Anomalous Magnetic Moment of W-boson at high temperature

A.V. Strelchenko
Department of Physics, Dniepropetrovsk National University
St. Naukova 13, Dniepropetrovsk 49050, Ukraine
strel@ff.dsu.dp.ua

Abstract

By the Schwinger proper-time method, the one-loop contribution to the anomalous magnetic moment of the W-boson is calculated at high temperature. It is shown that the value of AMM is positive and depends linearly upon temperature.

1 Introduction

The investigation of high-temperature radiative corrections to various physical values under the influence of external fields remains one of the important topics in high energy physics and cosmology. In particular, this makes it possible to explain some mechanism of the spontaneous generation of primordial magnetic field in the early Universe. Thus, in the paper [1] it was found that the radiative correction to the W-boson ground state, for $T \gg \sqrt{eH}$, stabilizes the vacuum of gauge bosons. Hence, one comes to self-consistent picture when, at high temperature, the spontaneously arisen magnetic field is stabilized by the radiative mass of charged gauge fields.

In the present paper we calculated a one-loop correction to the anomalous magnetic moment of the W-boson at high temperature. For this purpose the Schwinger proper-time method was applied which is quite appropriate at high-temperature region, where it is sufficient to take into account a static limit [2].

2 Mass Operator

Let us consider the Lagrangian of the electroweak theory (omitting fermions):

$$L_{WS} = L_{Gauge} + L_{FP} + L_{Scalar},$$

where
where it convenient to choose the external potential in the form

\[ \text{We introduced the following standard designations:} \]

\[ \begin{align*}
F & = \text{Higgs scalar,} \\
W^\pm & = \text{W-boson} \quad \text{and} \quad Z & = \text{Z-boson, respectively}, \\
\phi & = \text{masses of the W-boson and Z-boson, respectively,} \\
L & = \text{the exact expression of the scalar lagrangian} \\
\end{align*} \]

solid line corresponds to a neutral Higgs scalar

\[ \text{and the exact expression of the scalar lagrangian} \] \( L_{scalar} \) can be found, for example, in [3].

Here \( W^\pm, Z^\pm \) and \( A^\pm \) are the usual vector gauge boson fields, \( M \) and \( M_Z \) represent the masses of the W-boson and Z-boson, respectively, \( \phi^\pm \) and \( \varphi_Z \) are goldstone bosons, \( \phi \) is Higgs scalar, \( C_W, C_W^\dagger, C_Z, C_A \) are the Faddeev-Popov’s ghosts. The external magnetic field is introduced by dividing the potential \( A^\mu \) into radiative and classical parts, \( A^\mu = A^\mu_R + A^\mu_\mu \).

We introduced the following standard designations: \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \)

\[ \begin{align*}
P & = i \partial_\mu + e A_\mu \quad \text{and} \quad \bar{P} = i \partial_\mu + e \bar{A}_\mu \quad \text{are the covariant derivations,} \\
\theta & = g \sin \theta_W. \end{align*} \]

In the one-loop approximation, the W-boson mass operator is determined by the standard set of diagrams in Fig.1 [4], where double lines represent the Green’s function of charged particles: solid double lines shaded inside are for the W-boson Green’s function, solid double lines painted out inside are for the Green’s function of the Goldstone particle \( \varphi^\pm \), dashed double lines shaded inside are for the Green’s function of the charged ghost components \( C^\pm \). Thin wavy line corresponds to a radiative photon \( A^\mu_R \) and Z-boson, thin solid line corresponds to a neutral Higgs scalar \( \phi \), and thin dashed line corresponds to a neutral ghost component \( C_A \) and \( C_Z \), respectively. The W-boson mass operator in a magnetic field at high temperature can be written as (in the Feynman gauge)

\[ M_{\mu\nu} = \frac{e^2}{\beta} \int \frac{d^3 k}{(2\pi)^3} \left[ M^{\phi}_{\mu\nu}(k, p) + M^W_{\mu\nu}(k, p) + M^Z_{\mu\nu}(k, p) \right], \]

