Characterization of the vibration, stability and static responses of graphene-reinforced sandwich plates under mechanical and thermal loadings using the refined shear deformation plate theory

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Abstract
An analytical investigation was carried out to assess the free vibration, buckling and deformation responses of simply-supported sandwich plates. The plates constructed with graphene-reinforced polymer composite (GRPC) face sheets and are subjected to mechanical and thermal loadings while being simply-supported or resting on different types of elastic foundation. The temperature-dependent material properties of the face sheets are estimated by employing the modified Halpin-Tsai micromechanical model. The governing differential equations of the system are established based on the refined shear deformation plate theory and solved analytically using the Navier method. The validation of the formulation is carried out through comparisons of the calculated natural frequencies, thermal buckling capacities and maximum deflections of the sandwich plates with those evaluated by the available solutions in the literature. Numerical case studies are considered to examine the influences of the core to face sheet
thickness ratio, temperature variation, Winkler- and Pasternak-types foundation, as well as the volume fraction of graphene on the response of the plates. It will be explicitly demonstrated that the vibration, stability and deflection responses of the sandwich plates become significantly affected by the aforementioned parameters.

**Keywords**
Free vibration, buckling, graphene-reinforced polymer composite, temperature-dependent properties, refined shear deformation theory, analytical modelling

**Introduction**
Among the different types of carbon nanomaterials, graphene and its derivatives have received considerable attention in many different fields due to their exceptional chemical and physical properties. Graphene varieties such as graphene oxide (GO), reduced-graphene oxide (rGO) and graphene nanoplatelets (GNPs) are available as super lightweight nano-sized particles with large surface area and excellent thermo-electro-mechanical properties [1–3]. Several studies have investigated the effect of carbon nanotube on mechanical and thermal properties of polymer composites [4,5], as well as the static, dynamic and stability analyses of structural components formed by such nanocomposites [6–16]. A summary of the recent studies on graphene-reinforced composites is provided below.

Zhang et al. [17] investigated the bending, buckling, and vibration responses of functionally graded GO-reinforced composite beams based on the first-order shear deformation theory (FSDT). The effects of distribution and weight fraction of GO and slenderness ratio and boundary condition of the beams on their mechanical response were studied through a set of comprehensive numerical analyses. Rafiee et al. [18] developed a mathematical model for evaluating the large deflection, post-buckling and free nonlinear vibration responses of GNPs/fiber/polymer laminated multiscale composite beams. The model was developed based on the Euler-Bernoulli beam theory, which included the von Kármán geometric nonlinearity. Polit et al. [19] studied the effect of porosity distribution on the bending and stability responses of functionally graded graphene-reinforced porous nanocomposite curved beams by incorporating a higher-order theory, also accounting for the effect of thickness stretching. The analytical solution was developed using the Navier’s approach.

In other studies, the nonlinear bending and vibration, buckling and post-buckling responses of functionally graded graphene-reinforced laminated plates subject to thermal environments were studied by Shen et al. [20–23]. The governing equations were derived by accounting for the geometric nonlinearity using the von
Kármán strains and incorporation of a higher-order shear deformation plate theory. A two-step perturbation approach was incorporated to determine the nonlinear response of functionally graded GRC laminated plates. Kiani [24,25] analyzed thermal post-buckling and large amplitude free vibration behaviour of clamped and simply-supported composite laminated rectangular plates reinforced with GNPs in a thermal environment. The analysis was based on the non-uniform rational B-spline-based isogeometric finite element formulation. The resulting nonlinear eigenvalue problem was solved using a displacement-controlled strategy.

More recently, Gholami and Ansari [26] examined the free vibration and pre- and post-buckled responses of multilayer functionally graded graphene platelet-reinforced polymer composite rectangular plates under compressive in-plane mechanical loading. The parabolic shear deformation plate theory, von Kármán type nonlinearity and Hamilton’s principle in conjunction with the variational differential quadrature technique were used to achieve the weak form of the nonlinear equations of motion. The influences of GNPs distribution scheme, weight fraction, plate’s geometry and boundary conditions on the response were investigated. The buckling behaviour of trapezoidal corrugated multilayer functionally graded GNPs reinforced nanocomposites thin plates subjected to various mechanical loadings, including in-plane uniform shear, uniaxial compression and a combination of both was investigated by Yang et al. [27]. The unilateral and bilateral buckling responses were evaluated using an analytical and the Ritz methods, respectively. Fan et al. [28] presented and discussed the nonlinear dynamic behaviours of functionally graded GRC laminated plates resting on viscoelastic foundation under various loading conditions. The effects of ambient temperature and the interaction between the plate and its foundation were both considered. The load-deflection relationship was obtained using a two-step perturbation technique and the fourth-order Runge-Kutta numerical method. In another notable study, Liu et al. [29] used the three-dimensional theory of elasticity to investigate the static axisymmetric and asymmetric bending and free vibration of the multilayer annular plates reinforced with GNPs. A semi-analytical method, which combined the differential quadrature method and the state-space based differential quadrature method was incorporated in the analysis. Numerical results for the bending response and natural frequencies were presented.

The developments and progress made in advanced composite materials in recent years have created significant interest in their sandwich construction [30–38]. In modern sandwich composite plates, two thin face sheets made of fiber-reinforced composites or functionally graded materials or nanoparticle-reinforced composites are used to sandwich an appropriate structural foam or honeycomb. The resulting hybrid plates yield significantly stiffer and stronger responses compared to their monolithic 2D thin geometry counterparts in a cost-effective manner. As a result, the development of effective solutions for depicting the free vibration, stability and
static responses of such plates has attracted considerable interest. On the other hand, the recent advancements made in the development of more effective and accurate shear deformation theories that do not rely on the incorporation of a shear correction factor have also opened another avenue of interest [39–45].

In summary, the ever-increasing applications of advanced materials in primary structural components in recent decades have necessitated the development of more advanced and accurate theoretical models by which one could accurately predict the response of sandwich plates under various loading conditions. Therefore, the present study is conducted based on two main objectives. The first objective is to investigate the effect of inclusion of graphene on the natural frequencies, critical buckling capacity and maximum deflection of sandwich plates formed with graphene-reinforced polymer composite (GRPC) face sheets subjected to various mechanical and thermal loads. The second objective is to construct an admissible mathematical model of the problem and develop its solution based on a new refined shear deformation theory, which does not require any shear correction factors and compare its results against those obtained by the available solutions. It is worth mentioning that, to the best of author’s knowledge, the characterization of the vibration, stability and static responses of graphene-reinforced sandwich plates under mechanical and thermal loadings using the presented refined shear deformation plate theory is one of the novelties of this research and has not been conducted elsewhere.

**Problem formulation**

A symmetrical sandwich plate (symmetry with respect to mid-plane) made of a homogeneous core and two similar GRPC face sheets, as illustrated in Figure 1, is considered. The sandwich plate is within a thermal environment and is simply-supported on its edges and maybe resting on a Winkler or a two-parameter Pasternak foundations (the latter combines the Winkler springs and a shear layer), or be foundationless. The length, width and total height of the plate are designated as \(a\), \(b\) and \(h\), respectively. The thickness of each face sheet is assumed as \(h_f\) and thickness of the core is denoted by \(h_c\). Therefore, the total thickness of the plate is \(h = h_c + 2h_f\).

It should be mentioned that the assumptions and limitations of the present formulation are: (1) The displacements are small in comparison with the plate thickness; (2) The thickness stretching effect is not considered; (3) The transverse normal stress is negligible in comparison with the in-plane stresses; (4) The graphene reinforcement is aligned in the x-direction and is uniformly distributed through-the-thickness direction of the GRPC face sheets and (5) the present analytical solution is applicable to plates with simply-supported edges.
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**Materials properties of the sandwich plates**

The required effective material properties of GRPC face sheets can be expressed by the modified Halpin-Tsai micromechanical model, as follows [21]

\[
E_{11} = \eta_1 \frac{1 + 2\left(\frac{a_G}{h_G}\right)\gamma_{11}^G V_G}{1 - \gamma_{11}^G V_G} E_M
\]

\[
E_{22} = \eta_2 \frac{1 + 2\left(\frac{b_G}{h_G}\right)\gamma_{22}^G V_G}{1 - \gamma_{22}^G V_G} E_M
\]

\[
G_{12} = \eta_3 \frac{1}{1 - \gamma_{12}^G V_G} G_M
\]

where $a_G$, $b_G$ and $h_G$ are the length, width and the effective thickness of the graphene sheet, respectively, and

\[
\gamma_{11}^G = \frac{E_{11}^G}{E_{11}} - 1
\]

\[
\gamma_{22}^G = \frac{E_{22}^G}{E_M} - 1
\]

Figure 1. Coordinate system and geometry of sandwich plates with GRPC face sheets resting on elastic foundations.

