Acoustic radiation force and torque on red blood cells in viscous and viscoelastic fluids

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A blood vessel can be considered as a microfluidic system where the red blood cells (RBCs) are floating inside it. While an ultrasound wave is established, all microparticles are enforced by acoustophoretic forces. In this contribution, the acoustic radiation force and torque applied to each particle in one and two-particle systems including spheres and also RBCs are studied. The surrounding fluid in microfluidic devices is often water, while in our study is viscous and then viscoelastic fluid. In one–particle systems, we obtain that the radiation force on a sphere is greater than on an RBC of the same size. Furthermore, for both viscous and viscoelastic fluids the force increases (decreases) by fluid’s viscosity (relaxation time). In two–particle systems, the total radiation force applied to each particle consists of primary and interaction forces. The acoustic interaction forces between two spheres and also two RBCs in viscous fluids with various viscosities are examined. The force is a decreasing exponential function of the inter–particle distance. In the following, for a definite inter–particle distance, we obtain the functionality of the force between two RBCs in terms of viscosity and angle of rotation in a viscous fluid. The obtained equation indicates that the acoustic interaction force decreases linearly by the rotation angle. Then, the interaction force in viscoelastic fluid is obtained as a function of interparticle distance, rotation angle and rheological parameters such as viscosity and relaxation time. Our findings represent that the force is a power-law function of interparticle distance and an exponential function of rotation angle. Finally, the acoustic interaction torque on each RBC in viscous and viscoelastic fluids is obtained as a sinusoidal function of rotation angle.

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I. Introduction

Many studies have been done on particle manipulation which is essential for analyzing micro-sized particles in numerous biological and chemical applications [1–4]. The movement of micro-sized particles by ultrasound, known as acoustophoresis has received a lot of attention in the last decades [5–9]. Particles exposed to an external acoustic wave that is mainly a standing wave, are subjected to time-averaged forces from scattering waves called acoustic radiation forces. There are some computational and analytical works on the calculation of the acoustic radiation force exerted on a particle with an arbitrary shape, but most of these studies were done on particles with spherical geometry [5–8, 10–12]. In practice, there are many micro-sized particles in biological solutions. Therefore, the interaction force between particles plays a significant role which makes our study important. For two suspended micro-particles in an acoustic field, a nonlinear total force consist of primary and secondary forces is applied to each particle. The acoustic primary force is due to the incident and the scattered waves from the surface of the particle. The primary force pushes particles into the pressure node or antinode depending on their acoustic contrast factor [12]. The secondary (interaction) force is exerted on each particle owing to the effect of the scattered wave from the other one [13–20]. Most of contributions payed significant attention on the interaction force between two bubbles or sphere particles while there are few studies on radiation force between red blood cells.

At first, the acoustic interaction force is studied by Bjerknes [2] to investigate the acoustic inter-particle force between a pair of bubbles, based on the theory of rescattering of scattered waves. Then, Apfel and Embleton [3] obtained an approximation for the acoustic interaction force between two spheres using the King’s and Yosioka’s methods [1,6]. Doinikov and Zavtrak [21, 22] applied multipole re-expansion method. They used monopole and dipole terms of the multipole series expansion to calculate the primary and interaction forces. Doinikov [9] used five terms of this expansion to estimate the secondary force between two bubbles in the water. Bruus and Silva in 2014 indicated that the interaction force can be derived from the potential field just like the primary force [23]. Recently, Sepehrirahnama and his colleagues [24] investigated the acoustic radiation force and torque on nonspherical shape particles and studied the effect of Willis coupling. They presented a formalism of acoustic radiation force and torque based on the polarizability tensor, where Willis coupling terms are accounted.
To our knowledge, there is no study on interaction between RBCs in the presence of acoustic waves. Moreover, although particle movement in Newtonian fluids has been widely studied so far [9,18], there is little research on particle movement in non-Newtonian fluids. Non-Newtonian fluids such as blood and many other fluids are very ubiquitous in our daily life. Therefore it is important to invest our research focus on particle manipulation in non-Newtonian fluids to develop our understanding of cells behavior inside such fluids. In this paper, we have obtained the acoustic interaction force between RBCs in Newtonian and non-Newtonian fluids which have never been done before.

The rest of the manuscript is organized as follows. In Sec. II, the theoretical background is expanded by implementing a viscoelastic fluid model in the governing equations of acoustophoresis in the framework of first and second-order perturbation theory [25–29]. The numerical model and boundary conditions are described in Sec. III. The results are presented and discussed in Sec. IV. The last section concludes the paper.

