Physics Potential of the ICAL detector at the India-based Neutrino Observatory (INO)

The ICAL Collaboration
[The ICAL Collaboration]

Shakeel Ahmed, M. Sajjad Athar, Rashid Hasan, Mohammad Salim, S. K. Singh
*Aligarh Muslim University, Aligarh 202001, India*

S. S. R. Inbanathan
*The American College, Madurai 625002, India*

Venktesh Singh, V. S. Subrahmanyan
*Banaras Hindu University, Varanasi 221005, India*

Shiba Prasad Behera, Vinay B. Chandratre, Nitali Dash, Vivek M. Datar
*Bhabha Atomic Research Centre, Trombay, Mumbai 400085, India*

Animesh Chatterjee, Sandhya Choubey, Raj Gandhi, Anushree Ghosh
*Harish Chandra Research Institute, Jhunsi, Allahabad 211019, India*

Ali Ajmi, S. Uma Sankar
*Indian Institute of Technology Bombay, Powai, Mumbai 400076, India*

Prafulla Behera, Aleena Chacko, Sadiq Jafer, James Libby, K. Raveendrababu, K. R. Rebin
*Indian Institute of Technology Madras, Chennai 600036, India*

D. Indumathi, K. Meghna, S. M. Lakshmi, M. V. N. Murthy, Sumanta Pal
*Institute of Mathematical Sciences, Taramani, Chennai 600113, India*

Sanjib Kumar Agarwalla, Amina Khatun
*Institute of Physics, Sachivalaya Marg, Bhubaneswar 751005, India*

Poonam Mehta
*Jawaharlal Nehru University, New Delhi 110067, India*

Vipin Bhatnagar, R. Kanishka, A. Kumar, J. S. Shahi, J. B. Singh
*Panjab University, Chandigarh 160014, India*

Monojit Ghosh, Pomita Ghoshal, Srubabati Goswami, Chandan Gupta, Sushant Raut
*Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India*

Sudeb Bhattacharya, Suvendu Bose, Ambar Ghosal, Abhik Jash, Kamalesh Kar, Debasish Majumdar, Nayana Majumdar, Supratik Mukhopadhyay, Satyajit Saha
*Saha Institute of Nuclear Physics, Bidhannagar, Kolkata 700064, India*

B. S. Acharya, Sudeshna Banerjee, Kolahal Bhattacharya, Sudeshna Dasgupta
*Tata Institute of Fundamental Research, Colaba, Mumbai 400005, India*

C. D. Ravikumar, A. M. Vinodkumar
*University of Calicut, Kozhikode, Kerala 673635, India*
Gautam Gangopadhyay, Amitava Raychaudhuri
University of Calcutta, Kolkata 700009, India

Brajesh C. Choudhary, Ankit Gaur, Daljeet Kaur, Ashok Kumar, Sanjeev Kumar, Md. Naimuddin
University of Delhi, New Delhi 110021, India

Waseem Bari, Manzoor A. Malik
University of Kashmir, Hazratbal, Srinagar 190006, India

Jyotsna Singh
University of Lucknow, Lucknow 226007, India

S. Krishnaveni, H. B. Ravikumar, C. Ranganathaiah
University of Mysore, Mysuru, Karnataka 570005, India

Swapna Mahapatra
Utkal University, Vani Vihar, Bhubaneswar, 751004, India

Saikat Biswas\textsuperscript{SB}, Subhasis Chattopadhyay, Rajesh Ganai, Tapasi Ghosh\textsuperscript{TG}, Y. P. Viyogi\textsuperscript{YV}
Variable Energy Cyclotron Centre, Bidhannagar, Kolkata 700064, India

\textsuperscript{NM} Spokesperson, nkm@tifr.res.in
\textsuperscript{AC} Currently: University of Texas at Arlington, USA.
\textsuperscript{AG} Currently: Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro, Brazil.
\textsuperscript{AR} Currently: University of Bern, Switzerland.
\textsuperscript{DS} Currently: Central University of Karnataka, Gulbarga, India.
\textsuperscript{GR} Also at Chennai Mathematical Institute, Chennai, India.
\textsuperscript{MD} Currently: Weizmann Institute of Science, Rehovot, Israel.
\textsuperscript{PG} Currently: LNM Institute of Information Technology, Jaipur, India.
\textsuperscript{SR} Currently: KTH Royal Institute of Technology, Stockholm, Sweden.
\textsuperscript{SB} Currently: National Institute of Science, Education and Research, Bhubaneswar, India.
\textsuperscript{SD} Currently: University of Bristol, UK.
\textsuperscript{SP} Currently: University of Sheffield, UK.
\textsuperscript{TG} Currently: Universidade Federal de Goias, Goiania, Brazil.
\textsuperscript{TT} Currently: Louisiana State University, USA.
\textsuperscript{VD} Currently: Tata Institute of Fundamental Research, Mumbai, India.
\textsuperscript{YV} Raja Ramanna Fellow.
Abstract

The upcoming 50 kt magnetized iron calorimeter (ICAL) detector at the India-based Neutrino Observatory (INO) is designed to study the atmospheric neutrinos and antineutrinos separately over a wide range of energies and path lengths. The primary focus of this experiment is to explore the Earth matter effects by observing the energy and zenith angle dependence of the atmospheric neutrinos in the multi-GeV range. This study will be crucial to address some of the outstanding issues in neutrino oscillation physics, including the fundamental issue of neutrino mass hierarchy. In this document, we present the physics potential of the detector as obtained from realistic detector simulations. We describe the simulation framework, the neutrino interactions in the detector, and the expected response of the detector to particles traversing it. The ICAL detector can determine the energy and direction of the muons to a high precision, and in addition, its sensitivity to multi-GeV hadrons increases its physics reach substantially. Its charge identification capability, and hence its ability to distinguish neutrinos from antineutrinos, makes it an efficient detector for determining the neutrino mass hierarchy. In this report, we outline the analyses carried out for the determination of neutrino mass hierarchy and precision measurements of atmospheric neutrino mixing parameters at ICAL, and give the expected physics reach of the detector with 10 years of runtime. We also explore the potential of ICAL for probing new physics scenarios like CPT violation and the presence of magnetic monopoles.
Preface

The past two decades in neutrino physics have been very eventful, and have established this field as one of the flourishing areas of high energy physics. Starting from the confirmation of neutrino oscillations that resolved the decades-old problems of the solar and atmospheric neutrinos, we have now been able to show that neutrinos have nonzero masses, and different flavors of neutrinos mix among themselves. Our understanding of neutrino properties has increased by leaps and bounds. Many experiments have been constructed and envisaged to explore different facets of neutrinos, in particular their masses and mixing.

The Iron Calorimeter (ICAL) experiment at the India-based Neutrino Observatory (INO) [1] is one of the major detectors that is expected to see the light of the day soon. It will have unique features like the ability to distinguish muon neutrinos from antineutrinos at GeV energies, and measure the energies of hadrons in the same energy range. It is therefore well suited for the identification of neutrino mass hierarchy, the measurement of neutrino mixing parameters, and many probes of new physics. The site for the INO has been identified, and the construction is expected to start soon. In the meanwhile, the R&D for the ICAL detector, including the design of its modules, the magnet coils, the active detector elements and the associated electronics, has been underway over the past decade. The efforts to understand the capabilities and physics potentials of the experiment through simulations are in progress at the same time.

We present here the Status Report of our current understanding of the physics reach of the ICAL, prepared by the Simulations and Physics Analysis groups of the INO Collaboration. It describes the framework being used for the simulations, the expected response of the detector to particles traversing it, and the results we expect to obtain after the 50 kt ICAL has been running for about a decade. The focus of the physics analysis is on the identification of the mass hierarchy and precision measurements of the atmospheric neutrino mixing parameters. The feasibilities of searches for some new physics, in neutrino interactions as well as elsewhere, that can be detected at the ICAL, are also under investigation.

The first such report [2] had been published when the INO was being proposed, and the ICAL Collaboration was at its inception. Our understanding of the detector has now matured quite a bit, and more realistic results can now be obtained, which have been included in this report. The work on improving several aspects of the detector, the simulations, the reconstruction algorithms and the analysis techniques is in progress and will remain so for the next few years. This Report is thus not the final word, but a work in progress that will be updated at regular intervals.

In addition to the ICAL detector, the INO facility is designed to accommodate experiments in other areas like neutrinoless double beta decay, dark matter search, low-energy
neutrino spectroscopy, etc. Preliminary investigations and R&D in this direction are in progress. The special environment provided by the underground laboratory may also be useful to conduct experiments in rock mechanics, geology, biology etc. This Report focusses mainly on the ongoing physics and simulation related to the ICAL detector. The details of other experiments will be brought out separately.

The Government of India has recently (December 2014) given its approval for the establishment of INO. This is a good opportunity to present the physics capabilities of the ICAL experiment in a consolidated form.
3.2.2 The Magnetic Field ............................................. 30
3.2.3 The Resistive Plate Chambers ................................. 31
3.3 Event Simulation and Digitisation ............................... 32
3.4 Event Reconstruction ............................................. 32

4 ICAL Response to Muons and Hadrons ......................... 35
4.1 Simulations Response of ICAL to Muons ....................... 35
4.1.1 Momentum resolution in different azimuthal regions .... 37
4.1.2 Zenith Angle Resolution ..................................... 39
4.1.3 Reconstruction efficiency ................................... 40
4.1.4 Relative charge identification efficiency ................... 41
4.2 Simulations Response of ICAL to hadrons ..................... 41
4.2.1 Energy response to fixed-energy hadrons ................... 42
4.2.1.1 The $e/h$ ratio ............................................. 43
4.2.2 Analysis of the charged pion hit pattern .................... 45
4.2.2.1 Dependence of the energy resolution on hadron direction . 47
4.2.3 Response to hadrons produced by atmospheric neutrinos . 48
4.2.4 Hadron energy calibration .................................... 49
4.3 Salient features of the detector response ....................... 50

5 Neutrino Oscillation Physics at ICAL ............................ 51
5.1 Charged-current $\nu_\mu$ events in ICAL ....................... 51
5.2 Analysis with $E_\mu$ and $\cos \theta_\mu$ .............................. 55
5.2.1 Mass hierarchy sensitivity ................................... 56
5.2.1.1 Impact of priors .......................................... 57
5.2.1.2 Impact of systematic uncertainties ....................... 58
5.2.1.3 Impact of the true value of $\sin^2 \theta_{23}$ .................. 59
5.2.1.4 (In)sensitivity to the CP violating phase ................. 59
5.2.2 Precision measurement of $|\Delta m^2_{32}|$ and $\sin^2 \theta_{23}$ .... 61
5.2.2.1 Impact of finer binning ................................... 62
5.2.3 Sensitivity to the octant of $\theta_{23}$ ............................ 62
5.3 Analysis with $E_\nu$ and $\cos \theta_\mu$ ............................... 63
5.4 Analysis using correlated Information on $E_\mu$, $\cos \theta_\mu$, and $E'_\text{had}$ ................. 65
5.4.1 The three-dimensional binning ............................... 66
5.4.2 Mass hierarchy sensitivity ................................... 67
5.4.3 Precision Measurement of Atmospheric Parameters .......... 68
5.4.4 Non-maximal $\theta_{23}$ and its octant ......................... 70
5.5 ICAL physics potential: highlights ............................. 71

6 Synergy with Other Experiments ................................ 73
6.1 Combined mass hierarchy reach at $\delta_{CP} = 0$
for ICAL + T2K + NO$\nu$A ........................................ 73
6.2 Ensuring mass hierarchy sensitivity for all $\delta_{CP}$ .................. 75
6.3 Octant of $\theta_{23}$ from the ICAL, NO$\nu$A and T2K data .......... 76
6.4 ICAL data for improving CPV discovery potential of T2K and NOνA ........................................... 78

7 Exploring New Physics at ICAL ........................................ 81
  7.1 Probing Lorentz and CPT Violation ........................................ 81
     7.1.1 CPTV effects at the probability level ........................................ 81
     7.1.2 Simulation procedure and results ........................................ 84
  7.2 Search for Magnetic Monopoles ........................................ 85
     7.2.1 Monopole simulation for ICAL ........................................ 86
     7.2.2 Reach for limits on the monopole flux ........................................ 86

8 Concluding Remarks ........................................ 89

Acknowledgements ........................................ 91

Appendices ........................................ 93
  A Neutrino Fluxes at the Kamioka and INO sites ........................................ 93
  B Neutrino oscillation probabilities in matter ........................................ 99
     B.1 One Mass Scale Dominance approximation ........................................ 100
     B.2 Double expansion in $\alpha$ and $\sin\theta_{13}$ ........................................ 103
     B.3 Probability for Reactor neutrinos ........................................ 104
  C The Vavilov distribution function ........................................ 105
  D Hadron energy resolution as a function of plate thickness ........................................ 107
Executive Summary

The INO and the ICAL detector

The India-based Neutrino Observatory (INO) is proposed to be built in Bodi West Hills, in Theni district of Tamil Nadu in South India. The details of the location and the layout may be found in Chapter 1.

The main detector proposed to be built at the INO is the magnetised Iron CALorimeter (ICAL) with a mass of 50 kt. In ICAL, 5.6 cm thick iron plates act as the target, whose 151 layers are interleaved with the Resistive Plate Chambers (RPCs) that act as the active detector elements. See Table 1 for the specifications of the ICAL detector and the RPCs. The details of the detector, the RPCs and the magnetic field may be found in Section 3. The cavern for the detector will have enough space to accommodate another detector of the same size.

| ICAL                  |            |
|-----------------------|------------|
| No. of modules        | 3          |
| Module dimension      | 16 m × 16 m × 14.5 m |
| Detector dimension    | 48 m × 16 m × 14.5 m |
| No. of layers         | 151        |
| Iron plate thickness  | 5.6 cm     |
| Gap for RPC trays     | 4.0 cm     |
| Magnetic field        | 1.5 Tesla  |

| RPC                   |            |
|-----------------------|------------|
| RPC unit dimension    | 2 m × 2 m  |
| Readout strip width   | 3 cm       |
| No. of RPC units/Layer/Module | 64       |
| Total no. of RPC units| ∼ 30,000   |
| No. of electronic readout channels | 3.9 × 10^6 |

Table 1: Specifications of the ICAL detector.

The ICAL detector is optimised to be sensitive primarily to the atmospheric muon neutrinos in the 1–10 GeV energy range. The structure of the detector, with its horizontal layers of iron interspersed with RPCs, allows it to have an almost complete coverage to the direction of incoming neutrinos, except for those that produce almost horizontally traveling muons. This makes it sensitive to a large range of path lengths $L$ for the neutrinos travelling
through the Earth, while the atmospheric neutrino flux provides a wide spectrum in the neutrino energy $E_{\nu}$.

ICAL will be sensitive to both the energy and direction of the muons that will be produced in charged-current (CC) interactions of the atmospheric muon neutrinos (and antineutrinos) with the iron target in the detector. In addition, the fast response time of the RPCs (of the order of nanoseconds) will allow for a discrimination of the upward-going muon events and downward-going ones. This direction discrimination separates the neutrinos with short path lengths from those with longer ones. Such a separation is crucial since the neutrino oscillation probability is strongly dependent on the path length $L$.

Moreover, since ICAL is expected to be magnetised to about 1.5 T in the plane of the iron plates, it will be able to discriminate between muons of different charges, and hence will be capable of differentiating events induced by muon neutrinos and muon antineutrinos. Through this sensitivity, one can probe the difference in matter effects in the propagation of neutrinos and antineutrinos that traverse the Earth before they reach the detector. This in turn will allow for a sensitivity to the neutrino mass hierarchy, which is the primary goal of the ICAL experiment. In addition, the magnetic field improves the momentum resolution of the muons by measuring the extent of bending of the muon track in the local magnetic field.

The good tracking ability and energy resolution of ICAL for muons makes it very well suited for the study of neutrino oscillation physics through the observation of atmospheric neutrinos. In addition, ICAL is also sensitive to the energy deposited by hadrons in the detector in the multi-GeV range, a unique property that enables a significant improvement in the physics reach of ICAL, as will be clear in this report. In the present configuration the sensitivity of ICAL to electrons is very limited; however, this is still under investigation.

