The structure change of the $^{28}$Si core in $^{32}$S+$\alpha$ cluster structure

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Abstract. The $^{28}$Si+$\alpha$ cluster structure was studied. We construct the $^{28}$Si+$\alpha$ cluster model, where structure change of the $^{28}$Si core and the $\alpha$-cluster breaking are taken into account. It was found that, in the $^{28}$Si+$\alpha$ cluster system having $\alpha$-cluster at the surface of the $^{28}$Si core, the structure change of the $^{28}$Si core is energetically rather important while the $\alpha$-cluster breaking is not significant.

1. Introduction
The nuclear clustering plays important roles in ground and excited states of light nuclei. Excited states having core+$\alpha$ cluster structure called the $\alpha$-cluster excited state are known to exist in such nuclei as $^{20}$Ne [1] and $^{16}$O [2], and they are suggested also in $^{40}$Ca and $^{44}$Ti [3, 4]. Meanwhile, researches of the $\alpha$-cluster excited state in the middle of $sd$-shell are still limited, and the existence of $\alpha$-cluster excited states in this region is an open problem. Recently, $^{28}$Si+$\alpha$ excited states of $^{32}$S have been suggested in the experiments of $\alpha$ scattering on $^{28}$Si and inelastic scattering of $^{32}$S [5, 6]. In order to understand structure of the $^{28}$Si+$\alpha$ excited states by solving the core-$\alpha$ relative motion, it is important to take into account the structure change of the core nuclei and the breaking of the $\alpha$-cluster at the nuclear surface. However, in conventional cluster models, inner clusters are usually assumed and the cluster excitation effects are not considered. Our final goal is to study the $^{28}$Si+$\alpha$ excited states of $^{32}$S. To this end, for the first step, we investigate the structure change of the $^{28}$Si core and the $\alpha$-cluster breaking in the $^{28}$Si+$\alpha$ cluster structure.

2. Framework
We apply extended cluster model to take into account the structure change of the $^{28}$Si core and the $\alpha$-cluster breaking. We first explain Brink-Bloch $\alpha$-cluster model (conventional cluster model) [7], and later, we describe the extended model for treating the $\alpha$-cluster breaking and the structure change of the $^{28}$Si core.
2.1. Brink-Bloch α-cluster model

In the Brink-Bloch α-cluster model [7], each single-particle wave function $\varphi_i$ is expressed by a Gaussian wave packet:

$$
\varphi_i = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} \exp \left[-\nu (r - R_i)^2\right] \chi_\tau i,
$$

where $\chi_\tau i$ is the spin part, $\tau_i$ is the isospin part, $R_i$ is the center of the Gaussian wave packet, and $\nu$ is the width parameter. An α-cluster is described by four nucleons (spin up neutron and proton and spin down neutron and proton) expressed by the Gaussian wave packets at the same position. The total wave function $\Phi$ of an A-particle system is described by the antisymmetrized single-particle wave functions of α-clusters.

$$
\Phi = A [\varphi_1 \cdots \varphi_A],
$$

where $A$ is the antisymmetrizing operator.

2.2. Extended cluster model with α-cluster breaking

For description of the α-cluster breaking, we apply the method proposed by Itagaki et al [8] to introduce the effect of the spin-orbit potential. In this method, a parameter is introduced to break the α-cluster so as to gain the spin-orbit interaction. We add a momentum to nucleons of the α-cluster that fixes vertical direction with spin and $R_i$.

$$
R_i \to \hat{R}_i = R_i + i\lambda \left(\begin{array}{c}
\mathbf{e}_{\text{spin},i} \times (\mathbf{e}_{R_i}) \\
\end{array}\right),
$$

where $\lambda$ represents the degree of α-cluster breaking. If $\lambda = 0$, this model becomes the conventional cluster model and describes the intrinsic spin saturated state, where the expectation value of the spin-orbit interaction vanishes. When $\lambda$ is positive, spin-up and spin-down nucleons in the α-cluster have finite momenta with the opposite directions so as to gain the spin-orbit interaction, and the α-cluster has breaking. Indeed for a finite positive value of $\lambda$, the single nucleons in the α-clusters are boosted to have angular momentum parallel to their intrinsic spin, and therefore they feel the spin-orbit interaction from the core attractively.

2.3. Extended cluster model with structure change of $^{28}\text{Si}$ core

To describe $^{28}\text{Si}$ core and its structure change we adopt the extended 7α-cluster model where the parameter $\Lambda_c$ for the cluster breaking is incorporated to take into account the spin-orbit interaction effect. The present extended 7α-cluster Brink model is based on the studies of $^{28}\text{Si}$ with the Brink-Bloch 7α-cluster model [9] and also with the AMD (antisymmetrized molecular dynamics) method [10]. This extended 7α-cluster Brink model can describe both the $jj$-coupling shell model limit at $\Lambda_c = 1$ that is described the spherical $(0d_5/2)^{12}$ configuration, and the oblate deformation limit of the shell model configuration at $\Lambda_c = 0$.

