Thermal inflation and baryogenesis in heavy gravitino scenario

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Abstract

We present a thermal inflation model that incorporates the Affleck-Dine leptogenesis in heavy gravitino/moduli scenario, which solves the moduli-induced gravitino problem while producing a correct amount of baryon asymmetry and relic dark matter density. The model involves two singlet flat directions stabilized by radiative corrections associated with supersymmetry breaking, one direction that generates the Higgs $\mu$ and $B$ parameters, and the other direction that generates the scale of spontaneous lepton number violation. The dark matter is provided by the lightest flatino which might be identified as the axino if the model is assumed to have a $U(1)_{PQ}$ symmetry to solve the strong CP problem. We derive the conditions for the model to satisfy various cosmological constraints coming from the Big-Bang nucleosynthesis and the dark matter abundance.

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I. INTRODUCTION

Low energy supersymmetry (SUSY) is one of the most plausible candidates for physics beyond the Standard Model (SM) at the TeV scale \[1\]. In the context of supergravity, SUSY can be spontaneously broken in a hidden sector while giving a vanishing cosmological constant. In this framework, the gravitino mass is related to the scale of hidden sector SUSY breaking as \[M_{SB} \sim \sqrt{m_{3/2} M_{Pl}}\], where \[M_{Pl} = 2.4 \times 10^{18}\] GeV is the reduced Planck mass. In low energy SUSY scenario, the hidden sector SUSY breaking is transmitted to the supersymmetric standard model (SSM) to induce soft terms providing sparticle masses of \[\mathcal{O}(1)\] TeV. Then, for certain range of \[m_{3/2}\], gravitinos produced in the early Universe decay after the Big-Bang nucleosynthesis, which would destroy the successful prediction of the light element abundances \[2\]. This cosmological difficulty might be avoided if the gravitino is relatively heavy, e.g. \[m_{3/2} \gtrsim \mathcal{O}(10)\] TeV, so that the decay occurs before the nucleosynthesis.

Recent progress in string flux compactification \[3\] has provided a SUSY breaking scheme in which the hidden sector with a SUSY breaking scale \[M_{SB} \sim \sqrt{m_{3/2} M_{Pl}}\] is naturally sequestered from the SSM sector. String fluxes generically produce a warped throat in compact internal space, and then the SUSY breaking sector is stabilized at the IR end of throat. On the other hand, the high scale gauge coupling unification suggests that the SSM sector is located at the UV end, and thus is sequestered from the SUSY breaking at the IR end of throat \[4, 5\]. In such case, the effective contact interactions between the SUSY breaking hidden sector field and the SSM fields are so suppressed \[6\] that the conventional gravity mediation \[7\] which would give a soft mass of \[\mathcal{O}(m_{3/2})\] does not contribute to the SSM soft masses. As a result, the SSM soft masses can be much lighter than the gravitino mass, and \[m_{3/2} = \mathcal{O}(10)\] TeV is a natural outcome if one takes the SSM soft parameters to be near the weak scale. Moreover, this framework can stabilize moduli either by fluxes or by non-perturbative effects \[3\], and the resulting moduli masses are typically much heavier than the gravitino mass, \[m_{\phi} \sim 8\pi^2 m_{3/2}\] for moduli stabilized by non-perturbative effects and \[m_{\phi} \gg 8\pi^2 m_{3/2}\] for moduli stabilized by fluxes \[8\].

Nonetheless, the above scenario of heavy gravitino/moduli is not yet free from cosmological difficulty \[9\]. It has been noticed that there still arises a problem, the moduli-induced gravitino problem, due to that the gravitinos from moduli decays produce too many neutrali-
nos, which would overclose the Universe if the neutralino is the stable lightest supersymmetric particle (LSP). As another difficulty, in heavy gravitino scenario it is not straightforward at all to get the Higgs $\mu$ and $B$ parameters having a weak scale size, which would require a severe fine tuning for the correct electroweak symmetry breaking. Recently, an attractive solution to these problems has been proposed in [10], involving a singlet flat direction stabilized by radiative corrections associated with SUSY breaking. Once stabilized by radiative effects, the singlet flaton $S$ gets a loop-suppressed SUSY breaking $F$-component, $F_S/S \sim m_{3/2}/8\pi^2$, and then one can arrange the model to generate $\mu \sim B \sim F_S/S$, which would result in a weak scale size of $\mu$ and $B$ when $m_{3/2} = O(10)$ TeV. Such a scheme can solve the moduli-induced gravitino problem also. Now the LSP of the model is the fermionic component of $S$, the flatino, which is much lighter than the MSSM neutralino. Then the neutralinos produced by moduli/gravitino decays are not stable anymore, but decay into light flatinos with a relic mass density not overclosing the Universe. It is also natural to introduce a $U(1)_{PQ}$ symmetry spontaneously broken by the vacuum value of $S$, and then the pseudoscalar component of $S$ corresponds to the QCD axion solving the strong CP problem [11], and the flatino can be identified as the axino which has been proposed as a viable dark matter candidate in [12].

On the other hand, late time thermal inflation [13] is a natural consequence of generic flaton model, although its possibility was not explored in [10]. Thermal inflation also solves the moduli-induced gravitino problem by diluting away the coherent oscillation of moduli. A potential difficulty of thermal inflation is that any primordial baryon asymmetry is also washed away by the late time entropy production, so one needs a baryogenesis mechanism working after thermal inflation is over*. Recently, an interesting model of thermal inflation incorporating the Affleck-Dine (AD) leptogenesis [15] has been proposed in the context of the conventional gravity-mediated SUSY breaking scenario giving a weak scale size of $m_{3/2}$ [16, 17, 18].

* In gauge mediated SUSY breaking scenario, the baryon asymmetry produced by the Affleck-Dine mechanism can survive the dilution by thermal inflation if the initial amplitude of the AD flat direction is large enough [14]. In such case, the curvature of the potential at a field value larger than the messenger scale is determined by the small gravitino mass, thus the oscillation of the flat direction begins much later than the case of $m_{3/2} \gtrsim m_{\text{soft}}$, which would allow the generated baryon asymmetry large enough to survive the late time dilution. In our case of heavy gravitino scenario, this is not possible, so we need a baryogenesis after thermal inflation is over.
In this paper, we wish to examine the possibility of thermal inflation incorporating a similar AD leptogenesis in heavy gravitino/moduli scenario realized in the framework of sequestered SUSY breaking. As we will see, a heavy gravitino mass requires additional conditions for successful thermal inflation and AD leptogenesis, e.g. the model should involve two singlet flat directions both of which are stabilized by radiative effects, one flat direction that generates the Higgs $\mu$ and $B$ parameters, and the other flat direction that generates the scale of spontaneous lepton number violation. We present a viable model satisfying various cosmological constraints coming from the Big-Bang nucleosynthesis and the dark matter abundance.

This paper is organized as follows. In section 2, we briefly discuss the moduli-induced gravitino problem and also how to generate the weak scale size of $\mu$ and $B$ with a radiatively stabilized singlet flaton in heavy gravitino scenario. In section 3, we present a specific model of thermal inflation and AD leptogenesis in heavy gravitino scenario, and briefly analyze the phenomenological aspects of the model. A detailed discussion of the cosmological aspects of the model is provided in section 4, and section 5 is the conclusion.

II. LOW ENERGY SUSY WITH HEAVY GRAVITINO

A. Moduli-induced gravitino problem

A gravitino mass much heavier than the SSM soft mass $m_{\text{soft}}$ naturally appears in models with sequestered SUSY breaking. A SUSY breaking at the tip of throat in string flux compactification provides an attractive framework for sequestered SUSY breaking [4, 5]. Such string compactifications include moduli that are stabilized by flux or non-perturbative effect. The resulting moduli masses are comparable to $8\pi^2m_{3/2}$ for non-perturbative stabilization, and even much heavier than $8\pi^2m_{3/2}$ for flux stabilization [8]. When SUSY breaking effects are taken into account, those moduli $\phi$ develop a nonzero $F$-component given by

$$\frac{F^\phi}{\phi} \sim \frac{m_{3/2}^2}{m_\phi},$$

(1)

where $\phi$ and $m_\phi$ denote the modulus vacuum value and the modulus mass, respectively. Since $m_\phi \gtrsim 8\pi^2m_{3/2}$, the resulting moduli-mediated soft masses do not dominate over the
anomaly-mediated soft masses of \( \mathcal{O}(m_{3/2}/8\pi^2) \)\(^\dagger\) [8, 22, 23]. Thus, even when one takes into account the moduli-mediated soft masses of \( \mathcal{O}(F^\phi/\phi) \), it still holds true the order of magnitude relation \( m_{\text{soft}} \sim m_{3/2}/8\pi^2 \), and the gravitino generically has a heavy mass of \( \mathcal{O}(8\pi^2) \) TeV when \( m_{\text{soft}} \) is assumed to have a size of \( \mathcal{O}(1) \) TeV.

Moduli are expected to have a misalignment of \( \mathcal{O}(M_{\text{Pl}}) \) in the early Universe, and therefore their coherent oscillations would soon dominate the energy density of the Universe. Since moduli masses are much heavier than \( \mathcal{O}(10) \) TeV in sequestered SUSY breaking scenario, their decays take place well before the nucleosynthesis. Nevertheless, moduli can cause a cosmological problem unless the branching ratio of the moduli decay to gravitinos is highly suppressed. Indeed, non-thermal gravitino production by heavy moduli decay can lead to a severe over-abundance of dark matter in the Universe [9].

For a modulus \( \phi \) with \( m_{\phi} \gg m_{3/2} \), the number density of gravitinos produced by the modulus decay is given by

\[
\frac{n_{3/2}}{s} = \frac{3}{2} \frac{\Gamma_{\phi \rightarrow \tilde{G} \tilde{G}}}{\Gamma_{\phi \rightarrow \text{MSSM}}} \frac{T_R^\phi}{m_{\phi}},
\]

where \( s \) is the entropy density, \( \Gamma_{\phi \rightarrow \tilde{G} \tilde{G}} \) and \( \Gamma_{\phi \rightarrow \text{MSSM}} \) denote the decay width to the gravitino pair and to the MSSM particles, respectively, and \( T_R^\phi \) is the reheating temperature for the modulus decays:

\[
T_R^\phi = \left( \frac{90}{\pi^2 g_*(T_R^\phi)} \right)^{1/4} \sqrt{\Gamma_\phi M_{\text{Pl}}} \simeq 0.2 \left( \frac{10}{g_*(T_R^\phi)} \right)^{1/4} \left( \frac{C_\phi}{10^{-1}} \right)^{1/2} \left( \frac{m_\phi}{10^6 \text{GeV}} \right)^{3/2} \text{GeV},
\]

where \( g_*(T) \) is the number of relativistic degrees of freedom at \( T \), and \( \Gamma_\phi = C_\phi m_\phi^3/M_{\text{Pl}}^2 \) is the total modulus decay width\(^\ddagger\). Gravitinos produced by modulus decay do not interact with others, and promptly decay into the LSP. The decay temperature of the gravitino is given by

\[
T_{3/2} \simeq \left( \frac{90}{\pi^2 g_*(T_{3/2})} \right)^{1/4} \sqrt{\Gamma_{3/2} M_{\text{Pl}}} \sim \left( \frac{m_{3/2}}{m_\phi} \right)^{3/2} T_R^\phi,
\]

with \( \Gamma_{3/2} \sim 10^{-1} m_{3/2}^3/M_{\text{Pl}}^2 \) being the total decay width of the gravitino. Since \( m_\phi \gg m_{3/2} \), \( T_{3/2} \) is much lower than \( T_R^\phi \). Under the assumption of R-parity conservation, if the LSP

\(^\dagger\) In fact, moduli-mediation comparable to the anomaly mediation is phenomenologically desirable since the pure anomaly mediation [19] suffers from the tachyonic slepton problem in the minimal supersymmetric SM [20, 21].

