Electric dipole moments and $b$-$\tau$ unification in the presence of an intermediate scale in SUSY grand unification

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Abstract

We show that an intermediate gauge symmetry breaking scale can be a significant source of electric dipole moments for the electron and neutron in supersymmetric grand unified theories. New phases, similar to that of the CKM matrix, appear which do not arise from the supersymmetry (SUSY) breaking operators. To illustrate, we choose some grand unified SUSY models having an intermediate gauge symmetry breaking scale with some attractive features. We also show how well the $b$–$\tau$ unification hypothesis works in this class of models.
Supersymmetric grand unified theories (GUTs) with intermediate gauge symmetry breaking scales are attractive because they resolve a few longstanding problems and possess some desirable phenomenological features. For example, in models where the intermediate breaking scale $M_I \sim 10^{10} - 10^{12}$ GeV, one can naturally get a neutrino mass in the interesting range of $\sim 3 - 10$ eV, which could serve as hot dark matter to explain the observed large scale structure formation of the universe [1]. The window $\sim 10^{10} - 10^{12}$ GeV is also of the right size for a hypothetical PQ-symmetry to be broken so as to solve the strong CP problem without creating phenomenological or cosmological problems [2]. Models which allow even lower intermediate gauge symmetry breaking scale e.g $M_I \sim 1$ TeV are also interesting since they predict relatively light new gauge fields, as for example SU(2)$_R$ charged gauge bosons $W_R$. In all these intermediate scale models, lepton flavor violation is predicted [3] which may be close to the current experimental limit, and hence could provide a signal of such models.

In this letter, we point out another special feature of these intermediate scale models: they can give rise to detectable amounts of electric dipole moments (EDMs) to the electron and neutron. This feature does not depend on the nature of the ultimate unifying group. We will always assume that supersymmetry is broken via soft breaking terms introduced at a super high scale. We shall assume that the soft breaking terms at the high scale at which they are introduced are flavor blind and CP invariant. It has already been shown [4–6] for SUSY SO(10) models without an intermediate scale that there could be significant amounts of EDM for the electron and neutron. However for this to arise, the universal boundary condition for the soft SUSY breaking terms has to be implemented at a scale higher than the GUT scale $M_G$ such as the reduced Planck or string scales. Consequently, the EDMs for the electron and neutron are not expected to be produced in such a manner in SUSY GUTs with the attractive feature of gauge unification taking place at the string scale. The models with an intermediate gauge symmetry breaking scale however give rise to electron and neutron EDMs, irrespective of whether a universal soft SUSY breaking boundary condition appears at the GUT scale or above it. With the boundary conditions we have chosen, we
accurately calculate the intermediate scale effects on the EDMs. We also discuss the $b - \tau$ unification hypothesis for SUSY GUTs with an intermediate gauge symmetry breaking scale. Whenever in SUSY SO(10) grand unification without an intermediate scale a tau neutrino mass is desired in the interesting eV range, it is found that $b - \tau$ unification hypotheses has to be abandoned \[7\]. With the introduction of the intermediate scale, we examine how well that hypothesis works for the various models considered here.

We know that the intermediate scale gauge symmetry breaking theories with \(SU(2)_L \times SU(2)_R \times SU(4)_C\) or \(SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C\) as the intermediate scale gauge group, give rise to large lepton flavor violation which could be detected through processes like $\mu \rightarrow e\gamma$. The reason is quite simple. With intermediate gauge symmetry $SU(2)_L \times SU(2)_R \times SU(4)_C$, the quarks and leptons are unified. Hence, the $\tau$-neutrino Yukawa coupling is the same as the top Yukawa coupling. Through the renormalization group equations (RGEs), the effect of the large $\tau$-neutrino Yukawa coupling is to make the third generation sleptons lighter than the first two generations, thus mitigating the GIM cancellation in one-loop leptonic flavor changing processes involving virtual sleptons. Although the quarks and leptons are not unified beneath the GUT scale when the intermediate scale gauge group is $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$, the same effect is produced from the assumption that the top quark Yukawa coupling is equal to the $\tau$-neutrino Yukawa coupling at the GUT scale.

