Similarity measure on bipolar intuitionistic fuzzy soft set

S Anita Shanthi $^a$, Prathipa Jayapalan $^b$ and L Darwin Christdhas Henry$^c$

$^a,b$ Department of Mathematics
$^c$ Department of Plant Pathology, Faculty of Agriculture
Annamalai University
Annamalainagar-608002
Tamilnadu, India.

E-mail: shanthi.anita@yahoo.com
prathipajayapalan@gmail.com
lewin_darwin@yahoo.com

Abstract. This paper deals with, similarity measure on $BIF SS$. Decision making method on similarity measure of $BIF SS$ is constructed. Finally, an example is provided to show the applicability of this method.

1. Introduction
The theory of fuzzy set was introduced by Zadeh [1]. Atanassov [2] generalized the above concept to intuitionistic fuzzy set (IFS). Zhang [3] proposed the concept of bipolar fuzzy set. Bipolar fuzzy soft set was introduced by Saleem et al. [4].

The similarity measure on IFS are applied in pattern recognition, approximate reasoning and decision making. Deli and Cagman [5] proposed similarity measures on IFSS and defined some types of distances between two IFSS and developed a decision making method. Anita and Vadivel [6] defined similarity measure on IVIFSSRT.

This paper deals with notion of similarity measure on bipolar intuitionistic fuzzy soft sets ($BIF SS$) and a theoretical property of this measure is established. A decision making problem on similarity measure of bipolar IFSS is also developed.

2. Similarity measure on $BIF SS$
Definition of $BIF SS$ is given in [7].

In this section, similarity measure of $BIF SS$ is defined and it is established be a metric.

Definition: 2.1 Let $U = \{y_1, y_2, \ldots, y_m\}$ be an universal set, $E = \{\xi_1, \xi_2, \ldots, \xi_t\}$ be a set of parameters, $(BF, \xi)$ and $(BG, \xi)$ be two $BIF SS$ on $U$. Then the Hamming distance between $(BF, \xi)$ and $(BG, \xi)$ is defined as

\[
BH^d((BF, \xi), (BG, \xi)) = \frac{1}{4m} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{t} \left( |\mu^B_{\xi}(y_j) - \mu^B_{\xi}(y_j)| + |\mu^B_{\xi}(y_j) - \mu^B_{\xi}(y_j)| + |\nu^B_{\xi}(y_j) - \nu^B_{\xi}(y_j)| + |\nu^B_{\xi}(y_j) - \nu^B_{\xi}(y_j)| \right) \right\}
\]
The distance function $BH^d$ from $BIFSS(U)$, the set of all $BIFSS$ over $U$ to the set of non negative real numbers is a metric.

**Proof:**
Let $(BF, \xi)$, $(BG, \xi)$ and $(BH, \xi)$ be three $BIFSS$ over $U$.
(i) $BH^d((BF, \xi), (BG, \xi)) > 0$ follows from Definition 2.
(ii) $BH^d((BF, \xi), (BG, \xi)) = 0$

$$\Leftrightarrow |\mu_{BF(\xi)}(y_j) - \mu_{BG(\xi)}(y_j)| + |\nu_{BF(\xi)}(y_j) - \nu_{BG(\xi)}(y_j)|$$

$$= |\mu_{BF(\xi)}(y_j) - \mu_{BG(\xi)}(y_j)| + |\mu_{BF(\xi)}(y_j) - \mu_{BG(\xi)}(y_j)|$$

$$+ |\nu_{BF(\xi)}(y_j) - \nu_{BG(\xi)}(y_j)| + |\nu_{BF(\xi)}(y_j) - \nu_{BG(\xi)}(y_j)|$$

$\Rightarrow (BF, \xi) = (BG, \xi)$.

(iii) Clearly, $BH^d((BF, \xi), (BG, \xi)) = BH^d((BG, \xi), (BF, \xi))$.

