A Simple Model of Large Scale Structure Formation

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Abstract

We explore constraints on inflationary models employing data on large scale structure mainly from COBE temperature anisotropies and IRAS selected galaxy surveys, taking care not to apply linear perturbation theory to data in the non-linear regime. In models where the tensor contribution to the COBE signal is negligible, we find the spectral index of density fluctuations $n$ must exceed 0.7. Furthermore the COBE signal cannot be dominated by the tensor component, implying $n > 0.85$ in such models. The data favors cold plus hot dark matter models with $n$ close to unity and $\Omega_{HDM} \sim 0.20 - 0.35$. We present realistic grand unified theories, including supersymmetric versions, which produce inflation with these properties.

98.65 Dx,98.80 Cq,12.10 Dm
I. INTRODUCTION

The inflationary universe scenario [1] provides an attractive resolution of the well known “horizon”, “flatness”, and “causality” puzzles encountered in the standard big bang cosmology. This scenario is most easily realized within the framework of both ordinary and supersymmetric grand unified theories (GUTS). In the simplest realizations of inflation, the density fluctuations are Gaussian and close to the Harrison-Zeldovich form, and the background density equals the critical density $\rho_c$. Primordial nucleosynthesis implies that less than 10% of the background density is composed of baryons, and so the bulk of matter is “dark”, presumably in the form of relic elementary particles.

Almost a decade ago it was pointed out by Shafi and Stecker [2] that two types of dark matter, cold and hot, would provide, within the inflationary context, an elegant way of understanding observations which indicated a surprising amount of clustering on larger than cluster scales. Since only the cold component clustered on smaller scales, and the hot component only clusters on larger scales, such a universe would have an enhanced large scale clustering power. Examples of GUTS containing cold plus hot dark matter (C+HDM) were also presented. The implications of this picture for microwave background anisotropies were worked out in 1989 [3,4]. Mass functions [5], bulk streaming motions [3,4], cluster number densities and correlation functions, as well as “great attractors” were also found to be compatible [3,4] with data.

Recent data on large scale structure from a variety of sources, particularly COBE and IRAS selected galaxy surveys, provide additional strong support to this remarkably “simple” scenario for structure formation. The COBE group [8] found an anisotropy amplitude which they characterized by an averaged quadrupole moment amplitude $\delta T/T = Q_{rms-PS}/T = 6.2 \pm 1.5 \times 10^{-6}$. As we have emphasized [3], the earlier predictions [3,4] for C+HDM models (3/4 CDM, 1/4 HDM) with negligible baryonic matter content were $7.8 \times 10^{-6}/b_8$, while for CDM alone they were significantly smaller ($4.7 \times 10^{-6}/b_8$), where $b_8 = (\text{rms mass fluctuation on the scale } 8 \ h^{-1}\text{Mpc})^{-1}$. Including a 5% baryon density (implied by primordial nucleosyn-
thesis) modifies these predictions to $8.2 \times 10^{-6}/b_8$ for this C+HDM mixture and $5.1 \times 10^{-6}/b_8$ for CDM. Observationally derived values of $b_8$ seem to fall in the range $1.3 < b_8 < 2.5$, meaning the COBE result was in remarkable agreement with C+HDM models. COBE also lent some weak support for the inflationary power spectrum; the data are consistent with the $P(k) \propto k^n$ spectrum with $n = 1.1 \pm 0.5$. Following the COBE result other pieces of evidence pointing to the C+HDM model were discovered: compatibility with early quasar formation [10, 11], compatibility with galactic correlations and velocities, [12–14], APM correlations [14, 15] quiet local Hubble flow [16], “cosmic mach number” [17], and counts in cells [18, 19]. Thus, there seems no doubt that the C+HDM model is a candidate worthy of serious consideration.

While we know that the primordial density power spectrum $P(k) \propto k$ with C+HDM provides a good fit to the data, it has long been known that the inflationary scenario does not quite yield this simple form. In recent studies, these correction factors have been exploited to yield power spectra $P(k) \propto k^n$ with $n$ significantly less than 1 as a means to repair the relative large/small scale problems of the pure CDM model. In this paper, one of our tasks will be to undertake a study of the range of allowed $n$. [Note that in this work we will only consider $n \lesssim 1$, although inflationary models where $n$ exceeds unity can be constructed.] Since we do not precisely know the HDM fraction of the universe, we will explore the two dimensional parameter space $n - \Omega_{HDM}$, and examine the constraints imposed by the data.

We do this by comparing the model predictions with data from COBE, the power spectrum of IRAS galaxies, IRAS counts-in cells, and bulk velocities from the POTENT analysis. We will also consider requirements for the early formation of quasars. After fitting the normalization and the bias factor for IRAS galaxies, we calculate $\chi^2$ for each model. We then compare the results of the models and present $\chi^2$ contour plots in the $n - \Omega_{HDM}$ fraction plane. We also consider the possibility that some of the COBE signal was produced by the long wavelength gravity waves generated during inflation.

Because of uncertainties in the simulation of structure formation in the non-linear gravity regime, we will concentrate chiefly on structure which is still described by linear perturbation
theory (scales > 20h\(^{-1}\) Mpc). This avoids the difficulties inherent in identifying galaxies and clusters and separating out complicated dynamic effects, \(e.g.,\) the velocity bias of galaxies. Our main aim is to explore a large region of parameter space, and try to identify that part of it for which running more detailed computer simulations would make the most sense.

Finally, if we are to take seriously the whole picture of inflation and C+HDM we need to identify plausible models of inflation which are compatible with our analysis of the large scale data. Here we will consider only “realistic” models of inflation. By this, we mean models in which the inflating field is a part of a larger theory which contains the following elements:

- neutrinos with masses in the eV range.
- a cold dark matter candidate.
- a high energy particle physics structure which is compatible with low energy physics and its hints about higher symmetries.
- a successful baryogenesis following inflation.

Without these elements we find the inflation model to be somewhat \textit{ad hoc}.

The plan of the paper is as follows. In the next section, we will describe our testing of inflationary models with large scale structure data in the linear perturbation regime using a \(\chi^2\) analysis. This will turn out to strongly limit the power spectrum. In section III we will consider some general constraints from data which come from structure in the non-linear regime. This will limit mainly the HDM fraction. In section IV we consider models of inflation which satisfy all of our requirements for “realistic” inflation models. In particular we present some new examples of chaotic inflation based on supersymmetric grand unification. We end with some general conclusions in section V. We have attempted to make this paper somewhat self-contained for a more general audience.
II. CONSTRAINTS FROM LARGE SCALE STRUCTURE DATA IN THE LINEAR REGIME.

In this section, we will describe data and our $\chi^2$ analysis of theoretical predictions for these observations, but first we will discuss a few general issues concerning the models and the testing strategy employed here.

First of all, the problem with drawing strong conclusions about any given model of structure formation is that there are so many parameters to vary that we have a multi-dimensional parameter space to explore. We will choose to take best guess values of three of them, the baryonic fraction $\Omega_{\text{baryon}} = \rho_{\text{baryon}}/\rho_c$, the cosmological constant $\Lambda$, and the Hubble constant, $H_0 = 100h$ km sec$^{-1}$, with $h$ observationally constrained to the values $0.4 < h < 1.0$. Constraints on the age of the universe from globular star clusters, nuclear cosmochronology, and white dwarf ages imply that high values of the Hubble constant are forbidden, i.e. $h < 0.6$ in an $\Omega = 1$ universe. Thus these models will be allowed only if the observations eventually settle into a more restricted range $0.4 < h < 0.6$. Since many quantities vary as $h^2$ there is still some freedom despite this narrow range of Hubble constant. In the present analysis we will use only the central value, $h = 0.5$.

Primordial nucleosynthesis strongly constrains the baryon density \cite{20} as $0.010 \leq \Omega_{\text{baryon}}h^2 \leq 0.015$. Using $h = 0.5$, we can express this 95\% confidence constraint as

$$\Omega_{\text{baryon}} = 0.05 \pm 0.01 \hspace{1cm} (2.1)$$

We note that allowing for the uncertainty in $h$, the baryonic fraction could range from $\Omega_{\text{baryon}} \sim 0.03 - 0.9$. We intend to explore this uncertainty in the Hubble constant and baryon fraction and its implications in a future publication. Once we have fixed the baryon density, the other densities can be completely specified by the hot dark matter density $\Omega_{\text{HDM}}$ as

$$\Omega_{\text{CDM}} = 1 - \Omega_{\text{baryon}} - \Omega_{\text{HDM}}.$$ 

The hot dark matter fraction (combined with the hubble constant) also specifies the neutrino
mass. If we have one flavor of neutrino whose mass is in the eV range, usually taken to be \(\nu_\tau\), then

\[
\Omega_{HDM} = \left( \frac{m_{\nu_\tau}}{23 \text{ eV}} \right) \left( \frac{0.5}{h} \right)^2.
\]  

(2.2)

We will set the cosmological constant (\(\Lambda\)) to zero. There is evidence to support this choice. The local (within 60 \(h^{-1}\) Mpc) velocity field implies values of \(\Omega\) close to unity \([21]\). Furthermore, in a \(\Lambda\) dominated universe, there seems to be too few gravitational quasar lensing events, and the bulk streaming velocities are too small.

The growth of density fluctuations is affected by the dynamics of the matter content, producing a scale dependent modification of the density fluctuation power spectrum. The relative growth as a function of scale is discussed in Appendix A, and the results are summarized in Figure 1, where we present the transfer functions in Fourier space as a function of Fourier wavenumber \(k = 2\pi/\lambda\). In figure 1, we see that increasing the neutrino fraction decreases the amount of growth, and hence the amplitude, on small scales (large \(k\)). We also note that there are some modifications of the shape of the spectrum on quite large scales. For example the transfer function with \(\Omega_{HDM} = 0.3\) is slightly steeper at \(k \sim 0.07h\) Mpc\(^{-1}\) than that of either \(\Omega_{HDM} = 0.0\) (CDM) or \(\Omega_{HDM} = 0.5\).

