Heavy-light meson’s physics in Lattice QCD

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The possibility of revealing new physics by studying the flavor sector of the Standard Model strongly depends upon the accuracy that will be achieved in (near) future lattice QCD calculations and, in particular, on heavy-light meson’s observables. In turn, handling with heavy-light mesons on the lattice is a challenging problem, because of the presence of two largely separated energy scales, and at present it is impossible to extract matrix elements involving B mesons in external states without recurring to some approximation. In this note I give a fast overview of some of the methods that have been devised to handle such kind of problems, emphasizing those based on finite volume techniques, and briefly discuss some recent results obtained by their application.

Accurate and reliable theoretical calculations of many Standard Model processes are still lacking, particularly of the QCD contributions to decay rates and scattering amplitudes. Lattice QCD may eventually provide the required non-perturbative accuracy though numerical calculations are inevitably affected by systematics errors. Calculations are performed at finite lattice spacings, finite volumes and at unphysical values of the light quark masses but reliable estimates of these systematics can be obtained by performing simulations of several values of the cutoff, different physical volumes and of several values of the running quark masses. On a different ground have to be considered those systematics that cannot be reliably quantified like, for example, quenching, rooting etc. Quenching is not an issue anymore thank to several improvements that have been achieved in the field of simulation algorithms that opened the way toward simulations of full QCD (including the effects of sea quarks) with light quark masses falling within the range of applicability of chiral perturbation theory (see refs. [1, 2] for recent reviews). Concerning rooted staggered fermions, a proof in favor or against the validity of the ”fourth root trick” is still lacking while, at the same time, several hadronic observables have been calculated by using this formalism, ranging from \( f_\pi \) to \( M_\Upsilon \), and many of them are in fairly good agreement with available experimental determinations. Several authors have discussed this issue from different points of view and I will not enter in further details here (see for example refs. [3, 4, 5]).

When handling with heavy-light mesons on the lattice one is forced to introduce additional sources of systematics because of the simultaneous presence of two largely separated energy scales, the heavy quark mass and the confinement scale. A direct simulation of relativistic \( b \)-quarks would be possible by generating full QCD gauge ensembles on lattices with a number of points of the order of a few hundred per spatial direction, i.e. by taking under control at the same time cutoff effects \( \mathcal{O}(a m_b) \) (or \( \mathcal{O}(a m_b)^2 \) in improved theories) and finite volume effects \( \mathcal{O}(e^{-\Lambda_{\text{QCD}}L}) \). Presently available super computers allow the simulation of lattices with about 50 number of points per spatial direction (taking into account the necessity of continuum limit extrapolations) and several strategies have been devised to cope with such kind of two-scale problems. Different strategies can be classified in ”large volume” approaches (LVA) and ”small volume” approaches (SVA) depending on which one of the two scales is properly accommodated on the lattice.

By studying heavy-light mesons on lattices corresponding to physical volumes where pions as light as \( M_\pi \sim 300 \text{ MeV} \) can be simulated by meeting the bound \( M_{\pi L} \gtrsim 3 \), finite volume effects can be safely neglected in a first analysis since they are certainly smaller than the ones encountered in the study of light pseudoscalar mesons. On the contrary, having a limited number of lattice points per spatial direction, cutoff effects coming from relativistic propagating \( b \)-quarks would be too large (\( a m_b \sim 1 \)) and a proper handling of the heavy degrees of freedom requires the introduction of some approximation, typically effective field theories.

A first possibility consists in simulating relativistic heavy quarks with masses smaller or at best equal to the physical value of the charm quark mass and in extrapolating the observables of interest to the beauty mass by relying on analytical formulae derived by means of heavy quark effective theory (HQET). The extrapolations can be (and for several observables have been) turned into interpolations by introducing static quarks on the lattice through the
Eichten-Hill action [6]:

\[
\mathcal{L}_{\text{HQET}} = \bar{\psi}_h(D_0 + \delta m)\psi_h - \bar{\psi}_h \left( \omega_{\text{spin}} \vec{\sigma} \cdot \vec{B} + \frac{\omega_{\text{kin}}}{2} \vec{D}^2 \right) \psi_h + O(m_h^2), \quad \psi_h = \frac{1 + \gamma_0}{2} \psi
\]  

This action is renormalizable provided that the sub-leading terms \((\omega_X \propto m_h^{-1})\) are treated as insertions. Observables computed within the HQET framework need to be renormalized and matched to the corresponding QCD quantities at a matching scale \(\mu \simeq m_b\). If lattice simulations are performed on large volumes, the matching with QCD can be only performed perturbatively thus compromising the non-perturbative accuracy that a lattice calculation should provide.

