Heuristics for optimizing 3D mapping missions over swarm-powered ad-hoc clouds

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Received: 25 February 2021 / Revised: 4 May 2022 / Accepted: 24 May 2022 / Published online: 1 August 2022 © The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2022

Abstract

Drones have been getting more and more popular in many economy sectors. Both scientific and industrial communities aim at making the impact of drones even more disruptive by empowering collaborative autonomous behaviors—also known as swarming behaviors—within fleets of multiple drones. In swarming-powered 3D mapping missions, unmanned aerial vehicles typically collect the aerial pictures of the target area whereas the 3D reconstruction process is performed in a centralized manner. However, such approaches do not leverage computational and storage resources from the swarm members. We address the optimization of a swarm-powered distributed 3D mapping mission for a real-life humanitarian emergency response application through the exploitation of a swarm-powered ad hoc cloud. Producing the relevant 3D maps in a timely manner, even when the cloud connectivity is not available, is crucial to increase the chances of success of the operation. In this work, we present a mathematical programming heuristic based on decomposition and a variable neighborhood search heuristic to minimize the completion time of the 3D reconstruction process necessary in such missions. Our computational results reveal that the proposed heuristics either quickly reach optimality or improve the best known solutions for almost all tested realistic instances comprising up to 1000 images and fifteen drones.

Keywords Cloud computing · Swarm · 3D reconstruction · Workload optimization

This work was financial supported in part by the Natural Sciences and Engineering Research Council of Canada through the Canada Excellence Research Chair in data science for real-time decision-making, and in part by Prompt and MITACS (IT12973).

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Unmanned Aerial Vehicles (UAVs), which are also referred to as drones, are a remotely operated aircraft. Their aerial capabilities and low cost made them an attractive option for operations as building inspection, photo collection, and area surveillance. That explains their popularity and adoption in a multitude of sectors of the economy.

Besides being remotely operated by pilots, drones can also operate autonomously when obeying an on-board flight controller. This is handled by employing a collaborative intelligence program on each agent of the fleet while keeping them connected on the same wireless network. This collaborative capability that simulates natural swarms (Tan and Yang 2013) (e.g., ant colonies, bee swarms, and bird flocks) is further leveraged by applying swarm robotics to conceive a fleet of fully autonomous drones focusing on fulfilling a common mission. Swarm robotics studies how to coordinate, in a distributed and decentralized manner, a large group of simple embodied robots to perform collective tasks and maximize the swarm performance (Şahin 2005). Such swarm behavior is a powerful tool to foster UAV applications, given its capability to deliver both performance and resilience without requiring any centralized control. Swarming UAVs are deployed to support operations in a long list of domains (Nex and Remondino 2014), including forestry (Berni et al. 2009; Grenzdörffer et al. 2008; Martínez-de Dios et al. 2006), archaeology and architecture (Chiabrando et al. 2011; Lambers et al. 2007; Oczipka et al. 2009; Rinaudo et al. 2012; Verhoeven 2009), environmental monitoring (Hartmann et al. 2012; Manyoky et al. 2011; Niethammer et al. 2010; Smith et al. 2009; Zhang 2008), emergency management (Chou et al. 2010; Haarbrink and Koers 2006; Molina et al. 2012) and precision agriculture (Zarco-Tejada et al. 2014; Bendig et al. 2014; Díaz-Varela et al. 2015).

Likewise, the swarm robotics has drawn the attention of the operations research community as an attractive opportunity to improve the efficiency of swarm-powered missions (Otto et al. 2018; Coutinho et al. 2018). For instance, decentralized optimization methods have been leveraging search problems (Sujit and Ghose 2004; Oh et al. 2014, 2015; Lanillos et al. 2014; Gan et al. 2011; Ji et al. 2015), target assignment problems (Tang and Ozguner 2005; Karaman et al. 2008; Niccolini et al. 2010; Viguria et al. 2010; Choi et al. 2011; Barrientos et al. 2011; Moon et al. 2013, 2015; Turpin et al. 2014; Enright et al. 2015; Sadeghi and Smith 2017), node covering problems (Zorbas et al. 2016; Ladosz et al. 2018), scheduling problems (Caraballo et al. 2017), etc.

UAV swarming solutions are typically used in synergy with other technologies such as digital photogrammetry (Gomarasca 2009; Linder 2016), which focuses on extracting and displaying the relevant 2D/3D geometric information from the portrayed physical environment. Given a set of pictures that are fairly distributed across the area of interest—multiple shooting points and shooting perspectives should be considered for improved performances—it is possible to extract a 3D model of the region itself by performing a so-called three-dimensional reconstruction. In terms of synergy between the swarming and photogrammetry technologies, the 3D reconstruction process is typically included in a swarm-powered mission pipeline where swarming UAVs are responsible for collecting, as fast as possible, the set of aerial images required to properly build the 3D map of the desired area.
Although the aerial photo collection is typically carried out in parallel by multiple agents, current 3D mapping literature addresses the 3D-reconstruction process in a centralized manner (Doherty et al. 2016; Loianno et al. 2016; Schmiemann et al. 2017; Kobayashi et al. 2017; Golodetz et al. 2018; Milani and Memo 2016). Meyer et al. (2015) employed 3D reconstruction solutions which were conducted in a centralized base-station to survey a heritage site in Mexico, while other authors focused on multi-sensor data fusion to feed the 3D reconstruction algorithms with more accurate data (Hinzmann et al. 2016; Surmann et al. 2017).

Such approaches are susceptible to internet connectivity and network latency issues. Thus, the distributed power within the UAV swarm can be exploited establishing an ad hoc cloud infrastructure able to safely perform the 3D reconstruction of the considered 3D mapping mission (Chen et al. 2019; Hamdaqa et al. 2015; Malhotra et al. 2014; Yaqoob et al. 2016). This strategy profits from the computational power in the microcomputers installed on the swarming UAVs instead of the computational resources present in a powerful computer (e.g. OpenDroneMap 2020).

The motivation of this paper arises from a real-life problem in the emergency response field. The decision-making and situation awareness during a first response operation are highly boosted by 3D maps of the affected region. Such 3D models allow the first responders to detect relevant threats like damage in roads and buildings or insecure zones. Thus, creating 3D maps must be quickly done regardless internet connectivity to enhance the safety and efficiency of the first responders.

In Costa et al. (2020), the authors introduced the Covering-Assignment Problem for swarm-powered ad hoc clouds (CAPsac), whose objective, in the context of a 3D mapping UAV mission, is to optimally generate and place the multi-node computing workload that will be responsible for performing the 3D reconstruction process. Since each computing node is responsible for reconstructing a specific sub-region on a specific drone, the optimal solution of the problem describes how to minimize the processing time by optimally i) splitting the set of available photos to form multiple sub-regions and then ii) assigning each sub-region to a specific UAV.

During emergency field operations, each minute counts and nearly optimal solutions of the CAPsac problem must be computed as quickly as possible. In this way, 3D maps can be promptly put in the hands of the emergency responders, while leaving the UAVs available to perform additional critical tasks—including other 3D mapping missions. Furthermore, a fast CAPsac solution allows the flying UAVs to preserve the battery life by limiting the idle flying periods spent waiting for 3D processing instructions. Note that the autonomy of typical commercial drones does not go beyond one hour.

In this paper, we propose a Variable Neighborhood Search (VNS) heuristic (Hansen and Mladenović 2001) based on sub-tree reconstruction, splitting hyperplane reallocation, sub-region transfer, and sub-region swap neighborhoods for quickly optimizing a swarm-powered 3D mapping mission according to CAPsac. We also develop a mathematical programming-based heuristic, namely Decomposition-based heuristic, to assess the performance of matheuristics.

The paper is organized as follows. The next section describes how to cast a swarm-powered 3D mapping mission as CAPsac problem. The proposed mathematical programming-based heuristic is presented in Sect. 3. Likewise, the VNS fundamentals and the proposed VNS-based heuristic are exposed in Sect. 4. Finally, Sect. 5 shows
and analyzes the computational results obtained over realistic instances, while Sect. 6 outlines our concluding remarks.

2 Swarm-powered 3D mapping mission as a CAPsac

A swarm-powered sensing mission (e.g., an 3D mapping mission) is decomposed into two main independent phases (Mukherjee et al. 2020; Hoang et al. 2020):

1. **Photo collection** the UAVs of the swarm dynamically collaborate to collect all the necessary aerial pictures of the area of interest. Note that the set of required pictures is typically computed by dedicated mapping software and is merely an input of the mapping mission (Pepe et al. 2018).

2. **Data processing** (Tang et al. 2020) the collected pictures are collaboratively processed by the microcomputers installed on the UAVs to produce a 3D map. During this process, the computing workload can be parallelized over the available computing units. Furthermore, pictures can be transferred over the inter-drone wireless network to satisfy the input requirements of the distributed reconstruction tasks.

By casting a swarm-powered 3D mapping mission as a CAPsac, we aim to optimize the 3D processing phase only, with no direct control over the photo collection step. Therefore, we consider that the set of photos taken by the drones as well as their locations are input parameters for CAPsac solution. A swarm-powered 3D processing application employing a swarm of four UAVs is illustrated in Fig. 1 (Costa et al. 2020). In that example, the large “×”, “+”, “⋄”, and “◦” identify the drones. Only drones with powerful microcomputers able to sustain the 3D reconstruction methods are considered 3D-capable (Costa et al. 2020). The 3D-capable drones are represented...
by the $+$ and the $\times$ symbols. That is, $+$ and $\times$ are responsible for performing the 3D processing stage. The region portrayed by the set of photos $P$ is bounded by the continuous lines. Further, the photos are represented by the small “$\times$”, “$+$”, “$\Diamond$”, and “$\circ$”. Their symbols match with the UAV where the photo is stored, for instance, pictures identified by a small $+$ are stored in the UAV represented by the large $+$. A solution to the CAPsac problem describes how to parallelize optimally a massive 3D reconstruction task into smaller 3D reconstruction sub-tasks to be distributed across the 3D-capable drones. This means that a 3D reconstruction sub-task is associated with a specific sub-region of the target region. Therefore, the optimal solution of CAPsac minimizes the completion time (i.e., makespan) of the whole 3D reconstruction phase. When optimizing the CAPsac, three constraints cannot be neglected (Costa et al. 2020): the 3D reconstruction sub-regions must be a spatial-convex covering; the communication delays follow the Max-Min Fairness (MMF) paradigm (Bertsekas et al. 1992); the 3D reconstruction sub-tasks are distributed across the 3D-capable drones according to a reliability factor. More precisely, in this work, sub-regions (subsets of photos) form a partition of spatial-convex sets—all photos lying inside the set’s convex hull are allocated to that same sub-region. Figure 2 shows an example of a spatial-convex set, whereas Fig. 3 illustrates a set of photos which is not a spatial-convex set. In those figures, photos allocated to the set are represented by the filled “$\bullet$” and the empty “$\circ$” represent photos not allocated to the set. The assumption of non-overlapping sub-regions allows us to explore the solution space more efficiently by means of a spatial-partition tree (to be described in Sect. 4). Such simplification does not affect the quality of the resulting 3D models given enough photo footprint overlap in the mission planning phase (Pepe et al. 2018). Furthermore, sub-regions must be convex (see e.g., Frahm et al. 2010; Yang et al. 2016; Xie et al. 2018) since 3D reconstruction algorithms require that input photos overlap. Typically, 3D reconstruction algorithms comprise three steps: feature detection; feature matching; and reconstruction (Tang et al. 2020). The pixel patches—which represent visual features as corners, edges, and blobs—extracted from the photos during the feature detection step are matched across each other during the feature matching step. A pair of visual features (each from a distinct photo) is said to match if the same pixel patch is portrayed in the two photos under examination. Finally, the 3D model is built based on the photo 2D coordinates of those matched visual features. It is not possible to match visual features and build an 3D model if the photos assigned to a sub-region do not overlap. Hence, the sub-regions must be convex to allow photo overlapping and, consequently, the complete 3D model reconstruction. The communication delays must be taken into account since the photo collection stage is not optimized in the CAPsac, and the 3D reconstruction of a sub-region cannot start until its assigned drone has in its hard disk all photos belonging to that sub-region, that is, all 3D reconstruction input photos. Thus, a drone must request to the other drones the photos to complete its designed sub-region. A single tree network topology (dotted lines in Fig. 1), on top of which establishing TCP sessions—one per photo transfer, is adopted as inter-drone communication means. The MMF paradigm is considered to approximate the TCP-based rate allocation for multiple photo transfer.
sessions occurring within the swarm communication tree (Massoulié and Roberts 1999).

Finally, the sub-region (3D reconstruction sub-task) to drone assignments must be robust to drone malfunction. This is addressed by introducing a reliability factor, which dictates the minimal number of drones each sub-region must be assigned. For instance, a solution for a reliability factor equal to one is illustrated in Fig. 4. There, the
two spatial-convex sets (the same number as the 3D-capable drones) are represented by the dashed (left) and the dashed-and-dotted lines (right). The sub-region on the left is assigned to the UAV “+” and the sub-region on the right is assigned to the UAV “×”.

Let us now define the notation for CAPsac. We consider the region photographed by the set of photos $P$ as the region targeted by the 3D processing stage. The CAPsac considers that photo positioning was performed beforehand, and then, the locations of all captured photos in $P$ are fixed and known. Further, let $\lambda_p$ and $\mu_p$ be the estimated photo processing time and the data size of photo $p \in P$, respectively. Besides, a set of drones $D$ is at disposal but only a subset $\bar{D} \subseteq D$, where $|\bar{D}| = m$, is able to perform the 3D reconstruction. Therefore, the number of sub-regions constructed by the CAPsac is equal to $m$. Given a drone $d \in D$, the binary parameter $\theta_{dp}$ indicates if $d$ has the photo $p \in P$ in its hard disk.

The drones are disposed in an undirected tree topology $T = (N, A)$, in which $N$ corresponds to the set of nodes where each drone is located and $A$ comprises the sets of arcs linking the nodes in $N$. The set $F$ is composed by all transmission demands across pairs of drones, such that $F = \{f_{hd}|(h, d) \in D \times \bar{D}\}$, in which a specific transmission demand from the drone $h$ to the drone $d$ is represented by $f_{hd}$. For each demand $f_{hd}$, let $V^{hd}$ be the set of arcs $(i, j) \in A$ in the unique routing path between $h$ and $d$, and $F_{ij}$ the set of demands using the arc $(i, j)$, i.e., $F_{ij} = \{f_{hd} \in F|(i, j) \in V^{hd}\}$. Given an arc $(i, j) \in A$, let $c_{ij}$ be the capacity of the arc $(i, j)$. Also, denote by $c_{\min}^h$ the minimum $c_{ij}$ in $V^{hd}$. Finally, each transmission demand is allowed within a time limit $\hat{T}$, after which it is considered as infeasible.

All the notation of the presented parameters is presented in Table 1.
Table 1 CAPsac parameters for a 3D mapping mission. Adapted ©2020 IEEE (Costa et al. 2020)

| Parameters | Description |
|------------|-------------|
| $\lambda_p$ | Estimated processing time of photo $p$ |
| $\mu_p$ | Amount of data of photo $p$ |
| $\theta_{dp}$ | Equals 1 if drone $d$ has the photo $p$ stored in its memory |
| $F$ | Set of traffic demands between each pair of drones |
| $V_{ab}$ | Routing path of a demand $f_{ab} \in F$ from the drone $a$ to the drone $b$ |
| $c_{ij}$ | Transmission capacity of the link $(i, j) \in A$ |
| $\overline{c}_{ab}$ | Minimum $c_{ij}$ for $(i, j) \in V_{ab}$ |
| $\overline{F}_{ij}$ | Set of demands that use link $(i, j) \in A$ |
| $\sigma$ | Reliability factor |
| $m$ | Number of 3D-capable drones (equiv. number of sub-regions) |
| $\hat{T}$ | Maximum allowed time for transmitting photos between drones |

2.1 Mathematical formulation

Mathematical formulations to solve the CAPsac were proposed in Costa et al. (2020). In this work, we adopt the region-based formulation ($rCAPsac$) (Costa et al. 2020). The $rCAPsac$ formulation minimizes the completion time of the 3D processing stage $T_{\text{max}}$, and it exploits the set $S$ containing all rectangular spatial-convex sets of photos in $P$. Given a photo $p \in P$, the set $S_p$ comprises all sub-regions $S \in S$ that contain photo $p$. The binary variables $q_d^S$ indicate if $S \in S$ is selected and its 3D reconstruction is allocated to the drone $d \in D$. Remark that all selected sub-regions must be assigned to at least $\sigma$ (reliability factor) drones. Also, for each $S \in S$, the binary variable $o^S$ indicates (i.e., $o^S = 1$) if the sub-region $S$ is used in the solution; the $t^S$ represents the time required to perform the 3D reconstruction of $S$; the $\mu_{hd}^S$ expresses the amount of data added into the transmission demand $f_{hd}$ when $S$ is selected.

