**Calculation of nonzero-temperature Casimir forces in the time domain**

The MIT Faculty has made this article openly available. Please share how this access benefits you. Your story matters.

| **Citation**       | Pan, Kai et al. “Calculation of nonzero-temperature Casimir forces in the time domain.” Physical Review A 83 (2011). ©2011 American Physical Society. |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------|
| **As Published**   | http://dx.doi.org/10.1103/PhysRevA.83.040503                                                                                          |
| **Publisher**      | American Physical Society                                                                                                           |
| **Version**        | Final published version                                                                                                             |
| **Citable link**   | http://hdl.handle.net/1721.1/66137                                                                                                  |
| **Terms of Use**   | Article is made available in accordance with the publisher’s policy and may be subject to US copyright law. Please refer to the publisher’s site for terms of use. |
Calculation of nonzero-temperature Casimir forces in the time domain
Kai Pan,1,2 Alexander P. McCauley,1 Alejandro W. Rodriguez,1 M. T. Homer Reid,1,2 Jacob K. White,1,3 and Steven G. Johnson2,4
1Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
2Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
3Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
4Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 22 November 2010; revised manuscript received 25 March 2011; published 29 April 2011)
We show how to compute Casimir forces at nonzero temperatures with time-domain electromagnetic simulations, for example, using a finite-difference time-domain (FDTD) method. Compared to our previous zero-temperature time-domain method, only a small modification is required, but we explain that some care is required to properly capture the zero-frequency contribution. We validate the method against analytical and numerical frequency-domain calculations, and show a surprising high-temperature disappearance of a nonmonotonic behavior previously demonstrated in a pistonlike geometry.

doi:10.1103/PhysRevA.83.040503 PACS numbers: 31.30.jh, 12.20.—m, 02.70.—c, 42.50.Ct

In this paper, we show how to compute nonzero-temperature \((T > 0)\) corrections to Casimir forces via time-domain calculations, generalizing a computational approach based on the finite-difference time-domain (FDTD) method that we previously demonstrated for \(T = 0\) [1,2]. New computational methods for Casimir interactions [3–8] have become important in order to model nonplanar micromechanical systems where unusual Casimir effects have been predicted, and there has been increasing interest in \(T > 0\) corrections [9–19], especially in recently identified systems where these effects are non-negligible [12]. Although \(T > 0\) effects are easy to incorporate in the imaginary frequency domain, where they merely turn an integral into a sum over Matsubara frequencies [20], they are nontrivial in time domain because of the singularity of the zero-frequency contribution, and we show that a naive approach leads to incorrect results. We validate using both a one-dimensional (1D) system where analytical solutions are available and also a two-dimensional (2D) pistonlike geometry [3,21] where we compare to a frequency-domain numerical method. In the piston, we observe an interesting effect in which a nonmonotonic phenomenon previously identified at \(T = 0\) disappears for a sufficiently large \(T\).

The Casimir force arises from fluctuations at all frequencies \(\omega\), and the \(T = 0\) force can be expressed as an integral \(F(\omega) = \int_0^\infty f(\xi)\,d\xi\) over Wick-rotated imaginary frequencies \(\omega = i\xi\) [20]. At \(T > 0\), this integral becomes a sum over “Matsubara frequencies” \(\xi_n = n\pi\omega_T\) for integers \(n\), where \(\omega_T = 2k_B T/\hbar\) and \(k_B\) is Boltzmann’s constant [20]:

\[
F(T) = \pi\omega_T \left[ \frac{f(0^+)}{2} + \sum_{n=1}^\infty f(n\pi\omega_T) \right].
\]  

Equation (1) corresponds to a trapezoidal-rule approximation of the \(T = 0\) integral [8]. At room temperature, \(\xi = \pi\omega_T\) corresponds to a “wavelength” \(2\pi/\xi = 7\,\mu\text{m}\), much larger than most experimental separations, so usually \(T > 0\) corrections are negligible [20]. However, experiments are pushing toward \(> 1\,\mu\text{m}\) separations in attempts to measure this phenomenon [17,18], recently culminating in an experiment at several \(\mu\text{m}\) that appears to clearly observe the \(T > 0\) corrections [19]. In addition, we have recently predicted much larger \(T\) corrections with certain materials and geometries [12].

