A New Method for Atlanta World Frame Estimation

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Abstract

In this paper, we propose a new Atlanta frame estimation method by considering the relationship between vertical direction and horizontal directions. Unlike previous solutions, our method does not solve all the directions at one time. On the contrary, it estimates the directions sequentially. Concretely, our method first searches the vertical direction in $\mathbb{S}^2$ globally, then estimates the horizontal directions in one-dimension. As a consequence, the dimensionality of each subproblem problem is low and it can be solved efficiently. In other words, the running time of our method will not greatly increase as the number of horizontal directions increases. The advantages of our method are validated via testing on both synthetic and real-world data.

1 Introduction

In the man-made environments, the scenes usually have structural forms (e.g., the layout of buildings and many indoor objects such as furniture), which can be represented by a set of parallel and orthogonal planes or lines. Atlanta world makes an assumption that a vertical direction and multiple horizontal directions could represent the scene structure [27]. Therefore, it is a crucial step to estimate these vertical and horizontal directions, which is named Atlanta frame estimation, for the scene understanding [10, 17]. Furthermore, it could be utilized as key modules for various high-level vision applications such as scene representation [13, 31] and SLAM [32, 38].

Mathematically, an orientation in 3D Euclidean space is corresponding to a point in the 3D unit sphere. This means that the Atlanta frame estimation which estimates multiple orientations is a multiple-clustering (also multi-model fitting) problem in 3D unit sphere. There have been lots of general multiple-clustering algorithms [1, 3, 23] and some of them have been applied in world frame estimation [19, 33]. However, Atlanta frame estimation is not exactly the same as the general multiple-clustering problem. It has some special constraints that all horizontal directions are in a plane and the vertical direction is parallel to the normal of the plane. These special constraints reflect the essential properties of the Atlanta World assumption. If omitting these constraints, it will not only lead to a significant decrease in accuracy but also increase the dimensionality of the problem. Furthermore, most
of the multiple-clustering algorithms cannot guarantee global optimality when there are lots of outliers in observation [4, 5]. Therefore, recent developments in structural world frame estimation highlight the imminent need for robust and globally optimal methods with considering the above special constraints [17, 18].

Recently, Manhattan frame estimation [31], which is a special case of the Atlanta frame estimation, is solved efferently with considering orthogonal constraints by branch-and-bound method [18]. However, it will suffer the curse of dimensionality when coming to the Atlanta World [17]. The reason is obvious that there is a considerable number of horizontal directions, whose relationships that are different from Manhattan World are unknown. Consequently, the dimensionality of the problem will increase greatly with the number of the horizontal directions.

1.1 Related work

There is a large body of literature that are concerned with structural world frame estimation [17, 18, 20, 31]. Since it is a clustering problem in $S^2$ with some orthogonal constraints, we first review the works that apply the classical clustering or fitting method. With the definition of Atlanta World, Expectation Maximization (EM) type algorithms, which are popular to solve the chicken-and-egg problem [21], are applied in directions estimation [27]. However, the EM-type algorithms are local methods and have no guarantee of the global optimality. Therefore, there is an evident risk of local minima, and their performances rely heavily on a good initialization [2]. Besides, the RANdom SAmple Consensus (RANSAC) [26] based multi-structure estimation algorithms (e.g., T-linkage [22] and J-linkage [34]) are applied in structural directions estimation [19, 33]. These methods are fast, accurate and being one of the best performing in many cases, but the global optimality is not guaranteed due to their obvious heuristic nature [17]. Additionally, Straub, etc. [31] propose a real-time capable inference algorithm by considering the orthogonal constraints, which is useful for extracting the local structural orientation and segmentation of the scene from the data stream. However, when there are lots of outliers in the measurements, the above methods cannot guarantee the global optimality.

