Abstract: The paper deals with an architecture of fuel tanks onboard an aircraft. The fuel consumption mode has a great influence above the position of aircraft’s center of gravity; the authors have determined the position of each fuel tank center of gravity and have also determined its space trajectory during the fuel consumption. When a malfunction of the fuel consumption occurs, aircraft’s center of gravity might be displaced outside the safety envelope, which might badly influence the stability of the aircraft and, because of the autopilot intervention, it might also increase aircraft attitude, its total drag, as well as the specific fuel consumption of aircraft engines. The authors have designed a subroutine for restore the stability, by reordering the fuel consumption to rebalance the aircraft, to restore aircraft attitude and to reduce the engine’s specific fuel consumption initially increased by the drag.

Keywords: Center of gravity, fuel tank, position vector, balance, control;

1. INTRODUCTION

One of the most important parameters of the aircraft is its center of gravity position. This is determined beforehand in the design phase, but from the moment it is given to operation, it changes its position according to several factors (on-board crew, embarked passengers and their luggage, unequal fuel distribution or consumption etc.). Over time, monitoring and maintaining the center of gravity during the flight was a major challenge for flight safety and efficiency. Therefore, the problem of positioning the center of gravity on the aircraft must be approximated from the design phase, determining a safe volume within which it can move.[1,2]

One of the parameters that can be changed during the flight is the fuel mass on a particular tank or in a group of tanks; it might be generated by several causes, such as the damage or the malfunction of an engine, the uneven fuel consumption of aircraft’s engines during the flight, or mechanical/electrical disruption of some fuel system parts. If a major imbalance occurs, caused by the lack of fuel consumption from one of the tanks, aircraft’s autopilot must react and restore aircraft’s attitude; however, it involves aircraft’s aerodynamic commands architecture reconfiguration, in order to suppress the moment which has occurred because of the unsymmetrical distribution of the fuel. Consequently, the global drag of the aircraft might increase, which, obviously, increase the engines’ fuel consumption; in the mean time, passengers’ comfort might be affected, due to repeated autopilot’s small maneuvers to restore airplane’s flight attitude.
This work has as goal to determine a control method of the on-board fuel’s center of gravity, to keep it inside the safe volume, even when it occurs an important fuel consumption difference between the symmetrically arranged tanks (especially between the wing fuel tanks).

2. FUEL DISTRIBUTION CENTER OF GRAVITY

On the aircraft, the fuel consumption is made in a certain order, so that the center of gravity (noted in the paper with $C_g$) must be found in the safety envelope [1,3].

It is considered the placement of the fuel tanks according to Fig. 1, where it can be observed a number of five fuel tanks (which shapes are considered as random) left wing tank, right wing tank (symmetrically arranged with respect to the aircraft body), front fuselage tank, rear fuselage tank and the center tank (main fuel tank, usually meant to supply the engines). OXYZ is the aircraft frame, with the origin in the theoretical $C_g$, Y-axis on the left side, X-axis in front, following the flight direction, Z-axis upward, perpendicular to the other two. With $\vec{r}_l, \vec{r}_r, \vec{r}_f, \vec{r}_s, \vec{r}_m$, were noted the $C_g$ tanks position vectors for the left, right, front and rear respectively. The $C_g$ of the main tank is assumed to be imposed, from the design phase, inside of the envelope [4].

Starting from the hypothesis that any fuel tank is not symmetrical by any axis, it results that its inside fuel center of gravity varies during its consumption after a three-dimensional curve, presumed and illustrated in red on the figure.

The position vectors of $C_g$’s relative to the aircraft frame for each tank must be calculated using a vector description, as in [5,6,7]:

\[
\begin{align*}
\vec{r}_l &= \vec{r}_{L1} + \vec{r}_{L2} \\
\vec{r}_r &= \vec{r}_{R1} + \vec{r}_{R2} \\
\vec{r}_f &= \vec{r}_{F1} + \vec{r}_{F2} \\
\vec{r}_s &= \vec{r}_{S1} + \vec{r}_{S2} \\
\vec{r}_m &= \vec{r}_{M1} + \vec{r}_{M2}
\end{align*}
\] 

(1)

![FIG. 1 Location of aircraft fuel tanks](image)
where:
- \( \vec{r}_{i1}, \vec{r}_{i2}, \vec{r}_{i3}, \vec{r}_{i4}, \vec{r}_{i5} \), are the position vectors of the five fuel tanks frames, relative to the aircraft frame.
- \( \vec{r}_{g1}, \vec{r}_{g2}, \vec{r}_{g3}, \vec{r}_{g4}, \vec{r}_{g5} \), are the \( C_g \)’s position vectors of the five fuel tanks relative to their own frames, as Fig. 2 shows.

