Coherent radiation from a chain of charged particles on a circular orbit around a dielectric ball

L.Sh. Grigoryan¹, A.H. Mkrtchyan¹, S.B. Dabagov²,³, A.A. Saharian¹, V.R. Kocharyan¹, V.Kh. Kotanjyan¹, H.P. Harutyunyan¹, H.F. Khachatryan¹

¹Institute of Applied Problems of Physics NAS RA, 25 Hrachya Nersissyan Str., 0014 Yerevan, Armenia
²NR Nuclear University MEPhI, 31 Kashirskoe Sh., 115409 Moscow, Russian Federation
³INFN Laboratori di Frascati, 54 Via Enrico Fermi, I-00044 Frascati (Roma), Italy

January 20, 2023

Abstract

We investigate the spectral and angular distribution of the electromagnetic radiation from a chain of relativistic charged particles uniformly rotating along equatorial orbit around a dielectric ball. It is shown that, for weak absorption in the ball material and under relatively mild conditions on the distribution of the particles, the radiation intensity at specific rotation frequencies is essentially stronger than the corresponding radiation for a chain circulating in free space or in a homogeneous transparent medium with the same dielectric constant as that for the ball. We determine the values of parameters of the problem for which the charges in the chain emit coherently and the radiation intensity on a given harmonic increases in proportion to the square of the number of emitting charges. We also show that relative shifts in the particles locations up to 10% do not destroy the coherence properties of the radiation. It is demonstrated that the coherence effects may also dominate in the radiation intensity for chains with non-equidistant distributions of particles. The numerical results obtained for different dielectric balls have revealed the emitted radiation to be in the GHz/THz frequency ranges. The high-power radiation from the chain is confined near the rotation plane within the angular region determined by the Cherenkov angle for the velocity of the chain image on the ball surface. In the special case of an equidistant distribution of charged particles along the orbit the results of the present paper for angle integrated frequency distribution of the radiation are in agreement with those previously obtained by our group. We argue that similar coherence effects will be present in the radiation from a chain of bunches circulating around the ball.

Keywords: Synchrotron radiation, Cherenkov radiation, Coherent effects, Dielectric ball

*E-mail: saharian@ysu.am
1 Introduction

The interaction of charged particles with matter may essentially influence the characteristics of radiation processes and gives rise to new types of processes such as Cherenkov radiation (CR), transition radiation, channeling radiation, etc. [1]-[7]. In particular, the wide applications of synchrotron radiation (SR) in fundamental and applied sciences and in technology motivate the investigations of the influence of medium on spectral and angular characteristics of the radiation intensity. SR from a charge uniformly circulating in a homogeneous medium has been studied in [8]-[14]. It was shown that, under the Cherenkov condition for the velocity of the charge, as a consequence of the superposition of CR and SR the radiation features may differ significantly from those for the radiation in free space.

The interfaces separating two media with different electromagnetic properties may serve as an additional tool to control the spectral and angular characteristics of SR. As examples of exactly solvable problems of that kind, in our previous studies we have investigated the radiation from charges rotating around/inside dielectric ball and cylinder. Those investigations were based on the recurrence schemes for evaluation of the electromagnetic field Green tensors in spherically and cylindrically symmetric piecewise homogeneous media developed in [15, 16]. It has been shown that the presence of boundaries leads to additional new features that are absent in the case of homogeneous media. In particular, the investigations of SR from a charge moving along a concentric equatorial orbit around or inside a dielectric ball [17] [18] [19] have shown that, under the Cherenkov condition for the charge velocity and the ball material, the so-called ”resonance” radiation is generated. The flux of the resonant radiation in this case exceeds the corresponding value for the radiation in homogeneous and transparent medium by an order of magnitude. The radiation intensity for the cases of non-equatorial or shifted equatorial circular orbits around a dielectric ball has been investigated in [20, 21]. Similar features were observed for charges on circular and helical trajectories around or inside a dielectric cylinder (see [22]-[25] and references therein). SR from a charge rotating around a cylindrical grating has been studied in [26]. In [27] the conditions are specified under which strong narrow peaks appear in the spectral distribution of CR from a charged particle uniformly moving parallel to the axis of a dielectric waveguide.

The angular distribution of the high-power resonance radiation from a relativistic electron revolving around a dielectric ball is examined in [28], while the spectral distribution for a chain of equidistant electrons on a circular orbit has been studied in [29]. In the present paper, we investigate the spectral-angular distribution of the resonance radiation in the GHz and THz frequency ranges generated by a chain of uniformly moving particles unevenly distributed on a circular trajectory around a dielectric ball. The paper is organized as follows. In the next section we describe the problem setup and present the formula for the number of the radiated quanta. In section 3 numerical examples are given for fused silica and Teflon balls. Qualitative explanation is provided for coherence effects in the superposition of the radiations from separate charges. The main results are summarized in section 4.

2 Statement of the problem and the number of emitted quanta

We consider a chain of N electrons moving with velocity \( v = \text{const} \) along an equatorial circular trajectory around a homogeneous dielectric ball (see Fig. 1). The radii of the ball and rotation orbit will be denoted by \( r_b \) and \( r_e \), respectively. In accordance with the problem symmetry, the spherical coordinate system \((r, \theta, \varphi)\) will be used with the origin at the ball center and with the polar axis perpendicular to the rotation plane. Assuming that the chain rotates in the vacuum, for dielectric permittivity, as a function of the radial coordinate, one has \( \varepsilon(r) = \varepsilon_b = \varepsilon'_b + i\varepsilon''_b \) for \( r \leq r_b \) and \( \varepsilon(r) = 1 \) in the exterior region \( r > r_b \). Here, \( \varepsilon'_b \) and \( \varepsilon''_b \) are the real and imaginary
parts of the dielectric permittivity. The magnetic permeability of the ball material is taken to be equal to unity.

