A Combined Modal Split and Traffic Assignment Model With Capacity Constraints for Siting Remote Park-and-Ride Facilities

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This work was supported by the National Natural Science Foundation of China under Grant 71871103.

ABSTRACT From the perspective of urbanization, commuters’ long-distance and diversified trips are suitable for the connection between city and suburbs. The existing park-and-ride (P&R) facilities can influence user travel decisions, promoting the usage of public transportation and reducing the environmental pollution, but the P&R lots are usually located near train stations that the construction of lots at these locations is costly and complex. It is important to analyze the need for P&R and its capacity before its construction. Therefore, this paper proposes a bi-level model for determination of optimal location and capacity of a remote P&R (RPR) facility that allows a user to park a vehicle in a suburban area at lower cost and then take a bus to the nearby train station. A combined modal split and traffic assignment model with capacity constraints is developed as a lower-level model, and a nested-logit model is adopted to manage the mode similarity. The upper-level optimization is conducted based on the generalized cost of the transportation system according to the budget and environmental limits. The performance of the proposed model is experimentally verified that the optimal RPR scheme shifts commuters from the automobile mode to the transit and RPR modes, thus improving the social benefit dramatically. The model and analysis provide insights that may help operators determine how RPR facilities can be established to meet social cost improvement goals.

INDEX TERMS Bi-level programming model, combined modal split and traffic assignment, multimodal transportation network, remote park-and-ride.

I. INTRODUCTION
With the continuous expansion of urbanization and gradual refinement of urban functional land, commuters’ trips become long-distance and diversified [1], [2]. The demand for long-distance and diversified travel promotes the development of diversified and efficient transportation modes. Therefore, park-and-ride (P&R) facilities become an important way to allow commuters to use either their private vehicles or public transportation, depending on the geographic area they are situated. Using the P&R facilities can reduce traffic congestion and environmental pollution that the commuters should be motivated to switch from private-vehicle-only transportation mode to the combined transportation mode [3].

The P&R can be a bus-based P&R, which is commonly adopted in small cities where there is not developed rail system [4]–[6], and a rail-based P&R [7], [8], which is commonly adopted in highly-urban areas with an advanced rail system. Commuters prefer to use the rail-based P&R because its travel time is more accurate and stable compared to the bus-based P&R. However, the parking cost at train stations that are located in town centers is high, since the construction cost of such P&R services is relatively high. On the other hand, more parking spaces could reduce traffic congestion in the vicinity of the train stations and avoid delaying the commuters who travel by rail system. Hence, this paper focuses on a new P&R concept named the Remote P&R (RPR), which is also called the rail-based P&R with bus services. The RPR allows commuters to use their private vehicles until they reach the RPR facility located in a low-density suburban, and then transfer them to use feeder bus services to go to the train stations, and lastly transfer them to use rail services for the last portion of their trips, thus helping them to avoid traffic congestion and high parking costs while reaching major activity centers. Different from the work of Liu et al. [9] from 2018, in the RPR, the feeder
bus services are traditional bus lines because constructing dedicated feeder bus services to the train stations would be costly. The feeder bus service is one of the bus scheduling form called interval bus. The traditional bus lines can link to the train stations, and thus be even more practical. With a sufficient parking spaces and bus services, the RPR can promote the usage of public transport, alleviate traffic congestion in urban areas, and reduce environmental pollution. However, arbitrarily constructed RPR facilities could have an opposite effect, attracting more public transport riders to drive, rather than prompting more drivers to ride due to the convenience provided to driving. Besides, deficiently bus and rail services could also encourage more public transport riders to drive. Thus, a determination of optimal location and capacity of an RPR facility according to the approved budget and environmental limitations, considering multimodal transportation super network, is necessary.

In this article, we design a multimodal transportation super network that consists of a road network, including the urban road network and highway network, urban transit network, and rail network. In this super network, we propose a bi-level model for determination of optimal location and capacity of RPR facilities. We propose a combined modal split and traffic assignment model with capacity constraints as a lower-level model. Furthermore, we propose a system optimum model based on the generalized cost of the transportation system according to the budget and environmental limits as an upper-level model. Based on an Evans hybrid algorithm with an augmented Lagrange multiplier method, an improved genetic algorithm is applied to address the optimal solution of the RPR design.

The main contributions of this study can be summarized as follows.

1. The concept of the RPR scheme is modified, adopting the traditional bus service to the train stations to improve its applicability, and a multimodal transportation super network is considered to determine an optimal RPR scheme. The generalized cost functions that include the ticket cost, automobile exhaust treatment cost, and path congestion cost, which are important in user decision-making, are considered.

2. The parking capacity constraint and the limitation on on-street parking spot number are integrated into the Nested-Logit-user equilibrium (NL-UE) model to assign demand for a certain travel model in each travel mode in order to characterize the state of the transit system intensively. Due to the NL-UE capacity constraints, an augmented Lagrange multiplier method is combined with the successive weight average (MSWA) method.

The remainder of this paper is organized as follows. In Section II, we present a literature review closely related to our research. In Section III, the main problems are introduced, all the assumptions and constraints used in this work are provided, and feasible flow distributions across a multimodal transportation super network with RPR facilities are described. In Section IV, a combined modal split and traffic assignment model, and a third-stage shortest path searching approach in the multimodal transportation super network are presented. The optimal P&R facility and solution framework is introduced in Section V. The verification and applicability analysis of the proposed model using the RPR system is provided in Section VI. The conclusions, findings and directions for future research are given in Section VII.

II. LITERATURE REVIEW

Since the first introduction of the P&R in Detroit in the 1930s, the P&R has been recognized as an effective way to promote public transportation and reduce traffic externalities in urban areas. Many agencies have provided criteria for selecting the most suitable P&R facility locations, and many studies on the P&R have been focused on the policy side according to these criteria [10]–[12]. Dijk and Montalvo [13] explained how the deployment of P&R had been framed by policymakers within their broader transport policy based on the results of their survey that included 45 major cities in Europe. Wiseman et al. [14] concluded that the location of a P&R station is an important factor that could influence the performance of the P&R system; their conclusion was based on the established Adelaide Entertainment Centre P&R facility situated on the fringe of Adelaide’s CBD. Duncan and Christensen [15] demonstrated that P&R facilities had tended to be built in lower-density environments due to low land cost and high availability. The P&R schemes in the transportation network have great both theoretical and practical values. Therefore, the entire multimodal transportation network is considered in our paper.

The methods for solving the P&R facility location problem can be roughly categorized into three classes, the Geographic Information System (GIS)-based approach, the p-hub approach, and the equilibrium analysis with mathematical programming for location determination. From the perspective of the decision support system, in [16]–[18], the GIS-based methodologies were developed to evaluate potential locations of the terminal park-and-ride facilities and determine optimal locations of the park-and-ride facilities. In other studies, the focus was on the P&R schemes, and the p-hub approach was used for the determination of the P&R facility location. In other words, these studies considered the P&R facilities as hubs that attracted users that use car-to-public transportation to determine an optimal location of p-hub facilities from a set of location candidates. Farhan and Murray [19] proposed a multi-objective spatial optimization model that covered the potential demand as much as possible, located the park-and-ride facilities as close as possible to the major roadways, and sited such facilities in the context of an existing system. Aros-Vera et al. [20] proposed a mixed linear programming formulation to determine the location of a fixed number of P&R facilities such that to maximize their usage. Lu and Guo [21] developed a bi-objective programming model with spatial equity constraints to site P&R facilities in the existing traffic networks based on the maximal coverage and minimal resource utilization criteria. Cavadas and Antunes [22] proposed an optimization model that aimed
to determine the best possible locations for a given number of park-and-ride facilities under the objective of minimizing car usage in urban areas. Although numerous modeling analyses on P&R facilities can be found in the literature, different transit departure intervals and multimodal transportation network can lead to a different travel impedance, which can further result in different traveling behavior of network users. Meanwhile, the redistribution of network flows in the network allows the transportation authorities to re-balance the flow distribution, thus influencing the optimal location and capacity of the RPR facility. Hence, a systematic study is necessary to determine the optimal location and capacity of an RPR properly, considering both the modal split and the traffic assignment, which represents the main objective of this work.

