Chern-Simons Violation of Lorentz and PCT Symmetries in Electrodynamics

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Abstract

Recently proposed Lorentz-PCT noninvariant modifications of electromagnetism are reviewed. Their experimental consequences are described, and it is argued that available data decisively rules out their occurrence in Nature.

The principle of special relativity is very firmly established in the minds of physicists, and it is experimentally confirmed, without known exception. Nevertheless, today’s availability of high-precision instruments lets us ask whether this principle is only approximately true, and leads us to seek possible mechanism for its violation. Such an inquiry is not unreasonable, since we know that a relativity principle does not apply to the discrete transformations of time and space reversal.

Special relativity arose when the symmetry of Maxwell’s electrodynamical field theory, i.e., Lorentz invariance, was elevated to encompass particle mechanics, whose Newtonian, Lorentz noninvariant dynamics had consequently to be modified. Therefore violation of special relativity can be looked for in particle mechanics, in electromagnetism, or in both. I shall restrict my attention to possible nonrelativistic behavior in electromagnetism.

Let me record the conventional equations, both in compact Lorentz covariant, and in explicit vectorial notation. We are dealing with the electric and magnetic fields \( \mathbf{E}, \mathbf{B} \), that are components of a second-rank antisymmetric tensor \( F_{\mu\nu} = -F_{\nu\mu} \), or of its dual \( \ast F_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \):

\[
F_{\alpha\nu} = \frac{1}{2} \epsilon^{ij\alpha} F_{j\nu} = E_i \quad \text{and} \quad \frac{1}{2} \epsilon^{ijk} F_{jk} = \ast F_{\alpha\nu} = -B^i .
\]

They satisfy the homogeneous Maxwell equations

\[
\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 , \quad \nabla \cdot \mathbf{B} = 0
\]

or

\[
\partial_\mu \ast F^{\mu\nu} = 0 .
\]

*This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under contract #DE-FC02-94ER40818. MIT-CTP-2796 hep-ph/9811322 November 1998 Workshop on Lorentz violation, Bloomington, Indiana, November 1998 Orbis Scientia 1998, Coral Gables, Florida, December 1998
which permit writing the fields in terms of the potentials \( A^\mu = (\phi, A) \), by formulas that are invariant against gauge transforming the potentials:

\[
\phi \to \phi - \frac{1}{c} \frac{\partial}{\partial t} \alpha, \quad A \to A + \nabla \alpha
\]

\[
A_\mu \to A_\mu - \partial_\mu \alpha
\]

These formulas are

\[
E = -\nabla \phi - \frac{1}{c} \frac{\partial A}{\partial t}, \quad B = \nabla \times A
\]

\[
F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .
\]

The second set of Maxwell’s equations, which sees the sources of charge density \( \rho \) and current density \( j^\mu = (c\rho, j) \), reads

\[
\nabla \times B - \frac{1}{c} \frac{\partial E}{\partial t} = \frac{4\pi}{c} j, \quad \nabla \cdot E = 4\pi \rho
\]

or

\[
\partial_\mu F^{\mu \nu} = \frac{4\pi}{c} j^\nu
\]

and can be derived from the Lagrange density

\[
\mathcal{L} = \frac{1}{8\pi}(E^2 - B^2) - \rho\phi + \frac{1}{c} j \cdot A = -\frac{1}{16\pi} F_{\mu \nu} F^{\mu \nu} - \frac{1}{c} j_\mu A^\mu
\]

where the basic variables are the potentials, and the electromagnetic fields are expressed in terms of them. Consistency of the equations of motion and gauge invariance of the Lagrangian formalism require that the charge density and current satisfy a continuity equation

\[
\frac{\partial}{\partial t} \rho + \nabla \cdot j = 0 = \partial_\mu j^\mu .
\]

Let us now turn to modifications. In the most obvious departure from the standard formulas, we add a “photon mass term” by supplementing \( \mathcal{L} \) with \( \frac{\mu^2}{2} A^\mu A_\mu = \frac{\mu^2}{2} \phi^2 - \frac{\mu^2}{2} A^2 \) where \( \mu \) has dimension of inverse length. In the new equations of motion \(-\mu^2 A_\mu \) is added to \( j^\mu \), so that when the wave Ansatz \( e^{ik_\alpha x^\alpha} = e^{i(\omega t - k \cdot r)} \), \( k_\alpha = (\omega/c, k) \), \( k \equiv |k| \) is taken for fields in the source-free case \( (j^\mu = 0) \), the dispersion law reads

\[
k^\alpha k_\alpha = \mu^2 , \quad \omega = c\sqrt{k^2 + \mu^2}.
\]

