Microscopic calculation of the decay of Jaffe-Wilczek tetraquarks, and the Z(4433)

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Abstract. Here the tetraquarks are studied a la Jaffe and Wilczek. The decay width is fully computed with a microscopic quark model, using the resonating group method.

Keywords: Tetraquark, Confinement, Decay Width, Flip-flop

PACS: 12.39.Mk, 12.39.Pn

INTRODUCTION

Recently the discovery of the charged resonant state Z(4433) by the Belle Collaboration [1] has led to the proposal that this state is indeed a radial excitation of a tetraquark of the form uc\bar{d}c{\bar{c}} [2], being the particles X(3872) and X(3876), discovered by Belle and Babar, the fundamental tetraquark states (uc\bar{u}c and dc\bar{d}c).

Others interpret this state as a scattering resonance of the D∗(2010)\bar{D}(2420) system [3, 4] while still others interpret this as a threshold cusp in the same system [5].

The Z(4433) was detected by the decay Z(4433) → π±ψ, but if the model of the tetraquark is correct, the decay Z(4433) → D±D0 could also be possible.

SPECTRUM AND WAVEFUNCTIONS

We start to study the tetraquark system, by using the flux tube model, in which we have a potential given by

\[ V(r_1, r_2, r_3, r_4) = V_{Coulomb} + V_{Conf} \]  

The term \( V_{Coulomb} \) is the Coulomb part of the interaction corresponding to one gluon exchange, and \( V_{Conf} \) is the confining part of the potential.

The Coulomb interaction should be proportional to \( t_1^a t_2^a \), where \( t_1 \) and \( t_2 \) are the Casimir operators of the two particles. Since the two quarks are in an antitriplet state we get \( (t_1 + t_2)^{2} = t_1^2 = t_2^2 = \frac{4}{3} \), which gives \( t_1 \cdot t_2 = -2/3 \) so the quark-quark potential is

\[ V_{qq}(r) = -\frac{2}{3} \frac{\alpha_s}{r} \]
For the quark-antiquark Coulomb potential, since a quark and two antiquarks form an antitriplet, we get a factor \(-\frac{1}{3}\) for the quark-antiquark interaction

\[ V_{qq}(r) = -\frac{1}{3} \frac{\alpha_s}{r} \]  \hspace{1cm} (3)

As for the confining part of potential it is given by

\[ V_{\text{conf}}(r_1, r_2, r_3, r_4) = \sigma L_{\text{min}}(r_1, r_2, r_3, r_4) \]  \hspace{1cm} (4)

Where \(L_{\text{min}}\) is the minimum distance that links the four particles as shown in figure 1. The potential is given by

\[ V(r_1, r_2, r_3, r_4) = C - \frac{4}{3} \frac{\alpha_s}{r_1 - r_2} + \frac{1}{2} \frac{1}{r_2 + r_3} + \frac{1}{4} \frac{1}{r_3 - r_4} + \frac{1}{4} \frac{1}{r_2 - r_4} + \sigma L_{\text{min}}(r_1, r_2, r_3, r_4) \]  \hspace{1cm} (5)

We should solve the Schroedinger equation for this potential, but because the form of the confining part of the potential is cumbersome, we approximate our system by a system of a diquark and a di-antiquark, so that we only have the interaction between quarks (antiquarks) in the diquark (diantiquark) and the interaction diquark-diantiquark. Our simplified potential is given by

\[ V(r_u, r_c, r_{\bar{c}}, r_{\bar{d}}) = V_{\text{diqu}}(r_u, r_c) + V_{\text{diqu}}(r_{\bar{c}}, r_{\bar{d}}) + V_{\text{diqu-diqu}}(R_{uc}, R_{\bar{c} \bar{d}}) \]  \hspace{1cm} (6)

where \(R_{uc}\) and \(R_{\bar{c} \bar{d}}\) are the centers of mass of the \(uc\) and \(\bar{c} \bar{d}\) system.

\[ V_{\text{diqu}}(r_1, r_2) = C_{\text{diqu}} - \frac{2}{3} \frac{\alpha_s}{|r_1 - r_2|} + \sigma |r_1 - r_2| \]  \hspace{1cm} (7)

\[ V_{\text{diqu-diqu}}(r_1, r_2) = C_{\text{diqu-diqu}} - \frac{4}{3} \frac{\alpha_s}{|r_1 - r_2|} + \sigma |r_1 - r_2| \]  \hspace{1cm} (8)

So we define the coordinates

\[ R = \frac{m_u r_u + m_c r_c + m_d r_{\bar{d}} + m_{\bar{c}} r_{\bar{c}}}{2m_u + 2m_c} \]  \hspace{1cm} (9)

\[ \rho_{uc} = r_u - r_c \]  \hspace{1cm} (10)

\[ \rho_{\bar{c} \bar{d}} = r_{\bar{d}} - r_{\bar{c}} \]  \hspace{1cm} (11)

\[ r_T = R_{uc} - R_{\bar{c} \bar{d}} \]  \hspace{1cm} (12)

