Gravitational Microlensing and the Galactic Halo

Evalyn I. Gates, Geza Gyuk, and Michael S. Turner

1 Department of Astronomy & Astrophysics
Enrico Fermi Institute, The University of Chicago, Chicago, IL 60637-1433

2 NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory, Batavia, IL 60510-0500

3 Department of Physics
The University of Chicago, Chicago, IL 60637-1433

ABSTRACT

By means of extensive galactic modeling we study the implications of the more than eighty microlensing events that have now been observed for the composition of the dark halo of the Galaxy, as well as for other properties of the Galaxy. We take the Galaxy to be comprised of luminous and dark disk components, a bulge, and a dark halo consisting of both MACHOs and cold dark matter with each component being described by several observationally motivated parameters. We pare down an initial model space of millions of galactic models to viable models, those which are consistent with the observational data, including rotation curve, local projected mass density, and microlensing rates toward the LMC and bulge. On the basis of a conservative, minimal set of observational constraints an all-MACHO halo cannot yet be excluded, although in most viable models of the Galaxy the halo MACHO fraction is between 0% and 30%, consistent with expectations for a universe whose primary component is cold dark matter. An all-MACHO halo is required to be light, and when data on the local escape velocity and satellite-galaxy proper motions, which probe the extent of the dark halo, are taken into account, models which have a high MACHO mass fraction are ruled out. We also explore the possibility that there are no MACHOs in the halo. Finally, we point out several important tests that could definitively exclude an all-MACHO e.g., optical depth for microlensing which is less than $1.5 \times 10^{-7}$ toward the LMC or greater than $3 \times 10^{-6}$ toward the bulge.
1 Introduction

Paczynski’s bold proposal [1] to use microlensing to probe the Galactic halo for dark compact baryonic objects (referred to as MACHOs for Massive Astrophysical Compact Halo Objects) has become a reality. Three collaborations, EROS, OGLE and MACHO have reported over eighty microlensing events towards the Galactic Bulge and eight in the direction of the Large Magellanic Cloud (LMC) [2, 3, 4, 5]. (Preliminary analyses of the second year MACHO data toward the LMC indicate two new events [6].) This detection of microlensing has opened up a new window for exploring the dark halo of our galaxy. In this paper we use the existing data to shed light on the composition of the Galactic halo. Some of our key results have been summarized elsewhere [7, 8]; here we present the full details of our analysis.

There is compelling evidence that spiral galaxies are imbedded in extended non-luminous halos. This includes flat rotation curves measured for almost 1000 spiral galaxies, studies of binary galaxies including our own galaxy and M31, weak gravitational lensing, flaring of neutral hydrogen in the disks and studies of disk warping [9, 10]. While the halo of our own galaxy is in many respects more difficult to study, there is much important data here too; e.g., the rotation curve has been measured between 4 kpc and 18 kpc, the flaring of hydrogen gas has been studied, and the orbital motions of globular clusters and satellite galaxies have been determined [11]. All of these support the hypothesis of an extended dark halo.

Although there is strong evidence for the existence of a Galactic halo, there is little direct information concerning its composition. Since the halos of spiral galaxies are large and show little sign of having undergone dissipation they can be expected to reflect the composition of the Universe as a whole, though perhaps with some biasing (severe in the case of hot dark matter), and thus their composition is of more universal importance. X-ray observations rule out a hot, gaseous halo, and the Hubble Space Telescope has placed tight limits on the contribution of faint stars [12]. The most promising candidates for the halo material are baryons in the form of MACHOs and cold dark matter (CDM) particles.

A baryonic halo invokes the fewest hypotheses: Brown dwarves are known to exist. Further, substantial baryonic dark matter must exist given the robust nucleosynthesis lower bound on $\Omega_B$[13]. However, the success of CDM models in explaining the formation of large-scale structure and the appeal of a flat universe and the nucleosynthesis bound to $\Omega_B$ make a strong case for CDM. If the bulk of the Universe exists in the form of CDM, it is inevitable that our halo contains a significant CDM component [14]. (Even in the most radical scenario for the formation of the Galaxy, infall onto a baryonic seed mass, the amount of CDM accreted is at least equal to the total baryonic mass of the galaxy.) Conclusively demonstrating that the halo is not composed solely of baryons would comprise additional strong support, albeit circumstantial, for a halo comprised of CDM particles.

Gravitational microlensing provides a valuable tool for probing the baryonic contribution to the halo—and of the structure of the Galaxy itself. We shall focus on measurements of the optical depth for microlensing (the probability that a given distant star is being microlensed). The optical depth is determined by the amount and distribution of mass in microlenses along the line of sight. With sufficient lines of sight a sort of galactic tomography could in principle
be performed. At present only a few lines of sight have been probed: several in the direction of the LMC, which probe the halo, and several in the direction of the Bulge, which probe the inner galaxy. The small probability for microlensing, of order $10^{-6}$, means that millions of stars must be monitored. There are many fields of view available in the direction of the Bulge and so tomography of the inner galaxy is a realistic possibility. The situation for probing the halo is not as promising: with available resources only the direction toward the LMC has star fields of sufficiently high density to be useful. However, a space-based search should be able to target the Small Magellanic Cloud (SMC), perhaps some of the larger globular clusters and the closer galaxies such as M31.

Even with precise knowledge of the optical depths toward the LMC and bulge, it would still be difficult to interpret the results because of the large uncertainties in the structure of the Galaxy. As it is, small number statistics for the LMC lead to a range of optical depths further complicating the analysis. Detailed modeling of the Galaxy is essential to drawing reliable conclusions.

Thus, we adopt the following strategy for determining the MACHO composition of our galactic halo. We construct models of the Galaxy with five components: luminous and dark disks, baryonic and CDM halos, and a bulge. We describe each by parameters whose values are allowed to vary over a range motivated by previous modeling and observations. By simultaneously varying all the parameters we construct a very large space of models (more than ten million); from this we find a subspace of viable models consistent with the diverse set of observations that constrain the Galaxy—rotation curve, local projected mass density, measurements of the the amount of luminous matter in the disk and bulge, and measurements of the optical depth for microlensing toward the bulge and the LMC. The distribution of the MACHO halo fraction in these viable models allows us to infer its preferred value. Further, since it is difficult to exclude an all-MACHO halo we focus attention on models where the MACHO fraction is high to see what observations might be crucial in testing this possibility.

Our approach is not the only one that could be pursued. The MACHO Collaboration has focused on a handful of representative galactic models that are meant to span the larger range of possibilities [15]. This allows them to study each model in more detail and address not only the number of microlensing events, but also their durations (which are determined by a combination of the MACHO mass, distance and velocity across the sky). They reach a similar conclusion concerning the MACHO fraction of the halo—it is small in most models of the Galaxy—though they construct a model with an all-MACHO halo. While their approach allows them to address the question of the masses of MACHOs, they do not constrain their models with the totality of observations and thus they cannot address the viability of the models they consider. Indeed, we find their all-MACHO model incompatible with the observational data.

