Temperature-Induced Disorder-Free Localization

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Disorder-free localization is a paradigm of strong ergodicity breaking that has been shown to occur in global quenches of lattice gauge theories when the system is initialized in a superposition over an extensive number of gauge sectors. Here, we show that preparing the system in a thermal Gibbs ensemble without any coherences between different gauge sectors also gives rise to disorder-free localization, with temperature acting as a disorder strength. We demonstrate our findings by calculating the quench dynamics of the imbalance of thermal ensembles in both $U(1)$ and $Z_2$ lattice gauge theories through exact diagonalization, showing greater localization with increasing ensemble temperature. Furthermore, we show how adding terms linear in local pseudogenerators can enhance temperature-induced disorder-free localization due to the dynamical emergence of an enriched local symmetry. Our work expands the realm of disorder-free localization into finite-temperature physics, and shows counterintuitively that certain quantum nonergodic phenomena can become more prominent at high temperature. We discuss the accessibility of our conclusions in current quantum simulation and computing platforms.

FIG. 1. (Color online). A preparation Hamiltonian is used to prepare a system in a thermal ensemble, which is subsequently quenched to a LGT Hamiltonian. At zero temperature, the ensemble is a pure domain-wall state in a homogeneous gauge sector $g_{0}$, which is gapped by $V_{0}$ from other sectors, and the imbalance decays to zero at long times. At sufficiently high temperatures, the ensemble comprises nonnegligible weights over an extensive number of gauge superselection sectors [18, 19], which act as an effective disorder background leading to localization [22].

A particularly intriguing phenomenon in this vein is that of disorder-free localization (DFL) in lattice gauge theories (LGTs) [18–28]. Despite the absence of quenched disorder in the system, DFL can still arise in LGTs in the wake of quenching simple product states that comprise a superposition over an extensive number of gauge superselection sectors [18, 19], which as act an effective disorder background leading to localization [22].

A major incentive for building quantum simulation and computing platforms is the promise to simulate quantum many-body models and explore their rich physics [29–32]. Although most experimental works have been focused on ground-state properties and dynamics of pure quantum states, simulating finite-temperature physics is crucial for a multitude of applications such as, e.g., high-$T_{c}$ superconductivity, thermal phase transitions, and finite-temperature (de)confinement. Recently, there has been several experimentally relevant proposals for probing finite-temperature physics on quantum simulators and computers [33–37]. This motivates studying phenomena such as DFL at finite temperature, and probing whether localization can persist away from the paradigm of a pure initial state.

In this work, we show that quenching finite-temperature thermal ensembles in LGTs leads to non-ergodic dynamics even when the initial ensemble has no...
coherences between the different gauge sectors; cf. Fig. 1. Rather counterintuitively, this temperature-induced DFL (T-DFL), which is ultimately a quantum interference effect, becomes more prominent with increasing ensemble temperature. We showcase our findings in two paradigmatic LGTs that have been the focus of many recent experimental works, and also highlight how the use of local pseudogenerators \[38\] enhances T-DFL due to the dynamical emergence of an enriched local symmetry \[39\].

Models. — The principal property of a LGT is its gauge symmetry \([40, 41]\), which is introduced by the local generators \(\hat{G}_j\) and encoded in the commutation relations \(\left[ \hat{H}_0, \hat{G}_j \right] = 0, \forall j\), where \(j\) denotes a site in a system of size \(L\). Matter fields live on these sites, while gauge and electric fields reside on the links in between. The local generator \(\hat{G}_j\) imposes an intrinsic local relationship between the matter occupation on site \(j\) and the configurations of the electric fields on its adjacent links. Gauge-invariant states \(|\phi\rangle\) are simultaneous eigenstates of all generators: \(\hat{G}_j |\phi\rangle = g_j |\phi\rangle\). The eigenvalues \(g_j\) are so-called background charges, and a set for the entire system of \(L\) sites defines a unique gauge superselection sector \(g = (g_1, g_2, \ldots, g_L)\). We will add an appropriate superscript to \(\hat{H}_0\) and \(\hat{G}_j\) when referring to a specific LGT, but leave them without such a superscript when the discussion is general. Periodic boundary conditions are employed throughout this work.

