Simulation Analysis of Helicopter Ground Resonance
Nonlinear Dynamics

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Abstract. In order to accurately predict the dynamic instability of helicopter ground resonance, a modeling and simulation method of helicopter ground resonance considering nonlinear dynamic characteristics of components (rotor lead-lag damper, landing gear wheel and absorber) is presented. The numerical integral method is used to calculate the transient responses of the body and rotor, simulating some disturbance. To obtain quantitative instabilities, Fast Fourier Transform (FFT) is conducted to estimate the modal frequencies, and the mobile rectangular window method is employed in the predictions of the modal damping in terms of the response time history. Simulation results show that ground resonance simulation test can exactly lead up the blade lead-lag regressing mode frequency, and the modal damping obtained according to attenuation curves are close to the test results. The simulation test results are in accordance with the actual accident situation, and prove the correctness of the simulation method. This analysis method used for ground resonance simulation test can give out the results according with real helicopter engineering tests.

1. Introduction
The aeromechanical instability of a helicopter on the ground, commonly referred to as ground resonance, is a complex phenomenon involving both the rotor and body degrees of freedom in which the rotor lead-lag regressing mode which includes substantial body pitch and roll motion may become unstable. The dynamics problem of ground resonance must be considered in the development of helicopter is more important for a new model. For the analysis method of helicopter ground resonance, the linearization hypothesis method based on balance position of small disturbance is often used in engineering practice. It is assuming that the dynamic characteristics of landing gear system and rotor dampers are linear. In a range of small vibration amplitude, from microscopic side, the results will be correct when the actual vibration amplitude is not beyond the range, sometimes the assumption can lead to some incorrect results when beyond the range. In fact, the dynamic characteristics of the landing gear system including absorber with wheel and the rotor dampers are nonlinear, especially the stiffness and damping of landing gear are high nonlinear characteristics and very sensitive to environmental. The linearization will cause remarkable error to the stability of ground resonance at the taking off and landing conditions with large amplitude movement.

Since the 1950s, the stability problem of helicopter ground resonance had been studied in the domestic and abroad, the studies conducted give an insight into the mechanism of coupling, the influence of design parameters, and the test verification, as described in references [1,2], and obtained a lot of research achievements. A nonlinear analysis model of rotor and fuselage coupled had been established in references [3–6], the nonlinear damping characteristics of rotor dampers effect on the stability of rotor and fuselage coupled was evaluated. The use of a wheeled landing gear simulation
model, as described in reference [7], is one of considerable value in obtaining an understanding of the gear movement characteristics and the influence of gear design on ground resonance. Above studies only focus on the influence study of single component to ground resonance. The analysis method study of helicopter ground resonance considering the nonlinear dynamic characteristics of the landing gear wheel and absorber and rotor dampers has not been carried out.

The nonlinear dynamic characteristics of the landing gear wheel and absorber and rotor dampers were analyzed, referenced the modeling method of rotor dampers and landing gear components to the literatures [8–10], the calculation model of ground resonance considering the nonlinear dynamic characteristics of components was established, in this paper. The transient responses of the body and rotor blade were simulated under some disturbance using the numerical integral method. The frequency and damping of dynamic systems were obtained by the mobile rectangular window method. The nonlinear numerical simulation of ground resonance was carried out, and the stability margin or stability boundary of ground resonance was provided.

2. Nonlinear analytical model

2.1. Landing gear wheel nonlinear dynamics model

The nonlinear relationship of the static stiffness and displacement at the station of vertical compression can be expressed as:

\[ K(s_{z0}, \Delta s_x) = k_{w} + \frac{2k_{w} s_{z0}^{2}}{w} + k_{w} \frac{\Delta s_x}{w} + \frac{3k_{w} s_{z0}^{2}}{w} + \frac{3k_{w} s_{z0} \Delta s_x}{w} + k_{w} \frac{\Delta s_x^{2}}{w} | (1.08 p w \sqrt{d w}) \] (1)

The stiffness is the quadratic polynomial of compression displacement and the cubic polynomial of width w. The tire diameter and inflated-gas pressure are constant factors. When the wheel under static load compression, the static displacement is \( S_{z0} \).

