Few-nucleon systems interacting via non-local quark-model baryon-baryon interaction

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Abstract. Low- and medium-energy nucleon-deuteron scattering less than the incident energy 65 MeV per nucleon is examined by using the quark-model nucleon-nucleon interaction fss2. The off-shell effect of this non-local interaction yields a part of the attractive effect given by the three-nucleon force in the standard description by meson-exchange potentials. The triton binding energy and the spin-doublet scattering length are well described by fss2 without introducing the three-nucleon force. The nucleon analyzing power is slightly improved from the results by the AV18 potential, but the so-called $A_y$ puzzle in the low-energy region still remains. Similar reduction of the peak heights by the Coulomb force is also seen in the vector analyzing power of the deuteron, $iT_{11}(\theta)$. Other observables are rather similar to the predictions by meson-exchange potentials, including discrepancies of deuteron breakup differential cross sections from experiment for some particular kinematical configurations. The systematic KVI data of the $^1\text{H}(\vec{d}, 2p)n$ scattering at $E_d = 130$ MeV are reasonably reproduced for breakup differential cross sections and deuteron analyzing powers.

1. Introduction
The QCD-inspired spin-flavor $SU_6$ quark model (QM) for the baryon-baryon interaction, developed by the Kyoto-Niigata group, has achieved accurate description of available nucleon-nucleon ($NN$) and hyperon-nucleon experimental data [1]. These QM baryon-baryon interactions are characterized by the nonlocality and the energy dependence inherent to the framework of the resonating-group method (RGM) for two-cluster systems composed of three-quark clusters. In the strangeness sector, the Pauli forbidden state exists as the result of the exact antisymmetrization of six quarks. The short-range repulsion of the baryon-baryon interaction is mainly described by the quark-exchange kernel, which gives quite different off-shell properties from standard meson-exchange potentials. The energy dependence of the interaction is eliminated by the standard off-shell transformation [2], utilizing the square-root inverse of the normalization kernel. This procedure yields an extra nonlocality, whose effect was examined in detail for the three-nucleon ($3N$) bound state and hypertriton [3]. We found a large triton binding energy by the QM $NN$ interaction; namely, the deficiency of 350 keV, predicted by the model fss2, is almost half of the standard values 0.5 MeV - 1 MeV [4], given by the modern meson-exchange $NN$ potentials. A similar situation is also found in the recent Faddeev-Yakubovsky calculation for the binding energy of the $\alpha$-particle by fss2.

In this study, we apply our QM $NN$ interaction fss2 to the neutron-deuteron ($nd$) and proton-deuteron ($pd$) scattering in the Faddeev formalism for systems of composite particles [5].
The Alt-Grassberger-Sandhas (AGS) equations [6] are solved in the momentum representation, using the off-shell RGM $T$-matrix obtained from the energy-independent renormalized RGM kernel [2]. The Gaussian nonlocal potential constructed from the fss2 is essentially used in the isospin basis [7]. The singularity of the $NN$ $T$-matrix from the deuteron pole is handled in the Noyes-Kowalski method [8]. Another notorious moving singularity of the free three-body Green function is treated by the standard spline interpolation technique developed by the Böchum-Krakow group [9]. The Coulomb force in the $pd$ scattering is approximately treated by the screened Coulomb force, by introducing a finite cutoff radius $\rho$ [10]. We mainly use the channel-spin formalism with $S_c = 1 \times 1/2 = 1/2$ (doublet) $+ 3/2$ (quartet) channels, which is convenient to discuss the nucleon-deuteron ($ND$) elastic scattering. The $NN$ interaction up to $I_{\text{max}} = 4$ is included in the present calculation, which leads to the nucleon incident energies discussed in this paper up to about $E_N \sim 65$ MeV.

