Discussion of 16QAM with 3bit Information

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Abstract. Aiming at the improvement of the modulation scheme of the underlying link of the optical transport network, under the condition of 3 bits of information entropy, the possible improvement of the 16QAM modulation format is explored from the aspects of the number, position and probability distribution of constellation points. The effectiveness of the improved 16QAM modulation is illustrated by comparison with the original 16QAM and 8QAM modulation.

1. Questions Raised

The planning and construction of optical transport networks is a subject that operators, equipment vendors and government departments must consider. The basic rule of optical transmission is that under the same technical conditions, the transmission capacity will decrease as the transmission distance increases. In order to increase the transmission capacity and transmission distance, it is necessary to improve the transmission mode of the underlying link. For a given modulation scheme, the bit error rate (BER) and the signal-to-noise ratio (SNR) are one-to-one correspondence. The SNR corresponding to the error rate of threshold before error correction coding (pre-correction BER) is recorded as “SNR margin point”. Take the pre-correction BER to 0.02. When the pre-correction BER does not change, reducing the SNR margin can improve the anti-noise performance of the system. For the 16QAM and 8QAM modulation formats, try to arbitrarily change the number, position and probability of the points in the square constellation in the 16QAM modulation scheme, and explore a modulation scheme of lower SNR margin point than 8QAM (the distance between adjacent constellation points is equal). For comparison purposes, the information entropy of the modulation format needs to be kept at 3 bits. Information entropy is defined as:

$$\Omega = - \sum_{m=1}^{M} p_m \log_2 (p_m)$$  \hspace{1cm} (1)

$p_m$ -Probability of occurrence of each symbol state, $M$-Total number of states.

2. Theoretical Analysis

In the M-QAM system, the amplitude and phase of the signal are modulated simultaneously as two independent parameters [1]. Its signal waveform has the following form:

$$s_n(t) = A_n \cos(\omega t + \theta_n) \quad m = 1,2,L ,M$$

$$= A_{m} \cos(\omega t) + A_{ms} \sin(\omega t)$$  \hspace{1cm} (2)
$A_{kc}$ and $A_{ms}$ can take multiple discrete values to map the coordinates of constellation points in the constellation map.

If $s_{ms}$ is used to represent the ideal constellation point and the received symbol is $r_{ms}$, then the noise can be expressed as:

$$n_{ms} = r_{ms} - s_{ms}. \quad (3)$$

The noise typically obeys a normal distribution with the mean of 0, which is called the AWGN channel. The variance of the noise is equal to the average power of the noise, defined as:

$$P_n = \frac{1}{N} \sum_{k=1}^{N} |p_k|^2. \quad (4)$$

$N$ is the total number of symbols transmitted. The average signal power is defined as the mean of the square of the absolute value of the transmitted symbol:

$$P_s = \frac{1}{N} \sum_{k=1}^{N} |s_k|^2. \quad (5)$$

Define the ratio of signal to noise power as the signal to noise ratio. Using engineering representations:

$$SNR_{dB} = 10 \log \left( \frac{P_s}{P_n} \right). \quad (6)$$

For square QAM, since there is no closed-form solution for the theoretical error probability corresponding to some constellation points, the worst-case conditional error probability is used as the upper bound, and the close upper bound of the bit error rate [2] can be obtained as:

$$P_{[err]} \leq 4Q \left( \frac{3kE_b}{\sqrt{(M-1)N_0}} \right). \quad (7)$$

$Q(x)$ is a normal distribution function, $k = \log_2 M$, $E_b/N_0$ is the average SNR per bit.

In order to obtain a better modulation method against noise, it is necessary to consider the minimum Euclidean distance $d_{\text{min}}$ between constellation points, because $d_{\text{min}}$ directly determines the size of the noise margin [3]. By unifying different modulation patterns $d_{\text{min}}$ into equal values and correspondingly obtaining a constant noise power, it is convenient to compare. According to the known conditions, for the improvement of the 16QAM modulation method, two directions for reducing the noise margin can be considered:

1. Adjust the position of the 8QAM constellation point of the equal probability distribution.
2. Adjust the position and probability distribution of 16QAM constellation points.

