ON THE ORIGIN OF GeV EMISSION IN GAMMA-RAY BURSTS

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ABSTRACT

The most common progenitors of gamma-ray-bursts (GRBs) are massive stars with strong stellar winds. We show that the GRB blast wave in the wind should emit a bright GeV flash. It is produced by inverse-Compton cooling of the thermal plasma behind the forward shock. The main part of the flash is shaped by scattering of the prompt MeV radiation (emitted at smaller radii) which streams through the external blast wave. The inverse-Compton flash is bright due to the huge $e^\pm$ enrichment of the external medium by the prompt radiation ahead of the blast wave. At late times, the blast wave switches to normal synchrotron-self-Compton cooling. The mechanism is demonstrated by a detailed transfer simulation. The observed prompt MeV radiation is taken as an input of the simulation; we use GRB 080916C as an example. The result reproduces the GeV flash observed by the Fermi telescope. It explains the delayed onset, the steep rise, the peak flux, the time of the peak, the long smooth decline, and the spectral slope of GeV emission. The wind density required to reproduce all these features is typical of Wolf–Rayet stars. Our simulation predicts strong TeV emission 1 minute after the burst trigger; then a cutoff in the observed high-energy spectrum is expected from absorption by extragalactic background light. In addition, a bright optical counterpart of the GeV flash is predicted for plausible values of the magnetic field; such a double (optical+GeV) flash has been observed in GRB 130427A.

Key words: gamma-ray burst; general – relativistic processes – radiation mechanisms: non-thermal – radiative transfer – scattering – shock waves

Online-only material: color figures

1. INTRODUCTION

The luminosities of gamma-ray-bursts (GRBs) peak in the soft gamma-ray band around 1 MeV (e.g., Goldstein et al. 2012). Observations by the Large Area Telescope (LAT) onboard the Fermi satellite (Atwood et al. 2009) show that some GRBs also give rise to a longer GeV flash, with a distinct light curve (Ackermann et al. 2013). The energy emitted in the GeV band is smaller than that of the main ("prompt") MeV radiation, typically by a factor $\sim 10$. Nevertheless, as we argue in this paper, it can play a key role for understanding the nature of GRB explosions and their progenitors.

The GeV flash can shed light on the explosion picture only if its radiative mechanism is identified with some confidence. Ideally, one would hope for a model that reproduces the observed light curve and spectrum from a first-principle calculation. In search of such a model, one can consider various possibilities such as synchrotron emission from the blast wave (Zou et al. 2009; Kumar & Barniol Duran 2009; Ghisellini et al. 2010), hadronic processes (e.g., Asano et al. 2009; Razzaque et al. 2010), or inverse-Compton (IC) emission from internal shocks (e.g., Bošnjak et al. 2009; Toma et al. 2011). None of the proposed models, however, predict the observed light curve, and most models invoke extreme parameters (low external density and magnetic fields, or a huge explosion energy). The synchrotron mechanism of GeV emission is problematic as it requires extreme particle acceleration; even under most favorable conditions it cannot explain the observed spectrum which extends to 100 GeV (e.g., Piran & Nakar 2010; Sironi et al. 2013; Wang et al. 2013).

The radiative process capable of producing the observed flash is IC scattering; the seed photons for IC scattering can be provided by the prompt GRB or its afterglow radiation. In particular, Beloborodov (2005b) suggested that GRBs should be accompanied by GeV flashes due to IC scattering of the prompt MeV radiation streaming through the external blast wave. Observations by Fermi-LAT provide support to this picture:

1. In practically all GRBs detected by Fermi-LAT (except a few cases with poor photon statistics) the peak of the GeV flash overlaps with the prompt MeV radiation (Ackermann et al. 2013). The overlap implies that the GRB source experiences Compton cooling by the prompt MeV radiation (keV radiation in the rest frame of the source).
2. The GeV flash has a distinct light curve, different from the prompt MeV burst. It quickly rises and then shows a long monotonic decay, which lasts significantly longer than the prompt MeV emission. This is expected if the GeV flash is produced by the external blast wave. The blast wave has a larger radius and moves with a smaller Lorentz factor compared with the source of the prompt burst, and hence its emission can be spread over longer observational times.
3. The onset of GeV emission is slightly delayed with respect to the beginning of the prompt MeV burst. The arrival time of photons emitted by the blast wave at radius $R$ is roughly given by

$$t_{\text{obs}} \sim (1 + z) \left( \frac{R}{v_{\text{bw}}} - \frac{R}{c} \right) \approx (1 + z) \frac{R}{2\gamma^2 c}. \quad (1)$$

Here $t_{\text{obs}}$ is measured by the clock of a distant observer since the first light signal from the beginning of the explosion, $\Gamma = (1 - v^2_{\text{bw}}/c^2)^{-1/2} \gg 1$ is the Lorentz factor of the blast wave, and $z$ is the cosmological redshift. The delay
in the onset of GeV emission is expected if the blast-wave luminosity is suppressed at small radii. It equals the time it takes the explosion to reach the radius where the blast wave becomes a bright GeV source, which is typically a few seconds.

However, any model associating the GeV flash with the external blast wave faces the following puzzle. Many observed GeV flashes reach the peak and start to decay at time \( T_p \) much shorter than the duration of the prompt MeV burst, \( T_{\text{GRB}} \). For example, GRB 080916C has \( T_p \sim 0.1 \, T_{\text{GRB}} \) (Abdo et al. 2009). Why would the peak of blast-wave radiation be much shorter than the prompt burst itself? Consider the standard model where \( T_{\text{GRB}} \) corresponds to the duration of the ultra-relativistic ejecta that emits the prompt burst. Then \( c T_{\text{GRB}} (1+z)^{-1} \) is a measure of the ejecta thickness. The ejecta energy is transferred to the blast wave through the reverse shock, which may be relativistic and can cross the ejecta as quickly as \( T_{\text{cross}} \sim T_{\text{GRB}} \) (in observer time). The ejecta cannot transfer its energy to \( t_{\text{obs}} \ll T_{\text{GRB}} \), as this would require a supernalional motion of the reverse shock, and hence the self-similar deceleration of the blast wave should not begin until \( t_{\text{obs}} \sim T_{\text{GRB}} \). Then the GeV flash is not expected to decay until \( t_{\text{obs}} \sim T_{\text{GRB}} \) (e.g., Gao et al. 2009; He et al. 2011; Maxham et al. 2011). The problem becomes even more severe in explosion models with a non-relativistic reverse shock; then the deceleration/decay stage is not expected until \( t_{\text{obs}} \gg T_{\text{GRB}} \).

This puzzle is resolved by the fact that the blast wave propagates in a medium with a quickly changing composition. As discussed in detail below, at radii \( R \lesssim 10^{16} \) cm the medium is extremely rich in \( e^\pm \) pairs, with \( Z_\pm \gtrsim 10^4 \) pairs per proton. Pairs are inevitably produced by the prompt MeV radiation propagating ahead of the blast wave (Thompson & Madau 2000; Mészáros et al. 2001; Beloborodov 2002, hereafter B02; Kumar & Panaitescu 2004). The huge number of the prompt MeV photons (\( N_{\text{MeV}} \sim 10^{60} \) in isotropic equivalent for the brightest GRBs) implies exponential pair creation in a static optically thin medium. In addition, radiation exerts a strong force and significantly accelerates the external medium, which affects the strength of the forward shock and the evolution of its temperature.

We show in this paper that the forward shock propagating in the pre-accelerated pair-enriched medium is an extremely efficient producer of GeV emission, regardless of the details of the shock microphysics and its efficiency in nonthermal particle acceleration. This provides a robust mechanism for a GeV flash. This puzzle is resolved by the fact that the blast wave propagates in a medium with a quickly changing composition.

Ghisellini et al. (2010) associated GeV photons with synchrotron emission from nonthermal particles. We find that the GeV flash is produced by IC scattering of the prompt radiation by the thermal plasma behind the forward shock.

In this paper, we study explosions in the wind medium expected from a massive progenitor (e.g., Chevalier & Li 1999). We consider a Wolf–Rayet star with a typical mass-loss rate \( M \approx 10^{-5} \, M_\odot \, \text{yr}^{-1} \), which produces a wind with density profile \( \rho \propto R^{-2} \). We calculate the dynamics, \( e^\pm \) density, and temperature of the blast wave and show that it must generate an IC pair-dominated flash in the GeV band. Its light curve and spectrum can be calculated from first principles, using a direct simulation of radiative transfer.

Preliminary estimates explaining the proposed mechanism are presented in Section 2. Then in Sections 3 and 4 we describe the setup of our detailed calculations. The results are described in Section 5 using GRB 080916C as an example. In Section 6, we present analytical estimates for photon–photon (\( \gamma\gamma \)) opacity. Then, in Section 7, we discuss the expected synchrotron emission from the pair-loaded blast wave, and find that, in a broad range of the magnetization parameter \( \epsilon_B \), the GeV flash is accompanied by a bright and brief optical flash. In Section 8 we estimate the effect of the GeV flash on the external medium. Our results are summarized and discussed in Section 9.

2. PRELIMINARY ESTIMATES

2.1. Number of GeV Photons in the Flash

Consider a blast wave that sweeps up the external medium. Let \( \gamma_{\text{inj}} \) be the mean (thermal) Lorentz factor of hot electrons immediately behind the forward shock, and \( \Gamma \) be the bulk Lorentz factor of the shocked fluid. Subscript “inj” in \( \gamma_{\text{inj}} \) stands for “injection”—the hot plasma is injected at the shock front and cools down below it.

The plasma is Compton cooled by the prompt GRB radiation that gradually leaks out of the explosion ejecta and streams radially through the external blast wave.\(^4\) Let \( E_i \approx 1 \, \text{MeV} \) be the characteristic energy of the prompt photons in the lab frame; they serve as targets for IC scattering. Their energies in the fluid frame are

\[
E'_i = \frac{E_i}{2\Gamma}.
\]

The hot electrons injected at the shock front lose energy by upscattering the target photons. The typical energy of upscattered photons in the fluid frame is \( E'_{\text{IC}} \sim \gamma_e^2 E'_i \) (assuming Thomson scattering). The corresponding energy of IC photons in the lab frame is \( E_{\text{IC}} \approx (2/3)\Gamma E'_{\text{IC}} \), which gives

\[
E_{\text{IC}} \sim \frac{1}{3}\gamma_e^2 E_i.
\]

One can see that GeV photons are generated when

\[
\gamma_e \sim 50 \left(\frac{E_{\text{IC}}}{1 \, \text{GeV}}\right)^{1/2} \left(\frac{E_i}{1 \, \text{MeV}}\right)^{-1/2}.
\]

Then one can verify that the scattering is in the Thompson regime, \( \gamma_e E'_i/m_e c^2 < 1 \), although moderate Klein–Nishina corrections are beginning to appear at these energies.

\(^4\) The prompt photons are assumed to be emitted at a small radius \( R_{\text{MeV}} \ll R \), and their angles with respect to the radial direction are \( \theta \sim (R_{\text{MeV}}/R) \Gamma^{-1} \ll 1/\Gamma \).
As the electron injected with $\gamma_e = \gamma_{inj}$ cools down, it produces IC photons with decreasing $E_{IC} \propto \gamma_e^2$. Their number near a given energy $E_{IC}$ may be estimated as

$$M \sim \frac{\gamma_{inj} m_e c^2}{E_{IC}} \sim \frac{m_e c^2}{(E/E_{IC})^{3/2}} \sim \frac{\Gamma m_e c^2}{(E/E_{IC})^{3/2}}.$$  \hspace{1cm} (5)

The multiplicity of photons with $E_{IC} \sim 1$ GeV produced by an electron with $\gamma_{inj} \gg 50$ is $M \sim \Gamma / 60$.

The number of GeV photons (isotropic equivalent) produced by the shock wave is

$$N_{GeV} \sim MN_\pm,$$ \hspace{1cm} (6)

where $N_\pm$ is the number of electrons/positrons swept-up by the shock, proportional to the total swept-up mass $m$,

$$N_\pm = Z_\pm N_p, \hspace{1cm} N_p = \frac{m}{\mu_e m_p}.$$ \hspace{1cm} (7)

Here $Z_\pm$ is the pair loading factor of the external medium, $N_p$ is the number of swept-up protons, and $\mu_e$ depends on the chemical composition of the medium; $\mu_e = 1$ for hydrogen and $\mu_e = 2$ for heavier elements.

The medium is expected to be a wind from a massive progenitor, which is losing mass before the explosion with a rate $\dot{M}$. The mass of the wind medium contained in a sphere of radius $R$ is given by

$$m(R) = \frac{\dot{M} R}{w},$$ \hspace{1cm} (8)

where $w$ is the wind velocity. The likely GRB progenitors are Wolf–Rayet stars, whose observed winds have typical $\dot{M} \sim 10^{-5} M_\odot$ yr$^{-1}$, $w \sim 2 \times 10^8$ cm s$^{-1}$, and $\mu_e \approx 2$ (e.g., Hamann 1995; Lamers & Cassinelli 1999; Crowther 2007). This gives

$$N_p \sim 10^{52} R_{16} M_{-5}.$$ \hspace{1cm} (9)

The value of $Z_\pm$ can be exactly calculated using the observed luminosity and spectrum of the prompt GRB (Section 3.1); it has enormous values $Z_\pm \sim 10^5$ at the early stages of blast-wave expansion and then steeply decreases with radius. In particular, for GRB 080916C we will show below that the GeV flash peaks at a well-defined radius $R_p \approx 10^{16}$ cm where $Z_\pm \sim 10^4$.

