New low-$Q^2$ measurements of the $\gamma^* N \to \Delta(1232)$ Coulomb quadrupole form factor, pion cloud parametrizations and Siegert’s theorem

G. Ramalho

Universidade Cruzeiro do Sul, 01506-000, S˜ ao Paulo, SP, Brazil

1 Introduction

The first excited state of the nucleon, the $\Delta(1232)$, is an exceptional system in the context of the strong interactions (QCD). It dominates the electro-excitations of the nucleon and nucleon pion-production reactions [1,2,3]. The study of the $\Delta(1232)$ internal structure ruled by the quark-gluon degrees of freedom and quark-antiquark transitions [4,5] reveals that the quark-gluon degrees of freedom and quark-antiquark transition [1,2,4]. The magnetic dipole form factor $G_M$ can be parametrized by the relations $G_E \propto G_{E\gamma} / \left(1 + \frac{Q^2}{2M_{\Delta}(M_{\Delta} - M)}\right)$ and $G_C \propto G_{E\gamma}$, where $G_{E\gamma}$ is the neutron electric form factor, and $M, M_{\Delta}$ are the nucleon and $\Delta$ masses, respectively. These parametrizations are in full agreement with Siegert’s theorem, which states that $G_E = \frac{M_{\Delta} - M}{2M_{\Delta}} G_C$ at the pseudothreshold, when $Q^2 = -(M_{\Delta} - M)^2$, and improve previous parametrizations. Also a small valence quark component estimated by a covariant quark model contributes to this agreement. The combination of the new data with the new parametrization for $G_E$ concludes an intense period of studying the $\gamma^* N \to \Delta(1232)$ quadrupole form factors at low $Q^2$, with the agreement between theory and data.

PACS. 13.40.Gp Electromagnetic form factors – 14.20.Gk Baryon resonances with $S=0$ – 12.39.Ki Relativistic quark model

1 Introduction

The first excited state of the nucleon, the $\Delta(1232)$, is an exceptional system in the context of the strong interactions (QCD). It dominates the electro-excitations of the nucleon and nucleon pion-production reactions [1,2,3]. The study of the $\Delta(1232)$ internal structure ruled by the quark-gluon degrees of freedom and quark-antiquark transitions, interpreted as meson cloud, reveals that the $\gamma^* N \to \Delta(1232)$ transition is predominantly a magnetic transition [4,5]. The magnetic dipole form factor $G_M$ is dominated by valence quark effects, particularly at large momentum transfer squared, $Q^2$, and can be explained even by quark models based on a symmetric structure for the nucleon and the $\Delta(1232)$ [6,7,8,9,10,11].

The $\gamma^* N \to \Delta(1232)$ transition is also characterized by two sub-leading quadrupole form factors: the electric ($G_E$) and the Coulomb ($G_C$) form factors [12,13]. The non-zero values for the quadrupole form factors are interpreted as a consequence of the deviation of the $\Delta(1232)$ from a spherical shape [3,4,5,6,7,8,9,10,12,13]. Contrary to the case of the magnetic form factor, estimates of the electric and the Coulomb quadrupole form factors based on quark models predict only a small fraction of the values measured [13,5,14,22,23]. There are, however, evidence that the missing strength of the quadrupole form factors in quark models is due to meson cloud or quark-antiquark effects [12,23,24,25,26,27].

Estimates based on the limit of a large number of colors ($N_c$) and SU(6) quark models with symmetry breaking, indicate in fact, that in the low-$Q^2$ region the $\gamma^* N \to \Delta(1232)$ quadrupole form factors are dominated by pion cloud effects [12,23,24,31,32,33,34]. Parametrization of pion cloud contributions based on large $N_c$ have been proposed to explain the empirical data [29,34]. Those parametrization are very close to the empirical data [29,35]. In some cases, small valence quark contributions help to improve the description of the data [6,12,23,34].

There are, however, some limitations associated with those parametrizations. Until the last few years the parametrization for $G_C$ was in disagreement with the low-$Q^2$ data. In addition, there is a conflict between the pion cloud parametrizations and Siegert’s theorem [29,35], which relates the quadrupole form factors $G_E$ and $G_C$ at the pseudothreshold, when $Q^2 = -(M_{\Delta} - M)^2$.

