STRING THEORY AT SHORT DISTANCE AND
THE PRINCIPLE OF EQUIVALENCE

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ABSTRACT

Point particles fall freely along geodesics; strings do not. In string theory all probes of spacetime structure, including photons, are extended objects and therefore always subject to tidal forces. We illustrate how string theory modifies the behavior of light in weak gravitational fields and limits the applicability of the principle of equivalence. This gives in principle a window on the short-distance structure of geometry in quantum gravity where one can see in a model-independent way how some of its predictions differ from those of classical gravity. We compare this with the lessons of high-energy string scattering.

1. Introduction

In this talk I will discuss how one may get some insight into gravity and geometry at Planckian distances. String theory is a theory of quantum gravity, but we are far from being able to test it and have difficulty in facing some of the most basic issues. We will need a better understanding of the geometric structures if we are to fully uncover the symmetries and vacuum of the string. By the same token, clarifying how the coupling to geometry at these distances departs from general relativity may make it possible to identify low-energy effects which carry the signature of string physics. We examine how the extended structure of strings — crucial in cutting off the divergences of quantum gravity — leads to interesting qualitative departures from classical gravity as well as from local quantum field theory.

First we review why high-energy scattering of extended objects — as opposed to point particles — appears incapable of probing short-distance geometry. We then turn to low-energy manifestations of Planck scale physics by considering propagation of test strings in background gravitational fields. We attempt to highlight the most fundamental difference between particles and strings: particles fall freely while strings are subject to tidal forces. These tidal forces produce qualitatively new...
effects that are independent of most details of the particular string theory and its
class. We illustrate, therefore, using the open bosonic string with little loss of
generality. Moreover, it is not necessary to have an exact solution to the string
equations of motion for the metric for these effects to be manifest. Therefore these
issues may be largely decoupled from the problem of finding conformally invariant
string vacuum.

The importance of tidal forces on string propagation has been emp hasized in
the context of cosmic strings\[1\] and in connection with the issue of singularities in
string theory \[2,3,4\]. Since modes of a test string may couple to background fields
that are never seen by a test particle, the criteria for singular propagation and
geodesic completeness are different for strings and particles. This was studied in
the case of shock-wave type backgrounds that have the twin virtues of being exact
solutions to the string equations of motion and of being exactly solvable, with linear
equations for the string modes.

The latter 'virtue,' however, is peculiar to this highly restricted class of geome-
tries. For generic backgrounds, the non-linear interactions mix string modes. We
isolate the effect of this mixing on the string zero-modes and show that tidal forces
lead to a qualitative change in photon trajectories.

2. Short Distance and High Energy

The traditional route to fundamental physics of elementary particles is to explore
short distances through high energies. Forces become simpler, compositeness is
revealed, symmetries are restored. For point particles — and the local field theories
which describe them — short distance and high energy are almost syn onymous. For
strings they are not. High energy particles have short wavelengths, and one may
attempt to probe short distances $\Delta x$ with high enough energy $E$ such that

$$\Delta x \sim 1/E.$$  \hspace{1cm} (1)

Strings have an obstruction to resolving short distances: a string given extremely
high energy can stretch to large size, so that there may be an effective minimal
length that can be probed\[6\],

$$\Delta x \gtrsim c_1/E + c_2E,$$  \hspace{1cm} (2)

where $c_i$ are constants.

It should be noted, at least in passing, that this discussion does not apply to the fascinating
two-dimensional string theories that have been the focus of much recent attention. Without
transverse dimensions, strings are as much like particles as they can be, and the effects of
interest will have no counterpart unless it can be shown that some vestige of the transverse
dimensions has tidal couplings to background fields.
Recall how this comes about and how it is modified by the large quantum fluctuations of the string. The amplitude at each order is given by a path integral, or sum over string histories:

$$\mathcal{A}_G(p_i) = \int DgD\hat{X} e^{-I[X,g] + i \int P \cdot \hat{X}}$$  \hspace{1cm} (3)

where $X^\mu(\xi)$ is the spacetime trajectory of the string; $G$ is the genus of the surface; the string action is its invariant area,

$$I[X,g] = -\frac{1}{2\pi} \int d^2\xi \sqrt{g} g^{ab} \partial_\alpha X^\mu \partial_\beta X_\mu;$$  \hspace{1cm} (4)

