Multiobjective Simultaneous Topology, Shape and Sizing Optimization of Trusses Using Evolutionary Optimizers

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Abstract. This paper presents design of two-dimensional (2D) trusses to achieve their simultaneously optimal topology, shape and size. The optimization problems are posed to search for structural topology, shape and sizing such that multiobjective functions consisting of mass and compliance are minimized while stresses and displacements are assigned as design constraints. The design approach is based on a ground structure approach meaning that a structure having all possible truss element connection is initiated. Design variables determine how to remove or maintain those elements and at the same time nodal positions are varied. Truss optimization problems are assigned whereas a number of multiobjective evolutionary algorithms (MOEAs) are implemented to solve the problems. Based on the hypervolume indicator, it is found that Hybridization of Real-Code Population-Based Incremental Learning and Differential Evolution (RPBIL-DE) is the best performer. This study gives the baseline results for implementing MOEAs for simultaneous topology, shape and sizing optimization of trusses.

1. Introduction

Presently, design optimization of engineering structures [1-4] is essential since lower structural weight as well as higher strength structure is expected. For truss design [5, 6], design engineers need to find the best design that is realizable and practical. Design of such structures is carried out in such a way that their topology, shape and sizes are optimized [7-13]. Design objectives usually include structural weight and cost. Nevertheless, it has been realized that the consequence of having a structure with low weight is reduction of structural strength. Apart from that, structural reliability is one important issue that should be taken into consideration. Therefore, there should be another design objective used to cope with this concern. Structural compliance can be used to somewhat measure structural strength and reliability. Thus, to have effective truss design, the optimization problem should have at least two objectives i.e. mass (or cost) and compliance.

Since there are two objective functions, the optimizer that can deal with such a problem efficiently is multiobjective evolutionary algorithms (MOEAs) [14]. Using such optimizers is advantageous since they are simple to understand, code, and use. The methods are derivative-free leading them to be capable of solving many types of optimization problems. Most importantly, they can explore the set of
optimal solutions for multiobjective optimization which is usually called a Pareto optimal set within one optimization run. Over the past few decades, there have been numerous MOEAs proposed to solve a wide variety of engineering and other practical optimization problems. Some recent MOEAs are Multi-Objective Evolutionary Algorithm Based on Decomposition (MOEA/D) [15], Multiobjective Harmony Search (MOHS) [16], Unrestricted Population Size Evolutionary Multiobjective Optimisation Algorithm (UPS-EMOA) [17], Hybridisation of Real-Code Population-Based Incremental Learning and Differential Evolution (RPBIL-DE) [18], Differential Evolution for Multiobjective Optimization (DEMO) [19,20] and Multiobjective Particle Swarm Optimization (MPSO) [21]. The use of those MOEAs for tackling a design problem of trusses with simultaneous topological, shape and sizing design variables has not been widely investigated. As a consequence, their comparative performance for this type of structural design should be studied and reported.

This paper, therefore, presents the comparative studies of using a number of MOEAs for solving simultaneous topology shape and sizing design of two dimension trusses. The structure used as test problems are 15-bar truss, 39-bar truss and 45-bar truss. The problems are minimization of structural mass and compliance subject to stress and displacement constraints. MOEAs employed to explore Pareto fronts are MOEA/D, MOHS, UPS-EMOA, RPBIL-DE, DEMO and MPSO. The results obtained from the various optimizers are compared and discussed.

### 2. Optimization Problem Formulation

#### 2.1 Test problems

In this paper, the two-objective design problem for trusses with design variables of simultaneous topology, shape and sizing can be shown as follows (1):

\[
\text{Minimize: } f_1(x), f_2(x) \\
\text{subject to } |\sigma_{max}| \leq \sigma_{al} \\
\quad \quad \quad u_{max} \leq u_{alt}
\]

where \(f_1(x)\) is structural mass of truss, \(f_2(x)\) is structural compliance, \(\sigma_{max}\) is the maximum stress on truss elements, \(\sigma_{al}\) is the allowable stress relying on material used for building a truss, \(u_{max}\) is maximum nodal displacement of the structure due to applied forces and \(u_{alt}\) is allowable nodal displacement. The compliance value is a measure of structural global stiffness which can be computed as a dot product of structural displacement and external force vectors.

