Susceptibilities and correlation functions of the anisotropic spherical model

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Abstract. The static transverse and longitudinal correlation functions (CF) of a 3-dimensional ferromagnet are calculated for the exactly solvable anisotropic spherical model (ASM) determined as the limit $D \to \infty$ of the classical $D$-component vector model. The results are nonequivalent to those for the standard spherical model of Berlin and Kac even in the isotropic case. Whereas the transverse CF has the usual Ornstein-Zernike form for small wave vectors, the longitudinal CF shows a nontrivial behavior in the ordered region caused by spin-wave fluctuations. In particular, in the isotropic case below $T_c$ one has $S_{zz}(k) \propto 1/k$ (the result of the spin-wave theory) for $k \lesssim \kappa_m \propto T_c - T$.

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The spherical model (SM) of Berlin and Kac was believed many years to be the only model in the statistical theory of magnetism, which is exactly solvable in 3 dimensions. The other model, which is the limit $D \to \infty$ of the classical $D$-component vector model introduced by Stanley, has the same partition function in the homogeneous case, and for this reason the latter was not considered as an independent model by many researchers and received relatively little attention.

However, the $D = \infty$ model is nonequivalent to the standard spherical model of Berlin and Kac and goes beyond it in many respects. First, it can be easily generalized for anisotropic systems. Second, it does not use the global spin constraint, which leads to unphysical results in spatially inhomogeneous situations. In particular, in the SM the Curie temperature $T_c$ of a 4-dimensional ferromagnetic film with free boundary conditions, which is infinite in 3 dimensions and finite in the 4th dimension, has been found by Barber and Fisher to be a non-monotonous function of the number of its layers $N$. To the contrast, an improved version of the spherical model using separate spin constraints in each layer leads in the case mentioned above to the monotonically increasing $T_c(N)$, as it should be. However, even this improved version of the SM fails on physically meaningful 3-dimensional ferromagnetic films, since the latter are 2-dimensional objects and without anisotropy their $T_c$ is zero for any finite thickness. An adequate description of spatially inhomogeneous and low-dimensional ferromagnetic systems on the “spherical” level can be achieved only with the use of the anisotropic $D = \infty$ model, which can be called also anisotropic spherical model (ASM). This model was recently applied to study the dimensional crossover of $T_c(N)$ of anisotropic ferromagnetic films and the effects of thermal fluctuations in the Bloch-wall phase transition.

Here it will be shown that the ASM deviates from the standard SM even in the spatially homogeneous case and even in the isotropic limit, if the spin-spin correlation functions are concerned. Indeed, below $T_c$ in the $D = \infty$ model there are two different — longitudinal and transverse — correlation functions, whereas there is only one CF in the SM. A less trivial reason for the difference between two models relies on the fact that a wave-vector-dependent CF is proportional to the appropriate susceptibility, which is the linear response to a spatially inho-
mogeneous sinusoidal field. In this case, as was argued above, the global spin constraint used by the SM modifies the results. As we shall see, the k-dependent longitudinal CF of the ASM has a nontrivial non-Ornstein-Zernike form below $T_c$, which is similar to that following from the lowest-order spin-wave theory for $T \ll T_c$ \[12\]. For the uniaxially anisotropic ferromagnetic model the ASM yields finite longitudinal and transverse susceptibilities and correlation lengths in the ordered region.

The anisotropic generalization of the classical $D$-component vector model of Stanley \[3\] can be described by the Hamiltonian

$$\mathcal{H} = -\mathbf{H} \sum_i \mathbf{m}_i - \frac{1}{2} \sum_{ij} \sum_{\alpha=1}^D \eta_\alpha m_{\alpha i} m_{\alpha j},$$  \hspace{1cm} (1)$$

where $|\mathbf{m}_i| = 1$ and $\eta_\alpha \leq 1$ are anisotropy coefficients. Here we consider the uniaxial model with $\eta_1 \equiv \eta_2 = 1$ and $\eta_\alpha \equiv \eta \leq 1$ for $\alpha \geq 2$. The model \[1\] can be conveniently treated by the classical spin diagram technique \[12\], which allows classification of diagrams in powers of $1/D$ for $D \gg 1$. The equation of state $m(H,T)$ of the anisotropic spherical model, where $m \equiv \langle m_z \rangle$, is contained in the diagram series corresponding to the self-consistent Gaussian approximation (SCGA) \[12\], which becomes exact in the limit $D \to \infty$. In terms of the dimensionless variables $\theta \equiv T/T_{c}^{\text{MFA}}$ and $\eta \equiv H/J_0$, where $T_{c}^{\text{MFA}} = J_0/D$ is the mean-field transition temperature and $J_0$ is the zero Fourier component of the exchange interaction, one comes to the system of equations for the magnetization $m$ and the gap parameter $G$ \[12\]