Since 3D reconstruction cannot start until a drone has all its input pictures, the communication time required to exchange photos among the drones cannot be neglected. For each $f_{hd} \in F$, we denote by $z_{hd}$ the binary variable which indicates when the demand $f_{hd}$ is active—if there exists any data to be exchanged through $f_{hd}$. Accordingly, continuous variables $\phi_{hd}$ correspond to the transmission rate performed by the demand $f_{hd}$.

As mentioned, in the case of the CAPsac, the transmission rate allocation follows the MMF paradigm. Such rate allocation attends the MMF if and only if there is at least one bottleneck link $(i, j) \in A$ on the routing path $V_{hd}$ of each active demand $f_{hd} \in F$ (Nace and Pióro 2008). Furthermore, a link $(i, j)$ is a bottleneck of the demand $f_{hd}$ if and only if Nace and Pióro (2008)

(i) its capacity is saturated, i.e., $\sum_{f_{ab} \in \overline{F}_{ij}} \phi_{ab} = c_{ij}$ and
(ii) the transmission rate $\phi_{hd}$ of traffic demand $f_{hd}$ is the highest among the traffic demands routed over link $(i, j)$, i.e., $\phi_{hd} \geq \phi_{ab} \forall f_{ab} \in \overline{F}_{ij}$.
Given a demand \( f^{hd} \), let \( w_{ij}^{hd} \) be the binary variable equal to 1 if the link \((i, j)\) is a bottleneck of \( f^{hd} \). We denote by \( u_{ij} \) the highest transmission rate among the traffic demands carried by the link \((i, j) \in A\), that is, \( u_{ij} = \max_{f^{ab} \in F_{ij}} \{\phi^{ab}\} \).

The \( rCAPsac \) formulation of the \( CAPsac \) is described by the following MILP (Costa et al. 2020).

\[
\begin{align*}
\min_{q, o} & \quad T_{\text{max}} \\
\text{s.t.} & \quad T_{\text{max}} \geq \sum_{S \in S} t^S q_d^S \quad \forall d \in \tilde{D} \quad (1) \\
& \quad \hat{T} \cdot \phi^{hd} \geq \sum_{S \in S} \mu^S_q q_d^S \quad \forall f^{hd} \in F \quad (2) \\
& \quad \sum_{d \in \tilde{D}} q_d^S \geq \sigma o^S \quad \forall S \in S \quad (3) \\
& \quad \sum_{S \in S_p} o^S \geq 1 \quad \forall p \in P \quad (4) \\
& \quad \sum_{S \in S} o^S = m \quad (5) \\
& \quad z^{hd} \leq \sum_{S \in S} \mu^S_q q_d^S \quad \forall f^{hd} \in F \quad (6) \\
& \quad \phi^{hd} \leq \bar{\epsilon}^{hd} z^{hd} \quad \forall f^{hd} \in F \quad (7) \\
& \quad \sum_{(i, j) \in \mathcal{V}^{hd}} w_{ij}^{hd} \geq z^{hd} \quad \forall f^{hd} \in F \quad (8) \\
& \quad \sum_{f^{ab} \in F_{ij}} \phi^{ab} \leq c_{ij} \quad \forall (i, j) \in A \quad (9) \\
& \quad \sum_{f^{ab} \in F_{ij}} \phi^{ab} \geq c_{ij} w_{ij}^{hd} \quad \forall (i, j) \in A, \forall f^{hd} \in \mathcal{F}_{ij} \quad (10) \\
& \quad u_{ij} \geq \phi^{hd} \quad \forall (i, j) \in A, \forall f^{hd} \in \mathcal{F}_{ij} \quad (11) \\
& \quad \phi^{hd} \geq u_{ij} - c_{ij}(1 - w_{ij}^{hd}) \quad \forall (i, j) \in A, \forall f^{hd} \in \mathcal{F}_{ij} \quad (12) \\
& \quad o^S, q_d^S \in \{0, 1\} \quad \forall S \in S, \forall d \in \tilde{D} \quad (13) \\
& \quad w_{ij}^{hd} \in \{0, 1\} \quad \forall (i, j) \in A, \forall f^{hd} \in F \quad (14) \\
& \quad \phi^{hd} \geq 0, z^{hd} \in \{0, 1\} \quad \forall f^{hd} \in F \quad (15) \\
& \quad u_{ij} \geq 0 \quad \forall (i, j) \in A. \quad (16)
\end{align*}
\]

The objective function (1) minimizes the makespan of the 3D processing phase, i.e., the maximum processing time \( T_{\text{max}} \) across all 3D-capable drones in the swarm. Accordingly, \( T_{\text{max}} \) is computed by the constraints (2), in which, the summation \( \sum_{S \in S} t^S q_d^S \) determines the processing time of a given drone \( d \in \tilde{D} \). Constraints (3) restrict the
latency of all transmission demands $F$ up to $\hat{T}$. Constraints (4) enforce the reliability factor over all selected sub-regions. That is, when a sub-region $S$ is selected, it is assigned to at least $\sigma$ 3D-capable drones. The photos $p \in P$ are always covered given the constraints (5). The constraint (6) establishes $m$ (number of 3D-capable drones) as the number of sub-regions being selected. The MMF transmission rate allocation is computed by the constraints (7)–(13). Constraints (7) allow a transmission to be active only if there exist data to be sent from drone $h$ to drone $d$. Constraints (8) set the variables $\phi_{hd}$ to 0 when $f_{hd}$ is inactive. All active demands have at least one bottleneck link in their routing path according to inequalities (9). Constraints (10) ensure that all link capacities are respected. Constraints (10) and (11) jointly force that all bottleneck links will be saturated. That covers the first condition to a link $(i, j) \in A$ to be a bottleneck link. The second bottleneck link condition is ensured by constraints (12) and (13). Constraints (12) make $u_{ij}$ greater or equal to the highest transmission rate passing through the link $(i, j)$. If $(i, j) \in A$ is a bottleneck link of the demand $f_{hd}$, constraints (13) ensure that $\phi_{hd}$ will not be exceeded by any other transmission rate passing thought link $(i, j)$. The domain constraints are (14)–(17). The number of variables of the rCAPsac formulation can rapidly increase even though it is polynomial bounded by $O(|P|^4)$.

The CAPsac was proved NP-hard in Costa et al. (2020). A straightforward lower bound for the CAPsac is the perfect workload division among the available processing resources. In the context of a 3D mapping mission by UAVs, this means to equally spread the photo processing time of the whole target region by the number of 3D-capable drones. Given a set of photos $P$, a set of 3D-capable drones $\bar{D}$, and a reliability factor $\sigma$, the lower bound $Lb(P, \bar{D}, \sigma)$ is given by

$$Lb(P, \bar{D}, \sigma) = \sigma \sum_{p \in P} \lambda_p \frac{|\bar{D}|}{|D|}. \quad (18)$$

### 3 Decomposition-based heuristic

The formulation given in the previous section can be used to devise heuristics for the CAPsac. However, before that, let us define $\bar{n}$ as the perfect workload division w.r.t. the number of photos per sub-region

$$\bar{n} = \frac{|P|}{|D|}.$$

Inspired by the observation that good solutions have their workload w.r.t. the number of photos per sub-region close to $\bar{n}$, we decompose the problem by exploring first those solutions whose sub-regions have cardinality close to $\bar{n}$.

To that purpose, we restrict the solution space of rCAPsac to sub-regions whose photo cardinalities lie inside an interval $[n_\ell, n_u]$, which is iteratively increased. That interval is initialized with $n_\ell = \lfloor \bar{n} \rfloor$ and $n_u = \lceil \bar{n} \rceil$. As solving this restricted rCAPsac formulation leads to possibly infeasible or suboptimal solutions, one can iteratively increase the interval $[n_\ell, n_u]$, and solve the new restricted CAPsac until the objec-
tive function value \(T_{\text{max}}\) does not change between two consecutive iterations. This stopping condition is indeed sub-optimal. For example, in the presence of very contrasting photo-processing times \(\lambda_p\) among the photos \(p \in P\), the optimal solution might contain sub-regions with small processing times and large cardinalities.

Algorithm 1 summarizes the decomposition-based heuristic. It considers \(\Omega\), the set of distinct photo cardinalities from sub-regions in \(S\), and a mapping function \(\omega\) which maps the cardinalities in \(\Omega\) to their respective sub-regions in \(S\). The mapping function \(\omega\) is implemented as a hash table. Thus, for each hash table key \(n_i \in \Omega\), \(\omega(n_i)\) points to the set of sub-regions of \(S\) that contain \(n_i\) photos.

**Algorithm 1 Decomposition-based heuristic**

1: Enumerate rectangular sub-regions \(S\);
2: Construct and sort \(\Omega\);
3: Build \(\omega\) according to \(S\) and \(\Omega\);
4: \(n_\ell \leftarrow \lceil \frac{\bar{n}}{} \rceil ; n_u \leftarrow \lfloor \frac{\bar{n}}{} \rfloor ; S^0 \leftarrow \{\emptyset\}; T_{\text{max}}^0 \leftarrow \infty; i \leftarrow 0;
5: \text{repeat}
6: \quad i \leftarrow i + 1;
7: \quad S^i \leftarrow S^{i-1} + \omega(n_\ell) + \omega(n_u)
8: \quad T_{\text{max}}^i \leftarrow \text{solve rCAPsac encompassing sub-regions } S^i;
9: \quad \text{decrease } n_\ell \text{ to its closest smaller } n \in \Omega;
10: \quad \text{increase } n_u \text{ to its closest larger } n \in \Omega;
11: \text{until } T_{\text{max}}^{i-1} = T_{\text{max}}^i \text{ or } S^i = S

Algorithm 1 proceeds as follows. Step 1 enumerates the sub-regions \(S\) on \(O(|P|^4)\) operations (Costa et al. 2020). Step 2 constructs the sorted array \(\Omega\) whereas step 3 creates \(\omega\). The values of \(n_\ell, n_u, S^0, T_{\text{max}}^0\), and \(i\) are properly initialized at step 4. For each iteration (steps 5–11), \(i\) is incremented in step 6. Then, the set \(S^i\) is updated to include the sub-regions of cardinality equal to \(n_\ell\) and \(n_u\) in step 7. Then, step 8 solves the restricted \(\text{rCAPsac}\) formulation encompassing all sub-regions in \(S^i\). The new interval \([n_\ell, n_u]\) is obtained in steps 9-10 by updating \(n_\ell\) and \(n_u\). The algorithm iterates (lines 5-11) until the objective function value \(T_{\text{max}}\) does not change between two consecutive iterations, or that \(S^i\) contains all the sub-regions \(S\). We remark that the algorithm also iterates (lines 5-11) while no feasible solutions have been found yet. Accordingly, if the algorithm does not find a feasible solution and \(S^i = S\), the problem is proven infeasible. As a result, the heuristic takes at most \(O(|\Omega|)\) iterations to finish in the worst case. In such worst scenario, the full problem (i.e., the complete \(\text{rCAPsac}\) formulation) is solved once the interval \([n_\ell, n_u]\) encompasses all sub-regions in \(S\). It is worth pointing out that the optimal solution in one iteration remains feasible for the next one since increasing the interval \([n_\ell, n_u]\) is equivalent to simply adding new columns to the \(\text{rCAPsac}\) formulation. Hence, that solution is used as a feasible upper-bound solution for the restricted \(\text{rCAPsac}\) formulation solved in the next iteration.
4 Variable neighborhood search for CAPsac

Our VNS heuristic exploits a spatial partition tree as a spatial data representation to ensure the spatial-convexity of sub-regions in a feasible solution. Furthermore, our proposed VNS is based on four neighborhoods, namely: sub-tree reconstruction, splitting hyperplane reallocation, sub-region transfer, and sub-region swap. They are explained together with the VNS fundamentals and our VNS implementation in the following sections. Finally, we present how the VNS heuristic ensures communication delays feasibility during its search.

4.1 Variable neighborhood search fundamentals

Variable Neighborhood Search is a stochastic search metaheuristic, i.e., a local optimal evading framework based on heuristics, which aims at reaching optimal or near-optimal solutions for global combinatorial optimization problems. VNS has been successfully applied to a vast range of NP-hard problems (Hansen et al. 2019).

Likewise other search frameworks, e.g. simulated annealing and tabu search (Boussaïd et al. 2013), VNS keeps a single solution $x$ during the whole execution of the algorithm. However, it differs from other metaheuristics mainly by its search mechanism that searches for better solutions in increasingly wider neighborhoods of $x$, according to the parameters $k \in \mathbb{Z}^+$ and $k_{\text{max}} \in \mathbb{Z}^+$.

Given a set of neighborhoods $\mathcal{N} = \{ \mathcal{N}_1, \ldots, \mathcal{N}_{k_{\text{max}}} \}$ and an initial solution $x$, the VNS starts by applying a local descent method on a random neighboring solution from $\mathcal{N}_1(x)$. If the reached local minimum is better than $x$, it replaces $x$. Otherwise, the local descent method is performed again with the larger neighborhood $\mathcal{N}_{k+1}(x)$. The VNS iterates until the stopping condition (e.g., maximum CPU time) is met. The basic steps of VNS are given in Algorithm 2.

Algorithm 2 Variable Neighborhood Search - VNS

Select a set of neighborhoods $\mathcal{N}_k$, for $k = 1, \ldots, k_{\text{max}}$; Find an initial solution $x$;
repeat
$k \leftarrow 1$
repeat
(Diversification step): Choose a random neighboring solution $x'$ from $\mathcal{N}_k$;
(Intensification step): Apply a local descent method from $x'$, obtaining $x''$;
if $\text{cost}(x'') < \text{cost}(x)$ then
$x \leftarrow x''$;
$k \leftarrow 1$;
else
$k \leftarrow k + 1$;
end if
until $k > k_{\text{max}}$
until a stopping criterion is met
4.2 Spatial partition tree

The *spatial partition tree* allows the VNS to efficiently explore the solution space without directly handling spatial-convexity constraints. Inspired by the *k-dimensional trees* (Bentley 1975), the spatial partition tree is a (not necessarily complete) binary tree which recursively splits the metric space under consideration. In this kind of tree, all nodes are directly associated with a portion (sub-region) of the target region.

In the CAPsac context, the target region is discretized by the positions of the photos \( P \). Let us define \( L \) as the set of distinct photo latitudes and let \( C \) be the set of distinct photo longitudes, such that \( 1 \leq |C| \leq |P| \) and \( 1 \leq |L| \leq |P| \). The sets \( C \) and \( L \) offer a simple way to partition the space into rectangular sub-regions, such that any rectangular sub-region \( r \) can be represented by its left \( c_\leftarrow \in C \), right \( c_\rightarrow \in C \), inferior \( \ell_\wedge \in L \), and superior \( \ell_\vee \in L \) borders. As a result, constructing a spatial partition tree splits the target region and forms a partition of rectangular sub-regions.

A spatial partition tree is successively constructed by splitting the region of interest into new sub-regions as illustrated in Fig. 5. At first, the spatial partition tree contains only the root node corresponding to the whole target region as shown in Fig. 5a. The photos \( P \) (target region), identified by the "◦", are enclosed by a unique rectangular region \( r_1 \) (dashed lines). All nodes in the tree (round-cornered rectangles) have (i) their associated enclosed longitudes on axis \( C \) ("C"); (ii) their associated enclosed latitudes on axis \( L \) ("L"); (iii) the axis selected to guide the splitting ("Axis"); (iv) the index in the selected axis chosen to define the splitting hyperplane ("Splitting index"). It is worth to mention that the leaf nodes have the attributes (iii) and (iv) empty, and the photos inside a sub-region are implicitly given by the lists \( C \) and \( L \).

In each successive iteration (splitting step), a leaf node (rectangular sub-region) is selected to be split by means of an axis-aligned splitting hyperplane. The selected splitting hyperplane (denoted by "□" and represented by lines) is chosen from the location of the photos associated with a sub-region being split. The splitting step is repeated \( |\tilde{D}| - 1 \) times, that is, until \( |\tilde{D}| \) sub-regions are formed. For instance, starting from the root node in Fig. 5a, the construction of a partition with four sub-regions (i.e., \( |\tilde{D}| = 4 \)) is done after three iterations presented in Fig. 5b–d, respectively. Finally, Fig. 5d illustrates the final spatial-partition tree and its respective partition and splitting hyperplanes, where the photos \( P \) are split into the four rectangular sub-regions \( r_1, r_2, r_3, \) and \( r_4 \). This data structure guarantees the creation of sub-regions that are always spatial-convex sets.

