Article

The Impact of Lubricant Film Thickness and Ball Bearings Failures

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NAVAIR Public Release 2018-609 Distribution Statement A - "Approved for public release; distribution is unlimited"

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Abstract: An effort was made to find a relationship between the ratio of average asperities height and lubricant thickness at the point of contact of rolling element ball bearings, and empirical equations to predict the life for bearings under constant motion. Two independent failure mechanisms were considered, fatigue failure and lubricant failure resulting in seizing of the roller bearing. A theoretical formula for both of these methods was established for the combined probability of failure using both of these failure mechanisms. Fatigue failure was modeled with the empirical equations of Lundberg and Palmgren and standardized in DIN/ISO281. The seizure failure, which this effort sought to investigate, was predicted using Greenwood and Williamson’s theories on surface roughness and asperities during lubricated contact. These two mechanisms were combined, and compared to predicted cycle lives of commercial roller bearing, and a clear correlation was demonstrated. This effort demonstrated that the Greenwood-Williams theories on the relative height of asperities versus lubricant film thickness can be used to predict the probability of a lubricant failure resulting in a roller bearing seizing during use.

Keywords: Lubrication, Ball Bearings, Roller Bearings, Failures, L10, Film Thickness

1. Introduction

Ball bearings are used in countless mechanical applications to convert sliding mechanical contact into rolling contact [1–3], dramatically reducing friction energy losses. Sliding contact inherently has a high friction force, as random asperities can contact the surface and induce wear and damage to machined parts [4–9]. Rolling contact, however, has dramatically lower friction; the overwhelming majority of the friction loss is merely hysteresis from elastic deflections of the circular bearings.

Rolling element bearings are one of the most common configuration of ball bearings, with the bearings contained in a circular race to allow continued circular motion. So long as there is a minimum surface friction to enable the bearings to roll, there will be a dramatic reduction in circular friction for an object spinning inside or outside of the races. Bearings can be spherical, cylindrical, or a host of different configurations depending on the applications of the ball bearings.

A well built bearing can last indefinitely, however all mechanical objects have some risk of failure. Despite the previous assumptions that stresses less than half of yield have no significant risk of failure, there is always some risk of fatigue and fracture, which may manifest itself in the life of a ball bearing. The most likely bearing failure, however, is lubricant failure causing the bearings to seize. Ball bearings overwhelmingly use lubricant oils and greases to ensure there isn’t an excessive build-up of heat and friction between the races and the bearings. While a minimum amount of friction is necessary to ensure the bearings roll rather than slide (often specified as a minimum axial load), too much friction can cause the bearings to stick to the race and seize up, rather than allowing rolling.

Friction is inherently random and variable, as it is impacted by the different random surface asperities; as such it is incredibly difficult to model. The usual (but not exclusive) mechanism of lubricant failure is as followed: a high enough friction will heat the lubricant, which will reduce the
viscosity of the lubricant, which will increase the friction heating, and this feedback loop will continue until the friction between the bearing and the races is so great that the bearing seizes. If a bearing seizes during a critical application, the results can be catastrophic.

While it is impossible to truly know the exact nature of every bearing surface, empirical equations can be generated to determine the $L_{10}$ life from a known bearing load, lubricant cleanliness, lubricant viscosity, and continuous bearing speed. The $L_{10}$ life is defined as the number of revolutions a bearing can experience before a 10% chance of bearing failure. This effort is to study how tribological properties such as the lubricant film thickness \[10–12\] can serve to predict the change of failure after a single revolution, and thus estimate the $L_{10}$ life.

2. Empirical Equations for $L_{10}$ Life

In order to properly develop a numerical model for ball bearing failures, it is necessary to have empirical data on bearing failure to verify and validate it. In this aim, the $L_{10}$ empirical equations provided by Svenska Kullagerfabriken (SKF) will be used as a baseline \[13,14\]; SKF is a Swedish company founded in 1907 and is currently the world’s largest manufacturer of ball bearings. They have a bearing calculator that provides the $L_{10}$ life in revolutions before the bearings have a 10% chance of failure.

