Radially anisotropic ring-core optical fiber: towards vector-vortex guided transmission using the full modal space

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Abstract: The radially anisotropic ring-core fiber with cylindrical birefringence is theoretically and numerically investigated as a novel platform for the transmission of vector-vortex beams with unique modal properties. First, we elucidate the parametric conditions where such fiber enables modal substitution in which either the donut-shaped azimuthal TE01 or radial TM01 mode replaces the normal Gaussian-like HE11 mode as the fundamental mode of the waveguide. We also demonstrate that it is possible to significantly engineer the waveguiding properties of the fiber via the addition of small radial birefringence (∼10⁻⁴) so as to make the (hitherto non-degenerate) TE0m and TM0m modes fully degenerate. The latter property is used to create stable vortex modes of high purity (>99%) with the newly degenerate modal pair – a feat not possible with standard few-mode fibers—all without affecting the co-propagating hybrid HE/EH modes that remain available as an independent basis set to produce vortex beams of similarly high purity. These new insights are relevant to the topical fields of mode-division multiplexing (MDM), structured light, fiber modelling and fabrication. With respect to MDM applications, the newly available vortex modes created with the degenerate TE/TM basis set can now be concurrently used with the more common vortex modes created via the HE/EH modal basis set.

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1. Introduction

Radially anisotropic fibers are a special class of optical fibers, first studied by Black et al., where the fiber displays cylindrical birefringence [1]. Their theoretical analysis distinctively showed that a radially anisotropic optical fiber could be designed such as to have the fundamental mode to be either the azimuthal TE01 or radial TM01 mode instead of the usual linearly polarized HE11 mode [1]. Such fiber exhibits polar anisotropy, aka cylindrical material birefringence, where either the fiber core or cladding demonstrates different material refractive indices along the radial and azimuthal directions in polar coordinates [1–4]. This approach also opens the possibility to enhance one’s control over the selective excitation and modal separation of the radial and azimuthal (TE0m and TM0m) families of guided modes in such fibers [1,3]. Furthermore, ring-core fibers have shown to not only be a favorable structure for high-order mode guiding in radially anisotropic fibers [1,5], but was also more recently actively studied for the stable transmission of vector as well as vortex beams (that carry orbital angular momentum (OAM)) owing to their ability to lift the near modal degeneracy between adjacent eigenmodes [6]. Hence, radially anisotropic fibers with a ring-core structure offer a promising platform for studying structured light beams, which lends to further investigation.

Potential approaches towards the experimental demonstration of such anisotropic fibers include by means of liquid crystal structures [7], and more recently, metamaterial based optical fibers [8–10]. These metamaterial based anisotropic optical fibers, have spurred new opportunities for
research with possible applications in space-division multiplexing where, as we will show here, the fiber’s radial anisotropy could be tailored so as to enable a richer variety of co-propagating vector-vortex modes compared to conventional isotropic fibers. Aside from niche applications in radial/azimuthal polarized laser delivery and endoscopic imaging, the TE\textsubscript{om} and TM\textsubscript{om} vector modes of optical fibers have mostly been neglected in other areas such as optical communications where they are traditionally seen as sources of noise (via modal crosstalk) and loss rather than potential information channels. One important reason is that the TE\textsubscript{om} and TM\textsubscript{om} modes in classical isotropic fibers are not degenerate and thus cannot form a stable OAM mode basis set [11].

In this work, we propose and investigate the cylindrically anisotropic ring-core fiber design for the stable transmission of vector-vortex beams by means of a combination of either degenerate radially (TM\textsubscript{om}) and azimuthally (TE\textsubscript{om}) polarized vector modes, or the hybrid (HE / EH) modes. A full-vector finite element method (FEM) based fiber model in cylindrical coordinates was developed using the COMSOL Multiphysics 5.5 package. Further, our numerical model employed a user-controlled free triangular mesh that was optimized in the core region so as to respect a maximum node size of $\frac{\lambda}{10}$ (where $\lambda$ denotes the wavelength) and was delimited in space using perfectly matched layer outer boundary conditions. We also evaluated the mode purity and intermodal refractive index separation of the resulting OAM modes. Finally, we elucidate the waveguiding regimes where the proposed fiber can guide either the radially or azimuthally polarized vector mode as the fundamental mode.

