Are the majority of Sun-like stars single?

A. P. Whitworth* and O. Lomax

School of Physics and Astronomy, Cardiff University, Cardiff CF24 3AA, UK

Accepted 2015 January 14. Received 2015 January 7; in original form 2014 December 9

ABSTRACT

It has recently been suggested that, in the field, ~56 per cent of Sun-like stars (0.8 \(M_\odot \lesssim M_\ast \lesssim 1.2 \, M_\odot\)) are single. We argue here that this suggestion may be incorrect, since it appears to be based on the multiplicity frequency of systems with Sun-like primaries, and therefore takes no account of Sun-like stars that are secondary (or higher order) components in multiple systems. When these components are included in the reckoning, it seems likely that only ~46 per cent of Sun-like stars are single. This estimate is based on a model in which the system mass function has the form proposed by Chabrier, with a power-law Salpeter extension to high masses; there is a flat distribution of mass ratios; and the probability that a system of mass \(M\) is a binary is \(0.50 + 0.46 \log_{10}(M/M_\odot)\) for \(0.08 M_\odot \leq M \leq 12.5 M_\odot\), 0 for \(M < 0.08 M_\odot\), and 1 for \(M > 12.5 M_\odot\). The constants in this last relation are chosen so that the model also reproduces the observed variation of multiplicity frequency with primary mass. However, the more qualitative conclusion, that a minority of Sun-like stars are single, holds up for virtually all reasonable values of the model parameters. Parenthetically, it is still likely that the majority of all stars in the field are single, but that is because most M Dwarfs probably are single.

Key words: binaries – general – stars: formation – stars: low-mass.

1 INTRODUCTION

The multiplicity statistics of Sun-like stars in the field have recently been re-evaluated by Raghavan et al. (2010), using a volume-limited sample of 454 stars within 25 pc. They estimate that, for systems having a Sun-like primary, the multiplicity frequency is

\[
m_S(M_1 = M_\odot) = 0.44 \pm 0.02.
\]

The multiplicity frequency for systems having a primary of mass \(M_1\) is defined as

\[
m_S(M_1) = \frac{B + T + Q + \cdots}{S + B + T + Q + \cdots},
\]

where \(S\) is the number of single stars of mass \(M_1\), \(B\) is the number of binary systems having a primary of mass \(M_1\), \(T\) is the number of triple systems having a primary of mass \(M_1\), \(Q\) is the number of quadruple systems having a primary of mass \(M_1\), and so on. Implicit in the above statement is the fact that in reality one must consider a finite interval of mass, in order to have meaningful statistics.

Thus, the Raghavan et al. (2010) estimate of \(m_S(M_\odot)\) indicates that the number of single Sun-like stars exceeds, by a factor of \(1.28 \pm 0.08\), the number of Sun-like stars that are primaries in multiple systems. This is not quite the same as the inference made by Raghavan et al. (2010), ‘the majority. . . of solar-type stars are single’, and reiterated by Duchêne & Kraus (2013) ‘a slight majority of all field solar-type stars are actually single’, since it takes no account of Sun-like stars that are secondaries, tertiaries, etc. in multiple systems, only those that are primaries.

In the following, we show that the majority of Sun-like stars may, in fact, be in multiples. To keep the analysis simple we consider only single and binary systems, so we are concerned with evaluating the number of Sun-like stars that are secondaries in binary systems. Consideration of higher order multiple systems would only strengthen our conclusion by bringing into the reckoning additional components that might be Sun-like. Section 2 presents the model of binary statistics that we use, the ranges in which we allow the model parameters to vary, and the basic analysis. Section 3 presents the results, i.e. the fractions of Sun-like stars that are single, primary or secondary, and how these fractions depend on the parameter choices made. Section 4 summarizes our conclusions.

