The relation of extragalactic cosmic ray and neutrino fluxes: the logic of the upper bound debate

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Abstract. In a recent paper, Waxman and Bahcall [1] claimed that the present data on ultra-high energy cosmic rays imply a model-independent upper bound on extragalactic neutrino fluxes of $2 \times 10^{-8}$ GeV cm$^{-2}$ s$^{-1}$ sr$^{-1}$, independent of neutrino energy. Mannheim, Protheroe and Rachen [2, v2] have repeated this calculation and confirmed the WB-bound, within a factor of 2, only at a very limited energy range of $E_\nu \approx 10^{16}$–$10^{18}$ eV, while at other energies the neutrino flux is mainly limited by the extragalactic gamma ray background to a level about two orders of magnitude higher than the WB bound. In this paper we present a simple, (almost) no-math approach to the problem, and discuss under which astrophysical assumptions the WB-bound and the MPR-bound, respectively, apply. Then we discuss to which respect these assumptions apply to presently discussed models of extragalactic neutrino production. We note that, averaged over the observed luminosity function, blazars are sufficiently opaque to ultra-high energy neutrons that there is no conflict of the predicted neutrino fluxes with the cosmic ray data, and that these models are rather constrained by their contributions to the extragalactic gamma ray background. At present, no modifications are implied to the predicted neutrino events from these models in active or planned neutrino detectors.

1. Introduction

The most commonly assumed mechanism for the origin of cosmic rays is the Fermi acceleration of protons and ions at strong shock waves in magnetized plasmas. The same mechanism also accelerates electrons, which then emit their energy in synchrotron radiation with the magnetic field, and is therefore probably responsible for most of the non-thermal radiation in the universe. This has motivated theorists to search for the possible sources of cosmic rays up to the highest energies among the known objects which emit non-thermal radiation. A large fraction of the non-thermal power in the universe is provided by Active Galactic Nuclei (AGN), but also the enigmatic Gamma Ray Bursts, the most powerful explosive events in the universe, have obtained some attention as possible sources of energetic protons, i.e. cosmic rays. If cosmic rays exist in these sources, they can interact with the dense ambient photon field, producing secondary mesons (mostly pions), which decay and give rise to the emission of very high energy gamma-rays and neutrinos. Moreover, such interactions
can convert protons into neutrons, which are released from the magnetic confinement and can be emitted as cosmic rays.

The tight connection between cosmic ray, gamma ray and neutrino production has lead to some attempts to constrain the possible fluxes of one component by observations of the others. This is particularly important for the neutrino fluxes, for which large experiments are currently under construction. Its exploration can be expected to be one of the primary goals of the astrophysics in the next century.

2. Extragalactic cosmic rays, gamma rays and neutrinos

The origin of very high energy (>100 TeV) neutrinos in the universe is generally attributed to the decay of pions or other mesons (we discuss pions only for simplicity). Pions exist in three isospin states, \(\pi^+\), \(\pi^-\) and \(\pi^0\), which are in the gross average equally likely to be produced. The decay of charged pions leads to the production of neutrinos and charged leptons, while neutral pions decay electromagnetically into photons. Under the assumption that electrons deposit their energy into photons by radiative processes, equipartition of pion flavors leads immediately to an approximately equal luminosity in gamma rays and neutrinos from any pion-producing source.

\[
[\gamma-\text{luminosity}]^{\text{(cascade)}} \approx [\nu-\text{luminosity}]^{(E_{\nu})}
\]

One can show that this relation holds even when other mesons (e.g. kaons) are involved.

The production of pions usually involves the presence or the production of cosmic rays. In most models, pions are produced in \(pp\) and \(p\gamma\) interactions involving a pre-existing cosmic ray population in the source. To specify how cosmic rays are produced, most scenarios assume Fermi acceleration, which requires that cosmic ray protons are magnetically confined in the source. However, confinement breaks up when a proton interacts and changes into a neutron (isospin-flip), which can be ejected from the source. Charge conservation requires the production of at least one charged meson in this interaction, and therefore inevitably leads to the production of neutrinos. This can be expressed in the relation

\[
[\nu-\text{luminosity}]^{(E_{\nu})} = \langle \text{kinematics} \rangle [n - \text{luminosity}]^{(E_n = f E_{\nu})}
\]

For both \(pp\) and \(p\gamma\) interactions, the factor \(\langle \text{kinematics} \rangle\) is in the range \(0.2-1\), depending on the mean CMF interaction energy \(\mathbb{1} \mathbb{2}\). In some exotic models that involve the decay of topological defects in the universe, cosmic rays and mesons are produced in the same process and their relation can be predicted from QCD models of hadronic fragmentation \(\mathbb{3}\). Also this can be expressed in form of \((**)\), but with \(\langle \text{kinematics} \rangle \gg 1\).

