Pion-only, chiral light-front model of the deuteron

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We investigate the symptoms of broken rotational invariance, caused by the use of light front dynamics, for deuteron obtained using one- and two-pion-exchange potentials. A large mass splitting between states with $m = 0$ and $m = 1$ is found for the deuteron obtained from the one-pion-exchange (OPE) potential. The size of the splitting is smaller when the chiral two-pion-exchange (TPE) potential is used. When the TPE potential constructed without chiral symmetry is used, the deuteron becomes unbound. These results arise from significant relativistic effects which are much larger than those of the Wick-Cutkosky model because of the presence of the tensor force.

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I. INTRODUCTION

Recently, there has been much interest in effective field theories (EFT) for nuclear physics. In particular, the expansion of the nuclear forces about the chiral limit gives good results. Attempts have been made to apply pion-less EFTs to study deuteron properties. However, it has been shown that pions are required to obtain a well-controlled theory.

It is not surprising that the pions are necessary. Indeed, the gross features of the deuteron are determined principally by the one-pion-exchange (OPE) potential, a consequence of EFT. However, the short-distance behavior of the naive OPE potential is singular, and must be regulated. Physically, this regulation is provided by the exchange of more massive mesons and multi-meson exchanges. EFT tells us that these high-energy parts can be replaced by anything that has the correct form dictated by symmetry, and the low energy properties of the deuteron will be left unaffected.

The expectation that pionic effects are very important is related to work by Friar, Gibson, and Payne (FGP) who obtained non-relativistic deuteron wave functions using only the OPE potential. In their calculations a pion-nucleon form factor replaces the high-energy physics. The only influence of chiral symmetry in their model is to require that the pion-nucleon coupling be of the form $\tau \sigma \cdot \nabla$.

In this paper, we build upon the FGP model by performing a relativistic calculation using light-front dynamics. In addition, we go beyond the OPE potential and include the two-pion-exchange (TPE) potential. Chiral symmetry is known to have profound implications for the TPE interaction, so that we will be able to study the effect that ignoring chiral symmetry has on the deuteron.

Our plan is to introduce a model Lagrangian for nuclear physics using only nucleons and pions. A Lagrangian which includes chiral symmetry has been used to compute the OPE and TPE potentials for a new light-front nucleon-nucleon potential which involves the exchange six different mesons. We retain only the contributions arising from pionic exchanges here in Sect. II. The resulting pion-only potentials are then used to see if the deuteron state is bound and if so, to compute the binding energy in Sect. III. A final section is devoted to presenting a brief set of conclusions.

II. MODEL

We consider a pion-only light-front nucleon-nucleon potential derived from a nuclear Lagrangian. This model is inspired by the non-relativistic one-pion-exchange model used by Friar, Gibson, and Payne. We generalize their model to form the basis of our pion-only light-front model, which includes relativity automatically. This pion-only model is essentially the same as the model presented in Ref. restricted to pions only.

Our starting point is a nuclear Lagrangian which incorporates a non-linear chiral model for the pions. The Lagrangian is based on the linear representations of chiral symmetry used by Gursey. It is invariant (in the limit where $m_\pi \to 0$) under chiral transformations.

The pion-only model prescribes the use of nucleons $\psi$ and the $\pi$ meson, which is a pseudoscalar isovector. The Lagrangian $\mathcal{L}$ is given by

$$\mathcal{L} = \frac{1}{4} f^2 \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{4} m_\pi^2 f^2 \text{Tr}(U + U^\dagger - 2)$$

$$+ \bar{\psi} \left[ i \not{\partial} - U M \right] \psi,$$

where the bare mass of the nucleon is $M$ and the pion is $m_\pi$. The unitary matrix $U$ can be chosen to have one of the three forms $U_i$:

$$U_1 \equiv e^{i \gamma_5 \tau \cdot \pi / f},$$

$$U_2 \equiv \frac{1 + i \gamma_5 \tau \cdot \pi / 2f}{1 - i \gamma_5 \tau \cdot \pi / 2f^2},$$

$$U_3 \equiv \sqrt{1 - \pi^2 / f^2 + i \gamma_5 \tau \cdot \pi / f},$$
which correspond to different definitions of the fields. Note that each of these definitions can be expanded to give

\[
U = 1 + i\gamma_5 \frac{\tau \cdot \pi}{f} - \frac{\pi^2}{2f^2} + \mathcal{O} \left( \frac{\pi^3}{f^3} \right). \tag{5}
\]

In this paper, we consider at most two pion exchange potentials, so we consider \( U \) to be defined by Eq. (5).

