Quantum Seals

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Abstract

A quantum seal is a way of encoding a classical message into quantum states, so that everybody can read the message error-free, but at the same time the sender and all intended readers who have some prior knowledge of the quantum seal, can check if the seal has been broken and the message read. The verification is done without reading nor disturbing the sealed message.

1 Introduction

Before the age of electronic transfer of information, important letters and documents were often closed using a wafer of molten wax into which was pressed the distinctive seal of the sender [1]. This was meant to fulfill different purposes, namely the authentication of the sender, but it also enabled the receiver to verify that the letter or document had not been opened and read by a third party. This is referred to as a classical seal.

It is important to notice the following points. The sender is not committed to the content, since she can always change her mind, write a new letter and seal it. Moreover the receiver needs to have prior knowledge of the symbol of the seal in order to verify that it is actually authentic — but still it could be falsified and in principle the only person who can truly verify that the seal and the content is authentic is the sender. Even further, the seal can be broken by anyone who wishes to learn the content of the letter, by simply physically breaking the seal, and then open and read the letter.
In this paper the idea for making a Quantum Seal is presented. The protocol belongs to the same category as quantum cryptography [2], quantum bit commitment [3, 4], quantum signature [5] and authentication of quantum messages [6] etc. For the quantum seal a classical message is encoded into quantum states, in such a way that the quantum seal can be broken by anyone and the message read error-free. However it is possible to verify if the message has been read by checking if the quantum seal has been broken. The first goal is that this verification can be performed by the sender. Ultimately the quantum seal will possess the same properties as the classical seal, which means that anyone with some prior knowledge about the quantum seal will be able to check if it has been broken.

A first implementation of the protocol for quantum seals will be presented. The encoding of the message is performed by using the simplest form of classical error correction codes [7]. And the verification of the quantum seal is performed by using the so-called ‘SWAP-test’ [8]. Eventhough this implementation is only resistant to single qubit attacks (see section 5) it serves as a good illustration of the properties of quantum seals.

2 Encoding and verification by Alice

The first step in order to create the quantum seal is to find a way of encoding a classical message into quantum states so that the encoder, Alice, can always verify if the message has been read, i.e. she can check if the quantum seal has been broken. At the same time anyone can actually read the message. This can be achieved in a very simple way.

A classical message can be encoded into quantum states and read perfectly without errors by anyone if Alice writes the message using qubits states always prepared in the same basis and then announces which basis she used. For example, if she wants to write a string of bits, then she could use the states $|0\rangle$ and $|1\rangle$ corresponding to the $z$ (computational) basis to signify 0 and 1 respectively, and then announce the basis she used. This will allow anyone to measure the qubits in the correct basis and read the message. But in this way, naturally Alice have no way of checking if someone actually did read the message.

In order for Alice to be able to check if the message has been read, it is necessary to add states from a different basis, which if measured in the wrong basis will lead to errors — similarly to what is done in quantum cryptography. Still, since everybody should be able to read the message this has to be done in a clever way.

This can be done by encoding each classical bit into a block of three
qubits in such a way that when measured in the basis announced by Alice, the encoding is self-correcting, which means that the bit can be learned without error. However, at the same time Alice can check that it has been read. This can be achieved in the following way: Two of the qubits will be prepared in the same state in the computational basis according to the bit value Alice wants to write, the third qubit is a control qubit which is prepared in a state from one of the mutually unbiased bases $x$ and $y$. In other words two of the qubits are prepared in the message reading-basis ($\ket{0}$ and $\ket{1}$), whereas one control qubit will be in one of the four states $\ket{0_x}$, $\ket{1_x}$, $\ket{0_y}$ or $\ket{1_y}$. The state and position of the control qubit is chosen at random by Alice — the choice is done independently for each block of qubits. This means that in each block of three qubits two qubits are in the correct state, for example if Alice wants to write bit value 0, two of the qubits will be in the state $\ket{0}$, whereas the state of the last qubit will be chosen at random between the states from the other mutually unbiased bases.