The core equation for $L_{10}$ life is
\[
L_{10} = A_{SKF} \cdot \left( \frac{C}{P} \right)^{\hat{p}} \cdot 10^6, \quad (1)
\]
where $C$ (N) is the basic dynamic load rating, $P$ (N) is the equivalent load, and $A_{SKF}$ is the Life Modification Factor. The value of $\hat{p}$ was found empirically, and it is 3 for spherical bearings and $10/3$ for cylindrical bearings \[15–17\]. The value of $A_{SKF}$ is a function of the the combined influence of load and contamination on fatigue $\beta$; and the viscosity ratio $\kappa$, which represents the lubrication conditions and their influence on fatigue. The basis for the $\left( \frac{C}{P} \right)^{10/3}$ component of equation 1 is based on empirical research of Lundberg and Palmgren \[15–17\].

The dimensionless value of $\kappa$ is a ratio of the kinematic viscosity $\nu$ (m$^2$/s) over the rated viscosity $\nu_1$ (m$^2$/s)
\[
\kappa = \frac{\nu}{\nu_1}, \quad (2)
\]
where $\nu_1$ is a function of both the speed $\Omega_{rpm}$ and the average bearing diameter $d_m$ (m)
\[
\nu_1 = f(\Omega_{rpm}, d_m), \\
d_m = \frac{1}{2} (D + d),
\]
where $D$ (m) and $d$ (m) represent the diameter of the inner and outer bearing race. The value of $\kappa$ can range from 0.1 to 4.0, where $\kappa = 0.1$ represents total metal-on-metal contact, and $\kappa = 4.0$ represents a total lubricant coating. SKF did not publish their equation for $\nu_1$, but it can be determined from the SKF bearing calculator. A least squared analysis was performed, and an estimated function for the rated viscosity $\nu_1$ (m$^2$/s) is defined in equation 3
\[
\nu_1 = 689.2653 \cdot 10^{-6} \cdot d_m^{-0.52706} \cdot \Omega_{rpm}^{-0.7565}, \quad (3)
\]
where the mean diameter $d_m$ is in meters and the bearing speed $\Omega_{rpm}$ is in revolutions per minute. Calculated values of $\nu_1$ (m$^2$/s) are plotted in units of centistokes or mm$^2$/s in Figure 1.
The other term necessary to determine $A_{SKF}$ is the dimensionless coefficient $\beta$, which is the product of the cleanliness factor $N_c$ and the safety factor ratio of the fatigue load limit $P_u$ (N) over the equivalent bearing load $P$ (N)

$$\beta = N_c \frac{P_u}{P}.$$  \hspace{1cm} (4)

The cleanliness factor $N_c$ ranges from 0.2 to 1.0, with 0.2 representing the dirtiest possible lubricant, and 1.0 representing a perfectly clean lubricant. In this analysis, the lubricant will be assumed to be clean, with $N_c = 1$. The equivalent load $P$ (N) is a combination of radial and axial loads [18]

$$P = X_a F_a + X_r F_r,$$  \hspace{1cm} (5)

where $F_a$ (N) and $F_r$ (N) are the axial and radial loads, and $X_a$ and $X_r$ are bearing specific coefficients. For example, for spherical thrust bearings $X_a=1$ and $X_r=1.2$.

