Search for the $\chi'_c$ Charmonium States as Solution to the CDF $\psi'$ Puzzle.

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**Abstract**

The efforts of Roy-Sridhar-Close-Cho-Wise-Trivedi to resolve the CDF $\psi'$ anomaly with cascades from above-threshold $\chi'_c$ states require well defined signatures [a small total width and a large branching fraction for $\chi'_{cJ} \rightarrow \gamma + \psi'$] for the solution to be viable. Here we estimate the production of such states from $BR(B \rightarrow \chi'_{cJ} + X)BR(\chi'_{cJ} \rightarrow \gamma \psi')$ and $\gamma \gamma$ production of $\chi'_{c2}$ at CLEO II, and comment on the feasibility of testing the hypothesis in terms of current experimental capabilities.

**KEYWORDS.** CDF $\psi'$ anomaly; $\chi'_{cJ}$ search.

1 Introduction

The CDF measurement [1] of $J/\psi$ and $\psi'$ production at large transverse momentum ($P_T$) in 1.8 TeV $p\bar{p}$ collisions at the Tevatron has produced one of the most intriguing experimental results in recent times. At low transverse momentum the production of $\psi$’s in $p\bar{p}$ in collisions is expected to proceed via the production of $\chi$ states followed by their decay [2], i.e. fusion process $g + g \rightarrow \chi \rightarrow \psi + X$. The analogous process at larger transverse momentum is $gg \rightarrow g\psi$. This process generates a cross section that falls off at large $P_T$ much faster than, say, the jet rate. This observation led to an expectation that $\psi$’s produced at large $P_T$ came almost exclusively from b-quark decay. By detecting whether or not the $\psi$’s come from the primary event vertex, CDF has tested this expectation. The fraction of $\psi$’s produced directly
is almost independent of $P_T$ and the rate of direct $\psi$ production at large $P_T$ is (substantially) larger that had been expected. This expectation is thus now known to be false.

The dominant production mechanism of $J/\psi$'s at large tranverse momentum is now believed to be the fragmentation of light (and charm) quark and gluon jets into $\chi_c$ mesons that subsequently decay to $J/\psi$ via the radiative mode as stressed by Cho, Wise, and Trivedi where the gluon fragmentation is found to be particularly important. When this $J/\psi$ source is included, the theoretical prediction for $d\sigma(p\bar{p} \rightarrow J/\psi + X)/dP_T$ at $\sqrt{s} = 1.8$ TeV agrees within a factor of 2 with recent CDF data.

While the rate for $J/\psi$ production is in agreement with expectations, given the inherent theoretical uncertainties, the rate for $\psi'$ production at CDF is at least a factor of 20 above theoretical expectations. The calculation does not include the possibility of $\psi'$ production from the decay of $\chi_c'(2P)$ states. These $2P$ charmonium states are above $D\bar{D}$ threshold, however a branching ratio of a few $\%$ to $\psi'$ could be sufficient to explain the deficit. D.P. Roy and K. Sridhar as well as others investigated this possibility quantitatively.

