Gravitational Pressure and the Accelerating Universe

K. H. C. Castello-Branco 1*
and J. F. da Rocha-Neto 2†

1 Universidade Federal do Oeste do Pará,
Av. Marechal Rondon, 68040-070. Santarém, PA, Brazil.
2 Instituto de Física, Universidade de Brasília,
70910-900, Brasília, DF, Brazil.

Abstract

In the context of the teleparallel equivalent of General Relativity, which is an alternative geometrical formulation of General Relativity (rather than an alternative theory of gravity), which is based on torsion and tetrad fields (rather than on curvature and metric), we apply a definition for a stress-energy-momentum tensor that naturally arises in this framework. Specially, that quantity leads to a very simple definition of pressure that, for a Friedmann-Robertson-Walker (FRW) universe, includes the pressure of matter, the “vacuum pressure” (due to a cosmological constant) and, moreover, the pressure of the gravitational field itself. We have computed that total pressure over a spherical surface of the FRW universe, for any curvature index. In particular, when we considered a spatially flat universe, and by resorting to the current observational data on the cosmological observables, we have found that pressure is negative and the corresponding energy density associated to it is remarkably close to the actual critical density of the universe. We interpret this as the effect of the pressure of the gravitational field as being responsible for the present accelerated expansion of the universe, instead of assuming an exotic, totally unknown, ad hoc form of energy, namely, the so-called “dark energy”.

Keywords: Teleparallel Equivalent of General Relativity, tetrad field, energy-momentum of gravitational and matter fields, gravitational pressure

PACS numbers: 04.20.-q, 04.20.Cv, 04.70.Dy

* khccb@yahoo.com.br † rocha@fis.unb.br
1 Introduction

Direct evidence from precise measurements of the cosmological distance-redshift relation of Type-Ia supernovae has indicated with high confidence that the expansion of the universe is accelerating [1, 2, 3]. The data has also implied evidence for a universe endowed with a positive “vacuum energy” density (not necessarily constant). This sort of positive “cosmological constant” (or “vacuum energy”), whose origin is totally unknown, dominates (as compared to matter) the present expansion of the universe. That sort of energy is the so-called “dark energy” (see, e.g., [4]). Evidence that dark energy contributes with a large fraction to the energy content of the universe has also been given by measurements of the cosmic microwave background [5]. While dark energy has been invoked to account for observational data, its nature and origin remain totally unexplained. Among the most discussed possibilities for it are the cosmological constant (“vacuum energy”) or some sort of dynamical field, which is usually taken as a scalar field. However, for a true cosmological constant the corresponding (“vacuum”) energy density is constant throughout space-time, rather than slowly-varying with time as indicated by observational data. This means that, strictly speaking, a cosmological constant cannot be considered as the source of dark energy. The option for a varying dark energy (the so-called quintessence), as described by a dynamical scalar field, has been widely studied (for a list of references on this and other proposals for describing dark energy see, for instance, [6]). Nevertheless, as the origin of such a field is not known and in general fine-tunings must be invoked to justify why such field has not yet been detected [7], this is a largely ad hoc description of dark energy. There have also been proposals that the cosmic acceleration has its origin in modifications of General Relativity (see, for instance, [8]). However, the theoretical explanation of the cosmic speeding up remains a mystery and it constitutes a great challenge in physics since its discovery.

In this work, we consider the role that gravitational pressure may have in addressing the issue of the accelerated expansion of the present universe. For this, to a Friedmann-Robertson-Walker (FRW) universe, we apply the definition of gravitational energy-momentum which naturally arises in the context of the Teleparallel Equivalent of General Relativity (TEGR) [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. This is not a new theory of gravity, but rather it is an alternative geometrical description of it, based on
torsion and tetrad fields, rather than curvature and metric (although the Riemann curvature tensor is identically null in the TEGR, effects of curvature are still present in this framework). This theory is equivalent to General Relativity - hence its name - as the field equation of the TEGR reduces to the Einstein equation in terms of tetrad fields, which are the basic variables of the TEGR. Curiously, the first to formulate a theory of gravitation in a space-time manifold endowed with torsion and described by tetrad fields was Einstein himself, in his attempts to unify gravitation and electromagnetism (for the complete references of the several papers by Einstein on this issue and for a historical account, see Ref. [20]). For a recent review on TEGR, see Ref. [21]. In the framework of the TEGR, the concept of gravitational energy-momentum has been shown to be well defined [13, 18, 22, 23, 24, 25, 26]. This definition was firstly obtained from the Hamiltonian formulation of the theory [13], but later it has been shown that this definition can be directly obtained from the field equation of the TEGR [24]. More recently, the notion of gravitational pressure has also been shown to naturally arise in the context of the TEGR, from its energy-momentum (thus the definition of gravitational pressure also essentially derives from the field equation of the TEGR) [24, 25]. Therefore, as it happens with the electromagnetic field, the gravitational field is also endowed with pressure. Such definition of gravitational pressure has been investigate in the context of gravitational waves [17], of a Schwarzschild black hole [24], and, more recently, for Kerr [25] and Reissner-Nordström [27] black holes. In all those applications, the definition of gravitational pressure has led to physically consistent results.

We note that the difficulty at arriving at a consistently defined gravitational energy-momentum tensor has led to investigations on pseudo-tensors and quasi-local expressions, but it has become clear that such a difficulty is not due to the nature of gravity or, worse, that the equivalence principle forbids it. Rather, the origin of such a difficulty is in the traditional description of the gravitational field (see, e.g., Ref. [28] on this point of view, in which attempts to define the gravitational energy-momentum by means of the field-theoretical formulation of General Relativity are considered. However, earlier, in Ref. [29], it was already anticipated that the metrical description of the gravitational fields is not suitable for the purpose of describing gravitational energy). The geometrical formulation of the TEGR has proven to be more adequate for addressing the above mentioned difficulty.
In the present work, we have computed the total pressure due to gravitational and matter fields over a spherical surface in a general (i.e., for any curvature index) Friedmann-Robertson-Walker universe in the framework of the TEGR. Then, from the general expression so obtained, and bearing in mind the observational data available for the cosmological observables, we have paid attention to a spatially flat universe, in order to investigate how the gravitational pressure could affect the expansion of the present universe. We have found that a negative pressure, outwardly directed on any spherical surface, results. Then by resorting to observational data, we have achieved that the energy density associated to such a pressure is strikingly close to the actual critical density of the universe. This can be interpreted as the cosmic speed up of the actual universe being an effect which results from the pressure of the very gravitational field, without any need to resort to the idea of (totally unknown and arbitrary) dark energy. Interestingly, we note that modified teleparallel gravity also allows an alternative understanding of the cosmic acceleration problem, without resorting to the idea of dark energy (see, for instance, Ref. [30] and references therein). However, our approach here is totally different and we do not even consider $f(T)$ theories, i.e., modified teleparallel gravity, as well as in Ref. [30] the idea of gravitational pressure is not considered at all.

