An explicit high-efficiency algorithm for simulating high-cycle fatigue behavior of metals

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Abstract. This article presents a direct and efficient algorithm for simulating high-cycle fatigue failure effects within the framework of a recently proposed elastoplastic model, in which fatigue failure effects of metals are incorporated as inherent constitutive features. Toward bypassing very time-consuming procedures in conducting numerical integrations of the elastoplastic rate equations for extremely large number of loading cycles, from the new model it is possible to derive an explicit expression for the fatigue life directly in terms of the stress amplitude. Numerical examples show that the new algorithm thus established is much more efficient than usual direct numerical procedures and that is particularly the case in simulating fatigue failure effects with very high cycle number.

1. Introduction

Assessment of the reliability and safety issues for key engineering components made of metals is based upon effective methodologies of simulating complicated effects induced by fatigue failure. Numerous results in this respect have been obtained from various standpoints. Details may be found in survey articles [1-3]. Most recently, innovative elastoplastic models have been established [4-7], which automatically incorporate fatigue failure effects of metals into inherent response features.

Very time-consuming procedures have to be carried out in numerical integrations of elastoplastic rate equations. That is particularly the case for extremely large number of loading cycles. In reality, it is expected that the cycle number ranging from 10⁵ to 10⁸ may be involved in various cases of high and even ultrahigh cycle fatigue failure. Without an efficient algorithm, it appears unrealistic to perform numerical computations with very high loading cycles.

This contribution presents a direct and efficient algorithm for simulating high cycle fatigue failure within the framework of the new elastoplastic models in [4-6]. Toward bypassing the foregoing issue, from the new model an explicit expression for the fatigue life is derived directly in terms of the stress amplitude. Numerical examples show that the new algorithm thus established is much more efficient than usual numerical procedures and that is particularly the case in simulating fatigue failure effects with very high cycle number.
2. New elastoplastic equations incorporating fatigue effects

A reduced form of the self-consistent elastoplastic model established in [4-6] is used for our purpose. Let \( \mathbf{D} \) and \( \mathbf{\tau} \) be the stretching and the Kirchhoff stress. The deviatoric part of \( \mathbf{\tau} \) is signified by \( \mathbf{\tau}^\text{dev} \). Then, the elastoplastic \( J_2 \)-flow equations are presented as follows [4-6]:

\[
\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p, \\
\mathbf{D}^e = \frac{1}{2G} \mathbf{\tau}^\log + \frac{\nu}{E} \left( \text{tr} \mathbf{\tau}^\log \right) \mathbf{I}, \\
\mathbf{D}^p = \rho \left[ \frac{\zeta}{2} + \frac{\zeta}{2} \frac{\partial f}{\partial \mathbf{\tau}} \right], \\
\dot{\mathbf{\tau}} = \mathbf{\tau} : \mathbf{D}^p,
\]

In the above, \( G, \nu \) and \( E \) are the shear modulus, the Poisson ratio and Young’s modulus as in classical Hooke’s law. The \( \mathbf{\tau}^\log \) is used to denote the logarithmic rate of the Kirchhoff stress \( \mathbf{\tau} \). The quantities appearing in the above equations are given below:

\[
\rho = \frac{g}{2r} \left[ 1 + \tanh \left[ \beta \left( \frac{g}{\alpha r} - 1 \right) \right] \right] e^{-m(1-g/r)}, \\
\begin{cases}
f = g - r, \\
g = \frac{1}{2} J_2 = \frac{1}{2} \text{tr} \mathbf{\tau}^2, \quad r = \frac{1}{3} q^2, \\
\zeta = \frac{\bar{f}}{h}, \\
\bar{f} = 2G \mathbf{\tau} : \mathbf{D}, \quad \bar{h} = \frac{2}{3} J_2 \left[ 3G + q q'(\kappa) \right], \\
q = \frac{1}{2} q_0 \left[ 1 + \tanh \left[ \beta (\kappa_c - \kappa) \right] \right],
\end{cases}
\]

In the above, \( m, \alpha \) and \( \beta \) are dimensionless positive material parameters, and \( q_0 \) and \( \kappa_c \) are positive material parameters of the same dimension as the stress.

