Small instantons & the strong CP problem in composite Higgs models

R.S. Gupta, V.V. Khoze and M. Spannowsky

Institute for Particle Physics Phenomenology, Durham University, South Road, Durham, DH1 3LE

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We show that QCD instantons can generate large effects at small length scales in the ultraviolet in standard composite Higgs models that utilise partial compositeness. This has important implications for possible solutions of the strong CP problem in these models. First we show that in the simplest known UV completions of composite Higgs models, if an axion is also present, it can have a mass much larger than the usual QCD axion. Even more remarkable is the case where there are no axions, but the strong CP problem can be solved by generating the up quark mass entirely from the contribution of instantons thus reviving the massless up-quark solution for these models. In both cases no additional field content is required apart from what is required to realise partial compositeness.

I. INTRODUCTION

The strong CP problem is one of the five major particle physics puzzles that motivate the existence of new physics beyond the Standard Model (SM). The most elegant solution to this problem is the existence of a new $U(1)$ symmetry that is anomalous under QCD so that it is possible to rotate away the strong CP phase. The simplest possibility is that the up quark is massless which leads to the existence of an axial $U(1)$ symmetry that makes the strong CP phase unphysical. Another possibility is the existence of the Peccei-Quinn $U(1)$ symmetry that is spontaneously broken resulting in a Goldstone mode, the axion. As the Peccei-Quinn symmetry is anomalous under QCD, the axion gets a potential due to non-perturbative QCD effects and stabilises at a value that leads to a vanishing strong CP phase [1-3].

Both the above solutions are thought to have unambiguous low energy consequences. The massless up quark solution [4-8] can be tested by lattice simulations. Unfortunately the latest lattice studies indicate a non-zero up mass, that seemingly falsifies this possibility [9, 10]. This leaves the Peccei-Quinn solution which predicts the existence of the axion with a mass and coupling that is currently underway.

The above predictions, however, rely on the tacit assumption that any non-perturbative contribution to the up mass in the first case or to the axion in the second case, arises from the large instantons in the IR. If small instantons in the UV also become important it will completely alter the above experimental expectations. Previous attempts to enhance these UV contributions to the axion mass require additional elements—such as new coloured fermions [11, 12], extra dimensions [13] or a UV modification of the QCD gauge group [14-16].

In this work we show that small instanton contributions can become important in composite Higgs models with partially composite fermions [17, 18]. This can be achieved with no additional field content other than what is necessary to fully realise partial compositeness in standard UV completions of these models.

The enhancement of small instanton effects in composite model is possible because the two factors that suppress small instanton contributions in the SM , namely, the smallness of the strong coupling in the UV and the smallness of the product of the SM Yukawa couplings, can both be overcome in these models. The first suppression factor can be overcome because, as we will show, in order to generate composite partners for all SM fermions, many new coloured fermions need to be introduced. These new degrees of freedom alter the running of the QCD strong coupling in the UV where it grows again to non-perturbative values. As far as the suppression due to the Yukawa couplings is concerned, this can be overcome because in these models the effective SM Yukawa matrices can be anarchic and $O(1)$ in the UV.

We show that the enhancement of the small instanton contributions can be so effective in these models that it may be possible to generate the entire mass of the up quark from instanton effects. This leads to a solution of the strong CP problem as in the deep UV the up Yukawa is absent and indeed an additional $U(1)$ symmetry related to the axial rotation of left and right handed up quarks exists; such a chiral rotation can be used to completely rotate away the strong CP phase. We also show that in an alternative scenario where an axion field exists, its mass would lie outside the usual band for the QCD axion because of the enhancement of strong instanton effects in these models.

II. MODEL

We consider a straightforward extension of one of the simplest known UV completions of composite models [17, 18] by Ferretti [19] where the confining hypercolor gauge group is $SU(4)_HC$. The field content of our model is shown in Table I. The original model in Ref. [19] only has a single pair of fermions, $\chi_u$ and $\tilde{\chi}_u$; and it can generate partners only for one left handed doublet and one right handed up-type quark. In order to obtain partners for all SM fermions we have extended the field content of the original model by simply taking three copies, $\chi_u^i$ and $\tilde{\chi}_u^i$. 

| Field | Contents |
|-------|----------|
| $\chi_u$ | left-handed up quark |
| $\tilde{\chi}_u$ | left-handed down quark |
| $\chi_u^i$ | i-th right handed up quark |
| $\tilde{\chi}_u^i$ | i-th right handed down quark |
The condensates break indices under the UV fixed point. This is possible, for instance, if the employing fermionic partial compositeness [17, 18], we group in our model is no longer asymptotically free as in the notation above we have only made the index, \(i\), corresponding to the \(SU(3)_{c}^{p} \times SU(3)_{c}^{d}\) symmetry explicit.

