Gravitational Wave Detector for a Space Laboratory

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Abstract

We propose a new method of gravitational waves detection in the $10^{-1} \div 10^{-2}$ Hz band for a space laboratory based on the use of the Kozorez effect in the magnetic interaction of superconducting solenoids.

1 Introduction

As Thorne noted, it can be expected that the amplitude $h$ of gravitational wave bursts from astrophysical sources reaches the values of the order of $10^{-16} \div 10^{-17}$ in $10^{-1} \div 10^{-2}$ Hz band. Some authors proposed to use in this frequency band the Doppler tracking of interplanetary spacecraft, Skyhook-detector and the excitations of seismic motions in the Earth’s surface. A new type of the gravitational wave detector for the low frequency band is considered in this paper. The detector is based on the effect that the potential energy of a pair of magnetically interacting superconducting solenoids in weightless state, generally speaking, has the minimum at some nonzero distance. At this distance the solenoids are in a weak equilibrium condition. This system in vacuum is a weakly coupled nonlinear oscillator with very low energy dissipation and it can be used as a sensitive detector of tidal low-frequency accelerations of the solenoids. For measuring the tidal accelerations of the order of $10^{-13} \text{cm/s}^2$ in a satellite the gravity gradiometer
was proposed. However, the tidal acceleration of test bodies caused by low-frequency bursts is of the order of $10^{-16} \div 10^{-18} \text{ cm/s}$.

2 A Peculiarity of Magnetically Interacting Superconducting Solenoids

Consider a pair of the superconducting solenoids $P_1$ and $P_2$ in line, with points $x_1$ and $x_2$ as the centers in the weightless state. If the solenoids carry the persistent currents $I_1$ and $I_2$, their magnetic energies are $U_1 = I_1Q_1/2$ and $U_2 = I_2Q_2/2$, where $Q_1$ and $Q_2$ are magnetic fluxes in the solenoids. Since $Q_1 = L_1I_1 + MI_2$ and $Q_2 = L_2I_2 + MI_1$, where $L_1$ and $L_2$ are the inductances and $M$ is the mutual inductance of the solenoids, the currents are

$$I_1 = (Q_1L_2 - Q_2M)/D, \quad I_2 = (Q_2L_1 - Q_1M)/D,$$

where $D = L_1L_2 - M^2$. The fluxes $Q_1$ and $Q_2$ are constants and $M$ is a function of the distance $x = |x_1 - x_2|$ between the solenoids centers. Therefore, $I_1$ and $I_2$ are the function of $x$ and, when one of the conditions

$$M = L_1Q_2/Q_1, \quad M = L_2Q_1/Q_2$$

is satisfied, the currents change their signs. It follows from eqs. (1) that the energy $U = U_1 + U_2$ of the system is given by

$$U = (L_2(Q_1)^2 - 2MQ_1Q_2 - L_1(Q_2)^2)/(2D).$$

The energy of the solenoids magnetic interaction is

$$W = U - L_1Q_1^2/2 - L_2Q_2^2/2.$$

The Ampere force $F = -\partial W/\partial x$ affects the solenoids. It follows from the eq. (2) that $F = I_1I_2\partial M/\partial x$, where $I_1$ and $I_2$ are defined by eqs. (1).

The basic peculiarity of the interaction of superconducting circulating currents (The Kozorez effect) is that the function $W(x)$ has the minimum and the Ampere force is reduced to zero at the certain distance $x = x_0$, where one of the conditions is satisfied.
Indeed, $F = U'_M M'_x$, where $U'_M = \partial U / \partial M$ and $M'_x = \partial M / \partial x$. However, the function $U'_M = [Q_1 Q_2 M^2 + (L_2 Q_1^2 + L_1 Q_2^2) M - Q_1 Q_2 L_1 L_2]/D^2$ is equal to zero if one of the conditions (3) is satisfied.