Though the ICAL is yet to be built, its putative properties have been simulated using the CERN GEANT4 [3] package. The details of these simulations have been presented in Chapter 3. This report presents results on the response of ICAL to particles traversing through it in Chapter 4. The resultant physics potential of the detector, obtained from these simulations, is given in later chapters, where we focus on the identification of neutrino mass hierarchy, and the precise determinations of the atmospheric neutrino parameters: $|\Delta m^2_{3\ell}|$ and $\sin^2 2\theta_{23}$, as well as the octant of $\theta_{23}$. In addition, we also discuss some novel and exotic physics possibilities that may be explored at ICAL.

The simulation framework

For the results presented in this report, the atmospheric neutrino events have been generated with the NUANCE [4] neutrino generator using the Honda 3d fluxes [5] for the Kamioka site in Japan. The Honda atmospheric neutrino fluxes at Theni, the INO site, are expected to be finalised soon and will be used when available. A preliminary comparison of the fluxes at the two sites is also presented in Chapter 2 of this report. However, all results presented here are with the Kamioka Honda flux.

A typical CC interaction of $\nu_{\mu}$ in the detector gives rise to a charged muon that leaves a track, and single or multiple hadrons that give rise to shower-like features. The simulations of the propagation of muons and hadrons in the detector have been used to determine the response of the detector to these particles. This leads to the determination of detection efficiencies, charge identification efficiencies, calibrations and resolutions of energies and
In order to perform the physics analysis, we generate a large number (typically, an exposure of 1000 years) of unoscillated events using NUANCE, which are later scaled to a suitable exposure, and oscillations are included using a reweighting algorithm. The typical values of oscillation parameters used are close to their best-fit values, and are given in Table 2. Here $\Delta m_{32}^2 \equiv \Delta m_{32}^2 - (\cos^2 \theta_{12} - \cos \delta_{CP} \sin \theta_{32} \sin 2\theta_{12} \tan \theta_{23}) \Delta m_{21}^2$ is the effective value of $\Delta m_{atm}^2$ relevant for the two-neutrino analysis of atmospheric neutrino oscillations [6, 7]. The energies and directions of the relevant particles are then smeared according to the resolutions determined earlier. This approach thus simulates the average behaviour of the measured quantities. At the current stage of simulations, we also assume that the muon track and the hadron shower can be separated with full efficiency, and that the noise due to random hits near the signal events in the short time interval of the event is negligible. While these approximations are reasonable, they still need to be justified with actual detector, possibly by collecting data with a prototype. In the meanwhile, a complete simulation, which involves passing each of the generated events through a GEANT4 simulation of the ICAL detector, is in progress.

The different analyses then determine the relevant physics results through a standard $\chi^2$ minimization procedure, with the systematic errors included through the method of pulls, marginalizations over the allowed ranges of parameters, and including information available from other experiments using priors. The details of the analysis procedures for obtaining the neutrino mixing parameters have been given in Chapter 5, which presents the results using only the information on muon energy and angle, as well as the improvement due to the inclusion of information on hadron energies. Chapter 6 further includes combined analyses of the reach of ICAL with other current and near-future detectors such as T2K and NO\(\nu\)A, for the mass hierarchy and neutrino oscillation parameters. Chapter 7 discusses the reach of ICAL with respect to exotic physics possibilities such as the violation of CPT or Lorentz symmetries, the detection of magnetic monopoles, etc.

We now list the highlights of the results compiled in this report. Many of these results have appeared elsewhere [8, 9, 10, 11, 12, 13, 14, 15, 16], however some have been updated with more recent information.

### Detector response to propagating particles

#### Response to muons

The ICAL detector is optimised for the detection of muons propagating in the detector, identification of their charges, and accurate determination of their energies and directions. The energies and directions of muons are determined through a Kalman filter based algorithm.
The reconstructed energy (direction) for a given true muon energy (direction) is found to give a good fit to the Gaussian distribution, and hence the resolution is described in terms of the mean and standard deviation of a Gaussian distribution. The reconstruction efficiency for muons with energies above 2 GeV is expected to be more than 80%, while the charge of these reconstructed muons is identified correctly on more than 95% occasions. The direction of these muons at the point of their production can be determined to within about a degree.

The muon energy resolution depends on the part of the detector the muon is produced in, but is typically 25% (12%) at 1 GeV (20 GeV), as can be seen in the left panel of Fig. 1.

Response to hadrons

The detector response to hadrons is quantified in terms of the quantity $E'_{\text{had}} \equiv E_\nu - E_\mu$ for the CC processes that produce a muon, which is calibrated against the number of hits in the detector. The hit distribution for a given hadron energy is found to give a good fit to the Vavilov distribution. Hence $E'_{\text{had}}$ is calibrated against the mean of the corresponding Vavilov mean for the number of hits, and the energy resolution is taken to be the corresponding value of $\sigma$. The energy resolution is shown in Fig. 1. The complete description of Vavilov distributions needs a total of four parameters, the details of which may be found in Chapter 4.

The presence of different kinds of hadrons, which are hard to distinguish through the hit information, is taken care of by the generation of events through NUANCE, which is expected to produce the hadrons in the right proportions. The energy resolution of hadrons is found to be about 85% (36%) at 1 GeV (15 GeV). The information on the shape of the hadron shower is not used for extracting hadron energy yet; the work on this front is still in progress.
Physics reach of ICAL

Sensitivity to the Mass Hierarchy

In order to quantify the reach of ICAL with respect to the neutrino mass hierarchy, a specific hierarchy, normal or inverted, is chosen as the true (input) hierarchy. The CC muon neutrino events are binned in the quantities chosen for the analysis, and a $\chi^2$ analysis is performed taking the systematic errors into account and marginalising over the 3$\sigma$ ranges of the parameters $|\Delta m^2_{\text{eff}}|$, $\sin^2 \theta_{23}$ and $\sin^2 2\theta_{13}$. The significance of the result is then determined as the $\Delta \chi^2_{\text{ICAL-MH}}$ with which the wrong hierarchy can be rejected (see Chapter 5).

The analysis for mass hierarchy identification using only the muon momentum information \[11\] yields $\Delta \chi^2_{\text{ICAL-MH}} \approx 6.5$ with 10 years of exposure of the 50 kt ICAL, as can be seen from Fig. 2. However the same figure also shows that a considerable improvement in the physics reach is obtained if the correlated hadron energy information in each event is included along with the muon energy and direction information; i.e. the binning is performed in the 3-dimensional parameter space $(E_\mu, \cos \theta_\mu, E'_\text{had})$. The same exposure now allows the identification of mass hierarchy with a significance of $\Delta \chi^2_{\text{ICAL-MH}} \approx 9.5$ \[14\]. The significance depends on the actual value of $\theta_{23}$ and $\theta_{13}$, and increases with the values of these mixing angles. Depending on the values of these mixing angles, the significance may be $\Delta \chi^2_{\text{ICAL-MH}} \approx 7$–12 with an exposure of 500 kt-yr.

![Figure 2: The hierarchy sensitivity of ICAL with input normal (left) and inverted (right) hierarchy including correlated hadron energy information, with $|\Delta m^2_{\text{eff}}|$, $\sin^2 \theta_{23}$ and $\sin^2 2\theta_{13}$ marginalised over their 3$\sigma$ ranges \[14\]. Improvement with the inclusion of hadron energy is significant.](image)

Precision Measurements of oscillation parameters

The precision on the measurements of the neutrino oscillation parameters $\sin^2 \theta_{23}$ and $|\Delta m^2_{32}|$ is quantified in terms of $\Delta \chi^2_{\text{ICAL-PM}}(\lambda)$, where $\lambda$ is the parameter under consideration. The precision on the measurement of $\theta_{23}$ is essentially a function of the total number of events, and is expected to be about 12-14%, whether one includes the hadron energy information.
or not. The precision on $|\Delta m^2_{32}|$, however, improves significantly (from 5.4% to 2.9%) if the information on hadron energy is included.

Figure 3 shows the comparison of the 10-year reach of 50 kt ICAL in the $\sin^2 2\theta_{23} - \Delta m^2_{32}$ plane, with the current limits from other experiments. It is expected that the $\Delta m^2_{32}$ precision of ICAL would be much better than the atmospheric neutrino experiments that use water Cherenkov detectors, due to its better energy measurement capabilities. However the beam experiments will keep on accumulating more data, hence the global role of ICAL for precision measurements of these parameters will be not competitive, but complementary.

Figure 3: The precision reach of ICAL in the $\sin^2 2\theta_{23} - \Delta m^2_{32}$ plane, in comparison with other current and planned experiments [14]. Information on hadron energy has been included.

**Sensitivity to the octant of $\theta_{23}$**

While the best-fit value for $\sin^2 2\theta_{23}$ is close to maximal, it is not fully established whether it deviates from maximality, and if so, whether $\sin^2 \theta_{23}$ is less than or greater than 0.5, that is, whether it lies in the first or second octant. ICAL is sensitive to the octant of $\theta_{23}$ through two kinds of effects: one is through the depletion in atmospheric muon neutrinos (and antineutrinos) via the survival probability $P_{\mu\mu}$ and the other is the contribution of the atmospheric electron neutrinos to the observed CC muon events through the oscillation probability $P_{e\mu}$. Both effects are proportional to $\sin^2 2\theta_{13}$, however act in opposite directions, thereby reducing the effective sensitivity of ICAL to the $\theta_{23}$ octant. The reach of ICAL alone for determining the octant is therefore limited; it can identify the octant to a $2\sigma$ significance with 500 kt-yr only if $\sin^2 \theta_{23} < 0.37$ [14]. The information from other experiments clearly needs to be included in order to identify the octant.
Synergies with other experiments

Neutrino Mass Hierarchy determination

The ability of currently running long baseline experiments like T2K and NO\(\nu\)A to distinguish between the mass hierarchies depends crucially on the actual value of the CP-violating phase \(\delta_{\text{CP}}\). For example, if \(\delta_{\text{CP}}\) is vanishing, this ability is severely limited. However if one adds the data available from the proposed run of these experiments, a preliminary estimation suggests that even for vanishing \(\delta_{\text{CP}}\), the mass hierarchy identification of 3\(\sigma\) may be achieved with a run-time as low as 6 years of the 50 kt ICAL, for maximal mixing [17]. This may be observed in Fig. 4. Note that this improvement in the ICAL sensitivity is not just due to the information provided by these experiments on the mass hierarchy, but also due to the improved constraints on \(\Delta m^2_{32}\) and \(\theta_{23}\).

![Graph showing hierarchy sensitivity with input normal (left) and inverted (right) hierarchy when ICAL data is combined with the data from T2K and NO\(\nu\)A](image)

Figure 4: Preliminary results on the hierarchy sensitivity with input normal (left) and inverted (right) hierarchy when ICAL data is combined with the data from T2K (total luminosity of 8 \times 10^{21} \text{ pot in neutrino mode}) and NO\(\nu\)A (3 years running in neutrino mode and 3 years in antineutrino mode) [17].

Identifying mass hierarchy at all \(\delta_{\text{CP}}\) values

The large range of path length of the atmospheric neutrinos makes ICAL insensitive to the CP phase \(\delta_{\text{CP}}\), as a result its reach in distinguishing the hierarchy is also independent of the actual value of \(\delta_{\text{CP}}\) [15]. On the other hand the sensitivity of fixed-baseline experiments such as T2K and NO\(\nu\)A is extremely limited if \(0 < \delta_{\text{CP}} < \pi\) and the true hierarchy is normal. However adding of the ICAL information ensures that the hierarchy can be identified even in these unfavoured \(\delta_{\text{CP}}\) regions [11]. Of course, in the \(\delta_{\text{CP}}\) regions favourable to the long baseline experiments, the ICAL data can only enhance the power of discriminating between the two hierarchies.
Determination of the CP phase

Though ICAL itself is rather insensitive to $\delta_{CP}$, data from ICAL can still improve the determination of $\delta_{CP}$ itself, by providing input on mass hierarchy. This is especially crucial in the range $0 \leq \delta_{CP} \leq \pi$, precisely where the ICAL data would also improve the hierarchy discrimination of NO$\nu$A and other experiments [19].

Other physics possibilities with ICAL

ICAL is a versatile detector, and hence could be employed to test for a multitude of new physics scenarios. For example, the violation of CPT or Lorentz symmetry in the neutrino sector [15] can be probed to a great precision, owing to its excellent energy measurement capability. The passage of magnetic monopoles through the detector may be looked for by simply looking for slowly moving, undeflecting tracks [16]. Dark matter annihilation inside the Sun may be constrained by comparing the flux from the sun with the flux from other directions. Many such scenarios are under investigation currently.

Concluding remarks

A strong and viable physics program is ready for ICAL at INO. The simulations based on the incorporation of the ICAL geometry in GEANT4 suggests that the detector will have excellent abilities for detection, charge identification, energy measurement and direction determination for charged muons of GeV energies. The magnetic field enables separation of $\mu^-$ from $\mu^+$, equivalently that of $\nu_\mu$ from $\bar{\nu}_\mu$, thus increasing the sensitivity to the difference in matter effects on neutrino and antineutrino oscillations. It will also be sensitive to hadrons, an ability that will increase its physics reach significantly and will offer advantages over other atmospheric neutrino detectors. Apart from its main aim of identifying the neutrino mass hierarchy, ICAL can also help in precision measurements of other neutrino mixing parameters, and can probe exotic physics issues even beyond neutrinos.
Chapter 1

Introduction

The earth is just a silly ball
To them, through which they simply pass
Like dustmaids through a drafty hall
- John Updike

Many important developments have taken place in neutrino physics and neutrino astronomy in recent years. The discovery of neutrino oscillations and consequent inference about the non-vanishing mass of the neutrinos, from the study of neutrinos from the Sun and cosmic rays, have had far-reaching consequences for particle physics, astroparticle physics and nuclear physics. The observation of neutrinos from natural sources as well as those produced at reactors and accelerators have given us the first confirmed signals of physics beyond the Standard Model of particle physics. They have also enabled us access to the energy production mechanisms inside stars and other astrophysical phenomena.

Experimental observations of neutrino interactions began in the mid 1950s at Savannah river reactor by Reines and Cowan [20] followed by experiments deep in the mines of Kolar Gold Fields (KGF) in India [21] and in South Africa [22]. The pioneering solar neutrino experiments of Davis and collaborators in the USA [23, 24], the water Cherenkov detector Kamiokande [25] and its successor the gigantic Super-Kamiokande (SK) [26, 27], the gallium detectors SAGE [28] in Russia and Gallex [29], GNO [30] at the Laboratorio National di Gran Sasso (LNGS) in Italy, the heavy-water detector at the Sudbury Neutrino Observatory (SNO) in Canada [31, 32], the KamLAND [33] and K2K [34] experiments in Japan, etc. have together contributed in a very fundamental way to our knowledge of neutrino properties and interactions. The observation of solar neutrinos has given a direct experimental proof of the decades old hypothesis that the Sun and the stars are powered by thermonuclear fusion reactions that emit neutrinos. The recent results from reactor neutrino experiments, beginning with Double CHOOZ [35] in France and culminating in the results from Reno [36] in Korea and Daya Bay [37] in China, and from accelerator experiments like MINOS [38] and T2K [39] have further revealed properties of neutrinos that not only serve as windows to physics beyond the Standard Model of particle physics, but also provide possibilities of understanding the matter-antimatter asymmetry in the universe through the violation of the charge-parity (CP) symmetry in the lepton sector.
Impelled by these discoveries and their implications for the future of particle physics and astrophysics, plans are underway worldwide for new neutrino detectors to study such open issues as the hierarchy of neutrino masses, the masses themselves, the extent of CP violation in the lepton sector, the Majorana or Dirac nature of neutrinos, etc.. This involves R&D efforts for producing intense beams of neutrinos at GeV energies, suitable detectors to detect them at long baseline distances, and sensitive neutrinoless double beta decay experiments. A complementary approach to these is the use of atmospheric neutrinos, whose fluxes are more uncertain than beam neutrinos, but which provide a wider range of energies, and more importantly, a wider range of baselines.

The India-based Neutrino Observatory (INO) is one such proposal aiming to address some of the challenges in understanding the nature of neutrinos, using atmospheric neutrinos as the source. The unique feature of ICAL, the main detector in INO, will be the ability to distinguish neutrinos from antineutrinos, which enables a clearer distinction between the matter effects on neutrinos and antineutrinos travelling through the Earth, leading to the identification of neutrino mass hierarchy. In this Chapter, we shall introduce the INO laboratory, the ICAL detector, and describe the role of such a detector in the global context of neutrino physics experiments.