**A. Running of hypercolour coupling**

With the additional matter content, the hypercolour group in our model is no longer asymptotically free as in the original model in Ref. [19]. As is standard for models employing fermionic partial compositeness [17, 18], we assume instead, that the theory has a strongly coupled UV fixed point. This is possible, for instance, if the \(\beta\)-function of the hypercolor gauge coupling has the kind of dependance on the gauge coupling proposed in Ref. [20] and shown in Fig. 1. We will assume that our model lives in the region \(g > g_{*}\) and flows from the UV fixed point \(g = g_{*}\) to larger values \(g > g_{*}\) in the IR where it confines.

**B. Global symmetry breaking pattern**

When the hypercolor group \(SU(4)_{HC}\) confines \(\psi, \chi^{i}_{u,d}\) and \(\hat{\chi}^{i}_{u,d}\) form TeV-scale condensates,

\[
\langle \psi^{p} \bar{\psi}^{q} \rangle \sim \delta^{pq} f_{\psi}, \quad \langle \chi^{i}_{u} \bar{\chi}^{j}_{u} \rangle \sim \delta^{ij} f_{\chi_{u}}, \\
\langle \chi^{i}_{d} \bar{\chi}^{j}_{d} \rangle \sim \delta^{ij} f_{\chi_{d}},
\]

thus breaking the original global symmetry. Here \(p, q\) are the indices under the \(SU(5)\) global symmetry and \(i, j\) the indices under the \(SU(3)_{c}^{p}\) or \(SU(3)_{c}^{d}\) flavour symmetry. The condensates break \(SU(5)\) to \(SO(5)\) and \(SU(3) \times SU(3)\)' down to the diagonal \(SU(3)_{c}\) as in Ref. [19]. The coset space, \(G_{F}/H_{F}\), is given by,

\[
\frac{SU(5) \times SU(3) \times SU(3)_{c}^{p} \times SU(3)_{c}^{d} \times U(1)^{4}}{SO(5) \times SU(3)_{c} \times U(1)_{X} \times U(1)_{B}}
\]

(3)

where \(U(1)^{4} = U(1)_{X} \times U(1)_{B} \times U(1)_{A1} \times U(1)_{A2}\) and \(SU(3)_{c}^{p} = SU(3)_{c}^{d} \times SU(3)_{c}^{d}\)

The unbroken diagonal \(SU(3)_{c}\) is gauged to give the QCD Lagrangian. The electroweak group is also gauged. It is embedded in \(SO(5)\) as follows,

\[
\frac{SU(5) \times SU(3)_{c} \times U(1)_{X}}{G_{\text{cur}} \equiv SU(3)_{c} \times SU(2)_{L} \times SU(2)_{R} \times U(1)_{X}} \\
\supset G_{\text{SM}} \equiv SU(3)_{c} \times SU(2)_{L} \times U(1)_{Y} \\
\supset SU(3)_{c} \times U(1)_{\text{em.m.}},
\]

(4)

where hypercharge generator is given by \(Y = T_{3R} + X, T_{3R}\) being the diagonal \(SU(2)_{R}\) generator.

The Lagrangian in Eq. 1 is invariant under 5 independent \(U(1)\) symmetries, one associated with the rotation of each of the five fermion species \(\chi^{i}_{u}, \chi^{i}_{d}, \hat{\chi}^{i}_{u}, \hat{\chi}^{i}_{d}\) and \(\psi\). Two of these \(U(1)\)s are not spontaneously broken by the condensates in Eq. 2. The first is the linear combinations of these five symmetries that corresponds to the \(U(1)_{X}\) symmetry in Table I. The second is a linear combination of these \(U(1)\)s that can be identified with, \(U(1)_{B}\), the extension of the baryon number symmetry that includes the new fields. There are three remaining \(U(1)\)s that get broken spontaneously by the condensates. One linear combination of these three \(U(1)\)s is anomalous and thus not a true symmetry, which is why it does not appear in Eq. 3. This still leaves two \(U(1)\)s, namely \(U(1)_{A1}\) and \(U(1)_{A2}\), shown in Table I.