**II. Theoretical background**

In the absence of body forces, the fluid flow governing equations in the microfluidic systems for a viscous fluid are the kinematic continuity equation of \( \rho \) and the dynamic Navier-Stokes equation for the velocity field \( \mathbf{v} \), as

\[
\partial_t \rho = \nabla \cdot [-\rho \mathbf{v}] ,
\]

\[
\partial_t (\rho \mathbf{v}) = \nabla \cdot [\mathbf{p} \mathbf{I} + \sigma] .
\]

The shear stress, \( \sigma \), of viscous fluid is defined as,

\[
\sigma = \eta[\nabla \mathbf{v} + (\nabla \mathbf{v})^\mathsf{T}] + [\eta_b - \frac{2}{3}\eta](\nabla \cdot \mathbf{v}) \mathbf{I} ,
\]

where \( \eta \) and \( \eta_b \) are the dynamic and bulk viscosities, respectively. For viscoelastic fluids, a variety of models has been reported in the literature [25,26,31], to describe such fluids behavior. Here, we consider the Maxwell model [31] to investigate the effect of viscoelastic fluid on the acoustophoretic forces. In this model, the constitutive equation of fluid is expressed by

\[
\sigma + \frac{\eta}{G} \frac{\partial \sigma}{\partial t} = \eta \dot{\gamma} ,
\]
where $\dot{\gamma}$ and $G$ are the shear rate and dynamic modulus, respectively. The dot denotes the time derivative. The symmetric $\dot{\gamma}$ is defined as,

$$\dot{\gamma} = [\nabla v + (\nabla v)^T].$$  

(4)

Due to the frame invariance of the stress tensor, the time derivative of $\tau$, is replaced with the upper-convected time derivative as [30–32]

$$\nabla_T = \frac{\partial \tau}{\partial t} + v \cdot \nabla - (\nabla v) \cdot \tau - \tau \cdot (\nabla v)^T.$$  

(5)

Therefore, the Maxwell model is replaced with the upper-convected Maxwell model (UCM) [31]. In this way, the constitutive equation is represented by

$$\tau + \lambda \nabla_T = \eta \dot{\gamma},$$  

(6)

where $\lambda = \eta/G$ is the relaxation time. In this contribution, the governing equations are solved using the perturbation theory detailed by [32–34].

A. First-order perturbation

According to the perturbation theory, acoustic variables are considered as,

$$\rho(r,t) = \rho_0 + \rho_1(r,t) + \rho_2(r,t),$$  

(7a)

$$p(r,t) = p_0 + p_1(r,t) + p_2(r,t),$$  

(7b)

$$v(r,t) = v_0 + v_1(r,t) + v_2(r,t),$$  

(7c)

$$\tau(r,t) = \tau_0 + \tau_1(r,t) + \tau_2(r,t).$$  

(7d)

where subscript 0, 1 and 2 indicate the zeroth, first and second order of perturbation, respectively. The fluid is considered to be at rest in the unperturbed state, so the values of $v_0$ and $\tau_0$ is set to be zero. First order variables are considered time-harmonic with angular frequency $\omega = 2\pi f$; so that we have $v_1(r,t) = v_1(r)e^{-i\omega t}, \rho_1(r,t) = \rho_1(r)e^{-i\omega t}, p_1(r,t) = p_1(r)e^{-i\omega t}$, and $\tau_1(r,t) = \tau_1(r)e^{-i\omega t}$. In isentropic cases, we have $p_1 = c_0^2 \rho_1$, where $c_0$ is the speed of sound [35].

By substituting Eq. (7) into the Eq. (1), the first-order perturbation of the governing
equations are obtained as follow,

\[ i\omega \rho_1 = \rho_0 \nabla \cdot \mathbf{v}_1, \quad (8a) \]

\[ -i\omega \rho_0 \mathbf{v}_1 = -\nabla p_1 + \nabla \cdot \mathbf{\tau}_1. \quad (8b) \]

The first-order perturbation of shear stress, \( \mathbf{\tau}_1 \), of viscous fluid is written as,

\[ \mathbf{\tau}_1 = \eta \left[ \nabla \mathbf{v}_1 + ( \nabla \mathbf{v}_1 )^T \right] + \left[ \eta_b - \frac{2}{3} \eta \right] ( \nabla \cdot \mathbf{v}_1 ) \mathbf{I}, \quad (9) \]

and for a viscoelastic fluid, the equation (9) is replaced by the following equation,

\[ \mathbf{\tau}_1 = \eta^* \left[ \nabla \mathbf{v}_1 + ( \nabla \mathbf{v}_1 )^T \right], \quad (10) \]

where \( \eta^* = \eta/(1 - iDe) \) denotes the complex viscosity and \( De = \lambda \omega \) is called Deborah number. At \( \lambda = 0 \), the viscoelastic fluid behaves like a viscous fluid [26].