2. Fiber design parameters

In circular core optical fibers, it is generally known that vortex beams can be formed by the coherent superposition of $\pm \frac{\pi}{2}$ phase-shifted degenerate high-order hybrid modes:

$$CV_{\pm \ell, m} = HE_{\ell+1,m} \mp iHE_{\ell-1,m}$$

$$CV_{\pm \ell, m} = EH_{\ell-1,m} \mp iEH_{\ell+1,m}$$

Here, CV stands for “circular vortex” and $\ell$ represents the azimuthal number (or topological charge) of the corresponding OAM mode [12]. Vortex beams can additionally be created via a similar combination of the $\pm \frac{\pi}{2}$ phase-shifted azimuthally/radially polarized TE\textsubscript{om} and TM\textsubscript{om} modes [11,13] via:

$$UV_{\pm \ell, m} = TM_{0,m} \mp iTE_{0,m}$$

Because the TE\textsubscript{om} and TM\textsubscript{om} modes are normally non-degenerate in common isotropic circular optical fibers, the resulting OAM mode are known to be unstable vortices (UV) that quickly decays upon propagation in optical fiber [11]. Mode-division multiplexing (MDM) in cylindrical few-mode fibers (FMF) can be supported through either the well-known linearly polarized (LP) mode basis set or the stable vortex mode basis set described above. For a given normalized V-parameter value, let’s say $V = 3$ without loss of generality, the FMF can in theory support a greater number of independent LP modes ($LP_{10}^{x,y}$, $LP_{11a}^{x,y}$, $LP_{11b}^{x,y}$; total of 6 polarization mode channels) when compared to the vortex basis set: $CV_{a01}^{x,y}$ and $CV_{b11}^{x,y}$ (total of 4 polarization mode channels). However in practice, the near degeneracies in modal and polarization distribution tend to couple the high-order even “a” and odd “b” LP modes that result in significant crosstalk. The latter issue can be resolved using special polarization-maintaining FMFs that enable to lift these degeneracies so as to fully exploit the LP mode basis set [14,15].

In this work, we show that it is possible to resolve the phase mismatch between the TE\textsubscript{om} and TM\textsubscript{om} modes so as to make them degenerate by adding a small level of radial birefringence (i.e. radial anisotropy) in the high-index ring shaped core, all without affecting the remaining HE/EH hybrid modes. Such degenerate TE/TM modes then become available to complete the full vortex mode basis set [Eq. (2)] that can be utilized in MDM applications or otherwise. Figure 1 shows a
schematic diagram for the refractive index profile of the proposed radially anisotropic ring-core fiber, with $a$ and $b$ denoting the inner and outer core radii respectively, with their ratio defined as $\rho = \frac{a}{b}$. The birefringence in the fiber is defined along the two principal axes in polar coordinates ($n_r, n_\phi$) while the axial refractive index was here taken as $n_z = n_r$ as was similarly done in [5]. We note that the effect of the axial component ($n_z$) is negligible since the magnitude of the transverse guided fields ($E_r, E_\phi$) remained dominant compared to the longitudinal field ($E_z$) in our FEM simulations. In this study we assumed the inner and outer cladding material to be isotropic fused silica with a refractive index of $n_{\phi cl} = n_{r cl} = n_{cl} = 1.444$ at 1550 nm wavelength (used throughout this study) while the effect of the radial and azimuthal components of the core refractive index ($n_{rco}, n_{\phi co}$) was numerically investigated for values ranging from 1.450 to 1.500 with steps of 0.005. The latter fiber parameters yield a maximum core-cladding refractive index contrast of $\Delta n = 0.056$ that is close to practical attainable values reported in doped-core fused silica glass [16].

