Observer-based event-triggered adaptive containment control for multiagent systems with prescribed performance

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Abstract This paper focuses on the problem of the event-triggered adaptive containment control for a class of nonlinear multiagent systems (MASs) with prescribed performance and immeasurable states. First, the radial basis function neural networks (RBFNNs) are adopted to approximate the uncertain smooth nonlinear function, and the neural network-based state observer is designed to estimate the unmeasurable state. Besides, to reduce the control resource consumption and get a better balance between the system performance and network constraints, the switching threshold-based event-triggered control strategy is introduced. Based on this, the novel distributed containment controller is designed by utilizing the adaptive backstepping technique and the dynamic surface control (DSC) technique to guarantee that the output of each follower converges to the convex hull formed by multileader. Moreover, the containment errors can be converged to the prescribed boundary and all signals in closed-loop system are semi-global uniformly ultimately bounded (SGUUB) as well. Finally, the simulation examples are carried out to illustrate the efficiency of the proposed controller.

Keywords Adaptive containment control · Event-triggered · Prescribed performance · State observer

1 Introduction

Over the past decades, cooperative control of multiagent systems (MASs) has gained growing popularity on account of its extensive applications in various fields, such as sensor networks [1, 2] and marine vessels [3, 4] as well as spacecraft formation flying [5], to name just a few. According to the number of the leaders, cooperative control problem can be segmented into three categories including leaderless consensus control [6], leader-following consensus control [7, 8] and the containment control [9]. The fundamental characteristic of the containment control is that the MASs are composed of multiple leaders and all followers converge to the convex hull limited by multileader. In view of this, quite a few researches on containment control are designed for linear multiagent systems [10–13].

As is known to all, nonlinear systems [14–17] are more suitable for describing practical application scenarios than linear ones. However, the dynamics of nonlinear multiagent systems are often difficult to obtain, which means the traditional adaptive control method is not suitable any more. Recently, the approximate capacity of the neural network (NN) and the fuzzy logic system (FLS) have shed some insightful light on the investigations of the adaptive control problem for nonlinear multiagent systems. Mei et al. [18]...
applied the universal approximation property of radial basis function neural networks (RBFNNs) to tackle the obstacle of uncertain nonlinearities and proposed the NN-based distributed adaptive containment control scheme for second-order MASs with unknown nonlinear dynamics. Yoo [19] extended this approach to cope with consensus problems in high-order nonlinear strict-feedback multiagent systems under a directed graph topology, which can be better applied to the engineering systems. Besides, Wang and Tong [20] utilized FLSs to identify the unknown nonlinear functions and cope with the containment control problem for full-state-constrained nonlinear strict-feedback systems. However, the results described in [18–20] are based upon a quite ideal situation that the states of all followers are measurable, while it is inevitable that some states cannot be obtained directly because of economic constraints or technological limitations. In this case, Tong and Li [21] introduced the fuzzy adaptive observer to estimate unmeasured states for single-input and single-output (SISO) nonlinear systems. Subsequently, an adaptive fuzzy backstepping dynamic surface control (DSC) method was introduced for multiple-input and multiple-output (MIMO) nonlinear systems in [22]. Note that, the DSC technique shares the merit in overcoming the “explosion of complexity” issue inherent in each adaptive backstepping design step. The general idea of this technique is to introduce the first-order filter in each process of the backstepping design procedure.

Recently, as system performance has become significantly important for practical systems, a number of control schemes with prescribed performance, especially coordinated transformation-based prescribed performance, become major concerns as open for nonlinear multiagent systems [23–26]. The primary objective of prescribed performance is to guarantee that the control errors gather to the predefined performance bounds. In [26], the state feedback control approach was established to address the trouble of unknown pure-feedback nonlinear systems. Tong and Li [23] proposed the output feedback prescribed performance control method for SISO switched nonstrict-feedback nonlinear systems. The authors in [25] focused on the nonlinear large-scale systems consisting of immeasurable states and unknown time-varying delays. Ni et al. [27] investigated the fixed-time recurrent neural network control for uncertain strict-feedback nonlinear systems in the presence of unknown dead-zone output. More recently, by using FLSs, a quantized cooperative control strategy with the prespecified performance approach for MASs with unknown gains and input quantization was developed in [28]. Wang et al. [29] dealt with the issue of adaptive fuzzy containment control for nonlinear MASs with prescribed performance, unknown disturbance and unknown dead zone. However, the aforementioned results are time-triggered control, which will result in unnecessary resource consumption due to the continuously information transmission through the communication topology.

It should be mentioned that the researches of event-triggered control scheme for nonlinear systems are of extraordinary significance as it can further improve the control efficiency as well as decrease heavy communication burden and cost than classical time-triggered strategy [30]. Xing et al. [31] put forward the original event-triggered control method named the switching threshold strategy based on the framework of the fixed threshold strategy [32] and the relative threshold strategy [33, 34]. The proposed adaptive controller was able to compensate for the measurement errors so that the input-to-state stability (ISS) assumption is no longer needed. In [35], the event-triggered-based output feedback control mechanism was investigated for uncertain nonlinear systems. To decrease the communication channel bandwidth, the authors proposed the new 1-bit signal transmission rule which has been proved to be effective. Ma et al. [36] designed the observer-based adaptive event-triggered control method for the stochastic nonlinear systems along with full-state constraints and actuator faults. In order to obtain accurate tracking performance, it is essential to extend the event-triggered control from nonlinear systems to MASs. Consequently, Zhang et al. [37] developed the event-triggered tracking control scheme for nonlinear MASs with unknown disturbances that can be estimated by a disturbance observer. The problem of distributed adaptive output control for uncertain nonlinear MASs with unmeasured states and uncertain control gains was addressed, and the Nussbaum function was used to compensate for the uncertain control gains in [38]. Besides, Qiu et al. [39] applied the fixed threshold event-triggered control mechanism to pure-feedback nonlinear systems with prescribed performance and unavailable system states. It is worth noting that the problem of event-triggered adaptive containment control for nonlinear MASs with immeasurable states and predefined performance is still to be studied.
Motivated by the analysis mentioned above, this paper is concerned with the event-triggered adaptive containment control problem for a class of high-order uncertain nonlinear multiagent systems in strict feedback form with prescribed performance and immeasurable states. It aims to tackle with the following challenging issues:

1. how to analyze the influence induced by the immeasurable states as well as the event-triggered strategy on the tracking performance?
2. how to formulate the adaptive containment control scheme for the nonlinear MASs with the immeasurable states and prescribed performance?
3. how to design a threshold-based event-triggering condition for the adaptive tracking problem? Therefore, to solve this issues, RBFNNs are utilized to approximate the unknown nonlinear function of the system dynamic, as well as the NN-based observer is applied to estimate the unavailable states. Combined with the Lyapunov stability theorem and backstepping technique, the novel distributed adaptive containment control approach is proposed. The major contributions of this paper can be stated as follows: (1) By means of extending the switching threshold event-triggered strategy to the containment control of MASs, not only the communication burden can be alleviated but also the system performance can be ensured. In addition, the proposed control scheme can achieve the containment control objective. (2) Under the framework of prescribed performance control, the containment errors can converge to a predefined arbitrarily small compact set with rapid convergence speed and high accuracy. (3) The proposed control method can overcome the differentiation explosion by using the DSC technique, and thus, the designed controller becomes more effective than conventional adaptive backstepping controller, and the containment control also gains superior performance. All variables of the MASs are semi-global uniformly ultimately bounded (SGUUB) as well.

The rest of this paper is described as follows. In Sect. 2, the system formulation and preliminary knowledge are given. The controller design and stability analysis are shown in Sect. 3. The effectiveness of this proposed strategy is verified by the simulation results in Sect. 4. Section 5 finally draws the conclusion.