Now suppose Alice wants to write the classical bit sequence 0110..., then there are many ways for her to encode it, one way would be the following:

- $\ket{0}\ket{0}\ket{0_y}$ bit number 1, value = 0
- $\ket{0_x}\ket{1}\ket{1}$ bit number 2, value = 1
- $\ket{1}\ket{0_y}\ket{1}$ bit number 3, value = 1
- $\ket{0}\ket{0_x}\ket{0}$ bit number 4, value = 0

where each triplet of states corresponds to the encoding of one bit. Since Alice has announced in public which basis is the message reading-basis (here chosen to be the computational basis, i.e. $z$ basis), anyone can read it, but it will automatically introduce errors which can be identified by Alice. But since the encoding is self-correcting the message can nevertheless be read without errors.

To see this, assume that the reader is always measuring in the correct message reading-basis, then naturally all the states which are encoded in this basis will be read perfectly without error. But in each triplet there is one state which belongs to a basis which is mutually unbiased with the reading-basis, and for this state the bit value 0 or 1 is found with equal probability. For example $\ket{0}\ket{0}\ket{0_y}$, when read in the computational basis, could result in either 000 or 001 — with equal probability. However, since in each triplet there is only one control qubit, there can be at maximum one error and hence the correct bit value can be obtained by a simple majority vote. Which means that the message can be read without error.
On the other hand since the control qubit now has been measured in the reading-basis, a subsequent measurement in the correct preparation basis (in this case the $y$-basis) will lead to an error, with probability $1/2$. This will allow Alice at any stage to check if the message has been read, because she can at any time measure each qubit in its preparation basis and if she finds errors she will conclude that the seal has been broken and the message read.

Notice an interesting thing, namely that it is only with probability $1/2$ that the reader will learn the position of the control qubit. In the example described above, with probability $1/2$, the reader will find 000 from which the position of the control bit can not be identified, whereas with probability $1/2$ the result will be 001, from which it is obvious that the last bit corresponds to the position of the control qubit — but, of course, the reader learns nothing of the original state of the control qubit.

This part of the protocol enables everyone to read the message written by Alice error free, but at the same time it enables Alice to check if the quantum seal has been broken, because the message can not be read without giving rise to errors.

### 3 Intended reader verification

At this point in the protocol, Alice has written and sealed (encoded) her message into triplets of qubit states and stored them in a quantum memory which is accessible by anyone. Which means that the sealed message is now located in a public area. Notice that the sealed message is just a big product state of $N$ qubits, $|\phi_1\rangle|\phi_2\rangle|\phi_3\rangle\cdots|\phi_N\rangle$, where $|\phi_i\rangle$ is the state of the $i^{th}$ qubit and where $N = 3 \times$ (number of bits).

In order to have complete correspondence with the classical seal, not only Alice should be able to verify that the seal has not been broken, but all intended readers should be able to verify that the quantum seal is still intact, without reading, nor disturbing the message written by Alice.

In the case of the classical seal the receiver is familiar with the design of the symbol pressed into the wax, and uses this knowledge to identify the letter received as authentic. This means that prior knowledge is required in the classical case. For the quantum seal, this part of the protocol is obtained by Alice preparing additional qubits which she then distributes to the various intended readers, who can then use them to verify if the quantum seal is still intact.

The additional qubits prepared by Alice correspond to a subset of the message and control qubits. Alice does not reveal the state of any of the
qubits, but merely informs the receiver which qubits they correspond to in the sealed message. For example Bob-1, gets copies of qubit state number 1, 7, 16, 30, etc. but he doesn’t know the state, nor does he know if a given qubit is a message qubit or a control qubit (this issue will be addressed further below). Similarly, all the other intended readers (Bob-2 to Bob-L) likewise receive a set of qubits corresponding to a subset of the message and control qubits.

The verification of the quantum seal as such is done by performing a so-called SWAP test. A SWAP test can be used to evaluate if two states are identical, without any knowledge of the states themselves. This means that Bob-1, for example, can use his subset to make a SWAP test with the corresponding qubits in the sealed message. If he finds complete agreement, i.e. if all his qubits pass the SWAP test, he will conclude that the quantum seal is still intact and the message has not been read. Whereas if he finds that just one of his qubits fail the SWAP test he will conclude that the seal has been broken and the message has been read.

