Baryon and lepton number transport in
electroweak phase transition

J. Maalampi\textsuperscript{a}, J. Sirkka\textsuperscript{b} and I. Vilja\textsuperscript{b}

\textsuperscript{a} Department of Theoretical Physics, University of Helsinki
\textsuperscript{b} Department of Physics, University of Turku

May 1994

Abstract

We consider the baryon number generation by charge transport mechanism in the electroweak phase transition taking properly into account thermal fluxes through the wall separating true and false vacuum in the spatial space. We show that the diffusion from the true vacuum to the false one has a large diminishing effect on the baryon number unless the wall velocity is near to, but less than, the speed of sound in the medium and the ratio between the collision rate and wall thickness is about 0.3. The maximum net baryon density generated is $\rho_B/s \approx 0.2 \times 10^{-10}$, where $s$ is the entropy density of the Universe. If the wall proceeds as a detonation, no baryon number is produced.
It was pointed out by Sakharov \[1\] that the cosmological asymmetry between baryons and antibaryons can follow from the particle interactions in the early Universe if the following conditions are simultaneously satisfied: the baryon number $B$ is not conserved, $C$ and $CP$ are not exact symmetries and the Universe is out of thermal equilibrium. The electroweak theory has all these ingredients, if the weak phase transition is first order \[2\]. The baryon number non-conservation follows from the anomalous violation of the $B + L$ number ($L$ is the lepton number) due to non-perturbative sphaleron effects \[3\]. In a first order phase transition there will be local departure from thermal equilibrium in the vicinity of the walls between the symmetric and broken phases. In the standard electroweak model there also is built in $CP$ violation manifested as a complex phase in the Kobayashi-Maskawa quark mixing matrix. The resulting $CP$ violating effect is, however, at least two orders of magnitude too small to account for the observed baryon to entropy ratio $\simeq 10^{-10}$. One thus have to go beyond the minimal standard model.

One possible mechanism, so-called charge transport mechanism, that may produce a large enough baryon asymmetry was suggested by Cohen, Kaplan and Nelson \[4\]. The idea is that a net charge, e.g. lepton number, is reflected from the domain wall between broken and unbroken phase, and anomalous SU(2) effects in the unbroken phase transmute this into a net baryon number. Then the baryons produced pass thermally to the broken phase where they are stable. In order to produce a net charge, the reflected particles have to undergo $CP$ violating interactions in the wall. In \[4\] these are the Yukawa interactions of neutrinos which may involve unremovable phases. Such situation may be realized, for example, in the context of the singlet Majoron model \[5\] and the left-right symmetric model \[6\].

In this paper we shall reconsider some aspects of the charge transport mechanism. We derive coupled Boltzmann equations for the evolution of the baryon and lepton
numbers in the broken and unbroken phases. In particular, we shall take into account also the diffusion of particles from one phase to another, and we actually find out that the diffusion has in many cases quite large diminishing effect on the creation of the baryon number.

We are considering a system of an expanding bubble of the true vacuum, i.e. of the broken phase. The expansion starts with a critical bubble which is a spherical configuration of the order parameter \[7\]. We assume that the bubble remains spherical during the expansion, its radius being \( R(t) = R_0 + v_w t \), where \( v_w \) is the velocity of the moving wall and \( R_0 \) is the radius of the critical bubble. The time needed for completing the phase transition depends on the size of the critical bubble. In the following we will assume a weak phase transition in which case the critical bubble is large and the growth time of the bubble is of the same order than the bubble size. The growth time is given by \[8\]

\[
t_{\text{growth}} = \frac{0.073 M_{\text{Pl}}}{\sqrt{g_*} T_c^2} \left[ \ln \left( \frac{M_{\text{Pl}}^4}{T_c^4} \right) \right]^{-3/2},
\]

where \( g_* \) is the effective number of degrees of freedom (here \( g_* \simeq 110 \)), \( M_{\text{Pl}} \) is the Planck mass and \( T_c \) is the critical temperature. Using the result of \[8\] one finds out that the bubble radius reaches the value

\[
R_1 = (8\pi)^{1/3} v_w t_{\text{growth}} \simeq 3.2 \times 10^9 v_w / \text{GeV}
\]

upon completion of the phase transition. The critical bubble radius is thus

\[
R_0 = R_1 - v_w t_{\text{growth}} \simeq 2.1 \times 10^9 v_w / \text{GeV}.
\]

Strictly speaking, the results above are only for the Standard Model, but they are likely to give the correct order of magnitudes for a wide range of models not
too different from the Standard Model, like the Majoron model. Moreover, our final results are quite insensitive to the precise values of $R_0$ and $R_1$. It turns out that the changes on $R_0$ and $R_1$ between as large values as $10^4 v_w/T$ and $10^{11} v_w/T$ do not effect the value of the generated baryon number but by a factor 2, at most.

