Fulfilling entanglement’s optimal advantage via converting correlation to coherence

Haowei Shi, Bingzhi Zhang, Quntao Zhuang

Ming Hsieh Department of Electrical and Computer Engineering, University of Southern California, Los Angeles, California 90089, USA
email: qzhuang@usc.edu

Abstract: Entanglement boosts performance limits in sensing and communication even in noisy environments. We propose a conversion module to transform quantum correlation to coherent quadrature displacement, fulfilling the optimal receiver for quantum illumination and entanglement-assisted communication. [1] © 2023 The Author(s)

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Introduction. Quantum physics not only refreshes our understanding of the world but also brings unprecedented power in sensing and communication. In general, light propagation in these optical applications can be modeled as an overall quantum phase-shift thermal-loss channel \( \Phi_{q,b} \): upon an input mode described by the annihilation operators \( \hat{a}_S \), the output mode becomes \( \hat{a}_B = e^{i\theta} \sqrt{1 - \kappa} \hat{a}_B + \frac{1}{\sqrt{1 - \kappa}} \hat{a}_S \), where \( \kappa \) is the transmissivity, \( \theta \) is the phase shift, and \( \hat{a}_B \) models the thermal background. As shown in Fig. 1, in a sensing protocol, \( \hat{a}_B \) describes the probe signal sent out from the transmitter to interact with the physical system, and then unavoidably encounters noise \( \hat{a}_B \), before finally get detected. More specifically, in an ideal target detection scenario [2], when the target is present, it reflects \( \kappa \) portion of the signals embedded in noise, corresponding to the channel \( \Phi_{q,b} \), assumed known phase; while when the target is absent, only noise can be received, and the channel is modeled as \( \Phi_{q,b} \). In a phase-shift-keying (PSK) communication protocol, one encodes a classical message \( \theta \) to the phase of the signal \( e^{i\theta} \hat{a}_S \), and then \( \kappa \) portion of the signal is received in mixture with noise. From sender’s encoding to the receiver, light propagation is modeled as an overall channel \( \Phi_{q,b} \).

Despite entanglement’s benefit surviving noise in the above mentioned target detection (quantum illumination, QI) [2] and classical communication [3] scenarios, it is hard to actually design and build systems to fulfill such benefits optimally, as it requires extracting information that is delicately hidden in quantum correlations [4–6]. We resolve this open problem and fulfill entanglement’s optimal advantage in a surprisingly neat fashion—with a conversion module from intermodal correlation to intramodal coherence. We prove that such a correlation-to-displacement (’C-D’) conversion preserves all information of interest and therefore enables the optimal performance in a long list of EA sensing and communication protocols—QI target detection, phase sensing, classical communication, target ranging and arbitrary thermal-loss channel pattern classification. Moreover, the conversion module enables exact performance analyses and extends quantum advantages to the non-asymptotic region, unexplored due to the limitations of asymptotic tools. It also allows the proof of a folklore of a six-decibel error exponent advantage in an arbitrary thermal-loss channel pattern classification problem. Below, we describe two of the results in detail, while other parts can be found in Ref. [1].

Entanglement-assisted protocol. To benefit from entanglement, we consider \( M \) signal-idler pairs \( \{\hat{a}_{S_1, \hat{a}_{I_1}}\}_{m=1}^M \), where each pair is in a two-mode squeezed-vacuum (TMSV) state with mean photon number \( N_S \) [7], known to be optimal in target detection and communication [8, 9]. While the signals are sent through the channel \( \Phi_{q,b} \), the idlers are stored or pre-shared to the receiver side, leading to \( M \) return-idler pairs \( \{\hat{a}_{R_{m}, \hat{a}_{I_n}}\}_{m=1}^M \), each maintaining a phase-sensitive cross-correlation \( \langle \hat{a}_{R_{m}} \hat{a}_{I_n} \rangle = e^{i\theta} C_p \) with the amplitude \( C_p = \sqrt{\kappa N_S (N_S + 1)} \). When \( N_S \) is small, the amplitude of the correlation \( \propto \sqrt{N_S} \) in a EA protocol, while for a classical reference, the correlation \( \propto N_S \) and is therefore much smaller. In this regard, the crucial part of a measurement design to fulfill entanglement’s benefit is to detect the phase-sensitive cross correlation.