4.3 Sub-tree reconstruction neighborhood

Given a spatial partition tree \( T \), denote by \( D(T) \) the maximum depth of the nodes of \( T \). Let \( N(d) \) be the set of all nodes at the depth \( d \) of \( T \) where \( 0 \leq d \leq D(T) \). The sub-tree reconstruction neighborhood randomly reconstructs (as explained in Sect. 4.2) a sub-tree rooted at a non-leaf node \( n \in N(d) \). Figure 6 illustrates a neighboring solution in the right obtained from reconstructing the sub-tree rooted at the grey node (in depth 0) of the spatial partition tree in the left.
(a) Initial step. Spatial-convex tree (left) and the target region (right).

(b) Iteration 1. Spatial-convex tree (left) and its respective partition (right).

(c) Iteration 2. Spatial-convex tree (left) and its respective partition (right).

(d) Final iteration resulting in the spatial-partition tree (left) and its respective partition and splitting hyperplanes (right).

Fig. 5 Illustrative construction of a spatial-convex tree with four sub-regions.
4.4 Splitting hyperplane reallocation neighborhood

Given a spatial partition tree $T$ and a non-leaf node $n$ which is a parent of a leaf node, this neighborhood includes all solutions obtained by reallocating the splitting hyperplanes of the non-leaf nodes of the sub-tree rooted in $n$ (named $T_n$). Furthermore, such reallocation is made changing the splitting index (component “Splitting index”) and/or the axis orienting the hyperplane (component “Axis”). Unlike the sub-tree reconstruction neighborhood, the splitting hyperplane reallocation neighborhood does not restrict $n$ to be a node in a specific depth $0 \leq d \leq D(T)$. Moreover, it keeps the depth of the resulting sub-tree rooted in $n$ equal to that of the original sub-tree. Figure 7 illustrates the procedure in which the splitting index of the (colored) node $n$ (on the left) is changed to originate the neighboring tree on the right.
4.5 Sub-region transfer neighborhood

This neighborhood encompasses all solutions obtained by transferring one sub-region assignment \((r_\ell, d_i)\) from a drone \(d_i\) to a drone \(d_j\) such that \(i \neq j\). In Fig. 8, the transfer of the sub-region \(r_3\) from drone \(d_1\) to drone \(d_2\) generates a neighboring solution according to the sub-region transfer neighborhood for a reliability factor \(\sigma\) equal to two.

4.6 Sub-region swap neighborhood

Given a solution, its neighbors in the sub-region swap neighborhood are obtained by the swap of sub-regions assignments \((r_\ell, d_i)\) and \((r_m, d_j)\) between drones \(d_i\) and \(d_j\). Such swap yields the sub-region assignments \((r_\ell, d_j)\) and \((r_m, d_i)\). An example of neighboring solution (for \(\sigma = 2\)) in the sub-region swap neighborhood is given in Fig. 9, where the assignments \((r_2, d_3)\) and \((r_1, d_4)\) are swapped between drones \(d_3\) and \(d_4\).

![Fig. 8 Neighboring solution (right) obtained from a spatial partition tree (left) in the sub-region transfer neighborhood for \(\sigma = 2\)](image)

![Fig. 9 Neighboring solution (right) obtained from a spatial partition tree (left) in the sub-region swap neighborhood for \(\sigma = 2\)](image)
4.7 Computation of the processing time, transmission data, and transmission time

In order to efficiently explore the neighborhoods during the VNS local descent, it is important to adequately compute the photo processing time of a sub-region and the amount of data involved in the data transfers between drones. Given the boundaries of a sub-region and the drone \(d \in \tilde{D}\) dealing with the reconstruction of that sub-region, this information can be computed in constant time as follows.

The computation of the photo processing time of a sub-region is based on the sets of photos \(S_{c}^{\text{left}}, S_{c}^{\text{right}}, S_{\ell}^{\text{up}}, S_{\ell}^{\text{down}}\) for \(c \in C\) and \(\ell \in L\). Thus,

\[
S_{c}^{\text{left}} = \{ p \in P | \text{lng}(p) < \text{lng}(c) \},
\]
\[
S_{c}^{\text{right}} = \{ p \in P | \text{lng}(p) > \text{lng}(c) \},
\]
\[
S_{\ell}^{\text{up}} = \{ p \in P | \text{lat}(p) < \text{lat}(\ell) \},
\]
\[
S_{\ell}^{\text{down}} = \{ p \in P | \text{lat}(p) > \text{lat}(\ell) \},
\]

where \(\text{lng()}\) and \(\text{lat()}\) stand for the latitude and longitude respectively. Thus, \(S_{c}^{\text{left}}\) encompasses all photos on the left of \(c\) whereas \(S_{c}^{\text{right}}\) contains the photos on the right of \(c\). Similarly, \(S_{\ell}^{\text{up}}\) groups the photos above \(\ell\) and \(S_{\ell}^{\text{down}}\) the photos below \(\ell\).

We define \(T(\tilde{P})\) as the overall processing time of all photos in a set \(\tilde{P} \subseteq P\) such that

\[
T(\tilde{P}) = \sum_{p \in \tilde{P}} \lambda_p
\]

Finally, let \(c_\prec, c_\succ, \ell_\vee, \ell_\wedge\) represent, respectively, the left, the right, the bottom (inferior), and the top (superior) boundaries of a sub-region \(r\). The photo processing time \(T^{r}\) of a sub-region \(r\) delimited by \(c_\prec, c_\succ, \ell_\vee, \ell_\wedge\) is then given by

\[
T^{r} = T(\{ p \in P | \text{lng}(c_\prec) \leq \text{lng}(p) \leq \text{lng}(c_\succ), \text{lat}(\ell_\vee) \leq \text{lat}(p) \leq \text{lat}(\ell_\wedge) \}).
\]

Equivalently, one may subtract the processing times of the photos outside the sub-region \(r\) from the overall processing time \(T(P)\), i.e.,

\[
T^{r} = T(P) - T(S_{c_\prec}^{\text{left}} \cup S_{c_\succ}^{\text{right}} \cup S_{\ell_\vee}^{\text{down}} \cup S_{\ell_\wedge}^{\text{up}}).
\]
Thus, it is possible to express Eq. (24) as

\[ T^r = T(P) - T(S_{c<}^{\text{left}}) - T(S_{c>}^{\text{right}}) - T(S_{\ell_u}^{\text{down}}) - T(S_{\ell_u}^{\text{up}}) \]
\[ + T(Q_{c<,\ell_u}^1) + T(Q_{c>,\ell_u}^2) + T(Q_{c<,\ell_u}^3) + T(Q_{c>,\ell_u}^4). \]  
(29)

Given the boundaries of a sub-region \( r \), Eq. (29) allows to compute \( T^r \) based only on the processing times of \( S_{c<}^{\text{left}} \), \( S_{c>}^{\text{right}} \), \( S_{\ell_u}^{\text{down}} \), \( S_{\ell_u}^{\text{up}} \), \( Q_{c<,\ell_u}^1 \), \( Q_{c>,\ell_u}^2 \), \( Q_{c<,\ell_u}^3 \), \( Q_{c>,\ell_u}^4 \), and \( T(P) \). By precomputing these values in time \( O(|C| \cdot |L| \cdot |P|) \) (equiv. \( O(|P|^3) \)), there is no need of keeping in memory the index of photos belonging to \( r \). Thus, the photo processing time of any sub-region \( r \) can be computed by (29) in constant time \( O(1) \).

Likewise, the amount of data \( \mu_{r}^{\text{hd}} \) on a certain data transmission \( f^{\text{hd}} \in F \) concerning only the sub-region \( r \) can be computed by a closed form expression. Given the transmission demand \( f^{\text{hd}} \) and a set of photos \( P \), let \( \mu_{r}^{\text{hd}}(P) \) be the amount of data in \( P \) exchanged from drone \( h \) to the drone \( d \), where \( d \) is the photo-processing drone of \( P \), which is expressed as

\[ \mu_{r}^{\text{hd}}(P) = \sum_{p \in P} \theta_{hp} \mu_{p}. \]
(30)

Since the boundaries of the sub-region \( r \) and the transmission demand \( f^{\text{hd}} \) are known, the \( \mu_{r}^{\text{hd}} \) can be computed by the equation

\[ \mu_{r}^{\text{hd}} = \mu_{r}^{\text{hd}}(\{p \in P \mid \text{lng}(c_<) \leq \text{lng}(p) \leq \text{lng}(c_>) \land \text{lat}(\ell_u) \leq \text{lat}(p) \leq \text{lat}(\ell_u)\}) \]
\[ \mu_{r}^{\text{hd}} = \mu_{r}^{\text{hd}}(P) - \mu_{r}^{\text{hd}}(S_{c<}^{\text{left}} \cup S_{c>}^{\text{right}} \cup S_{\ell_u}^{\text{down}} \cup S_{\ell_u}^{\text{up}}) \]
\[ \mu_{r}^{\text{hd}} = \mu_{r}^{\text{hd}}(P) - \mu_{r}^{\text{hd}}(S_{c<}^{\text{left}}) - \mu_{r}^{\text{hd}}(S_{c>}^{\text{right}}) - \mu_{r}^{\text{hd}}(S_{\ell_u}^{\text{down}}) - \mu_{r}^{\text{hd}}(S_{\ell_u}^{\text{up}}) \]
\[ + \mu_{r}^{\text{hd}}(Q_{c<,\ell_u}^1) + \mu_{r}^{\text{hd}}(Q_{c>,\ell_u}^2) + \mu_{r}^{\text{hd}}(Q_{c<,\ell_u}^3) + \mu_{r}^{\text{hd}}(Q_{c>,\ell_u}^4). \]
(31)

The terms within the Eq. (31) can all be precomputed in time \( O(|D|^2 \cdot |C| \cdot |L| \cdot |P|) \) operations (equiv. \( |D|^2 \cdot |P|^3 \)). Thus, the amount of data transmitted can be obtained in constant time given the boundaries of the sub-region \( r \).

Note that both Eqs. (29) and (31) are suited to the way sub-regions are represented by a spatial partition tree. It only suffices to know the boundaries of a sub-region to query the terms in (29) and (31).

Finally, besides minimizing the latest processing time \( T_{\max} \), the VNS local descent must assure that that the maximum allowed transmission time \( \hat{T} \) is respected. For that, the transmission times \( t^{\text{hd}} \) of all active transmission demands \( f^{\text{hd}} \in F \) must be at most equal to \( \hat{T} \) (according to constraints (3)). This means that the algorithm needs to check whether

\[ t^{\text{hd}} \leq \hat{T} \quad \forall f^{\text{hd}} \in F \]
(32)
in order to certify a solution as feasible.
The transmission times $t^{hd}$, for all $(h, d) \in D \times \bar{D}$ are computed by

$$t^{hd} = \frac{\mu^{hd}}{\phi^{hd}} = \frac{\sum_{r \in R_d} \mu_r^{hd}}{\phi^{hd}},$$

(33)

where $R_d$ is the set of sub-regions assigned to the drone $d \in D$. Likewise the processing times $T^r$, it is possible to compute $\mu_r^{hd}$ in constant time. However, computing $\phi^{hd}$ requires the solution of an MMF rate allocation problem. Due to the fixed tree topology, one can solve an MMF rate allocation problem in $O(|D|^2 \times (|D| - 1))$ using e.g. the water filling method (Bertsekas et al. 1992).

### 4.8 Proposed VNS heuristic

According to Algorithm 2, we need to define how to create an initial solution as well as the diversification and intensification steps. These are described in the following.

#### Initial solution

The initial solution is created by (i) randomly building a spatial-partition tree with $\bar{D}$ sub-regions (as defined in Sect. 4.2); and by (ii) randomly assigning the obtained sub-regions to the set of 3D-capable drones $\bar{D}$ according to the reliability factor $\sigma$. Thus, each 3D-capable drone has at least one sub-region to process.

#### Diversification step

The diversification step uses the sub-tree reconstruction neighborhood. The VNS parameter $k$ establishes the depth from which a node will be selected in the spatial-partition tree $T$. More precisely, a node from depth $D(T) - k$ is randomly selected to be reconstructed. Finally, we adopt a $k_{\text{step}}$ equal to one and $k_{\text{max}}$ equal to the $D(T)$ of the current best solution.

#### Intensification step

Our local descent method is a Variable Neighborhood Descent (VND) (Hansen and Mladenović 2001). The proposed VND minimizes $T_{\text{max}}$ by sequentially exploring the sub-region transfer, sub-region swap, and splitting hyperplane reallocation neighborhoods. In order to accelerate the local searches, we limit neighborhood exploration to transfer/swapping/reallocation of sub-regions assigned to the set of drones, named $D_{\text{max}}$, holding the incumbent longest total processing time. That is, $D_{\text{max}}$ comprises the drones whose total photo-processing time is equal to the makespan ($T_{\text{max}}$). By denoting $R_{\bar{D}}$ as the set of all sub-regions assigned to drones $d \in \bar{D} \subseteq D$, the local searches are implemented in such a way that

i. N$_1$ - sub-region transfer: switches sub-regions $r \in R_{D_{\text{max}}}$ to other drones $d \in \bar{D} - D_{\text{max}}$. The size of this neighborhood is bounded by $\Theta(|R_{D_{\text{max}}}| \times |\bar{D} - D_{\text{max}}|)$. 

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ii $\mathcal{N}_2$ - sub-region swap: only considers sub-region swaps $(r_i, r_j)$ such that $r_i \in RD_{\text{max}}$. Consequently, this neighborhood has $\Theta(|RD_{\text{max}}| \times |RD_{\text{max}}^-|)$ neighboring solutions.

iii $\mathcal{N}_3$ - splitting hyperplane reallocation: for each $n \in ND_{\text{max}}$, where $ND_{\text{max}}$ is composed by the parent nodes of leaf nodes representing the sub-regions $r \in RD_{\text{max}}$, this neighborhood reallocates the splitting hyperplanes (i.e., their positions and/or orientations) of non-leaf nodes in the sub-tree rooted at $n$ (denoted $T_n$). Let us define $L_n$ and $C_n$ as the set of enclosed latitudes on axis $L$ and longitudes on axis $C$ at node $n$, respectively. In the worst case, the size of this neighborhood is limited by $O\left(\sum_{n \in ND_{\text{max}}} (|L_n| + |C_n|)^{|NT_n|}\right)$, where $NT_n$ is the set of non-leaf nodes in the sub-tree $T_n$.

Employing such sequence of neighborhoods grants higher priority to adjustments in the sub-region to drone assignments, which are encompassed in smaller neighborhoods that cause less changes to solution’s structure.

Algorithm 3 describes the local search used in this paper. It uses a first improvement exploration strategy, which pursues the first improvement direction in the local descent method. Limited computational experiments revealed that a best improving strategy in which the local descent used the best estimated improving direction was not better than its first improving counterpart. The algorithm requires an initial solution $x$ and a neighborhood $\mathcal{N}_t$ as input and return the local optimum w.r.t. $\mathcal{N}_t$. Also, let us define $\Phi$ as the hash table mapping a set of active transmission demands $\bar{F}$ to its respective MMF rate allocation $\phi$. This data structure accelerates the local search since it prevents the method from solving the same MMF rate allocation problem more than once for the same set of active transmission demands. Finally, let us define the function $T_{\text{max}}(\cdot)$ to return the makespan (i.e., latest completion time) of a solution in its argument.

Algorithm 3 Local search using first improvement search strategy.

