Tunable ground states in helical $p$-wave Josephson junctions

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Abstract
We study new types of Josephson junctions composed of helical $p$-wave superconductors with $k_x \pm k_y \hat{x}$ and $k_x \pm i k_y \hat{y}$-pairing symmetries using quasi-classical Green’s functions with generalized Riccati parametrization. The junctions can host rich ground states: $\pi$ phase, $0 + \pi$ phase, $j_0$ phase and $j$ phase. The phase transition can be tuned by rotating the magnetization in the ferromagnetic interface. We present the phase diagrams in the parameter space formed by the orientation of the magnetization or by the magnitude of the interfacial potentials. The selection rules for the lowest order current which are responsible for the formation of the rich phases are summarized from the current-phase relations based on the numerical calculation. We construct a Ginzburg–Landau type of free energy for the junctions with $d$-vectors and the magnetization, which not only reveals the interaction forms of spin-triplet superconductivity and ferromagnetism, but can also directly lead to the selection rules. In addition, the energies of the Andreev bound states and the novel symmetries in the current-phase relations are also investigated. Our results are helpful both in the prediction of novel Josephson phases and in the design of quantum circuits.

Keywords: Josephson junctions, helical superconductivity, ferromagnetism

(Some figures may appear in colour only in the online journal)

1. Introduction
Josephson junctions have been the subject of continuously growing interest because of rich ground states in these systems and their potential applications in superconducting electronics [1–5]. The ground states can be classified into 0 phase, $\pi$ phase, $\varphi_0$ phase and $\varphi$ phase according to the number and the position of the energy minimum within $2\pi$ intervals of the superconducting phase $\phi$ across the junctions. The junctions in the 0 phase and $\pi$ phase, which have been realized experimentally [6–9], have an energy minimum at $\phi = 0$ and $\phi = \pi$ [10, 11], respectively, while the $\varphi_0$ phases have a single energy minimum at $\phi = \varphi_0 = 0, \pi$ as predicted in [12]. The $\varphi$-phase, unlike the other phases, is a doubly degenerate state which possesses two energy minima at $\phi = \pm \varphi$ [13–19]. Several schemes to realize the $\varphi_0$ phase and the $\varphi$ phase have been proposed [20–22]. For example, it is expected that the latter...
phase can be realized in periodic alternating 0 and π junction structures [23]; recently the evidence of this phase has been found experimentally [24]. In fact, Josephson junctions can also host a mixture of the states, such as ϕ₀ ± ϕ phases proposed by Goldobin et al [21] more recently.

The formation of rich phases in Josephson junctions is based on the current-phase relations (CPRs). Generally, the Josephson current can be expressed as the composition of the harmonics sin nϕ and cos nϕ, in which the integer number n denotes the nth-order contribution. It is demonstrated that the lowest order current (LOC) with n = 1, sin ϕ or cos ϕ, is absent in spin-singlet superconductor|spin-triplet superconductor junctions due to the orthogonality of the Cooper-pair wave functions [25]. However, the situation will change, as predicted in [25], when the interface is magnetically active. For example, the interface which is a ferromagnetic barrier or of spin–orbit coupling can lead to a Josephson current proportional to cos ϕ when the triplet superconductor is in the chiral p-wave state [26, 27]. Furthermore, the dependence of CPRs on the magnetization in the barrier can bring different phases in spin-triplet Josephson junctions with p-wave pairing. The 0−π transition has been found when the superconductor is characterized by the d-vector with a uniform direction [28, 29]. Nevertheless, since the direction of the d-vector is independent of wavevectors, more phases cannot be expected in the junctions, although the interplay between ferromagnetism and triplet superconductivity can give many interesting and important physical results [30–32].

In this paper, we propose a concise scheme to realize rich ground states in Josephson junctions consisting of helical p-wave superconductors (HPSs) with pairing symmetries \( k_x \pm k_y \hat{y} \) and \( k_x \pm k_y \hat{x} \) and a ferromagnet (F). We are interested in these helical superconducting states for many reasons. The states, with d-vectors pinned in the crystallographic ab-plane, are candidates for pairing in Sr₂RuO₄ [33–35] and the triplet part of the order parameter in the non-centrosymmetric superconductor CePt₃Si [34, 36]. Further, \( k_x \hat{x} + k_y \hat{y} \) is the two-dimensional analog of the Balian–Werthamer state (B phase) in \(^3\)He [33, 34]; \( k_x \hat{x} - k_y \hat{y} \) is analogous to the quantum spin Hall system [37]. Recently, new symmetries of charge conductance in F|HPS junctions [38] and peculiar features of spin accumulation in spin-singlet superconductor|HPS junctions [39] were found. The selection rules for LOC in spin-singlet superconductor|HPS junctions are also summarized [40] and are distinct from those in the junctions involving a triplet superconductor described by a uniform d-vector [41]. As a result, it is reasonable to expect anomalous Josephson effects in the helical p-wave Josephson junctions. How CPRs depend on the orientation of the magnetization and which phases the junctions can host are questions which remain to be answered.

In the present work, we systematically study CPRs and ground states of HPS|HPS junctions using the method of quasi-classical Green’s functions with generalized Riccati parametrization [42, 43]. In order to conveniently describe the anisotropic superconductor in the junctions, we show explicitly the diagrammatic representation of the boundary conditions for this method. Through numerical calculations, we find the junctions can host the 0 phase, 0 + π phase, π phase, ϕ₀ phase and ϕ phase, where the 0 + π phase is a new ground state in which the free energy has two minima at ϕ = 0 and ϕ = π. The transition from one phase to another can be realized through controlling the direction of the magnetization with a weak external field. The phase diagrams are presented in the orientation space of the magnetization or in the space spanned by the magnitude of the magnetization and the non-magnetic potential. The selection rules for LOC are derived from CPRs which are responsible for the formation of rich phases. In order to explain the rules, we construct a Ginzburg–Landau type of free energy of the junctions with d-vectors in HPSs and the magnetization in F, which reveals the interaction mechanism between the helical p-wave superconductivity and ferromagnetism. We also clarify the Andreev bound states (ABS) formed at the interface and the novel symmetries in CPRs.

The paper is organized as follows. In section 2, we establish the theoretical framework which will be used to obtain the results. In section 3, we present the detailed numerical results for the junctions. The features of CPRs and phase diagrams are also covered. In section 4, we further discuss the selection rules for LOC from the viewpoint of free energy. Section 5 concludes the work.

**2. Quasi-classical Green’s function formalism**

We consider the Josephson junctions in the clean limit as shown in figure 1. The barrier, located at \( x = 0 \), with its interface along the \( y \)-axis, is modeled by a delta function \( U(x) = (U_0 + \mathbf{M} \cdot \hat{\mathbf{r}}) \delta(x) \) in which \( U_0 \) and \( \mathbf{M} \cdot \hat{\mathbf{r}} \) denote the non-magnetic potential and the ferromagnetic term, respectively. The magnetization \( \mathbf{M} = M \sin \theta_m \cos \phi_m \sin \theta_m \sin \phi_m \cos \theta_m \) where \( \theta_m \) is the polar angle and \( \phi_m \) is the azimuthal angle.

