Superfield Calculation of Loop Contribution in Extra Dimensional Theories

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Abstract

Superfields provide a compact description of supersymmetry representations. Loop corrections with superfield formalism are simpler and much more manageable than calculation in terms of component fields. In this paper we calculate the contribution of the Kaluza-Klein states, associated with extra dimensions, to the renormalization group beta function. These Kaluza-Klein particles circulate in the virtual loop, hence affecting the overall corrections at any order. We obtain the one-loop correction, which checks with the result previously obtained using the more laborious component field method. In addition, we calculate the two-loop correction coming from chiral KK states.

1 Introduction

Supersymmetry alleviates the hierarchy problem where the cancellation of the quadratic divergences occurs in loop-calculations at all orders of pertur-

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bation. This was first notice in the self-interacting model [1]. In modern language, for N=1 supersymmetry, the softening of quantum fluctuation is called non-renormalization of the superpotential. However, these results can be obtained more easily in the context of superfield calculations [2]. Superfields provide a compact description of supersymmetric representation and a useful theoretical tool for formulating the Lagrangians. In addition, superfields simplify the addition and multiplication of representations. This profound concept of superfield was proposed by [3] soon after the discovery of supersymmetry. Superfields are just functions on superspace, where ordinary space-time is enlarged to include fermionic coordinates. The quantum properties of supersymmetric field theories are best investigated using superfield perturbation theory or supergraph method. The advantage for the supergraph method is its calculational simplicity due to fewer Lorentz indices and simpler Dirac structure, as compare to the traditional component field approach. Also, the results are manifestly supersymmetric at all stages of calculation. The component content or fields can always be recovered by power series expansion in terms of the anti-commuting coordinates.

The study of superstring theory has revived interest in theories with extra dimensions [4][5][6][7]. This interest has been further stimulated in the recent years by the possibility that these hidden dimensions may be "large" — large enough that their phenomenological implications can be checked by experiments in the near future [8][9]. The dependency of gauge unification scale on the size of the compactified dimensions is such that larger compactified dimensions lead to lower unification scale [10]. In this paper I apply this superfield Feynman diagram method [11][12] to loop calculations in supersymmetry theory with extra dimensions. There are three basic standard approaches to perturbative calculations in higher dimensional theories: 1) by summation over the winding numbers using the Poisson resummation formula [13], 2) by using mixed propagators, where the coordinates of the uncompactified dimensions are Fourier transformed into momentum space.
while the compactified dimensions are kept in configuration space[14][15].

3) by summation over the Kaluza-Klein states, which are manifestation of fields confined in the compactified extra dimensions [16][10]. Each particle that can propagate in extra dimensions shows up as an infinite tower of particles with masses $\frac{n}{R}$ where $n \in \mathbb{Z}$ and the mass gaps is controlled by the size of the compactified scale.

In this paper, we follow method 3). Our model is a 4D super Yang-Mills theory with manifestation of extra dimensions as an infinite tower of Kaluza-Klein states. In this theory, non-chiral fields (Higgs, gauge and scalar bosons fields) can propagate freely in all dimensions therefore will have KK modes. Matter fields (Leptons, quarks,...), on the other hand, are restricted to the brane. Collectively, they are called the bulk and brane fields respectively. These Kaluza-Klein particles associated with the bulk fields are allowed to circulate in the virtual loop, hence affecting the overall corrections at any order. We obtained the one-loop correction with the inclusion of KK states, and as application we computed the Beta functions for the gauge coupling. In addition, we calculated the two-loop partial correction with KK states where pure vector contributions are not considered.

2 Kaluza-Klein Contribution at One-Loop

In this section, we consider 4D super Yang-Mills theory with manifestation of extra dimensions as an infinite tower of Kaluza-Klein states. Equivalently, a higher dimensional Super Yang-Mill theory where the extra dimensions are compactified on an orbifold. In this theory, gauge fields can propagate freely in all dimensions, but matter fields are confined to the branes or orbifold fixed points. This higher dimensional theory is non-renormalizable due to the increase in spacetime dimensionality. However, we can safely assume higher Kaluza-Klein excitations are decoupled from the theory at a given energy scale, giving rise to an approximate renormalized theory. We will assume
that there exist $\delta \equiv D - 4$ extra spacetime dimensions with compactified radius $R$, where $\mu_0 \equiv R^{-1}$ represents the energy scale of the compactified dimensions. Every non-chiral particle state in the MSSM with mass $m_0$ can have an infinite tower of Kaluza-Klein states with masses

$$m^2_n \equiv m_0^2 + \sum_{i=1}^{\delta} \frac{n_i^2}{R^2},$$

(1)

where each state mirrors the MSSM ground state and $n_i \in Z$ are the Kaluza-Klein excitation numbers.

Before embarking on the superfield calculation, we need to know exactly which objects we are calculating. Analogous to the self energy or vacuum polarization in component field approach, $\overline{\Gamma}$, called ”quantum correction to the effective action”, is the object which we need in our superfield calculation. The counter terms in the Lagrangian and subsequently the renormalization constants. To have a better understanding of $\overline{\Gamma}$, we write the generating functional for the Green’s functions as

$$Z[J] = e^{\frac{W[J]}{\hbar}} = \int D\varphi e^{\frac{i}{\hbar}(S[\varphi]+\int dx \varphi(x) J(x))}$$

(2)

where $W[J]$ is the generating functional for the connected Green’s functions and the classical action $S[\varphi]$, with $\pi(x)$ being the conjugate to $\varphi(x)$. The effective action is defined by the Legendre transformation

$$\Gamma[\pi] = W[J] - \int dx \pi(x) J(x),$$

(3)

is the central object of quantum field theories since it contains all necessary information about the theories. The effective action is also known as the generating functional for the n-point vertex function

$$\Gamma^n = \frac{\delta^n \Gamma}{\delta \pi(z_1) \cdots \delta \pi(z_n)} |_{J=0}.$$  

(4)
The n-point vertex function represents the one-particle irreducible Feynman diagram without the external lines. The generating functional for the Green’s functions becomes

\[ e^{i \frac{\hbar}{\hbar} \left( \Gamma[\pi] + \int dx \pi(x) J(x) \right)} = \int D\varphi e^{i \frac{\hbar}{\hbar} \left( S[\varphi] + \int dx \varphi(x) J(x) \right)}. \]

We make a change of variable \( \varphi \rightarrow \varphi + \pi \), the above becomes

\[ e^{i \frac{\hbar}{\hbar} \left( \Gamma[\pi] + \int dx \pi(x) J(x) \right)} = \int D\varphi e^{i \frac{\hbar}{\hbar} \left( S[\varphi + \pi] + \int dx (\varphi(x) + \pi(x)) J(x) \right)}, \]

since \( \frac{\delta \Gamma[\pi]}{\delta \pi(x)} = -\int dx' \delta(x - x') J(x') = -J(x) \).

\[ e^{i \frac{\hbar}{\hbar} \left( \Gamma[\pi] \right)} = \int D\varphi e^{i \frac{\hbar}{\hbar} \left( S[\varphi + \pi] - \int dx \varphi(x) \frac{\delta \Gamma[\pi]}{\delta \pi(x)} \right)}. \quad (5) \]

Next we Taylor expand \( S[\varphi + \pi] \) in \( \pi \)

\[ S[\varphi + \pi] = S[\pi] + \sum_{n=1}^{\infty} \frac{1}{n!} \int dx_1...dx_n \frac{\delta^n S[\pi]}{\delta \pi(x_1)...\delta \pi(x_n)} \varphi(x_1)...\varphi(x_n), \quad (6) \]

and plug this expansion into the above

\[ e^{i \frac{\hbar}{\hbar} \left( \Gamma[\pi] \right)} = \int D\varphi e^{i \frac{\hbar}{\hbar} \left( S[\pi] + \sum_{n=1}^{\infty} \frac{1}{n!} \int dx_1...dx_n \frac{\delta^n S[\varphi(x_1)...\varphi(x_n)]}{\delta \pi(x_1)...\delta \pi(x_n)} \varphi(x_1)...\varphi(x_n) - \int dx \varphi(x) \frac{\delta \Gamma[\pi]}{\delta \pi(x)} \right)}, \]

which reduces to

\[ e^{i \frac{\hbar}{\hbar} \left( \Gamma[\pi] - S[\pi] \right)} = \int D\varphi e^{i \frac{\hbar}{\hbar} \left( \sum_{n=1}^{\infty} \frac{1}{n!} \int dx_1...dx_n \frac{\delta^n S[\varphi(x_1)...\varphi(x_n)]}{\delta \pi(x_1)...\delta \pi(x_n)} \varphi(x_1)...\varphi(x_n) - \int dx \varphi(x) \frac{\delta \Gamma[\pi]}{\delta \pi(x)} \right)}. \]
Taking the natural logarithm on both sides,

$$\Gamma[\pi] - S[\pi] = \int D\varphi \left( \sum_{n=1}^{\infty} \frac{1}{n!} \int dx_1...dx_n \frac{\delta^n S[\pi]}{\delta \varphi(x_1)...\delta \varphi(x_n)} - \int dx(\varphi(x) \frac{\delta \Gamma[\pi]}{\delta \varphi(x)}) \right)$$

by inspection we have

$$\Gamma[\pi] - S[\pi] = \Gamma[\pi].$$  \hspace{1cm} (7)

$\Gamma[\pi]$ represents the quantum correction to the classical action $S[\pi]$. The effective action $\Gamma[\pi]$ is the classical action plus all quantum corrections.

