BEAT CEPHEID PERIOD RATIOS FROM OPAL OPACITIES

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ABSTRACT

The discovery of a large number of beat Cepheids in the Large Magellanic Cloud in the MACHO survey, provides an opportunity to compare the characteristics of such Cepheids over a range of metallicities. We produced a large grid of linear nonadiabatic pulsation models using the OPAL opacity tables and with compositions corresponding to those of the Milky Way, and the Large and Small Magellanic Clouds. Using the relationship between the period ratio and the main pulsation period, we are able to define a range of models which correspond to the observed beat Cepheids, and thereby constrain the physical characteristics of the LMC beat Cepheids. We are also able to make some predictions about the nature of the yet-to-be-discovered SMC beat Cepheids.

Subject headings: Cepheids: observations – galaxies: distances
1. Introduction

Beat Cepheids (hereafter, BC’s) provide us with valuable and precise tests of stellar atmosphere and pulsation models. Indeed, the observed period ratios in galactic BC’s provided one of the most secure pieces of evidence that the so-called ‘Cepheid mass discrepancy’ — the difference between the masses inferred from models and those found from observations of Cepheids in clusters or binary systems — was real.

The sample of known BC’s has grown recently as a result of analysis of the photometric data collected by the MACHO Project in their search for evidence of microlensing. Alcock et al. (1995) reported 45 new BC’s in an analysis of variables in the 22 fields near the bar of the Large Magellanic Cloud (LMC). (An additional 27 LMC BC’s are now known). This is in contrast to the 14 BC’s currently known in the Milky Way. The importance of this new sample arises from the expectation that the LMC BC’s are deficient in metals, compared to galactic BC’s, by a factor of 1.4–1.6 (Caldwell & Coulson, 1986) and the observational result that both the modal mix and period ratios found for the LMC BC’s are systematically different from their galactic counterparts.

The sensitivity of the observable properties of Cepheids to metallicity is still an open and important question with ramifications for both our understanding of the evolution of intermediate-mass stars and for the extragalactic distance scale. Period ratios, unlike most observationally derived quantities, can be determined with very high precision – 1 part in $10^5$ not being unusual. They are also direct measurements in the sense that no intermediate relationship with other stars are assumed and photometric calibration and reddening assumptions have no effect. Obviously, a calibration of metallicity based on period ratio would have great value. Such a relationship was seen by Andrievsky et al. (1993) who found a correlation between [Fe/H] and $P_1/P_0$, where $P_0$ and $P_1$ are the periods for the fundamental and first overtone modes, respectively. Comparison with Figure 6 of
Alcock et al. (1995) suggests that [Fe/H] can be predicted with even greater precision if it is a function of both log $P_0$ and $P_1/P_0$. In any case, the galactic sample by itself is not well-suited to this investigation because of its small number of stars and small range of metallicities.

Christensen-Dalsgaard & Petersen (1995) used existing M-L relations to fit pulsation models to the BCs, but they were not able to adequately match the models to the $P_2/P_1$ pulsators in the LMC. Buchler et al. (1996) also examined the characteristics of the LMC BCs to try to constrain the possible masses of the stars. In both of these studies, several assumptions were made, particularly about the form of the M-L relation.

The motivation for this paper is to provide a set of pulsation model results, based on current values for opacities, which can be used to interpret both the galactic and LMC BC data in a single framework, and to provide predictions for BC behavior in the yet-to-be-observed and even more metal-deficient Small Magellanic Cloud (SMC) Cepheid sample. We begin by describing our models, describe our results to date and what we believe we have learned from the LMC sample, and conclude with predictions for the SMC BC’s.

