Group velocity control in the ultraviolet domain via interacting dark-state resonances

Mohammad Mahmoudi\textsuperscript{1,2}, Robert Fleischhaker\textsuperscript{1}, Mostafa Sahrai\textsuperscript{3} and Jörg Evers\textsuperscript{1}

\textsuperscript{1} Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117 Heidelberg, Germany
\textsuperscript{2} Physics Department, Zanjan University, PO Box 45195-313, Zanjan, Iran
\textsuperscript{3} Research Institute for Applied Physics and Astronomy, University of Tabriz, Tabriz, Iran

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Abstract
The propagation of a weak probe field in a laser-driven four-level atomic system is investigated. We choose mercury as our model system, where the probe transition is in the ultraviolet region. A high-resolution peak appears in the optical spectra due to the presence of interacting dark resonances. We show that this narrow peak leads to the superluminal light propagation with strong absorption, and thus by itself is only of limited interest. But if in addition a weak incoherent pump field is applied to the probe transition, then the peak structure can be changed such that both sub- and superluminal light propagation or a negative group velocity can be achieved without absorption, controlled by the incoherent pumping strength. A suitable choice of laser propagation directions allows us to preserve these results under Doppler averaging.

1. Introduction
Optical properties of an atomic medium can be substantially modified by the application of external fields. In particular, atomic coherence induced by laser fields plays an important role in light–matter interaction and has found numerous implementations in optical physics [1]. One prominent application is the modification of the propagation of a light pulse through an atomic medium, which depends on the dispersive properties of the medium. The study of such pulse propagation phenomena has been triggered by a series of papers by Sommerfeld and Brillouin [2, 3] and continues to be of much interest [4–9]. It is well known that the group velocity of a light pulse can be slowed down [10, 11], can become faster than its value \( c \) in vacuum, or can even become negative [12–28]. Note that the superluminal light propagation with group velocity larger than \( c \) cannot transmit information faster than the vacuum speed of light, such that it is not at odds with causality [29]. The superluminal light propagation has been investigated for many potential uses, not only as a tool for studying a very peculiar state of matter, but also for developing quantum computers, high-speed optical switches and communication systems [30].

Both experimental and theoretical studies have been performed to realize super- and subluminal light propagation in a single system. For example, speed control in atomic systems has been achieved by changing the frequencies, amplitudes or phase differences of the applied fields. It has been shown that switching from subluminal to superluminal pulse propagation can be achieved by the intensity of the coupling fields [16–19], and the relative phase between two weak probe fields [20]. Morigi \textit{et al} [21] have compared the phase-dependent properties of the \( \diamond \) (diamond) four-level system with those of the double-\( \Lambda \) system. In [22], the gain-assisted superluminal light propagation was observed in a caesium vapour cell while in most other studies, superluminal light propagation is accompanied by considerable absorption. Sub- and superluminal light propagation together with the nonlinear optical gain or losses were observed in [23]. In [24], subnatural absorption resonances with positive and negative dispersion were demonstrated in a standing wave field. Steep negative dispersion is also possible in atomic media exhibiting electromagnetically induced absorption [25]. Two of the present authors suggested to use an incoherent pump field to control the light propagation from subluminal to superluminal [26, 27]. Recently, we have studied the light propagation of a...
probe pulse in a four-level double-lambda system, where the applied laser fields form a closed interaction loop [28]. In such systems, the finite frequency width of a probe pulse requires a time-dependent treatment of the light propagation. We have found both sub- and superluminal light propagation without absorption or with gain, controlled by the Rabi frequency of one of the coupling fields.

All these effects depend on the modification of the dispersive and absorptive properties of the atomic medium. A particular class of systems that allows us to modify the optical response to a great extent are those with the so-called interacting dark resonances [31]. A characteristic feature of such systems is the appearance of very sharp, high-contrast structures in the optical spectra. Resonances associated with double dark states can be made absorptive or transparent and their optical properties such as width and position can be manipulated by applying suitable coherent interactions. It was also shown that very weak incoherent excitation of the atoms can be sufficient to turn absorptive features into optical gain structures. This has been proposed as a model system to obtain a strong laser gain in the ultraviolet and vacuum ultraviolet regime by Fry et al [32].