Figure 1: A chain of relativistic electrons rotating around a ball in its equatorial plane.

In accordance with the periodicity of the motion the radiation is emitted on discrete angular frequencies $\omega_k = kv/r_e$, where $k = 1, 2, 3, \ldots$ determines the number of the radiated harmonic. Let $n_N(k, \theta)$ be the angular density for the number of quanta radiated by the chain on a given harmonic per rotation period $T = 2\pi r_e/v$. It can be presented in the factorized form

$$n_N(k, \theta) = F_N(k)n_1(k, \theta),$$

(1)

where $F_N(k)$ is the chain structure factor and $n_1(k, \theta)$ is the corresponding quantity for the radiation from a single electron. In the problem under consideration the factor $F_N(k)$ can be written as (see also [29])

$$F_N(k) = \left| \sum_{j=1}^{N} \exp(-ik\varphi_j) \right|^2,$$

(2)

where the angle $\varphi_j$ determines the relative position of the $j$-th particle inside the chain. The total number of the quanta on the $k$-th harmonic, emitted per rotation period, is expressed as

$$n_N(k) = \int_0^\pi n_N(k, \theta)d\theta.$$

(3)

We first analyze the conditions under which the chain generates high-power radiation at a given harmonic $k$, and then a qualitative explanation of that phenomenon will be given. We will also determine the dependence of the effect on random deviations of electrons locations with respect to the uniform distribution.

For a single electron rotating around a dielectric ball, the formula for the angular density of the number of quanta radiated per rotation period, $n_1(k, \theta) = n_1(\text{ball}, v, x, \varepsilon_b; k, \theta)$, is derived in [21]. For the discussion below it is convenient to write the corresponding expression in a different form given by

$$n_1(\text{ball}, v, x, \varepsilon_b; k) = 32\pi^3 \frac{e^2}{hc} k \sin\theta \sum_{s=0}^{\infty} \sum_{\mu=0,1} (-1)^s c_i^{(\mu)} \sqrt{(2l+1)^{2\mu-1} (l-k)!}$$

$$\times \left( \frac{i}{k} \partial_y \right)^\mu P_y^k(y) \left. \right|_{y=0} \left. \sum_{i,k}^{(\mu+2)} (\theta, 0) \right|^2,$$

(4)
where \( l = k + 2s + \mu \) and we use the notation \( x = r_b/r_e < 1 \). In (4), \( X_{l,m}^{(2)}(\theta, \varphi) \) and \( X_{l,m}^{(3)}(\theta, \varphi) = [n \times X_{l,m}^{(2)}(\theta, \varphi)] \) are the spherical vectors of electric and magnetic types \([30]\), respectively, and \( n \) is the unit vector along the direction determined by the angles \( \theta \) and \( \varphi \). The coefficients \( c_i^{(\mu)} \) are defined as

\[
\begin{align*}
    c_i^{(1)} &= iu \left[ j_i(u) - h_i(u) \frac{V_i^j(xu_b, xu)}{V_i^h(xu_b, xu)} \right], \\
    c_i^{(0)} &= (l + 1)c_{l-1}^{(1)} - l c_{l+1}^{(1)} + \frac{1 - \varepsilon_b l(l+1)u_b j_l(xu_b)}{x^2} \frac{z_l(xu_b, xu)}{(2l+1)} \\
    &\times \left[ \sum_{p=\pm 1} \frac{j_{l+p}(xu_b)}{V_{l+p}^h(xu_b, xu)} \right] \left[ \sum_{p=\pm 1} \frac{h_{l+p}(u)}{V_{l+p}^h(xu_b, xu)} \right],
\end{align*}
\]

with \( u = kv/c \) and \( u_b = kv\sqrt{\varepsilon_b}/c \). Here, \( j_i(y) \) and \( h_i(y) \) are the spherical Bessel and Hankel function (see, for example, \([31]\)). For the latter one has \( h_1(y) = j_1(y) + iyj_1(y) \), with \( yj_1(y) \) being the spherical Neumann function. In (5) we have used the notations

\[
V_l^f(xu_b, xu) = j_l(xu_b)\partial_x f_l(xu) - f_l(xu)\partial_x j_l(xu_b),
\]

for \( f = j \) and \( f = h \), and

\[
z_l(x, y) = 1 + \frac{\varepsilon_b - 1}{2} \sum_{p=\pm 1} \left( 1 + \frac{p}{2l+1} \right) \left[ 1 - \frac{xj_l(x)h_{l+p}(y)}{yj_l(x)h_l(y)} \right]^{-1}.
\]

In the absence of the dielectric ball \( (\varepsilon_b = 1) \) the formula (4) is reduced to the corresponding expression for SR in free space (for the radiation in a homogeneous medium see formula (11) below). Note that for given \( v \) and \( \varepsilon_b \), the angular density of the number of the radiated quanta, given by (1), is invariant under the rescaling \( r_b \to wr_b, r_e \to wr_e, \omega_k \to \omega_k/w \), with dimensionless scaling parameter \( w \).

