INTEGRAL OF SCALAR CURVATURE ON NON-PARABOLIC MANIFOLDS

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Abstract. Using the monotonicity formulas of Colding and Minicozzi, we prove that on any complete, non-parabolic Riemannian manifold \((M^3, g)\) with non-negative Ricci curvature, the asymptotic weighted scaling invariant integral of scalar curvature has an explicit bound in form of asymptotic volume ratio.

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1. Introduction

The study of integral of the curvature started from the well-known Gauss-Bonnet Theorem: for any compact 2-dim Riemannian manifold \((M^2, g)\), \(\int_{M^2} K \, d\mu = 2\pi \chi(M^2)\), where \(K\) is the Gaussian curvature of \(M^2\), \(d\mu\) is the element of area of \(M^2\), \(\chi(M^2)\) is the Euler characteristic of \(M^2\).

Cohn-Vossen [CV35] studied the integral of the curvature on complete 2-dim Riemannian manifold, obtained the so-called Cohn-Vossen’s inequality: If \((M^2, g)\) is a finitely connected, complete, oriented Riemannian manifold, and assume \(\int_{M^2} K \, d\mu\) exists as extended real number, then \(\int_{M^2} K \, d\mu \leq 2\pi \chi(M^2)\).

Motivated to get a generalization of the Cohn-Vossen’s inequality, Yau [Yau92, Problem 9] posed the following question: Given a \(n\)-dimensional complete manifold \((M^n, g)\) with \(Rc \geq 0\), let \(B_r(p)\) be the geodesic ball around \(p \in M^n\) and \(\sigma_k\) be the \(k\)-th elementary symmetric function of the Ricci tensor, is it true that \(\lim_{r \to \infty} r^{n+2k} \int_{B_r(p)} \sigma_k < \infty\)?

In 2013, Bo Yang [Yan13] constructed examples, which answered the above question for \(k > 1\) negatively. However, the interesting case \(k = 1\) is still open, where \(\sigma_1\) is the scalar curvature \(R\). We formulate it in the following question separately.

Question 1.1 (Yau). For any complete Riemannian manifold \((M^n, g)\) with \(Rc \geq 0\), any \(p \in M^n\), is it true that \(\lim_{r \to \infty} r^{2-n} \int_{B_r(p)} R < \infty\)?

Related to the above question, Shi and Yau [SY96] gave a scaling invariant upper bound estimate for the average integral of the scalar curvature, on Kähler manifolds with bounded, pinched, nonnegative holomorphic bisectional curvature.

On the other hand, if we relax the assumption \(Rc \geq 0\) to the non-negative sectional curvature \(K \geq 0\), among other things Petrunin [Pet08] proved: There exists
$C(n) > 0$, such that for any complete Riemannian manifold $(M^n, g)$ with sectional curvature $K \geq 0$ and any $p \in M^n$, $\int_{B_1(p)} R \leq C(n)$. Petrunin’s result implies that Question 1.1 has one partial affirmative answer when $K \geq 0$.

A complete Riemannian manifold is said to be non-parabolic if it admits a positive Green function, otherwise it is said to be parabolic. By a result of Varopoulos [Var81] a complete non-compact Riemannian manifold with $Rc \geq 0$ is non-parabolic if and only if

$$\int_1^{\infty} \frac{r}{Vol(B_r(p))} dr < \infty$$

For non-parabolic Riemannian manifolds, there exists a unique, minimal, positive Green function, denoted as $G(p, x)$, where $p$ is a fixed point on manifold. We define $b(x) = \frac{[n(n-2)\omega_n \cdot G(p, x)]^{\frac{1}{n}}}{\omega_n}$, where $\omega_n$ is the volume of the unit ball in $\mathbb{R}^n$. We also define the asymptotic volume ratio of the manifold $M^n$ as:

$$V_M = \lim_{r \to \infty} \frac{Vol(B_r(p))}{\omega_n r^n}.$$  

In this short note, we proved the following theorem, which makes some progress to Question 1.1 in 3-dim case.