Many of the recent techniques to compute Casimir forces in arbitrary geometries can be related to mature computational methods from classical electromagnetism (EM) [8]. One method is to use the fluctuation-dissipation theorem, by which the mean-square electric and magnetic fields \(\langle E^2 \rangle\) and \(\langle H^2 \rangle\) can be computed from classical Green’s functions [20], and the mean stress tensor can be computed and integrated to obtain the force [1–3]. In particular, at each \(\omega\), the correlation function \(\langle E^2 \rangle\) is

\[
\langle E_j(\omega,x)E_k(\omega,x') \rangle = -\frac{\hbar}{\pi} \text{Im} \left[ \omega^2 G^{E}_{jk}(\omega;x,x') \right] \coth \left( \frac{\omega}{\omega_T} \right),
\]

where \(G^{E}_{jk}(\omega;x,x') = (G^{E}_{kj}(\omega;x,x'))^*\) is the classical dyadic “photon” Green’s function, proportional to the electric field in the \(j\) direction at \(x\) due to an electric-dipole current in the \(k\) direction at \(x'\), and solves

\[
[\nabla \times \mu(\omega,x)^{-1} \nabla \times -\omega^2 \varepsilon(\omega,x)] G^{E}_{jk}(\omega,x,x') = \delta^3(x-x') \hat{e}_k,
\]

where \(\varepsilon\) is the electric permittivity tensor, \(\mu\) is the magnetic permeability tensor, and \(\hat{e}_k\) is a unit vector in direction \(k\). The magnetic-field correlation \(\langle H^2 \rangle\) has a similar form [1,2]. Note that the temperature dependence appears as a coth factor (from a Bose-Einstein distribution). If this is Wick rotated to imaginary frequency \(\omega = i\xi\), the poles in the coth function give the sum (1) over Matsubara frequencies [22]. In our EM simulation, what is actually computed is the electric or magnetic field in response to an electric or magnetic dipole current, respectively. This is related to \(G_{ij}\) by

\[
E_{jk}(\omega,x,x') = -i\omega G^{E}_{jk}(\omega,x,x'),
\]

where \(E_{jk}(\omega,x,x')\) denotes the electric field response in the \(j\)th direction due to a dipole current source \(J(\omega,x,x') = \delta(x-x') \hat{e}_k\) [1].

This equation can be solved for each point on a surface to integrate the stress tensor, and for each frequency to integrate the contributions of fluctuations at all frequencies. Instead of computing each \(\omega\) separately, one can use a pulse source in
time, whose Fourier transform contains all frequencies. As
derived elsewhere [1,2], this corresponds to a sequence of time-
domain simulations, where pulses of current are injected and
some function $\Gamma(t)$ of the resulting fields (corresponding to
the stress tensor) is integrated in time, multiplied by an appropriate
weighting factor $g(t)$. We perform these simulations using the
standard FDTD technique [23], which discretizes space and
time on a uniform grid. In frequency domain, Wick rotation
to complex $\omega$ is crucial to obtain a tractable frequency
integrand [3,8], and the analog in time domain is important
to obtain rapidly decaying fields (hence short simulations) [1,2].
In time domain, one must implement complex $\omega$ indirectly:
Because $\omega$ only appears explicitly with $\epsilon$ in Eq. (3), converting
$\omega$ to the complex contour $\omega(\xi) \equiv \xi \sqrt{1 + \frac{i \omega^2}{\xi}}$ is equivalent to
operating at a real frequency $\xi$ with an artificial conductivity
$\epsilon(\tau) \rightarrow \epsilon(\tau)(1 + \frac{i \omega^2}{\xi})$ [1,2]. (One cannot use purely imaginary
frequencies $\omega = i \xi$ in time domain, because the corresponding
material $\epsilon \rightarrow -\epsilon$ has exponentially growing solutions in time
[1].) Thus, by adding an artificial conductivity everywhere, and
including a corresponding Jacobian factor in $g(t)$, one obtains
the same (physical) force result in a much shorter time (with the
fields decaying exponentially due to the conductivity). (Note
that the resulting “time” is not physical time but is instead a
mathematical construction equivalent to a frequency-domain
calculation for equilibrium or quasistatic problems.)

