Signals of Supersymmetric Flavour Models in $B$ Physics

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Abstract

If the mechanism of Supersymmetry breaking is not flavour blind, some flavour symmetry is likely to be needed to prevent excessive flavour changing neutral current effects. We discuss two flavour models (based respectively on a $U(2)$ and on a $SU(3)$ horizontal symmetry) providing a good fit to fermion masses and mixings and particularly constraining the supersymmetry soft breaking terms. We show that, while reproducing successfully the Standard Model fit of the unitarity triangle, it is possible to obtain sizable deviations from the Standard Model predictions for three very clean $B$-physics observables: the time dependent CP asymmetries in $B_d \rightarrow J/\psi K^0$ and in $B_s \rightarrow J/\psi \phi$ and the $B_s - \bar{B}_s$ mass difference. Our analysis exhibits with two explicit realizations that in supersymmetric theories with a new flavour structure in addition to the Yukawa matrices there exist concrete potentialities for revealing supersymmetry indirectly in theoretically clean $B$-physics observables.
1 Introduction

For the last two decades, the indirect search for Supersymmetric (SUSY) signals through Flavour Changing Neutral Current (FCNC) and CP violating processes has proven to be a crucial complementary tool to direct accelerator search \[1\]. After the end of the LEP era, our hopes for a detection of SUSY particles focus on the upgraded Tevatron and even more on LHC, the resolutive machine for low-energy SUSY. In the years before LHC, the challenge for SUSY hints mostly relies upon virtual effects in FCNC and CP rare processes. After the intensive experimental and theoretical work on Kaon physics, and waiting for the important results on rare $K$ decays, the next frontier is represented by $B$ physics. Although all of us hope in some dramatic effect signaling the presence of new physics (for instance, had the CP asymmetry in $B \to J/\psi K (a_{J/\psi K})$ settled at the level of 10% there would be no doubt \[2\]), it is likely that we will have to face a more complicate situation where the information on new physics will be entangled with the hadronic uncertainties plaguing nonleptonic $B$ decays. In view of this fact, processes as $B_s - \bar{B}_s$ mixing acquire a crucial relevance in increasing the redundancy of the Unitarity Triangle (UT) determination, hence allowing for a possible discrimination among different SUSY extensions of the SM. In this respect, it proves quite useful to test various classes of low-energy SUSY models considering, in addition to the stringent constraints from $K$ physics, the joint information from the mixing and the CP asymmetries in $B$ physics.

On the other hand, just the severity of the present FCNC constraints \[3, 4, 5\] seems to point to two definite directions: either the mechanism of SUSY breaking is flavour blind, resulting in the so-called Minimal Supersymmetric Standard Model (MSSM) with Minimal Flavour Violation (MFV), or we need some mechanism (based on flavour symmetries, alignment, or heavy first generations sfermions, for instance) to forbid disastrously large SUSY contributions to FCNC and CP violating processes arising from the new flavour structure of the model. As for the former option, already several detailed analyses of the impact of these models on FCNC and CP violation have been performed \[6\]. Concerning the second possibility, much interesting work has recently focused on the construction of successful non-abelian flavour models \[7\]–\[14\], mainly concentrating on the prediction of fermion masses and mixing angles. However, with a few valuable exceptions, most of these works have not thoroughly investigated the impact of SUSY contributions to FCNC in relation with the UT determination. Such attitude was fully justified when the main objective was the prohibition of too large SUSY effects, but nowadays, since our goal is the detailed comparison of the SM and SUSY predictions on FCNC, it is mandatory to reconsider SUSY flavour models taking into account the specific SUSY contributions to rare processes.

As a first step in this direction, in this Letter we consider two promising models with nonabelian flavour symmetries, which particularly constrain the flavour structure of the SUSY soft breaking terms. We show that it is possible to successfully reproduce the SM fit of the UT while allowing for sizable deviations from the SM predictions for three interesting $B$ physics observables: $a_{J/\psi K}$, $a_{J/\psi\phi}$, the time-dependent CP asymmetry in $B_s \to J/\psi\phi$ decays, and $\Delta m_{B_s}$, the $B_s - \bar{B}_s$ mass difference. Our analysis shows the
importance of using theoretically clean $B$-physics observables in disentangling the SUSY effects in models with viable flavour structures.

