$\Theta^+$ in a chiral constituent quark model and its interpolating fields.

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Abstract

The recently discovered pentaquark $\Theta^+$ is described within the chiral constituent quark model. Within this picture the flavor-spin interaction between valence quarks inverts the $(1s)^4$ and $(1s)^3(1p)$ levels of the four-quark subsystem and consequently the lowest-lying pentaquark is a positive parity, $I=0$, $J=1/2$ state of the flavor antidecuplet, similar to the soliton model prediction. Contrary to the soliton model, however, the quark picture predicts its spin-orbit partner with $J=3/2$. Different interpolating fields intended for lattice calculations of $\Theta^+$ are constructed, which have a maximal overlap with this baryon if it is indeed a quark excitation in the 5Q system.

I. INTRODUCTION

The recent experimental discovery of the narrow $\Theta^+$ resonance around 1540 MeV with the strangeness +1 and minimal possible quark content $uudd\bar{s}$ [1–4] has sharpened the interest in the low energy QCD spectroscopy and phenomenology. So far its other quantum numbers are unknown, except for the isospin, which is probably $I=0$. This is because this resonance is not seen in the well studied $I=1 K^+p$ channel.

By itself the 5Q component in the baryon wave function is not something which is very surprising. For example, we know from the deep inelastic lepton scattering off nucleon that there is a significant nonstrange antiquark sea component in the nucleon wave function, implying that on the top of the valence $QQQ$ component, there are higher $QQQQ\bar{Q},...$ Fock components. Since we know that spontaneous breaking of chiral symmetry is a key phenomenon to understand the nucleon and other hadrons in the $u, d, s$ sector in the low-energy regime, the antiquarks in the nucleon sea are mostly correlated with quarks to form Goldstone bosons. Consequently the antiquark polarization in the nucleon sea should not be large [5]. This small, but non-zero antiquark polarization can be attributed to the small amplitude that $Q\bar{Q}$ are correlated into vector and higher mesons. The successful description of the low-energy baryon spectroscopy within the chiral constituent quark model [6] also suggests that effects of the higher Fock components in the baryon wave function are

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significant. Indeed, it is a coupling of the valence $QQQ$ component with the $QQQ\pi,...$ results in the effective flavor-spin interaction between the valence quarks, which is attributed to Goldstone boson exchange [6], two-pion-like exchange [7] or vector meson-like exchange [8] between the valence constituent quarks. This interaction is known to shift the excited octet of positive parity (Roper states) ($N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$,...) and decuplet states ($\Delta(1600)$,...) below the lowest excitations of negative parity. This physical picture has received a support from recent lattice calculations [9,10].

Yet, the $\Theta^+$ state is interesting since here the $5Q$ component is a minimal possible Fock component and this state can belong neither to octet nor decuplet baryons. The minimal possible representation that can accomodate $S = 1$ state is antidecuplet. If so we can expect also other antidecuplet members, which, however, can be strongly mixed with the octet states in those cases where the quantum numbers of octet and antidecuplet are similar. The mass and width of the $\Theta^+$ resonance have been strikingly predicted within the soliton picture [11]. Here the main assumption that fixes parameters of the antidecuplet is that the nonstrange member of the antidecuplet is $N(1710)$. However, both $N(1710)$ as well as $\Sigma(1880)$ (which is also considered to be a member of the antidecuplet in [11]) are well described within the chiral constituent quark model as octet states.

