Potential for the slow-roll inflation, mass-scale hierarchy and dark energy from type IIA supergravity

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Abstract. The magnetic fluxbrane solution with a strongly warped throat is studied in type IIA supergravity theory with a co-dimension one local source which is the \( Z_2 \)-symmetric UV boundary of the throat. The overall volume of extra space may be stabilized since the introduction of the local source breaks the no-scale structure of the theory and evades the no-go theorem. The radion field is defined as the position of the UV boundary ‘moved’ from its stable value fixed by the anisotropic Israel junction conditions. The analytical expression for the radion effective potential is found. The potential decreases exponentially (the exponent is equal to 0.21 in Planck units) in the slow-roll region and apparently meets other demands of the early inflation. Thus the radion potential serves an inflaton. Reissner–Nordstrom type deformation of the elementary fluxbrane solution permits us to construct the IR end of the throat and results in a tiny positive non-zero value of the radion potential in its extremal point seen today as dark energy density \( \rho_{DE} \). Expressions for the mass-scale hierarchy \( m/M_{Pl} \) and for the ‘acceleration hierarchy’ found in this paper give the physically interesting relation between two hierarchies: \( \rho_{DE} \sim m^8/M_{Pl}^4 \).

Keywords: dark energy theory, string theory and cosmology

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1. Introduction

This paper continues the research of [1] where the analytical expression for the radion effective potential capable to meet the demands of slow-roll inflation was found in a class of models with fluxbrane throat-like solutions. These studies develop the direction of thought of the papers [2]–[20] where the throat-like solutions in type IIB supergravity with warped Klebanov–Strassler conifold [21]–[24] were considered. In the present paper we consider the non-deformed and deformed throat-like solution in type IIA supergravity where six extra dimensions are given by the warped flat space whose base is a sphere [25]–[31].

Following the conventional approach, and in parallel with the Randall–Sundrum theory [32], we suppose that massive matter of the standard model (SM) is localized in the IR region near the tip of the throat whereas the volume of extra space is terminated by the co-dimension one local source—heavy ‘ultraviolet’ boundary where $\mathbb{Z}_2$ identification and corresponding junction conditions are imposed. We suppose that the dynamics of the boundary surface is described by the simplest positive tension Nambu–Goto action, hence the energy–momentum tensor of the boundary is isotropic. In section 3 it is shown that anisotropic Israel junction conditions stabilize the position of the isotropic boundary at the top of the throat.
This seemingly comes into conflict with the no-scale structure of supergravity theory ([4] and references therein) and with the no-go theorem [33, 4]. However there is no conflict at all. To fix the modulus of the overall volume of extra space is possible because the no-scale structure is spoiled here by the external local source (UV boundary), and because this co-dimension one local source also evades the no-go theorem. Indeed it is easy to show that the combination $\tilde{T}$ (defined by expression (34) of the paper [33]) of the components of the energy–momentum tensor does not meet demands of the no-go theorem in the case the positive tension co-dimension one local source is introduced in the action; this is not true, however, for the positive tension local sources of lower dimensions.

The Reissner–Nordstrom type deformation of the elementary extremal solution will be used as a tool to construct the IR end of the throat. This type of solution with a ‘bolt’ (in the terminology of [34]) was considered earlier in 6D models [35]–[37], where also the constant curvature of the four-dimensional space–time was introduced to make the Israel junction conditions consistent. As it was shown in [37] the value of this curvature must be extremely small and may correspond to the observed acceleration of the Universe (see the review [38]). In the present paper this approach is generalized in D10 type IIA supergravity theory. Contrary to [37] where the formula for dark energy $\rho_{\text{DE}} = G_N m^6$ (where $G_N$ is Newton’s constant, $m$ is the characteristic mass of matter) was found, the model considered in the present paper gives a more realistic result $\rho_{\text{DE}} \sim G_N^2 m^8$ (section 5.2).

The radion field is defined here as the position of the UV boundary which slowly depends on the space–time coordinates in four dimensions (cf [39]–[41]). The radion effective potential is calculated with the standard procedure of integrating out extra coordinates in the higher-dimensional action. Of course it is possible to rescale the isotropic radial coordinate and to make the position of the boundary fixed; then we come to the definition of radion as a factor field in a higher-dimensional metric [42]–[44]. Two approaches are basically equivalent when the radion effective potential is calculated.

The form of the radion potential depends only on the choice of the theory; in the type IIA supergravity considered in this paper the potential exponentially decreases upward in the throat with an exponent equal to 0.21, i.e. it is sufficiently flat to provide the slow-roll inflation [45, 46]. Thus the radion scalar field introduced in the paper may serve as an inflaton. The radion potential proves to be non-negative; its relatively flat region ends with a steep slope falling down to the minimum of the potential at the top of the throat.

In the present paper these results of [1] are repeated in a more transparent way. Also the validity of the hypothesis of [1] is proved: it is shown in section 5.3 that Reissner–Nordstrom type deformation of the elementary fluxbrane solution results in tiny positive deviation from zero of the minimal value of the radion effective potential. This deviation is seen today as dark energy density.

The idea to use a dynamical scalar associated with extra dimensions, interbrane distance in particular, as a candidate for inflaton is not a novel one (see, e.g., [19b, 44]). The very possibility to get in this way the exact analytical expression for the scalar field potential possessing qualitatively the basic features demanded by the astrophysical observations [47] looks attractive.

It must be noted that a physically meaningful radion effective potential may be found here only in the case where the bulk magnetic monopole fluxbrane solution is considered as a background, not the dual electric one. The nonequivalence of the two solutions is
immediately seen when the higher-dimensional consistency condition of [48] is applied, see the appendix in [1].

The formulae for the value of the electro-weak hierarchy presented in section 5 essentially develop the ideas of the works [49] where it was observed that, in the throat-like fluxbrane models, the hierarchy proves to be strongly dependent on the value of the \( n \)-form-dilaton coupling constant and on dimensionalities of the extra subspaces.

This paper is organized as follows. Basic action, bulk and junction equations are presented in section 2. In section 3 the stabilization of the volume modulus of the non-deformed throat-like solution in type IIA supergravity is demonstrated, the analytical expression for the radion effective potential is found and its compatibility with the demands of inflation is shown. In section 4 the generalization of the elementary solution induced by introduction of a non-zero ‘Maxwell’ field and non-zero curvature of the four-dimensional Universe is considered. Section 5 presents formulae for the mass-scale hierarchy and for the rate of acceleration of the Universe; the physically meaningful dependence of the two hierarchies is deduced. In section 6 results and problems are summarized and possible trends of future research are outlined.

2. Action, ansatz, dynamical equations

Let us consider the following action in \( D \) dimensions:

\[
S^{(D)} = M^{D-2} \left\{ \int \left[ R^{(D)} - \frac{1}{2} (\nabla \varphi)^2 - \frac{1}{2} \cdot n! e^{\alpha \varphi} F_{(n)}^2 - \frac{1}{2} \cdot 2! e^{\eta \varphi} F_{(2)}^2 \right. \\
\left. - \sigma e^{\gamma \varphi} \delta^{(1)} \frac{\sqrt{-h^{(D-1)}}}{\sqrt{-g^{(D)}}} \right] \sqrt{-g^{(D)}} \, d^{D} x + GH \right\},
\]

whose bulk part is an Einstein-frame truncated low-energy description of the string-based supergravity with dilaton and antisymmetric tensor; \( M, g_{AB}, R^{(D)} \) are ‘Planck mass’, metric and curvature in \( D \) dimensions; \( GH \) is the Gibbons–Hawking term; \( F_{(n)} \) is the \( n \)-form field strength; \( F_{(2)} \) is the 2-form ‘Maxwell’ field; \( \varphi \) is the dilaton field coupled to \( n \)-form, 2-form and to local source in (1) with coupling constants \( \alpha, \eta \) and \( \gamma \), respectively. The co-dimension one local source will serve as the UV boundary of the throat, its action is taken in the simplest Nambu–Goto form; mass parameter \( \sigma \) characterizes its tension equal to \( M^{D-2} \sigma \); \( h^{(D-1)} = \det h_{ab} \); \( h_{ab} \) is an induced metric on the boundary, \( a, b = \{0, 1 \ldots (D-2)\} \); \( \delta^{(1)} \) is the Dirac delta function fixing the position of the boundary.

