Vehicle chaos identification and intelligent suppression under combined excitation of speed bump and engine

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Abstract: For non-linear suspension, vehicle passes through a continuous speed bump, the chaos that may occur under the combined excitation of the speed bump and the engine. This study takes the five-degree-of-freedom vehicle model as the object of research, through the vehicle body poincaré section and the maximum Lyapunov index to identify the chaos produced by the vehicle under joint excitation, and utilises the feedback control of the optimal feedback gain coefficient based on the particle swarm optimisation (PSO) algorithm to suppress vehicle chaos. The results indicate that the vehicle is in a chaotic state in all speed range. Under low and medium speeds, the route to the chaos of the vehicle is the system coupling vibration under multi-frequency excitation, whereas in the high-speed condition, the vehicle approaches the chaos through the bifurcation. The chaos of the vehicle can be effectively suppressed by feedback control with the global optimal feedback gain searched by PSO. This study reveals the chaotic characteristics of non-linear suspension vehicles under combined excitation, which provides a new method for intelligent suppression of chaos.

1 Introduction

The continuous speed bump is set up on some special sections of expressways and its principle is to create vibration for passing vehicles and prompt drivers to decelerate [1]. Under the periodic excitation of the speed bump, the non-linear suspension vehicle may generate chaos. Chaos may cause adverse effects on the suspension model by simulation and experiment, used the vehicle was chaotic in the excitation frequency range. Naik and Borowiec [2] studied the chaotic vibration of a non-linear suspension system by the frequency bifurcations and phase trajectories, which also have a certain subjectivity in the identification of the vehicle. In the aspect of recognising vehicle chaos, the feedback gain coefficient has an obvious impact on the effect of the vehicle chaos suppression when the feedback control is used to suppress the vehicle chaos. It is difficult to determine the best feedback gain coefficient by the traditional method. Therefore, the five-degree-of-freedom vehicle model considering engine excitation is established in this paper, using the maximum Lyapunov index of the vehicle system to measure the chaos of the vehicle. The particle swarm optimisation (PSO) algorithm is introduced to search the optimal feedback gain coefficient to improve the search efficiency of the optimal feedback gain coefficient and to optimise the effect of the feedback control on the vehicle chaos.

2 Road excitation model of a continuous speed bump

A half sine function is used to establish the speed bump model. The height of the speed bump is $h$, the width of the single speed bump is $w_1$, and the intervals between the speed bumps are $w_2$ and $w_1 = w_2 = w$. The static model of the speed bump is shown in Fig. 1. The height of the speed bump is $h = 0.005$ m, the width of the speed bump is $w_1 = 0.5$ m, and the deceleration interval is $w_2 = 0.5$ m.

The vehicle passes the speed bump at the speed of $v$, the time required to pass a single speed bump is $t_1$, $t_1 = w_1/v$, the time required to pass the spacing of the speed bump is $t_2$, $t_2 = w_2/v$, and $w_1 = w_2$, so $t_1 = t_2$. The time of the vehicle passes through a speed bump and a speed bump spacing at the speed of $v$, that is, the dynamic model of a continuous sinusoidal speed bump is $T$

$$T = t_1 + t_2 = w_1 + w_2/v = 2w_1/v$$

(1)

and based on formula (1), the angular velocity $\omega$ of the dynamic model is...
\[ \omega = 2\pi/T = 2\pi/(2\pi/v) = \pi/v \]  

Thus, the dynamic model of continuous sinusoidal speed bump can be obtained as

\[
\begin{align*}
\dot{x}_2 &= vT
\end{align*}
\]

\[
\begin{align*}
\dot{y}_2 &= \left[ h \sin(\pi vT/\omega) + |h \sin(\pi vT/\omega)| \right] / 2
\end{align*}
\]

3 Non-linear vehicle suspension model

The vehicle structure is complex and needs to be simplified according to certain rules. Considering that the vehicle is basically the same on both sides of the road when the vehicle passes through the speed bump, there is a phase difference between the front and rear wheels, and the vehicle is regarded as the left and right symmetry structures, and the five-degree-of-freedom 1/2 vehicle model considering the engine excitation is set up, as shown in Fig. 2.