![Figure 1](image-url)
which span the orientation space of the magnetization functions. For the superconductors, we consider the following helical states,
\[
\begin{align*}
\mathbf{d}_1 &= \Delta_0(k_x \hat{x} + k_y \hat{y}), \\
\mathbf{d}_2 &= \Delta_0(k_x \hat{x} - k_y \hat{y}), \\
\mathbf{d}_3 &= \Delta_0(k_x \hat{x} + k_y \hat{y}), \\
\mathbf{d}_4 &= \Delta_0(k_x \hat{x} - k_y \hat{y}),
\end{align*}
\]
with \(\Delta_0\) the temperature-dependent gap magnitude which is determined by the Bardeen–Cooper–Schrieffer-type equation. For simplicity, we use \(HPS\) to denote the helical \(p\)-wave superconductor with the \(d\)-vector.

The HPS can be described by the quasi-classical Green’s function \(g\), a \(2 \times 2\) matrix in Keldysh space, which is solution of the Eilenberger equation with the normalization condition \(\mathbf{g} \otimes \mathbf{g} = -\pi^2 \mathbf{1}\). For the physical quantities involved in this paper, it is sufficient to obtain the retarded Green’s function \(g_R\) which is the upper-left element of \(g\). The retarded Green’s function \(g_R\), a \(4 \times 4\) matrix in spin\(\otimes\)particle-hole space, can be written as [42]
\[
\tilde{g}^R = -2\pi i \begin{pmatrix} g & f \\ -f & -\tilde{g} \end{pmatrix} + i\pi \tilde{\gamma}_n, 
\]
with the parametrization
\[
\begin{align*}
g &= (1 - \gamma \bar{\gamma})^{-1}, \\
\tilde{g} &= (1 - \bar{\gamma} \gamma)^{-1}, \\
f &= (1 - \gamma \bar{\gamma})^{-1} \tilde{\gamma} \\
f &= (1 - \bar{\gamma} \gamma)^{-1} \gamma, 
\end{align*}
\]
in which \(\gamma\) and \(\bar{\gamma}\) are the retarded coherence functions. Physically, \(\gamma\) (\(\bar{\gamma}\)) describes the probability amplitude for conversion of a hole (particle) to a particle (hole). The coherence functions, \(2 \times 2\) matrices in spin space, are a generalization of the so-called Riccati amplitudes. For simplicity, we have omitted the superscript ‘\(R\)’ for the retarded functions \(g, f, \tilde{g}, \tilde{f}\).

The coherence functions obey the Riccati-type transport equations
\[
\begin{align*}
(i\hbar \mathbf{v}_f \cdot \nabla + 2e\mathcal{E}) \gamma &= \gamma \Delta \gamma - \Delta, \\
(i\hbar \mathbf{v}_f \cdot \nabla - 2e\mathcal{E}) \bar{\gamma} &= \bar{\gamma} \Delta \bar{\gamma} - \bar{\Delta},
\end{align*}
\]
with boundary (initial) conditions, which are numerically stable. Here, \(v_f\) is the Fermi velocity, \(\mathcal{E}\) the quasi-particle energy measured from the Fermi energy, and \(\Delta\) the energy-gap matrix with the relation \(\Delta(k) = [\Delta(-k)]^T\). As in [42], we use \(\gamma, \bar{\gamma}, \Gamma, \hat{\Gamma}\) in the following to denote the incoming and outgoing quantities, respectively. The quasi-classical Green’s function characterized by the Fermi momentum \(\mathbf{p}_f\) is composed of both incoming and outgoing quantities. The solutions for \(\gamma_1, \bar{\gamma}_1\) and \(\gamma_2, \bar{\gamma}_2\) in the left (subscript 1) and the right (subscript 2) superconductor are stable when integrating the equations from the bulk to the interface; the initial conditions are their bulk values in the superconductor (see appendix A). The solutions for \(\Gamma_1, \hat{\Gamma}_1\) and \(\Gamma_2, \hat{\Gamma}_2\) are stable when integrating the equations from the interface to the bulk; the initial conditions are their values at the interface which can be expressed by the incoming quantities and the scattering matrix \(\hat{S}\) in the normal state. For example, \(\hat{\Gamma}_1\) can be written as
\[
\hat{\Gamma}_1 = \gamma_{11} + \gamma_{12} (1 - \gamma_2 \bar{\gamma}_2)^{-1} \gamma_2 \bar{\gamma}_2,
\]
Figure 2. Diagrammatic symbols of \(\gamma, \bar{\gamma}, S\) and \(\hat{S}\). \(\gamma\) describes the conversion of a hole (blue dashed line) to a particle (orange solid line); \(\bar{\gamma}\) describes the conversion of a particle to a hole. \(S(\hat{S})\) denotes the scattering of a particle (hole). Note, the arrow represents the momentum direction of a particle and the opposite direction of the momentum of a hole.

where the scattering processes are contained in \(\gamma_{\alpha\beta}\) with \(\alpha, \beta = 1, 2\).

The scattering matrix \(\hat{S}\) is diagonal in the particle-hole space, i.e. \(\hat{S} = \text{diag}(\hat{S}, \hat{S})\) with
\[
\hat{S} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}, \quad \hat{S} = \begin{pmatrix} \tilde{S}_{11} & \tilde{S}_{12} \\ \tilde{S}_{21} & \tilde{S}_{22} \end{pmatrix}
\]
where the parameters are given in \(\gamma_{\alpha\beta}\) with \(\alpha, \beta = 1, 2\).

