Hydrodynamic description of Hard-core Bosons on a Galileo ramp

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We study the quantum evolution of a cloud of hard-core bosons loaded on a one-dimensional optical lattice after its sudden release from a harmonic trap. Just after the trap has been removed, a linear ramp potential is applied, mimicking the so called Galileo ramp experiment. The non-equilibrium expansion of the bosonic cloud is elucidated through a hydrodynamical description which is compared to the exact numerical evolution obtained by exact diagonalization on finite lattice sizes. The system is found to exhibit a rich behavior showing in particular Bloch oscillations of a self-trapped condensate and an ejected particle density leading to two diverging entangled condensates. Depending on the initial density of the gas different regimes of Josephson-like oscillations are observed. At low densities, the trapped part of the cloud is in a superfluid phase that oscillates in time as a whole. At higher densities, the trapped condensate is in a mixed superfluid-Mott phase that show a breating regime for steep enough potential ramps.

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I. INTRODUCTION

It is of primary importance to understand the dynamical behavior of strongly correlated many-body quantum systems, especially since their behavior in many cases could show counter-intuitive effects with respect to the classical naive expectation. One of the early discovered such effect is the so-called Bloch oscillation regime occurring when a small constant force $F$ is applied to a quasi-particle living on a lattice [1]. In such a case, the particle momentum is drifted in time according to $q(t) = q(0) + Ft$ modulo the Brillouin zone, leading to a periodic motion with period $\tau_B = 2\pi/|F|$ (here and the lattice spacing are set to one). Such oscillatory behavior has been observed in many different physical contexts like semiconductors, thermal gases, photonics, cold atoms physics and with phonons [2]. At stronger forces, in multi-band systems, such oscillations can be suppressed by the Landau-Zener interband tunneling [3]. The Bloch oscillations can also be suppressed due to many-body effects with damping related to the integrability of the system [4][5].

On the other hand, experimental advances, notably on ultracold atomic gases have lead to a revival of the field of low-dimensional many-body systems [6], especially within non-equilibrium aspects [7][8]. Thanks to the very low dissipation rate and very long time phase coherence of such systems, it has become possible to realize experimentally almost the unitary dynamical evolution of ideal models like the one-dimensional Bose-Hubbard model, even in its hard-core limit [6]. In those systems, the release of the gas from a trap, which is a standard procedure for cold atoms experiments, can lead to interesting metastable states [9] or even to dynamical fermionization effects when the frequency of a parabolic trap is modified [10]. Recent studies have considered the dynamical behavior of hard-core bosons in tilted optical lattices [11][12], focusing in particular on the Bloch oscillations, Landau-Zener tunneling and entanglement properties.

In this study we pursue on the direction set by [12] by considering now a hard-core boson cloud initially localized on part of an optical lattice thanks to a harmonic trap. The trap is then suddenly removed and a linear ramp potential (a constant force $F$) is added to the region initially occupied by the cloud. The unitary expansion of the cloud leads to the escape of part of the condensate while the remaining part is self-trapped into the initial region and performs Bloch oscillations. Thanks to the initial correlations within the cloud the escaping particles are strongly entangled with the localized oscillating condensate. Depending on the initial density, two different situations arise. Namely, at low density the trapped condensate is in a superfluid phase that oscillates in time as a whole while at higher density it is in a mixed superfluid-Mott phase that show a richer behavior with, in particular, a breathing regime at large enough forces. The out-of-equilibrium dynamical behavior of the system is analyzed numerically by means of exact diagonalization following the methods developed in [13] and through an analytical hydrodynamical approximation already used in [12].

The paper is organized as follows: in section II, we present the model, its mapping to a Fermi system and the hydrodynamical description used in this paper. In section III, we study the dynamics after the sudden release of the bosonic cloud focusing first on the emitted wave-packet, then on the self-trapped condensate and finally discussing the entanglement between the emitted particles and the self-trapped ones. In section IV, a brief summary is given.