2. Attractive features of the spin doublet $ND$ interaction in fss2 The spin-doublet $^2S_{1/2}$ state corresponds to the triton ground state and the $ND$ interaction in this channel is especially attractive because of the strong distortion effect of the deuteron. On the other hand, the spin-quartet $^4S_{3/2}$ $ND$ interaction is strongly repulsive owing to the Pauli principle. The $3N$ force is only influential in the $J^\pi = 1/2^+$ channel, and the Coulomb force is more important in the low-energy region. These features are clearly seen from the $S$-wave eigenphase shifts at $E_N = 3$ MeV, listed in Table 1. The fss2 predictions are very similar not to AV18 but to the AV18 plus Urbana (UR) $3N$ potentials (AV18+UR3N) [11] shown in the parentheses. Our $nd$ and $pd$ $^2S_{1/2}$ nuclear phase shifts are more attractive than AV18 by $3^\circ - 5^\circ$ at the energy range of $E_N = 1 - 3$ MeV. In the same energy range, the maximum difference from the AV18+UR3N results is only $0.6^\circ$. This attractive behavior is correlated to the fact that fss2 predicts the almost correct triton binding energy. Table 1 also shows that our $^2S_{1/2}$ nuclear phase shift at $E_p=3$ MeV is not so attractive as to reproduce the result of the phase-shift analysis (PSA). This is apparently because the binding energy of $^3H$ is still insufficiently reproduced even by fss2.

In Table 2, the triton binding energy, spin doublet ($^2a_{nd}$) and quartet ($^4a_{nd}$) scattering lengths by fss2 are compared with some other extensive calculations using meson-exchange potentials and $3N$ forces [12]. Since the charge dependence is not included in our calculation, the effect is estimated form the Phillips line $-0.686 \text{ fm/MeV}$ for fss2, resulting in $\Delta(^2a_{nd}) \sim 0.10 - 0.14 \text{ fm}$. In the meson-exchange potentials, the experimental values of $^2a_{nd}$ and triton binding energy are reproduced only when the $3N$ force is included. The $3N$-force effect on the doublet scattering length $^2a_{nd}$ is more than 0.4 fm. The model fss2 almost reproduces the experimental values of the triton binding energy and $^2a_{nd}$ simultaneously without the $3N$ force.

### Table 1

The $nd$ and $pd$ eigenphase shifts at $E_N = 3$ MeV, predicted by fss2. For the $pd$ calculation, the cutoff Coulomb radius $\rho = 9$ fm is used. The corresponding parameters calculated by the Pisa group for the AV18 potential and AV18+UR3N potentials (in the parentheses) are also listed for comparison [11].

| Model     | fss2 ($nd$) | AV18(+UR3N) ($nd$) | fss2 ($pd$) | AV18(+UR3N) ($pd$) | PSA       |
|-----------|-------------|---------------------|-------------|---------------------|-----------|
| $^2S_{1/2}$ | 149.2       | 144.7 (149.2)       | 152.8       | 147.8 (152.2)       | 155.15 ± 0.23 |
| $^4S_{3/2}$ | -69.6       | -69.9 (-69.7)       | -62.5       | -63.1 (-63.1)       | -63.80 ± 0.11 |
Table 2. Comparison of the \( nd \) scattering lengths predicted by using \( \text{fss2} (I_{\text{max}} = 4) \) with other models \([12]\) \( (I_{\text{max}} = 5) \). In \( \text{fss2} \), the charge dependence of the \( NN \) force is neglected. The heading \( NN \) denotes calculations using only the \( NN \) force, and \( NN + \text{TM99} \) those including the Tucson-Melbourne 99 (TM99) \( 2\pi \)-exchange \( 3N \) force. The values of \( ^4a_{nd} \) are insensitive to the \( 3N \) force. The experimental values are taken from Ref. \([13]\).

| Model          | \( NN \)     | \( NN + \text{TM99} \) | \( NN \) | \( NN + \text{TM99} \) | \( NN (+\text{TM99}) \) |
|----------------|--------------|------------------|--------|------------------|----------------------|
| \( \text{fss2} \) | 8.307        | —                | 0.66   | —                | 6.30                 |
| CD-Bonn 2000   | 8.005        | 8.482            | 0.925  | 0.569            | 6.347                |
| AV18           | 7.628        | 8.482            | 1.248  | 0.587            | 6.346                |
| Nijm I         | 7.742        | 8.485            | 1.158  | 0.594            | 6.342                |
| exp.           | 8.482        | 0.65 ± 0.04      | 6.35 ± 0.02 |

3. \( nd \) and \( pd \) differential cross sections and polarization observables

Some examples for the elastic differential cross sections and polarization observables are shown in Fig. 1 for the nucleon incident energy \( E_N = 5 \) MeV. The solid curves represent the \( pd \) results with the Coulomb cutoff radius \( \rho = 8 \) fm, while the dashed curves \( nd \) without the Coulomb force.