In order to do our comparison in the most unambiguous way, we will confine our attention mostly to the regime where linear theory is appropriate. In the past, it was common to normalize linear power spectra by use of the fact that the rms optically selected galaxy density fluctuations are \(\delta N/N = 1\) on a scale of 8 \(h^{-1}\) Mpc. In the most naive case, where one assumes that galaxies (light) trace the mass distribution, one would set the rms mass fluctuation also equal to one on this scale. More recently, however, it has become apparent that the galaxies are more strongly clustered than the mass, i.e., they are biased tracers of the mass. The usual method for dealing with this complication is to assume that there is a linear relation between the optical galactic density and mass fluctuations using a bias parameter \(b\), which is independent of scale \(b = (\delta N/N)/(\delta M/M)\). The mass fluctuation \(\sigma(R)\) in a sphere of radius \(R\) is calculated in linear theory by
\[ \sigma^2(R) \equiv \frac{1}{2\pi^2} \int_0^\infty dk \, k^2 P_{th}(k) \left[ 3 \frac{j_1(kR)}{k} \right]^2, \]  

(2.3)

where \( j_1(x) \) is the first spherical Bessel function and the term in brackets is the Fourier transform of a sharp edged sphere of radius \( R \). Clearly, when a perturbation amplitude approaches unity, linear theory is no longer appropriate. Even with \( b \) as large as 2, this implies \( \sigma(8 \, h^{-1} \, \text{Mpc}) = 0.5 \), which is still quite non-linear. Using the spherical collapse approximation (see Appendix B) we estimate that for \( \sigma = 0.5 \), we are already highly contaminated by non-linear effects. If we consider scales for which \( \sigma(R) \leq 0.4 \) we estimate that the non-linear corrections will be \( \lesssim 30\% \).

A look at figure 1 shows that the greatest difference between the models considered here occurs on the small scales. Limiting our range to \( k \leq 0.3 \, h \, \text{Mpc}^{-1} \) implies that our testing will have a somewhat weakened ability to discriminate between models with different HDM fractions. In section III we will consider some constraints from non-linear structures to help us pin down the dark matter fraction.

We will first give a brief description of the large scale structure data, followed by a description of our calculations for different models.

A. Large Scale Structure Data

Here we will discuss the particulars of the data we are using. We explain our reasons for choosing the data and method of interpretation of this data.

1. COBE data

The large amplitude of the COBE measured temperature fluctuations is characterized by the extrapolated quadrupole moment \( Q_{rms-Ps} \). This amplitude was a factor of 2-3 larger than predicted in the usual CDM models fit to galactic structure, and so gave strong support for the C+HDM models [9]. However, various authors seem not content to use the COBE analysis of the amplitude. Instead they choose to use the sky variance at \( 10^\circ \) which, when
used with a Gaussian beam, implies a somewhat smaller amplitude for the fluctuations (e.
g., corresponding to $Q_{\text{rms}-PS} = 15.3 \mu K$). The COBE beam pattern, however, is only approximately a Gaussian shape. More careful analysis of the COBE results using the actual beam pattern seems to confirm the original higher value of the amplitude. In addition, use of the correct beam pattern reduces the variance of the fit amplitude, so the best fit $n = 1$ COBE amplitude now corresponds to $Q_{\text{rms}-PS} = 17.1 \pm 2.9 \mu K$. Although it is customary to quote the results in terms of the quadrupole moment, this corresponds to only the very small wavenumber end of the spectrum. The best fit amplitude of the quadrupole moment will be somewhat dependent on the value of $n$ used in the analysis. In order to find a better quantity than the quadrupole, Wright et al. recommend normalizing the amplitude to the hexadecupole $\delta T_4 = 12.8 \pm 2.3 \mu K$, when $n \neq 1$, as in the inflationary models we consider here. This yields a best fit amplitude less dependent on the value of $n$.

In passing, we note that the COBE results should be taken seriously, as a balloon experiment from the MIT/GSFC/Princeton collaboration sees the same temperature correlation function as COBE and find similar values for the fluctuation amplitude, which they specify with $Q_{\text{rms}-PS}$. However, they currently only have data from the northern hemisphere and their limits on $Q_{\text{rms}-PS}$ are not as constraining as those for COBE. In the future they plan to cover the southern hemisphere as well, which would improve the limits of the COBE power spectrum exponent, which are currently $n = 1.1 \pm 0.5$.

Ideally, we would like to include the detections of smaller scale anisotropy experiments in this analysis. There has been a wave of new detections of temperature anisotropies on $1^\circ - 2^\circ$ scales. It is not clear that all of the detections are giving a totally consistent picture, and even different scans with the same instrument give different results. A possible explanation is that the systematic errors these experiments face are quite complicated. We expect that these uncertainties will be resolved and that we will know the amount of degree scale anisotropy which exists with some precision. We also point out that C+HDM models, when normalized to COBE produce the same degree scale anisotropies as similarly normalized CDM models. The only differences occur for anisotropy measurements $\ll 1^\circ$. Since the
degree scale anisotropies are all in the right ballpark for Ω = 1, n ≈ 1 models normalized to COBE, we take this as an encouraging sign for the models we consider in this paper.

2. The IRAS Large Scale Survey of Galaxies

The IRAS survey of galaxies done by the QDOT (Queen Mary-Durham-Oxford-Toronto) collaboration [25], extends as deep as several hundred Mpc, approaching the smallest scales observable by the COBE satellite. Combining the COBE data with the IRAS survey of galaxies thus covers the whole range of scales where the fluctuations can be described by the linear theory. The QDOT IRAS survey has measured redshifts for 1-in-6 IRAS galactic sources (1824 galaxies) with IRAS fluxes > 0.6 Jy. The IRAS selected galaxies seem to be more uniformly distributed than optically selected galaxies and may therefore give a fairer representation of the universe. By concentrating on measuring only 1 in 6 galaxies, the QDOT collaboration obtained a deep sparse sample out to a depth larger than that of the Berkeley IRAS survey [26] which measured redshifts for all IRAS sources above a flux of 1.2 Jy. Using the redshift of the source as a distance indicator, combined with angular position data, one has a three dimensional picture of the galaxy distribution. This distribution can be analyzed to directly extract the power spectrum of density fluctuations [26,27]. The results of the Feldman, et al. [27] analysis is shown as the power spectrum data in Figure 2.

We will use this QDOT IRAS data to test models with theoretical power spectra given by

\[ P_{th}(k) = Ak^n[T(k)]^2. \]  (2.4)

For \( n \approx 1 \) and the transfer functions \( T(k) \) in figure 1, we can see that the power spectra go like \( P(k) \sim k \) on very large scales and like \( P(k) \sim k^{-3} - k^{-4} \) on very small scales, with a peak somewhere around \( k \sim 0.02 - 0.10 \). In order to get the proper normalization for the amount of structure on scales up to the power spectrum peak scale, one must measure the power on scales larger than the power spectrum peak scale. In the mass fluctuation
integral (equation 2.3) there are significant contributions to the mass fluctuations in 20 Mpc spheres coming from 100 Mpc scales ($k \sim 0.03 h/\text{Mpc}$). Thus we feel that to test power spectra of the type considered here, we require a survey out to a depth at least as large as the QDOT survey. A comparison of the QDOT and 1.2 Jy Berkeley power spectra, (see Ref \[27\]), suggests that the Berkeley survey may still be too small to be seeing all of the large scale power.

On large scales, the power spectrum is a better measure of clustering power (i.e., has smaller errors) than the galaxy correlation function \[28\]. However, for scales $< 100 h^{-1} \text{Mpc}$, the power spectrum is not as good an indicator. Since the largest differences between the models under consideration here occurs on smaller scales, we would like to supplement our analysis with a different estimator of power on small scales. To this end we use the “counts - in - cells” statistic from Efstathiou, et al., Ref. \[25\]. Here the redshift space of IRAS galaxies is cut up into near cubical cells of side length $\ell$ and the number of galaxies in each cell is counted. The cell to cell variance of galactic number is a direct indication of the underlying density fluctuations of length $\sim \ell$ in which the galaxies reside. The statistic is usually denoted by the symbol $\sigma^2(\ell)$. The values given in Ref. \[25\] are $\sigma^2(\ell) = 0.42 \pm 0.07$, $0.26 \pm 0.05$, $0.21 \pm 0.05$, and $0.047 \pm 0.024$, for $\ell = 20, 30, 40$, and $60 h^{-1} \text{Mpc}$, respectively. They also give a value for $\ell = 10 h^{-1} \text{Mpc}$, but this represents a fluctuation which is strongly contaminated by non-linear effects, and hence not appropriate for our analysis. The QDOT counts in cells data is presented in figure 3.

3. POTENT bulk flow velocities

We also use the bulk flow velocities from the POTENT analysis \[29\]. They represent the the rms velocities of spherical regions of radius $R$. The velocity has first been filtered with a Gaussian of width $R = 12 h^{-1} \text{Mpc}$ to reduce noise. This data is shown in figure 4. The POTENT analysis has been done using the IRAS galaxies of the Berkeley survey, and so suffers from the problem that enough of the universe has not been sampled to get a
fair estimate of the velocities. This is exacerbated by the fact that velocities are even more sensitive to the very large scale power than the mass fluctuation. However, we would still like to find a way to use these velocities because they do not depend on the bias. (Velocities are generated by the gravity of density fluctuations.) Since we have velocities from one local patch we will include the cosmic variance of the predicted velocity for any given patch of the universe in our analysis. Our treatment will be described in the next section.

