Flavour violation in supersymmetric SO(10) unification with a type II seesaw mechanism

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Abstract

We study flavour violation in a supersymmetric SO(10) implementation of the type II seesaw mechanism, which provides a predictive realization of triplet leptogenesis. The experimental upper bounds on lepton flavour violating processes have a significant impact on the leptogenesis dynamics, in particular they exclude the strong washout regime. Requiring successful leptogenesis then constrains the otherwise largely unknown overall size of flavour-violating observables, thus yielding testable predictions. In particular, the branching ratio for $\mu \to e\gamma$ lies within the reach of the MEG experiment if the superpartner spectrum is accessible at the LHC, and the supersymmetric contribution to $\varepsilon_K$ can account for a significant part of the experimental value. We show that this scenario can be realized in a consistent SO(10) model achieving gauge symmetry breaking and doublet-triplet splitting in agreement with the proton decay bounds, improving on the MSSM prediction for $\alpha_3(m_Z)$, and reproducing the measured quark and lepton masses.

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1 Introduction

Neutrino masses presumably arise at a scale much larger than the electroweak scale. If this is the case, a model-independent effective description of neutrino masses is possible at lower scales in terms of the dimension 5 operator $l_i l_j h_u h_u / \Lambda$ \[1\], where $l_i$ is the $i$-th family lepton doublet, $h_u$ is the $Y = +1/2$ Higgs doublet and $\Lambda$ is the scale at which the operator is generated, which can be as large as $10^{15}$ GeV. While this effective description is essentially unique, the high-energy mechanism leading to the above dimension 5 operator is not. Generally, it is assumed to arise from the tree-level exchange of SM singlet fermions (type I seesaw mechanism \[2\]). In this case, the low-energy information from lepton masses and mixing only determines 9 out of the 18 high-energy seesaw parameters. Due to this arbitrariness, it is not possible to make definite predictions for the lepton asymmetry generated in the decays of the heavy singlet neutrinos \[3\], nor, in supersymmetric models, for the lepton flavour violating (LFV) effects induced by their Yukawa interactions \[4\].

On the other hand, the exchange of singlet fermions is not the only possible origin of the $l_i l_j h_u h_u / \Lambda$ operator. In this paper, we consider the exchange of an SU(2)$_L$ triplet scalar with $Y = \pm 1$ (type II or triplet seesaw mechanism \[5\]). More precisely, we consider the SO(10) \[6\] implementation of the triplet seesaw mechanism proposed in Ref. \[7\], in which all flavour parameters contributing to low-energy observables are determined in terms of the SM fermion masses and mixings. This allows to make testable predictions for these observables, up to a few unknown flavour-blind parameters. Particularly interesting is the possibility \[7\] of accounting for the baryon asymmetry of the universe via triplet leptogenesis. In usual type II models, it is necessary to introduce additional triplets or singlets in order to provide a rich enough flavour structure to induce a non-vanishing CP asymmetry in triplet decays \[8\]. This brings back into the game a number of flavour unknowns. On the contrary, in the scenario that we consider, the additional states are heavy quarks and leptons whose masses and couplings are determined in terms of low-energy parameters through SO(10) relations. The generated baryon asymmetry then directly depends on the light neutrino parameters.

In this paper, we pursue the exploration of this scenario by studying the flavour- and CP-violating effects induced by the couplings of the heavy states to the MSSM squarks and sleptons. Assuming flavour-universal soft supersymmetry breaking terms at the GUT scale, flavour-violating observables are predicted up to a few unknown scale parameters and to a mild model-dependent uncertainty, thus allowing to test the scenario. Besides the contributions of the type II seesaw triplet and of its SU(5) partners already studied in Ref. \[9\], the presence of heavy quarks and leptons gives rise to additional contributions to the slepton and squark soft terms. In particular, they induce flavour and CP violation in the slepton singlet and squark doublet sectors, which were absent in the SU(5) type II seesaw model.

The paper is organized as follows. In Section 2, we present the main features of the SO(10) scenario we consider. Section 3 contains a qualitative discussion of its predictions for flavour and CP violation. Model-building aspects including gauge coupling unification, doublet-triplet splitting and proton decay are briefly discussed in Section 4 and addressed in greater detail in Appendices A, B and C where the main ingredients of a realistic model are given. Section 5 presents our numerical results. Finally, we give our conclusions in Section 6.

\[1\] Actually, the higher-dimensional operators needed to account for the measured masses of down quarks and charged leptons may affect this relation between high-energy and low-energy flavour parameters. We argue in Section 3.3 that the impact of this on physical observables is generally small, and neglect it in the following.
of the model, the boundary conditions for Yukawa couplings and soft terms and a subset of the renormalization group equations are displayed in Appendix D.

2 SO(10) unification with type II seesaw mechanism

We consider a supersymmetric SO(10) scenario with matter fields in $16$ and $10$ representations and type II realization of the seesaw mechanism, in which soft supersymmetry breaking terms arise at the GUT scale or higher. Since we are interested in the flavour-violating effects induced by the physics responsible for neutrino masses, we assume that the soft terms are flavour universal at the GUT scale.

Let us first describe the field content. In terms of SU(5) multiplets, the three families of SM fermions are described by $\bar{5}_{i}^{SU(5)} \oplus 10_{i}^{SU(5)}$, $i = 1,2,3$. In conventional SO(10) unification [6], $\bar{5}_{i}^{SU(5)}$ and $10_{i}^{SU(5)}$ are unified in a single $16_{i}^{SO(10)}$ together with a singlet (right-handed) neutrino participating in the generation of light neutrino masses through the type I seesaw mechanism. In this paper, we consider the alternative possibility that $10_{i}^{SU(5)}$ is embedded in a $16_{i}^{SO(10)}$, while $\bar{5}_{i}^{SU(5)}$ belongs to a $10_{i}^{SO(10)}$ (for similar or related approaches, see Ref. [10]). Having done this choice, we drop from now on the “SO(10)” superscript on SO(10) representations, and indicate the embedding of SU(5) representations into SO(10) representations by a superscript: for instance, $\bar{5}_{i}^{10}$ is contained in $10_{i}$, while $\bar{5}_{i}^{16}$ belongs to $16_{i}$. Besides the three $16_{i}$ and $10_{i}$ matter multiplets, the model also involves a $10$, a $16$ and a $54$ Higgs multiplets. These are needed in order to generate the quark and charged lepton masses, as well as the neutrino masses through the type II seesaw mechanism. The corresponding SO(10) superpotential reads:

$$W = W_{Yukawa} + W_{seesaw} + W_{GUT} + W_{nonren.},$$

$$W_{Yukawa} = \frac{1}{2} y_{ij} 16_{i} 16_{j} 10 + h_{ij} 16_{i} 10_{j} 16,$$

$$W_{seesaw} = \frac{1}{2} f_{ij} 10_{i} 10_{j} 54 + \frac{1}{2} \sigma 10 10 54 + \frac{1}{2} M_{54} 54^{2},$$

where $W_{GUT}$, whose explicit form is given in Appendices A and B, contains the terms responsible for SO(10) symmetry breaking and for the doublet-triplet splitting, and $W_{nonren.}$ includes the non-renormalizable operators needed to account for the measured ratios of down quark and charged lepton masses. The role of the $10$, $16$ and $54$ representations is the following. The $10$ contains the $Y = \pm 1/2$ MSSM Higgs doublet $h_{u}$ responsible for up quark masses. The $16$ plays a double role: it shares the $Y = -1/2$ MSSM Higgs doublet $h_{d}$ with the $10$ and (together with its $\bar{16}$ companion) it reduces the rank of the unified gauge group down to 4 through the vev of its SU(5)-singlet component; moreover, this vev pairs up the spare $\bar{5}_{i}^{16}$ and $\bar{5}_{i}^{10}$ and gives them a large mass, leaving only the $\bar{5}_{i}^{10}$ massless. The $54$ contains the SU(2)$_{L}$ triplet mediating the type II seesaw mechanism. The three singlet neutrinos contained in the $16_{i}$, on the other hand, do not couple to the light neutrinos at the renormalizable level. Assuming for definiteness that they acquire GUT-scale masses (e.g. from couplings $1_{i}16_{j}\bar{16}$ to three SO(10) singlets $1_{i}$), we can completely neglect their contributions to the light neutrino masses, if any, and we are left with a pure type II seesaw mechanism.

In writing Eqs. (2) and (3), we assumed that the superpotential is invariant under a matter parity, with the matter fields being the $16_{i}$ and the $10_{i}$. We also required the absence of mass terms of the form $10_{i}10_{j}$ and of additional interactions that would induce a vev for the $54$, in order to prevent a mixing between the $\bar{5}_{i}^{10}$ and the $\bar{5}_{i}^{16}$. The absence of such a mixing
ensures that the light neutrino masses arise from a pure type II seesaw mechanism, and is a crucial feature of the scenario. The scale of light neutrino masses requires $M_{54}$ to be smaller than the GUT scale; it is therefore natural to assume that this mass term arises at the non-renormalizable level, while it is forbidden or small at the renormalizable level. Furthermore, the leptogenesis scenario proposed in Ref. [7] requires that the $24^{54}$ component of the $54$ be heavier than the $10\oplus 15^{54}$ one. We refer the reader to Appendix [C] for the discussion of the splitting of the $54$.

The degrees of freedom surviving below the GUT scale are the MSSM states, which are massless before electroweak symmetry breaking, three heavy pairs of vector-like matter $5_{i}^{16} \oplus 1_{i}^{16}$, and the components of the $54$, which are also heavy. The light MSSM fields and the heavy ones are embedded in the SO(10) representations of the model as follows:

\[
\begin{align*}
16_{i} &= 10_{i}^{16} \oplus 5_{i}^{16} \oplus 1_{i}^{16} = (q_{i}, u_{i}^{c}, e_{i}^{c}) \oplus (D_{i}^{c}, L_{i}) \oplus n_{i}^{c}, \\
10_{i} &= 5_{i}^{10} \oplus 1_{i}^{10} = (d_{i}^{c}, l_{i}) \oplus (\bar{D}_{i}^{c}, \bar{L}_{i}), \\
54 &= 24^{54} \oplus 15^{54} \oplus \overline{15}^{54} = (S, T, O, V, \bar{V}) \oplus (\Delta, \Sigma, Z) \oplus (\bar{\Delta}, \bar{\Sigma}, \bar{Z}), \\
10 &= 5^{10} \oplus 1^{10} = (\cdots, \cos \theta_{H} h_{d}) \oplus (\cdots, h_{u}), \\
16 &= 10^{16} \oplus 5^{16} \oplus 1^{16} = \cdots \oplus (\cdots, \sin \theta_{H} h_{d}) \oplus \cdots,
\end{align*}
\]

dots stands for heavy fields which have been integrated out at the GUT scale (this includes in particular the coloured Higgs triplets mediating $D = 5$ proton decay). We have introduced an angle $\theta_{H}$ parametrizing the mixing among $Y = -1/2$ Higgs doublets ($0 < \theta_{H} < \pi/2$ with no loss of generality) and assumed that the $Y = +1/2$ MSSM Higgs doublet entirely resides in the $10$, so that all components of the $16$ are heavy (see Appendix [B] for an explicit realization of this). The $SU(3)_{C} \times SU(2)_{L} \times U(1)_{Y}$ quantum numbers of the $54$ components are $\Delta = (1, 3)_{+1}$, $\Sigma = (6, 1)_{-2/3}$, $Z = (3, 2)_{+1/6}$, $S = (1, 1)_{0}$, $T = (1, 3)_{0}$, $O = (8, 1)_{0}$ and $V \sim (3, 2)_{-5/6}$, while the heavy $L_{i}$ and $D_{i}^{c}$ obviously carry the same quantum numbers as the light $l_{i}$ and $d_{i}^{c}$. After breaking of the SO(10) gauge symmetry, the heavy quark and lepton fields acquire Dirac masses $(M_{D^{c}})_{ij} D_{i}^{c} \bar{D}_{j}^{c} + (M_{L})_{ij} L_{i} \bar{L}_{j}$, where, at the renormalizable level and neglecting the renormalization group running below the GUT scale:

\[
(M_{D^{c}})_{ij} = (M_{L})_{ij} = h_{ij} V_{1},
\]

in which $V_{1}$ is the vev of the SU(5)-singlet component of the $16$. As for the SM fermion masses, they are given by (again neglecting corrections from non-renormalizable operators and the RG running of the Yukawa couplings):

\[
\begin{align*}
(M_{u})_{ij} &= y_{ij} v_{u}, \\
(M_{c})_{ij} &= \sin \theta_{H} h_{ij} v_{d}, \\
(M_{d})_{ij} &= \sin \theta_{H} h_{ij}^{T} v_{d},
\end{align*}
\]

and the neutrino masses are given by the type II seesaw formula:

\[
(M_{\nu})_{ij} = -\frac{\sigma v_{u}^{2}}{2M_{\Delta}} f_{ij}.
\]

Note that the down quark and charged lepton masses, which satisfy the SU(5) relation $M_{d} = M_{e}^{T}$ at the renormalizable level, are proportional to $\sin \theta_{H}$, the $Y = -1/2$ Higgs mixing
Due to the way the SM fermions are embedded into SO(10) representations, the up quark mass matrix is not correlated with the down quark and charged lepton mass matrices as in conventional SO(10) models, which accounts in a natural way for the stronger mass hierarchy observed in the up quark sector.

The role of the SO(10) symmetry is to relate the masses and couplings of the heavy matter fields \((L_i, \bar{L}_i)\) and \((D^c_i, \bar{D}^c_i)\) to the ones of the light fermions, thus allowing to predict their contributions to observables such as the baryon asymmetry of the universe and the rates of flavour-violating processes. Indeed, Eqs. (2)–(3) and (7)–(9) show that all flavour parameters involved in the superpotential, as well as the masses of the heavy matter fields, are determined by the SM fermion masses and mixings, up to flavour-blind factors and to the non-renormalizable contributions needed to make \(M_d \neq M^T_d\). On the contrary, in conventional SO(10) models where neutrino masses arise from the type I seesaw mechanism, the Lagrangian below the GUT scale also depends on the flavour parameters encoded in the so-called \(R\) matrix \([11]\). The SO(10) scenario studied in this paper therefore has a higher predictive power. As for the unknowns associated with the non-renormalizable operators in \(W_{\text{nonren.}}\), we will argue in Section 3.3 that they are unlikely to affect our results in a sizable way.

## 3 Flavour and CP violation

### 3.1 General structure of radiative corrections to the MSSM soft terms

The \(f_{ij}10,10,54\) interactions introduce a new flavour structure, directly related to the light neutrino masses, on top of the MSSM one. As noted in Ref. \([12]\), this can be considered as a truly minimal extension to the lepton sector of the minimal flavour violation hypothesis in the quark sector, which does not rely on flavour basis dependent assumptions. In any case, the \(f_{ij}10,10,54\) interactions give rise to new flavour- and CP-violating effects at low energy because of their well-known impact on the MSSM soft terms through radiative corrections \([9]\). In addition, the \(y_{ij}16,16,10\) interactions induce new flavour and CP violation in the slepton singlet sector due to the presence of heavy lepton doublets in the \(16_i\), as we are going to see.

In order to compute these effects, we must write the renormalisation group equations (RGEs) for the MSSM parameters and integrate them. Before doing so, let us note that the \(l_i\) and \(L_i\) fields can mix, since they have the same quantum numbers after breaking of the SO(10) symmetry (and similarly for the \(d^c_i\) and \(D^c_i\) fields). Hence, the superpotential mass term for the lepton doublets reads, in full generality, \(L_i(M_{L})_{ij}\bar{L}_j + l_i(M_{LL})_{ij}\bar{L}_j\). The second term was omitted previously since \(M_{LL}\) vanishes at the tree level, while \(M_{L}\) is given by Eq. (7). At one loop, however, wave function renormalization induces a small \(l_i-L_j\) mixing:

\[
(M_{IL})_{ij} \approx -\frac{\sin 2\theta_H}{32\pi^2} \left( M_L y^h \right)_{ij} \ln \left( \frac{M_{\text{GUT}}}{\mu} \right).
\]

This in turn necessitates a redefinition of the heavy and light lepton doublets, which affects all couplings involving lepton doublets. However, the size of the effect is small (especially for small or moderate \(\tan \beta\)), and we shall neglect it in the following.