We can now count the number of potential Goldstone bosons that live in the above coset space. There are two Goldstone bosons, \(\eta^{\prime}_{1}\) and \(\eta^{\prime}_{2}\), corresponding to the spontaneous breaking of \(U(1)_{A1}\) and \(U(1)_{A2}\), respectively. In the electroweak sector the symmetry breaking pattern,

\[
\frac{SU(5)}{SO(5)}
\]

(5)

gives rise to 14 pseudo-Goldstone bosons. These include the Higgs doublet, \(H\), a real singlet, a hypercharge neutral \(SU(2)_{L}\) triplet and a complex \(SU(2)_{L}\) triplet charged under hypercharge. Finally, in the QCD sector the symmetry breaking pattern,

\[
\frac{SU(3) \times SU(3)_{c}^{p} \times SU(3)_{c}^{d}}{SU(3)_{c}}
\]

(6)

gives rise to 8 pseudo-Goldstone bosons. As we will discuss shortly, all these Goldstone modes become massive once we introduce other terms in the Lagrangian that explicitly break the original global symmetry, \(G_{F}\).
Table I: The two-component left-handed fermions of the UV theory. The confining hypercolor gauge group is $G_{1C}$ and $G_F = SU(5) \times SU(3) \times SU(3)^L \times SU(3)^R \times U(1)_X \times U(1)_B \times U(1)_{A1} \times U(1)_{A2}$ is the global symmetry group before symmetry breaking.

| $|SU(4)_{HC}$ | $SU(5)$ | $SU(3)$ | $SU(3)^L$ | $SU(3)^R$ | $U(1)_X$ | $U(1)_B$ | $U(1)_{A1}$ | $U(1)_{A2}$ |
|---|---|---|---|---|---|---|---|---|
| $\psi$ | 6 | 5 | 1 | 1 | 1 | 1 | 0 | 0 | $-18/5$ | 0 |
| $\chi_+^i$ | 4 | 1 | 3 | 1 | 3 | 1 | $-1/3$ | $-1/6$ | 1 | 1 |
| $\chi_-^i$ | 4 | 1 | 1 | 3 | 3 | 1 | $1/3$ | $1/6$ | 1 | 1 |
| $\chi_0^i$ | 4 | 1 | 1 | 3 | 3 | 3 | $1$ | $-1/6$ | 1 | -1 |
| $\tilde{\chi}_+^i$ | 4 | 1 | 3 | 1 | 3 | 1 | $-1/3$ | $-1/6$ | 1 | -1 |

C. Partial Compositeness

We will now discuss how partial compositeness can be realised in this model. We need a composite fermionic partner for each of the SM fermions. In the UV near the fixed point $g = g_\ast$ in Fig. 1 we identify the following baryonic operators,

$$
\mathcal{O}_{uL}^{c,i} = (x_u \bar{\psi} \chi_+^i) \quad \mathcal{O}_{uR}^{i} = (\bar{x}_u \psi \chi_+^i) \\
\mathcal{O}_{dL}^{c,i} = (x_d \bar{\psi} \chi_+^i) \quad \mathcal{O}_{dR}^{i} = (\bar{x}_d \psi \chi_+^i),
$$

which have components that have the right transformation properties to be partners of right-handed up-type quarks, left-handed up type quarks, right-handed down type quarks and left-handed down type quarks respectively. These are all left-handed two component spinor partners for the SM left handed up-type fermions.

The operator $\mathcal{O}_{uR}^{i}$ transforms as $(5, 3)_{2/3}$ under $SO(5) \times SU(3)^c \times U(1)_X$. It has components, that we call $u_{R}^{c,i}$, which transform as $(3, 1)_{2/3}$ under $SU(3)^c \times SU(2)_L \times U(1)_Y$. We will call these components, $U_{R}^{c,i}$, and they will serve as partners for the SM left handed up-type fermions.

As far as, $\mathcal{O}_{uR}^{i}$ and $\mathcal{O}_{dL}^{c,i}$ are concerned they transform respectively as $(5, 3)_{-1/3}$ and $(3, 1)_{1/3}$ under $SU(5) \times SU(3)_c \times U(1)_X$. They, have components that transform respectively as $(3, 1)_{-1/3}$ and $(3, 1)_{1/3}$ under $SU(3)^c \times SU(2)_L \times U(1)_Y$; we will call these $D_{U}^{i}$, the partners for the right-handed anti-down quarks and the left handed down quarks.

