Spatial Distributions of At-Many-Stations Hydraulic Geometry for Mountain Rivers Originated From the Qinghai-Tibet Plateau

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Abstract At-many-stations hydraulic geometry (AMHG) has provided a novel way to understand river network development, simulate water flow, and retrieve river discharge in data-scarce regions. Based on in-situ measurements of six major rivers originating from the Qinghai-Tibet Plateau (QTP), this study verifies the existence of AMHG relations along the rivers and explores AMHG relations for cross sections that are not located in the same river reach. The mainstreams and tributaries of the studied rivers in the southern and the eastern portions of the QTP have satisfactory AMHG relation strengths with $R^2 > 0.9$ for over 60% of the relations. For cross sections located in the same stream order or within a certain range of contributing area (CA), approximately 60% (9/15) and 53% (8/15) of the AMHG relations have an $R^2 > 0.6$. AMHG strength increases with increasing stream order and CA; this finding reflects the increasing coherence and maturity of the river networks associated with the geomorphic shaping power of increased discharge. Width-AMHG intercepts are larger than those of depth-AMHG and velocity-AMHG for all stream orders and CAs. Most of the congruent hydraulics generated from cross sections located in middle-scale rivers (orders 7–8) are within the observed range. Congruent hydraulics generally increase with an increase in in-situ measured hydraulics when the stream order or the CA increases. The AMHG relations existing among cross sections that are not located in the same reach, which is named as cross channel AMHG, indicate linear variability of cross-sectional geometric and hydraulic similarities in the same stream order or within a certain CA range. The results break the watershed divide boundary control on AMHG and have the potential to provide background knowledge for discharge estimation in mountain rivers located in the QTP.

1. Introduction

The power laws that hydraulic variables and flow discharges follow are referred to as hydraulic geometry (HG), and they describe the key role of river flow on river morphology (Leopold & Maddock, 1953). These geomorphic relationships were first discovered in the early 1910s by agricultural engineers in northern India and Pakistan to maintain fluvial equilibrium in constructed division channels. The HG theory was then constructed and proposed by Leopold and Maddock (1953). HG has many notable practical utilities, including geomorphological assessment, routing and optimizing in hydrologic models, flood monitoring, water resource management, design of channel restoration, habitat studies for aquatic organisms, and discharge estimates (Bieger et al., 2015; Gleason et al., 2018; Kebede et al., 2020; Neal et al., 2012; Parker et al., 2007; Parker & Wilkerson, 2011; Rosenfeld et al., 2007; Shields et al., 2003; Song et al., 2020). Theoretical research into the practical applications of HG has recently been a hotspot in river dynamics and fluvial geomorphology studies.

At-a-station hydraulic geometry (AHG) relates river width ($w$), water depth ($d$), flow velocity ($v$), and discharge ($Q$) over the range of discharges experienced at a cross section. Downstream hydraulic geometry (DHG) focuses on a reference discharge for a given flow recurrence frequency and the corresponding width, depth, and velocity for cross sections that are downstream along a river. Overall, both the AHG and DHG follow the same empirical equation set:

$$w = aQ^b$$  \hspace{1cm} (1)

$$d = cQ^f$$  \hspace{1cm} (2)
where \( a, c, \) and \( k \) are hydraulic geometry coefficients and \( b, f, \) and \( m \) are hydraulic geometry exponents. The coefficients and the exponents are constrained by \( ack = 1 \) and \( b + f + m = 1 \), respectively, in equation \( Q = w d v \).

Hydraulic geometry and river morphology are interrelated by the frequency of flows and the stream order considering all cross sections in a catchment boundary (Stall & Fok, 1970). Followers tried to explore basin-wide hydraulic geometry in account of the idea that “cross sections of a given stream system are interrelated” (Rhodes, 1977; Stall & Yang, 1970). Basin hydraulic geometry, defines the average values of \( w, d, \) and \( v \) for a given streamflow or for a given flow duration and drainage area, reveals general characteristics of a given stream network in a hydrologically homogeneous basin (Singh & Broeren, 1989). Following this chain, Gleason and Smith (2014) proposed AMHG, a geomorphic phenomenon that spatially and temporally links river cross sections along a river reach (AHG coefficients and exponents statistically relating \( w, d, \) and \( v \) to \( Q \)). Linear correlations were found between AHG exponents \((b, f, m)\) and log (AHG coefficients) \((a, c, k)\) (e.g., \( b = \alpha \log a + \beta, f = \gamma \log c + \mu, \) and \( m = \chi \log k + \eta \)). Therefore, the AMHG for a user-defined length of a river is defined as follows (Gleason & Smith, 2014):

\[
w_c = a_{x1, x2, ..., x_n} Q_{cw}^{b_{x1, x2, ..., x_n}}
\]

\[
d_c = c_{x1, x2, ..., x_n} Q_{cd}^{f_{x1, x2, ..., x_n}}
\]

\[
v_c = k_{x1, x2, ..., x_n} Q_{cv}^{m_{x1, x2, ..., x_n}}
\]

where subscript \( c \) refers to “congruent hydraulics,” the empirically fit river-specific constants that define AMHG, \( a/c/k \) and \( b/f/m \) are site-specific AHG coefficients and exponents in Equations 1–3 at each cross-section, and subscripts \( x1, x2, ... x_n \) correspond to spatially indexed cross-section locations (up to \( n \) total cross sections along a river reach).

Although not universally observed on natural rivers, AMHG is frequently verified and comprehensively reflects the relationship between the coefficients and exponents of AHG (Gleason et al., 2018). Some studies sought to verify the empirical relations across a wide range of physiographic settings, while other studies aimed to discover the theoretical basis of AMHG and apply this theory to discharge estimation. In summary, three key subjects related to AMHG research and can be summarized as follows: (1) the existence of AMHG as a geomorphological phenomenon for river reaches across various environments with differing climates, geologies, geomorphologies, soil, among others (Barber & Gleason, 2018; Gleason & Wang, 2015; Shen et al., 2016); (2) the theoretical basis for AMHG, including the calculation and proof of congruent hydraulics (Barber & Gleason, 2018; Brinkerhoff et al., 2019; Gleason & Wang, 2015); and (3) the development of river discharge and sediment estimates based on AMHG and further development of Bayesian AMHG-Manning (BAM) with and without prior in-situ measurements (Brinkerhoff et al., 2020; Feng et al., 2019; Flores et al., 2020; Gleason et al., 2014, 2018; Gleason & Hamdan, 2017; Hagemann et al., 2017; Zhao et al., 2019).