**Theorem 1.2.** For a complete non-compact Riemannian manifold $M^3$, which is non-parabolic with $Rc \geq 0$, we have

$$\lim_{r \to \infty} \frac{\int_{B_r(p)} R \cdot |\nabla b|}{r} \leq 8\pi [1 - V_M]$$

2. THE ESTIMATES OF GREEN FUNCTION AND ITS RELATIVES

In the rest of the paper, we always use $\rho(x) = d(p, x)$ unless otherwise mentioned. From the behavior of Green function $G(p, x)$ near singular point $p$, we get

(2.1) \[ \lim_{\rho(x) \to 0} \frac{b(x)}{\rho(x)} = 1 \]

(2.2) \[ \lim_{\rho(x) \to 0} |\nabla b| = 1 \]

Define $\theta_{\rho}(r) = \frac{Vol(\partial B_r(p))}{\rho_{n-1}}$ and $\theta = \lim_{r \to \infty} \theta_{\rho}(r)$, from Bishop-Gromov Volume Comparison Theorem, the limit always exists.

The following Lemma was essentially proved in [CM97] firstly, which used Gromov-Hausdorff convergence. Our statement followed from the intrinsic argument in [LTW97].

**Lemma 2.1.** If $M^n$ has $Rc \geq 0$ with $n \geq 3$ and maximal volume growth, for any $\delta \in (0, \frac{1}{2}]$, we have

$$(V_M)^\frac{1}{n-2} (1 + \tau)^\frac{1}{n} \rho(x) \leq b(x) \leq (V_M)^\frac{1}{n-2} (1 - \tau)^\frac{1}{n} \rho(x)$$

where $\tau = C(n)[\delta + (\theta_{\rho}(\delta \rho(x)) - \theta)]^{\frac{1}{n}}$. Especially, $\lim_{\rho(x) \to \infty} \frac{b(x)}{\rho(x)} = (V_M)^\frac{1}{n-2}$. □
Lemma 2.2. If $M^n$ has $Rc \geq 0$ with $n \geq 3$, and it is non-parabolic and not maximal volume growth, then $\lim_{r \to \infty} \sup_{\rho(x) = r} |\nabla b|(x) = 0$. Furthermore, if it has maximal volume growth, then $|\nabla b| \leq 1$.

Proof: By Li-Yau’s lower bound for the Green function [LY86],

$$C \int_{\rho(x)}^{\infty} \frac{s}{\text{Vol}(B_s(p))} ds \leq G(p, x)$$

Then for $\rho(x) = r_0$, apply Bishop-Gromov Volume Comparison Theorem,

$$G(p, x) \geq C \frac{r_0^n \text{Vol}(B_{r_0}(p))}{\rho(x)}$$

By Cheng-Yau [CY75] gradient estimate at such $x$,

$$|\nabla b|(x) = C(n) \frac{1}{G(p, x)} \cdot \left|\nabla G\right|(p, x) \leq C(n) \left(\frac{r_0^n \text{Vol}(B_{r_0}(p))}{\rho(x)}\right)^{\frac{1}{n-2}}$$

If $M^n$ has not maximal volume growth, we have the conclusion.

If $M^n$ has maximal volume growth, the conclusion follows from [Col12, Theorem 3.1].

The following lemma was implied by the argument in [CC96], and was used repeatedly in [CM97], [Col12] and [CMI14]. We give a direct proof of this result here for reader’s convenience.

Lemma 2.3. If $M^n$ has $Rc \geq 0$ with $n \geq 3$ and maximal volume growth, we have

$$\lim_{r \to \infty} \int_{\hat{b} \leq r} |\nabla b - \nabla \rho|^2 \frac{1}{V(\hat{b} \leq r)} = 0$$

where $\hat{b} = (\nabla M)^{\frac{1}{2n}} \cdot b$.

Proof: Firstly we recall that $\int_{b \leq r} |\nabla b|^2 = \omega_n r^n$, which implies

$$\int_{b \leq r} |\nabla \hat{b}|^2 = \nabla M \omega_n r^n$$

From the Green’s formula and $\Delta b = (n - 1) \frac{|\nabla b|^2}{b}$, we get

$$\int_{b \leq r} \nabla \hat{b} \cdot \nabla \rho = -\int_{b \leq r} \Delta \hat{b} \cdot \rho + \int_{b \leq r} |\nabla \hat{b}| \cdot \rho$$