II. THE MODEL AND ITS HYDRODYNAMIC DESCRIPTION

A. The model

We consider in this study the one-dimensional Bose-Hubbard model describing a set of bosons leaving on a lattice with a repulsive on-site interaction $U$ and submitted to an external potential $V(t)$ which may vary in time. The dynamics

\[ q(t) = q(0) + Ft \]
is generated by the Hamiltonian
\[ H = -J \sum_j (a_j^+ a_{j+1} + \text{h.c.}) + \frac{U}{2} \sum_j n_j (n_j - 1) + \sum_j V_j(t) n_j \]
with \( a_j \) and \( a_j^+ \) the usual destruction and creation bosonic operators, \( n_j = a_j^+ a_j \), the bosonic density at site \( j \). \( J \) is the hopping magnitude and will be set to \( J = 1/2 \) in the rest of this work (this will give a band width \( \Delta = 1 \)). In particular, we focus our attention on the hard-core boson limit of this model by taking the limit \( U \to \infty \). In this limit, the dynamics can be described by the new Hamiltonian
\[ H = -\frac{1}{2} \sum_j [b_j^+ b_{j+1} + \text{h.c.}] + \sum_j V_j(t) n_j \]
where the new bosonic creation and annihilation operators \( b_j^+ \) and \( b_j \) satisfy the on-site anticommutation rule \( \{b_j, b_j^+\} = 1 \), \( \{b_j, b_k\} = [b_j^+ b_k^+] = 0 \), avoiding a double occupancy of the same site, while commuting at distinct sites. The standard procedure to diagonalize this Hamiltonian is to fermionize it through a Jordan-Wigner mapping and then performing a canonical Bogoliubov transformation to new diagonal Fermi operators. Indeed, introducing the lattice Fermi creation operators \( c_j^+ = \prod_{\ell \leq j} (1 - 2n_{\ell})^{1/2} b_\ell \) and their adjoint annihilation operators \( c_j = (c_j^+)^\dagger \), rejecting the boundaries at infinity, the Hamiltonian is expressed as a tight-binding Fermi system
\[ H = -\frac{1}{2} \sum_j [c_j^+ c_{j+1} + \text{h.c.}] + \sum_j V_j(t) n_j \]
with \( n_j = c_j^+ c_j = b_j^+ b_j \) the occupation operator at site \( j \). Thanks to the quadratic form of this Hamiltonian, it is readily diagonalized through a standard canonical transformation [13] leading to the diagonal expression \( H = \sum_q \epsilon_q n_q^\dagger n_q \) where the \( \epsilon_q \) are the excitation energies associated to the free Fermi particles created by \( n_q^\dagger \) and destroyed by \( n_q \) and the problem is in principle solved. However, for a quite general inhomogeneous potential \( V_j(t) \) one has to compute numerically on a finite lattice the single-particle spectrum \( \epsilon_q \), while the relevant observables are expressed as determinants of the single particle Green functions \( \langle c_j^+ c_j \rangle \) which have also to be computed numerically in the general case. Nevertheless, in the limit of a very large system with a sufficiently smooth potential function \( V(x,t) \) one can well capture the features of the boson density dynamics by a hydrodynamic limit using a continuous description of the model [12].

