How to End Password Reuse on the Web

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Abstract—We present a framework by which websites can coordinate to make it difficult for users to set similar passwords at these websites, in an effort to break the culture of password reuse on the web today. Though the design of such a framework is fraught with risks to users’ security and privacy, we show that these risks can be effectively mitigated through careful scoping of the goals for such a framework and through principled design. At the core of our framework is a private set-membership-test protocol that enables one website to determine, upon a user setting a password for use at it, whether that user has already set a similar password at another website, but with neither side disclosing to the other the password(s) it employs in the protocol. Our framework then layers over this protocol a collection of techniques to mitigate the leakage necessitated by such a test. These mechanisms are consistent with common user experience today, and so our framework should be unobtrusive to users who do not reuse similar passwords across websites (e.g., due to having adopted a password manager). Through a working implementation of our framework and optimization of its parameters based on insights of how passwords tend to be reused, we show that our design can meet the scalability challenges facing such a service.

I. INTRODUCTION

The reuse of passwords is the No. 1 cause of harm on the internet.

– Facebook CSO Alex Stamos

Password reuse across websites remains a dire problem despite widespread advice for users to avoid it. Numerous surveys and studies over the past fifteen years indicate that 75%–93% of users set the same or similar passwords across different websites (e.g., [2], [3], [4], [5], [6], [7]). As such, breaches of a password database or a successful phish of a user’s password can often lead to the compromise of user accounts on other websites. Such “credential-stuffing” attacks are a primary cause of account takeovers [8], [9], allowing the attacker to drain stolen accounts of stored value, credit card numbers, and other personal information [9]. Moreover, this problem seems to be on the rise, with incidences of such attacks rising by 35% in the first quarter of 2016 alone [10]. Somewhat ironically, increasingly stringent password requirements contribute to password reuse, as users reuse strong passwords across websites to cope with the cognitive burden of creating and remembering them [11].

It is tempting to view password reuse as a mistake whose costs are borne only by those users who do, in fact, reuse passwords. However, preventing, detecting, and cleaning up compromised accounts and the value thus stolen is a significant cost for service providers, as well. A recent Ponemon survey [12] of 569 IT security practitioners who are responsible for the security of their companies’ websites estimated that credential-stuffing attacks incur costs in terms of application downtime, loss of customers, and involvement of IT security that average $1.7 million, $2.7 million and $1.6 million, respectively, per organization per year. Some companies go so far as to purchase compromised credentials on the black market to find vulnerable accounts proactively, either as a for-pay service to others (e.g., PasswordPing, https://www.passwordping.com) or for their own users (e.g., Facebook [1]). Beyond monitoring the black market, sites also must develop new technologies to identify overtaken accounts based on their use [1]. Even the sheer volume of credential-stuffing attacks is increasingly a challenge; e.g., in November 2017, a stunning 43% (3.6 out of 8.3 billion) of all login attempts served by Akamai involved credential abuse [13]. Finally, the aforementioned Ponemon survey estimated the fraud perpetrated using overtaken accounts could incur average losses of up to $54 million per organization surveyed [12]. As such, interfering with password reuse would not only better protect users, but would also reduce the considerable costs of credential abuse incurred by websites.

We thus argue in this paper that a technical mechanism to prevent password reuse across websites warrants consideration, and we propose and evaluate one such design here. At a high level, the framework we explore enables a server at which a user is setting a password, here called a requester, to ask of other websites, here called responders, whether the user has set a similar password at any of them. A positive answer can then be used by the requester to ask the user to select a different password (perhaps after suggesting alternatives that are likely to be different from what the user might have set elsewhere). As we will argue in Section III enlisting a surprisingly small number of major websites in our framework could substantially weaken the culture of password reuse.

We are under no illusions that our design, were it deployed, will be met with anything but contempt (at least temporarily) by the many users who currently reuse passwords at multiple websites. In this respect, the usability implications of our proposal are not unlike increasingly stringent password requirements, to which users have nevertheless resigned. However, options for password managers are plentiful and growing, with a variety of trustworthiness, usability, and cost properties (e.g., [14], [15]). Indeed, experts often list the use of a password manager that supports a different password per
website to be one of the best things a user can do to reduce her online risk [6]. While there might be a category of users who, despite having a rich online presence, cannot make use of a password manager for some reason, we expect those users to be few. And, of course, the old-fashioned practice of just writing down passwords should be available to nearly everyone who is capable of using a computer, as a last resort. Though historically maligned, the practice of writing down passwords is now more widely accepted, exactly because it makes it easier to not reuse passwords (e.g., [16], [17]).

Setting aside the debate of whether to prevent password reuse, there are many technical issues that need to be addressed to make a framework like the one we propose palatable. First, such a framework should not reduce the security of user accounts. Second, the framework should also not decay user privacy substantially, in the sense of divulging the websites at which a user has an account. Third, it is important that the protocol run between a requester and responders should scale well enough to ensure that it does not impose too much delay for setting a password at a website. As we will see, meeting these requirements is very challenging.

Our framework addresses these challenges as follows. To minimize risk to user accounts, we design a protocol that enables the requester to learn if a password chosen by a user is similar to one she set at a responder; neither side learns the password(s) the other input to the protocol, however, even by misbehaving. Our framework leverages this protocol, together with other mechanisms to compensate for leakage necessitated by the protocol’s output, to ensure that account security and privacy is not diminished. Among other properties, this framework ensures that the responders remain hidden from the requester and vice-versa. Scalability is met in our framework by carefully designing it to involve only a single round of interaction between the requester and responders. And, using observations about password reuse habits, we optimize our framework to detect similar password use with near-certainty while maximizing its scalability.

To summarize, our contributions are as follows:

- We initiate debate on the merits of preventing password reuse on the web, through coordination among websites. Our goal in initiating this debate is to question the zeitgeist in the computer security community that password reuse cannot be addressed by technical means without imposing unduly on user security or privacy. In particular, we show that apparent obstacles to a framework for interfering with password reuse across websites can be overcome through careful scoping of its goals and through reasonable assumptions (Section III).

- We propose a protocol for privately testing set membership that underlies our proposed framework (Section IV). We prove security of our protocol in the case of a malicious requester and against malicious responders (Appendix A).

- We embed this protocol within a framework to facilitate requester-responder interactions while hiding the identities of protocol participants and addressing risks that cannot be addressed by—and indeed, that are necessitated by—the private set-membership-test protocol (Section V). We evaluate implementations of our proposed framework with differing degrees of trust placed in it (Section VI and Appendix B).

- Using password-reuse tendencies, we illustrate how to configure our framework to minimize its costs while ensuring detection of reused passwords with high likelihood (Section VI). Finally, we demonstrate its scalability through experiments with a working implementation in deployments that capture its performance in realistic scenarios.

II. RELATED WORK

Despite the extent of password reuse, we are aware of no prior work to actively interfere with password reuse by the same user at multiple websites. Instead, approaches that implicitly or explicitly coordinate across sites to mitigate risks due to password reuse have set somewhat different goals.

Web single sign-on: Roughly speaking, web single sign-on (SSO) schemes such as OAuth (https://oauth.net/), OpenID (http://openid.net), OpenID Connect (http://openid.net/connect), and Facebook Login (https://developers.facebook. com/docs/facebook-login/), enable one website (an “identity provider”) to share a user’s account information with other websites (“relying parties”), typically in lieu of the user creating distinct accounts at those relying parties. As such, this approach mitigates password reuse by simply not having the user set passwords at the relying parties. However, if a user’s identity-provider account is compromised or unavailable, or if she simply chooses to close her identity-provider account, then the security or availability of her relying-party accounts can be impacted. In addition, the identity provider in these schemes typically learns the relying parties visited by the user [18]. Such issues have slowed their adoption [19].

Detecting leaked passwords: As discussed in Section I, some companies cross-reference account passwords against leaked passwords sold on the black market, either as a service to others or for their own users. This approach proactively detects risk to shared-password accounts that arises when one of those accounts is compromised. Other approaches detect attempts to reuse compromised passwords at honey accounts on other sites (e.g., [20]), thereby reactively detecting risk to accounts. Whether proactive or reactive, detecting stolen passwords is primarily useful if at-risk accounts at other sites can be located and protected, ideally before they are breached. In contrast, our goal is to discourage password reuse in the first place and, indeed, to eliminate the culture of password reuse altogether.

Prohibiting popular passwords: Schechter et al. [21] proposed a service by which sites could check whether a password chosen by a user is popular with other users or, more specifically, if its frequency of use exceeds a specified threshold. Our goals here are different—we seek to detect the use of similar
passwords by the same user at different sites, regardless of popularity.