$$= -(n - 1) \int_{b \leq r} \rho \frac{|\nabla \hat{b}|^2}{\hat{b}} + \int_{b \leq r} \nabla \hat{b} r + \int_{b \leq r} |\nabla \hat{b}| \cdot (\rho - r)$$

use $\int_{b \leq r} |\nabla b| = n \omega_n r^{n-1}$, we have

$$\int_{b \leq r} |\nabla \hat{b}| r = \nabla M n \omega_n r^n$$

Note by Lemma 2.1 and Lemma 2.2 the following holds

$$\lim_{r \to \infty} \frac{\int_{b \leq r} \rho \frac{|\nabla \hat{b}|^2}{\hat{b}} - |\nabla \hat{b}|^2}{V(\hat{b} \leq r)} = 0 \quad \text{and} \quad \lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b}| \cdot (\rho - r)}{V(\hat{b} \leq r)} = 0$$
By the above, using Lemma 2.1 again,
\[\lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b} \cdot \nabla \rho|}{V(b \leq r)} = -(n-1) \lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b}|^2}{V(b \leq r)} + \lim_{r \to \infty} \frac{V_M \omega_n r^n}{V(b \leq r)}\]
\[= \lim_{r \to \infty} \frac{V_M \omega_n r^n}{V(b \leq r)} = 1\]

Finally,
\[\lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b} - \nabla \rho|^2}{V(b \leq r)} = \lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b}|^2}{V(b \leq r)} + \lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \rho|^2}{V(b \leq r)} - 2 \lim_{r \to \infty} \frac{\int_{b \leq r} \nabla \hat{b} \cdot \nabla \rho}{V(b \leq r)}\]
\[= \lim_{r \to \infty} \frac{V_M \omega_n r^n}{V(b \leq r)} + 1 - 2 = 0\]

\[\square\]

**Corollary 2.4.** For a complete non-compact Riemannian manifold \(M^n\), which is non-parabolic with \(Rc \geq 0\), we have \(\lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b}|^3}{r^n} = (V_M)^{\frac{1}{n-1}} \omega_n\).

**Proof:** If \(M^n\) does not have the maximal volume growth, from Lemma 2.2, the conclusion follows directly.

In the rest of the proof, we assume that \(M^n\) has maximal volume growth. Let \(\tilde{r} = (V_M)^{\frac{1}{n-1}} r, \tilde{b} = (V_M)^{\frac{1}{n-1}} b\), we have
\[\frac{\int_{b \leq r} |\nabla \hat{b}|^3 - (V_M)^{\frac{3}{n-1}}}{r^n} = (V_M)^{\frac{3}{n-1}} \frac{\int_{b \leq r} |\nabla \hat{b}|^3 - 1}{r^n}\]

From Lemma 2.3, Lemma 2.2 and Lemma 2.1, use the Bishop-Gromov Volume Comparison Theorem,
\[\lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b}|^3 - 1}{r^n} \leq C(V_M) \lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b} - 1|}{r^n} \leq C \cdot \lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b} - \nabla \rho|}{r^n}\]
\[\leq C \cdot \lim_{r \to \infty} \left(\frac{\int_{b \leq r} |\nabla \hat{b} - \nabla \rho|^2}{r^n}\right)^{\frac{1}{2}} \cdot V(b \leq r)^{\frac{1}{2}}\]
\[\leq C \cdot \left(\lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b} - \nabla \rho|^2}{V(b \leq r)}\right)^{\frac{1}{2}} = 0\]

which implies \(\lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b}|^3 - 1}{r^n} = 0\).

Hence from the above and Lemma 2.1, we have
\[\lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla \hat{b}|^3}{r^n} = (V_M)^{\frac{3}{n-1}} \lim_{r \to \infty} \frac{V(b \leq r)}{r^n} = (V_M)^{\frac{3}{n-1}} \lim_{r \to \infty} \frac{V(b \leq r)}{r^n} = (V_M)^{\frac{3}{n-1}} \omega_n\]

\[\square\]
3. Integral of the Scalar Curvature by Dimension Reduction

Define
\[ \mathcal{A}(r) = r^{1-n} \int_{b=r} |\nabla b|^2 \quad \text{and} \quad \mathcal{V}(r) = r^{2-n} \int_{b \leq r} \frac{|\nabla b|^3}{b^2} \]
from the above definition, it is straightforward to get
\[ (3.1) \quad \mathcal{V}'(r) = r^{-1}(2-n)\mathcal{V}(r) + r^{-1}\mathcal{A}(r) \]

