Lepton flavor violating semileptonic $\tau$ decays
$\tau \to lP(V)$ in a topcolor scenario

Chong-Xing Yue, Li-Hong Wang, Wei Ma
Department of Physics, Liaoning Normal University, Dalian 116029, China

March 26, 2022

Abstract

The contributions of the neutral top-pion $\pi_1^0$ and the non-universal gauge boson $Z'$ predicted by topcolor scenario to the lepton flavor violating (LFV) semileptonic $\tau$ decays $\tau \to lP(V)$ ($P = \pi^0$, $\eta$, $\eta'$ and $V = \rho^0$, $\phi$) are discussed. We find that the contributions of $Z'$ to these decay processes are generally larger than those from $\pi_1^0$. $\pi_1^0$ can only make the value of the branching ratio $Br(\tau \to lP)$ in the range of $1\times10^{-11} \sim 1\times10^{-16}$, which is far below the sensitivity of foreseeable experiments. With reasonable values of the free parameters, the non-universal gauge boson $Z'$ can make the value of the branching ratio $Br(\tau^- \to \mu^-\phi)$ reach $1\times10^{-7}$, which might approach the observable threshold of near-future experiments.

PACS number(s):12.60.Cn, 13.35.Dx, 14.80.Cp.

*E-mail:cxyue@lnnu.edu.cn
I. Introduction

The flavor physics of quarks and leptons is one of the most important issue of current particle physics. Over the past decade years, the most surprising development in flavor physics is observation of neutrino oscillation, which can be seen as the first experimental clue for new physics beyond the standard model (SM) [1]. Observation of neutrino masses also provides the evidence for flavor violating in the lepton sector and gives the possibility of lepton flavor violating (LFV) among the charged leptons. It is well known that the tree-level LFV processes are absent in the SM, due to unitary of the leptonic analog of Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix and the masslessness of three neutrinos. Thus, observation of the LFV processes would be a clear signature of new physics beyond the SM. This fact has led to a great amount of theoretical effect for revealing the underlying new physics in the leptonic flavor sector.

The $\tau$ lepton is the most heavy particle in the leptonic sector of the SM, which is much more sensitive than the lepton $e$ or $\mu$ to new physics related to the flavor and mass-generation problems [2]. The leptonic or semileptonic character of $\tau$ decays provides a clean laboratory to test the structure of the weak currents and the universality of their couplings to the gauge bosons. Furthermore, its semileptonic decay is ideal tool for studying strong interaction effects in very clean conditions. Moreover, the sensitivity of probing the LFV semileptonic $\tau$ decays have been enhanced to $\mathcal{O}(10^{-7})$ [3]. Thus, in the framework of some popular models beyond the SM, studying the semileptonic $\tau$ decays related LFV is very interesting and needed.

The discovery of LFV in the neutrino oscillation experiments has opened a new era for flavor physics in the leptonic sector, where one can study the possible signatures of new physics via some LFV processes. The effects of new physics on the LFV semileptonic $\tau$ decays, such as $\tau \to lP$, $\tau \to lV$, and $\tau \to lPP$, have been extensively studied in Refs.[4,5,6,7], where $P(=\pi^0, \eta, \eta')$ and $V(=\rho^0, \phi)$ represent the pseudoscalar meson and vector meson, respectively. It has been shown that these decay processes are very sensitive to the new physics effects and the values of the branching ratios for some of these processes might be enhanced to the experimental interesting ranges. The constraints on the free
parameters of some specific models beyond the SM have been obtained.

To completely avoid the problems arising from the elementary Higgs field in the SM, various kinds of dynamical electroweak symmetry breaking (EWSB) models have been proposed, among which topcolor scenario is attractive because it can explain the large top quark mass and provide a possible EWSB mechanism [8]. Almost all of this kind of models propose that the underlying interactions, topcolor interactions, should be flavor non-universal. When one writes the non-universal interactions in the mass eigenbasis, it can induce the tree-level flavor changing (FC) couplings, which can generate rich phenomenology.

A common feature of the topcolor models, such as topcolor-assisted technicolor (TC2) models [9], flavor-universal TC2 models [10], top see-saw models [11] and top flavor see-saw models [12], is that the physical top-pions (\(\pi_{t}^{0,\pm}\)) and non-universal gauge boson \(Z'\) are predicted. These new particles treat the third generation fermions differently from those in the first and second generations and thus can lead to the tree-level FC couplings. The aim of this paper is to study the contributions of these new particles to the LFV semileptonic \(\tau\) decays \(\tau \to lP\) and \(\tau \to lV\) and see whether the values of their branching ratios can be significantly enhanced.