As it has been discussed in \([18, 28]\), under the condition \( \varepsilon''_b \ll \varepsilon'_b \) (weak absorption of the radiation in the ball material), for specific values of the parameter \( x = r_b/r_e \) and for large harmonics, \( k \gg 1 \), the number of quanta \( n_1(k) \), emitted by a single charge, significantly exceeds the number of the radiated quanta in the problems where the same charge is circulating in vacuum or in a transparent homogeneous medium with dielectric permittivity \( \varepsilon(r) = \varepsilon'_b \), \( 0 \leq r < \infty \). The increase of the radiation intensity is a consequence of coherent superposition of elementary waves for CR inside the ball emitted near the charge trajectory and multiply reflected from the ball surface. The fine tuning of the ratio \( x \) is required to ensure the condition for multiple reflections. The high-power radiation is mainly confined near the rotation plane in the angular region \([28]\) \( \pi/2 - \theta_{Ch} \leq \theta \leq \pi/2 + \theta_{Ch} \).

The mathematical reason for the appearance of the strong peaks can be understood as follows. We note that the argument of the spherical Hankel functions \( h_{l}(xu), h_{l+p}(xu) \) in the definition of the function \( z_l(xu_b, xu) \) is always real and, hence, those functions have imaginary parts. This means that the function \( z_l(xu_b, xu) \) is always imaginary and has no real zeros with respect to \( x \) for real values of the arguments. This correspond to that the ball has no electromagnetic spherical eigenmodes. Though the function \( z_l(xu_b, xu) \) is always different from zero, it can be extremely small for large values of \( l \). This is based on the observation that for \( 0 \leq y < 1 \) and for large \( v \) the Bessel function \( J_{\mu}(vy) \) is exponentially small with respect to the Neumann function \( Y_{\mu}(vy) \). This is seen from Debye’s asymptotic expansions \([31]\) with the leading order term

\[
\frac{J_{\mu}(vy)}{Y_{\mu}(vy)} \sim -\frac{1}{2}e^{-2\nu \zeta(y)}, \quad \zeta(y) = \ln \left( 1 + \frac{\sqrt{1 - y^2}}{y} \right) - \sqrt{1 - y^2}.
\]
By using this property, we write $h_l(y) = iy_l(y)[1 - i j_l(y)/y_l(y)]$ and for large $l$ expand the function \[7\] over the small ratio $j_l(y)/y_l(y)$:

$$z_l(x, y) \approx z_l^{(0)}(x, y) - i \frac{\varepsilon_b - 1}{2} \frac{x}{y} \sum_{p=\pm 1} \left( p + \frac{1}{2l+1} \right) \frac{j_l(x)j_{l+p}(x)}{[yj_{l+p}(y) - xj_l(x)y_{l+p}(y)]^2},$$ (9)

with the leading order term

$$z_l^{(0)}(x, y) = 1 + \frac{\varepsilon_b - 1}{2} \sum_{p=\pm 1} \left( 1 + \frac{p}{2l+1} \right) \left[ 1 - \frac{xj_l(x)y_{l+p}(y)}{yj_{l+p}(x)y_l(y)} \right]^{-1}.$$ (10)

Unlike the function $z_l(x, y)$, the function $z_l^{(0)}(x, y)$ is real for real values of the arguments and it may become zero. Now from \[9\] we see that at those zeros the function $z_l(x, y)$ is exponentially small and gives an exponentially large factor in the expression \[4\] for the angular density of the number of radiated quanta. This does not yet mean that the angular density will be exponentially large because the contributions of the other terms should be estimated as well. An important thing which should be emphasized here is that the values for the ratio $x$ for which strong peaks are present in the radiation intensity are determined by the zeros of the function \[10\] with high accuracy. We have checked that by numerical examples. Another important thing, based on the analytic estimates given above, is that the peaks come from the electromagnetic field modes of the electric type. The electromagnetic modes corresponding to the zeros of the function can be termed as "quasieingemodes" of the ball. They have a large confinement time inside the ball with multiple reflections from the ball surface (in the absence of absorption the exact eigenmodes would have an infinite confinement time).

The arguments presented are similar to those given in \[22\] for the radiation from a charge circulating inside or around a dielectric cylinder. A factor that is the analog of the function \[7\], with the spherical Bessel functions replaced by the cylindrical functions with an integer order, is present in the corresponding expression for the radiation intensity (the factor $\alpha_m$, $m = 1, 2, \ldots$, in \[22\]). The Hankel functions appear in the corresponding expression in the form $H_{m,\pm 1}(\rho_1 \sqrt{\varepsilon_1 \omega_0^2 \varepsilon_1/c^2 - k_z^2})$, where $\rho_1$ is the radius of the cylinder, $\omega_0$ is the angular velocity of the charge circulation, $\varepsilon_1$ is the dielectric permittivity of the medium surrounding the cylinder, and $k_z$ is the projection of the wave vector on the axis of the cylinder. An important difference from the case of dielectric ball is that now there are electromagnetic modes for which the argument of the Hankel functions becomes purely imaginary (modes with $|k_z| > m\omega_0\sqrt{\varepsilon_1/c}$) and they are transformed to Macdonald function. For those modes the function $\alpha_m$ becomes real and it may have zeros with respect to $k_z$. They are the eigenmodes of the dielectric cylinder. The corresponding radiation fields are exponentially suppressed in the region outside the cylinder and they propagate inside the cylinder.

### 3 Numerical analysis and qualitative explanation

Below we consider a chain of electrons with energy $E_e = 2$ MeV rotating around a ball made of quartz or Teflon and generating high-power radiation in the GHz or THz frequency ranges in dependence of the optimal choices for the values of the system parameters $N, \varphi_j, r_e, r_b, k, \varepsilon_b$.