Considering all fields time dependency in the form of \( e^{-i\omega t} \), the first-order acoustic wave equation for \( p_1 \) is obtained by insertion of Eqs. (8a) and (9) into the Eq. (8b) [12],

\[ \nabla^2 p_1 + \frac{\omega^2}{c_0^2} \left( 1 + i\omega \frac{\eta_b}{\rho_0} + \frac{1}{3} \eta \right) p_1 = 0. \quad (11) \]

For viscoelastic cases, the first-order acoustic wave equation is expressed by,

\[ \nabla^2 p_1 + \frac{\omega^2}{c_0^2} \left( 1 + i\omega \frac{2\eta^*}{\rho_0} \right) p_1 = 0. \quad (12) \]

**B. Second-order perturbation**

The time-averaged second-order perturbation approximation of governing equations for a viscous fluid are expressed by,

\[ \nabla \cdot \langle \mathbf{v}_2 \rangle + \kappa_s \langle \mathbf{v}_1 \cdot \nabla p_1 \rangle = 0, \quad (13a) \]

\[ \nabla \cdot \left[ \langle \mathbf{\tau}_2 \rangle - \langle p_2 \rangle \mathbf{I} - \rho_0 \langle \mathbf{v}_1 \mathbf{v}_1 \rangle \right] = 0, \quad (13b) \]

where \( \kappa_s = \frac{1}{\rho_s} \left( \frac{\partial \rho}{\partial p} \right)_s = \frac{1}{\rho_0 c_0^2} \) is the isentropic compressibility in classical fluid mechanics. In the
adiabatic limit $\rho_1 = \rho_0 \kappa_s p_1$. The time-averaged second-order perturbation of shear stress, $\tau_2$, of a viscous fluid is

$$\langle \tau_2 \rangle = \eta \langle \nabla v_2 + (\nabla v_2)^T \rangle + [\eta_b - \frac{2}{3} \eta] \langle \nabla \cdot v_2 \rangle I.$$  \hspace{1cm} (14)

For a viscoelastic fluid, Eq. (14) turns into

$$\langle \tau_2 \rangle = \eta \langle \nabla v_2 + (\nabla v_2)^T \rangle - \lambda \eta^* \langle (v_1 \cdot \nabla)(\nabla v_1 + (\nabla v_1)^T) \rangle$$
$$+ \lambda \eta^* \left\langle (\nabla v_1) \cdot [\nabla v_1 + (\nabla v_1)^T] + [\nabla v_1 + (\nabla v_1)^T] \cdot (\nabla v_1)^T \right\rangle.$$ \hspace{1cm} (15)

In the above equations the time average over full oscillation period, $T$, of each quantity, $Y(t)$, is defined as,

$$\langle Y \rangle = \frac{1}{T} \int_0^T Y(t) dt.$$ \hspace{1cm} (16)

The physical, real-valued time average of two harmonically varying fields with the complex representation, is given by

$$f(r, t) = f(r) e^{-i\omega t},$$ \hspace{1cm} (17a)
$$g(r, t) = g(r) e^{-i\omega t},$$ \hspace{1cm} (17b)
$$\langle fg \rangle = \frac{1}{2} Re[f^*g].$$ \hspace{1cm} (17c)

where the asterisk denotes complex conjugation.

C. Acoustic radiation force and torque

In this section, the acoustic radiation force and torque applied to microparticles, suspended in viscous and viscoelastic fluids in a standing plane wave, are studied. In our manuscript, the first-order scattering theory is used, because the particles' size is much smaller than the incident acoustic wavelength and they act as weak scattering points. The acoustic radiation force is obtained from the inviscid theory. This is a correct approximation for particles extremely larger than the viscous penetration depth $\delta = \sqrt{2\nu/\omega}$ ($\nu$ is the kinematic viscosity) [12]. At first, a single particle, and then a pair of particles suspended in viscous and viscoelastic fluids in a standing plane wave (at room temperature) are studied. The incoming wave is described by the velocity field $v_{in}$, and the outgoing wave from the particle is described by the velocity
field \( v_{sc} \). Therefore, the first-order velocity field \( v_1 \) is expressed by

\[
v_1 = v_{in} + v_{sc}.
\]

The acoustic radiation force acting on each particle is obtained by

\[
F_{rad} = - \int_{\partial \Omega} da \left( \langle p_2 \rangle n + \rho_0 \langle (n \cdot v_1) v_1 \rangle \right)
\]

\[
= - \int_{\partial \Omega} da \left( \frac{\kappa_s}{2} \langle p_1^2 \rangle - \frac{\rho_0}{2} \langle v_1^2 \rangle \right) n + \rho_0 \langle (n \cdot v_1) v_1 \rangle ,
\]

where \( \partial \Omega \) is the particle’s surface, and \( n \) is the unit-vector normal to the surface.

