Accretion driven turbulence in filaments I: Non-gravitational accretion

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ABSTRACT
We study accretion driven turbulence for different inflow velocities in star forming filaments using the code RAMSES. Filaments are rarely isolated objects and their gravitational potential will lead to radially dominated accretion. In the non-gravitational case, accretion by itself can already provoke non-isotropic, radially dominated turbulent motions responsible for the complex structure and non-thermal line widths observed in filaments. We find that there is a direct linear relation between the absolute value of the density weighted velocity dispersion and the infall velocity. The turbulent velocity dispersion in the filaments is independent of sound speed or any net flow along the filament. We show that the density weighted velocity dispersion acts as an additional pressure term supporting the filament in hydrostatic equilibrium. Comparing to observations, we find that the projected non-thermal line width variation depends strongly on the inclination of the filament due to the non-isotropic nature of the driven turbulence.

Key words: stars:formation – ISM:kinematics and dynamics – ISM:structure

1 INTRODUCTION
Turbulent motions are ubiquitous on all astrophysical scales. There is evidence for highly complex non-thermal motion from the intergalactic medium to the interstellar medium (ISM) and individual molecular clouds down to the smallest scales of protostellar discs. Likewise, as part of the ISM, star forming filaments are no exception to this observational fact. In contrast to its importance the origin of turbulence is still not fully understood and there are numerous potential sources for turbulent motions in the ISM (Mac Low & Klessen 2004; Elmegreen & Scalo 2004; Elmegreen & Burkert 2010; Klessen & Glover 2014). On cloud scale, molecular line observations are dominated by supersonic motions and show a direct correlation between size and line width (Larson 1981). This is usually interpreted as the direct result of a turbulent cascade from the scale of a tens of parsec sized molecular cloud down to the scale of parsec sized filaments (Kritsuk et al. 2013; Padoan et al. 2016). Filaments then inherit their internal velocity dispersion from the motions on larger scales. This model is also favoured by Herschel observations which show that filamentary structures are ubiquitous in molecular clouds (André et al. 2010; Arzoumanian et al. 2011, 2013; André et al. 2014). This picture has recently been challenged by the discovery that more massive filament are actually complex bundles of fibers whose line-of-sight superposition create the observed supersonic linewidths (Hacar et al. 2013) whose formation has also been found in numerical simulations (Smith et al. 2014; Moeckel & Burkert 2015). Independent of the formation process, we argue that accretion driven by the gravitational potential of the filaments alone can be enough to stimulate turbulent motions that are in agreement with the observations (see also Ibáñez-Mejía et al. (2016)).

Recent images of the high column density gas, traced by the usually optically thin C18O line of nearby filaments have shown that the non-thermal linewidth is predominantly sub- and transsonic along filaments. This is true for L1517 in Taurus (Hacar & Tafalla 2011), where the mean is about half the sound speed, as well as for the fibre-like substructure of the L1495/B213 region in Taurus (Hacar et al. 2013), where the mean is about the sound speed. Moreover, even in the Musca filament, a 6 pc long structure, subsonic non-thermal linewidths dominate along the filament (Hacar et al. 2016) contrarily to the prediction of Larson’s Law (Larson 1981) which predicts a much higher internal velocity dispersion. In this paper we explore the origin of turbulence in filaments.

In the following sections, we introduce the basic concepts we use to constrain our model (section 2). We then discuss the code and the numerical set-up (section 3). Thereafter, we present our results of the simulations and discuss them in detail (section 4). Additionally, we show that tur-
bulence plays a role in creating a hydrostatic equilibrium (section 5). Finally, we compare our data to the observations (section 6) and investigate the dependence on filament inclination.

2 BASIC CONCEPTS

In order to sustain turbulence inside a filament there has to be an external driving mechanism. Otherwise, turbulent motions decay on the timescale of a crossing time (Mac Low et al. 1998; Stone et al. 1998; Padoan & Nordlund 1999; Mac Low 1999; Mac Low & Klessen 2004). Here, we discuss a possible source of the external driving and the theoretical prediction.

2.1 Gravitational accretion onto a filament

Although there are different ways to accrete mass onto a filament, e.g. a converging flow, these processes are typically limited in time. A counterexample for a radial converging flow which is stable over longer timescales is the gravitational attraction of the filament itself. If we assume that the filament is isothermal and in hydrostatic equilibrium, then it has a density profile first described by Stodolkiewicz (1963) and Ostriker (1964):

\[ \rho(r) = \frac{\rho_0}{\left(1 + \frac{r}{H}\right)^2} \]  

where \( r \) is the cylindrical radius and \( \rho_0 \) is its central density. The radial scale height \( H \) is given by

\[ H^2 = \frac{2c_s^2}{\pi G \rho_c} \]  

where \( c_s \) is the isothermal sound speed and \( G \) the gravitational constant. We assume that the gas has an isothermal temperature of 10 K. Using a molecular weight of \( \mu = 2.36 \) gives the isothermal sound speed of \( c_s = 0.19 \) km s\(^{-1}\). One can integrate the profile to \( r \to \infty \) to get the critical line mass

\[ \left( \frac{M}{L} \right)_{\text{crit}} = \frac{2c_s^2}{G} \approx 1.06 \times 10^{16} \text{g cm}^{-1} = 16.4 \text{M}_\odot \text{pc}^{-1} \]  

above which a filament will collapse under its self-gravity. Following Heitsch et al. (2009), for a given line mass \( M/L \) the gravitational acceleration of the filament is:

\[ a = \frac{2GM/L}{r} \]  