2. Pulsation Models

Cepheid envelope models were analyzed for pulsational instability using a linear nonadiabatic pulsation code, similar to that of Castor (1971). The stability of the fundamental mode as well as the first and second harmonics were examined. For Cepheids in the Milky Way, abundances of $X = 0.7$, $Y = 0.28$, and $Z = 0.02$ were used. The corresponding abundances for the LMC and SMC Cepheids were $(0.7, 0.29, 0.01)$ and $(0.7, 0.296, 0.004)$, respectively. The models made use of the OPAL opacity tables using the
Grevesse and Noels solar composition mix (Iglesias & Rogers, 1996). Effective temperatures were chosen in the range 5500 to 8400 K at increments of 100 K. The envelopes for all models had 300 zones and were required to have a base zone temperature of at least $10^6$ K. Even though convection has only a minimal influence on the pulsation periods of the coolest models ($< 5800$ K), it was still included in the calculations with a mixing-length ratio of 1.0 used throughout.

The mass ranges studied were varied according to sample, with the galactic Cepheid models encompassing 3 to 5.75 $M_\odot$, while the LMC and SMC models had masses between 1.25 and 4.0 $M_\odot$. Rather than using a single mass-luminosity relation, we selected a range of luminosity values for each mass – typically 5 different luminosities per mass. The mass-luminosity ranges for the Milky Way Cepheid models cover the theoretically derived values obtained by Becker, Iben & Tuggle (1977), the convective overshoot models of Chiosi (1990), and the Wesselink masses derived by Simon (1990).

In total, there were 1500 models of galactic Cepheids, and 1470 models for the LMC and SMC Cepheids. A sample of the results are given in Tables 1 and 2 for the three compositions. As expected, the lower metallicity results in shorter periods and generally larger period ratios. The blue edges for the instability strips for each metallicity are given in Table 3. The blue edges were found to be hotter than those found by Chiosi, Wood & Capitano (1993), for LMC compositions. In the case of the fundamental mode models, the blue edge is approximately $\log T_{\text{eff}} = 0.07$ hotter, while the first overtone models are hotter by $\log T_{\text{eff}} = 0.05$. Typically, lower metallicities result in the blue edge moving to higher temperatures for both the fundamental and first overtone mode pulsators.

Several relations for the models can be derived, such as the period-mass-radius or ‘P-M-R’ relation. For the models with metallicities representative of the three different
galaxies, we find:

\[
P_0 = 0.027(M/M_\odot)^{-0.66}(R/R_\odot)^{1.68} \quad \text{Milky Way,} \tag{1}
\]

\[
P_0 = 0.027(M/M_\odot)^{-0.65}(R/R_\odot)^{1.67} \quad \text{LMC, and} \tag{2}
\]

\[
P_0 = 0.025(M/M_\odot)^{-0.64}(R/R_\odot)^{1.66} \quad \text{SMC.} \tag{3}
\]

These are very similar to the relations found by Moskalik (1995) as well as those of Fricke, Stobie, & Strittmatter (1972).

Another useful relation, and one which defines the parameter space for each set of models is the period-luminosity-mass-effective temperature or ‘P-L-M-T’ relation. We find:

\[
\log(L/L_\odot) = -13.19 + 1.19 \log P_0 + 4.00 \log T_{\text{eff}} + 0.78 \log(M/M_\odot) \quad \text{Milky Way,} \tag{4}
\]

\[
\log(L/L_\odot) = -13.41 + 1.20 \log P_0 + 4.06 \log T_{\text{eff}} + 0.78 \log(M/M_\odot) \quad \text{LMC, and} \tag{5}
\]

\[
\log(L/L_\odot) = -13.50 + 1.21 \log P_0 + 4.10 \log T_{\text{eff}} + 0.77 \log(M/M_\odot) \quad \text{SMC.} \tag{6}
\]

which are similar to those found by Chiosi (1990), using Los Alamos opacities (Huebner et al. 1977).