In this paper, we consider probe pulse propagation through a system which exhibits interacting dark resonances. The level configuration of our four-level scheme is based on the lasing system proposed in [32], and consists of three atomic states in a ladder configuration, with an additional fourth perturbing state coupled by a laser field to the upper state of the ladder system. The lower transition of the ladder system acts as the probe transition. This system can be realized, e.g., in mercury, where the probe transition has a low wavelength of 253.7 nm, i.e., in the ultraviolet region. We find that the medium susceptibility in dependence on the probe field detuning exhibits high-contrast structures characteristic of interacting dark states. These structures typically lead to superluminal probe field propagation with high absorption, and thus as such are of limited interest. If, however, a weak incoherent pumping is applied in addition to the probe field transition, then we find that in the region around a narrow structure both sub- and superluminal propagation as well as negative group velocities are possible without absorption, controlled by the incoherent pumping strength. Finally, we show that our main findings persist under averaging with Doppler averaging.

2. Analytical considerations

2.1. The model system

We consider an atomic four-level system as shown in figure 1(a). Transition |2⟩ ↔ |4⟩ is driven by a strong coherent field with frequency ω_{42} and the Rabi frequency Λ_{42}. A weak coupling field with frequency ω_{41} and the Rabi frequency Λ_{41} is applied to transition |1⟩ ↔ |4⟩. The weak probe field with frequency ω_{23} and the Rabi frequency Λ_{23} = g_p couples to transition |2⟩ ↔ |3⟩. Finally, an incoherent driving field with pump strength Λ is applied to the probe transition. We further include spontaneous decay with rates γ_{41}, γ_{42}, γ_{23} and γ_{13}, respectively, on the dipole-allowed transitions. The atomic transition frequencies are denoted by ω_{ij}, and the laser field detunings with respect to the atomic transition frequencies are Δ_{ij} = ω_{ij} − ω_{ij} (i, j ∈ 1, …, 4). A realization of our level scheme can be found, e.g., in mercury, see figure 1(b).

The density matrix equations of motion, in the rotating wave approximation, are

\[ \dot{\rho}_{11} = -2γ_{13}\rho_{11} + 2γ_{41}\rho_{44} - ig_{41}\rho_{14} + ig_{41}\rho_{41}, \]

\[ \dot{\rho}_{22} = -2γ_{23}\rho_{22} + 2γ_{24}\rho_{24} - 2Δ_{22} + 2Λ_{23} + ig_{23}\rho_{32} - ig_{23}\rho_{23} + ig_{42}\rho_{24} + ig_{42}\rho_{24}, \]

\[ \dot{\rho}_{33} = 2γ_{31}\rho_{11} + 2γ_{23}\rho_{22} + 2Δ_{33} - ig_{23}\rho_{32} + ig_{23}\rho_{23}, \]

\[ \dot{\rho}_{12} = -(Γ_{12} + iΔ_{41} - iΔ_{42} + 2Λ_{23})\rho_{12} - ig_{42}\rho_{14} - ig_{41}\rho_{21}, \]

\[ \dot{\rho}_{13} = -(Γ_{13} + iΔ_{41} - iΔ_{42} - iΔ_{23} + 2Λ_{23})\rho_{13} - ig_{42}\rho_{14} + ig_{41}\rho_{21}, \]

\[ \dot{\rho}_{14} = -(Γ_{14} + iΔ_{41})\rho_{14} - ig_{41}\rho_{11} + ig_{41}\rho_{44} - ig_{42}\rho_{12}, \]

\[ \dot{\rho}_{23} = -(Γ_{23} - iΔ_{23})\rho_{23} - ig_{23}\rho_{22} + 2Δ_{23} + ig_{42}\rho_{43}, \]

\[ \dot{\rho}_{34} = -(Γ_{34} + iΔ_{41} - iΔ_{42} + 2Λ_{23})\rho_{34} - ig_{42}\rho_{24} + ig_{41}\rho_{31}, \]

\[ \dot{\rho}_{41} = -(Γ_{41} + iΔ_{41})\rho_{41} - ig_{41}\rho_{14} + ig_{41}\rho_{44} - ig_{42}\rho_{24} + ig_{42}\rho_{24}, \]

\[ \dot{\rho}_{42} = -(Γ_{42} + iΔ_{42})\rho_{42} - ig_{42}\rho_{22} + 2Δ_{23} + ig_{42}\rho_{23}, \]

\[ \dot{\rho}_{43} = -(Γ_{43} + iΔ_{42} - iΔ_{23} + 2Λ_{23})\rho_{43} - ig_{42}\rho_{24} + ig_{42}\rho_{43}, \]