When we calculate the EDM of the electron the above stated principle applies, but we must also consider the phases at the gaugino-slepton-lepton vertices. Likewise, to generate the EDM for the neutron one needs the third generation down squark to be lighter than those of the other two generations, which occurs due to the large top Yukawa coupling, and new phases at the gaugino-squark-quark vertices. In fact, whenever there is an intermediate scale, irrespective of the intermediate gauge group (e.g. $SU(2)_L \times SU(2)_R \times SU(4)_C$ or $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$) such phases are generated. The reason for this is that right-handed quarks or leptons are unified in a multiplet in a given generation. The superpotential for an intermediate gauge symmetry breaking model can be written (in the
case of $SU(2)_L \times SU(2)_R \times SU(4)_C$:

\[ W_Y = \lambda_{F_u} F \Phi_2 \bar{F} + \lambda_{F_d} F \Phi_1 \bar{F}, \]  

(1)

where $F$ and $\bar{F}$ are the superfields containing the standard model fermion fields and transform as $(2, 1, 4)$ and $(1, 2, \bar{4})$ respectively and we have suppressed the generation and gauge group indices. We choose to work in a basis where $\lambda_{F_u}$ is diagonal in which $W_Y$ can be expressed as the following:

\[ W_Y = F \bar{\chi}_{F_u} \bar{F} \Phi_2 + F U^* \bar{\chi}_{F_d} U^\dagger \bar{F} \Phi_1. \]  

(2)

The matrix $U$ is a general $3 \times 3$ unitary matrix with 3 angles and 6 phases. It can be written as follows:

\[ U = S^* V S, \]  

(3)

where $V$ is the CKM matrix and $S$ and $S'$ are diagonal phase matrices. At the scale $M_I$ the superpotential for the Yukawa coupling can be expressed in the following manner:

\[ W_{\text{MSSM}} = Q \bar{\chi}_u U^c H_2 + Q V^* \bar{\chi}_d S^2 V^\dagger D^c H_1 + + E^c V^*_L S^2 V^\dagger L H_1, \]  

(4)

where the ability to reduce the number of phases by redefinition of fields has been taken advantage of to the fullest extent possible,

\[ S^2 \equiv \begin{pmatrix} e^{i\phi_d} & 0 & 0 \\ 0 & e^{i\phi_s} & 0 \\ 0 & 0 & 0 \end{pmatrix} \]  

(5)

is a diagonal phase matrix with two independent phases, and $V_L$ is the CKM matrix at the intermediate gauge symmetry breaking scale. It is not possible to do a superfield rotation on $D^c$ or $L$ to remove the right handed angle since at $M_I$ the third diagonal element of the scalar mass matrices $m^2_D$ and $m^2_L$ develop differently from the other two diagonal elements due to large top Yukawa coupling RGE effects. When the intermediate gauge symmetry group is $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$, the additional CKM-like phases will be generated in exactly the same way as described above.
The expressions from which we calculate EDMs are given below [4].

For the electron’s EDM we have:

$$|d_e| = e |F_2| |V_{td}/V_{ts}| \sin \phi,$$

(6)

where $F_2$ is given by

$$F_2 = \frac{\alpha}{4\pi \cos^2 \theta_W} m_\tau V_{\tau e} V_{\tau e}^* (V_{\tau \tau})^2 (A_e + \mu \tan \beta) \times$$

$$\times [G_2(m_{\tilde{e}}^2 L, m_{\tilde{e}}^2 R) - G_2(m_{\tilde{e}}^2 L, m_{\tilde{e}}^2 R) - G_2(m_{\tilde{e}}^2 L, m_{\tilde{e}}^2 R) + G_2(m_{\tilde{e}}^2 L, m_{\tilde{e}}^2 R)],$$

(7)

and $V_{ab}^e$ are the matrix elements of the matrix $V_I$ and the functions $G_2(a, b)$ are defined in Eqn. (20) in Ref. [5]. $\phi$ includes effects of all possible phases. For the neutron:

$$|d_n| = 4/9 e |F'_2| \sin \phi,$$

(8)

where $F'_2$ is given by

$$F'_2 = \frac{\alpha s}{4\pi} V_{td} V_{td} (V_{tb})^2 (A_d + \mu \tan \beta) \times$$

$$\times [G_2(m_{\tilde{d}}^2 L, m_{\tilde{d}}^2 R) - G_2(m_{\tilde{d}}^2 L, m_{\tilde{d}}^2 R) - G_2(m_{\tilde{d}}^2 L, m_{\tilde{d}}^2 R) + G_2(m_{\tilde{d}}^2 L, m_{\tilde{d}}^2 R)].$$