B. The $\chi^2$ Test

The reduced $\chi^2$ statistics is calculated by comparing the set of $N$ predictions $y_{th}^i$ and observations $y_{obs}^i$ according to the following formula

$$\chi^2 = \frac{1}{N_{d.o.f.}} \sum_{i=1}^{N} \left( \frac{y_{th}^i - y_{obs}^i}{\sigma_{obs}^i} \right)^2,$$

where $\sigma_{obs}^i$ are the standard observational errors. $N_{d.o.f.}$ equals the number of observations $N$ minus the number of fitted theoretical parameters in the model. In our analysis we use the 36 values of the IRAS $P(k)$, the 4 IRAS counts-in-cells, the 5 POTENT velocity values, the $b_I$ determination, and the COBE amplitude, which we will count as two points. In the $\chi^2$ analysis of $n = 1$ models in ref. [18], the COBE data is counted as two points, one for the sky noise at $10^5$ and another for the value of $Q_{rms-PS}$. Because of the complications of the non-gaussian beam pattern we have avoided this procedure, but we still choose to weight the hexadecupole amplitude as if it were two points in the analysis. This forces the amplitude of density perturbations to be slightly closer to the COBE normalized amplitude than by weighting it as only one point. The results of the analysis is quite similar if we were to weight COBE as only one data point. Thus we have a total of $N = 48$ data points which contribute to the $\chi^2$ sum.

For each value of $n$ and $\Omega_{HDM}$ we find the values of $b_8$ and $b_I$ which minimize $\chi^2(b_8, b_I)$ for each model. This is effectively a least squares fitting of $b_8$ and $b_I$ for each model, so the number of degrees of freedom is 2 less than the number of points, $N_{d.o.f.} = 46$ in our $\chi^2$
formula. The confidence levels of rejecting the hypothesis that the data fits the model are found by integrating the normalized $\chi^2$ distribution. The probability of getting $\chi^2 > 1.38$ is $< 5\%$, so we can reject such models with 95% confidence. We will now proceed to discuss how we compute the $y_i^{th}$.

C. Model Predictions From Linear Theory

Here we will explain how we compare our theoretical power spectra $P_{th}(k)$ to the data presented in the previous section. First of all, we can calculate the hexadecupole $\delta T_4$ (measured by COBE) with the formula given in Ref. [30] for the coefficient of the fourth spherical harmonic for each of the power spectra we are considering. However, we can find an effective wavenumber $k_{eff}$ for which the amplitude of $P(k_{eff})$ is directly proportional to the amplitude of the hexadecupole. We have found that the value $k_{eff} = 1.05 \times 10^{-3}h$/Mpc accurately characterizes the hexadecupole moment for the limited range of $n$ we are studying ($0.70 < n < 1.00$).

The COBE experiment cannot distinguish between temperature fluctuations generated by the Sachs-Wolfe effect of scalar (density) fluctuations and tensor (gravity wave) fluctuations, both of which are products of inflation. This signal confusion is worsened by the fact that the ratio of the moments generated (at least on COBE scales) for scalar and tensor contributions are nearly independent of scale [31–33]. The overall amplitudes of the moments for scalar and tensor modes, however, can be quite different, and one must construct a specific model of inflation and then evaluate the density perturbation and gravity wave amplitudes during inflation. In some models of inflation, such as “new” inflation [4], the contribution of gravity waves to the COBE signal is negligible, and the COBE signal relates only to the scalar density perturbations.

Models of inflation which produce a significant amount of gravity waves cannot be summarized with a universal formula. However, there is a relation derived from the toy model “power law inflation” [34] which relates the gravity wave (tensor) contribution to the COBE
signal \((\Delta T/T)_T\) to the contribution from the density (scalar) fluctuations signal \((\Delta T/T)_S\) via the spectral exponent of the density power spectrum \(n\) \(\text{[22,32,35]}\): \[
\frac{(\Delta T/T)_T^2}{(\Delta T/T)_T^2} \approx 7(1 - n). \tag{2.6}
\]

This relation also is reasonably accurate for chaotic inflation models. Since the COBE signal is a quadrature sum of the multipole moments, decreasing \(n\) decreases the fraction of COBE signal due to density waves, and thus implies a smaller amplitude for \(P_{th}(k)\). Thus the amplitude of the COBE quadrupole moment is \(\sqrt{8 - 7n}\) times larger than would be expected from the density component alone. We will consider models with gravity waves produced according to this formula.

To simulate the IRAS power spectrum \(P_I(k)\) we have to apply a couple of correction factors to our theoretical \(P_{th}(k)\). First of all the distribution of galaxies in redshift space on large scales appears more clustered because of the doppler contribution to the redshift from velocity perturbations \(\text{[36]}\). With a biased galaxy distribution \((b_I)\) this correction can be made by multiplying with the factor \[
P(k) \rightarrow \left[ 1 + \frac{2}{3b_I} + \frac{1}{5b_I^2} \right] P(k) \tag{2.7}
\]

However, on smaller scales there is the opposite effect; the doppler shifts from the peculiar velocities of galaxies become large in comparison to the Hubble velocities and in fact wash out this redshift clustering effect. This effect can be described with a velocity dependent factor \(\text{[37]}\)

\[
P(k) \rightarrow P(k) \sqrt{\frac{\pi}{2}} \frac{\text{erf}(kR_v)}{kR_v} \tag{2.8}
\]

where \(R_v = 4.4 \, h^{-1} \, \text{Mpc}\) for IRAS galaxies.

Applying these corrections to a linear power spectrum seems to give reasonable agreement with the power spectrum of “galaxies” in N-body simulations. For example, Feldman, et al. \(\text{[27]}\) present such a power spectrum (from ref. \(\text{[13]}\)) for a model with 30% HDM. The N-body spectrum becomes slightly higher than our linear spectrum for \(k > 0.15\) presumably due to
non-linear corrections. The worst disagreement ($\lesssim 30\%$), occurs at the small wavelength end $k = 0.2h$ Mpc$^{-1}$ of the QDOT IRAS $P(k)$. This error is still much smaller than the QDOT error bars, so we believe our procedure is relatively insensitive to non-linear effects.

To compare our predictions with the counts in cells data we calculate the mass variance in a spherical volume of radius $R = (3\ell^3/4\pi)^{1/3}$, i.e., the radius which encloses the same volume as $\ell^3$. We have checked that this procedure gives the same results as a cubic cell of length $\ell$ by computing the variance in a cube and a sphere of equal volume using a CDM, $n = 1$ power spectrum. The results were different by only a few percent.

We calculate the counts in cells directly for each model, using the same redshift space correction as appropriate for the IRAS power spectrum. Peacock [28] has suggested a different method for comparing counts-in-cells data to $P(k)$. He noted that one can write

$$2\pi^2\sigma^2(l) = P(k_{\text{eff}})k_{\text{eff}}^3$$

(2.9)

where the value of $k_{\text{eff}}$ depends on the spectrum. This is the method used by Taylor and Rowan-Robinson [18] for their $\chi^2$ analysis. It is instructive to calculate $k_{\text{eff}}$ for a typical model just to illustrate the wavenumbers being probed with the counts-in-cells measurements. For a $n = 1$, $\Omega_\nu = 0.25$ model, the values for $k_{\text{eff}}$ are given in Table 1.

Thus we see that for accurate count-in-cells numbers for these volumes, one needs to accurately get the power spectrum out to $k \lesssim 0.02$.

Note we are not considering the datum from cells of length $\ell = 10h^{-1}$ Mpc, as this point is strongly in the non-linear regime. In appendix B we estimate that even for $\ell = 20h^{-1}$ Mpc, the non-linear effects may be as large as $\sim 30\%$, which is roughly the same size as the disagreement between our linear $P(k)$ and the N-body simulation $P(k)$ at the smallest wavelength we consider, $k = 0.2h$/Mpc. For this reason we do not consider estimates of the power on scales much below $20h^{-1}$ Mpc. (The largest wavenumber from the IRAS QDOT $P(k)$ is $k_{\text{max}} = 0.195 h$ Mpc$^{-1}$.)

To do the testing, we first must calculate the linear theory power spectra for all models with a primordial power spectral index $0.70 < n < 1.00$ and $0 < \Omega_\nu < 0.5$. We have
limited $n$ to be $\leq 1$ because we are considering only grand unified models of “new” and “chaotic” inflation. We calculate the power spectra for these models in steps of $\delta n = 0.02$ and $\delta \Omega_\nu = 0.05$ using our form for $P(k)$:

$$P_{th}(k) = A k^n [T(k)]^2,$$

(2.10)

We will vary the amplitude $A$ by a factor of $2^{\pm 1}$ away from the COBE implied central value in 20 logarithmically spaced steps. However, since the definition of $A$ is author dependent, we will discuss our results in terms of the parameter $b_8$ for each model. Thus we are effectively varying $b_8$ in a range centered on the COBE best fit $b_8$.

The value of $b_I$ is somewhat more constrained than values of $b_8$. For the IRAS galaxies, separate determinations of $b_I$ have been made by comparing their velocities and distributions. These dynamical tests yield the 95% confidence values values $b_I = 1.16 \pm 0.42$ (Ref. [38]) and $b_I = 1.23 \pm 0.46$ (ref. [39]). However, the POTENT analysis [21] of the Berkeley IRAS galaxies finds that the 95% confidence interval for $b_I = 0.5 - 1.3$. We therefore combine these measurements to say that $b_I = 1.1 \pm 0.3$ to cover the 95% confidence overlap region of the $b_I$ determinations. We consider the measurement of $b_I = 1.1 \pm 0.3$ to be a bona fide data point which we add to our $\chi^2$ test. As is usual, we will assume a constant bias factor independent of scale and we will allow $b_I$ to vary in 7 linearly spaced steps between 0.8 and 1.4. We calculate the $\chi^2$ for each of the values of $b_I$ and $b_8$, and find the the set $(b_I, b_8)$ which gives the lowest value of $\chi^2$, and plot this minimum $\chi^2$ in our $\Omega_\nu - n$ contour plots. Thus we allow each model to put its best face forward in our test. Some of these models with $b_I = 1.00$ are shown in figure 5.

