Dense Wavelength Division Multiplexed Hyperentanglement

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Entanglement is a key resource in quantum information science and associated emerging technologies. Photonic systems offer a large range of exploitable entanglement degrees of freedom such as frequency, time, polarization, and spatial modes. Hyperentangled photons exploit multiple degrees of freedom simultaneously to enhance the performance of quantum information protocols. Here, we report a fully guided-wave approach for generating and analyzing polarization and energy-time hyperentangled photons at telecom wavelengths. Moreover, by demultiplexing the broadband emission spectrum of the source into five standard telecom channel pairs, we demonstrate compliance with fiber network standards and improve the effective bit rate capacity of the quantum channel by one order of magnitude. In all channel pairs, we observe a violation of a generalized Bell inequality by more than 27 standard deviations, underlining the relevance of our approach.

Introduction

Precise engineering and control of entanglement has led to remarkable advances in quantum information science. Notably, photonic entanglement has demonstrated advantages over classical means for provably securing communication\textsuperscript{[1]}, solving computational problems faster and/or with reduced resources\textsuperscript{[2, 3]}, and in optical sensing with increased resolution and precision\textsuperscript{[4–6]}. It was shown that a practical quantum advantage is reached with systems comprising a few tens of qubits\textsuperscript{[7]}. To engineer and access the resulting Hilbert space, one way is to coherently superpose $n \gg 10$ photons emitted from quantum dots\textsuperscript{[8, 9]} or from spontaneous parametric downconversion (SPDC) sources\textsuperscript{[10]}. In order to reduce the associated challenges on the photon generation side\textsuperscript{[11]}, one can alternatively use less photons that are entangled over more degrees of freedom (DOF) simultaneously, referred to as hyperentanglement\textsuperscript{[12, 13]}. In this sense, photons are pertinent carriers, offering a large variety of exploitable DOFs such as polarization, frequency, time-bin, spatial mode, and orbital angular momentum. Hyperentangled states are advantageous over their single-observable counterparts in many ways. They lead to a stronger violation of local realist theories, making them less sensitive against decoherence\textsuperscript{[14]}. From the applied side, complete photonic Bell state analysis can be implemented\textsuperscript{[17]}, having immediate repercussions for high-level quantum networking protocols such as teleportation. Detection failure on one DOF does not necessarily stop networking activity, as faithful entanglement transport is still achieved over all other DOFs. Additionally, as less particles need to be detected to access high-dimensional Hilbert spaces, efficiency is increased and experimental complexity is reduced.

The quality of a hyperentanglement source can be inferred through the violation of a generalized Bell inequality as defined by Barbieri \textit{et al.}\textsuperscript{[14]}. One starts with two Bell operators $\beta_{1, 2}$—one for each DOF, aiming at achieving a maximum violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality. For theories admitting local elements of reality, it is $|\langle \beta_{1, 2} \rangle| \leq 2$, while quantum physics permits reaching $|\langle \beta_{1, 2} \rangle| = 2\sqrt{2}$. Most previous experiments inferred $|\langle \beta_{1, 2} \rangle|$ through sequential measurements on each individual DOF with adapted analyzers\textsuperscript{[13, 18]}. However, to demonstrate a practical quantum networking advantage, all involved DOFs must be measured simultaneously. This allows violating a generalized CHSH inequality through the operator $\beta = \beta_1 \otimes \beta_2$\textsuperscript{[14]} for which local realist theories predict $|\langle \beta \rangle| \leq 4$, and quantum physics permits reaching a twice a high value, $|\langle \beta \rangle| = 8$.

In this work, we demonstrate a practical and fully guided-wave source of telecom-wavelength hyperentangled photon pairs. We exploit polarization and energy-time DOFs as they can be efficiently guided in standard fiber optical networks. In our experimental configuration (see Figure 1(a)) we infer hyperentanglement directly through the operator $\beta$ without replacing components on the setup. To further enhance the quantum channel networking capacity, we demonstrate also compliance with telecommunication standards by analyzing hyperentanglement in five dense wavelength division multiplexed channel pairs\textsuperscript{[19]}. By combining hyperentanglement and multiplexing, we achieve a one order of magnitude increase in the quantum channel capacity compared to ordinary entanglement distribution. This paves the way for more efficient quantum information protocols, notably in quantum communication and computation\textsuperscript{[20]}.

