Study of $B \to \pi \ell \nu$ and $B \to \rho \ell \nu$ decays and determination of $|V_{ub}|$

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We present an analysis of exclusive charmless semileptonic $B$-meson decays based on 83 million $B\bar{B}$ pairs recorded with the BABAR detector at the $\Upsilon(4S)$ resonance. Using isospin symmetry, we measure branching fractions $B(B^0 \rightarrow \pi^- \ell^+ \nu) = (1.38 \pm 0.10 \pm 0.16 \pm 0.08) \times 10^{-3}$ and $B(B^0 \rightarrow \rho^- \ell^+ \nu) = (2.14 \pm 0.21 \pm 0.48 \pm 0.28) \times 10^{-3}$, where the errors are statistical, experimental systematic, and due to form-factor shape uncertainties. We compare the measured distribution in $q^2$, the momentum-transfer squared, with theoretical predictions for form factors from lattice QCD and light-cone sum rules, and extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}| = (3.82 \pm 0.14 \pm 0.22 \pm 0.11^{+0.08}_{-0.52}) \times 10^{-3}$ from $B \rightarrow \pi \ell \nu$, where the fourth error reflects the uncertainty of the form-factor normalization.

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The parameter $|V_{ub}|$ is one of the smallest and least known elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1]. A precise determination of $|V_{ub}|$ would significantly improve the constraints on the Unitarity Triangle and provide a stringent test of the Standard Model mechanism for CP violation.

In this paper, we present a determination of $|V_{ub}|$ from charmless semileptonic decays of $B$ mesons with exclusively reconstructed final states, $B \rightarrow h_u \ell \nu$, where the hadronic state $h_u$ represents a $\pi^\pm$, $\pi^0$, $\rho^\pm$, or $\rho^0$ and $\ell$ represents $e$ or $\mu$. Exclusive decays allow for kinematic constraints and more efficient background suppression compared to inclusive decays, but must rely on theoretical form-factor predictions. Using isospin symmetry, we measure the branching fractions $B(B^0 \rightarrow \pi^- \ell^+ \nu)$ [2] and $B(B^0 \rightarrow \rho^- \ell^+ \nu)$ as a function of $q^2 = (p_\ell + p_\nu)^2$, the momentum-transfer squared, and extract $|V_{ub}|$ using recent form-factor calculations based on light-cone sum rules (LCSR) [2,8] and unquenched lattice QCD (LQCD) [2,8].

This measurement is based on a sample of 83 million $B\bar{B}$ pairs recorded with the BABAR detector [8] at the PEP-II asymmetric-energy $e^+e^-$ storage rings. The data correspond to an integrated luminosity of 75.6 fb$^{-1}$ collected at the $\Upsilon(4S)$ resonance and 8.9 fb$^{-1}$ recorded below it. Simulated $B\bar{B}$ events are used to estimate signal efficiencies and shapes of signal and background distributions. Charmless semileptonic decays are simulated as a mixture of three-body decays $B \rightarrow X_u \ell \nu$ ($X_u = \pi, \eta, \eta', \rho, \omega$) based on the ISGW II quark model [8]. Decays to non-resonant hadronic states $X_u$ with masses $m_{X_u} > 2m_\pi$ are simulated following a prescription of Ref. [8].

We identify charmless semileptonic decays by a charged lepton with momentum $|\vec{p}_\ell| > 1.3$ GeV [22], a $\pi$ or $\rho$ meson, and missing momentum $|\vec{p}_{miss}| > 0.7$ GeV in the event. We identify $\rho$ mesons via the decays $\rho^\pm \rightarrow \pi^\pm \pi^0$ and $\rho^0 \rightarrow \pi^+ \pi^- \pi^0$ with mass $0.65 < m_{\pi^+ \pi^-} < 0.85$ GeV, rejecting candidates in which a charged track is identified as a kaon; both $\pi^\pm$ and $\rho$ candidates are rejected if a charged track is identified as a lepton. The charged lepton is combined with an $\pi^0$, $\rho^0$ or $\rho^\pm$, opposite charge to form a "$Y$" candidate; $Y$ candidates are rejected if the lepton and an oppositely-charged track from the signal hadron are consistent with a $J/\psi \rightarrow \ell^+ \ell^-$ decay.