The above relations apply to the emission process only. To establish the corresponding relation for the observable luminosities of an astrophysical source, we have to take into account opacity factors that may reduce the ejected power in photons and neutrons through interactions with ambient matter or photons. No such modification applies to high-energy neutrinos in the cases of interest here. The interaction of neutrons generally leads to a reduction of the bolometric cosmic ray luminosity, since cosmic ray energy is converted into neutrinos or electromagnetic radiation. Of particular importance is the possibility of a neutron to flip back in such interactions into a proton, which keeps it confined over a long time and effectively
allows removing all its energy. Photons interacting with matter can convert their energy into heat and therefore reduce the energy content in non-thermal radiation. The interaction of energetic non-thermal photons with background radiation, however, does not in general lead to a reduction of the bolometric electromagnetic energy, since the $e^\pm$ pairs emit their energy at lower frequencies in synchrotron radiation or inverse-Compton scattering. Unless the cascade develops into saturated comptonization, this energy can be emitted in the MeV–GeV gamma ray regime, still having non-thermal characteristics.

To infer relations between observable fluxes, a further factor arising from the different propagation properties of gamma rays, neutrinos and cosmic rays has to be considered. Neutrinos, and gamma rays $\lesssim 100$ GeV, are not strongly affected by the presence of cosmic photon or matter backgrounds. The luminosity flux relation is therefore given by standard cosmological expressions in both cases. Cosmic rays $\gtrsim 10^{18}$ eV are affected by photohadronic pair and pion production with the local cosmic microwave background. Also at lower energies these interactions can modify the relation between cosmic ray and neutrino fluxes arriving from large redshifts. In addition, cosmic ray propagation can be influenced by the presence of magnetic fields in the source, our galaxy and on large scales.

Following this, we may write the relation of observable neutrino fluxes to observable and gamma ray and cosmic ray fluxes as

$$[\nu-\text{flux}](\text{bolometric}) < [\gamma-\text{flux}](\text{MeV–GeV})\langle\text{opacity}\rangle_\gamma \quad \text{(relation 1)}$$

and

$$[\nu-\text{flux}](E_\nu) \leq [\text{CR} - \text{flux}](E_{\text{CR}} = fE_\nu) \times \langle\text{opacity}\rangle_n \langle\text{kinematics}\rangle \langle\text{propagation}\rangle \quad \text{(relation 2)}$$

where the value of $\langle\text{propagation}\rangle$ is determined by the inverse ratio of the proton attenuation length to the Hubble radius, and by the dependence of the comoving source luminosity density on redshift. For non-evolving sources, it ranges from $\approx 1$ for $E_{\text{CR}} < 10^{17}$ eV, over $\sim 3$ at $E_{\text{CR}} = 10^{19}$ eV to $\gtrsim 100$ for $E_{\text{CR}} > 10^{20}$ eV. With source evolution on the level suggested by observations for AGN and starburst galaxies, the value at $E_{\text{CR}} \gtrsim 10^{19}$ eV is increased by about a factor of 5. The parameter $f \approx 0.01–0.05$ is determined from the interaction kinematics and depends on the CMF energy $[3, 4]$.

Relation 1 holds for the comparison of bolometric luminosities, and considers that a (maybe dominant) fraction of the non-thermal radiation of the source is not produced in photohadronic interactions, but by synchrotron-self Compton emission of co-accelerated primary electrons. The factor $\langle\text{opacity}\rangle_\gamma$ considers the fraction of energy reprocessed to energies below $\sim$ MeV by comptonization. In relation 2, which holds also for specific energies, we have considered the possibility that part of the cosmic ray ejection is due to the direct, non-adiabatic ejection of protons.