The partial compositeness Lagrangian can now be realised by linearly coupling the SM fermions to their partners,

$$
\mathcal{L}_{mix} = \frac{\lambda_{ij}^{U}}{4\pi} \frac{1}{A_{U(R)}} U_{ij}^{d} U_{ij}^{d} + \frac{\lambda_{ij}^{U}}{4\pi} \frac{1}{A_{U(L)}} U_{ij}^{d} U_{ij}^{d} + \frac{\lambda_{ij}^{D}}{4\pi} \frac{1}{A_{D(R)}} D_{ij}^{d} D_{ij}^{d} + \frac{\lambda_{ij}^{D}}{4\pi} \frac{1}{A_{D(L)}} D_{ij}^{d} D_{ij}^{d} + h.c.
$$

where $d_F$ is the conformal dimension of the corresponding operator, $F$. The conformal dimensions are independent of the flavour indices $i, j$ because of the $SU(3)^R \times SU(3)^L$ symmetry. The coupling of the SM fermions to the other possible baryonic operators—such as the right-handed SM quarks with $(\bar{x}_u \psi \chi_+^i)$ and $(\chi_+^i \psi \chi_+^i)$ or the left-handed quarks with $(\bar{x}_u \psi \chi_+^i)$ and $(\chi_+^i \psi \chi_+^i)$—is prohibited as we impose a $Z_2$ symmetry under which $x_{u,d}$ and the SM right-handed fermions are odd.

In the IR after confinement the above operators lead to composite states that pair with charge conjugates states (that can be obtained by interchanging $x_{u,d} \leftrightarrow x_{u,d}$) to form massive Dirac fermions. Once these are integrated out the SM Yukawa coupling between the SM fermions and the composite Higgs boson is generated (see for eg. Ref. [18]).

D. Explicit breaking by masses and four-$\psi$ interaction

Finally we add some additional terms not present in the original model of Ref. [19],

$$
\mathcal{L}_{new} = m_0 e^{i\theta_m} \psi_0 \psi_0 + \frac{g_4 e^{i\theta_4}}{\Lambda^2} (\bar{\psi}_+ - \bar{\sigma}_\mu \psi_0)(\bar{\psi}_+ - \bar{\sigma}_\mu \psi_0)
$$

where $m_0$ and $g_4$ are real, and the subscript $\{\alpha, \beta\}$ in the last term refers to the $T_{3R,3L}$ charges. To make our notation clear and to understand how each of these components of $\psi$ transform, recall that fermion $\psi$ transforms as a 5 of $SU(5)$. This decomposes into two $SU(2)_L$ doublets and an $SU(2)_L$ singlet,

$$
\begin{pmatrix}
\Psi_+ \\
\Psi_-
\end{pmatrix},
$$

$\Psi_{\pm}$ are $SU(2)_L$ doublets and the $\pm$ subscripts correspond to $T_{3L} = \pm 1/2$. We can explicitly write $\Psi^{T} = (\psi_+ \psi_-)$ where the second index now corresponds to the $T_{3L} = \pm 1/2$.

The first term breaks $U(1)_{A1}$ and thus gives a mass to $\eta_{1}^{T}$. The $\eta_{1}^{T}$ does not get a contribution form the mixing terms in Eq. 8 as we can extend the $U(1)_{A1}$ symmetry.
to the SM fermions in a way that is preserved by Eq. 8, i.e. by giving the $U(1)_{A1}$ charges, 8/5 and 18/5, respectively to the SM doublet and singlet fermions. The $\eta'_i$ actually would eventually get a contribution to its mass also from non-perturbative QCD effects as the above extended $U(1)_{A1}$ is anomalous under QCD. On the other hand, $U(1)_{A2}$ is already broken by the mixing terms in Eq. 8 which give $\eta'_2$ a mass.

The second term in Eq. 9 is a four-$\psi$ interaction between components of the fermion $\psi$ that explicitly breaks the original global symmetry $SU(5)$. This term would be essential in enhancing the QCD small instanton contributions in the next section.

### E. A minimally flavour and CP violating strong sector

Notice that the lagrangian in Eq. 1 is invariant under CP and the $SU(3)^u_F \times SU(3)^d_F$ flavour symmetry. These symmetries are broken only by the couplings $\lambda^{ij}_{uL}$, $\lambda^{ij}_{uR}$, $\lambda^{ij}_{dL}$, $\lambda^{ij}_{dR}$, $g_{4\psi} e^{i\theta_4}$, $m_\psi e^{i\theta_m}$ and the strong CP phases in the QCD and hypercolor sectors, $\theta_{QCD}$ and $\theta'$ respectively. Following, Ref. [21] here we will further assume that the mixings of the right handed quarks do not break the $SU(3)^u_F \times SU(3)^d_F$ symmetry so that,

$$
\lambda^{ij}_{uR,dR} \sim y_{uR,dR} e^{i\theta_B} \delta^{ij}.
$$

This implies that the SM Yukawa couplings would be proportional to the left handed mixings,

$$
\begin{align*}
Y^{ij}_u &\sim \frac{\lambda^{ik}_{uL} \lambda^{kj}_{uR}}{4\pi} \sim \lambda^{ij}_{uL} y_{uR} \\
Y^{ij}_d &\sim \frac{\lambda^{ik}_{dL} \lambda^{kj}_{dR}}{4\pi} \sim \lambda^{ij}_{dL} y_{dR}
\end{align*}
$$

which are the only spurions that break flavour symmetry, thus realising minimal flavour violation (MFV) [22].

