Analysis of the vertex $D^*D^*\rho$ with the light-cone QCD sum rules

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Abstract

In this article, we analyze the vertex $D^*D^*\rho$ with the light-cone QCD sum rules. The strong coupling constant $g_{D^*D^*\rho}$ is an important parameter in evaluating the charmonium absorption cross sections in searching for the quark-gluon plasmas. Our numerical value for the $g_{D^*D^*\rho}$ is consistent with the prediction of the effective $SU(4)$ symmetry and vector meson dominance theory.

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1 Introduction

The suppression of $J/\psi$ production in relativistic heavy ion collisions maybe one of the important signatures to identify the possible phase transition to the quark-gluon plasma \cite{1}. The dissociation of the $J/\psi$ in the quark-gluon plasma due to color screening can lead to a reduction of its production, on the other hand, the $J/\psi$ suppression maybe already present in the hadron-nucleus collisions. It is necessary to know absorption of the $J/\psi$ by the co-mover light mesons before we can make a definitive conclusion. The values of the $J/\psi$ absorption cross sections by the light hadrons are not known empirically, we have to resort to some theoretical approaches. Among existing approaches, the one-meson exchange model and the effective $SU(4)$ theory are typical \cite{2,3}. The detailed knowledge about the strong coupling constants which are basic parameters in the effective Lagrangian is of great importance. Furthermore, the strong coupling constants among the charmed mesons and light-mesons play an important role in understanding final-state interactions in the hadronic $B$ decays \cite{4}.

There have been many works dealing with the strong coupling constants concerning the charmed mesons, see e.g. Refs.\cite{5,6}. In Ref.\cite{7}, the authors calculate the strong coupling constant $g_{D^*D^*\rho}$ with the QCD sum rules, and obtain much larger value than what expected from the effective $SU(4)$ theory. In this article, we calculate the value of the $g_{D^*D^*\rho}$ with the light-cone QCD sum rules. The light-cone QCD sum rules carry out the operator product expansion near the light-cone $x^2 \approx 0$.

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instead of the short distance $x \approx 0$ while the non-perturbative matrix elements are parameterized by the light-cone distribution amplitudes (which classified according to their twists) instead of the vacuum condensates [3].

The article is arranged as: in Section 2, we derive the strong coupling constant $g_{D^* D^* \rho}$ with the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and in Section 4, conclusion.

2 Strong coupling constant $g_{D^* D^* \rho}$ with light-cone QCD sum rules

We study the strong coupling constant $g_{D^* D^* \rho}$ with the two-point correlation function $\Pi_{\mu\nu}(p, q)$,

$$\Pi_{\mu\nu}(p, q) = i \int d^4x e^{-iq \cdot x} \langle 0 | T \{ J_\mu(0) J_\nu^+(x) \} | \rho(p) \rangle , \quad (1)$$

$$J_\mu(x) = \bar{u}(x) \gamma_\mu c(x) , \quad J_\nu(x) = \bar{d}(x) \gamma_\nu c(x) , \quad (2)$$

where the currents $J_\mu(x)$ and $J_\nu(x)$ interpolate the vector mesons $D^{*0}$ and $D^{*+}$ respectively, the external state $\rho$ has the four momentum $p_\mu$ with $p^2 = m_\rho^2$.

According to the basic assumption of current-hadron duality in the QCD sum rules [9], we can insert a complete series of intermediate states with the same quantum numbers as the current operators into the correlation function to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the mesons $D^*$, we get the following result,

$$\Pi_{\mu\nu}(p, q) = -\frac{f_{D^*}^2 M_{D^*}^2 g_{D^* D^* \rho}}{\{M_{D^*}^2 - (q + p)^2\} \{M_{D^*}^2 - q^2\}^2} 2g_{\mu\nu} \epsilon \cdot q + \cdots , \quad (3)$$

where the following weak decay constant and phenomenological Lagrangian have been used,

$$\langle 0 | J_\mu(0) | D^*(p) \rangle = f_{D^*} M_{D^*} \epsilon_\mu ,$$

$$\mathcal{L} = \frac{i g_{D^* D^* \rho}}{\sqrt{2}} \left\{ \partial_\mu D^*_\nu \rho^\mu \bar{D}^* \nu - D^*_\mu \rho_\mu \partial^\nu \bar{D}^* \nu + D^*_\nu \partial^\mu \rho_\mu \bar{D}^* \nu - D^*_\mu \partial^\mu \rho_\nu \bar{D}^* \nu \right\} , \quad (4)$$

here $\rho = \sigma_i \rho_i$, $D^* = (D^{*0}, D^{*+})$ [2, 3].