One can calculate the potential energy which a gas parcel of mass \( m \) loses in free-fall starting with zero velocity at a distance \( R_0 \) to the filament radius \( R \) by integrating over \( r \):

\[ E_{\text{pot}} = 2\frac{G(M/L)m}{R} \ln \left( \frac{R_0}{R} \right) \]  

Therefore, the inflow velocity at the point of accretion \( R \) is:

\[ v_r = 2\sqrt{\frac{G(M/L)\ln \left( \frac{R_0}{R} \right)}{R}} \]  

Note that similar to the free fall velocity, the value does not depend on the mass of the gas parcel. It is also not sensitive to neither the starting position nor the line mass while depending stronger on the latter. As the filament accretes mass and increases in line-mass, the inflow velocity grows. However, as we want to analyse accretion driven turbulence in an equilibrium state we keep the inflow velocity constant. Therefore we choose to neglect the effects of gravity and use an artificial but constant mass inflow where we set the inflow velocity also to a constant value. The effects of gravity will be discussed in a subsequent paper. Assuming the extreme case of a filament with a gravitational influence of a hundred times the filament radius, which for a typical radius of 0.05 pc (Arzoumanian et al. 2011) is the size of a typical molecular cloud, we still need a line mass which is several times higher than the critical line-mass to achieve an inflow velocity of even Mach 10.0. Thus, we limit our maximum inflow velocity to Mach 10.0.

A consequence of a constant inflow velocity is a constant mass accretion rate. The absolute value is set by the radius of the inflow region \( R_0 \) and the density at that radius \( \rho_0 \):

\[ \dot{M} = \rho_0 v_r 2\pi R_0 L \]  

This should stay constant for every radial shell and thus leads to the following density profile outside of the filament:

\[ \rho(r) = \rho_0 \frac{R_0}{r} \]  

As there is an isothermal accretion shock formed at the filament boundary, pressure equilibrium requires that the density inside the filament is the outside density times a factor of the Mach number \( M \) squared. This leads to the following filament mass-radius relation:

\[ M(R) = \rho_0 M^2 \pi R R_0 L \]  

This has to be the same as the accreted mass given by the mass accretion rate given by Equation 7 times the time \( t \).

\[ \therefore t = \frac{M(R)}{\dot{M}} \]  

The radius of the filament evolves as:

\[ R(t) = \frac{2c_s^2 t}{v_r} \]  

2.2 Turbulence driven by accretion

Following Klessen & Hennebelle (2010), Heitsch (2013) derived an analytical expression for the velocity dispersion depending on the inflow velocity. We expect turbulence to decay on the timescale of a crossing time:

\[ \tau_d = \frac{L_d}{\sigma} \]  

where \( \sigma \) is the velocity dispersion in three dimensions and \( L_d \) is the driving scale of the system. Klessen & Hennebelle (2010) use the approach that the change in turbulent kinetic energy is given by the balance of the accretion of kinetic energy and the dissipation of turbulent energy:

\[ \dot{E}_t = \dot{E}_a - \dot{E}_d = (1 - \epsilon)\dot{E}_a \]  

with the energy accretion rate

\[ \dot{E}_a = \frac{1}{2} M v_r^2 \]  

and the loss by dissipation as

\[ \dot{E}_d = \frac{\dot{E}_t}{\tau_d} = \frac{1}{2} \frac{M v_r^3}{L_d} \]
They also introduce an efficiency factor $\epsilon$ as fraction of accreted energy which can sustain the turbulent motions:

$$\epsilon = \frac{\dot{E}_d}{\dot{E}}$$  \hspace{1cm} (15)

Thus, if the driving scale is the filament diameter $L_d = 2R$, Heitsch (2013) predicted a turbulent velocity dispersion of

$$\sigma = \left(2\epsilon R(t) \frac{\dot{M}}{\dot{M}(t)} \right)^{1/3}$$ \hspace{1cm} (16)

This assumes that $\epsilon$ is independent of the inflow velocity. In our simple case the radius depends on the constant inflow velocity as $1/v_r$ and the radial accretion leads to a radius and mass growing linear in time. Therefore, we expect a constant level of velocity dispersion which should behave as

$$\sigma \sim v_r^{1/3}.$$  \hspace{1cm} (17)

This is the relationship we want to confirm or disprove using numerical methods.