2 Problem formulation and preliminaries

2.1 Graph theory

The exchange of information among agents can be expressed as the directed graph $G = (\mathcal{V}, \mathcal{A}, \mathcal{W})$, with the set of vertexes $\mathcal{V} = \{1, 2, ..., N, N + 1, ..., N + M\}$ and the set of edges $\mathcal{A} = \{(i, j) | i \in \mathcal{V}, j \neq i\}$. The followers are labeled as $1, ..., N$, while “$N + 1, ..., N + M$” represent the leaders. $\mathcal{A} = [a_{i,j}] \in \mathbb{R}^{(N+M) \times (N+M)}$ stands for the adjacency matrix, where $(i, j) \in \mathcal{A}$ means that there exists an information flow from agent $j$ to agent $i$, and $a_{i,j} > 0$ when $(i, j) \in \mathcal{A}$, otherwise $a_{i,j} = 0$. The neighbors set of node $i$ is defined as $\mathcal{N}_i = \{j | (i, j) \in \mathcal{A}\}$. Let $L = [l_{i,j}] \in \mathbb{R}^{(N+M) \times (N+M)}$ be the Laplacian matrix, and $l_{i,j} = -a_{i,j}$ for all $j \neq i$, otherwise, $l_{i,j} = \sum_{j \in \mathcal{N}_i} a_{i,j}$. The Laplacian matrix equals to $L = D - A$, where $D = \text{diag}[d_1, ..., d_{N+M}]$ with $d_i = \sum_{j \in \mathcal{N}_i} a_{i,j}$ denotes the degree matrix of the digraph $G$. It is assumed that each follower owns at least one neighbor, while leaders own none. Then, the Laplacian matrix $L$ of the digraph $G$ can be partitioned into

$$
L = \begin{bmatrix}
L_1 & L_2 \\
0_{M \times N} & 0_{M \times M}
\end{bmatrix}
$$

Assumption 1 For each follower, there is at least one leader that possesses a directed path to it.

Remark 1 A directed path from node $i$ to node $j$ is a sequence of edges which is in the form of $(i, i_1), (i_1, i_2), ..., (i_l, j)$ in a directed graph. Assumption 1 is presented to ensure that each follower can obtain information from the leader.

Lemma 1 [40] Based on Assumption 1, each eigenvalue of matrix $L_1$ has a positive real part. Each row of $-L_1^{-1}L_2$ has a sum that equals to 1, and each entry of $-L_1^{-1}L_2$ is nonnegative.

Assumption 2 The outputs of the leaders $y_{li,r}$ ($l = N + 1, ..., N + M$) are adequately smooth function of $t$, and $y_{li,r}, \dot{y}_{li,r}, \ddot{y}_{li,r}$ are bounded, i.e., there exists a positive constant $\sigma$, such that

$$
||y_{li,r}||^2 + ||\dot{y}_{li,r}||^2 + ||\ddot{y}_{li,r}||^2 \leq \sigma
$$

Remark 2 Assumption 2 is presented to ensure that the leaders’ signals are bound, which is an important part
of illustrating that all signals in the closed-loop system are semi-global uniformly ultimately bounded.

**Definition 1** [19] A set $\mathcal{C}$ is convex, if for any $X_1, X_2 \in \mathcal{C}$ and any $\theta \in [0, 1], \theta X_1 + (1 - \theta) X_2 \in \mathcal{C}$. The convex hull of a set $\mathcal{C}$, denoted by $\mathcal{C}(X) = X_1, ..., X_n$, is the smallest convex set that contains all points in $X$. Concretely, $\mathcal{C}(X) = \{\sum_{k=1}^n \theta_k X_k | X_k \in X, \theta_k > 0, \sum_{k=1}^n \theta_k = 1\}$.

2.2 Model formulation

Consider a strict-feedback nonlinear multiagent system composed of $N$ agents. The dynamics of the $i$th ($i=1,...,N$) agent are described as follows:

$$
\dot{x}_{i,n} = K_i \tilde{x}_{i,n} + A_i y_i + \sum_{j=1}^n B_{i,j} f_i,j(\tilde{x}_{i,n}) + C_i u_i
$$

$$
y_i = D_i^T \tilde{x}_{i,n}, \quad i = 1, 2, ..., N
$$

where $\tilde{x}_{i,n} = [x_{i,1}, x_{i,2}, ..., x_{i,m}]^T \in \mathbb{R}^m$ are state vectors, $y_i \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ are the system output and the control input, respectively. $f_i,j(\cdot)$ denotes the unknown smooth nonlinear function. In this paper, it is assumed that only the output variables $y_i$ are available, while other states are unmeasurable during the controller design procedure.

Rewrite system (1) as:

$$
\tilde{x}_{i,n} = K_i \tilde{x}_{i,n} + A_i y_i + \sum_{j=1}^n B_{i,j} f_i,j(\tilde{x}_{i,n}) + C_i u_i
$$

$$
y_i = D_i^T \tilde{x}_{i,n}, \quad i = 1, 2, ..., N
$$

where

$$K_i = \begin{bmatrix}
-k_{i,1} & I_{(N-1) \times (N-1)} \\
-k_{i,2} & \vdots \\
-k_{i,n} & 0 & \cdots & 0
\end{bmatrix},
$$

$$A_i = \begin{bmatrix}
k_{i,1} & k_{i,2} & \cdots & k_{i,n}
\end{bmatrix}^T,$$

$$B_{i,j} = \begin{bmatrix}
0, ..., 0, 1, 0, ..., 0
\end{bmatrix}^T_{j-1},$$

$$C_i = [0, ..., 0, 1]^T, \quad D_i = [1, 0, ..., 0]^T.$$

The vector $A_i$ is selected so that $K_i$ is a strict Hurwitz matrix; thus, given any positive definite matrix $Q_i$ with $Q_i = Q_i^T > 0$, there is a positive symmetric matrix $P_i$ satisfying

$$K_i^T P_i + P_i K_i = -Q_i
$$

2.3 RBFNN and state-observer design

The universal approximation property of RBFNNs is used to approximate an unknown smooth function $\Psi(X) : \mathbb{R}^n \rightarrow \mathbb{R}^m$ defined on a compact set $\Omega_\varepsilon$; there exists an RBFNN as

$$\Psi(X) = W^*^T S(X)$$

where $X \in \Omega_\varepsilon \subset \mathbb{R}^n$ is the input vector, $W^* \in \mathbb{R}^{l \times m}$ is the ideal weight matrix, $l > 1$ represents the number of neurons, $S(X) = [S_1(X), S_2(X), ..., S_l(X)]^T$ is the Gaussian basis function vector, and $S_i(X) = \exp[\frac{-|X - \varrho_i|^2}{\varsigma_i}]$ for $i = 1, 2, ..., l$, where $\varrho_i = [\varrho_{i,1}, \varrho_{i,2}, ..., \varrho_{i,n}]^T$, $i = 1, 2, ..., l$ denotes the center of the receptive field, and $\varsigma_i$ represents the width of the Gaussian function.

As is known to all, RBFNNs can approximate any continuous nonlinear function to a desired accuracy on a compact space. On account of the excellent ability of RBFNNs, the unknown smooth function $f_i,j(\tilde{x}_{i,n}) \in \mathbb{R}^m$ can be approximated as

$$\hat{f}_i,j(\tilde{x}_{i,n})|\hat{W}_{i,j}) = \hat{W}_{i,j}^T S_{i,j}(\tilde{x}_{i,n})$$

where $\hat{W}_{i,j}$ represents the estimation of the optimal weight matrix $W_{i,j}^*$.

Let $\hat{x}_{i,j} = [\hat{x}_{i,1}, ..., \hat{x}_{i,n}]^T$ denote the estimation of the state vector $\tilde{x}_{i,j}$, one has

$$\hat{f}_i,j(\hat{x}_{i,n})|\hat{W}_{i,j}) = \hat{W}_{i,j}^T S_{i,j}(\hat{x}_{i,n})$$

The optimal weight matrix $W_{i,j}^*$ is expressed as

$$W_{i,j}^* := \arg \min_{\hat{W}_{i,j} \in \mathbb{R}^{l \times m}} \left\{ \sup_{\tilde{x}_{i,j} \in \mathbb{R}^{l \times m}} \left| \hat{f}_i,j(\hat{x}_{i,n})|\hat{W}_{i,j}) - f_i,j(\hat{x}_{i,n}) \right| \right\}$$
where $\mathcal{S}_{i,j}$, $\mathcal{S}_{i,j}$, and $\mathcal{S}_{i,j}$ are compact sets for $\hat{x}_{i,j}$, $\hat{x}_{i,j}$, and $\hat{W}_{i,j}$, respectively.