#### 3.1 Chain rotating around a ball of fused quartz

We start the discussion of the radiation features from the case of a chain on a circular orbit around a ball made of fused quartz.
3.1.1 Radiation from a single electron

Let us denote by $v_s = vx$ the velocity of the particle’s image on the surface of the ball and by $\theta_{Ch} = \arccos(c/v_s\sqrt{\varepsilon_b})$ the related Cherenkov angle, assuming that the Cherenkov condition $v_s\sqrt{\varepsilon_b}/c > 1$ is satisfied. In Fig. 2 we display the results of numerical calculations for the angular density of the number of quanta, $n_1 = n_1(\text{ball, } v, x, \varepsilon_b; k, \theta)$, radiated per rotation period by a single electron of energy 2 MeV in a circular orbit around a fused quartz ball with $\varepsilon_b = 3.78(1 + 10^{-4})$ [32, 33]. For the harmonic $k = 8$ the abovementioned amplification effect is obtained for the value $r_b/r_e = 0.9815$. We have numerically checked that this value coincides with the zero of the function $z_l^0(xu_b, xu)$ with rather good accuracy, in accordance of the analytic arguments given above. In order to see the influence of the dielectric ball on the radiation intensity, on the left panel of Fig. 3 we have plotted the angular density of the number of quanta, $n_1 = n_1(\infty, v, \varepsilon; k, \theta)$, for an electron rotating in vacuum, $\varepsilon = 1$ (the ball is absent). The right panel of figure 3 presents the corresponding quantity, $n_1 = n_1(\infty, v, \varepsilon; k, \theta)$, for the radiation from a single electron in a homogeneous transparent medium with dielectric permittivity $\varepsilon = \varepsilon_b' = 3.78$. The graphs in Fig. 3 are plotted by using the formula

$$n_1(\infty, v, \varepsilon; k, \theta) = \frac{2\pi e^2 k}{h c\sqrt{\varepsilon}} \left[ J_k^2(k\beta \sin \theta) \cot^2 \theta + \beta^2 J_k^2(k\beta \sin \theta) \right] \sin \theta,$$

(11)

for SR in a homogeneous transparent medium with dielectric permittivity $\varepsilon$. Here, $\beta = v\sqrt{\varepsilon}/c$ and $J_k(x)$ is the Bessel function. The radiation frequency is given by $\nu_k = kv/(2\pi r_e)$. For the resonant radiation on the harmonic $k = 8$, corresponding to Fig. 2 and for the ball radius $r_e = 1$ cm one gets $\nu_8 \approx 37$ GHz. The radiation frequency can be tuned by the choice of the ball radius for a given $r_e$ or by the choice of the latter for a given $r_b$.

Figure 2: The angular distribution of the number of quanta $n_1(k, \theta)$ generated by a single electron per rotation period as a function of the harmonic number $k$ and polar angle $\theta$ (in radians). The graphs are plotted for an electron orbiting in free space around a ball made of fused quartz.

As illustrated in Figs. 2 and 3 for a given value of the radiation harmonic $k$ (the harmonic $k = 8$ in the example at hand) and for fixed values of the other parameters, by tuning the ratio $r_b/r_e$ we can obtain an essential increase of the radiation intensity compared with the corresponding radiation for an electron circulating in free space or in a transparent homogeneous medium having dielectric permittivity equal to the real part of the dielectric permittivity for the
ball. We also see a significant increase of the radiation intensity in a homogeneous transparent medium compared with the radiation in vacuum.

3.1.2 Radiation from a chain

The spectral-angular distribution of the number of quanta radiated by a chain is given by the formula (1). For equidistant distribution of electrons in the chain one has \( \varphi_j = 2\pi(j - 1)/N \) and the expression (2) for the factor \( F_N(k) \) is simplified to (see also [29])

\[
F_N(k) = \begin{cases} 
N^2, & \text{for } k = mN, \quad m = 1, 2, \ldots \\
0, & \text{for } k \neq mN 
\end{cases}
\]  

(12)

This shows that the radiation takes place on the angular frequencies \( \omega_m = Nmv/r_e \). Of course, that is natural, because for an equidistant chain the period of the system motion \( T_N \) is given by \( T_N = T/N = 2\pi r_e/(Nv) \) and for the radiation frequencies we get \( \omega_m = 2\pi m/T_N \) with \( m = 1, 2, \ldots \). The electrons in the chain radiate coherently and the radiation intensity is amplified by the factor \( N^2 \).

In realistic situations the condition for the same separation between neighboring particles is obeyed approximately and it is of interest to see the sensitivity of the coherent superposition considering small variations in the equidistant distribution. Figure 4 shows the dependence of the chain structure factor on the relative maximal shift \( \sigma/d \) for each particle, where \( d = 2\pi r_e/N \) is the arc distance between neighboring particles. The shift from the equidistant position is taken as a randomly distributed quantity with the uniform distribution (of width \( \sigma \)) with the maximal shift equal to the half of the arc distance \( d \). From the data presented in Fig. 4 it follows that the chain radiates coherently, i.e. \( F_N \sim N^2 \), for up to 10% of relative shifts \( \sigma/d \). Thus, we can conclude that the replacement of a single charge by the chain formed by \( N = 8 \) equidistant charges rotating about the ball, additionally increases the radiation by a factor \( \sim N^2 \).