\(^\ddagger\) For \( \phi \) whose vacuum value determines the gauge coupling constants, it decays mainly into the gauge bosons and gauginos, and then \( C_\phi = \mathcal{O}(10^{-1}) \).
annihilation is not efficient, each gravitino will produce a stable LSP, yielding the LSP relic abundance given by

\[ \frac{\rho_\chi}{s} \approx \frac{m_\chi n_{3/2}}{s} \approx 0.2 \frac{\rho_{\text{cr}}}{s} \left( \frac{\alpha}{1.4 \times 10^{-5}} \frac{\Gamma_{\phi \to \tilde{G} \tilde{G}}}{\Gamma_{\phi \to \text{MSSM}}} \right) , \]  

(5)

where \( m_\chi \) denotes the LSP mass,

\[ \alpha = \left( \frac{C_\phi}{10^{-1}} \right)^{1/2} \left( \frac{10}{g_*(T^\phi_R)} \right)^{1/4} \left( \frac{m_\chi}{10^2 \text{GeV}} \right) \left( \frac{m_\phi}{10^6 \text{GeV}} \right)^{1/2} , \]  

(6)

and \( \rho_{\text{cr}}/s \approx 1.9 \times 10^{-9} \text{ GeV} \) is the ratio of the critical density to the entropy density in the present Universe.

Since \( \rho_\chi/\rho_{\text{cr}} \) in the present Universe cannot exceed about 0.25, the branching ratio of the modulus decay into the gravitino pair is bounded as

\[ \frac{\Gamma_{\phi \to \tilde{G} \tilde{G}}}{\Gamma_{\phi \to \text{MSSM}}} \lesssim 1.7 \times 10^{-5} \left( \frac{10^{-1}}{C_\phi} \right)^{1/2} \left( \frac{g_*(T^\phi_R)}{10} \right)^{1/4} \left( \frac{10^2 \text{GeV}}{m_\chi} \right) \left( \frac{10^6 \text{GeV}}{m_\phi} \right)^{1/2} . \]  

(7)

In view of that \( m_\phi \gtrsim 8\pi^2 m_{3/2} \) and \( m_{3/2} = \mathcal{O}(10) \text{ TeV} \) in sequestered SUSY breaking scenario, \( \Gamma_{\phi \to \tilde{G} \tilde{G}} \) should be highly suppressed if \( \chi \) corresponds to the MSSM neutralino having a mass of \( \mathcal{O}(10^2) \text{ GeV} \). However, it has been noticed that there is no suppression by \( m_{3/2}/m_\phi \) for the modulus decay into the helicity \( \pm 1/2 \) components of the gravitino \[9\]. As a consequence, even in the limit \( m_\phi \gg m_{3/2} \), the branching ratio is simply given by

\[ \frac{\Gamma_{\phi \to \tilde{G} \tilde{G}}}{\Gamma_{\phi \to \text{MSSM}}} \sim \frac{1}{4N_g} \sim 2 \times 10^{-2} , \]  

(8)

where \( N_g = 12 \) denotes the number of gauge bosons in the MSSM \[9\]. This branching ratio exceeds the above bound by several orders of magnitudes when \( \chi \) is identified as the MSSM neutralino, and this is the moduli-induced gravitino problem.

For a more careful analysis, one needs to include the effects of LSP annihilation. Including such effect, we find

\[ \rho_\chi/s = \frac{m_\chi n_\chi}{s} \approx \frac{1}{4} \left( \frac{90}{\pi^2 g_*(T^\chi_{3/2})} \right)^{1/2} \frac{m_\chi}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle T_{3/2} M_{Pl}} , \]  

(9)

where \( \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \) is the thermal average of the annihilation cross section times the relative velocity of \( \chi \). If the branching ratio of \( \phi \to \tilde{G} \tilde{G} \) exceeds the bound \[7\], in order not to overclose the Universe, the annihilation is required to be as efficient as

\[ \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \geq \frac{1.3 \times 10^{-4}}{\text{GeV}^2} \left( \frac{10}{g_*(T^\chi_{3/2})} \right)^{1/4} \left( \frac{m_\chi}{10^2 \text{GeV}} \right) \left( \frac{10^4 \text{GeV}}{m_{3/2}} \right)^{3/2} . \]  

(10)
On the other hand, even the Wino LSP has an annihilation cross section \( \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \ll 10^{-4} \text{GeV}^{-2} \), and thus it appears to be difficult to avoid the moduli-induced gravitino problem through the annihilation mechanism\(^6\).

**B. Higgs \( \mu \) and \( B\mu \) term**

In heavy gravitino scenario, the MSSM Higgs \( B \) parameter generically receives a contribution of \( \mathcal{O}(m_{3/2}) \) from SUGRA effect. Unless cancelled by other contribution, such a large \( B \) would make it difficult to realize the correct electroweak symmetry breaking. An attractive way to avoid this difficulty is to generate \( \mu \) and \( B \) by a vacuum value of flat direction which is stabilized by radiative corrections associated with SUSY breaking \([10, 26]\). Here we briefly discuss such a scheme using the model of \([10]\) as an example.

In addition to the canonical Kähler potential of the involved fields, the model includes the following Kähler mixing and the Yukawa interactions of the flaton field \( S \):

\[
\Delta L_{\text{int}} = \int d^4\theta \kappa S^* S' + \int d^2\theta \left[ y_H \Sigma H_u H_d + y_S \Sigma S S' \right] + \text{h.c.},
\]  

which are allowed by the global \( U(1) \) symmetry with the charge assignment: \( q(S) = q(S') = q(H_{u,d}) = 1, q(\Sigma) = -2 \). Then the direction along \( S \neq 0 \) with \( \Sigma = S' = 0 \) is flat in the supersymmetric limit. After integrating out the heavy fields \( \Sigma \) and \( S' \) under a large background value of \( S \), one obtains the effective theory of \( S \) described by

\[
\Delta L_{\text{eff}} = \int d^4\theta \left\{ Y_{S}^{\text{eff}} S^* S + \left( \kappa_{\text{eff}} \frac{S^*}{S} H_u H_d + \text{h.c.} \right) \right\},
\]  

with \( Y_S^{\text{eff}} = Y_S(Q = |S|) \) and \( \kappa_{\text{eff}} = \kappa y_H / y_S \). Here \( Q \) is the renormalization scale, and \( Y_S \) is the running wave function of \( S \). The potential is then determined by the running soft mass

\[
V = m_S^2(Q = |S|)|S|^2,
\]  

\(^6\) In fact, in other case that both \( m_\phi \) and \( m_{3/2} \) are of \( \mathcal{O}(10) \text{ TeV} \), the resulting Wino LSP abundance can be small enough not to overclose the Universe \([24, 25]\). However, in this case, the modulus \( \phi \) with \( m_\phi \sim m_{3/2} \) generically has \( F^\phi / \phi \sim m_{3/2} \), and then the modulus-mediated sfermion masses can be of \( \mathcal{O}(10) \text{ TeV} \) \([22]\). Here, we are focusing a different setup giving \( m_\phi \sim 8\pi^2 m_{3/2} \) (or heavier) with \( F^\phi / \phi \sim m_{3/2} / 8\pi^2 \) (or smaller), which is motivated by sequestered SUSY breaking realized in string flux compactification. In our case, even the Wino LSPs produced by moduli/gravitino decays overclose the Universe by about two orders of magnitudes.
where \( m_S^2 = - F^I F^J \partial_I \partial_J \ln Y_S \) for \( F^I \) denoting generic SUSY-breaking \( F \)-components in the model. The SUSY breaking \( F^I \) include first of all the \( F \)-component of the chiral compensator superfield

\[
C = C_0 + \theta^2 F^C,
\]

as well as \( F^S \) and the moduli \( F \)-component \( F^\phi \). For \( S_0 \equiv \langle S \rangle \gtrsim m_{3/2} \), the equation of motion for \( F^S \) reads

\[
\frac{F^S}{S_0} \simeq - F^I \partial_I \ln Y_S (Q = |S|).
\]

(14)

It is quite possible that \( Y_S \) is moduli-dependent at tree level, and then there will be a contribution of \( \mathcal{O}(F^\phi / \phi) \) to \( F^S / S_0 \). On the other hand, due to the super-Weyl invariance in the compensator formulation of SUGRA, the \( C \)-dependence of \( Y_S \) arises only through the RG running. As a result, the contribution from \( F^C \) is loop-suppressed, giving a contribution of \( \mathcal{O}(F^C / 8\pi^2 C_0) \) to \( F^S / S_0 \). In our case,

\[
\frac{F^C}{C_0} \simeq m_{3/2}, \quad \frac{F^\phi}{\phi} \sim \frac{m_{3/2}^2}{m_{\phi}} \lesssim \frac{m_{3/2}}{8\pi^2},
\]

and thus

\[
\frac{F^S}{S_0} = \mathcal{O}\left(\frac{m_{3/2}}{8\pi^2}\right).
\]

(15)

Now the \( \mu \) and \( B \mu \) terms arise from the effective Higgs bilinear operator in the effective Kähler potential (12):

\[
\mu = \kappa_{\text{eff}} \left\{ \left( \frac{F^S}{S_0} \right)^* + \mathcal{O}\left(\frac{m_{3/2}}{8\pi^2}\right) \right\}, \quad B = \frac{F^S}{S_0} + \mathcal{O}\left(\frac{m_{3/2}}{8\pi^2}\right),
\]

(16)

where the loop-suppressed contributions from \( F^C \) are included in \( \mathcal{O}(m_{3/2}/8\pi^2) \), together with the contributions from \( F^\phi \). Therefore, \( \mu \) and \( B \) generated by a radiatively stabilized flat direction can be naturally of \( \mathcal{O}(m_{3/2}/8\pi^2) \), so be the weak scale when \( m_{3/2} = \mathcal{O}(10) \text{ TeV} \).

C. Axino dark matter and thermal inflation

In heavy gravitino scenario, the addition of the singlet flaton \( S \) has important implications for cosmology. Using the relation \( Y_S^{\text{eff}} = Y_S (Q = |S|) \) and the stationary condition \( m_S^2 (Q = \)}
$S_0 \simeq 0$ for $S_0 \gtrsim m_{3/2}$ in the effective theory $\text{(12)}$, one can find that the radial flaton and the flatino\footnote{This flatino can be called also the axino which would be the right name if the global symmetry $U(1)_S$ is good enough to be a $U(1)_{PQ}$ symmetry solving the strong CP problem $\text{(11)}$.} acquire SUSY breaking masses

\begin{equation}
\begin{aligned}
m_{\sigma S}^2 & \simeq \frac{d m_3^2(Q)}{d \ln Q} \bigg|_{Q = S_0} = \mathcal{O} \left( \frac{m_{3/2}^2}{(8\pi^2)^3} \right), \\
m_{\psi S} & \simeq \frac{y_S^2(Q)}{16\pi^2} A_S(Q) \bigg|_{Q = S_0} = \mathcal{O} \left( \frac{y_S^2 m_{3/2}}{(8\pi^2)^2} \right),
\end{aligned}
\end{equation}

where

\begin{equation}
S = (S_0 + \sigma_S/\sqrt{2})e^{i a_S/\sqrt{2}S_0} + \sqrt{2}\theta\psi_S + \theta^2 F_S,
\end{equation}

and $A_S$ is the soft A-parameter associated with the Yukawa coupling $y_S$ in $\text{(11)}$. The angular flaton (= axion) $a_S$ remains massless before the explicit breaking of $U(1)_S$ is taken into account. As having a light mass suppressed by a loop factor relative to the MSSM sparticle masses, the flatino is the LSP and becomes a dark matter of the Universe under the usual assumption of R-parity conservation $\text{(12, 27)}$. Such a light dark matter is cosmologically favorable since the moduli-induced gravitino problem would be considerably alleviated as can be seen in $\text{(7)}$. This point has been used in $\text{(10)}$ to solve the moduli-induced gravitino problem by assuming $m_{\psi S} \sim 10^2$ MeV under the additional assumption that there is no thermal inflation triggered by the flaton $S$.