Now, we introduce the basic idea of how $T > 0$ is incor-
porated in the time domain, and explain where the difficulty
arises. The standard $T > 0$ analysis of Eq. (1) is expressed
in frequency domain, so we start there by exploiting the
fact that the time-domain approach is derived from a Fourier
transform of a frequency-domain approach. In particular, $g(t)$ is the Fourier transform of a weighting factor $g[\omega(\xi)]$ [1,2].
At real $\omega$, the effect of $T > 0$ is to include an additional factor $\coth\left(\frac{\omega(\xi)}{\omega_T}\right)$ in the $\omega(\xi)$ integral from Eq. (2). So, a naive
approximation is to replace $g[\omega(\xi)]$ with

$$g[\omega(\xi)] \rightarrow g[\omega(\xi)] \coth \left[ \frac{\omega(\xi)}{\omega_T} \right]$$

$$= -i \xi \left(1 + \frac{i \sigma}{\xi} \right) \left(1 + i \sigma/2 \xi \right) \coth \left[ \frac{\omega(\xi)}{\omega_T} \right], \quad (5)$$

using the $g[\omega(\xi)]$ expression for $T = 0$ from [1], and then
Fourier transform this to yield $g(t)$. However, there is an
obvious problem with this approach: The $1/\xi$ singularity in
$\coth\left(\frac{\omega(\xi)}{\omega_T}\right)$ means that Eq. (5) is not locally integrable around
$\xi = 0$, and therefore its Fourier transform is not well defined.
If we naively ignore this problem, and compute the Fourier
transform via a discrete Fourier transform, simply assigning
an arbitrary finite value for the $\xi = 0$ term, this unsurprisingly
gives an incorrect force for $T > 0$ compared to the analytical
Lifshitz formula for the case of parallel perfect-metal plates in
one dimension [24], as shown in Fig. 1 (green dashed line).

Instead, a natural solution is to handle $\omega \neq 0$ by the coth
factor as in Eq. (5), but to subtract the $\omega = 0$ pole and handle
this contribution separately. We will extract the correct $\omega = 0$
contribution from the frequency-domain expression Eq. (1),
convert it to time domain, and add it back in as a manual
correction to $g(t)$. In particular, the $\coth\left(\frac{\omega(\xi)}{\omega_T}\right)$ function has
poles at $\omega = n \pi \omega_T$ for integers $n$. When the $\omega$ integral is

![FIG. 1. (Color online) Comparison between FDTD (red circles)
and the analytical Lifshitz formula [25] (blue line) for the Casimir
force between perfect-metal plates in one dimension with separation
$a$. The $\omega = 0$ and $\omega \neq 0$ contributions to the Matsubara sum (1)
are plotted separately, in addition to the total force. The straightforward
method of including the $\coth(h\omega/2kT)$ Bose-Einstein factor in the
FDTD integration (green dashed line) gives an incorrect result
because the $\omega = 0$ pole requires special handling.]

Wick rotated, the residues of these poles give the Matsubara
sum Eq. (1) via contour integration [22]. If we subtract the
$n = 0$ pole from the coth, obtaining

$$g_{n>0}(\xi) = g[\omega(\xi)] \left\{ \coth \left[ \frac{\omega(\xi)}{\omega_T} \right] - \frac{\omega_T}{\omega(\xi)} \right\}, \quad (6)$$

the result of the time-domain integration of $g_{n>0}(t) \Gamma(t)$ will
therefore correspond to all of the $n > 0$ terms in Eq. (1), nor is
there any problem with the Fourier transformation to $g_{n>0}(t)$.
Precisely this result is shown for the 1D parallel plates in Fig. 1,
and we see that it indeed matches the $n > 0$ terms from the
analytical expression. To handle the $\omega = 0$ contribution, we
begin with the real-$\omega T = 0$ force expression, following our
notation from the time-domain stress-tensor method [1,2]:

$$F_i = \lim_{\omega \rightarrow 0} \frac{\hbar}{\pi} \int_0^\infty d\omega g_R(\omega) \Gamma_i(\omega), \quad (7)$$

where $g_R(\omega) = -i \omega$ is the weighting factor for the $\sigma = 0$
real $\omega$ contour and $\Gamma_i(\omega) = \Gamma_i^E(\omega) + \Gamma_i^H(\omega)$ is the surface-
integrated stress tensor (electric- and magnetic-field contri-
butions). From Eq. (1), the $\omega = 0$ contribution for $T > 0$ is then

$$F_i, (\omega=0) = \lim_{\omega \rightarrow 0} \frac{\hbar}{\pi} \int_0^\infty d\omega g_R(\omega) \Gamma_i(\omega) \rightarrow \frac{\hbar}{\pi} \int_0^\infty d\omega \left\{ \frac{1}{2} \left[ (-i \omega) \Gamma_i^E(\omega) - \frac{2 \pi}{\hbar} k_B T \right] \right\} \Gamma_i(\omega) \, \omega \, d\omega, \quad (8)$$

