Mathematical Modelling and Performance Analysis of Single Server Two-State Batch Arrivals and Batch Service Markovian Queue with Multiple Vacations

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Abstract Here a two-state batch arrivals and batch service Markovian queue with server vacation model has been taken into consideration for its mathematical modelling and performance analysis under transient state. The present Markovian queueing model considers a single server with multiple vacations and both arrivals and services in batches of variable size. Arriving customers are processed by a single server under FIFO queue discipline and arrival process follows Poisson distribution. In addition to this, both service time and vacation time of the server follows exponential distribution. Using Laplace transforms technique, solution of governing differential-difference equations of the model has been achieved and significant performance measures of the model have been explored and some special cases have been discussed as well. Finally, some valuable conclusive observations have been found and a few suggestions for future research of the model have been focused.

Keywords: Markovian queue; batch arrival and batch service, single server with multiple vacations; transient state analysis, performance measures

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1. Introduction

Markovian queueing models with bulk arrival and batch services have achieved considerable attention by several researchers. We come across many queueing situations in health services, manufacturing and production industries where both arrival and departure of customers/units occur in batches either fixed or variable sizes. It can be observed in the literature of queueing theory that A.K. Erlang’s solution of M/E_k/1 queue (Brockmeyer et al. 1948) [7] initiated the mathematical study of bulk queues. Later, some other noteworthy researchers, e.g. Gaver [16], Bailey [3] among others confined their attention in this direction. Rigorous study of literature reveals that Bhat [5] analyzed a single server bulk queue by using the imbedded Markov chain technique. Subsequently, some other pioneer research works explored on bulk queues can be found by Gupta and Goyal [18], Gaur [15], Borthakur and Medhi [6], Prabhu [32], Abolnikov et al. [1], Tadj and Ke [39,40], Chang and Choi [8], Suzuki [38], Chen A. et al. [11]. Moreover, Banerjee et al. [4] analysed a single server finite buffer bulk arrival and departure queueing model. Recently, Maurya [26,29,30] confined their attention to analyse queueing models with either bulk arrivals or bulk services or both in batches.

In earlier research works carried out by worth mentioning researchers as stated herein, it has been considered usually that server is available instantaneously to serve the customers demanding for their service. But the availability of a server in many queueing situations occurring in real life may not be instantaneous particularly in queue with machine repair model. In machine repair model, the service facility depends on machines and the service process is interrupted due to several reasons of occasional random failures including human and common-cause failures. In manually operated facilities the service may interrupt either due to stress or some other urgent work as per the requirement of the organization. These interruptions affect the different parameters of queueing systems. In the initial stage, some significant contributions in this connection are worth mentioning e.g. White and Christie [44], Heathcote [19], Thiruvengadam [42], Fischer [14]. Later, Jain et al. [21] focused their attention to deal a k-out-of-n system with dependent failure and standby support. Moreover, for further details in this connection we refer remarkable works of several researchers Chang Who [9], Gupta [17], Jain et al. [21]. Very recently Maurya [27,28] succeeded to present mathematical modelling for performance analysis and computational approach to cost and profit analysis of k-out-of-n repairable system involving human error and system failure constraints. All these studies the queueing...
situation under which the service is interrupted due to one or other reasons.

Sharda [35,36] explored the solution of bulk queuing models with the concept of intermittently available server. Chaudhry et al. [12] succeeded to find out both the transient and steady-state solution of a bulk arrival queue with intermittently available server. However, it has been keenly observed that the complexity of stochastic systems also increased due to the requirement of high tech system in mid of 20th century. The concept of dealing queuing models with server vacation was developed in 1970’ s by increasing the dimension of classical queuing theory. In queuing systems with server vacation, server can take vacations to perform other important jobs or to complete some additional work. Vacations represent server’s working on some supplementary jobs such as maintenance, inspection and repairs, or server’s failures that interrupt the service. It should be remarkable that allowing servers to take vacations makes queuing models more flexible in finding optimal service policies by sharing scare resources. Moreover, the server vacation policy reduces total cost of providing service. In this connection, some noteworthy researchers confined their attention e.g. Cooper [13] presented a study on queues served in a cycle order, in which the time period of serving other queues can be considered a service interruption of the queue under consideration. In multiple vacations policy server may keep taking vacations until it finds at least one customer waiting in the system at the time of vacation completion. Levy and Yechiali [25] who introduced the concept of server’s taking vacations that represent the durations of the server’s work on some supplementary projects, many research results on vacation models were published. Tian and Zhang [43] presented various vacation models with exhaustive and non-exhaustive service. Baba [2] also made a significant attempt in this direction. Jain et al. [22] examined $M^X/H_k/1$ queue with unreliable server and multiple vacation policy. Subsequently, bulk queue with multiple vacations was studied by various authors e.g. Lee and Srinivasan [24], Takine and Hasegawa [41], Reddy et al. [34], Sikdar and Gupta [37], Choudhury et al. [12], Tadj and Ke [40] and Kalyanaraman et.al. [23]. Very recently Maurya [29] focused to maximum entropy analysis of $M^Y/(G_1,G_2)/1$ retrial queueing model with second phase optional service and Bernoulli vacation schedule and succeeded to explore some significant performance measures.