To predigest our calculation, we will give our numerical results in the context of the TC2 models. In the next section, we will briefly summarize the relevant flavor-diagonal (FD) and FC coupling expressions of the new particles (the neutral top-pion \(\pi_{t}^{0}\) and non-universal gauge boson \(Z'\)) predicted by the TC2 models. The contributions of \(\pi_{t}^{0}\) and \(Z'\) to the LFV semileptonic \(\tau\) decays \(\tau \to lP\) and \(\tau \to lV\) are calculated in Sec.III and Sec.IV, respectively. Section V contains our conclusions.

II. The relevant couplings of the neutral top-pion \(\pi_{t}^{0}\) and the non-universal gauge boson \(Z'\)

In topcolor scenario [8], topcolor interactions, which are not flavor-universal and mainly couple to the third generation fermions, generally generate small contributions to EWSB and give rise to the main part of the top quark mass. Thus, the top-pions \((\pi_{t}^{0,\pm})\) have large Yukawa couplings to the third generation fermions, and can induce the
new FC couplings. In the TC2 models, the FD and FC couplings of the neutral top-pion \( \pi_t^0 \) to light fermions, which are related our calculation, can be written as [8,9,13,14]:

\[
\frac{m_f}{\nu} \bar{f} \gamma^5 f \pi_t^0 + \frac{m_\tau}{\nu} K \bar{\tau} \gamma^5 t \pi_t^0,
\] (1)

where \( \nu = \nu_W/\sqrt{2} \approx 174\text{GeV} \), \( f \) represents the light quark (\( u, d, c, \) or \( s \)), and \( l \) represents the first (second) generation lepton \( e(\mu) \). \( K \) is the lepton flavor mixing factor between the third- and the first- or second- generation leptons. Certainly, there is also the FC coupling \( \pi_t^0 \bar{\mu}e \). However, the topcolor interactions only contact with the third-generation fermions, and thus the flavor mixing between the first- and second-generation fermions is very small, which can be ignored [15].

An inevitable feature of topcolor scenario is that the SM gauge groups are extended at energy well above the weak scale. Breaking of the extended groups to their diagonal subgroups produces the non-universal massive gauge boson \( Z' \) [16]. This kind of new particles generally couple primarily to the third generation fermions and have large tree-level FC couplings.

The FD couplings of \( Z' \) to fermions, which are related our calculation, can be written as [8,9,17]:

\[
L_{Z'}^{FD} = -\sqrt{4\pi K_1} \left\{ Z'_\mu \left[ \frac{1}{2} \bar{\tau}_L \gamma^\mu \tau_L - \bar{\tau}_R \gamma^\mu \tau_R \right] - \tan^2 \theta' Z'_\mu \left[ \frac{1}{6} \bar{e}_L \gamma^\mu e_L + \frac{2}{3} \bar{\tau}_R \gamma^\mu \tau_R \right] 
+ \frac{1}{6} \bar{s}_L \gamma^\mu s_L - \frac{1}{3} \bar{\mu}_L \gamma^\mu \mu_L - \frac{1}{2} \bar{\mu}_R \gamma^\mu \mu_R + \frac{1}{6} \bar{u}_L \gamma^\mu u_L + \frac{1}{6} \bar{d}_L \gamma^\mu d_L 
+ \frac{2}{3} \bar{\mu}_R \gamma^\mu \mu_R - \frac{1}{3} \bar{d}_R \gamma^\mu d_R - \frac{1}{2} \bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R \right\},
\] (2)

where \( K_1 \) is the coupling constant and \( \theta' \) is the mixing angle with \( \tan \theta' = \frac{g_1}{\sqrt{4\pi K_1}} \). \( g_1 \) is the ordinary hypercharge gauge coupling constant. To obtain the top quark condensation and not form a \( b\bar{b} \) condensation, there must be \( \tan \theta' \ll 1 \) [9,10]. In above equation, we have assumed that there is no mixing between the SM gauge boson \( Z \) and the non-universal gauge boson \( Z' \). The FC couplings of \( Z' \) to leptons can be written as [17,18]:

\[
L_{Z'}^{FC} = \frac{1}{2} g_1 K' Z'_\mu [\bar{\tau}_L \gamma^\mu \mu_L + 2 \bar{\tau}_R \gamma^\mu \mu_R + \bar{\tau}_L \gamma^\mu e_L + 2 \bar{\tau}_R \gamma^\mu e_R],
\] (3)

where \( K' \) is the lepton flavor mixing factor. Since the non-universal gauge boson \( Z' \) treats the fermions in the third generation differently from those in the first and second
generation and treats the fermions in the first generation same as those in the second generation, so we have assumed $K'_{\tau\mu} = K'_{\tau e} = K'$ in above equation. In this case, the contributions of $Z'$ to the LFV semileptonic $\tau$ decays $\tau \to \mu P(V)$ are approximately equal to those for the decays $\tau \to e P(V)$.