Three truss optimization problems are selected, the first test problem is a simultaneous topology shape and sizing (TSS) optimization problem adapted from the 15-bar truss 2D structure problem from Ahrari, Atai and Deb in 2014 [8] and Rahami, Kaveh and Gholipour in 2008 [22]. Topological design variables are used to either remove or maintain those element while sizing variables are used to specify element sizes. Shape variables on the other hand will determine the nodal positions of some selected structural nodes. It should be noted that, during an optimization process, structural nodes of a ground structure in Figure 1 can be merged together.

The second test problem is the two-tier 39-bar truss problem, is TSS optimization by Ahrari, Atai and Deb in 2014 [8] and Deb and Gulati in 2001 [23] where the ground structure is shown in Figure 2. Topological, shape and sizing design variables are similar to the 15-bar truss case but this case has a symmetric ground structure which all possible ground members are illustrated.

45-bar truss is the third test problem shown by Ahrari, Atai and Deb in 2014 [8] and Deb and Gulati in 2001 of [23]. The 45-bar truss ground structure is depicted in Figure 3. It should be noted that all truss dimensions are given in inch unless otherwise specified. All nodes of the structure are connected with the all-to-all scheme or all pair-wise interconnection.
2.2 Design variables and boundary conditions

Table 1. Simulation data of two-dimensions for 15-bar truss problem.

| Design Variables | Shape(8) | x1=x6; x2=x7; y2; y3; y6; y7; y8 |
|------------------|----------|---------------------------------|
| Constraints      | Stress   | (σc)i ≤ 25 ksi; (σt)i ≤ 25 ksi, i=1,2,…,15 |
|                  | Displacement | None                             |
|                  | Shape Variables | 100 in ≤ x2, y2, y3 ≤ 140 in; 220 in ≤ x3 ≤ 260 in; 50 in ≤ y4 ≤ 90 in. |
|                  |            | ; 20 in ≤ y5 ≤ 60 in; -20 in ≤ y6, y7 ≤ 20 in. |
|                  | Size Variables | Ai, i=1,2,…,15 |
|                  | Search Range | A^i  is set of S, i=1,2,…,15 |
|                  |            | S = {0.111, 0.141, 0.174, 0.22, 0.27, 0.287, 0.347, 0.44, 0.539, 0.954, 1.081 |
|                  |            | 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.8, 3.131, 3.565, 3.813, 4.805 |
|                  |            | 5.952, 6.572, 7.192, 8.525, 9.3, 10.85, 13.33, 14.29, 17.17, 19.18 } (in^2) |
| Loading          | Case I    | Nodes | F_x (kips) | F_y (kips) |
|                  |           | 8     | -          | -10        |
| Mechanical       | Properties | Modulus of elasticity: E=10,000 ksi |
|                  |           | Density of the material is 0.1 lb/in^3 |

Table 1 shows the data required for optimization of the 15-bar truss. There are 23 design variables for all three variable types. Table 2 gives the details of design variables, structural loads and material properties for optimization of the two-tier 39-bar truss where the ground structure is shown in Figure 2. In the table x_i and y_i mean nodal positions of node i in x and y directions respectively. It should be noted that the structure is treated to be symmetric horizontally. There are 28 design variables for all three variable types. For the third case, details of design variables, structural loads and properties of material are presented in Table 3. There are 45 design variables for all two variable types. In this case considered topological and sizing design variables.

Figure 1. 15-bar truss ground structure.

Figure 2. 39-bar truss ground structure.

Figure 3. 45-bar truss ground structure.
Table 2. Simulation data of two-dimensions for 39-bar truss problem.

| Design Variables | Shape(7) | Topology, Size(21) | Constraints | Stress | Displacement |
|------------------|----------|-------------------|-------------|--------|--------------|
|                   | $\Delta x_1; \Delta x_2; \Delta x_3; \ldots \Delta x_{21} ; \Delta y_1; \Delta y_2; \Delta y_3; \ldots \Delta y_{19}$ | $A_i, i=1,2,\ldots,21$ (Geometry is symmetric in a-a axis) | $\sigma_c(i) \leq 25$ ksi; $\sigma_t(i) \leq 25$ ksi, $i=1,2,\ldots,39$ | $u_j \leq 2$ in., $j=1,2,\ldots,3(n_{node})$ |
| Search Range      | Shape Variables | Horizontal coordinates may many very within ± 120 in. of their initial value. | Vertical coordinates may many very within ± 120 in. of their initial value. |
| Size Variables    | $A_i$ is set of $[0.05, 2.25]$ (in$^2$), $i=1,2,\ldots,39$ |

| Loading | Case I | Nodes | $F_x$ (kips) | $F_y$ (kips) |
|---------|--------|-------|-------------|-------------|
|         |        | 2     | -           | -20         |
|         |        | 3     | -           | -20         |
|         |        | 4     | -           | -20         |

