Thermal instabilities in cooling galactic coronae: fuelling star formation in galactic discs

Alexander Hobbs,1* Justin Read,1,2 Chris Power3 and David Cole4

1Institute for Astronomy, Department of Physics, ETH Zurich, Wolfgang-Pauli-Strasse 16, CH-8093 Zürich, Switzerland
2Department of Physics, University of Surrey, Guildford GU2 7XH, Surrey, UK
3International Centre for Radio Astronomy Research, University of Western Australia, 25 Stirling Highway, Crawley, Western Australia 6009, Australia
4Jeremiah Horrocks Institute, University of Central Lancashire, Preston PR1 2HE, UK

ABSTRACT

We investigate the means by which cold gas can accrete on to Milky Way mass galaxies from a hot corona of gas, using a new smoothed particle hydrodynamics code, ‘SPHS’. We find that the ‘cold clumps’ seen in many classic SPH simulations in the literature are not present in our SPHS simulations. Instead, cold gas condenses from the halo along filaments that form at the intersection of supernovae-driven bubbles from previous phases of star formation. This positive feedback feeds cold gas to the galactic disc directly, fuelling further star formation. The resulting galaxies in the SPH and SPHS simulations differ greatly in their morphology, gas phase diagrams and stellar content. We show that the classic SPH cold clumps owe to a numerical thermal instability caused by an inability for cold gas to mix in the hot halo. The improved treatment of mixing in SPHS suppresses this instability leading to a dramatically different physical outcome. In our highest resolution SPHS simulation, we find that the cold filaments break up into bound, physically motivated clumps that form stars. The filaments are overdense by a factor of 10–100 compared to the surrounding gas, suggesting that the fragmentation results from a physical non-linear instability driven by the overdensity. This ‘fragmenting filament’ mode of disc growth has important implications for galaxy formation, in particular the role of star formation in bringing cold gas into disc galaxies.

Key words: Galaxy: evolution – Galaxy: formation – galaxies: evolution – galaxies: formation – galaxies: spiral – galaxies: structure.

1 INTRODUCTION

The star formation rate (SFR) of the Universe has been rapidly falling since a redshift of ~2–3 (e.g., Lilly et al. 1996; Madau, Pozzetti & Dickinson 1998; Hippelein et al. 2003), dominated by the most massive galaxies and galaxy groups. By contrast, the Milky Way – similarly to other disc galaxies – has been forming stars at a near-continuous or perhaps slightly declining rate for the past ~8 Gyr since z ~ 1 (e.g., Noh & Scalo 1990; Rocha-Pinto et al. 2000; Fuchs, Jahrreiß & Flynn 2009). The observed SFR of ~1–3 M⊙ yr−1 is hard to explain given the amount of cold gas present in the disc today; the Milky Way must have continuously accreted cold gas at a rate of ~1 M⊙ yr−1 over this time (Fraternali & Tomassetti 2012). Merger-driven accretion appears to account for just ~0.1 M⊙ yr−1 (Sancisi et al. 2008), and there is to date no evidence of a low-redshift ‘cold flow’ accretion mode (e.g. Stewart et al. 2011). This has led to two main solutions in the literature. The first is recycling of gas from existing stars in the disc through stellar winds (e.g., Roberts 1963; Sandage 1986; Kennicutt, Tamblyn & Congdon 1994). Leitner & Kravtsov (2011) estimate that this recycled gas could contribute at least half of the global SFR for a galaxy of Milky Way mass at low redshift (z < 0.5). The second is accretion from a massive hot halo, or corona, of gas surrounding the Galaxy. Such a hot halo has not yet been directly observed, but several indirect lines of evidence exist: observations of X-ray emitting gas (Gupta et al. 2012); absorption along sight lines to quasars (e.g., Williams et al. 2005; Fang et al. 2006; Kacprzak et al. 2008); pulsar dispersion measures (e.g., Gaensler et al. 2008; Anderson & Bregman 2010); a significant Galactic baryon deficiency when compared to the universal baryon fraction (e.g., Fukugita & Peebles 2006; Nicastro, Mathur & Elvis 2008); and evidence of ram pressure stripping of the Magellanic stream (e.g., Mastropietro et al. 2005, but see also Besla et al. 2007; Diaz & Bekki 2012) and other nearby dwarf galaxies (Greveich & Putman 2009). These studies give a hot halo mass of >5 × 109–1.5 × 1010 M⊙ assuming a Navarro, Frenk & White (1999) profile or >4 × 1010 assuming a flattened power-law profile (Anderson & Bregman 2011). Similar results are seen in other disc galaxies (e.g., Mo et al. 2005; Read & Trentham 2005; Sancisi et al. 2008; Anderson & Bregman 2011).
While it is likely that hot gaseous coronae surround disc galaxies, it is not clear how the gas cools and condenses out of these coronae and on to the disc to form stars. One particular mode of cold gas supply that has been seen in a number of numerical simulations (Kaufmann et al. 2006, 2007, 2009; Sommer-Larsen 2006; Putman, Peek & Heitsch 2009), but was initially proposed by Nulsen (1986), is that of direct cooling from the halo via thermal instability. However, doubt has been cast on this picture by Malagoli, Rosner & Bodo (1987) and Binney, Nipoti & Fraternali (2009) who show that hot haloes are linearly stable to density perturbations, making direct cooling unlikely. Joung, Bryan & Putman (2012) extend this treatment to the non-linear regime with dedicated numerical simulations, finding that while a runaway process of cooling and collapse is possible, it can only occur for overdensities of $\gtrsim 10$–$20$ with respect to the local background density. These results suggest that if gas is to cool from the hot coronae, then some mechanism is required to excite a thermal instability. Marinacci et al. (2011) suggest a mechanism whereby a galactic fountain seeds metal-rich gas into the metal-poor hot haloes, giving rise to a thermal instability that causes cold gas to rain down on to the disc in the form of $\sim 10^5 M_\odot$ clouds (see also Fraternali & Binney 2008). This model provides an excellent fit to both the kinematics and spatial distribution of warm H i gas in the Milky Way (Marasco, Fraternali & Binney 2012), although it cannot account for the high-velocity clouds (e.g., Sembach et al. 2003; Tripp et al. 2003; Collins, Shull & Giroux 2005). Alternative models include cooling stripped gas from dwarf galaxies or warm clouds at the disc-corona interface (e.g., Heitsch & Putman 2009; Peek 2009; Putman et al. 2012; Zavala et al. 2012).

The ‘direct cooling’ mode mentioned above is seen regularly in ‘classic’ smoothed particle hydrodynamics (SPH) simulations (e.g., Kaufmann et al. 2006, 2007, 2009; Sommer-Larsen 2006; Putman et al. 2009), where a thermal instability leads to a sudden and widespread condensation of gas from the halo in the form of cold, dense clumps. If correct, no special mechanism would be required to explain the continued star formation (SF) of disc galaxies over the past $\sim 8$ Gyr. However, ‘classic’ SPH is known to exhibit an artificial surface tension that inhibits mixing of different gaseous phases, leading to poor performance on a variety of hydrodynamic test problems (Agertz et al. 2007; Price 2008; Wadsley, Veeravalli & Couchman 2008; Read, Hayfield & Agertz 2010). The problem owes to two different effects that become significant at phase boundaries—a leading order error in the momentum equation and an entropy/pressure discontinuity (Read et al. 2010). Numerous solutions to these problems have been presented in the literature, leading to a welcome proliferation of different SPH ‘flavours’ (e.g., Ritchie & Thomas 2001; Price 2008; Wadsley et al. 2008; Read et al. 2010; Hopkins et al. 2012; Read & Hayfield 2012), and new Lagrangian fluid techniques (e.g., Inutsuka 2002; Cha, Inutsuka & Nayakshin 2010; Heß & Springel 2010; Springel 2010; Murante et al. 2011).

The new moving mesh code of Springel (2010) – AREPO – produces similar results to adaptive mesh refinement (AMR) codes, finding that ‘direct cooling’ of gas to form clumps does not occur (e.g., Agertz, Teyssier & Moore 2009; Vogelsberger et al. 2012; Torrey et al. 2012). However, both AREPO and AMR codes rely on similar finite difference methods, while the ‘direct cooling’ seen in classic SPH remains unexplained.

In this paper, we use a new flavour of SPH – ‘SPHS’ – recently developed by some of the authors (Read & Hayfield 2012) to revisit the problem of thermal instabilities in hot gaseous coronae. Our primary goals are to determine whether the instabilities seen in the classic SPH simulations are physical or numerical; and under what circumstances cold gas can condense out of a hot halo to fuel SF in disc galaxies.

This paper is organized as follows. In Section 2, we briefly review the SPHS algorithm and ‘classic’ SPH. We describe our initial conditions (ICs), and present our implementation of radiative cooling, SF and stellar feedback, and our treatment of a central supermassive black hole (SMBH). In Sections 3 and 4, we present our results, which we discuss in Section 5. Finally, in Section 6, we present our conclusions.