1.1 The ICAL detector at the INO facility

1.1.1 Neutrino experiments in India: past and present

Underground science in India has a long history. The deep underground laboratory at Kolar Gold Fields (KGF), where Indian scientists conducted many front ranking experiments in the field of cosmic rays and neutrinos, was a pioneering effort. The very first detection of neutrinos produced by cosmic rays was made at KGF in early 1965 [21] and paved the way for future experiments around the world. Not only Indian scientists, but scientists from Japan and UK also participated in experiments at KGF. Later this collaboration put together the first experiment that searched for proton decay. Nature did not oblige and experiments are still looking for proton decay. The KGF laboratory operated for nearly four decades, almost till the end of 1980’s, collecting data on atmospheric muon and neutrino interactions at various depths, starting from about 300 metres all the way down to 2700 metres. In the process they also detected some anomalous events which could not be attributed to neutrinos at depths around 2000 metres [40, 41, 42]. Such events have neither been proved wrong nor have they been confirmed by other experiments.

The INO project, the discussions about which were formally held first in the Workshop on High Energy Physics Phenomenology (WHEPP-VI) in Chennai [43] is an ambitious proposal to recapture this pioneering spirit and do experiments in neutrino physics at the cutting edge. The immediate goal of INO project is the creation of an underground laboratory which will house a large magnetised iron calorimeter (ICAL) detector to study the properties of naturally produced neutrinos in the earth’s atmosphere. Apart from experiments involving neutrinos, in the long term the laboratory is envisaged to develop into a full-fledged underground laboratory for studies in Physics, Biology and Geology as well. The INO is the first basic science laboratory planned on such a large scale in India.
1.1.2 Location and layout of the INO

The INO will be located at the Bodi West Hills (BWH), near Pottipuram village, in the Theni district of Tamil Nadu, India. The site has been chosen both for geotechnical reasons as well as from environmental considerations. It is near the historic city of Madurai, as shown in Fig. 1.1. Madurai is about 120 km from the INO site, and will also be the location for the Inter-Institutional Centre for High Energy Physics (IICHEP), where activities related to the INO will be carried out. The figure shows the location and also the features of the local terrain. The construction of the laboratory below the Bodi Hills involves building a horizontal tunnel, approximately 1900 m long, to reach the laboratory that is located under a mountain peak. One large and three small laboratory caverns are to be built with a rock burden of 1000 m or more all around (with a vertical overburden of \( \sim 1300 \) m) to house the experiments. The reduction of cosmic ray background at this site is almost the same as at the Gran Sasso laboratory, as can be seen from the bottom right panel, which shows the cosmic ray muon flux as a function of depth, with the locations of other major laboratories.

In addition to the main Iron CALorimeter (ICAL) detector whose prime goal is the determination of the neutrino mass hierarchy, the laboratory is designed to accommodate experiments in other areas like neutrinoless double beta decay, dark matter search, low-energy neutrino spectroscopy, etc. Preliminary investigations and R&D in this direction are in progress. The special environment provided by the underground laboratory may also be useful to conduct experiments in rock mechanics, geology, biology etc.

The present configuration of the laboratory caverns is shown in Fig. 1.2. The largest cavern that will house the main iron calorimeter detector (ICAL) is 132 m (L) \( \times 26 \) m (W) \( \times 32.5 \) m (H). This cavern, called “UG-Lab 1”, is designed to accommodate a 50 kt ICAL (planned) and a second possible ICAL-II neutrino detector of equal size. Each ICAL consists of three modules with dimensions of 16 m (L) \( \times 16 \) m (W) \( \times 14.5 \) m (H), so that the total footprint of both the detectors would be 96 m (L) \( \times 16 \) m (W). Fig. 1.2 shows three modules of the 50 kt ICAL.

1.1.3 The ICAL detector

The ICAL detector is similar in concept to the earlier proposed Monolith [44, 45] detector at Gran Sasso. The layout of the proposed ICAL detector is shown in Fig. 1.3. The detector will have a modular structure of total lateral size 48 m \( \times 16 \) m, subdivided into three modules of area 16 m \( \times 16 \) m. It will consist of a stack of 151 horizontal layers of 5.6 cm thick magnetized iron plates interleaved with 4 cm gaps to house the active detector layers, making it 14.5 m high. Iron spacers acting as supports will be located every 2 m along both X and Y directions; the 2 m wide roads along the transverse (Y) direction will enable the insertion and periodic removal of RPCs, when required.

The active detector elements, the resistive plate chambers (RPCs) made up of a pair of 3 mm thick glass plates of area 2 m \( \times 2 \) m separated by 2 mm spacers, will be inserted in the gaps between the iron layers. These will be operated at a high voltage of about 10 kV in avalanche mode. A high energy charged particle, passing through the RPCs, will leave signals that will be read by orthogonal X and Y pickup strips, about 3 cm wide, one on each side of an RPC. Detailed R & D has shown that the RPCs have an efficiency of around
Figure 1.1: The location of the INO site and the nearby major landmarks. The IICHEP is located about 120 km east of the INO site, in the city of Madurai, as shown in the top right panel. The photo in the bottom right panel shows the view of the hill under which the cavern will be located. The terrain is totally flat with minimal undergrowth as seen in the picture (Photo: M V N Murthy). The photo is taken before the start of any construction. The bottom left panel shows the suppression in intensity of atmospheric muon flux at various underground sites, compared to the INO cavern [2].
Figure 1.2: Underground caverns layout showing the footprints of proposed experiments and other components

Figure 1.3: Schematic view of the 50 kt ICAL detector
90–95% with a time resolution of about a nanosecond. This will allow the determination of the X and Y coordinates of the track of the charged particles passing through the RPC. The layer number of the RPC will provide the Z coordinate. The observed RPC time resolution of $\sim 1$ ns will enable the distinction between upward-going particles and downward-going particles. From the hit pattern observed in the RPCs, the energies as well as directions of the charged particles produced in the neutrino interactions can be reconstructed.

Each module will have two vertical slots cut into it to enable current-carrying copper coils to be wound around as shown in Fig. 1.3. Simulation studies [46] have shown that the iron plates can be magnetized to a field strength of about 1.5 T, with fields greater than 1 T over at least 85% of the volume of the detector. The bending of charged particles in this magnetic field will enable the identification of their charge. In particular, the sign of the charge of the muon produced by neutrino interactions inside the detector will help in identifying and studying the $\nu_\mu$ and $\bar{\nu}_\mu$ induced events separately. The magnetic field will also help in the measurement of the momentum of the final state particles, especially the muons.

With about 14000 iron plates of $2 \times 4$ m area and 5.6 cm thickness, 30000 RPCs of $2 \times 2$ m area, 4,000,000 electronic readout channels, and a magnetic field of 1.5 T, the ICAL is going to be the largest electromagnet in the world, and is expected to play a pivoting role in our understanding of neutrino properties.

### 1.2 Role of ICAL in neutrino mixing and beyond

In this section, we briefly discuss our present understanding of neutrino oscillation parameters and identify the fundamental issues in the neutrino sector that can be addressed by the ICAL detector.

#### 1.2.1 Current status of neutrino mixing parameters

The neutrino flavour states $|\nu_\alpha\rangle$ (where $\alpha = e, \mu, \tau$) are linear superpositions of the neutrino mass eigenstates $|\nu_i\rangle$ (with $i = 1, 2, 3$), with masses $m_i$:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle.$$  \hspace{1cm} (1.1)

Here $U$ is the $3 \times 3$ unitary mixing matrix. A physically motivated form of the mixing matrix that is conventionally used is [47, 48, 49]

$$U = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & c_{23}s_{13} \\
  s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$, and $\delta$ denotes the CP violating (Dirac) phase, also called $\delta_{CP}$. Note that the Majorana phases are not included in the above parameterization, since they do not play a role in neutrino oscillation experiments. The $U$ matrix diagonalises the
3×3 neutrino mass-squared matrix \( M_\nu M_\nu^\dagger \) in the basis of the charged leptons, which appears in the effective Hamiltonian for neutrino propagation, through

\[
U^\dagger M_\nu M_\nu^\dagger U = \text{diag}(m_1^2, m_2^2, m_3^2) .
\] (1.2)

The probability of an initial neutrino \( \nu_\alpha \) of flavour \( \alpha \) and energy \( E \) being detected as a neutrino \( \nu_\beta \) of the same energy but with flavour \( \beta \) after travelling a distance \( L \) in vacuum is

\[
P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}[U_{\alpha i} U_{\beta i}^* U_{\alpha j} U_{\beta j}^*] \sin^2(\Delta_{ij}) + 2 \sum_{i>j} \text{Im}[U_{\alpha i} U_{\beta i}^* U_{\alpha j} U_{\beta j}^*] \sin(2\Delta_{ij}) ,
\] (1.3)

where \( \Delta_{ij} = 1.27 \Delta m_{ij}^2 (eV^2) \times L \text{(km)}/E \text{(GeV)} \), with \( \Delta m_{ij}^2 = m_i^2 - m_j^2 \) the mass squared differences between the \( i \)th and \( j \)th neutrino mass eigenstates. Oscillation measurements are not sensitive to the individual neutrino masses, but only to their mass-squared differences. Note that the above expression is valid only for propagation through vacuum. In matter, the probabilities are drastically modified. The relevant expressions may be found in Appendix B.

The neutrino flavour conversion probabilities can be expressed in terms of the two mass squared differences, the three mixing angles, and the single CP-violating phase. Also of crucial importance is the mass ordering, i.e., the sign of \( \Delta m_{32}^2 \) (the same as the sign of \( \Delta m_{21}^2 \)). While we know that \( \Delta m_{21}^2 \) is positive so as to accommodate the observed energy dependence of the electron neutrino survival probability in solar neutrino experiments, at present \( \Delta m_{32}^2 \) is allowed to be either positive or negative. Hence, it is possible to have two patterns of neutrino masses: \( m_3 > m_2 > m_1 \), called normal ordering, where \( \Delta m_{32}^2 \) is positive, and \( m_2 > m_1 > m_3 \), called inverted ordering, where \( \Delta m_{32}^2 \) is negative. Determining the sign of \( \Delta m_{32}^2 \) is one of the prime goals of the ICAL experiment. **Note that, though the “mass ordering” is perhaps the more appropriate term to use in this context, the more commonly used term in literature is “mass hierarchy”**. In this report, therefore, we will use the notation “normal hierarchy” (NH) to denote normal ordering, and “inverted hierarchy” (IH) to denote inverted ordering. The word “hierarchy” used in this context has no connection with the absolute values of neutrino masses.

Table 1.1 summarises the current status of neutrino oscillation parameters [50, 51] based on the world neutrino data that was available after the NOW 2014 conference. The numbers given in Table 1.1 are obtained by keeping the reactor fluxes free in the fit and also including the short-baseline reactor data with \( L \leq 100 \text{ m} \) [50, 51].

Table 1.1 also provides the relative 1\( \sigma \) precision\(^1\) on the measurements of these quantities at this stage. The global fit suggests the best-fit value of \( \Delta m_{21}^2 = 7.5 \times 10^{-5} \text{ eV}^2 \) with a relative 1\( \sigma \) precision of 2.4\%. In a three-flavor framework, we have the best-fit values of \( \Delta m_{31}^2 = 2.42 \times 10^{-3} \text{ eV}^2 \) for NH and \( \Delta m_{32}^2 = -2.41 \times 10^{-3} \text{ eV}^2 \) for IH. A relative 1\( \sigma \) precision of 2.5\% has been achieved for the atmospheric mass-squared splitting.

As far as the mixing angles are concerned, \( \theta_{12} \) is now pretty well measured with a best-fit value of \( \sin^2 \theta_{12} = 0.3 \) and a relative 1\( \sigma \) precision of 4\% has been achieved for the solar mixing angle. Our understanding of the atmospheric mixing angle \( \theta_{23} \) has also improved a lot in recent years. Combined analysis of all the neutrino oscillation data available so far disfavours the maximal mixing solution for \( \theta_{23} \) at \( \sim 1.5\sigma \) confidence level [52, 50, 51, 53, 54]. This result

\(^1\)Here the 1\( \sigma \) precision is defined as 1/6th of the ±3\( \sigma \) variations around the best-fit value.
| Parameter | Best-fit values | 3σ ranges | Relative 1σ precision (in percent) |
|-----------|----------------|-----------|-----------------------------------|
| $\Delta m_{31}^2$ (eV$^2$) | $7.5 \times 10^{-5}$ | $[7.0, 8.1] \times 10^{-5}$ | $\sigma(\Delta m_{31}^2) = 2.4\%$ |
| $\Delta m_{32}^2$ (eV$^2$) | $2.46 \times 10^{-3}$ (NH) | $[2.32, 2.61] \times 10^{-3}$ (NH) | $\sigma(\Delta m_{32}^2) = 2.0\%$ |
| $\Delta m_{31}^2$ (eV$^2$) | $-2.45 \times 10^{-3}$ (IH) | $-[2.59, 2.31] \times 10^{-3}$ (IH) | $\sigma(\Delta m_{32}^2) = 1.9\%$ |
| $\sin^2 \theta_{12}$ | 0.3 | $[0.27, 0.34]$ | $\sigma(\sin^2 \theta_{12}) = 4.4\%$ |
| $\sin^2 \theta_{23}$ | 0.45 (NH), 0.58 (IH) | $[0.38, 0.64]$ | $\sigma(\sin^2 \theta_{23}) = 8.7\%$ |
| $\sin^2 \theta_{13}$ | 0.022 | $[0.018, 0.025]$ | $\sigma(\sin^2 \theta_{13}) = 5.3\%$ |
| $\delta_{CP}(^\circ)$ | 306 | $[0, 360]$ | – |

Table 1.1: Current status of neutrino oscillation parameters [50, 51]. The second column shows the central values of the oscillation parameters. The third column depicts the present 3σ ranges of the parameters with the relative 1σ errors being listed in the last column. Note that the parameter $\Delta m_{31}^2$ ($\Delta m_{32}^2$) is used while performing the fit with normal (inverted) hierarchy.

is mostly governed by the MINOS accelerator data in $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance modes [55]. The dominant term in $\nu_\mu$ survival channel mainly depends on the value of $\sin^2 2\theta_{23}$. Now, if $\sin^2 2\theta_{23}$ turns out to be different from 1 as suggested by the recent oscillation data, then it gives two solutions for $\sin^2 \theta_{23}$: one whose value is less than half, known as the lower octant (LO) solution, and the other whose value is greater than half, known as the higher octant (HO) solution. This creates the problem of octant degeneracy of $\theta_{23}$ [56]. At present, the best-fit value of $\sin^2 \theta_{23}$ in LO (HO) is 0.45 (0.58) assuming NH (IH). The relative 1σ precision on $\sin^2 \theta_{23}$ is around 8.7% assuming maximal mixing as the central value. Further improvement in the knowledge of $\theta_{23}$ and settling the issue of its octant (if it turns out to be non-maximal) are also important issues that can be addressed by observing atmospheric neutrinos.

For many years, we only had an upper bound on the value of the 1-3 mixing angle [57, 58, 59, 60]. A nonzero value for this angle has been discovered rather recently [35, 36, 37, 61, 62], with a moderately large best-fit value of $\sin^2 \theta_{13} = 0.022$, which is mostly driven by the high-statistics data provided by the ongoing Daya Bay reactor experiment [37, 62]. It is quite remarkable that already we have achieved a relative 1σ precision of 5.3% on $\sin^2 \theta_{13}$. On the other hand, the whole range of $\delta_{CP}$ is still allowed at the 3σ level.

1.2.2 Unravelling three-neutrino mixing with ICAL

As has been discussed earlier, the main advantage of a magnetised iron calorimeter is its ability to distinguish $\mu^+$ from $\mu^-$, and hence to study $\nu_\mu$ and $\bar{\nu}_\mu$ separately. This allows a cleaner measurement of the difference in the matter effects experienced by neutrinos and antineutrinos. However, this difference depends on the value of $\theta_{13}$. The recent measurement of a moderately large value of $\theta_{13}$ therefore boosts the capability of ICAL for observing these matter effects, and hence its reach in addressing the key issues related to the neutrino masses and mixing. In this section, we shall highlight the role that an iron calorimeter like ICAL...
will have in the context of global efforts to measure neutrino mixing parameters.

