Polarization Effects on the $e^+e^- \rightarrow W^+W^-$ process \\
with Large Extra Dimensions

(a) Kang Young Lee*, (a,b) H.S. Song†, and (a) JeongHyeon Song‡

(a) Center for Theoretical Physics, Seoul National University, Seoul 151-742, Korea
(b) Department of Physics, Seoul National University, Seoul 151-742, Korea

Abstract

We study large extra dimension effects on the polarizations of the $W$ pair and electron beam at the $e^+e^- \rightarrow W^+W^-$ process. It is shown that the measurements of the cross section for transversely polarized $W$ pair with the right-handed electron beam remarkably enhance the possibilities to see the low scale quantum gravity effects. Higher Linear Collider bounds on the string scale in this model can be obtained by using the left-handed electron beam.
In spite of the extraordinary successes of the Standard Model (SM) in explaining all the high energy experiments with large luminosity up to now [1], there have been various attempts to search for physics beyond the SM. Even though some astrophysical and collider measurements have intimated that the SM is not the whole story, the signals have been regarded as hints, not as facts. A major portion of the motivations to extend the SM comes from conceptual discomfort in the theoretical viewpoints, such as the hierarchy problem: the SM cannot provide a satisfactory answer to why the nature allows such an enormous ratio between two fundamental mass scales, the electroweak scale at $\sim 100$ GeV and the Planck mass scale at $\sim 10^{19}$ GeV. This problem has prompted the extensive studies of the supersymmetric models [2], the technicolor models [3], etc.

Recently Arkani-Hamed, Dimopoulos, and Dvali (ADD) have approached the hierarchy problem in a salient way by removing one prerequisite of the problem itself: the Planck mass is not fundamental, that is, there exists only one fundamental mass scale, $M_S$, in the nature [4]. The observed extremely small Newton’s constant $G_N$ or alternatively huge Planck mass scale can be obtained by introducing the existence of extra compact $N$ dimensions. Then the Planck mass is related to the $M_S$ and the size of extra dimensions. For example, if the extra dimensional space is the $N$-dimensional torus with the same compactification radii $R$, the relation is

$$\kappa^2 R^N = 16\pi(4\pi)^{\frac{N}{2}}\Gamma\left(\frac{N}{2}\right) M_S^{-(N+2)},$$

where $\kappa^2 \equiv 16\pi G_N$ [4]. The excellence of Newtonian mechanics in describing the solar system excludes the $N = 1$ case where the $R$ is order of $10^{13}$ cm. The $N = 2$ case which implies mm scale extra dimensions is not excluded by the current macroscopic measurement of gravitational force [4]. The cases of $N > 2$ are also acceptable but difficult to probe through macroscopic observations.

In order to be phenomenologically consistent with the SM at the electroweak scale, the model assumes that the SM fields are confined in our 4-dimensional brane while gravitons are freely propagating in the whole $(4 + N)$-dimensional bulk. Even though this discrimination
can be achieved by considering our world as a topological defect of a higher dimensional
theory and by localizing the SM fields at the vortex [4], its natural realization had been
already discussed in string theories [7,8]. Strong string coupling obtained by T-dualising
transforms the Kaluza-Klein modes of opens strings into the winding modes of open strings
of which the two ends are attached on a brane, which are good candidates of the SM particles.
And closed strings are still able to propagate orthogonal to the brane, identified as gravitons.

The existence of extra dimensions reveals through the interactions between gravitons
and the SM particles on our brane. Although the required invariance under general coordinate
transformations in the brane and bulk can, in principle, specify the interactions, the
presence of non-trivial metric in the extra dimension complicates them. Unless gravitons
take momentum compatible with or larger than $M_S$ at high energy collisions, the spacetime
region where collisions occur can be regarded as flat [4]. Then the linear approximation
is valid so that the metric in $(4 + N)$-dimensions can be expanded around the Minkowski
metric as

$$\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa \hat{h}_{\mu\nu},$$

where the $\hat{h}_{\mu\nu}$ is the canonical graviton fields and the hatted indices denote the $(4 + N)$-
dimensional spacetime. After the compactification on a sub-manifold of extra dimensions,
our brane effectively possesses Kaluza-Klein towers of massive spin 2 gravitons $\tilde{h}_{\mu\nu}$, massive
vector particles $\tilde{A}_{\mu\nu}$, and massive spin zero particles $\tilde{\phi}_{ij}$. The matter-graviton couplings are
specified by the minimal coupling of gravity, yielding the Feynman rules of the interactions
between the Kaluza-Klein gravitons and the SM particles [8,5,9]. Following the results of
Ref. [3], we use the effective action to the leading order in $\kappa = 1/M_{pl}$

$$I = -\frac{\kappa}{2} \sum_n \int d^4x \left[ \tilde{h}_{\mu\nu} \tilde{\phi}_{\mu\nu} + w \tilde{\phi}_{ij} T_{ij} \right],$$

where $w = \sqrt{2/3(n+2)}$ and $T^{\mu\nu}$ is the energy-momentum tensor. It is to be noted that
new kinds of interactions due to the low scale quantum gravity are neutral current ones.