Equations (6) and (7) with $Z_\pm \sim 10^4$ give a rough estimate for the number of GeV photons,

$$N_{GeV} \sim 10^{37},$$ \hspace{1cm} (10)

which is close to the isotropic equivalent of the bright GeV flashes observed by LAT. The high density of the progenitor wind and the huge pair enrichment is what makes the IC mechanism capable of emitting a bright flash; models neglecting pair creation would fall far short in $N_{GeV}$.

Note that the prompt GRB radiation plays a key role for the GeV flash in two ways: (1) it provides target photons for IC scattering and (2) its interaction with the external medium ahead of the shock ensures the $e^\pm$ enrichment of the medium. The $e^\pm$ pairs radiating GeV photons behind the shock are created by the prompt MeV photons propagating ahead of the shock. The total number of the prompt photons in a burst like GRB 080916C is huge, $N_{MeV} \sim 10^{90}$ (isotropic equivalent). Almost all these photons pass through the external medium unaffected, as the medium is optically thin. A small fraction of photons get scattered and converted to $e^\pm$ pairs, so the number of created pairs $N_\pm \ll N_{MeV}$. However, $N_\pm$ greatly exceeds $N_p$, by the factor $Z_\pm \gg 1$.

### 2.2. Radiative Efficiency in the GeV Band

As will be demonstrated with detailed calculations below, the external blast wave inevitably passes through a stage with the pair-loading factor $Z_\pm \sim 10^4$ and pre-acceleration Lorentz factor $\gamma \sim 10$. It is an extremely efficient producer of GeV emission at this stage. Three factors contribute to the high efficiency:

1. The high pair-loading factor $Z_\pm \sim 10^4 > m_p/m_e$ guarantees that most of the shock-dissipated energy is given to leptons.

2. At this stage, the shock-heated pairs have the thermal Lorentz factor $\gamma_{inj} \sim \Gamma/\gamma \sim 50$, so their IC emission peaks in the GeV band according to Equation (3). The relatively low value of $\gamma_{inj}$ is a result of pair loading and pre-acceleration of the external medium. Note that pre-acceleration reduces the strength of the forward shock: the fluid Lorentz factor jumps at the shock front from $\gamma \sim 10$ to $\Gamma$, which corresponds to electron heating to $\gamma_{inj} \sim \Gamma/\gamma$.\footnote{Energy transfer from the shocked ions to electrons is unable to significantly increase $\gamma_{inj}$ in the medium with $Z_\pm \sim 10^4$, since the ion abundance is smaller than $m_i/m_p$. This effect can, however, become significant soon after the peak of the flash, as $Z_\pm$ decreases.}

3. IC cooling of the shocked pairs is fast, so they efficiently radiate their energy. The cooling timescale of isotropic electrons with Lorentz factor $\gamma_e$ in the fluid frame is given by

$$t_{IC}' = \frac{3 m_e c}{4 \sigma_T U' \gamma_e},$$ \hspace{1cm} (11)

where $U' = (2\Gamma)^{-2} U$ is the energy density of the collimated prompt radiation in the fluid frame, and $U = L_{GRB}/4\pi R^2 c$. The cooling timescale should be compared with the expansion timescale of the blast wave, $t_{exp}' = R/c \Gamma$,

$$t_{IC}' < t_{exp}' \quad \text{for} \quad \gamma_e \gg 1.$$ \hspace{1cm} (12)

which gives $t_{IC}' < t_{exp}'$ for $\gamma_e \gg 1$. Compton cooling is fast for electrons emitting in the GeV band, $\gamma_e \gtrsim 50$. Electrons with $\gamma_e \gg 10^2$ scatter photons with a smaller rate due to the Klein–Nishina correction, but their cooling is still fast.

### 2.3. Lorentz Factor of the Blast Wave and Arrival Time of GeV Photons

The arrival time of IC photons emitted at radius $R_p \sim 10^{16}$ cm (peak of the GeV flash) depends on the Lorentz factor of the blast wave, $\Gamma$, according to Equation (1). Note that $R_p$ can be significantly smaller than the radius where the blast wave enters the self-similar deceleration. At this early stage, the blast-wave material is sandwiched between the forward and reverse shocks, and its Lorentz factor $\Gamma$ is regulated by the ram pressures in the two shocks, $P_f$ and $P_r$.

An estimate for $\Gamma$ may be obtained assuming pressure balance $P_f \sim P_r$. A convenient approximation for the shock pressure is
given by (Beloborodov & Ulm 2006)

$$P = \frac{4}{3}(\Gamma_{\text{rel}}^2 - 1)U_{\text{up}},$$  \hspace{1cm} (13)$$

where $\Gamma_{\text{rel}}$ is the relative Lorentz factor of the upstream and downstream, $U_{\text{up}} = \gamma_{\text{heat}}\rho_{\text{up}}c^2$ is the proper energy density of the upstream fluid, and $\rho_{\text{up}} = \gamma(1 + \beta)(1 + Z_\perp m_e/\mu m_p)\rho$; $\gamma_{\text{heat}} - 1$ is a measure of upstream heat relative to the rest mass, and we took into account that the pre-accelerated external medium is compressed by the factor of $\gamma(1 + \beta)$ as required by the continuity equation (B02). This gives

$$P_f \approx \frac{4}{3}\Gamma_{\text{rel}}^2\gamma_{\text{heat}}\frac{\rho_{\text{up}}c^2}{(1 + \frac{Z_\perp m_e}{\mu m_p})},$$  \hspace{1cm} (14)$$

where we used $\Gamma_{\text{rel}} \approx \Gamma/\gamma(1 + \beta) \gg 1$. In the absence of pre-heating and pre-acceleration ($\gamma_{\text{heat}} = \gamma = 1$) and moderate pair loading ($Z_\perp \ll m_p/\mu m_p$), Equation (14) reduces to the standard relation $P_f = (4/3)\Gamma_{\text{rel}}^2\rho_{\text{up}}c^2$.

For the reverse shock one can use Equation (13) with $\Gamma_{\text{rel}} \approx (1/2)(\Gamma_{\text{ej}}/\Gamma + \Gamma/\Gamma_{\text{ej}})$ and $U_{\text{up}} = \rho_{\text{ej}}c^2$,

$$P_r \approx \frac{1}{3}\left(\frac{\Gamma_{\text{ej}}}{\Gamma_{\text{ej}} - \Gamma_{\text{ej}}}\right)^2 \rho_{\text{ej}}c^2,$$  \hspace{1cm} (15)$$

where $\rho_{\text{ej}}$ and $\Gamma_{\text{ej}}$ are the fluid mass density and Lorentz factor of the ejecta. Then the pressure balance $P_f \approx P_r$ gives

$$\Gamma \approx \Gamma_{\text{ej}}\left\{1 + 2\Gamma_{\text{ej}}^{-2}\left[\frac{4\pi A^3(1 + Z_\perp m_e/\mu m_p)}{L_{\text{ej}}\gamma(1 + \beta)}\right]^{1/2}\right\}^{-1/2}.$$  \hspace{1cm} (16)$$

Here $L_{\text{ej}} = 4\pi R_{\text{ej}}^3\Gamma_{\text{ej}}^2\rho_{\text{ej}}c^3$ is the kinetic power of the ejecta (isotropic equivalent) and we used the external density profile $\rho = AR^{-2}$ where $A \equiv M/4\pi w \sim 10^{11}-10^{12}$ g cm$^{-3}$. In the case of a relativistic reverse shock, $\Gamma_{\text{ej}}^2 \gg \Gamma_{\text{ej}}^2$, the expression for $\Gamma$ simplifies and becomes independent of $\Gamma_{\text{ej}}$,

$$\Gamma \approx \left[\frac{L_{\text{ej}}\gamma(1 + \beta)}{16\pi A^3\gamma_{\text{heat}}(1 + Z_\perp m_e/\mu m_p)}\right]^{1/4}.$$  \hspace{1cm} (17)$$

This equation gives $\Gamma \sim 500$ for the parameters of GRB 080916C discussed in this paper. Note that $\gamma$ is determined by the force exerted by the prompt radiation front ahead of the blast wave. Our numerical calculations give $\gamma \sim 10$, $Z_\perp \sim 10^4$, and $\gamma_{\text{heat}} \approx 1$ at the peak radius of the GeV flash, $R_p \approx 10^{16}$ cm (see Section 5).

Using Equations (1) and (17), one finds the arrival time of the peak of the flash $t_{\text{obs}} \sim 1-10$ s, which is consistent with observations. The detailed calculations presented below will give a more accurate estimate for the arrival time of the peak. We will also calculate the light curve of the GeV flash and show that its decay after the peak extends over much longer times.

2.4. Energy Dissipated in the Forward Shock

As a final check, let us estimate the energy dissipated in the forward shock near the radius $R_p \sim 10^{16}$ cm. Since most of the dissipated energy $E_{\text{diss}}$ is radiated in GeV photons, one expects a GeV flash of energy $E_{\text{flash}} \sim E_{\text{diss}}$.

The dissipation rate in the forward shock is approximately given by

$$L_{\text{diss}} \approx 4\pi R^3(3P_f)\Gamma^2 c \sim 4\pi R^2(3P_f)\Gamma^2 c \sim L_{\text{ej}},$$  \hspace{1cm} (18)$$

where we used Equation (15) and assumed $\Gamma_{\text{ej}} \gg \Gamma$. The ejecta power $L_{\text{ej}}$ is comparable to or larger than the observed luminosity of the prompt GRB, $L_{\text{GRB}}$, depending on the prompt emission efficiency $\epsilon_{\text{rad}}$,

$$\frac{L_{\text{ej}}}{L_{\text{GRB}}} = \frac{1 - \epsilon_{\text{rad}}}{\epsilon_{\text{rad}}}. $$  \hspace{1cm} (19)$$

The peak luminosity of the flash $L_{\text{flash}}$ is below $L_{\text{diss}}$, because many photons emitted at $R_p$ arrive with delays due to large scattering angles. $L_{\text{flash}}$ scales with $L_{\text{GRB}}$ observed before the peak of the flash; their ratio $L_{\text{flash}}/L_{\text{GRB}}$ is sensitive to the prompt GRB efficiency $\epsilon_{\text{rad}}$.

2.5. Summary

As the blast wave passes through the radius $R_p \sim 10^{16}$ cm where $\gamma \sim 10$, the shock wave radiates most of the dissipated energy in the GeV band, and the emitted radiation arrives at $t_{\text{obs}} \sim 1-10$ s. This defines the peak of the GeV flash. Below we present detailed calculations that will give light curve and spectrum of the flash, before and after the peak.

3. SHOCK WAVE IN PAIR-LOADED MEDIUM

3.1. Pair Loading

The prompt MeV radiation is nearly perfectly beamed in the radial direction in the blast-wave region, as it is emitted at much smaller radii. Those prompt photons that have already overtaken the forward shock propagate in the external medium, which has not yet learned about the explosion. Some of these photons scatter off the ambient medium. Only a small fraction of photons are scattered (the medium is optically thin), however this fraction translates into a huge number of scattered photons per ambient electron. Many of these photons quickly convert to $e^\pm$ pairs. The conversion occurs because the scattered photons have large angles with respect to the primary (collimated) GRB radiation, and the large angle lowers the energy threshold for the $\gamma - \gamma$ reaction with the beam, $\gamma + \gamma \rightarrow e^+ + e^-$. The created pairs also scatter the prompt photons, which leads to exponential $e^\pm$ creation and a huge enhancement of the electron density ahead of the forward shock, by a factor $Z_\perp$ exceeding $10^4$ (B02). The $e^\pm$ loading factor $Z_\perp \gg 1$ at radii $R < R_{\text{lead}}$, where

$$R_{\text{lead}} \approx 10^{17} E_{\text{GRB},54}^{1/2} \text{ cm},$$  \hspace{1cm} (20)$$

and $E_{\text{GRB}}$ is the isotropic equivalent of the prompt GRB energy ahead of the forward shock.

The main dimensionless parameter that controls $Z_\perp$ at the forward shock is proportional to the column density of the GRB radiation ahead of the shock,

$$\xi = \frac{\sigma_T}{m_e c^2} \frac{E_{\text{GRB}}}{4\pi R^2} = 650 E_{\text{GRB},54} R_{16}^{-2}.$$  \hspace{1cm} (21)$$

At observer times $t_{\text{obs}} \ll T_{\text{GRB}}$, $E_{\text{GRB}}$ ahead of the shock is a fraction of the total prompt GRB energy (most of which is still behind the shock). The pair loading factor $Z_\perp(\xi)$ and the pre-acceleration Lorentz factor $\gamma(\xi)$ depend only on the prompt...
radiation field and not on the density of the ambient medium (B02).