Integrating out the non-universal gauge bosons $Z'$, Eq.(2) and Eq.(3) can give rise to the effective four fermion couplings $\tau\mu q q$ ($q = u, d, c$, and $s$):

$$L_{4f} = - \frac{\pi K' K' \tan^3 \theta'}{M^2_{Z'}} (\bar{\tau}_L \gamma^\mu \mu_L + 2 \bar{\tau}_R \gamma^\mu \mu_R) \left[ \frac{1}{6} (\bar{c}_L \gamma^\mu c_L + \bar{s}_L \gamma^\mu s_L + \bar{u}_L \gamma^\mu u_L + \bar{d}_L \gamma^\mu d_L) + \frac{2}{3} (\bar{c}_R \gamma^\mu c_R + \bar{u}_R \gamma^\mu u_R) - \frac{1}{3} (\bar{s}_R \gamma^\mu s_R + \bar{d}_R \gamma^\mu d_R) \right].$$

(4)

Where $M_{Z'}$ is the mass of the non-universal gauge boson $Z'$.

In the following sections, we will use above coupling expressions to calculate the branching ratios $Br(\tau \to \mu V)$ and $Br(\tau \to \mu P)$, and compare our numerical results with the corresponding experimental upper limits given in Table 1[3].

| Decay Process | Current Upper Limit (90% C.L.) |
|---------------|-------------------------------|
| $\tau^- \to \mu^- \pi^0$ | $4.1 \times 10^{-7}$ |
| $\tau^- \to \mu^- \eta$ | $2.3 \times 10^{-7}$ |
| $\tau^- \to \mu^- \eta'$ | $4.7 \times 10^{-7}$ |
| $\tau^- \to \mu^- \rho^0$ | $2.0 \times 10^{-7}$ |
| $\tau^- \to \mu^- \phi$ | $7.7 \times 10^{-7}$ |

Table 1: Current experimental upper limits on the branching ratios $Br(\tau \to \mu P(V))$.

III. The neutral top-pion $\pi^0_t$ and the LFV semileptonic $\tau$ decay $\tau \to \mu P$

It is well known that the LFV semileptonic decay $\tau \to lS$ ($S$ denotes a scalar meson) can only be generated by the scalar current, the decay $\tau \to lV$ ($V$ denotes a vector meson) can only be generated by the vector current, while the decay $\tau \to lP$ ($P$ denotes a pseudoscalar meson) can be induced by the axial-vector or pseudoscalar currents. The neutral top-pion $\pi^0_t$ is the CP-odd pseudoscalar particle, thus it can only have contributions to the decay $\tau \to lP$ via the $FC$ lepton couplings and the $FD$ light quark couplings.
To calculate the branching ratios of the LFV semileptonic $\tau$ decays $\tau^- \rightarrow \mu^- \pi^0$, $\mu^- \eta$, and $\mu^- \eta'$, we write the relevant pseudoscalar matrix elements as [4]:

$$
\langle 0|\bar{u}\gamma^5 u|\pi^0(p)\rangle = -\langle 0|\bar{d}\gamma^5 d|\pi^0(p)\rangle = \frac{i}{\sqrt{2}} \frac{m_u^2}{m_u + m_d} F_\pi, 
$$

$$
\langle 0|\bar{s}\gamma^5 s|\eta_8(p)\rangle = -i\sqrt{6} F^8_\eta \frac{m_{\eta_8}^2}{m_u + m_d + 4m_s},
$$

$$
\langle 0|\bar{s}\gamma^5 s|\eta_8'(p)\rangle = -i\sqrt{6} F^8_{\eta'} \frac{m_{\eta_8'}^2}{m_u + m_d + 4m_s}. 
$$

Where $F_\pi$, $F^8_\eta$, and $F^8_{\eta'}$ are the decay constants of the pseudoscalar mesons $\pi^0$, $\eta_8$, and $\eta_8'$, respectively.

Using Eq.(1), Eq.(5), Eq.(6), and Eq.(7), we can give the expressions of the branching ratios $Br(\tau^- \rightarrow \mu^- \pi^0)$, $Br(\tau^- \rightarrow \mu^- \eta)$, and $Br(\tau^- \rightarrow \mu^- \eta')$ generated by the neutral top-pion $\pi_t^0$ as:

$$
Br(\tau^- \rightarrow \mu^- \pi^0) = \frac{6K^2}{\cos^2 \theta_c} \left( \frac{m_\pi}{M_{\pi_t}} \right)^4 \left( \frac{m_u - m_d}{m_u + m_d} \right)^2 Br(\tau^- \rightarrow \nu_\tau \pi^-),
$$