Both soliton [13,14] (and quark-soliton [15–17]) as well as chiral constituent quark models [18,6,19] rely crucially on spontaneous chiral symmetry breaking as the most important phenomenon for the baryon physics. Yet, within the chiral constituent quark model the confinement of quarks is also considered to be important for radial and orbital motion of quarks. In the soliton (or quark-soliton) picture the octet, the decuplet and the antidecuplet members represent different rotational excitations of the chiral (pion) mean field, while excited states of positive and negative parity are considered as resonances in the pion-soliton system [20]. The one-particle (quark) motion is not considered at all within this picture. Within the chiral constituent quark model excitations of the nucleon are either spin-isospin excitations of quarks like in delta, or radial and orbital excitations of the quark motion like in $N(1440)$ and in $N(1535)$. That the confining interaction of quarks should be important for their orbital motion follows also from the lattice calculations. Indeed, at large current quark masses excited hadrons can be rigorously described as a system of quarks with orbital motion in a color-electric confining field. Lattice calculations show a very smooth evolution of the $N(1535) − N$ splitting versus current quark masses, see e.g. [21]. This splitting is large in the heavy quark limit and is described as the orbital excitation of the quark motion. This splitting very slowly increases towards the chiral limit, implying that near the chiral limit there is another mechanism, in addition to confinement, that contributes to this splitting. This is quite consistent with the chiral constituent quark model, where it follows that an appreciable part of this splitting is related to the flavor-spin interaction between valence quarks [6,8]. Another evidence in favour of the quark picture is that it provides a remarkably good description of nucleon electromagnetic and weak formfactors at not very large momenta transfer [22].

\[1\] The less consistent prediction for the pentaquark mass is given in ref. [12].
So it is interesting whether the chiral constituent quark picture can accommodate Θ⁺.

It has been realised some time ago [23,24] that the lowest pentaquark within this picture should be of positive parity, in contrast to pentaquarks within the naive model (where the residual interaction of constituent quarks is attributed to perturbative gluon exchange). On the first value it looks counterintuitive, since naively the ground state system is expected to be a collection of 1s quarks (since the intrinsic parity of the antiquark is negative, the ground state pentaquark must have a negative parity within this picture). However, if the dominant part of the N − Δ splitting is due to a flavor-spin interaction, then this interaction inverts some of the levels with positive and negative parity (like N(1440) and N(1535)). The physical reason for such an inversion is rather simple: The more symmetric the flavor-spin wave function of the baryon is, the more attractive contribution arises from the flavor-spin interaction. The Roper state N(1440) and other similar states belong to a completely symmetric 56 representation of SU(6), while N(1535) and other lowest states of negative parity are members of the SU(6) mixed symmetry 70 plet. Consequently the flavor-spin interaction shifts the Roper states strongly down with respect to the negative parity states.

Very similar reason explains why the lowest pentaquark is of positive parity. The orbitally excited pentaquark with L = 1 allows for a completely symmetric flavor-spin wave function of the four-quark subsystem, while such a subsystem can have only a mixed symmetry if all quarks are in the 1s state. Consequently the flavor-spin interaction shifts the orbitally excited four-quark state below the (1s)⁴ state. Under some fine tuning of interaction between the antiquark and four quarks (which is not constrained by the usual baryon spectroscopy) it is always possible to provide the necessary low mass and width of such a pentaquark [25].

It is interesting that both the soliton picture and chiral constituent quark picture predict the same quantum numbers for this antidecuplet state: I = 0, J⁺ = 1/2⁺. Lattice QCD calculations can potentially answer an important question about which physical picture is more relevant. Each picture must imply a very specific interpolator that optimally creates Θ⁺ from the vacuum in the lattice calculations. It is a purpose of this note to construct the most optimal interpolators for Θ⁺ if this resonance to be described as a quark (but not soliton) excitation.

II. THE QUANTUM NUMBERS AND WAVE FUNCTION OF THE LOWEST PENTAQUARK

In this section we consider in some detail the lowest pentaquark state within the chiral constituent quark model. Consider a 4Q subsystem within a pentaquark. The naive quark model predicts that in the ground state of the pentaquark all four quarks must be in the same 1s state of orbital motion and have positive parity. Consequently, keeping in mind the negative parity of the strange antiquark, the ground state pentaquark must have negative parity within the naive model. The orbital wave function of four quarks must be completely symmetric, i.e. is described by the [4]ₒ Young diagram. The color part of these four quarks
has a unique permutational symmetry $[211]_C$ in order to provide a color-singlet wave function of the pentaquark

$$[211]_C \times [11]_C = [222]_C + ...$$  \hspace{1cm} (1)