### 3. Can we impose “robust” or “model-independent” upper bounds?

Obviously, both relations 1 and 2 pose upper limits on the observable neutrino flux, if we chose proper upper limits for the parameters averaged over all contributing sources. $[\gamma-\text{flux}]$ and $[\text{CR} - \text{flux}]$ on the right hand sides are observables, so it is in principle possible to fix their values by observations. All other parameters have to be
determined by theory. We may distinguish, however, whether the theory used here is well established and supported sufficiently by observations, or whether we have to rely on weakly supported hypotheses. For example, the parameter \( \langle \text{kinematics} \rangle \) is, within narrow bounds, well known and little dependent on astrophysical model assumptions, as long we confine ourselves to neutrino production in \( p\gamma \) or \( pp \) interactions. More difficult is the situation for \( \langle \text{propagation} \rangle \) — while its value is well known for a given cosmic ray energy assuming straight-line propagation, since it only depends on well measured cross sections and the temperature of the microwave background, the influence of poorly known magnetic fields in the universe is difficult to determine. Obviously, the \( \langle \text{opacity} \rangle_\gamma \) and \( \langle \text{opacity} \rangle_n \) can hardly been constrained a priory without specifying a particular choice of sources.

As an additional complication, also the determination of \( [\gamma-\text{flux}] \) and \( [\text{CR-\text{flux}]} \) is not straightforward, since we have to distinguish the extragalactic contribution to these fluxes from the generally dominant galactic contribution. However, here we may remember the logic inherent to an “upper bound”: we do not need precise measurements for these quantities, it is sufficient to have upper limits for them. Obviously, a safe upper limit for any extragalactic flux contribution is, unless determined more precisely, given by the total observed flux. The limits determined by this minimal condition, however, may be very weak and of little practical relevance.

The extragalactic MeV – GeV gamma-ray background (hereafter EGRB) is fairly well determined. Relation 1 has therefore readily been used to normalize flux estimates for extragalactic neutrinos. A complication is only that there is little theoretical agreement in which energy range exactly the reprocessed electromagnetic cascade radiation emerges from the source. For example, normalizing to the EGRB above 1 MeV yields about one order of magnitude higher fluxes than normalizing to the EGRB above 100 MeV.

The extragalactic contribution to the cosmic ray spectrum is generally believed to dominate the total observed flux above \( 3 \times 10^{18} \) eV, where the cosmic ray spectrum shows a distinct feature, called “the ankle”. This belief is also somewhat supported by the absence of a signature of the galactic plane in the arrival direction distribution of the cosmic ray events, but it may be noted that models for a galactic halo origin of cosmic rays, which would not show such a signature, have been suggested. Another clear signature of an extragalactic origin would be the Greisen-Zatsepin-Kuzmin (GZK) cutoff expected above \( \sim 5 \times 10^{19} \) eV due to photoproduction losses of the cosmic rays at the microwave background. Unfortunately, the current experiments disagree about the presence or absence of this cutoff in the data, so that we have to wait for a better data statistics to achieve clarification.

Below the ankle, very little is known about the origin of the dominant cosmic ray component. Some experiments suggest a composition change from a dominantly heavy (iron) component to a light (proton) component at the ankle, i.e., at the same position where also the spectrum flattens. The result was recently corroborated by measurements at lower energies, which indicate an increasingly heavy composition of cosmic rays around “the knee” of the CR-spectrum at \( 10^{14} \)–\( 16 \) eV. This, and also some tentative results on a possible signature of the galactic plane in the arrival directions at \( 10^{17} \) eV, has lead to a common sense that cosmic rays below the ankle are dominantly

\[ \text{† For the convenience of a simple argument, we understand in this paper under EGRB the diffuse plus identified point source contribution to the extragalactic gamma ray background, thus the total density of ambient extragalactic gamma rays. We note that usually given experimental values for this quantities subtract extragalactic point sources.} \]
of galactic origin. Although this might be the case, and it is indeed expected from theoretical arguments, too, we have to be careful not to over-interpret these data. In particular the composition measurements at $10^{17}$ eV rely on Monte Carlo simulations using particle interaction cross section extrapolations into energy regions which are not explored by accelerators yet. It has been shown that the presence or absence of the composition change signature in different experiments is dependent on the Monte-Carlo code used for the data interpretation [7]. In summary, we may have good observational evidence that somewhere between “the knee” and “the ankle” of the cosmic ray spectrum the dominant origin changes from galactic to extragalactic. Any stronger statement on the exact shape of the extragalactic spectrum below the ankle, however, rather falls into the category of personal or public opinion. The only conservative upper limit on the extragalactic cosmic ray flux below the ankle and above the knee is therefore the measured total flux.

