NEUTRINO OSCILLATIONS IN THE FRAMEWORK OF THE TREE-NEUTRINO MIXING

S. M. Bilenky 1

Joint Institute for Nuclear Research, Dubna, R-141980, Russia
INFN, Sez. di Torino and Dip. di Fisica Teorica, Univ. di Torino, I-10125
Torino, Italy

Abstract

In the framework of the three-neutrino mixing the neutrino oscillations in the atmospheric and solar ranges of neutrino mass-squared differences are considered in the leading approximation. Neutrinoless double $\beta$-decay is also discussed.

1 Introduction

Convincing evidence in favor of neutrino oscillations, driven by small neutrino masses and neutrino mixing, were obtained in the Super-Kamiokande [1], SNO [2], KamLAND [3] and other neutrino experiments [4]. These findings opened a new field of research: physics of massive and mixed neutrinos.

In spite of big progress achieved in the recent years, there are many opened fundamental problems in the field. Some of them we will discuss later.

We will consider neutrino oscillations and neutrinoless double $\beta$-decay in the framework of the three neutrino mixing 2. Main features of neutrino oscillations in the three-neutrino case are determined by the smallness of two parameters $\frac{\Delta m^2_{21}}{\Delta m^2_{32}}$ and $|U_{e3}|^2$. We will consider here transition probabilities in the leading approximation. As we will see, in this approximation neutrino oscillations in the solar and atmospheric ranges of $\Delta m^2$ are described by two-neutrino type formulas and are decoupled (see [5]). This picture of neutrino oscillations corresponds to existing experimental data.

1Report at the 1st Yamada Symposium on Neutrinos and Dark Matter in Nuclear Physics, June 9-14, 2003, Nara, Japan.
2We will not consider here indications in favor of neutrino oscillations that were obtained in the LSND experiment. The results of the LSND experiment will be checked by the MiniBOONE experiment which is going on now at Fermilab [4].
2 Neutrino oscillations

The standard neutrino charged and neutral currents have the form

\[ j_{a}^{\text{CC}} = 2 \sum_{l} \bar{\nu}_{l} \gamma_{\alpha} l_{lL}; \quad j_{a}^{\text{NC}} = \sum_{l} \bar{\nu}_{l} \gamma_{\alpha} \nu_{lL}. \]  \tag{1}

We will assume that the flavor fields \( \nu_{lL} \) (\( l = e, \mu, \tau \)) are mixtures of the fields of neutrinos with definite masses

\[ \nu_{lL} = \sum_{i=1}^{3} U_{li} \nu_{iL}, \]  \tag{2}

where \( \nu_{i} \) is the field of neutrino (Dirac or Majorana) with mass \( m_{i} \) and \( U \) is 3×3 PMNS unitary matrix.

If neutrino mass-squared differences are so small that they can not be resolved in neutrino production and detection experiments, the state of the flavor neutrino \( \nu_{l} \) with momentum \( \vec{p} \) is given by the expression

\[ |\nu_{l}\rangle = \sum_{i=1}^{3} U_{li}^{*} |\nu_{i}\rangle. \]  \tag{3}

Here \( |\nu_{i}\rangle \) is the state of neutrino with momentum \( \vec{p}_{i} \) and energy

\[ E_{i} = \sqrt{p^{2} + m_{i}^{2}} \approx p + \frac{m_{i}^{2}}{2p} ; \quad (p^{2} \gg m_{i}^{2}) \]  \tag{4}

Let us assume that at \( t=0 \) in a weak decay flavor neutrino \( \nu_{l} \) was produced. At the moment \( t \) for the state vector we have

\[ |\nu_{l}\rangle_{t} = e^{-iH_{0}t} |\nu_{l}\rangle = \sum_{i=1}^{3} U_{li}^{*} e^{-iE_{i}t} |i\rangle, \]  \tag{5}

where \( H_{0} \) is the free Hamiltonian.