Mechanical Properties

- Modulus of elasticity: $E=10,000$ ksi
- Density of the material is 0.1 lb/in.$^3$

Table 3. Simulation data of two-dimensions for 45-bar truss problem.

| Design Variables | Topology, Size(15) | Constraints |
|------------------|-------------------|-------------|
|                   | $A_i, i=1,2,\ldots,45$ |
|                   | $\sigma_c(i) \leq 25$ ksi; $\sigma_t(i) \leq 25$ ksi, $i=1,2,\ldots,45$ |
|                   | $u_j \leq 2$ in., $j=1,2,\ldots,3(n_{node})$ |
| Search Range      | Size Variables | $A_i$ is set of $[0.09,1]$ (in.), $i=1,2,\ldots,45$ |
| Loading | Case I | Nodes | $F_x$ (kips) | $F_y$ (kips) |
|---------|--------|-------|-------------|-------------|
|         |        | 7     | -           | -10         |
|         |        | 8     | -           | -10         |
|         |        | 9     | -           | -10         |

Mechanical Properties

- Modulus of elasticity: $E=10,000$ ksi
- Density of the material is 0.1 lb/in.$^3$

2.3 Optimization Parameter Settings

The optimization parameter settings of MOEAs used to solve the test problems are assigned as:

- MOEA/D using number of neighboring weight vectors are 6 which is real codes, mutation probabilities and crossover being 0.1 and 1.0 respectively [15].
- UPS-EMOA using crossover probability, scaling factor for Differential Evolution operator, probability of choosing elements from an offspring in crossover, minimum population size and number of solution to be used as parents are set as 0.7, 0.8, 0.5, 10, and 25 respectively [16].
- MOHS using harmony memory considering rate being 0.5, min pitch adjustment rate being 0.2, max pitch adjustment rate being 2, min bandwidth rate being 0.45 and max bandwidth rate being 0.9 [17].
- RPBIL-DE using $N_p=40$ which real codes where each probability tray produces three design solutions. Crossover probability is set as 0.7, scaling factor is set as 0.8 and probability is set as 0.5 which is choosing elements from an offspring in crossover for Differential Evolution operators [18].
- DEMO using crossover probability, scaling factor for Differential Evolution (DE) operator, probability of choosing elements from an offspring in crossover are set as 0.7, 0.8 and 0.5 respectively [19,20].
- MPSO using the starting inertia weight is set as 0.5, ending inertia weight is set as 0.01, cognitive learning factor is set as 0.5 and social learning factor being 0.5. The best solution is randomly selected from Pareto archive [21].

Each optimizer is employed to solve each optimization problem 30 runs. The population size is set to be 100 while the total number of iterations is set to be 250 for all optimizers.

3. Results and discussions.

For multiobjective evolutionary algorithms (MOEAs), the non-dominated archive size is set as 100 which are equal to the population size. The hypervolume indicator will be used to measure the performance of MOEAs. The reference points used for computing the hypervolume for all test problems are given in Table 4.

Table 4. Reference points (Wmax) for hypervolume calculation.

|                  | 15-bar truss | 39-bar truss | 45-bar truss |
|------------------|--------------|--------------|--------------|
| \( f_1(x) \) (lb) | 3899.100     | 1960.900     | 906.254      |
| \( f_2(x) \) (kip.inch) | 51.256 | 76.909 | 25.993 |

The results of optimizing the 15-bar truss obtained from the various optimizers are compared in Table 5. According to the mean of the hypervolume from 30 runs, for the 15-bar truss, the best method is RPBIL-DE while the second best method is UPS-EMOA. The worst is MOEA/D. The best runs of all optimizers are found and the corresponding Pareto fronts are plotted as shown in Figure 4. From Table 5 the front that gives the highest hypervolume is from RPBIL-DE. This implies that RPBIL-DE gives a Pareto front with better front extension and front advancement more than other optimizers. Figure 5 shows some selected optimum solutions of the 15-bar truss problem obtained from the best Pareto front of RPBIL-DE. It can be seen that various structures can be obtained from running the optimizer once.

