Automatically Proving Microkernels Free from Privilege Escalation from their Executable

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Abstract—Operating system kernels are the security keystone of most computer systems, as they provide the core protection mechanisms. Kernels are in particular responsible for their own security, i.e. they must prevent untrusted user tasks from reaching their level of privilege. We demonstrate that proving such absence of privilege escalation is a pre-requisite for any definitive security proof of the kernel. While prior OS kernel formal verifications were performed either on source code or crafted kernels, with manual or semi-automated methods requiring significant human efforts in annotations or proofs, we show that it is possible to compute such kernel security proofs using fully-automated methods and starting from the executable code of an existing microkernel with no modification, thus formally verifying absence of privilege escalation with high confidence for a low cost. We applied our method on two embedded microkernels, including the industrial kernel AnonymOS†‡: with only 58 lines of annotation and less than 10 minutes of computation, our method finds a vulnerability in a first (buggy) version of AnonymOS and verifies absence of privilege escalation in a second (secure) version.

I. INTRODUCTION

Context. The security of many computer systems builds upon that of its operating system kernel. We define a kernel as a computer program that prevents untrusted code from performing arbitrary actions, in particular performing arbitrary hardware and memory accesses². Kernels that fail to prevent this are said to be vulnerable to privilege escalation attacks³. As this vulnerability is of the highest severity and can affect any kernel, formally verifying that kernels cannot be exploited by untrusted code to gain access to hardware privilege is of the uttermost importance. Actually, proving absence of privilege escalation (APE) is a mandatory step when attempting to formally verify a kernel: nothing can be proven unconditionally about a kernel unless this property holds.

Scope. Besides large well-known monolithic kernels (e.g., Linux, Windows, *BSD) whose size and complexity are currently out of reach of formal verification, there is a rich ecosystem of small-size kernels found in many industrial applications, such as security or safety-critical applications, embedded or IoT systems. This includes security-oriented kernels like separation kernels [7], microkernels [8], exokernels [9] and security-oriented hypervisors [10], [11] or enclave software [12]; but also kernels used in embedded systems, for example in microcontrollers [13], real-time [14], [15] or safety-critical [16], [17] operating systems.

We focus on such small-size kernels. To have a practical impact on these systems, a formal verification must be:

• Non-invasive: the verification should be applicable to the kernel as it is. Formal verification methods that require heavy annotations or rewrite in a new language are highly expensive and require developers with a rare combination of expertises (OS and formal methods);
• Automated: the cost and effort necessary to perform a formal verification should be minimized. Formal verification techniques that are manual (e.g., proof assistants) or semi-automated (e.g., deductive verification) require a large proof or annotation effort;
• Close to the running system: Verification should be performed on the machine code [18], [19] in order to remove the whole build chain (compiler, linker, compilation options, etc.) from the trust base. Machine code verification is all the more important on kernels, as they contain many error-prone low-level interactions with the hardware, not described by the source-level semantics.

Despite significant advances in the last decades [2], [8], [11], [12], [18], [20]–[26], existing kernel verification methods do not address these issues. In most cases, verification is applied to microkernels developed or rewritten for the purpose of formal verification (except [22]), and is performed only on source [21], [22] or assembly [2], [11], [25], [26], using highly expensive manual [8], [18], [21], [22] or semi-automated [2], [11], [12], [23], [24] methods. For example, the functional verification of the SeL4 microkernel [8] required 200,000 lines of annotations and still left parts of the code unchecked (boot, assembly).

Goal and challenges. We focus on the key property of privilege escalation, and seek to design a fully-automated program analysis able to prove the absence of privilege escalation...
in microkernels for embedded systems, directly from their executable. Besides the well-documented difficulty of static analysis of machine code [27], solving this goal poses two main technical challenges:

- Automatically proving absence of privilege escalation requires a formal definition that is generic (i.e. independent of the kernel) and suitable to the machine-code level. Indeed, absence of privilege escalation is usually established through higher-level properties such as safety of control-flow and memory together with preservation of invariants on protection mechanisms [11], but each of these properties requires an in-depth specification of the kernel behavior and knowledge of its source code—preventing automated machine-level verification. In addition, this definition should be amenable to static analysis, preferably using standard techniques;
- Most kernels are parameterized systems designed to run an arbitrary number of tasks. This is also true for microkernels in embedded systems: even if the number of tasks, size of scheduling tables and communication buffers often do not vary during execution, they depend on the application using the kernel. A flat representation of memory (enumerating all memory cells) [24] is no longer sufficient in such a setting, and we need more complex representations able to precisely summarize memory, like shape abstract domains [28], [29]. Unfortunately, they usually require a large amount of manual annotations, which defeats our goal of automation.

Contributions. We propose BINSEC/CODEX, a novel static analysis for proving absence of privilege escalation in microkernels from their executable. Our contributions include:

- An original formal model (Section III) suitable for defining privilege escalation attacks on parameterized kernel code and allowing to reduce the proof of absence of privilege escalation to a standard program analysis problem (finding non-trivial state invariants, Theorem 4), hence reusing the standard program analysis machinery. We also prove that absence of privilege escalation is the most fundamental kernel property, without which nothing can be proved (Theorem 3);
- A new 3-step methodology (Section IV) for proving absence of privilege escalation of parameterized kernels from their executable, featuring 1. automated extraction of most of the analysis (shape) annotations from kernel types; 2. parameterized fully-automated binary-level static analysis inferring an invariant of the kernel under a precondition on the user tasks, and 3. fully-automated method to check that the user tasks satisfy the inferred precondition;
- A novel weak shape abstract domain (Section V) able to verify the preservation of memory properties in parameterized kernels. This domain is efficient (thanks to a dual flow sensitive/flow insensitive representation), easily configurable (based on the memory layout of C types, most of the annotations are extracted automatically), and suitable to machine code verification (e.g., addressing indexing of data structures using numerical offsets);
- A thorough evaluation of our method on two different microkernels (Section VI), using two different instruction sets (x86 and ARMv7) and memory protection mechanisms (segmentation and pagination). This includes the study of an ARM Cortex-A9 port of AnonymOS\(^4\). The method is able to find a vulnerability in a beta version of this kernel, and to verify the absence of privilege escalation in a later version, in less than 450 seconds and with only 58 lines of manual annotations—several order of magnitudes less than prior verification efforts (Table IV, p. 13).

This work is the first OS verification effort to specifically address absence of privilege escalation. It is also the first to perform formal verification on an existing operating system kernel without any modification, on machine code, and the first to do so using a fully-automated technique able to handle parametrization. Finally, it is the first shape analysis performed on machine code.

We thus show that, contrary to a widespread belief [8], fully-automated methods like static analysis can be used to verify complex properties such as absence of privilege escalation in embedded microkernels, directly from their executable.

Limitations. Like any sound static analyzer, BINSEC/CODEX may be too imprecise on some code patterns, emitting false alarms. Currently, our analysis cannot handle dynamic task spawning nor dynamic modification of memory repartition, as well as self-modification or code generation in the kernel. Still, many microkernels and hypervisors fall in our scope [6], [7], [10], [11], [14]–[17], [22], [30].

Finally, while absence of privilege escalation is arguably the most important property of a kernel, verifying task separation is also of great importance. This is left as future work.

II. OVERVIEW

A. System description, attacker model and trust base

System description. We consider a computer system consisting in hardware running both untrusted software (user tasks) and security-critical software (including the kernel). Our goal is to ensure that the only code running as privileged is the uncompromised, security-critical code, where being privileged

\(^4\)AnonymOS is a concurrent industrial microkernel developed by AnonymFirm\(^1\), a leading tool provider for safety-critical real-time systems, with presence in the aerospace, automotive, and industrial automation markets.

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typically corresponds to having a hardware flag set (supervisor mode). Section III formalizes these notions.

**Attacker model.** The attack goal is to escalate privilege, either by running untrusted software with privilege or by injecting code into the security-critical software.

The attacker controls the user image — containing the user tasks code and data — loaded with the kernel before boot; can perform any software-based attack, such as modifying user task code and memory at runtime; but cannot make the hardware deviate from its specification, and thus cannot perform physical attacks nor exploit hardware backdoors or glitches [31].

**Trust base.** We want to trust a minimal number of components:

- the software used to load the kernel and user tasks in memory (bootloader, EEPROM flasher, etc.),
- the tools used to perform the formal verification.

Note that we do not need to trust the kernel source code nor any software producing the executable (e.g., compiler, assembler, linker, or build scripts).

**B. Illustrative example of an OS kernel**

To better understand privilege escalation attacks, how a kernel is structured to prevent these attacks, and how we verify that these attacks are impossible, we use, as an example, the barebone OS kernel of Figure 2. Although minimal, it handles an arbitrary number of tasks and features scheduling, memory protection and multiple levels of privilege. In particular, it should protect itself from the user tasks and be immune to privilege escalation attacks. The example is written in pseudo-C, but remember that our analysis is performed on machine code.

**Execution context of a task.** This kernel performs context switching between pre-defined tasks. Assuming a stack machine programming model, an execution context is defined here by a read-only executable code segment, a writable data segment holding the stack, a status register flags, and two registers pc (program counter) and sp (stack pointer) respectively pointing inside the code and data segments.

**Privilege level.** A bit inside the flags register indicates whether execution is privileged. Being privileged allows executing instructions that would change the privilege in the flags registers, or would change the values of the system registers (mpu1, mpu2, pc', sp' and flags'), using ' to denote what is sometimes called banked or shadow registers.

