Research on groundwater system identification in use of hydrological signals processing technique

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Abstract. Filtering techniques can be applied in processing hydrological signal identifying groundwater system. A series of groundwater table observation sequences in a coalmine was dealt with by use of recursive (IIR, Infinite Impulse Response) and non-recursive (FIR, Finite Impulse Response) filtering. The responding characteristics and dynamic patterns on the groundwater system were recognized by the correlation analysis. An identification model based on the Hammerstein-Wiener was applied to fit the data of measurement. The study shows that: (1) Three types of aquifer are divided according to their corresponding characteristics to the precipitation. Comparing the IIR and FIR filtering, the features of different scale filtering are summerized. (2) In nonlinear Hammerstein-Wiener model, the best fitting between simulation and measurement is up to 94.9%. The application of signals processing technique is a efficient approach to identify the groundwater system’s characteristics and patterns.

1. Introduction

Signal processing techniques, include sampling, filtering, analogue simulation, and system identification, are widely used in various branches of geoscience, e.g., remote sensing image processing [1], geophysical prospecting [2], hydrological information analysis. Zheng et al. [3] applied digital filtering technique to explore the explicit relationship between groundwater and earthquake preparation. Luo [4] used this technique to analysed the pattern of recharge and discharge in a confined groundwater system. Gao et al. [5] realized process monitoring and water resources system optimization in a water supply project.

Hydrological signals are processed by filtering technique to identify a groundwater system. This is a hotspot in hydrological research [6-8]. Wu et al. [9] analysed the linear time invariable (LTI) characteristics of underground confined aquifer system. Zhu et al. [10] further had proven that the confined aquifer system satisfied the conditions of incremental linear system (ILS), which provided a theoretical basis for the effective application of filtering technique on groundwater system analysis.

The IIR (Infinite Impulse Response) and FIR (Finite Impulse Response) filtering technique are applied to process the observation data of groundwater table and analyse groundwater system in the Zhaogezhuang coalmine, China. Several typical observation wells are selected to identify and simulate the groundwater system model. Through Hammerstein-Wiener model, the parameter information and simulink model which accord with the structural characteristics of the groundwater system model in the study site can be obtained. The results can provide basis for the establishment of the groundwater prediction model. It is valuable for prevention of the coalmine from groundwater inrush.
2. Theoretical basis

2.1. Recursive and non-recursive filtering

Recursive filtering has infinite impulse response (IIR) to the early signals. Non-recursive filtering does not have feedback in which the output signals have finite impulse response (FIR) to the input signals. For an n-order recursive filtering and symmetric non-recursive filtering, the equations are as follows:

\[
\text{IIR filtering: } y(i) = \sum_{n=0}^{N-1} a(n)x(i - n) + \sum_{n=1}^{N-1} b(n)y(i - n)
\]

\[
\text{FIR filtering: } y(i) = \sum_{n=-q}^{s} a(n)x(i - n)
\]

where \( y(i) \) is the filtering output at time \( i \); \( x(i-n) \) is the input at time \( i-n \); \( y(i-n) \) is the output at time \( i-n \); \( a(n) \) and \( b(n) \) are the filtering coefficient; \( s \) and \( q \) are the time domain parameters or sampling window (symmetric: \( s = q \)).

In equation (1), the input of IIR filtering includes not only the input signals but also the feedback of early signals. When as low-pass filtering, IIR filtering is an approximate smoothing method with a determined coefficient. In equation (2), the input signals of symmetric FIR filtering have effects on the output signals only in the window \((2q+1)\). When as low-pass filtering, FIR filtering is a moving average model with convolution form of weighting coefficient \( a[n] \).