We carry out the operator product expansion for the correlation function $\Pi_{\mu\nu}(p, q)$ in perturbative QCD theory, and obtain the analytical expressions at the level of quark-gluon degrees of freedom, then perform the double Borel transformation and match the quark-hadron duality below the threshold $s_0$, finally we obtain the sum
rule for the strong coupling constant $g_{D^*D^*\rho}$,

$$2g_{D^*D^*\rho}f_{D^*}^2M_{D^*}^2 \exp \left\{ \frac{M_{D^*}^2}{M_{D^*}^2 - M_{D^*}^2} \right\}$$

$$= f_\rho m_\rho M^2 \phi_\parallel(u_0) \left\{ \exp \left[ -\frac{m_\rho^2 + u_0(1 - u_0)m_\rho^2}{M^2} \right] - \exp \left[ -\frac{s_0^0}{M^2} \right] \right\}$$

$$+ \exp \left[ -\frac{m_\rho^2 + u_0(1 - u_0)m_\rho^2}{M^2} \right] \left\{ \left[ f_\rho - f_\rho \frac{m_u + m_d}{m_\rho} \right] m_\rho^2 h_{||}^{(s)}(u_0) \right\}$$

$$= \frac{-f_\rho m_\rho^2 A(u_0)}{4} \left[ 1 + \frac{m_\rho^2}{M^2} \right] - \frac{2f_\rho m_\rho^2 m_\rho^3}{M^2} \int_0^{u_0} \int_0^\tau d\tau \int_0^\tau dC(t) \right\}, \quad (5)$$

where

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2},$$

$$M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}. \quad (6)$$

Here we have neglected the contributions from the gluons, the contributions proportional to $G_{\mu\nu}$ can give rise to three-particle (and four-particle) meson distribution amplitudes with a gluon (and quark-antiquark pair), their corrections are always suppressed by the large Borel parameter and will not exceed 20%, for examples, one can consult the last two articles of Ref.[6]. In calculation, the two-particle $\rho$ meson light-cone distribution amplitudes up to twist-4 have been used, for explicit expressions, one can consult Ref.[10]. Due to the special tensor structure $g_{\mu\nu} \epsilon \cdot q$, some two-particle twist-4 light-cone distribution amplitudes have no contributions. The parameters in the light-cone distribution amplitudes are scale dependent, in this article, we take $\mu = 1$GeV. As the dominating contribution (about 90%) comes from the two-particle twist-2 term involving the $\phi_\parallel(u)$, for other terms, the continuum subtractions will not affect the result remarkably, we can neglect the subtractions. In some cases, the contributions from the two-particle twist-3 light-cone distribution amplitudes are very large (or dominating) [11], they depend heavily on the currents we choose to interpolate the mesons.

### 3 Numerical result and discussion

The input parameters are taken as $m_c = (1.35 \pm 0.10)$GeV, $m_u = m_d = (0.0056 \pm 0.0016)$GeV, $f_\rho = (0.216 \pm 0.003)$GeV, $f_{D^*} = (0.165 \pm 0.009)$GeV, $m_\rho = 0.775$GeV, $a_1^3 = 0.0$, $a_1^4 = 0.0$, $a_2^3 = 0.15 \pm 0.07$, $a_2^4 = 0.14 \pm 0.06$, $\lambda_3 = 0.030 \pm 0.010$, $\bar{\lambda}_3 = 0.0$, $\bar{\lambda}_3 = -0.09 \pm 0.03$, $\zeta_3 = 0.0$, $\zeta_3^T = 0.15 \pm 0.05$, $\lambda_4^3 = 0.0$, $\zeta_4^T = 0.15 \pm 0.05$ and $\zeta_4^T = -0.10 \pm 0.05$ [10]. The central value of the decay constant $f_{D^*}$ from lattice simulation is about $f_{D^*} = 0.23$GeV [12].
In this article, we take the value \( f_{D^*} = (0.22 \pm 0.02) \text{GeV} \) from the two-point QCD sum rules without perturbative \( \mathcal{O}(\alpha_s) \) corrections for consistency \(^3\). The duality threshold parameter \( s_0 \) is chosen to be \( s_0^0 = (6.5 \pm 0.5) \text{GeV}^2 \); the numerical (central) value of \( s_0 \) is taken from the QCD sum rules for the mass of the \( D^* \) \(^\text{[13]}\). The Borel parameters are chosen as \( M_1^2 = M_2^2 \) and \( M^2 = (3 - 7) \text{GeV}^2 \); in those regions, the value of the \( g_{D^*D^*\rho} \) is rather stable, the uncertainty from the Borel parameter is very small, less than 5%.