The same conclusion can be said in cases of the 39-bar truss problem as given in Table 6. The best runs of all optimizers are found and the corresponding Pareto fronts are plotted as shown in Figure 6. RPBIL-DE is the best optimizer based on the hypervolume value. For the 39-bar truss, the best method is RPBIL-DE while the second best is UPS-EMOA. The worst is MPSO. Figure 7 shows some selected optimum solutions of the 39-bar truss problem obtained from the best run of RPBIL-DE. Likewise, in case of 45-bar truss problem, similar conclusion can be said as given in Table 7. The best method still is RPBIL-DE while the second best still is UPS-EMOA and the worst is MPSO. The best runs of all optimizers are found and the corresponding Pareto fronts are plotted as shown in Figure 8. Figure 9 shows some selected optimum solutions of the 45-bar truss problem obtained from the best run of RPBIL-DE.

Table 5. Hypervolume of 15-bar truss.

| Optimizers | Mean        | (±)Std      | Minimum    | Maximum     |
|------------|-------------|-------------|------------|-------------|
| MOEA/D     | 128317.305  | 4201.075    | 119059.589 | 137192.826  |
| MOHS       | 160588.718  | 2379.642    | 156592.515 | 165291.290  |
| UPS-EMOA   | 170529.799  | 4514.161    | 159594.869 | 178442.317  |
| RPBIL-DE   | 178742.758  | 1166.760    | 176111.685 | 180326.015  |
| DEMO       | 168372.467  | 3828.054    | 157347.706 | 180176.778  |
| MPSO       | 130163.338  | 5136.202    | 119310.029 | 140318.421  |
Figure 4. 15-bar truss Pareto fronts from MOEAs.

Figure 5. Some design solutions of the 15-bar truss problem from RPBIL-DE.

Table 6. Hypervolume of 39-bar truss.

| Optimizers    | Mean (±)Std | Minimum   | Maximum   |
|---------------|-------------|-----------|-----------|
| MOEA/D        | 76300.680   | 1703.256  | 72775.722 | 80166.593 |
| MOHS          | 81153.007   | 941.083   | 79469.077 | 82961.035 |
| UPS-EMOA      | 95826.720   | 758.176   | 93729.321 | 97111.048 |
| RPBIL-DE      | 97079.814   | 303.243   | 96265.062 | 97593.684 |
| DEMO          | 93629.169   | 1987.434  | 88554.128 | 99218.717 |
| MPSO          | 75345.682   | 1809.990  | 71945.227 | 78828.173 |
Figure 6. 39-bar truss Pareto fronts from MOEAs.

Figure 7. Some design solutions of the 39-bar truss problem from RPBIL-DE.
Table 7. Hypervolume of 45-bar truss.

| Optimizers     | Mean   | (±)Std | Minimum   | Maximum   |
|---------------|--------|--------|-----------|-----------|
| MOEA/D        | 12243.775 | 241.263 | 11694.991 | 12701.859 |
| MOHS          | 11622.509 | 146.357 | 11268.308 | 11883.705 |
| UPS-EMOA      | 14322.093 | 236.615 | 13492.802 | 14626.433 |
| RPBIL-DE      | 14461.992 | 58.705  | 14271.463 | 14563.805 |
| DEMO          | 13531.746 | 560.496 | 12069.836 | 14973.128 |
| MPSO          | 12170.609 | 391.988 | 11552.289 | 13045.755 |

Figure 8. 45-bar truss Pareto fronts from MOEAs.

Figure 9. Some design solutions of the 45-bar truss problem from RPBIL-DE.

According to the mean value and standard deviation (Std) of the hypervolumes from 30 runs for all design problems, the best method is RPBIL-DE as shown in Table 8. The best parameter settings for this study are crossover probability as 0.7, a scaling factor as 0.8 and CR as 0.5, which are the same as those used in [18].
Table 8. The best MOEA based on the hypervolume indicator.

| Trusses | Optimizers | Mean     | (±)Std | Minimum    | Maximum    |
|---------|------------|----------|--------|------------|------------|
| 15-bar  | RPBIL-DE   | 178742.758 | 1166.760 | 176111.685 | 180326.015 |
| 39-bar  | RPBIL-DE   | 97079.814  | 303.243 | 96265.062  | 97593.684  |
| 45-bar  | RPBIL-DE   | 14461.992  | 58.705  | 14271.463  | 14563.805  |

4. Conclusions

A new design problem for truss TSS optimization is proposed. The optimization problem can optimize both structural mass (or cost) and reliability (compliance). The results in this study are set as the initial performance baseline for future development of multiobjective optimizers to be used with these design problems. Based on the hypervolume indicator and the best Pareto front obtained, it can be said that RPBIL-DE is the best performer in terms of Pareto front advancement and extension. Using the hypervolume alone, however, may not be sufficient for measuring the performance of MOEAs, as a result, other indicators should be implemented.