4. The Waxman-Bahcall upper bound

In a recent paper, Waxman and Bahcall [1] claimed that the relation between cosmic ray and neutrino fluxes (relation 2) sets a model-independent upper bound on extragalactic neutrino fluxes at all energies. This bound is about $1-2$ orders of magnitude stricter than previously assumed bounds from comparison with the EGRB (relation 1). Consequently, the authors claim to rule out most present models of neutrino production, in particular those connected to hadronic AGN models that have been generally normalized to the EGRB. As a corollary, they claim that this provides a model-independent proof that the EGRB is not completely produced by hadronic processes in AGN.

From the discussion above, this conclusion seems rather surprising, particularly regarding to the fact that their bound is energy independent in power per decade—a behavior that is not seen in the cosmic ray data, which they claim are the only pinpoint for their conclusion. It is therefore worth asking: (a) how exactly this bound was derived; (b) in which respect it is really model independent, which means that it affects any model suggested for extragalactic neutrino production in the past, present and future; (c) to which respect it really affects present models for neutrino production in AGN jets; and (d) whether it really rules out a hadronic production of the EGRB.

Indeed, Waxman and Bahcall (WB) use relation 2 to derive their bound, although this is somewhat hidden. Instead of writing down the relation of neutrino and cosmic ray mean free paths, they use a result obtained in an earlier paper for the local cosmic ray injection density at $10^{19}$ eV, where the propagation of ultra-high energy (UHE) cosmic rays was properly treated using a common transport approximation [8]. Applying then the trivial equations for cosmological neutrino transport, they derive the correct (straight-line) ⟨propagation⟩ factor used in relation 2. They also discuss properly the dependence of this factor on source evolution. Also their factor ⟨kinematics⟩ = 0.25 falls into the right range for photohadronic (or pp) neutrino production. Also, WB point out that no statement can be made on sources which are opaque to cosmic ray neutrons, which they exclude a priori from their treatment.

We may at this point summarize the assumptions that so far entered in the derivation of the WB-bound:

**Assumption 1:** Neutrinos are produced in interactions of cosmic rays with background photons or matter.
**Assumption 2:** The sources are transparent for neutrons of an energy $\sim 10^{19}$ eV.

**Assumption 3:** Cosmic rays of energy $10^{19}$ eV ejected by these sources are not affected by magnetic fields in or at the vicinity of the source, or on large scales.

While Assumption 2 is clearly stated in the paper, assumption 1 is rather implicitly understood. It is certainly justified since in fact most models of extragalactic neutrino production invoke this process. In contrast, WB devote an extensive discussion to the justification of assumption 3, where they show that (a) neutrons of this energy ($10^{19}$ eV) escape from most known strong field regions around putative neutrino sources before undergoing $\beta$-decay, and (b) protons of this energy cannot be confined in large scale fields (i.e. clusters of galaxies or superclusters) for time scales comparable to the Hubble time. From this, they conclude that magnetic fields cannot lead to inhomogeneities in the universal distribution of cosmic rays, which can be easily seen to be equivalent to the straight-line propagation assumption. Thus, according to WB, assumption 3 can be derived from our present observational upper limits on extragalactic magnetic fields. The authors neglect, however, that particles moving diffusively through an adiabatically decreasing magnetic field suffer energy losses due to expansion work towards the outer medium. We return to this issue below.

Under assumptions 1-3, the bound is only valid at one energy of the spectrum: it corresponds to a cosmic ray energy of $10^{19}$ eV, or a neutrino energy of $\sim 3 \times 10^{17}$ eV assuming standard kinematical relations. To extend it to other energies, WB introduce

**Assumption 4:** The overall cosmic ray injection spectrum in the universe has the spectral shape $dN/dE \propto E^{-2}$, and extends without break up to $10^{19}$ eV and beyond.

To support this assumption, WB refer to the theory of diffusive shock acceleration, which canonically predicts an $E^{-2}$ power law spectrum for particle acceleration at strong, non-relativistic shocks. They give no reason why the spectrum should extend to $10^{19}$ eV for all sources in the universe.

With assumption 4, the spectral shape of the WB bound becomes obvious: since the factor (kinematics) is only weakly dependent on energy (see Mücke et al. [4], these proceedings, for a more detailed discussion), the assumed flat (= constant power per decade) cosmic ray spectrum produces a flat neutrino spectrum. The WB upper bound is therefore the result of fitting a model spectrum to the observed cosmic ray flux at $10^{19}$ eV.