We calculate the POTENT rms velocities by the same procedure as described in ref. [10]. In our comparison of the rms POTENT velocities to theoretical rms velocities, we will incorporate the cosmic variance of the velocity field. The idea is that with a Gaussian density field, the velocity field will also be Gaussian. The magnitude of the velocity vector will have a $\chi^2$ distribution with 3 degrees of freedom. The variance of the rms velocity is much larger than the POTENT velocity errors. For example the 68% theoretical confidence range on
the average predicted velocity magnitude \( \langle v \rangle \) corresponds to \( (\langle v \rangle - 0.48\langle v \rangle, \langle v \rangle + 0.32\langle v \rangle) \), while the POTENT errors are less than \( \sim 15 \% \). This is plotted in figure 4 for the \( n = 0.96 \), 25% HDM model. If we normalize an \( n = 1.00 \) power spectrum to COBE then the predicted velocities are within the POTENT 1 \( \sigma \) error bars, regardless of the HDM fraction (see e.g., \( n = 1.00 \), 50% HDM model in figure 4). As we decrease \( n \) below 1.00, we will decrease the predicted velocities well below the POTENT values. Thus the upper limit on the predicted velocity will be the most relevant quantity for comparing theory to observations. We combine this upper limit in quadrature with the POTENT 1 \( \sigma \) error, and use this as a fairer estimate of the error bar in our \( \chi^2 \) analysis.

While adding errors in quadrature is strictly correct only for Gaussian errors, the velocity distribution is not too far from a Gaussian. We plot several theoretical predictions for \( \langle v \rangle \) against the POTENT velocities in figure 4. Even with these huge error bars, we find that the predicted velocities for models with significant gravity wave contributions will still have trouble matching the POTENT derived velocities.

**D. Results of Linear Data Analysis**

The results of our \( \chi^2 \) test are presented in figures 6 and 7 for models without and with gravity waves, respectively. Starting with our absolute best fitting model \( n = 1.00 \), 30% HDM and working outward, we plot 9 concentric curves with confidence levels of .5, 1, 5, 10, 25, 50, 68, and 95 %. (For reference, the best fit model has formally a probability \( 4 \times 10^{-5} \) for getting such a good fit by chance - although such small probabilities are meaningless for statistical analysis.) When one reaches the level of 1%, areas where the theory does not match the data become discernable in the data plots. The first thing to note about the graphs is that the overall level of \( \chi^2 \) for models with \( n \sim 1.00 \) is quite low. This result supports earlier claims that \( n = 1 \) models normalized to COBE have sufficient power to explain “large scale power” apparent in galactic clustering measures. If the measurement errors were smaller our test would be a much better discriminator between models.
We have two scenarios to discuss: models with and without gravity waves. First we point out one general trend which is common to all models, and which does not show up in the $\chi^2$ numbers. As we decrease $n$ we decrease the amount of mass clustering power on small scales. To increase the galactic clustering power to compensate for this, one needs to increase the amount of galactic biasing $b_I$. Since our program finds the best fit $b_I$, we will point out that small $n$ models correspond to highly biased models (large $b_I$). We could better limit the models if we had more precise information about $b_I$.

The $\chi^2$ contours for models with no gravity waves are shown in figure 6. As can be seen we can rule out models with $n \lesssim 0.7$ at 95% confidence. There is a slight dependence of $n$ on the HDM fraction, with the limit being $n \geq 0.67$ for models with no HDM and $n \geq 0.72$ for models with 50% HDM. This is easily understood since models with a lot of “tilt” already have little galaxy scale power, and large HDM fractions exaggerate this behavior. The region of best fits occurs in a roughly rectangular region $0.85 \lesssim n \lesssim 1.00$ and $0.1 \lesssim \Omega_{HDM} \lesssim 0.5$. That these fits are quite good can be seen from the direct comparisons to data shown in figures 3, 4, and 5 for a few models.

The $\chi^2$ contours for models with gravity wave contributions according to equation 2.5 are shown in figure 7. It is readily apparent that the allowed parameter space is much smaller. Here the 95% confidence limit on $n$ rules out models with $n \lesssim 0.85$. Again, there is a slight dependence of $n$ on the HDM fraction, with the limit being $n \geq 0.83$ for models with no HDM, and $n \geq 0.87$ for models with 50% HDM. The region of best fits occurs in a roughly rectangular region $0.94 \lesssim n \lesssim 1.00$, $0.1 \lesssim \Omega_{HDM} \lesssim 0.5$. Including significant amounts of gravity waves forces the normalization of the density power spectrum to be much lower, which depletes the amount of clustering power.

We note that for our 95% confidence limit an amount $\gtrsim 50\%$ of the COBE signal is attributed to the effect of density fluctuations. This tells us that to fit large scale structure one requires that the COBE signal cannot be dominated by inflation generated gravity waves. Our best fit models are those in which at least 80% of the COBE signal is due to density fluctuations.
Our two conclusions from this analysis are that the power spectrum must be close to the Harrison-Zeldovich form \((n = 1.00)\) and that the COBE signal must be mostly due to density fluctuations.

**III. CONSTRAINTS FROM DATA ON NON LINEAR STRUCTURES**

As noted in the previous section, we find the best models are those which have \(\Omega_{HDM} \gtrsim 0.1\). The statistical confidence of this conclusion is not very high, on the order of 10% for a given value of \(n\). This is not surprising, since we have confined ourselves to only the largest scales, \(k \lesssim 0.2h\, \text{Mpc}^{-1}\), where the transfer functions for all the models do not differ too much (see figure 1). The best place to discriminate between these models is on smaller scales, where we have data only on the non-linear part of the power spectrum.

**A. The Data**

The relation between non-linear structures and the amplitude of the linear power spectrum is quite complicated, and it is non-trivial to extract strong conclusions from this data. We will consider two such constraints which we feel can be used relatively safely - constraints on the amplitude of \(\sigma(8h^{-1}\, \text{Mpc})\) or equivalently \(b_8\), and the requirement that quasars form early enough to be compatible with observations. The quasar constraint is a lower bound on the power spectrum amplitude while \(\sigma(8h^{-1}\, \text{Mpc})\) is an upper bound (at a somewhat larger scale).

1. **High redshift quasars**

The discovery of quasars with high redshifts (about 20 with \(z \geq 4\)) was a direct challenge to theories of structure formation. One needs a minimum amplitude for density fluctuation on galactic scales to account for the quasar population. Efstathiou and Rees (ref. [10]) considered the formation of quasars in a highly biased \((b_8 = 2.5)\) \(n = 1\) CDM model.
The basic strategy is that in order for a massive black hole to form and power the quasar emission, one first requires a host dark matter halo to supply the gravitational potential to induce baryonic infall. The number density of structures for a given power spectrum can be computed using the Press-Schecter or BBKS techniques. The number density of structures depends exponentially on a parameter $\nu$ given by

$$\nu = \frac{\sigma_c}{\sigma(R)}$$  \hspace{1cm} (3.1)

where $R$ is the radius of the initial collapsing region appropriate for the objects in question and $\sigma_c$ is the linear theory value of $\sigma(R)$ which corresponds to the gravitational collapse and virialization of the object. Using the spherical collapse approximation (see Appendix A)

$$\sigma_c = 3(12\pi)^{2/3}/20 = 1.69.$$  \hspace{1cm} (3.1)

In this application we use the Gaussian filtered mass fluctuation $\sigma_G(R)$

$$\sigma_G(R)^2 = \frac{1}{2\pi^2} \int dk k^2 P(k) e^{-k^2R^2}$$  \hspace{1cm} (3.2)

where the subscript “G” is to remind us that we are using a Gaussian filtering function. The parameter $\nu$ then has the physical meaning of the ratio of the overdensity in a collapsed object to the ambient rms overdensities. Thus specifying $\nu$ tells us the amplitude of the density fluctuations on the scale $R$. Also, since the calculated number density depends exponentially on $\nu$, it is hoped that errors in the measured number density will not significantly change the value of $\nu$.

Efstathiou and Rees (ref. [40]) estimated that the minimum mass for a quasar halo was $\geq 2 \times 10^{12} M_\odot$. They found that a $b_8 = 2.5, n = 1$ CDM model could account for the number density of quasars at least out to a redshift of $z = 5$. Haehnelt and Rees (ref. [41]) improved on this treatment by showing that the $b = 2.5$ CDM could fit the quasar luminosity function at a variety of redshifts. Since both increasing the HDM fraction and reducing $n$ lessen the amount of power on small scales, it is obvious to ask whether the amplitude of quasar scale fluctuations has decreased below that required for making quasars. Haehnelt (ref. [11]) used the techniques of ref. [41] to limit the HDM fraction in $n = 1$ models and $n$ in CDM.
models, finding that \( n > 0.75 \) in CDM models and the HDM fraction must be \( \Omega_{HDM} \leq 0.3 \). Schaefer and Shafi (ref. [10]) showed that 25% HDM and \( n = 0.94 - 0.97 \) is compatible with the quasar density out to a redshift of \( z \sim 5 \).