The nonlinear interaction of ultrasound waves with a non-spherical particle may result in an acoustic radiation torque on the particle. The density of linear momentum flux transported by an acoustic wave is given by \( \langle P \rangle = - \langle \mathcal{L} \rangle I + \rho_0 \langle v_1 v_1 \rangle \), in which \( I, \mathcal{L} = \frac{\kappa_s}{2} \langle p_1^2 \rangle - \frac{\rho_0}{2} \langle v_1^2 \rangle \), and \( \rho_0 v_1 v_1 \) are unit tensor, Lagrangian density and Reynolds’ stress, respectively. The acoustic radiation torque is calculated using angular momentum flux which is defined as \( \langle L \rangle = r \times \langle P \rangle \).

Thus, the acoustic radiation torque on a particle is defined as [36],

\[
T = \int_{\partial \Omega} da \langle L \rangle \cdot \hat{n} = - \int_{\partial \Omega} da \left[ r \times \left( \langle \mathcal{L} \rangle I - \rho_0 \langle v_1 v_1 \rangle \right) \right] \cdot \hat{n},
\]

where \( r \) is the position vector that connect the origin of coordinate system to surface points.

### III. Numerical model and boundary conditions

As shown in Fig. 1, a microchannel with a square cross-section of the size, \( h = 500 \mu m \) in the xy plane is considered. The incident wave is a resonant, standing one-dimensional (1D) pressure wave of the form \( p = p_{amp} \sin(2\pi y/w) \), in which \( w \) and \( p_{amp} \) are the wave length and the amplitude of the wave. In our simulation, the diameter of the sphere is considered 8 \( \mu m \) and the diameter and thickness of the RBC are assumed 8 \( \mu m \) and 2.98 \( \mu m \) respectively.

The governing equations are solved for a one and two-particle systems of RBCs and spheres suspended in viscous and viscoelastic fluids using the finite element method. The weak-form-PDE is used to solve these equations [37, 39]. In the weak-form-PDE method, first, the flow equations are written as source-free flux formulation, \( \nabla \cdot J + F = 0 \). Then, they are converted to the weak-form. At the end, the weak-form equations are solved by finite element method.
Table 1: Physical parameters of fluids and incident acoustic wave at $T = 25^\circ C$ and $p_0 = 0.1013 \, MPa$. \( \eta = 16.9 \, mPa.s \) and \( \lambda = 7.8 \, ms \) are the real experimental data for whole blood [12,38].

| Parameter          | Symbol | Value | Unit  |
|--------------------|--------|-------|-------|
| Wave amplitude     | \( p_{amp} \) | 1     | bar   |
| Wave length        | \( w \)  | 1000  | \( \mu m \) |
| Actuation frequency| \( f \)  | 1.5   | MHz   |
| Mass density       | \( \rho_0 \) | 1000  | \( \text{kg/m}^3 \) |
| Speed of sound     | \( c_0 \)  | 1500  | m/s   |
| Bulk viscosity     | \( \eta_b \) | \( 2.485 \times 10^{-3} \, \text{Pa} \cdot \text{s} \) |
| Dynamic viscosity  | \( \eta \)  | 5, 9, 16.9, 25 | Pa·s |
| Relaxation time    | \( \lambda \) | 0, 0.6, 4, 7.8, 16 | ms |

Considering \( \mathbf{J} \) as a vector and \( F \) as a scalar (e.g. equations 8a and 13a), the weak form of a source-free flux equation is given by

$$
\int_{\Omega} \left[ -\nabla \tilde{\psi} \cdot \mathbf{J} + \tilde{\psi} F \right] \, d\mathbf{r} = 0 , \tag{21}
$$

where \( \tilde{\psi} \) is a test function. For \( \mathbf{J} \) as a tensor and \( \mathbf{F} \) as a vector (e.g. equations 8b and 13b), the weak form of a source-free flux equation is

$$
\int_{\Omega} \left[ -\nabla \tilde{\Psi}_m \cdot \mathbf{J} + \tilde{\Psi}_m \hat{m} \cdot F \right] \, d\mathbf{r} = 0 , \text{ for all } m \tag{22}
$$

where \( \tilde{\Psi}_m \) is the \( m \)-th component of a test vector \( \tilde{\Psi} \). In all cases, the zero-flux boundary condition, \( \mathbf{J} \cdot \mathbf{n} = 0 \), is considered. All boundaries of Fig. 1 including microchannel and particles, except the bottom wall, are considered as hard walls. Therefore, the velocity field for these boundaries is \( \mathbf{v} \cdot \mathbf{n} = 0 \). For the bottom wall the boundary condition is \( p = p_{amp} \sin(2\pi y/w) \).