**Memory protection.** A running task can access only the memory inside its code and data segment. This is enforced by the hardware through a Memory Protection Unit (MPU) controlled by two system registers mpu1 and mpu2. Each mpu register enables access to the memory addresses in a certain range for reading, writing or execution, depending on the contents of the register. When the processor is in unprivileged mode, all other memory accesses are forbidden, while in privileged mode the MPU is bypassed and the whole memory can be accessed.

**Kernel.** The kernel is the only program which is supposed to run privileged. Kernel execution proceeds as follows:

1) First, an interrupt occurs, which is the only way to switch from unprivileged to privileged execution. The hardware saves the context of the executing task and begins executing the kernel. In our example, the hardware: (a) saves the value of pc, sp, and flags; (b) restores the kernel stack pointer sp to the kernel stack and restores flags to allow privileged execution; and (c) sets pc to begin executing the kernel function;

2) Then, the kernel dispatches the execution according to the interrupt received. The only special case is the RESET interrupt, which boots (i.e. initializes) the system. Here, it consists in setting cur, a variable always pointing to the task that has been or will be executed. For all other interrupts the system is already executing a task, and its execution context must be saved in memory before we can switch to another task;

3) After that, the kernel chooses another task to be scheduled — here with a simple round-robin method, so it is sufficient to follow the next field in a circular list;

4) Then, the kernel switches to the context of the task being executed. Besides restoring the working values of the registers previously saved, the kernel correctly sets up the memory protection by updating system registers mpu1 and mpu2;

5) Finally the kernel returns from the interrupt, swapping the values of pc, sp, and flags with their primed counterparts. Here, the kernel relies on the invariant that flags' is such that after this instruction, the processor will be in unprivileged mode.

Note that interrupts are masked during kernel execution, i.e. kernel execution cannot be interrupted.

**User image.** System execution depends on the kernel code, but is also parameterized by the user (tasks) image, part of the initial state. Here, the image is made by C code statically allocating Task data structures, such that: (1) their pc and sp fields point into their respective code and stack sections; (2) the code_segment and data_segment fields should give the appropriate rights to these sections; (3) flags ensures that execution is not privileged; and (4) the next fields are such that all tasks in the system are in a circular list. Figure 9 (Appendix, p. 16) gives an example of C code for a user image.

Getting a working system simply requires to compile the user image and load it with the kernel. Such static system generation is common in embedded systems, where OS vendors and application developers are separate entities.

**C. Threats and mitigation**

**Attacks.** This kernel is secure in that it effectively prevents privilege escalation. However, slight changes in its behavior could allow a variety of attacks that we illustrate here:

- **Attacks targeting memory safety:** data corruption on one of the flags field of a Task would allow a user task to raise its privilege. Corruption can come e.g. from a stack overflow or following an invalid next or cur pointer;
Absence of privilege escalation is a fragile property. On the initially true property that holds for all reachable states, because it is:

- Belong to the invariant computed by our method of the memory protection mechanism. The properties below properties about memory, control-flow and correct working execute with hardware privilege (and that this code cannot finding an invariant implying that only the kernel code can based attack can lead to privilege escalation by automatically the problem of defining Absence of Privilege Escalation. Absence of privilege escalation does not per se imply memory safety nor control-flow safety—even if our approach proves these properties on the example kernel as a byproduct. Indeed, not all bugs are security critical, and a system suffering from a limited 1-byte stack overflow can still be secure while not respecting strict memory safety. Also, control-flow safety and memory-safety are very hard to define at machine code-level, as we lack information about code and data layout.

Hence, a first challenge here is to provide a formal definition of Privilege Escalation suitable to machine code analysis.

D. The case for automated verification of OS kernels

We can distinguish three classes of verification methods:

- Manual [8], [17], [18], [21], [22]: the user has to provide for every program point a candidate invariant, then prove via a proof assistant that every instruction preserves these candidate invariants;
- Semi-automated5 [2], [10]–[12], [23], [24]: the user has to provide the candidate invariants at some key program points (kernel entry and exit, loop and function entries and exits) and then use automated provers to verify that all finite paths between these points preserve the candidate invariants;
- Fully-automated: a sound static analyzer [32] automatically infers correct invariants for every program point. The user only provides invariant templates by selecting or configuring the required abstract domains.

Experience has shown that in OS formal verification “invariant reasoning dominates the proof effort”6 [8], [20], motivating our choice for fully-automated methods.

E. Challenges

Besides a suitable definition of Privilege Escalation, analyzing real microkernels adds extra challenges.

Machine code analysis. Binary-level static analysis is already very challenging on its own [27], [33], [34] as (1) the control-flow graph is not known in advance because of computed jumps7 (e.g. jmp @sp) whose resolution requires runtime values, (2) memory is a single large array of bytes with no

```
typedef struct {
  int32 pc, sp, flags;
  int64 code_segment, data_segment;
} struct

struct task *next;
} Task;
Task *cur;
extern Task task0;

register int32 sp, pc, flags, sp', pc', flags';
register int64 mpu1, mpu2;

void kernel(int32 interrupt_number) {
  /* Interrupt transition, done in hardware. */
  swap(sp,sp'); swap(flags',flags); swap(pc',pc); pc =&kernel;
  /* Save context unless during boot. */
  if(interrupt_number == RESET)
    { cur = &task0; }
  else
    { cur->sp = sp'; cur->pc = pc'; cur->flags = flags'; }
  /* Scheduler. */
  cur = cur->next;
  /* Context restore. */
  mpu = cur->code_segment; mpu2 = cur->data_segment;
  sp' = cur->sp; pc' = cur->pc; flags' = cur->flags;
  /* Return from interrupt (often done in hardware) */
  swap(sp,sp'); swap(flags,flags'); swap(pc,pc);
}
```

Fig. 2. Example: a minimalist OS kernel running on ideal hardware.

- Attacks targeting memory protection: data corruption on the code_segment or data_segment field of a Task would allow to extend the memory that the task can access, allowing further code injection or data corruption;
- Attacks targeting control-flow: changing the control-flow of the program to an unexpected execution path can lead to privilege escalation. For instance the attacker could use a stack smashing attack to jump to the instruction storing to cur->flags, with a wrong value.

Absence of privilege escalation is a fragile property. On the other hand, once verified, it implies that all of the above attacks are impossible or have only a limited impact.

Guarantee against attacks. An (inductive) invariant is a property that holds for all reachable states, because it is initially true and inductive (i.e. remains true after the execution of an instruction). Our method guarantees that no software-based attack can lead to privilege escalation by automatically finding an invariant implying that only the kernel code can execute with hardware privilege (and that this code cannot be modified). To be inductive, this invariant also contains properties about memory, control-flow and correct working of the memory protection mechanism. The properties below belong to the invariant computed by our method on the example kernel:

- Control-flow safety: All the instructions executed in privileged mode are those of the kernel function, whose code is never modified;
- Memory safety: All memory accesses done by the kernel are inside its stack, the cur global variable, or inside one of the Task. The stack never overflows, and the stack pointer at the kernel entry is constant;
- Working hardware protection: The flags’ register and the flags field in every Task ensure that execution is unprivileged. The two mpu registers, the code_segment and data_segment fields in every Task, do not contain write access to the kernel memory;
- Shape invariants: The cur variable (after boot) and the next field in every Task always point to a Task; each Task is separated from the others and from kernel data.

5Some authors call this technique automated; we use the word semi-automated to emphasize the difference with fully-automated methods.
6Klein et al. reports that 80% of the effort in SeL4 is spent stating and verifying invariants [8].
7Computed jumps are commonly introduced by compilers: return operations, function pointers, optimized compilation of C-like switch statements, dynamic method dispatch in OO-languages, etc.
prior typing nor partitioning, and (3) data manipulations are very low-level (masks, flags, legitimate overflows, low-level comparisons, etc.).

**Precondition.** Absence of privilege escalation may be true only for user images that match a given precondition. For instance our example kernel is vulnerable to privilege escalation if initially task0.next can point inside the kernel stack, or if task0.code_segment allows modifying the cur variable.

This suggests verifying the kernel assuming this precondition, and then checking that the user image satisfy the precondition. But writing this initial precondition would require a manual effort, going against having an automated analysis.

**Boot code.** Verifying boot code has its own difficulties: (1) type invariants holding at runtime may not hold in the initial state, so we have to verify their establishment rather than their preservation; (2) boot code often includes hard-to-analyze patterns such as dynamic memory allocation or creation of memory protection tables. Consequently, boot code is sometimes left unverified [8], and achieving perfect automatic analysis on boot code (0 false alarm) is very difficult.

**Parametrization.** If the number of tasks is not fixed, the kernel cannot find the memory location of data structures such as Task at fixed addresses in memory. A flat representation of memory [24] is no longer sufficient, and we need more complex representations able to precisely summarize memory (i.e. shape analyses [28], [29], [35]). Such analyses often require a tedious annotations, which defeats automation.

**Concurrency.** Operating system kernels are often concurrent because of nested interrupts, preemptive kernel threads, or (as in our case study) because the hardware provides several processors. Concurrency brings issues of analysis performance and precision.

### III. ABSENCE OF PRIVILEGE ESCALATION

We present here a formalization of absence of privilege escalation (APE) suitable to automated verification. Theorem 4 reduces this problem so that it can be tackled with standard methods—computation of state invariants.

The specific instantiation of this formalization to hardware-protected OS kernel can be found in Appendix B, p. 16.

**A. Privilege escalation**

We model a system (comprising the hardware, the operating system and the user tasks) as a transition system $(S, S_0, \rightarrow)$, where $S$ is the set of all possible states in the system, $S_0$ the set of possible initial states, and $\rightarrow \in S \times S$ represents the possible transitions: “$s_1 \rightarrow s_2$” means that executing one instruction starting from state $s_1$ can result in state $s_2$.