![Fig. 1](image)

**Fig. 1.** (a) Schematic diagram of the anisotropic ring-core fiber showing the radial and azimuthal axes along which the refractive index profile is defined ($n_r$ & $n_\phi$), and for two general cases of core anisotropy (b) $n_\phi > n_r$ and (c) $n_\phi < n_r$.

The fiber design parameters considered for this study have a core inner radius $a = 1$ um and outer radius $b = 4$ um giving an aspect ratio of $\rho = 0.25$. This aspect ratio was selected so that it is thin enough to have significant modal separation ($> 10^{-5}$) between adjacent vortex modes; but not too thin so as to become impractical to fabricate or to couple light. All while exhibiting modal cutoff properties that remain practically unchanged from a standard step-index fiber as was demonstrated for the isotropic ring-core fiber [17]. In order to make our analysis more general we also define two relative parameters, $\Delta_r = \frac{n_{r co}^2 - n_{cl}^2}{2n_{cl}^2}$ and $\Delta_\phi = \frac{n_{\phi co}^2 - n_{cl}^2}{2n_{cl}^2}$, that respectively denote the core’s radial refractive index contrast and azimuthal refractive index contrast.

3. Results and discussion

We investigated the modal properties of the first two mode families ($LP_{01}$ and $LP_{11}$) in the proposed radially anisotropic ring-core fibers for a range of absolute core refractive indices, $1.45 \leq n_{r co} \leq 1.50$ and $1.45 \leq n_{\phi co} \leq 1.50$, which translate into the following range of normalized refractive index contrasts: $0.42 \times 10^{-2} \leq \Delta_r \leq 3.9 \times 10^{-2}$ and $0.42 \times 10^{-2} \leq \Delta_\phi \leq 3.9 \times 10^{-2}$. 
The simulation results in Fig. 2(a) [and Fig. 2(c)] show that the value of effective refractive index \((n_{\text{eff}})\) for the fundamental HE\(_{11}\) mode [and the HE\(_{21}\) mode] increases with both the \(\Delta_r\) and \(\Delta_\phi\) refractive index contrasts. We note that the black dash line in all subfigures highlights the reference case when the fiber is isotropic: \(\Delta_r = \Delta_\phi\). Figure 2(b) shows the difference in the modal indices of the even and odd HE\(_{11}\) modes, which indicates that the HE\(_{11}\) mode retains a pure polarization degeneracy despite the significant radial birefringence (up to 3.9\%) introduced in the fiber. A similar behavior is shown in Fig. 2(d) for the even and odd HE\(_{21}\) modes. The reason for this sustained degeneracy is because the hybrid HE/EH modes have a completely mixed polarization such that their even and odd states have evenly distributed \(E\)-field components in both the radial and azimuthal directions. The latter behavior may be desirable in several applications - including for optical communications - where a perfectly polarization-degenerate fundamental HE\(_{11}\) mode is useful. Importantly, it also suggests that vortex modes created via the coherent superposition of the HE/EH hybrid modes [in Eq. (1)] still remain possible and unaffected by the addition of radial birefringence in the fiber.

![Fig. 2.](image-url)

**Fig. 2.** (a) Effective index \((n_{\text{eff}})\) of the HE\(_{11}\) and (c) HE\(_{21}\) modes, while (b) effective index separation \((\Delta n_{\text{eff}})\) between HE\(_{11}\) and HE\(_{21}\) and (d) HE\(_{21}\) and HE\(_{21}\) modes.

