0\(^+\) \rightarrow 2\(^+\) 0\(\nu\beta\beta\) decay triggered directly by the Majorana neutrino mass

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Abstract

We treat 0\(^+\) \rightarrow 2\(^+\) 0\(\nu\beta\beta\) decays taking into account recoil corrections to the nuclear currents. The decay probability can be written as a quadratic form of the effective coupling constants of the right-handed leptonic currents and the effective neutrino mass. We calculate the nuclear matrix elements for the 0\(^+\) \rightarrow 2\(^+\) 0\(\nu\beta\beta\) decays of \(^{76}\)Ge and \(^{100}\)Mo, and demonstrate that the relative sensitivities of 0\(^+\) \rightarrow 2\(^+\) decays to the neutrino mass and the right-handed currents are comparable to those of 0\(^+\) \rightarrow 0\(^+\) decays.

The neutrinoless double beta (0\(\nu\beta\beta\)) decay can take place through an exchange of neutrino between two quarks in nuclei if the electron neutrino is a Majorana particle and has a nonvanishing mass and/or right-handed couplings [1–3]. There may be other possible mechanisms such as those involving supersymmetric particles which also cause the decay of two neutrons into two protons and two electrons [4–6]. In the present work, however, we restrict ourselves to the conventional two-nucleon and \(\Delta\) mechanisms of 0\(\nu\beta\beta\) decay through light Majorana neutrino exchange. From the analyses of experimental data on 0\(^+\) \rightarrow 0\(^+\) 0\(\nu\beta\beta\) decays, stringent limits on the effective neutrino decay through the effective coupling constants of the right-handed leptonic currents have been deduced (see e.g. [3,7] and the references quoted therein). On the other hand it still seems to be believed widely that 0\(^+\) \rightarrow 2\(^+\) 0\(\nu\beta\beta\) decays are sensitive only to the right-handed currents. In view of the theorem that the electron neutrino should have a nonvanishing Majorana mass if 0\(\nu\beta\beta\) decay occurs anyway [8–10], an observation of 0\(\nu\beta\beta\) decay due to right-handed interactions would certainly mean also a nonvanishing Majorana mass of the electron neutrino. The purpose of the present work is, however, not to investigate the role of the

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Majorana neutrino mass in such a sense, but to demonstrate that it causes $0^+ \rightarrow 2^+ \; 0\nu\beta\beta$ decays directly.

A direct contribution of the neutrino mass to $0^+ \rightarrow 2^+ \; 0\nu\beta\beta$ decays was considered in [3] taking into account the nuclear recoil currents, and the inverse half-life was given as

$$[\tau_{1/2}(0^+ \rightarrow 2^+)]^{-1} = F_{1+}(Z_{1+})^2 + F_{1-}(Z_{1-})^2 + F_{2+}(Z_{2+})^2 + F_{2-}(Z_{2-})^2,$$

(1)

where $F_{j\pm} \; (j = 1, 2)$ are the phase space integrals and

$$Z_{1\pm} = M_\lambda\langle\lambda\rangle - M_\eta\langle\eta\rangle \pm M_m \frac{\langle m_\nu \rangle}{m_e},$$

$$Z_{2\pm} = M'_\eta\langle\eta\rangle \pm M_m \frac{\langle m_\nu \rangle}{m_e},$$

(2)

with the electron mass $m_e$ and

$$\langle m_\nu \rangle = \sum_j' U_{ej}^2 m_j,$$

$$\langle\lambda\rangle = \lambda \sum_j' U_{ej} V_{ej},$$

$$\langle\eta\rangle = \eta \sum_j' U_{ej} V_{ej}.$$  

(3)