2 MODEL PARAMETERS AND ANALYSIS

In order to model the binary statistics of Sun-like stars and estimate what fraction are secondaries in binaries, we need to specify the distribution of system masses, the fraction of systems that are binaries (as a function of system mass), and the distribution of mass ratios in binary systems.
We define the system mass to be \( M \), and the masses of stars to be \( M_i \), where \( i = 0 \) corresponds to a single star (hereafter a single), \( i = 1 \) corresponds to the primary in a binary (hereafter a primary), and \( i = 2 \) to the secondary in a binary (hereafter a secondary). In addition, we introduce the corresponding logarithmic variables

\[
\mu_i = \log_{10} \left( \frac{M_i}{M_{\odot}} \right), \quad \text{(systems)};
\]

\[
\mu_i = \log_{10} \left( \frac{M_i}{M_{\odot}} \right), \quad \text{(stars)}.
\]

Since

\[
M = M_{\odot} 10^\mu = M_{\odot} e^{\mu},
\]

where \( \ell \equiv \ln(10) \), we have

\[
dM = \ell M.
\]

The mass ratio of a binary system is \( q = M_2/M_1 \).

### 2.1 The distribution of system masses

We assume that low- and intermediate-mass systems (\( \mu < \mu_c \), see below) have a lognormal mass distribution, as proposed by Chabrier (2005),

\[
dN/d\mu LN = \frac{1}{(2\pi)^{1/2}/\sigma} \exp \left\{ -\frac{1}{2} \frac{(\mu - \mu s)^2}{\sigma^2} \right\},
\]

where the subscript LN is for lognormal. Chabrier (2005) estimates that \( \mu_s = -0.60 \) and \( \sigma = 0.55 \); the normalization in Chabrier (2005) is different because he uses physical units. We assume that these values are accurate to \( \pm 0.05 \), i.e., \( -0.65 \leq \mu_s \leq -0.55 \) and \( 0.50 \leq \sigma \leq 0.60 \).

For higher mass systems, \( M > M_c \), we adopt a power-law distribution of masses, with exponent \( \alpha \), i.e.

\[
dN/d\mu PL = \frac{K}{\ell M_{\odot}} \left( \frac{M}{M_{\odot}} \right)^{-\alpha},
\]

where the subscript PL is for power law. It follows that

\[
dN/d\mu PL = K \exp \left\{ -(\alpha - 1) (\ell) \mu \right\}.
\]

The default exponent is \( \alpha = 2.35 \), as first estimated by Salpeter (1955), and we assume this is accurate to \( \pm 0.35 \), i.e., \( 2.00 \leq \alpha \leq 2.70 \).

We require that the join between the two distributions be smooth, i.e. at \( \mu_c \) the distributions (equations 6 and 8) and their slopes should be equal, so

\[
\mu_c = \mu_s + (\alpha - 1) \ell \sigma_c^2,
\]

\[
K = \frac{1}{(2\pi)^{1/2}/\sigma} \exp \left\{ (\alpha - 1) (\ell) \mu_c - \frac{(\mu_c - \mu_s)^2}{2\sigma^2} \right\}.
\]

With the default values of the model parameters, the join occurs at \( \mu_c \simeq 0.34 \), corresponding to a system mass of \( M_c \simeq 2.2M_{\odot} \).

### 2.2 The fraction of systems that are binaries

Since the distribution of mass ratios appears to vary slowly – if at all – with primary mass in the range \( 0.2M_{\odot} \lesssim M_1 \lesssim 2M_{\odot} \) (Raghavan et al. 2010; Janson et al. 2012; Reggiani & Meyer 2013), the fraction of systems that are binaries must depend on system mass in a similar way to the dependence of multiplicity frequency on primary mass, but displaced to slightly higher masses. We therefore assume that the fraction of systems that are binaries is given by

\[
\beta(\mu) = \begin{cases} 
0, & \mu \leq -\beta_0/\beta_1; \\
\beta_0 + \beta_1 \mu, & -\beta_0/\beta_1 < \mu < (1 - \beta_0)/\beta_1; \\
1, & \mu \geq (1 - \beta_0)/\beta_1.
\end{cases}
\]

The justification for adopting this functional form is given in Section 2.6, where we also derive the default values of \( \beta_0 \) and \( \beta_1 \), and their ranges, by fitting the observed run of multiplicity frequency, \( m_s \) against primary log-mass, \( \mu_1 \) (see Fig. 1).
this is the fractional increase in the number of stars of mass $M_*$ in binaries that derives from taking account of secondaries. The subscripts $*$ on $b$ and $f$ record that these are stellar properties.