The basic premise is to recall that $\psi'$ is the heaviest $c\bar{c}$ bound state which lies below the $D\bar{D}$ threshold. Therefore, $n=1$ $\chi_{cJ}$ states cannot radiatively decay to $\psi'$ but their $n=2$ counterparts can. None of these $\chi_{cJ}'$ (or $\chi_{cJ}$ $(2P)$) states which lie above the $D\bar{D}$ threshold have been observed. Estimates of their masses yield $M(\chi_{c0}(2P)) = 3920$ MeV, $M(\chi_{c1}(2P)) = 3950$ MeV, and $M(\chi_{c2}(2P)) = 3980$ MeV. These mass values taken literally would kinematically allow the S-wave transitions $\chi_{c0}(2P) \rightarrow D\bar{D}$ and $\chi_{c1}(2P) \rightarrow D\bar{D}$ to occur. We therefore expect that the $J=0$ and perhaps the $J=1$ excited $\chi_{cJ}$ $(2P)$ states will be broad and have negligible branching fractions to lower $c\bar{c}$ bound states. However, angular momentum and parity considerations require the analogous decays $\chi_{c2}(2P)$ to proceed via $L=2$ partial waves. Although we cannot readily compute by how much these D-wave decays will be suppressed, it is possible that the branching fractions for $\chi_{c2}(2P)$ states to charmonium states below $D\bar{D}$ threshold could be significant. F. Close suggests that the $\chi_{c1}(2P)$ may also have suppressed hadronic widths due to quantum numbers or nodes in form factors manifested in decays near threshold, e.g. since $^3P_1 \rightarrow D\bar{D}^*$ is near threshold the $S$ and $D$ waves present are affected by radial wave function nodes which can conspire to reduce the width. Hence in section 2 below we will consider the search for both $\chi_{c1}(2P)$ and $\chi_{c2}(2P)$. Although the $^1D_2$ and $^3D_2$ charmonium states may be present in the CDF data and detectable, they are unlikely to explain the $\psi'$ (3685) enhancement. For instance $^1D_2$ production is suppressed, and it is expected to have a very small branching ratio to $\psi'\gamma$. 
On a quantitative basis Page [6] has calculated the total widths of radial \( \chi'_{cJ} \) states and found that they could be as small as 1-5 MeV. We need an estimate of the branching ratio \( BR(\chi'_{cJ} \to \gamma + \psi') \). A very rough estimate [7] can be obtained using the known experimentally measured branching ratio \( BR(\chi_{b2}(2P) \to \Upsilon(2S) + \gamma) \approx 16\% \) as follows:

\[
BR(\chi_{c2}(2P) \to \gamma \psi') = \left( \frac{Q_c}{Q_b} \right)^2 \left( \frac{k_c}{k_b} \right)^3 \frac{BR(\chi_{b2}(2P) \to \gamma \Upsilon(2S)) \Gamma_{tot}(\chi_{b2}(2P))}{\Gamma_{tot}(\chi_{c2}(2P))} \tag{1}
\]

Here \((Q_c/Q_b)^2 = \frac{4/3}{1/3} = 4\), and \((k_c/k_b)^3\) is the modification due to \( E_1 \) phase space; there will be some changes due to the actual size of \( b\bar{b} \) relative to that of \( c\bar{c} \) but this will be a factor of 2 or 3 and the estimate (1) may not be accurate to better than a factor of 5 to 10 anyway. For instance, taking \( M(\chi_{c2} (2P)) = 3980 \text{ MeV} \) and \( \Gamma_{tot}(\chi_{c2}(2P)) \approx \Gamma_{tot}(\chi_{c2}(2P)) \) gives \( BR(\chi_{c2}(2P) \to \gamma + \psi') \approx 100\% \). Thus the value \( BR(\chi_{c2}(2P) \to \gamma + \psi') \approx 10\% \) suggested by Roy and Sridhar [3] is by no means unreasonable. Another back of the envelope ansatz [6] is to take the \( 1P \to 1S \) charmonium data and assume the \( 2P \to 2S \) overlaps will have the same order of magnitude, then \( B(\chi_{c2}(2P) \to \gamma + \psi') \approx B(\chi_{c2}(1P) \to \gamma + J/\psi) = 13.5\% \), with a measured \( \Gamma_{c2}(1P) \) full width = 2 MeV [8]. Hence if one of the \( \chi_{cJ} \) (2P) states is calculated to have a total width in the range 1 MeV to 5 MeV [3], a branching ratio \( B(\chi_{cJ}(2P) \to \gamma + \psi') > 5\% \) can be expected (a value in the range 5-10% would be needed to explain the CDF \( \psi' \) anomaly [1, 3]). To summarize, the result is that in order of magnitude one expects the radiative transition to be \( O(100 \text{ KeV}) \) and hence the BR is \( O(1-10\%) \) if the hadronic width is \( O(10-1\text{MeV}) \). It would be surprising if the branching ratio is less than 1% or much greater than 10%. To give an adequate spread for illustration, we shall take in section 2,

\[
BR(\chi_{cJ}(2P) \to \gamma + \psi(2S)) = 1, 5, 10\%. \tag{2}
\]