\[ \dot{\rho}_{44} = -(Γ_{44} + iΔ_{42} + 2Δ_{23})\rho_{44} - ig_{42}\rho_{24} + ig_{42}\rho_{24}, \]
\[ \rho_{24} = -(\Gamma_4 + i\Delta_4 + \Lambda)\rho_{24} - ig_{42}\rho_{23} + ig_{24}\rho_{42} + ig_{p}\rho_{44} - ig_{41}\rho_{21}, \]  
\[ \rho_{34} = -(\Gamma_3 + i\Delta_3 + \Lambda)\rho_{34} + ig_{p}\rho_{24} - ig_{41}\rho_{31} - ig_{2}\rho_{32}, \]  
\[ \rho_{44} = 1 - \rho_{11} - \rho_{22} - \rho_{33}. \]  

In the above equations, \( \Gamma_{ij} = (2\gamma_{i} + 2\gamma_{j})/2 \) are the damping rates of the coherences with \( \gamma_{i} \) being the total decay rate out of state \( |i\rangle \), and \( \Delta_{p} = \Delta_{21} \) is the probe field detuning.

Our main observable is the response of the atomic medium to the probe field. As will be discussed in section 2.2, the linear susceptibility of the weak probe field is determined by the probe transition coherence \( \rho_{23} \). We therefore proceed by solving equations (1a)–(1i) in the steady state under the assumption of specific parameter relations.

First, in the absence of the incoherent pump field (\( \Lambda = 0 \)), an expansion of the steady-state coherence \( \rho_{23} \) to the leading order in the probe field Rabi frequency \( g_{p} \) yields

\[ \rho_{23} = \frac{-g_{p}(|g_{41}|^{2} - C_{13} \cdot C_{43})}{|g_{41}|^{2}C_{23} + C_{13}(|g_{42}|^{2} - C_{23} \cdot C_{34})}, \]  
\[ C_{13} = \Delta_{p} - \Delta_{41} + \Delta_{42} + i\Gamma_{13}, \]  
\[ C_{34} = \Delta_{p} + \Delta_{42} + i\Gamma_{34}, \]  
\[ C_{23} = \Delta_{p} + i\Gamma_{23}. \]  

It will turn out that an interesting parameter range for the present study is given by

\[ \Delta_{41} = \Delta_{42} = 0, \]  
\[ \Delta_{p} \ll \gamma_{41}, \gamma_{42}, \]  
\[ g_{41} \ll g_{42}, \]  
\[ \gamma_{13} = 0. \]  

In this limit, equation (2a) becomes

\[ \rho_{23} = \frac{-g_{p}(|g_{41}|^{2} - i\Delta_{p}\Gamma_{34})}{|g_{41}|^{2}\Delta_{p} + i\Gamma_{42}}. \]  

An inspection of equation (4) reveals that the imaginary part is strictly positive, and the half width of the absorption peak around \( \Delta_{p} = 0 \) is determined by

\[ w \simeq \left( \frac{g_{41}}{g_{42}} \right)^{2}\Gamma_{23} = \left( \frac{g_{41}}{g_{42}} \right)^{2}\gamma_{23}. \]  

Next, we seek the corresponding steady-state solution for \( \rho_{23} \) with incoherent pump field with pump intensity \( \Lambda \). The parameters are chosen to satisfy equations (3a)–(3d) as well as the new condition on the pump field

\[ \Lambda_{0} \ll \Lambda \ll \gamma_{41}, \gamma_{42}. \]  

Further, we assume the Rabi frequencies \( g_{ij} \) to be real in the following. We obtain in leading order of the probe field coupling \( g_{p} \)

\[ \rho_{23} = \frac{g_{41}g_{p}\Gamma_{23}}{(g_{42}\gamma_{23} + 2\Lambda\Gamma_{24}\gamma_{42})}\frac{\Delta_{p} - i\Lambda}{\Delta_{p}^{2} + \Lambda^{2}}. \]  

Here the parameter \( \Lambda_{0} \) is defined by

\[ \Lambda_{0} = \frac{g_{41}\gamma_{23}(\gamma_{41} + \gamma_{23})}{g_{42}^{2}\gamma_{41} + \gamma_{23}\Gamma_{42}(\gamma_{41} + \gamma_{23})} \simeq \left( \frac{g_{41}}{g_{42}} \right)^{2}\gamma_{23}. \]  

Since \( |g_{41}/g_{42}|^{2}\gamma_{23} \) can be made small, for a suitable combination of the Rabi frequencies \( g_{41} \) and \( g_{42} \), the condition \( \Lambda \gg \Lambda_{0} \) can be fulfilled even for incoherent pump strengths which are orders of magnitude smaller than those required, e.g., to saturate the optical transition.