### 2.6 Constraining $\beta_0$ and $\beta_1$

We identify the default values of $\beta_0$ and $\beta_1$ by requiring that the model (i) reproduce accurately the multiplicity frequency of Sun-like stars inferred from observation by Raghavan et al. (2010), and (ii) reproduce as closely as possible the run of multiplicity frequency with primary mass derived from other observational studies by Close et al. (2003), Basri & Reiners (2006), Fischer & Marcy (1992), Janson et al. (2012), Duquennoy & Mayor (1991), Kouwenhoven et al. (2007), Rizzuto et al. (2013), Preibisch et al. (1999) and Mason et al. (1998). The resulting default values are $\beta_0 = 0.50$ and $\beta_1 = 0.46$, so the probability that a system of mass $M$ is a binary becomes

$$
\beta(M) = \begin{cases} 
0, & M < 0.08 M_\odot; \\
0.50 + 0.46 \log_{10} \left( \frac{M}{M_\odot} \right), & 0.08 M_\odot \leq M \leq 12.5 M_\odot; \\
1, & M > 12.5 M_\odot.
\end{cases}
$$

(19)

The corresponding fit to the observational estimates is illustrated by the lower (bolder) line on Fig. 1. A brief discussion of this plot is appropriate.

The model equations and default parameters describing both the distribution of system masses (i.e. Chabrier lognormal at low and intermediate masses, with $\mu_S = -0.60$ and $\sigma_S = 0.55$, plus Salpeter power law, with negative slope $\gamma = 2.35$, at high masses), and the distribution of stellar mass ratios (flat, $\gamma = 0.00$, with no minimum, $q_{\text{MIN}} = 0.00$) appear to be the natural default choices; we will explore the consequences of varying the model parameters, both individually and collectively, in Section 3. The justification for adopting equation (11) for the fraction of systems that are binaries as a function of system mass, and hence the choice of the model parameters $\beta_0$ and $\beta_1$, is more ad hoc.

From the observational data presented in Fig. 1, it appears that, for $-1 \lesssim \mu_1 \lesssim 1$, $m_5$ increases approximately linearly with $\mu_1$, and hence a linear relation between $\beta$ and $\mu$ (i.e. equation 11) seems the simplest option to explore. However, it is also clear from Fig. 1 that equation (11) gives a much better fit when the Duquennoy & Mayor (1991) acceptance box is invoked for systems with Sun-like primaries (upper, fainter line) than when the Raghavan et al. (2010) acceptance box is invoked (lower, bolder line). Even if the Fischer & Marcy (1992) and Duquennoy & Mayor (1991) acceptance boxes are discounted, the Raghavan et al. (2010) acceptance box is hard to reconcile with the others without introducing a more complex function $m_5(\mu_1)$, for which the slope has a distinct minimum around $\mu_1 \sim 1$. Since – as far as we are aware – no physical explanation for such a minimum has been advanced, we avoid this complication. It would certainly be convenient if, when in future the multiplicity frequencies for systems with non-Sun-like primaries are evaluated more accurately, the values are reduced (say by $\sim 20\%$ per cent) to bring them into line with Raghavan et al. (2010), or if the Raghavan et al. (2010) estimate is revised upwards.

In this context, it is important to note that Sun-like singles derive from systems with $\mu = 0$, Sun-like primaries from systems with $0 < \mu \lesssim \log_{10}(2)$, and Sun-like secondaries from systems with $\log_{10}(2) < \mu < \infty$. In addition, with any sensible choice of the model parameters, the system mass function is falling quite rapidly with...
increasing $\mu$ for $\mu \geq 0$. Thus the important range of applicability of equation (11) is $0 \leq \mu \lesssim 0.6$.

At low $\mu$ ($\mu \lesssim -\beta_0/\beta_1, M < 0.08 M_\odot$), $\beta$ has to be set to zero to avoid non-physical predictions, and this has the consequence that the multiplicity frequency falls to zero for $\mu \lesssim -1.3$ (i.e. $M_\ast \lesssim 0.05 M_\odot$). The observed multiplicity frequency appears to be very low for such low-mass primaries (only one of the systems reported by Close et al. (2003) has $M_\ast \lesssim 0.05 M_\odot$), but it is probably not zero. We explore the effect of adopting model parameters that increase the multiplicity frequency of brown dwarfs and very low-mass stars in Section 3. However, it is important to note that the value of $\beta$ at these low masses is irrelevant to the statistics of Sun-like stars, because a low-mass core, $M < M_\odot$, cannot spawn a Sun-like star.

