Hubs and Clusters in the Evolving United States Internal Migration Network

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(Dated: October 16, 2008)

Abstract

Most nations of the world periodically publish $N \times N$ origin-destination tables, recording the number of people who lived in geographic subdivision $i$ at time $t$ and $j$ at $t + 1$. We have developed and widely applied to such national tables and other analogous (weighted, directed) socioeconomic networks, a two-stage–double-standardization and (strong component) hierarchical clustering–procedure. Previous applications of this methodology and related analytical issues are discussed. Its use is illustrated in a large-scale study, employing recorded United States internal migration flows between the 3,000+ county-level units of the nation for the periods 1965-1970 and 1995-2000. Prominent, important features such as “cosmopolitan hubs” and “functional regions”– are extracted from master dendrograms. The extent to which such characteristics have varied over the intervening thirty years is evaluated.

PACS numbers: Valid PACS 02.10.Ox, 02.10.Yn, 89.65.-s

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I. INTRODUCTION

A. L. Barabási, in his recent popular book, “Linked”, asserts that the emergence of hubs in networks is a surprising phenomenon that is “forbidden by both the Erdős-Rényi and Watts-Strogatz models” [1, p. 63] [2, Chap. 8]. Here, we indicate—and apply anew to extensive U. S. intercounty migration data—an analytical framework introduced in 1974 that the distinguished computer scientist R. C. Dubes, in a review of the compilation of multitudinous results [3], asserted “might very well be the most successful application of cluster analysis” [4, p. 142]. This two-stage methodology has proved insightful in revealing—in addition, to functional clusters—hub-like structures in networks of (weighted, directed) internodal flows. This approach, together with its many diverse socioeconomic applications, was documented in a large number of (subject-matter and technical) journal articles (among them [5–17, 18, 19, 20, 21, 22]), as well as in the research institute monographs [3, 23, 24]. It has also been the subject of various comments, criticisms and discussions [21, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38] (cf. [39, 40, 41]).

Though this procedure is applicable in a wide variety of social-science settings [3, 4], it has been primarily used, in a demographic context, to study the internal migration tables published at regular periodic intervals by most of the nations of the world. These tables can be thought of as $N \times N$ (square) matrices, the entries ($m_{ij}$) of which are the number of people who lived in geographic subdivision $i$ at time $t$ and $j$ at time $t+1$. (Some tables—but not all—have diagonal entries, $m_{ii}$, which may represent either the number of people who did move within area $i$, or simply those who lived in $i$ both at $t$ and $t+1$. It can sometimes be of interest to compare analyses with zero- and nonzero-diagonal entries [23]. However, this aspect will not be of any immediate concern to us here.) We will principally be considering the case below of U. S. migration tables for the periods 1965-1970 [23] and 1995-2000 based on 3,000+ county-level units.
II. TWO-STAGE METHODOLOGY

A. First Step: Double-Standardization of Raw Flows

In the first step (iterative proportional fitting procedure [IPFP] [42]), the rows and columns of the table of flows are alternately (biproportionally [43]) scaled to sum to a fixed number (say 1). Under broad conditions—to be discussed below—convergence occurs to a “doubly-stochastic” (bistochastic) table, with row and column sums all simultaneously equal to 1 [44, 45, 46, 47, 48, 49]. The purpose of the scaling is to remove overall (marginal) effects of size, and focus on relative, interaction effects. Nevertheless, the cross-product ratios (relative odds), $m_{ij}m_{kl}/m_{il}m_{kj}$, measures of association, are left invariant. Additionally, the entries of the doubly-stochastic table provide maximum entropy estimates of the original flows, given the row and column constraints [50, 51].

For large sparse flow tables, only the nonzero entries, together with their row and column coordinates are needed. Row and column (biproportional) multipliers can be iteratively computed by sequentially accessing the nonzero cells [52]. If the table is “critically sparse”, various convergence difficulties may occur. Nonzero entries that are “unsupported”—that is, not part of a set of $N$ nonzero entries, no two in the same row and column—may converge to zero and/or the biproportional multipliers may not converge [3, p. 19] [53] [54, p. 171]. The “first strongly polynomial-time algorithm for matrix scaling” was reported in [55].

The scaling was successfully implemented, in our largest analysis, with a 3,140 × 3,140 1965-70 intercounty migration table—having 94.5% of its entries, zero—for the United States [9, 23], as well as for a more aggregate 510 × 510 table (with State Economic Areas as the basic unit) for the US for the same period [14]. (Smoothing procedures could be used to modify the zero-nonzero structure of a flow table, particularly if it is critically sparse [56, 57]. If one takes the second power of a doubly-stochastic matrix [as we in fact do in Sec. V C], one obtains another such matrix—of predicted two-stage movements—but smoother in character. One might also consider standardizing the $i$th row [column] sum to be proportional to the number of non-zero entries in the $i$th row [column]—although we found considerable numerical difficulties when attempting this for the 1995-2000 U. S. intercounty migration table [Sec. V]. Another procedure—in line with the Google page-ranking [“teleporting random walk”] procedure [58, 59], that has been much studied and emulated—is to take some convex
combination of the doubly-stochastic table and the \( N \times N \) table all the off-diagonal entries of which are equal to \( \frac{1}{N-1} \).

Figs. 1 and 2 give a graphic display of the effect of the double-standardization (biproportional adjustment) on a 1962-68 French interprovincial migration table (the island of Corsica is omitted). Additionally, Figs. 3 and 4 are comparable displays for 1995-2000 U. S. interstate migration (Alaska, Hawaii and the District of Columbia are not included) (cf. [60]). The process–employed by Waldo Tobler–to produce these four figures was the following: (1) the average value \( (A) \) of all the entries in the (adjusted or unadjusted) table was found; (2) if the sum of the \( ij \) and \( ji \)-entries of the corresponding table exceeded \( 2A \), a bar, the thickness of which is proportional to this sum is drawn connecting area \( i \) to area \( j \). We see that in the two figures based on the double-standardization procedure, linkages between adjacent areas are much more (suiting our purposes) strongly stressed than using the raw flows themselves.

B. Second Step: Strong Component Hierarchical Clustering

In the second step of the two-stage procedure, the doubly-stochastic matrix is converted to a series of directed \((0,1)\) graphs (digraphs), by applying thresholds to its entries. As the thresholds are progressively lowered, larger and larger strong components (a directed path existing from any member of a component to any other) of the resulting graphs are found. This process (a simple variant of well-known single-linkage [nearest-neighbor or min] clustering [61]) can be represented by the familiar dendrogram or tree diagram used in hierarchical cluster analysis and cladistics/phylogeny (cf. [62, 63]). (The “CLASSIC” methodology proposed somewhat later by Ozawa–though couched in rather different terminology–appears to be fully equivalent to ours. Ozawa found the procedure to be useful in “the detection of gestalt clusters” [62].)