2 A model with a $U(2)$ flavour symmetry

Let us first consider a model based on a $U(2)$ symmetry acting on the two lighter families $[7]-[10]$. The pattern of fermion masses and mixing reveals an approximately symmetric structure under $U(2)$. This symmetry, in fact, suppresses (forbids, in the unbroken limit) the Yukawa couplings of the two lighter families and the non-degeneracy of their supersymmetric partners. Moreover, the $U(2)$ symmetry can be considered the residual symmetry unbroken after the large breaking of an $U(3)$ symmetry by the top Yukawa coupling. The fermions of the third, $\psi_3$, and of the first two families, $\psi_a$, $a = 1, 2$, have the obvious transformation properties under $U(2)$. The Higgs fields are assumed to be singlets. The Yukawa couplings involving the lighter families are associated to VEVs of SM singlets breaking the flavour symmetry and coupling to the SM fermions through non-renormalizable Yukawa interactions. Such VEVs can only transform as an antidoublet $\phi^a$, an antisymmetric tensor $A^{ab}$ or a symmetric tensor $S^{ab}$ under the flavour symmetry.$^1$ The two step breaking of the rank two group $U(2)$ can be accomplished by using only the first two of those representations: $\phi^a$ and $A^{ab}$. No assumption needs to be made on the orientation of the corresponding VEVs in the flavour space, since every choice is equivalent to $\langle \phi \rangle = (0, V)^T$, $V > 0$, $\langle A^{ab} \rangle = v \epsilon^{ab}, v > 0$ up to an $U(2)$ transformation. Notice that $\langle \phi \rangle$ leaves a residual $U(1)$ unbroken, which protects the mass of the lightest family. The asymmetric VEV $\langle A^{ab} \rangle$ then breaks the residual $U(1)$ and gives mass to the lightest family. The interfamily mass hierarchy is obtained if $V > v$, so that

$$U(2) \xrightarrow{V} U(1) \xrightarrow{v} 1.$$  \hspace{1cm} (1)

Within this framework, we now briefly describe a new model which is a variation of $^{[13]}$, to which we refer for a more detailed discussion of the general framework, and represents an example of how our understanding of flavour and CP-violation can be affected by new physics. We assume that the $U(2)$ breaking is communicated to the SM fermions $\psi_3, \psi_a$ through a Froggatt-Nielsen (FN) mechanism by an heavy $U(2)$ doublet $\chi^a$ in the same gauge representation as a whole fermion family. Since we want $\chi^a$ to be heavy in the $U(2)$ symmetric limit, we include the conjugated fields $\bar{\chi}_a$ in the messenger sector. We work in the context of a supersymmetric $SU(5)$ GUT. Once $U(2)$ is broken, the light (in the $U(2)$-symmetric limit) families $\psi_a$ and the heavy copies $\chi^a$ mix, thus giving rise to the light Yukawa couplings. We also take into account the possibility that the two $SU(5)$ multiplets $H_1$, $H_2$ containing the up and down light Higgses mix with heavy copies $H'_1$, $H'_2$, $U(2)$ singlets too.$^2$

$^1$Upper and lower indexes correspond to conjugated transformations.

$^2$If part of the hierarchy $m_b \ll m_t$ is due to a hierarchy between the corresponding Yukawa couplings, the latter can be accounted for by a mixing in the Higgs sector.
Let us now discuss the size of mass terms and VEVs. One simple possibility is to assume that the mass $M$ of the heavy doublets $\chi^a$, $\bar{\chi}_a$ is generated above the $SU(5)$ breaking scale, $M > M_{GUT}$, and is therefore $SU(5)$ invariant. A small ratio $V/M$ is then generated if the $U(2)$-breaking takes place at the $SU(5)$ breaking scale, $V \sim M_{GUT}$. $SU(5)$ breaking corrections to the heavy mass $M$ will also be correspondingly smaller than $M$. As for the mass $M'$ of the heavy multiplets possibly mixing with the Higgs multiplets, we will assume it to be of the order of the GUT scale. The $U(2)$-singlet, $SU(5)$ fiveplet messengers $H'_1$, $H'_2$ will therefore be lighter than the $U(2)$ doublet messengers $\chi^a$, $\bar{\chi}_a$. This at the same time accounts for the empirical relation $m_s/m_b \sim |V_{cb}|$ and for the hierarchy $m_e/m_t \ll m_s/m_b$, enhances the supersymmetric contributions to $B$-mixing, improves the agreement of the measured value of $|V_{ub}/V_{cb}|$ with the prediction of the model in terms of light quark masses \cite{14} and might be related to the large mixing in the neutrino sector indicated by the atmospheric neutrino anomaly \cite{14}. Finally, the breaking of the residual $U(1)$ occurs below the GUT scale, $v < M_{GUT}$. As for the transformation properties of the VEVs $A^{ab}, \phi^a$, under $SU(5)$, the only crucial assumption is that $A^{ab}$ is $SU(5)$ invariant, which accounts for the hierarchy $m_u/m_c/m_t \ll m_d/m_s/m_b$. By writing the most general superpotential and soft terms one then gets the following textures for quark and squark masses at the GUT scale:

