A New Approach in Decision of Pareto Efficient Concept in Probabilistic Vendor – Buyer Supply – Chain Problem for Imperfect Quality with Lead – Free Demand

R Setiawan
Mathematics Education, Sebelas Maret University, Surakarta, Indonesia
rubono.matematika@staff.uns.ac.id

Abstract. In this paper, the mathematical analysis of optimal solution or Economic Order Quantity (EOQ) of probabilistic vendor-buyer systems has been analyzed using Pareto optimal concept. It’s assumed that lead time length is limited. The quality of the product from vendor is not always 100% perfect, so imperfect quality product always exist due to specific probability rate. Pareto optimal concept has been used to find the weighted sum of the players’ objectives. Karush – Kuhn – Tucker (KKT) criteria is applied to determine the optimum value of decision variables and also minimum value of total cost as objectives function. Numerical example of appropriate simulation data is provided to compare the analytical result and to find how the effectiveness the Pareto Optimal concept to minimize the inventory cost in probabilistic assumptions rather than integrated scheme

1. Introduction
Inventory model is a mathematical model that concerns about supply chain management problem. The famous basic inventory model in the two-level supply chain is a vendor-buyer model, that was introduced by Goyal in his paper [6]. The main purpose of inventory model is to find the optimal decision of each party to get minimum total cost and optimum decision variables. Some researchers argue that product from the vendor is always 100 % perfect in quality. But, this assumption is not reliable to the fact that production process is not always 100 % perfect. Therefore, the process may deteriorate and produce poor – quality items. First, Salameh and Jaber [10] developed economic order quantity (EOQ) model where a random proportion of the items in a lot are defective. Furthermore, some author also use the assumption of imperfect quality items in their result, for example, Elyasi et.al.[4], Esmaeili et al.[5], Huang [7], Hsu and Hsu [8], Lin [9]. In the fact of reality, the deterministic model is not suitable for real process in supply chain system, so some researchers prefer to the probabilistic assumption, like uncertainty quantity demand and lead free demand, for example, Lin [9], Setiawan and Triyanto [11], and Setiawan [12].

The Lead time length in supply chain management maybe is one of the crucial problems that inflict many other problems in supply chain process, cost, and shortage condition, if they not handled properly. According to the Lin [9], lead time can be reduced, by additional crashing cost, customers service level improved, inventory in safety stocks reduced, and the competitive edge in business increased, in other words, it is controllable. Lin [9] study an integrated vendor-buyer inventory policy for a continuous review model with a random number of defective items and screening process gradually at a fixed screening rate in buyer’s arriving order lot. They assume that shortages are allowed and partially
backlogged on the buyer’s side and that the lead time demand distribution is unknown, except its first two moments.

In recent decades, the game theory approach introduced as an alternative way to find the optimal solution of supply chain system. Through these concept, economic order quantity of supply chain system can be analyzed in the sense of possibility of various strategy from each party in supply chain system. There are two main of strategy, cooperative and non-cooperative strategy. The first one is related to the Optimal Pareto scheme and the second is related to the Nash equilibrium and Stackelberg Equilibrium. According to the reference, Abad and Jaggi[1] and Elyasi et al [4], the optimal result of the non-cooperative approaches are not Pareto – efficient. To get a solution which is Pareto-efficient, a cooperative game applied. Through this paper, we analyze the EOQ result of probabilistic vendor-buyer inventory model, when each party agrees to use cooperative model and then under the condition that service level constraint is not active. In this paper, we propose some new result in probabilistic inventory using game theory approach. We arrange our paper as follows. In section 2 we explain the material and method including assumption and notation. In Section 3, we construct our inventory model including buyer’s expected average total cost and vendor’expected average total cost. We also give the mathematical result of EOQ based on Pareto optimal concept.

2. Material and Method

In this research, a mathematical analytical method is used to determine optimal result formula of decision variables, EOQ and optimal cost of the inventory system. First, algebraic manipulation and calculus optimization using partial derivative concept is used to produce a mathematical formula for total cost and its EOQ. Second, in order to get numerical and sensitivity analysis representation, we use the numerical example using appropriate data and we take into the mathematical formula of the optimal solution. We enforce all of the analytical processes regarding some basic assumptions which are detailed in the following subsection.