Recently, new data for the Coulomb quadrupole form factor became available in the region $Q^2 = 0.04–0.13$ GeV$^2$ from the experiments at Jefferson Lab/Hall A [36]. The new data compare extraordinarily well with an im-
proved large $N_c$ estimate of the pion cloud contributions to the quadrupole form factors, discussed in the present work, when valence quark contributions estimated by a covariant quark model are also included. The new parameterizations for the quadrupole form factors satisfy Siegert’s theorem \[29,30,31,32,33,34\]:

Siegert’s theorem states that when the \( \Delta(1232) \) and the nucleon are both at rest, one has \[29,30,31,32,33,34\]

\[
G_E(Q^2) = \kappa G_C(Q^2),
\]

where \( \kappa = \frac{M_A - M}{2M_A} \) and \( Q^2 = -(M_A - M)^2 \). The condition \( Q^2 = Q^2_{\text{pt}} \) defines the pseudothreshold, when the photon three-momentum \( q \) vanishes (\( |q| = 0 \)).

The remarkable agreement between the parameterizations for the quadrupole form factors (\( G_E \) and \( G_C \)) and the data can be observed in fig. 1. The data are from refs. \[21,36,42,43,44,45,46\]. Note in particular the excellent agreement with the new data from JLab/Hall A.\[30\]. The results for \( G_C \) are multiplied by \( \kappa \) for convenience. In the figure one can notice the convergence of the two lines at the lowest \( Q^2 \) point (pseudothreshold) proving the consistency with Siegert’s theorem. The pion cloud parameterizations for the form factors \( G_E \) and \( G_C \) discussed next, as well as the valence quark contributions discussed later, contribute to this success. The inclusion of the valence quark contributions compensates the underestimation associated with the pion cloud parameterizations.\[29,30,31,32,33,34\].

2 Framework

The internal structure of the baryons can be interpreted using as a combination of the large \( N_c \) limit, with \( SU(6) \) quark models with two-body exchange currents.\[32,34\]. The \( SU(6) \) symmetry breaking induces an asymmetric distribution of charge in the nucleon which generates non-zero results for the neutron electric form factor as shown in constituent quark models such as the Isgur-Karlon model \[16,17,18,19\] and others.\[33,34\]. Using the \( SU(6) \) symmetry breaking one can show that the \( \gamma^* N \to \Delta(1232) \) quadrupole moments are proportional to the neutron square charge radius (\( r^2_n \)).\[15,29,30,31,33,34,51\].

Using the low \( Q^2 \) expansion of the neutron electric form factor, \( G_E \), we can represent the \( Q^2 \) dependence of the quadrupole form factors in the form \[29,30,31,32,33,34\]:

\[
G_E^r(Q^2) = \left( \frac{M}{M_A} \right)^{3/2} \frac{M_A^2 - M^2}{2\sqrt{2}} \frac{\tilde{G}_E(Q^2)}{1 + \frac{Q^2}{2M_A(M_A - M)}},
\]

\( G_C^r(Q^2) = \left( \frac{M}{M_A} \right)^{1/2} \sqrt{2M_A M \tilde{G}_E(Q^2)}, \)

where \( \tilde{G}_E = G_E / Q^2 \).

The interpretation of the previous relations as the pion cloud contributions is the consequence of the relations between quadrupole form factors and \( r^2_n \) based on constituent quark models with two-body exchange currents. The effects of those currents can be interpreted as pion/meson contributions.\[31,32,33,34\]. The interpretation is also valid in large \( N_c \) limit, where the form factors \( G_E \) and \( G_C \) appear as higher orders in \( 1/N_c^2 \), compared to \( G_M \).\[29,32,33,34\].