$$
Br(\tau^- \rightarrow \mu^- \eta) = \frac{18K^2}{\cos^2 \theta_c} \left( \frac{F_\eta}{F_\pi} \right)^2 \left( \frac{m_\eta}{M_{\pi_t}} \right)^4 \left( \frac{m_u + m_d - 2m_s}{m_u + m_d + 4m_s} \right)^2 Br(\tau^- \rightarrow \nu_\tau \pi^-),
$$

$$
Br(\tau^- \rightarrow \mu^- \eta') = \frac{18K^2}{\cos^2 \theta_c} \left( \frac{F_{\eta'}}{F_\pi} \right)^2 \left( \frac{m_{\eta'}}{M_{\pi_t}} \right)^4 \left( \frac{m_u + m_d - 2m_s}{m_u + m_d + 4m_s} \right)^2 Br(\tau^- \rightarrow \nu_\tau \pi^-). 
$$

Where the meson decay constants are defined as: $F_\eta = F^8_\eta - \frac{1}{\sqrt{2}} F^0_\eta$ and $F_{\eta'} = F^8_{\eta'} + \frac{1}{\sqrt{2}} F^0_{\eta'}$. $
\theta_c$ is the Cabibbo angle. $M_{\pi_t}$ represents the mass of the physical top-pions ($\pi_t^{0,\pm}$), its value remains subject to large uncertainty [8]. However, it has been shown that its value is generally allowed to be in the range of a few hundred GeV depending on the models [19]. In our numerical estimation, we will take $M_{\pi_t}$ as a free parameter and assume that it is in the range of $150 GeV \sim 400 GeV$.

Certainly, the neutral top-pion $\pi_t^0$ can also generate contributions to the LFV semileptonic decay $\tau^- \rightarrow \mu^- P$ via the $Z$ penguin diagrams, i. e. the effective process $\tau^- \rightarrow \mu^- Z^* \rightarrow \mu^- f \bar{f}$. However, compared with those from $\pi^0_t$ exchange at the tree-level, the contributions are much small. So, we do not consider the one-loop contributions of $\pi^0_t$ to the decay $\tau^- \rightarrow \mu^- P$ in this paper.
Fig. 1: The branching ratio $Br(\tau^ - \rightarrow \mu^- \pi^0)$ as a function of the mixing parameter $K$ for three values of the mass parameter $M_{\pi_t}$.

Fig. 2: Same as Fig. 1 but for $Br(\tau^ - \rightarrow \mu^- \eta)$.  Fig. 3: Same as Fig. 1 but for $Br(\tau^ - \rightarrow \mu^- \eta')$.

Except the free parameter $M_{\pi_t}$, the branching ratio $Br(\tau^ - \rightarrow \mu^- P)$ depends on the mixing factor $K$. Topcolor scenario has not given any prediction about its value. In general, the experimental data about observables, such as $\mu$ anomalous magnetic moment $a_\mu$, the branching ratios $Br(\tau \rightarrow l_i \gamma)$ and $Br(\tau \rightarrow l_i l_j l_k)$, can give constraints on the values of the free parameter $K$. However, although the neutral top-pion $\pi^0_t$ can generate significant contributions to the LFV processes $\tau \rightarrow l_i \gamma$ and $\tau \rightarrow l_i l_j l_k$ via the FC couplings, the current experimental upper limits on $Br(\tau \rightarrow l_i \gamma)$ and $Br(\tau \rightarrow l_i l_j l_k)$ can not give severe constraints on the mixing factor $K$ [20]. Thus, in this paper, we will assume that the
value of the mixing factor $K$ is in the range of $0.1 \sim 0.9$.

In our numerical estimation, we will take $F_{\pi} = 131 MeV$, $F_{\eta}^{8} \approx 1.2F_{\pi}$, $F_{\eta}^{0} \approx 0.2F_{\pi}$, $F_{\eta'}^{8} \approx -0.45F_{\pi}$, $F_{\eta'}^{0} \approx 1.15F_{\pi}$ [21]. The other SM input parameters are taken as: $Br(\tau^{-} \rightarrow \nu_{\tau}\pi^{-}) \approx 11.06\%$, $\cos^{2}\theta_{c} \approx 0.95$, $m_{\tau} = 1.78 GeV$, $m_{u} \approx \frac{1}{2}m_{d} \approx 4 MeV$, $m_{s} = 115 MeV$, $m_{\eta} = 548 MeV$, $m_{\eta'} = 957 MeV$, and $m_{\pi} \approx 135 MeV$ [22].

