BAND GAP AND DOUBLE NEGATIVE PROPERTIES OF STAR-STRUCTURED SONIC METAMATERIAL

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Sonic metamaterials have a wide range of applications in wave control and super-resolution imaging, and are favored for several unique and advantageous properties. However, the structures of currently existing double-negative sonic metamaterials are highly complex, as they are composed of various materials, which limits their design function and application in practice. It is quite desirable, as such, to realize double-negative features by single materials simple in structure. Due to unique concave configurations and various resonances, star-shaped structures readily form band gaps and show superior material properties; as such they are considered ideal structures for single-phase metamaterials. In this proposal, single-phase metamaterials are designed based on star-shaped structure. The numerical results suggest that the star-shaped metamaterials have two band gaps, and double negative properties in specific frequency ranges.

Keywords: sonic metamaterials; star-shaped structure; single-phase; double negative

1. Introduction

Acoustic metamaterials [1] are specific artificial composite materials with extraordinary acoustic characteristics that natural materials do not have, e.g. negative effective mass density [2-4] and negative effective modulus [5,6]. Metamaterials with "double negative" property, as a hot research issue in metamaterials domain, can achieve negative refraction in certain frequency range [7], as well as inverse doppler effect [8], plate lens and super-resolution imaging [9-14]. By introducing a variety of resonant forms into the structural units, negative effective mass density and negative effective modulus are achieved simultaneously with the hybridization state formed by dipole resonance and unipolar resonance [15]. Therefore, the double negative acoustic metamaterials often have a complex structure and must been compounded by multiple materials, resulting in relatively difficult for functional design and practical application. The investigation on the key issue about
how to achieve double negative property by using single phase material with simple structure is significant for design and development of acoustic metamaterials device.

Using single-phase materials to achieve double negative property is challenged because these extraordinary properties depend on vibration characteristics of the artificial "atomic" (sub-wavelength locally resonance unit) in a specific frequency range. To achieve double negative property, it is necessary to introduce a resonant unit with both dipole and unipolar resonance into the structure, which means that negative effective mass density structure and negative effective modulus structure can be combined together [15-20]. For the negative mass structure, by introducing lumped mass into the sub-wavelength structure [16] and constituting the spring resonant unit, the material shows holistic negative equivalent mass density at the frequency of dipole resonance; as for the negative modulus structure, a helmholtz resonator or a rotating resonator [5] is usually introduced into the sub-wavelength structure, resulting in the existence of negative equivalent modulus near the unipolar resonant frequency. In addition, hybridization state and chiral structures are also used to design the double negative acoustic metamaterials [21,22]. Liu et al demonstrated that the equivalent mass density and the equivalent bulk modulus are both negative for the chiral structure in the frequency range with negative group velocity [21,22] and constructed a continuous chiral double negative metamaterial. By utilizing four resonant unit, Lai et al [15] formed hybridization elastomeric solid and obtained two dimensional double-negative metamaterial based on hybrid resonance state. Yang et al achieved double negative property using a double-layer film system [19]; when the double layer films vibrate in phase, the dipole vibration provides negative effective mass density; when the double layer films vibrate in reverse, the monopole vibration provides negative modulus. However, relying on single-phase material, these double resonant elements are difficult to achieve, while mostly composed of multiphase materials with complex structure. Hence, introducing new resonance mechanism in the acoustic metamaterials and using the single-phase material to structure the elements with both dipole and unipolar resonance is the key issue to the design of single-phase double negative acoustic metamaterials.

Due to the negative poisson-ratio property, the concave-star structure has many excellent mechanical and physical properties compared with the traditional structure, e.g. better designability, homodromous bendability, high energy absorption efficiency, etc [23-25]; by adjusting the beam size, concave angle, cell configuration and other parameters, scientists can control the macro properties of the concave-star structure. When considering the fluctuation, with volume changes and a variety of resonance behaviors, this special concave structure is conducive to the formation of low-frequency bandgap and extraordinary material properties. In this paper, based on numerical analysis method, the formation mechanism of the bandgap in star-shaped sonic metamaterials is systematically revealed. It is proved that the star-shaped structure with the special resonance properties can realize double negative property in the specific frequency range. The effects of the star-shaped structure parameters on the bandgap are investigated.