Before solving numerically the full 1-loop RGEs, let us illustrate the main features of the results by using the leading-log approximation for the exchange of the heavy degrees of freedom.\(^2\)

\(^2\)This offers the possibility of explaining the smallness of the bottom/top mass hierarchy for moderate values of \(\tan \beta\) in terms of a small Higgs mixing angle \(\theta_H\). For reasons explained in Section 4.1, however, we shall not use this possibility here.
In order to be able to identify each contribution, we assume that all components of the 54, namely $S$, $T$, $O$ and the vector-like pairs $(V, \bar{V})$ $(\Delta, \bar{\Delta})$, $(\Sigma, \bar{\Sigma})$ and $(Z, \bar{Z})$ have different masses (the mass $M_Z$ of the $(Z, \bar{Z})$ vector-like pair should not be confused with the $Z$ boson mass $m_Z$). We also consider the possibility that $M_D \neq M_L$, but ignore the effect of corrections to the mass relation $M_d = M_t^T$. Let us denote by $m_{16}^2$ and $m_{10}^2$ the universal soft terms for the three families of 16, and 10, matter fields at the GUT scale; by $m_{54a}$, $m_{10a}^2$ and $m_{10b}^2$ the soft terms for the 54, 16 and 10 SO(10) multiplets; and by $a_0$ the universal $A$-term (defined by $A_x = a_0 x$, where $x$ is any superpotential trilinear coupling $x$). Using the RGEs given in Appendix [D] we obtain (in matrix form):

$$
m_t^2 = (m_{t}^2)_{\text{MSSM}} - \frac{1}{(4\pi)^2} \left(2m_{16}^2 + m_{54}^2 + a_0^2 \right) f^\dagger \left[ \frac{3}{2} \ln \frac{M_{SUT}^2}{M_2^2} + \frac{3}{2} \ln \frac{M_{SUT}^2}{M_2^2} \right] f , \quad (11a)
$$

$$
m_{D}^2 = (m_{D}^2)_{\text{MSSM}} - \frac{1}{(4\pi)^2} \left(2m_{16}^2 + m_{54}^2 + a_0^2 \right) f^\dagger \left[ \frac{2}{3} \ln \frac{M_{SUT}^2}{M_2^2} + \frac{2}{3} \ln \frac{M_{SUT}^2}{M_2^2} \right] f , \quad (11b)
$$

In contrast to the well-known type I seesaw case, the flavour-violating corrections to $m_t^2$ are determined by the light neutrino mass matrix, with no ambiguity due to high-energy flavour parameters. Therefore, while the absolute rate of a given LFV process is not known, the correlations between different LFV channels are predicted with little uncertainty (at least if the contributions of the slepton singlet sector, to be discussed below, are subdominant). The first term in the squared brackets of Eq. (11b) is induced by the seesaw triplet interactions and is present in all type II seesaw models. The second term, as well as the first two terms in the squared brackets of Eq. (11b), corresponds to the contribution of the SU(5) partners of the triplet $H$. The additional terms in $m_t^2$ and $m_{D}^2$ are due to the presence of heavy leptons and quarks in the 16, and are characteristic of the model studied in this paper. In the limit $M_L = M_{D}^c$, $M_\Delta = M_\Sigma = M_Z$, $M_S = M_T = M_O = M_V$, the correction to the MSSM evolution is the same for $m_t^2$ and $(m_{D}^2)^2$, as dictated by the SU(5) invariance of the interactions from which they arise.

Another difference from other type II seesaw models comes from the corrections to the MSSM running of $m_{D}^2$ and $m_q^2$:

$$
m_{D}^2 = (m_{D}^2)_{\text{MSSM}} - \frac{\cos^2 \theta_H}{(4\pi)^2} \left(2m_{16}^2 + m_{54}^2 + a_0^2 \right) y \left[ 2 \ln \frac{M_{SUT}^2}{M_L^2} \right] y^\dagger , \quad (12a)
$$

$$
m_q^2 = (m_q^2)_{\text{MSSM}} - \frac{\cos^2 \theta_H}{(4\pi)^2} \left(2m_{16}^2 + m_{54}^2 + a_0^2 \right) y \left[ \ln \frac{M_{SUT}^2}{M_{D}^2} \right] y^\dagger , \quad (12b)
$$

where $m_{h_d}^2 = \cos^2 \theta_H m_{10a}^2 + \sin^2 \theta_H m_{10b}^2$. These are controlled by the up quark Yukawa couplings (evolved at the high scale). While the corrections to $m_q^2$ do not represent a deviation from the minimal flavour violation structure of the MSSM radiative corrections, the corrections to $m_{D}^2$ are similar to the ones induced above the GUT scale by the top quark Yukawa coupling in SU(5) models [13], although their origin is different.
3.2 Analytic approximations for the mass insertions and CP violation

In order to be able to quickly estimate the size of various flavour- and CP-violating observables, it is useful to provide analytic expressions for the mass insertions parameters \((i \neq j)\):  

\[
(\delta_{LL})_{ij} \equiv \frac{(m^2_{\tilde{L}})_{ij}}{m_{\tilde{e}_L}}, \quad (\delta_{RR})_{ij} \equiv \frac{(m^2_{\tilde{R}})_{ij}}{m_{\tilde{e}_R}}, \quad (\delta_{RL})_{ij} \equiv \frac{(A_e)_{ij}v_d}{m_{\tilde{e}_L}m_{\tilde{e}_R}},
\]

where \(m_{\tilde{e}_L} (m_{\tilde{e}_R})\) is an average doublet (singlet) slepton mass, and analogous quantities are defined in the up and down squark sectors. These parameters can be straightforwardly derived from Eqs. (11) and (12), but some care is needed regarding CP-violating phases. Even if we take real boundary conditions for the soft supersymmetry breaking parameters at \(M_{GUT}\), we will end up with complex soft terms at the weak scale because their RGEs involve complex couplings. These couplings contain, in addition to the CKM and PMNS phases, extra CP-violating phases inherited from the SO(10) structure. In order to identify the latter, it is useful to write the SO(10) Yukawa couplings in an appropriate basis for the SO(10) matter multiplets \(16_i\) and \(10_i\), namely:

\[
y = U^T_q D_y U_q, \quad h = D_h, \quad f = U^* D_f U^\dagger,
\]

where

\[
U_q = \text{Diag}(e^{i\Phi_u}, e^{i\Phi_d}, e^{i\Phi_e}) V \text{Diag}(e^{i\Phi_u^*}, e^{i\Phi_d^*}, e^{i\Phi_e^*}).
\]

In Eqs. (11) and (15), \(D_y, D_h, D_f\) are real diagonal matrices, \(U\) and \(V\) are the PMNS and CKM matrices in the standard PDG parametrization (extended to include two “Majorana” phases \(\rho\) and \(\sigma\) in the PMNS case), and \(\Phi^u_i, \Phi^d_i\) \((i = 1, 2, 3)\) are extra SO(10) phases, five of which are independent (one can impose e.g. \(\Phi^d_3 = 0\)). For simplicity, we neglect the effects of the non-renormalizable operators needed to correct the mass relation \(M_d = M^T_e\), and we therefore also assume \(M_{D,e} = M_L = D_h V_1\). Below the GUT scale, we are free to rephase independently the MSSM fields so that the Yukawa couplings \(\lambda_{u,d,e}\) and \(f_\Delta\) only contain CKM and PMNS phases; however, the extra SO(10) phases will reappear in other couplings, such as \(\lambda_d\) and \(f_Z\), which enter the RGEs for the MSSM soft terms (we refer to Appendix D for the definition of the superpotential couplings below the GUT scale). The outcome of this is that the mass insertion parameters depend on the phase differences \(\Phi^u_i - \Phi^d_i\) and \(\Phi^d_i - \Phi^d_j\) in addition to the 4 low-energy phases \(\delta_{CKM}, \delta_{PMNS}, \rho\) and \(\sigma\).

We are now ready to write the mass insertion parameters \((\delta_{MN})_{ij}\) \((M, N = L, R)\) in the leading-log approximation, assuming for simplicity \(M_\Delta = M_\Sigma = M_Z \equiv M_{15}, M_S = M_T = M_O = M_V \equiv M_{24}\), a common soft supersymmetry breaking mass \(m_0\) for all SO(10) chiral multiplets and, as before, a common A-terms \(a_0\) for all superpotential trilinear couplings. We obtain:

\[
16\pi^2(\delta_{LL})_{ij} \approx - \frac{3m^2_0 + |a_0|^2}{m^2_{\tilde{e}_L}} C^f_{ij},
\]

\[
16\pi^2(\delta_{RR})_{ij} \approx - \frac{2(3m^2_0 + |a_0|^2)}{m^2_{\tilde{e}_R}} e^{i(\Phi^d_i - \Phi^d_j)} (C^\lambda_{ij})^* ,
\]

\[
16\pi^2(\delta_{RL})_{ij} \approx - \frac{3a_0}{2m_{\tilde{e}_L}m_{\tilde{e}_R}} \left[ m_{\tilde{e}_L} C^f_{ij} + 3 e^{i(\Phi^d_i - \Phi^d_j)} (C^\lambda_{ij})^* m_{\tilde{e}_j} \right],
\]

\footnote{In the following discussion, we neglect the effect of radiative corrections in the Yukawa sector. This allows us to disentangle the CKM and PMNS phases from the extra SO(10) phases, which would otherwise be mixed by the renormalization group running.}
quarks and leptons, and the coefficients that involve the up quark Yukawa couplings read:

\[ 16\pi^2 (\delta_{LL})_{ij} \approx - \frac{3m_0^2 + |a_0|^2}{m_{dL}^2} \left[ (C_{\lambda u_{\text{MSSM}}}^a)_{ij} + C_{ij}^a \right], \quad (19) \]

\[ 16\pi^2 (\delta_{RR})_{ij} \approx - \frac{3m_0^2 + |a_0|^2}{m_{dR}^2} e^{i(\Phi_d^a - \Phi_d^j)} (C_{ij}^a)^*, \quad (20) \]

\[ 16\pi^2 (\delta_{RL})_{ij} \approx - \frac{3a_0}{2m_{dL} m_{dR}} \left\{ m_{di} \left[ (C_{\lambda u_{\text{MSSM}}}^a)_{ij} + 3C_{ij}^a \right] + e^{i(\Phi_d^a - \Phi_d^j)} (C_{ij}^a)^* m_{dij} \right\}, \quad (21) \]

where the dependence on the low-energy flavour parameters and phases is encapsulated in the coefficients \((C_{\lambda u_{\text{MSSM}}}^a)_{ij}, (C_{ij}^a, C_{ij}^d, C_{ij}^u, D_{ij}^u)\). The first two correspond to the CKM-induced MSSM contributions and are given by:

\[ (C_{\lambda u_{\text{MSSM}}}^a)_{ij} \approx \lambda_f^2 V_{li}^* V_{lj} \ln \left( \frac{M_{1\text{GUT}}^2}{m_{Z}^2} \right), \quad (C_{ij}^{\lambda u}) = \sum_k \lambda_k^{2} V_{lk} V_{jk}^* \ln \left( \frac{M_{5\text{a}}^2}{m_{Z}^2} \right), \quad (25) \]

where terms suppressed by \(\lambda_c/\lambda_t\) have been neglected in \((C_{\lambda u_{\text{MSSM}}}^a)_{ij}\). The other three coefficients correspond to the contributions of the heavy states present below the GUT scale and also contain a dependence on their masses. The one that involves the seesaw couplings is given by:

\[ C_{ij}^f = 3 \left( f^{\dagger} f \right)_{ij} \ln \left( \frac{M_{2GUT}^2}{M_{15}^2} \right) + \frac{12}{5} \sum_a f^a_{ai} f^a_{aj} \ln \left( \frac{M_{2GUT}^2}{M_{24}^2 + M_{5a}^2} \right) \]

\[ = 3 \sum_k f_k^{2} U_{ik} U_{jk}^* \ln \left( \frac{M_{2GUT}^2}{M_{15}^2} \right) + \frac{12}{5} \sum_{k,l} f_k f_l U_{ik} U_{jl}^* \sum_a U_{ak} U_{al}^* \ln \left( \frac{M_{2GUT}^2}{M_{24}^2 + M_{5a}^2} \right), \quad (26) \]

where \(f_i = (2M_\Delta/\sigma v_{\nu}^2) m_{\nu_i}\) and the \(M_{5a} (a = 1, 2, 3)\) are the common masses of the heavy quarks and leptons, and the coefficients that involve the up quark Yukawa couplings read:

\[ C_{ij}^{\lambda u} \approx \cos^2 \theta_H \lambda_f^2 V_{li}^* V_{lj} \ln \left( \frac{M_{2GUT}^2}{M_{5a}^2} \right), \quad D_{ij}^{\lambda u} = \cos^2 \theta_H \lambda_{ui} \lambda_{uj} \sum_a V_{ia} V_{ja}^* \ln \left( \frac{M_{2GUT}^2}{M_{5a}^2} \right), \quad (27) \]

where terms suppressed by \(\lambda_c/\lambda_t\) have been neglected in \(C_{ij}^{\lambda u}\). Due to this (very good) approximation, only the CKM and PMNS phases appear in Eqs. (25) to (27).

Let us have a closer look at the mass insertion parameters \((18)\) to \((24)\). In the up squark sector, the new contributions are subdominant with respect to the MSSM corrections, which are enhanced by \(\tan^2 \beta\) and by a large logarithm. The situation is more interesting in the down squark and charged slepton sectors. In the former, the new corrections controlled by the top quark Yukawa coupling are again subdominant with respect to the MSSM corrections, while they lead to RR flavour violation in the latter. The new corrections induced by the seesaw couplings, on the other hand, are not suppressed by small CKM angles and can give rise to large flavour violations in the RR down squark sector and in the LL slepton sector \([9]\). One observes the following correlations:

\[ M_{dR}^2 (\delta_{RR})_{ij} = \frac{m_d^2}{m_{dL}^2} (\delta_{RR})_{ij} e^{i(\Phi_d^a - \Phi_d^j)}, \quad (28) \]

\[ M_{dR}^2 (\delta_{RR})_{ij} = 2R \frac{m_d^2}{m_{dL}^2} (\delta_{RR})_{ij} e^{i(\Phi_d^a - \Phi_d^j)}, \quad (29) \]
where \( R \equiv \cos^2\theta_H \ln(M_{\text{GUT}}^2/M_{\text{GUT}}^2)/\left[ \ln(M_{\text{GUT}}^2/m_Z^2) + \cos^2\theta_H \ln(M_{\text{GUT}}^2/M_{\text{GUT}}^2) \right] \). The first one is characteristic of the SU(5) extension of the type II seesaw mechanism, while the second one is analogous (although from a different origin) to the one arising from the running between \( M_F \) and \( M_{\text{GUT}} \) in the minimal SU(5) model. In the case that the seesaw-induced corrections dominate, one further has:

\[
(\delta_{RR})_{ij} \ll (\delta_{LL})_{ij} , \quad (\delta_{LL})_{ij} \ll (\delta_{RR})_{ij} ,
\]

\[
\overline{m}_{d_R} \overline{m}_{d_L} (\delta_{RL})_{ij} = \overline{m}_{e_R} \overline{m}_{e_L} (\delta_{RL})_{ji} e^{i(\Phi^d - \Phi^d)} .
\]

Eqs. (28), (29) and (31) correlate the size of flavour-changing neutral currents (FCNCs) in the lepton and B/K sectors, as well as the charged lepton and quark electric dipole moments (EDMs), as we are going to see.

For illustration, we give in Table 1 numerical values for the leptonic \( \delta \)'s. These values were obtained by numerically solving the RGEs for the following choice of high-energy parameters, motivated by the leptogenesis scenario of Ref. [7]: \( M_{\Delta} = M_{\Sigma} = M_Z \equiv M_{15} = 10^{12} \text{GeV}, M_S = M_T = M_O = M_V = M_{24} = 10^{13} \text{GeV}, \sigma = 3.4 \times 10^{-2}, V_1 = M_{\text{GUT}} = 2 \times 10^{16} \text{GeV} \) and \( \tan \theta_H = 1 \). For the heavy (5, \( \tilde{5} \)) masses, we used \( M_{5_h} = m_{e_i} V_i/\sin \theta_i v_1 \), neglecting SU(5)-breaking corrections. The supersymmetric parameters were chosen to be \( m_0 = M_{1/2} = 1.5 \text{ TeV}, a_0 = 0 \), and \( \tan \beta = 10 \) (50), yielding the following superpartner masses (for \( \tan \beta = 10 \)): \( M_{\tilde{\chi}^0_1} = 112 \text{ GeV}, M_{\tilde{\chi}^0_1} = 218 \text{ GeV}, M_{\tilde{g}} = 723 \text{ GeV}, \) sleptons around \((1.5 - 1.6) \text{ TeV}\) and first two generation squarks around \(1.8 \text{ TeV}\) (as well as \( \mu = 562 \text{ GeV}\)). Finally, we took the best fit values of Ref. [15] for the measured neutrino oscillation parameters, together with \( m_1 = 0.005 \text{ eV} \) and \( \sin^2 \theta_{13} = 0 \) (0.05) for the yet unknown parameters. All phases were set to zero. The resulting values of the leptonic mass insertions shown in Table 1 can be compared with the bounds coming from the non-observation of LFV decays of charged leptons (see e.g. Refs. [16, 17, 18]). Although the sleptons are heavy in this example, the relatively large values of the \( \delta \)'s lead to a branching ratio for \( \mu \rightarrow e\gamma \) just below the experimental bound for \( \tan \beta = 10 \), and above it for \( \tan \beta = 50 \). Using Eqs. (28), (29) and (31), one can also compare the figures in Table 1 with the constraints on hadronic \( \delta \)'s coming from \( B \) and \( K \) physics (see e.g. Refs. [19, 18]).