To assure global optimality, J. Bazin, etc. propose global methods [4, 5, 17, 18] by applying branch-and-bound algorithm to solve a consensus set maximization. The fundamental theory of these global methods is rotation search [4]. Specially, the method in [4] is a natural application of rotation search. In [18], 2D-EGI (extended Gaussian image) and its integral image are applied to accelerate the calculation of the bounds in rotation search. Furthermore, rotation search is extended to Atlanta frame estimation in [17]. It is worth noting that rotation search usually means optimization in SO(3), which corresponds to $S^3$ [4, 5]. The rotation search theory in SO(3) has achieved great success in geometric vision problems (e.g., point set registration [4, 5], calibration [4, 5] and relative pose estimation [4, 5]). Besides, there have been several works focusing on improving the efficiency of the algorithm [4, 5]. However, the estimation of three-dimensional directions (i.e., Manhattan or Atlanta frame) is optimized in $S^2$. Unfortunately, there is still a lack of rigid theories regarding globally optimal optimization in $S^2$. In order to optimize the three-dimensional directions, we originally propose some new and solid mathematical conclusions about rotation search in $S^2$ in this paper.
1.2 Our contribution

In this paper, we propose a novel method for Atlanta frame estimation by considering the relationship between vertical direction and horizontal directions in Atlanta world. The contributions of this work are mainly as follows. (1) Our method decouples the vertical direction estimation and horizon directions estimation, which is different from the one-time Atlanta frame estimation. Consequently, the run time of our method will not greatly increase as the number of the horizontal directions increases. (2) We derive a global searching method in $\mathbb{S}^2$, which is different from conventional rotation search in $\text{SO}(3)$ (i.e., $\mathbb{S}^3$). Since the domain of the three-dimensional structural directions is inherently in $\mathbb{S}^2$, then our searching method is more efficient for Atlanta frame estimation. (3) Our method can guarantee the optimality of vertical direction estimation. We use the branch-and-bound algorithm, which is a global optimization method, to find the best solution.

2 Method

2.1 Problem formulation

In this paper, we denote the vertical direction as $v_0 \in \mathbb{S}^2$, $i$-th horizontal direction as $v_i \in \mathbb{S}^2$, $i = 1 \cdots M$, where $M$ is the number of horizontal directions. According to the Atlanta World assumption, their relationship is

$$v_0^T \cdot v_i = 0, i = 1 \cdots M \quad (1)$$

The input normal set is $N = \{n_j\}_{j=1}^N$, where $n_j \in \mathbb{S}^2$ is $j$-th effective unit normal, which represents two opposite directions, and $N$ is the number of input normals. Note that lines clustering and vanishing points detection are closely related to structural frames estimation, but we only consider normal-measurements as the input in the paper. Accordingly, the inlier set is defined as $S_I$.

$$S_I = \{(v_i, n_j)|\angle(v_i, n_j) < \tau, i = 0 \cdots M, j = 1 \cdots N\} \quad (2)$$

where $\tau$ is the inlier threshold. The popular robust objective function is maximizing the cardinality $E$ of the inlier set $S_I$ by optimize $v_i$.

$$E^*(v_i) = \max|S_I|, i = 0 \cdots M \quad (3)$$

However, the dimensionality of solving Eq.(3) will increase with the number of horizontal directions. We, in this paper, reformulate the original objective function with considering Eq.(1). The inlier set is reformulated as $S_I^v$.

$$S_I^v = \{(v_0, n_j)|\angle(v_0, n_j) < \tau \text{ or } |\angle(v_i, n_j)| < \tau, j = 1 \cdots N\} \quad (4)$$

Note that there is only vertical direction to be solved in Eq.(4). Accordingly, the objective function is

$$E^v_*(v_0) = \max|S_I^v| \quad (5)$$

In addition, we can rewrite the Eq.(3) as follows

$$E^v_*(v_0) = \max(\sum_{j=1}^N |\angle(v_0, n_j) - \frac{\pi}{2}| < \tau) \quad (6)$$
where \([\cdot]\) is an indicator function which returns 1 if the condition \(\tilde{A}u\) is true and 0 otherwise.

Our reformulation of Eq.(5) and Eq.(6) reduces the dimensionality of the original problem significantly, and it is enough effective to solve the vertical direction. Specifically, our reformulation remains constantly the dimensionality because there are no horizontal directions to be solved in this step. We solve the horizontal directions after obtaining the vertical direction, which is easier than solving the original problem at one time.