Also, much of the research on queuing models studied steady-state operation only. But we come across many queuing situations where performance measures based on time-dependent are greatly required in order to just getting optimum results. A number of systems starts operations and stopped operations at some time point say t. Business and service operations never operate under steady-state conditions. Furthermore, if there is no one for service initially, the busy or ideal time of the server & initial rate of output etc., will not be as per the steady state values. So, the steady-state solutions are not appropriate to represent the system accurately under those essential conditions of the systems. Thus to understand and improve the behaviour of customers and queuing systems effectively, transient analysis is must from the point of theory as well as its application.

Here a two-state batch arrivals and batch service Markovian queue with server vacation model has been taken into consideration for its mathematical modelling and performance analysis under transient state. The present Markovian queueing model considers a single server with multiple vacations and both arrivals and services in batches of variable size. Arriving customers are processed by a single server under FIFO queue discipline and arrival process follows Poisson distribution. In addition to this, both service time and vacation time of the server follows exponential distribution. In the present work, our key attempt is to examine the transient behaviour of a two-state bulk arrival bulk service queueing model with single server and multiple vacations. The effectiveness of exploring two-state transient probabilities lies in deliberating the following key facts:

i. How many units will arrive and depart simultaneously by a given time,

ii. How many units will arrive within a given time,

iii. How many units will be served during given time and other related information?

Here it should be notable that the application of queuing model taken into our present consideration can be found in manufacturing industries. For example, a practical situation of present queueing model can be found in a bakery. The products are made in batches in the oven. When raw material mixture for breads is not available then the baker will do maintenance or cleaning. Here, raw mixture corresponds to arrivals and baking of breads as service. The maintenance and cleaning represents vacation.

2. Assumptions and Notations of the Model

In order to establish mathematical modelling and performance analysis of the queueing model taken into present consideration, we use following assumptions and notations throughout the current study:

i. The customers/units are processed for their service under FIFO queue discipline.

ii. Arrivals occur in batches which follows Poisson distribution with mean arrival rate $\lambda$.

iii. Service times follow an exponential distribution with mean service rate $\mu$.

iv. Vacation time of the server is exponentially distributed with parameter $V$.

v. Different stochastic processes involved in the system are statistically independent.

vi. The server capacity is a random variable. Batch size is determined at the beginning of each service and either it is equal to the total number of customers waiting or the capacity of the service facility determined afresh before each service whichever is less.. The probability that the service channel can serve $k_2$ units, is $d_{k_2}$ , where $\sum_{k_2=1}^{Y} d_{k_2} =1$, $Y$ is the maximum capacity of the server.

vii. $\lambda a_{k_1} \Delta t (k_1 = 1,2,3,...)$ is the first order probability of $k_1$ arrivals in the short interval of time $\Delta t$, where
P(X = k_l) = a_{k_l} \cdot \sum a_{k_l} = 1 \text{ and } \lambda > 0 \text{ is the mean arrival rate.}

Vii. \ P_{i,j,V}(t) = \text{Probability that there are exactly i arrivals and j departures by time } t \text{ and the server is on vacation in relation to the queue; } i \geq j \geq 0

ix. \ P_{i,j,B}(t) = \text{Probability that there are exactly i arrivals and j departures by time } t \text{ and the server is busy in relation to the queue; } i > j > 0

x. \ P_{i,j}(t) = \text{Probability that there are exactly i arrivals and j departures by time } t; \ i \geq j \geq 0

3. Mathematical Formulation of the Model

i. \ \sum_{t=1}^{n} 1, \text{ The summation over all those permutations of } n \text{ objects taken } u (u=1,2,...,n) \text{ at a time, such that, } \sum_{t=1}^{n} r_t = n \text{ with } r_t > 0.