2.1. General Assumptions

In this paper we consider with a two-echelon supply chain consisting of a single vendor and single buyer as a model with a single product is considered. We follow the assumption that the lead time is lead-free demand, according to the reference in the works by Lin [9], lead time has n mutually independent component. The i - component has a minimum duration \(a_i\), a normal duration \(b_i\) and a crashing cost unit item \(c_i\) are assumed to be arranged such that \(c_1 \leq c_2 \leq c_3 \ldots \leq c_m\), then the lead time components are crashed one at a time with crashing cost \(c_i\), then \(c_2\) and so on, then \(L_{\text{min}} = \sum_{i=1}^{m} a_i \leq L \leq \sum_{i=1}^{m} b_i = L_{\text{max}}\), where \(L_i = L_{\text{max}} = \sum_{j=1}^{i} (b_j - a_j)\) and the lead time crashing cost per cycle \(C(L)\) for given \(L \in (L_i, L_{i-1}]\) is given by \(c_i(L_i, L) + \sum c_j(b_j - a_j)\). The defective items are always exist in lot size of \(Q\), which are sent from vendor to the buyer with a a percentage \(\gamma\). To ensure that the vendor has enough production capacity to fulfill the buyer’s demand, it must be assumed that the vendor’s production rate is greater than buyer’s demand rate or \((1 - \gamma)P > D\). The buyer will return all item which classified as defective and will be given a full price (discount) to the secondary market. The reorder point \(r\) is defined by the sum of expected demand during lead time and safety stock (SS) \(r\), that is \(r = DL + SS \Leftrightarrow r = DL + k\sigma \sqrt{L}\).

2.2. Notations.

In this paper, we use some following parameter with some specific notation

- **Q**: The size of the shipments from the vendor to the buyer, a decision variable.
- **n**: The number of shipments per batch production run, a decision variable.
- **D**: Expected demand per unit time on the buyer.
- **P**: The production rate at the vendor.
- **F**: The freight (transportation) cost per shipment.
- **S_b**: Buyer’s ordering cost per order.

...
$S_v$: Vendor’s setup cost per production.

$h_{b_1}$: Buyer’s holding cost for non defective item per unit per cycle.

$h_{b_2}$: Buyer’s holding cost for non defective item per unit per cycle.

$\pi$: Buyer’s shortage cost per unit short

$\delta_0$: Buyer’s marginal profit (cost of loss of demand) per unit.

$\beta$: Fraction of the demand during the stock-out period that will be backordered, $\beta \in [0,1]$.

$c_{ib}$: The vendor’s unit warranty cost per defective items cost per defective item.

3. Result and Discussion

3.1. Buyer’s expected average total cost per unit time.

Buyer’s total cost per cycle per unit time is the sum of the cost due to placing an order, transportation cost, screening cost, holding cost, expected shortage cost and lead time crashing cost. The mathematical formula of buyer’s total cost is given by the following equation

$$
TC_B(Q, k, n, L) = S_p + nF + c_{ib} Q + [\pi + \pi_0(1-\beta)]E[(X - r)^*] + C(L) +
$$

$$
\frac{h_{b_1} Q(1-\gamma)}{2x(1-\gamma)} \frac{Q(1-\gamma)}{D} + \frac{h_{b_2}}{2x(1-\gamma)} \frac{Q^2(1-\gamma)}{D} - \frac{Q^2(1-\gamma)}{2x}
$$

$$
(1)
$$

Using renewal – reward theorem, the expected average total cost per unit time for the buyer is

$$
ETC_B(Q, k, n, L) = \frac{D(S_B + nF + c_{ib} Q + \pi_0E[(X - r)^*] + C(L))}{Q(1-\gamma)} + h_{b_1} \frac{Q(1-\gamma)}{2x(1-\gamma)} + \frac{h_{b_1} k}{2x(1-\gamma)} + \frac{h_{b_2} Q Y + h_{b_2} Q(1-\gamma)}{2x(1-\gamma)}
$$

$$
(2)
$$

where $\pi + \pi_0(1-\beta) = \bar{\pi}$. Based on the Equation (2) we get the worst distribution of $ETC_B(Q, k, L)$

$$
ETC_B(Q, k, n, L) = \frac{D(S_B + nF + c_{ib} Q + \pi_0E[(X - r)^*] + C(L))}{Q(1-\gamma)} + h_{b_1} \frac{Q(1-\gamma)}{2x(1-\gamma)} + \frac{h_{b_1} k}{2x(1-\gamma)} + \frac{h_{b_2} Q Y + h_{b_2} Q(1-\gamma)}{2x(1-\gamma)}
$$

$$
(3)
$$

3.2. Vendor’s expected average total cost per unit time.

The vendor’s inventory per production cycle can be obtained by subtracting the accumulated buyer inventory level from the accumulated vendor inventory level.