Defining the corresponding minimal approximation errors $\lambda_{i,j}$ and the approximation errors $\mu_{i,j}$, respectively, as

$$
\lambda_{i,j} = f_{i,j}(\hat{x}_{i,j}) - \hat{f}_{i,j}(\hat{x}_{i,j}|W^*_{i,j})
$$

$$
\mu_{i,j} = f_{i,j}(\hat{x}_{i,j}) - \hat{f}_{i,j}(\hat{x}_{i,j}|\hat{W}_{i,j})
$$

Assumption 3 The corresponding minimal approximation errors $\lambda_{i,j}$ and the approximation errors $\mu_{i,j}$ are bounded, i.e., existing the constants $\lambda^0_{i,j} > 0$, $\mu^0_{i,j} > 0$ such that $||\lambda_{i,j}|| < \lambda^0_{i,j}$, $||\mu_{i,j}|| < \mu^0_{i,j}$. Defining

$$
r_i = \begin{cases} 
-(1 + \varepsilon_i) \left[ \alpha_i \tanh \left( \frac{s_{i,n} \alpha_i n}{\varepsilon} \right) + \tilde{a} \tanh \left( \frac{s_{i,n} \tilde{a}}{\varepsilon} \right) \right], & \varepsilon_i |u_i(t)| + a \geq m_i \\
\alpha_i n - \tilde{m}_i \tanh \left( \frac{s_{i,n} \tilde{m}_i}{\varepsilon} \right), & \varepsilon_i |u_i(t)| + a < m_i
\end{cases}
$$

By combining (1), (9), and (10), one obtains

$$
\hat{x}_{i,j} = \tilde{x}_{i,j} + 1 + k_{i,j}(x_{i,j} - \hat{x}_{i,j}) + \hat{f}_{i,j}(\hat{x}_{i,j} | \hat{W}_{i,j}),
$$

$$
\hat{x}_{i,n} = u_i + k_{i,n}(x_{i,n} - \hat{x}_{i,n}) + \hat{f}_{i,n}(\hat{x}_{i,n} | \hat{W}_{i,n})
$$

2.4 Event-triggered design

Different from the control strategy in [32, 33, 38, 39], the switching threshold event-triggered control strategy is given below, which can alleviate the communication burden while ensuring the system performance. The intermediate control function is defined as follows

$$
\tilde{u}_i(t) = \alpha_i \left[ u_i(t) + a \right] + \tilde{m}_i \tanh \left( \frac{s_{i,n} \tilde{m}_i}{\varepsilon} \right),
$$

where $\alpha_i$ is virtual control function to be designed later. $\varepsilon_i$, $\varepsilon_i$, $m_i$, $\tilde{m}_i, a$ and $\tilde{a} > a/(1 - \varepsilon_i)$ are positive design parameters, $0 < \varepsilon_i < 1$.

Define the triggering event as

$$
u_i(t) = r_i(t^k_i) \quad \forall t \in [t^k_i, t^{k+1}_i)
$$

$$
t^{k+1}_i = \inf \left\{ t \in \mathbb{R} | \tau_i(t) \geq \max \left\{ \varepsilon_i |u_i(t)| + a, m_i \right\} \right\}
$$

where $\tau_i(t) = r_i(t) - u_i(t)$ denotes the measurement error, $t^k_i, k \in \mathbb{Z}^+$ is the update time, i.e., when (14) is triggered, the time will be recorded by $t^{k+1}_i$, and the control value will be marked as $u(t^{k+1}_i)$. During the period $t \in [t^k_i, t^{k+1}_i)$, the control signal holds as $r_i(t^k_i)$.

From (13) and (14), we have

$$
\tau_i(t) = \begin{cases} 
\delta_{i,1}(t) \varepsilon_i u_i(t) + \delta_{i,2}(t) a, & \varepsilon_i |u_i(t)| + a \geq m_i \\
\delta_{i,3}(t) m_i, & \varepsilon_i |u_i(t)| + a < m_i
\end{cases}
$$

where $\delta_{i,1}(t), \delta_{i,2}(t)$ and $\delta_{i,3}(t)$ are continuous time-varying parameters with $\delta_{i,1}(t^k_i) = \delta_{i,2}(t^k_i) = \delta_{i,3}(t^k_i) = 0, \delta_{i,1}(t^{k+1}_i) = \pm 1, \delta_{i,2}(t^{k+1}_i) = \pm 1, \delta_{i,3}(t^{k+1}_i) = \pm 1, |\delta_{i,1}(t)| \leq 1, |\delta_{i,2}(t)| \leq 1, |\delta_{i,3}(t)| \leq 1, \forall t \in [t^k_i, t^{k+1}_i)$. Then, one has
\[ u_i(t) = \begin{cases} r_i, & 1 + \delta_{1,1}(t)\epsilon_i > m_i \\ \frac{\delta_{1,2}(t)a}{1 + \delta_{1,1}(t)\epsilon_i}, & \epsilon_i|u_i(t)| + a \geq m_i \\ \frac{\delta_{1,3}(t)m_i}{r_i}, & \epsilon_i|u_i(t)| + a < m_i \end{cases} \]

(16)

**Remark 3** This switching threshold-based controller has the advantages of the fixed threshold strategy and the relative threshold strategy. As there is a high magnitude of control signal \( u_i(t) \), the relative threshold strategy is chosen to obtain longer update intervals, while when \( u_i(t) \) is approaching to zero, the fixed threshold strategy is applied to obtain better system performance. The Zeno behavior can be avoided as well.

**Lemma 2** [31] The hyperbolic tangent function \( \tanh(\cdot) \) satisfies the following property

\[ 0 \leq |t| - t \tanh \frac{t}{\epsilon} \leq 0.2785\epsilon \]

where \( \epsilon \) is an arbitrary positive constant, and \( t \in \mathbb{R} \).

### 2.5 Prescribed performance

According to [26], the prescribed performance is accomplished by ensuring that the containment error surfaces \( v_{i,1} = \sum_{j=1}^{N} a_{i,j}(y_i - y_j) + \sum_{k=N+1}^{N+M} a_{i,k}(y_i - y_{i,r}) \) evolve strictly within predefined decaying bounds as follows

\[ -\eta_{i,\text{min}}\omega_i(t) < v_{i,1}(t) < \eta_{i,\text{max}}\omega_i(t), \quad \forall t \geq 0 \]

(17)

where \( \eta_{i,\text{min}} \) and \( \eta_{i,\text{max}} \) are positive parameters. The prescribed performance function \( \omega_{i}(t) = (\omega_{i,0} - \omega_{i,\infty})e^{-\zeta_{i}t} + \omega_{i,\infty} \) is a strictly decreasing smooth function satisfying \( \lim_{t \to \infty} \omega_{i}(t) = \omega_{i,\infty} > 0, \omega_{i,0} = \omega_{i}(0) > \omega_{i,\infty}, \zeta_{i} > 0 \) such that \( -\eta_{i,\text{min}}\omega_{i}(0) < v_{i,1}(0) < \eta_{i,\text{max}}\omega_{i}(0) \), and we can obtain \( |v_{i,1}(t)| < \max\{|\eta_{i,\text{min}}\omega_{i}(0), \eta_{i,\text{max}}\omega_{i}(0)\}\).

To achieve the desired performance, the constrained containment error is converted into an equivalent unconstrained one. Letting

\[ v_{i,1}(t) = \omega_{i}(t)\Theta_{i}(\beta_{i}(t)), \quad \forall t \geq 0 \]

(18)

where \( \beta_{i} \) is the converted error, and the smooth function \( \Theta_{i}(\beta_{i}) \) is strictly monotonic increasing, and

\[ \Theta_{i}(\beta_{i}) = \frac{\eta_{i,\text{max}}e^{\beta_{i}} - \eta_{i,\text{min}}e^{-\beta_{i}}}{e^{\beta_{i}} + e^{-\beta_{i}}} \]

The inverse function can be represented as

\[ \beta_{i}(t) = \Theta_{i}^{-1}\left( \frac{v_{i,1}(t)}{\omega_{i}(t)} \right) = \frac{1}{2} \ln \frac{\Theta_{i} + \eta_{i,\text{min}}}{\eta_{i,\text{max}} - \Theta_{i}} \]

(19)

and

\[ \dot{\beta}_{i}(t) = \pi_{i}\left( \dot{v}_{i,1} - \omega_{i}\dot{v}_{i,1} \right) \]

(20)

where \( \pi_{i} = \frac{1}{2\omega_{i}} \left[ \frac{1}{\Theta_{i} + \eta_{i,\text{min}}} - \frac{1}{\Theta_{i} - \eta_{i,\text{max}}} \right] \)

Define the following state transformation

\[ s_{i,1} = \beta_{i}(t) - \frac{1}{2} \ln \frac{\eta_{i,\text{min}}}{\eta_{i,\text{max}}} \]

(21)

the transformation state dynamics are

\[ \dot{s}_{i,1} = \pi_{i}\left( \dot{v}_{i,1} - \frac{\omega_{i}\dot{v}_{i,1}}{\omega_{i}} \right) \]

(22)

Similar to [26], it is obvious that \( v_{i,1} \) satisfies the prescribed performance expressed by (17) when \( s_{i,1} \) is bounded.