The \(U(1)\) quantum link model (QLM) is a formulation of lattice quantum electrodynamics where the infinite-dimensional gauge and electric fields are represented by spin-\(S\) operators \([42–46]\). Its Hamiltonian is given by

\[
\hat{\mathcal{H}}_0^{U(1)} = \frac{L}{2} \sum_{j=1}^{2L-1} J \left( \sigma_j^+ \hat{s}_{j,j+1}^+ + \sigma_j^- \hat{s}_{j,j+1}^- + H.c. \right) + \frac{\mu}{2} \sigma_j^z + \frac{\kappa^2}{2} \left( \hat{s}_{j,j+1}^z \right)^2 ,
\]

(1)

where the Pauli operator \(\sigma_j^z\) represents the matter field on site \(j\), and the spin-\(S\) operators \(\hat{s}_{j,j+1}^\pm\) and \(\hat{s}_{j,j+1}^z\) represent the electric and gauge fields, respectively, on the link between sites \(j\) and \(j+1\). The number of sites is denoted by \(L\), the fermionic mass by \(\mu\), the gauge coupling by \(\kappa\), and \(J=1\) sets the overall energy scale. The local generator of the \(U(1)\) gauge symmetry is given by

\[
\hat{G}_j^{U(1)} = (-1)^j \left( \hat{n}_j + \hat{s}_{j,j+1}^+ + \hat{s}_{j,j+1}^- \right),
\]

(2)

where \(\hat{n}_j = (\sigma_j^z + 1)/2\). The gauge invariance of Eq. (1) is encapsulated in the commutation relations \(\left[ \hat{H}_0^{U(1)}, \hat{G}_j^{U(1)} \right] = 0, \forall j\).

The \(\mathbb{Z}_2\) LGT is described by the Hamiltonian \([47–50]\)

\[
\hat{H}_0^{\mathbb{Z}_2} = \sum_{j=1}^{L} \left[ J (\hat{a}_{j,j+1}^+ \hat{a}_{j+1,j} + H.c.) - \hbar \phi_{j,j+1}^z \right],
\]

(3)

where \(\hat{a}_{j,j+1}\) are hard-core bosonic annihilation and creation operators with \(\hat{n}_j = \hat{a}_{j,j}^\dagger \hat{a}_{j,j}\) representing matter occupation on site \(j\), and the Pauli operator \(\phi_{j,j+1}\) represents the electric (gauge) field on the link between sites \(j\) and \(j+1\). The generator of the \(\mathbb{Z}_2\) symmetry is

\[
\hat{G}_j^{\mathbb{Z}_2} = (-1)^{\phi_{j,j+1}} \hat{P}_{j,j+1},
\]

(4)

where the gauge invariance of Hamiltonian (3) is manifest in the commutation relations \(\left[ \hat{H}_0^{\mathbb{Z}_2}, \hat{G}_j^{\mathbb{Z}_2} \right] = 0, \forall j\).

Both models (1) and (3) have recently been experimentally realized in synthetic quantum matter setups \([51–57]\).

Thermal ensemble and quench dynamics. — Let us consider the preparation Hamiltonian

\[
\hat{H}_{\text{prep}} = \frac{L}{2} \sum_{j=1}^{L} \left[ V_0 (\hat{a}_{j,j} + g_j) \right]^2 - \alpha_j \hat{n}_j,
\]

(5)

where \(\alpha_j = +1\) for \(j \leq L/2\) and \(\alpha_j = -1\) for \(j > L/2\), and \(V_0 > 0\) renders the homogeneous gauge sector \(gS = (g_1, \ldots, g_L)\) as the ground-state manifold, where \(g_j^g = 0, \forall j\), in the case of the \(U(1)\) QLM, and \(g_j^g = -1, \forall j\), in the case of the \(\mathbb{Z}_2\) LGT. We note that this choice of lowest-energy homogeneous gauge sector is not unique. As such, the ground state of \(\hat{H}_{\text{prep}}\) is a domain wall in the matter fields, with the left half of the chain fully occupied, while the right half is empty, residing in the lowest-energy homogeneous gauge sector.