We set our notation as the following: space-time indices $\mu, \nu, \ldots$ and $SO(3,1)$ indices $a, b, \ldots$ run from 0 to 3. Time and space indices are indicated according to $\mu = 0, i; \ a = (0), (i)$. Tetrad fields are denoted by $e^a_{\mu}$, and the torsion tensor reads $T_{a\mu\nu} = \partial_{\mu}e_{a\nu} - \partial_{\nu}e_{a\mu}$. The flat, Minkowski space-time metric tensor raises and lowers tetrad indices and is fixed by $\eta_{ab} = (-1, +1, +1, +1)$.

2 The Gravitational Energy-Momentum and Pressure in the TEGR

In this section we present a brief summary of the Teleparallel Equivalent of General Relativity (TEGR) (for a recent review, see [21]). In the TEGR the basic variables for the description of the gravitational field are tetrad fields, $e^a_{\mu}$, and the Lagrangian density is written in terms of a quadratic

\[\ldots\]
combination of the torsion tensor, $T_{a\mu\nu}$, which is related to the antisymmetric part of the Weitzenböck connection $\Gamma^\lambda_{\mu\nu} = e^a\lambda \partial_\mu e_{a\nu}$. The Riemann curvature tensor corresponding to this connection vanishes identically and, thus, one has a space-time manifold with distant (or absolute) parallelism, i.e., with teleparallelism (this means that it is possible to state that two vectors at distant points are said to be parallel if their components with respect to the (local) tetrads at the points considered are identical). The dynamics of the gravitational field in the realm of the TEGF is essentially the same as that of the usual metric theory. The equivalence of the TEGF with General Relativity is obtained by means of an identity that relates the scalar curvature $R_{e a\mu}$, obtained out of the tetrad field, with a combination of quadratic terms in the torsion tensor [9, 12, 13, 31] (see, also for details, the recent review [21])

$$eR_{e a\mu} \equiv -e \left( \frac{1}{4} T_{abc} T_{abc} + \frac{1}{2} T_{abc} T_{bac} - T^a T_a \right) + 2 \partial_\mu (e T^\mu),$$  \hspace{1cm} (1)

where $e \equiv \text{det}(e_{a\mu})$, $T_{abc} = e_b e_c T_{a\mu\nu}$, $T_a = T_{b a}$ and $T^\mu = T_{a\alpha}$. Therefore, in the TEGF the Lagrangian density for the gravitational and matter fields is defined as (see Ref. [21] and references therein)

$$L = -\kappa e \left( \frac{1}{4} T_{abc} T_{abc} + \frac{1}{2} T_{abc} T_{bac} - T^a T_a \right) - \frac{1}{c^2} L_m$$

$$\equiv -\kappa e \Sigma_{abc} T_{abc} - \frac{1}{c^2} L_m,$$  \hspace{1cm} (2)

in which $\kappa \equiv c^3/16\pi G$, $\Sigma_{abc}$ is defined by

$$\Sigma_{abc} = \frac{1}{4} (T_{abc} + T_{bca} - T_{cab}) + \frac{1}{2} (\eta^{a} T^{b} - \eta^{ab} T^{c}),$$  \hspace{1cm} (3)

and $L_m$ is the Lagrangian density for matter fields. The Lagrangian density $L$, given by Eq. (2), is invariant under the global $SO(3,1)$ transformations, since the absence of the divergent term on the right-hand side of Eq. (2) precludes the invariance of purely gravitational part of $L$ under arbitrary local $SO(3,1)$ transformations [15].

fields in space-time, $\{e_{(0)\mu}, e_{(1)\mu}, e_{(2)\mu}, e_{(3)\mu}\}$, which, as we will see, establish the local reference frame of an observer who moves along a trajectory represented by a worldline $x^\mu(\tau)$, where $\tau$ is the proper time of the observer. The components $e_{(0)\mu}$ and $e_{(1)\mu}$ are timelike and spacelike vectors, respectively.
The variation of \( L \) with respect to the tetrad field \( e^{a\mu} \) gives the field equation of the TEGR \[1\] (see also the review \[21\])

\[
e_{a\lambda}e_{b\mu}\partial_\nu(e^{a\Sigma^{b\lambda\nu}} - e^{(\Sigma^{b\nu}aT_{b\nu\mu} - \frac{1}{4}e_{a\mu}T_{bcd}^{\Sigma^{bcd}})}) = \frac{1}{4\kappa}eT_{a\mu},
\]

where \( T^{\lambda\mu} \) is defined by \( eT_{a\mu} = \delta L_m/\delta e^{a\mu} \). The left-hand side of the above equation may be identically rewritten in terms of the left-hand side of Einstein equation, as \( \frac{1}{2}e[R_{a\mu}(e) - \frac{1}{2}e_{a\mu}R(e)] \) (for details, see Ref. \[21\]). We remark that as the Lorentz indices in Eq. \((4)\) may be converted into space-time indices, then the left-hand side of the latter becomes proportional to \( (R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) \). This implies that a metric tensor, \( g_{\mu\nu} \), that is a solution of Einstein equation is also a solution of Eq. \((4)\). Hence, for a given space-time metric, that is a solution of Einstein equation, the physical results derived from considerations of it are also valid in the formulation of the TEGR (for further discussion, we refer the reader to the review \[21\]).

