Modal analysis of frameless arches made of thin-walled steel profiles

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Abstract. The article is devoted to the modal analysis of frameless arch structures based on lightweight gauge steel structures (LGSS). The relevance of the study is due to the need to take into account dynamic effects when calculating the load-bearing capacity. Dynamic loads are a crucial factor affecting the stress-strain state of arch structures. The paper presents theoretical studies of the influence of the main geometrical parameters of the arch structure made of cold-formed thin-walled steel profiles on the first eigenfrequency. The dependences between the dimensions of arched structures and eigenfrequencies are obtained. It is proved that to determine the load-bearing capacity of frameless arch structures with the span L > 12 m, it is necessary to determine the eigenfrequencies of vibrations of the structure.

1. Introduction
Throughout the last 15 years a new industrial field got formed and developed – the manufacture and implementation of lightweight gauge steel structures (LGSS) made of cold-formed profiles (in industrial and civil engineering). Lightweight gauge steel structures are vastly used today in the construction industry. These structures are broadly applied for the low-rise building construction in the private section and as well for the construction of structures with a span size up till 15 m. The application of this technology enables the manufacturing of profiles of effective geometry owing to low metal consumption balanced with sufficient cross-sectional stiffness.

For a long time arch structures have been the only structural form, allowing covering of large spans. Frameless arch structures are one of the fields in the construction of lightweight thin-walled and cold-formed steel profiles in the construction industry [1-2]. This technology represents a cylindrical arch made of steel profiles of the appropriate radius. The profiles are rolled in a firm lock, which leads to the creation of bearing and at the same time enclosing structure. The framework is absent, which reduces material consumption and mounting time. Frameless arch structures have a good perspective, as they are capable of providing the covering of significant areas without intermediate supports, they are easy to mount, have a low self-weight and sufficient thermal isolation [2].

One of the main advantages of the buildings made with the help of the lightweight gauge steel structure technology is considered the withstanding capacity against dynamic loads, although the groundings for such an assertion are not usually provided.

The paper [3] introduces a review about the conducted tests of a building out of lightweight gauge steel structures against seismic loads. The structural behavior showed that the real displacements of the columns were 9 times the calculated ones under the horizontal loading, characterized by 9 points, -14 sm against 1.72 sm. Herewith the authors indicate that roll-formed section profiles provided the
seismic load of 7, 8, 9 points perception. Besides, while conducting this analysis it should be accounted that each following load is applied after a previous 1 point lower load. The authors of the study claim that the failure of the columns is reached after the local stability loss of the profile walls.

The research on the dynamic instability of irregularly-shaped arch bridge based on Catastrophe Theory is discussed in [4]. Moreover, the issue of seismic impact for arch structures is studied in [5]. Using the incremental dynamic analysis, the behavior of concrete-filled steel tubular (CFST) arches under the seismic loads was studied. The numerical results received by the developing the nonlinear elastic-plastic finite element models were verified by the experiment with a shaking table test.

Since arch constructions work in a compression state, the problem of equilibrium stability is one of the most significant problems for engineers to consider. The paper [6] analyzes the equilibrium stability and the stress-strain state of plane round double-hinged arches, where the geometrically exact theory is used. Furthermore, the equilibrium stability of metal arches is studied in the [7], presenting the comparative analysis of theoretical, numerical, and experimental results. The influence on the stability of such parameters as vertical and horizontal support is discussed in [8, 9].

The issue of the out of plane stability of arches made of a thin-walled profile is studied in many papers. The paper [10] demonstrates the elastic buckling theory using the principle of minimum total potential energy, where the corresponding differential equations are derived. The principle of variation is used in [11] to derive the differential equations of stability. The significant issue of out-of-plane buckling of arches with carrying curvature is explained in [12]. This paper investigates arches with open thin-walled sections. To analyze the spatial stability of shear deformable thin-walled space frames and circular arches a consistent finite-element formulation is presented in [13].