We have extended the calculations of B02 in two ways: (1) B02 assumed a typical prompt GRB spectrum that peaks at \( E_{pk} = m_e c^2 \) while the bright bursts detected by LAT have higher than average \( E_{pk} \). We have extended the model to bursts with high \( E_{pk} \sim 1–10 \) MeV. (2) B02 used the “cold approximation” assuming that the loaded \( e^\pm \) pairs are quickly cooled to a non-relativistic temperature, so that the plasma ahead of the shock may be assumed to be cold. This approximation is accurate only for bursts with \( E_{pk} \ll 1 \) MeV. We have relaxed the cold approximation and included the thermal motions of pairs in our transfer simulations.

We performed our calculations for the prompt radiation with a broken power-law spectrum, whose spectral luminosity is given by

\[
L_E = L^\text{pk}_E \times \begin{cases} (E/E_{pk})^{-\alpha_1}, & E < E_{pk} \\ (E/E_{pk})^{-\alpha_2}, & E > E_{pk}. \end{cases}
\]

As a first test, we ran our code using the cold approximation and found excellent agreement with Figures 1–3 in B02. Note that Equation (4) in B02 misses the factor \( d\epsilon / d\epsilon_{\text{esc}} \) which should have canceled the factor of \( (1 + \beta)^{-1} \) in his Equations (42) and (43). However, the numerical results in B02 are based on the correct equations, the missing factor \( d\epsilon / d\epsilon_{\text{esc}} \) being a misprint that propagated to Equations (42) and (43).

Then we relaxed the cold approximation and obtained \( Z_{\pm}(\xi) \) and \( \gamma(\xi) \) for bursts with high \( E_{pk} \). Figure 1 shows sample models with \( \alpha_1 = 0 \) (photon index \(-1\)), and \( \alpha_2 = 1.5 \) (photon index \(-2.5\)) for various values of \( E_{pk} \). The obtained \( Z_{\pm}(\xi) \) and \( \gamma(\xi) \) do not depend on \( L^\text{pk}_E \).

For comparison, Figure 1 (left panel) also shows the results obtained with the cold approximation in the case of \( E_{pk} = 3 \) MeV. MeV radiation scattered by the cold plasma is preferentially directed along radius (a Klein–Nishina effect), which reduces the efficiency of pair creation. One can see that relaxing the cold approximation leads to significantly higher \( Z_{\pm} \), mainly because the hot plasma scatters photons through larger angles with respect to the primary collimated beam. The thermal Lorentz factor of the \( e^\pm \) plasma in the radiation front reaches \( \gamma_{th} \approx 3 \) in the “non-relativistic” zone where \( \gamma \approx 1 \); \( \gamma_{th} \) is reduced at larger \( \xi \) where \( \gamma \gg 1 \).

3.2. Forward Shock

The forward shock propagates in the pair-rich, pre-accelerated medium which is moving with \( \gamma < \Gamma \). The shock thermalizes the relative Lorentz factor,

\[
\Gamma_{\text{rel}} = \Gamma \gamma(1 - \beta_{bw} \beta) \approx \frac{\Gamma}{\gamma(1 + \beta)},
\]

where \( \beta_{bw} = (1 - \Gamma^{-2})^{1/2} \) and \( \beta = (1 - \gamma^{-2})^{1/2} \). If there is no energy exchange between \( e^\pm \) and ions, all shocked particles acquire the thermal Lorentz factor \( \gamma_{\text{inj}} \sim \Gamma_{\text{rel}} \) (assuming “cold” plasma ahead of the shock, \( \gamma_{th} \sim 1 \)). Some energy exchange is, however, expected. Let \( \delta \lesssim 1 \) be the fraction of ion energy that is immediately shared with \( e^\pm \) due to collective processes in the shock. Then the thermal Lorentz factor of shocked \( e^\pm \) is given by

\[
\gamma_{\text{inj}} = \Gamma_{\text{rel}} \left( \gamma_{th} + \delta \gamma_{th} \frac{\mu_e M_p}{Z_{el} m_e} \right),
\]

where \( \mu_e = 1 \) for hydrogen and \( \mu_e = 2 \) for heavier ions. The preheating by the prompt radiation gives \( \gamma_{th} \) comparable to unity (Section 3.1); in Section 8 we will discuss an extension of the model that can give \( \gamma_{th} \gg 1 \).

In the region of extremely strong pair loading, \( Z_{\pm} \gg 10^3 \), the second term on the right-hand side of Equation (24) is small compared with the first term, i.e., ions are energetically unimportant. In this zone, the shock emission is produced by
pairs with $\gamma_e \sim \Gamma_{FS}$ regardless of the value of $\varepsilon_e$; the $e^\pm$ pairs dominate the post-shock energy density and quickly radiate this energy away, leading to nearly 100% radiative efficiency.

The parameter $\varepsilon_e$ can become important where $Z_\pm \ll 10^4$. Numerical simulations of electron-ion shocks without pairs show $\varepsilon_e \sim 0.1$–0.3 (Sironi & Spitkovsky 2011). To our knowledge, there exist no calculations of $\varepsilon_e$ for pair-loaded electron-ion shocks; it is possible that $\varepsilon_e$ depends on $Z_\pm$.

The shock may also accelerate a small fraction of electrons/positrons to Lorentz factors much larger than $\gamma_m$ during the GeV flash; it can also be high at later times (McKee 1976).

The relativistic reverse shock becomes relativistic ($\Gamma$ pairs with $\gamma_e$) during the peak of the GeV flash; it can also be high at later times, i.e., the shock runs significantly faster, leaving more space for the post-shock material. As will be seen below, they are not needed to produce the GeV flash, and are not expected to dominate the flash energy output.

### 3.3. Blast-wave Dynamics

The Lorentz factor $\Gamma$ of the blast wave propagating in the pre-accelerated medium with a given Lorentz factor $\gamma(R)$ is calculated similarly to the standard blast wave propagating in a static medium. We are particularly interested in the early stage, before the reverse shock crosses the main part of the ejecta that carries most of the explosion energy. An estimate for $\Gamma$ at this stage was given in Section 2.3.

In our simulations we use a rather crude model for the blast-wave dynamics. Our approach is similar to the “mechanical” model of Beloborodov & Uhm (2006), where the blast-wave material is described by a single Lorentz factor $\Gamma$, and its evolution with time is derived from energy and momentum conservation. The pre-acceleration of the external medium by radiation reduces the pressure in the blast wave. The blast wave develops where $\gamma < \Gamma_{ej}$, closing the gap between the radiatively pre-accelerated external medium and the ejecta (B02).

The relativistic reverse shock crosses the ejecta on an observed timescale comparable to $t_{GRB}$. At later times the energy supply to the blast wave from the ejecta drops, and the explosion dynamics switches to the self-similar regime; we follow this transition in our simulation. The self-similar blast wave in a wind medium with a low radiative efficiency has $\Gamma \propto R^{-1/2}$, and with a high radiative efficiency $\Gamma \propto R^{-1}$ (Blandford & McKee 1976).

As discussed above, radiative efficiency is close to 100% during the peak of the GeV flash; it can also be high at later phases of the flash (see Section 8 below). The dynamics of radiative blast waves involves subtle effects. The large energy losses of the post-shock plasma imply its quick and significant compression. In this regime, the forward shock has the Lorentz factor $\Gamma_{FS} \approx \Gamma$. There is a thin shell of fluid immediately behind the shock with Lorentz factor $2^{-1/2}\Gamma_{FS}$ (as required by the jump conditions), so the true profile of the fluid Lorentz factor behind the shock is not flat—there must be a steep change from $2^{-1/2}\Gamma$ to $\Gamma$. The corresponding velocity profile is consistent with quick compression of the post-shock plasma—the expected result of strong radiative losses. The characteristic thickness of the compression layer behind the shock is set by the cooling length.

In the radiatively inefficient regime, the blast wave becomes nearly adiabatic and $\Gamma_{FS} \approx 2^{1/2}\Gamma$, i.e., the shock runs significantly faster, leaving more space for the post-shock material. Then the profile of the fluid Lorentz factor behind the shock is smooth and flat.

![Figure 2. Schematic illustration of the transfer problem. Red arrows show the prompt MeV radiation streaming from the ejecta and gradually overtaking the forward shock (FS). The prompt photons can be scattered in the external medium ahead of the shock (zone I) or in the shock-heated plasma (zone II). The coordinate $\sigma$ measures the distance from the leading edge of the radiation front; the unsaturated prompt radiation arrives to the observer at time $t_{obs} = (1+z)\sigma/c$. The scattered photons arrive with a delay.](image)

(A color version of this figure is available in the online journal.)

We model the transition between the radiative and adiabatic regimes in a crude way, switching from $\Gamma_{FS} = \Gamma$ to $\Gamma_{FS} = 2^{1/2}\Gamma$ when radiative efficiency drops below 1/2. Full hydrodynamical simulations will be needed in future accurate models.

### 4. Radiative Transfer

As long as the GeV flash is dominated by IC scattering of the prompt radiation streaming through the blast wave, its light curve can be obtained by solving radiative transfer for the prompt photons. The results will describe the main phase of the flash—its peak and early decay. Observations of GeV flashes by Fermi-LAT are typically limited to this early phase; e.g., in GRB 080916C it lasts until $t_{obs} \sim 400$ s (see below).

Pair loading described in Section 3.1 can also be thought of as a result of radiative transfer of the prompt photons, but scattered in the external medium ahead of the blast wave. One can think of both pair loading and flash emission as two parts of one global transfer problem for the prompt photons (Figure 2). To find an approximate solution to this problem, we divided it into two zones: ahead of the forward shock (zone I) and behind the shock (zone II). Scattering in zone I controls the pair loading of the blast wave (as it generates MeV photons with large angles). The GeV flash is produced by scattering in the shock-heated zone II.

The result of transfer in zone I was described in Section 3.1. The solution depends on the prompt radiation spectrum and should be obtained individually for a given GRB. For a given spectral shape (i.e., given $\alpha_1$, $\alpha_2$, $E_{MeV}$) the obtained $Z_\pm$ and $\gamma$ at the forward shock are functions of the GRB energy ahead of the shock,

$$E_{GRB} = \int_0^{\Gamma_{FS}} L_{GRB}(t) dt,$$  \hspace{1cm} (25)

where $t = (1+z)^{-1}t_{obs}$ and $\Gamma_{FS}$ is defined in Equation (27) below. $E_{GRB}$ determines the value of parameter $\xi$ (see Equation (21)) and thus determines $Z_\pm$ and $\gamma$. Note also that $\gamma$ and $Z_\pm$ enter our calculation of the blast-wave dynamics $\Gamma(R)$ (Section 2.3), thus
the two calculations are coupled and we perform them together, integrating over the history of the blast-wave expansion.

Once we obtain solutions for $\Gamma(R)$, $Z_\perp(R)$, and $\gamma(R)$, we turn to the calculation of photon scattering behind the shock (zone II). The blast wave is optically thin, so only a small fraction of the prompt GRB photons is involved in the radiative transfer. In addition, multiple IC scattering is strongly suppressed by the Klein–Nishina effect at high energies, so one can safely use the single scattering approximation. One must, however, follow the transfer of scattered photons through the radiation field, as many of them have high energies and can easily convert to $e^\pm$ pairs, even though they have small angles $\theta$ to the shock. These pairs are Compton cooled by the prompt radiation, increasing the multiplicity of IC photons.

Monte-Carlo technique is most suitable for this transfer problem. As the shock propagates distance $dR$ it sweeps up $dN_\pm = Z_\perp(R)n_p 4\pi R^2 dR$ electrons/positrons, where $n_p(R)$ is the proton number density of the external medium. The shocked particles are heated to $\gamma_{\text{inj}}$ given by Equation (24). Effectively, $dN_\pm$ hot particles are injected at the shock, and we follow their cooling behind the shock, track the produced IC photons, any secondary products that may result from photon absorption, and cooling of the secondary pairs.