Materials properties of the sandwich plates

The required effective material properties of GRPC face sheets can be expressed by the modified Halpin-Tsai micromechanical model, as follows [21]
where $E_M$ and $G_M$ refer to Young’s modulus and shear modulus of the matrix, while $E_{11}^G$, $E_{22}^G$ and $G_{12}^G$ indicate the longitudinal and transverse Young’s moduli and shear modulus of the graphene sheet, $V_M$ and $V_G$ are the volume fractions of the matrix and the graphene, respectively, which satisfy the condition $V_M + V_G = 1$. In order to account for the effects relating to the load transfer and interaction between the graphene and polymeric phases, the graphene efficiency parameters $\eta_j (j = 1, 2, 3)$ are introduced and incorporated in the original Halpin-Tsai model as presented in equation (1). The values of $\eta_1$, $\eta_2$ and $\eta_3$ are tuned by comparing the results from equation (1) against the ones from the molecular dynamics simulations [46].

The material properties of the matrix and graphene sheets are considered temperature-dependent in the present article. According to the Schapery model [47], the longitudinal and transverse thermal expansion coefficients of the GRPC face sheets can be given by

$$\alpha_{11} = \frac{V_G E_{11}^G \alpha_{11}^G + V_M E_M \alpha_M}{V_G E_{11}^G + V_M E_M}$$

$$\alpha_{22} = (1 + \nu_{12}^G) V_G \alpha_{22}^G + (1 + \nu_M) V_M \alpha_M - \nu_{12} \alpha_{11}$$

where $\alpha_{11}$, $\alpha_{22}$ and $\alpha_M$ are the thermal expansion coefficients, $\nu_{12}^G$ and $\nu_M$ are the Poisson’s ratios of the graphene sheet and the isotropic matrix, respectively. The Poisson’s ratio and mass density of the GRPC face sheets can also be obtained using the basic rule-of-mixture as follows:

$$\nu_{12} = V_G \nu_{12}^G + V_M \nu_M$$

$$\rho = V_G \rho_G + V_M \rho_M$$

where $\rho_G$ and $\rho_M$ represent the densities of the graphene and matrix, respectively.

**The four-variable shear deformation plate model and the constitutive equations**

In this study, the four-variable shear deformation plate model proposed by Zaoui et al. [42] is utilized in order to establish the kinematic relations of the plate. According to this theory, the displacement field of a sandwich plate can
be represented by

\[
\begin{align*}
  u(x, y, z, t) &= u_0(x, y, t) - z \frac{\partial w_0(x, y, t)}{\partial x} + g_1 f(z) \int \theta(x, y, t) dx \\
  v(x, y, z, t) &= v_0(x, y, t) - z \frac{\partial w_0(x, y, t)}{\partial y} + g_2 f(z) \int \theta(x, y, t) dy \\
  w(x, y, z, t) &= w_0(x, y, t)
\end{align*}
\] (6a–6c)

where \(u_0, v_0\) and \(w_0\) represent the mid-plane displacement components of the sandwich plate in the orthogonal \(x, y\) and \(z\) directions, respectively, \(g_1\) and \(g_2\) are geometric dependent coefficients, \(\theta\) represents the rotation of the mid-plane of the sandwich plate and \(f(z)\) is the shear strain shape function, which is represented by a sinusoidal function in the form of

\[
f(z) = \sin \left( \frac{\pi z}{h} \right)
\] (7)

It can be seen that the kinematic relations of the sandwich plate presented in equation (6) include four unknowns (\(u_0, v_0, w_0\) and \(\theta\)). For the aforementioned sandwich plate, the linear strain-displacement relationships derived from the kinematic equations (6a)–(6c), within the confines of the linear, small-strain elasticity theory, can be expressed as

\[
\begin{align*}
  e_x &= \frac{\partial u_0(x, y, t)}{\partial x} - z \frac{\partial^2 w_0(x, y, t)}{\partial x^2} + g_1 f(z) \theta(x, y, t) \\
  e_y &= \frac{\partial v_0(x, y, t)}{\partial y} - z \frac{\partial^2 w_0(x, y, t)}{\partial y^2} + g_2 f(z) \theta(x, y, t) \\
  \gamma_{xy} &= \frac{\partial u_0(x, y, t)}{\partial y} + \frac{\partial v_0(x, y, t)}{\partial x} - 2z \frac{\partial^2 w_0(x, y, t)}{\partial x \partial y} + g_1 f(z) \frac{\partial}{\partial x} \int \theta(x, y, t) dx \\
  &\quad + g_2 f(z) \frac{\partial}{\partial x} \int \theta(x, y, t) dy \\
  \gamma_{xz} &= g_1 \left( \frac{\partial}{\partial z} f(z) \right) \int \theta(x, y, t) dx
\end{align*}
\] (8a–8d)
\[
\gamma_{yz} = g_2 \left( \frac{\partial}{\partial z} f(z) \right) \int \theta(x, y, t) dy
\]  
(8e)

The integral terms used in the above equations may be resolved by a Navier-type method and can be expressed as follows [40]

\[
\frac{\partial}{\partial y} \int \theta(x, y, t) dx = A' \frac{\partial^2 \theta(x, y, t)}{\partial x \partial y}
\]  
(9a)

\[
\frac{\partial}{\partial x} \int \theta(x, y, t) dy = B' \frac{\partial^2 \theta(x, y, t)}{\partial x \partial y}
\]  
(9b)

\[
\int \theta(x, y, t) dx = A' \frac{\partial \theta(x, y, t)}{\partial x}
\]  
(9c)

\[
\int \theta(x, y, t) dy = B' \frac{\partial \theta(x, y, t)}{\partial y}
\]  
(9d)

in which the coefficients \(A'\) and \(B'\) are defined according to the type of solution used, which in this case would be for Navier’s method. Therefore, \(A'\), \(B'\), \(g_1\), and \(g_2\) are expressed as follows

\[
A' = \frac{-1}{\lambda^2}, \quad B' = \frac{-1}{\mu^2}, \quad g_1 = \lambda^2, \quad g_2 = \mu^2
\]  
(10)

where \(\lambda = m\pi/a\) and \(\mu = n\pi/b\) are the eigen frequency, \(m\) and \(n\) are the half-wave numbers. Substitution of equation (9) in equation (8) would yield the following linear strain expressions

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} + z \begin{bmatrix}
k_x^b \\
k_y^b \\
k_{xy}^b
\end{bmatrix} + f(z) \begin{bmatrix}
k_x^s \\
k_y^s \\
k_{xy}^s
\end{bmatrix}, \quad \begin{bmatrix}
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix} = \frac{df(z)}{dz} \begin{bmatrix}
\gamma_{yz}^s \\
\gamma_{xz}^s
\end{bmatrix}
\]  
(11)

where

\[
\begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} = \begin{bmatrix}
\frac{\partial u_0(x, y, t)}{\partial x} \\
\frac{\partial v_0(x, y, t)}{\partial y} \\
\frac{\partial u_0(x, y, t)}{\partial y} + \frac{\partial v_0(x, y, t)}{\partial x}
\end{bmatrix}, \quad \begin{bmatrix}
k_x^b \\
k_y^b \\
k_{xy}^b
\end{bmatrix} = \begin{bmatrix}
\frac{\partial^2 w_0(x, y, t)}{\partial x^2} \\
\frac{\partial^2 w_0(x, y, t)}{\partial y^2} \\
-2 \frac{\partial^2 w_0(x, y, t)}{\partial x \partial y}
\end{bmatrix}
\]
\[
\begin{align*}
\begin{bmatrix}
k_x^2 \\
k_y^2 \\
k_{xy}^2
\end{bmatrix} &= \begin{bmatrix}
g_1 A' \frac{\partial^2 \theta(x,y,t)}{\partial x^2} \\
g_2 B' \frac{\partial^2 \theta(x,y,t)}{\partial y^2} \\
(g_1 A' + g_2 B') \frac{\partial^2 \theta(x,y,t)}{\partial x \partial y}
\end{bmatrix},
\begin{bmatrix}
\gamma_{yz}^s \\
\gamma_{xz}^s
\end{bmatrix} &= \begin{bmatrix}
g_2 B \frac{\partial \theta(x,y,t)}{\partial y} \\
g_1 A \frac{\partial \theta(x,y,t)}{\partial x}
\end{bmatrix}
\end{align*}
\]