The SKF website provides tables for the value of $A_{SKF}$ as a function of $\beta$ and $\kappa$, as well as a calculator tool, but no specific formula was given. For this reason, the least squared method was used, and a close match all throughout the permissible range of $\beta$ and $\kappa$ yielded the empirical equation 6

$$A_{SKF} = (C_{1,1} \kappa^3 + C_{2,1} \kappa^2 + C_{3,1} \kappa + C_{4,1}) \cdot (C_{1,2} \beta^3 + C_{2,2} \beta^2 + C_{3,2} \beta + C_{4,2}),$$  \hspace{1cm} (6)

where the values of $C_{ij}$ are tabulated in Table 1. Values of $A_{SKF}$ calculated with equation 6 as a function of $\beta$ and $\kappa$ are plotted in Figure 2. Once the value of $A_{SKF}$ is determined, it can be used in equation 1 to find the $L_{10}$ life, defined as the number of revolutions the bearing can undergo before encountering a 10% chance of failure. If one were to determine the probability of failure during a single revolution of the bearings $P_f$, it can easily be defined as

$$P_f = 1 - 0.9^{1/L_{10}}.$$  \hspace{1cm} (7)

| $j$ | $i=1$ | $i=2$ | $i=3$ | $i=4$ |
|-----|-------|-------|-------|-------|
| 1   | -0.0559 | 0.3682 | -0.3540 | 0.1012 |
| 2   | 8.43233 | -8.2419 | 6.672284 | -0.043546 |
Figure 2. Values of $A_{SKF}$ calculated with equation 6 as a function of $\beta$ and $\kappa$. Circles represent data points obtained with the SKF bearing calculator [13].
3. Tribological Predictions of $L_{10}$ Life

Equation 1 can predict the $L_{10}$, but it gives no information as to the mechanics of the failure; it is a purely based on empirical data. In order to better understand the mechanism of failure, a model based on the tribological properties to find the values of $L_{10}$ needs to be developed, with equation 1 being used to verify and validate this model.

Regardless of the $L_{10}$ life, a ball bearing failure can happen; $L_{10}$ life is really a function of the probability of failure in the face of random conditions such as surface asperities. The most common form of bearing failure is seizure, where excessive friction can yield increased heating, which reduces the lubricant viscosity, increasing the friction, until eventually the friction increases till it is high enough that the bearing seizes. Another potential cause of failure is a failure in fatigue; this will increase exponentially with increasing load relative to fatigue life. For the purpose of the analysis, the driving cause of failure will be treated as an excessively high increase in friction from the approximated average friction.

Friction is never constant in practice, it constantly fluctuates about a given average, therefore, this failure prediction model will be normalized to a given quantity of standard deviations away from the mean friction

$$P_f = \frac{1}{2} \text{erfc}(\mu),$$

$$\text{erfc}(\mu) = 1 - \frac{2}{\sqrt{\pi}} \int_{0}^{\mu} e^{-t^2/2} dt,$$

where \( \text{erfc} \) represents the complementary error function, and \( \mu \) represents the Z-factor

$$\mu = \frac{x_i - x_m}{\sigma},$$

that corresponds to a given failure probability \( P_f \) for a single revolution. In equation 9, \( x_i \) (m) refers to a given asperities height over a given duration, \( x_m \) (m) refers to the mean asperities height of the roller bearing race, and \( \sigma \) (m) refers to the standard deviation of the asperities height for the given roller race.

The relationship between \( \mu \) and the $L_{10}$ life is thus

$$\mu = \text{erfc}^{-1}\{2 \cdot (1 - 0.9^{1/L_{10}})\}.$$

4. First Parametric Study

A parametric study was conducted, utilizing the SKF NUP 2304-ECP cylindrical roller bearing. The radii of the individual cylindrical bearings are \( R=4.455 \) mm, and the length is 13.267 mm; the mean radius that the bearings rotate at is 72 mm. The fatigue load limit \( P_u \) is 4,800 N, and the basic dynamic load rating \( C \) is 47,500 N. The bearing is made of steel, so the Young’s Modulus \( E_y \) will be 210 GPa,
120 and the poisson’s ratio \( p \) will be 0.3. The parametric study would calculate both the \( L_{10} \) life as defined
121 in equation 1, and compare it to the predicted lubricant film thickness [10–12,18,27–38], as well as the
122 relative fatigue load. The parametric study was conducted for a temperature ranging between 40°C
123 and 100°C, in increments of 2°C; an equivalent load ratio of 50 kN to 200 kN (in increments of 10
124 kN); and a bearing speed from 5,000 to 20,000 RPM, in increments of 1,000 RPM. With each of these
125 parameters, the \( L_{10} \) life was calculated with equation 1 and equation 6, and an equivalent \( \mu \) was found
126 with equation 10.