where

\[ \begin{align*}
M^{\phi}_{\mu\nu}(k, p) & = \frac{1}{4 \sin^2 \theta_W} (k^2 + m^2)^{-1} \left[ (2k - p)_\mu G(p - k)(2k - p)_{\nu} - 4M^2 G_{\mu\nu}(p - k) \right] \\
& + \frac{1}{4 \sin^2 \theta_W} (k^2 + M_Z^2)^{-1} [(2k - p)_\mu G(p - k)(2k - p)_{\nu}], \\
M^W_{\mu\nu}(k, p) & = k^{-2} \left\{ \Gamma_{\mu\alpha,\rho} G_{\alpha\beta}(p - k) \Gamma_{\nu\beta,\rho} + (p - k)_\mu \Delta(p - k) k_\nu + \\
& + k_\mu \Delta(p - k)(p - k)_\nu + M^2 \delta_{\mu\nu} G(p - k) + \\
& + \frac{1}{4 \sin^2 \theta_W} k^2 \left[ G_{\mu\nu}(p - k) - 2G_{\mu\nu}(p - k) + \delta_{\mu\nu} (G_{\rho\rho}(p - k) + 1) \right] \right\}, \end{align*} \]
\[
M_{\mu\nu}^Z(k, p) = (k^2 + M_Z^2)^{-1}\left\{ \cot^2 \theta_w \left[ \Gamma_{\mu, \rho} \delta_{G, \beta}(p - k) \Gamma_{\nu, \beta, \rho} + (p - k)_\mu \Delta(p - k) k_\nu + k_\mu \Delta(p - k)(p - k)_\nu \right] + \tan^2 \theta_w M^2 \delta_{\mu\nu} G(p - k) \right\},
\]
\[
\Gamma_{\mu, \rho} = \delta_{\mu\alpha}(2p - k)_\rho + \delta_{\alpha\rho}(2k - p)_\mu + \delta_{\mu\rho}(p + k)_\alpha,
\]
\[
G_{\mu\nu}(P) = -[P^2 + M^2 + 2i e F_{\mu\nu}]^{-1}
\]
is the W-boson Green’s function,
\[
G(P) = \Delta(P) = -[P^2 + M^2]^{-1}
\]
is the Green’s function of the Goldostone particle\(s \varphi^\pm\) and of the charged ghost components, respectively, \(\beta = \frac{1}{2}\). For static (high temperature) limit we put \(k_4 = 0\) [3].

In order to evaluate this expression we used the Schwinger proper time method, modified for the case of the high temperature (see for details [1]). Thus, the mass operator averaged over physical states of a vector particle can be obtained in the form

\[
\langle n, \sigma \mid M_{ij}^G \mid n, \sigma \rangle = \frac{2\alpha}{2\sinh \theta_W \pi \beta} \int_0^1 \frac{du}{\sqrt{u}} \int_0^\infty \frac{dx}{\sqrt{x}} (e H \Delta)^{-1/2} \exp \left[ -x u M_Z^2 e H \right] \times \exp \left\{ -(2n + 1) \left[ (\rho - x(1 - u)) - 2y(1 - u) \right] \right\} \times (\exp \left[ -\frac{x(1 - u) M_Z^2 e H}{u} \right] K(y) + \exp \left[ -\frac{x(1 - u) M_Z^2 e H}{u} \right] M(x, u), \tag{3}
\]

\[
\langle n, \sigma \mid M_{ij}^W \mid n, \sigma \rangle = \frac{\alpha}{2 \sqrt{\pi} \beta} \int_0^1 \frac{du}{\sqrt{u}} \int_0^\infty \frac{dx}{\sqrt{x}} (e H \Delta)^{-1/2} \exp \left[ -x u M_Z^2 e H \right] \times \exp \left\{ -(2n + 1) \left[ (\rho - x(1 - u)) - 2y(1 - u) \right] \right\} M(x, u), \tag{4}
\]

\[
\langle n, \sigma \mid M_{ij}^Z \mid n, \sigma \rangle = \frac{\alpha}{2 \sqrt{\pi} \beta} \int_0^1 \frac{du}{\sqrt{u}} \int_0^\infty \frac{dx}{\sqrt{x}} (e H \Delta)^{-1/2} \exp \left[ -x u M_Z^2 e H - \frac{x(1 - u) M_Z^2 e H}{u} \right] \times \exp \left\{ -(2n + 1) \left[ (\rho - x(1 - u)) - 2y(1 - u) \right] \right\} \times (\cot^2 \theta_W \tilde{M}(x, u) + \tan^2 \theta_W M^2 + 2 \cot^2 \theta_W K(y)), \tag{5}
\]

where \(x = euHs, y = x\sigma\),

\[
\tanh \rho = \frac{(1 - u) \sinh x}{(1 - u) \cosh x + u \sinh^2 x},
\]

\[
\Delta = (1 - u)^2 + 2u(1 - u) \frac{\sinh 2x}{2x} + u^2 \frac{\sinh^2 x}{x^2}.
\]