Particles and photons can be followed on the space-time diagram using lab-frame time $t_{\text{lab}}$ and radial position $R$ as coordinates. Note that $R$ is very close to $ct_{\text{lab}}$ everywhere in the relativistic blast wave (whose characteristic thickness $R/\Gamma^2 \ll R$). Therefore, instead of $t_{\text{lab}}$, it is convenient to use the coordinate $\sigma$ defined by

$$\sigma = ct_{\text{lab}} - R. \quad (26)$$

Then $\sigma = 0$ corresponds to the first GRB photons that will be received at $t_{\text{obs}} = 0$, and $\sigma_{\text{GRB}} = (1 + z)cT_{\text{GRB}}$ corresponds to the end of the prompt GRB, $t_{\text{obs}} = T_{\text{GRB}}$ (see Figure 2). As long as a particle has coordinate $\sigma < \sigma_{\text{GRB}}$, it is exposed to the prompt GRB photons and can scatter them. When coordinates $(R, \sigma)$ are used instead of $(R, ct_{\text{lab}})$, one can assert that all particles in the blast wave have the same radial position $R$, as the information about the small differences $\Delta R \sim R/\Gamma^2$ is carried by the coordinate $\sigma$. The blast-wave evolution is fully described by functions of $R, \Gamma, Z_\perp(R), \Gamma(R)$, etc. The growing radius of the expanding blast wave, $R \approx ct_{\text{lab}}$, now plays the role of a lab-time frame instead of coordinate $t_{\text{lab}}$. The coordinate $\sigma$ of the forward shock is given by

$$\sigma_{\text{PS}}(R) = ct_{\text{PS}} = \int_0^R \frac{dR'}{2\Gamma_{\text{PS}}(R')} \quad (27)$$

All shocked particles are advected by the expanding blast wave with Lorentz factor $\Gamma$, and their positions in the prompt radiation front, $\sigma$, evolve according to

$$d\sigma = \frac{dR}{\Gamma^2} \quad (28)$$

Next, consider an IC photon scattered at $R_{\text{sc}}, \sigma_{\text{sc}}$ through an angle $\theta_{\text{sc}}$ (measured in the lab frame). The scattered photon propagates along a straight line and its angle relative to the radial direction decreases,

$$\sin \theta(R) = \frac{R_{\text{sc}}}{R} \sin \theta_{\text{sc}}. \quad (29)$$

The photon coordinate $\sigma(R)$ grows according to

$$d\sigma = (1 - \cos \theta)dR. \quad (30)$$

As the IC photon propagates, we evaluate $\gamma - \gamma$ opacity along the ray (see below) and check for absorption. If the photon escapes, its arrival time is

$$t_{\text{obs}}(R_{\text{sc}}, \sigma_{\text{sc}}, \theta_{\text{sc}}) = (1 + z) \left[ \frac{\sigma_{\text{sc}}}{c} + \frac{R_{\text{sc}}}{c} (1 - \cos \theta_{\text{sc}}) \right]. \quad (31)$$

Every scattered photon is drawn from the prompt GRB radiation, which is assumed to be perfectly collimated at radii of interest, even when viewed from the rest frame of the blast wave. The luminosity $L_{\text{GRB}}(t_{\text{obs}})$ and spectrum of the prompt radiation are known from observations; in the simulations we approximate the prompt spectrum by a broken power law. One can directly calculate the prompt radiation flux at any $R$ and $\sigma$,

$$F(R, \sigma) = \frac{L_{\text{GRB}}(t_{\text{obs}})}{4\pi R^2}, \quad t_{\text{obs}} = (1 + z) \frac{\sigma}{c}. \quad (32)$$

The photon scattering by an electron with a given Lorentz factor $\gamma_e$ is simulated using the exact Klein–Nishina cross section and drawing the target photons from the prompt GRB spectrum. We assume that collective plasma effects maintain the isotropy of the electron distribution. This does not imply that the scattered radiation is isotropic in the fluid frame. The scattering rate for an electron moving with velocity $v$ is proportional to $1 - n \cdot v/c$, where $n$ is the unit vector in the radial direction (the photon direction before scattering). Thus, the electron has a higher probability to scatter a photon when $n \cdot v < 0$. As a result, IC radiation from isotropic relativistic electrons is significantly anisotropic. The scattered photons have a higher probability to carry a negative momentum in the fluid frame, which creates a "rocket effect" that tends to accelerate the blast wave. This effect is neglected in our dynamical model of the explosion (and should be included in future, more detailed models). However, the anisotropy of IC radiation is accurately calculated in our Monte-Carlo simulation as we follow all scattering events individually. The anisotropy impacts the distribution of photon arrival times measured by a distant observer, leading to an additional delay (see also Toma et al. 2009).

The IC photons can escape or get absorbed by another photon. The absorption opacity is discussed in detail in Section 6 below. Our Monte-Carlo simulation includes the opacity provided by the main (unscattered) beam of the prompt radiation,

$$\kappa_{\gamma\gamma}(\epsilon, \theta) \approx \frac{7}{12(1 + \alpha)^{5/3}} \frac{\sigma_z}{m_e c^3} F_e(\epsilon_{\text{th}}). \quad (33)$$

where $\theta$ is the angle of the IC photon, $\epsilon = E/m_e c^2$ is its dimensionless energy, and $\alpha = -d \ln F_e/d \ln \epsilon$ is the spectral slope of target radiation evaluated near the threshold $\epsilon_{\text{th}} = 2\epsilon_{\gamma\gamma}^{-1}(1 - \cos \theta)^{-1}$. As we follow each IC photon, we calculate the absorption opacity along its trajectory and check for absorption. If the photon gets absorbed at some $\sigma_{\text{abs}}$, we inject two new particles (an $e^\pm$ pair) sharing the energy of the absorbed photon. The absorbed photons indirectly contribute to the observed emission as they create secondary $e^\pm$ pairs whose IC emission may escape.

5. GeV FLASH

We have applied our transfer simulation to GRB 080916C, one of the first GRBs detected by LAT. It is an extremely bright burst, with isotropic energy equivalent $\sim 9 \times 10^{52}$ erg (Abdo et al. 2009). The burst duration is $T_{\text{GRB}} \approx 100$ s, which corresponds
to $\approx 20$ s when corrected for cosmological redshift $z \approx 4.35$. Abdo et al. (2009) fitted the prompt emission of GRB 080916C by the Band function in five consecutive time bins. We use the prompt emission described by these fits at $E < 100$ MeV as an input of our transfer simulation.

The main parameter of the problem is the external density. We consider the progenitor wind with mass density

$$\rho(R) = \frac{A}{R^2}, \quad A = \frac{\dot{M}}{4 \pi w}, \quad (34)$$

We find that $A \approx 3 \times 10^{11}$ g cm$^{-1}$ gives a GeV flash consistent with LAT observations, and therefore in all figures we show the explosion model with this $A$. The ejecta is assumed to have a high Lorentz factor $\Gamma_{ej} = 1200$ and carry energy five times that of the prompt GRB radiation, $L_{ej} = 5 L_{GRB}$. The blast wave is not sensitive to the value of $\Gamma_{ej}$ when $\Gamma \ll \Gamma_{ej}$ (Section 2.3).

Note that the blast wave is optically thin in the region of main interest, $R \gtrsim 10^{16}$ cm. Its Thomson optical depth at radius $R$ is given by

$$\tau_{\pm} \approx \frac{Z_{\pm} \sigma_T A}{\mu_e m_p R} \approx 2 \times 10^{-2} \left( \frac{Z_{\pm}}{10^4} \right) A_{11} R_{16}^{-1}. \quad (35)$$

Hereafter we assume $\mu_e = 2$ (a progenitor wind that is made of elements heavier than hydrogen).

5.1. Blast Wave Dynamics, Shock Heating and Cooling

Figures 3 and 4 show the blast-wave dynamics $\Gamma(R)$, pair loading $Z_{\pm}(R)$, and pre-acceleration Lorentz factor $\gamma(R)$. The displayed model assumes $\varepsilon_e = 1$; similar results are obtained for $\varepsilon_e = 0.1$ and 0. One can see the huge effect of the prompt radiation front on the external medium ahead of the blast wave. The medium is dominated by $e^\pm$ pairs at radii $R < 10^{17}$ cm; $Z_{\pm} \approx 10^4$ at $10^{16}$ cm. The prompt radiation accelerates the external medium to a relativistic speed at radii $R < 2 \times 10^{16}$ cm.

The Lorentz factor of the blast wave slowly decreases from 700 at $R = 10^{15}$ cm to 300 at $R \sim 10^{17}$ cm. One can notice jumps in the derivative $d\Gamma/dR$. These jumps are caused by the rough description of the observed prompt radiation taken from Abdo et al. (2009)—the burst was divided into five time bins of constant luminosities $L_{GRB}$. Our simulation assumes $L_{GRB} = 0.2 L_{ej}$ (which corresponds to a constant radiative efficiency, $\varepsilon_{rad} = 1/6$), and hence the ejecta is discretized into five shells with kinetic powers $L_{ej} = 5 L_{GRB}$. The pressure in the reverse shock jumps as it crosses the boundary of each shell, which affects the blast-wave dynamics. The reverse shock reaches the end of the ejecta at $R \sim 10^{17}$ cm and then the blast wave switches to the self-similar deceleration. At a comparable radius, Compton cooling of the forward shock becomes inefficient (as nearly all prompt radiation has overtaken the forward shock and decouples from it), and the blast wave becomes adiabatic. In this model, we neglected synchrotron self-Compton (SSC) cooling of the blast wave, because for GRB 080916C it becomes important only at late times $t_{\text{obs}} > 300$ s, where the LAT data ends.

Figure 5 shows the cooling tracks of the shock-heated particles on the $R_{\gamma_e}$ plane. The particles are cooling fast as long as the forward shock overlaps with the prompt radiation front, in agreement with Equation (11). Our simulation assumes that the prompt GRB ends at $\tau_{\text{prompt}}/c = (1 + z)^{-1}T_{GRB} \approx 19$ s. The last prompt photons overtake the forward shock at radius $R_1 \approx 1.2 \times 10^{17}$ cm, and Compton cooling by the prompt radiation ends.

5.2. Light Curve

Figure 6 shows the light curve of high-energy emission ($E_{\text{obs}} > 100$ MeV) predicted by the transfer simulation, and compares it with the LAT data. The peak of the GeV flash at $t_{\text{obs}} \sim 7$ s is dominated by IC emission near radius $R_p$ indicated in Figures 3 and 4.

The shock wave is a weak producer of GeV emission at radii $R < R_p$ because the shock is weak—it propagates in the medium pre-accelerated by the prompt radiation pressure to a
large Lorentz factor $\gamma$, which reduces the ram pressure in the shock and the thermal Lorentz factor of shocked particles $\gamma_{\text{m}}$ (Equation (24)). The IC emission of the forward shock appears in the GeV band when $\gamma$ decreases to $\sim 10$ and $\gamma_{\text{m}}$ reaches $\sim 50$. This condition determines the radius $R_p$ where the GeV flash peaks. As the shock expands to larger radii $R > R_p$, $\gamma_{\text{m}}$ becomes much greater than 50 and the multiplicity of GeV photons saturates at $M \lesssim 10$ (see Section 2.1). Then the decrease of the pair loading factor $Z_\Delta$ (Figure 4) leads to the decay of the GeV flash. The decay starts quickly at $R > R_p$, at $t_{\text{obs}} \ll T_{\text{GRB}}$, well before the reverse shock crosses the ejecta, i.e., well before the blast wave enters the stage of self-similar deceleration. This resolves the puzzle discussed in Section 1.

The production of GeV photons continues as long as the shock-heated plasma finds targets for IC scattering. Prompt photons serve as targets until $\sigma_{\text{FS}} = \sigma_{\text{GRB}}$, i.e., until the blast wave reaches the radius $R_1$ where the prompt emission completely overtakes the blast wave,

$$R_1 \approx 2\Gamma_{\text{FS}}^2 c T_{\text{GRB}} \frac{1}{1+z}.$$  

Photons scattered at radius $R_1$ arrive with a significant delay after the last prompt photons, depending on the scattering angle $\theta$,

$$t_{\text{obs}}(\theta) = T_{\text{GRB}} + (1+z)(1-\cos \theta) \frac{R_1}{c} \approx T_{\text{GRB}} \left[ 1 + 2\Gamma_{\text{FS}}^2 (1-\cos \theta) \right].$$  

Here $\Gamma_{\text{FS}} \approx \Gamma$ for a radiative forward shock and $\Gamma_{\text{FS}}^2 = 2\Gamma^2$ for a shock with a reduced radiative efficiency. The arrival time given by Equation (37) can be much longer than $T_{\text{GRB}}$. For isotropic scattering, the average scattering angle in the fluid frame $\bar{\theta} = \pi/2$ corresponds to $\cos \theta \approx \beta_{\text{em}}$ and $1-\cos \theta \approx (2\Gamma^2)^{-1}$. This would give $t_{\text{obs}} \approx 3 T_{\text{GRB}}$ if the shock is radiatively inefficient at $R_p$, and $t_{\text{obs}} \approx 2 T_{\text{GRB}}$ if it is efficient. In fact, even when the hot electrons are isotropic in the fluid frame, the scattering is anisotropic—the probability of “backward” scattering ($\theta > \pi/2$) is higher than the probability of “forward” scattering ($\theta < \pi/2$), as the backward-moving relativistic electron scatters the collimated prompt photons with a higher rate.

Thomson scattering would give a simple probability distribution $P(\cos \bar{\theta}) = (1-\cos \bar{\theta})/2$. Klein–Nishina corrections change this distribution, however it remains biased to large $\bar{\theta}$, delaying the average arrival time of scattered photons. As a result, a change in the GeV light curve associated with the end of the target prompt radiation at $R_1$ may be expected at observer time

$$t_1 \sim (3-4) T_{\text{GRB}}.$$  

The scattering regime significantly changes over the course of the flash. The peak at $t_{\text{obs}} \sim T_p$ is emitted in approximately Thomson regime. Indeed, at $R_p$ the shock wave heats the $e^\pm$ pairs to $\gamma_{\text{m}} \sim 50$ while the target radiation density in the fluid frame peaks at $E_p' \sim (2\Gamma^2)^{-1} E_p \sim 2$ keV; one can see that $\gamma_{\text{m}} E_p / m_e c^2 < 1$ and hence the Klein–Nishina corrections to the scattering cross section are moderate. At larger radii (and later observed times) $\gamma_{\text{m}}$ grows by a few orders of magnitude, and the scattering of photons with $E_i \sim E_p$ is suppressed by the Klein–Nishina effects. Then the shock wave is mainly cooled by softer photons of energy

$$E_i \lesssim E_{\text{KN}} \sim \frac{\Gamma}{\gamma_{\text{m}}} m_e c^2,$$

and cooling occurs in a regime that is intermediate between the Thomson and Klein–Nishina limits. In this regime, significant
luminosity is given to IC photons with energies $E_{\text{IC}}$ comparable to the electron energy, and hence the typical $E_{\text{IC}}$ weakly depends on the target radiation spectrum. As a result, the light curve shown in Figure 6 at $t_{\text{obs}} > T_p$ is not very sensitive to the spectrum of radiation that provides targets for IC scattering (we verified this by varying the target radiation in our transfer simulation). The remaining important condition is that the electrons have enough time to radiate their energy, i.e., cooling is faster than the expansion of the blast wave. This condition is satisfied (see Section 2.2 and Figure 5).