Different mass components in \( 5 \) have different phases. Thus, the flavor content of the state \( |\nu_{l}\rangle_{t} \) is different from the flavor content of the initial state. For the probability of the transition \( \nu_{l} \rightarrow \nu_{l}' \) from \( 5 \) we have

\[ P(\nu_{l} \rightarrow \nu_{l'}) = | \delta_{\nu_{l} \nu_{l}'} + \sum_{i \geq 2} U_{l' i}^{*} U_{l i}^{*} (e^{-i\Delta m_{i}^{2} \frac{t}{2E_{i}}} - 1) |^{2}, \]  \tag{6}
where $L \simeq t$ is the source-detector distance, $E$ is the neutrino energy and $\Delta m^2_{1i} = m^2_i - m^2_1$.  

Analogously, for the probability of the transition $\bar{\nu}_l \rightarrow \bar{\nu}'_l$ we have

$$P(\bar{\nu}_l \rightarrow \bar{\nu}'_l) = | \delta_{l' l} + \sum_{i \geq 2} U^*_{l' i} U_{li} (e^{-i\Delta m^2_{1i} \frac{L}{E}} - 1) |^2.$$  

(7)

We will make the following remarks.

1. In the case of the Dirac neutrinos the mixing matrix $U$ is characterized by three mixing angles $\theta_{12}, \theta_{23}$ and $\theta_{13}$ and one phase $\delta$. We will be interested in the elements of the first row and the first column of the matrix $U$. In the standard parameterization we have

$$U_{e1} = \sqrt{1 - |U_{e3}|^2} \cos \theta_{12}; \ U_{e2} = \sqrt{1 - |U_{e3}|^2} \sin \theta_{12}; \ U_{e3} = \sin \theta_{13} \ e^{-i \delta}$$  

(8)

For the $\mu, 3$ and $\tau, 3$ elements of the mixing matrix we have

$$U_{\mu 3} = \sqrt{1 - |U_{e3}|^2} \sin \theta_{23}; \ U_{\tau 3} = \sqrt{1 - |U_{e3}|^2} \cos \theta_{23}$$  

(9)

In the case of the Majorana neutrinos there are two additional phases in the matrix $U$. These phases do not enter, however, into expressions for the transition probabilities (6) and (7). Thus, in the general case the transition probabilities of neutrinos and antineutrinos depend on three angles, one phase and two mass-squared differences $\Delta m^2_{21}$ and $\Delta m^2_{32}$.

2. Neutrino oscillations can be observed if for at least one $\Delta m^2$ the condition $\Delta m^2 L \leq 1$ is satisfied. In this inequality $\Delta m^2$ is the neutrino mass-squared difference in eV$^2$, $L$ is the distance in m and $E$ is the neutrino energy in MeV.

3. If CP invariance in the lepton sector holds, the matrix $U$ is real in the Dirac case:

$$U = U^*$$  

(10)

In the case of the Majorana neutrino $\nu_i$ the matrix $U$ satisfies the condition

$$U_{ii} = U^*_{ii} \ \eta_i,$$  

(11)

We label neutrino masses in such a way that $m_1 < m_2 < m_3$.  

---

3We label neutrino masses in such a way that $m_1 < m_2 < m_3$.  

---
where $\eta_i = \pm i$ is the CP parity of the Majorana neutrino $\nu_i$.

From (6), (7), (10) and (11) it follows that in the case of the CP invariance in the lepton sector

$$P(\nu_i \rightarrow \nu_l) = P(\bar{\nu}_i \rightarrow \bar{\nu}_l).$$

(12)

4. In the case of the mixing of two types of neutrinos the index $i$ in (6) and (7) takes only one value $i = 2$ and indexes $l$ and $l'$ take two values ($\mu, \tau$ or $\mu, e$ or $e, \tau$). For the transition probability we have in this case

$$P(\nu_l \rightarrow \nu_{l'}) = |\delta_{l'l} + U_{l'} U_{l}^* (e^{-i\Delta m^2 L/2E} - 1)|^2,$$

where $\Delta m^2 = m_2^2 - m_1^2$. From this expression for the appearance probability ($l \neq l'$) we have

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos \Delta m^2 \frac{L}{2E}\right),$$

(13)

(14)

where $\theta$ is the mixing angle ($U_{l2} = \sin \theta$; $U_{l'2} = \cos \theta$).