Limiting password-based access: Takada\cite{22} proposed to interfere with the misuse of accounts with shared passwords by adding an “availability control” to password authentication. In this design, a user disables the ability to log into her website account at a third-party service and then re-enables it when needed. This approach requires that the attacker be unable to itself enable login, and so requires an additional authentication at the third-party service to protect this enabling.

Our approach is complementary to the above approaches to addressing password reuse, in the sense that it can be used alongside any or all of them. Unlike the above approaches, ours seeks to prevent the reuse of passwords by the same user across different websites, thereby strengthening the security of each account.

III. GOALS AND ASSUMPTIONS

In this section we seek to clarify the goals for our systems and the assumptions on which our design rests.

A. Deployment Goals

It is important to recognize that in order to break the culture of password reuse, we do not require universal adoption of the framework we propose here. Instead, it may be enough to enlist a (surprisingly small) number of top websites. To see this, consider just the 20 websites listed in Table I. For a back-of-the-envelope estimate, suppose that the users of each website in Table I are sampled uniformly at random from the 3.58 billion global Internet users. Then, in expectation an Internet user would have accounts at more than four of them. As such, if just these websites adopted our framework, it would force a large fraction of users to manage five or more dissimilar passwords, which is already at the limit of what users are capable of managing themselves: “If multiple passwords cannot be avoided, four or five is the maximum for unrelated, regularly used passwords that can be expected to cope with”\cite{23}. We thus believe that enlisting these 20 websites could already dramatically improve password-manager adoption, and it is conceivable that with modest additional adoption (e.g., the top 50 most popular websites), password reuse could largely be brought to an end.

A user who resists adopting a password manager will presumably continue using similar passwords across sites that do not participate in our framework. Each such reused password may also be similar to one she set at a site that does participate in our framework, but likely only one such site. If this reused password is compromised (e.g., due to a site breach), then the

| Website | Users (M) | Website | Users (M) |
|---------|----------|---------|----------|
| Facebook | 2167 | Taobao | 580 |
| YouTube | 1500 | Outlook | 400 |
| WhatsApp | 1300 | Sina Weibo | 376 |
| Yahoo | 1000 | Twitter | 330 |
| Gmail | 1000 | Amazon | 310 |
| WeChat | 980 | Baidu Tieba | 300 |
| QQ | 843 | LinkedIn | 260 |
| Instagram | 800 | Snapchat | 255 |
| Tumblr | 794 | Reddit | 250 |
| iCloud | 782 | Pinterest | 200 |

TABLE I: Websites and counts of active users for the estimates of Section III-A

attacker, acting as a requester in our framework, might be able (subject to limitations imposed by mechanisms described in Section V-A1) to confirm that this password is similar to that set by the same user at some participating site, but not which site (see Section III-C). If so, the attacker would need to try this password at the user’s account blindly at participating sites—essentially the same credential-stuffing attack that he would mount today in the absence of our framework. However, with the adoption of our framework, this attack would succeed at only one participating site, not many.

B. User Identifiers

A key assumption of our framework is that there is an identifier for a user’s accounts that is common across the requester and responders. An email address for the user would be a natural such identifier, and as we will describe in Section V-A1 this has other uses in our context, as well. Due to this assumption, however, a user could reuse the same password across different websites, despite our framework, if she registers a different email address for each of her accounts at those websites.

Several methods exist for a user to amass many distinct email addresses, but we believe they will interfere little with our goals here. First, some email providers support multiple addresses for a single account. For example, one Gmail account can have arbitrarily many addresses, since Gmail addresses are insensitive to capitalization, insertion of periods (‘.’), or insertion of a plus (‘+’) followed by any string, anywhere before ‘@gmail.com’. As another example, 33mail allows a user to receive mail sent to <alias>@<username>.33mail.com for any aliases string. Though these providers enable a user to provide a distinct email address to each website (e.g., 24), our framework could nevertheless extract a canonical identifier for each user. For Gmail, the canonical identifier could be obtained by normalizing capitalization and by eliminating periods and anything between ‘+’ and ‘@gmail.com’. For 33mail, you@<username>.33mail.com should suffice. Admittedly this requires customization specific to each such provider domain, though this customization is simple.

Second, various hosting services permit a customer to register a domain name and then support many email aliases

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1 User counts were retrieved on April 14, 2018 from https://www.statista.com/statistics/272014/global-social-networks-ranked-by-number-of-users/

http://blog.shuttlecloud.com/the-most-popular-email-providers-in-the-world/

https://expandedramblings.com/index.php/yahoo-statistics/

https://expandedramblings.com/index.php/taobao-statistics/

and https://www.statista.com/statistics/476196/number-of-active-amazon-customer-accounts-quarter/.

2 Estimate of Internet users was retrieved from https://www.statista.com/statistics/273018/number-of-internet-users-worldwide/ on April 14, 2018.
for that domain (e.g., <alias>@<domain>.com). For example, Google Domains (http://domains.google) supports 100 email aliases per domain. Since these domains are custom, it might not be tractable to introduce domain-specific customizations as above. However, registering one’s own domain as a workaround to keep using the same password across websites presumably saves the user little effort or money (registering domains is not free) in comparison to just switching to a password manager. Going further, a user could manually register numerous email accounts at free providers such as Gmail. Again, this is presumably comparable or more effort than alternatives that involve no password reuse. As such, we do not concern ourselves with such methods of avoiding password reuse detection.

This discussion highlights an important clarification regarding our goals: we seek to eliminate easy methods of reusing passwords but not ones that require similar or greater effort from the user than more secure alternatives, of which we take a password manager as an exemplar. That is, we do not seek to make it impossible for a user to reuse passwords, though we do seek to make reusing passwords about as difficult as not reusing them. We expect that even this modest goal, if achieved, will largely eliminate password reuse, since passwords are reused today almost entirely for convenience.

C. Security and Privacy Goals

The goals we take as more absolute have to do with the privacy of users and ensuring that our method of detecting password reuse does not substantially weaken users’ accounts. Specifically, we seek to ensure the following:

- **account location privacy**: Websites do not learn the identities of other websites at which a user has set up an account.
- **account security**: Our framework strengthens security of user accounts at a site that participates in our framework (as a requester and responder), by interfering with reuse of similar passwords at other participating sites. Moreover, it does not qualitatively degrade user account security in other ways.

As we will see, **account security** is difficult to achieve, since our framework must expose whether responders’ passwords are similar to the one chosen by the user at the requester. However, **account location privacy** hides from the requester each responder from which the requester learns this information. As such, if a user attempts to set the same password at a malicious requester that she has also set at some responder, or if a malicious requester otherwise obtains this password (e.g., obtaining it in a breach of a non-participating site), the malicious requester must still attempt to use that password blindly at participating websites, just as in a credential-stuffing attack today. (The attacker might succeed, but it would succeed without our framework, too.) Moreover, in Section V.A.I we will detail additional defenses against this leakage to further reduce the risk of attacks from malicious requesters to whom the user does not volunteer this password.

We conclude this section by summarizing some potential goals that we (mostly) omit from consideration in this paper. From a privacy perspective, we try to hide neither when a password is being set at some requester for an account identifier nor the number of responders at which an account has been established using that account identifier, simply because we are unaware of common scenarios in which these leakages would have significant practical ramifications. And, while we strive to guarantee **account location privacy** and **account security** even against a requester and responders that misbehave, we generally do not seek to otherwise detect that misbehavior. So, for example, each requester and responder has complete autonomy in determining the passwords that it provides to the protocol as the candidate password submitted by the user and the passwords similar to the one for the account with the same identifier, respectively. As we will see in Section VII, such misbehaviors can give rise to denial-of-service opportunities, for which we propose remedies there.

IV. Privately Testing Set Membership

A building block of our framework is a protocol by which a requester can inquire with a responder as to whether a password \( \pi \) chosen for an account identifier is similar to one already in use at some responder for the same identifier. If for an account identifier \( a \), the responder \( S \) has a set \( P(a) \) of passwords similar to that already set at \( S \), then the goal of this protocol is for the requester to learn whether the candidate password \( \pi \) is in \( P(a) \). However, any additional information leakage to the requester (about any passwords in \( P(a) \) or even the number of passwords in \( P(a) \)) or to the responder (about \( \pi \)) should be minimized.