B. Continuum limit and local equilibrium hypothesis

Consider the one-dimensional Hard-core bosons system on an infinite one-dimensional lattice with lattice spacing \( a \ll 1 \). Let the potential \( V(x) \) be a smooth real function (at least let us say \( V \in C^1 \)) on the lattice. We split the real line into regular intervals \([x, x + \Delta x]\), with \( \Delta x = aM \), containing a large number \( M \) of lattice sites, while keeping the width \( \Delta x \) small enough in the sense that \( \forall j a \in [x, x + \Delta x] \), \( V(ja) \approx V(x) \), that is the potential keeps almost a constant value on each interval. The Hamiltonian can be recast in the continuum limit \( a \to 0 \), \( \Delta x \to 0 \) while keeping \( \Delta x / a = M \gg 1 \) in the following form
\[ H = \int_{-\infty}^{\infty} dx \ H(x) \]
where the hamiltonian density \( \bar{H}(x) \) is given by
\[ \bar{H}(x) = \frac{1}{a \Delta x} \int_0^{\Delta x} dy \left[ -\frac{1}{2} \Psi^\dagger(x + y) \Psi(x + y - a) + \text{h.c.} \right] + V(x) \Psi(x + y) \Psi(x + y) \]
in terms of the continuous creation and annihilation Fermi field operators \( \Psi^\dagger(y) \) and \( \Psi(y) \). To achieve (3) with (4) we have supposed that the interaction contribution between different intervals, which is just a local boundary term, is very small compared to the contribution (4) within the interval. This is true if the potential variations are sufficiently small. The local Hamiltonian density \( \bar{H}(x) \) is simply a continuous version of (2) with a constant potential \( V(x) \), and it can be consequently easily diagonalized through the canonical mapping
\[ \Psi(x + y) = \int_0^\pi dq \ \phi_q(x + y) \eta(q, x) \]
\[ \Psi^\dagger(x + y) = \int_0^\pi dq \ \phi^*_q(x + y) \eta^\dagger(q, x) \]
where the field operators \( \eta(q, x) \) and \( \eta^\dagger(q, x) \) annihilates and respectively creates in region \([x, x + \Delta x]\) a particle with momentum \( q \) and satisfy the anti-commutation rules \( \{\eta(p,q), \eta^\dagger(p', q')\} = \delta(p - p') \delta(x - x') \). The exact form of the Bogoliubov functions \( \phi_q(x) \) entering into the definition of the new fields essentially depends on the boundary conditions imposed on the interval \([x, x + \Delta x]\). Using these new fields, the total Hamiltonian takes the diagonal form
\[ H = \int_{-\infty}^{\infty} dx \int_0^\pi dq \ [V(x) - \cos q] \eta^\dagger(q, x) \eta(q, x) \]

Therefore, with respect to this Hamiltonian, the \( N \)-particles ground state of the bosonic system is given by a local equilibrium state. That is, all the quasi-particles associated to each phase-space points \((q, x)\), with energies \( \epsilon(x,q) = V(x) - \cos q \) bellow the Fermi level \( \epsilon_F(N) \), are added to the vacuum state:
\[ |\Psi_0\rangle = \prod_{q,x} \eta^\dagger(q, x) |0\rangle \]
The Fermi energy is given by imposing the constraint \( \int dx \ \rho(x) = \int dx \ |\Psi(q, x)|^2 \eta(q, x) = N \) on the total number of particles. This readily implies for the ground-state density profile
\[ \rho(x) = \begin{cases} 0 & V(x) - \epsilon_F > 1 \\ \frac{1}{2} \arccos(V(x) - \epsilon_F) & \frac{|V(x) - \epsilon_F|}{\epsilon_F} < 1 \\ 1 & V(x) - \epsilon_F < -1 \end{cases} \]
In Figure 1 we compare the exact numerical diagonalization with the local equilibrium prediction on a system with \( L = 400 \) sites at half filling and, as an illustration, a potential \( V(x) = 2x/L + 1.5 \sin(8x/L) + \sin(16x/L) \). Notice the good matching between the continuum prediction \( \Psi(x) \) and the numerical data. In particular, we have also graphically reproduced the occupied energy levels (yellow region) at half filling factor. In practice, since \( \cos q \in [-1, 1] \), the energies \( \epsilon(q, x) \) fall in the gray strip which follows the shape of the potential. The local equilibrium approximation breaks down whenever the local number of particles is small and the potential varies sharply.

In a grand canonical situation, when the number of particles is not fixed, the Fermi level \( \epsilon_F \) in the ground state as to be set to 0 so that the ground state is build up by adding to the vacuum all excitations with negative energies:

\[
|\Psi_0\rangle = \prod_{\{(q, x): \epsilon(q, x) \leq 0\}} \eta(q, x)|0\rangle ,
\]

and consequently the density profile is given by \( \rho = \sum |\langle x| \Psi_0 \rangle|^2 \) with \( \epsilon_F = 0 \) and the total number of particles in that state is just \( \int dx \rho(x) \).