2 METHODS

2.1 SPHS

The SPHS method is described in detail in Read et al. (2010) and Read & Hayfield (2012). We give a brief summary of the main equations here. The discretized hydrodynamic equations of motion are chosen to minimize force errors as in Read et al. (2010):

$$\rho_i = \sum_j m_j W_{ij}(|r_{ij}|, h_i)$$

$$\frac{dv_i}{dt} = -\sum_j m_j \rho_j \rho_j \left[ P_i + P_j \right] \nabla W_{ij}$$

$$P_i = A_i \rho_i^\gamma,$$

where $m_i$ and $A_i$ are the mass and entropy, respectively, of particle $i$; $\nabla W_{ij} = \frac{1}{2} \left[ W_{ij}(h_i) + W_{ij}(h_j) \right]$; and $W$ is a symmetric kernel that obeys the normalization condition:

$$\int V W(|r - r'|, h) d^3r' = 1$$

and the property (for smoothing length $h$):

$$\lim_{h \to 0} W(|r - r'|, h) = \delta(|r - r'|)$$

and $r_{ij} = r_j - r_i$ is the vector position of the particle relative to the centre of the kernel.

We use a variable smoothing length $h_i$ as in Springel & Hernquist (2002) that is adjusted to obey the following constraint equation:

$$\frac{4\pi}{3} h_i^3 n_i = N_0; \quad \text{with} \quad n_i = \sum_j W_{ij},$$

where $N_0$ is the typical neighbour number (the number of particles inside the smoothing kernel, $W$). The above constraint equation gives fixed mass inside the kernel if particle masses are all equal. For the comparison with classic SPH, we employ the standard cubic spline (CS) kernel in both codes, whereas for the full SPHS runs (see Section 4) we use the ‘HOCT4’ kernel with 442 neighbours as

1 For a careful definition of ‘classic’ SPH see Section 2.2.

2 Strictly speaking $A$ is a function of the specific entropy $s$, i.e. $A \equiv A(s)$ – see Springel (2005) for more details.
this gives significantly improved force accuracy and convergence
(Read et al. 2010; Read & Hayfield 2012):
\[
W = \frac{N}{h^3} \left\{ \begin{array}{ll}
P x + Q & 0 < x \leq \kappa \\
(1 - x)^i + (\alpha - x)^i + (\beta - x)^i & \kappa < x \leq \beta \\
(1 - x)^i & \beta < x \leq \alpha \\
0 & x \leq 1
\end{array} \right.
\]
with \( N = 6.515, P = -2.15, Q = 0.981, \alpha = 0.75, \beta = 0.5 \) and \( \kappa = 0.214 \).

In addition to the above equations of motion, numerical dissipation
is switched on if particles are converging. This avoids multi-valued fluid quantities occurring at the point of convergent flow.

Regarding dissipation, the resulting multivalued pressures drive
waves through the fluid that propagate large numerical errors and
spoil convergence. The switch is given by
\[
\alpha_{\text{loc},i} = \frac{\min(\nabla \cdot \mathbf{v}_i, n_s, c_i)}{\max(\nabla \cdot \mathbf{v}_i, n_s, c_i)} \nabla \cdot \mathbf{v}_i < 0
\]

where \( \alpha_{\text{loc},i} \) describes the amount of dissipation for a given particle in the range \([0, \alpha_{\text{max}} = 1]\); \( c_i \) is the sound speed of particle \( i \) and \( n_s = 0.05 \) is a ‘noise’ parameter that determines the magnitude of velocity fluctuations that trigger the switch. Equation (8) turns on dissipation if \( \nabla \cdot \mathbf{v}_i < 0 \) (convergent flow) and if the magnitude of the spatial derivative of \( \nabla \cdot \mathbf{v}_i \) is large as compared to the local divergence (i.e. if the flow is going to converge). The key advantage as compared to most other switches in the literature is that it acts as an early warning system, switching on before large numerical errors propagate throughout the fluid (see also Cullen & Dehnen 2010).

The second derivatives of the velocity field are calculated using high-order polynomial gradient estimators described in Maron & Howes (2003) and Read & Hayfield (2012). We use the above switch to turn on dissipation in all advected fluid quantities – i.e. the momentum (artificial viscosity) and entropy (artificial thermal conductivity). Once the trajectories are no longer converging, the dissipation parameters are fully convergent and described in detail in Read & Hayfield (2012). The only free parameters are \( \alpha_{\text{max}} \) that describe the rate of dissipation that occurs when particle trajectories attempt to cross, and \( n_s \), the noise parameter. Owing to the nature of our switch in SPHS, the results converge with increasing resolution independently of the values chosen for these parameters (Read & Hayfield 2012).

In addition to the hydrodynamical modifications, the SPHS
method also includes an improved timestepping algorithm for strong
shocks. This was adapted from Saitoh & Makino (2009), who found that the evolution of a Sedov–Taylor blast wave (or similar) is captured incorrectly when the gas in the shock is on a very different timestep to the gas it is impinging upon.3 In extreme cases, the particles sitting ahead of the shock may not ‘feel’ the particles in the blast wave, as the stationary particles are on such long timesteps the shock has passed by before their next force update. Saitoh & Makino (2009) recommend restricting the timesteps between neighbours to be at most a factor of 4 in order to alleviate this problem; this is what we do also in SPHS.

3 We would like to thank Frazer Pearce & Stewart Muldrew for providing the first version of the timestepping code patch that we use.

### 2.2 Classic SPH

Throughout this paper, we will present comparisons with ‘classic’
SPH. We define this to be the fully conservative ‘entropy’ form of
SPH described in Springel & Hernquist (2002). The discretized Euler equations are the same as in SPHS, but with the momentum equation replaced by
\[
\frac{d\mathbf{v}_i}{dt} = -\sum_j m_j \left[ f_i \frac{P_i}{\rho_i} \nabla \cdot \mathbf{v}_j + f_j \frac{P_j}{\rho_j} \nabla \cdot \mathbf{W}_ij \right],
\]
where the function \( f_i \) is a correction factor that ensures energy conservation for varying smoothing lengths:
\[
f_i = \left( 1 + \frac{h_i}{\rho_i} \frac{\partial \rho_i}{\partial h_i} \right)^{-1}.
\]

We do not use the above conservative momentum equation in SPHS
since it leads to larger force errors with only a modest improvement
in energy conservation (at least when applied to galaxy and galaxy
cluster formation simulations; see Read et al. (2010) and Read &
Hayfield (2012) for further details).

Unlike in SPHS, there is no dissipation switching and \( \alpha = \alpha_{\text{max}} = \) const. = 1 always. There is also no dissipation in entropy; the only
numerical dissipation applied is the artificial viscosity. This prevents
multivalued momenta from occurring, but not multivalued entropy or pressure (e.g., Read et al. 2010).

For most simulations presented in this paper, we use a CS kernel
with 96 neighbours – both for SPH and SPHS. This is somewhat
larger than usually employed but is necessary for the high-order gradients required for the dissipation switch in SPHS (Read & Hayfield 2012). By using the same neighbour number for both hydrodynamic techniques, we ensure like spatial resolution. However, at this neighbour number, the CS kernel is prone to a pairing instability (Read et al. 2010) that could in principle seed numerical instabilities in a hot corona. We explicitly check that this is not the case by running
one of our classic SPH simulations with a more standard 42 neighbours, and with the HOCT4 kernel with 442 neighbours (that is manifestly stable to pairing). In all cases, we recover the result seen in the literature that the hot coronae break up into many cold clumps.

We therefore summarize the key changes in SPHS compared
to classic SPH as (i) modified artificial viscosity via a switch
that detects converging particle trajectories, (ii) the inclusion of
thermal conductivity, again with converging trajectories detection,
(iii) modified kernel shape, (iv) altered timestepping and (v) non-
conservative equation of motion to minimize force errors. We will
discuss which of these changes is most important for modelling
thermal instabilities in cooling galactic coronae in Section 5.

### 2.3 Initial conditions

Our setup is similar to the ‘cooling gaseous halo’ model of
Kaufmann et al. (2006, 2007). For our IC, we employ a live dark
matter (DM) halo of collisionless particles together with a gaseous
halo of SPH particles, with each component relaxed for many
dynamical times with an adiabatic equation of state (EOS) in order to
remove Poisson noise. The relaxation step is performed separately
for each resolution and kernel–neighbour number combination used,

---

3 We would like to thank Frazer Pearce & Stewart Muldrew for providing the first version of the timestepping code patch that we use.
in all cases using classic SPH (namely without thermal conductivity and with a fixed artificial viscosity) to ensure identical relaxed ICs for both codes. Both the DM and gas distributions follow a Dehnen–McLaughlin model (Dehnen & McLaughlin 2005) of the form
\[ \rho(r) = \frac{1}{(r/r_s)^{3/2} \left(1 + (r/r_s)^{3/2}\right)^{3/2}}. \] (11)
where the normalization constant \( C \) differs between the gas and DM profiles; the gas profile is normalized with \( C_{\text{gas}} = 13.4791 \text{ g cm}^{-3} \) to contain a mass of \( 1.5 \times 10^{11} \, \text{M}_\odot \) and the DM with \( C_{\text{dm}} = 134.791 \text{ g cm}^{-3} \) to contain a mass of \( 1.5 \times 10^{15} \, \text{M}_\odot \). The scale radius for the haloes is \( r_s = 40 \, \text{kpc} \), and both are truncated at a virial radius \( r_v = 200 \, \text{kpc} \).