### 2.2 Globally optimal estimating vertical direction

Finding the best \(v_0\) in \(S^2\) to maximize the cardinality \(E_v\) of the inlier set \(S^I_v\) is not a trivial problem \([9, 16]\). Additionally, the outlier observation, which is inevitable in the real application, increases the \(\tilde{A}\) difficulty of the estimation problem. Because it is well known that the robust estimation with outlier observation is an NP-hard problem \([8]\). To obtain the robust optimal vertical direction, we then use the branch-and-bound algorithm. The branch-and-bound algorithm is the most commonly used tool for solving NP-hard optimization problems and widely applied in many global optimization problems \([24]\). Briefly, the branch-and-bound algorithm recursively divides the search space into smaller spaces and estimates the upper bound and lower bound of the optimum in each subspace. Then, it removes the subspace which cannot produce a better solution than the best one found so far by the algorithm. The above process is repeated until the best optimum is found within the desired accuracy.

#### 2.2.1 Parametrization

Before applying the branch-and-bound algorithm, we must firstly parameterize the searching space. Geometrically, seeking a direction in 3D Euclidean space can be equivalent to the seeking of a point in the 3D unit sphere. However, two opposite directions could be represented by one effective normal. Therefore, the searching space can be expressed by a hemisphere. Nevertheless, how to represent the 3D hemisphere sphere elegantly is not an easy problem. In this paper, we use a compact expression, which has two parameters instead of three parameters with a unit constraint, to represent the 3D hemisphere sphere. Our idea is inspired by the relationship between the quaternion repression and the Angle-Axis repression for \(SO(3)\) \([11, 12]\).

Let \(s = (s_1, s_2, s_3) \in S^2\) denotes a direction in the 3D unit sphere and the unit constraint is \(s_1^2 + s_2^2 + s_3^2 = 1\). If \(s_1 \geq 0\) then it is a hemisphere, which can be expressed by \((r, \theta)\) in 2D polar coordinates as follows

\[

g_1 = \cos(r) \\
g_2 = \cos(\theta) \cdot \sin(r) \\
g_3 = \sin(\theta) \cdot \sin(r)
\]

where \(r \in [0, \pi/2]\) and \(\theta \in [-\pi, \pi]\). Let \(d = (d_1, d_2) \in \mathbb{R}^2\) denotes a point in 2D-disc. It can be expressed by \((r, \theta)\) in 2D polar coordinates.

\[

d_1 = \cos(\theta) \cdot r \\
d_2 = \sin(\theta) \cdot r
\]

The 2D disc and the 3D hemisphere are related by \((r, \theta)\). Specifically, the relationship can be interpreted as the 2D disc is mapped to the 3D hemisphere. Furthermore, the mapping
Figure 1: A visual interpretation of **Proposition 1**. The unit sphere in 3D represents the space of normal directions ($S^2$). The disc in 2D represents the space of compact parameters.

is one-to-one mapping, and it is worth noting that the relation between the disc and the hemisphere is similar to the relation between the quaternion repression and the Angle-Axis repression for $SO(3)$. We are inspired by a very famous theory that the angle between the two quaternions is less than their half Euclidean distance in Angle-Axis Space [11, 12], and propose a proposition as follows

**Proposition 1.** $d_a$ and $d_b$ are two points in 2D disc, their corresponding expressions in 3D hemisphere are $s_a$ and $s_b$. Then they have the relationship as follows

$$\angle(s_a, s_b) \leq \|d_a - d_b\|$$  \hspace{1cm} (12)

**Proof.** The complete derivation is in the appendix and Fig.1 shows the visual geometric interpretation.  

2.2.2 Bounds Estimation

In our branch-and-bound algorithm, the 2D square circumscribing the disc is used as the vertical direction domain for ease of manipulation. We recursively subdivide it into four smaller squares and calculate the estimation of the upper bound and lower bound for the optimum in each sub-branch. Since the success of a branch-and-bound algorithm is predicated on the quality of its bounds, we propose an elegant upper bound (the geometrical interpretation is illustrated in Fig.(2)) for the inlier set maximization problem based on our originally proposed proposition.