In this paper we shall consider the theory (1) for the following particular values of dimensionalities and coupling constants in (1):

\[
D = 10, \quad n = 4, \quad \alpha = \frac{1}{2}, \quad \eta = \frac{3}{2}, \quad \gamma = -\frac{1}{12}.
\]

With this choice the bulk part of the action (1) is a truncated Bose action of type IIA supergravity. It is worthwhile noting that D10 theory (1) with specific values of the constants given in (2) is just a compactification of the action of D11 \( M \)-theory where, in addition to the conventional bulk terms, the D10 local source is included:

\[
S_{(M)} = M^{10} \left\{ \int \left[ R^{(11)} - \frac{1}{2} \cdot 4! F_{(4)}^2 - \tilde{\sigma} \delta^{(1)} \frac{\sqrt{-h^{(10)}}}{\sqrt{-g^{(11)}}} \right] \sqrt{-g^{(11)}} \, d^{11} x + GH \right\},
\]
\( \hat{M}, \tilde{\sigma} \) are Planck mass and mass parameter of the local source in 11 dimensions. After reduction of action (3) to 10 dimensions the volume of the compact eleventh dimension becomes a dilaton field whereas the 2-form in (1) is a corresponding Kaluza–Klein field. However, we shall not refer any more to \( M \)-theory and consider the supergravity action (1), where dimensionalities and constants are given in (2), as the primary one throughout the paper.

The following ansatz for the bulk solution of the dynamical equations given by the action (1) and (2) will be used:

\[
ds_{(10)}^2 = b^2 \tilde{g}_{\mu\nu} \, dx^\mu \, dx^\nu + c^2 \, dz^2 + N^2 \, dr^2 + a^2 \, d\Omega_4^2, \quad \varphi = \varphi(r),
\]

\[
F_{(4)} = Q_{(4)} \, dy^1 \land dy^2 \land dy^3 \land dy^4, \quad F_{(2)zr} = dA_z(r)/dr = \frac{Q_{(2)}b^4 N}{a^4} e^{-3\varphi/2},
\]

where metric scale factors \( b, c, a \), ‘lapse function’ \( N \) and dilaton \( \varphi \) depend only on the isotropic coordinate \( r \), \( \tilde{g}_{\mu\nu} \) is the metric of the four-dimensional Universe \( M_{(3+1)} \), \( z \) is the coordinate of the torus \( S^1 \) of period \( T_z \), \( d\Omega_4^2 \) is the metric of the 4-sphere of unit radius; \( x^A = \{x^\mu, z, r, y^i\} \), \( A = 0, 1, \ldots, 9, \mu = 0, 1, 2, 3, i = 1, 2, 3, 4 \). \( Q_{(4)} \) is the charge of the magnetic monopole. \( A_z \) is the non-zero component of the vector potential of the 2-form field \( F_{(2)} \), \( Q_{(2)} \) is its ‘electric’ charge, the last equality for \( F_{(2)} \) in (4) is found from the ‘Maxwell’ equation for the 2-form written down for the metric ansatz (4).

Introduction of small \( F_{(2)} \neq 0 \) gives the Euclidean version of the Reissner–Nordstrom type deformation of the extremal fluxbrane solution. It will be shown in section 4 that this deformation provides the physical tool to terminate the throat at its IR end and also enforces dynamically to introduce the extremely small positive curvature \( \tilde{R}^{(4)} = 12\hat{h}^2 \) of the manifold \( M_{(3+1)} \); the auxiliary ‘Hubble constant’ \( \hat{h} \) is connected with the observed acceleration rate of the Universe \( \hat{h} = 10^{-60}M_{Pl} \) by scale transformation (30) below (see section 5 for more detail).

With ansatz (4) and \( \hat{h} \neq 0 \) action (1) with the parameters (2) in it gives the following gravity equations for scale factors \( b(r), c(r), a(r) \) (we do not need to put down the gravity constraint) and the equation for the dilaton field (prime means derivative over \( r \)):

\[
\frac{3\hat{h}^2}{b^2} + \frac{1}{N^2} \left[ -\frac{b''}{b} + \frac{b'^2}{b^2} + \frac{b'}{b} \left( \frac{N'}{N} - 4\frac{b'}{b} - \frac{c'}{c} - 4\frac{a'}{a} \right) \right] = \frac{3}{8} J_{(4)} - \frac{1}{8} J_{(2)} + \frac{1}{16} J_{(\sigma)},
\]

\[
\frac{1}{N^2} \left[ -\frac{c''}{c} + \frac{c'^2}{c^2} + \frac{c'}{c} \left( \frac{N'}{N} - 4\frac{b'}{b} - \frac{c'}{c} - 4\frac{a'}{a} \right) \right] = \frac{3}{8} J_{(4)} + \frac{7}{8} J_{(2)} + \frac{1}{16} J_{(\sigma)},
\]

\[
\frac{3}{a^2} + \frac{1}{N^2} \left[ -\frac{a''}{a} + \frac{a'^2}{a^2} + \frac{a'}{a} \left( \frac{N'}{N} - 4\frac{b'}{b} - \frac{c'}{c} - 4\frac{a'}{a} \right) \right] = \frac{5}{8} J_{(4)} - \frac{1}{8} J_{(2)} + \frac{1}{16} J_{(\sigma)},
\]

\[
\frac{1}{N^2} \left[ \varphi'' - \varphi \left( \frac{N'}{N} - 4\frac{b'}{b} - \frac{c'}{c} - 4\frac{a'}{a} \right) \right] = \frac{1}{2} J_{(4)} + \frac{3}{2} J_{(2)} - \frac{1}{12} J_{(\sigma)},
\]

where

\[
J_{(4)} \equiv \frac{e^{\varphi/2} F_{(4)}^2}{2 \cdot 4!} = \frac{e^{\varphi/2} Q_{(4)}^2}{2a^8}, \quad J_{(2)} \equiv \frac{e^{3\varphi/2} F_{(2)}^2}{2 \cdot 2!} = \frac{e^{-3\varphi/2} Q_{(2)}^2}{2b^8 a^8}, \quad J_{(\sigma)} \equiv e^{-\varphi/12} \sigma \frac{\delta(r - r_0)}{N}, \quad \hat{h}^2 = \frac{\tilde{R}^{(4)}}{12}.
\]
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We suppose that the local source is placed at some \( r = r_0 \) and that it terminates the throat ‘from above’, i.e. it forms the UV end of the throat. Thus we truncate space–time (4) at \( r = r_0 \), paste two copies of the inner region along the cutting surface and consider codimension one local source in the RHS of equations (5)–(8) as a heavy boundary where \( Z_2 \) symmetry is imposed. The space–time of the surface is a product

\[
M_{(3+1)} \times S^1 \times S^4,
\]

where physically this boundary may be an 8-brane which spans \( M_{(3+1)} \) and wraps around compact extra spaces \( S^1, S^4 \), or it may be viewed as a shell of branes of lower dimension uniformly distributed over compact subspaces. In any case we suppose, and this is the main hypothesis of the paper, that dynamics of the boundary surface is described by the simplest Nambu–Goto action, hence its energy–momentum tensor is isotropic as follows from the action (1). There are four junction conditions at the boundary: three Israel conditions for three subspaces in (10) and a jump condition for the dilaton field \( \varphi \). These conditions are immediately found by integrating equations (5)–(8) over \( r \) around \( r = r_0 \) (a factor 2 in the LHS in (11)–(14) reflects the \( Z_2 \) symmetry):

\[
\frac{2}{N^2} b' = \frac{\sigma e^{-\varphi/12}}{16N},
\]

\[
\frac{2}{N^2} c' = \frac{\sigma e^{-\varphi/12}}{16N},
\]

\[
\frac{2}{N^2} a' = \frac{\sigma e^{-\varphi/12}}{16N},
\]

\[
-\frac{2}{N^2} \varphi' = -\frac{1}{12} \frac{\sigma e^{-\varphi/12}}{N}.
\]

These relations must be valid at the position of the boundary \( r = r_0 \). Equations (11)–(14) are actually quite informative. We will see that they fix position \( r_0 \) of the UV boundary, i.e. determine the overall volume of the extra space, they ‘fine-tune’ magnetic monopole charge \( Q(4) \) and mass parameter \( \sigma \) in (1), and in the model with deformed extremal solution considered in section 4 the consistency of equations (11) and (12) demands the introduction of non-zero curvature of the Universe, the value of which, as it will be shown, may correspond to the observed acceleration of the Universe.