However, since the flaton couples to the thermal bath in the early Universe through the Yukawa interaction which is responsible for its stabilization, it is a more plausible possibility that $S$ is hold at the origin, $S = 0$, by its thermal mass in the early Universe. In that case, the Universe experiences a short period of thermal inflation $\text{(13, 16)}$ driven by the flaton potential energy $V_0 \sim m_{\sigma S}^2 S_0^2$ around the origin. After thermal inflation, $S$ would start to roll down towards its true minimum $S_0$ and oscillate around the true minimum with an amplitude of $\mathcal{O}(S_0)$. As it produces a tremendous amount of entropy, thermal inflation practically dilutes away all the unwanted relics, including the coherent oscillation of heavy moduli causing the moduli-induced gravitino problem.

Thermal inflation is a natural consequence of the flaton model providing a weak scale size of $\mu$ and $B$ in heavy gravitino scenario, and immediately solves the moduli-induced gravitino problem. However, there are certain cosmological constraints which require a considerable
extension of the model. After thermal inflation, the Universe is reheated by the decays of the oscillating radial flaton $\sigma_S$. If the axion $a_S$ is stable, the axion energy density produced by the decays of $\sigma_S$ is bounded by the Big-Bang nucleosynthesis, which requires that $\sigma_S$ decays dominantly into the SM particles \[28\]. In the model under consideration, to generate a weak scale size of $B$, it is designed that $S$ couples to the operator $H_uH_d$ in the low energy effective action \[12\] through the combination $S^*/S$ at tree level. With this structure of the model, the coupling of $\sigma_S$ to $H_uH_d$ is cancelled at tree level, so its strength is loop-suppressed. As the decays of $\sigma_S$ to the SM particles are mediated by the coupling to $H_uH_d$, this results in that $\sigma_S$ decays dominantly to the axions, so an overproduction of axions which would be in conflict with the Big-Bang nucleosynthesis \[28\].

Another difficulty to be overcome is that thermal inflation dilutes any pre-existing baryon asymmetry. Thus one needs to introduce a baryogenesis mechanism which works after thermal inflation is over. Recently, it has been noted that the observed baryon asymmetry can be generated via the Affleck-Dine (AD) leptogenesis after thermal inflation. The AD mechanism \[15\] can be implemented by a flat direction carrying nonzero lepton-number. A particularly interesting candidate for such a flat direction is the MSSM $LH_u$ as it can have a large nonzero value during the period of $\mu = 0$, then rolls back to the origin after a nonzero $\mu$ is induced by the flaton vacuum value. Here, $L$ denotes the lepton doublet superfield, and we do not specify the generation structure for simplicity. The dynamics of $LH_u$ is determined also by the dim $= 5$ neutrino mass operator:

$$
\Delta W_\nu = \frac{LH_uLH_u}{M_\nu},
$$

which might be generated by the seesaw mechanism \[29\]. Then, in order for the AD leptogenesis to work, the $A$-type soft parameter associated with the neutrino mass operator should satisfy \[16, 18\]

$$
|A_\nu|^2 < 6(m_L^2 + m_{H_u}^2 + |\mu|^2),
$$

where $m_i$ denote the soft scalar masses. Otherwise, $LH_u$ is trapped at a meta stable minimum with $LH_u \neq 0$ even after $\mu \neq 0$ is generated. In heavy gravitino scenario, this is a nontrivial requirement as $F_C$ of the SUGRA compensator generically gives a contribution of $O(m_{3/2})$ to $A_\nu$.

As we will see in the next section, all these difficulties of thermal inflation in heavy gravitino scenario can be naturally solved by introducing an additional singlet flat direction
Thermal inflation can successfully incorporate the AD leptogenesis if \( X \) is stabilized also by radiative effects associated with SUSY breaking, and its vacuum value gives the heavy right-handed neutrino mass inducing the neutrino mass operator \( \Delta W_\nu \) via the seesaw mechanism. With this additional flaton \( X \), the overproduction of axions is naturally avoided since now both flatons, \( S \) and \( X \), have an unsuppressed coupling to \( H_u H_d \), with which the flatons decay dominantly into the SM particles.

**III. THE MODEL**

To resolve the difficulties noticed in the previous section, we introduce an additional singlet flaton \( X \) together with the right-handed neutrino \( N \) generating the neutrino mass operator \( \Delta W_\nu \) via the seesaw mechanism, and an extra matter pair \( \Psi, \Psi^c \) providing a Yukawa coupling to stabilize the original flaton \( S \). The relevant part of the model is given by

\[
\Delta \mathcal{L}_{\text{int}} = \int d^4 \theta \kappa S^* S' + \int d^2 \theta \left[ y_H \Sigma H_u H_d + y_X X S' \Sigma \right] + \int d^2 \theta \left[ y_N N L H_u + \frac{1}{2} y_X^* X N N + \frac{1}{2} (\lambda_S S + \lambda_{S'} S') \Psi \Psi^c \right] + \text{h.c.}, \tag{20}
\]

where we have imposed two global \( U(1) \) symmetries, \( U(1)_S \) and \( U(1)_X \), with the following charge assignments

\[
\begin{align*}
U(1)_S & : \ (2, 2, 0, -2, 0, -1, 2, -2), \\
U(1)_X & : \ (0, 0, 2, -2, -1, 0, 2, 0)
\end{align*}
\]

for the fields \( S, S', X, \Sigma, N, L, H_u H_d \) and \( \Psi \Psi^c \), respectively. Obviously both \( X \) and \( S \) correspond to flat directions when \( \Sigma, S', N, \Psi \) and \( \Psi^c \) are all frozen at the origin. Here the extra matter fields \( \Psi, \Psi^c \) can be either gauge-singlet or gauge-charged. In case of gauge-charged \( \Psi, \Psi^c \), our model can be considered as a simple generalization of some models discussed in [30], incorporating thermal inflation and AD leptogenesis.

Assuming that both \( X \) and \( S \) get large vacuum values, one can integrate out the heavy \( \Sigma, S', N, \Psi \) and \( \Psi^c \). The resulting effective theory can be written as

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{MSSM}} + \int d^4 \theta \left[ Y_{\text{eff}}^X X^* X + Y_{\text{eff}}^S S^* S + \left( \frac{\kappa}{X} S^* H_u H_d + \text{h.c.} \right) \right] + \left( \int d^2 \theta \frac{1}{2} \lambda_\nu \frac{L H_u L H_u}{X} + \text{h.c.} \right), \tag{21}
\]
where \( \hat{\kappa} = \kappa y_H / y_X \) and \( \lambda_\nu = y_{\nu}^2 / y_{\nu}^7 \). Here the effective wave function coefficient of \( \varphi = S, X \) is given by \( Y_{\varphi}^{\text{eff}} = Y_{\varphi}(Q = |\varphi|) \), where the running wavefunction coefficient \( Y_{\varphi}(Q) \) can be computed from the underlying theory (20).

**A. Flaton stabilization**

For the underlying theory given by (20), the mixing between the two flatons, \( X \) and \( S \), in the effective potential is negligible. The flaton potential is again induced by radiative effects associated with SUSY breaking, thus can be written in terms of the running soft masses:

\[
V \simeq \sum_{\varphi = X, S} m_{\varphi}^2(Q = |\varphi|)|\varphi|^2,
\]

where

\[
m_{\varphi}^2(Q) = -F^I F^J \partial_I \partial_J \ln Y_{\varphi}(Q).
\]

Here we consider the case that \( m_{\varphi}^2(Q) \) is driven to be negative at certain low scales by the associated Yukawa interaction. Then the flaton field is stabilized at \( \varphi = \varphi_0 \) satisfying

\[
m_{\varphi}^2(Q = \varphi_0) \simeq 0,
\]

and one can find the flaton \( F \)-term is given by

\[
\frac{F^\varphi}{\varphi} \simeq -F^I \partial_I \ln Y_{\varphi}(Q = |\varphi|),
\]

where \( F^I \) denote the SUSY breaking \( F \)-components in the model, including \( F^C \simeq m_{3/2} \) and the moduli \( F \)-component \( F^\phi \sim m_{3/2}^2 / m_\phi \lesssim m_{3/2} / 8\pi^2 \). Again, for both \( S \) and \( X \), the resulting \( F^\phi / \varphi_0 \) is of the order of the MSSM soft mass \( m_{\text{soft}} \sim m_{3/2} / 8\pi^2 \) as desired.

Around the minimum of potential, the radial flaton \( \sigma_{\varphi} \) and the flatino \( \psi_{\varphi} \) acquire SUSY breaking masses

\[
m_{\sigma_{\varphi}}^2 \simeq \frac{dm_{\varphi}^2(Q)}{d\ln Q} \bigg|_{Q = \varphi_0} = \mathcal{O} \left( \frac{m_{\text{soft}}^2}{8\pi^2} \right),
\]

\[
m_{\psi_{\varphi}} \simeq \sum_{ij} \frac{y_{\varphi ij}^2(Q)}{16\pi^2} A_{\varphi ij}(Q) \bigg|_{Q = \varphi_0} = \mathcal{O} \left( \frac{m_{\text{soft}}}{8\pi^2} \right),
\]

where \( y_{\varphi ij} \) are the Yukawa couplings, \( A_{\varphi ij} \) are the soft \( A \)-parameters in the canonical basis. Here we are using the parametrization

\[
\varphi = \left( \varphi_0 + \frac{\sigma_{\varphi}}{\sqrt{2}} \right) e^{i\sigma_{\varphi}/\sqrt{2}\varphi_0} + \sqrt{2} \theta \psi_{\varphi} + \theta^2 F^\varphi,
\]
for \( \varphi = X, S \), and also the fact that the mixing between \( X \) and \( S \) is negligible. The axion component \( a_\varphi \) would remain massless unless the associated global \( U(1) \) symmetry is broken by higher dimensional operators or by non-perturbative effect. Because the flatinos have masses of \( \mathcal{O}(m_{\text{soft}}/8\pi^2) \), the lightest flatino will constitute the dark matter under the assumption of R-parity conservation.

**B. Higgs \( \mu \) and \( B \), and the effective potential of \( LH_u \)**

In the model in consideration, the Higgs \( \mu \) and \( B \) parameters are induced through the coupling of \( H_u H_d \) to the flaton field combination \( S^*/X \) in the effective lagrangian (21). Including the potentially possible contribution from the moduli \( F \)-component \( F_\phi/\phi \sim m_{3/2}/m_\phi \lesssim m_{3/2}/8\pi^2 \), the resulting \( \mu \) and \( B \) are given by

\[
\begin{align*}
\mu &= \hat{\kappa} \frac{S_0}{X_0} \left\{ \left( \frac{F_S}{S_0} \right)^* + \mathcal{O}(m_{\text{soft}}) \right\}, \\
B &= \frac{F_X}{X_0} + \mathcal{O}(m_{\text{soft}}),
\end{align*}
\]

where the contributions from \( F^\phi \) are in \( \mathcal{O}(m_{\text{soft}}) \). One then finds both \( \mu \) and \( B \) have a desirable size, i.e. \( \mathcal{O}(m_{\text{soft}}) \) when \( \hat{\kappa} S_0 \sim X_0 \), which is rather a natural possibility.