(Notice that $\hbar$ cancels in the $\omega = 0$ contribution: This term
dominates in the limit of large $T$ where the fluctuations can
be thought of as purely classical thermal fluctuations.) To
relate Eq. (9) to what is actually computed in the FDTD
method requires some care because of the way in which
we transform to the $\omega(\xi)$ contour. The quantity $\Gamma_i^E(\omega)$
is proportional to an integral of $E_{i1}(\omega) = -i \omega G_{i1}(\omega)$, from
Eq. (4). However, the $\omega(\xi)$ transformed system computes
$\Gamma_i^E(\xi) \sim \tilde{E}_{i1}(\xi) = -i \xi \tilde{G}_{i1}(\xi)$, where $G(\xi)$ solves Eq. (3)
with \( \omega^2 \varepsilon(\mathbf{r}) \to \xi^2 (1 + \frac{i \omega}{\xi}) \varepsilon(\mathbf{r}) \), but what we actually want is
\[-i \omega G_{ij}(\omega)_{|\omega=\xi} = -i \omega \xi \bar{G}_{ij}(\xi). \]
Therefore, the correct \( \omega = 0 \) contribution is given by
\[
\lim_{\omega \to 0^+} \Gamma_{T}(\omega) = \lim_{\xi \to 0^+} \frac{\alpha(\xi)}{\xi} \Gamma_{T}(\xi).
\]
(10)

Combined with \( \alpha(\xi) k_B T \) factor from Eq. (9), this gives an \( n \) = 0 contribution of \( \Gamma_{|\omega=0} \) multiplied by \(-\alpha(\xi)^2 k_B T / \xi|_{\omega=0}^\infty = \sigma k_B T \). This \( \omega = 0 \) term corresponds to a simple expression in the time domain, since \( \Gamma_{|\omega=0} \) simply is the time integral of \( \Gamma(t) \) and the coefficient \( \sigma k_B T \) is merely a constant. Therefore, while we originally integrated \( g_{n>0}(t) \Gamma(t) \) to obtain the \( n > 0 \) contributions, the \( n = 0 \) contribution is included if we instead integrate
\[
\{ g_{n>0}(t) + \sigma k_B T \} \Gamma(t).
\]
(11)
The term \( g_{n=0}(t) + \sigma k_B T \) generalizes the original \( g(t) \) function from [1] to any \( \Omega \geq 0 \).

We check Eq. (11) for the 1D parallel plate case in Fig. 1 against the analytical Lifshitz formula [25]. As noted above, the \( g_{n=0} \) term (6) correctly gives the \( n \geq 0 \) terms, and we also see that the \( \sigma k_B T \) term gives the correct \( n = 0 \) contribution, and hence the total force is correct.

As another check, we consider a more complicated pistonlike geometry [3], shown schematically in the inset of Fig. 2. Two square rods lie between two sidewalls, which we solve here for the 2D case of \( z \)-invariant fluctuations. At \( T = 0 \), such geometries exhibit an interesting nonmonotonic variation of the force between the two blocks as a function of sidewall separation \( d \) [3,21], which does not arise in the simple pairwise-interaction heuristic picture of the Casimir force. This can be seen in the solid lines of Fig. 2, where the nonmonotonicity arises from a competition between forces from transverse-electric (TE, \( \mathbf{E} \) in-plane) and transverse-magnetic (TM, \( \mathbf{E} \) out-of-plane) polarizations [26], which can be explained by a method-of-images argument [21]. In Fig. 2, the solid lines are computed by a \( T = 0 \) frequency-domain boundary-element method (BEM) [7], whereas the circles are computed by the \( T = 0 \) FDTD method [1,2], and both methods agree. We also compute the force at \( T = 1 \times 10^4 \pi c \bar{h} / k_B a \) where the \( \xi = 0^+ \) term dominates. We see that the FDTD method with the \( T > 0 \) modification Eq. (11) (diamonds) agrees with the frequency-domain BEM results (dashed lines), where the latter simply use the Matsubara sum (1) to handle \( T > 0 \).