Equations \[29\] - \[33\] were derived directly from the large \( N_c \) limit \[29\], apart from the denominator of the factor \( \tilde{G}_E \) in eq. \[29\]. This denominator is included in the present work in order to satisfy Siegert’s theorem.\[1\]. Note that in the limit \( Q^2 \to 0 \) the extra factor reduces to the unit, and we recover the original form as the pion cloud contributions discussed later, contribute to this success. The inclusion of the valence quark contributions compensates the underestimation associated with the pion cloud parameterizations.\[29,30,31,32,33,34\].

\( \text{I} \) Since we can write: 1 + \( \frac{Q^2}{2M_A(M_A - M)} \) as a combination of the large \( N_c \) limit, with \( SU(6) \) quark models with two-body exchange currents.\[32,34\]. The \( SU(6) \) symmetry breaking induces an asymmetric distribution of charge in the nucleon which generates non-zero results for the neutron electric form factor as shown in constituent quark models such as the Isgur-Karlon model \[16,17,18,19\] and others.\[33,34\]. Using the \( SU(6) \) symmetry breaking one can show that the \( \gamma^* N \to \Delta(1232) \) quadrupole moments are proportional to the neutron square charge radius (\( r^2_n \)).\[15,29,30,31,33,34,51\].

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level (one-body-currents) [50]. It is then necessary to include higher-order terms, such as two-body currents, in order to satisfy Siegert’s theorem [56]. As mentioned above, those currents are associated with the pion cloud contributions.

To describe the neutron electric form factor we consider the Galster parametrization [47]:

\[ G_{EN}(Q^2) = -\mu_n \frac{a\tau_N}{1 + d\tau_N} G_D, \]

where \( \mu_n = -1.913 \) is the neutron magnetic moment, \( \tau_N = \frac{Q^2}{2M} \), \( G_D = 1/(1 + Q^2/0.71)^2 \) is the dipole form factor, and \( a, d \) are two dimensionless parameters. In fig. 1 we use \( a = 0.9 \) and \( d = 2.8 \), a parametrization that describes very well the neutron electric form factor data. We assume that eq. (4) holds for \( Q^2 < 0 \), because we expect \( G_{EN} \) to be described by a smooth function near \( Q^2 = 0 \), and also because the range of extrapolation is small, since \( Q_{EM}^2 \simeq -0.1 \) GeV\(^2\). Similar extrapolations are considered in refs. [31,32,39].

We consider the Galster parametrization [47] because of its simplicity and also because of the limited precision of the \( G_E \) and \( G_C \) below 0.3 GeV\(^2\). Other phenomenological parametrizations with a similar number of parameters can also be considered [35,50,60,61,62,63]. More sophisticated parametrizations, with a larger number of parameters, have been derived based on dispersion relations and chiral perturbation theory [64-69]. In a separated work we study a new class of parametrizations for \( G_{EN} \) [70].

Some care should be taken with the use of the relations (2)-(3), since in principle they should not be interpreted exclusively as pion cloud contributions, because in the empirical parametrization of \( G_{EN} \) one includes all possible contributions, including also contributions due to valence quark effects. We note, however, that in an exact \( SU(6) \) model the contribution from the valence quarks associated with one-body currents vanishes [34-50,60,69]. Thus, in an approximated \( SU(6) \) symmetry, one can still expect that the quark-antiquark contributions are the dominant effect for \( G_{EN} \) and \( \tau^2 \) [34-31]. In those circumstances, one can use eqs. (2)-(3) to estimate the pion cloud contributions to the quadrupole form factors. Examples of models with pion cloud/sea quark dominance can be found in refs. [22,40,70,71,72].

3 Results

The theoretical estimates presented in fig. 1 are compared with data from Mainz [21,42], MIT-Bates [43] and Jefferson Lab [13,14] for finite \( Q^2 \), and the world average from the Particle Data Group at \( Q^2 = 0 \) [16] (empty diamonds and circles). The new data at \( Q^2 = 0.06, 0.13 \) GeV\(^2\) for \( G_E \) and \( Q^2 = 0.04, 0.06, 0.13 \) GeV\(^2\) for \( G_C \) are from JLab/Hall A [46] (solid diamonds and circles). To convert the new data for the electromagnetic ratios \( R_{EM} \equiv \frac{G_E}{G_D} \) and \( R_{SM} \equiv \frac{G_C}{2G_D} \) into \( G_E \) and \( G_C \), we use the MAID2007 parametrization for \( G_M: G_M = 3\sqrt{1 + \tau^2 (1 + a_1 Q^2)e^{-a_4 Q^2}} G_D \), where \( \tau = \frac{Q^2}{(3M_\Delta + M)^2}, a_1 = 0.01 \) GeV\(^{-2}\) and \( a_4 = 0.23 \) GeV\(^{-2}\) [39]. The larger error bars associated with the new data are mainly the consequence of the different model descriptions of the background [50].

