Effects of Fasteners Damping on Dynamical Characteristics of Track Structure

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Abstract. Track structure is modelled as an Euler beam that is supported by a series of discrete points. Further, the governing equations of the track structure subjected to moving harmonic excitation are established. On the basis of the dynamical characteristics and principle of superposition in frequency domain of periodic structure subjected to moving harmonic excitation, for any point, theoretical expression of the amplitude-frequency response is derived. Consequently, the nature characteristic and the effects of fasteners damping on dynamical characteristics of track structure subjected to moving harmonic excitation are investigated. The researches reveal that influences of damping on resonant peak bandwidth and peak value of the track structure are no obvious. While the effects of damping on dynamic response are very obvious on non-resonant high frequency band, and with the increasing of damping the dynamical responses of the track structure obviously decrease. The dynamic characteristics of the fastener bearing points and the midpoint on track structure are almost similarity, while there is a certain difference only in the frequency range far away from the excitation frequency.

1. Introduction

Track structure is modelled as an Euler beam that is supported by a series of discrete points. The governing equations of the track structure subjected to harmonic excitation are established. On the basis of the dynamical characteristics and principle of superposition in frequency domain of periodic structure subjected to moving harmonic excitation [1], for beam model of the track, the nature characteristic and the dynamical response of the damping are studied.
2. Dynamical Modelling

An Euler beam model is used to model track structure. A series of discrete points bear the beam, and space between two adjacent supporting points is \( L \) \([2-4]\), as shown in Figure 1.

![Dynamical modelling of track subjected to moving harmonic excitation](image)

**Figure 1.** Dynamical modelling of track subjected to moving harmonic excitation

3. Governing Equation of Motion

The governing equation of the track structure subjected to the moving harmonic excitation is derived \([5-6]\):

\[
EI \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} = P_i e^{i\omega t} \delta(x-x_i) - \sum_{i=1}^{N} F_i(t) \delta(x-x_i)
\]

where, \( w(x,t) \) is vertical vibration displacement, \( EI \) is the bending stiffness, \( \delta \) is the Dirac function, \( P_i e^{i\omega t} \) is harmonic load, \( v \) is velocity, \( F_i(t) \) is the support reaction force of sleeper \( i \) induced by the movement of the excitation along the rail.

Governing equation of motion of any two adjacent sleeper rails is written as follow:

\[
EI \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} = P_i e^{i\omega t} \delta(x-x_i-vt)
\]

Carrying out Fourier integral transform for Eq.\((2)\), governing equation of motion of any two adjacent sleeper rails is obtained in frequency domain

\[
EI \frac{\partial^4 \hat{w}}{\partial x^4} - \omega^2 m \hat{w} = \frac{P_i}{\omega} e^{i\omega x_i-v(x-x_i)}
\]

where, \( \hat{w} \) is the vertical displacement in frequency domain.

For track structure with periodic spacing \( L \) of sleepers, the amplitude-frequency responses of the track structure subjected to moving harmonic excitation satisfy the the infinite periodic structure theory.

\[
\hat{w}(x+L,\omega) = \int_{-\infty}^{\infty} w(x+L,t+L/v)e^{-i\omega (t+L/v)} dt \approx e^{i\omega L/v} \hat{w}(x,\omega)
\]

According to periodic structure theory, the dynamical response of system is transformed the dynamical response in one periodic scope \([7]\), shown as Figure 2.
Therefore, for $\omega > 0$, the generalized solution of Eq. (3) is defined as follows:

$$\hat{w}(x, \omega) = C_1 e^{jAx} + C_2 e^{-jAx} + C_3 e^{jAx} + C_4 e^{-jAx} + B e^{\frac{j\omega - \omega L}{v}}$$

(5)

where, $A = \frac{\omega^2 m}{(EI)}$, $B = \frac{P e^{\frac{j\omega - \omega L}{v}}}{v[E I ((\omega_1 - \omega)/v) - m \omega^2]}$.

