Charge dynamics of $t$-$J$ model and anomalous bond-stretching phonons in cuprates

P. Horsch$^*$ and G. Khaliullin$^*$

$^*$Max-Planck-Institut für Festkörperforschung, D-70569 Stuttgart, Germany

Abstract. The density response of a doped Mott-Hubbard insulator is discussed starting from the $t$-$J$ model in a slave boson $1/N$ representation. In leading order $O(1)$ the density fluctuation spectra $N(q, \omega)$ are determined by an undamped collective mode at large momentum transfer, in striking disagreement with results obtained by exact diagonalization, which reveal a very broad dispersive peak, reminiscent of strong spin-charge coupling. The $1/N$ corrections introduce the polaron character of the bosonic holes moving in a uniform RVB background. The resulting $N(q, \omega)$ captures all features observed in diagonalization studies, fulfills the appropriate sum rules, and apart from the broadening of the collective mode shows a new low energy feature at the energy $\chi J + \delta t$ related to the polaron motion in the spinon background. It is further shown that the low energy structure, which is particularly pronounced in $(\pi, 0)$ direction, describes the strong renormalization and anomalous damping of the highest bond-stretching phonons in La$_{2-x}$Sr$_x$CuO$_4$.

Dedicated to Ferdinando Mancini on the occasion of his 60th birthday

published in AIP Conference Proceedings 695: “Highlights in Condensed Matter Physics”, ed. A. Avella et al. (AIP, New York, 2003)

INTRODUCTION

High-temperature superconductors are doped Mott-Hubbard insulators, therefore the density response is expected to be very different from that of weakly correlated Fermi systems. The low-energy density response, i.e., in the frequency range inside the Mott-Hubbard or charge transfer gap, is proportional to the doping, i.e., described by transitions within the lower Hubbard band. For a complete understanding of the physics ruling these systems both the spin and the charge response must be considered in the full momentum $q$ and frequency $\omega$ domain. The spin response has been and still is extensively explored, mainly stimulated by a wealth of experimental data provided by neutron scattering. A similarly powerful experimental tool is missing for the charge response, with the exception of optical conductivity, which however provides information only in the limit $q \to 0$. In the absence of solid experimental information about the detailed structure of $N(q, \omega)$, the best we can do to test a theory is the computer experiment. This provides for small clusters an unbiased answer how $N(q, \omega)$ looks like, e.g., for the $t$-$J$ model[1, 2], and how this quantity depends on the key parameters, namely the magnetic exchange interaction $J$ and the doping $\delta$.

In one dimension the Hubbard physics is basically simple (although not mathematically), as it is characterized by charge and spin separation, which implies that the density response is basically that of non-interacting spinless fermions, i.e., showing vanishing excitation energy at $q = 0$ and $4k_F$. This is even true for the 1D $t$-$J$ model away from
$J = 0$ and $2t$, where the model is not exactly solvable\cite{1}. As we shall see the 2D model relevant for the cuprates shows a very different behavior compared to the 1D case.

Exact diagonalization studies for the $t$-$J$ model\cite{1,2} have revealed that the dynamical density response $N(q, \omega)$ for the 2D model is characterized by strong spin-charge coupling. These calculations show several features unexpected from the point of view of weakly correlated fermion systems which have to be explained by theory: (i) the strong suppression of low energy $2k_F$ scattering in the density response, (ii) a broad incoherent peak at high energy (several $t$) whose shape is rather insensitive to changes of hole concentration and exchange interaction $J$, (iii) the very different form of $N(q, \omega)$ compared to the spin response function $S(q, \omega)$, which share common features in weakly correlated fermionic systems, and finally (iv) there are at low energy ($\sim J$) some features that do depend on doping and $J$. It is worthwhile to note that finite temperature diagonalization studies\cite{3} show only weak temperature dependence for $T < 0.3t$ even at low energy.

While considerable analytical work has been done to explain the spin response of the $t$-$J$ model only few authors analysed $N(q, \omega)$. Wang et al.\cite{4} studied collective excitations in the density channel and found sharp peaks at large momenta corresponding to free bosons. Similar results were obtained by Gehlhoff and Zeyher\cite{5} using the X-operator formalism and by Foussats and Greco\cite{6} using a path integral representation for X-operators. Lee et al.\cite{7} obtained a broad incoherent density fluctuation spectrum by considering a model of bosons coupled to a quasistatic disordered gauge field, while Jackeli and Plakida\cite{8} employed the memory function approach.