The neutrino four-momentum, $p_\nu = (E_{miss}, \vec{p}_{miss})$, is inferred from the difference between the net four-momentum of the colliding-beam particles, $p_{beams} = (\vec{E}_{beams}, \vec{p}_{beams})$, and the sum of the four-momenta of all detected particles in the event. To reduce the effect of losses due to the detector acceptance, we require a total charge of the event of $|Q_{tot}| \leq 1$ and a polar angle of the missing momentum in the range $0.6 < \theta_{miss} < 2.9$ rad. In addition, the missing mass measured from the whole event should be compatible with zero. Because the missing-mass resolution varies linearly with the missing energy, we require $|m_{miss}^2/2E_{miss}| < 0.4$ GeV. We compute the angle between the $Y$ candidate and the $B$ meson, assuming zero missing mass, as $\cos \theta_{BY} = (2E_\ell E_Y - M_B^2 - M_Y^2) / (2|\vec{p}_\ell||\vec{p}_Y|)$. Here $M_B, M_Y, E_\ell^*, E_Y^*, \vec{p}_\ell^*, \vec{p}_Y^*$ refer to the masses, energies, and momenta of the $B$ and $Y$. Signal candidates are required to satisfy $|\cos \theta_{BY}| < 1.1$, allowing for detector resolution and photon radiation.

We restrict the momenta of leptons and hadrons in $Y$ candidates to enhance the signal over backgrounds. For $B \rightarrow \pi \ell \nu$, we require $|\vec{p}_\ell^*| + |\vec{p}_{miss}^*| > 2.6$ GeV; for $B \rightarrow \rho \ell \nu$, $1.5|\vec{p}_\ell^*| + |\vec{p}_{miss}^*| > 4.2$ GeV and $|\vec{p}_\nu^*| > 1.8$ GeV. These criteria keep 99.8% (75%) of true $B \rightarrow \pi(\rho) \ell \nu$ decays and reduce the $B \rightarrow X_u \ell \nu$ background by about 10% (80%) after all other selection criteria. To suppress backgrounds from $e^+ e^- \rightarrow q\bar{q} (q = u,d,s,c)$ and QED processes, we require at least five charged tracks in each event or, to increase the efficiency for $B^+ \rightarrow \pi^0 \ell^+ \nu$, four tracks and at least two photons. We also require

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\( L_2 = \sum_j |\vec{p}_j^*| \cos^2 \theta_j^* < 1.5 \) GeV. Here the sum is over all tracks in the event excluding the \( Y \) candidate, and \( \vec{p}_j^* \) and \( \theta_j^* \) refer to the momenta and the angles measured with respect to the thrust axis of the \( Y \). This requirement removes over 95\% of \( q\bar{q} \) and 80\% of \( B \to X_c \ell \nu \) background and retains about 50\% of the signal in all modes.

We discriminate against the remaining background using the variables \( \Delta E = (p_{\text{vis}} \cdot p_{\text{beams}} - s/2)/\sqrt{s} \) and \( m_{\text{ES}} = \sqrt{(s/2 + p_B \cdot p_{\text{beams}})^2 E_{\text{beams}} - p_B^2} \), where \( \sqrt{s} \) is the mass of the \( T(4S) \). Only candidates with \( |\Delta E| < 0.9 \) GeV and \( m_{\text{ES}} > 5.095 \) GeV are retained. The total signal selection efficiencies for the sum of electrons and muons are 3.5\% and 2.4\% for \( \pi^- \ell^+ \nu \) and \( \pi^0 \ell^+ \nu \), 0.53\% and 1.1\% for \( \rho^- \ell^+ \nu \) and \( \rho^0 \ell^+ \nu \).

To extract the signal yields, we perform a binned extended maximum-likelihood fit to \( \Delta E \) vs. \( m_{\text{ES}} \) distributions of the four signal modes simultaneously. The fit takes into account statistical fluctuations of both data and Monte-Carlo samples. We fit the relative proportions of the simulated signal and background samples to the data distributions in 5 GeV\(^2\) or 10 GeV\(^2\) intervals of \( q^2 \). To improve the \( q^2 \) resolution, we adjust \( |\vec{p}_j| \) so that \( \Delta E = 0 \). The resulting \( q^2 \) resolution is small compared to the chosen intervals of \( q^2 \) and can be described by the sum of two Gaussian functions of widths \( \sigma_1 \approx 0.2 \) GeV\(^2\) (containing about 75\% of signal events) and \( \sigma_2 \approx 0.5 \) GeV\(^2\).