We have also $U''_{xx} = U''_{MM}(M'_x)^2 + U'_M M''_{xx}$, where $U''_{xx} = \partial U'_x / \partial x$ and $M''_{xx} = \partial M'_x / \partial x$. The mutual inductance $M(x)$ is a monotonically decreasing function with an increasing distance $x$, and $M'(x) \neq 0$. Let $Q_2 < Q_1$. Since $M < L_1$, the first equation in (2) is satisfied and under this condition $U''_{MM}(x_0) = (Q_1)^2/[(L_1 L_2 - M^2)L_1] > 0$. Then $U''_{xx}(x_0) > 0$. If $Q_2 > Q_1$, the second equation in (2) is satisfied and we again arrive at this conclusion.

Therefore, the function $W(x)$ has a minimum at the distance $x_0$. If $Q_2 < Q_1$, the energy $W$ in the equilibrium position $x = x_0$ is $W_{\text{min}} = -Q_2^2/(2L_2)$. An illustrative example for the pair of the identical solenoids is given in Fig. 1. The parameters of the solenoids are: the inductances are $1.15Hn.$, the lengths are $5cm.$, the radiuses are $10cm.$, $Q_1 = 1.15Wb$, $Q_2 = Q_1/25$.

![Graph of the force $F(x)$](image_url)

**Fig.1.** The force $F(x)$
3 The Magnetically Coupled Solenoids as a Detector of Tidal Accelerations

Consider the properties of the system, formed by a pair of superconducting solenoids in weightless state, in the field of a gravitational wave with the frequency \( \nu \) extending during the time interval \( t_0 \) perpendicularly to \( P_1 P_2 \) - direction. This system can be regarded as an oscillator, the small oscillations of which in \( P_1 P_2 \) - direction are described by the nonlinear differential equation

\[
 m \ddot{q} + R(\dot{q}) + pq = f(t). \tag{4}
\]

In eq.(4) \( q = x - x_0 \) is a small deviation from the equilibrium position , \( \dot{q} = \partial q/\partial t \) , \( \ddot{q} = \partial \dot{q}/\partial t \) , \( R(\dot{q}) \) is the air resistance, \( p = U''(x_0) \) is the stiffness of the ”magnetic spring", \( f(t) = ma_g \sin(\omega t) \) at \( 0 < t < t_0 \) and \( f(t) = 0 \) at \( t > t_0 \) , \( m \) is the mass of the system , \( \omega = 2\pi \nu \) and \( a_g = \omega^2 hx_0/2 \) is the amplitude of the tidal acceleration, caused by the gravitational wave.

If 1-type superconductors are used in the solenoids, the air resistance \( R(\dot{q}) \) is the key cause of the oscillations damping in the given system. (This assertion is argued in Section 4). Such a detector can be named an ideal detector. For an ideal gas the function \( R(\dot{q}) = -b|\dot{q}| \), where \( b = \alpha \rho S/2 \) , \( \alpha \) is the aerodynamic factor of the resistance , \( \rho \) is the air density in the device, \( S \) is the crosssection of the solenoids orthogonally to the \( P_1 P_2 \) - direction. If the pressure is \( 10^{-10} \) Torr in the hydrogen atmosphere, the temperature \( T = 4.2K \) and \( S = 10cm \) , the magnitude of \( b \) reaches \( 10^{-14} \) gm/cm .

Since \( R(\dot{q}) \) and \( a_g \) are small quantities , we shall use the Bogolubov-Krilov method \( [5] \) for the analysis of nonlinear equation (4).

Suppose, \( \omega \) is close to the resonant frequency \( \omega_0 = (p/m)^{1/2} \). We shall seek an approximate solution of eq.(4) in the form

\[
 q(t) = A(t) \cos[\omega t + \vartheta], \tag{5}
\]

where \( A \) and \( \vartheta \) are slowly varied functions of time \( t \). Let us replace \( R(\dot{q}) \) in eq. (4) by the linear function \( \lambda e\dot{q} \) , where \( \lambda \) minimises the function

\[
 I(\lambda) = \int_0^T [R(\dot{q}) - \lambda e\dot{q}]dt \tag{6}
\]

\( (T = 2\pi) \). It is attained at \( \lambda e = \alpha b A \omega \), where \( \alpha = 8/(3\pi) \). Denote \( \lambda e/m \) by \( \beta \) . Then eq. (4) takes the form of a linear differential equation

\[
 \ddot{q} + \beta \dot{q} + (\omega_0)^2 q = a_g \sin(\omega t), \tag{7}
\]
where, however, \( \beta \) is a function of \( A \).