We now prepare the system in the canonical ensemble \((k_B = 1)\)

\[
\hat{\rho}_0 = \frac{e^{-\beta \hat{H}_{\text{prep}}}}{Z} = \sum_g p_g \hat{P}_g,
\]

(6)

where \(T = 1/\beta\) is the ensemble temperature, \(\hat{P}_g\) is the projector onto the gauge sector \(g\), \(\hat{\rho}_0 = \hat{P}_g e^{-\beta \hat{H}_{\text{prep}}} \hat{P}_g / Z_g\) and \(Z_g = \text{Tr} \{ \hat{P}_g e^{-\beta \hat{H}_{\text{prep}}} \} \) are the initial density operator and the partition function within sector \(g\), \(Z = \sum_g Z_g\) is the total partition function, \(p_g = Z_g / Z\) is the sector weight, and we have utilized \(\hat{H}_{\text{prep}} = \sum_g \hat{P}_g \hat{H}_{\text{prep}} \hat{P}_g\) since \(\left[ \hat{H}_{\text{prep}}, \hat{G}_j \right] = 0, \forall j\).

We now quench \(\hat{\rho}_0\) with \(\hat{H}_0\) at \(t=0\), which gives rise to the time-evolved density operator of the system \((h=1)\)

\[
\hat{\rho}(t) = e^{-i\hat{H}_0 t} \hat{\rho}_0 e^{i\hat{H}_0 t} = \sum_g p_g \hat{P}_g \hat{\rho}(g(t)) \hat{P}_g,
\]

(7)

where \(\hat{\rho}(g(t)) = \hat{P}_g e^{-i\hat{H}_0 t} \hat{\rho}_0 e^{i\hat{H}_0 t} \hat{P}_g\), as due to the gauge symmetry of \(\hat{H}_0\) we are able to write \(\hat{H}_0 = \sum_g \hat{P}_g \hat{H}_0 \hat{P}_g\).

Equation (7) allows for the calculation of the dynamics in each gauge sector separately, thereby allowing us to reach larger system sizes for the desired evolution times in our numerical simulations.

T-DFL is intuitively expected by examining Eq. (7), and noting that the dynamics of an observable \(A\) will be
imbalance (8) normalized by its value at $t=0$, which is conserved throughout the quench dynamics.

The gauge violation gives rise to a smaller gap $V$ at a fixed ratio $T/\beta$. Consequently, the gap $V$ of the ground-state sector also affects the degree of localization.

The gap $V_0$ of the ground-state sector also affects the degree of localization. The gap $V_0$ in Eq. (5) isolates the homogeneous sector by making it a ground-state manifold. At larger $V_0$, a higher-temperature ensemble is required to achieve localization, since the density of states is negligible at low energies $E_0-E_g \ll V_0$ above the ground state; see Fig. 1. Equivalently, the smaller $V_0$ is at a given temperature, the more likely is the system to localize, as shown in Fig. 2(b). Indeed, in the case of $V_0=0$, DFL can already occur at $T \approx 0$. In that case, there is no unique lowest-lying gauge sector, and the ground-state manifold of $H_{\text{prep}}$ will consist of $2^L$ degenerate domain-wall states residing in an extensive number of gauge sectors. In the inset of Fig. 2(b), we show the intimate connection between the gap $V_0$ and the ensemble temperature $T$, where the DFL behavior is identical at different values of these parameters at a fixed value of $V_0/T$. Even though we have chosen $S=1/2$ and $\mu=0.7J$ for the numerical simulations in Fig. 2, we have checked that our conclusions remain the same for different values of link spin and fermionic mass [58].