(9)

To calculate the squark, slepton and gaugino masses at the low scale we numerically run the RGEs from the GUT scale down to weak scale. We assume a universal boundary condition at the GUT scale $M_G$ i.e. all gaugino masses $M_i(M_G) = m_{\tilde{g}}$, all tri-linear scalar couplings $A_i(M_G) = A_0$, and all soft scalar masses $m_i^2(M_G) = m_{0i}^2$. We use the RGEs for a given intermediate gauge group from $M_G$ down to $M_I$ as given in Ref. [3,8]. From $M_I$ scale down to the weak scale, we use the MSSM RGEs (see, for example, Ref. [14]).

As examples to illustrate our point, we choose to use the following four intermediate scale models:

Model (1): It is based on the gauge group SU(2)$_R \times$SU(2)$_L \times$SU(4)$_C$ with gauge couplings that are found to be unified at a scale $M_G$ near the string unification scale. The model breaks to the minimal supersymmetric standard model at a scale $M_I \sim 10^{12}$ GeV and can have both large and small tan $\beta$ scenarios. For high tan $\beta$ scenario we use $M_I = 10^{12}$ GeV and
$M_G \approx 10^{18.26}$ GeV leading to $\alpha_s(M_Z) \approx 0.126$ and for low $\tan \beta$ scenario we use $M_I = 10^{12}$ GeV and $M_G \approx 10^{17.83}$ GeV leading to $\alpha_s(M_Z) \approx 0.119$.

Model (2): It is Case V of Ref. [9] where SO(10) is broken down to $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \times \text{SU}(3)_C$ gauge symmetry at the scale $M_G$. In this scenario, we use $M_I \approx 10^{12}$ GeV and $M_G \approx 10^{15.6}$ GeV leading to $\alpha_s(M_Z) \approx 0.129$.

Model (3): This is the model presented in Ref. [10] with SO(10) breaking down to $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \times \text{SU}(3)_C$. It is the only example we use for which D-parity is not broken at $M_G$ and hence left-right parity ($g_L = g_R$) is preserved in $G_I$. The field content allows $M_I \sim 1$ TeV with $M_G \approx 10^{16}$ GeV. We use MSSM below the scale $M_I$ for convenience although in the original work [10] the two Higgs doublet model has been used. The value of $\alpha_s$ with the MSSM below $M_I$ is about 0.129.

Model (4): This is the model discussed in Ref. [11]. Once again, SO(10) is broken down to $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \times \text{SU}(3)_C$ at $M_G$. In this model, $M_G$ is predicted to be exactly the same as in the conventional SUSY SO(10) breaking with no intermediate scale and the scale $M_I$ can have any value between the TeV and the GUT scales. Since there is only one Higgs bidoublet, this model prefers large values of $\tan \beta$ with $\lambda_t = \lambda_b$ at $M_I$. Nevertheless, the introduction of nonrenormalizable operators can allow for small $\tan \beta$.

In the Figs. 1(a)-1(d), we plot

$$d_r \equiv \log_{10}\left(\frac{d_e / \sin \phi}{4.3 \cdot 10^{-27}}\right),$$

where $d_e$ is the EDM of electron, as a function of the scalar mass $m_0$ for different values of $m_{1/2}$. Since the EDM of neutron is also of the same order, we do not plot them. Also, experimentally the EDM of electron is more constrained. The experimental bounds are given as: $d_n < 0.8 \cdot 10^{-25}\text{ecm}$ [12] and $d_e < 4.3 \cdot 10^{-27}\text{ecm}$ [13]. From the graphs it appears that one can use EDM as a signal for the intermediate scale in a grand unification scenario. The value of $|\sin \phi|$ could have arbitrary values from 0 to 1, but there is no reason to expect it to be suppressed.