Figure 1. The elastic scattering observables at \( E_N = 5 \) MeV, compared with the \( nd \) and \( pd \) (or \( dp \)) experimental data; (a) the elastic differential cross section \( (d\sigma/d\Omega) \), (b) the nucleon vector analyzing power \( A_y(\theta) \), (c) the deuteron vector analyzing power \( iT_{11}(\theta) \), (d) the deuteron tensor analyzing power \( T_{20}(\theta) \), (e) \( T_{21}(\theta) \) and (f) \( T_{22}(\theta) \). All the experimental data of the deuteron analyzing powers are for the \( dp \) scattering. See Ref. \([14]\) for the experimental data.
Here, the Coulomb force is very important to reproduce spin observables in the whole angular region. When the energy becomes higher, the Coulomb effect is confined only to the forward angles. The experimental data are reasonably reproduced except for the vector analyzing power of the nucleon \(A_y(\theta)\) and that of the deuteron \((iT_{11}(\theta))\) in the low-energy region \(E_N \leq 25\) MeV. These observables are very sensitive to the partial-wave truncation of the model space and to the Coulomb force, resulting in a large modification by a slight change of eigenphase shifts.

The long-standing \(A_y\) puzzle for the large discrepancy between the theory and experiment in the low-energy region \(E_N \leq 25\) MeV still persists even in our calculations. In order to examine the discrepancy more quantitatively, we calculate the relative magnitude at the maximum point according to \(RA_{max} = (A_{y}^{th})_{max}/(A_{y}^{exp})_{max}\), where \((A_{y}^{th})_{max}\) and \((A_{y}^{exp})_{max}\) are the maximum values of \(A_y(\theta)\) in the theoretical predictions and the experimental data, respectively. In Fig. 2, we show \(RA_{max}\) in the energy region \(E_N \leq 19\) MeV, and compare them with the AV18 predictions in Ref. [15]. The deficiency of the \(A_y(\theta)\) at the maximum point is about \(15 - 20\%\) in our case, which is smaller than \(25 - 30\%\) in the predictions by AV18. In our calculations, the \(A_y\) puzzle for the \(pd\) scattering is more serious than for the \(nd\) scattering. However, in the \(pd\) scattering, this discrepancy diminishes as the energy increases.

The energy dependence of the elastic differential cross sections at the minimum points (diffraction minima) is often discussed as Sagara discrepancy [16] and is very important to discuss the 3N force effect. In order to examine this quantitatively, we show in Fig. 3 the relative difference, which is defined by \(RD_{min} = [(d\sigma/d\Omega)^{cal}_{min} - (d\sigma/d\Omega)^{exp}_{min}] / (d\sigma/d\Omega)^{exp}_{min}\) in the energy region \(E_p < 23\) MeV. On the low-energy side \(E_p < 18\) MeV, fss2 reproduces the minimum values of the \(pd\) differential cross sections at accuracy less than 2%. In Ref. [17], the Urbana IX (URIX) 3N potential lessens the cross sections at the diffraction minima, resulting in a good agreement with the experimental data. In our calculations, fss2 takes into account this attractive effect at the \(NN\) level. This result is in accordance with the good reproduction of the triton binding energy and \(nd\) doublet scattering length [18].

![Figure 2](image1.png)  
**Figure 2.** The theoretical to experimental ratio \(RA_{max}\) for the maximum points of \(pd\) (open squares) and \(nd\) (filled squares) analyzing powers \(A_y(\theta)\).