As a geomorphic index, congruent hydraulics depict shared characteristics within a certain range of settings. AMHG suggests that there is a set of width-discharge, depth-discharge, and velocity-discharge values that are shared by all cross sections within a river (Gleason, 2015). How to depict the mutual characteristics that are shared between individual cross sections has attracted much attention. Gleason and Smith (2014) found that the relationship between \( \log (AHG \ coefficients) \) and \( AHG \ exponents \) of any pair of cross sections in a river reach is linear with a slope equivalent to:

\[
\text{AMHG slope} = \frac{b_{x2} - b_{x1}}{\log (a_{x2} / a_{x1})}
\]

Exponent \( b \) and coefficient \( a \) can be replaced with exponents \( f, m \) and coefficients \( c, k \), respectively. Equation 7 affirms that there should be theoretical \( w \sim Q, d \sim Q, \) and \( v \sim Q \) pairs that are shared by all cross
sections along a river reach. Gleason and Wang (2015) studied the mathematical basis and geomorphological implications of AMHG. They discovered that AMHG arises from the convergence of rating curves and can be represented by congruent hydraulic pairs \( w_c \sim Q_c, \ d_c \sim Q_c, \) and \( v_c \sim Q_c \), which are temporally and spatially invariable (Barber & Gleason, 2018). However, the existence of congruent hydraulics and their relationships to AMHG have not been thoroughly examined and are not well understood through a series of follow-up researches were conducted (Brinkerhoff et al., 2019). Barber and Gleason (2018) sought to establish correlations between common fluvial parameters and indices to investigate potential driving factors of AMHG for rivers with a strong AMHG presence, and they used 191 rivers in the United States to perform this test. However, no correlation between AMHG strength and congruent hydraulics, or average values of in-situ measured discharge, width, depth, and velocity, has been discovered (Barber & Gleason, 2018). Brinkerhoff et al. (2019) investigated the relationships between AMHG strength and local features (cross-section morphology, bed slope, and boundary roughness) of individual cross sections. They found that a strong AMHG is a result of a strong slope/resistance relationship between the stations used to define AMHG. Congruent hydraulics can either be within or outside the in-situ measured range (Gleason & Wang, 2015). In Barber and Gleason's (2018) research, 118 out of the 191 rivers show a strong AMHG with \( R^2 > 0.6 \). Specifically, over 77% of rivers with strong AMHG and only 30% of rivers with weak AMHG had a \( Q_c \) value that fell within the range of observed discharges.

More than 80% of the published papers regarding AMHG are focused on or related to the retrieval of river discharge and sediment data, relying on the applications of remote sensing or photogrammetry. AMHG allows reach-averaged discharge to be estimated where cross-sectional widths can be extracted from satellite data. Gleason et al. (2014) proposed an AMHG-based remote sensing discharge retrieval algorithm, which works for mass-conserved reaches and produced errors within 20%–30% of in-situ measured discharge in different types of rivers. Hagemann et al. (2017) developed the AMHG theory into BAM, which estimates river discharge using physical/empirical flow-law (Manning’s equation) and geometric/empirical flow law (AMHG) in a probabilistic manner. Feng et al. (2019) affirmed the possibility of estimating discharge independent of ground-based measurements with BAM. Recently, geo-BAM method proposed by Brinkerhoff et al. (2020) and the decile thresholding discharge estimation method developed by Mengen et al. (2020) contributes to a significant improvement in discharge estimation. However, estimation accuracy of the aforementioned method using only satellite-observed cross-sectional width data is not always guaranteed if no previous estimates were made (Brinkerhoff et al., 2020; Hagemann et al., 2017). The availability of gauge data and the accurate estimation of the AMHG can significantly improve the performance of discharge estimations (Bonnema et al., 2016).

As seen from the previous research regarding AMHG, less consideration has been taken into account for those cross sections across river reaches, although parameters across sites were linked and spatial relationships of cross-sectional morphologies were reflected in terms of the same river reach (Shen et al., 2016). This research gap prohibits a deeper understanding of river network distribution as well as the development and application of water flow simulations and estimates of river discharge in a broader scope. As the flowing water and sediment shape the river morphology to some extent, can we expect morphologies of cross sections across river reaches to be linked if they experience a similar stream order or contributing area? Does AMHG exist across rivers, regions, or other configurations? Therefore, it is necessary to expand the AMHG scope (e.g., across river reaches) to facilitate research on geomorphological assessment, routing and optimizing in hydrologic models, flood monitoring, and water resource management, among others.

The Qinghai-Tibet Plateau (QTP), known as Asia's Water Tower, is the origin of 10 major rivers flowing through the Asian continent. This region covers a wide range of climates and underlying surface characteristics, including differing geologies, geomorphologies, and soil and vegetation types. Rivers of different stream orders and located in different CA show a wide variety of stream patterns from single-thread to multi-thread and from rock-constrained to free-flow. Whether the rivers in the QTP conform to the AMHG theory or if any spatial distribution can be found for AMHG relations in this region are inquiries that need to be addressed. Width, depth, and velocity AMHG have been examined, with studies that have been mainly concentrated on US rivers encompassing a wide range of climate and geologic settings (Barber & Gleason, 2018; Gleason & Smith, 2014; Shen et al., 2016). Weak AMHG relations were mostly concentrated in the area of the western edge of the Rocky Mountains, while a total of 5 and 11 rivers along the
northwest and southeast coasts, respectively, were identified as having statistically strong AMHG (Barber & Gleason, 2018). However, the weak AMHG relations of Rock Mountains are based on limited data set, which is worth to be further verified in other mountainous regions (Barber & Gleason, 2018). For mountain rivers located in the QTP, AMHG research is nearly nonexistent according to our literature review. The existence and spatial distribution of AMHG, which has not been confirmed but is thought to be influenced by climate and geomorphology, can be further explored in the complex environments present in the QTP (Barber & Gleason, 2018).

In summary, following are some research gaps regarding AMHG theory and application: (a) AMHG knowledge based on in-situ measurements is very limited for data-scarce mountain rivers, which contributes less to understanding of river network distribution and development, and results in low accuracy of discharge estimation in these regions (Hagemann et al., 2017); (b) whether it is necessary to confine the AMHG relations to a given river reach might be questioned; (c) except for slope and flow resistance, how would the spatial features (stream order and CA) affect AMHG strength and congruent hydraulics. Due to the limitations of previous research, this article aims to (1) verify the existence of AMHG relations for mountain rivers originated from the QTP; (2) explore spatial distributions of AMHG exists for cross sections across river reaches; and (3) discuss the relationships between congruent hydraulics and stream order, CA as well as in-situ measured hydraulics.