The gas is initially (before relaxation) set up to be in hydrostatic equilibrium, with a temperature profile given by the relation
\[ T(r) = \frac{\mu m_p}{k_B} \frac{1}{\rho_{\text{gas}}(r)} \int_r^{\infty} \rho_{\text{gas}}(r) \frac{G M(r)}{r^2} \, dr, \] (12)
where \( \mu \) is the mean molecular weight, \( k_B \) is the Boltzmann constant, \( \rho_{\text{gas}}(r) \) is the radial gas density profile and \( M(r) \) is the enclosed mass of both components within a radius \( r \). Subsequent to relaxation, the gas is given a velocity field whereby the specific angular momentum profile follows a power law (Bullock et al. 2001; Kaufmann et al. 2007) such that
\[ j_{\text{gas}} \propto r^{1.0}, \] (13)
and, normalized by a rotation parameter \( \lambda = 0.038 \), defined by
\[ \lambda = \sqrt{\frac{j_{\text{gas}} |E_{\text{dm}}|^{1/2}}{GM_{\text{dm}}^{1/2}}}, \] (14)
where \( E_{\text{dm}} \) and \( M_{\text{dm}} \) are the total energy and mass of the DM halo. This normalization implicitly assumes negligible angular momentum transport between the DM halo and the gas, an assumption justified in Kaufmann et al. (2007). The rotational velocity field is implemented about the z-axis.

Relevant parameters for the simulations are given in Table 1. The DM haloes were constructed using variable particle masses in order to minimize computational expense, as described in section 3.1 of Cole, Dehnen & Wilkinson (2011). Gravitational softening lengths for the collisionless N-body components (DM and stars) were fixed, with the values given in Table 1 (\( \epsilon_{\text{dm}} \), which also applies to the softening lengths of any stars formed during the simulation). For the gas, we employed adaptive gravitational softening lengths which were set equal to the gas smoothing length at all times so as to ensure that there was no artificial bias towards pressure support or gravitational collapse in Jeans-unstable regions, as per Bate & Burkert (1997). We did not employ the conservative correction terms as suggested in Price & Monaghan (2007); we will explore these in future work. To explicitly check that our variable softening does not affect our results, we re-ran the SPH-96-res1 and SPHS-96-res1 simulations with fixed gravitational softening in the gas, set to the minimum smoothing length in the latter two simulations. Our results were largely unchanged in terms of the SPH/SPHS comparison.

### 2.4 Radiative cooling

For all of the simulations, we employ radiative cooling in the gas with a cooling floor of \( T_{\text{cool}} = 100 \, \text{K} \). We use a toy cooling curve that follows Katz, Weinberg & Hernquist (1996) above \( 10^4 \, \text{K} \), assuming primordial abundance, and Mashchenko, Wadsley & Couchman (2008) below \( 10^4 \, \text{K} \) assuming solar abundance. This crudely models a low-metallicity cooling halo that rapidly reaches approximate solar abundance in cooling star-forming regions. We will consider a more realistic cooling curve that self-consistently responds to the injection of metals from star-forming regions in future work.

In addition to a cooling floor, we also prevent gas from cooling to the point at which the Jeans mass for gravitational collapse becomes unresolved. A given SPH/SPHS simulation has a mass resolution equal to \( M_{\text{res}} = N_{\text{res}} m_{\text{gas}} \), where \( m_{\text{gas}} \) is the mass of a gas particle and \( N_{\text{res}} \) is the number of gas particles that constitutes a resolvable mass, i.e. a single resolution element. Bate & Burkert (1997) find this to be \( \sim 2N_{\text{neigh}} \), where \( N_{\text{neigh}} \) is the number of neighbours of a gas particle; this number is somewhat of a rule of thumb and depends on the resolving scale of the particular kernel employed. For all of the tests, we use \( N_{\text{res}} = 128 \). The resolving scales of our CS and HOCT4 kernels (see Section 2.1) are such that this number is reasonable (for more details on the resolving power of these kernels, see Read et al. 2010; Dehnen & Aly 2012; Read & Hayfield 2012).

Our ‘dynamic’ cooling floor effectively ensures that the Jeans mass is always resolved within our simulation. For a given mass element \( M_{\text{res}} \), we can write a Jeans density, namely
\[ \rho_J = \left( \frac{\pi k T}{\mu m_p G} \right)^{3/2} \left( \frac{1}{M_{\text{res}}} \right)^2, \] (15)
which manifests in the simulation as a polytropic \( P = A(s) \rho^{4/3} \). Gas is not allowed to collapse (for a given temperature) to densities higher than given by equation (15), and we identify gas that lies on the polytrope as ‘star-forming’, allowing it to form stars above a fixed density threshold of 100 atoms cm\(^{-3}\) (see Section 2.5). We emphasize that the polytrope is not physical, but a purely numerical device to prevent star-forming gas from becoming unresolved in our simulations (see, e.g., Truelove et al. 1997; van de Voort et al. 2011; Dubois et al. 2012).

### 2.5 SF and feedback

SF in our simulations is modelled in a ‘sub-grid’ fashion according to observational constraints on the SFR and efficiency. We allow gas that lies on the polytrope to form stars (N-body particles) above a

| ID          | \( N_{\text{gas}} \) | \( N_{\text{dm}} \) | \( m_{\text{gas}} \) (\( \text{M}_\odot \)) | \( m_{\text{dm}} \) (\( \text{M}_\odot \)) | \( M_{\text{res}} \) (\( \text{M}_\odot \)) | \( h_{\text{min}} \) (kpc) | \( h_{\max} \) (kpc) | \( \epsilon_{\text{dm}} \) (kpc) |
|-------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| SPH-96-res0 | \( 1.5 \times 10^3 \) | \( 1.8 \times 10^3 \) | \( 1 \times 10^6 \) | \( 1\text{–}14 \times 10^6 \) | 1.28 \times 10^8 | 0.1 | 1790 | 0.45 |
| SPH-96-res0 | \( 1.5 \times 10^3 \) | \( 1.8 \times 10^3 \) | \( 1 \times 10^6 \) | \( 1\text{–}14 \times 10^6 \) | 1.28 \times 10^8 | 0.08 | 295 | 0.45 |
| SPH-96-res1 | \( 7.5 \times 10^3 \) | \( 9 \times 10^3 \) | \( 2 \times 10^5 \) | \( 2\text{–}29 \times 10^5 \) | 2.56 \times 10^7 | 0.03 | 310 | 0.2 |
| SPH-96-res1 | \( 7.5 \times 10^3 \) | \( 9 \times 10^3 \) | \( 2 \times 10^5 \) | \( 2\text{–}29 \times 10^5 \) | 2.56 \times 10^7 | 0.03 | 230 | 0.2 |
| SPH-442-res2 | \( 3.75 \times 10^3 \) | \( 4.5 \times 10^3 \) | \( 4 \times 10^4 \) | \( 4\text{–}59 \times 10^4 \) | 5.12 \times 10^6 | 0.008 | 200 | 0.09 |
fixed density threshold, with an efficiency of 0.1 as per observations (e.g., Lada & Lada 2003) of giant molecular clouds (GMCs). The SFR follows the Schmidt volume density law for SF (Schmidt 1959), namely

$$\rho_{\text{SFR}} \propto \rho_{\text{gas}}^{1.5},$$

which is implemented in the simulation by employing the dynamical time as the relevant SF time-scale, i.e.

$$\frac{d\rho_{\text{gas}}}{dt} = \frac{\rho_{\text{gas}}}{t_{\text{dyn}}},$$

where $\eta$ is the SF efficiency.

In all the simulations, we include feedback from supernovae (SNe), which takes the form of an injection of thermal energy from the star particle into nearby gas particles. Due to our finite resolution, each star particle is not an individual star but is instead representative of a stellar distribution, and so we integrate over a Salpeter initial mass function (IMF; Salpeter 1955) between 8 and 100 $M_\odot$ in order to determine the number of SNe that should be feeding back at any particular time. We treat only Type II SNe, with the energy of each SN event set to $E_{\text{SN}} = 10^{51}$ erg. To determine the time of injection, we use the standard relation:

$$\frac{t_{\text{MS}}}{t_\odot} \sim \left(\frac{M}{M_\odot}\right)^{-2.5},$$

where $t_{\text{MS}}$ is the main-sequence time of $t_{\text{MS}} \approx 11.8$ Myr after the initial formation, at which the energy injection from the number of SNe determined from integration over the IMF is implemented as a delta function in time. We couple the feedback energy to a given mass of gas, rather than just the neighbours of the star particle – this mass is set to be our resolvable mass $M_{\text{res}}$ (see Section 2.4). In doing so, we ensure that both the SF and the feedback is resolved. The actual injection of energy is convolved with a CS kernel to ensure a smoothing of the thermal coupling over the mass scale receiving the SN feedback.

Th polytropic pressure floor, together with the fixed SF density threshold, is crucial to our sub-grid modelling strategy for SF; it means that the gas that is able to form stars is (i) undergoing Jeans collapse and (ii) at the typical density of GMC star-forming regions. By constraining ‘star-forming gas’ to lie on the polytrope we are ensuring that SF occurs in collapsing gas regions, rather than just in regions of high density. We note that there has been some recent evidence that GMCs and GMC complexes may not need to be globally gravitationally bound for regions within them to undergo SF (Dobbs, Burkert & Pringle 2011) but that even in this picture the dense sub-structures actually forming stars are both (i) in a state of gravitational collapse and (ii) at a density of $\gtrsim 10^2$ atoms cm$^{-3}$.

A further aspect of our polytropic pressure floor is to maintain consistency with the parameters of the SF sub-grid model. These are tailored to star-forming gas at approximately a GMC mass scale, and employ the corresponding global observational constraints, rather than including any of the finer details of the SF process (e.g., metal-line cooling, non-equilibrium chemistry, etc.). To allow gas to collapse to higher and higher densities as it cools would lead to a situation whereby not only would this collapse become dominated by numerical effects but also reach a state that was inconsistent with the global constraints of our sub-grid prescription. We plan to explore our sub-grid modelling strategy in this context in a forthcoming paper (Hobbs et al., in preparation), with particular importance placed on convergence with increasing resolution. In the current paper, we do not expect convergence of the SF properties owing to the fact that the polytrope line shifts to lower mass scales as the resolution is increased; this however is not a problem for the comparison we make between the two numerical methods, since this is done at identical resolution.

