Scotogenic model with $B - L$ symmetry and exotic neutrinos

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Abstract

We consider a model with three Higgs doublet in a discrete $B - L \times Z_3$ discrete symmetries. Two of the scalar doublets are inert due to the $Z_3$ symmetry. We calculated all the mass spectra in the scalar and lepton sectors and accommodate the leptonic mixing matrix as well.

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The existence of inert scalar fields comes from very long ago, in particular the inert doublet \[1\]. Here we consider a model in which the scotogenic mechanism \[2\] is at work generating neutrino masses and the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. The model has three doublets with the same quantum number as the standard model Higgs doublet: $H \equiv S, D_1, D_2$, being the latter two doublets inert because of an unbroken (at the tree level) $Z_3$ symmetry. We also include three right-handed neutrinos but with nos-standard assignment of $B - L$. See the Table \[1\] for the transformation under $B - L$ and $Z_3$ of the lepton and scalar fields. Although neutrinos with this $B - L$ assignment make this symmetry free of anomalies \[3, 4\], we will consider it a global symmetry.

The Yukawa interactions are given by

$$-L_{\text{Yukawa}}^{\text{leptons}} = G^l_{ij} \bar{L}_i l_R S + G^\nu_{ij} \bar{L}_i N_{1R} \tilde{D}_1 + G^\nu_{2ik} \bar{L}_i L N_{kR} \tilde{D}_2$$

$$+ M_1 \bar{N}_{1R} N_{1R} + M_{kl} \bar{N}_{kR} N_{lR} + H.c.,$$

where $i, j = e, \mu, \tau$ and $k, l = 2, 3$.

The more general $SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$ invariant scalar potential for the three doublets, is given by:

$$V(S, D_1, D_2) = \mu^2_{SM} |S|^2 + \mu^2_{D1}|D_1|^2 + \mu^2_{D2}|D_2|^2 + \lambda_1 |S^\dagger S|^2 + \lambda_2 |D_1^\dagger D_1|^2 + \lambda_3 |D_2^\dagger D_2|^2$$

$$+ \lambda_4 |S^\dagger D_1|^2 + \lambda_5 |S^\dagger D_2|^2 + \lambda_6 |D_1^\dagger D_2|^2 + \lambda_7 |S^\dagger D_1|^2$$

$$+ \lambda_8 |S^\dagger D_2|^2 + [\mu^2_{12} D_1^\dagger D_2 + \lambda_9 (S^\dagger D_1)^2 + \lambda_{10} (S^\dagger D_2)^2 + H.c.]$$

Notice that the term $\mu^2_{12}$ and $\lambda_{9,10}$ break $B - L$ softly and hard, respectively. We will assume these parameters are real. If $\mu^2_{12} \neq 0$ and $\lambda_{9,10} = 0$ the doublets are not inert anymore and we have a mechanism as in Ref. \[5\] in which the smallness of the neutrino masses is due to the smallness of the VEV of the doublet(s) $D_1(D_2)$. On the other hand, if $\mu^2_{12} = 0$ and $\lambda_{9,10} \neq 0$ we have the scotogenic mechanism with two inert doublet as in Ref. [fortes]. Here we will consider only the latter case.

Doing as usual the shifted as $S^0 = \frac{1}{\sqrt{2}}(v_{SM} + h + iG)$ with $v_{SM} = 246$ GeV, and $D^0_{1,2} = \frac{1}{\sqrt{2}}(R_{1,2} + iI_{1,2})$, so that the constraint equations are given by:

$$v_{SM}(2\mu^2_s + 2v^2_{SM}\lambda_1) = 0$$

The masses matrices are all diagonal and the eigenvalues are:
• For CP even scalars:

\[ m_{R1} = 2\lambda_1 v_{SM}^2 \]
\[ m_{R2} = \mu_{d1}^2 + \frac{v_{SM}^2}{2} (2\lambda_{10} + \lambda_5 + \lambda_8) \]
\[ m_{R3} = \mu_{d2}^2 + \frac{v_{SM}^2}{2} (2\lambda_9 + \lambda_5 + \lambda_8) \] (4)