Require: $x$ and $\mathcal{N}_t$
1: $\text{Stop} \leftarrow \text{false}$;
2: while not $\text{Stop}$ do
3: \hspace{1em} $\text{Stop} \leftarrow \text{True}$;
4: \hspace{2em} for all $x' \in \mathcal{N}_t(x)$ do
5: \hspace{3em} if $T_{\text{max}}(x') < T_{\text{max}}(x)$ then
6: \hspace{4em} identify the active demands $\bar{F}(x')$;
7: \hspace{3em} if $\not\exists \Phi(\bar{F}(x'))$ then
8: \hspace{4em} $\Phi(\bar{F}(x')) \leftarrow \text{MMF}(\bar{F}(x'))$;
9: \hspace{3em} end if
10: \hspace{3em} $\bar{T} \leftarrow \text{compute communication delays ($\Phi(\bar{F}(x'))$, $\mu(x')$)}$;
11: \hspace{3em} if $\forall h \in [0, \bar{T}] \forall h' \in [0, \bar{T}]$ then
12: \hspace{4em} $\text{Stop} \leftarrow \text{False}$;
13: \hspace{4em} $x \leftarrow x'$;
14: \hspace{4em} break;
15: \hspace{3em} end if
16: \hspace{2em} end if
17: \hspace{1em} end for
18: end while
19: return $x$;
Regarding Algorithm 3, step 1 initializes the flag Stop to false. While the local optimum is not reached (steps 2–18), the local search explores the neighboring solutions \( x' \in \mathcal{N}_t(x) \) (steps 4–17). If for neighboring solution \( x' \) \( T_{\text{max}}(x') < T_{\text{max}}(x) \) (steps 5–16), the algorithm proceeds to checking whether \( x' \) has feasible transmission times associated with it according to expression (32). First, it identifies the active demands \( \bar{F} \) in \( x' \) (step 6). A demand between a pair of drones is active if there exists any data to be exchanged between them (Sect. 2.1). Thus, step 6 checks whether \( \mu_{hd} = \sum_{r \in R_d} \mu_{r}^{hd} > 0 \) for each \( f^{hd} \in F \). Then, it computes the MMF rate allocation problem for \( \bar{F} \) if and only if \( \bar{F} \) is not already a stored key of \( \Phi \) (steps 7–9).

The set of communication delays \( \bar{T} = \{t^{hd} | f^{hd} \in F\} \) is computed in step 10 according to the Eq. (33) given the transmission rates \( \Phi(\bar{F}(x')) \) and the amount of data \( \mu = \{\mu_{hd} | f^{hd} \in F\} \) in solution \( x' \). If \( x' \) has all communication delays \( t^{hd} \in \bar{T} \) smaller or equal to \( \hat{T} \) (steps 11–15), the local search (i) marks \( x \) as sub-optimum (step 12); (ii) sets \( x' \) as the new \( x \) (step 13), and (iii) breaks the for loop to explore the neighborhood w.r.t. the new incumbent \( x \) (step 14). Once reached, the local optimum concerning \( \mathcal{N}_t \) is returned in step 19.

Finally, the proposed VND is summarized in Algorithm 4.

**Algorithm 4 Variable Neighborhood Descend(VND) for the CAPsac**

1: \( \mathcal{N} = \{\text{sub-region transfer, sub-region swap, splitting hyperplane reallocation}\} \)
2: \( t \leftarrow 1 \)
3: if \( \sigma = 1 \) then
4: \( t \leftarrow 2 \)
5: end if
6: repeat
7: \( x'' \leftarrow \text{perform LocalSearch}(x', \mathcal{N}_t) \);
8: if \( T_{\text{max}}(x'') < T_{\text{max}}(x') \) then
9: \( x' \leftarrow x'' \);
10: \( t \leftarrow 1 \)
11: if \( \sigma = 1 \) then
12: \( t \leftarrow 2 \)
13: end if
14: else
15: \( t \leftarrow t + 1 \);
16: end if
17: until \( t > 3 \)

In the steps 2-5 of the algorithm, we choose the initial value of \( t \) according to the reliability factor (\( \sigma \)) used. It is set to \( t = 1 \) for \( \sigma > 1 \) and \( t = 2 \) otherwise. That is, the local search applying the sub-region transfer neighborhood (\( \mathcal{N}_1 \)) is used only for instances with \( \sigma > 1 \) since it cannot find better solutions for \( \sigma = 1 \). This is due to the fact that each 3D-capable drone must have at least one sub-region assigned to it. The VND proceeds by applying the local searches according to the current neighborhood \( \mathcal{N}_t \) until it reaches the local optimum w.r.t. all neighborhoods in \( \mathcal{N} \) (steps 6–17). Each new solution \( x'' \) (obtained at step 7 by Algorithm 3) with better \( T_{\text{max}} \) than \( x' \) is kept and \( t \) is reset likewise steps 2-5 (steps 8–13). Otherwise, \( t \) is increased by one (steps 14–16), and the algorithm iterates.
5 Computational experiments

Our experimental analysis aims to evaluate (i) the effectiveness of the decomposition strategy; (ii) the effectiveness of employing different neighborhoods in our variable neighborhood method; and (iii) the performance of the proposed methods and sensitivity regarding both the reliability factor $\sigma$ and the maximum transmission time allowed $\hat{T}$. We used CPLEX v12.10 as general-purpose integer linear programming solver and C++ as programming language compiled with gcc v5.4.0. All experiments were carried exploiting a single core on a machine powered by an Intel E5-2683 v4 Broadwell 2.1GHz with 20Gb of RAM, and running the CentOS Linux 7.5.1804 OS.

5.1 Instances and notation adopted

Our experimental analysis is carried on the instances proposed in Costa et al. (2020) (available at https://github.com/ds4dm/CAPsac). All instances are based on realistic data and comprise two scenarios: (i) one in which all photos are processed in the same amount of time $\lambda$, named *unweighted*; and (ii) another one in which each photo has a different processing time $\lambda_p$, named *weighted*. The name of the instances follows the notation $X$-$P$-$Y$-$D$-$Z$-$\%$-$W$ where “$X$” is “u” for the unweighted instances and “w” for the weighted instances, “$Y$” stands for the number of photos in the instance, “$Z$” specifies the number of drones in the swarm, and “$W$” informs the percentage of drones able to perform 3D reconstruction. Remark that the number of 3D-capable drones ($|\bar{D}|$) is always equal to $\lfloor Z \times \frac{W}{100} \rfloor$. The characteristics of the tested instances are listed in Table 2.

Optimal makespans are obtained by solving the CAPsac formulation (Costa et al. 2020) with CPLEX within one day of execution. Whenever CPLEX is not able to prove optimality within this time horizon, the best feasible solution obtained in Costa et al. (2020) is reported instead, which is indicated by an “*” in the tables. Average and best results are then computed according to that best known solution.

5.2 Effectiveness of the decomposition strategy

This section assesses the effectiveness of the decomposition strategy exploited by the proposed decomposition-based heuristic.

Tables 3 and 4 report for each instance (named under column “Instance”) the following values regarding the optimal (or best known) solutions:

- the smallest cardinality found among the sub-regions (column $n^*_\ell$);
- the largest cardinality found among the sub-regions (column $n^*_u$);
- the cardinality of the sub-region which yields the makespan (column $n^*_{mks}$);
- the associated makespan value (column OPT).

| Table 2 | Characteristics of the tested instances |
|---------|----------------------------------------|
| Images (P) | 200, 400, 500, 750, 1000 |
| Drones (D) | 5, 7, 10, 15 |
| %3D-capable drones ($\%\bar{D}$) | 50%, 70%, 90% |
Table 3 Percentage deviations of the complete rCAP sac formulation and the decomposition method when solving unweighted instances

| σ = 1, \( \hat{T} = \infty \), stop condition= 86400s | rCAP sac | Decomposition |
|-----------------------------------------------------|----------|---------------|
| Instance | \( n^*_{\ell} \) | \( n^*_{u} \) | \( n^*_{mks} \) | \( \text{opt} \) | \( \text{Dev.(%)} \) | \( \text{sec.} \) | \( n^*_{\ell} \) | \( n^*_{u} \) | \( \text{Dev.(%)} \) | \( \text{sec.} \) |
| u-P200D5% \( \hat{D} \)70 | 65 | 70 | 70 | 1870.40 | 0.00 | 179.37 | 64 | 72 | 0.00 | 0.28 |
| u-P200D7% \( \hat{D} \)50 | 60 | 70 | 70 | 1870.40 | 0.00 | 548.55 | 64 | 72 | 0.00 | 0.50 |
| u-P400D5% \( \hat{D} \)70 | 132 | 135 | 135 | 3607.20 | 0.00 | 6996.32 | 128 | 140 | 0.00 | 1.04 |
| u-P400D7% \( \hat{D} \)50 | 130 | 135 | 135 | 3607.20 | 0.00 | 11,355.80 | 128 | 140 | 0.00 | 1.55 |
| u-P200D5% \( \hat{D} \)90 | 50 | 50 | 50 | 1336.00 | 0.00 | 244.25 | 49 | 51 | 0.00 | 0.11 |
| u-P200D7% \( \hat{D} \)70 | 50 | 50 | 50 | 1336.00 | 0.00 | 256.03 | 49 | 51 | 0.00 | 0.23 |
| u-P400D5% \( \hat{D} \)90 | 100 | 100 | 100 | 2672.00 | 0.00 | 9346.54 | 99 | 102 | 0.00 | 0.46 |
| u-P400D7% \( \hat{D} \)70 | 100 | 100 | 100 | 2672.00 | 0.00 | 45,059.71 | 99 | 102 | 0.00 | 0.55 |
| u-P200D10% \( \hat{D} \)50 | 40 | 40 | 40 | 1068.80 | 0.00 | 1838.53 | 39 | 42 | 0.00 | 1.46 |
| u-P400D10% \( \hat{D} \)50 | 80 | 80 | 80 | 2137.60 | 0.00 | 43171.40 | 78 | 81 | 0.00 | 5.44 |
| u-P200D7% \( \hat{D} \)90 | 30 | 34 | 34 | 908.48 | 0.00 | 1901.86 | 28 | 38 | 0.00 | 8.00 |
| u-P400D7% \( \hat{D} \)90 | 60 | 68 | 68 | *1816.96 | 32.35 | 86,400.00 | 64 | 70 | 1.47 | 13.28 |
| u-P200D10% \( \hat{D} \)70 | 24 | 30 | 30 | *801.60 | 33.33 | 86,400.00 | 27 | 32 | 0.00 | 115.18 |
| u-P400D10% \( \hat{D} \)70 | 50 | 60 | 60 | *1603.20 | 300.00 | 86,400.00 | 54 | 63 | – 3.33 | 2013.98 |
| u-P500D15% \( \hat{D} \)50 | 70 | 72 | 72 | 1923.84 | – | 86,400.00 | 69 | 75 | 0.00 | 333.59 |
| u-P750D15% \( \hat{D} \)50 | 108 | 117 | 117 | *3126.24 | – | 86,400.00 | 104 | 110 | – 7.69 | 262.73 |
| u-P1000D15% \( \hat{D} \)50 | 143 | 144 | 144 | *3847.68 | – | 86,400.00 | 136 | 145 | 0.00 | 7434.67 |
| u-P200D10% \( \hat{D} \)90 | 18 | 24 | 24 | *641.28 | 8.33 | 86,400.00 | 21 | 25 | 0.00 | 3366.98 |
| u-P400D10% \( \hat{D} \)90 | 42 | 45 | 45 | 1202.40 | – | 86,400.00 | 42 | 46 | 0.00 | 3262.06 |
Table 4 Percentage deviations of the complete rCAPsac formulation and the decomposition method when solving weighted instances

| Instance    | $n^*_e$ | $n^*_u$ | $n^*_mks$ | OPT  | rCAPsac Dev.(%) | sec. | Decomposition | $n^*_e$ | $n^*_u$ | Dev.(%) | sec. |
|-------------|---------|---------|-----------|------|----------------|------|---------------|---------|---------|---------|------|
| w-P200D5%D70 | 65      | 70      | 70        | 1886.98 | 0.00          | 242.54 | 64            | 72      | 0.00    | 0.36    |      |
| w-P200D7%D50 | 60      | 70      | 70        | 1885.31 | 0.00          | 784.93 | 64            | 72      | 3.26    | 0.39    |      |
| w-P400D5%D70 | 130     | 135     | 135       | 3735.32 | 0.00          | 10,209.88 | 128           | 140     | 0.00    | 1.21    |      |
| w-P400D7%D50 | 130     | 135     | 135       | 3773.12 | 0.00          | 5987.39 | 128           | 140     | 0.00    | 1.93    |      |
| w-P200D5%D90 | 50      | 50      | 50        | 1409.48 | 0.00          | 212.44 | 49            | 51      | 0.00    | 0.30    |      |
| w-P200D7%D70 | 50      | 50      | 50        | 1399.82 | 0.00          | 802.39 | 49            | 51      | 0.00    | 0.56    |      |
| w-P400D5%D90 | 100     | 100     | 100       | 2812.93 | 0.00          | 21,189.45 | 99            | 102     | 0.00    | 0.57    |      |
| w-P400D7%D70 | 100     | 100     | 100       | 2791.66 | 0.00          | 40,872.49 | 99            | 102     | 0.00    | 1.49    |      |
| w-P200D10%D50 | 40      | 40      | 40        | 1121.89 | 0.00          | 9106.51 | 39            | 42      | 0.00    | 13.92   |      |
| w-P400D10%D50 | 72      | 88      | 88        | 2290.48 | 0.00          | 40,443.02 | 78            | 81      | 1.15    | 34.01   |      |
| w-P200D7%D90 | 30      | 35      | 35        | 944.06  | 0.00          | 8669.90 | 28            | 38      | 0.00    | 17.77   |      |
| w-P400D7%D90 | 60      | 75      | 75        | *1909.18 | 19.17 | 86,400.00 | 63          | 72      | −0.94   | 25.17   |      |
| w-P200D10%D70 | 26      | 30      | 30        | 820.40  | 0.00          | 62,142.01 | 27            | 32      | 0.29    | 1086.76 |      |
| w-P400D10%D70 | 56      | 63      | 60        | *1671.12 | 123.18 | 86,400.00 | 55          | 62      | −2.30   | 7232.05 |      |
| w-P500D15%D50 | 66      | 78      | 72        | *2028.11 | –    | 86,400.00 | 69          | 75      | −2.40   | 1791.89 |      |
| w-P750D15%D50 | 96      | 119     | 117       | *3138.73 | –    | 86,400.00 | 102         | 112     | −4.91   | 2562.77 |      |
| w-P1000D15%D50 | 132     | 160     | 160       | *4343.26 | –    | 86,400.00 | 135         | 147     | −9.80   | 7762.67 |      |
| w-P200D10%D90 | 18      | 24      | 24        | *665.54  | 20.03 | 86,400.00 | 19          | 27      | −2.85   | 15,655.03 |      |
| w-P400D10%D90 | 40      | 50      | 50        | *1302.82 | –    | 86,400.00 | 39          | 49      | −2.89   | 9645.66 |      |
Further, we report the percentage deviations (columns “Dev.(%)”) achieved by the complete rCAPsac formulation and the decomposition method with respect to the best known makespan. We also report the total computing CPU times spent by both methods under column (“sec”). Finally, concerning the decomposition-based heuristic, we report the last interval \([n_L, n_u]\) used before the algorithm is halted by its stopping condition. For all experiments, we set a time limit of 2 hours for the MIP formulation solved by CPLEX in each iteration of the decomposition-based heuristic, and 24 hours as maximum time for solving rCAPsac by CPLEX. A trace (“−”) is reported whenever CPLEX is not able to find a feasible solution within this time limit.

The decomposition strategy proves effective and yields equivalent or better deviation values than solving the complete rCAPsac formulation by CPLEX within the defined time limit. In fact, it is able to find a best known makespan for several instances, as indicated by the negative deviations in the table. However, for a couple of instances, i.e., (“u−P400D7%D90”, “w−P200D7%D50”, “w−P400D10%D50”, “w−P200D10%D70”), the decomposition algorithm is not able to attain the best known solutions obtained in Costa et al. (2020). For those cases, we observe that the intervals \([n_L, n_u]\) must be increased to allow subsets of photos whose cardinalities are out of the defined bounds. For example, for instance u−P400D7%D90, the best known solution contains a sub-region with 60 photos while the last iteration of the decomposition method was executed for \([n_L, n_u] = [64, 70]\). Finally, the decomposition method demands significantly smaller execution times than solving the complete rCAPsac formulation by CPLEX as well as, in general, the formulations in Costa et al. (2020).

### 5.3 Effectiveness of the neighborhoods

To assess the sequence of neighborhoods in our proposed VND, we compare the performance of distinct neighborhood configurations—\(N_1, N_1 + N_2, N_1 + N_3, N_1 + N_2 + N_3\)—within our proposed VNS method, where \(N_1\) refers to the sub-region transfer neighborhood, \(N_2\) refers to sub-region swap neighborhood, and \(N_3\) refers to the splitting hyperplane reallocation neighborhood.

Tables 5 and 6 report the results of the unweighted and weighted instances, respectively. For each “Instance” and reliability factor “\(\sigma\)”, the column “MIN.” presents the shortest makespan found across all neighborhood configurations. The average percentage deviation from “MIN.” achieved by each neighborhood configuration is reported in columns “Avg(%)”.