### 5. Critique of the Waxman-Bahcall bound as a general upper limit

Obviously, Waxman and Bahcall made no “mistake” in deriving their bound — for extragalactic neutrino sources which comply with assumptions 1-4, it is indeed a valid upper limit for the observable flux, based on observations. The question we have to ask is whether this justifies the claim of “model-independence”: is it in fact reasonable to believe that assumptions 1-4 are all of general validity for any possible neutrino source?

Assumption 1 is certainly the one that can most easily be accepted; it is the only mechanism predicting high-energy neutrinos so far which is not entirely speculative. Nevertheless, we may remind the reader that other models have been suggested. Such models, which are based on string hadronization (the so-called “topological defect”
models for cosmic ray origin [3], predict for the given cosmic ray flux a neutrino flux about two orders of magnitude larger than the WB bound. This is based on a relation similar to our relation 2, but with \( \langle \text{kinematics} \rangle \sim 100 \), rather than \( 0.25 \) as used by WB. It has been shown that these models are strongly constrained by the more general relation 1, i.e. the requirement not to over-produce the EGRB [4].

Regarding assumption 2, we may just follow Waxman and Bahcall and restrict our consideration a priori to sources fulfilling it. However, we disagree with WB in stating that such sources cannot be identified or constrained by any other emission than neutrinos. For a large subclass of them, i.e., those who emit gamma rays in the MeV–GeV (but not up to the TeV) regime, non-thermal gamma emission produced from \( \pi^0 \)-induced unsaturated synchrotron-pair cascades can emerge from the source. As correctly noted by Waxman and Bahcall, there is a strict connection between \( \gamma \gamma \) and \( n\gamma \) opacity [1, 2]. While sources transparent to TeV gamma rays can be shown to be transparent to UHE neutrons, too, one can use the same relation to show that sources which show an opacity break in the gamma ray spectrum at a few GeV must be opaque to neutrons at \( 10^{19} \) eV. For example, this is the case for most high luminosity gamma ray blazars. In fact, the observed non-thermal gamma-ray emission from such blazars did motivate the assumption that they are strong neutrino sources, and at the same time it restricts the maximum neutrino flux from such sources by relation 1 to a level about two orders of magnitude above the WB bound. We discuss below in which respect the WB bound still affects the expected neutrino fluxes from blazars.

Before discussing assumption 3, we turn to assumption 4, which is certainly the one with the weakest observational support. In fact, even the theoretical support WB give has to be questioned. For the large variety of shocks of different speeds and compression ratios in astrophysical sources, shock acceleration theory predicts a large range of spectral indices; the common value of 2 is just as a canonical assumption, applying to non-relativistic, strong shocks. Even more questionable is the assumption that in all sources the spectrum extends to \( 10^{19} \) eV. Obviously, even if all accelerators would have a spectral index of 2, the contribution of sources with cutoffs below \( 10^{19} \) eV can locally produce an overall spectrum much steeper. This would allow higher associated neutrino fluxes at energies below \( 10^{17} \) eV, without being in conflict with the cosmic ray data at \( 10^{19} \) eV where only a few sources contribute. Obviously, the existence of Fermi accelerating sources with a proton maximum energy below \( 10^{19} \) eV cannot be ruled out from first principles. Rather, it is suggested by a consistent interpretation of gamma ray observations of blazars within the hadronic model.

Without assumption 4, which allows to restrict the comparison of neutrino and cosmic ray transport properties to the energy \( 10^{19} \) eV, the validity of assumption 3 has to be revised. Taking the results of WB for confinement in cores of rich galaxy clusters and large scale supergalactic filaments, we obtain confinement over a Hubble time for protons below \( 10^{18} \) eV and \( 10^{16} \) eV, respectively. We could apply the same calculation to galactic halos of magnetic field strength \( 1 \) \( \mu \)G on a variation scale of \( 10 \) kpc, extending to \( \sim 300 \) kpc, and again obtain confinement for cosmic rays below \( 10^{16} \) eV. At the same energy, we can also expect the halo of our own galaxy to modify the incoming extragalactic proton flux, similar to the modifications of the solar wind observed in cosmic rays around 1 GeV. Since \( 10^{16} \) eV protons are connected to the production of \( \sim 300 \) TeV neutrinos, it would be unreasonable to propose an upper bound on their extragalactic flux based on cosmic ray observations directly related to their energy, regardless what the limits on the extragalactic contribution to the observed cosmic ray flux at \( \sim 10^{16} \) eV are.
Moreover, protons which migrate through a gradually decreasing magnetic field, with a gyro-radius much smaller than the scale over which the field changes, lose energy towards adiabatic expansion if there is some interaction with the cold plasma which allows them to keep an isotropic distribution in the rest frame of the bulk flow. The detailed energy loss depends on the exact field configurations, but for the simplest case of a largely chaotic field, the energy loss follows the same rule as the adiabatic expansion of a relativistic gas, i.e. $E_{\text{CR}} \propto R^{-1}$. If this applies to the putative outflows in galactic halos (galactic winds), and if we assume that most neutrino sources reside in galaxies having winds, then the cosmic ray bound can be relaxed by one order of magnitude or more for neutrino energies below $10^{17}$ eV. A similar effect can be obtained by the likely assumption that a considerable fraction of neutrino sources reside in galaxies belonging to more or less dense clusters or groups with stronger-than-average magnetic fields between galaxies, leading to a (partial) large scale confinement of cosmic rays below $10^{18}$ eV.