A few caveats should be kept in mind. Because the acceptance of the MACHO and EROS experiments to event duration are limited, the present data address only the halo component made up of MACHOS with masses from about $10^{-7}M_\odot$ to $10^2M_\odot$. It has been argued that objects of mass outside this range are unlikely: MACHOs of mass $10^{-7}M_\odot$ evaporate on a time scale less than the age of the galaxy [16]; Black holes of greater than
would disrupt the globular clusters \cite{17}. However, there remains the possibility that the halo baryons are in the form of either molecular clouds with a fractal distribution \cite{18} or very massive ($m \sim 10^2 M_\odot - 10^4 M_\odot$) black holes \cite{17}. Neither of these options is particularly compelling—molecular clouds should have collapsed by the present and the massive progenitors of such black holes would likely have produced $^4$He or heavy elements—however, they cannot be ruled out conclusively at this time.

In our analysis we also assume that MACHOs are smoothly distributed rather than clumped. If they were strongly clumped the microlensing rate could vary significantly across the sky, which might appear to allow a smaller or larger optical depth toward the LMC for a given MACHO halo fraction. However, if more than one clump were on average expected in a patch of sky the size of the LMC then the optical depth would be again close to its average. Thus, for clumping to significantly affect the optical depth there must be at most a few clumps in the solid angle subtended by the LMC. But if this is the case, then we can expect no more than a few thousand such clumps over the entire sky out to the distance of the LMC. To be a significant fraction of the total halo mass ($\sim$ a few $\times 10^{11} M_\odot$) each clump must be of order few $\times 10^8 M_\odot$, far greater than the mass of a globular cluster. A few thousand of these objects residing in the halo would seem to be ruled out firmly by dynamical constraints based upon the stability of the disk \cite{17}.

Our paper is organized as follows: In the next Section we discuss galactic modeling and the minimal constraints we impose on models. In Section 3 we discuss the implications of microlensing on galactic modeling. We also consider additional reasonable constraints, the local escape velocity and satellite galaxy proper motions, which preclude any model with an all-MACHO halo. In Section 4 we examine more closely the few models that allow an all-MACHO halo (within the minimal constraints) as well as those models that allow a no-MACHO halo. In the final Section we summarize our results and discuss future observations—from measurements of galactic parameters to strategies for the microlensing measurements—that can sharpen conclusions concerning the MACHO fraction of the halo.

2 Galactic Modeling

Modeling of the Galaxy is an established subject—the basic features and dimensions of the Galaxy were determined early in this century—but also one that is still undergoing significant change. Evidence for a dark halo has accumulated over the past two decades (see, e.g. \cite{19}) and over the past five years or so a strong case has for a bar-like, rather than axisymmetric, bulge has developed \cite{20}. Microlensing has the potential for contributing significantly to our understanding of the structure of the Galaxy, both of the composition of the halo and the mass distribution interior to the solar circle.

The current picture of the Galaxy is a barred spiral, consisting of three major components: a central bulge (bar), a disk and a dark halo. The luminous components are a thin, double exponential disk with a vertical scale height of about 0.3 kpc and a radial scale length of about 3.5 kpc, a smaller (few percent of the disk mass) “thick” disk with vertical scale height of about 1 kpc to 1.5 kpc \cite{21}, and a central bulge region, which recent observations indicate
is a triaxial bar \[20\].

Evidence for the dark halo is less direct, but firm nonetheless. It comes from the rotation curve, which is flat out to at least 18 kpc (and probably out to 50 kpc) and the approach of Andromeda and the Galaxy toward one another. At the solar circle about 40% of the centripetal acceleration is provided by the gravitational force of the halo, and beyond that the fraction is even greater. The mass of the Galaxy inferred from the approach of Andromeda is at least a factor of ten greater than that which can be accounted for by stars alone \[1\]. Moreover, the evidence for dark halos associated with spiral galaxies in general is very secure. A recent survey of the rotation curves of more than 900 spiral galaxies indicates flat or slightly rising rotation curves at the limit of the observations, providing strong evidence for their massive dark halos \[22\]. From a completely different direction, Brainerd, Blandford and Smail \[10\] have mapped the dark halos of several spiral galaxies by means of their weak-gravitational lensing of very distant galaxies. Their results indicate that the halos studied have radial extent of at least 100h\(^{-1}\)kpc and total masses in excess of 10\(^{12}\)M\(_\odot\).

The values of the parameters that describe the components of the Galaxy are not well determined; this is especially true for the halo whose presence is only known by its gravitational effects. In addition, there is interplay between the various components as the observations typically constrain the totality of the model, rather than a given component. Modeling uncertainties introduce significant, irreducible uncertainties in the determination of the MACHO content of the halo. In order to understand these uncertainties we explore a very wide range of models that are consistent with all the data that constrain the Galaxy.

We consider two basic models for the bulge, the first following Dwek et al. \[23\] who have utilized DIRBE surface brightness observations to construct a triaxial model for the bulge:

\[
\rho_{\text{BAR}} = \frac{M_0}{8\pi abc} e^{-s^2/2}, \quad s^4 = \left[ \frac{x^2}{a^2} + \frac{y^2}{b^2} \right]^2 + \frac{z^4}{c^4},
\]

where the bulge mass \(M_{\text{Bulge}} = 0.82M_0\), the scale lengths \(a = 1.49\) kpc, \(b = 0.58\) kpc and \(c = 0.40\) kpc, and the long axis is oriented at an angle of about 10° with respect to the line of sight toward the galactic center. While we do not take the axes and inclination angles to be modeling parameters, we later explore the sensitivity of our results to them. We also consider an axisymmetric Kent model for the bulge \[24\]. The rotation curve contribution was calculated in the point mass approximation. At \(r = 5\) kpc this approximation is accurate to better than 10%.

The bulge mass is not well determined, and we consider \(M_{\text{Bulge}} = (1 - 4) \times 10^{10}M_\odot\), in steps of \(0.5 \times 10^{10}M_\odot\). Previous estimates have been in the range \((1 - 2) \times 10^{10}M_\odot\) \[13 24 25\], although a recent study by Blum \[26\] which utilized the tensor virial theorem found a bar mass closer to \(3 \times 10^{10}M_\odot\) (assuming a bar orientation of 20 degrees – smaller (larger) angles of orientation imply larger (smaller) bulge masses).