Using above given values of the relevant parameters, we present the branching ratios $Br(\tau^{-} \rightarrow \mu^{-}\pi^{0})$, $Br(\tau^{-} \rightarrow \mu^{-}\eta)$, and $Br(\tau^{-} \rightarrow \mu^{-}\eta')$ as functions of the mixing factor $K$ for three values of the mass $M_{\pi}$ in Fig.1, Fig.2, and Fig.3, respectively. From these figures, we can see that the values of the branching ratios $Br(\tau^{-} \rightarrow \mu^{-}\pi^{0})$, $Br(\tau^{-} \rightarrow \mu^{-}\eta)$, and $Br(\tau^{-} \rightarrow \mu^{-}\eta')$ increase as the mixing parameter $K$ increasing and the mass parameter $M_{s_{t}}$ decreasing. However, in all of the parameter space, the values of these branching ratios are much smaller than the corresponding experimental upper limits given in Table 1. Thus, we have to say that the possible signatures of the neutral top-pion $\pi^{0}_{t}$ can not be detected via the LFV process $\tau^{-} \rightarrow l^{-}P$ in the future experiments.

IV. The non-universal gauge boson $Z'$ and the LFV semileptonic $\tau$ decays $\tau \rightarrow lP(V)$

The new physics models beyond the SM generally predict the existence of extra neutral gauge boson $Z'$. If discovered it would represent irrefutable proof of new physics, most likely that the SM gauge group should be extended [23]. If these extensions are associated with flavor symmetry breaking, the gauge interactions will not be flavor-universal which predict the existence of non-universal gauge boson $Z'$ [16]. This kind of new particles can lead to rich phenomenology [for review see [24]]. In this section, we will consider the contributions of the non-universal gauge boson $Z'$ predicted by the TC2 models to the LFV semileptonic $\tau$ decays $\tau^{-} \rightarrow \mu^{-}P$ and $\tau^{-} \rightarrow \mu^{-}V$.

Using the expressions of the effective four fermion couplings $\tau\mu qq$ ($q = u$, $d$, $c$, and $s$) given in Eq.(4), the effective interactions, which are related our calculation, can be written as:

$$L_{\tau\mu\pi}^{eff} = F_{\pi}[A_{L}^{\tau}\bar{L}_{\mu}\gamma^{\mu}\mu_{L} + A_{R}^{\tau}\bar{R}_{\mu}\gamma^{\mu}\mu_{R}]\partial_{\mu}\pi^{0} + h. c. ,$$ (11)
\[ L_{\tau H}^{\text{eff}} = F_\nu [A_L^0 \bar{\tau}_L \gamma^\mu \mu_L + A_R^0 \bar{\tau}_R \gamma^\mu \mu_R] \partial_\mu \eta + h. \ c. , \] (12)

\[ L_{\tau H}^{\text{eff}} = \frac{m_\eta^2}{g_\rho} [A_L^0 \bar{\tau}_L \gamma^\mu \mu_L + A_R^0 \bar{\tau}_R \gamma^\mu \mu_R] \rho_\mu^0 + h. \ c. , \] (13)

\[ L_{\tau H}^{\text{eff}} = \frac{m_\phi^2}{g_\phi^2} [A_L^0 \bar{\tau}_L \gamma^\mu \mu_L + A_R^0 \bar{\tau}_R \gamma^\mu \mu_R] \phi^0 + h. \ c. \] (14)

with

\[ A_L^0 = A_L^\tau = \frac{A}{2}, \quad A_R^0 = A_R^\tau = A; \] (15)

\[ A_L^\eta = \frac{A}{2\sqrt{3}}, \quad A_R^\eta = \frac{A}{\sqrt{3}}; \] (16)

\[ A_L^\phi = \frac{A}{3}, \quad A_R^\phi = \frac{A}{3}. \] (17)

Where \( A = \frac{g^2 K' \tan \theta}{4 M_{Z'}^2}, \frac{1}{g_\rho} \approx 0.2, \) and \( \frac{1}{g_\phi} \approx 0.25 \) [21].

In the context of the TC2 models, the expressions of the corresponding branching ratios induced by the non-universal gauge boson \( Z' \) can be written as:

\[ Br(\tau^- \rightarrow \mu^- \pi^0) = \frac{5g_1^6 K^2}{1024 G_F^2 \pi K_1 M_{Z'}^4 \cos^2 \theta_c} Br(\tau^- \rightarrow \nu_\tau \pi^-), \] (18)

\[ Br(\tau^- \rightarrow \mu^- \eta) = \frac{5g_1^6 K^2}{3072 G_F^2 \pi K_1 M_{Z'}^4 \cos^2 \theta_c (\frac{F_\eta}{F_\pi})^2 (1 - \frac{m_\eta^2}{m_\tau^2})^2 Br(\tau^- \rightarrow \nu_\tau \pi^-), \] (19)

\[ Br(\tau^- \rightarrow \mu^- \rho^0) = \frac{5g_1^6 K^2}{1024 G_F^2 \pi K_1 M_{Z'}^4 \cos^2 \theta_c} Br(\tau^- \rightarrow \nu_\tau \rho^-), \] (20)

\[ Br(\tau^- \rightarrow \mu^- \phi) = \frac{5g_1^6 K^2}{1152 G_F^2 \pi K_1 M_{Z'}^4 \cos^2 \theta_c (\frac{m_\phi}{F_\pi})^2 (1 - \frac{m_\phi^2}{m_\tau^2})^2 (1 + \frac{2m_\phi^2}{m_\tau^2})^2 Br(\tau^- \rightarrow \nu_\tau \pi^-). \] (21)