Recently the idea of the existence of large extra dimensions has drawn quite explosive
attentions of particle physicists. Above all, attractive is that the quantum gravity scale
can be as low as TeV so to be testable. It would be worthwhile to search for collider tests which can confirm or exclude the validity of the model. Various phenomenological studies have been performed to constrain the scale of $M_S$ or the number of extra dimensions $N$ from the existing data of collider experiment [10, 12], and in the future experiments [11, 16].

Two kinds of processes at colliders are computable by using the effective action in Eq.(3), single graviton emission and virtual exchange of gravitons. For the first case, the presence of extra dimension induces the Kaluza-Klein multiplicity of phase factor, proportional to $(\Delta E \cdot R)^N$, where $\Delta E$ is the energy of emitting graviton. Thus any process involving single graviton emission shows the large dependence on $N$ since the branching ratio is proportional to $(\Delta E/M_S)^{N+2}$ [4]. Studies of various processes show that even in the future NLC or LHC, the $N \geq 5$ cases are practically impossible to distinguish from the SM background [11, 14].

Virtual exchange of gravitons does not have such dependence; the scattering amplitudes, suppressed by $\kappa^2$ and mediated by Kaluza-Klein towers, are proportional to $1/M_S^{4}$. Their dependences on the $N$, according to compactification models, may not be as critical as in the single graviton emission case [11,12]. Moreover in this case, the polarizations of incoming and outgoing particles are affected by the fact that gravitons are of spin two while photons or $Z$ bosons, mediating the SM neutral currents, are of spin one. Therefore, one of the most useful and significant measurements to probe the existence of the extra dimensions would be the measurements of the polarizations at virtual graviton exchange processes.

In this paper we concentrate on the process $e^+e^- \rightarrow W^+W^-$ in the future Linear Colliders (LC) [17]. The spin-one nature of the $W$ bosons provides more channels to signal new physics, and the measurements of the $W$ polarizations are attainable from the decay angular distributions [18]. Therefore it is desirable to derive in detail the effects of large extra dimensions on the $W$ and beam polarizations.
Figure 1: Feynman Diagrams contributing to the process $e^+e^- \rightarrow W^+W^-$ including large extra dimension effects.

For the process

$$e^-(p_1, \kappa) + e^+(p_2, \kappa) \rightarrow W^-(q_1, \lambda) + W^+(q_2, \bar{\lambda}),$$

there are four Feynman diagrams, one $t$-channel diagram mediated by the neutrino and three $s$-channel ones mediated by the photon, Z boson, and spin-2 gravitons as depicted in Fig.1. At high energies where the electron mass is negligible, there are 18 different helicity amplitudes $M^\pm(\lambda, \bar{\lambda})$, where superscripts denote the electron helicities. Since the CP-invariance relates some amplitudes such as

$$M^\pm(\lambda, \bar{\lambda}) = M^\pm(-\lambda, -\bar{\lambda}),$$

thus only 12 amplitudes are independent [19]. By using the helicity formalism in Ref. [20] and the polarization convention in Ref. [19], we calculate the helicity amplitudes for the left-handed electron beam as

$$M^+_{++} = s_\theta [f_L \beta_W + \frac{g^2 s}{4t} (c_\theta - \beta_W) - 2f_D c_\theta (1 - \beta_W^2)],$$

$$M^+_{+-} = \frac{-\sqrt{2}(1 + c_\theta)}{m_W}[f_L \beta_W + \frac{g^2 s}{8t}(2c_\theta - 1 - 2\beta_W + \beta_W^2) - f_D (2c_\theta - 1)(1 - \beta_W^2)],$$

$$M^-_{++} = 2s_\theta (1 + c_\theta) \langle \frac{g^2 s}{8t} - f_D \rangle,$$

$$M^-_{0+} = \sqrt{2}(1 - c_\theta)[f_L \beta_W + \frac{g^2 s}{8t}(2c_\theta + 1 - 2\beta_W - \beta_W^2) - f_D (2c_\theta + 1)(1 - \beta_W^2)],$$

$$M^-_{00} = -\frac{s_\theta}{m_W} [f_L \beta_W (3 - \beta_W^2) + \frac{g^2 s}{4t}(2c_\theta - 3\beta_W + \beta_W^3) - 2f_D c_\theta (2 - 3\beta_W^2 + \beta_W^4)],$$

