Flow model in the impeller of a centrifugal pump

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Abstract. The calculation of the fluid flow in the interscapular channel of a centrifugal impeller with a finite number of blades, the construction of the energy characteristics of the impeller and its optimization by the number of vanes are considered. The calculation consists of two parts: firstly, the determination of the theoretical pump head taking into account the effect of a finite number of vanes based on the analysis of the force interaction, and, secondly, the determination of hydraulic losses in the impeller by integrating the friction stresses over the limiting surfaces. The results from both parts are used to optimize the number of vanes in the pump impeller.

Based on the results of the analysis of the experimental visualization of the flow in the impeller (Figure 1) [1], it can be concluded that the difference between the circumferential component of the fluid velocity and the circumferential velocity of the leading disk at the unpressurized side of the vane, and their equality on the pressure side.

Considering that the force interaction in a liquid propagates only in the form of a longitudinal elastic wave, it can be concluded that the instantaneous direction of the liquid velocity should coincide with the rectilinear direction of wave propagation (force interaction). If we neglect the dissipation of the motion energy and the damping of the wave on the scale of the considered geometric forms, we can assume that the value of the instantaneous velocity along the line of elastic force interaction is a constant value. The initial (generating) point of the line of elastic force interaction is located on the surface of the vane and sets the value of the velocity along the line. Therefore, it is possible to define a family of characteristic lines for the transferable movement, along which the value of the transferable velocity is determined and constant.

Based on the foregoing, it is possible to write down an expression for the pump head at each point at the outlet of the impeller for various types of vanes.

The calculation scheme for a wheel with rectilinear tangential vanes is shown in Figure 2. In this case, the expression for the pump head at an arbitrary point \(j\) takes the form:

\[
H_j = U_2 C_{2uj} = U_2 U_{transf} \cos \gamma = U_2 U_i \cos \gamma .
\] (1)

The circumferential velocity is determined by the expression:

\[
U = R \omega \quad \Rightarrow \quad H_j = R_i R_2 \omega^2 \cos \gamma .
\] (2)

The radius at any point of the vane is determined by the expression:

\[
R_i = R_2 \cos \gamma .
\] (3)
Figure 1. Visualization of the flow in the impeller of a low-flow pump with rectilinear vanes: \( \dot{V} = 5 \cdot 10^{-6} \text{ m}^3/\text{s} \); \( D_2 = 0.0405 \); \( b_2 = 0.003 \); \( D_1 = 0.0155 \).

The final expression for the pump head will take the form:

\[
H_j = R^2 \omega^2 \cos^2 \gamma
\]  
(4)

The theoretical head in case a finite number of vanes is determined step by step according to the following algorithm. At each step, the increment of angle \( \gamma \) is set. The head value at the current point is determined by the equation (4). The theoretical head is defined as the averaging of the obtained values:

\[
H_T = \frac{\sum_{j=1}^{n} H_j}{n}.
\]  
(5)

Influence coefficient of a finite number of vanes:

\[
k_z = \frac{H_T}{H_{T\infty}}.
\]  
(6)

The value of the theoretical head in an impeller with an infinite number of vanes is determined by the classical expression of Euler's streamjet theory [2].

Figure 3 shows the change in the theoretical head for a finite number of vanes, depending on the number of vanes.

Given the complexity of viscous flows in the impellers, the real characteristics differ from the theoretical ones by the amount of hydraulic losses:

\[
H = H_T - \Delta H = \eta_z H_T
\]  
(7)

Hydraulic losses depend on the magnitude and direction of the relative velocities in the flow part of the impeller and are determined by the integral of the friction stresses along the limiting surfaces: the cylindrical surfaces of the vanes and the end surfaces of the cover disks.
To date, the assessment of the actual values of the head depending on the main geometric and operating parameters of the impeller in most practical applications is based on semi-empirical expressions obtained as the result of generalization and analysis of experimental data [3; 4; 5; 6].

A method for determining hydraulic losses, which is the integration of friction stresses along the limiting surface of a curved channel, is presented in the work [1]. Here, when integrating the finite-difference analogs of the equations of impulses of the spatial boundary layer, the thickness of the impulse loss $\delta^*_{\rho}$ and the bevel angle of the bottom streamline $\varepsilon$ are calculated at each step.

The total head drop is written as an integral [7]

$$\Delta H = \frac{\Delta P}{\rho} = \frac{P_2 - P_1}{\rho} = \frac{1}{\rho} \int \tau_0 dS = \frac{1}{\rho} \int \tau_0 d\phi d\psi ;$$

Expanding $\tau_0$ into projections, we obtain

$$\tau_0 = \sqrt{\tau_{0\phi}^2 + \tau_{0\psi}^2} ;$$

$$\Delta H = \frac{1}{\rho} \int \sqrt{\tau_{0\phi}^2 + \tau_{0\psi}^2} d\phi d\psi .$$

Considering $\tau_{0\psi} = \varepsilon \tau_{0\phi}$, we get
\[ \Delta H = \frac{1}{\rho} \int_{S} \sqrt{\tau_{0\varphi}^{2} + \varepsilon^{2} \tau_{0\varphi}^{2}} \, d\varphi d\psi = \frac{1}{\rho} \sqrt{[1 + \varepsilon^{2}]} \int_{S} \tau_{0\varphi} \, d\varphi d\psi. \] (11)

From the law of friction [8]

\[ \tau_{0\varphi} = 0.01256 \rho U^{2} \left( \frac{U \varphi^{**}}{v} \right)^{-0.25} . \] (12)

Substituting in (11), we obtain the expression for the head loss

\[ \Delta H = \frac{1}{\rho} \sqrt{[1 + \varepsilon^{2}]} \int_{S} \tau_{0\varphi} \, d\varphi d\psi = 0.01256 \sqrt{[1 + \varepsilon^{2}]} \int_{S} U^{2} \left( \frac{U \varphi^{**}}{v} \right)^{-0.25} \, d\varphi d\psi. \] (13)

Substituting the found values of \( \varepsilon \) and \( \varphi^{**} \), we obtain the value of the head losses.

The presented method for calculating the spatial boundary layer is quite simple and intuitive. However, there is a need for further elaboration of the method to bring it to a form that makes it possible to calculate the three-dimensional flow of a working fluid in a channel of arbitrary shape.

It should be noted that the results of comparison of the numerical and experimental visualization show a similar flow pattern, which indirectly confirms the validity of the assumptions and the correctness of the conclusions when constructing a method for determining the field of transferable velocity in the interscapular channel of the impeller.

Based on the results of theoretical studies, an algorithm was developed, which allows calculating local values of head along the length of the interscapular channel using the known geometric parameters of the wheel, angular velocity and flow rate.

Sample calculation results are presented in Figures 4 and 5.

Figure 4 shows the hydraulic losses of the head along the channel length.

![Figure 4. Hydraulic head losses along the channel length.](image-url)
Figure 5 shows the dependences of the coefficient of influence of the finite number of vanes \( k_z \), hydraulic efficiency \( \eta_z \) and optimization parameter \( k_z \cdot \eta_z \) on the number of impeller vanes.

![Diagram showing dependences of \( k_z \), \( \eta_z \) and \( k_z \cdot \eta_z \) on the number of vanes.]

Having carried out a series of calculations for a different number of vanes, it is possible to select the most optimal option according to the criterion \( k_z \cdot \eta_z \rightarrow \text{max} \).

References
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