From the perspective of equilibrium analysis with mathematical programming, some studies focus on the P&R schemes in a linear corridor. Wang et al. [23], [24] proposed optimization models considering the parking charge for profit maximization and social cost minimization subject in the linear city. Liu et al. [25] performed a bi-modal analysis in a competitive railway and highway system to characterize commuters’ modal choices and park-and-ride transfer behaviors. Du and Wang [26] further extended the park-and-ride transfer behaviors to the case of commuter heterogeneity and travel time uncertainty with correlation. Other studies focused on the P&R schemes in the entire transportation network. For instance, Wang et al. [27] performed a bi-level model in which the upper-level problem was to find the optimal parking fee for improving network performance, and the lower-level problem was to evaluate network performance in equilibrium. Pineda et al. [28] proposed an integrated stochastic equilibrium model that considered both private automobile traffic and transit networks to incorporate the interactions between them in terms of travel time and generalized costs. Song et al. [29] proposed an integrated planning framework for P&R facility location and its optimal capacity as well as its transit service frequency. The users’ decision-making in the previous studies was based on the travel time. However, under the combined travel mode, the commuter choice of a travel mode is based on many factors. Additionally, in the existing studies, the parking capacity constraint has never been a major concern in the development of traffic assignment models. Further, once the parking spots are all occupied, other cars cannot be parked until the parking spots become available again.

III. PROBLEM STATEMENT
A. PROBLEM DESCRIPTION
In this section, we explain an RPR scheme in a transportation network that includes the urban road network, urban transit network, highway network, and rail network. The urban road network and highway network can be used only by automobiles. The urban transit network can be used by buses, and the rail network can be used by trains, performing service trips. All the services involve picking up and dropping off passengers at a sequence of stops. The scenarios are generated referring to the urbanization background that commuters’ city-to-city travels are characterized by a long distance and diversification. When there is no P&R facility, commuters can finish their trips by using only an automobile or a bus and then rail services, or by using only rail services if the trip origin is close to a train station. However, commuters can first travel by automobile and then transfer to rail service when there is a rail-based P&R facility. The RPR schemes are introduced into the transportation system network to avoid traffic congestion and high parking cost near the train station. The RPR schemes allow commuters to use automobiles for the initial portion of their trips to the RPR facility located in a low-density suburban, then transfer to feeder bus services for the second portion of the trips to the train stations, and finally, transfer to rail services for the last portion of the trips to destination. Because of the long-distance highway network and rail network, commuters should first decide how to travel in intercity transportation, and then decide how to travel in urban transportation. Considering the independence from irrelevant alternation (IIA) characteristics, we construct a tree structure travel model, which is shown in Fig. 1.

As shown in Fig. 1, there are four types of travel modes from suburbs to downtowns. Travel mode $m_1$ represents travel by an automobile in urban and intercity transportation. Travel mode $m_2$ represents travel by rail services. Travel mode $m_3$ represents travel by a bus first, and then transfer to rail services. Travel mode $m_4$ represents the RPR scheme. The notations used in this work are listed in Table 1.

The optimization of the location and capacity of an RPR facility is based on the following assumptions:

(i) The link generalized cost functions are separable, i.e., \( c(x^m) \) is only a function of its own flow \( x^m \).

(ii) The generalized path cost $f^k$ is additive to its links, i.e., \( \delta^k_{aw} = 1 \) if a path $k$ uses a link $a$; otherwise, \( \delta^k_{aw} = 0 \).

(iii) Commuters should first determine the travel mode in intercity transportation before starting their trips and then the travel mode in urban transportation.

(iv) Total OD demands are stable, so commuter’s arrivals follow uniform distribution while vehicle arrivals follow an exponential distribution.
TABLE 1. Notations.

| Set | Description |
|-----|-------------|
| G   | Set of networks that can be specified by transportation modes |
| A   | Set of links |
| K   | Set of origin-destination (OD) pairs |
| N   | Set of travel modes in intercity transportation |
| M_r | Set of travel modes in urban transportation that conducts intercity transportation by rail services |
| M   | Set of travel modes from suburbs to downtowns |
| l   | Set of transportation modes in the network, e.g., automobile, bus service, or rail service |

| Variable | Description |
|----------|-------------|
| c(x_a)  | Generalized cost of a link a |
| l_a      | Distance on a link a |
| v        | Average walking speed to the station |
| F_{i,i}  | Service frequency of transit service on a link a in a mode i |
| φ_j      | Per unit time cost of a generalized cost function j |
| e_i      | Ticket cost to the transit service in a transportation mode i |
| c_{i,a}  | Travel time on a link a in a mode i when the link flow is zero |
| C_a      | Capacity of the road on a link a |
| S_r      | Passenger capacity of a vehicle |
| S_b      | Passenger capacity of a bus |
| S_p      | Passenger capacity of a train |
| α_j, β_j | Parameters of generalized cost function j |
| P_c      | Parking fee in the downtown |
| P_i      | Parking fee at the RPR facility |
| φ        | Unit fuel cost per kilometer |
| y_{i,a}  | Capacity of the on-street parking spot on an RPR link a |
| q_{a,w}  | Generalized cost on a path k of an OD pair w |
| σ, θ     | Parameters of NL function |
| μ_{a,w}  | Generalized cost of the minimum generalized cost path in the road network of an OD pair w |
| χ_{a,m}  | The attractiveness measure of the automobile mode based on the unit of measure equivalent to the generalized cost |
| μ_{a,m}  | Generalized cost of the minimum generalized cost path in rail services of an OD pair w |
| χ_{a,m}  | The attractiveness measure of rail services based on the unit of measure equivalent to the generalized cost |
| μ_{a,m}  | Generalized cost of the minimum generalized cost path in a travel mode m |
| χ_{a,m}  | The attractiveness measure of a travel mode m based on the unit of measure equivalent to the generalized cost |
| δ_{a,m}  | A binary coefficient equals one if a path a uses link a of an OD pair w; otherwise, it is equal to zero. |
| λ_1, λ_2 | Parameters of carbon monoxide emission reaction function |
| ζ       | Minimum number of RPR facilities |
| h       | Unit construction cost of the RPR facility |
| b       | Budget limit of the construction cost of the RPR facility |
| e        | Environmental limit on the carbon monoxide emission |

Decision variable | Description |
|------------------|-------------|
| f_{k,a}          | Flow of a path k of an OD pair w |
| x_a              | Flow of link a |
| q_{a,m}          | Travel demand of the automobile mode of an OD pair w |
| q_{b,m}          | Travel demand of rail services of an OD pair w |
| q_{p,m}          | Travel demand of travel mode m of an OD pair w |
| δ_{k,a}          | A binary decision variable equals one if the capacity of the RPR facility is equal to \( r \); otherwise, it is equal to zero. |
| v_{i,a}          | Capacity of the RPR facility on an RPR link a |

In order to present the modeling framework, we consider a multimodal transportation super network that consists of a road network, including the urban road network and highway network, urban transit network, and rail network used by automobiles, buses, and trains, respectively [30]. The structure of the road network is basically the same as the actual road network structure. Following the actual road network topology, in the considered network, nodes represent intersections, and links represent roads between adjacent intersections. The structures of urban transit and rail networks are not the same as that of the road network but that of the service path. In the urban transit network and rail network, nodes represent stations or transfer stations, and links represent service paths between adjacent stations. In the independent network of the urban transit network and the rail network, any nodes can be passed by multiple service paths. In addition, independent networks are connected by constructing the multimodal transportation super network model with an RPR. The transfer nodes are not only the transfer points between urban transit and rail network, but also park-and-ride. In addition, commuters usually need to travel for a certain time (online process) before arriving at the first network node. After arriving at the destination node, they also need to travel for a certain time before arriving at the final destination (offline process). Additionally, in the multimodal transportation super network construction (v). All OD pairs can start a trip by automobile or a bus service near the bus lines, and a part of OD pairs located near the train station can start a trip by using rail services directly.