At the start of the simulation there is no gas above the SF density threshold. The central peak in density in the ICs is $\sim 1$ atoms cm$^{-3}$, two orders of magnitude below the density at which the gas is allowed to form stars. Typically, this SF density is reached (as the gas begins to collapse towards the centre) by approximately 30–40 Myr into the simulation.

### 2.6 SMBH sub-grid model

We model the SMBH at the centre of the halo as a sink particle (see, e.g., Bate, Bonnell & Price 1995). The accretion of gas on to the SMBH is implemented using an ‘accretion radius’ approach; gas that makes it inside a distance of $r_{\text{acc}} = 0.1$ kpc from the black hole is removed from the simulation, while its mass is then added to the SMBH mass $M_{\text{bh}}$. We start the SMBH as a seed of negligible mass since even the accretion of a single gas particle grows it to a mass of $\sim 10^4$–$10^6 M_\odot$ depending on the resolution (see Table 1). As a result of this mass being negligible before any accretion occurs, it was necessary to tie the black hole to the centre of the potential in order to stop it wandering excessive distances during the early phases of accretion. We therefore re-position the SMBH on the centre of the potential at every timestep.

It is important to clarify here that due to the size of the accretion radius, when we refer to ‘SMBH accretion’ we are in fact very far away from actually resolving the accretion on to the central SMBH itself. Instead, the SMBH accretion should be thought of as ‘sink particle accretion’ in that it is a numerical necessity that simplifies this aspect of the simulation and allows it to run in a reasonable time. The gas entering $r_{\text{acc}}$ would likely not add to the mass of the black hole – most of it would simply be captured into an accretion disc around the SMBH, with only the lowest angular momentum part of it actually growing the hole (Hobbs et al. 2011; Power, Nayakshin & King 2011). When dealing with the terms ‘SMBH accretion rate’ or ‘SMBH mass’, the reader should therefore view these as the accretion rate into the central 0.1 kpc, and the mass inside the central 0.1 kpc, respectively. We are of course adding this accreted mass to a point particle (which can therefore reach quite a high mass $\gtrsim 10^4 M_\odot$), but since it is anchored to the centre of the potential at all times, this will not have a significant numerical dynamical effect (as it would if an SMBH of this mass was allowed to wander/be kicked out in one particular direction).

For the simulations presented in this paper, we do not model feedback from the SMBH, using it as a sink for accreting gas only. Our prescription for SMBH growth in this paper is somewhat crude, but we stress that this is because our focus is on the differences between the SPH and SPHS hydrodynamic methods; keeping our SMBH sub-grid model simple allows us to present a cleaner numerical test between the two. We plan to investigate the role of black hole feeding and feedback in similar galaxy formation simulations with a more advanced sub-grid model in future papers.

### 3 RESULTS

We run four main simulations, two in classic SPH and two in SPHS, each at two different resolutions (see Table 1). The global properties between the different runs are similar due to the identical ICs; as the gaseous halo cools, it loses hydrostatic support and begins to collapse, with the small net rotation velocity about the z-axis causing a disc to form initially in the x–y plane. We start by
describing the overall behaviour of the simulations, and then discuss the differences in the results between the two numerical methods and any resolution-dependent departures from this overall behaviour. Our fiducial comparison is made between the two higher resolution runs, SPH-96-res1 and SPHS-96-res1.

As the gaseous halo collapses, it falls towards the centre of the computational domain. Within the first $\sim 10$ Myr, some of the gas that has reached the central $\sim 1$ kpc undergoes a starburst, while the rest is accreted by the SMBH. The feedback associated with this SF event is strong enough to drive a near-spherical overpressurized bubble outwards into the infalling gas, temporarily evacuating the central region of most of its gas and halting the infall within the expanding cavity that is created. SF largely ceases at this point, while the gas that has formed into a small-scale disc in the central kiloparsec continues to accrete at a significantly lower rate. The expanding shell sweeps up mass and slows down, eventually allowing further infall to the central regions, and in most cases, subsequent starbursts and feedback events.

While the initial starburst was largely spherical, driving a very symmetric bubble into the surrounding gas, the subsequent feedback events occur with a greater degree of asymmetry, and so hot bubbles are generated at a variety of different orientations at different times. In these later events, gas is able to flow in more readily along the intersections between bubbles, where it is able to build up the disc in the central few kiloparsecs.

### 3.1 Differences between SPH and SPHS

#### 3.1.1 Lower resolution (res0)

We start by comparing the low resolution runs, SPH-96-res0 and SPHS-96-res0. These are shown at a time of $t = 2$ Gyr in Fig. 1.

The initial evolution is similar, with some of the gas that reaches the central kiloparsec being accreted by the SMBH, and some undergoing a starburst that drives a hot bubble into the gaseous halo. In the SPHS case, this initial feedback is slightly stronger, with the gas that has received the thermal ‘kick’ reaching higher temperatures – a maximum of $5 \times 10^8$ K versus $8 \times 10^7$ K – and expanding slightly faster than in the SPH case. Immediately after the feedback event, the accretion rate drops by several orders of magnitude (see Fig. 2). The expanding shell slows down as it sweeps up mass from the gaseous halo, and it is at this point that the two simulations differ; in the SPH run, the gas is able to make its way back inside the central few kiloparsecs, where it cools and forms stars, giving rise to a second starburst at $\sim 1.2$ Gyr. In the SPHS case, however, the gas has not collapsed back into the central regions by the end of the simulation at $t = 2$ Gyr, and so there is no associated subsequent starburst event by this time.

We can understand some of these differences by looking at the SF history between the two low-resolution runs, which can be seen in Fig. 2 along with the accretion history. The SMBH accretion rate largely follows the SFR, and so in the SPH run there are two to three major starburst and accretion events that take place to later times after the initial one. In the SPHS case, the initial starburst lasts for longer – rather than undergoing a sudden event and then dropping to zero, as it does in SPH, the SFR shows a more sustained period of activity for the first $\sim 0.5$ Gyr. This is the reason for the lack of the second starburst in the SPHS run – the sustained initial SF and feedback has driven the gas to higher temperatures and lower densities and so it takes a much longer time to cool and condense back into the central regions, failing to do so by the end of the simulation at $t = 2$ Gyr. We have run this particular simulation on for longer, and the second starburst does indeed occur after $\sim 2.5$ Gyr.

**Figure 1.** Projected surface density plots for SPH-96-res0 (left) and SPHS-96-res0 (right), both with $N_{\text{gas}} = 1.5 \times 10^5$, at $t = 2$ Gyr. There is a clear difference in the amount of structures present in the hot halo due to SNe feedback – in SPH the shape of the overpressurized bubbles is more defined and a number of spherical clumps have condensed out of the ambient gas. In SPHS, the gas distribution is more homogeneous, and no such clumps have formed.
Crucially, even at this low resolution, we can see that by far the main difference in the projected gas density maps (Fig. 1) between the SPH and SPHS simulations is the presence of overdense clumps that have condensed from the ambient halo gas between ~10 and 50 kpc. These clumps are not present in the SPH run, and we go into more detail on this particular result for the higher resolution case in Section 3.1.3.

### 3.1.2 Higher resolution (res1)

The higher resolution runs constitute our fiducial comparison between the two methods. Gas surface density projections at $t = 1.2$ Gyr are shown in Fig. 3. Once again the initial starburst and feedback events are similar, although the initial feedback is slightly stronger in SPHS than in SPH, with the kicked gas reaching a maximum of $10^5$ K versus $2 \times 10^5$ K and again expanding faster. At this resolution the peak of the SFR for this starburst is slightly higher in SPHS. Due to the stronger feedback, more of the gas is evacuated in the initial overpressurized bubble, and takes longer to make its way back inside the expanding shell. The accretion rate in SPH is therefore reduced for the period immediately following the first starburst (between ~0.2–0.7 Gyr) compared to SPH (see Fig. 4).

Also at this higher resolution, there are subsequent major starbursts in both the SPH and SPHS runs. The second large starburst occurs at ~1 Gyr, and takes place slightly earlier in SPH than in SPHS. This time the second SPH starburst is more powerful than the SPHS one. We attribute this to the fact that immediately before the second starburst the SPH run saw multiple condensing gas clumps entering the central few kiloparsecs, which was not seen in SPHS. The expanding bubbles in the second starburst also differ noticeably between the two methods – in SPH the boundaries of the cavities are thinner whereas in SPHS these outer ‘walls’ are more mixed in with the surrounding gas (see Fig. 3).

The SF and accretion histories for the higher resolution runs can be seen in Fig. 4. Most noticeable is the presence of three peaks in the SFR to later times in the SPHS run, compared to two peaks in SPH. Once again the accretion rate largely follows the SFR, although not precisely, and in particular we notice that whenever there is a strong dip in the SFR, the corresponding dip in the $M_{bh}$ is not as extreme. This is also true for the peaks, with the exception of the first peak of the later starburst/accretion events in SPHS at $t \sim 1$ Gyr. We have already mentioned how the second SPH starburst is more powerful than its SPHS equivalent, and this is seen also in the SFR, with the SPH peaking at a higher rate and falling off more slowly.

