Evolution of electromagnetic and Dirac perturbations around a black hole in Hořava gravity

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(Dated: May 17, 2011)

Abstract

The evolution of electromagnetic and massless Dirac perturbations in the space-time geometry of Kehagias-Sfetsos(KS) black hole in the deformed Hořava-Lifshitz(HL) gravity is investigated and the associated quasinormal modes(QNMs) are evaluated from time domain integration data. We find a considerable deviation in the nature of field evolution in HL theory from that in the Schwarzschild space-time and QNM region extends over a longer time in HL theory before the power-law tail decay begins. The dependence of the field evolution on the HL parameter $\alpha$ is also studied. In the time domain picture we find that the length of QNM region increases with $\alpha$. But the late time decay of field follows the same power-law tail behavior as in the case of Schwarzschild black hole.

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I. INTRODUCTION

Recently a renormalizable theory of gravity in 3+1 dimensions was proposed by Hořava, inspired from the Lifshitz theory in solid state physics, now known as Hořava-Lifshitz (HL) theory [1–3]. The theory is a potential candidate of quantum field theory of gravity. It assumes a Lifshitz-like anisotropic scaling between space and time at short distances, characterized by a dynamical critical exponent $z = 3$ and thus breaking the Lorentz invariance. While in the IR limit it flows to $z = 1$, retrieving the Einstein’s General Relativity (GR). The discussions on the consequences of theory are going on [4, 5]. As a new theory, it is interesting to investigate its various aspects in parallel. Since the theory has the same Newtonian and post Newtonian corrections as those of GR, systems of strong gravity, like black holes, are needed to get observable deviation from the standard GR. Various black hole solutions are found in HL theory [6–25] of which one with asymptotically flat Minkowski spacetime is the KS black hole obtained by applying deformation in the original theory [26]. Various aspects of KS black hole were explored in the past [27–39].

Perturbation of black hole is the best way to study the properties of these exotic objects. As it is well established, the evolution of perturbations in black hole space times involves three stages [40–42]. The first one is an initial response containing the information of the initial form of the original wave field followed by a region dominated by damped oscillation of the field called quasinormal modes, which depends entirely on the background black hole spacetimes. Finally the field decays at late time as a power-law fall off of the field. QNMs play an important role in astrophysical processes involving black holes. These interests are associated with their relevance in gravitational wave analysis [43]. Apart from the observational interest of detection of quasinormal ringing by gravitational wave detectors [44], study of QNM of AdS black holes are of particular interests in the study of AdS/CFT correspondence [45, 46]. It was conjectured that a large static black hole in AdS space corresponds to a thermal state in conformal field theory (CFT). The fundamental link between QNMs of a large AdS black hole and the dual field theory were shown in the work of Horowitz who computed the QNMs of AdS black holes [47]. Motivated by this, a series of studies on the QNMs in asymptotically AdS backgrounds were done for various fields and dimensions [48–51]. See Ref. [52] for a recent review of QNMs of the black hole. The late-time relaxation of field perturbation is also an interesting topic of study since it reveals the actual physical
mechanism by which a perturbed black hole sheds its hairs. It was first demonstrated by Price that in the background of the Schwarzschild spacetime the massless neutral field at late time dies off as $\Psi \sim t^{-(2\ell+2)}$ or $\Psi \sim t^{-(2\ell+3)}$ depending on the initial conditions and the multipole order $\ell$ \[53\]. The evolution of field and the spectra of QNMs may be different in various theories of gravity and would help us to distinguish these theories.