For anisotropic superconductors, the pair potential and hence the bulk solutions of \(\gamma_{12}\) and \(\gamma_{12}\) are also dependent on the direction of the momentum of the quasi-particles. In order to show the scattering processes at the interface clearly and to write the momentum-dependent quantities conveniently and correctly, it is necessary to explicitly give the diagrammatic representation of \(\gamma_{\alpha\beta}\) and \(\bar{\gamma}_{\alpha\beta}\), in which the directions of the momenta contained in the coherence functions and the scattering matrices are specific. We adopt the diagrammatic symbols for \(S, \hat{S}, \gamma\) and \(\bar{\gamma}\) defined in [42] as shown in figure 2. The diagrams for \(\gamma_{\alpha\beta}\) are given in figure 3. For simplicity, we do not show the diagrams for \(\gamma_{\alpha\beta}\) which can be given in a similar way. Along the reverse direction of the arrow, we can write the expressions of \(\bar{\gamma}_{\alpha\beta}\) as
\[
\begin{align*}
\bar{\gamma}_{11} &= \tilde{S}_{11} \gamma_1 S_{11} + \tilde{S}_{12} \gamma_2 S_{21}, \\
\bar{\gamma}_{12} &= \tilde{S}_{11} \gamma_1 S_{12} + \tilde{S}_{12} \gamma_2 S_{22}, \\
\bar{\gamma}_{21} &= \tilde{S}_{21} \gamma_1 S_{11} + \tilde{S}_{22} \gamma_2 S_{21}, \\
\bar{\gamma}_{22} &= \tilde{S}_{21} \gamma_1 S_{12} + \tilde{S}_{22} \gamma_2 S_{22}.
\end{align*}
\]

Ignoring proximity effect, the retarded Green’s function \(\tilde{g}^R\) in the left-hand superconductor can be obtained by
substituting $\tilde{g}$ into equation (3). The Josephson current density can be found from

$$\rho_{\text{eff}} = \frac{\hbar}{\pi} \gamma g_{\text{eff}} f_{\text{FS}}$$

with $f_{\text{FS}}$ the density of states at the Fermi level in the normal state; the Fermi surface average is only over positive directions. The Matsubara frequency

$$\omega_n = \frac{\pi kT}{\theta_D / 2}$$

with $\theta_D$ the angle between the normal to the interface and the momentum of the incident particle. The dimensionless Josephson current denoted by $I_J$ can be expressed as

$$I_J = \frac{8\pi k^2 \gamma^2 \sin \phi}{Z^2(1 + \gamma^2)^2 + k^2 \gamma^2 (1 + \gamma^4) - 2k^2 \gamma^2 \cos \phi}.$$  

3. Results and discussion

3.1. HP$_1$S|HP$_2$S junction

In our calculations, the temperature is taken as $T = 0.3T_C$. Firstly, we consider the CPRs for $X = 0$. There is no magnetic potential in the interfacial barrier. The HP$_1$S|HP$_2$S junction degenerates into the HP$_1$S|HP$_2$S one. The effective expression of $\gamma w_n, \theta$ in this case can be written as

$$j(w_n, \theta) = \frac{8\pi k^2 \gamma^2 \sin \phi}{Z^2(1 + \gamma^2)^2 + k^2 \gamma^2 (1 + \gamma^4) - 2k^2 \gamma^2 \cos \phi}.$$  

with $\gamma$ defined in appendix A, which gives the sinusoidal form of the CPRs as shown in figure 4(a) with $Z = 0, 1$ and 5. When writing the effective expression of $j(w_n, \theta)$, we used the relation $\gamma w_n = 1/\gamma (-w_n)$, with $\gamma^*$ the complex conjugate of $\gamma$, and canceled the terms which have no contribution to the current density $J$. The critical current for the tunneling limit with $Z = 5$ is larger than that of the transparent limit where $Z = 0$. The dependence of the critical current on the barrier height is different from that of the $s$-wave Josephson junction which possesses the sinusoidal CPR, and the suppressed critical current with increasing $Z$ [10]. For the $s$-wave situation, the energies of the ABS are $E = \pm \Delta_0 \sqrt{1 - D \sin^2 \phi / 2}$ with $D$ the transmission coefficient, which applies to the point contact or short junction [44]. The zero-energy level appears when $D = 1$ for the transparent limit and will disappear where $D < 1$. However, this is not the case for the HP$_1$S|HP$_2$S junction, as shown in figure 4(b). When $\theta = 0$, the zero-energy level always exists irrespective of the barrier height. The energies of ABS can be expressed as

$$E_{\text{ABS}} = \frac{8\pi k^2 \gamma^2 \sin \phi}{Z^2(1 + \gamma^2)^2 + k^2 \gamma^2 (1 + \gamma^4) - 2k^2 \gamma^2 \cos \phi}.$$  

Figure 3. The scattering processes involved in $\gamma_{ij}$ which conserve the momentum component parallel to the interface. $\gamma_{11/21}$ gives two processes where an incident particle from the left-hand (right-hand) superconductor is converted into a hole moving into the same superconductor. $\gamma_{12/22}$ gives two processes where an incident particle from the right-hand (left-hand) superconductor is converted into a hole moving into the superconductor on the opposite side.

Figure 4. (a) The CPRs of the HP$_1$S|HP$_2$S junction for $X = 0$ with $Z = 0, 1$ and 5. (b) The corresponding energies of the ABS for $\theta = 0$. **With kind permission from Springer Nature Group.**
\[ E = \pm \Delta_0 \sqrt{D} \cos \phi/2 \] with \( D = \frac{1}{1 + \alpha^2} \), the transition coefficient for the normal incidence of the quasi-particles, which is simply the square of the modulus of the diagonal element of \( S_{12} \) or \( S_{21} \).

When \( X = 0 \), the CPR strongly depends on the orientation of the magnetization. Figure 5(a) gives the CPRs for \( X = 1 \) and \( Z = 0 \). We take the azimuthal angle \( \phi_m = 0 \). For \( \theta_m = 0 \) (\( \mathbf{M} \parallel \mathbf{e}_z \)), we have the \( \phi \)-dominated CPR. The free energy of the junction, given by

\[ \frac{A}{2e^2} \int_0^\infty I_j(\psi) d\psi, \]

has a minimum at \( \phi = 0 \) with no current across the junction. When the relative angle between the magnetization and the \( z \)-axis is increased, such as \( \theta_m = 0.3\pi \) or \( 0.5\pi \) (\( \mathbf{M} \parallel \mathbf{e}_z \)), the current curve crosses the horizontal line with \( I_j = 0 \) at a position in between \( \phi = 0 \) and \( \phi = \pi \). The free-energy-phase relation has two minima at \( \phi = 0 \) and \( \phi = \pi \); the junction is in the \( 0 \pm \pi \) phase. The energies of ABS for \( \theta_m = 0 \) and \( \theta_m = 0.5\pi \) are presented in figure 5(b). The presence of the \( x \)-component of the magnetization leads to the splitting of the energies.

From figure 5(a), we can find that the rotation of the magnetization can tune the HPSFHP junction between two states: the \( 0 \) phase in which the free-energy minimum is obtained at \( \phi = 0 \) and the \( 0 \pm \pi \) phase in which the free-energy minima are obtained at \( \phi = 0 \) and \( \pi \). For clarity, we show in figure 5(c) the phase diagram for the states in the orientation space of the magnetization. There are two characteristics: (a) The \( 0 \pm \pi \) phase can be realized in two circle-like zones with centers located at the points \( (\theta_m, \phi_m) = (0.5\pi, 0) \) and \( (0.5\pi, \pi) \), respectively. The ‘diameter’ of the zones is about \( 0.36\pi \) long. (b) The phase diagram is symmetric about the axes \( \theta_m = 0.5\pi \), \( \phi_m = 0.5\pi \) and \( \phi_m = \pi \), which is a reflection of the symmetries of the CPRs about the direction of the magnetization. They are \( I_j(\theta_m, \phi_m) = I_j(\pi - \theta_m, \phi_m) \) \( = I_j(\theta_m, \pi - \phi_m) = I_j(\theta_m, \pi + \phi_m) \). It is interesting to compare the CPRs with those of spin-triplet Josephson junctions characterized by \( d \)-vectors with uniform directions \[ 45 \]. There, when the \( d \)-vectors are both along the \( z \)-axis, \( I_j \) is independent of the azimuthal angle of the magnetization. As a result, the orientation space will be divided into rectangular zones by different phases.