The SWAP test works in the following way: Bob chooses the qubit he wants to test, $|\phi_{test}\rangle$, and takes the corresponding qubit from the sealed message, $|\phi_{seal}\rangle$, (remember Bob knows which number it corresponds to, for example, qubit number 1). Then he couples the two qubits with an additional qubit, an ancilla, in the state $|0\rangle$, i.e. Bob now has the three qubit quantum state $|0\rangle|\phi_{test}\rangle|\phi_{seal}\rangle$, and he performs the following operations one the full system

\[
(H \otimes 1 \otimes 1) (c - \text{SWAP}) (H \otimes 1 \otimes 1) |0\rangle|\phi_{test}\rangle|\phi_{seal}\rangle = \frac{1}{2} |0\rangle (|\phi_{test}\rangle|\phi_{seal}\rangle + |\phi_{seal}\rangle|\phi_{test}\rangle) \\
+ \frac{1}{2} |1\rangle (|\phi_{test}\rangle|\phi_{seal}\rangle - |\phi_{seal}\rangle|\phi_{test}\rangle)
\]

where $H$ is the Hadamard transformation, $1$ the identity operator and SWAP is the transformation: $|\phi_{test}\rangle|\phi_{seal}\rangle \rightarrow |\phi_{seal}\rangle|\phi_{test}\rangle$ and $c$-SWAP means that it is a SWAP which is controlled by the ancilla (for a more detailed description of the SWAP test see [8]). These transformations are followed by a measurement of the ancilla in the $|0\rangle$, $|1\rangle$ basis. The state $|1\rangle$ is found with probability $\frac{1}{2} - \frac{1}{2} (|\phi_{test}\rangle|\phi_{seal}\rangle)^2$, from which it is seen that if $|\phi_{test}\rangle = |\phi_{seal}\rangle$, the ancilla will never be found in the state $|1\rangle$, but $|0\rangle$ will always be obtained. In other words if Bob obtains $|1\rangle$, he will conclude that the states were not identical, hence they did not pass the SWAP test.

\[\text{Alice can, of course, produce any number of copies of a given state, since she knows the state of any given qubit. So this does not go against the non-cloning theorem.}\]
If the sealed message has not been read, all of Bob's qubits will pass the SWAP test, whereas if the message has been read some of Bob's qubit will fail the SWAP test and he will consequently conclude that the quantum seal has been broken and assume that the message has been read.

Suppose that Bob’s test qubit is in one of the four states from the $x$ or the $y$ basis, which means that it is one of the control qubits; and assume that the message has been read in the $z$ basis (as announced by Alice) then the control qubit is no longer in its correct state, but in one of the states from the $z$ basis. If Bob performs a SWAP test, then, since all the involved states are mutually unbiased, the probability that the SWAP test will fail is $\frac{1}{2} - \frac{1}{2} |\langle \phi_{\text{test}} | \phi_{\text{seal}} \rangle|^2 = \frac{1}{2} - \frac{1}{2} \frac{1}{2} = \frac{1}{4}$. This means that if Alice supplies Bob with sufficiently many qubits, so that he can perform many SWAP tests, statistically Bob should obtain an error and conclude that the seal has been broken.

Notice one important point namely that the SWAP test does not give Bob any information about the state, nor will it destroy the seal if it is intact. Which exactly satisfies the intended reader verification requirements: the quantum seal can be verified without reading nor disturbing the sealed message.

4 Some security aspects

A fully detailed security analysis of the protocol for quantum seals is beyond the scope of the present paper. Indeed there may exist many different implementations of the idea and, most probably, each implementation must be analyzed independently. Here I mention a few security aspects related to the presented implementation of the protocol, which may also prove important to future implementations, and in the next section discuss the limitation of the proposed implementation.