Petersen Diagrams (\(\log P_0 - P_0/P_1\)) for the models are shown in Figures 1, 2, and 3, as well as the location of observed BC’s in the Milky Way and the LMC. The concentrations of models in the figures are due to identical luminosities, but varied masses. Models near the local maxima also tend to have similar values of effective temperature, typically \(T_{\text{eff}} = 5900K\) for the Milky Way models and \(T_{\text{eff}} = 6100K\) for the LMC models. The temperature for the Milky Way models is similar to that found for BCs by Barrell (1981), while the lower metallicity LMC models have both higher temperatures and higher period ratios for identical masses and luminosities, as well as slightly shorter periods. In some cases the \(P_2/P_1\) models extend into the region where the observed \(P_1/P_0\) pulsators lie.
While it may appear that there these stars could possibly be pulsating in the $P_2/P_1$ mode, it should be noted that these are the lowest temperature models ($T_{\text{eff}} \approx 5500K$), and are much cooler than the observed effective temperature for BC's.

3. Analysis

One distinction of the BC population in the LMC, compared to that in the Milky Way, is the number of stars found at short periods ($\log P_0 < 0.4$). Nearly half of the LMC BC’s are in this period range, as opposed to one galactic Cepheid. With such short periods, there are also correspondingly lower luminosities and masses for these stars. In the set of LMC metallicity models discussed above, luminosities as low as $\log(L/L_\odot) = 2.25$ are needed to reproduce periods and period ratios for these extremely short-period Cepheids. The masses that correspond to these models are typically less than $2M_\odot$. Larger masses for comparable luminosities would result in lower values of the period ratio.

The pulsation models which fell closest to the location of the LMC BC’s on the Petersen diagram were from the LMC metallicity group. The parameter space these models covered corresponded to several different mass-luminosity relations.

To determine which form of the mass-luminosity relation best describes the LMC Cepheids, further sets of models were produced using three different $M - L$ relations,

\begin{align}
\log(L/L_\odot) &= 3.43 \log M/M_\odot + 0.675 \\
\log(L/L_\odot) &= 3.52 \log M/M_\odot + 1.0 \\
\log(L/L_\odot) &= 1.64 \log M/M_\odot + 2.30
\end{align}

where the first relation is from Becker, Iben & Tuggle (1977), using the composition of
$X = 0.7$, $Y = 0.29$ and $Z = 0.01$, the second is from Chiosi (1990) for full convective overshooting, and the last is from Simon (1990) based on a fit to Wesselink masses for galactic Cepheids. Models were produced to cover the range of observed periods. The resulting Petersen diagrams are presented in Figures 4, 5, and 6 for both the F/1H and 1H/2H mode pulsators. For both the Becker, Iben & Tuggle and Chiosi $M - L$ relations, the models are able to only fit the longest-period F/1H Cepheids. Both $M - L$ relations fail to predict period ratios for the majority of the 1H/2H pulsators. On the other hand, the Simon $M - L$ relation does reproduce the period ratios of most of the pulsators in both groups. The obvious difference is the slope of the $M - L$ relation which is relatively small in the Simon relation. As was pointed out by Christensen-Dalsgaard & Petersen (1995) and Buchler et al. (1996), there are several aspects of LMC Cepheids which are still puzzling. In general, many types of evolutionary models that are used to fit the characteristics of Cepheids in the LMC are not always able to adequately satisfy all criteria. The fact that an $M - L$ relation, which was not based upon evolutionary models, appears to provide the best fit for both the F/1H and 1H/2H BC’s seems to further amplify this point. This may indicate the need for further revisions in low metallicity evolutionary models.

A second possible interpretation of the deviation between observations and theoretical period ratios is that progressively lower metallicity BC’s are found in the instability strip as one goes to shorter periods (and, hence, lower masses and luminosities).

4. Predictions for the Beat Cepheids of the SMC

Twelve field centers in the SMC are being monitored by the MACHO Project. At this writing, these fields have not yet been searched for variable stars. However, there are at least two indications that these fields contain additional BC’s. Wesselink & Shuttleworth (1965) reported “a few Small Magellanic Cloud Cepheids, with periods near three days,
which exhibit considerable irregularities in their light curves,” which they then suggested were the SMC counterparts of galactic BC’s. Second, Payne-Gaposchkin & Gaposchkin (1966) list numerous instances of short-period Cepheids with unusually large scatter in their phased lightcurves - a notation which Alcock et al. (1995) found correlated with BC activity in the similar catalogue of Payne-Gaposchkin (1971).

