Parametrization of the $\bar{p}p$ Elastic Scattering Differential Cross Section Between $2 \text{ GeV}/c \leq P_{\text{lab}} \leq 16 \text{ GeV}/c$

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A parameterization of the $\bar{p}p$ differential elastic scattering cross section in the beam momentum range from 2 to 16 GeV/c is proposed. The parameterization well describes the existing data including the observed diffraction pattern at four-momentum transfer $|t|$ up to 1.5-2.0 GeV$^2$. It can be used for detailed calculations of the radiation load on the detectors being designed for the PANDA detector at the future FAIR facility in Darmstadt.

1 Introduction

The science goals underlying the international FAIR project [1] that is being realized in Darmstadt span a broad range of research activities on the structure of matter. One component of this facility is directed towards studies of hadronic matter at the sub-nuclear level with beams of antiprotons. These studies focus on two key aspects: confinement of quarks and the generation of the hadron masses. They are intimately related to the existence (and spontaneous breaking) of chiral symmetry, a fundamental property of the strong interaction. These goals will be persuaded by performing precision measurements of charged and neutral decay products from antiproton-proton annihilation in the charmonium mass region. These high rate experiments place demanding requirements on the materials and detectors employed. Since up to one half of the total antiproton-proton reaction cross section in the beam momentum range of interest is elastic scattering, a good description of the $\bar{p}p$ elastic scattering differential cross section is required in order to quantitatively assess the detectors being designed. This parameterization is additionally required since the beam monitoring and luminosity control at the experiment will be implemented via measuring the elastic $\bar{p}p$ scattering.

1.1 The High Energy Storage Ring (HESR) for antiprotons

The future Facility for Antiproton and Ion Research - FAIR - in Darmstadt will include a storage ring for beams of phase space cooled antiprotons with unprecedented quality and intensity. The antiproton beam will be produced by a primary proton beam from the planned fast cycling, superconducting 100 T-m ring. The antiprotons will be collected with an average rate of about $10^7$/s and then stochastically cooled and stored. After $5 \times 10^{10}$ antiprotons have been produced, they will be transferred to the HESR where internal experiments in the beam momentum range $1.5 - 15 \text{ GeV}/c$ can be performed.

The HESR is designed as a racetrack shaped storage ring with a maximum magnetic bending power of 50 Tm. The storage ring will have a circumference of 574 m, including

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two 132 m long straight sections. One of these sections will be mainly used for the installation of an internal target in combination with a large detector system. The opposite long straight section is used for beam injection, beam acceleration and electron cooling. In addition a stochastic cooling system for transverse and longitudinal cooling is foreseen to be installed at the entrance and exit of straight sections. The latter has to be designed to allow for experiments with either high momentum resolution of about $10^{-5}$ at reduced luminosity or at high luminosity up to $2 \times 10^{32}/cm/s$ with enlarged momentum spread.

1.2 Hadron Spectroscopy with Antiproton Annihilation at the PANDA detector

The PANDA experiment, located at an internal target position of the High Energy Storage Ring for anti-protons, is one of the large installations at the future FAIR facility [2]. It is being planned by a multi-national collaboration, currently consisting of about 350 physicists from 50 institutions in 15 countries. The PANDA detector is designed as a multi-purpose setup. The cornerstones of the PANDA physics program are:

- Study of narrow charmonium states at so far unprecedented precision;
- Search for gluonic excitations such as hybrids and glueballs in the charmonium mass region;
- Investigate the properties of mesons with hidden and open charm in the nuclear medium;
- Spectroscopy of double strange hypernuclei.

The experiment will use internal targets. It is conceived to use either pellets of frozen $H_2$ or cluster jet targets for the $\bar{p}p$ reactions, and wire targets for the $\bar{p}A$ reactions. Pellet targets such as the one in operation at WASA at COSY shoot droplets of frozen $H_2$ with radii 20-40 $\mu$m with typical separations of 1 mm transversely through the beam.

This detector facility must be able to handle high rates ($10^7$ annihilations/s), with good particle identification and momentum resolution for $\gamma$, $e$, $\mu$, $\pi$, $K$, and $p$. Furthermore, the detector must have the ability to measure $D$, $K_S$, and $\Lambda$ which decay at displaced vertices. Finally, a large solid angle coverage is essential for partial wave analysis of resonance states.