2. Data and Methods

2.1. Studied Area

The studied region, which includes basins of the upper Yellow River (YR), the Yalong River (YLR), the upper Jinsha River (JSR), the Lantsang River (LCR), the Nu River (NR), and the Yarlung Zangbo River (YLZBR), is mainly located in the southern and the eastern portions of the QTP, which is within the Qinghai, Sichuan, Gansu, and Yunnan provinces and the Tibet Autonomous Region of China (Figure 1). The total area of the studied region is 130.787 × 10^4 km². To maintain the integrity of the upper Yellow River,
portions of the connecting regions between the QTP and the Loess Plateau are included. Similarly, marginal regions that belong to the upper Jinsha River, the Lantsang River, and the Nu River are also included in this study, though they are outside the southern and eastern portions of the QTP (Figure 1). The landscape of the studied region, especially for transient regions, is highly fragmented and heavily influenced by tectonic movement and geological disasters (Qin et al., 2020). Hillslope-channel coupling (high potential of hillslope sediment delivery to streams) affects channel forms in mountainous regions (Hassan et al., 2019). But the studied cross sections of our research are mainly located in straight reaches where rarely suffered from landslides, debris flows, glacial outbursts, and other extreme events. The elevation shows a decreasing trend from >7,300 m in the inland QTP to <150 m in the southernmost area. The annual precipitation increases from 150 mm in the inland QTP to >4,000 mm near the town on Pasighat in the Yarlung Zangbo River (Ding et al., 2007). The main discharge sources include the melting of the persistent winter snowpack, glacial deposits acting as groundwater reservoirs of the inland QTP, and summer rainstorms in the remaining area.

2.2. Data Collection

The data used in this study were acquired from Annual Hydrological Reports of PR China (1967–2017). The data ranges that were used in this research (numbers in parentheses represent the year) are as follows: upper Yellow River (2007–2017), Yalong River and upper Jinsha River (2007–2017), Lantsang River and Nu River (1976–1985), and Yarlung Zangbo River (1967–1982). For different river basins, data at different time periods were used due to the following reasons: (1) hydrological data of the Lantsang River, the Nu River, and the Yarlung Zangbo River in recent 35 years are not available to us. In-situ measures conducted in recent 20 years were far less than those conducted in the 1960s–1980s for the above three river basins; (2) AMHG depicts spatial relationships between AHG coefficients and exponents of different cross sections along a river reach. This study focuses on large-scale spatial distributions but not the temporal variations in AMHG in the QTP. About ten years’ data were used and small interannual variations of AHG were ignored.

The measurements of flow characteristics (river width, water depth, flow velocity, and flow discharge) and cross-section morphology were conducted strictly in accordance with the national standard of China, “Code for water flow measurement in open channels” (Ministry of Water Resources, P. R. China, 2016). Outlines of the above standard are: selection and setting up of measuring cross section; measurements of cross section morphology; measurements of flow hydraulics in flood period, dry period, and ice period; inspection and precision evaluation of measuring results. The data set has certain representativeness of mountain rivers. CA, river width, flow depth, flow velocity, and flow discharge were within the ranges of 83–259,177 m², 1.4–21.2 m, 0.08–21.2 m, 0.05–5.02 m s⁻¹, and 0.03–10,400 m³ s⁻¹, respectively. The data set consisted of 119 river cross sections and covered a wide range of stream patterns from single-thread (represented by straight and meandering rivers) to multi-thread (represented by braided rivers) (Table S1). Sixty-six cross sections are located within the area of the QTP, while another 73 cross sections are located outside the southeastern and northeastern margins of the QTP (Figure 1 and Table S2).

The SRTM 90 m DEM (can be downloaded from “http://www.gscloud.cn/”) and ArcGIS 10.5 (ESRI Inc.) were used to generate river networks. Threshold filtering method was used to generate river networks (Mark, 1984; Martz & Garbrecht, 1992). We set a flow accumulation threshold of 40 (equals to 0.324 km²) after trial-and-error and found that the generated river networks matched the rivers presented in Google Earth images well. The stream orders of the studied region were 1st through 10th, and only the cross sections located in order 5–9 streams were presented in this study because the available measurements were mainly concentrated in rivers of these orders (Figure 1 and Table 1).
2.3. Data Processing

2.3.1. Criteria for Data Screening

A total of 201 measured river cross sections (obtained from national and regional hydrological gauge stations) are located in the studied area, and 139 of them were selected for analysis in this study. Percentages of the selected cross sections to the total cross sections are 71%, 76%, 74%, 74%, 56%, and 50% for the upper Yellow River, the Yalong River, the upper Jinsha River, the Lantsang River, the Nu River, and the Yarlung Zangbo River, respectively (Table 1). The criteria for the filtering of river cross sections were as follows: the cross sections should (1) have a consecutive hydrological record length exceeding 10 years; (2) have had relatively low anthropogenic influence (e.g., no hydropower station and artificial diversion 10 km upstream and downstream of the measured cross section; located outside the backwater zone of a dam) during the study time period to maximum remove the external disturbance; (3) keep away from the region where might be affected by extreme events like glacial outbursts, landslides, etc.; and (4) act as a natural riverway with perennial drainage. After initial filtering based on the above four criteria, cross sections were then re-filtered based on their AHG strength, evaluated by \( R^2 > 0.6 \) of the power regression for \( w/d/v \sim Q \) relations to generate reliable and representative AMHG relations and avoid possible bias of data limitations. For example, the \( w \sim Q \) rating curve of the CJR cross section (Station ID 80) was not used in the corresponding analysis because its \( R^2 \) was only 0.248 (Table S2). All the calculated AHG relations for 139 cross sections are provided in Table S2. Due to the significant effects of ice on AHG relations (Qin et al., 2020), data that were affected by border ice, slush ice run, and ice cover were excluded. Finally, unrealistic fittings (cross sections with fitted AHG exponents \( (b, f, m) \) that were <0) were removed.

2.3.2. Calculation of the AMHG

Power functions depicting the relationships of \( Q \sim w, Q \sim h, \) and \( Q \sim v \) were first fitted using the least-squares method in MATLAB 2018b. The AHG coefficients, exponents, and \( R^2 \) of the fittings were recorded (Table S2). Referring to the original definition of AMHG and results from subsequent studies (Barber & Gleason, 2018; Brinkerhoff et al., 2019; Gleason & Smith, 2014; Gleason & Wang, 2015), a minimum number of 4 cross sections were used to calculate the AMHG relations in this study. This minimum number not only accounts for the very limited number of in-situ measured cross sections from the Annual Hydrological Reports, but it also ensures a maximum number of the fitted AMHG relations in different scenarios. Considering the whole studied region, the AMHG fitted by four cross sections (Yalong River, Yarlung Zangbo River) can be regarded as a reference in analyzing spatial distributions when the AMHG relation is strong. Additionally, Barber and Gleason (2018) indicated that no correlation was found between the number of cross sections (>6) used to calculate AMHG and the AMHG strength. Finally, the AMHG of five main streams (excluding the Nu River as there were only two measured cross sections) and one secondary tributary of the Yellow River (Datong River) were fitted (Figure 2). The lengths of these six reaches range from 370 to 2,744 km, which are longer than those used for discharge retrievals based on remote sensing images (Feng et al., 2019). Mass was not conserved, and discharge increased downstream with the confluence of tributaries.