For the disappearance probability from the condition of the conservation of the probability we find

$$P(\nu_l \rightarrow \nu_l) = 1 - P(\nu_l \rightarrow \nu_{l'}) = 1 - \frac{1}{2} \sin^2 2\theta \left(1 - \cos \Delta m^2 \frac{L}{2E}\right).$$

(15)

Let us stress that in the case of the mixing of two types of neutrinos the following relations holds

$$P(\nu_l \rightarrow \nu_l) = P(\nu_{l'} \rightarrow \nu_{l'}) \quad (l \neq l')$$

(16)

5. All existing atmospheric neutrino oscillation data are perfectly described if we assume that $\nu_\mu \rightarrow \nu_\tau$ oscillations take place. From the analysis of the Super-Kamiokande data the following best-fit values of the oscillations parameters were obtained

$$\Delta m^2_{\text{atm}} = 2.5 \cdot 10^{-3} \text{eV}^2; \quad \sin^2 2\theta_{\text{atm}} = 1.0 \quad (x^2_{\text{min}} = 163.2/170 \text{ d.o.f.})$$

(17)

The data of all solar neutrino experiments are well described if we assume that the probability of the solar neutrinos to survive is given
by the two-neutrino MSW expression. From the analysis of the data of all solar experiments for the best-fit values of the oscillation parameters in the LMA region the following values were found \[2\]

\[
\Delta m^2_{\text{sol}} = 5 \cdot 10^{-5}\text{eV}^2; \quad \tan^2 \theta_{\text{sol}} = 0.34; \quad \chi^2_{\text{min}} = 57/72 \text{ d.o.f.}
\] (18)

Finally the data of the KamLAND experiment \[3\] are well described if we assume that the probability of the reactor $\bar{\nu}_e$ is given by the two-neutrino expression (15). The best-fit values of the oscillation parameters

\[
(\Delta m^2)_{\text{KamLAND}} = 6.9 \cdot 10^{-5}\text{eV}^2; \quad (\sin^2 2\theta)_{\text{KamLAND}} = 1
\] (19)

obtained from the analysis of the KamLAND data, are compatible with the solar LMA values (18). The results of the KamLAND experiment perfectly confirm effect of the oscillations of electron neutrinos discovered in the solar neutrino experiments.

3 Neutrino oscillations in the atmospheric and LBL experiments

We will consider now vacuum neutrino oscillations in the atmospheric and long baseline (LBL) accelerator and reactor neutrino experiments with $L_E \simeq 10^{-3}$ (see \[5\]). From analysis of all existing neutrino oscillation data it follows that two-neutrino mass-squared differences $\Delta m^2_{\text{sol}}$ and $\Delta m^2_{\text{atm}}$ satisfy the inequality

\[
\Delta m^2_{21} \simeq \Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{atm}} \simeq \Delta m^2_{32}
\]

Thus, for $L_E \simeq 10^{-3}$ the phase $\Delta m^2_{21} \frac{L}{2E}$ in Eq.(6) and Eq.(7) is small and the contribution of $\nu_2$ into the expressions for the neutrino and antineutrino transition probabilities can be neglected. We have

\[
P(\nu_l \to \nu_{l'}) \simeq | \delta_{l' l} + U_{l'3} U_{l3}^* (e^{-i\Delta m^2_{32} \frac{L}{2E}} - 1) |^2
\] (20)

From this expression for $l \neq l'$ we have

\[
P(\nu_l \to \nu_{l'}) = \frac{1}{2} A_{l' l} (1 - \cos \Delta m^2_{32} \frac{L}{2E})
\] (21)
where the oscillation amplitude is given by

\[ A_{l';l} = 4 \left| U_{l3} \right|^2 \left| U_{l3}' \right|^2 = A_{l'v} \]  (22)