$1$ Of course, if $T$ is infinite, the imbalance will be zero at all times: $\mathcal{I}(0)=\mathcal{I}(t)=0$, and normalized imbalance is not defined. Hence, by $T \rightarrow \infty$ we denote asymptotically large temperatures and do not mean infinite temperature.
ensemble. However, one can enhance the symmetry of effective disorder over their background charges in a thermal of gauge sectors, which then allows for a greater effect initial state [39, 59]. The origin of this difference is for the $Z$ Tiance starting in the canonical thermal ensemble (6) with temperature $T=1/\beta$ and quenching with the $Z_2$ LGT Hamiltonian (3). Results are computed in exact diagonalization for $L=8$ sites. (a) The normalized imbalance settles at long times into a plateau whose value increases with temperature. (b) Adding a term linear in the local pseudogenerators (10) to the quench Hamiltonian leads to an enhancement of disorder-free localization for a fixed temperature.

Next, we look at $T$-DFL in the $Z_2$ LGT (3) in Fig. 3. The picture is qualitatively identical to that of the U(1) QLM in Fig. 2(a), where we see a direct connection between the ensemble temperature $T$ and the steady-state value of the normalized-imbalance plateau in Fig. 3(a). The larger $T$ is, the more prominent DFL is, with a larger value for long-time-normalized-imbalance plateau. We also checked that a thermal ensemble does not faithfully describe this plateau, predicting instead a zero imbalance in the long-time limit regardless of the value of $T$. As in the case of the U(1) QLM, the steady-state value of the normalized imbalance shows a direct monotonic relation with the gauge violation, see inset of Fig. 3(a), indicating that $T$-DFL occurs only when $\varepsilon > 0$ in the initial thermal ensemble.

It is worth noting that the $T$-DFL is markedly weaker for the $Z_2$ LGT compared to the U(1) QLM. This is in agreement with results on DFL starting in a superposition initial state [39, 59]. The origin of this difference is that the $Z_2$ LGT can locally admit only two charges $\pm 1$ due to the underlying $Z_2$ gauge symmetry, thus leading to a restricted form of discrete binary disorder. In the case of the U(1) QLM with a spin-1/2 representation, a local constraint admits four different charges $g_j = (-1)^j q_j$, with $q_j \in \{-1, 0, 1, 2\}$. This naturally leads to a wider variety of gauge sectors, which then allows for a greater effective disorder over their background charges in a thermal ensemble. However, one can enhance the symmetry of the $Z_2$ LGT by introducing the term $V \hat{H}_W=V \sum_j \hat{W}_j$, where the local pseudogenerator (LPG) is defined as [38]

$$
\hat{W}_j = \hat{\tau}_{j-1} \hat{\tau}_{j+1} + 2 g_j h_j.
$$

The LPG is identical to the local full generator $G_{Z_2}^j$ in the homogeneous sector $g_{ss}$:

$$
G_{Z_2}^j |\phi\rangle = g_{ss}^j |\phi\rangle \iff \hat{W}_j |\phi\rangle = g_{ss}^j |\phi\rangle.
$$

It was initially developed as an experimentally feasible scheme for stabilizing $Z_2$ LGTs [38, 60], but has recently been shown to lead to an effective Hamiltonian that enhances DFL when starting in a superposition initial state [39, 61]. This effective Hamiltonian hosts an enriched local symmetry associated with $\hat{W}_j$ that contains the $Z_2$ gauge symmetry of Eq. (3). This means that now there are more local-symmetry superselection sectors over which the effective disorder can be generated, and this leads to more prominent DFL in the case of a superposition initial state [39]. It would be interesting to see if this enhancement also occurs in the case of a system prepared in a thermal ensemble. For this purpose, we consider a canonical ensemble with an asymptotically large temperature $T \to \infty (\beta \to 0)$ and quench it with $H_{Z_2}^{2} + V \hat{H}_W$. The ensuing imbalance dynamics is shown in Fig. 3(b). As the LPG-term strength $V$ is increased, the DFL is drastically enhanced. It is important to note here that the addition of the LPG term does not alter the value of $\varepsilon$ because $[G_{Z_2}^j, \hat{W}_j]=0$. The enhancement of DFL occurs strictly from the local symmetry that emerges due to the LPG term. This behavior is also present at finite temperatures, where if DFL is present at a given temperature $T$ for $V=0$, it will get enhanced for $V>0$. In case the system initialized at temperature $T$ is delocalized for $V=0$, it will remain delocalized for $V>0$.