$$M_d = m^D \left( \begin{array}{ccc} 0 & e^\epsilon & 0 \\ -e^{-\epsilon} & 0 & e^{i\phi} \\ 0 & \rho & 1 \end{array} \right)$$  \hspace{1cm} (2)$$

$$M_u = m^U \left( \begin{array}{ccc} 0 & cee' & 0 \\ -ceee' & 0 & ae \\ 0 & bee^{i\psi} & 1 \end{array} \right)$$ \hspace{1cm} (3)$$

$$m_Q^2 = m_{3/2}^2 \left( \begin{array}{ccc} 1 & 0 & \alpha e e' \\ 0 & 1 & 0 \\ \alpha^* e e' & 0 & r_3 \end{array} \right)$$ \hspace{1cm} (4)$$

$$m_d^2 = m_{3/2}^2 \left( \begin{array}{ccc} 1 & 0 & \alpha' e e' \\ 0 & 1 + \lambda \rho |^2 & \beta \rho^* \\ \alpha^* e e' & \beta^* \rho & r_3' \end{array} \right)$$ \hspace{1cm} (5)$$

$$m_u^2 = m_{3/2}^2 \left( \begin{array}{ccc} 1 & 0 & \alpha'' e e' \\ 0 & 1 & 0 \\ \alpha''* e e' & 0 & r_3'' \end{array} \right)$$ \hspace{1cm} (6)$$

where $\epsilon = O(V/M)$, $\epsilon' = O(v/M)$, $\rho = O(V/M')$ and all other coefficients arise from couplings of order one. The parameters $r_3, r_3', r_3''$ differentiate the third sfermion family masses from the $U(2)$ invariant masses of the first two families. They can differ from one since the flavour symmetry does not constrain this ratio. For simplicity, from now on we will assume $r_3 = r_3' = r_3''$.

Some comments are in order. Since $(M_d)_{22} = 0$, an asymmetry $(M_d)_{32} > (M_d)_{23}$ is required in order to agree with the relation $(m_s/m_b)_{GUT} \sim |V_{cb}|_{GUT}$ without invoking
cancellations between the contributions to \( V_{cb} \) from \( M_d \) and \( M_u \). Such an asymmetry is obtained here because \((M_d)_{32}\) is generated by exchange of the \( U(2) \) singlets \( H'_1, H'_2 \) at the scale \( M' \sim V \), whereas \((M_d)_{23}\) is generated by the exchange of the \( U(2) \) doublets \( \chi^a, \bar{\chi}^a \) at the higher scale \( M \gg V \). The same singlet exchange splits the masses of the first two families in the down-right sector. Since the \( U(2) \) singlets \( H'_1, H'_2 \) are \( SU(5) \) singlets, they do not contribute at first order to the up-quark mass matrix: both \((M_u)_{23}\) and \((M_u)_{32}\) are of order \( \epsilon \). The larger hierarchy \( m_c/m_t \ll m_s/m_b \) follows. As for the further suppression of \( m_u m_c/m_t^2 \) with respect to \( m_d m_s/m_b^2 \), it is due here to the invariance of \( A^{ab} \) under \( SU(5) \) \[9, 13\]. The operator \( A^{ab} T_a T_b H \), in standard \( SU(5) \) notations, does in fact vanish due to the antisymmetry of \( A^{ab} \). \( SU(5) \) breaking effects must be included in order to generate a non-vanishing \((M_u)_{12}\) entry, thus giving the extra \( \epsilon \) there. Finally, the factor \((1 + \rho^2 k^2)^{-1/2}\) in the \((M_d)_{12}\) entry comes from the diagonalization of the kinetic terms. Notice that, thanks to rephasing invariance, we have the freedom to have all real entries apart from \((M_d)_{23}\) and \((M_u)_{32}\). We choose to work with real parameters, and so explicitly write these phases in terms of two angles \( \phi \) and \( \psi \).

We do not discuss here the A-terms. The flavour symmetry constrains them to have the same structure of the Yukawa couplings. Once the constraints from \( \Delta F = 1 \) processes (and EDMs) have been taken into account\[^3\], the contributions to the \( \Delta F = 2 \) transitions relevant to the UT fit are negligible \[^4\]. We can therefore safely drop these terms in the following.

