Solvability and stability of the inverse Sturm–Liouville problem with analytical functions in the boundary condition

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1. In Theorem 2.3, the first sentence should be read as follows: “Let \( f_j(\lambda), j = 1, 2, \) be entire functions, let \( \{\lambda_n\}_{n=1}^{\infty} \) and \( \omega \) be complex numbers such that the sequence \( \{v_n\}_{n=0}^{\infty} \), constructed by them, is an unconditional basis in \( H \) and \( \left\{ \frac{w_n}{\|v_n\|_H} \right\}_{n=0}^{\infty} \in L_2. \)” The latter condition is important for convergence of the series at step 3 of Algorithm 2.1.

2. In Proposition 3.7, the second sentence should be read as follows: “Then there exists \( \varepsilon > 0 \) (depending on \( q \)) such that, for any functions \( K, \bar{N} \in L_2(0, \pi) \), satisfying the estimate

\[
\Theta := \max \left\{ \|K - \bar{K}\|_{L_2(0, \pi)}, \|N - \bar{N}\|_{L_2(0, \pi)} \right\} \leq \varepsilon
\]

and the condition

\[
\int_0^\pi (K(t) - \bar{K}(t)) \, dt = 0, \quad (*)
\]

there exists a unique function \( \tilde{q} \in L_2(0, \pi) \) such that \( \omega = \tilde{\omega} \) and \( \{\bar{K}, \bar{N}\} \) are the Cauchy data of \( \tilde{q}. \)” The reason of condition (*) is that the Cauchy data are related with the constant \( \omega \) by the relation \( \int_0^\pi K(t) \, dt = \omega. \) This relation holds, since the function \( \eta_1(\lambda) \) defined by formula (9) in the original paper is analytical at \( \lambda = 0. \) Adding condition (*) does not influence on the application of Proposition 3.7 in the proof of Theorem 3.1, because relation (28) for \( n = 0 \) implies \( \int_0^\pi \bar{K}(t) \, dt = \omega. \)

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