![FIGURE 1.](image.png)

FIGURE 1. Comparison of $N(q, \omega)$ obtained by the slave-boson theory\cite{9} (solid lines) with diagonalization data\cite{1} for a periodic $4 \times 4$ cluster with $J/t = 0.4$ and doping $\delta = 0.25$. The dashed line in the $(\pi, \pi)$ spectrum indicates the $\delta$-function collective peak obtained when polaron effects are neglected.

Starting from a slave-boson representation we show that the essential features observed in the numerical studies can be obtained in the framework of the Fermi-liquid phase of the $t$-$J$ model at zero temperature\cite{9}. Our main findings are: (i) at low momenta the main effect of strong correlations is to transfer spectral weight from particle-hole excitations into a pronounced collective mode. Because of the strong damping of
this mode (linear in $q$) due to the coupling to the spinon particle-hole continuum, this collective excitation is qualitatively different from a sound mode. (ii) At large momenta we find a strict similarity of $N(q, \omega)$ with the spectral function of a single hole moving in a uniform RVB spinon background. In this regime $N(q, \omega)$ consists of a broad peak at high energy whose origin is the fast, incoherent motion of bare holes. (iii) The polaronic nature of dressed holes leads to the formation of a second peak at lower energy, which is more pronounced in $(\pi, 0)$ direction in agreement with diagonalization studies\cite{1, 2}.

We show that the anomalous renormalization of certain phonon modes as observed in inelastic neutron scattering provides a sensitive test of the peculiarities of the low-energy density response. In this contribution we shall analyse the strong, doping-dependent renormalization of the highest breathing phonon modes which is a generic feature in the high-$T_c$ compounds.

### SLAVE BOSON THEORY OF DENSITY RESPONSE

We start from the $N$-component generalization of the slave-boson $t$-$J$ Hamiltonian\cite{10, 4} $H_{tJ} = H_t + H_J$, which is obtained by replacing the constrained electron creation operators $c_{i, \sigma}^+ = c_{i, \sigma}^+(1 - n_{i, -\sigma}) \rightarrow f_{i, \sigma}^+ h_i$:

$$H_t = \frac{-2t}{N} \sum_{<i,j> \sigma} (f_{i\sigma}^+ h_j^+ h_i f_{j\sigma} + h.c.),$$

$$H_J = \frac{J}{N} \sum_{<i,j> \sigma \sigma'} f_{i\sigma} f_{i\sigma'} f_{j\sigma'}^+ f_{j\sigma} (1 - h_i^+ h_i)(1 - h_j^+ h_j),$$

where $f_{i, \sigma}^+$ is a fermionic (spinon) operator, $\sigma = 1, \cdots, N$ is the fermionic flavor index, and $h_i$ denotes the bosonic holes. These operators obey standard commutation rules, yet the number of these auxiliary particles must obey the constraint $\sum_{\sigma} f_{i\sigma} f_{i\sigma} + h_i^+ h_i = N/2$. The original $t$-$J$ model is recovered for $N = 2$.

The slave boson parametrization provides a straightforward description of the strong suppression of density fluctuations of constrained electrons through the representation of the density response in terms of a dilute gas of bosons. A common treatment of model (1) is the density-phase representation (“radial” gauge\cite{11}) of the bosonic operator $h_i = r_i \exp(i \theta_i)$ with the subsequent $1/N$-expansion around the Fermi-liquid saddle point. While this gauge is particularly useful to study the low energy and momentum properties, it is not very convenient for the study of the density response in the full $\omega$ and $q$ space. Formally the latter follows in the radial gauge from the fluctuations of $r_i^2$. However, if one considers for example convolution type bubble diagrams, one realizes that their contribution to the static structure factor is correctly of order $1/N$, but is not proportional to the density of holes $\delta$ as it should be. According to Arrigoni et al\cite{12} such unphysical results originate from a large negative pole contribution in the $\langle r_{-q} r_q \rangle_\omega$ Green’s function of the real field $r$, which is hard to control by a perturbative treatment of phase fluctuations.

