New localization method of $U(1)$ gauge vector field on flat branes in (asymptotic) $AdS_5$ spacetime

Zhen-Hua Zhao$^1$, Qun-Ying Xie$^{2,3}$ and Yuan Zhong$^{3,4}$

$^1$Department of Applied Physics, Shandong University of Science and Technology, Qingdao, 266590 People’s Republic of China
$^2$School of Information Science and Engineering, Lanzhou University, Lanzhou 730000, People’s Republic of China
$^3$Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, People’s Republic of China
$^4$IFAE, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

E-mail: zhaozhh09@lzu.edu.cn, xieqy@lzu.edu.cn and yzhong@ifae.es

Received 9 August 2014, revised 3 November 2014
Accepted for publication 17 November 2014
Published 14 January 2015

Abstract

It is well known that the $U(1)$ gauge vector field, with standard five-dimensional (5D) action, cannot be localized on Randall–Sundrum-like braneworlds with an infinite extra dimension. In this paper, we propose a modified 5D action to localize the $U(1)$ gauge vector field on flat branes with an infinite or finite extra dimension. The localization method is realized by adding a dynamical mass term into the standard 5D action of the vector field, which is proportional to the 5D scalar curvature. It is shown that the vector zero mode is localizable if the 5D spacetime is (asymptotic) $AdS_5$. Moreover, the massive tachyonic modes can be excluded.

Keywords: extra dimensions, braneworld, localization of vector field

1. Introduction

Braneworld theories have received much attention since the success of the Randall–Sundrum (RS) thin brane models [1, 2]. In braneworld theories, the localization of gravity and all kinds of matter fields on the brane is always an important issue. It is well known that the four-dimensional graviton can be localized in the RS thin brane scenario [1, 3] and in thick brane scenarios [4–10]. The localization of the real scalar field is the same as gravity in general relativity [11], but may be different from or even opposite to gravity in some modified gravity [12, 13]. By introducing the usual Yukawa coupling between the background scalar field and
the fermion field, the fermion can also be localized on the brane generated by an odd scalar field [11, 14–19]. However, if the brane is generated by an even scalar field, we need to introduce a new localization mechanism in order to localize the fermion on the brane [20].

In five-dimensional (5D) spacetime, the $U(1)$ gauge field $A_M$ with the usual action

$$S \sim \int d^5x \sqrt{-g} F_{MN} F^{MN}, \quad (1.1)$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength, can be localized on the brane in some special braneworld models, for example, in the standing-wave braneworld model [21], in braneworld models with finite extra dimension [22–26], and in a 6D model [27]. But it cannot be localized in an RS-like braneworld model with an infinite extra dimension [11, 28, 29].

In order to localize the $U(1)$ gauge field on branes in 5D RS-like models with an infinite extra dimension, the typical localization mechanism is to reform the action (1.1). In the thin brane scenario, many ideas have been proposed for this issue [30–33]. In [30], the author added a topological term and a three-form gauge potential into the action (1.1). In [31], a bulk mass term of the vector and a coupling between the vector potential and the brane were introduced. In [32], the action (1.1) was changed to

$$S \sim \int d^5x \sqrt{-g} e^{2\alpha(y)} F_{MN} F^{MN}, \quad (1.2)$$

where $e^{2\alpha(y)}$ is the warp factor. In this model the vector zero mode can be localized on the negative tension brane. In [33], gauge field kinetic terms induced by localized fermions were added into the gauge field action.

In order to localize $U(1)$ vector fields in thick brane models, Kehagias and Tamvakis (KT) proposed a coupling between the gauge field and an extra dilaton field. The KT mechanism has been applied in many different braneworld scenarios [34–40]. Recently, Chumbes, Holf da Silva and Hott (CHH) proposed a new mechanism [29], in which gauge and tensor fields directly couple to a functional of the background scalar field. By introducing a Stueckelberg-like action, Vaquera-Araujo and Corradini realized the localization of a vector in a thick brane model [41].

In this paper, we propose a new localization method to localize the $U(1)$ gauge field on the brane. We assume that the 5D gauge field has a dynamic mass term, which is proportional to the 5D scalar curvature. Our localization method can be used both in thin and thick braneworld models with an infinite or finite extra dimension. To localize the gauge field zero mode, the only assumption we need is that the 5D spacetime is (asymptotic) AdS$_5$. With the same assumption, we can further prove that there is no tachyonic mode in the KK spectrum.

