In this paper we explore metallic CNTs as a novel class of analog current amplifiers, which are core components of nanoscale electronic design [1,2]. Applying quantum kinetics to the observed current response of single-wall nanotubes, we map the current as a function not simply of the driving voltage along the tube but also, crucially, as a function of carrier density.

The touchstones of performance in high-current-gain transistors have been devices taking advantage of the properties of the quantum-confined metallic electron gas formed at epitaxial III-V heterojunctions [3]. These two-dimensional quantum devices dominate millimeter-wave signal processing technology thanks to three superior features: (a) the high carrier density attainable in a quantum-well confined metallic channel relative to conventional bulk doping; (b) the ability to sensitively tune the density by simple gate control; and (c) reliable low-resistance contacts at the channel's source and drain.

Carbon nanotubes hold promise for a novel class of nanoscopic amplifier, reliant upon the unique behavior of one-dimensionally confined carriers. In broad concept, metallic CNTs share much with their heterojunction predecessors. Physically, however, they differ markedly in much stronger confinement and larger energy scale, far smaller dimensions, and unmatched stability [4].

Transistor operation in semiconducting CNT structures is already well demonstrated [1,4,5,6]. While both metallic and semiconducting nanotubes share, as fundamental attributes, quantum confinement and mechanical and thermal toughness, they differ greatly with respect to the criteria (b) and (c) above.

One reason why semiconducting CNT devices do not perform to their theoretical potential is the difficulty of fabricating source and drain contacts that are both reproducible and have intrinsically low access resistance. This degrades the maximum achievable current-voltage response as well as the signal gain [1]. In metallic CNTs, it is known that these drawbacks can be largely resolved. This invites a sharper theoretical analysis of their possibilities.

As far as analog signal detection and amplification are concerned, developments in metallic CNT transport offer solid evidence that metallic nanotubes can outperform their semiconducting counterparts. In so doing, they should certainly also surpass the best established III-V quantum-well approaches. Further, III-Vs and similarly complex technologies cannot be integrated into true sub-mesoscopic structures; but the CNTs’ rugged simplicity overcomes this obstacle [1].

After many experimental studies on transport in CNTs, it is now practical to fabricate reproducibly good ohmic contacts to the channel. These developments have yielded new and reliable data on electrical response in metallic samples [3,6,7,8]. At the same time, kinetic models for CNTs have begun to probe phonon scattering effects at voltages well above the linear regime [3,6,9,10].

To assess metallic CNTs as super-sensitive analog transistors, we must predict their current gain. We do this through a detailed kinetic approach. The description (i) is easily integrated with current-voltage measurements [3,6,9,10]. (ii) allows straightforward extraction of basic scattering parameters, and (iii) makes no ad hoc assumptions foreign to the known behavior of the electron gas.

Consider the electron population in the lowest one-dimensional conduction subband of a uniform, metallic single-wall nanotube. Its steady-state momentum distribution $f_k(\mu)$ depends on chemical potential $\mu$ and bath thermal energy $k_BT$. The quantum transport equation, at external driving voltage $V$ over length $L$, is

\[ \frac{eV}{\hbar L} \frac{\partial f_k(\mu)}{\partial k} = \frac{1}{\tau_{\text{in}}(E_k)} \left( f_k(\mu) - \frac{\langle f^{-1}\rangle}{\tau_{\text{in}} f_{\text{eq}}(\mu, k_BT)} f_{\text{eq}}^{\text{eq}}(\mu, k_BT) \right) - \frac{1}{\tau_{\text{el}}(E_k)} \frac{f_k(\mu) - f_{-k}(\mu)}{2}. \]  

\[ (1) \]
The right-hand-side collision terms encode the scattering dynamics through two relaxation rates: $1/\tau_{\text{in}}$ (inelastic) and $1/\tau_{\text{el}}$ (elastic). In general, these vary with subband energy $E_k$. Traces $\langle \cdot \cdot \cdot \rangle$ represent expectations over wavevector space, spin, and a twofold “valley” degeneracy. Thus, the expectation of $f_k/\tau_{\text{in}}(E_k)$ is $\langle \tau_{\text{in}}^{-1} f \rangle = 4 \int (dk/2\pi) \tau_{\text{in}}(E_k)^{-1} f_k$. The equilibrium function $f^0(\mu, k_BT)$ has the usual Fermi-Dirac form, explicitly dependent on $\mu$ and $k_BT$.

Equation (1) automatically respects detailed balance. It is unconditionally gauge invariant. This is due to the explicit form of the collision terms. For collision times independent of $E_k$, Eq. (1) has an analytic solution at both low and high fields.