In order to present the modeling framework better, we assume the link travel time functions are symmetric based on assumption (i). In this work, it is assumed that the urban transit network has bus lanes, and there are no interactions between the flows on different links. Besides, we consider that OD demands, path-flows, link-flows, and link travel times are consistent with each other so that the path travel time is additive to its links which proves the assumption (ii). Assumption (iii) is very critical to our model because ignoring the mode choice could lead to significant differences. In this paper, we consider that commuter travel choice is an NL model in assumption (iii); then, the traffic assignment is extended to a combined modal split and traffic assignment model. Since this work mainly discusses the standard deterministic user equilibrium assignment, we consider an environment where travelers are not sensitive to uncertainty in assumption (iv). In addition, the implementation of geographical information systems and wireless communication systems in public transportation systems makes it possible to understand traffic conditions timely. In this work, assumption (v) has significant importance because the problem of complex OD pairs leads to another challenging topic in optimization problems.
network, there are two transfer types, transfer between modes (such as transfer between automobile and train or transfer between bus and train), and transfer within modes (such as transfer between different bus or train lines). Thus, this paper builds multimodal transportation super network model, which is shown in Fig. 2. Commuters are moving from node 1 in the suburb to node 16 in the downtown.

The structure of the multimodal transportation super network is illustrated in Fig. 2. Let the road network \( G_{cr} \) denote the actual road network, and the urban transit network \( G_{bs} \) and two rail networks \( G_{tr} \) denote the service path network (e.g., links \( a_3, a_5 \) and \( a_9 \)). Besides, nodes of urban transit and rail service are expanded into multiple virtual nodes according to the number of lines passing through the node. Considering the same travel modes, let the transfer nodes with dashed lines denote transfer process (e.g., links \( a_{10} \) and \( a_{11} \)) that travelers can transfer lines at a node in the same mode. Besides, two virtual nodes represent the same station. Considering the different travel modes, the transfer nodes with dashed lines represent transfer process (e.g., links \( a_4, a_6, a_7 \), and \( a_8 \)) that two nodes can transfer between different modes. Note that link \( a_4 \) represents the park process on an RPR site at node 6. The boarding and alighting arcs represent online and offline processes respectively (e.g., links \( a_1 \) and \( a_2 \)).

As shown in Fig. 2, an RPR site is constructed at node 6, and it is adjacent to the train stations at nodes 10 and 11. One bus service (link \( a_5 \)) is provided to connect the RPR site and railway station at node 10. The RPR users drive from their homes located at node 1, park their cars at node 6, next take the bus service at node 6, and then take train service at the station at node 10. In addition, if the origin is near to a train station, the commuters have direct access to the rail network. Based on the constructed multimodal transportation super network system, each travel mode consists of online links, travel links, transfer links, and offline links. A travel link in the travel mode \( m_1 \) is denoted as \( G_{cr} \), in the travel mode \( m_2 \) it is denoted as \( G_{bs} \), in the travel mode \( m_3 \) it is denoted as \( G_{tr} \), and finally, in the travel mode \( m_4 \) it is denoted as \( G_{ct} + G_{bs} + G_{tr} \). We translate the comfort cost into additional in-vehicle time cost so that the total in-vehicle cost increases with the passenger flow. With the aim to simplify our model, we assume the additional in-vehicle time cost follows the BPR function.

1) GENERALIZED COST OF ONLINE AND OFFLINE LINKS
For commuters who choose bus or train service as an initial transportation mode, the generalized cost of the corresponding online link \( A_{on}^{bs} \) or \( A_{on}^{tr} \) consists of the walking-time cost to the station, the waiting-time cost at the station, and the ticket cost of the chosen service. Regarding the waiting-time cost, since it is assumed that commuter’s arrivals follow the uniform distribution and vehicle arrivals follow an exponential distribution, the waiting time can be estimated as a frequency of the corresponding transit line [31], and the offline time is zero. For the commuters who choose an automobile as the initial transportation vehicle, the generalized cost of the online time is zero, and the offline link \( A_{on}^{tr} \) refers to the parking fee. The functions (1)–(3) are shown as follows:

\[
\begin{align*}
    c_a &= \varphi_1(l_a + \frac{1}{F_{bs}^{cr}}) + g_{bs} \quad \forall a \in A_{on}^{bs} \\
    c_a &= \varphi_1(l_a + \frac{1}{F_{tr}^{cr}}) + g_{tr} \quad \forall a \in A_{on}^{tr} \\
    c_{on}^{m} &= p_c \quad \forall a \in A_{on}^{tr}
\end{align*}
\]

2) GENERALIZED COST OF TRAVEL LINKS
The travel links can be divided into links related to rail service \( A_{tr}^{cr} \), bus service \( A_{bs}^{cr} \), and automobile \( A_{tr}^{cr} \). As for the rail service links, the generalized cost includes only the in-vehicle time cost because the ticket cost is allocated to the online and transfer links. The generalized cost of bus service is commonly the same as that of rail service. Passenger capacity of a vehicle is used to express as a BPR-type function with regard to the constant travel time, passenger volume on the transit service link. The in-vehicle travel time on the transit service link can be given and fixed because transit service has an exclusive right-of-way without congestion interactions with other transportation modes. However, as the passenger volume inside the vehicles increases, passengers’ comfort will be affected by the degree of crowding within the vehicles. To capture this crowding effect, a modified in-vehicle travel time is used to model the passengers’ discomfort within the vehicles. Furthermore, the generalized cost of using an automobile consists of in-vehicle time cost and fuel cost. The functions (4)–(6) are shown as follows:

\[
\begin{align*}
    c(a) &= \varphi_2 t_a^{cr}(1 + \alpha_1(x_a F_{tr}^{cr} S_{tr})^\beta_1) \quad \forall a \in A_{tr}^{cr} \\
    c(a) &= \varphi_3 t_a^{bs}(1 + \alpha_2(x_a F_{bs}^{cr} S_{bs})^\beta_2) \quad \forall a \in A_{bs}^{cr} \\
    c(a) &= \varphi_4 t_a^{tr}(1 + \alpha_3(x_a C_{tr}^{cr} S_{tr})^\beta_3 + l_a \times \phi) \quad \forall a \in A_{tr}^{cr}
\end{align*}
\]

3) GENERALIZED COST OF TRANSFER LINKS
In order to determine the generalized cost of transfer links, we analyze both the RPR process and the transfer process between different public transportation services. As for the RPR process, the generalized cost of an RPR link \( A_{tr} \) consists of the parking-time cost, parking fee, the waiting-time cost at the station, and the ticket cost of the service. Besides, the total number of P&R users who transfer from automobile to
public transportation at the P&R candidate nodes should not exceed the parking lot capacity at the corresponding nodes. With the aim to be more realistic, it is assumed that all P&R candidate nodes have small initial parking capacity denoted by \( y_0^a \), which represents the limited capacity of the on-street parking spot. Then, the capacity of a P&R facility is equal to \( y_a \). Thus, the total parking capacity of a candidate link is \( y_0^a + y_a \). As for the transfer process between different public transportation services, the generalized cost of link consists of the waiting-time cost at the station and service ticket cost, based on functions (8). “bs-tr” denotes the transfer process between bus service and the rail service. As for the transfer process between the same public transportation services, the generalized cost of link consists of the waiting-time cost at the station and service ticket cost, based on functions (9). “bs-bs” denotes the transfer process between bus services. “tr-tr” denotes the transfer process between rail services. The functions (7)–(9) are shown as follows:

\[
c(x_a) = \varphi_4 a^b(1 + \alpha_d(\frac{x_a}{S_{cr}(\frac{y_0^a}{y_a} + y_a)})^b) + p_s \\
+ \varphi_1(\frac{L_a}{v^a} + \frac{1}{F_{bs}^a}) + g_{bs} y_a \forall a \in A_p \tag{7}
\]

\[
c_a = \varphi_1(\frac{L_a}{v^a} + \frac{1}{F_{ma}^a}) + g_{ma} y_a \in A_m^p, m \in \{bs - tr \} \tag{8}
\]

\[
c_a = \frac{\varphi_1}{F_{ma}^a} + g_{ma} y_a \in A_m^p, m \in \{bs - bs, tr - tr \} \tag{9}
\]

