Effect of dark energy sound speed and equation of state on CDM power spectrum

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Abstract. We have studied the influence of equation of state $w$, and effective sound speed $c_e$ of the dark energy perturbations on the cold dark matter (CDM) power spectrum. We have considered different cases of the equation of state and the effective sound speed, and the CDM power spectrum is found to be generically suppressed in these cases as compared to the ΛCDM model. The suppression at different length scales depends on the value of $w$ and $c_e$, and the effect of different $w$ is profoundly seen at all length scales. The influence of sound speed is significantly seen only at the intermediate length scales, and is negligible at scales very much larger and smaller than the Hubble scale.

1. Introduction
Recent supernova (SN) type 1a supernova observations have confirmed that the universe is expanding at an accelerated rate at the present epoch [1]. Understanding the nature of the component which drives this accelerated expansion is one of the major challenges of present day cosmology. One of the possible candidates could be a dynamical scalar field, with a negative pressure, known as dark energy (DE). Several models of scalar field dark energy like the quintessence, k-essence, Chaplygin gas, etc. have been studied extensively [2, 3, 4]. In general, these models can lead to degenerate evolution of the scale factor [5]. The scalar field models of dark energy are described by the two parameters equation of state (EOS) $w$ and speed of sound (ESS) $c_e$. Since, DE is a scalar field, it not only contributes in the background, but also through the perturbations, so we need to take into account perturbations to the DE [6, 7]. The ESS, which relates the density perturbations to pressure perturbation is useful in studying the effects of perturbations in DE. So considering the DE, perturbations can provide vital information in understanding these models, hence, distinguish and possibly ruling out some of the models. Our aim here is to study how perturbation in such models of DE will effect the CDM power spectrum. We have focused on evolving EOS and ESS, whose evolution is not fixed by a particular model of DE, and seen its effects on observable quantities such as CDM power spectrum.

We have considered a scalar field model of DE, whose action is given by:

$$S[\phi] = \int \sqrt{-g} \mathcal{L}(X, \phi) d^4 x,$$

where $\mathcal{L}(X, \phi)$ is the Lagrangian density of the scalar field. Also, we have assumed the universe to be spatially flat consisting of CDM and DE. The EOS parameter for the DE with above
The percentage suppression in matter power spectrum in DE models in comparison to ΛCDM model is plotted for a constant $w$ and for different values of $c_e^2$. It can be seen that suppression is more for DE models with $w = -0.8$. The power spectrums shown in this figure correspond to its value at the present epoch.

mentioned action is given by: $w = \frac{\mathcal{L}(X, \phi)}{2(\frac{\rho_d}{3a^2})X - \mathcal{L}(X, \phi)}$, the corresponding effective ESS of DE is $c_e^2 = \frac{(\frac{\partial L}{\partial X})}{(\frac{\partial L}{\partial X}) + 2X(\frac{\partial^2 L}{\partial X^2})}$. From these definitions of EOS and ESS, it is obvious that their evolution is governed by the Lagrangian density of the scalar field DE. However, it is also possible to obtain the form of Lagrangian density from a given evolution of $w$ parameter and $c_e^2$ (see [8]). Here, we have used this fact, and chosen a parameterized form of $w$ and $c_e^2$ to study the influence of perturbations in DE on CDM power spectrum. We have started by presenting the necessary equations to study the evolution of perturbations, and the perturbed Friedmann-Robertson-Walker (FRW) metric in the longitudinal gauge is given by:

$$ds^2 = (1 + 2\Phi) dt^2 - a^2(t) (1 - 2\Phi) \left[ dx^2 + dy^2 + dz^2 \right],$$

where $\Phi$ is the gravitational potential describing the scalar metric perturbations. In a system consisting of CDM and DE, using the linearized Einstein’s equations and the covariant conservation equations for DE, we can obtain the following set of equations for the evolution of perturbations:

$$\Phi'' = \frac{\Phi'}{a} \left( \frac{\ddot{a}}{aH^2} + 4 \right) - \frac{\Phi}{a^2} \left( \frac{2\ddot{a}}{aH^2} + 1 \right) + \frac{4\pi G \bar{\rho}_d}{a^2H^2} \left( c_e^2 \delta_d - (c_e^2 - c_a^2) \sigma_d \right),$$

$$\sigma_d' = \left( \frac{\ddot{a}}{aH^2} + 3 \frac{c_e^2 - c_a^2}{a} + (3w - 1) \right) \frac{\sigma_d}{a} - \frac{3c_e^2 \delta_d}{a} - \frac{3\Phi(1 + w)}{a}, \text{ and}$$

$$\delta_d' = \left( \frac{3 \delta_d}{a} \right) (w - c_e^2) + \frac{3\sigma_d}{a} (c_e^2 - c_a^2) + \frac{k^2 \Phi}{3a^2H^2} + 3(1 + w) \Phi'. \quad (4)$$

The quantities appearing in the above equations are as follows: (i) $\delta_d$ is the density contrast given by $\delta_d = \frac{\delta \rho_d}{\rho_d}$, (ii) $\sigma_d$ is a dimensionless variable defined via the equation: $\delta T_d^0 = (\rho_d/3Ha^2) \sigma_d$, and (iii) $c_a^2 \equiv \dot{\rho}_d/\ddot{\rho}_d$ is the adiabatic ESS of the DE. Here, dot and prime denote the derivatives.
Figure 2. Figure shows the suppression in matter power spectrum compared to ΛCDM model for the case when \( w \) is evolving, but \( c^2_e \) is constant. In the left panel, \( w \) is initially \(-1\) and asymptotically approaches \(-0.8\), whereas in the right panel, \( w \) is initially \(-0.8\) and asymptotically approaches \(-1\). It can also be verified that in both the panels \( w \) at the present epoch is \(-0.9\).