### C. Initial state

In the following we will fix the initial state of the bosonic condensate as the grand canonical ground state associated to a harmonic trap potential. The harmonic potential is parameterized as

\[
V(x, t < 0) = \alpha \left( x - \frac{A}{2} \right)^2 - \mu_0 , \quad \alpha = \frac{4}{A^2}(1 + \mu_0) ,
\]

with \( A < 0 \) and \( \mu_0 > -1 \) such that there is at least few particles loaded on the lattice. This parametrization insures that the bosonic condensate is centered at \( A/2 \) with a spacial extension of width \( |A| \) such that outside the region \( x \in [A, 0] \) the density (in the hydrodynamic limit) is vanishing (see figure 2). For a given width \( |A| \) of the condensate, we have two qualitative distinct situations that are controlled by the value of the chemical potential \( \mu_0 \). Indeed, for \( \mu_0 < 1 \), the potential is never smaller than \(-1\) and the condensate is fully in its superfluid phase with a density profile

\[
\rho(x) = \frac{\theta(-x) \theta(x + A) \arccos \left( \frac{x - A}{2} - \mu_0 \right)}{\pi},
\]

where \( \theta(x) \) is the Heaviside function. We will refer to this state as the pure superfluid phase (SF-phase). On the other hand, for \( \mu_0 > 1 \), there is in the middle of the condensate a Mott insulating phase, with \( \rho = 1 \), that extends spatially over the region \( x \in [\frac{A}{2} - \Delta_{Mott}, \frac{A}{2} + \Delta_{Mott}] \) with

\[
\Delta_{Mott} = \frac{|A|}{2} \sqrt{\frac{\mu_0 - 1}{\mu_0 + 1}} .
\]

This Mott phase is surrounded by two superfluid phases, of spatial extensions

\[
\Delta_{SF} = \frac{|A|}{2} - \Delta_{Mott} = \frac{|A|}{2} \left( 1 - \sqrt{\frac{\mu_0 - 1}{\mu_0 + 1}} \right) ,
\]

with a profile given by \( \rho = \sum |\langle x| \Psi_0 \rangle|^2 \) with the harmonic potential \( \Psi(x) \), as illustrated on figure 2. As the chemical potential \( \mu_0 \) is getting larger and larger, the Mott phase is growing at the expense of the superfluid phases, shrinking them to the boundaries close to \( x = A \) and \( x = 0 \). We call this state the mixed superfluid-Mott phase (Mixed SF-Mott phase).

### III. DYNAMICS AFTER THE SUDDEN QUENCH

#### A. Sudden release of the trap and loading of the linear ramp

At time \( t = 0^+ \), starting from the previous initial state, we suddenly release the gas from the parabolic trap and load an
external constant force $F$ on the negative side of the real axis. The force is described by the linear ramp potential
\[ V(x, t > 0) = -Fx \theta(-x) \]
and the system dynamics is governed by a new Hamiltonian $H$ with potential $V(x, t > 0) = -Fx \theta(-x)$. As we will see in the following, the main features of the dynamics are well described in the hydrodynamic limit. As seen previously, the initial state has approximately a coarse-grained phase-space density
\[ \rho(x, q) \approx \frac{1}{\pi} \theta(-x) \theta(A) \theta(q) \theta(qF(x) - q) \]
with a local Fermi wave-vector \( qF(x) = \pi \rho(x) \) where \( \rho(x) \) is given by (8). The local initial density is then simply given by adding all the local quasiparticles (associated to each phase-space point) with different wave-vectors \( q \): \( \rho(x) = \int dq \ w(x, q) \). Just after the sudden quench, the population living initially on the tilted band with energies within the interval \( \pi \theta(qF(x) - q) \)

B. Wave-packet emission

1. Density profile

The typical evolution of the bosonic density, just after the sudden quench, is shown on figure 3 for several forces and for the two distinct typical initial states, that is the pure SF-phase and the mixed SF-Mott state. As we see clearly on figure 3, only part of the bosons are ejected to the right with a corresponding density that is spreading in time (notice that since the numerical results are obtained on a finite lattice with open boundary conditions the ejected particles are reflected when they reach the right boundary wall). The remaining particles are self-trapped into the initial region, performing Bloch oscillations (we will come back on this point in the next section). The reason for the escape of the particles to the right is best understood from the hydrodynamical description. Indeed, the population living initially on the tilted band with energies within the interval \( \left[ -1, 1 \right] \) are connected at \( t = 0^+ \) to the propagating states of the energy band at \( x > 0 \) (see figure 2a). Those particles will consequently escape from there initial position and propagate to the right toward \(+\infty\). Right movers will escape directly while left movers will be first reflected on the left tilted band edge and then propagate toward the right direction. From energy conservation, within the tilted band, as for Bloch oscillations, the momentum of a right(left) mover with energy \( \epsilon \) evolves as \( q^\pm(t) = \pm q + Ft \),