For linear thermoelastic materials, the stress field is defined as a linear function of the mechanical and thermal strain fields

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{yz} \\
\tau_{xz} \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix}
Q_{11} & Q_{12} & 0 & 0 & 0 \\
Q_{21} & Q_{22} & 0 & 0 & 0 \\
0 & 0 & Q_{44} & 0 & 0 \\
0 & 0 & 0 & Q_{55} & 0 \\
0 & 0 & 0 & 0 & Q_{66}
\end{bmatrix} \begin{bmatrix}
ev_x \\
ev_y \\
\gamma_{yz} \\
\gamma_{xz} \\
\gamma_{xy}
\end{bmatrix} - \begin{bmatrix}
z_{11} \Delta T \\
z_{22} \Delta T \\
0 \\
0 \\
0
\end{bmatrix}
\]

In the above equation, \(\Delta T = T - T_0\) is the temperature differential, where \(T\) and \(T_0\) are the final and reference temperatures, respectively. Moreover, \(Q_{ij}(i,j = 12,456)\) are the reduced material stiffness coefficients compatible with the plane-stress conditions and are defined as follows [33]

\[
\begin{align*}
Q_{11} &= \frac{E_{11}}{1 - \nu_{12} \nu_{21}}, & Q_{22} &= \frac{E_{22}}{1 - \nu_{12} \nu_{21}}, & Q_{12} &= \frac{\nu_{12} E_{22}}{1 - \nu_{12} \nu_{21}}, \\
Q_{44} &= G_{23}, & Q_{55} &= G_{13}, & Q_{66} &= G_{12}
\end{align*}
\]

The equations of motion of the aforementioned sandwich plate under thermo-mechanical loadings derived on the basis of the stationary potential energy [48] are mathematically represented as

\[
\begin{align*}
\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + J_0 g_1 A' \frac{\partial \ddot{\theta}}{\partial x} \\
\delta v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} &= I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + J_0 g_2 B' \frac{\partial \ddot{\theta}}{\partial y}
\end{align*}
\]

\[
\begin{align*}
\delta w_0 : \frac{\partial^2 M_b^0}{\partial x^2} + 2 \frac{\partial^2 M_{bx}^0}{\partial x \partial y} + \frac{\partial^2 M_y^0}{\partial y^2} + q(x,y) - K_w w_0 + K_s \nabla^2 w_0 + N_x \frac{\partial^2 w_0}{\partial x^2} \\
+ N_y \frac{\partial^2 w_0}{\partial y^2} + 2N_{xy} \frac{\partial^2 w_0}{\partial x \partial y}
\end{align*}
\]

\[
= I_0 \ddot{w}_0 + I_1 \left( \frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - I_2 \nabla^2 \ddot{w}_0 + J_1 \left( g_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + g_2 B \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right)
\]
\[ \delta \theta : -g_1 A' \frac{\partial^2 M^s_x}{\partial x^2} - (g_1 A' + g_2 B') \frac{\partial^2 M^s_{xy}}{\partial x \partial y} - g_2 B' \frac{\partial^2 M^s_{y}}{\partial y^2} + g_1 A' \frac{\partial S^s_{x}}{\partial x} + g_2 B' \frac{\partial S^s_{yz}}{\partial y} \]

\[ = -J_0 \left( g_1 A' \frac{\partial \ddot{u}_0}{\partial x} + g_2 B' \frac{\partial \ddot{v}_0}{\partial y} \right) + J_1 \left( g_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + g_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right) \]

\[ - J_2 \left( (g_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + (g_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) \]  

(15d)

in which \( I_0, I_1, I_2, J_0, J_1 \) and \( J_2 \) are the mass moments of inertia defined by

\[ (I_0, I_1, I_2, J_0, J_1, J_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, f(z), z^2, zf(z), f^2(z)) \rho dz \]  

(16)

The terms \( K_W \) and \( K_s \) denote the stiffnesses of the normal (Winkler) springs and the shear layer, respectively (see Figure 1). The force and moment components to be used in equation (15) can be obtained by incorporating the following constitutive relations

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy} \\
M_x^b \\
M_y^b \\
M_{xy}^b \\
M_x^s \\
M_y^s \\
M_{xy}^s \\
S_{xz}^s \\
S_{yz}^s \\
S_{xy}^s
\end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} dz
\]

(17)

By substituting equation (11) in equation (13) and the obtained results into equation (17) one would obtain the resultant forces and moments in the following matrix form

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy} \\
M_x^b \\
M_y^b \\
M_{xy}^b \\
M_x^s \\
M_y^s \\
M_{xy}^s \\
S_{xz}^s \\
S_{yz}^s \\
S_{xy}^s
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 \\
A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 \\
0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s \\
0 & 0 & B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 \\
0 & 0 & B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 \\
0 & 0 & 0 & 0 & D_{66} & 0 & 0 & 0 & D_{66}^s & 0 & 0 \\
B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\
B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{66}^s & 0 & H_{12}^s & H_{22}^s & 0 \\
0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & H_{66}^s
\end{bmatrix}
\]
where \( A_{ij}, B_{ij}, D_{ij} \) and \( H_{ij} \) are the equivalent sandwich plate’s stiffness, defined by

\[
\begin{pmatrix}
A_{11} & B_{11} & D_{11} & B_{11}^i & D_{11}^i & H_{11}^i \\
A_{22} & B_{22} & D_{22} & B_{22}^i & D_{22}^i & H_{22}^i \\
A_{12} & B_{12} & D_{12} & B_{12}^i & D_{12}^i & H_{12}^i \\
A_{66} & B_{66} & D_{66} & B_{66}^i & D_{66}^i & H_{66}^i \\
\end{pmatrix}
\]

\[
= \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{pmatrix}
Q_{11} \\
Q_{22} \\
Q_{12} \\
Q_{66} \\
\end{pmatrix}
\begin{pmatrix}
1, z, z^2, f(z), zf(z), f^2(z) \\
\end{pmatrix} dz,
\]

\[
\begin{pmatrix}
A_{44}^i \\
A_{55}^i \\
\end{pmatrix}
= \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{pmatrix}
Q_{44}^i \\
Q_{55}^i \\
\end{pmatrix}
\frac{(df(z))^2}{dz} dz
\]

The resultant forces and moments due to the thermal loading \( (N_x^T = N_y^T, M_x^bT = M_y^bT \) and \( M_x^sT = M_y^sT) \) are obtained by

\[
\begin{bmatrix}
N_x^T \\
N_y^T \\
M_x^bT \\
M_y^bT \\
M_x^sT \\
M_y^sT \\
\end{bmatrix}
= \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix}
Q_{11} \\
Q_{12} \\
Q_{12} \\
Q_{22} \\
\end{bmatrix}
\begin{bmatrix}
\alpha_{11} \\
\alpha_{12} \\
\alpha_{22} \\
\end{bmatrix}
(\Delta T)(1, z, f(z)) dz
\]

The equations of motion represented by equation (15) can be expressed in terms of the four unknowns \( (u_0, v_0, w_0 \) and \( \theta \) by substituting for the resultants forces and moments from equation (18). Therefore, the governing equations of motion for a
general sandwich plate resting on an elastic foundation, take the form

\[ A_{11} \frac{\partial^2 u_0}{\partial x^2} + A_{66} \frac{\partial^2 u_0}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 v_0}{\partial x \partial y} - B_{11} \frac{\partial^3 w_0}{\partial x^2 \partial y} - (B_{12} + 2B_{66}) \frac{\partial^3 w_0}{\partial x \partial y^2} \]

\[ + g_1 A' B_{11} \frac{\partial^3 \theta}{\partial x^3} + (g_2 B' B_{12} + (g_1 A' + g_2 B')) B_{66} \frac{\partial^3 \theta}{\partial x \partial y^2} \]

\[ = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + J_0 g_1 A' \frac{\partial \ddot{\theta}}{\partial x} \]  