We note, in agreement with Waxman and Bahcall, that none of the above effects can strongly influence the propagation of cosmic rays at $\gtrsim 10^{19}$ eV. One reason for this is, that neutrons of this energy jump out of the confinement of most of the structures we discuss before undergoing $\beta$-decay. Our critique is rather directed against the connection of the justification of assumption 3 on the validity of assumption 4. Dropping assumption 4, i.e. the application of a model spectrum, the influence of magnetic fields can no longer be neglected. On the basis of relation 2, together with assumptions 1 and 2, we have therefore derived a neutrino upper bound which is truly based on the observed cosmic ray flux $[^3]$. We have also discussed the possible influence of magnetic fields on this bound. The result is that, as expected, we confirm the WB bound for a neutrino energy of $\sim 3 \times 10^{17}$ eV, but find much less restrictive limits at lower energies. At neutrino energies below about $10^{15}$ eV, the flux is only limited by the EGRB, regardless of the choice of parameters.

Both, the cosmic ray data we use and the magnetic fields we assume, suffer from difficulties in the interpretation of the data, and can therefore be disputed. However, at this point we may remind again in the logical meaning of the term “upper bound”: to derive a true upper bound, we can only use the observational upper limits on both the extragalactic cosmic ray flux and the magnetic fields connected to extragalactic sources and large scales. Everything else would not comply with the standards of a reliable scientific result. It is needless to say that we do not want to propose neutrino fluxes of this strength — we only state that they cannot be ruled out by general theoretical arguments and current observations. It is also worth to note that the qualitative feature of our result, i.e., that our bound is nowhere as strict as at $3 \times 10^{19}$ eV, is independent whether we use or don’t use the cosmic ray composition data, or whether we assume or don’t assume an effect of magnetic fields.

At last, we may also have a look on neutrino energies higher than $3 \times 10^{17}$ eV, which are produced by cosmic rays above $10^{19}$ eV. Here, as we see immediately from relation 2, the factor (propagation) rises from 3 to about 100 (for the no-evolution case). Assuming that the observed quantity $[\text{CR – flux}]$ does not drastically change, this would imply a strong increase of the upper bound. In fact, when we look at the data, a continuation of the cosmic ray spectrum as a power law $dN/dE \propto E^{-2.7}$ beyond $10^{20}$ eV is suggested by one of three large exposure experiments $[^4]$, and consistent with the combined result of all experiments (including the ones with lower exposure), and can therefore not be ruled out. The fact that the WB bound does not show this increase again goes back on assumption 4: The assumption of a flat injection spectrum...
implies a drastical change in the slope of the observed CR spectrum, i.e. the existence of the GZK-cutoff. This is also consistent with present data, but we note that it is the current lower limit on the observed CR flux at this energy, and can therefore not be used for upper limit estimates. We also point out that the common assumption that the post-GZK cosmic rays origin from a strong, local source, would not imply a increase of the neutrino flux: For a local source, the factor (propagation) in relation 2 obviously approaches unity, since both cosmic rays and neutrinos propagate (approximately) loss-free and in straight lines. An increase of the neutrino flux correlated to our bound would imply that the non-observation of the GZK cutoff is due to the increased activity of all CR sources in the universe, rather than to one local source. Although this scenario is currently not favored by theoretical arguments, it cannot be ruled out a priori. Only an observational upper limit excluding the associated neutrino flux would so this. We note that the present theoretical upper limit on the UHE neutrino flux is set by the observed EGRB through relation 1.