Figure 3(a)-(b) show the effective refractive index of the TM\(_{01}\) and TE\(_{01}\) modes as a function of the core relative index contrast in the radial (\(\Delta_r\)) and azimuthal (\(\Delta_\phi\)) directions. As expected, we see in Fig. 3(a) that the TM\(_{01}\) mode with radial polarization \((E_r,0)\) depends only on changes in \(\Delta_r\), while in Fig. 3(b) we observe the reverse behavior: the azimuthally polarized TE\(_{01}\) mode \((0, E_\phi)\) solely depends on \(\Delta_\phi\). Here we note that simulations are performed for a range of relative core index contrasts of \(\Delta_r\) or \(\Delta_\phi\) \(\geq 0.53 \times 10^{-2}\), because the cut-off limit for the TE\(_{01}\)/ TM\(_{01}\) mode was found to be \(\Delta_{r/\phi} \approx 0.53 \times 10^{-2}\). We also confirmed that the latter cut-off values are validated with a theory of ring-core fibers based on the exact solution of Maxwell equations and using the dimensionless \(V\)-parameter [17]. For a radially anisotropic ring-core
fiber this parameter must be defined separately for TM modes as $V_{TM} = \left(\frac{2\pi b}{\lambda}\right)\left(\sqrt{n_{co}^2 - n_{cl}^2}\right)$, and for TE modes: $V_{TE} = \left(\frac{2\pi b}{\lambda}\right)\left(\sqrt{n_{co}^2 - n_{cl}^2}\right)$. Through this formalism we numerically found, as expected, the cut-off condition for both the TE$_{01}$ and TM$_{01}$ modes as $V_{cut}(r, \phi) = 2.412$, a value that is in very close quantitative agreement with a similar isotropic ring-core fiber of ratio $\rho = 0.25$ [17]. Owing to their mixed E-field polarization distribution, the definition of the V-parameter for the HE/EH hybrid modes, $V_{HE/EH} = \left(\frac{2\pi b}{\lambda}\right)\left(\sqrt{\bar{n}_{co}^2 - \bar{n}_{cl}^2}\right)$, depends on the transverse areal average of the radial and azimuthal indices, $\bar{n}_i = \frac{\int_i (n_i + n_\phi) r d\phi dr}{2 \int_i r d\phi dr}$, inside the respective core and cladding regions ($i = \{co, cl\}$). One can then readily determine the corresponding cut-off conditions of the high-order HE/EH hybrid modes using the exact relations derived for the isotropic ring-core fiber [17].

![Fig. 3.](image)

Fig. 3. (a) Effective index ($n_{eff}$) of the TM$_{01}$ and (b) TE$_{01}$ mode (c) Effective index separation ($\Delta n_{eff}$) between TM$_{01}$ and TE$_{01}$ (d) Anisotropic fiber condition for degeneracy ($|\Delta n_{eff}(TM_{01} - TE_{01})| < 10^{-10}$) of radially and azimuthally polarized mode propagation for four different fibers (with $\rho = 0.25, 0.50, 0.75$ and $b = 4$ um as well as $\rho = 0.55$ and $b = 2.7$ um) based on FEM calculation.

One of the prime objectives of the current study is to find a fiber design that enables the TE$_{01}$ and TM$_{01}$ modes to become degenerate so as to form stable OAM modes via Eq. (2). Figure 3(c) displays the effective refractive index separation ($\Delta n_{eff}$) between TE$_{01}$ and TM$_{01}$ mode within the fiber design space. The latter plot allows to identify fiber parameters for which the condition of TE/TM modal degeneracy occurs: $|\Delta n_{eff}| < 10^{-10}$. On inspection of Fig. 3(c) one realizes that the condition for TE/TM modal degeneracy ($\Delta n_{eff} \rightarrow 0$) is found in proximity to the normal isotropic fiber case. A zoom-in on the region of interest [Fig. 3(d)] indicates that TE/TM modal degeneracy can effectively be obtained when $\Delta_r > \Delta_\phi$ and that only a small amount of radial...
birefringence ($\Delta_r \sim 10^{-4}$) is required, and increases quasi-linearly with the core-cladding refractive index contrast of the fiber. Here we also investigated the effect the core’s aspect ratio $\rho$ on the radial birefringence for $\rho = 0.25, 0.50, 0.75$ and $b = 4 \mu$m as shown in Fig. 3(d). Figure 3(d) also includes the fiber design ($\rho = 0.55, b = 2.7 \mu$m) for which we found the highest modal separation (to be discussed in Fig. 5). The latter result indicates that the proposed radially anisotropic ring-core fiber could in principle be realized using weakly birefringent metamaterials arranged along a cylindrical symmetry [8–10].

