Existence of sigma meson in pi-pi scattering

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Abstract

In this talk I summarize a recently proposed mechanism to understand \( \pi\pi \) scattering to 1 GeV. The model is motivated by the \( 1/N_C \) expansion to QCD, and includes a current algebra contact term and resonant pole exchanges. Chiral symmetry plays an important role in restricting the form of the interactions. The existence of a broad low energy scalar (\( \sigma \)) is indicated.

1 Introduction

The \( \pi\pi \) scattering has been studied as an important test of the strong interaction. Now QCD is known to be the fundamental theory of the strong interaction. However, it is very difficult to reproduce the experimental data directly from QCD. One clue is given by the structure of the chiral symmetry, which approximately exists in the QCD Lagrangian and is broken by the strong interaction of QCD. Another clue is given by the \( 1/N_C \) expansion to QCD. In the large \( N_C \) limit, QCD becomes a theory of weakly interacting mesons, and the \( \pi\pi \) scattering is expressed as an infinite sum of tree diagrams of mesons.

The experimental data in the low energy region near \( \pi\pi \) threshold can be reproduced by using the information from chiral symmetry. This situation is easily understood by using a chiral Lagrangian which includes pions only. In addition, by including the higher derivative terms together with one-loop effects, the applicable energy region is enlarged. This systematic low energy expansion is called the chiral perturbation theory.

In the higher energy region, however, the one-loop amplitude of chiral perturbation theory violates the unitarity bound around 400 – 500 MeV in the \( I = 0 \) S-channel. For the \( P \)-wave amplitude, we have the \( \rho \) meson, and chiral perturbation theory may break down at the resonance position. The explicit inclusion of resonances in the high energy region easily reproduces the amplitude, of course.

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| $\sigma(400 - 1200)$ | $0^+ (0^{++})$ | $400 - 1200$ | $600 - 1000$ |
| $\rho(770)$ | $1^+ (1^{--})$ | $769.9$ | $151.2$ |
| $f_0(980)$ | $0^+ (0^{++})$ | $980$ | $40 - 100$ |

Table 1: Resonances included in the $\pi\pi \rightarrow \pi\pi$ channel as listed in the PDG.

When we apply the large $N_C$ argument to the practical $\pi\pi$ scattering, we cannot actually include an infinite number of resonances. Moreover, the forms of interactions are not fully determined in the large $N_C$ limit. Nevertheless, some encouraging features were previously found in an approach which truncated the particles appearing in the effective Lagrangian to those with masses up to an energy slightly greater than the range of interest. Moreover, the chiral symmetry played an important role to restrict the form of interaction, i.e., the effective Lagrangian was constructed by using the information of chiral symmetry. This seems reasonable phenomenologically and is what one usually does in setting up an effective Lagrangian.

In this talk I concentrate on the energy region below 1 GeV. For the established resonances lighter than 1 GeV, $\rho$ and $f_0(980)$ are contained in the particle data group (PDG) list (see Table 1). However, the width of $f_0(980)$ is not well determined. Moreover, the existence of a light scalar $\sigma$ is suggested by several authors. Here I will determine these resonance parameters by fitting to the $I = 0$ S-wave $\pi\pi$ scattering amplitude.

This paper is organized as follows. In section 2 I will briefly show the interaction terms. Section 3 is the main part of this talk, where I will show how to regularize the amplitude, and fit the resonance parameters to the experimental data of the $I = 0$, $J = 0$ partial wave amplitude. Finally, a summary is given in section 4.

## 2 Interaction Terms

In this section I will briefly show the interactions between resonances and pions. First I include the vector meson as a gauge field of chiral symmetry, which is equivalent to the hidden local gauge method (See, for a review, Ref. 9.) at tree level. The $\rho\pi\pi$ interaction is given by

$$\mathcal{L}_\rho = g_{\rho\pi\pi} \vec{p}_\mu \cdot (\partial^\mu \vec{\pi} \times \vec{\pi}) ,$$

(1)
where $g_{\rho\pi\pi}$ is the $\rho\pi\pi$ coupling constant. Next, I include scalar resonances, $\sigma$ and $f_0(980)$. These are iso-singlet fields; the interaction with two pions is given by

$$\mathcal{L}_f = -\frac{\gamma_f}{\sqrt{2}} f \partial_\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} \quad (f = \sigma, f_0(980)).$$

Here I should note that the chiral symmetry requires derivative-type interactions between the scalar field and pseudoscalar mesons.