In the case of a weak first order phase transition nucleated bubbles grow as deflagrations. The advancing phase boundary is preceded by a supersonic shock front which hits the fluid in the false vacuum and makes it to move to the same direction as the boundary. The shock front discontinuity is formed within a time period of $t_0 = O(10) 1/T_c$ which is very short compared with $t_{\text{growth}}$. The velocity of the shock front can be estimated as 

$$ v_{\text{shock}} = \frac{1}{\sqrt{3}} \sqrt{\frac{3T_q^4 + T_f^4}{3T_q^4 + T_f^4}} \approx \frac{1}{\sqrt{3}}. \quad (4) $$

since the difference between the temperature of the symmetric phase $T_f$ and the temperature in the region between phase and shock fronts $T_q$ is estimated to be smaller than one percent. Hence the shock front moves essentially with the speed of sound. In front of the shock wall the particles are at rest in average; there is no net flux in this region. Inside the shock wave particles are moving with a slow velocity outwards causing a uniform particle flux. Behind the wall in the broken phase particles are again at rest in average.

Let us now derive the evolution equations for the lepton numbers $L_1$ and $L_2$, and for the baryon numbers $B_1$ and $B_2$, where the subscript 1 refers to the symmetric region and the subscript 2 to the broken one. We start by writing the equation of continuity for the lepton and baryon number densities, $l_i$ and $b_i$, and the radial fluxes, $\lambda_i$ and $\beta_i$, in the symmetric and broken regions. Assuming spherical symmetry the
The source terms in the symmetric region are due to anomalous $B + L$-violating processes, characterized by a time scale $\tau_B \simeq 1\text{GeV}^{-1}$ [4]. The parameter $x$ is defined as the equilibrium value of the ratio $(B + L)/(B - L)$. In the broken region the source terms can be neglected, since there the $B + L$ violation effects are expected to be small [4].

We now integrate the first two equations of (5) over the volume between the phase boundary with a radius $R(t)$ and the shock front with a radius $R_s(t) = R_{s0} + v_s t$. Knowing the exact value of the initial shock front radius $R_{s0}$ is not that important as the shock wave is formed, as mentioned, in a very short time period in comparison with the whole span of the time evolution.

The ensuing equation for the lepton number $L_1$ in the symmetric phase reads ($\dot{L}$ stands for time derivative)

$$\dot{L}_1 = 4\pi R_s^2 l_1(R_s, t)v_s - 4\pi R^2(t)l_1(R, t)v_w + 4\pi R^2(t)\lambda_1(R, t) - \frac{1}{2\tau_B}[(1 - x)B_1 + (1 + x)L_1].$$

An analogous equation can be derived for the baryon number $B_1$. In deriving Eq. (6) it was assumed that the lepton number flux at the shock front can be neglected.

\[^1\text{We assume that the sphaleron energy is large enough [11] to prevent the baryon number dilution in the broken phase.}\]
From the latter two equations of (5) one obtains similarly the time evolution equation for the lepton number in the broken phase:

\[ \dot{L}_2 = 4\pi R^2 l_2(R, t)v_w - 4\pi R^2 \lambda_2(R, t), \]  

(7)

and an analogous equation for the \( B_2 \). In deriving the equations for \( L_2 \) and \( B_2 \) it was assumed that the flux of the lepton number is negligible at \( r = R_0 \) for all times. Strictly speaking this is not true but serves as a good approximation.