Furthermore, the renormalization procedure is particularly cumbersome on the lattice because of the presence of power divergences that arise as a consequence of the mixing of operators of lower dimensions with the observable of interest. A solution to these problems has been found by Heitger and Sommer [7] and it is based on a finite volume technique. In the following I will just illustrate the basic idea behind this method remanding to ref. [8] for an exhaustive review and to ref. [9] for a practical application of the method to the calculation of the leptonic decay constants of the heavy-strange mesons. The small volume approach to HQET consists in simulating lattices on progressively large volumes and by calculating, within the effective theory, the so-called “step scaling functions” (SSF), formally defined by

\[
\mathcal{O}^{\text{QCD}}(m_b; L_0) = C(m_b)\mathcal{O}^{\text{HQET}}(L_0)
\]  

where \(\mathcal{O}\) is the observable, \(L_0\) the physical extension of the volume, that in practical application is of the order of 0.5 fm, and \(C(m_b)\) is the matching coefficient. Physical results are subsequently obtained by performing simulations on progressively large volumes and by calculating, within the effective theory, the so-called ”step scaling functions” (SSF), formally defined by

\[
\mathcal{O}^{\text{HQET}}(2L) = \sigma_\mathcal{O}(\mathcal{O}^{\text{HQET}}(L))
\]  

At each step of the calculation the continuum limit can and must be taken and at the end of the game one ends up with the observable calculated within HQET, renormalized and matched with non-perturbative accuracy.

A second large volume approach consists in simulating heavy quarks on lattices with spacings such that \(am_b \sim 1\) by using the so-called Fermilab (FNAL) lagrangian [10, 11, 12],

\[
\mathcal{L}_{\text{FNAL}} = \bar{\psi} \left[ m_0 + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - r_l \frac{aD_0^2}{2} - r_s \frac{a\vec{D}^2}{2} + e_B \frac{i\sigma_{ij} F_{ij}}{4} + e_E \frac{\sigma_{0l} F_{0l}}{2} \right] \psi
\]  

i.e. the Symanzik effective action for quarks at small spatial momenta \(|q\vec{p}| \ll 1\) with mass dependent coefficients. The improvement coefficients are calculated by matching FNAL observables to QCD ones at \(m_h \simeq m_b\). As in the previous case, by making simulations on large volumes the matching can be only performed perturbatively. Recently it has been observed [13] that the number of independent coefficients can be reduced to three and that they can be determined non-perturbatively by making, again, simulations on a small physical volume but with a fine lattice spacing [14].

By this fast (and incomplete) review of large volume approaches to heavy-light meson’s physics on the lattice it comes out that the preservation of (full) non-perturbative accuracy requires simulations to be performed at fine lattice spacings \((am_b \ll 1)\) and, for the time being, on small physical volumes.

In ref. [13] we have proposed a different finite volume method to handle with two-scale problems on the lattice, subsequently applied in refs. [15, 16, 17] to the quenched calculation of the \(b\)-quark mass and of the heavy-strange pseudoscalar leptonic decay constants, the so-called ”step scaling method” (SSM). Within the SSM approach the small volume calculations are needed in order to resolve the dynamics of the heavy quarks without recurring to any approximation but introducing, at intermediate stages, finite volume effects (FVE).
\[ \sigma(B \rightarrow D^{(*)} \ell\nu) = (\text{kin. fact.}) |V_{cb}|^2 (w - 1)^{3/2} \left[ G_{B \rightarrow D}(w) \right]^2 \]  

where \( w = p_i \cdot p_f / M_i M_f \). The proper definition of the form factors on the lattice in terms of three-point correlation functions can be found in refs. \[18, 19, 20\]. The final result is obtained by further evolving the volume from \( L_1 \) to \( L_2 = 4L_0 \) by calculating a second step scaling function and by the following identity

\[ O^{B \rightarrow D^{(*)}}(w; L_0, L_1) = O^{B \rightarrow D^{(*)}}(w; L_0, L_1) \sigma^{B \rightarrow D^{(*)}}(w; L_0, L_1) \sigma^{B \rightarrow D^{(*)}}(w; L_0, L_1) \]  

The crucial point is that the step scaling functions are calculated by simulating heavy quark masses \( m_P \) smaller than the \( b \)-quark mass. The physical value \( \sigma^{B \rightarrow D^{(*)}}(w; L_0, L_1) \) is obtained by a smooth extrapolation in \( 1/m_P \) that relies on the HQET expectations (see ref. \[16\]) and upon the general idea that finite volume effects, measured by the \( \sigma \)'s, are almost insensitive to the high energy scale. The final result is obtained by further evolving the volume from \( L_1 \) to \( L_2 = 4L_0 \) by calculating a second step scaling function and by the following identity

\[ O^{B \rightarrow D^{(*)}}(w; L_2) = O^{B \rightarrow D^{(*)}}(w; L_0) \sigma^{B \rightarrow D^{(*)}}(w; L_0, L_1) \sigma^{B \rightarrow D^{(*)}}(w; L_0, L_1) \sigma^{B \rightarrow D^{(*)}}(w; L_0, L_1) \]  