In order to cope with the variety of final states and the large range of particle momenta and emission angles, associated with the different physics topics, the detector has almost $4\pi$ detection capability both for charged particles and photons. It is divided into two sub-components, a central target spectrometer and a forward spectrometer with an overall length of 12 m of the total detector.

The beam monitoring and control at the experiment will be implemented via measurement of elastic $\bar{p}p$ scattering. Thus, it is very important to have a good systematic description of known experimental data. It is a subject of the paper. In Sec. 1 we give the main formulae. The fitting procedure is described in Sec. 2. A short conclusion is presented at the end of the paper.
2 $\bar{p}p$ Elastic Scattering

A collection of the differential cross section data for $\bar{p}p$ elastic scattering with beam momentum above 1 GeV/c is presented in Fig. 1 [5]–[22]. As seen from the figure, at low beam momenta ($1 - 2$ GeV/c) Coulomb scattering dominates at low 4-momentum transfer ($|t| < 0.05$ GeV$^2$). At higher energies a dip appears in the region $|t| \sim 0.4$ GeV$^2$. Above $P_{lab} > 4$ GeV/c an additional diffractional dip appears near $|t| \sim 2$ GeV$^2$.

At low momentum transfer the cross section is usually parameterized as

$$\frac{d\sigma}{dt} = \frac{\pi}{k^2} \left| f_C e^{i\delta} + f_H \right|^2 = \frac{d\sigma_C}{dt} + \frac{d\sigma_{int}}{dt} + \frac{d\sigma_H}{dt},$$

where

$$\frac{d\sigma_C}{dt} = \frac{4\pi\alpha^2_{EM} G^4(t)}{\beta^2 t^2};$$

$$\frac{d\sigma_{int}}{dt} = \frac{\alpha_{EM} \sigma_{Total}}{\beta |t|} G^2(t) e^{\frac{1}{2}Bt} (\rho \cos \delta + \sin \delta)$$

$$\frac{d\sigma_H}{dt} = \frac{\sigma_{Total}^2 (1 + \rho^2)}{16\pi} e^{Bt}.$$

Here, $d\sigma_C/dt$ and $d\sigma_H/dt$ are the Coulomb and hadronic parts of the cross section, respectively. $d\sigma_{int}/dt$ represents the interference term. $\alpha_{EM}$ is the fine structure constant.
Table 1: Fit parameters to the measured data at the given beam momenta.

| $P_{lab}$ | $A_1$         | $t_1$        | $A_2$         | $t_2$        | $A_3$ | $\chi^2$ | $N$ |
|---------|---------------|-------------|---------------|-------------|------|----------|-----|
| 2.33    | 528.4 ± 14.0 | 0.085 ± 0.002 | 0.137 ± 0.014 | 0.430 ± 0.028 | 2.21 ± 0.27 | 48.6 | 66 |
| 2.85    | 443.1 ± 13.3 | 0.094 ± 0.002 | 0.172 ± 0.016 | 0.377 ± 0.020 | 1.82 ± 0.21 | 74.9 | 88 |
| 5.00    | 268.0 ± 15.7 | 0.113 ± 0.003 | 0.350 ± 0.033 | 0.221 ± 0.006 | 2.11 ± 0.12 | 250.8 | 86 |
| 5.70    | 237.9 ± 11.4 | 0.091 ± 0.003 | 0.106 ± 0.026 | 0.366 ± 0.409 | 0.92 ± 0.23 | 21.9 | 47 |
| 6.20    | 279.1 ± 11.0 | 0.086 ± 0.002 | 0.136 ± 0.009 | 0.264 ± 0.004 | 1.00 ± 0.02 | 339.2 | 70 |
| 10.10   | 276.9 ± 41.1 | 0.096 ± 0.006 | 0.218 ± 0.080 | 0.193 ± 0.018 | 3.11 ± 0.74 | 50.8 | 35 |
| 10.40   | 173.2 ± 6.9  | 0.090 ± 0.001 | 0.079 ± 0.020 | 0.284 ± 0.033 | 0.97 ± 0.30 | 42.9 | 61 |
| 15.95   | 140.3 ± 54.7 | 0.099 ± 0.010 | 0.171 ± 0.182 | 0.203 ± 0.065 | 2.13 ± 2.27 | 7.4 | 23 |
| 16.00   | 108.4 ± 6.5  | 0.094 ± 0.004 | 0.034 ± 0.039 | 0.615 ± 0.683 | 0.17 ± 0.26 | 8.6 | 34 |

The proton dipole form factor $G(t) = (1 + \Delta)^{-2}$, where $\Delta = |t|/0.71$. The Coulomb phase

$$
\delta(t) = \alpha_{EM} \left[ 0.577 + \ln \left( \frac{B|t|}{2} - 4\Delta \right) + 4\Delta \ln(4\Delta + 2\Delta) \right].
$$

$\sigma_{Total}$ is the total hadronic cross section, and $B$ is the so-called slope parameter. $\rho$ is the ratio of real to imaginary parts of the hadronic scattering amplitude at zero momentum transfer. The hadronic part of the amplitude can be parameterized as a simple exponent, $f_H \propto e^{4\Delta t}$.

