An Efficient Algorithm for Reliability Evaluation of the Bus Network

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ABSTRACT The bus network is widely used in industrial automation and avionics systems due to its many advantages. Network reliability is an important indicator of the bus network design and analysis. However, the widely used reliability evaluation method is not capable of dealing with the bus network. In this paper, we model the bus network without redundancy and the bus network with redundancy, propose the algorithms to calculate the two-terminal reliability and the k-terminal reliability of the bus network with redundancy, and verify the effectiveness of the algorithms in three scenarios. The experimental results show that when the reliability of the link reaches 0.9, the two-terminal reliability of the bus network with redundancy of six terminal nodes, eight terminal nodes, and ten terminal nodes is 40.95%, 52.17%, and 61.26% higher than that of the bus network without redundancy, respectively; the k-terminal reliability is 61.26%, 74.58%, and 83.32% higher than that of the bus network without redundancy, respectively. The bus network with redundancy increases the communication paths for data exchange between devices and has higher reliability than the bus network without redundancy. The algorithms proposed in this paper provide an effective solution for the reliability evaluation of the bus network. It perfects the reliability evaluation system of the network with different topology architectures.

INDEX TERMS Minimal path sets, minimal cut sets, reliability evaluation, sum-of-disjoint products, the bus network.

I. INTRODUCTION

In the communication network, the network topology architecture is the interconnection layout between devices in the network [1]. The network topology architectures mainly include bus topology [2], star topology [3], ring topology [4], tree topology [5], and their hybrid topology [6]. The bus topology is often used in the local area network. It has a simple and flexible structure, and it is easy to install and use. In addition, devices can be easily inserted and deleted to facilitate network expansion [7]. In the bus network, all devices share a single cable, so the required cables are minimal and look neat. Since the bus network uses fewer cables and does not require expensive network equipment such as switches, wiring costs and workload are greatly reduced [7]. Furthermore, the failure of a device does not affect the entire network in the bus network. This means that failures, additions, and deletions of devices do not affect communication between other devices. Therefore, the bus network has high reliability. Moreover, the bus network has a strong ability to share resources. All devices in the bus network are directly connected to the bus, and the messages sent by any device can be received by other devices on the bus. At the same time, the bus network reduces the burden of network communication processing, and the communication processing is distributed at each site.

The bus topology uses a single cable to connect all devices, that is, the transmission medium connected to the devices is shared by all the devices. All devices have the equal position, and there is no central node to achieve control. It is currently the most widely used and the most traditional mainstream network architecture. The bus network is mainly used in the...
backbone network part. It is not only suitable for the industrial automation field, but also widely used in aerospace, automobile, and other fields [8]. Ethernet is the most famous bus network and the most widely used local area network technology. It has the advantages of simplicity and convenience, good compatibility, low cost, great potential for sustainable development, and high communication rate [9]. The CAN has been widely used in the automotive and industrial networks, aviation industry, industrial control, and other fields, offering simple and low-cost installation and implementation, high real-time performance, and high reliability [10], [11]. It is one of the most widely used field buses in the world. The 1553B is the most widely used data transmission bus for military aircraft, transport aircraft, and avionics systems due to its reasonable topology architecture and dual redundancy mode, which has high reliability and high real-time performance [12], [13], [14].

With the rapid increase in the complexity of communication networks and the amount of transmitted data, the dependability of the network has received more and more attention [8], [15]. The dependability of the system is the ability to provide services that can be reasonably trusted [16]. Dependability is a comprehensive concept that includes the following attributes: reliability, availability, safety, security, performability, maintainability, and testability [16]. Among them, network reliability has received the most attention [17], [18]. Reliability is defined by the IEEE as “the ability of a system or component to perform its required functions under stated conditions for a specified period of time” [19]. Reliability based on connectivity is a fundamental requirement of network design. Therefore, network reliability usually refers to the probability that the network satisfies the required connectivity. When analyzing the reliability of the network, we use three indicators, namely two-terminal reliability, k-terminal reliability, and all-terminal reliability [4]. The two-terminal reliability refers to the probability that there is at least one fault-free communication path for communication between two nodes in the network, where one node is the source node and the other node is the destination node. The k-terminal reliability is defined as the probability that there is at least one fault-free communication path for communication between k nodes in the network, where k ranges from 2 to the total number of nodes in the network. The all-terminal reliability refers to the probability that there is at least one fault-free communication path for communication between all nodes in the network.

Reliability has become a key factor in the design and operation of the bus network. In the actual application of the bus network, the first thing to do is to evaluate the reliability of its topology architecture. Specifically, it is to evaluate how the network reliability changes as nodes or links experience random failures. And more specifically, the three reliability indicators described above are used to evaluate the bus network. This can guide the actual layout of the bus network. Currently, for the bus network, researchers mostly focus on the performance of the network based on the bus topology architecture [20], [21]. Although Chaturvedi et al. [22], [23] proposed methods for calculating network reliability, these methods fail for the bus network. Therefore, to our best knowledge, no literature proposes a method suitable for evaluating the reliability of the bus network. This paper models the bus network without redundancy and the bus network with redundancy, proposes the algorithms for calculating the reliability of the bus network with redundancy, and verifies the effectiveness of the algorithms. The algorithms proposed in this paper fill the gap in the reliability calculation method of the bus network and perfect the reliability evaluation system of the network with different topology architectures.

Therefore, in this paper, our main contributions are as follows:

- We model the bus network without redundancy and the bus network with redundancy by placing virtual nodes.
- The algorithms for calculating the two-terminal reliability and the k-terminal reliability of the bus network with redundancy are proposed. Furthermore, we verify the effectiveness of the proposed algorithms in three scenarios.
- The algorithms proposed in this paper provide an effective solution for the reliability analysis of the bus network. It perfects the reliability evaluation system of the network with different topology architectures. At the same time, it has certain directive significance for the actual layout of the bus network.