$$M^-_{+-} = -2s_\theta (1 - c_\theta) \langle \frac{g^2 s}{8t} - f_D \rangle.$$
and for the right-handed electron beam we have

\[
\begin{align*}
\mathcal{M}_{++}^+ &= s_\theta [f_R \beta_W - 2 f_D c_\theta (1 - \beta_W^2)], \\
\mathcal{M}_{+0}^+ &= \frac{\sqrt{2}(1 - c_\theta)}{m_W} [f_R \beta_W - f_D (2 c_\theta + 1) (1 - \beta_W^2)], \\
\mathcal{M}_{+-}^+ &= 2 f_D s_\theta (1 - c_\theta), \\
\mathcal{M}_{0+}^+ &= -\frac{\sqrt{2}(1 + c_\theta)}{m_W} [f_R \beta_W - f_D (2 c_\theta - 1) (1 - \beta_W^2)], \\
\mathcal{M}_{00}^+ &= -\frac{s_\theta}{m_W} [f_R \beta_W (3 - \beta_W^2) - 2 f_D c_\theta (2 - 3 \beta_W^2 + \beta_W^4)], \\
\mathcal{M}_{+-}^+ &= -2 f_D s_\theta (1 + c_\theta).
\end{align*}
\]

The CP-invariance implies

\[
\mathcal{M}_{+-} = \mathcal{M}_{++}, \quad \mathcal{M}_{+0} = \mathcal{M}_{0+}, \quad \mathcal{M}_{0-} = \mathcal{M}_{+-}.
\]

In Eqs. (6) and (7) we denote \( s_\theta = \sin \theta, \ c_\theta = \cos \theta \), and

\[
\epsilon_L = -\frac{1}{2} + \sin^2 \theta_W, \quad \epsilon_R = \sin^2 \theta_W,
\]

\[
f_{L,R} = \frac{g^2 \epsilon_{L,R}}{1 - M_Z^2/s} - e^2,
\]

\[
\hat{m}_W = \frac{M_W}{\sqrt{s/2}} = \sqrt{1 - \beta_W^2}.
\]

The terms proportional to \( f_{L,R} \) denote the contributions from the \( \gamma \)- and \( Z \)-mediated diagrams, and those proportional to the \( g^2 s/t \) from the neutrino-mediated one. It can be easily seen that the preparation of right-handed electron beam switches off the \( t \)-channel \( \nu \)-mediated contributions. These SM results are consistent with those in Ref. [19,14]. The low scale quantum gravity effects are included in \( f_D \), defined by

\[
f_D = -\frac{\pi s^2}{2M_{pl}^2} \sum_i \frac{1}{s - m_i^2} \simeq \frac{\pi s^2}{2M_S^4} \ln \left( \frac{M_S^2}{s} \right) \quad \text{for } N = 2
\]

\[
\simeq \frac{\pi s^2}{(N-2)M_S^4} \quad \text{for } N > 2.
\]

It is to be noted that the two helicity amplitudes \( \mathcal{M}_{+-}^+ \) and \( \mathcal{M}_{+-}^+ \) vanish at the tree level in the SM, but retain extra dimension effects as sizable as the other amplitudes.
In Fig. 2 and 3, we plot the differential cross sections for the \( W \) pair production at \( e^+e^- \) collisions with respect to the \( W^- \) scattering angle against the electron beam at \( \sqrt{s} = 1 \) TeV in the case of \( N = 2 \) and \( M_S = 2.5 \) TeV, broken down to the transverse and longitudinal helicity components of the \( W \) bosons. The cases with \( N > 2 \) unless \( N \) is too large shows similar behaviors. The effects of large extra dimensions with respect to the SM background can be enhanced by using the right-handed electron beam and selecting the \( W^+W^- \) polarizations to be both transverse. This is because the employment of the right-handed electron beam eliminates the dominant SM contributions of the \( t \)-channel \( \nu \)-mediated diagram in Fig.1. And the SM background from the \( s \)-channel with the right-handed electron beam is proportional to

\[
f_R = e^2 \left[ \frac{1}{1 - M_Z^2/s} - 1 \right] \propto \frac{M_Z^2}{s} \quad \text{for} \quad s \gg m_Z,\]

which decreases as the beam energy becomes larger, while the extra dimension correction proportional to \( f_D \) in Eq. (10) increases. Furthermore, at the tree-level, \( \mathcal{M}^{+(-)}_{++}^{(SM)} = \mathcal{M}^{-+}_{++}^{(SM)} = 0 \) in the SM. The \( \sigma_{TT} \) including large extra dimension effects is about \( \sim 10^4 \) times the SM background.