So far, we have used the observed period ratios in the Milky Way and LMC BC’s to determine the $M - L$ relation which matches the data for the assumed metallicities. If our fits are correct, we can successfully predict the period ratios for the even more metal-deficient SMC. As before, we assume that the metallicity of the SMC is $X = 0.7$, $Y = 0.296$, and $Z = 0.004$ for our models.

The Petersen diagram of the SMC BC’s is shown in Figure 3, which when compared to that found for the LMC composition (Figure 2), reveals the familiar trend of systematically shorter periods and higher period ratios for lower metallicities. If we assume that the BC’s in the SMC have a similar mass, luminosity and temperature distribution as those in the LMC, we would find the relationship between the period ratio and the fundamental mode period for the F/1H pulsators to be:

\[
P_{1}/P_{0} = 0.740 - 0.033 \log P_{0}, 0.1 \leq \log P_{0} \leq 0.69. \tag{10}
\]

This relation assumes a similar slope as seen in the LMC, an average increase in the F/1H period ratio of about 0.01, and a slight decrease in the pulsation period.

The 1H/2H pulsators do not appear to be as sensitive to metallicity changes as do the F/1H mode pulsators, so it is unlikely that their location on the Petersen diagram would change significantly from one galaxy to another.
5. Conclusions

We find that the relatively shallow Simon mass-luminosity relation produces pulsation models which most closely reproduce the MACHO Project BC period ratios reported in Alcock et al. (1995). We note the possibility that the shortest-period BC’s are systematically metal-poor relative to their longer-period counterparts. We predict that the SMC BC’s will:

- display BC behavior to shorter periods and will consequently be more numerous,
- have higher period ratios for the F/1H stars than the LMC by 0.01 on the average, and
- have proportionally more 1H/2H pulsators due to the hotter edge of the excitation region and the more extended blue loops typical of the lower metallicity of the SMC Cepheids.

Further work will include the modeling of Cepheids over a greater range of mass, luminosity and metallicity. The models described in this paper can be accessed via anonymous FTP at nitro9.earth.uni.edu (in /pub/Cepheids) and at the WWW site [http://nitro9.earth.uni.edu/Cepheids/].

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REFERENCES

Alcock, C., Allsman, R.A., Axelrod, T.S., Bennett, D.P., Cook, K.H., Freeman, K.C., Griest, K., Marshall, S.L., Peterson, B.A., Pratt, M.R., Quinn, P.J., Reimann, J., Rodgers, A.W., Stubbs, C.W., Sutherland, W., and Welch, D.L. 1995, AJ, 109, 1653.

Andrievsky, S. M., Kovtjukh, V. V., Makarenko, E., N., and Usenko, I. A. 1993, MNRAS, 265, 257.

Barrell, S. L. 1981, MNRAS, 196, 357.

Becker, S. A., Iben, I. Jr., and Tuggle, R. S. 1977, ApJ, 218, 633.

Buchler, J. R., Kollath, Z., Beaulieu, J. P., and Goupil, M. J. 1996, ApJ, 462, L83.

Caldwell, J.A.R., and Coulson, I.M. 1986, MNRAS, 218, 223.

Castor, J. I. 1971, ApJ, 166, 109.

Chiosi, C. 1990, in Confrontation Between Stellar Pulsation and Evolution, ed. C. Cacciari, and G. Clementini, ASP Confernce Series, 158.

Chiosi, C., Wood, P. R., and Capitano, N. 1993, ApJS, 86, 541.

Christensen-Dalsgaard, J. and Petersen, J. O. 1995, AA, 299, L17.

Fricke, K., Stobie, R.S., and Strittmatter, P.S. 1972, ApJ, 171, 593.

