Route to Coherent Supercontinuum Generation in the Long Pulse Regime
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Abstract—In this paper, we investigate the possibility of generating supercontinuum in highly nonlinear fibers with improved stability characteristics through modulation of the input pulse. Specifically, we show that broadband coherent supercontinuum can be generated in the long pulse regime, provided the modulation parameters and nonlinear fiber length are carefully tuned. Based on physical arguments, we derive general guidelines for the choice of modulation frequency and fiber length. Significantly, the guidelines are valid for arbitrary long pulses.

Index Terms—Optical fibers, optical modulation, optical propagation in nonlinear media, optical solitons.

I. INTRODUCTION

Coherent supercontinuum (SC) generation in highly nonlinear fibers typically relies on relatively complex and expensive pump sources that can produce pulses of femtoseconds duration [1]. As there is a strong interest in developing and refining applications, the development of inexpensive, low-footprint, and phase-stable SC sources that avoid the use of such pump sources is highly desirable. In this context, it is also important to note that many compact pump sources producing longer (picosecond and greater) pulse durations are also suitable to the generation of ultra-broadband spectra in nonlinear fibers. However, SC generation in the long pulse regime has been shown to be initiated by noise-seeded modulation instability (MI) processes, and therefore, associated with large shot-to-shot variations both in amplitude and phase [2]. The resulting poor stability and coherence characteristics of the SC in this case may be detrimental to potential applications. Thus, there is a significant interest in developing techniques that can introduce a degree of systematic control into the shaping and stability of SC generation in this regime.

The idea of using a continuous-wave (CW) dual-frequency field to produce high-repetition-rate pulse trains through induced MI in optical fibers was initially presented by Hasegawa [3], and has attracted significant interest, with several experimental approaches being demonstrated [4]–[10]. In this regard, criteria for minimizing the noise amplification in the generation of gigahertz pulse train have also been recently derived by Kobtsev and Smirnov based on the numerical expression of the MI gain in optical fibers [11]. The CW dual-frequency field technique has been further applied successfully to the generation of frequency combs in highly nonlinear fibers spanning more than 100 nm around 1550 nm [12], [13], where the generation of the frequency comb was shown to arise from a seeded MI process [12] or multiple four-wave mixing products [13].

In the context of SC generation using long (picosecond) pump pulses, two different pumping methods based on a dual-frequency input field have been proposed recently in order to circumvent the effect of noise amplification and seed the SC generation process in a coherent manner [14], [15]. In particular, Solli et al. demonstrated the possibility of selectively amplifying a short (femtosecond) duration, small amplitude probe pulse obtained from a filtered SC by a much longer (picoseconds) pump pulse with larger amplitude in order to reduce the influence of noise [14], [16]. Launching simultaneously the pump and probe in a highly nonlinear fiber, they could amplify and seed the dynamics of the probe, yielding a broadband SC through usual anomalous pumping soliton dynamics. Although the SC generated had much improved coherence characteristics, it is essentially incoherent for spectral components located further away from the pump arising from large temporal jitter in the Raman red-shifted solitons. Another approach was presented in [16], where the use of a modulated input field with a high contrast was suggested in order to seed the MI and control the SC generation process. However, although it was shown that improved stability can be obtained in this way for a certain range of modulation frequency, high coherence across the whole SC bandwidth was not obtained. Furthermore, the results were purely numerical with no attempt to develop detailed analytic insight into how this process occurs.

In this paper, we revisit the idea introduced in [16] and use physical arguments based on the soliton dynamics of the subpulse components associated with the modulated envelope. This allows us to derive specific analytic conditions on the input modulation parameters and fiber lengths required to obtain a coherent SC over a wide range of input pulse durations in the long pulse regime. The results presented in this paper therefore suggest an approach to the systematic design of coherent broadband spectra for specific applications. Significantly, we show that a proper choice of initial modulation and fiber length can, in fact, lead to highly coherent SC across the full bandwidth for

1In fact, this appears to be a fiber optics example of a coherent-photon seeding or phase-wave suppression process studied earlier in the context of mode-locked laser stabilization.

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arbitrary long pulses with large peak power. Numerical simulations based on the generalized nonlinear Schrödinger equation are used to validate our physical arguments and illustrate the degree to which stability of the output SC spectra can be controlled. The results here are the first that examine applications in SC spectral shaping and stability. This technique could have high impact in the spectral stabilization and control of SC generation in the long pump pulse regime.