The hot electrons see a significant scattering optical depth in the target photons of energies $E_t \sim E_{\text{KN}}$. Note that the same photons are near the threshold for $\gamma - \gamma$ reaction with the IC photons of energy $E_{\text{IC}} \sim \Gamma \gamma_e m_e c^2$. This implies that the IC photons see an interesting optical depth to $\gamma - \gamma$ absorption (the $\gamma - \gamma$ cross section $\sigma_{\gamma\gamma} \gtrsim 0.1 \sigma_{\gamma e}$ is comparable to Compton cross section). In our simulation, we observed significant absorption of IC photons and emission from secondary pairs at $t_{\text{obs}} > T_p$, which has a modest impact on the light curve in Figure 6. It more significantly affects the emission at energies $E \gg 1$ GeV (see below).

5.3. Spectrum

Figure 7 shows the spectrum of high-energy emission predicted by the transfer simulation at $t_{\text{obs}} \sim 2, 8,$ and 70 s. The spectrum is shaped by fast Compton cooling of the shock-heated $e^\pm$, partial absorption of IC photons by photon–photon collisions, $\gamma + \gamma \rightarrow e^+ + e^-$, and IC cooling of the secondary pairs. The spectrum received near the peak of the flash ($t_{\text{obs}} \sim 8$ s) is quite flat in the GeV band, $E L_E \sim$ const, mainly because of the fast evolution of $\gamma_{\text{inj}}$ with radius, which implies a quick growth of the maximum IC photon energy from $\lesssim 1$ GeV to $\gtrsim 100$ GeV. As the blast wave expands by a factor of 2 around $R_p \approx 10^{16}$ cm, $\gamma_{\text{inj}}$ changes by a factor of $\sim 30$ (see Figure 5). Photons scattered in this region have a broad and flat energy distribution in the GeV band, and arrive at comparable times $t_{\text{obs}}$ (which vary with the photon angles).

After the peak, $t_{\text{obs}} > T_p$, a large fraction of the blast-wave power is emitted at energies $E \gtrsim 100$ GeV. Absorption is significant for photons with energies $E \gtrsim 10$ GeV; however, it never strongly suppresses the high-energy emission. This is an interesting feature of radiative transfer through the pair-loaded blast wave. It is related to the fact that the flash peaks when the radiation front has a well defined value of $\xi \sim 300$ (see Section 5.4) and $\xi$ gradually decreases after the peak. The parameter $\xi$ is a measure of the column density of prompt photons, and its preferred value $\xi \sim 300$ corresponds to a preferred value of the optical depth to $\gamma - \gamma$ absorption, $\tau_{\gamma\gamma}$, which turns out to be comparable to unity. The opacity seen by the high-energy IC photons is dominated by the unscattered, beamed prompt radiation with photon index close to $-1$ (energy index $\alpha_1 \approx 0$). The resulting optical depth is roughly constant with photon energy $E$ at $E \gg 10$ GeV, and its dependence on the emission angle $\theta$ is given by

$$\tau_{\gamma\gamma}(\theta) \approx 0.06 x^2 \epsilon_{pk}^{-1} \xi, \quad (40)$$

where $x = \theta \Gamma \sim 1$ and $\epsilon_{pk} = E_{pk}/m_e c^2 \sim 10$ in GRB 080916C. We used Equation (60) derived in Section 6 below and substituted $\alpha = \alpha_1 = 0$ and $\alpha_2 = 1.5$. A significant fraction of the high-energy photons are emitted within the “escape cone” $\theta \lesssim x_{\text{esc}}/\Gamma$ where $\tau_{\gamma\gamma} \lesssim 1$.

Photons that do not escape produce an additional component of “reprocessed” high-energy emission from the secondary pairs. This component creates the flat “knee” in the spectrum at $1-100$ GeV at $t_{\text{obs}} \sim 10^{-10^5}$ s (Figure 7) and leads to the overall two-hump appearance of the high-energy spectrum.

The high-energy spectrum in Figure 7 cuts off at energy $E_{\text{max}}$ which increases with time and reaches the TeV band at $t_{\text{obs}} \sim 1$ minute. The cutoff is the result of our assumption that only thermal heating occurs in the shock wave. The Lorentz factor of thermal particles (given by Equation (24)) reaches $\gamma_{\text{th}} \gtrsim 10^5$ at late stages of the flash when $Z_\pm$ is reduced. The thermal particles produce IC photons of maximum energy $E_{\text{max}} \sim \Gamma \gamma_{\text{th}} m_e c^2 \gtrsim 30$ TeV. Emission above $E_{\text{max}}$ is possible if the post-shock plasma contains a nonthermal component accelerated at the shock; it would not, however, make a large contribution to the flash energy and would not significantly change the GeV emission observed by LAT.

Figure 7 also shows the prompt emission observed by Gamma-ray Burst Monitor (GBM) below 100 MeV. Recent analysis of the GBM and LAT data shows clear evidence for two separate spectral components that dominate below and above 100 MeV (Ackermann et al. 2013; S. Guiriec et al., in preparation). This agrees with the theoretical expectation that the prompt MeV emission comes from a separate (internal) source at small radii. Note that its spectrum may extend to high energies and contribute to the flux detected by LAT, mixing with the IC emission from the external shock wave. However, the external shock is the stronger source in the GeV band, especially at late times when the prompt emission declines.

As seen in Figure 7, the predicted GeV emission from the pair-loaded external shock starts very soft and quickly hardens as the flash reaches its peak. The average spectral slope between
is the time coordinate of the forward shock, which is related to the arrival time of the GeV photons by

\[ t_{\text{obs}} \sim (1 + z) \frac{R}{c} \approx 2 (1 + z) \frac{t_{\text{FS}}}{c}. \]  

(43)

The main parameter \( \xi \) that governs pair loading and pre-acceleration of the external medium (Equation (21)) is

\[ \xi \approx 650 L_{54} t_{\text{FS}} R_{16}^{-2} \approx 570 L_{54} \left( \frac{t_{\text{obs}}}{1 + z} \right)^{-1} \left( \frac{\Gamma}{500} \right)^{-4}. \]  

(44)

where \( L_{54} = L_{\text{GRB}} / 10^{54} \text{ erg s}^{-1} \).

The value of \( \xi \) at the peak of the flash can be estimated using the approximate relation (see B02 and Figure 1),

\[ \gamma \approx \left( \frac{\xi}{\xi_{\text{acc}}} \right)^{3/2}, \quad \xi_{\text{acc}} \approx 100-200, \]  

(45)

valid in the region of main interest, \( 1 < \xi/\xi_{\text{acc}} < 3 \). IC emission from the shocked electrons peaks at \( E_{\text{IC}} \sim 1 \text{ GeV} \) when \( \gamma_{\text{em}} \sim 2(E_{\text{IC}}/E_{\text{ph}})^{1/2} \) (Section 2), which corresponds to

\[ \frac{\Gamma}{\gamma} \sim 50, \]  

(46)

yielding

\[ \xi \approx 2 \xi_{\text{acc}} \left( \frac{\Gamma}{500} \right)^{-1/3}. \]  

(47)

Combining Equations (44) and (47), we obtain the radius and Lorentz factor of the blast wave when it emits the peak of the GeV flash \( t_{\text{obs}} = T_p \),

\[ R_p \approx 10^{16} L_{54}^{6/13} \left( \frac{T_p}{(1 + z)s} \right)^{7/13} \text{ cm}, \]  

(48)

\[ \Gamma(R_p) \approx 500 L_{54}^{3/13} \left( \frac{T_p}{(1 + z)s} \right)^{-3/13}. \]  

(49)

where \( T_p \) is the observed arrival time of the peak.

Using the obtained \( \Gamma \) and Equation (17) one can estimate the parameter \( A = M / 4 \pi w \) of the wind medium,

\[ A \approx \frac{L_{\text{GRB}} \gamma}{8 \pi c^3 \Gamma^4} \left( 1 + \frac{Z \mu_e \mu_p}{\mu_e \mu_p} \right)^{-1} \approx 10^{11} \frac{1 - \epsilon_{\text{rad}}}{\epsilon_{\text{rad}}} L_{54}^{4/13} \left( \frac{T_p}{(1 + z)s} \right)^{9/13} \text{ g cm}^{-1}. \]  

(50)

These estimates assume that the reverse shock is ultrarelativistic \( (\Gamma_{\text{ej}} \gg \Gamma) \); it is straightforward to obtain a more general estimate of \( A \) using Equation (16) instead of Equation (17).

### 6. PHOTON–PHOTON ABSORPTION

Prompt GRB radiation partially absorbs the GeV flash. The target photons providing opacity for the GeV flash can be divided into two categories: (1) the almost perfectly collimated prompt radiation (Section 6.1), and (2) scattered prompt photons (Sections 6.2 and 6.3). The density of scattered radiation is relatively small—the external medium and the blast wave are optically thin even after \( e^\pm \) loading.—however, it may provide an interesting contribution to the \( \gamma - \gamma \) opacity, because the scattered photons have larger angles and higher energies.
6.1. Unscattered Prompt Radiation

Let us first evaluate the $\gamma - \gamma$ opacity provided by the unscattered prompt radiation, which we assume to be perfectly collimated at radii where the GeV flash is produced. The absorption optical depth seen by a high-energy photon of dimensionless energy $\epsilon = E/m_e c^2$ propagating at some angle $\theta$ along its path $s$ is given by

$$\tau_{\gamma \gamma}(\epsilon) = \int \frac{F_s(\epsilon)}{4\pi R^2} \sigma_{\gamma \gamma}(\epsilon_{\text{cm}})(1 - \mu) d\mu ds,$$  

where $\sigma_{\gamma \gamma}$ is the cross section for reaction $\gamma + \gamma \rightarrow e^+ + e^-$ in the center-of-momentum frame of the two colliding photons, $\epsilon_{\text{cm}}$ is the photon energy in this frame, and $\mu = \cos \theta$ describes the angle between the two photons in the lab frame. The spectral flux of the target photons is

$$F_s(\epsilon_i) = \frac{L_s(\epsilon_i)}{4\pi R^2},$$  

where

$$L_s(\epsilon_i) = L_{pk}^s \left( \frac{\epsilon_i}{\epsilon_{pk}} \right)^{-\alpha},$$

is the spectral luminosity of the prompt radiation and $\epsilon_{pk}$ is the peak/break energy of the prompt GRB spectrum. For a broken power-law spectrum with indices $\alpha_1$ and $\alpha_2$, $L_{pk}^s$ is related to the bolometric luminosity $L_{GRB}$ by

$$L_{GRB} = \frac{(\alpha_2 - \alpha_1)}{(1 - \alpha_1)(\alpha_2 - 1)} L_{pk}^s \epsilon_{pk}.$$

Using the relation $2\epsilon_{\text{cm}}^2 = \epsilon_i(1 - \mu)$ to express $\epsilon_i$ in terms of $\epsilon_{\text{cm}}$ and evaluating the integral over $\epsilon_{\text{cm}}$, one finds

$$\tau_{\gamma \gamma} = \psi \sigma_T \int \frac{L_{pk}^s}{4\pi m_e c^3 R^2} \left( \frac{\epsilon_{\text{thr}}}{\epsilon_{pk}} \right)^{-\alpha} (1 - \mu) d\mu ds,$$

where

$$\epsilon_{\text{thr}} = \frac{2}{\epsilon(1 - \mu)},$$

and the numerical factor $\psi(\alpha)$ can be approximated as (Svensson 1987)

$$\psi(\alpha) \approx \frac{7}{12(1 + \alpha)^{5/3}},$$

which is accurate to within 0.3% in the range $0 < \alpha < 6$. The quantity $\psi\sigma_T$ has the meaning of effective cross section for absorption. The spectral slope $\alpha = \alpha_1$ if $\epsilon_{\text{thr}} \ll \epsilon_{pk}$ and $\alpha = \alpha_2$ if $\epsilon_{\text{thr}} \gg \epsilon_{pk}$.