For example, when } n=3\text{ 3 \text{ is the sum of permutation of 3 taken one (u=1) at a time i.e., } r_1 = 3; \text{ permutations of 3 taken two (u=2) at a time i.e., } r_1 + r_2 = 3; \text{ and permutations of 3 taken (u=3) at a time, i.e. } r_1 + r_2 + r_3 = 3.

ii. The inverse Laplace transform of } \frac{Q(p)}{P(p)} \text{ is given as following;}

\[ \sum_{k=1}^{n} \sum_{\substack{k}} t^{m_k} e^{-p_k} \frac{d^{m_k-1} Q(p)(p-a_k)^{m_k}}{p^{m_k}} \] 

where,

\[ P(p) = (p-a_1)^{m_1}(p-a_2)^{m_2}(p-a_3)^{m_3}...(p-a_n)^{m_n} \text{ and } Q(p) \text{ is polynomial of degree } m_1 + m_2 + m_3 + ... + m_n - 1\]

iii. If \ L^{-1}\{f(s)\} = F(t) \text{ and } L^{-1}\{g(s)\} = G(t) \text{ , then } \frac{F(u)G(t-u)}{} = F*G,

\[ F*G \text{ is called the convolution of } F \text{ and } G.\]

iv. The inverse Laplace of \ \frac{B_{a,b}^h(t)}{B_{a,b}^h(s)} = \frac{1}{(s+a)^\alpha(s+b)^\beta} \text{ using (ii) is}

\[ e^{-at} \frac{e^{-bt}}{(b-a)^\beta} \text{ for } \alpha, \beta > 0 \text{ and } a < b. \]

v. The inverse Laplace of \ \frac{B_{a,b}^h(s)}{B_{a,b}^h(s)} = \frac{1}{(s+a)(s+b)^\beta} \text{ using (ii) is}

\[ e^{-at} \frac{e^{-bt}}{(a-b)^\beta} \text{ for } \alpha, \beta > 0 \text{ and } a < b. \]

vi. \ \sum_{\alpha}^a = 1 \text{ and } \sum_{\beta}^\beta = 0 \text{ for } \beta < \alpha \text{ and } \delta_{i,j} = \begin{cases} 1, \ i = j \\ 0, \ i \neq j \end{cases} \text{ Initially, for the present queueing model we have}

\[ P_{0,0,V}(0) = 1 \quad P_{0,0,B}(0) = 0 \]

4. Governing Differential-Difference Equations of the Model

The governing differential-difference equations of the queueing model taken into consideration are as following:
\[
\frac{d}{dt} P_{i,j,V}(t) = -\lambda P_{i,j,V}(t) + \mu \sum_{e=1}^{\text{Y}} \sum_{g=1}^{\text{Y}} d_e P_{i-e,j,B}(t); \\
Y > i \geq 0
\]  

\[
\frac{d}{dt} P_{i,j,V}(t) = -\lambda P_{i,j,V}(t) + \mu \sum_{e=1}^{\text{Y}} \sum_{g=1}^{\text{Y}} d_e P_{i-e,j,B}(t); \\
i \geq Y
\]  

\[
\frac{d}{dt} P_{i,j,V}(t) = -(\lambda + v)P_{i,j,V}(t) \\
+ \mu \sum_{e=1}^{i-j} a_e P_{i-e,j,V}(t); \\
i > j \geq 0
\]  

\[
\frac{d}{dt} P_{i,j,B}(t) = -(\lambda + \mu)P_{i,j,B}(t) \\
+ \mu \sum_{e=1}^{i-j} d_e P_{i-e,j,B}(t) + \lambda \sum_{e=1}^{i-j} a_e P_{i-e,j,B}(t) (1 - \delta_{i-j}) \\
+ vP_{i,j,V}(t); \\
i > j \geq 0, j < Y
\]  

\[
\frac{d}{dt} P_{i,j,B}(t) = -(\lambda + \mu)P_{i,j,B}(t) \\
+ \mu \sum_{e=1}^{i-j} d_e P_{i-e,j,B}(t) + \lambda \sum_{e=1}^{i-j} a_e P_{i-e,j,B}(t) (1 - \delta_{i-j}) \\
+ vP_{i,j,V}(t); \\
i > j, j \geq Y
\]  

\[
P_{i,j}(t) = P_{i,j,V}(t) + P_{i,j,B}(t)(1 - \delta_{i,j})
\]  