Interestingly, Fig. 2 shows that the nonmonotonic effect disappears for \( T = 1 \times 10^4 \pi c \bar{h} / k_B a \), despite the fact that the method-of-images argument of [21] ostensibly applies to the \( \xi = 0^+ \) quasistatic limit (which dominates at large \( T \)) as well as to \( \xi > 0 \). The argument used the fact that TM fluctuations can be described by a scalar field with Dirichlet boundary conditions (vanishing at the metal), and in this case the sidewalls introduce opposite-sign mirror sources that reduce the interaction as \( d \) decreases; in contrast, TE corresponds to a Neumann scalar field (vanishing slope), which requires same-sign mirror sources that increase the interaction [21]. In Fig. 2, however, while the \( T = 1 \times 10^4 \pi c \bar{h} / k_B a \) TM force still decreases as \( d \) decreases, the TE force no longer increases for decreasing \( d \) at \( T = 1 \times 10^4 \pi c \bar{h} / k_B a \). The problem is that the image-source argument most directly applies to \( z \)-directed dipole sources in the scalar-field picture—electric \( J^E \) currents for TM and magnetic \( J^H \) currents for TE—while the situation for in-plane sources (corresponding to derivative of the scalar field from dipolelike sources) is more complicated [27]. For a sufficiently large \( T \) dominated by the \( \xi = 0^+ \) contribution (as here), we find numerically that the \( J^H \) sources no longer contribute to the force as \( \xi \to 0^+ \). Intuitively, as \( \xi \to 0^+ \) a magnetic dipole produces nearly constant (wavelength \( \to \infty \)) field, which satisfies the Neumann conditions and hence is not affected by geometry. Instead, numerical calculations show that the TE \( \xi = 0^+ \) contribution is dominated by \( J^E \) sources and the corresponding electric stress-tensor terms, which turn out to slightly decrease in strength as \( d \) decreases. (A related effect is that, for small \( d \), it can be observed in Fig. 2 that the \( T = 1 \) force is actually smaller than the \( T = 0 \) force, again due to the suppression of the TE contribution. Since the force diverges as \( T \to \infty \), this means that the force changes nonmonotonically with \( T \) at small \( d \); a similar nonmonotonic temperature dependence was previously observed for Dirichlet scalar-field fluctuations in a sphere-plane geometry [15].)

In contrast, if we consider the three-dimensional constant cross-section problem with \( z \)-dependent fluctuations, corresponding to integrating \( e^{ikz} \) fluctuations over \( k_z \) [20]; then we find that the nonmonotonic effect is preserved at all \( T \). This is easily explained by the fact that, for perfect metals, \( k_z \neq 0 \) is mathematically equivalent to a problem at \( k_z = 0 \) and \( \xi \to \sqrt{\xi^2 + k_z^2} \) [3,28], and so the \( n = 0 \) Matsubara term still contains contributions equivalent to \( \xi > 0 \) in which the \( J^H \) mirror argument applies and the situation is similar to \( T = 0 \). In any case, this 2D disappearance of nonmonotonicity seems unlikely to be experimentally relevant, because we find that it only occurs for \( T \geq 0.7 \times 10^4 \pi c \bar{h} / k_B a \), which for \( a = 1 \mu m \) corresponds to \( T \geq 5000 \) K.

Thus, a simple (but not too simple) modification to our previous time-domain method allows off-the-shelf FDTD
software to easily calculate Casimir forces at nonzero temperatures. Although the disappearance of nonmonotonicity employed here as a test case appears unrealistic, recent predictions of other realistic geometry and material effects [12], combined with the fact that temperature effects in complex geometries are almost unexplored at present, lead us to hope that future work will reveal further surprising temperature effects that are observable in micromechanical systems. Although various authors have debated which model $\varepsilon(\omega)$ most accurately reflects experiments [10,11,17–19], computational methods such as the one here are independent of such debates, since any desired material can be inserted in the computation (using standard techniques to express material dispersion in FDTD [23]). For example, although the perfect metals used in our examples are obviously an artificial material, it has been argued that if the perfect metal is viewed as the limit of a Drude model then the $n = 0$ contribution should be omitted [10] for the TM mode (using our TE/TM convention)—in our computational method, this is accomplished simply by dropping the second term from Eq. (11) during the TM calculation (and it turns out that nonmonotonicity still disappears in a similar manner).