For the disk component we take the sum of a “fixed,” thin luminous disk whose constituents (bright stars, gas, dust, etc.) are not expected to serve as lenses,

\[
\rho_{\text{LUM}}(r, z) = \frac{\Sigma_{\text{LUM}}}{2h} \exp[-(r - r_0)/r_d]e^{-|z|/h},
\]
with scale length $r_d = 3.5 \text{kpc}$, scale height $h = 0.3 \text{kpc}$, and local projected mass density $\Sigma_{\text{LUM}} = 25 M_\odot \text{pc}^{-2}$ \cite{27}, and a “variable” disk component whose constituents are assumed to be lenses. For the variable component we consider first a distribution similar to that of the luminous matter but with varying scale lengths $r_d = 3.5 \pm 1 \text{kpc}$, and thicknesses $h = 0.3 \text{kpc}$, and $1.5 \text{kpc}$. We also consider a model where the projected mass density varies as the inverse of galactocentric distance (Mestel model) \cite{28}.

The motions of stars perpendicular to the galactic plane have been used to infer the total local projected mass density within a distance of $0.3 \text{kpc} - 1.1 \text{kpc}$ of the galactic plane \cite{29}. The values so determined are between $40 M_\odot \text{pc}^{-2}$ and $85 M_\odot \text{pc}^{-2}$. As a reasonable range we require that $\Sigma_{\text{TOT}}(1 \text{kpc}) = \int_{-1 \text{kpc}}^{1 \text{kpc}} \rho(r_0, z)dz = 35 - 100 M_\odot \text{pc}^{-2}$, which constrains the local projected mass density of the dark disk to be $10 M_\odot \leq \Sigma_{\text{VAR}} \leq 75 M_\odot \text{pc}^{-2}$. (We also include the contribution of the halo to $\Sigma_{\text{TOT}}(1 \text{kpc})$, which for flattened halo models can be significant, about $20 M_\odot \text{pc}^{-2}$, and reduces the mass density that the variable disk can contribute.)

The dark halo is assumed to be comprised of two components, baryonic and non-baryonic, whose distributions are independent. We first assume independent isothermal distributions for MACHOs and cold dark matter with core radii $a_i = 2, 4, 6, \ldots, 18, 20 \text{kpc}$,

$$\rho_{\text{HALO},i} = \frac{a_i^2 + r_0^2}{a_i^2 + R^2} \rho_{0,i}, \quad (3)$$

where $i = \text{MACHO}, \text{CDM}$ and $\rho_{0,i}$ is the local mass density of component $i$.

There are indications from both observations \cite{30, 31} and CDM simulations \cite{32} that halos are significantly flattened. In order to explore the effects of flattening we also consider models with an axis ratio $q = 0.4$ (an E6 halo) for both the baryonic and non-baryonic halos with distributions of the form

$$\rho_{\text{HALO},i} = \frac{a_i^2 + R_0^2}{a_i^2 + R^2 + (z/q)^2} \rho_{0,i}, \quad (4)$$

where $(R, z)$ are cylindrical coordinates. While flattening does affect the local halo density significantly, increasing it by roughly a factor of $1/q$ (see Ref. \cite{8}), it does not affect the halo MACHO fraction significantly.

Finally, we consider the possibility that the MACHOs are not actually in the halo, but instead, due to dissipation, are more centrally concentrated. To describe this we use the distribution in Eq. (3) but with $r^2$ replaced by $r^n$, for $n = 3, 4$ and core radii $a_{\text{MACHO}} = 1, 2 \text{kpc}$. Such a distribution approximates models of a spheroidal component \cite{19, 33} (note, in these models we also explicitly include a Dwek bar).

We construct our models of the Galaxy by letting the parameters describing the various components vary independently. By doing so we consider millions of models. We pare down the space of models to a smaller subset of viable models by requiring that observational constraints be satisfied. The kinematic requirements for our viable models are: circular rotation speed at the solar circle ($r_0 = 8.0 \text{kpc} \pm 1 \text{kpc}$) $v_c = 220 \text{km s}^{-1} \pm 20 \text{km s}^{-1}$; peak-to-trough variation in $v(r)$ between $4 \text{kpc}$ and $18 \text{kpc}$ of less than $14\%$ (flatness constraint \cite{14});
and circular rotation velocity at 50 kpc greater than 150 km s\(^{-1}\) and less than 307 km s\(^{-1}\).

We first impose this minimal set of constraints in order to be as conservative as possible in our conclusions; later we impose additional reasonable, but less secure constraints, involving the rotation curve at large distances and the local escape velocity.

We also impose constraints from microlensing, both toward the bulge and toward the LMC. The optical depth for microlensing a distant star by a foreground star is

\[
\tau = \frac{4\pi G}{c^2} \int_0^\infty ds \rho_s(s) \int_0^s dx \rho_l(x) x(s-x)/s \int_0^\infty ds \rho_s(s),
\]

where \(\rho_s\) is the mass density in source stars, \(\rho_l\) is the mass density in lenses, \(s\) is the distance to the star being lensed, and \(x\) is the distance to the lens. In calculating the optical depth toward the bulge, we consider lensing of bulge stars by disk, bulge and halo objects; for the LMC we consider lensing of LMC stars by halo and disk objects. Except where we are constructing microlensing maps of the bulge (see Section 5) we define the direction of the bulge to be toward Baade’s window, \((b, l) = (-4^\circ, 1^\circ)\).

We adopt the following constraints based upon microlensing data: (a) \(\tau_{\text{BULGE}} \geq 2.0 \times 10^{-6}\) and (b) \(0.2 \times 10^{-7} \leq \tau_{\text{LMC}} \leq 2 \times 10^{-7}\). The bulge constraint is based upon the results of the OGLE Collaboration [3] who find \(\tau_{\text{BULGE}} = (3.3 \pm 1.2) \times 10^{-6}\), as well as the results of the MACHO Collaboration who find \(\tau_{\text{BULGE}} = 3.9^{+1.8}_{-1.2} \times 10^{-6}\) [5]. To be sure, there are still important uncertainties, e.g., detection efficiencies and whether or not the stars being lensed are actually in the bulge; however, we believe this to be a reasonable bound to the optical depth. The optical depth to the LMC is based upon the MACHO Collaboration’s measurement [37], \(\tau_{\text{LMC}} = 0.80 \times 10^{-7}\), as well as the results of the EROS Collaboration [4]. Here too there are uncertainties. In addition to the obvious small number statistics, the events might not all be microlensing. As a reasonable first cut we have taken the 95% Poisson confidence interval based upon the MACHO results.

Bulge microlensing provides a crucial constraint to galactic modeling and eliminates many models. It all but necessitates a bar of mass at least \(2 \times 10^{10} M_\odot\), and, as has been emphasized by others [25], provides additional evidence that the bulge is bar-like. Because of the interplay between the different components of the Galaxy, the bulge microlensing optical depth indirectly constrains the MACHO fraction of the halo. On the other hand, LMC microlensing only constrains the MACHO fraction of the halo.