Our calculation is similar to the method used in reference [10]. We use the supergraph technique in calculating the chiral correction of the vector superfield where computation are being carried out in a larger space called superspace coordinates. Superspace contains the ordinary space-time coordinates $x^\mu$ and four additional anticommuting numbers $\{\theta_\alpha\}_{\alpha=1,2}$ and $\{\bar{\theta}_\alpha\}_{\alpha=1,2}$. Superfields are then defined to be functions of superspace. According to our model, chiral superfields are confined to the brane, being prohibited to propagate in the extra dimensions, will not have KK excitations. Therefore, no KK contribution coming from the chiral superfields. The effects of the extra dimensions through the manifestation of KK excitations can only come from the radiative correction of the vector superfield to itself. With the extensive use of the super Feynman rules derived in reference [11], the chiral correction of the vector superfield takes on the form in configuration space

$$\Gamma_\Phi(V) = \frac{1}{2} \int d^8z_1 d^8z_2 V^a(x_1, \theta_1, \bar{\theta}_1)$$

$$\times (igT^a_{ij})(igT^b_{ji})V_b(x_2, \theta_2, \bar{\theta}_2)[D^2(z_1) \frac{i\delta^8(z_1 - z_2)}{16(\partial_1^2 - m^2)} D(z_2)]$$

$$\times [D^2(z_2) \frac{i\delta^8(z_2 - z_1)}{16(\partial_2^2 - m^2)} D(z_1)] \hat{\rightarrow}$$

$$\times [D^2(z_2) \frac{i\delta^8(z_2 - z_1)}{16(\partial_2^2 - m^2)} D(z_1)]$$  \hspace{1cm} (8)
where $V^a$ is a vector superfield, the $z'$s are the super-coordinates

$$z^M = (x^\mu, \theta^\alpha, \bar{\theta}_\dot{\alpha})$$  \hspace{1cm} (9)

$T^a$ is the generator of the Lie group $G$ which span the Lie algebra under the commutator

$$[T^a, T^b] = if^{abc}T^c.$$  \hspace{1cm} (10)

The $D'$s are the super spinorial derivatives and $g$ is the coupling constant between chiral and vector superfields. The super-diagram in momentum space corresponds to figure 1.

Using covariant algebra, the correction becomes

$$\Gamma_\Phi(V) = \frac{1}{2} \sum A \delta^{ab} T_A(R) g^2 \int d^8 z_1 d^8 z_2 V^a(z_1, \theta_1, \bar{\theta}_1) \times V_0(z_2, \theta_2, \bar{\theta}_2) [D^2(z_1)D^2(z_1) \delta^8(z_1 - z_2)] \times \frac{\delta^8(z_1 - z_2)}{16(\partial_1^2 - m_n^2)}.$$  \hspace{1cm} (11)

where $\sum A \delta^{ab} T_A(R) = T_i^{ab}T_{ji}^{ab}$ depends on the representation $R$ on which the fields are chosen. Next, we integrate by parts on $z_1$, and using the properties of the super covariant derivatives, the correction becomes

$$\Gamma_\Phi(V) = \frac{1}{2} \sum A T_A(R) g^2 \int d^4 x_1 d^4 \theta_1 d^4 x_2 d^4 \theta_2 V^a(z_2, \theta_2, \bar{\theta}_2) \delta^8(z_1 - z_2) \times \left\{ 16^2 \Box \int \frac{d^4q}{(2\pi)^4} e^{iq(x_1 - x_2)} + 16^2 i\partial_{\alpha} \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} e^{iq(x_1 - x_2)} \overline{D^\alpha} D_{\alpha} \right\} \times \frac{V^a(z_1, \theta_1, \bar{\theta}_1)}{16(\partial_1^2 - m_n^2)16(\partial_2^2 - m_n^2)}.$$  \hspace{1cm} (12)

Fourier transforms the vector superfield $V$ and all propagators to momentum.
space yield quantities such as \( p, p', q \) and \( k \) in the correction. Then we perform the following integrations: \( \int d^4\theta_2, \int d^4x_1, \int d^4x_2, \int d^4p', \int d^4q \). The correction reduces to

\[
\Gamma_\Phi(V) = \frac{1}{2} \sum_A T_A(R) g^2 \int d^4\theta_1 \frac{d^4p}{(2\pi)^4} \tilde{V}(p, \theta_1, \bar{\theta}_1) \]  

\[
\times \left\{ \int \frac{d^4k}{(2\pi)^d} \left[ -(k-p)^2 + \frac{1}{2} (k-p)^2 \frac{D^\alpha D_\beta}{\beta} D^\alpha D_\beta + \frac{1}{4} D^2 D^2 \right] \tilde{V}(-p, \theta_1, \bar{\theta}_1) \right\}
\]

The above expression contain quadratic, linear and logarithmic divergences. However, we have cancellation of the quadratic and linear divergences coming from other similar supergraphs. The overall divergences due to chiral superfield give

\[
\Gamma_\Phi(V) = \frac{1}{2} \sum_A T_A(R) g^2 \int d^4\theta_1 \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \tilde{V}(p, \theta_1, \bar{\theta}_1) \]  

\[
\times \left\{ \frac{1}{2} \tilde{V}^a(-p) p^2 P_T + 2((k-p)^2 + m_n^2) - 2(k-p)^2 - 2m_n^2 \tilde{V}^a(p) \right\}
\]

where \( P_T = \frac{D^\alpha D^\beta D_\alpha D_\beta}{8p^2} \). We are then left with only logarithmic divergence.

\[
\Gamma_\Phi(V) = \frac{1}{2} \sum_A T_A(R) g^2 \int d^4\theta_1 \frac{d^4p}{(2\pi)^4} \tilde{V}^a(-p) p^2 P_T \tilde{V}^a(p) \]  

\[
\times \left\{ \int \frac{d^d k \mu^{4-d}}{(2\pi)^d} \frac{1}{[k^2 + m_n^2][(k-p)^2 + m_n^2]} \right\}
\]

With the aid of dimensional reduction and method used by reference [10], the loop integration can be evaluated in d-dimension. The correction reduces
\[
\Gamma_\Phi(V) = \frac{g^2}{2(4\pi)^2} \sum_A T(R) \int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \\
\times \int_0^\infty dt \frac{dt}{t} e^{-t[pz(1-z)+m_0^2]} \int d^4\theta \left\{ \frac{1}{2} \tilde{V}^a(-p)p^2 P_T \tilde{V}^a(p) \right\}.
\] (16)

The vector superfield can propagate in the extra dimensions, hence will have KK contribution. Their contribution to the effective action is figure 2, takes on the form

\[
\Gamma_{V,KK}(V) = -\frac{g^2}{2} \sum_{n_i=-\infty}^{\infty} C_2(g) \int \frac{d^4p}{(2\pi)^4} \int d^4\theta \left\{ \frac{1}{2} \tilde{V}^a(-p)p^2 P_T \tilde{V}^a(p) \right\} \\
\times \left\{ \int \frac{d^d k \mu^{4-d}}{(2\pi)^d} \frac{1}{[k^2 + m_{n_i}^2]|(k-p)^2 + m_{n_i}^2|} \right\}
\] (17)

where \(\sum_{n_i=-\infty}^{\infty} = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \cdots \sum_{n_\delta=-\infty}^{\infty}\) is the summation over KK states and \(\delta\) is the number of extra dimensions. \(C_2(g)\) is the second Casimir coefficient. After reparametrization of the denominator, the correction becomes

\[
\Gamma_{V,KK}(V) = -\frac{g^2}{2} \sum_{n_i=-\infty}^{\infty} C_2(g) \int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \int d^4\theta \left\{ \frac{1}{2} \tilde{V}^a(-p)p^2 P_T \tilde{V}^a(p) \right\} \\
\times \left\{ \int \frac{d^d k \mu^{4-d}}{(2\pi)^d} \frac{1}{[k^2 - 2k \cdot p z + p^2 z + m_{n_i}^2]^2} \right\}
\] (18)

We can perform the loop integration in d-dimension. Using

\[
\int \frac{d^d p}{[p^2 + 2p \cdot q - M^2]^{\alpha}} = \frac{(-1)^d \pi^\frac{d}{2} \Gamma(\alpha - \frac{d}{2})}{\Gamma(\alpha) [-q^2 - M^2]^{\alpha - \frac{d}{2}}}
\] (19)
and let \( q = -pz \) and \( -M^2 = p^2 z + m_{n_i}^2 \). The correction reduces to

\[
\Gamma_{V,KK}(V) = -\frac{g^2}{2} \sum_{n_i = -\infty}^{\infty} C_2(g) \int_0^1 dz \int \frac{d^4 p}{(2\pi)^4} \int d^4 \theta \left\{ \frac{1}{2} \tilde{V}^\alpha(-p) p^2 P_T \tilde{V}^\alpha(p) \right\} \\
\times \left\{ (-1)^{\frac{d}{2} \mu^{d} - d} \frac{\pi \frac{d}{2} \Gamma(2 - \frac{d}{2})}{(2\pi)^d} \right\},
\]

(20)

letting \( \varepsilon = 4 - d \) the correction reduces to

\[
\Gamma_{V,KK}(V) = -\frac{g^2}{2} \sum_{n_i = -\infty}^{\infty} C_2(g) \int_0^1 dz \int \frac{d^4 p}{(2\pi)^4} \int d^4 \theta \left\{ \frac{1}{2} \tilde{V}^\alpha(-p) p^2 P_T \tilde{V}^\alpha(p) \right\} \\
\times \left\{ (-1)^{\frac{d}{2} \mu^{d} \varepsilon} \frac{\pi \frac{d}{2} \Gamma(\frac{\varepsilon}{2})}{(2\pi)^d} \right\}.
\]

(21)

By using the identity

\[
\int_0^\infty x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{n+1}}
\]

(22)

let \( a = p^2 z(1 - z) + m_{n_i}^2 \) and \( \frac{\varepsilon}{2} = n + 1 \), the correction becomes

\[
\Gamma_{V,KK}(V) = -\frac{g^2}{2} \sum_{n_i = -\infty}^{\infty} C_2(g) \int_0^1 dz \int \frac{d^4 p}{(2\pi)^4} \int d^4 \theta \left\{ \frac{1}{2} \tilde{V}^\alpha(-p) p^2 P_T \tilde{V}^\alpha(p) \right\} \\
\times \left\{ (-1)^{\frac{d}{2} \mu^{d} \varepsilon} \frac{\Gamma(\frac{\varepsilon}{2})}{(4\pi)^\frac{d}{2}} \int_0^\infty t^{\frac{d}{2} - 1} e^{-[p^2 z(1 - z) + m_{n_i}^2]t} dt \right\}.
\]

(23)

Analytically continue from \( d \to 4 \) or \( \varepsilon \to 0 \), the correction becomes

\[
\Gamma_{V,KK}(V) = -\frac{g^2}{2} \sum_{n_i = -\infty}^{\infty} C_2(g) \int_0^1 dz \int \frac{d^4 p}{(2\pi)^4} \int d^4 \theta \left\{ \frac{1}{2} \tilde{V}^\alpha(-p) p^2 P_T \tilde{V}^\alpha(p) \right\} \\
\times \left\{ \frac{1}{(4\pi)^2} \int_0^\infty t^{-1} e^{-[p^2 z(1 - z) + m_{n_i}^2]t} dt \right\}.
\]