We find that the imaginary part of equation (7) is negative if the condition \( \Lambda \gg \Lambda_{0} \) is fulfilled. Thus, \( \Lambda_{0} \) indicates the incoherent pumping rate at which the absorption peak turns into a gain structure, if the conditions in equations (3a)–(3d) and (6) are fulfilled.

2.2. Observables

Our main observable is the response of the atomic medium to the probe field. The linear susceptibility of the weak probe field can be written as [33]

\[ \chi (\omega_{p}) = \frac{N\eta_{p}}{e_{0}E_{p}}\rho_{23}(\omega_{p}), \]  

where \( N \) is the atom number density in the medium, \( \eta_{p} \) is the probe transition dipole moment and \( \chi = \chi^{r} + i\chi^{i} \).

The susceptibility \( \chi (\omega_{p}) \) is related to the index of refraction \( n = n^{r} + in^{i} \) via \( n^{i}(\omega_{p}) = 1 + \chi (\omega_{p}) \), and the real and imaginary parts of \( \chi (\omega_{p}) \) correspond to the dispersion and absorption, respectively.

The slope of the dispersion with respect to the probe detuning has a major role in the calculation of the group velocity. We introduce the group index, \( n_{g} = c/v_{g} \), where the group velocity \( v_{g} \) of the probe field for vanishing absorption is given by [10, 22]

\[ v_{g} = \frac{c}{n^{r}(\omega_{p}) + \alpha_{p}/[2n^{i}(\omega_{p})]^{2}\frac{n^{i}(\omega_{p})}{\alpha_{p}}}. \]  

Equation (10) implies that, for a negligible real part \( \chi^{r}(\omega_{p}) \), the group velocity can be significantly reduced via a steep positive dispersion. Strong negative dispersion, on the other hand, can lead to an increase in the group velocity and even to a negative group velocity.

Substituting equations (7) and (9) in equation (10), the group index of the probe field evaluates to

\[ n_{g} - 1 = \frac{g_{42}^{2}g_{p}\gamma_{23}}{g_{42}^{2}\gamma_{23} + 2\Lambda\Gamma_{42}\gamma_{42}}\frac{\Delta_{p}^{2} - \Lambda^{2}}{(\Delta_{p}^{2} + \Lambda^{2})^{2}}. \]  

It can be expected from equation (11) that for suitable parameters, the group index around \( \Delta_{p} = 0 \) is negative and accompanied by the gain, and this is indeed what we find below.

The relation between the coherence and susceptibility equation (9) can be rewritten as

\[ \chi (\omega_{p}) = \frac{N\eta_{p}}{e_{0}E_{p}}\rho_{23}(\omega_{p}) = \frac{3N\lambda_{1}^{2}\gamma_{23}\rho_{23}(\omega_{p})}{8\pi^{2}\gamma^{2}g_{p}/\gamma}, \]  

where we have used \( \gamma_{23} = \left( \frac{\gamma_{23}^{2}}{\lambda_{42}^{2}\lambda_{34}^{2}} \right)(3\pi e_{0}c^{3}) \) and \( g_{p} = \eta_{23}E_{p}/\hbar \) as well as \( \omega_{23} = 2\pi c/\lambda_{23} \) with the probe transition

\[ \lambda_{23} \]
parameters $\gamma$, $\gamma_0$, added a weak decay rate to the case found in mercury, see figure 1(b). We have

$$\Delta_1 = 0.01 \gamma, g_p = 10^{-4} \gamma, g_{41} = 0, g_{42} = 4 \gamma, \Lambda = 0, \Delta_{42} = \Delta_{41} = 0.$$ 

Throughout our discussion of numerical results, we will assume these parameters in order to evaluate the susceptibility.