From Table 1 we can see that the LL mass insertions are the dominant source of lepton flavour violation in our numerical example, i.e. we are in the situation described by Eq. (30). This is due to the fact that the choice made for the model parameters leads to rather large \( f_{ij} \)'s (namely \( f_{33} \sim 0.1 \)): a smaller value of the ratio \( M_{\Delta}/\sigma \) would yield smaller \( f_{ij} \) couplings,

| \( \sin^2 \theta_{13} \) | \( \tan \beta \) | 0.05 | 0 | 0.05 | 0 |
|---|---|---|---|---|---|
| \( \delta^L_{e1} \) | 2.0 \times 10^{-3} | 1.7 \times 10^{-4} | 2.0 \times 10^{-3} | 1.5 \times 10^{-4} |
| \( \delta^L_{e2} \) | 1.8 \times 10^{-3} | 1.0 \times 10^{-4} | 1.9 \times 10^{-3} | 1.4 \times 10^{-4} |
| \( \delta^L_{e3} \) | 5.7 \times 10^{-3} | 5.9 \times 10^{-3} | 6.1 \times 10^{-3} | 6.3 \times 10^{-3} |
| \( \delta^R_{e1} \) | 2.1 \times 10^{-6} | 2.1 \times 10^{-6} | 2.8 \times 10^{-8} | 4.6 \times 10^{-7} |
| \( \delta^R_{e2} \) | 5.8 \times 10^{-5} | 5.4 \times 10^{-5} | 1.4 \times 10^{-4} | 1.2 \times 10^{-6} |
| \( \delta^R_{e3} \) | 4.3 \times 10^{-4} | 4.3 \times 10^{-4} | 2.9 \times 10^{-4} | 2.9 \times 10^{-4} |

Table 1: Numerical values of the leptonic mass insertion parameters \( \delta^L_{LL}, \delta^R_{RR} \) for two different values of \( \sin^2 \theta_{13} \) and \( \tan \beta \) and for the choice of parameters mentioned in the text.
enhancement. As for the up quark EDM, since the couplings: electron and neutron EDMs, which depend on the same combination of phases and flavour down quark EDM. Together with Eqs. (28) and (29), this implies a correlation between the are suppressed by small Yukawa couplings or CKM angles, making it much smaller than the in which the suppression due to the double flavour change is compensated for by the tan \( \beta m f_3 \) enhancement. As for the up quark EDM, since \( \langle \delta_{RR}^i \rangle_{ij} \approx 0 \) its flavour-violating contributions are suppressed by small Yukawa couplings or CKM angles, making it much smaller than the down quark EDM. Together with Eqs. (28) and (29), this implies a correlation between the electron and neutron EDMs, which depend on the same combination of phases and flavour couplings:

\[
\text{Im} \left[ e^{i(\Phi_3^d - \Phi_3^u)} C_{13}^f C_{13}^\lambda \right].
\]  

Using formulae available in the literature (see e.g. Ref. [23]), one can estimate the size of the charged lepton and neutron EDMs for the choice of parameters made in Table 11. With the superpartner masses given above and an average slepton (down squark) mass \( \overline{m}_\tau = 1.56 \text{ TeV} \) \( (\overline{m}_d = 1.8 \text{ TeV}) \), one obtains, for \( \tan \beta = 10 \):

\[
d_{e} \simeq \left( -1.3 \times 10^{-24} \text{ e cm} \right) \text{ Im} \left[ (\delta_{LL}^e)_{i3} (\delta_{RR}^e)_{3i} \right],
\]

\[
d_{n} \simeq \left( 1 \pm 0.5 \right) \left( -2.9 \times 10^{-22} \text{ e cm} \right) \text{ Im} \left[ (\delta_{LL}^d)_{13} (\delta_{RR}^d)_{31} \right].
\]
For the neutron EDM, we used the formula $d_n = (1 \pm 0.5) [1.4(d_d - 0.25d_u) + 1.1e(d_d^2 + 0.5d_u^2)]$ [24], in which we only kept the dominant gluino contributions to down quark electric dipole moments, while we neglected the up quark dipole moments. The values of the $\delta$’s given in Table I, together with $|\langle \delta_{LL}^d \rangle_{12}(\delta_{RR}^d)_{31}| = 2.2 \times 10^{-7}$ (for $\theta_{13} = 0$) and $3.5 \times 10^{-6}$ (for $\sin^2 \theta_{13} = 0.05$), lead to $d_c \lesssim (7 \times 10^{-33} - 10^{-31})$ cm, $d_\mu \lesssim 3 \times 10^{-30}$ cm and $d_\nu \lesssim (1 \pm 0.5) (6 \times 10^{-29} - 10^{-27})$ cm, where the upper ranges correspond to varying $\theta_{13}$ from 0 to its experimental upper limit. These bounds are well below the present experimental upper bounds (respectively $1.6 \times 10^{-27}$ cm [25], $1.9 \times 10^{-19}$ cm [26] and $2.9 \times 10^{-26}$ cm [27]), while some tension with the constraints from LFV processes is already present for the same values of the $\delta$’s. Therefore, taking into account the FCNC constraints, we do not expect significant flavour-violating contributions to EDMs in the scenario studied in this paper\[4].

A more promising CP-violating observable is the indirect CP violation parameter in the kaon sector, $\varepsilon_K$. A recent evaluation of the Standard Model prediction for $\varepsilon_K$ suggests that its measured value still allows for a significant supersymmetric contribution [28]. In the mass insertion language, the relevant contributions are $\text{Im} [\langle \delta_{MN} \rangle_{12} (\delta_{PQ})_{12}]$, with $(M, N), (P, Q) \in \{(L, L), (R, R)\}$ or $(L, R), (R, L)$. Since the LR mass insertions are suppressed by the down and strange quark masses and $|\langle \delta^d_{LL} \rangle_{12}|$ is suppressed by $|V_{td}V_{ts}|$, the leading contributions to $\varepsilon_K$ (for $f_{33} \gtrsim 0.01 - 0.03$) are proportional to $\text{Im} [\langle \delta_{LL}^d \rangle_{12} (\delta_{RR}^d)_{31}] \propto R^{-1} \text{Im} [e^{i(\phi_d^1 - \phi_d^2)} (C_{12}^d) \lambda_{12}^d]$ and to $\text{Im} [(\delta_{RR}^d)_{31}] \propto \text{Im} [e^{i(\Phi_{d}^1 - \Phi_{d}^2)} (C_{12}^{d*})^2]$, which depend both on CKM/PMNS phases and on the unknown extra SO(10) phases. As discussed at the end of Section 3, this might account for a significant part of the measured value of $\varepsilon_K$.

### 3.3 Model dependence related to higher-dimensional operators

Throughout the above discussion, we assumed that Eqs. (7) and (8) hold at the GUT scale, yielding the following relations between the masses of the heavy $(L_i, L_{\bar{i}})$ and $(D^c_i, \bar{D}^c_i)$ fields and of the SM quarks and leptons:

$$M_L = \frac{V_1}{\sin \theta_H v_d} M_e, \quad M_{D^c} = \frac{V_1}{\sin \theta_H v_d} M_d^T, \quad (38)$$

as well as $M_d = M_e^T$. The last relation is known to be in gross contradiction with the measured masses of the first two generation fermions, but since the corresponding Yukawa couplings are small, this can easily be cured by higher-dimensional operators involving fields with SU(5)-breaking vevs. These operators also affect, in a model-dependent way, the relations [38] between the masses of the heavy and light matter fields. Moreover, they generally introduce a mismatch between the SO(10) bases in which $M_d$ and $M_e$ are diagonal, as well as between the SO(10) bases in which $M_d$ and $M_{D^c}$ ($M_e$ and $M_L$) are diagonal. All these effects have an impact on the running of the MSSM soft terms and on the resulting flavour violation. However we argue below that, under reasonable assumptions, their impact on physical observables should remain small.

The first effect, i.e. the fact that the masses of the $(L_i, L_{\bar{i}})$ and $(D^c_i, \bar{D}^c_i)$ fields are no longer determined by the light fermion masses, introduces only a mild model dependence. Indeed, the heavy masses $M_{L_i}$ and $M_{D^c_i}$ enter Eqs. (11) and (12) [or equivalently Eqs. (26) and (27)] only logarithmically. On top of that, in the case that $M_{24} > M_{L_2, t_d}$, $m_T^d$ and $m_T^e$ only depend on

\[4\] Of course, “flavour-independent” phases carried by diagonal A-terms or by the mu term may still induce large EDMs.
$M_{L3}$ and $M_{D_{ij}^q}$, which are presumably little affected by the higher-dimensional operators, while in the case that $M_{24} > M_{L1,D_{ij}^l}$ some model dependence arises from the contributions of these operators to $M_{L3}$ and $M_{D_{ij}^l}$. As for $m_{2c}^2$, it is dominated by $M_{L3}$-dependent contributions.

The second effect, i.e. the mismatch between the mass eigenstate basis of the heavy and light quark (lepton) fields, introduces new flavour parameters in the model in the form of unitary matrices connecting a given SO(10) basis to another. As a result, the couplings of the components of the 10 (namely the $l_i$, $d_i^c$, $\bar{L}_i$ and $D_i^c$ fields) to the components of the 54 are no longer equal at the GUT scale, when expressed in terms of mass eigenstates. For instance, $f_\Delta$ and $f_Z$ (see Appendix D for the definition of these couplings) now differ by a (model-dependent) unitary matrix. It follows that the radiative corrections to the MSSM soft terms do not only depend on low-energy flavour parameters, but are also sensitive to these high-energy flavour parameters. However, since the unitary matrices originate from the higher-dimensional operators correcting the masses of the first two generation down quark and charged lepton, it is natural to expect that they are characterized by small mixing angles, thus only mildly affecting the flavour structure of the $f_{ij}$ couplings, which on the contrary is characterized by large $\theta_{12}$ and $\theta_{23}$ angles. We shall therefore neglect this model dependence in the following, and simply input the $SU(5)$ relations \[\text{(38)}\] in our numerical study.

For completeness, we list below the D=5 operators which can correct the mass relation $M_d = M_{L}^T$, assuming the field content and pattern of vevs of Appendix A

\[a_{ij}^{(1,2)} \left( \frac{16,16}{\Lambda} \right)_{10} \left( \frac{10,45_1,2}{\Lambda} \right)_{10}, \ b_{ij}^{(1,2)} \left( \frac{16,45_1,2}{\Lambda} \right)_{16} \left( \frac{10,16}{\Lambda} \right)_{16}, \ c_{ij} \left( \frac{16,16}{\Lambda} \right)_{10} \left( \frac{10,54_2}{\Lambda} \right)_{10}, \ (39)\]

where the subscripts specify the contraction of SO(10) indices and $\Lambda$ is the cutoff. In Eq. \[\text{(39)}\], the non-vanishing vevs of $45_1$ and $45_2$ are along the $T_{3R}$ and $B - L$ directions, respectively, and $54'$ has a vev in the Pati-Salam singlet direction. The operator $b_{ij}^{(2)}$, for instance, yields the following corrections to the mass matrices of the heavy and light matter fields:

\[\begin{align*}
(M_L)_{ij} &= \sin \theta_{H} v_d \left( h_{ij} - 3c b_{ij}^{(2)} \right), \quad (M_{L})_{ij} = V_1 \left( h_{ij} + 3c b_{ij}^{(2)} \right), \\
(M_D^T)_{ij} &= \sin \theta_{H} v_d \left( h_{ij} - c b_{ij}^{(2)} \right), \quad (M_{D})_{ij} = V_1 \left( h_{ij} + c b_{ij}^{(2)} \right),
\end{align*}\]

where $c = 2\sqrt{2/3} V_{B-L}/\Lambda$. Even in the simple case where a single operator ($b_{ij}^{(2)}$ in this example) is present, the experimental values of the down quark and charged lepton masses are not sufficient to fully determine the couplings $h_{ij}$ and $b_{ij}^{(2)}$. Hence, the heavy field masses and the mixing patterns in the four mass matrices are model dependent. As argued above, however, this is unlikely to affect the radiative corrections to the MSSM soft terms in a sizable way if the couplings $h_{ij}$ and $b_{ij}^{(2)}$ have a hierarchical flavour structure.

4 Model-building aspects

Before presenting our numerical results for flavour violation in the next section, let us briefly present the main ingredients necessary to promote the SO(10) scenario studied in this paper to a realistic model. This includes the dynamics of gauge symmetry breaking, a viable doublet-triplet splitting mechanism and the generation of the intermediate scales associated with the

\[\text{(40)}\]
components of the 54 multiplet consistently with gauge coupling unification. More details can be found in Appendices A, B and C.

4.1 Gauge symmetry breaking and doublet-triplet splitting

In order to break the SO(10) gauge group down to \( SU(3)_C \times SU(2)_L \times U(1)_Y \), we introduce two adjoint Higgs representations \( 45_1 \) and \( 45_2 \) with non-vanishing vevs in the \( T^3_R \) and \( B-L \) directions, respectively, and a \( 54' \) which breaks SO(10) down to its Pati-Salam subgroup (we also introduce an SO(10) singlet). The rank of the gauge group is broken by the \( (16, 16) \) pair. A superpotential that enforces this pattern of vevs is given in Appendix A. In this example, all vevs are proportional to a single mass parameter, \( M_{16} \). Hence, if all superpotential couplings are of order one, SO(10) is broken down to \( SU(3)_C \times SU(2)_L \times U(1)_Y \) in one step. Note that the \( 54' \) cannot be identified with the \( 54 \) multiplet involved in the seesaw mechanism, since a vev of the latter would induce an unwanted \( \bar{5}_{10} \) \( / \bar{5}_{16} \) mixing (see Section 2), and also for reasons related to the doublet-triplet splitting mechanism.

In order to avoid proton decay at an unacceptable rate, we must split the electroweak doublets from the colour triplets in the \( 10 \) and \( 16 \) Higgs multiplets. This can be achieved by a generalization of the missing vev mechanism [29] involving an additional \( 10' \) Higgs multiplet and an adjoint Higgs representation with a vev aligned along the \( B-L \) direction, which is precisely the role of the \( 45_2 \) (see Appendix B for details). After SO(10) symmetry breaking, all colour triplets acquire GUT-scale masses, and a single pair of Higgs doublets \( (h_u, h_d) \) remains massless, the \( Y = +1/2 \) electroweak doublet in \( 10 \) and a combination of the \( Y = -1/2 \) electroweak doublets in \( 10 \) and \( 16 \):

\[
h_u = H_{u10}^{10}, \quad h_d = \cos \theta_H H_d^{10} + \sin \theta_H H_d^{16},
\]

where the Higgs mixing angle is given by \( \tan \theta_H = \bar{\eta} V_{ij} / M_{16} \), with \( \bar{\eta} \) a superpotential coupling. As discussed in Section 2, \( \sin \theta_H \neq 0 \) is necessary to give masses to the down quarks and charged leptons, which live in the \( 10_i \)'s. Furthermore, the leptogenesis scenario of Ref. [7] assumes that \( h_d \) contains a non-negligible \( H_{d10}^{10} \) component (\( \cos \theta_H \neq 0 \)).

Since the down quark and charged lepton masses are proportional to \( \sin \theta_H \) at the renormalizable level, the top/bottom mass hierarchy could in principle be explained by a small Higgs mixing, rather than by a hierarchy in the Yukawa couplings or a large value of \( \tan \beta \). This, however, would imply rather heavy vector-like matter fields, since according to Eq. (38) their masses are inversely proportional to \( \sin \theta_H \). On the contrary, leptogenesis requires a relatively light \( (L_1, \bar{L}_1) \) pair in order to satisfy the condition for a non-vanishing CP asymmetry (\( M_{1} > 2M_{L_1} \)). We shall therefore stick to the case \( \sin \theta_H \sim 1 \).

4.2 Proton decay

Although all colour Higgs triplets acquire GUT-scale masses through the doublet-triplet splitting mechanism, their contribution to the proton decay mode \( p \rightarrow K^+ \bar{\nu} \) can still exceed the experimental limit. To suppress it further, one must impose additional constraints on the combination of doublet-triplet splitting parameters which enters the proton decay rate. More precisely, one must either arrange some cancellation among these parameters, or allow a pair of Higgs doublets to lie at an intermediate scale \( M_H \lesssim 10^{14} \text{ GeV} \) (see Appendix B.2 for details). In this paper, we choose the latter option, which in turn affects the running of the gauge cou-
plings and leads us to split the \((15 \oplus 15)^{54}\) and \(24^{54}\) multiplets in order to restore successful gauge coupling unification, as we discuss below.

### 4.3 Intermediate scales and gauge coupling unification

The leptogenesis scenario of Ref. [7] requires the \(15 \oplus \overline{15}\) component of the \(54\) to be lighter than its \(24\) component. Actually the full \(SU(5)\) multiplets are not needed for leptogenesis: only the \((\Delta, \bar{\Delta})\) pair in \(15 \oplus \overline{15}\), which is responsible for the type II seesaw mechanism, and the \(S\) and \(T\) fields in \(24\) (or just one of them) are really necessary. The main reason for using complete \(SU(5)\) representations in Ref. [7] was gauge coupling unification. However, we have seen that the experimental constraint on the \(p \to K^+\bar{\nu}\) rate can be satisfied by allowing a pair of Higgs doublets to lie at an intermediate scale, which in turn spoils unification. As shown in Appendix C, this can be cured by splitting the masses of the components of \((15 \oplus \overline{15})^{54}\) and \(24^{54}\) in an appropriate manner. This also has the advantage of maintaining perturbativity above the GUT scale, and of avoiding too strong flavour-violating effects in the region of the parameter space relevant for leptogenesis, where the \(f_{ij}\) couplings are large and induce important radiative corrections to the MSSM soft terms.