The Press-Schecter technique is, however, not without errors. Even if we knew the quasar number density perfectly, and hence could deduce the exact value of \( \nu \), there would still be uncertainties. These uncertainties have been discussed before in a variety of places. Here we follow the discussion of reference [22]. The uncertainties in \( \nu \) can be found by

\[
\Delta \ln \nu = \Delta \ln \sigma_c - \frac{d\sigma(R)}{dR} \frac{dR}{dM} \Delta \ln M
\]  

(3.3)

First of all it is not clear what value of \( \sigma_c \) to use. While most authors use the value 1.69, comparison of Press-Schecter results to N-body simulations imply values of \( \sigma_c \) anywhere from 1.33 [40] to 1.69 [42]. To bracket these values we can assume \( \sigma_c = 1.5 \pm 0.2 \). Second there are errors in the quoted mass of the object. The theoretical value for the quasar halo mass is somewhat uncertain, and the mass of the object derived from the luminosity function comes with an additional error. To evaluate the total uncertainty one needs to know the value of \( (d\sigma(R)/dR)(dR/dM) \). For the quasar halo mass of \( 2 \times 10^{12} M_\odot \), the Gaussian filter radius is \( R = 0.6 h^{-1} \) Mpc. For \( n = 1 \) models \( (d\sigma(R)/dR)(dR/dM) \) ranges from 0.14 for CDM models to 0.033 for 50% HDM. Assuming a factor of 3 error in the observational and theoretical masses, we then have

\[
\Delta \ln \nu = \pm 0.13 \pm 0.04 \pm 0.04
\]  

(3.4)

for 50%HDM \( (n = 1) \) which has the smallest uncertainties of the \( n = 1 \) models. We conclude that there could be an uncertainty of about 20% in the amplitude of the density fluctuation \( \sigma(R) \) derived from the quasar density.

In order to have the proper quasar density at early redshifts, one needs to have \( \sigma(0.6 h^{-1} \) Mpc) \( \geq 1.1 \) today (using \( \sigma_c = 1.5 \)). The effect of the hot dark matter on the growth of density fluctuations at these scales changes the above threshold value by only a few percent [11]. To forbid models which are unlikely to have quasars form early enough, we make the replacement
\[ \chi^2 \rightarrow \chi^2 + 20 \left[ \frac{\sigma(0.6h^{-1}\text{Mpc}) - 1.1}{0.22} \right]^2 \]  

(3.5)

whenever \( \sigma(0.6h^{-1}\text{Mpc}) \leq 1.1 \). We have weighted this contribution to \( \chi^2 \) by 20 to strongly penalize models without sufficient small scale power. Since there are about 40 data points in our \( \chi^2 \) analysis an amplitude \( \sigma(0.6h^{-1}\text{Mpc}) \) which is 40% \((2 \times 20\%)\) smaller than our threshold amplitude will cause \( \chi^2 \) to be so large that the model will be ruled out by the quasar data alone.

2. \( \sigma(8h^{-1} \text{Mpc}) \)

Since it has become traditional to specify the amplitude of linear theory by \( b_8 \), some attention has been paid to determining the value of \( b_8 \) from observations. There are several ways of getting at this quantity which seem to be converging on the range \( b_8 = 1.5 - 2.0 \) for the \( \Omega = 1 \) models. We will consider some of the more recent attempts to constrain this parameter.

The easiest way to estimate the density fluctuations is by finding the variance of the galactic number density. For the IRAS galaxies we have two estimates of the number density fluctuations: one from the QDOT survey \[43\]

\[ b_I \sigma_{\text{non-linear}}(8h^{-1}\text{Mpc}) = 0.69 \pm 0.09 \]  

(3.6)

which is in perfect agreement with the estimate from the Berkeley survey \[44\]

\[ b_I \sigma_{\text{non-linear}}(8h^{-1}\text{Mpc}) = 0.69 \pm 0.04 \]  

(3.7)

The error on the averaged combined measurement is essentially the same as the Berkeley error, \( b_I \sigma_{\text{non-linear}}(8h^{-1}\text{Mpc}) = 0.69 \pm 0.04 \). The 95% confidence upper limit is then \( b_I \sigma_{\text{non-linear}}(8h^{-1}\text{Mpc}) \leq 0.77 \) If we take an extremely small value for \( b_I = 0.5 \) which is the POTENT 95% confidence lower limit, this would imply that \( \sigma_{\text{non-linear}}(8h^{-1}\text{Mpc}) \leq 1.54 \). If we use the spherical collapse model (see Appendix B) to determine what that means in terms of the linear density fluctuation, this implies
To be extra cautious, we will adopt the constraint

$$\sigma_{\text{linear}}(8h^{-1}\text{Mpc}) \leq 0.71, \text{ or } b_8 \geq 1.40.$$  \hfill (3.8)

for a new round of $\chi^2$ testing.

A few remarks are in order here concerning the value of $b_8$. Early investigations found that CDM models with $n = 1$ required values of $b_8 \sim 2 - 3$ \[15\] to get agreement with galactic velocity dispersion data and correlation functions. However, large scale structure data required that CDM have smaller values of $b_8$. It was postulated that dynamic effects, especially “velocity bias” \[16\] might accommodate smaller values of $b_8$. High resolution simulations \[17\] however, find $b_8 \geq 1.4$, despite their confirmation of the existence of the “velocity bias” effect. This conclusion seems to also hold for $n < 1.00$ \[18\]. Ref. \[18\] also suggests that this is true even in models with C+HDM. However, they did not consider the effect of the dynamical effects of HDM in their study, and this seems to allow us to use $b_8 < 2$ with significant amounts of HDM \[12\] - \[14\]. No systematic study has been done to determine what values of $b_8$ are allowed, although $b_8 = 1.5$ seems to work with $n = 1.00, \Omega_{HDM} = 0.30$. Our constraint $b_8 \geq 1.25$ seems to be easily consistent with these case studies.

Another constraint on the mass fluctuation amplitude comes from cluster properties. Since clusters are a few Mpc in size, they are an almost ideal choice for determining $b_8$. However, to determine the number density of cluster mass structures analytically, one must use a procedure such as the “BBKS” method \[19\] or the Press-Schechter \[50\] model. There are some uncertainties associated with this technique however, which we have discussed in the previous section on quasars. In ref. \[6\] it was found that $b_8 = 1.12 - 0.96$ for models with 0% - 50% HDM, based on $R \geq 0$ Abell cluster number abundance. To arrive at this number van Dalen and Schaefer (ref. \[6\]) used the spherical collapse model. The $R = 0$ Abell clusters are the poorest Abell clusters and represent initially weaker density fluctuations. The smaller amplitude density fluctuations tend to be highly asymmetric \[13\] and collapse faster than spherical perturbations (see e.g. ref. \[51\]), so a spherical collapse model probably
gives $b_8$ values which are too small. It was estimated that more accurate values for $b_8$ would be at least 30\% larger, i.e., $b_8 = 1.46 - 1.25$, consistent with the adopted restriction $b_8 \geq 1.25$.

Perhaps a more reliable way to study the mass fluctuations in clusters is to select them by their X-ray temperatures, as this gives a direct indication of the gravitational mass potential. These studies tend to give results consistent with much higher $b_8$, implying $b_8 \sim 2.0 - 2.5$ (\cite{32}, $b_8 = 1.6 - 1.9$ \cite{42} and $b_8 = 2.0$, \cite{53}). Thus we find our restriction $b_8 \geq 1.25$ is, if anything, too conservative.

**B. Results of Non-Linear Analysis**

In Figures 8 and 9 we again plot the $\chi^2$ contours for models with and without significant gravity wave temperature anisotropies. The effect of our non-linear constraints is clear. The restriction $b_8 \geq 1.25$ forces the normalization of small $\Omega_{HDM}$, $n \sim 1$ models to be too low to match the large scale structure data. This is symptomatic of the CDM models which, when normalized to COBE, have too much small scale power. The effect of enforcing the lower limit on the amplitude from quasars is to cut out a triangle of parameter space corresponding to large $\Omega_{HDM}$ and small $n$. This is symptomatic of $\Omega_{HDM} \sim 1$ model problems, there being not enough small scale power to explain the early epoch of quasar formation.

What we are left with is a patch of parameter space which has $n \sim 1$ and $\Omega_{HDM} = 0.3 \pm 0.2$. This is in agreement with earlier studies and shows that models with $\Omega_{HDM} \sim 1/4$ are significantly better fits to the data than with no HDM. The range of $n$ is roughly the same as in the linear analysis, although the non-linear quasar constraint has effectively chopped off the low $n$, high HDM fraction corner of parameter space of the previous best fits. Our allowed region of parameter space overlaps the allowed region found by Liddle and Lyth, (ref. \cite{54}). However, their analysis took the non-linear constraints of refs. \cite{11,42} at face value so their allowed region is somewhat smaller than ours. They noted however, that their allowed region was meant to be suggestive of trends in the data and should not be taken literally. On the other hand we are taking pains to be overconservative with non-linear
constraints in the hope that our limits will be firmer.

Thus we find the following properties which it is desirable for inflation to have. We require a density perturbation spectrum which is quite close to a Harrison-Zeldovich spectrum as the data do not seem to favor much “tilting”. For models with a negligible gravity wave contribution to the COBE signal, the best fits occur for $0.9 \lesssim n \lesssim 1.0$. In models with some gravity wave contribution, we find an even tighter range of best fit $n$ values, namely $0.94 \lesssim n \lesssim 1.00$. Since we used equation 2.5 to determine this, we see that for $n = 0.94$ the gravity wave contribution to COBE is only 20%. Thus the data favor models for which density perturbations are $\gtrsim 80\%$ responsible for the COBE signal. We also find $0.15 \lesssim \Omega_{HDM} \lesssim 0.45$ gives the closest fits to the data, which implies we would nominally like one flavor of neutrino with a mass $m_{\nu} = 3 - 10$ eV. (The best fits imply a narrower range $\Omega_{HDM} = 0.20 - 0.35$. With these attributes in mind we proceed to explore possible models for inflation.