The moderately large $\theta_{13}$ value has opened the door to the fundamental measurements of (i) the neutrino mass ordering, (ii) the deviation of 2-3 mixing angle from its maximal value and hence the correct octant of $\theta_{23}$, and (iii) the CP phase $\delta_{\text{CP}}$ and to look for CP violation in the lepton sector, for several experiments which would have had limited capability to address these questions had this parameter been significantly smaller. Central to all these measurements are effects which differ between neutrinos and antineutrinos. These could either be matter related effects which enhance or suppress the oscillation probabilities (relevant for i and ii above), or those induced by a non-zero value of $\delta_{\text{CP}}$ (relevant for iii). The ICAL will be sensitive to the matter effects, but will have almost no sensitivity to the actual value of $\delta_{\text{CP}}$.

Prior to summarizing the role of ICAL, it is useful to mention several other experiments which are underway or will come online in the next decade and a half with the aim of making the three important measurements mentioned above. Mass hierarchy measurements are slated to be performed by the proposed atmospheric neutrino detectors Hyper-K [63] and PINGU [64], as well as by the accelerator based long-baseline beam experiments NOvA [65, 66, 67], LBNE [68, 69], and LBNO [70, 71, 72]. Additionally, the medium-baseline reactor oscillation experiments [73], JUNO [74] and RENO-50 [75] aim to determine the hierarchy by performing a very precise, high statistics measurement of the neutrino energy spectrum. The CP phase $\delta_{\text{CP}}$ can be measured (in principle) by accelerator experiments like T2K [76, 77], NOvA [78, 79, 80], T2HK [63], LBNE [69], and LBNO [72]. These experiments, if they run in both the neutrino and the antineutrino mode, would additionally be sensitive to the octant of $\theta_{23}$ [81, 82], and so would the large-mass atmospheric experiments like ICAL [14] and Hyper-K [83].

The ICAL detector at the INO cavern will provide an excellent opportunity to study the atmospheric neutrinos and antineutrinos separately with high detection efficiency and good enough energy and angular resolutions in the multi-GeV range in the presence of the Earth’s matter effect. This feature of the ICAL detector is quite unique in the sense that the detection efficiency and the energy resolution of the currently running or upcoming water or ice based atmospheric neutrino detectors are quite limited in the multi-GeV range where the Earth’s matter effect plays an important role. There is no doubt that the rich data set which would be available from the proposed ICAL atmospheric neutrino experiment will be extremely useful to validate the three flavor picture of the neutrino oscillation taking into account the Earth’s large matter effect in the multi-GeV range. The first aim of the ICAL detector would be to observe the oscillation pattern over at least one full period, in order to make a precise measurement of the atmospheric oscillation parameters. The ICAL detector performs quite well in a wide range of $L/E$ variations compared to the water Cherenkov detectors and can confirm the evidence of the sinusoidal flavor transition probability of neutrino oscillation already observed by the Super-Kamiokande detector by observing the dips and peaks in the event rate versus $L/E$ [84]. Unlike in case of Super-Kamiokande where the sub-GeV events have played an important role to perform this $L/E$ analysis, in the ICAL detector, multi-GeV events will dominate this study. In its initial phase, the ICAL experiment will also provide an independent measurement of $\theta_{13}$ by exploring the Earth’s matter effect using the atmospheric neutrinos. This will certainly complement the ongoing efforts of the reactor and the accelerator experiments to learn about the smallest lepton
mixing angle $\theta_{13}$.

The relevant neutrino oscillation probabilities that the ICAL will be sensitive to are $P_{\mu\mu}$, $P_{\mu\bar{\mu}}$, $P_{e\mu}$, and $P_{\bar{e}\mu}$, especially the former two. These probabilities have a rich structure for neutrinos and antineutrinos at GeV energies, traveling through the Earth for a distance of several thousands of km (for a detailed description, see Appendix B). The matter effects on these neutrinos and antineutrinos lead to significant differences between these oscillation probabilities, which may be probed by a detector like ICAL that can distinguish neutrinos from antineutrinos. This feature of ICAL would be instrumental in its ability to distinguish between the two possible mass hierarchies.

Detailed simulations of the ICAL detector performance, as discussed in the following chapters, show that ICAL would be an excellent tracker for muons. The energy and direction of a muon would also be reconstructed rather accurately, with the muon direction resolution of better than a degree at high energies. Furthermore, the capability of ICAL to study the properties of the final state hadrons in multi-GeV neutrino interactions would be one of its unique features. This would allow the reconstruction of the neutrino energy in every event, albeit with large error. (Note that the extraction of hadronic information at multi-GeV energies in currently running or upcoming water or ice based atmospheric neutrino detectors is quite challenging; the efficiency of reconstruction of multi-ring events is rather small in such detectors.) As a result, the ICAL would have a significant stand-alone sensitivity to the mass hierarchy, which, when combined with data from experiments like NOvA and T2K, would significantly enhance the overall sensitivity to this important quantity.

Although the ICAL would not be sensitive to the value of $\delta_{CP}$, this very feature would make it an important supporting experiment for others that are sensitive to $\delta_{CP}$, in a unique manner. Note that in experiments where event rates are sensitive simultaneously to both matter and CP phase effects, disentangling one from the other restricts the sensitivities to individual and unambiguous measurements of each of the three quantities (i), (ii) and (iii) mentioned above. The virtue of ICAL here would lie in its ability to offer a data-set that is free of entanglements between matter enhancements and dynamical CP violating effects due to a non-zero $\delta_{CP}$. Thus, when used in combination with other experiments, the ICAL measurements will facilitate the lifting of degeneracies which may be present otherwise. In particular, the ICAL data, when combined with that from NOvA and T2K, would make a significant difference to their discovery potential of CP violation [19].

1.2.3 Addressing new physics issues with ICAL

The full role of an iron calorimeter in the global scenario of neutrino physics is rich and complex. In addition to what is described here, it can add to our knowledge on very high energy muons [85, 86] on hitherto undiscovered long range forces [87], on CPT violation [15, 88] and on non-standard interactions [89], among other issues. The future of neutrino physics is, in our opinion, crucially dependent on the synergistic combination of experiments with differing capabilities and strengths. A large iron calorimeter brings in unique muon charge identification capabilities and an event sample independent of the CP phase. Both these aspects will play an important role in our concerted global effort to understand the mysteries of neutrino physics and consequently understand physics beyond the Standard Model.
Though the ICAL has been designed mainly with neutrino physics in view, it is expected that many non-neutrino issues may find relevance with this detector. For example, a few decades ago, both in the cosmic ray neutrino experiments [40, 41] and later in the proton decay experiment [42] at Kolar Gold Fields (KGF) in south India, some unusual events were seen. These so-called Kolar events were multi-track events with some unusual features which could not be explained away by any known processes of muons or neutrinos. Recently it has been speculated that such events may be caused by the decay of unstable cold dark matter particles with mass in the range of 10 GeV with a life time approximately equal to the age of the universe [90]. Such an interpretation may be easily tested with ICAL at INO, even without further modifications [91]. Signals of dark matter annihilation inside the Sun can also be detected at ICAL. The possible observation of GUT monopoles is another such issue that can be addressed at ICAL with its current setup.
Chapter 2

Atmospheric Neutrino Fluxes

Everything is in a state of flux, Even the status quo.
- Robert Byrne

2.1 Introduction to Atmospheric Neutrinos

Atmospheric neutrinos are produced in the cosmic ray interactions with the nuclei of air molecules in the atmosphere. The first report of cosmic ray induced atmospheric neutrinos was from the deep underground laboratories at Kolar Gold Field (KGF) in India by TIFR, Osaka University and Durham University [21], and immediately afterwards by Reines et al. [22] in an experiment conducted in South African mines in 1965. Atmospheric neutrinos have been studied since then in several other underground laboratories, and important discoveries such as the evidence for neutrino oscillations [27], have been made. We will briefly review the atmospheric neutrinos in this section.

Primary cosmic rays are high energy particles impinging on the Earth from galactic and extragalactic sources. Their origins are still clouded in mystery. In the GeV energy range, the cosmic ray particles are made up of mainly protons and about 9% helium nuclei, with a small fraction of heavy nuclei. Although the energy spectrum of cosmic rays extends to very high energies, beyond even $10^{10}$ GeV, it falls rapidly as energy increases. When cosmic rays enter the atmosphere, interactions with the nuclei in air molecules produce secondary particles. These secondary particles are mainly pions with a small admixture of kaons. These mesons decay mainly to muons and their associated neutrinos following the decay chain

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu),$$
$$\mu^\pm \rightarrow e^\pm + \bar{\nu}_\mu (\nu_\mu) + \nu_e (\bar{\nu}_e).$$

(2.1)

Kaons also decay in a similar manner producing the two neutrino flavours, but their contribution to the atmospheric neutrino flux is small compared to the pions for neutrinos of a few GeV. We call the neutrinos produced in this manner as atmospheric neutrinos. It may be noted that only the $\nu_e$ and $\nu_\mu$ neutrinos, along with their antiparticles, are produced in the atmosphere. The flux of $\nu_\tau$ requires the production of mesons with heavy quarks, as a result their flux is extremely small and we do not consider these neutrinos here. A schematic illustration of this cascading neutrino production is shown in Fig. 2.1.
From Eq. 2.1 it is clear that the ratio

\[ R = \frac{\Phi(\nu_\mu) + \Phi(\bar{\nu}_\mu)}{\Phi(\nu_e) + \Phi(\bar{\nu}_e)} \approx 2 , \]

where \( \Phi \) denotes the flux of neutrinos. The ratio is only approximate, since at high energies muon may not decay before reaching the surface of the Earth. It, however, remains greater than 2, which may be observed from Fig. 2.2. The figure displays the direction-integrated neutrino fluxes in various models, as well as the ratios of fluxes of different kinds of neutrinos.

An important property of the atmospheric neutrino flux is that it is symmetric about a given direction on the surface of the Earth, that is

\[ \Phi_\nu(E, \cos \theta) = \Phi_\nu(E, -\cos \theta) , \]

where \( \theta \) is the zenith angle. This result is robust above 3 GeV, though at lower energies the geomagnetic effects result in deviations from this equality. Therefore, at higher energies, any asymmetry in the fluxes of the upward-going and downward-going neutrinos can be attributed to the flavour changes during propagation. Even at lower energies, large deviations from the above equality are not expected, except from neutrino oscillations. This up-down asymmetry is thus the basis of atmospheric neutrino analysis, and it was effectively used by the SuperKamiokande collaboration to establish the first confirmed signal of neutrino oscillations [27]. Of course detailed analyses need the calculations of atmospheric neutrino fluxes as functions of energies, zenith angles as well as azimuthal angles.

The atmospheric neutrinos not only provide neutrinos in two distinct flavours, but also over a whole range of energies from hundreds of MeV to TeV and beyond. Yet another advantage over the conventional accelerator neutrino beams is the fact that the atmospheric neutrinos traverse widely different distances in different directions: from \( \sim 10 \) km on the
way downwards to more than 12000 km on the way upwards through the centre of the earth. They also traverse matter densities varying from very small (essentially air) to almost 13 g/cc when passing through the earth’s core.

These facts make the analysis of atmospheric neutrinos not only interesting but also unique. The ICAL is expected to exploit the advantages of the freely available atmospheric neutrino flux, not only to explore the neutrino oscillation parameters but also determine the hierarchy of neutrino masses, and perhaps probe new physics.

2.2 Calculations of atmospheric neutrino fluxes

The neutrino oscillation studies with atmospheric neutrinos can be put on a firm foundation provided the atmospheric neutrino flux estimates and their interaction cross sections are known as precisely as possible. The main steps in the determination of atmospheric neutrino fluxes are:

- The energy spectrum of primary cosmic rays: The flux of primary cosmic rays decreases approximately as \( E^{-2.7} \) in the 10 GeV to TeV region. Consequently the flux of neutrinos decreases rapidly in the high energy region. The flux of cosmic rays outside the atmosphere is isotropic and constant in time. These are well measured experimentally up to tens of GeV. The primary spectrum of cosmic ray protons can be fitted to a form

\[
\Phi(E) = K [E + b \exp(-c\sqrt{E})]^{-\alpha},
\]

where \( \alpha = 2.74, \ K = 14900, \ b = 2.15, \ c = 0.21 \) [95].

- The energy spectrum of secondary muons: The interactions of primary cosmic rays with the air nuclei produce in pions and kaons, which in turn yield muons. An important input needed for this calculation is the hadronic cross sections. These are well measured
Figure 2.3: Zenith angle dependence of the neutrino flux averaged over azimuthal angles are shown for three different energies. This has been calculated for the Kamioka location. Figures are taken from Honda et al. [96]. Note that the flux scales are different for different energies.

in accelerator experiments from low energy up to hundreds of GeV. Beyond the range of accelerator energies these cross sections are model-dependent. Hence the composition of the secondary cosmic rays and their energy spectrum is well known up to TeV energies. The muons are produced by the decay of these mesons.

- The energy spectrum of neutrinos: This needs modelling the altitude dependence of interactions in the atmosphere, the geomagnetic effect on the flux of cosmic rays and secondaries, and the longitudinal dependence of extensive air showers.

Uncertainties in each one of the above steps limit the precision in the determination of neutrino fluxes on the surface. Typically these introduce an uncertainty of the order of 15-20% in the overall normalization.

Some typical features of the zenith angle distributions of atmospheric neutrino fluxes may be seen in Fig. 2.3, which show the fluxes (averaged over azimuthal angles) for three neutrino energies, calculated for Kamioka, the location of the Superkamiokande detector. The figure shows that the flux is typically maximum near $\cos \theta = 0$, i.e. for horizontal neutrinos, where the muons have had the maximum proper time to decay. Also, the ratio of muon to electron neutrino flux is observed to increase at higher energies and at more vertical (down-going or up-going) neutrinos, where muons have less proper time to decay, so the second reaction in Eq. 2.1 is less efficient. It can also be seen that at $E > 3$ GeV, the fluxes are essentially
symmetric in zenith angle. However at lower energies, there is some asymmetry, arising mainly from the bending of muons in the geomagnetic field.

Recently Honda et al. [97] have calculated the atmospheric neutrino spectrum at the INO location (Theni). The analyses presented in this report use fluxes at the Kamioka location. We plan to use the recently compiled fluxes [96, 97] for the Theni site in our future analysis.
Chapter 3

The ICAL Simulation Framework

A good simulation ... gives us a sense of mastery over experience.
Heinz R. Pagels

The broad simulation framework for the ICAL, starting with event generation, is indicated schematically in Table 3.1. The events in the detector are generated using the NUANCE Monte Carlo generator [4]. This uses the atmospheric neutrino fluxes as described in Chapter 2 along with various possible neutrino-nucleus interaction cross-sections to generate the vertex and the energy-momentum of all final states in each event; these are then propagated through the virtual ICAL detector using the GEANT4 simulation tool. The GEANT4 simulates the propagation of particles through the detector, including the effects of the iron, the RPCs, and the magnetic field. The information in the events is then digitised in the form of \((X,Y,Z)\) coordinates of the hits in the RPCs and the timing corresponding to each of these “hits”. This is the information available for the event reconstruction algorithms, which attempt, from the hit pattern, to separate the muons tracks from the showers generated by the hadrons, and reconstruct the energies and directions of these particles. The process is described in detail below.

3.1 Neutrino interactions and event generation

Neutrino and antineutrino interactions in the ICAL detector are modelled using NUANCE neutrino generator version 3.5 [4]. Some preliminary studies and comparisons have also been initiated using the GENIE neutrino generator [98], but are not a part of this Report. The interactions modelled in NUANCE include (i) quasi-elastic scattering (QE) for both charged and neutral current neutrino interactions with nucleons, which dominate below neutrino energies of 1 GeV, (ii) resonant processes (RES) with baryon resonance production mainly from neutrinos with energy between 1 and 2 GeV, (iii) deep inelastic scattering (DIS) processes with considerable momentum (squared) transfer from the neutrino to the target, with the nuclei breaking up into hadrons, which is the dominant contribution in the multi-GeV region, (iv) coherent nuclear processes on nuclei, and (v) neutrino-electron elastic scattering and inverse muon decay. These are the main neutrino interaction processes
### NUANCE

**Neutrino Event Generation**

\[ \nu + N \rightarrow \ell + X \]

Generates particles that result from a random interaction of a neutrino with matter using theoretical models for both neutrino fluxes and cross-sections.

**Output:**

(i) Reaction Channel  
(ii) Vertex and time information  
(iii) Energy and momentum of all final state particles

### GEANT

**Event Simulation**

\[ \ell + X \text{ through simulated ICAL} \]

Simulates propagation of particles through the ICAL detector with RPCs and magnetic field.