IV. USER EQUILIBRIUM PROBLEM

In order to evaluate the RPR scheme in a multimodal transportation super network, the network equilibrium flows should be analyzed. The travel plan of a network user consists of the limited capacity of the on-street parking spot [33], [34]. Thus, the total parking capacity of a candidate link \( y_0^a + y_a \). As for the transfer process between different public transportation services, the generalized cost of link consists of the waiting-time cost at the station and service ticket cost, based on functions (8). “bs-tr” denotes the transfer process between bus service and the rail service. As for the transfer process between the same public transportation services, the generalized cost of link consists of the waiting-time cost at the station and service ticket cost, based on functions (9). “bs-bs” denotes the transfer process between bus services. “tr-tr” denotes the transfer process between rail services. The functions (7)–(9) are shown as follows:

\[
f_m = q_m = q_m \geq 0 \tag{10}
\]

\[
f_m = q_m \times e^{-\theta (\mu_m^a - \chi_m^a)} \forall W \in W \tag{11}
\]

\[
\mu_m^a = \frac{1}{\sigma} \sum_{m \in M} e^{-\sigma (\mu_m^a - \chi_m^a)} \forall W \in W \tag{12}
\]

\[
q_m = q_m \times e^{-\sigma (\mu_m^a - \chi_m^a)} \forall W \in W, m \in M \tag{13}
\]

Equations (10)–(13) refer to the modal split, and the probability of choosing travel mode is defined by the generalized cost of the minimum generalized cost path. Based on the utility maximization theory, we can get that \( 1 \geq \theta/\sigma \geq 0 \). Equations (14)–(15) denote user equilibrium conditions expressing that all commuters under mode \( m \) have identical path generalized cost, and no traveler can unilaterally reduce the generalized cost by changing routes. Subsequently, (16) represents the parking queue penalty \( \phi_m^a \) condition, which shows that \( \phi_m^a \) is positive when the flow in a transfer link is larger than the capacity of the parking spot [33], [34].

A. MATHEMATICAL MODEL

The mathematical formulation of the NL-UE problem in a multimodal transportation super network with P&R facilities can be expressed as:

\[
\min F_2 = \sum_{a \in A_e} \int_{x_a} q_{wa}^a d(x_a) dx + \sum_{w \in W} q_{wa}^m \left[ \frac{1}{\theta} (\ln \frac{q_{wa}^m}{q_{wa}} - 1) - \chi_{wa}^n \right] \tag{14}
\]

Subject to

\[
q_{wa}^m = \sum_{k \in K_{wa}^m} f_{wk}^k \forall W \in W(\mu_{wa}^m) \tag{15}
\]

\[
q_{wa}^m = \sum_{k \in K_{wa}^m} f_{wk}^k \forall W, m \in M_w(\mu_{wa}^m) \tag{16}
\]

\[
q_{wa}^m = \sum_{m \in M} q_{wa}^m \forall W \in W(\mu_{wa}^m) \tag{17}
\]

Equations (18)–(25) denote constraints of the objective function, where Greek letters in brackets represent the corresponding dual variables. The constraints given by (18)–(21) represent the flow conservation equations of the path-flows. The constraint given by (22) denotes the relationship function between the link flow and the path flow. The constraints given by (23)–(24) guarantee positive values of the corresponding
parameters. Equation (25) ensures that the sum of flows at different modes on the link \( a \) is less than the maximum limit on the capacity of the parking lot.

**Proposition 1 (Equivalence):** The NL-UE problem defined by (14)–(16) is equivalent to the mathematical formulation given by (17).

**Proof:** Use the Lagrange multiplier method to construct function with constraints \( z_1 \) and \( z_2 \), and Lagrange multipliers, as follows:

\[
\begin{align*}
    z_1 &= \sum_{a \in A_n} \int_0^{\alpha_a} c(x_a)dx + \sum_{m \in M_R} \sum_{a \in A_n} \int_0^{\alpha_a} c(x_a)dx \\
    z_2 &= \sum_{w \in W} q_w^m \left( \ln \frac{q_w^m}{q_w} - 1 - \chi_w^m \right) + \sum_{w \in W} \sum_{m \in M_R} q_w^m \left( \ln \frac{q_w^m}{q_w} - 1 - \chi_w^m \right) + \sum_{w \in W} q_w^m \left( \ln \frac{q_w^m}{q_w} - 1 - \chi_w^m + \frac{1}{\sigma} \right)
\end{align*}
\]

Derive the Karush–Kuhn–Tucker (KKT) conditions with a path flow in mode \( m \) as follows:

\[
(x_a - y_a^0 - y_a) \phi_a^m = 0, \quad \phi_a^m \geq 0 \quad (28)
\]

\[
\sum_{a \in A_n} (c(x_a) + \phi_a^m) \delta_{aw}^k - \mu_w^m \geq 0 \forall k \in K_w^m, \quad w \in W \quad (29)
\]

\[
\sum_{a \in A_n} (c(x_a) + \phi_a^m) \delta_{aw}^k - \mu_w^m \geq 0 \forall k \in K_w^m, \quad w \in W, \quad m \in M_R \quad (30)
\]

\[
\left( \sum_{a \in A_n} (c(x_a) + \phi_a^m) \delta_{aw}^k - \mu_w^m \right) \delta_{aw}^k = 0 \forall k \in K_w^m, \quad w \in W \quad (31)
\]

\[
\left( \sum_{a \in A_n} (c(x_a) + \phi_a^m) \delta_{aw}^k - \mu_w^m \right) \delta_{aw}^k = 0 \forall k \in K_w^m, \quad w \in W, \quad m \in M_R \quad (32)
\]

In (28)–(32), \( \phi_a^m \) denotes the additional cost of exceeding the maximal parking lot capacity; \( \mu_w^m \) and \( \mu_w^m \) are the generalized costs of the minimum generalized cost path on a path \( W \) in intercity transportation mode \( n_c \) and urban transportation mode \( m \), respectively. Similarly, derive the KKT conditions with demand in mode \( m \) as follows:

\[
\frac{1}{\theta} \ln \frac{q_w^m}{q_w} + \mu_w^m - \xi_w - \chi_w^m q_w^m = 0 \forall w \in W \quad (33)
\]

\[
\frac{1}{\theta} \ln \frac{q_w^m}{q_w} + \mu_w^m - \xi_w - \chi_w^m q_w^m = 0 \forall w \in W \quad (34)
\]

\[
\frac{1}{\theta} \ln \frac{q_w^m}{q_w} + \mu_w^m - \xi_w - \chi_w^m q_w^m \geq 0 \forall w \in W \quad (35)
\]

\[
\frac{1}{\theta} \ln \frac{q_w^m}{q_w} + \mu_w^m - \xi_w - \chi_w^m q_w^m \geq 0 \forall w \in W \quad (36)
\]

\[
\frac{1}{\theta} \ln \frac{q_w^m}{q_w} + \mu_w^m - \mu_w^m - \chi_w^m q_w^m = 0 \forall w \in W, \quad m \in M_R \quad (37)
\]