We use the isospin relations \( \Gamma(B^0 \to \pi^- \ell^+ \nu) = 2\Gamma(B^+ \to \pi^0 \ell^+ \nu) \) and \( \Gamma(B^0 \to \rho^- \ell^+ \nu) = 2\Gamma(B^+ \to \rho^0 \ell^+ \nu) \) to reduce the number of fit parameters to nine: five for the signal yields in the five \( q^2 \) intervals for \( B \to \pi \ell \nu \) decays, three for the signal yields in the three \( q^2 \) intervals for \( B \to \rho \ell \nu \) decays, plus one scale parameter, shared among all \( q^2 \) intervals and signal modes, to fit the overall normalization of the \( B \to X_c \ell \nu \) background. We classify signal candidates as “combinatorial signal” if the reconstructed lepton comes from the isospin-conjugate decay or the hadron is incorrectly selected. The fit uses common parameters for combinatoric signal and signal. The normalization of the simulated non-\( B \bar{B} \) background is scaled separately for events with \( \ell^\pm \) and \( \mu^\pm \) to match the off-resonance data. We smooth the distributions for this low-statistics background to reduce single-bin statistical fluctuations.

Figures 1 and 2 show projections of the fitted \( \Delta E \) vs. \( m_{\text{ES}} \) distributions for each \( q^2 \) interval for \( B \to \pi \ell \nu \) and \( B \to \rho \ell \nu \), respectively. Integrated over the whole \( q^2 \) range, we observe 396 \( \pi^- \ell^+ \nu \), 137 \( \pi^0 \ell^+ \nu \), 95 \( \rho^- \ell^+ \nu \), and 98 \( \rho^0 \ell^+ \nu \) decays. The resulting partial and total branching fractions are given in Table I. The fitted normalization of the \( B \to X_c \ell \nu \) background is consistent with the measured total branching fraction.

### Table I: Partial and total branching fractions

| \( q^2 \) Range | \( \Delta B_\pi \) | \( \epsilon_\pi \) | \( q^2 \) Range | \( \Delta B_\rho \) | \( \epsilon_\rho \) |
|-----------------|-------------|-------------|-----------------|-------------|-------------|
| (GeV\(^2\))     | (10\(^{-4}\)) | (%)         | (GeV\(^2\))     | (10\(^{-4}\)) | (%)         |
| 0–5             | 0.30 ± 0.05 | 2.1         | 0–10            | 0.73 ± 0.17 | 0.7         |
| 5–10            | 0.32 ± 0.05 | 2.9         | 10–15           | 0.82 ± 0.10 | 0.97        |
| 15–20           | 0.27 ± 0.05 | 3.5         | 15–25           | 0.59 ± 0.07 | 0.44        |
| 20–25           | 0.26 ± 0.03 | 3.3         | 25–30           | 2.14 ± 0.21 | 0.72        |

The goodness-of-fit is evaluated using a \( \chi^2 \)-based comparison of the fitted \( \Delta E \) vs. \( m_{\text{ES}} \) distributions and data, yielding \( \chi^2/\text{dof} = 1.27 \). As a check, we have performed the fit for \( \epsilon_\pi \) and \( \epsilon_\rho \) separately and obtain consistent results.

The fit also allows us to study the \( q^2 \) dependence of the form factors. In decays to pseudoscalar mesons there is only one form factor, \( f_+ \) (for low-mass leptons), and we can extract the shape of \( f_+(q^2) \) directly from the measured \( q^2 \) spectrum. We perform a \( \chi^2 \) fit with a function proposed by Becirevic and Kaidalov (BK) \( f_+(q^2) = \frac{c_B(1 - \alpha)}{(1 - q^2/m_B^2)(1 - \alpha q^2/m_B^2)} \), where \( m_{B^*} = 5.32 \) GeV is the mass of the \( B^* \) resonance, \( c_B \) is a normalization factor, and \( \alpha \) is a shape parameter. Since we cannot measure the normalization, only \( \alpha \) is meaningful. Leaving both \( c_B \) and \( \alpha \) free, we fit \( \alpha = 0.61 \pm 0.09 \), in agreement with LQCD results. For decays to vector mesons, there are three form factors. The experimental uncertainties for \( B \to \rho \ell \nu \) are still too large to measure these. Thus we have to rely on theoretical predictions.