Hence the combined color-orbital permutational symmetry of four quarks within the naive model is

$$[211]_C \odot [4]_O = [211]_{CO}. \hspace{1cm} (2)$$

In eq. (1) and below $[k_1, k_2, ...]$ (with all $k_i$ being the non-negative integers $k_1 \geq k_2 \geq k_3 ...$) is a notation for the Young diagram with $k_1$ boxes in the first row, $k_2$ boxes - in the second row, etc, $\times$ means outer product of two representations, which is constructed according to Littlewood’s rule, while $\odot$ denotes inner product of two representation of the symmetric group, i.e. product of different wave functions for the same group of particles. The Pauli principle requires that the total color-orbital-flavor-spin wave function of four quarks must be antisymmetric, $[1111]_{COFS}$. This restricts the flavor-spin wave function to be $[31]_{FS}$.

Within the chiral constituent quark model the most attractive contribution from the flavor-spin residual interaction between valence quarks, 

$$-\vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j,$$  \hspace{1cm} (3)

arises if the flavor-spin Young diagram is completely symmetric, $[4]_{FS}$. Such a flavor-spin symmetry can be obtained only if we allow one of the quarks to be in the $1p$ state, i.e. the orbital momentum of four quarks is $L = 1$. Clearly, the kinetic energy of the $(1s)^3(1p)$ configuration is larger than that of $(1s)^4$. However, the attraction from the flavor-spin interaction, which is fixed by $N - \Delta$ splitting, is so strong in the $[4]_{FS}$ case, that it overcomes larger kinetic energy and the four-quark subsystem with the quantum numbers $L = 1, [4]_{FS}$ becomes the ground state of four quarks. In addition, with the given flavor-spin symmetry of a few-quark system, the most attractive contribution from the interaction (3) arises when the flavor permutational symmetry is the most "antisymmetric" among a few possibilities [6]. This uniquely fixes the flavor and spin symmetries of the ground state four-quark subsystem to be $[22]_F$ and $[22]_S$, respectively. Since the required pentaquark must have strangeness +1, then the four-quark subsystem can consist only of $u, d$ quarks and hence $[22]_F$ symmetry uniquely determines isospin of four quarks to be $I = 0$. Hence, the quantum numbers of the four-quark subsystem within the $\Theta^+$ pentaquark are

$$P = -, [211]_C, [31]_O, [1111]_{CO}, [22]_F, [22]_S, [4]_{FS}, L = 1, S = 0, J = 1, I = 0. \hspace{1cm} (4)$$

It is clear from the flavor symmetry of four quarks and their isospin $I = 0$, that the pentaquark must belong to the flavor antidecuplet, because both flavor antidecuplet and octet are contained in the outer product of $[22]_F$ (four quarks) and $[11]_F$ (antiquark), but only the antidecuplet is compatible with the $I = 0, S = +1$ quantum numbers.
III. INTERPOLATING FIELD FOR THE PENTAQUARK

Now our task is to construct such a local interpolating field which would have maximal overlap with the wave function (4). The four-quark interpolator can be constructed as a product of two diquark interpolators.\(^2\) The \(P = -, [31]_O\) orbital wave function of four quarks can be obtained in two different ways. The first way is to construct such a wave function as a system of two scalar (spatially symmetric, \([2]_O\)) diquarks with \(L = 1\) relative motion orbital momentum. The corresponding interpolator then will consist of the product of two isoscalar-scalar bilinears (see below). However, such an interpolator will also couple well to the four-quark subsystem with two strongly clustered isoscalar-scalar diquarks. Hence it will be difficult, if impossible, to distinguish in lattice calculations with such an interpolator between the present picture and the picture suggested in ref. [26]. The second way to obtain \(P = -, [31]_O\) four-quark wave function is to use one diquark which is spatially symmetric, \([2]_O\) (i.e. it has positive parity), and the other diquark which is spatially antisymmetric, \([11]_O\), with negative intrinsic parity. The corresponding interpolator will strongly couple to the wave function (4), but will not couple at all to the system of two strongly clustered scalar diquarks. Hence, the strong signal obtained with such an interpolator would mean that one indeed observes the four-quark subsystem in the state (4). Below we will consider in detail such interpolators.