Wavelength multiplexed hyperentanglement generation and analysis

Our hyperentanglement source generates a two-photon state of the form

$$|\Psi \rangle = \prod_{k=1}^{5} \left( |H\rangle |H\rangle + |V\rangle |V\rangle \right)_{k} \otimes |E\rangle |E\rangle + |L\rangle |L\rangle_{k}. \quad (1)$$
Figure 1. Setup for the generation and analysis of hyperentanglement. (a) The photon pair source is based on a fully fibred nonlinear Sagnac interferometer [21]. Blue lines indicate PMFs. After deterministic separation of signal and idler photons, they are sent to their respective hyperentanglement analyzers. Each of them is composed of an unbalanced MI for energy-time entanglement analysis and a polarization state analyzer made of a HWP and PBS. Thereafter, the photons are spectrally demultiplexed using DWDMs and detected using standard SPDs. Hyperentanglement in wavelength anticorrelated channel pairs is revealed through coincidence detection. (b) Photon pair emission spectrum centred at 1560 nm. The insets show how Alice’s and Bob’s photons are spectrally demultiplexed accordingly to the ITU 100 GHz channel grid.

Here, $H$ and $V$ represent horizontal and vertical photon polarization modes, and $E$ and $L$ denote early and late emission times of an energy anticorrelated photon pair [22]. The index $k$ labels the wavelength channel pair in which the pair is generated.

As depicted in Figure 1(a), our scheme is based on a fully guided-wave nonlinear Sagnac interferometer (see also [21]). A wavelength-stabilized fiber-coupled 780 nm continuous-wave laser is sent through a wavelength division multiplexer ($WDM_1$) to a nonlinear fiber Sagnac loop in which a fiber polarizing beam-splitter (f-PBS) define the input and output. After the f-PBS, horizontally (vertically) polarized light propagates in the clock-wise (counter-clockwise) direction in polarization maintaining fibers (PMF). One of the PMFs is physically rotated by $90^\circ$ such that vertically polarized 780 nm light pumps a $3.8$ cm long periodically poled lithium niobate waveguide (PPLN/w) from both sides simultaneously. Inside the PPLN/w, pump photons are converted to vertically polarized signal ($s$) and idler ($i$) photon pair contributions in both directions through type-0 SPDC. As shown in Figure 1(b), the photon pair emission spectrum shows a bandwidth of about 40 nm centered at the degenerated wavelength of 1560 nm, and we now define that signal (idler) photons are above (below) degeneracy. After the PPLN/w, the pair contributions are coupled back into the PMF, subsequently overlapped at the f-PBS, and separated from the pump at WDM$_1$. By precisely adjusting the polarization of the pump laser, a maximally polarization entangled Bell state is generated: $|\Phi^+_{\text{pol}}\rangle = \frac{1}{\sqrt{2}} (|H_s\rangle|H_i\rangle + |V_s\rangle|V_i\rangle)$ [21]. The pairs are further deterministically separated as a function of their wavelengths using a standard telecom C/L-band splitter ($WDM_2$), and sent to Alice and Bob. We measure the photons’ polarization state using a half-wave plate (HWP), a PBS, and single-photon detectors (SPD, IDQ220) in each output arm. Coincidence counting allows then to reveal non local correlations.

The second DOF of our photon pairs is energy-time entanglement which is mediated through the energy conservation of the SPDC process. As the energy of each generated photon pair must equal the energy of one pump laser photon, the (vacuum) wavelengths of the involved photons are related by: $\lambda_{p}^{-1} - \lambda_{s}^{-1} + \lambda_{i}^{-1}$. Here, the subscript p stands for the pump photon. Energy-time entanglement is revealed using unbalanced Michelson interferometers (MI) in the Franson configuration [22]. For optimal stability, we choose MIs made of fused fiber beam-splitters (BS), and Faraday mirrors (FM) at which photons are reflected back to the BS [23]. Each MI has a travel time difference of $\Delta t \approx 300$ ps which is much larger than the single-photon coherence time $\tau_c \approx 5$ ps, and
therefore single photon interference is prohibited. However, we adjust both interferometers to have the same travel time difference within ±0.03 ps, such that higher-order interference is observed [10, 24]. This is enabled by the spontaneous character of the SPDC process and the use of a long coherence time continuous-wave pump laser. In consequence, a photon pair contribution that is generated at an early instant (E), where both photons propagate through the long MI arms, is indistinguishable from a pair that is created at a 300 ps later instant (L) in which both photons propagate along the short paths [22]. Through coincidence detection those interfering contributions are post-selected from the others in which photons take opposite paths in the MIs, interfering contributions are post-selected from the others in which photons take opposite paths in the MIs, such that higher-order interference is prohibited. How-


