D-branes in type I string theory

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Abstract: We review the boundary state description of D-branes in type I string theory and show that the only stable non BPS configurations are the D-particle and the D-instanton. We also compute the gauge and gravitational interactions of the non-BPS D-particles and compare them with the interactions of the dual non-BPS particles of the heterotic string, finding complete agreement.

1 D-branes of type II theories

The Dirichlet branes \cite{DFT} can be described, in the weak coupling regime of string theory, by a boundary conformal field theory and admit a two-fold interpretation: on the one hand, they are objects on which open strings can end, and on the other hand they can emit or absorb closed strings. Therefore, introducing D-branes in a theory of closed strings amounts to extend their conformal field theory by introducing world-sheets with boundaries and imposing appropriate boundary conditions on the closed string coordinates $X^\mu$. In the operator formalism these boundary conditions are implemented through the so called boundary state $|Dp\rangle$ \cite{Gubser:2000}, whose bosonic part is defined by the following eigenvalue problem

$$\partial_\tau X^\alpha(\sigma, 0) |Dp\rangle_X = 0\ , \quad \left( X^i(\sigma, 0) - x^i \right) |Dp\rangle_X = 0 \ ,$$

where $\alpha = 0, \ldots, p$ labels the longitudinal directions, $i = p+1, \ldots, 9$ labels the transverse directions and the $x^i$'s denote the position of the brane in the transverse space. World-sheet supersymmetry requires that analogous equations must be also imposed on the left and right moving fermionic fields $\psi^\alpha$ and $\bar{\psi}^\alpha$, thus defining the fermionic part of the boundary state

$$\left( \psi^\alpha(\sigma, 0) - i \eta \bar{\psi}^\alpha(\sigma, 0) \right) |Dp, \eta\rangle_\psi = 0 \ , \quad \left( \psi^i(\sigma, 0) + i \eta \bar{\psi}^i(\sigma, 0) \right) |Dp, \eta\rangle_\psi = 0 \ ,$$

where $\eta = \pm 1$. Notice that there are two consistent implementations of the fermionic boundary conditions corresponding to the sign of $\eta$, and consequently there are two different boundary states

$$|Dp, \eta\rangle = |Dp\rangle_X |Dp, \eta\rangle_\psi$$

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both in the NS-NS and in the R-R sectors. The overlap equations (1) and (2) allow to determine the explicit structure of the boundary states (3) up to an overall factor. This normalization can then be uniquely fixed by factorizing amplitudes with closed strings emitted from a disk and turns out to be given by (one half of) the brane tension measured in units of the gravitational coupling constant, \( T_p = \sqrt{\pi} (2\pi \sqrt{\alpha'})^{3-p} \). We would like to remark that even if each boundary state \( |Dp,\eta\rangle \) is perfectly consistent from the conformal field theory point of view, not all of them are acceptable in string theory. In fact, to describe a physical D-brane a boundary state has to satisfy three requirements [3]:

\( i \)) to be invariant under the closed string GSO projection (and also under orbifold or orientifold projections if needed);
\( ii \)) the tree level amplitude due to the exchange of closed strings between two boundary states, after modular transformation, has to make sense as a consistent open string partition function at one-loop;
\( iii \)) the open strings introduced through the D-branes must have consistent couplings with the original closed strings and eventually, have to be compatible with the orbifold/orientifold projection.

Using these prescriptions, it is rather simple to find the boundary state for the supersymmetric BPS \( D_p \)-branes of type II. In particular, the GSO projection of the type II theories forces us to retain only the following linear combinations

\[ |Dp\rangle_{NS} = \frac{1}{2} \left[ |Dp, +\rangle_{NS} - |Dp, -\rangle_{NS} \right] , \quad |Dp\rangle_{R} = \frac{1}{2} \left[ |Dp, +\rangle_{R} + |Dp, -\rangle_{R} \right] \]  