The lateral buckling behavior of arches is observed in [14, 15]. These papers provide the analytical and numerical solutions. The influence of the buckling loads and modes are researched. The comprehensive information about analytical solutions for buckling of arches is presented in [16]. The flexural-torsional buckling of shallow arches with the open thin-walled section under uniform radial loads is studied in [17]. The paper [18] deals with nonlinear buckling confined thin-walled functionally graded material (FGM) arch subjected to external pressure. The numerical results show very close agreement with the present analytical solutions.

The paper [19] presents a method of analyzing the results of a numerical investigation into plastic hinge formation in arched corrugated thin-walled profiles. This method based on the identification of a perturbation component of displacement in selected points of the profile. The numerical analysis of thin-walled free-form arches was used in paper [20]. To realize it this paper is concerned with the development and application of two curved beam elements.

The evaluation of the exact natural frequencies and vibration modes of arch structures is presented in [21], where spatial Timoshenko arches were studied. In the work of Rudak [22] an attempt to estimate the lowest eigenfrequency of the frameless cylindrical coatings with the help of the Riley method was made. Wherein only radial displacements of the coating elements and connected masses (snow load) are considered.

As radial and longitudinal displacements of the frameless cylindrical coatings under static load are commensurable, the conclusion can be made that longitudinal displacements considerably affect the eigenfrequencies of the covering.

An essential part of the design is the definition and estimation of the frameless arch structure’s eigenfrequencies. Eigenfrequencies are requisite for the calculation of frameless arch structures considering wind pulsation, seismic, and also for the resonance vortex excitation calculations.

Low eigenfrequencies can lead to the coinciding with the excitation frequencies, resulting in intensive oscillations related to resonance, this, in turn, leads to the structural collapse. That is why it is necessary to define the eigenfrequencies of the structure while calculating the bearing capacity of the frameless arch constructions [22].

The object of the research is the stress-strained state of the thin-walled cold-formed profiles. The subject of the research is the frameless arch structure made of thin-walled cold-formed steel profiles.
The goal of the present work is to make a comparison of the eigenfrequencies of the arches depending on the different parameters of the frameless structure (span, arch rise, profile thickness).

2. Methods

The modal analysis of the frameless arch structure was implemented in the software package SCAD Office 21.1.

The paper considers a double-hinge symmetric arch with the span – $L$ (m) and the arch rise – $f$ (m), which is presented in Figure 1.

The computational model is represented as a flat arch in the software package (Figure 1). The support joints of the computational model are fixed against movements along the axes X, Y, Z, other joints are fixed against movement along the Y axis. All the profiles of the arch structure are connected with the help of the rolling machine, therefore it is assumed that the structure is totally decoupled from the plane.

The parameters of the arch profile are taken as for the arch profile C-110 with the thickness $t = 1.0; 1.2; 1.5$ mm produced by «Metal profile factory «Avangard» in Krasnoyarsk according to the technical specifications TU 1120-001-82913322-2009 «Profiled sheetings for frameless arch structures». The scheme of the profile with linear dimensions is demonstrated in Figure 2.
For the calculation the cross-section of the arch profile was designed in the software TONUS, which is a satellite of the software package SCAD Office 21.1 (Figure 3).

Table 1 provides the main stiffness properties of the cross-sectional profile C-110 with the thickness $t = 1.2$ mm.

| Characteristics              | Value | Unit of Measurement |
|------------------------------|-------|---------------------|
| Steel density                | $\rho$ | 7850                | kg/m$^3$            |
| Young's modulus              | E     | 2·105               | MPa                 |
| Poisson’s ratio              | $\nu$ | 0.3                 |                      |
| Longitudinal stiffness       | EF    | 16110050            | kg                  |
| Bending stiffness            | EIV   | 49540               | kg·m$^3$            |
| Shear stiffness              | GFV   | 3827770             | kg                  |

Evaluation of the lowest eigenfrequency and determination of mode was performed by the method of iteration of subspaces in the software package SCAD Office 21.1. Radial and longitudinal displacements of structural elements are taken into account. The modal analysis of the eigenfrequencies was made under the action of the gravity and snow loads.

