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From integer to real numbers: students’ obstacles in understanding the decimal numbers

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Abstract. The various difficulties and obstacles that students face when they are introduced to decimal numbers are well documented and described in the relevant bibliography. This research is a preliminary qualitative research. The participants of the study are 60 elementary school students from the fifth and sixth grade. The data were collected using the test and interview. The data analysis showed that the elementary school student performance on number sense of decimal was still weak on the component of understanding the meaning and concept of numbers.

1. Introduction

The knowledge about decimal numbers is very important for the students, especially to assist students in measuring. Measuring with integers does not show accurate results, therefore decimals are required. The concept of decimal numbers is used in many ways. The importance of decimal computation is not limited to mathematics courses, but also ubiquitous in biology, physics, chemistry, engineering, economics, sociology, psychology, and many other areas [1]. In a survey of a nationally representative sample of US workers according to Handel [2], 68% of participants from diverse job categories reported using rational numbers at work. Hence, a good understanding of real numbers, including decimal numbers, is very important to support the students to deal with their job when they become an adult.

The decimal number is studied starting from elementary school. In general, decimal numbers are introduced in fourth grade. Students’ knowledge about decimals and the use of decimals in the base-ten system should be very secure before high school [3]. Learning decimal is critical for students’ pre-algebraic readiness for learning further mathematics skills [4]. So, the students should have a good understanding of decimal concept for getting a good understanding of mathematics.

The decimal number is a part of real numbers. Decimals refer to rational numbers expressed in base-10 notation [1]. Many challenges to be faced in learning about real numbers. Students seem to be unaware of their thinking about the fundamental difference between integer and rational numbers. The result of National Assessment of Educational Progress in 2004, showed only 29.2% of 17-year-olds correctly write 29/1000 as the fractional equivalent of .029 [5]. Studies concerning decimals, fractions, and percentages reveal a substantial lack of conceptual understanding by children that extends across all three rational number symbolic notations [6].

Steinle and Stacey [7] concluded that the cumulative effect of instruction for many years is that some misconceptions are covered over, instead of being overcome. Misconceptions about decimal numbers
will not be overcome without developing instruction which considers the obstacles that may arise to students in the study of decimals. Therefore, this study aims to show learning obstacles which faced by students in understanding decimals concept.

There are three kinds of learning obstacle, epistemological obstacle, didactical obstacle, and ontogenical obstacle [8-11]. Each obstacle is caused by different factors. The epistemological obstacle is a limitation of students' understanding of something that is only related to a particular context according to their learning experience [9]. The didactical obstacle is an obstacle that arises as a result of the instructional choice in learning, so it can be avoided through the development of alternative instructional approaches [8, 11]. Ontogenical obstacle arises from student limitations, associated with neurophysiology, related to students’ mental stage [8, 10, 11]. This study describing and documenting the various difficulties and obstacles faced by students when they were introduced to decimal numbers. The students’ answers are also analyzed to be classified according to the types of learning obstacle. Each learning obstacle indicates an inaccurate didactical design that causes obstacles to students. The result of this study could be a consideration for developing instruction about decimal numbers. Furthermore, it could be used as a reflection for teachers to re-personalize and recontextualize in order to facilitate students’ way of thinking and to design the learning path that suitable with students need.

2. Experimental Method
This research was using interpretive paradigm which was part of Didactical Design Research. This research was conducted at the elementary school in Sumedang, West Java Province, Indonesia. There were 60 students of six and five graders as participants. The participants were the students who had learned about decimal numbers. The selection of the participants was done by choosing the grade 5 and 6 students who already learned the decimal number from the same teacher. With the same learning experience, the background of the obstacles experienced by students is easier to trace. The data was collected from the written test and interview about their difficulty. The interview only carried out on 15 students who were the representative of the answers that appeared most often and students who had answers that need to be followed up to get a deeper information. The problems given to students were four problems of decimal numbers concept related to everyday life.

3. Results and Discussion
Four problems were given to students to discover learning obstacle. The purpose of the problems was to determine the ability of students in interpreting the decimal number, including how to declare decimal numbers, compare decimal values, convert decimals into fractions, convert fractions into decimals, represent the decimal numbers, and perform decimal computation tasks (addition and subtraction).

Difficulties faced by students in solving the problems were documented. In general, most of the students were facing difficulties caused by their inability to understand the place value in decimal number. The students thought that the place value in decimal number was the same one with the place value in integers. As a consequence, students mistakenly compare decimal number values and were facing difficulties remembering the addition and subtraction procedure using longhand calculation.

The difficulties were analyzed and classified into three kinds of learning obstacle, there were an epistemological obstacle, didactical obstacle, and ontogenical obstacle. See the problems which were given to students in figure 1 below.
Figure 1. The first problem that was given to the students.

Based on students’ answer about problem number 1a, 1b, and 1c, about 80% of the students did not understand the value of each number shown in the context of the problem. Students gave the wrong answer in naming 0.66, they answer it as zero point sixty-six. This showed that students’ conception about place value in decimal numbers was not correct, because they thought that the numbers after the comma were integers. The student did not understand about decimal numbers concept. Consequently, students were getting wrong in sorting the decimals from the largest to the smallest in problem 1b. See figure 2 for more detail about students’ answer.

| Day      | Weight (kg) |
|----------|-------------|
| Sunday   | 0.606       |
| Monday   | 0.0666      |
| Tuesday  | 0.6         |
| Wednesday| 0           |
| Thursday | 0.66        |
| Friday   | 0.060       |

a. On Thursday, Hana obtained strawberries 0.66 kg. Write 0.66 numbers using letters!
b. Arrange the weight of strawberries obtained by Hana for 6 days, from the heaviest to the lightest!
c. On Thursday, Hana got 0.66 kg strawberry. So, Hana had 0 kg + \( \frac{6}{100} \) kg, and + \( \frac{6}{100} \) kg strawberry.
d. On Tuesday, Hana got 0,6 kg strawberry, while her friend, Citra, got \( \frac{1}{5} \) kg strawberry. Are the weight of Hana and Citra’s strawberries different or not?
e. How much is the total weight of Hana’s strawberry on Monday and Tuesday?
f. How much is the gain of Hana’s strawberry on Thursday and Friday?