The structure of the paper is as follows. Related work is reviewed in Section II. Models of the bus network without redundancy and the bus network with redundancy are presented in Section III. Problems of the reliability calculation of the bus network without redundancy and the bus network with redundancy are presented in Section IV. In Section V, the algorithms for evaluating the two-terminal reliability and the k-terminal reliability for the bus network with redundancy are proposed. Section VI evaluates the two-terminal reliability and the k-terminal reliability of the bus network without redundancy and the bus network with redundancy in three scenarios. Finally, we conclude this paper in Section VII.

II. RELATED WORK

Recently, research on the bus network has mainly focused on qualitative analysis of network reliability or fault injection to analyze the extent of error propagation or improvements for the specific network with bus topology architecture to improve its performance. Rushby [2] only qualitatively analyzed the bus topology and did not quantitatively analyze it. Ademaj et al. [24] first observed the effect of errors on a time-triggered architecture (TTA) bus topology system by applying two fault injection methods. Finally, the error containment mechanisms of TTA bus topology were analyzed through the obtained error propagation degree. Dehbashi et al. [6] investigated the impact of faults on the network and the propagation of errors in the network by using 43500 bit-flip fault injection. Fault injection describes some reliability-related characteristics but ultimately fails to quantify what effect they have on
the reliability of the network. Barranco et al. [25] evaluated the reliability of the bus network from the perspective of the influence of system fault-tolerance coverage, cabling failure rate, and other factors on system reliability quantitatively. Yan et al. [26] presented a startup algorithm for the time-triggered architecture (TTA) with the bus topology and discussed the temporal boundary for the system of arbitrary number of nodes that integrates the proposed algorithm by using formal derivation. Kaur et al. [27] designed a completely passive remote node (RN) based on fiber bragg grating (FBG) for the high split NG-PON2 system based on the bus topology and analyzed two performance indicators of the proposed node, namely the bit error rate (BER) and optical signal to noise ratio (OSNR). Sundaram et al. [28] designed the bus topology by applying a novel linear algebraic theory. After a series of verifications, the results showed that the proposed method has better performance than other traditional methods.

Choi et al. [20] proposed methods to improve the performance of IVNs based on the bus topology. Simulations showed that the methods can improve the data transmission speed and reduce the transmission delay. Rios et al. [21] proposed a new idea to use orthogonal frequency division multiplexing (OFDM) in bus topology. The proposed method can improve the speed of data transmission during long-distance transmission in industrial sites. Liu Yu-Liang [29] proposed the embedding method and the wavelength allotment algorithm to solve the routing and wavelength allotment problems for embedding exchanged folded hypercube communication patterns in WDM optical networks based on the bus topology. The above-mentioned research has ignored the most basic point, that is, the impact of the topology architecture of the bus network itself on the communication reliability between devices in the network. They did not quantitatively evaluate the reliability of the bus network from the perspective of communication reliability between terminal nodes. Chaturvedi et al. [22], [23], [30], [31] proposed some methods for the calculation of network reliability. They proposed the algorithms for calculating network reliability [22], [23]. After that, they proposed an algorithm to evaluate the reliability of the network with heterogeneous link capacities [30]. Since then, they proposed an algorithm for calculating the reliability of time evolving delay tolerant networks [31]. However, these reliability calculation methods are not applicable to the bus network.

To the best of our knowledge, no researchers have yet performed the quantitative analysis of communication reliability between devices in the bus network. To address this problem, we improved the algorithms proposed by Chaturvedi et al [22], [23]. Therefore, we modeled the bus network, proposed the algorithms for calculating the reliability of the bus network, and verified the effectiveness of the algorithms.

III. NETWORK MODEL

The bus network is different from the networks with other topology architectures such as star topology and tree topology. Any two terminal nodes in the bus network can communicate with each other directly without forwarding by other nodes. The reliability evaluation of the bus network in this paper is based on the methods of [22] and [23], which believed that if a segment of the transmission medium fails, the whole transmission medium will collapse. Nevertheless, in practical applications, the failure of a certain segment of the transmission medium will only cause the communication of the nodes connected to the transmission medium to be blocked, and the communication between other nodes will not be affected. Therefore, it is necessary to model the bus network when analyzing its reliability. To improve the reliability of the bus network, a redundant design is necessary. Thus, the bus network is classified as the bus network without redundancy and the bus network with redundancy. The bus network without redundancy is the network that has only one common transmission medium, and the bus network with redundancy is the network that has two common transmission media. We model these two types of the bus network respectively as follows.

A. THE BUS NETWORK WITHOUT REDUNDANCY

In the bus network without redundancy, all terminal nodes share a single transmission medium, and any terminal node can directly communicate with other terminal nodes. The bus network without redundancy of four terminal nodes is shown in Fig. 1, where TD1, TD2, TD3, and TD4 are four terminal nodes. TD1 can communicate directly with TD2. Similarly, TD2 and TD4 can communicate with each other directly without passing through TD3. The red numbers 1 to 4 represent the links from TD1-TD4 to the transmission medium, respectively. The red numbers 5 to 7 represent the three link segments of the transmission medium. The failure of any link from link 1 to link 7 does not affect the transmission of other links. If link 6 fails for some reason, TD1 will not be able to continue communicating with other terminal nodes, but the communication between TD2, TD3, and TD4 will not be affected. Similarly, if link 3 fails, TD3 cannot communicate with other terminal nodes, but TD1, TD2, and TD4 can still communicate with each other normally.