As far as CP phases, $\theta_m$, $\theta_g$, $\theta_R$, $\theta_{QCD}$ and $\theta'$ are concerned, we can transfer all of them to $\lambda^{ij}_{uL}$ and $\lambda^{ij}_{dL}$ by taking the following steps:

1. First, the phase $\theta_m$ can be rotated away by $\psi_0 \rightarrow \psi_0 e^{-i\theta_m/2}$ which redefines $\theta_g$, $\theta_R$ and $\theta'$.
2. Next, the phase $\theta_g$ associated to $g_{4\psi}$ can be rotated to $\lambda^{ij}_{uL}$ and $\lambda^{ij}_{dL}$ by making the transformation $\psi_+ \rightarrow \psi_+ e^{i\theta_4}$. This also redefines $\theta'$.
3. Then $\theta'$ can be eliminated by an equal rotation of all $\chi_i$ and $\bar{\chi}_i$, which also redefines $\theta_{QCD}$.
4. Finally $\theta_R$ can be eliminated by an equal but opposite rotation of the $\chi_i$ relative to the $\bar{\chi}_i$.

This still leaves $\theta_{QCD}$ which can be entirely shifted to $\lambda^{ij}_{uL}$ and $\lambda^{ij}_{dL}$ by chiral rotations of the SM quarks while keeping the combination,

$$
\bar{\theta}_{QCD} = \theta_{QCD} + \text{ArgDet}[\lambda_{u\lambda_d}]
$$
unchanged. Because we have a MFV like structure, as in the SM, there is only one more physical phase in our theory, the CKM phase,

$$\theta_{CKM} = \text{ArgDet}[\lambda_u \lambda_d - \lambda_d \lambda_u].$$ \quad (14)

III. EFFECT OF SMALL INSTANTONS

The effect of QCD instantons at high energies are suppressed due to two reasons. (1) the suppression factor $$\kappa_s = e^{-2\pi/\alpha_s}$$ is small as QCD is asymptotically free, and, (2) there is a suppression factor that goes as the product of the Yukawa couplings of all the SM quarks, all of which are active at high energies. Both these effects can be overcome in the model we are considering because, (1) the new coloured fermions $$\chi^{i}_{u,d}$$ and $$\tilde{\chi}^{i}_{u,d}$$ that form the composite fermionic partners can lead to large UV values of the QCD coupling and (2) the mixings, $$\lambda^{ij}_{uL,dL}$$ and $$\tilde{\lambda}^{ij}_{uL,dL}$$, and thus the Yukawas in Eq. 12 can run to higher values in the UV in these models as we will now show.

A. Running of $$\alpha_s$$

In our model there are 8 new flavours of fermions for every generation once we take into account the 4 hypercolour degrees of freedom of $$\chi_{u,d}$$ and $$\tilde{\chi}_{u,d}$$. Including the SM fermions, there are $$n_f = 30$$ flavours. Using the usual expression,

$$\frac{dg_s}{d\log \mu} = -(11 - 2n_f/3) \frac{g_s^3}{16\pi^2}$$ \quad (15)

we find that the QCD beta function is positive. Assuming that the new flavours become active at 1 TeV we find that, $$g_s = 4\pi$$ for $$\mu \sim 2000$$ TeV where the instanton vertex will become unsuppressed.

We will assume that some UV degrees of freedom cut-off this growth of this coupling above a scale $$M \sim 2000$$ TeV such that $$\kappa_s$$ has a maximal value at this scale. We will treat this maximal value as a free parameter that can vary from $$\kappa_s = 10^{-94}$$ for $$g_s = 1$$ to $$\kappa_s \sim 1$$ for $$g_s = 4\pi$$. We will also assume that at a scale $$M' > M$$, the QCD gauge coupling growth is tamed, $$\kappa_s$$ becomes negligible and the QCD instantons are again highly suppressed.