Since the color-orbital wave function of four quarks is \([1111]_{CO}\), both diquark interpolators must have antisymmetric color-orbital structure, \(d \sim [11]_{CO}\). Hence, one of the diquarks must be color-antisymmetric, \([11]_C\), and the other - color-symmetric, \([2]_C\). Both diquarks must be symmetric in the flavor-spin space, \([2]_{FS}\). This can be provided if each diquark has the same symmetry in flavor and spin spaces. Both diquarks must also have equal isospin in order that a total isospin can be constructed to be 0. Hence there are only two possibilities:

(i)

\[
d_1 \sim |P = -, [2]_C, [11]_O, [11]_{CO}, [11]_F, [11]_S, [2]_{FS}, L = 1, S = 0, J = 1, I = 0 >, \tag{5}
\]

\[
d_2 \sim |P = +, [11]_C, [2]_O, [11]_{CO}, [11]_F, [11]_S, [2]_{FS}, L = 0, S = 0, J = 0, I = 0 > . \tag{6}
\]

(ii)

\[
d_1 \sim |P = -, [2]_C, [11]_O, [11]_{CO}, [2]_F, [2]_S, [2]_{FS}, L = 1, S = 1, J = 0, 1, 2, I = 1 >, \tag{7}
\]

\[
d_2 \sim |P = +, [11]_C, [2]_O, [11]_{CO}, [2]_F, [2]_S, [2]_{FS}, L = 0, S = 1, J = 1, I = 1 > . \tag{8}
\]

Now we will translate the language of orbital, flavor and spin symmetries into the language of covariant bilinears, which is required for lattice calculations. The local interpolator

\(^2\)Here and below under diquark we understand only a subsystem of two quarks without implying a diquark clustering.
for the $|P = -, [11]_O, [11]_F, [11]_S, L = 1, S = 0, J = 1, I = 0\rangle$ diquark must be isoscalar-vector diquark bilinear field

$$\frac{1}{\sqrt{2}} \left[ u^T(x) C \gamma_\mu \gamma_5 d(x) - d^T(x) C \gamma_\mu u(x) \right].$$  \hspace{1cm} (9)

Here and below $C$ is charge conjugation matrix and $T$ denotes transpose of the Dirac spinor. Clearly, for the interpolator one may use only either one of the terms in (9).

The $|P = +, [2]_O, [11]_F, [11]_S, L = 0, S = 0, J = 0, I = 0\rangle$ diquark is to be interpolated by the isoscalar-scalar bilinear

$$\frac{1}{\sqrt{2}} \left[ u^T(x) C \gamma_5 d(x) - d^T(x) C \gamma_5 u(x) \right].$$  \hspace{1cm} (10)

The other possible diquarks, $|P = -, [11]_O, [2]_F, [2]_S, L = 1, S = 1, J = 0, I = 1\rangle$, $|P = -, [11]_O, [2]_F, [2]_S, L = 1, S = 1, J = 1, I = 1\rangle$ and $|P = -, [11]_O, [2]_F, [2]_S, L = 1, S = 1, J = 2, I = 1\rangle$ can be interpolated by the isovector-pseudoscalar

$$\frac{1}{\sqrt{2}} \left[ u^T(x) C \gamma_5 d(x) + d^T(x) C \gamma_5 u(x) \right]; \ u^T(x) C u(x); \ d^T(x) C d(x),$$  \hspace{1cm} (11)