The explicit expressions for \(M(x, u), \tilde{M}(x, u)\) and \(K(y)\) are given in Appendix.
3 Anomalous Magnetic Moment

Now consider the case of weak magnetic fields which are characterized by the condition \( \frac{eH}{M^2} \ll 1 \). As is known, the one-loop contribution to the anomalous magnetic moment of the W-boson is defined as \[4\]

\[
\Delta \kappa = -\frac{1}{2} \frac{\partial \Re \langle M \rangle}{\partial e H \sigma} \bigg|_{H=0} .
\] (6)

To perform the integrations in the Eqs. (3) and (4) it is necessary to divide the integral region over \( u \) into the two parts: \( 0 \leq u < u_0 \) and \( u_0 < u \leq 1 \), where \( \frac{eH}{M^2} \ll u_0 \ll 1 \). In the region \( u_0 < u \leq 1 \) only small values of \( x \) contribute, and one can expand the integrands on the r.h.s. into the power series of \( x \). In a similar manner, in the region \( 0 \leq u < u_0 \), one can use the expansion into the power series of \( u \). The result should not depend on \( u_0 \). After carrying out the integrations over \( u \) and \( x \) we obtain for scalar sector (diagrams a.- b.)

\[
\Delta \kappa_{\phi} = 0.212 \frac{T}{M} \alpha ,
\] (7)

and for gauge sector (diagrams c.- h.)

\[
\Delta \kappa_{W} = 1.048 \frac{T}{M} \alpha ,
\] (8)

\[
\Delta \kappa_{Z} = 2.121 \frac{T}{M} \alpha .
\] (9)

Eqs. (7), (8) and (9) yield the bosonic contribution to the W-boson AMM in the high-temperature approximation.
4 Conclusion

The anomalous magnetic moment of the W-boson at zero temperature was calculated in [4] and can be written in the following way:

\[ \Delta \kappa_W = \frac{7}{16} \frac{e^2}{\pi^2}, \]  
\[ (10) \]

\[ \Delta \kappa_\phi = \frac{g^2}{16 \pi^2} \int_0^1 \frac{du u^2}{u^2 + \frac{m^2}{M^2} (1 - u)} (u^2 - u + 2 + \frac{m^2}{2M^2} (1 - u)), \]  
\[ (11) \]

\[ \Delta \kappa_Z = \frac{g^2 \cos^4 \theta_W}{16 \pi^2} \int_0^1 \frac{du u^2}{u^2 \cos^2 \theta_W + (1 - u)} (8u - 10u^2 + 12u^3 - \frac{1}{\cos^2 \theta_W} (2 - 4u + 9u^2 - u^3) - \frac{1}{2 \cos^4 \theta_W} (4 - 5u - u^2)). \]  
\[ (12) \]

The result obtained in the present paper represents a high-temperature correction to these quantities. As is directly followed from Eqs. (7), (8) and (9), at high temperature and sufficiently weak magnetic fields, when \( \sqrt{eH} \ll T \ll M \), the value of the anomalous magnetic moment is positive and increases linearly with the growth of temperature.

It is interesting to compare this result with the AMM of the fermion, calculated in [6] (see also review article [7]). Therein it is found that the temperature correction to the gyromagnetic ratio of the electron has the form \( a_T = \alpha T (\frac{T}{m_e})^2 \). As one can see, the temperature contribution to the fermion AMM depends upon \( T^2 \) term, whereas in the W-boson case the AMM is proportional to \( T \). In contrast to the fermion case, where contribution to the magnetic moment can be separated at any values of the field \( H \), the radiative correction to the W-boson AMM can be defined only in weak fields.