### 3. Results

#### 3.1. Without interacting dark-state resonance

In figure 2, we show the real (blue dashed) and imaginary (red solid) parts of the probe field susceptibility $\chi$ versus the probe detuning $\Delta_p$, which correspond to the dispersive and absorptive properties of the medium, respectively. In this figure, the perturbing laser field is switched off, $g_{41} = 0$. The other parameters are $\gamma_{41} = \gamma, \gamma_{23} = 0.14 \gamma, \gamma_{42} = 0.79 \gamma, \gamma_{13} = 0.01 \gamma, g_p = 10^{-4} \gamma, g_{42} = 4 \gamma, \Lambda = 0, \Delta_{42} = \Delta_{41} = 0$. Note that the ratios of the decay rates correspond to the case found in mercury, see figure 1(b). We have added a weak decay rate $\gamma_{13}$, since otherwise in the steady state all population is trapped in $|1\rangle$. The driving field with the Rabi frequency $g_{42}$ leads to an Autler–Townes doublet with a dip in the absorption at zero detuning, i.e., partial electromagnetically induced transparency (EIT). The slope of the real part of the susceptibility in the region of reduced absorption is positive. We thus find that the subluminal light propagation occurs around zero detuning with reduced absorption as it is common for EIT. If the state $|4\rangle$ was long-lived, then the EIT leading to the partial transparency would be more pronounced such that the absorption would vanish at zero detuning.

#### 3.2. With interacting dark-state resonance

In figure 3, in addition we apply the weak perturbing field with the Rabi frequency $g_{41} = 0.04 \gamma$, and assume negligible decay on transition $|3\rangle \leftrightarrow |1\rangle$, since a trapping in this state is now avoided by the additional laser field. The results are identical to figure 2 except for a narrow absorption spike at around zero detuning. The shape and width of the absorption spike are determined by equation (2a) and (5), respectively. In particular, the width is much less than the natural linewidth. Again, for a long-lived state $|4\rangle$, the transparency regions on each side of the absorption spike would become two points of EIT, i.e., a double dark state [31]. In terms of the light propagation, the slope of the real part of the susceptibility around zero detuning is negative such that the superluminal light propagation could be observed, albeit with high absorption.

#### 3.3. With interacting dark-state resonance and incoherent pumping

We now in addition apply a weak incoherent pumping field on the probe transition $|2\rangle \rightarrow |3\rangle$. Figure 4 shows the corresponding results. The incoherent pump field rate is chosen as $\Lambda = 4 \times 10^{-5} \gamma$. In this case, the superluminal light propagation found in figure 3 at $\Delta_p = 0$ switches to subluminal propagation, and the absorption spike at zero detuning becomes a gain spike. The shape of the gain spike is determined by the imaginary part of equation (7) which is Lorentzian with half width equal to $\Lambda$. A dressed-state analysis of the transition to an amplifying medium will be given in section 3.4. It can be seen from figure 4 that at $\Delta_p \approx \pm 3.1 \times 10^{-4} \gamma$ (indicated by the purple vertical lines), the imaginary part of the susceptibility vanishes together with a negative slope of the real part. At these probe parameters, the real part of the susceptibility is as a function of the probe detuning $\Delta_p$. The other parameters are the same as in figure 2; (b) is a closeup on the central part of (a).
field detunings, the real part of the susceptibility itself is nonzero, and is negative (positive) for $\Delta_p \approx -3.1 \times 10^{-4} / \gamma$ ($\Delta_p \approx 3.1 \times 10^{-4} / \gamma$). In the following, we discuss the two cases of interest with resonant or non-resonant probe field separately.

We start with the resonant case $\Delta_p = 0$. In figure 5(a), we study the effect of the incoherent pumping strength $\Lambda$ on the magnitude of the imaginary part of the susceptibility $\chi$ at resonance $\Delta_p = 0$. It can be seen that depending on the coupling field Rabi frequency $g_{42}$, the transition from absorption to gain occurs at different values of the incoherent pumping. For the parameters of figure 3, which correspond to the long-dashed green curve in figure 5(a), the transition is at about $\Lambda \approx 2 \times 10^{-4} / \gamma$. This explains why the gain could be observed for the parameters in figure 4. On increasing the incoherent pumping further, the imaginary part approaches zero again.

After the discussion of the absorption, we now turn to a discussion of our main observable, the group velocity. Since the real part of $\chi$ itself vanishes at $\Delta_p = 0$, the group velocity is determined by the slope of the real part of the susceptibility at $\Delta_p = 0$, see equation (10). This quantity is shown in figure 5(b). It can be seen that, for no or small incoherent pumping, the system exhibits a negative slope, which leads to a superluminal or even negative group velocity. On increasing $\Lambda$, the slope can be adjusted to large positive values, where subluminal light can be expected. Thus in principle the system allows for a wide range of group velocities, controlled via the incoherent pump rate $\Lambda$. But from a comparison of figures 5(a) and 5(b) it can be seen that typically negative slopes are accompanied by absorption, while positive slopes occur together with the gain. Thus, at $\Delta_p \approx 0$, only a reduction of the group velocity is accessible in experiments without absorption. The different curves in figure 5 further show that the precise response of the system to the incoherent pumping can be controlled by varying the coupling field Rabi frequency $g_{42}$. In particular, stronger coupling fields $g_{42}$ may be favourable, since then the range of possible slopes is increased, as can be seen from figure 5(b).

