Quenching effects in strong penguin contributions to $\epsilon'/\epsilon$

Jack Laiho$^a$

$^a$Department of Physics, Princeton University, Princeton, NJ 08544

Quenching effects in strong penguin matrix elements are investigated. A lattice determination of $\alpha^{NS}_{q}$, the constant that appears in the quenched ChPT relevant for the lattice analysis of $K \rightarrow \pi\pi$ matrix elements, shows that this constant is large. The original RBC analysis of $Q_6$ matrix elements is revisited in light of this result. Also, the numerical effects of choosing the singlet Golterman-Pallante method of quenching $Q_6$ is investigated.

1. Introduction

There have been several recent lattice attempts to calculate $\text{Re}(\epsilon'/\epsilon)$, the direct $CP$ violating parameter in $K \rightarrow \pi\pi$ decays. These include the attempts using domain wall fermions by the CP-PACS $^1$ and RBC $^2$ collaborations. A notable feature of both of these calculations is that their central values differ drastically from experiment, though the approximations made were rather severe. In particular, the quenched approximation and leading order chiral perturbation theory were employed in both calculations.

Although the quenched approximation is uncontrolled, where quenched lattice results have been compared to experiment, at least for simple quantities such as masses of flavored mesons and decay constants, the agreement is at or better than $\sim 25\%$. However, there is no good reason to think that this agreement should hold for all low-energy hadronic phenomena. In fact there is a particular difficulty with quenched $\epsilon'/\epsilon$ that is not present in other quenched calculations.

Since the original RBC $\epsilon'/\epsilon$ result was reported, it was shown by $^3$$^4$ that there is an ambiguity in defining the quenched strong penguin four-quark operators. This ambiguity affects the operator,

$$Q_6 = \bar{s}_a \gamma_\mu (1 - \gamma^5) d_b \sum_q \bar{q}_b \gamma^\mu (1 + \gamma^5) q_a,$$  \hspace{1cm} (1)

whose $K \rightarrow \pi\pi$ matrix element has a large contribution to $\epsilon'/\epsilon$. There are at least two ways of defining this operator in the quenched theory, and we dub the calculations done with these different definitions the quenched singlet and non-singlet methods. The non-singlet method corresponds to the choice of the operator made in the standard (original RBC) approach, while the singlet choice corresponds to the Golterman-Pallante proposal. It is not known a priori whether either of these two methods gives results close to those of the full theory, and debates on this matter are likely to be resolved only when we have the results of dynamical calculations with the physical number of light flavors. It is possible that an appropriate linear combination of the two methods may give a reasonable approximation to the full theory; see the contribution of Norman Christ to these proceedings for such a proposal.

For the non-singlet method used (implicitly) in RBC’s previous work, it was shown that $K \rightarrow \pi\pi$ may have a large contribution from an additional low energy constant (LEC), as suggested by the large $N_c$ approximation, where $N_c$ is the number of colors $^5$. Since for all $\Delta I = 1/2$ amplitudes, the leading order ChPT analysis employed in $^2$ was that of the full theory, it seems desirable to revisit the analysis of $Q_6$ matrix elements in light of the new results in quenched ChPT. We emphasize that this ambiguity is present only for the strong penguin operators, and so does not affect the $Q_8$ contribution to $\epsilon'/\epsilon$. This quenching ambiguity also does not affect $\text{Re} A_0, \text{Re} A_2$ and, therefore, the $\Delta I = 1/2$ rule.

2. Singlet and non-singlet methods

We discuss the situation for $Q_6$ in order to illustrate the subtlety involved in quenching the
strong penguin operators. From Eq (1) it can be seen that the right part of $Q_6$ is a sum over active light flavors, so that in the full theory the right part is a flavor singlet under the symmetry group SU(3)$_R$. In the quenched theory one has at least two options. In the first option one chooses to sum over all the quarks, including valence and ghost. In this case the right component of the operator transforms as a singlet under the extended symmetry group; this is called the quenched singlet option.

In the second option, one may choose to sum in Eq (1) over only the valence quarks. In this case the operator can be decomposed into two operators, one of which transforms as a singlet under the irrep of the extended symmetry group, while the other does not (rather, for $Q_6$, it transforms in the adjoint representation); we choose to call this the quenched non-singlet method. We have shown that the matrix element to which the contribution to the fermion mass from chiral symmetry breaking. Figure 1 shows that the value of $\alpha_q^{NS}$ so obtained is in rough agreement with the large $N_C$ calculation [5], implying that this constant must be taken into account in the chiral limit according to ChPT.

is the contribution to the fermion mass from chiral symmetry breaking. Figure 1 shows that the size of this ambiguity is of the same order of the chiral limit we are trying to extract.