A predicate privileged : $S \rightarrow \text{Bool}$ tells whether a state has access to the privilege under study. In a typical OS kernel, this predicate corresponds to the value of a hardware register containing the hardware privilege level. The privilege level restricts how a state can evolve in a $\rightarrow$ transition. For instance on usual processors, system registers cannot be modified when in an unprivileged state.

Two entities are sharing their use of the system, called the kernel and the attacker. A predicate $A$-controlled : $S \rightarrow \text{Bool}$ tells which entity controls the execution (returning true if it is the attacker, and false if it is the kernel). In the kernels that we consider, a state is kernel-controlled if the next instruction that it executes comes from the kernel executable file and was never modified; all the other states are considered attacker-controlled.

This formalization corresponds to an attack model where the attacker chooses the untrusted software running on a system, but cannot change the security-critical software nor the hardware behavior.

**Definition 1 (Privilege escalation).** We define privilege escalation of a transition system $(S, S_0, \rightarrow)$ as reaching a state which is both privileged and controlled by the attacker:

$$\text{privilege escalation} \triangleq \exists s : \bigwedge \{ \text{privileged}(s), \text{A-controlled}(s) \} \land \exists s_0 \in S_0 : s_0 \rightarrow^* s$$

where $\rightarrow^*$ is the transitive closure of the $\rightarrow$ relation.

Thus, an attacker can escalate its privilege by either gaining control over privileged kernel code (e.g., by code injection), or by leading the kernel into giving it its privilege (e.g., by corrupting memory protection tables).

**B. Parameterized verification**

The previous definition cannot be used for parameterized verification, because the execution of the system depends on what is executed by the attacker. We solve this problem by defining a new semantics for machine code which is independent from the attacker’s execution.

1) **Regular and interrupt transitions:** We partition the transition relation into regular transitions and interrupt transitions:

$$s \rightarrow s' \triangleq s \overset{\text{regular}}{\rightarrow} s' \lor s \overset{\text{interrupt}}{\rightarrow} s'$$

The regular transition $\overset{\text{regular}}{\rightarrow}$ from states $s$ to $s'$ corresponds to the execution of an instruction $i \in I$, the set of all instructions.

$$s \overset{\text{interrupt}}{\rightarrow} s' \triangleq \exists i \in \text{exec}(s, \text{next}(s))$$

next : $S \rightarrow I$ fetches and decodes the next instruction, while exec : $S \times I \rightarrow P(S)$ executes this instruction (which may be non-deterministic). We assume that regular transitions $s \overset{\text{regular}}{\rightarrow} s'$ either preserve the current privilege level or evolve from a privileged state to an unprivileged state, but cannot evolve from an unprivileged state to a privileged one. The interrupt transition $\overset{\text{interrupt}}{\rightarrow}$ is thus the only way to evolve from unprivileged to privileged. In OS kernels, it corresponds to the reception of hardware or software interrupts.
2) **Empowering the attacker**: Note that the \( \rightarrow \) transition is defined only when the instruction under execution is known, which prevents parameterized verification. That is why we define a new transition system \( (S, S_0, \rightarrow) \) with the same sets \( S \) and \( S_0 \), but with a new transition relation \( \rightsquigarrow \).

We first define the \( \Downarrow \) relation which over-approximates the transitions that an attacker can effectively perform (i.e., we make the attacker more powerful). Instead of being able to execute only one known instruction \( \text{next}(s) \), the attacker will now be able to execute sequences of arbitrary instructions:

\[
s \Downarrow s'' \triangleq s'' = s \lor (\exists i, s' : s \Downarrow s' \land s'' \in \text{exec}(s', i))
\]

The \( \rightsquigarrow \) relation restricts the ability to execute arbitrary instructions to attacker-controlled states; when a state is kernel-controlled, the normal transition apply. In addition, interrupt transitions are also possible in attacker-controlled states.

\[
s \rightsquigarrow s' \triangleq s \rightarrow s' \lor (A\text{-controlled}(s) \land s \Downarrow s')
\]

**Theorem 1.** The set of reachable states for the \( (S, S_0, \rightarrow) \) transition system is included in the set of states reachable for \( (S, S_0, \rightsquigarrow) \).

**Proof.** This follows directly from the fact that for every \( s \), \( \{s' : s \rightarrow s'\} \subseteq \{s' : s \rightsquigarrow s'\} \)

**Corollary 1.** If there are no privilege escalation in the transition system \( (S, S_0, \rightsquigarrow) \), there are also no privilege escalation in the transition system \( (S, S_0, \rightarrow) \)

**Proof.** This is the contrapositive of the fact that if a state exists in \( (S, S_0, \rightarrow) \) where privilege escalation happens, this state also exists in \( (S, S_0, \rightsquigarrow) \).

Thanks to Corollary 1, the \( (S, S_0, \rightsquigarrow) \) transition system can be used to prove absence of privilege escalation instead of \( (S, S_0, \rightarrow) \), with the benefit of establishing this proof independently from a particular concrete attacker.

**C. Proof strategy**

We now show that we can recast absence of privilege escalation to ease its formal verification.

1) **Absence of privilege escalation as a state property**: A state property \( \in S \rightarrow \text{Bool} \) is a predicate over states. A state property is satisfied if it is true in every reachable state.

**Theorem 2.** A transition system does not have privilege escalation if, and only if, it satisfies the secure property, where

\[
\text{secure}(s) \triangleq \neg(A\text{-controlled}(s) \land \text{privileged}(s))
\]

Recasting absence of privilege escalation as a state property provides a method to prove this property. The idea is to find a state invariant: a property \( p \) which holds on every initial state \( s \in S_0 \) and is inductive (i.e. \( p(s) \land s \rightarrow s' \Rightarrow p(s') \)), and thus holds on each reachable state. If this invariant \( p \) is stronger than secure, this proves that we cannot reach a state which is both attacker-controlled and privileged.

2) **Reasoning about consequences of privilege escalation**: Given how we empowered the attacker, we can also reason about the consequences of a privilege escalation: indeed the definition of \( \rightsquigarrow \) implies that the attacker will do everything that it can do. Thus, to prove absence of privilege escalation in \( (S, S_0, \rightarrow) \), one can prove that a bad consequence of privilege escalation never happens in \( (S, S_0, \rightsquigarrow) \).

In the rest of the paper, we will consider that bad consequence is given by the following assumption:

**Assumption 1.** Running an arbitrary sequence of privileged instructions allows to reach any possible state.

This assumption is reasonable for every system that confines an adversarial code in some kind of container, as escaping such a container means getting rid of any restrictions on the possible actions. In the case of hardware protected OSes, executing an arbitrary sequence of instructions with hardware privilege allows changing any register or memory location, thus reaching any state. Using this assumption we can prove the following theorem:

**Theorem 3.** If a transition system \( (S, S_0, \rightsquigarrow) \) is vulnerable to privilege escalation, then the only satisfiable state property in the system is the trivial state invariant \( \top \), true for every state.

**Proof.** If a privilege escalation vulnerability exists, then any state can be reached, as executing an arbitrary sequence of instructions starting from a privileged state can lead to any state. The only state invariant that is true on every state is \( \top \).

We define as non-trivial any state invariant different from \( \top \).

**Theorem 4.** If a transition system satisfies a non-trivial state invariant, then it is invulnerable to privilege escalation attacks.

**Proof.** By contraposition, and the fact that state invariants are satisfiable state properties.

Theorems 3 & 4 have two crucial practical implications:

- If privilege escalation is possible, the only state property that holds in the system is \( \top \), making it impossible to prove definitively any useful property. Thus, proving absence of privilege escalation is a necessary first step for any formal verification of an OS kernel;
- The proof of any state property different from \( \top \) implies as a byproduct the existence of a piece of code able to protect itself from the attacker, i.e. a kernel with protected privileges. In particular, we can prove absence of privilege escalation automatically, by successfully finding any non-trivial state invariant with a sound static analyzer.

**IV. BINSEC/CODEX FOR OS VERIFICATION**

In Section III we have reduced the problem of proving absence of privilege escalation to the problem of finding a non-trivial system invariant. We now detail our methodology to find such an invariant.
A. Background: general principles

Abstract interpretation [32] is a method for building sound static analyzers that can infer program invariants and verify program properties. Using it, we can compute a set of states guaranteed to be larger than the set of reachable states for the \((\mathcal{S}, \mathcal{S}_0, \rightarrow)\) transition system. If this computed set is different from \(\mathcal{S}\), this proves that a non-trivial invariant exists.

Abstract interpretation works by computing abstract values — elements of abstract domains — representing a set of states. Abstract domains are iteratively computed until a (post-)fixpoint is reached, yielding an over-approximation of the set of reachable states. This over-approximation allows to trade precision for termination. Such abstract domains range from simple (intervals) to complex (polyhedra), offering various trade-offs between precision and scalability, and domains can be combined together [36].

In practice, designing an abstract interpreter amounts to defining or choosing a combination of abstract domains tailored to the problem at hand.