From (9) and (22) for the amplitudes of \( \nu_\mu \rightarrow \nu_\tau \) and \( \nu_\mu \rightarrow \nu_e \) transitions we obtain the following expressions

\[ A_{\tau;\mu} = (1 - \left| U_{e3} \right|^2) \sin^2 \theta_{23}; \quad A_{e;\mu} = 4 \left| U_{e3} \right|^2 (1 - \left| U_{e3} \right|^2 \sin^2 \theta_{23}) \]  (23)

For the survival probability \( P(\nu_l \rightarrow \nu_l) \) from the condition of the conservation of the probability we have

\[ P(\nu_l \rightarrow \nu_l) = 1 - \sum_{l' \neq l} P(\nu_l \rightarrow \nu_{l'}) \]  (24)

where

\[ B_{l;\ell} = \sum_{l' \neq l} A_{l';l} = 4 \left| U_{l3} \right|^2 (1 - \left| U_{l3} \right|^2). \]  (25)

As it is seen from (21) and (22), the CP phase \( \delta \) does not enter into the expressions for the transition probabilities. This means that in the leading approximation the relation

\[ P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \]

is satisfied automatically.

Thus, the violation of the CP invariance in the lepton sector can be revealed in neutrino oscillations only if the contribution of all three massive neutrinos are involved. Because the parameter \( \frac{\Delta m^2_{32}}{\Delta m^2_{31}} \) is small, effects of the violation of the CP invariance in the lepton sector in the case of the mixing of three massive neutrinos are strongly suppressed.

The transition probabilities Eq.(21) and Eq.(24) in every channel have two-neutrino form. This is obvious consequence of the fact that we took into account only the major contribution of the largest neutrino mass-squared difference \( \Delta m^2_{32} \). Due to the unitarity condition \( \sum_{l} |U_{l3}|^2 = 1 \) the transition probabilities Eq.(21), Eq.(22), Eq.(24) and Eq.(25) are characterized by three parameters. We can choose \( \Delta m^2_{32}, \sin^2 \theta_{23}, |U_{e3}|^2 \).

The parameter \( |U_{e3}|^2 \) is small. This follows from the results of the CHOOZ and Palo Verde reactor neutrino experiments [6].
No indications in favor of neutrino oscillations were found in these experiments. For the probability of the reactor antineutrino to survive from (24) we have

\[ P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} B_{\text{e,e}} \left( 1 - \cos \Delta m_{32}^2 \frac{L}{2E} \right), \tag{26} \]

where

\[ B_{\text{e,e}} = 4 |U_{e3}|^2 \left( 1 - |U_{e3}|^2 \right) \tag{27} \]

From exclusion plots obtained from the analysis of the CHOOZ and Palo Verde data we have

\[ B_{\text{e,e}} \leq B_{\text{e,e}}^0, \tag{28} \]

where the upper bound \( B_{\text{e,e}}^0 \) depends on \( \Delta m_{32}^2 \). From Eq.(27) and Eq.(28) we obtain the bound

\[ |U_{e3}|^2 \leq \frac{1}{4} B_{\text{e,e}}^0 \tag{29} \]

For the S-K best-fit value \( \Delta m_{32}^2 = 2.5 \cdot 10^{-3} \text{eV}^2 \) from the CHOOZ exclusion plot we have

\[ |U_{e3}|^2 \leq 4 \cdot 10^{-2} \text{ (95% CL)}. \tag{30} \]

Taking into account accuracies of the present day experiments we can neglect \( |U_{e3}|^2 \) in the expressions for the transition probabilities. In this approximation \( \Delta m_{32}^2 \approx \Delta m_{\text{atm}}^2 \) and neutrino oscillations in the atmospheric range of \( \Delta m^2 \) are pure vacuum two-neutrino \( \nu_\mu \rightarrow \nu_\tau \) oscillations. The S-K and other atmospheric neutrino data are in agreement with such picture.