Then, (39)–(41) can be obtained, and they are equivalent to the multinomial logit model given by (10)–(11).

\[
\frac{q_w^m}{q_w} = e^{\theta \xi_w} \times e^{-\theta (\mu_w^m - \chi_w^m)} \forall w \in W \quad (39)
\]

\[
\frac{q_w^m}{q_w} = e^{\theta \xi_w} \times e^{-\theta (\mu_w^m - \chi_w^m)} \forall w \in W \quad (40)
\]

Furthermore, based on the KKT conditions, (42)–(43) can be obtained.

\[
\frac{q_w^m}{q_w} = e^{\theta \mu_w^m} \times e^{-\theta (\mu_w^m - \chi_w^m)} \forall w \in W \quad (42)
\]

\[
\mu_w^m = -\frac{1}{\theta} \ln \left( \sum_{m \in M_R} e^{-\theta (\mu_w^m - \chi_w^m)} \right) \forall w \in W \quad (43)
\]

Considering the flow conservation in (19), we can get (44) by summing (43) for \( M_R \) which is equivalent to the multinomial logit model given by (13).

\[
1 = e^{\sigma \mu_w^m} \times \sum_{m \in M_R} e^{-\sigma (\mu_w^m - \chi_w^m)} \forall w \in W, \quad m \in M_R \quad (44)
\]

**Proposition 2 (Existence):** If the generalized link costs given by (1)–(9) are continuous, then there exists a solution to the problem defined by (17).

**Proof:** According to (17), the Hessian matrix corresponding to \( z_1 \) is positive definite when \( \alpha \) and \( \beta \) are positive, so it is a strictly convex function. Besides, in the Hessian matrix corresponding to \( z_2 \), the elements on the diagonal are positive, and the non-diagonal elements are equal to zero. Therefore, the Hessian matrix is positive definite, and strictly convex. Accordingly, the objective function of the model is strictly convex. The constraints are linear and nonnegative, and the feasible region denotes a convex set. The mathematical model is convex programming. Thus, the local optimal solution represents the global optimal solution, and it is unique.

**B. SHORTEST PATH SEARCHING APPROACH**

For the development of multimodal transport super network model with RPR schemes, it is crucial to solve the shortest path problem first. In the multimodal network with the RPR, there are four shortest path problems related to the auto, rail, bus-rail, and RPR models, respectively. The shortest path problems of auto and rail modes are quite straightforward and can be directly solved by the existing label-based algorithms in the corresponding sub-network. However, solving the shortest path problems of the bus-rail and RPR modes is difficult because, in a multimodal network \( G \), it cannot be guaranteed that the shortest path between OD of this pair is a multimodal path. In order to find the shortest path of bus-rail and RPR modes efficiently, we use the second-stage and third-stage shortest path searching approaches to obtain
the shortest bus-rail and RPR paths within 2 or 3 searching rounds, respectively. An illustrative example presented in Fig. 3 is used to elaborate on the third-stage shortest path searching approach.

As shown in Fig. 3, in the first stage of the urban transportation, the label-correcting shortest path algorithm is executed to obtain the shortest paths from origin node O to each RPR site in the auto sub-network simultaneously. Further, \( k(1, P(N)) \) represents the itinerary of the shortest path from the origin to the RPR site \( P(N) \), and \( c(1, P(N)) \) denotes the generalized cost of this path. In the second stage, in the bus sub-network, a dummy link is added between the origin and each RPR site, which results in a modified network. The dummy link represents the shortest path \( k(1, P(N)) \), and the generalized cost of the corresponding dummy link is \( c(1, P(N)) \). Then, the second-round shortest path searching in the modified network is executed, using \( k(1, B(N)) \) to denote the itinerary of the shortest path from the origin to the bus stop \( B(N) \) in the vicinity of the train station \( T(N) \), and \( c(1, B(N)) \) to denote the generalized cost of this path. Similarly, in the third stage, a dummy link is added between the origin and each train station, and the bus-rail mode shortest path is obtained. In addition, travelers can choose among many super paths in the multimodal transportation super network, but not every super path will be considered by travelers.

Therefore, this paper defines constraints on the feasible super paths as follows.

1. Transfer times should be less than three times. In other words, a feasible super patch should have maximal three transfer links.

2. Two continuous transfers should be at different stations, that is, feasible super paths should have only one transfer link for the transfer between the same number of nodes in different networks.

3. Feasible super paths should be traveled by paths that are suitable for a travel mode \( m \). For instance, paths cannot transfer from the urban transit network to the road network.

4. The generalized cost of feasible super paths should be within the range that travelers can pay. The generalized cost \( f_w^k \) of the feasible super paths should be less than \((1 + H_w)\mu_w^k\). Considering the intercity and urban transportation, the path extension coefficient is 1.3 in urban transportation and 1.6 in intercity transportation.

V. RPR DESIGN OPTIMIZATION

This section designs the bi-level model to optimize RPR design. The equilibrium model proposed in Section IV can be used as a lower model to determine the traffic assignment under the RPR schemes. In the upper level, transportation planning should minimize the generalized cost of the transportation system and maximize the social benefits by locating the RPR facilities reasonably. The project construction should minimize the construction cost of the RPR facility in order to maximize its function and value. Besides, environmental protection should locate RPR facilities reasonably in order to attract more automobile travelers to public transport to reduce the environmental pollution caused by automobile exhaust [35], [36]. Therefore, an optimal RPR design should consider the generalized cost of the transport system, considering the budget and environmental limits of the upper model.

A. PROBLEM FORMULATION

The objective of the existing P&R facility location problem is usually taken as a system optimum. However, it is necessary to consider limits on both the construction cost and the carbon monoxide emission of vehicle exhaust profoundly. Furthermore, the carbon monoxide emission of vehicle exhaust of each link [37], [38] can be expressed as:

\[
e_{a}(x_{a}) = \lambda_1 \times t(x_{a}) \times \exp(\frac{\lambda_2 \times \iota_a}{t(x_{a})}) \quad \forall a \in A^{rf} \tag{45}
\]

Based on the equilibrium model proposed in Section IV, the upper-level objective function (46) can be defined as follows:

\[
\min F_1(x) = \sum_{a} x_{a} e_{a}(x_{a}) \tag{46}
\]

Subject to

\[
\sum_{a \in A^{p}} \sum_{i \in I_{a}} z_{a}^{i} \geq \xi \tag{47}
\]

\[
\sum_{a=1}^{A^{p}} by_{a} \leq B \tag{48}
\]

\[
\sum_{a} x_{a} e_{a}(x_{a}) \leq C \tag{49}
\]

\[
y_{a} = \sum_{i \in I_{a}} i \times z_{a}^{i}, \quad \forall a \in A^{p} \tag{50}
\]

\[
\sum_{i \in I_{a}} z_{a}^{i} = 1, \quad \forall a \in A^{p} \tag{51}
\]

\[
z_{a}^{i} \in \{0, 1\}, \quad \forall a \in A^{p}, \quad i \in I_{a} \tag{52}
\]
The lower-level objective function (53) subjected to constraints (18)–(25) is expressed as follows:

\[
\min F_2(x) = \sum_{a \in A_p} \int_0^{x_a} c(x_a)dx + \sum_{w \in W} \sum_{m \in M_t} q_w^m \left[ \frac{1}{\theta} \left( q_w^m - q_w^m \right) - y_{w}^m \right] + \sum_{a \in A_p} \left( \sum_{m \in M_t} \int_0^{x_a} c(x_a)dx \right) + \sum_{w \in W} \left[ \frac{1}{\sigma} \left( q_w^m - q_w^m \right) - y_{w}^m + \frac{1}{\sigma} \right]
\]

Equations (47)–(52) denote the constraints of the upper-level objective function, which is used to improve the solution method. Constraint (47) ensures that the total number of RPR facilities should not be less than their minimum number. Constraint (48) reflects the budget limit that the construction cost cannot exceed \( B \). Constraint (49) reflects the environmental limit that the carbon monoxide emission of vehicle exhaust cannot exceed \( C \). The capacity limit of the RPR facilities is given by (25). Different from (47)–(49), the constraint defined by (25) represents the constraint of flow allocation and does not represent the optimal solution constraint. Constraints given by (50)–(51) ensure that all the feasible values are grouped in a set \( I_a \) for any \( y_a \). Finally, the constraint given by (52) guarantees that \( z_a \) is a binary variable.