$$G = \frac{m}{m + h},$$

$$\theta G \eta G = 1 - m^2.$$ 

Here \[\eta P'(\eta)/P(\eta) \leq 1\]

$$P(X) \equiv v_0 \sum_{\mathbf{q}} \frac{1}{(2\pi)^3} \frac{1}{1 - X \lambda_\mathbf{q}},$$  \hspace{1cm} (3)$$

where $v_0$ is the unit cell volume and $\lambda_\mathbf{q} \equiv |J_\mathbf{q}|/J_0$. In the long-wavelength limit $\lambda_\mathbf{q} \equiv 1 - aq^2$, where $a \sim a_0^2$ and $a_0$ is the lattice spacing. For 3-dimensional lattices with the nearest neighbour interactions the integral $P(X)$ has the following properties:

$$P(X) \approx \begin{cases} \frac{1 + X^2}{z}, & X \ll 1 \\ W - c_0 (1 - X)^{1/2}, & 1 - X \ll 1, \end{cases}$$  \hspace{1cm} (4)$$

where $z$ is the number of nearest neighbors, $W$ is the Watson integral, and $c_0 = v_0/(4\pi a^3)^{3/2}$. For the simple cubic (sc) lattice $v_0 = a_0^3$ and $\alpha = a_0^2/6$, hence $c_0 = (2/\pi)(3/2)^{3/2}$. The solution of the system of equations \[\eta P'(\eta)/P(\eta) \leq 1\] simplifies for zero field, where below $T_c$ one has $G = 1$ (the zero spin-wave gap) and

$$m = (1 - \theta/\theta_c)^{1/2}, \quad \theta \leq \theta_c = 1/P(\eta).$$  \hspace{1cm} (5)$$

It can be seen that in the isotropic case, $\eta = 1$, the value of the phase transition temperature $\theta_c$ reduces to the well-known result $\theta_c = 1/W$. Using \[\eta P'(\eta)/P(\eta) \leq 1\] one can calculate the longitudinal susceptibility $\chi_z \equiv \partial m / \partial H$. The zero-field reduced susceptibility, $\tilde{\chi}_z \equiv J_0 \chi_z$, has the form

$$\tilde{\chi}_z = \begin{cases} \frac{G}{1 - G}, & \theta > \theta_c \\ \frac{\theta}{2m^2} [\eta P'(\eta) + P(\eta)], & \theta < \theta_c, \end{cases}$$  \hspace{1cm} (6)$$

where $P'(X) \equiv dP(X)/dX$ and $G$ satisfies the equation $\theta G \eta G = 1$ above $\theta_c$. Solving this equation near $\theta_c$ in the linear approximation in $1 - G \ll 1$, using \[\eta P'(\eta)/P(\eta) \leq 1\], and introducing the reduced temperature variable

$$\epsilon \equiv \frac{\theta_c}{\theta} - 1,$$  \hspace{1cm} (7)$$

one can rewrite \[\eta P'(\eta)/P(\eta) \leq 1\] in the form

$$\tilde{\chi}_z \approx I(\eta) \begin{cases} (-\epsilon)^{-1}, & \theta > \theta_c \\ (2\epsilon)^{-1}, & \theta < \theta_c, \end{cases}$$  \hspace{1cm} (8)$$

where

$$I(\eta) = 1 + \eta P'(\eta)/P(\eta) \approx \begin{cases} 1 + 2\eta^2/z, & \eta \ll 1 \\ c_0 1/\sqrt{1 - \eta}, & 1 - \eta \ll 1. \end{cases}$$  \hspace{1cm} (9)$$