The paper is structured as follows. In section 2, we show the setup of our localization method of the gauge field on the brane and prove that the zero mode is localizable under the assumption that the five-dimensional spacetime is (asymptotic) $AdS_5$. Further, in section 3, we prove that tachyon modes can be excluded. We conclude our results in the final section.

2. The localization method of the vector field on the branes

The line-element describing a flat (Minkowski) braneworld embedded in five-dimensional spacetime is assumed to be [1]

$$ds^2 = G_{MN} dx^M dx^N = e^{2\alpha(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$  \quad (2.1)

where $e^{2\alpha(y)}$ is the warp factor, $\alpha(y)$ is a function of the extra dimension $y$, $G_{MN}$ is the metric of 5D bulk spacetime and $M, N = 0, 1, 2, 3, 4$ stand for the bulk coordinate indices, and the
Minkowski metric $\eta_{\mu\nu}$ on branes with signature $(-1, +1, +1, +1)$ and $\mu, \nu = 0, 1, 2, 3$ correspond to the brane coordinate indices.

In order to localize vector fields on branes, we introduce a dynamical mass term and the action reads

$$S = \int d^4x dy \sqrt{-G} \left( -\frac{1}{4} G^{MN} G^{RS} F_{MR} F_{NS} - \frac{1}{2} M^2 G^{MN} A_M A_N \right).$$  \hfill (2.2)$$

where $M^2$ is a function of the 5D scalar curvature. Here, we set

$$M^2 = -\frac{1}{16} R, \quad \hfill (2.3)$$

With the metric in equation (2.1), the scalar curvature is given by

$$R = -4 \left( 5a^2 + 2a' \right).$$  \hfill (2.4)$$

In this paper, the prime stands for the derivative with respect to $y$. Because the brane is flat, $R$ is only a function of the extra-dimension coordinate $y$.

Here, we parametrize $A_M$ in the following way [31, 42, 43]:

$$A_M = \left( \hat{A}_\mu + \partial_\mu \phi, \ A_4 \right), \quad \hfill (2.5)$$

where $\hat{A}_\mu$ is the transverse component, which satisfies the transverse condition $\partial_\mu \hat{A}^\mu = 0$, and $\phi$ is the longitudinal component. Substituting equation (2.5) into the action (2.2), with the transverse condition, one can find that the transverse vector $\hat{A}_\mu$ decouples from the scalar fields $\phi$ and $A_4$, and the action (2.2) can be split into two parts,

$$S = S_V (\hat{A}_\mu) + S_S (\hat{A}_4, \ \phi),$$  \hfill (2.6)$$

where [44, 45]

$$S_V = \int d^4x dy \sqrt{-\tilde{G}} \left( -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{2} \tilde{\partial}_\mu \tilde{A}_\mu \tilde{\partial}_\nu \tilde{A}_\nu G^{\nu\mu} - \frac{1}{2} M^2 \tilde{A}_\mu \tilde{A}^\mu G^{\mu\nu} \right).$$  \hfill (2.7)$$

$$S_S = \int d^4x dy \sqrt{-\tilde{G}} \left( -\frac{1}{2} \tilde{\partial}_\mu (\tilde{A}_\nu \tilde{\partial}_\mu (\tilde{A}_\nu \tilde{\partial}_\nu \phi) G^{\nu\mu} - \frac{1}{2} \tilde{M}^2 \tilde{\partial}_\mu \phi \tilde{\partial}_\nu \phi G^{\mu\nu} \right.$$

$$- \frac{1}{2} \tilde{\partial}_\mu \tilde{A}_\nu \tilde{\partial}_\nu \tilde{A}_4 \tilde{G}^{\mu\nu} - \frac{1}{2} \tilde{A}_4 \tilde{A}_4 \tilde{M}^2 \tilde{G}^{\mu\nu} + \tilde{\partial}_\mu \tilde{A}_4 \tilde{\partial}_\nu (\tilde{A}_\nu \tilde{\partial}_\nu \phi) G^{\nu\mu} \right).$$  \hfill (2.8)$$

and $\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$.