**Discussion and outlook.**—We have demonstrated temperature-induced disorder-free localization, where a LGT initialized in a thermal canonical ensemble at sufficiently high temperature will exhibit DFL in its quench dynamics up to all accessible evolution times. No polarizing field is employed to create any kind of coherent superposition over the gauge sectors, and the preparation Hamiltonian is chosen such that at zero temperature the system will reside only in a homogeneous gauge sector where no DFL is possible.

We have illustrated our findings in two paradigmatic systems: the U(1) quantum link formulation of the Schwinger model and the $Z_2$ lattice gauge theory. The qualitative picture in both is the same: the larger the temperature of the initial ensemble, the more localized will the dynamics of the system be at late times. Additionally, we have shown how the gap of the preparation Hamiltonian can also influence DFL, with smaller gaps leading to more prominent DFL at a given temperature. A given ratio of the gap and ensemble temperature will always lead to the same degree of localization. In the
case of the $Z_2$ LGT, linear weighted sums in the local pseudogenerator can be employed to dynamically induce an emergent enriched local symmetry that leads to enhanced T-DFL. Furthermore, the choice of the preparation Hamiltonian is not unique to Eq. (5). In fact, for the $Z_2$ LGT one can replace the gap term with $V_0 \sum_j W_j$, which offers greater experimental feasibility and leads to T-DFL, although the temperature-dependence may become different. It is also worth noting that T-DFL can be stabilized against gauge-breaking errors using linear gauge protection [39, 58, 59].

Intriguingly, even though DFL is a quantum interference phenomenon, here we show it gets more pronounced at high temperatures. Hence, one might expect that in contrast to common intuition that low-temperature physics probes quantum effects, there might be other phenomena of quantum nonergodicity becoming prominent at high temperature.

Our findings can be tested in ultracold-atom platforms [52–55, 62] and on digital quantum computers [63], which recently have become a viable framework for simulating various gauge-theory phenomena [51, 56, 57]. One way to generate a thermal ensemble on a quantum computer is via a purification called a thermofield double state [33–35]. The latter can be constructed on two identical subsystems of the circuit in an enlarged Hilbert space. The desired Gibbs ensemble on one subsystem is then realized by tracing out the second subsystem.

Our numerical results indicate that T-DFL in our 1+1D setting does not include a finite-temperature delocalization-localization transition but only a crossover. It would be interesting to explore such a possible transition in 2+1D, e.g., in Rydberg tweezer arrays [60].

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NONTHERMAL DFL STEADY STATE

Here we show that the steady state reached in the imbalance at finite temperature cannot be described by a thermal ensemble, even though the system is initially in a thermal ensemble at $t = 0$. Let us assume by way of contradiction that the late-time steady state can be described by the thermal Gibbs ensemble

$$\hat{\rho}_{ss}^{CE} = \frac{e^{-\beta_{ss}\hat{H}}}{\text{Tr} \{ e^{-\beta_{ss}\hat{H}} \}},$$

(S1)

where $T_{ss} = 1/\beta_{ss}$ is its temperature, and $\hat{H}$ is the quench Hamiltonian, which in our case is either $\hat{H}_0$ or $\hat{H}^{Z_2}_0 + V \hat{R}_W$. The global quenches considered in our work are unitary, and thus the quench energy must be conserved at all times:

$$E_{\text{quench}} = \text{Tr} \{ \hat{H} \hat{\rho}_0 \} = \text{Tr} \{ \hat{H} \hat{\rho}_{ss}^{CE} \}.$$  