Dynamical evolution of scalar field in Horava black hole spacetimes is studied using Horowitz-Hubeny approach\[54\]. The spectrum of entropy/area is discussed from the viewpoint of QNMs of scalar field in HL gravity\[55, 56\]. QNMs of various fields around KS black hole spacetime were calculated using WKB method\[57–60\]. In earlier work\[61\] we have studied time evolution of scalar field using the time domain integration method\[50, 62, 63\]. A noticeable deviation of the evolution behavior of scalar field in the ringdown region from the standard Schwarzschild black hole was found. However the late time evolution of field fails to show any distinction from the Schwarzschild case. The late time behavior of massless scalar field depends only on the multipole order $\ell$ and decays as $t^{-(2\ell+3)}$. Where as massive field in the intermediate range the decays as $t^{-(\ell+3/2)} sin(mt)$, but in the asymptotic late time the decay is dominated by $t^{-5/6} sin(mt)$ tail.

It is interesting to see how various other field perturbations decay in KS black hole spacetime. In this paper we study the evolution of electromagnetic and massless Dirac fields in the KS black hole spacetime and probe the signature of the now theory by comparing the present results with those of the standard theory of GR. The paper is organized as follows. In Sec II we derive the radial wave equation for various field perturbations around KS black hole and discuss the numerical method used. In Sec III the evolution of electromagnetic and massless Dirac field are studied using the method of time domain integration and the QNMs are evaluated and the results are discussed in Sec IV.

### II. PERTURBATION OF記者 AROUND KS BLACK HOLE

The IR vacuum of pure HL gravity is found to be anti-de Sitter\[6, 7\]. Even though HL gravity could recover GR in IR at the action level for a particular value of the parameter $\lambda = 1$, there found a significant difference between these black hole solutions and the usual Schwarzschild AdS solution. The asymptotic fall-off of the metric for these black hole solutions is much slower than that of usual Schwarzschild AdS black holes in GR. Meanwhile
Kehagias and Sfetsos could find a black hole solution in asymptotically flat Minkowski spacetimes by applying deformation in HL theory by adding a term proportional to the Ricci scalar of three-geometry, $\mu^4 R^{(3)}$ while the cosmological constant $\Lambda_W \to 0$. This will not alter the UV properties of the theory but it does the IR ones leading to Minkowski vacuum analogous to Schwarzschild spacetime in GR. KS solution is given as,

$$ds^2 = -N(r)^2 dt^2 + f(r)^{-1} dr^2 + r^2 d\Omega^2,$$  

where $f(r)$ has the form,

$$N(r)^2 = f(r) = \frac{2(r^2 - 2Mr + \alpha)}{r^2 + 2\alpha + \sqrt{r^4 + 8\alpha Mr}}.$$  

The event horizons are at,

$$r_\pm = M \pm \sqrt{M^2 - \alpha}.$$  

When $\alpha = 0$ the solution reduces to the Schwarzschild spacetime case.

The evolution of massless scalar field $\Phi$, electromagnetic field $A_\mu$ and massless Dirac field $\psi$ in the spacetime $g_{\mu\nu}$, specified by Eq.(1) are governed by the Klein-Gordon, Maxwell’s and the Dirac equations respectively,

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) \Phi = 0,$$

$$F_{\mu\nu} = 0, \quad \text{with} \quad F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu},$$

$$[\gamma^a e^\mu_a(\partial_\mu + \Gamma_\mu)] \psi = 0.$$  

The radial part of the above perturbation equations can be decoupled from their angular parts and reduced to form,

$$\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2}\right) \Psi_\ell(t, r) = -V(r) \Psi_\ell(t, r) = 0,$$  

where $r_*$ is the tortoise coordinate defined by $dr_* = \frac{1}{f} dr$ and the effective potentials $V(r)$ for different fields are given by,

$$V_S(r) = f(r) \left[\frac{\ell(\ell + 1)}{r^2} + \frac{1}{r} \frac{\partial f(r)}{\partial r}\right], \quad \ell = 0, 1, 2, \ldots,$$
\[ V_{EM}(r) = f(r) \left[ \frac{\ell(\ell + 1)}{r^2} \right], \quad \ell = 1, 2, 3, \ldots, \quad (9) \]

\[ V_{D\pm}(r) = \sqrt{\frac{f(r)|k|}{r^2}} \left[ |k|\sqrt{f(r)} \pm \frac{r \partial f(r)}{2} \pm f(r) \right], \quad |k| = 1, 2, 3, \ldots, \quad (10) \]