**Theorem 1** (upper bound). Given a square $B$ in 2D square as a branch, whose half side is $\sigma$. The center is $d_0$ which is corresponding to $s_0$ in hemisphere. The upper bound of inlier set cardinality can be chosen as

$$\overline{E}(B) = \sum_{j=1}^{N} \left| \angle(s_0, n_j) < \tau + \sqrt{2} \cdot \sigma \right| + \sum_{j=1}^{N} \left| \|s_0 - n_j\| - \frac{\pi}{2} \right| < \tau + \sqrt{2} \sigma$$  \hspace{1cm} (13)
The complete proof is in the appendix. □

**Theorem 2** (lower bound). Given a square $B$ in 2D square as a branch, whose half side is $\sigma$. The center is $d_0$ which is corresponding to $s_0$ in hemisphere. The lower bound of inlier set cardinality can be chosen as

$$E_v(B) = \sum_{j=1}^{N} \lfloor \angle(s_0, n_j) < \tau \rfloor + \sum_{j=1}^{N} \lfloor \|\angle(s_0, n_j) - \frac{\pi}{2}\| < \tau \rfloor$$

(14)

**Proof.** It is very simple that the function value at a specific point within the domain is less than or equal to the maximum. □

Now, we have the upper bound and lower bound for objective function within a certain feasible domain. Then we can apply the branch-and-bound algorithm to search the best vertical direction. The algorithm is summed up as Algorithm 1.

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**Algorithm 1** Branch-and-bound algorithm to obtain vertical direction

**Input:** Normals $n_i \in S^2, i = 1 \cdots N$ and inlier threshold $\tau$.

**Output:** Optimal vertical direction $v_0^\ast$.

1: Initialize $B \leftarrow$ square of $\pi$, and insert $B$ into a priority queue $q$.
2: while $q$ is not empty do
3: Subdivide $B$ into four cubes $\{B_d\}_{d=1}^4$.
4: For each $B_d$ calculate upper bound and lower bound $\{\bar{E}_d, E_d\}_{d=1}^4$.
5: Update the best solution so far: $E_v^\ast(v_0^\ast) = \max_{i} E_i, i$ for all branches.
6: Remove the branches that $\bar{E}_i < E^\ast_i, i$ for all branches.
7: Update highest priority cube $B$ with upper bound $\bar{E}$ for next loop.
8: if $\bar{E} = E^\ast_v$ then
9: Terminate and return $v_0^\ast$.
10: end if
11: end while

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### 2.3 Horizontal directions clustering

After obtaining the vertical direction, the Atlanta frame estimation problem becomes very easy, because all horizontal directions are in the horizontal plane whose normal is parallel to the vertical direction. Therefore, estimating the horizontal directions with known vertical direction is a one-dimensional clustering or multi-model fitting problem. There have been lots of methods could solve the problem [15, 35](e.g., Gaussian Mixture Models, $k$-Means Clustering and Hierarchical Clustering). It is also worth noting that if the number of horizontal directions is known, we can apply the branch-and-bound algorithm to solve the horizontal directions sequentially, and we give the bounds and its rigid proof in the appendix for the one-dimensional searching methods.

Nevertheless, we give a simpler method to solve the subproblem in this paper. The method does not need the number of the horizontal directions. Firstly, according to the Atlanta World assumption, we can significantly remove the outliers, which are not in the horizontal plane.
Theorem 3. In Atlanta World frame estimation, given the vertical direction \( \mathbf{v}_0 \), the horizontal direction inlier \( \mathbf{n} \) must satisfy the following inequation.

\[
|\angle(\mathbf{v}_0, \mathbf{n}) - \frac{\pi}{2}| < \tau \tag{15}
\]

Proof. The horizontal inlier can be defined as

\[
S^h_i = \{(\mathbf{v}_i, \mathbf{n}_j) | \angle(\mathbf{v}_i, \mathbf{n}_j) < \tau, i = 1 \cdots M, j = 1 \cdots N \} \tag{16}
\]

According to Eq.(1) which is the essential constraints of the Atlanta World assumption, we have

\[
|\angle(\mathbf{v}_0, \mathbf{n} - \frac{\pi}{2})| = |\angle(\mathbf{v}_0, \mathbf{n}) - \angle(\mathbf{v}_0, \mathbf{v}_i)| \leq \angle(\mathbf{v}_i, \mathbf{n}) \leq \tau \tag{17}
\]

the last two inequations follow from the triangle inequality in spherical geometry. \( \square \)

Eq. (15) gives the rule to reduce the outliers significantly and reduces the dimensionality of the problem to one. Note that we cannot remove all the outliers in this step. After filtering the outliers, all the remaining normals are in an equatorial patch of the sphere as Fig. 3. We then project them into the equatorial plane and use the angles to represent these normals. Concretely, we first define the 0-degree direction \( \mathbf{v}_0^h \) and 90-degree direction \( \mathbf{v}_90^h \) and calculate the angles \( \{\theta_j\}_{j=1}^k \) corresponding to the remaining \( k \) normals. We then solve the horizontal directions estimation problem in the angle-space by finding the peaks of the angle-histogram as shown in Fig. 3.