Now we discuss the viability of $b - \tau$ unification hypothesis in the models we have
The value for $m_b^{\text{pole}}$ from the existing data is $m_b^{\text{pole}} = 4.75 \pm .05$ calculated in Ref. [12], and we use $m_\tau = 1.777$ GeV. The predicted $m_b$ mass in these intermediate scale models mainly depend on 3 factors: the value of $\lambda_t$, $\alpha_s(M_Z)$ and the location of the intermediate scale $M_I$. Using larger values of $\lambda_t$ of course lowers the $m_b$ mass, while using larger values of $\alpha_s$ increases it. For these models at the scale $M_G$, we have used the maximum perturbative value for the top Yukawa coupling which is about 3.54. For model (1), since leptons and down quarks are unified in the same multiplet at the intermediate scales we have $\lambda_b = \lambda_r \neq \lambda_t = \lambda_{\nu_r}$ for the low tan $\beta$ scenario. We find $m_b^{\text{pole}} = 4.78$ GeV. For the large tan $\beta$ version of that model, we have $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_r}$ instead, and find $m_b^{\text{pole}} = 4.80$ GeV. We find that model (1) is able to provide very reasonable $b$-quark mass predictions since $\alpha_s$ is of moderate values and since down to the scale $M_I$ the relation $\lambda_b = \lambda_\tau$ exists intact. For models (2) and (3) with low values for tan $\beta$, we have $\lambda_t = \lambda_{\nu_r}$ and $\lambda_b = \lambda_\tau$ only at the GUT scale and find $m_b$ pole masses of 5.76 GeV and 6.20 GeV, respectively. However if we had used smaller values of $\alpha_s$ as used in the original references [9,10] for those models or had we assumed large values for tan $\beta$, these masses would be much closer to the desired range. As in Ref. [16], one could purposefully construct models with $M_I \sim 10^{12}$ GeV and lower values for $\alpha_s$ so as to improve the $b$-quark mass prediction. For model (4) with $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_r}$ at $M_G$, in Fig. 2(a) we choose to plot the $m_b^{\text{pole}}$ mass as a function of $M_I$ since the intermediate gauge symmetry breaking scale in that model can lie anywhere between the weak scale and the GUT scale. Notice that the $b$-quark mass at first increases as the intermediate gauge symmetry breaking scale moves away from the GUT scale. But, it then reaches a peak value when the intermediate scale is about $10^8$ GeV, and then for $M_I$ less than that scale it decreases. The reason for this behavior can be found in the RGEs for $\lambda_b$ and $\lambda_\tau$. The RGE for $\lambda_b$ feels the influence of the large top Yukawa coupling while $\lambda_\tau$ instead feels the influence of the $\tau$ neutrino coupling. Though the magnitude of the top and the $\tau$ neutrino couplings are same at the GUT scale, the $\tau$-neutrino coupling decreases faster than the top Yukawa coupling and reaches its fixed point sooner. If the Intermediate
breaking scale is decreased $\lambda_b(M_I)$ would also decrease, however $\lambda_\tau(M_I)$ would not decreases as much. So, effectively the mass of $m_b$ decreases, since $m_b$ mass depends on the ratio of $\lambda_b$ to $\lambda_\tau$. We further note that the interesting values for the intermediate gauge symmetry breaking scale $M_I \sim 1$ TeV and $M_I \sim 10^{12}$ GeV can both give good values for the $b$-quark mass. Effects of this low intermediate scale could be observed in the future colliders. In Fig. 2(b), we assume the possibility of model (4) allowing a range of values for $\tan \beta$ in order to plot the $m_b^{\text{pole}}$ as a function of $\tan \beta$ for the interesting case of $M_I = 10^{12}$ GeV. We see that larger values of $\tan \beta$ are preferred and give very reasonable values for the $b$-mass. In both Figs. 2(a) and 2(b), we show results for two different values of $\alpha_s$ as explained in the figure caption.