• For CP odd scalars:

\[ m_{I1} = 0 \]
\[ m_{I2} = \mu_{d1}^2 + \frac{v_{SM}^2}{2} (-2\lambda_{10} + \lambda_5 + \lambda_8) \]
\[ m_{I3} = \mu_{d2}^2 + \frac{v_{SM}^2}{2} (-2\lambda_9 + \lambda_5 + \lambda_8) \] (5)

• For charged scalars:

\[ m_{c1} = 0 \]
\[ m_{c2} = \frac{1}{4} (2\mu_{d1}^2 + \lambda_4 v_{SM}^2) \]
\[ m_{c3} = \frac{1}{4} (2\mu_{d2}^2 + \lambda_5 v_{SM}^2) \] (6)

The charged leptons has the following Yukawas: \( G^l_{11} = 0.000412, G^l_{12} = G^l_{21} = 0.00050, G^l_{22} = 0.0036553, G^l_{13} = G^l_{31} = -0.0007823, G^l_{23} = G^l_{32} = -0.00348, G^l_{33} = 0.0035874 \), and we obtain \( m_e = 0.510 \text{ MeV}, m_\mu = 105.658 \text{ MeV} \) and \( m_\tau = 1776.86 \text{ MeV} \) and the matrix \( V_L^l \) defined as \( l'_i = V_L^l l_L \) (primed fields are symmetry eigenstates and unprimed ones mass eigenstates):

\[
V_L^l = \begin{pmatrix}
0.564898 & 0.527065 & 0.634897 \\
-0.814496 & 0.479468 & 0.326661 \\
-0.132241 & -0.701651 & 0.700142
\end{pmatrix}
\] (7)

Defining the lepton mixing matrix as \( V_{PMNS} = V_L^l V_L^{l\dagger} \), it means that this matrix appears in the charged currents coupled to \( W^- \).

Notice that neutrinos are still massless at tree level as in SM. However, in the scalar potential there are the interactions like \( \lambda_4 \) and \( \lambda_5 \) in the scalar potential in (2), that induce the sort of diagrams as those in Fig. [1] With these interactions, it is possible to implement the mechanism of Ref. [2] for the radiative generation of neutrinos masses.
In fact, the diagram in Fig. 1 are exactly calculable from the exchange of \(ReD_{1,2}^0\) and \(Im\phi^0_{1,2}\) \[2\]

\[
(M_{\nu})_{ij} = \sum_{a,k} \frac{Y_{ia}Y_{jk}M_k}{32\pi^2} \left[ \frac{m_{Ra}^2}{m_{Ra}^2 - M_k^2} \ln \frac{m_{Ra}^2}{M_k^2} - \frac{m_{Ia}^2}{m_{Ia}^2 - M_k^2} \ln \frac{m_{Ia}^2}{M_k^2} \right]
\]

where \(m_{Ra}\) and \(m_{Ia}\) with \(a = 1, 2\) are the masses of \(ReD_{1,2}^0\) and \(ImD_{1,2}^0\), respectively. In the present model the \(Y_{ij}\) corresponds to \(G_{2ij}^\nu\) when the coupling is with the \(N_{2,3R}\) and \(Y_{ij}\) corresponds to \(G_{1i}^\nu\) when the coupling is with the \(N_{1R}\). We can define \(\Delta^2 = m_{Ra}^2 - m_{Ia}^2 = 2\lambda_g v_{SM}^2\), and \(m_{0a}^2 = (m_{Ra}^2 + m_{Ia}^2)/2\), \(a = 1, 2\). If \(\lambda_g \ll 1\), we can write

\[
(M_{\nu})_{ij} = \frac{v_{SM}^2}{8\pi^2} \left[ \lambda_{10} \frac{G_{1i}^\nu G_{ij}^\nu M_1}{m_{R1}^2 - M_1^2} \left( 1 - \frac{M_1^2}{m_{R1}^2 - M_1^2} \right) + \lambda_9 \frac{G_{2i}^\nu G_{2j}^\nu M_k}{m_{R2}^2 - M_k^2} \left( 1 - \frac{M_k^2}{m_{R2}^2 - M_k^2} \right) \right]
\]

where we have omitted a sum in \(k = 2, 3\).