Because we focus on the localization of the vector field, and the scalar sector has been discussed in [43], we will therefore not discuss the localization of the scalar particles in this paper. The transverse vector field can be decomposed as

$$\hat{A}_\mu (x, y) = \sum_n A^{(n)}_\mu (x) \chi_n (y),$$  \hfill (2.9)$$

where $A^{(n)}_\mu (x)$ is the 4D vector KK mode and $\chi_n (x)$ is called the KK wave function [46], here and the following $'\Sigma_n'$ is a shorthand for the summation and integration over discrete and continuum KK modes.

By means of the KK decomposition (2.9), the action (2.7) can be further reduced to

$$S_V = -\frac{1}{4} \sum_n \int d^4x \int d^4y (\eta^{\mu\nu} \eta^{\rho\sigma} F_{\mu\rho}^{(n)} F_{\nu\sigma}^{(n)} + 2m_n^2 \eta^{\mu\nu} A^{(n)}_\mu A^{(n)}_\nu).$$  \hfill (2.10)$$

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where \( F_{\mu\nu}^{(4)} = \partial_{\rho} A_{\nu}^{(4)} - \partial_{\nu} A_{\mu}^{(4)} \) is the four-dimensional vector field strength tensor. In order to obtain the action (2.10), the KK wave function \( \chi_n(y) \) is required to satisfy the following equation
\[
-\partial_y \left( e^{2\alpha} \partial_y \chi_n \right) + \chi_n e^{2\alpha} \mathcal{M}^2 = m_n^2 \chi_n, \tag{2.11}
\]
and the orthonormalization condition
\[
\int \chi_m(y) \chi_n(y) dy = 0, \quad (m \neq n). \tag{2.12}
\]
From the actions (2.10), the localization of the 4D vector KK mode requires
\[
I \equiv \int_{-\infty}^{+\infty} \chi_n^2(y) dy < \infty. \tag{2.13}
\]
By using the following field transformation
\[
\chi_n = e^{-\alpha} \tilde{\chi}_n, \tag{2.14}
\]
Equation (2.11) can be rewritten as
\[
-\tilde{\chi}_n'' + \left( \alpha'' + \alpha'^2 + \mathcal{M}^2 - e^{-2\alpha} m_n^2 \right) \tilde{\chi}_n = 0. \tag{2.15}
\]
For the vector zero mode, \( m_0 = 0 \), equation (2.15) can be written as
\[
-\tilde{\chi}_0'' + \left( \alpha'' + \alpha'^2 + \mathcal{M}^2 \right) \tilde{\chi}_0 = 0. \tag{2.16}
\]
Substituting equations (2.3) and (2.4) into equation (2.16), we can obtain
\[
-\tilde{\chi}_0'' + \left( \frac{3\alpha'}{2} + \frac{9\alpha^2}{4} \right) \tilde{\chi}_0 = 0, \tag{2.17}
\]
which can be further factorized as
\[
\left( -\frac{d}{dy} - \frac{3}{2} \alpha' \right) \left( \frac{d}{dy} - \frac{3}{2} \alpha' \right) \tilde{\chi}_0 = 0. \tag{2.18}
\]
The solution of the above equation is
\[
\tilde{\chi}_0(y) = c_0 e^{\frac{3}{2} \alpha(y)}, \tag{2.19}
\]
and further one can get
\[
\chi_0 = e^{-\alpha} \tilde{\chi}_0 = c_0 e^{\frac{3}{2} \alpha}. \tag{2.20}
\]
With the above solution, the integration (2.13) reads
\[
I = \int_{-\infty}^{+\infty} \chi_0^2 dy = c_0^2 \int_{-\infty}^{+\infty} e^{3\alpha} dy. \tag{2.21}
\]
Because the integrand in equation (2.21) is continuous at the interval \( y \in (-\infty, +\infty) \), the convergency of the above integrations is determined by the asymptotic behaviors of integrands at the infinity.

Now, we prove that if the five-dimensional spacetime is asymptotic AdS, the vector zero mode is always localizable. The condition of asymptotic AdS means that, when \( y \rightarrow \pm \infty \), the warp factor has the following asymptotic behavior
\[
\alpha(y) \big|_{y \rightarrow \pm \infty} \rightarrow -k |y|, \tag{2.22}
\]
where $k$ is a positive constant and usually relates with the five-dimensional fundamental scale in RS-like braneworld scenarios. With this condition, the asymptotic behavior of the integrand in equation (2.21) reads as

$$e^{\alpha y} \mid_{y \to \pm \infty} \to e^{-k|y|},$$

(2.23)

Therefore, the integration (2.21) is convergent, namely, the 4D vector zero mode is localizable. If the extra dimension is finite, the conclusion is also valid.