Here $m_j$ is the mass of the eigenstate Majorana neutrino $N_j$, $U_{ej}$ and $V_{ej}$ are the amplitudes of $N_j$ in the left- and right-handed electron neutrinos, and the summations should be taken over light neutrinos ($m_j \ll 100 \text{ MeV}$). The nuclear matrix elements $M_\alpha \; (\alpha = \lambda, \eta, m)$ are defined by

$$M_\alpha = \langle 2^+_F\| \frac{1}{2} \sum_{n,m} \tau_n^+ \tau_m^+ (M_\alpha)_{nm}\|0^+_I \rangle.$$  

(4)

The explicit forms of the two body operators $M_\lambda$, $M_\eta$ and $M'_\eta$ were given in [11] including the contribution of the $\Delta$ mechanism, in which the $0\nu\beta\beta$ decay proceed through an exchange of a Majorana neutrino between two quarks in the same baryon in a nucleus. On the other hand the operator $M_m$ was derived in [3] as

$$(M_m)_{nm} = -\frac{1}{2} i m_e \left\{ [\mathbf{r}_{nm} \otimes (\mathbf{\sigma}_n C_m - \mathbf{\sigma}_m C_n)]^{(2)} \right\}$$
\[ + (g_V / g_A)^2 \left[ r_{nm} \otimes (D_n - D_m) \right]^{(2)} \} H(r_{nm}), \]

where \( r_{nm} = r_n - r_m \), \( H(r) \) is the neutrino propagation function, \( g_V \) and \( g_A \) the vector and axial vector coupling constants. \( C_n \) and \( D_n \) are the recoil correction terms to the axial vector and vector nuclear currents \([2,12]\) given by

\[
C_n = (p_n + p'_n) \cdot \sigma_n / 2M, \\
D_n = [p_n + p'_n - i\mu_\beta \sigma_n \times (p_n - p'_n)] / 2M,
\]

where \( p_n \) and \( p'_n \) are the initial and final nucleon momenta, \( M \) the nucleon mass, and \( \mu_\beta = 4.7 \). The above expression for \( M_m \) is, however, not suitable for numerical calculations as it stands. Therefore, as was done for \( M_\lambda, M_\eta \) and \( M'_\eta \) in \([11]\), we expand it in terms of the operators \( M_{inm} \) with simpler spin and orbital structures,

\[ (M_m)_{nm} = \sum_i C_{mi} M_{inm}. \]

We define the matrix element \( M_i \) of the operator \( M_{inm} \) analogously to Eq. (4). The coefficients \( C_{mi} \) and the two-body operators \( M_{inm} \) are listed in Table 1, where

\[
h = r_{nm} H(r_{nm}), \\
h' = -r_{nm} H'(r_{nm}), \\
S_{\lambda nm} = [\sigma_n \otimes \sigma_m]^{(\lambda)}, \\
S_{\pm nm} = \sigma_n \pm \sigma_m, \\
y_K nm = i[\hat{r}_{nm} \otimes \hat{r}_{nm}]^{(K)}, \\
y'_{K nm} = i[\hat{r}_{nm} \otimes \hat{P}_{nm}]^{(K)}, \\
Y_{K nm} = [\hat{r}_{nm} \otimes \hat{r}_{nm} + \hat{r}_{nm}]^{(K)} (r_{+nm}/r_{nm}), \\
Y'_{K nm} = i[\hat{r}_{nm} \otimes \hat{P}_{nm}]^{(K)}, \\
\hat{r}_{+nm} = r_n + r_m, \\
\hat{a} = a / |a|, \\
p_{nm} = \frac{1}{2} (p_n - p_m), \\
P_{nm} = p_n + p_m. \tag{8}
\]

As was described in detail in \([11]\), \( M_\lambda \) and \( M_\eta \) can be expanded in terms of \( M_{inm} \) with \( 1 \leq i \leq 5, 8 \leq i \leq 13 \), and \( M'_\eta \) in terms of \( M_{inm} \) with \( i = 6, 7 \) (for the definition of \( M_{inm} \) with \( 6 \leq i \leq 13 \), which do not appear in Table 1, see \([11]\)). Of these operators, \( M_{inm} \) with \( 8 \leq i \leq 13 \) are related to the \( 0 \nu \beta \beta \) transitions which involve virtual \( \Delta \) particles in nuclei, and they are induced by the operator \( M_{2nm} \) interpreted as acting on two quarks in a nucleon or a \( \Delta \) particle.