The coherence effect discussed above for the equidistant distribution may take place for more general distributions of charges on a circular trajectory. Let us consider the distribution of \( N \) charges given by

\[
\varphi_1 = 0, \quad \varphi_j = 2\pi r_j, \quad j = 2, \ldots, N,
\]  

(13)

at \( t = 0 \), where \( r_j \) are positive rational numbers such that \( r_2 < r_3 < \cdots < r_N < 1 \). In the special case of the equidistant distribution one has \( r_j = (j - 1)/N \). Let \( r_j = p_j/q_j \) be
the standard form of the rational number $r_j$. For the sum in the structure factor (4) we get 
$$1 + \sum_{j=2}^{N} \exp(-2\pi ikp_j/q_j).$$ From here it follows that for the harmonics of the radiation with 
k = mk_0, where $m$ is a positive integer and $k_0$ is the least common multiple of the numbers 
$q_2, q_3, \ldots, q_N$, the chain radiates coherently with $F_N(mk_0) = N^2$. However, unlike the case of 
equidistant distribution, in general, the radiation on harmonics $k \neq mk_0$ is present as well. The 
radiation on those harmonics can be of the order $N^2$, though $F_N(k) < N^2$. To illustrate this 
point, in Fig. 5 the function $F_N(k)$ is displayed for the distribution of $N = 8$ particles with the 
locations described by 
$$\{r_j\}_{j=2\ldots N} = \left\{ \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, 3, \frac{3}{5}, \frac{3}{4} \right\}.$$ 
For this example $k_0 = 60$. Note that the function $F_N(k)$ is periodic with the period equal 
to $k_0$. We can have a more general case of distribution when the condition for the coherent 
superposition is obeyed only for a subsystem of particles in the chain. In this case one has 
a partial coherence with the radiation intensity proportional to the square of the number of 
particles in that subsystem.

### 3.2 Radiation for a Teflon ball

Similar results are obtained for a ball made of Teflon. In this case the parameters of the system 
have the following values: $\varepsilon_b = 2.2(1 + 0.0002i)$, $r_b/r_e = 0.9616$, $k_0 = 20$. The corresponding 
graphs are presented in Figs. 6 and 7. As one can see from presented figures, the amplification 
effect takes place in this case as well. Taking, for example, $r_e = 0.15\text{cm}$, for the frequency of 
the resonance radiation we get $\omega_{k_0}/2\pi = 6 \cdot 10^{11}\text{Hz}$ which is in the THz range.

### 3.3 Qualitative explanation

Here we provide a qualitative explanation of the radiation intensity enhancement. As it already 
has been mentioned above, compared with the case of a single particle, for an equidistant 
distribution of $N$ particles on a circle the period of motion is reduced by the factor $N$ and, as 
a consequence, the radiation frequencies are given as $\omega_m = Nmv/r_e$, $m = 1, 2, \ldots$, i.e., only
Figure 5: The structure factor of the chain of $N = 8$ particles distributed in accordance with (14).

Figure 6: The same as in figure 2 for a Teflon ball. The graphs are plotted for a charge rotating in vacuum around a Teflon ball (left panel) and in a transparent and homogeneous medium with permittivity $\varepsilon = \varepsilon'_b = 2.2$ (right panel).
the harmonics $k = Nm$ are radiated with the power $N^2$ times larger than the corresponding radiation for a single particle.

First, we consider the case of a single particle with the angular coordinate $\varphi = vt/r_e$. In accordance with the problem symmetry, for the corresponding electric field we can write the Fourier expansion \[ E(r, \theta, \varphi, t) = \sum_{s=-\infty}^{+\infty} E_s(r, \theta) e^{is(\varphi - vt/r_e)}, \] where $E_{-s}(r, \theta) = E_s^*(r, \theta)$.

Ignoring the absorption in the material of the ball, the total radiation intensity can be evaluated in terms of the work done by the electromagnetic field on the charge:

\[
I_1 = -qv \cdot E(r_e, \pi/2, vt/r_e) = -2qv \sum_{s=1}^{\infty} \text{Re} \left[ E_{s,\varphi}(r_e, \pi/2) \right],
\]

where $E_{s,\varphi}(r_e, \theta)$ is the azimuthal component of the Fourier coefficient. For a chain of point charges with the locations $\varphi = \varphi_j + vt/r_e$, the corresponding Fourier expansion reads

\[
E^{(N)}(r, \theta, \varphi, t) = \sum_{j'=1}^{N} \sum_{s=-\infty}^{+\infty} E_s(r, \theta) \exp \left[ is(\varphi - \varphi_{j'} - vt/r_e) \right].
\]

The energy radiated per unit time by the $j$-th particle in the chain is expressed as

\[
I_j^{(N)} = -2qv \sum_{j'=1}^{N} \sum_{s=1}^{\infty} \text{Re} \left[ E_{s,\varphi}(r_e, \pi/2) e^{is(\varphi_j - \varphi_{j'})} \right],
\]

