A new parameter adaptive control of four-dimensional hyperchaotic system

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Abstract. By adding a state feedback controller to a three-dimensional chaotic system, a new four-dimensional hyperchaotic autonomous system is constructed. The nonlinear dynamic behavior of the system is deeply investigated, including power spectrum, Poincare cross section, Lyapunov exponential spectrum and bifurcation graph. The analysis shows that the new four-dimensional hyperchaotic system has complex chaotic motions with different parameters, and the related analog chaotic circuits are designed. Finally, a parameter adaptive controller is designed to achieve global stability control of chaotic systems. The effectiveness of the proposed method is verified by numerical simulation.

1. Introduction

Chaos is a unique form of motion in non-linear dynamic systems. It exists widely in nature, such as biology, physics, chemistry, geological science and other research fields. Since Lorenz discovered the first chaotic system in 1963, researchers have been searching for new chaotic systems in various ways, and have made great progress [1]. In 1999, Chen Guanrong discovered a new chaotic attractor Chen system which is not topologically equivalent to Lorenz system in anti-control of chaotic system [2]. In 2002, Lü Jinhu and others further discovered Lü system [3].

The chaotic system has at least one positive Lyapunov exponent, and its dynamic behavior is complex. It can be used in secure communication, image encryption and lossless detection [4-5]. Compared with chaotic system, hyperchaotic system has two or more positive Lyapunov exponents. Its dynamic behavior is more complex. It can improve the confidentiality of communication, the degree of image encryption and the accuracy of lossless detection [6-8].

The research of chaos control and synchronization is one of the hot topics in non-linear science. It has wide application prospects in military communications, aviation, biomedicine and other fields [9-11]. In this paper, a new hyperchaotic system is proposed for the traditional Lü system, and the nonlinear dynamic behavior of the system is deeply studied, including power spectrum, Poincare cross section, Lyapunov exponential spectrum and bifurcation diagram. Finally, by constructing a new Lyapunov function, a parameter adaptive controller is designed to achieve global stability control of chaotic systems. The effectiveness of the proposed method is verified by numerical simulation.
2. New hyperchaotic system model

The state equation of three-dimensional Lü system is

\[
\begin{align*}
\dot{x}_1 &= a(x_2 - x_1) \\
\dot{x}_2 &= c x_2 - x_1 x_3 \\
\dot{x}_3 &= -b x_1 + x_1 x_2
\end{align*}
\]

Suppose \( a = 35, \ b = 3, \ c = 20 \), Lü system has three unstable equilibrium points \( O(0,0,0) \), \( P_+(2\sqrt{15}, 2\sqrt{15}, 20) \), \( P_-(2\sqrt{15}, -2\sqrt{15}, 20) \). The phase diagram of the three-dimensional chaotic Lü system is shown in Fig. 1.

![Fig. 1 the phase diagram of the three-dimensional chaotic Lü system](image)

Two necessary conditions must be satisfied in order to produce a hyperchaotic system: For autonomous systems, at least four-dimensional system; there are at least two positive Lyapunov exponents and the sum of all Lyapunov exponents is less than zero. Accordingly, a nonlinear controller \( x_4 \) is applied to the first equation of the three-dimensional Lü system and the third equation \( x_1 x_2 \) is rewritten to \( x_1^2 \) to produce a new chaotic system.
Choosing $a = 35$, $b = 3$, $c = 20$, $d = 10$, then calculate the Lyapunov exponents of a new four-dimensional chaotic system $LE_1 = 0.2248$, $LE_2 = 0.5808$, $LE_3 = -1.4920$ and $LE_4 = -17.3136$. The system has two positive Lyapunov exponents and two negative Lyapunov exponents, and the sum of Lyapunov exponents is less than 0, which indicates that the system is in hyperchaotic state. The planar phase diagrams of the new four-dimensional hyperchaotic system on each plane are shown in Fig. 2.

Fig. 2. The planar phase diagrams of the new four-dimensional hyperchaotic system on each plane

### 3. Basic characteristics of new hyperchaotic systems

#### 3.1. Dissipation

For a new four-dimensional hyperchaotic system, there exists Eq(3).

$$\nabla V = \frac{\partial \dot{x}_1}{x_1} + \frac{\partial \dot{x}_2}{x_2} + \frac{\partial \dot{x}_3}{x_3} + \frac{\partial \dot{x}_4}{x_4} = -a + c - b = -18 < 0$$

According to the Eq(3), the system (2) is a dissipative system and converges at an exponential rate $e^{-18t}$ to zero, the existence of attractors is proved.
3.2. Equilibrium Point and Stability
Let the right side of the equation of the new four-dimensional hyperchaotic system (2) be 0, then we can get that there is only one equilibrium point $(0,0,0)$ for the system. The Jacobian matrix of the analytic system at the equilibrium point is

$$J = \begin{bmatrix}
-a & a & 0 & 1 \\
0 & c & 0 & 0 \\
0 & 0 & -b & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

(4)

Let $|\lambda I - J| = 0$, we get four eigenvalues at the equilibrium point $\lambda_1 = -35.2834$, $\lambda_2 = -3$, $\lambda_3 = 0.2834$, $\lambda_4 = 20$. It can be seen that the equilibrium point of the system is an unstable saddle point.