3 Implications of Microlensing for Galactic Modeling

In this Section we discuss the characteristics of the viable models, focussing particularly on the composition of the halo (MACHO fraction and local halo mass density), but also paying attention to the other parameters in our galactic models. We display our results in histograms of the number of viable models as a function of various modeling and derived parameters. These plots resemble likelihood functions that are marginalized with respect to those parameters. They are in fact not likelihood distributions; because the most important
uncertainties in modeling the Galaxy are systematic in character, e.g., the model of the Galaxy itself, the rotation curve, the shape of the halo, and even the galactocentric distance and local speed of rotation, we resisted the urge to carry out a more rigorous statistical analysis which might have conveyed a false level of statistical significance.

We first discuss the features of the models that satisfy our minimal constraints and then go on to discuss the models that survive when we impose additional constraints that better serve to define the extent of the dark halo (escape velocity and rotation curve at large distances as defined by satellite galaxy proper motions). In these discussions we rely heavily upon histograms which detail the characteristics of the acceptable galactic models. However, before we do, let us summarize our main results:

- In most viable models the halo MACHO fraction is between 0% and 30%, though when only the minimal constraints are applied there are models with MACHO fraction greater than 60%. When the additional constraints are applied there are no viable models with halo MACHO fraction greater than 60% (see Fig. 1). (Halo MACHO fraction $f_B$ is defined to be the MACHO mass fraction of the halo interior to 50 kpc).

- In viable models the local MACHO mass density is sharply peaked around $10^{-25}$ g cm$^{-3}$ (see Fig. 2) and the total MACHO mass (within 50 kpc) is peaked around $1 \times 10^{11} M_{\odot}$.

- In viable models with a flattened halo the total local halo mass density is between about $4 \times 10^{-25}$ g cm$^{-3}$ and $1.5 \times 10^{-24}$ g cm$^{-3}$ (see Fig. 2). Flattening increases the local halo mass density by factor of order the axis ratio.

- The bulge microlensing constraint precludes any model with a Kent (axisymmetric) bulge, and the bar mass in most viable models is between $2 \times 10^{10} M_{\odot}$ and $3 \times 10^{10} M_{\odot}$. The necessity of a relatively heavy galactic bar plays an important role constraining the halo MACHO fraction to a small fraction.

3.1 Minimal constraints

There are several features that are generic to most models that satisfy the minimal set of constraints (see Figs. 3-8). The most important of these is that independent of almost all the model parameters, the peak of the MACHO fraction occurs for $f_B \lesssim 20\%$ (the only exception being a spherical halo model with a very small core radius for the non-baryonic component, which peaks at $f_B \sim 30\%$). While the range of MACHO fraction extends from 0% to 90%, most models have $f_B < 30\%$. (We discuss the handful of high MACHO-fraction models in the next Section). No model with a thick dark disk (either exponential or $1/r$ profile) and $f_B > 60\%$ survives our constraints, and the distribution for these thick disk models peaks at $f_B \sim 0$. The absence of MACHOs in the halo is allowed because a thick disk can contribute up to $0.5 \times 10^{-7}$ to the optical depth toward the LMC [7], which allows the LMC microlensing constraint to be satisfied without recourse to MACHOs in the halo.

The bulge mass in most models is between $2 \times 10^{10} M_{\odot}$ and $3 \times 10^{10} M_{\odot}$, which is consistent with estimates from recent efforts to model the bar [25, 26]. Models with a Kent bulge do
not provide sufficient microlensing toward the bulge, and as pointed out in previous work by the authors and others [7, 38], the disk cannot provide more than about $1 \times 10^{-6}$ to the optical depth toward the bulge. A heavy bar is necessary to obtain optical depths to the galactic bulge in excess of $3 \times 10^{-6}$, as currently suggested by the experimental data. The distribution of galactocentric distance ($r_0$) is somewhat dependent on the disk model, with thick disk models generally favoring smaller $r_0$. The distribution for the local circular velocity is relatively broad, but it is generally peaked at the low end of the range, around $210 \text{ km s}^{-1} - 220 \text{ km s}^{-1}$. The trend for all dark disk models is toward larger scale length ($r_d$). The value of the disk surface density depends on the disk model, although lighter disks are favored in all cases (i.e., little mass in the dark disk).

The distribution of optical depths toward the LMC and the bulge are shown in Figs. 3-8. In general, $\tau_{\text{LMC}}$ is relatively flat. This is easily understood: for a given model, the microlensing optical depth is sensitive only to the MACHO fraction, which is unaffected by the kinematic cuts. For thick-disk models (both exponential and $1/r$) there is also a relatively large bin at the smallest allowed value of $\tau_{\text{LMC}}$. This is due to additional allowed models with very small halo MACHO fraction where the LMC lensing is done by the disk (lensing toward the LMC is negligible in thin-disk models [7]). The bulge optical depth is somewhat peaked toward the low end of the acceptable range, mainly due to the difficulty of achieving $\tau_{\text{bulge}} > 3 \times 10^{-6}$.

The local MACHO mass density peaks at about $10^{-25} \text{ g cm}^{-3}$ in all models and the mass of MACHOs in the halo peaks at about $1 \times 10^{11} M_\odot$. However, the total local halo mass density is more dependent on the halo model, in particular on whether or not the halo is flattened; see Fig. 2. (Since the MACHO fraction of the halo is small, this also applies to the local mass density of CDM particles.) Flattening of the halo, for which there is good evidence, increases the local halo density by a factor of order the axis ratio $q$. In a flattened halo model, the local halo density is larger by a factor

$$\frac{\rho_{0,\text{flattened}}}{\rho_{0,\text{spherical}}} = \frac{\sqrt{1 - q^2}}{q \sin^{-1}(\sqrt{1 - q^2})},$$

relative to a spherical halo model with the same asymptotic rotation velocity and core radius (for the E6 halo, this factor is about 2). This has important implications for the direct detection of non-baryonic dark matter, and is discussed in detail elsewhere [8]. However, our results for the MACHO fraction of the halo are essentially independent of the amount of halo flattening as can be seen in figures 3-11. Both the total mass of the halo and the MACHO halo mass shift slightly toward smaller values in a flattened halo model.

### 3.2 Additional constraints

The models we have considered viable thus far have been subject to a very minimal set of constraints – that is, we have tried to be as generous as possible in admitting models, probably too generous. There are additional constraints which bear on the size and extent of the dark halo. They are especially crucial to the issue of the MACHO fraction of the
halo: Microlensing toward the LMC closely constrains the mass of the MACHOs in the halo, and therefore the halo MACHO fraction depends sensitively upon the total halo mass. The models with high MACHO fraction are characterized by light halos; the additional constraints place a stringent lower bound to the halo mass and thus upper bound to the MACHO fraction, eliminating all models with MACHO fraction greater than 60%.