where the domain of integration \( Q(\epsilon) \) over the momenta is shown on figure 4(a). In figure 4(b) we show the exact numerical results obtained by exact diagonalization of the escaping density compared to the hydrodynamic prediction (18). We see a very good agreement up to small interference effects that are obviously neglected in the continuum limit. It is interesting to notice that the rightmost front of the escaping wave-packet shows, beyond the hydrodynamic envelop, a staircase structure as already noticed in a different context [14, 15]. Each of the stairs corresponds to a particle, the integrated density over each such stair being equal to one, ejected to the right and moving ballistically with velocity \( v(\epsilon) = \sqrt{1 - \epsilon^2} \). Behind the front the exact structure is a bit more complicated and it is difficult to discriminate between different particles.
The left and right movers densities are given by

$$\rho(x, t) = \rho^+(x, t) + \rho^-(x, t).$$

(19)

The left and right movers densities are given by

$$\rho^\pm(x, t) = \int_0^{\pi} \frac{dq}{2\pi} \Pi[f_1(q), f_2(q)](g^\pm(x, q, t))$$

(20)

where

$$g^\pm(x, q, t) = x - \frac{1}{F} \cos(q \pm Ft)$$

(21)

and

$$f_1(q) = \begin{cases} \frac{1 + \cos q - \sqrt{\cos q + F_0}}{2} & q \in [0, \tilde{q}] \\ \frac{1 + \sqrt{\cos q + F_0}}{2} & q \in [\tilde{q}, \pi] \end{cases}$$

(22)

$$f_2(q) = \frac{A}{2} - \alpha \cos \frac{q}{2}$$

(23)

with $\tilde{q} = \arccos(-1 - F_M x_M)$ where $x_M$ is the right-most locus of the initial trapped condensate (see figure [4a]).

On figure [5] we have plotted over a half-period the oscillating density profile of the self-trapped particles for the two initial states (SF-Mott and SF phases) as extracted from the exact numerical diagonalization results obtained on a chain of 400 sites with a force $F = 0.04$ and compared to the hydrodynamical prediction (19), together with a schematic representation of the initial state. First of all we see a very good agreement of the hydrodynamical prediction with the exact numerical results. In these figures we clearly see the two different oscillation regimes in the SF-Mott case (see figures [2a] and [2c]). The reason for that is easily understood from the schematic pictures of the initial states. Indeed at low enough forces, as seen on the schematic picture of the initial state, the energy of the local density at the separation point $x_{SF-M}$ between the SF phase and the Mott phase is higher than the energy at the left-most initial locus $x = A$ and consequently the density will explore regions on the left of $A$ up to the locus $x_A = x_{SF-M} - 2/F$. This phenomenon disappears when the energy at the locus $x_{SF-M}$ is getting smaller than the energy at the point $x = A$. This precisely appears above a threshold force $F^*$ given by $F^* = 0.0689$, where we see almost no evolution of the profile at the left side. Beyond that value, as seen on figure [3c] for $F = 0.1$, the left and right sides of the density profile are moving in phase. With a SF initial state, the self trapped density is always globally oscillating as a whole (see figure [3d]).

2. Current density profile

This remarkable periodic motion of the trapped density is naturally associated to a flow of particles giving rise to a periodic current density $j(x, t) = \langle J(x, t) \rangle$ which is defined through the continuity equation $i[H, n(x)] = -\nabla \cdot j(x)$ where $n(x)$ is the occupation operator at site $x$. In figure [4b] we show a snapshot of the current density obtained from exact diagonalization for a SF state at $F = 0.05$ (top) and a SF-Mott initial state at low forces ($F = 0.04$) and high forces ($F = 0.1$). In both cases, in the trapped region we see clearly the Bloch oscillations with a strip structure for the pure SF phase indicating the collective motion of the superfluid condensate while in the mixed SF-Mott situation we observe the oscillations (in phase at $F = 0.04$ and in opposite phase at $F = 0.1$) of the two SF phases surrounding a stationary Mott plateau.