(iv) For all $i \in \{1, 2, ..., m\}$, $j \in \{1, 2, ..., t\}$

$$|\mu_{BF(\xi)}(y_j) - \mu_{BG(\xi)}(y_j)| + |\mu_{BF(\xi)}(y_j) - \mu_{BG(\xi)}(y_j)|$$

$$\leq BH^d((BF, \xi), (BG, \xi)) \leq BH^d((BH, \xi), (BG, \xi)) + BH^d((BH, \xi), (BG, \xi)).$$

Hence, $BH^d$ satisfies the triangle inequality.

Therefore, $BH^d$ is a metric.

**Definition: 2.3** Let $(BF, \xi)$ and $(BG, \xi)$ be two $BIFSS$ over $U$. Then, by using the Hamming distance, a similarity measure between $(BF, \xi)$ and $(BG, \xi)$ denoted by $BS^m((BF, \xi), (BG, \xi))$ is defined as

$$BS^m((BF, \xi), (BG, \xi)) = \frac{1}{1 + BH^d((BF, \xi), (BG, \xi))}.$$
Step 1. Construct a \( BIFSS(BF, \xi) \) over \( U \) based on experts evaluation.

Step 2. Construct a \( BIFSS(BG, \xi) \) over \( U \) based on available data.

Step 3. Compute the Hamming distance between \( (BF, \xi) \) and \( (BG, \xi) \).

Step 4. Determine the similarity measure between \( (BF, \xi) \) and \( (BG, \xi) \).

Step 5. Conclude using the value of similarity measure.

Example: 3.3 A rural area is affected by cyclone. A team of three members \( U = \{S_1, S_2, S_3\} \) from governmental organization inspected the cyclone affected area and decided to provide them the recommended relief measures, that are described by the parameter set \( \xi = \{\xi_1, \xi_2, \xi_3, \xi_4\} \) where \( \xi_1 = \) supply of water and food, \( \xi_2 = \) provision of alternate livelihood, \( \xi_3 = \) reconstruction of roads and rails and \( \xi_4 = \) reconstruction of houses. Based on these recommendations the governmental organization has to provide funds for the rehabilitation work for the needy people who suffer due to this natural calamity.

Step 1. A \( BIFSS(BF, \xi) \) over \( U \) based on the previous records of relief measures on similar situations is given in Table 1 as follows:

| \( U \) | \( S_1 \) | \( S_2 \) | \( S_3 \) |
| --- | --- | --- | --- |
| \( \xi_1 \) | \((-0.45, 0.75), (-0.5, 0.25))\) | \((-0.28, 0.22), (-0.01, 0.2))\) | \((-0.12, 0.62), (-0.54, 0.03))\) |
| \( \xi_2 \) | \((-0.2, 0.9), (-0.7, 0.1))\) | \((-0.43, 0.8), (-0.03, 0.12))\) | \((-0.4, 0.76), (-0.6, 0.3))\) |
| \( \xi_3 \) | \((-0.5, 0.87), (-0.1, 0.07))\) | \((-0.75, 0.5), (-0.02, 0.38))\) | \((-0.35, 0.9), (-0.62, 0.08))\) |
| \( \xi_4 \) | \((-0.2, 0.83), (-0.8, 0.06))\) | \((-0.15, 0.62), (-0.41, 0.04))\) | \((-0.5, 0.8), (-0.3, 0.16))\) |

Step 2. \( BIFSS(BG, \xi) \) over \( U \) based on the reports of the team which visited area-1 is given in Table 2 as follows:

| \( U \) | \( S_1 \) | \( S_2 \) | \( S_3 \) |
| --- | --- | --- | --- |
| \( \xi_1 \) | \((-0.09, 0.11), (-0.02, 0.08))\) | \((-0.2, 0.7), (-0.5, 0.18))\) | \((-0.14, 0.3), (-0.25, 0.1))\) |
| \( \xi_2 \) | \((-0.18, 0.26), (-0.03, 0.11))\) | \((-0.05, 0.85), (-0.75, 0.02))\) | \((-0.11, 0.4), (-0.04, 0.01))\) |
| \( \xi_3 \) | \((-0.35, 0.8), (-0.5, 0.21))\) | \((-0.27, 0.63), (-0.42, 0.14))\) | \((-0.15, 0.38), (-0.1, 0.09))\) |
| \( \xi_4 \) | \((-0.16, 0.3), (-0.02, 0.12))\) | \((-0.22, 0.14), (-0.06, 0.01))\) | \((-0.12, 0.2), (-0.1, 0.07))\) |

Step 3. Hamming distance between \( (BF, \xi) \) and \( (BG, \xi) \) calculated using Definition 2.1 is \( BH_d((BF, \xi), (BG, \xi)) = 1.14 \).