3 Experiments

To validate our new method and highlight our contribution, we have conducted experiments with both synthetic and real-world data. All experiments are performed in MATLAB2018b on a laptop with Intel(R) i5 CPU 1.60GHz and 8 GB RAM.

3.1 Synthetic data

We generate a set of vertical planes as Fig.4(a) simulating a street corner of Atlanta World to test our method. Firstly, we show the convergence of our global searching method in \( S^2 \), and then we show the robustness to outlier and noise. Lastly, we show the time complexity...
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Figure 4: A demo for synthetic experiments. (a) Six planes with outlier and noise are simulating a street corner in Atlanta World; (b) Atlanta fame(red directions) in normal space; (c) Evolution over iteration of the upper and lower bounds; (d) Evolution over iteration of the remaining area in 2D square.

Figure 5: The robustness experiments. (a) Error($^\circ$) under different levels of noise; (b) Run time(s) under different levels of noise; (c) Error($^\circ$) under different levels of outlier; (c) Run time(s) under different levels of outlier.

of our method and confirm our contribution that the time will not increase badly with the number of horizontal directions in searching for vertical directions.

**Convergence** We generate six vertical planes, and their relationship of horizontal directions are unknown. The total number of 3D points is 10k. To make a realistic simulation, we corrupt the data with a certain percentage $p$ of the input data and a certain level of noise. Concretely, we add $p = 20\%$ outliers and Gaussian noise whose standard deviation is 0.001. The inlier threshold of our method is set to $1^\circ$. To validate the global optimality, a random rotation is applied to this system, and we run our global searching method in $S^2$ to recover the vertical direction. The evolution process of searching in $S^2$ is shown in Fig.4 (c) and (d).

**Robustness to noise and outlier** To test the robustness of our vertical direction searching method, we use the six vertical planes which are added different levels of noise, in which the standard deviation is from 0.001 to 0.01. The total number of the 3D points is 5000. The outlier rate is still kept at 20%. We run our global searching method in two different inlier thresholds and each trail is executed 100 times repeatedly with random rotated vertical direction and calculate the vertical direction error. Fig 5.(a) and(b) demonstrates the results. Besides, different levels of outliers are added to the six planes to verify the robustness to the outlier of our method. The outlier rate is from 0 to 80%, and the noise level is 0.005. Similarly, each trail is repeated 100 times and different inlier thresholds are tested. The results are demonstrated in Fig. 5(c) and (d).

**Time profiling** To show the computational efficiency, we test the vertical searching method with different numbers of normals, from 1000 to 8000. There are also 20% outlier and Gaussian noise with 0.001 standard deviation in the six vertical planes. The median
time of 100 trails is all shown in Fig. 6(a). In addition, it is well known that getting the exact solution to robust estimation with outlier measurements can be solved faster than $O(N^d)$, where $N$ is the number of measurements; $d$ is the dimensionality of the problem [8]. Therefore, reducing the dimensionality of the original problem is very important to speed up the estimating directions and is one of our main contributions. We test our method in different numbers of planes from 2 to 5, in the meanwhile, the outlier percentage is 20% and the standard deviation of the noise is 0.001, and each trial is repeated 100 times. Fig. 6(b) shows the run time of the entire algorithm.

3.2 Real-world data

We test our method on the NYUv2 Dataset[29], which contains 1449 RGB images, along with the corresponding depths, as well as camera information. In our experiments, we utilize the data to estimate the vertical direction of the scene. Concretely, we generate the normals from the depth image and estimate the vertical direction from the downsampled normal data for all scenes. The threshold is set to $1^\circ$ in branch-and-bound algorithm. The results show that our methods could find the vertical direction for almost all the scenes. The running time of each scene is also shown in Fig. 7. In addition, we give more visual results in appendix.