Use tire static compression characteristic test data for fitting, the coefficients \( k_{aw}, k_{aw}, k_{aw}, k_{aw} \) in above equations can be confirmed. When calculating the dynamic stiffness, the stiffness formula should be multiplied by a tire inflation pressure factor \( c \), then, dynamic stiffness is given by \( K'(s_{z0}, \Delta s_x) = c \cdot K(s_{z0}, \Delta s_x) \). Modified with the tire dynamic vertical stiffness test results, the inflation pressure factor \( c \) can be concluded.

For ground resonance analysis of helicopter, the lateral stiffness of wheel should be considered. In development of model, the tire lateral stiffness test should be done, and the rigidity and damping corresponding vibration amplitude and frequency can be obtained. Then, wheel lateral stiffness is not only the function of design parameters of tire, such as tire width \( w \), diameter \( d \) and inflation pressure \( p \), but also the function of displacement of static compression and lateral vibration, expressed as

\[ K_y(s_z, s_y) = 1.24 p w [a_{w0} + a_{w1} \frac{s_z}{w} + a_{w2} (\frac{s_z}{w})^{2}] (1 + c_y s_y) \] (2)

Tire provides heading stiffness while the mainwheel is braking. For the ground resonance analysis of braking condition, the heading stiffness test of wheel is needed. The expression of heading stiffness is similar to the lateral stiffness.

\[ K_y(s_z, s_y) = 5 p d [a_{w0} + a_{w1} \frac{s_z}{d} + a_{w2} (\frac{s_z}{d})^{2}] (1 + c_y s_y) \] (3)

With wheel landing gear, whether nosewheel or tailwheel, or other configurations, the helicopter must be designed to landing gear steering wheel can be free. When a helicopter sliding or running, take-off or landing on the ground, the rotation function of the steering wheel can’t be locked. The lateral stiffness will be reduced with the steering wheel, and the influence is big, especially the static compression displacement of tire is small, the lateral stiffness to reduce is the largest. The expression of torsional stiffness is similar to the heading stiffness.
\[ K_{d}(s_{z}, \theta) = 1.8p_{w}^{3}\left[ a_{p_{w}} + a_{p_{w}} \frac{s_{z}}{d} + a_{p_{w}} \frac{s_{z}^2}{d^2} \right] \] (4)

Stiffness and damping of a wheel are the same as the elastomeric dampers, expressed in elastic stiffness and damping stiffness respectively. However, damping stiffness is usually expressed as the ratio of the elastic stiffness, known as the loss angle, the ratio of the damping stiffness and elastic stiffness is equal to loss tangent. In fact, tire stiffness is changing with frequency, but this change is small shown from same test results. The change is obvious for damping, however, is usually loss angle increases with the increase of frequency. From the results of experiment and processed into equivalent viscous damping, equivalent viscous damping has little change with frequency. Therefore, tire damping expression constructed as irrelevant to the frequency.

\[ F_{y}^{e}(s_{z}, \Delta s) = C(s_{z}, \Delta s) \Delta s \]
\[ C(s_{z}, \Delta s) = |K(s_{z}, \Delta s)|C_{d}(s_{z}, \Delta s) \]
\[ C_{d}(s_{z}, \Delta s) = (d_{d} + d_{s} s_{z} + d_{j} s_{z}^{-1}) (1 + c_{d} \Delta s) \] (5)

The ratio coefficient changed with the quadratic of static compression displacement, and the linear relationship with \( \Delta s \) as corresponding vibration direction, are considered in the expression.

2.2. Landing gear shock-absorber nonlinear dynamics model
According to the literature [9], the axial force of the shock-absorber is expressed as, \( F_{s} = F_{a} + F_{f} + F_{b} \),

Where \( F_{a} \) is the air spring force; \( F_{f} \) is the internal friction force; \( F_{b} \) is the damping force for oil.