$$
\left[nQ_p + (n - 1)T\right] - \frac{nQ(1-\theta)}{2} - T[Q + 2Q + \cdots + (n - 1)Q] = \frac{nQ^2_p}{2p} - \frac{n^2Q^2}{2p} + \frac{n(n - 1)Q^2(1-\gamma)}{2D}
$$

$$
(4)
$$

Thereby, defining holding cost for the vendor as $h_v\left[\frac{n(n - 1)Q^2(1-\gamma)}{2D}\right]$. After adding warranty cost, the vendor expected average total cost per cycle per unit time $TC_v(Q, n)$ is formulated by

$$
ETC_v(Q, n) = \frac{E[TC_v(Q, n)\mid nT]}{E[\pi T]} = \frac{D}{nQ(1-\theta)(1-\theta)} [S_v + c_{vw} Q Y + h_v \left(\frac{n(n - 1)Q^2(1-\gamma)}{2D}\right)]
$$

$$
(5)
$$

3.3. EOQ using Pareto Optimal

According to the Abad & Jaggi [1] and Elyasi et al.[4], the optimal result of the non-cooperative approaches is not Pareto – efficient. So in order to get the Pareto-efficient solution, a cooperative game is applied. We use the cooperative game introduced by Abad & Jaggi [1] and also used by Elyasi et al.
The weighted sum of the players’ objectives is optimized with the assumption that the players (vendor and buyer) agree on the result of the Nash equilibrium

\[ Z = \lambda_p (ETC_b) + (1 - \lambda_p)ETC_v, \quad 0 < \lambda < 1 \]  

so

\[ Z = \lambda_p \left[ \frac{D\left(s_b + nF + c_{ib}Q + \frac{\pi \sigma L}{2} (\sqrt{1 + k^2} - k) + C(L)\right)}{Q(1 - \gamma)} + h_{b1} \sigma \sqrt{L} \left(k + \frac{(1 - \beta)\pi \sqrt{1 + k^2} - k}{2}\right) + h_{b2} Q \gamma + \frac{(h_{b1} D - h_{b2})Q \gamma}{2x(1 - \gamma)} + \frac{h_{b2} Q(1 - \gamma)}{2} \right] + (1 - \lambda_p) \left[ \frac{D}{nQ(1 - \gamma)(1 - \theta)} \left[ S_v + c_{pv} Q \gamma \theta + h_v \left[ \frac{n(n-1)Q^2(1-\gamma)}{2D} \right] \right] \right] \]  

(7)

It’s easy to prove \( Z \) is convex function. To get optimal solution for decision variables, we take the first partial derivative of \( Z \) with respect to \( n \) and then set this equal to zero to get optimal value of parameter \( \lambda_p \) as follow

\[ n^* = \frac{(1 - \lambda_p^*)D(S_v + c_{pv} Q^* \gamma \theta)}{Q^*(1 - \gamma)(1 - \theta) \left( \frac{\lambda_p^* F}{Q^*(1 - \gamma)} + \frac{(1 - \lambda_p^*) h_v Q^*^2 (1 - \gamma)}{2D} \right)} \]  

(8)

with \( \lambda_p^* = \frac{1}{(1 + B)} \). For fixed value of \( L_i \in [L_{i-1}, L_i], i = 1, 2, 3, \ldots, N \), optimal shipment lot size \( Q \) and it’s safety factor, which yield the minimum of \( Z \) are solutions to the set of following equations \( \frac{\partial Z}{\partial Q} = 0, \frac{\partial Z}{\partial k} = 0 \) corresponding to the optimal value of \( \lambda_p^* \). Therefore, we have

\[ Q^* = \frac{\frac{\lambda_p^* D}{(1 - \gamma)} \left( S_b + nF + \frac{\pi \sigma L}{2} (\sqrt{1 + k^2} - k) + C(L) \right) - \frac{(1 - \lambda_p^*) D S_v}{n(1 - \gamma)(1 - \theta)}}{\lambda_p^* h_{b2} \gamma + \frac{(h_{b1} D - h_{b2})Q \gamma}{2x(1 - \gamma)} + \frac{h_{b1}(1 - \gamma)}{2} + \frac{(1 - \lambda_p^*) h_v (n - 1)}{2}} \]  

(9)

\[ Q^* = \sqrt{\frac{B - 1}{(B - 1)^2}} \]  

(10)

where \( B = \frac{2h_{b1}}{n(1 - \beta) + D \frac{D}{2Q(1 - \gamma)}} \). As we know from (9) and (10), explicit form solution of \( k, Q, \) and \( n \) are not possible, so we use a numerical example to get an optimal approximation of decision variables.