We now turn to a discussion of the non-resonant case, $\Delta_p \neq 0$, and focus on the regions with vanishing absorption, such as $\Delta_p \approx 3.1 \times 10^{-4} / \gamma$ in figure 4. It can be seen that around these probe field detunings, the imaginary part of the susceptibility vanishes, such that the probe field passes unattenuated through the medium. At the same time, the real part of the susceptibility is nonzero, and has a negative slope. Therefore, at these frequencies, superluminal or negative group velocities are accessible without absorption. In order to study this result in more detail, in figure 6(a) we show the probe field detuning $\Delta_0$ at which the imaginary part of the susceptibility vanishes as a function of the incoherent pumping rate $\Lambda$. It can be seen that for no or small incoherent pumping $\Lambda$, there is always an absorption such that no $\Delta_0$ can be found. Once $\Lambda$ is large enough for a root in the imaginary part of the susceptibility to occur, the position of the root first increases rapidly with $\Lambda$, and then saturates. The required value of $\Lambda$ also depends on the strength of the coupling field $g_{42}$ as can be seen from figure 6(a).
The corresponding figure 6(b) depicts the slope of the real part of the susceptibility as a function of the pumping field strength $\Lambda$. It can be seen that by varying the pump field strength $\Lambda$, both positive and negative slopes can be achieved at frequencies where the medium absorption is zero. After passing through a maximum positive slope, the slope drops to a minimum negative slope and negative slopes can be achieved at frequencies where the values of the slope are of order $10^{-7}$ and either positive or negative. The maximum absolute value of the slope are of order $10^{-9}$ s$^{-1}$, such that the term proportional to $\partial^2 \chi'/\partial \omega^2$ in the denominator of equation (10) for our probe transition varies between approximately $-10^7$ and $+10^7$. Therefore, strongly sub- and superluminal propagation as well as a large range of negative group velocities occur without absorption in our sample, controlled by the magnitude of the incoherent pumping. It should be noted that only very weak incoherent pumping is required, as can be seen from the scaling of the $x$-axes in figure 6.

3.4. Dressed-state analysis

We now introduce the dressed states generated by the strong driving field acting on transition $|2\rangle \leftrightarrow |4\rangle$ and the coupling field acting on transition $|1\rangle \leftrightarrow |4\rangle$, in order to demonstrate the presence of interacting dark resonances due to the perturbing coupling, there are transitions between $|0\rangle$ and $|3\rangle$. In the bare state picture, these transitions correspond to three photon resonances $|1\rangle \rightarrow |4\rangle \rightarrow |2\rangle \rightarrow |3\rangle$ [32].

The transition from absorption to gain under the influence of the incoherent pump field $\Lambda$ can be interpreted in terms of transitions between the different bare and dressed states. As explained above, the narrow double dark-state resonance arises due to transitions from the dressed state $|0\rangle$ to the bare state $|3\rangle$. In figure 7, we show the populations of these two states together with the imaginary part of the susceptibility for parameters as in figure 3. The probe field detuning is chosen as $\Delta_p = 0$, i.e., we look at the centre of the narrow structure. It can be seen that, as expected, virtually all population is in the bare state $|3\rangle$ for vanishing pumping field. The reason for this is that the weak probe field can only transfer a very small amount of population into the other states. With increasing pumping field $\Lambda$, however, the population transfers out of the state $|3\rangle$ into the dressed state $|0\rangle$. The population in $|0\rangle$ dominates among the different dressed states because it

![Figure 6](image_url)

Figure 6. (a) Position $\Delta_0$ of the root of the imaginary part of the susceptibility $\chi$ as a function of the pumping field strength $\Lambda$. At this frequency, light passes unattenuated through the medium. (b) Slope of the real part $\chi'$ of the susceptibility at the detuning $\Delta_0$ with vanishing absorption. The parameters are as in figure 3 but with $g_{42} = 10\gamma$ (blue short-dashed), $g_{42} = 7\gamma$ (purple solid) and $g_{42} = 4\gamma$ (green long-dashed).