Huebner, W. F., Mertz, A. L., Magee, N. H. Jr., and Argo, M. F. 1977, Astrophysical Opacity Library, UC-34b.

Iglesias, C. A., and Rogers, F. J. 1996, ApJ, 464, 943.

Moskalik, P. 1995, in Astrophysical Applications of Powerful New Atomic Databases, ed. S. J. Adelman, ASP Conference Series.

Payne-Gaposchkin, C. 1971, Smithsonian Contr. Ap., 13.

Payne-Gaposchkin, C., and Gaposchkin, S. 1966, Smithsonian Contr. Ap., 9.
Simon, N. R. 1990, in *Confrontation Between Stellar Pulsation and Evolution*, ed. C. Cacciari, and G. Clementini, ASP Conference Series, 193.

Wesselink, A.J., and Shuttleworth, M. 1965, MNRAS, 130, 443.
Fig. 1.— Petersen diagram for galactic composition models. The lower sequence shows the location of the F/1H mode pulsators, while the upper sequence shows the dashed 1H/2H pulsators. The symbols indicate the different values for log($L/L_\odot$). The progression of the models from the left to the right is due to the increase in the primary period with decreasing temperature. 13 F/1H mode Beat Cepheids are shown (filled circles), as well as a likely 1H/2H pulsator, CO Aur (open circle).

Fig. 2.— Petersen diagram for LMC composition models. Symbols are the same as in Figure 1. LMC F/1H mode Beat Cepheids (Alcock et al., 1995) are shown (filled circles), as well as 1H/2H beat Cepheids (open circles).

Fig. 3.— Petersen diagram for SMC composition models. Symbols are the same as in Figure 1.

Fig. 4.— Petersen diagram for LMC models using the Becker, Iben & Tuggle (1977) $M - L$ relation. The different masses for the models are indicated by the symbols, with the F/1H mode pulsation models on the lower region of the graph and the 1H/2H models on the upper region. Also shown are the observed F/1H LMC beat Cepheids (large filled circles) and the 1H/2H LMC beat Cepheids (large open circles).

Fig. 5.— Petersen diagram for LMC models using the Chiosi (1990) $M - L$ relation. Symbols are the same as in Figure 4.

Fig. 6.— Petersen diagram for LMC models using the Simon (1990) $M - L$ relation. The different masses for the models are indicated by the symbols. Also shown are the observed F/1H LMC beat Cepheids (large filled circles) and the 1H/2H LMC beat Cepheids (large open circles).
TABLE 1. Fundamental Mode Models with $T_{\text{eff}} = 6000$ K.

TABLE 2. First Harmonic Mode Models with $T_{\text{eff}} = 6000$ K.

TABLE 3. Blue Edges for Milky Way, LMC and SMC Models, in units of 100 K.
Table 1. Fundamental Mode Models with $T_{\text{eff}} = 6000$ K