C. Computer implementations

A FORTRAN implementation of the two-stage process was given in [64], as well as a realization in the SAS (Statistical Analysis System) framework [65]. Subsequently, the noted computer scientist R. E. Tarjan [66] devised an \( O(M(\log N)^2) \) algorithm [67] for strong
FIG. 1: Salient flows based on unadjusted 1962-68 French interprovincial migration table

component hierarchical clustering, and, then, a further improved $O(M(\log N))$ method \cite{68}, where $N$ is the number of nodes and $M$ the number of edges of a directed graph. (These substantially improved upon the earlier works \cite{64, 65}, which required the computations of transitive closures of graphs—in terms of which the analysis of Ozawa \cite{62} is phrased—and were $O(MN)$ in nature.) A FORTRAN coding—involving linked lists—of the improved Tarjan algorithm \cite{68} was presented in \cite{69}, and applied in the aforementioned 1965-70 US intercounty study \cite{23}. If the graph-theoretic (0,1)-structure of a network under study is not strongly connected \cite{70}, independent two-stage analyses of the subsystems of the network would be appropriate.

The goodness-of-fit of the dendrogram generated to the doubly-stochastic table itself can be evaluated—and possibly employed, it would seem, as an optimization criterion (cf. \cite{71} p. 210 \cite{72}, Sec. 3 \cite{73}). Distances between nodes in the dendrogram satisfy the (stronger
FIG. 2: Salient flows based on doubly-standardized 1962-68 French interprovincial migration table. Note the increase in linkages between adjacent areas.

FIG. 3: Salient flows based on 1965-70 unadjusted U. S. interstate migration table
FIG. 4: Salient flows based on 1965-70 doubly-standardized U. S. interstate migration table. Note the increase in linkages between adjacent areas.

than triangular ultrametric inequality, \( d_{ij} \leq \max (d_{ik}, d_{jk}) \) [74, p. 245] [75, eq. (2.2)]. We will examine issues pertaining to ultrametric fit and residuals from such fits in Sec. V A. (Costa and colleagues have studied hierarchical aspects of “complex networks” 76 77.)

III. EMPIRICAL RESULTS

A. Cosmopolitan or Hub-Like Units

1. Internal migration flows

Geographic subdivisions (or groups of subdivisions) that enter into the bulk of the dendrograms produced by the two-stage procedure at the weakest levels are those with the broadest ties. These are “cosmopolitan”, hub-like areas, a prototypical example being the French capital, Paris 3 Sec. 4.1 6. Similarly, in parallel analyses of other internal migration tables, the cosmopolitan/non-provincial natures of London 78, Barcelona 16 3 Sec. 6.2, Figs. 36, 37, Milan 12 3 Sec. 6.3, Figs. 39, 40] (cf. 13), Amsterdam 3 p. 78 25, West Berlin 3 p. 80, Moscow (the city and the oblast as a unit) 19 3 Sec. 5.1 and Figs. 6, 7, Manila (coupled with suburban Rizal) 79, Bucharest 18, Île-de-Montréal 3 p. 87], Zürich, Santiago, Tunis and Istanbul 80 were–among others–highlighted in the respective dendrograms for their nations 3 Sec. 8.2 15 pp. 181-182 8 p. 55. In the 1965-70 inter-
county analysis for the US, the most cosmopolitan entities were: (1) the centrally-located paired Illinois counties of Cook (Chicago) and neighboring, suburban DuPage; (2) the nation’s capital, Washington, D. C.; and (3) the paired South Florida (retirement) counties of Dade (Miami) and Broward (Ft. Lauderdale) [9, 23, 81]. In general, counties with large military installations, large college populations or state capitals also interacted broadly with other areas [23, p. 153]. Application of the two-stage methodology to 1965-66 London inter-borough migration [25] indicated that the three inner boroughs of Kensington and Chelsea, Westminster, and Hammersmith acted–as a unit–in a cosmopolitan manner [3, Sec. 5.2, Fig. 10]. (In Sec. 8.2 and Table 16 of the anthology of results [3], additional geographic units and groups of units found to be cosmopolitan with regard to migration, are enumerated.)

It should be emphasized that although the indicated cosmopolitan areas may generally have relatively large populations, this can not, in and of itself, explain the wide national ties observed, since the double-standardization, in effect, renders all areas of equal overall size. (However, to the extent that larger areas do have fewer zero entries in their corresponding rows and columns, a bias to cosmopolitanism may in fact be present, which should be carefully considered. Possible corrections for bias were discussed above in Sec. II A.) If one were to obtain a (zero-diagonal) doubly-stochastic matrix, all the entries of which were simply \( \frac{1}{N-1} \), it would indicate complete indifference among migrants as to where they come from and to where they go. A maximally cosmopolitan unit would be one for which all the corresponding row and column entries were \( \frac{1}{N-1} \) (if all the diagonal entries, \( m_{ii} \), are \( a priori \) zero). (It seems interesting to note that cosmopolitan areas appear to have a certain minimax character, that is, the maximum doubly-stochastic entry for the corresponding row and column tends to be minimized.)

2. **Trade and interindustry flows**

The nation of Italy possessed the broadest ties in a two-stage analysis of the value of 1974 trade between 113 nations, followed by a closely-bound group composed of the four Scandinavian countries [17] [3, Sec. 5.6, Fig. 22]. In a two-stage study (but using weak rather than strong components of the associated digraphs) of the 1967 U. S. interindustry transaction table, the industry with the broadest (most diffuse) ties was found to be Other Fabricated Metal Products [10, 82] [24, pp. 13-18].
3. Journal citations

In a two-stage analysis of 22 mathematical journals, the *Annals of Mathematics* and *Inventiones Mathematicae* were strongly paired, while the *Proceedings of the American Mathematical Society* was found to possess the broadest, most diffuse ties [8].

In a recent, large-scale (\(N > 6000\)) journal-to-journal citation analysis, decomposing “the network into modules by compressing a description of the probability flow”, Rosvall and Bergstrom preliminarily omitted from their analysis the prominent journals *Science*, *Nature* and the *Proceedings of the National Academy of Sciences* [83, p. 1123]. (Those are precisely the ones that would be expected to be “cosmopolitan” or hub-like in character, and to be highlighted in a corresponding two-stage analysis.) Their rationale for the omission was that “the broad scope of these journals otherwise creates an illusion of tighter connections among disciplines, when in fact few readers of the physics articles in *Science* also are close readers of the biomedical articles therein”. (In [24, pp. 125-153], we reported the results of a partial hierarchical clustering—not a two-stage analysis, but one originally designed and conducted by Henry G. Small and William Shaw—of citations between more than 3,000 journals. The clusters obtained there were compared with the actual subject matter classification employed by the Institute for Scientific Information.)