The results are computed for 20 executions of the VNS-based heuristics within 300s of CPU execution time. These experiments are conducted over different reliability factors \(\sigma > 1\) so as that the sub-region transfer neighborhood could be evaluated. Moreover, we do not present results for instances with fewer than five 3D-capable drones since all neighborhood configurations performed equivalently.

We can observe that the splitting hyperplane reallocation neighborhood (\(N_3\)) is an important neighborhood for improving the quality of the solutions obtained by our VNS. This is not surprising given that it is the only one able to change photo-processing times of sub-regions. The best results are usually obtained when including
Table 5  Percentage deviations of employing distinct neighborhoods in the VND when solving unweighted instances

| Instance | $\sigma$ | MIN. | $T = \infty$, stop condition= 300s | N1 $\bar{D}$ | $N_1 + N_2$ $\bar{D}$ | $N_1 + N_3$ $\bar{D}$ | $N_1 + N_2 + N_3$ $\bar{D}$ |
|----------|---------|------|---------------------------------|----------------|-----------------|-----------------|-----------------|
| u-P200D10% $\bar{D}50$ | 2       | 2137.60 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 3       | 3206.40 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 4       | 4275.20 | 0.00 | 0.00 | 0.00 | 0.00 |
| u-P400D10% $\bar{D}50$ | 2       | 4275.20 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 3       | 6412.80 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 4       | 8550.40 | 0.00 | 0.00 | 0.00 | 0.00 |
| u-P200D7% $\bar{D}90$ | 2       | 1790.24 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 3       | 2672.00 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 4       | 3580.48 | 0.00 | 0.00 | 0.00 | 0.00 |
| u-P400D7% $\bar{D}90$ | 2       | 3580.48 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 3       | 5344.00 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 4       | 7134.24 | 0.00 | 0.00 | 0.00 | 0.00 |
| u-P200D10% $\bar{D}70$ | 2       | 1549.76 | 0.86 | 0.09 | 0.00 | 0.00 |
|          | 3       | 2297.92 | 0.64 | 0.35 | 0.35 | 0.06 |
|          | 4       | 3072.80 | 0.04 | 0.17 | 0.09 | 0.04 |
| u-P400D10% $\bar{D}70$ | 2       | 3072.80 | 0.43 | 0.52 | 0.04 | 0.00 |
|          | 3       | 4595.84 | 0.52 | 1.98 | 0.23 | 0.12 |
|          | 4       | 6118.88 | 0.39 | 0.44 | 0.39 | 0.31 |
| u-P500D15% $\bar{D}50$ | 2       | 3847.68 | 0.42 | 0.62 | 0.00 | 0.00 |
|          | 3       | 5744.80 | 0.56 | 1.23 | 0.23 | 0.05 |
|          | 4       | 7641.92 | 0.89 | 0.77 | 0.70 | 0.61 |
| u-P750D15% $\bar{D}50$ | 2       | 5744.80 | 1.02 | 1.26 | 0.44 | 0.47 |
|          | 3       | 8630.56 | 1.24 | 0.46 | 0.26 | 0.20 |
|          | 4       | 11498.60 | 0.64 | 0.42 | 0.23 | 0.16 |
| u-P1000D15% $\bar{D}50$ | 2       | 7641.92 | 1.17 | 1.01 | 0.65 | 0.52 |
|          | 3       | 11462.88 | 1.42 | 0.65 | 0.37 | 1.25 |
|          | 4       | 15283.84 | 0.62 | 0.64 | 0.52 | 0.43 |
| u-P200D10% $\bar{D}90$ | 2       | 1202.40 | 0.56 | 0.11 | 0.00 | 0.11 |
|          | 3       | 1790.24 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 4       | 2378.08 | 1.12 | 1.01 | 1.12 | 0.96 |
| u-P400D10% $\bar{D}90$ | 2       | 2404.80 | 0.22 | 0.11 | 0.00 | 0.00 |
|          | 3       | 3580.48 | 0.00 | 0.00 | 0.00 | 0.00 |
|          | 4       | 4756.16 | 0.79 | 0.56 | 0.53 | 0.45 |
| Average  |       |        | 0.41 | 0.38 | 0.19 | 0.17 |
Table 6  Percentage deviations of employing distinct neighborhoods in the VND when solving weighted instances

| Instance  | σ | MIN. | $\hat{T} = \infty$, stop condition= 300s | VNS | $N_1$ | $N_1 + N_2$ | $N_1 + N_3$ | $N_1 + N_2 + N_3$ |
|-----------|---|------|-----------------------------------------|-----|--------|------------|------------|------------------|
|            |   |      |                                         | VNS | Avg. (%) | Avg. (%)    | Avg. (%)    | Avg. (%)         |
| w-P200D10% $\bar{D}50$ | 2 | 2212.64 | 0.18 | 0.16 | 0.11 | 0.06 |
|            | 3 | 3314.91 | 0.16 | 0.18 | 0.15 | 0.16 |
|            | 4 | 4436.74 | 0.03 | 0.02 | 0.02 | 0.02 |
| w-P400D10% $\bar{D}50$ | 2 | 4433.53 | 0.19 | 0.22 | 0.08 | 0.09 |
|            | 3 | 6648.11 | 0.22 | 0.24 | 0.13 | 0.15 |
|            | 4 | 8901.19 | 0.01 | 0.03 | 0.03 | 0.02 |
| w-P200D7% $\bar{D}90$ | 2 | 1816.30 | 0.14 | 0.16 | 0.04 | 0.03 |
|            | 3 | 2724.19 | 0.00 | 0.00 | 0.00 | 0.00 |
|            | 4 | 3632.45 | 0.06 | 0.06 | 0.03 | 0.02 |
| w-P400D7% $\bar{D}90$ | 2 | 3697.18 | 0.11 | 0.11 | 0.02 | 0.02 |
|            | 3 | 5545.48 | 0.00 | 0.00 | 0.00 | 0.00 |
|            | 4 | 7394.18 | 0.06 | 0.06 | 0.03 | 0.03 |
| w-P200D10% $\bar{D}70$ | 2 | 1577.87 | 0.97 | 0.84 | 0.40 | 0.30 |
|            | 3 | 2365.78 | 0.63 | 0.49 | 0.43 | 0.26 |
|            | 4 | 3155.85 | 0.66 | 0.40 | 0.34 | 0.33 |
| w-P400D10% $\bar{D}70$ | 2 | 3164.05 | 0.65 | 0.91 | 0.47 | 0.33 |
|            | 3 | 4747.60 | 0.52 | 0.35 | 0.37 | 0.18 |
|            | 4 | 6326.53 | 0.57 | 0.40 | 0.35 | 0.32 |
| w-P500D15% $\bar{D}50$ | 2 | 3902.67 | 0.79 | 0.68 | 0.27 | 0.27 |
|            | 3 | 5852.50 | 1.33 | 1.18 | 0.27 | 0.21 |
|            | 4 | 7801.65 | 0.72 | 0.54 | 0.31 | 0.22 |
| w-P750D15% $\bar{D}50$ | 2 | 5843.52 | 0.67 | 0.99 | 0.28 | 0.27 |
|            | 3 | 8762.21 | 0.58 | 0.44 | 0.32 | 0.24 |
|            | 4 | 11679.59 | 0.70 | 0.55 | 0.32 | 0.28 |
| w-P1000D15% $\bar{D}50$ | 2 | 7724.25 | 1.08 | 1.04 | 0.34 | 0.37 |
|            | 3 | 11590.54 | 0.58 | 1.11 | 0.32 | 0.21 |
|            | 4 | 15451.40 | 0.58 | 0.36 | 0.19 | 0.26 |
| w-P200D10% $\bar{D}90$ | 2 | 1230.42 | 1.10 | 1.08 | 0.66 | 0.74 |
|            | 3 | 1838.98 | 0.13 | 0.16 | 0.02 | 0.02 |
|            | 4 | 2454.44 | 0.51 | 0.28 | 0.42 | 0.22 |
| w-P400D10% $\bar{D}90$ | 2 | 2463.59 | 1.05 | 0.69 | 0.62 | 0.48 |
|            | 3 | 3688.59 | 0.10 | 0.13 | 0.01 | 0.01 |
|            | 4 | 4922.46 | 0.56 | 0.26 | 0.30 | 0.17 |
| Average   |   |       | 0.47 | 0.43 | 0.23 | 0.19 |
the neighborhoods $N_1$ and $N_2$ before $N_3$ (i.e., $N_1 + N_2 + N_3$ configuration). This happens since the $N_1 + N_2 + N_3$ configuration adjusts both the sub-region to drone assignments and the sub-region’s photo processing times. In fact, this configuration yields the best average deviations for 57 out of 66 cases (including unweighted and weighted instances), i.e., 86.36% of the cases. Therefore, we will consider hereafter the configuration $N_1 + N_2 + N_3$ within our VNS heuristic.

5.4 Comparison of the proposed methods

This section evaluates the performance of the proposed heuristics as well as their sensitivity with respect to the reliability factor $\sigma$ and to the maximum allowed transmission time $\hat{T}$. For the decomposition-based heuristic (named “Decomposition”), we set a time limit of 2 hours for the MIP formulation solved in each iteration and the overall execution time of the decomposition is limited to 24 hours (i.e., 86,400s). In contrast, the VNS was allowed to run for 300 seconds. These time limit values were chosen to provide the methods with enough time to achieve near-optimal solutions for the comparison analysis.

The makespan values in seconds (columns “$T_{\text{max}}$”) obtained by the proposed methods for the unweighted and weighted instances are reported in Tables 7 and 8, respectively. Results are reported for each instance for various values of $\sigma$, ranging from 1 to $|\overline{D}| - 1$ (presented under column labelled “$\sigma$”). A trace (“−”) is placed whenever a method cannot find a feasible solution until before being halted. For this first series of experiments the communication constraints were relaxed, i.e., $\hat{T}$ is set to a value sufficiently large. Both tables report the instance employed at each row (column “Instance”), the optimal/best known makespan for an instance configuration (column “OPT”), the CPU time (in seconds) spent by CPLEX and the decomposition heuristic when solving an instance (columns “sec.”), the best and average makespan values found by the VNS heuristic out of 20 executions (column “Best $T_{\text{max}}$” and “Avg $T_{\text{max}}$”, respectively), and the average CPU time (in seconds) spent by the VNS heuristic to reach the solution returned at the end of its execution (column “Avg.(s)”).

We observe from the tables that both decomposition and VNS methods performed well for all instances tested. In fact, both methods find solutions when the exact method fails. In general, it appears that increasing the reliability factor does not affect the performance of the proposed heuristic methods.

Furthermore, the decomposition could reach the optimum or improve (values indicated in bold) the best known makespan (cases with “∗” in the column “OPT”) in several instances, except for the cases “u-P400D7%$D90” ($\sigma = 1$), “w-P200D7%$D50” ($\sigma = 1$), “w-P400D5%$\overline{D}70$” ($\sigma = 2$), “w-P200D5%$\overline{D}90$” ($\sigma = 2$), “w-P400D5%$\overline{D}90$” ($\sigma = 2$), “w-P400D5%$\overline{D}90$” ($\sigma = 2$), “w-P400D10%$D50$” ($\sigma = 1$), “w-P200D7%$\overline{D}70$”($\sigma = 2$), “w-P400D7%$\overline{D}70$”($\sigma = 2$), and “w-P200D10%$D70$” ($\sigma = 1$) for which more iterations without improving the objective function need to be done by the method in order to be able to obtain best known solutions (as discussed in Sect. 5.2). Otherwise, it kept the percentage deviations w.r.t. the optimum below 1.47% (“u-P400D7%$\overline{D}90$ and $\sigma = 1$”) for the unweighted instances and below 3.26% (“w-P200D7%$\overline{D}50$” and $\sigma = 1$) for the weighted cases. Moreover, the decomposition method usually requires
Table 7 Makespan($T_{\text{max}}$) in seconds when solving unweighted instances and varying $\sigma$