![Figure 1. The bus network without redundancy of four terminal nodes.](image-url)
one link. That is, if any one of them fails, the other two cannot transmit data normally. However, in the actual bus network, these three links do not affect each other, and the failure of any link will not affect the functions of other links. Therefore, we place virtual nodes VN1 and VN2 at the junctions of TD2, TD3, and the transmission medium, respectively. TD2 can communicate with TD1 located on its left, and can also communicate with TD3 and TD4 located on its right. TD3 can communicate with TD1 and TD2 located on its left, and can also communicate with TD4 located on its right. TD1 and TD4 are edge terminal nodes of the network and can only communicate with terminal nodes on one side of them. On the other hand, TD1 in Fig. 1 communicates with other terminal nodes through links 1 and 6. If any one of these two links fails, TD1 cannot communicate with other terminal nodes normally. Therefore, link 1 and link 6 are actually one link. Similarly, link 4 and link 7 are actually one link. Therefore, virtual nodes are not placed at the junctions of TD1, TD4 and the transmission medium. This way of modeling is consistent with the actual communication of the bus network without redundancy. In Fig. 2, the red number 1 represents the link between TD1 and VN1, the red number 2 represents the link between TD2 and VN1, the red number 3 represents the link between TD3 and VN2, and the red number 5 represents the link between VN1 and VN2. When TD1 communicates with TD3, it is assumed that TD1 is the sending node and TD3 is the receiving node. The link-wise communication path between TD1 and TD3 is 1-5-3. We make the following assumptions: the nodes in the network are completely reliable and the links may fail. Therefore, the communication between TD1 and TD3 will be affected by link 1, link 5, and link 3. If any one of these links fails, the communication between TD1 and TD3 cannot continue.

A. THE BUS NETWORK WITHOUT REDUNDANCY

Similar to the bus network without redundancy, any terminal node of the bus network with redundancy can also communicate directly with other terminal nodes. All terminal nodes share two transmission media, which are redundant to each other. When one fails, the other can still transmit data. Therefore, the bus network with redundancy improves the reliability of data transmission compared to the bus network without redundancy. The bus network with redundancy of four terminal nodes is shown in Fig. 3, where TD1, TD2, TD3, and TD4 are four terminal nodes. There are two communication paths between TD1 and TD2, which can communicate with each other directly. Similarly, TD2 and TD4 also have two paths to communicate with each other directly without passing through TD3. The red numbers 1 to 4 represent the links from TD1-TD4 to transmission medium A, respectively. The red numbers 5 to 8 represent the links from TD1-TD4 to transmission medium B, respectively. The red numbers 9 to 11 represent the three link segments of transmission medium A. The red numbers 12 to 14 represent the three link segments of transmission medium B. Transmission medium A and transmission medium B are mutually redundant. If any one of link 9, link 10, and link 11 is faulty, transmission medium A is faulty and cannot be used for data transmission. However, transmission medium B can still normally transmit data. The failure of transmission medium A does not cause the paralysis of the entire network.

According to the above analysis of the data transmission characteristics of the bus network with redundancy, we model the network in Fig. 3 as shown in Fig. 4. Similar to the bus network without redundancy, the methods of [22] and [23] cannot be used directly for reliability evaluation considering the characteristics of actual communication in the bus network with redundancy. Because it treats links 9, 10, and 11 as one link, and links 12, 13, and 14 as one link. But in fact, these six links do not affect each other. Therefore, we also need to place virtual nodes in the bus network with redundancy. Similarly, virtual nodes VN1, VN2, VN3, and VN4 are placed at the junctions of TD2, TD3 and transmission medium A, transmission medium B, respectively, where TD2 and TD3 are able to communicate with the terminal nodes on their left and right sides. For TD1 and TD4 located at the edge of the network, no virtual nodes are placed at the junctions between them and the transmission media. Because they can only communicate with terminal nodes on their one side. On the other hand, TD1 in Fig. 3 communicates with other terminal nodes through link 1 and link 10 or link 5 and link 13. If either link 1 or link 10 fails, TD1 cannot communicate with other terminal nodes through transmission medium A. Therefore, in the actual communication, link 1 and link 10 are equivalent to one link. Similarly, if either link 5 or link 13 fails, TD1 cannot communicate with other terminal nodes through transmission medium B. Therefore, link 5 and link 13 are
also equivalent to one link. Likewise, link 4 and link 11 are equivalent to one link, and link 8 and link 14 are equivalent to one link. Hence, we model links 1 and 10, links 5 and 13, links 4 and 11, and links 8 and 14 as one link, respectively. This way of modeling is consistent with the actual communication of the bus network with redundancy. In Fig. 4, the red numbers 1 to 8 represent the links between TD1 to TD4 and VN1 to VN4, respectively. The red number 9 represents the link between VN1 and VN2, that is, transmission medium A. The red number 10 represents the link between VN3 and VN4, that is, transmission medium B. When TD1 communicates with TD3, it is assumed that TD1 is the sending node and TD3 is the receiving node. There are two communication paths between TD1 and TD3, and the link-wise paths are 1-9-3 and 5-10-7. We still assume that nodes in the network are completely reliable and the links may fail. When any one of link 1, link 9, and link 3 fails, transmission medium A cannot be used to transmit data. However, due to the existence of redundant transmission medium B, TD1 and TD3 can still communicate with each other normally.

IV. PROBLEM STATEMENT

We have modeled the bus network, then we can enumerate the minimal path sets and minimal cut sets of the network using minimal path set-based and minimal cut set-based approaches proposed by Chaturvedi et al. [22], [23]. Finally, we calculate the reliability of the network using Multiple Variable Inversion-Sum-of-Disjoint Products (MVI-SDP) algorithm [32].