The pion cloud contributions for the \( \gamma^* N \rightarrow \Delta(1232) \) quadrupole form factors given by eqs. (2)-(3) can be complemented by small valence quark contributions to the respective form factors (around 10%, near \( Q^2 = 0 \)). As discussed in ref. [55], those contributions are naturally consistent with Siegert’s theorem. The valence quark contributions to the \( \gamma^* N \rightarrow \Delta(1232) \) quadrupole form factors are produced by the high angular momentum components in the nucleon and/or \( \Delta(1232) \) wave functions. As a consequence of the orthogonality between the nucleon and \( \Delta(1232) \) states, the valence quark contributions to the quadrupole form factors vanish at the pseudothreshold and the Siegert’s theorem condition is trivially satisfied [39,12,34]. The validity of Siegert’s theorem depends then only on the pion cloud contribution. It is for that reason that the parametrizations (2)-(3) are particularly useful.

To estimate the valence quark contribution we use the covariant spectator quark model [47,48,12,13,74], because it is covariant and it is also consistent with lattice QCD simulations [75]. In ref. [12] the valence quark contributions are calculated using an extrapolation from lattice QCD results with large pion masses, to the physical point [51,12,74]. The contributions to the quadrupole form factors are the consequence of quark \( D \)-wave components on the \( \Delta(1232) \) wave function [12,74]. The free parameters of the model associated with the two possible \( D \)-states are the mixture coefficients and parameters associated with the shape of the radial wave functions. All the free parameters of the model are fixed by the lattice data [12]. In the lattice QCD simulations with large pion masses the meson cloud effects are very small, and the physics associated with the valence quarks can be better calibrated [3,12,74]. The results of the valence quark contributions to
the form factors $G_E$ and $G_C$ estimated by the model from ref. 12 are presented in fig. 2. The results displayed correspond to a 0.72% mixture for both $D$-states. Note, in the figure, that the quadrupoles form factors follow the approximated relation $G_E \simeq nG_C$.

The present estimates of the valence quark contributions to the quadrupole form factors can be compared with other estimates from the bare core contributions, such as the Sato-Lee 25 and DM models 26. It is important to note, however, that those parametrizations are not consistent with the constraints from quark models, since the quadrupole form factors must vanish at the pseudothreshold, as a consequence of the orthogonality between the states 35.

The final results for the electric and Coulomb quadrupole form factors, presented in fig. 1, are then the sum of the pion cloud parametrizations 27–29 and the valence quark contributions from the covariant spectator quark model 12. It is worth mentioning that the combination of the two effects is fundamental to the agreement between theory and data. This happens because the modified form of $G_{E2}^\pi$ decreases the estimate of $G_E$, which is compensated by the inclusion of the valence quark component. Also for $G_C$, the addition of the valence quark component is essential to reproduce the magnitude of $G_C$ below 0.5 GeV$^2$ 40.

In fig. 1 one can also notice that there are some discrepancies between the new JLab/Hall A data and the previous measurements from MAMI and MIT-Bates 21, 22, 41 below 0.15 GeV$^2$. The magnitude of the form factor $G_C$ is smaller in previous measurements. The result at $Q^2 = 0.06$ GeV$^2$ from MAMI 21 is inconsistent with the new results at $Q^2 = 0.04$ and 0.09 GeV$^2$. This discrepancy has been identified as a consequence of the procedure used to calculate the resonant amplitudes from the cross sections 36.