![Figure 7](image_url)

Figure 7. Populations of the dressed state $\rho_{00} = \langle 0\rangle \langle 0\rangle$ defined in equation (14a) (red solid), bare state $\rho_{33} = \langle 3\rangle \langle 3\rangle$ (blue short-dashed) and imaginary part of the susceptibility (green long-dashed) as a function of the pumping field strength $\Lambda$. The parameters are as in figure 3 with $\Delta_p = 0$. The purple vertical line indicates the transition from absorption to gain.
contains a large contribution of the bare state \[ |1 \rangle \], which is populated by spontaneous emission, but emptied only by the weak perturbing field \( g_{41} \). Together with this population transfer, the absorption decreases, until the imaginary part of the susceptibility vanishes where the populations of \( |3 \rangle \) and \( |0 \rangle \) are equal. A further increase of the incoherent pumping generates a population inversion on transition \( |0 \rangle \rightarrow |3 \rangle \), such that the medium amplifies. Thus, the absorption or amplification properties of the narrow double dark resonance can be directly explained by the population difference between the initial and the final state. A modification of this difference via the incoherent pump field allows us to control the medium absorption.

3.5. Doppler averaging

Finally, we discuss the influence of Doppler broadening on the results obtained so far. One might be tempted to judge that the narrow structure arising from the interacting dark resonances will be washed out if broadening is taken into account. In order to clarify this question, we averaged our results for the susceptibility over a Maxwellian velocity distribution

\[
f(\Delta) \, d\Delta = \frac{1}{\sqrt{\pi} v_{\text{ms}}} \, e^{-\frac{\Delta^2}{v_{\text{ms}}^2}} \, d\Delta
\]

of the atoms in order to simulate a Doppler-broadened gas. The distribution (16) is characterized by the mean absolute velocity of the atoms \( v_{\text{ms}} = \sqrt{\frac{k_B T}{m}} \), where \( k_B \) is the Boltzmann constant, \( m \) is the mass of the atoms and \( T \) is the temperature. Effectively, the Doppler shift yields modified laser frequencies

\[
\omega_{ij}^{\text{eff}} = \omega_{ij} \left( 1 + \frac{V}{c} \right) \quad (i, j \in \{1, \ldots, 4\}) \tag{17}
\]

for all laser-driven transitions. In the following numerical analysis, we assume a particle density of \( 10^{12} \text{ cm}^{-3} \). For mercury, this corresponds to a temperature of about 256 K [34], a mean velocity \( v_{\text{ms}} = 145.68 \text{ m s}^{-1} \), and a Doppler width of 3.6 GHz. All laser fields are co-propagating in the same direction.

In figure 8(a), we compare the susceptibility with and without Doppler averaging for parameters as in figure 4(b). Without averaging, for this set of parameters the medium exhibits gain, and the real part of the susceptibility has a positive slope around \( \Delta_p = 0 \). Moving away from the resonance, the real part approaches zero. A comparison with the averaged results shows that, interestingly, the width of the narrow double dark-state resonance is largely unaffected by the broadening. But the averaged medium absorbs, and the slope of the real part of the susceptibility is negative. Away from the resonance, the real part tends to a positive value. It turns out that due to the Doppler broadening, the two wide resonances at \( \Delta_p \approx \pm 4 \gamma \) shown in figure 4(a) move together. This causes an increase of the baseline of the imaginary part of the susceptibility around \( \Delta_p = 0 \). It is possible to counter this effect by increasing the Rabi frequency \( g_{42} \), which is responsible for most of the splitting of these two resonances.

In figure 8(b), we show unbroadened curves as in subfigure (a) together with the curves for the Doppler-averaged case with