The first additional constraint on the galactic potential that we consider comes from the local escape velocity. Based upon the velocity of the fast moving stars Leonard and Tremaine [39] have determined that the local escape velocity lies in the range $450 \, \text{km s}^{-1} < v_{\text{ESC}} < 650 \, \text{km s}^{-1}$ (with 90% confidence level), with a stronger lower limit of $430 \, \text{km s}^{-1}$. Kochanek [40] obtains a slightly higher range of $489 \, \text{km s}^{-1} < v_{\text{ESC}} < 730 \, \text{km s}^{-1}$. Based on these values we adopt $v_{\text{ESC}} > 450 \, \text{km s}^{-1}$.

Next we consider the information about the galactic rotation curve at large distances ($50 \, \text{kpc} - 100 \, \text{kpc}$) based upon the proper motions of satellites of the Milky Way. Recently Jones, Klemola and Lin [41] have measured the proper motion of the LMC. They find a total galactocentric transverse velocity of $215 \pm 48 \, \text{km s}^{-1}$. Proper motions for Pal 3 [42] (galactocentric distance 79 kpc) and Sculptor [43] (galactocentric distance 95 kpc) have also been measured, yielding $252 \, \text{km s}^{-1} \pm 85 \, \text{km s}^{-1}$ and $199 \, \text{km s}^{-1} \pm 58 \, \text{km s}^{-1}$ respectively. Assuming that these satellite galaxies are bound to our Galaxy, they provide strong evidence that the galactic halo is massive and extended.

Finally, a study of the rotation curves of over 900 spiral galaxies [22] indicates that for all of these galaxies the rotation curves are flat, rising or only gently falling at twice the optical radius ($r_{\text{opt}} \equiv 3.2 r_d$), depending on the luminosity. Based on rotation curves of galaxies similar to the Galaxy ($L/L_\star = 1.4 h^2, r_d \approx 3.5 \, \text{kpc}$), the rotation velocity at $2 r_{\text{opt}} \sim 22 \, \text{kpc}$ should be within a few percent of $v_c$, and further, at a galactocentric distance of $50 \, \text{kpc}$ the rotation velocity should be at least $200 \, \text{km s}^{-1}$. Combining this with the satellite proper motions we require $180 \, \text{km s}^{-1} \leq v_c(50 \, \text{kpc}) \leq 280 \, \text{km s}^{-1}$.

We impose these additional constraints on our “canonical” model—E6 halo, thin, double-exponential disk, and Dwek bar—with all other parameters allowed to vary as before. The results are displayed in Fig. 9. The most striking consequence of the additional kinematic constraints is the exclusion of all models with a baryon fraction greater than 60%, and essentially all models with a baryon fraction greater than 50%. It is worth noting that this result follows from either constraint alone. That is, models with an all-MACHO halo are characterized by both $v_{\text{ESC}} < 450 \, \text{km s}^{-1}$ and $v_c(50 \, \text{kpc}) < 180 \, \text{km s}^{-1}$. The results for a spherical halo are similar.

The halo MACHO fraction for these models is strongly peaked around 10% to 20%. This result is independent of the bar mass, local disk surface mass density, disk scale length and our galactocentric distance. It is also insensitive to the optical depth for microlensing toward the galactic bulge. It is, as one would expect, sensitive to the optical depth for microlensing toward the LMC.

These additional constraints also narrow the estimate for the total mass of the Galaxy

\footnote{The escape velocity from an isothermal halo increases logarithmically; to compute $v_{\text{ESC}}$, we truncate the halo at a distance of 100 kpc.}
(within 50 kpc) to \((5 \pm 1) \times 10^{11} M_\odot\). This is consistent with the value obtained recently by Kochanek [10], who used similar constraints on the extent of the dark halo, although a much more restricted set of galactic models.

4 Very MACHO and No-MACHO Halos

4.1 Very-MACHO halos

In Figs. 3 to 8 the characteristics of galactic models with MACHO fraction \(f_B \geq 0.75\) are shown as dotted lines. (It should be noted that the histograms for these models with very-MACHO halos have been multiplied by a factor of 50 relative to the other models.) The crucial common feature of very-MACHO models is a light halo (total mass less than \(4 \times 10^{11} M_\odot\)). Only thin-disk models allow \(f_B \geq 0.75\). The reason for this illustrates how the bulge microlensing constraint also influences other aspects of the galactic model. Models with an exponential thick disk require a heavier bar to account for microlensing toward the bulge: A thick disk contributes far less to microlensing toward the bulge than does a thin disk [4]. On the other hand, the rotation curve from our position outward requires a heavy disk for support if the halo is light. Therein lies the rub: the inner part of the rotation curve cannot tolerate both a heavy disk and a heavy bar.

Because very-MACHO models are characterized by light halos they are also characterized by: (i) a small local rotation speed, \(v_c \leq 215 \text{ km s}^{-1}\); (ii) large (total) local surface mass density, \(\Sigma_0 \geq 60 M_\odot \text{ pc}^{-2}\); (iii) light bar, \(M_{\text{Bulge}} = 2.0 \times 10^{10} M_\odot\) in most of these models; (iv) a rotation curve that falls to a small asymptotic value, \(v(50 \text{ kpc}) \lesssim 180 \text{ km s}^{-1}\); and (v) a local escape velocity that is less than \(420 \text{ km s}^{-1}\). Further, because the bar is the most efficient source of lensing, a lighter bar results in a low optical depth toward the bulge, \(\tau_{\text{bulge}} \approx 2 \times 10^{-6}\). Finally, to avoid having a halo that is too light, these models are necessarily characterized by high optical depth toward the LMC, \(\tau_{\text{LMC}} \sim 2 \times 10^{-7}\).

4.2 No-MACHO halos

Because the optical depth for microlensing toward the LMC is so much smaller than it would be for an all-MACHO halo one should also consider the possibility that there are no MACHOs in the halo. Further, the optical depth toward the LMC is based on only three events seen by the MACHO Collaboration and two by the EROS Collaboration. Not only are the numbers small, so that Poisson fluctuations alone are large, but it is not impossible that some of the events are not even due to microlensing. In that regard, the MACHO Collaboration refers to their events as two candidates and one microlensing event (the amplitude 7 event) [15], while the EROS Collaboration has established that one of their events involves a binary star (of period much shorter than the event duration) [44]. Thus, the actual optical depth could be quite small.

If the optical depth for microlensing toward the LMC is much less than \(10^{-7}\) (the current central value), it could be explained by a combination of microlensing of LMC stars by LMC
stars and a thick disk component (a thick disk can contribute up to $0.5 \times 10^{-7}$, though it should be noted that a thick disk cannot also account for the large microlensing rate toward the bulge).