B. SOLUTION FRAMEWORK

The proposed bi-level model denotes an NP-hard problem, including the NL model and multimodal subnetwork, which further increases the complexity and introduces challenges to solving the problem. The bi-level model represents a mixed-integer nonlinear program that has been used for the transport network design [39], [40]. In order to reduce the complexity and solving of this NP-hard problem, this paper transforms the mixed-integer nonlinear program from a continuous problem into a discrete problem. Thus, the continuous variable \( I_a \) is defined as a discrete variable expressed as \( \{1, 2, 3, 4, 5, \ldots, I\} \). Each discrete variable represents a construction grade of the RPR facility. In order to solve the upper- and lower-level problems, two model-solution optimization algorithms are required. Due to the fact that the proposed model is a complex and NP-hard problem, exact solutions of large-scale experiments are not practical in the large sizes. The feasible approaches would be classified into decomposition-based algorithm, gradient-based algorithm, and intelligent algorithm. Genetic algorithm as an evolutionary optimization algorithm conducts a search through the space of solutions by exploiting a population of points in parallel rather than a single point. To assure the applicability and generality of the proposed model to realistic size cases, genetic algorithm has shown to be effective in solving this type of complex problems [41], [42]. In this work, an improved genetic algorithm is used to determine the initial RPR scheme, and the Evans hybrid algorithm is employed to determine link flows under the RPR scheme and then obtain the value of the upper-level model under the RPR scheme. The RPR scheme is constantly modified by the improved genetic algorithm to find an optimal scheme. As for capacity constraint, we consider only a link \( a, a \in A_p \), which the RPR mode can travel by. Besides, the augmented Lagrange multiplier method is used to construct the function defined by (54) that transforms the capacity inequality constraint into equality constraint by using a relaxation variable \( s (s \geq 0) \). The mentioned function is given by:

\[
F_3(x) = F_2(x) + \sum_{a=1}^{A_p} \sigma_a h_a(x_a, s_a) + \frac{1}{2\gamma} \sum_{a=1}^{A_p} h_a^2(x_a, s_a)
\]

\[
g_a(x_a) = x_a - (y_a + y_a^0) \leq 0 \quad \forall a \in A_p
\]

\[
h_a(x_a, s_a) = g_a(x_a) + s_a = 0
\]

where \( \gamma \) denotes the penalty parameter, and \( \sigma \) denotes the Lagrange multiplier vector that reflects the congestion degree of the RPR transfer process. Then, the extreme value is evaluated using function \( F_3(x) \). Furthermore, equation (54) can be re-formulated, resulting in (58).

\[
s_a = \max[0, -\frac{\sigma_a}{\gamma} - g_a(x_a)]
\]

\[
F_a(x) = F_2(x) + \frac{1}{2\gamma} \sum_{a=1}^{A_p} \left( \max[0, \sigma_a + \gamma(g_a(x_a))]^2 - \sigma_a^2 \right)
\]

In terms of the OD and flow iteration, the descent directions are determined by the functions (59)–(60), and the iteration step size is determined by the function (61).

\[
q_{m+1}^k = q_m^k + \alpha^k(q_m^k - q_m^k)
\]

\[
x_{a+1}^k = x_a^k + \alpha^k(x_a^k - x_a^k)
\]

\[
\alpha^k = \frac{1 + 2 + 3 + \ldots + k}{k}
\]

In terms of the convergence criteria, the contribution of the penalty to the function decreases by approximating to the optimal solution. Thus, the proportion of penalty in the enhanced Lagrange functions (62)–(69) is used as a convergence criterion, and the penalty term has the same magnitude as the original objective function.

\[
y_1 = \frac{\tau(x)}{\frac{1}{2} \sum_{a=1}^{A_p} h_a^2(x_a, s_a)}
\]

\[
\gamma_{a+1} = \begin{cases} 
\rho \gamma_a \|g_a(x_a^a)\| \geq \beta \|g_a(x_a^{a-1})\| \quad \gamma_a \|g_a(x_a^a)\| < \beta \|g_a(x_a^{a-1})\| \\
\gamma_a \|g_a(x_a^a)\| \gamma_a \|g_a(x_a^{a-1})\| \max[0, g_a(x_a)]
\end{cases}
\]

\[
\sigma_a^1 = \max[0, c(x_a) - c(y_a + y_a^0) + \gamma \max[0, g_a(x_a)]]
\]
population generation logical block, we use binary-coding for \([a_1, a_2, \ldots, a_i]\) and real-coding for \([b_1, b_2, \ldots, b_i]\); \([a_1, a_2, \ldots, a_i, b_1, b_2, \ldots, b_i]\) denotes the value range of a gene in each chromosome. Besides, the value of \(b_i\) is set to zero if the value of \(a_i\) is zero. The lengths of \([a_1, a_2, \ldots, a_i]\) and \([b_1, b_2, \ldots, b_i]\) are equal to the numbers of RPR candidate sites. In the link-flow computation logical block, the lower objective function is solved based on the principle of chromosomes. The algorithm is as follows.

Algorithm Pseudo-Code of the Lower Objective Function Algorithm

**Input:** set of networks \(G\); set of travel modes \(N, M_v, M_r\); set of OD pairs \(K\); set of transportation modes \(I\); capacity of the RPR facility \(\gamma_a\) and \(\sigma^a_n\); other parameters

**Initialize:** \(k \leftarrow 0; n \leftarrow 0; e_1 \leftarrow \infty; e_2 \leftarrow \infty\)

\(e_1 \leftarrow \infty; \delta \leftarrow 0.001; \delta \leftarrow 0.001\)

While \(e_1 > \delta\)

\(do k \leftarrow k + 1\)

calculate:

i. \(c(x^k_a)\) according to (1)-(9)

ii. \(\tilde{q}_{m}^k\) according to (10)-(13)

iii. \(\chi^k_a\) with all or none assignment model

iv. \(q^k\) according to (61)

update: \(q_{m}^{k+1}\) and \(x_a^{k+1}\) according to (59)-(60)

\(e_1 \leftarrow \sqrt{\sum_a(x_a^{k+1} - x_a^k)^2 / \sum_a(x_a^k)^2}\)

end while

calculate:

i. \(\gamma_1\) and \(\sigma^{1}_n\) according to (62) and (64)

ii. \(g_a(x^k_a)\) according to (55)

While max(\(\varepsilon_1, \varepsilon_2\)) > \(\delta_3\)

\(do n \leftarrow n + 1; k \leftarrow 0; e_2 \leftarrow \infty; \delta_3 \leftarrow 0.001\)

While \(e_2 > \delta_3\)

\(do k \leftarrow k + 1\)

calculate: \(c(x^k_a)\) according to (1)-(9)

If \(a \in A_p\) then

\(do c(x^k_a) \leftarrow c(x^k_a) + \max(0, \sigma^a_n) + \gamma_a(x^k_a)\)

end if

calculate:

i. \(\tilde{\eta}_m^k\) according to (10)-(13)

ii. \(\tilde{x}_a^k\) with all or none assignment model

iii. \(\delta^k\) according to (61)

update: \(\tilde{q}_{m}^{k+1}\) and \(x_a^{k+1}\) according to (59)-(60)

\(e_1 \leftarrow \sqrt{\sum_a(x_a^{k+1} - x_a^k)^2 / \sum_a(x_a^k)^2}\)

end while

update \(\gamma_a\) and \(\sigma^{n}_n\) according to (63) and (65)

update: \(\kappa(x^a), \tau(x^a)\) and \(g_a(x^a)\) according to (66), (67), (55)

\(\varepsilon_1 \leftarrow \kappa(x^a)/(\kappa(x^a) + \tau(x^a))\)

\(\varepsilon_2 \leftarrow \|g_a(x^a)\|\)

end while

The solution algorithm to the above objective function is presented in Fig. 4.