6. The impact of the Waxman-Bahcall upper bound on present models

In the last section we have shown how the special selection of parameters and assumptions allowed Waxman and Bahcall to set their cosmic ray upper bound to the lowest value possible for any model assumption. We have also shown that there is a large freedom to invent models which evade this bound at energies different from those chosen by WB in their derivation. Here we want to discuss in which respect their bound affects present models which are already discussed in the literature. Clearly, since such models make a clear prediction about the global source spectrum, the bound may be of more relevance here since we can compare in the most restrictive energy regime, $E_{\text{CR}} \approx 10^{19}$ eV or $E_{\nu} \sim 3 \times 10^{17}$ eV.

We start with AGN models. Waxman and Bahcall have already noted that there is one class of AGN related neutrino models for which no bound whatsoever can be stated, except by direct neutrino observations: the so called AGN core model \cite{11}, which is opaque to both neutrons and gamma rays. (Actually, there is a bound also on this models, since the energy in gamma rays is converted to X-rays by saturated comptonization, and can be compared to the total flux of observed extragalactic X-ray point sources and the diffuse background. This is the way how this model indeed has been normalized.) Here we concentrate, like Waxman and Bahcall, on the discussion of AGN jet models.

First of all, it should be noted that Waxman and Bahcall did not discover that such models may be constrained by cosmic ray data. Mannheim (1995, \cite{12}) already pointed out this problem, and suggested two models: Model A, which was constructed to explain both the cosmic ray data (assuming neutron transparency and straight-line propagation) and the EGRB above 100 MeV (however, using an incorrect relation of gamma ray and neutrino fluxes, see Mücke et al. \cite{4}, these proceedings), and Model B, normalized to the gamma ray background above 1 MeV, which was at that time overestimated by the incorrect Apollo measurements by one order of magnitude. It was noted in ref. \cite{12} that for Model B, in order to evade overproduction of cosmic rays above the ankle, one has to assume some mechanism preventing their cosmic rays from reaching us at these energies.

Citing Model B only, and two similar models by Protheroe \cite{13}, and Halzen and Zas \cite{14}, Waxman and Bahcall claimed that all these models violate the cosmic ray bound by two orders of magnitude and can therefore be ruled out. In fact, using the
correct bound derived for the cosmological evolution observed in AGN, the discrepancy is reduced to a factor $\sim 30$, and by another factor of 2 when we recalculate the WB bound at $E_{\text{CR}} = 10^{19}$ eV using a precise Monte-Carlo simulation of extragalactic transport, rather than the approximate treatment performed by Waxman (1995, [8]).

Obviously, hadronic blazar models follow assumption 1, and since we have a model spectrum given we can chose an energy in the spectrum where assumption 3 applies as well. To support the validity of assumption 2 (transparency) for AGN jets, WB refer to the observed TeV emission of Mrk 421 and Mrk 501. Unfortunately, they misinterpret the TeV data in stating that the observed emission at 10 TeV proves that blazar jets are optically thin at this energy. In fact, the observed break of the gamma ray spectral index between the EGRET regime ($< 30$ GeV) and the Whipple/HEGRA data ($> 300$ GeV) [15] implies that these sources become optically thick at $< 300$ GeV.

It should be pointed out that in an homogeneous emitter, a $\gamma\gamma$-opacity larger than one does not lead to an exponential cutoff, as sometimes erroneously assumed, but to a spectral break by the amount of the low energy flux spectral index. For Mrk 421 and Mrk 501, the observed GeV–TeV break matches this prediction very well. Moreover, a spectral break of this kind is not expected in the emission spectrum of the hadronic scenario, so a consistent interpretation of the data within this model requires the assumption that the observed break is due to opacity. Therefore, the $\gamma\gamma$-opacity of Mrk 501 at 10 TeV is $\sim 30$, rather than $\ll 1$ as assumed by WB. Correcting this in the estimate, we obtain an neutron-opacity of Mrk 501 at $10^{19}$ eV of $\sim 0.1$. Mrk 501 is therefore indeed optically thin for neutrons, but we have to note that it is a low-luminosity blazar, and that the opacity is directly proportional to the blazar luminosity. Averaging the neutron opacity of blazars over the flat blazar luminosity function, we obtain an average value $\langle \text{opacity} \rangle_n \sim 10$, which reduces the cosmic ray ejection per given neutrino flux (or, in other words, increases the bound) by the same factor (see [3] v2 for details). This removes the discrepancy with the WB bound — existing AGN jet models are not at any energy in conflict with the cosmic ray data, because they do not fulfill assumption 2. Obviously, this also shows that a hadronic production of the EGRB is not ruled out by the cosmic ray constraint, as Waxman & Bahcall claim. Rather, improved determinations of the EGRB and its origin may set the strongest constraints on the possible neutrino fluxes. We note that this result was obtained without any modifications to the models, and without invoking other energy loss processes for cosmic rays expected in the extended halos of radio-loud AGN.

