First Signs for String Breaking in Two-Flavor QCD

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We have been examining the phenomenon of string breaking in QCD with two flavors of dynamical staggered quarks. We construct a transfer matrix from a combination of “string” and “two-meson” channels. Preliminary results with low statistics show the expected signs of string breaking.

1. INTRODUCTION

During the past few years there has been renewed interest in the important phenomenon of string breaking (SB), which is predicted by QCD, but which lattice QCD simulations for a long time have failed to show conclusively.

String breaking is observed as a leveling off of the static quark-antiquark potential at large separation. The potential is the separation-dependent ground-state eigenvalue of the QCD hamiltonian in the presence of the static quark-antiquark pair. Traditionally, this eigenvalue was sought in the Wilson loop observable, which is the expectation value of the transfer matrix on a “string” state with the string of color flux connecting the static quark and antiquark. Experience has shown, not surprisingly, that the string state (S) is a very poor variational ansatz for a state that looks more like two static-light mesons. Including an admixture of a two-meson component (M) should help \cite{1-8}.

In the following we report on preliminary results that have been obtained using the relevant string-string and string-meson operators for QCD.

2. STUDYING STRING BREAKING ON THE LATTICE

Our conventional Wilson loop is computed with APE smearing of the space-like gauge links. In hamiltonian language the expectation value of this operator is the transfer matrix $G_{SS}(R,T)$ between an initial and final state $S$ consisting of a static quark-antiquark pair separated by a fat string of color flux. We enlarge the space by including a meson-antimeson state $M$ with an extra light quark in the vicinity of the static antiquark and an extra light antiquark in the vicinity of the static quark. Thus we also compute the additional transfer matrix elements $G_{MM}(R,T), G_{MS}(R,T)$ and $G_{SM}(R,T)$. They are diagrammed in Figures 1 and 2.

![Figure 1. The static-light meson-antimeson pair contribution to the full QCD propagator. The wiggly lines denote the light quark propagator. Shown are the ‘direct’ and ‘exchange’ terms respectively.](image)

In principle, both channels couple to a common set of eigenvalues. At large $T$ we expect to reach the ground state, defined by the largest generalized eigenvalue

$$G(R, T + 1)u(R, T) = \lambda(R, T)G(R, T)u(R, T)$$

The potential is then given by $V(R) =$
\[ -\lim_{T \to \infty} \log |\lambda(R, T)|. \]

The vector \( u(R, T) \) defines the variationally optimum admixture of \( S \) and \( M \) with the largest overlap with the ground state.

A key unitarity condition is that the eigenvalues and eigenstates approach a constant with increasing \( T \). This condition together with a demonstration of a smooth transition with increasing \( R \) between a string-dominated state and two-meson-dominated state constitutes a true test of string breaking and should distinguish a quenched calculation from a proper calculation with dynamical quarks.

3. NUMERICAL METHOD

To maximize statistics we generate “all-to-all” propagators for the light quark, using a Gaussian random source method. Results reported here are based on 15 such sources per gauge configuration, but we plan to increase this number\(^9\).

Figure 2. The string-meson transition correlator \( G_{SM} \) (and its hermitian conjugate \( G_{MS} \)). The wiggly line again denotes the light quark propagator.

When constructing operators involving both staggered fermions and gauge links, one has to pay careful attention to staggered fermion phases. A consistent treatment results from interpreting the static quark as an infinitely massive staggered quark. For example, a hopping parameter expansion around an on-axis \( R \times T \) rectangular path gives, in addition to the Wilson-loop gauge-link product, a net phase factor \((-1)^{RT} \times (-1)^{R+T}\), independent of the Dirac phase conventions. We use a similar construction to get the phases for the nonclosed gauge link products in the diagrams of Figures 1 and 2. In that case the Dirac phase convention for the gauge link products must be consistent with that of the light quark. A peculiar consequence of this construction is that the transition matrix elements must vanish for off-axis displacements \( \vec{R} \) that have more than one odd Cartesian-displacement component.

4. LATTICE SIMULATION

We use a set of stored configurations of size \( 20^3 \times 24 \) at \( \beta=5.415 \) generated with two flavors of dynamical staggered fermions of mass \( m_a = 0.0125 \). This set gives a ratio \( m_\pi/m_\rho \approx 0.358 \) and has a lattice spacing of about 0.17 fm, which gives a spatial lattice size of \( \approx 3.4 \) fm, and a temporal size of \( \approx 4 \) fm.

For the light fermions in the static-light mesons we use the same parameters as for the dynamical fermions. With the choice of 15 random sources we have currently analyzed about 70 out of the 200 archived configurations.

Figure 3. The behavior of the two-meson to two-meson matrix element \( G_{MM} \) as a function of the spatial separation \( R \). Results were obtained from fits to the data as a function of \( T \) for \textit{fixed} \( R \).

5. RESULTS

In Figure 3 we show the results for the two-meson to two-meson transition \( G_{MM} \). For short
distances the operator behaves like the Wilson loop, with a linearly rising value, while for distances greater than 3 in lattice units it starts to level off. The dashed line is twice the mass of the heavy-light meson.

The results for the transition matrix element $G_{MS}$ shown in Figure 4 are similar within large errors, with the energy leveling off at $R_\alpha \approx 5$ in lattice units.

In Figure 5 we show the combined results — including those for the pure Wilson loop correlator. The string-like behavior of the $G_{MM}$ and, especially, of the $G_{MS}$ correlators at short distances is clearly visible.

**Figure 4.** The same as in Fig. 2, but for the $G_{MS}$ operator.

**Figure 5.** Combining the results for the $G_{MM}$ ($\square$), $G_{MS}$ ($\diamond$) and Wilson loop ($G_{SS}$) ($\times$) correlators.

### 6. CONCLUSIONS

Our present results with low statistics show a behavior expected with string breaking. The cross-over region is found to be at a distance of $R_\alpha = 5 - 6$, or about $0.8 - 1.1$ fm. However, to demonstrate string breaking convincingly, one must find a smooth transition with increasing $R$ between a string-dominated state and two-meson-dominated state and demonstrate that the eigenvalues and eigenvectors of the transfer matrix approach a constant at large $T$. Work in this direction is currently underway.

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