Air spring force for the low-pressure chamber and high-pressure chamber:

\[ F(s) = a \frac{1}{1 - b s}, \quad 0 \leq S \leq S_{\text{max}}^{L} \]
\[ F'(s) = a' \frac{1}{1 - b'(s - S_{\text{max}}^{L})}, \quad S_{\text{max}}^{L} < S < S_{\text{max}}^{H} \] (6)

When calculate the dynamic stiffness, the force is multiplied by temperature coefficient \( c \). The displacement integration method is to solve stiffness. Using the least square method, with the results of buffer static compression feature test and stiffness test, the coefficients \( a, b, a', b' \) and \( c \) can be obtained.

According to the fluid mechanics of the classic theory of local pressure loss, the main factors affecting oil damping force is speed \( \dot{S} \). Using experimental modeling method, the mathematical model of oil damping force can be established, with the relationship of movement speed and static compression position, \( F_{o} = a(s) \dot{S}^{2} \). According to the principle of equal work, the work of friction and oil damping force with steady-state forced vibration in a cycle is equal to the work of damping forces by test measured.

2.3. Rotor lag damper nonlinear model
There are two kinds of dampers used for helicopter rotor system usually, the elastomeric damper and the hydraulic shimmy damper. According to the types and the main design parameters, model for the two kinds of dampers respectively

The nonlinear physical model of elastomeric damper mentioned in the literature [6], is written as the following analytical form of the load and displacement.

\[ F_{b}(s, \dot{s}) = G_{b}(s)s + G_{d}(s) \dot{s} \]
\[ G_{b}(s) = k_{s} s + k_{b} s^{2} + k_{j} s^{3} + k_{s} s^{4} + k_{s} s^{5} + k_{s} s^{6} + k_{s} s^{7} \]
\[ G_{d}(s) = c_{s} s + c_{b} s^{2} + c_{j} s^{3} + c_{s} s^{4} + c_{s} s^{5} \] (7)

Where \( k_{s}, k_{b}, \ldots, c_{s}, c_{b}, \ldots \) are model parameters. The damping force of elastomeric damper depends on only its displacement, in the opposite direction with the relative movement velocity, and the relevance with frequency is very small essentially, shown by experimental data. The damping modeling method of elastomeric lag damper is the same as wheel.
The nonlinear curve of damping force of hydraulic damper and the axial velocity, as shown in literature [3], is divided into three periods for modelling. Its formula is expressed as:

\[ F_d(v) = S_{10} + S_{12}v + S_{13}v^2 | v \leq v_1 \]
\[ F_d(v) = S_{20} + S_{22}v + S_{23}v^2 | v_1 < v \leq v_2 \]
\[ F_d(v) = S_{30} + S_{32}v | v_2 < v \leq v_{\text{max}} \]  

(8)

Experimental data show that the damping force of hydraulic damper related to displacement, speed and frequency also. The opening force and speed of the PORV of hydraulic damper can be determined according to the test data. With the experimental data of amplitude and frequency and sampling time, and other known quantity, the relationship between damping force and speed can be deduced.

2.4. Equations of ground resonance

Using the spatial dynamic model, the helicopter dynamic system is divided into two parts of the fuselage and rotor system. The fuselage as a rigid body with six degrees of freedom: \( x, y, z, \phi, \phi', \phi'' \). For each blade, the freedom degree of lead-lag motion is only considered, denote by \( \xi_k \).

The movements of body and landing gear and rotor blade are described on basis of analysis of these dynamic models. The time domain nonlinear force of rotor dampers for blades lead-lag motion and landing gear for body motion can be determined, using the established nonlinear dynamic models of landing gear and rotor dampers, the inertial loads acting on the blade and rotor hub and body can also be obtained. Then, the equation of lead-lag motion of each blade in the rotor rotating-coordinate system and the equation of body motion in the fixed-coordinate system can be carry out respectively by Lagrange theorem. Finally, the analysis model of ground resonance considering the nonlinear dynamic characteristics of these parts can be set up.