### 3 Fit to $\pi\pi$ scattering to 1 GeV

In this section, I will calculate the $S$-wave $\pi\pi$ scattering amplitude by including resonances as explained in the previous section.

The most problematic feature involved in comparing the leading $1/N_C$ amplitude with experiment is that it does not satisfy unitarity. Since the mesons have zero width in the large $N_C$ limit, the amplitude diverges at the resonance position. Thus in order to compare the $1/N_C$ amplitude with experiment we need to regularize the resonance contribution. The ordinary narrow resonances such as $\rho$ meson are regularized by including the width in the denominator of the propagator (the Breit-Wigner form):

$$\frac{M \Gamma}{M^2 - s - iM \Gamma}. \quad (3)$$

This is only valid for a narrow resonance in a region where the background is negligible. Note that the width in the denominator is related to the coupling constant.

For a very broad resonance there is no guarantee that such a form is correct. A suitable form turned out to be of the type

$$\frac{MG}{M^2 - s - iMG'}, \quad (4)$$

where the parameter $G'$ is a free parameter which is not related to the coupling constant.

Even if the resonance is narrow, the effect of the background may be rather important. This seems to be true for the case of the $f_0(980)$. Demanding local unitarity in this case yields a partial wave amplitude of the well known form:

$$e^{2i\delta}M \Gamma + e^{i\delta} \sin \delta, \quad (5)$$

where $\delta$ is a background phase (assumed to be slowly varying). I will adopt a point of view in which this form is regarded as a kind of regularization of the model. Of course, non zero $\delta$ represents a rescattering effect which is of higher order in $1/N_C$.  

The quantity $e^{2i\delta}$, taking $\delta = constant$, can be incorporated into the squared coupling constant connecting the resonance to two pions. In this way, crossing symmetry can be preserved. The non-pole background term in Eq. (3) and hence $\delta$ is to be predicted by the other pieces in the effective Lagrangian.

Another point which must be addressed in comparing the leading $1/N_C$ amplitude with experiment is that it is purely real away from the singularities. The regularizations mentioned above do introduce some imaginary pieces but these are clearly more model dependent. Thus it seems reasonable to compare the real part of the predicted amplitude with the real part of the experimental amplitude.

Let me start from the current algebra $+ \rho$ contribution. The predicted curve is shown in Fig. 1 of Ref. [1]. Although the introduction of $\rho$ dramatically improves unitarity up to about 2 GeV, $R_0^0$ violates unitarity to a lesser extent starting around 500 MeV. To recover unitarity, we need a negative contribution to the real part above this point, while below this point a positive contribution is preferred by experiment. Such behavior matches with the real part of a typical resonance contribution. The resonance contribution is positive in the energy region below its mass, while it is negative in the energy region above its mass. Then I include a low mass broad scalar resonance, $\sigma$. The $\sigma$ contribution to the real part of the amplitude component $A(s, t, u)$ is given by

$$A_{\sigma}(s, t, u) = \frac{\gamma_{\sigma}^2}{2} \frac{(s - 2m_{\pi}^2)^2}{M_{\sigma}^2 - s - iM_{\sigma}G'} ,$$

(6)

where the factor $(s - 2m_{\pi}^2)^2$ is due to the derivative-type coupling required for chiral symmetry in Eq. (2). $G'$ is a parameter which we introduce to regularize the propagator. It can be called a width, but it turns out to be rather large so that, after the $\rho$ and $\pi$ contributions are taken into account, the partial wave amplitude $R_0^0$ does not clearly display the characteristic resonant behavior.

A best overall fit is obtained with the parameter choices; $M_{\sigma} = 559$ MeV, $\gamma_{\sigma} = 7.8$ GeV$^{-1}$ and $G' = 370$ MeV. The result for the real part $R_0^0$ due to the inclusion of the $\sigma$ contribution along with the $\pi$ and $\rho$ contributions is shown in Fig. 1. It is seen that the unitarity bound is satisfied and there is a reasonable agreement with the experimental points [11, 12] up to about 800 MeV.

Next, let me consider the 1 GeV region. Reference to Fig. 1 shows that the experimental data for $R_0^0$ lie considerably lower than the $\pi + \rho + \sigma$ contribution between 0.9 and 1.0 GeV and then quickly reverse sign above this point. This is caused by the existence of $f_0(980)$. As we can see easily, a naive inclusion of $f_0(980)$ does not reproduce the experimental data, since the real part of the typical resonance form gives a positive contribution in the energy region below its mass, while it gives a negative contribution
in the energy region above its mass. However, we need a negative contribution below 1 GeV and a positive contribution above 1 GeV.