5. Solution of the Differential-Difference Equations of the Model

Applying Laplace transforms on equations (2) to (6) along with equation (1) and solving recursively, we get following algebraic equations:

\[
\tilde{P}_{0,0,V}(s) = \left( \frac{1}{s + \lambda} \right)
\]  

\[
\tilde{P}_{0,0,J}(s) = \sum_{h=1}^{\text{Y}} \left( \sum_{\eta=i}^{\text{Y}} a_{\eta} \right) \left( \frac{\lambda}{s + \lambda + \mu} \right)^{(1-\delta_{i,J})} \left( \frac{\mu}{s + \lambda + \mu} \right)^{(0)}
\]  

\[
\tilde{P}_{i,j,V}(s) = \sum_{i=1}^{\text{Y}} \left( \sum_{g=1}^{\text{Y}} \frac{ud_g}{s + \lambda} \right) \tilde{P}_{i-g,j,B}(s); \\
Y > i > 0
\]  

\[
\tilde{P}_{i,j,V}(s) = \sum_{i=1}^{\text{Y}} \left( \sum_{g=1}^{\text{Y}} \frac{ud_g}{s + \lambda} \right) \tilde{P}_{i-g,j,B}(s); \\
i \geq Y
\]  

\[
\tilde{P}_{i,j,B}(s) = \sum_{h=1}^{\text{Y}} \left( \sum_{\eta=i}^{\text{Y}} a_{\eta} \right) \left( \frac{\lambda}{s + \lambda + \mu} \right)^{(1-\delta_{i,h})} \left( \frac{\mu}{s + \lambda + \mu} \right)^{(0)}
\]  

\[
\tilde{P}_{j,j,B}(s) = \sum_{h=1}^{\text{Y}} \left( \sum_{\eta=i}^{\text{Y}} a_{\eta} \right) \left( \frac{\lambda}{s + \lambda + \mu} \right)^{(1-\delta_{j,h})} \left( \frac{\mu}{s + \lambda + \mu} \right)^{(0)}
\]  

\[
\tilde{P}_{j,j,B}(s) = \sum_{h=1}^{\text{Y}} \left( \sum_{\eta=i}^{\text{Y}} a_{\eta} \right) \left( \frac{\lambda}{s + \lambda + \mu} \right)^{(1-\delta_{j,h})} \left( \frac{\mu}{s + \lambda + \mu} \right)^{(0)}
\]
In order to solve algebraic equations (7) to (15), taking inverse Laplace transforms of equations (7) to (15); from which it is fairly easy to get their solutions respectively as given in following equations:

$$P_{i,j,V}(t) = e^{-\lambda t}$$  \hspace{1cm} (16)

$$P_{i,j,V}(t) = \sum_{\eta=0}^{\infty} \mathbb{P}_{g=1} \left\{ \left( t \right)^{\eta} \mathbb{B}_{i,j,v}(t) \right\}$$  \hspace{1cm} (17)

$$P_{i,j,V}(t) = \sum_{\eta=i}^{\infty} \mathbb{P}_{g=1} \left\{ \left( t \right)^{\eta} \mathbb{B}_{i,j,v}(t) \right\}$$  \hspace{1cm} (18)

$$P_{i,j,V}(t) = \sum_{\eta=0}^{\infty} \mathbb{P}_{g=1} \left\{ \left( t \right)^{\eta} \mathbb{B}_{i,j,v}(t) \right\}$$  \hspace{1cm} (19)

$$P_{i,j,V}(t) = \sum_{\eta=0}^{\infty} \mathbb{P}_{g=1} \left\{ \left( t \right)^{\eta} \mathbb{B}_{i,j,v}(t) \right\}$$  \hspace{1cm} (20)