II. PHYSICAL GUIDELINES

Our approach is based on considering a temporal modulation imposed across the entire pump pulse envelope in a standard way by adding to the input field a replica downshifted by $f_m$, such that the total field can be written as $A_m(t) = A(0, t) \times [1 + \alpha_0 \exp(-i2\pi f_m t)]$. Here, $A(0, t) = \sqrt{P_0} \exp(-2\ln 2 \sigma^2 / T_{\text{FWHM}})$ is the envelope of the original pump pulse with peak power $P_0$ and duration $T_{\text{FWHM}}$. The intensity modulation contrast (visibility) is given by $2\alpha_0 / (1 + \alpha_0^2)$. This type of dual-frequency field can be readily generated using optical frequency shifting techniques.

In the presence of significant modulation contrast, the input pulse can be considered as a series of ultrashort subpulses of duration $T_i \approx (1/f_m) \times (1 + \alpha_0^2) / (8\alpha_0) / 1.763$ superimposed onto a broad lower amplitude background envelope. Under these conditions, the SC spectral broadening is expected to be dominated by the dynamics of these ultrashort subpulses, with only a minor contribution from the background envelope. We consider the case of pumping in the anomalous dispersion regime, which is known to yield broader SC spectra for constant peak power [1]. In order to determine conditions on the modulation characteristics associated with coherent SC generation, we adapt previously developed criteria in the single pulse regime [1] to the individual subpulses of the modulated envelope. More specifically, if we consider the central subpulse of amplitude $P_i = (1 + \alpha_0^2)^2 P_0$ and soliton order $N_i = (\gamma P_i T_i^2 / |\beta_2|)^{1/2}$, we can expect coherent SC seeded by this subpulse provided $N_i < N_{cr}$, where $N_{cr} \approx 10$ generally ensures coherent SC [1]. This condition, which applies for the case of anomalous dispersion pumping regime, can be recast into a more useful form in terms of the initial modulation frequency

$$f_m \geq \frac{(1 + \alpha_0)^3}{8\alpha_0} \frac{1}{T_{\text{FWHM}}} \frac{N}{N_{cr}}$$

(1)

and by postulating that the propagation length is equal to the fission length of the central subpulse $L_{\text{fiss}} \approx N_i / \gamma P_i$ [1], we can estimate a useful guideline for the optimal fiber length

$$L \approx \frac{N_{cr}}{(1 + \alpha_0)^2 \gamma P_0}.$$  

(2)

Generally, one can use propagation distance exceeding the fission length and still obtain coherent SC [1], but propagating over a distance around the fission length represents a useful criterion to minimize the fiber length. In all these results, $N$ is the input soliton number of the unmodulated pulse, and $\beta_2$ and $\gamma$ represent the dispersion and nonlinearity at the pump wavelength. Although these results are based on the central subpulse parameters, of course, neighboring subpulses would be expected to induce themselves similar coherent SC generation, with their presence manifested in the overall output spectrum in the form of spectral modulation (sometimes referred to as spectral “channeling”) or comb-like sub-structure at the frequency $f_m$. These considerations are based on the fact that the subpulse dynamics dominate the overall evolution of the spectral broadening process, which requires that the amplitude of the initial modulation is large enough to prevent noise-seeded MI phenomena from the broad background to develop. We have found that an initial modulation strength $\alpha_0 = 0.05$ corresponding to a 10% contrast is typically sufficient to be in such regime. Note that this value agrees with the results presented in [11], where a modulation depth exceeding 10% was found to lead to the generation of a low-noise periodic pulse train from a CW signal through stimulated MI.

III. NUMERICAL MODELING

To illustrate the benefit of our approach, we consider the propagation of Gaussian 1-ps, 10-kW peak power pulses at 1060 nm in a photonic crystal fiber with a zero-dispersion wavelength at 1025 nm (see Fig. 1) and nonlinear coefficient $\gamma = 11 \text{ W}^{-1} \cdot \text{km}^{-1}$. The input pulse parameters correspond to an average power of 1 W at a repetition rate of 100 MHz, readily produced by unamplified Yb$^{3+}$ lasers while the fiber envisaged here is available commercially.

The input pulse parameters correspond to a soliton number $N = 98$, and under these conditions, one would a priori expect strongly incoherent SC generation seeded by initial MI dynamics. To model SC generation, we use the well-established generalized nonlinear Schrödinger equation, which includes the noninstantaneous part of the nonlinear response of silica, the frequency dependence of the nonlinear response, and the quantum noise on the input pulse envelope as a one-photon-per mode background [1]. Although noise has been implemented in a different manner, especially for the case of quasi-CW input [17], [18], the issue is really how noise influences the competition between MI and the soliton dynamics of the subpulses on the modulated pulse envelope. And this case corresponds, in fact, to the regime, where noise modeling using either a one photon per mode model or a coherent state representation yield equivalent results that have also been quantitatively confirmed by experiment [19]. In addition, the width of the temporal window of