Of course, this does not violate Lorentz invariance – the mathematical expression of the special relativity principle – but it destroys Einstein’s physical reasoning that led him to special relativity: light no longer travels with a universal velocity in all reference frames, and “c” becomes a mysterious limiting velocity that is not attained by any physical particle. Gauge invariance appears to be violated, but today we know that the gauge principle can be obscured by subtle symmetry-breaking mechanisms, for example, the mass \( \mu \) could arise from a feeble Higgs effect. After solving the modified field equations with prescribed sources, one finds electromagnetic fields that are distorted by the mass term. Comparison to experiment is made with geomagnetic data, leading to the limit \( \mu < 3 \times 10^{-24} \) GeV [A. Goldhaber and M. Nieto, Rev. Mod. Phys. 43, 277 (1971)] while observations of the galactic magnetic field
give $\mu < 3 \times 10^{-36}$ GeV [G. Chibisov, Usp. Fiz. Nauk 119, 551 (1976); Sov. Phys. Usp. 19, 624 (1976)] (1 GeV $\sim 10^{13}$ cm$^{-1}$).

Lorentz invariance and the relativity principle, but not rotational invariance, disappear if the Lagrange density is modified by the addition of a further $\frac{1}{8}\pi B^2$ term, proportional to $\epsilon$. However, by rescaling $A$, one sees that this is equivalent to redefining the velocity of light from $c$ to $c_\epsilon \neq c$, in the electromagnetic part of the theory while retaining $c$ as a parameter in the (unspecified) matter kinematics. Hence this modification can be exported into the matter sector and I shall not discuss it further. It has been studied by S. Coleman and S. Glashow [Phys. Lett. B 405, 249 (1997)], who use cosmic ray data to bound the magnitude of the addition by $10^{-23}$.

I now come to yet another modification, introduced by S. Carroll, G. Field, and me almost a decade ago, which recently came again to attention [Phys. Rev. D 41, 1231 (1990)]. To begin, let us note that in addition to $-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} = E^2 - B^2$, another Lorentz scalar, quadratic in the field strengths, can be constructed: $-\frac{1}{4} * F_{\mu\nu} F^{\mu\nu} = E \cdot B$. However, adding this to the electromagnetic Lagrange density does not affect the equations of motion, because that quantity, when expressed in terms of potentials – the dynamical variables in a Lagrangian formulation – involves total derivatives, which do not contribute to equations of motion:

$$\frac{1}{2} * F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} \partial_\mu (e^{\mu\alpha\beta\gamma} A_\alpha F_{\beta\gamma})$$

$$-2E \cdot B = \frac{\partial}{\partial t} \left( \frac{1}{c} A \cdot B \right) + \nabla \cdot (\phi B - A \times E) .$$

However, when the $E \cdot B$ quantity is multiplied by another space-time–dependent field $\theta(t, r)$, the total derivative feature disappears and such an addition would affect dynamics. Once again using the freedom to modify a Lagrange density by total derivatives, we see that the addition of $\theta * FF$ to the Lagrange density is equivalent to adding $-\partial_\mu \theta \epsilon^{\mu\alpha\beta\gamma} A_\alpha F_{\beta\gamma}$. If $\theta$ is a dynamical field, then the extended electromagnetism $+ \theta$ system remains Lorentz invariant. We shall however posit that neither $\theta$ nor $\partial_\mu \theta$ are dynamical quantities; rather $\partial^\mu \theta$ is a constant 4-vector $p^\mu = (m, p)$ that picks out a direction in space-time, thereby violating Lorentz invariance. Thus we are led to consider an electromagnetic theory, where the conventional Maxwell Lagrange density is modified by

$$\Delta L = -\frac{1}{4} p_\mu (e^{\mu\alpha\beta\gamma} A_\alpha F_{\beta\gamma}) = -\frac{1}{2} p_\mu * F^{\mu\nu} A_\nu$$

$$= -\frac{1}{2} m A \cdot B + \frac{1}{2} p \cdot (\phi B - A \times E) .$$

The sourceless Maxwell equations are unchanged (the fields continue to be expressed by potentials). Only the equations with sources are changed, and the change can be viewed as a field-dependent addition to the source current.