### 3 NUMERICAL SET-UP

We executed the numerical simulations with the code RAMSES (Teyssier 2002). The code uses a second-order Godunov scheme to solve the conservative form of the discretised Euler equations on an Cartesian grid. For the simulations we applied the MUSCL scheme (Monotonic Upstream-Centred Scheme for Conservation Laws, van Leer (1977)) together with the HLLC-Solver (Toro et al. 1994) and the multidimensional MC slope limiter (van Leer 1979).

We simulate a converging radial flow onto a non-gravitating filament in order to study the generated turbulence. We use a 3D box with periodic boundary conditions in the x-direction and outflow boundaries in the other directions. The periodic boundary prohibits the loss of turbulent motions in x-direction. As RAMSES cannot use a radial inflow boundary we define a cylindrical inflow zone which lies at the edges of the box and has a thickness of two cells from where material flows onto the central x-axis of the box. The initial gas density inside of the box is set to a mean of $3.92 \times 10^{-21}$ g cm$^{-3}$ corresponding to about $10^7$ particles per cubic centimeters for a molecular weight of $\mu = 2.36$. Additionally, a random perturbation is added inside of the inflow zone at the beginning of the simulation. This is illustrated in Figure 1 where we show a density slice through the y-z plane. The inflow has a constant density of $3.92 \times 10^{-21}$ g cm$^{-3}$ and a constant velocity. Thus, it leads to a build-up of material in form of a filament with a radius that grows over time as it is not restricted by gravity. Consequently, one has to ensure a big enough box that an equilibrium can be established. The surrounding cells around the inflow zone are given a constant density with the same value as the inflow zone and pressure and do not affect the simulation.

The complete box is set to be isothermal with a temperature of 10 K and a molecular weight of $\mu = 2.36$. In general, the boxsize is 0.8 pc For the control runs with a higher temperature the boxsize is doubled to ensure enough space to reach a velocity dispersion equilibrium.

As the inflow initially leads to a thin, compressed central filament with a high density, we employ adaptive mesh refinement (AMR) to resolve high density regions while keeping the resolution low in the rest of the box. We enforce a minimum refinement in the whole box to ensure a resolution of at least 256$^3$. The filament gas is resolved up to a resolution of 512$^3$, which is enough to fulfill the Truelove criterion for the maximum occurring density within a factor of 16 in all simulations (Truelove et al. 1997) except for the Mach 10.0 case. For a short time the maximum density reaches values that are within a factor of 8 of the Truelove criterion but later on decreases quickly to values that are at least a factor of 16 of the Truelove criterion. We also test the same case with a lower resolution of a factor of 2 and see no difference in the behavior.

### 4 SIMULATIONS

In this section we present the outcome of our simulations. In order to measure the velocity dispersion and the mass of the filament one has to distinguish between filament material and ambient medium. As the inflowing material is shocked at the filament surface there is a clear increase in density and a clear drop in radial velocity. As the internal density of the filament decreases over time due to Equation 9, we use the radial velocity to distinguish inflowing material from filament material instead of a density threshold. To measure the filament radius we use the position of the highest density gradient which traces the general shock position.

#### 4.1 Initial perturbation

As a reference test case we set up a converging flow with a velocity of Mach 5.0 without any perturbation of the initial density field. As material streams in it is compressed on the central axis of the box. In order to avoid unphysical densities in the initial phase of this case we add an already
Figure 2. Projection of a test case of a filament without initial perturbation on the left compared to one including an initial perturbation on the right at 1.0 Myr. Both cases have an inflow of Mach 5.0. The horizontal dashed lines indicate the analytical prediction for the radius. Turbulence leads to a mean radius which lies above the radius predicted for thermal pressure.

Figure 3. Evolution of the generated velocity dispersion for the reference case without an initial density perturbation. Volume weighted values are given in cyan and density weighted values in orange. In both cases the longitudinal velocity dispersion is zero and as no motions are generated along the filament the velocity dispersion is dominated by radial motions together with a small contribution of angular motions. After a short settling phase an equilibrium is established where the value of the velocity dispersion is constant.

Figure 4. Evolution of the mass accretion rate (blue) and the filament radius (red) for the reference case without an initial density perturbation. The analytical predictions are given by the light blue and orange dashed lines respectively. The mass accretion rate is somewhat higher and the radius consequently evolves somewhat faster than predicted. Correcting both by taking into account a radial turbulent pressure term gives a much better correspondence as shown by the dashed-dotted lines.