This general specification can be met with a private set-membership-test protocol. Though several such protocols exist (e.g., [25], [26], [27], [28]), we develop a new one here with an interaction pattern and threat model that is better suited for our framework. In particular, existing protocols require special hardware [27] or more rounds of interaction [25], [26], or leak more information in our threat model [25], [26], [28] than the one we present. Our protocol is built on principles similar to a set-intersection cardinality protocol due to Egert et al. [29] Section 4.4], though we (i) reduce the information it conveys to only the results of a membership test, versus the cardinality of a set intersection, and (ii) analyze its privacy properties in the face of malicious behavior by a requester or responder (versus only an honest-but-curious participant in their work), accounting for leakage intrinsic in the application for which we use it here.

A. Partially Homomorphic Encryption

Our protocol builds upon an IND-CPA-secure [30] multiplicatively homomorphic encryption scheme \( \mathcal{E} = \langle \text{Gen}, \text{Enc}, \text{Dec}, \times_{\{1\}} \rangle \) with the following algorithms. Below, \( z \leftarrow Z \) denotes random selection from set \( Z \) and assignment to \( z \), and \( Y \overset{\$}{\leftarrow} Y' \) denotes that random variables \( Y \) and \( Y' \) are distributed identically.
Gen is a randomized algorithm that on input $1^k$ outputs a public-key/private-key pair $(pk, sk) \leftarrow \text{Gen}(1^k)$. The value of $pk$ uniquely determines a plaintext space $\mathbb{G}$ where $\langle \mathbb{G}, \times \rangle$ denotes a multiplicative, cyclic group of order $r$ with identity $1_\mathbb{G}$, and where $r$ is a $k$-bit prime. The randomized function $\$G(\mathbb{G})$ returns a new, random $m \leftarrow \mathbb{G}$. $Z_r = \{0, \ldots, r-1\}$ and $Z_r^* = \{1, \ldots, r-1\}$, as usual.

Dec is a deterministic algorithm that on input a private key $sk$ and ciphertext $c \in C(pk) (m)$, for $m \in \mathbb{G}$ and $pk$ the public key corresponding to $sk$, produces $m \leftarrow \text{Dec}_sk(c)$. If $c \notin C(pk)$, then $\text{Dec}_sk(c)$ returns \( \bot \).

$\times [\cdot]$ is a randomized algorithm that on input a public key $pk$ and ciphertexts $c_1 = C(pk)(m_1)$ and $c_2 = C(pk)(m_2)$ produces a ciphertext $c \leftarrow c_1 \times \times pk c_2$ chosen uniformly at random from $C(pk)(m_1m_2)$. If $c \notin C(pk)$ or $c_2 \notin C(pk)$, then $c_1 \times \times pk c_2$ returns $\bot$. We use $\prod_{i=1}^n$ and $\exp_{pk}$ to denote multiplication of a sequence and exponentiation using $\times pk$, respectively, i.e.,

$$\prod_{i=1}^n c_i \overset{d}{=} c_1 \times \times pk c_2 \times \times pk \ldots \times pk c_z$$

$$\exp_{pk}(c, z) \overset{d}{=} \prod_{i=1}^n c$$

B. Protocol Description

Our protocol is shown in Figure 1 with the actions by the requester $R$, with $R$ listed on the left (lines 1–6), and by the responder $S$ listed on the right (8–24), and messages between them in the middle (7). In Figure 1 and below, $[z]$ for integer $z > 0$ denotes the set $\{0, \ldots, z-1\}$.

At a conceptual level, our private set-membership-test protocol works as follows. The requester $R$ takes as input an account identifier $a$, the user’s chosen password $\pi$, a Bloom-filter length $\ell$, and the hash functions $(h_i)_{i \in [k]}$ for the Bloom filter (i.e., each $h_i : \{0, 1\}^* \rightarrow [\ell]$). $R$ computes its Bloom filter containing $\pi$, specifically a set of indices $J_R \leftarrow \bigcup_{i \in [k]} \{h_i(\pi)\}$ (line 10). The responder $S$ receives as input a set $P(a')$ of passwords similar to the password for each local account $a' \in A$ (i.e., $A$ is a set of local account identifiers), and upon receiving message $m_1$ computes its own $\ell$-sized Bloom filter containing $P(a)$, i.e., indices $J_S \leftarrow \bigcup_{a \in P(a)} \bigcup_{i \in [k]} \{h_i(\pi')\}$ (line 19). The protocol should return $true$ to $R$ if $\pi \in P(a)$, which for a Bloom filter is indicated by $J_R \subseteq J_S$ (with some risk of false positives, as will be discussed in Section V-B).

Our protocol equivalently returns a value to $R$ that indicates whether $[\ell] \setminus J_S \subseteq [\ell] \setminus J_R$, without exposing $J_S$ to $R$ or $J_R$ to $S$. To do so, the requester $R$ encodes $J_R$ as ciphertexts $c_j \leftarrow C(pk)(1_\mathbb{G})$ if $j \in [\ell] \setminus J_R$ and $c_j \leftarrow C(pk)(m)$ for a randomly chosen $m \leftarrow \mathbb{G}$ if $j \in J_R$ (23). In this way, when $S$ computes $\rho$ in line 24—i.e., by homomorphically multiplying $c_j$ for each $j \in [\ell] \setminus J_S$ and then exponentiating by a random $z \leftarrow Z_r^*$ $\rho \leftarrow \exp_{pk}\left(\prod_{j \in [\ell] \setminus J_R} c_j \right) \cdot z$ (23) —is in $C(pk)(1_\mathbb{G})$ if $[\ell] \setminus J_S \subseteq [\ell] \setminus J_R$ and otherwise is almost certainly not in $C(pk)(1_\mathbb{G})$. As such, $R$ returns $true$, indicating that $\pi$ is similar to the password set at $S$ for account $a$, if and only if $\text{Dec}_{sk}(\rho) = 1_\mathbb{G}$ (25).

It is important that both $S$ and $R$ check the validity of the ciphertexts they receive (lines 21 and 34) respectively. For $S$, implicit in this check is that $pk$ is a valid public key (i.e., capable of being output by Gen). For our implementation described in Section V-B, these checks are straightforward.

C. Security

We now reason about the security offered by the protocol of Figure 1 against malicious requesters (Section IV-C.1) and against malicious responders (Section IV-C.2). More specifically, our focus in this section is properties that underlie account security as informally described in Section III account location privacy will be discussed in Section V All proofs of propositions in this section are in Appendix A.

1) Security against malicious requester: A malicious requester learns nothing more from executing the protocol in Figure 1 besides the result $m \overset{\circ}{=} 1_\mathbb{G}$ in line 6 where no other information is encoded in $\rho$ if the responder follows the
protocol (i.e., unconditional security). First, if \( \rho \not\in C_{pk}(1_G) \) then \( \rho \) is a ciphertext of any \( m \in G \setminus \{1_G\} \) with equal probability:

**Proposition 1.** If the responder follows the protocol, then
\[
P(\rho \in C_{pk}(m) \mid \rho \not\in C_{pk}(1_G)) = \frac{1}{|G|} \text{ for any } m \in G \setminus \{1_G\}.
\]

Second, if \( \rho \in C_{pk}(m) \), then it is uniformly distributed in \( C_{pk}(m) \):

**Proposition 2.** If the responder follows the protocol, then
\[
P(\rho = c \mid \rho \in C_{pk}(m)) = \frac{1}{|C_{pk}(m)|} \text{ for any } m \in G \text{ and any } c \in C_{pk}(m).
\]

2) **Security against malicious responder:** The system of which the protocol in Figure 1 is a component will typically leak the result of the protocol run to the responder. Specifically, if a run of the protocol is immediately followed by another run of the protocol, then this is an indication that the protocol returned true, i.e., that \( \pi \in P(a) \). As such, for the purposes of this section, we will assume that the result of the protocol is leaked to the responder reliably.

The implications of this leakage to the requirements for the encryption scheme \( E \) are that the requester serves as an oracle for the responder to learn whether a ciphertext \( \rho \) of its choosing satisfies \( \rho \in C_{pk}(1_G) \). However, the requester does so for only a single ciphertext \( \rho \), in that the public key \( pk \) changes every time the protocol is run (line 11). Still, the responder could potentially use this oracle to determine which of the ciphertexts \( \{c_j\}_{j \in [\ell]} \) that it receives in line 11 satisfy \( c_j \in C_{pk}(1_G) \) and, in turn, gain information about the password \( \pi \) that the user is trying to set. Indeed, some leakage of this form is unavoidable; e.g., the responder could simply set \( \rho = c_0 \) and, in doing so, learn whether \( c_0 \in C_{pk}(1_G) \). Similarly, the responder could set \( \rho = c_0 \times pk c_1 \); if the protocol returns true, then the responder can conclude with near certainty that both \( c_0 \in C_{pk}(1_G) \) and \( c_1 \in C_{pk}(1_G) \).