(21a)

\[ A_{22} \frac{\partial^2 v_0}{\partial x^2} + A_{66} \frac{\partial^2 v_0}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 u_0}{\partial x \partial y} - B_{22} \frac{\partial^3 w_0}{\partial y^3} - (B_{12} + 2B_{66}) \frac{\partial^3 w_0}{\partial x^2 \partial y} \]

\[ + g_2 B' B_{22} \frac{\partial^3 \theta}{\partial y^3} + (g_1 A' B_{12} + (g_1 A' + g_2 B')) B_{66} \frac{\partial^3 \theta}{\partial x^2 \partial y} \]

\[ = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + J_0 g_2 B' \frac{\partial \ddot{\theta}}{\partial y} \]  

(21b)

\[ B_{11} \frac{\partial^3 u_0}{\partial x^3} + (B_{12} + 2B_{66}) \left( \frac{\partial^3 u_0}{\partial x \partial y^2} + \frac{\partial^3 v_0}{\partial x^2 \partial y} \right) + B_{22} \frac{\partial^3 v_0}{\partial y^3} - D_{11} \frac{\partial^4 w_0}{\partial x^4} \]

\[ - 2(D_{12} + 2D_{66}) \frac{\partial^4 w_0}{\partial x^2 \partial y^2} - D_{22} \frac{\partial^4 w_0}{\partial y^4} + g_1 A' D_{11} \frac{\partial^4 \theta}{\partial x^4} + g_2 B' D_{22} \frac{\partial^4 \theta}{\partial y^4} \]

\[ + (g_1 A' + g_2 B')(D_{12} + 2D_{66}) \frac{\partial^4 \theta}{\partial x^2 \partial y^2} + q(x, y) - K w w_0 + K_s \nabla^2 w_0 + N_y \frac{\partial^2 w_0}{\partial x^2} \]

\[ + N_y \frac{\partial^2 w_0}{\partial y^2} + 2N_{xy} \frac{\partial^2 w_0}{\partial x \partial y} \]

\[ = I_0 \ddot{w}_0 + I_1 \left( \frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - I_2 \nabla^2 \ddot{w}_0 + J_1 \left( g_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + g_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) \]  

(21c)

\[ - g_1 A' B_{11} \frac{\partial^3 u_0}{\partial x^3} - (g_2 B' B_{12} + (g_1 A' + g_2 B')) B_{66} \frac{\partial^3 u_0}{\partial x \partial y^2} \]
\[
- (g_1 A' B_{12}^s + (g_1 A' + g_2 B') B_{66}^s) \frac{\partial^3 v_0}{\partial x^2 \partial y} - g_2 B' B_{22}^s \frac{\partial^3 v_0}{\partial y^3} + g_1 A' D_{11}^s \frac{\partial^4 w_0}{\partial x^4}
\]
\[
+ (g_1 A' + g_2 B') (D_{12}^s + 2D_{66}^s) \frac{\partial^4 w_0}{\partial x^2 \partial y^2} + g_2 B' D_{22}^s \frac{\partial^4 w_0}{\partial y^4} - (g_1 A')^2 H_{11}^s \frac{\partial^4 \theta}{\partial x^4}
\]
\[
- 2(g_1 A' H_{12}^s - (g_1 A' + g_2 B') H_{66}^s) \frac{\partial^4 \theta}{\partial x^2 \partial y^2} - (g_2 B')^2 H_{22}^s \frac{\partial^4 \theta}{\partial y^4} + (g_1 A')^2 A_{55}^s \frac{\partial^2 \theta}{\partial x^2}
\]
\[
+ (g_2 B')^2 A_{44}^s \frac{\partial^2 \theta}{\partial y^2}
\]
\[
= -J_0 \left( g_1 A' \frac{\partial v_0}{\partial x} + g_2 B' \frac{\partial v_0}{\partial y} \right) + J_1 \left( g_1 A' \frac{\partial^2 w_0}{\partial x^2} + g_2 B' \frac{\partial^2 w_0}{\partial y^2} \right)
\]
\[
- J_2 \left( (g_1 A')^2 \frac{\partial^2 \theta}{\partial x^2} + (g_2 B')^2 \frac{\partial^2 \theta}{\partial y^2} \right)
\]  \hspace{1cm} (21d)

**Solution procedure**

The Navier method is implemented to formulate the closed-form solutions of equation (21) for obtaining the temperature-dependent free vibration, buckling and deflection responses of simply-supported sandwich plates with GRPC face sheets resting on elastic foundation subject to mechanical and thermal loadings. The applied simply-supported boundary conditions are of the following form

\[
v_0 = w_0 = \theta = N_x = M_x^b = M_x^s = 0 \text{ at } x = 0, a, \hspace{1cm} (22a)
\]
\[
u_0 = w_0 = \theta = N_y = M_y^b = M_y^s = 0 \text{ at } y = 0, b. \hspace{1cm} (22b)
\]

Here, on the basis of the Navier method, the solution of equation (21), which automatically satisfies the boundary conditions in equation (22), can be represented by

\[
\begin{align*}
&\begin{pmatrix}
  u_0(x, y, t) \\
v_0(x, y, t) \\
w_0(x, y, t) \\
\theta(x, y, t)
\end{pmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{pmatrix}
  U_{mn} \cos(\lambda x) \sin(\mu y) e^{i\omega t} \\
  V_{mn} \sin(\lambda x) \cos(\mu y) e^{i\omega t} \\
  W_{mn} \sin(\lambda x) \sin(\mu y) e^{i\omega t} \\
  \Theta_{mn} \sin(\lambda x) \sin(\mu y) e^{i\omega t}
\end{pmatrix}
\end{align*}
\]  \hspace{1cm} (23)
where $U_{mn}$, $V_{mn}$, $W_{mn}$, $\Theta_{mn}$ are unknown coefficients. The transverse distributed load, $q(x,y)$, can also be represented by a double-Fourier sine series as

$$q(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin(\lambda x) \sin(\mu y)$$  \hspace{1cm} (24)

In which

$$q_{mn} = \frac{4}{ab} \int_0^a \int_0^b q(x,y) \sin(\lambda x) \sin(\mu y) \, dx \, dy$$  \hspace{1cm} (25)

It should be pointed out that $q_{mn}$ of a plate subjected to a uniformly distributed transverse load $q_0$ can be defined as

$$q_{mn} = \frac{16q_0}{mn\pi^2} (m = n = 1, 3, 5, \ldots)$$  \hspace{1cm} (26)

Upon substitution of the displacement functions of equation (23) into the equations of motion (equation (21)), and rearranging the terms, one can obtain the closed-form solutions through the following matrix.

$$\left( [K]_{4 \times 4} - \omega^2 [M]_{4 \times 4} \right) \begin{pmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Theta_{mn} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \end{pmatrix}$$  \hspace{1cm} (27)

For a deflection analysis, the natural frequency ($\omega$) and the buckling capacities ($N_0^x$, $N_0^y$, $N_0^{xy}$) in equation (27) are set to zero and the resulting simultaneous equations are solved for deflections of the sandwich plates. For vibration and buckling analyses, the determinant of the coefficient matrix of equation (27) is set to zero and the non-trivial solution is obtained. Also, the following dimensionless quantities for Pasternak type foundation is introduced in the numerical results

$$k_1 = K \frac{E_0h^3}{b^4}, \quad k_2 = K_s \frac{E_0h^3}{b^2}$$  \hspace{1cm} (28)