In the limit where \( m_\pi \to 0 \), the Lagrangian in Eq. (1) is invariant under the chiral transformation

\[
\psi \to e^{-i\gamma_5 \tau \cdot a} \psi, \quad U \to e^{-i\gamma_5 \tau \cdot a} U e^{-i\gamma_5 \tau \cdot a}. \tag{6}
\]

We use the Lagrangian to obtain the OPE and TPE light-front nucleon-nucleon potentials \([9, 10, 11, 13, 14]\). We find that there are three classes of TPE potentials. The first class, the TPE box diagrams, consists of diagrams where the pion lines do not cross. The second class, the TPE contact diagrams, consists of diagrams where at least one of the vertices is a two-pion contact vertex, as demanded by chiral symmetry. The third class is the TPE crossed diagrams, where the pion lines cross.

As discussed in Refs. \([10,11,15,16]\), we have some freedom in deciding which TPE diagrams to include in our potential. In particular, we may neglect just the crossed diagrams, or both the crossed and contact diagrams. Although neglecting these diagrams may affect the exact binding energy calculated, we should find a partial restoration of rotational invariance as compared to using just the OPE potential.

We also want to keep the potentials chirally symmetric as well. Whereas rotational invariance of the potential is partially restored by including higher-order potentials, chiral symmetry is restored by including the \( \pi \pi \) contact interaction graphs to the same order as the non-contact graphs.

Chiral symmetry tells us that for pion-nucleon scattering at threshold, the time-ordered graphs approximately cancel. Furthermore, upon closer examination, we find that all the light-front time-ordered graphs for the scattering amplitude vanish except for the two graphs with instantaneous nucleons and the contact graph. These graphs are shown in Fig. 1. Using the Feynman rules Refs. \([10,11,15,16]\), and denoting the nucleon momentum by \( k \) and the pion momentum by \( q \), we find that

\[
\mathcal{M}_U = C \frac{\tau_i \tau_j}{2(k^+ + q^+)} u(k') \gamma^j u(k), \tag{7}
\]

\[
\mathcal{M}_X = C \frac{\tau_i \tau_i}{2(k^+ - q^+)} u(k') \gamma^j u(k), \tag{8}
\]

\[
\mathcal{M}_C = C \frac{-\delta_i \tau_j}{M} u(k') u(k), \tag{9}
\]

where the factors common to all amplitudes are denoted by \( C \).

For threshold scattering, we take \( k^+ = M \) and \( q^+ = m_\pi \). In that limit, we find

\[
\mathcal{M}_U = C' \frac{\delta_i \tau_j - i\epsilon_{i,j,k} \tau_k}{2(M - m_\pi)}, \tag{10}
\]

\[
\mathcal{M}_C = C' \frac{-\delta_i \tau_j}{M}, \tag{11}
\]

where \( C' = C \pi(k') u(k) \). In the limit that \( m_\pi \to 0 \), the sum of these terms vanishes. The term in these equations proportional to \( \tau_k \) is the famous Weinberg-Tomazawa term \([17,18]\).

The fact that the amplitudes cancel only when the contact interaction is included demonstrates that chiral symmetry can have a significant effect on calculations. In terms of two-pion-exchange potentials, this result means that the contact potentials cancel strongly with both the iterated box potentials and the crossed potentials. This serves to reduce the strength of the total two-pion-exchange potential, which should lead to more stable results.