$g_{ab}$ is the metric on the punctured worldsheet; and $P$ is a source representing the incoming particles for the scattering of momentum eigenstates, $P^\mu(\xi) = \sum p_i^\mu \delta^{(2)}(\xi, \xi_i)$. When all of the momenta get large (i.e., fixed-angle scattering), the path integral is dominated by a classical trajectory and we can evaluate it semiclassically. This is easy to see: define rescaled string coordinates and momenta by

$$X = \sqrt{s} \hat{X}, \quad p_i = \sqrt{s} \tilde{p}_i,$$  \hspace{1cm} (5)

where $s = 4E^2$. Then

$$\mathcal{A}_G(s; \tilde{p}_i) = \int DgD\hat{X} e^{s(-I[\hat{X},g] + i \int \hat{P} \cdot \hat{X})}.$$  \hspace{1cm} (6)

Thus the $s \to \infty$, or equivalently $M_{\text{Planck}} \to 0$, limit corresponds to the semiclassical limit of string theory[7], with the dominant contribution coming from the trajectory

$$\hat{X}^\mu(\xi) = \sum_i \tilde{p}_i^\mu \mathcal{G}(\xi, \xi_i)$$  \hspace{1cm} (7)

where $\mathcal{G}(\xi, \xi')$ is the Green function for the action $I[\hat{X},g]$. In the coordinates of Refs. [8], this can be written as

$$X^\mu(\xi) = \frac{\sqrt{s}}{G+1} \sum_{j=1}^4 \tilde{p}_j^\mu \ln |\xi - a_j| + O(1/s).$$  \hspace{1cm} (8)

Observe from Eq. (5) that the mean size of the string histories contributing to the integral grows with energy as $\sqrt{s}$. This observation leads to uncertainty relation of Eq. (2).
Using the stationary point Eq. (7) to evaluate the amplitude, one finds

\[ A_G \sim e^{-s/(G+1)}. \]  

This behavior is extremely soft at high energies, so soft in fact that it violates bounds for local quantum field theories. The source of the behavior is not hard to find: it is characteristic of an extended object, and it is precisely the extended structure which enters at high energies that is responsible for the finite ultraviolet behavior that is the raison d’être of string theory as a theory of quantum gravity. Certainly we need to understand better how the theory can be non-local without spoiling causality, but can we deduce directly from these results any information about how string theory modifies spacetime structure at the Planck scale?

Because the high-energy limit corresponds to the semi-classical limit, it is tempting to try to interpret the stationary trajectories as “the” path of the string, and the size of the worldsheet as “the” size of the interaction region to see, for instance, if one can probe arbitrarily short distances. The full interpretation reveals subtle surprises.

The fixed-angle, fixed-genus calculation shows that the mean size of the region probed by the string is of order

\[ \langle X \rangle \sim \sqrt{s}/G. \]  

This shows that strings stretch at high energy (for fixed \( G \)), but it also suggests high orders processes in perturbation theory probe arbitrarily short spacetime distances for a given energy \( \sqrt{s} \). In fact this hope is dashed by large quantum fluctuations which render the perturbation series divergent. This divergence signals that the theory is unstable at such short distances. The series may be Borel resummed\(^9\), with the result that the physically meaningful “master trajectory” has a size that varies only logarithmically with energy, so we can crudely write

\[ \langle X \rangle \sim O(\ell_{\text{Planck}}), \]  

while the amplitude behaves as \( A_{\text{resum}} \sim \exp(-\sqrt{s}) \), just on the edge of what would be needed to restore locality.

What does this tell us about gravity and about geometry?

If short distances are unobservable, are they irrelevant? Not necessarily. We have seen that at Planck energies there is a limit to extracting short-distance information with a readily available spacetime interpretation. Fundamental though this regime is, we have to look elsewhere to get a handle on how geometry is revealed in string interactions over Planck distances. We do see that it is crucial, when the probes in the theory are themselves extended, to ask only questions that can be answered with test strings.
3. Tidal Forces on Strings

Photons fall freely along geodesics. Strings do not. If string theory indeed describes how gravity acts on elementary particles at the quantum level, we should explore the consequences of this simple fact.

The first statement is an immediate consequence of the principle of equivalence. It leads to testable and tested observations: bending of light by the sun, gravitational redshift, cosmological redshift, etc. To a structureless point particle in free fall, no gravitational forces are present.