| Composition | Mass $M/M_\odot$ | $\log(L/L_\odot)$ | $\log(L/L_\odot)$ | $\log(L/L_\odot)$ | $\log(L/L_\odot)$ | $\log(L/L_\odot)$ | $\log(L/L_\odot)$ |
|-------------|------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| LMC         | 1.25             | 0.1996            | 0.4143            | 0.6333            |
| SMC         | 1.25             | 0.1928            | 0.4080            | 0.6276            |
| LMC         | 1.50             | 0.1469            | 0.3588            | 0.5757            |
| SMC         | 1.50             | 0.1397            | 0.3518            | 0.5690            |
| LMC         | 1.75             | 0.1029            | 0.3127            | 0.5272            |
| SMC         | 1.75             | 0.0952            | 0.3052            | 0.5200            |
| LMC         | 2.00             | 0.0655            | 0.2739            | 0.4866            | 0.7029            |
| SMC         | 2.00             | 0.0574            | 0.2660            | 0.4789            | 0.6957            |
| LMC         | 2.25             | 0.0327            | 0.2402            | 0.4515            | 0.6666            |
| SMC         | 2.25             | 0.0250            | 0.2324            | 0.4436            | 0.6589            |
| LMC         | 2.50             | 0.0033            | 0.2102            | 0.4205            | 0.6344            | 0.8509            |
| SMC         | 2.50             | -0.0044           | 0.2023            | 0.4124            | 0.6263            | 0.8432            |
| LMC         | 2.75             | -0.0234           | 0.1832            | 0.3928            | 0.6056            | 0.8217            |
| SMC         | 2.75             | -0.0311           | 0.1752            | 0.3845            | 0.5972            | 0.8135            |
| MW          | 3.00             | -0.0417           | 0.1654            | 0.3752            | 0.5882            | 0.8037            |
| LMC         | 3.00             | -0.0479           | 0.1585            | 0.3676            | 0.5797            | 0.7951            |
| SMC         | 3.00             | -0.0557           | 0.1504            | 0.3592            | 0.5710            | 0.7866            |
| MW          | 3.25             | -0.0645           | 0.1425            | 0.3520            | 0.5644            | 0.7796            |
| LMC         | 3.25             | -0.0706           | 0.1357            | 0.3444            | 0.5559            | 0.7707            |
| SMC         | 3.25             | -0.0786           | 0.1275            | 0.3359            | 0.5471            | 0.7619            |
| MW          | 3.50             | 0.1211            | 0.3304            | 0.5424            | 0.7574            |
| LMC         | 3.50             | 0.1143            | 0.3229            | 0.5339            | 0.7482            |
| SMC         | 3.50             | 0.1060            | 0.3143            | 0.5250            | 0.7392            |
| MW          | 3.75             | 0.1010            | 0.3103            | 0.5220            | 0.7367            |
| LMC         | 3.75             | 0.0943            | 0.3028            | 0.5136            | 0.7274            |
| SMC         | 3.75             | 0.0858            | 0.2941            | 0.5045            | 0.7182            |
| MW          | 4.00             | 0.0821            | 0.2913            | 0.5029            | 0.7174            |
| LMC         | 4.00             | 0.0756            | 0.2838            | 0.4945            | 0.7080            |
| SMC         | 4.00             | 0.0667            | 0.2750            | 0.4853            | 0.6986            |
| MW          | 4.25             | 0.2752            | 0.4860            | 0.6998            | 0.9151            |
| MW          | 4.50             | 0.2585            | 0.4692            | 0.6827            | 0.8980            |
| MW          | 4.75             | 0.2431            | 0.4535            | 0.6668            | 0.8820            |
| MW          | 5.00             | 0.4387            | 0.6517            | 0.8668            | 1.0820            |
| MW          | 5.25             | 0.4250            | 0.6376            | 0.8524            | 1.0677            |
| MW          | 5.50             | 0.4117            | 0.6240            | 0.8386            | 1.0541            |
| MW          | 5.75             | 0.3989            | 0.6109            | 0.8254            | 1.0412            |
Table 2. First Harmonic Mode Models with $T_{\text{eff}} = 6000$ K