Sum-of-Disjoint Products (SDP) can tackle reliability evaluation problems more efficiently and effectively by producing a compact reliability expression. It expresses the network reliability as the union of minimal path sets or minimal cut sets of the network, then converts this union into the sum of disjoint terms, and finally calculates the corresponding reliability. If the minimal path sets of the network are known, it is assumed that the \( m \) minimal path sets are \( P_1, P_2, \ldots, P_m \). The theoretical formula of network reliability calculated by SDP is shown in (1).

\[
R = \Pr \left( \bigcup_{i=1}^{m} P_i \right) = \Pr(P_1 \cup P_2 \cup \ldots \cup P_m)
\]

where \( R \) is the reliability of the network, \( P_i \) is the i-th minimal path, \( \Pr(P_i) \) is the reliability of \( P_i \), and \( m \) is the total number of minimal path sets.

If the minimal cut sets of the network are known, it is assumed that the \( k \) minimal cut sets are \( C_1, C_2, \ldots, C_k \). The theoretical formula of network reliability calculated by SDP is shown in (2).

\[
R = 1 - \Pr \left( \bigcup_{i=1}^{k} C_i \right) = 1 - \Pr(C_1 \cup C_2 \cup \ldots \cup C_k)
\]

\[
= 1 - \left( \sum_{1 \leq i \leq k} \Pr(C_i) - \sum_{1 \leq i < j \leq k} \Pr(C_i \cap C_j) \right) + \sum_{1 \leq i < j < l \leq k} \Pr(C_i \cap C_j \cap C_l) + \ldots + (-1)^{k-1} \Pr(C_1 \cap C_2 \cap \ldots \cap C_k)
\]

where \( R \) is the reliability of the network, \( C_i \) is the i-th minimal cut, \( \Pr(C_i) \) is the reliability of \( C_i \), and \( k \) is the total number of minimal cut sets.

SDP can be further classified into Single Variable Inversion (SVI) and Multiple Variable Inversion (MVI) according to the method of inverting the variable. MVI can obtain a more compact reliability expression than SVI. MVI-SDP method in [32] can not only obtain a minimized and compact reliability expression, but also save CPU time. The pseudocode of MVI-SDP algorithm in [32] is shown in Fig. 5. First, each minimal path \( P_i, i = 2, 3, \ldots, m \) or each minimal cut \( C_i, i = 2, 3, \ldots, k \) is represented by the equivalent decimal integer value. Then, it is used (3) to compute the conditional integer set.

\[
E_i(j) = \text{bitxor} (\text{bitor}(D_i, D_j), D_i), \quad j = 1, 2, \ldots, (i - 1)
\]

where \( E_i(j) \) is the conditional integer set, \( D_i \) is the equivalent decimal integer value corresponding to the i-th minimal path or i-th minimal cut, \text{bitxor} \) is the bitwise exclusive OR operation, and \text{bitor} \) is the bitwise OR operation.

\( E_i(j) \) describes an event that \( D_i \) is operational while the minimal path sets or minimal cut sets prior to \( D_j \) have failed. Next, a minimal conditional integer set is obtained by performing some operations in the conditional integer set. Specifically, if a conditional integer set \( E_a \) is a subset of another conditional integer set \( E_i \), then \( E_i \) is discarded. Then, the bitwise AND operation forms the minimal conditional integer set into two subsets, namely independent groups (IG) and dependent groups (DG). When the result of the AND operation is 0, the integer is placed into the IG. Otherwise, it is placed into the DG. In the IG, we replace each 0 entry with –1 and each nonzero entry with the path sets number.
or the cut sets number they come from. The IG is then combined to produce a single disjoint term, $IG_n$. The DG sets are formed according to the indexes of minimal path sets in $P_i, j = 1, 2, \ldots, (i-1)$ or minimal cut sets in $C_j, j = 1, 2, \ldots, (i-1)$. Then, the disjoint items about the minimal path $P_i$ or the minimal cut $C_i$ are generated. Finally, $IG_n$ is combined with each disjoint term obtained above. Thus, we obtain the exclusive and mutually disjoint terms (emds) for each minimal path or minimal cut. There is a corresponding relationship between each emd and the reliability expression.

For each emd, the following operations are performed. Indexes with a value of 0 in the emd are found, and their link reliability is multiplied by each other. For indexes with values greater than 0 in the emd, the indexes of the same link reliability is multiplied by each other. For indexes with a value of 0 in the emd are found, and their reliability is calculated directly using the above method. In this way, the value of the reliability expression corresponding to each emd is obtained, and then these values are added to each other to obtain the final network reliability. So far, the reliability calculation of the network is completed.