In the presence of the neutrino mass operator \( \Delta W_\nu = \lambda_\nu LH_u LH_u/2X \), the effective potential for the \( LH_u \) flat direction is given by

\[
V_{LH_u} = \frac{1}{2} (m_L^2 + m_{H_u}^2 + |\mu|^2) |\ell|^2 + \left( \frac{A_\nu \lambda_\nu}{8} \frac{\ell^4}{X_0} + \text{c.c.} \right) + \frac{|\lambda_\nu|^2}{4} \frac{|\ell|^6}{X_0^2},
\]

where \( \ell \) is the field variable parameterizing the \( LH_u \) flat direction, and \( A_\nu \) is the soft \( A \)-parameter for \( \Delta W_\nu \), which is given by

\[
A_\nu = \frac{F_X}{X_0} + \mathcal{O}(m_{\text{soft}}).
\]

During the period before a nonzero \( \mu \) is induced by the flaton vacuum value, if \( m_L^2 + m_{H_u}^2 < 0 \), which will be assumed in the following, \( \ell \) is stabilized at

\[
\ell_0 \sim \sqrt{|A_\nu X_0/\lambda_\nu|}.
\]

If the condition (19) is satisfied, which is easily done in our model, \( \ell \) is correctly rolling back to the origin after implementing the AD leptogenesis. Note that it is crucial for the AD
leptogenesis to work that $A_\nu$ is of the order of $m_{\text{soft}} \sim m_{3/2}/8\pi^2$, which is achieved in our model by generating the scale of lepton number violation by a radiatively stabilized vacuum value $X_0$.

### C. Flaton decay

In this subsection, we examine the decay of the radial flaton $\sigma_\phi$ ($\phi = S, T$). Since $m_{\sigma_\phi} = \mathcal{O}(m_{\text{soft}}/4\pi)$, these radial flatons are kinematically forbidden to decay into the MSSM sparticles, the gravitino, or the moduli. The axion components $a_\phi$ can acquire a small mass from non-perturbative effect or higher dimensional operator breaking $U(1)_\phi$, but they will be taken to be almost massless in the following. In the low energy effective theory, the decays of $\sigma_\phi$ into axions or flatinos are mediated by the interactions

$$L_{\text{int}} = \frac{\sigma_\phi}{2\sqrt{2}\varphi_0} (\partial^\mu a_\phi)(\partial_\mu a_\phi) + \frac{\lambda_{\psi_\phi}}{2\sqrt{2}\varphi_0} \sigma_\phi \psi_\phi \psi_\phi + \text{h.c.},$$

where $\lambda_{\psi_\phi}$ is determined by the running $A$-parameters, $A_{\phi ij}$, associated with the Yukawa couplings, $y_{\phi ij}$, responsible for the flaton stabilization:

$$\lambda_{\psi_\phi} \simeq \frac{1}{\sum_{kl} y_{\phi kl} A_{\phi kl} \sum_{ij} \left( \frac{dy_{\phi ij}^2}{d\ln Q} A_{\phi ij} + y_{\phi ij}^2 \frac{dA_{\phi ij}}{d\ln Q} \right)|_{Q=\varphi_0},$$

where $i, j$ and $k, l$ run over the fields having a nonzero Yukawa coupling with the flaton $\phi = X, S$. One then finds that generically $\lambda_{\psi_\phi}$ has a value of $\mathcal{O}(10^{-1})$ or less. Then the decay rates are estimated as

$$\Gamma_{\sigma_\phi \rightarrow a_\phi a_\phi} = \frac{1}{64\pi} \frac{m_{\sigma_\phi}^3}{\varphi_0^2},$$

$$\Gamma_{\sigma_\phi \rightarrow \psi_\phi \psi_\phi} = \frac{\lambda_{\psi_\phi}^2 m_{\psi_\phi}^2 m_{\sigma_\phi}}{32\pi} \frac{m_{\sigma_\phi}}{\varphi_0^2},$$

where we have taken $\hat{\kappa}S_0 \sim X_0$.

The radial flatons decay also to the SM particles, mainly through the effective interactions

$$L_{\text{int}} = \frac{1}{\sqrt{2}} \left( \frac{\sigma_X}{X_0} - \frac{\sigma_S}{S_0} \right) \left( |\mu|^2 |H_u^0|^2 + |\mu|^2 |H_d^0|^2 - B_\mu H_u^0 H_d^0 \right) + \text{h.c.},$$

which would induce a mass mixing between $\sigma_\phi$ and the neutral Higgs fields after the electroweak symmetry breaking. The flaton-Higgs mixing then allows $\sigma_\phi$ to decay directly into the SM fermions, which is dominated by the bottom quark channel:

$$\Gamma_{\sigma_\phi \rightarrow b\bar{b}} \sim \frac{3}{4\pi} \left( 1 - \frac{|B|^2}{m_A^2} \right)^2 \left( 1 - \frac{4m_b^2}{m_{\sigma_\phi}^2} \right)^{3/2} \left( \frac{|\mu|^2}{m_h^2} \right)^2 \frac{m_b^2 m_{\sigma_\phi}}{\varphi_0^2},$$

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where $b$, $h$, and $A$ are the bottom quark, the lightest neutral CP even Higgs boson, and the neutral CP odd Higgs boson, respectively.

After thermal inflation is over, the Universe is dominated by the energy density of coherently oscillating radial flaton $\sigma_\phi$. If $\sigma_\phi$ decays dominantly to axions, it would be in conflict with the Big-Bang nucleosynthesis \[28\]. With the above results, the branching ratio to the decay into axions is given by

$$\frac{\Gamma_{\sigma_\phi \rightarrow a a}}{\Gamma_{\sigma_\phi \rightarrow b \bar{b}}} \sim \frac{1}{48} \left( \frac{m_{\sigma_\phi}}{m_b} \right)^2 \left( \frac{m_\mu}{|\mu|^2} \right)^2,$$

(36)

which can be easily smaller than $O(10^{-1})$ to satisfy the bound from the Big-Bang nucleosynthesis.

Meanwhile, assuming $X_0 > S_0$, the following interaction between two flatinos is induced by the $X$ dependence of $Y_{S}^{\text{eff}}$:

$$\mathcal{L}_{\text{flatino}} = \frac{\lambda_\psi}{\sqrt{2}X_0} \bar{\psi} X \sigma^\mu \tilde{\psi}_S (\partial_\mu a_S) + \text{h.c.},$$

(37)

where $\lambda_\psi = \langle X \partial_X \ln Y_{S}^{\text{eff}} \rangle$. Then the heavier flatino $\psi_1$ decays into the lighter flatino $\psi_0$ plus an axion with the decay rate

$$\Gamma_{\psi_1 \rightarrow \psi_0} \simeq \frac{|\lambda_\psi|^2}{32\pi} \left( 1 - \frac{m_{\psi_0}^2}{m_{\psi_1}^2} \right)^3 \frac{m_{\psi_1}^3}{X_0^2}.$$

(38)

Since $\lambda_\psi$ is induced at higher than two-loop level, its magnitude is suppressed as $|\lambda_\psi| < O(1/(8\pi^2))^2$, so that $\psi_1$ decays with a long lifetime while producing an axion together with the axino dark matter. Cosmological implication of such a late decay of the heavier flatino will be discussed in the next section.

**IV. COSMOLOGY**

The singlet flaton fields in our model allow a natural realization of thermal inflation and baryogenesis, and provide light flatinos one of which is the LSP. We assume, as the boundary condition of our cosmology, that radiation is dominant before the commencement of moduli coherent oscillation, and the temperature was high enough to hold all fields except moduli near the origin. As temperature drops down, the flaton field $\varphi$ starts to roll down towards its true minimum $\varphi = \varphi_0$ at the critical temperature $T_\varphi$:

$$T_\varphi = \frac{\dot{m}_\varphi}{\beta_\varphi} \quad (\varphi = X, S),$$

(39)
where $\hat{m}_\varphi^2 = -m_\varphi^2(Q \sim m_{\text{soft}}) > 0$ is the soft scalar mass squared of $\varphi$ renormalized at $Q \sim m_{\text{soft}}$, and $\beta_\varphi$ determines how strongly $\varphi$ couples to thermal bath \footnote{31}

$$\beta_\varphi^2 = \frac{1}{8} \sum_{ij} |y_{\varphi ij}|^2. \quad (40)$$

Before the Higgs $\mu$ term is induced by the flaton vacuum value, $\ell$ parameterizing the $LH_u$ flat direction is also thermally trapped at the origin, and its critical temperature is given by

$$T_\ell = \frac{\hat{m}_\ell}{\beta_\ell}, \quad (41)$$

where $\hat{m}_\ell^2 = -(m_L^2 + m_{H_u}^2)/2 > 0$, and $\beta_\ell$ is given by

$$\beta_\ell^2 = \frac{1}{8} \left( \sum_{ij} |y_{Lij}|^2 + 4 \sum_a C_2^a(L) g_a^2 \right) + (L \leftrightarrow H_u), \quad (42)$$

where $C_2^a(\Phi)$ denotes the quadratic Casimir invariant of $\Phi$. The pattern of thermal inflation and relevant cosmological contents depend on in what order the fields $X$, $S$ and $\ell$ roll down to the minimum. Here we consider the case that the underlying theory \footnote{20} leads to

$$T_S < T_\ell < T_X, \quad (43)$$

which is a natural possibility.

Meanwhile, the vanishing cosmological constant at the true vacuum $\langle \varphi \rangle = \varphi_0$, $\langle \ell \rangle = 0$ implies a nonzero potential energy near the origin:

$$V_0 = \sum_{\varphi=X,S} V_\varphi = \sum_{\varphi=X,S} \alpha_\varphi^2 \hat{m}_\varphi^2 \varphi_0^2, \quad (44)$$

with

$$\alpha_\varphi \approx \frac{1}{\sqrt{2}} \frac{m_{\sigma_\varphi}}{m_\varphi}, \quad (45)$$

which has a value of $\mathcal{O}(10^{-1})$. Hence, during the epoch of the thermal confinement, $V_0$ plays the role of vacuum energy. If necessary to be definite, we fix the vacuum values of the flaton fields as

$$X_0 \sim 10^{13} \text{GeV}, \quad S_0 \sim 10^{12} \text{GeV}, \quad (46)$$

where we have taken into account that neutrino masses in the range of $10^{-2}$ eV are obtained for $X_0 \sim \lambda_\nu \times 10^{15} \text{GeV}$, and $\mu$ is of $\mathcal{O}(m_{\text{soft}})$ for $\hat{\kappa} S_0 \sim X_0$. 

16
A. Thermal inflation

When Hubble parameter $H$ becomes comparable to the mass of the modulus $m_\phi$ at a time $t = t_\phi$, the modulus $\phi$ starts its coherent oscillation with Planck scale amplitude, and the epoch of moduli domination begins.

Thermal inflation begins at $t = t_1$ when the modulus energy density $\rho_\phi$ becomes comparable to $V_0$, and thus one finds

$$\frac{\rho_\phi}{\rho_r} \sim \frac{a(t_1)}{a(t_\phi)} \sim \left(\frac{H(t_\phi)}{H(t_1)}\right)^{2/3} \sim \left(\frac{m_\phi^2 M_{Pl}^2}{V_0}\right)^{1/3}, \quad (47)$$

where $\rho_r$ is the energy density of radiation, and the temperature at $t_1$ is

$$T_1 \sim \left(\frac{\rho_r}{\rho_\phi}\right)^{1/4} V_0^{1/4} \sim \left(\frac{V_0^2}{m_\phi M_{Pl}}\right)^{1/6}. \quad (48)$$

This epoch of thermal inflation continues at least until $X$ becomes unstable when the temperature drops to $T_X$ at $t = t_X$.

The thermal history after $t_X$ crucially depends on the ratio between $V_X$ and $V_S$. If $V_X \gg V_S$, the Universe is dominated by the non-relativistic $X$ particles. In this case, if $V_S$ becomes dominant before $S$ rolls down to its true minimum, a second epoch of thermal inflation is driven by $V_S$ before or after $\ell$ becomes unstable. Otherwise, matter domination by $X$ particles would continue until the particles decay and reheat the Universe for the Big-Bang nucleosynthesis. On the other hand, if $V_X \lesssim V_S$, thermal inflation is driven essentially by $V_S$, continuing until $S$ becomes unstable, and the Universe is reheated by the decay of $S$.

1. $V_X \gg V_S$

Since the commencement of the oscillation of $X$, its energy density $\rho_X$ dominates the Universe as matter. As the Universe expands, $\rho_X$ becomes comparable to $V_S$ at $t = t_2$.