(24)
Then we sum over the Kaluza-Klein states with the help of the Jacobi Theta function

\[ \Gamma_{V,\text{KK}}(V) = -\frac{g^2}{2} \frac{1}{(4\pi)^2} C_2(g) \int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \times \int_0^\infty dt \left\{ \Theta_3 \left( \frac{it}{\pi R^2} \right) \right\} \delta e^{-p^2 z(1-z)t} \int d^4\theta \left\{ \frac{1}{2} \tilde{V}^a(-p) p^2 P_T \tilde{V}^a(p) \right\} \]

where \( \Theta_3 \) is the Jacobi Theta function

\[ \Theta_3 \left( \frac{it}{\pi R^2} \right) = \sum_{n_i=-\infty}^{\infty} e^{-m_{n_i}^2 t}, \] (26)

and \( f_{\text{ast}} f_{\text{bst}} = \delta_{ab} C_2(g) \). Since the ghost superfields are non-physical, it will not have ghost Kaluza-Klein contribution. The ghost correction is figure 3

\[ \Gamma_c(V) = -\frac{g^2}{2} \frac{1}{(4\pi)^2} C_2(g) \int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \int_0^\infty dt e^{-|p^2 z(1-z)+m_0^2|t} \times \int d^4\theta \left\{ \frac{1}{2} \tilde{V}^a(-p) p^2 P_T \tilde{V}^a(p) \right\}. \] (27)

Therefore, the overall divergence at first order approximation can be written as

\[ \Gamma_\text{loop}^{1\text{loop}}_{\text{KK}, V, \Phi, c}(V) = \frac{g^2}{8\pi^2} \int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \int_0^\infty dt \frac{e^{-|p^2 z(1-z)+m_0^2|t}}{t} \times \left\{ e^{-m_0^2 t} \sum T_A(R) - \frac{5}{2} C_2(g) \left\{ \Theta_3 \left( \frac{it}{\pi R^2} \right) \right\} \right\} \delta \times \int d^4\theta \left\{ \frac{1}{2} \tilde{V}^a(-p) p^2 P_T \tilde{V}^a(p) \right\}, \] (28)

where \( \frac{D^6\mathcal{T}_D^3}{8} = p^2 P_T \). The first term in the bracket is the contribution of the chiral superfields, the second term comes from the vector superfield, and the third term is from the unphysical ghost superfield.
We are now in position to obtain the counter term and the renormalization constant for our model.

\[ \Gamma_{\text{effective}} = \int \! d^8p \tilde{V}^a \frac{D^\beta D^2 D^\beta \tilde{V}^a}{8} + \Gamma^{\text{1-loop}}_{KK,V,\Phi,c} \] (29)

which is infinite. We must add a counter term to the effective action to render it finite.

\[ \Gamma = \int \! dp \tilde{V}^a (-p) \frac{D^\beta D^2 D^\beta \tilde{V}^a (p)}{8} + \Gamma^{\text{1-loop}}_{KK,V,\Phi,c}(V) \]

\[ + \Delta Z_2 \int \! dp \tilde{V}^a \frac{D^\beta D^2 D^\beta \tilde{V}^a}{8} \]

\[ = \int \! dp (1 + \Delta Z_2) \tilde{V}^a (-p) \frac{D^\beta D^2 D^\beta \tilde{V}^a (p)}{8} + \Gamma^{\text{1-loop}}_{KK,V,\Phi,c}(V) \] (30)

by inspection, \( \Delta Z_2 \) must be

\[ \Delta Z_2 = -\frac{g^2}{8\pi^2} \int_0^1 dz e^{-p^2 z(1-z)t} \int_0^\infty dt \frac{1}{t} \left\{ e^{-m_0^2 t} \sum T_A(R) - \frac{5}{2} C_2(g) \left\{ \Theta_3\left(\frac{it}{\pi R^2}\right) \right\}^\delta - \frac{1}{2} e^{-m_0^2 t} C_2(g) \right\}. \] (31)

At zero momentum transferred \( p = 0 \), we have

\[ \Delta Z_2 = -\frac{g^2}{8\pi^2} \int_0^\infty dt \frac{1}{t} \left\{ e^{-m_0^2 t} \sum T_A(R) - \frac{5}{2} C_2(g) \left\{ \Theta_3\left(\frac{it}{\pi R^2}\right) \right\}^\delta - \frac{1}{2} e^{-m_0^2 t} C_2(g) \right\} \] (32)

let \( Z_2 = 1 + \Delta Z_2 \), \( g_r = Z_g g \), and \( Z_2 Z_2^{\dagger} = 1 \). This implies \( Z_g = Z_2^{-\frac{1}{2}} \)

\[ Z_g = \left[ 1 - \frac{g^2}{8\pi^2} \int_0^\infty dt \frac{1}{t} \left\{ -\frac{5}{2} C_2(g) \left\{ \Theta_3\left(\frac{it}{\pi R^2}\right) \right\}^\delta - \frac{1}{2} e^{-m_0^2 t} C_2(g) \right\} \right]^{-\frac{1}{2}} \]
we then have \( g^2 = Z_2 g_r^2 \implies \)

\[
\frac{1}{g_r^2} = \frac{1}{g^2} Z_2 \]

\[
= \frac{1}{g^2} \left[ \frac{1 - \frac{g^2}{8\pi^2} \int_0^\infty \frac{dt}{t}}{\frac{5}{2} C_2(g) \left\{ \Theta_3 \left( \frac{it}{\pi R^2} \right) \right\}^\delta - \frac{1}{2} e^{-m_0^2 t} C_2(g)} \right] \] (33)

with \( \alpha \equiv \frac{g^2}{4\pi} \), we then have the following

\[
\alpha_r^{-1} = \alpha^{-1} - \frac{1}{2\pi} \int_0^\infty \frac{dt}{t} \left\{ e^{-m_0^2 t} \sum T_A(R) - \frac{5}{2} C_2(g) \left\{ \Theta_3 \left( \frac{it}{\pi R^2} \right) \right\}^\delta - \frac{1}{2} e^{-m_0^2 t} C_2(g) \right\} .
\]

To obtain an explicit expression for the beta function, operate \( t \frac{\partial}{\partial t} \) on both sides of

\[
\frac{1}{g^2} = \frac{1}{g_r^2} + \frac{1}{8\pi^2} \int_0^\infty \frac{dt}{t} \left\{ e^{-m_0^2 t} \sum T_A(R) - \frac{5}{2} C_2(g) \left\{ \Theta_3 \left( \frac{it}{\pi R^2} \right) \right\}^\delta - \frac{1}{2} e^{-m_0^2 t} C_2(g) \right\} .
\] (34)

we have

\[
-\frac{2}{g^3} \frac{\partial g}{\partial t} = \frac{1}{8\pi^2} \int d \left\{ e^{-m_0^2 t} \sum T_A(R) - \frac{5}{2} C_2(g) \left\{ \Theta_3 \left( \frac{it}{\pi R^2} \right) \right\}^\delta - \frac{1}{2} e^{-m_0^2 t} C_2(g) \right\} .
\] (35)

The beta function becomes

\[
\beta(g) = t \frac{\partial g}{\partial t} = -\frac{g^3}{16\pi^2} \left\{ e^{-m_0^2 t} \sum T_A(R) - \frac{5}{2} C_2(g) \left\{ \Theta_3 \left( \frac{it}{\pi R^2} \right) \right\}^\delta - \frac{1}{2} e^{-m_0^2 t} C_2(g) \right\} .
\] (36)

In order to obtain results which agree with ref.[10], we must modify our model by allowing the chiral or matter superfields to propagate freely in the
extra-dimensions. The correction to the effective action at one-loop becomes

\[
\Gamma^{1\text{loop}}_{KK,V,\Phi,c}(V) = \frac{g^2}{8\pi^2} \int_0^1 dz \int \frac{d^d p}{(2\pi)^d} \int_0^\infty \frac{dt}{t} e^{-p^2 z(1-z)t} \times \left\{ \left[ \sum T_A(R) - \frac{5}{2} C_2(g) \right] \left\{ \Theta_3 \left( \frac{it}{\pi R^2} \right) \right\}^\delta \right. \\
\left. - \frac{1}{2} e^{-m_0^2 t} C_2(g) \right\} \times \int d^4 \theta \left\{ \frac{1}{2} \bar{V}^a(-p) p^2 P_T \bar{V}^a(p) \right\}, \quad (37)
\]

The renormalization constant is

\[
Z_g = \left[ 1 - \frac{g^2}{8\pi^2} \int_0^\infty \frac{dt}{t} \left\{ \left[ \sum T_A(R) - \frac{5}{2} C_2(g) \right] \left\{ \Theta_3 \left( \frac{it}{\pi R^2} \right) \right\}^\delta \right. \\
\left. - \frac{1}{2} e^{-m_0^2 t} C_2(g) \right\} \right]. \quad (38)
\]

The beta function becomes

\[
\beta(g) = t \frac{\partial g}{\partial t} = -\frac{g^3}{16\pi^2} \left\{ \left[ \sum T_A(R) - \frac{5}{2} C_2(g) \right] \left\{ \Theta_3 \left( \frac{it}{\pi R^2} \right) \right\}^\delta \right. \\
\left. - \frac{1}{2} e^{-m_0^2 t} C_2(g) \right\}. \quad (39)
\]

The explicit agreement of the results to ref.\cite{10} is transparent when we expand the effective action into component form\footnote{Inverse Fourier transforming \( \int dp^8 \bar{V}^a(-p) D^{\mu\nu\rho\sigma}_8 \bar{V}^a(p) \) to configuration space and expanding into component fields by the machinery of supersymmetry algebra.}.