To capture this leakage and the assumptions underlying our protocol more formally, we define a responder-adversary \( B \) to be a pair \( B = \langle B_1, B_2 \rangle \) of probabilistic algorithms. \( B_1 \) takes as input \( pk \) and \( \{c_j\}_{j \in [\ell]} \) and outputs a ciphertext \( \rho \) and a state \( \phi \); \( B_2 \) is provided the oracle response (i.e., whether \( \rho \in C_{pk}(1_G) \)) and the state \( \phi \) and then outputs a set \( J_B \subseteq [\ell] \). \( B \) is said to succeed if \( J_B = J_R \), where \( J_R \) is the set of indices the requester “set” in its Bloom filter by encrypting a random group element (line 13). More specifically, we define the experiment \( \text{Expt}_E^S(B_1, B_2) \) as follows:

**Experiment \( \text{Expt}_E^S(B_1, B_2) \):**
\[
\langle pk, sk \rangle \leftarrow \text{Gen}(1^n) \\
\mathbb{J}_R \leftarrow \{ J \subseteq [\ell] \mid |J| = k \} \\
\forall j \in [\ell] : c_j \leftarrow \begin{cases} \text{Enc}_{pk}(S(G)) & \text{if } j \in J_R \\ \text{Enc}_{pk}(1_G) & \text{if } j \not\in J_R \end{cases} \\
\langle \rho, \phi \rangle \leftarrow B_1(\langle pk, \{c_j\}_{j \in [\ell]} \rangle) \\
J_B \leftarrow B_2(\langle \phi, \rho \not\in C_{pk}(1_G) \rangle) \\
\text{return } (J_B \neq J_R)
\]

Then, we analyze the security of our protocol against responder-adversaries \( B \) that run in time polynomial in \( k \) by bounding \( P(\text{Expt}_E^S(B) = \text{true}) \).

**ElGamal encryption:** In order to better analyze the security of the protocol in Figure 1 we instantiate the encryption scheme \( E \). We do so with ElGamal encryption [31], which is implemented as follows.

- \( \text{Gen}(1^n) \) returns a private key \( sk = \langle u \rangle \) and public key \( pk = \langle g, U \rangle \), where \( u \leftarrow \mathbb{Z}_k \), \( g \) is a generator of the (cyclic) group \( \langle G, \times_G \rangle \), and \( U \leftarrow g^u \). We leave it implicit that the public key \( pk \) and private key \( sk \) must include whatever other information is necessary to specify \( G \), e.g., the elliptic curve on which the members of \( G \) lie.
- \( \text{Enc}_{(g,U)}(m) \) returns \( \langle V, W \rangle \) where \( V \leftarrow g^u \), \( v \leftarrow \mathbb{Z}_k \), and \( W \leftarrow mU^v \).
- \( \text{Dec}_{(u)}(\langle V, W \rangle \rangle \) returns \( VW^{-u} \) if \( \{V, W \} \subseteq G \) and returns \( \perp \) otherwise.
- \( \langle V_1, W_1 \rangle \times_{(g,U)} \langle V_2, W_2 \rangle \) returns \( \langle V_1V_2g^u, W_1W_2U^v \rangle \) for \( y \leftarrow \mathbb{Z}_k \) if \( \{V_1, V_2, W_1, W_2 \} \subseteq G \) and returns \( \perp \) otherwise.

**Generic group model:** We prove the security of our protocol against a responder-adversary \( B \) in the generic group model as presented by Maurer [32]. The generic group model allows modeling of attacks in which the adversary \( B \) cannot exploit the representation of the group elements used in the cryptographic algorithm. For some problems, such as the discrete logarithm problem on general elliptic curves, generic attacks are currently the best known (though better algorithms exist for curves of particular forms, e.g., [33]). Somewhat like the random oracle model [34], the generic group model is idealized, and so even an algorithm proven secure in the generic group model can be instantiated with a specific group representation that renders it insecure. Still, and also like the random oracle model, it has been used to provide assurance for the security of designs in numerous previous works; e.g., see Koblitz and Menezes [35] for a discussion of this methodology and how its results should be interpreted.

A function \( f : \mathbb{N} \rightarrow \mathbb{R} \) is said to be **negligible** if for any positive polynomial \( \phi(\kappa) \), there is some \( \kappa_0 \) such that \( f(\kappa) < \frac{1}{\phi(\kappa)} \) for all \( \kappa > \kappa_0 \). We denote such a function by \( \text{negl}(\kappa) \).
Proposition 3. If $E$ is ElGamal encryption, then in the group model,

$$\Pr \left( \text{Expt}_E^B(B = \text{true}) \leq 2 \left( \frac{\ell}{k} \right)^{-1} + \text{negl}(\kappa) \right)$$

for any responder-adversary $B$ that runs in time polynomial in $\kappa$.

The proof of Proposition 3 is in Appendix A. The bound in Proposition 3 is tight, in the sense that there is a generic responder-adversary that achieves it (to within a term negligible in $\kappa$). This adversary $B = \langle B_1, B_2 \rangle$ performs as follows: $B_1$ outputs, say, $\rho \leftarrow c_0$ and, upon learning $\rho \in C_{pk}(1_\ell)$, $B_2$ guesses $J_B$ to be a $k$-element subset of $[\ell]$ containing 0 iff $\rho \notin C_{pk}(1_\ell)$.

V. PREVENTING PASSWORD REUSE

In this section, we introduce how to build a password reuse detection framework based on the private set-membership-test protocol introduced in Section IV.

A. Design

Our password reuse detection framework enables a requester $R$ to inquire with multiple responders as to whether the password $\pi$ chosen by a user for the account at $R$ with identifier $a$ is similar to another password already set for $a$ at some responder. The requester does so with the help of a directory, which is a (possibly replicated) server that provides a front-end to requesters for this purpose. The directory stores, per identifier $a$, a list of addresses (possibly pseudonymous addresses, as we will discuss below) of websites at which $a$ has been used to set up an account. We stress that the directory does not handle or observe passwords in our framework.

The requesters and responders need not trust each other in our framework, and employ the protocol described in Section IV to interact via the directory. More specifically, a user for the account at $R$ with identifier $a$ is similar to another password already set for $a$ at some responder. The requester does so with the help of a directory, which is a (possibly replicated) server that provides a front-end to requesters for this purpose. The directory stores, per identifier $a$, a list of addresses (possibly pseudonymous addresses, as we will discuss below) of websites at which $a$ has been used to set up an account. We stress that the directory does not handle or observe passwords in our framework.

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hash function $H$, making $P(a)$ more costly to exploit if the site is breached \[42\]. In particular, this hash function need not be the same as that used to hash the real password during normal login, and so can be considerably more time-consuming. Going further, the responder could proactively generate the set $J_S$ when the password for $a$ is set at $S$, and dispense of $P(a)$ altogether. However, if the Bloom filter size $\ell$ selected by the requester is not fixed and known to the responder in advance (see Section \[V-C\]), then the responder would need to store multiple sets $J_S$ reflecting differently sized $P(a)$ sets.

**Protecting $J_S$ from disclosure:** As shown in Section \[IV-C\] the only information leaked to the requester is the result of the protocol in Figure \[1\] i.e., $\rho \in C_{pk}(1_G)$. Regardless of the behavior of the requester (Propositions \[1\][2]). Still, however, this information can erode the security of $J_S$ for account $a$ over multiple queries. For example, if a malicious requester selects $c_j \sim \text{Enc}_{pk}(m)$ for one Bloom-filter index $j$ and $c_{j'} \sim \text{Enc}_{pk}(1_G)$ for $j' \neq j$, then the result of $m \sim 1_G$ reveals whether $j \in J_S$. After $\ell$ such queries, the requester can learn the entirety of $J_S$ and then search for the items stored in the Bloom filter offline.

