Bulk viscosity in nuclear and quark matter: a short review

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Abstract. The history and recent progresses in the study of bulk viscosity in nuclear and quark matter are reviewed. The constraints from baryon number conservation and electric neutrality in quark matter on particle densities and fluid velocity divergences are discussed.

The time scale for damping of the vibration and of the gravitational radiation driven instability in neutron stars is crucial to the stellar stability, which is controlled by the shear and bulk viscosities. The bulk viscosity originated from the re-establishment of chemical equilibrium is important in some circumstances. In this short note, we will give an overview on the history and recent progresses in the study of bulk viscosity in nuclear and quark matter.

1. Nuclear matter

Sawyer [1] pointed out that at temperatures higher than $10^9$ K the bulk viscosity for neutron matter is much larger than the shear viscosity. This is determined by opposite temperature behavior of the bulk and shear viscosities that the bulk viscosity increases while the shear decreases with growing temperature. Since then, the bulk viscosities in nuclear matter have been widely investigated.

*npe*(µ) matter. Low density neutron star matter is composed of neutrons $n$ and a small admixture of protons $p$, electrons $e$, and possibly muons $µ$. The bulk viscosity is mainly determined by the modified Urca processes,

$$N + n \rightarrow N + p + l + ν_l, \quad N + p + l \rightarrow N + n + ν_l,$$

(1)

where $N$ denotes the spectator nucleon for energy and momentum conservation, $l$ the electron or muon, and $ν_l$ the associated neutrino. In late 1960s, Finzi and Wolf analyzed the damping of neutron star pulsations via modified Urca processes in *npe* matter [2], followed by Sawyer [1] and Haensel *et al.* [3] who investigated the bulk viscosity for *npe* and *npeµ* matter respectively. The relaxation time for the modified Urca processes is of order $τ ∝ T^{-6}$. 
At densities of a few times normal nuclear matter density, the direct Urca processes,
\[ n \rightarrow p + l + \nu, \quad p + l \rightarrow n + \nu, \]  
may also be allowed provided the proton fraction exceeds the Urca limit of about 1/9 [4]. The bulk viscosity was calculated for the direct Urca processes by Haensel and Schaeffer [5] in npe matter and by Haensel et al. [6] in npeµ matter. A weaker temperature dependence for the relaxation time, \( \tau \propto T^{-4} \), is found due to a smaller number of particles involved. Consequently the bulk viscosity for the direct Urca is about 4-6 orders of magnitude larger than that from the modified Urca at typical neutron star temperatures, \( T \sim 10^9\text{--}10^{10} \text{K} \). The large difference occurs at low temperatures.

There may be superfluids in neutron star matter due to attractive part of baryon-baryon potential, for reviews, see e.g., Refs. [7, 8]. The bulk viscosity with superfluidity for the direct and modified Urca processes in npeµ matter was investigated by Haensel et al. [3, 6], which is substantially suppressed compared to normal state matter.

**Hyperon matter.** Hyperons may appear in the neutron star core [9, 10, 11, 12]. With increasing densities, \( \Sigma^- \) and \( \Lambda \) hyperons appear, followed by \( \Xi^0, \Xi^- \) and \( \Sigma^+ \). Most authors only considered \( \Sigma^- \) and \( \Lambda \) hyperons which have the lowest threshold densities. Weak nonleptonic hyperon processes such as
\[ n + n \leftrightarrow p + \Sigma^-, \quad n + p \leftrightarrow p + \Lambda, \quad n + n \leftrightarrow n + \Lambda \]  
dominate the bulk viscosity, while their direct Urca processes [13] have negligible contributions although they are comparable to nucleons’ Urca. The relaxation time from processes (3) is of order \( T^{-2} \).

Langer and Cameron introduced the subject of bulk viscosity in hyperonic matter by estimating the damping of neutron star vibrations [14]. Jones carried out the semi-quantitative calculation of bulk viscosity in hyperonic matter [15, 16], where the weak nonleptonic process \( nn \leftrightarrow p\Sigma^- \) through \( W \)-exchange was calculated. Recently, Jones [17] studied the reaction \( nn \leftrightarrow n\Lambda \), the dominant channel for \( \Lambda \) production in hypernuclei experiments in general, which cannot be mediated by single \( W \) boson.

The rate of the reaction \( nn \leftrightarrow n\Lambda \) from the data is found to be several orders of magnitude larger than \( nn \leftrightarrow p\Sigma^- \) via the \( W \)-exchange. All of these works are the order-of-magnitude estimates.