Another interesting feature of the radially anisotropic ring-core fiber relates to a parametric space where either the TE$_{01}$ or TM$_{01}$ mode becomes the fundamental mode of the fiber, thus replacing the conventional HE$_{11}$ mode in standard optical fibers. The latter feature could benefit the fiber delivery of cylindrical vector beams and their applications, including: super-resolution imaging modalities [18–20], laser material processing [21,22] and mode-division multiplexing [23,24].

The azimuthally polarized TE$_{01}$ mode becomes the lowest-order mode within the region bordered by the white line in Fig. 4(a) which delineates the criterion for fundamental TE$_{01}$ mode: $\Delta n_{eff} \left(\text{TE}_{01} - \text{HE}_e^{c11}\right) \geq 0$. Similarly, Fig. 4(b) identifies the region where the radially polarized TM$_{01}$ mode becomes fundamental ($\Delta n_{eff} \left(\text{TM}_{01} - \text{HE}_e^{c11}\right) \geq 0$). We note that the ability to tune
the phase-matching conditions between the generic fundamental HE\textsubscript{11} mode of the fiber and the radial/azimuthal (TM\textsubscript{0n}/TE\textsubscript{01}) modes could be exploited in optical sensing applications by means of modal interferometric measurements.

As described in Eq. (1,2), the proposed fiber can be designed for the stable transmission of vortex beams formed by the combination of either hybrid HE/EH cylindrical vector modes or the radially (TM\textsubscript{0m}) and azimuthally (TE\textsubscript{0m}) polarized modes. In Fig. 5 we examine the particular case where both the CV\textsubscript{11} and UV\textsubscript{11} vortex modes are supported by a properly designed radially anisotropic ring-core fiber. We subsequently study the guided stability of CV and UV modes in fiber, which largely depends on the intermodal separation between co-propagating mode, as low intermodal separation leads to high energy coupling between different OAM modes resulting in lower mode stability and purity. Henceforth, to study the stability of CV\textsubscript{11} (HE\textsubscript{e21} \pm i HE\textsubscript{o21}) and UV\textsubscript{11} (TM\textsubscript{01} \pm i TE\textsubscript{01}) modes in fiber, we calculated the minimum absolute refractive index separation between adjacent eigenmodes, namely, the TE\textsubscript{01} and HE\textsubscript{21} \textsuperscript{r} pair (\textit{\Delta}n\textsubscript{eff}(TM\textsubscript{01}−HE\textsubscript{21})), and the TM\textsubscript{01} and HE\textsubscript{01} \textsuperscript{o} pair (\textit{\Delta}n\textsubscript{eff}(TM\textsubscript{01}−HE\textsubscript{01})) in Fig. 5(a). Looking at Fig. 5(a) one realizes that the parametric region with sufficient minimum absolute effective index separation (\textit{\min} \textit{\mid} \textit{\Delta}n\textsubscript{eff}(UV\textsubscript{11}−CV\textsubscript{11}) \textit{\mid} ≥ 5 \times 10^{-5}) appears in principle quite broad. Figure 5(b) allows to further refine this investigation by showing that the UV\textsubscript{11} mode created by the superposition of TE\textsubscript{01} and TM\textsubscript{01} modes can reach levels of minimum intermodal separation \textit{\mid} \textit{\Delta}n\textsubscript{eff}(UV\textsubscript{11}−CV\textsubscript{11}) \textit{\mid} ≥ 5 \times 10^{-5} that are close to what has been previously reported in isotropic ring-core fibers [25]. Moreover, our simulations indicate that for fiber parameters \( \rho = 0.55 \) and \( b = 2.7 \) um one can theoretically