(S2)

Equation (S2) is now an implicit equation with the only unknown being $\beta_{ss}$, which can be solved using, e.g., Newton’s method. Hence, $\hat{\rho}_{ss}^{CE}$ can be numerically determined, and one can check whether $\text{Tr} \{ \hat{A} \hat{\rho}_{ss}^{CE} \} \approx \text{Tr} \{ \hat{A} \hat{\rho}(t \to \infty) \}$, for any local observable $\hat{A}$ [S1]. In case this condition is satisfied, then the system will have thermalized [S2, S3], and otherwise it means the dynamics is nonergodic [S4]. We numerically find that $\hat{\rho}_{ss}^{CE}$ only predicts a zero imbalance for all the quenches considered in our work, which is in contradiction to the exact dynamics for sufficiently large temperatures when $T$-DFL is present. As such, we conclude that the steady state arising at long times is nonthermal when there is $T$-DFL.

When it comes to the steady state, one can alternatively look at the prediction from a microcanonical ensemble. The latter is constructed from the eigenstates $|\epsilon_m \rangle$ of the quench Hamiltonian that lie within the energy shell $S = [E_{\text{quench}} - \delta \epsilon, E_{\text{quench}} + \delta \epsilon]$, where $E_{\text{quench}} = \text{Tr} \{ \hat{H} \hat{\rho}_0 \}$ is the quench energy. The microcanonical ensemble then takes the form [S1]

$$\hat{\rho}_{ss}^{ME} = N_{E_{\text{quench}}, \delta \epsilon}^{-1} \sum_{m; \epsilon_m \in S} |\epsilon_m \rangle \langle \epsilon_m|,$$

(S3)

where $N_{E_{\text{quench}}, \delta \epsilon}$ is the number of quench-Hamiltonian eigenstates with eigenenergies within the shell $S$. In our numerical calculations, we have set $\delta E = 0.1J$, though we have checked that our qualitative conclusions do not strongly depend on this choice. In our numerical simulations, $\hat{\rho}_{ss}^{ME}$ also predicts a zero imbalance at long times for all quenches considered, even when $T$-DFL is prominent. This further shows that the finite plateau is described by a nonthermal steady state.

TEMPERATURE-INDUCED DFL IN THE SPIN-3/2 U(1) QLM WITH $S > 1/2$

In the main text, our analysis of the U(1) QLM has focused on a spin-1/2 representation of the local gauge fields. However, $T$-DFL is not restricted to that case, and higher-$S$ representations will also exhibit it.

To demonstrate this, we show the quench dynamics of the normalized imbalance for the spin-1 and spin-3/2 U(1) QLM in Fig. S1(a,b), respectively, for $L = 4$ matter sites, $\mu = 0.7J$ and $\kappa = 0.1\sqrt{J}$. The qualitative picture is the same as the spin-1/2 case, where $T$-DFL arises at sufficiently high ensemble temperatures, and the degree of localization displays a monotonic relationship with the ensemble temperature. We have also checked that the finite plateau arising in case of $T$-DFL is not described by a thermal ensemble, whereas the latter always predicts a zero imbalance at late times. As in the case of the spin-1/2 U(1) QLM, we have checked that the specific values of $\mu$ and $\kappa$ do not alter the qualitative picture of $T$-DFL.

STABILITY OF TEMPERATURE-INDUCED DFL

It has been shown that DFL arising from a superposition initial state is unstable in the presence of gauge-breaking perturbations [S5]. However, there has recently been experimentally feasible proposals based on linear gauge protection
FIG. S1. (Color online). Temperature-induced DFL in the (a) spin-1 and (b) spin-3/2 U(1) QLM. The qualitative behavior is the same as in the case of the spin-1/2 U(1) QLM, with DFL getting enhanced with initial-ensemble temperature $T$.