**Algorithm:** Calculation of network reliability using MVI-SDP algorithm

```
1: Input: Minimal path sets $P_i, i = 1, 2, \ldots, m$ or minimal cut sets $C_i, i = 1, 2, \ldots, k$ that need to be calculated. Reliability of each link ($R_{ij}$)
2: Output: Network reliability
3: for $(P_i, P_j = 2, 3, \ldots, m) \text{ or } (C_i, C_j = 2, 3, \ldots, k)$ do
4:   Convert $P_i$ or $C_i$ to the equivalent decimal integer value $D_i$
5:   $R(I_j) = \text{bitxor (bitwise OR of $D_i$, $D_j$), } j = 1, 2, \ldots, (i-1)$
6:   Obtain the conditional integer set
7:   $m = \text{Obtain the minimal conditional integer set}$
8:   for $(E_j, j = 1, 2, \ldots, (i-1))$ do
9:     if $(R_{ij}(E_j))$ is a subset of $E_i$ (another one of $E_i$) then
10:        Discard $E_i$
11:     else
12:        end
13:     end
14: end
15: Using bitwise AND, forming $E_j$ into two subsets: IG and DG
16: In the IG, replace each 0 with $-1$ and each nonzero with the path sets number or the cut sets number they come from
17: Combine IG to form $IG_n$ //Produce a single disjoint term
18: DG sets
19: Generate the disjoint terms about $P_i$ or $C_i$
20: $IG_n$ is combined with each disjoint term obtained above //Obtain the emds for $P_i$ or $C_i$
21: Calculate the reliability for each emd using $R_{ij}$, then add them up
22: Calculate the reliability for $P_i = 1$ or $C_i = 1$
23: Add the above-obtained reliability values up to obtain the network reliability
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**FIGURE 5.** Pseudocode of MVI-SDP algorithm.

The methods of [22] and [23] are performed under the following assumptions:

The nodes in the network are completely reliable, and only the links in the network may fail.

The link failure in the network is a permanent failure.

All links of the network have the same reliability.

All link failures in the network are statistically independent.

There are only two states on the link in the network: working or failing.

This paper analyzes the two-terminal reliability and k-terminal reliability of the network. Since we placed virtual nodes in the modeling of the bus network, they are taken into account when evaluating the all-terminal reliability of the network. Therefore, it does not make sense to evaluate the all-terminal reliability of the bus network. On the other hand, when analyzing the k-terminal reliability of the network in this paper, the k terminal nodes we select are all the terminal nodes in the network. Therefore, evaluating the k-terminal reliability of the network is also evaluating the reliability of communication between all terminal nodes in the bus network.

When the methods of [22] and [23] are used to enumerate the minimal path sets and minimal cut sets of the bus network, erroneous results are produced. Section IV-A and Section IV-B describe the problems encountered in evaluating the two-terminal reliability and the k-terminal reliability of the bus network without redundancy and the bus network with redundancy, respectively.

**A. THE BUS NETWORK WITHOUT REDUNDANCY**

First, we evaluated the two-terminal reliability of Fig. 2, selecting TD1 as the sending node and TD4 as the receiving node. Then we enumerated the minimal path sets from TD1 to TD4 using the method of [22]. A minimal path between two terminal nodes is a series of links without loops from the source node to the destination node. The minimal path set-based method starts from the source node, finds the nodes connected to the source node, continues this process, and finally finds the destination node, which is a minimal path between the source node and the destination node. In this way, all the minimal path sets between the source node and the destination node can be found. In Fig. 2, the link-wise communication path between TD1 and TD4 is 1-5-4. There is only one path from TD1 to TD4, which is consistent with the actual data exchange. Then, we calculated the two-terminal reliability using MVI-SDP approach when TD1 communicated with TD4. The two-terminal reliability is 0.7290 when the reliability of the link reaches 0.9.

Next, we evaluated the k-terminal reliability of Fig. 2. The k nodes we selected were TD1 to TD4. What we actually analyzed is the reliability of communication between all terminal nodes attached to the bus network. A minimal cut refers to one link or several links of the network. The condition that these links must satisfy is that when they fail, the nodes in the network cannot exchange data with each other. We enumerated the minimal cut sets between k nodes using the method of [23]. In Fig. 2, there are five link-wise k-terminal minimal cut sets, namely 1, 2, 3, 4, and 5. This is consistent with actual communication. When any one of the five links fails, at least one terminal node cannot communicate with other terminal nodes. Then, we calculated the k-terminal reliability using MVI-SDP approach. The k-terminal reliability is 0.5905 when the link reliability is 0.9.

From the above analysis of the two-terminal reliability and the k-terminal reliability of the bus network without redundancy, we can see that after modeling the bus network without redundancy, the two-terminal reliability and the k-terminal reliability can be obtained using minimal path set-based and
minimal cut set-based approaches of [22] and [23]. The results are consistent with the actual communication between terminal nodes.

B. THE BUS NETWORK WITH REDUNDANCY

Similar to the bus network without redundancy, we evaluated the two-terminal reliability of Fig. 4, selecting TD1 as the sending node and TD4 as the receiving node. We enumerated the minimal path sets from TD1 to TD4 using the method of [22]. There are 8 paths when TD1 communicates with TD4. This is different from the actual communication between the two terminal nodes. In the actual communication, there are only two paths from TD1 to TD4. The link-wise paths are 1-9-4 and 5-10-8. This is because TD1 can only directly transmit data to TD4 through transmission medium A or B, terminal nodes other than TD1 and TD4 cannot forward data. The reason why there are 8 paths is that the method of [22] considers that TD2 and TD3 are nodes that can forward data, and the data of TD1 can be forwarded by TD2 and TD3, and then sent to TD4. The method of [22] considers that as long as there is a non-repeating path from the source node to the destination node regardless of whether the transmitted data is forwarded by other terminal nodes, this path is the minimal path. However, the data transmission of the bus network has its characteristics. When the terminal nodes communicate with each other, the data is directly transmitted from the source node to the destination node without being forwarded by other terminal nodes. After the above analysis, we conclude that the method of [22] is not suitable for enumerating the minimal path sets when two devices in the bus network with redundancy exchange data with each other.