Here \( k \) is positive or negative nonzero integer related to the total orbital angular momentum by \( \ell = |k + \frac{1}{2} - \frac{1}{2} \) and \( V_{D+} \) and \( V_{D-} \) are the super symmetric partners and give same spectra. So we choose \( V_{D+} \) by omitting the subscript. The perturbation equations (Eq.(7)) can be written in the null coordinates \( u = t - r^* \) and \( v = t + r^* \) as,

\[-4 \frac{\partial^2}{\partial u \partial v} \Psi(u, v) = V(u, v)\Psi(u, v). \quad (11)\]

In order to get the time evolution picture of the field we integrate this equation numerically using the following finite difference scheme suggested in [62],

\[ \Psi_N = \Psi_W + \Psi_E - \Psi_S - \frac{h^2}{8} V(S)(\Psi_W + \Psi_E) + O(h^4) \quad (12) \]

where the points N, S, E and W form a null rectangle with an overall grid scale factor of \( h \) having relative positions as \( N(u + h, v + h), W(u + h, v), E(u, v + h) \) and \( S(u, v) \). Since we are interested in the late-time behavior of the wave function which is found to be independent of the initial shape, we set \( \psi(u, v = 0) = 0 \) and a Gaussian profile \( \Psi(u = 0, v) = \exp \left[ -\frac{(v-v_c)^2}{2\sigma^2} \right] \).

In all our calculations we set the initial Gaussian with width \( \sigma = 3 \) centered at \( v_c = 10 \). In order to progress on the above numerical scheme one has to get the value of the potential at \( r(r_s) = r((v - u)/2) \) at each step. Here we use the Runge-Kutta method to numerically integrate the equation for the tortoise coordinate and find the values of \( r(r_s) \) at each step by cubic spline interpolation as suggested in Ref.[63]. After the integration is completed the values of \( \Psi \) on line \( u = v - 2c \) is extracted and plotted as a function of \( t = v - c \); here, \( c \) is the value of \( r_s \) at which the field is evaluated.

### III. TIME EVOLUTION OF FIELDS AROUND KS BLACK HOLE

The evolution of scalar field around KS black hole was studied in Ref. [61]. The typical nature of time evolution of a massless scalar field is shown in Fig[1]. The plot shows how the generic behavior of wave function (initial outburst, quasi-normal oscillations, and power-law
decay) in HL theory is differed from pure Schwarzschild spacetime. The QNM phase ends at a later time in HL theory and the oscillation frequency and the damping time have a higher values in HL theory. We can calculate the QNM frequencies from the numerically integrated data by nonlinear $\chi^2$ fitting. The calculated values of QNMs are given in Table III and Table III.

![FIG. 1: Time evolution of massless scalar field around KS black hole with $\alpha = 0.8$ (top curve) and the Schwarzschild black hole (bottom curve) for $\ell = 1$.](image)

| $\alpha$ | $Re(\omega)$ | $Im(\omega)$ | $Re(\omega)$ | $Im(\omega)$ |
|------|-------------|-------------|-------------|-------------|
| 0    | 0.29111     | -0.09800    | 0.29224     | -0.09758    |
| 0.2  | 0.29564     | -0.09424    | 0.29750     | -0.09418    |
| 0.4  | 0.30071     | -0.09008    | 0.30154     | -0.09027    |
| 0.5  | 0.30345     | -0.08777    | 0.30474     | -0.08817    |
| 0.6  | 0.30636     | -0.08526    | 0.30850     | -0.08564    |
| 0.8  | 0.31260     | -0.07937    | 0.31416     | -0.07979    |
| 1    | 0.31918     | -0.07162    | 0.32057     | -0.07247    |