3.4. EOQ using Integrated Scheme We will use the integrated scheme to find joint’s expected average total cost per unit time and to compare the previous result of EOQ

\[ JETC_u(Q, k, L, n) = \frac{D}{nQ(1 - \gamma)(1 - \theta)} \left[ S_v + c_{pv} Q \gamma \theta + h_v \left[ \frac{n(n-1)Q^2(1-\gamma)}{2D} \right] \right] + \frac{D\left(s_b + nF + c_{ib}Q + \frac{\pi \sigma L}{2} (\sqrt{1 + k^2} - k) + C(L)\right)}{Q(1 - \gamma)} + h_{b1} \sigma \sqrt{L} \left(k + \frac{(1 - \beta)\pi \sqrt{1 + k^2} - k}{2}\right) + \frac{(h_{b1} D - h_{b2})Q \gamma}{2x(1 - \gamma)} + \frac{h_{b2} Q(1 - \gamma)}{2} \]  

(11)
It’s easy to prove that function $JETC_{ni}(Q,k,L,n)$ is convex function fixed value of $L_i \in [L_{i-1}, L_i]$, $i = 1,2,3,\ldots,N$. After apply optimum condition to the Equation (11) we get the optimal shipment lot size $Q$ and it’s safety factor as follows:

$$k^* = \frac{(1-A)}{\sqrt{A(2-A)}} \quad A = \frac{2h_b}{\pi \left(\frac{D}{Q(1-\gamma)}+h_b(1-\beta)\right)}, \quad n^* = \frac{D(S_v+c_{mv}Q^*\gamma\theta)}{\frac{1}{2}h_bQ^*2(1-\gamma)+DF(1-\theta)}$$

$$Q^* = \sqrt{\frac{2D}{n^*(1-\gamma)(1-\theta)}} \frac{S_b+n^*(1-\beta)\left(S_b+n^*F+\frac{\pi\sigma_\tau^2}{2}(\sqrt{1+k^*2}-k^*)+C(L)\right)}{h_bn^*(n^*-1)(1-\gamma)} + \frac{h_bD-h_bk^*\gamma}{x(1-\gamma)} + h_bk^*(1-\gamma)+2h_b\gamma$$

(12)

Like the previous result in Pareto optimal result, explicit form solution of $k$, $Q$ and $n$ are not possible in analytic sense, because the evaluation of Equations (12) requires the information of the value of the other decision variable, so we have to use a numerical example to get an optimal approximation of decision variables.

3.5. Numerical examples and Sensitivity Analysis

We use the algorithm, which proposes by Lin [9], to find the approximation of the optimal value of decision variables and also the total cost of inventory system by Pareto optimal concept and integrated system. We consider an integrated inventory with a single vendor, single buyer and single product with following example data: $D = 1200$ unit/year, $\sigma = 8$ unit/week, $S_b = $ 400/order, $S_v = $ 3000/setup, $P = 3000$ units/year, $h_v = $ 14/unit/year, $h_{b1} = $ 15/unit/year, $h_{b2} = $ 11/unit/year, $F = $ 20/shipment, $c_{wp} = $ 5/unit, $c_{ib} = $ 0.5/unit, $x = 175200$ unit/year, $\pi_0 = $ 50/unit. We use the lead time has three components with the lead time data used in reference by Lin [9]. We propose the numerical result of simulation data with vary in parameter $\beta$ and $\gamma$ value. we investigate the effects of parameters $\beta$ and $\gamma$ to the to the inventory total cost and decision variables. The optimum value result of decision variable and total cost function value as an objective function and also integrated scheme as comparation. We propose our numerical result in Table 1. According to the numerical example result, we get some valuable information for our analytical result. Total cost of inventory system based on Pareto optimal scheme is more economically profitable to each party rather than integrated scheme. Although, they are included in the same category of strategy that is cooperative strategy, according to the numerical result about the effect of defective item rate ($\gamma$), the larger value of defective item rate, then it will be result in smaller value of optimum order quantity $Q$ and it’s total cost. But, there is an interesting result about the value of defective item rate, we can say that an “bifurcation value” in the value of about 0.05. If the defective item rate is greater than those critical point, then the total value will increase than before. From this result, it’s recommended that tolerable value of defective item rate of this supply chain model is less than 0.05. In the other-hand, for fixed value of defective item, then the changes value of $\beta$ also resulted in changes in the optimal value of total inventory cost. In average, the greater value fraction of the demand in the stock out period $\beta$ then it will the result of the smaller value of total cost of inventory system.
Table 1. Numerical Simulation