2. Modal and numerical calculation method

The star-shaped honeycombs were widely studied due to its special property as negative Poisson's ratio, and advantages in mechanical and physical performances, such as low weight, high strength and high energy absorption. However, its wave characteristics especially the effective parameters were rarely investigated. According to the type of cell structure, the two-dimensional star-shaped structure can be divided into four-point and six-point. The four-point star-shaped structure was focused in this paper, and the structural layout is shown in Fig. 1. It can be seen the four-point star-shaped structure is a simple square lattice, and the unit cell is shown in Fig. 1(b). It composed of square re-entrant corners of equal length $L_2$, joined by four straight beams with equal length $L_1$ and thickness $t$. The corner angle between the adjacent cell walls is denoted by $\theta$, and the lattice constant is denoted by $a$. The selected structural parameters were straight ribs $L_1 = 0.882\text{cm}$, re-entrant corners length $L_2 = 1\text{cm}$, thickness $t = 0.05\text{cm}$, corner angle $\theta = 70$ degree, and lattice con-
The constant $a = 2.793\text{cm}$. The whole structure was made of steel, and the materials parameters were density $\rho = 7.78\ \text{g/cm}^3$, Young’s modulus $E = 210.6\ \text{GPa}$, and shear modulus $G = 81\ \text{MPa}$. In this paper, the band structure and effective parameters of star-shaped stature are firstly investigated, then a solid super-lens is designed based on its negative parameters.

Finite element methods (FEMs) are commonly used to calculate the band structure and effective medium parameters of sonic metamaterials. A distinct advantage of the FEM is the flexibility of modeling various materials with complex structure, good convergence and high precision. The FEM software COMSOL Multiphysics was used to calculate the band structure and design the super-lens. The calculation model of the band structure can be reduced to a single unit by applying the Bloch boundary on the two opposite boundaries and the whole band structure can be obtained by letting the wave vector sweep the edges of the irreducible Brillouin zone. The effective medium parameters were calculated by pickup of the displacement, strain, stress and force on the boundaries, and the calculation methods were referred to the previous study.

3. Results and discussion

3.1 Bandgap structure and double negative properties of star-shaped structure

Fig. 2 The band structure of a star-shaped structure of the infinite crystal (solid line), and the dispersion curve of the water (dashed line); (b) the displacement distributions of the unit cell corresponding to the eigenstate marked in (a) with "A"; the retrieved effective mass density $\rho_{eff}$ (c) and effective bulk and shear modulus $\kappa_{eff}$ and $\mu_{eff}$ (d).

Fig. 2 shows the band structure along the M\GammaXM path of the irreducible Brillouin zone of the simple square lattice. It can be seen two low-frequency band gaps appear at frequency ranges of 5591 to 6610Hz, and 9574 to 18653Hz. The seventh and eighth branches with negative slop are observed in the frequency range of 8760 to 9574Hz, where the refraction indexes of star-shaped structure are negative according to the metamaterials theories. In this paper, the eighth branch
(green line in Fig. 2(a)) is focused due to the longitudinal mode, which can easily couple to the incident longitudinal sound in the water. Fig. 2(b) gives the displacement distributions of the unit cell corresponding to eigenstate of "A" as shown in Fig. 2(a). The eigenstate indicates the deformation is mainly depended the bend of the slender beam, which is different with the traditional sonic metamaterials. The deformation can be divided into two different ways, which are bends of star corner beams and translational motions of the straight beams. And the translational motions of the straight beams similar to the eigenstate of three-component metamaterials are typical dipole oscillations, which can offer negative mass density. The bend deformations of corner beams can be equivalent to the four rotation modes at star corner, which can offer negative effective modulus. The translational mode combine with the rotation modes can constitute a hybrid state, which has double negative properties during the specific frequency ranges.