Fig. 6. Simulated phase distribution and intensity profile of (a) UV\textsuperscript{−11} and CV\textsuperscript{−11} modes for \( \rho = 0.25 \) (b) 0.50 and (c) 0.75, while fiber core radius \( b = 4 \) um and (d) for \( \rho = 0.55 \) and \( b = 2.7 \) um.
achieve large intermodal separation ($\Delta n_{\text{eff}} \sim 10^{-4}$) that is generally desirable for the long-distance and low-crosstalk fiber transmission of optical vortex modes. We expect the intermodal separation between adjacent eigenmodes to become even wider ($>10^{-4}$) for larger OAM charge weights ($\ell>1$) as reported in similar isotropic ring-core fiber designs [25]. We further note that the previous systematic approach could be extended to find the appropriate design parameters of the radially anisotropic ring-core fiber that will support higher-order ($\ell \geq 2$) vortex modes enabled by both CV and UV modes.

Figure 6(b)-(c) presents the phase distribution and intensity profiles of OAM $^{\pm}_{11}$ modes formed by the coherent superposition of hybrid HE modes ($\text{C}_{11}^{-} = \text{HE}_{21}^e + i \text{HE}_{21}^o$) as well as the radially and azimuthally polarized vector modes ($\text{UV}_{11}^{+} = \text{TE}_{01} + i \text{TM}_{01}$) for fiber parameters: $\rho=0.25, 0.50, 0.75, b=4 \mu\text{m}$, $\Delta_r = 0.0360$ and $\Delta_\phi = 0.0359$. In addition, Fig. 6(d) plots the intensity and phase distributions for the fiber design ($\rho=0.55, b=2.7 \mu\text{m}$) with which we achieved the highest intermodal separation in our simulations. One can observe in Fig. 6 that the calculated OAM mode purities qualitatively appear very good.

The corresponding exact OAM purities of the CV $^{\pm}_{11}$ and UV $^{+}_{11}$ modes were calculated and the results for the exemplar fiber design $\rho=0.25$ and $b=4 \mu\text{m}$ are shown in Fig. 7(a). As expected, the modal purities for the dominant topological charges ($\ell = \pm 1$) are greater than 99.9%, with minor weights distributed to neighboring charge numbers [Fig. 7(a)]. Mode purity was here evaluated by calculating the normalized power weight ($P_{\ell}$) of each OAM charge number (i.e. the integer order of azimuthal harmonic) represented by $\ell$, and performing the projection of the transverse electric field distribution $u(\rho, \theta)$ on the “spiral harmonics” $\exp(-i \ell \theta)$. The power $C_{\ell}$ carried by each $\ell$-th order topological charge number was then obtained [13,26]:

$$C_{\ell} = \frac{1}{2\pi} \int_0^\infty \left| \int_{0}^{2\pi} u(\rho, \theta) \cdot \exp(-i \ell \theta) \right|^2 \rho d\rho$$

(3)

The normalized power weight for each spiral harmonic was then determined simply:

$$P_{\ell} = \frac{C_{\ell}}{\int_{-\infty}^{\infty} C_{\ell}}$$

(4)

We extended the investigation of OAM mode purity in Fig. 7(b) to the high-order CV $^{\pm}_{2,1}$ modes and for the multiple fiber designs that support both stable CV and UV modes, previously

![Fig. 7. (a) OAM charge weights of generated OAM beams (CV $^{-}_{1,1}$ and UV $^{+}_{1,1}$) in fiber with $\rho = 0.25, b = 4 \mu\text{m}$ and $\Delta_\theta = 0.036, \Delta_\phi = 0.0359$ (for design point with highest $\Delta_\phi$ in Fig. 3(d)) (b) Vortex mode purity of (UV $^{+}_{1,1}$, CV $^{-}_{1,1}$, CV $^{-}_{2,1}$, and CV $^{+}_{2,1}$) for all fiber designs (red curve in Fig. 3(d)).]
identified and studied in Fig. 3(d) and Fig. 5(b). Our results [Fig. 7(b)] indicate that high modal purity (>99%) is similarly achieved in these other cases of interest for the radially anisotropic ring-core fiber. We nonetheless observe a very slow decline in the purities of the \( CV_{-1,1} \), \( CV_{-2,1} \) and \( CV_{+2,1} \) modes with higher refractive index contrast \( \Delta_\phi \) values. In contrast, the modal purity of the \( UV_{+11} \) vortex mode created by the degenerate radial and azimuthal eigenmodes remained fairly constant, irrespective of changes in the core’s refractive index contrast.

