Switching management by adiabatic passage in two periodically modulated nonlinear waveguides

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We theoretically investigate light propagation in two periodically modulated nonlinear waveguides with certain propagation constant detuning between two guides. By slowly varying the amplitude of modulation, we can steer the light to the desired output waveguide when equal amounts of lights are launched into each waveguide. We also reveal that the light propagation dynamics depends sensitively on the detuning between two guides. Our findings can be explained qualitatively by means of adiabatic navigation of the extended nonlinear Floquet states.

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In recent years, a lot of interests have been made to the theoretical and experimental advances in the field of engineered photonic structures[1,2]. Two illuminating examples are periodically modulated waveguide arrays and directional couplers. They not only provide an ideal platform for investigating a wide variety of coherent quantum effects including coherent enhancement and destruction of tunneling[3,4], Zener tunneling[5,6] and dynamical localization[7], but also open up exceptional opportunities for the control of light propagation such as discrete diffraction-managed solitons[8,9], all-optical switching of polychromatic or monochromatic light[10,11], soliton switching[12], and so on. In addition to periodic modulation of a photonic lattice by periodically curving waveguide or varying its refractive index along the propagation direction, adiabatic passage scheme in optical waveguide system by slowly varying its geometry or refractive index is also an attractive alternative for control of light tunneling and demonstration of adiabatic light transfer such as Landau-Zener tunneling[13], stimulated Raman adiabatic passage in linear[14] and nonlinear regimes[15], and autoresonant dynamics[16].

Combination of periodic modulation and adiabatic management of the system parameters provides an additional possibility for control of light propagation. Recently, some proposals have been suggested independently for transition of a superfluid to a Mott-insulator[17,18] and generation of coherent matter waves[19]. Two illuminating examples in many-body systems of driven Bose-Einstein condensates (BECs), and for realization of wave packet dichotomy[20] and adiabatic quantum state transfer[21] in modulated linear waveguide systems, both by slowly tuning the amplitude of modulation in these periodically driven systems. In two subsequent works, the mean-field dynamics of driven BECs has been investigated by extending the conventional Floquet states of linear systems to non-linear Floquet states, and it is found that atomic population can be precisely manipulated by adiabatically controlling nonlinear Floquet state on condition that the nonlinear strength is slowly changed[22,23]. In view of the analogy between the mean-field dynamics of BEC and optics of Kerr media, such two methods for adiabatic control of nonlinear Floquet state proposed in Refs. [22,23] may be applied to the modulated nonlinear waveguide systems. However, adiabatic management of Kerr nonlinearity is not so accessible as management of other system parameters (for example, the linear refractive index profile) in optical waveguide systems.

In this article, we consider light propagation in two periodically modulated nonlinear waveguides with certain propagation constant detuning between two guides. We find that through adiabatical increase of the amplitude of modulation, the light becomes concentrated in a single waveguide when equal amounts of lights are launched into each waveguide, and that the final light intensity distribution is highly determined by the detuning between two waveguides. Our results can offer benefits for all-optical switching and navigation of nonlinear Floquet state in the nonlinear waveguide systems.

We consider the simplest possible arrangement which consists of two coupled asymmetric waveguide elements with Kerr nonlinearity and with the linear refractive index periodically modulated along the propagation direction. We also suppose that each of the waveguides is single moded and excitation of radiation modes is neglected. Under these conditions, and with the use of coupled-mode theory, the evolution of the electric field for the two-channel coupler is described by the following set of equations:

\[
\frac{dc_1}{dz} = \frac{E_0}{2} c_1 + \frac{E(z)}{2} c_1 - \chi |c_1|^2 c_1 - \frac{v}{2} c_2, \quad (1)
\]

\[
\frac{dc_2}{dz} = -\frac{E_0}{2} c_2 - \frac{E(z)}{2} c_2 - \chi |c_2|^2 c_2 - \frac{v}{2} c_1, \quad (2)
\]

where \(c_1\) and \(c_2\) represent respectively field amplitudes in the first waveguide and the second waveguides, \(\chi\) is the strength of Kerr nonlinearity, \(v = \pi/L_c\) is the coupling constant with coupling length \(L_c\), \(E_0\) denotes the detuning between two waveguides, and \(z\) represents a dimensionless propagation distance. Here we take the form of \(E(z) = E_1 \cos(\omega z)\) with \(E_1\) being the amplitude and

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frequency of the modulation. It is clear that if we view \( z \) as time \( t \), the above equations can be regarded as describing the system of a quantum wave under periodic driving.