TABLE I: QNM frequencies of massless scalar field for $\ell = 1$, calculated using WKB and numerical integration data
In order to study the evolution of electromagnetic field in HL black hole spacetime we numerically integrate the perturbation equation Eq. (11) with effective potential Eq. (9) using the time domain method. In Fig. 2(a) the time evolution of the electromagnetic field \( \Psi(t, r) \) at a fixed radius \( r_* = 10 \) for \( \ell = 1 \) is plotted in comparison with the corresponding Schwarzschild case. We can see that the time length of the QNM phase increases in HL theory. The late time tail starts at \( t \approx 300 \) for KS black hole with \( \alpha = 0.8 \) whereas it is at \( t \approx 230 \) for the Schwarzschild case. The oscillation frequency and the damping time have a higher values in HL theory. The variation of oscillatory region of \( \Psi(t, r) \) with the parameter \( \alpha \) is shown in Fig. 2(b). The oscillation frequency increases with \( \alpha \) and shows a slower damping for higher values of \( \alpha \).

The late-time behavior of the field around black hole spacetime is also an important topic of study in black hole physics. It was found that during the late-time a massless neutral field dies off as an inverse power of time by the factor \( t^{(2\ell+3)} \) depending on the multipole order of the perturbation \([53, 64]\). It interesting to see how the field at late time behaves in the HL theory.

Fig. 3(a) shows the late time behavior of wave function for different values of \( \alpha \), with multipole index \( \ell = 1 \). We find that the late time behavior is independent of \( \alpha \) and follows the behavior of the Schwarzschild case with \( \Psi \sim t^{-5.1} \). In Fig. 3(b) field evolution for different multipole index is shown with \( \alpha = 0.5 \). The perturbation dies off at late time as \( t^{-(2\ell+3)} \).
FIG. 2: Evolution of electromagnetic field for $\ell = 1$. (a) Around KS black hole with $\alpha = 0.8$ (top curve) and the Schwarzschild black hole (bottom curve). (b) QNM region of the time evolution for different values of $\alpha$. Curves from bottom to top is for $\alpha = 0, 0.4, 0.8$ and 1.

as in the case of Schwarzschild case. The field falls off as $\Psi \sim t^{-5.08}$, $t^{-7.09}$ and $t^{-9.09}$ for $\ell = 1, 2$ and 3 respectively. The predicted values are -5, -7 and -9 respectively.

FIG. 3: Late time decay of electromagnetic field. (a) for different values of $\alpha$ with $\ell = 1$. The Field decay as an inverse power of time with $t^{-5.08}$, for all values of $\alpha$. (b) Decay of field with different multipole order $\ell$ for $\alpha = 0.5$. The field decay as $t^{-5.08}$, $t^{-7.09}$ and $t^{-9.09}$ for $\ell = 1, 2$ and 3.

We have seen that quasinormal ringing phase is a dominated form of decay in the evolution of perturbations after the initial transient phase. The QNMs calculated from the numerically integrated data are given in Table III and Table III. We can find a good agreement with the earlier results obtained using WKB method in Ref. [60]. Results show that the oscillation frequency and the damping time increase with $\alpha$. 
TABLE III: QNM frequencies of electromagnetic field for $\ell = 1$, calculated using WKB and numerical integration data

| $\alpha$ | WKB $Re(\omega)$ | WKB $Im(\omega)$ | Time domain $Re(\omega)$ | Time domain $Im(\omega)$ |
|----------|-------------------|--------------------|--------------------------|--------------------------|
| 0        | 0.24587           | -0.09311           | 0.24401                  | -0.09051                 |
| 0.2      | 0.25152           | -0.08942           | 0.25225                  | -0.08706                 |
| 0.4      | 0.25796           | -0.08530           | 0.25733                  | -0.08447                 |
| 0.5      | 0.26151           | -0.08299           | 0.26383                  | -0.08135                 |
| 0.6      | 0.26532           | -0.08047           | 0.26575                  | -0.08009                 |
| 0.8      | 0.27375           | -0.07435           | 0.27771                  | -0.07454                 |
| 1        | 0.28308           | -0.06569           | 0.28478                  | -0.06581                 |