It turns out that Eq. \((4)\) may be rewritten as \[21, 24\]

\[
\partial_\nu(e^{a\Sigma^{\lambda\nu}}) = \frac{1}{4\kappa}e^{a\mu}(t^{a\mu} + T^{\lambda\mu}),
\]

where \( T^{\lambda\mu} \) (which is given by \( T^{\lambda\mu} = e^a\Sigma^{\lambda\mu}T^{a\mu} \)) is the energy-momentum tensor of the matter fields and \( t^{\lambda\mu} \), defined by

\[
t^{\lambda\mu} = \kappa(4\Sigma^{bc\lambda}T_{bc\mu} - g^{\lambda\mu}\Sigma^{bcd}T_{bcd}),
\]

is identified as the energy-momentum tensor of the gravitational field \[21, 24\].

Because of the antisymmetry in the last two indices of \( \Sigma^{a\mu\nu} \), it follows from Eq. \((5)\) that \[21, 24\]

\[
\partial_\lambda[e^{a\mu}(t^{a\mu} + T^{\lambda\mu})] = 0.
\]

Therefore, from the above equation it is possible to write a continuity (or balance) equation \[21, 24\],

\[
\frac{d}{dt}\int_V d^3x e^{a\mu}(t^{0\mu} + T^{0\mu}) = -\int_S dS_j [e^{a\mu}(t^{j\mu} + T^{j\mu})],
\]
in which the integration is carried out in the three-dimensional volume $V$, bounded by the surface $S$. We remark that on Eqs. (7) and (8) the tensors $t^{\lambda\mu}$ and $T^{\lambda\mu}$ appear on equal footings.

On the left-hand side of the above equation we have the time derivative of the total (gravitational and matter fields) energy-momentum, $P^a$, enclosed by a volume $V$ of the three-dimensional space \[^24\] \[^21\].

\[
P^a = \int_V d^3x e e^a_{\mu} (t^0{}_{\mu} + T^0{}_{\mu}).
\]  \hspace{1cm} (9)

And, on the right-hand side of Eq. (8), we identify the quantities

\[
\Phi^a_g = \oint dS_i (ee^a_{\mu} t^{i\mu})
\]  \hspace{1cm} (10)

and

\[
\Phi^a_m = \oint dS_j (ee^a_{\mu} T^{j\mu})
\]  \hspace{1cm} (11)

as the fluxes of energy-momentum of the gravitational and matter fields, respectively.

Using Eq. (5), Eq. (9) may be written in terms of $\Pi^{ai} = -\frac{4}{\kappa e} \Sigma^{ai0}$, which is the density of momentum canonically conjugate to $e_{ai}$, as

\[
P^a = -\int_V d^3x \partial_i \Pi^{ai} = -\oint dS_i \Pi^{ai}.
\]  \hspace{1cm} (12)

The definition of the energy-momentum $P^a$, in the above equation, was obtained for the first time in the context of Hamiltonian formulation of the TEGR in the vacuum \[^32\]. This definition is invariant under coordinate transformations of the three-dimensional space and under time reparametrizations. In vacuum, Eq. (12) represents the gravitational energy-momentum vector $P^a = (E/c, \mathbf{P})$, whereas in non-empty space-times it represents the total energy-momentum of the gravitational and matter fields (see Ref. \[^21\] and references therein).

If we now substitute Eq. (9) in the left-hand side of Eq. (8), and use Eq. (5), we may rewritten Eq. (8) simply as

\[
\frac{dP^a}{dt} = -\oint dS_j \phi^{aj},
\]  \hspace{1cm} (13)
where
\[ \phi^{aj} = 4\kappa \partial_{\nu}(\Sigma^{aj\nu}) \]. (14)

Recall that the Lorentz index \(a\) in Eq. (13) assumes the values \((0), (i)\). Thus, if now one restricts it to assume only spatial values, \(i.e.,\) for \(a = (i) = (1), (2), (3)\), one has
\[ \frac{dP^{(i)}}{dt} = -\int_S dS_j \phi^{(ij)} \], (15)
where
\[ \phi^{(ij)} = 4\kappa \partial_{\nu}(\Sigma^{(ij)\nu}) \]. (16)

On the left-hand side of Eq. (15) we have the total momentum of the gravitational and matter fields divided by time, that is, it has dimension of force. And since on the right-hand side \(dS_j\) is an element of area, it follows that the quantity \(-\phi^{(ij)}\) has dimension of force per unit of area, \(i.e.,\) it is a pressure density. Thus, \(-\phi^{(ij)}\) precisely represents the pressure density along the \((i)\)-direction, over a unit area element whose normal is oriented along the \(j\)-direction [24]. To obtain the radial pressure, in spherical-type coordinates, we fix \(j = r, \theta, \phi\) and therefore \(j = 1\) is associated with the radial direction. In spherical-type coordinates, \(\phi^{(r)1} = \phi^{(i)1}n_i\), where \(n_i = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)\) is the unit radial vector, the pressure density along the radial direction is
\[ -\phi^{(r)1} = -(\sin \theta \cos \phi \phi^{(1)1} + \sin \theta \sin \phi \phi^{(2)1} + \cos \theta \phi^{(3)1}) \] (17)
and then the radial pressure \(times unit area, p(r)\), is given by
\[ p(r) = \int_0^{2\pi} d\phi \int_0^\pi d\theta [-\phi^{(r)1}] \]. (18)

The expression for \(-\phi^{(r)1}\) for a FRW universe will be given in the next Section.

The expression given by Eq. (18) has recently been applied to the study of the thermodynamics of Kerr [25] and Reissner-Nordström [27] black holes. In Ref. [25] the effect of the gravitational pressure on the Penrose process was to decrease its efficiency. In Ref. [27] the investigation of the gravitational entropy of the black hole has led to the result that the gravitational pressure over the event horizon of the black hole modify the standard, Bekenstein-Hawking entropy-area relation, even if one assumes that the (gravitational,
i.e., classical) thermodynamic temperature of the black hole is given by the (quantum) Hawking temperature.

We close this section with the remark that for static (co-moving) observers the radial pressure density, $-\phi^{(r)}_1$, turns out to be the total pressure density, simply due to the fact that for a spherical type metric the components $\phi^{(r)}_2$ and $\phi^{(r)}_3$, associated with the pressure density in the directions perpendicular to the radial both identically vanish, as can be directly checked by means of their expressions. This matches with our intuition, due to the symmetry of spatial homogeneity and isotropy of the FRW universe.