Then we could also write the kinetical energy operator as

\[ \hat{T} = \frac{p_u^2}{2M} + \frac{p_c^2}{2\mu_1} + \frac{p_{\bar{d}}^2}{2\mu_2} + \frac{p_{\bar{c}}^2}{2\mu} + \frac{p_T^2}{2\mu} \]  \hspace{1cm} (13)

Writing the wavefunction as

\[ \Psi(r_u, r_c, r_{\bar{c}}, r_{\bar{d}}) = \psi_{CM}(R) \psi_{uc}(\rho_{uc}) \psi_{\bar{d} \bar{c}}(\rho_{\bar{d} \bar{c}}) \phi_T(r_T) \]  \hspace{1cm} (14)
We arrive at three independent Schrödinger equations

\[-\frac{\hbar^2}{2m}\nabla^2 \psi + V \psi = \varepsilon \psi\]  \hspace{1cm} (15)

In this work we only care about s wave particles, so we expand the wavefunctions in the s-wave solutions of the harmonic oscillator

\[\psi_i(\rho_i) = \sum_n A_n \frac{H_{2n+1}(\rho_i)}{\rho_i} e^{-\frac{\rho_i^2}{2a}}\] \hspace{1cm} (16)

We do the same expansion to calculate the meson wavefunctions, which we use latter to calculate the decay width. We use the parameters \(\sigma = (440\text{MeV})^2\), \(\frac{4}{3}\alpha_s = 0.27\) and fix the constant so that we get a mass of 4433\(\text{MeV}\) for the first radial excitation of the tetraquark, and we get the value \(C = -2328\text{MeV}\). This is a standard procedure for mesons, we choose the potential constant to get the correct masses. The results are given in table 1.

| Particle | \(C(\text{MeV})\) | \(M(\text{MeV})\) |
|----------|-----------------|-----------------|
| \(\pi\)  | -1557           | 140             |
| \(\eta_c\) | -680            | 2980            |
| \(J/\Psi\)  | -797            | 3097            |
| \(D\)    | -976            | 1867            |
| \(Z(1S)\) | -2328           | 3913            |
| \(Z(2S)\) | -2328           | 4433            |
| \(Z(3S)\) | -2328           | 4830            |

**DECAY WITH MICROSCOPIC QUARK EXCHANGE**

Now we study the decay of the tetraquark system into a two meson system. Note that the potential is different in the tetraquark and in the meson-meson channels. We assume the "true" potential of the system is the flip-flop potential

\[V_{FF}(r_u, r_c, r_d, r_c) = \min(V_T, V_{\pi\psi}, V_{DD}) = \min(V_T, V_\pi + V_{\psi}, V_{D^0} + V_{D^+})\] \hspace{1cm} (17)
Where $V_T$ is the tetraquark potential, given above, $V_\pi$, $V_\psi$, $V_{D^0}$ and $V_{D^+}$ are the quark-antiquark potentials in the respective mesons, which is of the funnel type

$$V_i(r_i) = C_i - \frac{4 \alpha_s}{3 r_i} + \sigma r_i$$  \hspace{1cm} (18)$$

The difference $V_{FF} - V_T$ is the perturbation that causes the decay of the tetraquark into two mesons. The full hamiltonian is given by

$$\hat{H} = \sum_i T_i(p_i) + V_{FF}(r_u, r_c, r_d, r_\epsilon)$$  \hspace{1cm} (19)$$

We could rewrite the hamiltonian as $H = H_1 + V_1$ with $V_1 = V_{FF} - V_T$ and $V_2 = V_{FF} - V_{\pi\psi}$. Now we calculate the matrix elements

$$\langle T|H|T \rangle = M_T^0 + \langle T|V_1|T \rangle$$  \hspace{1cm} (20)$$

Note that we need to redefine the potential so that $M_T = M_T^0 + \langle V_1 \rangle$. We have, also

$$\langle \pi\Psi|H|T \rangle = M_T^0 \langle \pi\Psi|T \rangle + \langle \pi\Psi|V_1|T \rangle$$  \hspace{1cm} (21)$$

$$\langle \pi\Psi|H|\pi\Psi \rangle = M_\pi + M_\psi + \langle \pi\Psi|V_2|\pi\Psi \rangle$$  \hspace{1cm} (22)$$

To compute the decay of the tetraquark in two mesons we have to solve the equation

$$
\begin{pmatrix}
M_T^0 + \langle V_1 \rangle & M_T^0 \rho^* \langle \phi | + \nu^* \langle \chi | \\
M_T^0 \rho \langle \phi | + \nu \langle \chi | & M_\pi + M_\psi + T + \langle V_2 \rangle \pi \psi
\end{pmatrix}
\begin{pmatrix}
1 \\
\rho - \psi
\end{pmatrix}
= E S
\begin{pmatrix}
1 \\
\rho - \psi
\end{pmatrix}
$$  \hspace{1cm} (23)$$

where

$$S = \begin{pmatrix}
1 \\
\rho \langle \phi |
\end{pmatrix}$$  \hspace{1cm} (24)$$