IV. MODELS OF INFLATION

Grand unified theories (GUTS) provide the simplest framework for implementing the inflationary scenario. Although supersymmetric GUTS are currently more popular, for completeness we will also consider the ordinary non-supersymmetric versions. The simplest example of the latter is provided by $SO(10)$ with an intermediate mass scale. The minimal non-supersymmetric $SU(5)$ model is excluded both by the precise determination of $\sin^2 \theta_W$ and by proton decay experiments. This is just as well from our point of view since, as observed a decade ago, [55], non-SUSY $SO(10)$ models with an intermediate (B-L breaking) scale $M_{B-L}(\sim 10^{12} \text{ GeV})$ strongly suggest that the tau neutrino mass is in the eV range. The presence of a $U(1)$ axion symmetry not only resolves the strong CP problem but also provides the cold dark matter component.

Two versions of the inflationary scenario, ‘new’ and ‘chaotic’, are readily realized in GUTS. The spectral index $n$ of density fluctuations in the simplest realistic models typically lies between 0.96 and 0.92, although in some versions of chaotic inflation with SUSY GUTS,
\( n \) could be as low as 0.88. Values of \( n \) much smaller than this are not particularly well motivated, both from the point of view of model building as well as observations of the large scale structure.

A. Inflation with Non-Supersymmetric SO(10)

For definiteness, let us consider the following breaking:

\[
SO(10) \longrightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y
\]

\[ M_X \quad M_{B-L} \]

A recent two loop renormalization group calculation involving the gauge couplings gives \([56]\) \( M_X \sim 10^{15} - 10^{16} \) GeV and \( M_{B-L} \sim 10^{12\pm1} \) GeV, consistent with the measured values of \( \alpha_c(M_Z) \) and \( \sin^2 \theta_W(M_Z) \).

A simple version of the see saw mechanism for neutrino masses \([57]\) suggests the hierarchy:

\[
m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \approx m_u^2 : m_c^2 : \mathcal{O}(10^{-1})m_t^2
\]  

(4.1)

Here the three mass eigenstates \( \nu_1, \nu_2, \nu_3 \) primarily consist of \( \nu_e, \nu_\mu \) and \( \nu_\tau \) respectively, and we have assumed (see later) that the heavy (right handed Majorana) neutrino associated with the third family is a factor 10 (or so) heavier than the other two.

The MSW interpretation of the solar neutrino data suggests \([58]\) that the \( \nu_2 \) mass is \( \sim 10^{-2.7} - 10^{-2.5} \) eV. With \( m_t(m_t) \sim 130 - 150 \) GeV, as suggested by recent analyses of the electroweak data, we expect the \( \nu_3 \) (essentially \( \nu_\tau \), with some admixture of \( \nu_\mu \) and \( \nu_e \)) mass to be \( \sim 5 - 10 \) eV. (Without the numerical factor in equation (4.1), this mass would exceed the cosmological bound.) Note that according to this simple \( SO(10) \) example, unless the \( \nu_\tau - \nu_\mu \) mixing happens to be tiny, the two neutrino oscillation experiments CHORUS and NOMAD should determine whether or not the ‘tau’ neutrino is cosmologically significant.

The \( SO(10) \) model with the above symmetry breaking chain suggests the existence of some dark matter in the form of tau neutrino. However, two essential ingredients are still
missing, implementation of inflation and a candidate for cold dark matter (CDM). [Recall
that inflation with only hot dark matter (HDM) does not seem compatible with the observed
large scale structure, especially galaxy formation.] The simplest way to incorporate CDM
here is to invoke a $U(1)$ axion symmetry \cite{59} broken at a scale around $10^{11} - 10^{12}$ GeV.
Both the axions and neutrinos are then cosmologically significant. It may be useful to
reiterate how this has come about. In non-supersymmetric $SO(10)$, an intermediate scale
is needed to bring about consistency with the measured value of $\sin^2 \theta(M_Z)$ as well as with
the lower limits on the proton lifetime. This forces the gauged $B - L$ symmetry to break
at an intermediate scale which, for the above chain, is about $10^{12\pm1}$ GeV. Coupled with the
see saw, this strongly suggests that the tau neutrino mass ($\sim m^2_H/M_{B-L}$) is in the eV range.
That is, the neutrino is a significant component of the dark matter. Cold dark matter is
then needed to reconcile the inflationary scenario with observations related to large scale
structure.

As far as inflation is concerned, the most straightforward scenario is realized by intro-
ducing a weakly coupled gauge singlet field $\phi$ a la Shafi and Vilenkin \cite{59}. The part of the
potential which drives (new) inflation is given by

$$V(\phi) = \lambda \phi^4 \ln \left( \frac{\phi^2}{M^2} - \frac{1}{2} \right)$$

(4.2)

where $M$ denotes the vacuum expectation value (vev) of $\phi$. The quantity $\lambda$ can be reliably
estimated by considering the contribution of the scalar perturbations to the microwave back-
ground quadrupole anisotropy and identifying it with COBE’s determination of $Q_{rms-PS}$.
[Note that for the potential in (4.2), the spectral index $n \approx 0.94$, and the tensor contribution
to the anisotropy is negligible.]

One has (the subscript $S$ denote the scalar contribution)

$$\left( \frac{\Delta T}{T} \right)_S^2 \simeq \frac{32\pi}{45} \frac{V^3}{V'^2 M_P^2} \left| k^{-H} \right|$$

(4.3)

where $M_P(= 1.2 \times 10^{19}$ GeV) denotes the Planck scale, and the right hand side is to be
evaluated when the scale $k^{-1}$, corresponding to the present horizon size, crossed outside the
horizon during inflation. Equation (4.3) can be rewritten as
\[
|\Delta T/T|_S \simeq 0.067\sqrt{\lambda N_H^3/2}\ln(\phi_H^2/M^2)^{1/2}
\]

(4.4)

where \(N_H(\approx 55)\) denotes the number of e-foldings experienced by this scale, and \(\phi_H\) is the field value when the scale crossed outside the horizon.

The logarithmic factor in (4.4) is of order 1 - 10, assuming that the vacuum energy density that drives inflation is comparable to \(M_X^4\). For \(M_X \sim 10^{15.5} \text{GeV} \), \(M \sim M_P = 1.2 \times 10^{19} \text{GeV}\), and taking \((\Delta T/T)_{\text{COBE}} \approx 6 \times 10^{-6}\), the fundamental quantity \(\lambda\) is estimated to be

\[
\lambda \approx 2.2 \times 10^{-14}
\]

(4.5)

Any inflationary scenario is incomplete without an explanation of the origin of the observed baryon asymmetry in the universe. In the present case the inflaton mass \(m_\phi \approx 10^{12.5} \text{GeV}\), and so the basic idea is to create an initial lepton asymmetry via inflaton decay into one or more species of the heavy (‘right handed’) majorana neutrinos. The appearance of ‘sphaleron’ induced processes at the electroweak scale converts a specified fraction of this asymmetry into the observed baryon asymmetry. Details of how a satisfactory scenario is realized along these lines can be found in ref. [61]. We mention here a few salient features:

(i) The reheat temperature \(T_r \sim 10^{8.5} \text{GeV}\) so that the out of equilibrium condition on the ‘heavy’ (\(\sim 10^{12} \text{GeV}\)) neutrinos is readily satisfied;

(ii) The requirement that for temperature below \(T_r\) the rates for lepton number violating processes (\(\nu \nu \leftrightarrow H^0*H^0*\) and \(\nu H^0 \leftrightarrow \bar{\nu}H^0*\), where \(H^0\) is the electroweak scalar higgs) be smaller than the expansion rate (\(H \sim 20T^2/M_P\), where \(H\) denotes the Hubble constant) of the Universe, leads to the following constraint on the light neutrino masses [32]:

\[
m_\nu \lesssim \frac{4 \text{eV}}{(T_r/10^{10} \text{GeV})^{1/2}} \approx 20 \text{eV}
\]

(4.6)

This is fully consistent with a cold plus hot dark matter scenario where neutrinos in the mass range \(3 - 10 \text{eV}\) are needed.
(iii) In the present approach the colored scalar triplets which mediate proton decay are not needed for baryogenesis and consequently are allowed to have masses \( \sim M_X \).

(iv) Finally, it is possible, following ref. [63], to identify the inflaton with the field that spontaneously breaks the axion symmetry. This would make for a more economical approach.

(v) With an appropriate re-interpretation, the chaotic inflationary scenario can be realized within the framework of this \( SO(10) \) model. The ratio of the scalar to the tensor contribution in this case is

\[
(\Delta T/T)_T^2/(\Delta T/T)_S^2 \approx 0.22. \tag{4.7}
\]

B. Supersymmetric Inflation

The presently measured gauge couplings of the standard model, when extrapolated to higher energies with supersymmetry (SUSY) broken at scales around \( 10^3 \text{ GeV} \) [64], appear to merge at scales around \( 10^{16} \text{ GeV} \). This is a boost for supersymmetric GUTS, with \( SU(5) \) or \( SO(10) \) being the obvious gauge groups. In the presence of unbroken matter parity, either of them can provide a cold dark matter candidate in the form of LSP (lightest supersymmetric particle). However, hot dark matter in the form of massive neutrinos most naturally appear in the \( SO(10) \) model. The supersymmetric \( SO(10) \) scheme has some additional features which make it attractive from the particle physics viewpoint. For instance,

(a) A \( Z_2 \) subgroup of the center of \( SO(10) \) (more precisely Spin (10)) is left unbroken if tensor representations are employed to do the symmetry breaking. This \( Z_2 \) symmetry [65], which is not contained in \( SU(3)_c \times SU(2)_L \times U(1)_Y \), acts precisely as matter parity!