The various weak nonleptonic hyperon processes have been recalculated by several authors with a modern equation of state. Haensel et al. [18] studied the \( nn \leftrightarrow p\Sigma^- \) process within the nonrelativistic limit, they found the bulk viscosity to be several orders of magnitude larger than that of the direct and modified Urca processes. Lindblom and Owen [19] computed the contribution of the \( np \leftrightarrow p\Lambda \) in addition to the \( nn \leftrightarrow p\Sigma^- \) process. The superfluidity case had also been studied in both works.

Most of the weak hadronic processes involved in the \( nn \leftrightarrow n\Lambda \) process also contribute to the \( nn \leftrightarrow p\Sigma^- \) one, so the rates of these two processes should be of the same order, opposite to Jones’ estimate. Dalen and Dieperink [20] studied the bulk viscosity from all three processes in [3] using one-pion-exchange (OPE), which can well describe the rates of these processes in hypernuclei. Their results showed that the bulk
viscosity in the OPE picture is about 1-2 orders of magnitude smaller than that with \( W \)-exchange.

Recent hypernuclei data showed the potentials for \( \Lambda \) and \( \Xi \) are attractive but that for \( \Sigma \) is repulsive in normal nuclear matter \([21, 22]\). Chatterjee and Bandyopadhyay \([23]\) took this fact into account and found the disappearance of \( \Sigma \) in nuclear matter. They calculated the bulk viscosity from \( np \leftrightarrow p\Lambda \). Their results showed that the bulk viscosity incorporating the hyperon potentials implied by the data is larger than that without them.

2. Quark matter

When the baryon density is above 5-10 times the normal nuclear matter density, the deconfinement phase transition will take place marked by the formation of quark matter. Initially there are only \( u \) and \( d \) quarks, as the density grows the \( s \) quarks will appear. Quark matter with \( u, d, \) and \( s \) quarks can be self-bound and be called strange quark matter \([24, 25]\). The star made of strange quark matter is called quark star or strange star \([26, 27]\).

Nonleptonic processes. The importance of dissipation due to the nonleptonic reaction,

\[
s + u \leftrightarrow u + d. \tag{4}
\]

was first observed by Wang and Lu \([28]\). They showed that stellar pulsations would be strongly damped in quark matter. Saywer \([29]\) then formulated the damping in terms of the bulk viscosity based the linear expansion of the reaction rate of \((1)\) in \( \delta \mu = \mu_s - \mu_d \), where \( \mu_i \) with \( i = s, d \) are the quark chemical potentials. In the temperature range characteristic of young neutron stars, the bulk viscosity arising from the reaction \((1)\) is orders of magnitude larger than that for normal nuclear matter. However the above linear assumption is not proper at low temperatures \( (T \ll \delta \mu) \), where the rate is proportional to \( \delta \mu^3 \). Madsen \([30]\) showed that this nonlinearity effect leads to much larger the bulk viscosity than previously assumed. Note that strong interactions can also influence the rate of the reaction \((1)\) and the bulk viscosity of strange quark matter significantly \([31]\). At low temperatures, the viscosity is strongly suppressed, while at high temperatures it is slightly enhanced. The above calculations are based on MIT bag model where all quark masses are taken to be constants. Alternatively quark massess can be assumed to depend on baryon density \([32]\). By employing this density dependent quark model, the bulk viscosity at low temperatures and high relative perturbations increases 2-3 orders of magnitude, while at low perturbations the enhancement is 1-2 orders of magnitude compared to the results obtained in other approaches \([33]\).

Sufficiently dense and cold quark matter is expected to be a color superconductor \([34]\). A recent interest in the study of the bulk viscosity in color superconducting phases is growing, for example, the bulk viscosity in the 2-flavors color superconducting phase (2SC) from the reaction \((1)\) has been computed by Alford and Schmitt \([35]\). The bulk viscosity due to kaons in color-flavor-locked (CFL) phase has also been calculated \([36]\).
Leptonic processes. Due to phase space restrictions \[37, 38\] the rates of the leptonic processes,

\[ u + e \rightarrow q + \nu_e, \quad q \rightarrow u + e + \bar{\nu}_e, \quad (q = d, s) \]  \hspace{1cm} (5)

are much smaller than nonleptonic ones at low temperatures. These Urca processes are the most efficient way to cool the neutron stars. As shown by Anand et al. \[33\], when the temperature increases the contribution of leptonic processes to the bulk viscosity will exceed that from nonleptonic ones. So more careful studies are needed to explore the dependence of bulk viscosity on the broader ranges of temperature. The energy emissivity in Urca processes in various color superconducting phases have been studied by several groups \[39, 40, 41, 42\]. The bulk viscosity from the Urca processes for d quarks in a spin-one color superconductor has been calculated by Sa’d, Shovkovy and Rischke \[43\].