| Composition | Mass M/M$_\odot$ | log($L/L_\odot$) | log($L/L_\odot$) | log($L/L_\odot$) | log($L/L_\odot$) | log($L/L_\odot$) | log($L/L_\odot$) |
|-------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| LMC         | 1.25             | 0.0583           | 0.2696           | 0.4758           |
|             | 1.25             | 0.0555           | 0.2655           | 0.4708           |
|             | 1.50             | 0.0052           | 0.2170           | 0.4271           |
|             | 1.50             | 0.0027           | 0.2135           | 0.4224           |
|             | 1.75             | -0.0397          | 0.1710           | 0.3830           |
|             | 1.75             | -0.0421          | 0.1679           | 0.3789           |
|             | 2.00             | -0.0771          | 0.1314           | 0.3440           | 0.5535           |
|             | 2.00             | -0.0801          | 0.1284           | 0.3402           | 0.5488           |
|             | 2.25             | -0.1094          | 0.0969           | 0.3089           | 0.5205           |
|             | 2.25             | -0.1114          | 0.0946           | 0.3058           | 0.5161           |
|             | 2.50             | -0.1381          | 0.0663           | 0.2773           | 0.4899           | 0.6982           |
|             | 2.50             | -0.1403          | 0.0641           | 0.2744           | 0.4859           | 0.6929           |
|             | 2.75             | -0.1640          | 0.0390           | 0.2487           | 0.4617           | 0.6720           |
|             | 2.75             | -0.1667          | 0.0367           | 0.2460           | 0.4581           | 0.6670           |
|             | 3.00             | -0.1851          | 0.0160           | 0.2242           | 0.4377           | 0.6506           |
|             | 3.00             | -0.1880          | 0.0141           | 0.2227           | 0.4356           | 0.6474           |
|             | 3.00             | -0.1912          | 0.0117           | 0.2201           | 0.4322           | 0.6427           |
|             | 3.25             | -0.2072          | -0.0065          | 0.2003           | 0.4132           | 0.6271           |
|             | 3.25             | -0.2105          | -0.0087          | 0.1987           | 0.4113           | 0.6241           |
|             | 3.25             | -0.2143          | -0.0116          | 0.1961           | 0.4081           | 0.6198           |
|             | 3.50             | -0.2262          | 0.1783           | 0.3904           | 0.6049           |
|             | 3.50             | -0.302           | 0.1765           | 0.3886           | 0.6022           |
|             | 3.50             | -0.3035          | 0.1737           | 0.3854           | 0.5981           |
|             | 3.75             | -0.0473          | 0.1578           | 0.3691           | 0.5840           |
|             | 3.75             | -0.0504          | 0.1557           | 0.3673           | 0.5814           |
|             | 3.75             | -0.0541          | 0.1527           | 0.3642           | 0.5775           |
|             | 4.00             | -0.0662          | 0.1385           | 0.3492           | 0.5642           |
|             | 4.00             | -0.0691          | 0.1361           | 0.3472           | 0.5617           |
|             | 4.00             | -0.0738          | 0.1329           | 0.3440           | 0.5579           |
|             | 4.25             | 0.1243           | 0.3324           | 0.5463           | 0.7601           |
|             | 4.50             | 0.1078           | 0.3151           | 0.5286           | 0.7431           |
|             | 4.75             | 0.0933           | 0.2995           | 0.5121           | 0.7269           |
|             | 5.00             | 0.2848           | 0.4965           | 0.7115           | 0.9235           |
|             | 5.25             | 0.2716           | 0.4821           | 0.6968           | 0.9098           |
|             | 5.50             | 0.2591           | 0.4683           | 0.6827           | 0.8964           |
|             | 5.75             | 0.2467           | 0.4550           | 0.6690           | 0.8834           |
Table 3. Blue Edges for Milky Way, LMC and SMC Models, in units of 100 K.