The first line of \[\eta P'(\eta)/P(\eta) \leq 1\] is valid for the weakly-anisotropic model, $1 - \eta \ll 1$, in the narrow temperature interval

$$-\epsilon \ll \epsilon^* \equiv \frac{c_0}{4W} \sqrt{1 - \eta} \ll 1,$$  \hspace{1cm} (10)$$

whereas the second one is valid in the whole region below $\theta_c$. The latter is the reason to define $\epsilon$ in the non-standard form \[\eta P'(\eta)/P(\eta) \leq 1\]. It can be seen from \[\eta P'(\eta)/P(\eta) \leq 1\] that the critical behaviour of the longitudinal susceptibility in the ASM is the same as that in the mean field approximation (MFA), including the famous ratio of critical amplitudes $2$. This equivalence holds for all the critical indices \[\eta P'(\eta)/P(\eta) \leq 1\], since for $\eta < 1$ the square-root singularity of $P(X)$ at $X = 1$ is suppressed. In the extreme case of the “spherical Ising model”, $\eta = 0$, the mean field approximation for the ASM becomes exact, since the fluctuations of the transverse spin components die out with the transverse coupling $\eta$ and the influence of longitudinal fluctuations vanishes in the spherical limit, $D \to \infty$. For the anisotropic model, $\eta < 1$, the quantity $\tilde{\chi}_z$ is finite in the ordered region, vanishing at $\theta = 0$ and diverging at $\theta = \theta_c$. On the contrary, the longitudinal susceptibility of the isotropic model, $\eta = 1$, diverges as $\tilde{\chi}_z \propto (-\epsilon)^{-2}$ above $\theta_c$ and is infinite in the whole region below $\theta_c$ due to the Goldstone-mode fluctuations. In this case for $h \ll 1$ one has
\[ \Delta m = m(h) - m(0) \approx \frac{\theta_0}{m h^{1/2}} h^{1/2}, \]  

which coincides with the result of the standard spherical model and is in accord with the prediction of the lowest-order spin-wave theory for stationary systems with spontaneous breaking of a continuous symmetry. A similar expression for a general D-component vector model, which contains the additional factor \( 1 - 1/D \), was derived by Fisher and Privman from the scaling arguments. Measuring the square-root singularity of magnetization by Fisher and Privman from the scaling arguments.

\[ \chi_\perp \equiv \beta S_\perp(k), \]  

\[ \chi_\parallel \equiv \chi_\perp(0), \]  

which diverges in the SCGA fluctuations of the molecular field; (c) ladder equation for the transverse four-spin correlation line \( V_k \); (d) Dyson equation for the renormalized transverse interaction \( \eta \beta^2 J_k \). Unlabeled small circles denote transverse spin components.

\[ \chi_\perp \approx \chi_\perp(0), \]  

\[ \chi_\parallel \approx \chi_\parallel(0), \]  

in the whole temperature range \( \eta \theta \). One can see that even a small anisotropy, as \( 1 - \eta = 10^{-2} \), has a profound influence on \( \chi_\parallel \) below \( \theta_\perp \).

Unlike the transverse CF given by (13), the exact longitudinal CF, \( S_{zz}(k) \), cannot be determined from the SCGA in the spherical limit, since for \( D \to \infty \) the fluctuations of the single longitudinal spin component become nonessential and \( S_{zz}(k) \) disappears from the SCGA equations. Therefore, the wave-vector-dependent longitudinal CF in the anisotropic spherical model should be considered separately, which is the main purpose of this paper. With the help of the classical spin diagram technique \( S_{zz}(k) \) determined by (13) can be represented as

\[ S_{zz}(k) = \frac{\tilde{\Lambda}_{zz}(k)}{1 - \tilde{\alpha}_{zz}(k) \beta J_k}, \]  

where \( \tilde{\Lambda}_{zz}(k) \) is the compact (irreducible) part of \( S_{zz}(k) \) given by the diagrams, which cannot be cut by the one longitudinal interaction line \( \beta J_k \). The quantity \( \tilde{\Lambda}_{zz}(k) \) is in turn given in the limit \( D \to \infty \) by the set of diagrams represented in Fig. 2b. Such a choice of diagrams is based on the arguments of [18]. More technical details can be found in [19], where the transverse CF, \( S_{zz}(k) \), \( \alpha \geq 2 \), was calculated up to the first order in \( 1/D \) for low-dimensional ferro- and antiferromagnets in magnetic field. The analytical form of \( \tilde{\Lambda}_{zz}(k) \) of Fig. 2b reads

\[ \tilde{\Lambda}_{zz}(k) = \tilde{\Lambda}_{zz} + \tilde{\alpha}_{zz} V_k. \]  