3. Results

Using the extended cluster model, the energy expectation value of a $^{28}\text{Si} + \alpha$ system is calculated as a function of the inter-cluster distance ($R$). By analyzing the $\Lambda_c$ and $\lambda$ dependence of the energy, we investigate how the core structure change and the α-cluster breaking affect to the energy of the $^{28}\text{Si} + \alpha$ system. The total wave function of the extended cluster model is projected to the $J^\pi = 0^+$ state. As for the effective nuclear interaction, the Volkov No. 2 (Majorana parameter $M = 0.67$) [11] for the central force and G3SR ($V = 2000$ MeV) [12] for the spin-orbit force are adopted. The Coulomb force is approximated seven range Gaussians.
3.1. Structure change of $^{28}\text{Si}$ core

We here discuss the calculation without the $\alpha$-cluster breaking ($\lambda_\alpha = 0$) to focus only on the structure change of the $^{28}\text{Si}$ core. We show the energy expectation value of $^{28}\text{Si}+\alpha$ as a function of the parameter $\Lambda_c$ for the structure change of the $^{28}\text{Si}$ core in Fig.1. It is found that, in the case of the large inter-cluster distance $R = 8, 10$ fm, the energy almost degenerates in a wide $\Lambda_c$ region of $\Lambda_c \geq 0.3$ because the oblate and spherical states degenerates in the isolate $^{28}\text{Si}$ without an $\alpha$-cluster. On the other hand, in the small $R$ region, the oblately deformed $^{28}\text{Si}$ core (around $\Lambda_c = 0.38$) tends to be energetically favored than the spherical $^{28}\text{Si}$ core. This suggests that the core structure change occurs when the $\alpha$-cluster exists at the surface of the $^{28}\text{Si}$ core. The origin structure change of the $^{28}\text{Si}$ core can be understood by the Pauli blocking effect. Namely, the $^{28}\text{Si}$ core favors to deform oblately to avoid overlapping with the $\alpha$-cluster at the surface.

We also show the energy expectation value of $^{28}\text{Si}+\alpha$ as a function of the inter-cluster distance $R$ in Fig.2. Here the energies for the fixed $^{28}\text{Si}$ core cases of the oblale $^{28}\text{Si}$ with $\Lambda_c = 0.38$ and the spherical $^{28}\text{Si}$ with $\Lambda_c = 0.80$ are shown. As mentioned above, the oblately deformed $^{28}\text{Si}$ core is favored than the spherical state in the small $R$ region, while they almost degenerate to each other at the large distance region.

If the $\alpha$-cluster excited states exist in $^{32}\text{S}$, they are expected to have the $\alpha$-cluster amplitude at the nuclear surface. The present result suggests that the structure change of the $^{28}\text{Si}$ core may occur because of the $\alpha$-cluster at the surface and it may give some effect on the $\alpha$-cluster excited state of $^{32}\text{S}$.

3.2. $\alpha$-cluster breaking

We investigate the $\alpha$-cluster breaking effect on the energy expectation value of $^{28}\text{Si}+\alpha$ by analyzing the $\lambda_\alpha$ dependence of the energy. For $^{28}\text{Si}+\alpha$ with a fixed inter-cluster distance $R$, the optimized $\lambda_\alpha$ for the minimum energy is small as $\lambda_\alpha \sim 0$ in $R > 3$ fm region indicating that the $\alpha$-cluster breaking may be minor effect in $^{28}\text{Si}+\alpha$ system (hardly energy difference at $R \geq 4$ region). The reason for the unfavored $\alpha$-cluster breaking is as follows. In general, in the mechanism of the $\alpha$-cluster breaking caused by the spin-orbit potential at the nuclear surface, nucleons in the $\alpha$-cluster occupy $ls$-favored orbits rather than to form the $\alpha$-cluster. However, in the $^{28}\text{Si}+\alpha$ system, nucleons in the $^{28}\text{Si}$ core already fills the $ls$-favored orbits, $0d_5/2$ orbits, and
they block the $\alpha$-cluster breaking. In other words, the $\alpha$-cluster breaking tends to be suppressed at the surface of the $^{28}\text{Si}$ core because of the Pauli blocking effect from the core nucleons.

4. Summary and outlook
We calculate the energy expectation value of a $^{28}\text{Si}+\alpha$ system using the extended cluster model. By analyzing the core structure change and the $\alpha$-cluster breaking in the $^{28}\text{Si}+\alpha$ system, we found that the structure change of the $^{28}\text{Si}$ core is energetically rather important while the $\alpha$-cluster breaking is not significant when the $\alpha$-cluster exists at the surface of the $^{28}\text{Si}$ core. In the $^{28}\text{Si}$ core, the oblately deformed state tends to be favored than the spherical sub-shell closed state because of the $\alpha$-cluster at the surface though they almost degenerate in the isolate $^{28}\text{Si}$ without an $\alpha$-cluster. We also found that the degree of $\alpha$-cluster breaking depends on the core nuclei and it is minor in $^{28}\text{Si}+\alpha$ system compared with $^{16}\text{O}+\alpha$ system. The present result suggest that the structure change of the $^{28}\text{Si}$ cluster may give important effect on the $\alpha$-cluster excited state of $^{32}\text{S}$ and that the $\alpha$-cluster breaking effect may be relatively minor. We are now proceeding to perform further calculations of the generator coordinate method by superposing the $^{28}\text{Si}+\alpha$ wave functions to study the $\alpha$-cluster excited states of $^{32}\text{S}$.

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