For this reason we have proposed an alternative method for obtaining $\alpha_q^{NS}$ that does not suffer from this problem. Using the graded CPS symmetry we have shown that for degenerate quark masses, the amplitude $K \rightarrow \pi$ does not have divergences. This matrix element calculated on the same configurations is plotted in Figure 1, and the value of $\alpha_q^{NS}$ so obtained is in rough agreement with the large $N_C$ calculation [5], implying that this constant must be taken into account in the RBC analysis.

4. Effects of $\alpha_q^{NS}$ in the non-singlet analysis

The LO ChPT prediction for the $K \rightarrow 0$ matrix element including the $\alpha_q^{NS}$ does not produce a good fit to the data (uncorrelated $\chi^2$/dof $\approx 30$), and it is possible that higher order effects need to be included in the analysis. The data appears quite linear, and cannot accommodate the large

\[ \langle 0|Q_6^{QNS}|K \rangle + \langle 0|Q_6^{QNS}|K \rangle/|\pi|d|K \rangle d \approx m_d + m_{res} \text{ (squares), and } f_s^2/(2\alpha_q^{(3)}) \langle \pi|Q_6^{QNS}|K \rangle \text{ (small circles) as a function of } m_d \text{ equal to the degenerate quark mass } m_f. \]
log in the analytical expression. Unfortunately, the NLO contributions for the non-singlet method are not currently known, so it is not possible to do a systematic analysis. The corrections to $K \to 0$ for the singlet part of the $Q_6$ operator required a one-loop calculation to go to NLO $[O(p^4)]$ \cite{7}. The non-singlet contribution to $O(p^4)$, however, would require a two-loop calculation, but this is not likely to be available anytime soon.

It is clearly difficult to separately resolve nonlinearities using the numerical data if there is, in fact, a cancellation between logarithmic and quadratic terms over the range of quark masses used in our simulations. Although there is reasonable agreement between the results for the subtracted $K \to \pi$ matrix element leaving out the heaviest mass data points in $\langle 0|Q_6|K^0 \rangle$ versus including a quadratic term in the fit and using all ten data points, we do not believe we have enough control over the subtraction in order to obtain the needed LEC’s from $K \to \pi$. We, therefore, cannot claim a result for $K \to \pi \pi$ in the non-singlet method.

5. Results for the singlet method

Although the quenched uncertainties make it unclear as to which method is best to approximate the full theory, the singlet method has the practical advantage that the NLO formulas are known, making it a useful test of the whole formalism, including a comparison to (quenched) continuum methods. The quenched NLO formulas do indeed produce good fits to the data. We are able to obtain a subtracted $K \to \pi$ amplitude which is shown in Fig 2, along with the original RBC data reported for subtracted $K \to \pi$ (where the $\alpha_{NS}^Q$ term was not taken into account and a linear fit was used), for comparison. The slope of this plot as a function of $m_f$ gives the LEC needed for LO $K \to \pi \pi$ determinations, and one can see that the slope is enhanced by roughly a factor of two in the singlet method over the original RBC result. On the other hand, a complete NLO fit shows that the linear part of the singlet fit (the slope in the chiral limit) is roughly parallel with the slope quoted for the original RBC data. Thus, it is extremely important to have the

![Figure 2. The lower data set is the matrix element $\langle \pi^+|Q_6^Q|K^+ \rangle_{sub}$ where the divergent piece has been subtracted. This was obtained using the singlet method, and the fit is to the known NLO form. The upper data set is the original RBC result for $\langle \pi^+|Q_6|K^+ \rangle_{sub}$ with a linear fit shown for comparison.](image_url)

NLO formulas as the result can be affected by a factor of two or more.

6. Outlook

The large quenching effects show that dynamical calculations will be necessary in order to significantly reduce this large systematic uncertainty in $\epsilon'/\epsilon$. Also, if the reduction method of \cite{6} is used, it appears necessary to have NLO ChPT in order to extract the needed LEC’s from lattice data.

REFERENCES

1. CP-PACS Collaboration, J.I. Noaki et al., hep-lat/0108013.
2. RBC Collaboration, T. Blum et al., hep-lat/0110075.
3. M. Golterman and E. Pallante, JHEP 0110 (2001) 037.
4. Ibid, hep-lat/0212008.
5. M. Golterman and S. Peris, hep-lat/0309101.
6. C. Bernard et al., Phys. Rev. D 32, 2343 (1985).
7. M. Golterman and E. Pallante, JHEP 0008 (2000) 023.