Intermediate representation for machine code analysis. Intermediate Representations (IR) [37]–[39] are the cornerstone of modern binary-level code analyzers, used to lift the different binary Instruction Set Architectures into a single and simple language. We rely on the IR of BINSEC [38], [40], called DBA — other IRs are similar. Its syntax is shown Figure 4.

```
(stmt) ::= store(e) e | (reg) e ← (e) | goto(e)
    | (stmt);(stmt) | if(e)(stmt) else(stmt)
(e) ::= (cast) | (reg) | load (e) | (unop) | (e) | (binop) (e)
(unop) ::= + | − | × | ÷ | udiv | urem | sdiv | srem
(binop) ::= | and | xor | or | shl | shr | sar
(arith) ::= + | − | × | ÷ | udiv | urem | sdiv | srem
(bitwise) ::= and | or | xor | shl | shr | sar
(cmp) ::= = | ≠ | > | < |
```

Fig. 4: Low-level IR for binary code

DBA is a minimalist language comprising only a single type of elements (bitvector types) and four types of instructions: register assignments, memory stores, conditionals, and jumps (static and computed). Expressions contain memory load, as well as standard modular arithmetic and bit-level operators. Values are stored in registers or in memory (a single byte-level array), the imperative semantics is standard. This is enough to encode the functional semantics of major instructions sets — including x86 and ARM.

B. Key ingredients

In Section III we have simplified the problem of proving absence of privilege escalation, to the problem of finding a non-trivial system invariant. We intend to use binary-level static analysis to infer and verify this invariant automatically.

In practice, this requires developing a static analysis for machine code and run it on the system loop (Figure 1). We define a special transfer function to handle the \(\Delta\) (user code) transition, that removes knowledge about the contents of registers and memory accessible from the loaded memory protection table. This approach should work if the system is not parameterized, i.e. when the system is completely known, or at least the memory layout of user tasks is known [18], [24].

Yet, we face two problems here: (1) binary-level static analysis is already very challenging on its own [27], [33], [34] (Section II-E) — see, e.g., the boot code analysis in our case study; (2) the systems we are interested in are parameterized.

Our methodology builds upon three key ingredients.

Key 0: An up-to-date binary-level static analysis. We build a state-of-the-art sound static analyzer for machine code (Section IV-D), picking among the best practices from the literature [27], [33], [34], [41]–[43]. Interestingly, while prior works is partitioned [27] into raw binary analysis (no assumption, adequate to adversarial analysis such as malware but extremely difficult to get precise) and standard-compilable code analysis (with hard-coded extra assumptions\(^8\)), our own method does target standard-compilable code but the extra assumptions are explicit (shape annotations) and fully checked.

Key 1: A type-based weak shape abstract domain. We propose a novel weak shape abstract domain (Section V) based on the layout of types in memory. This fulfills many roles:

- Being a shape domain, it can be used to summarize the memory and allows representation of addresses which is scalable (no enumeration) and precise;
- Types and type invariants can encode the precondition on (the shape of) user images in a simple way;
- Being based on types, most of the annotations can be extracted from the type declarations of the kernel code. This provides a base set of annotations for the shape domain, that is easy to strengthen using type invariants.

Key 2: Differentiated handling of boot and runtime. Contrary to runtime code, the boot code does not preserve data structure invariants but establishes them. Thus, parameterized verification (with 0 false alarms) of boot code is complex. On the other hand, when the user image is known, boot code execution is almost deterministic (except from small sources of non-determinism: multicore handling, device initialization, etc.), mostly analyzable by simple interpretation.

We propose an asymmetric treatment of boot code and runtime: our parameterized analysis completely ignores the alarms in boot code, meaning that when reaching 0 alarm in the runtime, we get a system invariant I under the precondition that the state after boot is in I. The latter is then verified by analyzing (mostly, interpreting) the boot code with a given user tasks image.

C. Putting things together: the 3-step methodology

Our methodology (Figure 5) consists in three steps:

1. Automated annotation. We automatically extract type declarations from the kernel (currently using the DWARF debug section in either the kernel or user image executable, but extraction from source code is also feasible), and (optionally) strengthen them with simple type invariant annotations

\(^8\)Typically, assumptions on control flow (trust in an external disassembler) or memory partitioning (e.g., stack is separated from the rest of memory).
2. Parameterized static analysis. Then, we launch the static analyzer on the kernel (boot code and runtime code) using the shape annotations configuring the shape abstract domain. The result is an invariant for the runtime \( T \) under the precondition that the state after boot satisfies \( I \).

3. User tasks checking. Given a user image, we then interpret the boot code and check that its final state indeed satisfies the invariant precondition on the kernel runtime. If it does, we have a system invariant—if not trivial it guarantees absence of privilege escalation.

D. Binary-level static analysis

Outside of our type-based weak shape abstract domain (Section V), our analyzer builds on state-of-the-art techniques for machine code analysis:

Control flow. Control and data are strongly interwoven at binary level, since resolving computed jumps (e.g., jmp @sp) requires to precisely track values in registers and memory, while on higher-level languages retrieving a good approximation of the CFG is simple. Following Kinder or Védrine et al. [33], [41], our analysis computes simultaneously the CFG and value abstractions;

Values. We mainly use efficient non-relational abstract domains (reduced product of the signed and unsigned meaning of bitvectors [34], and congruence information [44]), complemented with symbolic relational information [34], [45], [46] for local simplifications of sequences of machine code;

Memory. Our memory model is ultimately byte-level in order to deal with very low-level coding aspects of microkernels. Yet, as representing each memory byte separately is inefficient and imprecise, we use a stratified representation of memory caching multi-byte loads and stores, like Miné [47]. In addition, the tracked memory addresses are found on demand;

Precision. To have sufficient precision, notably to enable strong updates to the stack, our analysis is flow and fully context-sensitive (i.e. inline the analysis of functions), which is made possible by the small size and absence of recursion typical of microkernels. Moreover we unroll the loops when the analysis finds a bound on their number of iterations;

Concurrency. We handle shared memory zones through a flow-insensitive abstraction making them independent from thread interleaving [48]. Our weak shape abstract domain (Section V) represents one part of the memory in a flow-insensitive way. For the other zones we identify the shared memory zones by intersecting the addresses read and written by each thread [49], and only perform weak updates on them. More formal details are given in Appendix C (p. 17).

V. A TYPE-BASED WEAK SHAPE ABSTRACT DOMAIN

The weak shape abstract domain is designed to verify the preservation of memory layouts expressed using a particular dependent type system, tailored to the analysis of machine code (e.g., using subtyping to access field offsets implicitly).

A. Types and labeling of memories

Our memory abstraction considers only memories that follow constraints coming from how C compilers lay out values in memory. We interpret a C declaration like Task task0 as having two distinct meanings: first, that the memory addresses occupied by task0 is labeled as being a Task; second, that the contents of task0 are in the set of values that a Task should contain. In particular, task0.next should only contain addresses with a Task label.

Formally, we will define \( \text{types } T \), \( \text{labelings } \mathcal{L} \in A \rightarrow T \) of addresses, \( \text{interpretations } \mathcal{L}(a) \) of types as set of values, and memories \( m \in A \rightarrow V \); and we will consider only \( (m, \mathcal{L}) \) pairs such that \( \mathcal{L} \) is a labeling for \( m \), a relation defined as

\[
\forall a \in A : \ m[a] \in (\mathcal{L}(a))_V
\]

and meaning that each address \( a \) in a memory \( m \) should contain a value \( m[a] \) whose type is \( \mathcal{L}(a) \).

Types. We build a new type system upon that of C, with the following syntax for types:

\[
T \ni t ::= \text{word} \mid \text{int} \mid s_k \mid t^* \quad \text{where, informally, word represents any value, int represents any value used as an integer (i.e., not as a pointer), } s_k \text{ represents the } k\text{th byte in a C structure } s, \text{ and } t^* \text{ represents a pointer to } t. \quad \text{Note that } s^* \text{ in C (a pointer to a structure } s) \text{ is translated in our system into } s_0^* \text{ (a pointer to the 0th byte of } s).\]

Labeling and memory. Consider the C code of Figure 6(a)⁹. Compiling and running it produces (Figure 6(c)) both a memory \( m \in A \rightarrow V \) (mapping from addresses to values) and a labeling \( \mathcal{L} \in A \rightarrow T \) (mapping from addresses to types). The address 0x01 corresponds to \( \text{foo.data} \), which should hold the byte at offset 1 in a value of type \( \text{foo} \); thus the label of 0x01, \( \mathcal{L}(0x01) \), is \( \text{foo} \).

Subtyping. Following the C types definition, we know that addresses at offset 1 in \( \text{foo} \) structures are used to store pointers to a \( \text{bar} \) structure (i.e. addresses whose label is \( \text{bar0} \)). Thus, we want to say that 0x01 is also labeled by \( (\text{bar0})^* \).

⁹To simplify the presentation, we assume that every word, including pointers, has size one byte.
Types as set of values. In addition, labels also constrain the values that a memory cell can hold are constrained. The address space of a memory cell is a labeling, results in a new memory cell, for which \( L \) is still a labeling. This is a useful property: it means that cells holding pointers to \( \ast foo \) structures will always contain those, and cannot point to other data structures (such as page tables). For instance, if there are no memory stores to addresses whose label is a supertype of \( t \), we have proved that addresses whose label is \( t \) are read-only.

The reason why the type domain \( T^d \) is an efficient shape abstraction is that it does not track the contents of addresses in the user image (the parameterized part, \( \Lambda_F \)) at all. It only tracks the types of the values in registers \( R \) and memory cells of the rest of the kernel (the fixed part, \( \Lambda_F \)), ensuring that they point to appropriate addresses in \( \Lambda_F \).