The new operators \( M_{inm} \) with \( 14 \leq i \leq 25 \) appear only in the expansion of \( M_m \). In the derivation of \( C_{mi} \) listed in Table 1, we have not taken into ac-
functions obtained in [11]. In order to calculate all $M_i$ of the products the calculated matrix elements $M_i$ constructed in the same manner as in the case of the $76^\text{Ge}$ decay, the calculation of the matrix elements $M_i$ with $1 \leq i \leq 13$ has been performed in [11]. In the present work we calculate only the new ones with $14 \leq i \leq 25$ using the nuclear wave functions obtained in [11]. In order to calculate all $M_i$ with $1 \leq i \leq 25$ for the $100^\text{Mo}$ decay, the nuclear wave functions of $100^\text{Mo}(0^+_1)$ and $100^\text{Ru}(2^+_1)$ are constructed in the same manner as in the case of the $76^\text{Ge}$ decay. Table 2 shows the calculated matrix elements $M_i$ for the $76^\text{Ge}$ and $100^\text{Mo}$ decays as a sum of the products $C_{mi}M_i$. It should be noted that the matrix elements of the operators with rank 0 spin part, *i.e.* $M_1$, $M_4$, $M_{14}$ and $M_{15}$ have the dominant contributions to $M_i$. Table 3 summarizes the calculated matrix elements $M_i$, $M'_\eta$, $M'_{\eta'}$ and $M_m$ for the $76^\text{Ge}$ and $100^\text{Mo}$ decays.

The differential rate for $0^+ \rightarrow 2^+ 0\nu\beta\beta$ decay with the energy of one of the emitted electrons $\epsilon_1$ and the angle between the two electrons $\theta_{12}$ can be written as

$$\frac{d^2W_{0\nu}}{d\epsilon_1 d\cos\theta_{12}} = a^{(0)}(\epsilon_1) + a^{(1)}(\epsilon_1)P_1(\cos\theta_{12}) + a^{(2)}(\epsilon_1)P_2(\cos\theta_{12}).$$

Each of the angular correlation coefficients $a^{(k)}(\epsilon_1)$ ($k = 0, 1, 2$) can be expressed as a sum of the products of an electron phase space factor and a second order monomial of $Z_{j\pm}$ defined in Eq. (2). The explicit form of $a^{(0)}(\epsilon_1)$, which yields $(\ln 2)/2$ times the right hand side of Eq. (1) upon integration over $\epsilon_1$, can be readily obtained by combining the relevant equations in [3]. Since the expressions for $a^{(1)}(\epsilon_1)$ and $a^{(2)}(\epsilon_1)$ are rather complicated, they will be given elsewhere. Numerical calculations show that $a^{(1)}(\epsilon_1)$ is dominated by a term with the factor $-(Z_{1+})^2 + Z_{2+}Z_{2-}$ times a positive function of $\epsilon_1$, whereas $a^{(2)}(\epsilon_1)$ by a term with the factor $2Z_{1+}Z_{1-} - (Z_{2+})^2 - (Z_{2-})^2$. For later reference we denote these two factors as $z^{(1)}$ and $z^{(2)}$, respectively.