127 The next step was to predict the film thickness of the lubricant at the point of contact between the
128 bearings and the rollers during elastohydrodynamic contact [1,39–46]. In 1974, empirical equations by
129 Hamrock & Dowson [32] characterized the minimum \( h_0 \) (m) and central \( h_c \) (m) film thickness

\[
\begin{align*}
(h)_{min} & = 3.63R'(U_n^{0.68})(C_n^{0.49})(W_n^{-0.073})(1 - \exp[-0.68\kappa_{ellipse}]) \\
(h) & = 2.69R'(U_n^{0.67})(C_n^{0.53})(W_n^{-0.067})(1 - 0.61\exp[-0.73\kappa_{ellipse}]) \\
U_n & = \frac{\mu_0 U}{E'R'}, \\
G_n & = \alpha_{PVC}E', \\
W_n & = \frac{W}{E'R'^2}.
\end{align*}
\]

where \( h_{min} \) (m) is the minimum film thickness, \( h_c \) (m) is the central film thickness, \( U_n \) is the
130 dimensionless speed parameter, \( G_n \) is the dimensionless material parameter, \( W_n \) is the dimensionless
131 load parameter, \( \kappa_{ellipse} \) is the ellipticity of the contact area, \( \mu_0 \) (Pa-s) is the dynamic viscosity of the
132 lubricant at atmospheric pressure, \( \alpha_{PVC} \) (Pa\(^{-1}\)) is the pressure viscosity coefficient, and \( U \) (m/s) is the
133 velocity of contact. The reduced Young’s Modulus \( E' \) (Pa) and reduced radius \( R' \) (m) are for Hertz
134 contact equations for elastic deflection [1,47]. Assuming cylindrical rollers and a consistent material is
135 used, the equations for \( E' \) and \( R' \)

\[
\begin{align*}
R' & = \frac{R}{2}, \\
E' & = \frac{E_y}{1 - p^2}.
\end{align*}
\]

where \( R \) (m) is the radius of the cylindrical bearing, \( E_y \) (Pa) and \( p \) is the Young’s Modulus and
137 Poisson’s ratio of the bearing material.
138
139
140 If there is a given friction force that will cause the bearings to seize, and the friction is affected by
141 the ratio of the height of the surface asperities (which follow a normal distribution) over the lubricant
142 film thickness, an accurate equation for \( \mu \) as a function of \( h_c \) (m) was realized with equation 19

\[
\mu = X_1 + X_2 \exp(-\frac{h_c}{\sigma}) + X_3 \sqrt{\frac{p}{P_f}},
\]

where \( \sigma \) was predicted as 1 \( \mu m \) RMS for the surface asperities, and \( P_f \) was defined as 4,800 N. Equation
142 19 incorporated two separate failure mechanisms, where \( X_2 \) is a coefficient for the lubricant seizure
143 based on friction (originating from Greenwood Williams theory [19–26]), and \( X_3 \) is a coefficient for the
144 rolling contact fatigue failure [15–17]. The fatigue life theory is an entirely different and independent
145 failure mechanism from lubricant seizure; equation 19 combines both potential failures into \( \mu \) to obtain
146 an overall probability of bearing failure \( P_f \) during a given revolution.