**Output:**

(i) \(x, y, z, t\) of the particles as they propagate through detector  
(ii) Energy deposited  
(iii) Momentum information

### DIGITISATION

**Event Digitisation**

\((X, Y, Z, T)\) of final states on including noise and detector efficiency

Add detector efficiency and noise to the hits.

**Output:**

(i) Digitised output of the previous stage

### ANALYSIS

**Event Reconstruction**

\((E, \vec{p})\) of \(\ell, X\) (total hadrons)

Fit the muon tracks using Kalman filter techniques to reconstruct muon energy and momentum; use hits in hadron shower to reconstruct hadron information.

**Output:**

(i) Energy and momentum of muons and hadrons, for use in physics analyses.

---

Table 3.1: The simulation frame-work as implemented in the ICAL simulation package.
of relevance for our simulation frame work, with the contribution of the last two being the least in the few GeV energy region of interest. A simple ICAL geometry has been described within NUANCE, including mainly the iron and glass components of the detector, as most of the interactions will occur in these two media. NUANCE identifies these bound nucleons (with known Fermi energies) differently from free nucleons and also applies final state nuclear corrections.

The NUANCE generator calculates event rates by integrating different cross sections weighted by the fluxes for all charged current (CC) and neutral current (NC) channels at each neutrino energy and angle. Some typical total cross sections for different CC processes are illustrated in Fig. 3.1. Based on the interaction channel, there can be 10-40% uncertainty in cross sections in the intermediate energy ranges [99].

![Figure 3.1: Muon neutrino (left) and muon antineutrino (right) total charged current cross sections in (cm$^2$/GeV), obtained from NUANCE, are shown (smooth lines) as a function of incident neutrino energy, $E_\nu$, in comparison with the existing measurements of these cross sections along with their errors [100]. Note that the $y$-axis scale of the two panels is different.](image)

As mentioned in Chapter 2, for the present we have used the Honda fluxes [5] generated at the location of Kamioka. This will be changed soon to that at the actual location of INO. We choose to generate only unoscillated neutrino events using NUANCE for the simulations, even though there is a provision for generating oscillated events in it. The oscillations are applied externally, separately in each analysis.

3.2 Simulation of the ICAL Detector

We now describe the ICAL detector geometry within the GEANT4 [3] simulation framework. This includes the geometry itself, and the magnetic field map and the RPC characteristics that are inputs to the simulation.

3.2.1 The Detector Geometry

The simulations have been performed for the 50 kt ICAL detector, which has a modular structure with the full detector consisting of three modules, each of size 16 m (length) ×
16 m (width) × 14.5 m (height), with a gap of 20 cm between the modules. The ICAL coordinates are defined as follows. The direction along which the modules are placed is labelled as the \( x \)-direction with the remaining horizontal transverse direction being labelled \( y \). At present, \( x \) is also considered to coincide with South, since the final orientation of the INO cavern is not yet decided. The \( z \)-axis points vertically upwards so that the polar angle equals the zenith angle \( \theta \) while the zero of the azimuthal angle \( \phi \) points South. The origin is taken to be the centre of the second module. Each module comprises 151 horizontal layers of 5.6 cm thick iron plates. The area of each module is 16 m \( \times \) 16 m, while the area of each iron plate is 2 m \( \times \) 4 m. There is a vertical gap of 4 cm between two layers. The iron sheets are supported every 2 m in both the \( x \) and \( y \) directions, by steel support structures. The basic RPC units have dimensions of 1.84 m \( \times \) 1.84 m \( \times \) 2.5 cm, and are placed in a grid format within the air gaps, with a 16 cm horizontal gap between them in both \( x \) and \( y \) directions to accommodate the support structures.

Vertical slots at \( x = x_0 \pm 4 \) m (where \( x_0 \) is the central \( x \) value of each module) extending up to \( y = \pm 4 \) m and cutting through all layers are provided to accommodate the four copper coils that wind around the iron plates, providing a magnetic field in the \( x-y \) plane, as shown in Fig. 3.2. The detector excluding the coils weighs about 52 kt, with 98% of this weight coming from iron where the neutrino interactions are dominantly expected to occur, and less than 2% from the glass of the RPCs. In the central region of each module, typical values of the field strength are about 1.5 T in the \( y \)-direction, as obtained from simulations using MAGNET6.26 software [101].

### 3.2.2 The Magnetic Field

Fig. 3.2 depicts the magnetic field lines in the central iron plate near the centre of the central module. The arrows denote the direction of magnetic field lines while the length of the arrows (and the shading) indicates the magnitude of the field. The maximum magnitude of the magnetic field is about 1.5 T. Notice that the field direction reverses on the two sides of the coil slots (beyond \( x_0 \pm 4 \) m) in the \( x \)-direction. In between the coil slots (an 8 m \( \times \) 8 m square area in the \( x-y \) plane) the field is maximum and nearly uniform in both magnitude (to about 10%) and direction; we refer to this as the central region. Near the edges in the \( x \) direction (outside the coil slots) the field is also fairly uniform, but in the opposite direction; this is called the side region. Near the edges in the \( y \) direction, i.e., in the regions \( 4 \) m \( \leq |y| \leq 8 \) m, both the direction and magnitude of the magnetic field vary considerably; this region is labelled as the peripheral region.

In our simulations, the field has been assumed to be uniform over the entire thickness of an iron plate at every \((x, y)\) position, and has been generated in the centre of the iron plate, viz., at \( z = 0 \). In the 4 cm air gap between the iron plates, the field is taken to be zero since it falls off to several hundred gauss in these regions, compared to more than 1 T inside the iron plates. The magnetic field is also taken to be zero in the (non-magnetic) steel support structures. These support structures, along with the coil slot, form the bulk of the dead spaces of the detector.

The side and peripheral regions are beset by edge effects as well as by non-uniform and lower magnetic field. We confine ourselves, in the present study, to tracks generated only in the central region (\(-4 \) m \( \leq x - x_0 \leq 4 \) m and \(-4 \) m \( \leq y \leq 4 \) m), although the particle may
subsequently travel outside this region or even exit the detector.

3.2.3 The Resistive Plate Chambers

In order to appreciate the hit pattern in the simulated detector it is necessary to describe the active detector elements, the RPCs. These are glass chambers made by sealing two 3 mm thick glass sheets with a high DC voltage across them, with a uniform gap of 2 mm using plastic edges and spacers through which a mixture of R134A (∼95%), isobutane, and trace amounts of SF₆ gas continually flows. In brief, the working principle of an RPC is the ionisation of the gaseous medium when a charged particle passes through it. The combination of gases keeps the signal localised and the location is used to determine the trajectory of the charged particle in the detector. For more details, see Ref. [102].

A 150 micron thick copper sheet is the component most relevant to the simulation and track reconstruction as it inductively picks up the signal when a charged particle traverses the chamber. This copper sheet is pasted on the inside of a 5 mm thick foam (used for structural strength and electrical insulation) placed both above and below the glass chamber. It is pasted on the side of the foam facing the glass and is insulated from the glass by a few sheets of mylar. This layer is scored through with grooves to form strips of width 1.96 cm in such a way that the strips above and below are transverse to each other, that is, in the x and y directions. These pick-up strips thus provide the x and y location of the charged

---

1Note that the strip width in the current ICAL design is 3 cm; however this is subject to change.
particle as it traverses the RPC while the RPC layer number provides the $z$ information. A timing resolution of about 1.0 ns is assumed as also an efficiency of 95\%, consistent with the observations of RPCs that have been built as a part of the R&D for ICAL [102].

3.3 Event Simulation and Digitisation

Muons and hadrons, generated in neutrino interactions with the detector material, pass through dense detector material and an inhomogeneous magnetic field. Simulation of such particles through the detector geometry is performed by a package based on the GEANT4 toolkit. Here the ICAL geometry is written to a machine readable GDML file that can be read off by other associated packages, like the event reconstruction package.

When a charged particle, for example, a muon, passes through an RPC, it gives a signal which is assigned $x$ or $y$ values from the respective pick-up strip information, a $z$-value from the layer information, and a time stamp $t$. The global coordinates of the signals are then translated through digitisation into information of the $X^{th}$ $x$ strip and the $Y^{th}$ $y$ strip at the $Z^{th}$ plane. These digitized signals along with the time stamp form what are called “hits” in the detector as this is precisely the information that would be available when the actual ICAL detector begins to take data.

The spatial resolution in the horizontal plane is of the order of cm (due to the strip width) while that in the $z$ direction is of the order of mm (due to the gas gap between the glass plates in the RPCs). The effect of cross-talk, i.e., the probability of either or both adjacent strips giving signals in the detector, is also incorporated, using the results of the on-going studies of RPCs [102]. Finally, since the $X$ and $Y$ strip information are independent, all possible pairs of nearby $X$ and $Y$ hits in a plane are combined to form a cluster.

A typical neutrino CC interaction giving rise to an event with a muon track and associated hadron shower is shown in Fig. 3.3. It can be seen that the muon track is clean with typically one or two hits per layer, whereas the hadron hits form a diffused shower.

3.4 Event Reconstruction

The reconstruction of individual hadrons in the hadronic showers is not possible since the response of the detector to different hadrons is rather similar. Only an averaged information on the energy and direction of the hadrons is in principle possible; furthermore, hadrons, due to the different nature of their interactions, propagate relatively short distances in the detector. The response of ICAL to hadrons as determined by simulations studies is described in Sec. 4.2.

Muons, on the other hand, being minimum ionizing particles, leave long clean tracks, and hence the ICAL detector is most sensitive to them. The muon momentum can be determined from the curvature of its trajectory as it propagates in the magnetized detector, and also by measuring its path length. The nanosecond time resolution of the RPCs also allows the distinction between up-going and down-going muons. The muon momentum reconstruction is achieved by using a track finder package, followed by a track fitting algorithm that reconstructs both the momentum and charge of the muon, using the information on the local magnetic field.
The track finder uses clusters, i.e., the combinations of all possible pairs of nearby $X$ and $Y$ hits in a $Z$-plane, as its basic elements. A set of clusters generated in three successive layers is called a tracklet. The track finder algorithm uses a simple curve fitting algorithm to find possible tracklets by finding clusters in three adjacent planes. It includes the possibility of no hit (due to inefficiency) in a given plane, in which case the next adjoining planes are considered. Typically, charged current muon neutrino interactions in ICAL have a single long track due to the muons and a shower from the hadrons near the vertex. Since typically muons leave only about one or two hits per layer they traverse ($\sim 1.6$ on average) as opposed to hadrons that leave several hits per layer, the hadron showers are separated by using criteria on the average number of hits per layer in a given event.

Ends of overlapping tracklets are matched to form longer tracks, and the longest possible track is found by iterations of this process. The track finder package thus forms muon tracks as an array of three dimensional clusters. In the rare cases when there are two or more tracks, the longest track is identified as the muon track. The direction (up/down) of the track is calculated from the timing information which is averaged over the $X$ and $Y$ timing values in a plane. For muon tracks which have at least 5 hits in the event, the clusters in a layer are averaged to yield a single hit per layer with $X$, $Y$ and timing information; the coordinates of the hits in the track are sent to the track fitter for further analysis. (This translates to a minimum momentum of about 0.4 GeV/c for a nearly vertical muon, below which no track is fitted.)

The track fitter, a Kalman-filter based algorithm, is used to fit the tracks based on the bending of the tracks in the magnetic field. Every track is identified by a starting vector $X_0 = (X, Y, X/Z, Y/Z, q/p)$ which contains the position of the earliest hit $(X, Y, Z)$ as recorded by the finder, with the charge-weighted inverse momentum $q/p$ taken to be zero. Since the tracks are virtually straight in the starting section, the initial track direction (the slopes $X/Z, Y/Z$) is calculated from the first two layers. This initial state vector is then extrapolated to the next layer using a standard Kalman-filter based algorithm, which
calculates the Kalman gain matrix using the information on the local magnetic field and
the geometry, the composition of the matter through which the particle propagates, and the
observed cluster position in that later.

In the existing code, the state prediction is based upon the swim-swimmer package and
the corresponding error propagation is performed by a propagator matrix \[103\]. The state
extrapolation takes into account process noise due to multiple scattering as described in
\[104\] and energy loss in matter, mostly iron, according to the Bethe formula \[105\]. A new
improved set of formulae for the propagation of the state and errors \[106\], optimised for
atmospheric neutrinos with large energy and range, have also been developed, and are being
used in the Kalman filter. The extrapolated point is compared with the actual location of a
hit in that layer, if any, and the process is iterated.

The process of iteration also obtains the best fit to the track. The track is then extrap-
olated backwards to another half-layer of iron (since the interaction is most likely to have
taken place in the iron) to determine the vertex of the interaction and the best fit value of
the momentum at the vertex is returned as the reconstructed momentum (both in magnitude
and direction). Only fits for which the quality of fit is better than $\chi^2/\text{ndf} < 10$ are used in
the analysis.

While $q/p$ determines the magnitude of the momentum at the vertex, the direction is
reconstructed using $X/Z$ and $Y/Z$, which yield the zenith and the azimuthal angles, i.e., $\theta$
and $\phi$. The results on the quality of reconstruction are presented in the next chapter.
Chapter 4

ICAL Response to Muons and Hadrons

“Muons are clean because they leave a trail, Hadrons are dirty because they shower.”

– M. V. N. Murthy

In this chapter, we discuss the simulations response of ICAL to the final state particles produced in neutrino-nucleus interactions as discussed in the previous chapter. Being minimum ionising particles, muons typically register clean long tracks with just about one hit per RPC layer in the detector while hadrons produce a shower with multiple hits per layer, due to the very different nature of their interactions. Multiple scattering further affects the quality of the track.

First we discuss the detector response with respect to single particles (muons or hadrons) with fixed energies. In order to simulate the neutrino events fully, we then use the particles generated in atmospheric neutrino events using the NUANCE [4] event generator, for calibration. For the case of single muons, we study the response of the detector to the energy/momentum, direction and charge of muons propagated with fixed energy/momentum and direction (θ, φ) from the central region of the detector (described in Chapter 3). Next we propagate the hadrons, mainly single pions, also with fixed energy and direction, through the central region of the detector and determine the energy response of the detector with respect to hadrons. Amongst the particles generated via NUANCE, the muons tracks can be separated while the hits from all the hadrons in the event are treated as just one shower.

4.1 Simulations Response of ICAL to Muons

In this Section, we present the results of the reconstruction of the charge, energy and direction of muons [3]. For this study, we confine ourselves to tracks generated only in the central region of the ICAL detector, i.e. −4 m ≤ x ≤ 4 m, −4 m ≤ y ≤ 4 m and −4 m ≤ z ≤ 4 m, with the origin taken to be the center of the central detector module. The particle may subsequently travel outside this region or even exit the detector: both fully contained and partially contained events are analyzed together. At low energies, the tracks are fully
contained while particles start to leave the detector region for \( P_{\text{in}} \gtrsim 6 \text{ GeV/c} \), depending on the location of the vertex and the direction of the particles.

The \( \mu^- \) and \( \mu^+ \) are analysed separately for fixed values of the muon momentum \( P_{\text{in}} \) and direction: while \( \cos \theta \) is kept fixed for a set of typically 10000 muons, the azimuthal angle \( \phi \) is smeared over all possible values from \(-\pi\)–\(\pi\). The distribution of reconstructed muon momentum for the particular choice \( (P_{\text{in}}, \cos \theta) = (5 \text{ GeV/c}, 0.65) \) is shown in the left panel of Fig. 4.1. Since the results are almost identical, as can be seen from the figure, only the results for \( \mu^- \) are presented in the further analysis.

The mean and the rms width \( \sigma \) are determined by fitting the reconstructed momentum distribution; the momentum resolution is defined as \( R \equiv \sigma / P_{\text{in}} \). Apart from the intrinsic uncertainties due to particle interactions and multiple scattering effects, the distribution—especially its width—is also sensitive to the presence of detector support structures, gaps for magnetic field coils, etc., that have been described in the previous chapter.

![Figure 4.1](image-url)

**Figure 4.1:** The left panel shows the reconstructed momentum distributions for \( (P_{\text{in}}, \cos \theta) = (5 \text{ GeV/c}, 0.65) \) smeared over the central volume of the detector for \( \mu^- \) and \( \mu^+ \) particles [8]. The right panel shows the same distribution, but for \( (P_{\text{in}}, \cos \theta) = (1 \text{ GeV/c}, 0.65) \), for \( \mu^- \), fitted with the Landau-convoluted-with-Gaussian distribution.