B. Functional Clusters of Units

1. Internal migration regions

Geographically isolated (insular) areas—such as the Japanese islands of Kyushu and Shikoku [5]—emerged as well-defined clusters (regions) of their constituent (seven and four, respectively) subdivisions (“prefectures” in the Japanese case) in the dendrograms for the two-stage analyses, and similarly the Italian islands of Sicily and Sardinia [12], the North and South Islands of New Zealand, and the Canadian islands of Newfoundland and Prince Edward Island [3, p. 90] (cf. [84, 85]). The eight counties of Connecticut, and other New England groupings, as further examples, to be elaborated upon below, were also very prominent in the highly disaggregated U. S. analysis [23]. Relatedly, in a study based solely upon the 1968 movement of college students among the fifty states, the six New England states were strongly clustered [11, Fig. 1]. Employing a 1963 Spanish interprovincial migration
table, well-defined regions were formed by the two provinces of the Canary Islands, and the four provinces of Galicia \[16\ \[3\ Sec. 6.2.1, Fig. 37\]. The southernmost Indian states of Kerala and Madras (now Tamil Nadu) were strongly paired on the basis of 1961 interstate flows \[22\]. A detailed comparison between functional migration regions found by the two-stage procedure and those actually employed for administrative, political purposes in the corresponding nations is given in Sec. 8.1 and Table 15 of \[3\].

It should be noted that it is rare that the two-stage methodology yields a migration region composed of two or more noncontiguous subregions—even though no contiguity information, of course, is explicitly present in the flow table nor provided to the algorithm (cf. \[57\ \[86\]). A notable exception to this rule was the uniting of the northern Italian region of Piemonte—the location of industrial Turin, where Fiat is based—with (poor) southern regions, before joining with central regions, in an aggregate 18-region 1955-70 study \[13\ \[3\ p. 75\] (cf. \[12\]).

2. Intermarriage and interindustry clusters

In a two-stage analysis of a $32 \times 32$ table of birthplace of bridegroom versus birthplace of bride of 1947 Australian intermarriages \[87\], Greece and Cyprus were the strongest dyad \[3\ Sec. 5.7, Fig. 25\].

In the 1967 US interindustry two-stage (weak component) analysis, two particularly salient pairs of functionally-linked industries were: (1) Stone and Clay Products, and Stone and Clay Mining and Quarrying; and (2) Household Appliances and Service Industry Machines (the latter industry purchases laundry equipment, refrigerators and freezers from the former) \[10\ \[82\ \[24\ pp. 13-18\].

IV. STATISTICAL ASPECTS

It would be of interest to develop a theory—making use of the rich mathematical structure of doubly-stochastic matrices—by which the statistical significance of apparent hubs and clusters in dendrograms produced by the two-stage procedure could be evaluated \[23\ pp. 7-8\ \[88\]. In the geographic context of internal migration tables, where nearby areas have a strong distance-adversion predilection for binding, it seems unlikely that most clustering results generated could be considered to be—in any standard sense—“random” in nature. On
the other hand, other types of “origin-destination” tables, such as those for occupational mobility [89], journal citations [8] [24 pp. 125-153], interindustry (input-output) flows [10] [82], brand-switches [3 Sec. 9.6] [90], crime-switches [3 Sec. 9.7] [91 Table XII], and (Morse code) confusions [3 Sec. 9.8] [92], among others, clearly lack such a geographic dimension (cf. [93]). An efficient algorithm—considered as a nonlinear dynamical system—to generate random bistochastic matrices has recently been presented [46] (cf. [94, 95]).

In the 1965-70 US 3,140-county migration study, a statistical test of Ling [96] (designed for undirected graphs), based on the difference in the ranks of two edges, was employed in a heuristic manner [23, pp. 7-8]. For example, the 3,148th largest doubly-stochastic value, 0.12972 (corresponding to the flow from Maui County to Hawaii County), united the four counties of the state of Hawaii. The (considerably weaker) 7,939th largest value, 0.07340 (the link from Kauai County, Hawaii, to Nome, Alaska), integrated the four-county state of Hawaii into a much larger 2,464-county cluster. (Data for the additional [fifth] very small county of Kalawao were only given in the 1995-2000 analysis.) The difference of these two ranks, 4,192 = 7,340 - 3,148, is a measure of isolation (“survival time”) of this state as a cluster. Reference to Table 1 in [23] showed the significance of the state of Hawaii as a functional internal migration unit at the 0.01 level [23, p. 7]. (In the computation of this table, the approximation was used that the number of edges in the relevant digraphs was a negligible proportion of all possible 3,140 \times 3,139 edges.)

A. Random digraphs

Also, the possibility of employing the asymptotic theory of random digraphs [97, 98] for statistical testing purposes was raised in [23]. In this regard, it was necessary to consider the 38,815-th largest entry of the doubly-stochastic matrix to complete the hierarchical clustering of the 3,140 counties. The probability is 0.973469 that a random digraph with 3,140 nodes and 38,414 links is strongly connected [98 p. 361], where 0.973469 = e^{-2e^{-4.30917}}, and 38,814 = 3140(\log 3140 + 4.30917). Evidence of systematic structure in the migration flows can, thus, be adduced, since the digraph based on the 38,814 greatest-valued links was not strongly connected [23 p. 8] (cf. 99). (For our 1995-2000 analysis [Sec. V, the counterpart of the probability 0.973469 is 0.107134.)

In a random digraph with a large number of nodes, the probability is close to one that
all nodes are either isolated or lie in a single (“giant”) strong component. The existence of intermediate-sized clusters is thus evidence of non-randomness, even if such groups are not themselves significant according to the isolation (difference-of-ranks) criterion of Ling [96]. With randomly-generated data and many taxonomic units, one would expect the two-stage procedure to yield a dendrogram exhibiting complete chaining. So, although single-linkage clustering is often criticized for producing chaining, chains can also be viewed simply as indications of inherent randomness in the data. In contrast to single-linkage clustering, strong component hierarchical clustering can merge more than two clusters (children) into one (parent) node. This serves to explain why fewer clusters (2,245) were generated in the intercounty migration study than the 3,139 that single-linkage (in the absence of ties) would produce.

B. A cluster-analytic isolation criterion

Dubes and Jain [100] provided “a semi-tutorial review of the state-of-the-art in cluster validity, or the verification of results from clustering algorithms”. Among other evaluative standards, they discussed isolation criteria, which “measure the distinctiveness or separation or gaps between a cluster and its environment”. Such a statistic was developed and applied in [101] in order to extract a small proportion of 5,385 clusters (3,140 of them single units, 673 pairs, 230 triples, 104 quartets, . . . ) for detailed examination based on the two-stage analysis of the 1965-1970 United States intercounty migration tables [23].

