ABSTRACT: We study the injection of air in a granular suspension. We use a linear Hele-Saw cell filled with a suspension which is displaced by air, leading to a Saffman-Taylor (fingering) instability. For the suspension, we use an iso-dense mixture where the fluid and the particles have the same density. The volume fraction of particles can thus be adjusted over a wide range. We discuss the question of an effective rheology inside the cell as well as the pattern formation as a function of the granular compacity. We finally report results on the finger width for stable fingers and the thresholds for their destabilization.

INTRODUCTION

Viscous fingering has received much attention as an archetype of pattern-forming systems and was studied intensively for Newtonian fluids (Saffman & Taylor 1958, Bensimon et al. 1986, Homsy 1987) and non-Newtonian fluids (Lindner et al. 2002). It is an important model system suited for a better understanding of fluid injection in porous media with important practical applications such as crude oil recovery.

Here, we consider a case where the fluid to be displaced is an isodense non-Brownian suspension. The volume fraction of the grains can be easily controlled and thus, the rheology of the suspension can be varied over a wide range: going from an effective viscous fluid up to a jammed dense paste. This system can be seen as a fundamental tool to understand many complex injection problems such as those existing in weakly consolidated porous media. Its potential interest would lie in a better understanding of many practical processes such as the stabilization of saturated soils by air injection or the consolidation of oil extraction wells.

A difficult question is still to understand the interfacial dynamics between a suspension and another fluid. Viscous fingering is mainly governed by the local viscosity at the finger tip and is also very sensitive to the presence of particles. We thus expect that studying the pattern formation will lead to a better understanding of particles dynamic at the interface between suspension and air (or another injection fluid).

The paper is organized as follows. In Sec.2 we will recall the basic equations for the Saffman-Taylor instability. Sec. 3 describes the set-up and experimental methods. In Sec. 4, the experimental results concerning the rheology of suspensions and the Darcy’s law are presented and discussed. In Sec. 5, we interest ourselves at the finger shape as well as the finger width. Sec. 6 gives a summary of the obtained results.

2 THE SAFFMAN-TAYLOR INSTABILITY

2.1 Presentation of the instability

The Saffman-Taylor instability is typically studied in a thin linear channel or Hele-Shaw cell (Fig. 1). The width of the cell $W$ is chosen to be large compared to the channel thickness $b$. The cell is filled with a viscous fluid which is then pushed by air. The properties of the viscous fluid are its viscosity $\eta$, its surface tension $\gamma$, and its density $\rho_f$. The viscosity and the density of air are neglected.

When air pushes the viscous fluid (for example due to an imposed pressure gradient $\nabla P$), the interface between the two fluids destabilizes. This destabilization leads to the formation of a viscous finger of width $w$, propagating at velocity $U$. In general, one considers the relative finger width: $\tilde{w} = w / W$.

![Figure 1: Schematic drawing of the experimental set-up.](image-url)
For Newtonian fluids, the motion in the Hele-Shaw cell is described by the two-dimensional gap averaged velocity field $\mathbf{u}$. It is given by the local Darcy’s law, which relates the local pressure gradient to the fluid velocity.

Away from the interface the flow can be considered as uniform and one can relate the mean flow velocity $V$ to the pressure gradient $r \, \Delta P$:

$$ V = \frac{b^2}{12} \, r \, \Delta P : \quad (1) $$

### 2.3 Finger selection

The relative finger width $w$ is determined by the capillary number $Ca = \frac{U}{r \, \Delta P}$ which represents the ratio of viscous forces and capillary forces. The viscous forces tend to narrow the finger, whereas the capillary forces tend to widen it. One thus anticipates that the relative width of the viscous fingers decreases with increasing finger velocity. For large values of the capillary parameter, $Ca$, reaches a limiting value of about half the channel width.

The control parameter of the instability is $1 = B = \frac{C}{a}$ with $a = b$ the aspect ratio of the Hele-Shaw cell. When scaled on $1 = B$, measurements for different systems all fall on the same universal curve (Saffman & Taylor 1958, McLean & Saffman 1981).

### 3 EXPERIMENTAL SET-UP

We use polystyrene spheres of 40 $\mu$m in diameter in a modified silicone oil DC704 of viscosity $\eta = 59.5$ mPa.s and surface tension $\gamma = 31.0$ mN/m. The density of the fluid matches the one of the grains ($1.05$ g/cm$^3$) closely leading to what we call an iso-dense suspension. This allows us to control the fraction of grains over a wide range of shear rates (1 s$^{-1}$ < $\dot{\gamma}$ < 100 s$^{-1}$). For all suspensions one observes Newtonian behavior over a wide range of shear rates ($1 < \dot{\gamma} < 100$ s$^{-1}$). Our fingering experiments are indeed performed in this range of shear rate and we can thus define a constant viscosity $\eta_h$ from the rheological measurements for each suspension.