One important property of the flavour structure in eq. (2) is the presence of a large mixing between the second and third generation in the right-handed sector. This is irrelevant for SM contributions to flavour-changing processes, but has a large impact in the sfermionic sector. Indeed, squark exchange with this mixing can generate large coefficients for the Left-Right four-fermion operators in the \( \Delta F = 2 \) effective Hamiltonian, which are then enhanced both by the QCD running and by the matrix elements. Therefore, we are in the interesting situation in which there is a complementary sensitivity of SUSY contributions to those features of the flavour structure that cannot be probed considering only SM-induced amplitudes. This explains why in this case it is very important to include SUSY effects when testing the flavour structure of the model. The same considerations apply, as we shall see in the following, to the model based on a \( SU(3) \) flavour symmetry.

**Unitarity Triangle Analysis**

As discussed in the Introduction, our aim here is to show how SUSY effects can modify the predictions of flavour models, and in particular how the shape of the UT depends on the contributions from the SUSY sector. In general, some of the parameters of the flavour model can be determined using only SM-dominated (tree-level) processes. However, the CP-violating phases and the sfermion mass parameters can only be extracted from loop processes. In principle, one should proceed by simultaneously fitting all these parameters. Notice that indeed the saturation of \( \epsilon'/\epsilon \) can be obtained even for tiny values of the corresponding A-parameters \[^14\].
parameters. Unfortunately, at present this is not possible since the only relevant quantities that have been measured are $\varepsilon_K$ and $\Delta m_{B_d}$, together with the lower bound on $\Delta m_{B_S}$. When, hopefully in the near future, more experimental data will be available (a more precise measurement of $a_{J/\psi K}$, CP-asymmetries in other channels, rare decays, etc.), a global fit will be feasible. For the purpose of illustrating the potentially large effects due to SUSY contributions, we can however proceed by fixing the CP phases in the Yukawa couplings to some representative values. We then scan over the sfermion parameter space imposing $\varepsilon_K$, $\Delta m_{B_d}$ and $\Delta m_{B_s}$ constraints and obtain predictions for other observables as a function of SUSY parameters. Once new measurements are available, these predictions can be turned into further constraints on the SUSY parameter space.

For our numerical analysis, we first run with SUSY one-loop renormalization group equations the mass matrices from the GUT to the electroweak scale \cite{16}. We then use the NLO QCD running \cite{17, 18} from the electroweak scale to the hadronic scale for the $\Delta F = 2$ amplitudes and take the relevant $B$-parameters from lattice QCD, whenever they are available. In particular, we use the NLO $\Delta S = 2$ effective Hamiltonian in the Landau RI scheme (LRI) as given in ref. \cite{5} and the corresponding $B$ parameters from ref. \cite{19}. Concerning the $\Delta B = 2$ $B$-parameters, only one of the three we need is available at present, and we have taken it from ref. \cite{20}.

The first step of the analysis is to fit the parameters entering the fermionic matrices for fixed values of the phases, to reproduce the experimental values for fermion masses and $|V_{ub}|$, $|V_{us}|$ and $|V_{cb}|$, which can be determined using tree-level weak decays. In table 1 we report some numerical examples for different choices of the phase. The fit uses the values in table 2 as input parameters.

The second step is to constrain the SUSY parameters making use of $\varepsilon_K$ and $\Delta m_{B_d}$. We can then predict $\Delta m_{B_s}$, $a_{J/\psi K}$ and $a_{J/\psi \phi}$ for each given set of SUSY masses compatible with the constraints. First of all, we note that for vanishing phases in the Yukawa couplings, once the $\varepsilon_K$ and $\Delta m_{B_d}$ constraints are imposed, the predicted value of $\Delta m_{B_s}$ is below the present lower bound for almost any choice of SUSY parameters. The reason for this is the following. For vanishing CKM phase, the UT collapses to the positive $\bar{\rho}$ axis, which implies that the SM contribution to $\Delta m_{B_d}$ is about one half of the experimental value. While this can be compensated by a large SUSY contribution, the flavour structure then forces the SUSY contribution to $\Delta m_{B_s}$ to interfere destructively with the SM one, resulting inevitably in a too low value for the $B_s - \bar{B}_s$ mass difference (see fig. 1).

Once we introduce CP violation in the CKM matrix this anticorrelation between SUSY contributions to $\Delta m_{B_d}$ and $\Delta m_{B_s}$ is lost, and good fits can be obtained also for relatively small values of the CKM phase. This is interesting since, as we anticipated in the introduction, not only can we successfully reproduce all the observed CP violation,