For extended objects, life is quite different. Even in free fall they sense the presence of a gravitational field through tidal forces. These are, to be sure, very weak in the case of a body whose characteristic size $\ell$ is much much less than the scale $R$ over which the field varies. Yet small though these forces may be, they are never negligible compared to zero and they may give the leading signature for the existence of internal structure.

Consider deflection of light by the sun. By the equivalence principle, we know that light of all frequencies is deflected by an identical angle. Indeed, the equation of motion for a classical, massless particle is

\[
\frac{d^2 x^{\mu}}{d\tau^2} + \Gamma^{\mu}_{\nu\lambda} \frac{dx^{\nu}}{d\tau} \frac{dx^{\lambda}}{d\tau} = 0,
\]

where $\tau$ is an affine parameter and $\Gamma^{\mu}_{\nu\lambda}$ is the Christoffel symbol for the Schwarzschild metric appropriate to the region outside the sun. A change in energy can be absorbed in a change of affine parameter, so that the spacetime path is independent of energy. This is true regardless of the field equations satisfied by the metric.

Now for a classical string $x^{\mu}(\tau)$ becomes $X^{\mu}(\tau, \sigma)$, Eq. (12) generalizes to

\[
\Box X^{\mu} + \Gamma^{\mu}_{\nu\lambda}(X(\tau, \sigma)) \partial_{\alpha} X^{\nu} \partial^{\alpha} X^{\lambda} = 0,
\]

so there are always forces, even in a frame falling freely with the center of mass. This is as expected for an extended body. In Eq. (12) one can find coordinates in which the connection vanishes along the trajectory and the equation of motion is $\ddot{x}^{\mu}(\tau) = 0$, but in Eq. (13), the connection cannot be made to vanish along the whole world sheet. The equations of motion are non-linear (except for such special cases as plane-wave backgrounds [3]), mixing all string modes. The center-of-mass energy does not scale out and the geodesics are no longer frequency-independent. For photons in string theory, then, one would expect to see colors bent by different amounts, and in general a host of other effects, such as a frequency-dependence to spectral redshifts.
Though these effects are extraordinarily small (typically suppressed by $\ell/R \sim M_{\text{Planck}}/M_{\text{star}}$), they represent the leading deviation from classical gravity due to finite structure at the Planck scale. They are worth exploring. Moreover, one sees that the string scale naturally limits the principle of equivalence because there are no “arbitrarily small,” local, inertial frames.

To see stringy corrections to general relativity, one usually considers how Einstein’s field equations,

$$R_{\mu \nu} = 0,$$

are modified by a series of correction terms, $\alpha' RR + \ldots$, in powers of $\alpha'$, which arise from imposing conformal invariance on the two-dimensional sigma model. The new metric which solves the full string equations naturally changes the point-particle trajectories of Eq. (12) by terms of order $\alpha'$, but it does not change the fact that the new trajectories are still independent of energy. The changes themselves are not only small; they are model-dependent and they are difficult even in principle to distinguish from possible non-string modifications to Einstein’s equations.

The string-corrections predicted by Eq. (13), on the other hand, offer the hope of observing effects which have zero background in classical gravity. We concentrate, therefore, on how the geodesics depend on the finite string size, rather than on how the metric and trajectories differ from those of general relativity. One even obtains a good qualitative view by ignoring all corrections to the metric equations, and this is how we shall illustrate these issues. It almost goes without saying that the type of string theory, the compactification, the presence of supersymmetry can change only the quantitative details, not the overall picture. For clarity we discuss the massless vector states of the open bosonic string but the heterotic string could be just as easily used.

Of course string theory predicts many departures from classical gravity as well as from local point-particle quantum field theory. If we could execute in practice any thought experiment at all, we might look directly to the physics of high energies and temperatures. But more subtle, indirect low-energy effects of the underlying theory are perhaps more likely to be seen first and may well be essential in giving direction to further exploration.