Consider a high-energy photon generated by IC scattering at radius $R_{IC}$ with angle $\theta_{IC}$ relative to the radial direction. As the photon propagates, its angle changes according to Equation (29). This change is related to the path element $d\theta$ by $d\theta = -R d\theta / \sin \theta$, and one can express the integral in Equation (55) as an integral over $0 < \theta < \theta_{IC}$, which yields (in the small-angle approximation $\theta_{IC} \ll 1$)

$$\tau_{\gamma \gamma}(\epsilon, \theta_{IC}) = \frac{\sigma_T L_{pk}^s}{4\pi m_e c^3} \psi(\alpha) \left( \frac{\epsilon_{pk} \epsilon}{\epsilon_{thr}} \right)^{\alpha \theta_{IC}^{2\alpha+2}} R_{IC}.$$

Note that $\tau_{\gamma \gamma} \rightarrow 0$ if $\theta_{IC} \rightarrow 0$. The condition $\tau_{\gamma \gamma} < 1$ defines an escape cone $\theta_{IC} < \theta_{esc}(\epsilon)$ for IC photons of a given energy $\epsilon$. It is useful to rewrite Equation (58) as

$$\tau_{\gamma \gamma} = \frac{\sigma_T L_{pk}^s}{8\pi m_e c^3 R_{IC}} \psi(\alpha) \left( \frac{\epsilon_{\text{thr}}}{\epsilon_{pk}} \right)^{-\alpha},$$

where $\epsilon_{\text{thr}} = 4(\epsilon_{pk}^2 \gamma^{-1})$ is the threshold energy evaluated at the emission radius $R_{IC}$. High-energy photons produced by the plasma moving with a bulk Lorentz factor $\Gamma$ have the characteristic beaming angle $\theta_{BL} \sim \Gamma^{-1}$ (or somewhat larger, because of the anisotropy effect discussed after Equation (32)). It is convenient to describe the photon angle using the variable $x = \Gamma \theta_{IC}$, which is comparable to unity for a typical IC photon. Then the optical depth may be written as

$$\tau_{\gamma \gamma} \approx \xi x^2 \frac{L_{pk}^s}{L_{GRB}} \psi(\alpha) \left( \frac{\epsilon_{\text{thr}}}{\epsilon_{pk}} \right)^{-\alpha}.$$

Here $\xi$ is the main physical parameter of the prompt radiation front given by Equation (21), and we estimated $L_{GRB}$ ahead of the forward shock as $L_{GRB} \approx L_{GRB}/f_{GRB}$ with $f_{GRB} \approx R/2\Gamma^2 c$. The peak of the GeV flash occurs at $\xi \sim 300$ (Section 5.4).

IC photons of energy $\epsilon < \epsilon_1 = 4\Gamma^2\epsilon_{pk}x^2$ interact with prompt photons $\epsilon_1 > \epsilon_{thr} > \epsilon_{pk}$ and $\alpha = \alpha_1$; this gives $\tau_{\gamma \gamma} < 1$. Absorption is significant for IC photons with $\epsilon > \epsilon_1$. These photons can interact with the low-energy part of the prompt spectrum $\epsilon_1 < \epsilon_{pk}$ where $\alpha = \alpha_1$. Note that $\epsilon_1 \approx 0$ (photon index $-1$) is typical for GRBs, including GRB 080916C. Then $\tau_{\gamma \gamma}$ weakly varies with $\epsilon$ for $\epsilon > \epsilon_1$, and its value is close to unity for $\xi \sim 300$.

For GRBs with $\alpha_1 < 0$, $\tau_{\gamma \gamma}$ is maximum at $\epsilon = \epsilon_1$ and decreases at higher energies. For GRBs with $\alpha_1 > 0$, $\tau_{\gamma \gamma}$ continues to grow with $\epsilon > \epsilon_1$ and becomes well above unity. Then the size of the escape cone $\theta_{esc}$ decreases as a power-law with $\epsilon$, and so does the fraction of escaping photons. This implies a steeper spectrum where $\tau_{\gamma \gamma} \gg 1$ (but not an exponential cutoff).

6.2. Prompt Radiation Scattered Ahead of the Forward Shock

High-energy photons from the forward shock have to pass through the prompt radiation that has been scattered ahead of the shock by the pair-loaded and pre-accelerated ambient medium. The specific intensity of the scattered radiation can be expressed as

$$I_{sc}(\epsilon_{sc}, \mu_{sc}, \sigma) = \int_0^\sigma d\sigma' F_s(\epsilon_0) Z_{\mu} n_0 \frac{d\sigma}{2\pi d\mu_{sc} d\epsilon_{sc}}.$$

Here $F_s$ is the spectral flux of prompt radiation, $\epsilon_0$ is the prompt photon energy (before scattering), $\mu_{sc} = \cos \theta_{sc}$ describes the scattering angle, and $\epsilon_{sc}$ is the photon energy after scattering; $Z_{\mu}(\sigma')$ is the pair loading factor, and $n_0$ is the external electron density before $e^\pm$ loading. The integral is taken over the Lagrangian coordinate $\sigma' = ct - R$ that measures the distance inside the prompt radiation front; $d\sigma/(1 - \mu_{sc})$ is the elementary path length along the scattered photon trajectory in the lab frame.

We are interested in the optical depth $\tau_{\gamma \gamma}$ created by the scattered radiation, as seen by a high-energy photon of energy

\[\text{\footnotesize{\textsuperscript{7} The factor } d\sigma_0/d\epsilon_{sc} \text{ is missing in Equation (4) in B02.}}\]
\[ \epsilon \text{ emitted by the shock wave. The photon has an angle } \theta \sim \Gamma^{-1}, \]
which is much smaller than the typical angles of the target photons \( \theta_{\text{ac}} \sim \gamma^{-1} \) (where \( \gamma = (1 - \beta^2)^{-1/2} \) is the Lorentz factor of the pair-loaded medium accelerated by the radiation front). Therefore, here the high-energy IC photon may be approximated as perfectly collimated in the radial direction, \( \theta = 0 \). Then,

\[ \tau_{\gamma\gamma}(\epsilon) = 2 \pi R \int \int \frac{L_{\gamma}^e(\epsilon_{\text{ac}}, \mu_{\text{ac}})}{\epsilon_{\text{ac}} \mu_{\text{ac}} c^3} \sigma_{\gamma\gamma}(\epsilon_{\text{cm}})(1 - \mu_{\text{ac}}) d\mu_{\text{ac}} d\epsilon_{\text{ac}}. \]  

(62)"
by the prompt radiation; this would weaken the GeV flash. (2) Synchrotron losses give emission in softer bands, e.g., optical or X-rays, providing an additional test for the pair-dominated flash mechanism. (3) Synchrotron photons may become the main targets for IC scattering by the high-energy electrons in the blast wave, which can affect the observed light curve and spectrum of high-energy emission.

7.1. Cooling Rate and the Characteristic Photon Energy

The competition between synchrotron cooling and Compton cooling by the prompt radiation was discussed by Beloborodov (2005b). The two contributions to the cooling rate of isotropic electrons with a thermal Lorentz factor $\gamma_e \gg 1$ are given by

$$\dot{E}_{\text{syn}} = -\frac{4}{3} \sigma_T U'_f c \gamma_e^2, \quad \dot{E}_{\text{IC}} \approx -\frac{4}{3} \sigma_T U'_f c \gamma_e^2,$$  

where $U'_f$ is the magnetic energy density, and $U'_f$ is the energy density in the prompt photons of energy $E < E_{\text{KN}}$ (Equation (39)) which can be scattered with approximately Thomson cross section; $U'_f$ and $U_f$ are measured in the fluid frame. We include only the unscattered prompt radiation in $U'_f$, assuming that it dominates Compton cooling of the blast wave; the density of synchrotron radiation from the blast wave itself is assumed to be relatively small. Then,

$$U'_f \approx f_T U', \quad f_T \approx \left\{ \begin{array}{ll} 1 & E_{\text{KN}} \gg E_p, \\ \frac{E_{\text{KN}}}{E_p} & E_{\text{KN}} < E_p, \end{array} \right.$$  

where

$$U' = \frac{L_{\text{GRB}}}{16 \pi c R^2 T^2}.$$  

is the energy density of the prompt radiation in the fluid frame, and $\sigma_T$ is the spectral index of radiation at photon energies $E < E_p$.

The magnetic energy density behind the shock may be expressed in the standard form using the parameter $\varepsilon_B$,

$$U'_B = 3 \varepsilon_B P_f = \varepsilon_B \frac{4 \rho c^2 T^2}{\gamma (1 + \beta)},$$  

where $\rho$ is the mass density of the external medium and $\gamma = (1 - \beta^2)^{-1/2}$ is its pre-acceleration Lorentz factor; we neglected the increase in $\rho$ due to $e^\pm$ pairs loaded ahead of the shock. We focus here on the main phase of the GeV flash before the reverse shock has crossed the ejecta. Then Equation (17) may be used to obtain another expression for $U'_B$,

$$U'_B \approx \frac{\varepsilon_B L_{\text{ej}}}{4 \pi c R^2 T^2}.$$  

The ratio of synchrotron and Compton cooling rates is then given by

$$\frac{\dot{E}_{\text{syn}}}{\dot{E}_{\text{IC}}} \approx \frac{U'_B}{U'_T} \approx \frac{4 \varepsilon_B L_{\text{ej}}}{f_T L_{\text{GRB}}},$$  

The numerical factor $f_T$ is comparable to unity at the peak of the GeV flash, when the forward shock heats the plasma to $\gamma_{\text{max}} \sim 10^2$. After the peak, $\gamma_{\text{max}}$ increases, however the flash light curve shown in Figure 6 is still dominated by particles cooled to $\gamma_c \sim 10^7$, with $f_T \sim 1$.

The characteristic energy of synchrotron photons is given by

$$E_s \approx 0.2 \Gamma \gamma_e^2 \frac{eB'}{m_p c},$$  

where $B' = (8 \pi U'_B)^{1/2}$ is the magnetic field measured in the fluid frame. Using Equation (73) one obtains

$$E_s \sim 20^{-3/2} \left( \frac{\gamma_e}{100} \right)^2 \left( \frac{L_{\text{ej}}}{10^{54} \text{ erg s}^{-1}} \right)^{1/2} R_{16}^{-3} \text{ eV}.$$  

Most of the synchrotron power is emitted by particles with $\gamma_e \sim \gamma_{\text{max}}$. As the blast wave expands from $R \sim 10^{15}$ cm to $10^{17}$ cm, $\gamma_{\text{max}}(R)$ evolves from low values $\sim 1$ to $\sim 10^2$ (at the peak of the GeV flash) to $\sim 10^{-2}$, see Equation (24) and Figure 5. As a result, $E_s(\gamma_{\text{max}})$ evolves by a huge factor $\sim 10^9$, and hence the blast wave must produce broad-band synchrotron radiation. The emitted synchrotron power may be estimated using Equation (74) with $f_T$ that corresponds to $\gamma_{\text{max}}$. Moderately high $\varepsilon_B \gtrsim 10^{-5}$ would imply strong synchrotron emission in the hard X-ray band. It can easily conflict with the observed radiation spectrum, which can be used to infer an upper limit $\varepsilon_B \lesssim 10^{-5}$ for GRB 080916C.

7.2. Optical Flash

If one is interested in radiation in a fixed spectral band, e.g., optical $E \sim 2 (1 + z)$ eV, the observed emission will be dominated by particles that have cooled behind the shock to Lorentz factor $\gamma_e = \gamma_{\text{opt}}$ such that $E_s(\gamma_{\text{opt}}) \sim 2 (1 + z)$ eV. From Equation (76) one finds

$$\gamma_{\text{opt}} \sim 10^3 \left( \frac{\varepsilon_B}{10^{-6}} \right)^{-1/4} \left( \frac{L_{\text{ej}}}{10^{54} \text{ erg s}^{-1}} \right)^{1/4} R_{16}^{1/2} (1 + z)^{1/2}.$$  

A more accurate expression for $\gamma_{\text{opt}}$ may be obtained from Equation (75) using Equation (72),

$$\gamma_{\text{opt}} \approx \frac{10^4}{\Gamma} \left( \frac{\varepsilon_B \rho c^2}{1} \right)^{1/4} (1 + z)^{1/2}.$$  

In the blast wave with pure thermal heating, optical emission remains negligible until $\gamma_{\text{opt}}(R)$ exceeds $\gamma_{\text{opt}}$, the optical light curve is expected to reach its peak at this point. This happens soon after the peak of the GeV flash.