3. Non-deformed fluxbrane solution, stabilization of the volume modulus and potential for the slow-roll inflation

3.1. Bulk solution and stabilization of the volume modulus

In this section the way of thought of paper [1] is repeated for the particular case of type IIA supergravity given by action (1) with constants (2) in it where we discard the ‘Maxwell’ field and consider \( M_{(3+1)} \), the Minkowsky space–time (i.e. put \( F_{(2)} = 0, \tilde{h} = 0 \) in (5)–(8)).
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Then ansatz (4) for the elementary magnetic fluxbrane solution of equations (5)–(8) looks like [26]–[31]

\[ \text{d}s^2 (10) = H^{-3/8} (\tilde{g}_{\mu \nu} \, dx^\mu \, dx^\nu + dz^2) + H^{5/8} (dr^2 + r^2 \, d\Omega_4^2), \quad e^\varphi = e^{\varphi_\infty} H^{-1/4}, \]

\[ F(4) = Q(4) \, dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4, \quad H = 1 + \left( \frac{L}{r} \right)^3, \quad L^3 = \frac{1}{3} Q(4) e^{\varphi_\infty / 4}, \]

where \( \varphi_\infty \) is the value of the dilaton field at \( r = \infty \).

The metric (15) describes a warped ‘throat’ of proper length \( \int_0^L H^{5/16} \, dr \cong 16L \) with an integrable singularity at \( r = 0 \) where curvature \( R^{(10)} \to \infty \). The low-energy action (1) makes sense only if curvature components are small as compared to \( M^2 \). For the space–time (15) the condition formulated, for example, for the scalar curvature in 10 dimensions is

\[ R^{(10)} = \frac{45}{32} \frac{1}{L^2} \left( \frac{L}{r} \right)^{1/8} < M^2, \]

where this inequality is written inside the throat where \( r \ll L \).

From (16) the minimal permitted value \( r_{\text{min}} \) of the isotropic coordinate is immediately determined:

\[ r > r_{\text{min}} = k^8 (ML)^{-16} L, \]

where coefficient \( k \) is equal to 45/32 when \( R^{(10)} \) is used in the estimate inequality (16). In what follows we shall consider \( k \) some number of order one.

The large value of the exponent in the RHS of (17) reflects its non-analytical dependence on the 4-form-dilaton coupling constant \( \alpha \). In the general case this exponent is equal to \( \Delta / \alpha^2 \) [1][1] (where \( \Delta \) is a well-known parameter of the elementary fluxbrane solutions determined by \( \alpha \) and dimensionalities; \( \Delta = 4 \) in the type IIA and type IIB supergravities). For \( \alpha = 0 \) in (1), as it takes place in type IIB supergravity where there is \( AdS_5 \times S^5 \) asymptotic inside the throat, \( r_{\text{min}} = 0 \), there is no singularity of curvature at any \( r \).

Comparison of the metrics (15) and (4) gives \( b = c = H^{-3/16}, a = H^{5/16} r \), hence the jump conditions (11) and (12) coincide; also the dilaton jump condition (14) identically follows from (11) for the dilaton field given in (15). Compatibility of jump conditions (11) and (13) demands \( b'/b = a'/a \), i.e. \( (H r^2)' = 0 \) at \( r = r_0 \) which gives

\[ r_0 = \frac{L}{2^{1/3}}, \]

whereas (11) taking account of (18) connects \( L, \sigma \) and \( \varphi_\infty \). We shall write down this relation multiplying it by the higher-dimensional Planck mass \( M \):

\[ M L = 12 \left( \frac{2}{3} \right)^{1/3} \frac{M}{\sigma} e^{\varphi_\infty / 12} \equiv 10.5 \, g, \]

where the dimensionless constant

\[ g = \frac{M}{\sigma} e^{\varphi_\infty / 12} \]

There is an unfortunate mistake in [1] where a factor 2 is omitted in the denominator of the exponent in the RHS of expression (14) of [1]: this, however, does not change essentially the conclusions of [1]. The present paper gives the corrected formulae when type IIA supergravity is considered.

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is an important parameter which determines the physical predictions of the model. \( g \) is an invariant of scale transformation \( g_{AB} \rightarrow \text{e}^{2\lambda}g_{AB} \), \( \varphi \rightarrow \varphi + 12\lambda \), \( M \rightarrow \text{e}^{-\lambda}M \) (\( \lambda = \text{const} \)) which is an invariance of the action (1) when \( F_{(2)} = 0 \) and constants (2) are used in (1). The whole approach makes sense only if \( ML \gg 1 \), hence it is necessary that \( g \geq 1 \) as follows from (19).

From (19) and (15) it also follows the ‘fine-tuning’ condition:

\[
Q_{(4)}^{1/3} = 12 \cdot 2^{1/3} \cdot \sigma^{-1}.
\]

This is a direct analogy of the fine-tuning of the bulk cosmological constant and the brane’s tension demanded in the Randall–Sundrum model [32]. However, here the bulk magnetic 4-form charge \( Q_{(4)} \) is not an input parameter in the action but a free constant of the bulk solution of the dynamical equations. Hence relation (21) is by no means a fine-tuning but a constraint determining magnetic charge through the constant \( \sigma \) of the action (1).

We will see below that position (18) of the UV boundary of the throat determining the overall volume of extra space is a point of zero minimum of the corresponding effective potential. As was already noted in section 1 dynamical stabilization of the modulus of the volume of extra space is possible because in the model under consideration the local source in (1) breaks the no-scale structure of the theory and because this codimension one local source also violates the conditions of the no-go theorem.

Thus dynamics of the model terminates the extra space of the space–time (15) at the top of the throat, at its UV end (18). In section 4 the IR end of the throat, where supposedly the standard model resides, will be fixed near the tip of the throat by the use of a small deformation of the extremal solution (15). This deformation will not influence essentially the form of the radion effective potential calculated in section 3.2 for the non-deformed background (15), but will result in a tiny positive deviation (seen today as dark energy density) from the minimal zero value of the potential found for the non-deformed background (see section 5.3).

3.2. Radion as inflaton. Potential for the slow-roll inflation

Effective action \( S^{(3+1)} \) in four dimensions is conventionally calculated by integrating out extra coordinates in a higher-dimensional action. Taking into account the radion field this will give the effective scalar-tensor Brans–Dicke type action (see, e.g., [40b, 42]). To calculate the effective action \( S^{(3+1)} \) we shall use in (1) the bulk solution (15) but move the UV boundary (and hence change the upper limit of the integration over the isotropic coordinate \( r \) in (1)) from \( r = r_0 \) (18) fixed by junction conditions (11)–(14) to the arbitrary position \( \rho(x) \), slowly depending on coordinates \( x^\mu \):

\[
r_0 \rightarrow \rho(x),
\]

where \( \rho(x) \) is called the radion field [19b], [39]–[41]. This definition of the radion field is equivalent to the more conventional one where the radion is considered as depending on the \( x^\mu \) factor of the lapse function \( N \) in the metric (4) [42]–[44]. The gradient terms of \( \rho(x) \) contribute to the induced metric of the UV boundary:

\[
h_{ab} = g_{ab} + \rho, a \rho, b g_{rr},
\]
where \( x^a = \{ x^\mu, z, y^i \} \) and \( g_{ab}, g_{rr} \) are the corresponding components of the bulk metric (15). Then, considering that \( \rho(x) \) does not depend on \( z, y^i \) and depends on \( x^\mu \) slowly as compared to the scales of the bulk solution, the Lagrangian \( L_{\text{bs}} \) of the local source in (1) (with constants (2) in it and taking account of (15)) takes the form

\[
L_{\text{bs}} = -M^8 \sigma e^{-\varphi/12} \delta(r - \rho) \sqrt{-g^{(4)}} \frac{\sqrt{-h^{(9)}}}{\sqrt{-g^{(10)}}} \approx \frac{M^8 \sigma e^{-\varphi/12} \delta(r - \rho)}{H^{14/48}} \left[ 1 + \frac{1}{2} H g^{\mu\nu} \rho_{,\mu} \rho_{,\nu} \right].
\]

In calculating the radion effective potential we shall substitute \( r_{\text{IR}} \to r = 0 \) in the lower limit of the integration over \( r \) in (1). This will not change \( S^{(3+1)} \) essentially since it is supposed that \( r_{\text{IR}} \ll L \) and since all integrals in (1) are convergent at \( r = 0 \). We also postulate that \( Z_2 \) symmetry at the ‘moved’ UV boundary surface is preserved, i.e. bulk integration must be fulfilled over two pasted copies of the solution. Direct calculation shows that for the magnetic monopole fluxbrane bulk solution (15) this procedure gives zero value of the radion potential at \( \rho = r_0 \) (18) where jump conditions (11)–(14) are valid; this corresponds to the consistency conditions [48]. This is not the case for the dual bulk electric fluxbrane solution—see Note 2 below.