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How good is the hypothesis of low sensitivity of the FVE with respect to the high energy scale can be appreciated by looking at the plot in the left panel of Figure 1 in which the step scaling function \( \sigma^{P \rightarrow D^*}(w; L_0, L_1) \) is plotted at a fixed value of \( w \) as a function of the inverse mass of the heavy quark of the pseudoscalar meson. Similar plots are shown for the other step scaling functions together with continuum extrapolations in refs. \[18, 19, 20\].
physical results for $|V_{cb}| F^{B \rightarrow D^*}(w)$ are compared with some of the available experimental determinations in the plot in the right panel of Figure 1. These results have been obtained within the quenched approximation with the aim of checking the validity of the method and to explore the feasibility of a future full QCD calculation. Nevertheless they may have some phenomenological relevance since they allow the extraction of $V_{cb}$ without extrapolating experimental data. In fact no other results are presently available for the form factors beyond the point at zero recoil ($w \geq 1$).

The step scaling method can be combined with the finite volume approach to HQET in order to turn the already mild extrapolations of the step scaling functions into interpolations and further improve the accuracy of the results \cite{21}. So far, these two methods have been used only in quenched applications. Unquenched results for the leptonic decay constants of the heavy-light mesons, the $b$-quark mass and the semileptonic form factors will be produced in the near future by using the large physical volumes gauge ensembles generated by the Coordinated Lattice Simulations (CLS) effort and by producing small volume full QCD gauge ensembles satisfying Schrödinger Functional boundary conditions \cite{22}. Some preliminary results have already been obtained and can be found in refs. \cite{23,24} together with a presentation of the CLS initiative.

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References

[1] L. Giusti, PoS LAT2006 (2006) 009 [arXiv:hep-lat/0702014].
[2] K. Jansen, plenary talk at the LAT2008 conference.
[3] S. R. Sharpe, PoS LAT2006 (2006) 022 [arXiv:hep-lat/0610094].
[4] M. Creutz, PoS LAT2007 (2007) 007 [arXiv:0708.1295 [hep-lat]].
[5] A. S. Kronfeld, PoS LAT2007 (2007) 016 [arXiv:0711.0699 [hep-lat]].
[6] E. Eichten and B. R. Hill, Phys. Lett. B 234 (1990) 511.
[7] J. Heitger and R. Sommer [ALPHA Collaboration], JHEP 0402 (2004) 022 [arXiv:hep-lat/0310035].
[8] R. Sommer, arXiv:hep-lat/0611020.
[9] M. Della Morte et al., JHEP 0802 (2008) 078 [arXiv:0710.2201 [hep-lat]].
[10] A. X. El-Khadr, A. S. Kronfeld and P. B. Mackenzie, Phys. Rev. D 55 (1997) 3933 [arXiv:hep-lat/9604004].
[11] S. Aoki, Y. Kuramashi and S. i. Tominaga, Prog. Theor. Phys. 109 (2003) 383 [arXiv:hep-lat/0107009].
[12] M. B. Oktay and A. S. Kronfeld, arXiv:0803.0523 [hep-lat].
[13] N. H. Christ, M. Li and H. W. Lin, Phys. Rev. D 76 (2007) 074505 [arXiv:hep-lat/0608006].
[14] H. W. Lin and N. Christ, Phys. Rev. D 76 (2007) 074506 [arXiv:hep-lat/0608005].
[15] M. Guagnelli, F. Palombi, R. Petronzio and N. Tantalo, Phys. Lett. B 546 (2002) 237 [arXiv:hep-lat/0206023].
[16] G. M. de Divitiis et al., Nucl. Phys. B 675 (2003) 309 [arXiv:hep-lat/0305018].
[17] G. M. de Divitiis et al., Nucl. Phys. B 672 (2003) 372 [arXiv:hep-lat/0307005].
[18] G. M. de Divitiis, R. Petronzio and N. Tantalo, JHEP 0710 (2007) 062 [arXiv:0707.0587 [hep-lat]].
[19] G. M. de Divitiis, E. Molinaro, R. Petronzio and N. Tantalo, Phys. Lett. B 655 (2007) 45 [arXiv:0707.0582 [hep-lat]].
[20] G. M. de Divitiis, R. Petronzio and N. Tantalo, accepted on Nucl. Phys. B, [arXiv:0807.2944 [hep-lat]].
[21] D. Guazzini, R. Sommer and N. Tantalo, JHEP 0801 (2008) 076 [arXiv:0710.2229 [hep-lat]].
[22] M. Della Morte et al., PoS LAT2007 (2007) 246 [arXiv:0710.1188 [hep-lat]].
[23] G. von Hippel et al., PoS LATTICE2008 (2008) 227 [arXiv:0810.0214 [hep-lat]].
[24] M. Della Morte, P. Fritzsch, J. Heitger and R. Sommer, [arXiv:0810.3166 [hep-lat]].