It follows from eq.(3) that at \( Q_1 > Q_2 \) the stiffness \( p \) is given by

\[
p = \left[ Q_1 M'_x(x_0) \right]^2 / (L_1 D)
\]  \( \text{(8)} \)

The typical values of \( \omega \) are much less than 1Hz. Substituting eq. (5) into (4), ignoring the terms \( \ddot{A} \) and \( \beta \dot{A} \) and setting the coefficients of \( \cos(\omega t) \) and \( \sin(\omega t) \) equal to zero, we obtain the differential equations system for \( A \) and \( \vartheta \):

\[
\dot{A} + \frac{\alpha \beta \omega^2 A}{2m} + \frac{a_g}{2\omega} \cos(\vartheta) = 0
\]  \( \text{(9)} \)

\[
\dot{\vartheta} + \frac{\omega^2 - \omega_0^2}{2\omega m} - \frac{a_g}{2Am} \sin(\vartheta) = 0
\]  \( \text{(10)} \)

Setting \( \dot{A} = 0 \) and \( \dot{\vartheta} = 0 \) and eliminating the phase \( \vartheta \), we obtain the equation

\[
A[(\omega^2 - \omega_0^2)^2 + \alpha b^2 \omega^4 m^{-2} A^2] = (\omega^4 h x_0)/4
\]  \( \text{(11)} \)

This equation gives the implicit function \( A = A(\omega) \) at stationary oscillations. At the resonance (\( \omega = \omega_0 \)) the amplitude is given by

\[
A = \left[ mh x_0 / (2ab) \right]^{1/2}.
\]  \( \text{(12)} \)

At \( \omega = \omega_0 \) and zero initial condition there exists solution \( A = A(t) \) with a constant phase \( \vartheta = \pi \):

\[
A(t) = A_m \tanh(t/\tau_0),
\]  \( \text{(13)} \)

where \( \tau_0 = \omega^{-1}[8m/(\alpha bh x_0)]^{1/2} \) is the detector relaxation time. If in eq. (12) \( m = 10^4 \text{gm} \), \( x = 50 \text{cm} \), \( b = 10^{-3} \text{gm/cm} \), \( h = 10^{-22} \) and \( \omega = 0.1 \text{Hz} \) we find that the stationary amplitude of the detector response \( A_m = 1.5 \times 10^{-7} \text{cm} \).

However, it is impossible to observe such high amplitudes since relaxation time of the ideal detector (\( \tau_0 \) is proportional to \( h^{-1/2} \)) is too high. However, \( A(t) \) reaches \( 2 \times 10^{-15} \text{cm} \) already at the observation time \( t = 2.6 \times 10^6 \text{s} \) (one month).

At \( t \ll t_0 \) we obtain \( A(t) = (A_m/\tau_0)t \). Consequently, if the interval of a resonant gravitational-wave burst is \( T = 2\pi/\omega \), the detector response is

\[
q(t) = (\pi h x_0/2) \cos(\omega_0 t)
\]  \( \text{(14)} \)
at $t > T$. Thus, the gravitational wave resonant burst gives rise to the long-duration, poorly damped oscillations with the frequency $\omega$ and the amplitude $\pi h x_0/2$. At $h = 10^{-17}$ and $x_0 = 50$ cm the amplitude is $7.8 \cdot 10^{-16}$ cm.

If the resonant frequency $\omega_0$ is much less than $\omega$, then a numerical solution of eq. (5) shows that the response $q(t)$ reaches $1.7 \cdot 10^{-15}$ cm at $t = 10$ s and further on slightly varies in time.