The maximum mesh size on particles’ boundaries is equal to \( 0.01 \mu m \), and \( 2 \mu m \) in bulk region. The mesh element growth rates on particles’ boundaries and in bulk are 1.0003 and 1.1, respectively. A sketch of spatial mesh of computational domain is presented in Fig. 2. The fluid inside the microchannel is supposed to be a quiescent viscous and viscoelastic fluid. The physical parameters of fluids and the incident acoustic wave at temperature $T = 25^\circ C$ and pressure $p_0 = 0.1013 \, MPa$ are presented in Table 1.

By solving the governing equations, the pressure and velocity fields are obtained. Then the
acoustic radiation forces and interaction torque are calculated by Eqs. (19) and (20).

IV. Results and discussion

In this study, extensive numerical calculations were carried out to study the acoustic radiation forces and torques applied to particles in one-particle and two-particle systems consisting of RBCs and spheres. The particles are suspended in viscous and viscoelastic fluids which are exposed to an external standing plane wave. The results of RBCs and spheres are compared. Our results show that the effective area of particles against the incoming wave is important in calculating the acoustic forces. Also the results indicates that the viscosity and in the viscoelastic fluid case, the relaxation time, play important roles in the magnitude of acoustic forces experienced by particles. In the following, results are presented and their implications are discussed.

A. One-particle system

The configurations of a one-particle system for a sphere and RBC are shown in Fig. 2. The standing wave is considered in y-direction. Figure 3 indicates the acoustic radiation force versus the particle distance from the pressure node, $d$, which is applied to an RBC with $\theta = 0^\circ$, suspended in a viscous fluid with $\eta = 16.9$ mPa.s. The value of radiation force at $d = 0$ is equal to zero which means that the pressure node is an equilibrium point. The results show that the period of radiation force is twice of the pressure distribution in compatible with Eq. 19.

The behavior of acoustic radiation force on the RBC in a viscous fluid with various viscosities is compared in Fig. 4(a). The results show that for definite $d$ value, the radiation force increases with fluid viscosity. In the next step, the radiation forces on the RBC as a function of $d$ for viscous and viscoelastic fluids are presented in Fig. 4(b). Our simulation data shows that the radiation force in a viscous fluid is more than in viscoelastic one. This result is compatible with relation $\eta^* = \eta/(1 - iDe)$. By this relation for definite $\eta$ value, the effective viscosity of viscoelastic fluid is less than the viscous fluid. Therefore, using the results of Fig. 4(a), it is reasonable that the radiation force on an RBC in a viscoelastic fluid is less than viscous fluid.

The comparison between the radiation forces acting on the sphere and RBC is shown in Fig. 5. Figures 5(a) and 5(b) are the results for viscous fluid; while the Figs. 5(c) and 5(d)
comapare the radiation forces for the viscoelastic case. The results indicate that the sphere always experiences a greater force than the RBC. The reason is that for a same radius of these two particles (see Fig. 2), the effective cross-section of sphere against the incident wave is more than RBC.

B. Two-particle system

For two-particle microfluidic systems, the total radiation force applied to each particle is divided into the primary and secondary forces. The primary force applied to each particle is due to the waves incoming on and the scattered from that particle. The secondary force is caused by the scattered waves from the other particle. In this section, first, the total and primary forces applied to each particle are calculated and then the interaction force between them is examined in detail. At the end, the acoustic interaction torque imposed on each particle is discussed. The schematic of system configuration is presented in Fig. 6.

1. Total and primary forces

In this section, the total and primary forces applied to each particle is calculated. First, a two-particle system consist of two RBCs with \( \theta = 0^\circ \) suspended in a viscous fluid is considered. Figure 7(a) shows the total and primary forces applied to each RBC in a viscous fluid. In semi-logarithmic scale, the total force has a discontinuity for an intermediate value of \( L \) which is due to the zero value of total force at this point. For small \( L \) values, the acoustic interaction force between two RBCs is strong, that is why the total radiation force is large at small distances. By increasing the inter-particle distance, the interaction force decreases while the primary force increases. At a definite \( L \), these two forces neutralize each other; therefore, the total force becomes zero. For large \( L \) values, the interaction force is negligible, so the total and primary forces are almost equal. The comparison of total forces for various viscous fluids show that the decay of interaction force is faster for smaller viscosities. So the zero value of total force occurs in small \( L \) values for small viscosities.