Another possibility is that the MACHOs responsible for microlensing toward the LMC are in a more centrally condensed component, e.g., the spheroid. In Figs. 10 and 11 we show the characteristics of models with a no-MACHO halo and MACHO spheroid with density profiles $r^{-n}$ ($n = 3, 4$) and core radii $b = 1, 2$ kpc. The viable models are characterized by: (i) very small MACHO fraction, spheroid mass/halo mass less than 0.2; (ii) very low optical depth, $\tau_{\text{LMC}} \lesssim 5 \times 10^{-8}$; and (iii) spheroid mass which peaks at $5 \times 10^{10} M_\odot$ for $n = 3$ and $3 \times 10^{10} M_\odot$ for $n = 4$, consistent with independent dynamical measurements.

5 Discussion and Summary

5.1 Microlensing and the bulge

The number of microlensing events detected in the direction of the galactic bulge is currently more than eighty and will continue to grow. As the statistics improve, the optical depth along different lines of sight toward the bulge can be determined, allowing tomography of the inner galaxy, in turn providing information about the shape, orientation, and mass of the bulge, and indirectly about the Galaxy as a whole.

Already the unexpectedly high optical depth towards the galactic center provides further evidence that the bulge is more bar-like than axisymmetric. Much more can be learned. In Fig. 12 we present microlensing maps of the bulge for several different models. The first panel shows contours of constant $\tau_{\text{bulge}}$ for a massive ($M_{\text{Bulge}} = 4.0 \times 10^{10} M_\odot$) Kent bulge with a light disk. Even with this very high bulge mass, the microlensing rates are not high enough to account for the observations. The second panel shows a microlensing map for a slightly less massive ($M_{\text{Bulge}} = 3.0 \times 10^{10} M_\odot$) Dwek bar oriented almost directly towards us, $\theta = 10^\circ$. Despite the lower mass which makes this model more likely to pass kinematic cuts, the optical depths are much higher, with bulge-bulge events clearly dominating. A slight asymmetry in galactic longitude is apparent, but it may be too small to be detected. The third panel shows a microlensing map for the same mass bar, but oriented at 45$^\circ$. The optical depths for microlensing are much smaller, the contours are considerably less steep and more elongated along the longitude axis. For comparison, the the effect of a heavier disk is shown in panels four and five, for a models similar to those in panels two and three. The additional microlensing provided by the disk results in higher optical depths and an elongation of the microlensing contours along the direction of galactic longitude.

While the orientation of the bar provides a strong signature in the microlensing maps, the overall rate is an important constraint by itself. The models shown in panels three and five with an orientation of 45$^\circ$ are already excluded by our constraint, $\tau_{\text{bulge}} \geq 2.0 \times 10^{-6}$. Figure 13a shows the number of viable models with a thin disk and flattened halo as a function of bar orientation. Clearly a bar pointing towards us is preferred, with bar orientations of greater than 30$^\circ$ almost entirely excluded.
The modeling we have described here has already indicated the necessity of a relatively massive bar, \((2 - 3) \times 10^{10} M_\odot\), even in the case of a bar oriented at \(10^\circ\) from our line of sight. This, together with the results shown in Fig. 13 suggest that the bar has a mass of \((2 - 3) \times 10^{10} M_\odot\) and is oriented at an angle of less than \(20 - 30^\circ\) from our line of sight. As discussed earlier, considering rate alone there is a degeneracy between bar mass and orientation: lower mass can be traded for smaller angle. As can be seen in Fig. 12 mapping can break this degeneracy.

5.2 Future directions

While the results of the microlensing experiments to date seem to strongly indicate that the primary component of the halo is not MACHOs, as we have emphasized here it is not yet possible to exclude this hypothesis with any certainty. Since the question is of such importance, it is worth considering future measurements that could lead to more definite conclusions. Based upon our extensive modeling we can identify a number of key measurements.

Recall that the models with all-MACHO halos had a number of distinctive features: (i) large optical depth toward the LMC, \(\tau_{\text{LMC}} \simeq 2 \times 10^{-7}\); (ii) small optical depth toward the bulge, \(\tau_{\text{bulge}} \simeq 2 \times 10^{-6}\); (iii) a small local rotation speed, \(v_c \leq 215 \text{ km s}^{-1}\); (iv) large (total) local surface mass density, \(\Sigma_0 \geq 60 M_\odot \text{ pc}^{-2}\); (v) light bar, \(M_B \simeq 2.0 \times 10^{10} M_\odot\); (vi) a rotation curve that falls to a small asymptotic value, \(v(50 \text{ kpc}) \lesssim 180 \text{ km s}^{-1}\); and (vii) a local escape velocity that is less than 420 km s\(^{-1}\).

What then are the prospects for falsifying the all-MACHO halo hypothesis? Because \(\tau_{\text{LMC}}\) is apparently so small, it may be difficult to accumulate sufficient statistics over the next few years to exclude the possibility that \(\tau_{\text{LMC}}\) is as large as \(2 \times 10^{-7}\). It may be more promising to establish that \(\tau_{\text{bulge}}\) is greater than \(2 \times 10^{-6}\), due to the higher microlensing rate toward the bulge, Or, other observations could establish that the mass of the bulge is in excess of \(2 \times 10^{10} M_\odot\), which cannot be tolerated in models where the halo is entirely comprised of MACHOs.

Several characteristics of an all-MACHO halo involve parameters of the galactic model and the galactic rotation curve. Improvements here could be equally decisive. The study of the proper motions of satellite galaxies will further constrain the rotation curve at large distances, and the recent observation of a dwarf galaxy at a galactocentric distance of 16 kpc [16] presents yet another opportunity. Continued efforts to deduce the local escape velocity might well rule out all-MACHO scenarios. A more precise determination of the local circular velocity and position would also help limit the range of viable models. Precision measurements of the pulse arrival times for the binary pulsar PSR 1913+16 are reaching the level of precision where the effects of solar acceleration, which depends upon both \(r_0\) and \(v_c\), can be accurately determined [17].

Equally interesting is testing the hypothesis of a no-MACHO halo. Measurements of the event duration and light-curve distortions due to parallax effects could help discriminate between MACHOs in the halo and disk and/or LMC. Likewise, the distribution of events in the LMC provides an important test of whether or not the lenses are part of the LMC.
It is probably more difficult to determine whether or not the lenses are in the spheroid (as opposed to the halo).

5.3 Summary

Microlensing has already proven its utility as a probe of the structure of the Galaxy. Based upon the existing data—which is likely to represent but a small fraction of what will be available over the next few years—and the extensive modeling discussed here important conclusions can already be drawn.

First and foremost, the MACHO fraction of the galactic halo in most viable models of the Galaxy is small—between 0% and 30% (see Fig. 1). The few models with a halo MACHO fraction of greater than 60% are characterized by a very light halo. When additional reasonable constraints that define the minimal extent of the halo (such as local escape velocity and proper motions of satellite galaxies) are taken into account none of these models remain viable. The apparent elimination of the promising baryonic candidate for the dark matter halo of our own galaxy further enhances the case for cold dark matter and provides further impetus for the efforts to directly detect cold dark matter particles (e.g., neutralinos and axions).