(4)
in the NS-NS and in the R-R sectors respectively, with \( p = 0, 2, 4, 6, 8 \) for IIA, \( p = -1, 1, 3, 5, 7, 9 \) for IIB. To read the spectrum of the open strings living on the \( D_p \)-brane (called \( p\)-\( p \) strings), one has first to evaluate the closed string exchange amplitude \( \langle Dp | P | Dp \rangle \) where \( P \) is the closed string propagator, and then perform a modular transformation to the open string channel. Applying this procedure, one finds the following relations

\[ \begin{align*}
\text{NS} \langle Dp, \eta | P | Dp, \eta \rangle_{NS} &= \int_{0}^{\infty} \frac{ds}{s} \text{Tr}_{NS} q^{2L_0 - 1}, \\
\text{NS} \langle Dp, \eta | P | Dp, -\eta \rangle_{NS} &= \int_{0}^{\infty} \frac{ds}{s} \text{Tr}_{R} q^{2L_0}, \\
r \langle Dp, \eta | P | Dp, \eta \rangle_{R} &= \int_{0}^{\infty} \frac{ds}{s} \text{Tr}_{NS} (-1)^F q^{2L_0 - 1}, \\
r \langle Dp, \eta | P | Dp, -\eta \rangle_{R} &= \int_{0}^{\infty} \frac{ds}{s} \text{Tr}_{R} (-1)^F q^{2L_0} = 0 ,
\end{align*} \]

(5)

where \( q = e^{-\pi s} \). It is then clear that in order to obtain the supersymmetric (\( i.e. \) GSO projected) open string amplitude

\[ Z_{open,BPS} \int_{0}^{\infty} \frac{ds}{s} \left[ \text{Tr}_{NS} \left( \frac{1 + (-1)^F}{2} \right) q^{2L_0 - 1} - \text{Tr}_{R} \left( \frac{1 + (-1)^F}{2} \right) q^{2L_0} \right] , \]

(6)

one must consider the following boundary state

\[ |Dp\rangle = |Dp\rangle_{NS} \pm |Dp\rangle_{R} \]

(7)

where the sign ambiguity is related to the existence of branes and anti-branes.

The criteria \( i \) - \( iii \) defining physical D-branes do not rely at all on space-time supersymmetry, and thus one may wonder whether in type II theories there may exist also
non-supersymmetric branes. This problem has been systematically addressed in a series of papers by A. Sen [4], who constructed explicit examples of non-BPS branes. In particular, he considered the superposition of a D-string of type IIB and an anti-D-string (with a $Z_2$ Wilson line) and by suitably condensing the tachyons of the open strings stretching between the brane and the anti-brane, he managed to construct a new configuration of type IIB which behaves like a D-particle, does not couple to any R-R field and is heavier by a factor of $\sqrt{2}$ than the BPS D-particle of the IIA theory. The boundary state for this non-BPS D-particle has been explicitly constructed in Ref. [6]. This construction can be obviously generalized to the case of a pair formed by two BPS D($p+1$)-branes with opposite R-R charge (and with a $Z_2$ Wilson line) which, after tachyon condensation, becomes a non-BPS Dp-brane. Alternatively, this same non-BPS configuration can be described starting from a superposition of two BPS Dp branes with opposite R-R charge and modding out the theory by the operator $(-1)^{F_L}$ whose effect is to change the sign of all states in the R-R and R-NS sectors. In this second scheme, a superposition of a Dp-brane and anti-Dp-brane of type IIA (IIB) becomes in the reduced theory a non-BPS Dp-brane of type IIB (IIA). In either way we therefore find that there exist non-BPS Dp-branes for $p = -1, 1, 3, 5, 7, 9$ in IIA and $p = 0, 2, 4, 6, 8$ IIB. For reviews on this subject, see Refs. [7, 3].

A boundary state interpretation can be given to these non BPS branes provided they satisfy conditions i) - iii) mentioned above. Being manifestly non supersymmetric, the spectrum of open strings on their world volume is itself non supersymmetric. The corresponding vacuum amplitude in the open channel is in fact given by

$$Z_{\text{open, nonBPS}}^{p\rightarrow p} = \int_0^\infty \frac{ds}{s} \left[ \text{Tr}_\text{NS} q^{2L_0-1} - \text{Tr}_\text{R} q^{2L_0} \right],$$

where there is no GSO projection on the open string spectrum. Condition ii) and relations (5) then imply that the corresponding boundary state is, for any value of $p,$

$$|Dp\rangle = \sqrt{2} |Dp\rangle_{\text{NS}},$$

thus confirming the facts that non BPS branes are heavier by a factor of $\sqrt{2}$ than the BPS branes of same dimension and are neutral under R-R forms. Moreover, for a given theory, not any value of $p$ is acceptable. Indeed the closed string amplitude between a BPS and a non BPS brane of same dimension reads