**Numerical results and discussion**

In the present paper, the natural frequencies, buckling capacity and deflection of sandwich plates with GRPC face sheets subject to various mechanical and thermal loading scenarios are investigated using the four-variable shear deformation plate
model presented above. In doing so, one should first establish the effective material properties of GRPCs, which is used to form the face sheets of the sandwich plates. Poly (methyl methacrylate) thermoplastic polymer (i.e., PMMA), is adopted as the matrix, and the material properties of it are provided in Table 1, where \( T = T_0 + \Delta T \) and \( T_0 = 300K \) (room temperature). Titanium alloy is adopted as the homogeneous core layer, and the material properties of it are assumed to be nonlinear functions of temperature as shown in Table 1. The zigzag graphene sheets with an effective thickness of \( h_G = 0.188 \) nm and \( \rho_G = 4118 \) kg/m\(^3\), \( a_G = 14.76 \) nm, \( b_G = 14.77 \) nm are selected as reinforcement. The graphene efficiency parameters are presented in Table 2, and it is assumed that \( G_{13} = G_{23} = 0.5G_{12} \) [22]. The temperature-dependent material properties of zigzag graphene sheets are extracted from reference [46]. For \( 300 K \leq T \leq 1000 K \) the temperature-dependent thermo-mechanical properties of graphene sheets such as Young’s and shear moduli and thermal expansion coefficients are defined by the following relationships

\[
E_{11}^G = (2.16637 - 0.00193T + 2.93701 \times 10^{-6}T^2 - 1.51775 \times 10^{-9}T^3)TPa
\]

\[
E_{22}^G = (2.16868 - 0.00193T + 2.85954 \times 10^{-6}T^2 - 1.45145 \times 10^{-9}T^3)TPa
\]

\[
G_{12}^G = (0.53514 + 8.22436 \times 10^{-4}T - 1.2932 \times 10^{-6}T^2 + 5.78507 \times 10^{-10}T^3)TPa
\]

\[
\alpha_{11}^G = (-3.83788 + 0.01416T - 1.63355 \times 10^{-5}T^2 + 6.33589 \times 10^{-9}T^3) \times 10^{-6}/K
\]

\[
\alpha_{22}^G = (-3.73997 + 0.01296T - 1.35033 \times 10^{-5}T^2 + 4.60392 \times 10^{-9}T^3) \times 10^{-6}/K
\]
In the following, the variations of different configurations of the sandwich plate (i.e., as per GRPC face sheets types) are stated:

- The (1-1-1) configuration: in this configuration, the sandwich plate is made of three equal thickness layers (i.e., $h_f = h_c = 1$ mm).
- The (1-2-1) configuration: in this configuration, the thickness of the core is twice of the face sheets (i.e., $h_f = 1$ mm and $h_c = 2$ mm).
- The (1-4-1) configuration: in this configuration, the thickness of the core is four times of the face sheets (i.e., $h_f = 1$ mm and $h_c = 4$ mm).
- The (2-1-2) configuration: in this configuration, the thickness of the core is half of the face sheets (i.e., $h_f = 1$ mm and $h_c = 0.5$ mm).

**Free vibration analysis**

The free vibration responses of the aforementioned square sandwich plates with GRPC face sheets subject to different temperatures obtained based on the above-mentioned solution are compared against some numerical results available in the literature and the results are presented in Tables 3 and 4. All plates examined in these two tables had GRPC face sheets containing 7% (by volume) graphene. Note that the natural frequency values in this paper have been presented by the following relation: $\omega = \frac{\omega_0^2}{h} \sqrt{\frac{V_0}{E_0}}$, as this parameter was used to present the results by other researchers. 

### Table 2. The efficiency parameters for different volume fraction of graphene and various thermal environment.

| $T[K]$ | $V_G$ | $\eta_1$ | $\eta_2$ | $\eta_3$ |
|--------|-------|----------|----------|----------|
| 300    | 3%    | 2.929    | 2.855    | 11.842   |
|        | 5%    | 3.068    | 2.962    | 15.944   |
|        | 7%    | 3.013    | 2.966    | 23.575   |
|        | 9%    | 2.647    | 2.609    | 32.816   |
|        | 11%   | 2.311    | 2.260    | 33.125   |
| 400    | 3%    | 2.977    | 2.896    | 13.928   |
|        | 5%    | 3.128    | 3.023    | 15.229   |
|        | 7%    | 3.060    | 3.027    | 22.588   |
|        | 9%    | 2.701    | 2.603    | 28.869   |
|        | 11%   | 2.405    | 2.337    | 29.527   |
| 500    | 3%    | 3.388    | 3.382    | 16.712   |
|        | 5%    | 3.544    | 3.414    | 16.018   |
|        | 7%    | 3.462    | 3.339    | 23.428   |
|        | 9%    | 3.058    | 2.936    | 29.754   |
|        | 11%   | 2.736    | 2.665    | 30.773   |

$$\nu_{12}^G = 0.177$$ (29)
The (1-1-1) configuration: in this configuration, the sandwich plate is made of GRPC face sheets (i.e., as per GRPC face sheets types) are stated:

- The (1-2-1) configuration: in this configuration, the thickness of the core is twice of the face sheets (i.e., $h_c = 2h_f$).
- The (2-1-2) configuration: in this configuration, the thickness of the core is half of the face sheets (i.e., $h_c = \frac{1}{2}h_f$).
- The (1-4-1) configuration: in this configuration, the thickness of the core is four times of the face sheets (i.e., $h_c = 4h_f$).

In the following, the variations of different configurations of the sandwich plate are examined with the results presented in Table 3. All plates examined in the present study, the thickness of each GRPC face sheet is identical and $V_G = 9\%$ (by volume). Note that the natural frequencies of the sandwich plates with GRPC face sheets subject to different temperatures obtained based on the above-mentioned solution are compared against some numerical results available in the literature and the results are presented in Tables 3 and 4.

In the present study, the thickness of each GRPC face sheet is identical and $h_f = 1\ mm$, whereas the total thickness of the sandwich plates is taken to be $h = 2.5, 3, 4$ and $6\ mm$; therefore, the examined core to face sheet thickness ratios are taken to be $h_c/h_f = 0.5, 1, 2$ and $4$ [49,50]. In other words, the plates have the same planar dimensions and only their thicknesses are different. As expected, it can be observed researchers, whose results will be used throughout for comparison. Therefore, hereafter, the statement “natural frequency” refers to this parameter. It should be noted that $\rho_0$ and $E_0$ are the reference values of $\rho_C$ and $E_C$ at $T = 300K$.

Table 3. Comparison of the first six natural frequencies ($\Omega = \omega^2 h/\sqrt{\rho_0 E_0}$) of square GRPC plates under different temperatures ($b/h = 10, h = 2\ mm, V_G = 7\%$).

| $T/\text{K}$ | Source     | $\Omega_{11}$ | $\Omega_{12}$ | $\Omega_{21}$ | $\Omega_{22}$ | $\Omega_{13}$ | $\Omega_{31}$ |
|------------|------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 300        | Shen et al. [21] | 28.0982       | 64.9584       | 65.0364       | 95.6904       | 116.1739      | 116.3356      |
|            | Kiani [25]  | 28.0794       | 64.8676       | 64.9378       | 95.4811       | 116.8946      | 117.0263      |
|            | Present     | 28.0855       | 64.8455       | 65.0837       | 95.6278       | 115.9986      | 116.4499      |
| 400        | Shen et al. [21] | 21.9591       | 54.8037       | 56.4669       | 84.0640       | 101.1662      | 105.3867      |
|            | Kiani [25]  | 21.9430       | 55.8754       | 57.6844       | 83.8599       | 102.5441      | 103.4163      |
|            | Present     | 21.9515       | 54.7910       | 57.6719       | 84.0228       | 101.1841      | 105.4034      |
| 500        | Shen et al. [21] | 15.0773       | 45.2749       | 52.0752       | 74.6301       | 88.3236       | 97.9272       |
|            | Kiani [25]  | 15.0636       | 47.9939       | 49.4060       | 74.4269       | 91.9083       | 93.9191       |
|            | Present     | 15.0680       | 45.2891       | 52.0856       | 74.6908       | 88.4061       | 98.0032       |

Table 4. Comparison of the first six natural frequencies ($\Omega = \omega^2 h/\sqrt{\rho_0 E_0}$) of square GRPC plates resting on elastic foundations under temperature of 300K with various values of $(k_1, k_2)$; $(b/h = 20, h = 2\ mm, V_G = 7\%)$.

| $(k_1, k_2)$ | Source     | $\Omega_{11}$ | $\Omega_{12}$ | $\Omega_{21}$ | $\Omega_{22}$ | $\Omega_{13}$ | $\Omega_{31}$ |
|-------------|------------|---------------|---------------|---------------|---------------|---------------|---------------|
| (0,0)       | Shen et al. [21] | 29.5854       | 72.7627       | 73.0926       | 112.3927      | 140.7286      | 141.4999      |
|             | Present     | 29.5811       | 72.7445       | 73.0714       | 112.3420      | 140.6734      | 141.4383      |
| (100,0)     | Shen et al. [21] | 30.9790       | 73.3379       | 73.6652       | 112.7646      | 141.0252      | 141.7949      |
|             | Present     | 30.9745       | 73.3193       | 73.6437       | 112.7136      | 140.9697      | 141.7331      |
| (10,010)    | Shen et al. [21] | 33.5605       | 76.1127       | 76.4282       | 115.6587      | 143.9198      | 144.6742      |
|             | Present     | 33.5554       | 76.0926       | 76.4053       | 115.6057      | 143.8615      | 144.6098      |
that the natural frequencies of the sandwich plates with GRPC face sheets decreases with an increase in the temperature, as the stiffness of the materials become adversely affected by the applied temperature. As an example, the first natural frequency of (1–1-1) sandwich plates is reduced by approximately 56%, when the temperature rises from $T = 300 \text{ K}$ to $T = 400 \text{ K}$. This is due to the fact that the rise in temperature reduces the stiffness of the sandwich plates, in turn reducing the frequencies of the panel.