The total decay width of the radial flaton $\sigma_\varphi$ ($\varphi = X, S$) can be written as

$$\Gamma_\varphi = C_\varphi \frac{\tilde{m}_\varphi^3}{\varphi_0^2}, \quad (49)$$

where $C_\varphi$ has a value of $O(10^{-3})$ because $m_\varphi^2 = O(m_{\text{soft}}^2/8\pi^2)$. For $\ell$, one finds

$$\Gamma_\ell = C_\ell \frac{\tilde{m}_\ell^3}{\ell_0^2}, \quad (50)$$
with \( C_\ell = \mathcal{O}(10^{-1}) \) and \( \ell_0 \sim \sqrt{|A_{\nu}X_0/\lambda_{\nu}|} \) being the vacuum value of \( \ell \) in the absence of the \( \mu \) term. For \( H \gg \Gamma_\nu \), the contribution of \( \rho_X \) to radiation at \( t = t_2 \) leads to**

\[
T_{\rho_X}^4(t_2) \approx \left( \frac{\pi^2}{30} g_*(T_{\rho_X}(t_2)) \right)^{-1} \left( \frac{\Gamma_X}{H(t_2)} V_S \right) \sim 0.1 V_S^{1/2} \Gamma_X M_{Pl} \left( \frac{100}{g_*(T_{\rho_X})} \right),
\]

which implies

\[
\frac{T_{\rho_X}^4(t_2)}{T_\ell^4} \sim 10^{-6} M_{Pl} \left( \frac{100}{g_*(T_{\rho_X}(t_2))} \right) \left( \frac{C_X}{10^{-3}} \right) \left( \frac{\alpha_S}{10^{-1}} \right) \left( \frac{\hat{m}_S \hat{m}_X^3}{\bar{m}_\ell} \right) \left( \frac{10 S_0}{X_0} \right). \tag{52}
\]

For \( X_0 \gtrsim 10^{12} \) GeV, \( T_{\rho_X}(t_2) \) is lower than \( T_\ell \) and thus the second thermal inflation can take place after \( \ell \) becomes unstable. Meanwhile, in case of \( T_{\rho_X}(t_2) < T_\ell \), the temperature due to the radiation contribution of \( \rho_\ell \) at \( t = t_2 \) is

\[
T_{\rho_\ell}^4(t_2) \approx \left( \frac{\pi^2}{30} g_*(T_{\rho_\ell}(t_2)) \right)^{-1} \left( \frac{\Gamma_\ell}{H(t_2)} \rho_\ell(t_2) \right) \sim 0.1 \frac{V_S^{1/2} \Gamma_\ell \Gamma_X^2 M_{Pl}^3}{T_\ell^8} \left( \frac{100}{g_*(T_{\rho_\ell}(t_2))} \right), \tag{53}
\]

and therefore, by using \( V(\ell = 0) - V(\ell = \ell_0) \sim \hat{m}_\ell^2 \ell_0^3 \) for \( \mu = 0 \), we find

\[
\frac{T_{\rho_X}^4(t_2)}{T_{\rho_\ell}^4(t_2)} \sim 10^4 \frac{X_0^2}{M_{Pl}^2} \left( \frac{10^{-1}}{C_\ell} \right) \left( \frac{10^{-3}}{C_X} \right) \left( \frac{\hat{m}_\ell}{\bar{m}_X} \right)^3,
\]

\[
\frac{T_{\rho_\ell}^4(t_2)}{T_S^4} \sim 10^{-10} \beta_S^4 \frac{M_{Pl}^3}{X_0^3} \left( \frac{100}{g_*(t_2)} \right) \left( \frac{\alpha_S}{10^{-1}} \right) \left( \frac{C_\ell}{10^{-3}} \right) \left( \frac{C_X}{10^{-3}} \right)^2 \left( \frac{\hat{m}_\ell^2 \hat{m}_X}{\bar{m}_\ell \bar{m}_S} \right)^3 \left( \frac{10 S_0}{X_0} \right). \tag{54}
\]

For \( X_0 \lesssim 10^{15} \) GeV with \( \beta_S \sim \mathcal{O}(1) \), the partial decay of \( \ell \) at \( t = t_2 \) thus contributes dominantly to radiation with temperature larger than \( T_S \). This implies that, after \( \ell \) decouples from thermal bath, a second epoch of thermal inflation begins at \( t = t_2 \) with a background temperature

\[
T_2 = T_{\rho_\ell}(t_2). \tag{55}
\]

The thermal inflation ends as \( S \) rolls away from the origin, and then the decay of \( S \) eventually reheats the Universe for the Big-Bang nucleosynthesis††

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** We assume that the energy loss of the oscillating field due to the parametric resonance is small.

†† The contribution to radiation from the late decay of \( X \) is negligible for our choice of the flaton vacuum values.
The number of e-folds for each epoch of thermal inflation is estimated as
\[
N_1 \simeq \ln \left( \frac{T_1}{T_X} \right) \simeq 6.8 + \ln \left[ \beta_X^3 \left( \frac{\alpha_X}{10^{-3}} \right)^4 \left( \frac{X_0}{10^{13}\text{GeV}} \right)^4 \left( \frac{10^3\text{GeV}}{m_X} \right)^2 \left( \frac{10^6\text{GeV}}{m_\phi} \right) \right],
\]
\[
N_2 \simeq \frac{1}{3} \ln \left( \frac{\rho_\ell(t_2)}{\rho_\ell(t_S)} \right) \sim \frac{1}{3} \ln \left[ \frac{V_\ell \Gamma_\ell M_{Pl}}{T_\ell^4 \Gamma_X^2 M_{Pl}^2} \frac{V_S \Gamma_X^2 M_{Pl}^2}{T_\ell^4 \Gamma_X^2 M_{Pl}^2} \right]
\]
\[
\simeq 5.5 + \frac{1}{3} \ln \left( \frac{\alpha_S}{10^{-1}} \right) \left( \frac{C_\ell}{10^{-1}} \right) \left( \frac{C_X}{10^{-3}} \right)^2 \left( \frac{\hat{m}_S \hat{m}_X}{\hat{m}_\ell^2} \right) \left( \frac{10S_0}{X_0} \right) \left( \frac{10^{13}\text{GeV}}{X_0} \right)^3. \tag{56}
\]

These e-folds, which are additive to the standard primordial inflation, do not interfere directly with CMB or cosmic 21 cm fluctuations‡‡, but may have an observable effect on the primordial density perturbation though it may be difficult to disentangle from uncertainties in the model of primordial inflation.

The entropy productions from the decays of \( \varphi \) and \( \ell \) provide the dilution factors
\[
\Delta_X \sim \left( \frac{T_\ell}{T_X} \right)^3 \frac{V_X}{\rho_X(t_\ell)} \sim \left( \frac{T_\ell}{T_X} \right)^3 \frac{V_X \Gamma_X^2 M_{Pl}^2}{T_\ell^3}, \tag{57}
\]
\[
\Delta_\ell \sim \left( \frac{T_S}{T_\ell} \right)^3 \frac{V_\ell}{\rho_\ell(t_S)} \sim \left( \frac{T_S}{T_\ell} \right)^3 \frac{V_\ell \Gamma_\ell M_{Pl}}{T_\ell^4 V_S^{1/2}} \tag{58}
\]
\[
\Delta_S \sim \frac{V_S}{T_S^3 T_d}, \tag{59}
\]
where \( T_d \) is the decay temperature of the flaton field \( S \) after thermal inflation. Therefore, the total dilution is given by
\[
\Delta_{\text{tot}} = \Delta_X \Delta_\ell \Delta_S \sim \frac{V_X V_\ell V_S^{1/2} \Gamma_X \Gamma_\ell \Gamma_\ell M_{Pl}^3}{T_\ell^3 T_d T_S^3} \sim 10^{28} \beta_X^3 \beta_\ell^3 \frac{\alpha_X^2 \alpha_S}{10^{-3}} \left( \frac{C_\ell^2 C_X^4}{10^{-11} C_S} \right)^{1/2} \left( \frac{10S_0}{X_0} \right)^2 \left( \frac{\hat{m}_X}{\hat{m}_\ell \hat{m}_S} \right) \left( \frac{10^3\text{GeV}}{\hat{m}_S} \right)^{5/2} \tag{60}
\]
where we have used \( T_d \sim (\Gamma_S M_{Pl})^{1/2} \) though we will redefine it later in a more definite way.

Thermal inflation should dilute enough the abundance of moduli to avoid dark matter over-production described in Section II. The bound on the abundance at the time of the

‡‡ For primordial inflation at \( V^{1/4} \sim 10^{10} \text{ GeV} \) and assuming radiation domination after inflation, if the reheating temperature of thermal inflation is \( T_d = \mathcal{O}(1) \text{ GeV} \), the observable Universe leaves the horizon around 40 e-folds before the end of inflation. The observed CMB covers only around 6 e-folds after the horizon exit of the observable Universe \[32\], and the potentially observable cosmic 21 cm fluctuations cover around 9 additional e-folds \[33\].
decay of moduli is
\[
\frac{n_{\phi}}{s} \lesssim 0.25 \frac{\Gamma_{\phi}}{\Gamma_{\phi \to \tilde{G}} m_{\chi}} \left( \frac{\rho_{\phi}}{s} \right)_{\text{present}} \approx 5.0 \times 10^{-12} \frac{\Gamma_{\phi}}{\Gamma_{\phi \to \tilde{G}}} \left( \frac{100 \text{ GeV}}{m_{\chi}} \right).
\]  (61)

For modulus particles produced before the thermal inflation, the late time abundance is estimated as
\[
\frac{n_{\phi}}{s} \sim \left( \frac{M_{\text{Pl}}}{m_{\phi}} \right)^{1/2} \frac{1}{\Delta_{\text{tot}}} \sim 10^{-22} \left( \frac{10^6 \text{ GeV}}{m_{\phi}} \right)^{1/2} \left( \frac{10^{28}}{\Delta_{\text{tot}}} \right).
\]  (62)

If produced at the end of the first thermal inflation, the modulus abundance is given by
\[
\frac{n_{\phi}}{s} \sim \frac{m_{\phi} M_{\text{Pl}}^{2} H(t_{X})^{4}}{T_{X}^{3}} \frac{1}{m_{\phi}^{4}} \frac{V_{X}^{2}}{\Delta_{\text{tot}}} \sim \frac{V_{X}^{2}}{m_{\phi}^{2} M_{\text{Pl}}^{2} T_{X}^{2} \Delta_{\text{tot}}} \sim 10^{-32} \beta_{X}^{3} \left( \frac{\alpha_{X}}{10^{-1}} \right)^{4} \left( \frac{\tilde{m}_{X}}{10^{3} \text{ GeV}} \right) \left( \frac{10^{6} \text{ GeV}}{m_{\phi}} \right)^{3} \left( \frac{10^{28}}{\Delta_{\text{tot}}} \right),
\]  (63)

whereas, for those produced at the end of the second thermal inflation, we find
\[
\frac{n_{\phi}}{s} \sim \frac{m_{\phi} M_{\text{Pl}}^{2} H(t_{S})^{4}}{T_{S}^{3}} \frac{1}{m_{\phi}^{4}} \frac{V_{S}^{2} T_{d}}{m_{\phi}^{2} M_{\text{Pl}}^{2}} \sim 10^{-26} \left( \frac{\alpha_{S}}{10^{-1}} \right)^{4} \left( \frac{C_{S}}{10^{-3}} \right)^{1/2} \left( \frac{\tilde{m}_{S}}{10^{3} \text{ GeV}} \right)^{7/2} \left( \frac{10^{6} \text{ GeV}}{m_{\phi}} \right)^{3} \left( \frac{S_{0}}{10^{28} \text{ GeV}} \right). \]  (64)

All of these contributions to the moduli abundance are far below the safe level of (61).

