Locality problem, Bell’s inequalities and EPR experiments

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Abstract

The question has been solved whether Bell’s inequalities cover all possible kinds of hidden-variable theories. It has been shown that the given inequalities can be hardly derived when the changing space position of photon-pair source together with the microscopic space structure of measuring devices are taken into account; and when corresponding impact parameters (i.e., exact impact points) of photons in individual measuring devices (polarizers) influence measured values, in addition to usually considered characteristics.

1 Introduction

The interpretation problem of the quantum-mechanical mathematical model is still open. While the original controversy between A. Einstein and N. Bohr [1, 2] started from different philosophical attitudes of these two physicists to understanding the notion of reality it was expected that the problem would be solved on the physical basis after J. Bell [3] derived the known inequalities. These inequalities should be fulfilled by experimental data if a hidden-variable theory corresponded to physical reality.

Experiments initiated by these inequalities and performed in 1972-82 (see, e.g., [4]) ended with the conclusion that Bell’s inequalities are surely violated by experimental data. And this violation has been usually interpreted as a decisive argument against any theory of hidden variables and, consequently, against the locality of microscopic objects.

Such a conclusion might have seemed to be strongly supported also by finding of d’Espagnat [5] that Bell’s inequalities have been practically in a conflict with Schroedinger equation itself. It was shown, however, already earlier by Bohm [6] that a kind of hidden variables has been contained already in this equation. Therefore, the following question should be raised: Are such hidden variables excluded by experimental data, too? Or: Are these hidden variables covered by Bell’s inequalities or not? And/or: How to proceed to remove such a dilemma?

Necessity of such an analysis is strongly supported also by the fact that it is difficult to understand the nonlocality of microscopic objects on macroscopic distances or a mechanism acting faster than light (one or the other) as required
if one accepts the standard quantum-mechanical model and its Copenhagen interpretation.

In the following we will attempt to give an answer to the given questions. A new kind of hidden variables will be introduced and the impossibility of deriving the usual Bell’s limit under these new conditions will be demonstrated with the help of two standard approaches, i.e., those of Bell and of Clauser and Horn (see, e.g., [7]).

2 EPR experiment and a new kind of hidden variables

The contemporary EPR experiments consist in establishing coincidence transmissions of two photons (having the same polarization and going in opposite directions) through two polarizers under different angles between polarizer axes:

\[ \begin{array}{c}
\alpha \\
| \leftrightarrow \alpha \leftrightarrow|
\end{array} \]

Coincidence probability in individual events depending on common hidden variables \( \Lambda \) may be written as

\[ p_{\alpha,\beta}(\Lambda) = p^{(1)}_{\alpha}(\Lambda) \cdot p^{(2)}_{\beta}(\Lambda). \]  

Due to incidental values of \( \Lambda \) the measured values equal

\[ P_{\alpha,\beta} = \int d\Lambda p_{\alpha,\beta}(\Lambda) \]  

where \( \int d\Lambda \) represents statistical average over all possible quantities of \( \Lambda \) set.

The set \( \Lambda \) may involve practically four different kinds of parameters:

\[ \Lambda \equiv \{ \lambda, \lambda', \lambda_a, \lambda_b \} \]

where \( \lambda \) represents proper spin (or polarization) characteristic, \( \lambda' \) other hidden variables determining the state of the photon pair, and \( \lambda_a \) and \( \lambda_b \) - instantaneous characteristics (including microscopic fluctuations - see, e.g., [8]) of individual macroscopic measuring devices. The last two parameters must be denoted as redundant now as they have been practically included in probability functions when Bell modified his original approach [9]. We will omit them in the following analysis and limit our considerations to the other two.

While the meaning of \( \lambda \) is clear we should ask what characteristics may be represented by \( \lambda' \) if they should not be covered by Bell’s inequalities (see the last
paragraph of Sec. 1). And further, whether such a characteristic may belong to a realistic description of the microscopic world.

Let us assume that \( \lambda' \) represents the source position of a photon pair and its momentum direction; i.e., three coordinates and two components of unit vector (energy of both the photons going in opposite directions being defined). These characteristics cannot be omitted in any realistic approach. However, they make it possible to determine the exact impact point of each photon in the corresponding polarizer plane, which may be represented by a two-dimensional vector \( \hat{b}^{(j)} \) where \( j = 1, 2 \) denotes the corresponding measuring device. Consequently, the properties of any individual photon in the polarizer plane are defined by \( \lambda \) and \( \hat{b}(j) \) quite independently of the setting of measuring devices.