Where \( Br(\tau^- \rightarrow \nu_\tau \rho^-) \approx 25\%, \) \( m_\phi = 1.019 \text{ GeV}, \) and \( G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2} \) [22]. By replacing \( F_\eta \rightarrow F_\eta' \) and \( m_\eta \rightarrow m_\eta', \) we can easily give the expression of the branching ratio \( Br(\tau^- \rightarrow \mu^- \eta') \) from Eq.(19). However, because of the cancellation between the decay constants for the singlet and octet components in the \( \eta' \) meson, there is \( F_\eta' \approx \frac{1}{3} F_\eta, \) so the value of \( Br(\tau^- \rightarrow \mu^- \eta') \) is approximately smaller than that of \( Br(\tau^- \rightarrow \mu^- \eta) \) by one order of magnitude. Thus, we have not given the expression for \( Br(\tau^- \rightarrow \mu^- \eta') \) in above equations.

It has been shown that vacuum tilting and the constraints from Z-pole physics and \( U(1) \) triviality require \( K_1 \leq 1 \) [10]. The lower limits on the \( Z' \) mass \( M_{Z'} \) can be obtained
via studying its effects on various observables, which have been precisely measured in the present high energy collider experiments [8]. For example, the lower bounds on $M_{Z'}$ can be obtained from dijet and dilepton production in the Tevatron experiments [25] or $B\bar{B}$ mixing [26]. However, these bounds are significantly weaker than those from the precision electroweak data. Ref.[16] has shown that, to fit the precision electroweak data, the $Z'$ mass $M_{Z'}$ must be larger than $1 TeV$. In our numerical estimation, we will assume that the value of $M_{Z'}$ is in the range of $1 TeV \sim 2 TeV$.

![Fig.4: The branching ratios as functions of the mass parameter $M_{Z'}$ for $K' = \frac{1}{\sqrt{2}}$ and three values of the coupling parameter $K_1$.](image)

The branching ratios $Br(\tau^- \rightarrow \mu^- P)$ and $Br(\tau^- \rightarrow \mu^- V)$ contributed by the non-universal gauge boson $Z'$ are plotted as functions of the mass parameter $M_{Z'}$ for $K' = \frac{1}{\sqrt{2}}$.
and three values of the coupling parameter $K_1$ in Fig.4a $\sim$ Fig.4d. From these figures, one can see that the contributions of $Z'$ to the LFV process $\tau^- \rightarrow \mu^- P$ are larger than those of the neutral top-pion $\pi^0_t$ in most of the parameter space of the TC2 models. For $K' = \frac{1}{\sqrt{2}}, M_{Z'} = 1\text{TeV},$ and $K_1 = 0.2,$ the values of the branching ratios $Br(\tau^- \rightarrow \mu^- \pi^0)$ and $Br(\tau^- \rightarrow \mu^- \eta)$ can reach $6.8 \times 10^{-9}$ and $6.3 \times 10^{-9},$ respectively. However, these values are not large enough to be detected in the future high energy experiments [3]. For the LFV processes $\tau^- \rightarrow \mu^- \rho^0$ and $\tau^- \rightarrow \mu^- \phi,$ the values of the branching ratios are larger than those for the LFV processes $\tau^- \rightarrow \mu^- \pi^0$ and $\tau^- \rightarrow \mu^- \eta.$ For $K' = \frac{1}{\sqrt{2}}, 0.1 \leq K_1 \leq 0.8,$ and $1\text{TeV} \leq M_{Z'} \leq 2\text{TeV},$ the branching ratios $Br(\tau^- \rightarrow \mu^- \rho^0)$ and $Br(\tau^- \rightarrow \mu^- \phi)$ are in the ranges of $3.1 \times 10^{-8} \sim 2.4 \times 10^{-10}$ and $5.5 \times 10^{-7} \sim 4.3 \times 10^{-9},$ respectively. We expect that the value of $Br(\tau^- \rightarrow \mu^- \phi)$ might approach the corresponding experimental upper limits [3].