To verify the double negative properties of star-shaped structure, the effective parameters were further calculated. The lattice constant of the unit cell is about 2.8 cm, and the wavelength is about 6 times bigger than the lattice constant in the frequency range of 8760 to 9574 Hz. According to the effective medium theories, its wave behaviors can be described by effective parameters, which can be obtained via surface integration method. Both the effective mass density and modulus of eighth branch along ΓX direction are shown in Fig. 2 (c) and (d). It can be seen that the effective density \( \rho_{\text{eff}} \), bulk modulus \( \kappa_{\text{eff}} \), and shear modulus \( \mu_{\text{eff}} \) are all negative during 8760 to 9574 Hz. Compared with the traditional sonic metamaterials, the star-shaped metamaterials only need single-phase material, and the structure is more simple. It is noted that the values of the effective density \( \rho_{\text{eff}} \), bulk modulus \( \kappa_{\text{eff}} \) are close to water in the frequency range of 9300 Hz to 9430 Hz, which is interested in this work to realize superlens.

Fig. 3 Pressure field distribution obtained for negative refraction simulation with a plane wave at 9380 Hz.

In order to further check this, we considered a plane wave at 9380 Hz incident from the left on a finite slab with five star-shaped unit cells, as shown in Fig. 3. The plane wave makes angle of 45 degree with the vertical axis, incident on the center of the slab and is 5 cm in width. It can be seen both the incident and transmitted waves are located on the left-hand side of vertical axis, and the transmitted waves follow inverted Snell's law in star-shaped structure. In propagating process, the wave beams are negatively refracted in two times. According to the location of incident and transmitted waves, the negative refractive index can be calculated and is close to -1. The simulations further check the negative refractive properties of the star-shaped metamaterials.

### 3.2 Influence of concave angle on bandgap structure and double negative properties of star-shaped structure

The star-shaped structure has many structure parameters, including the concave angle “\( \alpha \)" in the joint, the length of the straight beams “\( L_1 \)”, the length of the cant beam “\( L_2 \)”, the thickness of beams “\( t \)” and so on. These parameters can change the shape and overall mechanical properties, achieve the regulation of band structure. The previous investigations suggested that the change of the concave angle \( \alpha \) will influence the Poisson's ratio and the effective static elastic modulus. To study the effect of the concave angle “\( \alpha \)” on the fluctuation characteristics of the star-shaped structure, the band structure of the star-shaped structure with the concave angle \( \alpha \) from 50 ° to 120 ° is calculated,
and the influence of the concave angle for the width and position of band gap is analyzed. In calculations, only the concave angle is changed, and “L,” “I” and “I” is constant. Figure 4 shows the change curves of the first and second band gap cut-off frequency with variable concave angle.

![Figure 4](image)

**Fig. 4** Dependence of the cut-off frequency for the band gaps on concave angle.

We can find that only the star-shaped structure with the concave angle between 50-70 degrees can generate the first band gap, as shown in Fig. 4. And when the concave angle is 70°, the band gap is wider than that of the star-shaped structure with other concave angles. With the increase of concave angle, low cut-off frequency of the second band gap decreases firstly and then increases gradually, while the upper cut-off frequency increases firstly and then decreases gradually, and the band gap width is maximum at 60°.

### 4. Conclusion

In this paper, the band gap formation mechanisms and effective parameters of the star structure are investigated based on the FEM. It can be seen that the star-shaped structure metamaterials can generate two broadband gaps in low frequency, due to the existence of abundant resonance mode caused by bending deformation of beams. The calculated parameters verify that the single-phase star-shaped structure can achieve double negative properties owing to the hybrid state resulted from bending formation. Moreover, the position and width of band gap can be adjusted by the concave angle, and other structure parameters. The results in this paper proved that the double negative property and lower band gap can be obtained by single-phase material and simple structure, which provides a new idea for the new acoustic lens, sub-wavelength sound focusing and imaging.

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