The risk of such an attack can be largely mitigated by involving the user. For example, if $a$ is an email address or can be associated with one at the directory, then the directory can email the user upon being contacted by a requester, to confirm that she is trying to (re)set her password at some website. This email could be presented to the user much like an account setup confirmation email today, containing a URL at the directory on which the user could click to confirm her attempt to (re)set her password. The directory would simply hold message $m_1$ or each message $m_2$ until receiving this confirmation, discarding the message if it times out awaiting the user confirmation. (Presumably the requester website would alert the user to check her inbox for this confirmation email.) To avoid requiring the user to click confirmation URLs over the course of multiple attempts to select a password and so multiple runs of the protocol in Figure \[1\] (which should occur only if the user is still not using a password manager), the directory could allow one such confirmation to permit protocol runs for a short window of time (e.g., one minute), at a risk of allowing a few extra queries from a malicious requester. However, except during this window, attempts by malicious requesters to query responders will be dropped by the directory. For a requester at which a user $a$ has no account, such queries to responders will be dropped except when concurrent with password (re)set attempts for $a$ at another requester, which presumably would be difficult to anticipate.

**Limiting utility of a $J_S$ disclosure:** The risk that the adversary finds the password, even with $J_S$, is small if the user is already using a password manager, since state-of-the-art password managers can generate and auto-fill passwords that resist even an offline attack. Even if the user is not already using a password manager, obtaining the account password using this attack should again be difficult if the cryptographic hash function $H$ is costly to compute. Moreover, the attacker can utilize a guessed account password only if it can determine the responder $S$ at which it is set for account $a$, which it is prevented from doing by account location privacy.

Still, to counter this risk for those not yet using a password manager and just in case the attacker finds $S$, we advocate that $S$ form its set $P(a)$ to include honey passwords \[43, 44, 45\]. That is, when the password is set (or reset) at a website for account $a$, a collection of $d$ honey passwords should be chosen, as well, via a state-of-the-art method of doing so. Similar passwords should then be generated for each of the $d+1$ passwords (the real password and the honey passwords) using an identical algorithm, and the similar passwords for each should together be used to constitute $P(a)$. In this way, even if the attacker learns the entire contents of $J_S$ for a responder $S$, the set $J_S$ will contain at least $d+1$ passwords that appear to be roughly equally likely. If any of the honey passwords are then used in an attempt to log into the account, the website can lock the account and force the user to reset her password after successfully authenticating via a fallback method. The primary cost of using honey passwords is a linear-in-$d$ growth in the size of $P(a)$, which linearly reduces the number of passwords similar to the real password that can be accommodated by the Bloom-filter size $\ell$ (which is determined by the requester). We will show in Section \[VI\] however, that this cost has little impact on preventing password reuse.

2) **Security for the requester:** Security for the requester in our framework is more straightforward, given Proposition\[3\] that proves the privacy of $J_R$ against a malicious responder (and from the directory) in the generic group model. Moreover, the requester’s identity is hidden from responders either by the directory (in the trusted-directory model) or because the requester contacts the directory anonymously (in the untrusted-directory model).

As discussed in Section \[IV-C\] and accounted for in Proposition\[3\] each responder (and the directory) learns the outcome of the protocol (i.e., $\rho \in C_{pk}(1_G)$), since it sees whether the requester runs the protocol again. That is, a true result will presumably cause the requester to reject the password and ask the user for another, with which it repeats the protocol. Because the password is changed each time, the information gained by each responder does not accumulate over these multiple runs.

Still, if the false result returned for the password $\pi$ finally accepted at the requester is of concern, it is possible to hide even this information, at some extra expense. To do so, the requester follows the acceptance of $\pi$ with a “decoy” run of the protocol if necessary to ensure that two runs of the protocol are conducted (i.e., only if $\pi$ was the first password offered by the user), and then with another decoy run with probability $\frac{1}{2}$. The requester ignores the result of each decoy run, and the user need not be delayed while it is conducted. The decoy(s) hide whether the second-to-last run of the protocol pertains to
the password \( \pi \) that the requester finally accepted. That said, because these decoy runs add overhead and, again, because the responder is limited to learn information about \( \pi \) in only a single protocol run (and to learn a limited amount, per Proposition 3), we do not consider decoys further here.

B. Implementation

We built a prototype implementation of our framework to evaluate its performance and scalability, and to inform its parameterization (see Section VI). We realized the cryptographic part of our protocol in C and other parts using the Go language.

1) Cryptography: Our implementation uses the ElGamal cryptosystem in an elliptic-curve group (EC-ElGamal) as the underlying multiplicatively homomorphic scheme \( \mathcal{E} \) leveraged in Figure 1 owing to its shorter keys and ciphertexts for comparable security in comparison to ElGamal over other groups [46]. We realized all cryptographic operations using MIRACL [https://github.com/miracl/MIRACL], an efficient C software library for elliptic-curve cryptography.

Our implementation includes four standardized elliptic curves: secp160r1, secp192r1 (NIST P-192), secp224r1 (NIST P-224) and secp256r1 (NIST P-256) [47], [48]. Elliptic-curve cryptosystems based on these curves can provide security roughly equivalent to RSA with key lengths of 1024, 1536, 2048 and 3072 bits, respectively. The generator \( g \) used with each curve has a cofactor of 1 [47], so that the group includes all curve points. This allows the requester and responders to check the validity of ciphertexts (i.e., lines 31 and 74 in Figure 1) by checking if each ciphertext component is a valid point on the elliptic curve (or the point at infinity).

To make messages shorter and save bandwidth, we enable point compression in our implementation. Point compression (e.g., [49] Section A.9.6)) is a technique that compresses each elliptic-curve point to half its original size by using only \( y \) mod 2 in place of its \( y \) coordinate value. Correspondingly, point decompression reconstructs the point by recovering the \( y \) coordinate based on the \( x \) coordinate and \( y \) mod 2.

2) Bloom filters: A Bloom filter has a false positive rate of \( \approx (1 - e^{-k \ell})^k \) where \( n = |P(a)| \) denotes the number of elements to be inserted into the Bloom filter by the responder, \( \ell \) denotes the length of the Bloom filter and \( k \) denotes the number of hash functions (e.g., see [50] pp. 109-110)). As such, the number of hash functions that minimizes false positives is

\[
k_{\text{opt}} = \frac{\ell}{n} \cdot \ln 2
\]

and in this case, the minimized false positive rate is

\[
\left( \frac{1}{2} \right)^{k_{\text{opt}}} = \left( \frac{1}{2} \right)^{\frac{\ell}{n} \cdot \ln 2} \approx (0.6185)^{\frac{\ell}{n}} \quad (1)
\]

In our framework, \( k \) and \( \ell \) are decided by the requester, while \( n \) is determined by each responder with the knowledge of \( k \) and \( \ell \) received from the requester. In our implementation, the requester chooses \( k = 20 \) by default, and so each responder then generates a set \( P(a) \) of size \( n \leq \frac{\ell}{k} \cdot \ln 2 \) to ensure a false positive rate of \( \approx 2^{-20} \).

3) Precomputation: We use precomputation to optimize the creation of ciphertexts \( c_j \) by the requester in our protocol. Specifically, the requester precomputes private key \( u \), public key \( U \), and values \( \{V_j\}_{j \in \ell} \) and \( \{W_j\}_{j \in \ell} \), where each \( \langle U, V_j, W_j \rangle \) is a valid Diffie-Hellman triple, i.e., \( \langle V_j, W_j \rangle \in C_{(g,U)}(1_G) \). To create a ciphertext \( c_j \) of a different group element \( m \neq 1_G \), the requester need only multiply \( W_j \) by \( m \); thus, line 3 is completed in at most one multiplication per \( j \in [\ell] \). In practice, this precomputation could begin once the user enters the account registration web page and continue during idle periods until a password is successfully set.

C. Response Time

In this section, we evaluate the response time of our prototype system as seen by the requester (and in the absence of any user interaction, such as that described in Section V-A1), with two goals in mind. First, we want to systematically measure the effects of various parameter settings on our prototype implementation, to inform the selection of these parameters through an optimization process discussed in Section VI. We mainly explore two different parameters of our framework:

- The maximum number of similar passwords \( n = |P(a)| \) per responder (as determined by setting the Bloom filter size \( \ell = \left\lceil \frac{20m}{\ln 2} \right\rceil \) in the protocol), and the number \( m \) of responders.
- In Appendix B we also explore the impact of EC-ElGamal key length on the protocol response time and bandwidth.

The second main goal of our experiments here is to compare the performance of our prototype with and without leveraging Tor for implementing account location privacy, i.e., the untrusted-directory and trusted-directory models, respectively. In doing so, we hope to shed light on the performance costs of adopting a more pessimistic trust model in which the directory is not trusted to protect the websites where each account identifier \( a \) has been used to register an account.