As I discussed around Eq. (5), the effect of the background is important in this $f_0(980)$ region. In this case the background is given by the $\pi + \rho + \sigma$ contribution. Figure 1 shows that the real part of the background is very small so that the background phase $\delta$ in Eq. (5) is expected to be roughly $90^\circ$. This background effect generates the extra minus sign in front of the $f_0(980)$ contribution, as we can see from Eq. (5). Thus the $f_0(980)$ gives a negative contribution below the resonance position and gives a positive contribution above it. This is clearly exactly what is needed to bring experiment and theory into agreement up till about 1.2 GeV.

The actual amplitude used for the calculation properly contains the effects of the pions’ derivative coupling to the $f_0(980)$:

$$A_{f_0(980)}(s, t, u) = e^{2i\delta} \frac{\gamma_{f_0\pi\pi}}{2} \frac{(s - 2m_\pi^2)^2}{M_{f_0}^2 - s - iM_{f_0}\Gamma_{f_0}},$$

where $\delta$ is a background phase parameter and the real coupling constant $\gamma_{f_0\pi\pi}$ is related to the $f_0(980) \to \pi\pi$ width. The background phase parameter $\delta$ is predicted by

$$\frac{1}{2} \sin(2\delta) \equiv \tilde{R}_0^0(s = M_{f_0}^2),$$

where $\tilde{R}_0^0$ is computed as the sum of the current algebra, $\rho$, and sigma pieces.
A best fit of our parameters to the experimental data results in the curve shown in Fig. 2. Only the three parameters $\gamma_{\sigma}, G'$ and $M_{\sigma}$ are essentially free. The others are restricted by experiment. Since the total width of $f_0(980)$ has a large uncertainty (40 – 100 MeV in PDG list), we also fit this. In addition we have considered the precise value of $M_{f_0}$ to be a parameter for fitting purpose. The best fitted values are shown in Table 2 together with the predicted background phase $\delta$ and the $\chi^2$ value. The predicted background phase is seen to be close to 90°, and the low lying sigma has a mass of around 560 MeV and a width of about 370 MeV.

| $M_{f_0(980)}$ | $\Gamma_{f_0(980)}$ | $M_{\sigma}$ | $G'$ | $\gamma_{\sigma}$ | $\delta$ (deg.) | $\chi^2$ |
|----------------|---------------------|--------------|------|-------------------|-----------------|--------|
| 987            | 64.6                | 559          | 370  | 7.8               | 85.2            | 2.0    |

Table 2: The best fitted values of the parameter together with the predicted background phase $\delta$ and the $\chi^2$ value. The units of $M_{f_0(980)}$, $\Gamma_{f_0(980)}$, $M_{\sigma}$ and $G'$ are MeV and that of $\gamma_{\sigma}$ is GeV$^{-1}$.

4 Summary

In this talk I showed the main mechanism of the analysis done in Ref. [1]: (1) motivated by the large $N_C$ approximation to QCD, we include the resonances with masses up to an energy slightly greater than the range of interest, and use the chiral symmetry to restrict the forms of the interactions; (2) the current algebra + $\rho$ contribution violates the unitarity bound around 560 MeV region but it is recovered by including the low mass broad resonance sigma; (3) the $\pi + \rho + \sigma$ contribution gives an important background effect to the $f_0(980)$ contribution, i.e., the sign in front of the $f_0(980)$ contribution is reversed by the background effect. The third mechanism, which leads to a sharp dip in the $I = J = 0$ partial wave contribution to the $\pi\pi$-scattering cross section, can be identified with the very old Ramsauer-Townsend effect [13] which concerned the scattering of 0.7 eV electrons on rare gas atoms. The dip occurs because the background phase of $\pi/2$ causes the phase shift to go through $\pi$ (rather than $\pi/2$) at the resonance position. (Of course, the cross section is proportional to $\sum_{I,J}(2J + 1)\sin^2(\delta_J^I)$.) This simple mechanism seems to be all that is required to understand the main feature of $\pi\pi$ scattering in the 1 GeV region.

The detailed analysis, which includes the effects of the inelasticity ($\pi\pi \rightarrow K\bar{K}$ channel opens at 990 MeV.) and the next group of resonances, is done in Ref. [1]. The results show that those effects only fine-tune the best fitted values shown in Table 2. Finally I should
stress that the consistent neglect of the crossed-channel $\rho$ exchange does not destroy the existence of $\sigma$ meson. The best fitted values are $M_\sigma = 378$ MeV and $G' = 836$ MeV.

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