However, in a realistic picture we must admit necessarily that the response of a measuring device (in a single event) will not depend directly on \( \hat{b}^{(j)} \) but on an exact impact point in the proper microscopic structure of a polarizer consisting of a periodical grid of atoms. And it is not possible to substitute individual results by statistical averages in deriving Bell’s inequalities for coincidence measurements.

Then of course, the response of the measuring device in an individual event will depend on the proper effective impact parameter that will be obtained by projecting the point given by \( \hat{b}^{(j)} \) into the polarizer two-dimensional grid structure formed by individual interaction centers (atoms). And it is evident that such an effective impact parameter will depend necessarily also on the setting \( \gamma \) \( (\gamma \) representing \( \alpha \) or \( \beta \) due to changing correlation of the two given plane structures. We will denote such an effective impact parameter as \( b^{(j)}_{\gamma} \). It must be related always to an individual interaction center, as it is done in all collision experiments. However, at difference to collision processes at higher energies the orientation of impact parameters in polarizer plane structure cannot be neglected as the photon must interact necessarily with more than one interaction center.

Respecting, therefore, the realistic space structure of the whole measuring problem we must conclude that there are always three different and in principle independent (two-dimensional) vectors characterizing the situation in each measuring device in the plane perpendicular to the photon tracks: \( \lambda, b^{(1)}_{\alpha}, \alpha; \) and \( \lambda, b^{(2)}_{\beta}, \beta; \) i.e., the spin, the effective impact parameter and the vector representing angle orientation of the given measuring devices.

It will be shown in the following that in derivations of Bell’s inequalities some simplifying conditions have been used, which may be applied to only if the corresponding probabilities do not depend on the mentioned vector triples; e.g., if the dependence on \( b^{(j)}_{\gamma} \) is neglected.

While the statistical distributions of \( \lambda' \) and of \( \hat{b}^{(j)} \) are fully independent of the settings of individual measuring devices it is not more true for the distributions of \( b^{(j)}_{\gamma} \) obtained by projection into the actual polarizer structure. They may be \( \gamma \)-dependent due to changing space orientations of the microscopic structures of macroscopic objects. We will demonstrate this new situation now on two exam-
ple (the approaches of Bell and of Clauser and Horn) to a greater detail.

3 Bell’s approach

Let us assume that the situation in each measuring device is characterized by the mentioned triple of two-dimensional vectors: $\lambda, b^{(j)}_\gamma$. The expression for the experimentally established probabilities may be then written as

$$P_{\alpha,\beta} = \int d\lambda \int db^{(1)}_\alpha db^{(2)}_\beta \ p_{\alpha,\beta}(\lambda, b^{(1)}_\alpha, b^{(2)}_\beta)$$

(4)

where

$$p_{\alpha,\beta}(\lambda, b^{(1)}_\alpha, b^{(2)}_\beta) = p^{(1)}_\alpha(\lambda, b^{(1)}_\alpha) \cdot p^{(2)}_\beta(\lambda, b^{(2)}_\beta).$$

(5)

The quantity $p^{(j)}_\gamma(\lambda, b^{(j)}_\gamma)$ represents the probability that a photon characterized by $\lambda$ and $\lambda'$ went through the corresponding polarizer when it was set to $\gamma$; $b^{(j)}_\gamma$ being a unique function of $\lambda'$ for a given $\gamma$.

Bell’s inequalities are then represented by the condition

$$P_{\alpha,\beta} + P_{\alpha',\beta'} + P_{\alpha',\beta} - P_{\alpha,\beta'} \leq 2$$

(6)

for any values of four angles. The inequalities (6) should be violated in the case of the standard quantum-mechanical model. They have been proven with the help of different theoretical approaches to hold for a hidden-variable theory. However, in none of them the influence of the mentioned effective impact parameters has been taken into account.

There is a fundamental difference between the parameters $b^{(j)}_\gamma$ and the previously used parameters $\lambda_A$ and $\lambda_B$. The latter ones represented time fluctuations (or other characteristics) of individual measuring devices being fully independent of other parameters characterizing a photon pair, while the former ones follow from the space structure of the whole process in each event, combining the space orientations of the microscopic system and of measuring devices. An important question then arises, how it is with the derivation of Bell’s inequalities for coincidence measurement in such a case when the averaging over $\lambda'$ must be substituted by averaging over two different effective impact parameters $b^{(j)}_\gamma$. It means that while the statistical distributions of both the parameters $\lambda$ and $\lambda'$ may be regarded as independent of other parameters the statistical distributions of effective impact parameters must be expected to depend significantly on settings $\alpha$ and $\beta$; being influenced by the corresponding orientations of space structures of measuring devices. The $\alpha, \beta$ dependence of measured values is then given by two different and in principle mutually independent factors; one coming from $\lambda$ and the other coming from $b^{(j)}_\gamma$. 
Bell’s approach has started practically (see, e.g., Ref. [7], Eqs. (3.11-3.12)) from the equation