4 For our choice of SUSY parameters the gluino exchange represents the dominant SUSY contribution. We performed the actual computation of $\Delta F = 2$ amplitudes in the mass insertion approximation (MIA) \cite{3}. Given the particular textures we are using for sfermion soft mass terms, to obtain a reliable result in MIA for $\Delta S = 2$ observables, multiple mass insertions have been included.
Table 1: Results of the fit of fermionic parameters for different choices of the phases $\psi$ and $\phi$ (see text for details) in the $U(2)$ case. The values in the first half of the table correspond to the fitted parameters, and the results in the second half correspond to the purely SM contributions to $\Delta F = 2$ processes. The mass differences are given in $ps^{-1}$. $\bar{\rho}$ and $\bar{\eta}$ appear in the Wolfenstein parameterization of the CKM matrix [24]. The definition of the asymmetries is according to ref. [25].

| $\phi$ | 0  | -0.25 | -0.25 | -0.5 |
| $\psi$ | 0  | 0     | -0.25 | -0.25 |
| $\epsilon$ | 0.059 | -0.055 | 0.073 | 0.064 |
| $\epsilon'$ | 0.0064 | -0.0058 | -0.0054 | -0.0065 |
| $\rho$ | 0.49 | 0.49 | -0.33 | -0.46 |
| $a$ | 1.13 | 1.11 | 1.03 | 0.88 |
| $b$ | -3.34 | -3.23 | 1.91 | -2.46 |
| $c$ | 1.03 | 0.87 | 0.71 | -0.82 |
| $k$ | -0.75 | -0.46 | -1.07 | -0.77 |

| $\rho$ | 0.428 | 0.357 | 0.253 | 0.246 |
| $\bar{\eta}$ | 0 | 0.168 | 0.164 | 0.365 |
| $\epsilon_{K}^{SM}$ | 0 | 0.00103 | 0.00124 | 0.00255 |
| $a_{J/\psi K}^{SM}/\eta_{CP}$ | 0 | 0.489 | 0.418 | 0.784 |
| $a_{J/\psi K}^{SM}/\eta_{CP}$ | 0 | -0.016 | -0.017 | -0.038 |
| $|\Delta m_{B_{s}}^{SM}|$ | 0.196 | 0.249 | 0.358 | 0.409 |
| $|\Delta m_{B_{s}}^{SM}|$ | 16.0 | 16.1 | 16.3 | 15.5 |
| | Value | Error | Ref. |
|---|---|---|---|
| $|V_{us}|$ | 0.2237 | 0.0037 | [24] |
| $|V_{ub}|$ | 35.5$^{-4}$ | 3.6 $\times 10^{-4}$ | [24] |
| $|V_{cb}|$ | $41.0 \times 10^{-3}$ | 1.6 $\times 10^{-3}$ | [24] |
| $m_t(m_t)$ | 167 | 5 | [25] |
| $m_c(2\text{GeV})$ | 1.48 | 0.28 | [26] |
| $m_b(M_b)$ | 4.26 | 0.09 | [27] |
| $m_s(2\text{GeV})$ | 0.120 | 0.009 | [28] |
| $Q \equiv \frac{m_s/m_d}{\sqrt{1-(m_u/m_d)^2}}$ | 22.7 | 0.8 | [29] |
| $m_s/m_d$ | 21 | 4 | [30] |
| $\tan \beta$ | 3 | | |
| $\sin^2 \theta_W$ | 0.23117 | | |
| $M_Z$ | 91.188 | | |
| $M_{\text{GUT}}$ | $2 \times 10^{16}$ | | |
| $M_g(M_Z)$ | 500 | | |
| $m_{3/2}$ | 200 | | |
| $\alpha_{\text{QCD}}(M_Z)$ | 0.119 | | |
| $|\varepsilon_K|$ | $2.271 \times 10^{-3}$ | $0.017 \times 10^{-3}$ | [30] |
| $\Delta m_{B_d}$ | 0.487 | 0.014 | [23] |
| $\Delta m_{B_s}$ | $> 14.5$ (95% c.l.) | | [23] |
| $a_{J/\psi K/\eta_{\text{CP}}}$ | 0.48 | 0.16 | [31] |
| $\Delta m_K$ | $3.495 \times 10^{-15}$ | $0.013 \times 10^{-15}$ | | |
| $m_{B_d}$ | 5.279 | 0.002 | | |
| $m_{B_s}$ | 5.369 | 0.002 | | |
| $m_{K^0}$ | 0.497672 | 0.000031 | | |
| $f_{B_d}$ | 0.174 | 0.022 | [20] |
| $f_{B_s}$ | 0.204 | 0.015 | [20] |
| $f_K$ | 0.161 | 0.0015 | | |
| $\hat{B}_{B_d}^{Q_1}$ | 1.38 | 0.11 | [20] |
| $\hat{B}_{B_s}^{Q_1}$ | 1.35 | 0.05 | [20] |
| $B_{K_s}^{\text{MS}}(2\text{GeV})_{Q_1}$ | 0.61 | 0.06 | [5] |
| $B_{K_s}^{\text{LR}}(2\text{GeV})_{Q_4}$ | 1.04 | 0.06 | [5] |
| $B_{K_s}^{\text{LR}}(2\text{GeV})_{Q_5}$ | 0.73 | 0.10 | [5] |