To explore AMHG relations across river reaches, five variants of AMHG were tested to show how it might manifest in other forms than originally proposed. These include two tests within a basin (along a river reach, different river reaches but the same stream order) and three tests across basins (same stream order, similar contributing area, two geomorphic units including inside the QTP and in transient regions) (Figure 3). Five stream orders (orders 5–9) and five levels of CA were considered in account of different clustering modes of cross sections (Figure 3). The following are the AMHG relations of different stream orders and ranges of CAs (revised from Gleason and Smith [2014]):

\[
w_c = a_{x1x2...xn}Q_{cw1,cw2...cn}^b
\]

\[
w_v = a_{x1x2...xn}Q_{cv1,cv2...cn}^b
\]

where the subscripts \( x_1, x_2, ..., x_n \) correspond to spatially indexed cross-section locations (up to \( n \) total cross sections at each stream order or range of CA), \( O_i \) represents the stream order \( i (i = 5, 6, 7, 8, \) and 9), and \( A_i \)
represents the five ranges of CA ($i = 1, 2, 3, 4, 5$ and represents CA $< 1,000$ km$^2$, $1,000$ km$^2 \leq CA \leq 5,000$ km$^2$, $5,000$ km$^2 < CA \leq 10,000$ km$^2$, $10,000$ km$^2 < CA \leq 100,000$ km$^2$ and CA $> 100,000$ km$^2$, respectively). Similarly, depth-AMHG and velocity-AMHG can be formulated in the same way. Cross channel AMHG, generated from cross sections that are located across river reaches but in the same stream order (Equation 8) or a certain range of CA (Equation 9), is defined to reflect the hydraulic self-similarity of cross sections and the consistency of AHG coefficients and exponents across reaches.

Gleason and Wang (2015) proposed that the linearity ($R^2$) of AMHG should be interpreted as a geomorphic index indicating the degree of convergence of AHG curves, the cross-sectional geometric variability, and the hydraulic self-similarity of a given river reach. Therefore, the strength of the AMHG can be represented by $R^2$ of the fitted AMHG curve. Congruent hydraulic pairs ($Q_{cw}, w_c; Q_{cd}, d_c; Q_{cv}, v_c$) represent the intersections

![Figure 2](image2.png)

**Figure 2.** AMHG of five main streams (a–e) and one tributary of the Yellow River (f). AMHG, at-many-stations hydraulic geometry.

Figure 3. Flow chart differentiating the original AMHG (red) and the extended (black) AMHG. AMHG, at-many-stations hydraulic geometry.
of the AHG rating curves (Gleason & Wang, 2015). For rivers that exhibit strong AMHG, $Q_{c_w}$, $Q_{c_d}$, $Q_{c_v}$, $w_c$, $d_c$, and $v_c$ are given by the spatial mode of the time mean of each of these cross-sectional quantities (Gleason & Wang, 2015). To reach the minimum divergence of the rating curves, two conditions needed to be met: (1) sufficient variability of the AHG exponents and (2) AMHG where $R^2 > 0.5$ (congruent hydraulics without superscript* in Table 2).

### 3. Results

#### 3.1. AMHG of the Cross Sections in a Given River Reach

Data from the six reaches, which include five main streams and one tributary of the Yellow River, were used to explore the relationships between AHG coefficients and exponents along a river reach. In terms of three AMHG relations (width-, depth-, and velocity-) for all reaches, there is at least one relation for each reach that has an $R^2 > 0.9$ (Figure 2). For the four reaches (Yellow River, Jinsha River, Lantsang River, and Datong River) that have $>5$ in-situ measured cross sections, the $R^2$ of the fitted AMHG relations are $>0.8$, excluding the width-AMHG of the Jinsha River (Figures 2a, 2b, 2d, and 2f). The outlier of the width-AMHG of the Jinsha River (shown as a blue circle in Figure 2b) corresponds to the Tuotuo River cross-section (JSR1) which is located in a braided reach, while all other cross sections of the Jinsha River (JSR2 to JSR11, Qin et al., 2020) are located in single thread reaches. Stream pattern might be a factor that influences the AMHG relation. However, due to very few cross sections are located in braided reaches, we are unable to generate AMHG relations of braided rivers alone. Cross sections located in all stream patterns are combined in the analysis. Our future work will focus on the classification of stream patterns.

#### Table 2

| Items | Log ($Q_{c_w}$)$^a$ | Log ($Q_{c_d}$)$^a$ | Log ($Q_{c_v}$)$^a$ | Log ($d_c$)$^a$ | Log ($v_c$)$^a$ | $\text{Mean}$ | $\text{Mean}$ | $\text{Mean}$ | $\text{Mean}$ |
|-------|-------------------|-------------------|-------------------|----------------|----------------|---------------|---------------|---------------|---------------|
| Upper Yellow River | 6.8 | 7.8 | 8.3 | 5.3 | 1.4 | 1.3 | 725.4 | 161.9 | 2.66 | 1.50 |
| Upper Jinsha River | 7.3 | 10.5$^b$ | 7.5 | 5.1 | 2.8 | 0.7 | 2,029.0 | 165.3 | 5.81 | 1.89 |
| Yalong River | 12.5$^c$ | 11.0$^c$ | 9.6$^b$ | 5.6$^c$ | 3.1$^c$ | 1.9$^b$ | 1,619.2 | 115.6 | 5.97 | 1.88 |
| Lantsang River | 16.1$^b$ | 12.3$^b$ | 10.1$^b$ | 5.9$^b$ | 4.2$^b$ | 1.9$^b$ | 1,857.3 | 140.5 | 6.41 | 2.10 |
| Yarlung Zangbo River | 15.4$^b$ | 5.4 | 19.6$^b$ | 6.4$^b$ | 1.2 | 7.7$^b$ | 1,815.3 | 196.6 | 5.83 | 1.24 |
| Datong River | 5.8 | 6.5 | 6.4 | 4.3 | 0.9 | 1.1 | 172.6 | 54.9 | 1.53 | 1.54 |
| Order 5 rivers | 8.6$^c$ | 7.5$^c$ | 11.8$^c$ | 3.9$^c$ | 1.3$^c$ | 4.4$^c$ | 10.1 | 13.7 | 0.40 | 0.94 |
| Order 6 rivers | 7.3$^c$ | 8.2$^b$ | 8.2$^c$ | 4.1$^c$ | 1.6$^c$ | 2.4$^b$ | 47.1 | 28.9 | 0.86 | 1.08 |
| Order 7 rivers | 8.0 | 6.4 | 7.8 | 4.7 | 0.9 | 1.7$^b$ | 139.2 | 62.8 | 1.25 | 1.26 |
| Order 8 rivers | 9.4$^b$ | 7.8 | 8.8 | 5.3 | 1.5 | 1.8$^b$ | 561.7 | 115.2 | 2.51 | 1.45 |
| Order 9 rivers | 17.9$^b$ | 9.2$^b$ | 9.8 | 6.1$^b$ | 1.5 | 2.7$^b$ | 2,139.0 | 166.1 | 6.41 | 1.92 |
| CA < 0.1 $\times$ 10$^4$ km$^2$ | 7.4$^c$ | 9.8$^c$ | 11.6$^c$ | 3.8$^c$ | 2.3$^c$ | 4.1$^c$ | 14.3 | 16.0 | 0.51 | 0.94 |
| 0.1 $\times$ 10$^4$ ≤ CA < 0.5 $\times$ 10$^4$ km$^2$ | 5.8 | 7.4$^b$ | 7.1$^b$ | 3.9 | 1.2 | 1.8$^b$ | 45.9 | 31.4 | 0.89 | 1.17 |
| 0.5 $\times$ 10$^4$ < CA ≤ 1 $\times$ 10$^4$ km$^2$ | 9.7$^b$ | 6.5$^b$ | 8.5$^b$ | 5.0 | 0.9 | 2.0$^c$ | 140.8 | 66.4 | 1.17 | 1.25 |
| 1 $\times$ 10$^4$ < CA < 10 $\times$ 10$^4$ km$^2$ | 8.3$^c$ | 6.8 | 6.9 | 5.1$^c$ | 1.0 | 1.1 | 426.7 | 101.3 | 2.16 | 1.48 |
| CA > 10 $\times$ 10$^4$ km$^2$ | 14.5$^b$ | 9.4$^b$ | 8.7 | 5.9 | 2.5 | 1.3 | 1,936.6 | 169.0 | 6.01 | 1.78 |