Figure 2. Ordering decimal numbers.

As shown in figure 2, students thought that the largest decimal on the problem was the decimal that had the most digit numbers after the comma. This conception was not correct and affects the inhibition of the ability of students in interpreting the decimal number. Based on the interview with a few students, they thought that 0.0666 was bigger than 0,606 because six hundred and sixty-six was bigger than six hundred and six. This conception named “longer is larger”behaviour, it is choosing decimals with the most digits after the decimal point as the largest [12]. One of the ways of thinking behind “longer is larger” behaviour is referred to as whole number thinking [12].

On problem 1c, the students answered that 0.66 was the same as \( 0 \) kg + \( \frac{6}{100} \) kg + \( \frac{6}{100} \) kg. Based on the interview result, students got the answer from their learning experience in class. The teacher said that one digit after comma was same as tenth, two digits after comma was same as hundredth, and so on. The students did not understand why the rule was like that. Students did not learn decimal numbers through multiple representations, so the students did not understand how to interpret the value of the decimal position.

On problem 1d, there was a student who answered that 0,6 and \( \frac{1}{6} \) was different, but her reason was not correct. See figure 3 for more detail.
She thought $\frac{1}{6}$ was the same as 1,6. She was unable to connect between fractions and decimal numbers, consequently, the student could not changing fraction to decimal or decimal to fraction. A similar understanding also shown by a pre-service teacher, she held misconceptions about decimals as evident in her association of decimals with reciprocals [13]. Learn fraction before decimals without using concrete context and exploring the meaning of numerator and denominator can make a concept “decimals are identical to fractions”.

On problem 1e and 1f, some students gave the wrong answer in positioning the decimal comma when they adding the decimals in a downward way. Students’ mistake was caused by their conception which considers a decimal number was the same as an integer. Consequently, they applied addition and subtraction rules of integers arranged starting from the unit (left). There were some students who answered problem 1f by using the concept of fractions, but he got the problem when asking to interpret $\frac{6}{1000}$ to decimals.

The problem 2 and 3 were about decimal representation. See figure 3 and figure 4 about students’ answer on problem 2 and 3.

**Figure 3.** Reciprocal thinking decimal numbers.

**Figure 4.** The second problem that was given to the students.

**Figure 5.** The third problem that was given to the students.
Based on the interview about problem 2, students thought that each small square was 0.1. There were 25 little squares, so the students answered 0.25. The answer was true, but the reason was incorrect. The students did not relate fraction concept for representing the decimal on the grid. Students’ answer about problem 3 indicated evidence that students were not able to use fractional concepts to represent decimals on the grid. Students thought that each small square was 0.1, so they coloured four little square to representing 0.4.

Problem 4 was about line number. No one gave a true answer. All of the students thought the answer was 0.10. The student did not understand that decimal was different from integer number concept. See figure 6 for more detail.

**Figure 6.** The fourth problem that was given to the students.

Figured 6 shows “long is larger” behaviour. In this case, such student considers a decimal number as two separate whole numbers separated by a dot [12]. So, 0.10 was thought to be the next number after 0.9. 

Based on students’ answer, the ontogenical obstacle was visible in a few students who gave the wrong answer although they had already studied the concept. Ontogenical obstacles arise because students can not follow the situation that occurs in learning as a result of the lack of understanding about the technical key of a learning process [14]. As shown in Figure 3, the student did not understand about technical keys of converting fraction to decimal. So, the student thought $\frac{1}{6}$ was the same as 1.6. Moreover, ontogenical obstacles also arise because of incompatibility of conceptual level in learning design and students condition [14]. In problem 1d and 1e about decimal computation tasks, students made some mistakes in positioning decimal comma. It was caused by the previous learning experience that was not enough to make the students understand the rule of positioning comma in decimal computation tasks.

The didactical obstacle was the most visible obstacle in this study. The sequence and stages of the curriculum including its presentation in the class can lead to didactical obstacles [14]. In problem 1a, 1b, 1c, and 4, students could not give the correct answer because they were not learning decimal through a proper didactical situation. The student learned decimals without exploring the meaning of decimals. So, the students did not understand about place value of decimals.

The epistemological obstacle was visible when the students could not give a correct answer because their learning experience was different from the context of the problem given. The epistemological obstacle was caused by the limitations of the context used on the first time a concept learned by students [14]. In problem 2 and 3, students could not give a correct answer about the representation of decimal in the grid. It was an epistemological obstacle, because students have already studied about fraction, but they were never using grid or area to representing decimals.

4. **Conclusion**

Based on the result, the difficulties that faced by most of the students were about the fundamental difference between integer and rational numbers. Many students could not understand about place value after the comma in decimal numbers. They thought that the numbers after the comma were like integers (whole numbers). Learning obstacles that students faced can be reduced by developing a didactical design which considering student obstacles.
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