The existing filament to the initial conditions with a radius of 0.05 times the boxlength and with a constant density of ten times the ambient density corresponding to a total mass of 0.23 M_☉. Note that we do not need to include an initial filament for the more realistic simulations shown later with includes an initial perturbation as the generated turbulence prevents the density from reaching values which cannot be resolved. If Equation 16 holds we expect that some of the accreted kinetic energy is converted into turbulent motions. However, the first impression of the density projection on the left hand side of Figure 2 does not exhibit any indication of turbulence being present. The motions are purely angular and radial and the little substructure which is generated is washed out due to the projection. This impression is confirmed by the velocity dispersion evolution in Figure 3 where we show the total three-dimensional velocity dispersion as well as its components. The volume weighted velocity dispersion is defined as the standard deviation of the spatial velocity distribution. We calculate the total by taking the square root of the sum of the variances of the components. For the density weighted velocity dispersion we normalise
the velocities with the density of its respective cell divided by the average density before calculating the variance. Note that we use the box centre on the filament axis to define the centre of the filament. As we use a cartesian grid code this can lead to a wrong split-up in the cylindrical components if the filament axis lies not exactly in the centre of the box. We analysed the error in the total velocity dispersion using a cartesian and a cylindrical calculation and conclude that the error is negligible and at most of one per cent. As one can see in Figure 3, an equilibrium is established after about a Myr where the velocity dispersion becomes almost constant. We observe this behavior in all our simulations and it is also found in turbulent smooth-particle-hydrodynamical simulations of filaments forming in a turbulent medium (Clarke et al. 2017). In our reference case only motions in radial and angular directions are generated. Motions along the filament are completely missing. Therefore, we see substantially less substructure.

We also evaluate the radial evolution and mass accretion rate of the filament in order to verify if Equation 7 and Equation 10 hold. In Figure 4 we show both of them together with the analytical expectations. The filament radius \( R(t) \) closely follows what we would expect but grows slightly faster. Moreover, the measured mass accretion rate has a small but existing constant offset to the analytical value. Both effects can be explained by the contribution of the internal turbulence. The turbulent motions act as an additional pressure increasing the size of the filament. The enhanced growth of the filament radius against the inflow slightly increases the gas accretion rate. Correcting the velocity for the radial expansion given by Equation 10 as \( 2c^2_s/v_r \) by including the contribution of the density weighted radial turbulence as \( 2(c^2_r + \sigma^2_r)/v_r \) and adding this term to the inflow velocity in the mass accretion rate Equation 7, gives a much better prediction for the radial growth and a mass accretion rate which lies exactly on the measured value (dashed-dotted lines in Figure 4). At about 4.6 Myr the filament reaches the limits of the box and its maximum numerically resolved extent. This means that mass cannot be effectively accreted as seen in the mass accretion rate which leads to a decay in velocity dispersion seen in later evolution plots.

The simulation so far is academic as the interstellar medium is not completely smooth and will always contain density fluctuations. If one includes such density perturbations the visual impression of the generated turbulence changes completely as can be seen on the right hand side of Figure 2. As the symmetry in x-direction is now broken, longitudinal motions are generated. This leads to a considerable amount of substructure which resemble much more observed filaments. We show a detailed analysis in the next subsections where we present the cases of an inflow velocity generating a super- and subsonic internal velocity dispersion. In order to validate if the generated velocity dispersion depends on the initial density perturbation we vary the initial density perturbation in strength and form. Using a flat, "white noise" perturbation on the one hand and a Gaussian perturbation on the other shows no quantitative difference in magnitude of the intrinsic filament velocity dispersion. Varying the perturbation strength also shows no dependence on the initial perturbation. In Figure 5 we show that even changing the perturbation amplitude over five orders of magnitude the resulting value of velocity dispersion varies only minimally. This leads to the following conclusion: As long as one includes a symmetry breaking perturbation the amount of generated turbulence is robust. In addition this experiment demonstrates that including perturbations it is vital to break the symmetry of the simulation.

4.2 Supersonic turbulence

In Figure 6 we plot the evolution of a Mach 5.0 inflow with an initial density perturbation. The observable structure is in a stark contrast to the reference case missing the initial perturbation. The filament shows every indication of turbulent motions rearranging material constantly. The bubbling and sloshing in the filament forms temporary ridges and overdensities, the most prominent on the central line at the beginning of the simulation. Over time the central overdensity weakens as the lack of gravity lets the material to spread freely. Nevertheless, the visual impression is that of an observable filament albeit the filament broadens to an unrealistic width due to the neglect of gravity. Similar to the reference case the velocity dispersion settles to an equilibrium as shown in Figure 7. The equilibrium level is considerably higher as in the case without perturbations. It is in the supersonic regime and dominated by the radial velocity dispersion. Interestingly, the longitudinal and angular velocity dispersions, both volume and density weighted, settle to the same level of about half the sound speed. Furthermore, their values are at about 2/3 of the radial density weighted and about half the volume weighted three-dimensional velocity dispersion. This relation between the radial velocity dispersion and the other components remains robust and true for all cases of generated velocity dispersion, not only in the supersonic case. As the filament reaches the boundary of the box earlier than the reference case, the velocity dispersion decays after 3.0 Myr.