Abbreviations: AMHG, at-many-stations hydraulic geometry; CA, contributing area.

$^a$Represents natural log. $^b$Represents the congruent value does not occur within the range of observed values. $^c$Represents the $R^2$ of the fitted AMHG is smaller than 0.5.

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patterns with the help of remote sensing imagery interpretation. Then, cross sections will be clustered by single-thread and multi-thread river reaches to generate AMHG relations.

Additionally, the $R^2$ of the width-AMHG is usually smaller than those of the depth-AMHG and velocity-AMHG (Figure 2). These results agree with previous studies conducted by Gleason and Smith (2014), Gleason and Wang (2015), and Shen et al. (2016). This finding is likely due to the fact that river width is more sensitive to the variations of discharges and boundary conditions than water depth and flow velocity. Random variations of river width are evident while water depth and flow velocity show relatively stable status. Less consistent variations of river widths lead to a weaker strength of width-AMHG.

The relative relations between the depth-AMHG and the velocity-AMHG differ between the Yellow River (including its tributary) and other rivers. The curves of the depth-AMHG and the velocity-AMHG for the Yellow River and Datong River nearly overlap as shown in Figures 2a and 2f. The two curves have similar slopes and intercepts. However, the curves of the depth-AMHG for the remaining river reaches (excluding the Yarlung Zangbo River) are located to the upper right of the velocity-AMHG curves. In conclusion, AHG coefficients and exponents in the same river reach show high correlation on a spatial scale under the original definition for AMHG in the southern and eastern portions of the QTP. Both our study and Barber and Gleason's research (Barber & Gleason, 2018) have focused on mountain regions. Strong AMHGs were verified in the QTP region (AMHG strength $R^2 > 0.8$, Figure 2) while relatively weak AMHGs were observed in the Rocky Mountain area (AMHG strength $R^2 < 0.4$). One possible explanation might be attributed to that the rivers located in the west Rocky Mountains own lower stream orders than those of the rivers located in the QTP. Of course, more information on local climate, geology, geomorphology, soil and vegetation, and so on, are needed for further comprehensive explanations.

3.2. AMHG of the Cross Sections in the Same Stream Order

3.2.1. AMHG for Cross Sections of Individual River Basins

AMHG relations are fitted at different stream orders to explore the existence of AMHG for individual river basins and study the spatial distribution of the AMHGs. The Yalong River is the largest tributary of the upper Jinsha River and merges into the upper Jinsha River in Panzihua City (26.61°N, 101.81°E) (Figure 1). Therefore, these two rivers are combined and analyzed in Section 3.2.1.

When considering the same river basin but different stream orders (Figures 3 and 4), the strengths of the three AMHG relations for individual river basins generally increase with increasing stream order, excluding order 8 streams of the Jinsha River and Yalong River, as shown in Figure 4g. When considering the same stream order but different river basins, the AMHG exhibits varied spatial distribution. The Yellow River, Jinsha River, Yalong River, and Lantsang River are located from north to south in the eastern part of the QTP. Absolute values for depth-AMHG slopes for these river basins decrease from north to south within the same stream order (Figure 4). This pattern is potentially because the mean annual discharge generally increases from north to south, which is determined by the local climate and landscape. This result also verifies the increasing trend of $Q_w$ for all studied reaches from north to south of the eastern QTP (Table 2).

Additionally, when compared with the AMHG strength of the mainstream of the Yarlung Zangbo River, the AMHG strength of the cross sections located in order 8 streams of the Yarlung Zangbo River Basin exhibit stronger correlations (Figures 2e and 4h). Specifically, the depth-AMHG and velocity-AMHG strengths of the order 8 streams of the Yarlung Zangbo River Basin are much stronger than those of the mainstream of the Yarlung Zangbo River. This finding implies that stream order should be an appropriate clustering mode for AHG coefficients and exponents.

3.2.2. AMHG for All Cross Sections in the Studied Area

Section 3.2.1 determined that AMHG relationships exist in the same stream order of individual river basins but are not belong to a single river reach. This section will extend the range and further explore AMHG relations existing in the same stream order for all cross sections in the studied area (Figures 3 and 5). Generally, the AMHG strength increases with increasing stream order and is mostly $R^2 > 0.6$ (excluding $R^2$ of order 8 streams) for cross sections located at order 7 and higher streams (Figure 5). This result can be attributed to the variability of AHG exponents ($f$ and $m$), which generally increase when the stream order increases from
However, the width exponents of order 9 streams plot within a relatively small range (0.032–0.192) compared to those of lower-order streams (Figure 6d), which also result in an $R^2 > 0.6$.

Figure 4. AMHG for cross sections located in the Upper Yellow River Basin, the Upper Jinsha River + Yalong River Basin, the Lantsang River Basin, the Yarlung Zangbo River Basin at stream orders 6 (a–c), 7 (d, e), 8 (f–h), and 9 (i, j). AMHG, at-many-stations hydraulic geometry.

5 to 8 (Figures 6e and f). However, the width exponents of order 9 streams plot within a relatively small range (0.032–0.192) compared to those of lower-order streams (Figure 6d), which also result in an $R^2 > 0.6$. 