4 Measurement noises

The detector under study is a nonlinear oscillator. Let us find the variance of its thermal fluctuations that is other than the one of a linear oscillator. Consider first the linear oscillator described by the differential equation of the form $\ddot{q} + \beta \dot{q} + \omega^2 q = f(t)$. Suppose, the oscillator relaxation time is much longer than the measurements time, i.e. $\beta t \ll 1$. In that case, according to a rigorous solution by Chandrasekhar [6] the position $q$ variance $(\sigma_q)^2$ is given by

$$\sigma_q^2 = \frac{kT}{m \omega^2 \beta t} \left[ 1 - \frac{\sin(2\omega_0 t)}{2\omega_0 t} \right] + o(\beta t^2) \quad (15)$$

and the velocity $v = \dot{q}$ variance is given by

$$\sigma_v^2 = \frac{kT}{m} \beta t \left[ 1 + \frac{\sin(2\omega_0 t)}{2\omega t} + o(\beta t^2) \right] \quad (16)$$

The expressions between the brackets are essential at low resonant frequencies of the oscillator.

In the case under consideration for an approximate analysis of the thermal fluctuations it is expedient to take the statistic linearization of the function $R(\dot{q})$. Namely, we put $R(\dot{q}) = \lambda_1 e \dot{q}$, where $\lambda_1$ minimises the function $I_1(\lambda_1) = \langle [R(\dot{q}) - \lambda_1 \dot{q}] \rangle$. (The symbol $\langle \rangle$ denotes an ensemble mean). Now the random oscillations of the detector, caused by the thermal noise, are given approximately by the differential equation $\ddot{q} + b \dot{q} + \omega^2 q = \xi(t)$, where $\beta = \lambda_1 e / m$ and $\xi(t)$ is a Gaussian noise.

The desired value of $\lambda_1 e$ is $\lambda_{1e} = \langle \dot{q} R(\dot{q}) \rangle / \langle \dot{q}^2 \rangle$.

Assuming that the distribution function is approximately Gaussian, we find that $\beta = 4(2\pi)^{-1/2} m^{-1} \sigma_v$. Thus, $\beta$ is the function of $\sigma_v$.

Now we are considering eq. (15) as an equality whence the function $\sigma_v(t)$ can be found. Substituting the above expression of $\beta$ into eq.(16) and found
function $\sigma_v(t)$ - into eq.(13) we obtain, finally, that the value of the variance $\sigma^2_q$.

\[
\sigma^2_q(t) = \frac{4b^2}{(2\pi)^{1/2}m\omega_0^2} \frac{kT}{m} t^2 \left[ 1 - \frac{\sin^2(2\omega_0 t)}{2\omega_0^2 t^2} \right]
\]

(17)

For example, if $m = 10^4 \text{gm}$, $\nu_0 = 0.01 \text{s}$, $t = 100 \text{s}$, $b = 10^{-4} \text{gm/cm}$, then the root-mean-square magnitude of the thermal fluctuations is $<q^2>_1/2 = \sigma_q = 10^{-20} \text{cm}$. Thus, the detector has a very low level of the thermal noise, that is much less than the expected detector response to the gravitational wave bursts.

The inhomogeneity of the Earth and spacecraft gravitational fields lead to more serious problems. The difference in the solenoids gravitational accelerations approximately equal to $U_{ik} x_0$, where $U_{ik} = \partial^2 U / \partial x^i \partial x^k$ and $U$ is the Earth gravitational potential. The variations of $U_{ik}$ during the orbital motion of the spacecraft are well beyond the tidal accelerations $a_g$ in eq.(4). This problem can be solved by choosing a geostationary or very distant from the Earth spacecraft orbit.

It is necessary to take into account the fluctuations in the gravity gradient within the spacecraft too. For example, the 1cm shift in the position of the 5gm mass at its distance 2m from the solenoids along the $P_1P_2$-direction causes the variation $4 \cdot 10^{-14} \text{cm/s}^2$ in the relative acceleration of the solenoids.

A little part of the inductance (the motional inductance) is a function of the temperature $T$. So, variations in $T$ cause the variations $\delta L$ in the inductance $L$. (In the above illustrative example at the wire radius 0.05cm $dL/L$ is about $10^{-16}$ at $T = 2K$ and $10^{-20}$ at $T = 0.1K$).