Two simple possibilities emerge from the analysis of gauge coupling unification done in Appendix C (the ratio \(M_T/M_\Delta = 10\) is motivated by leptogenesis):

- **model (i):** intermediate \((\Delta, \bar{\Delta}), (\Sigma, \bar{\Sigma})\) and \(T\), with
  
  \[
  M_\Delta = M_\Sigma , \quad M_T = 10M_\Delta , \quad M_H = 10^{14} \text{ GeV} , \quad (42)
  \]

- **model (ii):** intermediate \((\Delta, \bar{\Delta}), (\Sigma, \bar{\Sigma}), S, T\) and \(O\), with \((S\) is actually irrelevant for gauge coupling unification)
  
  \[
  M_\Sigma^3/M_\Delta^2 \approx 10^{115} \text{ GeV} , \quad M_S = M_T = M_O = 10M_\Delta , \quad M_H = 10^{14} \text{ GeV} , \quad (43)
  \]

all other components of the \(54\) having GUT-scale masses. The desired splittings can be achieved by appropriate higher-dimensional operators (see Appendix C). In model (i), the unification scale lies almost one order of magnitude lower than in the MSSM, which is at odds with the experimental limit on \(p \to \pi^0e^+\). While this might be cured by 2-loop running and GUT threshold corrections, we prefer to adopt model (ii) for the numerical study of the next section. We note in passing that both models give similar (although quantitatively different) results for flavour violation. For illustration, let us consider model (ii) with \(M_\Delta = 10^{12} \text{ GeV}\), \(M_\Sigma = M_S = M_T = M_O = 10^{13} \text{ GeV}\) and \(M_H = 10^{14} \text{ GeV}\) (the other parameters are chosen to be \(V_1 = M_{\text{GUT}}, \lambda_H \equiv |\sigma_\Delta(\mu = M_\Delta)| = 0.045, \tan \beta = 10\) and \(\tan \theta_H = 1\)). The spectrum of heavy states below the GUT scale is shown in Fig. 1(a) and the renormalization group running of the gauge couplings in Fig. 1(b). At the 1-loop level, one finds:

\[
M_{\text{GUT}} = 1.2 \times 10^{16} \text{ GeV} , \quad \alpha_{\text{GUT}} = 1/12.5 . \quad (44)
\]

The prediction for \(\alpha_3(m_Z)\), including supersymmetric thresholds and the 2-loop MSSM running, agrees with the measured value within \(1\sigma\). Finally, the Landau pole lies one order of magnitude above \(M_{\text{GUT}}\).
5 Numerical results

In this section, we perform a numerical analysis of flavour violation in the SO(10) model defined above. Since the flavour-violating effects we want to study arise from radiative corrections to the sfermion soft terms, we numerically solve the full 1-loop RGEs from the unification scale \( M_{\text{GUT}} \) down to the low-energy scale \( M_{\text{EWSB}} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \), taking into account the presence of additional states and interactions below the GUT scale, which modify the running of the MSSM soft terms as outlined in Section 3. A subset of these RGEs is given in Appendix D, where the definitions of the various superpotential couplings and soft terms can also be found.

In order to set the initial conditions at the GUT scale for the superpotential couplings and to compute the masses of the heavy \((5_i, \bar{5}_i)\) pairs (which depend on the SO(10) couplings \(h_{ij}\)), we first evolve the Yukawa matrices and the low-energy neutrino mass operator from low energy up to the seesaw scale \(M_\Delta\), where the couplings \(f_{ij}\) are computed according to the seesaw formula \((M_\nu)_{ij} = -\lambda_H f_{ij} v_u^2 / 2 M_\Delta\), in which \(\lambda_H \equiv |\sigma_{\Delta}(\mu = M_\Delta)|\). Then we evolve the \(f_{ij}\) together with the other Yukawa couplings up to the GUT scale. This procedure is iterated until convergence of the heavy \((5_i, \bar{5}_i)\) masses \(M_{5_i}\) is reached. The other mass scales are taken as inputs and varied in order to study their impact on leptogenesis and on the flavour-violating observables listed in Table 2. Moreover, to satisfy the proton decay constraint consistently with gauge coupling unification, we split the components of the \(54\) as indicated in Eq. (43). Thus, specifying the triplet mass \(M_\Delta\) is enough to fix the masses of all \(54\) components. The value of \(M_H\) has a weaker impact on gauge coupling unification, and we keep it to be \(10^{14}\) GeV. The other inputs in the procedure, apart from the SM and supersymmetric parameters, are the seesaw coupling \(\lambda_H\), \(V_1\) and \(\tan \theta_H\), which are needed to compute the heavy \((5_i, \bar{5}_i)\) masses.

In order to single out the flavour-violating effects arising from radiative corrections, we assume universal boundary conditions for the soft supersymmetry breaking terms at the GUT
Table 2: Flavour-violating observables studied in the numerical analysis and their experimental values or upper bounds.

| Observable                  | Bound                              | Ref. |
|-----------------------------|------------------------------------|------|
| BR(μ → eγ)                  | < 1.2 × 10^{-11}                  | 30   |
| BR(μ → eee)                 | < 1.0 × 10^{-12}                  | 31   |
| CR(μ → e in Ti)             | < 4.3 × 10^{-12}                  | 32   |
| BR(τ → µγ)                  | < 4.4 × 10^{-8}                   | 33   |
| BR(τ → eγ)                  | < 3.3 × 10^{-8}                   | 33   |
| εK                          | (2.229 ± 0.012) × 10^{-3}         | 34   |
| ΔmK                         | (3.483 ± 0.006) × 10^{-12} MeV    | 34   |
| ΔmBs                        | (17.77 ± 0.12) ps^{-1}            | 34   |
| BR(b → sγ)                  | 2.77 × 10^{-4} − 4.33 × 10^{-4} (3σ) | 35   |
| BR(Bs → µµ)                 | < 4.7 × 10^{-8}                   | 36   |
| ΔmB                         | (0.507 ± 0.005) ps^{-1}           | 34   |
| ΔmD                         | (0.0237^{+0.006}_{-0.0071}) ps^{-1} | 34   |

Figure 2: Feynman diagrams responsible (together with the CP-conjugated diagrams) for the CP asymmetry in the decays of the scalar triplet into Standard Model lepton doublets.

scale, with common gaugino mass parameter $M_{1/2}$, scalar soft mass $m_0$ and $A$-term $a_0$. For definiteness, we set $a_0 = 0$ in the following. In a later stage, we shall also comment about the effect of relaxing the equality of the soft mass parameters for different SO(10) multiplets, still assuming flavour-blind supersymmetry breaking. After having performed the running from the GUT scale to low energy, we check that electroweak symmetry breaking does take place and that no tachyonic states are present in the spectrum. We then compute the masses of all Higgs bosons and superpartners and impose the mass limits coming from direct searches at LEP and at the Tevatron. For the rates of LFV processes, we use the expressions of Ref. [37]. The supersymmetric contributions to BR($B_{d,s}^0 → μ^+μ^-$) are estimated using the formulae of Ref. [38], while BR($b → sγ$) is computed with the help of the routine SusyBSG [39]. The supersymmetric contributions to the meson mass splittings $Δm_K$, $Δm_D$, $Δm_B$, $Δm_{Bs}$ and to the indirect CP violation parameter $ε_K$ are computed in the mass insertion approximation, using the formulae of Ref. [40]. We recall in Table 2 the experimental values and upper limits for these observables.

As observed in Section 3.2 and illustrated in Fig. 4, the predictions of the model for LFV processes span many orders of magnitude, due to their strong dependence on the ratio $M_Δ/λ_H$. In order to obtain definite predictions, we shall restrict the seesaw parameter space to the region favoured by the leptogenesis scenario of Ref. [7], which is built in
the SO(10) model considered here. The source of the lepton asymmetry is the CP asymmetry in triplet decays, which arises from the interference between the tree-level and one-loop diagrams shown in Fig. 2 and is given by (in the limit $M_{L_1} \ll M_\Delta < M_{L_1} + M_{L_2}$): 

$$
\varepsilon_\Delta \simeq (0.1/10\pi) \frac{\text{Im}[f_{11}(f^* f^*)_{11}]}{\text{Tr}(f f^*) + |f_{11}|^2 + |\lambda_H|^2 (1 + \cos^4 \theta_H)},
$$

where $M_S = M_T = 10M_\Delta$ has been used. The presence of heavy lepton fields with hierarchical masses is crucial for generating a non-vanishing CP asymmetry. In particular, the condition $M_\Delta > 2M_{L_1}$ must be satisfied for the loop integral to have an imaginary part.

Let us identify the region of the $(M_\Delta, \lambda_H)$ parameter space in which successful leptogenesis is possible. One can distinguish between two regimes [7]: the first one, characterized by a large value of the CP asymmetry in triplet decays and by a strong washout, requires order one values of the $f_{ij}$ couplings. This however is in conflict with the experimental upper bounds on lepton flavour violation, unless the supersymmetric spectrum is very heavy. In the second regime, the CP asymmetry is smaller, but the efficiency factor accounting for the dilution of the generated lepton asymmetry by washout processes can be of order one. Given that the observed baryon asymmetry is reproduced for $\eta\varepsilon_\Delta \simeq 10^{-8}$, where $\eta$ is the efficiency factor, successful leptogenesis is possible for $\varepsilon_\Delta \gtrsim 2 \times 10^{-8}$. In Fig. 3 we plot $\varepsilon_\Delta$ as a function of the seesaw parameters $M_\Delta$ and $\lambda_H$. The light neutrino parameters are chosen as in Ref. [7], with the best fit values for the measured oscillation parameters taken from Ref. [15] and $m_1 = 0.005 \text{ eV}$, $\sin^2 \theta_{13} = 0.05$, $\rho = \pi/4$, $\sigma = \pi/2$ and $\delta = 0$. In the region $M_\Delta \lesssim 6 \times 10^{11} \text{ GeV}$, the condition for a non-vanishing CP asymmetry, $M_\Delta > 2M_{L_1}$, is not satisfied, while in the small $\lambda_H$ region some of the $f_{ij}$ couplings become non-perturbative below the GUT scale, making the computation of $\varepsilon_\Delta$ non reliable. The white line separates the weak and strong washout
Figure 4: Dependence of $\text{BR}(\mu \rightarrow e \gamma)$ on $M_\Delta$ and $\lambda_H$, for the following choice of mSUGRA parameters: $m_0 = M_{1/2}/2 = 700$ GeV, $a_0 = 0$, $\tan \beta = 10$ and $\mu > 0$. The other parameters are chosen as in Fig. 3. The white dots indicate the area where both $\text{BR}(\mu \rightarrow e \gamma) < 1.2 \times 10^{-11}$ and $\varepsilon_\Delta > 2 \times 10^{-8}$.

regimes: below this line, the conditions for a large efficiency, $\Gamma(\Delta \rightarrow \tilde{L}_1 \tilde{L}_1) < H(M_\Delta)$ and $\Gamma(\Delta \rightarrow H_u H_u) > H(M_\Delta)$, are not satisfied (see Ref. [7] for details). As shown by Fig. 3, $\varepsilon_\Delta$ can reach sizable values in the large efficiency region.

Let us now see how the LFV decay $\mu \rightarrow e \gamma$ constrains the parameter space of Fig. 3. This is shown in Fig. 4 for a point of the MSSM (mSUGRA) parameter space giving a rather light superpartner spectrum: $m_0 = M_{1/2}/2 = 700$ GeV, $a_0 = 0$, $\tan \beta = 10$ and $\mu > 0$. The brown (dark) area is excluded by the experimental upper limit $\text{BR}(\mu \rightarrow e \gamma) < 1.2 \times 10^{-11}$. Comparing Fig. 4 with Fig. 3, one observes a tension between the requirement of successful leptogenesis and the experimental constraint on $\mu \rightarrow e \gamma$, due to the fact that both $\varepsilon_\Delta$ and $\text{BR}(\mu \rightarrow e \gamma)$ grow with the ratio $M_\Delta/\lambda_H$ (to which the $f_{ij}$ couplings are proportional). Indeed, the region of the $(M_\Delta, \lambda_H)$ parameter space consistent with both $\text{BR}(\mu \rightarrow e \gamma) < 1.2 \times 10^{-11}$ and $\varepsilon_\Delta > 2 \times 10^{-8}$, marked with white dots in Fig. 4, is rather small for the chosen MSSM parameters.

An heavier superpartner spectrum and/or a smaller value of $\tan \beta$ would increase the size of this region by relaxing the $\mu \rightarrow e \gamma$ constraint, without affecting leptogenesis. A smaller value of $\theta_{13}$ would also relax the $\mu \rightarrow e \gamma$ constraint, but it would simultaneously reduce $\varepsilon_\Delta$. Nevertheless, the requirement of successful leptogenesis (together with the non-observation of $\mu \rightarrow e \gamma$) considerably restricts the seesaw parameter space, thus allowing us to make testable predictions for flavour-violating observables.

In the following, we present our results for a set of seesaw parameters belonging to the region

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6 In passing, this shows that the supersymmetric version of the leptogenesis scenario proposed in Ref. [7] can be excluded on the basis of low-energy flavour physics measurements, if the supersymmetric partners are accessible at the LHC (and barring cancellation with other sources of flavour violation in the soft terms).
Figure 5: Predictions for BR(\(\mu \rightarrow e\gamma\)) in the \((m_0, M_{1/2})\) plane, assuming \(a_0 = 0\), \(\tan \beta = 10\) and \(\mu > 0\). The parameters of the SO(10) model are: \(M_\Delta = 10^{12}\) GeV, \(\lambda_H = 0.045\), \(V_1 = M_{\text{GUT}}\) and \(\tan \theta_H = 1\). The neutrino parameters are chosen as in Fig. 3. Where successful leptogenesis is possible, namely we take \(M_\Delta = 10^{12}\) GeV and \(\lambda_H = 0.045\), together with \(V_1 = M_{\text{GUT}}\) and \(\tan \theta_H = 1\) (which gives \(\varepsilon_\Delta \simeq 2.5 \times 10^{-8}\)). The corresponding spectrum of heavy states is shown in Fig. 1(a). As for the neutrino parameters, we choose them as in Fig. 3. We are now ready to present the predictions of the model for various flavour-violating observables as a function of the MSSM parameters. In Fig. 5 the contours of BR(\(\mu \rightarrow e\gamma\)) are plotted in the \((m_0, M_{1/2})\) plane for \(a_0 = 0\), \(\tan \beta = 10\) and \(\mu > 0\) (the contours corresponding to different values of \(\tan \beta\) can be easily deduced from Fig. 5 by noting that BR(\(\mu \rightarrow e\gamma\)) approximately scales as \(\tan^2 \beta\)). The parameter space is bounded by the LEP limits on superpartner and Higgs boson masses for low values of \(M_{1/2}\), and by the absence of radiative electroweak symmetry breaking for large values of \(m_0\) and moderate \(M_{1/2}\). BR(\(b \rightarrow s\gamma\)) gives a constraint similar to the Higgs mass bound, while the other hadronic observables of Table 2 do not significantly restrict the parameter space. This implies that the flavour physics signatures of the model are expected to show up in the lepton sector rather than in the hadronic sector (with the possible exception of \(\varepsilon_K\) discussed at the end of this section). One can see from Fig. 5 that the on-going experiment MEG [41], which aims at a sensitivity of \(\mathcal{O}(10^{-13})\) on BR(\(\mu \rightarrow e\gamma\)), will probe the model over a large portion of the MSSM parameter space. Interestingly, this region approximately corresponds to the one that is accessible at the LHC. This can be seen in Fig. 6(a) where the contours of the gluino and of the lightest stop masses are plotted in the \((m_0, M_{1/2})\) plane, and the area that will be probed by MEG is marked with dots. For completeness, we show in Fig. 6(b) the full supersymmetric spectrum corresponding to the mSUGRA point of Fig. 4, i.e. \(m_0 = M_{1/2} = 700\) GeV, \(a_0 = 0\), \(\tan \beta = 10\) and \(\mu > 0\). Note that this spectrum is characterized by rather heavy sfermions compared to neutralinos and charginos. This is due to the relatively large value of the unified gauge coupling, which enhances the gauge contributions to the running of sfermion masses at high energy. As a result, the lightest neutralino is found to be the LSP over the whole \((m_0,\)
which explains why the blue dotted line stops at $m_{\tilde{t}_1} \approx 1750$ GeV.

In Section 3.2, it is generally smaller than BR($\tau \to \mu \gamma$). Given the experimental upper limits shown in Table 2, BR($\tau \to e \gamma$) is out of reach of super B factories, $\mu - e$ conversion looks more promising, given that proposed experiments at Fermilab [42] and at J-PARK [43] aim at respective sensitivities of $10^{-16}$ and $10^{-18}$ on CR($\mu \to e$ in Ti). If approved, these experiments would test the model well beyond the MEG reach.

In order to estimate the impact of non-universal (but flavour-blind) boundary conditions on the predictions for LFV observables, we performed a random scan of the soft scalar masses for different SO(10) multiplets between 0 and 3 TeV, assuming a fixed value $M_{1/2} = 700$ GeV of the common gaugino mass parameter. The results for BR($\mu \to e \gamma$) are shown in Fig. 7(b) where the blue dotted line[7] corresponds to the universal case $m_{16} = m_{10} = m_{16\mu} = m_{10\mu} = m_{54} \equiv m_0$. Relaxing the universality of soft scalar masses can enhance or suppress BR($\mu \to e \gamma$) by up to 2 orders of magnitude, but most of the points still remain within the reach of MEG (unless the lightest slepton is very heavy).

As mentioned earlier, LFV observables provide much stronger constraints on the model
Figure 7: (a) BR($\tau \rightarrow \mu \gamma$), BR($\mu \rightarrow eee$) and CR($\mu \rightarrow e$ in Ti) versus BR($\mu \rightarrow e\gamma$) for the same choice of parameters as in Fig. 5, scanning over $m_0$ and $M_{1/2}$ in the ranges $0 < m_0 < 3$ TeV and $0 < M_{1/2} < 2$ TeV. The grey area is excluded by the experimental upper limit on BR($\mu \rightarrow e\gamma$). (b) BR($\mu \rightarrow e\gamma$) as a function of the lightest slepton mass for non-universal soft scalar masses $m_{16}$, $m_{10}$, $m_{16\mu}$, $m_{10\mu}$ and $m_{54}$ in the [0, 3 TeV] range, and $M_{1/2} = 700$ GeV. The blue dotted line corresponds to the universal case $m_{16} = m_{10} = m_{16\mu} = m_{10\mu} = m_{54} \equiv m_0$.

studied in this paper than hadronic observables. A possible exception is represented by the indirect CP violation parameter in the kaon sector, $\varepsilon_K$. It is well known that $\varepsilon_K$ is very sensitive to new sources of flavour and CP violation in the 1-2 down squark sector. In the present model, radiative corrections generate large off-diagonal entries both in the LL slepton mass matrix (which leads to large LFV rates) and in the RR down squark mass matrix. Moreover, order one phases in the PMNS matrix, hence in the $f_{ij}$ couplings, are needed to account for the baryon asymmetry of the universe. Unfortunately, unknown extra SO(10) phases spoil the link between CP violation in the neutrino sector and the RG-induced CP violation in the sfermion mass matrices (see the discussion in Section 3.2). Nevertheless, barring cancellations between contributions carrying different phases, it is reasonable to expect a large imaginary part of $(\delta_{RR}^d)_{12}$. In Fig. 8 the contours of the supersymmetric contribution to $\varepsilon_K$ are plotted in the $(m_0, M_{1/2})$ plane for the same choice of parameters as in Fig. 5 assuming $\arg[(\delta_{RR}^d)_{12}] = 0.5$. While low values of $M_{1/2}$ would give too large a contribution to $\varepsilon_K$, a rather light superpartner spectrum can account for up to $(10-20)\%$ of its experimental value. According to Ref. [28], this is precisely what is needed in order to reconcile the SM prediction for $\varepsilon_K$ with experiment.