and for the radiation intensity from the chain we get $I^{(N)} = -2qv \sum_{s=1}^{\infty} F_N(s) \text{Re} \left[ E_{s,\varphi}(r_e, \pi/2) \right]$, where the structure factor is given by (2). In the case of equidistant distribution with $\varphi_j = 2\pi(j - 1)/N$, from (17) one finds $I_j^{(N)} = -2Nqv \sum_{s=1}^{\infty} \text{Re} \left[ E_{Nm,\varphi}(r_e, \pi/2) \right]$ and this quantity is the same for all particles in the chain. As a consequence, for the total radiation intensity from the equidistant chain we get $I^{(N)} = -2N^2qv \sum_{s=1}^{\infty} \text{Re} \left[ E_{Nm,\varphi}(r_e, \pi/2) \right]$. As seen from (16), for an equidistant distribution and for harmonics $k = |s| = Nm$ the electric fields of the charges with $j' \neq j$ at the location of the $j$-th charge are added coherently and the radiation from a single charge in the chain is increased by $N$ times. Another factor in enhancement of the total radiation comes from the number of particles in the chain.
Similar effect of coherent superposition of radiations from separate sources will also be present if instead of separate charges we take bunches of electrons. Note that in the problem with quasi-equidistant distribution of the bunches on a circle one can have two types of coherence. The first one is the coherence between separate bunches and the second one corresponds to the coherent effects in the radiation of a separate bunch. The coherent synchrotron radiation (CSR) from a separate bunch has been widely investigated in the literature both theoretically and experimentally (see, for example, [36]-[33] and reviews [6] [14] [45]). For a single bunch, CSR is emitted in the spectral range where the radiation wavelength is larger than the bunch length. It has been observed in relatively wide range of wavelengths, from microwaves to far infrared region. In many cases the theoretical results obtained within the framework of the simplest model of a linear bunch with a vanishing transverse size are in good agreement with observational data. More complicated 2D and 3D models have been developed as well (see, for example, [46] [47] [48] and references therein).

In the problem under consideration the radiation intensity for a single linear bunch on a circular orbit can be obtained from the results given above. The angular density of the number of quanta radiated by a single bunch, \( n_{\text{ch}}^{(b)}(k, \theta) \), is given by the formula \( n_{\text{ch}}^{(b)}(k, \theta) = F_{\text{ch}}^{(b)}(k) n_1(k, \theta) \) (compare with (1)), with \( N_\text{ch} \) being the number of particles in the bunch. Similar to (2), for the bunch factor one has \( F_{\text{ch}}^{(b)}(k) = \left| \sum_{j=1}^{N_\text{ch}} \exp(-ik \varphi_j^{(b)}) \right|^2 \), where \( \varphi_j^{(b)} \) determines the angular location of the j-th particle in the bunch at \( t = 0 \). Assuming that the coordinates of separate particles are independent random variables we introduce the distribution function \( f(\varphi) \). The product \( f(\varphi) d\varphi \) is the probability to find an electron of the bunch in the angular interval \( (\varphi, \varphi + d\varphi) \). After averaging over the positions of a charge in the bunch we find \( F_{\text{ch}}^{(b)}(k) = N_\text{ch} \left[ 1 + (N_\text{ch} - 1)g^{(b)}(k) \right] \), where the bunch form factor is given by \( g^{(b)}(k) = \left| \int_0^{2\pi} f(\varphi)e^{-ik\varphi}d\varphi \right|^2 \). Having this result, for the radiation of a chain of bunches one gets \( n_N(k, \theta) = F_N(k)F_{\text{ch}}^{(b)}(k)n_1(k, \theta) \). Note that CSR in free space radiated by a chain of electron bunches in submillimeter and millimeter wavelength range has been observed in [49] [50] [51].

4 Conclusion

The coherence effects may essentially increase the radiation intensity emitted by a system of charged particles. In the present paper we have discussed those effects for the radiation from a chain of particles on a circular orbit around a dielectric ball. In our previous research it has already been demonstrated that in the problem with the same geometry and for a single charge, under specific conditions on the parameters the influence of the ball may increase the radiation intensity on a given harmonic by orders of magnitude, compared with the radiation from the same charge rotating in free space or in a transparent homogeneous medium. It has been argued that the increase of the radiation intensity is related to the constructive interference of the electromagnetic oscillations of the CR generated inside the ball near the entire trajectory of the particle and partially confined inside the ball by multiple reflections from its surface [13]. The radiation is propagating in the angular range \( \theta \lesssim \theta_{\text{cr}} \) near the rotation plane. Here we have provided an analytical procedure for the determination of the location of the peaks in the radiation intensity as a function of the ratio \( r_\text{b}/r_e \).

For a chain of equidistant charges rotating around the ball, the constructive interference of the waves generated by separate charges in the chain gives rise to an additional increase in the radiation intensity by the factor \( N^2 \) with \( N \) being the number of particles in the chain. The corresponding waves are radiated on the harmonics \( k = mN, m = 1, 2, \ldots \). We have shown that the relative shifts in the particles locations up to 10% do not destroy the coherence properties of the radiation. One can have coherent radiation from a chain where the distribution
of the particles is not equidistant. An example is provided by the distribution \( r_j \) where \( r_j \) are rational numbers. In this case, radiation amplification by the factor \( N^2 \) also takes place on certain harmonics \( k = mk_0 \). However, unlike the case of equidistant distribution, in general, the radiation on harmonics \( k \neq mk_0 \) is present as well. Similar coherence effects will be present in the radiation from a chain of bunches circulating around the ball. This phenomenon can be utilised in developing high-power monochromatic sources of electromagnetic radiation in the GHz/THz frequency ranges. We have demonstrated that considering the radiation from a chain rotating around balls made of fused quartz and Teflon. The choice of those materials is motivated by the weak absorption of the radiation in the frequency range under consideration. The latter is an essential point to have multiple circulations of CR inside the ball. The radiation frequency can be controlled by tuning the parameters \( r_e, r_b \) or the energy of particles in the chain. We have argued that similar coherent effects will also appear in the radiation from a chain of bunches. In the special case of an equidistant distribution of particles in the chain the results of the present paper for the angle integrated frequency distribution of the radiation are in agreement with those previously obtained in \cite{29} for a dielectric ball made of fused quartz. Another special case with a single charge circulating around a ball has been recently considered in \cite{28}.