In addition, the reconstructed momentum distribution of low energy muons has a clear asymmetric tail, as can be seen in the right panel of Fig. 4.1 for muons with \( (P_{\text{in}}, \cos \theta) = (1 \text{ GeV/c}, 0.65) \). It can be seen that the distribution at low energies is significantly broader, and there is also a shift in the mean. It is therefore fitted with a convolution of Landau and Gaussian distributions. For muons with \( P_{\text{in}} > 2 \text{ GeV/c} \), the distributions are fitted with purely Gaussian distributions. In the case of Landau-Gaussian fits, the width is defined as \( \sigma \equiv \text{FWHM}/2.35 \), in order to make a consistent and meaningful comparison with the Gaussian fits at higher energies, where the square root of the variance, or the rms width, equals FWHM/2.35. Before we present results on the muon resolution, we first discuss the impact on the resolutions of the muon angle and location within the detector.
4.1.1 Momentum resolution in different azimuthal regions

The number of hits in the detector by a muon with a fixed energy will clearly depend on the zenith angle, since muons traversing at different angles travel different distances through each iron plate. As a result, the momentum resolution would depend on the zenith angle. However, it also has a significant dependence on the azimuthal angle for two different reasons. One is that the magnetic field explicitly breaks the local azimuthal symmetry of the detector geometry. There is an additional effect due to the coil gaps that are located at $x = x_0 \pm 4$ m where $x_0$ is the centre of each module. The second reason is that the support structures are also not azimuthally symmetric; moreover, the length of track “lost” within these dead spaces is also a function of the location from where the muon was propagated and the zenith angle at which it traverses these spaces. The cumulative dependence on the azimuthal angle is a complex consequence of all these dependences and impacts low momentum and large zenith angle muons more than higher energy, small angle ones.

For instance, a muon initially directed along the $y$-axis experiences less bending since the momentum component in the plane of the iron plates (henceforth referred to as in-plane momentum) is parallel to the magnetic field. Furthermore, upward-going muons that are in the negative (positive) $x$ direction experience a force in the positive (negative) $z$ direction (the opposite is be true for $\mu^+$) and so muons injected with $|\phi| > \pi/2$ traverse more layers than those with the same energy and zenith angle but with $|\phi| < \pi/2$ and hence are better reconstructed. This is illustrated in the schematic in Fig. 4.2 which shows two muons ($\mu^-$) injected at the origin with the same momentum magnitude and zenith angle, one with positive momentum component in the $x$ direction, $P_x > 0$ and the other with negative $x$ momentum component. The muon with $P_x > 0$ (initially directed in the positive $x$ direction) bends differently than the one with $P_x < 0$ (along negative $x$ direction) and hence they traverse different number of layers, while having roughly the same path length. Hence, muons with different $\phi$ elicit different detector response. Because of these effects, the momentum resolution is best studied in different azimuthal angle bins. We separate our muon sample into four regions/bins: bin I with $|\phi| \leq \pi/4$, bin II with $\pi/4 < |\phi| \leq \pi/2$, bin III with $\pi/2 < |\phi| \leq 3\pi/4$, and bin IV with $3\pi/4 < |\phi| \leq \pi$. The resolutions of $P_m$ in the above $\phi$-regions, for six values of the zenith angle, are shown in Fig. 4.3.

It may be noticed that, at lower energies the resolution improves as $\cos \theta$ increases, as expected, but at higher energies the behaviour is rather complicated. At higher energies, the muons injected at larger zenith angles in $\phi$ regions I and IV, have better resolutions than their counterparts at more vertical angles (for instance, $\cos \theta = 0.45$ versus $\cos \theta = 0.85$) because larger portions of the tracks, being more slanted, are still contained within the detector. In contrast in $\phi$ regions II and III, muons with larger zenith angles have worse resolution than those at smaller zenith angles because the former exits the detector from the $y$ direction and are partially contained. In general, at angles larger than about 75$^\circ$ ($\cos \theta = 0.25$), the resolution is relatively poor since there are several times fewer hits than at more vertical angles.

Finally, note that the simulations studies of the physics reach of ICAL discussed in the later chapters use the azimuth-averaged resolutions for muons. This is because the main focus of ICAL is the study of neutrino oscillations using atmospheric neutrino fluxes that are azimuthally symmetric for $E_\nu \geq 2$ GeV. While studying the neutrinos from fixed
Figure 4.2: Schematic showing muon tracks (for $\mu^-$) in the $x$-$z$ plane for the same values of $(P_{\text{in}}, \cos \theta)$ but with $|\phi| < \pi/2$ and $> \pi/2$ (momentum component in the $x$ direction positive and negative, respectively). The different bending causes the muon to traverse different number of layers in the two cases.

Figure 4.3: Muon resolution as a function of input momentum and $\cos \theta$, in bins of $\phi$ as described in the text [8].
astrophysical sources, for example, the azimuthal dependence of the detector needs to be taken into account.

In the rest of this section, we present simulations studies of the azimuth-averaged response of ICAL to the muon direction and its charge, since these are not very sensitive to the azimuth. We also present the overall reconstruction efficiency for muons; this determines the overall reconstruction efficiency of the entire neutrino event as there may or may not be an associated hadron shower in the event.

### 4.1.2 Zenith Angle Resolution

The events that are successfully reconstructed for their momenta are analysed for their zenith angle resolution. The reconstructed zenith angle distribution for muons with $P_{\text{in}} = 1 \text{ GeV/c}$, at zenith angles $\cos \theta = 0.25$ and $\cos \theta = 0.85$, are shown in Fig. 4.4 (Muons with $\cos \theta = 1$ are up-going). It can be seen that a few events are reconstructed in the opposite (downward) direction i.e., with zenith angle ($\theta_{\text{rec}} \sim \pi - \theta$). For muons with $P_{\text{in}} = 1 \text{ GeV/c}$ at large (small) angles with $\cos \theta = 0.25 (0.85)$, this fraction is about 4.3 (1.5) %. As energy increases, the fraction of events reconstructed in the wrong direction drastically comes down. For example, this fraction reduces to 0.3% for muons with $P_{\text{in}} = 2 \text{ GeV/c}$ at $\cos \theta = 0.25$.

![Figure 4.4: Reconstructed theta distribution for $P_{\text{in}} = 1 \text{ GeV/c}$ at two values of $\cos \theta$.](image)

The events distribution as a function of the reconstructed zenith angle is shown in the left panel of Fig. 4.5 for a sample $(P_{\text{in}}, \cos \theta) = (5 \text{ GeV/c}, 0.65)$. It is seen that the distribution is very narrow, indicating a good angular resolution for muons. The right panel of Fig. 4.5 shows the $\theta$ resolution as a function of input momentum for different zenith angles.

As noted earlier, due to multiple scattering and the smaller number of layers with hits, the momentum resolution is worse at lower energies. This is also true for the zenith angle, whose resolution improves with energy. For a given energy, the resolution is worse for larger zenith angles since again the number of layers with hits decreases. Even so, it is seen that the angular resolution for $\cos \theta \geq 0.25$ (i.e. $\theta \lesssim 75^\circ$) is better than 1° for muon momenta greater than 4 GeV/c.
Figure 4.5: The left panel shows the reconstructed $\theta$ distribution for input $(P_{\text{in}}, \cos \theta) = (5 \text{ GeV}/c, 0.65)$. The right panel shows the $\theta$ resolution as a function of input momentum.

### 4.1.3 Reconstruction efficiency

The reconstruction efficiency for muons is defined as the ratio of the number of reconstructed events $n_{\text{rec}}$ (irrespective of charge) to the total number of events, $N_{\text{total}}$ (typically 10000). We have

$$
\epsilon_{\text{rec}} = \frac{n_{\text{rec}}}{N_{\text{total}}} \quad (4.1)
$$

$$
\delta\epsilon_{\text{rec}} = \sqrt{\epsilon_{\text{rec}}(1 - \epsilon_{\text{rec}})/N_{\text{total}}}.
$$

The left panel of Fig. 4.6 shows the muon reconstruction efficiency as a function of input momentum for different $\cos \theta$ bins.

Figure 4.6: The left (right) panel shows the reconstruction efficiency (the relative charge identification efficiency) as a function of the input momentum for different $\cos \theta$ values.

When the input momentum increases, the reconstruction efficiency also increases for all angles, since the number of hits increases as the particle crosses more number of layers. At
larger angles, the reconstruction efficiency for small energies is smaller compared to vertical angles since the number of hits for reconstructing tracks is less. But as the input energy increases, above almost 4 GeV since the particle crosses more number of layers, the efficiency of reconstructing momentum also increases and becomes comparable with vertical angles. At higher energies the reconstruction efficiency becomes almost constant. The drop in efficiency at high energies for vertical muons is due to the track being partially contained, their smaller bending in the magnetic field, as well as the impact of the detector support structure. It is expected that this may improve as the track recognition algorithms are refined and better tuned.

4.1.4 Relative charge identification efficiency

The charge of the particle is determined from the direction of curvature of the track in the magnetic field. Relative charge identification efficiency is defined as the ratio of number of events with correct charge identification, \( n_{\text{cid}} \), to the total number of reconstructed events:

\[
\epsilon_{\text{cid}} = \frac{N_{\text{cid}}}{N_{\text{rec}}},
\]

\[
\delta\epsilon_{\text{cid}} = \sqrt{\epsilon_{\text{cid}}(1 - \epsilon_{\text{cid}}) / N_{\text{rec}}}. 
\]

Figure 4.6 shows the relative charge identification efficiency as a function of input momentum for different \( \cos \theta \) bins. As seen earlier, there is a very small contribution to the set with the wrongly identified charge from the events where the track direction is wrongly identified \((\theta \rightarrow \pi - \theta)\); such events will also reconstruct with the wrong charge as there is a one-to-one correspondence between the up-down identification and the muon charge.

When a low energy muon propagates in the detector it undergoes multiple scattering. So the number of layers with hits is small, and the reconstruction of charge goes wrong, which results in poor charge identification efficiency as can be see from Fig. 4.6. As the energy increases, the length of the track also increases, due to which the charge identification efficiency also improves. Beyond a few GeV/c, the charge identification efficiency becomes roughly constant, about 98%.

4.2 Simulations Response of ICAL to hadrons

An important feature of ICAL is its sensitivity to hadrons over a wide energy range. This allows the reconstruction of the energy of the incoming muon neutrino in a charged-current event by combining the energies of the muon and the hadrons. It also enables the detection of neutral-current events, charged-current DIS events generated by \( \nu_e \) interactions, and charged-current \( \nu_\tau \) events where the \( \tau \) decays hadronically. The information contained in all these events adds crucially to our knowledge of neutrino oscillations. The charged-current events is a direct measure of the oscillation probabilities among the three active neutrino species. On the other hand, the neutral-current events are not affected by active neutrino oscillations, and hence help in flux normalization, as well as in the search for oscillations to sterile neutrinos. It is therefore important to characterize the response of the ICAL to hadrons.
The hadrons generated from the interactions of atmospheric neutrinos consist mainly of neutral and charged pions, which together account for about 85% of the events. The rest of the events consist of kaons and nucleons, including the recoil nucleons that cannot be distinguished from the remaining hadronic final state. The neutral pions decay immediately giving rise to two photons, while the charged pions propagate and develop into a cascade due to strong interactions. For the neutrino-nucleon interaction $\nu_\mu N \rightarrow \mu X$, the incident neutrino energy is given by

$$E_\nu = E_\mu + E_{\text{hadrons}} - E_N,$$

where $E_N$ is the energy of the initial nucleon which is taken to be at rest, neglecting its small Fermi momentum. The visible hadron energy depends on factors like the shower energy fluctuation, leakage of energy, and invisible energy loss mechanisms, which in turn affect the energy resolution of hadrons. We choose to quantify the hadron response of the detector in terms of the quantity $\frac{3}{9}$

$$E'_{\text{had}} = E_\nu - E_\mu.$$

As the first step in understanding the ICAL response to hadrons, single charged hadrons of fixed energies are generated via Monte Carlo and propagated through the detector to compare its response to them. The response to charged pions is then studied in more detail, and the pion energy is calibrated against the number of hits. Next, the multiple hadrons produced through atmospheric neutrino interactions are generated using NUANCE [4], and the quantity $E'_{\text{had}}$ is used to calibrate the detector response. This should take care of the right combination of the contributions of different hadrons to the hits, on an average. It would of course be dominated by neutral and charged pions, and hence we expect it to be similar to the response to fixed-energy pions.

### 4.2.1 Energy response to fixed-energy hadrons

In an RPC, the X- and Y-strip information on a hit is independently obtained from the top and bottom pick-up strips, as described in Sec. 1.1.3 and then combined to give the $(X,Y)$ coordinates of the hit. For a muon, the $(X,Y)$ positions of the hits in a given layer can easily be identified since a muon usually leaves only one or two hits per layer. However a hadron shower consists of multiple hits per layer, and combining all possible X and Y strip hits leads to overcounting, resulting in what are termed as “ghost hits”. To avoid the ghost hit counts, the variables “x-hits” and “y-hits”—the number of hits in the X and Y strips of the RPC, respectively—can be used. We choose to perform the energy calibration with the variable “orig-hits”, which is the maximum of x-hits or y-hits.

Figure 4.7 shows the comparison of these three types of hit variables for $\pi^\pm$ of energy 3 GeV. Clearly, there is no significant difference among these variables, however orig-hits has been used as the unbiased parameter. It is also observed that the detector response to the positively and negatively charged pions is identical, so we shall not differentiate between them henceforth.

Fixed-energy single pion events in the energy range of 1 to 15 GeV were generated using the particle gun in GEANT4. The total number of events generated for each input energy value is 10000 in this section, unless specified otherwise. Each event is randomly generated to
Figure 4.7: The comparison of the distributions of x-hits, y-hits and orig-hits for π− (left) and π+ (right) of energy 3 GeV [9].

have vertices over a volume 2 m × 2 m × 2 m in the central region of the ICAL detector. As in the earlier section, the reference frame chosen has the origin at the centre of the detector, the z-axis pointing vertically up, and the x-y plane along the horizontal plates, with the three modules lined up along the x axis. The hadron direction is uniformly smeared over the zenith angle 0 ≤ θ ≤ π and azimuth of 0 ≤ φ ≤ 2π. This serves to smear out any angle-dependent bias in the energy resolution of the detector by virtue of its geometry which makes it the least (most) sensitive to particles propagating in the horizontal (vertical) direction.

Figure 4.8 shows the hit distributions in the detector for pions, kaons, and protons at various energies in the range of 1 to 15 GeV. It is observed that the hit patterns are similar for all these hadrons, though the peak positions and spreads are somewhat dependent on the particle ID. Hence the detector cannot distinguish the specific hadron that has generated the shower. The large variation in the number of hits for the same incident particle energy is mainly a result of different strong interaction processes for different hadrons (for π0 the interactions are electromagnetic since it decays immediately to a γγ pair), and partly an effect of angular smearing.

4.2.1.1 The e/h ratio

The NUANCE [4] simulation suggests that the fraction of the different types of hadrons produced in the detector is π+:π−:π0::0.38:0.25:0.34, with the remaining 3% contribution coming mainly from kaons [10]. While the response of the detector to π+ and π− is almost identical as seen earlier, its response to the electromagnetic part of the hadron shower that originates from π0 is different. This may be quantified in terms of the e/h ratio, i.e. the ratio of the electron response to the charged-pion response. This ratio would help us characterise the effect of neutral hadrons on the energy resolution.

In order to study this ratio, we generated 100,000 electron events at fixed energies in the energy range 2–15 GeV, propagating in arbitrary directions (with θ smeared from 0−π and φ from 0−2π) from vertices within a volume of 2 m × 2 m × 2 m in the central region of the ICAL detector. The hit distributions averaged over all directions for 2, 5, 10 and 14 GeV electrons is shown in Fig. 4.9. This may be compared with the hit distributions shown
Figure 4.8: The direction-averaged hit distributions at various energies for $\pi^\pm$, $\pi^0$, $K^\pm$ and protons propagated from vertices smeared over the chosen detector volume [9].

in Fig. 4.8. The response is almost the same as that for $\pi^0$, with narrower high-energy tails than those for charged pions.