![Figure 1. Dispersion profile of the fiber used in the numerical simulations. The zero-dispersion wavelength is at 1025 nm.](image-url)
the simulations was adjusted in order to avoid subpulses wrapping around the time window due to the periodic boundary conditions. Specifically, for the 1 ps case considered here and the 20 ps case envisaged next, the time window used was 7 and 130 ps, respectively. We carry out 1000 simulations using different noise seeds in order to determine average spectral characteristics and coherence properties. Fig. 2 illustrates the spectral and coherence characteristics of the generated SC after 14 cm in the absence of modulation on the input pulse, yet in the presence of quantum noise. The length is chosen to be 14 cm as it is approximately 16 times the nonlinear length, which is approximately the fiber length required for SC generation in the long pulse regime. Clearly, the noise-seeded MI initial dynamics leads to an SC with large spectral variations from shot-to-shot and poor stability.

We next impose a modulation on top of the input pulse with \( a_0 = 0.15 \), which ensures that the central subpulse contribution to the SC spectrum is significant. Imposing the soliton number of the central subpulse \( N_s \) to be 10, this value of \( a_0 \) gives a minimum value of 12.5 THz for the modulation frequency \( f_m \) in order to obtain coherent SC generation. Although the necessary fiber length given by (2) for SC generation is 7 cm, we consider propagation over 14 cm to allow a more direct comparison with the case of the unmodulated pump pulse. Fig. 2 shows the SC spectrum and associated coherence properties when the input pulse is modulated with the aforementioned characteristics. In comparison with the unmodulated pump case (also shown in Fig. 2), the SC spectrum exhibits a dramatic increase in coherence as well as a slightly larger bandwidth. As expected, the contribution of the multiple subpulses manifests itself in the form of a highly structured spectrum. Additional numerical simulations show that the SC coherence is, in fact, not only even higher for shorter fiber lengths, but also remains high across the SC bandwidth for longer lengths, exceeding several times the minimum value given by (2).

Additional insight into the propagation dynamics of the modulated pulse can be obtained by plotting (using a false gray scale representation) the corresponding spectral and temporal evolutions, and comparing these results to that of a single initial input pulse with peak power \( P_2 \) and duration \( T_2 \) (with \( N_s = 10 \)), as shown in Fig. 3. The change in the envelope of the SC spectrum generated by the modulated pulse consists of a first stage of slow symmetrical spectral broadening followed by an abrupt transition into SC and further stabilization of the bandwidth. These dynamics are consistent with nonadiabatic temporal compression experienced by a higher order soliton (here, corresponding to the central subpulse). After the distance of maximum compression \( L_{\text{fiber}} \approx 7 \) cm, the higher order soliton pulse breaks up into multiple subpulses. And this is precisely what is also observed for each of the individual subpulses arising from the modulation of the input pulse, showing clear evidence that the ultrashort pulse dynamics indeed dominate the propagation with minor influence of the broad temporal background. A direct consequence is that the spectral envelope and overall bandwidth of the generated SC by the modulated pulse are primarily determined by the parameters of the central subpulse and therefore the applied modulation. We also note in the spectral evolution of the modulated pulse the clear contribution of the multiple subpulses through spectral modulation (or “channeling”). Of additional interest is the fact that, for propagation distance exceeding the fission point of the individual subpulses, the comb-like structure of the SC spectrum loses contrast due to nonlinear interactions between the pulses ejected by the multiple subpulses themselves [see Fig. 3(a)].

Significantly, additional simulations carried out over a wider parameter range show that the approach discussed before is not only restricted to pulses on the order of picosecond or below, but also valid for much longer pulses. We have verified numerically that the guidelines are valid for pulses even exceeding 1 ns. In fact, we believe that our approach applies for arbitrary input
IV. Conclusion

We have investigated the possibility of generating broadband SC with high degree of coherence in the long pulse regime. Specifically, we have shown that imposing an initial modulation with appropriate characteristics on top of the broad pulse can lead to a fully coherent continuum, which is in marked contrast with incoherent spectra generated by unmodulated pulses. These results are remarkable for the pulse durations and level of peak power envisaged here, and to our knowledge, represents the first example of how coherent SC can be generated in such a long pulse regime. In fact, the recipe given here for the generation of coherent SC in the long pulse regime is rather general and can be further applied to even much longer initial pulse duration exceeding the nanosecond range. However, as the initial pulse duration increases, the comb-like structure of the SC becomes inherently more apparent, which may also be desirable for certain applications. These findings could help solving the long-standing problem of coherent SC generation in the long pulse regime.

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Fig. 4. (a) Simulation results of SC generated by 20 ps pulses with modulation at 12.5 THz. Individual simulation results (gray traces) and the calculated mean from the ensemble (black line) are shown. The associated calculated coherence is plotted on the right axis. (b) Zoomed portion of the central part of the comb-like SC spectrum is displayed.
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