Thus, our type domain \( T^d = (\Lambda_F \cup R) \rightarrow T \) consists in a mapping, tracking the type of the content of each register or fixed-address memory cell. The meaning this abstract domain is given using its concretization function \( \gamma_{TT} \), which is a mapping from abstract values to the set of states they represent:

\[
\gamma_{TT} : T^d \rightarrow P(S)
\]

\[
\gamma_{TT}(t) = \{(mem, regs) \mid \exists L : \forall r \in R : \text{regs}[r] \in t^f[r] \land \forall a \in \Lambda_F : \text{mem}[a] \in t^f[a] \land L \text{ is a labeling for } mem \}
\]

In practice, the main operations of the domain consist in handling pointer arithmetic (by communicating with the numeric domain) to precisely track offsets inside structures; using type information to retrieve the type of pointers for load operations (e.g., loading from an address of type \( \ast foo \) returns an address of type \( \ast bar \)); and verifying the preservation of the type upon stores (e.g., storing a value of type \( \ast int \) into an address of type \( \ast foo \) makes the analysis fail with an error).

C. Additional features

Several extensions are required in the actual analysis to successfully run our case study:

### (a) Type and globals definitions in C

```c
typedef struct { foo *next; bar *data; } foo;
typedef struct { int x; int y; foo f; } bar;
foo u; bar v;
```

### (b) Subtyping relations derived from the C definitions

### (c) Example concrete memory (\( m \)) and labeling (\( L \))

![Fig. 6. Typing a small memory](image)

We express this property as a subtyping relationship: \( foo \) is a subtype of \( (\ast bar) \) (written \( \text{foo} \subseteq (\ast \text{bar}) \)). The inclusion \( t \subseteq u \) means that addresses labeled by \( t \) are also labeled by \( u \). This subtyping relationship is very important for programs written in machine languages, and deals in particular with the following issue. While accessing the field of a structure (e.g., \( p = & (p->\text{next}) \)) is an explicit operation in C, it is implicit in machine code (the previous statement is a no-op when \( \text{next} \) has offset 0), thus requiring pointers to a structure to simultaneously point to its first field.

Figure 6(b) provides the subtyping relations derived from Figure 6(a). Formally, if \( s \) contains a \( t \) at offset \( o \), then for all \( k \) such that \( 0 \leq k < \text{sizeof}(t) \): \( s_{o+k} \subseteq t_k \) (where \( t_0 = t \) in the case of scalar types).

### (c) Example concrete memory (\( m \)) and labeling (\( L \))

![Fig. 6. Typing a small memory](image)
• We need to handle numerical properties over scalar types; for instance values contained in flags field in Figure 2 should always have the “UNPRIVILEGED” bit set, or data_segment does not intersect with the range of kernel addresses (Figure 8 p.16);
• Another important extension is to handle null pointers. Our type system distinguishes never-null and maybe-null pointer types (the former being a subtype of the latter) and issues alarms on null pointer dereferences;
• Lastly, the domain is extended to support arrays in order to implement bound checking and to support type annotations expressing the fact that the length of an array is contained somewhere else in memory.

Appendix A (p.16) presents our complete annotation language.

VI. CASE STUDY & EXPERIMENTAL EVALUATION

We seek to answer the following Research Questions:

RQ1: Effectiveness Can BINSEC/CODEX automatically verify absence of privilege escalation on a real microkernel?
RQ2: Internal evaluation What are the respective impacts of the different elements of our analysis method?
RQ3: Scalability and analyzer performance How does BINSEC/CODEX scale?
RQ4: Genericity Does BINSEC/CODEX work on different kernels, hardware architectures and toolchains?

A. Experimental setup

Implementation. Our static analysis technique has been implemented in BINSEC/CODEX, a plugin of the BINSEC [50] framework for binary-level semantic analysis. We reuse the intermediate-representation lifting and executable file parsing of the platform, but reimplement a whole static analysis on top of it, with adequate domains, including weak shapes. Development is done in OCaml, the plugin counts around 41 kloc.

Case study. We consider an industrial case study: the microkernel of AnonymOS, a part of an industrial solution for implementing security- and safety-critical real-time applications, used in industrial automation, automotive, aerospace and defense industries. The kernel is being developed by AnonymFirm—an SME whose engineers are not formal method experts—using standard compiler toolchains. The system is parameterized: the kernel and the user tasks establish the precondition; (2) the invariant for the whole system, i.e., computes an invariant under precondition for the kernel runtime and checks that the user tasks establish the precondition; (2) the precision of the analysis, measured by the number of alarms (i.e., assumptions that the analyzer cannot prove); (3) the effort necessary to setup the analysis, measured by the number of lines of manual annotations; and (4) the performance of the analysis, measured in CPU time and memory utilization.

We consider both kernel versions and two configurations (i.e., sets of shape annotations):

- Generic contains types and parameter invariants which must hold for all legitimate user image;
- Specific further assumes that the stacks of all user tasks in the image have the same size. This is the default for AnonymOS applications, and it holds on our case study.

B. Effectiveness (RQ1)

Protocol. The goal of our first experiment is to evaluate the effectiveness of the approach, measured by: (1) the fact that the method indeed succeeds in computing a non-trivial invariant for the whole system, i.e., computes an invariant under precondition for the kernel runtime and checks that the user tasks establish the precondition; (2) the precision of the analysis, measured by the number of alarms (i.e., assumptions that the analyzer cannot prove); (3) the effort necessary to setup the analysis, measured by the number of lines of manual annotations; and (4) the performance of the analysis, measured in CPU time and memory utilization.

We consider both kernel versions and two configurations (i.e., sets of shape annotations):

- Generic contains types and parameter invariants which must hold for all legitimate user image;
- Specific further assumes that the stacks of all user tasks in the image have the same size. This is the default for AnonymOS applications, and it holds on our case study.

Results. The main results are given in Table I. The generic annotations consist in only 57 lines of manual annotations, in addition to 1057 lines that were automatically generated (i.e., 5% of manual annotations). When analyzing the BETA version with these annotations, only 3 alarms are raised in the runtime:

- One is a true vulnerability: in the supervisor call entry routine (written in manual assembly), the kernel extracts the system call number from the opcode that triggered the call, sanitizes it (ignoring numbers larger than 7), and uses it as an index in a table to jump to the target system call function; but this table has only 6 elements, and is
followed by a string in memory. This off-by-one error allows a \texttt{svc 6} system call to jump to an unplanned (constant) location, which can be attacker-controlled in some user images. The error is detected as the target of the jump goes to a memory address whose content is not known precisely, and that we thus cannot decode:

- One is a false alarm caused by debugging code temporarily violating the shape constraints: the code writes the constant 0xdeadbeef in a memory location that should hold a pointer to a user stack (yielding an alarm as we cannot prove that this constant is a valid address for this type), and that memory location is always overwritten with a correct value further in the execution;
- The last one is a false alarm caused by an imprecision in our analyzer when user stacks can have different sizes.

When analyzing the \texttt{v1} version, the first two alarms disappear (no new alarm is added). Analyzing the kernel with the specific annotations makes the last alarm disappear. In all cases user tasks checking succeeds.

Analyzing the \texttt{v1} kernel with the specific annotations allows to reach 0 alarms, meaning that we have a fully-verified invariant and a proof of absence of privilege escalation.

Computation time is always very low: less than 11 minutes for the static analysis and 35 seconds for user tasks checking.

Conclusions. Experiments show that verifying absence of privilege escalation of an industrial microkernel using only fully-automated methods is feasible (albeit with a very slight amount of manual annotations). Especially:

- The analysis is effective, in that it identifies real errors in the code, and verifies their absence once removed;
- The analysis is very precise, we manage to reach 0 false alarm on the correct code, and we had no more than 2 false alarms on each configuration of the analysis;
- The annotations burden is very small (58 simple lines), as most of the annotations are extracted fully automatically;
- Finally, the analysis time, for a kernel whose size is typical for microkernels, is very small (between 406 and 647 seconds). Analysis time would be even smaller (86 seconds) if the system had used a single core.

An additional finding is that verifying low-level code such as the assembly parts of the kernel is essential: this code is small but prone to errors, as witnessed by the one we found.

C. Evaluation of the methodology (RQ2)

Protocol. The goal of this experiment is to evaluate our 3-step methodology (Section IV), in particular (1) whether our shape domain is needed, (2) what is the nature and impact of the shape annotations, and (3) whether differentiated handling of boot code is mandatory.

We experiment on the \texttt{v1} kernel version, and report results for both the boot code and the runtime, using different sets of annotations with an increasing amount of annotations:

- No annotation (equivalent to having no shape domain);
- Generated annotations (without any manual annotations);
- Minimal annotations with which the analysis terminate;
- Generic and Specific are the annotations defined above;
- Dedicated hardcodes some parameters, such as the number of tasks, with values of the sample user tasks.

Results. Table II shows the result of this evaluation. The analysis does not succeed in finding an invariant without the shape domain or without manual annotations — the analysis is too imprecise and aborts in boot, denoted \textbf{✗}. Only 10 lines of manual annotations are necessary for the analysis to complete (minimal), albeit with many alarms in both boot code and runtime. These annotations mainly limit the range of array indices to prevent overflows in pointer arithmetic. The generic configuration adds 47 lines indicating which pointer types or structure fields may be null, which fields hold array indices, and relating array lengths with memory locations holding these lengths. This configuration eliminates most alarms in the runtime, but 60 alarms remain in the boot code. The specific annotations reach 0 alarms in runtime, but still 59 alarms in boot code. Even the dedicated annotations cannot eliminate all alarms in boot code.

Interestingly, we also found that some of the invariants in the generated annotations do not hold before boot.