The encoding: Someone may attempt to read the sealed message by reading only two out of three qubits in a triplet, hoping to obtain the same result twice and hence know the bit value without reading all three qubit states. The hope would be to avoid reading the control qubit and hence avoid making any errors. But if the sealed message is sufficiently long, errors should be guaranteed by statistics.

The distribution of subsets of qubits: If Alice provides the intended readers with the number of the qubits in public this could be exploited by others to avoid reading exactly those qubits. This means that each reader should know only his subset of qubits and not the subset of any other. However, this problem could easily be solved by Alice sending this
information to each of the intended readers using quantum cryptography.

The nature of the subsets: In each subset some of the qubits should correspond to message qubits and some will correspond to control qubits. In principle the message qubits can not be used to check the quantum seal (they will always pass the SWAP test) only the control qubits can be used for that. However each intended reader receives both message qubits and control qubits to avoid that they can use the knowledge of the position of the control qubits to cheat. The number of qubits supplied to each reader, should be such that by simply reading the subset he has received he can not not learn the message, but at the same time he must possess enough qubits in order to actually check if the quantum seal is still intact (in terms of good statistics).

5 Limitations of the proposed implementation

Above it was shown how someone trying to read the sealed message by measuring each qubit independently will automatically introduce errors, hence break the quantum seal and unavoidedly be detected.

However, for the proposed implementation, it is possible to completely avoid detection by making collective measurement on each triplet of qubits, hence reading the sealed message bit by bit (remember that each bit is encoded into three qubits). This is due to the fact that the twelve different three qubit states which encode bit value 0 are orthogonal to the twelve three qubit states which encode bit value 1. Indeed all the 0-states lie in the subspace spanned by the following states (in the z-basis), |000⟩, |001⟩, |010⟩ and |100⟩, whereas the 1-states all lie in the orthogonal subspace spanned by |111⟩, |110⟩, |101⟩ and |011⟩. This means that a measurement of the corresponding projectors:

\[
P_0 = |000⟩⟨000| + |001⟩⟨001| + |010⟩⟨010| + |100⟩⟨100|
\]

\[
P_1 = |111⟩⟨111| + |110⟩⟨110| + |101⟩⟨101| + |011⟩⟨011|
\]

will distinguish perfectly between the 0 and 1-states without disturbing the states. Which means that this kind of collective measurement will allow someone to read the sealed message without introducing any errors and hence will avoid detection.

In other words the presented implementation of the protocol is not robust against collective measurements, but only against single qubit attacks. However the described implementation serves as a perfect example and illustration of the properties of the protocol for quantum seals.
6 Concluding remarks

Quantum seals have been introduced, they can be used to verify if a classical message encoded into quantum states have been read. A first implementation of this idea in terms of classical error correcting codes and a so-called SWAP test has been presented. It has been shown that someone trying to read the sealed message by measuring each qubit independently will automatically introduce errors and hence break the quantum seal and be detected.

However, the proposed implementation is resistant only against single qubit attacks. Someone who is able to make a collective measurement on three qubits will be able to read the sealed message without making any errors. This means that it is an open problem to find an implementation of the protocol for quantum seals which offers protection against any kind of attack. A completely secure implementation of quantum seals may very well require different tools than the classical error correcting codes and the SWAP test which is used here, for example one could imagine the use of entanglement and Bell Inequalities.

It should be emphasized that Alice is not committed to the content of her message, as for the classical seal she is absolutely free to change her mind, write a new message and seal it. Moreover, one of the basic ideas is that anyone should be able to read the message written by Alice, which means that it is not the security of the content which has to be accounted for, but instead the guarantee that anyone reading the message will actually leave a trace, i.e. some errors.

Finally notice the following interesting points; The protocol for quantum seals require no classical communication between Alice and the intended readers contrary to most other protocols in the area of quantum cryptography. A priory it only requires that Alice is able to send quantum states and make public (classical) announcements. More, the classical seal is a physical object namely a wax wafer, whereas the quantum seal is somehow a way of encoding. I do not know of any protocol which can be implemented, for example, on a classical computer which works as a seal.

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