In above figures, we have taken the flavor mixing parameter $K'$ as a fixed constant. In fact, for the TC2 models, the extended gauge groups are broken at the TeV scale, which proposes that $K'$ is an $\mathcal{O}(1)$ free parameter. Its value can be generally constrained by the current experimental upper limits on the LFV processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_l.$ However, from the numerical results of Ref.[18], we can see that the LFV processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_l$ can not give severe constraints on the mixing factor $K'.$ Thus, we expect that $K' = \frac{1}{\sqrt{2}}$ is consistent with theoretically-allowed parameter regions and also with current experimental data.

The non-universal gauge boson $Z'$ can also induce the effective coupling $\tau^- \mu^- f \overline{f}$ via the off-shell photon penguin diagrams, i.e. the effective process $\tau^- \rightarrow \mu^- \gamma^* \rightarrow \mu^- f \overline{f},$ which can contribute to the LFV semileptonic $\tau$ decay processes $\tau^- \rightarrow \mu^- \rho^0$ and $\tau^- \rightarrow \mu^- \phi.$ However, the contributions of the off-shell photon penguin diagrams induced by $Z'$ exchange to the $\tau^- \mu^- f \overline{f}$ coupling are much smaller than those of $Z'$ exchange at tree level[18]. Thus, in this paper, we have neglected the contributions of the off-shell photon penguin diagrams to the LFV processes $\tau^- \rightarrow \mu^- \rho^0$ and $\tau^- \rightarrow \mu^- \phi.$

V. Conclusions and discussions

The evidence for the neutrino masses and flavor mixing, which can be seen as the
first experimental clue of new physics beyond the SM, implies the non-conservation of
the lepton flavor symmetry. Thus, the LFV processes in the charged lepton sector are
expected, which are very sensitive to new physics beyond the SM. Considering the
sensitivity of probing the LFV semileptonic \( \tau \) decays have been enhanced to \( \mathcal{O}(10^{-7}) \), we
calculate the branching ratios for the LFV processes \( \tau^- \rightarrow \mu^- P \) (\( P = \pi^0, \eta, \eta' \)) and
\( \tau^- \rightarrow \mu^- V \) (\( V = \rho^0, \phi \)) in the context of the TC2 models.

A common feature of topcolor scenario is that it predicts the existence of the neutral
top-pion \( \pi_t^0 \) and the non-universal gauge boson \( Z' \), which have the tree-level FC couplings
to ordinary leptons. Thus, these new particles can generate significant contributions to
the LFV processes. In this paper, we have calculated the contributions of \( \pi_t^0 \) and \( Z' \)
predicted by the TC2 models to the LFV processes \( \tau^- \rightarrow l^- P \) and \( \tau^- \rightarrow l^- V \). Our
numerical results show that, in most of the parameter space, the neutral top-pion \( \pi_t^0 \)
can only make the values of the branching ratios \( Br(\tau^- \rightarrow l^- \pi^0) \) and \( Br(\tau^- \rightarrow l^- \eta(\eta')) \)
in the range of \( 1 \times 10^{-11} \sim 1 \times 10^{-16} \), which are still several orders of magnitudes below
the accessible current experimental bounds. For the non-universal gauge boson \( Z' \), its
contributions to the LFV semileptonic \( \tau \) decays are generally larger than those of the
neutral top-pion \( \pi_t^0 \). For example, with reasonable values of the free parameters in the
TC2 models, \( Z' \) exchange can make the value of the branching ratio \( Br(\tau^- \rightarrow \mu^- \phi) \) reach
\( 1 \times 10^{-7} \), which might approach the detectability threshold of near future experiments.
Certainly, our numerical results are strongly depend on the values of the mixing parameter
\( K' \) and the mass parameter \( M_{Z'} \).

Some popular models beyond the SM, such as SUSY, little Higgs models, and ex-
tra dimension models, predict the existence of the extra neutral gauge boson \( Z' \), which
generally has the LFV coupling to leptons and might produce significant contributions
to the LFV semileptonic \( \tau \) decays. One can use these decay processes to measure the
coupling strength of \( Z' \) with leptons and to distinguish the topcolor models from other
new physics models via definition of an angular asymmetry [7]. More studying about the
effects of the extra gauge boson \( Z' \) on the LFV semileptonic \( \tau \) decays is needed and it
will be helpful to discriminate various specific models beyond the SM in the future high
energy experiments.

Acknowledgments

C. X. Yue would like to thank the Abdus Salam International Centre for Theoretical Physics (ICTP) for partial support. This work was supported in part by Program for New Century Excellent Talents in University (NCET-04-0290), the National Natural Science Foundation of China under the Grants No. 10475037 and 10675057.

References

[1] For recent review, M. C. Gonzalez-Garcia, Y. Nir, Rev. Mod. Phys. 75, 345(2003); A. Strumia and F. Vissani, hep-ph/0606054.

[2] A. Pich, Nucl. Phys. Proc. Suppl. 98, 385(2001); M. Davier, A. Hocker, Z.-Q. Zhang, hep-ph/0507078.

