Fully automated calculation in fermion scattering *

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The package aiTALC has been developed for fully automated calculations of two fermion production at $e^+e^-$ collider and other similar reactions. We emphasize the connection and interoperability between the different modules required for the calculation and the external tools DIANA, FORM and LOOPTOOLS. Results for $e^+e^- \rightarrow ff, e^+e^-$ are presented.

1. INTRODUCTION

The proposed $e^+e^-$ international linear collider (ILC), is expected to open a new era of precision physics and help understanding many of the fundamental questions that the discoveries of the forthcoming large hadron collider (LHC) will arise. With energies up to at least 500 GeV in its first phase [1], the ILC will uniquely provide a level of experimental precision not reached so far yet in particle physics, demanding an equivalent understanding of the theoretical predictions and uncertainties which, for the two fermion production, should not be larger than a few permill.

In order to fulfill such theoretical duty, the calculations within the present standard model must include typically the next or next-to-next to leading order perturbative corrections, depending on the process. The case of two fermion production is specially interesting since it plays a key role for

1. luminosity monitoring (Bhabha scattering),

2. directly measuring precise distributions and asymmetries,

3. establishing limits on New Physics as contact interaction, $Z'$, leptoquarks, or gravitons in extra dimensions,

4. quantifying backgrounds for other processes.

Thanks to the recent progress in computer technologies and software applications, particle physics has greatly profited from an indeterminate number of tools and packages developed for the specific task of automating perturbative calculations. Globally speaking, today’s available calculational tools do not individually meet the next properties:

• Performing higher order corrections beyond leading order.

• Free-availability and free-based code (i.e. neither the package nor its software requirements have to be purchased).

• Comprehensibility, both in documentation and installation.

• Flexibility, being able to deal with different processes, theoretical models and to allow some modifications.

Next, it will be shortly described some features of the package aiTALC showing some interesting immediate applications in two fermion production.

2. AUTOMATION

The tool aiTALC [2] is designed with the intention of matching the desired requirements pointed out above. Its goal is to perform automated perturbative calculations of cross sections in high energy physics, restricted to $2 \rightarrow 2$ fermion processes within the electroweak standard model (EWSM) and quantum electrodynamics (QED).
The tool integrates only free-of-charge packages existing on the market. As the calculation proceeds, in a modular fashion, it profits from an adaptation of the DIANA package (based itself on the QGRAF code) for the generation and analysis of Feynman graphs, the FORM language when dealing analytically with the large expressions in the amplitudes and, finally, the LOOPTOOLS library (also integrating the FF package) for the numerical calculation of the loop integrals.

In order to simplify and guarantee a wide portability during installation, it proceeds conforming to the standards of the UNIX packages. The operations include a customization of your system settings via configure and the later extraction of individual packages (if needed) and examples by a make instruction. At this point the installation is finished.

A detailed discussion of the algorithms used, the strategy of the calculation and internal work of each of the modules has been already presented elsewhere. Here I will shortly discuss the working environment and the examples provided to the user.

2.1. Makefile as environmental language

Automation requires effective intercommunication and a smart way to organize the tasks. This second concept has been exploited through the Makefile environment, providing a chain dependency between files and modules. The Makefile environment accomplishes the running of the whole process, from letting the user only the responsibility of modifying the driver file to building the tree, loop and fortran sections. At the end, the compilation of the program leading to the final results is achieved without further interaction.

The main directory into a process contains a main Makefile that organize the calculation in a generic footing. Under a generic unique make command, it calls the submakefiles Level.mk and Fortran.mk, providing them the necessary variables to carry out the tree level, the loop level and the compilation of the generated code for numerical output. The Makefile will consider the inclusion of extra files in the main process directory as new settings for the computation instead of the default ones appearing in the directories form/kitform3/ and diana/prg/.

As a result of the previous call the directories tree/, loop and fortran are created and filled with the log files of the diagram generation, symbolic amplitudes, eps drawings of the diagrams (for the two levels) and the complete FORTRAN programs, subroutines with the form factors grouped by topologies and other kinematical functions. The elapsed time will always be printed in the file Makefile.log.

The numerical executable program main.out is located inside fortran and on its default running returns a sample of differential cross sections for five different angles at a given energy of 500 GeV. There are 7 printout columns: first and second remind the energy and scattering angle. Born cross section, virtual loop corrections and soft photon radiation appear in the columns 3 to 5 respectively, while the sum of all them corresponds to the column number 6 (the expected result). The last column is reserved for the squared of the loop amplitude serving as a rough estimator of the order of the error at next perturbative order, or main result in cases where the tree level is not physically possible.

Alternative integrated cross sections and/or forward-backward asymmetries are available when the logical variables lprintics, lprintfba become ‘true’. For them a second group of columns show the estimated integration error following Richardson extrapolation to Romberg integration.