1) Experimental setup: In our evaluations, we set up one requester, one directory, and up to 128 responders, spread across six machines located in our department. The six machines were on the same 1Gb/s network. The requester and the directory ran on separate machines with the same specification: 2.67GHz × 8 physical cores, 72GiB RAM, Ubuntu 14.04 x86_64. The (up to) 128 responders were split evenly across four other, identical machines: 2.3GHz × 32 physical cores with hyper-threading enabled (and so 64 logical cores), 128GiB RAM, Ubuntu 16.04 x86_64. Each of the responders sharing one machine was limited to using two logical cores, and had its own exclusive data files, processes, and network sockets.

Parameters were set to the following defaults unless otherwise specified: \( n = 1000, m = 64 \), and elliptic-curve key length of 192 bits. In particular, \( m = 64 \) is conservative based on recent studies. For example, a 2017 study with 154 participants found that users have a mean of 26.3 password-
protected web accounts \cite{7}, which is quite consistent with other studies (e.g., 51, 52).

Because the public Tor network is badly under-provisioned for its level of use and so its performance varies significantly over time, in our tests for the untrusted-directory model, we utilized a private Tor network with nodes distributed across North America and Europe. Our private Tor network (see Figure 3) consisted of three Tor authorities, eight normal onion routers, and two special onion routers. The eight normal onion routers were running on eight different Amazon EC2 (m4.large) instances, one located in each of the eight Amazon AWS regions in North America and Europe. Among these onion routers, three were also running as Tor authorities, with one in Europe, one in U.S. West, and the other in U.S. East. Two special onion routers were running on the machine in our department hosting the directory; one (“Exit” in Figure 3) exclusively served as the exit node of Tor circuits from requesters, and the other (“RP” in Figure 3) served exclusively as the “rendevous point” picked by the directory to communicate with Tor hidden services, i.e., the responders. As shown in Figure 3 each circuit additionally included two more onion routers (“OR” in Figure 3) chosen at random from among the eight normal onion routers already described.

All datapoints reported in the graphs below are averaged from 50 executions. Relative standard deviations, denoted $\chi$, are reported in figure captions.

2) Results: A measure of primary concern for our framework is the response time witnessed by the requester, since this delay will be imposed on the user experience while setting her password. Figure 4a shows the response time in the trusted-directory model, where the requester connects directly to the directory and the directory connects directly with each responder. In contrast, Figure 4b shows the response time in the untrusted-directory model, and so connections are performed through Tor. Precomputation costs (see Section V.B3) are not included in Figure 4 as these costs are expected to be borne off the critical path of interacting with the user. Tor circuit setup times are amortized over the 50 runs contributing to each datapoint in Figure 4b. In practice, we expect this setup cost to be similarly amortized over attempts needed by the user to choose an acceptable (not reused) password, or relegated to a precomputation stage when the user first accesses the requester’s account creation/password reset page.

One observation from Figure 4 is that the response-time cost of mistrusting the directory and so of relying on Tor to implement account location privacy, is typically $\geq 2\times$ for the parameters evaluated there. Recall that in Figure 4b both the requester–directory and directory–responder communications were routed through two onion routers chosen randomly from Amazon datacenter locations in North America and Europe (see Figure 3), in contrast to LAN communication in Figure 4a. The costs of these long-haul hops and Tor-specific processing increased as $n$ grew, due to the corresponding growth in query message size (see Appendix B).

Another effect illustrated in Figure 4 is the impact of additional responders (i.e., growth of $m$) on the response time witnessed by the requester. The main underlying cause of this effect is the variance in the speeds with which the responders return responses to the directory. This variance is small when communication is direct, but it grows substantially when Tor is used, due primarily to the differences in routes taken between the directory and each responder.

These effects are also illustrated in Figure 5 which shows the response time observed by the requester when the directory returned the proportion of $m = 64$ responses on the vertical axis as soon as that proportion was available to it. So, for example, Figure 5a shows that when $n = 2^{10}$, if the directory waited for 75% of the responses (i.e., 48 responses) before returning them to the requester, the requester observed an average response time of 9.55s (since (9.55, 0.75) is a point on the $n = 2^{10}$ curve).

When Tor is being used, the delays before the directory receives responses, and their variance, could presumably be reduced with improvements to Tor that leverage the multicast nature of the communication pattern in our framework. Specifically, the directory forwards the same message $m1$ to all responders. An anonymous communication system that exploits this one-to-many multicast pattern to gain efficiencies while still hiding the multicast recipients (e.g., 53) could presumably reduce these delays and their variance.

VI. PARAMETER OPTIMIZATION

At first glance, the results of Section V.C are perhaps discouraging, since they suggest that the response time of
testing with a large number \( n \) of similar passwords and at a large number \( m \) of queried responders is potentially large, especially if the directory is untrusted (Figure 4b). In this section we describe an approach to select optimal parameters for use in our framework, specifically parameter values \( m \) and \( n \) that maximize the likelihood of detecting the use of a similar password, subject to a response-time goal. As we will see, the results are not discouraging at all—a high true detection rate can be achieved within reasonable response-time limits with a surprisingly small \( n \) and while querying a modest number \( m \) of responders from among the total number of responders \( M_a \) registered at the directory for account \( a \).

The reason behind this initially surprising result is the typical manner in which people create new passwords by applying simple, predictable transforms to existing passwords. Numerous studies (e.g., [41], [39]) have found very low variation in the patterns that users leverage to modify their passwords (when they modify their passwords at all). Provided that responder \( S_i \) populates \( P_i(a) \) based on current knowledge about these transforms, the probability that the user’s candidate password \( \pi \) is contained within \( P_i(a) \) at a randomly chosen responder \( S_i \) is approximately as shown in Figure 6 (cf., [39] Fig. 5), as a function of the \( \frac{n}{d+1} \) passwords in \( P_i(a) \) that are similar to the actual password set for account \( a \) (i.e., ignoring those similar to the \( d \) honey passwords; see Section V-A1). As we can see, this probability is already substantial for very small \( \frac{n}{d+1} \). For example, this probability is \( \approx 0.34 \) for even \( \frac{n}{d+1} = 1 \); in other words, users on average employ the same password at \( \approx 34\% \) of the websites where they have accounts. Moreover, this probability grows quickly as \( \frac{n}{d+1} \) is increased only slightly.

The key insight here is that if a user’s candidate password \( \pi \) is chosen in the way that users typically do, then using a large \( n \) provides little additional power (Figure 6) but imposes substantially greater cost (Figure 4) than using a small \( n \) to detect that \( \pi \) is similar to another one already chosen. Moreover, if we model the true detection rate when querying \( m \) randomly chosen responders \( S_i \) as \( \text{tdr} = 1 - (\mathbb{P}(\pi \notin P_i(a)))^m \) (ignoring the probability of false detections due to the use of a Bloom filter), increasing \( m \) provides more detection power, in contrast. To more precisely balance these parameters and the response time of the protocol, we model the response time using

\[
    t(m, n) = \beta_0 + \beta_1 \cdot n + \beta_2 \cdot m + \beta_3 \cdot n \cdot m
\]

Regression analysis using the data in Section V-C yields \( \beta_0 = 1.5507, \beta_1 = 5.8834 \times 10^{-3}, \beta_2 = 2.6209 \times 10^{-3} \) and \( \beta_3 = 4.7135 \times 10^{-5} \) in the trusted-directory case (\( \text{RMSE} = 0.4547 \)) and \( \beta_0 = 6.4595 \times 10^{-3}, \beta_1 = 2.2885 \times 10^{-3}, \beta_2 = 1.0271 \times 10^{-3} \) and \( \beta_3 = 2.0336 \times 10^{-5} \) in the untrusted-directory case (\( \text{RMSE} = 0.1276 \)). Then, the requester chooses \( m \) and \( n \) using the following optimization:

\[
    \begin{align*}
    &\text{maximize } \text{tdr} = 1 - (\mathbb{P}(\pi \notin P_i(a)))^m \\
    &\text{subject to } t(m, n) \leq t_{\text{goal}} \\
    &\quad 1 \leq n \\
    &\quad 1 \leq m \leq M_a
    \end{align*}
\]

where \( t_{\text{goal}} \) is the response time requested by the requester and \( M_a \) is the number of responders registered at the directory as having an account for identifier \( a \). The directory can send
configuration was the same as in Section VI-C, no two requests were allowed to use the same Tor circuit, since they would be unable to do so in a real deployment, where different addresses for the same responder are stored for different user accounts at the directory. (The exception is if the requests were for the same user and at the same responder.) So, each request necessitated construction of new Tor circuits to its responders, which increased response times commensurately.