Here in the limit \( D \to \infty \) one has [19]
\[ \tilde{\Lambda}_{zz} = \tilde{\Lambda}_{\alpha\alpha} + \tilde{\Lambda}_{\alpha\alpha\beta\beta} \xi^2, \quad \tilde{\Lambda}_{\alpha\alpha z} = \tilde{\Lambda}_{\alpha\alpha\beta\beta} \xi, \]

where \( \alpha \neq \beta \neq z, \)

\[ \tilde{\Lambda}_{\alpha\alpha} = \frac{\theta G}{D}, \quad \tilde{\Lambda}_{\alpha\alpha\beta\beta} = -\left( \frac{\theta G}{D} \right)^3 \frac{1}{1 - \theta G/2}. \quad (17) \]

\( G \) satisfies the system of equations (2), and

\[ \xi \equiv \beta (H + mJ_0) = \frac{D}{\theta} (h + m) = \frac{Dm}{\theta G} \quad (18) \]

is the temperature-normalized molecular field. The quantity \( \tilde{V}_k \) in (13) is the solution of the ladder equation Fig. 2, and has the form

\[ \tilde{V}_k = \frac{V_k}{1 - \tilde{\Lambda}_{\alpha\alpha\beta\beta} V_k}, \quad (19) \]

where \( \tilde{\Lambda}_{\alpha\alpha\beta\beta} \) is given by (13),

\[ V_k = \frac{D - 1}{2} \int \frac{d\eta}{(2\pi)^3} \eta \tilde{\beta} \tilde{J}_q \eta \tilde{J}_{k-q}, \quad (20) \]

and the renormalized transverse interaction \( \eta \tilde{\beta} \tilde{J}_q \) determined by Fig. 2I reads

\[ \eta \tilde{\beta} \tilde{J}_q = \frac{\eta \beta J_q}{1 - \tilde{\alpha}_{\alpha\eta} \eta \beta J_q}. \quad (21) \]

The factor \( D - 1 \) in (20) results from the summation over transverse spin components in Fig. 2I, or in Eq. (13), as well as in the ladder equation Fig. 2C. Such a factor does not appear, if one tries to take into account the similar diagrams for the transverse CF \( S_{\perp}(k) \equiv S_{\alpha\alpha}(k), \) \( \alpha \geq 2, \)

and thus these diagrams vanish in the spherical limit and \( S_{\perp}(k) \) has the trivial Ornstein-Zernike form (13).

Combining now Eqs. (13)-(21), one comes after simplifications to the final result

\[ \tilde{\chi}_z^{-1}(k) = G^{-1} - \lambda_k + \frac{2m^2}{\theta G^2} \frac{1}{r_k}, \quad (22) \]

where

\[ r_k = \int \frac{d\eta}{(2\pi)^3} \frac{1}{1 - G\eta\lambda_k - G\eta\lambda_{k-q}}. \quad (23) \]

The result similar to (22), in which, however, only the last term is present, was obtained earlier [1] within the lowest-order spin-wave theory well below \( T_c \). On the other hand, for the standard spherical model the correlation function has the trivial Ornstein-Zernike for all temperatures [12]. The integral in (23) can be easily calculated for \( k = 0 \) and in the corner of the Brillouin zone, \( k = b, \) for the sc lattice, where \( \lambda_{b-q} = -\lambda_q. \) The result has the form

\[ r_k = P(\eta G) \left\{ \begin{array}{ll} I(\eta G), & k = 0 \\ 1, & k = b, \end{array} \right. \quad (24) \]

where \( I(X) \) is given by (9). One can see that \( \tilde{\chi}_z^{-1}(0) \) of (23) is in accord with the previously obtained expression (1) for \( h = 0 \) below \( \theta_c, \) where \( G = 1. \) Under the same conditions \( \tilde{\chi}_z^{-1}(b) = \theta/(2\theta_c), \) which can be used to control numerical calculations. For the weakly anisotropic model in the case \( 1 - \eta G \ll 1 \) the integral (24) can be calculated analytically for small wave vectors, \( a_0k \ll 1, \) which results in

\[ r_k \approx \frac{c_0}{\sqrt{\alpha k^2}} \arctan \left( \frac{1}{2} \sqrt{\frac{\alpha k^2}{1 - \eta G}} \right). \quad (25) \]