Second, it is not impossible that the halo of the Galaxy contains no MACHOs. If the optical depth for microlensing toward the LMC is at the low end of the credible range, say less than about $0.5 \times 10^{-7}$, the microlensing events seen could be due to microlensing by objects in the disk and/or LMC. Or, it could be that the lenses are not halo objects, but rather exist in a more centrally condensed component of the Galaxy (e.g., the spheroid).

Third, based upon our modeling we conclude that the plausible range for the local density of dark halo material is between $6 \times 10^{-25}$ g cm$^{-3}$ and $13 \times 10^{-25}$ g cm$^{-3}$, most which is not in the form of MACHOs (see Fig. 2). This estimate is about a factor of two higher than previous estimates because we have taken the flattening of the halo into account \[8\].

Fourth, it is not possible to account for the large microlensing rate in the direction of the bulge with an axisymmetric bulge; a bar of mass $(2 - 3) \times 10^{10} M_\odot$ is required to meet our minimal constraint $\tau_{\text{bulge}} \geq 2 \times 10^{-6}$.

Finally, while we are not able to rule out an all-MACHO halo with certainty, our modeling points to future measurements that could be decisive. The very few models with very-MACHO halos ($f_B \geq 0.75$) that survive our minimal set of constraints have distinctive features that make allow them to be falsified: $\tau_{\text{LMC}} \approx 2 \times 10^{-7}$; $\tau_{\text{bulge}} \approx 2 \times 10^{-6}$; $v_c \leq 215$ km s$^{-1}$; $\Sigma_0 \geq 60 M_\odot$ pc$^{-2}$; $M_B \approx 2.0 \times 10^{10} M_\odot$; $v_c(50 \text{kpc}) \lesssim 180$ km s$^{-1}$; $v_{\text{esc}} \lesssim 420$ km s$^{-1}$.

Acknowledgments

We thank C. Alcock, K. Cudworth and D. Bennett for helpful conversations. This work was supported in part by the DOE (at Chicago and Fermilab) and the NASA (at Fermilab through grant NAG 5-2788).
References

[1] B. Paczynski, *Astrophys. J.* 304, 1 (1986).

[2] E. Aubourg et al., *Nature* 365, 623 (1993).

[3] A. Udalski et al., *Astrophys. J.* 426, L69 (1994); *ibid*, in press (1994); *Acta Astron.* 43, 289 (1993); *ibid* 44, 165 (1994); *ibid* 44, 227 (1994).

[4] C. Alcock et al., *Nature* 365, 621 (1993).

[5] K. Griest et. al., (MACHO collaboration), To appear in the proceedings of the Pas- cos/Hopkins Symposium, Baltimore, Maryland, World Scientific, 1995; C. Alcock et al., *Astrophys. J.* 445, 133 (1995).

[6] C. Alcock, private communication

[7] E. Gates, G. Gyuk, and M.S. Turner, *Phys. Rev. Lett.* 74, 3724 (1995).

[8] E. Gates, G. Gyuk, and M.S. Turner, *Astrophys. J. Lett.* 449, L000 (1995).

[9] some refs on the existence of a halo: Rubin, V. 1993, *Proc. Natl. Acad. Sci. USA* 90, 4814; Knapp, J., & Kormendy, J., eds. 1987, *Dark Matter in the Universe* (IAU Symposium #117), (Dordrecht: Reidel); Zaritsky, D. & White S. D. M. 1994, ApJ, 435, 599; Fich, M., & Tremaine, S. 1991, *Annu. Rev. Astron. Astrophys.* 29, 409; Persic, M. & Salucci P. 1995, *Astrophys. J. Supp.*, in press; Olling, R.P., to appear in *Astron. J.* (1995); Charlton & Salpeter, *Astrophys. J.* 375, 517 (1991); Sparke & Casertano, *Mon. Not. R. Astron. Soc.* 234, 873 (1988).

[10] Brainerd, T. G., Blandford, R. D. & Smail I. 1995, submitted to *Astrophys. J.*

[11] M. Fich, L. Blitz & A. A. Stark, *Astrophys. J.* 342, 272 (1989); Norris & Hawkins, *Astrophys. J.* 380, 104 (1991); M.R.Merrifield, *Astron. J.* 103 (5), 1552 (1992).

[12] J. Bahcall et al., *Astrophys. J. Lett.* 435, L51 (1994).

[13] C. Copi, D. N. Schramm, and M. S. Turner, *Science* 267, 192 (1995).

[14] E. Gates and M. S. Turner, *Phys. Rev. Lett.* 72, 2520 (1994).

[15] C. Alcock et al., submitted to *Astrophys. J.* (1995).

[16] A. De Rujula, Ph. Jetzer, and E. Masso, *Astron. Astrophys.* 254, 99 (1992).

[17] B. Carr, *Annu. Rev. Astron. Astrophys.* 32, 531 (1994).

[18] F. De Paolis, G. Ingrosso, Ph. Jetzer, and M. Roncadelli, astro-ph/9504080 (1995).
[19] J. N. Bahcall, M. Schmidt, and R.M. Soneira, *Astrophys. J.* **265**, 730 (1983); J. A. R. Caldwell and J. P. Ostriker, *ibid* **251**, 61 (1981).

[20] L. Blitz, in *Back to the Galaxy* (AIP Conference Proceedings 278), eds. S. S. Holt and F. Verter, (AIP, New York, 1993), p. 98 (1993); J. Binney, *ibid*, p. 87 (1993); K. Z. Stanek et al., astro-ph/9508008 (1995).

[21] G. Gilmore, R. F. G. Wyse, and K. Kuijken, *Ann. Rev. Astron. Astrophys.* **27**, 555 (1989).

[22] M. Persic, P. Salucci and F. Stel, *Mon. Not. R. Astron. Soc.* **000**, 000 (1995); Persic, M. & Salucci P. 1995, Ap. J. Supplement, in press.

[23] E. Dwek et al., *Astrophys. J.* **445**, 000 (1995).

[24] S. M. Kent, *Astrophys. J.* **387**, 181 (1992).

[25] H. Zhao, D.N. Spergel and R.M. Rich, *Astrophys. J.* **440**, L13 (1996).

[26] Blum, R., *Astrophys. J.* **444**, L89 (1995).

[27] See e.g., J.N. Bahcall, *Astrophys. J.* **276**, 169 (1984); K. Kuijken and G. Gilmore, *ibid* **376**, L9 (1991). Taking a smaller value for $\Sigma_{\text{LUM}}$ would allow for larger $\Sigma_{\text{VAR}}$ and hence larger $\tau_{\text{BULGE}}$; however, taking $\Sigma_{\text{LUM}} = 0$ only increases $\tau_{\text{BULGE}}$ by $3 \times 10^{-6}$.