(b) In some versions of SUSY \( SO(10) \), the important parameter \( \tan \beta (\equiv \phi^u/\phi^d) \), the ratio of the two vevs which provide masses to ‘up’ type and ‘down’ type
quarks) is predicted to lie close to \(m_t/m_b\) [66]. One consequence of this is the identification of the ‘bino’ (the supersymmetric partner of the \(U(1)_Y\) gauge boson) as the LSP, with mass \(\sim 200 - 300\) GeV.

(c) Fermion Mass Ansatzes have recently attracted a fair amount of attention and are most simply realized within the framework of \(SO(10)\) [67].

To summarize, particle physics considerations as well as observations of large scale structure which favor a cold plus hot dark matter scenario, together suggest SUSY \(SO(10)\) as an attractive way to proceed. Inflation, either ‘new’ or ‘chaotic’, can be implemented by introducing a suitable singlet superfield. Remarkably enough, singlets are typically employed to achieve the breaking of the GUT symmetry, (without breaking SUSY) and we exploit one of them to induce inflation!

Let \(\Phi\) denote the \(SO(10)\) singlet (inflaton) superfield, \(\chi(\bar{\chi})\) are the higgs superfields in the \(126(\bar{126})\) representations whose vevs provide Majorana masses to the right handed neutrino, and \(16_i (i = 1, 2, 3)\) are the matter superfields. To simplify the discussion, we restrict attention to the sector involving an interplay only between these superfields. This allows us to discuss the salient features of the (chaotic) inflationary scenario including baryogenesis.

Consider the renormalizable superpotential

\[
W = \alpha \Phi(\chi \bar{\chi} - M_X^2) + \frac{\beta}{2} \Phi^2 + \frac{\gamma}{3} \Phi^3
\]

\[
+ \gamma_{ij} 16_i 16_j \bar{\chi}
\]

(4.8)

Note that \(\Phi \rightarrow \Phi\) under the matter parity contained in \(SO(10)\) (similarly \(\chi, \bar{\chi} \rightarrow \chi, \bar{\chi}\), and \(16_i \rightarrow -16_i\)).

The superpotential \(W\) gives rise to a supersymmetric ground state in which (vevs refer to the scalar components of the superfields)

\[
<\phi> = 0, \ <16_i> = 0, \quad (4.9)
\]

\[
<\chi> = <\bar{\chi}>^* \neq 0
\]
It is clear that matter parity is unbroken and we expect the LSP to contribute to the cold dark matter component.

Even though $B - L$ is now broken at $M_X \sim 10^{16}$ GeV, the right handed ‘tau’ neutrino mass must be of order $10^{12} - 10^{13}$ GeV, if the ‘light’ tau is to be the dark matter component. The inflaton must be at least twice as heavy, and one simple way to have $m_\phi \sim 10^{13}$ GeV is to arrange the coefficients $\alpha$ and $M$ in (4.8) to be of order $m_\phi/M_X$ and $m_\phi$ respectively. The decay rate of the inflaton into right handed neutrinos is given by

$$\Gamma \sim \frac{1}{4\pi} \left( \frac{m_\phi}{M_X} \right)^6 m_\phi \text{ GeV}$$

(4.10)

With $m_\phi \sim 10^{13}$ GeV, the reheat temperature $T_r$ is of order $10^6$ GeV. Baryogenesis via leptogenesis now proceeds along the lines given in ref. [61]. Note that because of the relatively low $T_r$, the otherwise vexing gravitino problem is neatly avoided in this approach.

Depending on the details, the spectral index $n$ lies between 0.94 (if the quartic potential dominates) and 0.96 (with a quadratic potential dominant during the chaotic inflationary phase). The ratio $(\Delta T/T)_r^2/(\Delta T/T)_5^2$ is $0.22(0.11)$, respectively.

C. Inflation Without the Singlet

The question we wish to address here is the following: Is it possible to implement inflation with GUTS without the gauge singlet? Surprisingly perhaps [68], an affirmative answer appears possible for a special class of supersymmetric GUTS in which, up to a normalization constant, the GUT scale is determined in terms of $M_S$ and $M_P$, the SUSY breaking scale and the Planck scale respectively. Moreover, the normalization constant is fixed from the quadrupole anisotropy. Such models [69] naturally arise after compactification of the ten dimensional $E_8 \times E_8$ heterotic string theory [70], and models based on $G \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$ or its subgroups provide some elegant examples. The scalar fields needed to spontaneously break $G$ to $SU(3)_c \times SU(2)_L \times U(1)$ can be used to drive inflation!

The key ideas are relatively straightforward and perhaps best illustrated by a simplified example. Consider a rank five gauge symmetry $H \equiv SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$, 

30
where the extra factor $U(1)'$ is to break at some superheavy scale $M$ (below $M_P$). Let $\phi, \bar{\phi}$ denote the pair of higgs scalars whose vevs do this breaking. Note that $<\phi> = <\bar{\phi}>^*$ so that the $D$ term vanishes. Since the only independent dimensionful parameters are $M_S$ and $M_P$ ($M$ is determined in terms of them), the $\Phi \bar{\Phi}$ terms in the superpotential are either absent or carry coefficients of order $M_S$. (Here $\Phi, \bar{\Phi}$ denote the corresponding superfields.) Moreover, in order to ensure $F$ flatness, the cubic couplings $\Phi^3, \bar{\Phi}^3$ are also absent (otherwise $U(1)'$ would break at scales $\sim M_S$).

In the absence of SUSY breaking the superpotential $W$ is taken to be

$$W = h\chi\phi + \frac{\kappa}{M_P} (\Phi \bar{\Phi})^2 + \cdots$$

(4.11)

where $\chi$ denotes some matter superfield with the coupling $h$ of order unity. Assuming a radiative breaking scenario along the lines envisaged in supergravity models with electroweak breaking, the effective potential takes the generic form

$$V(\phi, \bar{\phi}) \sim -M_S^2 |\phi|^2 + \kappa^2 \frac{|\phi|^6}{M_P^2}$$

(4.12)

Minimization of (4.12) leads to the result

$$M \equiv |<\phi>| = |<\bar{\phi}>| \sim \kappa^{-\frac{1}{2}} (M_SM_P)^{\frac{1}{2}}$$

(4.13)

Provided that $\kappa^2$ is sufficiently small, the potential in (4.12) will yield a satisfactory (chaotic) inflationary scenario. It turns out that $\kappa \sim 10^{-7}$ (from COBE), which gives $M \approx 10^{15} \text{ GeV}$. The spectral index $n \approx 0.92$, while the ratio of the tensor to the scalar quadrupole anisotropy is $(\Delta T/T)_T^2/(\Delta T/T)_S^2 \approx 0.4$. Values of $n \approx 0.88$ (but no lower!) can be entertained within this framework. This is just as well since our analysis in the earlier sections seems to favor the range $1.0 > n > 0.9$.

To summarize, grand unification provides an elegant framework for implementing both the 'new' and 'chaotic' inflationary scenarios. Some popular models based on $SO(10)$ or $SU(3)_c \times SU(3)_L \times SU(3)_R$ predict the value of the spectral index $n$ in the range 0.96 to 0.92. Cold dark matter, in axions and/or LSP, as well as hot dark matter in massive neutrinos are readily incorporated in these schemes.
V. CONCLUSIONS

We have performed a $\chi^2$ goodness of fit test of the predicted linear theory power spectra against data on scales ranging from 1 to $10^4$ Mpc, mainly from the COBE satellite and the QDOT IRAS survey of galaxies. We find that the inflation based scenario of large scale structure formation, in which the dark matter consists of cold plus hot components, can provide a good fit to large scale structure data.

Taking the primordial power spectrum to have spectral exponent $n$, we find with 95% confidence, respectively, that $n \geq 0.7$, $(n \geq 0.85)$, in models with (without) significant gravity wave contributions to the COBE anisotropy. The precise bound depends slightly on the HDM fraction. We find in models with a significant gravity wave anisotropy, that the COBE signal must not be dominated by the gravity wave contribution.

If one insists on only one type of dark matter, i.e., CDM, the best fits are for $n = 0.84$ ($n = 0.92$) in models without (with) gravity waves, respectively.

The best fit region for all data, including some constraints from non-linear structure, is an roughly an ellipse (see figures 8 and 9). For models with small amplitude gravity wave anisotropies, the focii of the ellipse are approximately at $(\Omega_{HDM} = 0.20, n = 0.92)$ and $(\Omega_{HDM} = 0.35, n = 0.98)$. For models with a large tensor COBE anisotropy, the ellipse is more eccentric, with the focii at roughly $(\Omega_{HDM} = 0.2, n = 0.96)$ and $(\Omega_{HDM} = 0.35, n = 0.98)$. Thus, the best fits occur for $\Omega_{HDM} \sim 0.20 - 0.35$, with $n$ very close to unity.

Realistic examples of inflation from grand unification theories, including both supersymmetric and ordinary GUTS, which have these properties have been presented. These models are also consistent with other cosmological and particle physics constraints.