Thus symbolically the Brans–Dicke type effective action \( S^{(3+1)} \) depending on the general metric \( \tilde{g}_{\mu\nu}(x) \) of the manifold \( M_{(3+1)} \) and on the radion field \( \rho(x) \) is found when extra coordinates are integrated out in the action (1):

\[
S^{(4)} = 2 \int_0^\rho L_{\text{bulk}} + \int L_{\text{bs}} = \int \left[ \Phi(\rho) \tilde{R}^{(4)} - \frac{1}{2} \omega(\rho) \tilde{g}^{\mu\nu} \rho_{,\mu} \rho_{,\nu} - \tilde{V}(\rho) \right] \sqrt{-\tilde{g}^{(4)}} d^{(4)}x,
\]

where \( L_{\text{bulk}} \) sums up all bulk terms in (1) including the Gibbons–Hawking term, \( L_{\text{bs}} \) is given in (24); \( \tilde{R}^{(4)} \) is the scalar curvature of the \((3 + 1)\)-dimensional space–time described by metric \( \tilde{g}_{\mu\nu}(x) \) slowly depending on \( x^\mu \). The Brans–Dicke field \( \Phi(\rho) \), kinetic term function \( \omega(\rho) \) and auxiliary radion potential \( \tilde{V}(\rho) \) in (25) are calculated when the bulk metric (15) is used in (1) where it is taken that \( \tilde{g}_{\mu\nu} = \eta_{\mu\nu} \) is the Minkowski metric in four dimensions; \( Q_{(4)}, \sigma, \varphi_\infty \) may be expressed through the characteristic length of the throat \( L \) with the use of dependences given in (15), (19); \( T_z \) is the period of torus \( S^1 \) in (15), \( \Omega_4 \) is the volume of the 4-sphere of unit radius. Simple calculations finally give:

For the Brans–Dicke field:

\[
\Phi(\rho) = 2M^8 \Omega_4 T_z \int_0^\rho H r^4 dr = 2M^2(ML)^8(MT_z)\Omega_4 \left[ \frac{1}{5} \left( \frac{\rho}{L} \right)^5 + \frac{1}{2} \left( \frac{\rho}{L} \right)^2 \right].
\]

For the kinetic term function:

\[
\omega(\rho) = M^8 \Omega_4 T_z \int \sigma e^{-\varphi/12} \delta(r - \rho) H^{4/3} r^4 dr
\]

\[
= M^4(ML)^8(MT_z)\Omega_4 12 \left( \frac{2}{3} \right)^{1/3} \left( \frac{\rho}{L} \right)^4 \left[ 1 + \left( \frac{L}{\rho} \right)^3 \right]^{4/3}.
\]
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And for potential in (25) (expression in square brackets includes the GH term; gravity constraint was used in finding it):

\[
\tilde{V}(\rho) = M^8 \Omega_4 T_z \left\{ -2 \int_0^\rho \left[ \frac{24}{H^{5/8} r^2} - \frac{Q^2 \phi_{\infty}^2 H^{-1/8}}{H^{5/2} r^2} \right] H^{5/8} r^4 \, dr \\
+ \int \sigma e^{-\phi_{\infty}/12} H^{1/24} \delta(r - \rho) H^{5/16} r^4 \, dr \right\} = 4 M^4 (ML)^3 (MT_z) \Omega_4 F \left( \frac{r}{L} \right),
\]

(28)

where the function \( F(y) \) is given by the formula

\[
F(y) = y^3 \left[ 3 \left( \frac{2}{3} \right)^{1/3} (1 + y^3)^{1/3} + \frac{3}{2(1 + y^3)} - 4 \right],
\]

(29)

and the value of \( y_0 \) is found from (18). It is easy to see that \( F(y) \) possesses a minimum at \( y = y_0 \) and \( F(y_0) = 0 \). The same is true for the potential \( \tilde{V}(\rho) \) (28) at \( \rho = r_0 \).

Note 1. Although the Gibbons–Hawking term is a full divergence and we consider a compact extra space it would be a mistake to discard the GH term in (25) when the radion effective potential is calculated. Direct calculation of the GH term in (25), when step functions reflecting mirror \( Z_2 \) jumps of \( b(r), c(r), a(r), \phi(r) \) are taken into account, shows that it really vanishes at the solution of the dynamical equations, i.e. at \( \rho = r_0 \). But the GH contribution to the radion effective potential is by no means equal to zero when the upper limit of integration in (25) is changed from \( r_0 \) to an arbitrary value \( \rho \).

Note 2. It is impossible to calculate from the action (1) the physically meaningful radion effective potential in the case when the dual electric 6-form \( F_6 \) is used in (1). Although the electric 4-brane extremal solution is given by the same formulae (15), (18)–(21) as a magnetic one, the values of action (1), \( S_m \) and \( S_e \), calculated at the magnetic and electric fluxbrane solutions as a background, differ drastically. General consistency conditions [48] say that \( S_m \) must vanish at the solution of the dynamical equations but these conditions are not applicable to \( S_e \) (see the appendix in [1]).

According to (28) and (29) \( \tilde{V}(\rho) \to 0 \) at \( \rho \to 0 \) and \( \tilde{V}(\rho) \to \infty \) at \( \rho \to \infty \). However, this behavior is of no physical interest since a similar behavior possesses the Brans–Dicke field (26). To get the physical radion effective potential the low-dimensional Brans–Dicke effective action (25) must be written in the Einstein-frame metric and the radion field must be transformed in a way providing the canonical form of its kinetic term.

Thus let us rescale the metric \( \tilde{g}_{\mu\nu} \) in the Brans–Dicke action in the RHS of (25) to the Einstein-frame metric \( g_{\mu\nu} \):

\[
\tilde{g}_{\mu\nu} = \frac{M_{Pl}^2}{\Phi(\rho)} g_{\mu\nu},
\]

(30)

where \( M_{Pl} = 10^{19} \) GeV is the Planck mass.

The effective action (25) being expressed as a functional of the Einstein-frame metric \( g_{\mu\nu} \) introduced in (30) and of the canonical radion field \( \psi \) (defined below) takes the
\begin{equation}
S^{(4)} = \int \left[ M_{Pl}^2 R^{(4)} - \frac{1}{2} M_{Pl}^2 (\nabla \psi)^2 - \mu^4 V(\psi) \right] \sqrt{-g^{(4)}} \, d^{(4)}x. \tag{31}
\end{equation}

\(\mu\) is a calculable constant of the dimensionality of mass—the characteristic of the radion potential, \(V(\psi)\) is taken dimensionless for convenience. Also the dimensionless (normalized to the Planck mass) canonical radion field \(\psi\) is introduced in (31):

\begin{equation}
\psi(\rho) = \frac{1}{L} \int_{r_0}^{\rho} \epsilon(\rho) \, d\rho = \int_{y_0}^{y} \epsilon(y) \, dy, \quad y = \frac{\rho}{L}, \tag{32}
\end{equation}

here the point (18) of the stable extremum of the radion effective potential is chosen at \(\psi = 0; \ y_0 = 2^{-1/3}\); \(\epsilon(\rho)\) is expressed through functions \(\Phi(\rho)\) and \(\omega(\rho)\) given in (26) and (27):

\begin{equation}
e^2(y) = L^2 \left[ \frac{\omega(\rho)}{\Phi(\rho)} + 3 \left( \frac{1}{\Phi(\rho)} \frac{d\Phi}{d\rho} \right)^2 \right]
= 6 \left( \frac{2}{3} \right)^{1/3} y^4 \left( \frac{y^5}{5} + \frac{y^2}{2} \right)^{-1} \left( 1 + \frac{1}{y^3} \right)^{4/3} + 3y^8 \left( \frac{y^5}{5} + \frac{y^2}{2} \right)^{-2} \left( 1 + \frac{1}{y^3} \right)^2 \tag{33}\end{equation}