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Similar to the AMHG relations of a given river reach (Figure 2), the strength of the width-AMHG is generally weaker than the strengths of the depth-AMHG and velocity-AMHG relations. Compared with the AMHG strengths of the individual river basins (Figure 4), the AMHG of all studied reaches generally shows equal or weaker strengths (Figure 5b–5e), with a few exceptions (Figures 4a, 4c, and 4g). Variability of the background environment within a basin is less than that of the whole studied region, which contributes to the increase in AMHG strength of the same river basin.

Relative relationships between the width-AMHG, depth-AMHG, and velocity-AMHG change regularly with increasing stream order. Absolute values of AMHG slopes decrease with increasing stream order for all width, depth, and velocity curves when the cross sections are located in the order 7–9 rivers (Figure 5). This phenomenon can be used to explain the increase in congruent hydraulics with increased stream order (Table 2). Specifically, the depth-AMHG curve, which is located in the lower-left direction relative to the velocity-AMHG curve, moves toward the upper right and finally plots to the upper right of the velocity-AMHG curve as the stream order increases from 5 to 9 (similarly, the velocity-AMHG curve moves toward the lower-left direction with increasing stream order). Similar changes are found for the AMHG relations of individual river basins (Figure 4). In addition, width-AMHG intercepts are larger than those of depth-AMHG and velocity-AMHG for all stream orders (Figure 5). Width-AMHG slopes of the order 5 and 6 streams are larger than those of velocity-AMHG slopes, but are smaller for the 7–9 order streams.

3.2.3. AMHG for Cross Sections Located in Transient Regions or Inside the QTP

For the cross sections that are located in different river reaches but in the same stream order, we further classified them into two types according to geomorphic units: cross sections inside the QTP and cross sections in transition regions (Figure 3 and Table 3).

We compared AMHG strengths in the same stream order for: (1) the whole studied region, (2) inside the QTP, and (3) outside the QTP but in transient regions connecting the QTP and other geomorphic units.
Results indicate that AMHG strength overall increases after taking the geomorphic units into account. Compared with those generated by all cross sections, AMHG strengths generated by cross sections located in order 5, 6, 8, and 9 streams increase after taking the geomorphic units into account. For cross sections located in order 7 streams, no significant difference has been observed before and after taking the geomorphic units into account (total difference of AMHG strength ($R^2$) before and after taking the geomorphic units into account is $-0.064$). Overall, subdividing cross sections into two geomorphic units (inside the QTP or in transient regions) contributes to the increase of AMHG strength (Table 3).

### 3.3. AMHG of the Cross Sections Within a Certain Limit of Contributing Area

CA is another clustering mode for AHG coefficients and exponents to generate possible AMHGs (Figure 3). AMHG exists within a certain CA range; however, the cross sections do not belong to the same stream...
order or tributary/mainstream (Figure 7). Generally, AMHG strength increases with increasing CA. In terms of the three AMHG relations (width-, depth- and velocity-), cross sections located in the area with CA ≥ 0.1 × 10^4 km^2 have at least one AMHG strength with R^2 > 0.6. Specifically, all the depth-AMHGs have an R^2 > 0.6, except for those that belong to CA < 0.1 × 10^4 km^2 (Figure 7). In addition, the width-AMHG has the weakest strength, followed by velocity-AMHG. Depth-AMHG has the strongest strength in most CA ranges. Similar to the relative positions among the width-AMHG, depth-AMHG, and velocity-AMHG curves of different stream orders, the depth-AMHG curve moves toward the upper right and the velocity-AMHG curve moves toward the lower left with increasing CA. Width-AMHG slopes are larger than those of depth-AMHG and velocity-AMHG when CA < 0.5 × 10^4 km^2, and vice versa when CA > 0.5 × 10^4 km^2. Width-AMHG intercepts are larger than those of depth-AMHG and velocity-AMHG intercepts for all CAs.

### 3.4. Congruent Hydraulics for AMHG and Their Geomorphic Characteristics

#### 3.4.1. Characteristics of Congruent Hydraulics

Stronger AMHG relationships contribute to improved estimations of congruent hydraulics (Gleason & Wang, 2015). AMHGs with an R^2 > 0.5 are used for the variation analysis discussed in this section (congruent hydraulics with no superscript^c mark in Table 2). Generally, congruent discharges represented by width, depth, and velocity (Q_c_w, Q_c_d and Q_c_v) show similar variation trends in different item groups (Table 2). The mainstreams of the Yellow River, the Jinsha River, the Yalong River, the QIN ET AL. 10.1029/2020WR029090

| Stream order | Clustering modes | Width-AMHG | Depth-AMHG | Velocity-AMHG |
|--------------|------------------|------------|------------|---------------|
| 5 All cross sections | 0.316 | 0.148 | 0.312 |
| Inside the QTP | 0.540 | 0.431 | 0.406 |
| In transient regions | 0.256 | 0.013 | 0.277 |
| 6 All cross sections | 0.482 | 0.537 | 0.676 |
| Inside the QTP | 0.348 | 0.705 | 0.618 |
| In transient regions | 0.548 | 0.535 | 0.723 |
| 7 All cross sections | 0.654 | 0.856 | 0.702 |
| Inside the QTP | 0.492 | 0.811 | 0.409 |
| In transient regions | 0.848 | 0.914 | 0.885 |
| 8 All cross sections | 0.509 | 0.852 | 0.857 |
| Inside the QTP | 0.511 | 0.894 | 0.822 |
| In transient regions | 0.567 | 0.895 | 0.910 |
| 9 All cross sections | 0.674 | 0.932 | 0.945 |
| Inside the QTP | 0.694 | 0.976 | 0.924 |
| In transient regions | 0.698 | 0.937 | 0.955 |

Abbreviations: AMHG, at-many-stations hydraulic geometry; QTP, Qinghai-Tibet Plateau.

![Figure 7](image_url) AMHG for cross sections located within a certain CA range from <1,000 km^2 to >100,000 km^2. AMHG, at-many-stations hydraulic geometry; CA, contributing area.
Lantsang River, and the Yarlung Zangbo River were successively distributed from north to south, excluding the Datong River (Figure 1). $Q_{c,w}$ shows an evident increasing trend from north to south, while $Q_{c,d}$ and $Q_{c,v}$ exhibit exceptions to this increase in the Yellow River and the Yarlung Zangbo River (Table 2). The Yarlung Zangbo River has the largest $w_c$, while the Lantsang River has the largest $d_c$ and $v_c$. Additionally, variations in $w_c$ are smaller than those of $d_c$ and $v_c$ on a spatial scale. Excluding stream orders 5 and 6, all congruent hydraulics strictly increases with increasing stream order. However, congruent hydraulics do not show a strict increasing trend as the CA increases, particularly for $d_c$ and $v_c$ (Table 2).