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} j^\nu + p_\mu * F^{\mu\nu}$$

$$\nabla \times B - \frac{1}{c} \frac{\partial E}{\partial t} = \frac{4\pi}{c} j - m B + p \times E , \quad \nabla \cdot E = 4\pi \rho - p \cdot B .$$

Note that the field equations are gauge invariant, even though the Lagrange density is not. The quantities $m$ and $p$ have dimension of inverse length; the latter breaks rotational invariance by selecting a direction in space; $m$ breaks the invariance of the theory against Lorentz boosts. Presumably for the interesting case we should select vanishing $p$ (rest frame of $p^\mu$, which is taken to be time-like) so that rotational isotropy is maintained. Parity is
also broken, since the pseudoscalar $B$ mixes with vector $E$, but charge conjugation and time inversion remain intact, hence PCT is broken.

Before describing the consequences of our model, let me recall some facts about the various quantities that we have introduced. $F^{\mu\nu}F_{\mu\nu}$ is the so-called Chern-Pontryagin density; its non-Abelian generalization plays an important role in the “standard” particle physics model, where it is a measure of the anomalous (quantum mechanical) nonconservation of the axial vector current. It is responsible for the decay of the neutral pion to two photons, and for proton decay. The 4-vector whose divergence gives $F^{\mu\nu}F_{\mu\nu}$ is called the Chern-Simons density. Both objects are templates for the topologically nontrivial behavior of non-Abelian gauge fields.

In some extensions of the standard model, $F^{\mu\nu}F_{\mu\nu}$ is coupled to a further dynamical field, like the $\theta$-field mentioned above, which describes a hypothetical particle – the axion – whose role is to ensure CP symmetry of strong interactions. However, no evidence for such a particle has been found thus far.

Note further that if we were living in $(2 + 1)$-dimensional space-time, that is on a plane rather than in a three-dimensional volume, the $\epsilon$-tensor would have only three indices and we could introduce the Chern-Simons term into $(2 + 1)$-dimensional electrodynamics without the external 4-vector $p^\mu$, i.e., $(2 + 1)$-dimensional Lorentz invariance would be preserved by $\Delta L = me^{\alpha\beta\gamma}A_\alpha F_{\beta\gamma}$. Chern-Simons modified electrodynamics plays a role in planar electromagnetic phenomena, as in the quantum Hall effect, and perhaps also in high-$T_c$ superconductivity.

Finally we remark that with vanishing $p$ and absence of sources, so that $E = 0$, the remaining modified Maxwell equations read $\nabla \times B = -mB$, $\nabla \cdot B = 0$. These have arisen previously in magnetohydrodynamics. They coincide with the conventional Maxwell equations in the presence of neutral sources and steady currents ($\rho = 0, \nabla \cdot j = 0$) and are seen to be equivalent to the conventional Ampère’s law, when the further condition is imposed that $j$ is proportional to $B$.

What is the consequence of our Lorentz invariance violating modification?

Let us examine wave solutions in the absence of sources. ($\rho = j = 0, j^\mu = 0$). We again make the Ansatz that fields behave as exponentials of phases, $e^{i(\omega t - k \cdot r)} = e^{ik_\alpha x^\alpha}$, $k^\alpha = (\omega/c, k)$, $k \equiv |k|$, and find the dispersion law

$$(k^\alpha k_\alpha)^2 + (k^\alpha k_\alpha)(p^\beta p_\beta) = (k^\alpha p_\alpha)^2.$$ 

From this one can show that introducing $p^\alpha$ has the consequence of splitting the photons into two polarization modes, each traveling with different velocities $\omega/k$ – forceful evidence of Lorentz and parity violation. This is very easily seen in the rotation invariant case, $p^\alpha = 0$. One finds

$$\omega^2 = ck(ck \pm mc).$$

Note that $\omega$ can become imaginary for modes with $k < m$. This means there are unstable runaway solutions. These do not contradict energy conservation, because the energy $E$ is no longer the positive expression of the Maxwell theory, $\frac{1}{2} \int d^3r(E^2 + B^2)$. Rather we now have

$$E = \frac{1}{2} \int d^3r \left[ E^2 + (B + \frac{m^2}{2} A)^2 \right] - \frac{m^2}{8} \int d^3r A^2.$$

With unstable solutions each of the two terms contributing to $E$ grows without bound, yet $E$ stays finite and time-independent owing to a cancellation between the two. (The energy is
gauge invariant – in spite of appearances.) However, runaway, exponentially growing modes can be avoided by allowing for noncausal propagation for well-behaved sources (similar to the way runaway solutions are eliminated from the Abraham-Lorentz equation of conventional electrodynamics).

