Interaction-Free Measurements, Atom Localisation and Complementarity

Anders Karlsson, Gunnar Björk and Erik Forsberg
Laboratory of Photonics and Microwave Engineering, Department of Electronics, Royal Institute of Technology (KTH), Electrum 229, 164 40 Kista, Sweden
electronic mail:andkar@ele.kth.se
(April 1, 2022)

We analyse interaction-free measurements on classical and quantum objects. We show the transition from a classical interaction free measurement to a quantum non-demolition measurement of atom number, and discuss the mechanism of the enforcement of complementarity in atom interferometric interaction-free measurements.

PACS numbers: 03.65.Bz, 42.50.-p

I. INTRODUCTION

One of the conceptual foundations of quantum mechanics states that one cannot obtain information about an object without the object being disturbed by the measurement process. Also the non-observance of a result represents additional information and hence modifies the wavefunction, as described 1960 by Renninger, who used the notion of "negative result measurement" [1], and later Dicke who analysed the change of an atomic wavefunction by the non-scattering of a photon [2]. Elitzur and Vaidman (denoted EV in the following) [3] pointed out the notion of "negative result measurement" [1], and later analyses [3], optimised the fraction of interaction-free measurements, given that an object was in the path. We also show the enforcement of complementarity, which follows solely from entanglement [4] using the IFM as a delicate "which path" ("welcher weg") detector in an atom interferometric [5] Young’s setup in Fig. 1, sticking to the 50 % photon, 50 % atom case. The joint state vector after the beamsplitters is $|g\rangle$ denotes the ground state atom in the upper interferometer arm, and $|e\rangle$ the atomic inversion. We also show the enforcement of complementarity, which follows solely from entanglement [4] using the IFM as a delicate "which path" ("welcher weg") detector in an atom interferometric [5] Young’s double slit experiment similar to the micromasers discussed in [6].

Let us first discuss "which-path" detection in the scheme of Elitzur-Vaidman (EV) [3], applied to the IFM as a delicate "which path" detector in an atom interferometric [5] Young’s experiment similar to the micromasers discussed in [6].

We briefly discuss the quantum properties of the setup in Fig. 1 sticking to the 50 % photon, 50 % atom beamsplitter case, the latter constructed via one $\pi/2$ and one $\pi$-pulse for the atom. The input is a single photon and a single ground-state two level atom. Let $|g\rangle$, $|e\rangle$ denote a ground or excited atom in the upper path, and $|g\rangle$, $|e\rangle$ the ground state atom in the lower path, i.e. the atom is never excited when taking the lower path. The joint state vector after the beam-splitters is $(|g, g\rangle + i|e, -\rangle)/\sqrt{2}$, where trivial, overall phase-factors are omitted. The free evolution Hamiltonian reads

$$H_0 = \hbar \omega [\hat{a}^\dagger \hat{a} + \frac{1}{2}] + \frac{1}{2} (1 + \sigma_z), \quad (1.1)$$

where $\omega$ is the atom transition and light (angular) frequency, and $\sigma_z = |e\rangle \langle e| - |g\rangle \langle g|$ is the atomic inversion. The lower photon arm and the upper atom arm in the interferometers subsequently interact under the (rotating wave) interaction Hamiltonian

$$\hat{H}_i = \hbar \Omega_R (\hat{a}^\dagger \sigma_- + \hat{a} \sigma_+). \quad (1.2)$$