B. Running of $$\lambda^{ij}_f$$

We will work in the mass basis where $$\lambda^{ij}_{uL}(M') = \text{diag}[y_{uL}, y_{cL}, y_{tL}]$$ and $$\tilde{\lambda}^{ij}_{dL}(M') = \text{diag}[y_{dL}, y_{sL}, y_{bL}]$$. The couplings $$y_f$$ run between the UV and IR scale, $$m_s \sim 1$$ TeV, of the composite masses,

$$\mu \frac{dy_f}{d\mu} = (d_F - 5/2)y_f + b \frac{N_{HC} y_f^3}{16\pi^2}$$ \quad (16)

where $$b$$ is an $$\mathcal{O}(1)$$ factor, $$N_{HC} = 4$$ is the number of colours for the hypercolour group and $$d_F$$ is the conformal dimension of the operator corresponding to the fermionic partner, $$F$$, that couples to the SM fermion, $$f$$. The first term allows an anarchic and $$\mathcal{O}(1)$$ valued matrix $$\lambda^{ij}_f(M')$$ to generate a hierarchical $$\lambda^{ij}_f(m_s)$$ thus explaining the SM masses and mixings. This can be seen if we solve the above equation by ignoring the second term,

$$y_f(m_s) = y_f(M) \left(\frac{m_s}{M}\right)^{d_F - 5/2},$$ \quad (17)

which shows that $$\mathcal{O}(1)$$ differences in the $$d_j$$ can lead to exponential hierarchies in the IR. Including the second term does not change this qualitative feature, in fact it can lead to hierarchies between couplings involving operators with the same $$d_F$$. In particular it results in a fixed point at $$y_f = 4\pi/\sqrt{bN_{HC} \gamma_F}$$ where $$\gamma_F = d_F - 5/2$$.

IV. UP QUARK MASS FROM SMALL INSTANTONS

In this section we will consider the model defined in Sec. II and assume that one of the eigenvalues of $$\lambda^{ij}_{uL}$$ vanishes at the scale $$M'$$ where instanton effects are negligible, i.e., $$y_{uL}(M') = 0$$. Instanton effects around the scale $$M$$ then generate a non-zero value for the up quark mixing, $$y_{uL}$$, via the 't Hooft vertex.

A. Non-perturbative generation of $$y_{uL}$$

The 't Hooft instanton vertex due to the QCD anomaly in this model is an interaction including all the coloured fermion species. In Fig. 2 (a) we show only the first generation fermions, $$u_L, u_R, d_L, d_R, c_L, c_R, t_L, t_R, \chi_{uL}^{1}, \chi_{dL}^{1}$$ and $$\tilde{\chi}_{uL}^{1}$$ explicitly. Working in the mass basis we start from the anomaly vertex to generate the $$y_{uL} u_L U^{c\dagger}_{L}$$ term as shown in Fig. 2 (a). The fermions of the other generations have not been shown for space constraints but a identical topology exists for them with the only difference that now the $$c_L U_{L}^{c\dagger}$$ and $$t_L U_{L}^{c\dagger}$$ lines are also closed by the couplings $$y_{dL}$$ and $$y_{bL}$$.

To get the NDA estimate for $$y_{uL}$$ we can redraw the same diagram but now in terms of QCD and hypercolour singlets as shown in Fig. 2 (b). If one considers the pairs, $$U^{c\dagger}_{L} U^{c\dagger}_{R}, U^{c\dagger}_{L} U^{c\dagger}_{R}, D^{c\dagger}_{L} D^{c\dagger}_{R}$$ and $$D^{c\dagger}_{L} D^{c\dagger}_{R}$$ as QCD singlet scalars, this diagram becomes very similar to the one considered in Ref. [11] where new scalars connect fermion pairs, such as $$u_L u_R^{\dagger}$$ to $$d_L d_R^{\dagger}$$ in the 't Hooft vertex. The result of a full calculation in Ref. [11] is that the only suppression factor is given by, $$\prod_f Y_f^4/4\pi$$, where the $$Y_f$$ are the Yukawa couplings of the scalars to the fermion pairs. In our case the coupling of the SM fermions to the scalars, formed from the composite partners, reaches its perturbative limit for $$y_{fL} = y_{fR} = 4\pi$$. Thus adapting the result of Ref. [11] to our case, and including a
that this effect is at least 7-loop suppressed \[28\], the SM in Ref. \[23\] can be adapted to our model to show is because the arguments based on spurion analysis for in the SM this is expected to be highly suppressed. This $θ$ the scale 

$$\frac{y_{uL}}{4π} \sim \kappa_s \left( \frac{g_4 ψ}{16\pi^2} \right)^3 \sum_{f=d,s,c,b,t} y_{fL}^2 \frac{y_{fR}^2}{4π},$$ \hspace{1cm} (18)

where all the above couplings are at the scale $M$, and following Eq. 11, all the $y_{fR} = y_{uR,dR}$ depending on whether $f$ is an up or down type fermion. The white circles in Fig. 2(b) represent vertices arising from the strong sector whereas the dark circles denote vertices external to the strong sector. Here we have assumed a suppression only due to the former couplings.