In Figure 12 we show the volume weighted probability density function (PDF). As expected for fully develop-
**Figure 6.** Density cut through the centre of the filament with a Mach 5.0 inflow. The velocities in the plane are given by the arrows with the lengths showing the log-scaled magnitude. The filament shows indications of ongoing turbulent motions.

**Figure 7.** Evolution of the generated velocity dispersion of a Mach 5.0 inflow with an initial density perturbation. The volume weighted values are given by the cyan lines, the density weighted values by the orange lines. As in the reference case, an equilibrium is established after two Myr. The dotted vertical line shows the timestep when the filament radius reaches the domain boundary.

**Figure 8.** The kinetic energy power spectrum of the filament with Mach 5.0 inflow at different times. The black dashed-dotted line shows the expectation from Kolmogorov’s theory and the black dotted line the expectation of Burger’s turbulence. The vertical dashed lines show the respective filament radius and the vertical dotted lines the filament diameter.
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Figure 9. Density cut through the centre of the filament with a Mach 3.0 inflow. The velocities are again given by log-scaled arrows. Only the surface of the filament is mildly perturbed.

Figure 10. Evolution of the velocity dispersion of a Mach 3.0 inflow. As in the case of a Mach 5.0 inflow an equilibrium is established where the velocity dispersion is quasi constant. After the filament reaches the domain boundary (dotted vertical line) the velocity dispersion decays.

Figure 11. The kinetic energy power spectrum of a filament with Mach 3.0 inflow at different times.

oped, supersonic turbulence it follows a log-normal distribution (Vazquez-Semadeni 1994; Padoan et al. 1997; Passot & Vázquez-Semadeni 1998). The width of the distribution
is correlated to the Mach number as
\[
\sigma_s^2 = \ln \left( 1 + b^2 M^2 \right). \tag{18}
\]
The parameter \( b \) depends on the ratio of solenoidal to compressional driving of the turbulent motions, varying smoothly from \( b = 1/3 \) for purely solenoidal to \( b = 1 \) for purely compressive driving with a value of \( b \approx 0.4 \) for a natural mix of modes \( F_{\text{comp}}/(F_{\text{sol}} + F_{\text{comp}}) = 1/3 \) (Federrath et al. 2008, 2010). All of our simulations with supersonic turbulence show about the same value of \( b \approx 0.5 \) but there is also some variation over time. We also show the build-up of the kinetic energy power spectrum of the filament in Figure 8. As the filament grows, the maximum of the distribution shifts to bigger scales. It always follows closely the filament radius, given by the dashed vertical lines. Thus, the radius is the scale for the driving mechanism. We also show the predicted power spectrum for supersonic Burger’s turbulence with a power law of \(-2\) as well as the expectation of Kolmogorov’s theory (Kolmogorov 1941) which predicts a decay of subsonic turbulence to smaller scales with a power law of \(-5/3\). As one can see, the geometrical form of the filament limits the power on large scales. Nevertheless, the power spectrum follows the expectation quite well on intermediate scales. However, it is impossible to distinguish between supersonic and Kolmogorov decay as both of them lie too close together.

### 4.3 Subsonic turbulence

In contrast to the turbulent motions of the Mach 5.0 inflow, a slower inflow velocity is only capable of generating subsonic turbulent motions despite itself being supersonic. In Figure 9 we show the visual impression of a Mach 3.0 inflow. It is not strong enough to generate substructure inside the filament. Only the surface is mildly perturbed. The lack of internal motions can also be seen in the velocity dispersion evolution in Figure 10. At all times it is below the sonic line. Nevertheless, it again reaches an equilibrium after about 2.0 Myr which decays after reaching the size of the box at about 3.4 Myr. We also show the kinetic energy power spectrum for the subsonic turbulent case in Figure 11. The power spectrum shows a similar behavior to the supersonic case and again the maximum of the cascade, the indicator of the driving scale, corresponds to the filament radius. As in the supersonic case, it is not easy to distinguish between a Kolmogorov and an \( k^{-2} \) cascade but at later times the power law seems more similar to the former one which is to be expected for subsonic turbulence. In contrast to the supersonic case, we see a strong lack of compressional modes in the split-up of the power spectrum. This is also confirmed by Equation 18 where we get a value of \( b = 1/3 \).