We follow therefore Popov\cite{13} using the density-phase treatment only for small momenta $q < q_0$, while keeping the original particle-hole representation of the den-
sity operator, \( h^+ h \), at large momenta. More precisely \( h_i = r_i \exp(i\theta_i) + b_i \), where \( b_i = \sum_{|q| > q_0} h_q \exp(i q R_i) \). The cutoff \( q_0 \) is introduced dividing “slow” (collective) variables represented by \( r \) and \( \theta \) from “fast” (single-particle) degrees of freedom \( b_i \) and \( b_i^\dagger \). As discussed by Popov[13] this “mixed” gauge is particularly useful for finite temperature studies to control infrared divergences. We start formally with the “mixed” gauge and keep only terms of order \( \delta \) and \( 1/N \) in the bosonic self energies. In this approximation our zero temperature calculations become quite straightforward: The cutoff \( q_0 < \delta \) actually does not enter in the results and we arrive finally at the Bogoliubov theory for a dilute gas of bosons moving in a fluctuating spinon background.

The Lagrangian corresponding to the model (1) is then given by (the summation over \( \sigma \) is implied)

\[
L = \sum_i \left( f_i^\dagger \left( \frac{\partial}{\partial \tau} - \mu_f \right) f_i \sigma + b_i^\dagger \left( \frac{\partial}{\partial \tau} - \mu_b \right) b_i \right) + H_f + H_J
\]

\[
+ \frac{i}{\sqrt{N}} \sum_i \lambda_i \left( f_i^\dagger f_i + (r_i + b_i^\dagger)(r_i + b_i) - \frac{N}{2} \right),
\]

\( H_f = -\frac{2t}{N} \sum_{<ij>} f_i^\dagger f_j \sigma (b_j^\dagger b_i + r_i r_j + r_j b_i + b_j^\dagger r_i) + h.c. \) (3)

Here the \( \lambda \) field is introduced to enforce the constraint, and \( \mu_f, \mu_b \) are fixed by the particle number equations \( \langle n_f \rangle = \frac{N}{2}(1 - \delta) \) and \( \langle r_i^2 + b_i^\dagger b_i \rangle = \frac{N}{2} \delta \), respectively. The uniform mean field solution \( r_i = r_0 \sqrt{N/2} \) leads in the large \( N \) limit to the renormalized narrow fermionic spectrum \( \tilde{\varepsilon}_k = -i \gamma_k - \mu_f \), with \( \tilde{\gamma} = J \chi + t \delta \), \( \gamma_k = \frac{1}{2} \left( \cos k_x + \cos k_y \right) \), \( \chi = \sum_{\sigma} \langle f_i^\dagger f_j \sigma \rangle / N \), and \( z = 4 \) the number of nearest neighbors. In the \( N = \infty \) limit \( \chi_\infty \approx 2/\pi^2 \) is given by that of free fermions, while for the original \( t-J \) model its value should be larger[14] due to Gutzwiller projection. In the following \( \chi = \frac{3}{2} \chi_\infty \) will be used. Distinct from the finite-temperature gauge-field theory of Nagaosa and Lee[15] the bond-order phase fluctuations acquire a characteristic energy scale in this approach[4], and the fermionic (“spinon”) excitations can be identified with Fermi-liquid quasiparticles. The mean field spectrum of bosons is \( \omega_0 = 2 z \chi t (1 - \gamma_q) \). Thus the effective mass of holes \( m^0_h \approx 1/t \) is much smaller than that of the spinons.

Due to the diluteness of the bosonic subsystem, \( \delta \ll 1 \), the density correlation function \( \chi_{q,\omega} = \langle \delta n^h \delta n^h \rangle_{q,\omega} \) is mainly given by the condensate induced part which is represented by the Green’s function \( \langle (b_q^+ + b_{-q})(b_q + b_{-q}^+) \rangle_{\omega} \) for \( q > q_0 \), and \( 2 \langle r_{-q} r_q \rangle_{\omega} \) for \( q < q_0 \), respectively:

\[
\chi_{q,\omega} \approx \frac{N}{2} \bar{\gamma}_0 \left( \langle (b_q^+ + b_{-q})(b_q + b_{-q}^+) \rangle_{q > q_0} + 2 \langle r_{-q} r_q \rangle_{q < q_0} \right).
\] (5)