With the new model, fatigue failure effects are characterized jointly by \( \rho \) in equation (5) and \( q \) in equations (6) and (9), which are two reduced forms of the broad case in [4-6].

3. Explicit formula for fatigue life

In the uniaxial loading-unloading cases, the elastoplastic equations in the last section are reduced to the following forms:

\[
\frac{dh}{d\tau} = \frac{1}{E} + \frac{\rho}{3G(1-\rho)}, \\
\frac{d\kappa}{d\tau} = \frac{\rho \tau}{3G(1-\rho)},
\]

with
\[
\rho = \frac{\tau^2}{2q_0^2} \left[ 1 + \tanh \beta \left( \frac{\tau}{\alpha r} - 1 \right) \right] e^{-m(1-\tau/q_0)}, \tag{12}
\]

Here, the axial stress and the axial logarithmic strain are designated by \(\tau\) and \(h\), respectively.

The plastic work produced in the \(i\)-th loading cycle from the minimum stress \(A_i\) to the maximum stress \(\Delta\), denoted as \(\Delta\kappa_i\), may be derived as follows:

\[
\Delta\kappa_i = \int_{\Delta}^{\kappa_i} \frac{\rho\tau}{3G(1-\rho)} d\tau,
\tag{13}
\]

Then, the total plastic work after \(N\) loading cycles is as follows:

\[
\kappa_N = \sum_{i=1}^{N} \Delta\kappa_i = \int_{\Delta}^{\kappa_N} \frac{\rho\tau}{3G(1-\rho)} d\tau.
\tag{14}
\]

When the total plastic work \(\kappa_N\) approaches the critical plastic work \(\kappa_c\), the stress limit \(q\) goes rapidly down to vanish and, as such, fatigue failure will emerge. Hence, a unified failure criterion for loading-unloading cycles of varying amplitudes may be derived, as given below:

\[
\kappa_c = \kappa_N = \sum_{i=1}^{N} \int_{\Delta}^{\kappa_i} \frac{\rho\tau}{3G(1-\rho)} d\tau.
\tag{15}
\]

For usually treated constant stress amplitudes, the following simple formula for the accumulated plastic work and the fatigue life are obtainable:

\[
\kappa_N = N \int_{\Delta}^{\kappa} \frac{\rho\tau}{3G(1-\rho)} d\tau,
\tag{16}
\]

\[
N = \int_{\Delta}^{\kappa_c} \frac{\kappa_c}{3G(1-\rho)} d\tau.
\tag{17}
\]

The new and direct criterion leads to an algorithm of high efficiency in treating high cycle fatigue cases as the cycle number ranges from \(10^5\) to \(10^8\) times. It is worthwhile to note that, for such high cycle numbers, the time consumption with usual numerical integration procedures may far exceed what could be afforded.

4. Numerical examples for model validation

For the purpose of model validation, we compare the model simulation with the test data for EN AW-6063, a kind of aluminum alloy, reported in [7] for both uniaxial monotonic extension till failure and high cycle fatigue with the amplitude ratio \(R=0.1\). The parameter values are listed in table 1, and the model simulations with the new algorithm are compared with the test data, as depicted in figures 1 and 2.

| Parameter values for simulation of monotonic and fatigue failures. |
|-------------------|-----|-----|-----|-----|-----|-----|
| \(E\) (GPa) | \(\nu\) | \(q_0\) (MPa) | \(\kappa_c\) (MPa) | \(\beta\) | \(m\) | \(\alpha\) |
| 70 | 0.3 | 222 | 23 | 7 | 2.3 | 0.48 |
5. Summary
With the new algorithm, less than 30 seconds are needed in calculating the S-N curve for a number of stress amplitudes. In this case, $10^7$ and even much higher cycles are involved. However, it is expected that more than 100 hours would be consumed with usual numerical integration procedures.

Here, the direct algorithm of high efficiency is provided for the uniaxial case. It is of much significance to extend this algorithm to the broad case of multiaxial fatigue effects. Also, medium high and even low cycle fatigue effects need be treated, in addition to high cycle fatigue cases treated here. That is particularly the case for multiaxial effects. Results in these respects will be reported elsewhere.

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