## 3 Kaluza-Klein Contribution at Two-Loop

In this section, we use supergraph technique to calculate three super-diagrams contributing to the effective action. These diagrams contain the massive chiral-multiplet correction to the massive vector-multiplet at two-loop with Kaluza-Klein states. In the next section, we evaluate the two-loop integral.
3.1 Supergraph calculation

In momentum space, the correction of figure 4 is

\[
\Gamma^{2\text{loop}}_{KK,1}(V) = \sum_{n_{i}=-\infty}^{\infty} g^{4} \int d^{8}z_{1}d^{8}z_{2}d^{8}z_{3}d^{8}z_{4} V(z_{1}, \theta_{1}, \bar{\theta}_{1}) V(z_{4}, \theta_{4}, \bar{\theta}_{4}) \tag{40}
\]

\[
\times \left\{ \frac{\delta^{8}(z_{1}-z_{2})}{16(\delta_{1}^{2}-m_{n}^{2})} \frac{\delta^{2}(z_{2})}{D^{2}(z_{2})} \right\} \left[ \frac{\delta^{8}(z_{2}-z_{3})}{16(\delta_{2}^{2}-m_{n}^{2})} \frac{\delta^{2}(z_{3})}{D^{2}(z_{3})} \right]
\times \left[ \frac{-\delta^{8}(z_{3}-z_{4})}{16(\delta_{3}^{2}-m_{n}^{2})} \frac{\delta^{2}(z_{4})}{D^{2}(z_{4})} \right] \right\}.
\]

using \( D^{2}_{1} \delta^{8}(z_{1}-z_{2}) \frac{\delta^{2}}{D^{2}} = D^{2}_{1} \frac{\delta^{2}}{D^{2}} \delta^{8}(z_{1}-z_{2}) \), the correction yields

\[
\Gamma^{2\text{loop}}_{KK,1}(V) = \sum_{n_{i}=-\infty}^{\infty} g^{4} \int d^{8}z_{1}d^{8}z_{2}d^{8}z_{3}d^{8}z_{4} V(z_{1}, \theta_{1}, \bar{\theta}_{1}) V(z_{4}, \theta_{4}, \bar{\theta}_{4}) \tag{41}
\]

\[
\times \left\{ \frac{\delta^{8}(z_{1}-z_{2})}{16(\delta_{1}^{2}-m_{n}^{2})} \frac{\delta^{2}(z_{2})}{D^{2}(z_{2})} \right\} \left[ \frac{\delta^{8}(z_{2}-z_{3})}{16(\delta_{2}^{2}-m_{n}^{2})} \frac{\delta^{2}(z_{3})}{D^{2}(z_{3})} \right]
\times \left[ \frac{-\delta^{8}(z_{3}-z_{4})}{16(\delta_{3}^{2}-m_{n}^{2})} \frac{\delta^{2}(z_{4})}{D^{2}(z_{4})} \right] \right\}.
\]

Using the result at one-loop,

\[
D^{2}_{1} \frac{\delta^{2}}{D^{2}} \delta^{8}_{12} D^{2}_{2} \frac{\delta^{2}}{D^{2}} \delta^{8}_{23} = \delta^{8}_{12} \left[ 16^{2} \Box_{23}^{\delta} + \frac{16^{2}}{2} \partial_{\sigma}^{\delta} \delta_{23}^{\delta} D_{\alpha 2} + 16 \delta_{23}^{\delta} \frac{\delta^{2}}{D^{2}} \frac{\delta^{2}}{D^{2}} \right] \tag{42}
\]
where $\delta^8_{12} = \delta^8(z_1 - z_2) = \delta^4(x_1 - x_2)\delta^4(\theta_1 - \theta_2)$, the correction reduces to

$$\Gamma_{KK,1}^{loop}(V) = \sum_{n_i=-\infty}^{\infty} -g^4 \int [\prod_{j=1}^{4} dx_j d^4\theta_j] V(z_1, \theta_1, \overline{\theta}_1) V(z_4, \theta_4, \overline{\theta}_4)$$

$$\times \{ \delta^4(x_1 - x_2)\delta^4(\theta_1 - \theta_2)[16^2 \Box \delta^4(x_2 - x_3)$$

$$+ i\frac{16^2}{2} \partial^\alpha D^\beta D\alpha + 16 \delta^4(x_2 - x_3) D^2 D^2] \}$$

$$\times \{ \delta^8(z_3 - z_2)\delta^8(z_3 - z_4)[16^2 \Box \delta^4(x_4 - x_1)$$

$$+ i\frac{16^2}{2} \partial^\beta D^\alpha D\alpha + 16 \delta^4(x_4 - x_1) D^2 D^2] \}$$

$$\times \frac{16^{-5}}{(\Box_1 - m^2_{n_i})(\Box_2 - m^2_{n_i})(\Box_3 - m^2_{n_i})(\Box_4 - m^2_{n_i})(\Box_5 - m^2_{n_i})},$$

$$\Gamma_{KK,1}^{loop}(V) = \sum_{n_i=-\infty}^{\infty} -Sg^4 \int [\prod_{j=1}^{4} dx_j d^4\theta_j] V(z_1, \theta_1, \overline{\theta}_1) V(z_4, \theta_4, \overline{\theta}_4)$$

$$\times \delta^4(x_1 - x_2)\delta^4(\theta_1 - \theta_2)\delta^4(x_2 - x_3)$$

$$\times \{ [16^2(-q^2) + \frac{16^2}{2} q^\alpha D^\alpha D\alpha + 16 D^2 D^2]$$

$$\times \delta^8(z_3 - z_2)\delta^8(z_3 - z_4)\delta^4(x_4 - x_1)$$

$$\times [16^2(-k^2) + \frac{16^2}{2} k^\beta D^\alpha D\alpha + 16 D^2 D^2] \}$$

$$\times \frac{16^{-5}}{(\Box_1 - m^2_{n_i})(\Box_2 - m^2_{n_i})(\Box_3 - m^2_{n_i})(\Box_4 - m^2_{n_i})(\Box_5 - m^2_{n_i})}. $$
The bracket above yields

$$\delta^8_{12} \delta^4_{23}\{}$$

$$\begin{align*}
16^4 q^2 k^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} & - \frac{16^4}{2} q^2 k^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} \\
-16^4 q^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} D^2 D^2 & + \frac{16^4}{2} q^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} D^2 D^2 \\
\frac{16^4}{4} q^2 k^2 D^2 D^2 & + \frac{16^4}{2} q^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} D^2 D^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} D^2 D^2 \\
-16^3 k^2 & + \frac{16^4}{2} q^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} D^2 D^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} D^2 D^2 \\
+16^4 q^2 D^2 & \delta^8_{32} \delta^8_{34} \delta^4_{41} D^2 D^2 \\
\end{align*}$$

$$\delta^8_{12} \delta^4_{23} = \delta^8_{12} \delta^4_{23} \{$$

In order to have non-vanishing result, we must have an even number of $D'$s and $\overline{D}'s$ between the delta functions. The fourth, fifth, and sixth term in the bracket vanish as consequence of the identity. Next, we Fourier transform the vector-multiplet superfield. With integration by parts and various properties of the covariant derivatives and $\delta$-functions, the 2-loop correction reduces to

$$\Gamma_{KK;1}^{\text{2loop}} = \sum_{n_i = -\infty}^{\infty} -g^4 \int \left[ \Pi^4_{j=1} d^4 x_j d^4 \theta_j \right] \frac{d^4 p}{(2\pi)^4} \frac{d^4 p'}{(2\pi)^4} \tilde{V}(-p, \theta_1, \overline{\theta}_1) \tilde{V}(p', \theta_4, \overline{\theta}_4)$$

$$e^{-ip \cdot x_1} e^{ip' \cdot x_4}$$

$$\times \left\{ \begin{align*}
\delta^8_{12} \delta^4_{23} \delta^8_{32} \delta^8_{34} \delta^4_{41} & \left( 16^4 (q \cdot k)^2 - \frac{16^4}{2} i q^2 k^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} D^2 D^2 - 16^3 q^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} D^2 D^2 \right) \\
+ \delta^8_{12} \delta^4_{23} \delta^8_{32} \delta^8_{34} \delta^4_{41} & \left( -16^4 k^2 + \frac{16^4}{2} r^2 k^2 \delta^8_{32} \delta^8_{34} \delta^4_{41} D^2 D^2 - 16^3 D^2 D^2 \right) \\
\times \frac{16^{-5}}{(h^2 + m^2_i)(q^2 + m^2_i)(r^2 + m^2_i)(t^2 + m^2_i)(k^2 + m^2_i)} \right\}$$

by inspection the second term in {} contains only two $\delta^4(\theta_i - \theta_j)$, hence it vanishes upon $\int d^4 \theta_2 d^4 \theta_3 d^4 \theta_4$. The correction reduces to a single point in superspace (i.e. $\theta_1$). Writing the explicit form of the delta functions, the
correction becomes

\[ \Gamma^{2\text{loop}}_{KK,1}(V) = \sum_{n_i = -\infty}^{\infty} -g^4 \int d^4\theta_1 d^4x_1 d^4x_2 d^4x_3 d^4x_4 \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p', \theta_1, \bar{\theta}_1) \]
\[ \times \int \frac{d^4h}{(2\pi)^4} e^{ih(x_1-x_2)} \int \frac{d^4q}{(2\pi)^4} e^{iq(x_2-x_3)} \int \frac{d^4r}{(2\pi)^4} e^{ir(x_3-x_2)} \]
\[ \times \int \frac{d^4t}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} e^{ik(x_4-x_1)} \]
\[ \times \left( 16^4(q \cdot k)^2 - \frac{16^4}{2} i q^2 k^2 \beta D^2 D_{\beta} - 16^3 q^2 \beta D^2 D^2 \beta \right) \]
\[ \times \frac{16^{-5}}{(h^2 + m_{n_i}^2)(q^2 + m_{n_i}^2)(r^2 + m_{n_i}^2)(t^2 + m_{n_i}^2)(k^2 + m_{n_i}^2)}. \]  