3 Gravitational Pressure in a FRW Universe

A homogeneous and isotropic universe is described by the (standard) Friedmann-Robertson-Walker (FRW) metric in co-moving coordinates (see, e.g., [33])

$$ds^2 = -c^2 dt^2 + a^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right\}, \quad (19)$$

in which $k = 0, 1$, or $-1$ is the curvature constant of a three-dimensional spatial hypersurface, $a(t)$ is the “scale factor” and $t$ is the “cosmic” time. Fortunately, observations of the actual universe can provide information on the values of the parameters $k$ and $a$. Observational data indicates that the actual universe is endowed with a small, but non-vanishing cosmological constant (or some sort of dynamical cosmological “constant”). Hence, in order to properly address the dynamics of the universe (and specially the issue of its accelerated expansion), one has to consider the cosmological term (or simply the $\Lambda$-term) in the Einstein field equation, namely

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \quad (20)$$

For the energy-momentum tensor of the cosmic fluid in its rest-frame one has

$$T_{\mu\nu} = p g_{\mu\nu} + \left( \rho + \frac{p}{c^2} \right) u_\mu u_\nu, \quad (21)$$

where $\rho$ and $p$ are its total mass density and pressure, respectively, and $u_\mu = (-c, 0, 0, 0)$ is its four-velocity. Hence, the total energy density of the cosmic fluid is given by $c^2 \rho$.  

8
Since Einstein equation (20) is non-linear, the effect of plugging the $\Lambda$-term is, in principle, independent on which side it enters into the equation. Therefore, as usual, we will consider the effect of a cosmological constant through the right-hand side of the Einstein equation. This means we will consider that besides the cosmic fluid, a contribution given by

\[
T_{\mu \nu}^{\text{vac}} = -\frac{\Lambda c^4}{8\pi G}g_{\mu \nu}
\]

will enter in the right-hand side of the Einstein equation, keeping the left side without the $\Lambda$-term. We note that the quantity $\Lambda c^4/G$ has dimension of pressure.

From (19), (20) and (21) one has

\[
\frac{2}{c^2} \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \Lambda = -\frac{8\pi G\rho}{c^4}
\]

and

\[
\frac{\dot{a}^2}{a^2c^2} + \frac{k}{a^2} - \frac{1}{3} \Lambda = \frac{8\pi G\rho}{3c^2}.
\]

It is convenient to rewrite Eq. (24) as

\[
\left( \frac{\dot{a}}{a} \right)^2 = -\frac{k c^2}{a^2} + 2\frac{1}{3}\Lambda c^2 + \frac{8\pi G \rho}{3},
\]

which is the Friedmann equation.

Subtracting (24) from (23) one has

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) + \frac{1}{3} \Lambda c^2.
\]

Eqs. (25) and (26) will be used later in this Section.

In order to investigate the effect of the gravitational pressure on Friedmann universe models, we will compute the radial pressure over a spherical surface. For this, as a first step, let us construct a tetrad field related to the FRW metric (19). A set of tetrad fields adapted to static observers and which corresponds to the metric (19) can be directly derived through the relationship $g_{\mu \nu} = e^\alpha_\mu e_{\alpha \nu}$. Firstly, let us note that a tetrad field, which is
the basic field variable of the TEGR, is naturally interpreted as a reference frame adapted to an observer in space-time [34]. This interpretation has been explored in investigations on about both the stress-energy-momentum and angular momentum of the gravitational field defined in the framework of the TEGR (see [7], [25], [27]). To every observer in space-time a tetrad field can be adapted, in the following way [34]: let \( x^\mu(s) \) be the world-line, \( C \), of an observer in space-time, where \( s \) is the observer’s proper-time. Thus, the observer’s four-velocity along \( C \), defined by \( u^\mu(s) = dx^\mu / ds \), is identified with the \( a = (0) \) component of \( e^\mu_a \), that is, \( u^\mu(s) = e_{(0)}^\mu \) along \( C \). In this way, each set of tetrad fields defines a class of reference frames in space-time. In what follows we will consider a set of tetrad fields adapted to a static observer in space-time. Given a metric \( g_{\mu \nu} \), the tetrad field related to it can be straightforwardly obtained through \( g_{\mu \nu} = \eta^{ab} e^a_{\mu} e^b_{\nu} \). A set of tetrad fields adapted to static observers is achieved by imposing on \( e^\mu_a \) the following conditions: (i) \( e_{(0)}^i = 0 \), which implies that \( e_{(k)0} = 0 \), and (ii) \( e_{(0)i} = 0 \), which implies that \( e_{(k)}^0 = 0 \). The physical meaning of condition (i) is easily seen (the translational velocity of the observer is null, \( i.e. \), the three components of the frame velocity in the three-dimensional space are null). In its turn, the condition (ii) is a condition on the rotational state of motion of the observer. It implies that the observer (more precisely the three spatial axes of the observer’s local spatial frame) is (are) not rotating with respect to a nonrotating frame (for details, we refer the reader to Ref. [23] and references therein). Consequently, conditions (i) and (ii) are six conditions one has to impose on the tetrad field in order to completely fix its structure. From the conditions (i) and (ii), one can easily obtain the set of tetrad fields which corresponds to static observers and is related to the metric (19). It reads

\[
e^a_{\mu} = \begin{pmatrix}
-c & 0 & 0 & 0 \\
0 & A \sin \theta \cos \varphi & a(t) r \cos \theta \cos \varphi & -a(t) r \sin \theta \sin \varphi \\
0 & A \sin \theta \sin \varphi & a(t) r \cos \theta \sin \varphi & a(t) r \sin \theta \cos \varphi \\
0 & A \cos \theta & -a(t) r \sin \theta & 0
\end{pmatrix}, \tag{27}
\]

where \( A = \frac{a}{\sqrt{1 - kr^2}} \). It follows that the determinant \( e = det(e^a_{\mu}) \) is \( e = cAa^2(t) r^2 \sin \theta \), which will be needed later.