The known value of the up quark Yukawa can be reproduced in the strongly coupled regime when the couplings in Eq. 18 saturate their perturbative limit. For instance we obtain for the up Yukawa,

$$Y_u(m_+) \sim \frac{y_{uL}(m_+) y_{uR}(m_+)}{4π} \sim 1.5 \times 10^{-5} \kappa_s \left( \frac{g_4 ψ}{16\pi^2} \right)^3,$$ \hspace{1cm} (19)

if we solve the RG equations in Eq. 16 assuming $b = 1/4$, taking $d_{UL,DL} = 7/2$, $y_{uR,dR}(M) = 4π$ in Eq. 11, $y_{uR,dR}(m_+) = y_{uR,dR}(M)/10$, and other boundary conditions fixed by the measured value of the SM fermions masses.

### B. Solution to the strong CP problem

Let us first consider the scale $M'$ at and above which QCD instanton effects are suppressed so that $y_{uL}(M') = 0$. At this scale the phase $θ_{QCD}$ can be removed simply by a chiral rotation of the up quark fields.

It is still instructive to see how, leading order, $θ_{QCD}$ vanishes just below the scale $M$ where QCD instanton effects become important. These effects generate a non-zero $y_{uL}$ given by Eq. 18. As explained in Sec. II E, we work in a convention where chiral rotations are used to transfer the couplings to the mixing matrices $λ^j_i$ and there is no $GG$ coupling to start with. A contribution to $θ_{QCD}$ from the fermionic phases can arise at this scale from closing the ‘t Hooft vertex completely, which gives,

$$θ_{QCD}(M) = \text{Arg}(y_{uL}^* y_{uR} \prod_{f=d,s,c,b,t} y_{fL}^* y_{fR}^*)$$

$$= \text{Arg}(\prod_{f=u,d,s,c,b,t} |y_{fL}|^2 |y_{fR}|^2) = 0.$$ \hspace{1cm} (20)

As explained in Sec. II E there is only one more physical phase in our theory, the CKM phase, $θ_{CKM}$. From the scale $M$ to the experimental scale, $θ_{CKM}$ can induce $θ_{QCD}$ due to Renormalisation Group (RG) effects, but as in the SM this is expected to be highly suppressed. This is because the arguments based on spurion analysis for the SM in Ref. [23] can be adapted to our model to show that this effect is at least 7-loop suppressed [28].

### V. A HEAVY AXION FROM SMALL INSTANTONS

Now we consider a different scenario from Sec. IV, taking $y_{uL}(M') \neq 0$, but introduce a new pseudoscalar field, $φ$. In addition to all the Lagrangian terms in Sec. II, we consider the new term,

$$L_φ = \frac{g_4^2}{32\pi^2} φ G_{µν} \tilde{G}^{µν}.$$ \hspace{1cm} (21)

where we take $f \geq M \sim 10^6$ GeV. We assume that above this scale is the only one that breaks the shift symmetry of $φ$ via anomalous QCD effects so that it can be identified with the QCD axion.

Non-perturbative effects at the scale $M$ in the UV as well as the usual QCD scale in the IR will contribute to the axion potential. The UV contribution can be estimated by first shifting $θ_{QCD}$ to the ‘t Hooft vertex in Fig. 2(a) and 2(b) and then completely closing all the fermion lines which is possible now that $y_{uL} \neq 0$. This then gives a new contribution to the axion potential,

$$V(φ) = κ M^4 \cos \left( φ \frac{f}{f} + \bar{θ}_{QCD} \right) + m^2 f^2 φ \left( \frac{φ}{f} + \bar{θ}_{QCD} \right)$$

where,

$$κ \sim \kappa_s \left( \frac{g_4 ψ}{16\pi^2} \right)^3 \prod_{f=u,d,s,c,b,t} \frac{y_{fL}^2 y_{fR}^2}{4π}.$$ \hspace{1cm} (22)

Again, all the above couplings are at the scale $M$ and all the $y_{fR} = y_{uR,dR}$ depending on whether $f$ is an up or down type fermion (see Eq. 11). The second term above is the usual large instanton contribution in the IR, with $m_+$ being the pion mass and $f_+$, the pion decay constant. Both the terms are aligned in phase because both the contributions arise from closing the same ‘t Hooft vertex, but at different scales.