6 Conclusions

We have studied flavour violation in a supersymmetric SO(10) implementation of the type II seesaw mechanism, which provides a predictive realization of triplet leptogenesis. In this scenario, the high-energy flavour parameters involved in the computation of the baryon asymmetry of the universe and of flavour-violating observables are determined in terms of the Standard
Model fermion masses and mixing, up to mild model-dependent uncertainties. The overall size of the FCNC effects is then controlled by a few unknown flavour-blind parameters, while the ratios of FCNC rates for different flavour channels, such as \( \text{BR}(\tau \to \mu \gamma)/\text{BR}(\mu \to e \gamma) \), mainly depend on low-energy parameters.

The features of flavour violation in the SO(10) scenario studied in this paper present some interesting differences with the SU(5) implementation of the type II seesaw mechanism [9], because of additional contributions coming from the heavy matter fields crucial for leptogenesis. These give rise, for example, to radiative corrections to the soft mass matrices \( m^2_{ee} \) and \( m^2_{q} \) controlled by the top quark Yukawa coupling. Moreover, the presence of a built-in leptogenesis mechanism provides a criterion for fixing the values of the unknown flavour-blind parameters, thus yielding testable predictions for the rates of flavour-violating processes. Imposing the conditions for successful leptogenesis together with the experimental constraints on FCNCs, we found that the predicted branching ratio for \( \mu \to e \gamma \) lies within the sensitivity of the MEG experiment if the superpartner spectrum is accessible at the LHC, while \( \tau \to \mu \gamma \) is out of reach of future super B factories. \( \mu - e \) conversion on Titanium is also a promising process, with a predicted rate within the reach of proposed experiments at Fermilab and J-PARK even in regions of the parameter space where \( \text{BR}(\mu \to e \gamma) \) lies below the MEG sensitivity. Hadronic observables only receive small contributions once the experimental bounds on LFV processes are imposed, with the possible exception of \( \varepsilon_K \).

We have also studied flavour-violating contributions to leptonic and hadronic EDMs, as well as to the \( \varepsilon_K \) parameter. The CKM and PMNS phases, together with new CP-violating phases associated with the SO(10) structure, can in principle induce sizable contributions to these observables, even if the soft terms are real at the high scale. The experimental bounds on LFV processes, however, prevent significant contributions to the EDMs, while a sizable contribution to \( \varepsilon_K \) (which might be necessary to account for its measured value) is still possible.

The predictivity of the scenario studied in this paper relies on the SO(10) relations between
the flavour structures of the SM and heavy matter fields. These relations, however, only hold at the tree level and may be affected by model-dependent corrections from the non-renormalizable operators necessary to account for the measured quark and lepton masses. Nevertheless, the impact of these corrections on our results can be estimated to be mild.

The presence of heavy states below the GUT scale, besides inducing low-energy flavour and CP violation through radiative corrections, also has an impact on the superpartner spectrum. Indeed, their contribution to the beta functions leads to a relatively large value of the unified gauge coupling, which enhances the gauge contribution to the running of sfermion masses. One of the consequences of this is that the lightest neutralino turns out to be the LSP in an unusually large portion of the parameter space.

Finally, we have shown that the SO(10) implementation of the type II seesaw mechanism studied in this paper can be promoted to a consistent model including the dynamics of gauge symmetry breaking, doublet-triplet splitting consistent with the present experimental bounds on proton decay, and gauge coupling unification. The doublet-triplet splitting is achieved through a generalization of the missing vev mechanism. The proton decay rate is suppressed by arranging, without fine-tuning, a pair of Higgs doublets to lie at an intermediate scale. Gauge coupling unification is then restored by an appropriate splitting of the SU(5) components of the $54$ containing the type II seesaw triplet, which in turn brings $\alpha_3(m_Z)$ within 1σ from its experimental value, thus improving on the MSSM prediction. In passing, we provided an updated, comprehensive analysis of proton decay from D=5 and D=6 operators.

Acknowledgments

We thank F. Joaquim and A. Rossi for useful comments. The work of MF was supported in part by the Marie-Curie Intra-European Fellowship MEIF-CT-2007-039968. AR acknowledges partial support from the RTN European Program “UniverseNet” (MRTN-CT-2006-035863).
A SO(10) gauge symmetry breaking

The purpose of this appendix is to provide an explicit sector breaking the SO(10) gauge symmetry down to the SM gauge group. It is well known that this breaking can be realized by the most general renormalizable superpotential involving the following Higgs representations: one $45$, one $54$ and one $(16, \overline{16})$ pair. In this case, both SM singlets in the $45$ acquire a nonzero vev. However, we need a $45$ multiplet with a vev aligned in the $B-L$ direction to implement the missing vev mechanism for doublet-triplet splitting (see Appendix A). In order to obtain such a vacuum alignment, we must introduce additional fields and consider a non-generic superpotential. A simple realization of this is provided by the following superpotential:

$$W_{SB} = \frac{1}{2} f_1 54' 45_1 45_1 + (\lambda_1 S + f_{12} 54') 45_1 45_2 + \frac{1}{3} \lambda S 54' 54' 54' + \overline{16} (M_{16} + g 45_1) 16 \ ,$$ (A.1)

where $S$ is an SO(10) singlet and the contractions of SO(10) indices are understood. As shown below, the $54'$ acquires a GUT-scale vev and therefore cannot be identified with the $54$ multiplet involved in the type II seesaw mechanism.

Altogether, eight SM singlets can acquire a nonzero vev at the GUT scale. We normalize their vevs as follows: $\langle \text{Tr}(54'^4 45'_i) \rangle = |V_{PS}|^2$, which breaks SO(10) down to its Pati-Salam subgroup, $\langle \text{Tr}(54'_i 45) \rangle = |V_{B-L}|^2 + |V_R|^2$ ($i = 1, 2$), $(16')16 = |V_{12}|^2$, $(\overline{16} \overline{16}) = |\overline{V}_1|^2$ and $\langle S \rangle = S$. In order to preserve supersymmetry, these vevs must satisfy F-flatness and D-flatness conditions. The solution to these constraints reads

$$V_{B-L}^{(1)} = V_R^{(2)} = 0 \ , \quad V_R^{(1)} = \frac{1}{2g} M_{16} \ , \quad V_{B-L}^{(2)} = -\frac{3\sqrt{3} f_1}{10 \sqrt{2} f_{12} g} M_{16} \ ,$$

(A.2)

together with $|V_1| = |\overline{V}_1|$. If the dimensionless couplings are of order one, then all nonzero vevs are of order $M_{16}$, the unique mass parameter in $W_{SB}$. Therefore SO(10) is broken down to the SM gauge group in one step. The two $45$ vevs are aligned along the $T_{3R}$ and $B-L$ directions, respectively; in the rest of the paper we will rename them $V_R \equiv V_R^{(1)}$ and $V_{B-L} \equiv V_{B-L}^{(2)}$.

The SM vacuum defined by Eq. (A.2) provides all uneaten chiral superfields with a GUT-scale mass, except for a pair of charged states with $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers $(1, 1, \pm 1)$, which remains massless. This problem can be cured by adding the following terms to $W_{SB}$:

$$\frac{1}{2} \lambda_3 S 45_3 45_3 + \lambda_{123} 45_1 45_2 45_3 \ .$$ (A.3)

This does not modify the vacuum alignment discussed above and does not introduce extra mass parameters either. The F-flatness conditions imply that the $45_3$ does not acquire a vev. The coupling $\lambda_{123}$ is sufficient to make the unwanted massless states heavy, while the coupling $\lambda_3$ guarantees that all $45_3$ components are also massive.\footnote{Let us mention for completeness that the D-flatness and F-flatness conditions admit another solution characterized by $V_{B-L}^{(2)} = V_R^{(1)} = 0$ and all other vevs nonzero. This solution would also be satisfactory for our purposes, but we stick to the first one for definiteness.}

\footnote{In the alternative vacuum characterized by $V_{B-L}^{(2)} = V_R^{(1)} = 0$, the massless states are $(3, 1, -2/3) \oplus (3, 1, 2/3)$.}
For completeness, we give the masses of the $X$ and $Y$ gauge bosons, which can mediate proton decay through $D = 6$ operators (see Appendix B.3):

$$M_X^2 = M_Y^2 = g_{\text{GUT}}^2 \left( \frac{5}{6} |V_{PS}|^2 + \frac{1}{3} |V_{B-L}|^2 + \frac{1}{2} |V_R|^2 \right),$$

(A.4)

where $g_{\text{GUT}}$ is the SO(10) gauge coupling.

We remark that the desired vev alignment is obtained because several couplings allowed by the SO(10) gauge symmetry are absent in Eq. (A.1). In fact, one can add mass terms for $45_1$ and $54'$ without upsetting the vev alignment. On the contrary, it is crucial to forbid all $45_2$ couplings except $\lambda_{12}$ and $f_{12}$. We do not try to justify the non-generic form of $W_{SB}$ by global symmetries, which would require a more complicated set of fields and interactions.

Note finally that it is possible to align the vev of one of the two $45$'s without introducing the SO(10) singlet $S$. In fact, if one replaces $\lambda_{12}S$ in Eq. (A.1) by a bare mass $M_{12}$, the F-flatness equations still have a solution with $V_R^{(1)} = 0$ and a second one with $V_{B-L}^{(1)} = 0$, all other vevs being nonzero. This is sufficient to realize the doublet-triplet splitting, but this requires the presence of two mass parameters ($M_{16}$ and $M_{12}$) in $W_{SB}$.

B Doublet-triplet splitting and proton decay

In this appendix, we provide an explicit doublet-triplet splitting mechanism for the SO(10) scenario studied in this paper, and derive the conditions imposed by the non-observation of proton decay on the heavy spectrum.

B.1 Doublet-triplet splitting

In the SO(10) implementation of the type II seesaw mechanism considered in this paper, up quark masses arise from the superpotential couplings $16_116_{10}$, while down quark and charged lepton masses arise from the couplings $16_{10}16$. The $Y = +1/2$ MSSM Higgs doublet $h_u$ should therefore reside mainly in the $10$ in order to account for the large top quark mass, while the $Y = -1/2$ Higgs doublet $h_d$ should contain a significant $H_d^{16}$ component, and we require that it also has a non-vanishing $H_{d'}^{10}$ component\footnote{Indeed, the leptogenesis scenario of Ref. \cite{leptogenesis} assumes that the dominant decay modes of the heavy matter fields preserve $B - L$, which is guaranteed if $h_d$ contains a non-negligible $H_d^{10}$ component.}. The doublet-triplet splitting mechanism, besides rendering all colour triplets coupling to light matter fields heavy, should therefore leave $H_u^{10}$ and an admixture of $H_d^{10}$ and $H_{d'}^{16}$ massless.

This can be achieved by a generalization of the missing vev mechanism \cite{missing_vev} involving an additional $10'$ Higgs multiplet and an adjoint Higgs field with a non-vanishing vev in the $B - L$ direction (which motivates the choice made for $45_2$ in Appendix A). The superpotential that accomplishes the desired doublet-triplet splitting reads:

$$W_{DT} = \frac{1}{2} M_{10'}10'10 + h_{10'}45_210 + \overline{16}(M_{16} + g45_1)16 + \frac{1}{2} \overline{16}16(\eta_{10} + \rho 10'),$$

(B.1)

where the third term also plays a role in SO(10) symmetry breaking (and therefore appears in Eq. (A.1)), while the coupling $\rho \overline{16}1610'$ is optional. As in the case of Eq. (A.1), this is not the most general superpotential for the fields involved. After SO(10) symmetry breaking, one
obtains the following doublet and triplet mass matrices, written in the \((\mathbf{10}, \mathbf{10}', \mathbf{16})\) (left) and \((\mathbf{10}, \mathbf{10}', \mathbf{16})\) (right) bases:

\[
M_D = \begin{pmatrix}
0 & 0 & -\eta V_1 \\
0 & M_{10}' & -\rho V_1 \\
0 & 0 & M_{16}
\end{pmatrix},
\quad
M_T = \begin{pmatrix}
0 & h_{V_{B-L}} & -\eta V_1 \\
\sqrt{6} V_{B-L} & M_{10}' & -\rho V_1 \\
0 & 0 & M_{16} + 2g V_R
\end{pmatrix}.
\]

From Eq. \((\text{B.2})\) we can see that all colour triplets and two pairs of Higgs doublets acquire GUT-scale masses, while \(H_{u_{10}}\) and a combination of \(H_{d_{10}}\) and \(H_{d_{16}}\) remain massless:

\[
h_u = H_{u_{10}}, \quad h_d = \cos \theta_H H_{d_{10}} + \sin \theta_H H_{d_{16}},
\]

where the Higgs mixing angle \(\theta_H\) is given by (with no loss of generality, we assume \(\eta V_1\) and \(M_{16}\) to be real):

\[
\tan \theta_H = \frac{\eta V_1}{M_{16}}.
\]

Note that the alignment of the \(45_2\) vev along the \(B - L\) direction is crucial for the splitting of the doublet and triplet components.

A few comments are in order about the above doublet-triplet mechanism. First, the SO(10)-breaking sector contains a \(54'\) representation with a vev in the Pati-Salam direction, whose couplings to \(10 10\) and \(10 10'\) must be forbidden as they would make all Higgs doublets heavy. On the contrary, the coupling \(10 10 54\) involved in the seesaw mechanism is harmless since \(54\) does not acquire a vev. Second, the mass term \(M_{16}1616\) in Eqs. \((\text{A.1})\) and \((\text{B.1})\) is crucial both for GUT symmetry breaking and for the doublet-triplet splitting: if it were absent, F-flatness would imply either \(V_1 = 0\), leaving the SO(10) rank unbroken, or \(V_R = 0\), leaving a pair of colour triplets massless.

### B.2 Higgs-mediated proton decay

In general, giving GUT-scale masses to the colour Higgs triplets is not enough to suppress the proton decay rate below its experimental upper limit. To achieve this, one must impose additional constraints on the triplet mass matrix. We show below that this can be done in a simple way in the doublet-triplet splitting scenario presented above.