The largest value of the isolation criterion, for all clusters of fewer than 2,940 units, was attained by a region formed by the eight constituent counties of the state of Connecticut. (Groups formed by the application of the two-stage procedure to interareal migration data are, as a strong rule, composed of contiguous areas [3, 15]. This occurs even in the absence of contiguity constraints, reflecting the distance decay of migration.) The 11,080th largest doubly-standardized entry, 0.05666, corresponding to movement from New Haven to (New York City suburban) Fairfield, unified these eight counties. Not until the 16,047th largest doubly-standardized value, 0.04085 (the functional linkage from Litchfield, Connecticut to Berkshire, Massachusetts), viewing the clustering procedure as an agglomerative one, was Connecticut absorbed into a larger region. The isolation criterion \(i\) for Connecticut is set
equal to
\[ i = 25.3175 = -\log \left[ \left( \frac{8 \times 7 + 3132 \times 3131}{3140 \times 3139} \right)^{16047 - 11080} \right] \]  \hspace{1cm} (1)

The term in large parentheses is the proportion of cells in the 3,140 × 3,140 table associated with either movement within (8-county) Connecticut or within the set of 3,132 complementary counties (since intracounty flows are not available, a diagonal correction is made). This term, raised to the power shown, is the probability (unadjusted for occupied cells) that none of 4,967 = 16,047 – 11,080 consecutive doubly-standardized values would correspond to movement between Connecticut and its complement. (In our 1995-2000 analysis [Sec. V], we find analogously the result, 3,132 = 12,107–8,975, yielding \( i = 16.1339 \), still the most significant of any of the fifty-two 8-county clusters there.) Such a Connecticut-complement linkage could possibly result in a merger: an unobserved phenomenon. (For further details, including maps, discussion and extensive applications of the isolation criterion developed to the U. S. intercounty analysis, see [101].) This isolation score \( i \) for the cluster formed by the four counties of Hawaii—discussed above—was 12.21, while the District of Columbia had the highest score, 23.81, for any single county [101, Table I].

V. TWO-STAGE ANALYSES OF U. S. INTERCOUNTY MIGRATION FLOWS

A 3,107 × 3,107 migration table for the United States for the period 1995-2000 can be readily constructed from freely available data at the website, [http://www.census.gov/population/www/cen2000/ctytocflow/index.html](http://www.census.gov/population/www/cen2000/ctytocflow/index.html). We have been able to conduct a two-stage analysis of this table. In the 1965-70 analysis [23], 3,140 units of 3,141 had been utilized—with Loving County, TX, the smallest US county, being omitted since it had no recorded in-migrants. The reduction to 3,107 units in the 1995-00 table is due to the administrative amalgamation now of 34 independent cities of Virginia with neighboring counties. (Loving County is included in the later analysis, as well as now the second smallest—and poorest—US county, Kalawao County, HI.) The 1965-70 table was 94.5% sparse (zero entries), and the 1995-00 table, 92.3% sparse (the difference perhaps largely being due to the Census sampling design). In Figs. 5 and 6 we present matrix plots of the unadjusted and adjusted tables. (The states are ordered alphabetically—not in terms of the postal [“zip”] code—and the counties, alphabetically within states. County No. 1 is Au-
FIG. 5: Unadjusted 1995-2000 intercounty U. S. migration table. The large square near the end—for alphabetical reasons—of the diagonal corresponds to the state with the most (254) counties, Texas. tauga, AL; County No. 1000 is Boyd, KY; County No. 2000 is Dunn, North Dakota; and County No. 3,107 is Weston, WY.) The largest square on the diagonal in both these figures corresponds to the state with the most number of counties (254), that is, Texas. (The double-standardization—giving a more pronounced block-diagonal structure—brings out more strongly intrastate movements which would clearly tend to be favored over interstate ones due to effects of distance and possible state loyalties and ties.)

A. Most cosmopolitan units

In Fig. 7 we show the most cosmopolitan counties or groups of counties based on the doubly-standardized values themselves, while in (the less flat) Fig. 8, the ordinal rank of the doubly-standardized value is used instead. The doubly-standardized values associated
with the evolution from beginning to end of the hierarchical clustering range from 0.530385 to 0.0225427, while the ranks extend from 7 to 25,329. The comparable statistics for the 1965-70 analysis based on 3,140 units were 0.47730 to 0.01659 and 24 to 38,815. So, ignoring any possibly necessary corrections due to the slightly different sizes (3,140 vs. 3,107) in the two periods and different degrees of sparsity (94.5% vs. 92.5%), one might conclude—since 0.0225427 > 0.01659—that the most cosmopolitan counties in the earlier analysis were more so (less "provincial") than the most cosmopolitan counties in the later period. (For the choice of most appropriate locations at which to truncate dendrograms so as to distinguish cosmopolitan from provincial units, see Sec. V B 3.)

The non-truncated (master) versions of these two figures (along with their counterparts based on the [smoothed] second power or square of the doubly-stochastic table) are given in the Electronic-only material and will be examined in Sec. V B. In Fig. 9 we show the ultrametric fit to the doubly-stochastic table generated by the hierarchical clustering pro-
FIG. 7: Truncated dendrogram–showing the most cosmopolitan and groups of cosmopolitan counties–based upon doubly-standardized 1995-2000 intercounty migration flows. To obtain a distance-like (dissimilarity) measure, we subtract the doubly-stochastic values from the largest such value, 0.530385

PROCEDURE, and in Fig. [10], the residuals from this fit. As a measure of goodness-of-fit, let us take the ratio of the sum of squares of the residuals from the largest $k = 25,329$ entries (the number needed to complete the hierarchical clustering process) to the sum of squares of the 25,329 entries themselves. This ratio was 0.30861. In Fig. [11], we show this measure of fit as a function of $k$. The minimum (best fit) of 0.0964163 is reached for the 7,229-th largest doubly-stochastic entry, 0.0766761.

The list of most cosmopolitan counties is much more “Sunbelt”-oriented in nature than
FIG. 8: Truncated dendrogram–showing the most cosmopolitan and groups of cosmopolitan counties–based upon ordinal rankings of doubly-standardized 1995-2000 intercounty migration flows

in the 1965-70 analysis [23] discussed above. (Let us note a technical point: even though our strong component hierarchical clustering procedure can and often does unite more than two smaller clusters, in order to fit within the Mathematica hierarchical clustering framework, we have to map our results into an equivalent hierarchical clustering in which only binary mergers occur–though these may occur at equal thresholds. In our actual 1995-00 clustering, there were 2,497 mergers, as opposed to 3,106. The comparable figures for 1965-70 were 2,245 and 3,139.)