As a second step in order to evaluate the radial pressure over a spherical surface of the FRW universe class, one needs to compute the quantities \( \phi^{(i)} \) given by Eq. (16) and which appear in Eq. (17). For the set of tetrad fields
given by Eq. (27), one can compute the components of the torsion tensor, \( T_{a\mu\nu} \), and of the tensor \( \Sigma_{a\mu\nu} \), from what will follow the computation of \( \phi^{(i)1} \). All these calculations are long but straightforward and they lead to

\[
\begin{align*}
\phi^{(1)1} &= 4\kappa \sin^2 \theta \cos \varphi \left( \frac{\partial_0 (a\dot{a})}{c^2} r^2 - 1 + \sqrt{1 - kr^2} \right), \\
\phi^{(2)1} &= 4\kappa \sin^2 \theta \sin \varphi \left( \frac{\partial_0 (a\dot{a})}{c^2} r^2 - 1 + \sqrt{1 - kr^2} \right), \\
\phi^{(3)1} &= 4\kappa \sin \theta \cos \theta \left( \frac{\partial_0 (a\dot{a})}{c^2} r^2 - 1 + \sqrt{1 - kr^2} \right). 
\end{align*}
\] (28)

Inserting now the above relations into Eq. (17) and performing the integration in Eq. (18), one obtains that the radial (which is also the total) pressure times unit area over a surface of radius \( r \) is given by (recall that \( x^0 = ct \))

\[
p(r) = -\frac{c^4}{G} \left\{ -1 + \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a} \right) \frac{r^2}{c^2} + \sqrt{1 - kr^2} \right\}. 
\] (29)

Let us now consider a spatially flat universe, what corresponds to take \( k = 0 \). In this case, Eq. (29) reduces to

\[
p(r) = -\frac{c^2}{G} \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a} \right) r^2. 
\] (30)

Substituting into the latter its corresponding terms which appear in Eqs. (25) and (26), one obtains

\[
p(r) = \left( -\frac{4\pi}{3} \rho c^2 + 4\pi p - \frac{2 \Lambda c^4}{3 G} \right) r^2. 
\] (31)

Recall that \( p \), on the right-hand side of (31), is the pressure of matter and it should not be confused with \( p(r) \) which figures on the left-hand side of the above equation. Strictly speaking the quantity \( (31) \) represents the radial force on the spherical surface of radius \( r \) (pressure multiplied by unit area). Therefore, defining the pressure, \( P \), over a spherical surface of radius \( r \), as \( p(r) \) divided by the surface area, \( 4\pi r^2 \), one obtains that the total pressure is

\[
P = - \left( \frac{1}{3} \rho c^2 + \frac{1}{6\pi} \frac{\Lambda c^4}{G} - p \right). 
\] (32)
From this result, as it does not depend on the distance, \( r \), it follows that \( P \) does not depend on the corresponding spherical surface of radius \( r \) either. Thus, the pressure \( P \) is spatially homogeneous and isotropic, what complies with the fact that the universe is homogeneous and isotropic [and that we have chosen a set of static, co-moving observers to deal with, according to the set of tetrad fields given by Eq. (27)]. Nevertheless, in order to investigate the effect of the pressure given by (32) on the expansion of the universe as a whole, we will focus our attention on the apparent horizon of the universe, which is defined as the surface where the ingoing radial null geodesic congruences have vanishing expansion and the outgoing radial null geodesic congruences have positive expansion (see, e.g., [35],[36] and references therein). As opposed to the event horizon, the apparent horizon is not a global one, but rather it is defined (quasi-)locally (for details, we refer the reader to Ref. [36]). Besides, the apparent horizon always exists in all FRW space-times, as opposed to event and particle horizons [35],[37]. As we are considering a spatially flat FRW universe, the apparent horizon coincides with the “Hubble horizon” (see below). We also remark that, as compared to event horizons, apparent horizons seem to be more suitable for formulating a consistent thermodynamics in cosmological space-times (see, e.g., [35] and references therein). From now on, we will consider the pressure (32) as simply the pressure over the apparent horizon. The location of the apparent horizon can be found directly if it is characterized as a momentarily stationary light-front in terms of its area coordinate [37], \( \tilde{r}_{AH} = ar_{AH} \) (\( r_{AH} \) is the radius in co-moving coordinates). The apparent horizon is located at \( \tilde{r}_{AH}^2 = \frac{a^2c^2}{\dot{a}^2 + k} \) (see, e.g., [35] and [37]), that is, at the radius

\[
\tilde{r}_{AH} = \frac{c}{\sqrt{H^2 + k/a^2}},
\]

(33)

where \( H = \dot{a}/a \) is the Hubble parameter. Thus, for a spatially flat FRW universe, the apparent horizon coincides with the Hubble horizon.

As the present universe is in an era in which radiation pressure is negligible and matter is non-relativistic (“cold matter”), we take \( p = 0 \) in Eq. (32). Thus, one is left with

\[
P = - \left( \frac{1}{3} \rho + \frac{1}{6\pi} \frac{\Lambda c^2}{G} \right) c^2.
\]

(34)
Therefore, it results that the total pressure on the apparent horizon is negative, directed outward over it, like a tension (we pay attention that, according to (32), this conclusion is valid irrespective of the spherical surface chosen). Consequently, as according to observations the actual universe is dominated by some sort of energy whose corresponding pressure is negative, then instead of a unknown, mysterious dark energy the accelerated expansion of the universe can be in fact a result of a basic property of the gravitational field itself, namely, its pressure (one must recall that the classical electromagnetic field is endowed with pressure, or, more generally, stress-energy-momentum, and thus one would expect the same for the classical gravitational field). We will analyze more on this possibility in the following.