$$\text{BPS} \langle Dp | P | Dp \rangle_{\text{nonBPS}} = \frac{1}{\sqrt{2}} \int_0^\infty \frac{ds}{s} \left[ \text{Tr}_\text{NS} q^{2L_0-1} - \text{Tr}_\text{R} q^{2L_0} \right],$$

which is not a sensible open string partition function due to the global irrational factor $1/\sqrt{2}.$ Thus, whenever a BPS Dp-brane exists, a non BPS one cannot, hence we conclude that non BPS branes exist for $p = -1, 1, 3, 5, 7, 9$ in IIA and $p = 0, 2, 4, 6, 8$ IIB.

It is clear from (8) that the non-BPS branes of type II are not stable, because the absence of the GSO projection on the open strings leaves the NS tachyon on their world-volume. However, they could become stable in an orbifold of type II theory, say IIA(B)/$P,$ provided that the tachyon be odd under the projection $P.$ In the orbifold theory in fact, the non-BPS vacuum amplitude of the $p-p$ open-strings reads

$$Z_{\text{open}} = \int_0^\infty \frac{ds}{s} \left[ \text{Tr}_\text{NS} \left( \frac{1+P}{2} q^{2L_0-1} \right) - \text{Tr}_\text{R} \left( \frac{1+P}{2} q^{2L_0} \right) \right],$$

and the natural question to ask is to which boundary state it corresponds. In the case of a space-time orbifold, the perturbative spectrum of the bulk theory contains only closed
strings which can be untwisted or twisted under the orbifold. Therefore, there are several sectors to which the bosonic states belong, and there exist different types of boundary states depending on which components in those sectors they have. The twisted boundary states then account for the piece depending on $P$ in (11). In the case of a world-sheet orbifold, however, this simple picture does not hold. To illustrate this point, we shall consider in the next section the specific case of the type I theory.

### 2 D-branes of type I theory

The type I theory is the orbifold of the IIB theory by the world-sheet parity $\Omega$. The distinctive feature of this model is that the perturbative states of the twisted sector of the bulk theory now correspond to unoriented open strings which should then be appropriately incorporated in the boundary state formalism. Let us briefly summarize how this is done [4]. The starting point is the projection of the closed string spectrum onto states which are invariant under $\Omega$. The corresponding closed string partition function is obtained by adding a Klein bottle contribution to the modular invariant (halved) torus contribution. The Klein bottle is a genus one non-orientable self-intersecting surface which may be seen equivalently as a cylinder ending at two crosscaps. A crosscap is a line of non-orientability, a circle with opposite points identified, and thus the associated crosscap state $|C\rangle$ is defined by

$$X^\mu(\sigma + \pi, 0) |C\rangle = X^\mu(\sigma, 0) |C\rangle, \quad \partial_\tau X^\mu(\sigma + \pi, 0) |C\rangle = -\partial_\tau X^\mu(\sigma, 0) |C\rangle,$$

and by the analogous relations appropriate for world-sheet fermions. As is clear from these equations, the crosscap state is related to the boundary state of the BPS space-time filling D9 brane through $|C\rangle \propto i^{L_0+L_0} |D9\rangle$, and its normalization turns out to be 32 times the normalization of the boundary state for the D9-brane. Consequently, the (negative) charge for the unphysical 10-form R-R potential created by the crosscap must be compensated by the introduction of 32 D9 branes. In this way we then introduce unoriented open strings starting and ending on these 32 D9 branes, whose vacuum amplitude is given by

$$Z_{9-9}^{open} = \frac{1}{2} \left(2^0 \langle D9|P|D9\rangle + 2^5 \langle D9|P|C\rangle + 2^5 \langle C|P|D9\rangle\right).$$

(12)

By adding to (12) the contribution of the Klein bottle we obtain an expression, in which the tadpoles for the massless unphysical states cancel. A moment thought shows that this corresponds to choose the open string gauge group to be $SO(32)$. Thus, we can say that the type I theory possesses a “background” boundary state given by

$$\frac{1}{\sqrt{2}} (|C\rangle + 32|D9\rangle),$$

(13)

where the factor of $1/\sqrt{2}$ has been introduced to obtain the right normalization of the various spectra. Performing a modular transformation, we can rewrite the amplitude of eq. (12) in the open string channel as follows

$$Z_{9-9}^{open} = \int_0^\infty \frac{ds}{s} \left[ \text{Tr}_{NS} \left( \frac{1+(-1)^F}{2} \frac{1+\Omega}{2} q^{2L_0-1} \right) - \text{Tr}_{R} \left( \frac{1+(-1)^F}{2} \frac{1+\Omega}{2} q^{2L_0} \right) \right],$$