Figure 3 compares the \( q^2 \) distributions for \( B \to \pi \ell \nu \) and \( B \to \rho \ell \nu \) with the various form-factor calculations, which we implement by reweighting simulated signal events. We use \( \chi^2 \) probabilities to quantify the agreement: for \( B \to \pi \ell \nu \) we obtain good agreement with the BK fit to the data, \( P(\chi^2) = 35\% \), and the predictions of LCSR1 \( 38\% \), LQCD1 \( 14\% \), and LQCD2 \( 35\% \); but only marginal agreement with the prediction of ISGW II \( < 1\% \). For \( B \to \rho \ell \nu \) all calculations are compatible with the data within the large experimental uncertainties.

The systematic errors in the extraction of the branching fractions are listed in Table II. The contributions from each \( q^2 \) interval are conservatively treated as fully correlated and added linearly to obtain the uncertainty of the total branching fractions. Part of the \( q^2 \) variation of the stated errors may be due to statistical variations.
FIG. 1: (color online) Projected $m_{ES}$ (a-e) and $\Delta E$ (f-j) distributions in five intervals of $q^2$ for the combined $B \to \pi \ell \nu$ modes. The projections are shown for signal bands $-0.15 < \Delta E < 0.25$ GeV and $m_{ES} > 5.255$ GeV, respectively. The error bars on the data points represent the statistical uncertainties. The histograms show simulated distributions for signal (white), combinatoric signal (white, dotted), cross feed from other $B \to X_u \ell \nu$ decays (hatched), $B \to X_c \ell \nu$ decays (light shaded/yellow) and non-$B\bar{B}$ background (dark shaded/blue). The normalizations of the signal and $B \to X_c \ell \nu$ background simulations have been scaled to the results of the maximum-likelihood fit.

TABLE II: Relative systematic uncertainties of the partial and total branching fractions $B(B^0 \to \pi^- \ell^+ \nu)$ ($\Delta B_{\pi}$) and $B(B^0 \to \rho^- \ell^+ \nu)$ ($\Delta B_{\rho}$) in the various $q^2$ bins. The total uncertainty in each column is the sum in quadrature of the listed contributions.

| $q^2$ Range (GeV$^2$) | $\delta \Delta B_{\pi}/\Delta B_{\pi}$ (%) | $\delta \Delta B_{\rho}/\Delta B_{\rho}$ (%) |
|------------------------|--------------------------------------|--------------------------------------|
| 0–5                   | 7.4 5.7 9.2 3.5 8.7 6.8             | 16.7 10.8 15.9 14.2                  |
| 5–10                  | 8.8 5.0 7.3 2.5 4.8 5.7             | 12.7 4.7 10.9 9.1                    |
| 10–15                 | 2.3 2.3 2.3 2.3 2.3 2.3             | 2.3 2.3 2.3 2.3                      |
| 15–20                 |                                     |                                      |
| 20–25                 |                                     |                                      |
| 0–10                  | 5.0 3.9 3.9 4.3 3.6 4.2             | 7.2 1.8 3.8 4.2                      |
| 10–15                 | 0.5 1.5 0.6 2.2 5.7 2.1             | 10.9 9.1 19.2 12.5                   |
| 15–20                 | 13.5 2.4 1.0 7.8 5.6               | 11.2 0.9 1.6 4.6                     |
| 20–25                 |                                     |                                      |
| $N_{B\bar{B}}$         | 1.1 1.1 1.1 1.1 1.1 1.1             | 1.1 1.1 1.1 1.1                      |
| $B$ Lifetimes          | 1.1 1.4 0.2 0.5 2.1 1.1             | 0.7 0.6 0.9 0.7                      |
| $f_+/-f_0$             | 0.7 0.8 0.3 0.4 1.0 0.7             | 1.6 0.6 0.4 0.9                      |
| Isospin Breaking       | 0.1 1.1 1.8 0.1 0.1 0.6             | 6.4 4.5 0.9 4.2                      |
| Radiative Corrections  | 0.8 0.3 0.2 0.3 1.5 0.6             | 0.5 0.1 0.9 0.5                      |
| Total Error            | 18.7 9.5 12.8 7.3 14.8 11.8          | 28.1 15.9 27.7 22.5                  |

in simulated samples.

Uncertainties in the simulation of the reconstruction of charged particles and photons are evaluated by varying the reconstruction efficiencies and the photon-energy resolution and are added in quadrature. In addition, most $K_L^0$ escape detection. The impact of $K_L^0$ interactions in the calorimeter is estimated by varying in simulation their detection efficiency and energy deposition. To assess the uncertainty of the $K^0$ production rate, we vary the inclusive branching fractions of $D^+ \to K^0 X$, $D^0 \to K^0 X$, and $D_s^+ \to K^0 X$ within their published errors [13]. All these constitute the total uncertainty of the
neutrino reconstruction, which is dominant. For lepton identification we use relative uncertainties of ±2% and ±4% for electrons and muons, respectively.