**Remark 4** Different form the control strategy in [25] and [28] which cannot be applied to the MASs with multileader, this paper attempts to deal with the containment control problems for MASs. Moreover, the unmeasured states and the switching threshold-based event-triggered strategy are incorporated in this work.

### 3 Controller design and stability analysis

In this section, an adaptive neural network-based controller and parameter adaptive laws are presented by the backstepping technique and DSC technique. All signals of closed-loop system (1) are SGUUB, and the containment error \( v_{i,1} \) is as small as possible.

To design n-step adaptive backstepping consensus controller, the coordinate transformations are defined as

\[ s_{i,j} = \hat{s}_{i,j} - v_{i,j}, \quad z_{i,j} = v_{i,j} - \alpha_{i,j-1}, \quad i = 1, \ldots, N, \quad j = 2, \ldots, n \]

(23)
where $s_{i,j}$ is an error surface, $v_{i,j}$ is a filtered virtual control, which is obtained through a first-order filter on virtual control function $a_i \cdot j-1$, and $z_{i,j}$ is the output error of the first-order filter.

**Lemma 3** [29] Letting $y_i = [y_1, ..., y_N]^T$, and $s_{i,n} = [s_{1,1}, ..., s_{N,1}]^T$. Then, the following inequality holds

$$|| y_i + \mathcal{L}_1^{-1} \mathcal{L}_2 y_i, r || \leq || s_{i,n} || / || \tilde{\sigma}(\mathcal{L}_1) ||$$

where $|| \tilde{\sigma}(\mathcal{L}_1) ||$ is the minimum singular value of $\mathcal{L}_1$.

**Step 1:** Combining (1), (7), (10) with (23), the time derivative of $y_{i,1} = \sum_{j=1}^N a_i \cdot j(y_i - y_j) + \sum_{l=N+1}^{N+M} a_i \cdot l(y_i - y_{l,r})$ is

$$\dot{v}_{i,1} = \sum_{j=1}^N a_i \cdot j(y_i - y_j) + \sum_{l=N+1}^{N+M} a_i \cdot l(y_i - y_{l,r})$$

$$= \sum_{j=1}^N \dot{x}_{i,j} - \sum_{l=N+1}^{N+M} a_i \cdot l\dot{y}_{i,l}$$

$$= \sum_{j=1}^N \dot{x}_{i,j} + f_i(x_i, 1, e_{i,1})$$

$$= \sum_{j=1}^N \dot{x}_{i,j} + \tilde{W}_1 \cdot S_i (\tilde{x}_{i,j}) + \tilde{W}_1 T \cdot S_i (\tilde{x}_{i,j}) + \dot{\lambda}_{i,1} + e_{i,2}$$

$$= \sum_{j=1}^N \dot{x}_{i,j} + \tilde{W}_1 \cdot S_i (\tilde{x}_{i,j}) + \tilde{W}_1 T \cdot S_i (\tilde{x}_{i,j}) + \dot{\lambda}_{i,1} + e_{i,2}$$

where $\tilde{W}_1 \cdot = W^*_1 - \tilde{W}_1$, $j = 1, ..., n$.

Consider the following Lyapunov function candidate

$$V_{i,1} = e^T P_i e_i + \frac{1}{2} \dot{y}_{i,1}^2 + \frac{1}{2} \tilde{W}_1 T \cdot \tilde{W}_1$$

$$+ \sum_{j=1}^N \frac{a_i \cdot j}{2 \chi_{j,1}} \tilde{W}_{j,1}^T \cdot \tilde{W}_{j,1} \quad (25)$$

where $y_{i,1}$, $\chi_{j,1}$ are both positive design constants.

The time derivative of $V_{i,1}$ along (3), (10) and (22) is

$$\dot{V}_{i,1} = -e_i^T Q_i e_i + 2 e_i^T P_i \mu_i + s_{i,1}^T \pi_i \left( \dot{y}_{i,1} - \frac{\hat{\omega}_i v_{i,1}}{\omega_i} \right)$$

$$- \frac{1}{\gamma_{i,1}} \tilde{W}_{i,1}^T \cdot \tilde{W}_{i,1} - \sum_{j=1}^N \frac{a_i \cdot j}{\chi_{j,1}} \tilde{W}_{j,1}^T \cdot \tilde{W}_{j,1} \quad (26)$$

Using the Young’s inequality $2a^T b \leq ||a||^2 + ||b||^2$, one has

$$2e_i^T P_i \mu_i \leq ||e_i||^2 + ||P_i||^2 ||\mu_i||^2 \quad (27)$$

$$s_{i,1}^T \pi_i \left( d_i(\lambda_{i,1} + e_{i,2}) - \sum_{j=1}^N a_i \cdot j(\lambda_{j,1} + e_{j,2}) \right) \leq \pi_i^2 ||s_{i,1}||^2 + \frac{1}{2} ||\mathcal{L}_1||^2 ||\lambda_{i,1}||^2 + \frac{1}{2} ||\mathcal{L}_1||^2 ||e_{i,2}||^2 \quad (28)$$

Substituting (24), (27), (28) into (26) results in

$$\dot{V}_{i,1} \leq -I_{min} ||e_i||^2 + ||P_i||^2 ||\mu_i||^2$$

$$\sum_{j=1}^N \frac{a_i \cdot j}{\chi_{j,1}} \tilde{W}_{j,1}^T \cdot \tilde{W}_{j,1}$$

$$- \frac{1}{\gamma_{i,1}} \tilde{W}_{i,1}^T \cdot \tilde{W}_{i,1} - \sum_{j=1}^N \frac{a_i \cdot j}{\chi_{j,1}} \tilde{W}_{j,1}^T \cdot \tilde{W}_{j,1}$$

$$+ s_{i,1}^T \pi_i \left( d_i(\lambda_{i,1} + e_{i,2}) - \sum_{j=1}^N a_i \cdot j(\lambda_{j,1} + e_{j,2}) \right) - \pi_i^2 ||s_{i,1}||^2 + \frac{1}{2} ||\mathcal{L}_1||^2 ||\lambda_{i,1}||^2 + \frac{1}{2} ||\mathcal{L}_1||^2 ||e_{i,2}||^2$$

$$- \sum_{j=1}^N a_i \cdot j(\lambda_{j,1} + e_{j,2}) - \sum_{j=1}^N a_i \cdot j(\lambda_{i,1} + e_{i,2})$$

$$\quad \left(29\right)$$
where \( l_{\text{min}} = \min(\lambda_{\text{min}}(Q_i) - 1 - \frac{1}{2}||L_1||_F^2) \), and \( \lambda_{\text{min}}(Q_i) \) is the smallest eigenvalue of matrix \( Q_i \).

Design the virtual control function \( \alpha_{i,1} \) and the parameter adaptive law \( \dot{W}_{i,1}, \dot{W}_{j,1} \) as follows

\[
\alpha_{i,1} = \frac{1}{d_i} \left[ -\frac{c_{i,1}}{\pi_i} s_{i,1} + \pi_i s_{i,1} \right] + \sum_{j=1}^{N} a_{i,j} (\dot{x}_{j,2} + \dot{W}_{j,1}^T S_{j,1}(\dot{x}_{j,1}) \right) + \sum_{l=N+1}^{N+M} a_{i,j} \dot{y}_{l,r} + \dot{\omega}_{i,j} v_{i,1} \left( \frac{\omega_i}{\omega_i} \right) - \dot{W}_{i,1}^T S_{i,1}(\dot{x}_{i,1})
\]

\[
\dot{W}_{i,1} = \gamma_{i,1} \left( \pi_i d_i s_{i,1} S_{i,1}(\dot{x}_{i,1}) - \alpha_{i,1} \dot{W}_{i,1} \right)
\]

\[
\dot{W}_{j,1} = \chi_{j,1} \left( -\pi_i s_{j,1} S_{j,1}(\dot{x}_{j,1}) - \varphi_{j,1} \dot{W}_{j,1} \right)
\]

where \( c_{i,1} > 0, \sigma_{i,1} > 0 \) and \( \varphi_{j,1} > 0 \) are design parameters.