From this point on, our comparisons are made with the higher resolution set of runs. We outline a few specific differences that we wish to highlight between the results obtained with the SPH and SPHS methods.

### 3.1.3 Clumps versus filaments

Of particular interest is the behaviour of the two methods just after $t = 1$ Gyr. As we have already mentioned, the SPH run has started to see condensed clumps form out of the halo gas, which fall into the centre and form stars, giving rise to a powerful second starburst.

As the bubble expands through the surrounding gas, there appears to be a sudden condensation of multiple dense clumps from the previously low-density gas. These clumps scatter off each other as they fall to the centre, but remain seemingly protected from the ambient gas through which they travel. The clumps condense to the point that they reach the polytrope (refer to Sections 2.4 and 2.5) and therefore undergo SF and subsequent feedback. The result is that the flow in the inner ~100 kpc becomes very disordered, with continued asymmetric feedback events of varying strengths. The clumps that reach the central few kiloparsecs either add to or remain in orbit of the disc, although some are accreted. In the SPH run, however, this condensation of cold, dense clumps does not occur. Rather, the structures that form are filamentary in nature, forming at the intersection between bubbles, with gas infalling through the filaments to grow the disc (or to be accreted by the SMBH). The very few spherical overdensities that do begin to form are quickly disrupted by travelling through the ambient gas before they can reach the densities required to lie on the polytrope. This quite striking comparison can be seen clearly in Fig. 3, where the structure formation and mode of feeding is clearly

---

**Figure 2.** SFR and SMBH accretion histories for $N_{gas} = 2 \times 10^5$, with SPH (left) and SPHS (right). The solid line represents SFR, dotted $M_{bh}$. The initial starbursts and accretion events start off identical, but in SPHS both the SFR and $M_{bh}$ are prolonged compared to SPH, with a secondary peak just as the main starburst is shutting off. The SPH run shows a second major starburst not present in the SPHS run. In both simulations, the shape of the SMBH accretion rate curve largely follows that of the SFR, albeit with a small offset in time.

---

**Figure 3.** Where the structure formation and mode of feeding is clearly...
different between the two methods. In Figs 5 and 6 (second from top, left-hand panels), we isolate the overdense regions through a series of cuts in the phase diagram. The differences between the cold clumps (SPH) and filaments (SPHS) can clearly be seen. In Section 3.3, we discuss the reasons for these differences and demonstrate that the clumps are a numerical artefact.

3.1.4 Disc formation

Fig. 7 shows the surface density projection of the gaseous disc that forms in the higher resolution SPH and SPHS runs. Both discs have a similar mass of $\sim 2 \times 10^7 \, M_\odot$, and are of a similar size, extending out to $\approx 2.5 \, \text{kpc}$. These similarities are understandable
Figure 5. Projected surface density plots (left), phase diagrams (middle) and projected temperature plots (right) in SPH-96-res1 at $t = 1.2$ Gyr. The white dot at (0, 0, 0) marks the SMBH. The blue dotted line indicates the cooling floor, while the green and brown dotted lines mark the fixed SF threshold and polytropic pressure floor, respectively. The two thicker diagonal lines indicate the standard Sedov–Taylor solution (magenta) and the post-Sedov–Taylor adiabatic expansion (black) that fit the orientation of the feedback peaks and their evolution in time, respectively. The red outlined boxes indicate the regions of the phase diagram being plotted. Contours are binned logarithmically, starting at the minimum value for $\rho$ or $T$ and increasing with a factor of $d\rho/\rho = dT/T = 0.1$. The colours correspond to the number of gas particles, with cyan representing individual particles and subsequent colours going up in factors of 5. In this simulation, we see a great deal of structure, in the form of multiple overdense clumps, distributed over a large part of the computational domain and lying within the ‘isothermal contraction’ region of the phase diagram. We also see a broadened spike along the Sedov–Taylor gradient that corresponds to many different SN feedback events.
Figure 6. Projected surface density plots (left), phase diagrams (middle) and projected temperature plots (right) in SPHS-96-res1 at $t = 1.2$ Gyr. Linetyles, etc., are as per Fig. 5. There are striking differences in the structures that have formed compared to the SPH case (Fig. 5) — in the latter we saw multiple clumps whereas here we see one or two filaments. Again, the structure occupies the isothermal contraction region of the phase diagram, with the gas flowing down the filaments to feed the disc and form stars. Similar to the SPH case, we see a broadened spike along the Sedov–Taylor gradient corresponding to multiple feedback events, but reduced in magnitude.
Thermal instabilities in galactic coronae

Figure 7. Projected surface density plots of the central disc that forms in the SPH-96-res1 (left) and SPHS-96-res1 (right) runs, at \( t = 2 \) Gyr. Both discs are of a similar size, although the stochastic mode by which they have grown is very dependent on the random orientations of the SNe feedback, and they are therefore at different orientations despite the same initial velocity field for the gaseous halo. The SPH disc is surrounded by multiple clumps that formed in the halo whereas the clumps present in the SPHS case have formed from the disc itself.

given the identical IC used for both runs. However, the formation of the disc is strongly influenced by the asymmetries present in the SF and feedback events, which eject hot bubbles in a variety of directions into the gaseous halo. It is therefore not surprising that the discs in the SPH and SPHS runs are at different orientations, as the SF and feedback histories of the two are somewhat different. The stochastic nature of the feeding to the central regions is so significant that neither disc is in the preferred \( x - y \) plane set by the angular momentum of the ICs. The two discs do in fact start out in the \( x - y \) plane as they are initially formed in the first starburst, but undergo precession and mid-plane rotation as they are torqued by subsequent infalling gas at a variety of different orientations.

One important distinction between the discs in the two methods is the presence of the clumps orbiting outside the disc in the SPH case, which are not there in SPHS. We note that this was also seen in the study of gas discs building up in cosmological simulations by Torrey et al. (2012). This difference relates to the clumpy versus filamentary mode of feeding the disc as discussed above in Section 3.1.3. There are clumps within the disc in the SPH run, but these are physical, having formed out of the disc itself through gravitational instability, rather than being formed further out in the halo and migrating to the centre.

3.1.5 Galaxy morphology and stellar content

When we look in more detail at the kinematics of the gas and stars in the inner region, we find that the morphologies of both the stars and the gas actually differ by considerably more than a visual inspection of the disc (see Fig. 7) would suggest. In particular, the SPH-96-res1 run shows a larger amount of low angular momentum material within the central 8 kpc than is present in the SPHS equivalent. This can be seen clearly in Fig. 8, which shows plots of the \( J_z/J_c \) ratio (often referred to as the ‘disc/bulge’ ratio), where \( J_z \) is the magnitude of the angular momentum vector along the rotation axis of the inner 2 kpc of each disc, and \( J_c \) is the magnitude of the angular momentum for a circular orbit at a radius \( r \), namely \( J_c = rv_c \), where \( v_c^2 = GM(r)/r \) is the circular velocity from the enclosed mass of all species (gas + stars + DM + SMBH). This ratio was calculated for each star and for each gas particle and the histograms for both shown in the figure (top left and right, respectively). It is clear from these plots that the SPHS run shows a kinematically more pronounced disc than SPH, particularly in the gas but also in the stars as well. The many cold clumps that are formed in SPH provide the inner 8 kpc with a greater amount of low angular momentum material (gaseous and stellar) as well as a greater randomness of orbits as they fall in. The top-right plot tells us further that the gas disc seen in SPH in Fig. 7 is more disturbed and warped compared to the SPHS one, being strongly torqued out of its plane and heated by the infalling gas clumps at different orientations. We note that since it is this gas that tends to lie on the polytrope and is therefore ‘star-forming’, the stars that form here will likely inherit the \( J_z/J_c \) ratio seen for the gas in the top-right plot. This means that to later times (e.g., 3–4 Gyr), the stellar kinematic disc/bulge ratio will likely become even more disparate between the SPH and SPHS simulations, with SPH favouring a bulge and SPHS favouring a disc.

Also in Fig. 8, we see a plot of the enclosed mass in stars with radius (bottom left) and enclosed mass of gas with radius (bottom right), showing that the stellar content of the galaxies in SPH and SPHS differ considerably outside of \( \sim 2 \) kpc. The SPH run has a greater amount of mass in stars at all radii, and in particular further out in the halo, where the cold clumps are forming stars out to...
Figure 8. Plots of the kinematic ‘disc/bulge’ ratio at $t = 2$ Gyr for the inner 8 kpc in the stars (top left) and the gas (top right) together with the enclosed mass in stars (bottom left) and gas (bottom right) out to 100 kpc for the SPH-96-res1 (black solid) and SPHS-96-res1 (blue dashed) simulations. The SPHS run has a more disc-like morphology inside the central 8 kpc, with the $J_z/J_c \approx 1$ signal (a ‘disc’) being a factor of $\approx 4$ higher than SPH in the stars and a factor of $\approx 5$ higher than SPH in the gas. The SPH run shows a stronger peak at lower angular momentum with a $J_z/J_c \approx 0$ signal (a ‘bulge’) that also corresponds to more randomized orbits than in SPHS.

20–30 kpc. In the gas, the profiles are similar, but the SPHS run has a greater mass of gas at large radii in the hot halo.

3.2 Features in $\rho$–$T$ space

In order to follow the relevant behaviour and understand evolution of the simulations better, we analyse a number of aspects of them in terms of their locations on a phase diagram $\rho$ versus $T$. We isolate a few key regions and follow the behaviour of the gas that inhabits these regions on the phase diagram. The corresponding plots for this section are Figs 5 and 6, in which the phase diagrams are plotted along with the projected surface density and projected temperature for the isolated phase regions.