In the present work, we restrict our study to the low $Q^2$ region, because we cannot expect the pion cloud parametrizations 27–29 to be valid for arbitrary large values of $Q^2$, since they are derived from the low-$Q^2$ relation $G_{E0} \simeq -\frac{1}{4}\pi^2 Q^2$. One can then assume that for large values of $Q^2$, eqs. 27–29 are modified according to $G_{E2}^\pi \rightarrow G_{E2}^\pi/(1 + Q^2/A_E^2)$ and $G_{C2}^\pi \rightarrow G_{C2}^\pi/(1 + Q^2/A_C^2)^2$, where $A_E$ and $A_C$ are large momentum cutoff parameters. In those conditions, the form factors $G_E$ and $G_C$ would be, at large $Q^2$, dominated by the valence quark contributions, as predicted by perturbative QCD, with falloffs: $G_E \propto 1/Q^4$ and $G_C \propto 1/Q^6$ 70–74.

Using the previous results for $G_E$ and $G_C$ we can also calculate the electromagnetic ratios $R_{EM}$ and $R_{SM}$, and compare the results with the measured data. To estimate $G_M$ we use the MAID2007 parametrization 39. The comparisons are presented in fig. 3 up to 1.5 GeV$^2$. In that region one can observe some deviation between the model and the $R_{SM}$ data for $Q^2 = 0.3–0.8$ GeV$^2$. This result seems to indicate that the pion cloud parametrization for $G_C$ may not be very accurate as the parametrizations presented in other works 29, 31. It is worth mentioning, however, that those works use parametrizations of $G_M$ based on relations with the nucleon form factors, and not the empirical parametrizations of $G_M$, as in the present study.

Another important point to discuss is the value of the functions $R_{EM}$ and $R_{SM}$ when $Q^2 = 0$. In fig. 3 the two functions are very close at $Q^2 = 0$. The numerical result is $R_{SM} - R_{EM} \simeq 0.05\%$. This result corroborates the large $N_c$ estimate: $R_{EM}(0) = R_{SM}(0)$, apart from terms $O(1/N_c^2)$ 29. In our framework, the previous result is the consequence of the combination between the relation for the pion cloud parametrizations at $Q^2 = 0$: $G_E^\pi = \frac{M^2 - M^2_D}{4M^2_D} G_C^\pi$, and the approximated relation between the valence quark contributions: $G_E \simeq \frac{M^2 - M^2}{4M^2_D} G_C^\pi$ as observed in fig. 2.

The consistence of the new data can be tested in the near future by lattice QCD simulations with pion masses near the physical point 78. Meanwhile, for simulations not too far from the pion physical mass, one can test the compatibility between lattice QCD simulations and empirical data using chiral effective field theories and chiral quark models 79–81, 32.

4 Outlook

To summarize, in this work we present parametrizations for the pion cloud contributions to the $\gamma^*N \rightarrow \Delta(1232)$ quadrupole form factors that are fully consistent with Siegert’s theorem. When we combine those parametrizations with a consistent calculation of the valence quark contributions, we obtain an excellent description of the available data, including in particular, the most recent measurements of $G_E$ and $G_C$ at low $Q^2$. Since the valence quark components are extrapolated from lattice QCD, and the

\[ M^2 - M^2_D \approx \frac{M^2 - M^2}{4M^2_D}, \]

apart from relative corrections of the order $O(\frac{1}{N_c^2})$.
pion cloud parametrizations are determined by $G_{E,n}$, our final results are genuine predictions.

The understanding of the properties of the quadrupole form factors at low $Q^2$ has been a challenge, since the derivation of the relations between $\gamma^*N \rightarrow \Delta(1232)$ quadrupoles and the neutron square charge radius in the context of constituent quark models, $SU(6)$ symmetry and large $N_c$ limit, and since the first measurements of the quadrupole ratios in the modern accelerators [33,41,53]. Combining two features, namely, the new data for the quadrupole form factors and a new parametrization for the pion cloud contribution for $G_E$, we have achieved at least a consistent description of the $\gamma^*N \rightarrow \Delta(1232)$ quadrupole form factors at low $Q^2$.

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