The \( 1/N \) self-energy corrections to these functions are calculated in a conventional way[11,10] expanding \( r_i = (r_0 \sqrt{N} + (\delta r)_i)/\sqrt{2} \) and considering Gaussian fluctuations around the mean field solution. Neglecting all terms of order \( \delta / N \) and \( q_0^2 / N \), only one relevant \( 1/N \) contribution remains which corresponds to the dressing of the slave-boson Green’s function by spinon particle-hole excitations. Within this approximation and at
zero temperature no divergences occur at low momenta, thus one can take the limit \( q_0 \to 0 \). The final result for the dynamic structure factor (normalized by the hole density) is:

\[
N_{q,\omega} = \frac{2}{\pi} \text{Im} \left( \left( \omega q^a + S^{(1/N)}_{q,\omega} - \mu_b \right) / D_{q,\omega} \right),
\]

(6)

\[
D_{q,\omega} = (\omega q^a + S^{(1/N)}_{q,\omega} - \mu_b)(\omega q + S^{(1/N)}_{q,\omega} - S^{(1/N)}_{q,\omega} - \mu_b) - (\omega q - A^{(1/N)}_{q,\omega})^2.
\]

(7)

The origin of the contribution

\[
S^{(1)}_{q,\omega} = ztr_0^2 \left( \frac{(1 + \Pi_2)^2}{\Pi_1} - \Pi_3 \right)_{q,\omega},
\]

(8)

\[
\Pi_m = zr \sum_k \frac{n(\xi_k) - n(\xi_{k+q})}{\xi_k - \xi_{k+q} - \omega - i0^+} (\gamma_k + \gamma_{k+q})^{m-1},
\]

(9)

is the indirect interaction of bosons via the spinon band due to the hopping term (which gives \( \Pi_3 \) in (4)) and due to the coupling to spinons via the constraint field \( \lambda \). The latter channel provides a repulsion between bosons, making \( S^{(1)}(\omega = 0) \) positive and therefore ensuring the stability of the uniform mean-field solution.

The \( 1/N \) self energies \( S^{(1/N)} \) and \( A^{(1/N)} \) are essentially a single boson property. They are given by the symmetric and antisymmetric combinations (with respect to \( \omega + i0^+ \to -\omega - i0^+ \)) of the self energy

\[
\Sigma^{(1/N)}_{q,\omega} = \frac{4}{N} \sum_{|k|<k_F<|k'|} \left( zr \gamma_{k'-q} \right)^2 G^0_{q+k-k'} (\omega + \xi_k - \xi_{k'}).
\]

(10)

Here \( G^0_q(\omega) = (\omega - \omega_q - \Sigma^{(1/N)}_{q,\omega} + \mu_b)^{-1} \) is the Green’s function for a single boson moving in a uniform RVB background. Although in the context of \( 1/N \) theory the \( G^0 \) function in (5) should be considered as a free propagator, we shall use here the selfconsistent polaron picture for a single hole[16]. This is crucial when comparing the theory for \( N = 2 \) with diagonalization studies. Finally, the constants \( a \) and \( \mu_b \) in (3) are given by \( (1 - tr_0^2/i) \) and \( S^{(1/N)}(\omega = q = 0) \), respectively. The parameter \( r_0^2 \) in Eq.8 which formally corresponds to the condensate fraction in our theory, is determined selfconsistently from \( r_0^2 = \delta - \sum_{q\neq0} \bar{n}_q \). The momentum distribution \( \bar{n}_q = \langle b_q^+ b_q \rangle \) is calculated from the corresponding bosonic Green’s function for finite hole-density. The interactions implied in the polaron formation lead to a significant reduction of the number of bosons in the condensate[9], and affects the balance of the selfenergies \( S^{(1)} \) and \( S^{(1/N)} \) in Eq. (7).