All of the exponential terms can be rewritten so that we can integrate in configuration space

\[ \Gamma^{2\text{loop}}_{KK,1}(V) = \sum_{n_i = -\infty}^{\infty} -g^4 \int d^4\theta_1 d^4x_1 d^4x_2 d^4x_3 d^4x_4 \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p', \theta_1, \bar{\theta}_1) \]
\[ \times \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4p'}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4r}{(2\pi)^4} \int \frac{d^4t}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \]
\[ \times e^{ix_1(p-h)} e^{ix_2(q-h)} e^{ix_3(r-t)} e^{ix_4(p'+k-t)} \]
\[ \times \left( 16^4(q \cdot k)^2 - \frac{16^4}{2} i q^2 k^2 \beta D^2 D_{\beta} - 16^3 q^2 \beta D^2 D^2 \beta \right) \]
\[ \times \frac{16^{-5}}{(h^2 + m_{n_i}^2)(q^2 + m_{n_i}^2)(r^2 + m_{n_i}^2)(t^2 + m_{n_i}^2)(k^2 + m_{n_i}^2)}, \]
so we can perform $\int d^4x_1 d^4x_2 d^4x_3 d^4x_4$. This will yield four delta functions

\[ \Gamma_{K,K,1}^{2\text{loop}}(V) = \sum_{n=\infty}^{\infty} g^4 \int d^4\theta_1 \tilde{V}(-p, \theta_1, \overline{\theta}_1) \tilde{V}(p, \theta_1, \overline{\theta}_1) \]

\[ \times \frac{d^4p}{(2\pi)^4} \frac{d^4p'}{(2\pi)^4} \frac{d^4h}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{d^4t}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \]

\[ \times \delta^4(h - p - k) \delta^4(q - h - r) \delta^4(r + p - q) \delta^4(p' + k - t) \]

\[ \times \left\{ \frac{16^4(q \cdot k)^2 - \frac{16^3}{2} i q^2 k^\beta D^\beta D - 16^3 q^2 D^2 D^2}{16^5 (h^2 + m^2_{n_i}) (q^2 + m^2_{n_i}) (r^2 + m^2_{n_i}) (t^2 + m^2_{n_i}) (k^2 + m^2_{n_i})} \right\}. \]

Integrating over the delta functions one at a time, the correction yields

\[ \Gamma_{K,K,1}^{2\text{loop}}(V) \]

\[ = \sum_{n=\infty}^{\infty} g^4 \int d^4\theta_1 \tilde{V}(-p, \theta_1, \overline{\theta}_1) \tilde{V}(p, \theta_1, \overline{\theta}_1) \frac{d^4p}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \]

\[ \times \left\{ \frac{16^4(r + p + k)^2 k^2 - \frac{16^3}{2} i (r + p + k)^2 k^\beta D^\beta D}{16^5 ((p + k)^2 + m^2_{n_i}) ((r + p + k)^2 + m^2_{n_i}) (r^2 + m^2_{n_i})} \right\} \times \left\{ \frac{16^5 (r^2 + m^2_{n_i}) (k^2 + m^2_{n_i})}{16^5 (h^2 + m^2_{n_i}) (q^2 + m^2_{n_i}) (r^2 + m^2_{n_i}) (t^2 + m^2_{n_i}) (k^2 + m^2_{n_i})} \right\}. \]

The above expression is the contribution of figure 4, to the effective action. The contribution of other diagrams can be calculated the same way.
The contribution of figure 5 is

\[
\Gamma_{KK,2}^{\text{loop}}(V) = \sum_{n_i = -\infty}^{\infty} g^4 \int d^8 z_1 d^8 z_2 d^8 z_3 V(z_1, \theta_1, \overline{\theta}_1) V(z_3, \theta_3, \overline{\theta}_3) \quad (50)
\]

\[
\left\{ \begin{aligned}
[D^2(z_1) & \frac{\delta^8(z_1-z_2)}{16(\partial_1^2-m_{1i}^2)} \overline{D^2}(z_2)] [\frac{-\delta^8(z_2-z_1)}{16(\partial_2^2-m_{2i}^2)}] \\
[D^2(z_2) & \frac{\delta^8(z_2-z_3)}{16(\partial_2^2-m_{2i}^2)} \overline{D^2}(z_3)] [D^2(z_3) \frac{\delta^8(z_3-z_1)}{16(\partial_3^2-m_{3i}^2)} \overline{D^2}(z_1)]
\end{aligned} \right.
\]

again using result at one loop, the correction reduces to

\[
\Gamma_{KK,2}^{\text{loop}}(V) = \sum_{n_i = -\infty}^{\infty} g^4 \int d^8 z_1 d^8 z_2 d^8 z_3 V(z_1, \theta_1, \overline{\theta}_1) V(z_3, \theta_3, \overline{\theta}_3) \quad (51)
\]

\[
\times \left\{ \begin{aligned}
[D^2(z_1) & \frac{\delta^8(z_1-z_2)}{16(\partial_1^2-m_{1i}^2)} \overline{D^2}(z_2)] [\frac{-\delta^8(z_2-z_1)}{16(\partial_2^2-m_{2i}^2)}] \\
[D^2(z_2) & \frac{\delta^8(z_2-z_3)}{16(\partial_2^2-m_{2i}^2)} \overline{D^2}(z_3)] [D^2(z_3) \frac{\delta^8(z_3-z_1)}{16(\partial_3^2-m_{3i}^2)} \overline{D^2}(z_1)]
\end{aligned} \right.
\]

With the aid of identity \((D_1^2 \overline{D}_1^2 \delta_{12}^8) \delta_{21}^8 \delta_{23}^8 = \delta_{12}^8 \overline{D}_1^2 D_1^2 \delta_{21}^8 \delta_{23}^8\), the correction reduces to

\[
\Gamma_{KK,2}^{\text{loop}}(V) = \sum_{n_i = -\infty}^{\infty} g^4 \int d^8 z_1 d^8 z_2 d^8 z_3 V(z_1, \theta_1, \overline{\theta}_1) V(z_3, \theta_3, \overline{\theta}_3) \delta_{12}^8 \overline{D}_1^2 D_1^2 \delta_{21}^8 \delta_{23}^8
\]

\[
\times \left\{ \begin{aligned}
\left[ \frac{D_1^2 \overline{D}_1^2 \delta_{12}^8 \delta_{31}^8 \delta_{31}^8}{16^2(\partial_1^2-m_{1i}^2)(\partial_2^2-m_{2i}^2)(\partial_3^2-m_{3i}^2)} \right] \\
\frac{D_1^2 \overline{D}_1^2 D_1^2 \delta_{21}^8 \delta_{23}^8}{16^4(\partial_1^2-m_{1i}^2)(\partial_2^2-m_{2i}^2)(\partial_3^2-m_{3i}^2)(\partial_4^2-m_{4i}^2))}
\end{aligned} \right. \quad (53)
\]
Performing the expression reduces to a single point in superspace. We then sys-

$$\Gamma_{KK,2}^{2\text{loop}}(V)$$

$$= \sum_{n_1=\infty}^{\infty} g^4 \int d^8 z_1 d^8 z_2 d^8 z_3 V(z_1, \theta_1, \bar{\theta}_1) V(z_3, \theta_3, \bar{\theta}_3) \delta_{12}^8 \delta_{23}^8$$

$$\times \left\{ \frac{[16^2 \Box_\delta^4 + i \frac{16^2}{2} \partial^\mu \partial^\nu \delta_{\mu \nu} \frac{\partial^4}{\partial x^4} D_\alpha + 16 \delta_{31}^4 \frac{\partial^2}{\partial x^2} D^2]}{16^3 ([\partial_1^2 - m_1^2] [\partial_2^2 - m_2^2] [\partial_3^2 - m_3^2] [\partial_4^2 - m_4^2])} \right\},$$

where we use $\delta_{12}^8 \frac{\partial^2}{\partial x^2} D^2_1 \delta_{21}^8 = 16 \delta_{12}^8 \delta_{21}^8$. Next we Fourier transform to momentum space

$$\Gamma_{KK,2}^{2\text{loop}}(V) = \sum_{n_1=\infty}^{\infty} g^4 \int d^8 z_1 d^8 z_2 d^8 z_3 \bar{V}(-p, \theta_1, \bar{\theta}_1) \bar{V}(p', \theta_3, \bar{\theta}_3)$$

$$\times \frac{d^4 p \ d^4 p' \ d^4 h \ d^4 q \ d^4 r \ d^4 k}{(2\pi)^4 (2\pi)^4 (2\pi)^4 (2\pi)^4 (2\pi)^4}$$

$$\times e^{ix_1(h-r+k-p)} e^{ix_2(r-h+q)} e^{ix_3(p'+k-q)} \delta^4(\theta_1 - \theta_2) \delta^4(\theta_2 - \theta_3)$$

$$\times \left\{ \frac{[\frac{1}{16} k^2 + i \frac{1}{32} \partial^\mu \partial^\nu D_\alpha + \frac{i}{16} \frac{\partial^2}{\partial x^2} D^2]}{([h^2 + m_1^2]) (r^2 + m_2^2) (q^2 + m_3^2) (k^2 + m_4^2)} \right\}.$$

Performing $\int \int d^4 \theta_2 d^4 \theta_3$, the correction yields

$$\Gamma_{KK,2}^{2\text{loop}}(V) = \sum_{n_1=\infty}^{\infty} g^4 \int d^8 z_1 d^8 z_2 d^8 z_3 \bar{V}(-p, \theta_1, \bar{\theta}_1) \bar{V}(p', \theta_1, \bar{\theta}_1)$$

$$\times \frac{d^4 p \ d^4 p' \ d^4 h \ d^4 q \ d^4 r \ d^4 k}{(2\pi)^4 (2\pi)^4 (2\pi)^4 (2\pi)^4 (2\pi)^4}$$

$$\times e^{ix_1(h-r+k-p)} e^{ix_2(r-h+q)} e^{ix_3(p'+k-q)}$$

$$\times \left\{ \frac{[\frac{1}{16} k^2 + i \frac{1}{32} \partial^\mu \partial^\nu D_\alpha + \frac{i}{16} \frac{\partial^2}{\partial x^2} D^2]}{([h^2 + m_1^2]) (r^2 + m_2^2) (q^2 + m_3^2) (k^2 + m_4^2)} \right\},$$

where the expression reduces to a single point in superspace. We then sys-
tematically integrate over the coordinate space \( \int d^4x_1 d^4x_2 d^4x_3 \). This will give us three delta functions in momentum space

\[
\Gamma_{KK,2}^{2\text{loop}}(V) = \sum_{n_i = -\infty}^{\infty} g^4 \int d^8\theta_1 \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p', \theta_1, \bar{\theta}_1) \\
\times \frac{d^4 p}{(2\pi)^4} \frac{d^4 p'}{(2\pi)^4} d^4 r d^4 q d^4 k \delta^4(h - r - k - p) \\
\times \delta^4(r - h + q) \delta^4(p' + k - q)
\]