Acknowledgments

The work was supported by the Science Committee of RA, in the frames of the research project No. 21AG-1C069.

References

[1] Zrelov, V.P., 1970. Vavilov-Cherenkov Radiation in High-Energy Physics. Israel Program for Scientific Translations, Jerusalem.

[2] Ter-Mikaelian, M.L., 1972. High Energy Electromagnetic Processes in Condensed Media. Wiley Interscience, New York.

[3] Kumakhov, M.A., 1986. Radiation of Channeled Particles in Crystals. Energoatomizdat, Moscow (in Russian).

[4] Akhiezer, A.I., Shulga, N.F., 1996. High Energy Electrodynamics in Matter. Gordon and Breach Publishers, Luxemburg.

[5] Rullhusen, P., Artru, X., Dhez, P., 1998. Novel Radiation Sources Using Relativistic Electrons. World Scientific, Singapore.

[6] Afanasief, G.N., 2004. Vavilov-Cherenkov and Synchrotron Radiation. Springer, Netherlands.

[7] Potylitsin, A.P., 2011. Electromagnetic Radiation of Electrons in Periodic Structures. Springer, Berlin.

[8] Tsytovich, V.N., 1951. To the question of the radiation of fast particles in a magnetic field. Westnik MGU 11, 27-36 (in Russian).

[9] Kitao, K., 1960. Energy loss and radiation of a gyrating charged particle in a magnetic field. Prog. Theor. Phys. 23, 759-775.
[10] Erber, T., White, D., Latal, H.G., 1976. Inner bremsstrahlung processes. 2. Acta Phys. Austriaca 45, 29-64.

[11] Schwinger, J., Tsai, W.-Y., Erber, T., 1976. Classical and quantum theory of synergic synchrotron-Cerenkov radiation. Ann. Phys. 96, 303-332.

[12] Erber, T., White, D., Tsai, W.-Y., Latal, H.G., 1976. Experimental aspects of synchrotron-Cerenkov radiation. Ann. Phys. 102, 405-447.

[13] Rynne, T.M., Baumgartner, G.B., Erber, T., 1978. The angular distribution of synchrotron-Cerenkov radiation. J. Appl. Phys. 49, 2233-2240.

[14] Giani, S., Bagulya, A.V., Grichine, V.M., 1999. X-ray synchrotron radiation in medium. Phys. Lett. B 460, 467-473.

[15] Arzumanyan, S.R., Grigoryan, L.Sh., Saharian, A.A., 1995. On the theory of radiation of charged particles in stratified spherically-symmetric medium. Izv. Nats. Akad. Nauk Arm., Fiz. 30, 99-105 (Engl. Transl.: J. Contemp. Phys.).

[16] Grigoryan, L.Sh., Kotanjyan, A.S., Saharian, A.A., 1995. Green function of an electromagnetic field in cylindrically symmetric inhomogeneous medium. Izv. Nats. Akad. Nauk Arm., Fiz. 30, 239-244 (Engl. Transl.: J. Contemp. Phys.).

[17] Arzumanyan, S.R., Grigoryan, L.Sh., Saharian, A.A., Kotanjian, Kh.V., 1995. On the theory of radiation of charged particles in stratified spherically-symmetric medium. II. Izv. Nats. Akad. Nauk Arm., Fiz. 30, 106-113 (Engl. Transl.: J. Contemp. Phys.).

[18] Grigoryan, L.Sh., Khachatryan, H.F., Arzumanyan, S.R., Grigoryan, M.L., 2006. High power Cherenkov radiation from a relativistic particle rotating around a dielectric ball. Nucl. Instrum. Methods Phys. Res., Sect. B 252, 50-56.

[19] Arzumanyan, S.R., Grigoryan, L.Sh., Khachatryan, H.F., Grigoryan, M.L., 2008. The features of synchrotron radiation from a relativistic particle rotating inside a spherical cavity. Nucl. Instrum. Methods Phys. Res., Sect. B 266, 3715-3720.

[20] Grigoryan, L.Sh., Khachatryan, H.F., Arzumanyan, S.R., Grigoryan, M.L., 2006. Radiation from a particle rotating along a not equatorial orbit of a dielectric ball. Izv. Nats. Akad. Nauk Arm. Fiz. 41, 163-169 (Engl. Transl.: J. Contemp. Phys.).

[21] Grigoryan, L.Sh., Khachatryan, H.F., Grigoryan, M.L., 2014. Intense Cherenkov radiation from a charged particle revolving along a shifted equatorial orbit about a dielectric ball. J. Phys. Conf. Ser. 517, 012006.

[22] Saharian, A.A., Kotanjyan, A.S., 2005. Synchrotron radiation from a charge moving along a helical orbit inside a dielectric cylinder. J. Phys. A 38, 4275-4292.

[23] Saharian, A.A., Kotanjyan, A.S., 2009. Synchrotron radiation from a charge moving along a helix around a dielectric cylinder. J. Phys. A 42, 135402.

[24] Saharian, A.A., Kotanjyan, A.S., 2012. Synchrotron radiation inside a dielectric cylinder. Int. J. Mod. Phys. B 26, 1250033.