Next, we evaluated the k-terminal reliability of Fig. 4. Similar to the bus network without redundancy, the k nodes we selected were TD1 to TD4. The results show that the obtained k-terminal minimal cut sets do not match the actual communication. For example, a link-wise minimal cut is 3-4-10. But in fact, 4-10 is already a minimal cut. Therefore, link 3 does not need to be added. Therefore, 3-4-10 is not a minimal cut. The reason for the erroneous result is that the algorithm considers that in addition to these two terminal nodes, other terminal nodes can also forward data. Therefore, the method of [23] is not suitable for enumerating the minimal cut sets of the bus network with redundancy.

Based on the above analysis, we can conclude that after modeling the bus network, we can use minimal path set-based and minimal cut set-based approaches [22], [23] to analyze the two-terminal reliability and the k-terminal reliability of the bus network without redundancy. However, the algorithms of [22] and [23] are no longer applicable to enumerate minimal path sets and minimal cut sets of the bus network with redundancy. Therefore, it is necessary to propose new methods to enumerate the minimal path sets and the minimal cut sets of the bus network with redundancy. The proposed algorithms are described in detail in Section V.

V. ALGORITHMS FOR EVALUATING THE RELIABILITY OF THE BUS NETWORK WITH REDUNDANCY

We describe the proposed algorithms in detail for evaluating the two-terminal reliability and the k-terminal reliability of the bus network with redundancy in this section.

A. ALGORITHM FOR EVALUATING THE TWO-TERMINAL RELIABILITY OF THE BUS NETWORK WITH REDUNDANCY

In the actual communication, there are only two paths when TD1 communicates with TD4 in Fig. 4, and TD2 and TD3 cannot forward data. But in the method of [22], TD2 and TD3 can forward data. Therefore, we need to further process the minimal path sets obtained using the method of [22]. First, we remove the path sets that contain terminal nodes other than the two terminal nodes that need to be evaluated for two-terminal reliability, and obtain the actual minimal path sets between two terminal nodes. Then we evaluate the two-terminal reliability using MVI-SDP method. The pseudocode for calculating the two-terminal reliability of the bus network with redundancy is shown in Fig. 6. The flowchart of Algorithm 1 is shown in Fig. 7.

Algorithm 1 Evaluation of two-terminal reliability of the bus network with redundancy

1: Input: Two terminal nodes that need to be evaluated for two-terminal reliability,
2:  R: Reliability of each link (R_{ij}),
3: Output: Two-terminal reliability
4: Generate all minimal path sets between two terminal nodes.
5: Remove the path sets that contain terminal nodes other than the two terminal nodes.
6: nodes that need to be evaluated for two-terminal reliability, and obtain the actual minimal path sets between two terminal nodes (pth).
7: sRel = MVI-SDP (p(θ), R_{ij})
8: Two-terminal reliability = sRel

FIGURE 6. Pseudocode for calculating the two-terminal reliability of the bus network with redundancy.

We use Algorithm 1 to analyze the two-terminal reliability between TD1 and TD4 in Fig. 4. When TD1 communicates with TD4, data is sent from TD1, passed through virtual nodes VN1 and VN2 or VN3 and VN4, and is finally received by TD4. There are 2 paths from TD1 to TD4. The link-wise paths are 1-9-4 and 5-10-8. This corresponds to the actual data exchange between two devices of the bus network with redundancy. Then we use MVI-SDP method to calculate that the two-terminal reliability is 0.9266 when the link reliability reaches 0.9. We conclude that the bus network with redundancy has higher two-terminal reliability than the bus network without redundancy. This is because the bus network with redundancy has two mutually redundant transmission media. The bus network with redundancy has more paths for data transmission than the bus network without redundancy when two devices exchange data with each other. Therefore, the bus network with redundancy has higher two-terminal reliability.

B. ALGORITHM FOR EVALUATING THE K-TERMINAL RELIABILITY OF THE BUS NETWORK WITH REDUNDANCY

The k-terminal reliability is analyzed using the minimal cut set-based approach. The minimal cut sets obtained by using
FIGURE 7. The flowchart of Algorithm 1.

Algorithm 2 Evaluation of k-terminal reliability of the bus network with redundancy

1: Input: k terminal nodes that need to be evaluated for k-terminal reliability,
2: Reliability of each link (R_{ij})
3: Output: k-terminal reliability
4: Generate all minimal cut sets (cut) of every two terminal nodes in k terminal nodes that need to be evaluated for k-terminal reliability.
5: Ncut = cut
6: Generate the union of Ncut minimal cut sets (cutsets).
7: sysRel = MVI-SDP (cutsets, R_{ij})
8: k-terminal reliability ← sysRel

FIGURE 8. Pseudocode for calculating the k-terminal reliability of the bus network with redundancy.

The method of [23] are inconsistent with the actual communication. Therefore, we need to propose an algorithm for enumerating the minimal cut sets of the bus network with redundancy. The pseudocode for calculating the k-terminal reliability of the bus network with redundancy is shown in Fig. 8. The flowchart of Algorithm 2 is shown in Fig. 9. First, all minimal cut sets of every two terminal nodes in k terminal nodes that need to be evaluated for k-terminal reliability are enumerated. Since there are only two paths when any two terminal nodes communicate with each other, any link on one path and any link on the other path form a minimal cut. For example, when TD1 and TD4 in Fig. 4 transmit data, the link-wise minimal cut 1-10 is one of the minimal cut sets. The minimal cut sets of k terminal nodes can be obtained by calculating the union of the minimal cut sets of all two terminal nodes. Then we arrange the obtained minimal cut sets by cardinality and lexicographically. Finally, the k-terminal reliability of the network can be obtained by using MVI-SDP method.

FIGURE 9. The flowchart of Algorithm 2.

The k-terminal reliability of TD1 to TD4 in Fig. 4 is analyzed by using Algorithm 2. We first obtain 44 minimal cut sets for communication between TD1 and TD2, TD1 and TD3, TD1 and TD4, TD2 and TD3, TD2 and TD4, and TD3 and TD4. Then, we calculate the union of these minimal cut sets obtained, and arrange them by cardinality and lexicographically. Table 1 shows the 25 minimal cut sets when the four terminal nodes in Fig. 4 communicate with each other. Finally, we get the k-terminal reliability of 0.8323 using MVI-SDP method when the link reliability reaches 0.9. We conclude that the bus network with redundancy has higher k-terminal reliability than the bus network without redundancy. This is because the bus network with redundancy has two mutually redundant transmission media. The bus network with redundancy has more paths for data transmission than the bus network without redundancy in the case where k terminal nodes in the network exchange data with each other. Therefore, the bus network with redundancy has higher k-terminal reliability.