Now, we turn to the CPRs for \( X = 0 \) and \( Z = 0 \). Figure 6(a) shows the currents with \( \phi_m = 0 \) at \( X = 1 \) and \( Z = 1 \). The CPR for \( \theta_m = 0.5\pi \), see figure 5(a), evolves into the \( \sin \phi \)-dominated line shape with a negative critical current, see figure 6(a), as the non-magnetic potential \( Z \) increases from 0 to 1. The free energy of the junction in this case has a minimum at \( \phi = \pi \); the junction is in the \( \pi \) phase. The energy of the ABS for \( \theta_m = 0.5\pi \) is given in figure 6(b). From the phase diagram in figure 6(c), we can find the zones for the \( \pi \) state are located in the ellipse-like zones for the \( 0 \pm \pi \) state. They possess the same centers: the ‘diameter’ of the \( \pi \) zones is about \( 0.36\pi \) long; the major (minor) axis of the \( 0 \pm \pi \) zones is about \( 0.54\pi \) (0.4\( \pi \)) long. If one continues to increase the values of \( X \) and \( Z \) and simultaneously keeps \( X = Z \), another new state will emerge at the upper and the lower edges of the \( 0 \pm \pi \) zones. Figure 7(a) and (b) plot the CPRs and the free energies for the edge point \( (\theta_m, \phi_m) = (0.5\pi, 0.25\pi) \) at various values of \( X \) and \( Z \). As shown in the figures, the energy minima of the new state are realized at the location in between \( \phi = 0 \) and \( \phi = \pi \) and its symmetric location in between \( \phi = \pi \) and \( \phi = 2\pi \). This new state is the so-called \( \varphi \) phase. From

![Figure 5](image-url)
Figure 6. (a) The CPRs of the HP1S∥HP2S junction for \( Z = 1, X = 1 \) and \( \phi_m = 0 \). (b) The corresponding energies of the ABS. (c) The phase diagram for 0 phase, 0 + \( \pi \) phase and \( \pi \) phase in the orientation space at \( Z = 1 \) and \( X = 1 \).

Figure 7. (a) The CPRs of the HP1S∥HP2S junction for \( Z = X \) when \( \phi_m = 0.25\pi \) and \( \theta_m = 0.5\pi \). (b) The corresponding free-energy-phase relations. (c) The phase diagram for 0 phase, 0 + \( \pi \) phase, \( \pi \) phase and \( \varphi \) phase in the orientation space at \( Z = X = 3 \).
There are two main features: (a) the sin φ-type current always exists, both for the non-magnetic interface and the magnetic case; (b) no matter how one changes the magnitude of the potentials and the direction of the magnetization, the cos φ-type current will not be obtained. The two features will be further analyzed in section 4.

3.2. HP₁S|HP₃S junction

For \( X = 0 \), the effective expression of \( j(w_m, \theta) \) can be written as

\[
j(w_m, \theta) = -4\pi k_s^2 \cos \phi \times \left[ \frac{\gamma^2}{Z^2(1 + \gamma^2)^2 + k_s^2(1 + \gamma^4) - 2k_s^2\gamma^2 \sin \phi} \right. \\
- \frac{\gamma^2}{Z^2(1 + \gamma^2)^2 + k_s^2(1 + \gamma^4) + 2k_s^2\gamma^2 \sin \phi} \right]
\]

(10)

The CPRs are shown in figure 9(a) with \( Z = 0, 1 \) and 5. In contrast to the HP₁S|HP₃S junction, there is no LOC in junction HP₁S|HP₃S. The current with the sin \( 2\phi \) form dominates the CPRs. The energies of ABS with \( \theta = 0 \) are given by \( E = \pm \Delta_0 \sqrt{(1 + \sin \phi)/2(1 + Z^2)} \) as shown in figure 9(b). One must remember that \( I_j \propto \sin 2\phi \) is the typical CPR for spin-singlet/spin-triplet superconductor junctions. The absence of LOC in these junctions originates from the orthogonality of the order parameters. For the junctions with the chiral-p-wave state in triplet superconductors [26], the energies of the ABS are given by \( E = \pm \Delta_0 \sqrt{(1 + Z^2 \pm \sqrt{(1 + Z^2)^2 - \sin^2 \phi^2)}/2(1 + Z^2)} \).

Figure 10(a) plots the CPRs for \( X = 1 \) and \( Z = 0 \). For \( \phi_m = 0 \), the variation of the polar angle \( \theta_m \) only changes the value of the critical current; the CPRs keep the sin \( 2\phi \) form. That is to say, when \( M \) is in the \( xz \)-plane, one cannot expect the presence of LOC. The situation will be changed when \( \phi_m \) deviates from 0 as given in figure 10(b) with \( \phi_m = 0.25\pi \). As \( \theta_m \) is increased from zero, the harmonic sin \( \phi \) emerges and soon dominates the CPR. The junction changes its state from the \( 0 + \pi \) phase to the \( \pi \) phase accordingly. The phase diagram in the orientation space is presented in figure 10(c) which is invariant under a reflection about \( \theta_m = 0.5\pi \) or under a \( \pi \) translation of \( \phi_m \). The invariances of the diagram are the results of symmetries of the current, i.e. \( I_j(\theta_m, \phi_m, \phi) \).
For phase to the 0 phase accordingly, as shown in figure 10(a).

The CPRs for $X = 0$ and $Z = 0$ are presented in figure 11 with $X = 1$ and $Z = 1$. It is found from figure 11(a) that for $\theta_m = 0$, LOC with the harmonic $\cos \phi$ dominates the CPR. The corresponding free energy has a single minimum at $\phi \approx 1.5\pi$, as given in figure 11(d), which indicates the junction is in the so-called $\varphi_0$ phase. As $\theta_m$ is increased, LOC is weakened and will disappear when $\theta_m = 0.5\pi$. The junction changes its state from the $\varphi_0$ phase to the $0 + \pi$ phase accordingly. For $\phi_m = 0.25\pi$ in figure 11(b), as $\theta_m$ is increased to $0.5\pi$, $I_J \propto \sin \phi$ with negative critical current will dominate the CPR. The junction changes its state from the $\varphi_0$ phase to the $\pi$ phase accordingly as shown in figure 11(e). In contrast, for $\phi_m = 0.75\pi$, $I_J \propto \sin \phi$ with positive critical current will dominate the CPR when $\theta_m$ is increased to $0.5\pi$. The junction changes its state from the $\varphi_0$ phase to the $0$ phase accordingly, as shown in figure 11(f).