[S6, S7] that stabilize and even enhance DFL [S8–S10]. To put things on a formal footing, let us consider the gauge-breaking terms

$$\lambda \hat{H}_U^{(1)} = \lambda \sum_{j=1}^{L} \left[ \hat{s}_j^+ \hat{s}_{j+1}^- + \hat{s}_j^- \hat{s}_{j+1}^+ + \frac{\hat{s}_{j+1}^+ \hat{s}_{j+1}^-}{2\sqrt{S(S+1)}} \right],$$  

(S4a)

$$\lambda \hat{H}_{Z_2} = \lambda \sum_{j=1}^{L} \left[ \hat{a}_j^+ \hat{a}_{j+1}^- + \hat{a}_j^- \hat{a}_{j+1}^+ + \hat{\tau}_{j,j+1} \right],$$  

(S4b)

at strength $\lambda$, relevant to synthetic quantum matter implementations of U(1) QLMs and $Z_2$ LGTs with both dynamical matter and gauge fields [S11–S14]. These error terms involve tunneling of matter without a concomitant change in the electric field to preserve Gauss’s law, or vice versa. To protect against them, we add the terms [S6, S7, S10]

$$V \hat{H}_G = V \sum_{j=1}^{L} j G_j^{(1)},$$  

(S5a)

$$V \hat{H}_W = V \sum_{j=1}^{L} j \hat{W}_j,$$  

(S5b)

where the full local generator of the U(1) gauge symmetry is given in Eq. (2) and the local pseudogenerator $\hat{W}_j$ is defined in Eq. (10) for the case of the $Z_2$ LGT.

FIG. S2. (Color online). Linear gauge protection against gauge-breaking errors in (a) the spin-1/2 U(1) QLM at $T = 10J$ using Eq. (S5a), and (b) the $Z_2$ LGT at $T = 20J$ using Eq. (S5b). In both cases we see that without protection, the gauge-breaking errors (S4) completely destroy T-DFL. At sufficiently large $V$, however, T-DFL is restored, and even enhanced in the case of the $Z_2$ LGT due to an emergent enriched local symmetry associated with the LPGs $\hat{W}_j$, defined in Eq. (10).

We now quench our initial thermal ensemble with $\hat{H} = \hat{H}_0 + \lambda \hat{H}_1 + V \hat{H}_{\text{pro}}$, where $\hat{H}_{\text{pro}} = \hat{H}_G$ for the case of the U(1) QLM and $\hat{H}_{\text{pro}} = \hat{H}_W$ for the case of the $Z_2$ LGT. Due to the gauge-breaking term $\lambda \hat{H}_1$, the time-evolved density operator $\hat{\rho}(t) = e^{-i\hat{H}t} \hat{\rho}_0 e^{i\hat{H}t}$ cannot be written in the form (7), as now $\hat{H}$ includes gauge-breaking terms $\lambda \hat{H}_1$ that couple different gauge sectors. This renders the numerical simulations more tasking, and as such we restrict our results to $L = 4$ matter sites. As shown in Fig. S2(a) for the case of the U(1) QLM, a finite value of $\lambda$ will completely destroy T-DFL in the absence of protection ($V = 0$), where the imbalance will go to zero in agreement
with a thermal-ensemble prediction (see first section). However, upon turning on linear gauge protection ($V > 0$), we see a stabilization of the imbalance, where the temperature-induced DFL is qualitatively restored, with greater quantitative agreement with the ideal case the greater $V$ is.

Similarly in the case of the $Z_2$ LGT, shown in Fig. S2(b), unprotected gauge-breaking errors destroy DFL. Upon adding the LPG term (S5b), however, the DFL is restored and also enhanced. This stems from the dynamical emergence of an enriched local symmetry due to the LPGs $\hat{W}_j$. This enriched local symmetry generates a greater number of local-symmetry sectors, leading to a greater effective disorder in the thermal average of the imbalance, and hence enhanced localization [S9, S10].