### 4.4 Dependence on inflow velocity

The crucial question of accretion driven turbulence is how much internal velocity dispersion in a filament is generated in dependence of the inflow velocity. Therefore, we repeat the simulation for inflow velocities of Mach number 1.5, 2.0, 2.15, 2.25, 2.5, 3.0, 3.5, 4.0, 5.0, 6.0, 7.5 and 10.0. We show the resulting values of the velocity dispersion are in Figure 13 where we plot them against their inflow velocity. The errorbars are given by the inherent variance of the velocity dispersion for different initial seeds of the perturbation spectrum and is about five per cent of the measured turbulence. In Figure 14 we also show the dependence of the Mach number squared on the inflow Mach number squared as this is the measure of the turbulent and inflowing energy. Both the accreted kinetic energy and the turbulent energy depend on the total accreted mass of the filament. Nevertheless, one does not have to account for mass as all the accreted material also has to be set into turbulent motions and thus the mass term cancels from both energy terms. This is illustrated later in Equation 24. In both figures we show the volume weighted velocity dispersion on the left hand side and the density weighted velocity dispersion on the right hand side. One can see that the measurement method has a major impact on the results. First, we look at the volume weighted velocity dispersion in Figure 13. We observe three distinct regimes: A high inflow velocity (light blue) leads to super sonic turbulence, an intermediate inflow velocity (medium blue) generates subsonic turbulence and a low inflow velocity (dark blue) results in nearly no turbulence. All of these regimes can be well fitted by linear relationships which do not necessarily go through the zero point. The fitting parameters are always given in the legend. Striking is the break at the sonic line (dashed horizontal line) going from the intermediate subsonic regime to the high velocity supersonic regime. This break does not appear in the density weighted velocity dispersion on the right hand side of Figure 13. Now the high and medium inflow velocity values follow one and the same relation. Thus, we now can fit the data by one single linear relationship that connects both regimes and that is shown by the dashed orange line with the parameters:
\[
\sigma = 0.29 v_T - 0.45 c_s. \tag{19}
\]

The fact that we can fit a single line from the high supersonic end down to low subsonic turbulent velocities shows that there is no difference in the physics at work in both regimes. The break in the volume weighted relation also occurs for the squared volume weighted velocity in Figure 14. Although the slope stays roughly the same there is a gap
between the supersonic and the subsonic regime. Contrarily, the density weighted velocity dispersion squared shows a smooth transition from nearly no turbulence to a constant slope. The density weighted velocity dispersion is also a measurement of the kinetic energy in the turbulence of the filament, a fact that we have confirmed by calculating the kinetic energy. The break in the volume weighted velocity dispersion results from the fact that the nature of the velocity and density distribution changes from the subsonic to the supersonic regime. Supersonic turbulence is shock dominated and due to compression most of the kinetic energy is in high densities. The information about the high densities is lost if one only measures the volume weighted velocity dispersion.

In the right panel of Figure 14 we also overplot Equation 19. Despite being a linear correlation between velocity dispersion and infall speed it fits the numerical results remarkably well and deviates only slightly for very low Mach numbers. We also attempt to fit a parabola to all of the density weighted data but are not able to get a good match. Therefore, we conclude that the linear relation of Equation 19 provides the best analytical description of the underlying physics. We stress that a linear relation is in a stark contrast to the prediction of Heitsch (2013) with a power law of 1/3rd, given by Equation 16. We discuss the implication of this in section 7.

We repeat a subset of the simulations for a higher temperature of 40 and 100 K and get similar results, all lying on the analytical relation given by Equation 19. Thus our results are independent of the temperature. We also repeat a
subset of the simulations with an additional longitudinal velocity component in the inflow. The result in velocity dispersion is unchanged and the only difference is that the whole filament now begins to move in x-direction with constant velocity.

5 PRESSURE EQUILIBRIUM

In order to see the impact of turbulence on the pressure we analyse its components as function of radius and time. In Figure 15 we show the different contributions to the pressure calculated in radial bins for the Mach 5.0 case with an initial density perturbation. We calculate the velocity dispersion, the average density and average thermal pressure in each slice along the filament and in radial bins which are chosen to be four cells wide. We use these values to calculate the respective pressure terms and take the mean along the filament in order to determine an average value for the whole filament. There is an overlap region where filament gas and environment gas mix due to the filament not being completely straight and round. Therefore, we use only the filament gas to calculate the velocity dispersion and only the environment gas to calculate the ram pressure and plot them both simultaneously. We determine the turbulent pressure component with the density weighted velocity dispersion. Thus, the turbulent pressure is given by

\[ P_{\text{turb}} = \langle \rho \rangle \sigma^2, \]

where the braket notation represents the expectation value. The average ram pressure is calculated by the average density and radial velocity as:

\[ P_{\text{ram}} = \langle \rho \rangle (v_r)^2 \]

One can see in Figure 15 that outside of the filament the turbulent pressure (dashed-dotted line) is zero and the thermal pressure (dashed line) is negligible. As material streams towards the filament the ram-pressure (dotted line) increases as the density increases. When the accretion flow reaches the filament the ram-pressure breaks down and some of the energy is converted into turbulent motions giving rise to a turbulent pressure component. In the overlap region of filament and environment gas we see both a contribution of turbulent pressure and ram pressure due to our split-up of components. As one can see the average pressure inside the filament given by the average thermal pressure together with the average turbulent pressure (solid line) has nearly the same value as the average ram pressure. The filament is compressed and restrained by the accretion flow. Before the filament has settled there is quite an overpressure in the centre of the filament. As the filament grows outward, it adjusts to the outside ram-pressure and a constant pressure inside the filament is established.