Integrating over the three delta functions \( \int d^4 h d^4 q d^4 p' \), The contribution of diagram yields

\[
\Gamma_{KK,2}^{2\text{loop}}(V) = \sum_{n_i = -\infty}^{\infty} -g^4 \int d^4\theta_1 \frac{d^4 p}{(2\pi)^4} \frac{d^4 r}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p, \theta_1, \bar{\theta}_1)
\]

\[
\times \left\{ \frac{\left[ -\frac{1}{16} k^2 + \frac{i}{32} \partial_\alpha D^\alpha D_\alpha \right] \left[ -(\partial_2^2 - m_{n_i}^2)(r^2 + m_{n_i}^2)(p^2 + m_{n_i}^2)(k^2 + m_{n_i}^2) \right]}{((r + p + k)^2 + m_{n_i}^2)(r^2 + m_{n_i}^2)(r^2 + m_{n_i}^2)(k^2 + m_{n_i}^2)} \right\}.
\]

The contribution of figure 6 is

\[
\Gamma_{KK,3}^{2\text{loop}}(V) = \sum_{n_i = -\infty}^{\infty} g^4 \int d^8 z_1 d^8 z_2 V(z_1, \theta_1, \bar{\theta}_1) V(z_2, \theta_2, \bar{\theta}_2)
\]

\[
\left\{ \frac{D_2^2 D_1^{2\delta_8}}{16(\partial_1^2 - m_{n_i}^2)} \left[ \frac{-\delta_2^8}{16(\partial_2^2 - m_{n_i}^2)} \right] \frac{D_2^2 D_1^{2\delta_8}}{16(\partial_1^2 - m_{n_i}^2)} \right\}.
\]

Using properties of super-spinor derivatives and integration by parts and the
following
\[ (D_1^2 D_1^2 \delta_{12}^8) \delta_{21}^8 (D_2^2 D_2^2 \delta_{21}^8) = 16^2 \delta_{12}^8 \delta_{21}^4, \]
the correction becomes
\[
\Gamma_{KK,3}^{loop}(V) = - \sum_{n=\infty}^\infty g^4 \int d^8 z_1 d^8 z_2 V(z_1, \theta_1, \theta_1) V(z_2, \theta_2, \theta_2)
\times \left\{ \frac{\delta_{12}^8 \delta_{21}^4 \delta_{21}^4}{16(\delta_1^2 - m_{n_1}^2)(\partial_2^2 - m_{n_2}^2)(\partial_3^2 - m_{n_3}^2)} \right\}.
\] (60)

Fourier transforms into momentum space and integrate \( \int d^4 \theta_2 \)
\[
\Gamma_{KK,3}^{loop}(V) = - \sum_{n=\infty}^\infty g^4 \int d^4 \theta_1 d^4 x_1 d^4 x_2 \tilde{V}(-p, \theta_1, \theta_1) \tilde{V}(p', \theta_1, \theta_1)
\times \frac{d^4 p' d^4 h' d^4 q}{(2\pi)^4 (2\pi)^4 (2\pi)^4 (2\pi)^4} \frac{d^4 r}{(2\pi)^4} \frac{e^{ix_1(h-p-r-q)}}{16(h^2 + m_{n_1}^2)(r^2 + m_{n_2}^2)(q^2 + m_{n_3}^2)}
\times e^{ix_2(p' + q - h + r)} \left\{ \frac{1}{16(h^2 + m_{n_1}^2)(r^2 + m_{n_2}^2)(q^2 + m_{n_3}^2)} \right\}.
\] (61)

Performing \( \int d^4 x_1 d^4 x_2 \)
\[
\Gamma_{KK,3}^{loop}(V) = - \sum_{n=\infty}^\infty g^4 \int d^4 \theta_1 \tilde{V}(-p, \theta_1, \theta_1) \tilde{V}(p', \theta_1, \theta_1)
\times \frac{d^4 p d^4 p' d^4 h d^4 q}{(2\pi)^4 (2\pi)^4 (2\pi)^4 (2\pi)^4} d^4 r d^4 q
\times \left\{ \frac{\delta^4(h - p - r - q)\delta^4(p' + q - h + r)}{16(h^2 + m_{n_1}^2)(r^2 + m_{n_2}^2)(q^2 + m_{n_3}^2)} \right\},
\] (62)
and integrate $\int d^4h d^4p'$ yields

$$
\Gamma_{K,K,3}^{\text{loop}}(V) = - \sum_{n_4 = -\infty}^{\infty} g^4 \int d^4\theta_1 d^4p \frac{d^4r}{(2\pi)^4 (2\pi)^4} d^4q \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p, \theta_1, \bar{\theta}_1)
$$

$$
\times \left\{ \frac{1}{16((p + q + r)^2 + m_n^2)(r^2 + m_n^2)(q^2 + m_n^2)} \right\}.
$$

The contribution of figure 7 is

$$
\Gamma_{K,K,4}^{\text{loop}}(V) = \sum_{n_4 = -\infty}^{\infty} g^4 \int d^8z_1 d^8z_2 d^8z_3 d^8z_4 V(z_1, \theta_1, \bar{\theta}_1) V(z_3, \theta_3, \bar{\theta}_3)
$$

$$
\times \left\{ \frac{[D_2^2 \delta_{42}^8][16(\delta_2^2 - m_n^2)]}{16(\delta_2^2 - m_n^2)} \right\}.
$$

$$
\Gamma_{K,K,4}^{\text{loop}}(V) = - \sum_{n_4 = -\infty}^{\infty} g^4 \int d^8z_1 d^8z_2 d^8z_3 d^8z_4 V(z_1, \theta_1, \bar{\theta}_1) V(z_3, \theta_3, \bar{\theta}_3)
$$

$$
\times (D_1^2 D_1^2 \delta_{12}^8 \delta_{24}^8 (D_2^2 D_2^2 \delta_{23}^8) \delta_{34}^8)
$$

$$
\times \left\{ \frac{[16^2 \Box \delta_{41}^8 + i 16^2 \partial_\sigma \delta_{41}^8 D_\sigma + 16 \delta_{41}^8 D^2 D^2]}{16^5[(\delta_1^2 - m_n^2)(\delta_2^2 - m_n^2)(\delta_3^2 - m_n^2)(\delta_4^2 - m_n^2)]} \right\}.
$$

Using $\delta_{24}^8 D_2^2 D_2^2 \delta_{23}^8 = 16 \delta_{24}^8 \delta_{23}^8$, the correction becomes

$$
\Gamma_{K,K,4}^{\text{loop}}(V) = - \sum_{n_4 = -\infty}^{\infty} g^4 \int d^8z_1 d^8z_2 d^8z_3 d^8z_4 V(z_1, \theta_1, \bar{\theta}_1) V(z_3, \theta_3, \bar{\theta}_3)
$$

$$
\times (D_1^2 D_1^2 \delta_{12}^8 \delta_{24}^8 \delta_{23}^8)
$$

$$
\times \left\{ \frac{[16^2 \Box \delta_{41}^8 + i 16^2 \partial_\sigma \delta_{41}^8 D_\sigma + 16 \delta_{41}^8 D^2 D^2]}{16^5[(\delta_1^2 - m_n^2)(\delta_2^2 - m_n^2)(\delta_3^2 - m_n^2)(\delta_4^2 - m_n^2)]} \right\}.
$$

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With the properties of the super-spinor derivatives and integration by parts, and the following
\[
(D_1^2 \overline{D}_1^2 \delta_{12}) \delta_{24} \delta_{34} = 32 \delta_{12} \delta_{24} \delta_{34},
\]
the correction reduces to
\[
\Gamma_{KK,4}^{2\text{loop}}(V) = - \sum_{n_i = -\infty}^{\infty} g^4 \int d^8 z_1 d^8 z_2 d^8 z_3 d^8 z_4 V(z_1, \theta_1, \overline{\theta}_1) V(z_3, \theta_3, \overline{\theta}_3) \ 
\]
\[
\times 32 \delta^8_{12} \delta^8_{24} \delta^8_{34} \left\{ \frac{[16^2 \Box \delta_{11}^4 + i \frac{16^2}{2} \partial^\alpha \delta_{41}^4 \overline{D}_D D_\alpha + 16^4 \delta_{41}^4 \overline{D}_D^2 D^2]}{16^5[(\partial^2_1 - m_{n_i}^2)(\partial^2_2 - m_{n_i}^2)(\partial^2_3 - m_{n_i}^2)(\partial^2_4 - m_{n_i}^2)(\partial^2_5 - m_{n_i}^2)]} \right\}.
\]

By inspection, the above expression contain only \(\delta^4(\theta_1 - \theta_2) \delta^4(\theta_3 - \theta_4)\). Thus upon \(\int d^4 \theta_2 d^4 \theta_3 d^4 \theta_4\) integration, the correction vanishes
\[
\Gamma_{KK,4}^{2\text{loop}}(V) = 0
\]
(68)

These are all possible diagrams of chiral correction to the vector superfield.

### 3.2 Evaluation of Two-Loop Integrals

In this section, we compute the two-loop integrals of the non-vanishing super-diagrams:

The evaluation of figure 4 begins with the result of the previous section where superfield formalism was used to manipulate the contribution in to
manageable form. The correction is

$$\Gamma_{KK,1}^{2\text{loop}}(V)$$

$$= \sum_{n_i=-\infty}^{\infty} -g^4 \int d^4\theta_1 \tilde{V}(-p, \theta_1, \overline{\theta}_1) \tilde{V}(p, \theta_1, \overline{\theta}_1) \frac{d^4p}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{d^4k}{(2\pi)^4}$$

$$\times \left\{ 16^4(r + p + k)^2k^2 - \frac{16^4}{2}i(r + p + k)^2k^2 \overline{\mathcal{D}} \mathcal{D} \beta \right\}$$

$$- 16^3(r + p + k)^2D^2 \overline{D}^2$$

$$16^5 \left[ (p + k)^2 + m^2_{\eta_i} + (r + p + k)^2 + m^2_{\eta_i} + (r^2 + m^2_{\eta_i}) + (p + k)^2 + m^2_{\eta_i} + m^2_{\eta_i} + (1 - y_1 - y_2 - y_3 - y_4)(k^2 + m^2_{\eta_i}) \right] \right\} . \quad (69)$$