The motion of a quantized test string in a background gravitational field, described by the metric $G_{\mu \nu}$ is given by the Polyakov path integral

$$\int \mathcal{D}X \mathcal{D}g \, \exp \left( \frac{i}{2\pi \alpha' \hbar} \int d^2 \sigma \sqrt{g} g^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu \nu}(X) \right).$$

We work at string tree level and assume for the moment that the metric $G_{\mu \nu}$ is a conformally invariant background, such that the $\beta$-function vanishes to all orders;
we comment briefly below on other possible couplings in this action. Latin indices represent world-sheet coordinates, \((\sigma^0, \sigma^1) = (\tau, \sigma)\) and Greek indices denote target space-time coordinates. Both spacetime and worldsheet have Minkowski signature.

Initial and final states are imposed as boundary conditions on the trajectories included in the sum or by inserting vertex operators. Alternatively, one may derive the Bogoliubov transformation between the in and out states. The coefficients which give the transition amplitude can be extracted as usual from the time evolution equations for the quantized modes. This has been discussed in detail for the non-zero modes of the string in Refs. [5,3,2]. We will only discuss here the zero-mode dynamics, which is given by the solving for the zero-mode propagation equation, subject to the normal-ordered Virasoro constraints.

From Eq. (15), the equation of motion for a string \(X(\tau, \sigma)\), generalizing (12) is

\[
\Box X^\mu + \Gamma^\mu_{\nu\lambda}(X) \partial_\alpha X^\nu \partial^\alpha X^\lambda = 0. \tag{16}
\]

For a pointlike string trajectory, \(i.e.,\) when \(X\) is independent of \(\sigma\), Eq. (16) reduces to Eq. (12).

The best way to see what is going on is to consider the motion of a small string. By small, we always mean small compared to the typical scale of gravitational variations. One expects the string’s center of mass to follow at least approximately the same motion as a test particle, so let us expand the string trajectory in the following way[5]:

\[
X^\mu(\tau, \sigma) = x^\mu(\tau) + \eta^\mu(\tau, \sigma) + \xi^\mu(\tau, \sigma). \tag{17}
\]

The first term, \(x^\mu(\tau)\), is geodesic satisfying Eq. (12). The merit of this expansion is that the connection \(\Gamma^\mu_{\nu\lambda}\) and its derivatives are all evaluated along \(x^\mu(\tau)\).

We define the second term, \(\eta\), to be the solution to the linearized equation of motion expanded about \(x^\mu\):

\[
\frac{D^2\eta^\mu}{D\tau^2} - \frac{\partial^2\eta^\mu}{\partial\sigma^2} + R^\mu_{\alpha\beta\gamma} \dot{x}^\alpha \dot{x}^\beta \eta^\gamma = 0, \tag{18}
\]

where \(D/D\tau\) represents the usual covariant derivative along the curve \(x^\mu(\tau)\). The meaning of \(\eta\) is clear from this equation: The first two terms generalize the flat-space harmonic motion of the string to include parallel transport, and the curvature term represents the usual force of geodesic deviation on an extended object.
The remaining piece of the expansion, $\xi$, carries the interesting physics. It satisfies
\[
\frac{D^2 \xi^\mu}{D \tau^2} - \frac{\partial^2 \xi^\mu}{\partial \sigma^2} + R_{\alpha\beta}^\mu \partial^\alpha \partial^\beta \xi^\nu =
- \Gamma_{\nu\lambda}^\mu \partial_\nu \eta^\lambda - \frac{1}{2} \left( \partial_\nu \partial_\lambda \Gamma_{\nu\lambda}^\mu \right) \dot{x}^\nu \dot{x}^\lambda \eta^\alpha \eta^\beta - \left( \partial_\alpha \Gamma_{\nu\lambda}^\mu \right) \dot{x}^\nu \eta^\lambda \eta^\alpha + \cdots ,
\]
where we have only shown terms up to second order in the perturbative expansion around $x^\mu(\tau)$. The equation is linear in $\xi^\mu$ but not homogeneous. The left-hand side has the same form as Eq. (18). After solving Eq. (18), the right-hand side, quadratic in $\eta$, acts as a source term that alters the center-of-mass zero-mode motion of the string.

Indeed, Fourier expand $\eta(\tau, \sigma)$,
\[
\eta^\mu(\tau, \sigma) = \sum \eta_m^\mu(\sigma) e^{im\sigma},
\]
and similarly for $\xi$. The center of mass of the string is then follows
\[
x^\mu(\tau) + \eta_0^\mu(\tau) + \xi_0^\mu(\tau).
\]
Modulo reparametrizations, it is $\eta_0 + \xi_0$ that departs from the point-particle path.