However, since we do not use the crossed graphs for the nucleon-nucleon potential, we must come up with a prescription which divides the contact interactions into two parts which cancel the box and crossed potentials separately. We do this by formally defining two new contact interactions, so that

\[
\mathcal{M}_{Cu} \equiv \frac{M}{2(M + \lambda m_\pi)} \mathcal{M}_C, \tag{13}
\]

\[
\mathcal{M}_{Cx} \equiv \mathcal{M}_C - \mathcal{M}_{Cu}, \tag{14}
\]

and the value of \( \lambda \) is of order unity. With these definitions, we find that at threshold and in the chiral limit,

\[
\mathcal{M}_U + \mathcal{M}_{Cu} = 0, \tag{15}
\]

\[
\mathcal{M}_X + \mathcal{M}_{Cx} = 0. \tag{16}
\]

Since the prescription given by equations (13) and (14) leads to the correct results in the chiral limit for pion-nucleon scattering, we use it to factor the TPE potentials so that chiral symmetry can be approximately maintained while neglecting the crossed potentials.

We find that the results of our TPE calculation which include approximate chiral symmetry have a very weak dependence on the value of \( \lambda \), provided that \( 0 < \lambda < 1 \). As such, we show only the results for \( \lambda = 1 \). These features indicate that we can incorporate approximate chiral symmetry without including crossed graphs simply by weighting each contact interaction graph with a factor of \( \lambda \).

### III. RESULTS

To numerically calculate the bound states for these pion-only potentials, we must choose values for the potential parameters. As a starting point, we look for inspiration from the non-relativistic one-pion-exchange model used by Friar, Gibson, and Payne (FGP) \([8]\). The basic parameters of the model are the mass of the nucleon \( M = 938.958 \text{ MeV} \), the mass of the pion \( m_\pi = 938 \text{ MeV} \), the pion-nucleon scattering length \( a_\pi = 0.4 \cdot 10^{-3} \text{ FGP units} \), the nucleon-nucleon scattering length \( a_N = -0.6 \cdot 10^{-3} \text{ FGP units} \), the pion-nucleon coupling constant \( g_\pi = 8 \), and the nucleon-nucleon coupling constant \( g_N = \frac{1}{2} \).
and the parameter $\Lambda$ is fit to reproduce the deuteron binding energy for a given value of $n$. We can obtain the FGP model from our light-front nucleon-nucleon potential by considering only the OPE potential and performing a non-relativistic reduction. To understand our results, it is important to first consider how well the light-front potential and wave functions approximate the non-relativistic potential and wave functions.

It is useful to start by reviewing some of the properties of the deuteron. First, we note that the deuteron is very lightly bound. Second, although a majority of the deuteron wave function resides in the non-relativistic regime, it has a high-momentum tail that falls off rather slowly, as $1/k^4$. Recalling our experience with the Wick-Cutkosky model, where we found that mass splitting between states with different $m$ values is small for lightly bound states, it may appear plausible that masses of the $m = 0$ and $m = 1$ states of the deuteron are approximately the same when calculated with the light-front OPE potential. However, the high-momentum tail of the deuteron enhances the effects of the potential’s relativistic components. Since the breaking of rotational invariance is a relativistic effect, this implies that the mass splitting may be large for the deuteron calculated with the light-front potential.

To clearly understand how a large mass splitting might arise in the light-front pion-only model, we take a step back and consider a scalar version of the pion-only model, in which we assume that the pion has a scalar coupling to the nucleon. This allows for a more direct comparison with the Wick-Cutkosky model, since the main difference between the scalar-pion-only potential and the Wick-Cutkosky potential (aside from an isospin factor) is the pion-nucleon form factor. Since the denominator of the form factor has the same form as the denominator of the potential, the form factors do not significantly change the rotational properties of the scalar-pion-only potential. Another difference is that factors of $\overline{m}u$ appear in the numerator of the scalar-pion-only potential, but the effect of those is small.

The bound state calculations are performed using techniques detailed in Refs. [10, 11], so we concern ourselves here only with the results. In Table I, we show the binding energies for deuterons calculated with the scalar-pion-only model. Two pion-nucleon form factors are considered, the first one has $\Lambda = 1.0$ GeV, for which the coupling constant was fit to give the correct binding energy for the non-relativistic potential, and the second form factor uses $\Lambda = 1.915$ GeV, which was fit to give the correct binding energy for the light-front potential. Those form factors are used to calculate the binding energies for the non-relativistic and light-front potentials, as well as the instantaneous and retarded potentials, which are relativistic and defined by analogy with instantaneous and retarded potentials defined for the Wick-Cutkosky model in Ref. [12]. We find that the binding energies for all of the potentials have the same order of magnitude, and that the light-front potentials have consistently lower binding energies, which confirms the behavior observed in Ref. [10]. In addition, the binding energies of the $m = 0$ and $m = 1$ light-front potentials are essentially degenerate.