The different contributions to the matter-energy content in the universe are usually taken as baryonic matter (ordinary matter), dark matter, cosmological constant (“vacuum energy”) and the completely unknown dark energy (see, e.g., [4]). Contributions from the stress-energy-momentum of gravity are completely ignored when addressing the nature of the cosmic accelerated expansion. Measurements of the cosmological observables show that presently the sum of the fractions of all contributions to the energy density in the universe is close to 1 (see, for instance, Ref. [38] and references therein), that is,

$$\Omega_{m0} + \Omega_{\Lambda0} \approx 1,$$

in which the dimensionless density parameters $\Omega_{m0}$ and $\Omega_{\Lambda0}$ are defined by (see, e.g., [37])

$$\Omega_{m0} \equiv \frac{8\pi G \rho_m}{3H_0^2},$$

and

$$\Omega_{\Lambda0} \equiv \frac{\Lambda c^2}{3H_0^2} = \frac{8\pi G \rho_{\text{vac}}}{3H_0^2}.$$

The index zero in all the above relations means that these quantities are evaluated at the present time.

In view of the result given by Eq. (34), let us now define an effective density

$$\rho_{\text{eff}} \equiv \frac{1}{3} \rho + \frac{1}{6\pi} \frac{\Lambda c^2}{G},$$
with which (34) can be rewritten as

\[ P = -\rho_{\text{eff}} c^2. \]  

(39)

Since we have been identifying the \( \rho \) which appears in (38) as the density corresponding only to matter and radiation, and considering that presently the contribution from radiation can be neglected, we thus take \( \rho = \rho_m \) (density of matter). In this way, using definitions (36) and (37) in (38), one is led to

\[ \rho_{\text{eff}} = \frac{H_0^2}{8\pi G} (\Omega_{m0} + 4\Omega_{\Lambda0}). \]  

(40)

As the critical density at present time, which is defined by \( \rho_{\text{cr}} \equiv \frac{3H_0^2}{8\pi G} \), is important for comparing the contributions of all the possible forms of energy densities in the universe, let us consider it and rewrite (40) as

\[ \rho_{\text{eff}} = \frac{1}{3} \rho_{\text{cr}} (\Omega_{m0} + 4\Omega_{\Lambda0}). \]  

(41)

Now, using the observational result (35), one can write

\[ \rho_{\text{eff}} = \frac{1}{3} \rho_{\text{cr}} (4 - 3\Omega_{m0}). \]  

(42)

Observational data also gives \( \Omega_{m0} \approx 0.3 \), such that we are left with

\[ \rho_{\text{eff}} \approx 1.033 \rho_{\text{cr}}. \]  

(43)

Hence, we have obtained that the energy density associated with the (negative) pressure (39) has a value strikingly close to the critical density of the actual universe. This complies with a universe which is presently spatially flat. As a result, the contribution of the gravitational pressure to the pressure given by (39), which can be considered as responsible for the cosmic acceleration of the actual universe, can be considered as the origin of the effect that is currently attributed to the totally unknown dark energy \( \Lambda \). Instead of being due to a mysterious form of energy or exotic matter, the accele-

---

\( ^2 \)It should be remarked that we are considering, as is widely assumed, that dark matter is cold, i.e., that it is pressureless, what implies that, in a fluid perspective, its equation of state is \( w_{\text{dm}} = 0 \). The issue if dark matter is cold or warm is still an open one. Nevertheless,
rated expansion can thus be described as an effect of a basic property of the gravitational field itself, namely, that it is endowed with pressure (or stress).

In view of Eq. (43), it is instructive to rewrite Eq. (39) as

\[ P = w \rho c^2, \]

(44)

where \( w \approx -1.033 \). According to the 7-year WMAP data, \( w = -1.10 \pm 0.14(1\sigma) \) [43], while from data mainly on Type-Ia supernovae from the SNLS3 sample, \( w = -1.069^{+0.091}_{-0.092} \) [44]. Thus, the value of \( w \) we have found is close to those provided from the available data.

We note that for computing the pressure we have considered a spatially flat universe. But for this kind of universe one has that the total density of all contributions to the energy density in the universe is just equal to the critical density. Moreover, the observational result (35) has its best fitting corresponding to the case of a spatially flat universe [38]. In fact, that is why one considers that the actual universe is spatially flat.