As a side remark, we note that due to the very flat neutrino spectrum AGN models expect (and always expected) neutrino fluxes in the interesting PeV regime which are much below the WB bound. Even if the bound would be perfectly valid, a direct discrepancy at these energies never existed.

We now may add a few remarks on neutrinos from Gamma Ray Bursts. Here, no discrepancy with the cosmic ray bound has been found by Waxman and Bahcall. We may remark, that due to the property of this model to expect an optically thin, $E^{-2}$ cosmic ray emission spectrum, and cutoffs generally above $10^{19}$ eV, it is the only model to which all assumptions of Waxman and Bahcall, thus also their bound, fully apply. However, we may point out an interesting turn of the argument: In highly relativistic flows like GRB (and AGN also), we have good reason to assume that the direct ejection of protons is strongly suppressed, because the adiabatic loss time is of the order of the crossing time of the shell. Thus, it is likely that only neutrons can be ejected from a GRB shell [17]. If this is the case, then we can use the WB bound for
an $E^{-2}$ neutron spectrum, as expected to be produced by GRB, as a test flux for the hypothesis that GRB are the dominant sources of ultra-high energy cosmic rays [18]. If it could be independently confirmed that GRBs follow the evolution pattern observed in star-formation, an observed neutrino flux correlated with GRB events “only” on the level predicted by Waxman and Bahcall (1997 [19]), which is then about one order of magnitude below the appropriate bound, would provide evidence against rather than in favor of this scenario.

7. Conclusions

The relation of cosmic ray and neutrino fluxes has been shown to be an important, so far not sufficiently noticed measure for the viability of models of extragalactic neutrino production. It is thanks to Waxman and Bahcall that this point has now found attention by the community. Unfortunately, the way their result was presented, namely as a model-independent upper bound on any kind of extragalactic neutrino production, could impose to some severe misunderstandings. The most serious could be that this result might shatter our confidence in the object of very-high and ultra-high energy neutrino observatories.

In this paper we have presented the case that the Waxman-Bahcall upper bound is not model-independent, but rather relies on very special model assumptions. We have also shown that present models for extragalactic neutrino fluxes, which have provided one motivation for the construction of the experiments mentioned above, are not seriously affected by their result and need no modifications. However, it is also clear that the consistency of these models with cosmic ray observations is marginal, so that cosmic ray data can be regarded an important constraint for their parameter space.

With respect to the motivation of experiments, we may make one point very clear: The debate whether the Waxman-Bahcall bound is valid or not is a purely theoretical dispute. It is the dispute whether assumptions 1-4 stated in Section 3 generally apply to nature, or whether they do not. The decision can only be made by experiment. Although theories are necessary to understand our data, they can never replace them. Truly model-independent bounds are only observational upper limits.

The discussion in this paper also made clear how many important questions regarding the origin of cosmic rays can be decided by neutrino observations. The prediction of neutrinos above the WB-bound at energies $10^{16}-10^{18}$ eV is an important test of the viability of hadronic blazar models, and of the total contribution of hadronic blazar emission to the extragalactic gamma ray background. Setting upper limits to neutrinos from Gamma-Ray Burst below the WB-bound may enable us to limit their contribution to the ultra-high energy cosmic ray spectrum — or, finding them on the level of the bound, would provide strong evidence that they are indeed the dominant sources of these cosmic rays. Finally, in case that the non-existence of the GZK cutoff in the cosmic ray spectrum is further supported by observations, searching for neutrinos in excess of the WB-bound at ultra-high energies ($>10^{19}$ eV) can test whether this is due to an increased overall activity of the cosmic ray/neutrino sources in the universe, or rather due to the contribution from one, local source (or even our own galactic halo). All in all, these are only a few reasons why the tight connection between extragalactic cosmic ray and neutrino fluxes provides a strong additional motivation for VHE and UHE neutrino observatories.
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