Now we ready to consider the (pseudoscalar) pion-only model. The only difference between this potential and the scalar-pion-only potential is that the numerator contains factors of $\overline{m}v/\Lambda$ instead of $\overline{m}u$. Although this seems to be only a small change, it has a large effect on the binding energies. The pseudoscalar coupling generates a tensor force, which is more sensitive to the relativistic components of the wave function than the scalar force. In general, this means that the differences in binding energies obtained with different potentials, such as those shown in Table I, will be larger for the pion-only potential.

In Table II, we show the binding energies for deuterons computed with the pion-only model. Two pion-nucleon form factors are considered, the first one has $\Lambda = 1.01$ GeV, which was fit for the non-relativistic pion-only model, and the second form factor uses $\Lambda = 1.9$ GeV, which was fit to give the most reasonable binding energy for the light-front potentials. We find that the binding energies vary greatly depending on which potential is used. In fact, the light-front potentials do not bind the deuteron with the first form factor, and with the second form factor, the mass splitting between the $m = 0$ and $m = 1$ states is very large. These facts indicate that the deuteron wave functions are very sensitive to subtle changes in the relativistic structure of the pion-only potentials.

The reason for this mass splitting is that the OPE potential breaks rotational invariance. To reduce this splitting, higher-order potentials must be used. Naively, one might think that we can choose any set of TPE graphs which is a truncation of a rotationally-invariant infinite series of graphs. In particular, one might think that using the TPE box graphs, which we denote as the non-chiral-TPE (ncTPE) potential, would be adequate for our analysis of rotational invariance.

Another choice for the TPE potential is to take sum of the TPE box graphs and the TPE contact graphs. To incorporate chiral symmetry as accurately as possible without including the crossed TPE potentials, we weight the contact vertex with a factor of $M/(2(M + m_u))$, as explained in earlier. Note that in the sum, which we call the TPE potential, chiral symmetry provides a cancelation between the box diagrams and the contact diagrams. This indi-
icates that the results obtained with the TPE potential will be more stable than those obtained with the ncTPE potential.

To check this stability and the restoration of rotational invariance of the pion-only model, we calculate the energy of the deuteron using the OPE, OPE+TPE, and OPE+ncTPE potentials. We verify that the results are independent of the choice of the pion-nucleon form factor by considering three different choices for the form factor.

For the first pion-nucleon form factor, we choose \( n = 1 \) and find that \( \Lambda = 1.9 \text{ GeV} \) gives a reasonable range of binding energies. We use the form factor to calculate the lowest energy bound state with arbitrary total angular momentum.

The results for the first pion-nucleon form factor are shown in Table \( \text{III} \). Note that the results show that both the mass splitting and the difference in the percentage of the D-state wave function decrease when TPE diagrams are included. In addition, the wave functions are almost completely in the \( J = 1 \) state. As for the D-state probability, we first note that it increases with the binding energy. It is consistent (given the range of values it and the binding energy take) with the value of 6% reported in Ref. [1], for the FGP model.

Table \( \text{III} \) also shows that numerically the states are almost angular momentum eigenstates, since the percentage of each state with \( J = 1 \) is almost 100%. This happens even though the eigenstates of the light-front Hamiltonian are not in principle eigenstates of the angular momentum. However, this does not mean that rotational invariance is unbroken. Rotations relate the different \( m \) states, and from the mass splittings, we see that the states do not transform correctly.

Another thing to note in Table \( \text{III} \) are the effects of using the ncTPE potential. As mentioned earlier, this potential is greater in magnitude than the TPE potential and should have a larger effect on the binding energy. In fact, the effect is so large that it serves to unbind the deuteron. (Strictly speaking, this indicates only that the binding energy is very small or zero; the error in the binding energy calculation increases as the binding energy approaches zero.) Because the ncTPE potential has such a large effect by itself, it is impossible to determine what effect it has on the rotational properties of the state. Only the TPE potential can be used to analyze the restoration of the state’s rotational invariance.