For $Z = 1$ and $X = 1$, we also have symmetries of $I_J$ such as $I_J(\theta_m, \phi_m, \phi) = - I_J(\pi - \theta_m, \phi_m, 2\pi - \phi)$, $I_J(\theta_m, \phi_m, \phi) = I_J(\theta_m, \pi + \phi_m, \phi)$, and $I_J(\theta_m, n\pi/2, \phi) = I_J(\theta_m, (n + 1)\pi/2, \phi)$. From the numerical results, we find the $\varphi_0$ phase can exist in the HP$\Sigma$S S junction except for $\theta_m = 0.5\pi$. It is worth noting that the phase cannot be achieved in the HP$\Sigma$S S junction due to the absence of the $\cos \phi$-type current in their CPRs.

3.3. HP$\Sigma$S S junction

The CPRs for $X = 0$ are presented in figure 12; they also satisfy $I_J \propto \sin 2\phi$ as those in the HP$\Sigma$S S junction do. One cannot obtain LOC when the magnetic potential is absent in the interface. The effective expression of $j(w_m, \theta)$ is given by

\begin{equation}
\label{eq:11}
j(w_m, \theta) = \frac{-8\pi k^4 |\gamma|^{10} \sin 2\phi}{k^2(1 + |\gamma|^2) + Z^2[1 + |\gamma|^2]^2 - 4k^2|\gamma|^2 \sin^2 \phi}
\end{equation}

We do not show the energies of the ABS because they are the same as those for the HP$\Sigma$S S junction. Figure 13 plots the CPRs for $X = 2$ and $Z = 0$. For $\phi_m = 0$, as shown in figure 13(a), the increment in the value of $\theta_m$ only suppresses the critical current. The magnetization in the $xz$-plane will not bring about the LOC. For $\phi_m = 0.25\pi$, as shown in figure 13(b), as $\theta_m$ is increased from $0$, the $\sin \phi$-type current soon begins to dominate the CPRs. The junction changes its state from the $0 + \pi$ phase to the $0$ phase. Since we have $I_J(\theta_m, \phi_m, \phi) = -I_J(\theta_m, \pi - \phi_m, \pi - \phi)$, the harmonic $\sin\phi$ with negative critical current will dominate the CPRs for $\phi_m = 0.75\pi$ when $\theta_m$ is increased from $0$. In this case, the junction changes its state from the $0 + \pi$ phase to the $\pi$ phase. The phase diagram for 0 phase, $0 + \pi$ phase and $\pi$ phase are presented in figure 13(c). The symmetries of the diagram are the results of the relations $I_J(\theta_m, \phi_m, \phi) = I_J(\pi - \theta_m, \phi_m, \pi - \phi)$ and $I_J(\theta_m, \phi_m, \phi) = I_J(\theta_m, \pi + \phi_m, \pi + \phi)$. There are also some black lines with $\theta_m = n\pi/2$ or $\phi_m = n\pi$ in the diagram. For these values, we have $I_J \propto \sin 2\phi$ with no LOC. Figures 14(a)–(c) show the CPRs for $X = 1$ and $Z = 2$. For $\phi_m = 0$ in figure 14(a), the cos-\phi-type CPR evolves into the sin$0\phi$ form as $\theta_m$ is increased. The junction changes its state from the $\varphi_0$ phase with $\varphi_0 \approx 0.5\pi$ to the $0 + \pi$ phase accordingly as shown in figure 14(d). However, for $\phi_m = 0.25\pi$ in figure 14(b), the cos-\phi-type CPR will evolve into the sin$0\phi$ form as $\theta_m$ is increased. The junction changes its state from the $\varphi_0$ phase to the $0$ phase accordingly, as shown in figure 14(e). For $\theta_m = 0.75\pi$ in figure 14(c), the CPR will evolve into the sin$\phi$ form with a negative critical current. The junction changes its state from the $\varphi_0$ phase to the $\pi$ phase accordingly, as given in figure 14(f). For $Z = 1$ and $X = 2$, we have the symmetry relations which are $I_J(\theta_m, \phi_m, \phi) = I_J(\theta_m, \pi + \phi_m, \phi) = - I_J(\theta_m, \phi_m, \pi - \phi)$ and $I_J(\theta_m, \phi_m, \phi) = I_J(\theta_m, (n + 1)\pi/2, \phi)$. The $\varphi_0$ phase can exist in the junction except for $\theta_m = 0.5\pi$. The features of CPRs in the HP$\Sigma$S S junction are the same as those in the HP$\Sigma$S S junction which have been summarized in section 3.2. Finally, we briefly discuss...
the CPRs in other types of helical junctions. For the junctions with the symmetric geometry such as the HP$_1$S|HP$_2$S junction, we have trivial CPRs which are dominated by the harmonic $\sin \phi$. For other asymmetric junctions, their CPRs can be derived from the junctions we have considered. For example, $J_F(\phi)$ in junction HP$_2$S|HP$_3$S is identical to $J_F(\pi - \phi)$ in the HP$_1$S|HP$_2$S junction.

4. Free energy and selection rules

Now, we explain the features of the CPRs of the helical Josephson junctions through constructing the free energy of junctions. The selection rules for LOC will be obtained. Firstly, we consider the non-magnetic junctions with $X = 0$. In this case, there are two relevant vectors in each junction, i.e. $\mathbf{d}_1$ and $\mathbf{d}_3$ with $\alpha = 2, 3$ or 4. We calculate the scalar product of the vectors:

$$
\langle \mathbf{d}_1 \cdot \mathbf{d}_2 \rangle_{k_s} = \frac{1}{3} \Delta^2_0,
$$

$$
\langle \mathbf{d}_1 \cdot \mathbf{d}_3 \rangle_{k_s} = 0,
$$

$$
\langle \mathbf{d}_3 \cdot \mathbf{d}_4 \rangle_{k_s} = 0,
$$

in which $\langle \cdot \rangle_{k_s}$ denotes the average over the momentum parallel to the interface. The vanishing of the average value implies the ‘orthogonality’ of the superconducting states. As a result, LOC will be absent in the non-magnetic junctions HP$_2$S|HP$_3$S and HP$_3$S|HP$_4$S. In contrast, the harmonic $\sin \phi$ dominates the CPR in the HP$_1$S|HP$_2$S junction due to the finite average value. This indicates a contribution to the free energy, $\langle \mathbf{d}_3 \cdot \mathbf{d}_3 \rangle_{k_s} \cos \phi$, for the non-magnetic junctions. The Josephson current, as the derivative of the free energy with respect to $\phi$, is proportional to $\langle \mathbf{d}_3 \cdot \mathbf{d}_3 \rangle_{k_s} \sin \phi$. Hence, the selection rule is just the non-zero condition for the current, i.e. $\langle \mathbf{d}_3 \cdot \mathbf{d}_3 \rangle_{k_s} \neq 0$.