Conclusions. Parameterized verification of the kernel cannot be done without the shape domain. The ability to extract the shape configuration from types is extremely useful, as 95\% of the annotations are automatically extracted, requiring only 57 lines of simple manual annotations. Finally, differentiated handling of boot code is necessary as both the boot code is much harder to analyze than the runtime, and the shape invariants holds only after boot code terminates.

D. Scalability and analyzer performance (RQ3)

Protocol. We now turn to scalability and performance. (1) First, we specialize the generic shape annotations to fix
the number of tasks in the system to different constant values, in order to assess its scalability w.r.t. the number of tasks. 

(2) Second, we examine the interpretation of the boot code and measure the number of instructions executed deterministically – i.e. instructions whose inputs and outputs are singleton values. This is important regarding user tasks checking, as such instructions can be analyzed exactly using efficient concrete (non-abstract) interpretation.

Results. Results of the first experiment are given in Table III. As expected, the execution time variations between different numbers of tasks are very low. Actually, fixing the number of tasks provides only a slight speedup compared to the case where this number is unknown.

| TABLE III | SCALABILITY EXPERIMENT |
|-----------|-------------------------|
| Number of tasks | 10 | 10^3 | 10^7 | unknown |
| Static analysis time (s) | 380 | 387 | 388 | 406 |
| Static analysis memory (GB) | 5.3 | 5.6 | 5.6 | 5.8 |

For the second experiment, out of 111,491 executed instructions on the boot code, 111,447 (99.96%) were deterministic. The remaining instructions were all related to low-level device driver initialization (reading MMIO values), and were independent from the user image.

Conclusion. Our static analysis scales well to an arbitrary number of user tasks. Moreover, almost every instruction in the boot code is executed deterministically and can thus be executed using efficient concrete interpretation. Finally, our technique performance is in line with the fastest OS verification methods (Table IV).

E. Genericity (RQ4)

Protocol. We now assess the genericity of our approach. We apply our tool to EducRTOS, [51] a small academic OS developed for teaching purpose. It is both a separation kernel (with task isolation) and a real-time OS. Interestingly, EducRTOS differs significantly from AnonymOS: it runs on x86 (32 bit) and memory protection relies on segmentation. We consider 6 variants, using two compilers (clang-8.0.1 and gcc-9.2.0) and three different optimization levels (O1, O2, O3). The code contains between 2.7 and 6.3 kbytes of instructions.

Results. We verify absence of privilege escalation on all variants, in less than 3 seconds each. The same annotations (12 manual lines, 144 automatically extracted) are used to verify all variants. There is no alarm in the runtime, and 5 alarms in the boot code, that our method allows to ignore.

Conclusion. Together with the AnonymOS case study, these results show that BINSEC/CODEX can be used in a variety of OS projects, and is robust to small variations in the OS binary.

VII. DISCUSSION

Threats to validity. Perhaps the main limitation of our work is the need to trust our static analyzer and user tasks checker. We mitigate this problem the following way. Our prototype is implemented as part of BINSEC [50] whose robustness have been demonstrated in prior large scale studies [52]–[55]. Especially, the IR lifting part has been exhaustively tested [56] and positively evaluated in an independent study [39]. Moreover, the user tasks checker is simple and share many components with the analyzer, allowing to crosschecks results between interpretation and full analysis. Finally, results have been crosschecked for consistency between versions, and all alarms on runtime have been manually investigated. It would be possible to further reduce the trust base by using a verified static analyzer [57], albeit with an important performance penalty.

Representativity of the case studies. We have verified an industrial kernel designed for safety-critical and hard real-time applications, whose size and complexity are in line with the general practice of these fields. While this kernel is indeed more restricted than a general-purpose OS [8], [20], [58] – no dynamic task creation nor dynamic re-purpose of memory, fixing memory partitioning is standard in embedded [6], [14]–[17], [24] and highly-secure [7] systems.

Regarding the verification itself, the analysis does not sidesteps any major difficulty: the kernel was unmodified, it runs concurrently, it initializes itself and creates its memory protection tables dynamically, we have verified all of the kernel code – including boot and the assembly parts, and our verification is parameterized. Finally, we have shown that the analysis works on differing setups (architecture, protection).

Scope of verification. The property that we target, absence of privilege escalation, is weaker than, e.g., task separation or full functional correctness [8], [12], [18], [20], [22]–[24], [58]. Indeed, most existing OS verification efforts (Table IV), if completed, would imply APE as a byproduct. On the other hand, APE is a universal property over OS kernels, is essential to security (Theorem 4) and must be proved in any complete formal verification effort. In some systems it is even the main property of interest [10].

Degree of automation. While we do use only fully-automated methods, we still had to write a small amount of manual annotations (58 lines for AnonymOS and 12 lines for EducRTOS) to complete our main analysis. These annotations are not state invariants, but configure the analysis so that state invariants can be inferred. Some annotation is unavoidable in a parameterized verification as the AnonymOS kernel does not ensure APE for arbitrary user tasks images. Still, the annotation effort is extremely low compared to prior work.

Limits. As already stated, our binary-level static analysis cannot handle dynamic task spawning, dynamic memory re-partitioning, code self-modification, code generation and recursion. The last limitation could be overcome with state of the art techniques (possibly at the price of precision), the other ones lay at the forefront of program analysis research.

VIII. RELATED WORK

Formal verification of OS kernels. Table IV presents a comprehensive overview of prior OS verification efforts. We
already discussed the scope of verification in Section VII. Note that most existing works leave unchecked hypotheses about the code, assuming for instance control flow integrity [10], semantics preservation of the compilation [11], [23] or correctness of some unverified parts of the code [19], [20], [22]—in particular assembly and boot code.

**Fully-automated kernel verification:** We qualified as fully-automated the techniques that are able to infer invariants automatically. Vasudevan et al. [11] used abstract interpretation in combination with deductive verification and runtime monitoring to verify manual annotations. Despite not inferring the invariants, some verification techniques feature an advanced level of automation. Vasudevan et al. [10] uses bounded model checking to verify hand-written proof obligations, with the help of manual verification statements. Dam et al. [24], [30] designed their kernel so that it requires assertions only at the beginning and end of the kernel, and most of these assertions are generated automatically from a formal model.

**Machine-level kernel verification:** Only two previous efforts target machine code [18], [30]. Both approaches are based on a flat memory model, and thus cannot handle parameterized number of tasks.

We propose BINSEC/CODEX, the first fully automated approach able to verify absence of privilege escalation on microkernels. Our approach is also the first fully automated OS verification method enabling binary-level reasoning and parameterized reasoning. While we focus on the key property of APE, we are confident that the method can be extended to stronger properties such as task separation. Finally, our method can complement existing—more manual—techniques to OS verification, either by automatically inferring parts of the annotations and discharging parts of the proof obligations, or by helping to prove unchecked low-level assumptions.

**Static analysis of machine code.** Relevant sound techniques were discussed in Section IV-D. Most of them either make no assumption at all (raw binary) [33], [41] at the price of precision, or rely on implicit extra assumptions (standar-dly-compiled code) [27].

We pick best practices from existing sound analyses—any progress there can be directly reused in our method. Our two main novelties are (1) to make checked explicit assumptions about data layout described by C types, and (2) to specialize our approach to the typical kernel architecture, computing an invariant (of the runtime) under precondition (established by boot code) and adding an extra precondition checking step.

Several existing works drop soundness and only compute “best effort invariants” [59], [60] in order to gain robustness [59] or to guide latter analysis [55]. This is not an option for us as we look for formal verification of APE. Finally, symbolic execution is widely used on machine code [50], [61]–[65] but aims to find bugs rather than to prove their absence.

**Fully-automated memory analyses.** Points-to and alias analyses are fast and easy to setup but are too imprecise for formal verification, and generally assume that the code behaves nicely, e.g., type-based alias analyses [66] assume that programs comply with the strict aliasing rule—while kernel codes often do not conform to C standard [19]. On the other hand, shape analyses [28], [29] can fully prove memory invariants, but require heavy parametrization and are generally too slow to scale to a full microkernel.

### TABLE IV

**Comparison of kernel verification efforts**

| Verified Kernel | Verified Property | Degree of automation | Verif. Level | Parame-terized | Multi-core | Infers inv. | Manual Annotations (LoC) | Case Study | Unproved code (LoC) | Non-invasive Analysis time (s) |
|----------------|-----------------|---------------------|--------------|----------------|-----------|-----------|------------------------|------------|----------------------|-------------------------------|
| This work      | Absence of priv. escalation | ✓ | Fully automated | Machine | ✓ | ✓ | ✓ | 58✓ | ✓ | ✓ | 406 | 0 | ✗ | ✗ | ✗ |
| XMHF [10]      | Memory integrity | ✗ | Semi automated | Source | ✗ | ✗ | ✗ | N/A | 32 (C) | 76 | 388 (asm) | ✗ |
| üXMHF [11]     | Security properties | ✓ | Semi automated | Source + assembly | ✓ | ✓ | ✗ | 5,544 | 0 ✓ ✗ | 3,739 | 272 |
| Verve Nucleus [2] | Type Safety | ✓ | Semi automated | Assembly | ✗ | ✓ | ✗ | 4,309 | 0 ✓ | 14,400 | 272 |
| Prosper [24], [30] | Compliance with specification | ✓ | Semi automated | Machine | ✓ | ✓ | ✗ | 6,400 | 0 ✓ | ≤ 28,800 |
| Baby hypervisor [23], [25] | Compliance with specification | ✓ | Semi automated | Source + assembly | ✓ | ✓ | ✗ | 8,200 tokens | 0 ✓ | 4,571 | 272 |
| Komodo [12] | Compliance with specification | ✓ | Semi automated | Assembly | ✓ | ✗ | ✗ | 18,655 | 0 ✓ | 14,400 | 272 |
| UCLA Secure Unix [20] | Compliance with specification | ✓ | Manual | Source | ✓ | ✓ | ✗ | N/A | 80% ✓ | N/A | ✗ |
| µCOS-II [22] | Compliance with specification | ✓ | Manual | Source | ✓ | ✓ | ✗ | 34,887 | 37% ✓ | 57,600 | 272 |
| SeL4 [8], [19] | Compliance with specification | ✗ | Manual | Source | ✓ | ✓ | ✗ | 200,000 | 1,200 (C, boot) + 500 (asm) | N/A | ✗ |
| CertIKOS [58] | Compliance with specification | ✓ | Manual | Source + assembly | ✓ | ✓ | ✗ | 100,000 | 0 ✓ | N/A | ✗ |