Let us first adapt the standard computation of the Higgs-mediated proton decay rate \([44, 45, 46, 47]\) to our case. Upon integrating out the heavy Higgs triplet superfields, one obtains the following D=5 operators:

\[
\frac{1}{2} \kappa_{ijkl} (Q_i Q_j)(Q_k L_l) \bigg|_{g_2} + \frac{1}{2} \kappa'_{ijkl} u^c_i u^c_j d^c_k e^c_l \bigg|_{g_2} + \text{h.c.},
\]

where the contraction of gauge indices is understood, and the dimensionful coefficients \(\kappa_{ijkl}\) and \(\kappa'_{ijkl}\) generated by the superpotential \([11]\) are given by \([11]\):

\[
\kappa_{ijkl} = -y_{ij} h_{kl} \left(M_T^{-1}\right)_{T^{10}T^{16}}, \quad \kappa'_{ijkl} = -(y_{il}h_{jk} - y_{jl}h_{ik}) \left(M_T^{-1}\right)_{T^{10}T^{16}},
\]

where \(\left(M_T^{-1}\right)_{T^{10}T^{16}}\) is the \((T^{10}, T^{16})\) entry of the inverse triplet mass matrix. Since colour invariance implies \(\kappa_{iiil} = 0\) and \(\kappa'_{iiil} = 0\), the dominant proton decay modes arising from

\[\text{Note that the flavour structure of } \kappa_{ijkl} \text{ and } \kappa'_{ijkl} \text{ is the same as in the minimal SU(5) model. This is an obvious consequence of the way the SM matter fields are embedded into SO(10) representations.}\]
the above operators involve a kaon, and in practice $p \to K^+ \bar{\nu}$ dominates. The corresponding amplitude is obtained by “dressing” the D=5 operators of Eq. \ref{eq:B.5} with gaugino/higgsino loops. If the first two generation squarks are close in mass, as the observed weakness of hadronic flavour-violating processes may suggest, the dominant contributions come from the wino dressing of the QQQL operator and from the charged higgsino dressing of the $u^c u^c d^c e^c$ operator. Here we consider only the former contribution; the latter (which is significant for the $K^+ \bar{\nu}_e$ channel only) would give a stronger constraint on $M_T$ only for large values of $\tan \beta$. The proton partial decay width then reads:

$$
\Gamma(p \to K^+ \bar{\nu}) = \frac{(m_p^2 - m_{K^+}^2)^2}{32\pi f_\pi^2 m_p^2} |\beta_H|^2 \\
\times \sum_i \left[ 1 + \frac{m_p}{3m_A} (D + 3F) \right] C_{112i} + \left[ \frac{m_p}{2m_{\Sigma^0}} (D + F) + \frac{m_p}{6m_A} (D + 3F) \right] C_{121i} \right]^2 \quad (B.7)
$$

where $f_\pi = 130$ MeV is the pion decay constant; $\beta_H$ is the hadronic parameter defined by $\beta_H P_L u_p = \langle 0 | (u_L d_L) u_L | p \rangle$, where $u_p$ is the proton spinor and the parenthesis indicates the contraction of Lorentz indices; D and F are chiral Lagrangian parameters; and $C_{ijkl}$ is the Wilson coefficient of the four-fermion operator $(u_L d_L)(\bar{d}_L k_L \nu_l)$. The most recent lattice determination of $\beta_H$ is $|\beta_H| = 0.0120(26)$ GeV$^3$ \footnote{See Ref. \ref{48}, while an analysis of hyperon decay measurements gives $F + D = 1.2670 \pm 0.0030$ and $F - D = -0.341 \pm 0.016$ \footnote{See Ref. \ref{49}}. The Wilson coefficients $C_{ijkl}$ read:

$$
C_{ijkl} = \frac{\alpha_2}{4\pi} A_R \sum_{m,n} V_{mj} V_{nk} (\kappa_{imnl} - \kappa_{mnil}) \left[ f(\tilde{d}_{Lm}, \tilde{u}_{Lm}) + f(\tilde{u}_{Ln}, \tilde{e}_{Li}) \right], \quad (B.8)
$$

where $A_R$ is a renormalization factor, $V$ is the CKM matrix, the $\kappa$ coefficients are expressed in the basis $q = (u, d') \equiv (u, V d)$, and the loop function $f$ is given by ($M_2$ is the wino mass):

$$
f(a, b) = \frac{M_2}{m_a^2 - m_b^2} \left[ \frac{m_a^2}{m_a^2 - M_2^2} \ln \left( \frac{m_a^2}{M_2^2} \right) - \frac{m_b^2}{m_b^2 - M_2^2} \ln \left( \frac{m_b^2}{M_2^2} \right) \right]. \quad (B.9)
$$

For degenerate fermion masses ($m_a = m_b \equiv \tilde{m}$) and a hierarchy $M_2^2 \ll \tilde{m}^2$, $f(a, b)$ reduces to $M_2^2/\tilde{m}^2$ to a good approximation. In Eq. \ref{eq:B.5}, the quantity $(\kappa_{imnl} - \kappa_{mnil})$ is evaluated at the GUT scale, where the operator QQQL is generated. The renormalization factor $A_R = A_{SD} A_{LD}$ contains a short-distance piece $A_{SD}$ which accounts for the renormalization of the superpotential operator QQQL from $M_{\text{GUT}}$ to the supersymmetry breaking scale (here identified with $m_Z$), and a long-distance piece $A_{LD}$ which encodes the renormalization of the four-fermion operator $(u_L d_L)(\bar{d}_L k_L \nu_l)$ from $m_Z$ to $\mu_{\text{had}} = 1$ GeV. The latter is given by:

$$
A_{LD} = \left( \frac{\alpha_3(\mu_{\text{had}})}{\alpha_3(m_c)} \right)^{2/9} \left( \frac{\alpha_3(m_c)}{\alpha_3(m_b)} \right)^{6/25} \left( \frac{\alpha_3(m_b)}{\alpha_3(m_Z)} \right)^{6/23} \simeq 1.4, \quad (B.10)
$$

and the former by (neglecting the Yukawa contributions):

$$
A_{SD} = \prod_I \left( \frac{\alpha_1(M_I)}{\alpha_1(M_{I+1})} \right)^{-1/b_I^1} \left( \frac{\alpha_2(M_I)}{\alpha_2(M_{I+1})} \right)^{-3/b_I^2} \left( \frac{\alpha_3(M_I)}{\alpha_3(M_{I+1})} \right)^{-4/b_I^3}, \quad (B.11)
$$

where $I$ runs over all mass thresholds $M_I$ between $m_Z$ and $M_{\text{GUT}}$, and $b_I^{1,2,3}$ are the beta function coefficients between $M_I$ and $M_{I+1}$. Given the hierarchy among the quark Yukawa...
where we made use of the constraint \( \tau(p \rightarrow K^+\bar{\nu}) > 2.3 \times 10^{33} \) yrs (90\% C.L.) \[50\]. For definiteness, we consider the model (ii) of Section \ref{sec:models} with \( M_A = 10^{12} \) GeV, \( \tan \beta = 10 \) and a spectrum close to the one of Fig. \[6\](b), with squark masses in the TeV range (except for the lightest stop), sleptons masses around 750 GeV and \( M_2 \approx 280 \) GeV. This spectrum gives \( f(\tilde{d}_L, \tilde{c}_L) + f(\tilde{e}_L, \tilde{\nu}_L) \approx 0.45 \text{ TeV}^{-1} \).

Evaluating the Yukawa couplings at the GUT scale, we find \( \lambda_u V_{ud} V_{us} = 5.2 \times 10^{-7} \), \( \lambda_e V_{cd} V_{cs} = -2.6 \times 10^{-4} \), \( \lambda_l |V_{td} V_{ts}| = 7.5 \times 10^{-5} \), and \( \lambda_d V_{ud}, \lambda_s V_{us}, \lambda_b V_{ub} = (0.43, 1.9, 2.0) \times 10^{-4} \). Depending on the values of the high-energy phases, a partial cancellation between the \( i_L \) and the \( \tilde{c}_L \) contributions is possible. Here we do not consider this possibility and keep only the dominant \( \tilde{c}_L \) contribution, which gives:

\[
\sqrt{\sum_i |C_{11i}|^2} \simeq 9.9 \times 10^{-13} \text{ GeV}^{-1} \left( A_{SD} / 8.1 \right) \left( \frac{1}{\sin \theta_H} \right) \left( \frac{2f}{0.45 \text{ TeV}^{-1}} \right) (M^{-1}_T)_{T_{10^{30^{16}}}, \text{B.13}}
\]

where we have replaced \( f(\tilde{d}_L, \tilde{c}_L) + f(\tilde{e}_L, \tilde{\nu}_L) \) by an average value \( 2f \), and \( A_{SD} \approx 8.1 \) (to be compared with 7.8 in the MSSM with \( \tan \beta = 10 \)) takes into account the various thresholds shown in Fig. \[1\](a). From Eqs. \[B.7\] and \[B.13\], we finally obtain:

\[
\frac{\tau(p \rightarrow K^+\bar{\nu})}{2.3 \times 10^{33} \text{ yrs}} = \left( \frac{0.012 \text{ GeV}^3}{|eta_H|} \right)^2 \left( \frac{\sin \theta_H}{1} \right)^2 \left( \frac{0.45 \text{ TeV}^{-1}}{2f} \right)^2 \left( \frac{(4.4 \times 10^{18} \text{ GeV})^{-1}}{(M^{-1}_T)_{T_{10^{30^{16}}}}} \right)^2, \text{B.14}
\]

which yields the upper bound \( (M^{-1}_T)_{T_{10^{30^{16}}}} \lesssim 4.4 \times 10^{18} \text{ GeV} / (\sin \theta_H) / (\tan \beta / 10)^{-1} \), where we have restored the approximate \( \tan \beta \) dependence of the proton decay amplitude (one can indeed see from Eq. \[B.12\] that \( C_{112i} \) and \( C_{121i} \) scale as \( 1 / \sin 2\beta \)). This bound is a conservative one, and several effects could relax it: there are still large uncertainties on the hadronic parameter \( \beta_H \); \( A_{SD} \) overestimates the running of the \( \kappa \) coefficients, since it does not include the Yukawa contribution; a superpartner spectrum characterized by heavier sfermions and/or a stronger sfermion/gaugino mass hierarchy would reduce the proton decay amplitude; corrections to the mass relation \( M_d = M_T \) could affect the couplings of the Higgs triplets to quarks and leptons; and finally, some sfermion mixing patterns compatible with the observed level of flavour violation can significantly reduce the proton decay amplitude \[51\].

Let us now translate this bound into constraints on the doublet-triplet splitting parameters.

From Eq. \[B.2\] we have:

\[
(M^{-1}_T)_{T_{10^{30^{16}}}} = \frac{\sqrt{3} \sqrt{\nu_1 (\sqrt{6} \eta M_{10'} - \rho h V_{B-L})}}{\sqrt{2} M_{16} (h V_{B-L})^2} = \frac{3 \tan \theta_H M_{10'} (h V_{B-L})^2}{\sqrt{2} M_{16} h V_{B-L}}, \text{B.15}
\]

where we made use of \( V_R = M_{16}/2g \) and \( \tan \theta_H = \eta \sqrt{\nu_1}/M_{16} \). Since the SO(10) gauge symmetry is broken in one step (see Appendix \[A\]), \( \sqrt{\nu_1} \sim V_{B-L} \sim M_{16} \sim M_{GUT} \) and the desired suppression of \( (M^{-1}_T)_{T_{10^{30^{16}}}} \) with respect to its natural value \( M^{-1}_{GUT} \) can be achieved by a partial cancellation.
in the combination \((\sqrt{6}\eta M_{10'} - \rho h V_{B-L})\), or by taking \(\rho \ll 1\) and \(M_{10'}\) in the \((10^{13} - 10^{14})\) GeV range, where the lower value corresponds to the conservative bound on \((M_T^{-1})_{T \to T'^{16}}\). Note that \(\tan \theta_H \ll 1\) would not help, since the upper bound on \((M_T^{-1})_{T \to T'^{16}}\) becomes stricter for smaller values of \(\theta_H\). In this paper we take\(^{12}\) \(M_{10'} = 10^{14}\) GeV in order to avoid a strong cancellation among unrelated superpotential parameters. One can check that all colour Higgs triplets acquire GUT-scale masses in this case, while one pair of Higgs doublet sits at the intermediate scale \(M_{10'}\). The consequences for gauge coupling unification are discussed in Appendix C and summarized in Section 4.3.

### B.3 Gauge-mediated proton decay

Let us now discuss the contribution of SO(10) gauge interactions to proton decay. Due to the way the SM matter fields are embedded into SO(10) representations, there are no new gauge contributions with respect to the SU(5) case, i.e. only the X and Y gauge bosons mediate proton decay. The dominant decay mode from \((X,Y)\) exchange is \(p \to \pi^0 e^+\), and its rate is given by (neglecting the non-gauge contributions, which are subdominant):

\[
\Gamma(p \to \pi^0 e^+) = \frac{(m_p^2 - m_{\pi^0}^2)^2}{64\pi f_{\pi}^2 m_p^3} |\alpha_H|^2 (1 + D + F)^2 (|C_{RL}|^2 + |C_{LR}|^2),
\]

where \(\alpha_H\) is the hadronic parameter defined by \(\alpha_H P_L u_p = \langle 0 | (u_R d_R) u_L | p \rangle\), and D and F are the same chiral Lagrangian parameters as above, and \(C_{RL}\) and \(C_{LR}\) are the Wilson coefficients of the four-fermion operators \((u_R d_R)(u_L e_L)\) and \((u_L d_L)(u_R e_R)\), respectively. The most recent lattice determination of \(\alpha_H\) is \(|\alpha_H| = 0.0112(25)\) GeV\(^3\). The Wilson coefficients take the same form as in minimal SU(5):

\[
C_{RL} = A_R \frac{g_{\text{GUT}}^2}{M_V^2}, \quad C_{LR} = A_R \frac{g_{\text{GUT}}^2}{M_V^2} (1 + |V_{ud}|^2),
\]

where \(M_V = M_X = M_Y\) is the mass of the heavy \((X,Y)\) gauge bosons. The long-distance piece of the renormalization factor \(A_R = A_{SD} A_{LD}\) is given by Eq. (B.10), while its short-distance piece reads:

\[
A_{SD} = \prod_I \left( \frac{\alpha_1(M_I)}{\alpha_1(M_{I+1})} \right)^{-\frac{23}{364}} \left( \frac{\alpha_2(M_I)}{\alpha_2(M_{I+1})} \right)^{-\frac{3}{32}} \left( \frac{\alpha_3(M_I)}{\alpha_3(M_{I+1})} \right)^{-\frac{4}{33}},
\]

where, as in Eq. (B.11), the Yukawa contributions have been neglected, and \(I\) runs over all mass thresholds \(M_I\) between \(m_Z\) and \(M_{\text{GUT}}\). Together with Eq. (B.17), Eq. (B.10) gives:

\[
\tau(p \to \pi^0 e^+) = (8.2 \times 10^{34}\ \text{yrs}) \left( \frac{0.0112\ \text{GeV}^3}{|\alpha_H|} \right)^2 \left( \frac{2.4}{A_{SD}} \right)^2 \left( \frac{1/24}{\alpha_{\text{GUT}}} \right)^2 \left( \frac{M_V}{10^{16}\ \text{GeV}} \right)^4,
\]

to be compared with the experimental upper bound \(\tau(p \to \pi^0 e^+) > 8.2 \times 10^{33}\ \text{yrs}\) (90\% C.L.)\(^{52}\).

In Eq. (B.19), \(A_{SD} = 2.4\) and \(\alpha_{\text{GUT}} = 1/24\) are the reference MSSM values. Consider now the model (i) of Section 4.3 with \(M_\Delta = M_\Sigma = 10^{12}\ \text{GeV},\ M_T = 10^{13}\ \text{GeV},\ M_H = 10^{14}\ \text{GeV}\)

\(^{12}\)Note that the mass scale \(M_{10'} \sim 10^{14}\ \text{GeV}\) can be generated by an interaction term \(\lambda S^{10'} 10'\) with \(\lambda \sim 10^{-2}\), where \(S\) is the SO(10) singlet introduced in Appendix A.
and the same heavy fermion spectrum as in Fig. 1(a). At the one-loop level and omitting both low-energy and GUT thresholds, one obtains $\alpha_{\text{GUT}} = 1/14$, $M_{\text{GUT}} = 3.5 \times 10^{15}$ GeV and $A_{\text{SD}} = 2.4$. These values are at odds with the experimental limit on the proton lifetime (assuming $M_V = M_{\text{GUT}}$ leads to $\tau(p \to \pi^0 e^+) = 4.2 \times 10^{32}$ yrs). On the contrary, for model (ii) with the spectrum of Fig. 1(a) (with $M_\Delta = 10^{10}$ GeV, $M_\Sigma = M_S = M_T = M_O = 10^{13}$ GeV and $M_H = 10^{14}$ GeV), one finds $\alpha_{\text{GUT}} = 1/12$, $M_{\text{GUT}} = 1.2 \times 10^{16}$ GeV and $A_{\text{SD}} = 2.5$, leading to $\tau(p \to \pi^0 e^+) = 3.9 \times 10^{34}$ yrs for $M_V = M_{\text{GUT}}$. We shall therefore adopt model (ii) as our reference model, although model (i) may be viable if 2-loop running and threshold effects conspire to increase the GUT scale. Also, the corrections needed to depart from the minimal SU(5) relation $M_d = M_e^T$ will in general modify Eq. (B.17) by introducing non-trivial fermion mixing angles in the heavy gauge boson couplings, and this could significantly reduce the proton decay rate [53].

B.4 Proton decay from non-renormalizable operators

For completeness, we mention that proton decay could also be induced by non-renormalizable superpotential operators of the form:

$$d_{ijkl} \frac{\Lambda^2}{16} \sqrt{16\cdot16\cdot16\cdot10\cdot16},$$  

(B.20)

where $\Lambda$ is the cutoff, and the SU(5)-singlet component of the $16\bar{16}$ acquires a GUT-scale vev. These operators, if present, will generate the $D = 5$ operators of Eq. (B.5) after SO(10) symmetry breaking. In order to avoid a conflict with the experimental bound on proton lifetime, they must either be forbidden by some symmetry, or the ones involving light generation fields must be suppressed by small coefficients $d_{ijkl}$. This is actually what one would expect in a theory of flavour capable of explaining the hierarchy of fermion masses. Note that the required suppression of the coefficients is less severe than in conventional SO(10) models, where the dangerous operators, of the form $16\cdot16\cdot16\cdot10\cdot16$, have dimension 5.

C Intermediate scales and gauge coupling unification

In this appendix, we study the constraints imposed by the requirement of successful gauge coupling unification on the extra heavy states present below the GUT scale, using 1-loop renormalization group equations.

As shown in Appendix B.2 the experimental constraint on the $p \to K^+ \bar{\nu}$ rate can easily be satisfied by allowing a pair of Higgs doublets to lie at an intermediate scale. This tends to spoil unification, but we will show that the problem can be cured by splitting the components of the $54$. In fact, it is also desirable to give GUT-scale masses to some components of the SU(5) multiplets $(15, 15)$ and $24$ contained in the $54$ in order to preserve perturbative unification. Indeed, the presence of additional chiral superfields at intermediate scales increases the value of the unified gauge coupling $\alpha_{\text{GUT}}$ with respect to the MSSM. One has to check that a Landau pole is not reached soon above (or even below) the GUT scale, because this would make the predictions of the model very sensitive to the effects of higher-dimensional operators. In the SO(10) scenario studied in this paper, keeping the full $54$ close to the seesaw scale would give $\alpha_{\text{GUT}} \simeq 1/4$, and the Landau pole would be reached at a scale $\Lambda \approx 2M_{\text{GUT}}$. Thus, motivated both by perturbativity and by proton decay, we are led to split the $(15, 15)$ and $24$ multiplets, keeping below the GUT scale only the components that are necessary for the
seesaw mechanism and for leptogenesis, together with possible additional components needed to achieve unification.