6 OBSERVABLE VELOCITY DISPERSION

In order to compare our simulations to observations we derive the line-of-sight velocity by sending a ray through the computational domain. We vary the inclination of the filament and treat every crossed volume element with its respective density as an discrete emitter of a line-of-sight velocity.
value. These are converted to density weighted Gaussian line profiles with a dispersion width corresponding to the thermal linewidth of \( \text{C}^{18}\text{O} \) with a given value of \( \sigma = 0.0526 \) km s\(^{-1}\). We bin the resulting line profiles into histograms with a bin width of 0.05 km s\(^{-1}\) to get a complete line emission for each observed spatial pixel. We then measure the velocity centroid and the linewidth as an observer would do by fitting a Gaussian to the complete line. In order to get the non-thermal velocity dispersion we have to subtract the contribution of the thermal gas motions of the full width half maximum (FWHM) of the line as e.g. Myers (1983):

\[
\sigma_{\text{NT}} = \sqrt{\frac{\text{FWHM}^2}{8 \ln 2} - \frac{k_B T}{m}}
\]  

We show the non-thermal linewidth along a filament through its centre in Figure 16 for the case of subsonic turbulence forming due to a Mach 3.0 and for the supersonic turbulent case due to a Mach 5.0 inflow in Figure 17 both including an initial perturbation. At a first glance it is important to note the striking similarity to real observations (Hacar & Tafalla 2011; Hacar et al. 2013; Tafalla & Hacar 2015; Hacar et al. 2016). As in the actual measurements we see an inherent spatial variation due to the non-homogeneous nature of the turbulence. The variation is also not stationary but changes with time, mimicking the sloshing motions of the total filament. However, the mean of the data distribution stays relatively stable, only varying by about 10%.

Figure 16. The measured non-thermal linewidth along the central axis of the filament for the case with a Mach 3.0 inflow for different inclinations. Data points are given by the blue squares and their mean by the blue dotted line. The solid, dashed and dashed-dotted black lines show the total, the radial and the longitudinal density weighted velocity dispersion respectively.

Figure 17. The same as in Figure 16 but for an inflow of Mach 5.0.

Figure 18. The same as in Figure 16 but for an inflow of Mach 10.0.
Comparing the mean to the analytical value of the velocity dispersion it becomes clear that an observer cannot see the full picture of the internal motions of the filament. The observer will preferentially see the density weighted radial velocity dispersion (dashed line) as filament are more likely to be detected for small inclinations. Thus, the line-of-sight is parallel to the radial motions. While the subsonic case shows mean values which lie below any velocity dispersion typically observed, it is also the only one where the mean of the observed distribution lies above the expected value of the radial velocity dispersion. Going to higher inflow speeds, the mean of the observed distribution generally lies below the expected value. The situation becomes even worse for cases of high turbulence as shown in 18 where the discrepancy can be even as large as a factor of two. Moreover, the discrepancy between the observed mean and the total velocity dispersion is even larger. Despite having a supersonic total density weighted velocity dispersion, most of the data points of the Mach 5 inflow show a subsonic or at best a transonic non-thermal line width. This means that the interpretation of the inherent motions in an observed filament can be severely flawed. A filament dominated by supersonic motions will be classified as only having subsonic motions. 

With the addition of gravity the problem will become even more severe. The collapse of the filament or its condensation into collapsing clumps. The formation of turbulent filament, including self-gravity will be discussed in a subsequent paper. We find that there is a linear dependency of the density weighted velocity dispersion on the inflow velocity. Below Mach 2.0 the relationship flattens as shown in the right panels of Figure 13 and Figure 14. Why there is a break in the relation is not completely clear. One possibility is that the low inflow velocity region is an artifact of numerical noise. In this case it must be dominated by perturbations on small scales. Therefore, we smooth our data to remove the small scale signal and recalculate the velocity dispersion. The result shows that the velocity dispersion is unchanged. Therefore, the measured velocity dispersion is in the large scale structure. Even for a high degree of smoothing the velocity dispersion remains the same. Thus, there must be a physical reason for the break in the slope. An explanation could be that filaments have a bottom level of minimal velocity dispersion comparable to a basic eigenmode which is exited by the constant inflow. In the subsonic regime sound waves are supposed to be very inefficient in dissipating energy and indeed if one sets up a filament with a random velocity perturbation there is a level of velocity dispersion below which the kinetic energy does not decay. Its dependence on form and density of the filament is out of the scope of this paper and will be discussed elsewhere.

We now want to focus the form of the linear fit. As shown in Figure 13 we measure a linear relationship of $\sigma = 0.29 v_r - 0.45 c_s$.