Our conclusion can easily be verified in the RS-II model [3], in which $\alpha(y) = -k |y|$. Substituting it into equation (2.21), we obtain

$$I = 2e_{0}^{2} \int_{0}^{+\infty} e^{-k y} dy = 2e_{0}^{2}/k.$$  

(2.24)

Normalizing $I$ results in $e_{0} = \sqrt{k/2}$. Therefore, the vector zero mode is localizable in the RS-II model.

### 3. Excluding 4D vector tachyonic modes

Further, we will discuss if this new localization method can exclude the tachyonic modes of the 4D vector field. To discuss the massive modes and mass spectrum, it is more convenient to work with the conformal metric:

$$ds^{2} = e^{2\alpha(z)} \left( \eta_{\mu\nu} dx^{\mu} dx^{\nu} + dz^{2} \right).$$

(3.1)

The form of dynamic mass with above metric (3.1) is

$$M^{2} = -\frac{1}{16} \mathcal{R} = \frac{1}{4} e^{-2\alpha} \left( 3\dot{\alpha}^{2} + 2\ddot{\alpha} \right).$$

(3.2)

where the symbol $\cdot$ means the derivative with respect to $z$. Making use of the KK decomposition

$$\hat{A}_{\mu}(x, z) = \sum_{n} A^{(n)}_{\mu}(x) \chi_{n}(z),$$

(3.3)

and the transformation

$$\chi_{n}(z) = \rho_{n}(z) e^{-\alpha/2},$$

(3.4)

the action (2.7) reduces to

$$S_{V} = -\frac{1}{4} \sum_{n} \int d\tau \rho_{n}^{2}(\tau) \int d^{4}x \left( F_{\mu\nu}^{(n)} F^{(n),\mu\nu} + 2m_{n}^{2} A^{(n)}_{\mu} A^{(n),\mu} \right).$$

(3.5)

The localization of KK modes means

$$I \equiv \int_{-\infty}^{+\infty} \rho_{n}^{2}(y) dy < \infty.$$  

(3.6)

At the same time, the function $\rho(z)$ is required to satisfy a Schrödinger-like equation

$$\left( -\ddot{\rho} + V(z) \right) \rho_{n}(z) = m_{n}^{2} \rho_{n}(z).$$

(3.7)
and the orthonormalization condition
\[ \int \rho_m(z) \rho_n(z) \, dz = 0 \quad (m \neq n). \] (3.8)
The effective potential is given by,
\[ V(z) = \frac{1}{2} \ddot{\alpha}(z) + \frac{1}{4} \dot{\alpha}(z)^2 + e^{2\alpha} M^2. \] (3.9)
Substituting the expression of \( M^2 \) in (3.2) into equation (3.9), we obtain
\[ V(z) = \ddot{\alpha}(z) + \dot{\alpha}(z)^2. \] (3.10)
With the above potential equation (3.7) can be further factored as
\[ \left( -\partial_z - \dot{\alpha} \right) \left( \partial_z - \dot{\alpha} \right) \rho_k(z) = m_n^2 \rho_k(z). \] (3.11)
This is the supersymmetry mechanics form of the Schrödinger equation (3.7) [47], which guarantees the positivity of \( m_n^2 \). Therefore, the tachyonic vector modes are excluded. This result is independent of the asymptotic behavior of spacetime.

4. Conclusion

In this paper, we proposed a new localization method of the \( U(1) \) gauge field. In our method a dynamical mass term was added into the 5D action of the vector field. The dynamical mass term is proportional to the 5D scalar curvature. It was shown that, if the brane is embedded in a 5D \( AdS_5 \) spacetime, then the vector zero mode is localized on the brane. Moreover, we also proved that there is no vector tachyonic mode with our method.

Acknowledgments

We thank Professor Y-X Liu for helpful discussions. ZHZ is supported by the National Natural Science Foundation of China (grant no. 11305095), the Natural Science Foundation of Shandong Province, China (grant no. ZR2013AQ016), and the Scientific Research Foundation of Shandong University of Science and Technology for Recruited Talents (grant no. 2013RCJJ026). QYX is supported by the National Natural Science Foundation of China (grant no. 11375075). YZ is supported by the scholarship granted by the Chinese Scholarship Council (CSC).

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