Neglecting $\rho$ and integrating $d\sigma_H/dt$, one finds $B$ to be

$$
B = \frac{\sigma^2_{Total}}{16\pi \sigma_{Elastic}}.
$$

Using the Particle Data Group’s parameterization for the total and elastic $\bar{p}p$ scattering cross section [23], it is easy to calculate $B$ at a given energy.

$$
\sigma_{Total} = 38.4 + 77.6 P_{lab}^{-0.64} + 0.260 \ln^2(P_{lab}) - 1.20 \ln(P_{lab}) \text{ (mb),} v
$$

$$
\sigma_{Elastic} = 10.2 + 52.7 P_{lab}^{-1.16} + 0.125 \ln^2(P_{lab}) - 1.28 \ln(P_{lab}) \text{ (mb).}
$$

This simple parameterization is indicated below by the solid line in Figs 3–5. As seen in Figs. 3–5, such parameterization can be applied at $P_{lab} \geq 1.1$ GeV/c and $|t| \leq 0.2 - 0.3$ GeV$^2$.

In order to describe the differential cross section in a wider range of $t$, a $\chi^2$ minimization of the following expression has been applied to the data:

$$
\frac{d\sigma}{dt} = A_1 \cdot \left[ e^{t/2t_1} - A_2 \cdot e^{t/2t_2} \right]^2 + A_3 \cdot e^{t/t_3}.
$$

The parameters are presented in Table 1 and Fig. 2. This parameterization well describes most part of the data, with the following notable exceptions. The data at $P_{lab} = 3.55$ and 3.66 GeV/c were not included in the table 1, because there were only few points at small $|t|$. The data at $P_{lab} = 6.2$ GeV/c [15] gave the large value of $\chi^2$ due to the same reason. The fit of the data at $P_{lab} = 16$ GeV/c [17] resulted in too small value for $A_3$, because the points at large $t$ were not presented. The situation with the data at $P_{lab} = 5$ and 10.1 GeV/c [13, 19] was not so clear.
Figure 2: Energy dependencies of the fitted parameters. The light points are the data of the tabl. 1. Dark points are from the tabl. 2.

Figure 3: Differential elastic cross sections of $\bar{p}p$ scattering at $P_{lab} = 2 - 4$ GeV/c. Points are experimental data [7, 8, 10]. Dashed lines are results of single exponential parametrizations. Histograms represent our parametrization.
Figure 4: The same as in Fig. 3 for $P_{\text{lab}} = 4 - 8$ GeV/c. Points are experimental data [13, 14, 15, 16].

Figure 5: The same as in Fig. 3 for $P_{\text{lab}} = 8 - 16$ GeV/c. Points are experimental data [19, 20, 16, 17].
Table 2: Summary of the parameters of the constrained fit to the differential cross section data.

| $P_{lab}$ | $A_1$   | $A_2$   | $t_2$   | $A_3$   | $\chi^2$ | $N$ |
|-----------|---------|---------|---------|---------|----------|-----|
| 2.33      | 582 ± 17.0 | 0.196 ± 0.008 | 0.322 ± 0.012 | 2.72 ± 0.38 | 57.6 | 66 |
| 2.85      | 426 ± 8.5   | 0.153 ± 0.005  | 0.394 ± 0.012 | 1.78 ± 0.19 | 84.9 | 88 |
| 3.55      | 382 ± 50.2  | 0.137 ± 0.021  | 0.392 ± 0.060 | 1.33 ± 0.42 | 10.7 | 12 |
| 5.7       | 232 ± 7.4   | 0.110 ± 0.010  | 0.351 ± 0.028 | 0.77 ± 0.20 | 28.4 | 47 |
| 10.40     | 171 ± 1.9   | 0.074 ± 0.004  | 0.293 ± 0.015 | 0.89 ± 0.12 | 42.9 | 61 |
| 15.95     | 113 ± 4.2   | 0.049 ± 0.013  | 0.289 ± 0.049 | 0.69 ± 0.32 | 8.47 | 23 |

The parameterization did not allow to determine a regular dependence of the parameters on the beam momentum. The next step was to redo the minimization of the function to the data, excluding the data at $P_{lab} = 3.66, 5, 6.2, 10.1,$ and $16$ GeV/c. It was assumed that $t_1 = 0.0899$ (an average value of $t_1$ in Table 1) in order to reduce the number of the parameters. The results are presented in Table 2 and Fig. 2. The energy dependence of the parameters becomes more regular.