It is seen from (33) that in the \(\rho \ll L \ (y \ll 1)\) limit, i.e. inside the throat, \(\epsilon(y) \sim y^{-1}\) and in the \(\rho \gg L \ (y \gg 1)\) limit we have \(\epsilon \sim y^{-1/2}\). Hence it follows from (32) that in these two limits:

\begin{equation}
\psi(y) = c \cdot \ln y, \quad c = 2(18^{1/3} + 3)^{1/2}, \quad 0 < y = \frac{\rho}{L} \ll 1, \tag{34}
\end{equation}

and

\begin{equation}
\psi(y) = 2(10 \cdot 18^{1/3})^{1/2} y^{1/2}, \quad 1 \ll y = \frac{\rho}{L} < \infty. \tag{35}
\end{equation}

The radion potential \(\mu^4 V(\psi)\) in (31) is expressed through the auxiliary potential \(\tilde{V}(\rho)\) (28) and Brans–Dicke field \(\Phi(\rho)\) (26):

\begin{equation}
\mu^4 V(\psi) = M_{Pl}^4 \frac{\tilde{V}(\rho)}{\Phi(\rho)^2} = \frac{M_{Pl}^4}{(MT_x)(ML)^7 \Omega_4} K(y(\psi)), \tag{36}
\end{equation}

where the function \(F(y)\) is given in (29) and the dependence \(y(\psi)\) must be found from (32) and (33). The characteristic density \(\mu^4\) of the radion effective potential is defined in (36). It depends on the period \(T_x\) of torus \(S^1\) in the metric (15) and on the length of the throat \(L\); the expression for \(ML\) is given in (19), while the value of \(MT_x\) will be calculated in section 4. Because of the strong inequality \(ML \gg 1\) demanded by the applicability of the low-energy string approximation \(\mu^4\) in (36) proves to be suppressed as compared to the Planck density \(M_{Pl}^4\). This is important since an effective action approach is valid only if the radion potential in (31) is essentially below the Planck density

\begin{equation}
\mu^4 V(\psi) \ll M_{Pl}^4. \tag{37}
\end{equation}
We will see in section 4 that, although $V(\psi)$ is growing down the throat, inequality (37) is valid everywhere in the region of applicability of the low-energy string approximation given by the condition (17).

The form of the dimensionless potential $V(\psi)$ (36) depends only on the choice of the theory. For type IIA supergravity with the co-dimension one local source (choice 2 of dimensionalities and coupling constants) $V(\psi)$ is drawn in [1] (curve ‘D’ in figure 1 of [1]). The potential is non-negative: as expected it possesses zero minimum at $\psi = 0$ where junction conditions (11)–(14) are valid. To the right of this point $V(\psi)$ increases, reaches a maximum and then again falls down to zero at infinity. Thus the stable state $\psi = 0$ where supposedly our Universe ‘lives’ is protected from the runaway decompactification by a certain potential barrier. This situation is typical for all theories with compactified extra dimensions [50]. It is not without interest to study in the framework of the considered model to what extent this ‘protection’ is reliable. But we will leave this work for the future.

Asymptotic behavior of the dimensionless radion potential $V(\psi)$ in the limits $\psi \ll -1$ and $\psi \gg 1$ immediately follows from (36), taking account of expression (29) for $F(y)$ and asymptotic (34) and (35) for $\psi(y)$:

For $\psi \ll -1$:

$$V_-(\psi) = (2^{7/3}3^{2/3} - 10)e^{-\psi/c} \approx 0.48 \cdot e^{-0.21\psi}, (38)$$

where $c$ is given in (34).

And at $\psi \gg 1$:

$$V_+(\psi) = 2^{20} \cdot 3^5 \cdot 5^8 \cdot \left(\frac{2}{3}\right)^{1/3} \cdot \psi^{-12} \approx 8.7 \times 10^{13} \psi^{12}. (39)$$

Now we can look shortly at the possibility to apply these results to the description of inflation in the early Universe [51,45,46]. The radion field introduced above hopefully may serve as an inflaton field (cf [19b,44]). We may suppose that initially the ‘heavy lid’ boundary of the extra space was located somewhere deep in the throat ($\rho_n \ll L$ or $\psi_n \ll -1$) and after that, obeying the dynamics determined by the action (31), it rolls down the exponential asymptotic $V_-(\psi)$ (38) of the radion potential (36) to the steep slope leading to a stable brane’s position (18) ($\psi = 0$, see (32)) at the top of the throat.

The following questions must be answered. Does the radion potential $V(\psi)$ (36) meet the necessary flatness and slow roll conditions? Can this scenario provide the number of e-foldings $N_e$ during inflation demanded by the astrophysical observations ($N_e \approx 80–100$) [45–47]? For the exponentially decreasing potential $V(\psi) \sim e^{-k\psi}$ flatness and slow roll conditions demand $k^2 \ll 1$, which seemingly is true for $k = 0.21$ as in (38). The number of e-foldings during inflation is given by the simple formula [45,46] (prime means derivative over $\psi$ which, we remind ourselves, is dimensionless—in Planck units):

$$N_e = \int_{\psi_n}^{\psi_{fin}} \frac{V(\psi)}{V'(\psi)} d\psi = \frac{\psi_{fin} - \psi_n}{k}, (40)$$

where the last equality is found for the exponential potential; $\psi_n$ and $\psi_{fin}$ are the values of the radion (inflaton) field in the beginning and at the end of the inflation. Thus for the value of $k = 0.21$ (38) it follows from (40) that the necessary number of e-foldings is reached if $\psi_{fin} - \psi_n > 20$. 

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The end of inflation where reheating begins is expected at the beginning of the steep slope of the radion potential. Analyses of the exact analytical expression of $V(\psi)$ (36) shows that the steep slope begins somewhere at $\psi_{\text{fin}} \approx -20$. Hence to achieve the sufficiently long period of inflation the initial value of the radion (inflaton) field must satisfy $\psi_{\text{in}} \leq -40$, i.e. the initial position of the ‘heavy lid’ boundary must be sufficiently deep in the throat.

Let us look at the validity of the inequality $\psi_{\text{in}} \leq -40$ from the point of view of applicability of the low-energy string approximation. The permitted values of the isotropic coordinate $r$ must obey inequality $r > r_{\text{min}}$ where $r_{\text{min}}$ is determined in (17). The corresponding minimal permitted value $\psi_{\text{min}}$ may be calculated from the asymptotic expression (34) (where it is taken that $y_{\text{min}} = \rho_{\text{min}} / L = r_{\text{min}} / L$). If we express $(ML)$ in (17) from (19) then the value of $\psi_{\text{min}}$ is found from (34):

$$\psi_{\text{min}} = 38 \cdot \ln k - 180 - 76 \cdot \ln g.$$  \hspace{1cm} (41)

As long as $k$ in (17) is of order one and the parameter $g \geq 1$ (according to (19) this is demanded by the condition of validity of the approach $ML \gg 1$) this value of $\psi_{\text{min}}$ is essentially below the value providing the necessary number of e-foldings during inflation ($\psi_{\text{in}} \approx -40$). Thus permitted length of the throat does not come into conflict with demands of early inflation.

To be sure that the effective action approach of this section is consistent the validity of inequality (37) must be established. This will be done in section 4.3.

4. Deformation of the elementary fluxbrane solution

4.1. General formulae

In this section equations (5)–(8) will be analysed for the case of non-zero ‘Maxwell’ field $F_{(2)}$ given in ansatz (4) and non-zero small constant positive curvature of the Universe, i.e. when $Q_{(2)} \neq 0, \tilde{h} \neq 0$ in (5)–(8). Let us rewrite the metric of ansatz (4) in the form

$$ds_{(10)}^2 = b^2(g_{\mu\nu} \, dx^\mu \, dx^\nu + U \, dz^2) + f^2 \left( \frac{dr^2}{U} + r^2 \, d\Omega_4^2 \right),$$  \hspace{1cm} (42)

this is always possible with a transformation of the isotropic coordinate $r$ in (4). The non-deformed solution (15) of equations (5)–(8) looks for the metric (42) as

$$b = \bar{b} = H^{-3/16}, \quad U = \bar{U} = 1, \quad f = \bar{f} = H^{5/16}, \quad e^\varphi = e^{\tilde{\varphi}} = e^{\varphi_{\infty}} H^{-1/4},$$  \hspace{1cm} (43)

where we included for convenience the expression (15) for the ‘non-deformed’ dilaton field.