In the hydrodynamical limit, the current density of the self-trapped condensate is simply given by summing over all the quasiparticles current contributions $\rho(q) v(q)$ with the veloci-
FIG. 5. (Color online) (a) Top: Self-trapped density profile for an initial SF-Mott state at different times for $F = 0.04$. The hydrodynamical prediction is given by the dashed line. (b) Middle-up: Same as (a) for $F = 0.0689$. (c) Middle-down: Same as (a) for $F = 0.1$. (d) Bottom: Self-trapped density profile for an initial SF state at different times for $F = 0.05$. The hydrodynamical predictions are represented in each case by the dashed lines.

ties $v(q) = \pm \sin(q(0) \pm Ft)$ leading to

$$j(x,t) = \int_0^{\pi} \frac{dq}{2\pi} \sin(q + Ft) \Pi_{f_2(q),f_1(q)}(g^+(x,q,t))$$

$$- \int_0^{\pi} \frac{dq}{2\pi} \sin(q - Ft) \Pi_{f_2(q),f_1(q)}(g^-(x,q,t)) . \quad (24)$$

We have plotted in figure 7 the current density (24) compared to the exact numerical one at different positions as a function of time for the two cases, SF and SF-M. We observe a perfect matching of the hydrodynamical prediction (24) with the exact numerical values. We also observe a very nice sinus behavior of the current in the middle of the superfluid condensate reminiscent of a Josephson-type oscillations which is explicitly given from (24) as $j(A/2, t) = 1/\pi \sin(Ft)$.

D. Entanglement between the trapped and the escaping particles

Just after the sudden unloading of the parabolic trap and the quench of the linear ramp, the initial condensate is split into two disjoint parts: the escaping particles and the remaining self-trapped ones. Due to the initial correlations in the starting state (10), these two well separated condensates are entangled. This entanglement between propagative modes (into the right propagative energy band) and bound states (into the self-trapping region) can be quantified through the bi-partite von Neuman entropy $S(x,t) = -\text{Tr} \{ \rho(x,t) \ln \rho(x,t) \}$ where $\rho(x,t) = \text{Tr}_{y>x} \{|\Psi(t)\rangle \langle \Psi(t)|\}$ is the reduced density matrix associated to the left of position $x$ on the lattice, deduced from the time-evolved pure state $|\Psi(t)\rangle$ by tracing out the degrees of freedom on the right of $x$ (see for example the review [16]).

As seen on figure 8 we have qualitatively two different situations depending on whether the initial state is a pure SF...
FIG. 7. (Color online) Top: Current density at $x = -100$ for $F = 0.1$ (shortest period) and $F = 0.05$ in the SF case. Bottom: Current density at $x = -185$ for $F = 0.1 > F^*$ (shortest period) and $F = 0.04 < F^*$ in the SF-Mott case. Full lines are the corresponding hydrodynamic predictions.

FIG. 8. (Color online) Top: Entanglement entropy as a function of space and time for $F = 0.3$ with a SF initial state generated with $A = 50$ and $\mu_0 = 0$. Bottom: Entanglement entropy as a function of space and time for $F = 0.5$ with a SF-M initial state generated with $A = 50$ and $\mu_0 = 2$.

IV. CONCLUSION

We studied the dynamical behavior of a hard core boson gas initially prepared in a parabolic trap and subject to the sudden quench of a linear ramp potential just after the release of the trapping potential (like in the so-called Galileo ramp experiment with classical particles). The study was based on exact numerical diagonalization methods and on a hydrodynamic description which allowed the very good understanding of the dynamical behavior at the level of local density and local current density. The dynamics of this setup is rich, showing an escape of particles and Bloch oscillations for the self-trapped remaining condensate. Depending on the initial trap, the state of the condensate is either in a purely superfluid phase or in a mixed state with a Mott phase surrounded by two superfluid phases. In these two cases the behavior of the self-trapped condensate is different, showing in particular in the SF-M phase the possibility for a breathing condensate at high forces while in the SF situation the condensate is always oscillating as a whole. Notice finally that this setup can be used to create entangled many-body wave-packets by coupling the Bloch bound states to propagative modes.

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