The leading cosmopolitan counties found (and some of their apparently migration-relevant
features), in decreasing order, are:

1. Brevard, FL (the “Space Coast”, the Kennedy Space Center);
2. Mohave, AZ (Lake Havasu, Grand Canyon);
3. Clark, NV (Las Vegas);
4. Hillsborough and Pinellas, FL, which are grouped with the pair (represented by the topmost box with “2” inside it), Pasco and Hernando, FL. (This quartet–having an isolation index of 11.9717–is completely coterminous with the governmentally designated Tampa-St. Petersburg-Clearwater Metropolitan Statistical Area [MSA]. Additionally, Pasco and Hernando have the greatest isolation index, 14.6413, of any pair in the entire analysis [(Table V B 2)];
5. The next lower box with “2” in it, stands for the southern Gulf Coast dyad formed by Collier County (East Naples) and Lee County (Fort Myers, a single-county MSA), FL;
6. San Diego, CA;
FIG. 10: Residuals (mostly negative) from the fit of the ultrametric structure (Fig. 9) to the 1995-2000 doubly-stochastic table internal migration flow table (Fig. 6)

(7) Dallas, TX;
(8) Maricopa, AZ (Phoenix);
(9) Cook, IL (Chicago);
(10) Orange, Seminole, and Osceola, FL (corresponding to the upper box with “3” in it) (these three counties, along with Lake County, form the Orlando-Kissimmee MSA);
(12) Sumter and Lake, FL (the next box with “2” in it);
(13) Monroe, FL (Key West);
(14) Cumberland, NC (giant Fort Bragg and Pope Air Force Base);
(15) Mecklenburg, NC (Charlotte);
(16) Martin, St. Lucie and Indian River, FL (the lower box containing “3”) (Indian River borders Brevard County, the most cosmopolitan nationally);
(17) Palm Beach, FL together with the pair Miami-Dade and Broward, FL (the lowest
FIG. 11: Goodness-of-fit of the ultrametric structure (Fig. 9) to the $k$ largest 1995-2000 doubly-stochastic values. The best fit, 0.0964163, is attained at rank 7,229, corresponding to a doubly-stochastic value of 0.0766761. At that threshold, there are 388 clusters (strong components).

box with “2” in it); (this southeastern Florida triad comprises the Miami-Fort Lauderdale-Pompano Beach MSA, highlighted in gray in the master dendrograms [Electronic-only material]);

(18) Hennepin, MN (Minneapolis);
(19) Marion, FL (bordering the (17) cluster on the north);
(20) Bell, TX (Fort Hood);
(21) Polk, FL (Lakeland);
(22) Citrus, FL (formerly part of Hernando County);
(23) Weld, CO (Greeley);
(24) Larimer, CO (Fort Collins);
(25) Kenai Peninsula, AK (Seward);
(26) Los Angeles, CA; and
(27) Pierce, WA (Fort Lewis and McChord Air Force Base).

The comparable list for the earlier period 1965-70 takes the form [SI, Table 1]: (1) Cook and DuPage, IL; (2) District of Columbia; (3) Dade and Broward, FL; (4) Pierce, WA; (5) Harris, TX (Houston); (6) Riverside and San Bernadino, CA; (7) Orange, CA; (8) Lake,
IL (lying in the Chicago metropolitan area); (9) Monroe, FL; (10) Los Angeles, CA; (11) Pinellas, FL; (12) Brevard, FL; (13) Polk, FL; (14) Pulaski, Mo (Fort Leonard Wood); (15) Geary, KS (Fort Riley); (17) Wayne, MI (Detroit); (18) Bell, TX; (19) Hillsborough, FL; (20) El Paso, CO (Air Force Academy); (21) Ventura, CA; (22) Cumberland, NC; (23) St. Louis County and City, MO; (24) Norfolk, VA (Atlantic Fleet headquarters); (25) Arlington County and Alexandria City, VA; and (26) Sedgwick, KS (Wichita, McConnell Air Force Base).

We see that this 1965-70 list is relatively weaker than the 1995-00 list in terms of Sunbelt counties, but relatively stronger in terms of counties with large military installations (though Brevard County, Florida does have Patrick Air Force Base). We note, in particular, that the first non-Sunbelt county in the 1965-70 list (that is, Cook, IL) is ninth here, but (coupled with DuPage, IL) was most cosmopolitan in the earlier analysis. Also, the District of Columbia (colored pink in the associated master dendrograms [Electronic-only material], p. 3), which had the lowest threshold of isolation of any single county in the 1965-70 analysis \[23, p. 31\], slips very substantially.

The most immediate explanation for the relative decrease in cosmopolitanism of counties with large military installations would appear to be the elimination of the draft in 1973–so, it would seem, military installations became less relatively populated by transient, recently migrant individuals (draftees)–as well as the downsizing of the military since the Vietnam War. (The peak of 2.4 million troops was reached in 1969, while in 2000, there were some 1.384 million military personnel.)

B. Migration regions

1. Selected features

We find—in the first two (searchable) master (non-truncated) dendrograms (Electronic-only material)—that the states of Hawai‘i (red, \(i = 14.121\), p. 2), Connecticut (blue, \(i = 16.1339\), p. 2) and Rhode Island (green, \(i = 11.8384\), p. 3) are reconstituted from their respective counties. (The most cosmopolitan county in Hawai‘i is Honolulu, and in Connecticut, Fairfield [a “bedroom suburb”] of New York City. Both Hawai‘i and Connecticut emerged as clusters in the 1965-70 analysis, while all the counties of Rhode Island, but
for historic Newport, were grouped.) In both analyses, the fifteen southern counties (colored black) of Maine are clustered (p. 8). (The northernmost, omitted county, Aroostook is agricultural, Canadian-oriented, and well-recognized as highly anomalous in terms of the general character of Maine [23, p. 48, pp. 118-119].) In the 1995-00 dendrograms (Electronic-only material), these fifteen counties immediately merge with six of the ten Lake Region counties of New Hampshire. The five counties of Rhode Island are also strongly linked with seven (or eight) Massachusetts counties (Table I).

The five Pennsylvania counties (colored orange) of the Philadelphia-Camden-Wilmington MSA (p. 5) were grouped in both analyses, and similarly the four New York metropolitan counties (colored brown) of Long Island (p. 2). (Their isolation indices in the 1995-2000 analyses are relatively weak, that is, 3.55119 and 2.50869, respectively.)

In the 1965-70 analysis, the dyad forming first in the agglomerative process was comprised of the South Dakota counties of Dewey and Ziebach (p. 17), which together form the Cheyenne Indian Reservation. It is the sixth such couple in the later analysis, with the four pairs now forming first being: (1) Stewart and Webster, Georgia (p. 17) (these two counties will be found to have the largest associated diagonal entries when the doubly-stochastic table is squared in Sec. V.C; (2) Garfield and Petroleum, Montana (p. 32); (3) the Eastern Shore of Virginia, that is, Accomack and Northampton Counties (p. 3 and Table I); (4) and Cassia and Minidoka Counties, Idaho, which form the Burley Micropolitan Statistical Area (p. 2 and Table I). Further, the interstate Jackson Micropolitan Statistical Area–formed by Teton County, Idaho and Teton County, Wyoming–also comprises a strongly bound pair (p. 5). Our master dendrograms end with the pair of Alabama counties–lying in the Montgomery MSA–of Autauga and Elmore.

The (strongly black-populated) Mississippi Delta is defined by Wikipedia as consisting of seventeen counties. Thirteen of these counties can be found in a certain fifteen-county 1995-00 cluster (colored magenta, having \( i = 2.8686, \) p. 23). (In the 1965-70 analysis, we noted a six-county subcluster [23, p. 57].) The southernmost member of the seventeen-county group, Warren County (Vicksburg), is omitted from the thirteen-member cluster (along with Washington, Carroll and Holmes Counties).