The subsequent decay of the optical flash can be described using the following estimate for the optical luminosity,

$$L_{\text{opt}} \sim E L \sim \frac{dN_\pm}{dt} \Gamma \frac{\gamma_{\text{opt}} m_e c^2}{2} f_{\text{syn}},$$  

where $t = (1 + z)^{-1} t_{\text{obs}}$, $N_\pm$ is the number of electrons/positrons cooling behind the shock, and

$$f_{\text{syn}} = \frac{\dot{E}_{\text{syn}}(\gamma_{\text{opt}})}{\dot{E}_{\text{IC}}(\gamma_{\text{opt}}) + \dot{E}_{\text{syn}}(\gamma_{\text{opt}})} \approx \frac{U'_B}{U'_T(\gamma_{\text{opt}})}.$$  

Equation (79) states that each particle emits in the optical band a fraction $f_{\text{syn}}/2$ of its energy in the lab frame, \( \Gamma \gamma_{\text{opt}} m_e c^2 \), as $\gamma_e$ decreases from $\gamma_{\text{opt}}$ to $\gamma_{\text{opt}}/2$. The emitted energy $\sim \Gamma \gamma_{\text{opt}} m_e c^2/2$ is shared by IC and synchrotron photons; in our case the IC losses dominate and the synchrotron fraction $f_{\text{syn}} \ll 1$ is given by Equation (74). Then we obtain

$$L_{\text{opt}} \sim 10^{49} R_{16} Z_\pm \left[ \gamma(1 + \beta) \varepsilon_B A_{12} \right]^{1/2} \times \frac{L_{\text{ej}}}{L_{\text{GRB}}} \varepsilon_B (1 + z) \text{ erg s}^{-1}.$$  


Here we used $dN_{\nu}/dt \sim Z_{\pm}(4\pi R^4 / \mu m_{\nu} t)$ and $t \sim R/cT^2$. Equation (81) shows that the decay of the optical flash is controlled by the evolution of the factor $Z_{\pm}R[\gamma(1 + \beta)]^{1/2}$ with time $t$. This evolution is fast; when approximated by a power law $t^{-\alpha}$ its slope is $\alpha = -2$. One can also see from Equation (81) that the optical flash can be extremely bright even for a modest $\epsilon_\nu \sim 10^{-6}$. Its peak occurs where $Z_{\pm} \sim 10^2$.

In summary, the peak luminosity of the optical flash is achieved when $\gamma_{\rm inj}$ exceeds $\gamma_{\rm op}$. This typically happens at $\tau_{\rm obs} \sim 10(1 + \zeta)$ s. The optical flash can be extremely bright, but it quickly decays. We find that its luminosity drops by a factor of $10^{-2}$ as $\tau_{\rm obs}$ grows by a factor of 10, mainly because of the decreasing pair loading factor $Z_{\pm}$. At later times $Z_{\pm}$ drops to unity, which implies the end of the fast decay; then the optical flash should evolve to normal optical afterglow.

Note that the $e^\pm$ pairs collected at $R \lesssim 10^{17}$ cm are Compton cooled to a low temperature and do not contribute to the afterglow emission at late times. This is in contrast to explosion models where the prompt radiation quickly decouples from the blast wave and Compton cooling is inefficient; in this case the blast wave would carry slowly cooling pairs and the synchrotron afterglow would have a long “memory” of pair loading (Beloborodov 2005a).

8. IMPACT OF THE GeV FLASH ON THE EXTERNAL MEDIUM

Our transfer simulations described in Section 5 show that some of the produced high-energy photons do not escape—they are absorbed by the prompt radiation beam and convert to $e^\pm$ pairs. Most of the conversion events occur behind the forward shock and join the shocked plasma moving with Lorentz factor $\Gamma$, however a small fraction convert ahead of the shock and join the external medium, which moves with a much smaller Lorentz factor $\gamma$. These rare events create particles of very high energies ($\text{GeV}-\text{TeV}$) in the external medium, depositing their energy and momentum. Thus, the GeV flash itself creates additional pre-heating and pre-acceleration of the external medium, which was not taken into account in our model of the radiation front in Section 3.1. We now estimate this effect and its implications.

8.1. Fraction of the Flash Power Deposited Ahead of the Shock

First, let us roughly estimate the fraction of the flash power that converts to $e^\pm$ pairs ahead of the shock wave. Only photons with sufficiently small angles can overtake the forward shock,

$$\theta < \theta_{\max} = \Gamma^{-1}_{\text{FS}}.$$  

(82)

For the simplest estimate, we picture the flash source as an infinitesimally thin shell behind the shock (the fast-cooling limit) and assume that only photons emitted with $\theta < \theta_{\max}$ have a chance to convert ahead of the shock. The absorption optical depth $\tau_{\gamma\gamma}$ seen by these photons is given by Equation (58); it increases with $\theta$ and is maximum at $\theta_{\max}$. The deposited power ahead of the shock may be written as

$$L_{\pm} = \zeta \int \tau_{\gamma\gamma}(\epsilon, \theta_{\max}) L_{\epsilon} d\epsilon,$$  

(83)

where

$$\tau_{\gamma\gamma}(\epsilon, \theta_{\max}) \approx \frac{\psi(\alpha)}{2^{2\alpha+1}(2\alpha + 3)} \frac{\sigma_T L_{\epsilon}^{pk}(\epsilon_{pk})^{\alpha}}{4\pi m_e c^3 R^{2\alpha+2}}.$$  

$L_{\epsilon}$ is the flash spectrum, and $\zeta = 0.01-0.1$ is a numerical factor determined by the angular distribution and spectrum of the flash radiation. The spectral slope of the target radiation, $\alpha$, is determined as follows. The main target photons contributing to $\tau_{\gamma\gamma}$ have energies

$$\epsilon_\gamma \approx 2\epsilon_{\gamma\gamma} = \frac{4}{\epsilon(1 - \cos \theta)} \approx \frac{8 G^2 M}{\epsilon},$$  

(84)

which should be compared with $\epsilon_{pk} \sim 10$. This gives

$$\alpha = \begin{cases} \alpha_2, & \epsilon < 8 \epsilon_{pk}^{-1} \Gamma^2_{\text{FS}} \\ \alpha_1, & \epsilon > 8 \epsilon_{pk}^{-1} \Gamma^2_{\text{FS}}, \end{cases}$$  

(85)

with the characteristic $\epsilon_1 = 8 \Gamma^2_{\text{FS}} / \epsilon_{pk}$ corresponds to photon energy $\epsilon_{pk} m_e c^2 \sim 10^2$ GeV. The flash spectrum extends above $\epsilon_1$ after the peak time $T_p$, when $\gamma_{\rm inj}$ exceeds $\Gamma_{\text{FS}}$; then photons with $\epsilon > \epsilon_1$ make the main contribution to the integral in Equation (83), and $\tau_{\gamma\gamma}$ should be evaluated with $\alpha = \alpha_1$. In particular, for $\alpha_1 = 0$ we obtain

$$\frac{L_{\pm}}{L_{\text{flash}}} \sim 0.1 \frac{\sigma_T L_{\epsilon}^{pk}}{4\pi m_e c^3 R^2 \Gamma_{\text{FS}}^2},$$  

(86)

where we assumed that a large fraction of the flash luminosity $L_{\text{flash}}$ is emitted above $10^2$ GeV; this assumption is satisfied in the self-consistent model, as we show below.

8.2. Pre-heating and Pre-acceleration

The injection of power $L_{\pm}$ into the external medium can be described as inelastic collision which heats and accelerates the medium. Consider an external mass shell

$$dm = 4\pi R^2 \rho dR = 4\pi A dR.$$  

(87)

It first interacts with the prompt radiation and then it is exposed to the high-energy flash photons, which deposit energy,

$$dE_{\pm} \sim L_{\pm} \frac{dR}{2\Gamma_{\text{FS}}^2 c^2}.$$  

(88)

This energy is deposited in the form of ultra-relativistic $e^\pm$ pairs, which are expected to immediately share their momentum $dE_{\pm}/c$ with the medium through collective processes (B02). The GeV flash accelerates the medium to a high Lorentz factor $\gamma' \gg 1$ if

$$G \equiv \frac{dE_{\pm}}{dm c^2} = \frac{L_{\pm}}{8\pi c^3 \Gamma_{\text{FS}}^2 A} \gg 1.$$  

(89)

The deposited energy $dE_{\pm}$ is shared between the bulk kinetic energy of the accelerated medium and its internal energy (i.e., heat). The ultra-relativistic pairs can scatter the prompt radiation ahead of the forward shock; however, since the pairs are isotropic in the fluid frame, the produced high-energy photons have large angles and quickly convert to $e^\pm$ pairs, which join the medium.8

For simplicity, let us consider radii where the pre-acceleration by the prompt radiation is not significant ($R > 2 \times 10^{16}$ cm, see Figure 3), so that we can isolate the effect of the GeV flash. We can evaluate the Lorentz factor gained by the shell, $\gamma'$, and

8 This cascade in the external medium has a moderate effect on pair multiplicity $Z_{\pm}$. The high-energy particles injected by the flash radiation are relatively close to the forward shock and have time for a moderate number of scatterings before they are swept by the shock. A dedicated numerical simulation will be needed to quantify this effect.
its new rest-mass \( dm' \) (which includes the deposited heat) from the energy and momentum conservation laws,
\[
dm + \frac{dE_{\pm}}{c^2} = \gamma' dm',
\]
(90)
\[
\frac{dE_{\pm}}{c^2} = \gamma' \beta' dm'.
\]
(91)
This gives
\[
\gamma_{\text{heat}} \equiv \frac{dm'}{dm} = (2G + 1)^{1/2},
\]
(92)
\[
\gamma' \beta' = \frac{G}{\gamma_{\text{heat}}}. \tag{93}
\]
Also note the relation,
\[
\gamma_{\text{heat}} = \gamma'(1 + \beta'). \tag{94}
\]

One can show that \( G \gg 1 \) is expected, which implies a strong impact of the flash on the external medium, \( \gamma_{\text{heat}} \gg 1 \) and \( \gamma' \gg 1 \). Indeed, substituting Equation (86) into Equation (89) and using the simple estimate for the blast-wave Lorentz factor \( \Gamma^4 \sim L_{\text{ej}}/16\pi c^3 A \) (see Equations (17) and (96) below), one obtains
\[
G \sim \frac{0.1\xi \sigma_T L_{\text{ej}}^{pk}}{2\pi m_e c^3 R} \frac{\Gamma_{\text{FS}}^4}{\Gamma_{\text{ej}}^4} L_{\text{flash}}^4 \sim 0.4\xi \frac{\Gamma_{\text{FS}}^4}{\Gamma_{\text{ej}}^4} \frac{L_{\text{ej}}^{pk}}{L_{\text{GRB}} L_{\text{flash}}} \tag{95}
\]
which gives a typical \( G \sim 10^2 - 10^3 \). The value of \( G \) is strongly reduced at smaller radii where the prompt radiation pre-accelerates the external medium to \( \gamma > 1 \). The effect of \( G \gg 1 \) should develop soon after the peak of the GeV flash, when \( \gamma_{\text{ej}} > 10^2 \) and \( \gamma < 10 \).

8.3. Effect on the Blast Wave Lorentz Factor

We now estimate the effect of pre-acceleration and pre-heating by the flash radiation on the blast-wave Lorentz factor \( \Gamma \). Similar to Section 2.3 we consider sufficiently early times \( t_{\text{obs}} < t_{\text{GRB}} \) and use the pressure balance between the forward and reverse shock, \( P_f \sim P_r \), for a rough estimate. On the other hand, to isolate the effect of the flash, we consider late enough times when the prompt radiation does not significantly pre-accelerate the medium, \( \gamma \approx 1 \). Then Equation (17), with \( \gamma \) replaced by \( \gamma' \) and \( Z_{\perp} \ll \mu_e m_p/m_e \), gives
\[
\Gamma^4 \approx \frac{L_{\text{ej}}}{16\pi c^3 A}. \tag{96}
\]
The result is the same as if there were no effect of the flash on the external medium—the terms \( \gamma'(1 + \beta') \) and \( \gamma_{\text{heat}} \) cancel (see Equation (94)). The enhancement of the shock pressure due to the increased fluid mass by the factor of \( \gamma_{\text{heat}} \) is compensated by the reduction of pressure due to the fluid pre-acceleration to \( \gamma' \).

We conclude that the blast-wave dynamics should not be strongly changed by the flash impact on the external medium. More detailed calculations will, however, be needed at smaller radii where the effect of the flash radiation on the external medium interferes with that of the prompt radiation, increasing the pre-acceleration Lorentz factor from \( \gamma > 1 \) to a new \( \gamma' \).

8.4. Effect on Radiative Efficiency

The deposited heat implies a huge energy per electron ahead of the shock, \( \gamma_{\text{heat}} m_e c^2 \). In the region of interest, where \( G \gg 1 \) and \( \mu_e m_p/m_e \gg Z_{\perp} \gg 1 \), one finds
\[
\gamma_{\text{heat}} \approx \gamma_{\text{heat}} \frac{\mu_e m_p}{Z_{\perp} m_e} \gg 1. \tag{97}
\]
When the hot fluid passes through the shock, the thermal Lorentz factor of particles increases to \( \gamma_{\text{heat}} \) given by Equation (24). Using Equation (94), one obtains
\[
\gamma_{\text{ej}} \approx \Gamma \frac{\mu_e m_p}{Z_{\perp} m_e}. \tag{98}
\]
This relation shows that all the energy available for dissipation in the blast wave \( (Z_{\perp} \gamma_{\text{heat}} m_e c^2 / \mu_e m_p \approx \Gamma c^2 \) per unit external mass) has been converted into the heat of pairs behind the shock. It implies the effective \( \varepsilon_e = 1 \), regardless of the efficiency of energy transfer from the ions to pairs at the shock front. The high-energy particles behind the shock radiate most of their energy and produce radiation beamed within angle \( \theta \sim \Gamma^{-1} \).

Our transfer simulations in Section 5 and analysis in Section 6 show that a large fraction of this radiation avoids \( \gamma - \gamma \) absorption and escapes, leading to a high radiative efficiency of the blast wave.