The single gear balance equation:

\[ K_n \ddot{Z}_n + C_n \dot{Z}_n + m_n \dot{Z}_n = K_{m_n} \ddot{Z}_{m_n} + C_{m_n} \dot{Z}_{m_n} \quad (n = 1,2,3) \]  

(9)

The force and moment equilibrium equations of rotor and body coupling:

\[ M_f \ddot{X} + \sum_{k=1}^{3} \left[ K_n \ddot{X}_n + C_n \dot{X}_n \right] - S_{n} \sum_{k=1}^{\infty} \xi_k \sin \psi_k = F_x' \]
\[ M_f \ddot{Y} + \sum_{k=1}^{3} \left[ K_n \ddot{Y}_n + C_n \dot{Y}_n \right] + S_{n} \sum_{k=1}^{\infty} \xi_k \cos \psi_k = F_y' \]
\[ M_f \ddot{Z} + \sum_{k=1}^{3} \left[ K_n \ddot{Z}_n + C_n \dot{Z}_n \right] + T - M_g = F_z' \]

(10)

(11)

(12)

\[ I_x \ddot{\phi}_x + I_y \ddot{\phi}_y + \sum_{k=1}^{3} \left[ K_n \ddot{\phi}_x + C_n \dot{\phi}_x \right] - Z_{a} \left( K_n \ddot{X}_n + C_n \dot{X}_n \right) - S_{n} Z_{a} \sum_{k=1}^{\infty} \xi_k \cos \psi_k = M_{\phi x} \]
\[ I_y \ddot{\phi}_y + \sum_{k=1}^{3} \left[ K_n \ddot{\phi}_y + C_n \dot{\phi}_y \right] - X_{a} \left( K_n \ddot{X}_n + C_n \dot{X}_n \right) - S_{n} X_{a} \sum_{k=1}^{\infty} \xi_k \sin \psi_k = M_{\phi y} \]
\[ I_z \ddot{\phi}_z + I_x \ddot{\phi}_x + \sum_{k=1}^{3} \left[ K_n \ddot{\phi}_z + C_n \dot{\phi}_z \right] - Y_{a} \left( K_n \ddot{X}_n + C_n \dot{X}_n \right) + S_{n} Y_{a} \sum_{k=1}^{\infty} \xi_k (\sin \psi_k + \cos \psi_k) = M_{\phi z} \]

(13)

(14)

(15)

The blade lead-lag behavior equation:

\[ I_s \ddot{\xi}_k + F_{x_k}'' + M_s \left( \ddot{\xi}_k, \dot{\xi}_k \right) + S_{s} \Omega^2 \xi_k e_{ll} + S_{s} \left( \dot{Z}_a - Z_{a} \ddot{\phi}_a \right) (\cos \phi_k - \sin \phi_k \dot{\xi}_k) - S_{s} \left( \ddot{X}_a + Z_{a} \dot{\phi}_a \right) \sin \phi_k + \cos \phi_k \dot{\xi}_k = M_{\xi_k} \quad (k = 1,2,\ldots,N_b) \]

(16)

Where \( M, K \) and \( C \), respectively, represent mass, stiffness and damping. The index \( n \) is corresponding to the numbered gear, the index \( h \) represents absorber, \( f \) represents the body center of gravity. \( I_x, I_{yz}, I_Y, I_{xz} \) are the body moments of inertia. \( I_s, S_{s}, S_{s}, F_{x_k}, F_{y_k}, F_{z_k} \) represent the moment of inertia and static moment of blade around the vertical hinge, respectively.
represent aerodynamic force and moment. $F_x^e$ is aerodynamic drag. $M_{\dot{\theta}_1, \dot{\phi}_1}$ is restraint moment of blade root spring. $M_{\dot{\phi}_1}$ is Coriolis moment of blade lagging.

3. Analysis and discussions for simulation test

To investigate the ground resonance test simulation with the same conditions of a certain type helicopter. The calculation values of simulation are converted to close to the locations of test points, then to compare with the test curves. From the aspects such as frequency and reduction damping ratio, analysis the accuracy of the simulation calculation results, analysis the reason of the differences.