The primary and total forces for viscoelastic fluids with various \( \lambda \) values are presented in Fig. 7(b). The results show that the decay rate of interaction force increases by \( \lambda \) in compatible with the relation \( \eta^* = \eta/(1 - i\lambda\omega) \) and the results of Fig. 7(a).

2. Acoustic interaction force in a viscous fluid
In this section, the acoustic interaction forces between two spheres and two RBCs with \( \theta = 0 \), suspended in a viscous fluid are computed as functions of fluid viscosity \( \eta \) and inter–particle distance \( L \). The forces between a pair of spheres and a pair of RBCs have been plotted in Figs. 8(a) and 8(b), respectively. The data show that the value of \( \ln F \) decreases linearly by \( L \) as 
\[
\ln F = A(\eta)L + B(\eta).
\]
Furthermore, for a definite inter–particle distance, \( F \) increases by \( \eta \). The functionality of \( A \) and \( B \) are presented in Figs. 8(c) and 8(d) for both spheres and RBCs. Thus, according to these data one may write the interaction forces for two spheres and two RBCs, respectively as
\[
F(L, \eta) = \begin{cases} 
\exp \left[ (0.006 \ln \eta - 0.15)L + (0.14 \ln \eta - 10.95) \right], & \text{spheres} \\
\exp \left[ (0.004 \ln \eta - 0.14)L + (0.24 \ln \eta - 11.65) \right], & \text{RBCs}
\end{cases}
\]
These relations in Eq. (23) show that the functionality of forces is similar in both cases, but the coefficients are different which is due to the effect of particles geometry. For definite \( \eta \) and \( L \) values, the interaction force between spheres is bigger than RBCs.

In the next step, we study the behavior of \( F \) between RBCs as a function of \( \theta \) and \( \eta \) for a definite \( L \) value. Figure 9 (a) indicates \( F \) versus \( \theta \) for various viscosities. The results show that \( F \) is a linear decreasing function of \( \theta \). The slopes and intercepts of \( F(\theta, \eta) \), are plotted in Figs. 9(b) and 9(c), respectively. Using these data, the functionality of \( F \) is obtained as,
\[
F(\theta, \eta) = -(0.0026 \ln \eta + 0.01)\theta + (9.81 \times 10^{-7})\eta^{0.32}.
\]
Equation (24) in agreement to the results of Fig. 9 (a) indicate that the maximum value of the interaction force is at \( \theta = 0^\circ \). The reason is that for this configuration, particles are exposed to the incident waves with their largest effective cross-section.

### 3. Acoustic interaction force in a viscoelastic fluid

Following our calculations in previous section, we obtain the acoustic interaction force between a pair of spheres and RBCs suspended in a viscoelastic fluid. The interaction force between two RBCs as a function of \( L \) for various \( \eta \) and \( \lambda \) is plotted in Figs. 10(a) – 10(d). The results show that for definite \( \eta \) and \( \lambda \) values, \( F \) behaves as a decreasing power-law function of \( L \). Therefore, we can consider the interaction force as \( F = m_2 L^{m_1} \). The functionality of \( m_1 \) and \( m_2 \) versus \( \lambda \)
for various $\eta$ are plotted in Figs. 10(e) and 10(f), respectively. Insets are the slopes of $m_1$ and $m_2$ as functions of $\eta$. By fitting all these data the interaction force is obtained as,

$$F(L, \eta, \lambda) = [(-31 \ln \eta - 73.44) \ln \lambda + 166.1 \ln \eta + 12.8]L^{(0.31 \eta^{0.83} - 0.04 \ln \eta - 2.4)}.$$ (25)

By repeating all these steps, one may obtain the interaction force between two spheres as follows,

$$F(L, \eta, \lambda) = [(-79.1 \ln \eta - 107.23) \ln \lambda + 347.46 \ln \eta + 36.84]L^{(0.76 \eta^{0.33} - 0.07 \ln \eta - 2.34)}.$$ (26)

Equations (25) and (26), in agreement with the data of Fig. 10 show that $F$ increases (decreases) by viscosity (relation time). These two equations show that the functionality of $F(L, \eta, \lambda)$ in both cases is similar, but the constant coefficients are different. For definite $L$, $\eta$, and $\lambda$ values the interaction force between two spheres is bigger than two RBCs.