We begin by considering the light tunneling behavior when the amplitude \( E_1(z) \) of the modulation is adiabatically increased from zero to a constant value. For simplicity, we consider a linear ramp \( E_1(z) = Kz \). The working amplitude \( E_1/\omega \) is held constant at 2.4 after it reaches the point. We have solved the two coupled equations \( 1 \) and \( 2 \) numerically with the initial state \( \{c_1 = 1/\sqrt{2}, c_2 = 1/\sqrt{2}\} \). Two different scenarios of the beam dynamics are identified in Fig. 1, for two different values of the detuning \( E_0 \). For small detuning \( E_0 = 0.01 \), we see that the light is finally localized in the second waveguide. As the detuning is changed to \( E_0 = -0.01 \), the light becomes concentrated in the first waveguide. The numerical results clearly indicate that we can steer the light to the desired output waveguide in an adiabatic manner in the periodically modulated nonlinear couplers. Likewise, under the circumstance that the amplitude \( E_1(z) \) of the modulation is adiabatically decreased from \( E_1/\omega = 2.4 \) to zero, when light is launched into one waveguide, it will equally split into the two output waveguides in a reverse process.

To shed light on the underlying physics, we turn to the Floquet theory for a periodically-driving system. Though our system is nonlinear, its Floquet state and quasienergies can be similarly defined. That is, Eqs. 1 and 2 possess Floquet states in the form of \( \{c_1, c_2\}^T = \{\tilde{c}_1, \tilde{c}_2\}^T \exp(-i\varepsilon z) \), where \( \varepsilon \) is the quasi-energy and amplitudes \( \{\tilde{c}_1, \tilde{c}_2\}^T \) are periodic with modulation period \( T = 2\pi/\omega \).

Adopting the numerical method developed in Refs. 4 and 27, we have computed the Floquet states and corresponding quasienergies \( \varepsilon \). The results are plotted in Fig. 2, which shows that two extra quasienergy levels will emerge within a certain range of \( E_1/\omega \) for both cases of \( E_0 = \pm 0.01 \), in stark contrast to the linear case where the number of quasienergy levels is fixed by the size of the chosen basis. Different from the zero-detuning case [i.e., \( E_0 = 0 \) in Eqs. 1 and 2], where nonlinear Floquet states display degeneracy in the lowest quasi-energy level at the bottom of the triangular structure[4, 27], degeneracies are lifted for a nonzero detuning \( E_0 \neq 0 \). We also display in the inset figures the time-averaged population probability \( \langle |c_1|^2 \rangle = \int_0^T dz |c_1(z)|^2/T \) for the Floquet state corresponding to the lowest quasienergy. The insets show that the lowest Floquet state with nearly symmetric population distribution can undergo a strictly continuous evolution to a state with strong population imbalance. It is interesting to note that the population imbalance of the lowest Floquet state for detuning \( E_0 = 0.01 \) is almost the opposite to that for detuning \( E_0 = -0.01 \). Thus, by choosing different \( E_0 \) signs, we can realize the strong localization of light intensity in different waveguides through adiabatic navigation of the lowest nonlinear Floquet states when the modulation amplitude \( E_1 \) is increased slowly from zero to a constant value [see Fig. 1].

The induced localization versus detuning \( E_0 \) is more clearly demonstrated in Fig. 3. The figure shows the time-averaged intensity \( \langle |c_1|^2 \rangle \) (black squares) for the lowest Floquet state at the working amplitude \( E_1/\omega = 2.4 \) with a scan of the detuning \( E_0 \) across zero, which indicates that states with opposite population imbalances can be reached through choosing different detuning signs. To describe the dynamical process, by choosing the initial state \( \{c_1, c_2\} = (1/\sqrt{2}, 1/\sqrt{2}) \) which is close to the ground

FIG. 1: (color online) Light localization induced from a linearly ramped modulation \( E_1(z) = Kz \), where \( K = 0.01T^{-1} \) with \( T = 2\pi/\omega \) being the modulation period, for the coupled-mode equation \( 1 \) and \( 2 \) with two different values of \( E_0 \). The initial condition is \( \{c_1 = 1/\sqrt{2}, c_2 = 1/\sqrt{2}\} \). The other parameters are \( \varepsilon = -0.4, v = 1, \omega = 10 \). After the working amplitude \( E_1/\omega = 2.4 \) has been reached, holding \( E_1(z) \) constant keeps the light localization at a constant level.

FIG. 2: (color online) The quasi-energies versus \( E_1/\omega \) at (a) \( E_0 = 0.01 \) and (b) \( E_0 = -0.01 \). The top-right inset is the time-averaged intensity \( \langle |c_1|^2 \rangle \) for the Floquet state in the lowest quasi-energy level. The other parameters are \( \chi = -0.4, v = 1, \omega = 10 \).
state of the undriven system and by slowly increasing the modulation amplitude $E_1/\omega$ from zero to the working amplitude $E_1/\omega = 2.4$, we record quantity $|c_1|^2$ at the end of process, illustrated as red triangles in Fig. 3. In this process, the system will adiabatically follow the lowest Floquet state and thus achieve the targeted state with complete light localization in a desired waveguide when positive or negative detuning $E_0$ is chosen. Our numerical results show that the light localization persists for moderate values of detuning $E_0$, which implies it is easier to realize experimentally the light switching managements.