a Assuming that the proof is completed to cover all the code.  
b Control flow integrity is assumed.  
c Generated from a 21,000 lines of HOL4 manual proof.  
d Plus 181,054 LoC of specification and support libraries.  
e The reported compilation time includes the support libraries.  
f The verification is concurrent because of in-kernel preemptions.  
g The translation to assembly is also verified.  
h The hypervisor supports a single guest  
i The hypervisor supports two guests
Our weak type-based shape abstract domain hits a middle ground: it is fast, precise, handles low-level behaviors (outside of the C standard) and requires little configuration. This is also the first shape analysis performed on machine code.

Marron [67] also describes a weak shape domain performing only weak updates, but on a type-safe language with no implicit type casts, pointer arithmetic, nor nested data structures.

Type-based verification of memory invariants. Walker et al. [20] already observes in the 1980’s that reasoning on type invariants is well suited to OS kernel verification. Several systems build around this idea [2], [12], leveraging a dedicated typed language. Cohen et al. [68] describe a typed semantics for C with additional checks for memory typing preservation, similar to our own checks on memory accesses. While they use it in a deductive verification tool for C (to verify an hypervisor [23]), we build an abstract interpreter for machine code.

IX. CONCLUSION

Operating system kernels are the keystones of computer system security, leading to several efforts towards their formal verification. Yet, while these prior works were overall successful, they often require a huge amount of manual efforts, and verification was often led only at source level, on crafted kernels. We focus in this paper on the key requirement that a kernel should protect itself, coined as absence of privilege escalation, and provide a methodology to verify it from the kernel executable only, using fully automated methods with a very low amount of manual configuration. Our methodology handles parameterized systems thanks to a novel type-based weak shape abstract domain. The technique has been successfully demonstrated on two embedded microkernels, including an industrial one: with less than 60 lines of manual annotations, we were able to find a vulnerability in a preliminary version and to verify absence of privilege escalation in a secure version, without any remaining false alarm.

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C. the abstract domains used in our analysis.

A. Annotation language

Annotations take the form of type declarations similar to those of the C language, except that numerical properties may be specified on values and array sizes. The annotation language is presented in Figure 7, and the shape annotations required for the example kernel of Figure 2 is presented in Figure 8. An example of code for an user image is given in Figure 9.

Fig. 7. Annotation language for configuring the shape domain

Fig. 8. Shape annotations configuring the analysis for the example kernel

B. Semantics of an architecture with memory protection

Here, we instantiate the formalization of Section III by providing a formal model of a computer using operating system software controlling the use of hardware privilege. Such a definition is necessary to define and prove correct a static analysis which computes sets of possible concrete execution described by this model.

States. Values \( \mathbb{V} \) are machine integers (e.g. \( \mathbb{V} = \{0, 2^{32} - 1\} \)). \( \mathbb{A} \subseteq \mathbb{V} \) is the set of memory addresses. A memory \( m \in \mathbb{M} \) is just a mapping from addresses to values, i.e., \( \mathbb{M} = \mathbb{A} \rightarrow \mathbb{V} \). We note \( \mathcal{R} \) the set of register names in the system; in the example of Section II, this includes for instance “\( \text{mpu}_1 \)” and “\( \text{sp} \)” or “\( r0 \)” and “\( r11 \)” on ARMv7 processors.

void code(void){
    while(true){
        printf("Hello\n");
        asm("software_interrupt");
    }
}

void after_code(void) {}

#define STACK_SIZE 256
char stack0[STACK_SIZE], stack1[STACK_SIZE];

Task task0 = {
    .sp = &stack0[STACK_SIZE-\( \text{sizeof}(\text{int})\)];
    .pc = &code;
    .flags = 0 | UNPRIVILEGED;
    .code_segment =
        &code \< 32 | &after_code | EXEC;
    .data_segment =
        &stack0 \< 32 | &stack0[STACK_SIZE] | READ | EXEC;
    .next = &task1;
};

Task task1 = {
    .sp = &stack1[STACK_SIZE-\( \text{sizeof}(\text{int})\)];
    .pc = &code;
    .flags = 0 | UNPRIVILEGED;
    .code_segment =
        &code \< 32 | &after_code | EXEC;
    .data_segment =
        &stack1 \< 32 | &stack1[STACK_SIZE] | READ | WRITE;
    .next = &task0;
};

Fig. 9. Example of code producing a user image for the example kernel

States \( \mathbb{S} \) are tuples of \( \mathbb{M} \times (\mathcal{R} \rightarrow \mathbb{V}) \) where each state \( s \) has a memory \( s.\text{mem} \in \mathbb{M} \), and a mapping \( s.\text{regs} \in \mathcal{R} \rightarrow \mathbb{V} \) of registers’ names to their values.

Registers \( \mathcal{R} = \mathcal{R}_s \uplus \mathcal{R}_u \) are partitioned into system registers \( \mathcal{R}_s \) and user registers \( \mathcal{R}_u \). In a secure system, only the kernel is able to modify system registers, using the following mechanism.

Execution level and privilege. States are partitioned between privileged (i.e., supervisor-level) or unprivileged (i.e., user-level) states. Generally, privileged corresponds to the value of a bit in a system register like \( \text{flags} \) in Figure 2.

Transitions between unprivileged states cannot change the values in the system registers. Moreover when a state is unprivileged, the only way for its successor to be privileged is by performing a interrupt transition, detailed below.

Transitions. The transitions (regular and interrupt) have already been described in Section III-B1. In a (standard, Von Neumann) machine execution, getting the next instruction correspond to fetching in memory the opcode pointed by the program counter, then decoding it:

\[
\text{next}(s) \triangleq \text{decode}(s.\text{mem}[s.\text{regs}[\text{pc}]])
\]

We do not detail the format of instructions, as it is standard; in our tool it is encoded as a sequence of basic instructions of DBA intermediate language of the BINSEC tool [38]. The interrupt transition \( \rightarrow^{\text{interrupt}} \) corresponds to the reception of a hardware or software interrupt. This transition 1. makes \( s' \) privileged, 2. changes \( s'.\text{regs}[\text{pc}] \) to a specific label that
we call the kernel entry point, and 3. possibly performs other operations, such as saving the values of registers in system registers or memory.

**Memory protection.** A key component of any OS kernel invariant consists in ensuring that the memory protection is properly set up. Indeed memory protection and hardware privilege are the two mechanisms that the kernel must use to protect itself from the user code, and a vulnerable memory protection generally leads to a possible privilege escalation.

The memory protection is modeled as follows. A predicate accessible : $\mathbb{S} \times \mathcal{A} \rightarrow \mathcal{P}\{R, W, X\}$ returns the access rights for an address $a$ in a state $s$. A state can perform a regular transition only if it can access all the memory addresses that are needed for fetching and executing this instruction; otherwise it must perform an interrupt transition.

The accessible predicates generalizes every memory protection mechanism found in usual hardware (note that memory translation can be encoded if needed in the semantics of instructions). In some systems (e.g., based on Memory Protection Units), accessible depends only on the value of some system registers. This means that unprivileged code cannot change the set of accessible addresses directly (an indirect attack is still possible, by having the kernel load corrupt data in system registers). In other systems (e.g., based on Memory Management Units), accessible also depends on addresses in memory pointed by a system register (the memory protection tables, for instance page tables). This makes possible a direct attack where unprivileged code modifies memory protection tables directly, if some of the addresses of the memory protection tables in use are accessible.

**Multiprocessor.** In multiprocessor systems, the state is a tuple of $\mathbb{M} \times (\mathbb{R} \rightarrow \forall)^n$, where $n$ is the number of processors. For simplicity’s sake we focus our explanation on the single-processor case: the system is assumed to have only one processor unless where explicitly mentioned.

### C. Description of the static analysis

In this section we present (with some simplifications) a formal description of our binary-level static analyzer.

**Background.** When given a transition system $\langle \mathbb{S}, S_0, \rightarrow \rangle$, static analysis by abstract interpretation [32] allows to compute a finite representation of a super-set of the reachable states. As a set of states is isomorphic to a property over states, this finite representation also corresponds to a state property. Abstract interpretation is sound — the computed invariants are correct by construction —, and can thus be used as a proof technique.

Abstract interpretation works by combining abstract domains, i.e. partially-ordered sets of abstract values, each representing a set of concrete values. Formally, the meaning of an abstract value $d^a$ belonging to an abstract domain $\mathbb{D}^a$ is given by its concretization, which is a function $\gamma^a : \mathbb{D}^a \rightarrow \mathcal{P}(\mathbb{D})$ mapping abstract value to the set of concrete values it represent. For instance, intervals $[a, b]$ are finite representations of (possibly infinite) sets of integers, e.g., $\gamma([3, +\infty)) = \{x \in \mathbb{Z} \mid 3 \leq x\}$.