The $e/h$ ratio is obtained as

$$e/h = e_{\text{mean}}/\pi_{\text{mean}}^+,$$  \hspace{1cm} (4.5)

where $e_{\text{mean}}$ is the arithmetic mean of the electron hit distribution and $\pi_{\text{mean}}^+$ is the arithmetic mean of the hit distribution for $\pi^+$, for a given fixed energy of the two particles. If $e/h = 1$, then the detector is said to be compensating. The variation of the $e/h$ ratio with incident energy is shown in the right panel of Fig. 4.9.

It can be seen that the value of $e/h$ decreases with energy. However, it should be noted that there is no direct measurement of the energy deposited in ICAL. Here the energy of a shower is simply calibrated to the number of hits, and electrons which travel smaller distances in a high Z material like iron have lower number of hits compared to charged pions. At lower energies the electron as well as pion shower hits are concentrated around a small region. The mean of the electron hit distribution is roughly the same or slightly larger than that of the $\pi^+$ hit distribution. With the increase in energy, the charged pions travel more distance and hence give more hits (as they traverse more layers) since the hadronic interaction length
The left panel shows the hit distributions of fixed energy single electrons at 2, 5, 10 and 14 GeV, averaged over all directions. The right panels shows the variation of the $e/h$ ratio with the particle energies [10].

is much more than the electromagnetic interaction length at higher energies and hence the ratio of hits in the two cases drops with energy.

In a neutrino interaction where all types of hadrons can be produced (although the dominant hadrons in the jet are pions), the response of ICAL to hadrons produced in the interaction depends on the relative fractions of charged and neutral pions. Using the relative fractions $\pi^+ : \pi^- : \pi^0 :: 0.38 : 0.25 : 0.34$ as mentioned above, the average response of hadrons obtained from the charged current muon neutrino interaction can be expressed as:

$$R_{\text{had}} = [(1 - F_0) \times h + F_0 \times e], \quad (4.6)$$

$$= h \left[ (1 - F_0) + F_0 \times \frac{e}{h} \right],$$

where $e$ is the electron response, $h$ the charged hadron response and $F_0$ is the neutral pion fraction in the sample.

The atmospheric neutrino events of interest in ICAL are dominated by low energy events with hadrons typically having energies $E < 10$ GeV for which the average value of $e/h$ is $e/h \approx 0.9$. Using $F_0 = 0.34$ in Eq. (4.7), we get the average hadron response for NUANCE-generated events to be $R_{\text{had}} \approx 0.97h$ which is not very different from $h$. For this reason, the analysis of response with multiple hadrons in NUANCE-generated events sample is not expected to be very different from that of the single pions sample. However we shall confirm this by first focusing on the detector response to fixed-energy charged pions in Sec. 4.2.2, and then moving on to a more general admixture of different hadrons in Sec. 4.2.3.

### 4.2.2 Analysis of the charged pion hit pattern

The hit distributions for charged pions, at sample values of $E = 3, 8$ GeV, are shown in Fig. 4.10. The distributions are asymmetric with long tails, with a mean of about two hits per GeV. In addition, at low energies several events yield zero hits in the detector.

A search for a good fitting function for the distribution was made, and it was found that the Vavilov distribution function gives a good fit for all energies, as is illustrated in Fig. 4.10.
This distribution (see Appendix C) is described by the four parameters $P_0$, $P_1$, $P_2$ and $P_3$, which are energy-dependent [9]. The Vavilov distribution reduces to a Gaussian distribution for $P_0 \geq 10$, which happens for $E > 6$ GeV. However at lower energies, it is necessary to use the full Vavilov distribution.

The mean $\bar{n}(E)$ of the number of hits from the Vavilov fit at different energies is shown in the left panel of Fig. 4.11. It increases with increasing pion energy, and saturates at higher energies. It may be approximated by

$$\bar{n}(E) = n_0[1 - \exp(-E/E_0)], \quad (4.7)$$

where $n_0$ and $E_0$ are constants. This fit has to be interpreted with some care, since $n_0$ and $E_0$ are sensitive to the energy ranges of the fit. The value of $E_0$ is found to be $\sim 30$ GeV when a fit to the energy range 1–15 GeV is performed. Since the energies of interest for atmospheric neutrinos are much less than $E_0$, Eq. (4.7) may be used in its approximate linear form $\bar{n}(E) = n_0 E/E_0$. A fit to this linear form is also shown in Fig. 4.11.

Since in the linear regime ($E \ll E_0$) one has

$$\frac{\bar{n}(E)}{n_0} = \frac{E}{E_0}, \quad (4.8)$$

The energy resolution may be written as

$$\frac{\sigma}{E} = \frac{\Delta n(E)}{\bar{n}(E)}, \quad (4.9)$$

where $(\Delta n)^2$ is the variance of the distribution. The notation $\sigma/E$ will be used for energy resolution throughout, and Eq. (4.9) will be taken to be valid for the rest of the analysis.

The energy resolution of pions may be parameterized by

$$\frac{\sigma}{E} = \sqrt{\frac{a^2}{E} + b^2}, \quad (4.10)$$

Figure 4.10: The hit distributions at 3 GeV (left) and 8 GeV (right), for pions propagating in the detector, starting from randomized vertices over a volume of 2 m × 2 m × 2 m in the detector. The red curve denotes a fit to the Vavilov distribution [9].
where $a$ and $b$ are constants. The energy resolutions for charged pions as functions of the pion energy are shown in the right panel of Fig. 4.11. The parameters $a$ and $b$ extracted by a fit to Eq. (4.10) over the pion energy range 1–15 GeV are also shown.

### 4.2.2.1 Dependence of the energy resolution on hadron direction

Since the number of layers traversed by a particle would depend on the direction of the particle, it is expected that the energy calibration and energy resolution for hadrons will depend on the direction of the hadron. To check this dependence, we simulate pions of fixed energies in the detector, which travel in different directions. The directions are binned into 5 zenith angle bins, and the distributions of number of hits is recorded. The ratio of the rms width of the distribution and its mean is used as a measure of the energy resolution [10]. Figure 4.12 shows the zenith angle dependence of the hadron energy resolution.

Figure 4.11: The mean hit distribution (left) and the energy resolution (right) for fixed-energy charged pion events, as a function of pion energy. The right panel also shows a fit to Eq. (4.10) [9].

Figure 4.12: The dependence of the pion energy resolution on the zenith angle [10].
Since there is only a mild dependence on the hadron direction, and the direction of hadron itself cannot be determined yet with a good confidence, we continue to use the direction-averaged results in the future analyses in this Report.

### 4.2.3 Response to hadrons produced by atmospheric neutrinos

Atmospheric neutrino interactions in the detector may contain no hadrons (for quasi-elastic scattering events), one hadron or multiple ones (in resonance scattering and DIS events). While the former events dominate for $E_{\nu} \sim 1$ GeV, at higher energies the DIS events dominate. In this section we focus on the charged-current $\nu_\mu$ interactions in the detector that produce hadrons in addition to the charged muons.

**We assume here that the $\nu_\mu$ CC events can be clearly separated from the NC as well as $\nu_e$ CC events, and that the muon and hadron shower may be identified separately. Preliminary studies show that this is a reasonable assumption for $E_{\nu} \gtrsim 1$ GeV. At lower energies where the number of hits is small, the misidentification of a muon hit as a hadron one, or vice versa, can significantly affect the hadron energy calibration.**

The atmospheric neutrino ($\nu_\mu$) and antineutrino ($\overline{\nu}_\mu$) events in ICAL have been simulated using the neutrino event generator NUANCE (v3.5) [4]. The hadrons produced in these interactions are primarily pions, but there are some events with kaons (about 3%) and a small fraction of other hadrons as well. As discussed earlier, it is not possible to identify the hadrons individually in ICAL. However since the hit distribution of various hadrons are similar to each other (see Fig. 4.8), and the NUANCE generator is expected to produce a correct mixture of different hadrons at all energies, it is sufficient to determine the hadron energy resolution at ICAL through an effective averaging of NUANCE events, without having to identify the hadrons separately.

![Figure 4.13: The mean hit distribution (left) and the energy resolution (right) for hadron events generated by NUANCE, as a function of $E'_{\text{had}}$. The right panel also shows a fit to Eq. (4.10). The bin widths are indicated by horizontal error bars [9].](image)

A total of 1000 kt-years of “data” events (equivalent to 20 years of exposure with the 50 kt ICAL module) were generated with NUANCE. The events were further binned into the various $E'_{\text{had}}$ energy bins and the hit distributions (averaged over all angles) in these bins are fitted to the Vavilov distribution function. The mean values ($\text{MeanVavilov}$) of these
distributions as a function of $E'_{\text{had}}$ are shown in the left panel of Fig. 4.13. As expected, these are similar to the mean values obtained earlier with fixed energy pions. Since the mean hits grow approximately linearly with energy, the same linearized approximation used in section 4.2.2 can be used to obtain the energy resolution $\sigma/E = \Delta n/\bar{n}$. The energy resolution as a function of $E'_{\text{had}}$ is shown in Fig. 4.13. The energy resolution ranges from 85% (at 1 GeV) to 36% (at 15 GeV).

The effective energy response obtained from the NUANCE-generated data is an average over the mixture of many hadrons that contribute to hadron shower at all energies. The fractional weights of different kinds of hadrons produced in neutrino interactions may, in principle, depend upon neutrino oscillations. In addition, the relative weights of events with different energy that contribute in a single energy bin changes because neutrino oscillations are energy dependent. In order to check this, events with oscillations using the best-fit values of standard oscillation parameters (mixing angles and mass-squared differences) were also generated. The resolutions obtained without and with oscillations are very close to each other. Thus, the hadron energy resolution can be taken to be insensitive to oscillations.

### 4.2.4 Hadron energy calibration

To calibrate the hadron energy $E'_{\text{had}}$ against the hit multiplicity, hadrons from the simulated NUANCE “data” were divided into bins of different hit multiplicities $n$. Even here, a good fit was obtained for the Vavilov distribution function at all values of $n$. We show the mean, $\text{Mean}_{\text{Vavilov}}$, and the standard deviation, $\sigma_{\text{Vavilov}}$, obtained from the fit in the calibration plot in Fig. 4.14.

![Calibration plot for $E'_{\text{had}}$ from hadron hits](image)

Figure 4.14: Calibration plot for $E'_{\text{had}}$, where mean and $\sigma$ from the Vavilov fits are represented by the black filled circles and error bars, respectively.

For charged-current $\nu_\mu$ events, the energy of the incident neutrino can be reconstructed through

$$E_\nu = E_\mu + E'_{\text{had}},$$  \hspace{1cm} (4.11)

where $E_\mu$ is reconstructed from the Kalman filter algorithm and $E'_{\text{had}}$ is calibrated against the number of hadronic hits. The neutrino energy resolution will in principle depend on the energy and direction of the muon as well as that of the hadron shower. The poor energy
resolution of hadrons also makes the energy resolution of neutrinos rather poor, and loses
the advantage of an accurate muon energy measurement. Therefore, reconstructing neutrino
energy is not expected to be the most efficient method for extracting information from the
ICAL analysis. Indeed, we expect to use the muon and hadron information separately, as
will be seen in Sec. 5.4.

4.3 Salient features of the detector response

The ICAL detector is mainly sensitive to muons produced in the charged-current interactions
of atmospheric $\nu_\mu$ or $\bar{\nu}_\mu$. We have studied the response of the detector to muons generated
in the central volume of a module of the ICAL detector where the magnetic field is uniform.
The momentum, charge and direction of the muons are determined from the curvature of the
track in the magnetic field using Kalman filter algorithm. The response of the detector to
muons in the energy range 1–20 GeV with $\cos \theta > 0.25$ is studied in the different azimuthal $\phi$
regions. The momentum resolution, reconstruction efficiency, charge identification efficiency
and direction resolution are calculated. The momentum resolution is about 20% (10%) for
energies of 2 GeV (10 GeV), while the reconstruction efficiency is about 80% for $E_\mu > 2$
GeV. The relative charge identification efficiency is found to be 98% for almost all energies
above the threshold. The direction resolution is found to be better than a degree for all
angles for energies greater than about 4 GeV.

The hadron events of interest in the ICAL detector primarily contain charged pions. The
hit pattern of pions and kaons in the detector is similar; hence it is not possible to separate
different hadrons in the detector. Similarly, neutrino-nucleus interactions produce events
with multiple hadrons in the final state (generated by the NUANCE neutrino generator),
whose energies cannot be reconstructed individually. However, the total energy deposited in
hadrons can be determined by a calibration against the hit multiplicity of hadrons in the
detector.

The hit patterns in single and multiple hadron events are roughly similar, and may be
described faithfully by a Vavilov distribution. Analyses, first with fixed-energy pions, and
later with a mixture of hadrons from atmospheric $\nu_\mu$ interaction events, show that a hadron
energy resolution in the range 85% (at 1 GeV) – 36% (at 15 GeV) is obtainable. The
parameters of the Vavilov fit presented here as a function of hadron energy can be used for
simulating the hadron energy response of the detector, in order to perform physics analyses
that need the hadron energy resolution of ICAL. We have also presented the calibration for
the energy of the hadron shower as a function of the hit multiplicity. This analysis will be
improved upon by incorporating edge effects and noise in a later study, after data from the
prototype detector is available.

The reconstruction of hadrons allows us to reconstruct the total visible energy in NC
events. Combined with the information on the muon energy and direction in the CC events,
it will also allow one to reconstruct the total neutrino energy in the CC events. As we shall
see in Chapter 5, the correlated information in muon and hadron in a CC event will also
help to enhance the capabilities of the ICAL. The ICAL will be one of the largest neutrino
detectors sensitive to the final state muons as well as hadrons in neutrino interactions at
multi-GeV energies, and this advantage needs to be fully exploited.
Chapter 5

Neutrino Oscillation Physics at ICAL

The pendulum of mind oscillates between sense and nonsense, not between right and wrong.
– Carl Gustav Jung

In this chapter we will present the physics capabilities of ICAL for the mixing parameters within the three-generation flavor oscillation paradigm. We shall restrict ourselves to the charged-current events produced in the ICAL from $\nu_\mu$ and $\bar{\nu}_\mu$ interactions, which produce $\mu^-$ and $\mu^+$, respectively. We shall start by describing our analysis method in Sec. 5.1, and then proceed to present the results showing the physics reach of this experiment for various quantities of interest. We shall focus on the identification of the neutrino mass hierarchy, as well as on the precision measurements of $|\Delta m^2_{32}|$ and $\theta_{23}$.

The results will be presented using three different analyses. First in Sec. 5.2 we use only the information on the measured muon energy and muon direction ($E_\mu, \cos \theta_\mu$), both of which should be rather precisely measured in this detector, as described in detail in Chapter 4. We next show the improvement expected in the precision measurement of the atmospheric mass squared difference and the mixing angle if we use the information on the hadron energy in addition, to reconstruct the neutrino energy in each event. In this analysis, first in Sec. 5.3 we analyse the data in terms of the reconstructed neutrino energy and the measured muon angle ($E_\nu, \cos \theta_\mu$). However the reconstruction of neutrino energy involves the addition of the rather coarsely known hadron energy information to the measured muon energy, which results in a dilution of the muon energy information, which is more accurately known due to the good tracking capabilities of the ICAL. In order to retain the benefits of the accurately measured muon energy, we separately use the information on the measured muon energy, muon direction and the hadron energy ($E_\mu, \cos \theta_\mu, E'_{\text{had}}$) corresponding to each atmospheric neutrino event at the ICAL detector. The results of this final analysis, which leads to the best physics reach for ICAL at this stage, are presented in Sec. 5.4.

5.1 Charged-current $\nu_\mu$ events in ICAL

We focus on the charged-current events from the atmospheric $\nu_\mu$ interactions, that produce muons in the ICAL. We shall start by dividing them into bins of energy and momenta, taking into account the efficiencies and resolutions obtained in Chapter 4. As its output,
the generator provides the 4-momentum ($p^\mu$) of the initial, intermediate and the final state particles for each event. To reduce the Monte Carlo fluctuations in the events obtained, we generate an event sample corresponding to 1000 years of running of ICAL and scale it down to the desired exposure for the $\chi^2$ analysis.