According to the exploration direction given above, when the position, the probability distribution of the constellation point and the transmission channel are determined, the Monte Carlo method can be used to simulate the relationship between BER and SNR. The simulation flow chart is shown in Figure 1.
3. 8QAM Constellation Map Improvement Exploration

In the QAM modulation with $M=8$, it is required to obtain a smaller SNR margin point than in the case of Figure 2 (a), because the information entropy of the equal probability 8QAM is 3 bits, so only the distribution of the constellation point position is discussed.

![Figure 1. Monte Carlo simulation flow chart.](image1)

![Figure 2. Constellation distribution of 8QAM.](image2)
First consider the transposition of its star constellation distribution, which is b in Figure 2, to explore whether the rotation of the constellation can achieve the requirement of reducing the SNR margin. Secondly, change the shape of the constellation. Common shapes include rectangles, circles, triangles, etc. Since the circular distribution average power is much larger when the minimum Euclidean distance is equal, the symbol power is larger, so it is not considered. Therefore, the constellation diagram changes shape to a rectangular constellation. Figure 2 (c) and (d) show the constellation of rectangular 8QAM and rectangular transposed 8QAM. Finally, Monte Carlo simulation is carried out on the scheme of the equal probability 8QAM constellation, and the relationship between BER and SNR is shown in Fig. 3. The abscissa represents SNR and the ordinate represents BER, the same below.

It can be seen from Figure 3 that the SNR-BER curves of (a) and (b), (c) and (d) are very similar, indicating that the anti-noise performance of the two is almost the same, and the method of rotating the constellation does not improve the anti-noise performance. In addition, the curve of the rectangular 8QAM is above the star 8QAM, indicating that the method of using the rectangular constellation does not improve the anti-noise performance, and it will make the anti-noise performance worse.

4. 16QAM Constellation Map Improvement Exploration
In the QAM modulation of M=8, it is considered in two steps, first fix the constellation point probability, select the possible constellation map distribution, and then improve the anti-noise performance by modifying the probability of the constellation point.

4.1. Constellation point position distribution
For the constellation point position distribution, first assume that the probability of each constellation point is the same and fixed. Then, under the premise of ensuring the smallest Euclidean distance between two adjacent points, search for the point closest to the origin from circle to circle, on account of the closer to the origin, the smaller the symbol power.
In this way, the overall symbol power can be reduced, thereby reducing the noise power and the bit error rate. Thus, it obtains four kinds of constellation point distributions of 12QAM and 16QAM. Because 12QAM has redundancy in encoding and wastes resources, the distribution of two constellation points of 16QAM is given, as shown in Figure 4. In Figure 4, the star constellation of the case (a) has a smaller average power, while the square constellation of the (b) case is easier to implement in practice.

4.2. Probability improvement

For the distribution of constellation point probabilities, the following constraints are met according to the conditional constraints:

\[
\sum_{m=1}^{16} p_m \log_2 (p_m) = 3 \quad \text{s.t.} \quad 0 < p_m < 1 \quad \sum_{m=1}^{16} p_m = 1
\]  

And according to the nature of the constellation map, increasing the probability of inner circle point can reduce the noise power, and increasing the probability of outer circle point can increase the judgment accuracy. In order to minimize the overall error rate, the following algorithm for solving the probability is proposed:

Step1: Generate 16 random numbers with the probability sum of 1
Step2: Take out a sequence of Information entropy is 3-bit
Step3: Use the penalty function to assign the probability of inner and outer circle points, and generate raw data based on probability and coding
Step4: Using Monte Carlo method for solving BER
Step5: Taking the smallest BER corresponding sequence is the optimal solution
According to the above algorithm, the SNR-BER relationship curve with improved probability can be obtained as shown in Figure 5. As can be seen from the figure: under the same bit error rate BER=0.02, the SNR of the star 16QAM and the square 16QAM after the improved probability is smaller than the original, that is, the noise margin is reduced. It is reduced by 0.5dB relative to the simulated value, and is reduced by 1dB from the theoretical value, indicating that the modulation mode after adjusting the probability has better anti-noise performance than the original. But, compared to 8QAM, the improved probability of 16QAM does not reduce the noise margin to a level lower than 8QAM.

5. Conclusion
Based on the realistic requirements of optical transport network planning, this paper explores the anti-noise performance of 16QAM under 3bit information entropy from three aspects: the number, location and probability of constellation star points. Simulation experiments show that improving the position and probability distribution of 16QAM constellation points can reduce the bit error rate and improve the anti-noise performance, but it cannot make the tolerance point lower than that of 8QAM.

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