with respect to time and scale factor respectively. Now, solving the Eqs. (2), (3) and (4), we have obtained expressions for \( \Phi(a, k) \) and \( \delta_d(a, k) \). Then, the density contrast \( \delta_m \), describing the evolution of perturbations in the CDM can be evaluated from the temporal component of the linearized Einstein’s equation, and is given by:

\[
\delta_m(a, k) = -\left( \frac{2}{\Omega_m(a)} \right) \left( \Phi + a \Phi' + \left( \frac{k^2}{3H^2a^2} \right) \phi \right) - \left( \frac{\Omega_d(a)}{\Omega_m(a)} \right) \delta_d(a, k),
\]

where \( \Omega_m \) and \( \Omega_d \) are the dimensionless density parameters. It is evident from the Eq. (5) that the matter power spectrum is dependent on the effective ESS and EOS. The corresponding expression for matter perturbation in the ΛCDM is independent of \( c^2_e \) and \( \delta_d \), as there are no perturbations to the cosmological constant. To see how the perturbations in DE effect the CDM power spectrum, we have compared the growth of CDM perturbation in DE models with respect to ΛCDM model.

2. Effect of EOS and ESS parameters on CDM power spectrum

We have now investigated, how varying the EOS \( w \) and ESS \( c^2_e \) parameters of DE will influence CDM power spectrum. The most commonly studied scalar field DE models are with a constant value of \( w \) and \( c^2_e \). We have considered a class of models, in which \( c^2_e \) is constant, and allowed for evolving EOS. The EOS can have a parametric form, which depends on the scale factor, and can evolve from a initial value to asymptotic value, spanning the entire evolution of the universe. Then, there are classes of models, where \( c^2_e \) is epoch dependent and \( w \) is constant. Again, we can use a similar parameterizations for \( c^2_e \), which allow for all possible values of \( c^2_e \). Finally, we can have the most general case, where both \( w \) and \( c^2_e \) are epoch dependent (for details of parameterization used, see [8]). Next, we have quantitatively seen how the ESS and EOS influence the behaviour of power spectrum, by taking the difference in the growth of CDM perturbations in the scalar field models of DE with respect to the ΛCDM. The percentage
difference can be found by numerically evaluating the following quantity:

\[ \Delta \% \equiv \frac{P_{QCDM}(k) - P_{\Lambda CDM}(k)}{P_{\Lambda CDM}(k)} \times 100, \]

where \( P_{QCDM}(k) \) is the CDM power spectrum at the present epoch in DE models, and \( P_{\Lambda CDM}(k) \) is the corresponding power spectrum in ΛCDM model. Figure 1 shows \( \Delta \) for a constant value of \( c_e^2 \) and \( w \), it can be seen that at all length scales of perturbations percentage difference is negative. It can be inferred that the CDM power spectrum is suppressed in DE models in comparison to ΛCDM model. Figure 2 shows the percentage difference in models, where \( w \) is epoch dependent and \( c_e^2 \) is constant, again there is a suppression in CDM power spectrum at all length scales. Similarly, in the cases where \( c_e^2 \) is evolving or where both \( c_e^2 \) and \( w \) are evolving with time, the CDM power spectrum is found to be suppressed compared to ΛCDM model [8].

There are certain key points to be noted from the Figures, the suppression depends on the values of \( c_e^2 \) and \( w \). For instance, a bigger value of \( w = -0.8 \), gives more suppression than a smaller value \( w = -0.9 \), implying that the more EOS deviates from the value ΛCDM (\( w = -1 \)) the suppression in CDM power spectrum increases. Also, the effect of different values of \( c_e^2 \) (as seen in Figures 1 and 2) is profound only at the intermediate length scales of \( k \), whereas for scales much higher and smaller than the Hubble scale, the value of \( c_e^2 \) is irrelevant. The reason why we see the effect of \( c_e^2 \) at intermediate scales is that the perturbation in DE grows only at scales greater than the sound horizon (which, for the case of \( c_e^2 = 0.001 \), it is \( k = 0.07 \, h/Mpc \), for other cases of \( c_e^2 \) considered here, sound horizon will be greater than this value). For scales much smaller than Hubble scale, say \( (k \gg 0.001 \, h/Mpc) \), the effect of \( c_e^2 \) is inappreciable, as the perturbation in DE is negligibly smaller in comparison to that in dark matter. The DE can be approximated to be homogeneously distributed at these scales. On the other hand, at scales much larger than the Hubble radius \( (k \ll 0.001 \, h/Mpc) \), it is the adiabatic perturbations of DE, which contribute to the CDM power spectrum, hence, the effective ESS of DE does not influence the CDM perturbations. Also, it turns out that the variation of an evolving \( c_e^2 \) does not have any considerable effect on CDM power spectrum, whereas the different evolution of \( w \) will have much more significant influence [8]. Hence, exact information about the EOS, and ESS will help in better understanding the nature of DE.

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