of continuous environments (Moscow: Physics mathematics literature) p 352
[8] Trubnikov B A 1991 Theory of plasma (Moscow: Energy atom publishing) p 464