Here $\sigma_- = |g\rangle \langle e|$, and $\sigma_+ = |e\rangle \langle g|$ are the Pauli spin-flip operators, $\hat{a}$ the photon annihilation operator, and
\( \Omega_R \) the vacuum Rabi frequency. Propagating through the rest of the photon interferometer, generally renders a partially entangled superposition of photon, ground- and excited atom states. However, if \( \Omega_R \tau = \pi, 3\pi, \ldots \), the joint photon-atom state becomes perfectly entangled \( |\psi_{\text{at, det}}\rangle = (-i|0, 1, g, -\rangle + i|1, 0, -\rangle, g\rangle)/\sqrt{2} \), whereas if \( \Omega_R \tau = 0, 2\pi, \ldots \), the joint state becomes a product state \( |\psi_{\text{at, det}}\rangle = |1, 0\rangle \otimes (|g, -\rangle + i|g, -\rangle, g\rangle)/\sqrt{2} \). Assuming perfect photodetection, when \( \Omega_R \tau = \pi, 3\pi, \ldots \), the photon-atom interferometer makes a sharp (stroboscopic) quantum non-demolition measurement \( \mathbf{0} \) of the ground state atom number \( \hat{N}_g = |g\rangle\langle g|, \) using an absorptive coupling instead of a dispersive coupling usually employed. Note that the entanglement is an absolute necessity for any information extraction. In the product state, there is no atom ”which-path” information in the photon state. Furthermore, note that when specifying the interaction Hamiltonian, the spatial overlap of the object and the probe system by necessity enters. Any non-zero readout eliminates all atom modes whose overlap is zero, even though the measurement is not an explicit position measurement. This is why the IFM and other quantum measurements also becomes a measurement of the ”precense” of an object. Adding losses (atom damping) to the system, translates the result towards the classical IFM.

The single Mach-Zender IFM scheme features a relatively large probability of absorption. Using a folded Mach-Zender interferometer \[ \mathbf{3} \) or a cavity resonator \[ \mathbf{2} \), the likelihood may approach unity, i.e. the ”which-path” information can asymptotically be extracted without any photon absorption-”interaction”, or classical photon “precense” at the object. It could be noted that also the folded Mach-Zender or the resonator can be further optimised \[ \mathbf{12} \). Let us discuss one such scheme, the interaction free measurement in a symmetric resonator, replacing the photon interferometer by a cavity detector system as in Fig. \[ \mathbf{3} \).

In the cavity IFM, a symmetric cavity is put on resonance to transmit all photons when empty. With an object present, the cavity interference is suppressed and the photons are (mostly) reflected (hence not entering the cavity), indicating the presence of the object. For an opaque object, standard resonator formulas gives \( \mathcal{L}_m = 1 \) and \( \mathcal{A} = 1 - \mathcal{R} \), where \( \mathcal{R} \) is the resonator (power) reflectance. Thus \( \mathcal{L}_m - \mathcal{A} = \mathcal{R} - 1 \) is asymptotically feasible. Furthermore, it is easily shown that in the cavity IFM, the average dissipated energy \( \hbar \omega A \) times the observation time \( T_{\text{meas}} \) fulfills \( \hbar \omega A \times T_{\text{meas}} \geq \hbar/2 \), Bohr’s energy-time uncertainty relation. We believe this is a lower limit for any IFM scheme.

To generate a single photon input we use an auxiliary high Q cavity with a single photon inside that is unidirectionally coupled to the IFM cavity, i.e. a cascaded quantum system \[ \mathbf{14,15} \). The Hamiltonian becomes

\[
\hat{H}_0 = \hbar \omega \left( \frac{1}{2} (\hat{a}^\dagger \hat{a} + 1) + (\hat{b}^\dagger \hat{b} + \frac{1}{2}) + \hat{\sigma}_z \right)
\]

where \( \hat{b} \) is the photon annihilation operator in the auxiliary cavity, and the last term in the Hamiltonian describes the direct cavity coupling. The irreversible photon transfer to the detectors is described by the Lindblad operators \( \hat{C}_R = (\sqrt{\gamma_a} \hat{a} + \sqrt{\gamma_b} \hat{a}^\dagger )_{\delta R} \), and \( \hat{C}_T = \sqrt{\gamma_a} \hat{a}^\dagger_{\delta T} \), where \( \gamma_b \) is the decay rate from the auxiliary cavity, \( \gamma_a \) is the single sided decay rate from the IFM cavity, and \( \delta R \) and \( \delta T \) represent the detector states.