Figure 1 shows the single electron spectra $dW_{0\nu}/d\epsilon_1 = 2a^{(0)}$ and the ratios of
the angular correlation coefficients $a^{(1)}/a^{(0)}$ and $a^{(2)}/a^{(0)}$ for the three limiting cases, (a) $\langle \lambda \rangle \neq 0$, (b) $\langle \eta \rangle \neq 0$ and (c) $\langle m_\nu \rangle \neq 0$. Since the coefficients $a^{(k)}(\epsilon_1)$ depend on the parameters $\langle \lambda \rangle$, $\langle \eta \rangle$ and $\langle m_\nu \rangle$ through $Z_{j\pm}$, the results shown in Fig. 1 are independent of nuclear models for the cases (a) and (c). We can also easily understand the signs of $a^{(1)}$ and $a^{(2)}$ from the relations $z^{(1)} = -(M_\lambda \langle \lambda \rangle)^2$ and $z^{(2)} = 2(M_\lambda \langle \lambda \rangle)^2$ for the case (a), and $z^{(1)} = -2(M_m\langle m_\nu \rangle/m_e)^2$ and $z^{(2)} = -4(M_m\langle m_\nu \rangle/m_e)^2$ for the case (c). On the other hand for the case (b), we obtain $z^{(1)} = -(M_\eta \langle \eta \rangle)^2 + (M'_\eta \langle \eta \rangle)^2$ and $z^{(2)} = 2(M_\eta \langle \eta \rangle)^2 - 2(M'_\eta \langle \eta \rangle)^2$, and consequently a cancellation between the contributions of $M_\eta$ and $M'_\eta$ occurs when these are of comparable magnitudes. This is just the case for the $^{100}$Mo decay, but not for the $^{76}$Ge decay where $M'_\eta$ is much smaller than $M_\eta$ so that there is no significant difference between the cases (a) and (b) in the angular correlation. It should also be noted in Fig. 1 that the single electron spectra for all the three cases (a), (b) and (c) have approximately the same shape. This is in contrast with the $0^+ \rightarrow 0^+$ decays where the spectrum for $\langle \lambda \rangle \neq 0$ is very different from those for $\langle m_\nu \rangle \neq 0$ or $\langle \eta \rangle \neq 0$ [2,3].

Using the matrix elements in Table 3 and the phase space integrals $F_{j\pm}$ calculated in [3], we can deduce from the experimental data $\tau_{1/2}^{0\nu}(0^+ \rightarrow 2^+_1) > 8.2 \times 10^{23}$ yr (90% C.L.) [14] for the $^{76}$Ge decay the constraints on the right-handed current couplings and the effective neutrino mass listed in Table 4. As for the $^{100}$Mo decay, the Osaka group has obtained the limit $\tau_{1/2}^{0\nu}(0^+ \rightarrow 2^+_1) > 1.4 \times 10^{22}$ yr (68% C.L.) [15] assuming $\langle \lambda \rangle \neq 0$. Because of the differences in the angular correlation as we see from Fig. 1, an analysis of the same raw experimental data might yield a half-life limit significantly different from the above value especially for the case $\langle \eta \rangle \neq 0$. However we assume here just the same half-life limit also for the cases $\langle \eta \rangle \neq 0$ and $\langle m_\nu \rangle \neq 0$ in order to compare the resulting constraints with those from the $^{76}$Ge data.

The limits which can be deduced from the experimental bound $\tau_{1/2}^{0\nu}(0^+ \rightarrow 0^+) > 5.7 \times 10^{25}$ yr (90% C.L.) [16] on the $0^+ \rightarrow 0^+$ decay of $^{76}$Ge using the nuclear matrix elements of [17] are $|\langle \lambda \rangle| < 3.8 \times 10^{-7}$, $|\langle \eta \rangle| < 2.2 \times 10^{-9}$ and $|\langle m_\nu \rangle| < 0.19$ eV. Comparing these limits with those of Table 4, we notice the considerable difference in the absolute sensitivities between the $0^+ \rightarrow 0^+$ and $0^+ \rightarrow 2^+$ decays, which reflects the smaller $Q$-value as well as the higher electron partial waves associated with the latter. However, it should be stressed here that the relative sensitivities to $\langle m_\nu \rangle$ and $\langle \eta \rangle$ are comparable in both cases. In other words, $\langle m_\nu \rangle = 1$ eV would give roughly the same decay rate as $\langle \eta \rangle = 10^{-8}$ in the $0^+ \rightarrow 2^+$ as well as in the $0^+ \rightarrow 0^+$ decays. At the same time it should also be noted that the $0^+ \rightarrow 2^+$ decay is relatively more sensitive to $\langle \lambda \rangle$.