3. FUEL TANK’S CENTER OF GRAVITY DETERMINING

For any of the aircraft’s fuel tanks, its center of gravity is “moving” after a curve, as in Fig. 2, with respect to the fuel level inside. One has to determine this curve’s equation and to emphasize its correlation to the fuel level (which is given by the fuel level sensor).

In order to determine the \( C_g \) for any tank fuel, a meshing method is used, with \( n \) volumes (samples) of known geometric shapes, as shown in Fig. 3. The tank level sensor is the one who gives information on the volume fuel, and also on its level by a vertical component \( h \) (fuel height in the tank). For \( k \) volumes (samples), the height of the fuel level in the tank is:

\[
h_k = \sum_{j=1}^{k} z_j,
\]

where \( z_j \) is the height of each item of fuel volume.

The \( C_g \) for each sample, depending on its geometry (the samples may differ from one sample to another) may be easy calculated, so that the coordinates of the fuel \( C_g \) result as a function of the height \( h \) (the fuel level, given by the sensor). Thus, for each fuel level, it corresponds a \( C_g \) (eg. for \( h_1 \) corresponds \( C_{g1} \), for \( h_2 \) corresponds \( C_{g2} \), and so on).

FIG. 2 Center of gravity’s position vector for a fuel tank (the right wing tank)

FIG. 3 Meshing method for center of gravity’s position vector determining
The equation for the center of gravity’s position vector for the geometric composed shape of the tank is, according to the equations in [5,6,7]:

\[ \vec{r}_{Cgk} (x_{Cgk}, y_{Cgk}, z_{Cgk}) = \frac{\sum_{j=1}^{k} \vec{r}_j (x_{Cgj}, y_{Cgj}, z_{Cgj}) \cdot m_j (x_{Cgj}, y_{Cgj}, z_{Cgj})}{\sum_{j=1}^{k} m_j (x_{Cgj}, y_{Cgj}, z_{Cgj})}, \]  

where \( x_{Cgj}, y_{Cgj}, z_{Cgj} \), are the coordinates of the level \( j \), and for the level \( k \) can be written as:

\[ x_{Cgk} = \frac{\sum_{j=1}^{k} x_{Cgj} \cdot m_j (x_{Cgj}, y_{Cgj}, z_{Cgj})}{\sum_{j=1}^{k} m_j (x_{Cgj}, y_{Cgj}, z_{Cgj})}, \]

\[ y_{Cgk} = \frac{\sum_{j=1}^{k} y_{Cgj} \cdot m_j (x_{Cgj}, y_{Cgj}, z_{Cgj})}{\sum_{j=1}^{k} m_j (x_{Cgj}, y_{Cgj}, z_{Cgj})}, \]

\[ z_{Cgk} = \frac{\sum_{j=1}^{k} z_{Cgj} \cdot m_j (x_{Cgj}, y_{Cgj}, z_{Cgj})}{\sum_{j=1}^{k} m_j (x_{Cgj}, y_{Cgj}, z_{Cgj})}. \]

The equations (3) to (6) can be written [5,6,7] for each tank, and it results the position vectors relative to each fuel tank frame:

\[ \vec{r}_L = \vec{r}_{L1} (x_{L1}, y_{L1}, z_{L1}) + \frac{\sum_{k=1}^{L} \vec{r}_{Lk} (x_{CgLk}, y_{CgLk}, z_{CgLk}) \cdot m_{Lk} (x_{CgLk}, y_{CgLk}, z_{CgLk})}{\sum_{k=1}^{L} m_{Lk} (x_{CgLk}, y_{CgLk}, z_{CgLk})}, \]

\[ \vec{r}_R = \vec{r}_{R1} (x_{R1}, y_{R1}, z_{R1}) + \frac{\sum_{k=1}^{R} \vec{r}_{Rk} (x_{CgRk}, y_{CgRk}, z_{CgRk}) \cdot m_{Rk} (x_{CgRk}, y_{CgRk}, z_{CgRk})}{\sum_{k=1}^{R} m_{Rk} (x_{CgRk}, y_{CgRk}, z_{CgRk})}, \]

\[ \vec{r}_P = \vec{r}_{P1} (x_{P1}, y_{P1}, z_{P1}) + \frac{\sum_{k=1}^{P} \vec{r}_{Pk} (x_{CgPk}, y_{CgPk}, z_{CgPk}) \cdot m_{Pk} (x_{CgPk}, y_{CgPk}, z_{CgPk})}{\sum_{k=1}^{P} m_{Pk} (x_{CgPk}, y_{CgPk}, z_{CgPk})}, \]