As is customary, we define $M_{\text{GUT}}$ as the scale where $\alpha_1$ and $\alpha_2$ meet. Denoting by $M_n$ the masses of the intermediate-scale states and by $b_i^{(n)}$ ($i = 1, 2, 3$) their contributions to the beta-function coefficients, the 1-loop predictions for $\alpha_3(m_Z)$, $M_{\text{GUT}}$ and $\alpha_{\text{GUT}}$ read:

$$
\frac{1}{\alpha_3(m_Z)} - \frac{1}{\alpha_3^0(m_Z)} = \frac{1}{2\pi} \sum_n \frac{b_{32}^{(n)} b_{21} - b_{21}^{(n)} b_{32}}{b_{21}} \ln \frac{M_n^{\text{GUT}}}{M_n},
$$

$$
\ln \frac{M_{\text{GUT}}}{M_n^{\text{GUT}}} = -\sum_n \frac{b_{21}^{(n)}}{b_{21}} \ln \frac{M_n^{\text{GUT}}}{M_n},
$$

$$
\frac{1}{\alpha_{\text{GUT}}} - \frac{1}{\alpha_{\text{GUT}}^0} = \frac{1}{2\pi} \sum_n \frac{b_{2}^{(n)} b_{21}^{(n)} - b_{21}^{(n)} b_2}{b_{21}} \ln \frac{M_n^{\text{GUT}}}{M_n},
$$

where the superscript "0" refers to MSSM quantities, with $(b_1^0, b_2^0, b_3^0) = (\frac{43}{5}, 1, -3)$, $M_{\text{GUT}}^0 \simeq 2 \times 10^{16}$ GeV, $\alpha_{\text{GUT}}^0 \simeq 1/24$, and

$$
b_{ij} \equiv b_i - b_j, \quad b_i \equiv b_i^0 + \sum_n b_i^{(n)}. \tag{C.4}
$$

Given the fact that the MSSM prediction for $\alpha_3(m_Z)$ significantly deviates from the measured value (using 2-loop RGEs and including low-energy supersymmetric thresholds, one finds $[1/\alpha_3^{\text{exp}}(m_Z) - 1/\alpha_3^0(m_Z)] \simeq +4.3/(2\pi)$, which corresponds to a $5\sigma$ deviation), a positive contribution of the extra states to Eq. (C.1) would be welcome.

Replacing Eq. (C.2) into Eq. (C.3) one finds:

$$
\frac{1}{\alpha_{\text{GUT}}} = \frac{1}{\alpha_{\text{GUT}}^0} - \frac{b_2}{2\pi} \ln \frac{M_{\text{GUT}}}{M_{\text{GUT}}^0} - \sum_n \frac{b_2^{(n)}}{2\pi} \ln \frac{M_n^{\text{GUT}}}{M_n}. \tag{C.5}
$$

The third term always increases $\alpha_{\text{GUT}}$, while the second one can decrease or increase it, depending on whether $M_{\text{GUT}}$ is smaller or larger than $M_{\text{GUT}}^0$. However, the contribution of the second term is bounded by the requirement $M_{\text{GUT}} \gtrsim 5 \times 10^{15}$ GeV coming from proton decay (see Appendix B.3). Therefore, in order to avoid a too large value of $\alpha_{\text{GUT}}$, the additional intermediate-scale fields besides the ones needed to realize leptogenesis and to suppress proton decay [namely $(\Delta, \Sigma)$, $S$ and/or $T$, $(\Phi_1, \Phi_2)$ and $(H, \overline{H})$] should better be SU(2)$_L$ singlets. There are two such 54 components: $(\Sigma, \overline{\Sigma})$ and $O$. Since $O$ is an electroweak singlet, it does not affect $M_{\text{GUT}}$ nor $\alpha_{\text{GUT}}$ (but it corrects the prediction for $\alpha_3$). Adding only $O$ would push $M_{\text{GUT}}$ above the Planck scale. We are thus left with two possibilities: (i) intermediate $(\Sigma, \overline{\Sigma})$; (ii) intermediate $(\Sigma, \overline{\Sigma})$ and $O$. The right-hand side of Eq. (C.1) reads, for each of the two cases:

(i) $\frac{1}{10\pi} \left[ 41 \ln \frac{M_{\text{GUT}}}{M_\Sigma} - 22 \ln \frac{M_{\text{GUT}}}{M_\Delta} - 20 \ln \frac{M_{\text{GUT}}}{M_T} - 7 \ln \frac{M_{\text{GUT}}}{M_H} \right]$, \tag{C.6a}

(ii) $\frac{1}{10\pi} \left[ 33 \ln \frac{M_{\text{GUT}}}{M_\Sigma} - 21 \ln \frac{M_{\text{GUT}}}{M_\Delta} - 15 \ln \frac{M_{\text{GUT}}}{M_T} - 6 \ln \frac{M_{\text{GUT}}}{M_H} + 15 \ln \frac{M_{\text{GUT}}}{M_O} \right]$, \tag{C.6b}
while \( M_{\text{GUT}} \) and \( \alpha_{\text{GUT}} \) are given by the same expression in both cases:

\[
\ln \frac{M_{\text{GUT}}}{M_{\text{GUT}}^0} = \frac{1}{15} \left[ -8 \ln \frac{M_{\text{GUT}}^0}{M_\Sigma} + \ln \frac{M_{\text{GUT}}^0}{M_\Delta} + 5 \ln \frac{M_{\text{GUT}}^0}{M_T} + \ln \frac{M_{\text{GUT}}^0}{M_H} \right],
\]

\[
\frac{1}{\alpha_{\text{GUT}}} - \frac{1}{\alpha_{\text{GUT}}^0} = \frac{1}{30\pi} \left[ 88 \ln \frac{M_{\text{GUT}}^0}{M_\Sigma} - 71 \ln \frac{M_{\text{GUT}}^0}{M_\Delta} - 85 \ln \frac{M_{\text{GUT}}^0}{M_T} - 26 \ln \frac{M_{\text{GUT}}^0}{M_H} \\
- 15 \ln \left( \frac{M_{\text{GUT}}^0}{M_{51}M_{52}M_{53}} \right) \right],
\]

where we assumed \( M_{L_i} = M_{D_i}^\pm \equiv M_5 \). From the above equations, we can see that lowering \( M_\Sigma \) decreases \( \alpha_{\text{GUT}} \) as desired (and improves the prediction for \( \alpha_3 \)), but decreases \( M_{\text{GUT}} \).

Let us first consider case (i). Assuming \( M_\Sigma = M_\Delta \equiv M_{15} = 10^{12} \) GeV, \( M_T \equiv M_{24} = 10^{13} \) GeV and \( M_H = 10^{14} \) GeV (as well as \( \lambda_H = 0.045 \), \( V_1 = M_{\text{GUT}} \), \( \tan \beta = 10 \) and \( \tan \theta_H = 1 \) to fix the masses of the heavy \( (5, \bar{5}) \) pairs), we obtain, at the 1-loop level:

\[
\frac{1}{\alpha_3(m_Z)} - \frac{1}{\alpha_3^0(m_Z)} = \frac{0.19}{2\pi} , \quad M_{\text{GUT}} = 3.5 \times 10^{15} \text{ GeV} , \quad \frac{1}{\alpha_{\text{GUT}}} = 14.1 .
\]

In this case, the prediction for \( \alpha_3(m_Z) \) is only slightly larger than in the MSSM, and the value of the unified coupling remains reasonable. Unfortunately, the unification scale lies almost one order of magnitude below the MSSM prediction, which leads to a too fast proton decay rate (see Appendix B.3). Gauge coupling unification is approximately preserved if \( M_{15} \) and \( M_{24} \) are varied while keeping the ratio \( M_{24}/M_{15} \) fixed. In particular, increasing \( M_{15} \) while keeping \( M_{24}/M_{15} = 10 \) slightly increases \( M_{\text{GUT}} \) (and decreases \( \alpha_3 \) and \( \alpha_{\text{GUT}} \)). For instance, for \( M_{15} = 10^{13} \) GeV, one obtains \( 1/\alpha_3(m_Z) - 1/\alpha_3^0(m_Z) = +0.27/(2\pi) \), \( M_{\text{GUT}} = 4.8 \times 10^{15} \) GeV and \( \alpha_{\text{GUT}} = 1/15.9 \). Still the unification scale is dangerously close to the proton decay bound.

Let us now consider case (ii). As anticipated, only \( \alpha_3(m_Z) \) is affected by the presence of \( O \) below \( M_{\text{GUT}} \). This gives the possibility of increasing \( M_{\text{GUT}} \) with respect to case (i) by increasing \( M_\Sigma \), while correcting the prediction for \( \alpha_3(m_Z) \) by adjusting \( M_O \). For instance, the choice \( M_\Sigma = 10^{12} \) GeV, \( M_T = M_O = M_\Sigma = 10^{13} \) GeV and \( M_H = 10^{14} \) GeV gives, at the 1-loop level:

\[
\frac{1}{\alpha_3(m_Z)} - \frac{1}{\alpha_3^0(m_Z)} = \frac{2.2}{2\pi} , \quad M_{\text{GUT}} = 1.2 \times 10^{16} \text{ GeV} , \quad \frac{1}{\alpha_{\text{GUT}}} = 12.5 .
\]

In this case, unification works better than in the MSSM. Indeed, we checked that the prediction for \( \alpha_3(m_Z) \), including low-energy thresholds and the 2-loop MSSM running, matches the measured value within 1\( \sigma \). Moreover, there is no conflict between the value of \( M_{\text{GUT}} \) and proton decay, and the Landau pole lies one order of magnitude above \( M_{\text{GUT}} \). From Eq. (C.6), we can see that the contributions of \( T \) and \( O \) cancel if \( M_T = M_O \). Therefore, unification is still preserved if the masses of the various states are varied while keeping \( M_T = M_O \) and \( M_\Sigma^3/M_\Delta^2 \approx 10^{15} \) GeV.

The splitting of the masses of the 54 components needed to realize case (ii) can be achieved with the set of operators shown in Table B, where \( V_{PS} \) is the Pati-Salam invariant vev of the 54, \( V_{B-L} \) is the vev of the 45, which is aligned along the \( B-L \) direction, and 45 has no vev (see Appendix A). At the renormalizable level, only \( (Z, Z) \) and \( (V, V) \) acquire a (GUT-scale) mass. The dimension-5 operators provide \( M_S = M_T = M_O \sim M_{\text{GUT}}^2/\Lambda \) as well as \( M_\Sigma \sim M_{\text{GUT}}^2/\Lambda \) (more precisely, when the GUT-scale masses of the 45 and 54 are taken into account, \( M_{S,T,O} \) and \( M_\Sigma \) are further suppressed by a mild seesaw-like mechanism). Finally, the dimension-6 operator generates \( M_\Delta \sim M_{\text{GUT}}^3/\Lambda^2 \).
Below $M_{\text{GUT}}$, the superpotential terms [2] and (3) read, in terms of the SM components (4):

$$W = (\lambda_u)_{ij} \tilde{u}_i^c q_j h_u + (\lambda_d)_{ij} \tilde{d}_i^c q_j h_d + (\lambda_e)_{ij} \tilde{e}_i^c l_j h_d$$

$$+ (\hat{\lambda}_d)_{ij} D_i^c q_j h_d + (\hat{\lambda}_e)_{ij} \tilde{e}_i^c L_j h_d + (M_L)_{ij} L_i \tilde{L}_j + (M_D^c)_{ij} D_i^c \tilde{D}_j^c$$

$$+ \frac{1}{2} (f_{\Delta})_{ij} l_i \Delta l_j + \frac{1}{2} (f_{\Delta})_{ij} \tilde{L}_i \Delta \tilde{L}_j + \frac{1}{\sqrt{2}} (f_Z)_{ij} \tilde{d}_i^c Z l_j + \frac{1}{\sqrt{2}} (f_Z)_{ij} \tilde{D}_i^c Z \tilde{L}_j$$

$$+ \frac{1}{2} (f_{\Sigma})_{ij} d_i^c \Sigma d_j + \frac{1}{2} (f_{\Sigma})_{ij} \tilde{D}_i^c \Sigma \tilde{D}_j^c + \frac{1}{\sqrt{2}} (f_{\bar{V}})_{ij} \tilde{d}_i^c \bar{V} \tilde{L}_j + \frac{1}{\sqrt{2}} (f_{\bar{V}})_{ij} \tilde{D}_i^c \bar{V} \tilde{l}_j$$

$$+ \frac{1}{\sqrt{2}} (f_{O})_{ij} d_i^c O \tilde{D}_j^c + \frac{1}{\sqrt{2}} (f_{T})_{ij} \tilde{L}_i T \tilde{l}_j + \sqrt{\frac{3}{20}} (f_{S_i})_{ij} \tilde{L}_i S_l \tilde{l}_j - \frac{1}{\sqrt{15}} (f_{S_d})_{ij} \tilde{d}_i^c S \tilde{D}_j^c$$

$$+ \frac{1}{\sqrt{2}} \sigma_T h_u T h_d + \sqrt{\frac{3}{20}} \sigma_S h_u S h_d + \frac{1}{2} \sigma_\Delta h_d \Delta h_d + \frac{1}{2} \sigma_\Delta h_u \Delta h_u$$

$$+ M_\Delta \Delta \Delta + M_Z \bar{Z} \bar{Z} + M_\Sigma \Sigma \Sigma + M_V \bar{V} \bar{V} + \frac{1}{2} (M_S S^2 + M_T T^2 + M_O O^2) \ , \quad (D.1)$$

where Clebsch-Gordan coefficients have been factorized out, and contractions of $SU(3)_C$ and $SU(2)_L$ indices are understood. The interactions of fields with GUT-scale masses, namely the right-handed neutrinos and the components of 16 and 10 other than the light Higgs doublets, have been omitted.\(^\text{13}\) The boundary conditions at the GUT scale for the superpotential couplings and mass parameters are:

$$\lambda_u = y \ , \ \lambda_e = \lambda_d^T = \sin \theta_H h \ , \ \hat{\lambda}_d = \lambda_d = \cos \theta_H y \ ,$$

$$f_X = f \ \text{for} \ X = \Delta, \bar{\Delta}, Z, \bar{Z}, \Sigma, \bar{\Sigma}, V, \bar{V}, O, T, S, S_d \ ,$$

$$\sigma_\Delta = \cos^2 \theta_H \sigma \ , \ \sigma_{\bar{\Delta}} = \sigma \ , \ \sigma_T = \sigma_S = \cos \theta_H \sigma \ , \quad (D.2)$$

where the Higgs mixing angle $\theta_H$ is defined by Eq. (B.3), and $V_1$ is the vev of the $SU(5)$ singlet component of the 16. We did not write boundary conditions for the masses of the 54 components with GUT-scale masses should not appear in Eq. (D.1), nor in the RGEs.

\(^{13}\)Similarly, the interactions of the 54 components with GUT-scale masses should not appear in Eq. (D.1), nor in the RGEs.
components, since, as discussed in Appendix C, they are assumed to be split by operators not included in the superpotential.

The soft supersymmetry breaking terms for the SO(10) fields are defined by:

$$
-L_{\text{soft}} = (m^2_{16})_{ij} \mathbf{16}^* \mathbf{16} + (m^2_{10})_{ij} \mathbf{10}^* \mathbf{10} + m^2_{16} \mathbf{16}^* + m^2_{10} \mathbf{10}^* + m^2_{54} \mathbf{54}^* \\
+ \left( \frac{1}{2} (A_y)_{ij} \mathbf{16}^* \mathbf{10} + (A_h)_{ij} \mathbf{16}^* \mathbf{10} + \frac{1}{2} (A_f)_{ij} \mathbf{10}^* \mathbf{54} + \frac{1}{2} A_5 \mathbf{10} \mathbf{10} \mathbf{54} + \text{h.c.} \right) \\
+ \left( \frac{1}{2} M_{1/2} \lambda^a \lambda^a + \text{h.c.} \right),
$$

(D.3)

where we used the same notation for the chiral multiplets and for their scalar components, $\lambda^a$ are the SO(10) gauginos, and we omitted the $B$-terms. The soft terms of the SM components are given, at the GUT scale, by the following boundary conditions:

$$
m_q^2 = m_{w^c}^2 = m_{e^c}^2 = m_L^2 = m_{D^c}^2 = m_{16}^2, \\
m_t^2 = m_{d^c}^2 = m_H^2 = m_{D^c}^2 = m_{10}^2, \\
m_{u^2}^2 = m_{16}^2, \\
m_{h_d}^2 = \cos^2 \theta_W m_{10}^2 + \sin^2 \theta_W m_{10}^2, \\
m_{X}^2 = m_{54}^2 \text{ for } X = \Delta, \breve{\Delta}, \Sigma, \breve{\Sigma}, S, T, O, V, \breve{V}, \\
M_1 = M_2 = M_3 = M_{1/2}.
$$

The boundary conditions for the $A$-terms, not included in the above list, are analogous to the ones for the corresponding Yukawa couplings. One has for instance:

$$
A_u = A_y, \quad A_e = A_d^T = \sin \theta_W A_h, \quad \hat{A}_e = \hat{A}_d = \cos \theta_W A_y.
$$

(D.5)

In the numerical study of Section 5 we solved the 1-loop RGEs for all superpotential couplings and soft terms below $M_{\text{GUT}}$. For brevity, we only list below the RGEs for the MSSM parameters, which are sufficient for a leading-log analysis of flavour-violating effects. Let us first recall the 1-loop RGEs for gauge couplings and gaugino masses:

$$
\frac{d}{dt} \alpha^{-1}_a = -\frac{1}{2\pi} b_a, \quad b_a = -3 C_2(G_a) + \sum_R T_a(R),
$$

(D.6)

$$
\frac{d}{dt} M_a = \frac{1}{2\pi} b_a \alpha_a M_a,
$$

(D.7)

where $t = \log(\mu/\mu_0)$, $\mu$ being the renormalization scale and $\mu_0$ a reference scale, $C_2(G_a)$ is the second Casimir invariant of the group $G_a$, $T_a(R)$ is the Dynkin index of the representation $R$, and the sum in $b_a$ runs over all chiral superfields with mass smaller than $\mu$.