In Fig. 2, \( z_c \) is the vertical displacement of the engine; \( z_b \) is the vertical displacement of the body; \( z_c \) and \( z_b \) are the vertical displacements of the front and rear suspensions, respectively; \( q_f \) and \( q_r \) are vertical road excitations for front and rear tyres, respectively. \( m_e \) is the engine quality; \( f_b \) is the damping coefficients of the front and rear tyres, respectively. The engine excitation can be expressed as \( \Delta \sin(\omega t + \phi) \) \( \Delta \). The speed range of a general vehicle through a continuous speed bump is 10–20 m/s. Taking the vehicle speed as the abscissa and the body displacement as the vertical coordinate, the speed bifurcation diagram of the body displacement is made, as shown in Fig. 3.

The bifurcation diagram cannot effectively distinguish the quasi-periodic motion and the non-periodic motion at each speed of the vehicle. The chaotic state of the vehicle system and the approach to chaos need to be further analysed by using the poincaré section diagram. For the poincaré section, a phase point on the plane is taken as the starting point, and the phase trajectory goes back to the plane after a periodic motion, and the intersection point of the phase trajectory and the plane is the point of the poincaré section. After several cycles, the number of points on the section reflects the motion state of the system. When the poincaré section has only one fixed point and a few discrete points, it can be determined that the motion is periodic; when the poincaré section

4 Analysis of chaotic characteristics of the vehicle

4.1 Analysis of chaotic characteristics of the image method

The speed range of a general vehicle through a continuous speed bump is 10–20 m/s. Taking the vehicle speed as the abscissa and the poincaré section point of the body displacement at each speed as the vertical coordinate, the speed bifurcation diagram of the body displacement is made, as shown in Fig. 3.

Vehicle model parameters, as shown in Table 1 [9]:

**Table 1** Model parameters of vehicle

| Symbol | Parameter values | Symbol | Parameter values |
|--------|------------------|--------|------------------|
| \( m_1 \), kg | 45 | \( c_{e1} \), N/(m/s) | 10,000 |
| \( m_2 \), kg | 1180 | \( c_{e2} \), N/(m/s) | 1500 |
| \( m_3 \), kg | 45 | \( c_{e2d} \), N/(m/s) | 1200 |
| \( m_4 \), kg | 50 | \( c_{e2u} \), N/(m/s) | 1500 |
| \( J \), kg m² | 633.615 | \( c_{g2d} \), N/(m/s) | 1200 |
| \( a \), m | 1.123 | \( \vartheta_{1u} \), N/(m/s) | 8 |
| \( b \), m | 1.377 | \( \vartheta_{1d} \), N/(m/s) | 5 |
| \( d \), m | 1.34 | \( c_{1u} \), N/(m/s) | 8 |
| \( k_e \), N/m | 122,000 | \( c_{r1d} \), N/(m/s) | 5 |
| \( k_{r2} \), N/m | 36,925 | \( n_2 \) | 1.5 |
| \( k_{r2} \), N/m | 30,310 | \( n_3 \) | 1.5 |
| \( k_{r1} \), N/m | 140,000 | \( n_1 \) | 1.25 |
| \( k_{r1} \), N/m | 140,000 | \( n_{1} \) | 1.25 |
is a closed curve, it can be determined that the motion is quasi-periodic; when the poincaré section is a densely packed point and there is a hierarchical structure, it can be determined that the motion is in a chaotic state. Three speeds 10, 15, and 20 m/s are selected, and the poincaré section diagrams of the vehicle body corresponding to the vehicle speed are made, which are, respectively, shown in Figs. 4–6. The chaotic state of the vehicle and the ways of a vehicle entering chaos are analysed at low speed, medium speed, and high speed.

Fig. 4 shows that the poincaré section point of the vehicle body consists of two separate limit cycles, indicating that the vehicle is in a non-periodic motion, and the way that the vehicle enters the chaos is the coupling of the vibration of the system under the multi-excitation frequency.

The two separate limit cycles of the poincaré section of the vehicle body of Fig. 5 fuse into a limit cycle and have a broken trend. It shows that the vehicle is in a non-periodic motion, and the way that the vehicle enters the chaos is still coupled with the vibration of the system under the multi-excitation frequency.

In Fig. 6, the vehicle phase trajectories are a group of spiral lines. The limit cycle of the poincaré section diagram is completely broken, indicating that the vehicle is in a non-periodic motion, the limit cycle ruptures, and the vehicle enters chaos by the way of bifurcation.

4.2 Analysis of chaotic characteristics of the numerical method

The maximum Lyapunov index of the vehicle system is used to identify the chaotic state of the vehicle system. The Lyapunov index is the most reliable method of identifying chaos [6], which represents the numerical characteristics of the average exponential divergence rate of the adjacent trajectories in the phase space, reflecting the sensitivity of the chaotic system to the initial value.