4 Conclusion

In this paper, we propose a novel method to estimate the Atlanta world frame. In contrast to existing methods, we decouple the estimation of vertical direction and horizontal directions. Concretely, we first propose a novel globally optimal searching method in $S^2$ to solve the
vertical direction. Then, we filter the outliers that are not in the horizontal plane and estimate the horizontal directions by searching peaks of the histogram in one-dimension angle space. Furthermore, our method has been validated on both synthetic data and real-world data. It is worth noting that our globally optimal searching method in $S^2$ can be extended to work on other structural worlds (e.g., Mixture of Manhattan World [31]). Moreover, GPU can be used to accelerate the speed of our method, since the calculation of the bounds can be paralleled.

**References**

[1] Paul Amayo, Pedro Piniés, Lina M Paz, and Paul Newman. Geometric multi-model fitting with a convex relaxation algorithm. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 8138–8146, 2018.

[2] Michel Antunes and Joao P Barreto. A global approach for the detection of vanishing points and mutually orthogonal vanishing directions. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 1336–1343, 2013.

[3] Daniel Barath and Jiri Matas. Multi-class model fitting by energy minimization and mode-seeking. In *Proceedings of the European Conference on Computer Vision (ECCV)*, pages 221–236, 2018.

[4] Jean-Charles Bazin, Yongduek Seo, Cédric Demonceaux, Pascal Vasseur, Katsushi Ikeuchi, Inso Kweon, and Marc Pollefeys. Globally optimal line clustering and vanishing point estimation in manhattan world. In *2012 IEEE Conference on Computer Vision and Pattern Recognition*, pages 638–645. IEEE, 2012.

[5] Jean-Charles Bazin, Yongduek Seo, and Marc Pollefeys. Globally optimal consensus set maximization through rotation search. In *Asian Conference on Computer Vision*, pages 539–551. Springer, 2012.

[6] Parra Bustos and Álvaro Joaquín. Robust rotation search in computer vision. 2016.

[7] Dylan John Campbell, Lars Petersson, Laurent Kneip, and Hongdong Li. Globally-optimal inlier set maximisation for camera pose and correspondence estimation. *IEEE transactions on pattern analysis and machine intelligence*, 2018.

[8] Tat-Jun Chin, Zhipeng Cai, and Frank Neumann. Robust fitting in computer vision: Easy or hard? In *ECCV*, 2018.

[9] N. I. Fisher. *Statistical Analysis of Circular Data*. 1993.

[10] Alex Flint, David Murray, and Ian Reid. Manhattan scene understanding using monocular, stereo, and 3d features. In *2011 International Conference on Computer Vision*, pages 2228–2235. IEEE, 2011.

[11] Richard Hartley, Jochen Trumpf, Yuchao Dai, and Hongdong Li. Rotation averaging. *International journal of computer vision*, 103(3):267–305, 2013.

[12] Richard I Hartley and Fredrik Kahl. Global optimization through rotation space search. *International Journal of Computer Vision*, 82(1):64–79, 2009.
[13] Varsha Hedau, Derek Hoiem, and David Forsyth. Recovering the spatial layout of cluttered rooms. In 2009 IEEE 12th international conference on computer vision, pages 1849–1856. IEEE, 2009.

[14] Jan Heller, Michal Havlena, and Tomas Pajdla. Globally optimal hand-eye calibration using branch-and-bound. IEEE Transactions on Pattern Analysis and Machine Intelligence, 38(5):1027–1033, 2016.

[15] Anil K. Jain, M. Narasimha Murty, and Patrick J. Flynn. Data clustering: A review. ACM Comput. Surv., 31:264–323, 1999.

[16] D S Johnson and F P Preparata. The densest hemisphere problem. Theoretical Computer Science, 6(1):93–107, 1977.

[17] Kyungdon Joo, Tae-Hyun Oh, In So Kweon, and Jean-Charles Bazin. Globally optimal inlier set maximization for atlanta frame estimation. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 5726–5734, 2018.

[18] Kyungdon Joo, Tae-Hyun Oh, Junsik Kim, and In So Kweon. Robust and globally optimal manhattan frame estimation in near real time. IEEE transactions on pattern analysis and machine intelligence, 41(3):682–696, 2019.

[19] Seongdo Kim and Roberto Manduchi. Multi-planar fitting in an indoor manhattan-world. In 2017 IEEE Winter Conference on Applications of Computer Vision (WACV), pages 11–19. IEEE, 2017.