In practice, the generation of 100% polarized electron beam is infeasible. We consider the expected polarization of the electron beam as 90%. Figure 4 shows the differential cross sections against the \( W \) scattering angle according to the \( W \) polarizations at \( \sqrt{s} = 1 \) TeV when \( N = 2 \) and \( M_S = 2.5 \) TeV. The dominant \( t \)-channel SM background contaminates the unique behavior of \( d\sigma_{TT} \), however, substantial corrections to the SM background still remain. We do not consider the polarization of the positron beam since it is more difficult to generate, expected presumably in the range of 60 % to 65% \([17]\).

No radiative corrections are included here. The SM radiative corrections have the effects of the same orders of magnitude on the results with or without considering this model since the effects of the extra dimensions mainly come from the interference with the SM amplitudes.
\[
\sqrt{s} = 0.5 \text{ TeV ( } \int L = 50 \text{ fb}^{-1}) \quad \sqrt{s} = 1 \text{ TeV ( } \int L = 200 \text{ fb}^{-1})
\]

|               | \(N = 2\) | \(N = 6\) | \(N = 2\) | \(N = 6\) |
|---------------|-----------|-----------|-----------|-----------|
| \(\sigma_{\text{tot}}^{\text{unpol}}\) | 4.6       | 2.6       | 9.3       | 5.3       |
| \(\sigma_{\text{TT}}^{\text{unpol}}\) | 4.6       | 2.6       | 9.3       | 5.4       |
| \(\sigma_{\text{tot}}^{90\% \text{ RH}}\) | 3.4       | 2.0       | 6.8       | 4.1       |
| \(\sigma_{\text{TT}}^{90\% \text{ RH}}\) | 3.4       | 2.0       | 7.0       | 4.1       |
| \(\sigma_{\text{tot}}^{90\% \text{ LH}}\) | 5.1       | 2.9       | 10.1      | 5.8       |
| \(\sigma_{\text{TT}}^{90\% \text{ LH}}\) | 5.1       | 2.9       | 10.2      | 5.8       |

Table 1. The LC bounds of \(M_S\) in TeV at 95% confidence level according to the beam or \(W\) pair polarizations.

The LC bounds on the \(M_S\) are derived by the statistical errors with the angular cut \(|\cos \theta| < 0.95\) at 95% confidence level at \(\sqrt{s} = 0.5, 1.0\) TeV and \(N = 2, 6\) from six different observables according to the beam and \(W\) pair polarizations. As for the beam polarization effects on the \(M_S\) bounds, the preparation of the left-handed electron beam is expected to yield higher bounds. With a given beam polarization, the \(\sigma_{\text{TT}}\)’s are likely to give higher \(M_S\) bounds. On account of the smaller numbers of transversely polarized \(W\) pair events, these results imply that large extra dimension corrections in the \(\sigma_{\text{TT}}\)’s are much larger than those in the \(\sigma_{\text{tot}}\).

It is concluded that valuable information about the models with large extra dimensions can be obtained by observing the \(W\) pair and beam polarizations at the \(e^+e^- \rightarrow W^+W^-\) process. In particular, the measurements of the cross section for transversely polarized \(W\) pair with the right-handed electron beam highly enhances the possibilities to see the low scale quantum gravity effects. The current inability to generate purely polarized beam at \(e^+e^-\) colliders contaminates this feature. Almost purely polarized beams are possible in the future \(\mu^+\mu^-\) colliders [21], since the muons are prepared through the pion decays accompanied by purely chiral neutrinos. We expect definite signal of the large extra dimension effects through the observations of transverse polarizations of \(W\) pair with the right-handed muon beam at
the muon colliders. It has been shown that for the LC bounds of the string scale $M_S$ the use of left-handed electron beam is preferred, and for the probe of large extra dimension effects the measurements of the cross section for transversely polarized $W$ pair are.

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FIG. 2. The differential cross section with respect to the $W^-$ scattering angle when unpolarized beam with $\sqrt{s} = 1$ TeV is used, broken down to the transverse and longitudinal helicity components of the $W$ bosons. The solid line includes large extra dimension effects when $N = 2$ and $M_S = 2.5$ TeV. The dashed line denotes the SM background.
FIG. 3. The differential cross section with respect to the $W^-$ scattering angle when the right-handed electron beam with $\sqrt{s} = 1$ TeV is used, broken down to the transverse and longitudinal helicity components of the $W$ bosons. The solid line includes large extra dimension effects when $N = 2$ and $M_S = 2.5$ TeV. The dashed line denotes the SM background.
FIG. 4. The differential cross section with respect to the $W^-$ scattering angle with the beam polarization $+90\%$ at $\sqrt{s} = 1$ TeV, broken down to the transverse and longitudinal helicity components of the $W$ bosons. The solid line includes large extra dimension effects when $N = 2$ and $M_S = 2.5$ TeV. The dashed line denotes the SM background.