4. Conclusion

We propose the radially anisotropic ring-core fiber as a new class of optical fiber tailored for the efficient guided transmission of vector-vortex beams. This new fiber is characterized by a core that exhibits cylindrical birefringence along the principal polar coordinates (in contrast to along the Cartesian coordinates for the usual linear birefringence). In particular, by means of theoretical analysis and FEM based numerical simulations, we elucidate the waveguiding conditions for which the fiber supports either the radially (TM\(_{01}\)) or the azimuthally (TE\(_{01}\)) polarized modes so the lowest-order fundamental mode. Also probably of higher practical interest, we show that the insertion of relatively small radial birefringence (\( \Delta_\gamma \sim 10^{-4} \)) in the fiber core allows the hitherto non-degenerate TE\(_{0m}\) and TM\(_{0m}\) modes to become degenerate so as to enable the creation of stable optical vortex beams of high modal purity (>99%) through their coherent superposition. This unique feat can be achieved without affecting the remaining hybrid HE/EH modes of mixed polarization. These results are of particular relevance to OAM based mode-division multiplexing since the newly available TE/TM based vortex modes can be added on top of the commonly studied OAM channels created by the HE/EH modal basis set. The work is relevant to the areas of space-division multiplexing, transmission of structured light, fiber modelling and fabrication.

Appendix

Appendices include data for effective index values of \( CV_{+2,1} \), \( CV_{+2,1}^+ \) modes: Fig. 8(a) shows the \( n_{\text{eff}} \) values of EH\(_{11}^e\) mode. Results display that the value of \( n_{\text{eff}} \) increases with increase in either \( \Delta_\gamma \) or \( \Delta_\phi \) value, and which region shows the cut-off for EH\(_{11}^e\) mode. Figure 8(b) indicates \( \Delta n_{\text{eff}} \) between EH\(_{11}^e\) and EH\(_{11}^o\) modes. The maximum intermodal separation \( \Delta n_{\text{eff}} < 9 \times 10^{-11} \), demonstrate degeneracy for EH\(_{11}^e\) and EH\(_{11}^o\) modes irrespective of anisotropic, ideal for stability of OAM (\( \ell = 2 \)) beam in fiber. Similarly, \( n_{\text{eff}} \) values for HE\(_{31}^e\) can be found in Fig. 8(c) and subsequently \( \Delta n_{\text{eff}} \) is shown in Fig. 8(d) for HE\(_{31}^c\) and HE\(_{31}^o\) modes, where \( \Delta n_{\text{eff}} < 1.5 \times 10^{-10} \) for all fiber design parameters. Finally, Fig. 8(e & f) shows the \( n_{\text{eff}} \) and \( \Delta n_{\text{eff}} \) for HE\(_{12}^e\) mode and subsequently Fig. 8(g & h) displays the \( n_{\text{eff}} \) and \( \Delta n_{\text{eff}} \) values of HE\(_{41}^e\) modes. All plots clearly indicated that modal degeneracy still remains preserve in higher order modes thus these modes can be used for stable vortex beam creation in such fiber.
Fig. 8. (a) Effective index ($n_{eff}$) of EH$^e_{11}$, (c) HE$^e_{31}$ modes, (e) HE$^e_{12}$ modes, and (g) HE$^e_{41}$ modes, while (b) effective index separation ($\Delta n_{eff}$) between EH$^e_{11}$ and EH$^o_{11}$, (d) between HE$^e_{31}$ and HE$^o_{31}$ modes, (f) between HE$^e_{12}$ and HE$^o_{12}$ modes, and (h) between HE$^e_{41}$ and HE$^o_{41}$ modes.

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