4 Neutrino oscillations in the solar and KamLAND experiments

In the framework of the three-neutrino mixing we will consider now neutrino oscillations in the solar range of \( \Delta m^2 \) (see \cite{5}). The \( \nu_e (\bar{\nu}_e) \) vacuum survival probability can be written in the form

\[ \text{The large values of } |U_{e3}|^2 (|U_{e3}|^2 \geq 1 - \frac{1}{4} B_{\text{e,e}}^0) \text{ are excluded by the solar neutrino data.} \]
\[ P(\nu_e \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_e) = \left| \sum_{i=1,2} |U_{ei}|^2 e^{-i\Delta m_{i1}^2 \frac{L}{2E}} + |U_{e3}|^2 e^{-i\Delta m_{31}^2 \frac{L}{2E}} \right|^2 \] (31)

We are interested in the survival probability averaged over the region where neutrinos are produced, over neutrino spectrum, energy resolution etc. Because of the inequality \( \Delta m_{31}^2 \gg \Delta m_{21}^2 \), in the expression for the averaged probability the interference between the first and the second terms of (31) disappears. The averaged survival probability can be presented in the form

\[ P(\nu_e \to \nu_e) = |U_{e3}|^4 + (1 - |U_{e3}|^2)^2 \, P^{(1,2)}(\nu_e \to \nu_e). \] (32)

Here

\[ P^{(1,2)}(\nu_e \to \nu_e) = 1 - \frac{1}{2} A^{(1,2)} \left(1 - \cos \Delta m_{21}^2 \frac{L}{2E}\right), \] (33)

where

\[ A^{(1,2)} = 4 \frac{|U_{e1}|^2 |U_{e2}|^2}{(1 - |U_{e3}|^2)^2} = \sin^2 2\theta_{12} \] (34)

The expression (32) is also valid in the case of matter. In this case \( P^{(1,2)}(\nu_e \to \nu_e) \) is the two-neutrino \( \nu_e \) survival probability in matter, calculated under the condition that the density of electrons \( \rho_e(x) \) in the effective Hamiltonian of the interaction of neutrino with matter is changed by \( (1 - |U_{e3}|^2) \rho_e(x) \).

Thus, \( \nu_e (\bar{\nu}_e) \) survival probability is characterized in the solar range of \( \Delta m^2 \) by three parameters \( \Delta m_{21}^2, \tan^2 \theta_{12}, |U_{e3}|^2 \). The only common parameter for the atmospheric and solar ranges of \( \Delta m^2 \) is \( |U_{e3}|^2 \). In the approximation \( |U_{e3}|^2 \to 0 \) oscillations in the solar range of \( \Delta m^2 \) are described by the two-neutrino expression

\[ P(\nu_e \to \nu_e) = P^{(1,2)}(\nu_e \to \nu_e) \] (35)

and \( \Delta m_{21}^2 \simeq \Delta m_{\text{sol}}^2 \tan^2 2\theta_{12} \simeq \tan^2 2\theta_{\text{sol}} \).

Thus, in the leading approximation \( (|U_{e3}|^2 \to 0, \Delta m_{21}^2 \frac{L}{2E} \to 0 \text{ in the atmospheric and LBL experiments}) \) neutrino oscillations in the atmospheric and solar ranges of \( \Delta m^2 \) are decoupled (see [5]).
5 Neutrinoless double $\beta$-decay

The search for neutrinoless double $\beta$-decay $((\beta\beta)_{0\nu}$ -decay)

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-$$  \hspace{1cm} (36)

of some even-even nuclei is the most sensitive and direct way of the investigation of the nature of neutrinos with definite masses $\nu_i$: the process (36) is allowed only if massive neutrinos are Majorana particles.