In order to solve Eqs. (6) and (7) and the corresponding equations for the baryon number, one has to know the number densities and the fluxes of leptons and baryons at the boundary of the two phases. There are different effects one should take into account. The CP violating scattering of neutrinos in the advancing wall produce a net lepton number flux on the both sides of the wall. This effect was inspected in detail in [4]. They found numerically that the thermally averaged net lepton number flux \( f_L \), in units of entropy density, reflected from the wall to the symmetric phase is

\[ -f_L = 2.0 \times 10^{-10} - 4.2 \times 10^{-12} \]

(8)

at \( T = 200 \) GeV, depending on the wall velocity and thickness.

There is also a flux of leptons and baryons due to thermal transport of particles from one phase to another. The driving force of the thermal flux is the particle density differences over the phase boundary. The thermal flux of the lepton number in the symmetric phase, to be denoted by \( \lambda_1 \), can be presented in the form \( \lambda_1 = -D \nabla l_1 \). Here \( \nabla l_1 \) is the radial lepton number gradient and \( D \) is the diffusion coefficient. A simple estimate for the diffusion coefficient is \( D = \langle v \rangle^2 \tau / 3 \), where \( \tau \) is the average time between particle collisions [12]. Because all particles in the unbroken phase are massless and they are still relativistic in the broken phase (except
the heavy right–handed neutrinos) the average particle velocity $\langle v \rangle$ is about unity. The collision time $\tau$ can be calculated from the model given. In the Standard Model one can estimate $\tau \simeq 0.1\text{GeV}^{-1}$.

If the wall between the broken and unbroken phase were at rest we could approximate thermal flux at the boundary as $D(l_1(R(t), t) - l_2(R(t), t))/\delta$, where $\delta$ is the thickness of the wall. The wall is, however, moving with velocity $v_w$. Taking that into account the formula modifies to

$$\lambda_1 = \frac{D}{2\delta} [l_1(R(t), t)(1 + v_w) - l_2(R(t), t)(1 - v_w)].$$

(9)

The factors $(1 + v_w)/2$ and $(1 - v_w)/2$ give the fractions of light particles which are caught by the wall in the symmetric region or catch the wall in the broken region, respectively. Those low-energy leptons, which reflect back to the symmetric region because of becoming too heavy to penetrate the wall, produce a flux $2r_\nu Dl_1(R(t), t)(1 + v_w)/(2\delta)$. Here the quantity $r_\nu$ gives the fraction of the heavy neutrinos that are reflected, and the overall factor of two is there due to the fact that in the reflection the lepton number changes by two units. In our numerical calculations we will use $r_\nu = 1/4$ corresponding to the situation in the Majoron model [4]. Note that there is no contribution of heavy leptons coming from the broken region to the symmetric region, because they are, in average, much slower than the wall. The thermal fluxes for the baryon number obtained similarly. We have approximated the collision time $\tau$ to be the same for the both cases.

About the net lepton and baryon number densities we assume a uniform distribution inside the expanded bubble and a linearly decreasing densities inside the shock front, i.e. between the radii $R(t)$ and $R_0$, so that $l_1(R_s) = b_1(R_s) = 0$. This yields
\[ L_1 = \frac{\pi}{3} l_1(R, t) (R_s - R)(R_s^2 + 2RR_s + 3R^2), \]
\[ L_2 = \frac{4\pi}{3} l_2(R, t) (R^3 - R_0^3). \] (10)

Using these approximations the evolution of the lepton and baryon numbers is described by the following set of equations:

\[
\frac{1}{3u^2} \frac{dL_1}{du} = 1 - \frac{\tau}{6\delta v_w} [(1 + v_w)(1 + r_\nu)\alpha_1(u)L_1 - (1 - v_w)(1 - r_\nu)\alpha_2(u)L_2] - \alpha_1 L_1 - \frac{R_0}{2v_w\tau_B}[(1 - x)B_1 + (1 + x)L_1],
\]
\[
\frac{1}{3u^2} \frac{dB_1}{du} = -\frac{\tau}{6\delta v_w} [(1 + v_w)\alpha_1(u)B_1 - (1 - v_w)\alpha_2(u)B_2] - \alpha_1 B_1 - \frac{R_0}{2v_w\tau_B}[(1 - x)B_1 + (1 + x)L_1],
\]
\[
\frac{1}{3u^2} \frac{dL_2}{du} = -1 + \frac{\tau}{6\delta v_w} [(1 + v_w)(\alpha_1(u)L_1 - (1 - v_w)\alpha_2(u)L_2)(1 - r_\nu) + \alpha_2 L_2],
\]
\[
\frac{1}{3u^2} \frac{dB_2}{du} = \frac{\tau}{6\delta v_w} [(1 + v_w)(\alpha_1(u)B_1 - (1 - v_w)\alpha_2(u)B_2) + \alpha_2 B_2]. \] (11)