The modes of $\eta$ are decoupled:
\[
\frac{D^2 \eta_m^\mu}{D \tau^2} + m^2 \eta_m^\mu + R_{\alpha\beta}^\mu \partial^\alpha \partial^\beta \eta_m^\nu =
\dot{\eta}_m^\mu + m^2 \eta_m^\mu + 2 \Gamma_{\nu\lambda}^\mu \dot{x}^\nu \eta_m^\lambda + \left( \partial_\beta \Gamma_{\nu\lambda}^\mu \right) \dot{x}^\nu \dot{x}^\lambda \eta_m^\beta = 0
\]
\[
\eta \text{ describes the transverse string oscillator modes, so that } \eta_0 = 0 \text{ and } \eta_m \cdot x = 0 \text{ in an asymptotic flat space region. For a scattering interaction this initial condition on Eq. (18) ensures the } \eta_0(\tau) = 0 \text{ throughout for all } \tau; \text{ the first-order excitations of the oscillators do not affect the center of mass. This is intuitively clear: tidal forces stretch an object but they do not shift its center of mass, to leading order.}
\]

On the other hand, the higher order term $\xi_0$ does couple to the oscillator non-zero modes of $\eta$,
\[
\frac{D^2 \xi_0^\mu}{D \tau^2} + R_{\alpha\beta}^\mu \dot{x}^\alpha \dot{x}^\beta \xi_0^\nu =
- \Gamma_{\nu\lambda}^\mu \left( \dot{\eta}_m^\nu \eta_m^\lambda + m^2 \dot{\eta}_m^\nu \eta_m^\lambda \right) - \frac{1}{2} \left( \partial_\nu \partial_\lambda \Gamma_{\nu\lambda}^\mu \right) \dot{x}^\nu \dot{x}^\lambda \eta_m^\alpha \eta_m^\beta - \left( \partial_\alpha \Gamma_{\nu\lambda}^\mu \right) \dot{x}^\nu \eta_m^\alpha \eta_m^\lambda.
\]
It is $\xi_0(\tau)$, therefore, which gives the leading deviation between the path followed by a string and the path followed by a particle in a given gravitational field.
It is easy to see now that whether we use an exact metric solution or an approximate one, say tree-level in sigma-model perturbation theory, the source terms for $\xi_0$ will persist. The coefficients of the equations will just get small corrections.

What about other background fields? Obviously a Brans-Dicke scalar field just changes the equations determining the metric, not the string propagation. A massive dilaton, as one might hope for from non-perturbative effects, poses no threat, and even a strictly massless dilaton can be absorbed into the metric by a spacetime conformal rescaling, although this can only be done for one type of massless matter.

For first-quantized strings, these classical equations characterize the dominant contribution to the path integral, Eq. (15), but must of course be supplemented by the constraint equations

$$T_{ab} = 0 \quad (24)$$

to select the physical states. To treat low-lying states at the level of these calculations is necessary to impose the correct quantum, normal-ordered, Virasoro conditions in the asymptotic initial region. A massless ‘photon’ state, for instance, is given by exciting a single mode of the lowest oscillator level:

$$\zeta_\mu \alpha_{-1}^\mu |0;p\rangle, \quad (25)$$

where $\alpha_{-1}$ is the standard mode creation operator, $\zeta_\mu$ is the polarization vector and $|0;p\rangle$ is the vacuum state with no oscillator excitations and momentum $p$. To compute the transition coefficients, then, one needs to know $\eta_{-1}(\tau)$ and $\xi_0(\tau)$.

3.1 String motion in a Schwarzschild metric

Now we illustrate these effects with an example: we apply these equations to motion in a background Schwarzschild metric,

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = \left(1 - \frac{2GM}{r}\right)dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1}dr^2 - r^2d\Omega^2. \quad (26)$$

We specialize to the case of four non-compact dimensions for concreteness. It is

\[\text{Note that the classical version of this state (for circular polarization)}\]

$$\left(X^0, X^1, X^2, X^3\right) = (\alpha'E, \cos \tau \cos \sigma, \sin \tau \cos \sigma, \alpha'E)$$

\[\text{is forbidden by the classical constraint } \dot{X}^2 + \dot{X}^2 = 0, \text{ which lacks the normal-ordering constant that renders the spin-one state massless.}\]
convenient to denote

\[ a(r) \equiv 1 - 2GM/r, \]  
(27)

and to work in units where the Schwarzschild radius \( R_S = 2GM = 1. \)