## 2 Search Method for \( \chi'_{cJ} \) States

The optimal method for accumulation of \( \chi'_{cJ} \) events at CLEO II is to take advantage of the inclusive decays of B mesons to Charmonium. Hence we seek to estimate

\[
B_I \equiv BR(B \to \chi'_{cJ} + X) \times B[\chi'_{cJ} \to \gamma \psi'] \tag{3}
\]

The branching ratio \( BR(B \to \chi'_{cJ} + X) \) is given by Bodwin et al. [8] as

\[
R(\chi_{cJ}) \times 10.7\% \times | R'_{\chi'_{cJ}}(0)/ R'_{\chi'_{cJ}}(0) |^2 . \tag{4}
\]
where $R(\chi_{cJ}) = \Gamma(b \to \chi_{cJ} + X)/\Gamma(b \to e^-\bar{\nu}_e + X)$, 10.7% is the observed semileptonic branching ratio for the B-meson, and multiplicative last term $| R'_{\chi_c}(0)/R'_{\chi_c}(0) |^2$ is Braaten’s correction factor [10] for estimating $BR(B \to \chi_c' + X)$ from $BR(B \to \chi_c + X)$. The derivatives of the wave functions at the origin can be obtained from potential models. This procedure is certainly correct for the color-singlet matrix element (the $c\bar{c}$ contribution) and it may also be correct for the color octet matrix element (the $c\bar{c}g$ contribution) if the latter is dominated by the radiation of soft gluons from the $c\bar{c}$ state. Whether or not this is the case remains to be seen [10].

The expression $R(\chi_{cJ})$ in (4) is given in terms of the nonperturbative parameters $H_1$ and $H'_8$ (proportional to the probabilities for a $c\bar{c}$ pair in a color-singlet P-wave and a color-octet S-wave state, respectively, to fragment into a color singlet P-wave bound state) and takes the following form [9]

\begin{equation}
R(\chi_{c1}) \approx 12.4 \left( 2C_+ - C_- \right)^2 H_1/M_b + 9.3 \left( C_+ + C_- \right)^2 H'_8(M_b)/M_b
\end{equation}

\begin{equation}
R(\chi_{c2}) \approx 15.3 \left( C_+ + C_- \right)^2 H'_8(M_b)/M_b.
\end{equation}

Here $C_+$ and $C_-$ are Wilson coefficients that arise from evolving the effective 4-quark interactions mediated by the W boson from the scale $M_W$ down to the scale $M_b$. Numerically $C_+(M_b) \approx 0.87$, $C_-(M_b) \approx 1.34$, $H_1 \approx 15$ MeV, $H'_8(M_b) \approx 2.5$ MeV, and $M_b = 5.3$ GeV.

For the value $| R'_{\chi_c}(0)/R'_{\chi_c}(0) |^2$, we use a recent quark potential model [11] which takes into account that the value of $R'(0)$ for P-state charmonium is sensitive to the short distance behavior of the potential, so that it is better to use values obtained from potentials whose short distance behavior is more reliable. The potential [11] (an improved version of the Buchmüller-Grunberg-Tye potential) approaches the 2-loop QCD result at short distance, leads to energy spectra and leptonic widths in very good agreement with experiment. The values for $R'(0)$ for the P-wave charmonium states from this potential model Program [12] are:

\begin{equation}
\begin{array}{ll}
\text{State} & R'(0) \text{ in GeV}^{5/2} \\
\chi_c & 0.20 \\
\chi_c' & 0.23
\end{array}
\end{equation}

For comparison, the value of $R'(0)$ for $\chi_c$ [10] obtained directly from the measured widths of $\chi_{c1}$ and $\chi_{c2}$ was about 0.15 GeV$^{5/2}$, hence one should ascribe an error of not less than 30% to any of these values. It is nevertheless reassuring that a recent compilation [13] of first nonvanishing derivative at zero $c\bar{c}$ separation for radial Schrödinger wave function of earlier
potential models, give for $| R'_{\chi_c}(0)/R'_{\chi_c}(0) |^2$ values 1.36 (Buchm"uller-Tye), 1.05 (Power-law), 0.97 (Logarithmic), and 1.42 (Cornell), quite compatible with

$$| R'_{\chi_c}(0)/R'_{\chi_c}(0) |^2 = 1.32$$

of the recent potential model [11].