It is possible to show that this is equivalent to an energy balance. In order to get a kinetic energy we take the square of the equation and multiply it with half of the mass accretion rate which gives

$$\frac{1}{2} M c_s^2 = 0.09 \cdot \frac{1}{2} M v_r^2 - 0.13 M c_s v_r + 0.1 M c_s^2$$

with the restriction that $v_r \geq 1.55 c_s$.

For a constant velocity dispersion the term on the left hand side is the change in turbulent energy and the first term on the right hand side the rate of accreted kinetic energy. To show the meaning of the other terms we use the equation of change in turbulent energy given by

$$\dot{E}_t = a \dot{E}_a - \dot{E}_d + \dot{E}_0$$

where $\dot{E}_t$ is the change in turbulent energy, $\dot{E}_a$ is the kinetic energy accretion rate, $\dot{E}_d$ is a constant energy change and $\dot{E}_d$ is the energy dissipation rate. We also consider that energy is lost in the isothermal accretion shock and is radiated away. As the fraction of the lost energy should be roughly constant per time we introduce a constant efficiency $\alpha$ which has a value between zero and one. The similarity with Equation 26 is striking and leads to an accretion efficiency of

$$\alpha = 0.09.$$ 

The constant energy rate is given by

$$\dot{E}_0 = 0.1 M c_s^2$$

and the energy lost due to dissipation is

$$\dot{E}_d = 0.13 M c_s v_r.$$ 

The latter can be explained by taking the definition of the dissipation rate in Equation 14, assuming that the dissipation scale is a factor $l$ times the filament radius and that the energy dissipates with the sound speed. Therefore,

$$\dot{E}_d = \frac{1}{2} \frac{M c_s^3}{l R} = \frac{1}{4} \cdot \frac{M c_s v_r}{l t} = \frac{1}{4 \alpha} M c_s v_r$$

7 DISCUSSION AND CONCLUSIONS

We have presented a numerical study on accretion driven turbulence in filaments. The focus of this paper was to analyze the dependence of the velocity dispersion on the inflow velocity. We deliberately neglected the effects of gravity in order to have a constant inflow velocity and to be able to follow the driving of turbulence long enough without the radial collapse of the filament or its condensation into collapsing clumps. The formation of turbulent filament, including self-gravity will be discussed in a subsequent paper. We find that there is a linear dependency of the density weighted velocity dispersion on the inflow velocity. Below Mach 2.0 the relationship flattens as shown in the right panels of Figure 13 and Figure 14. Why there is a break in the relation is not completely clear. One possibility is that the low inflow velocity region is an artifact of numerical noise. In this case it must be dominated by perturbations on small scales. Therefore, we smooth our data to remove the small scale signal and recalculate the velocity dispersion. The result shows that the velocity dispersion is unchanged. Therefore, the measured velocity dispersion is in the large scale structure. Even for a high degree of smoothing the velocity dispersion remains the same. Thus, there must be a physical reason for the break in the slope. An explanation could be that filaments have a bottom level of minimal velocity dispersion comparable to a basic eigenmode which is exited by the constant inflow. In the subsonic regime sound waves are supposed to be very inefficient in dissipating energy and indeed if one sets up a filament with a random velocity perturbation there is a level of velocity dispersion below which the kinetic energy does not decay. Its dependence on form and density of the filament is out of the scope of this paper and will be discussed elsewhere.
This means that the dissipation scale is \( l = (4 \cdot 0.13)^{-1} \approx 1.92 \) times the radius. Thus, energy is dissipated on the scale of the diameter of the filament in contrast to the driving scale which is shown in section 4 to be the radius of the filament. The constant gain of energy is harder to explain but could be an indication of a base energy in the filament which is independent of the inflow velocity as discussed above. Nevertheless, at the lower boundary of our fit of \( v_r = 1.55 c_s \) it is of the same order as the accreted energy rate and rapidly loses influence for higher inflow velocities.

Our results can be summarized as follows:

(i) The accretion of material leads to non-isotropic, sub- and supersonic turbulence depending on the energy accretion rate as long as the symmetry is broken. The efficiency of transferred energy is independent of the accretion rate and equal to 9%.

(ii) The amount of turbulence generated is independent of the perturbation strength of the initial density field and independent on an additional velocity component parallel to the filament.

(iii) The turbulence reaches an equilibrium level which scales linearly with the accretion rate.

(iv) The filament radius grows linearly with an additional turbulent pressure from the density weighted radial velocity dispersion. It is also the driving length of the turbulence.

(v) The density weighted velocity dispersion contributes to the pressure equilibrium inside the filament.

(vi) The turbulent energy is dissipated on the scale of the filament diameter with a rate given by the sound speed.

(vii) An observer often cannot measure the true velocity dispersion and will even misinterpret the level of turbulence to be subsonic while the intrinsic velocity dispersion could still be supersonic.

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