Table 2: Experimental data and fixed parameters in the analysis. The $B$ mass differences are given in $\text{ps}^{-1}$, the $K$ mass difference and all other masses in $\text{GeV}$. $M_{\text{g}}(M_{Z})$ is the gluino mass at the electroweak scale and $m_{3/2}$ is the mass of the first two generations of sfermions at the GUT scale. $a_{J/\psi K/\eta_{\text{CP}}}$ is the world average of asymmetry measurements (normalized for CP-even final states). $\hat{B}_{B_d}^{Q_1}$ and $\hat{B}_{B_s}^{Q_1}$ are the renormalization group invariant $B$-parameters for the SM $\Delta B = 2$ operators. $B_{K_s}^{\text{MS}}(2\text{GeV})_{Q_1}$ is the $B$-parameter in the $\overline{\text{MS}}$ scheme for the SM $\Delta S = 2$ operator, and $B_{K_s}^{\text{LR}}(2\text{GeV})_{Q_{4,5}}$ are the $B$-parameters in the Landau RI scheme for the SUSY $\Delta S = 2$ operators $Q_{4,5}$ (see ref. [5] for details).
Figure 1: Dependence of $\Delta m_{B_s}$ (in ps$^{-1}$) on $m_{G3}$ (in TeV), the GUT scale mass of the third family. Here the Yukawa couplings are assumed to be real ($\phi = \psi = 0$). The line represents the lower bound from experiments $\Delta m_{B_s} > 14.5$ ps$^{-1}$ [23].

Figure 2: Dependence of the time dependent CP asymmetries in $B_d$ system on the phase of $\beta'$, for $(\phi = -0.25, \psi = 0)$ ($\circ$), $(\phi = -0.25, \psi = -0.25)$ ($\bullet$) and $(\phi = -0.5, \psi = -0.25)$ ($\times$). The thick line with the shadowed region corresponds to the SM prediction $a_{J/\psi K}/\eta_{CP} = 0.692 \pm 0.065$ [32].
Figure 3: Dependence of the time dependent CP asymmetries in $B_s$ system on the phase of $\beta'$, for $(\phi = -0.25, \psi = 0)$ ($\circ$), $(\phi = -0.25, \psi = -0.25)$ ($\bullet$) and $(\phi = -0.5, \psi = -0.25)$ ($\times$). The thick line is the SM prediction $\sim -3\%$ [33].

Figure 4: Dependence of $\Delta m_{B_s}$ (in $ps^{-1}$) on $\lambda$, for $(\phi = -0.25, \psi = 0)$ ($\circ$), $(\phi = -0.25, \psi = -0.25)$ ($\bullet$) and $(\phi = -0.5, \psi = -0.25)$ ($\times$). The thick line with the shadowed region is the SM prediction $\Delta m_{B_s} = 16.3 \pm 3.4$ $ps^{-1}$ [24].
but thanks to SUSY contributions we can obtain values for $\Delta m_{B_s}$, $a_{J/\psi K}$ and $a_{J/\psi \phi}$ that can considerably differ from the SM predictions. As an example, we report in figs. 2, 3 and 4, the scatter plots for $\Delta m_{B_s}$, $a_{J/\psi K}$ and $a_{J/\psi \phi}$ for non-vanishing CKM phases, to be compared with the predictions of the standard UT analysis (see for example ref. [24] for up-to-date results) and the SM prediction $a_{J/\psi \phi} \simeq 0$. Notice that, as expected, for increasing phases $\phi$ and $\psi$ the prediction tends to reproduce the SM ones, due to the fact that SUSY is playing a weaker role. Indeed, it is possible to show that this model can reproduce $\varepsilon_K$ and $\Delta m_{B_d}$ also with vanishing SUSY contributions [34].

3 A model with an SU(3) flavour symmetry

In this case quark Superfields are assigned to transform as a triplet under $SU(3)$ to be denoted by $\psi_i \sim 3$. This model is very similar to the one discussed in [11]. The flavons in the model are $S^{ij} \sim \bar{6}$ and $\phi_i \sim 3$. Another singlet $T^{ij} \sim 8$, not directly coupled to matter Superfields, is required to get phenomenologically acceptable textures (it is responsible for the appearance of the parameter $b$, see below). The breaking pattern associated to $SU(3)$ breaking fields directly coupled to SM fermions is

$$SU(3) \xrightarrow{<S^{33}>} SU(2) \xrightarrow{<\phi>} \emptyset.$$ 

The symmetry violating operators involving the lighter families are suppressed by the flavons VEVs over the scale of symmetry breaking messengers in the FN mechanism. The suppression factors we get are $1 > \eta > \epsilon > \epsilon'$ in the equations below.