3. Baryon number conservation and enforced charge neutrality for bulk viscosity in quark matter

A more realistic case for the bulk viscosity in quark matter is to take both of nonleptonic and leptonic processes into account. There must be electrons to compensate the net positive charge in a three-flavor quark system because the mass of s quarks is much larger than those of light quarks. The bulk viscosity should be calculated for charge neutral quark matter. In nonleptonic and leptonic reactions, the electron number and flavors are not conserved, while the baryon number and electric charge are conserved. From the continuity equations, baryon number conservation and charge neutrality, we can derive the following constraints for particle number densities and velocity divergences \[44\],

\[ n_B \nabla \cdot \mathbf{v}_B = \sum_{i=u,d,s} \frac{1}{3} n_i \nabla \cdot \mathbf{v}_i, \quad n_e \nabla \cdot \mathbf{v}_e = \sum_{i=u,d,s} Q_i n_i \nabla \cdot \mathbf{v}_i, \]  \hspace{1cm} (6)

where \( n_j, Q_j \) and \( \mathbf{v}_j \) are number densities, electric charges and fluid velocities for particle \( j = B, e, u, d, s \) respectively. The indices \( B \) and \( e \) denote baryons and electrons. Here the first equation is from baryon number conservation and the second one from charge neutrality. All previous treatments in the literature used the conditions \( n_B dX_i/dt = J_i \) to determine the bulk viscosity, where \( X_i \equiv n_i/n_B \) are fractions of baryon number for particle \( i = u, d, s, e \) and \( J_i \) their sources. Using \( dn_B/dt = -n_B \nabla \cdot \mathbf{v}_B \), we can get \( n_B dX_i/dt = dn_i/dt - X_i dn_B/dt = dn_i/dt + n_i \nabla \cdot \mathbf{v}_B = J_i \). Comparing it with the continuity equation for particle \( i \), \( dn_i/dt + n_i \nabla \cdot \mathbf{v}_i = J_i \), we obtain \( \nabla \cdot \mathbf{v}_B = \nabla \cdot \mathbf{v}_i \), which obey Eq. (6) obviously. Hence the usage of \( n_B dX_i/dt = J_i \) in literature implies the unique value of velocity divergences for all particle species. However, considering the special role of strange quarks due to their large mass, we propose a new possibility that the velocity divergence for s quarks is different from those of light quarks and electrons, which corresponds to a new oscillation pattern for the bulk viscosity. As an extreme case, we assume \( \nabla \cdot \mathbf{v}_s = 0 \) and \( \nabla \cdot \mathbf{v}_u = \nabla \cdot \mathbf{v}_d \). From Eq. (6), we derive \( \nabla \cdot \mathbf{v}_e = n_p (2n_u - n_d)/n_e (n_u + n_d) \nabla \cdot \mathbf{v}_B \) and \( \nabla \cdot \mathbf{v}_{u,d} = 3n_p n_u/n_u + n_d \nabla \cdot \mathbf{v}_B \). We can use \( \delta n_B, \delta n_e \) and \( \delta n_s \) as independent variables for the
density oscillation. Then $\delta n_{u,d}$ and $\delta P$ can be expressed in terms of these independent variables. In order to close the system of equations for the bulk viscosity, we need two additional inputs, e.g. $d\delta n_{e}/dt + n_{e}\nabla \cdot \mathbf{v}_{e}$ and $d\delta n_{s}/dt$ (note that $\nabla \cdot \mathbf{v}_s = 0$), given by the reaction rates which can also be expressed in terms of independent variables. The former can be written as $d\delta n_{e}/dt + n_{e}\nabla \cdot \mathbf{v}_{e} = d\delta n_{e}/dt - (2n_u - n_d)/(n_u + n_d)d\delta n_{B}/dt$. Then the bulk viscosity is determined from the definition $\delta P = -\zeta \nabla \cdot \mathbf{v}_{B}$. The results are shown in Fig. 3 of Ref. [41]. The values of the bulk viscosity in this solution is 1-2 orders of magnitude larger than the conventional ones.

Note that the assumption $\nabla \cdot \mathbf{v}_s = 0$ reflects the consideration that the $s$ quarks are much heavier and may respond to the density oscillation more reluctantly than light particles. If the strange quark mass is small, one can investigate many other solutions which are close to the conventional solution $\nabla \cdot \mathbf{v}_{B} = \nabla \cdot \mathbf{v}_{s}$, for example, one can assume velocity divergences of particles deviate from that of baryons by a small amount.

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