In summary, we have calculated $0^+ \rightarrow 2^+$ $0\nu\beta\beta$ decay rates taking into account the recoil corrections to the nuclear currents. As a result, the expression for the decay probability becomes a quadratic form of not only the effective
coupling constants $\langle \lambda \rangle$ and $\langle \eta \rangle$ of the right-handed leptonic currents but also the effective neutrino mass $\langle m_\nu \rangle$ which would be totally absent without the inclusion of the recoil corrections. In other words, the recoil corrections give the lowest order contribution to the $0^+ \rightarrow 2^+ 0\nu\beta\beta$ decay for the case where $\langle \lambda \rangle = \langle \eta \rangle = 0$ and $\langle m_\nu \rangle \neq 0$. Furthermore, by the numerical calculation of the relevant nuclear matrix elements, we have demonstrated that the relative sensitivities of $0^+ \rightarrow 2^+$ decays to $\langle m_\nu \rangle$ and $\langle \eta \rangle$ are comparable to those of $0^+ \rightarrow 0^+$ decays.

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Fig. 1. Single electron spectrum $dW_{0\nu}/d\epsilon_1$ in arbitrary units and the ratios of the angular correlation coefficients $a^{(1)}/a^{(0)}$ and $a^{(2)}/a^{(0)}$ for the $0^+ \rightarrow 2^+_1$ $0\nu\beta\beta$ decay of $^{100}$Mo. They are all plotted against the kinetic energy fraction of one of the two emitted electrons, where $Q_{\beta\beta}(0^+ \rightarrow 2^+_1) = 2.494$ MeV. Only one of the three lepton number violating parameters is assumed to be nonvanishing for each of the three cases: (a) $\langle \lambda \rangle \neq 0$, (b) $\langle \eta \rangle \neq 0$ and (c) $\langle m_\nu \rangle \neq 0$. 

$\langle \epsilon_1 - m_e c^2 \rangle / Q_{\beta\beta}$
Table 1
The operators $M_{inm}$ and the coefficients $C_{mi}$, the latter in units of the electron-nucleon mass ratio $m_e/M$.