$$\nu \langle \rho_{\pi\psi} | \chi \rangle = \frac{1}{8} \int d^3 \rho \pi d^3 \rho \psi \psi_{uc}(\rho_{uc})^* \psi_{d\bar{c}}(\rho_{d\bar{c}})^* \phi_T(r_i) V(\rho_{\pi}, \rho_{\psi}, r_{\pi\psi}) \psi_{\pi}(\rho_{\pi}) \psi_\psi(\rho_{\psi})$$  \hspace{1cm} (25)$$

$$\nu \langle \rho_{\pi\psi} | \phi \rangle = \frac{1}{8} \int d^3 \rho \pi d^3 \rho \psi \psi_{uc}(\rho_{uc})^* \psi_{d\bar{c}}(\rho_{d\bar{c}})^* \phi_T(r_i) \psi_\pi(\rho_{\pi}) \psi_\psi(\rho_{\psi})$$  \hspace{1cm} (26)$$

The term $\nu \langle \phi |$ comes from the nonorthogonality between the tetraquark state and the meson-meson state. If the states where orthogonal the $S$ matrix would be the identity.

We have the equations

$$M_T - E + (e^* M_T^0 - e^* E) \langle \phi | \rho - \psi \rangle + \nu^* \langle \chi | \rho - \psi \rangle = 0$$  \hspace{1cm} (27)$$

$$(M_T^0 - E) e \langle \phi | + \nu \langle \chi | + (M_\pi + M_\psi + T + \langle V_2 \rangle \pi \psi - E) \rho - \psi = 0$$  \hspace{1cm} (28)$$

Now eliminating $\rho - \psi$ we get

$$E - M_T = - \int \frac{d^3 p}{(2\pi)^3} \frac{\chi^*(p) \chi(p)}{M_\pi + M_\psi + T(p) + \langle V_2 \rangle \pi \psi - E - i\epsilon}$$  \hspace{1cm} (29)$$
where
\[ \chi(p) = (M_T^0 - E)e\langle p\pi\psi|\phi\rangle + \nu\langle p\pi\psi|\chi\rangle \] (30)

To calculate the decay width, we make the replacement \( E \rightarrow E - i\frac{\Gamma}{2} \) in the left side of the equation and make \( E \simeq M_T \) in the right side. Also we neglect the term \( \langle V_2\rangle\pi\psi \) which corresponds to a short range interaction between the mesons.

For the terms \( \nu\langle p\pi\psi|\chi\rangle \) and \( e\langle p\pi\psi|\phi\rangle \) we use the eq. 16 for all the wavefunctions, so that \( \nu\langle p\pi\psi|\chi\rangle \) is given by
\[
\nu\langle p\pi\psi|\chi\rangle = N \int d^3r_{\pi\psi} d^3\rho_{\pi} d^3\rho_{\psi} e^{-\frac{1}{2}C_{\pi\psi}\rho_{\pi}^2 e^{-\frac{1}{2}C_{\psi\psi}\rho_{\psi}^2}} e^{-\frac{1}{2}C_{\pi\psi}\rho_{\pi}\rho_{\psi} e^{-i\rho_{\pi}\rho_{\psi}}} \]
\[
P_1(\rho_{uc}) P_2(\rho_{\bar{d}\bar{c}}) P_3(\rho_{\pi}) P_4(\rho_{\psi}) P_5(\rho_T) V(\rho_{\pi},\rho_{\psi},\rho_T) \]
where the \( P_i \) are polynomials. \( \rho_{uc}, \rho_{\bar{d}\bar{c}} \) and \( \rho_T \) are given in terms of \( \rho_{\pi}, \rho_{\psi} \) and \( r_{\pi\psi} \), by
\[
\rho_{uc} = \frac{1}{2} (\rho_{\pi} - \rho_{\psi}) + r_{\pi\psi} \] (32)
\[
\rho_{\bar{d}\bar{c}} = \frac{1}{2} (\rho_{\psi} - \rho_{\pi}) + r_{\pi\psi} \] (33)
\[
\rho_T = R_{uc} - R_{\bar{d}\bar{c}} = \frac{m_u}{m_u + m_d} \rho_{\pi} + \frac{m_c}{m_u + m_c} \rho_{\psi} \] (34)

For \( e\langle p\pi\psi|\phi\rangle \), we just replace the potential by 1. Since the weight of the integral is gaussian, we use the Monte Carlo method to evaluate this nine-dimensional integral. So we calculate the width for the decay of the \( Z(4433) \) (assuming it is a tetraquark in the \( 2S \) state) into \( \pi\psi(1S) \) and \( \pi\psi(2S) \), and obtain (preliminary results) \( \Gamma(\pi\psi(1S)) = 0.2MeV \) and \( \Gamma(\pi\psi(2S)) = 4.6MeV \).

**CONCLUSIONS**

The result is somewhat smaller than the experimental result for the decay of the \( Z(4433) \) which has a width of \( \Gamma = 44^{+17}_{-13} MeV \) [4], but this result is only preliminary. Also, we don’t include the spin effects, and it is known that the hyperfine splitting effect could increase the decay width.

Even though, we think that our method is quite powerful and general, and we expect to get more accurate and conclusive results using this method.

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