The San Joaquin Valley of California is defined by Wikipedia as comprised of seven counties. These seven, plus Madera County, are clustered (light green, p. 6).

The six California counties that form the North Coast American Viticultural Area also
function as a migration region (light brown, p. 3).

The three New York counties (Chautauqua, Cattaraugus and Allegany) forming the “Southern Tier” are highlighted in light blue (p. 7).

2. Most well-defined 1995-2000 migration regions

“South Jersey” is—according to Wikipedia—composed of eight New Jersey counties. With the omission of its most northern member (in fact, classified for governmental purposes as in the New York metropolitan area), Ocean County, the seven (Philadelphia-oriented) counties form a very well-defined migration region (colored light orange, \( i = 28.7301 \), p. 5, while for 1965-70, \( i = 20.8996 \) [23, p. 64] [101]). In fact, arranged in terms of decreasing values of \( i \) for the period 1995-2000, this region emerges as the most well-defined in the entire analysis (Tables I-II). (Many of the values of \( i \) given in the tables for the 1965-70 period are available in [101].) Since all the values of \( i \) for 1995-2000 listed are larger than \(- \log \frac{1}{100000} = 11.5129\), we can infer that all these regions are significant at the 0.00001 level.

*French Louisiana* is defined by Wikipedia as the amalgamation of Acadiana/”Cajun Country” (22 parishes) and Greater New Orleans (7 parishes)–St. Charles and St. John the Baptist being common to both–giving a 27-parish region. Our analysis yields a well-defined \((i = 16.7764)\) 27-parish region also, having 24 parishes in common with the Wikipedia definition. Our candidate region contains the three parishes, Allen, (the Mississippi-bordering pair of) East Feliciana and West Feliciana, but lacks those of Avoyelles (immediately adjacent, however, in the dendrogram), Orleans (coextensive with the City of New Orleans) and St. Tammany (also in the New Orleans metropolitan area). (The last two parishes–located on pp. 9 and 15 of the first two master dendrograms–are relatively cosmopolitan–as might be anticipated from their wide [pre-Hurricane Katrina] renown.)

The Northern New England region is composed of the three states of Maine, New Hampshire and Vermont, plus the two (mutually well-separated) Massachusetts counties of (western) Berkshire and (northeastern) Essex.

Our Northern Lower Michigan region is composed of twenty-six counties, twenty-two of which are contained in the twenty-seven county Wikipedia definition. Our region, however, extends further to the Southeast around Saginaw Bay, with Isabella, Midland, Bay and Saginaw counties, and omits five of Wikipedia’s southwesternly situated ones (Wexford,
| Region                                | States | Page | no. counties | \(i\) (1995-00) | \(i\) (1965-70) |
|---------------------------------------|--------|------|--------------|------------------|------------------|
| South Jersey                          | NJ     | 6    | 7            | 28.7301          | 20.8996          |
| Glades + Hendry + Okeechobee          | FL     | 1    | 3            | 23.474           |                  |
| “Delmar” + Baltimore                  | DE,MD  | 5    | 15           | 20.283           |                  |
| Western Ohio + Randolph, IN           | OH,IN  | 25   | 14           | 20.0938          |                  |
| Western New York                      | NY     | 7    | 18           | 19.4948          |                  |
| Rhode Island + S. E. Mass.            | RI,MA  | 3    | 12           | 18.6991          |                  |
| Greater Orlando                       | FL     | 1    | 3            | 17.6523          |                  |
| Northern Lower Michigan               | MI     | 8,9  | 26           | 17.2098          |                  |
| French Louisiana                      | LA     | 30,31| 27           | 16.7764          |                  |
| Brevard                               | FL     | 1    | 1            | 16.3097          | 19.6942          |
| Golden Triangle (Beaumont +)          | TX     | 4    | 6            | 16.1803          |                  |
| Connecticut                           | CT     | 2    | 8            | 16.1339          | 25.3175          |
| Mohave (Kingman)                      | AZ     | 1    | 1            | 15.463           | 6.39121          |
| Clark (Las Vegas)                     | NV     | 1    | 1            | 15.1784          | 6.23128          |
| Rexburg, ID + Jackson, WY MSAs        | ID,WY  | 5    | 4            | 15.0882          |                  |
| Eastern Rust Belt                      | NJ,OH,PA,WV | 24 | 82          | 15.0412          |                  |
| Burley MSA                            | ID     | 2    | 2            | 14.8809          |                  |
| Pasco + Hernando                      | FL     | 1    | 2            | 14.6413          |                  |
| San Diego                             | CA     | 1    | 1            | 14.2408          | 12.5938          |
| Maysville MSA + 3 counties            | KY     | 19   | 5            | 14.1822          |                  |
| Hawaii                                | HI     | 2    | 5\(^{a}\)    | 14.121           | 12.21            |
| Northern High Plains                  | MT,ND,NE,SD | 36,37 | 55 | 13.8799 | |
| Middle Ohio Valley                    | IN,KY  | 24,25| 27           | 13.821           |                  |
| Eastern Shore                         | VA     | 3    | 2            | 13.7051          |                  |

\(^{a}\)A fifth county, Kalawao, was included in the 1995-00 data, but not in 1965-70

TABLE I: Most well-defined 1995-2000 migration regions and their isolation indices
| Region                                      | States                        | Page | no. counties | $i_{(1995-00)}$ | $i_{(1965-70)}$ |
|--------------------------------------------|-------------------------------|------|--------------|----------------|----------------|
| Dallas                                     | TX                            | 1    | 1            | 13.5473        | 14.8557        |
| Maine + 7 NH counties                      | ME,NH                         | 8    | 22           | 13.4716        |                |
| Southeastern Arizona                       | AZ                            | 2    | 3            | 13.3503        |                |
| Maricopa (Phoenix)                         | AZ                            | 1    | 1            | 13.2608        | 12.5479        |
| Eastern Upstate New York                   | NY                            | 7    | 28           | 13.3052        |                |
| Michigan Thumb                             | MI                            | 6    | 6            | 13.2208        |                |
| Wasatch Back                               | UT                            | 11   | 8            | 13.1616        |                |
| N. Vermont + Coos, NH                      | NH,VT                         | 11   | 10           | 13.0778        |                |
| S. Central Tennessee                       | TN                            | 22   | 10           | 13.3092        |                |
| Northeast South Carolina                   | SC                            | 15   | 8            | 13.0276        |                |
| Northern New England                       | MA,ME,NH,VT                   | 9,10 | 42           | 12.8446        |                |
| Cook (Chicago)                             | IL                            | 1    | 1            | 12.7682        | 16.8933        |
| Southeastern Indiana                       | IN                            | 25   | 10           | 12.7172        |                |
| Northwestern Lower Michigan                | MI                            | 9,10 | 9            | 12.6567        |                |
| High Colorado Rockies                      | CO                            | 3    | 3            | 12.5892        |                |
| Joplin Area                                | MO                            | 5    | 3            | 12.3071        |                |
| Central Savannah River                     | GA                            | 22   | 4            | 12.2086        |                |
| Southern Maryland                          | MD                            | 3    | 3            | 12.1217        |                |
| Amarillo (Potter + Randall)                | TX                            | 1    | 2            | 12.0528        | 8.16948        |
| Tampa MSA                                  | FL                            | 1    | 4            | 11.9717        |                |
| York+Adams                                 | PA                            | 3    | 2            | 11.9433        | 13.7789        |
| Lake + Sumter                              | FL                            | 1    | 2            | 11.8635        |                |
| Rhode Island                               | RI                            | 3    | 5            | 11.8384        | 11.7668\(^a\) |
| Central Appalachia                         | MD,NC,TN,VA,WV                | 27,28| 77           | 11.7459        |                |