time entanglement simultaneously, the resulting overall quantum state reads

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|E_1\rangle |E_1\rangle + e^{i(\phi_0 + \phi_1)} |L_1\rangle |L_1\rangle) \otimes (|H_1\rangle |H_1\rangle + |V_1\rangle |V_1\rangle), \]

(2)

which covers a 16-dimensional Hilbert space.

Besides using hyperentanglement for doubling the quantum channel capacity, we additionally exploit standard telecom wavelength division multiplexing to achieve a further five times enhancement. As shown in Figure 1(b), we exploit the fact that the generated photon pairs are energy anticorrelated, i.e. generated pairwise symmetrically around the degenerate wavelength of 1560 nm. This allows demultiplexing the emission spectrum into several anticorrelated wavelength channels using standard telecom dense wavelength division multiplexers (DWDM). Accordingly to the International Telecommunication Union (ITU) standards in the 100 GHz grid [24], we demultiplex the spectrum into the channel pairs ITU10–33, ITU11–32, ITU12–31, ITU13–30, and ITU14–29, leading to the state |Ψ⟩ (cf. equation 1).

We measure the quality of the hyperentanglement by inferring ⟨β⟩ through coincidence detection at four measurement settings for each photon (two for each DOF), i.e. 16 settings in total. Alice projects the polarization state of the signal photon on two complementary angles \(\alpha_s\) and \(\alpha'_s = \alpha_s + 45^\circ\), and sets the phase of her MI to \(\phi_s\) and \(\phi'_s = \phi_s + \frac{\pi}{2}\). On the other side, Bob’s four settings for the idler photon are \(\alpha_i, \alpha'_i = \alpha_i + 45^\circ, \phi_i\) and \(\phi'_i = \phi_i + \frac{\pi}{2}\) [11].

\[\langle \beta \rangle = 45^\circ, \phi_s = 0\), and \(\alpha'_i = 45^\circ, \phi'_i = \frac{\pi}{2}\).\]

Figure 2 shows the obtained coincidence count rates in the channel pair ITU10–33 for four fixed settings at Alice’s analyzers: \(\{\alpha_s = 0^\circ, \phi_s = 0\}\), \(\{\alpha_s = 0^\circ, \phi'_s = \frac{\pi}{2}\}\), \(\{\alpha'_i = 45^\circ, \phi'_i = 0\}\), and \(\{\alpha'_i = 45^\circ, \phi'_i = \frac{\pi}{2}\}\).

\[\text{Figure 2. Coincidences count rates for four fixed settings at Alice’s analyzers: (a) } \{\alpha_s = 0^\circ, \phi_s = 0\}, \text{ (b) } \{\alpha_s = 0^\circ, \phi'_s = \frac{\pi}{2}\}, \text{ (c) } \{\alpha'_i = 45^\circ, \phi'_i = 0\}, \text{ and (d) } \{\alpha'_i = 45^\circ, \phi'_i = \frac{\pi}{2}\}.\]

For all measurements, Bob’s settings \(\alpha_i\) and \(\phi_i\) at Bob’s site. The results demonstrate the rotation invariance of the correlations as all measurements are essentially similar, except a polarization and/or phase offset equal to the settings \{\alpha'_i, \phi'_i\} of Alice’s analyzers. Experimental data are fitted with a 2-dimensional sine wave function through which interference fringe visibilities of 98.0%±1.5% are inferred for both polarization and energy-time observables.