[25] Kotanjyan, A.S., Mkrtchyan, A.R., Saharian, A.A., Kotanjyan, V.Kh., 2018. Radiation of surface waves from a charge rotating around a dielectric cylinder. JINST 13, C01016.
[26] Saharian, A.A., Kotanjyan, A.S., Mkrtchyan, A.R., Khachatryan, B.V., 2017. Synchrotron and Smith-Purcell radiations from a charge rotating around a cylindrical grating. Nucl. Instrum. Methods Phys. Res., Sect. B 402, 162-166.

[27] Saharian, A.A., Grigoryan, L.Sh., Grigorian, A.Kh., Khachatryan, H.F., Kotanjyan, A.S., 2020. Cherenkov radiation and emission of surface polaritons from charges moving paraxially outside a dielectric cylindrical waveguide. Phys. Rev. A 102, 063517.

[28] Grigoryan, L.Sh., Saharian, A.A., Khachatryan, H.F., Grigoryan, M.L., Sargsyan, A.V., Petrosyan, T.A., 2020. Angular distribution of high power radiation from a charge rotating around a dielectric ball. JINST 15, C04035.

[29] Arzumanyan, S.R., 2012. Features of radiation from a chain of relativistic charged particles rotating about a dielectric ball. J. Contemp. Phys. 47, 11-16.

[30] Berestetskii, V.B., Lifshitz, E.M., Pitaevskii, L.P., 2012. Quantum Electrodynamics. Butterworth-Heinemann, UK.

[31] Handbook of Mathematical Functions, edited by Abramowitz, M., Stegun, I.A., 1972. Dover, New York.

[32] Voronkova, E.M., Grechushnikov, B.N., Distler, G.I., Petrov, I.P., 1965. Optical Materials for Infrared Technology. Nauka, Moscow (in Russian).

[33] Chudpooti, N., et al., 2021. Wideband dielectric properties of silicon and glass substrates for terahertz integrated circuits and microsystems. Mater. Res. Express 8, 056201.

[34] Grigoryan, L.Sh., Khachatryan, H.F., Arzumanyan, S.R., Grigoryan, M.L., 2006. Features of radiation from a particle rotating around a dielectric ball, Preprint IAPP NAS RA N1-06, Yerevan (in Russian).

[35] Arzumanyan, S.R., Grigoryan, L.Sh., Khachatryan, H.F., Grigoryan, M.L., 2010. Some features of electromagnetic field of charged particle revolving about dielectric ball. J. Phys. Conf. Ser. 236, 012007.

[36] Nakazato, T., et al., 1989. Observation of coherent synchrotron radiation. Phys. Rev. Lett. 63, 1245-1248.

[37] Andersson, A., Johnson, M.S., Nelander, B., 2000. Coherent synchrotron radiation in the far-infrared from a 1 mm electron bunch. Opt. Eng. 39, 3099-3105.

[38] Abo-Bakr, M., Feikes, J., Holldack, K., Wustefeld, G., Hubers, H.-W., 2002. Steady-state far-infrared coherent synchrotron radiation detected at BESSY II. Phys. Rev. Lett. 88, 254801.

[39] Byrd, J.M., et al., 2002. Observation of broadband self-amplified spontaneous coherent terahertz synchrotron radiation in a storage ring. Phys. Rev. Lett. 89, 224801.

[40] Sannibale, F., et al., 2004. A model describing stable coherent synchrotron radiation in storage rings. Phys. Rev. Lett. 93, 094801.

[41] Wang, F., et al., 2006. Coherent THz synchrotron radiation from a storage ring with high-frequency RF system. Phys. Rev. Lett. 96, 064801.
[42] Di Mitri, S., et al., 2018. Coherent THz emission enhanced by coherent synchrotron radiation wakefield. Sci. Rep. 8, 11661.

[43] Evain, C., et al., 2019. Stable coherent terahertz synchrotron radiation from controlled relativistic electron bunches. Nature Physics 15, 635-639.

[44] Hartemann, F.V., 2002. High-Field Electrodynamics. CRC Press, Boca Raton.

[45] Müller, A.S., Schwarz, M., 2015. Accelerator-Based THz Radiation Sources. In: Jaeschke, E., Khan, S., Schneider, J., Hastings J. (eds), 2015. Synchrotron Light Sources and Free-Electron Lasers. Springer, Cham. https://doi.org/10.1007/978-3-319-04507-8_6-1.

[46] Huang, C., Kwan, T.J.T., Carlsten, B.E., 2013. Two dimensional model for coherent synchrotron radiation. Phys. Rev. ST Accel. Beams 16, 010701.

[47] Stupakov, G., Tang, J., 2021. Calculation of the wake due to radiation and space charge forces in relativistic beams. Phys. Rev. Accel. Beams 24, 020701.

[48] Stupakov, G., 2022. Transverse force in a relativistic beam moving along a curved trajectory. Phys. Rev. Accel. Beams 25, 014401.

[49] Ishi, K., et al., 1991. Spectrum of coherent synchrotron radiation in the far-infrared region. Phys. Rev. A 43, 5597-5604.

[50] Shibata, Y., et al., 1991. Observation of interference between coherent synchrotron radiation from periodic bunches. Phys. Rev. A 44, R3445-R3448.

[51] Shibata, Y., et al., 1991. Observation of coherent synchrotron, Čerenkov, and wake-field radiation at millimeter wavelengths using an L-band linear accelerator. Phys. Rev. A 44, R3449-R3451.