isovector-vector

$$\frac{1}{\sqrt{2}} \left[ u^T(x) C \gamma_\mu \gamma_5 d(x) + d^T(x) C \gamma_\mu \gamma_5 u(x) \right]; \ u^T(x) C \gamma_\mu \gamma_5 u(x); \ d^T(x) C \gamma_\mu \gamma_5 d(x),$$  \hspace{1cm} (12)

and isovector-pseudotensor

$$\frac{1}{\sqrt{2}} \left[ u^T(x) C \sigma_{\mu \nu} d(x) + d^T(x) C \sigma_{\mu \nu} u(x) \right]; \ u^T(x) C \sigma_{\mu \nu} u(x); \ d^T(x) C \sigma_{\mu \nu} d(x)$$  \hspace{1cm} (13)

bilinears, respectively.

Finally, the $|P = +, [2]_O, [2]_F, [2]_S, L = 0, S = 1, J = 1, I = 1\rangle$ diquark must be described via isovector-axialvector bilinear field

$$\frac{1}{\sqrt{2}} \left[ u^T(x) C \gamma_\mu d(x) + d^T(x) C \gamma_\mu u(x) \right]; \ u^T(x) C \gamma_\mu u(x); \ d^T(x) C \gamma_\mu d(x).$$  \hspace{1cm} (14)

Note that each quark field in the bilinears above carries a color index, which is omitted in this section. Clearly all these color indices must be contracted into a color-singlet pentaquark, which will be done in the next section.

**IV. THE COLOR PART OF THE INTERPOLATOR**

The next step is to specify color indices of quarks and to construct a four-quark subsystem with the $[211]_C$ symmetry. This can be done with the help of the Clebsch-Gordan coefficients
of the $SU(3)_C$ group. To specify each representation (wave function) we will use the following chain of subgroups

$$SU(3)_C \supset O(3)_C \supset O(2)_C.$$  

Hence the color wave function of one particle (or of a few particles) is characterised by the permutational symmetry $[f]_C$ (or by the symbol $(pq)$ which is uniquely connected to $[f]_C$), by "color orbital momentum" $L_C$ which specifies representation of $O(3)_C$, and by its projection $m_C$ that determines representation of $O(2)_C$. For example, the one-quark field belongs to the fundamental triplet and is completely specified by $[1]_C$, $L_C = 1, m_C = -1, 0, 1$. In the following it will be denoted as $|1m_C\rangle$. The antiquark color-antitriplet field is specified by $[11]_C$, $L_C = 1, m_C = -1, 0, 1$. The Clebsch-Gordan coefficient for the $SU(3)_C$ is given as a product of its scalar factor (which is independent of index $m_C$) and the Clebsch-Gordan coefficient for $O(3)_C$:

$$\langle [f]_C, L_C, M_C|[f']_C, L'_C, M'_C; [f'']_C, L''_C, M''_C \rangle = \langle [f]_C, L_C|[f']_C, L'_C; [f'']_C, L''_C \rangle C^{L, M, M'C, L'M'_C, L'_C, M''_C, M'_C, M''_C}_C.$$  

Then the color-antisymmetric diquark is constructed as antisymmetrized product of two quarks

$$d_{CA} \equiv |[11]_C, L_C = 1, M_C\rangle = \sum_{M'_{C}, M''_{C}} C_{11M'_{C}, 1M''_{C}}^{1M_C} |1M'_C\rangle |1M''_C\rangle,$$

while the two different color-symmetric diquarks are

$$d'_{CS} \equiv |[2]_C, L_C = 0, M_C = 0\rangle = \sum_{M'_{C}, M''_{C}} C_{1M'_{C}, 1M''_{C}}^{00} |1M'_C\rangle |1M''_C\rangle,$$

$$d''_{CS} \equiv |[2]_C, L_C = 2, M_C\rangle = \sum_{M'_{C}, M''_{C}} C_{2M'_{C}, 1M''_{C}}^{2M_C} |1M'_C\rangle |1M''_C\rangle,$$

where $M'_C$ and $M''_C$ are color indices of the first and second quarks within the given diquark.