2.2. Identification technique and theoretical model

Identification technique has a long development history. Some researches can be found from decades of literature [6-8]. Hammerstein-Wiener Model, a typical nonlinear model, consists of static nonlinear input, dynamic linear and static nonlinear output in series [11,12]. The fundamental structure is shown in Figure 1 and the equations are as follows:

\[
\begin{align*}
  w(t) &= N(u(t)) = \sum_{i=1}^{m} a_i g_i(u(t)) \\
  x(t) &= \begin{bmatrix} B(q) \\ F(q) \end{bmatrix} w(t - n_k) + e(t) \\
  y(t) &= N(x(t)) = \sum_{j=1}^{n} \beta_j f_j(x(t))
\end{align*}
\]

where \( y(t) \) is the output at time \( t \) (e.g., groundwater table); \( u(t) \) is the control input at time \( t \) (e.g., precipitation, pump drainage); \( w(t) \) and \( x(t) \) are intermediate immeasurable variables; \( e(t) \) is zero mean white noise interference; \( a_i \) and \( \beta_j \) are coefficients of base functions \( g_i(\cdot) \) and \( f_j(\cdot) \) in the nonlinear input and output, respectively; \( B(q) \) and \( F(q) \) are the polynomials of unit lag operator \( q \); \( q \) is order or unit lag operator, and \( q^t u(t) = u(t-t) \); \( n_k \) is the lag of controlling the influential factor; \( m \) and \( n \) are respectively the order of nonlinear input and the one of output.

Hammerstein-Wiener (H-W) model processes input and output signals under the nonlinear methods. It is taking the intermediate variable as the input and output of dynamic linear. The dynamic linear can be any one of known linear valuation models, e.g., auto regressive moving average controlled by X-factor (ARMAX) model and state space model.

3. Overview of the study site
3.1. Geological setting
The study site is a Zhaozezhuang coalmine of Kailuan (i.e., a Group), Tangshan City, Hebei province (figure 2). The Coalmine, about 15.96 km², is located in coal accumulating area of Kaiping syncline (i.e., extending to the NE-SW). There are Sinian (Z), Cambrian (€), Ordovician (O), Carboniferous (C), Permin (P) and other strata distributed in the area. The main coal accumulating strata include Carboniferous Zhaozezhuang (C₂2) and Permin Damiaozhuang (P₁₁). Total thickness is about 150 m, containing 15-20 layers of coal, 7th and 9th coal bed are available [13].

![Figure 2. Study site and geological setting.](image)

According to literature, Kaiping Syncline is a deflection Syncline by multi-phase tectonics, the total axial direction is NE30° ~60° , while in Guye district located in Zhaozezhuang coalmine, the axial direction is strong tilting to EW, the axial plane is tilting to NW and the two wings are asymmetric [14]. It is forming a complex structural pattern that the dip angle of the northwest wing is steep and even reversed, the strata of the southeast wing are flat (i.e. dip angle: 10° ~25° ) and destroyed by a series of EW-distributed compressor-shear faults and folds. Under the influence of mining operation, the fault has potentially to connect the Ordovician limestone aquifer and coal accumulating area. Figure 3 shows the I-I' profile map of the groundwater system in the study site.

![Figure 3. I-I' profile map of the groundwater system in the study site.](image)

3.2. Hydrogeological conditions of mineral deposit
The main aquifer is Ordovician limestone fractured confined aquifer with the thickness of 700 m exposed in the northern part of the mining area. The study site received precipitation infiltration recharge in a good conditions and in a large area. The aquifer has the stronger water-yield property with single-well water inflow is 6.0 m³/min. Thirteen long-time groundwater table observation wells are set up in the study site for different layers in limestone aquifers.

The coalmine occurred several times of water inrush disasters [15]. The lastest two water inrush
incidents occurred in 13th level in 2005 [16] and in 2015, respectively. The exploitation of coal bed is still threatened by water inrush. Previous research was carried out a simulation study for the water inrush incident in 2005 [17], which basically solved the groundwater process mechanism problem under the small engineering scale. But the dynamic characteristics of groundwater system in the whole study site under the large scale have unsolved.