| Instance   | $\sigma$ | OPT  | sec. | Decomposition | VNS          |
|------------|----------|------|------|---------------|--------------|
|            |          |      |      | $T_{\text{max}}$ | Stop condition = 300s |
|            |          |      |      | $T_{\text{max}}$ | Best $T_{\text{max}}$ | Avg.$T_{\text{max}}$ | Avg.$T_{\text{max}}$(s) |
| u-P200D5%D70 | 1        | 1870.40 | 15.05 | 1870.40 | 1870.40 | 1870.40 | 0.26 | 0.01 |
|            | 2        | 3607.20 | 112.83 | 3607.20 | 3607.20 | 3607.20 | 0.31 | 0.01 |
| u-P200D7%D50 | 1        | 1870.40 | 10.25 | 1870.40 | 1870.40 | 1870.40 | 0.42 | 0.01 |
|            | 2        | 3607.20 | 100.99 | 3607.20 | 3607.20 | 3607.20 | 0.59 | 0.01 |
| u-P400D5%D70 | 1        | 3607.20 | 41.22 | 3607.20 | 3607.20 | 3607.20 | 1.03 | 0.01 |
|            | 2        | 7214.40 | 539.52 | 7214.40 | 7214.40 | 7214.40 | 1.14 | 0.02 |
| u-P400D7%D50 | 1        | 3607.20 | 68.28 | 3607.20 | 3607.20 | 3607.20 | 1.50 | 0.02 |
|            | 2        | 7214.40 | 1637.54 | 7214.40 | 7214.40 | 7214.40 | 1.54 | 0.03 |
| u-P200D5%D90 | 1        | 1336.00 | 7.06  | 1336.00 | 1336.00 | 1336.00 | 0.10 | 0.01 |
|            | 2        | 2672.00 | 1680.46 | 2672.00 | 2672.00 | 2672.00 | 0.12 | 0.01 |
|            | 3        | 4008.00 | 870.76  | 4008.00 | 4008.00 | 4008.00 | 0.11 | 0.05 |
| u-P200D7%D70 | 1        | 1336.00 | 6.41  | 1336.00 | 1336.00 | 1336.00 | 0.19 | 0.04 |
|            | 2        | 2672.00 | 2296.34 | 2672.00 | 2672.00 | 2672.00 | 0.26 | 0.01 |
|            | 3        | 4008.00 | 1150.97 | 4008.00 | 4008.00 | 4008.00 | 0.21 | 0.07 |
| u-P400D5%D90 | 1        | 2672.00 | 35.02  | 2672.00 | 2672.00 | 2672.00 | 0.47 | 0.04 |
|            | 2        | *5344.00 | 86400.00 | 5344.00 | 5344.00 | 5344.00 | 0.44 | 0.01 |
|            | 3        | 8016.00 | 4593.69  | 8016.00 | 8016.00 | 8016.00 | 0.46 | 4.66 |
| u-P400D7%D70 | 1        | 2672.00 | 59.71  | 2672.00 | 2672.00 | 2672.00 | 0.58 | 0.12 |
|            | 2        | *5344.00 | 86400.00 | 5344.00 | 5344.00 | 5344.00 | 0.76 | 0.01 |
|            | 3        | 8016.00 | 5954.75  | 8016.00 | 8016.00 | 8016.00 | 0.79 | 6.29 |
| u-P200D10%D50 | 1        | 1068.80 | 38.94  | 1068.80 | 1068.80 | 1068.80 | 1.46 | 0.08 |
|            | 2        | 2137.60 | 20602.10 | 2137.60 | 2137.60 | 2137.60 | 0.71 | 0.15 |
| Instance | σ   | OPT  | sec.  | Decomposition | VNS     |
|----------|-----|------|-------|---------------|---------|
|          |     |      |       | Time limit = 86400s | Stop condition = 300s |
|          |     |      |       | $T_{\text{max}}$ | $T_{\text{max}}$ | Avg. $T_{\text{max}}$ | Avg. (s) |
| 3        | *3286.56 | 86400.00 | 3206.40 | 1.79 | 3206.40 | 3206.40 | 0.17 |
| 4        | 4275.20 | 56226.05 | 4275.20 | 8.87 | 4275.20 | 4275.20 | 2.21 |
| u-P400D10% $\bar{D}$50 |     |       |       |       |       |       |       |
| 1        | 2137.60 | 107.68 | 2137.60 | 5.75 | 2137.60 | 2137.60 | 0.14 |
| 2        | *4275.20 | 86400.00 | 4275.20 | 2.87 | 4275.20 | 4275.20 | 0.24 |
| 3        | *6519.68 | 86400.00 | 6412.80 | 3.64 | 6412.80 | 6412.80 | 0.90 |
| 4        | *8550.40 | 86400.00 | 8550.40 | 2.93 | 8550.40 | 8550.40 | 5.27 |
| u-P200D7% $\bar{D}$90 |     |       |       |       |       |       |       |
| 1        | 908.48 | 49.52 | 908.48 | 8.12 | 908.48 | 908.48 | 1.73 |
| 2        | *1816.96 | 86400.00 | 1790.24 | 17.56 | 1790.24 | 1790.24 | 0.12 |
| 3        | *2672.00 | 86400.00 | 2672.00 | 14.76 | 2672.00 | 2672.00 | 0.01 |
| 4        | *3607.20 | 86400.00 | 3580.48 | 23.89 | 3580.48 | 3580.48 | 0.46 |
| 5        | *4542.40 | 86400.00 | 4488.96 | 444.98 | 4488.96 | 4526.37 | 21.96 |
| u-P400D7% $\bar{D}$90 |     |       |       |       |       |       |       |
| 1        | *1816.96 | 86400.00 | 1843.68 | 13.32 | 1816.96 | 1816.96 | 1.11 |
| 2        | *3580.48 | 86400.00 | 3580.48 | 14.69 | 3580.48 | 3580.48 | 0.31 |
| 3        | *5477.60 | 86400.00 | 5344.00 | 15.52 | 5344.00 | 5344.00 | 0.01 |
| 4        | *7748.80 | 86400.00 | 7160.96 | 2449.49 | 7134.24 | 7134.24 | 8.05 |
| 5        | *10688.00 | 86400.00 | 8924.48 | 64.40 | 8924.48 | 9018.00 | 63.95 |
| u-P200D10% $\bar{D}$70 |     |       |       |       |       |       |       |
| 1        | *801.60 | 86400.00 | 801.60 | 108.54 | 801.60 | 801.60 | 1.85 |
| 2        | *1603.20 | 86400.00 | 1549.76 | 1136.86 | 1549.76 | 1549.76 | 10.43 |
| 3        | –      | –     | 2297.92 | 30.52 | 2297.92 | 2297.92 | 74.77 |
| 4        | –      | –     | 3072.80 | 55.08 | 3072.80 | 3074.14 | 40.66 |
| 5        | *4328.64 | 86400.00 | 3847.68 | 597.09 | 3847.68 | 3847.68 | 17.27 |
| 6        | *5718.08 | 86400.00 | 4595.84 | 184.89 | 4649.28 | 4670.66 | 51.14 |
| Instance          | $\sigma$ | OPT  | Time limit | $T_{\text{max}}$ | Stop condition | $T_{\text{max}}$ | Avg. $T_{\text{max}}$ | Avg. (s) |
|-------------------|----------|------|------------|------------------|----------------|------------------|-----------------------|----------|
| u-P400D10% $\tilde{D}$70 | 1        | *1603.20 | 86400.00   | **1549.76**      | 2183.17        | **1549.76**      | 1563.12                | 64.66    |
|                   | 2        | –     | 3072.80    | 16065.91         | 3072.80        | 3072.80         | 3072.80                | 56.34    |
|                   | 3        | –     | 4595.84    | 296.30           | 4595.84        | 4602.52         | 70.87                 |          |
|                   | 4        | –     | 6118.88    | 7915.51          | 6118.88        | 6137.58         | 34.18                 |          |
|                   | 5        | –     | 7668.64    | 21612.06         | 7641.92        | 7686.01         | 62.77                 |          |
|                   | 6        | –     | 9191.68    | 5877.11          | 9218.40        | 9352.00         | 79.74                 |          |
| u-P500D15% $\tilde{D}$50 | 1        | 1923.84 | 16185.19   | 1923.84          | 329.20         | 1923.84         | 1959.91                | 97.38    |
|                   | 2        | –     | 3847.68    | 14417.75         | 3847.68        | 3847.68         | 3847.68                | 13.77    |
|                   | 3        | –     | 5744.80    | 21621.88         | 5744.80        | 5744.80         | 5744.80                | 76.45    |
|                   | 4        | –     | 7641.92    | 14412.51         | 7641.92        | 7688.68         | 36.03                 |          |
|                   | 5        | –     | 9565.76    | 14413.03         | 9565.76        | 9611.18         | 27.35                 |          |
|                   | 6        | –     | 11489.60   | 8106.20          | 11489.60       | 11696.68        | 58.85                 |          |
| u-P750D15% $\tilde{D}$50 | 1        | *3126.24 | 86400.00   | **2885.76**      | 270.72         | **2885.76**      | **2933.86**              | 52.65    |
|                   | 2        | –     | 5744.80    | 17315.08         | 5744.80        | 5771.52         | 78.88                 |          |
|                   | 3        | –     | 8657.28    | 14419.11         | 8630.56        | 8649.26         | 91.33                 |          |
|                   | 4        | –     | 11462.88   | 14419.09         | 11489.60       | 11508.30        | 76.75                 |          |
|                   | 5        | –     | 14348.64   | 14421.91         | 14428.80       | 14434.14        | 35.91                 |          |
|                   | 6        | –     | 17234.40   | 14418.32         | 17234.40       | 17485.57        | 37.64                 |          |
| u-P1000D15% $\tilde{D}$50 | 1        | *3847.68 | 86400.00   | 3847.68          | 7424.05        | 3847.68         | 3950.55                | 64.74    |
|                   | 2        | –     | 7695.36    | 14439.34         | 7641.92        | 7682.00         | 36.38                 |          |
|                   | 3        | –     | 86400.00   | –               | 11489.60       | 11605.83        | 67.56                 |          |
|                   | 4        | –     | 86400.00   | –               | 15283.84       | 15349.30        | 76.41                 |          |
Table 7 continued

Parameters: $\hat{t} = \infty$

| Instance | $\sigma$ | OPT | sec. | $T_{max}$ | sec. | $T_{max}$ Avg. | VNS | Stop condition = 300s | Avg. $T_{max}$ | Avg. (s) |
|----------|---------|-----|------|-----------|------|----------------|-----|----------------------|---------------|---------|
| u-P200D10% $\bar{D}90$ | 1 | *641.28 | 86400.00 | 641.28 | 3201.67 | 641.28 | 641.28 | 17.94 |
| 2 | – | – | 1202.40 | 3589.18 | 1202.40 | 1203.74 | 10.09 |
| 3 | *2030.72 | 86400.00 | 1790.24 | 297.44 | 1790.24 | 1790.24 | 1790.24 | 1.15 |
| 4 | – | – | 2404.80 | 10993.74 | 2378.08 | 2400.79 | 1.89 |
| 5 | – | – | 2992.64 | 1200.84 | 2992.64 | 2992.64 | 12.98 |
| 6 | – | – | 3580.48 | 7224.15 | 3580.48 | 3580.48 | 0.21 |
| 7 | – | – | 4168.32 | 2382.04 | 4168.32 | 4217.75 | 43.90 |
| 8 | – | – | 4782.88 | 7820.39 | 4809.60 | 4817.62 | 13.23 |
| u-P400D10% $\bar{D}90$ | 1 | 1202.40 | 17470.88 | 1202.40 | 3376.48 | 1202.40 | 1241.14 | 131.36 |
| 2 | – | – | 2378.08 | 903.16 | 2404.80 | 2404.80 | 8.97 |
| 3 | – | – | 3580.48 | 262.56 | 3580.48 | 3580.48 | 0.08 |
| 4 | – | – | 4756.16 | 991.95 | 4756.16 | 4777.54 | 58.81 |
| 5 | – | – | 5958.56 | 255.96 | 5958.56 | 5975.93 | 59.92 |
| 6 | – | – | 7134.24 | 862.01 | 7134.24 | 7134.24 | 1.96 |
| 7 | – | – | 8336.64 | 183.60 | 8336.64 | 8379.39 | 69.29 |
| 8 | – | – | 9512.32 | 240.45 | 9619.20 | 9619.20 | 31.49 |
Table 8: Makespan ($T_{max}$) in seconds when solving weighted instances and varying $\sigma$

| Instance    | $\sigma$ | OPT | sec. | Decomposition | VNS |
|-------------|---------|-----|------|---------------|-----|
|             |         |     |      | $T_{max}$     |     |
|             |         |     |      | sec.          |     |
|             |         |     |      | Best $T_{max}$|     |
|             |         |     |      | Avg. $T_{max}$|     |
|             |         |     |      | Avg.(s)       |     |
| w-P200D5%$\tilde{D}$70 | 1 | 1886.98 | 15.84 | 1886.98 | 0.39 | 1886.98 | 1886.98 | 0.01 |
|             | 2 | 3699.67 | 172.46 | 3699.67 | 0.37 | 3699.67 | 3699.67 | 0.02 |
| w-P200D7%$\tilde{D}$50 | 1 | 1885.31 | 22.74 | 1946.84 | 0.41 | 1885.31 | 1885.31 | 0.03 |
|             | 2 | 3740.19 | 653.34 | 3740.19 | 0.47 | 3740.19 | 3740.19 | 0.04 |
| w-P400D5%$\tilde{D}$70 | 1 | 3735.32 | 79.99 | 3735.32 | 1.20 | 3735.32 | 3735.32 | 0.01 |
|             | 2 | 7457.00 | 1988.79 | 7458.55 | 1.22 | 7458.55 | 7458.55 | 0.19 |
| w-P400D7%$\tilde{D}$50 | 1 | 3773.12 | 94.42 | 3773.12 | 2.47 | 3773.12 | 3773.12 | 0.03 |
|             | 2 | 7436.46 | 1464.54 | 7436.46 | 1.53 | 7436.46 | 7436.46 | 0.26 |
| w-P200D5%$\tilde{D}$90 | 1 | 1409.48 | 212.44 | 1409.48 | 0.26 | 1409.48 | 1409.48 | 0.08 |
|             | 2 | 2735.45 | 3429.09 | 2736.79 | 0.33 | 2735.45 | 2736.23 | 0.90 |
| w-P200D7%$\tilde{D}$90 | 1 | 1399.82 | 453.18 | 1399.82 | 0.55 | 1399.82 | 1399.82 | 0.10 |
|             | 2 | 2724.20 | 2647.32 | 2726.87 | 0.50 | 2724.20 | 2725.04 | 0.40 |
| w-P400D5%$\tilde{D}$90 | 1 | 2812.93 | 925.85 | 2812.93 | 0.56 | 2812.93 | 2812.93 | 0.14 |
|             | 2 | *5555.98 | 86400.00 | 5568.43 | 0.75 | 5555.12 | 5555.30 | 13.20 |
| w-P400D7%$\tilde{D}$70 | 1 | 2791.66 | 605.48 | 2791.66 | 1.74 | 2791.66 | 2791.66 | 0.15 |
|             | 2 | *5545.75 | 86400.00 | 5555.17 | 1.37 | 5545.49 | 5545.51 | 3.73 |
| w-P400D7%$\tilde{D}$90 | 1 | 2791.66 | 605.48 | 2791.66 | 1.74 | 2791.66 | 2791.66 | 0.15 |
|             | 2 | *5545.75 | 86400.00 | 5555.17 | 1.37 | 5545.49 | 5545.51 | 3.73 |
|             | 3 | *8346.83 | 86400.00 | 8346.83 | 1.43 | 8346.83 | 8346.83 | 13.77 |
### Table 8 continued

| Instance     | \( \sigma \) | OPT    | sec.   | decomposition | \( T_{\max} \) | \( T_{\max} \) Avg. | \( T_{\max} \) Avg. (s) | VNS            | Stop condition = 300s | Best \( T_{\max} \) | Avg. \( T_{\max} \) | Avg. \( \bar{t} \) |
|--------------|--------------|--------|--------|---------------|----------------|---------------------|----------------------|----------------|----------------------|----------------------|----------------------|----------------------|
| w-P200D10% \( \bar{D}50 \) | 1            | 1121.89| 3611.52| 1121.89       | 1121.89       | 1121.89             | 42.04                | 1121.89       | 1121.89       | 1121.89       | 1121.89       |
|              | 2 *2222.97   | 86400.00| 2216.51| 99.52         | 2216.51       | 2212.64             | 126.59               | 2213.97       | 2213.97       | 2213.97       | 2213.97       |
|              | 3 *3329.75   | 86400.00| 3314.91| 69.04         | 3314.91       | 3314.91             | 54.75                | 3320.12       | 3320.12       | 3320.12       | 3320.12       |
|              | 4 *4509.50   | 86400.00| 4436.74| 19.78         | 4436.74       | 4436.74             | 76.78                | 4437.46       | 4437.46       | 4437.46       | 4437.46       |
| w-P400D10% \( \bar{D}50 \) | 1            | 2290.48| 40443.02| 2316.78       | 2316.78       | 2290.48             | 6.50                 | 2290.48       | 2290.48       | 2290.48       | 2290.48       |
|              | 2 *4591.18   | 86400.00| 4445.09| 16.37         | 4445.09       | 4434.52             | 78.96                | 4437.33       | 4437.33       | 4437.33       | 4437.33       |
|              | 3 *6765.02   | 86400.00| 6650.29| 19.40         | 6650.29       | 6648.11             | 86.10                | 6657.81       | 6657.81       | 6657.81       | 6657.81       |
|              | 4 *9938.25   | 86400.00| 8901.19| 73.87         | 8901.19       | 8901.19             | 58.79                | 8902.89       | 8902.89       | 8902.89       | 8902.89       |
| w-P200D7% \( \bar{D}90 \) | 1            | 944.06 | 8669.90| 944.06        | 944.06        | 944.06              | 58.91                | 944.06        | 944.06        | 944.06        | 944.06        |
|              | 2 *1838.27   | 86400.00| 1818.19| 315.24        | 1818.19       | 1816.30             | 94.62                | 1816.84       | 1816.84       | 1816.84       | 1816.84       |
|              | 3 *2737.73   | 86400.00| 2724.26| 193.49        | 2724.26       | 2724.19             | 151.52               | 2724.19       | 2724.19       | 2724.19       | 2724.19       |
|              | 4 *3784.47   | 86400.00| 3633.31| 2826.81       | 3633.31       | 3632.45             | 149.25               | 3633.19       | 3633.19       | 3633.19       | 3633.19       |
|              | 5 *4952.71   | 86400.00| 4582.94| 2834.41       | 4582.94       | 4585.65             | 133.66               | 4600.59       | 4600.59       | 4600.59       | 4600.59       |
| w-P400D7% \( \bar{D}90 \) | 1 *1909.18   | 86400.00| 1891.23| 54.26         | 1891.23       | 1891.23             | 125.80               | 1904.24       | 1904.24       | 1904.24       | 1904.24       |
|              | 2 *3922.21   | 86400.00| 3701.03| 319.12        | 3701.03       | 3697.34             | 113.25               | 3697.90       | 3697.90       | 3697.90       | 3697.90       |
|              | 3 *5567.01   | 86400.00| 5545.72| 49.86         | 5545.72       | 5545.48             | 78.77                | 5545.48       | 5545.48       | 5545.48       | 5545.48       |
|              | 4 *9094.33   | 86400.00| 7398.95| 7395.50       | 7398.95       | 7394.18             | 130.84               | 7396.18       | 7396.18       | 7396.18       | 7396.18       |
| w-P400D7% \( \bar{D}90 \) | 5 -          | -      | 9302.92| 15918.34      | 9302.92       | 9300.46             | 113.56               | 9361.46       | 9361.46       | 9361.46       | 9361.46       |
| w-P200D10% \( \bar{D}70 \) | 1 820.40     | 62142.01| 822.78 | 1015.08       | 822.78        | 825.06              | 81.79                | 825.06        | 825.06        | 825.06        | 825.06        |
|              | 2 -          | -      | 1577.97| 11209.34      | 1577.97       | 1577.87             | 115.51               | 1582.31       | 1582.31       | 1582.31       | 1582.31       |
|              | 3 *2446.63   | 86400.00| 2372.45| 14413.80      | 2372.45       | 2365.78             | 149.33               | 2371.93       | 2371.93       | 2371.93       | 2371.93       |
|              | 4 *4168.27   | 86400.00| 3159.91| 8257.01       | 3159.91       | 3157.05             | 107.74               | 3165.93       | 3165.93       | 3165.93       | 3165.93       |
### Table 8 continued