3.2.1 Isothermal contraction phase

As the gas cools out of hydrostatic equilibrium, it experiences a variable cooling rate that is set by the combined cooling curve...
of Katz et al. (1996) and Mashchenko et al. (2008) used in our simulations. At \( T = 10^7 \) K (the switch over between the two curves), the cooling becomes inefficient, and so gas tends to ‘pile up’ at around this temperature. We have isolated a region on the phase diagram (second from top) that corresponds to gas that has cooled to this temperature and is condensing, moving nearly isothermally across the marked region to higher densities and eventually on to the polytrope. This we refer to as an ‘isothermal contraction phase’, and has particular relevance to the many spurious clumps that are seen in the SPH runs, as it marks the region on the phase diagram that they inhabit. Although still present in the higher resolution SPHS runs, this region is significantly less occupied, and does not extend as far to low densities as in the SPH case.

### 3.2.2 Polytrope phase

As mentioned in Sections 2.4 and 2.5, the diagonal line shown on the phase diagram corresponding to a polytropic EQS with an adiabatic index of 4/3 functions as a dynamic pressure floor, ensuring that the Jeans mass is always resolved in the gas. The polytrope ‘fills in’ from gas that either cools directly on to it or from gas that reaches it through the isothermal contraction phase discussed above. Once on the polytrope, the gas can move along it to higher densities (and higher temperatures as per the EQS) and of course can move off it either by an increase in temperature for a given density or by a decrease in density for a given temperature. Gas can also leave the polytrope by being converted into stars, which is allowed to occur beyond the fixed threshold of \( 10^7 \) atoms cm\(^{-3}\), denoted by the green dotted line. The presence of gas on the polytrope past this fixed threshold is therefore transitory – it fills in and subsequently disappears as stars form.

### 3.2.3 Hot bubble phase

The filling in of the polytrope beyond the fixed density threshold for SF is often followed by an ejection event that pushes gas into the ‘overpressurized bubble’ section of the phase diagram. Within this region there we can identify both the \( \rho - T \) trend for a given bubble, as well as the evolution of this trend with time, through analytic arguments. To start with, the similarity solution for a Sedov–Taylor blast wave due to an energy deposition \( E \) in a uniform medium (Sedov 1959) is given by

\[
r(t) \propto \left( \frac{E}{\rho_{\text{ISM}}} \right)^{1/5},
\]

where \( \rho_{\text{ISM}} \) is the average (constant) density of the surrounding medium. We can define an overdensity parameter \( \delta_\iota \equiv \rho_\iota / \rho_{\text{ISM}} \) for the shocked gas, which under the assumption of a strong adiabatic shock, is a constant – given by \( \delta_\iota \approx 4 \). The shock velocity, \( v_s \equiv dR/dr \) is therefore:

\[
v_s \propto \frac{2}{5} E^{1/4} (4 \rho_\iota)^{1/5} \rho_{\text{ISM}}^{3/5} \rho_\iota = \frac{2r}{S},
\]

which we can write as

\[
v_s \propto E^{1/2} r^{-3/2} \rho_{\text{ISM}}^{-1/2}
\]

The post-shock temperature for an adiabatic shock with velocity \( v_s \) is

\[
T_s \propto \frac{\mu m_p v_s^2}{k_B},
\]

which gives us

\[
T_s \propto E r^{-3} \rho_{\text{ISM}}^{-1}.
\]

The trend \( T \propto \rho^{-1} \) is shown on the top phase plot (dotted magenta line) in Figs 5 and 6. The shape of this feature remains constant but evolves along the phase diagram as the bubbles expand. Such evolution can be described analytically by a post Sedov–Taylor solution for adiabatically expanding hot gas. If we assume that the expansion is sufficiently fast that radiative cooling can be neglected, the EQS is \( P \propto \rho^{\gamma} \), with \( \gamma = 5/3 \). We can write this in terms of the sound speed by noting that

\[
c_s^2 = \frac{\gamma P}{\rho},
\]

and therefore that

\[
P = \frac{\rho kT}{\mu m_p},
\]

which we equate to obtain

\[
T \propto \rho^{2/3}
\]

for the time evolution of the bubble. This trend is also plotted in the top phase diagram (dotted black line) in Figs 5 and 6.

Even though the analysis described above corresponds to the solution for a uniform medium, it fits the SNe ejecta in our simulations remarkably well.\(^4\) The reason for this is that the two departures from the standard Sedov–Taylor solution that are present in our model exert competing effects on the \( \rho - T \) profile of our hot bubbles.

First, we have a non-uniform medium, where \( \rho \) falls off according to equation (11). This has the effect of reducing the velocity fall-off as the shell propagates and sweeps up mass. Secondly, we have an inwardly directed velocity field as the gas cools and inflows under the influence of the gravitational potential, which has the effect of enhancing the velocity fall-off. The infall velocity arises self-consistently from the potential, which is a function of the density profile \( \rho(r) \). The two departures from standard Sedov–Taylor therefore balance out, and the majority of the SN-driven bubbles behave according to the analytical arguments above (with small additional departures due to radiative cooling and the modification of the density profile due to previous SNe explosions).

### 3.3 Numerical clump formation

So far, it is clear that the presence of the overdense clumps is a feature of the SPH method, but the question remains as to what causes them. The fact that an identical simulation run with SPHS (CS kernel with 96 neighbours) does not yield these structures suggests that the prevention of multivalued fluid quantities plays a role in avoiding their formation, as this is the main difference between the methods when the kernel and neighbour number are not sufficient to have improved force accuracy (refer to Read & Hayfield 2012). In this section, we show that this is indeed the case; it is the removal of pressure blips in an otherwise smooth flow that prevents the condensation of the clumps.

To identify bound gaseous clumps, we ran the Amiga halo finder (AHF; Gill, Knebe & Gibson 2004; Knollmann & Knebe 2009) on each simulation output. We focus here on the most gas-rich clump in the SPH run. Tagging the clump particles, we traced the evolution

\(^4\)To see the time evolution clearly the reader is directed to http://www.phys.ethz.ch/~ahobbs/movies.html.
of the clump back in time to its initial formation. This particular clump forms from the merging of three distinct regions of gas, one of which has a significantly higher density and lower temperature than the other two. As the three regions merge, a small density peak forms with a corresponding entropy dip caused by gas cooling in the peak. The flow at this point is strongly shearing and the density peak rapidly shears away. By contrast, however, the entropy dip remains due to the lack of multiphase mixing in SPH. The presence of an entropy dip with no corresponding density peak drives a pressure dip at the centre of the merging gas. This can be seen on the left-hand side of Fig. 9, where the entropy and pressure dips are marked by the red and magenta circles, respectively. The central pressure dip drives an inward collapse and the formation of a bound clump (see Fig. 10).

In order to be sure that this multivalued pressure problem is not present in the new SPH method, we took the starting conditions from the SPH run at $t = 0.4$ Gyr – just before the ‘clump gas’ began to merge – and ran it with SPH (CS kernel, 96 neighbours). Once again, we tracked the evolution of the gas which, in the SPH case, ended up in the clump. The initial merging occurred in exactly the same way, but as the density peak sheared away, so too did the dip in the entropy. As a result, there was no pressure dip and therefore no initial seed for collapse. This can be seen on the right-hand side of Fig. 9, where the features marked by the circles on the left-hand side are not present.

Fig. 10 shows the state of the same ‘clump-identified’ gas in the SPH and the SPHS cases. The former, as a result of the ‘pressure-dip seed’, has collapsed to form a near-spherical clump with multiple overdensities. In the SPHS run, however, the gas has been drawn out into a filament, or stream, having not been artificially pulled inwards due to a central dip in the gas pressure.

The filament/stream seen in Fig. 10 for the SPH evolution of the merging gas shows individual density peaks forming along the central axis – however, at this resolution these are dissimilar structures from the clumps seen in SPH, as they are transient, being mixed in with the surrounding gas before they can form stars. In the next section we discuss the formation of bound structures (which we term ‘physical clumps’) within the filament at higher resolution, which form due to the density enhancement created as two or more evacuated bubbles intersect.

4 ‘PHYSICAL CLUMP’ FORMATION WITH SPHS: FRAGMENTATION OF OVERDENSE FILAMENTS

We now move on from the comparison between the two methods at identical spatial resolution to a simulation that employs the full SPHS algorithm: in addition to the dissipation for advected fluid quantities, we also have improved force accuracy through the use of the HOCT4 kernel with 442 neighbours. We perform this simulation at significantly higher resolution than our fiducial comparison runs, reaching a gas particle mass of a few times $10^4 M_\odot$ (see Table 1).

The evolution of the higher resolution simulation is globally similar to those at lower resolution – an initial central starburst sends a strong Sedov–Taylor bubble into the surrounding gas, which shuts off SF and reduces SMBH accretion until such time as the gas makes its way back to the central regions. The subsequent starbursts are then highly asymmetric, with multiple cavities created from individual SNe feedback events. Again, there is no sign of the many artificial clumps condensing from the ambient halo gas, and the structure that forms as the gas cools is filamentary in nature, caused by the merging of the outer walls of two or more of the feedback-driven bubbles.