We prove the correctness of Algorithms 1 and 2 using mathematical induction. Next, we analyze the worst-case...
TABLE 1. Link-wise Minimal Cut Sets of Four Terminal Nodes Communicating with Each Other.

| Cut No. | Link-wise minimal cut sets | Cut No. | Link-wise minimal cut sets |
|---------|---------------------------|---------|---------------------------|
| 1       | 1-5                       | 14      | 8-9                       |
| 2       | 1-6                       | 15      | 4-5                       |
| 3       | 2-5                       | 16      | 4-10                      |
| 4       | 2-6                       | 17      | 4-8                       |
| 5       | 1-10                      | 18      | 2-10                      |
| 6       | 1-7                       | 19      | 2-7                       |
| 7       | 5-9                       | 20      | 6-9                       |
| 8       | 9-10                      | 21      | 3-6                       |
| 9       | 7-9                       | 22      | 2-8                       |
| 10      | 3-5                       | 23      | 4-6                       |
| 11      | 3-10                      | 24      | 3-8                       |
| 12      | 3-7                       | 25      | 4-7                       |
| 13      | 1-8                       |         |                           |

The worst-case complexity of Algorithms 1 and 2 would be as shown in (4) and (5), respectively.

\[
O((3n - 5) \times (3n - 4) \times 2) + O(1) = O(18n^2 - 54n + 40) + O(1) \approx O(n^2) \quad (4)
\]

\[
O(n(n - 1) \times 2) + O(1) = O\left(\frac{1}{2}n^2 - \frac{1}{2}n\right) + O(1) \approx O(n^2) \quad (5)
\]

VI. RELIABILITY EVALUATION

In this section, we use the proposed algorithms to evaluate the two-terminal reliability and the k-terminal reliability of the bus network without redundancy and the bus network with redundancy in three scenarios. Fig. 10, Fig. 11, and Fig. 12 show the bus network without redundancy and the bus network with redundancy of six terminal nodes, eight terminal nodes, and ten terminal nodes, respectively. TD1 to TD6 in Fig. 10 are six terminal nodes, TD1 to TD8 in Fig. 11 are eight terminal nodes, and TD1 to TD10 in Fig. 12 are ten terminal nodes. These nodes can communicate with each other directly. There is only one transmission medium in Fig. 10(a), Fig. 11(a), and Fig. 12(a), and there are two transmission media in Fig. 10(b), Fig. 11(b), and Fig. 12(b). Transmission media A and B represent two transmission media that are redundant with each other. One of the transmission media fails, and the other transmission medium can still transmit data.

We model the networks in Fig. 10, Fig. 11, and Fig. 12 as shown in Fig. 13, Fig. 14, and Fig. 15, respectively. In Fig. 13(a), virtual nodes VN1 to VN4 are placed. The red numbers 1 to 9 are the link numbers. In Fig. 13(b), virtual nodes VN1 to VN8 are placed. The red numbers 1 to 18 are the link numbers. In Fig. 14(a), virtual nodes VN1 to VN6 are placed. The red numbers 1 to 13 are the link numbers. In Fig. 14(b), virtual nodes VN1 to VN12 are placed. The red numbers 1 to 26 are the link numbers. In Fig. 15(a), virtual nodes VN1 to VN8 are placed. The red numbers 1 to 17 are...
FIGURE 14. Modeling diagram of (a) the bus network without redundancy; (b) the bus network with redundancy of eight terminal nodes.

FIGURE 15. Modeling diagram of (a) the bus network without redundancy; (b) the bus network with redundancy of ten terminal nodes.

the link numbers. In Fig. 15(b), virtual nodes VN1 to VN16 are placed. The red numbers 1 to 34 are the link numbers.

We also need to make some assumptions when evaluating the bus network that has been modeled. In an actual network, links may fail due to aging, external force damage, cable connector failure, and other reasons. Generally, a link fails when it cannot transmit data generated by nodes directly connected to the link. Furthermore, a failure of the node inevitably leads to the failure of the link and introduces statistically dependent failure. Therefore, the statistically independent assumption is based on the assumption of completely reliable nodes. In this paper, we focus on the reliability evaluation of the network topology, which should prevent other factors such as link failure rate from interfering with the results. Therefore, it is necessary to assume that all links of the network have the same reliability. From the above analysis, the assumptions we make need to be consistent with those made when using the methods of [22] and [23].

First, we use Algorithm 1 to evaluate the two-terminal reliability of Fig. 13, Fig. 14, and Fig. 15, selecting TD1 as the sending node and TD6 as the receiving node in Fig. 13, TD1 as the sending node and TD8 as the receiving node in Fig. 14, and TD1 as the sending node and TD10 as the receiving node in Fig. 15, respectively. The networks of Fig. 13(a), Fig. 14(a), and Fig. 15(a) all have only one communication path when two terminal nodes communicate with each other. The link-wise communication paths in Fig. 13(b), Fig. 14(b), and Fig. 15(b) are 1-13-14-15-6 and 7-16-17-18-12, 1-17-18-19-20-21-8 and 9-22-23-24-25-26-16, and 1-21-22-23-24-25-26-27-10 and 11-28-29-30-31-32-33-34-20, respectively. Then, we calculate the two-terminal reliability by using MVI-SDP method. At the same time, the impact of link reliability on the two-terminal reliability was considered. We considered link reliability ranging from 0.9 to 0.99 with an interval of 0.01, for a total of 10 cases in this paper. The detailed results of the changes in the two-terminal reliability with different link reliability in Fig. 13, Fig. 14, and Fig. 15 are shown in Fig. 16.

FIGURE 16. The two-terminal reliability comparison diagram of the bus network without redundancy and the bus network with redundancy in three scenarios.