9. DISCUSSION

9.1. Mechanism of the GeV Flash

The external shock of the GRB explosion in a dense progenitor wind generates a bright GeV flash due to IC cooling of the shock-heated plasma. We showed that scattering of the prompt MeV radiation streaming through the external blast wave is the key mechanism during the main phase of the flash, shaping its peak and early decay. Most MeV photons stream without any interaction, however a small fraction get scattered, and many of the scattered photons (in particular those scattered in the external medium ahead of the forward shock) collide with other MeV photons and convert to \( e^\pm \) pairs. This leads to a dramatic enhancement of electron density in the blast wave, by a factor of \( Z_{\perp} \sim 10^2 \) at radii \( R \sim 10^{16} \) cm, and hence a dramatic increase in the number of prompt photons scattered in the blast wave. In addition, the GRB radiation pressure significantly pre-accelerates the external medium ahead of the forward shock. This effect reduces the strength of the shock and regulates the spectrum of its IC radiation.

We have examined the IC pair-dominated flash using a direct radiative transfer simulation. As an example, we calculated the flash expected from GRB 080916C, one of the few brightest GRBs well observed by LAT. When the reverse shock is relativistic, the dynamics and emission of the forward shock are indifferent to the precise Lorentz factor of the ejecta \( \Gamma_{\text{ej}} \); only the ejecta power \( L_{\text{ej}} \) is important. \( L_{\text{ej}} \) can be estimated from the observed GRB luminosity assuming a plausible radiative efficiency of the prompt emission \( \varepsilon_{\text{rad}} < 1 \). The main remaining parameter of the blast wave is the density of the external medium which depends on the progenitor mass-loss rate \( M \). We find that \( M \approx 10^{-5} M_\odot \) yr\(^{-1} \), which is typical for Wolf–Rayet stars, gives a GeV flash in striking agreement with observations (Figure 6).

Our results explain the previously puzzling features of the GeV light curve including the early peak and the long decay.
The light curve is shaped by the pre-acceleration and pair-loading effects; the peak is reached where $\gamma \sim 10$ and $Z_\perp \sim 10^4$, when most of the shock energy is emitted in IC photons of energy $E_{IC} \sim (\Gamma/\gamma)^2$ MeV, in the GeV band. The predicted spectrum in the GeV band has the photon index $\sim -2$ (Figure 7), which is consistent with observations (Ackermann et al. 2013). At the high-energy end, $E \gg 10$ GeV, the spectrum is affected by $\gamma - \gamma$ absorption. However, absorption does not strongly suppress the emission even at very high energies $E > 100$ GeV. Our analysis in Section 6 shows that the main source of $\gamma - \gamma$ opacity seen by the GeV photons is the unscattered prompt radiation; the corresponding optical depth $\tau_{\gamma \gamma}$ is given by Equation (58), which is self-regulated to a moderate value comparable to unity. As a result, we predict escaping gamma-rays at energies $E \gg 10$ GeV, up to the TeV range, where the flash can be detected by the atmospheric Cherenkov telescopes.

When comparing the model with the LAT data we assumed that all observed GeV emission comes from the blast wave. In fact, at early times, the high-energy tail of the prompt emission may contribute to the observed GeV light curve near the peak of the flash. Variability detected at early times provides evidence for such a contribution. After subtraction of the prompt emission, the true light curve of the GeV flash may have a somewhat lower peak, perhaps by a factor $\sim 2$. Then our best-fit model will need to be revised, resulting in moderate changes in $A$, $R_\gamma$, and $\Gamma$.

Given the similar light curves of the GeV flashes in many GRBs, it appears likely that all of them are produced by the same mechanism. This includes GRB 090510 that was attributed to the short GRB class, which is usually associated with a different type of progenitors. It could be that GRB 090510 is an “impostor” and its progenitor had a significant wind before the explosion. A wind medium was also suggested by Panaitescu (2011) based on the afterglow properties of GRB 090510. Our preliminary analysis of the GeV flash in GRB 090510 confirms the requirement of a high external density at $R \sim 10^{16}$ cm, suggesting a wind medium. However, the formal constraints on the density profile in this case are not tight and will be investigated in a future work. In contrast, the IC flash in GRB 080916C requires the density profile to be close to $R^{-2}$; a uniform medium would give a GeV light curve much flatter than observed.

9.2. Approximations Used and Possible Extensions

From a technical point of view, this paper examined the coupled problem of radiative transfer and blast-wave dynamics in a wind medium. The problem can be solved exactly from first principles, although in this paper we used some approximations. Below we summarize our approximations, discuss the accuracy of our results, and outline directions for future work.

1. We conservatively assumed that the post-shock plasma is dominated by the thermal $e^\pm$ population. This assumption is broadly consistent with observations of collisionless shocks in the solar system and supernovae, as well as numerical simulations of relativistic shocks (e.g., Sironi & Spitkovsky 2009). Our calculations made no additional assumptions concerning particle acceleration in the shock wave. The likely presence of a small number of nonthermal particles would weakly change the predicted light curve shown in Figure 6 (as discussed in Section 5) except possibly at the earliest stages, before the peak of the flash. We used the simplest possible approximation where the shocked particles acquire the mono-energetic distribution $\delta(\Gamma_e - \gamma_{inj})$ with $\gamma_{inj}$ given by Equation (24). Detailed future models can use a more realistic distribution, e.g., Maxwellian, and include nonthermal particles.

2. Our calculations had to invoke one phenomenological parameter $\epsilon_e$. The shock wave heats ions and electrons/positrons, and $\epsilon_e$ is the fraction of the ion energy that is immediately (due to collective plasma effects) passed to $e^\pm$. This parameter is not relevant at the peak of the flash, however its value can affect the decay after the peak (see Figure 6). Future particle-in-cell simulations of pair-loaded shocks may provide an estimate for $\epsilon_e$. In Section 8, we showed that the blast wave after the peak of the GeV flash enters a peculiar radiative regime which can be described as emission with effective $\epsilon_e = 1$. For comparison, Figure 6 also presents the GeV flashes obtained with $\epsilon_e = 0$ and 0.1; it shows that variations in $\epsilon_e$ would have a modest effect on the light curve. Comparison with the LAT data in Figure 6 gives no preference to any $\epsilon_e$ at times $t_{obs} < 40$ s. At later times, the data favors $\epsilon_e > 0.1$. The value of $\epsilon_e$ makes a significant difference for the flash spectrum at very high energies $E \gg 1$ GeV (see Figure 8).

3. The numerical models presented in this paper focused on the main phase of the GeV flash and did not include possible IC emission at radii $R > R_1$, where $R_1$ is given by Equation (36). In reality, some target photons are available for the blast wave even at $R > R_1$ (they are provided by a weaker/softer tail of the prompt radiation and by the synchrotron emission from the blast wave). The high-energy emission will continue as long as the target radiation field is able to drain an interesting fraction of the shock energy via Compton cooling. Thus, the observed light curve of the GeV flash can extend to much longer observational times than shown in Figure 6. As the radiation density decreases behind the prompt radiation front, the transition from fast to slow cooling regime will affect the GeV light curve.

4. We used a simplified “mechanical” model for the blast-wave dynamics, which treats the shocked gas as one body. It is equivalent to assuming a flat profile of the fluid Lorentz factor behind the forward shock. Future detailed models of GeV flashes will be based on full hydrodynamical simulations. We found that the light curve of the GeV flash near its maximum is quite sensitive to small refinements in $\Gamma(R)$, even when these refinements are at $\sim 10\%$ level. Thus, careful hydrodynamical simulations will help improve the accuracy of the explosion reconstruction from the observed GeV emission.

5. We calculated in detail how the scattering of GRB radiation and pair creation in the external medium impacts the forward shock. However, we did not study the dynamical effect of pair creation behind the shock. Many of the photons scattered in the external medium propagate into the blast wave and the unshocked ejecta, and create pairs there with a rate similar to that ahead of the blast wave. As these pairs are picked up by the relativistic flow, they exert a significant drag and heat it. Our preliminary estimates suggest that this effect is important for the blast-wave dynamics at early times, and will reduce the Lorentz factor $\Gamma$ at small radii $R \approx 10^{15}$–$10^{16}$ cm. It can strongly affect the rise of the GeV light curve. We defer the full calculation to a future work; it will also include the “rocket effect” due
to anisotropy of IC emission, which will give a push to the blast wave. All these effects will likely change the rise to the peak and possibly the peak itself. Therefore, we only trust our best-fit value of the wind density parameter \( A \) within a factor of \( \sim 2 \).

6. The full non-linear calculation of radiative transfer is challenging and was not completely done in this paper. In particular, we saw in our simulations that some rare IC photons (with highest energies and smallest angles) convert to \( e^\pm \) ahead of the blast wave and deposit huge energy and momentum. Thus, the full non-linear problem must include the impact of the GeV flash on the external medium, not only the impact of the prompt radiation. Our analysis of this effect in Section 8 suggests that it does not significantly change the ram pressure in the forward shock. However, it has another important implication: it leads to the effective \( \varepsilon_d \equiv 1 \) and enforces the high radiative efficiency of the blast wave. Detailed nonlinear simulations of this effect are deferred to a future work.

Such simulations will also allow one to explore the following possibility. The high-energy pairs created in the external medium by the IC flash photons may not be completely cooled before the shock reaches them and boosts their energy even more, producing extremely energetic particles. These particles in turn produce more energetic photons, some of which can again convert ahead of the shock, injecting new very high energy pairs. Thus, the following cycle is possible for a small number of particles/photons: shock-heating \( \rightarrow \) emission of high-energy photons \( \rightarrow \) photon conversion to \( e^\pm \) ahead of the shock \( \rightarrow \) shock heating. As a result, ultra-high-energy particles could be generated. This bootstrap mechanism is similar to “photon breeding” proposed by Stern & Poutanen (2006).

9.3. Future Observational Tests

The predicted peak time of the GeV flash, \( T_p \), depends on the density parameter \( A \) (Section 5.4). Although many bursts detected by LAT have \( T_p \ll T_{\text{GRB}} \), some may have \( T_p \sim T_{\text{GRB}} \). It will be useful to study such bursts for the following reason. Our calculations predict that the flash peaks in the GeV band, and its emission below 100 MeV is weak and has a hard spectral slope (Figure 7). This weak emission can only be seen when the bright prompt emission turns off. A flash with \( T_p \sim T_{\text{GRB}} \) would still be near its peak at \( t_{\text{obs}} > T_{\text{GRB}} \), and the measurement of its spectrum could be extended below 100 MeV to test our prediction in this energy band.

Future analysis of the entire sample of LAT bursts will allow one to estimate the wind density for a number of GRBs. Our preliminary analysis of the published LAT catalogue of 35 bursts (Ackermann et al. 2013) suggests that the density parameter \( A \approx 10^{11} \text{--} 10^{12} \text{ g cm}^{-1} \) is typical for GRBs with detected GeV flashes.

The total energy of the GeV flash is roughly proportional to the product of its peak luminosity \( L_p \) and its peak time \( T_p \), which scales with \( A \). We conclude that the flash is likely to be detected in GRBs that are bright and exploding in dense stellar winds. This may explain why only \( \sim 10\% \) of GRBs are found to produce strong emission in the GeV band. Note also that a relatively low wind density is suggested by the analysis of optical afterglows in a sample of bursts, none of which was detected by LAT (Hascoët et al. 2014).

9.4. Optical Flash

We argued in Section 7.2 that the magnetic field in the blast wave may be measured through observations of the low-energy (synchrotron) counterpart of the GeV flash, in particular in the optical band. A small magnetization parameter \( \varepsilon_B \) would not affect the GeV flash and still give bright optical emission which scales as \( \varepsilon_B^{1/2} \).

For instance, for GRB 080916C \( \varepsilon_B \approx 10^{-6} \) gives an optical counterpart that peaks at \( \sim 10^{50} \text{ erg s}^{-1} \text{ in } \sim 10(1+z) \text{ s} \), followed by a steep decay phase, roughly as \( t_{\text{obs}}^{-2} \). This fast decay is mainly controlled by the quickly decreasing pair-loading of the external medium as the blast wave expands past \( \sim 10^{16} \text{ cm} \). Most of the shock energy is lost to the fast Compton energy, and only a small fraction is given to the optical synchrotron emission.

The expected optical flash is very similar to the flash observed in GRB 990123 (Ackeroft et al. 1999). Note that it reached its peak well before the end of the prompt emission, which is consistent with efficient Compton cooling of the flash-producing electrons (Beloborodov 2005b). Unfortunately, GRB 990123 could not be observed at high energies (it was too far off axis for EGRET, the only available GeV telescope at the time). If our interpretation of the optical flash in GRB 990123 is correct, it should have been accompanied by a bright GeV flash.

Such double (optical+GeV) flashes may be detected by future simultaneous observations by Fermi and optical robotic telescopes at times \( t_{\text{obs}} \sim (10-100)(1+z) \text{ s} \) after the burst trigger. Our calculations predict that the peak of the optical flash is slightly delayed compared with the GeV peak and decays faster.

When this work was completed, the first detection of a double optical+GeV flash was reported in GRB 130427A (Vestrand et al. 2014). It confirms the predictions of our model. A detailed study of the flash in GRB 130427A and its implications will be published elsewhere (I. Vurm et al., in preparation).

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