4 Conclusions and Final Remarks

In this work, the accelerated expansion rate of the actual universe has been addressed by resorting to the basic concept of gravitational pressure, which naturally arises in the framework of the Teleparallelism Equivalent to General Relativity (TEGR) [24],[21]. This is not a new theory of gravity, but rather an alternative, but equivalent, geometrical description of Einstein’s
General Relativity. Such equivalence takes place at the level of field equations. Instead of most usual approaches, in the present work no assumption about scalar fields which would originate some mechanism responsible for dark energy has been done. On the contrary, in trying to arrive at a simple description of the presently accelerated universe, we have applied the definition of the stress-energy-momentum of the gravitational and matter fields, which, as we have reviewed in this work, is directly identified from the field equation of the TEGR, which is equivalent to Einstein field equation. As is well-known, such a stress-energy-momentum is not directly available in General Relativity. From the definition given by Eq. (13), it is possible to define an expression for the total pressure of gravitational and matter fields. We have computed the total pressure (of gravitational and matter fields) over a spherical surface of the FRW universe. We have found that it is independent of the radius of the surface [see Eq. (32)], what complies with the homogeneity and isotropy of the universe, as well as with the choice we have made use of the tetrad fields adapted to co-moving observers [see Eq. (27)]. From Eq. (32), as a particular case, it follows that for an empty de Sitter universe (for which $\rho = p = 0$), the total pressure reduces simply to $P = -\Lambda c^4/6\pi G$. Hence, $P$ is negative (for $\Lambda > 0$) and proportional to $\Lambda$ (as one might intuitively expect). Interestingly, we remark that the latter value is $4/3$ greater than the value of the pressure $p_{\Lambda} = -\Lambda c^4/8\pi G$, which is the pressure that corresponds to an otherwise empty universe, but interpreted as being filled with a cosmic fluid whose energy-momentum is given by Eq. (22), i.e., $T_{\mu\nu}^{\text{vac}} = - (\Lambda c^4/8\pi G) g_{\mu\nu}$. We thus conclude that even if the cosmological constant enters as a fluid component in the cosmic budget of an empty de Sitter universe, the effect of the gravitational pressure is to negatively increase the (total, gravitational plus “$\Lambda$-fluid”) pressure of the space-time. Furthermore, according to Eq. (32), in the epochs of the universe when $p$, the pressure of matter and radiation, is non-negligible the total pressure, $P$, is less negative than in other epochs (in the radiation-dominated era, $p = \rho c^2/3$, and then $P$ achieves its lower absolute value,
what corresponds to the lowest tension on, e.g., the apparent horizon, or any other spatial spherical two-surface of the universe). And for negligible $p$, as is the case in the present universe, it follows that $P$ is given by Eq. (34), that is, the total pressure over any spherical two-surface in the universe is most negative in this epoch, what corresponds to the greatest tension over such a surface. In order we could study the effect of the pressure of the gravitational field on the present expansion of the universe as a whole, we have thought of that surface as the cosmological apparent horizon (although this was not a mandatory condition, since the result given by Eq. (34) is independent of the radius of the surface) and we have also considered the case of a spatially flat FRW universe, as the cosmological data favors it. As already pointed out, this pressure is negative (see Eq. (39)) and it is like a surface tension over the apparent horizon - the pressure is outwardly directed on the horizon. Moreover, by resorting to the observational data currently available, we have obtained that the energy density associated with the pressure (39) has a value strikingly close to the critical density of the actual universe. Consequently, one can think of that pressure as acting to cause the repulsive effect which is responsible for the accelerated expansion of the universe. Hence, the universe is under an accelerated expansion due basically to the effect of (the pressure of) its own gravitational field, rather than due to some exotic, totally unknown sort of energy (“dark energy”). We further remark that the pressure $P$, given by Eq. (39), increases negatively with time. In fact, since for a perfect fluid the energy conservation law for matter, given by $\nabla_\mu T^{\mu 0} = 0$, implies that $\dot{\rho} + 3H(\rho + p/c^2) = 0$, it follows that, considering that for the actual universe $p$ is negligible (cold matter), we are left with $\dot{\rho} = -3H\rho < 0$ for an expanding universe (for which $H > 0$). Thus, since $\dot{\rho}_{\text{eff}} = \frac{1}{3}\dot{\rho}$ [see Eq. (38)], from Eq. (39), we have $\dot{P} > 0$, what implies that as the universe expands the (negative) pressure $P$ increases. It should be noted that the quantity $\rho_{\text{eff}}$, given by Eq. (38), is not simply $\rho_m + \rho_{\text{vac}}$, that is, it is not simply the sum of the contributions only of matter and “vacuum energy” (cosmological constant), but rather $P$ takes also into account the gravitational pressure, as implied by the result (39). Equivalently, this can also be viewed from Eqs (37) and (38), which imply that $\rho_{\text{eff}} = \frac{1}{3}(\rho_m + 4\rho_{\text{vac}})$, from what we can see that $\rho_{\text{eff}} - (\rho_m + \rho_{\text{vac}}) = \frac{1}{3}\rho_{\text{vac}} - \frac{2}{3}\rho_m$. This clearly shows that something more than matter and cosmological constant plays a role in the accelerated expansion of the universe, namely, the gravitational field itself. We recall that $P$, given by Eq. (39), stems from the total momentum of
the gravitational and matter fields, as defined in the realm of the TEGR. In summary, our results (39) and (44) take into account not only the usual contributions to the dynamics of the universe but also the contribution of the gravitational field itself. In other words, the contribution of the pressure of the gravitational field is really playing a role in the expansion of the universe. As far as we know, that the gravitational field itself could play a role in the actual accelerated expansion of the universe has not been addressed in the literature.

We recall that it is widely argued in the literature that as the present universe is accelerated ($\ddot{a} > 0$), it follows from Eq. (26) that $(\rho + p/c^2) < 0$, for $\Lambda > 0$. This implies that $p$ would be negative, and then an exotic sort of matter would have to be taken into account ("dark energy") accordingly. But, as we have shown, if we consider that the cosmic fluid is composed only of normal, rather than exotic matter, we can describe the accelerated expansion of the present universe by means of the total pressure of the gravitational and matter fields, without the need of evoking any exotic type of matter. Instead, we have evoked a basic property of the gravitational field itself, namely, its pressure. The latter stems from the definition of the energy-momentum of the gravitational field, which in turn is directly derived from the field equation of the TEGR. We recall that in General Relativity the notion of energy-momentum of the gravitational field is a cumbersome issue and is not generally well-defined. On the other hand, from the continuity equation of the TEGR [see Eq. (8)], the contribution of the gravitational field for the total energy-momentum of the space-time is taken into account on equal footings with that of matter fields.

During the development of this work, we have found the interesting proposal of an alternative interpretation for dark energy discussed in Ref. [45]. In the latter, the authors have taken into account the entropy and temperature intrinsic to the horizon of a spatially flat FRW universe. Those entropy and temperature were considered to be due to the information that is holographically stored on the apparent horizon (in this case, given by the Hubble radius). As argued in Ref. [15], the cosmic acceleration would be due to an entropic force that would arise from the information stored on the horizon surface screen. We note that, in the context of our approach, it is quite possible to introduce an entropy of gravitational origin, as one considers that this would be defined by means of the thermodynamic relation $TdS = dE + PdV$,
where \( T \) could be identified with the temperature of the apparent (or Hubble) horizon, \( S \) is the entropy, \( E \) is the energy (necessarily including the gravitational energy) and \( P \) is the pressure as we have computed in the present work. The thermodynamics of the apparent horizon of the FRW universe is an important issue and is currently under investigation.