According to preliminary studies of the stress-strain state of the frameless arch structure, it was found that the greatest internal force factors are obtained under the action of snow load according to the second variant of the load distribution. According to annex B.2 of the Set of Rules SP 20.13330.2016 (Loads and Impacts), the second variant of snow load distribution is shown in Figure 4. For the calculation according to the Set of Rules the snow load area is III.
Modal analysis was performed with the following load combination:

\[(L_1) \cdot 0.9 + (L_2) \cdot 0.5\]  

(1)

3. where: L1 is the gravity load (  

4. ); L2 is the snow load (  )

Figure 4. Variants of the distribution of snow load

Figure 5. Scheme of application of the gravity load in SCAD Office 21.1
5. Results and Discussion

This section provides the results of the modal analysis of the frameless arch structure. Table 2 presents the results of the calculation of the eigenfrequencies in software package SCAD Office 21.1 for the arch with the next geometrical parameters: the arch rise \( f_0 = 4 \text{ m} \); the span \( L = 11 \text{ m} \); the arch profile C-110 with the thickness \( t = 1.2 \text{ mm} \). Moreover, Table 2 demonstrates the eigenvalues and the modal masses for the corresponding shape modes.

| Mode | Frequency | Period | Modal masses (%) |
|------|-----------|--------|------------------|
| 1    | 0.067     | 14.816 | 2.358 78.68 0 1.15 |
| 2    | 0.02      | 49.21  | 7.832 0.128 0.422 0.58 0.16763 |
| 3    | 0.011     | 90.563 | 14.421 10.195 0 1.017 |
| 4    | 0.007     | 136.51 | 21.737 0.007 0 0.58 0.25798 |
| 5    | 0.004     | 223.63 | 35.609 3.727 0 1.017 |
| 6    | 0.003     | 314.02 | 50.003 0.007 0 0.58 0.25798 |
| 7    | 0.002     | 414.82 | 66.055 2.037 0 1.017 |
| 8    | 0.002     | 479.21 | 76.308 0.013 0.008 0 1.9285 |
| 9    | 0.002     | 617.47 | 98.323 0.01 0.131 0 7.254 |
| 10   | 0.002    | 617.47 | 98.323 0.01 0.131 0 7.254 |
|      | Sum of modal masses | 97.542 | 100 | 84.176 |

According to p.11.1.8 of the Set of Rules SP 20.13330.2016 (Loads and Impacts), it is necessary to conduct the calculation of the frameless arch structures considering the dynamic contributions, as:

\[
f_1 < f_{\text{lim}} < f_2
\]  

where: \( f_1 \) (Hz) is the first (lowest) eigenfrequency; \( f_2 \) (Hz) is the second eigenfrequency; \( f_{\text{lim}} \) (Hz) is the limiting eigenfrequency.

The limiting eigenfrequency \( f_{\text{lim}} \) (Hz) is determined by the formula (3):

\[
f_{\text{lim}} = \frac{w_0 k(z_e) \gamma_f}{940 T_{\text{e,lim}}}
\]

where: \( w_0 \) (Pa) is the value of wind pressure; \( k(z_e) \) is the coefficient taking into account the change in
wind pressure depending on height $z_e$ (m) ($z_e = 0.8h$, $h$ (m) – the height of a building); $\gamma_f$ is the load safety factor; $T_{g,\text{lim}}$ is the limiting period.

According to the Set of Rules SP 20.13330.2016 (Loads and Impacts): $w_0 = 300$ Pa – depending on the wind load area (II); $\gamma_f = 1.4$; $T_{g,\text{lim}} = 0.0077$ – depending on the logarithmic decrement $\delta$, which is adopted in this paper $\delta = 0.15$.

$k(z_e)$ for heights $z_e < 300$ m is determined by the formula (4):

$$k(z_e) = k_{10}(\frac{z_e}{10})^{2\alpha}$$  \hspace{1cm} (4)

where: $k_{10} = 0.65$, $\alpha = 0.2$ are the parameters determined by the table 11.3 in the Set of Rules SP 20.13330.2016 (Loads and Impacts), depending in the type of terrain (the type «B» for this calculation).