In order to obtain the active neutrinos masses we assume a normal hierarchy and, without loss of generality, that \(M_1 \sim M_{2,3}\) and will be represented from now on by \(M_R\). \(M^\nu\) is diagonalized with a unitary matrix \(V_L^\nu\) i.e., \(\hat{M}^\nu = V_L^\nu M^\nu V_L^\nu\), where \(\hat{M}^\nu = \text{diag}(m_1, m_2, m_3)\).

In the charged lepton sector we assume their masses at the central values in PDG \(M^l = (0.510, 105.658, 1776.86)\) GeV. It is important to note from these considerations, that there exist a multitude of other possibilities which satisfy also the masses squared differences and the astrophysical limits in the active neutrino sector. Each one corresponds to different parametrization of the unitary matrices \(V^l_{L,R}, V^\nu_{L}\).

We will obtain the neutrinos masses from Eq. (9), we have as free parameters \(\lambda, M_R, M_{D^+_1}, M_{D^+_2}\) and the Yukawas. In the Fig. 2 we show the dependence of \(\lambda\) respect the main Yukawas \(G_{\nu e}^{\nu d}\) in (a) and \(G_{\nu e}^{\nu e}\) in (b) for fixed \(M_R\) values. Notice that \(G_{e1}\) are essentially of the same order of magnitude, while the rest are suppressed by four orders of magnitude when comparing with any specific value of those \(G_{\nu e}^{\nu e}\).

The mass matrices in the charged sector \(M^l\) are diagonalized by a bi-unitary transformation \(\hat{M}^l = V_{L}^l M^l V_{R}^l\) and \(\hat{M}^l = \text{diag}(m_e, m_\mu, m_\tau)\). The relation between symmetry eigenstates (primed) and mass (unprimed) fields are \(l'_{L,R} = V_{L,R} l_{L,R}\) and \(\nu'_L = V_{L}^\nu \nu_L\), where \(l'_{L,R} = (e', \mu', \tau')_{L,R}, l_{L,R} = (e, \mu, \tau)_{L,R}\) and \(\nu'_L = (\nu_e, \nu_\mu, \nu_\tau)_{L}\) and \(\nu_L = (\nu_1, \nu_2, \nu_3)_{L}\). Defining the lepton mixing matrix as \(V_{PMNS} = V_{L}^l V_{L}^{\nu}\), it means that this matrix appears in the charged currents coupled to \(W^\pm\). From Eq.(A1) and Eq.(A5) to Eq.(??) to Eq.(??) we obtain Eq.(??).
TABLE I: Masses of the scalars in Scotogenic model (in GeV).

| Sol. | Masses in TeV | $G^i_{11}$ | $G^i_{12}$ | $G^i_{13}$ | $G^i_{22}$ | $G^i_{23}$ | $G^i_{33}$ |
|------|---------------|-------------|------------|------------|------------|------------|------------|
| P1   | $M_R = 0.5, M_0 = 2.2$ | 0.000421836 | 0.000514741 | -0.000800772 | 0.00374767 | -0.00356758 | 0.00367645 |
| P2   | $M_R = 1.5, M_0 = 2.2$ | 0.000421836 | 0.000514741 | -0.000800772 | 0.00374767 | -0.00356758 | 0.00367645 |
| P3   | $M_R = 2.5, M_0 = 2.2$ | 0.000421836 | 0.000514741 | -0.000800772 | 0.00374767 | -0.00356758 | 0.00367645 |
| P4   | $M_R = 3, M_0 = 2.2$  | 0.00042195  | 0.000515086 | -0.000801103 | 0.00374772 | -0.00356751 | 0.00367645 |

\[
|V_{PMNS}| \approx \begin{pmatrix} 0.815 & 0.565 & 0.132 \\ 0.479 & 0.527 & 0.702 \\ 0.327 & 0.635 & 0.700 \end{pmatrix}, \quad (10)
\]

which is in agreement within the experimental error data at 3\(\sigma\) given by [7]

\[
|V_{PMNS}| \approx \begin{pmatrix} 0.795 - 0.846 & 0.513 - 0.585 & 0.126 - 0.178 \\ 0.4205 - 0.543 & 0.416 - 0.730 & 0.579 - 0.808 \\ 0.215 - 0.548 & 0.409 - 0.725 & 0.567 - 0.800 \end{pmatrix}, \quad (11)
\]

and we see that it is possible to accommodate all lepton masses and the PMNS matrix. Here we do not consider CP violation.