The percentage of rating curves intersecting within the observed discharge range can be used to predict the strength of AMHG (Barber & Gleason, 2018; Gleason & Wang, 2015). Among 96 congruent hydraulics (Table 2), 25% (24 out of 96 congruent hydraulics) are generated from weak AMHG strength items with $R^2 < 0.5$ (with superscript $^a$ mark in Table 2), while 50% (48 out of 96 congruent hydraulics) are not within the observed discharge range (with superscript $^b$ mark in Table 2). For those weak AMHG items, 79% (19 out of 24 congruent hydraulics) are not within the observed discharge range. For those items with strong AMHG, only 40% (29 out of 72 congruent hydraulics) of them are not within the observed discharge range. These results indicate the consistency between the AMHG strength and the percentage of congruent hydraulics falling within the range of observed values for cross sections clustered by different items. Congruent hydraulics of the lower order streams (orders 5 and 6) and the higher-order streams (order 9) often plot outside the observed range (Table 2 and Figures S1–S3). Most of the congruent hydraulics of cross sections located in stream orders 7–8 are within the observed range (Figures S1–S3). Meanwhile, the AMHG strength increases while the exponent $b$ decreases with increasing discharge (Figures 6 and 4).

### 3.5. Relationships Between Congruent Hydraulics and Observed Hydraulics

To further study how the congruent hydraulics vary with observed hydraulics, the congruent hydraulics and corresponding in-situ measured mean hydraulics in Table 2 were plotted as linear functions with stream order and CA (Figure S5). Congruent $w$ increases significantly with an increase in in-situ measured $w$ when stream order and CA increase. Though this exhibits an increasing trend, the increasing degrees for congruent $Q$ and $d$ are not as strong as that of congruent $w$ (Figure S5). The above findings verify the slight positive trend ($R = 0.51$) observed between mean $Q$ and $Q_c$ found by Barber and Gleason (2018), who determined that $Q_c$ generally increases as the mean $Q$ increases, though the mean $Q$ cannot provide a reliable prediction of $Q_c$ ($R^2 = 0.26$).

### 4. Discussions

#### 4.1. Discussions on AMHG Characteristics

Section 3.1 indicates an irregular array of AHG exponents and coefficients (Figure 2). Specifically, AHG exponents or coefficients along a river reach do not show a strict increasing or decreasing trend from upstream to downstream (Figures 2d and 2f). Set exponents $m$ and coefficients $k$ of the velocity-AHG as an example, $m$ generally decrease with the increase of $k$. Based on this framework, however, increase of coefficients $k$ and decrease of exponents $m$ conform to the following sequence: DTR3, DTR5, DTR4, DTR1, and DTR2 for the Datong River and LCR4, LCR3, LCR5, LCR2, and LCR1 for the Lantsang River, respectively (Figures 2d and 2f). Not strict but overall trends from downstream to upstream can be observed for the increase of coefficients $k$ and decrease of exponents $m$ along a river reach. Similar patterns can be found for coefficients ($a$, $c$) and exponents ($b$, $f$) of other river reaches (upper Yellow River and upper Jinsha River). Possible explanations might be: with the increased CA and the convergence of tributaries, $Q$ increases downstream, which contributes to overall increasing trends of river width, water depth, and flow velocity. Due to complex geology and geomorphology background for mountain rivers in the studied region, adjustments of $w$, $d$, and $v$ with $Q$, therefore, do not obey strict increasing law as the lowland rivers do, which leads to some irregularities in the distributions of the coefficients and exponents.

Combining Sections 3.2.1–3.2.3, AMHG does exist in cross sections of the same stream order; however, these cross sections do not belong to the same tributary or mainstream. The Jinsha, the Yalong, the Lantsang, the Nu, and the Yarlung Zangbo River Basins exhibit narrow drainage basin shapes while the upper Yellow River Basin, located in northeast QTP, shows a relatively circular shape. Drainage basin shapes in the QTP...
and its boundary regions are largely controlled by geology and determined by geomorphology background. Large variations of the cross section morphologies in such complex regions are expected, which might contribute to weak AMHG strengths. However, the results in Sections 3.2 and 3.3 provide evidence for AMHG as a widespread fluvial phenomenon in the same stream order or within a certain CA range. The existence of the AMHG that does not belong to a river reach indicates a hydraulic self-similarity of cross sections and a consistency of AHG coefficients and exponents that possibly exist in the same stream order or within a certain CA range. This consistency is not only shaped by flowing water but also possibly determined by the consistency of local climate, landscape, soil, and vegetation. Cross channel AMHG reflects these consistency to a great extent. More tests should be conducted to see whether these AMHGs can exist in a wider geographical range and various climate types.

Compared with those of the same stream order, the AMHG strengths ($R^2$) within a certain CA range are relatively weak. Though stream order was found to be well related to CA (Stall & Fok, 1968), it owns a more rich physical meaning than that of CA. Stream order is one of the intrinsic attributes of a natural river system (Figures 2, 4, 5, and 7). It is a comprehensive reflection of the local geology, geomorphology, soil, vegetation, and climate. However, CA represents only the partial aspects of geomorphology that facilitate the shaping of local cross sections that impact the AHG and AMHG. Therefore, stream order has an improved cluster effect on AHG coefficients and exponents than that of the one-sided CA and is reflected in AMHG strength.

### 4.2. Factors Affecting AMHG Strength

AMHG strength generally increases with increasing stream order and CA, although some exceptions can be found in width-AMHG and velocity-AMHG (Figure S4). Chi-square tests were used to verify the statistical significance of the increase in AMHG strength. In terms of the relationship between stream order and AMHG strength, the $p$-values of the $R^2$ for river width, water depth and flow velocity using the chi-square test are 0.094, 0.069, and 0.014, respectively. In terms of the relationship between CA and AMHG strength, the $p$-values of the $R^2$ of river width, water depth, and flow velocity are 0.537, 0.073 and 0.092, respectively. The increasing trend of velocity-AMHG strength for stream order shows statistical significance with a $p$-value < 0.05, while the other five increasing trends show no statistical significance ($p > 0.05$). Additionally, the width-AMHG strength shows a less significant increasing trend than those of the depth-AMHG and velocity-AMHG strengths. Moreover, the significance ($p$-values of chi-square tests) of the increasing AMHG strength with increasing stream order is larger than that for CA, which might be explained by the better fittings for the AMHGs for stream orders than those for CAs.

Barber and Gleason (2018) indicated that the AMHG strength did not correlate to any available observed congruent hydraulics ($Q$, $w$, $d$, and $v$) or the number of cross sections used to fit the AMHG relation. Stream order and CA are not hydraulic parameters, but they are inherent attributes of a fluvial system and comprehensively synthesize characteristics of climate, landscape, soil, and tectonic movement in the studied region. Several reasons contribute to the increase in AMHG strength with increased stream order and CA:

1. Cross-section morphology is shaped by the geology, geomorphology, fluvial processes (water and sediment), and even tectonic movement present in the studied region. Lower-order streams (order 5 and 6 streams), representing an integrated record of past head cutting, bed incision and bank collapsing, have small CAs, which result in a small driving force and a large resistance. Discharges and stream power are small, while the boundary constraint is strong. The shaping power (river morphology and channel geometry that are shaped by flowing water and sediment) of the flowing water on the cross-section morphology is relatively small when compared to that of the higher-order streams (order 7–9 streams). Geological control under a complex local landscape dominates the morphology of lower-order streams, which contributes to the high variability of cross-section morphology and less consistent variation in AHG coefficients and exponents. Outliers shown in Figure 8 represent this inconsistency and are mainly located in lower-order streams.