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6 Appendix

The explicit expressions obtained upon averaging the mass operator of the W-boson over physical states have the forms

\[ M(x, u) = \tilde{M}(x, u) - 2K(y) + \frac{1}{\sin^2 \theta_W} (\cosh^2 x - 2e^{-2y}) \left\{ up_3^2 + (2n + 1) eH \frac{u^2 \sinh^2 x}{x^2 \Delta} \right\}, \]

\[ + eH \frac{u^2}{4x^2 \Delta} \left[ 2 \sinh 2x + \frac{4x(1-u)}{u} \right] + \frac{ueH}{2x}, \]

\[ \tilde{M}(x, u) = 2e^{2y} \left\{ \sinh^2 x - \frac{u}{x^2 \Delta} \left( \frac{4x(1-u)}{u} \right) \cosh 3x \sinh x + 8 \sinh 2x \sinh^2 x \right\} \]

\[ + eH \left[ 2A(x) - \frac{u^2 A(x)}{2x^2 \Delta} \right] - 2up_3^2 + 2\bar{p}^2 \}

\[ + 2e^{2y} \left\{ up_3^2 + (2n + 1) eH \frac{u^2 \sinh^2 x}{x^2 \Delta} + eH \frac{u^2}{4x^2 \Delta} \left[ 2 \sinh 2x + \frac{4x(1-u)}{u} \right] + \frac{ueH}{2x} \}

\[ + eH \left[ 2A(-y) \frac{u(1-u)}{x} \cosh 2x \Delta + \frac{u^2 (8 \sinh^2 x + 1)}{x^2 \Delta} \left[ \sinh 2x + \frac{4x(1-u)}{u} \right] + \frac{u}{2x} \}

\[ + neH \left[ 4 \sinh^2 x + N(x) + N(-x) + \frac{u^2 \sinh^2 x}{x^2 \Delta} \left[ 2 \cosh 2x + \frac{4x(1-u)}{u} \sinh 2x \right] \}

\[ + (2n + 1 - \sigma) eH \left\{ \frac{A^2(y) + (1 - 4 \sinh^2 y)e^{-2y}D(-y) + 2\bar{p}(1-u)A(y)}{D(y)} \left[ \frac{A(y)}{D(y)} - u \right] \right\} \]

\[ + \frac{u^2 \sinh^2 y}{y^2 \Delta(y)} \left[ \frac{A(y)}{D(y)} + u \left[ 1 + e^{2y} + \frac{u^2 \sinh^2 y}{y^2 \Delta(y)} D(-y) \right] \right] \]

\[ + \left[ \frac{A(1-u)u \sinh^2 y}{y^2 \Delta(y)} \right] \left[ \frac{A(y)}{D(y)} \right] \left[ A(y) + e^{2y} \frac{\sinh \theta_W}{y^2 \Delta(y)} \right] \]

\[ - \left[ (1 + e^{2y}) A(-y) + A(-y) \frac{u^2 \sinh^2 y}{y^2 \Delta(y)} \left[ \frac{1}{2} - \frac{u}{u} \right] \right] + u(2 + u)p_3^2 + \bar{p}^2 \]

\[ + (10 + 8 \sinh^2 y) K(y) + \frac{eH \sigma}{\Delta(-y)} \left[ 8A(-y) - 2A(-y) - A^2(-y) - 2 \bar{A}^2(-y) \right] \]

\[ + eH \left\{ A(x) - \frac{A(x)}{D(-x)} + e^{-2x} \left[ 1 - \frac{2x}{u} (1-u) \right] \left[ \frac{A(x)}{D(x)} + \frac{u^2 \sinh^2 x}{x^2 \Delta} \right] \right\}, \]

where

\[ \Delta(y) = (1-u)^2 + 2u(1-u) \frac{\sinh 2y}{2y} + \frac{u^2 \sinh^2 y}{y^2}, \quad \Delta \equiv \Delta(x), \]

\[ K(y) = \frac{1}{2} (2n + 1 - \sigma) \left[ -u^2 e^{-2y} \frac{D(-y)}{D(y)} - 2u \frac{A(y)}{D(y)} + \frac{u^2 \sinh^2 y}{y^2 \Delta(y)} \right] \left[ \frac{A(y)}{D(y)} \right] + \frac{eH \sigma}{\Delta(-y)}, \]

\[ N(x) = \frac{A(x)}{D(x)} \left[ 2 \cosh 2x - \frac{e^{-2x}2x}{u} (1-u) \right], \]

\[ A(y) = e^{-2y} - 1, \quad D(y) = A(y) - \frac{2y}{u} (1-u). \]
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Figure 1: W-boson mass operator in the one-loop approximation.