TABLE IV: QNM frequencies of electromagnetic field for $\ell = 2$, calculated using WKB and numerical integration data

| $\alpha$ | WKB $Re(\omega)$ | WKB $Im(\omega)$ | Time domain $Re(\omega)$ | Time domain $Im(\omega)$ |
|----------|-------------------|--------------------|--------------------------|--------------------------|
| 0        | 0.45713           | -0.09506           | 0.45088                  | -0.09471                 |
| 0.2      | 0.46531           | -0.09179           | 0.46239                  | -0.09117                 |
| 0.4      | 0.47453           | -0.08799           | 0.47153                  | -0.08711                 |
| 0.5      | 0.47961           | -0.08582           | 0.47835                  | -0.08502                 |
| 0.6      | 0.48507           | -0.08339           | 0.48332                  | -0.08183                 |
| 0.8      | 0.49736           | -0.07743           | 0.49756                  | -0.07439                 |
| 1        | 0.51197           | -0.06887           | 0.52823                  | -0.06897                 |

Now we study the evolution of Dirac field in HL black hole spacetime. We numerically integrate the perturbation equation Eq. (11) with the effective potential Eq. (10) using the time domain method. Fig. 4(a) displays the time evolution of the Dirac wave function $\Psi(t, r)$ at a fixed radius $r_\ast = 10$, for $k = 2$. The deviation of generic time dependence of wave function in the HL theory from pure Schwarzschild spacetime is clear in the plot. QNM region lasts for a longer time in HL theory. The late time tail starts at $t \approx 310$ for $\alpha = 0.8$. 


whereas it is at \( t \approx 240 \) for the Schwarzschild case. Also the oscillation frequency and the damping time have a higher values in HL theory. The variation of oscillatory region of \( \Psi(t, r) \) with the parameter \( \alpha \) is shown in Fig.4(b). The oscillation frequency increases with \( \alpha \) and shows a slower damping for higher values of \( \alpha \).

![Fig. 4: Time evolution of Dirac field with \( k = 2 \). (a) Around KS black hole(top curve) and the Schwarzschild black hole(bottom curve). (b)QNM region of the time evolution of Dirac field for different values of \( \alpha \). Curves from bottom to top is for \( \alpha = 0, 0.4, 0.8 \) and 1.](image)

Fig.5(a) shows the late time behavior wave function for different values of \( \alpha \), with \( k = 2 \). The late time behavior is independent of \( \alpha \) and field decays in the inverse power of time as \( t^{-5.08} \). In Fig.5(b), field evolution for different multipole indices are shown with \( \alpha = 0.5 \). The perturbation dies off at late time as \( \Psi \sim t^{-3.08}, t^{-5.08} \) and \( t^{-7.09} \) for \( k = 1, 2 \) and 3 respectively. The predicted values are -3, -5 and -7 respectively.

The quasinormal ringing phase of the perturbation can be seen clearly in these figures. The calculated values of QNMs from the time domain data are given in TableIII and TableIII. We can find a good agreement of the values obtained in Ref.59 from the WKB scheme. Results show that the oscillation frequency and the damping time increase with \( \alpha \).