\[ P_{\alpha,\beta} - P_{\alpha,\beta'} = \oint d\lambda \left\{ \oint db_\alpha db_\beta p_{\alpha,\beta} - \oint db_\alpha db_\beta' p_{\alpha,\beta'} \right\} \]

by adding to its right side the expression

\[ \oint d\lambda \left\{ \oint db_\alpha db_\beta p^{(1)}_\alpha p^{(2)}_\beta p^{(1)}_{\alpha'} p^{(2)}_{\beta'} - \oint db_\alpha db_\beta' p^{(1)}_\alpha p^{(2)}_\beta' p^{(1)}_{\alpha'} p^{(2)}_{\beta'} \right\} , \]

equaling zero if impact-parameter dependence is neglected. Then taking into account that it holds \( p^{(j)}_\gamma \leq 1 \) and inserting the values of one instead of two probability factors (differently in different terms) the inequalities (7) might be derived.

However, it is evident that the given approach cannot be applied to when the influence of impact parameters is taken into account as the expression (8) cannot be equal zero. The expression (8) is well defined and may equal zero for any \( \alpha, \alpha', \beta \) and \( \beta' \) only if the statistical distributions of all parameters \( b_\gamma \) are independent of \( \gamma \); it means if it holds

\[ b^{(j)}_\gamma \equiv b^{(j)}_{\gamma'} \]

for any \( \gamma \) and \( \gamma' \).

Any dependence of measured values on impact parameters must be, therefore, excluded if Bell’s inequalities are to be derived for experiments with two photons. Bell’s inequalities would be, however, derived if the parameter \( \lambda \) were not represented by a vector (e.g., if instead of photons two scalar particles were emitted in opposite directions and measurement consisted in establishing simple coincidence detection), even if Eq. (9) were not fulfilled. The individual probability factors would not depend more on mutual orientations of two vectors \( \lambda \) and \( b_\gamma \) in such a case.

4 Approach of Clauser and Horn

The same assumption was involved practically in the other approaches presented in Ref. [7], too. We will demonstrate it yet on the approach proposed by Clauser and Horn. Taking into account the influence of corresponding impact parameters one can write:

\[ P^{(1)}_\alpha = \oint d\lambda \oint db^{(1)}_\alpha p^{(1)}_\alpha (\lambda, b^{(1)}_\alpha) , \]
\[ P^{(2)}_\beta = \oint d\lambda \oint db^{(2)}_\beta p^{(2)}_\beta (\lambda, b^{(2)}_\beta) , \]
\[ P_{\alpha,\beta} = \oint d\lambda \oint db^{(1)}_\alpha db^{(2)}_\beta p_{\alpha,\beta} (\lambda, b^{(1)}_\alpha, b^{(2)}_\beta) . \]
It is not possible then to obtain Eq. (3.19) of Ref. [7] simply from Eq. (3.18) as individual members in Eq. (3.18), i.e., in the expression

\[ p_{\alpha,\beta} - p_{\alpha,\beta'} + p_{\alpha',\beta} + p_{\alpha',\beta'} - p^{(1)}_{\alpha} - p^{(2)}_{\beta}, \]

would have to be integrated and averaged over different variables \( b_{\gamma}^{(j)} \) (having different statistical distributions). The given approach could be applied to and Bell’s inequalities would be obtained if it held again

\[ b_{\gamma}^{(j)} \equiv b_{\gamma'}^{(j)}. \]

The additional condition (9) or (11) relates to one-particle states only, which is in agreement with the recent results of Revzen and Mann [10]. Any influence of internal space structure of macroscopic measuring objects on measured results seems to be fully neglected in such a case.

5 Concluding remark

The effective impact parameters \( b_{\gamma}^{(j)} \) must be necessarily included in the description of physical processes if a hidden variable theory is to correspond fully to physical reality. The influence of internal structures of macroscopic measuring devices on measured results cannot be then omitted. However, in such a case it is not more possible to derive Bell’s inequalities for probabilities measured in EPR polarization experiments. Consequently, their violation by the corresponding experimental data can be hardly regarded as a proof of non-locality (or inseparability) of microscopic objects. A more detailed analysis of the whole problem should be performed and physical meaning of EPR experiments newly analyzed.

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