2. Higher-order streams have relatively large driving forces and small boundary constraints. Higher discharges and stream power indicate a greater contribution of contemporary fluvial processes to the shaping of cross-section morphology. This effect results in a relatively wide U-shaped cross section (Figure 8 and Figures 6a and 6d) indicate an increase in coefficient $a$ and a decrease in exponent $b$ with
increasing stream order, respectively). The wide U-shaped cross section corresponds to a flat channel bottom, steep banks, and a relatively large shape exponent $r$ in Dingman’s cross-section geometry model (Dingman, 2007). In addition, the stream power increases along the river reach, which facilitates the regular changes in river width, water depth and flows velocity. As a result, AHG exponents vary consistently with the form of $b + f + m = 1$ in higher-order streams (Figure 8b presents an inclined plane with some outliers located in order 5–7 streams).

In summary, the stability of cross sections increases with increasing stream order and CA, which then contributes to the increase in AMHG strength. It can then be deduced that the AHG of cross sections located in the same stream order and shaped by discharges within similar ranges are dependent on each other but are not site specific, as previously theorized. These findings extend the AMHG theory and support Rhodes’ view (Rhodes, 1977), who argued that “all cross sections of a given stream system are interrelated.”

### 4.3. Indications of Congruent Hydraulics

The percentage of rating curves intersecting within the observed discharge range can be used to predict the strength of AMHG (Barber & Gleason, 2018; Gleason & Wang, 2015). Barber and Gleason (2018) studied 191 rivers on the US continent. The authors found that over 77% of rivers with strong AMHG ($R^2 > 0.6$, 118 out of 191 rivers) had a value of $Q_c$ that was within the range of observed discharges, while only 30% of rivers with weak AMHG ($R^2 < 0.6$, 73 out of 191 rivers) had $Q_c$ values that were within the range of observed discharges. They also pointed out that rivers with strong AMHG but a $Q_c$ value outside the observed $Q$ range can be considered “low $b$ value streams,” which may result in an unsuccessful discharge estimation. This study verified most of these previous results. The results in Section 3.4.1 confirm the view of Barber and Gleason (2018) and further reveal that low $b$ values contribute to congruent hydraulics outside the observed range only for cross sections located in higher-order streams (Figures 5, S1, S2, and S3).

Previous studies (Barber & Gleason, 2018; Gleason & Wang, 2015) focused on AMHG relations generated from cross sections in a single river reach. River reaches are the clustering mode for cross sections. For each reach, there is one corresponding AMHG and three sets of coefficient and exponent pairs. It is not clear whether rivers with weak AMHGs that had $Q_c$ values within the range of observed discharges or rivers with strong AMHGs that had $Q_c$ values outside the range of observed discharges belong to a certain CA range or stream order. This study attempts to cluster cross sections in the same stream order or within a certain CA range.
range. Results may contribute to a deep understanding of intrinsic features of rivers and the effects of local backgrounds (geology, geomorphology, and climate) on AMHG relations from another side. Strong AHG relations and a strong river-wide slope function that well describes hydraulics everywhere along a river reach yield the intersection of AHG curves and strong AMHG relations (Brinkerhoff et al., 2019). Following the work of Brinkerhoff et al. (2019), future works will be focused on hydraulically and geomorphically reconciling the extended cross channel AMHG theory proposed by this research with AHG relations. Whether the existence of slope/resistance relation be a prerequisite for a strong AMHG is worth to be further verified across reaches and basins.

5. Summary and Conclusions

Based on in-situ measurements of six major rivers and their tributaries originating from the QTP, this study first verified the existence of AMHG relations in the main streams of these rivers and then explored cross channel AMHG relations in the same stream order and within a certain CA range. Relationships among stream order, CA and AMHG strength were studied. Congruent hydraulics and their relations to AMHG as well as in-situ measured hydraulics were tested. The following are the findings and implications of this research:

(1) AMHG exists in both main streams and cross sections located within the same stream order and certain ranges of CA: (a) Main streams of the Yellow River, the Datong River, the Yalong River, the Jinsha River, the Lantsang River, and the Yarlung Zangbo River have at least one AMHG relation (width-AMHG, depth-AMHG, or velocity-AMHG) with an $R^2 > 0.9$, which supports the existence of AMHG in mountain rivers located in the southern and eastern portions of the QTP and demonstrates the power of using AMHG to predict AHG across the study river reaches. (b) AMHG strength ($R^2$) increases with increasing stream order and CA. The $R^2$ of cross sections located within order 7 and higher streams is largely > 0.6. Cross sections located in streams with CA $\geq 0.1 \times 10^4$ km$^2$ have at least one AMHG strength where $R^2 > 0.6$. (c) Width-AMHG intercepts are larger than those of depth-AMHG and velocity-AMHG for all stream orders and CAs. (d) Congruent hydraulics generated from cross sections located in middle-scale rivers (orders 7–8) are mostly within the observed range. The congruent $w$ increases significantly with increasing in-situ measured $w$, while the increases of $Q$ and $d$ are relatively gradual as stream order or CA increases.

(2) This study covers a large area of the eastern and southern portions of the QTP, including various environmental factors (such as climate, geology, landscape, vegetation, and soil) and stream patterns (single-thread rivers and multi-thread rivers). The findings from such complex areas indicate that AHG coefficients and exponents are functionally related and dependable between cross sections when they are in the same stream order or within a certain CA range. The cross channel AMHG is defined to reflect the hydraulic self-similarity of cross sections and the consistency of AHG coefficients and exponents across reaches. The results of this research have the following implications: (a) breaking the watershed divide boundary for AMHG; (b) providing a basis for water and sediment simulation and research on river network structure and development; and (c) providing the potential for using the AMHG theory for discharge estimation across the watershed divide, especially for data-scarce mountain rivers.

(3) More testing is needed to verify the existence of AMHGs that are not located in a single river reach in complex climate, geologic, and geomorphologic environments, especially for rivers in semi-arid and arid areas. The theoretical basis behind the mathematical artifact for AMHGs that existed in the same stream order or certain CA range should be studied further in the future.

Data Availability Statement

In-situ measured river width, water depth, flow velocity, and flow discharge data were provided by the Bureau of Hydrology at the Ministry of Water Resources of China, which is acknowledged here. Data provided by the Bureau of Hydrology at the Ministry of Water Resources of China were in the form of hard copy but not electronic copy, therefore, no link (URL or DOI) can be presented here.
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