To put Table III in context, a throughput of 50 qualifying responses per second is enough to enable each of the 287 million Internet users in the U.S. to setup or change passwords on more than 5 accounts per year. Moreover, we believe the numbers in Table III to be pessimistic, in that in each request, the $m$ responders were chosen from only $M_n = 64$ responders in total, versus from likely many more in practice. Still, based on Table IIIb, a deployment using the untrusted-directory model would presumably require adaptations of Tor for our use-case (e.g., [53]) and distribution of the directory.

We note, however, that even a non-replicated directory should easily handle the storage requirements of our design. With 3.58 billion active Internet users worldwide and an average of 26 password-protected accounts per user, the storage of a Tor hidden service address for each user account at each website amounts to only $\approx 1.5$TB of state. In the trusted-directory model, the storage requirements would be even less.

VII. DENIALS OF SERVICE

Our design introduces denial-of-service opportunities for misbehaving requesters, responders, or the directory. We discuss these risks here, as well as methods to remedy them.

Perhaps the most troubling is a responder who returns $\rho \in C_{pk}(1_G)$ regardless of the request ciphertexts $\langle c_j \rangle_{j \in [k]}$ in message $m_1$ thereby giving the requester reason to reject the user’s chosen password even when the user’s chosen password is not similar to others she set elsewhere. This denial-of-service attack would primarily serve to frustrate users, but fortunately a responder that misbehaves in this way can be caught by simple audit mechanisms. For example, at any point, the directory could generate a message $m_1$ in which each $c_j \in C_{pk}(1_G)$ and for which it knows the private key $sk$ corresponding to $pk$; if a responder responds to this query with $\rho \in C_{pk}(1_G)$, then the directory has proof that the responder is lying and, e.g., can simply remove the responder from future requests.

---

**TABLE III: Maximum qualifying responses per second**

| $n$   | 1   | 6   | 11  | 16  | 21  | 26 |
|-------|-----|-----|-----|-----|-----|-----|
| $m_1$ | 1   | 4   | 10  | 19  | 26  | 26  |
| $m_2$ | 1   | 10  | 19  | 26  | 26  | 26  |
| $tdr$ | 343 | .985| $\approx$1 | $\approx$1 | $\approx$1 | $\approx$1 |

(a) Trusted directory

(b) Untrusted directory

---

**TABLE II: Choices for $m$ and $n$ computed using optimization in Section VII with $M_n = 26$**

In light of this finding, it is clear that the full range of parameter settings explored in Section VII-C will very rarely be needed in practice. This is fortunate, since small values of $n$ greatly improve the throughput of requester-responder interactions, especially in the trusted-directory model. To see this, Table III shows the throughput of our implementation, measured as the largest number of qualifying responses achieved as the requests per second were increased, as a function of $n$ and $m$. In Table IIIa, a response was qualifying if its response time was $\leq 5s$.

In contrast, in Table IIIb a response was qualifying if its response time was $\leq 8s$. This $3s$ difference between the standards for qualifying in the two tests was needed because we constructed the untrusted-directory test to capture as faithfully as possible the Tor costs that a real deployment would incur. Notably, even though the $m$ responders queried per request were chosen from only 64 responders in total (the
queries. In principle, a requester could also generate such audit queries, though doing so would require the directory to suspend the user-consent mechanism in Section [V-A1] In this case, a detection would enable the requester to learn that either one of the responders is misbehaving (but it would need help from the directory to figure out which one) or that the directory is misbehaving (in which case it would need to report it to some managing authority).

Other misbehaviors can render our framework silently ineffective while they are allowed to persist. For example, a malicious directory could simply not query responders at all, instead forging their responses to indicate no password reuse (i.e., each $p_i \notin C_{pk} \setminus C_{pk} \langle 1 \rangle$). Again, a simple audit (knowingly attempting to reuse a password at a requester) can detect such misbehavior. Presuming such misbehaviors will occur rarely and be remedied quickly, we believe our framework will suffice to discourage password reuse even if it usually works.

As our framework enables the requester to perform pre-computation to reduce its costs on the critical path of protocol execution, the critical-path computation cost of the protocol is greater for the responder than it is for the requester (see Appendix B). This is even more true for misbehaving requesters that replay the same request, in an effort to merely occupy directory and responder resources. Of course, this concern is not unique to our framework, and various techniques to stem such denials of service exist that would be amenable to adoption in our framework (e.g., [54], [55]). In addition, steps detailed in Section [V-A1] to require user consent (through clicking on a confirmation URL) to complete the protocol could interfere with such attacks. In the worst case, however, responders and the directory can refuse requests until the flood subsides, albeit temporarily reducing the utility of our framework to the status quo today.

VIII. CONCLUSION

Adams and Sasse famously declared, “Users are not the enemy” [23]. While we do not mean to suggest otherwise, it has also long been understood in a variety of contexts that users must be compelled to adhere to security policies, as otherwise they will not do so. Despite decades of haranguing users to stop reusing passwords, their adoption of methods to manage passwords more effectively has been painfully slow. In combination with advances in credential-stuffing attacks that leverage breached password databases, this problem has become so severe as to prompt some organizations to take dramatic and arguably counterproductive actions such as buying black-market passwords (e.g., [1]), thereby rewarding the breach.

We believe it is now time to consider the possibility of imposing technical measures to prevent the use of similar passwords across websites. In this paper we have presented one possible method for doing so, by coordinating password selection across websites so that similar passwords cannot be used for the same account identifier. An ingredient in this framework is a set-membership-test protocol, which we designed to fit well with our framework and that we proved satisfies desired security properties. Our framework builds from this protocol to address the leakage necessitated by our goals, and specifically to implement two desirable properties: account security and account location privacy. Finally, we leveraged tendencies of how users reuse passwords to optimize the parameters for our framework, enabling it to be effective with surprisingly modest costs.

Our intention with this paper is to initiate a debate around whether technical means to interfere with password reuse are appropriate and whether an acceptable technical approach is possible, toward the end of changing password-reuse culture. We look forward to contributing to the debate.

IX. ACKNOWLEDGMENTS

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APPENDIX A

PROOFS

Proof of Proposition 1. Note that ρ ∈ Cpk because each cj ∈ Cpk, by line 51 if

\[
\left( \prod_{j \in \mathcal{J}_c} c_j \right) \in C_{pk}(m')
\]

in line 54 then ρ ∈ C_{pk}(m'z) for z chosen in line 56. So, if ρ /∈ C_{pk}(1c) or, in other words, (m'z) /∈ 1c, then z such that (m'z) = m for a specific m ∈ G \ {1c} is chosen in line 59 with probability \( \frac{1}{1-z} \).

Proof of Proposition 2. This follows immediately since for c1 ∈ C_{pk}(m1) and c2 ∈ C_{pk}(m2), the value c ← c1 × pk c2 is chosen uniformly at random from C_{pk}(m1m2).

For the proof of Proposition 3 we leverage the generic group model as presented by Maurer [32], which Jager and Schwenk [56] have shown to be equivalent to the other
common generic model, due to Shoup [57]. In the Maurer model, $B$ (i.e., $B_1$ in experiment $\text{Exp}_{\mathcal{E}}^3$) is provided only the group order $r$ and black-box access (i.e., oracle access) to the group elements $g, U, \{V_j\}_{j \in [t]}, \{W_j\}_{j \in [t]}$, where each $c_j = (V_j, W_j)$, rather than receiving these group elements as inputs. Because the group representation is never exposed to $B$, each group element is equivalently represented as its base-$g$ discrete logarithm. So, the oracle holds integers $1, u, \{v_j\}_{j \in [t]}, \{w_j\}_{j \in [t]}$ to represent $g, U, \{V_j\}_{j \in [t]}, \{W_j\}_{j \in [t]}$, respectively, where $U = g^u$, each $V_j = g^{v_j}$, and each $W_j = g^{w_j}$. The oracle stores each of these values in an array at an index known to the adversary. Moreover, the oracle supports creation of new values via computation queries reflecting the application of the group operation $\times_G$ to two existing group elements represented at indices specified in the query; the values so created are appended to the array but not returned. Specifically, in each computation query, $B$ specifies two indices, and the oracle applies the $\times_G$ operator to the group elements $g^x, g^{x'}$ represented by the values $x$ and $x'$ at those indices, resulting in $x + x'$ being stored in the array to represent $g^{x+x'}$.