Secondly, we consider the magnetic case. There are three relevant vectors, i.e. $\mathbf{M}$, $\mathbf{d}_1$ and $\mathbf{d}_3$, with $\alpha = 2, 3$ or 4, in each junction. In order to include the interaction between the magnetization and the helical superconductivity, we calculate the following scalar product of the vectors,

$$
\langle \mathbf{d}_1 \cdot \mathbf{d}_2 \rangle_{k_s} = \frac{1}{4} \Delta^2_0 \left[ \frac{1}{3} (7 + \cos \theta_m) - 2 \sin^2 \theta_m \cos 2\phi_m \right],
$$

$$
\langle \mathbf{d}_1 \cdot \mathbf{d}_3 \rangle_{k_s} = -\frac{1}{2} \Delta^2_0 \sin^2 \theta_m \sin 2\phi_m,
$$

$$
\langle \mathbf{d}_3 \cdot \mathbf{d}_4 \rangle_{k_s} = \frac{1}{6} \Delta^2_0 \sin^2 \theta_m \sin 2\phi_m.
$$

in which $\langle \cdot \rangle_{k_s}$, denotes the average over the momentum parallel to the interface. The vanishing of the average value implies the ‘orthogonality’ of the superconducting states. As a result, LOC will be absent in the non-magnetic junctions HP$_2$S|HP$_3$S and HP$_3$S|HP$_4$S. In contrast, the harmonic $\sin \phi$ dominates the CPR in the HP$_1$S|HP$_2$S junction due to the finite average value. This indicates a contribution to the free energy, $\langle \mathbf{d}_3 \cdot \mathbf{d}_3 \rangle_{k_s} \cos \phi$, for the magnetic junctions. Accordingly, the Josephson current is proportional to $\langle \mathbf{d}_3 \cdot \mathbf{d}_3 \rangle_{k_s} \sin \phi$. 

Figure 10. (a) The CPRs of the HP$_1$S|HP$_2$S junction for $Z = 0, X = 1$ and $\phi_m = 0$. (b) The CPRs for $Z = 0, X = 1$ and $\phi_m = 0.25\pi$. (c) The phase diagram for 0 phase, 0 + $\pi$ phase and $\pi$ phase in the orientation space at $Z = 0$ and $X = 1$. 

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Thirdly, for the magnetic case, we can also construct another scalar quantity involving both the magnetization $M$ and two $d$-vectors. The averages of the quantity for different junctions are given by

$$\langle M \cdot (d_1 \times d_2) \rangle_{\varphi_m} = 0,$$
$$\langle M \cdot (d_1 \times d_2) \rangle_{\varphi_m} = \frac{1}{3} \Delta_0^2 \cos \theta_m,$$
$$\langle M \cdot (d_1 \times d_2) \rangle_{\varphi_m} = -\Delta_0^2 \cos \theta_m. \quad (14)$$

For the junction HP,$\tilde{\Sigma}$HP,$\tilde{\Sigma}$, the value of the average is zero for all $\theta_m$ and $\phi_m$; one cannot find the $\cos \varphi$-type current in the junction. For the junction HP,$\tilde{\Sigma}$HP,$\tilde{\Sigma}$, the vanishing of the average happens only at $\theta_m = \pi/2$; one can obtain the $\cos \varphi$-type current so long as $\theta_m$ and $\phi_m$ are chosen. This implies another contribution to the free energy, $\langle M \cdot (d_1 \times d_2) \rangle_{\varphi_m} \sin \varphi$, for the magnetic junctions. The term $\langle M \cdot (d_1 \times d_2) \rangle_{\varphi_m} \cos \varphi$ contributes to the Josephson current accordingly. The selection rules for the magnetic case are also the non-zero conditions for the current.

The complete expressions of the free energy and the Josephson current are very complicated: they are functions of temperature, the non-magnetic potential, the magnitude and the direction of magnetization and the superconducting phase $\phi_m$.

Here, we try to give qualitative explanations of the formation of various phases in helical junctions on the basis of the constructed free energy and the corresponding current. For the HP,$\tilde{\Sigma}$HP,$\tilde{\Sigma}$ junction, there is no $\cos \varphi$-type LOC. The current can be expressed as a composition of $\langle d_1 \cdot (d_2) \rangle_{\varphi_m} \sin \varphi$ and $\langle d_1 \cdot (d_2) \rangle_{\varphi_m} \sin 2\varphi$.

The second order harmonic $\sin 2\varphi$ originates from the coherent tunneling of an even number of Cooper pairs. In this case, the smaller the value of $\langle d_1 \cdot (d_2) \rangle_{\varphi_m}$, the more easily the $0 + \pi$ phase comes into being. As shown in figure 15(a), $\langle d_1 \cdot (d_2) \rangle_{\varphi_m} \sin \varphi$ obtains its minimum value at two points in the orientation space of magnetization. The orientation specified by $\theta_m, \phi_m$ in the zones around the points will lead to the formation of the $0 + \pi$ phase which corresponds to the phase diagram given in figure 5. The $\pi$ phase and the $\varphi$ phase are results of the sign reversal of the current when $X$ or $Z$ is changed. Note, the $\varphi_0$ phase does not exist in the junction due to the absence of the $\cos \varphi$-type LOC.