Furthermore, we extract ⟨\(\beta\)⟩ for different polarization and phase settings \(\alpha_i\) and \(\phi_i\) at Bob’s site, for which the complementary settings are \(\alpha'_i = \alpha_i + 45^\circ\) and \(\phi'_i = \phi_i + \frac{\pi}{2}\). The results in Figure 3 show several local extrema of ⟨\(\beta\)⟩. The maximum at \(\{\alpha_i = 22.5^\circ, \phi_i = \frac{\pi}{4}\}\) amounts to \(\langle \beta \rangle = 7.73 ± 0.12\), thus violating the generalized CHSH Bell inequality by 31 standard deviations [14]. The inferred correlation strength for all the 16 different combinations of settings are shown in Table 1. Let us stress that for six (ten) combinations of settings, negative (positive) correlations are observed, similarly to standard Bell inequality tests where usually three positive and one negative correlators are found.

To further demonstrate that hyperentanglement is compatible with wavelength demultiplexing, we repeat the measurements for the same settings in four additional channel pairs. The summary of the experimental results...
In summary, we have demonstrated wavelength division multiplexing and therefore validating our approach. Entanglement is compatible with dense wavelength division multiplexing and therefore validating our approach.

### Conclusion

In summary, we have demonstrated wavelength division multiplexed hyperentanglement generation and analysis with high quality. In addition to the fundamental character of this work, our scheme can be straightforwardly applied to practical fiber-based quantum key distribution with up to one order of magnitude increased bit rates compared to ordinary schemes [19]. In this perspective, it has already been shown that wavelength division multiplexed quantum key distribution is possible with only a moderate increase in resources [23].

In view of such an implementation, the presented fully guided-wave scheme further allows ultra-compact and stable design, e.g., including a fiber pigtailed PPLN/w module and an integrated PBS. Additionally, the ability to perform complete photonic Bell state measurements allows high-efficiency implementation of complex quantum communication protocols, such as teleportation and entanglement swapping [17]. The performance of such protocols can be further enhanced using quantum memories capable of storing hyperentanglement [20]. The robustness of quantum networks is also increased through hyperentanglement [27]. For example, if one entanglement analyzer fails, secure quantum key distribution is still possible by exploiting the other DOFs. Besides potential applications in quantum communication, the very way that hyperentanglement is generated in our scheme allows it to be straightforwardly adapted to the needs of various experiments across all fields of quantum information science. As examples, a similar source has been used for a fundamental quantum physics experiment [28] and for the quantum-enhanced determination of fibre chromatic dispersion [21].

We therefore believe that our approach has the potential to become a working horse solution in a large variety of photonics applications where quantum enhancement is sought.

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**Table I. Measured correlation strengths** for the 16 measurement setting combinations in the wavelength channel pair ITU10–ITU33. The four polarization settings are \( \alpha_s = 0^\circ \), \( \alpha_s' = 45^\circ \), \( \alpha_i = 22.5^\circ \), and \( \alpha_i' = 67.5^\circ \). The phase-settings are \( \phi_s = 0 \), \( \phi_s' = \frac{\pi}{2} \), \( \phi_i = \frac{\pi}{4} \), and \( \phi_i' = \frac{3\pi}{4} \). Typical uncertainties are of about 0.03.

| Settings \( \{\alpha_s, \alpha_i\} \) | \( \{\alpha_s', \alpha_i'\} \) | \{\(\alpha_s', \alpha_i\)\} | \{\(\alpha_s, \alpha_i'\)\} |
|---|---|---|---|
| \( \{\phi_s, \phi_i\} \) | 0.51 | -0.33 | 0.46 | -0.41 |
| \( \{\phi_s', \phi_i\} \) | -0.57 | 0.34 | 0.50 | 0.54 |
| \( \{\phi_s', \phi_i\} \) | -0.36 | 0.30 | 0.62 | 0.52 |
| \( \{\phi_s', \phi_i\} \) | -0.69 | 0.58 | 0.55 | 0.46 |

**Table II. Results in different DWDM channels.** In all channel pairs, a strong violation of the generalized CHSH Bell inequality is observed [14].

| Signal channel | Idler channel | Maximum \(\langle \beta \rangle\) | Standard deviations |
|---|---|---|---|
| ITU | ITU |
| 10 | 33 | 7.73 ± 0.12 | 31 |
| 11 | 32 | 7.25 ± 0.12 | 27 |
| 12 | 31 | 7.63 ± 0.12 | 30 |
| 13 | 30 | 7.61 ± 0.11 | 32 |
| 14 | 29 | 7.78 ± 0.13 | 29 |

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