We first expand the numerator and reparametrizing the denominator

$$\Gamma_{KK,1}^{2\text{loop}}(V)$$

$$= \sum_{n_i=-\infty}^{\infty} -g^4 \int d^4\theta_1 \tilde{V}(-p, \theta_1, \overline{\theta}_1) \tilde{V}(p, \theta_1, \overline{\theta}_1) \frac{d^4p}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} N$$

$$\times 4! \int_0^1 dy_1 dy_2 dy_3 dy_4 \left[ y_1((p + k)^2 + m^2_{\eta_i}) + y_2((r + p + k)^2 + m^2_{\eta_i}) \right.$$

$$+ y_3(r^2 + m^2_{\eta_i}) + y_4((p + k)^2 + m^2_{\eta_i})$$

$$\left. + (1 - y_1 - y_2 - y_3 - y_4)(k^2 + m^2_{\eta_i}) \right]^{-5} ,$$

where $N$ is the expanded numerators. Using $\int_0^\infty t^n e^{-at} dt = \frac{\Gamma(n+1)}{a^{n+1}}$, and
completing the square in \( r \), the above becomes

\[
\Gamma_{KK,1}^{2\text{loop}}(V) = \sum_{n_i=-\infty}^{\infty} -g^4 \int d^4\theta_1 \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p, \theta_1, \bar{\theta}_1) \frac{d^4p}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} N! \int_0^{\infty} t^4 dt \]

\[
\int_0^1 dy_1 dy_2 dy_3 dy_4 e^{\frac{i}{\pi} \left( \frac{y_2 (p+k)}{y_2+y_3} \right)^2} \int_0^{\Gamma(5)} \left( \frac{1}{\sin^2 \theta} \right) \left( \begin{array}{c}
(y_2 + y_3) \left[ r + \frac{y_2 (p+k)}{y_2+y_3} \right] - \frac{y_2^2 (p^2+k^2+2pk)}{y_2+y_3} \\
+ k^2 (1-y_1-y_3-y_4) + 2kp(y_1+y_2+y_4) + p^2(y_1+y_2+y_4) + m_n^2 
\end{array} \right)^t \]

Shifting the integration variable to \( r' = r + \frac{y_2 (p+k)}{y_2+y_3} \equiv r + U \), the numerator becomes

\[
N = \frac{1}{16^2 \cdot 2} \left[ \begin{array}{c}
\frac{r^2 (32k^2 - ik^2 D^3 D_{\beta} - 32D^2 D_{\beta})}{\beta} \\
\frac{+ 32(U^2 k^2 + p^2 k^2 + k^4 - 2pU k^2 + 2pk^3 - 2Uk^3)}{\beta} \\
\frac{+ 32(-U^2 - p^2 - k^2 + 2pU - 2pk + 2Uk) D^2 D_{\beta}}{\beta} \\
\frac{-i(U^2 + p^2 + k^2 - 2pU + 2pk - 2Uk) k^2 D^3 D_{\beta}}{\beta} 
\end{array} \right].
\]

The integration w.r.t. \( r' \) is carried out using \( \int \frac{dr'd}{(2\pi)^4} e^{-ar'^2} = (4\pi a)^{-\frac{d}{2}} \) and

\[
\int \frac{dr'd}{(2\pi)^4} e^{-ar'^2} = \frac{1}{\Gamma(5)} \int_0^1 dy_1 dy_2 dy_3 dy_4 \]

\[
\Gamma_{KK,1}^{2\text{loop}}(V) = \sum_{n_i=-\infty}^{\infty} -Sg^4 \int d^4\theta_1 \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p, \theta_1, \bar{\theta}_1) \frac{d^4p}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} N! \int_0^{\Gamma(5)} \left( \begin{array}{c}
\frac{(y_2+y_3+y_4)^2}{y_2+y_3+y_4} + k^2 (1-y_1-y_3-y_4) + 2kp(y_1+y_2+y_4) + p^2(y_1+y_2+y_4) + m_n^2 
\end{array} \right)^t
\]

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where
\[
16^2 \cdot 2N' = \frac{(32k^2 - i k^\beta D^\beta D_\beta - 32D^2 D^2)}{32\pi^2(y_2 + y_3)^3t^3} + \frac{1}{16\pi^2(y_2 + y_3)^2t^2} \times \left[ 32(U^2k^2 + p^2k^4 + k^4 - 2pUk^2 + 2pk^3 - 2Uk^3) 
+ 32(-U^2 - p^2 - k^2 + 2pU - 2pk + 2Uk)D^2 D^2 \right] \right].
\]

(74)

Completing the square in \(k\) and then shift integration variable to
\[
k' = k + \frac{P}{32} \left( y_1 + y_2 + y_4 - \frac{y_2^2}{y_2 + y_3} \right) \equiv k + M
\]

(75)

The correction becomes
\[
\bar{\Gamma}^{K,K,1}_{K,K,1}(V)
= \sum_{n_1 = -\infty}^{\infty} -g^4 \int d^4\theta_1 \bar{V}(-p, \theta_1, \bar{\theta}_1) \bar{V}(p, \theta_1, \bar{\theta}_1) \frac{d^4p}{(2\pi)^4} N'! \int_0^1 \int_0^1 dy_1 dy_2 dy_3 dy_4
\times \frac{1}{\Gamma(5)} \int_0^1 t^4 dt \int_0^\infty \frac{d^4k'}{(2\pi)^4}
\times \exp \left\{ \begin{array}{c}
k'^2 \left( 1 - y_1 - y_3 - y_4 - \frac{y_2^2}{y_2 + y_3} \right) + m_p^2 \\
+ p^2 \left( \frac{(y_1 + y_2 + y_4 - \frac{y_2^2}{y_2 + y_3})^2}{1 - y_1 - y_3 - y_4 - \frac{y_2^2}{y_2 + y_3}} \right) + \left( y_1 + y_2 + y_3 - \frac{y_2^2}{y_2 + y_3} \right) \end{array} \right\} (-t).
\]

(76)
Integration w.r.t. $k'$ yields

$$
\Gamma_{KK,1}^{2\text{loop}}(V)
= \sum_{n_1 = -\infty}^{\infty} -g^4 \int d^4\theta_1 \tilde{V}(-p, \theta_1, \vec{\theta}_1) \tilde{V}(p, \theta_1, \vec{\theta}_1) \frac{d^4p}{(2\pi)^4} N''4!
\int_0^1 \frac{dy_1 dy_2 dy_3 dy_4}{\Gamma(5) 16^2 \cdot 2\pi^2 (y_2 + y_3)^2} \int_0^\infty t^2 dt
\times e \left[ p^2 \left( \frac{y_1 + y_2 + y_4 - \frac{y_2^2}{y_2^2 + y_3^2}}{1 - y_1 - y_3 - y_4 - \frac{y_2^2}{y_2^2 + y_3^2}} \right) + \left( y_1 + y_2 + y_3 - \frac{y_2^2}{y_2^2 + y_3^2} \right) \right] t,
\right]
\tag{77}
$$

where

$$
N'' = \frac{3}{64\pi^2 Y^4 t^4}
\left[
\frac{2 + \frac{2y_2^2}{(y_2 + y_3)^2}}{y_2 + y_3}
\right]
\left[
\frac{1}{(y_2 + y_3)^2} + 2R^2 + 2M^2 y_2^2 + 12M^2 + 2M^4 - 4pR
\right]
\left[
\frac{4pM y_2}{(y_2 + y_3)^2} - 12pM + 12RM - \frac{12M^2 y_2}{(y_2 + y_3)^2} - 2\overline{D}^2 D^2
\right]
\left[
\frac{1}{32\pi^2 Y^3 t^3} + \frac{1}{16\pi^2 Y^2 t^2}
\right]
\left[
\frac{M^2}{(y_2 + y_3)^2} + \frac{M^2 \overline{D}^2 D\beta}{(y_2 + y_3)^2} - \frac{\overline{D}^2 D^2}{(y_2 + y_3)^2}
\right]
\left[
+ 2M \left( \frac{R^2}{2} - 2RM y_2 \right) + \frac{M^2 y_2^2}{(y_2 + y_3)^2} \right] - 2pM + 2M^4
\left[
- 4pRM + 4pM^2 y_2 \right] - 4pM^3 - 4RM^3 + \frac{4M^4 y_2}{(y_2 + y_3)}
\left[
- \frac{4pM y_2}{(y_2 + y_3)} + 4pM - 4RM + \frac{4M^2 y_2}{(y_2 + y_3)} \overline{D}^2 D^2
\right]
\left[
\frac{1}{16} i - \frac{16(y_2 + y_3)}{y_2 M^\beta} \right] + \frac{16}{8} + \frac{8}{8}
\left[
\frac{p y_2 M^\beta}{8(y_2 + y_3)} + \frac{p M^\beta}{y_2 M^\beta} + \frac{p R M^\beta}{y_2 M^\beta} + \frac{p R M^\beta}{y_2 M^\beta}
\right]
\right].
\tag{78}
$$
Here
\[
Y = \left(1 - y_1 - y_3 - y_4 - \frac{y_2^2}{y_2 + y_3}\right),
\] (79)
\[
M = \frac{p \left( y_1 + y_2 + y_4 - \frac{y_2^2}{y_2 + y_3}\right)}{(1 - y_1 - y_3 - y_4 - \frac{y_2^2}{y_2 + y_3})},
\] (80)
\[
U = \frac{(p + k)y_2}{y_2 + y_3} \equiv R + \frac{ky_2}{y_2 + y_3},
\] (81)
\[
R = \frac{py_2}{y_2 + y_3}.
\] (82)

Correction of figure 5 is
\[
\Gamma_{KK,2}^{loop}(V) = \sum_{n_i = -\infty}^{\infty} -g^4 \int d^4\theta_1 \frac{d^4p}{(2\pi)^4} \frac{d^4r}{(2\pi)^4 \xi} \frac{d^4k}{(2\pi)^4} \bar{V}(-p, \theta_1, \theta_1) \bar{V}(p, \theta_1, \theta_1)
\]
\[
\times \left\{ \begin{array}{c}
\frac{-1}{16} k^2 + \frac{i}{32} k^6 D^\beta D_\beta - \frac{1}{16} D^2 D^2 \\
((r + p + k)^2 + m_{n_i}^2)(r^2 + m_{n_i}^2)((p + k)^2 + m_{n_i}^2)(k^2 + m_{n_i}^2)
\end{array} \right\}.
\] (83)