Parameters: \( \hat{t} = \infty \)

| Instance     | \( \sigma \) | OPT | sec. | Decomposition | Time limit = 86400s | VNS Stop condition = 300s |
|--------------|--------------|-----|------|---------------|----------------------|--------------------------|
| w-P400D10\% \( \bar{D} \) | 1 * | 1671.12 | 86400.00 | 3945.45 | 22388.35 | 3947.86 | 3958.33 | 68.83 |
| w-P400D10\% \( \bar{D} \) | 2 * | 3303.16 | 86400.00 | 1632.77 | 7249.23 | 1633.77 | 1639.62 | 114.80 |
| w-P400D10\% \( \bar{D} \) | 3 | – | – | 3166.70 | 27629.18 | 3164.05 | 3175.29 | 134.49 |
| w-P400D10\% \( \bar{D} \) | 4 * | 7080.11 | 86400.00 | 6335.64 | 28821.90 | 6334.85 | 6347.79 | 101.82 |
| w-P400D10\% \( \bar{D} \) | 5 * | 8989.29 | 86400.00 | 7918.13 | 21613.88 | 7911.42 | 7943.12 | 96.72 |
| w-P400D10\% \( \bar{D} \) | 6 | – | – | 9529.57 | 21616.77 | 9534.57 | 9630.75 | 73.83 |
| w-P500D15\% \( \bar{D} \) | 1 * | 2028.11 | 86400.00 | 1979.39 | 1826.05 | 1988.39 | 2007.95 | 55.76 |
| w-P500D15\% \( \bar{D} \) | 2 | – | – | 3905.71 | 11820.59 | 3902.44 | 3913.18 | 128.80 |
| w-P500D15\% \( \bar{D} \) | 3 | – | – | 5851.96 | 21620.22 | 5855.56 | 5864.11 | 144.98 |
| w-P500D15\% \( \bar{D} \) | 4 | – | – | 7800.26 | 14410.70 | 7804.04 | 7818.67 | 135.90 |
| w-P500D15\% \( \bar{D} \) | 5 | – | – | 9753.65 | 28826.40 | 9750.66 | 9790.69 | 95.56 |
| w-P500D15\% \( \bar{D} \) | 6 | – | – | 11730.98 | 21619.57 | 11776.53 | 11904.97 | 112.61 |
| w-P750D15\% \( \bar{D} \) | 1 * | 3138.73 | 86400.00 | 2984.54 | 2474.51 | 3008.45 | 3047.97 | 105.02 |
| w-P750D15\% \( \bar{D} \) | 2 | – | – | 5849.27 | 7556.80 | 5844.06 | 5858.75 | 101.48 |
| w-P750D15\% \( \bar{D} \) | 3 | – | – | 8767.62 | 19072.32 | 8762.21 | 8784.29 | 126.76 |
| w-P750D15\% \( \bar{D} \) | 4 | – | – | 11680.18 | 21624.97 | 11679.59 | 11712.72 | 144.81 |
| w-P750D15\% \( \bar{D} \) | 5 | – | – | 14602.86 | 19282.36 | 14610.90 | 14665.37 | 80.64 |
| w-P750D15\% \( \bar{D} \) | 6 | – | – | – | 86400.00 | 17665.34 | 17820.95 | 130.71 |
| Instance     | $\sigma$ | OPT   | sec. | $T_{\text{max}}$ | $T_{\text{max}}$ | $T_{\text{max}}$ | $T_{\text{max}}$ |
|--------------|---------|-------|------|------------------|------------------|------------------|------------------|
| **w-P1000D15%** |         |       |      |                  |                  |                  |                  |
| D50          | 1       | *4343.26 | 86400.00 | 3917.65 | 7657.63 | 3962.55 | 4020.12 | 86.24 |
|              | 2       | –      | –    | 7740.00 | 14407.20 | 7724.25 | 7752.53 | 142.33 |
|              | 3       | –      | –    | 86400.00 | 11594.43 | 11614.93 | 122.14 |
|              | 4       | –      | –    | 86400.00 | 15467.11 | 15493.21 | 112.32 |
|              | 5       | –      | –    | 19306.31 | 11764.35 | 19315.50 | 19383.26 | 112.98 |
|              | 6       | –      | –    | 23253.94 | 14438.73 | 23309.74 | 23647.27 | 108.75 |
| **w-P2000D%** |         |       |      |                  |                  |                  |                  |
| D90          | 1       | *665.54 | 86400.00 | 646.58 | 15579.50 | 659.08 | 662.94 | 121.04 |
|              | 2       | –      | –    | 1237.80 | 21616.32 | 1230.42 | 1239.56 | 74.07 |
|              | 3       | –      | –    | 1840.77 | 28840.56 | 1838.98 | 1839.37 | 104.40 |
|              | 4       | –      | –    | 2460.09 | 21622.26 | 2454.44 | 2459.90 | 81.69 |
|              | 5       | –      | –    | 3069.42 | 28825.47 | 3067.49 | 3076.62 | 90.21 |
|              | 6       | –      | –    | 3678.99 | 36027.84 | 3678.08 | 3678.69 | 92.09 |
|              | 7       | –      | –    | 4305.23 | 21622.02 | 4300.25 | 4330.28 | 105.68 |
|              | 8       | –      | –    | 4943.87 | 21621.77 | 4954.66 | 4978.82 | 73.89 |
| **w-P400D10%** |         |       |      |                  |                  |                  |                  |
| D90          | 1       | *1302.82 | 86400.00 | 1265.11 | 9770.29 | 1270.19 | 1309.43 | 105.74 |
|              | 2       | –      | –    | 2468.59 | 14732.29 | 2467.42 | 2475.52 | 89.36 |
|              | 3       | –      | –    | 3690.58 | 21646.61 | 3688.64 | 3689.12 | 145.47 |
|              | 4       | –      | –    | 4925.60 | 21659.91 | 4924.38 | 4930.98 | 105.31 |
|              | 5       | –      | –    | 6151.24 | 21651.88 | 6158.54 | 6169.61 | 98.90 |
|              | 6       | –      | –    | 7378.95 | 21647.51 | 7377.22 | 7378.39 | 89.97 |
|              | 7       | –      | –    | 8615.73 | 19407.21 | 8611.91 | 8655.26 | 101.35 |
|              | 8       | –      | –    | 9893.10 | 17639.44 | 9929.49 | 9981.36 | 75.65 |
considerable smaller execution times than the time spent by CPLEX (first column “sec”) to find the optima (or the best known solution for the cases with “∗”) reported in the column “OPT”.

Regarding the VNS-based heuristic, it also presented good performance. It is the fastest proposed method, with average makespan deviation w.r.t. the best known solution always inferior to 3.22% (“u-P400D10% доб" and $\sigma = 1$) for unweighted instances, and inferior to 0.57% (“w-P200D10% доб" and $\sigma = 1$) for weighted ones. Besides, VNS solutions are in average better than the best known makespan in 42 instances of the instances tested (values in bold).

Those average makespan values (column “Avg. $T_{\text{max}}$”) improvements yield average reduction of 6.14% and 5.17% for the unweighted and weighted cases, respectively. In particular, the VNS heuristic is the unique method reaching feasible solutions for the instances “u-P1000D15% доб" ($\sigma \in \{3, 4, 6\}$), and “w-P1000D15% доб" ($\sigma \in \{3, 4\}$). Finally, the VNS heuristic is more reliable in terms of finding near-optimal solutions within a small time horizon when compared to the decomposition method.

The choice of representing a solution through a spatial partition tree as shown in Sect. 4.2 is very important to simplify the exploration of neighboring solutions. However, such representation is not able to represent all the possible spatial-convex covering of the solution space. For instance, Fig. 10 illustrates an example of covering (dashed lines) which cannot be represented by the adopted spatial-partition tree data structure. Representing such covering would require the use of another data structure. Our VNS explores rather efficiently the solution space, although it is proven not capable of visiting all the feasible solutions for the CAPsac.

Fig. 10 Spatial-convex covering which cannot be represented by the adopted spatial-partition tree data structure
5.5 Impact of the communication constraints

Regarding the sensitivity analysis of the proposed methods to the parameter $\hat{T}$, that is done by progressively decreasing the value of $\hat{T}$ by 1 second until the communication delays turn the problem infeasible. In particular, the experiments are concentrated upon the instances “u-P200D5%D70” and “w-P200D5%D70” with $\sigma = 1$. In these instances, the longest communication delays found when executing the decomposition heuristic and the VNS (for 20 runs), when no constraints are imposed to the maximum transmission time (i.e., $\hat{T} = +\infty$), vary between 33.6s and 48s. For that reason, the first value of $\hat{T}$ used in our experiments was 60 seconds, being decreased by 1 second for each tested $\hat{T}$ down to $\hat{T} = 23s$, when both instances are actually proved infeasible by CPLEX. An extended analysis when varying $\hat{T}$ for more instances is reported at https://github.com/ds4dm/CAPsac. The observed results are similar to the analysis reported next.

Figure 11 presents the results of instances “u-P200D5%D70” and “w-P200D5%D70” obtained by the decomposition-based and the VNS heuristics for $\hat{T} \in \{24s, 25s, 26s, \ldots, 59s, 60s\}$. The figure shows how decreasing $\hat{T}$ values affect the ability of the proposed methods to achieve optimal or near-optimal solutions. We used distinct shaded regions in the figure to represent the different optimal $T_{max}^*$ values obtained within the interval of tested $\hat{T}$ values. Thus, we can observe for instance “u-P200D5%D70” two shaded regions before the problem becomes infeasible for $\hat{T} = [24s, 25s, 26s, \ldots, 59s, 60s]$. The vertical axis “Dev.(%)” in the figure reports the percentage deviation w.r.t. the optimum $T_{max}^*$ in the shaded region associated with each value of $\hat{T}$ tested. Green lines and “•” symbols represent the deviations obtained by the decomposition-based method (named “Decomp.”) whereas the average deviation achieved by 20 runs of the VNS method (denoted by “VNS”) are represented by the orange lines and empty “◦”.

It can be observed in Fig. 11 from the unweighted instance that the decomposition-based method achieves optimality for most of the tested $\hat{T}$ values except for $27s \leq \hat{T} \leq 33s$ where it reaches percentage deviations up to 27.27%. Those high deviations are mainly due to the $T_{max}^*$ increase for $\hat{T} \leq 33s$, from 1870.40 to 2939.20. We observed that for $27s \leq \hat{T} \leq 33s$ the initial bounds $n_\ell$ and $n_u$ of the decomposition method are not good estimates for the cardinalities of the sub-regions that compose the optimal solutions, and thus, the heuristic is likely to finish before reaching a near-optimal solution. For $24s \leq \hat{T} \leq 26s$, the initial limits $n_\ell$ and $n_u$ lead to infeasible problems until they are large enough to encompass feasible solutions, which were in this case, close to optima. In contrast, the VNS method achieves optimality for all tested $\hat{T}$ values when solving the unweighted case.

Regarding the weighted case, the decomposition method presents similar behavior as when solving the unweighted case, and its solution deviation goes up to 28.42% for $27s \leq \hat{T} \leq 33s$. The first $T_{max}^*$ increase from 1886.98 to 1934.39 (i.e. +2.5%) was not sufficient to affect the quality of the solution obtained by the decomposition. The VNS only presents small increases in the average deviation for $36s \leq \hat{T} \leq 47s$ but they never surpass 1.25%, otherwise the VNS reaches optimality.
We remark that the VNS method constructs its initial solution in our experiments using $\hat{T} = +\infty$. This means that the VNS initial solutions may be infeasible in terms of the transmission delays. Our experiments allowed us to observe that the proposed VNS was able to find feasible solutions even from infeasible starting points.
6 Conclusions

We addressed the optimization of the 3D reconstruction step within a swarm-powered 3D mapping mission according to the so-called Covering-Assignment Problem for swarm-powered ad hoc clouds - CAPsac. It minimizes the completion time of the 3D reconstruction photo-processing phase by exploiting the distributed computational power of the swarm of drones. This was achieved by integrating the photo covering (workload) optimization along with the assignment of the drones to the photo processing subtasks (i.e., sub-regions).

Since time is a very precious resource in emergency field operations supported by UAVs, the limited computation time - in the order of a couple of minutes for the biggest and more complex instances, we proposed two heuristic algorithms to solve this NP-hard problem in a limited amount of time (i) a mathematical programming-based heuristic as well as a (ii) Variable Neighborhood Search method.

Despite the fact that we had focused on a swarm-powered distributed 3D reconstruction for humanitarian emergency response application, the proposed methods can be deployed to any application in the CAPsac’s scope. Finally, the methods can be adapted to consider auxiliary computing resources in addition to those offered by the UAV swarm.

Computational experiments were conducted with the unweighted and weighted realistic instances available online at https://github.com/ds4dm/CAPsac to assess the performance of the proposed heuristic methods. The experiments exposed that the VNS heuristic either quickly achieves near-optimal solutions or rapidly improves the best known makespan for a vast number of instances. The decomposition-based heuristic is also effective in most of the tested cases and is an efficient method when compared to solving the CAPsac formulations by commercial solvers. Although the performance of decomposition-based heuristic deteriorates for some values of $\hat{T}$ close to infeasibility, the sensitivity analysis done for the decomposition and VNS methods demonstrated that those methods still perform well when varying $\sigma$ and $\hat{T}$.

Our current work with Humanitas Solutions (https://www.humanitas.io/) is focused on embedding the heuristics proposed in this paper into real deployed UAV swarms.

Acknowledgements The authors would like to thank all support from Humanitas Solutions and the NSERC Canada Excellence Research Chair in Data Science for Real-Time Decision-Making. This work was also partially funded by Prompt, and in part by MITACS. The computing resources were made available by Compute Canada.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

Barrientos, A., Colorado, J., Cerro, Jd., Martinez, A., Rossi, C., Sanz, D., Valente, J.: Aerial remote sensing in agriculture: a practical approach to area coverage and path planning for fleets of mini aerial robots. J. Field Robot. 28(5), 667–689 (2011)
Heuristics for optimizing 3D mapping missions over…

Bendig, J., Bolten, A., Bennertz, S., Broscheit, J., Eichfuss, S., Bareth, G.: Estimating biomass of barley using crop surface models (CSMS) derived from UAV-based RGB imaging. Remote Sens. 6(11), 10395–10412 (2014). https://doi.org/10.3390/rs6110395

Bentley, J.L.: Multidimensional binary search trees used for associative searching. Commun. ACM 18(9), 509–517 (1975). https://doi.org/10.1145/361002.361007

Berni, J.A.J., Zarco-Tejada, P.J., Suarez, L., Fereres, E.: Thermal and narrowband multispectral remote sensing for vegetation monitoring from an unmanned aerial vehicle. IEEE Trans. Geosci. Remote Sens. 47(3), 722–738 (2009). https://doi.org/10.1109/TGRS.2008.2010457

Bertsekas, D.P., Gallager, R.G., Humblet, P.: Data Networks, vol. 2. Prentice-Hall International, New Jersey (1992)

Boussaïd, I., Lepagnot, J., Siarry, P.: A survey on optimization metaheuristics. Inf. Sci. 237, 82–117 (2013). https://doi.org/10.1016/j.ins.2013.02.041

Caraballo, L., Díaz-Bañez, J., Mazá, I., Ollero, A.: The block-information-sharing strategy for task allocation: a case study for structure assembly with aerial robots. Eur. J. Oper. Res. 260(2), 725–738 (2017). https://doi.org/10.1016/j.ejor.2016.12.049

Chen, W., Liu, B., Huang, H., Guo, S., Zheng, Z.: When UAV swarm meets edge-cloud computing: the QoS perspective. IEEE Netw. 33(2), 36–43 (2019). https://doi.org/10.1109/MNET.2019.1800222

Chiabrando, F., Nex, F., Piatti, D., Rinaudo, F.: UAV and RPV systems for photogrammetric surveys in archaeological areas: two tests in the piedmont region (Italy). J. Archaeol. Sci. 38(3), 697–710 (2011). https://doi.org/10.1016/j.jas.2010.10.022

Choi, H., Kim, Y., Kim, H.: Genetic algorithm based decentralized task assignment for multiple unmanned aerial vehicles in dynamic environments. Int. J. Aeronaut. Space Sci. 12(2), 163–174 (2011)

Chou, T.Y., Yeh, M.L., Chen, Y.C., Chen, Y.H.: Disaster monitoring and management by the unmanned aerial vehicle technology. Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci. 38, 137–142 (2010)

Costa, L.R., Alonso, D., Gianoli, L.G., Lodi, A.: The covering-assignment problem for swarm-powered ad-hoc clouds: a distributed 3d mapping use-case. IEEE Internet Things J. (2020). https://doi.org/10.1109/JIOT.2020.3039261

Coutinho, W.P., Battarra, M., Fliege, J.: The unmanned aerial vehicle routing and trajectory optimisation problem, a taxonomic review. Comput. Ind. Eng. 120, 116–128 (2018). https://doi.org/10.1016/j.cie.2018.04.037