Here, however, we are in a new regime whereby bound, physical clumps are in fact able to form from the hot halo, but only within the filament(s). The latter structure provides a strong density enhancement within which collapse and fragmentation can occur through non-linear thermal instability (Fernández, Joung & Putman 2012; Joung et al. 2012), rather than through a numerical pressure discontinuity as was the case for the clumps in SPH. Fig. 11 shows the state of the inner 30 kpc at a time when the filaments have recently formed. We used AIF to identify the locations of any self-bound physical clumps; these are shown in blue. One or two have already collapsed and are forming stars. As can clearly be seen from the figure, the only locations where these physical clumps arise are in the streams and in the disc at the centre. The overdensity here is $\sim 10^{−10}$, which is consistent with the instability threshold for a non-linear perturbation in Joung et al. (2012).

The gas flowing through the filaments ends up contributing to the disc, as do the physical clumps that form from the fragmentation of the filament. Some of these, however, form stars before they can reach the disc, and therefore end up as orbiting stellar clusters, with masses of $\sim 10^7 M_\odot$. The stellar clusters, while they are still star-forming, contribute to the continued feedback events and can therefore aid the formation of subsequent streams that may feed the disc.

In order to quantify the ability of the cold gas in the filaments to grow the disc, in Fig. 12 we plot the gas mass contained in particular phase regions (the same regions marked in Figs 5 and 6 but for our highest resolution full SPHS run). We also plot the rate of increase or decrease in mass within this region to measure the feeding rate through the filaments. The main period of interest is after the second starburst, and we see from Fig. 12 (top plot) that the streams (and the physical clumps formed in the streams) are feeding the central disc at a rate of $\sim 1 M_\odot$ yr$^{-1}$ for the first Gyr after the starburst. This rate is gradually declining over time, and by $t = 2$ Gyr has dropped to $\sim 0.1 M_\odot$ yr$^{-1}$. Looking at the middle plot, we see further that the gas on the polytrope (most of which is in the disc) exhibits a similar rate of decrease in mass; this can be explained by referring to the SFR (bottom) plot in Fig. 11, which shows that the feeding rate of the disc by the filaments is approximately matched by the SFR, since most of the SF is occurring in the disc at this time.

The third (bottom) plot of Fig. 12 shows the amount (and rate of increase/decrease) of gas in the region of the phase diagram that we have identified as corresponding to the gas recently ejected from SNe explosions. We see here that this is by far the dominant repository for the gas that has already fallen into the central regions – the mass contained in stars, disc, filaments and physical clumps combined adds up to $\lesssim 1/10$ of the mass that has been ejected into the hot halo, the latter being on average $\sim 1–2 \times 10^{10} M_\odot$.

5 DISCUSSION

5.1 Clump formation via numerical instability

We find a clear difference in how cold gas condenses from a hot halo in SPH versus ‘classic’ SPH. The formation of approximately hundreds of cold clumps from relatively homogeneous regions of the hot halo in SPH owes to a numerical thermal instability. As hot SNe bubbles are driven into the low-density gas, there is a compression due to the shock front that excites cooling for some of the gas particles. This cooled gas should mix in entropy with the surrounding, hotter gas, but does not due to the lack of multiphase
Thermal instabilities in galactic coronae

Figure 9. Plots of density (top), pressure (middle) and entropy (bottom) for the most gas-rich overdense clump in the SPH-96-res1 run, at $t = 0.63$ Gyr. The gas that formed the clump was identified and tracked back earlier in the simulation to before it formed. On the left-hand side are the plots showing the evolution of this particular clump with the SPH method, while on the right-hand side the plots show the evolution of the clump with the SPHS method (although starting from the same SPH-96-res1 snapshot from which the clump was taken – at $t = 0.4$ Gyr). Each property ($\rho, P, A$) is plotted in the centre of mass frame of the clump gas at a time in its evolution where a density peak that occurred just previously has been smoothed out. This density peak had a corresponding entropy dip at the same location. When evolved with SPHS, the smoothing of the density peak coincides with a diffusion of the entropy, removing the dip and allowing the pressures to remain smooth; however, when exactly the same IC is evolved with SPH, the entropy dip remains (red circle), driving an equivalent dip in the pressures (magenta circle). This central pressure dip drives the contraction of the gas and the subsequent formation of the clump. For the full evolution and a clear picture of how this occurs the reader is directed to the corresponding movie at http://www.phys.ethz.ch/~ahobbs/movies.html.
Figure 10. Visualization of the numerical experiment discussed in Section 3.3 and Fig. 9. The IC (top plot) at \( t = 0.4 \) Gyr is identical (taken from the SPH simulation of an earlier state of the gas in one of the bound clumps) but was evolved both with SPH (bottom left) and with SPHS (bottom right) to a time of \( t = 1 \) Gyr. The differences are striking: the SPH evolution of the gas has formed a near spherical, dense clump while the SPHS evolution shows a stretched out filament of lower density. The point at which the two diverge is shown in Fig. 9, where a pressure discontinuity at the centre of the merging gas in the SPH case causes an artificial contraction that leads to the formation of the clump; this feature is not present in the SPHS case.
Thermal instabilities in galactic coronae

Figure 11. Projected surface density plot of the SPHS-442-res2 run at $t = 0.73$ Gyr (top) and SFR/SMBH histories (bottom; linestyles as per Fig. 2). As in the lower resolution case, we see the formation of one or more cold filaments that have formed out of the surrounding gas as a result of the interaction of SN-driven hot bubbles. The filaments are funnelling gas down onto the disc at the centre of the computational domain. The progenitors of gas-rich ‘physical clumps’ that are formed in the filaments and in the disc are marked in blue. The SFR history shows three to four distinct starburst phases, closely followed by the SMBH accretion rate.

mixing. Some of the gas is therefore artificially kept at low entropy, leading to pressure dips (refer to equation 3) when the densities are smoothed across the neighbours while the entropies are not. These pressure dips seed the formation of dense clumps of gas.

Figure 12. Plots of the gas mass contained within a particular region of the phase diagram (black) along with the rate of change of the gas mass into (blue; an ‘inflow’) and out of (red; an ‘outflow’) the phase region. The regions are as per Fig. 6 but in this case for the SPHS-442-res2 run. The ‘inflow’ and ‘outflow’ rates are plotted in bins of 80 Myr. The plots correspond to: (i) the gas in the ‘isothermal contraction’ region (top) – this comprises the temporary disc created in the initial starburst and then later the filaments that form and grow a new disc; (ii) the gas in the ‘polytrope’ region (middle) – this comprises the star-forming gas, i.e. the physical clumps that form in the disc and the filaments; and (iii) the gas in the ‘hot bubble’ region, corresponding to gas that has been ejected into the hot halo by SN feedback.
By contrast, in SPHS the gas is allowed to mix both in density and entropy and the numerical clumps do not form; this allows the gas to subsequently form into cold filaments at the intersection between colliding SN-driven bubbles. These cold filaments then feed the disc.

In the current paper, the SPH clump formation is therefore a direct result of the SNe feedback seeding artificially unmixed, low-entropy gas into the surrounding medium. Indeed, if we run the simulations with negligible feedback, we find that clump formation does not occur even in classic SPH. This demonstrates how low-noise our ICs are. By contrast, the Poisson noise in the Kaufmann et al. (2006) ICs means that it is all too easy for clumps to form. In Kaufmann et al. (2009), the ICs are relaxed to be low-noise, and as such clump formation is suppressed for the typical steep entropy profile used here. Noise will act similarly to SNe feedback, driving patches of high density that cool, seeding the same numerical thermal instability that we find seeds clump formation in our classic SPH simulations. It is clear then that classic SPH is numerically thermally unstable, and any small perturbation (however it may be caused) will seed artificial clump formation. While we can, in the current paper, carefully craft our ICs to be extremely smooth for these idealized simulations, in real calculations featuring cosmological setups this is unlikely to be straightforward.

There may be a situation where the formation of such clumps from thermal instability in low-density gas is physical, as indeed is suggested by Kaufmann et al. (2009) and explored analytically by Binney et al. (2009), which is when the entropy profile across the region is nearly completely flat and the cooling time-scale can become smaller than the oscillation time-scale for small, isotropic perturbations. However, such an entropy profile is a somewhat extreme setup for a galaxy formation simulation (both cosmological and non-cosmological) and indeed is certainly not the case in our simulations, where the entropy gradient is quite steep both initially and after multiple SNe feedback events. Even in this situation, Binney et al. (2009) find that thermal conduction suppresses linear thermal instabilities through the damping of small-scale perturbations, even for very small fractions of Spitzer’s value (Spitzer 1962).

It is clear then that although SPHS has a number of important changes relative to classic SPH that make it an improved algorithm for modelling galaxy formation (refer back to Section 2.2), in the simulations performed in this paper it is the inclusion of thermal conductivity that matters most. The ability of the code to share entropy between neighbouring particles eliminates artificial pressure dips that cause numerical conduction of the gas into clumps. It is therefore important to note that any SPH code that implements some form of thermal conductivity – e.g. GASOLINE (Wadsley, Stadel & Quinn 2004), or employs a density-independent formulation that removes artificial surface tension (Richie & Thomas 2001; Saitoh et al. 2009; Hopkins et al. 2012) – is likely to perform much better than classic SPH on this particular galaxy formation problem.

5.2 Physical clump formation along filaments

The ability of the SPHS simulations to prevent artificial condensation allows us to follow the formation of structures that were not previously seen in SPH simulations due to the dominance of the numerical clump mode of structure formation. The gas in-between hot bubbles (created by SNe feedback) is found to condense out as the bubbles begin to cool and collapse back. This condensation takes the form of filaments, through which cold gas is funneled down to the galactic disc. In our highest resolution SPHS run, the filaments further undergo fragmentation to form bound, ‘physical clumps’. Both the condensing of the filaments and the subsequent fragmentation are physically motivated, with the latter owing to a non-linear instability caused by the $\gtrsim 10$ overdensity in the filament (Joung et al. 2012).