2. \( V_{X} \lesssim V_{S} \)

In this case, we have only single epoch of thermal inflation driven essentially by \( V_{S} \), but it is extended by the radiation contribution of \( X \) and \( \ell \) when those fields decouple from thermal bath. The \( e \)-folding of the thermal inflation is given as
\[
N \simeq \ln \left[ \left( \frac{T_{1}}{T_{X}} \right) \left( \frac{a_{X}}{a_{1}} \right) \left( \frac{a_{S}}{a_{1}} \right) \right] \simeq \frac{1}{3} \ln \left[ \frac{V_{0}}{m_{\phi}^{1/2} M_{\text{Pl}}^{1/2} T_{X}^{3} \Gamma_{X} M_{\text{Pl}} \Gamma_{\ell} M_{\text{Pl}}} \frac{V_{X} \Gamma_{X} M_{\text{Pl}} \Gamma_{\ell} M_{\text{Pl}}}{T_{X}^{4} V_{S}^{1/2} T_{S}^{4} V_{S}^{1/2}} \right] \simeq 12 + \frac{1}{3} \ln \left[ \beta_{X}^{3} \beta_{S}^{2} \left( \frac{\alpha_{X}}{10^{-1}} \right)^{2} \left( \frac{C_{X}}{10^{-1}} \right) \left( \frac{\tilde{m}_{X} m_{X}^{2}}{\tilde{m}_{S} S_{0}} \right) \left( \frac{10^{3} \text{ GeV}}{m_{\phi}} \right) \left( \frac{10^{6} \text{ GeV}}{m_{\phi}} \right)^{1/2} \right].
\]  (65)

The dilution factor from the decay of \( X \) is obtained as
\[
\Delta_{X} \sim \left( \frac{T_{\ell}}{T_{X}} \right)^{3} \frac{V_{X}}{\rho_{X}(t_{\ell})} \sim \left( \frac{T_{\ell}}{T_{X}} \right)^{3} V_{X} \frac{\Gamma_{X} M_{\text{Pl}}}{\Gamma_{\ell} M_{\text{Pl}}} \frac{T_{X}^{4} V_{S}^{1/2}}{T_{S}^{4} V_{S}^{1/2}},
\]  (66)
while those from $\ell$ and $S$ have the same forms as (58) and (59), respectively. Hence the total dilution factor is

$$
\Delta_{\text{tot}} \sim \frac{V_X}{T_X^3 T_d} \frac{V_\ell}{T^4 T^4_S} \sim \frac{\alpha_X^2 C_X C_X}{C^4_S} \beta_X^2 \beta^4_S \left( \frac{\hat{m}_X \hat{m}_X}{\hat{m}_S} \right) \left( \frac{S_0}{\hat{m}_S} \right) \left( \frac{M_{Pl}}{\hat{m}_S} \right)^{3/2} \\
\sim 10^{28} \beta_X^3 \beta^4_S \left( \frac{\alpha_X}{10^{-1}} \right)^2 \left( \frac{C^2_X}{C^2_S} \right)^{1/2} \left( \frac{\hat{m}_\ell \hat{m}_X}{\hat{m}_S} \right) \left( \frac{10^3 \text{GeV}}{\hat{m}_S} \right)^{5/2} \left( \frac{S_0}{10^{12} \text{GeV}} \right),
$$

and thus, compared to the bound in (61), the abundance of moduli becomes totally negligible in this case too.

After thermal inflation, the Universe is reheated by the decay of the flaton that ends the last thermal inflation. Since there is no unique decay temperature, we instead define the flaton decay temperature $T_d$ by

$$
\rho_r(T_d) \equiv \frac{1}{2} \Gamma^2 \varphi M_{Pl}^2,
$$

or

$$
\pi^2 \frac{g_*(T_d)}{30} T_d^4 = \rho_{SM}(T_d) = \frac{1}{2} \Gamma_{\varphi \to \text{SM}} \Gamma \varphi M_{Pl}^2,
$$

where $\Gamma_{\varphi \to \text{SM}}$ is the decay width to the SM particles, and $\varphi$ is either $X$ or $S$. This corresponds to a time

$$
t_d \simeq \frac{1}{\Gamma \varphi},
$$

and at the moment we have

$$
\rho_\varphi(T_d) \simeq \frac{1}{2} \Gamma^2 \varphi,
$$

and the entropy increases after $t_d$ by a factor $S_t/S_d \simeq 2$ where $S_d$ and $S_t$ are the entropy at $t = t_d$ and a late time, respectively [18].

**B. Baryogenesis**

For $T_X > T_\ell > T_S$, the flaton field $X$ decouples first from thermal bath and settles to its true minimum $X = X_0$. The scalar potential implementing the AD leptogenesis is then determined by the effective theory (21), which involves

$$
\mathcal{L}_{\text{AD}} = \int d^4 \theta Y_S \left( S S^* + \left( \int d^4 \theta \hat{\kappa} \frac{S^*}{X_0} H_u H_d + \int d^2 \theta \frac{1}{2} \lambda \frac{L H_u L H_u}{X_0} + \text{h.c.} \right) \right),
$$

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where $Y_{S}^{\text{eff}} = Y_{S}(Q = |S|)$. To analyze the dynamics induced by $m_{L}^{2}(Q) + m_{H_{u}}^{2}(Q) < 0$ at $Q \sim m_{\text{soft}}$, we parameterize the associated flat directions\footnote{We set quark and lepton directions to be zero since they are expected to be held at the origin throughout the dynamics. This will become clear from the subsequent argument.} as

$$L = (0, l)^{T}, \quad H_{u} = (h_{u}, 0)^{T}, \quad H_{d} = (0, h_{d})^{T}$$

with the $D$-term constraint for the $SU(2)_{L}$ gauge symmetry

$$D_{2} = |h_{u}|^{2} - |h_{d}|^{2} - |l|^{2} = 0.$$  \hspace{1cm} (74)

The relevant part of the potential is then written as

$$V_{\text{AD}} = V_{S} + \frac{1}{2} g_{2}^{2} D_{2}^{2} + m_{S}^{2} |S|^{2} + m_{L}^{2} |l|^{2} + m_{H_{u}}^{2} |h_{u}|^{2} + m_{H_{d}}^{2} |h_{d}|^{2}$$

$$+ \left( \frac{1}{2} A_{\nu} \lambda_{\nu} l^{2} h_{u}^{2} - B \hat{\mu} \frac{S^{*}}{X_{0}} h_{u} h_{d} + \text{c.c.} \right)$$

$$+ |\lambda_{\nu} l h_{u}^{2}|^{2} + |\lambda_{\nu} l^{2} h_{u} + \hat{\mu} \frac{S^{*}}{X_{0}} h_{d}|^{2} + |\hat{\mu} \frac{S^{*}}{X_{0}} h_{u}|^{2},$$

with $\hat{\mu}$ given by

$$\hat{\mu} = \hat{\kappa} \left( \frac{F_{S}^{*}}{S} \right)^{*} + \mathcal{O}(m_{\text{soft}}),$$

(75)

where the loop-suppressed contribution from $F^{C}$ is included in $\mathcal{O}(m_{\text{soft}})$. Note that $B$, $A_{\nu}$, and $\hat{\mu}$ have values of $\mathcal{O}(m_{\text{soft}})$ and are nearly independent of $\varphi = X, S$ for $|\varphi| \gg m_{\text{soft}}$ since the equation of motion for $F^{\varphi}$ is given by (25).

Initially, all fields are held at the origin by their finite temperature potential. As the temperature drops down, one of the unstable directions, $S$ or $\ell = l h_{u}$, will roll away from the origin. We assume $l h_{u}$ rolls away first, i.e. $T_{\ell} < T_{S}$. Then the potential term $\frac{1}{2} A_{\nu} \lambda_{\nu} l^{2} h_{u}^{2}$ fixes the phase of $l h_{u}$, while $|\lambda_{\nu} l h_{u}^{2}|^{2}$ and $|\lambda_{\nu} l^{2} h_{u}|^{2}$ stabilize its magnitude. The $l h_{u}$ field may partially reheat the thermal bath and so prolong the thermal inflation, but eventually $S$ will also roll away from the origin, ending thermal inflation. As $S$ rolls away, the term $B \hat{\mu} S^{*} h_{u} h_{d}/X_{0}$ will force $h_{d}$ to become non-zero. This provides temporarily a large mass to quark and lepton directions and constrains those directions to zero, shielding the dynamics from the dangerous non-MSSM vacuum in the direction associated quark and lepton $^{18,34}$. Then, $B \hat{\mu} S^{*} h_{u} h_{d}/X_{0}$ fixes the phase of $S^{*} h_{u} h_{d}$. As $S$ nears its minimum, the cross term
from $|\lambda_u l^2 h_u + \tilde{\mu} S^* h_d / X_0|^2$ rotates the phase of $lh_u$ generating a lepton asymmetry, and at the same time $|\tilde{\mu} S^* h_u / X_0|^2$ gives an extra contribution to the mass squares of $lh_u$ and $h_u h_d$, bringing them back in towards the origin. Thus, we have a type of the Affleck-Dine (AD) leptogenesis. Preheating then damps the amplitude of the $lh_u$ and $h_u h_d$ fields keeping them in the lepton preserving region near the origin [17]. The $lh_u$ and $h_u h_d$ fields then decay, at a temperature in the MSSM sector above the electroweak scale, and their lepton number is converted to baryon number by sphaleron processes. Finally, the flatons $S$ and $X$ decay, diluting the baryon density to the value required by observations, $n_B / s \sim 10^{-10}$.

One may think that, if $X$ decays later than $S$, the baryon asymmetry generated due to the dynamics of $S$ may be significantly diluted. However, the dilution is possible only when $V_X \gg V_S$ with $X_0$ different from $S_0$ by more than several order, which is very unnatural in our model.

Comparing to the model considered in [18], the Higgs $\mu$ term in our model has a linear dependence on the triggering field $S$ rather than quadratic:

$$\mu = \tilde{\mu} \frac{S^*}{X_0}, \quad (77)$$

where $\tilde{\mu}$ does not depend on $S$ for $|S| \gg m_{soft}$. This may weaken the strength of torque responsible for the angular momentum and preheating, but would only make a difference of a factor of $\mathcal{O}(1)$. We therefore expect that the AD baryogenesis would work well in our model too.

C. Dark matter

The model [21] provides the lightest flatino as the dark matter under the assumption of R-parity conservation since flatinos $\psi_\varphi$ ($\varphi = X, S$) have small masses of $\mathcal{O}(m_{soft}/8\pi^2)$. In addition, the axions $a_\varphi$ can also become a dark matter of the Universe if they are light enough to be stable. These dark matter components can be cold, warm or even hot, depending on their masses and how they are produced. In this subsection, we will derive cosmological constraints on the dark matter.
1. Cold dark matter

As a cold dark matter, the lightest flatino is dominantly produced by the decay of the corresponding flaton if it eventually reheats the Universe for the Big-Bang nucleosynthesis. Another source is the decay of heavier sparticles in thermal bath, which is dominated by the decay of the lightest MSSM sparticle. In addition, axion misalignment and strings also contribute to the energy density of cold dark matter.