However, this margin of reduction in the frequency as a result of the applied temperature is reduced when the change in higher frequencies is considered. Moreover, it can also be observed that as expected, the natural frequencies of the sandwich plates decrease with an increase in the core to face sheet thickness ratio $h_c/h_f$. This would have been expected, since in sandwich plates a relatively small change in the core thickness would result in a significant change in plate’s stiffness (for instance, stiffness of 1-4-1 configuration is almost larger than 1-1-1 configuration). It is also worth mentioning that by increasing the core to face sheet thickness ratio, the decreasing effect of the temperature rise on the natural frequency is decreased.

The results presented in Table 6 reveals the effects of foundation stiffness and the core to face sheet thickness ratio $h_c/h_f$ on the natural frequencies of the sandwich plates with $b/h = 20$, $V_G = 9\%$. Plates resting on no foundation (i.e., $(k_1, k_2) = (0,0)$), as well as those resting on two different foundation types are considered. The two foundations considered are (i) the Winkler elastic foundation with $(k_1, k_2) = (100, 0)$ and (ii) the Pasternak elastic foundation with $(k_1, k_2) = (10, 010)$. As expected, the natural frequencies are increased with an increase in the supporting foundation’s stiffness. It can be seen that the first natural frequency of (1-1-1) sandwich plate with $(k_1, k_2) = (10,010)$ is 226.8% higher than those of the foundationless sandwich plate. Since the elastic foundation is assumed to be compliant, the introduction of elastic foundation will provide an added stiffness to the sandwich plate.

The effect of the volume fraction of graphene on the fundamental frequency of square sandwich plates with GRPC face sheets is demonstrated in Figure 2.
Table 6. The first six natural frequencies \(\Omega = \frac{\omega a^2}{h} \sqrt{\frac{h}{E}}\) of square sandwich plates with GRPC face sheets resting on elastic foundations under temperature of 300 K with various values of \((k_1, k_2)\); \((b/h) = 50, V_G = 9\%\).

| Configuration | \((k_1, k_2)\) | \(\Omega_{11}\) | \(\Omega_{12}\) | \(\Omega_{21}\) | \(\Omega_{22}\) | \(\Omega_{13}\) | \(\Omega_{31}\) |
|---------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \((1-1-1)\)   | \((0,0)\)      | 7.4941         | 18.6610        | 18.7371        | 29.8463        | 37.1699        | 37.3708        |
|               | \((100,0)\)    | 15.4603        | 23.0430        | 23.1047        | 32.7634        | 39.5495        | 39.7384        |
|               | \((10,010)\)   | 24.4943        | 37.8524        | 37.8900        | 50.1546        | 58.0173        | 58.1463        |
| \((1-2-1)\)   | \((0,0)\)      | 6.8684         | 17.1102        | 17.1727        | 27.3672        | 34.0956        | 34.2611        |
|               | \((100,0)\)    | 14.0938        | 21.0743        | 21.1251        | 30.0041        | 36.2454        | 36.4011        |
|               | \((10,010)\)   | 22.3070        | 34.5123        | 34.5434        | 45.7679        | 52.9738        | 53.0805        |

Figure 2. Fundamental frequency \(\Omega = \frac{\omega a^2}{h} \sqrt{\frac{h}{E}}\) of square sandwich plates with GRPC face sheets in thermal environment \(b/h = 20, T = 300 K\).

All tested four configurations of the sandwich plates have \(b/h = 20\) at \(T = 300 K\). The results shown in the figure reveals the significant influence of graphene on the fundamental frequency of the sandwich plates. The sandwich plate whose face sheets contains 11% graphene produced the highest gain in the fundamental
frequency. For instance, the fundamental frequency of (2-1–2) configuration containing 11% graphene in its face sheets reaches a maximum value of 8.21, an increase by 55.5% compared to (2-1-2) configuration containing 3% graphene. This significant enhancement in the fundamental frequencies of the sandwich plates with GRPC face sheets is due to the inherent stiffness and reinforcing effect contributed to the plate by graphene. Moreover, the results illustrated in the figure reveal that 9% volume fraction would be the optimal graphene volume content. Additionally, since the variation of change in the frequencies as a function of volume fraction attains somewhat of a plateau, one could expect that the inclusion of additional graphene would not improve the fundamental frequency of the plates by an appreciable margin.

**Buckling analysis**
In the following section, the effects of various parameters such as temperature, graphene volume fraction, core to face sheet thickness ratio $h_c/h_f$, foundation stiffness and different loading on the critical mechanical and thermal buckling capacities of the aforementioned sandwich plate will be evaluated by examining the numerical results.

**Buckling response under thermal load.** Before presenting the numerical results of the buckling analyses, it is of paramount importance to establish the integrity of the presented formulation. For this purpose, the results obtained from the present study are compared with those obtained through the literature. Table 7 presents the critical buckling temperatures of square plates resting on elastic foundations subjected to a uniform temperature rise, which have $h = 2\text{mm}$, $T = 300\text{K}$, $V_G = 7\%$, compared with the results presented from Refs. [23,51]. As seen, the results obtained by the proposed formulation match very closely to those in the literature, thus validating the integrity of the proposed formulation.

The variation of the critical buckling temperatures against graphene volume fraction for different configurations of sandwich plates is shown in Figure 3. The results reveal that the buckling strength increases as a function of increasing
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For instance, the fundamental frequency of (2-1-2) configuration containing 11% graphene in its face sheets reaches a maximum value of 8.21, an increase by 55.5% compared to (2-1-2) configuration containing 3% graphene. This significant enhancement in the fundamental frequencies of the sandwich plates with GRPC face sheets is due to the inherent stiffness and reinforcing effect contributed to the plate by graphene. Moreover, the results illustrated in the figure reveal that 9% volume fraction would be the optimal graphene volume content. Additionally, since the variation of change in the frequencies as a function of volume fraction attains somewhat of a plateau, one could expect that the inclusion of additional graphene would not improve the fundamental frequency of the plates by an appreciable margin.

Buckling analysis

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Buckling response under thermal load.

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The variation of the critical buckling temperatures against graphene volume fraction for different configurations of sandwich plates is shown in Figure 3. The results reveal that the buckling strength increases as a function of increasing graphene volume fraction of GRPC face sheets. Moreover, increasing the amount of graphene volume fraction from 3% to 11% generated a maximum enhancement of approximately 7% in the critical buckling temperatures in (1-1-1) configured sandwich plate. Moreover, according to the results, the sandwich plates with lower

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**Figure 3.** Critical buckling temperatures (in [K]) of square sandwich plates with GRPC face sheets under uniform temperature rise ($b/h = 20$, $T = 300\text{K}$).