In order to interpolate the parameters presented here to other beam momenta in the range $2 < P_{Lab} < 16$ GeV/c, the results in Table 2 have been parameterized as follows:

$$A_1 = 115.0 + 650.0 \cdot e^{-P_{lab}/4.08},$$
$$t_1 = 0.0899,$$
$$A_2 = 0.0687 + 0.307 \cdot e^{-P_{lab}/2.367},$$
$$t_2 = -2.979 + 3.353 \cdot e^{-P_{lab}/0.67009},$$
$$A_3 = 0.11959 + 3.86474 \cdot e^{-P_{lab}/0.765}.$$  

This parameterization is indicated by the solid lines in Fig. 2. A description of the experimental data is presented in Figs. 3–5. As seen, we have a good description of main part of the data in the region of $t \leq 1.5 – 2.0$ GeV/c². However, there is a regular discrepancy between the data at $P_{lab} = 5$ GeV/c [13] and the parameterization in the region of the second maximum. We suppose that the data at $P_{lab} = 6.2$ GeV/c [15] are distorted too strong, and are not in an agreement with common regularity. The data at $P_{lab} = 10.1$ [19] are reproduced only roughly in the second maximum. It would be well to re-measure the data at pointed momenta.

For Monte Carlo simulation of the elastic scattering, $d\sigma/dt/\sigma_{elastic}$ was presented as a sum of two distributions:

$$\frac{1}{\sigma_{elastic}} \frac{d\sigma}{dt} = d_1(t) + d_2(t),$$
$$d_1(t) = A_1 \cdot \left[ e^{t/2t_1} - A_2 \cdot e^{t/2t_2} \right]^2 / \int \frac{d\sigma}{dt} dt',$$
$$d_2(t) = A_3 \cdot e^{t/t_2} / \int \frac{d\sigma}{dt} dt'.$$

Sampling of $t$ according to the second distribution was performed by the formulae

$$t = t_2 ln \left[ 1 - \xi(1 - e^{t_{max}/t_2}) \right],$$

where $\xi$ is random number uniformly distributed in the interval [0,1].
The first distribution was re-written as

\[ d_1(t) = \left[ e^{t/t_1} + A^2 \cdot e^{t/t_2} \right] \frac{\left[ e^{t/2t_1} - A^2 \cdot e^{t/2t_2} \right]^2}{\left[ e^{t/t_1} + A^2 \cdot e^{t/t_2} \right]} / \int \frac{d\sigma}{dt} dt'. \]

The expression in the first brackets was considered as a distribution, and the fraction was taken as rejection function.

![Figure 6: Hit positions in the Straw Tube Tracker detector of the PANDA experiment due to the elastic scattering.](image)

Figure 6: Hit positions in the Straw Tube Tracker detector of the PANDA experiment due to the elastic scattering.

Now, the proposed parameterization and the Monte Carlo algorithm are implemented in PANDA computational framework which allows one to estimate an influence of the elastic scattering on the PANDA sub-detectors. To understand expected results, a simple consideration can be applied. In PANDA experiment, the target will be surrounding by beam pipe. Thus, the scattering anti-protons with \( \theta < 10^\circ \), and the recoil protons with energy \( T < 20 \text{ MeV} \) flying with polar angle near to \( 90^\circ \) will not be registered. The differential cross-section falls down very quickly with decreasing of emission angle of the recoil protons. So, the recoil protons will be registered by central tracking detector as a broad jet with main axis depending on the beam energy. Results of direct simulation of hit positions in Straw Tube Tracker detector due to the elastic scattering are in agreement with the above given consideration (Fig. 6). According to it, the elastic scattering will produce non-uniform radiation load in the central part of the detector.

Effect of the elastic scattering has to be taken into account at design of the PANDA sub-detectors, and for creation of the beam monitor.

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