The uncertainty of the $B \to X_s \ell \nu$ background is evaluated by varying the $B \to D/D^*/D^{**} \ell \nu$ branching fractions \cite{13} and the $B \to D^* \ell \nu$ form factors \cite{14}. For the $B \to X_u \ell \nu$ background, we independently vary the branching fractions of $B^+ \to \omega \ell^+ \nu$ and $B^+ \to \eta' \ell^+ \nu$ within their published errors \cite{13,15}. We assume equal branching fractions for $\eta \ell \nu$ and $\eta' \ell \nu$ and use a relative uncertainty of 100% for the latter. We also vary the non-resonant contribution within the range allowed by the uncertainty of the total $B \to X_u \ell \nu$ branching fraction \cite{10}.

The impact of quark-hadron duality violation or weak annihilation effects have not been considered. We estimate the uncertainty of the small remaining non-$B \bar{B}$ background by comparing simulation with off-resonance data and extract a normalization error of ±70% for electrons and ±25% for muons.

The overall uncertainty of the number of produced $B$ mesons is 1.1%. We take into account the uncertainties of the ratio of $B$ lifetimes, $\tau_{B^+}/\tau_{B^0} = 1.081 ± 0.015$ \cite{13}, the charged-to-neutral $B$ production ratio $f_+ / f_0 = 1.044 ± 0.050$ \cite{13}, and the potential effect of isospin breaking due to $\rho^0 - \omega$ mixing \cite{13}. We assign an uncertainty of 20% to the radiative corrections based on PHOTOS \cite{13}.

The impact of the uncertainties of the $B \to \pi \ell \nu$ form-factor shape on the measured branching fractions is negligible, whereas for the different $B \to \rho \ell \nu$ form-factor calculations we see variations of up to ±6% in $B(B^0 \to \pi^- \ell^+ \nu)$ and ±13% in $B(B^0 \to \rho^- \ell^+ \nu)$. We take the full spread between calculations as the uncertainty of the $q^2$ dependence of the form factors.

\begin{table}[h]
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{q}^2 \textbf{Range} & \textbf{Δζ} \textbf{Range} & \textbf{V}_{ub} \textbf{Range} \\
(GeV^2) & (ps) & (10^{-3}) \\
\hline
\textbf{π FF} & & \\
\textbf{LCSR1} & 0 – 15 & 5.1±1.3 & 3.27 ± 0.16 ± 0.19 ± 0.10^{+0.53}_{-0.36} \\
& 15 – 25 & 1.5±0.4 & 4.92 ± 0.25 ± 0.29 ± 0.15^{+0.76}_{-0.52} \\
\textbf{LQCD1} & 0 – 15 & 2.0±0.5 & 4.16 ± 0.22 ± 0.24 ± 0.15^{+0.72}_{-0.47} \\
& 15 – 25 & 7.7±2.3 & 3.40 ± 0.13 ± 0.20 ± 0.10^{+0.42}_{-0.29} \\
\textbf{LCSR2} & 0 – 15 & 5.7±1.7 & 4.00 ± 0.14 ± 0.23 ± 0.12^{+0.78}_{-0.49} \\
& 15 – 25 & 6.1±2.1 & 3.82 ± 0.14 ± 0.22 ± 0.11^{+0.85}_{-0.52} \\
\textbf{LQCD2} & 0 – 15 & 12.7 & 2.82 ± 0.15 ± 0.30 ± 0.18 \\
& 15 – 25 & 14.2 & 2.91 ± 0.12 ± 0.33 ± 0.19 \\
\hline
\textbf{ρ FF} & & \\
\textbf{LCSR2} & 0 – 15 & 17.2 & 2.85 ± 0.14 ± 0.32 ± 0.19 \\
& 15 – 25 & 17.2 & 2.85 ± 0.14 ± 0.32 ± 0.19 \\
\hline
\end{tabular}
\caption{\label{tab:Vub}The $|V_{ub}|$ derived for $B \to \pi \ell \nu$ and $B \to \rho \ell \nu$ signal for various $q^2$ regions and form-factor (FF) calculations: LCSR1, LQCD1, LQCD2, LCSR2, ISGW II. For the cross feed from the other mode, we have quoted the BK fit to data for $\pi \ell \nu$ and LCSR2 for $\rho \ell \nu$. Quoted errors are statistical, experimental, systematic, uncertainties of form-factor shape and form-factor normalization $\Delta \zeta$ (no form-factor normalization uncertainties are available for $\rho \ell \nu$).}
\end{table}