The population elements are randomly generated in the initial population generation logical block. The generated population is first used to determine link flows under the RPR scheme in the link-flow computation logical block. Based on the link-flow solution, the quality of a genome is evaluated in the fitness computation logical block. Then the generated population repeatedly undergone a series of genetic operators in the genetic manipulation logical block until new “better” genomes are produced. In the initial
In the fitness computation logical block, the fitness value of a genome reflects the quality of the coded solution, and the value of the objective function $F_1(x)$ is used as a fitness. If the constraints (47)–(49) cannot be met for the current RPR scheme, the fitness value update to a large number $M$. Because the minimal fitness of the RPR scheme is the main objective, this large number can be a penalty for inappropriate RPR scheme. If the RPR scheme can meet all the constraints, the optimal RPR scheme should be used to determine the corresponding fitness value. In the genetic manipulation logical block, the roulette wheel selection method is used to select better genomes from generated population. The improved single-point crossover method is used to exchanging bits of two parent genomes according to a predefined crossover probability. If the selected points are $a_i$ and $a_i'$, and the selected points’ values are the same, then $b_i$ and $b_i'$ are selected to crossover; if the selected points’ values are different, we select $a_i$ and $a_i'$, $b_i$ and $b_i'$ to crossover separately. Further, if the selected points are $b_i$ and $b_i'$, and the selected points’ values are zero and positive number, respectively, we select $a_i$ and $a_i'$, $b_i$ and $b_i'$ to crossover separately; otherwise, we select $b_i$ and $b_i'$ to crossover. The improved Gaussian mutation method is used to switch bits in a genome according to predefined mutation probability. If the selected point is $a_i$, and the selected point’s value is zero, we mutate $a_i$ to 1 and mutate $b_i$ by the Gaussian mutation method to crossover them separately. If the selected point’s value is 1, we mutate $b_i$ by the Gaussian mutation method. If the selected point is $b_i$, and the selected point’s value is zero, we mutate $a_i$ to 1 and mutate $b_i$ by the Gaussian mutation method. If the selected point’s value is a positive number, we mutate $b_i$ by the Gaussian mutation method. In addition, the termination criterion is that the best fitness value does not change in ten successive generations.

**VI. MODEL VERIFICATION**

In this section, a numerical example built based on the Sioux-Falls network was used to illustrate the performance of the proposed model and algorithm. The network used in this study was from Liu et al. [9] from 2018. The data used included the network topology information, the link performance function parameters, and the OD demands. However, commuters’ long-distance and diversified trips were suitable for the connection between city and suburbs. For test purposes, numerical example built from the Sioux-Falls network was then adjusted for the optimal design of RPR facilities. The area highlighted by the red ellipse was the downtown. The subway lines between the suburb areas and city loop were designed, and the distances around the city center were extended, as shown in Fig. 5.

The link attributes of the Sioux-Falls network are given in Appendix A.1. Unlike the free-flow travel time by automobile, the travel time on the transit service link could be given and fixed because transit service had an exclusive right-of-way without congestion interactions with other transportation modes. Moreover, average speed of a train could be different because section speed of a train was defined by the timetable at stations. The transit lines were round trips, and the link attributes of the transit network given in Appendix A.2. In order to reduce the problem-solving complexity, it was assumed that the ticket costs of the bus and rail services of different lines were the same, and they were denoted as $g_{bs}$ and $g_{tr}$, respectively. The frequencies of different lines of bus and rail services were also the same, and they were denoted as $F_{bs}$ and $F_{tr}$, respectively. Besides, the walking distances of online links, offline links, and transfer links were assumed to be the same, and it was denoted as $l_w$. The parameters of the BPR type function adopted for the generalized link cost were the same for different link types. The related information relating to the transit network is presented in Table 2. The focus was on the travel demands in commuting hours, where the majority of travel demands originated from the suburbs and went into business districts. There were 40 OD pairs, and the OD demand data are provided in Table 3. Four sites (nodes 3, 6, 14, and 19) outside the city center were RPR car park candidates. The feasible construction grade of each RPR site was assumed to be from 1 to 5, respectively representing 100, 200, 300, 400, and 500 parking spaces at the RPR nodes 4 and 6. Moreover, the construction grade of each RPR site was assumed to be from 1 to 5, respectively representing 200, 400, 600, 800, and 1000 parking spaces at the RPR nodes 14 and 19. The passenger capacity of a vehicle was assumed to be one person of a vehicle. The number of on-street parking spaces at RPR nodes 4, 6, 14 and 19 was assumed to be 40, 30, 50, and 50, respectively. The initial parking time at each RPR node was 3 min. Other parameters are set as $\rho = 1.2$, $\beta = 0.8$, $\varepsilon_1 = 0.001$, and $\varepsilon_2 = 0.001$.

The MATLAB software version R2016a was used to determine the optimal RPR facility scheme on a laptop with a 2.2 GHz Dual Core and 8 GB of RAM. The crossover probability and mutation probability were 0.6, and 0.05, respectively. The comparison of the parameters of the optimal RPR
facility scheme obtained by the model without capacity constraints and that obtained by the proposed model is presented in Table 4, where it can be seen that the proposed model outperformed the other model. Regarding the utilization ratio of RPR facilities, the utilization rate of the model without capacity constraints was more than 100%. However, the RPR facilities were different from the conventional parking facilities, and most of the car parking places were occupied for a long time, so other cars were not allowed to park until the parking spots were vacant. The utilization rate of the proposed model was up to 100% because the enhanced Lagrange multiplier method would increase the cost of the overloaded link until the links met the capacity constraints.

Based on the results presented in Table 4, the grade of the RPR nodes in the proposed model was different from that of another model. The total cost generated by the proposed model increased, unlike another model. Therefore, it can be concluded that the commuters who chose RPR transportation mode could effectively decrease the total travel cost. Moreover, the proposed model shifted 100 parking spaces from node 6 to node 4, which was because the budget limit allocated parking spaces with a budget limit, and node 4 could attract more commuters to choose RPR mode than node 6. The nodes 14 and 19 were allocated well so that they could serve the maximum number of commuters who chose the RPR mode with a budget limit. Besides, the grade of node 14 was higher than that of node 19 for both models. Although origins 14, 23, and 24 were closer to node 14 compared to origin 20, commuters who were close to the station preferred to take the bus service to the train station because travel modes $m_3$ and $m_4$ had the same waiting times and transfer times for the bus and rail services. According to the user equilibrium theory, the generalized travel cost of the path that passed through links 14-15, 23-14, and 24-23 was the same as that of the path that passed through link 20-19. Therefore, node 19 could attract more commuters to choose the RPR transportation mode than node 19 with a limit budget.