(14)

where the part depending on $\Omega$ comes from the Möbius contribution. Thus, we see that in the type I theory the crosscap state plays the same role that the twisted part of the
boundary state had in the space-time orbifolds. This means that the vacuum amplitude for a generic $D_p$-brane is given by the half-sum of a cylinder and Möbius strip contribution as follows

$$Z_{\text{open}}^{p-p} = \frac{1}{2} \left( \langle Dp|P|Dp \rangle + \langle Dp|P|C \rangle + \langle C|P|Dp \rangle \right).$$  \hspace{1cm} (15)

In addition, the presence of background $D_9$-branes allows for open strings stretching between a $D_p$ and one of the 32 $D_9$-branes, whose partition function is given by the mixed amplitude

$$Z_{\text{open}}^{p-9\oplus 9-p} = \frac{32}{2} \left( \langle Dp|P|D_9 \rangle + \langle D_9|P|Dp \rangle \right).$$  \hspace{1cm} (16)

In the following, we shall use this remark in order to classify and obtain the boundary states for BPS and stable non-BPS branes of type I theory.

We have seen before that the type IIB theory contains BPS $D_p$-branes with $p = -1, 1, 3, 5, 7, 9$. The BPS $D$-branes of type I are then BPS branes of type II even under the $\Omega$ projection, a fact which occurs for $p = 1, 5, 9$, the R-R part of $|Dp\rangle$ being odd under $\Omega$ for the other values, $p = -1, 3, 7$. We thus have in principle $D_p$-branes with the following boundary state

$$\frac{1}{\sqrt{2}} |Dp\rangle = \frac{1}{\sqrt{2}} (|Dp\rangle_{\text{NS}} + |Dp\rangle_{\text{R}}),$$  \hspace{1cm} (17)

where the factor of $1/\sqrt{2}$ is there as in (13) in order to obtain the right normalization of vacuum amplitudes. A moment thought shows that the $\Omega$ projection has the effect to reduce the world volume gauge group on $N$ branes from $SU(N)$ to $SO(N)$, accordingly with the vacuum amplitude here obtained. However, there is a subtlety in the case of the $D_5$-brane. Indeed, due to the particular form of the Möbius strip in that case, it is not possible to interpret the vacuum amplitude obtained from a boundary state of the type (17) with $p = 5$ as a sensible partition function of open strings, e.g. containing a gauge vector in the adjoint representation of some group. In fact, the $D_5$-brane of type I is the projection of 2 $D_5$-branes of type II so that its boundary state is

$$\frac{2}{\sqrt{2}} (|D5\rangle_{\text{NS}} + |D5\rangle_{\text{R}}).$$  \hspace{1cm} (18)

In particular the R-R density charge of the $D_5$-brane is doubled so that the Dirac quantization rule relating the $D1$ and $D5$ R-R densities is still valid. From the vacuum amplitude we directly see that the gauge group on the world volume of $N$ $D5$-branes is $Sp(2N)$.

The type IIB theory contains unstable non-BPS $D_p$-branes with $p = 0, 2, 4, 6, 8$ which are described by the boundary state (1). Now, we address the question whether these $D$-branes become stable in the type I theory, i.e. we examine whether the tachyons of the $p-p$ open strings are removed by $\Omega$. As explained in [4], the world-sheet parity can be used to project the spectrum of the $p-p$ strings only if $p = 0, 4, 8$. Indeed, only in these cases $\Omega^2 = 1$ on the $p-p$ open strings, which are thus compatible with the orbifold projection (condition ii). Thus, the non-BPS $D2$ and $D6$ branes will not be further considered. However, we must take into account also another kind of configuration, namely the superposition of a BPS $Dp$-brane and an anti-$Dp$-brane of type IIB which clearly does not carry any R-R charge. This configuration is unstable due to the presence of tachyons in the open strings stretching between the brane and the anti-brane, but in the type I theory these tachyons might be projected out. A systematic analysis [8, 6] shows that in this case $\Omega$ can be used as a projection only if $p = -1, 3, 7$. 
In conclusion, we have to analyze the stability of the non-BPS D$p$-branes of type I with $p = -1, 0, 3, 4, 7, 8$ whose corresponding boundary states are given by

$$\frac{\mu_p}{\sqrt{2}} |Dp\rangle_{NS}$$

with suitable positive values of $\mu_p$. To address this problem, we need to consider the spectrum of the unoriented strings living on the brane world-volume (the $p$-$p$ sector), and also the spectrum of the open strings stretched between the D$p$-brane and each one of the 32 D9-branes of the background (the $p$-$9 \oplus 9$-$p$ sector), in which tachyonic modes could develop.