References

[1] S. P. Perlmutter et al., *Nature* **391**, 51 (1998) [astro-ph/9712212].
[2] S. Perlmutter et al., *Astrophys. J.* **517**, 565 (1999) [astro-ph/9812133].
[3] G. Riess et al., *Astron. J.* **116**, 1009 (1998) [astro-ph/9805201]. B. Schmidt et al., *Astrophys. J.* **507**, 46 (1998) [astro-ph/9805200].
[4] P. J. E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75**, 559 (2003).
[5] S. Dodelson, *Modern Cosmology* (Academic Press. Amsterdam, 2003).
[6] E. Linder, *Am. J. Phys.* **76**, 197 (2008) [astro-ph/0705.4102v1].
[7] M. Trodden and S. M. Carroll, *TASI Lectures: Introduction to Cosmology* [astro-ph/0401547v1].
[8] S. M. Carroll et al., *Phys. Rev.* **D70**, 043528 (2004) [astro-ph/0306438].
[9] F. W. Hehl, *in Proceedings of the 6th School of Cosmology and Gravitation on Spin, Torsion, Rotation and Supergravity, Erice, 1979*, edited by P. G. Bergmann and V. de Sabbata (Plenum, New York, 1980).
[10] J. M. Nester, *Int. J. Mod. Phys.* **A4**, 1755 (1989).
[11] J. W. Maluf, *J. Math. Phys.* **35**, 335 (1994).
[12] F. W. Hehl, J. D. McCrea, E. W. Mielke, and Y. Ne’eman, *Phys. Rep.* **258**, 1 (1995) [arXiv:gr-qc/9402012].
[13] J. W. Maluf, J. F. da Rocha-Neto, T. M. L. Torbrio, and K. H. Castello-Branco, *Phys. Rev.* **D65**, 124001 (2002) [arXiv:gr-qc/0204035v1].
[14] Y. N. Obukhov and J. G. Pereira, Phys. Rev. D67, 044016 (2003) [arXiv:gr-qc/0212080].

[15] Y. N. Obukhov and G. F. Rubilar, Phys. Rev. D73, 124017 (2006) [arXiv:gr-qc/0605045].

[16] C. Chen, J. Liu, and J. M. Nester, Mod. Phys. Lett. A22, 2039 (2007) [arXiv:gr-qc/0705.1080v2].

[17] J. W. Maluf and S. C. Ulhoa, Phys. Rev. D78 (2008) 047502, Erratum-ibid. D78 (2008) 069901.

[18] K. H. C. Castello-Branco and J. F. da Rocha-Neto, Gen. Rel. Grav. 44 (2012) 3017.

[19] R. Aldrovandi and J. G. Pereira, Teleparallel Gravity: An Introduction (Springer. Heidelberg, 2013).

[20] T. Sauer, Historia Math. 33, 399 (2006) [arXiv:physics/0405142v1].

[21] J. W. Maluf, Ann. Phys. (Berlin) 525, 339 (2013) [arXiv:1303.3897].

[22] J. W. Maluf, F. F. Faria, and K. H. Castello-Branco, Class. Quant. Grav. 20, 4683 (2003) [arXiv:gr-qc/0307019v2].

[23] J. W. Maluf, F. F. Faria, and S. C. Ulhoa, Class. Quant. Grav. 24, 2743 (2007).

[24] J. W. Maluf, Ann. Phys. (Berlin) 14, 723 (2005) [arXiv:gr-qc/0504077v2].

[25] J. W. Maluf, S. C. Ulhoa, and J. F. da Rocha-Neto, Phys. Rev. D85, 044050 (2012).

[26] J. F. da Rocha-Neto, J. W. Maluf, and S. C. Ulhoa, Phys. Rev. D82, 124035 (2010).

[27] K. H. C. Castello-Branco and J. F. da Rocha-Neto, Phys. Rev. D88, 024045 (2013).

[28] S. V. Babak and L. P. Grishchuk, Phys. Rev. D61, 024038 (1999).

[29] J. W. Maluf, J. Math. Phys. 36, 4242 (1995).
[30] G. R. Bengochea and R. Ferraro, *Phys. Rev.* **D79**, 124019 (2009); E. V. Linder, *Phys. Rev.* **D81**, 127301 (2010); Erratum-ibid. **D82**, 109902(E) (2010); S. Capozziello, V. F. Cardone, H. Farajollahi, and A. Ravanpak, *Phys. Rev.* **D84**, 043527 (2011); J. B. Dent, S. Dutta, and E. N. Saridakis, *JCAP* 1101:009, 2011; K. Bamba, R. Myrzakulov, S. Nojiri, and S. D. Odintsov, *Phys. Rev.* **D85**, 104036 (2012); M. Jamil, D. Momeni, and R. Myrzakulov, *Eur. Phys. J.* **C72**, 2122 (2012); C. Xu, E. N. Saridakis, and G. Leon, *JCAP* 1207 (2012) **005**; K. Bamba, S. D. Odintsov, and D. Sáez-Gómez, *Phys. Rev.* **D88**, 084042 (2013); S. Nesseris *et al.*, *Phys. Rev.* **D88**, 103010 (2013).

[31] M. Blagojevic, *Gravitation and Gauge Symmetries* (IOP. Bristol 2002).

[32] J. W. Maluf and J. F. da Rocha-Neto, *Phys. Rev.* **D64**, 084014 (2001).

[33] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, W. H. Freeman and Company (1973).

[34] F. W. Hehl, J. Lemke, and E. W. Mielke, *Two Lectures on Fermions and Gravity*, in *Geometry and Theoretical Physics*, edited by J. Debrus and A. C. Hirshfield (Springer Berlin, Heidelberg) (1991).

[35] V. Faraoni, *Phys. Rev.* **D84**, 024003 (2011).

[36] S. Hayward, *Phys. Rev.* **D49**, 6467 (1994).

[37] W. Rindler, *Relativity: Special, General, and Cosmological* (2nd ed., Oxford University Press, Oxford, 2006).

[38] S. Weinberg, *Cosmology* (Oxford University Press, Oxford, 2008).

[39] L. Xu and Y. Chang, *Phys. Rev.* **D88**, 127301 (2013).

[40] Planck Collaboration, *Planck 2013 results. I. Overview of products and scientific results*, [arXiv:1303.5062v1](http://arxiv.org/abs/1303.5062) [astro-ph.CO].

[41] A. Schneider *et al.*, [arXiv:1309.5960v2](http://arxiv.org/abs/1309.5960) [astro-ph.CO].

[42] A. L. Serra and M. J. L. D. Romero, *Mon. Not. R. Astron. Soc. Lett.* (2011), [arXiv:1103.5465v2](http://arxiv.org/abs/1103.5465) [gr-qc].

[43] E. Komatsu *et al.*, *Astrophys. J. Suppl. Ser.* **192**, 18 (2011).
[44] M. Sullivan et al., Astrophys. J. 737, 102 (2011).

[45] D. A. Easson, P. H. Frampton, and G. F. Smoot, Phys. Lett. B 696, 273 (2011) [arXiv:1002.4278v3].