The textures we get are (neglecting higher order terms)

$$M_d = m^D \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & c\eta & b\epsilon \\ 0 & \epsilon & \eta \end{pmatrix}$$ (7)

$$M_d = m^U \begin{pmatrix} 0 & 0 & 0 \\ 0 & c\eta & 0 \\ 0 & 0 & \eta \end{pmatrix}$$ (8)

$$m^2_Q = m^2_{3/2} \begin{pmatrix} 1 & 0 & \alpha \epsilon \epsilon' \\ 0 & 1 + \lambda \epsilon^2 & \beta \epsilon \eta \\ \alpha^* \epsilon \epsilon' & \beta^* \epsilon \eta & r_3 \end{pmatrix}$$ (9)

$$m^2_d = m^2_{3/2} \begin{pmatrix} 1 & 0 & \alpha' \epsilon \epsilon' \\ 0 & 1 + \lambda' \epsilon^2 & \beta' \epsilon \eta \\ \alpha'^* \epsilon \epsilon' & \beta'^* \epsilon \eta & r_3 \end{pmatrix}$$ (10)

$$m^2_u = m^2_{3/2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & r_3 \end{pmatrix}.$$ (11)

5The auxiliary fields in [11] modify the breaking pattern but as far as the observable sector is concerned the effective breaking is the one shown.
where \( c \approx m_c/m_t, m^U \) and \( m^D \) are proportional to the masses of top and bottom quark respectively. As in the \( U(2) \) case, \( r_3 \) denotes the ratio \( m^2_{G_3}/m^2_{3/2} \). Although for unbroken \( SU(3) \) one has \( r_3 = 1 \), the large breaking can generate a mass splitting between the third and first two generations of order one.

Comparing the Yukawa couplings to the ones in ref. [11], one sees that the \((1,3)\) and \((3,1)\) entries are missing in our case. This implies that the CKM phase in the present model is negligibly small (proportional to \( m_c/m_t \)). However, as we shall see in the following, we are able to explain \( \varepsilon_K \) with SUSY contributions and fit the UT, and therefore we do not need to introduce these additional entries. Notice that the reality of the fermionic mass matrices is not another assumption added by hand, but just a consequence of the structure of the textures, that always allows to redefine the fermionic fields in such a way as to remove all the complex phases (an explicit check of this property can be achieved with the Jarlskog determinant [35]). The \( U(2) \) model presented in the previous section does not share this property, due to the non-trivial structure of the up-type quark mass matrix, and indeed the fit of the model required a sizable complex phase in the CKM matrix, as discussed before. The possibility of fitting all CP violating observables with a real CKM matrix is indeed an interesting property of this \( SU(3) \) model.

Just as in the case of \( U(2) \), we have a large mixing between second and third generation in the right-handed sector, due to the presence of the asymmetry parameter \( b \). Therefore, also in this case one can have large SUSY contributions to \( \Delta F = 2 \) processes induced by sfermion mixing in the right-handed sector (see the discussion below eq. (2)).

**Unitarity Triangle Analysis**

Since in this case we can neglect the CKM phase, we can separately fit the Yukawa couplings to the SM-dominated quantities, and the SUSY parameters to \( \Delta F = 2 \) amplitudes. In this case, the UT collapses to a line, but in the region of negative \( \bar{\rho} \). This means that the SM contribution to \( \Delta m_{B_d} \) is exceedingly large (1.04 ps\(^{-1}\)). This is compensated by SUSY contributions. The predicted amplitude for \( \Delta m_{B_s} \) can be much larger than given by the standard UT analysis, and the CP asymmetries \( a_{J/\psi K} \) and \( a_{J/\psi \phi} \) can also differ in sizable way from the SM prediction.

In table 3 we report the fitted values of the fermionic parameters and the purely SM contributions to \( \Delta F = 2 \) processes. The parameter \( b \), responsible for the large asymmetry between the entries \( M_{23}^D \) and \( M_{32}^D \), is generated, as explained in detail in ref. [11], by a \( SU(3) \) breaking in the adjoint representation, which is however not directly coupled to matter fields. We assume \( SU(3) \) breaking to take place at a scale near the fundamental one, and we take \( \eta = 0.7 \), compatibly with this assumption.

We notice that all the solutions we find correspond to relatively small phases also in the SUSY sector. One may then think that this model could be embedded in some “approximate CP” scenario [36].