![Figure 3](image2.png)  
**Figure 3.** Deviation of the diffraction minimum from the experimental data measured by \(RD_{min}\) (see the text.)
4. Deuteron breakup processes

Figure 4 shows the breakup differential cross sections for the reactions $d(p,2p)n$ and $d(n,2n)p$ at $E_N = 13$ MeV. The $nd$ results are shown by the dashed curves and should be compared with the experimental data with error bars. We find that the Coulomb effect is rather small in these examples and our results are very similar to the predictions by meson-exchange potentials, given in Refs. [19, 20, 21, 22]. A slight overestimation of the peak in the quasi-free scattering (QFS) is reduced by the Coulomb effect and the agreement with the pd experimental data [23] or more recent one [24] is improved for $\rho = 16$ fm or 8 fm. The np final state interaction (FSI) peaks are well reproduced. In the collinear (COLL1, COLL2) and coplanar star (CST1) configurations, the nd data [25] agree well with theoretical predictions, but our CST1 result is at least 10 - 20% too small. The space star result is located just between the lower pd data and the higher nd data, which is the same feature as other predictions by meson-exchange potentials. This disagreement of breakup differential cross sections at $E_n = 13$ MeV is still an unsolved problem called the space star anomaly [25].

Figure 5 shows the systematic change of the deuteron breakup differential cross sections at $E_d = 130$ MeV for kinematical configurations with $\theta_1 = \theta_2 = 13^\circ$ and $\phi_{12} = \phi_1 - \phi_2 = 20^\circ - 180^\circ$. For the pp final-state interaction with $\phi_{12} = 20^\circ$, the choice $\rho = 8$ fm (dotted curve) is not good enough to reproduce the oscillatory structure, although a large peak in the no-Coulomb case (dashed curve) is strongly suppressed. The characteristic behavior of oscillation is only reproduced if $\rho$ is taken to be sufficiently large like $\rho \geq 16$ fm. The analyzing power in the breakup reaction is even more sensitive to the treatment of the Coulomb force. (Not shown.) In most cases, the results with $\rho = 8$, 16 and 20 fm are rather similar to each other, but for some

![Figure 4](image-url)

**Figure 4.** Breakup differential cross sections for the reactions $d(p,2p)n$ (solid curve) and $d(n,2n)p$ (dashed curve) at $E_N = 13$ MeV, compared with the experimental data [26].
kinematical configurations with $\theta_1 \sim \theta_2$ and $\phi_{12} = 0^\circ$ unpleasant oscillation appears for $\rho \geq 20$ fm. A larger $\rho$-value requires more partial waves and makes the solutions of AGS equations more singular. In spite of these limitations for the choice of large $\rho$, the results of $\rho = 8$ fm are rather stable and show that the Coulomb effect is not so large for $E_N = 65$ MeV except for the $pp$ final-state interaction region.

5. Summary
In spite of some apparent disagreement between the theory and experiment in breakup observables, our QM $NN$ interaction fss2 is still very successful in reproducing the essential features of almost all other experimental data in the three-nucleon system without reinforcing it with the three-body force. These include: 1) a nearly correct binding energy of the triton [3], 2) reproduction of the doublet and quartet $S$-wave scattering lengths, $^2a$ and $^4a$ [18], 3) low-energy differential cross sections of the $pd$ elastic scattering up to $E_p \sim 35$ MeV at the diffraction minima [8], 4) improved maximum heights of the nucleon analyzing power $A_p(\theta)$ in the low-energy region $E_n \leq 25$ MeV, although still insufficient [14], 5) breakup differential cross sections with many kinematical configurations [26]. Many of these improvements are related to the sufficiently attractive $nd$ interaction in the $^2S$ channel, in which the strong distortion effect of the deuteron is very sensitive to the treatment of the short-range repulsion of the $NN$ interaction. In our QM $NN$ interaction, this part is described by the quark exchange kernel of the color-magnetic quark-quark interaction. In the strangeness sector involving the $\Lambda N$ and $\Sigma N$ interactions, the effect of the Pauli repulsion on the quark level appears in some baryonic channels. It is therefore interesting to study $\Sigma^-\Lambda$-deuteron scattering in the present framework to find the repulsive effect directly related to the quark degree of freedom.

Figure 5. Breakup differential cross sections for the reaction $H(\bar{d},2p)n$, with the energy $E_d = 130$ MeV. The experimental data are taken from [27].
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