By invoking (30)–(32), we can obtain

\[
\dot{V}_{i,1} \leq -l_{\text{min}} ||e_i||^2 - c_{i,1} s_{i,1}^2 + \pi_i d_i s_{i,1} (z_{i,2} + s_{i,2}) + \sigma_{i,1} \dot{W}_{i,1}^T \dot{W}_{i,1} + \sum_{j=1}^{N} a_{i,j} \dot{v}_{j,1} + \rho_{i,1}
\]

\[
V_{i,1} = V_{i,1} + \frac{1}{2} s_{i,2}^2 + \frac{1}{2} \dot{z}_{i,2}^2 + \frac{1}{2} \gamma_{i,2} \dot{W}_{i,1}^T \dot{W}_{i,2} - \gamma_{i,2} \dot{v}_{i,2}
\]

where \( \gamma_{i,2} \) is a positive design constant.

The time derivative of \( V_{i,2} \) along (7), (8), (23), (33)–(35) is

\[
\dot{V}_{i,2} \leq -l_{\text{min}} ||e_i||^2 - c_{i,1} s_{i,1}^2 + \pi_i d_i (z_{i,2} + s_{i,2}) + \sigma_{i,1} \dot{W}_{i,2}^T \dot{W}_{i,1} + \rho_{i,1}
\]

\[
\dot{V}_{i,2} + \dot{W}_{i,2} S_{i,2} (\dot{\hat{x}}_{i,2}) + \dot{W}_{i,2}^T S_{i,2} (\dot{\hat{x}}_{i,2}) - \dot{v}_{i,2} \leq -l_{\text{min}} ||e_i||^2 - c_{i,1} s_{i,1}^2 + \pi_i d_i (z_{i,2} + s_{i,2}) + \sigma_{i,1} \dot{W}_{i,2}^T \dot{W}_{i,1} + \rho_{i,1}
\]

\[
\frac{1}{2} z_{i,2}^2 + \dot{z}_{i,2}^2 + \dot{\hat{x}}_{i,2}^2 + \dot{\hat{x}}_{i,1}^2 + \dot{\hat{x}}_{i,2}^2 + \dot{\hat{x}}_{i,3}^2 + \dot{\hat{x}}_{i,4}^2 + \dot{\hat{x}}_{i,5}^2 + \dot{\hat{x}}_{i,6}^2 \leq -l_{\text{min}} ||e_i||^2 - c_{i,1} s_{i,1}^2 + \pi_i d_i (z_{i,2} + s_{i,2}) + \sigma_{i,1} \dot{W}_{i,2}^T \dot{W}_{i,1} + \rho_{i,1}
\]

According to Young’s inequality, we have

\[
s_{i,j} (\lambda_{i,j} - \mu_{i,j}) \leq \frac{1}{2} s_{i,j}^2 + \frac{1}{2} ||n_{i,j}||^2
\]

By invoking (38), we can obtain

\[
\dot{V}_{i,2} \leq -l_{\text{min}} ||e_i||^2 - c_{i,1} s_{i,1}^2 + \pi_i d_i (z_{i,2} + s_{i,2}) + \sigma_{i,1} \dot{W}_{i,2}^T \dot{W}_{i,1} + \rho_{i,1}
\]

\[
\dot{V}_{i,2} + \dot{W}_{i,2} S_{i,2} (\dot{\hat{x}}_{i,2}) + \dot{W}_{i,2}^T S_{i,2} (\dot{\hat{x}}_{i,2}) - \dot{v}_{i,2} \leq -l_{\text{min}} ||e_i||^2 - c_{i,1} s_{i,1}^2 + \pi_i d_i (z_{i,2} + s_{i,2}) + \sigma_{i,1} \dot{W}_{i,2}^T \dot{W}_{i,1} + \rho_{i,1}
\]
\[
- \frac{1}{\tau_{i,2}} \dot{z}_{i,2}^2 + z_{i,2}M_{i,2}(-) - \frac{1}{\gamma_{i,2}} \dot{W}_{i,2}^T \dot{W}_{i,2} \\
+ \sum_{j=1}^{N} a_{i,j} \dot{Q}_{j,1} \dot{W}_{j,1} + \rho_{i,1} + \frac{1}{2} ||\eta_{i,2}^0||^2 (39)
\]

Construct the virtual control function \(\alpha_{i,2}\) and the parameter adaptive law \(\dot{W}_{i,2}\) as

\[
\alpha_{i,2} = -c_{i,2} s_{i,2} - \tau_{i,1} d_{i,1} s_{i,1} - \frac{1}{2} s_{i,2} + k_{i,2} \hat{x}_{i,1} - k_{i,2} x_{i,1} \\
- \dot{W}_{i,2}^T \dot{s}_{i,2}(\hat{x}_{i,1}) + \ddot{v}_{i,2} (40)
\]

\[
\dot{W}_{i,2} = \gamma_{i,2} \left(s_{i,2} \dot{s}_{i,2}(\hat{x}_{i,1}) - \sigma_{i,2} \dot{W}_{i,2}\right) (41)
\]

where \(c_{i,2} > 0\) and \(\sigma_{i,2} > 0\) are design parameters.

From (39), (40) and (41), one has

\[
\dot{V}_{i,2} \leq -\min \{||e||^2 - c_{i,1} s_{i,1}^2 + s_{i,1}^T \tau_{i,1} d_{i,1} z_{i,2} \\
- c_{i,2} s_{i,2}^2 + s_{i,2}^T \tau_{i,3} z_{i,3} + \sigma_{i,2} \dot{W}_{i,2}^T \dot{W}_{i,2} + \sum_{j=1}^{N} a_{i,j} \dot{Q}_{j,1} \dot{W}_{j,1} + \rho_{i,2} \\
- \frac{1}{\tau_{i,2}} \dot{z}_{i,2}^2 + z_{i,2}M_{i,2}(-)\} (42)
\]

where \(\rho_{i,2} = \rho_{i,1} + \frac{1}{2} ||\eta_{i,2}^0||^2\).

**Step j** (\(3 \leq j < n\)): Similar to the derivation in Step 2, from (23), we obtain

\[
\dot{s}_{i,j} = s_{i,j+1} + z_{i,j+1} + \alpha_{i,j} - k_{i,j} \hat{x}_{i,j} \\
+ k_{i,j} x_{i,j} + \dot{W}_{i,j}^T \dot{s}_{i,j}(\hat{x}_{i,j}) - \ddot{v}_{i,j} (43)
\]

Choose the Lyapunov function candidate as

\[
V_{i,j} = V_{i,j-1} + \frac{1}{2} \dot{s}_{i,j}^2 + \frac{1}{2} \dot{Z}_{i,j}^2 + \frac{1}{2} \gamma_{i,j} \dot{W}_{i,j}^T \dot{W}_{i,j} (44)
\]

where \(\gamma_{i,j}\) is a positive design constant.

The time derivative of \(V_{i,j}\) along (42) and (43) is

\[
\dot{V}_{i,j} \leq -\min \{||e||^2 + s_{i,1}^T \tau_{i,1} d_{i,1} z_{i,2} \\
- \sum_{k=1}^{j-1} c_{i,k} s_{i,k}^2 + \sum_{k=1}^{j-1} \sigma_{i,k} \dot{W}_{i,k}^T \dot{W}_{i,k} \\
+ \sum_{j=1}^{N} a_{i,j} \dot{Q}_{j,1} \dot{W}_{j,1} + \sum_{k=1}^{j-1} s_{i,k} z_{i,k+1} + \sum_{k=2}^{j-1} \rho_{i,j} + \frac{1}{2} \dot{Z}_{i,j}^2\} (45)
\]

where \(\rho_{i,j} = \rho_{i,j-1} + \frac{1}{4} ||\eta_{i,j}^0||^2\).