Returning now to our plane wave solutions, we observe that a plane-polarized wave – a superposition to two circularly polarized modes traveling at different velocities – will be rotated when it travels through space. Since \( p^\alpha \) is small, we can solve for \( \omega \) to first order in \( p^\alpha \), and find, (without setting \( p \) to zero)

\[
k = \frac{\omega}{c} \pm \frac{1}{2} (m - \mathbf{p} \cdot \hat{k})
\]

so the change \( \Delta \) in the polarization, as the wave travels a distance \( L \) is

\[
\Delta = -\frac{1}{2} (m - \mathbf{p} \cdot \hat{k}) L.
\]

This is similar to the Faraday effect, where a polarization change is induced by ambient magnetic fields. However our phase change is wavelength independent, while the Faraday effect rotation is proportional to wavelength squared. So the two effects can be distinguished.

When comparing predictions of this theory to experimental data, Carroll, Field and I assumed that rotation invariance holds, we set \( p = 0 \), and the entire effect is parameterized by \( m \) (time-like \( p^\alpha \)). Geomagnetic data can be confronted with the distorted magnetic field that solves the modified equations in the presence of sources. But the data is somewhat difficult to interpret in our context, and the most plausible limit is

\[
m \leq 6 \times 10^{-26} \text{GeV}.
\]

However, examining the polarization of light from distant galaxies and removing the rotation due to the Faraday effect, yields a much more stringent result

\[
m \leq 10^{-42} \text{GeV}
\]

[see also M. Goldhaber and V. Trimble, J. Astrophys. Astr. 17, 17(1996)]. Since effects of nonzero \( m \) can appear only at distances greater than the associated Compton wavelength, which for the above is the distance to the horizon, astrophysical data apparently rules out nonvanishing \( m \).

How about vanishing \( m \) and nonvanishing \( \mathbf{p} \) (space-like \( p^\mu \))? Our formula for polarization change indicates that here too there should be a non-Faraday rotation. Moreover, one can show that space like \( p^\alpha \) produce no instability. However, Carroll, Field, and I believed that such a violation of rotational symmetry is unlikely.

Thus, we were very surprised when there appeared a Physical Review Letter by B. Nodland and J. Ralston [Phys. Rev. Lett. 78, 3043 (1997)] alleging that precisely this kind of anisotropy exists. Evidence for this startling assertion was drawn from the same galactic data that we used in our analysis, which gave us the null result.

We were not the only ones surprised. Here is a sampling of news stories about this “discovery” in the popular and semipopular press, following a report of the Nodland-Ralston “result” issued by the American Institute of Physics.

American Institute of Physics

“Is the universe birefringent?”, 17 April 1997
Interest in the result also evoked humorous reactions, in the form of a syndicated cartoon by Hilary Price depicting existential anguish engendered by life in an anisotropic universe, a statement by Lyndon La Rouche that he knew it all the time (interview, 7 May 1997 with A. Papert), and a claim for extraterrestrial life (http://www.enterprisemission.com).

Unfortunately, it appears that Nodland and Ralston made a mistake in their data analysis. Carroll and Field [Phys. Rev. Lett. 79, 234 (1997)] reanalyzed the data, identified their error, and found no anisotropy. Thus our original conclusion that there is no evidence for a Chern-Simons modification to electromagnetism stands, and has
been confirmed by several other investigations. The entire matter is nicely reviewed on [http://ITP.UCSB.edu/~carroll/aniso.html](http://ITP.UCSB.edu/~carroll/aniso.html).

In spite of the negative results, we can nevertheless draw an interesting conclusion. We know that in Nature parity P, charge conjugation C, and time reversal T are violated. While local field theory and Lorentz invariance guarantee that PCT will be conserved, it remains an experimental and interesting question whether PCT is valid in Nature, which perhaps does not make use only of local field theoretic dynamics. But PCT violation in any corner of a grand unified theoretical structure would induce PCT violation in electrodynamics, so the stringent limits that we put on the PCT-violating Chern-Simons term, also limit some forms of PCT violation anywhere else in the “final theory”.