Let us first analyze the $p$-$p$ sector, whose total vacuum amplitude is given by Eq. (15). Performing the modular transformation to the open string channel, and expanding the resulting expression in powers of $q = e^{-\pi s}$, one can see [6] that

$$Z_{open}^{p-p} \sim \int_0^{\infty} \frac{ds}{2s} s^{-\frac{2p+1}{2}} q^{-1} \left[ \mu_p^2 - 2 \mu_p \sin \left( \frac{\pi}{4} (9 - p) \right) \right].$$

The $q^{-1}$ behavior of the integrand signals the presence of tachyons in the spectrum; therefore, in order not to have them, we must require that

$$\mu_p = 2 \sin \left( \frac{\pi}{4} (9 - p) \right).$$

Since $\mu_p$ has to be positive, the only possible solutions are

| $p$ | $-1$ | $0$ | $7$ | $8$ |
|-----|-----|-----|-----|-----|
| $\mu_p$ | $2 \sqrt{2}$ | $2$ | $2 \sqrt{2}$ |

From this table we see that in the type I theory there exist two even non-BPS but stable D$p$-branes: the D-particle and the D8-brane. Moreover, there exist also two odd non-BPS but stable D$p$-branes of type I: the D-instanton and the D7-brane. Their tension is twice the one of the corresponding type IIB BPS branes, in accordance with the fact that, as mentioned above, they can be simply interpreted as the superposition of a BPS brane with an anti brane. This classification of the stable non-BPS D-branes of type I based on the table (22) is in complete agreement with the results of Refs. [8] derived from the K-theory of space-time.

Let us now analyze the $p$-$9 \oplus 9$-$p$ sector by considering the ”mixed” cylinder amplitude (16) which, after the modular transformation, reads

$$Z_{open}^{9-p \oplus p-9} = 2^5 \mu_p V_{p+1} (8 \pi^2 \alpha')^{-\frac{p+1}{2}} \int_0^{\infty} \frac{ds}{2s} s^{-\frac{p+1}{2}} \left[ \frac{f_3^{p-1}(q) f_2^{9-p}(q)}{f_1^{p-1}(q) f_4^{9-p}(q)} - \frac{f_3^{p-1}(q) f_4^{9-p}(q)}{f_1^{p-1}(q) f_4^{9-p}(q)} \right].$$

where the first and second term in the square brackets account respectively for the NS and R sector, and the $f_i$'s are the standard one-loop functions [1]. This expression needs some comments. First, for $p = -1, 0$ we see that there are no tachyons in the spectrum; moreover, the values of $\mu_p$ for the D-instanton and D-particle are crucial in order to obtain a sensible partition function for open strings stretching between the non-BPS objects and the 32 D9-branes. Secondly, for $p = 7, 8$ we directly see the existence of a NS tachyon, so that the corresponding branes are actually unstable [6]. Hence, only the D-instanton and the D-particle are fully stable configurations of type I string theory. Nevertheless, the strict relation connecting the D0-brane and the D(-1)-brane to the D8-brane and the D7-brane respectively, suggests that also the latter may have some non trivial meaning.
Finally, we observe that the zero-modes of the Ramond sector of these $p$-$9 \oplus 9$-$p$ strings are responsible for the degeneracy of the non-BPS $D_p$-branes under the gauge group $SO(32)$; in particular the $D$-particle has the degeneracy of the spinor representation of $SO(32)$. Thus the $D$-particle accounts for the existence in type I of the non-perturbative non-BPS states required by the heterotic/type I duality.