### 3.2 Radial infall

The simplest case is radial infall of a massless photon from infinity. For a point particle, the motion determined from \( ds^2 = 0 \) is

\[ \dot{x}^0 = \alpha' E/a(r), \quad \dot{r} = -\alpha' E. \]  
(28)

A string photon of energy \( E \) in the asymptotic region \( r \to \infty \) has, in addition to this center of mass motion, a single transverse oscillator excitation. This determines the initial conditions on \( \eta \), and from the equation of motion (cf. Eq. (18)),

\[ \Box \eta^\theta + 2\Gamma^\theta_{\tau\rho} \dot{\eta}^\theta = 0 \]  
(29)

one finds the solution

\[ \eta^\theta(\tau, \sigma) = \frac{\cos \tau \cos \sigma}{\alpha' E \tau} \tilde{\eta}, \quad \eta^\sigma(\tau, \sigma) = \frac{\sin \tau \cos \sigma}{\alpha' E \tau} \tilde{\eta}, \]  
(30)

where \( \tilde{\eta} \propto \ell_{\text{Planck}} \) is a constant. Now we can substitute this into the equation of motion for the zero mode of \( \xi \) to obtain

\[ \ddot{\xi}_0 + \dot{\xi}_0 \left( \frac{\alpha' E a'}{a} \right) + \dot{\xi}_0^3 \left( \frac{\alpha' E a'}{a} \right)^2 = \frac{\tilde{\eta}^2 r a(r)}{2\tau^4} \alpha' E, \]

\[ \ddot{\xi}_0^0 - \dot{\xi}_0^0 \left( \frac{\alpha' E a'}{a} \right) + \dot{\xi}_0^0 \left( \frac{\alpha' E a'}{a} \right)^2 - \left( \frac{a'}{a} \right)^2 = 0. \]  
(31)

For radial motion, the function \( r(\tau) \) is known, \( r = -\alpha' E \tau \). These equations become clearer if we eliminate the parameter \( \tau \) in favor of \( r \), since then the energy \( E \) scales out of the equations! One can solve for the scaling function, \( \xi_0(r) \), numerically. It evolves smoothly from infinity, at least in the weak field region where we trust the formalism. Note further that \( \tilde{\eta} \propto \ell_{\text{Planck}} \), so \( \xi_0(r) \propto \ell_{\text{Planck}} \).
The result of the second-order tidal effect $\xi_0$ is to change the energy of the photon. A static observer at radius $r$ sees the string-photon with energy

$$E' = \frac{1}{\pi\alpha'} \frac{d}{d\tau} \int_0^\pi d\sigma X^0(r, \sigma)$$

$$= \frac{E}{a(r)} + \frac{1}{\alpha'} \frac{d\xi_0}{d\tau}$$

$$= E \left( \frac{1}{1 - R_s/r} - \frac{d\xi_0}{dr} \right).$$

So the gravitational redshift is modified by a new term, whose size is of order $\ell_{\text{Planck}}^2/R_s^2$ — as expected and extremely small.

The importance of this result is that the shift $\Delta E$ is proportional to the energy $E$, as is the usual redshift. Had this not been the case, we would have predicted that the redshifts of spectral lines from stars would depend on their frequency, in contrast with classical gravity and giving — in principle — a small deviation to signal the presence of string effects and quantum gravity.

Is this true in general? It would be disappointing if tidal effects all entered with the same energy dependence as point particles. Luckily this is not the case. The energy-dependence for radial infall is a consequence of symmetry, and it no longer holds for scattering. The heuristic picture of classical spinning strings is helpful here. In flat space, two strings with different energies moving with the speed of light differ in their rate of rotation. High energy strings rotate more slowly when seen by an inertial observer, as required by Lorentz invariance. Now for two strings which fall radially inward and rotate transversely, the tidal forces of Eq. (16) are also transverse. Since they are independent of the string’s orientation in the transverse plane, they are therefore independent of the rate of rotation and hence independent of the energy.