The characteristic velocity of the system at density $n = \langle f^0(\mu, k_BT) \rangle$ is its group velocity at the characteristic wave vector $k_{av} \equiv \langle 2|k| f^0(\mu, k_BT)/n \rangle$. This yields $v_{av}(\mu) = \hbar^{-1}dE/dk|_{k=k_{av}}$. Note that $k_{av}$ and $v_{av}$ go to their Fermi values $k_F$ and $v_F$ at large $n$. The elastic collision time is then fixed by $\tau_{\text{el}} = \lambda_{el}/v_{av}(\mu) = L/v_{av}(\mu)$. Elastic scattering is insensitive to field effects.

If inelastic collisions act solely at the leads, transport is ballistic. However, electrons energized by the field emit longitudinal optical phonons copiously. The inelastic MFP, $\lambda_{in}(\mu, eV)$, is not just shorter than $L$ at room temperature (even at low fields) but depends sensitively on how the carriers’ energy gained in transit is dissipated: the more the carriers draw from the voltage, the more they shed by enhanced emission.

To extract the physics of $\lambda_{in}(\mu, eV)$ we start with the solution for the current, following from Eq. (1). A metallic single-wall nanotube (MSWNT) has linearly dispersive bands. For electrons, $E_k = hv_{in}k$ with $v_{in} = 5 \times 10^3\text{cm}^{-1}$; for holes, $E_k = -hv_{in}k$. The expectation $I = \langle -e \nu f \rangle$ is best evaluated in the Fourier space dual to $k$. Some algebra yields

$$I = 4G_0V \frac{\tau_{in}}{\lambda_{el}k_{av}} \left[1 - \exp \left( -\frac{hLk_{av}}{eV\sqrt{\tau_{in}}} \right) \right]$$

where $G_0 = e^2/\pi \hbar$. The total relaxation time is $\tau^{-1} = \tau_{el}^{-1} + \tau_{in}^{-1}$. The expression for $I$ is exact in the degenerate regime, and remains accurate at low density.

It is interesting to look at the behavior of Eq. (2) at high fields. Suppose $\tau_{in} \sim V^{-\beta}$. At large $V$, inelasticity dominates the Matthiessen rate: $\tau^{-1} \sim \tau_{in}^{-1}$. When $eV \gg E_{av}$, there are two possibilities. If $\beta \leq 1$ then for $A$ a constant

$$I = nev_{in} \sqrt{\frac{\tau_{in}}{\lambda_{in}}} \left[1 - \exp \left( -\frac{AV_{\beta-1}}{1 - A\nu_{\beta-1}} \right) \right] \rightarrow nev_{in} \sqrt{\frac{\tau_{in}}{\lambda_{in}}}$$

and the current saturates. Otherwise, for $\beta > 1$ we get

The current, which must vanish linearly as $V \rightarrow 0$, will also die off at large $V$. There is a critical field at which $I$ peaks, and then falls again.

Exactly that feature marks the response of a suspended MSWNT. Figure 1 exemplifies this remarkable behavioral change in an actual device. Two current-voltage ($I-V$) characteristics measured by Pop et al. (2010) for the heat-sunk tube, circles for the suspended tube. Curves are our kinetic simulations: dashed lines for the heat-sunk and full lines for suspended tubes. The radical changes in $I-V$ are faithfully reproduced.

$$I \sim V^{1-\beta} \rightarrow 0.$$
the term C

strong field-dependent inelastic effects by the analytic fit each sample goes to a constant. C

is 2(−δµ/e − 2(−eδn) ln(d/2R)/e where e is the dielectric constant and −eδn is the induced change in electron charge density.

The bias loop’s inverse total differential capacitance is C

−1 = −δVg/(εLδn) = (2/εL) ln(2R/d) + C

−1 and the term C

= ε2Lδn/∂µ is the “quantum capacitance” from which τ

= λ

(μ, eV)/v

av(μ). The low-field MFP λ

(0) is set by the low-field slope of the current-voltage curve; parameters α, β are set by I(V) measured at large V. The characteristic energy is E

av ≡ E

k

av.

There is a close match between our model λ

and its empirical behavior. At high field, the asymptotic tail of Eq. 3 correctly reproduces the experimental I−V results (see Fig. 3). At low field, it goes to the appropriate inelastic MFP determining the linear slope at V → 0. This leaves us to consider the density dependence; a critical factor.

Knowledge of the density (or chemical-potential) dependence, essential to transconductance, must be inferred from the Fermi-liquid nature of the carriers. In a degenerate channel the field acts bodily on every carrier in the Fermi sea, but only the very few at its surface can scatter freely. Most are in lock-step. Thus the Fermi liquid as a whole resists the tendency to emit phonons, on par with its characteristic energy E

F.