The unknown $C_1 \sim C_4$ of Eq.(5) are written as vector form as follows:

$$c = \{C_1, C_2, C_3, C_4\}^T$$

(6)

For one periodic scope, the boundary conditions are obtained as follows:

$$\left.\frac{d^n w(x, \omega)}{dx^n}\right|_{x=L} = \left. e^{j(\omega - \omega L/v)} \left(\frac{d^n w(x, \omega)}{dx^n}\right)\right|_{x=0}, \quad n = 0, 1, 2,$$

$$\left.\frac{d^3 w(x, \omega)}{dx^3}\right|_{x=L} = \left. e^{j(\omega - \omega L/v)} \left[\frac{d^3 w(x, \omega)}{dx^3} + (K_c(\omega) \cdot W(x, \omega))\right]\right|_{x=0}$$

(7)

where, $K_c(\omega)$ is the composite stiffness, $K_c(\omega) = K + iC\omega$.

Substituting Eq.(5) into Eq.(7), the following matrix equation is obtained.

$$D c = S$$

(8)

where, $D$ is $4 \times 4$ matrix, $c$ is constant vector, $S$ is $4 \times 1$ vector.

Using by Eq.(4), the amplitude-frequency response of anyone point outside the structural period is obtained.

$$\hat{w}(\bar{x}, \omega) = e^{j(\omega - \omega) n_i L/v} \cdot \hat{w}(x, \omega)$$

(9)

where, $\bar{x} = x + n_i L$, $x(0 \leq x \leq L)$, $n_i$ is the number of period in scope between point $\bar{x}$ and point $x$.

Based on the amplitude-frequency response of the track, the dynamical responses of fastener damping are studied numerically.

4. Dynamic Response of the Damping

In this section, the nature characteristic and the dynamical response of damping for track beam model are studied [8]. Considering $P = 1 kN$ and $v = 120 km/h$, the maximum vibration displacement of track structure under different excitation frequency are shown in Fig.3.
From Figure 3, with the increasing of frequency the peak responses of the track structure decrease gradually. And there are peak responses on excitation frequency of 54Hz, 214Hz and 374Hz.

According to nature characteristic of track structure[9], under unit moving harmonic excitation with excite frequency 130Hz and 300Hz respectively, the dynamic responses of fastener damping for fastener support and midspan points (x=21.5625m, x=21.875m) on rail are numerically studied, as shown as Figs.4 and 5.

![Figure 3](image-url)

**Figure 3.** The maximum vibration displacement under different excitation frequency \((x = 21.875m)\)

(a) The midspan point                         (b) The fastener support point

**Figure 4.** Amplitude frequency response of the fastener damping as the excitation frequency \(f_l=130\)Hz

(a) The midspan point                         (b) The fastener support point

**Figure 5.** Amplitude-frequency response of the fastener damping as excitation frequency \(f_l=300\)Hz
Form Figures 4 and 5, under moving harmonic excitation, the peak displacement of track occurs near the excitation frequency, and as far away from excitation frequency the dynamical response of system rapidly decrease. The influences of damping on resonant peak bandwidth and peak value of the track structure are no obvious. While in non-resonant high frequency band, the effects of damping on dynamic response are very obvious and with the increasing of damping the dynamic responses of the track structure obviously decrease. On non-resonant high frequency band 300Hz~400Hz, the fastener damping can obviously decrease response vibration of the track structure. Under the same load, the dynamic responses of the fastener support and the midspan points on rail are similarity on the whole. But there are some differences far away from excite frequency.

In non-resonant low frequency band, the response peak bandwidth of the midspan point is wider than that of the fastener support point. In non-resonant high frequency band, the effect of damping on the response of the midspan point is also more sensitive.

5. Conclusion

In this paper, the harmonic excitation is considered, according to the dynamical characteristics and superposition principle of the periodic structure, the nature characteristic and the dynamical response of damping for track beam model are studied. For the midspan point and the fastener support point, the dynamical characteristics of two points are comparatively studied.

Furthermore, the effects of the fastener damping on the dynamical characteristics of the track structure are investigated numerically. Within the range of 0 ~ 500Hz, there occur the displacement response peaks as excitation frequency 54Hz, 214Hz and 374Hz. The fasteners damping has no significant influence on the resonant peak bandwidth and peak value of the system. But in non-resonant high frequency band, the influence of the fastener damping on the amplitude response is significant, and the fastener damping can effectively restrain vibration of track structure.

Acknowledgement

This research was financially supported by Beijing Natural Science Foundation (1182010), Innovation Team Foundation of Beijing Academy of Science and Technology (IG201503C2) and Open Project Foundation of Jiangsu Key Laboratory of Advanced Food Manufacturing Equipment & Technology (FM-201802).

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