We also compared the two suburb areas in the northern and southern areas around the downtown, and tested the travel mode share of different OD pairs; the obtained results are shown in Fig. 6. For different OD pairs and the same origin, the longer the distance from the origin was, the smaller the
share of a travel mode $m_1$ was. Thus, the rail service could effectively decrease the generalized travel cost of the long-distance travel. Besides, the rail service has a disadvantage of long waiting and transfer times for short-distance travel. For the given different OD pairs on the same destination in the northern suburb area, the share of travel mode $m_1$ was scarcely smaller, and the longer the distance from the origin to the train stations was, the larger the share of a travel mode $m_4$ was. This was because the distances of the travel in intercity transportation were scarcely shorter, and the distances in the urban transportation affected the share of travel modes $m_3$ and $m_4$. Compared to the travel mode $m_3$, the RPR mode shortened the travel time of both the bus service and the rail service.

In order to evaluate the performance of the proposed model further, the budget limit of the proposed model was analyzed. We tested the proposed model for different discrete values of the budget limit, which were from $1 \times 10^5$ to $2 \times 10^5$ with an increment of $1 \times 10^4$, whereas the values of the other parameters remained unchanged. The MATLAB software was used to obtain the system total cost and RPR scheme under different budget limits, and results are shown in Fig 7.

When the budget limit increased, the generalized cost of the transportation system decreased, as it was expected. Namely, at a higher budget limit, more parking spaces could be built, and system cost could be improved based on the transfer link function. Moreover, the capacity of RPR facilities in the northern suburb area were basically stable, and the grade of node 6 was instable to make full use of the budget when the budget limit increased. Thus, the capacity of RPR facilities in the northern suburb area basically satisfied the travel demand of the RPR mode. The capacity of RPR facilities in the southern suburb area increased with the budget limit. Hence, the potential travel demand of the RPR mode in the southern suburb area could still be excavated.

In order to analyze the effect of different environmental limits, the environmental limit of the proposed model was analyzed. We tested the proposed model for different discrete values of the environmental limit, which were from 4700 to 5000 with an increment of 50, whereas the values of the other parameters remained unchanged. The MATLAB software was used to obtain the system total cost and RPR scheme under different environmental limits, and results are shown in Fig 8.

When the environmental limit increased, the generalized cost of the transportation system was basically stable, as it was expected. Namely, the generalized cost of the transportation system was kept to a minimum, and the carbon monoxide emission of vehicle exhaust not exceeded the limit. However, RPR schemes allowed commuters to use automobiles for the initial portion of their trips to the RPR facility. Compared with travel mode $m_3$, the exhaust emission of vehicle for the initial portion had an opposite effect in environmental pollution. When the environmental limit decreased, the capacity of RPR facilities in the northern suburb area changed because the carbon monoxide emission of vehicle exhaust exceeded the limit. Moreover, the generalized cost of the transportation system increased because the in-vehicle travel time cost on the transit service increased affected by the degree of crowding within the vehicles. Therefore, it could be concluded that arbitrarily RPR facilities could have an opposite effect in environmental pollution, attracting more public transport riders to drive, rather than prompting more drivers to ride due to the convenience provided to driving.

In terms of the limitations of the proposed method, we could see that the reasonable selection of environmental limit affected the RPR scheme, based on the results presented in Fig 8. Moreover, there was no solution to the proposed model when the environmental limit was set below 4700 g/h.
TABLE 5. Link attributes of sioux-falls network.

| Tail node | Head node | Free-flow travel time (min) | Length (km) | Capacity (number of vehicles) |
|-----------|-----------|-----------------------------|-------------|------------------------------|
| 1         | 2         | 30                          | 27          | 600                          |
| 1         | 3         | 8                           | 7           | 900                          |
| 2         | 1         | 30                          | 27          | 600                          |
| 2         | 6         | 8                           | 7           | 900                          |
| 3         | 4         | 9                           | 8           | 1200                         |
| 3         | 12        | 60                          | 100         | 1200                         |
| 4         | 5         | 10                          | 9           | 900                          |
| 4         | 11        | 70                          | 110         | 600                          |
| 5         | 6         | 12                          | 10          | 900                          |
| 5         | 9         | 50                          | 80          | 900                          |
| 6         | 5         | 12                          | 10          | 900                          |
| 6         | 8         | 60                          | 100         | 900                          |
| 7         | 8         | 50                          | 80          | 300                          |
| 8         | 9         | 30                          | 50          | 300                          |
| 8         | 16        | 30                          | 50          | 300                          |
| 9         | 8         | 30                          | 50          | 300                          |
| 9         | 10        | 30                          | 50          | 300                          |
| 10        | 9         | 30                          | 50          | 300                          |
| 10        | 16        | 30                          | 50          | 300                          |
| 11        | 10        | 60                          | 100         | 600                          |
| 11        | 12        | 50                          | 80          | 600                          |
| 13        | 12        | 70                          | 140         | 900                          |
| 13        | 24        | 9                           | 8           | 600                          |
| 14        | 11        | 50                          | 80          | 300                          |
| 14        | 15        | 10                          | 9           | 1200                         |
| 15        | 10        | 80                          | 130         | 900                          |
| 15        | 14        | 10                          | 9           | 900                          |
| 15        | 19        | 12                          | 10          | 600                          |
| 16        | 8         | 30                          | 50          | 300                          |
| 16        | 10        | 30                          | 50          | 300                          |
| 17        | 10        | 50                          | 110         | 300                          |
| 17        | 16        | 30                          | 65          | 600                          |
| 18        | 7         | 40                          | 65          | 600                          |
| 18        | 16        | 50                          | 80          | 600                          |
| 19        | 15        | 12                          | 10          | 600                          |
| 19        | 17        | 40                          | 65          | 1200                         |
| 20        | 18        | 80                          | 160         | 600                          |
| 20        | 19        | 16                          | 14          | 600                          |
| 20        | 21        | 12                          | 10          | 600                          |
| 20        | 22        | 15                          | 13          | 600                          |
| 21        | 20        | 12                          | 10          | 600                          |
| 21        | 22        | 10                          | 8           | 600                          |
| 21        | 24        | 10                          | 9           | 600                          |
| 22        | 15        | 7                           | 6           | 900                          |
| 22        | 23        | 11                          | 9           | 600                          |
| 23        | 14        | 7                           | 6           | 600                          |
| 23        | 22        | 11                          | 9           | 600                          |
| 24        | 13        | 10                          | 8           | 600                          |
| 24        | 21        | 11                          | 9           | 600                          |
| 24        | 23        | 10                          | 8           | 600                          |

This was because the carbon monoxide emission of vehicle exhaust within all feasible RPR schemes exceeded the environmental limit. In view of this, a goal programming approach could be adopted to determine how RPR facilities can be established to meet social cost improvement goals as well as minimize vehicular emission [42].

VII. CONCLUSION

In this study, a bi-level model of a new P&R service scheme named the RPR is presented. The scenarios are generated based on the urbanization background such that commuters' trips are long-distance, going from a suburban area to the downtown. In addition, the RPR schemes are added to the multimodal transportation super network that includes the urban road network, urban transit network, highway network, and rail network. We use the NL-UE model to assign demand to each mode to characterize the transit system state. Furthermore, the main objective is to minimize the generalized cost of the transportation system according to the budget and environmental limits to determine an optimal RPR scheme. An Evans hybrid algorithm with an augmented Lagrange multiplier method and an improved genetic algorithm are employed to solve bi-level model for determination of optimal location and capacity of RPR facilities.

The proposed model was compared with the model without capacity constraints. Regarding the utilization ratio of RPR facilities, the proposed model was more reasonable in allocating the RPR facilities. By comparing different OD pairs and budget limits of the proposed model, it was implied that the RPR services can significantly influence network users’ travel decisions, promote the usage of public transportation, and improve the social benefits. More generally, the model and analysis provide insights that may help operators determine how RPR facilities can be established to meet social cost improvement goals.

In our future work, we will incorporate uncertain demand and uncertain correlated path-flows, link-flows, and link travel times in our model. In addition, we will consider how to locate RPR facilities strategically, as well as minimize vehicular emission.

APPENDIX

See Table 5 and 6 here.

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