The structure of \( N(q,\omega) \) (6) in the small \( \omega, q \) limit is mainly controlled by the interaction of bosons represented by the \( S^{(1)} \) term \( (\sim r_0^2) \), while the internal polaron structure of the boson determined by \( S^{(1/N)} \) is less important. \( N(q,\omega) \) consists of a weak spinon particle-hole continuum with cutoff \( \sim v_F q \), and a very pronounced linear collective mode which nearly exhausts the sum rule. The velocity of this mode is always somewhat smaller than the spinon Fermi velocity, \( v_s \leq v_F \simeq z^s \), which implies a strong damping \( \sim \omega \) (or \( q \)) of this mode (Fig.2).
FIGURE 2. Density fluctuation spectra $N(q, \omega)$ for $J/t = 0.4$ and $\delta = 0.15$ along $(\pi, 0)$ (left) and $(\pi, \pi)$ directions (right). Energy in units of $t$.

FIGURE 3. Low energy density response at $(\pi, 0)$ for $J = 0.3$ and $\delta = 0.18$. The peak centered at $\sim (\chi J + \delta t)$ due to the polaron motion appears in the $1/N$ order on top of the spinon particle-hole continuum with edge at energy $z(\chi J + \delta t)$. Small oscillations in the data result from the discretization of the energy scale in the numerical solution and have no physical meaning.

The density response $N(q, \omega)$ (Fig. 2) at large momenta, $q > \delta$, which we can compare with diagonalization results (Fig. 1), is dominated by the properties of the single boson self-energy $S^{(1/N)}$. The calculated density response of the $t$-$J$ model has three characteristic features on different energy scales: (i) The main spectral weight of the excitations at large momenta is located in an energy region of order of several $t$. This high energy peak is very broad and incoherent as a result of the strong coupling of bosons to low-energy spin excitations. The position of this peak and its shape are rather insensitive to the ratio $J/t \leq 1$ in agreement with conclusions of [1, 2]. This is simply due to the fact that the high-energy properties of the $t$-$J$ model are controlled by $t$. (ii) The theory predicts also a second peak at lower energy which is more pronounced in the direction $(\pi, 0)$ (see also Fig. 3), while its weight is strongly suppressed for $q$ near $(\pi, \pi)$. The origin of this excitation is due to the formation of a polaron-like band of dressed bosons. The relative weight of this contribution increases with $J$ as a result of the increasing spinon bandwidth. (iii) In addition there is the spinon particle-hole continuum which is generated by $S^{(1)}$ with relatively small weight ($\propto \delta^2$). At $(\pi, 0)$ the high energy cutoff of the spinon continuum is at $z(\chi J + \delta t)$ (Fig. 3), while the polaron peak is at about $(\chi J + \delta t)$. We
FIGURE 4. Comparison of static structure factor $N(q)$ (solid line) calculated for $J = 0.4$ with the result for a Gutzwiller projected wave function [17] (squares) and spinless fermions (dash-dotted) for $\delta = 0.213$. The comparison with the result obtained for non-interacting fermions with spin (dashed line) shows the large reduction of the density response due to the constraint.

note that the polaron peak in $N(q, \omega)$ at optimal doping is in the same energy range as the high energy longitudinal optical phonons in cuprates, and as we shall discuss below leads to a strong damping and anomalous renormalization of these modes for certain wave numbers.

Figure 4 provides a comparison of the static charge structure factor $N(q)$ with that of a Gutzwiller projected wave function and with that for uncorrelated electrons. The latter comparison shows the strong suppression of the density response due to correlations. Finally we note that our calculated structure $N(q)$ factor satisfies the sum rule for constrained fermions $\sum_q N(q) = \delta(1-\delta)$ within 1% [9].

RENORMALIZATION OF BREATHING PHONONS

Inelastic neutron scattering experiments on high-$T_c$ superconductors have shown that in particular the highest energy longitudinal optic phonon branch near $(\pi,0)$ softens and broadens strongly as holes are doped into the insulating parent compound. Whereas the corresponding breathing mode at $(\pi,\pi)$ shows much smaller softening and no anomalous broadening (Fig.5). This effect appears to be generic for cuprates and detailed neutron scattering studies have been reported for La$_{2-x}$Sr$_x$CuO$_4$ [18, 19, 20, 21], YBa$_2$Cu$_3$O$_{6+\delta}$ [19, 22, 23] and also for Bi-based cuprates [24]. This certainly shows that there are phonons which couple rather strongly to the charge carriers that contribute to superconductivity, and one may ask what is special about the interaction of strongly correlated electrons and phonons.