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APPENDIX A: TRANSFER FUNCTION CALCULATIONS

As described in ref. [3], we integrate the Fourier space evolution equations in (conformal) time using the gauge-invariant variables for the density $\Delta_{ca}$ and velocity $V_a$ perturbations in each energy density component (CDM, neutrinos, photons, baryons) as given in ref. [71]. Here we integrate the equations in conformal time instead of the scale factor as we had done previously. We begin the integration well before the matter dominated era ($z = 8.3 \times 10^6$) and integrate up until the present time ($z = 0$). We have used only one flavor of massive neutrino, with the other two flavors essentially massless. For the baryon equations we use the equations for the difference between the baryon and photon density and velocity perturbations (the variables $S_{br} \equiv \Delta_{c,\text{baryon}} - (3/4)\Delta_{cr}$ and $V_{br} \equiv V_b - V_r$ given in ref. [71] (section II-5). For the massive neutrinos we use the imperfect fluid treatment of ref. [72]. We numerically integrate the equations using a Hamming predictor-corrector routine as we have found it tracks the oscillations of the relativistic components more accurately than Bulirsch-Stoer or Runge-Kutta routines. After recombination is completed, ($z = 900$) we switch from integrating the baryon-photon difference equations to simply integrating the baryon and photon component ($\Delta_{c,\text{baryon}}, \Delta_{cr}$ and $V_{\text{baryon}}, V_r$) equations separately.

We have checked the results of our code by comparing our baryonic transfer functions against those of ref. [4] who gives values for $\Omega_{\text{baryon}} = 0.1$ and 0.01 and found good agreement, although here we present results only for $\Omega_{\text{baryon}} = 0.05$. We have fit the transfer functions to an inverse 5th order polynomial and the results are given in table 2. We find a 5th order inverse polynomial works a little better for C+HDM models than a fourth order as is more usual for CDM models. The transfer functions given here are not baryonic transfer functions, but rather are fits to the total density perturbation (i.e., $\Delta = \Omega_{\text{CDM}}\Delta_{c,\text{CDM}} + \Omega_\nu\Delta_{c\nu} + \Omega_{\text{baryon}}\Delta_{c,\text{baryon}}$). We define our transfer function as

$$T(k) = \frac{\Delta(k, t_0) \Delta(k = 0, t_i)}{\Delta(k, t_i) \Delta(k = 0, t_0)}$$
where $t_i$ and $t_0$ are the initial and present times, respectively. The transfer functions are accurate to a few percent down to $k = 1 \, h$/Mpc.

$$T(k) = \frac{1}{1 + t_1k^{0.5} + t_2k^1 + t_3k^{1.5} + t_4k^2 + t_5k^{2.5}}.$$

(A2)

In table 2, all coefficients are for $k$ in Mpc (with $h=0.5$).

**APPENDIX B: ESTIMATING THE CONTRIBUTION OF NON-LINEAR EFFECTS.**

In the previous section we alluded to the fact that non-linear effects were becoming important at $8h^{-1}$ Mpc. We would like to estimate the size of the non-linear effects. To get a crude estimate we use the “spherical collapse model” treatment (see, e.g., ref. [73]). This approximates a spherical overdensity in a flat universe locally as a miniature closed collapsing universe, so one can follow the collapse into the non-linear regime. We can define the non-linear overdensity as $\sigma_{\text{non-lin}}$ which is given by the following equation

$$\sigma_{\text{non-lin}} = \frac{\rho}{\rho_b} - 1 = \frac{9(\theta - \sin \theta)^2}{2(1 - \cos \theta)^3},$$

(B1)

where $\rho_b$ is the background density and $\theta$ is the conformal time coordinate which parametrizes a closed spacetime. For the same value of $\theta$ the linear theory predicts $\sigma_{\text{lin}}$

$$\sigma_{\text{lin}} = \frac{3}{20}[6(\theta - \sin \theta)]^{2/3},$$

(B2)

Thus if $\sigma_{\text{non-lin}} = 0.6$, we can estimate $\sigma_{\text{lin}} = 0.4$, a 50% correction. To keep within a range where the perturbations are linear we must restrict ourselves to scales where $\sigma \leq 0.4$, where non-linear corrections are estimated to be $\leq 30\%$.

To estimate what value of $\sigma_m$ the counts in cells at $\ell = 20h^{-1}$ Mpc implies, we must first correct for redshift space effects

$$\sigma^2(\ell) = b_I^2 \left[ 1 + \frac{2}{3b_I} + \frac{1}{3b_I^2} \right] \sigma_m^2(\ell).$$

(B3)

We will assume for now that $b_I = 1.2$, which implies $\sigma_m = 0.4$. Thus we should not consider data on scales $\leq 20h^{-1}$ Mpc.
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FIGURES

FIG. 1. Transfer functions for the cold plus hot dark matter models. The curves represent the relative growth of density fluctuations as a function of scale. The curves represent the present time (z=0) transfer functions and in a universe with $\Omega_{CDM} + \Omega_{HDM} + \Omega_{baryon} = 1$, where we have taken the canonical value of $\Omega_{baryon} = 0.05$ from primordial nucleosynthesis.

FIG. 2. We show the calculated model power spectra using our best fit model parameters for $n = 1.00$ only. The values of $b_8$ and $b_I$ have been fit as described in the text. The values of $b_I$ are close to 1.0, with the values $b_I = 1.1, 1.1, 0.9, 0.9,$ and 1.0 for $\Omega_{HDM} = 0.0, 0.1, 0.2, 0.3,$ and 0.4, respectively.

FIG. 3. The counts in redshift space cells data from the IRAS QDOT survey [25]. Illustrated are the curves for some selected models. Low mass fluctuation amplitudes of some models are compensated by using high values of the IRAS bias $b_I$ as can be seen here.

FIG. 4. The values of the bulk streaming velocities extracted by the POTENT analysis program with $1 \sigma$ error bars [29]. Also shown are predicted velocity curves for selected models. The models with low $n$ and gravity waves can be ruled out by this data even though the cosmic variance for the velocity predictions in a single region are quite large. To illustrate this we have plotted the 68% confidence limits on the prediction of large scale streaming velocities for the 25% HDM model.

FIG. 5. Here we show the IRAS QDOT power spectrum data [27] and the COBE amplitude constraint [23] (converted to a redshift space power spectrum constraint for $b_I = 1.0$). The models shown have negligible gravity wave contributions to the COBE anisotropy, and have best fit normalizations $b_8 = 1.26, 1.41, \text{ and } 1.54$ for the models shown with 0%, 25%, and 45% HDM, respectively. We have shown only half of the QDOT data so the plot is easier to read. The value of $n = .84$ yields the best fitting CDM model without gravity waves. Note that simply tilting a CDM spectrum cannot reproduce the large scale “bump” in the IRAS power spectrum, which is generally why CDM models do not fare as well as C+HDM models.
FIG. 6. Contour plot for $\chi^2$ - No gravity waves $\chi^2$ contours in the $n$-$\Omega_{HDM}$ plane for models with a negligible gravity wave content. Moving outward from the center of the graph are contours corresponding to .5% (dotted), 1% (dash-dotted), 5% (dashed), 10% (heavy dashed) 25% (long dashed), 50% (heavy long dashed), 68% (solid), and 95% (heavy solid) confidence levels.

FIG. 7. Contour plot for $\chi^2$ - With gravity waves $\chi^2$ contours in the $n$-$\Omega_{HDM}$ plane for models with gravity waves. Moving outward from the center of the graph are contours corresponding to .5% (dotted), 1% (dash-dotted), 5% (dashed), 10% (heavy dashed) 25% (long dashed), 50% (heavy long dashed), 68% (solid), 95% (solid), 99% (heavy solid) confidence levels.

FIG. 8. Same as figure 6, but we have added the constraints from non-linear data. We have added the restriction that $b_8 \geq 1.25$ and that we have sufficient power for quasar formation. The procedure for adding these constraints in a $\chi^2$ analysis is described in the text. The significance of the contours is here not well defined because of the way we have added the non-linear constraints.

FIG. 9. Same as figure 7, but we have added the constraints from non-linear data. We have added the restriction that $b_8 \geq 1.25$ and that we have sufficient power for quasar formation. The significance of the contours is here not precise because of the way we have added the non-linear constraints.
TABLES

TABLE I. $k_{\text{eff}}$ probed by counts in cells of volume $\ell^3$

| $\ell$   | $k_{\text{eff}}$ |
|----------|------------------|
| 20 Mpc$/h$ | 0.120 $h$/Mpc  |
| 30 Mpc$/h$ | 0.081 $h$/Mpc  |
| 40 Mpc$/h$ | 0.031 $h$/Mpc  |
| 60 Mpc$/h$ | 0.022 $h$/Mpc  |

TABLE II. Transfer functions with $\Omega_{\text{baryon}} = 0.05$, $h = 0.5$

| $\Omega_\nu$ | $t_1$  | $t_2$  | $t_3$  | $t_4$  | $t_5$  |
|---------------|--------|--------|--------|--------|--------|
| 0.00          | -1.150 | 29.60  | 48.49  | -43.17 | 132.4  |
| 0.05          | -0.8654| 17.65  | 165.1  | -277.1 | 343.4  |
| 0.10          | -0.2942| 1.393  | 274.6  | -472.6 | 538.2  |
| 0.15          | 0.1157 | -8.820 | 330.0  | -541.4 | 660.8  |
| 0.20          | 0.3176 | -12.69 | 334.9  | -495.8 | 726.1  |
| 0.25          | 0.3128 | -10.60 | 296.3  | -361.6 | 756.5  |
| 0.30          | 0.1363 | -3.540 | 219.1  | -142.9 | 771.2  |
| 0.35          | -0.1454| 6.144  | 127.6  | 78.64  | 840.3  |
| 0.40          | -0.4276| 15.15  | 53.35  | 193.1  | 1084.  |
| 0.45          | -0.6522| 21.44  | 15.98  | 131.2  | 1582.  |
| 0.50          | -0.7882| 24.04  | 25.49  | -147.3 | 2395.  |