A set of time series is obtained by solving formula (4) by fourth-order five-level Runge-Kutta algorithm, and a set of one-dimensional time series is formed by the first and end connection of the time series. Phase space reconstruction of the time series by using the coordinate delay method

\[ Y_k = (x_k, x_{k+\tau}, ..., x_{k+(m-1)\tau})^T \]  

In (8), \( Y_k \) is the \( m \)-dimensional phase space, \( k = 1, 2, ..., N - (m - 1)\tau \) is the delay time, and \( m \) is the embedding dimension. To ensure the fidelity of the phase space reconstruction and the accuracy of the calculation of the Lyapunov exponent, the selection of these two parameters is very important. The mutual information method is used to obtain the relationship between the delay time \( \tau \) and the information entropy \( I(\tau) \) [10]. So the conclusion is that the optimal delay time is 14 sampling points backward, and the sampling compensation \( \tau = 0.001 \) means that the delay time is \( \tau = 0.014 \) s. Using the artificial neighbouring point method to determine the embedding dimension, using MATLAB to calculate the relationship between the embedding dimension and the proportion of false neighbour points, it can be concluded that when the embedding dimension is \( m = 29 \), if the embedding dimension will continue to expand, the proportion of false neighbour points no longer change. Therefore, when the road excitation frequency is 10 Hz, the embedding dimension of the vehicle system time series should be 29 [10]. Then, the wolf method is used to calculate the maximum Lyapunov index of time series after phase space reconstruction. The maximum Lyapunov index of vehicle system at different speeds is shown in Fig. 7.

The maximum Lyapunov index of the vehicle is greater than zero at all speeds, and the vehicle is in chaos. There is a great difference between the pavement excitation frequency and the engine excitation frequency at low speed, and under multi-frequency excitation, the vehicle chaos is more obvious. The difference between the excitation frequency of the road surface and the engine frequency is reduced at medium speed, the limit cycle of vehicle gradually fuses, and the vehicle chaos degree decreases. At high speed, the limit cycle of the vehicle is broken, and the vehicle enters into chaos through the bifurcation route, and the degree of chaos increases dramatically. It is noted that at 15.69 m/s, the maximum Lyapunov index of the vehicle system increases sharply, which is due to the complete fusion of the limit ring near the speed of the vehicle, and the way that the vehicle enters the chaos is transformed from multi-frequency excitation to bifurcation, and the vehicle is chaotic at this time.

5 Vehicle chaos intelligent suppression

5.1 Feedback control

Direct variable feedback control [7] can achieve chaos suppression by adjusting the feedback gain coefficient on the premise that the unsteady period orbit of the system target is unknown. Using the piecewise quadratic function \( x|\tau \) as the form of feedback control, this control method is easy to implement and has little effect on the system [11]. The controller of the design system is in the form of
The feedback gain coefficient determines the effect of the chaos suppression. An improved PSO algorithm is introduced to minimise the maximum Lyapunov index of the vehicle system as the control target and search for the best feedback gain coefficient \([k_1, k_2, k_3, k_4]\). Different from the traditional feedback control, the feedback gain coefficient in this paper keeps changing in the solution space of the chaos system at each vehicle speed after feedback control is calculated. This constant \(C\) is set to 0.7 in this paper.

\[
\dot{z}_i = z_i - k_i \varepsilon_i[z_i] = (F_{1i} + \cdots + F_{4i})/m_0 - k_i \varepsilon_i[z_i] \tag{9}
\]

5.2 Intelligent searches for optimal feedback control coefficient

The feedback gain coefficient determines the effect of the chaos suppression. An improved PSO algorithm is introduced to minimise the maximum Lyapunov index of the vehicle system as the control target and search for the best feedback gain coefficient \([k_1, k_2, k_3, k_4]\). Different from the traditional feedback control, the feedback gain coefficient in this paper keeps changing in the solution space of the chaos system at each vehicle speed after feedback control is calculated. This constant \(C\) is set to 0.7 in this paper.

\[
\dot{z}_i = z_i - k_i \varepsilon_i[z_i] = (F_{1i} + \cdots + F_{4i})/m_0 - k_i \varepsilon_i[z_i] \tag{9}
\]

\[
L = \begin{cases} 
\max_{1 \leq i \leq m} |L_{ya_i} - L_{ya_{avg}}| & \max_{1 \leq i \leq m} |L_{ya_i} - L_{ya_{avg}}| > 1 \\
1 & \text{other}
\end{cases} \tag{11}
\]

When \(\sigma^2 < C\), the particle swarm is considered to be in a precociously and precociously treated; when \(\sigma^2 > C\), the particle swarm is assumed to be in a non-premature state, a next-generation particle swarm is generated, and the resulting next-generation particle swarm is returned to step (iii) to continue the particle swarm calculation. This constant \(C\) is set to 0.7 in this paper.