At high $\mu$ ($\mu \geq (1 - \beta_0)/\beta_1, M > 12.5 M_\odot$), $\beta$ has to be set to one to avoid non-physical consequences. In fact, the acceptance boxes due to Kouwenhoven et al. (2007), Rizzuto et al. (2013), Preibisch et al. (1999) and Mason et al. (1998) are all lower limits. At these high primary masses, the multiplicity frequency is an inadequate measure of multiplicity, because higher order multiple systems become increasingly important at high primary mass, and the multiplicity frequency does not distinguish between higher order multiples and binaries (Hubber & Whitworth 2005). The pairing factor (number of orbits per system) or companion frequency (mean number of companions) would be more appropriate measures. In addition, the notion of high-mass field stars is fraught, because the highest-mass stars do not live long enough for their birth clusters to dissolve completely into the field, and therefore many high-mass stars in the field are runaways, which have been ejected in violent $N$-body interactions, and therefore seldom, if ever, have companions. The multiplicity statistics for high-mass stars, which point to very high pairing factors and companion frequencies, tend actually to pertain to stars that are still intimately involved with their birth clusters.

3 RESULTS

3.1 The default solution

For Sun-like stars and the default parameter set, we obtain $m_S(M_\odot) = 0.440$, $b_S(M_\odot) = 0.535$ and $f_S(M_\odot) = 0.42$. In other words, although for every $\sim 56$ Sun-like stars, there are only $\sim 44$ Sun-like primaries in binary systems, there are also $\sim 20$ Sun-like secondaries in binary systems, hence a total of $\sim 66$ Sun-like stars in binaries.

3.2 The effect of varying the model parameters

Figs 2 and 3 represent the ($m_S(M_\odot)$, $b_S(M_\odot)$)-plane in the vicinity of the solution obtained with the default parameters (hereafter the default solution). The black dot at $m_S(M_\odot) = 0.440$ and $b_S(M_\odot) = 0.535$ on Fig. 2 represents the default solution. The vertical dashed lines demark the limits on $m_S(M_\odot)$ obtained by Raghavan et al. (2010), viz. $0.42 < m_S(M_\odot) < 0.46$. The horizontal dotted line is at $b_S(M_\odot) = 0.50$; for solutions below (above) this line more (less) than 50 per cent of Sun-like stars are single.

In Fig. 2, the lines passing through the default solution show how the solution changes if one of the model parameters is varied. The arrow at one end of a line indicates the direction in which the parameter increases, and the symbol for the parameter in question is given beside this arrow. The lines for $\mu$, $\sigma_S$ and $\beta_1$ lie almost on top of one another; each of these lines extends approximately the same distance on either side of the default solution, and therefore the length can be inferred from the position of the corresponding arrow (the line for $\sigma_S$ is the longest, and that for $\beta_1$ the shortest).

Over the range of system log-masses that contributes Sun-like stars, $0 \leq \mu < \infty$, the mass function is decreasing with increasing $\mu$, i.e. $dN/d\mu < 0$. Increasing $\mu_S$ and/or increasing $\sigma_S$ and/or decreasing $\alpha$ increases $dN/d\mu$, i.e. reduces the downward slope of the system mass function, so there are then more Sun-like primaries relative to singles (increased $m_S(M_\odot)$) and more Sun-like secondaries relative to primaries (increased $b_S(M_\odot)$). Conversely, decreasing $\mu_S$ and/or decreasing $\sigma_S$ and/or increasing $\alpha$ decreases $m_S(M_\odot)$ and $b_S(M_\odot)$. Increasing $\alpha$ from its minimum value ($\alpha = 2.00$) has an ever diminishing effect, because the switch from the lognormal system mass distribution to the power-law distribution occurs at ever increasing $\mu_S$ (see equation 9); consequently the line for increasing $\alpha$ does not extend far beyond the default solution on Fig. 2, once $\alpha > 2.35$.