In conclusion, we find that intermediate gauge symmetry breaking can be a significant source of electric dipole moments for the electron and neutron. We have illustrated this effect for four different models. One of which has the gauge couplings unified at the string scale and the others at the usual GUT scale ($\sim 10^{16}$ GeV). In all the models, the universal SUSY soft breaking boundary condition is assumed to be introduced at the GUT scale $M_G$. Of course for the models where the gauge unification scale is of order $10^{16}$ GeV, if we had assumed the boundary condition at the reduced Planck scale or string scale the EDM predictions would have been further enhanced. We also examined how well the $b-\tau$ unification hypothesis works for these models, and find that sometimes it works very well especially when the intermediate gauge symmetry unifies quarks and leptons or when $\tan \beta$ is large.

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Figure captions

Fig. 1(a) : \( d_r \equiv \log_{10}(d_e/(4.27 \times 10^{-27}\sin \phi)) \) for the low \( \tan \beta \) version of model (1) is plotted as a function of of the universal soft SUSY breaking mass \( m_0 \).

The solid lines correspond to \( \mu > 0 \), while the dashed lines correspond to \( \mu < 0 \).

The upper two lines in the vicinity of \( m_0 = 100 \) are for \( m_{1/2} = 100 \text{ GeV} \), and the lower two lines are for \( m_{1/2} = 150 \text{ GeV} \).

\( \lambda_{tG} = 3.54 \) for all the lines.

Fig. 1(b) : \( d_r \) for model (2) is plotted as a function of of the universal soft mass \( m_0 \).

The solid lines correspond to \( \mu > 0 \), while the dashed lines correspond to \( \mu < 0 \).

The upper two lines in the vicinity of \( m_0 = 150 \text{ GeV} \) are for \( m_{1/2} = 160 \text{ GeV} \), and the lower two lines are for \( m_{1/2} = 200 \text{ GeV} \).

\( \lambda_{tG} = 3.54 \) for all the lines.

Fig. 1(c) : \( d_r \) for Model (3) is plotted as a function of the universal soft SUSY breaking gaugino mass \( m_{1/2} \).

The solid lines correspond to \( \mu > 0 \), and the dashed lines correspond to \( \mu < 0 \).

The upper two lines around \( m_0 = 150 \text{ GeV} \) are for \( m_{1/2} = 190 \text{ GeV} \), and the lower two lines in that region are for \( m_{1/2} = 220 \text{ GeV} \).

\( \lambda_{tG} = 3.54 \) for all the lines.

Fig. 1(d) : \( d_r \) for Model (4) is plotted as a function of \( \log_{10}(M_I/\text{GeV}) \).

The solid lines correspond to \( \mu > 0 \), and the dashed lines correspond to \( \mu < 0 \).

The upper two lines around \( M_I = 10^8 \text{ GeV} \) correspond to \( \lambda_{tG} = 3.54 \), and the lower two lines in the same region correspond to \( \lambda_{tG} = 1.38 \). \( m_0 = m_{1/2} = 180 \text{ GeV} \) for all the lines.

Fig. 2(a) : The \( m_b^\text{pole} \) values are plotted as a function of \( \log_{10}(M_I/\text{GeV}) \) in model(4) with complete third generation Yukawa coupling unification. The solid line corresponds to \( \alpha_s = 0.117 \), \( \sin^2\theta_W = 0.2332 \) (within 2-\( \sigma \) of the experimental mid-value) and \( \alpha = 1/127.9 \) at the \( M_Z \).
scale, the dashed line corresponds to $\alpha_s = 0.122$, $\sin^2 \theta_W = .2321$ (the experimental mid-value) and $\alpha = 1/127.9$ at the $M_Z$ scale.

$\lambda_{t_G} = 3.54$ for both of the lines.

Fig. 2(b) : The $m_b^{\text{pole}}$ values are plotted as a function of $\tan \beta$ in model(4) with $M_I = 10^{12}$ GeV. The solid and dashed lines correspond to the same values of $\alpha_s$ as in Fig. 2(a). $\lambda_{t_G} = 3.54$ for both of the lines.
Fig. 1a

$dr$

$m_0$/GeV

Fig. 1b

$dr$

$m_0$/GeV
Fig. 1c

$\frac{d_r}{m_0/\text{GeV}}$

Fig. 1d

$\frac{d_r}{\log_{10}(M_T/\text{GeV})}$
Fig. 2a

Log$_{10}$($M_I/$GeV)

Fig. 2b

$\tan[\beta]$

$m_b^{pole}/$GeV

$m_b^{pole}/$GeV