The matrix element of the $((\beta\beta)_{0\nu}$ -decay is the product of the effective Majorana mass

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i.$$ \hspace{1cm} (37)

and nuclear matrix element. Taking into account different calculations of the nuclear matrix elements, for the effective Majorana mass from the data of the most precise $^{76}$Ge experiments [7] the following upper bound was obtained

$$|m_{\beta\beta}| \leq (0.3 - 1.2) \text{ eV}.$$ \hspace{1cm} (38)

Many new experiments on the search for the neutrinoless double $\beta$-decay are in preparation at present [4]. In these experiments the sensitivities

$$|m_{\beta\beta}| \simeq (1 \cdot 10^{-1} - 1.5 \cdot 10^{-2}) \text{ eV}$$ \hspace{1cm} (39)

are expected to be achieved.

The evidence for neutrinoless double $\beta$-decay would be a proof that neutrinos $\nu_i$ are Majorana particles. We would like to stress here that the value of the effective Majorana mass $|m_{\beta\beta}|$, combined with the values of the neutrino oscillation parameters, would enable us to obtain an information about the character of the neutrino mass spectrum, minimal neutrino mass $m_1$ and possibly Majorana CP phase (see [3]). Let us consider three typical neutrino mass spectra.

1. The hierarchy of neutrino masses $m_1 \ll m_2 \ll m_3$.

We have in this case the following upper bound

$$|m_{\beta\beta}| \leq \sin^2 \theta_{\text{sol}} \sqrt{\Delta m_{\text{sol}}^2} + |U_{e3}|^2 \sqrt{\Delta m_{\text{atm}}^2}.$$ \hspace{1cm} (40)
Using the best-fit values of the oscillation parameters and the CHOOZ bound on \( |U_{e3}|^2 \) (see (17), (18) and (30)), we obtain the bound

\[
| < m > | \leq 3.8 \cdot 10^{-3} \text{eV},
\]

(41)

which is significantly smaller than the expected sensitivities of the future \((\beta\beta)_{0\nu}\) experiments.

2. Inverted hierarchy of neutrino masses: \( m_1 \ll m_2 < m_3 \).

The effective Majorana mass is given in this case by the expression

\[
|m_{\beta\beta}| \simeq (1 - \sin^2 2\theta_{\text{sol}} \sin^2 \alpha)^{1/2} \sqrt{\Delta m^2_{\text{atm}}},
\]

(42)

where \( \alpha = \alpha_3 - \alpha_2 \) is the the difference of the Majorana CP phases of the elements \( U_{e3} \) and \( U_{e2} \).

From this expression it follows that

\[
\frac{1}{2} \sqrt{\Delta m^2_{\text{atm}}} \cos 2\theta_{\text{sol}} \lesssim |m_{\beta\beta}| \lesssim \sqrt{\Delta m^2_{\text{atm}}},
\]

(43)

where the upper and lower bounds correspond to the case of the CP conservation with the equal and opposite CP parities of \( \nu_3 \) and \( \nu_2 \).

Using the best-fit value of the parameter \( \tan^2 \theta_{\text{sol}} \) (see (18)), we have

\[
\frac{1}{2} \sqrt{\Delta m^2_{\text{atm}}} \lesssim |m_{\beta\beta}| \lesssim \sqrt{\Delta m^2_{\text{atm}}},
\]

(44)

Thus, in the case of the inverted mass hierarchy the scale of \( |m_{\beta\beta}| \) is determined by \( \sqrt{\Delta m^2_{\text{atm}}} \). If the value of \( |m_{\beta\beta}| \) is in the range (44), which can be reached in the future experiments it would be an argument in favour of inverted neutrino mass hierarchy.

3. Practically degenerate neutrino mass spectrum \( m_2 \simeq m_3 \simeq m_1 \gg \sqrt{\Delta m^2_{\text{atm}}} \).

For the effective Majorana mass we have in this case

\[
|m_{\beta\beta}| \simeq m_1 \left| \sum_{i=1}^{3} U^2_{ei} \right|.
\]

(45)
Neglecting small contribution of $|U_{e3}|^2$, for $|m_{\beta\beta}|$ we obtain the relations (42)-(44), in which $\sqrt{\Delta m^2_{\text{atm}}}$ must be changed by $m_1$. For neutrino mass $m_1$ we have in this case the range

$$|m_{\beta\beta}| \leq m_1 \leq \frac{|m_{\beta\beta}|}{|\cos 2\theta_{\text{sol}}|} \simeq 2|m_{\beta\beta}| \quad (46)$$