The calculated value of \( \mu \) found with equation 19 closely matches the value of \( \mu \) found with
150 equation 10 (utilizing empirical equations 1 and 6), and is observed to match in Figure 3. The
151 coefficients for this particular design is \( X_1 = 8.1130 \), \( X_2 = -3.1285 \), and \( X_3 = -1.0173 \). By taking the
Figure 3. Calculated values of the $L_{10}$ life, utilizing theoretical equation 19 and empirical equation 10, all for a parametric series of loads, speeds, and lubricant temperatures. The data is placed in ascending order of $L_{10}$ life with the experimentally validated SKF empirical equations.

value of $\mu$ defined in equation 19, and using equations 7 and 8 one can predict the $L_{10}$ life of a roller bearing

$$L_{10} = \log(0.9)/\log\{1 - 0.5\cdot\text{erfc}(\mu)\}.$$  \hspace{1cm} (20)

5. Second Parametric Study

A second parametric was conducted to see if varying the bearing size would affect the coefficients for equation 19. The mean bearing radius was modeled from 30 mm to 500 mm. With a changing bearing diameter, the radius of the rollers $R (m)$ was consistently adjusted so 25 rollers in the bearings would consistently fit within the roller bearing circumference

$$R = \frac{d_m \pi}{2 \cdot N_r},$$

where $N_r = 25$ represents the number of cylindrical roller bearings. As observed in Figure 4, the first two coefficients, $X_1$ and $X_2$ are a clear function of the inverse of the diameter

$$X_1 = 5.1358 + \frac{0.2130}{d_m}, \hspace{1cm} (21)$$

$$X_2 = 0.2071 - \frac{0.2330}{d_m}.$$
Figure 4. Coefficients of equation 19 as a function of average bearing diameter $d_m$.

The third coefficient $X_3$ changes very little for changing average diameters $d_m$ (m), it is predominantly an average of -1.0. By plugging the values of equation 21 into equation 19, one can get a comprehensive equation for the effective bearing life

$$
\mu = 5.1358 + \frac{0.2130}{d_m} + (0.2071 - \frac{0.2330}{d_m}) \cdot \exp\left(-\frac{h_c}{\sigma}\right) - \sqrt{\frac{P}{P_{uf}}},
$$

$$
L_{10} = \log(0.9)/\log\{1 - 0.5 \cdot \text{erfc}(\mu)\}.
$$

The correlation coefficient $R$ was predicted for all values of $d_m$ (m) in equation 22, and consistently the correlation coefficient $R$, as plotted in Figure 5, exceeded 0.97.

6. Conclusion

In conclusion, a validated model to predict the probability of failures for roller bearings was developed. Empirical equations from SKF were developed from available data on commercial bearings to predict the $L_{10}$ life based on known bearing conditions (lubricant viscosity, bearing speed, loads). These conditions were used, along with the roller bearing geometry, to predict the lubricant film thickness at the central point of contact. A thicker film thickness is expected to inherently have lower friction, and therefore a lower chance of lubricant failure, and a clear trend of lubricant thickness impacting the probability of bearing failure per revolution is observed. The relative load to the fatigue load is also taken into consideration; fatigue is considered a minor yet calculable influence on determining the bearing $L_{10}$ life. This model demonstrates how the lubricant film thickness can be
Figure 5. The correlation coefficient $R$ of the SKF calculator data using equation 22, versus the SKF calculator results [13].
used to obtain a reasonable approximation for the life and probability of failure in seizing of a roller bearing.

Acknowledgments: The author would like to acknowledge Lou Vocaturo, Clay Merkel, Kevin Larkins, and Glenn Shevach for useful discussions.

Author Contributions: M.M. is the sole author of this manuscript.

Conflicts of Interest: The founding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, and in the decision to publish the results.

Abbreviations

The following abbreviations are used in this manuscript:

- MDPI: Multidisciplinary Digital Publishing Institute
- DOAJ: Directory of open access journals
- BB: Ball Bearing
- L_{10}: Number of revolutions before 10% chance of failure
- COF: Coefficient of Friction

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