There is a well-known [25]–[31] exact Schwarzshild type bulk solution generalizing the metric (15) in the way of (42) where

$$U = U_{\text{Sch}} = 1 + \frac{\text{const}}{r^3}$$  \hspace{1cm} (44)

and $b, f, \varphi$ are like in (43). This solution was used in [1] to build the IR end of the throat in the ‘bolt’ point where $U_{\text{Sch}} = 0$. However, as was estimated in [1] and was shown exactly in [37] in a 6D generalization of the Randall–Sundrum model, the value of dark energy density found from the Schwarzshild type deformation of the elementary throat-like solution is about 60 orders above the observed value $10^{-120} M_{\text{Pl}}^4$. That is why in [37] not a
Schwarzschild type but a Reissner–Nordstrom type deformation of the Randall–Sundrum AdS model was used, and it was shown that in this case the calculated value of the dark energy density may be in accordance with observations.

Before starting the analyses of the solution of equations (5)–(8) when \( Q_{(2)} \neq 0 \) and \( \tilde{h} \neq 0 \) it is worthwhile to outline shortly the logic of the introduction in these models of the extremely small positive curvature of the Universe. The point is that the presence of \( U(r) \neq \text{const} \) in the metric (42) results in a discrepancy of the Israel junction conditions at the UV boundary of space–time (the discrepancy appears since it is supposed that the energy–momentum tensor of the boundary is isotropic). Because of the rapid decrease of the additional term, depending on \( r \), in \( U(r) \) with increase of \( r \) from the IR end to the UV end of the warped extra space this discrepancy at the UV end proves to be quite small. The remedy may be the introduction of small non-zero positive curvature of the Universe which will give an additional term in \( U(r) \) repairing the Israel junction conditions. However, the decrease with growth of \( r \) of the Schwarzschild term (44) proves to be insufficiently quick and, as was said above, does not give the observed value of the dark energy density; a more satisfactory result may be expected when the Reissner–Nordstrom type deformation is used.

To our knowledge the exact solution of equations (5)–(8) when \( Q_{(2)} \neq 0 \) and \( \tilde{h} \neq 0 \) is not found yet. If \( Q_{(2)}, \tilde{h} \) are small as compared to the scales of the non-deformed solution (15), and this is the case under consideration, then induced variations of \( b(r), f(r), \varphi(r) \) in (42) are also small as compared to their ‘non-deformed’ values (43) and may be studied in a linear approximation of equations (5)–(8). Also the variations of position of the UV boundary and of the ‘fine-tuning’ condition, i.e. variations of \( r_0, Q_{(4)} \) whose ‘non-deformed’ values are given in (18) and (21), will be small. However, in the context of the present paper there is no need to calculate all these small variations.

The peculiarity of the situation is that a change of \( U(r) \) in (42) does not need to be small as compared to \( U = \bar{U} = 1 \) of (43). To find \( U(r) \) it is sufficient to subtract equations (5) and (6) where, in accordance with (42), we put \( c^2 = Ub^2, N^2 = f^2 / U, a = rf \). The resulting equation for \( U(r) \) looks as follows:

\[
U'' + U' \left( 5 \frac{b'}{b} + 3 \frac{f'}{f} + \frac{4}{r} \right) = -2f^2 \frac{e^{-3\varphi_{\infty}/2} Q_{(2)}^2}{2b^8 f^8 r^8} - f^2 \frac{6\tilde{h}^2}{b^2}.
\]  

(45)

Since \( Q_{(2)}, \tilde{h} \) in the RHS of (45) are supposed to be small, the other functions in (45) \( (b(r), f(r), \varphi(r)) \) may be taken in zero approximation. Substitution of their expressions (43) into (45) gives

\[
U'' + \frac{4}{r} U' = -\frac{Q_{(2)}^2 e^{-3\varphi_{\infty}/2}}{r^8} - 6\tilde{h}^2 \left( 1 + \frac{L^3}{r^3} \right).
\]  

(46)

The free solution of (46) is, as expected, the Schwarzschild potential (44); in what follows we shall discard this term of \( U(r) \) for the reasons explained above in this section.

Subtraction of junction conditions (11) and (12) taking into account \( c^2 = Ub^2 \) gives the simple condition of their consistency:

\[
U'(r_0) = 0.
\]  

(47)

Strictly speaking (47) must be valid at the location of the \( Z_2 \)-symmetric UV boundary slightly shifted from its ‘non-deformed’ position (18). But in the lowest approximation we may take in (47) \( r_0 = L/2^{1/3} \) given in (18). From (47) the value of \( \tilde{h} \) will be determined.
4.2. Case $F_{(2)} \neq 0$, $\tilde{h} = 0$. Determination of modulus $T_z$

As will be seen the value of $\tilde{h}$ determined from condition (47) is extremely small; hence at the IR end of the throat and practically everywhere inside the throat the $\tilde{h}$ term is essentially below the $Q_{(2)}$ term in the RHS of (46). Thus let us first put down the solution of equation (46) in the case $\tilde{h} = 0$:

$$U = 1 - \left( \frac{l}{r} \right)^6, \quad r^6 = \frac{Q^2_{(2)} e^{-3\phi_{\infty}/2}}{18},$$

(48)

it is supposed that $l \ll L$, which means that deformation (48) of the elementary solution (15) is small; we remind ourselves that the Schwarzshild term $\sim r^{-3}$ (see (44)) is deliberately omitted in (48). The metric (42) with $U(r)$ given in (48) is a Euclidean ‘time’ version of the Reissner–Nordstrom generalization of the elementary throat-like solution.

The ‘bolt’ point $r = l$, where $U = 0$ is the IR end of the throat, is topologically equivalent to the pole of a 2-sphere [34]–[36]. Space–time (42) may possess conical singularity at this point in the case a ‘matter trapping’ co-dimension two IR brane is placed there (see, e.g., [36,52,53]). This will produce a deficit angle $\delta_d$ depending on the tension of the IR brane which will influence the value of the period $T_z$ of the Euclidean ‘time’ $S^1$ calculated from (42) and (48). We shall not consider this option in the present paper and postulate that $\delta_d = 1$, hence the IR end of the throat is supposed to be smooth.

Then taking the zero-order dependences (43) for $b(r)$, $f(r)$ in the metric (42) and (48) for $U(r)$ the following expression for the period of the torus $S^1$ of space–time (15) (or (42)) is found:

$$T_z = \frac{2\pi}{3} H^{1/2} l \approx \frac{2\pi}{3} L \left( \frac{L}{l} \right)^{1/2},$$

(49)

where $H(r)$ is given in (15) and it is taken at $r = l$; the last approximate equality is valid since $l \ll L$. Thus the period $T_z$ of the extra torus of space–time (42) is not an arbitrary modulus of the solution but is determined by (49) through the characteristic lengths of the throat $L, l$. From (49) it follows that $T_z \gg L$.

Substitution of modulus (49) in expression (26) for the Brans–Dicke field $\Phi$, taking into account (18) and (19), gives the important quantity, entering the formulae for hierarchies in section 5, as a function of $l$ and the parameter $g$ (20):

$$\frac{M}{\sqrt{\Phi(r_0)}} = 10^{-4} g^{-3} \left( \frac{l}{L} \right)^{1/4},$$

(50)

where the coefficient $10^{-4}$ absorbs numbers of formulae (18) for $r_0$, (19) for $ML$, (26) for $\Phi$ and (49) for $T_z$, including the value of the volume of the 4-sphere of unit radius $\Omega_4 = 8\pi^2/3$ in (26).