3.3 **Deflection at large distance**

To see what happens in a more general situation, consider deflection of a massless string-photon at large impact parameter.

In classical gravity, any metric theory predicts that the deflection of light by the sun, or other massive body, depends only on the impact parameter, $b = L/E$, where $L$ is the angular momentum. Light of all frequencies is deflected by the same angle. The precise angle depends on the field equations of the metric, but this fact does not; it depends on the principle of equivalence and on the pointlike nature of the test particle. Does light in string theory get dispersed as well as getting deflected?
Physically, two limiting cases may be distinguished. At extremely high energies, the string rotates very slowly as it passes a star or black hole and tidal forces stretch the string along a fixed direction. At very low energies, the string rotates rapidly and is subjected to alternate pulling and pushing. The motion can then be treated adiabatically. The net tidal forces do not cancel; but because the string undergoes many oscillations before the background field changes appreciably, the net effect is independent of just how fast the string is rotating. That is, at low energies the tidal effect on the deflection angle is energy-independent. This is how the point particle limit is recovered.

Now the first case, a string at large enough energy that it rotates slowly during the time of the interaction, requires alas that the string energy exceed the rest mass of the black hole, rendering the test-string treatment obviously inappropriate, and suggesting where the calculation matches on to the physics of high-energy string scattering.

In the low-energy case, we just argued physically that the leading correction to the deflection angle is energy independent, so we must go to higher adiabatic order and study the equations of motion derived above.

Again, the center of mass starting trajectory for the expansion is given (up to a quadrature) by

\[ \dot{r} = \pm \alpha' E \left( 1 - \frac{b^2 a(r)}{r^2} \right)^{1/2}, \]
\[ \dot{x}^0 = \alpha' E / a(r), \]
\[ \dot{\phi} = \alpha' E \frac{b}{r^2}. \]

(33)

Rather than use the string decomposition of Eq. (17), it proves slightly simpler to redefine \( \eta^\mu + \xi^\mu \rightarrow A^\mu + B^\mu \), where \( A^\mu(\tau, \sigma) \) is defined to be the parallel transported oscillation of the string, so that

\[ \frac{D^2 A^\mu}{D\tau^2} + \frac{\partial^2 A^\mu}{\partial\sigma^2} = 0, \]

(34)

and the geodesic deviation term present in Eq. (18) now will appear on the right-hand side as a source term for \( B^\mu \).

The resulting equations are still coupled and complicated. The equation for
The terms on the right-hand side with \( \cos \tau \) and \( \sin \tau \) vary extremely rapidly; if one averages over several string oscillations, only the coefficient of the \( \cos^2 \tau \) survives. Since it is quadratic in \( \tau \) derivatives of the coordinates, the resulting equation can be rescaled, as in the radial infall case, to show that the net change in the deflection angle \( B_0^\varphi \) is independent of energy.

This is again smaller than terms we have ignored, so we must work harder to obtain the leading energy-dependent contribution to \( B_0^\varphi \). This comes in next order of the adiabatic expansion, giving a source term proportional to \( (\alpha' E)^2 \), and hence an energy-dependent contribution to the scattering angle. This is as expected from the discussion above. We therefore expand the source term in time derivatives (and \( B \) in powers of \( \alpha' \)) to separate the overall cumulative shift in the source term from the rapid oscillations. These equation are lengthy and, in any event, no easier to solve analytically. Having identified the leading energy-dependent term in the scattering angle, going further requires a more detailed analysis of the equations than we will undertake here. It is straightforward to check numerically how it behaves and to verify that the source term is indeed physical.

4. Discussion

String theory predicts departures from general relativity not only because conformal invariance modifies the field equations but because strings themselves are extended objects. This gives a characteristic signature of stringy effects at energies high and low.

It should be noted that a related type of classical behavior was considered long ago. In Refs. [10] it was noted that the equations of motion for a spinning particle differ from Eq. (12) and that this would lead, for instance, to an energy dependent scattering angle in the light-deflection problem, if only the spin orientation were perpendicular to the plane of motion — which it is not.

Of course even if one had a small black hole and effects were much bigger, there are numerous other effects that would have to be computed to isolate the tidal physics; some of these are very large. Loop effects of course make real photons much much larger that the Planck length, even before considering wave optics effects. To
find any measurable effects it would be desirable to find systems on which the tidal
effects are cumulative.

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