The last term in (22) modifies the \( k \)-dependence of \( \chi_z(k) \) below \( \theta_c \) in the gapless case \( \eta G = 1 \) in the range of small wave vectors, which shrinks if \( \theta_c \) is approached from below. Instead of the Ornstein-Zernike form \( \chi_z(k) \propto 1/k^2 \) one has (cf. [10,11])

\[ \chi_z(k) \propto \frac{1}{k}, \quad k \ll \kappa_m = \frac{2m^2}{\theta c_0 a_0^2} \quad (26) \]

The latter defines a length scale \( \xi_m, \) which exists only in the ordered region \( (m > 0) \) and diverges at \( \theta_c \) due to the vanishing of magnetization (5):

\[ \xi_m \equiv \frac{1}{\kappa_m} = \frac{\theta}{2m^2} c_0 a_0^2 \quad (27) \]

For the sc lattice \( c_0 a_0^2 = \frac{1}{2\pi^2} \) and hence \( \xi_m = \frac{3\theta}{4\pi m^2} a_0. \) The length \( \xi_m \) is analogous to the “bare”, i.e., the mean-field correlation length below \( T_c, \) which follows, in particular, from the Landau-Ginzburg phenomenological free energy. This analogy is, however, not complete, since \( \xi_m \) diverges at the actual transition temperature \( T_c \) and not at \( T_{SC}^{MFA} \) \( (\theta = 1). \) The crossover of \( \chi_z(k) \) at \( k \sim \kappa_m \) for temperatures slightly below \( T_c, \) was described earlier with the help of the renormalization group approach [2].

The \( k \)-dependences of \( \tilde{\chi}_z^{-1}(k) \) of (22) in the isotropic case obtained with the help of numerical integration in (23) are represented in Fig. 3 for different temperatures below \( T_c. \) Curves of such a type for Heisenberg model systems such as EuO and EuS could be observed, in principle, in neutron scattering experiments, but such experiments were not carried out up to now.

Our next task is to calculate explicitly the real-space correlation functions (12) in the small-anisotropy case \( 1 - \eta \ll 1. \) Since the CFs themselves are proportional to \( 1/D \) and thus vanish in the spherical limit [see, e.g., (3)], it is more convenient to deal with the appropriate susceptibilities, which are given by

\[ \tilde{\chi}_{\alpha\alpha}(r) = \int \frac{d\eta}{(2\pi)^3} e^{ikr} \tilde{\chi}_{\alpha\alpha}(k). \quad (28) \]

In the transverse case using (13) one comes to the well-known result

\[ \tilde{\chi}_{\perp}(r) \approx \frac{c_0 a_0^{1/2}}{r} e^{-r/\xi_{c\perp}}, \quad \xi_{c\perp} \approx \sqrt{\frac{\alpha}{1 - \eta G}} \quad (29) \]
for \( r \gg a_0 \). One can see that in the isotropic case, \( \eta = 1 \), the transverse correlation length, \( \xi_{c,\perp} \), if infinite in the whole region below \( \theta_c \), where \( G = 1 \). In more complicated situations, as for the longitudinal susceptibility below \( \theta_c \), the correlation length can be determined as \( \xi_c = 1/\kappa_c \), where \( \kappa_c \) is the singularity point of \( \tilde{\chi}_z(k) \) on the imaginary axis \( k = i\kappa_c \), which is closest to the origin. This leads with the use of (32) to the transcendental equation for \( u_c \equiv \xi_{c,\perp}/(2\xi_{c,\perp}) \) having the form

\[
\ln \left( 1 + \frac{u_c}{1 - u_c} \right) = \alpha \equiv \frac{\xi_{c,\perp}}{\xi_m} = \frac{2m^2}{\theta_c \sqrt{1 - \eta}}, \tag{30}
\]

where \( \xi_m \) is given by (27). This equation coincides with Eq. (4.19) of [8], which determines the temperature-dependent width \( \delta \lambda = 2\xi_{c,\perp} \) of the (linear) domain wall in the uniaxial spherical model. The asymptotic solutions of (31) read

\[
u_c \approx \begin{cases} \sqrt{a/2}, & a \ll 1 \quad \text{(slightly below } \theta_c) \\ 1 - 2e^{-a}, & a \gg 1 \quad \text{(far below } \theta_c). \end{cases} \tag{31}
\]