[3] Y. Enari et al. [Belle Collaboration], Phys. Lett. B622, 218(2005); Y. Yusa et al. [Belle Collaboration], Phys. Lett. B640, 138(2006).

[4] D. Black, T. Han, H.-J. He, and M. Sher, Phys. Rev. D66, 053002(2002); A. Brignole, A. Rossi, Nucl. Phys. B701, 3(2004).

[5] M. Sher, Phys. Rev. D66, 057301(2002); S. Kanemura, T. Ota, and K. Tsumura, Phys. Rev. D73, 016006(2006); C.-H. Chen and C.-Q. Geng, Phys. Rev. D74, 035010(2006).

[6] A. Llakovac, B. A. Kniehl and A. Pilaftsis, Phys. Rev. D52, 3993(1995); A. Llakovac, Phys. Rev. D54, 5653(1996); A. Atre, V. Barger and T. Han, Phys. Rev. D71, 113014(2005); T. Fukuyama, A. Llakovac and T. Kikuchi, hep-ph/0506295; V. Cirigliano and B. Grinstein, Nucl. Phys. B752, 18(2006); W. J. Li, Y. D. Yang, and X. D. Zhang, Phys. Rev. D73, 073005(2006).
[7] A. Goyal, *hep-ph/0609095*; A. G. Akeroyd, M. Aoki, Y. Okada, *hep-ph/0610344*.

[8] C. T. Hill and E. H. Simmons, *Phys. Rept.* **381**, 235(2003); **390**, 553(E)(2004).

[9] C. T. Hill, *Phys. Lett. B* **345**, 483(1995); K. D. Lane and E. Eichten, *Phys. Lett. B* **352**, 382(1995); K. D. Lane, *Phys. Lett. B* **433**, 96(1998); G. Cvetic, *Rev. Mod. Phys.* **71**, 513(1999).

[10] M. B. Popovic, E. H. Simmons, *Phys. Rev. D* **58**, 095007(1998); G. Burdman and N. J. Evans, *Phys. Rev. D* **59**, 115005(1999).

[11] B. A. Dobrescu, C. T. Hill, *Phys. Rev. Lett.* **81**, 2634(1998); R. S. Chivukula, B. A. Dobrescu, H. Georgi, C. T. Hill, *Phys. Rev. D* **59**, 075003(1999).

[12] H.-J. He, T. Tait, C.-P. Yuan, *Phys. Rev. D* **62**, 011702(2000).

[13] D. Kominis, *Phys. Lett. B* **358**, 312(1995); G. Burdman and D. Kominis, *Phys. Lett. B* **403**, 101(1997).

[14] Chong-Xing Yue, Lan-Jun Liu, and Dong-Qi Yu, *Phys. Rev. D* **68**, 035002(2003).

[15] H.-J. He, C.-P. Yuan, *Phys. Rev. Lett.* **83**, 28(1999); G. Burdman, *Phys. Rev. Lett.* **83**, 2888(1999); H.-J. He, S. Kanemura, C.-P. Yuan, *Phys. Rev. Lett.* **89**, 101803(2002).

[16] R. S. Chivukula and E. H. Simmons, *Phys. Rev. D* **66**, 015006(2002).

[17] G. Buchalla, G. Burdman, C. T. Hill, and D. Kominis, *Phys. Rev. D* **53**, 5185(1996).

[18] Chong-Xing Yue, Yan-Ming Zhang, Lan-Jun Liu, *Phys. Lett. B* **547**, 252(2002).

[19] W. Loinaz and T. Takeuchi, *Phys. Rev. D* **60**, 015005(1999); Chong-Xing Yue, Yu-Ping Kuang, Xue-Lei Wang, and Wei-Bin Li, *Phys. Rev. D* **62**, 055005(2000).

[20] Chong-Xing Yue, Dong-Qi Yu, Lan-Jun Liu, *Phys. Rev. D* **69**, 095003(2004).

[21] T. Feldmann, *Int. J. Mod. Phys. A* **15**, 159(2000).
[22] S. Eidelman et al. [Particle Data Group], *Phys. Lett. B592*, 1(2004).

[23] J. Erler, P. Langacker, *Phys. Lett. B456*, 68(1999); *Phys. Rev. Lett. 84*, 212(2000).

[24] A. Leike, *Phys. Rept. 317*, 143(1999); G. Weiglein et al.[LHC/LC Study Group], *Phys. Rept. 426*, 47(2006).

[25] A. A. Andrianov et al., *Phys. Rev. D58*, 075001(1998); K. R. Lynch et al., *Phys. Rev. D63*, 035006(2001).

[26] E. H. Simmons, *Phys. Lett. B526*, 365(2002).