For definiteness, assume that $Q_1 < Q_2$. Then under the equilibrium position $Q_2L_1 - Q_1M = 0$ and $D \approx L_1L_2$. If at moment $t = 0$ a temperature variation begins, then, according to eqs. (2), at $t > 0$ we have $I_1 \approx I_1^0 + I_1^0(\delta L_2/L_2)$ and $I_2 \approx I_2^0(\delta L_1/L_1)$, where $I_1^0 = Q_1/L_1$ and $I_2^0 = Q_2/L_2$. Now the force of the interaction between the solenoids in absence of exterior forces is other than zero:

\[
F_T = I_1^0 I_2^0 (\delta L_1/L_1) M'(x_0),
\]

(18)

where $M'(x_0) = \partial M/\partial x$. The force $F_T$ gives rise to slow oscillations of the solenoids relative to the equilibrium position $x_0$.

Denote the acceleration $F_T/m$ by $a_T$. Obviously, it is necessary to select the detector parameters so that during the measurements time the inequality $a_g < a_T$ is valid. We do not analyze the problem of optimization of the
detector parameters here. However, it should be noted that it is easiest of all to obey the above inequality if the detector resonant frequency $\omega_0$ is much less than the frequency $\omega$ of the wave. This happens to be the case for the considered above illustrative example (Fig.1) at $Q_1 = 1.15 \cdot 10^{-1} Wb$. In that case $\omega_0 = 2.6 \cdot 10^{-3} s^{-1}$, $I_1^0 = 0.1 A$ and $I_2^0 = I_1^0/25$). At $T = 2K$ and $\delta L/L = 10^{-16}$ the acceleration $a_T = 5 \cdot 10^{-18} cm/s$. Meanwhile, at $\nu = 0.1 Hz$ and $h = 10^{-17}$ the gravitational acceleration $a_g = 4 \cdot 10^{-17} cm/s$. (The detector response to short bursts found by the numerical solution of eq.(4) is about $7 \cdot 10^{-16} cm$ at $t_0 = 10s$).

It follows from eq.(8) that the stiffness $p = [I_1^0 M'(x_0)]^2/L_2$. Hence, the temperature fluctuations cause the resonant frequency variations $\delta \omega_0$. They are given by $\delta \omega_0/\omega_0 = -\delta L_1/(2L_1)$. These variations may be essential at the resonant detection of a continuous signal.

Under oscillations of the solenoids relative to the equilibrium position the persistent currents in the solenoids are not constants ( $\dot{I} = dI/dt \neq 0$ ). From the viewpoint of the two-fluid model the intensity of the supercurrent is $J = en_sv_s$, where $e$ is the charge of the electron, and $n_s$ and $v_s$ are the concentration and velocity of the superconducting electrons correspondingly. As a result of the acceleration of the superconducting electrons an electric field $E$ appears inside the superconductor that can be found from the equality $m\dot{v}_s = -eE$, where $\dot{v}_s = dv_s/dt$. If the time -depending current is of the form $J \exp(-i \omega t)$, then $E = -[im\omega J/(e^2n_s)]$. This electric field causes a motion of the normal electrons according to the following equation [7]

$$md\langle v_n \rangle/dt + (m/\tau_0)\langle v_n \rangle = -eE,$$

where $\langle v_n \rangle$ is the mean velocity of the normal electrons and $\tau_0$ is the relaxations time (usually about $10^{-13}s$). Then, there is also a normal current in the solenoids with the intensity $J_n = -en_n\langle v_n \rangle$, where $n_n$ is the normal electrons concentration. Solving the above equation of motion we find that at low frequencies $\omega$ the normal current is given by

$$I_n = (n_n/n_s)\omega\tau I_0^0 \sin(\omega t).$$  \hspace{1cm} (19)

Let $Q_1 < Q_2$. The amplitude $I_2^{max}$ of the supercurrent in the solenoid $P_2$ caused by gravitational-wave bursts is equal to $I_2^0 A$, where $A$ is the amplitude of the detector response and $I_2 = dI_2/dq$. It follows from eq.(1) that $I_1' = (I_2^0/L_1)M'$ and $I_2' = (I_1^0/L_2)M'$. The typical value of $I_2'$ is of the
order of $10^{-2} \div 10^{-4} A/cm$. Since $A$ is about $10^{-16} cm$, the typical value of $I_{s,\text{max}}$ proves to be about $10^{-19} A$. Setting in (19) $n_n/n_s = 1/5$ and $\nu = 0.1 Hz$ we find that the amplitude of the normal current is $10^{-34} A$. During time $dt$ the energy dissipation is $RI_n^2 dt$, where $R$ is the superconductor wire normal resistance. If $R = 1 \Omega$, then during the time $T = 2\pi/\omega$ the energy dissipation is about $10^{-67} J$. At the same time the tidal forces work is about $ma_g A = 10^{-36} J$.