Similarly, we reparametrized the denominator and cast in terms of an exponential function. The correction yields
\[
\Gamma_{KK,2}^{loop}(V) = \sum_{n_i = -\infty}^{\infty} -g^4 \int d^4\theta_1 \frac{d^4p}{(2\pi)^4} \frac{d^4r}{(2\pi)^4 \xi} \frac{d^4k}{(2\pi)^4} \bar{V}(-p, \theta_1, \theta_1) \bar{V}(p, \theta_1, \theta_1)
\]
\[
\times \frac{3!}{\Gamma(4)} \int_0^1 \Pi_{i=1}^3 dy_i N \int_0^\infty dt \frac{d^3t}{t^3}
\]
\[
\times \exp \left\{ - \left[ \begin{array}{c}
(y_1 + y_2)r^2 - \frac{y_1^2(p + k)^2}{y_1 + y_2} + (1 - y_2)k^2 + (y_1 + y_3)p^2 \\
+ 2kp(y_1 + y_3) + 2ry_1(p + k) + m_{n_i}^2
\end{array} \right] t \right\},
\] (84)
where \( N = \left[ -\frac{1}{16}k^2 + \frac{i}{32}k^2D^2 D_\beta - \frac{i}{16}D^2 D^2 \right] \). Integration w.r.t. \( r' \) yields

\[
\Gamma_{K,2}^{\text{loop}}(V) = \sum_{n_i = -\infty}^{\infty} -g^4 \int d^4\theta_1 \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p, \theta_1, \bar{\theta}_1)
\]

\[
\times \left( \frac{3!}{\Gamma(4)} \right) \int_0^1 \frac{\Pi^3_{i=1} dy_i N}{16\pi^2(y_1 + y_2)^2} \int_0^\infty dt |D_\alpha(n_i) + m^2_n| t^{\frac{1}{2}} \right) \}
\]

Completing the square in \( k' \) and shift to \( k' = k + \left( \frac{y_1 y_2 + y_3 y_4}{y_1 + y_2 - y_3 - y_4} \right) p \equiv k + T \). Integrating w.r.t \( k' \) yields

\[
\Gamma_{K,2}^{\text{loop}}(V) = \sum_{n_i = -\infty}^{\infty} -g^4 \int d^4\theta_1 \frac{d^4p}{(2\pi)^4} \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p, \theta_1, \bar{\theta}_1) \frac{3!}{\Gamma(4)16\pi^2} (85)
\]

\[
\times \left( \frac{3!}{\Gamma(4)} \right) \int_0^1 \frac{\Pi^3_{i=1} dy_i N}{16\pi^2(y_1 + y_2)^2} \int_0^\infty dt \exp \left\{ - \left( \frac{y^2 + y_1 y_2}{(y_1 + y_2)(y_1 y_2 - y_3 y_4)} \right) + m^2_n \right\} t^{\frac{1}{2}} \right) \}
\]

\[
\times \left\{ \int_0^1 \frac{\Pi^3_{i=1} dy_i (y_1 + y_2)}{16\pi^2(y_1 + y_2)^2(y_1 y_2 - y_3 y_4)} \left[ -T^2 - \frac{i\gamma^0 D^\alpha(n_i) + m^2_n}{\sigma} + \frac{T^2 y_1^2}{16} \right] \right\}.
\]

The contribution from figure 6 is

\[
\Gamma_{K,3}^{\text{loop}}(V) = -\sum_{n_i = -\infty}^{\infty} g^4 \int d^4\theta_1 \frac{d^4p}{(2\pi)^4} \frac{d^4\theta_1}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p, \theta_1, \bar{\theta}_1)
\]

\[
\times \left\{ \frac{1}{16((p + q + r)^2 + m^2_n)^2 + m^2_n(r^2 + m^2_n)(q^2 + m^2_n)} \right\}.
\]

(87)
The evaluation of this diagram yields

\[ \Gamma_{KK,3}^{2\text{loop}}(V) = - \sum_{n_1=-\infty}^{\infty} \frac{g^4}{16^3\pi^4\Gamma(3)} \int d^4\theta_1 \frac{d^4p}{(2\pi)^4} \tilde{V}(-p, \theta_1, \bar{\theta}_1) \tilde{V}(p, \theta_1, \bar{\theta}_1) \]

\[ \times \int_0^1 \frac{dy_1dy_2}{(1-y_2)^2 \left( y_1 + y_2 - \frac{y_1^2}{1-y_2} \right)^2} \int_0^\infty \frac{dt}{t^2} \]

\[ \times \exp \left\{ - \left[ p^2 \left( y_1 - \frac{y_1^2}{1-y_2} - \frac{y_1 - \frac{y_1^2}{1-y_2}}{y_1 + y_2 - \frac{y_1^2}{1-y_2}} \right) + m_n^2 \right] t \right\} . \]  

(88)

Integration w.r.t. Feynman parameters \( \{y_i\} \) are carried out using maple. The correction to the effective action yields

\[ \Gamma_{total}^{2\text{loop}} = \Gamma_1^{2\text{loop}} + \Gamma_2^{2\text{loop}} + \Gamma_3^{2\text{loop}}. \]  

(89)

Similar to the one-loop correction, we define \( Z_2^{2\text{loop}} = 1 + \Delta Z_2^{2\text{loop}} \). At zero
momentum transferred \( p = 0 \), we calculated \( Z_2^{2\text{loop}} \) to be

\[
Z_2^{2\text{loop}} = 1 + \sum_{n_i = -\infty}^{\infty} \frac{g^{44!}}{16^3 \Gamma(5) 2\pi^4} \left\{ \begin{aligned}
&\frac{3}{2} \int_0^\infty \frac{dt}{t^2} e^{-m_n^2 t} \left[ -\frac{27}{70} - \frac{1}{2\epsilon^2} \left( \frac{1}{24} + \frac{1}{6\epsilon^2} + \frac{6}{5} \right) \right] \\
&\quad + \frac{1}{2} \int_0^\infty \frac{dt}{t} e^{-m_n^2 t} \left[ \left( -\frac{3}{10} \right) + \frac{1}{5\epsilon^2} \right] - 2 \mathcal{D}^2 D^2 \left( \frac{-7}{9} + 4 \ln 2 - \frac{3}{2\epsilon^2} - \frac{8}{9\epsilon^3} - \frac{1}{6\epsilon^4} \right) \\
&\quad + \int_1^\infty \frac{dy}{y} L(y_1) \\
&\quad - \frac{1}{2} + \frac{23}{24} \arctan h \left( \frac{4}{25} \sqrt{14} \right) - \frac{33}{24} \arctan h \left( \frac{1}{25} \sqrt{14} \right) \\
&\quad + \frac{1}{2\epsilon} + 3 \ln \epsilon - \int_0^1 K(y_1) dy_1 \\
&\quad \left( \frac{-1}{\epsilon} - 2 \ln \epsilon + 1 - \frac{9}{4} \ln 4 \right) + \int_0^1 F(y_1) dy_1 \end{aligned} \right\} \tag{90}
\]

...
Summing over the Kaluza Klein modes yields

\[
Z_2^{2\text{loop}} = 1 + \frac{g^4 4!}{16^3 \Gamma(5) 2\pi^4} \left\{ \frac{3}{2} \int_0^{\infty} \frac{dt}{t^2} \left\{ \Theta_3\left( \frac{it}{\pi R^2} \right) \right\}^\delta \left[ -\frac{27}{70} - \frac{1}{7} \left( \frac{1}{2} \frac{1}{2} + \frac{6}{5} \right) \right] \\
+ \frac{1}{2} \int_0^{\infty} \frac{dt}{t} \left\{ \Theta_3\left( \frac{it}{\pi R^2} \right) \right\}^\delta \left[ \left( -\frac{3}{10} - \frac{1}{5i\pi} \right) \frac{1}{t} - 2D^2 D^2 \left( -\frac{3}{20} - \frac{1}{4} \left( \frac{1}{10} - \frac{1}{4} \right) \right) \right] \\
- \int_0^{\infty} \frac{dt}{t} \left\{ \Theta_3\left( \frac{it}{\pi R^2} \right) \right\}^\delta \left\{ \partial D^2 D^2 \left[ -\frac{7}{9} + 4 \ln 2 - \frac{3}{2} + \frac{8}{9} - \frac{1}{6 \pi^2} + \frac{1}{3 \pi} + \frac{1}{3} \ln \varepsilon \right] \right\} \right\}. \\
\]

\[
- \frac{g^4 3!}{16^3 \Gamma(5) \pi^4} \left\{ \frac{1}{2} \int_0^{\infty} \frac{dt}{t^2} \left\{ \Theta_3\left( \frac{it}{\pi R^2} \right) \right\}^\delta \left[ \frac{9877}{1030} + \frac{3}{64 \pi^2} + \frac{1645}{1024} - \frac{1277713}{107912} \right] \ln \varepsilon \\
+ \int_0^{1} L(y_1) dy_1 \right\} \\
\times \left\{ \frac{23}{24} \left[ \frac{4}{28} \tan \left( \frac{4}{28} \sqrt{14} \right) - \frac{23}{24} \tan \left( \frac{4}{28} \sqrt{14} \right) \right] \right\} \\
\times \left\{ \frac{1}{2} + \frac{3}{2 \pi} + 3 \ln \varepsilon - \int_0^{1} K(y_1) dy_1 \right\} \\
- \frac{g^4 2!}{16^3 \Gamma(3) \pi^4} \int_0^{\infty} \frac{dt}{t^2} \left\{ \Theta_3\left( \frac{it}{\pi R^2} \right) \right\}^\delta \left\{ \frac{1}{\pi} - 2 \ln \varepsilon + 1 \right\} \\
\times \int_0^{1} F(y_1) dy_1 \right\}. \tag{91} \]

4 Summary

In this paper, we have embedded Kaluza-Klein excitations directly into \( N = 1, D = 4 \) MSSM via superfield formulation, where many component fields diagrams are calculated simultaneously. The effects of the extradimensions are transparent through the computation of the one-loop correction to the vector superfield. As application, we calculated the Beta functions and evolution of the gauge couplings derived here agree with the component field approach. In addition, we calculate the two-loop partial corrections to vector superfield where pure virtual vector loops corrections are not considered.
5 Appendix

Figure 1: Chiral Correction.
Figure 2: Vector Correction.
Figure 3: Ghost Correction.
Figure 4: Quantum Correction of Diagram #4 to the Effective Action.
Figure 5: Quantum Correction of Diagram #5 to the Effective Action.
Figure 6: Quantum Correction of Diagram #6 to the Effective Action.
Figure 7: Quantum Correction of Diagram #7 to the Effective Action.

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Figure 1
Figure 5
Figure 6