Díaz-Varela, R.A., De la Rosa, R., León, L., Zarco-Tejada, P.J.: High-resolution airborne UAV imagery to assess olive tree crown parameters using 3d photo reconstruction: application in breeding trials. Remote Sens. 7(4), 4213–4232 (2015). https://doi.org/10.3390/rs70404213

Doherty, P., Kvarnström, J., Rudol, P., Wzorek, M., Conte, G., Berger, C., Hinzmann, T., Stastny, T.: A collaborative framework for 3d mapping using unmanned aerial vehicles. In: International Conference on Principles and Practice of Multi-Agent Systems. Springer, pp. 110–130 (2016)

Enright, J.J., Frazzoli, E., Pavone, M., Savla, K.: UAV Routing and Coordination in Stochastic, Dynamic Environments, Springer Netherlands, Dordrecht, pp 2079–2109 (2015) https://doi.org/10.1007/978-90-481-9707-1_28

Frahm, J.M., Fite-Georgel, P., Gallup, D., Johnson, T., Raguram, R., Wu, C., Jen, Y.H., Dunn, E., Clipp, B., Lazebnik, S., Pollefeys, M.: Building Rome on a cloudless day. In: Daniilidis, K., Maragos, P., Paragios, N. (eds.) Computer Vision—ECCV 2010, pp. 368–381. Springer, Heidelberg (2010)

Gan, SK., Sukkarieh, S.: Multi-UAV target search using explicit decentralized gradient-based negotiation. In: 2011 IEEE International Conference on Robotics and Automation (ICRA), IEEE, pp. 751–756 (2011)

Golodetz, S., Cavallari, T., Lord, N.A., Prisacariu, V.A., Murray, D.W., Torr, P.H.: Collaborative large-scale dense 3d reconstruction with online inter-agent pose optimisation. arXiv preprint arXiv:1801.08361 (2018)

Gomarasca, M.A.: Elements of Photogrammetry, vol. 3, Springer, Netherlands, Dordrecht, pp 79–121 (2009). https://doi.org/10.1007/978-1-4020-9014-1_3

Grenzdörffer, G., Engel, A., Teichert, B.: The photogrammetric potential of low-cost UAVs in forestry and agriculture. Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci. XXXVI I(B1), 1207–1214 (2008)

Haarbrink, R., Koers, E.: Helicopter UAV for photogrammetry and rapid response. In: 2nd International Workshop “The Future of Remote Sensing”, ISPRS Inter-Commission Working Group I/V Autonomous Navigation, Citeseer, vol. 1 (2006)
Hamdaqa, M., Sabri, M.M., Singh, A., Tahvildari, L.: Adoop: MapReduce for ad-hoc cloud computing. In: Proceedings of the 25th Annual International Conference on Computer Science and Software Engineering, pp. 26–34. http://dl.acm.org/citation.cfm?id=2886444.2886449 (2015)

Hansen, P., Mladenović, N., Brimberg, J., Pérez, JAM.: Variable Neighborhood Search. Springer International Publishing, Cham, pp. 57–97 (2019). https://doi.org/10.1007/978-3-319-91086-4_3

Hansen, P., Mladenović, N.: Variable neighborhood search: principles and applications. Eur. J. Oper. Res. 130(3), 449–467 (2001). https://doi.org/10.1016/S0377-2217(00)00100-4

Hartmann, W., Tillo, S., Eisenbeiss, H., Schindler, K.: Determination of the UAV position by automatic processing of thermal images. In: ISPRS: International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, vol. XXXIX-B6, pp. 111–116 (2012). https://doi.org/10.5194/isprsarchives-XXXIX-B6-111-2012

Hinzmann, T., Stastny, T., Conte, G., Doherty, P., Rudol, P., Wzorek, M., Galceran, E., Siegwart, R., Gilitschenski, I.: Collaborative 3d reconstruction using heterogeneous UAVs: system and experiments. In: International Symposium on Experimental Robotics, Springer, pp. 43–56 (2016)

Hoang, V.T., Phung, M.D., Dinh, T.H., Ha, Q.P.: System architecture for real-time surface inspection using multiple UAVs. IEEE Syst. J. 14(2), 2925–2936 (2020). https://doi.org/10.1109/JYST.2019.292299

Ji, X., Wang, X., Niu, Y., Shen, L.: Cooperative search by multiple unmanned aerial vehicles in a nonconvex environment. Math. Probl. Eng. 2015, 1–19 (2015). https://doi.org/10.1155/2015/196730

Kargman, S., Inalhan, G.: Large-scale task/target assignment for UAV fleets using a distributed branch and price optimization scheme. In: IFAC Proceedings, vol. 41, no. 2, pp. 13310–13317 (2008). https://doi.org/10.3182/20080706-5-KR-1001.02255, 17th IFAC World Congress

Kobayashi, K., Shishido, H., Kameda, Y., Kitahara, I.: Method to generate disaster-damage map using 3d photometry and crowd sourcing. In: 2017 IEEE International Conference on Big Data (Big Data), IEEE, pp. 4397–4399 (2017)

Ladors, P., Oh, H., Chen, W.H.: Trajectory planning for communication relay unmanned aerial vehicles in urban dynamic environments. J. Intell. Rob. Syst. 89(1), 7–25 (2018). https://doi.org/10.1007/s10846-017-0484-y

Lambers, K., Eisenbeiss, H., Sauerbier, M., Kupferschmidt, D., Gaiseca, T., Sotoodeh, S., Hanusch, T.: Combining photogrammetry and laser scanning for the recording and modelling of the late intermediate period site of pinchango alto, palpa, peru. J. Archaeol. Sci. 34(10), 1702–1712 (2007). https://doi.org/10.1016/j.jas.2006.12.008

Lanillos, P., Gan, S.K., Besada-Portas, E., Pajares, G., Sukkarieh, S.: Multi-UAV target search using decentralized gradient-based negotiation with expected observation. Inf. Sci. 282, 92–110 (2014). https://doi.org/10.1016/j.ins.2014.05.054

Linder, W.: Digital Photogrammetry. Springer, Berlin (2016). https://doi.org/10.1007/978-3-662-50463-5

Loianno, G., Mulgaonkar, Y., Brunner, C., Ahuja, D., Ramanandan, A., Chari, M., Diaz, S., Kumar, V.: A swarm of flying smartphones. In: 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), IEEE, pp. 1681–1688 (2016)

Malhotra, A., Dhurandher, S.K., Kumar, B.: Resource allocation in multi-hop Mobile Ad hoc cloud. In: 2014 Recent Advances in Engineering and Computational Sciences (RAECS), IEEE, pp. 1–6, (2014). https://doi.org/10.1109/RAECS.2014.6799578. http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=6799578

Manyoky, M., Theiler, P., Steudler, D., Eisenbeiss, H.: Unmanned aerial vehicle in cadastral applications. In: ISPRS-International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, vol. XXXVIII-1/C22, pp. 57–62 (2011). https://doi.org/10.5194/isprsarchives-XXXVIII-1-C22-57-2011

Martínez-de Dios, J., Merino, L., Caballero, F., Ollero, A., Viegas, D.: Experimental results of automatic fire detection and monitoring with UAVs. For. Ecol. Manag. 234(1), S232 (2006)

Massoulié, L., Roberts, J.: Bandwidth sharing: objectives and algorithms. In: Proceedings of INFOCOM’99. Eighteenth Annual Joint Conference of the IEEE Computer and Communications Societies, vol 3, IEEE, pp. 1395–1403 (1999)

Meyer, D., Fraiho, E., Lo, E., Rissolo, D., Kuester, F.: Optimizing UAV systems for rapid survey and reconstruction of large scale cultural heritage sites. In: Digital Heritage, vol. 1, IEEE, pp. 151–154 (2015)

Milani, S., Memo, A.: Impact of drone swarm formations in 3d scene reconstruction. In: 2016 IEEE International Conference on Image Processing (ICIP), IEEE, pp. 2598–2602 (2016) https://doi.org/10.1109/ICIP.2016.7532829
Molina, P., Colomina, I., Victoria, T., Skaloud, J., Kornus, W., Prades, R., Aguilera, C.: Searching lost people with UAVs: the system and results of the close-search project. Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci. 39, 441–446 (2012)

Moon, S., Shim, D.H., Oh, E.: Cooperative task assignment and path planning for multiple UAVs, Springer Netherlands, Dordrecht, pp. 1547–1576 (2015). https://doi.org/10.1007/978-90-481-9707-1_82

Moon, S., Oh, E., Shim, D.H.: An integral framework of task assignment and path planning for multiple unmanned aerial vehicles in dynamic environments. J. Intell. Robot. Syst. 70(1), 303–313 (2013). https://doi.org/10.1007/s10846-012-9740-3

Mukherjee, A., Misra, S., Sukrutha, A., Raghuvanshi, N.S.: Distributed aerial processing for IoT-based edge UAV swarms in smart farming. Comput. Netw. 167, 107038 (2020). https://doi.org/10.1016/j.comnet.2019.107038

Nace, D., Piëro, M.: Max-min fairness and its applications to routing and load-balancing in communication networks: a tutorial. IEEE Commun. Surv. Tutori. 10(4), 5 (2008)

Nex, F., Remondino, F.: UAV for 3d mapping applications: a review. Appl. Geomat. 6(1), 1–15 (2014). https://doi.org/10.1007/s12518-013-0120-x

Nicolini, M., Innocenti, M., Pollini, L.: Multiple UAV task assignment using descriptor functions. IFAC Proc. Vol. 43(15), 93–98 (2010)

Niethammer, U., Rothmund, S., James, M., Travellletti, J., Joswig, M.: UAV-based remote sensing of landslides. Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci. 38, 496–501 (2010)

Oczypka, M., Bemmann, J., Piezonka, H., Munkabayar, J., Ahrens, B., Achtelik, M., Lehmann, F.: Small drones for geo-archaeology in the steppes: locating and documenting the archaeological heritage of the Orkhon valley in Mongolia. In: Michel, U., Cívco, D.L. (eds.), Remote Sensing for Environmental Monitoring, GIS Applications, and Geology IX, International Society for Optics and Photonics, SPIE, vol. 7478, pp. 53 – 63 (2009) https://doi.org/10.1117/12.830404

Oh, H., Shin, H.S., Kim, S., Tsourdos, A., White, B.A.: Cooperative Mission and Path Planning for a Team of UAVs, Springer Netherlands, Dordrecht, pp. 1509–1545 (2015) https://doi.org/10.1007/978-90-481-9707-1_14

Oh, H., Kim, S., Tsourdos, A., White, B.A.: Coordinated road-network search route planning by a team of UAVs. Int. J. J. Syst. Sci. 45(5), 825–840 (2014). https://doi.org/10.1080/00207721.2012.737116

OpenDroneMap (2020) CloudODM. https://github.com/OpenDroneMap/CloudODM

Otto, A., Agatz, N., Campbell, J., Golden, B., Pesch, E.: Optimization approaches for civil applications of unmanned aerial vehicles (UAVs) or aerial drones: a survey. Networks (2018). https://doi.org/10.1002/net.21818

Pepe, M., Fregonese, L., Scaioni, M.: Planning airborne photogrammetry and remote-sensing missions with modern platforms and sensors. Eur. J. Remote Sens. 51(1), 412–436 (2018). https://doi.org/10.1080/22797254.2018.1444945

Rinaudo, F., Chiabrando, F., Lingua, A.M., Spanò, A.: Archaeological site monitoring: Uav photogrammetry can be an answer. In: ISPRS—International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, vol. XXXIX-B5, pp. 583–588 (2012). https://doi.org/10.5194/isprsarchives-XXXIX-B5-583-2012

Sadeghi, A., Smith, S.L.: Heterogeneous task allocation and sequencing via decentralized large neighborhood search. Unmanned Syst. 5(02), 79–95 (2017). https://doi.org/10.1142/S2301385017500066

Şahin, E.: Swarm robotics: from sources of inspiration to domains of application. In: Şahin, E., Spears, W.M. (eds.) Swarm Robotics, pp. 10–20. Springer, Heidelberg (2005)

Schmiemann, J., Harms, H., Schattenberg, J., Becker, M., Batzdorfer, S., Frerichs, L.: A distributed online 3d- lidar Mappin system. In: ISPRS: International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, vol. XLII-2/W6, pp. 339–346 (2017). https://doi.org/10.5194/isprs-archives-XLII-2-W6-339-2017

Smith, J.G., Dehn, J., Hoblitt, R.P., LaHusen, R.G., Lowenstern, J.B., Moran, S.C., McClelland, L., McGee, K.A., Nathenson, M., Okubo, P.G., Pallister, J.S., Poland, M.P., Power, J.A., Schneider, D.J., Sisson, T.W.: Volcano monitoring. In: Geological Monitoring, Geological Society of America, pp. 273–305 (2009). https://doi.org/10.1130/2009.monitoring(12)

Sujit, P.B., Ghose, D.: Search using multiple UAVs with flight time constraints. IEEE Trans. Aerosp. Electron. Syst. 40(2), 491–509 (2004). https://doi.org/10.1109/TAES.2004.1310000

Surmann, H., Berninger, N., Worst, R.: 3d mapping for multi hybrid robot cooperation. In: 2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), IEEE, pp. 626–633 (2017)
Tan, Y., Yang Zheng, Z.: Research advance in swarm robotics. Defence Technol. 9(1), 18–39 (2013). https://doi.org/10.1016/j.dt.2013.03.001

Tang, P., Vick, S., Chen, J., German, Paal S.: Chapter 2: surveying, geomatics, and 3D reconstruction. In: Brilakis, I., Haas, C. (eds.), Infrastructure Computer Vision, Butterworth-Heinemann, pp. 13 – 64 (2020). https://doi.org/10.1016/B978-0-12-815503-5.00002-4. http://www.sciencedirect.com/science/article/pii/B9780128155035000024

Tang, Z., Ozguner, U.: Motion planning for multtarget surveillance with mobile sensor agents. IEEE Trans. Rob. 21(5), 898–908 (2005). https://doi.org/10.1109/TRO.2005.847567

Turpin, M., Michael, N., Kumar, V.: Capt: concurrent assignment and planning of trajectories for multiple robots. Int. J. Robot. Res. 33(1), 98–112 (2014). https://doi.org/10.1177/0278364913515307

Verhoeven, G.J.J.: Providing an archaeological bird’s-eye view: an overall picture of ground-based means to execute low-altitude aerial photography (laap) in archaeology. Archaeol. Prospect. 16(4), 233–249 (2009). https://doi.org/10.1002/arp.354

Viguria, A., Maza, I., Ollero, A.: Distributed service-based cooperation in aerial/ground robot teams applied to fire detection and extinguishing missions. Adv. Robot. 24(1–2), 1–23 (2010). https://doi.org/10.1163/016918609X12585524300339

Xie, X., Yang, T., Li, J., Ren, Q., Zhang, Y.: Fast and seamless large-scale aerial 3d reconstruction using graph framework. Assoc. Comput. Mach. N. Y. ICIGP 2018, 126–130 (2018). https://doi.org/10.1145/3191442.3191448

Yang, Y., Chang, M.C., Wen, L., Tu, P., Qi, H., Lyu, S.: Efficient large-scale photometric reconstruction using divide-recon-fuse 3d structure from motion. In: 2016 13th IEEE International Conference on Advanced Video and Signal Based Surveillance (AVSS), pp. 180–186 (2016). https://doi.org/10.1109/AVSS.2016.7738070

Yaqoob, I., Ahmed, E., Gani, A., Mokhtar, S., Imran, M., Guizani, S.: Mobile ad hoc cloud: a survey. In: Wireless Communications and Mobile Computing, pp. 421–430 (2016). https://doi.org/10.1002/wcm.2709

Zarco-Tejada, P., Diaz-Varela, R., Angileri, V., Loudjani, P.: Tree height quantification using very high resolution imagery acquired from an unmanned aerial vehicle (UAV) and automatic 3d photoreconstruction methods. Eur. J. Agron. 55, 89–99 (2014). https://doi.org/10.1016/j.eja.2014.01.004

Zhang, C.: An UAV-based photogrammetric mapping system for road condition assessment. In: ISPRS: International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, vol. XXXVII-B5, pp. 627–632 (2008)

Zorbas, D., Pugliese, L.D.P., Razafindralambo, T., Guerriero, F.: Optimal drone placement and cost-efficient target coverage. J. Netw. Comput. Appl. 75, 16–31 (2016). https://doi.org/10.1016/j.jnca.2016.08.009

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