**Step n**: In the final design step, from (11) and (23), one obtains

\[
\dot{s}_{i,n} = u_{i} - k_{i,n} \hat{x}_{i,1} + k_{i,n} x_{i,1} + \dot{W}_{i,n}^T s_{i,j}(\hat{x}_{i,n}) - \ddot{v}_{i,n} (49)
\]
Choose the Lyapunov function candidate as
\[ V_{i,n} = V_{i,n-1} + \frac{1}{2} \beta_i^2 + \frac{1}{2} z_{i,n}^2 + \frac{1}{2} y_{i,n} \tilde{W}_{i,n}^T \tilde{W}_{i,n} \] (50)
where \( y_{i,n} \) is a positive design constant.

The time derivative of \( V_{i,n} \) along (48) and (49) is
\[
\dot{V}_{i,n} \leq -l_{\min} |\varepsilon_i|^2 + s_{i,1}^T \pi_i d_i z_{i,2} - \sum_{k=1}^{n-1} c_{i,k} \beta_i^2_k \\
+ \sum_{k=1}^{n} \sigma_{i,k} \tilde{W}_{i,k}^T \tilde{W}_{i,k} + \sum_{j=1}^{N} a_{i,j} \eta_j 1 \tilde{W}_{j,1}^T \tilde{W}_{j,1} \\
+ s_{i,n-1} s_{i,n} + \sum_{k=2}^{n-1} s_{i,k} \tilde{z}_{i,k+1} + \frac{1}{2} s_{i,n} |\varepsilon_i|^2 \\
+ \rho s_{i,n-1} + \frac{1}{2} |\eta_{i,n}|^2 \\
- \frac{1}{y_{i,n}} \tilde{W}_{i,n}^T \tilde{W}_{i,n} + s_{i,n} \left( u_i - k_{i,n} \hat{x}_{i,1} + k_{i,n} x_{i,1} \right) \\
+ \tilde{W}_{i,n}^T \tilde{W}_{i,n} \tilde{x}_{i,n} + \frac{1}{2} s_{i,n} - \dot{\varepsilon}_i, 
\] (51)

Construct the virtual control function \( \alpha_{i,n} \) and the parameter adaptive law \( \tilde{W}_{i,n} \) as
\[
\alpha_{i,n} = -c_{i,n} s_{i,n} - s_{i,n-1} - \frac{1}{2} \beta_i^2 s_{i,n} + k_{i,n} \tilde{x}_{i,1} - k_{i,n} x_{i,1} \\
- \dot{W}_{i,n}^T s_{i,n} \tilde{x}_{i,n} + \dot{\varepsilon}_i, \quad \tilde{W}_{i,n} = y_{i,n} \left( s_{i,n} s_{i,n-1} \tilde{W}_{i,n} - \sigma_{i,n} \tilde{W}_{i,n} \right) \] (52)

where \( c_{i,n} > 0 \) and \( \sigma_{i,n} > 0 \) are design parameters.

From (12) and (16), one has
\[
u_i(t) = \begin{cases} \\
- \frac{1}{1 + \delta_{i,1}(t) \xi_i + \delta_{i,2}(t) \eta_i} \left( \alpha_{i,n} \tanh \left( \frac{s_{i,n} \alpha_{i,n}}{\epsilon} \right) + \tilde{a} \tanh \left( \frac{s_{i,n} \tilde{a}}{\epsilon} \right) \right) - \frac{\delta_{i,2}(t) a}{1 + \delta_{i,1}(t) \xi_i}, \\
\varepsilon_i |u_i(t)| + a \geq m_i \\
\alpha_{i,n} - \tilde{m}_i \tanh \left( \frac{s_{i,n} \tilde{m}_i}{\epsilon} \right) - \delta_{i,3}(t) m_i, \\
\varepsilon_i |u_i(t)| + a < m_i 
\end{cases} 
\] (54)

Since \( \forall a \in \mathbb{R} \) and \( \varepsilon > 0 \), \( -\tanh(\xi_i/\epsilon) \leq 0 \), in the interval \([t_k, t_{k+1}]\), from (12) we have \( s_{i,n} r_i \leq 0 \). As \( \delta_{i,1}(t) \in [-1, 1] \), \( \delta_{i,2}(t) \in [-1, 1] \), we have \( s_{i,n} r_i (1 + \delta_{i,1}(t) \xi_i) \leq s_{i,n} r_i (1 + \xi_i) \), and \( \delta_{i,2}(t) a (1 + \delta_{i,1}(t) \xi_i) \leq a (1 - \xi_i) \).

From (51)–(54), when \( \varepsilon_i |u_i(t)| + a \geq m_i \), we have
\[
\dot{V}_{i,n} \leq -l_{\min} |\varepsilon_i|^2 + s_{i,1}^T \pi_i d_i z_{i,2} - \sum_{k=1}^{n-1} c_{i,k} \beta_i^2_k \\
+ \sum_{k=1}^{n} \sigma_{i,k} \tilde{W}_{i,k}^T \tilde{W}_{i,k} + \sum_{j=1}^{N} a_{i,j} \eta_j 1 \tilde{W}_{j,1}^T \tilde{W}_{j,1} \\
+ s_{i,n-1} s_{i,n} + \sum_{k=2}^{n-1} s_{i,k} \tilde{z}_{i,k+1} - \frac{1}{2} s_{i,n} |\varepsilon_i|^2 \\
+ \rho s_{i,n-1} + \frac{1}{2} |\eta_{i,n}|^2 \\
- \frac{1}{y_{i,n}} \tilde{W}_{i,n}^T \tilde{W}_{i,n} + s_{i,n} \left( u_i - k_{i,n} \hat{x}_{i,1} + k_{i,n} x_{i,1} \right) \\
+ \tilde{W}_{i,n}^T \tilde{W}_{i,n} \tilde{x}_{i,n} + \frac{1}{2} s_{i,n} - \dot{\varepsilon}_i \] (55)

when \( \varepsilon_i |u_i(t)| + a < m_i \), we have
\[
\dot{V}_{i,n} \leq -l_{\min} |\varepsilon_i|^2 + s_{i,1}^T \pi_i d_i z_{i,2} - \sum_{k=1}^{n-1} c_{i,k} \beta_i^2_k \\
+ \sum_{k=1}^{n} \sigma_{i,k} \tilde{W}_{i,k}^T \tilde{W}_{i,k} + \sum_{j=1}^{N} a_{i,j} \eta_j 1 \tilde{W}_{j,1}^T \tilde{W}_{j,1} \\
+ s_{i,n-1} s_{i,n} + \sum_{k=2}^{n-1} s_{i,k} \tilde{z}_{i,k+1} - \frac{1}{2} s_{i,n} |\varepsilon_i|^2 \\
+ \rho s_{i,n-1} + \frac{1}{2} |\eta_{i,n}|^2 \\
- \frac{1}{y_{i,n}} \tilde{W}_{i,n}^T \tilde{W}_{i,n} + s_{i,n} \left( u_i - k_{i,n} \hat{x}_{i,1} + k_{i,n} x_{i,1} \right) \\
+ \tilde{W}_{i,n}^T \tilde{W}_{i,n} \tilde{x}_{i,n} + \frac{1}{2} s_{i,n} - \dot{\varepsilon}_i \] (55)

when \( \varepsilon_i |u_i(t)| + a < m_i \).
According to Young’s inequality, we have

\[ |\dot{\tilde{z}}_{i,k+1} + M_{i,k+1}(\cdot)| \leq \frac{z_{i,k+1}^2}{2} + \frac{1}{2}\tilde{z}_{i,k+1}^2 \]

where \( \rho_i = \rho_{i,n} + \frac{\xi}{2}(n-1) + \sum_{k=1}^{n} \frac{1}{2} \sigma_{i,k} W_{i,k}^* T W_{i,k}^* + \sum_{j=1}^{N} \frac{1}{2} a_{i,j} 1 \ W_{j,1}^* T W_{j,1}^* \).

**Assumption 4** For all initial conditions in the closed-loop system, there is a positive constant \( b \) satisfying \( V(0) \leq b \).