Let \( \bar{n} = \frac{V_\bar{b}}{\overline{\omega_n}} \) and \( \overline{\mathbf{B}} = \nabla^2(b^2) - 2|\nabla b|^2 \cdot g \), where \( g \) is the Riemannian metric on \( M^n \). Let \( \langle \overline{\mathbf{B}}(\bar{n}), \nu \rangle = \overline{\mathbf{B}}(\bar{n}, \nu) \) for any \( \nu \), then \( |\overline{\mathbf{B}}(\bar{n})|^2 = |\overline{\mathbf{B}}(\bar{n})|^2 + \overline{\mathbf{B}}(\bar{n}, \bar{n})^2 \).

The following lemma is the analogue of Theorem 2.12 of [Co112].

**Lemma 3.1.** For a complete non-compact Riemannian manifold \( M^n \), which is non-parabolic with \( \mathcal{R}c \geq 0 \), we have
\[ \lim_{r \to \infty} \mathcal{A}(r) = (n - 2) \lim_{r \to \infty} \mathcal{V}(r) = n\omega_n(V_M)^{\frac{1}{n-2}} \]

**Proof:** Let \( \beta = 1 \) in [CMI14] Theorem 3.2, we obtain
\[ (3.2) \quad \mathcal{A}'(r) = -\mathcal{V}(r) \]
From [CMI14] Theorem 1.1, we have
\[ (3.3) \quad \mathcal{A}'(r) \leq 0 \]
Combining (3.2), we get
\[ (3.4) \quad \mathcal{V}'(r) \leq \mathcal{A}'(r) \leq 0 \]
From \( \mathcal{A}(r), \mathcal{V}(r) \geq 0 \) and (3.4), we know that \( \lim_{r \to \infty} \mathcal{A}(r), \lim_{r \to \infty} \mathcal{V}(r) \) exists. By L’Hôpital’s rule,
\[ \lim_{r \to \infty} \mathcal{V}(r) = \lim_{r \to \infty} \frac{\int_{b \leq r} b^{-2} |\nabla b|^3}{r^{n-2}} = \lim_{r \to \infty} \frac{\int_{b \leq r} b^{-2} |\nabla b|^2}{r^{n-2}(n-2) r^{n-3}} = \lim_{r \to \infty} \frac{1}{n-2} \mathcal{A}(r) \]
On the other hand, from the L’Hôpital’s rule,
\[ (3.5) \quad \lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla b|^3}{r^n} = \lim_{r \to \infty} \frac{\int_{b \leq r} |\nabla b|^2}{nr^{n-1}} = \lim_{r \to \infty} \frac{1}{n} \mathcal{A}(r) \]
The conclusion follows from Corollary 2.4 and (3.3). \( \Box \)

**Proposition 3.2.** For a complete non-compact Riemannian manifold \( M^n \), which is non-parabolic with \( \mathcal{R}c \geq 0 \), we have
\[ \lim_{r \to \infty} \int_{b \leq r} |\nabla b|[\Pi_0]^2 + \mathcal{R}(\bar{n})] + \frac{|\overline{\mathbf{B}}(\bar{n})|^2 + (n-2)|\overline{\mathbf{B}}(\bar{n})|^2}{4(n-1)b^2|\nabla b|} = 0 \]
\[ \lim_{r \to \infty} r^{2-n} \int_{b \leq r} |\nabla b| \cdot H^2 = \frac{(n-1)^2 n}{n-2} \omega_n(V_M)^{\frac{1}{n-2}} \]
where $H$ is the mean curvature of the level set of $b$ with respect to the normal vector $\nabla b$.