To quantify the information extraction and the back-action, we use the general interferometric quantities of Englert \[ \mathbf{4} \) of distinguishability \( D \), maximum likelihood \( \mathcal{L}_{\text{opt}} \), and fringe visibility \( \mathcal{V} \). Let \( \hat{\rho} = \hat{\rho}_a + \hat{\rho}_b \) be the density matrix of the coupled atom-photon system, where \( \hat{\rho}_a = \langle g | \hat{\rho} | g \rangle + \langle e | \hat{\rho} | e \rangle \) and \( \hat{\rho}_b = \langle -g | \hat{\rho} | g \rangle \), are the (unnormalised) density matrices for the atom taking the upper or lower path, respectively. The distinguishability \( D \) can be defined \[ \mathbf{12} \), slightly generalised from \[ \mathbf{1} \), as

\[
D = \text{Tr}_{\text{det}} \{ (|\hat{\rho}_a - \hat{\rho}_b|) \}, \tag{1.4}
\]

where \( |\hat{A}| \equiv \sqrt{\hat{A}^\dagger \hat{A}} \) denotes the absolute value of the operator \( \hat{A} \) and the trace is taken over the detector states. From this, the optimum maximum likelihood \( \mathcal{L}_{\text{opt}} \), i.e. making the optimum use of the probe states is given from \[ \mathbf{1} \), \( \mathcal{L}_{\text{opt}} = (1 + D)/2 \). Note, that what is measured in an actual experiment is the Hilbert space distance in a chosen detector basis \( |\psi_i\rangle \)

\[
\mathcal{D}_m \equiv \sum_i |\langle \psi_i | (\hat{\rho}_a - \hat{\rho}_b) | \psi_i \rangle| \leq \mathcal{D}, \tag{1.5}
\]

giving the corresponding likelihood estimate \( \mathcal{L}_m \equiv (1 + \mathcal{D}_m)/2 \leq \mathcal{L}_{\text{opt}} \). To quantify the backaction on the atom state we use the atomic fringe visibility \( \mathcal{V} \) given from the reduced density operator \( \hat{\rho}_{\text{at}} \equiv \text{Tr}_{\text{det}} \{ \hat{\rho} \} \) as \( \mathcal{V} \equiv \sqrt{\langle g | (\hat{\rho}_{\text{at}} - \langle g | \hat{\rho}_{\text{at}} | g \rangle ) | g \rangle} = |\mathcal{V}_{\text{at}} - a| \). The quantities \( \mathcal{D} \) and \( \mathcal{V} \) satisfies the duality relation \[ \mathbf{1} \),

\[
\mathcal{D}^2 + \mathcal{V}^2 \leq 1, \tag{1.6}
\]

with equality only for a detector initially in a pure state, and the entangling interaction being unitary. This relation is a fundamental statement of complementarity.

In Fig. \[ \mathbf{3} \) the fringe visibility \( \mathcal{V} \), the maximum likelihood \( \mathcal{L}_m \) and the square sum \( \mathcal{D}_m^2 \) are plotted as a function of the normalised interaction time \( \Omega_R \tau \). The parameters are \( \gamma_a/\Omega_R = 0.4, \gamma_b/\Omega_R = 0.04 \), where \( \gamma_a >> \gamma_b \) is needed in order for the photon to fully enter the IFM cavity, and \( \Omega_R > 2 \gamma_a \) is needed to have a reflection when the atom is inside. As seen, the cavity IFM localises the atom with a high efficiency. Note that, classically the photon (asymptotically with increasing reflectivity) does not enter the cavity, yet a measurement is performed. The mechanism is that the impinging photon sets up the dressed atom-photon states.
\(|\pm\rangle = 1/\sqrt{2}|g, 1\rangle \pm |e, 0\rangle\) with a large reflection for the incident light at the cold cavity resonance. In this case, the single pass phase shift (energy dressing) is very small, on the order of \(\Delta \propto (1 - R)\), compared to \(\pi\) for the atom-photon interferometer. The reason why \(\mathcal{V}^2 + D^2_\text{a} < 1\) stems from the choice of detection basis and the continuous non-unitary observation by the detectors. For a discrete measurement, by a proper unitary interaction before the detection, one may reach \(D_m = D\) \([2]\). For the continuous measurement case, it remains a bit unclear whether that is feasible. However, as \(D_m \rightarrow 1\) when the reflectivity approaches unity, we believe one should be able to reach also \(D_m \rightarrow D\) in this case. For short interaction times \(t \ll 1/\Omega_R\), \(D_m \propto t^2\) and the absorption \(A \propto t^5\) (if losses are inserted). This ability to extract information without absorption in a (repeated) weak measurement is at the heart of the high efficiency IFM. It does not, however, imply the absence of backaction (decrease in fringe visibility), as is evidenced from the duality relation Eq. 1.6. Note that the visibility decreases according to the potentially available information \(\mathcal{D}\), not with that which actually is extracted \(D_m \leq \mathcal{D}\).