$$P_{i,j,B}(t) = \sum_{\eta=0}^{\infty} \mathbb{P}_{g=1} \left\{ \left( t \right)^{\eta} \mathbb{B}_{i,j,v}(t) \right\}$$  \hspace{1cm} (21)

$$P_{i,j,B}(t) = \sum_{\eta=0}^{\infty} \mathbb{P}_{g=1} \left\{ \left( t \right)^{\eta} \mathbb{B}_{i,j,v}(t) \right\}$$  \hspace{1cm} (22)

$$P_{i,j,B}(t) = \sum_{\eta=0}^{\infty} \mathbb{P}_{g=1} \left\{ \left( t \right)^{\eta} \mathbb{B}_{i,j,v}(t) \right\}$$  \hspace{1cm} (23)
The probability of exactly \( n \) customers in the system at time \( t \), denoted by \( P_n(t) \), in terms of \( P_{ij,B}(t) \) as

\[
\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} P_{ij,B}(t) = \sum_{i=1}^{\infty} \left( \sum_{j=1}^{i-1} P_{ij,B}(t) \right) (1-\delta_{ij}) + \sum_{j=1}^{\infty} \left( \sum_{i=1}^{j-1} P_{ij,B}(t) \right) (1-\delta_{ij}) = 1
\]

It represents the probability of arrivals is not affected by the vacation time of the server.

The Laplace transform of the mean number of arrivals is as following: \( \sum_{i=0}^{\infty} \bar{P}_{i,V}(s) = \frac{\lambda}{s^2} \) and the Laplace inverse of the mean number of arrivals by time \( t \) is as following: \( \sum_{i=0}^{\infty} \bar{P}_{i,V}(t) = \lambda t \)

The probability that exactly \( j \) customers have been served by time \( t \) ( \( P_{j}(t) \) ) can be obtained by

\[
P_{j}(t) = \sum_{i=0}^{j} P_{ij,B}(t)
\]

The density function of \( T \), the time for the \( j \)th departure, can be obtained as

\[
f_T(t) = \sum_{k=j}^{\infty} \left( \frac{dP_k(t)}{dt} \right). \text{ Since } P(T \leq t) = \text{ Prob. } \{ \text{at least } j \text{ departures in } (0,t) \} = \sum_{k=j}^{\infty} P_k(t)
\]

The probability of exactly \( n \) customers in the system at time \( t \), denoted by \( P_n(t) \), in terms of \( P_{ij,B}(t) \) as

\[
P_n(t) = \sum_{j=0}^{\infty} P_{j+n,j}(t)
\]

The waiting time distribution for a customer can be derived as \( P(W > T/t) \), the probability that a customer waits more than \( T \) time units in the system, given that the customer arrives at time \( t \).

\[
P(W > T/t) = \sum_{n=0}^{\infty} P(W > T/n \text{ customers in the system at time } t) P_n(t)
\]

\[
= \sum_{n=0}^{\infty} P(\text{number of service by time } t < n+1) P_n(t)
\]

\[
= \sum_{n=0}^{\infty} \sum_{g=0}^{\infty} e^{-ut} \frac{\mu^g}{g!} P_n(t)
\]

The system utilization can also be obtained in terms of \( P_{ij,B}(t) \). The probability that the system is empty at some time \( t_1 \) is \( \sum_{j=0}^{\infty} P_{j,j}(t_1) \). The idle time of server is given by

\[
I(t) = \frac{1}{t_1} \int_{0}^{t_1} \sum_{j=0}^{\infty} P_{j,j}(t_1) \, dt_1 \text{ and therefore the server utilization time is given by } U(t) = 1 - \frac{1}{t} \int_{0}^{t} \sum_{j=0}^{\infty} P_{j,j}(t_1) \, dt_1.
\]