a. Flatinos produced by the flaton decay: For the flaton $\varphi$ that ends the thermal inflation, its decay reheats the Universe. The late time flatino abundance from the flaton decay is determined by

$$\frac{n_{\psi}}{s} = \frac{2\Gamma_{\varphi \rightarrow \psi \psi}}{m_{\sigma \varphi}} a^3 \int_0^t dt' a^3(t') \rho_\varphi(t') \sim \frac{2 T_d}{m_{\sigma \varphi}} \frac{\Gamma_{\varphi \rightarrow \psi \psi}}{\Gamma_{\sigma \varphi \rightarrow \text{SM}}},$$

where the decay temperature $T_d$ is roughly given by

$$T_d \sim 1\text{GeV} \left[ \left( \frac{100}{g_*(T_d)} \right) \left( \frac{C_{\varphi}}{10^{-3}} \right)^2 \left( \frac{m_{\varphi}}{10^3\text{GeV}} \right)^6 \left( \frac{10^{12}\text{GeV}}{\varphi_0} \right)^4 \right]^{1/4},$$

which follows from (69). Using (33) and (78), one can find the current abundance of flatino dark matter $\psi_0$

$$\Omega_{\psi_0} \simeq 5.6 \times 10^8 \left( \frac{m_{\psi_0}}{1\text{GeV}} \right) \frac{n_{\psi_0}}{s}$$

$$\simeq 3.6 \left( \frac{100}{g_*(T_d)} \right)^{1/2} \left( \frac{\lambda_{\psi_0}}{10^{-1}} \right)^2 \left( \frac{m_{\psi_0}}{1\text{GeV}} \right) \left( \frac{m_{\psi_0}}{1\text{GeV}} \right)^2 \left( \frac{1\text{GeV}}{T_d} \right) \left( \frac{10^{11}\text{GeV}}{\varphi_0} \right)^2,$$

where we have used that one flatino dark matter is produced per each heavier flatino $\psi_1$ if $\psi_0 = \psi_1$, and that the total decay width is given by $\Gamma_\varphi \simeq \Gamma_{\sigma \varphi \rightarrow \text{SM}}$. Therefore, the requirement $\Omega_{\psi_0} \leq \Omega_{\text{CDM}} \simeq 0.2$ translates into

$$m_{\psi_0} \lesssim 1.8\text{GeV} \left[ \left( \frac{g_*(T_d)}{100} \right)^2 \left( \frac{T_d}{1\text{GeV}} \right) \left( \frac{10^{-1}}{\lambda_{\psi_0}} \right)^2 \left( \frac{\varphi_0}{10^{12}\text{GeV}} \right)^2 \right]^{1/3},$$

which is at the lower end of its expected range (26).

b. Flatinos produced by the decay of thermally generated MSSM sparticles: The flaton coupling to $H_u H_d$ in the effective Kähler potential (21) induces

$$\mathcal{L}_{\text{int}} = \lambda_S \frac{\mu}{S_0} H_{u,d} \tilde{H}_{d,u} \psi_S + \frac{\lambda_S}{S_0} H_{u,d} \bar{H}_{d,u} \sigma^\mu \partial_\mu \bar{\psi}_S - \frac{\mu}{X_0} H_{u,d} \tilde{H}_{d,u} \psi_X + \text{h.c.},$$

(82)
where \( \lambda = \langle S \partial S \ln \hat{k} \rangle = \mathcal{O}(1/8\pi^2) \), \( \lambda' = \hat{k}S_0/X_0 = \mathcal{O}(1) \), and \( \hat{H}_{u,d} \) are the Higgsino fields.

The above interactions lead the lightest MSSM sparticle \( \chi \) to decay into flatinos with the decay rate \[ \Gamma_{\chi \to \psi \varphi} = \frac{C_{\psi \varphi} m_\chi^3}{16\pi \varphi_0^2}, \tag{83} \]

where \( C_{\psi \varphi} = \mathcal{O}(10^{-2}C_{\psi X}) \) for \( \mu \) of the soft mass scale and \( S_0 \sim 10^{-1}X_0 \). Here \( C_{\psi X} \sim 1 \) may contain a factor of \( m_Z^2/m_\chi^2 \), and we have neglected the masses of decay products.

The thermal bath generates \( \chi \) with the number density \[ n_\chi = \frac{1}{\pi^2} \int_0^\infty dk \frac{k^2}{\exp \sqrt{k^2 + m_\chi^2} + 1}, \tag{84} \]

and they subsequently decay into flatinos, dominantly into \( \psi X \). The late time flatino abundance is thus estimated as

\[ \frac{n_{\psi X}}{s} = g_1^{1/4}(T_d)g_5^{5/4}(T_\chi) \left( \frac{\Gamma_{\sigma_\psi \to \text{SM}}}{\Gamma_{\chi}} \right)^{1/2} \frac{\Gamma_{\chi \to \psi X M_{\text{Pl}}}}{m_\chi^2} F_\psi(x), \tag{86} \]

where \( F_\psi(x) \) is approximated as

\[ F_\psi(x) \sim \begin{cases} 5.3x^7 & \text{for } x \ll 1 \\ 5.4 & \text{for } x \gg 1 \end{cases}, \tag{87} \]

with

\[ x = \frac{2}{3} \left( \frac{g_5(T_d)}{g_5(T_\chi)} \right)^{1/4} \frac{T_d}{T_\chi}, \tag{88} \]

and \( T_\chi \simeq 2m_\chi/21 \) being the temperature at which the flatino production rate is maximized.

Using (83) and (86), the current abundance of flatino dark matter is obtained

\[ \Omega_{\psi_0} \simeq 5.6 \times 10^8 \left( \frac{m_{\psi_0}}{1 \text{GeV}} \right) \frac{n_{\psi X}}{s} \]

\[ \simeq 2.7 \times 10^{-2}C_{\psi X} \left( \frac{10^3}{g_5^{3/4}(T_d)} \right) \left( \frac{m_\chi}{10^2 \text{GeV}} \right) \left( \frac{m_{\psi_0}}{1 \text{GeV}} \right) \left( \frac{10^{13} \text{GeV}}{X_0} \right)^2 F_\psi(x), \tag{89} \]

where we have used \( \Gamma_{\varphi} \simeq \Gamma_{\sigma_\psi \to \text{SM}} \), and that the number density of \( \psi_0 \) is the same as that of \( \psi X \) even when \( \psi X = \psi_1 \). Therefore, for \( x \ll 1 \), one finds

\[ \Omega_{\psi_0} \simeq 1.2 \times 10^5 C_{\psi X} \left( \frac{T_d}{m_\chi} \right)^7 \left( \frac{10^3 g_5^{3/2}(T_d)}{g_5^2(T_\chi)} \right) \left( \frac{m_\chi}{10^2 \text{GeV}} \right) \left( \frac{m_{\psi_0}}{1 \text{GeV}} \right) \left( \frac{10^{13} \text{GeV}}{X_0} \right)^2, \tag{90} \]
which does not exceed $\Omega_{\text{CDM}}$ if the flaton decay temperature satisfies

$$T_d \lesssim \frac{m_\chi}{6.7} \left( \frac{g_s(T_\chi)}{g_s(T_d)} \right)^{1/4} \left[ C_{\psi_X}^{-1} \left( \frac{g^{1/4}_s(T_d) g^{5/4}_s}{10^3} \right) \left( \frac{10^2 \text{GeV}}{m_\chi} \right) \left( \frac{1 \text{GeV}}{m_{\psi_0}} \right) \left( \frac{X_0}{10^{13} \text{GeV}} \right)^2 \right]^{1/7}. \quad (91)$$

Since $T_d$ is expected to be a few GeV, the above cosmological bound is well satisfied for $m_\chi = \mathcal{O}(10^2)$ GeV. Note that the flaton decay temperature can be less than or similar to the freeze-out temperature of $\chi$, which is about $m_\chi/20$ [32]. Therefore, in order to avoid direct production of $\chi$ from the flaton decay, we may require $m_{\sigma_\varphi} < 2m_\chi$, which is quite plausible in our model.

Flatinos will also be produced by the decay of $\chi$ after they freeze out. However, the standard Big-Bang neutralino freeze-out abundance is good match to the dark matter abundance, our freeze-out abundance of $\chi$ will typically be less than the standard abundance, and $m_{\psi_\varphi} \ll m_\chi$, therefore the flatino abundance generated after the freeze-out should be safe.

c. **Axion misalignment:** The cold dark matter contributions of misalignment and axionic strings from QCD-axion are well known. Thus we consider only the cases of non-QCD axion under the assumption that the mass of the axion appears due to a tree-level symmetry breaking term which is small enough not to disturb the radiative stabilization of the associated flaton field. In this case, the mass of axion is not connected with its coupling constant.

The energy density of an axion misalignment when oscillation commences is

$$\rho_a = \frac{m^2_a \Theta^2 \varphi_0^2}{2 N^2}, \quad (92)$$

where $a = a_\varphi$, $m_a$ is the axion mass, $\Theta$ is the misalignment angle, and $N$ is the vacuum degeneracy. The current energy density is

$$\frac{\rho_a}{\rho_\sigma} = \left( \frac{a_0}{a_{\text{osc}}} \right) \frac{g_s(T_{\text{osc}})}{g_s(T_0)} \left( \frac{g_{ss}(T_0)}{g_{ss}(T_{\text{osc}})} \right)^{4/3} \left( \frac{\rho_a}{\rho_\sigma} \right)_{\text{osc}} \left( \frac{\pi^2}{9} \right)^{1/4} \frac{g^{5/4}_s(T_{osc})}{g_s(T_0)} \left( \frac{g_{ss}(T_0)}{g_{ss}(T_{osc})} \right)^{1/3} \frac{\varphi^2}{N^2} \frac{m^2_a \varphi^2}{(3H_{\text{osc}} M_P)^{3/2} T_0}. \quad (93)$$

Therefore, $\Omega_a < \Omega_{\text{CDM}}$ requires

$$m_a < 6.0 \times 10^{-5} \text{eV} \left( \frac{100}{g_s(T_{\text{osc}})} \right)^{11/6} \frac{N^4}{\Theta^4} \left( \frac{10^{12} \text{GeV}}{\varphi_0} \right)^4, \quad (94)$$

where we have used $3H_{\text{osc}} = m_a$ and $g_s(T_{\text{osc}}) = g_{ss}(T_{\text{osc}})$. 

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Thermal inflation can dilute those contributions if the reheating temperature is lower than the temperature at which the mass of axion becomes comparable to the expansion rate. We do not analyze the dilution in this paper, instead we refer the reader to [18] for the case.

d. **Axionic strings:** The axion produced by the strings is

\[ \frac{n_a}{s} \sim A \frac{\varphi_0^2}{T_{osc} M_{Pl}}, \]  

(96)

where \( A \equiv [1 \text{ or } \ln (\hat{\mu}/H_{osc})] \). The current energy density is

\[ \frac{\rho_a}{\rho_c} = \frac{4}{3} A \left( \frac{\pi^2}{10^2 g_*(T_{osc})} \right)^{1/4} \left( \frac{g_* (T_0)}{g_*(T_{osc})} \right) \frac{m_a \varphi_0^2}{3 H_{osc} M_{Pl}^{3/2} T_0}. \]

(97)

Therefore, in order for \( \Omega_a \) not to exceed \( \Omega_{CDM} \), it is required

\[ m_a \lesssim 2.5 \times 10^{-6} \text{eV} \left( \frac{50}{A} \right)^2 \left( \frac{100}{g_*(T_{osc})} \right)^{1/2} \left( \frac{10^{12} \text{GeV}}{\varphi_0} \right)^4, \]

(98)

where we have again used \( 3H_{osc} = m_a \).

The mass bound (98) implies that tree level symmetry breaking should be very small. For instance, the higher dimensional superpotential term

\[ \Delta W \propto \frac{\varphi^n}{M_{Pl}^{n-3}}, \quad (n > 3) \]

(99)

provides the axion a mass

\[ m_a^2 \sim |A\varphi_0| \left( \frac{\varphi_0}{M_{Pl}} \right)^{n-3}, \]

(100)

where \( A \sim m_{3/2} \) (or \( m_{\text{soft}} \)). In this case, the requirement (98) is satisfied for

\[ n \gtrsim 3 + \frac{\ln(m_a^2/|A\varphi_0|)}{\ln(\varphi_0/M_{Pl})} \]

\[ \gtrsim 7.4 - \frac{1}{15} \ln \left[ \left( \frac{m_a}{10^{-6} \text{eV}} \right)^2 \left( \frac{|A|}{10^4 \text{GeV}} \right) \left( \frac{10^{12} \text{GeV}}{\varphi_0} \right) \right] - \frac{1}{3} \ln \left( \frac{\varphi_0}{10^{12} \text{GeV}} \right). \]

(101)

2. **Warm/hot dark matter**

The LSP \( \psi_0 \) and axion can be warm or hot when they are produced by the decay of the next to LSP (NLSP) \( \psi_1 \) (for LSP and axion) and the radial flaton \( \sigma_\varphi \) (for axion). The constraint on hot dark matter comes from CMBR and structure formation [35, 36, 37, 38].