Let us discuss the conditioned dynamics of the successfull IFM, i.e. as given from a readout in the \(d_R\) detector. The atom density matrix \(\hat{\rho}_{at,\text{cond}}\) contingent on the readout of a photon in the \(d_R\) detector is given from \(\hat{\rho}_{at,\text{cond}} = \langle g, d_R|\hat{\rho}|g, d_R\rangle/\langle g, d_R|\hat{\rho}|g, d_R\rangle\). In Fig. 1, is shown the conditioned evolution of the reduced atomic density matrix initially prepared in the even superposition state \([|\psi_{at,e}\rangle = 1/\sqrt{2}(|g-\rangle + |g+\rangle)]\). The numerical parameters are \(\gamma_a/\Omega_R = 0.4, \gamma_b/\Omega_R = 0.04\). Two cases are illustrated \(\gamma_c = 0\) corresponding to no spontaneous emission, and \(\gamma_c/\Omega_R = 1.2\) corresponding to strong atom damping, i.e. through the damping operator \(C_c = \sqrt{\gamma_c}\sigma_-.\) Here there is still some probability of the photon first entering the cavity, and then exiting to detector \(d_R\). However, increasing \(\gamma_c\) (and \(\Omega_R\), the probability of the photon being in the cavity and not being absorbed decreases, but the conditioned evolution remains similar.

Let us finally discuss the enforcement of complementarity in IFM on quantum objects. The cavity IFM scheme is very similar to the micromaser "which path" detector/quantum eraser \([3,4]\) where "which path" information is obtained without scattering or introducing large uncontrolled phase factors into the interfering beams. The post IFM atom density matrix in the ideal case of zero visibility can be written as a mixture of the complementary interference patterns \(\hat{\rho}_{at,out} = 1/2|\psi_{at,e}\rangle\langle \psi_{at,e}| + 1/2|\psi_{at,o}\rangle\langle \psi_{at,o}|\), where \(|\psi_{at,e}\rangle = 1/\sqrt{2}(|g-\rangle + |g+\rangle)\) and \(|\psi_{at,o}\rangle = 1/\sqrt{2}(|g-\rangle - |g+\rangle)\). To extract the pure states, the detection basis must be shifted (before the detection) by a unitary transformation, i.e. in practice by adding the reflected and transmitted beams on a \(50\%\) beamsplitter to give the new detection basis \(|d_R\rangle \rightarrow |1/\sqrt{2}(d_R + d_T), 1/\sqrt{2}(d_R - d_T)|\). In the conditioned dynamics of selecting the outcomes from one of the two new detection basises, the interference is once again retrieved. This is how the cavity IFM can be turned into a quantum eraser \([3]\). General conditions for the possibility to retrieve quantum interference in conditioned dynamics will be given elsewhere \([2]\). The interesting difference compared to previous quantum erasers is the classical absence of interference in the IFM between the probe photon and the atom. In the micromaser "which path" detector \([3,4]\), the atom enters in the excited state and exits in the ground state. Here the atom essentially remain in the ground state throughout the interaction. The coupled atom-photon interferometers also displays these features \([2]\), i.e. the upper path atom wavefunction makes a deterministic \(\pi\) phase shift if and only if the photon takes the same path. We like to stress that \(\text{no net energy and no net momentum is imparted on the atoms in the IFM schemes.} \ |\psi_{at,e}\rangle\) and \(|\psi_{at,o}\rangle\) have essentially the same mean momenta (=zero) and roughly the same variance. Complementarity is enforced, not refering to the position-momentum uncertainty relation, but is due to the entanglement in the atom-photon state established by the unitary interaction on the initial atom superposition state, in accordance with the view of Scully et. al. \([1]\).