The vibration response time histories of the nose from the ground resonance test of a certain helicopter with a certain weight are shown in figure 1. Figure 1(a) is the nose lateral vibration response, in the condition of zero lift and in ground idle speed, the attenuation frequency is 1.67 Hz, the damping ratio is 4.8955%, and the frequency of the exciting period of the experimental curve is 1.67 Hz, by the experimental data analysis. It shows that the motion of blade lead-lag natural model has not been excited, and is not the body pitch frequency also. The nose vertical vibration response is shown in figure 1(b), 50% lift, in nominal rotor speed, the excitation frequency is 3.33 Hz, attenuation frequency is 3.33 Hz, and damping ratio is 1.307357%. The data results show that the motion of blade lead-lag natural model has not been excited, and is not the body yaw frequency also. Therefore, the accurate frequency excitation is hard to do in the actual model test.

Figure 1. Vibration response time histories of the nose from the ground resonance test of a certain helicopter, (a) The nose lateral, zero lift, (b) The nose vertical, 50% lift.

The simulation results of ground resonance are shown in figure 2. For simulation calculation, the nose lateral vibration response is coupled by the lateral movement of body center of gravity and rolling motion, and the nose vertical vibration response is coupled by the vertical movement of body center of gravity and pitch motion. The nose lateral vibration response time histories of simulation test is shown in figure 2(a), in ground idle speed. Figure 2(c) is the nose vertical vibration response time histories of simulation test, in nominal rotor speed. At the same time, the response of blade lead-lag motion under the same conditions is simulated respectively, as shown in figures 2 (b) and 2(d).

Figure 2. Vibration response time histories of ground resonance simulation test, (a) The nose lateral, zero lift, (b) Lead-lag motion, zero lift, (c) The nose vertical, 50% lift, (d) Lead-lag motion, 50% lift.

An exciting force was applied to the rotor at some time, lasting t seconds, in a simulation test of ground resonance. After the excitation had been removed, the system was free vibration. For the simulation test of ground resonance in figures 2(a) and 2(b), an exciting force was applied by simulated at 160s, and stopped the excitation after ten seconds, the rotor lift was zero, in ground idle
speed. The attenuation frequency of blade was 1.2994 Hz, damping ratio was 4.21531%, calculated according to figure 2(b). Switched these to the fixed coordinate system, the frequency was 2.9 Hz, was just the frequency of lead-lag regressing mode in this state, and the damping ratio was 1.88875%.

For figures 2(c) and 2(d), an excitation was simulated at 250s, and stopped the excitation after ten seconds also, the rotor lift was 50% of the weight, in nominal speed. The attenuation frequency of blade was 2.9485 Hz, damping ratio was 2.41647%, calculated according to figure 2(d). Switched these to the fixed coordinate system, the frequency was 2.9682 Hz, was just the frequency of lead-lag regressing mode in this situation, and the damping ratio was 2.4%.

The comprehensive analysis of the frequency and damping in figures 1 and 2 shows that the excited frequency of simulation test is not the same as model experiment. The frequency of actual experiment is not accurate, is not the nature frequency of body, nor the rotor lead-lag regressing frequency, but the attenuation frequency is close to the simulation results, the corresponding attenuation damping ratio is similar. This indicates the simulation test results are in accordance with the practice. The large fluctuation of the simulation curve shows the influence of nonlinear dynamic characteristics of the components on the stability of the helicopter. The large blade attenuation damping of simulation test and the fast response attenuation indicate that the system has a high stability.

4. Conclusion
As the effects of nonlinearities existed in systems, there will probably be errors from engineering analysis when the linearized hypothesis is taken into account. In order to present an accurate analysis for helicopter ground resonance rotor/fuselage coupled dynamic systems, nonlinear differential equations are derived in this paper. Time-domain analyses is applied to predict dynamic instabilities and compared with test data of type helicopter. By comparing the frequency and damping ratio, concluded that the simulation test can exactly lead up the blade lead-lag regressing mode frequency, and the modal damping obtained according to attenuation curves closed to test results, the correctness of the analytical model had been validated. This analysis method used for ground resonance simulation test can give out the results according with real helicopter tests.

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