\(^a\)Newport County was not directly clustered with the other four counties of the state in 1965-70

TABLE II: Most well-defined 1995-2000 migration regions and their isolation indices (cont.)
Missaukee, Osceola, Lake and Mason).

Adams County, PA was created from part of York County, PA (Table II).

We did omit from Table I the rather anomalous twelve-county, four-state (ID, OR, UT, WA) cluster found on p. 19 of the dendrograms, even though it has $i = 18.1505$. It is essentially composed of two rather remote noncontiguous (OR-WA and ID-UT) sets of areas united by the links Clark, ID $\rightarrow$ Sherman, OR (having a doubly-stochastic value of 0.270946, the 261-st largest) and Skamania, WA $\rightarrow$ Bear Lake, ID (0.135147, the 2,655-th largest). (We have no immediate explanation for these apparently surprisingly relatively large values.)

3. Cosmopolitan/Provincial Boundary

There is a large 2,423-county cluster (lacking all of the New England and Hawaiian counties)–having the high value $i = 27.7726$–stretching in the dendrogram from Navarro, TX (p. 9) until the very end (Autauga, AL). (This might be considered to be a domain of lesser cosmopolitan counties or groups of counties.) It is a subcluster of a 2,588-county cluster ($i = 27.7304$) stretching from Quay, NM (p. 7), again to the end. Still larger, but somewhat weaker, is a 3,069-county cluster, $i = 20.2582$, extending from Salt Lake, UT [p. 1] until the end. Further, starting with Wayne, NC, but excluding Androscoggin, ME (p. 8), there is a 2,483-county cluster extending to the end with $i = 19.7518$. If we were to maps such results, the cosmopolitan counties–it would seem–would comprise “archipelagos” in the “sea” of provincial counties.

C. Master dendrograms based on the square of the doubly-stochastic table

Matrix multiplying the doubly-stochastic form of the 1995-2000 (zero-diagonal) inter-county migration table by itself, we obtain another doubly-stochastic table (Fig. 12), but now one with non-zero diagonal entries–which ranged from a high of 0.314164 and 0.3060604 for the members of a pair of small Georgia counties–Webster and Stewart (we recall that these two counties were the first cluster formed in the hierarchical [agglomerative] process–to lows of 0 and 0.0000253087 for the Hawaii counties of Kalawao and Kauai, respectively. (Kalawao County–once the site of a leprosy colony–was in many respects anomalous because of its very small size, and might in retrospect been readily omitted from the analyses.
FIG. 12: Square of the doubly-stochastic form (Fig. 6) of the 1995-2000 intercounty U. S. migration table. Only 2.82% of the entries of the matrix are 0, while 92.3% are in the unsquared matrix.

Of course, if a county has a large associated diagonal entry in the newly derived doubly-stochastic table, its off-diagonal entries, which are the only ones which affect its clustering properties, will tend to be reduced in size.) The resulting master dendrograms (presented in the Electronic-only material, along with the ultrametric [ordinal] fit in Fig. 13, the residuals from this fit in Fig. 14 and the goodness-of-fit measure in Fig. 15)–again employing the strong component hierarchical clustering methodology–are even more biased in cosmopolitanism to Sunbelt counties. (The largest 163,341 doubly-stochastic values were required to complete the strong component hierarchical clustering, much more than the 25,329 needed in the original [unsquared] analysis.)

There are obviously many interesting significant features in these figures, as the many long strings of counties within single states, apparent upon examination, would indicate. The isolation index for the seven-county “South Jersey” migration region (p. 7) has now
climbed to 49.8337, while the five clustered counties of Hawaii (p. 12) have an index of 34.4687. There is a still higher index of 64.2316 for a 39-county region (pp. 6-7) composed of all the counties of Maine, New Hampshire and Vermont, but for Bennington County, VT (p. 6), which is adjacent to New York and clustered with counties of that (non-New England) state instead. This 39-county tri-state region is included along with Eastern Upstate New York counties in a 66-county region (with a very high $i = 118.237$). There was now also a 7-county Connecticut region (lacking New York City “bedroom”-suburban Fairfield County), having $i = 32.913$ (p. 5).

Additionally, to identify some of the other prominent clusters, we have a 15-county New York region ($i = 34.0272$, p. 13), a 12-county Arkansas region ($i = 32.9012$, pp. 8-9), a 20-county (northeastern) North Carolina region ($i = 32.5555$, pp. 14-15), a 32-county Ohio region ($i = 29.8344$, pp. 18-19), a 16-county (western) South Carolina region ($i = 29.9287$, p. 19).
FIG. 14: Residuals (mostly negative) of the fit of the ultrametric structure (Fig. 13) to the square of the 1995-2000 doubly-stochastic table internal migration flow table (Fig. 12), an 83-county NJ-NY-PA-WV (“Rust Belt”) region ($i = 26.9231$, pp. 12-13), a 17-county tri-state “Delmarva” region ($i = 26.1837$, p. 13), and a joining of seven northern Florida counties with seventy-seven of Georgia ($i = 24.5441$, pp. 29-30).

D. Use of teleporting random walk

Motivated by the widespread interest in and emulation of the PageRank algorithm used by popular search engines such as Google [58, 59], we took a weighted combination of the doubly-stochastic 1995-2000 U. S. intercounty migration table and the zero-diagonal $3,107 \times 3,107$ doubly-stochastic table with all its off-diagonal entries equal to $\frac{1}{3106}$. A weight of 0.9 was applied to the former table, and 0.1 to the latter table.

Again, precisely (the largest) 25,329 entries of the resultant table were needed to com-
FIG. 15: Goodness-of-fit of the ultrametric structure to the $k$ largest values in the square of the 1995-2000 doubly-stochastic table. The best fit, 0.130718, is attained at rank 12,603, corresponding to a doubly-stochastic value of 0.0206195. At that threshold, there are 377 clusters (strong components).