| Mass $M/M_\odot$ | log($L/L_\odot$) | log($L/L_\odot$) | log($L/L_\odot$) | log($L/L_\odot$) | log($L/L_\odot$) |
|------------------|------------------|------------------|------------------|------------------|------------------|
| 1.25 F           | -/79/80          | -/78/79          | -/77/78          | -/77/78          | -/77/78          |
| 1.25 1H          | -/84/84          | -/83/84          | -/82/83          | -/82/83          | -/82/83          |
| 1.5 F            | -/79/80          | -/78/79          | -/77/78          | -/77/78          | -/77/78          |
| 1.5 1H           | -/83/84          | -/83/84          | -/82/83          | -/82/83          | -/82/83          |
| 1.75 F           | -/79/79          | -/78/78          | -/77/78          | -/77/78          | -/77/78          |
| 1.75 1H          | -/83/83          | -/83/83          | -/82/83          | -/82/83          | -/82/83          |
| 2.0 F            | -/79/79          | -/78/78          | -/77/77          | -/76/77          | -/76/77          |
| 2.0 1H           | -/82/83          | -/82/83          | -/81/83          | -/81/83          | -/81/83          |
| 2.25 F           | -/79/79          | -/78/78          | -/77/77          | -/76/77          | -/76/77          |
| 2.25 1H          | -/82/82          | -/82/82          | -/81/82          | -/81/82          | -/81/82          |
| 2.5 F            | -/79/79          | -/78/78          | -/77/77          | -/76/76          | -/75/76          |
| 2.5 1H           | -/82/82          | -/81/82          | -/81/82          | -/80/82          | -/80/82          |
| 2.75 F           | -/79/79          | -/78/78          | -/77/77          | -/76/76          | -/75/76          |
| 2.75 1H          | -/81/82          | -/81/82          | -/81/82          | -/80/82          | -/80/82          |
| 3.0 F            | 79/79/79         | 77/78/78         | 76/77/77         | 75/76/76         | 74/75/76         |
| 3.0 1H           | 80/81/82         | 80/81/81         | 79/80/81         | 79/80/81         | 79/80/81         |
| 3.25 F           | 79/79/79         | 77/78/78         | 76/77/77         | 75/76/76         | 74/75/76         |
| 3.25 1H          | 80/81/82         | 80/81/81         | 79/80/81         | 79/80/81         | 79/80/81         |
| 3.5 F            | 77/78/78         | 76/77/77         | 75/76/76         | 74/75/76         | 74/75/76         |
| 3.5 1H           | 80/81/81         | 79/80/81         | 79/80/81         | 79/80/81         | 79/80/81         |
| 3.75 F           | 78/78/78         | 76/77/77         | 75/76/76         | 74/75/75         | 74/75/75         |
| 3.75 1H          | 80/80/81         | 79/80/81         | 79/80/81         | 79/80/81         | 79/80/81         |
| 4.0 F            | 78/78/78         | 76/77/77         | 75/76/76         | 74/75/75         | 74/75/75         |
| 4.0 1H           | 80/80/81         | 79/80/81         | 79/80/80         | 79/80/80         | 79/80/80         |
| 4.25 F           | 76/-/-           | 75/-/-           | 74/-/-           | 74/-/-           | 74/-/-           |
| 4.25 1H          | 79/-/-           | 78/-/-           | 78/-/-           | 78/-/-           | 78/-/-           |
| 4.5 F            | 76/-/-           | 75/-/-           | 74/-/-           | 74/-/-           | 74/-/-           |
| 4.5 1H           | 79/-/-           | 78/-/-           | 78/-/-           | 78/-/-           | 78/-/-           |
| 4.75 F           | 76/-/-           | 75/-/-           | 74/-/-           | 74/-/-           | 74/-/-           |
| 4.75 1H          | 79/-/-           | 78/-/-           | 78/-/-           | 78/-/-           | 78/-/-           |
| 5.0 F            | 75/-/-           | 74/-/-           | 74/-/-           | 74/-/-           | 74/-/-           |
| 5.0 1H           | 78/-/-           | 78/-/-           | 78/-/-           | 78/-/-           | 78/-/-           |
| 5.25 F           | 75/-/-           | 74/-/-           | 74/-/-           | 74/-/-           | 74/-/-           |
| 5.25 1H          | 78/-/-           | 78/-/-           | 78/-/-           | 78/-/-           | 78/-/-           |
| 5.5 F            | 75/-/-           | 74/-/-           | 74/-/-           | 74/-/-           | 74/-/-           |
| 5.5 1H           | 78/-/-           | 78/-/-           | 78/-/-           | 78/-/-           | 78/-/-           |
| 5.75 F           | 75/-/-           | 74/-/-           | 74/-/-           | 74/-/-           | 74/-/-           |
| 5.75 1H          | 78/-/-           | 77/-/-           | 77/-/-           | 77/-/-           | 77/-/-           |