For E

F ≫ eV, Pauli blocking strongly inhibits any and all increases in scattering rates. We include the strong Fermi-liquid effects in Eq. 3 by normalizing eV to E

av → E

F. Pauli blocking is the main source of variation of λ

with µ. In the low-density limit (pinchoff), we have E

av → k

B T, so λ

sheds its dependence on density.

Figure 3 compares our results from Eqs. 1–3 directly with experimental data for different length channels. The quality of fit to the measured I−V curves is good. The asymptotic tails are faithfully rendered. In the shortest channels we see some departure from measured data. We ascribe this to (a) the proximity of the bulk leads, which provide efficient heat sinking and thus qualitatively change the hot-phonon behavior, and (b) nonuniform densities in very short CNTs.

In signal detection, the transconductance is central. We now address g

, starting at Eq. 2 and the in-channel response g

≡ e(∂I/∂µ). Since I is a very strong function of τ

(μ, eV), we see that g

has a rich structure wherein µ and V enter more than once:

\[ \tilde{g}_m(\mu, eV) = e \frac{\partial I}{\partial \mu} = eI e \frac{\partial}{\partial \mu} \left[ \ln \left( \frac{\tau(\mu, eV)}{\tau_{\alpha}(\mu)} \right) + \ln \left( \frac{kF(\mu)}{kF(\mu)} \right) + \ln \left( 1 - \exp \left( -\frac{hLkF(\mu)}{eV\tau(\mu, eV)} \right) \right) \right]. \]

Next we map \( \tilde{g}_m \) to g

. Our bias gate is a cylinder of radius R = 20nm coaxial with the CNT of diameter d = 2.4nm. The capacitive loop comprises the bias source V

, the tube, and the gap to the gate. Thus \( \delta V_g = -\delta \mu/e - 2(-e\delta n) \ln(d/2R)/e \) where e is the dielectric constant and \( -e\delta n \) is the induced change in electron charge density.

In metallic CNTs the large group velocity greatly enhances C

so that \( |C_g/C_q - 1| \lesssim 10^{-2} \). Hence g

= \( \tilde{g}_m \).

A metallic CNT has zero band gap. For good signal gain, however, we need the operating region of maximal g

; that point lies well away from threshold.

FIG. 2: Functional behavior of the inelastic mean free path in suspended metallic nanotubes. Chain lines: λ

extracted from data [6]. Full lines: simulation (see Eq. 3 in text). All curves show a clear V−1.5 falloff at high field. The inset shows λ

for the heat-sunk sample of Fig. 1: full dots are points extracted from data [6]; full line is our simulation.

FIG. 3: Current response in suspended MSWNTs of varying lengths; see also Fig. 2. Symbols are measured data points [Fig. 3(a), Ref. [6]]. Full lines are our simulation.

the suspended samples studied in Ref. [6], there is a very clear falloff with β = 1.5. In the low-field regime, λ

for each sample goes to a constant.

With the extracted results we can reliably model the strong field-dependent inelastic effects by the analytic fit

\[ \lambda_m(\mu, eV) = \lambda^{(0)}_m \tanh(u^2)/u^2; \quad u = \alpha eV/E_{av}. \]

\[ (3) \]

\[ \tilde{g}_m(\mu, eV) = e \frac{\partial I}{\partial \mu} = eI e \frac{\partial}{\partial \mu} \left[ \ln \left( \frac{\tau(\mu, eV)}{\tau_{\alpha}(\mu)} \right) + \ln \left( \frac{kF(\mu)}{kF(\mu)} \right) + \ln \left( 1 - \exp \left( -\frac{hLkF(\mu)}{eV\tau(\mu, eV)} \right) \right) \right]. \]
and each is described by Eq. (2). Thus, in terms of\( g_\sim \) on nanosecond time scales, far longer than transit times\( -f \). Moreover, since electron-hole recombination occurs\( -V \). The rapid rise and fall of the transconductance with driving\( -I \) peak is 50GHz at\( V_g = \pm 200mV, V = 0.35V \).

In sum: we have used a microscopic Fermi-liquid kinetic analysis to predict the transconductance of a metallic CNT, a property vital to the design of high-gain nanotube transistors. Our theory is strictly conserving, self-contained, and readily implemented. To our knowledge, no detailed account of the critical density dependence of\( g_m \) has been available until now. Our predictions are directly testable in gated devices. Systematic measurements of\( g_m \) will allow new insights into the dynamics of one-dimensional metallic channels.

Finally we note that suspended metallic nanotubes are extremely unusual, in that both their high\( g_m \) and their negative differential conductance coexist in the same operating region. Such a combination holds the clear possibility of a radically new kind of active structure, with signal processing capabilities previously unseen at nanoscopic scales.

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