In the limit of large Borel parameter \( M^2 \), the strong coupling constant \( g_{D^*D^*\rho} \) takes up the following behavior,

\[
g_{D^*D^*\rho} \propto \frac{M^2 f_\parallel(u_0)}{f_{D^*}^2} \propto \frac{M^2 f_\parallel a_2}{f_{D^*}^2} \quad . \tag{7}\]

It is not unexpected, the contribution from the twist-2 light-cone distribution amplitude \( \phi_\parallel(u) \) is greatly enhanced by the large Borel parameter \( M^2 \), its contribution is dominating, about 90%, (large) uncertainties of the relevant parameters presented in above equation have significant impact on the numerical result. The main uncertainties come from the two parameters \( f_{D^*} \) and \( a_2^\parallel \), variations of the two parameters can lead to large uncertainties, about (10 − 20)%.

Taking into account all the uncertainties, finally we obtain the numerical value for the strong coupling constant \( g_{D^*D^*\rho} \), which is shown in Fig.1,

\[
g_{D^*D^*\rho} = 2.6 \pm 0.7 . \tag{8}\]

If we take the replacement \( \exp \left[ -\frac{m_c^2 + u_0(1-u_0)m_2^2}{M^2} \right] \rightarrow \exp \left[ -\frac{m_c^2 + u_0(1-u_0)m_2^2}{M^2} \right] - \exp \left[ -\frac{s_0^0}{M^2} \right] \)

to subtract the continuum contributions of the terms besides the twist-2 light-cone distribution amplitude, the central value \( g_{D^*D^*\rho} = 2.6 \) will decrease about 5%. Comparing with the values from the effective \( SU(4) \) symmetry and vector meson dominance theory, \( g_{D^*D^*\rho} = g_{DD\rho} = 2.52 \) \(^3\), our numerical value \( g_{D^*D^*\rho} \rightarrow 2.6 \sqrt{2} = 1.8 \pm 0.5 \) is much smaller. In the vector meson dominance theory, \( g_{DD\rho} = \frac{m_\rho}{\sqrt{2}f_\rho} \), the contributions from the radial excited states \( \rho(1450), \rho(1700), \rho(1900), \cdots \) are neglected. We can make a crude estimation with the simple replacement \( m_\rho \rightarrow \frac{m_\rho m_{1450}}{m_\rho + m_{1450}} \) to take into account the contribution from the \( \rho(1450) \), \( g_{D^*D^*\rho} = g_{DD\rho} = 1.66 \), our numerical result is reasonable, the \( SU(4) \) symmetry breaking effect for the strong coupling constants is small. In Ref.\(^7\), the authors introduce some functions to parameterize the form-factor for small spacelike \( Q^2 \), then extrapolate to the mass shell, much larger value is obtained, \( g_{D^*D^*\rho} = 6.6 \pm 0.3 \). Although the model functions have solid theoretical foundation at large \( Q^2 \), extrapolation to the mass shell may have good or bad behaviors, which correspond to the systematic errors. It is not unexpected that the values from the QCD sum rules and light-cone QCD sum rules are different.

\(^2\)One can consult the last article of Ref.\(^9\) or Ref.\(^{13}\) for the explicit expression.
4 Conclusion

In this article, we analyze the vertex $D^* D^* \rho$ with the light-cone QCD sum rules. The strong coupling constant $g_{D^* D^* \rho}$ is an important parameter in evaluating the charmonium absorption cross sections in searching for the quark-gluon plasmas. Our numerical value for the $g_{D^* D^* \rho}$ is consistent with the prediction of the effective $SU(4)$ symmetry and vector meson dominance theory.

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