I. THE BOUNDEDNESS OF THE POTENTIAL

The boundedness of the potential from below has to be a criteria defined allowing the greatest number of the parameter space. In the earlier work in Ref.[10] was computed some set of vacuum stability conditions following the Copositive Criteria for the Boundedness of the Scalar Potential, in short the basic idea is construct the quartic couplings as a pure square of the combinations of bilinear scalar fields and set their coefficients could be non-negative, with this we can certainly makes the vacuum stable. However, for scalar potential more complicated certain amount of ambiguities may arise, for more details see Ref.[10].

So in the base $|S|^2, |D_1|^2, |D_2|^2$, we have:
$$A = \begin{pmatrix}
\lambda_1 & \lambda_4 & \lambda_5 + \lambda_9 r_1^2 \\
\lambda_4 & \lambda_2 & \lambda_6 + \lambda_{10} r_2^2 \\
\lambda_5 + \lambda_9 r_1^2 & \lambda_6 + \lambda_{10} r_2^2 & \lambda_3
\end{pmatrix}.$$  \hspace{1cm} (12)

The values for $r_i^2$ are those to minimize the entries of the matrix. We have two relevant cases for the off-diagonal elements with sums: if both coupling constants are positive/negative, the minimum comes from choosing $r_i^2 = 0$; if the constants have opposite signs, the minimum comes from $r_i^2 = 1$.

For a symmetric matrix $A$ of order 3 the copositivity criteria are summarized as follows: $a_{ii} > 0$ and $v_{ij} = a_{ij} + \sqrt{a_{ii} a_{jj}} > 0$ and $\sqrt{a_{11} a_{22} a_{33}} + a_{12} \sqrt{a_{33}} + a_{13} \sqrt{a_{22}} + a_{23} \sqrt{a_{11}} + \sqrt{v_{12} v_{13} v_{23}} > 0$. Explicitly we obtain:

\begin{align*}
\lambda_1 &> 0, \\
\lambda_2 &> 0, \\
\lambda_3 &> 0, \\
\lambda_4 + \sqrt{\lambda_1 \lambda_2} &> 0, \\
\lambda_5 -\lambda_9 + \sqrt{\lambda_1 \lambda_3} > 0 \text{ or } \lambda_5 + \sqrt{\lambda_1 \lambda_3} > 0, \\
\lambda_6 -\lambda_{10} + \sqrt{\lambda_2 \lambda_3} > 0 \text{ or } \lambda_6 + \sqrt{\lambda_2 \lambda_3} > 0
\end{align*}

(13)

and

\begin{align*}
\sqrt{(\lambda_1 \lambda_2 \lambda_3)} + \lambda_4 \sqrt{\lambda_3} \\
+ (\lambda_5 + \lambda_9) \sqrt{\lambda_2} + (\lambda_6 + \lambda_{10}) \sqrt{\lambda_3} > 0,
\end{align*}

(14)

It is easy to verify that if the conditions in Eqs. (13) are satisfied the conditions in Eq. (14) are automatically satisfied. Hence, the positivity of the scalar potential is guarantee just by the conditions in Eq. (13).

II. CONCLUSIONS

The scotogenic mechanism is based on the existence of inert doublets (they do not couple with charged leptons and do not contribute to the masses of the vector bosons) and this character is due the existence of a discrete symmetry, usually $Z_2$ or $S_3$ [6]. However, It
has been noted in Refs. [8, 9] that the running of the parameters in model may induce a spontaneously breakdown of this symmetry and the doublets lose its inert character. This occurs also in the present model in which the discrete symmetry is $Z_3$. The renormalization group equation of the $\mu_d^2 > 0$ parameter is

$$16\pi^2 \frac{d}{dt} \mu_d^2 = H(\mu_x^2, \{a\}, T, T^\nu, g^2_i) - 4M_1^2 \text{Tr}(G_1^\nu G_1^{\nu\dagger}) - 4 \sum_{k=2}^3 M_k^2(G_2^\nu G_2^{\nu\dagger})_{kk},$$