\[ \vec{r}_B = \vec{r}_{B1} (x_{B1}, y_{B1}, z_{B1}) + \frac{\sum_{k=1}^{B} \vec{r}_{Bk} (x_{CgBk}, y_{CgBk}, z_{CgBk}) \cdot m_{Bk} (x_{CgBk}, y_{CgBk}, z_{CgBk})}{\sum_{k=1}^{B} m_{Bk} (x_{CgBk}, y_{CgBk}, z_{CgBk})}. \]
The final equation of the center of gravity for the fuel system that encompasses all the tanks is:

\[
\vec{r}_M = \frac{\sum_{j=1}^{k_M} \overrightarrow{r}_{Mj}(x_{CGMj0}, y_{CGMj0}, z_{CGMj0}) \cdot m_{Mj}(x_{CGMj1}, y_{CGMj1}, z_{CGMj1})}{\sum_{j=1}^{k_M} m_{Mj}(x_{CGMj0}, y_{CGMj0}, z_{CGMj0})},
\]

(11)

The final equation of the center of gravity for the fuel system that encompasses all the tanks is:

\[
\vec{r}_C(x_{Cg}, y_{Cg}, z_{Cg}) = \frac{\overrightarrow{r}_L \cdot m_L + \overrightarrow{r}_R \cdot m_R + \overrightarrow{r}_F \cdot m_F + \overrightarrow{r}_g \cdot m_g + \overrightarrow{r}_M \cdot m_M}{m_L + m_R + m_F + m_g + m_M},
\]

(12)

where \(m_L, m_R, m_F, m_B, m_M\) are also presented in the denominators of the equations (7) to (11) and represent the amount of fuel, determined for each tank.

According to the successive course of equations (2) to (12), it results the dependence between \(\vec{r}_C\) and \(h\).

4. FUEL MANAGEMENT CONTROL ALGORITHM

For a more complex system, following the calculation method, several tanks can be added to the system, thus obtaining other position (positions) of the center of gravity, depending on the case and on the specific design. Furthermore, the above determined mathematical equations can be later used in computer algorithms, which can further be used as part of the fuel management system and also part of the automatic flight control system [8].

A subroutine than runs as a background process, for the fuel management algorithm that is responsible for maintaining the fuel center of gravity inside the safety envelope, can be described as a simplified pseudocode (Table 1).

| Table 1. Pseudocode model |
|---------------------------|
| if the coordinates of X_Cg can be found in the coordinate stack of X_CgImposed take no action |
| else if X_CgL’s modulus is greater than X_CgR’s modulus turn on transfer pump_L until X_CgL modulus is equal to X_CgR modulus |
| else turn on transfer pump_R until X_CgR modulus is equal to X_CgL modulus |
| if the coordinate of Y_CgCb can be found in the coordinate stack of Y_CgImp take no action |
else if \( Y_{\text{CgF}} \)’s modulus is greater than \( X_{\text{CgB}} \)’s modulus

\[
\text{turn on transfer pump}_F \text{ until } X_{\text{CgF}} \text{ modulus is equal to } X_{\text{CgB}} \text{ modulus}
\]

else

\[
\text{turn on transfer pump}_B \text{ until } X_{\text{CgB}} \text{ modulus is equal to } X_{\text{CgF}} \text{ modulus}
\]

Supposing that, because of a malfunction of aircraft’s engine 1 (left wing engine), the fuel consumption from the left wing tank is delayed (or even stopped); the left wing becomes more weighty and the center of gravity might left the safety envelope. Consequently, the autopilot reacts and compensates the bank of the aircraft, which means that ailerons and rudder control becomes active; however, if the fuel management system is active, it commands the cross-feed valve opening, the right wing tank valve closing and the consumption only from the left tank, until the balance is reestablished, that means that until the center of gravity rejoins the safety envelope. A similar situation might occur with the front and the rear fuel tanks, but the issue may be easier solved, in the same manner.

**CONCLUSIONS**

The fuel control system plays an important role in maintaining the center of gravity of the airplane. Thus, the estimation of the fuel weight center becomes part of the fuel management system and also the automatic flight control system. By determining the center of gravity of each fuel tank, it is possible to make a variable mass calculation system, that can estimate the center of gravity of the total fuel on board the aircraft. For future use, with these kind of methods, computer comparison algorithms may be used to analyze data, from the fuel system sensors (such as flowmeter, fuel pressure, fuel gauge), and compare it with the airplane's allowed tolerance of center of gravity.

The algorithm of fuel center of gravity calculation may be implemented as a subroutine of the flight control system, only after the safety envelope of the global center of gravity was specified and mathematically defined.

The study in this paper might be extended with an effective simulation for an existing airplane, with a complex fuel system (a complex distribution of the on-board fuel tanks).

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