Increasing $\beta_0$ and/or $\beta_1$ above the default values increases the number of primaries and secondaries, relative to singles, and hence increases both $m_S(M_\odot)$ and $b_S(M_\odot)$; increasing $\beta_0$ has a larger effect than increasing $\beta_1$ because the binary statistics of Sun-like stars are dominated by systems with $0 \leq \mu \lesssim 0.6$, i.e. relatively small $\mu$. Conversely, decreasing $\beta_0$ and/or $\beta_1$ below their default values decreases both $m_S(M_\odot)$ and $b_S(M_\odot)$.
value, but decreasing $\gamma$ below the default value augments $m_s(M_\odot)$ and reduces $b_s(M_\odot)$.

Table 2 gives the parameter symbols; their default values and prescribed ranges; and the partial derivatives, $\partial m_s(M_\odot)/\partial X$, $\partial b_s(M_\odot)/\partial X$ and $\partial f_s(M_\odot)/\partial X$, where $X$ is one of the model parameters (i.e. $X \equiv \mu_0, \sigma_X, \beta_0, \beta_1, \varphi_{\text{MIN}}, \gamma$), and all the derivatives are evaluated for the default parameter set.

Fig. 3 displays the distribution of solutions when the model parameters are varied simultaneously and randomly. This plot is produced by generating $\sim 1.8 \times 10^7$ different solutions ($\sim 10^7$ of which fall within Fig. 3). For each solution, the value of parameter $X$ is chosen by generating a random Gaussian deviate, $\mathcal{G}$, from a distribution with mean 0 and standard deviation 1, and then putting $X = X_0 + \mathcal{G} \Delta X$, where the values of $X_0$ and $\Delta X$ for each model parameter are given in Table 2 (Columns 3 and 4); with this procedure, $\sim 32$ per cent of parameter values fall outside the range $X_0 \pm \Delta X$.

If we consider all the solutions generated in this way (including those that fall outside Fig. 3), $\sim 21$ per cent of them give $f_s < 0.50$. However, the majority of these solutions involve very low values of $\beta_0$, and therefore they also deliver $m_s(M_\odot) < 0.42$ (i.e. below the range estimated by Raghavan et al. 2010).

If we limit consideration to solutions that satisfy the constraints calculated by Raghavan et al. (2010), i.e. $0.42 < m_s(M_\odot) \leq 0.46$ (including those that fall outside Fig. 3), then $\sim 3$ per cent of the allowed solutions give $f_s < 0.50$, because these solutions require both very low $\beta_0$ and very low $\gamma$. We conclude that it is rather unlikely that the majority of Sun-like stars are single.

3.3 Other masses

If we accept the default parameters as a reasonable representation of the binary statistics of low- and intermediate-mass stars, we can estimate the fraction of such stars that are single. For example, for $M_* \leq 0.70 M_\odot$, $\geq 50$ per cent of stars are single. Thus most M Dwarfs are single.

4 CONCLUSIONS

We have developed a model to estimate the fraction of Sun-like stars that are secondaries in binary systems. Our model (which only considers singles and binary systems, and therefore gives a lower limit to the fraction of Sun-like stars that are in multiples) invokes a lognormal distribution of system masses (Chabrier 2005) with a power-law tail at high masses (Salpeter 1955); a flat distribution of mass ratios (e.g. Raghavan et al. 2010; Janson et al. 2012; Reggiani & Meyer 2013); and the probability that a system with mass $M$ is a binary,

$$\beta = 0.50 + 0.47 \log_{10}(M/M_\odot).$$

| SOURCE | $m_s(M_\odot)$ | $\beta_0$ | $\beta_1$ | $b_s(M_\odot)$ | $f_s(M_\odot)$ |
|--------|----------------|----------|-----------|----------------|----------------|
| Raghavan et al. (2010) | 0.44 | 0.50 | 0.46 | 0.535 | 0.461 |
| Duquennoy & Mayor (1991) | 0.58 | 0.64 | 0.52 | 0.667 | 0.450 |
Table 2. Column 1 gives the model parameters. Columns 2 through 4 give their symbols, their default values and their ranges. Columns 5 through 7 give the derivatives of, respectively, the multiplicity frequency of Sun-like stars, \( m_\text{S}(M_\odot) \), the fraction of Sun-like stars that are in binaries, \( b_\star(M_\odot) \), and the ratio of Sun-like secondaries to Sun-like primaries, \( f_\star(M_\odot) \), with respect to each model parameter in turn; all the derivatives are evaluated for the default values of the model parameters.