To make sure that the results found are independent of the pion-nucleon form factor, we performed the same calculation with \( n = 1 \) and \( \Lambda = 2.1 \text{ GeV} \) and with \( n = 2 \) and \( \Lambda = 2.9 \text{ GeV} \). The results (shown in Ref. [1]) are qualitatively the same as those in Table \( \text{III} \) demonstrating that these results are robust.

IV. CONCLUSIONS

In this paper, a light-front pion-only model was used to investigate the effects that relativity and chiral symmetry have on the deuteron. We used OPE, OPE+TPE, and OPE+ncTPE potentials to calculate the binding energy and wave function for the \( m = 0 \) and \( m = 1 \) states of the deuteron. We find that the splitting between the \( m = 0 \) and \( m = 1 \) states is smaller for the OPE+TPE potential as compared to what the OPE potential gives. We also find that chiral symmetry must be included to obtain sensible results when using two-pion-exchange potentials.

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TABLE I: Binding energies for deuterons calculated with several potentials and two different pion-nucleon form factors for the scalar-pion-only model. The $n$ parameter of the form factor is 1, the coupling constant is $\frac{g^2}{4\pi} = 0.424$, and the pion mass is used as the mass of the exchanged meson.

| Potential      | $\Lambda = 1.0$ GeV | $\Lambda = 1.915$ GeV |
|----------------|---------------------|-----------------------|
| Non-relativistic | 2.2236 MeV          | 4.2539 MeV            |
| Instantaneous   | 2.1581 MeV          | 4.0862 MeV            |
| Retarded        | 2.3111 MeV          | 4.5224 MeV            |
| Light-front, $m = 0$ | 1.2027 MeV          | 2.2296 MeV            |
| Light-front, $m = 1$ | 1.2027 MeV          | 2.2294 MeV            |

TABLE II: Binding energies for deuterons calculated with several potentials and two different pion-nucleon form factors for the (pseudoscalar) pion-only model. The $n$ parameter of the form factor is 1, the coupling constant is $\frac{g^2}{4\pi} = 14.6$, and the pion mass is used as the mass of the exchanged meson. The negative binding energy for the light-front potentials in the first column indicates that the states are not bound.

| Potential      | $\Lambda = 1.01$ GeV | $\Lambda = 1.9$ GeV |
|----------------|----------------------|---------------------|
| Non-relativistic | 2.2244 MeV           | 227.019 MeV         |
| Instantaneous   | 0.0146 MeV           | 28.192 MeV          |
| Retarded        | 1.3590 MeV           | 130.472 MeV         |
| Light-front, $m = 0$ | -0.0269 MeV          | 0.788 MeV           |
| Light-front, $m = 1$ | -0.0246 MeV          | 8.856 MeV           |

TABLE III: The values of the binding energy for the $m = 0$ and $m = 1$ states, the difference of those binding energies ($\Delta$), percentage of the wave function in the D state, and the percentage of the wave function in the $J = 1$ state for the $m = 0$ and $m = 1$ states for different potentials. The pion-nucleon form factor uses $n = 1$ and $\Lambda = 1.9$ GeV.

| Potential       | Binding Energy (MeV) | D state (%) | $J = 1$ (%) |
|-----------------|----------------------|-------------|-------------|
|                 | m=0 | m=1  | $\Delta$ | m=0 | m=1  | m=0 | m=1  |
| OPE             | 0.7884 | 8.8561 | -8.0677 | 4.09 | 12.27 | 99.99 | 99.78 |
| OPE+TPE         | 0.6845 | 2.5606 | -1.8761 | 3.98 | 8.08  | 99.98 | 99.88 |
| OPE+ncTPE       | -0.0107 | -0.0087 | -0.0020 | 0.15 | 0.66  | 100.00 | 100.00 |

FIG. 1: The non-vanishing diagrams for pion-nucleon scattering at threshold: (a) $M_V$, (b) $M_X$, and (c) $M_C$. 