Finally we compare the time evolution of different fields in HL gravity. In Fig.6(a) massless scalar, Dirac and electromagnetic fields are plotted for \( \alpha = 0.5 \). The only difference between evolution of these three fields is in the QNM phase whereas the late time tails follow the same decaying pattern with same power law exponent. In Fig.6(b) the variation of real and imaginary part of QNM versus \( \alpha \) are plotted. All the three fields show the same dependence on the HL parameter \( \alpha \).
FIG. 5: Late time decay of Dirac field. (a) For different values of $\alpha$ with $k = 2$. The field decay as an inverse power law of time with $t^{-5.08}$ for all values of $\alpha$. (b) Decay of Dirac field for different $k$ with $\alpha = 0.5$. The field decay as $t^{-3.08}, t^{-5.08}$ and $t^{-7.09}$ for $k = 1, 2$ and 3 respectively.

| \(\alpha\) | \(Re(\omega)\) | \(Im(\omega)\) | \(Re(\omega)\) | \(Im(\omega)\) |
|-----------|----------------|----------------|----------------|----------------|
| 0         | 0.17645        | -0.10011       | 0.17830        | -0.10744       |
| 0.2       | 0.18187        | -0.09614       | 0.17952        | -0.09481       |
| 0.4       | 0.18709        | -0.09167       | 0.18756        | -0.09312       |
| 0.5       | 0.18963        | -0.08916       | 0.19040        | -0.09249       |
| 0.6       | 0.19212        | -0.08639       | 0.19333        | -0.09172       |
| 0.8       | 0.19681        | -0.07981       | 0.19635        | -0.07572       |
| 1         | 0.20053        | -0.07113       | 0.200101       | -0.06939       |

TABLE V: QNM frequencies of Dirac field for $k = 2$, calculated using WKB and numerical integration data

IV. SUMMARY

The evolution of electromagnetic and massless Dirac perturbations in KS black hole in the deformed Hořava-Lifshitz gravity is studied in this work. Time domain integration method is used to obtain the evolution picture. The question raised was whether we can distinguish the fundamental nature of the HL theory from a knowledge of the evolution of fields around corresponding black holes. Comparing with the Schwarzschild case, we have found that
TABLE VI: QNM frequencies of Dirac field for $k = 2$, calculated using WKB and numerical integration data

| $\alpha$ | $Re(\omega)$ | $Im(\omega)$ | $Re(\omega)$ | $Im(\omega)$ |
|----------|---------------|---------------|---------------|---------------|
| 0        | 0.37863       | -0.09654      | 0.37348       | -0.08850      |
| 0.2      | 0.38489       | -0.09317      | 0.38126       | -0.09488      |
| 0.4      | 0.39195       | -0.08932      | 0.38935       | -0.09014      |
| 0.5      | 0.39583       | -0.08712      | 0.39126       | -0.08816      |
| 0.6      | 0.39998       | -0.08468      | 0.39153       | -0.08388      |
| 0.8      | 0.40920       | -0.07869      | 0.40337       | -0.07758      |
| 1        | 0.41978       | -0.07021      | 0.41415       | -0.07239      |

FIG. 6: (a) Evolution of massless scalar, Dirac and electromagnetic fields around KS black hole with $\alpha = 0.5$ and $\ell = 1$. (b) QNMs of different field as a function of $\alpha$ with $\ell = 1$.

The QNM phase has been extended for a longer time in HL theory before the power-law tail begins. Also the oscillation frequency and the damping time have higher values in HL theory. The variation of the field evolution on the Horava parameter $\alpha$ are also studied. In the time domain picture we find that the length of QNM region increases with $\alpha$. The late time decay of field is found to be independent of nature of field and the HL parameter $\alpha$ and follows the same power-law tail behavior as in the case of Schwarzschild black hole. These results will help us to check the correctness of the theory provided our gravitational
detectors can trap these signals with high accuracy.

**Acknowledgments**

NV wishes to thank UGC, New Delhi for financial support under RFSMS scheme. VCK is thankful to CSIR, New Delhi for financial support under Emeritus Scientistship scheme and wishes to acknowledge Associateship of IUCAA, Pune, India. The authors are grateful to Dr. Alexander Zhidenko for the help and clarifications received in developing the code for the numerical integration.

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