Formulas (5) and (6) demonstrate the example of calculation the limiting frequency for the arch with the arch rise $f_0 = h = 0.4$ m:

$$k(z_e) = 0.65 \cdot \left(\frac{0.8 \cdot 4}{10}\right)^{2 \cdot 0.2} = 0.412$$  \hspace{1cm} (5)

$$f_{\text{lim}} = \frac{300 \cdot 0.412 \cdot 1.4}{940 \cdot 0.0077} = 1.82$$  \hspace{1cm} (6)

The values of the first (lowest) eigenfrequency $f_1$ for the arch the profile C-110 with the thickness $t = 1.0$ mm (Table 3); $t = 1.2$ mm (Table 4); $t = 1.5$ mm (Table 5) are presented below. In addition to the diverse profile thicknesses, the next parameters were varied: span ($L_0$), arch rise ($f_0$). Some of the cells in the tables are empty because in these cases the ratio of the arch rise to the span is too high.

### Table 3. Values of the first (lowest) eigenfrequency $f_1$, Hz for arch the profile C-110 with the thickness $t = 1.0$ mm

| Arch rise $f_0$, m | Span $L_0$, m | First (lowest) eigenfrequency $f_{1\text{, Hz}}$ | Limiting eigenfrequency $f_{\text{lim\, Hz}}$ |
|-------------------|---------------|------------------------------------------|------------------------------------------|
|                   | 11  | 12  | 15  | 16  | 18  | 24  | 30  |                   |
| 4                 | 2.19*| 1.9*| 1.58| 1.34| 1.1 | 0.89| 0.78| 1.82               |
| 5                 | 2.12*| 1.836| 1.51| 1.286| 1.03 | 0.82| 0.734| 1.90               |
| 6.5               | -   | -   | 1.327| 1.124| 0.932 | 0.78| 0.65 | 2.00               |
| 7.5               | -   | -   | 0.812| 0.79 | 0.74 | 0.634| 0.6 | 2.06               |
| 9                 | -   | -   | -   | -   | 0.564| 0.54| 0.53 | 2.14               |
| 12                | -   | -   | -   | -   | -   | 0.52 | 0.5 | 2.26               |
| 15                | -   | -   | -   | -   | -   | -   | 0.48| 2.37               |

*value of the eigenfrequency, which is less than the corresponding limiting frequency

### Figure 7. First (lowest) eigenfrequency at the arch rise $f_0 = 4$ m and $f_0 = 5$ m for the arch profile C-110 with the thickness $t = 1.0$ mm
Table 4. Values of the first (lowest) eigenfrequency \( f_1 \), Hz for the arch profile C-110 with the thickness \( t = 1.2 \) mm

| Arch rise \( f_0 \), m | Span \( L_0 \), m | First (lowest) eigenfrequency \( f_1 \), Hz | Limiting eigenfrequency \( f_{lim} \), Hz |
|------------------------|----------------|---------------------------------|-------------------|
|                        | 11             | 12           | 15           | 16           | 18           | 24           | 30           |
| 4                      | 2.358*         | 2.05*       | 1.72        | 1.42        | 1.2      | 0.98        | 0.85        | 1.82        |
| 5                      | 2.28*          | 1.984*      | 1.64        | 1.36        | 1.12    | 0.9         | 0.78        | 1.90        |
| 6.5                    | -              | -           | 1.386       | 1.129       | 0.932   | 0.78        | 0.67        | 2.00        |
| 7.5                    | -              | -           | 0.89        | 0.82        | 0.74    | 0.65        | 0.62        | 2.06        |
| 9                      | -              | -           | -           | -           | -       | 0.607       | 0.583       | 0.55        | 2.14        |
| 12                     | -              | -           | -           | -           | -       | 0.561       | 0.52        | 2.26        |
| 15                     | -              | -           | -           | -           | -       | 0.5         | 0.5         | 2.37        |