Here $p$ describes the refractive index amplitude, the profile of individual waveguides with widths $w_x$. The refractive index change $\mu_0$ mainly defines the propagation constant mismatch $E_0$ in the coupled-mode equation.

To elucidate a more rigorous dynamic, we conducted simulations with the nonlinear Schrödinger equation for the dimensionless electric field amplitude $\psi(x,z)$, which describes the propagation of monochromatic light waves along $z$ direction.

\[
\frac{i}{\hbar}\frac{\partial \psi}{\partial z} = -\frac{1}{2}\frac{\partial^2 \psi}{\partial x^2} - |\psi|^2\psi - pR(x,z)\psi. \tag{3}
\]

Here $x$ and $z$ are the normalized transverse and longitudinal coordinates, while $p$ describes the refractive index amplitude. The refractive index distribution of the waveguide coupler is given by

\[
R(x,z) = [1 - \mu_0 - \mu \cos(\omega z)] \exp\left[-\left(\frac{x - w_x/2}{w_x}\right)^6\right] + [1 + \mu_0 + \mu \cos(\omega z)] \exp\left[-\left(\frac{x + w_x/2}{w_x}\right)^6\right], \tag{4}
\]

with $w_x$ being the waveguide spacing, $w_x$ the channel width, $\mu$ the longitudinal modulation amplitude, and $\omega$ the modulation frequency. The super-Gaussian function $\exp(-x^6/w_x^6)$ describes the profile of individual waveguides with widths $w_x$. The refractive index change $\mu_0$ mainly defines the propagation constant mismatch $E_0$ in the coupled-mode equation.

![FIG. 3: (color online) The time-averaged intensity (black squares) for every Floquet state in the lowest quasienergy level at the working modulation amplitude $E_1/\omega = 2.4$. The red triangles are for the recorded quantities $|c_1|^2$ at $E_1/\omega = 2.4$ according to the adiabatic process outlined in Fig. 1 (more details can be seen in text). The other parameters are the same as the ones in Figs. 1 and 2.](image)

We numerically simulate the modulated waveguide coupler by integrating the continuous wave equation. In our simulation, the initial states are chosen as $\psi(x, 0) = A\phi_g(x)$ with $\phi_g(x)$ being the shape of the fundamental linear mode of the unmodulated symmetric coupler and $A$ the input amplitude, and the dimensionless parameters are set as $w_x = 0.3$, $w_s = 3.2$, $p = 2.78$ and $\omega = 3.45 \times (2\pi/100)$. As in the current experimental setup, $w_x$ and $w_s$ are in units of $10 \mu m$, and $p = 2.78$ corresponds to a refractive index of $3.1 \times 10^{-4}$. When $\mu_0 = \mu = 0$, the light periodically switches between channels with beating frequency $\Omega_0 = 2\pi/T_b$, where $T_b = 100$ for those parameters. The amplitude $\mu$ of the modulation is adiabatically increased from zero to a constant value $\mu = 0.2$ for systems sizes up to $z = 16T_b$. With the given system parameters, we first use the imaginary time evolution method to find the lowest state $\phi_g(x)$ for the symmetric linear coupler ($\mu_0 = \mu = 0$ in Eq. 1) which can be constructed as $\phi_g(x) = (1/\sqrt{2}) [u_1(x) + u_2(x)]$, where $u_1$ and $u_2$ are the localized waves in the two individual waveguides. In all simulations we used the input $\psi(x, 0) = 0.3\phi_g(x)$.

The behaviors of the light propagation are visualized in Fig. 4, which illustrates strong light localization for slight detuning $E_0$. As the amplitude $\mu$ of the modu-
lution is adiabatically increased from zero to a constant value $\mu = 0.2$, which corresponds to the working amplitude $E_1/\omega = 2.4$ in the coupled-mode equations, the light becomes confined in a single waveguide centered at $w_x/2$ when $\mu_0 = -0.001$, whereas it is finally confined in the other waveguide centered at $-w_x/2$ if the detuning is changed to $\mu_0 = 0.001$. The numerical result shows that strong confinement of light in a single waveguide with relatively higher static refractive index can be achieved by slowly increasing the amplitude $\mu$ of the modulation. It is in good agreement with the predictions based on the coupled-mode theory.

In summary, we have suggested a method for controlling light propagation in a periodically modulated nonlinear coupler by adiabatically varying the amplitude of modulation instead of varying the nonlinear strength proposed in some previous works. We find that induced light localization depends sensitively on the sign of detuning between two waveguides and thus it is possible to control the distribution of light among the output guides. The findings may offer a great potential for all-optical beam shaping and switching.

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