| β  | θ   | Q   | k | n | L | λ   | ETC_b | ETC_v | JETC | Z   |
|----|-----|-----|---|---|---|-----|-------|-------|------|-----|
| γ = 0.050 |
| 0.00 | 0.2 | 382 | 4.92 | 1 | 3 | 0.9836 | 8,377 | 9,947 | 18,324 | 8,403 |
| 0.50 | 0.2 | 386 | 3.03 | 1 | 3 | 0.9836 | 7,646 | 9,837 | 17,483 | 7,682 |
| 0.75 | 0.2 | 340 | 1.99 | 1 | 3 | 0.9836 | 7,295 | 9,689 | 16,984 | 7,334 |
| 1.00 | 0.2 | 422 | 0.58 | 1 | 3 | 0.9836 | 7,112 | 8,994 | 16,106 | 7,143 |
| γ = 0.2 |
| 0.00 | 0.2 | 192 | 5.26 | 1 | 3 | 0.9836 | 11,036 | 23,457 | 34,493 | 11,240 |
| 0.50 | 0.2 | 194 | 3.40 | 1 | 3 | 0.9836 | 10,307 | 23,292 | 33,599 | 10,520 |
| 0.75 | 0.2 | 195 | 2.42 | 1 | 3 | 0.9836 | 9,950 | 23,110 | 33,060 | 10,166 |
| 1.00 | 0.2 | 200 | 1.34 | 1 | 3 | 0.9836 | 9,625 | 22,602 | 32,227 | 9,838 |
| γ = 0.5 |
| 0.00 | 0.2 | 123 | 6.03 | 1 | 3 | 0.9836 | 21,232 | 58,442 | 79,674 | 21,842 |
| 0.50 | 0.2 | 124 | 4.23 | 1 | 3 | 0.9836 | 20,503 | 58,364 | 78,867 | 21,124 |
| 0.75 | 0.2 | 124 | 3.32 | 1 | 3 | 0.9836 | 20,141 | 58,290 | 78,431 | 20,767 |
| 1.00 | 0.2 | 124 | 2.39 | 1 | 3 | 0.9836 | 19,785 | 58,150 | 77,935 | 20,414 |

4. Conclusion and Remarks

In this paper we consider with the probabilistic vendor-buyer inventory model for imperfect quality with lead-free demand, it’s mean that lead time demand is uncertainty and information about lead time distribution are limited only in first and two moments. It is considering a partial back ordering process with certain rate. We investigate the optimal order quantity under the condition that the vendor and the buyer agree to play cooperative strategy, so we use Pareto optimal scheme to get the optimal decision of each party. Based on the analytical result, it is difficult to get an explicit form of each decision variable, so to get more information about that variable we use numerical simulation with some appropriate data. Regarding the numerical example result, we get the result that total cost of inventory system based on Pareto Optimal scheme is more economically profitable to each party rather than integrated scheme.

References

[1] Abad P L and Jaggi C K 2003 Int. J. of Prod. Econ., 83, 115-122.
[2] Abad P L 1994 Eur. J. of Oper.R., 78, 334-354.
[3] Bylka S 2003 Int. J. of Prod Econ, 82, 533-544.
[4] Elyasi M, Khoshalhan F, and Khanmirzae M, Int. J. Eng. Comp. 5, No.2,211-222.
[5] Esmaeili M, Aryanezhad M B, and Zeephongsekal P 2009 Eur.J.Oper.Res.195
[6] Goyal S K 1977 Int.J.Prod.Res.15 107-11.
[7] Huang C K 2002 Prod.Plan.Control 13 355-61.
[8] Hsu J T and Hsu L F 2012 Int.J.Eng.Comp.3 703-20.
[9] Lin H J 2012 Yugosl.J.Oper.Res.23 87-109.
[10] Salameh M K and Jaber M Y 2000 Int.J.Prod.Econ.64 59-64.
[11] Setiawan R and Triyanto 2016 Far East J. Math.Sci.99 109-132.
[12] Setiawan R 2016 Far East J.Math.Sci.100 1695-704.

Acknowledgment
The author would say to the anonymous referees for their valuable comments and suggestions. This work is supported by PNBP Sebelas Maret University Grant for Fundamental Research 2018, No.543/UN27.21/PP/2018, Institute for Research and Community Service (LPPM), Sebelas Maret University, Surakarta, Indonesia.