*value of the eigenfrequency, which is less than the corresponding limiting frequency

Figure 8. First (lowest) eigenfrequency at the arch rise \( f_0 = 4 \) m and \( f_0 = 5 \) m for the arch profile C-110 with the thickness \( t = 1.2 \) mm

Table 5. Values of the first (lowest) eigenfrequency \( f_1 \), Hz for the arch profile C-110 with the thickness \( t = 1.5 \) mm

| Arch rise \( f_0 \), m | Span \( L_0 \), m | First (lowest) eigenfrequency \( f_1 \), Hz | Limiting eigenfrequency \( f_{lim} \), Hz |
|------------------------|----------------|---------------------------------|-------------------|
|                        | 11             | 12           | 15           | 16           | 18           | 24           | 30           |
| 4                      | 2.571*         | 2.23*       | 1.86*        | 1.5          | 1.253       | 1.03        | 0.9          | 1.82        |
| 5                      | 2.48*          | 2.155*      | 1.76         | 1.41         | 1.15        | 0.95        | 0.83         | 1.90        |
| 6.5                    | -              | -           | 1.336        | 1.231        | 0.932      | 0.78        | 0.76         | 2.00        |
| 7.5                    | -              | -           | 0.968        | 0.84         | 0.78       | 0.7         | 0.68         | 2.06        |
| 9                      | -              | -           | -            | -            | 0.76       | 0.67        | 0.66         | 2.14        |
| 12                     | -              | -           | -            | -            | -          | 0.612       | 0.612        | 2.26        |
| 15                     | -              | -           | -            | -            | -          | -            | 0.54         | 2.37        |

*value of the eigenfrequency, which is less than the corresponding limiting frequency
Figure 9. First (lowest) eigenfrequency at the arch rise $f_0 = 4$ m and $f_0 = 5$ m for the arch profile C-110 with the thickness $t = 1.5$ mm.

The graphs (Figure 7 – Figure 9) demonstrate that the eigenfrequency decreases, when:
1. The span $L_0$ increases;
2. The arch rise $f_0$ increases;
3. The cross-sectional profile thickness $t$ rises.

Figure 10 illustrates the comparison of the dependence graphs of the eigenfrequency on the arch span with the constant arch rise $f_0 = 4$ m with the diverse cross-sectional profile thicknesses. Based on the obtained results the lowest eigenfrequency is provided by the smallest profile thickness.

Figure 10. First (lowest) eigenfrequency at the arch rise $f_0 = 4$ m for the arch profile C-110 with the thickness $t = 1.0; 1.2; 1.5$ mm.

6. Conclusions
The key question studied in the current paper – definition, and comparison of the structural eigenfrequencies under the influence of different parameters of the frameless arch structure.

As a result of the study the following linear dependencies were stated:
1. With the enlargement of the arch rise and span the natural eigenfrequency of the system reduces;
2. With decreasing the thickness of the cross-section the value of the natural eigenfrequency gets lower.

According to p.11.1.8 of the Set of Rules SP 20.13330.2016 (Loads and Impacts), it is necessary to conduct the calculation of the frameless arch structures considering the dynamic contributions, as $f_{lim}$
\[ f_1 < f_2, \quad f_{\text{lim}} = (1.82\ldots2.37) \text{ Hz.} \]

For the frameless arch structures with \( L_0 > 12 \text{ m} \) and \( f_0 > 1/3 \ L_0 \) eigenfrequencies need to be determined. Low eigenfrequency can coincide with the excitation frequencies, resulting in intensive oscillations related to resonance, which leads to the structural collapse.

It is proved that with the reduction of the profile cross-sectional thickness and also with increasing the arch rise and span natural eigenfrequencies of the system decreases. It is proved that for the determination of the bearing capacity of the frameless arch structures with \( L_0 > 12 \text{ m} \) and \( f_0 > 1/3 \ L_0 \) it is crucial to determine the natural eigenfrequencies of the structure.

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