(v) The chaos optimisation algorithm makes use of the random, ergodicity, and regularity characteristics of chaotic variables to optimise the search in the solution space and it is easy to jump out of the local optimal solution [12]. Selecting the logistic map to generate the chaotic variable \(y_q^{\prime}\)

\[
y_q^{\prime} = \mu y_q (1 - y_q) \quad q = 1, 2, \ldots, n \tag{12}
\]

These \(n\) variables form the solution space of the chaos optimisation algorithm, and these \(n\) new feedback gain coefficients are brought into (13) to calculate the maximum Lyapunov index of the vehicle system. This solution space makes the feedback gain coefficient of the maximum Lyapunov index minimum of the vehicle system as \(k_{ig}\) update \(k_{ig}\) with \(k_{ig}^{\prime}\), and return to step (iii) to continue particle swarm calculation.

(vi) Generating the \(i\)th generation particle group and its ‘migration’ speed according to the equations below:

\[
k_{ig}^{\prime} = k_{ig}^{\prime} + \alpha v_{im}^{\prime} \tag{14}
\]

\[
v_{ig}^{\prime} = \alpha v_{im}^{\prime} + c_1 r_1 (k_{ig} - k_{im}) + c_2 r_2 (k_{ig} - k_{avg}) \tag{15}
\]

where \(i = 1, 2, 3, 4; d = 1, 2, \ldots, M; \alpha\) is the constancy coefficient, which controls the weight of the ‘migration’ speed of the particle swarm; \(c_1, c_2 \geq 0\) are learning factors; \(r_1, r_2\) are random numbers between random factors [0,1].

The process is shown in Fig. 8.

5.3 Control effect analysis

To analyse the effect of feedback control on chaos suppression, optimal feedback gain coefficient obtained by the PSO algorithm is brought into (9), and the vehicle body poincaré section is taken at a controlled vehicle speed of 20 m/s, as shown in Fig. 9.

Comparing Fig. 6 with Fig. 9, it is found that the state of motion of the vehicle after the control is transformed from chaotic motion to periodic motion. Feedback control can effectively suppress vehicle system chaos.

Further analyse the feedback control of vehicle chaos at various vehicle speeds, the maximum Lyapunov index of the vehicle system at each vehicle speed after feedback control is calculated. The maximum Lyapunov indexes of the vehicle system before and after feedback control are shown in Fig. 10. The maximum Lyapunov index of the vehicle system is significantly reduced at each vehicle speed, indicating that feedback control can effectively suppress vehicle chaos at each vehicle speed.
The limit cycle of the vehicle is broken at high-speed, the vehicle enters chaos through the bifurcation approach at high excitation are analysed, and the feedback control based on PSO is used to suppress vehicle chaos. The way of vehicle entering chaos in the middle-to-low-speed range is the coupling of system vibration under multi-frequency excitation and the vehicle enters chaos through the bifurcation approach at high speed.

(ii) According to the maximum Lyapunov index at each vehicle speed, the difference between the road excitation frequency and the engine excitation frequency at low speed is obvious, and the vehicle enters chaos through the bifurcation route, and the degree of chaos increases sharply. At the speed of 15.69 m/s, the maximum Lyapunov index of the vehicle system increases dramatically, the vehicle limit ring is completely integrated near the speed, and the vehicle enters the chaotic way from multiple frequency excitation to bifurcation, which means that the vehicle undergoes paroxysmal chaos.

(iii) The feedback control is used to suppress vehicle chaos. The PSO algorithm is used to search the global optimal feedback gain coefficient of feedback control. Moreover, chaotic search is utilised to improve the PSO algorithm of avoiding 'premature' phenomenon of the particle swarm and to make the calculation jump out of the local optimal solution as soon as possible. The results show that the feedback control can effectively reduce the maximum Lyapunov index of the vehicle system at various vehicle speeds, restrain vehicle chaos, and convert the vehicle motion state from chaotic motion to periodic or quasi-periodic motion.

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8 References

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