Consider another question: ought we to include in eq.(10) in addition to the term $R(q)$ an effective force $F_{ef}$, which describes the energy dissipation caused by the normal current? In principle, it can be done since according to eq.(19) $I_n$ is of the form $(n_n/n_s)\tau\dot{I}_s(t)$. Hence, $RI^2 dt$ can be written as $F_{ef} \dot{q} dt$, where $F_{ef} = R\tau^2 (\partial I/\partial q)^2 \dot{q}$. Let us compare $F_{ef}$ with the force $R(q)$. The value of $\dot{q}$ is approximately $\omega A$. At low frequencies and at $b \gg 10^{-10} gm/cm$ the following inequality is fulfilled: $R(q) \ll F_{ef}$, since $R\tau^2 (I)^2 \ll bA\omega$. (Because of too longe relaxation time $\tau_0$, small values of $b$ ought not to be used).

Thus, at very small amplitude oscillations and low frequencies the energy dissipation caused by the normal current is insignificant.

Consider briefly the physical principles of the solenoids relative shift measurement caused by the gravitational-wave bursts. For definiteness, assume that $Q_2 < Q_1$. At the equilibrium position the supercurrent $I_2 = 0$. The deviations in the equilibrium position cause the change in the proper magnetic flux of the solenoid $P_2$ is $\delta Q = \delta I_2 L_2$, where $\delta I_2 = I'_2 q$. Thus, the value $\delta Q_2$ is given by

$$\delta Q_2 = I'_2 M' q$$  \hspace{1cm} (20)

Suppose that the distance between the near solenoids ends is less than 1cm at the distance between the solenoids centers of 50cm. In that case the value $M'$ reaches $10^{-2} Gn/cm$ or more than that. At $I'_1 = 0.1A$ $dQ_2$ is about $10^{-3}\Phi_0$, where $\Phi_0 = 210^{-15} Wb$ is the magnetic flux quantum.

A method of $dQ_2$ measuring is shown in Fig.2. In this schematic diagram $S$ is a superconducting quantum-interferometer device (SQUID) that is attached to the solenoid $P_2$ and coupled with $P_2$ inductively by a flux transformer $T$. 

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The minimal magnetic flux measured by the SQUID with the $T$ is given by 

$$\delta Q_{min} = 2[2L_a\mathcal{E}(\nu)]^{-1/2}/N_a,$$

where $L_a$ is the antenna inductance, $N_a$ is its turns number and $\mathcal{E}(\nu)$ is the SQUID noise expressed as an input energy resolving at the frequency $\nu$. At the frequencies $\nu \leq 0.1 Hz$ the value $\mathcal{E}(\nu) = (10^{-31}/\nu n)J/(Hz)^{-1/2}$. At $\nu = 0.1 Hz$ $\delta Q_{min} = 10^{-4}\Phi_0$ or less than that. Thus, $\delta Q_{min} < \delta Q_2$.

We mean that the solenoids $P_1$ and $P_2$ are inside a superconducting shield. The results of the SQUID measurements are transmitted by radio by means of a conversion ”voltage - frequency” and by using an isotropic active antenna. Such a method of the solenoids shift measuring is insensitive to micrometeorites impacts and other forces affecting the spacecraft.

In the SQUID circuit there exists noise magnetic flux $\Phi_n$ of the order of $10^{-5}\Phi_0$ or less than that. Because of the inductive coupling between the SQUID and the solenoid $P_2$ a noise current appears in the latter that is less than $\Phi/L_2 = 10^{-20}A$. It is much less than the noise effect upon the detector is a small value.

A number of the other disturbing forces in the space laboratory is considered in [11]. The author would like to thank C.W.F. Everitt and P.W. Worden Jr. for the materials on the STEP.
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