**Proof:** From (3.2), (3.3) and (3.1), we have

$$\lim_{r \to \infty} \int_{b \leq r} |\nabla b| [\Pi_0^2 + Rc(\vec{\eta})] + \frac{\|B(\vec{\eta})\|^2 + (n-2)\|\nabla b\|^2}{2(1-\omega_h)}$$

$$\leq \lim_{r \to \infty} r \left[ A'(r) - 2(n-2)\lambda_1^2(r) \right]$$

(3.6)

the last equation above follows from Lemma [3.1]

It is straightforward to compute the mean curvature of the level set of $b$ with respect to the normal vector $\nabla b$ as the following:

$$H = \frac{(n-1)|\nabla b|}{b} - \frac{B(\vec{\eta}, \vec{\eta})}{2b|\nabla b|}$$

From (3.7) and (3.6), also note $|B(\vec{\eta}, \vec{\eta})| \leq |B(\vec{\eta})|^2$, we get

$$\lim_{r \to \infty} r^{2-n} \int_{b \leq r} |\nabla b| \cdot H^2 = \lim_{r \to \infty} r^{2-n} \int_{b \leq r} \left\{ (n-1)^2 \frac{|\nabla b|^3}{b^2} - (n-1) \frac{|\nabla b| \cdot B(\vec{\eta}, \vec{\eta})}{b^2} + \frac{|B(\vec{\eta}, \vec{\eta})|^2}{4b^2|\nabla b|} \right\}$$

$$\leq (1 + \epsilon) \lim_{r \to \infty} r^{2-n} \int_{b \leq r} (n-1)^2 \frac{|\nabla b|^3}{b^2} + (4 + 1) \lim_{r \to \infty} r^{2-n} \int_{b \leq r} \frac{|B(\vec{\eta})|^2}{4b^2|\nabla b|}$$

$$\leq (1 + \epsilon) \lim_{r \to \infty} r^{2-n} \int_{b \leq r} (n-1)^2 \frac{|\nabla b|^3}{b^2}$$

let $\epsilon \to 0$ in the above, we have

$$\lim_{r \to \infty} r^{2-n} \int_{b \leq r} |\nabla b| \cdot H^2 \leq \lim_{r \to \infty} r^{2-n} \int_{b \leq r} (n-1)^2 \frac{|\nabla b|^3}{b^2} = (n-1)^2 \lim_{r \to \infty} \lambda_1^2 (r)$$

Similar as the above, we can get $\lim_{r \to \infty} r^{2-n} \int_{b \leq r} |\nabla b| \cdot H^2 \geq (n-1)^2 \lim_{r \to \infty} \lambda_1^2 (r)$. The conclusion follows from the above argument and Lemma [3.1].

**Proposition 3.3.** For a complete non-compact Riemannian manifold $M^n$, which is non-parabolic with $Rc \geq 0$, we have

$$\lim_{r \to \infty} r^{2-n} \int_{b \leq r} |\nabla b| R - \int_0^r dt \int_{b= \eta} R(b^{-1}(t)) = -(n-1)\omega_n(V_M)^{\frac{1}{n-2}}$$

**Proof:** From the Gauss equation, we have

$$R(M^n) = R(b^{-1}(t)) + 2Rc(\vec{\eta}) - 2 \sum_{i \neq j} A_i A_j$$

$$= R(b^{-1}(t)) + 2Rc(\vec{\eta}) - \frac{n-2}{n-1} H^2 - |\Pi_0|^2$$

$$= R(b^{-1}(t)) + 2Rc(\vec{\eta}) + |\Pi_0|^2 - \frac{n-2}{n-1} H^2$$
The conclusion follows from Proposition 3.2.

In the rest of this section, we will apply the general results obtained before, to study the curvature behavior on 3-dim Riemannian manifolds and their applications.

**Lemma 3.4.** For a complete non-compact Riemannian manifold $M^3$, which is non-parabolic and diffeomorphic to $\mathbb{R}^3$, if $b^{-1}(t)$ is a smooth surface, then it is connected.

**Proof:** If $\Omega_1, \Omega_2$ are two connected components of $b^{-1}(t)$, then the base point of $b$, denoted as $p$, is enclosed by the unique surface $\Omega_i$ (otherwise $G \equiv r^{-1}$ on the region enclosed by $\Omega_1$ and $\Omega_2$ from the maximum principle, from the unique continuation of harmonic function, we get the contradiction).

Note $M^3$ is diffeomorphic to $\mathbb{R}^3$, hence the other $\Omega_{i+1}$ encloses one region $\Omega \subset M^3$, and $p \notin \Omega$. By the maximum principle again $G \equiv r^{-1}$ on $\Omega$, the contradiction follows from the unique continuation of harmonic function again.

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