7. Special Cases of the Model

**Case I:** When arrivals and departures both are occurring one by one, i.e. by substituting \( \alpha_1 = 1 \) and \( \alpha_2, \alpha_3, \alpha_4, \ldots = 0 \) and \( d_1 = 1 \), for \( Y = 1 \); \( d_Y = 0 \), for \( Y > 1 \) in equations (16) to (24), we obtain
\[ P_{0,0,V}(t) = e^{-\lambda t} \]  
\[ P_{i,0,V}(t) = \left( \frac{\lambda^i}{i!} \right) \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i ; \quad i > 0 \]  
\[ P_{i,j,Y}(t) = \left( \frac{\lambda^i}{i!} \right) \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i \mathbf{P}_{i-j,B}(t) ; \quad i > j > 0 \]  
\[ P_{i,i,Y}(t) = \left( \frac{\mu e^{-\lambda t}}{i!} \right) \mathbf{P}_{i-i,B}(t) ; \quad i > 0 \]  
\[ P_{i,0,B}(t) = \left( \frac{\lambda^i}{i!} \right) \sum_{h=1}^{Y} \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i \mathbf{P}_{i-h,B}(t) ; \quad i > 0 \]  
\[ P_{i,j,B}(t) = \left( \frac{\lambda^i}{i!} \right) \sum_{h=1}^{Y} \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i \mathbf{P}_{i-h,B}(t) ; \quad i > 0 \]  
\[ P_{i,j,B}(t) = \left( \frac{\lambda^i}{i!} \right) \sum_{h=1}^{Y} \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i \mathbf{P}_{i-h,B}(t) ; \quad i > j > 0 \]  
\[ P_{i,i,B}(t) = \left( \lambda^i \right) \sum_{h=1}^{Y} \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i \mathbf{P}_{i-h,B}(t) ; \quad i > 0 \]  
\[ P_{i,j,B}(t) = \left( \lambda^i \right) \sum_{h=0}^{Y} \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i \mathbf{P}_{i-h,B}(t) ; \quad i > 0 \]

The results (25) to (30) coincide with the results (1.2.15) to (1.2.20) of Indra [20].

**Case II:** When server is instantaneously available, i.e. the mean vacation time is zero. By taking \( v \to \infty \) in equations (25) to (30), the results will coincide with the findings of Pegden and Rosenshine [31].

**Case III:** The results for the case, when departures are occurring in batches of variable size and arrivals are occurring one by one, are obtained by substituting \( a_1 = 1 \) and \( a_2, a_3, a_4, \ldots = 0 \) in equations (16) to (24).

\[ P_{0,0,V}(t) = e^{-\lambda t} \]  
\[ P_{i,0,V}(t) = \left( \frac{\lambda^i}{i!} \right) \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i ; \quad i > 0 \]  
\[ P_{i,i,V}(t) = \left( \frac{\lambda^i}{i!} \right) \sum_{h=1}^{Y} \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i \mathbf{P}_{i-h,B}(t) ; \quad i > 0 \]  
\[ P_{i,j,B}(t) = \left( \lambda^i \right) \sum_{h=0}^{Y} \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i \mathbf{P}_{i-h,B}(t) ; \quad i > 0 \]  
\[ P_{i,j,B}(t) = \left( \lambda^i \right) \sum_{h=0}^{Y} \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i \mathbf{P}_{i-h,B}(t) ; \quad i > 0 \]  
\[ P_{i,j,B}(t) = \left( \lambda^i \right) \sum_{h=0}^{Y} \left( \mathbf{B}_{\lambda,\lambda,V}(t) \right)^i \mathbf{P}_{i-h,B}(t) ; \quad i > 0 \]

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**Case IV:** When the mean vacation time is zero, i.e. by letting \( v \to \infty \) in equations (31) to (39). The results will coincide with the findings of Prem Chand [33].

### 8. Discussions and Conclusive Observations

A single server Markovian queueing model with several features (i) bulk arrival and bulk service policy (ii) two-state and multiple vacations, has been dealt here extensively under transient state conditions and its significant performance measures have been explored. Various useful performance measures of following four special cases have also been investigated:

i. When arrivals and departures of units both are occurring one by one,

ii. When server is instantaneously available,

iii. When departures occur in batches of variable size and arrivals occur one by one,

iv. When the mean vacation time is zero.

Moreover, under our present study it has also been examined that the results carried out herein agree with corresponding results of aforesaid mentioned models explored by previous researchers. In addition to this, mean while demonstration for the justification of two-state transient model in many practical situations has been focused. This model can be applied in the design of computer networks. Simulated results can be explored by means our analytical results explored herein for its numerical illustration. Finally, it is suggested here that the present study can be extended with other concepts such as non-Markovian assumptions etc.

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