The renormalization of these phonons can be calculated in the framework of the $t$-$J$ model, since the breathing oxygen motion of these modes (see Fig.5) modulates the energy of a hole in a Zhang-Rice singlet state, and therefore couples directly to the density of doped holes. Expanding the Zhang-Rice energy $E_{ZR} = 8 \frac{t_{pd}^2}{\Delta E}$ with respect to the oxygen displacements $u'_\alpha = u_\alpha(R_i + \delta^O_\alpha)$, $\alpha = x, y$, of the four O-neighbors at $R_i + \delta^O_\alpha$
FIGURE 5. Doping dependence of the high energy bond-stretching phonons along \((\zeta,0,0)\) and \((\zeta,\zeta,0)\) directions for undoped \(\text{La}_2\text{CuO}_4\) (open circles) and \(\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4\) (filled circles) as obtained by neutron scattering. The lower panel gives the line width at half maximum. The displacements of oxygen ions (a) for the \(q=(\pi,\pi)\) breathing phonon distortion and (b) for the \((\pi,0)\) half-breathing mode are indicated by arrows in the right panel. Here \((\pi,0)\) corresponds to \((0.5,0,0)\) reduced lattice units. (reprinted from Pintschovius and Reichardt\[19\] with permission from World Scientific)

FIGURE 6. Calculated phonon spectral function \(B_{ph}\) for \((\pi,0)\) and \((\pi,\pi)\) breathing phonons for \(\delta=0.15, t=0.4\text{ eV}, J=0.12\text{ eV}\) and \(\xi=0.25\) (solid lines) compared to undoped system (dash-dotted lines) and the inelastic neutron scattering data\[20\] for \(\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4\). (reprinted from Khaliullin and Horsch\[25\] with permission from Elsevier)

We assume that the resonance integral obeys the Harrison relation \(t_{pd} \propto r_0^{-7/2}\), where \(r_0\) is the Cu-O distance, and obtain \(g=7E_{ZR}/4r_0\), i.e., \(g \approx 2\text{eV/Å}\). The lattice part of the Hamiltonian is determined by the force constant \(K \approx 25\text{eV/Å}^2\) for the longitudinal O-motion. Due to the structure of \(H_{e-ph}\) the breathing modes couple directly to \(\chi_{\mathbf{q},\omega}\).
We have studied the renormalization of the phonon Green’s functions along $(\pi, 0)$ and $(\pi, \pi)$ directions

\[
D_{q,\omega}^{ph} = \frac{\omega_{q,0}}{\omega^2 - \omega_{q,0}^2 (1 - \alpha_q \chi_{q,\omega})},
\]

where $\omega_{q,0}$ is the bare phonon frequency, i.e. measured in the undoped parent compound, and $\alpha_q = \frac{4e^2}{\kappa} (\sin^2 q_x/2 + \sin^2 q_y/2)$. Based on the parameters of the $pd$-model we estimate for the dimensionless coupling constant $\xi = g^2/tK \sim 0.3 - 0.5$. The coupling constant $\alpha_q^2$ vanishes at the $\Gamma$ point and becomes maximal at the zone edges. This explains the marginal changes of renormalized phonon frequency $\omega_q$ at $q = (0, 0)$. The particularly strong increase of the renormalization for $q$ along $(\pi, 0)$ is a combined effect of the strong increase of $\alpha_q^2$ and the low energy polaron structure in the density response.

![Doping dependence of low-energy density response at $(\pi, 0)$ (solid lines). As a consequence of the scaling of the polaron structure $\propto (\chi J + \delta t)$ there is a strong change in the renormalization and damping of the $(\pi, 0)$ optical phonon (dashed lines), which is at $\omega_0 = 0.2t$ in the undoped system.](image)

Figure 6 calculated for typical parameters for cuprate superconductors, shows the strong renormalization of the $(\pi, 0)$ half-breathing mode for La$_{1.85}$Sr$_{0.15}$CuO$_4$ with a twice as large shift as for the $(\pi, \pi)$ breathing phonon. The large damping of the $(\pi, 0)$ phonon results from the hybridization with the large polaron peak in $N(q, \omega)$ at this momentum and is consistent with the experimental data\cite{20}. The phonon energies of the undoped parent compound $\omega_{q,0} = 80(90)$ meV for $(\pi, 0)$ and $(\pi, \pi)$, respectively, are taken from Ref.\cite{19}.