Solving the RG equations in Eq. 16 assuming $b = 1/4$, taking $d_{UL,DL} = 7/2$, $y_{uR,dR}(M) = 4π$ in Eq. 11, $y_{uR,dR}(m_+) = y_{uR,dR}(M)/10$, and other boundary conditions fixed by the measured value of the SM fermions masses, we obtain,

$$κ \sim \kappa_s \left( \frac{g_4 ψ}{16\pi^2} \right)^3 4.2 \times 10^{-4}.$$ \hspace{1cm} (23)

The above numerical value is the maximal possible one corresponding to the case when all the couplings saturate their perturbative limit. The factor, $κ_s$, can vary over a large range from unity to exponentially small values as the strong coupling is varied; the axion mass can thus vary from its minimum value due to the IR contribution to values as large as 10 TeV when the suppression factor, $κ$, approaches the maximal value in Eq. 23.

We show the allowed region in the coupling-mass parameter space in Fig. 3, where the $y$-axis is $g_φ$, the axion coupling to photons, defined by the coupling,

$$-\frac{g_φ}{4} F_{µν} \tilde{F}^{µν}.$$ \hspace{1cm} (24)
HB Stars CAST
Cosmology
Haloscopes
γ-rays
Axion
-10 -5 0 5 10
-20
-18
-16
-14
-12
-10
Log10mϕ [eV]
Log10 $g_{a\gamma}$ [GeV$^{-1}$]

Figure 3: The allowed parameter space for the QCD axion of Sec. V is shown in red. The other bounds have been adapted from Ref. [14, 24] where a detailed discussion of these can be found.

where,

$$g_{a\gamma} \sim \frac{\alpha_{em}}{2\pi f}.$$  \hspace{1cm} (25)

We see that a huge area in the parameter space is allowed for the QCD axion in this model. The usual QCD axion band is the left edge of the area shown in Fig. 3. While parts of this area are ruled out by existing constraints— in particular cosmological ones in the region where a thermal population of the axion can exist [24]— large parts of the allowed region still remain unconstrained.

VI. CONCLUSIONS

In this work we showed that small instanton effects can become very important in standard UV completions of composite Higgs models with partially composite fermions. This is possible because both the QCD gauge coupling and effective Yukawa interactions run to larger values with energy resulting in unsuppressed instanton contributions in the UV. As far as the Yukawa interactions are concerned, it is well-known that in partially composite models, the hierarchical nature of the SM fermion masses and mixings can arise from anarchic and $\mathcal{O}(1)$ interactions in the UV. The QCD coupling grows because in order to have fermionic partners for all SM fermions, many new fermions in the hypercolour sector need to be introduced. These fermions are also charged under QCD and this generates a positive $\beta$-function which results in the QCD gauge coupling running to non-perturbative values in the UV. The only modification of the lagrangian required to achieve this effect are the explicit breaking terms in Eq. 9.

As a consequence of this enhancement of small instanton contributions, we show that the up quark mass can arise entirely from instanton contributions. This implies that the up quark mass vanishes in the deep UV and is only generated additively by instanton effects. In the deep UV one can thus rotate away the strong CP phase. In an alternative scenario where an axion field exists, we show that its mass can be as large as 10 TeV. The allowed parameter space is much larger than the usual QCD band as shown in Fig. 3. Our model thus opens up new areas in the coupling-mass parameter space that are still unconstrained by existing bounds. This motivates the development of new experimental strategies to probe these regions.

Note added: While we were in the process of completing this project, Ref. [25] appeared, that also provides a heavy axion candidate in composite models. While this work also utilises the set-up of Ref. [19], there is no overlap with Sec. V. This is because Ref. [25] does not use the same mechanisms for enhancing small instanton contributions, namely running of the QCD gauge coupling and the $\lambda_{ij}^2$, that have been used in this work. The other important differences include the fact that Ref. [25] has an in-built axion candidate and, unlike this work, it requires a hypercolour condensation scale of at least 1000 TeV resulting in a tuned Higgs sector.

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[28] The diagrams shown in Ref. [23] for the SM would apply for our model as long as the Higgs fermion couplings in the SM diagram are appropriately replaced by strong dynamics. Apart from RG effects there are also finite contributions to the strong CP phase from the CKM phase, but in Ref. [23] these were also shown to be much smaller than the experimental bound, $\theta_{QCD} \lesssim 10^{-10}$, from neutral electric dipole moment experiments [26, 27].