**Remark 5** It is worth noting that the proposed containment control scheme has the following advantages by means of the DSC technique. In [31] and [39], as the neighbors’ observer state information and the approximation term of the neighbors’ nonlinearities and the leaders’ nonlinearities are presented in the first virtual controller, the actual controller requires repeated differentiation of the virtual controllers during the backstepping design step. Hence, the complexity of the actual controller sharply increases. With the increasing of agents’ number and systems’ order, this problem becomes more acute. However, the proposed approach based on the DSC technique does not require such repeated derivative terms owing to the introduction of first-order filter (34). Therefore, the proposed controller is much simpler than the traditional backstepping one.

**Remark 6** Different from the classical sample-data control framework, the condition that the controller should trigger at any time instant in [30] is changed here. In this article, the controller will be triggered only when the measurement error meets a certain condition, which can further improve the control efficiency as well as decrease unnecessary resource consumption.
Based on the proposed event-triggered control, the main results can be concluded as follows.

**Theorem 1** Consider nonlinear MASs (1) in the fixed directed communication topology with prescribed performance and unmeasured states with adaptive updating laws (31) (32) (41) (47) (53) and proposed adaptive controller (30) (40) (46) (52) (54) subject to triggered signal (13) (14). Based on Assumption 1–4, all signals of the closed-loop system are SGUUB and the containment control objective is achieved with the containment errors converge to a small residual set nearing the origin with prescribed performance. Moreover, the Zeno behavior is thoroughly excluded.

**Proof** Take into account the Lyapunov function candidate for the whole system as

\[
V = \sum_{i=1}^{N} V_{i,n}
\]

Consider the set \( \Gamma_i = \{ \sum_{k=1}^{n} [s_{i,k}^2 + \frac{1}{\eta\sqrt{k}} \tilde{W}_i^T \tilde{W}_i, k] + \sum_{k=1}^{n-1} e_i k + \sum_{j=1}^{1} \frac{a_{i,j}}{\tau_{i,j}} e_i k \tilde{W}_i, j \tilde{W}_i, j + \epsilon_i^T P_i e_i, \leq 2b, \}
\]

\( i = 1, \ldots, N. \) Since \( \Gamma_i \) is a compact set, and \( M_i, j+1 \) is a continuous function, there exists a positive constant \( Q_{i,j+1} \) so that \( |M_{i,j+1}(\cdot)| \leq Q_{i,j+1} \) on \( \Gamma_i \).

Therefore, the time derivative of \( V \) is

\[
\dot{V} \leq \sum_{i=1}^{N} \left\{ -l_{min} ||e_i||^2 - c_{i,1}s_{i,1}^2 + \frac{\pi_i^2 d_i^2}{2} - s_i^2, -c_{i,1} \right\} - \sum_{k=2}^{n} (c_{i,k} - \frac{1}{2}) s_i k - \sum_{k=1}^{n} \frac{1}{2} \sigma_i k \tilde{W}_i, k \tilde{W}_i, k - \sum_{j=1}^{1} \frac{1}{2} a_{i,j} \tilde{W}_i, j \tilde{W}_i, j - \sum_{k=1}^{n-1} \left( \frac{1}{\tau_{i,k+1}} - \frac{Q_i^2, k+1}{2|l|} - \frac{1}{2} \right) s_{i,k+1}^2 + \rho_i \leq -\kappa V + R
\]

where

\[
\kappa = \min \left\{ l_{min}, \frac{\pi_i^2 d_i^2}{2} \right\},
\]

\[
2(c_{i,j} - \frac{1}{2}) \frac{\eta_{i,j}}{\sigma_{i,j}} \frac{\chi_{i,j}}{\tau_{i,j}} + 2(\frac{1}{\tau_{i,k+1}} - \frac{Q_i^2, k+1}{2\xi} - \frac{1}{2}) \right\}
\]

\[ i = 1, \ldots, N, \ j = 2, \ldots, n, \ p = 1, \ldots, n, \ k = 1, \ldots, n - 1. \]

\[
R = \sum_{i=1}^{N} \rho_i
\]

Inequality (65) implies \( \dot{V} < 0 \) on \( V = b \) when \( \kappa > R/b. \) Hence, \( V \leq b \) is an invariant set, i.e., if \( V(0) \leq b, \) for all \( t \geq 0, \) one has \( V(t) \leq b. \)

Integrating (65) over \([0,t], \) it is easy to obtain that

\[
V(t) \leq e^{-\kappa t} V(0) + \frac{R}{\kappa} \left[ 1 - e^{-\kappa t} \right]
\]

Using \( \frac{||s_{i,1}||^2}{2} \leq V, \) we have \( ||s_{i,1}||^2 \leq 2e^{-\kappa t} V(0) + \frac{2R}{\kappa} \left[ 1 - e^{-\kappa t} \right]. \) As time increases, \( e^{-\kappa t} \rightarrow 0, \) \( ||s_{i,1}||^2 \leq \frac{1}{||\sqrt{2\xi}||} \sqrt{2R/\kappa} \) holds; we can conclude that the containment errors are bounded with prescribed performance. According to Assumption 2–4, it can be obtained that all signals of the closed-loop system are SGUUB.

Furthermore, from \( \tau_{i}(t) = r_{i}(t) - u_{i}(t), \) we have \( d||\tau_{i}(t)||/dt = sign(\tau_{i}(t)) ||\tau_{i}(t)|| \leq ||\tau_{i}(t)||. \) According to (12), it is known that \( \dot{r}_{i}(t) \) is bounded, which means there exists positive constant \( H \) with \( \dot{r}_{i}(t) \leq H. \) From \( \tau_{i}(t) = 0, \) and \( \lim_{t \rightarrow \infty} \tau_{i}(t) = m_{i}, k \in \mathbb{Z}^{+}, \) one can obtain that there exists \( t^{*}, \) and \( \dot{t}_{i}^{k+1} - \dot{t}_{i}^{k} \geq t^{*} \geq m_{i}/H, \) so we conclude that\( Zeno \) behavior is avoided.

The proof is completed. \( \blacksquare \)

**4 Simulation results**

In this section, the effectiveness of the proposed approach is validated via two examples.

**A. Numerical example**

To validate the effectiveness of the proposed containment control method, an example of three followers and two leaders is utilized, and the fixed communication graph \( G \) is depicted in Fig. 1; we choose \( a_{i,j} = 1 \) on \( (i, j) \in \mathbb{R}, \) \( a_{i,j} = 0 \) otherwise. The dynamics of
three followers are modeled as

\[
\begin{align*}
\dot{x}_{i,1} &= x_{i,2} + \frac{1}{4} x_{i,1} \sin(x_{i,1}) \\
\dot{x}_{i,2} &= u_i + x_{i,2} \sin(x_{i,1}) \quad i = 1, 2, 3 \\
y_i &= x_{i,1}
\end{align*}
\]

The dynamics of the leaders are described as

\[
\begin{align*}
y_{4d} &= \sin(t + 1.86) - 0.6 \\
y_{5d} &= \sin(t + 1.86) + 0.6
\end{align*}
\]