The strong doping dependence of this effect, shown in Fig 7, is due to the scaling $\propto (\chi J + \delta t)$ of the polaron peak position in $N(q, \omega)$. In particular our results imply that the large damping of the $(\pi, 0)$ optical phonon should disappear at larger doping concentrations.
SUMMARY

We have outlined a 1/N slave-boson theory for the density response of the \( t-J \) model, which explains the data obtained by exact diagonalization. We demonstrated that the predicted low energy polaron structure in the density response, which is particularly pronounced along \((\pi,0)\), explains the anomalous doping induced line width and shift of the longitudinal planar \((\pi,0)\) phonon. The energy of the polaron peak is determined by the spinon energy scale, therefore we predict a nontrivial doping dependence for the phonon renormalization. In that respect further neutron scattering studies of the doping dependence of phonons would provide a sensitive test for the low energy density response as well as for the spin structure in the different doping regimes.

ACKNOWLEDGMENTS

We are grateful to L. Pintschovius for several illuminating discussions about the interpretation of neutron scattering data and for permission to reprint Figure 5.

REFERENCES

1. T. Tohyama, P. Horsch, and S. Maekawa, Phys. Rev. Lett. 74 980 (1995).
2. R. Eder, Y.Ohta, and S. Maekawa, Phys. Rev. Lett. 74 5124 (1995).
3. J. Jaklic and P. Prelovsek, Adv. Phys. 49, 1 (2000).
4. Z. Wang, Y. Bang, and G. Kotliar, Phys. Rev. Lett. 67, 2733 (1991).
5. L. Gehlhoff and R. Zeyher, Phys. Rev. 52, 4635 (1995).
6. A. Foussats and A. Greco, Phys. Rev. B 65, 195107 (2002).
7. D.K.K. Lee, D.H. Kim and P.A. Lee, Phys. Rev. Lett. 76, 4801 (1996).
8. G. Jackeli and N.M. Plakida, Phys. Rev. B 60, 5266 (1999).
9. G. Khaliullin and P. Horsch, Phys. Rev. B 54, R9600 (1996).
10. G. Kotliar and J. Liu, Phys. Rev. B 38, 5142 (1988).
11. N. Read and D.M. Newns, J. Phys. C 16, 3273 (1983).
12. E. Arrigoni et al., Physics Reports 241, 291 (1994).
13. V.N. Popov, Functional Integrals in Quantum Field Theory and Statistical Physics, (D. Reidel, Dordrecht, 1983).
14. F.C. Zhang, C. Gros, T.M. Rice, and H. Shiba, Supercond. Sci. Technol. 1, 36 (1988); R.B. Laughlin, J. Low Temp. Phys. 90, 443 (1995).
15. N. Nagaosa and P. Lee, Phys. Rev. Lett. 64, 2450 (1990).
16. C.L. Kane, P.A. Lee and N. Read, Phys. Rev. B 39, 6880 (1989).
17. C. Gros and R. Valenti, Phys. Rev. B 50, 11313 (1994).
18. L. Pintschovius et al., Physica C 185-189, 156 (1991).
19. L. Pintschovius and W. Reichardt, Physical Properties of High Temperature Superconductors IV, edited by D. Ginsberg (World Scientific, Singapore, 1994), p. 295.
20. R. J. McQueeney, T. Egami, G. Shirane, and Y. Endoh, Phys. Rev. B 54, R9689 (1996); R. J. McQueeney et al., Phys. Rev. Lett. 82, 628 (1999).
21. L. Pintschovius and M. Braden, Phys. Rev. B 60, R15039 (1999).
22. W. Reichardt, J. Low Temp. Phys. 105, 807 (1996).
23. L. Pintschovius, W. Reichardt, M. Kläser, T. Wolf, and H. v. Löhneisen, Phys. Rev. Lett. 89, 037001 (2002).
24. B. Renker et al., Physica C 162-164, 462 (1989).
25. G. Khaliullin and P. Horsch, Physica C 282-287, 1751 (1997).