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**Figure 8.** Doppler-averaged susceptibilities. The particle density is \( 10^{12} \text{ cm}^{-3} \), which corresponds to a temperature of about 256 K. The blue short-dashed and the red solid curves are the real and imaginary parts of unbroadened reference data. The black dotted and the green dash-dotted curve are the corresponding Doppler-averaged results. (a) Parameters are as in figure 4(b). (b) As in (a), but with \( g_{42} = 12 \gamma \) for the Doppler-averaged susceptibility. (c) Reference: parameters as in (a) except for the negative slope of the corresponding green long-dashed curve in figure 6(b). The averaged curves are drawn for the same parameters except for \( g_{42} = 8 \gamma \). (d) Reference: parameters as in (a) except for \( \Delta = 16.45 \times 10^{-6} \gamma \), which corresponds to the largest positive slope of the corresponding green long-dashed curve in figure 6(b). The averaged curves are drawn for the same parameters except for \( g_{42} = 7.32 \gamma \).
$g_{42} = 12\gamma$. It can be seen that qualitatively similar results can be obtained in the averaged case simply by increasing $g_{42}$, even though in this particular case the magnitude of the response of the averaged medium is smaller than in the reference case. In subfigures (c) and (d), we turn to parameters where in the unaveraged case the slope of the real part of the susceptibility has maximum positive or negative values, as these are the most interesting points in terms of group velocity control. In (c), the parameters of the reference curves are as in (a), except for the incoherent pumping rate $\Lambda = 21.0 \times 10^{-6}\gamma$, which is the rate at which the largest negative slope is obtained for the corresponding long-dashed green curve in figure 6(b). The Doppler-averaged curves are drawn for the same parameters except for an increase of $g_{42}$ to $g_{42} = 8\gamma$, as explained above. It can be seen that, both for the averaged and the unaveraged case, the medium absorption vanishes at a certain probe-field frequency together with the negative dispersion of comparable slope. In (d), the parameters are as in (a) except for $\Lambda = 16.45 \times 10^{-6}\gamma$. At this pumping rate, the largest positive slope of the corresponding long-dashed green curve in figure 6(b) occurs. The averaged curves are drawn for the same parameters except for $g_{42} = 7.32\gamma$. For these parameters, a comparable negative dispersion is achieved at the point of transparency.

In figure 8(d), the slope of the dispersion around the point of vanishing absorption is almost constant over a frequency range roughly given by the width of the narrow resonance. This is an important requirement in order to achieve undistorted pulse propagation through the medium [35]. The case of the maximum negative dispersion in figure 8(c) has a small curvature of the dispersion in the transparency region. This curvature can be minimized by moving the transparency region to a suitable position via a change of the incoherent pump rate $\Lambda$.

It should be noted that we only modified the coupling field Rabi frequency $g_{42}$ in order to recover large positive or negative dispersion without absorption depending on the applied incoherent pump rate $\Lambda$. This change was motivated by the need to compensate the shifting of the broad resonances due to the Doppler averaging. An additional optimization of the other system parameters may further improve the result. A different setup of the laser propagation directions was discussed in [32]. Experimental observations of narrow double dark resonances in the Doppler-broadened media were reported, e.g., in [36].

3.6. Numerical verification of the analytical results

Throughout this section, figures 2–8 have been obtained from a numerical solution of the full density matrix equations (1a)–(1f). In the following, we verify our approximate analytical expressions, equations (2a)–(7), by a comparison to the exact numerical calculations. The result is shown in figure 9, where the solid red curves correspond to the approximate analytical solutions, whereas the blue dashed curves represent our numerical results. The approximate result equation (4) for the case without incoherent pump field is shown in comparison to the numerical data in figures 9(a) and (b). It turns out that in this case, the results from equation (4) are virtually identical to the corresponding numerical results. Equation (7) for the case with incoherent pumping is compared to the numerical results in figures 9(c) and (d). Here, the analytic results only describe the qualitative behaviour of the curves. The reason for this is that in this figure, we chose parameters for which the condition $\Lambda \gg \Lambda_0$ in equation (6) is not well satisfied. If the incoherent pumping $\Lambda$ is increased, the agreement of the approximate results with the numerical calculation improves. Thus we conclude that our analytical results describe the system well enough to allow for an optimization of the parameters towards a desired peak structure, as long as the conditions on the parameters are satisfied.

4. Conclusion

We have discussed the dispersive and absorptive properties of a four-level atomic medium that exhibits interacting dark-state resonances. In our numerical analysis, we have focused on mercury atoms with an ultraviolet probe field wavelength of 243.7 nm. Due to the interacting resonances, a high-resolution structure appears in both the absorption and the dispersion spectra. A weak probe field tuned to this resonance usually experiences superluminal propagation with absorption. But if in addition a weak incoherent pump field is applied to the probe transition, then the superluminal light propagation changes to the subluminal light propagation accompanied by no absorption or gain. Slightly off resonance, the probe field experiences a vanishing imaginary part of the susceptibility. At these off-resonant frequencies, the real part of the susceptibility itself is nonzero and has a slope depending on the incoherent pumping strength. Thus, both sub- and superluminal light propagation as well as negative group velocities can be achieved without absorption. The control
via the incoherent pump fields suggests potential applications, e.g., in optical switching devices or in controllable pulse delay lines for the ultraviolet frequency region.

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