The lepton numbers \( L_i \) and baryon numbers \( B_i, i = 1, 2, \) are presented in units of \( f_L \) and they are scaled with a reference volume \( V_0 = 4\pi/3R_0^3. \) We have defined a dimensionless variable \( u = R(t)/R_0 \) and the functions

\[
\alpha_1(u) = 4[(\bar{u} - u)(\bar{u}^2 + 2u\bar{u} + 3u^2)]^{-1}, \]
\[
\alpha_2(u) = 1/(u^3 - 1), \] (12)

where \( \bar{u} = R_s/R_0 = R_{s0}/R_0 + v_s/v_w(u - 1). \) The baryon density in the broken phase, the quantity we are interested in, is then given by

\[ b_2 = f_L \frac{R_0^3}{R_1^3} B_2(u = R_1/R_0). \] (13)
Before moving to the numerical solutions of our final equations (11), we will make some general observations. It is quite easy to deduce from the equations that there are solutions with no baryon number generation. This depends on the values of the unknown parameters like the wall thickness $\delta$ and the wall velocity $v_w$, as well as the diffusion time scale $\tau$. It turns out that the decisive quantity is

$$Q = \frac{\tau}{6\delta v_w} (1 - v_w)(1 - r_v).$$

(14)

This can be seen as follows. First make the ansatz that the solution is "almost trivial", i.e. $B_1$, $B_2$ and $L_1$ are zero. Then $L_2$ of the following form solves (11):

$$L_2 = \frac{1}{Q} (u^3 - 1) + C(u^3 - 1)^{1-Q}.$$  

(15)

When $Q > 1$ the initial condition $L_2(u = 1) = 0$ forces the integration constant $C$ equal to zero. By substituting the solution (15) to eqs. (11) one then finds that, as consistency requires, the derivatives of $L_1$, $B_1$ and $B_2$ vanish. In order to have baryon number generated one has therefore to require $Q < 1$. Physically this means that the wall have to be thick enough or it must advance into the unbroken phase rapidly enough to avoid dilution of the net baryon number due to particle diffusion through the wall. Note that this is also the asymptotic solution for the Boltzmann equations when $R_1/R_0 \gg 10^9$. Physically that is, however, not a favoured case.

Let us now present our numerical results. The theoretical estimates for the values of the wall velocity $v_w$ and the wall thickness $\delta$ vary with the assumptions made of the strength of the phase transition and the nature of the bubble expansion. We keep these quantities as free parameters. As we have found, the wall thickness in fact appears in our expressions always along with the free time $\tau$, the results depending on their ratio $\tau/\delta$ only.

In Fig. 1 we present the values of the generated baryon density $b_2$ as a function
of $v_w$ and $\tau/\delta$. The maximum value of $b_2$ achieved inside the $b_2 = 0.2$ contour is about 0.23, and it corresponds to the wall velocity close to the velocity of sound and the wall thickness close to, but slightly larger than, the neutrino free path. We have plotted also the $Q = 1$ contour, to right of which no baryon number is generated, because there the wall moves so slowly or is so thin that the particle diffusion through the wall dilutes the baryon number. On the other hand, the decreasing of the net baryon number with decreasing $\tau/\delta$ follows from the fact that in the case of a thick wall less baryon number moves into the broken phase from the unbroken phase where it is created. This decrease is of course quite independent on the wall velocity as the baryon number flux goes towards the advancing wall.

In Fig. 1 we have integrated the evolution equations from the initial bubble radius $R_0$ to the final radius $R_1$, whose values were given in eqs. (3) and (2). As mentioned, this corresponds to a weak phase transition where the initial bubbles are quite large and the expansion time small. We have also investigated the sensitivity of our result on this assumption. This may be relevant since it is also possible that the initial bubbles are very small and that the bubble radius increases several orders of magnitude upon the completion of the phase transition. In Fig. 2 we present the generated baryon number as a function of the ratio $R_1/R_0$ with $R_1$ fixed to the value (2). One can conclude from this figure that the net baryon number does not depend significantly on the initial size of the bubble in the physical region of the ratio $R_1/R_0$.