In addition to computation queries, $B$ can also perform equality queries, where it asks whether the group elements represented by two indices it specifies are the same. Finally, in accordance with our protocol, the adversary is permitted to ask just one DDH query, i.e., whether $uv \equiv_r w$ for $u$ the second value in the array (representing $U$) and $v$ and $w$ at specified indices in the array representing $g^v$ and $g^w$, respectively. This corresponds to providing $B_2$ with the answer to whether $\rho \in C_{pk}(1_G)$, to the protocol, the adversary is permitted to ask just one DDH query, i.e., whether $uv \equiv_r w$ for $u$ the second value in the array (representing $U$) and $v$ and $w$ at specified indices in the array representing $g^v$ and $g^w$, respectively. This corresponds to providing $B_2$ with the answer to whether $\rho \in C_{pk}(1_G)$, where $pk = (g, U)$ and $\rho = (g^v, g^w)$.

**Proof of Proposition 2.** Through the computation operations available to $B$, every value stored in the oracle is of the form

$$\alpha u + \sum_{j \in [t]} \beta_j v_j + \sum_{j \in [t]} \gamma_j w_j + \delta (2)$$

for constants $\alpha, \{\beta_j\}_{j \in [t]}, \{\gamma_j\}_{j \in [t]}, \delta \in \mathbb{Z}_r$ known to $B$.

Each equality query tests whether $g^z \equiv_r g^{z'}$ or, in other words, whether $z \equiv_r z'$, for values $z$ and $z'$ at the specified indices. These values are of the form in (2), i.e.,

$$z \equiv_r \alpha u + \sum_{j \in [t]} \beta_j v_j + \sum_{j \in [t]} \gamma_j w_j + \delta$$

and $z' \equiv_r \alpha' u + \sum_{j \in [t]} \beta'_j v_j + \sum_{j \in [t]} \gamma'_j w_j + \delta'$

and so the test $z \equiv_r z'$ is equivalent to

$$\left[ (\alpha - \alpha') + \sum_{j \in [t]} (\gamma_j - \gamma'_j) \right] u + \sum_{j \in [t]} (\beta_j - \beta'_j) v_j + \sum_{j \in [t]} (\gamma_j - \gamma'_j) w_j + (\delta - \delta') \equiv_r 0 \tag{3}$$

where $u, \{v_j\}_{j \in [t]}, \{w_j\}_{j \in [t]}$ are chosen independently at random from $\mathbb{Z}_r$. As such, ignoring queries that return true with probability 1 (and so teach the adversary nothing), the probability that each oracle query returns true is $1/r$. So, letting $E$ denote the event that at least one equality query returns true, if $B$ makes $q$ equality queries, then

$$\mathbb{P}(E) \leq \frac{q}{r} \tag{4}$$

$B_2$ is eventually provided the answer to whether $\rho \in C_{pk}(1_G)$ for a $\rho$ of its choosing. In this case $\rho$ is represented by a pair $(v, w)$ where $v$ and $w$ are of the form in (2), i.e.,

$$v \equiv_r \alpha u + \sum_{j \in [t]} \beta_j v_j + \sum_{j \in [t]} \gamma_j w_j + \delta$$

$$w \equiv_r \alpha' u + \sum_{j \in [t]} \beta'_j v_j + \sum_{j \in [t]} \gamma'_j w_j + \delta'$$

and the test $\rho \in C_{pk}(1_G)$ is equivalent to $uv \equiv_r w$ or, in other words,

$$\alpha u^2 + (\delta - \delta') u + \sum_{j \in [t]} (\beta_j u - \beta'_j) v_j + \sum_{j \in [t]} (\gamma_j u - \gamma'_j) w_j - \delta' \equiv_r 0 \tag{5}$$

Let $J = \{j \in [t] \mid \gamma_j u - \gamma'_j \not\equiv_r 0\}$. The number of possible sets $J_R$ such that $J \cap J_R = \emptyset$ is $\binom{|t|-|J|}{k}$, and so

$$\mathbb{P}(J \cap J_R = \emptyset \mid \neg E) = \frac{\binom{|t|-|J|}{k}}{\binom{|t|}{k}} \tag{6}$$

Since the answer to $\rho \not\in C_{pk}(1_G)$ is not computed using $w_j$ for any $j \in [t] \setminus J$, $B_2$ must choose which of these indices are in $J_R$ blindly. So, if $J \cap J_R = \emptyset$, then

$$\mathbb{P}(J_B = J_R \mid J \cap J_R = \emptyset \wedge \neg E) \leq \frac{1}{\binom{|t|-|J|}{k}} \tag{7}$$

On the other hand, now suppose $J \cap J_R \not= \emptyset$, and recall that $w_j$ for each $j \in J \cap J_R$ is distributed uniformly and independently in $\mathbb{Z}_r$. In the event $\neg E$, at least $r - q$ values remain equally possible from the adversary’s point of view for each $w_j$, $j \in J \cap J_R$, and so $\rho \in C_{pk}(1_G)$ with probability

$$\mathbb{P}(\rho \in C_{pk}(1_G) \mid J \cap J_R \not= \emptyset \wedge \neg E) \leq \frac{1}{r - q} \tag{8}$$

Moreover, if $\rho \not\in C_{pk}(1_G)$, $B_2$ can succeed with choosing $J_R$ with probability

$$\mathbb{P}(J_B = J_R \mid \rho \not\in C_{pk}(1_G) \wedge J \cap J_R \not= \emptyset \wedge \neg E) \leq \frac{1}{\binom{|t|}{k} - \binom{|t|-|J|}{k}} \tag{9}$$
So,
\[ P \left( \text{Expt}^S_B = \text{true} \right) \]
\[ = P \left( J_B = J_R \right) \]
\[ \leq P \left( J_B = J_R \mid J \cap J_R = 0 \wedge -E \right) + P \left( E \right) \]
\[ \leq P \left( J_B = J_R \mid J \cap J_R = 0 \wedge -E \right) P \left( \neg \left( J \cap J_R = 0 \mid -E \right) \right) + P \left( E \right) \]
\[ = \left( \left[ P \left( J_B = J_R \mid \rho \notin C_{pk}(1G) \wedge J \cap J_R \neq 0 \wedge -E \right) \right] + P \left( \rho \notin C_{pk}(1G) \mid J \cap J_R \neq 0 \wedge -E \right) \right) \cdot P \left( J \cap J_R \neq 0 \mid -E \right) + P \left( E \right) \]
\[ \leq P \left( J_B = J_R \mid J \cap J_R = 0 \wedge -E \right) P \left( J \cap J_R = 0 \mid -E \right) \]
\[ \left( \left[ P \left( J_B = J_R \mid \rho \notin C_{pk}(1G) \wedge J \cap J_R \neq 0 \wedge -E \right) \right] + P \left( \rho \notin C_{pk}(1G) \mid J \cap J_R \neq 0 \wedge -E \right) \right) \cdot P \left( J \cap J_R \neq 0 \mid -E \right) + P \left( E \right) \]

Filling in the values from 41 and (6)–(9) gives the result. □

**APPENDIX B**

**RESOURCE UTILIZATION MICROBENCHMARKS**

In this appendix we evaluate the resource utilization imposed by our protocol in Figure 1. Figure 7a shows the computational burden for computing the query (line \( m_1 \)) and response (line \( m_2 \)) messages in the protocol of Figure 1. (In comparison, the computational cost on the requester to process the response is minimal and so is omitted here.) Recall from Section V-B3 that our protocol implementation leverages precomputation; precomputation costs are not included in Figure 7a. Tor was not used in these tests. Figure 7b shows the size of the query message (message \( m_1 \)), which is the cost that dominates the bandwidth use of the protocol, since the response (message \( m_2 \)) is only a single ciphertext.

We caution the reader in interpreting these figures that the resource costs for large values of \( n \) are included for completeness and to inform the optimization in Section VI. For reasons we discuss in Section VI, such large values of \( n \) will generally not be necessary in our protocol.

One peculiarity evident in Figure 7a is that the responder’s computational cost is better when using the 256-bit elliptic curve than using the 224-bit one. This anomaly is caused by the point compression technique (see Section V-B1): to recover the points’ \( y \) coordinates from received EC-ElGamal ciphertexts \( \{ c_j \}_{j \in \mathcal{S}} \), the responder needs to calculate square roots of \( y_j^2 = x_j^2 + ax_j + b \) over the field \( \mathbb{F}_p \) for prime \( p \). If \( p \equiv 3 \mod{4} \), then \( (y_j^2)^{p+1} \) immediately gives the solution. However, if \( p \equiv 1 \mod{4} \), then one needs to use other less efficient algorithms to find the solution and, unfortunately, secp224r1 (NIST P-224) happens to be this case. Query generation involves no point decompression and so is not subject to this peculiarity.