Note that the transition between two regimes in (31) occurs at \( e \sim \epsilon^* \sim \sqrt{1 - \eta} \) [see (19)]. The explicit results for the correlation length \( \xi_{c,\perp} \) itself, including that above \( \theta_c \), read

\[
\xi_{c,\perp} \approx \begin{cases} \sqrt{\frac{\alpha}{1 - G}}, & 0 < -\epsilon \ll \epsilon^* \\ \sqrt{\xi_m \xi_{c,\perp}}/2, & 0 < \epsilon \ll \epsilon^* \\ \frac{\xi_{c,\perp}}{2} \left( 1 + \exp \left( \frac{-\xi_{c,\perp}}{\xi_m} \right) \right), & \epsilon^* < \epsilon, \end{cases} \tag{32}
\]

where \( I(\eta) \) is given by the second line of (19). The numerically calculated temperature dependencies of the longitudinal and transverse correlation lengths are illustrated in Fig. 3.

\[\text{FIG. 3. The wave vector dependences of the inverse longitudinal susceptibility } \tilde{\chi}_z^{-1}(k) \text{ of the isotropic spherical ferromagnet along the [111] direction of } k \text{ for different temperatures in the ordered region.}\]

To calculate the Fourier-transform \( \tilde{\chi}_z(r) \) below \( \theta_c \) with the help of (23), (22), and (21), it is convenient to perform at first the integration over the angle variables and then to deform the \( k \)-integration contour in the complex plane. In this way one comes to the well-behaved expression

\[
\tilde{\chi}_z(r) \approx \frac{2\alpha_0 a^{1/2}}{r} \left[ e^{-r/\xi_{c,\perp}} - \frac{1 - u_c^2}{\alpha(1 - u_c^2)} + 2u_c^2 \right]
\]

\[
+ \int_1^\infty \frac{du}{u \ln \left( \frac{u + a}{u - a} \right) + \pi^2 u^2}, \tag{33}
\]

where the first term is the contribution of the pole determining the longitudinal correlation length \( \xi_{c,\perp} \) [see (31)], and the second one is the integral along the cut of the arctan-function of (24). The expression above simplifies in various limiting cases. For \( a \ll 1 \), i.e., much faster than the first term of (33). Thus, in this case only the pole term in (33) is relevant, and with the use of (31) one obtains

\[
\tilde{\chi}_z(r) \approx \frac{c_0 a^{1/2}}{r} e^{-r/\xi_{c,\perp}}, \tag{34}
\]

where \( \xi_{c,\perp} \) is given by the middle line of (22). In the opposite case, \( a \gg 1 \), the pole sticks to the beginning of the cut, \( u = 1 \), and the amplitude of the pole term becomes exponentially small because of the factor \( 1 - u_c^2 \) [see (31)]. Thus, in this case only the cut term in (33) is essential, and the result can be simplified to

\[
\tilde{\chi}_z(r) \approx \frac{2c_0 a^{1/2}}{\pi r} e^{-r/\xi_{c,\perp}} \int_0^\infty \frac{dx}{1 + x^2} \exp \left( \frac{2r}{\pi \xi_m} x \right), \tag{35}
\]

\[\text{FIG. 4. Temperature dependences of the transverse, } \xi_{c,\perp}, \text{ and longitudinal, } \xi_{c,\perp}, \text{ correlation lengths, as well as the characteristic length } \xi_m, \text{ in the uniaxial spherical model.}\]
where $\xi_{cz}$ is given by the the lower line of (32). The asymptotic forms of (35) are (34) for $r \ll \xi_m$ and

$$\tilde{\chi}_z(r) \cong \frac{2m^2}{\theta} \left( \frac{\xi_m}{r} \right)^2 e^{-r/\xi_{cz}} \tag{36}$$

for $r \gg \xi_m$. One can say that for the isotropic model, where the true correlation length $\xi_{cz}$ is infinite below $\theta_c$, the “bare” one, $\xi_m$ of (27), still plays its role in some sense: For $r \gtrsim \xi_m$ the slow decay $\tilde{\chi}_z(r) \propto 1/r$ changes to a faster one, $\tilde{\chi}_z(r) \propto 1/r^2$.

As a conclusion, the anisotropic spherical model (ASM) considered here is a good exactly solvable “toy” model for classical spin systems, which can be successfully applied in many situations where the standard spherical model of Berlin and Kac fails. More realistic models of phase transitions possess, naturally, the other, nonspherical, values of the critical indices, but this is, however, only a quantitative effect, which is less important in comparison to the profound role played by the gapless spin waves in the isotropic model below $T_c$. The latter are properly taken into account in the ASM, which is thereby a very important step beyond the mean field approximation.

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