4. Dynamic filtering analysis of groundwater

4.1. FIR filtering analysis during precipitation

The annual statistical data of No. 1, No. 5, No. 8 and No. 9 wells are processed by FIR filtering (q=2.5, 5-year window period) and compared with the annual precipitation in Tangshan section (figure 4) There are three types of response characteristics of the groundwater table in precipitation Conditions.

- **Type of delay with a wide amplitude.** The well No. 1 is the typical observation well of this type. Filtering curve of 5-year is fluctuating between -275.0 m ~ -175.0 m with an amplitude of 100.0 ± 10.0 m. There is a lag period of 8-9 months between the peak of the water table and the precipitation.

- **Type of gradual delay with a narrow amplitude.** The well No. 5 to monitoring northern recharge located in the periphery of mining area is representative. Fluctuation range of the filtering curve is -15.0 m ~ 35.0 m, with an amplitude of 20.0 ± 2.0 m. The fluctuation is weaker than the former one and the response lag time with trend of a gradual change. According to the relevant literature [18-20], the stratum of study site is Cambrian stratum, the main rocks are mudstone and argillaceous limestone.

- **Type of transition with a wide amplitude.** This type is distributed in the intermediate transition zone of above two types—the fault zone. The representative wells are No. 8 and No. 9 with a large fluctuation and a rapid flow of fractured water. According to 5-year filtering curve, the water table in the well No. 8 shows in a decreasing trend and a lagging peak value (i.e. 7 months for lagging). The fluctuation amplitude of No. 9 filtering curve is -190.0 m ~ -75.0 m, the maximum is 115.0 ± 10.0 m, the peak value is about 7 months in advance, however.

The three types of groundwater responding to precipitation signal reveal implicitly that there are
two groundwater sub-systems and a transitional subsystem in the study site. (1) Sub-system for Ordovician fractured limestone water with better hydraulic connection, faster water velocity and larger amplification for input signals. (2) Sub-system for strong weathered argillaceous limestone water. This system is represented by the gradual delay with narrow amplitude, slower flow velocity, low fluctuation, gradually expansion of lag period for input signals. (3) Transitional system for fracture-pore limestone water. This system is between above two sub-systems.

4.2. Comparison of FIR and IIR results in groundwater subsystem

Well No. 1 is the representative of Ordovician limestone fractured water system. This groundwater table observation sequence is analysed by FIR filtering technique under different time windows. Figure 5(a) shows the FIR filtering results from January 1992 to December 2006. The time domain window parameters are taken \( q=0.25 \) (half a year), \( q=0.5 \) (1 year), \( q=1.0 \) (2 years), \( q=2.5 \) (5 years).

![Figure 5](image.png)

**Figure 5.** Results comparison between (a) FIR filter and (b) IIR filter with different filter coefficients in the same groundwater observation sequence. The phase-shifting feature could be observed in the IIR filter results.

The well No. 1 is reanalysed by IIR filtering technique. First-order recursive filtering is used for easy to compare with another filtering. The formula is as follows:

\[
y[i] = (1 - \alpha) \cdot x[i] + \alpha \cdot y[i - 1]
\]

where \( \alpha \) is filter coefficient.

Figure 5(b) shows the FIR output results of filtering with the coefficients of 0.5, 0.75, 0.85 and 0.99. With increase linearly of the filtering coefficient, the phase-shifting feature can be observed obviously. The increase of the filtering coefficient also makes the sensitivity reduction and the filtering curve smoothing. The larger filtering coefficient is, the more obviously the low-pass filtering effect has.

5. Identification and analogue simulation model in precipitation driving effect

5.1. Data preparation

Base on the above three groundwater sub-systems, the wells of No. 1, No. 5 and No. 9 as representative observation wells were selected to establish the identification and analogue simulation model. This model was taking the FIR filtering results of 5-year precipitation as the system input, while the FIR filtering results with same period in the representative observation wells as the system output.

Re-sample technique was adopted to regularize the irregular data by adjusting the data sequence to a new interval. For null values generating during the re-sampling process, interpolation methods (e.g., linear or spline interpolation) was used to reconstruct the observation sequence. By the above data
processing, the error under the time interval for 2 or 3 times per month in a long time span of 15 years was so small that they could be neglected.