**Table 8.** Critical buckling temperatures (in [K]) of square sandwich plates with GRPC face sheets resting on elastic foundations under uniform temperature rise ($b/h = 50$, $V_G = 9\%$, $T = 300\text{K}$).

| Configuration | $(k_1, k_2)$ | (1-1-1) | (1-2-1) | (2-1-2) | (1-4-1) |
|---------------|-------------|---------|---------|---------|---------|
| (0,0)         | 322.8836    | 326.7197| 320.6098| 332.3618|
| (100,0)       | 369.0194$^a$| 382.5218$^a$| 361.1872$^a$| 403.1388$^a$|
| (10,010)      | 508.6825$^a$| 554.3198$^a$| 482.8418$^a$| 627.4313$^a$|

$^a$Mode for sandwich plate is $(m, n) = (1, 2)$. 
core to face sheet thickness ratio \( \frac{h_c}{h_f} \) exhibit higher range of critical buckling temperatures. More specifically, the critical buckling temperature of (1–4–1) sandwich plate is 15.5% more than (2–1–2) sandwich plate which contains 11% volume fraction of graphene. Additionally, the effects of foundation stiffness and core to face sheet thickness ratio \( \frac{h_c}{h_f} \) on the critical buckling temperature of the sandwich plates with \( b/h = 50 \), \( V_G = 9\% \) at \( T = 300\,K \) are tabulated in Table 8. The results in this Table show that the critical thermal buckling temperatures of the sandwich plates increase as the foundation’s stiffness increases.

**Table 9. Critical buckling capacities \( N_{cr} \) (in kN) of square sandwich plate with GRPC face sheets subjected to uniaxial compression in thermal environments \( (b/h = 20) \).**

| \( T[K] \) | \( V_G \) | (1-1-1) | (1-2-1) | (2-1-2) | (1-4-1) |
|---|---|---|---|---|---|
| 300 | 3% | 60.5215 | 125.3314 | 39.6560 | 359.1412 |
| | 5% | 89.5521 | 172.3044 | 60.3264 | 444.0628 |
| | 7% | 122.3607 | 225.4225 | 83.7046 | 540.2013 |
| | 9% | 146.0921 | 263.7958 | 100.6802 | 609.6724 |
| | 11% | 153.0198 | 275.9204 | 105.6175 | 630.0185 |
| 400 | 3% | 56.3340 | 117.0996 | 36.8607 | 337.2190 |
| | 5% | 79.1775 | 154.0658 | 53.1176 | 404.0470 |
| | 7% | 107.9508 | 200.6531 | 73.6133 | 488.3506 |
| | 9% | 123.9348 | 226.5154 | 85.0315 | 535.1695 |
| | 11% | 133.1793 | 241.5017 | 91.6115 | 562.3297 |
| 500 | 3% | 56.0655 | 115.2032 | 36.8551 | 326.7349 |
| | 5% | 75.3048 | 146.3476 | 50.5367 | 383.0484 |
| | 7% | 100.9911 | 187.9431 | 68.8271 | 458.3228 |
| | 9% | 116.7739 | 213.4927 | 80.0904 | 504.5843 |
| | 11% | 126.8427 | 229.8193 | 87.2540 | 534.1772 |

**Buckling response under mechanical load.** In this section, the resulting mechanical buckling capacities are presented. Tables 9 and 10 present the effects of the core to face sheet thickness ratio \( \frac{h_c}{h_f} \), the temperature variation \( (T = 300, 400 \text{ and } 500K) \) and graphene volume fractions of 3% to 11% on the critical buckling capacity of variously configured sandwich plates with \( b/h = 20 \) under uniaxial and biaxial compression, respectively. It is observed that the critical buckling capacity of the sandwich plates with GRPC face sheets decreases with increase in the applied temperature. Since the material properties for both core and face sheets are assumed to be temperature-dependent, the variation in the temperature would cause a reduction in the elastic moduli and strengths of the sandwich plates. Moreover, the critical buckling capacity of the sandwich plates increases with an increase in the core to face sheet thickness ratio \( \frac{h_c}{h_f} \). This was expected since the flexural rigidity of the plate increases as its thickness increases. Furthermore, the
Table 10. Critical buckling capacities $N_{cr}$ (in kN) of square sandwich plate with GRPC face sheets subjected to biaxial compression in thermal environments ($b/h = 20$).

| $T[K]$ | $V_G$ | Configuration |          |          |          |          |
|--------|-------|---------------|----------|----------|----------|----------|
|        |       | (1-1-1)       | (1-2-1)  | (2-1-2)  | (1-4-1)  |
| 300    | 3%    | 30.2608       | 62.6657  | 19.8280  | 179.5706 |
|        | 5%    | 44.7761       | 86.1522  | 30.1632  | 222.0314 |
|        | 7%    | 61.1804       | 112.7113 | 41.8523  | 270.1007 |
|        | 9%    | 73.0461       | 131.8979 | 50.3401  | 304.8362 |
|        | 11%   | 76.5099       | 137.5102 | 52.8088  | 315.0093 |
| 400    | 3%    | 28.1670       | 58.5498  | 18.4304  | 168.6095 |
|        | 5%    | 39.5888       | 77.0329  | 26.5588  | 202.0235 |
|        | 7%    | 53.9754       | 100.3266 | 36.8067  | 244.1753 |
|        | 9%    | 61.9674       | 113.2577 | 42.5158  | 267.5848 |
|        | 11%   | 66.5897       | 120.7509 | 45.8058  | 281.1649 |
| 500    | 3%    | 28.0328       | 57.6016  | 18.4276  | 163.3675 |
|        | 5%    | 37.6524       | 73.1738  | 25.2684  | 191.5242 |
|        | 7%    | 50.4956       | 93.9716  | 34.4136  | 229.1614 |
|        | 9%    | 58.3870       | 106.7464 | 40.0452  | 252.2922 |
|        | 11%   | 63.4214       | 114.9097 | 43.6270  | 267.0886 |

Table 11. Critical buckling capacities $N_{cr}$ (in kN) of square sandwich plate with GRPC face sheets resting on elastic foundations subjected to uniaxial and biaxial compression ($b/h = 50$, $V_G = 9\%$, $T = 300K$).

| Load   | $(k_1, k_2)$ | Configuration |          |          |          |          |
|--------|---------------|---------------|----------|----------|----------|----------|
|        |               | (1-1-1)       | (1-2-1)  | (2-1-2)  | (1-4-1)  |
| Uniaxial|(0,0)          | 59.2060       | 106.7431  | 40.9075  | 246.5755 |
|        | (100,0)       | 140.7763a     | 252.5935a | 97.3991a | 578.0669a|
|        | (10,010)      | 378.5972a     | 675.3863a | 262.5525a| 1529.3507a|
| Biaxial|(0,0)          | 29.6030       | 53.3716   | 20.4537  | 123.2878 |
|        | (100,0)       | 112.6210a     | 202.0748a | 77.9193a | 462.4535a|
|        | (10,010)      | 302.8778a     | 540.3090a | 210.0420a| 1223.4805a|

$^a$Mode for this sandwich plate is $(m, n) = (2, 1)$.

results presented in these tables show that the increase in the volume fraction of graphene results in an increase in the buckling capacity of the sandwich plates. This is due to the fact that the modulus of elasticity of graphene is much larger than that of the matrix; thus, the addition of graphene results in a significant increase in the maximum flexural rigidity of the plate, despite the relatively low volume fraction content of graphene. A more significant enhancement is observed in (2-1-2) configured plates at room temperature; the buckling capacity of these plates was enhanced by 166.3% when the graphene content was increased from 3%
Moreover, by increasing the temperature from $T = 300K$ to $T = 500K$ the enhancement in the critical buckling capacity is reduced from 166.3% to 136.7%.

Table 11 presents the critical buckling capacities of simply-supported foundationless sandwich plates and those resting on an elastic foundation. As can be seen, the buckling capacities for the sandwich plate resting on Winkler and/or Pasternak elastic foundations are much higher than the foundationless plates. This is due to the buckling mode changing from first mode in the foundationless plates (i.e., $(m,n) = (1,1)$) to mode $(m,n) = (2,1)$ in the plates resting on a foundation. It should also be stated that with the presence of an elastic foundation, the critical buckling capacity of all types of sandwich plates is increased at approximately the same rate.

**Deflection analysis**

For verification purposes, the dimensionless maximum central deflections obtained by the proposed solution in this study are compared with those obtained by the recently developed existing solutions. Table 12 reports the comparison of the dimensionless maximum central deflections of a simply-supported square PmPV/CNT composite plates reinforced with carbon nanotube (PmPV/CNT) subjected to a uniformly distributed load of $q_0 = -1 \times 10^5N/m^2$ under temperature of $T = 300K$. The obtained results agree well with the results obtained by the closed-form solution of Wattanasakulpong and Chaikittiratana [52]. It should be noted that the material properties considered in this case study are listed in the aforementioned reference.