Then we can construct the required $|[211]_C, L_C = 1, M_C\rangle$ tetraquark out of two diquarks:

$$|[211]_C, L_C = 1, M_C\rangle = \sqrt{1/6} C_{11M'_{C}, 1M''_{C}}^{1M_C} |[2]_C, L'_C = 0, M'_C = 0\rangle |[11]_C, L''_C = 1, M''_C = M_C\rangle + \sqrt{5/6} \sum_{M'_{C}, M''_{C}} C_{2M'_{C}, 1M''_{C}}^{1M_C} |[2]_C, L'_C = 2, M'_C\rangle |[11]_C, L_C = 1, M''_C = M_C\rangle.$$  

(15)

Finally, we have to combine the color wave function of the tetraquark with the antiquark into a color-singlet pentaquark:

$$|[222]_C, L^{5Q}_C = 0, M^{5Q}_C = 0\rangle = \sum_{M_C} C_{1M_C, 1-M_C}^{00} |[211]_C, L_C = 1, M_C\rangle |[11]_C, L_C = 1, -M_C\rangle.$$  

A final step is to combine two diquarks according to the possibilities (i) and (ii) in (5) - (8) and strange antiquark into a few possible interpolators for a pentaquark. As an example, we present below one of these interpolators
\[ I_1 = \sum_{M_C, M'_C, M''_C, m'_C, m''_C} C_{1M_C}^{00} C_{1M'_C}^{00} C_{1M''_C}^{1M_C} C_{1M'_C}^{1M_C} C_{1M''_C}^{1M_C} \]
\[ + \sqrt{5/6} C_{2M_C}^{1M_C} C_{2M'_C}^{2M_C} \left[ u_{M_C}^T C \gamma_5 d_{M'_C} \right] \left[ u_{M'_C}^T C \gamma_5 d_{M''_C} \right] \bar{s}_{-M_C}. \]  

(16)

Other possible interpolator can be obtained, e.g. by substituting of the vector diquark in the first square brackets in eq. (16) by the pseudoscalar one, \[ \left[ u_{M_C}^T C \gamma_5 d_{M'_C} \right] \] and of the scalar diquark in the second brackets - by the axial vector one, \[ \left[ d_{M_C}^T C \gamma_5 d_{M''_C} \right]. \]

V. DISCUSSION

We have shown that if the discovered \( \Theta^+ \) state is to be described within the chiral constituent quark picture, then the lowest lying pentaquark must have positive parity, in contrast with the negative parity of the naive quark model. Also within our picture the lowest pentaquark will have exactly the same other quantum numbers as within the soliton picture: \( I = 0, J = 1/2 \). Contrary to the soliton picture, the quark picture predicts also its spin-orbit partner with \( P = +, J = 3/2, I = 0, S = 1 \), since the coupling of \( L = 1 \) tetraquark with the strange antiquark produces both \( J = 1/2 \) and \( J = 3/2 \) states. Keeping in mind that typically the spin-orbit splittings in baryon spectroscopy are of the order of 100 MeV and less, it would be interesting to perform an experimental search of the \( J = 3/2, S = +1 \) pentaquark in the region 1400 - 1700 MeV.

We have constructed a few interpolating fields intended for a lattice search of \( \Theta^+ \), that would have a maximal overlap with \( \Theta^+ \) if this state is to be described within the chiral constituent quark picture. The optimal strategy would be to use simultaneously a few interpolators and to calculate a cross-correlation matrix. We anticipate however a difficulty in these lattice calculations. Usually the signal from the given state is first detected at rather large quark masses and then traced towards chiral limit. It is a-priori not clear, however, whether the analog of \( \Theta^+ \) exists in the heavy quark region. If not, the signal from \( \Theta^+ \) can appear only below some critical current quark mass.

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