5.2. Identification and analogue simulation model

Identification and analogue simulation model of the system could be established by inputting the generated data into MATLAB (R2011b) System Identification Tool, selecting Hammerstein-Wiener model in the nonlinear model and setting relevant parameters. Table 1 shows the detailed information of H-W model and the parameters. Figure 6 shows the fitting results of simulation and measurement.

Table 1. Information of H-W identification model and parameters on the groundwater system in the study site.

| System          | lithology                  | type                              | well | lag | identification model                                      | nonlinear model |
|-----------------|----------------------------|-----------------------------------|------|-----|----------------------------------------------------------|-----------------|
| fractured aquifer | Ordovician limestone      | delay with wide amplitude         | No.1 | 19  | $B(q) = q^{-19} - 0.9336 \cdot q^{-20}$                  | piecewise linear |
|                 |                            |                                   |      |     | $F(q) = 1 - 2.998 \cdot q^{-1} + 2.998 \cdot q^{-2}$     | $m, n$ are 10   |
|                 |                            |                                   |      |     | $- q^{-3}$                                               |                 |
| fractured-porosity transition | Ordovician limestone | transition with wide amplitude     | No.9 | 1   | $B(q) = -0.2004 \cdot q^{-1} + q^{-2} - 0.7965 \cdot q^{-3}$ | piecewise linear |
|                 |                            |                                   |      |     | $F(q) = 1 - 2.796 \cdot q^{-1} + 2.592 \cdot q^{-2}$     | $m, n$ are 10   |
|                 |                            |                                   |      |     | $- 0.7961 \cdot q^{-3}$                                 |                 |
| porous aquifer  | Cambrian argillaceous limestone | gradual delay with narrow amplitude | No.5 | 31  | $B(q) = q^{-31} - 1.011 \cdot q^{-32}$                   | piecewise linear |
|                 |                            |                                   |      |     | $F(q) = 1 - 1.887 \cdot q^{-1} + 0.8129 \cdot q^{-2}$     | $m, n$ are 10   |
|                 |                            |                                   |      |     | $+ 0.07417 \cdot q^{-3}$                                 |                 |

*Note: 1 unit of the input response lag is about 2 weeks.

Table 1 shows that the type of transition with no lag time and wide amplitude fracture system. There is an inputting lag time of one unit sample, named as the dead time, due to the demand of difference between output and the previous input in the discrete system. In this system, coefficient $B(q)$ is a third-order (i.e., three zeros) polynomial needs to consider more input of early stage precipitation to obtain better simulation results. The coefficient $B(q)$ in the other two systems is second-order with input response lags of 19 and 31, respectively. The coefficient $F(q)$ in all systems are third-order (i.e., three poles) polynomials. Nonlinear sections of input and output are fitted by piecewise linear for 10 sections.

Figure 6. Simulation results of groundwater system identification model and their analysis in study site. (a) Fractured aquifer system. (b) Fractured-porosity transition system. (c) Porous aquifer system.

The fittings are 89.2% (No. 1), 94.9% (No. 9) and 87.9% (No. 5) for the three systems (figure 6), corresponding to the limestone fractured aquifer system, the limestone fracture-porosity transition system, and the argillaceous limestone porous aquifer system, respectively.
6. Conclusion

- Filtering technique is not only an effective method for signal processing but also an effective way to obtain groundwater system information. This technique can provide basic information for prevention mining areas from groundwater inrush disaster.
- Water table fluctuation of groundwater system is significantly affected by precipitation. The three types correlation to three systems can be divided according to their responding amplitude, delay and transition characteristic to precipitation.
- Base on nonlinear Hammerstein-Wiener model, three representative observation wells of the groundwater systems are selected, to establish an identification model and analyse the correlation parameters. The fitting between the identification and the measurement are 86.1%, 94.9%, and 87.9%, respectively.

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