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Currently allowed hot dark matter fractional contribution to the present critical density is \( \Omega_{HDM} \lesssim 10^{-2} \) \[39\]. The constraint may be more stringent as suggested from the analysis of the early re-ionization of the Universe at high redshift \[40\]. Taking into account the recent analysis of WMAP 5-year data \[41\], the allowed warm/hot dark matter fractional contribution to the present critical density is likely to be

\[
\Omega_{WHDM} \lesssim 10^{-3}.
\] (102)

We will take this as the upper bound on the fractional energy density of our warm/hot dark matter.

a. Hot axion from the flaton decay: Axions produced by the flaton decay have a current momentum

\[
p_a = \frac{a}{a_0} \frac{m_{\sigma\phi}}{2},
\] (103)

where \( a \) is the scale factor at the time they were created and \( a_0 \) is the scale factor now. Whereas, the current momentum of an axion produced at \( t_d \) is

\[
p_d = \frac{a_d}{a_0} \frac{m_{\sigma\phi}}{2} = \frac{S_d^{1/3} g_s^{1/3} (T_0) T_0}{S_t^{1/3} g_s^{1/3} (T_d) T_d} \frac{m_{\sigma\phi}}{2} \approx 1.48 \times 10^{-4} \, \text{eV} \left( \frac{m_{\sigma\phi}}{g_s^{1/3} (T_d) T_d} \right),
\] (104)

so it may be relativistic now. The current number density spectrum is given by

\[
p_a \frac{dn_{\text{hot}}}{dp_a} = \left( \frac{a}{a_0} \right)^3 \frac{2 \rho_{\phi} \Gamma_{\sigma\phi \rightarrow aa}}{m_{\sigma\phi} H} = 16 p_d^3 \frac{\Gamma_{\sigma\phi \rightarrow aa}}{p_a^3} \rho_{\phi},
\] (105)

which may provide an observational test of our model in the future. The energy density of the axions is

\[
\frac{\rho_a^{\text{hot}}}{\rho_{\text{SM}}} = \frac{g_*(T_d) g_s^{4/3} (T)}{g_*(T) g_s^{4/3} (T_d)} \frac{\Gamma_{\sigma\phi \rightarrow aa}}{\Gamma_{\sigma\phi \rightarrow \text{SM}}},
\] (106)

Therefore, assuming that the hot axions are still relativistic now, their current energy density is estimated as

\[
\Omega_a^{\text{hot}} \simeq 2 \times 10^{-5} \left( \frac{100}{g_*(T_d)} \right)^{\frac{4}{3}} \frac{\Gamma_{\sigma\phi \rightarrow aa}}{\Gamma_{\sigma\phi \rightarrow \text{SM}}}.
\] (107)

\* The energy density of thermally produced axions is \( \Omega_a \sim m_a/131\text{eV}, \) hence it will be subdominant \[42\].

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b. **Hot axion from the decay of the NLSP:** Because of the coupling (37), axions can be produced also by the decay of the NLSP, i.e. the heavier flatino $\psi_1$, which is originated from the decays of flaton and $\chi$. Neglecting axion mass, the current momentum of the axion from the NLSP*** is given by

$$ p_a = a \frac{m_{\psi_1}^2 - m_{\psi_0}^2}{2m_{\psi_1}}. \quad (108) $$

In particular, the current momentum of the axion produced at $t'_d = \Gamma_{\psi_1 \rightarrow \psi_0}^{-1}$ is

$$ p'_d = a'_d \frac{m_{\psi_1}^2 - m_{\psi_0}^2}{2m_{\psi_1}} = \frac{g_{sS}^{1/3}(T_0)T_0}{g_{sS}^{1/3}(T'_d)T'_d} \frac{m_{\psi_1}^2 - m_{\psi_0}^2}{2m_{\psi_1}} \approx 1.87 \times 10^{-4} \text{eV} \left(\frac{m_{\psi_1}}{g_{sS}^{1/3}(T'_d)T'_d} \right) \left(1 - \frac{m_{\psi_0}^2}{m_{\psi_1}^2}\right), \quad (109) $$

where $T'_d$ is the background temperature when the NLSP decays

$$ T'_d \sim \left(\frac{\pi^2}{90} g_{s}(T'_d)\right)^{-1/4} \left(\Gamma_{\psi_1 \rightarrow \psi_0} M_{Pl}\right)^{1/2} \sim 10 \text{eV} \left(\frac{\Gamma_{\psi_1 \rightarrow \psi_0}}{10^{-35} \text{GeV}}\right)^{1/2}. \quad (110) $$

In this case, the axion would thus be ultra-relativistic now for any plausible axion mass.

The current number density spectrum is

$$ p_a \frac{dn^\text{hot}_a}{dp_a} = \left(\frac{a}{a_0}\right)^3 \frac{2\rho_{\psi_1}}{m_{\psi_1}} \frac{\Gamma_{\psi_1 \rightarrow \psi_0}}{H} = \frac{16p^3_a}{m_{\psi_1}^4} \frac{\rho_{\psi_1}}{H} \frac{\Gamma_{\psi_1 \rightarrow \psi_0} p^3_a}{H p^3_d}, \quad (111) $$

which again may provide an observational test of our model in the future. The late time energy densities of the axions are

$$ \rho^\text{hot}_a \bigg|_{\text{flaton}} \simeq \left(\frac{g_{s}(T_d)g_{sS}^{4/3}(T)}{g_{s}(T)g_{sS}(T_d)g_{sS}^{1/3}(T'_d)}\right) \frac{T_d}{g_{sS}(T)} \frac{m_{\psi_1}}{T'_d} \frac{1}{m_{\psi_1}} \left.\rho_{\psi_1}\right|_{T=t'_d}, \quad (112) $$

$$ \rho^\text{hot}_a \bigg|_{\chi} \simeq \left(\frac{g_{sS}^{4/3}(T)}{g_{sS}(T)a_{T'}^{1/3}(T'_d)}\right) \frac{m_{\psi_1}}{T'_d} \frac{1}{m_{\psi_1}} \left.\rho_{\psi_1}\right|_{T=t'_d}, \quad (113) $$

where $(n_{\psi_1}/s)'_d$ is given by (86), and their current energy densities are

$$ \Omega^\text{hot}_a \bigg|_{\text{flaton}} \simeq 4 \times 10^{-5} \left(\frac{T_d}{g_{s}(T_d)}\right)^{\frac{4}{3}} \frac{m_{\psi_1} \Gamma_{\psi_1 \rightarrow \psi_0}}{T_d} \frac{\Gamma_{\psi_1 \rightarrow \psi_0}}{T'_d} \frac{1}{m_{\psi_1}} \left.\rho_{\psi_1}\right|_{T=t'_d} \frac{1}{H p^3_d}. \quad (114) $$

$$ \Omega^\text{hot}_a \bigg|_{\chi} \simeq 1.8 \times 10^{-4} \left(\frac{T_d}{g_{s}(T'_d)}\right)^{\frac{4}{3}} \frac{10^3}{g_{sS}^{1/4}(T_d)g_{sS}^{3/4}(T')} \frac{m_{\psi_1} \Gamma_{\psi_1 \rightarrow \psi_0} M_{Pl}}{T_d} \frac{1}{m_{\psi_1}^2} \left.\rho_{\psi_1}\right|_{T=t'_d} \frac{1}{H p^3_d}. \quad (115) $$

The energy densities in (107), (114) and (115) are typically well below the bound of (102) in our model.

*** We ignore the effect of NLSP’s momentum, since NLSPs from the decay of flatons and $\chi$ will be highly non-relativistic during most of its life.
c. Warm/hot LSP from the decay of NLSP: Eqs. (108) - (111) are applicable in this case too. Note that, although it is heavy and non-relativistic now, the LSP may be warm or even hot unless the NLSP ($\psi_1$) and LSP ($\psi_0$) are highly degenerate, and may have observable astrophysical effects. The late time energy density of the LSP is simply given by

$$\Omega_{\psi_0} = \frac{m_{\psi_0}}{m_{\psi_1}} \Omega_{\psi_1}$$

(116)

where $\Omega_{\psi_1}$ is the would-be energy density of the NLSP $\psi_1$ if it did not decay. Therefore, from (81), (89), (90), (102) and (118), $\Omega_{\psi_0} \lesssim \Omega_{\text{WHDM}}$ requires for NLSPs from flaton

$$m_{\psi_1} \lesssim 0.3 \text{GeV} \left[ \left( \frac{m_{\psi_1}}{m_{\psi_0}} \Gamma_{\psi_0}^{1/2} \right) \left( \frac{g_{0.4}(T_d)}{10} \right) \left( \frac{T_d}{1 \text{GeV}} \right) \left( \frac{10^{-1}}{10^{12} \text{GeV}} \right) \right]^{1/4},$$

(117)

while for those from $\chi$

$$T_d \lesssim \frac{m_{\chi}}{26} \left( \frac{g_{0.4}(T_d)}{A} \right)^{1/4} \left[ \frac{1}{A} \left( \frac{g_{0.4}(T_d)}{10^3} \right) \left( \frac{10^2 \text{GeV}}{m_{\chi}} \right) \left( \frac{1 \text{GeV}}{m_{\psi_0}} \right) \left( \frac{10^{12} \text{GeV}}{\varphi_0} \right)^2 \right]^{1/2},$$

(118)

where $\Gamma_{\varphi} \simeq \Gamma_{\psi_0}^{0.2} \text{SM}$ has been used.

V. CONCLUSION

Heavy gravitino with $m_{3/2} = \mathcal{O}(10) \text{ TeV}$ is a generic prediction of sequestered SUSY breaking scenario. String flux compactification provides a natural setup for sequestered SUSY breaking, i.e. SUSY breaking at the IR end of warped throat, and also can stabilize all moduli by fluxes or nonperturbative effects. The resulting moduli masses are much heavier than the gravitino mass, $m_\phi \sim 8\pi^2 m_{3/2}$ or even heavier, which might be considered as an attractive feature in view of the cosmological moduli problem. However such heavy moduli still cause a cosmological difficulty, the moduli-induced gravitino problem, producing too many LSPs which would overclose the Universe if the LSP is given by the MSSM neutralino. Another potential difficulty of sequestered SUSY breaking scenario is that it is not straightforward to get a weak scale size of the Higgs $\mu$ and $B$ parameters.

An attractive way to get a weak scale size of $\mu$ and $B$ in heavy gravitino scenario is to generate them by a singlet flat direction which is stabilized by radiative effects associated with SUSY breaking. Unless one assumes an unusual type of initial condition, such a flat direction generically triggers a late thermal inflation. Thermal inflation would immediately
solve the moduli-induced gravitino problem, however it requires a baryogenesis mechanism to work after thermal inflation is over.

In this paper, we have presented a model of thermal inflation in heavy gravitino scenario, which successfully incorporates the Affleck-Dine leptogenesis while producing a correct amount of relic dark matter density. The model involves two singlet flat directions stabilized by radiative effects, one direction that triggers thermal inflation and generates a weak scale size of $\mu$ and $B$, and the other direction that generates the scale of spontaneous lepton number violation. The dark matter is provided by the lightest flatino which might be identified as the axino if the model is assumed to have a $U(1)_{PQ}$ symmetry to solve the strong CP problem. The collider signal of the model highly depends on the flaton vacuum values $\langle \varphi \rangle$ ($\varphi = X, S$) which determine the rate of the decay of the lightest sparticle in the MSSM to the lighter flatinos. If $\langle \varphi \rangle$ are well above the intermediate scale $\sim 10^{10}$ GeV, the collider phenomenology of the model would be almost the same as the conventional MSSM. For $\langle \varphi \rangle \sim 10^{10}$ GeV or lower, the model can give a distinct signal associated with the decay of the lightest sparticle in the MSSM [10], and this will be the subject of future work [43].

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