Assembling the pieces together from (3), (4), (5), and (7), we have for (3) and J=1.

$$B_1 = 2.89 \times 10^{-5}, 1.45 \times 10^{-4}, 2.89 \times 10^{-4}$$

for the respective values of $BR[\chi'_c \to \gamma \psi']$ given in (2). The corresponding values for J=2 are

$$B_2 = 3.77 \times 10^{-5}, 1.89 \times 10^{-4}, 3.77 \times 10^{-4}$$

The world average (WA) and CLEO-II measurement [14] for $BR(B \to \chi_c + X)$ are

$$BR(B \to \chi_{c1} + X) = 0.42 \pm 0.07\% \ (WA)$$
$$BR(B \to \chi_{c2} + X) = 0.25 \pm 0.10\% \ (CLEO)$$

For central values of (10), multiplying the above experimental numbers by correction factor (7), we have

$$BR(B \to \chi'_{c1} + X) = 5.54 \times 10^{-3}$$
$$BR(B \to \chi'_{c2} + X) = 3.30 \times 10^{-3}$$

This compares with the values obtained by scaling the predictions of Bodwin et al. [9] for $BR(B \to \chi_cJ + X)$ using the correction factor (7) of

$$BR(B \to \chi'_{c1} + X) = 2.89 \times 10^{-3}$$
$$BR(B \to \chi'_{c2} + X) = 3.77 \times 10^{-3}.$$  

The agreement seems good for the $\chi'_{c2}$ case.

The final step is to estimate the number of observed events, $N_{JcJ}^{obs}$. First we note that at CLEO-II, the number of produced B-mesons is given by

$$N_B^{\text{produced}} = (\int L dt) \times \sigma(e^+e^- \to Y(4S) \to B\bar{B}) \times 2$$
where on a good year the integrated luminosity \( \int \mathcal{L} \, dt \) is 2 \( fb^{-1} \) of data on tape, the \( \sigma(e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}) \) cross section is about 1.07 nb, and the factor of 2 takes into account productions of pairs of B mesons in \( \Upsilon(4S) \) decays. The number of observed events \( N_{J}^{\text{obs}} \) is then given by

\[
N_{J}^{\text{obs}} = N_B^{\text{produced}} \times B_J \times B(\psi' \rightarrow J/\psi \pi^+\pi^-) \times \left( \sum_{\ell=e,\mu} B(J/\psi \rightarrow \ell^+\ell^-) \right) \times \epsilon \quad (14)
\]

where \( B_J \) is defined in (3), and \( \epsilon \) is the efficiency for detecting \( \psi' \rightarrow J/\psi \pi^+\pi^- \) in the dilepton mode (about 20% in the CLEO-II detector \([14]\)). Hence using theory (8), (9), or (12), we have

\[
N_{1}^{\text{obs}} = 0.95, 4.77, 9.51 \quad \text{events}
\]
\[
N_{2}^{\text{obs}} = 1.24, 6.22, 12.40 \quad \text{events}
\]

for the respective values of \( BR(\chi'_{cJ} \rightarrow \gamma\psi') \) given in (2). If we take advantage of the experimentally known branching ratios (10) in deducing (11), we have

\[
N_{1}^{\text{obs}} = 1.82, 9.11, 18.23 \quad \text{events}
\]
\[
N_{2}^{\text{obs}} = 1.09, 5.43, 10.86 \quad \text{events}
\]

for the respective values of \( BR(\chi'_{cJ} \rightarrow \gamma\psi') \). The theoretical mass estimates \([5]\) \( M(\chi_{c1}(2P)) = 3950 \text{ MeV} \) and \( M(\chi_{c2}(2P)) = 3980 \text{ MeV} \) are also useful. The approximate locations of these resonances are needed to conduct the experimental search and reduce background.