ACKNOWLEDGMENTS

This work was supported in part by the Singapore-MIT Alliance Computational Engineering Flagship program, by the Army Research Office through the ISN under Contract No. W911NF-07-D-0004, by US Department of Energy Grant No. DE-FG02-97ER25308, and by the Defense Advanced Research Projects Agency (DARPA) under Contract No. N66001-09-1-2070-DOD.

[1] A. W. Rodriguez, A. P. McCauley, J. D. Joannopoulos, and S. G. Johnson, Phys. Rev. A 80, 012115 (2009).
[2] A. P. McCauley, A. W. Rodriguez, J. D. Joannopoulos, and S. G. Johnson, Phys. Rev. A 81, 012119 (2010).
[3] A. Rodriguez, M. Ibanescu, D. Iannuzzi, J. D. Joannopoulos, and S. G. Johnson, Phys. Rev. A 76, 032106 (2007).
[4] S. J. Rahi, T. Emig, N. Graham, R. L. Jaffe, and M. Kardar, Phys. Rev. D 80, 085021 (2009).
[5] A. Lambrech, P. A. Maia Neto, and S. Reynaud, New J. Phys. 8, 243 (2006).
[6] S. Pasquali and A. C. Maggs, Phys. Rev. A 79, 020102(R) (2009).
[7] M. T. H. Reid, A. W. Rodriguez, J. White, and S. G. Johnson, Phys. Rev. Lett. 103, 040401 (2009).
[8] S. G. Johnson, [http://arxiv.org/abs/1007.0966] (2010).
[9] M. Bostrom and B. E. Seremius, Phys. Rev. Lett. 84, 4757 (2000).
[10] K. A. Milton, J. Phys. A 37, R209 (2004).
[11] J. S. Hoye, I. Brevik, J. B. Aarseth, and K. A. Milton, J. Phys. A: Math. Gen. 39, 6031 (2006).
[12] A. W. Rodriguez, D. Woolf, A. P. McCauley, F. Capasso, J. D. Joannopoulos, and S. G. Johnson, Phys. Rev. Lett. 105, 060401 (2010).
[13] C. Genet, A. Lambrech, and S. Reynaud, Int. J. Mod. Phys. A 17, 761 (2002).
[14] H. Haakh, F. Intravaia, and C. Henkel, Phys. Rev. A 82, 012507 (2010).
[15] A. Weber and H. Gies, Phys. Rev. Lett. 105, 040403 (2010).
[16] M. Antezza, L. P. Pitaevski, S. Stringari, and V. B. Svetovoy, Phys. Rev. Lett. 97, 222303 (2006).
[17] R. S. Decca, D. Lopez, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, and V. M. Mostepanenko, Phys. Rev. D 75, 077101 (2007).
[18] M. Masuda and M. Sasaki, Phys. Rev. Lett. 102, 171101 (2009).
[19] A. O. Sushkov, W. J. Kim, D. A. R. Dulvit, and S. K. Lamoreaux, Nat. Phys. 7, 230 (2011).
[20] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, Statistical Physics Part 2, 3rd ed. (Pergamon, Oxford, 1980), Vol. 9.
[21] S. J. Rahi, A. W. Rodriguez, T. Emig, R. L. Jaffe, S. G. Johnson, and M. Kardar, Phys. Rev. A 77, 030101(R) (2008).
[22] S. K. Lamoreaux, Rep. Prog. Phys. 68, 201 (2005).
[23] A. Taflove and S. C. Hagness, Computational Electrodynamics: The Finite-Difference Time-Domain Method, 3rd ed. (Artech, Norwood, 2005).
[24] L. D. Landau and E. M. Lifshitz, Statistical Physics: Part I, 3rd ed. (Butterworth-Heinemann, Oxford, 1980).
[25] K. A. Milton, The Casimir Effect: Physical Manifestations of Zero-Point Energy (World Scientific, River Edge, 2001).
[26] A. Rodriguez, M. Ibanescu, D. Iannuzzi, F. Capasso, J. D. Joannopoulos, and S. G. Johnson, Phys. Rev. Lett. 99, 080401 (2007).
[27] P. Rodriguez-Lopez, S. J. Rahi, and T. Emig, Phys. Rev. A 80, 022519 (2009).
[28] T. Emig, R. L. Jaffe, M. Kardar, and A. Scardicchio, Phys. Rev. Lett. 96, 080403 (2006).