Correlation-to-displacement conversion. Given the return-idler pairs, \( \{\hat{a}_{R_{m}, \hat{a}_{I_n}}\}_{m=1}^M \), as shown in Fig. 1(b), the module first performs individual heterodyne measurement on each \( \hat{a}_{R_{m}} \), producing the complex measurement result \( M_{m} \), which obeys a circularly-symmetric complex Gaussian distribution with variance \( v_m \equiv (N_B + \kappa N_S + 1)/2 \), where \( N_B/(1 - \kappa) \) is the mean photon number of noise \( \hat{a}_B \). Conditioned on the output \( M_{m} \), each \( \hat{a}_{I_n} \) is in a displaced thermal state \( \hat{\rho}_{d_{m,e}} \), with mean \( d_{m,e} = (C_p/2\sqrt{v_m}) e^{i\theta} M_{m} \) and mean thermal photon number \( E = N_S (N_B + 1 - \kappa)/2v_m \leq N_S \). At this stage, various measurement strategies for the idler...
modes are possible for information extraction. For example, as shown in Fig. 1(b), a beam splitter array with proper weights $w_m \propto d_m^2$, can combine all outputs into a single mode in state $\hat{\rho}_{dE}$, with mean $d_t = |d_t| e^{i\theta}$ and thermal noise $E$. Here the amplitude square $|d_t|^2 = \sum_{m=1}^{2^M-1} |d_m|^2$ satisfies the $\chi^2$ distribution of $2M$ degrees of freedom $P_{M}(x) \sim \frac{1}{2} \left( \frac{M}{2} \right)^{1/2} e^{-x/(2M)}$ with mean $2M\xi^2$ and variance $4M\xi^2$, where $\xi \equiv \sqrt{E}/4\sqrt{\bar{N}}$.

This module converts phase-sensitive cross-correlation between each signal-idler pair to the complex displacement amplitude of a coherent state, thereby the quantum problem of receiver design is mapped to a semi-classical one.

Quantum illumination. QI for target detection considers the discrimination between two channels $\Phi_{0,0}$ and $\Phi_{\kappa,0}$. In this case, the conversion module produces two displaced thermal state $\hat{\rho}_{0,N_0}$ (target absent) and $\hat{\rho}_{\eta\kappa,TE}$ (target present), where $x \sim P_{M}(\cdot)$ obeys the $\chi^2$ distribution. We find that the Helstrom limit of C-D conversion module, which can be achieved by concatenating the optimum measurement, reaches the Nair-Gu (NG) lower bound in the error exponent $[1, 8]$. And the optimality holds as long as $N_S \ll 1$ and $\kappa \ll 1 + N_B$. At the $N_S \ll 1$ limit, the states output from the conversion module are coherent states with low noise, and therefore the Dolinar receiver [10] completes the optimum measurement design. In Fig. 2(a), we evaluate the performance of the Dolinar receiver combined with a C-D conversion module (green), which indeed achieves the optimal error probability (red). Some discrepancy can be found when $M$ is too large, due to the small noise $E \approx N_S$ being significant at low error probability. As expected, the legacy EA receiver designs, OPAR and PCR [9, 11] are outperformed.

Entanglement assisted communication. Consider PSK with repetitions, where $M$ signal modes are modulated by the same phase $\theta$ uniformly randomly chosen from $[0,2\pi)$. The output of the conversion module, conditioned on $\theta$, is in state $\int d\theta P_{M}(\theta)\hat{\rho}_{\eta\kappa,\theta,TE}$. To compare with the ultimate performance, we consider the EA classical capacity $C_E$ [3]. In Ref. [1], we show via numerical evaluation for small $N_S$ that the Holevo information $\chi_{C-D}$ of C-D module output approaches $C_E$, therefore verifying the optimality of the conversion module to fulfill the EA advantage in communication. Indeed, at the limit of low brightness, $N_S \to 0$, we derive $\chi_{C-D} \sim \kappa N_S \ln(1/N_S)/(N_B + 1)$, which achieves the scaling of the EA capacity. The optimality holds for the binary PSK modulation. We find $\chi_{C-D}$ achievable by a near-term measurement design: binary PSK combined with the Hadamard code and Green machine [12]. Its performance is shown in Fig. 2(c) in magenta, which achieves the optimal $\ln(1/N_S)$ scaling of $\chi_{C-D}$ and outperforms OPAR and PCR, while only relying on linear optics and photon counting.

Discussions on experimental realizations. In terms of microwave QI, a recent experiment has eventually realized a 20% advantage in the error exponent, utilizing the sub-optimal OPAR [13]. The C-D module is practical in that it does not require the return signals and the stored idlers to interact—one heterodyne detects the returned signals and perform operations on all idler modes conditioned on measurement results, leading to tremendous simplification in experimental realizations [13].

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