3.3. Power Spectrum and Poincare Cross Section
The power spectrum of periodic signal is discrete, and non-periodic signal (chaotic signal) is continuous. Fig. 3 shows the power spectrum of hyperchaotic system with variable $x_3$. It can be seen that the chaotic attractor is non-periodic, and there are relatively unstable limit cycles in the chaotic region. Besides obvious noise, the power spectrum of chaotic system also has the peak power spectrum line when it reaches the periodic orbit.

Fig. 3. The power spectrum of hyperchaotic system with variable $x_3$

In order to facilitate the analysis of the nonlinear dynamic behavior of the hyperchaotic system, the Poincare map of the hyperchaotic system on the cross section $x_3 = 0$ is selected as the cross section, as shown in Fig.4. The cross section shows a dense point with fractal structure, which indicates that the system is in chaotic state.

Fig.4. Poincare map with Section $x_3 = 0$
3.4. Lyapunov exponential spectrum and bifurcation graph
Lyapunov exponential spectra and bifurcation diagrams are used to characterize the non-linear dynamic characteristics of chaotic systems. The influence of key parameters on the state of motion of chaotic systems can be obtained by analysis.

When the parameters $a = 35, b = 3, c = 20$ are fixed, change the value of the parameter $d$. The Lyapunov exponential spectrum and bifurcation diagram of the system are shown in Fig. 5 and Fig. 6.

![Fig. 5. Lyapunov exponential spectrum of the system varying with parameters $d$](image1)

![Fig. 6. Bifurcation graph of the system varying with parameters $d$](image2)

4. Circuit Implementation of New Hyperchaotic System
The most direct way of physical realization of chaotic and hyperchaotic systems is to rely on analog electronic circuits. Before designing the circuit, the differential equation of hyperchaotic system (2) must be transformed appropriately. The purpose of this transformation is to make the state variables of the chaotic system change in the operating voltage range of integrated circuits through linear proportional compression, and to simplify the circuit as much as possible. Based on this analysis, we design an analog electronic circuit using linear resistor, linear capacitor, operational amplifier (LM741), analog multiplier (AD633), as shown in Fig 7.
The state equation of Fig.7(a)
\[
\frac{dx_1}{dt} = \frac{R_1}{R_1 C_1} (x_1 - x_1 + x_1 - x_1)
\]

(5)

The state equation of Fig.7(b)
\[
\frac{dx_2}{dt} = \frac{R_2}{R_2 C_2} (x_2 - x_2 + x_2 - x_2)
\]

(6)

The state equation of Fig.7(c)
\[
\frac{dx_3}{dt} = \frac{R_3}{R_3 C_3} (-x_3 + x_3 + x_3 - x_3)
\]

(7)

The state equation of Fig.7(d)
\[
\frac{dx_4}{dt} = \frac{R_4}{R_4 R_5 R_6} x_4
\]

(8)

The new chaotic circuit can be obtained by choosing the resistance value of the parameters \( R_1 \sim R_{20} \) corresponding to the equation of system (2).

5. Adaptive Control of New Hyperchaotic Systems

Assuming that the parameters of the system are perturbed slightly and deviate from the predetermined target value \( a = 35 \), in order to return the parameters in the system to the reference values in the reference model, and to transform the disturbed system from the unstable state to the desired stable state, a parameter adaptive controller is designed.

\[
\dot{a} = -(a - 35)(x_2 - x_2)^2
\]

(9)

Selecting positive definite Lyapunov function \( V = \frac{1}{2}(a - 35)^2 \), then we find

\[
\dot{V} = (a - 35) \dot{a} = -(a - 35)^2 (x_2 - x_2)^3 \leq 0
\]

(10)

So, \( V \) is a semi-negative definite function. According to Lyapunov stability theory, the adaptive controller (9) can guarantee the global stability of the system and realize the stability control of the disturbed system.

To verify the above analysis results, the fourth-order Runge-Kutta algorithm is used to simulate the controlled system (10). When the parameters \( a \) of the controlled system are disturbed slightly and deviate from the pre-set target value, the controller will be applied for 6.5 seconds and then return to the original orbit to achieve stability in a local scope, the simulation results are shown in Fig 8.
6. Conclusion
In this paper, a new four-dimensional hyperchaotic autonomous system is constructed based on a three-dimensional chaotic system. The nonlinear dynamic behavior of the system is investigated in depth, including power spectrum, Poincare cross section, Lyapunov exponential spectrum and bifurcation graph. The analysis shows that the new hyperchaotic system has complex chaotic motions with different parameters, and relevant analog electronic circuits are designed. Finally, a parameter adaptive controller is designed to make the chaotic system return to an asymptotically stable state quickly after the parameters are disturbed. The global stability of the controlled system is proved by using Lyapunov direct method. The numerical simulation results show that the controller can still guarantee the stability of the system when the parameters are greatly disturbed. The validity of the parameter adaptive controller is verified.

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