We extract $|V_{ub}|$ (see Table \ref{tab:Vub}) from the partial branching fractions $\Delta S$ using the relation $|V_{ub}| = \sqrt{\Delta S/\tau_{B^0} \Delta \zeta}$, where $\tau_{B^0} = (1.536 ± 0.014)$ ps \cite{13}, is the $B^0$ lifetime and $\Delta \zeta$ denotes the predicted form-factor normalization in each $q^2$ interval. For $q^2 < 15$ GeV$^2$ we derive $|V_{ub}|$ using LCSR calculations; for $q^2 > 15$ GeV$^2$ we use unquenched LQCD. To extract $|V_{ub}|$ from this measurement over the whole $q^2$ range, we extrapolate the LQCD results to low $q^2$ using the fits of the BK parametrization in Ref. \cite{4,5} and the LCSR results to high $q^2$ using the parametrization given in Ref. \cite{3}. We adopt the uncertainties of the form-factor normalization

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3a}
\caption{(color online) Comparison of the differential decay rates as functions of $q^2$ for $B \to \pi \ell \nu$ (a) and $B \to \rho \ell \nu$ (b) with various form-factor predictions. The data are background subtracted and corrected for efficiency and radiative effects. The error bars are statistical (inner) and statistical plus systematic (outer).}
\end{figure}
estimated in Refs. \[1\], \[3\], \[4\].

In conclusion, we have measured the exclusive branching fractions \(B(B^0 \rightarrow \pi^-\ell^+\nu)\) and \(B(B^0 \rightarrow \rho^-\ell^+\nu)\) as a function of \(q^2\), and have extracted \(|V_{ub}|\) using recent form-factor calculations. We measure the total branching fractions,

\[
\begin{align*}
B(B^0 \rightarrow \pi^-\ell^+\nu) &= (1.38 \pm 0.10 \pm 0.16 \pm 0.08) \times 10^{-4}, \\
B(B^0 \rightarrow \rho^-\ell^+\nu) &= (2.14 \pm 0.21 \pm 0.48 \pm 0.28) \times 10^{-4},
\end{align*}
\]

where the errors are statistical (data and simulation), experimental systematic, and uncertainties of the form-factor calculations. As a consistency check, we have also measured the branching fractions for the charged and neutral \(\pi\ell\nu\) samples separately, \(B(B^0 \rightarrow \pi^-\ell^+\nu) = (1.41 \pm 0.17 \pm 0.17 \pm 0.08) \times 10^{-4}\), \(B(B^+ \rightarrow \pi^0\ell^+\nu) = (0.70 \pm 0.10 \pm 0.08 \pm 0.04) \times 10^{-4}\). The ratio \(\Gamma(B^0 \rightarrow \pi^-\ell^+\nu)/\Gamma(B^+ \rightarrow \pi^0\ell^+\nu) = 2.21 \pm 0.41\) is consistent with the assumed isospin relation within the quoted statistical uncertainty.

For \(B \rightarrow \pi\ell\nu\), the \(q^2\) distribution agrees well with calculations based on LCSR \[2\] and unquenched LQCD \[4\], \[5\], but the data disfavor ISGW II \[6\]. Instead of averaging results based on different calculations, we choose the measured form-factor shape and normalization of LQCD2 and quote

\[|V_{ub}| = (3.82 \pm 0.14 \pm 0.22 \pm 0.11^{+0.88}_{-0.52}) \times 10^{-3},\]

where the additional fourth error reflects the uncertainty of the form-factor normalization. The results are consistent with previous measurements \[15\], \[16\], but have higher statistical accuracy, are less dependent on theoretical form-factor predictions, and benefit from recent advances in theoretical calculations \[2\], \[3\], \[4\], \[5\].

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[23] These efficiencies and signal yields depend upon \(q^2\)-dependent form factors which, unless otherwise stated, are fit to the data for \(B \rightarrow \pi\ell\nu\) and calculated using LCSR \[4\] for \(B \rightarrow \rho\ell\nu\).