[28] Such a model (referred to as a Mestel disk [48]) produces a flat rotation curve in the plane of the galaxy; however, in order to account for a rotation velocity of 220 km s$^{-1}$, a local surface mass density of 220$M_{\odot}$ pc$^{-2}$ is required. P. Sackett has recently revisited these models, because a thin Mestel disk ($h \sim 0.3$ kpc) whose local surface density is about 220$M_{\odot}$ pc$^{-2}$ can: (i) marginally account for the bulge rate; (ii) marginally account for the LMC rate; and (iii) produce a flat rotation curve ($v = 220$ km s$^{-1}$) in the galactic plane without a halo. However, such a model would not produce a flat rotation curve outside the galactic plane nor would it explain the warping of the disk and the flaring of neutral gas; moreover, it is in severe conflict with kinematic studies that indicate $\Sigma_{\text{TOT}}$ (1 kpc) is at most 100$M_{\odot}$ pc$^{-2}$ [29].

[29] J.N. Bahcall, C. Flynn, and A. Gould, *Astrophys. J.* **389**, 234 (1992); K. Kuijken and G. Gilmore, *ibid* **367**, L9 (1991) and Refs. [19].

[30] P. Sackett, H. Rix, B.J. Jarvis, K.C. Freeman, *Astrophys. J.* **436**, 629 (1994).

[31] See, e.g. H. Rix, astro-ph/9501068.

[32] C.S. Frenk, S.D.M. White, M. Davis, and G. Efstathiou, *Astrophys. J.* **327**, 507 (1988); J. Dubinski and R.G. Carlberg, *Astrophys. J.* **378**, 496 (1991).

[33] G.F. Giudice, S. Mollerach, and E. Roulet, *Phys. Rev.* **D50** 2406 (1994).
Our expression for $\tau$ when applied to the bulge implicitly assumes that all the mass density in the disk and bulge is available for lensing. However, the bulge lenses are certainly not bright stars. Zhao et al.\cite{zhao1995} find that correcting for this is a small effect. To account for the fact that the lenses are unlikely to be bright stars we simply do not include the contribution of the fixed luminous disk to $\tau_{\text{BULGE}}$.

The range we adopt for $\tau_{\text{LMC}}$ is the 95\% confidence range derived from the likelihood function based upon 3 events assuming Poisson statistics. However, the issue of uncertainties is far more complicated and the true 95\% confidence range is probably larger; see C. Han and A. Gould, Astrophys. J. in press \cite{han1995}. For example, the measured optical depth $\tau_{\text{MEAS}} = \frac{\pi}{4E} \sum \hat{t}_i / \varepsilon_i$, where the sum $i$ runs over events, $\hat{t}_i$ is the duration of event $i$ and $\varepsilon_i$ is the \textquotedblleft efficiency\textquotedblright of detecting an event of duration $\hat{t}_i$. The MACHO efficiency drops significantly for $\hat{t}_i \ll 10$ da and $\tau_{\text{LMC}}$ could be larger than reported due to short duration events. However, because of the null results of the EROS CCD search for short-duration events and the MACHO spike analysis it seems unlikely that this is the case.
Figure Captions

**Figure 1:** The number of viable models as a function of halo MACHO fraction for our minimal set of constraints (solid line) and additional constraints based upon the local escape speed and proper motions of satellite galaxies (dashed line).

**Figure 2:** The local mass density of halo matter, (a) total and (b) in MACHOs, for models with a Dwek bar, thin double-exponential disk and spherical halo (dotted line) or flattened (E6) halo (solid line).

**Figure 3:** Galactic model with a Dwek bar, thin double-exponential disk, and spherical halo. (a) Number of viable models as a function of the halo MACHO fraction, \( f_B \), for various values of the model input parameters. For the histograms labeled \( \tau_{LMC} \) and \( \tau_{BULGE} \) only it is assumed that these optical depths are known to precision of 10%. (b) Histograms of the number of viable models as a function of various model input parameters and optical depths. (c) Histograms of the number of viable models corresponding to selected model output parameters, from left to right and top to bottom, local mass density of CDM particles, local mass density of halo MACHOs, asymptotic rotation velocity, local escape velocity, total mass of MACHOs in the halo (within 50 kpc), and total halo mass (within 50 kpc). The dotted lines correspond to the number of models with \( f_B \geq 0.75 \), scaled upward by a factor of 50.

**Figure 4:** As in Fig. 3 for a galactic model with a Dwek bar, thick double-exponential disk, and spherical halo.

**Figure 5:** As in Fig. 3 for a galactic model with a Dwek bar, \( 1/r \) disk, and spherical halo.

**Figure 6:** As in Fig. 3 for a galactic model with a Dwek bar, thin double-exponential disk, and flattened (E6) halo.

**Figure 7:** As in Fig. 3 for a galactic model with a Dwek bar, thick double-exponential disk, and flattened (E6) halo.

**Figure 8:** As in Fig. 3 for a galactic model with a Dwek bar, \( 1/r \) disk, and flattened (E6) halo.

**Figure 9:** As in Fig. 3 for a galactic model with a Dwek bar, thin double-exponential disk, and E6 halo, where in addition we have imposed the constraints based upon the local escape speed and proper motions of satellite galaxies.

**Figure 10:** As in Fig. 3 for a galactic model with a Dwek bar, thin double-exponential disk, \( 1/r^3 \) baryonic spheroid and spherical CDM halo.

**Figure 11:** As in Fig. 3 for a galactic model with a Dwek bar, thin double-exponential disk, \( 1/r^4 \) baryonic spheroid and spherical CDM halo.

**Figure 12:** Contours of constant optical depth for microlensing in the direction of the bulge for (a) Kent model with \( M_B = 4.0 \times 10^{10} M_\odot \) and \( \Sigma_{V,AR} = 60 M_\odot \text{pc}^{-2} \); and models
with $M_B = 3.0 \times 10^{10} M_\odot$ and (b) $\theta = 10^\circ$, $\Sigma_{VAR} = 10 M_\odot \text{pc}^{-2}$; (c) $\theta = 45^\circ$, $\Sigma_{VAR} = 10 M_\odot \text{pc}^{-2}$; (d) $\theta = 10^\circ$, $\Sigma_{VAR} = 60 M_\odot \text{pc}^{-2}$; (e) $\theta = 45^\circ$, $\Sigma_{VAR} = 60 M_\odot \text{pc}^{-2}$. Contours are $0.5, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0 \times 10^{-6}$ from the outside inwards.

**Figure 13:** (a) The number of viable models as a function of bar orientation for models with a thin double-exponential disk and E6 halo. (b) The microlensing optical depth to the bar as a function of the bar orientation to our line of sight.