[20] Gim Hee Lee. Line association and vanishing point estimation with binary quadratic programming. In 2017 International Conference on 3D Vision (3DV), pages 584–592. IEEE, 2017.

[21] Hongdong Li and Richard Hartley. The 3d-3d registration problem revisited. In 2007 IEEE 11th international conference on computer vision, pages 1–8. IEEE, 2007.

[22] Luca Magri and Andrea Fusiello. T-linkage: A continuous relaxation of j-linkage for multi-model fitting. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 3954–3961, 2014.

[23] Luca Magri and Andrea Fusiello. Multiple model fitting as a set coverage problem. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 3318–3326, 2016.

[24] David R. Morrison, Sheldon H. Jacobson, Jason J. Sauppe, and Edward C. Sewell. Branch-and-bound algorithms: A survey of recent advances in searching, branching, and pruning. Discrete Optimization, 19:79–102, 2016.

[25] Alvaro Parra Bustos, Tat-Jun Chin, and David Suter. Fast rotation search with stereographic projections for 3d registration. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 3930–3937, 2014.

[26] Rahul Raguram, Ondrej Chum, Marc Pollefeys, Jiri Matas, and Jan-Michael Frahm. Usac: a universal framework for random sample consensus. IEEE transactions on pattern analysis and machine intelligence, 35(8):2022–2038, 2013.
[27] Grant Schindler and Frank Dellaert. Atlanta world: An expectation maximization framework for simultaneous low-level edge grouping and camera calibration in complex man-made environments. In Proceedings of the 2004 IEEE Computer Society Conference on Computer Vision and Pattern Recognition, 2004. CVPR 2004., volume 1, pages I–I. IEEE, 2004.

[28] Yongduek Seo, Young-Ju Choi, and Sang Wook Lee. A branch-and-bound algorithm for globally optimal calibration of a camera-and-rotation-sensor system. In 2009 IEEE 12th International Conference on Computer Vision, pages 1173–1178. IEEE, 2009.

[29] Nathan Silberman, Derek Hoiem, Pushmeet Kohli, and Rob Fergus. Indoor segmentation and support inference from rgbd images. In ECCV, 2012.

[30] Julian Straub, Trevor Campbell, Jonathan P. How, and John W. Fisher, III. Efficient global point cloud alignment using bayesian nonparametric mixtures. In The IEEE Conference on Computer Vision and Pattern Recognition (CVPR), July 2017.

[31] Julian Straub, Oren Freifeld, Guy Rosman, John J Leonard, and John W Fisher. The manhattan frame modelâ ˘AˇTmanhattan world inference in the space of surface normals. IEEE transactions on pattern analysis and machine intelligence, 40(1):235–249, 2018.

[32] Niko Sünderhauf and Peter Protzel. Switchable constraints for robust pose graph slam. In 2012 IEEE/RSJ International Conference on Intelligent Robots and Systems, pages 1879–1884. IEEE, 2012.

[33] Jean-Philippe Tardif. Non- iterative approach for fast and accurate vanishing point detection. In 2009 IEEE 12th International Conference on Computer Vision, pages 1250–1257. IEEE, 2009.

[34] Roberto Toldo and Andrea Fusiello. Robust multiple structures estimation with j-linkage. In European conference on computer vision, pages 537–547. Springer, 2008.

[35] Rui Xu and Donald C. Wunsch. Survey of clustering algorithms. IEEE Transactions on Neural Networks, 16:645–678, 2005.

[36] Jiaolong Yang, Hongdong Li, and Yunde Jia. Optimal essential matrix estimation via inlier-set maximization. In European Conference on Computer Vision, pages 111–126. Springer, 2014.

[37] Jiaolong Yang, Hongdong Li, Dylan Campbell, and Yunde Jia. Go-icp: A globally optimal solution to 3d icp point-set registration. IEEE transactions on pattern analysis and machine intelligence, 38(11):2241–2254, 2016.

[38] Huizhong Zhou, Danping Zou, Ling Pei, Rendong Ying, Peilin Liu, and Wenxian Yu. Structslam: Visual slam with building structure lines. IEEE Transactions on Vehicular Technology, 64(4):1364–1375, 2015.