At CLEO II, the \( \chi'_{c2} \) state can also be searched for via the two photon production of this \( J=2 \) state. For instance, the number of events \( N_{\chi'_{c2}} \) from \( e^+e^- \rightarrow e^+e^-\gamma\gamma \rightarrow e^+e^-\chi'_{c2} \) is estimated to be

\[
N_{\chi'_{c2}} = N_{\chi_{c2}} \times \frac{\sigma(\gamma\gamma \rightarrow \chi'_{c2})}{\sigma(\gamma\gamma \rightarrow \chi_{c2})} \times \frac{BR(\chi'_{c2} \rightarrow \psi'\gamma)}{BR(\chi_{c2} \rightarrow J/\psi\pi^+\pi^-)} \times \frac{BR(\psi' \rightarrow J/\psi \pi^+\pi^-) \times BR(J/\psi \rightarrow e^+e^-)}{BR(J/\psi \rightarrow \ell^+\ell^-)} \quad (17)
\]

In a recent paper on measurement of two-photon production of the \( \chi_{c2} \), J. Dominick et al. \([15]\), using 1.5 \( fb^{-1} \) of data taken with beam energies near the \( \Upsilon(4S) \), 25.4 ± 6.9 \( N_{\chi_{c2}} \) events were obtained, with efficiency \( \epsilon(\ell^+\ell^-) = 0.187 \pm 0.003 \). Taking into account that data accumulation is now 2 \( fb^{-1} \) on \( \Upsilon(4S) \) and 1 \( fb^{-1} \) in the continuum, the number \( N_{\chi_{c2}} \) can be doubled and we assume that \( \epsilon(\ell^+\ell^-\pi^+\pi^-) \cong \epsilon(\ell^+\ell^-)/2 \). For \( BR(\chi'_{c2} \rightarrow \psi'\gamma) = 10\% \) and taking central values for experimental numbers to illustrate, we have

\[
N_{\chi'_{c2}} \cong 6 \times \frac{\sigma(\gamma\gamma \rightarrow \chi'_{c2})}{\sigma(\gamma\gamma \rightarrow \chi_{c2})} \quad \text{events}. \quad (18)
\]
The cross section ratio is equal to the $BR(\chi'_{c2} \rightarrow \gamma\gamma)/BR(\chi_{c2} \rightarrow \gamma\gamma)$ and is not yet known since the total width $\Gamma(\chi'_{c2})$ has not yet been measured.

3 Conclusions

We have presented above in section 2 event rates for observing the $\chi'_{cJ}(J = 1, 2)$ in B decays at CLEO-II. Though the event rates $N^j_{\text{obs}}$ given by (15) and (16) are not large even with a year’s accumulation of $B\bar{B}$, they can be steadily increased by extending the $B\bar{B}$ accumulation over a period of several years. Furthermore we have been surprised by how large the branching ratios $BR(\chi'_{cJ} \rightarrow \gamma\psi')$ can be in section 1, given the known and sizable $BR(\chi_{b1} \rightarrow \gamma + \Upsilon(2S)) \approx (21 \pm 4\%)$ and $BR(\chi_{b2} \rightarrow \gamma + \Upsilon(2S)) \approx (16.2 \pm 2.4\%)$. Hence optimistically we can expect $N^j_{\text{obs}}$ to be in the range of 10-20 events for an integrated luminosity of $2fb^{-1}$, as needed to explain the CDF $\psi'$ anomaly [1]. Estimates for $2\gamma$ production of $\chi'_{c2}$ are given in (17) and (18). At CDF the invariant mass spectrum of $\gamma\psi'$ combination can be studied. This possibility should be explored in parallel with the CLEO effort. I wish to thank D.P. Roy for encouragement, my colleagues X. Tata and T. Browder for their comments and reading of the manuscript, and E. Braaten, F. E. Close, Y. P. Kuang, and P. Page for very helpful communications. This work was supported in part by the US Department of Energy under Grant DE-FG-03-94ER40833.

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