5.3 Overdense filaments versus cosmological ‘cold drizzle’

Our finding that at sufficient resolution the SPHS simulations show resolved, physical clumps forming in the overdense filament(s) that feed the disc shows striking similarities to the break-up of the ‘cold mode’ cosmological streams seen in the larger scale high-resolution simulations by Kereš et al. (2009) and Kereš & Hernquist (2009). In both cases, the seeding of an overdensity that subsequently allows for a non-linear mode of collapse is responsible for the formation of individual, pressure confined clouds – but the nature of the overdensity differs. In our case, filaments are formed from the intersection of collapsing, SN-generated hot bubbles, at scales of $\sim 20–30$ kpc, and penetrate all the way down to the forming galaxy. In the Kereš & Hernquist (2009) picture, the progenitors of the clouds are large-scale flows that originate outside the halo, reaching down to $\sim 30–40$ kpc from the galaxy. Despite being the ‘cold mode’ of halo accretion, these streams are hotter than the filaments we see in our simulations, having a temperature $T \sim 10^4$ K versus $T \sim 10^5$ K.

It would seem, therefore, that our SN-created filaments represent a smaller scale copy of a (likely physical) mode of clump formation that has been seen to occur at larger scales but that does not reach all the way down to the galaxy, although in general the seeded clouds possess preferentially lower angular momentum than the surrounding, hotter gas, and are able to ‘rain down on’ the galactic disc. Our smaller scale mode, on the other hand, is able to feed the disc directly, with the physical clumps that form being funneled down through the filaments to, in most cases, contribute significantly to the mass budget of the galaxy.

It is also important to note that while the initial clump formation in the cosmological ‘cold drizzle’ may be trustworthy, in SPH simulations of this process the survival time of these structures will be artificially long due to their inability to mix with the ambient medium (cf. Agertz et al. 2007).

5.4 Implications for feeding the Milky Way disc

The action of the SNe feedback in driving the filamentary mode of disc feeding has particular relevance to the recent work by Fraternali & Binney (2008) and Marinacci et al. (2011), where the authors suggest a model of gas entrainment from the hot halo through the motion of feedback-driven galactic fountain clouds. These colder clouds mix with the coronal gas and invoke a significant amount of mass transfer from the hot phase to the cold phase, before sinking back again to the galaxy. The key concept here is that the cooling of gas from the halo is dependent on the presence of star-forming gas deeper inside the potential well. Our picture is slightly different, but still within the same vein, as it is the interaction of the walls of multiple SN-driven bubbles that cause the condensing of the filaments, which in turn may break up and form cold, physical clumps through non-linear instability as per Joung et al. (2012). Naturally, therefore, we require that SF occurs in the disc in order for cold gas to be able to accrete – a form of positive feedback.

Observations by Boomsma et al. (2005) lend support to the mode of structure formation and galaxy feeding that we have found; X-ray emission from hot gas is seen above the plane of the nearby spiral galaxy NGC253, along with an overdense filament-like structure
detected in HI at the edge of the hot region, that stretches out for approximately 12 kpc. These features are associated with a significant amount of SF activity and indeed a recent starburst and superwind from the galaxy (Heckman, Armus & Miley 1990).

In our highest resolution full-SPHS simulation, we find that the fragmenting filaments formed from overlapping SN-driven bubbles feed the disc (and subsequently, SF in the disc) at a rate of \(\sim 1 \, \text{M}_\odot \, \text{yr}^{-1}\), similar to what is required by observations to fuel SF in the Milky Way (Noh & Scalo 1990; Rocha-Pinto et al. 2000). We also find that the gas mass ejected from SNe explosions remains near-constant around \(1 - 2 \times 10^{10} \, \text{M}_\odot\) for the duration of the simulation (2 Gyr). If one assumes that the inner hot halo is composed primarily of gas from SNe feedback processes then this is in excellent agreement with the mass of the hot halo as determined by pulsar dispersion measures (e.g., Gaensler et al. 2008; Anderson & Bregman 2010). We emphasize, however, that our goal in this paper is to study the physics of thermal instabilities in cooling haloes rather than to build a faithful model of Milky Way-mass disc galaxies. Our results are likely sensitive to our sub-grid model parameters, to our resolution, and to our ICs that are idealized and non-cosmological. Thus, while we can identify an important thermal instability in the halo that can in principle supply significant cold gas to the disc, it is unclear how important it is with respect to other mechanisms like gas-rich mergers, low-redshift cold streams and gas recycling from stars in the disc. We will explore this in future work.

5.5 Implications for disc morphology

A further aspect of the comparison between the SPH and SPHS runs at identical resolution that is particularly interesting is the difference in the kinematic morphologies of the galaxies due to the different cooling mechanisms, i.e. clumps versus filaments. In the SPH case the cold, numerical clumps contributed to a stronger galactic bulge-like feature with a greater amount of material at low angular momentum and on random orbits, while in SPHS the growth of the disc from the cold streams led to a stronger disc feature that was less warped and undergoing a lesser torque from infalling material at different orientations. This result naturally has implications for the so-called angular momentum problem in galaxy formation, namely the presence of bulges that are overdominant and discs that are underdominant compared to observations (see, e.g., Maller & Dekel 2002; Burkert & D’Onghia 2004; Governato et al. 2010).

5.6 To converge or not to converge

As we have mentioned in Section 2.5, we do not expect the SF histories in our simulations to converge as we go up in resolution, since as we raise the resolution we lower our polytropic cooling floor. This allows us to resolve ever smaller scale physical processes that are not captured at the resolution of the res0 and res1 simulations. For example, at res0 we do not resolve the cooling filaments in SPHS. These appear at res1, leading to the sudden appearance of a second peak in SF (compare Figs 2 and 4, right-hand panels). In Fig. 11, we show the results of an even higher resolution simulation – res2 – that resolves yet another scale: the break up of the filaments into clumps. This has a smaller effect on the global SF (Fig. 11, lower panel), but nonetheless does not produce smoothly converging results. Only when all such key scales are resolved, can we freeze our polytropic cooling floor to prevent collapse to even smaller scales and achieve numerical convergence. We discuss this in more detail in a forthcoming paper on numerical convergence in SF simulations.

6 CONCLUSIONS

We have presented simulations of a cooling gaseous halo in a Milky Way mass galaxy, using this particular problem to perform the first scientific investigation with a new hydrodynamics code, SPHS. We have compared the results obtained (at identical spatial resolution) with that of the standard (‘classic’) SPH method employed in the literature, finding significant differences in the mode of gas cooling in the halo and subsequently the mode of disc feeding. The puzzle of the many cold clumps seen forming from the halo in many SPH simulations of galaxy formation is attributed to a numerical inability driven by unresolved mixing of different gas phases. We demonstrate both with our full simulations (Section 3.1) and with a more idealized test (Section 3.3) that the removal of pressure blips in an otherwise smooth flow prevents the formation of the clumps, leading instead to the formation of cold filaments that feed the disc. The resulting galaxies in the SPH and SPHS simulations differ greatly in their morphology, gas phase diagrams, and stellar and gaseous disc/bulge ratio.

We have explored in more detail the mode of disc feeding seen in our SPHS simulations, going to higher resolution and employing a kernel that gives improved force accuracy. We find a new way of bringing cold gas to the galactic disc, namely the fragmentation and collapse through non-linear thermal instability of filament(s) formed at the intersection of SN-driven bubbles. The feeding rate of cold gas (\(T \lesssim 10^4\) K) to the disc is found to be approximately a solar mass per year, which suggests this is a promising model for fuelling late-time SF in real spiral galaxies.

We emphasize that our focus in this paper was on understanding what drives thermal instabilities in hot halo gas, and in particular performing a comparison between the ‘classic’ SPH and the SPHS numerical methods. Our numerical experiments are idealized and do not present a complete picture of galaxy formation. Nonetheless, by employing a hydrodynamics method that resolves the mixing of different gas phases, we find a novel mode of cold gas accretion and disc growth that may be very relevant for galaxy formation.

ACKNOWLEDGEMENTS

We are grateful to the referee for providing a detailed report that helped improve the paper. We thank Volker Springel for the use of GADGET-3 in this work. We acknowledge useful discussions with Filippo Fraternali, Walter Dehnen, Tobias Kaufmann and Javiera Guedes. AH would like to thank Alexander Knebe for the use of, and guidance on, the Amiga halo finder (AHF). JR would like to acknowledge the support from SNF grant PP00P2_128540/1. The simulations were performed on the NCI and iVEC facilities at the University of Western Australia (UWA).

REFERENCES

Agertz O. et al., 2007, MNRAS, 380, 963
Agertz O., Teyssier R., Moore B., 2009, MNRAS, 397, L64
Anderson M. E., Bregman J. N., 2010, ApJ, 714, 320
Anderson M. E., Bregman J. N., 2011, ApJ, 737, 22
Bate M. R., Burkert A., 1997, MNRAS, 288, 1060
Bate M. R., Bonnell I. A., Price N. M., 1995, MNRAS, 277, 362
Besla G., Kallivayalil N., Hernquist L., Robertson B., Cox T. J., van der Marel R. P., Alcock C., 2007, ApJ, 668, 949
Binney J., Nipoti C., Fraternali F., 2009, MNRAS, 397, 1804