Step 4. Similarity measure between \( (BF, \xi) \) and \( (BG, \xi) \) calculated using Definition 2.3 is \( BS^m((BF, \xi), (BG, \xi)) = \frac{1}{1+BH_d((BF, \xi), (BG, \xi))} = 0.46 \).

Step 5. Similarity measure of \( (BF, \xi) \) and \( (BG, \xi) \) is < \( \frac{1}{2} \). Since the two \( BIFSS \) are not significantly similar, it is concluded that the area is not affected by cyclone.

Based on the inspection of the governmental organization team, the data collected for the rural area-2 is given in Table 3 below.
Table 3. $BIFSS (BH, \xi)$ gives the data collected by the team for rural area-2

| $u$ | $S_1$ | $S_2$ | $S_3$ |
|-----|-------|-------|-------|
| $\xi_1$ | ($(-0.16, 0.3), (-0.2, 0.35)$) | ($(-0.19, 0.7), (-0.4, 0.14)$) | ($(-0.27, 0.5), (-0.32, 0.18)$) |
| $\xi_2$ | ($(-0.18, 0.46), (-0.15, 0.12)$) | ($(-0.35, 0.6), (-0.3, 0.15)$) | ($(-0.22, 0.38), (-0.16, 0.13)$) |
| $\xi_3$ | ($(-0.3, 0.64), (-0.27, 0.12)$) | ($(-0.14, 0.56), (-0.32, 0.16)$) | ($(-0.25, 0.3), (-0.13, 0.2)$) |
| $\xi_4$ | ($(-0.46, 0.72), (-0.15, 0.14)$) | ($(-0.17, 0.58), (-0.18, 0.13)$) | ($(-0.47, 0.6), (-0.14, 0.23)$) |

Hamming distance between $(BF, \xi)$ and $(BH, \xi)$, $BH^d((BF, \xi), (BG, \xi)) = 0.84$.

Similarity measure between $(BF, \xi)$ and $(BH, \xi)$

$$BS^m((BF, \xi), (BH, \xi)) = \frac{1}{1 + BH^d((BF, \xi), (BH, \xi))} = 0.55.$$ 

Similarity measure of $(BF, \xi)$ and $(BH, \xi)$ is $> \frac{1}{2}$. Since the two $BIFSS$ are significantly similar, it is concluded that the area is affected by cyclone.

4. Conclusion

In this paper, Similarity measure on bipolar intuitionistic fuzzy soft sets is proposed. Moreover, a decision making method on similarity measure has been developed.

References

[1] Zadeh L A (1965) Fuzzy sets, *Information and control*, 8, pp: 338-353.
[2] Atanassov K (1986) Intuitionistic fuzzy sets, *Fuzzy sets and systems*, 20(1), pp: 87-96.
[3] Zhang W R (1998) Bipolar fuzzy sets, *Proceeding of FUZZ-IEEE*, pp: 835-840.
[4] Saleem Abdullah, Muhammad Aslam and Kifayat Ullah (2014) Bipolar fuzzy soft sets and its applications in decision making problem, *Journal of intelligent and fuzzy systems*, 59(10), pp: 443-457.
[5] Deli I and Cagman N (2013) Similarity measures of intuitionistic fuzzy soft sets and their decision making, *arXiv:1304.0456*, 2, pp: 1-15.
[6] Anita Shanthi S and Vadivel Naichu J (2015) A decision making method based on similarity measure of interval valued intuitionistic fuzzy soft set of root type, *The Journal of fuzzy mathematics*, 23(2), pp: 443-457.
[7] Anita Shanthi S and Prathipa Jayapalan (2019) A VIKOR method based on bipolar intuitionistic fuzzy soft set, *Advances in mathematics*, 9(4), pp: 1511-1519.