| $i$ | $M_{inm}$ | $C_{mi}$ |
|-----|-----------|----------|
| 1   | $-\sqrt{3}h'S_0y_2$ | $-\frac{1}{2}[\mu_\beta(g_V/g_A) + \frac{1}{2}]$ |
| 2   | $h'S_2$ | $\frac{1}{6}[\mu_\beta(g_V/g_A) - 1]$ |
| 3   | $h'[S_2 \otimes y_2]^{(2)}$ | $-\frac{\sqrt{3}}{4\sqrt{3}}[\mu_\beta(g_V/g_A) - 1]$ |
| 4   | $h'y_2$ | $\frac{1}{2}(g_V/g_A)^2$ |
| 5   | $h'[S_+ \otimes y_2]^{(2)}$ | $\frac{\sqrt{3}}{4\sqrt{2}}[\mu_\beta(g_V/g_A)^2 - (g_V/g_A)]$ |
| 14  | $-\sqrt{3}hS_0y'_2$ | $\frac{1}{3}$ |
| 15  | $hy'_2$ | $-(g_V/g_A)^2$ |
| 16  | $HS_2$ | $-\frac{1}{2}[\mu_\beta(g_V/g_A) - 1]$ |
| 17  | $hS_2y'_0$ | $-\frac{1}{\sqrt{2}}$ |
| 18  | $h[S_2 \otimes y'_1]^{(2)}$ | $-\frac{\sqrt{3}}{2}$ |
| 19  | $h[S_2 \otimes y'_2]^{(2)}$ | $-\frac{\sqrt{7}}{2\sqrt{3}}$ |
| 20  | $h[S_+ \otimes y'_1]^{(2)}$ | $\frac{1}{2\sqrt{2}}(g_V/g_A)$ |
| 21  | $h[S_+ \otimes y'_2]^{(2)}$ | $\frac{\sqrt{3}}{2\sqrt{2}}(g_V/g_A)$ |
| 22  | $h[S_1 \otimes Y'_1]^{(2)}$ | $-\frac{1}{4}$ |
| 23  | $h[S_1 \otimes Y'_2]^{(2)}$ | $-\frac{\sqrt{3}}{4}$ |
| 24  | $h[S_- \otimes Y'_1]^{(2)}$ | $-\frac{1}{4\sqrt{2}}(g_V/g_A)$ |
| 25  | $h[S_- \otimes Y'_2]^{(2)}$ | $-\frac{\sqrt{3}}{4\sqrt{2}}(g_V/g_A)$ |
Table 2
Calculated matrix elements $M_m$ for the $^{76}\text{Ge}$ and $^{100}\text{Mo}$ decays. The entries are the values of the products $C_m M_i$ and their sum $M_m$ in units of $10^{-3}\text{fm}^{-1}$.

| $i$ | $^{76}\text{Ge}$ | $^{100}\text{Mo}$ |
|-----|-----------------|-----------------|
| 1   | -0.0229         | -0.0077         |
| 2   | 0.0013          | 0.0003          |
| 3   | 0.0002          | 0.0008          |
| 4   | -0.0017         | -0.0011         |
| 5   | -0.0003         | 0.0007          |
| 14  | -0.0191         | -0.0227         |
| 15  | -0.0128         | -0.0112         |
| 16  | -0.0033         | -0.0006         |
| 17  | -0.0017         | 0.0000          |
| 18  | -0.0028         | 0.0019          |
| 19  | -0.0005         | 0.0001          |
| 20  | 0.0006          | 0.0005          |
| 21  | 0.0002          | -0.0001         |
| 22  | 0.0020          | -0.0000         |
| 23  | 0.0020          | -0.0026         |
| 24  | -0.0047         | -0.0001         |
| 25  | 0.0012          | -0.0010         |
| sum | -0.0624         | -0.0427         |

Table 3
Calculated matrix elements for the $0^+ \rightarrow 2_1^+ 0\nu\beta\beta$ decays of $^{76}\text{Ge}$ and $^{100}\text{Mo}$ in units of $10^{-3}\text{fm}^{-1}$.

|       | $M_\lambda$ | $M_\eta$ | $M'_\eta$ | $M_m$  |
|-------|-------------|----------|-----------|--------|
| $^{76}\text{Ge}$ | 1.81 a      | 13.37 a  | 0.18 a    | -0.0624 |
| $^{100}\text{Mo}$| -6.33       | 3.38     | 5.17      | -0.0427 |

$^a$ Ref. [11].
Table 4
Constraints on the right-handed current couplings and the effective neutrino mass.

|        | $^{76}$Ge | $^{100}$Mo |
|--------|-----------|------------|
| $|\langle \lambda \rangle |$  | $< 8.9 \times 10^{-4}$ | $< 3.9 \times 10^{-4}$ |
| $|\langle \eta \rangle |$  | $< 1.2 \times 10^{-4}$ | $< 4.3 \times 10^{-4}$ a |
| $|\langle m_\nu \rangle |$ [eV] | $< 1.0 \times 10^4$ | $< 2.2 \times 10^4$ a |

a Assuming the same limit on $\tau_{1/2}^{0\nu}$ as the $\langle \lambda \rangle$ mode.