Subsequently, we study the acoustic interaction force between two RBCs as a function of $\theta$, $\eta$, and $\lambda$ for a definite $L$ value. Figures 11(a) – 11(d) indicates the behavior of $F$ as a function of $\theta$ for various viscosities and relaxation times. The data show that $\ln F$ is a linear decreasing function of $\theta$. The slopes and intercepts are plotted in Figs. 11(e) and 11(f), respectively. Insets are the slopes of $s_1$ and $s_2$ versus $\eta$. Using these data, the interaction force is obtained as follows,

$$F(\theta, \eta, \lambda) = \exp((0.002\eta^{-1.17}\lambda - 0.015\eta^{0.012})\theta + (0.23 \ln \eta - 1.06) \ln \lambda + 0.3 \ln \eta - 15.12).$$ (27)

Equation (27) in compatible with Fig. 11 shows that the maximum value of the force occurs at $\theta = 0^\circ$ and as mentioned before, it is due to the fact that in this angle particles are exposed to the acoustic waves with their largest effective cross-section.

### 4. Acoustic interaction torque

In this section, the acoustic interaction torque applied to each particle in a two-particle system of RBCs suspended in viscous and viscoelastic fluids is studied. The configuration of system has been shown in Fig. 1.

For viscous fluids, the interaction torque is obtained as a function of $\eta$ and $\theta$. The value of
torque $T$ exerted on a particle has been plotted in Fig. 12 (a). These data indicate that the functionality of $T$ is sinusoidal. Therefore, we have fitted the results by $T = T_0(\eta) \sin(m\theta + b_0(\eta))$. The functionality of $T_0$ and $b_0$ are presented in Figs. 12 (b) and 12 (c). For all cases, the coefficient $m$ is constant. After performing the fitting process, we find the following functionality for the interaction torque to $\theta$ and $\eta$,

$$T(\theta, \eta) = (0.65 \times 10^{-10}\eta^{0.22}) \sin [0.035\theta + \exp(0.036\eta - 3.77)] .$$  \hspace{1cm} (28)

Our simulation results in compatible with Eq. (28) show that the maximum value of $T$ is at $\theta = 45^\circ$. The torque on each RBC at $\theta = 0^\circ$ is zero; because the configuration in this situation is symmetric. For $\theta = 90^\circ$, the effective cross-section of particles and consequently the interaction torque are very small. Moreover, the value of $T$ increases by $\eta$.

For viscoelastic fluid, $T$ is obtained as a function of $\eta$, $\theta$, and $\lambda$. The results of $T$ is shown in Figs. 13 (a)–13 (d). The functionality of the torque is similar to what we obtained for viscous fluid. The functionality of $T_0$ and $b_0$ are shown in Figs. 13 (e) and 13 (f), respectively. The coefficient $m$ is independent of $\eta$ and $\lambda$. Insets are the slopes of $T_0$ and $b_0$ as functions of $\lambda$. As clearly seen, the results suggest the following relation for $T$,

$$T(\theta, \eta, \lambda) = (-0.24 \ln\lambda + 1.2) \ln \eta \times 10^{-11} \sin [0.035\theta - (0.018\lambda + 0.014)\eta^{-0.17\ln\lambda - 0.12}] ,$$ \hspace{1cm} (29)

which shows that $T$ decreases by $\lambda$ and increases by $\eta$.

**V. Conclusion**

In this paper, the acoustic radiation forces and torques applied to a sphere and also RBC in one and two-particle systems were studied. The surrounding fluid was considered viscous and then viscoelastic. In one-particle system, our results have shown that the sphere was enforced by greater forces than the RBC with the similar diameter. This is due to the bigger effective cross-section of sphere. In addition, the force increased (decreased) by viscosity (relaxation time).

In two-particle system, at first, total and primary forces were represented. We realized that the particles in a viscous fluid experience larger forces than in viscoelastic fluid with same viscosity values. In the next step, we investigated the interaction force between a pair of spheres
and also two RBCs with \( \theta = 0^\circ \) surrounded by a viscous fluid. By fitting of simulation data, the interaction force was obtained as a decreasing exponential function of inter-particle distance. Then, for a definite value of inter-particle distance, the functionality of the force applied to an RBC in terms of the angle of rotation and viscosity was obtained. Our data demonstrated that the interaction force decreases linearly by \( \theta \). By repeating all these calculations in viscoelastic fluids, the mathematical equation for interaction force on each sphere and RBC with a definite \( \theta \) value was found as a power-law function of \( L \). Furthermore, for a constant \( L \) value, the force on each RBC was decreasing exponential function of \( \theta \).

At the end, the acoustic interaction torque on each RBC was studied. The results showed that the torque is a sinusoidal function of the rotation angle for both viscous and viscoelastic fluids.