The parameter $\sin^2\alpha$, which characterize the violation of the CP invariance in the lepton sector, is given by the following relation

$$\sin^2\alpha \simeq \left(1 - \frac{|m_{\beta\beta}|^2}{m_1^2}\right) \frac{1}{\sin^2 2\theta_{\text{sol}}} \quad (47)$$

If the mass $m_1$ is measured in the future $\beta$-decay experiments [9] and the value of the parameter $\sin^2 2\theta_{\text{sol}}$ is determined in the solar and KamLAND experiments, from the results of the future $(\beta\beta)_{0\nu}$-experiments an information on the Majorana CP phase can be inferred.

The determination of the Majorana mass $|m_{\beta\beta}|$ from the measurement of the life-time of the $(\beta\beta)_{0\nu}$-decay requires the knowledge of the nuclear matrix elements. At present there are large uncertainties in the calculation of these quantities: different calculations of the lifetime of the $(\beta\beta)_{0\nu}$-decay differ by about one order of magnitude.

If $(\beta\beta)_{0\nu}$-decay of different nuclei will be observed, from the ratios of the life-times (which depend on the ratios of the nuclear matrix elements and known space factors) model independent conclusions on different calculations of the nuclear matrix elements can be inferred [10].

6 Conclusion

There are many unsolved problems in the physics of massive and mixed neutrinos. From our point of view the most urgent ones are the following:

- What is the value of the parameter $|U_{e3}|^2 = \sin^2 \theta_{13}$?

The answer to this question possibly will be obtained in LBL accelerator experiments searching for $\nu_\mu \rightarrow \nu_e$ oscillations and LBL reactor experiments searching for disappearance of reactor $\nu_e$ [11].
What is the nature of neutrinos with definite masses? Are they Dirac or Majorana particles?
The answer to this question possibly will be obtained in future experiments on the search for neutrinoless double \(\beta\)-decay [4].

What is the value of the minimal neutrino mass \(m_1\)?
The answer to this question possibly will be obtained in future tritium experiment KATRIN [9] and/or from cosmology [4].

References

[1] Super-Kamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett. 85, 3999-4003 (2000).

[2] SNO collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 87, 071301 (2001). Phys.Rev.Lett. 89, 011301 (2002); Phys.Rev.Lett 89, 011302 (2002); nucl-ex/0204009

[3] KamLAND collaboration, K. Eguchi et al., Phys. Rev. Lett. to be published, hep-ex/0212021

[4] See Proceedings of the 1st Yamada Symposium on Neutrinos and Dark Matter in Nuclear Physics (June 9–14, 2003, Nara Japan).

[5] S.M. Bilenky, C. Giunti and W. Grimus. Prog. Part. Nucl. Phys. 43, 1 (1999); hep-ph/9812360

[6] CHOOZ Collaboration, M. Apollonio et al., Phys. Lett. B 466 415 (1999). Palo Verde collaboration, F. Boehm, J. Busenitz et al., Phys. Rev. Lett. 84, 3764 (2000); Phys. Rev. D 62 (2000) 072002.

[7] HEIDELBERG-MOSCOW collaboration, H. V. Klapdor-Kleingrothaus et al., Eur. Phys. J. A 12,(2001) 147; IGEX Collaboration, C. E. Aalseth et al., Physics of Atomic Nuclei 63 (2000) 1225; hep-ex/0202026

[8] S. Petcov in Proceedings of the 1st Yamada Symposium on Neutrinos and Dark Matter in Nuclear Physics (June 9–14, 2003, Nara Japan).
[9] KATRIN collaboration, A. Osipowicz et al., hep-ex/0109033

[10] S.M. Bilenky, J.A. Grifols, Phys. Lett. B 550 (2002) 154.