with $t = \ln \mu$ where $\mu$ is the energy scale at which $\mu_d^2$ are calculated, $\mu_x^2 = \mu_s^2, \mu_d^2, T$ and $T^\nu$ involve the Yukawa matrices in the charged leptons and neutrinos, respectively; $g_i^2 = g_{1,2}^2$ are the gauge couplings, and $\{a\}$ denotes the dimensionless couplings in the scalar potential. $\mu_{SM}^2$ and the Yukawa couplings have a similar expression but without the last term. The important point is that $H$ is positive. We see from (??) that if $M_1$ and/or $M_k$ are large enough, the scalars $D_{1,2}^0$ lose its inert character, $\mu_d^2 < 0$, in a given scale $t^*$. [8, 9]. For small neutrino Yukawa couplings and $|a| \ll 1$, the positive term is dominated by $\mu_d^2$ [8]

$$16\pi^2 \frac{d}{dt} \mu_d^2 \approx \mu_d^2 - 4 \sum_{k=2}^3 M_k^2(G_2^\nu G_2^{\nu\dagger})_{kk},$$

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**Appendix A: $V_L^j$ parameterizations**

The five $V_L^j$ parameterizations:

$$V_L^j(P1) = \begin{pmatrix} 
0.564875 & 0.527096 & 0.634891 \\
0.814545 & -0.479284 & -0.326809 \\
-0.132033 & -0.701753 & 0.700079 
\end{pmatrix},$$

$$V_L^j(P2) = \begin{pmatrix} 
-0.564882 & -0.527082 & -0.634896 \\
0.814538 & -0.479297 & -0.326806 \\
-0.13205 & -0.701754 & 0.700075 
\end{pmatrix},$$
\[ V_L(P3) = \begin{pmatrix} 0.653815 & -0.520669 & -0.549026 \\ 0.756087 & 0.477658 & 0.447409 \\ -0.0292947 & 0.707634 & -0.705971 \end{pmatrix}, \quad (A3) \]

\[ V_L(P4) = \begin{pmatrix} -0.564901 & -0.527041 & -0.634913 \\ 0.814516 & -0.479339 & -0.3268 \\ -0.132102 & -0.701756 & 0.700062 \end{pmatrix}, \quad (A4) \]

The five \( V_L^\nu \) parameterizations:

\[ V_L^\nu(P1) = \begin{pmatrix} 0.0000963407 & 0.00005628 & 1 \\ -1 & -0.000135305 & 0.0000963483 \\ 0.0013531 & -1 & 0.000056267 \end{pmatrix}, \quad (A5) \]

\[ V_L^\nu(P2) = \begin{pmatrix} 0.0000914927 & 0.0000534808 & 1 \\ -1 & -0.000128688 & 0.0000914996 \\ 0.00128693 & -1 & 0.0000534691 \end{pmatrix}, \quad (A6) \]

\[ V_L^\nu(P3) = \begin{pmatrix} 0.0000859045 & 0.0000502026 & 1 \\ -1 & -0.000120763 & 0.0000859106 \\ 0.00120767 & -1 & 0.0000501922 \end{pmatrix}, \quad (A7) \]

\[ V_L^\nu(P4) = \begin{pmatrix} 0.0000765121 & 0.0000447061 & 1 \\ 1 & 0.000107519 & -0.0000765169 \\ 0.000107522 & -1 & 0.0000446979 \end{pmatrix}, \quad (A8) \]

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TABLE II: Transformation properties of the fermion and scalar fields under $(B - L)$. We do not include quarks because they are singlet under $Z_3$ and $B - L = +1/3$ as usual.

|       | $L_1$ | $l_{jR}$ | $N_{1R}$ | $N_{2,3R}$ | $S$ | $D_1$ | $D_2$ |
|-------|-------|----------|----------|------------|-----|-------|-------|
| $(B - L)$ | -1    | -1       | -5       | +4         | 0   | 6     | 3     |
| $Z_3$  | w     | 1        | 1        | w          | w   | w     | w     |
FIG. 1: one-loop for neutrinos masses. The vertices A, B, C and D are given in Eq.(??)
FIG. 2: Dependence of $\lambda$ respect the main Yukawa $G_{e1}$ (a) for fixed Values de $M_R$. In (b) the other Yukawas provide the same value for any $M_R$.