In summary, interaction-free measurements are interaction-free in that the average absorption can be much less than one energy quanta \([16]\). For quantum objects we may always (in principle) observe the modification of the wavefunction also for the successfull IFMs. Yet, the more opaque and classical the object gets, i.e. the stronger the coupling to a reservoir becomes, the smaller is the detectable trace of interaction, in the limit of a completely opaque classical object to the point where there cannot have been an interaction for the successfull IFM \([38]\). The mechanism of the enforcement of complementarity is subtle, but follows from the (physical) process of the entangling unitary interaction.

II. ACKNOWLEDGEMENTS

The authors would like to thank Edgard Goobar, Tedros Tsegaye and G"oran Lindblad at KTH, Yoshihisa Yamamoto at Stanford University, Paul G. Kwiat at Los Alamos National Laboratory, and Harald Weinfurther and Anton Zeilinger at Innsbruck University for useful discussions and for sending preprints. This work was supported by NFR–the Swedish Natural Science Research Council and TFR–the Swedish Technical Science Research Council.

[1] M. Renninger, Z. Phys., 158, 417 (1960)
[2] R.H. Dicke, Am. J. Phys., 49, 925 (1981)
[3] A. Elitzur and L. Vaidman, Found. Phys., 23, 987 (1993).
[4] B.G. Englert, Phys. Rev. Lett., 77, 2154 (1996).
[5] O. Carnal and J. Mlynek, Phys. Rev. Lett., 66, 2689 (1991).
[6] M. O. Scully, B. G. Englert and H. Walther, Nature (London), 351, 111 (1991).
[7] V. Braginsky, Yu. I. Vorontsov and K. S. Thorne, Science 209, 547 (1980).
[8] P. Kwiat, H. Weinfurther, T. Herzog, A. Zeilinger and M. Kasevich, Phys. Rev. Lett., 74, 4763 (1995);
[9] Y. Yamamoto, private communication, ESF Quantum Optics Conference, Davos, 1994.
[10] This was mentioned by S. Haroche in his "Cavity Quantum Electrodynamics" lectures at the 1995 Les Houches summer school on "Quantum Fluctuations".
[11] A. Karlsson, E. Goobar, T. Tsegaye and G. Björk, to be presented at the QELS’97 conference.
[12] G. Björk and A. Karlsson, to be submitted.
[13] M.O. Scully and K. Drühl, Phys. Rev. A, 25, 2208 (1982).
[14] C.W. Gardiner, Phys. Rev. Lett., 70, 2269 (1993).
[15] H.J. Carmichael, Phys. Rev. Lett., 70, 2273 (1993).
[16] D. Gabor, "Light and Information", Progress in Optics, E. Wolf ed., vol. 1:122 (1966).

FIG. 1. Coupled atom-photon interferometer using 50 % atom- and 50 % photon beamsplitters. The box $H_i$ denotes the interaction region.

FIG. 2. Atom interferometer with a cavity IFM detector in one of the atom arms.

FIG. 3. Fringe visibility $V$, maximum likelihood $L_m$ and (non-optimum) duality $D_m^2 + V^2$ plotted as a function of normalised time.

FIG. 4. Time evolution of the conditioned atomic density operator elements $\hat{\rho}_{at,g\bar{g}}$ (probability of atom in IFM path) and $|\hat{\rho}_{at,g\bar{g}}| = V/2$, conditioned on the detection in $d_R$ versus normalised time.