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Figure 1: Configuration of a pair of RBCs in a microchannel. $L$ is the center to center distance between two RBCs. $\theta$ is the angle of RBCs' symmetry axis and the wave direction.
Figure 2: An RBC (a), and a sphere (b) in a microchannel. The standing plane wave is along the $y$–direction. $d$ is the particle distance from the pressure node.
Figure 3: Acoustic radiation force on an RBC in a viscous fluid with $\eta = 16.9 mPa.s$. 
Figure 4: (a) Acoustic radiation force on an RBC in a viscous fluid with $\eta = 5 \text{ mPa.s}$ (●) and $\eta = 25 \text{ mPa.s}$ (■). (b) Acoustic radiation force on an RBC in a viscoelastic fluid with $\eta = 16.9 \text{ mPa.s}$, $\lambda = 0 \text{ ma}$ (■), and $\lambda = 16 \text{ ms}$ (●).
Figure 5: Radiation forces $F$ versus $d$ for the sphere (•) and RBC (■). The two top figures data are for viscous fluid with viscosity values equal to (a) 5, and (b) 25 mPa.s. (c) and (d) are the data for viscoelastic fluid with $\eta = 16.9 \text{ mPa.s}$, $\lambda = 0.6 \text{ ms}$ and 16 ms respectively.
Figure 6: Schematic of two particle in microchannel under a standing plane wave for (a) RBCs and (b) spheres. The standing wave is in $y$–direction, and $L$ is the distance between the center of two particles.
Figure 7: Total (□) and primary (○) forces applied to each RBC in a two-particle system for (a) viscous fluids with $\eta = 5$ (−−) and 25 $mPa.s$ (−), and (b) viscoelastic fluids with $\eta = 16.9$ $mPa.s$, $\lambda = 0.6$ (−) and 7.8 $ms$ (−−).
Figure 8: Acoustic interaction force between (a) two spheres and (b) two RBCs for viscous fluids with $\eta = 5$ ($\blacktriangleright$), 9 ($\blacktriangleleft$), 16.9 ($\bullet$) and 25 ($\blacksquare$) mPa.s. (c) The slopes and (d) intercepts of $\ln F(L, \eta)$ for spheres ($\circ$) and RBCs ($\square$).
Figure 9: (a) Acoustic interaction force between two RBCs versus $\theta$ with $L = 15 \, \mu m$ for viscous fluids with $\eta = 5$ (▲), 9 (▲), 16.9 (●) and 25 (■) mPa.s. (b) The slopes and (c) intercepts of $F(\theta, \eta)$. 
Figure 10: Acoustic interaction force between a pair of RBCs ($\theta = 0^\circ$) versus $L$ for viscoelastic fluid with viscosities (a) $\eta = 5$, (b) 9, (c) 16.9, and (d) 25 mPa.s. The data are for $\lambda = 0.6$ (■), 4 (●), 7.8 (▲) and 16 (►) ms. (e) The slopes and (f) intercepts of $F(L, \eta, \lambda)$ for viscosities $\eta = 5$ (■), 9 (●), 16.9 (▲), 25 (►) mPa.s. Insets show slopes of $m_1$ and $m_2$. 
Figure 11: Acoustic interaction force between a pair of RBCs ($L = 15 \, \mu m$) versus $\theta$ for viscoelastic fluid with viscosities (a) $\eta = 5$, (b) 9, (c) 16.9, and (d) 25 mPa.s. The data are for $\lambda = 0.6$ (■), 4 (●), 7.8 (▲) and 16 (◆) ms. (e) The slopes and (f) intercepts of $F(L, \eta, \lambda)$ for viscosities $\eta = 5$ (■), 9 (●), 16.9 (▲), 25 (◆) mPa.s. Insets show slopes of $s_1$ and $s_2$. 
Figure 12: (a) Acoustic interaction torque applied to an RBC versus $\theta$ for $L = 15 \, \mu m$ in viscous fluid with viscosities $\eta = 5$ (■), 9 (●), 16.9 (▲), and 25 (▷) mPa.s. The coefficient (b) $T_0$ and (c) $b_0$ of $T(\theta, \eta)$. 
Figure 13: Acoustic interaction torque applied to an RBC ($L = 15 \mu m$) versus $\theta$ for viscoelastic fluid with relaxation times (a) $\lambda = 0.6$, (b) 4, (c) 7.8, and (d) 16 ms. The data are for $\eta = 5$ (■), 9 (●), 16.9 (▲) and 25 (▾) mPa.s. The coefficients (e) $T_0$ and (f) $b_0$ of $T(\theta, \eta, \lambda)$ for relaxation times $\lambda = 0.6$ (■), 4 (●), 7.8 (▲), 16 (▾) ms. Insets show slopes of $T_0$ and $b_0$. 