We noted that “South Jersey” was again a 7-county cluster with precisely the same isolation index of 28.7301 (Table I). Overall, our original clustering appeared to be totally robust in its qualitative features to the effect of the teleporting random walk, at least with the particular weights (0.9,0.1) we employed. Potentially, if the square of the doubly-stochastic table were similarly teleported, the associated hierarchical clustering results might not be so robust.

VI. CONCLUDING REMARKS

A. Aggregation issues

One might—using the indicated two-stage procedure—compare the hierarchical structure of geographic areas using internal migration tables at different levels of geographic aggregation (counties, states, regions...) (cf. [93]). To again use the example of France, based on a 21 × 21 interregional table for 1962-68, Région Parisienne was the most hub-like [3, Sec. 4.1]
while using a finer 89 × 89 interdepartmental table for 1954-62, the dyad composed of Seine (that is Paris and its immediate suburbs) together with the encircling Seine-et-Oise (administratively eliminated in 1964) was most cosmopolitan [7, 8, Sec. 6.1]. (In [93], “two distinct approaches to assessing the effect of geographic scale on spatial interactions” were developed.)

We, in fact, can directly compare the results of our U. S. 1965-70 migration between 3,140 counties study [23] with a highly detailed study [14] for the very same period conducted on the more aggregate level of 510 State Economic Areas (SEAs, collections of counties). In terms of relative cosmopolitan characteristics, the list based on the SEAs does have some different emphases than that given above in terms of the counties. According to Fig. 2 of [14], the most cosmopolitan SEAs, in decreasing order, were Alaska; Hawaii (2 SEAs); Southeast Florida (3 SEAs); Southwest Florida; North Florida; the Chicago SMSA; the New York SMSA; Norfolk-Portsmouth SMSA; San Bernadino and Riverside SMSA; the District of Columbia; and the Maryland suburbs of D. C. (2 SEAs). (As previously noted, in the county-level 1965-70 analysis, the two most cosmopolitan entities were the Chicago metropolitan pair of Cook and DuPage, and the District of Columbia—while in the 1995-00 intercounty analysis, Brevard, FL and Mohave, AZ played these roles.) Let us also bring to the reader’s attention a 2005 discussion paper in which the 1995-2000 U. S. interstate migration table is studied using both double-standardization and “social network analysis” [60]. (Hierarchical clustering is also employed, but apparently not that form based on strong components.)

B. Max-flow/Min-cut application

In [102], Newman applied the famous Ford-Fulkerson max-flow/min-cut theorem [103] Chap. 22] to weighted networks (which he mapped unto unweighted multigraphs). Earlier, this theorem had been used to study Spanish [85], Philippine [104], and Brazilian, Mexican and Argentinian [105] internal migration, US interindustry flows [24, pp. 18-28] [106, 82, Sec. III] and the international flow of college students [21] (cf. [107])—all the corresponding flows now being left unadjusted, that is not (doubly- nor singly-) standardized.

In this “multiterminal” approach, the maximum flow and the dual minimum edge cut-sets, between all ordered pairs of nodes are found. Those cuts (often few or even null in
number) which partition the \( N \) nodes nontrivially—that is, into two sets each of cardinality greater than 1—are noted. The set in each such pair with the fewer nodes is regarded as a nodal cluster (region, in the geographic context). It has the interesting, defining property that fewer people migrate into (from) it, as a whole, than into (from) its node. In the Spanish context, the (nodal) province of Badajoz was found to have a particularly large out-migration sphere of influence, and the (Basque) province of Vizcaya (site of Bilbao and Guernica), an extensive in-migration field \[85\]. In an analysis of 1967 US interindustry transactions based on 468 industries, among the industries functioning as nodes of production complexes with large numbers of members were: Advertising; Blast Furnaces and Steel Mills; Electronic Components; and Paperboard Containers and Boxes. Conversely, among those serving as nodes of consumption complexes were Petroleum Refining and Meat Animals \[82, 106\].

C. Subdominant eigenvalue

Pentney and Meila have extended spectral clustering algorithms to “asymmetric affinities” \[108\]. In line with their approach, we computed the subdominant eigenvalue (0.906253) of the \( 3,107 \times 3,107 \) doubly-stochastic 1995-2000 intercounty migration table, and the associated eigenvector. (Trivially, the dominant eigenvalue is 1, and the components of the corresponding eigenvector all equal.) Interestingly, the largest (most positive) seventy-eight components of this vector all corresponded to counties of Georgia, while the smallest (most negative) one hundred and ten components were all from one or another of the contiguous triad of Great Plains states, North Dakota, South Dakota and Nebraska. (The most negative two values are for Dewey and Ziebach Counties of South Dakota, which as we have previously indicated form the Cheyenne Indian Reservation, and was the first cluster to form in the 1965-70 [agglomerative] hierarchical clustering \[23\].) In Fig. \[16\] we present a “list plot” of these components. (The counties are listed alphabetically within states, and the states themselves alphabetically also.) A rough gestalt estimate might yield some thirty to forty clusters. Dorogovtsev and Mendes have reviewed “the recent rapid progress in the statistical physics of evolving networks” \[109\].
FIG. 16: Components of the subdominant eigenvector of the doubly-stochastic form of the 1995-2000 U. S. intercounty migration table. The highest-situated evident cluster is composed of counties of the state of Georgia

D. U. S. internal migration network

We have presented (Electronic material) master dendrograms descriptive of the rich geographical and sociological evolving tapestry of the United States–as reflected in the 1965-1970 and 1995-2000 migration flows between the 3,000+ county-level units. Our results have been derived using a demonstratedly-insightful two-stage methodology–double-standardization of the recorded flows followed by (strong component) hierarchical clustering–applicable to (weighted, directed) socioeconomic networks, in general. Applying a graph-theoretic isolation criterion, we extracted particularly distinct large multicounty migration regions, well describable as “French Louisiana”, “Northern Lower Michigan”, “Northern New England”, et al. Certain tightly-knit functional clusters–for example, the states of Connecticut, Hawaii, as well as “South Jersey”–are invariant over the thirty-year study period. Broad “cosmopolitan” or “hub-like” migration to and from “Sunbelt” counties (Clark County, Nevada [Las Vegas], for instance) became relatively more conspicuous and migration associated with counties with large military installations (Pierce County, Washington [Fort Lewis and McChord Air Force Base], for example), less so. Further, the most cosmopolitan units for 1965-70 (the paired Chicago metropolitan counties of Cook and DuPage, Illinois, and the District of Columbia heading the list) were more cosmopolitan in character than the leading ones in the later analysis. We supplemented these analyses by studying both the square and a “teleported random walk” form of the doubly-stochastic table, as well as its subdominant eigenvalue. The hierarchical clustering obtained is robust against teleportation.
Acknowledgments

I would like to express appreciation to the Kavli Institute for Theoretical Physics (KITP) for technical support, as well as Waldo Tobler for granting permission to use Figures 1-4.

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