4.3. Consistency of the effective action approach

Now when we determined the modulus $T_z$ (49) it is possible to check the inequality (37) which is the condition of applicability of the low-dimensional effective action approach of section 3. Since the potential (36) grows down the throat it is sufficient to verify (37) at the IR end of the throat, i.e. at $\rho = l$. 
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Substitution in (37) of the asymptotic expressions (38) and (34) for $V(\psi)$, $\psi(\rho)$ (and taking into account formulae (19) and (49) for $ML$, $MTz$ entering in the definition of $\mu^4$ in (36)) inequality (37) at $\rho = l$ is expressed through location $l$ of the IR end of the throat and the parameter $g$ defined in (20):

$$\frac{\mu^4}{M^4_{Pl}} V(\psi(l)) = 6 \times 10^{-11} g^{-8} \left( \frac{L}{l} \right)^{1/2} < 1. \quad (51)$$

The location of the IR end of the throat must meet the demand $l > r_{min}$ of validity of the low-energy action (1), where $r_{min}$ is estimated in (17). It is interesting to note that, even at this depth, inequality (51) remains to be valid. More: its validity does not depend on the value of the parameter $g$ which drops out from the expression (51). In fact, substitution of $l = r_{min}$ in (51) taking into account expression (17) for $r_{min}$ gives

$$\frac{\mu^4}{M^4_{Pl}} V(\psi(r_{min})) = 10^{-2} \frac{k^4}{k^4} < 1, \quad (52)$$

where this inequality is valid since the coefficient $k$ introduced in (17) is supposed to be of the order of 1.

### 4.4. Adjustment of Israel conditions and determination of $\tilde{h}$

Condition (47) of consistency of the Israel junction equations for subspaces $M_{(3+1)}$ and $S^1$ of the boundary of space–time (42) cannot be fulfilled for $U(r)$ given in (48). To repair the Israel conditions the necessary anisotropy of the energy–momentum tensor of the boundary was introduced in [35, 36] in a 6D model. But perhaps it would be more natural to escape the arbitrary modifications of the action of the local source in (1) and to resolve the problem with the introduction of a small positive curvature of space–time $M_{(3+1)}$ [37]. We shall go this way. Thus taking $\tilde{h} \neq 0$ in the RHS of (46) and taking account of (48) the following expression for $U(r)$ is found from equation (46):

$$U = 1 - \left( \frac{l}{r} \right)^6 - \frac{3}{5} \tilde{h}^2 r^2 + \frac{3\tilde{h}^2 L^3}{r}. \quad (53)$$

Then $\tilde{h}$ is immediately determined from (47) (where $r_0 = L/2^{1/3}$ (18)):

$$\tilde{h} = \sqrt{\frac{2^{5/3} \cdot 5}{3} \frac{1}{L} \left( \frac{L}{L} \right)^3}. \quad (54)$$

Condition $U = 0$ gives the location of the IR end of the throat. The presence of the $\tilde{h}$ terms in the expression for $U(r)$ (53) will make a shift of this position from the value $r = l$ determined from (48). This shift is, however, extremely small since it follows from (54) that at $r = l$ the main (second) $\tilde{h}$ term in the RHS of (53) is suppressed by the factor $(l/L)^5$ as compared to the $Q_{(2)}$ term. Since $l \ll L$ we may consider $r = l$ the location of the IR end of the throat. Actually the ‘curvature’ $\tilde{h}$ terms may be neglected in (53), as well as in the RHS of (46), practically everywhere inside the throat; they become comparable with the ‘Maxwell’ $Q_{(2)}$ terms only in the vicinity of the top of the throat $r \approx L$, although both remain quite small there.
An auxiliary ‘Hubble constant’ $\tilde{h}$ (54) characterizes the curvature $\tilde{R}^{(4)}$ of the manifold $M_{(3+1)}$ (see (9)). To find the observed rate of acceleration of the Universe $h$ we must rescale $\tilde{R}^{(4)}$ with transformation (30) to the Einstein-frame curvature $R^{(4)}$. This will be done in section 5.

5. Calculation of the mass-scale hierarchy and of the ‘acceleration hierarchy’

5.1. Formula for mass scale hierarchy

Following the Randall and Sundrum approach [32] we take mass parameters of matter action written in the primordial metric of the action (1) equal to the fundamental scale $M$. Also it is conventionally supposed that massive matter of the standard model is concentrated near the IR end of the strongly warped space–time—let it be because of trapping at the IR brane or because of pure gravitational accretion at the IR end. Then, in the case the warped throat-like solution (15) is considered, the mass of the visible matter is decreased as compared to $M$ by the value of the warp factor $H^{-3/16}$ at $r = r_{IR}$:

$$M \rightarrow M H^{-3/16}(r_{IR}) \approx M \left(\frac{r_{IR}}{L}\right)^{9/16},$$

(55)

where it was taken into account that $H \approx (L/r)^3$ at $r \ll L$.

In the previous section the IR end of the throat was built as a ‘bolt’ point $r = l$ of the deformed metric (42) and (48). In what follows we shall put

$$r_{IR} = l > r_{min},$$

(56)

where $r_{min}$ (17) is the point in the depth of the throat where effective low-energy string-induced action (1) is not valid any more.

To find the observed electro-weak scale $m$ it is necessary to write down the effective matter action in lower dimensions in the Einstein-frame metric $g_{\mu\nu}$ introduced in (30). The Brans–Dicke field $\Phi(\rho)$ in (30) must be taken at $\rho = r_0$ (18), i.e. in the minimum of the radion effective potential (28) (or equivalently in the minimum of the potential (36) at $\psi = 0$) where supposedly our Universe is stabilized after inflation and reheating.

In calculating the mass-scale hierarchy it is possible to use expression (26) for $\Phi(r_0)$ found by integration out of extra coordinates in the action (1) and (2) when the non-deformed solution (15) is taken as a background. The only impact of deformation upon these calculations is dynamical fixation of the IR end of the throat at $r = l$ and determination of the period of the torus $T_z$ (49) performed in section 4. Thus, from (55), (30) and (50) the following expression is found for the mass-scale hierarchy as a function of the location of the IR end of the throat $l$ and the parameter $g$ (20) of the fluxbrane solution:

$$\frac{m}{M_{Pl}} = H^{-3/16} \frac{M}{\sqrt{\Phi(r_0)}} = 10^{-4} \cdot g^{-3} \cdot \left(\frac{l}{L}\right)^{13/16}. $$

(57)

In the case $g = 1$ to get the observed value of the mass hierarchy it is necessary to place the IR end of the throat sufficiently deep: $l/L \approx 10^{-16}$. This value of $l$ practically coincides with the limit of validity of the low-energy approximation $r_{min}$ (17). For $g > 1$ the situation becomes less dangerous.
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For the limiting depth of the throat, i.e. when we substitute in (57) \( l = r_{\text{min}} \) from (17) (where the formula (19) for \( ML \) is used) expression (57) is

\[
\frac{m}{M_{\text{Pl}}} = 10^{-17} k^{13/2} g^{-16}.
\]

(58)

It is seen that (58) gives the observed value of the mass-scale hierarchy \( m/M_{\text{Pl}} = 10^{-16} \) for \( g \approx 1, \ k \approx 1 \). Of course this game in numbers should not be taken seriously since the RHS of (58) is strongly dependent on free parameters \( k, \ g \). It is interesting, however, that a large value of the mass hierarchy may be found without the introduction of large numbers ‘by hand’.

The large number \( 10^{17} \) in (58) appeared here ‘from nothing’, i.e. from coefficient 10.5 in (19) (in a general case equal to \( 4(n - 1)[\Delta/2(n - 1)]^{1/(n-1)} \), see (26) of [1], \( \Delta \) is given in (9) of [1]) and from the exponent 16 in (17) (in a general case equal to \( \Delta/\alpha^2 \)). Here we have \( n = 4, \ \Delta = 4, \ \alpha = 1/2 \).

Physically, expression (58) for the mass-scale hierarchy follows from the ‘bold’ hypothesis of [1, 49c] that SM resides at the brink of existence of the target space–time, i.e. that massive matter falls down the very ‘bottom’ of the throat and concentrates there, being stopped by unknown higher-curvature terms not included in the low-energy action (1). In any case the unambiguous result of the paper not depending on these speculations is given by expression (57) for the mass-scale hierarchy.

5.2. Values of the acceleration rate and of dark energy density

In section 4.4 expression (54) for the auxiliary ‘Hubble constant’ \( \tilde{h} \) was deduced from the Israel junction conditions at the UV boundary of the throat. To find the observed rate of acceleration of the Universe \( h \) (equal to \( 10^{-60} M_{\text{Pl}} \) according to the observations) it is necessary to perform a scale transformation (30) taken at the point \( \rho = r_0 \) of the extremum of the radion potential (as was done for \( m \) in expression (57) above):

\[
\frac{h}{M_{\text{Pl}}} = \frac{\tilde{h}}{M} \frac{M}{\sqrt{\Phi(r_0)}} = 10^{-5} \cdot g^{-4} \cdot \left( \frac{l}{L} \right)^{13/4}.
\]

(59)

The last equality was found from (54), (50) and (19).