For the HP,$\tilde{\Sigma}$HP,$\tilde{\Sigma}$ junction with $Z = 0$, the current is the composition of $\langle d_1 \cdot (d_2) \rangle_{\varphi_m} \sin \varphi$ and $\langle d_1 \cdot (d_2) \rangle_{\varphi_m} \sin 2\varphi$. The positive (negative) value of $\langle d_1 \cdot (d_2) \rangle_{\varphi_m}$ is favorable to the formation of the $0 + \pi$ phase.
formation of the 0 (π) phase. $(\mathbf{d}_m \cdot \mathbf{d}_M)_k$ possesses two peaks with the positive maximum value and two valleys with the negative minimum value, as shown in figure 15(b). The values of $(\mathbf{d}_M \cdot \mathbf{d}_M)_k$ around the peaks and the valleys help to form the π phase and the 0 phase, respectively, which leads to the phase diagram in figure 10. The black lines in the diagram are the results of the absence of $\sin \phi$ when $(\mathbf{d}_M \cdot \mathbf{d}_M)_k = 0$. For the HP$_1$S|HP$_4$S junction with $Z = 0$, the presence of the cos $\phi$-type LOC for $\theta_m = \pi/2$ is helpful in the formation of the $\varphi_0$ phase. In this situation, the current is
the composition of $(\mathbf{M} \cdot (\mathbf{d}_L \times \mathbf{d}_R))_k\cos \phi$ and $\sin \phi$ when $\theta_m = n\pi$ or $\phi_m = n\pi/2$ with $(\mathbf{d}_L \cdot \mathbf{d}_R)_k = 0$, or the composition of $(\mathbf{M} \cdot (\mathbf{d}_L \times \mathbf{d}_R))_k\cos \phi$ and $\sin \phi$ when $\theta_m = n\pi$ and $\phi_m = n\pi/2$ with $(\mathbf{d}_L \cdot \mathbf{d}_R)_k = 0$. The former composition corresponds to the free-energy-phase relations in figure 11(d); the latter composition corresponds to the relations in figures 11(e) and (f). In figure 11, we have taken $\theta_m \leq \pi/2$ which results in the $\varphi_0$ phase with $\pi < \varphi_0 < 2\pi$. When $\theta_m > \pi/2$, $(\mathbf{M} \cdot (\mathbf{d}_L \times \mathbf{d}_R))_k$ becomes negative, the value of $\varphi_0$ will shift from $\pi < \varphi_0 < 2\pi$ to $0 < \varphi_0 < \pi$. For the HP1S-HP2S junction, the explanations are similar to those for the HP1S-HP3S junction.

5. Conclusions

In this paper, we calculate the current in the helical $p$-wave Josephson junctions using the quasi-classical Green’s function method with diagrammatic representation of the boundary conditions. Various CPRs are found in the junctions due to the interfacial potential-dependent current, which lead to rich phase diagrams. The presence of LOC plays an important role in the formation of different phases. In order to reveal the laws for the occurrence of LOC, we construct two kinds of scalar quantities with magnetization and $d$-vectors which reflect the interplay of ferromagnetism and helical superconductivity. The non-zero condition for the averages of the quantities will directly lead to the selection rules for LOC. In fact, from our analysis, we can also infer some results for the CPRs in the junctions described by $d$-vectors with uniform directions. For example, one will not find LOC in the non-magnetic junctions when two $d$-vectors are perpendicular to each other; LOC will not be found in the junctions in which one vector is proportional to $k_i$ and the other is proportional to $k_f$.

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Appendix A. Bulk solutions and the scattering matrix

The bulk values of $\gamma_1$ and $\tilde{\gamma}_1$ in the left-hand superconductor with the HP$_1$S-wave symmetry are written as

$$\gamma_1 = \begin{pmatrix} \gamma^* & 0 \\ 0 & \gamma \end{pmatrix} e^{i\theta}, \quad \tilde{\gamma}_1 = \begin{pmatrix} \gamma^* & 0 \\ 0 & \gamma \end{pmatrix} e^{-i\theta},$$

with $\gamma = \frac{i\Delta e^{i\theta}}{\omega_0 + \sqrt{\omega^2 + \Delta^2}}$.

The bulk values of $\gamma_2$ and $\tilde{\gamma}_2$ in the right-hand superconductor are given by

$$\gamma_2 = \begin{pmatrix} \gamma^* & 0 \\ 0 & \gamma \end{pmatrix}, \quad \tilde{\gamma}_2 = \begin{pmatrix} \gamma^* & 0 \\ 0 & \gamma \end{pmatrix} \text{ for HP}_2\text{S},$$

$$\gamma_2 = i \begin{pmatrix} \gamma^* & 0 \\ 0 & -\gamma \end{pmatrix}, \quad \tilde{\gamma}_2 = i \begin{pmatrix} \gamma^* & 0 \\ 0 & -\gamma \end{pmatrix} \text{ for HP}_3\text{S},$$

$$\gamma_2 = i \begin{pmatrix} \gamma^* & 0 \\ 0 & -\gamma \end{pmatrix}, \quad \tilde{\gamma}_2 = i \begin{pmatrix} \gamma^* & 0 \\ 0 & -\gamma \end{pmatrix} \text{ for HP}_3\text{S} \text{ (A.2)}.$$

When we write the expressions, we have taken the directions of wavevectors (as shown in figure 3) into account.

For the interface with the ferromagnetic potential, the explicit expressions of the scattering matrices can be given by

$$S_{11} = \begin{pmatrix} \frac{Z^2 - X^2 - ik'((Z - X)\cos\theta_m)}{X^2 + (k'_e + iZ)^2} & \frac{ik'X\sin\theta_m e^{-i\theta}}{X^2 + (k'_e + iZ)^2} \\ \frac{ik'X\sin\theta_m e^{i\theta}}{X^2 + (k'_e + iZ)^2} & \frac{Z^2 - X^2 - ik'((Z + X)\cos\theta_m)}{X^2 + (k'_e + iZ)^2} \end{pmatrix},$$

$$S_{22} = S_{11}, \quad S_{12} = S_{21} = \tilde{S}_{12} = \tilde{S}_{21} = \tilde{S}_{11} = S_{11}^*,$$

and $\tilde{S}_{12} = \tilde{S}_{21} = \tilde{S}_{12}$. 

Figure 15. (a) The normalized $(\mathbf{d}_L \cdot \mathbf{d}_R)_k$ as a function of $\theta_m$ and $\phi_m$. (b) The normalized $(\mathbf{d}_L \cdot \mathbf{d}_R)_k$ as a function of $\theta_m$ and $\phi_m$. 

\[\begin{align*}
S_{11} &= \begin{pmatrix}
\frac{Z^2 - X^2 - ik'((Z - X)\cos\theta_m)}{X^2 + (k'_e + iZ)^2} & \frac{ik'X\sin\theta_m e^{-i\theta}}{X^2 + (k'_e + iZ)^2} \\
\frac{ik'X\sin\theta_m e^{i\theta}}{X^2 + (k'_e + iZ)^2} & \frac{Z^2 - X^2 - ik'((Z + X)\cos\theta_m)}{X^2 + (k'_e + iZ)^2}
\end{pmatrix}, \\
S_{22} &= S_{11}, \quad S_{12} = S_{21} = \tilde{S}_{12} = \tilde{S}_{21} = \tilde{S}_{11} = S_{11}^*, \quad \text{and} \quad \tilde{S}_{12} = \tilde{S}_{21} = \tilde{S}_{12}.
\end{align*}\]
Appendix B. The transformation of d-vectors

The energy-gap matrix in the coordinate of spin space in F can be obtained by performing unitary transformation:

$$\Delta_m = U^\dagger \Delta U^*$$  \hspace{1cm} (B.1)

with

$$U = \begin{pmatrix}
\cos \frac{\theta_m}{2} e^{-i\omega_m/2} & -\sin \frac{\theta_m}{2} e^{-i\omega_m/2} \\
\sin \frac{\theta_m}{2} e^{i\omega_m/2} & \cos \frac{\theta_m}{2} e^{i\omega_m/2}
\end{pmatrix}$$  \hspace{1cm} (B.2)