For large shear rates ($\dot{\gamma} > 100$ s$^{-1}$) and high grain fraction ($\phi = 30\%$) weak shear thinning is observed. For low shear rates ($\dot{\gamma} < 1$ s$^{-1}$) and high grain fractions a decrease of the viscosity is observed which could be due to the existence of a yield stress, whereas for low grain fractions an increase of viscosity is observed.

Note that these results are in qualitative and quantitative agreement with previous work (Zarraga et al. 2000).

### 4 RESULTS: RHEOLOGY AND DARCY’S LAW

#### 4.1 Rheology

Figure 2 shows the viscosity of the suspensions as a function of the shear rate for different grain fractions.

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### 4.2 Darcy’s law

Assuming the flow far away from the finger to be uniform, we expect that the classical Darcy’s law (Equation 1) linking the gap averaged fluid velocity $V$ to the imposed pressure gradient $r \, \Delta P$ in our Hele-Shaw cell remains valid.

The imposed pressure gradient is calculated by:

$$ r \, \Delta P = \frac{p}{L} $$

where $p$ is the applied pressure drop and $L$ the distance between the finger tip and the exit of the cell. Mass conservation allows to obtain the velocity $V$ of the fluid far away from the interface from the finger velocity $U$ simply by using $V = U$ (Lindner et al. 2000).
Following equation 1, results are described on Figure 3, representing \( V : (12 R_h \rightarrow \beta ) \) as a function of \( r P \). If one compares with what is obtained from Darcy’s law (Fig. 3), one observes systematic deviations towards higher slope values, which increase with increasing volume fraction. This might indicate that there exists an effective viscosity \( C \) in the cell which is lower than \( R_h \) obtained with the rheometer.

5 RESULTS: FINGERS AND PATTERNS
5.1 Experimental observations

Figures 5a,b,c,d,e,f present the typical evolution of patterns observed for increasing velocity, valid for most grain fractions (up to 40%, not presented here).

For low velocities, the finger is stable and symmetrical with respect to the cell central axis (Fig. 5a). For higher velocity, it becomes asymmetric (Fig. 5b), although it remains stable (i.e., we could measure its width).

The first destabilization occurs on one side of the finger like a sinusoidal wave (Fig. 5c). Thereafter, for increasing velocity, this type of destabilization occurs on both sides (Fig. 5d).

At ever larger velocity, we observe classical destabilization of Saffman-Taylor fingers like side-branching (Fig. 5e) or tip-splitting (Fig. 5f).

5.2 Finger width

For stable fingers (Figs 5a,b), we measured the relative finger width. When representing as a function of the control parameter \( 1 = B \) of the instability one has to decide which viscosity to use.

Figures 6 and 7 represent as a function of \( 1 = B \) calculated with \( R_h \) (Fig. 6) and with \( C \) (Fig. 7). The line represents a fit to the data for the pure fluid and gives thus the master curve of the instability.

We observe an increasing dispersion of the results with grain fraction. However, on Figure 6, the data is scattered around the master curve whereas on Figure 7, all data seem to be below this curve.

As the finger width depends on the viscosity at the finger tip, one expects that a non-uniform flow and grain distribution will affect the finger width. Thus, one could interpret these results in terms of an effective viscosity at the finger tip which would vary around \( R_h \) with \( C \) being the lower limit.
5.3 Instability thresholds

Following the evolution of the finger shapes (Figs 5), one observes several instability thresholds. On Figure 8 the first threshold (apparition of oscillations on one side Fig. 5c) and the threshold for side branching (Fig. 5e) are plotted. Here  is used to obtain the value of  for .

We observe little variations of the first threshold with which is found to be  whereas in the case of pure fluid (i.e. = 0), we observe stable fingers up to 10,000.

Following the work of Bensimon et al. (1986), one can obtain a critical amplitude of the perturbations leading to a finger destabilization at a value  for a finite amplitude 5 m. Furthermore, this work predicts the destabilization to be independent on the wavelength of the perturbation. These predictions are in good agreement with our experimental data if we consider the size of the grains (40 m) to be of the order of  and the threshold to be weakly dependent of fraction of the grains (i.e. the concentration of grains at the finger tip) which should be proportional to the wavelength of the perturbation.

To test this further experiments are planned changing the particle diameter.

6 CONCLUSION AND PERSPECTIVES

In summary we have reported results on the injection of air in granular suspensions. The dynamics of the air/suspension interface can be described by a Saffman-Taylor like instability. Rheological measurements and the use of Darcy’s law for flow in the Hele-Shaw cell lead to different values for the viscosities of the suspensions. This could be explained by a non-homogeneous flow in the cell.

Furthermore we report the evolution of the observed patterns with finger velocity. For stable fingers, an increasing scatter of the data of the relative finger width with the grain fraction was observed, which might be linked to the local viscosity at the finger tip.

Finally, an analysis of the instability thresholds leads us to underline the importance of the grain size for the destabilization of the fingers.

In further studies of the dynamics of the interface between a pure fluid and a suspension, two cases seem to be of interest: the case of dense suspensions (> 50 %) and the case of miscible flow where the pushing pure fluid and the suspending one are identical.

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