It can be seen from the results in Fig. 16 that when the link reliability reaches 0.9, the two-terminal reliability of the bus network without redundancy of six terminal nodes is 0.5905 compared with 0.8323 for the bus network with redundancy, the two-terminal reliability of the bus network without redundancy of eight terminal nodes is 0.4783 compared with 0.7278 for the bus network with redundancy, and the two-terminal reliability of the bus network without redundancy of ten terminal nodes is 0.3874 compared with 0.6247 for the bus network with redundancy. When the link reliability is 0.9, the two-terminal reliability of the bus network with redundancy of six terminal nodes, eight terminal nodes, and ten terminal nodes is 40.95%, 52.17%, and 61.26% higher than that of the bus network without redundancy, respectively. This is because the bus network with redundancy has two mutually redundant transmission media. When two terminal nodes communicate with each other, there are two transmission paths. When one path fails, the entire network cannot communicate. From Fig. 16, it can be seen

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that as the reliability of the link increases, the two-terminal reliability of the bus network without redundancy and the bus network with redundancy both increase. We can conclude that the reliability of the link has an important influence on the two-terminal reliability of the network. Therefore, when two devices communicate with each other in the actual bus network, we can improve the two-terminal reliability of the network by increasing the link reliability. From the above comprehensive evaluation, we can conclude that the two-terminal reliability of the bus network with redundancy is higher than that of the bus network without redundancy. It is especially noticeable when the reliability of the link is reduced. By using Algorithm 1, the calculated two-terminal reliability is consistent with the actual communication, which verifies the effectiveness of Algorithm 1.

Next, we use Algorithm 2 to evaluate the k-terminal reliability of Fig. 13, Fig. 14, and Fig. 15. The k nodes we select in Fig. 13, Fig. 14, and Fig. 15 are TD1 to TD6, TD1 to TD8, TD1 to TD10, respectively. In fact, evaluating the k-terminal reliability is evaluating the reliability of communication between all terminal nodes in the bus network. First, we enumerate all k-terminal minimal cut sets. Fig. 13(a), Fig. 13(b), Fig. 14(a), Fig. 14(b), Fig. 15(a), and Fig. 15(b) have 9, 81, 13, 169, 17, and 289 minimal cut sets, respectively. Meanwhile, we considered the influence of link reliability on the k-terminal reliability. We still considered link reliability ranging from 0.9 to 0.99 with an interval of 0.01, for a total of 10 cases in this paper. The detailed results of the changes in the k-terminal reliability with different link reliability in Fig. 13, Fig. 14, and Fig. 15 are shown in Fig. 17.

![FIGURE 17. The k-terminal reliability comparison diagram of the bus network without redundancy and the bus network with redundancy in three scenarios.](image)

From the results in Fig. 17, we can see that when the link reliability reaches 0.9, the k-terminal reliability of the bus network without redundancy of six terminal nodes is 0.3874 compared with 0.6247 for the bus network with redundancy, the k-terminal reliability of the bus network without redundancy of eight terminal nodes is 0.2542 compared with 0.4438 for the bus network with redundancy, and the k-terminal reliability of the bus network without redundancy of ten terminal nodes is 0.1668 compared with 0.3057 for the bus network with redundancy. When the link reliability is 0.9, the k-terminal reliability of the bus network with redundancy of six terminal nodes, eight terminal nodes, and ten terminal nodes is 61.26%, 74.58%, and 83.32% higher than that of the bus network without redundancy, respectively. This is because the bus network with redundancy has more transmission paths than the bus network without redundancy when all devices in the network exchange data with each other. As can be seen from Fig. 17, with the increase of link reliability, the k-terminal reliability of the bus network without redundancy and the bus network with redundancy both increase. We can conclude that the reliability of the link has an important influence on the k-terminal reliability.

Therefore, when all devices communicate with each other in the actual bus network, we can improve the k-terminal reliability of the network by increasing the link reliability. Through the above analysis, it can be concluded that when all devices of the network exchange data with each other, the reliability of the bus network with redundancy is higher than that of the bus network without redundancy. By using Algorithm 2, the calculated k-terminal reliability is consistent with the actual communication, which verifies the effectiveness of Algorithm 2.

According to the above quantitative evaluation of the two-terminal reliability and the k-terminal reliability of the bus network, we verify the effectiveness of the proposed algorithms. At the same time, we conclude that the bus network with redundancy has undeniable advantages over the bus network without redundancy. It is especially noticeable when the reliability of the link is reduced.

### VII. CONCLUSION

Based on the minimal path set-based approach and the minimal cut set-based approach, we proposed the reliability analysis algorithms for the bus network. First, we modeled the bus network without redundancy and the bus network with redundancy. Then, we proposed algorithms to calculate the two-terminal reliability and the k-terminal reliability of the bus network with redundancy. Finally, we evaluated the two-terminal reliability and the k-terminal reliability of the bus network without redundancy and the bus network with redundancy in three scenarios to verify the effectiveness of the algorithms. At the same time, we considered the impact of ten different link reliability scenarios on the two-terminal reliability and the k-terminal reliability. The experimental results show that when the reliability of the link reaches 0.9, the two-terminal reliability of the bus network with redundancy of six terminal nodes, eight terminal nodes, and ten terminal nodes is 40.95%, 52.17%, and 61.26% higher than that of the bus network without redundancy, respectively; the k-terminal reliability is 61.26%, 74.58%, and 83.32% higher than that of the bus network without redundancy, respectively.
The evaluations show that the bus network with redundancy has higher two-terminal reliability and k-terminal reliability than the bus network without redundancy. The bus network with redundancy increases the communication paths between devices. Therefore, it enables the reliable transmission of data. The algorithms proposed in this paper provide an effective solution for the reliability evaluation of the bus network. It perfects the reliability evaluation system of the network with different topology architectures. At the same time, it has certain directive significance for the actual layout of the bus network.

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