We now present the basic ideas and results about the gravitational and gauge interactions of two stable non-BPS $D$-particles of type I string theory (the detailed calculations and analysis of these interactions can be found in [9]). In the type I theory, $D$-branes interact via exchanges of both closed and open bulk strings. Since the dominant diagram for open strings has the topology of a disk, it gives a subleading (in the string coupling constant) contribution to the diffusion amplitude of two branes which is thus dominated by the cylinder diagram, i.e. by the exchange of closed strings. In the long distance limit, this accounts for the gravitational interactions. Let us now use this observation to calculate the dominant part of the scattering amplitude between two $D$-particles of type I moving with a relative velocity $v$. What we need to compute is the cylinder amplitude between the boundary state of a static $D$-particle $|D_0\rangle$ and the boundary state of a moving $D$-particle $|D_0, v\rangle$ (the latter is simply obtained by acting with a Lorentz boost on $|D_0\rangle$ [10]). Thus, the amplitude we are looking for is $A = \langle D_0|P|D_0, v\rangle + \langle D_0, v|P|D_0 \rangle$. From this expression, we can extract the long range gravitational potential energy, which, in the non relativistic limit, reads $[9]$

$$V^{\text{grav}}(r) = (2\kappa_{10})^2 \frac{M_0^2}{7\Omega_8 r^7} \left( 1 + \frac{1}{2} v^2 + o(v^2) \right),$$

(23)

where $r$ is the radial coordinate, $\Omega_8$ is the area of the unit 8-dimensional sphere, $M_0 = T_0/\kappa_{10}$ is the $D$-particle mass and $\kappa_{10}$ is the gravitational coupling constant in ten dimensions. Hence the boundary state calculation correctly reproduces the gravitational potential we expect for a pair of $D$-particles in relative motion.

Although they are subdominant in the string coupling constant, the interactions of the $D$-particle with the open strings of the bulk are nevertheless interesting because they account for the gauge interactions. Since the non-BPS $D$-particles of type I are spinors of $SO(32)$, their gauge coupling is fixed by the spinorial representation they carry. The stringy description of such a coupling has been provided in [9] where we have shown that it is represented by an open string diagram with the topology of a disk with two boundary components, one lying on the D9-branes from which the gauge boson is emitted, and the other lying on the $D$-particle. At the points where the two boundary components join, we thus have to insert a vertex operator $V_{90}$ (or $V_{09}$) that induces the transition from Neumann to Dirichlet (or from Dirichlet to Neumann) boundary conditions in the nine space directions. As we have mentioned before, the $SO(32)$ degeneracy of the $D$-particle is due to the fermionic massless modes of the open strings stretching between the $D$-particle and each of the 32 D9-branes; therefore it is natural to think that the boundary changing operators $V_{90}$ and $V_{09}$ are given by the vertex operators for these massless fermionic modes. By construction, these operators carry Chan Paton factors in the fundamental representation of $SO(32)$, while the vertex operator $V_{\text{gauge}}$ for the gauge boson carries a Chan-Paton factor in the adjoint. As a consequence, this diagram must be considered as the one point function of the gauge boson in the background formed by a $D$-particle seen as an object in the bi-fundamental representation of $SO(32)$. Hence, we do not see the entire gauge degeneracy of the $D$-particle because the degrees of freedom we use to describe it are not accurate enough. This is reminiscent from the fact that,
in the boundary state formalism, also the Lorentz degeneracy of a D-brane is hidden. Using this result, we can easily compute the Coulomb potential energy \( V_{\text{gauge}}(r) \) for two D-particles placed at a distance \( r \). Indeed, this is simply obtained by gluing two 1-point functions with a gauge boson propagator, and reads

\[
V_{\text{gauge}}(r) = -\frac{g_{\text{YM}}^2}{2} \frac{1}{7} \Omega_8 r^7 \left( \delta^{AB} \delta^{CD} - \delta^{AC} \delta^{DB} \right),
\]

(24)

where \( g_{\text{YM}} \) is the gauge coupling constant in ten dimensional type I string theory, and \( A, B, C \) and \( D \) are indices in the fundamental representation of \( SO(32) \). We conclude by recalling that the non-BPS D-particles of type I are dual to perturbative non-BPS states of the \( SO(32) \) heterotic string which also have gravitational and gauge interactions among themselves. These can be computed using standard perturbative methods and if one takes into account the known duality relations and renormalization effects, one can explicitly check that they agree with the expressions (23) and (24). This agreement provides further dynamical evidence of the heterotic/type I duality beyond the BPS level.

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