2.2. Examples with loop corrections

The package contains three examples located in the examples/ directory which the user may run without further considerations for test and learning purposes. Two of them contain radiative corrections:

- Bhabha scattering in QED \((e^-e^+ \rightarrow e^-e^+)\) in Bhabha_QED,
- flavour changing through neutral current example \((e^-e^+ \rightarrow b\bar{s} in leLe.bS)\).

\(^{2}\)As in the flavour changing neutral currents processes
With the purpose of giving a flavour of execution time, the examples were run on two different machines, representing today’s typically available computers. The results are shown in Table timings and take from some seconds to minutes. Only when the size of the produced Fortran code begins to be large (i.e. more than ∼200KB for a single subroutine to be compiled), then the GNU compiler gets extremely overloaded slowing down significantly the overall time.

Table 1

Typical timings for the different modules. The technical specifications for system ‘Desk(top)’: Intel Pentium III 853MHz cpu, 256MB RAM, GNU g77 version 3.3.3 compiler. For ‘Lap(top)’ are: Intel Centrino 1.5GHz cpu, 512MB RAM, Intel ifort version 8.1 compiler.

| Module  | 1eLe.bS | bhabha_QED |
|---------|---------|------------|
|         | Desk    | Lap        | Desk    | Lap    |
| tree    | 4”      | 1”         | 30”     | 9”     |
| loop    | 2’20”   | 52”        | 37”     | 12”    |
| fortran | 4:35’56” | 33”   | 26”     | 8”     |
| Total   | 4:38’20” | 1’26” | 1’33”   | 29”    |

3. Selected results

3.1. Fermion anti-fermion production

The $e^-e^+ \rightarrow b\bar{b}$ and $e^-e^+ \rightarrow t\bar{t}$ are two of the processes where it is expected to measure, with relatively high statistics, the indirect impact of the Higgs sector due to the large couplings involved through the top quark mass.

Two plots are presented in Fig. trying to emphasize the magnitude of the corrections with increasing energy for the $t\bar{t}$ and $b\bar{b}$ production. The figure on top shows the expected integrated cross section, for the whole phase space. Continuous lines represent the calculation at Born level while the others, dotted and dash-dotted lines, account for the corrections with two different cuts on the maximum energy available for the soft-photon, which is related with the acollinearity of the two fermions in the final state. The lower graph, scans the expected forward-backward asymmetry, up to 1 TeV. Note the phenomenological implications of the $Z$-boson threshold, inverting the asymmetry for the $b$ quarks and also leading to the well-known resonance on the integrated cross section.

For the top, the threshold energy for production is above the $Z$ mass leading to positive asymmetries.

3.2. Massive Bhabha scattering

Besides the phenomenological implications of Bhabha scattering ($e^+e^- \rightarrow e^+e^-$) in accelerator physics, it has also been considered here for ‘calibration’ in automated computations. The fact of comparing the massive case introduce most of the potential sources for discrepancies between two different implementations.

In Table 2, we present values for differential cross sections at 500 GeV for different $\cos \theta$. The expectations of the Born level and $O(\alpha)$ radiative corrected values are shown for the package $aTALC^3$, another totally independent automated package composed out of FeynArts+FORMCalc+LoopTools, and the same calculation dropping out terms proportional to electron mass. The results are impressive since the agreement between both packages are far from the massless electron limit, and they are limited by the roundoff of double-precision in the table. This increases from ∼11 to 15 digits the previous agreement reported by different groups in such a technical comparisons [12,13,14], performing therefore a strong check of the correctness of the calculations.

4. Conclusions

We have focussed on the development and immediate applicability of computer algebra and numerical tools to the scattering of two by two fermions, resulting into a major initiative for fully automated calculations, $aTALC$, involving the feasibility of integrating the work performed by others and to adapt it to some specific needs.

The direct application of this tool led to the calculation of practically all $2 \rightarrow 2$ fermion processes

3Quadruple precision is turned on here, setting 33 digits of precision.
Table 2: Numerical comparison of Bhabha scattering at \( \sqrt{s} = 500 \text{ GeV} \) against the package \textsc{FeynArts} + \textsc{FormCalc} and the massless electron case. Numbers for \textsc{atTalc} are shown in quadruple precision.

\[
\begin{array}{cccccccc}
  \text{Group} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
  \text{F A} + \text{F C} & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 \\
  \text{atTalc} & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\
  \text{F A} + \text{F C} & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\
  \text{atTalc} & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 \\
  \text{F A} + \text{F C} & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\
  \text{atTalc} & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 \\
  \text{F A} + \text{F C} & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\
  \text{atTalc} & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 & 0.9999 \\
\end{array}
\]
Figure 1. Integrated cross section (above) and Forward-backward asymmetry (below) for $b\bar{b}$ (thin lines) and $t\bar{t}$ (thick ones) production at high energies. The corrected results ($B+l+s$) are computed with two alternatives soft-photon maximum energy fraction, $\omega = 0.1$ and $0.05$. 