**State domain** $\mathbb{S}^d = \mathbb{D}^s \times \mathbb{D}^d$

**Control flow domain** $\mathbb{C}^d = \mathcal{P}(\mathbb{L} \times \mathbb{L})$

**Data flow domain** $\mathbb{D}^d = \mathbb{L} \rightarrow \mathbb{M}^d$

**Storage domain** $\mathbb{M}^d = \mathbb{N}^d \times \mathbb{P}^d$

**Numeric domain** $\mathbb{N}^d = \text{any conjunction of numeric constraints over the values bound to } \mathcal{A}_F$ and $\mathcal{R}$

**Type domain** $\mathbb{T}^d = (\mathcal{A}_F \sqcup \mathcal{R}) \rightarrow \mathcal{T}$

$\mathcal{T}$ is the set of types, $\mathcal{A}_F$ the set of fixed addresses in the kernel, and $\mathcal{R}$ the set of register names, $\mathcal{L}$ the set of program locations.

Fig. 10. Implementation of the abstract domains.

The set of reachable states in a $\langle \mathbb{S}, S_0, \rightarrow \rangle$ transition system is formally defined as follows. We define a function $F$:

$$F : \mathcal{P}(\mathbb{S}) \rightarrow \mathcal{P}(\mathbb{S})$$

$$F(S) = S_0 \cup S \cup \{s' \mid \exists s \in S : s \rightarrow s'\}$$

such that the set of states reachable from $S_0$ is defined as the least fixpoint of $F$ (noted $\text{lfp}(F)$), i.e., is the smallest set $S$ such that $F(S) = S$.

The computation of an invariant using abstract interpretation mimics this definition. Given an abstract domain $\mathbb{S}^d$ representing the set of states $\mathbb{S}$, a concretization function $\gamma : \mathbb{S}^d \rightarrow \mathbb{S}$, and a sound approximation $F^d : \mathbb{S}^d \rightarrow \mathbb{S}^d$ of $F$, i.e. such that

$$\forall S^d \in \mathbb{S}^d : F(\gamma(S^d)) \subseteq \gamma(F^d(S^d))$$

then every postfixpoint $P^d$ of $F^d$, i.e. such that $F^d(P^d) \subseteq P^d$ will be a sound approximation of $\text{lfp}(F)$, i.e.:

$$\text{lfp}(F) \subseteq \gamma(P^d)$$

A postfixpoint of $F^d$ can be computed by upward iteration sequences with widening [32], which consists in growing the abstract value until it cannot grow any more (which means we found a postfixpoint).

In general, abstract domains are proved and designed in a modular way by composition of abstract domains [36]. The concretizations functions are used to establish the soundness of the transfer functions, describing how the abstract state is modified by the $\rightarrow$ transition.

In the remainder of this section we will explain the main abstractions we use, but will give only informal presentation of the transfer functions (as they follow from the semantics and from the concretization), and we will omit the soundness proofs. For the sake of simplicity, all of our abstract domains concretize into sets of states $\mathcal{P}(\mathbb{S})$.

**Main state abstraction.** Static analysis of machine code needs to analyse simultaneously the control flow and the data flow, as each depends on the other. To this end, our main abstraction combines a control-flow abstraction $\mathbb{C}^d$ with a data-flow abstraction $\mathbb{D}^d$, in a manner similar to [41].

Central to our abstractions is the notion of program locations $\mathcal{L}$. What is a program location is an implementation choice of the analyzer: a natural choice is to consider that a program locations is a kernel address (i.e., a precise address in the kernel code segment). In our case study we chose a more precise abstraction: a program location consists in a kernel
address with a call stack. We denote by $\mathcal{L}(s)$ the program location of a state $s$.

Then, our control flow abstraction $\mathcal{C}$ is a graph between program locations $L$. This graph is represented as the set of its edges $\mathcal{P}(L \times L)$. The meaning of a graph $\mathcal{C}$ is that (1) the only reachable program locations are the nodes in the graph, and (2) the only possible location after executing a state whose location is $\ell_1$ is a location $\ell_2$ where $\ell_2$ is a successor of $\ell_1$ in the graph. Formally:

$$\gamma_{\mathcal{C}}(c^2) = \{ s_1 \in S \mid \exists (\ell_1, \ell_2) \in c^2 : \exists s_2 \in S : s_1 \rightarrow s_2 \wedge \mathcal{L}(s_1) = \ell_1 \wedge \mathcal{L}(s_2) = \ell_2 \}$$

The data-flow abstraction $\mathbb{D} : L \rightarrow \mathbb{M}^2$ maps each program location $\ell$ to a storage abstraction $\mathbb{M}^2$—described hereafter—representing the values in the memory and registers. It is the standard abstraction for flow-sensitive analyses [32], [69], [70]. Its meaning is that the memory and value of registers for a state with program location $\ell$ must correspond to what is described in the storage abstraction. Formally:

$$\gamma_{\mathbb{D}}(d^2) = \{ s \in S \mid \exists \ell \in L : \ell = \mathcal{L}(s) \wedge s \in \gamma_{\mathbb{M}}(d^2[\ell]) \}$$

The main state abstraction $\mathbb{S}^2$ simply consists in a product [36] of these previous abstractions. It represents a set of states that must match both abstractions. Formally:

$$\gamma_{\mathbb{S}}(c^2, d^2) = \gamma_{\mathcal{C}}(c^2) \cap \gamma_{\mathbb{D}}(d^2)$$

The analysis works by performing multiple rounds of the following steps in sequence:
1) Perform a standard data-flow analysis using the current CFG $c^2$, to compute a new $d^2 \in \mathbb{D}^2$.
2) Iterate over all locations $l \in c^2$ to compute all possible outgoing edges, given the possible memories at the instruction entry $d^2[l]$ (this uses the same resolve function than [41]). Newly-discovered edges are added to the CFG $c^2$.

The iteration sequence starts with an abstraction $\left( c_0^2, d_0^2 \right)$ of the initial states $S_0$. In our case study, this corresponds to $c_0^2$ containing only the first instruction in the kernel, and $d_0^2$ representing a mapping from this instruction to every possible initial values of memory and registers (obtained from the kernel executable file, plus system registers indicating that the interrupt received is a RESET). The analysis terminates when the fixpoint is reached, i.e., no new edge is discovered in the CFG. In practice, several small optimisations are used to reuse results between rounds (e.g., caching the results), and to have fewer rounds (by early exploration of the newly-discovered CFG nodes).

**Theorem 5.** If the transfer functions for $\mathbb{M}^2$ are sound, the result $s^{\text{final}}_i$ of the analysis is a sound abstraction of all the reachable states in the system (and thus a state invariant).

In our case study we use the checked hypothesis that kernel-controlled code is at a fixed location and is not modified (i.e., we have no self-modifying nor dynamic loading of code). This hypothesis is checked by verifying that no memory store can modify a read-only region (see Figure 11), and reporting an error if this happens.

**Memory storage abstraction.** The memory storage abstraction $\mathbb{M}^2$ represents the contents of all the storage in the system, i.e., memory cells and registers. Its concretization has signature:

$$\gamma_{\mathbb{M}} : \mathbb{M}^2 \rightarrow \mathcal{P}(s)$$

where $S = (A \rightarrow V) \times (R \rightarrow V)$ is a pair of a memory (map from addresses to values) and values of registers (map from register names to values).

![Fig. 11. Partitioning of addresses $A$](image)

The structure of this abstraction is derived from how we can partition the memory in the system into the following parts (see Figure 11):

- **Unprotected memory:** addresses that may be modified by non-kernel code.
- **Protected memory:** protected addresses, i.e., memory protection should be set up so that these addresses cannot be modified by non-kernel code. This part is sub-partitioned into:
  - **Fixed part ($A_F$),** containing data structures whose addresses do not depend on the running application (and whose addresses are at a fixed location in the kernel executable file). Addresses in this part can be further distinguished; in particular writable addresses (containing the stack, and global variables like cur in the example kernel) are distinguished from read-only addresses, containing the code, jump tables, strings, etc.
  - **Parameterized part ($A_P$),** containing the data structures whose number and size depends on the application; like the circular list of tasks in the example kernel.

Following this partition, our main storage abstraction $\mathbb{M}^2$ is a product $\mathbb{N}^2 \times \mathbb{T}^2$ of two different storage abstractions corresponding to the different memory parts that we need to track:

- A precise numerical abstraction $\mathbb{N}^2$, whose purpose is to track precisely the numerical values contained in the memory cells of the fixed address part, and in the registers; obtained by standard [71] lifting of a numerical domain into a domain handling numerical properties of a fixed number of memory locations;
- A typed abstraction $\mathbb{T}^2$ representing the contents of the parameters part, and its relation with the registers and fixed-address part, detailed in Section V.
Note that the unprotected memory is supposed to be subject to arbitrary modifications from an attacker, so no abstraction needs to keep track of its possible contents.

$\mathcal{M}$ is a standard product of the $\mathcal{N}$ and $\mathcal{T}$ abstractions, so its concretization is defined simply as:

$$\gamma_{\mathcal{M}}(n, t) = \gamma_{\mathcal{N}}(n) \cap \gamma_{\mathcal{T}}(t)$$

The above is a simplified formalization of our real abstraction, and additional extensions are necessary to make it work in practice (see Section IV-D).
This figure "memory-regions.png" is available in "png" format from:

http://arxiv.org/ps/2003.08915v1
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