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References

[1] Bureerat S 2013 Optimization for Mechanical Engineering (Khon Kaen: Khon Kaen University Press)
[2] Srisompong S and Bureerat S 2008 Geometrical design of plate-fin heat sinks using hybridization of MOEA and RSM IEEE Transaction on Components and Packaging Technologies 31 351-9
[3] Kunakote T, Sombudha S and Bureerat S 2009 Symp. Conf. on ME-NETT23 (Chiang Mai) vol 23 (Khon Kaen: ME-NETT) p 373
[4] Zitzler E and Thiele L 1999 Multiobjective Evolutionary Algorithms : A Comparative Case Study and the Strength Pareto Approach IEEE Tran.on Evolutionary Computation 3 257-71
[5] Bureerat S 2013 Truss Finite Elements Analysis and Design (Khon Kaen: Khon Kaen University Press)
[6] Chadchotsak P 2005 Analysis of Roof Truss using Timoshenko Beam Model KKU Engineering Journal 32 829-40
[7] Noilublao C and Bureerat S 2009 Simultaneous Topology Shape and Sizing Optimisation of Skeletal Structures Using Multiobjective Evolutionary Algorithms Evolutionary Computation ISBN 978-953-307-008-7 pp 487–580
[8] Ahrari A, Atai A A and Deb K 2014 Simultaneous Topology Shape and Size Optimization of Truss Structures by Fully Stressed Design Based on Evolution Strategy Engineering Optimization 1-22
[9] Zhou M, Pagaldipiti N, Thomas H L and Shyy Y K 2004 An integrated approach to topology, sizing, and shape optimization 26 308-17
[10] Noilublao N and Bureerat S 2011 Simultaneous topology, shape and sizing optimisation of a three-dimensional slender truss tower using multiobjective evolutionary algorithms Computers and Structures 89 2531-8
[11] Greiner D and Hajela P 2012 Truss topology optimization for mass and reliability considerations—co-evolutionary multiobjective formulations Struct Multidisc Optim 45 589-613
[12] Pholdee N and Bureerat S 2012 Performance enhancement of multiobjective evolutionary optimisers for truss design using an approximate gradient Computers and Structures 106
115-124

[13] Noiluplao N and Bureerat S 2013 Simultaneous Topology, Shape and Sizing Optimisation of Plane Trusses with Adaptive Ground Finite Element Using MOEAs Hindawi Publishing Corporation Mathematical Problems in Engineering 1-9

[14] Zhou A, Qu B Y, Li H Zhao S Z, Suganthan P N and Zhang Q 2011 Multiobjective evolutionary algorithms: A survey of the state of the art Swarm and Evolutionary Computation 1 32-49

[15] Ma X, Liu F, Li Y, Li L, Jiao L, Liu M and Wu J 2014 MOEA/D with Baldwinian learning inspired by the regularity property of continuous multiobjective problem Neurocomputing 145 336-52

[16] Mohamed F, Santawy E and Ahmed A N 2012 A Multi-Objective Chaotic Harmony Search Technique for Structural Optimization Computing Science 1 9-12

[17] Aittokoski T and Miettinen K 2008 Int. Conf. on Engineering Optimization (Rio de Janeiro) (Rio de Janeiro: EngOpt )

[18] Pholdee N and Bureerat S 2013 Hybridisation of real-code population-based incremental learning and differential evolution for multiobjective design of trusses Information Sciences 223 136-52

[19] Das S and Suganthan P N 2011 Differential evolution :a survey of the state-of-the-art IEEE Transactions on Evolutionary Computation 15 4-31

[20] Mlakar M, Petelin D, Tušar T and Filipi B 2015 GP-DEMO :Differential Evolution for Multiobjective Optimization based on Gaussian Processmodels Operational Research 243 347-61

[21] Sierra M R and Coello C A C 2006 Multi-Objective Particle Swarm Optimizers: A Survey of the State-of-the-Art Computational Intelligence Research 2 287-308

[22] Rahami H, Kaveh A and Gholipour Y 2008 Sizing, geometry and topology optimization of trusses via force method and genetic algorithm Engineering Structures 30 2360–9

[23] Deb K and Gulati S 2001 Design of truss-structures for minimum weight using genetic Algorithms Finite Elements in Analysis and Design 37 447-65