Finally it is instructive to express the ‘acceleration hierarchy’ \( h/M_{\text{Pl}} \) not through \( l/L \) but through the value of the mass hierarchy \( m/M_{\text{Pl}} \) (57):

\[
\frac{h}{M_{\text{Pl}}} = 10^{11} g^8 \left( \frac{m}{M_{\text{Pl}}} \right)^4.
\]

(60)

The simple, fourth power, dependence of two hierarchies is a real gift after many of the cumbersome exponents above.

Dark energy density \( \rho_{\text{DE}} \) responsible for acceleration of the Universe (60) is equal to

\[
\rho_{\text{DE}} = \frac{1}{2} M_{\text{Pl}}^2 R^{(3+1)} = 6h^2 M_{\text{Pl}}^2 = 6 \times 10^{22} g^{16} \left( \frac{m}{M_{\text{Pl}}} \right)^8.
\]

(61)

It is seen that in the case parameter \( g = 1 \) (\( g \) is defined in (20)) the observed value of the dark energy \( 10^{-120} M_{\text{Pl}}^4 \) is found from (61) for \( m = 10 \text{ GeV} \).
In the limiting depth of the throat substitution of $r_{IR} = l = r_{\text{min}}$ (17) into (59) gives
\[ \frac{h}{M_{\text{Pl}}} = 10^{-57} g^{-56} \kappa^{26}. \] (62)

The drawback of this expression, as of (58) for the mass-scale hierarchy, is strong dependence of the RHS on the values of arbitrary parameters of order one. We outline, however, that possibly the main result of the paper $\rho_{\text{DE}} \sim G_{\text{N}}^2 m^8$ (61) ($G_{\text{N}} = M_{\text{Pl}}^{-2}$ is Newton’s constant) does not depend on the ‘bold’ hypothesis described at the end of the previous subsection.

5.3. $\rho_{\text{DE}}$ as a value of the radion potential in its extremum

In section 3 the exact analytical form of the radion effective potential was found for the non-deformed background solution; it was shown that the potential possesses zero minimum at the value of the radion field where all dynamical equations, including junction conditions (11)–(14), are fulfilled. To repeat the same calculations for the deformed background (42) is not a simple task, the more so in that we do not know the exact bulk solution when $F_{(2)} \neq 0$ in ansatz (4) and when the curvature of the manifold $M(3+1)$ is not equal to zero. It is possible to show, however, that deformation of the background will result in a tiny shift of the value of the potential $\mu^4 V_{\text{extr}}$ (36) in its extremum from zero to the value equal to the dark energy density $\rho_{\text{DE}}$ (61).

The tool for calculation of $\mu^4 V_{\text{extr}}$ is the general consistency condition of paper [48] which is valid at the solution of the dynamical equations. In our case expression (12) of [48], written when the parameter $\alpha_{[49]}$ of this paper is taken equal to $p = 3$, gives
\[ \oint b^4 (T^m_m - T^\mu_\mu) = -2 \oint b^2 \tilde{R}^{(3+1)}, \] (63)
here $T^m_m$, $T^\mu_\mu$ are traces of the energy–momentum tensor of matter fields ($F_{(4)}$, $F_{(2)}$, $\varphi$) in (1) in internal subspaces and in four dimensions correspondingly; $b$ is the warp factor in metric (4) ($W$ in [48]); $\oint$ symbolizes the integration over compact internal space which is deciphered in the LHS of (25) where the upper limit of integration $\rho$ is to take the value $\rho = r_0$ (18) determined by the dynamical jump conditions (11)–(14). $\tilde{R}^{(3+1)}$ is the curvature of the manifold $M(3+1)$ of space–time (4) which is equal to zero for the non-deformed space–time (15) and is equal to $12\tilde{h}^2$ in the case the deformed metric (42) is considered.

Also, as was shown in the appendix in [1], in the case the form fields of the action (1) ‘live’ only in internal space the combination of components of the energy–momentum tensor in the LHS of (63) is proportional to the Lagrangian $L$ of the action (1) calculated on the solution of dynamical equations:
\[ T^m_m - T^\mu_\mu = -4L. \] (64)

Hence from (63), (64) it follows that at $\rho = r_0$ in (25):
\[ \oint b^4 L = \frac{1}{2} \Phi_{\text{extr}} \tilde{R}^{(3+1)}. \] (65)
We took into account here that $\oint b^2 = \Phi_{\text{extr}}$ is the value of the Brans–Dicke field in (25) at the point of the extremum of the potential.
According to definition (25) effective action in four dimensions in the point of extremum where the radion field is constant is equal to

$$\int b^4 L = \Phi_{\text{extr}} \tilde{R}^{(3+1)} - \tilde{V}_{\text{extr}} = \tilde{V}_{\text{extr}},$$  

(66)

where the last equality is valid because the value of the action is calculated at the solution of the Einstein equations in four dimensions.

Thus finally from (65) and (66) it follows that $\int b^4 L = \tilde{V}_{\text{extr}}$. The same is true for the extremal value of the potential in the Einstein frame action (31):

$$\mu^4 V_{\text{extr}} = \frac{1}{2} M_{Pl}^2 \tilde{R}^{(3+1)} = 6h^2 M_{Pl}^2 = \rho_{\text{DE}},$$

(67)

where $\rho_{\text{DE}}$, see in (61).

This rather strong result does not depend on details of the solution and follows from the consistency condition (63) in the case the proportionality (64) is fulfilled. To check up (64) is a simple task, whereas (63) was found in [48] after a certain integral of full divergence over the compact internal space was put equal to zero. And on this point special caution is demanded as it was noted in [48] as well. It would be important to check up with direct calculation the consistency condition (63) for the space–time of type (42) with the ‘bolt’ point (where $U = 0$) topologically equivalent to the 2-sphere.

6. Conclusion

This paper presents three apparently physically interesting results:

(1) Exact expression (36) for the scalar field potential $\mu^4 V(\psi)$ in the effective action (31) is calculated for the non-deformed fluxbrane solution as a background. Asymptotic (38) of the potential describes slow-roll inflation, where the potential possesses a steep slope for reheating and zero minimum where matter-dominating evolution of the Universe begins.

(2) Formula (67) for the tiny positive deviation (seen today as dark energy density $\rho_{\text{DE}}$ (61)) of the extremal value of the radion effective potential calculated for the ‘deformed’ background.

(3) Expressions (57) for the mass-scale hierarchy $m/M_{Pl}$, (59) for ‘acceleration hierarchy’ $h/M_{Pl}$ and their relation (60) which gives the non-trivial dependence $\rho_{\text{DE}} \sim G_N^2 m^8$ (61), where $G_N$ is Newton’s constant. This dependence is progress as compared to Zeldovich’s ‘numerology’ where $\rho_{\text{DE}} \sim G_N m^6$ [54].

Also it was demonstrated that under a natural additional hypothesis (that SM resides at the border of space–time where low-energy string approximation stops being valid) large numbers ($10^{17}$ and $10^{57}$ in (58) and (62)) may be found ‘from nothing’, i.e. from dimensionalities $D = 10$, $n = 4$ and the value of the 4-form-dilaton coupling constant $\alpha = 1/2$.

All quantitative results of the paper depend solely on the choice of the theory—the type IIA supergravity in this paper. It would be interesting to trace the logic of the paper for some other theories.

Since the canonical radion field $\psi$ is associated with position $\rho(x)$ of the UV boundary terminating the throat (see (22) and (32)) it would be interesting to find the description of the non-trivial effective dynamics in four dimensions given by the action (31) with
Potential for the slow-roll inflation, mass-scale hierarchy and dark energy from type IIA supergravity potential (36) in the language of a heavy boundary moving in a higher-dimensional non-stationary background formed taking account of the gravitational back-reaction of this $\mathbb{Z}_2$-symmetric co-dimension one local source.

The basic difficulty of the approach of this paper is the lack of physical grounds for the very appearance of the UV boundary surface of the throat and for the choice of its dynamics. The simplest Nambu–Goto choice taken in the action (1) is crucial for the calculations of this paper. But ‘simplest’ does not mean ‘well grounded’.

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