Using the relation between the d-vector and the energy-gap matrix given by

$$\Delta = \begin{pmatrix} -d_x & id_y & d_z \\ d_z & d_x - id_y & 0 \\ 0 & 0 & d_x + id_y \end{pmatrix}$$  \hspace{1cm} (B.3)

we obtain the vectors $d_{\alpha M}$ which can be written as

$$d_{1M} = \Delta_0 [\cos \theta_m \cos(\theta - \phi_m) \hat{\chi} + \sin(\theta - \phi_m) \hat{y} + \sin \theta_m \cos(\theta - \phi_m) \hat{z}],$$

$$d_{2M} = \Delta_0 [\cos \theta_m \cos(\phi_m + \theta_m) \hat{\chi} - \sin(\phi_m + \theta_m) \hat{y} + \sin \theta_m \cos(\phi_m + \theta_m) \hat{z}],$$

$$d_{3M} = \Delta_0 [\cos \theta_m \sin(\theta + \phi_m) \hat{\chi} + \cos(\theta + \phi_m) \hat{y} + \sin \theta_m \sin(\theta + \phi_m) \hat{z}],$$

$$d_{4M} = \Delta_0 [\cos \theta_m \sin(\theta - \phi_m) \hat{\chi} - \cos(\theta - \phi_m) \hat{y} + \sin \theta_m \sin(\theta - \phi_m) \hat{z}].$$  \hspace{1cm} (B.4)

References

[1] Ioffe L B, Geshkenbein V B, Feigel'man M V, Fauchere A L and Blatter G 1999 Nature (London) 398 679
[2] Blatter G, Geshkenbein V B and Ioffe L B 2001 Phys. Rev. B 63 174511
[3] Blais A and Zagoskin A M 2000 Phys. Rev. A 61 042308
[4] Goldberg E, Sickinger H, Weides M, Ruppelt N, Kohlstedt H, Kleiner R and Koelle D 2013 Appl. Phys. Lett. 102 242602
[5] Linder J and Robinson J W A 2015 Nat. Phys. 11 307
[6] Jiang J S, Davidovic D, Reich D H and Chien C L 1995 Phys. Rev. Lett. 74 314
[7] Ryazanov V V, Oboznov V A, Rusanov A Y, Veretennikov A V, Golubov A A and Aarts J 2001 Phys. Rev. B 64 24227
[8] Robinson J W A, Piano S, Burnell G, Bell C and Blamire M G 2006 Phys. Rev. Lett. 97 177003
[9] Shelukhin V et al 2006 Phys. Rev. B 73 174506
[10] Golubov A A, Kupriyanov M Y and Il'ichev E 2004 Rev. Mod. Phys. 76 411
[11] Bulaevskii L N, Kuzii V V and Sobyanin A A 1977 Pis'ma Zh. Eksp. Teor. Fiz. 25 314
[12] Bulaevskii L N, Kuzii V V and Sobyanin A A 1977 JETP Lett. 25 290
[13] Blank A 2008 Phys. Rev. Lett. 101 107005
[14] Tanaka Y and Kashiwaya S 1996 Phys. Rev. B 53 R11957
[15] Tanaka Y and Kashiwaya S 1997 Phys. Rev. B 56 892
[16] Yip S 1995 Phys. Rev. B 52 3087
[17] Goldobin E, Koelle D, Kleiner R and Buzdin A 2007 Phys. Rev. B 76 224523
[18] Lipman A, Mints R G, Kleiner R, Koelle D and Goldobin E 2014 Phys. Rev. B 90 184502
[19] Goldobin E, Kleiner R, Koelle D and Mints R G 2013 Phys. Rev. Lett. 111 057004
[20] Alidoust M and Linder J 2013 Phys. Rev. B 87 060503
[21] Goldobin E, Koelle D and Kleiner R 2015 Phys. Rev. B 91 214511
[22] Goldobin E, Mironov S, Buzdin A, Mints R G, Koelle D and Kleiner R 2016 Phys. Rev. B 93 134511
[23] Buzdin A and Koshelev A E 2003 Phys. Rev. B 67 222050(R)
[24] Sickinger H, Lipman A, Weides M, Mints R G, Kohlstedt H, Koelle D, Kleiner R and Goldobin E 2012 Phys. Rev. Lett. 109 107002
[25] Pals J A, Van Haerden W and Van Maaren M H 1977 Phys. Rev. B 15 2592
[26] Tanaka Y and Kashiwaya S 2000 J. Phys. Soc. Jpn. 69 1152
[27] Asano Y, Tanaka Y, Sigrist M and Kashiwaya S 2003 Phys. Rev. B 67 184505
[28] Brydon P M R and Manske D 2009 Phys. Rev. Lett. 103 147001
[29] Brydon P M R, Inotakics C and Manske D 2009 New J. Phys. 11 055055
[30] Bujnowski B, Timm C and Brydon P M R 2012 J. Phys.: Condens. Matter 24 045701
[31] Brydon P M R, Kastening B, Morr D K and Manske D 2008 Phys. Rev. B 77 104504
[32] Brydon P M R, Manske D and Sigrist M 2008 J. Phys. Soc. Jpn. 77 103714
[33] Mackenzie A P and Maeno Y 2003 Rev. Mod. Phys. 75 657
[34] Maeno Y, Kittera S, Nomura T, Yonezawa S and Ishida K 2012 J. Phys. Soc. Jpn. 81 011009
[35] Zhang J, Lorschisch C, Gu Q and Kienna R A 2014 J. Phys.: Condens. Matter 26 252011
[36] Bauer E, Hilser G, Michor H, Paul C, Scheidt E W, Gribov A, Serpegin Y, Noel H, Sigrist M and Rowl P 2004 Phys. Rev. Lett. 92 027003
[37] Qi X L, Hughes T L, Raghu S and Zhang C S 2009 Phys. Rev. Lett. 102 187001
[38] Cheng Q, Jin B and Yu D Y 2015 Phys. Lett. A 379 1172
[39] Lu C K and Yip S 2009 Phys. Rev. B 80 024504
[40] Cheng Q and Jin B 2016 Europhys. Lett. 113 17007
[41] Brydon P M R, Chen W, Asano Y and Manske D 2013 Phys. Rev. B 88 054509
[42] Eschrig M 2009 Phys. Rev. B 80 134511
[43] Eschrig M 2000 Phys. Rev. B 61 9061
[44] Bagwell P F 1992 Phys. Rev. B 46 12573
[45] Brydon P M R, Inotakics C and Manske D 2009 New. J. Phys. 11 055055