After the validation of the proposed solution, the influences of temperature variation and elastic foundation on the deflection response of all the variously configured GRPC-reinforced sandwich plates are examined. The results presented in Table 13 show the comparison of the maximum dimensionless central deflection
Table 13. Dimensionless maximum central deflections $\hat{w} = -\frac{w}{h}$ of square sandwich plate with GRPC face sheets resting on elastic foundations subjected to uniformly distributed load ($b/h = 50$, $V_G = 9\%$, $q_0 = -1 \times 10^5 N/m^2$).

| $T[K]$ | $(k_1, k_2)$ | (1-1-1) | (1-2-1) | (2-1-2) | (1-4-1) |
|--------|-------------|---------|---------|---------|---------|
| 300    | (0,0)       | $322.1771 \times 10^{-3}$ | $317.6698 \times 10^{-3}$ | $323.8420 \times 10^{-3}$ | $309.4159 \times 10^{-3}$ |
|        | (100,0)     | $82.3922 \times 10^{-3}$  | $82.0213 \times 10^{-3}$  | $82.5451 \times 10^{-3}$  | $81.3481 \times 10^{-3}$  |
|        | (10,010)    | $33.7870 \times 10^{-3}$  | $33.7125 \times 10^{-3}$  | $33.8208 \times 10^{-3}$  | $33.5802 \times 10^{-3}$  |
| 400    | (0,0)       | $379.7260 \times 10^{-3}$ | $369.433 \times 10^{-3}$  | $383.2698 \times 10^{-3}$ | $352.5023 \times 10^{-3}$ |
|        | (100,0)     | $86.2698 \times 10^{-3}$  | $85.448 \times 10^{-3}$   | $86.5197 \times 10^{-3}$  | $84.4782 \times 10^{-3}$  |
|        | (10,010)    | $34.5010 \times 10^{-3}$  | $34.164 \times 10^{-3}$   | $34.5513 \times 10^{-3}$  | $34.1691 \times 10^{-3}$  |
| 500    | (0,0)       | $402.7050 \times 10^{-3}$ | $392.2888 \times 10^{-3}$ | $406.5017 \times 10^{-3}$ | $373.7063 \times 10^{-3}$ |
|        | (100,0)     | $87.5476 \times 10^{-3}$  | $86.9346 \times 10^{-3}$  | $87.7951 \times 10^{-3}$  | $85.8195 \times 10^{-3}$  |
|        | (10,010)    | $34.7247 \times 10^{-3}$  | $34.6129 \times 10^{-3}$  | $34.7741 \times 10^{-3}$  | $34.4124 \times 10^{-3}$  |

Figure 4. Dimensionless maximum central deflections $\hat{w} = -\frac{w}{h}$ of square sandwich plates with GRPC face sheets subjected to uniformly distributed load ($b/h = 20$, $T = 300$K, $q_0 = -1 \times 10^5 N/m^2$).
of the plates subjected to a uniformly distributed load. The results reveal that the deflections of the variously configured sandwich plates evaluated at room temperature are significantly less than those tested at higher temperatures. This is because the increase in temperature leads to a decrease in the stiffness in the face sheets of the plates. Moreover, it can be seen that as expected, the dimensionless maximum central deflection values of the sandwich plate resting on elastic foundations are lower than those of the foundationless sandwich plate.

Next, the effect of volume fraction of graphene and the core to face sheet thickness ratio $h_c/h_f$ on the dimensionless maximum central deflections of the sandwich plates are compared and the results are presented in the graph in Figure 4. As can be seen, the increase in the volume fraction of graphene from 3% to 11% improved (decreased) the value of deflection of the plates; however, the rate of change in the improvement becomes insignificant once the volume fraction surpasses 9%. Hence, it can be evidently deduced that graphene reinforcement plays a very important role in improving the overall stiffness of the sandwich plates.

**Summary and conclusion**

This paper investigated the influence of the addition of graphene on the vibration, stability and static responses of sandwich plates. In addition, it presented an analytical solution for analyzing the natural frequencies, buckling capacity and elastic deformation of plates under various mechanical and thermal loading scenarios. Various configurations of the simply-supported sandwich plates formed with nanoparticle-reinforced face sheets, resting on different elastic foundations subject to the combined loads were investigated. Temperature-dependent material properties were considered for both the core and face sheets of the sandwich plates. The governing differential equations of motion were derived on the basis of a recently developed refined shear deformation plate theory and were subsequently solved analytically using the Navier method.

The results revealed that the natural frequencies and critical mechanical and thermal buckling capacities increased with the addition of nanoparticles, and that the plates experienced lower deflection when carrying a uniformly distributed load. Moreover, it was found that graphene volume contents beyond 9% could not further improve the natural frequencies, critical buckling and elastic deformation of the sandwich plates in a significant way. Moreover, the fundamental frequencies and buckling capacities of sandwich plates having a fixed volume fraction of graphene in their face sheets examined at lower temperatures were higher than those considered under higher temperatures. As also expected, the presence of an elastic foundation improved the buckling performances of the reinforced plates, resulting in the change of their buckling mode. As for the influence of nanoparticles on the natural frequencies, the plate with (2–1-2) configuration, which had the lowest thickness, exhibited the highest natural frequencies.
In conclusion, the refined shear deformation plate theory incorporated in this study, which includes a lower number of unknowns compared to the other pertinent theories that are involved with a greater number of unknowns, produced as accurate results. The future work will extend the formulation and assess the response of circular and skewed plates.

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**Appendix**

**Notation**

- $a, b$ length and width of the sandwich plate
- $a_G, b_G, h_G$ length, width and the effective thickness of the graphene
- $A_{ij}, B_{ij}, D_{ij}, H_{ij}$ equivalent sandwich plate’s stiffness components
- $E_{11}, E_{22}, G_{12}$ longitudinal and transverse Young’s moduli and shear modulus of the face sheets
- $E_C$ Young’s modulus of core of the sandwich plate
- $E_{11}^G, E_{22}^G, G_{12}^G$ longitudinal and transverse Young’s moduli and shear modulus of the graphene
\(E_M, G_M\) Young’s modulus and shear modulus of the matrix of face sheets

\(f(z)\) shear strain shape function

\(g_1, g_2\) geometric dependent coefficients

\(h_c, h_f, h\) thickness of the core, thickness of the face sheets and total thickness of the sandwich plate

\(I_0, I_1, I_2, J_0, J_1, J_2\) mass moments of inertia components

\(K_W, K_s\) stiffnesses of the Winkler springs and the shear layer

\(m, n\) half-wave numbers

\(N_x^T, N_y^T, M_x^bT, M_y^bT, M_x^cT, M_y^cT\) resultant components of forces and moments due to the thermal loading

\(N_x^0, N_y^0, N_{xy}^0\) In-plane normal and shear buckling capacities

\(Q_{ij}(i,j = 12,456)\) material stiffness components

\(q(x, y)\) transverse distributed load

\(V_M, V_G\) volume fractions of the matrix of face sheets and the graphene

\(u_0, v_0, w_0\) mid-plane displacement components of the sandwich plate in the orthogonal x, y and z directions

\(T, T_0\) final and reference temperatures

\(\Theta_{mn}\) temperature differential

\(\Delta T\) unknown coefficients

\(U_{mn}, V_{mm}, W_{mn}, \Theta_{mn}\) thermal expansion coefficients of the graphene and the matrix of face sheets

\(\chi_{11}^G, \chi_{22}^G, \chi_M\) longitudinal and transverse thermal expansion coefficients of the face sheets

\(\chi_C\) Thermal expansion coefficients of core of the sandwich plate

\(\eta_{j}(j = 1,2,3)\) graphene efficiency parameters

\(\nu_{12}^G, \nu_{12}, \nu_M\) Poisson’s ratios of the graphene and the matrix of face sheets

\(\nu_{12}\) Poisson’s ratio of the face sheets

\(\nu_C\) Poisson’s ratios of core of the sandwich plate

\(\rho\) density of the face sheets

\(\rho_G, \rho_M\) densities of the graphene and matrix of face sheets

\(\rho_C\) density of core of the sandwich plate

\(\theta\) rotation of the mid-plane of the sandwich plate

\(\lambda, \mu\) eigen frequency

\(\omega\) natural angular frequency