The prescribed performance function is \(w_i(t) = (4.5 - 1.2)e^{-1st} + 1.2\). The RBFNN contains 36 nodes, which centers spaced in the range of [-6,6]. The Gaussian basis function vectors is \(\varsigma = \left[\exp(\eta \sigma), \eta \sigma, \eta \sigma^2, \eta \sigma^3, \eta \sigma^4, \eta \sigma^5, \eta \sigma^6, \eta \sigma^7, \eta \sigma^8, \eta \sigma^9, \eta \sigma^{10}, \eta \sigma^{11}, \eta \sigma^{12}, \eta \sigma^{13}, \eta \sigma^{14}, \eta \sigma^{15}, \eta \sigma^{16}, \eta \sigma^{17}, \eta \sigma^{18}, \eta \sigma^{19}, \eta \sigma^{20}, \eta \sigma^{21}, \eta \sigma^{22}, \eta \sigma^{23}, \eta \sigma^{24}, \eta \sigma^{25}, \eta \sigma^{26}, \eta \sigma^{27}, \eta \sigma^{28}, \eta \sigma^{29}, \eta \sigma^{30}, \eta \sigma^{31}, \eta \sigma^{32}, \eta \sigma^{33}, \eta \sigma^{34}, \eta \sigma^{35}, \eta \sigma^{36}\right]^T\), and the width of Gaussian function is \(\varsigma_k = 2, k = 1, 2, ..., 36\). Choose correlative design parameters as \(k_{1,1} = k_{2,1} = k_{3,1} = 15, k_{1,2} = k_{2,2} = 13, k_{3,2} = 14, \gamma_{1,1} = 1.6, \gamma_{1,2} = 8, \gamma_{2,1} = 3.3, \gamma_{2,2} = 8, \gamma_{3,1} = 1.6, \gamma_{3,2} = 8, \sigma_{1,1} = \sigma_{1,2} = \sigma_{2,1} = \sigma_{2,2} = \sigma_{3,1} = \sigma_{3,2} = 3, \chi_{1,1} = \chi_{2,1} = \chi_{3,1} = 8, \epsilon_{1,1} = \epsilon_{2,1} = \epsilon_{3,1} = 3, \eta_{1 \text{min}} = \eta_{2 \text{min}} = \eta_{3 \text{min}} = 0.6, \eta_{1 \text{max}} = \eta_{2 \text{max}} = \eta_{3 \text{max}} = 0.9, \tau = 0.02, c_{1,1} = 16, c_{2,1} = 16, c_{3,1} = 15, c_{1,2} = 110, c_{2,2} = 100, c_{3,2} = 30, \mu_1 = \mu_2 = \mu_3 = 0.8, \hat{m}_1 = \hat{m}_2 = \hat{m}_3 = 1, a = 0.1, \alpha = 0.5, \epsilon = 8, \epsilon_1 = 0.5, \epsilon_2 = 0.4, \epsilon_3 = 0.5\).

Choose initial conditions as \(x_{1,1}(0) = 0.5, x_{2,1}(0) = 0.5, x_{3,1}(0) = 0.4, x_{1,2}(0) = 0.5, x_{2,2}(0) = 0.5, x_{3,2}(0) = 0.4, \hat{x}_{1,1}(0) = 0.5, \hat{x}_{2,1}(0) = 0.5, \hat{x}_{3,1}(0) = 0.4\).

Figures 2, 3, 4, 5, 6 and 7 describe the simulation results. Figure 2 shows the containment result that the outputs of all followers asymptotically converge to the convex hull formed by the two leaders. Figure 3 plots the second-order states of three followers. Figure 4 presents the states \(x_{i,1}\) and \(\dot{x}_{i,2}(i = 1, 2, 3)\) and their corresponding observers, and it is shown that the system outputs can be well estimated by NN-based observer. The containment errors and performance bounds are shown in Fig. 5, which indicate the control scheme can impose \(v_{ij}\) to remain within the prescribed bounds. Figure 6 shows the trajectories of the event-triggered control signals \(u_i\). The event-triggered times for three followers are expressed in Fig. 7. From the simulation results, the efficiency of the proposed control scheme can be verified.

### B. Multiple single-link robot systems

Consider a multiagent system consisting of three followers and two leaders. Each follower is modeled by the following single-link robot system.

\[
M \ddot{\phi}_i + \frac{1}{2} mgl \sin(p_i) = R_i
\]

where \(i = 1, 2, 3\), \(p_i\) is the angle position, \(R_i\) is the control force of the link, \(g\) is the acceleration of gravity.
$m$ is the mass, and $l$ is the length of the robot. The parameters are chosen as $M = 0.5, m = 1, l = 1$. Let $x_{i,1} = p_1, x_{i,2} = \dot{p}_1$ and $u_{i} = R_i$. Then, the system can be rewritten as

$$
\begin{align*}
\dot{x}_{i,1} &= x_{i,2} \\
\dot{x}_{i,2} &= 2u_{i} + f_{i,2} & i = 1, 2, 3 \\
y_{i} &= x_{i,1} 
\end{align*}
$$

where $f_{i,2} = -\frac{1}{2}mglsin(x_{i,1})$.

The dynamics of the leaders are described as

$$
\begin{align*}
y_{4d} &= 0.2 \sin(1.5t) + 5.6 \\
y_{5d} &= 6.5 - \exp(-t)
\end{align*}
$$

The communication topology and the RBFNNs are same as the numerical one. The prescribed performance function is $\omega_{i}(t) = (8.8 - 1.8)e^{-0.46t} + 1.8$. Choose correlative design parameters as $k_{1,1} = k_{2,1} = k_{3,1} =$

---

Fig. 4 States and their observers

Fig. 5 Curves of containment errors $v_{i,1}(i = 1, 2, 3)$ and performance bounds

Fig. 6 Control signals $u_{i}(i = 1, 2, 3)$
Based on the above results, we can consider that the proposed control scheme is feasible for single-link robot systems.

Figure 7 The triggered number under the switching threshold strategy

\[ 15, k_{1,2} = k_{2,2} = k_{3,2} = 15, \gamma_{1,1} = 1.6, \gamma_{1,2} = 8, \gamma_{2,1} = 3.3, \gamma_{2,2} = 8, \gamma_{3,1} = 1.6, \gamma_{3,2} = 8, \sigma_{1,1} = \sigma_{1,2} = \sigma_{2,1} = \sigma_{2,2} = \sigma_{3,1} = \sigma_{3,2} = 3, \chi_{1,1} = \chi_{2,1} = \chi_{3,1} = 8, \vartheta_{1,1} = \vartheta_{2,1} = \vartheta_{3,1} = 3, \eta_{1\min} = \eta_{2\min} = \eta_{3\min} = 0.6, \eta_{1\max} = \eta_{2\max} = \eta_{3\max} = 0.9, \tau = 0.015, c_{1,1} = 11, c_{2,1} = 11, c_{3,1} = 10, c_{1,2} = 110, c_{2,2} = 120, c_{3,2} = 30, m_1 = m_2 = m_3 = 0.8, \tilde{m}_1 = \tilde{m}_2 = \tilde{m}_3 = 1, a = 0.1, \bar{a} = 0.5, \epsilon = 8, \varepsilon_1 = 0.4, \varepsilon_2 = 0.4, \varepsilon_3 = 0.4. \]

Choose initial conditions as \( x_{1,1}(0) = 5.5, x_{2,1}(0) = 5.5, x_{3,1}(0) = 5.5, x_{1,2}(0) = 5.5, x_{2,2}(0) = 5.5, x_{3,2}(0) = 5.5, \hat{x}_{1,1}(0) = 0, \hat{x}_{2,1}(0) = 0, \hat{x}_{3,1}(0) = 0. \)

The leader-following consensus performance is shown in Figure 8, and it can be seen that the followers converge into the convex hull limited by the leaders, which means that the containment control is achieved. Figure 9 plots the second-order states of three followers. The local neighborhood containment errors and performance bounds are shown in Figure 10, which indicated the containment errors can converge into the specified boundary. The trajectories of the event-triggered control input signals \( u_i \) are described in Figure 11. The event-triggered times for three followers are expressed in Figure 12. Based on the above results, we can consider that the proposed control scheme is feasible for single-link robot systems.

Figure 8 Trajectories of \( x_{i,1}(i = 1, 2, 3) \) and leaders \( y_{4d} \) and \( y_{5d} \)

Figure 9 Trajectories of \( x_{i,2}(i = 1, 2, 3) \)

Figure 10 Curves of containment errors \( v_{i,1}(i = 1, 2, 3) \) and performance bounds
5 Conclusion

In this paper, the adaptive event-triggered containment control scheme has been investigated for a class of strict-feedback nonlinear MASs with prescribed performance and unmeasured states which can be estimated by NN-based observers. Based on the backstepping control and DSC technique, the adaptive consensus controllers are developed. The proposed control method can figure out not only the problem of the explosion of complexity inherent in the backstepping design step but also the problem of unavailable states. Different from the classical time-triggered strategy, the event-triggered strategy can decrease the heavy communication burden and improve the control efficiency. It is proved that the containment control purpose has been accomplished. In particular, the control signals have been updated by switching threshold event-triggered mechanism, which provides more elasticity in the balance of network constraints and system performance. On account of the prescribed performance function, the local neighborhood containment errors converge into a prescribed set. According to the Lyapunov stability theory, it is demonstrated that all signals of the closed-loop system are SGUUB. Finally, the effectiveness of the proposed containment control approach is confirmed by simulation examples. Future research topics could include considering the updating of both the control signal and the adaptive parameters.

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Data availability All data generated or analyzed during this study are included in this published article.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any experiments with human or animal participants performed by any of the authors.

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