We have presented our results in terms of the reflected lepton number flux $f_L$. This quantity has been estimated in [4] for some values of the parameters. For $\delta \simeq 0.1$ GeV$^{-1}$, $v_w = 0.87$ and the critical temperature $T_c = 200$ GeV they give $f_L = -1.2 \times 10^{-10} s$, where $s$ is the present entropy density in the Universe. Applying this value to our results would yield the maximum net baryon number density $\rho_B/s \simeq$
$0.24 \times 10^{-10}$. This is quite close but slightly below the allowed range $\rho_B/s = (0.4 - 1.0) \times 10^{-10}$ obtained from the standard nucleosynthesis \cite{13}. The maximum value of $\rho_B/s$ corresponds the values $\tau/\delta \simeq 0.3$ and $v_w = 0.55$. It is relatively sensitive to the actual values of wall velocity and ratio $\tau/\delta$. The present theoretical estimates for these are $\delta \simeq (10 - 40)/T_c$ and $v_w$ lies between 0.2 and 0.8 \cite{8}. Note, however, that if the bubble wall proceeds as a detonation, i.e. $v_w > v_s$, no baryon number is created. In that case there is no region preceding the bubble wall where lepton number conversion to the baryon number could take place.

Our result seems to give some indication that the baryon number production by charge transport mechanism may not be powerful enough to create the observed baryon number. Because there is some uncertainty in the actual value of $f_L$, the baryogenesis via this scenario is not completely ruled out. The value of $f_L$ has been calculated only for two wall velocities and for a few wall thicknesses using critical temperature $T_c = 200$ GeV. Also the baryon number production might be enhanced if there existed relatively large initial baryon and/or lepton number density created before electroweak phase transition. Such could be created in the Majoron model by decays of of right–handed neutrinos \cite{14} or, in e.g. left–right symmetric model, by some mechanism during the symmetry breaking of the right–handed sector. These issues are currently under study \cite{13}. 

11
References

[1] A.D. Sakharov, JETP Lett. 5 (1967) 24.

[2] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. B155 (1985) 36.

[3] N.S. Manton, Phys. Rev. D28 (1983) 2019.

[4] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Nucl. Phys. B349 (1991) 727.

[5] Y. Chikashige, R.M. Mohapatra and R.D. Peccei, Phys. Lett. B98 (1981) 265.

[6] J.C. Pati and A. Salaam, Phys. Rev. D10 (1974) 275.

[7] S. Coleman, V. Gleiser and A. Martin, Commun. Math. Phys. 58 (1978) 211.

[8] K. Enqvist, J. Ignatius, K. Kajantie and K. Rummukainen, Phys. Rev. D45 (1992) 2415.

[9] J. Ignatius, K. Kajantie H. Kurki-Suonio and M. Laine, HU-TFT-93-43.

[10] L. Landau and E. Lifshitz, Fluid Mechanics, 2nd edition (Pergamon Press, Oxford, 1987).

[11] K. Enqvist, K. Kainulainen and I. Vilja, Nucl. Phys. B403 (1993) 749.

[12] See e.g., L.E. Reichl, A Modern Course in Statistical Physics (Edward Arnold Ltd, Austin, 1980).

[13] See e.g., E. Kolb and M. Turner, The Early Universe (Addison Wesley, Reading, MA, 1990, and references therein.

[14] I. Vilja, Phys. Lett. B324 (1994) 197.

[15] Work in progress.
FIGURE CAPTION

Figure 1. The net generated baryon density in the \((\log(\tau/\delta), \nu_w)\) -plane in units of \(-f_L\). With the parameter values corresponding to the area below the \(Q = 1\) curve no baryon number is generated. \(v_s\) is the sound velocity.

Figure 2. Dependence of the generated baryon density on the ratio \(R_1/R_0\) with \(\tau/\delta = 0.2, \nu_w = 0.5\) and \(R_1\) fixed to the value \(2\).
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9405378v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9405378v1