| Model function | Model parameter, \( X \) | Default value, \( X_0 \) | Range \( \Delta X \) | \( \frac{dm_\text{S}}{dX} \) | \( \frac{db_\star}{dX} \) | \( \frac{df_\star}{dX} \) |
|----------------|--------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| DISTRIBUTION   | \( \mu_\text{S} \)       | -0.60           | \( \pm 0.05 \)  | 0.13            | 0.18            | 0.27            |
| OF SYSTEM      | \( \sigma_\text{S} \)    | +0.55           | \( \pm 0.05 \)  | 0.32            | 0.44            | 0.75            |
| LOG-MASSES     | \( \alpha \)             | +2.35           | \( \pm 0.35 \)  | 0.00            | -0.02           | -0.14           |
| BINARY         | \( \beta_0 \)            | +0.50           | \( \pm 0.05 \)  | 0.90            | 0.90            | -0.20           |
| FRACTION       | \( \beta_1 \)            | +0.46           | \( \pm 0.05 \)  | 0.07            | 0.09            | 0.15            |
| DISTRIBUTION   | \( q_{\min} \)           | +0.00           | \( +0.10, -0.00 \) | -0.07           | 0.03            | 0.59            |
| OF MASS RATIOS | \( \gamma \)             | +0.00           | \( +2.00, -0.60 \) | -0.03           | 0.01            | 0.26            |

We find that, even if the multiplicity frequency is as low as the recent estimate of Raghavan et al. (2010), \( 0.42 \leq m_\text{S}(M_\odot) \leq 0.46 \), and even if we vary the model parameters significantly from their default values, when account is taken of secondaries (and higher order components in multiple systems), the majority of Sun-like stars are probably in multiple systems. Our model predicts that the fraction of stars that are in multiple systems is a monotonically increasing function of stellar mass, and that the majority of stars with mass \( M_\star > 0.7 M_\odot \) are in multiple systems. Conversely, most M Dwarfs, and hence most stars overall, are single.

ACKNOWLEDGEMENTS

APW and OL gratefully acknowledge the support of a consolidated grant (ST/K00926/1) from the UK Science and Technology Funding Council. We thank the referee, Mike Simon, for his useful comments.

REFERENCES

Basri G., Reiners A., 2006, AJ, 132, 663
Chabrier G., 2005, in Corbelli E., Palla F., Zinnecker H., eds, Astrophysics and Space Science Library, Vol. 327, The Initial Mass Function 50 Years Later. Springer, Dordrecht, p. 41
Close L. M., Siegler N., Freed M., Biller B., 2003, ApJ, 587, 407
Duchêne G., Kraus A., 2013, ARA&A, 51, 269
Duquennoy A., Mayor M., 1991, A&A, 248, 485
Fischer D. A., Marcy G. W., 1992, ApJ, 396, 178
Hubber D. A., Whitworth A. P., 2005, A&A, 437, 113
Janson M. et al., 2012, ApJ, 754, 44
Kouwenhoven M. B. N., Brown A. G. A., Portegies Zwart S. F., Kaper L., 2007, A&A, 474, 77
Lucy L. B., 2006, A&A, 457, 629
Mason B. D., Gies D. R., Hartkopf W. I., Bagnuolo W. G., Jr, ten Brummelaar T., McAlister H. A., 1998, AJ, 115, 821
Preibisch T., Balega Y., Hofmann K.-H., Weigelt G., Zinnecker H., 1999, New Astron., 4, 531
Raghavan D. et al., 2010, ApJ, 190, 1
Reggiani M., Meyer M. R., 2013, A&A, 553, A124
Rizzuto A. C. et al., 2013, MNRAS, 436, 1694
Salpeter E. E., 1955, ApJ, 121, 161

This paper has been typeset from a \( \text{LaTeX} \) file prepared by the author.