For illustrative purposes, we report in figures 5, 6 and 7 the scatter plots for the \( a_{J/\psi K} \) and \( a_{J/\psi \phi} \) asymmetries and for \( \Delta m_{B_s} \). Similar plots can be obtained as a function of the
Table 3: Results of the fit of fermionic parameters for different in SU(3) with real CKM in. The values in the first half of the table correspond to the fitted parameters, and the results in the second half correspond to the purely SM contributions to $\Delta F = 2$ processes. The mass differences are given in ps$^{-1}$.

| Parameter | Value |
|-----------|-------|
| $\epsilon$ | -0.31 |
| $\epsilon'$ | -0.0053 |
| $b$ | 0.10 |
| $\bar{\rho}$ | -0.35 |
| $\bar{\eta}$ | 0 |
| $\bar{\rho}^\text{SM}_K$ | 0 |
| $\bar{\eta}^\text{SM}_K$ | 0 |
| $a_{J/\psi K}^{SM}/\eta_{CP}$ | 0 |
| $a_{J/\psi \eta}^{SM}/\eta_{CP}$ | 0 |
| $|\Delta m_{B_s}^{SM}|$ | 1.04 |
| $|\Delta m_{B_s}^{SM}|$ | 14.0 |

other parameters. We see that large values of both $a_{J/\psi K}$ and $\Delta m_{B_s}$ can be obtained, which would unambiguously signal new physics contributions. Also small values of $a_{J/\psi K}$ are possible.

4 Conclusions

We have studied SUSY virtual effects in two nonabelian flavour models, in which both the flavour structure of the fermionic and the sfermionic sectors are tightly constrained. We have explicitly shown the relevance of SUSY corrections, and discussed how these may modify the UT fit in these models and generate significant deviations from SM predictions for three theoretically clean observables: $a_{J/\psi K}$, $a_{J/\psi \psi}$ and $\Delta m_{B_s}$. In the model based on a $U(2)$ flavour symmetry, where CP violation is present in the CKM matrix and a good fit can also be obtained in the limit of negligible SUSY contributions, the shape of the UT can be sizably modified for SUSY masses around 500 GeV, resulting in large values of the CP asymmetry in $B_s \to J/\psi \phi$ decays and of $\Delta m_{B_s}$. In the SU(3) model, the CKM matrix is real to a very good approximation, and the UT collapses to a line with negative $\bar{\rho}$; however, for SUSY masses around 500 GeV, particle contributions can account for all of the observed CP violation, while large deviations from the SM predictions for $a_{J/\psi K}$, $a_{J/\psi \psi}$ and $\Delta m_{B_s}$ are possible.

In conclusion, the role played by SUSY in FCNC and CP violating processes crucially depends on the nature of the mechanism which originates the SUSY breaking and transmits the information to the observable sector. A first, plausible option is that such mechanism has nothing to do with what gives rise to the flavour structure of the theory. The MFV situation is encountered in classes of SUSY models: anomaly, gauge, gaugino mediated SUSY breaking mechanisms constitute interesting examples. In these cases
Figure 5: Dependence of the time dependent CP asymmetries in $B_d$ system on the phase of $\beta'$ in SU(3) with real CKM. The thick line with the shadowed region corresponds to the SM prediction $a_{J/\psi K}/\eta_{CP} = 0.692 \pm 0.065$ \cite{72}.

Figure 6: Dependence of the time dependent CP asymmetries in $B_s$ system on the phase of $\beta'$ in SU(3) with real CKM. The thick line is the SM prediction $\sim -3\%$ \cite{73}.
the hopes to indirectly observe SUSY manifestations in FCNC are rather slim: the CP asymmetry in $b \to s\gamma$ or the $\gamma$ angle of the UT are certainly interesting possibilities, but overall the general impression is that we will have to wait for direct detection to have a SUSY signal. On the contrary, if one turns to gravity mediated SUSY breaking, there is no particular reason for such flavour blindness. As soon as a new flavour structure arises in the sfermionic sector, SUSY allows for quite conspicuous new contributions to FCNC, which in general are even too large for the tight FCNC experimental constraints. Among the adopted solutions to this flavour problem, the presence of an additional non-abelian flavour symmetry stands up as one of the most attractive possibilities. In this context, our analysis has considered a couple of interesting examples. The message which emerges from them is twofold. On one hand it appears that SUSY plays a major role in the fit of the UT. On the other hand it emerges that SUSY flavour models have concrete potentialities to exhibit sizable departures from the SM in particularly clean B-physics observables, while keeping under control all the other dangerous FCNC threats. Here the “competition” between direct and indirect searches to give the first hint for SUSY remains still open.

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