The 1-loop RGEs for the MSSM Yukawa couplings are:

$$
(4\pi)^2 \frac{d}{dt} \lambda_u = \lambda_u \left( 3 \lambda_u^\dagger \lambda_u^\dagger \lambda_u + \lambda_d^\dagger \lambda_d^\dagger \lambda_d + \lambda_d^\dagger \lambda_d + \lambda_u^\dagger \lambda_u + \lambda_d^\dagger \lambda_d \right) \\
+ \lambda_u \left( \frac{3}{4} |\sigma_T|^2 + \frac{3}{20} |\sigma_S|^2 + \frac{3}{2} |\sigma_D|^2 \right) \lambda_u - \left( \frac{13}{15} g_1^2 + 3 g_2^2 + \frac{16}{3} g_3^2 \right) \lambda_u,
$$

(D.8)
\[
(4\pi)^2 \frac{d}{dt} \lambda_d = \lambda_d \left( 3\lambda_d^\dagger \lambda_d + \lambda_u^\dagger \lambda_u + 3\hat{\lambda}_d \lambda_d^\dagger \right) + \text{Tr} \left( 3\lambda_d \lambda_d^\dagger + \lambda_e \lambda_e^\dagger + \lambda_e \hat{\lambda}_d + 3\hat{\lambda}_d \lambda_d^\dagger \right) \lambda_d \\
+ \left( fZ f_Z^\dagger + fV f_V^\dagger + 2f_Z f_Z^\dagger f_d f_d^\dagger + \frac{4}{3} fO f_O^\dagger + \frac{1}{15} f_s f_s^\dagger \right) \lambda_d \\
+ \left( \frac{3}{4} |\sigma T|^2 + \frac{3}{20} |\sigma T|^2 \right) \lambda_d - \left( \frac{7}{15} g_1^2 + 3g_2^2 + \frac{16}{3} g_3^2 \right) \lambda_d, \quad (D.9)
\]

\[
(4\pi)^2 \frac{d}{dt} \lambda_e = \left( 3\lambda_e^\dagger \lambda_e + 3\hat{\lambda}_e \lambda_e^\dagger \right) \lambda_e + \text{Tr} \left( 3\lambda_d \lambda_d^\dagger + \lambda_e \lambda_e^\dagger + \lambda_e \hat{\lambda}_d + 3\hat{\lambda}_d \lambda_d^\dagger \right) \lambda_e \\
+ \left( \frac{3}{2} f_d^\dagger f_d + \frac{3}{2} f_Z f_Z^\dagger f_d + \frac{3}{4} f_V f_V^\dagger f_d + \frac{3}{20} f_s f_s^\dagger f_d \right) \lambda_e \\
+ \left( \frac{3}{4} |\sigma T|^2 + \frac{3}{20} |\sigma T|^2 \right) \lambda_e - \left( \frac{9}{5} g_1^2 + 3g_2^2 \right) \lambda_e. \quad (D.10)
\]

The 1-loop RGEs for the MSSM soft fermion masses are:

\[
(4\pi)^2 \frac{d}{dt} m_q^2 = \left( m_q^2 \lambda_q^\dagger \lambda_q + \lambda_q^\dagger \lambda_q m_q^2 + 2\lambda_q^\dagger \lambda_q m_{\tilde{q}}^2 \lambda_q + 2A_{\tilde{q},A_u} \right) \\
+ \left( m_q^2 \lambda_d^\dagger \lambda_d + \lambda_d^\dagger \lambda_d m_{\tilde{d}}^2 + 2\lambda_d^\dagger \lambda_d m_{\tilde{d}}^2 \lambda_d + 2A_{\tilde{d},A_d} \right) \\
+ \left( m_t^2 \lambda_t^\dagger \lambda_t + \lambda_t^\dagger \lambda_t m_t^2 + 2\lambda_t^\dagger \lambda_t m_t^2 \lambda_t + 2A_{\tilde{t},A_t} \right) \\
- \left( \frac{2}{15} |M_1|^2 g_1^2 + 6|M_2|^2 g_2^2 + \frac{32}{3} |M_3|^2 g_3^2 - \frac{1}{5} g_1^2 \right), \quad (D.11)
\]

\[
(4\pi)^2 \frac{d}{dt} m_{\tilde{e}}^2 = \left( m_{\tilde{e}}^2 \lambda_{\tilde{e}}^\dagger \lambda_{\tilde{e}} + \lambda_{\tilde{e}}^\dagger \lambda_{\tilde{e}} m_{\tilde{e}}^2 + 2\lambda_{\tilde{e}}^\dagger \lambda_{\tilde{e}} m_{\tilde{e}}^2 \lambda_{\tilde{e}} + 2A_{\tilde{e},A_{\tilde{e}}} \right) \\
+ \left( m_{\tilde{e}}^2 f_Z f_Z^\dagger f_e f_e^\dagger m_{\tilde{e}}^2 + 2f_Z f_Z^\dagger f_e f_e^\dagger m_{\tilde{e}}^2 + 2f_Z m_{\tilde{e}}^2 f_Z^\dagger + 2A_{f_Z} A_{f_Z} \right) \\
+ 2 \left( m_{\tilde{e}}^2 f_S f_S^\dagger m_{\tilde{e}}^2 + 2f_S f_S^\dagger m_{\tilde{e}}^2 + 2f_S m_{\tilde{e}}^2 f_S^\dagger + 2A_{f_S} A_{f_S} \right) \\
+ \frac{1}{15} \left( m_{\tilde{e}}^2 f_s f_s^\dagger f_{\tilde{e}} + f_s f_s^\dagger f_{\tilde{e}} m_{\tilde{e}}^2 + 2f_s f_s^\dagger m_{\tilde{e}}^2 + 2f_s m_{\tilde{e}}^2 f_s^\dagger \right) \lambda_{\tilde{e}} \\
+ \left( \frac{4}{3} \left( m_{\tilde{e}}^2 f_O f_O^\dagger m_{\tilde{e}}^2 + 2f_O f_O^\dagger m_{\tilde{e}}^2 + 2f_O m_{\tilde{e}}^2 f_O^\dagger + 2A_{f_O} A_{f_O} \right) \\
+ \left( \frac{2}{3} m_{\tilde{e}}^2 f_V f_V^\dagger m_{\tilde{e}}^2 + 2f_V f_V^\dagger m_{\tilde{e}}^2 + 2f_V m_{\tilde{e}}^2 f_V^\dagger + 2A_{f_V} A_{f_V} \right) \\
+ 2 \left( \frac{2}{15} |M_1|^2 g_1^2 + \frac{32}{3} |M_3|^2 g_3^2 - \frac{2}{5} g_1^2 \right), \quad (D.12)
\]

\[
(4\pi)^2 \frac{d}{dt} m_{\tilde{u}}^2 = \left( m_{\tilde{u}}^2 \lambda_{\tilde{u}}^\dagger \lambda_{\tilde{u}} + \lambda_{\tilde{u}}^\dagger \lambda_{\tilde{u}} m_{\tilde{u}}^2 + 2\lambda_{\tilde{u}}^\dagger \lambda_{\tilde{u}} m_{\tilde{u}}^2 \lambda_{\tilde{u}} + 2A_{\tilde{u},A_{\tilde{u}}} \right) \\
- \left( \frac{32}{15} |M_1|^2 g_1^2 + \frac{32}{3} |M_3|^2 g_3^2 + \frac{4}{5} g_1^2 \right), \quad (D.13)
\]
\[ (4\pi)^2 \frac{d}{dt} m_t^2 = \left( m_t^2 \lambda_t^e \lambda_e + \lambda_t^e \lambda_e m_t^2 + 2 \lambda_t^e \lambda_e m_{h_d}^2 + 2 \lambda_t^e \lambda_e m_{h_u}^2 + 2 A_t^e \lambda_t^e \right) \\
+ \frac{3}{2} \left( m_t^2 f_{\Delta} f_{\Delta} + f_{\Delta}^2 m_{t}^2 + 2 f_{\Delta} f_{\Delta} m_{\Delta}^2 + 2 f_{\Delta} m_{\Delta}^2 T f_{\Delta} + 2 A_{f_{\Delta}}^e A_{f_{\Delta}}^e \right) \\
+ \frac{3}{2} \left( m_z^2 f_{\Delta} f_{\Delta} + f_{\Delta} f_{\Delta} m_{t}^2 + 2 f_{\Delta} f_{\Delta} m_{\Delta}^2 + 2 f_{\Delta} m_{\Delta}^2 T f_{\Delta} + 2 A_{f_{\Delta}}^e A_{f_{\Delta}}^e \right) \\
+ \frac{3}{2} \left( m_t^2 f_{\Delta} f_{\Delta} + f_{\Delta} f_{\Delta} m_{t}^2 + 2 f_{\Delta} f_{\Delta} m_{\Delta}^2 + 2 f_{\Delta} m_{\Delta}^2 T f_{\Delta} + 2 A_{f_{\Delta}}^e A_{f_{\Delta}}^e \right) \\
+ \frac{3}{4} \left( m_t^2 f_{T} f_{T} + f_{T} f_{T} m_{t}^2 + 2 f_{T} f_{T} m_{\Delta}^2 + 2 f_{T} m_{\Delta}^2 T f_{T} + 2 A_{f_{T}}^e A_{f_{T}}^e \right) \\
+ \frac{3}{20} \left( m_t^2 f_{S} f_{S} + f_{S} f_{S} m_{t}^2 + 2 f_{S} f_{S} m_{\Delta}^2 + 2 f_{S} m_{\Delta}^2 T f_{S} + 2 A_{f_{S}}^e A_{f_{S}}^e \right) \\
+ \left( \lambda_t^e \lambda_e m_{L}^2 + m_{L}^2 \lambda_t^e \lambda_e \right) - \left( \frac{6}{5} |M_1|^2 g_1^2 + 6 |M_2|^2 g_2^2 + \frac{3}{5} g_1^2 S \right), \tag{D.14} \]

\[ (4\pi)^2 \frac{d}{dt} m_{e^c}^2 = 2 \left( m_{e^c} \lambda_e \lambda_{e^c} + \lambda_{e^c} \lambda_{e^c} m_{e^c}^2 + 2 \lambda_{e^c} \lambda_{e^c} m_{h_u}^2 + 2 \lambda_{e^c} \lambda_{e^c} m_{h_d}^2 + 2 A_{e^c} \lambda_{e^c} \right) \\
+ \frac{2}{5} \left( m_{e^c} \lambda_e \lambda_{e^c} + \lambda_{e^c} \lambda_{e^c} m_{e^c}^2 + 2 \lambda_{e^c} \lambda_{e^c} m_{h_u}^2 + 2 \lambda_{e^c} \lambda_{e^c} m_{h_d}^2 + 2 A_{e^c} \lambda_{e^c} \right) \\
- \left( \frac{24}{5} |M_1|^2 g_1^2 + \frac{6}{5} g_1^2 S \right), \tag{D.15} \]

where the hypercharge D-term contribution \( S \) is given by:

\[ S = m_{h_u}^2 - m_{h_d}^2 + \text{Tr}(-m_t^2 + m_{e^c}^2 - 2 m_{e^c}^2 + m_{d^c}^2 + m_q^2) + \text{Tr}(-m_L^2 + m_{D^c}^2 + m_{L^c}^2 - m_{D^c}^2) \\
+ 3(m_{\Delta}^2 - m_{\Delta}^2) + (m_{Z}^2 - m_{Z}^2) + 4(m_{\Delta}^2 - m_{\Delta}^2) + 5(m_{\Delta}^2 - m_{\Delta}^2). \tag{D.16} \]

The mixing terms \( m_{D^c}^2 d^c D^c + m_{L^c}^2 l^c L + \text{h.c.} \) appearing in the RGEs for \( m_{t}^2 \) and \( m_{e^c}^2 \), although absent at \( M_{\text{GUT}} \), are generated by the RGE evolution and must be included for consistency.

Finally, the 1-loop RGEs for the MSSM A-terms are:

\[ (4\pi)^2 \frac{d}{dt} A_u = A_u \left( 5 \lambda_u^t \lambda_u + \lambda_d^t \lambda_d + \lambda_d^t \lambda_d \right) + 4 \lambda_u \lambda_u A_u + 2 \lambda_u \left( \lambda_d^t A_d + \lambda_d^t \lambda_d \right) \\
+ \text{Tr} \left( 3 \lambda_u \lambda_u \right) A_u + \left( \frac{3}{4} |\sigma_T|^2 + \frac{3}{20} |\sigma_S|^2 + \frac{3}{2} |\sigma_D|^2 \right) A_u \\
+ \text{Tr} \left( 6 A_u \lambda_u \right) \lambda_u + \left( \frac{3}{2} A_{\sigma_T} \sigma_T^* + \frac{3}{10} A_{\sigma_S} \sigma_S^* + 3 A_{\sigma_D} \sigma_D^* \right) \lambda_u \\
+ \frac{13}{15} g_1^2 \left( 2 M_1 \lambda_u - A_u \right) + 3 g_2^2 \left( 2 M_2 \lambda_u - A_u \right) + \frac{16}{3} g_3^2 \left( 2 M_3 \lambda_u - A_u \right), \tag{D.17} \]
\[(4\pi)^2 \frac{d}{dt} A_d = A_d \left( 5\lambda_d^\dagger \lambda_d + \lambda_u^\dagger \lambda_u + 5\lambda_d^\dagger \tilde{\lambda}_d \right) + 4\lambda_d \lambda_d^\dagger A_d + 2\lambda_d \left( \lambda_u^\dagger A_u + 2\lambda_d^\dagger \tilde{A}_d \right) \\
+ \left( f_Z f_Z^\dagger + 2f_v f_v^\dagger + 2f_S f_S^\dagger + \frac{4}{3} f_O f_O^\dagger + \frac{1}{15} f_{s_d} f_{s_d}^\dagger \right) A_d \\
+ \left( 2A f_z f_z^\dagger + 2A f_v f_v^\dagger + 4A f_s f_s^\dagger + \frac{8}{3} A f_o f_o^\dagger + \frac{2}{15} A f_{s_d} f_{s_d}^\dagger \right) \lambda_d \\
+ \text{Tr} \left( 3\lambda_d \lambda_d^\dagger + \lambda_c \lambda_c^\dagger + \lambda_e \lambda_e^\dagger + 3\lambda_d \lambda_d^\dagger \right) A_d + \left( \frac{3}{4} |\sigma_T|^2 + \frac{3}{20} |\sigma_S|^2 + \frac{3}{2} |\sigma_D|^2 \right) A_d \\
+ 2 \text{Tr} \left( 3A_d \lambda_d^\dagger + A_e \lambda_e^\dagger + \tilde{A}_e \lambda_e^\dagger + 3\tilde{A}_d \lambda_d^\dagger \right) \lambda_d + \left( \frac{3}{2} A_{\sigma_T} \sigma_T^* + \frac{3}{10} A_{\sigma_S} \sigma_S^* + 3A_{\sigma_D} \sigma_D^* \right) \lambda_d \\
+ \frac{7}{15} g_1^2 (2M_1 \lambda_d - A_d) + 3g_2^2 (2M_2 \lambda_d - A_d) + \frac{16}{3} g_3^2 (2M_3 \lambda_d - A_d) , \quad (D.18) \]

\[(4\pi)^2 \frac{d}{dt} A_e = 4 \left( \lambda_c \lambda_c^\dagger + \lambda_e \lambda_e^\dagger \right) A_e + 5A_e \lambda_e^\dagger \lambda_e + 5\tilde{A}_e \lambda_e^\dagger \lambda_e \\
+ A_e \left( \frac{3}{2} f^\dagger f_A + \frac{3}{2} f_Z f_Z^\dagger + \frac{3}{2} f_v f_v^\dagger + \frac{3}{4} f_T f_T^\dagger + \frac{3}{20} f_{s_l} f_{s_l}^\dagger \right) \\
+ A_e \left( 3f^\dagger f_A + 3f_Z^\dagger f_Z + 3f_v^\dagger f_v + \frac{3}{2} f_T^\dagger f_T + \frac{3}{10} f_{s_l}^\dagger f_{s_l} \right) \\
+ \text{Tr} \left( 3\lambda_d \lambda_d^\dagger + \lambda_c \lambda_c^\dagger + \lambda_e \lambda_e^\dagger + 3\lambda_d \lambda_d^\dagger \right) A_e + \left( \frac{3}{4} |\sigma_T|^2 + \frac{3}{20} |\sigma_S|^2 + \frac{3}{2} |\sigma_D|^2 \right) A_e \\
+ 2 \text{Tr} \left( 3A_d \lambda_d^\dagger + A_c \lambda_c^\dagger + \tilde{A}_{\sigma_T} \lambda_c^\dagger + 3\tilde{A}_d \lambda_d^\dagger \right) \lambda_e + \left( \frac{3}{2} A_{\sigma_T} \sigma_T^* + \frac{3}{10} A_{\sigma_S} \sigma_S^* + 3A_{\sigma_D} \sigma_D^* \right) \lambda_e \\
+ \frac{9}{5} g_1^2 (2M_1 \lambda_e - A_e) + 3g_2^2 (2M_2 \lambda_e - A_e) , \quad (D.19) \]
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