In the oscillated event sample, the total number of $\mu^-$ events come from the combination of the $\nu_\mu \rightarrow \nu_\mu$ and the $\nu_e \rightarrow \nu_\mu$ channels as

$$
\frac{d^2N}{dE_\nu \, d(\cos \theta_\nu)} = N_T \times N_D \times \sigma_{\nu_\mu} \times \left[ P_{\mu\mu} \frac{d^2\Phi_{\nu_\mu}}{dE_\nu \, d(\cos \theta_\nu)} + P_{e\mu} \frac{d^2\Phi_{\nu_e}}{dE_\nu \, d(\cos \theta_\nu)} \right],
$$

(5.1)

where $N_D$ is the number of targets and $N_T$ is the exposure time of the detector. Here $\Phi_{\nu_\mu}$ and $\Phi_{\nu_e}$ are the fluxes of $\nu_\mu$ and $\nu_e$, respectively, and $P_{\alpha\beta}$ is the $\nu_\alpha \rightarrow \nu_\beta$ oscillation probability. The first term in Eq. (5.1) corresponds to the number of $\mu^-$ events from $\nu_\mu$ that have survived oscillations, while the second term corresponds to the oscillated $\nu_e$ flux into $\nu_\mu$.

The oscillation probabilities $P_{\mu\mu}$ and $P_{e\mu}$ are calculated numerically for any given set of oscillation parameters for each event, corresponding to the neutrino energy and zenith angle associated with it. The net number of muon events are thus obtained by adding the “survived” and the “oscillated” $\nu_\mu$ events. Both $\mu^-$ and $\mu^+$ events in the detector are generated using a similar procedure. This final data sample is then binned in energy and zenith angle bins.

Figure 5.1 shows the zenith angle distribution of $\mu^-$ events in the muon energy bin $E_\mu = 2 - 3$ GeV, before and after invoking oscillations. We use the oscillation parameters described in Table 1.1 and take the exposure to be 50 kt $\times$ 10 years.

![Figure 5.1: Zenith angle distribution of $\mu^-$ events for the bin $2$ GeV $\leq E_\mu < 3$ GeV, without and with flavor oscillations. The detector efficiencies have not been included here. The error bars shown are statistical [11].](image-url)
These are the events in an ideal detector. To proceed further, we apply the muon reconstruction efficiencies and resolutions (in both energy and direction) obtained in the previous chapter. The reconstruction efficiency ($\epsilon_{R-}$) and the charge identification (CID) efficiencies ($\epsilon_{C-}$ for $\mu^-$ and $\epsilon_{C+}$ for $\mu^+$ event sample) are applied as follows:

$$N_{\mu^-}^C = \epsilon_{C-} \times N_{\mu^-}(E_\mu, \cos \Theta_\mu) + (1 - \epsilon_{C+}) \times N_{\mu^+}(E_\mu, \cos \Theta_\mu),$$

with

$$N_{\mu^-}(E_\mu, \cos \Theta_\mu) = \epsilon_{R-}(E_\mu, \cos \Theta_\mu) \times N_{\mu^-}^{\text{true}}(E_\mu, \cos \Theta_\mu),$$

where $N_{\mu^-}^{\text{true}}$ is the number of $\mu^-$ events in a given $(E_\mu, \cos \theta_\mu)$ bin. All the quantities appearing in Eq. (5.2) are functions of $E_\mu$ and $\cos \theta_\mu$. The same procedure is applied for determining the $\mu^+$ events.

Figure 5.2 shows the zenith angle distribution of events obtained before and after applying the reconstruction and CID efficiencies. Compared to Fig. 5.1, one can notice that the number of events fall sharply for the almost horizontal ($\cos \theta_\mu \approx 0$) bins. This is because the reconstruction efficiency for muons falls as we go to more horizontal bins since the iron slabs and RPCs in ICAL are stacked horizontally. As a result there are hardly any events for bins with $-0.2 \leq \cos \theta_\mu < 0.2$.

![Figure 5.2: Zenith angle distribution of oscillated $\mu^-$ events for the bin $2 \text{ GeV} \leq E_\mu < 3 \text{ GeV}$, after taking into account detector efficiencies. The error bars shown are statistical.](image)

Finally, the muon resolutions $\sigma_E$ and $\sigma_{\cos \theta}$ are applied as follows:

$$(N_{\mu^-}^D)_{ij}(E, \cos \theta) = \sum_k \sum_l N_{\mu^-}^C(E_k^i, \cos \theta_l^j) K_l^i(E_k^i) M_j^l(\cos \theta_l^j),$$

where $(N_{\mu^-}^D)_{ij}$ denotes the number of muon events in the $i^{th}$ $E$-bin and the $j^{th}$ $\cos \theta$-bin after applying the energy and angle resolutions. Here $E$ and $\cos \theta$ are the measured muon
energy and zenith angle. The summation is over the true energy bin $k$ and true zenith angle bin $l$, with $E_\mu^k$ and $\cos \theta_\mu^l$ being the central values of the $k^{th}$ true muon energy and $l^{th}$ true muon zenith angle bin. The quantities $K^k_i$ and $M^l_j$ are the integrals of the detector resolution functions over the bins of $E$ and $\cos \theta$, the measured energy and direction of the muon, respectively. These are evaluated as

$$K^k_i(E^k_\mu) = \int_{E_{L_i}}^{E_{H_i}} dE \frac{1}{\sqrt{2\pi}\sigma_{E^k_\mu}} \exp \left( -\frac{(E^k_\mu - E)^2}{2\sigma_{E^k_\mu}^2} \right), \quad (5.5)$$

and

$$M^l_j(\cos \theta^l_\mu) = \int_{\cos \theta_{L_j}}^{\cos \theta_{H_j}} d\cos \theta \frac{1}{\sqrt{2\pi}\sigma_{\cos \theta^l_\mu}} \exp \left( -\frac{(\cos \theta^l_\mu - \cos \theta)^2}{2\sigma_{\cos \theta^l_\mu}^2} \right), \quad (5.6)$$

where $\sigma_{E^k_\mu}$ and $\sigma_{\cos \theta^l_\mu}$ are the energy and zenith angle resolutions, respectively, in these bins, as obtained in Chapter [3]. We perform the integrations between the lower and upper boundaries of the measured energy ($E_{L_i}$ and $E_{H_i}$) and the measured zenith angle ($\cos \theta_{L_j}$ and $\cos \theta_{H_j}$). For the extreme $\cos \theta$ bins, the bins are taken to be $(-\infty, -0.9)$ and $[0.9, +\infty)$ while integrating, and the events are assigned to the bins $[-1, -0.9]$ and $[0.9, 1]$, respectively. This ensures that no event is lost to the unphysical region and the total number of events does not change after applying the angular resolution. For $E^k_\mu < 1$ GeV, the integrand in Eq. (5.5) is replaced with the Landau distribution function. Figure 5.3 shows the zenith angle distribution of $\mu^-$ events before and after folding in the resolution functions. The angular dependence seems to get only slightly diluted. This is due to the good angular resolution of the detector.

![Figure 5.3: Zenith angle distribution of $\mu^-$ events for the bin $2$ GeV $\leq E_\mu < 3$ GeV before and after including energy and zenith angle resolution function. Here $E$ and $\theta$ are the measured energy and measured zenith angle, respectively. The error bars shown are statistical [11].](image-url)
Table 5.1 shows the total number of muon events with the measured energy range 0.8–10.8 GeV at various stages of the analysis for an exposure of 50 kt × 10 years. Note the sharp fall in statistics due to the reconstruction efficiencies. The reconstruction efficiencies are particularly poor for the near-horizontal bins where the reconstruction of the muon tracks is very hard. The small increase in the number of events after applying the energy resolution function is due to the spillover of events from the low-energy part of the spectrum to measured energies greater than 0.8 GeV. The spillover to the energy bins with $E_\mu > 10.8$ GeV is comparatively small. The zenith angle resolution leaves the number of muon events nearly unchanged.

|                      | $\mu^-$ | $\mu^+$ |
|----------------------|---------|---------|
| Unoscillated         | 14311   | 5723    |
| Oscillated           | 10531   | 4188    |
| After Applying Reconstruct and CID Efficiencies | 4941   | 2136    |
| After Applying ($E, \cos \theta$) Resolutions | 5270   | 2278    |

Table 5.1: Number of muon events produced in CC $\nu_\mu$ interactions at various stages of the analysis for an exposure of 50 kt × 10 years in the energy range 0.8–10.8 GeV.

5.2 Analysis with $E_\mu$ and $\cos \theta_\mu$

In this analysis, we use only the information in muon energy and muon direction. These two quantities can be measured with a good precision, much better than the precision on hadron energy. Therefore we ignore the latter for now. We shall include the hadron information in our analysis in the next two sections.

We generate the data at the benchmark true values for oscillation parameters given in Table 1.1. We define a $\chi^2$ for the ICAL data as

$$
\chi^2(\mu^-) = \min_{\{\xi_k\}} \sum_{i,j} \left[ 2 \left( N^\text{theory}_{ij}(\mu^-) - N^\text{data}_{ij}(\mu^-) \right) + 2 N^\text{data}_{ij}(\mu^-) \ln \left( \frac{N^\text{data}_{ij}(\mu^-)}{N^\text{theory}_{ij}(\mu^-)} \right) \right] + \sum_k \xi_k^2,
$$

(5.7)

where

$$
N^\text{theory}_{ij}(\mu^-) = N^\text{theory}_{ij}^{0}(\mu^-) \left( 1 + \sum_{k=1}^{l} \pi_{ij}^k \xi_k \right).
$$

(5.8)

Here we use the linearized approximation while using the method of pulls. We have assumed a Poissonian distribution for the errors in this definition of $\chi^2$. The reason is that the number of events falls sharply with energy due to the falling flux (cf. Fig. 2.3) and for small exposure times these bins could have very few events per bin. Since ICAL will have separate data in $\mu^-$ and $\mu^+$, we calculate this $\chi^2(\mu^-)$ and $\chi^2(\mu^+)$ separately for the $\mu^-$ sample and the $\mu^+$ sample respectively and then add the two to get the total $\chi^2$ as

$$
\chi^2 = \chi^2(\mu^-) + \chi^2(\mu^+).
$$

(5.9)
In the above equations, $N^\text{data}_{ij}(\mu^-)$ and $N^\text{data}_{ij}(\mu^+)$ are the observed number of $\mu^-$ and $\mu^+$ events respectively in the $i^{th}$ energy and $j^{th}$ angle bin and $N^\text{theory}_{ij}(\mu^-)$ and $N^\text{theory}_{ij}(\mu^+)$ are the corresponding theoretically predicted event spectrums. This predicted event spectrum could shift due to the systematic uncertainties, which is taken care of by the method of pulls [108, 109]. The shifted spectrum $N^\text{theory}_{ij}$ is given by Eq. (5.8), where $\pi_k$ is the $k^{th}$ systematic uncertainty in the $ij^{th}$ bin and $\xi_k$ is the pull variable corresponding to the uncertainty $\pi_k$.

The $\chi^2$ is minimized over the full set of pull variables $\{\xi_k\}$. In our analysis we have considered the muon energy range 0.8 GeV to 10.8 GeV with 10 bins of bin size 1 GeV. The zenith angle range in $\cos\theta$ is taken from $-1$ to $+1$, with 80 bins of bin size 0.025. Note that the zenith angle resolution of ICAL is $\sim 0.01$ in $\cos\theta$ over the entire parameter range of interest, the number of zenith angle bins is limited to ensure enough number of events in individual bins.

The index $k$ in Eqs. (5.7) and (5.8) runs from 1 to $l$, where $l$ is the total number of systematic uncertainties. We have included the following five systematic uncertainties in our analysis. An overall flux normalization error of 20% and an overall cross-section normalization error of 10%. A 5% uncertainty on the zenith angle dependence of the fluxes is included. An energy dependent “tilt factor” is considered according to the following prescription. The event spectrum is calculated with the predicted atmospheric neutrino fluxes and then with the flux spectrum shifted according to

$$
\Phi_\delta(E) = \Phi_0(E) \left( \frac{E}{E_0} \right)^\delta \simeq \Phi_0(E) \left( 1 + \delta \ln \frac{E}{E_0} \right),
$$

(5.10)

where $E_0 = 2$ GeV and $\delta$ is the 1\(\sigma\) systematic error which we have taken as 5\%. The difference between the predicted events rates for the two cases is then included in the statistical analysis. Finally, an over all 5\% systematic uncertainty is included.

### 5.2.1 Mass hierarchy sensitivity

Figure 5.4 shows the discovery potential of ICAL alone for the neutrino mass hierarchy, as a function of the number of years of data taking of the 50 kt ICAL. The data is generated for the values of the oscillation parameters given in Table 1.1 and for $\sin^2 \theta_{23}^{(\text{true})} = 0.5$ for a definite hierarchy. This simulated data is then fitted with the wrong mass hierarchy to check the statistical significance with which this wrong hierarchy can be disfavoured.

The three lines in Fig. 5.4 correspond to the different values of $\sin^2 2\theta_{13}^{(\text{true})}$ shown in the legend boxes in the figure. The left-hand panel is for true normal hierarchy while the right-hand panel is for true inverted hierarchy. In Fig. 5.4 the plots show the sensitivity reach of ICAL when $\chi^2$ is marginalized over oscillation parameters $|\Delta m^2_{\text{eff}}|$, $\sin^2 \theta_{23}$ and $\sin^2 2\theta_{13}$, meaning these oscillation parameters are allowed to vary freely in the fit within the ranges shown in Table 1.1 and the minimum of the $\chi^2$ taken. The CP phase $\delta_{\text{CP}}$ does not significantly impact the ICAL mass hierarchy sensitivity. (This will be discussed in some detail later.) Therefore, we keep $\delta_{\text{CP}}$ fixed at 0 in the fit. The parameters $\Delta m^2_{21}$ and $\sin^2 \theta_{12}$ also do not affect $\chi^2$ and hence are kept fixed at their true values given in Table 1.1. From the figure we see that for full marginalization within the current 3\(\sigma\) allowed range for $|\Delta m^2_{\text{eff}}|$, $\sin^2 \theta_{23}$ and $\sin^2 2\theta_{13}$, the sensitivity reach of ICAL with 10 (5) years data would be only about $2.2\sigma$ ($1.6\sigma$) for $\sin^2 \theta_{23}^{(\text{true})} = 0.5$ and $\sin^2 2\theta_{13}^{(\text{true})} = 0.1$, for true normal hierarchy. The impact for the inverted hierarchy case is seen to be marginally worse.

56
Figure 5.4: Left panel shows the $\Delta \chi^2$ for the wrong hierarchy when normal hierarchy is taken to be true, while the right panel shows the corresponding reach when inverted hierarchy is taken as true. The three lines are for three different values of $\sin^2 2\theta_{13}^{\text{(true)}} = 0.08, 0.1$ and 0.12 as shown in the legend box, while $\sin^2 2\theta_{23}^{\text{(true)}} = 0.5$ for all cases. We take only ICAL muon data into the analysis and marginalize over oscillation parameters $|\Delta m^2_{\text{eff}}|$, $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$, which are allowed to vary freely within their current 3$\sigma$ ranges given in Table 1.1.

We shall further explore the effects of priors, systematic uncertainties, and the true value of $\theta_{23}$ in some detail. The lessons learnt from this study will be applied directly to later analyses.

5.2.1.1 Impact of priors

All values of the oscillation parameters are not allowed with equal C.L. by the current data. Moreover, all oscillation parameters are expected to be measured with much better precision by the ongoing and upcoming neutrino experiments. In fact, by the time ICAL is operational, all of the current accelerator-based and reactor experiments would have completed their scheduled run and hence we expect that by then significant improvements in the allowed ranges of the oscillation parameters would have been made. In particular, we expect improvement in the values of $\sin^2 2\theta_{13}$, $|\Delta m^2_{\text{eff}}|$ and $\sin^2 2\theta_{23}$ from the data coming from current accelerator and reactor experiments. A projected combined sensitivity analysis of these experiments shows that the 1$\sigma$ uncertainties on the values of $\sin^2 2\theta_{13}$, $|\Delta m^2_{\text{eff}}|$ and $\sin^2 2\theta_{23}$ are expected to go down to 0.1%, 2% and 0.65%, respectively [78]. Since marginalization over these parameters makes a difference to $\chi^2$ for the wrong hierarchy, better measurement on them from the current experiments will improve the mass hierarchy sensitivity reach of ICAL. One could incorporate this information into the analysis by including “priors” on these parameters, through

$$\chi^2_{\text{ICAL}} = \chi^2 + \sum \chi^2_{\text{prior}}, \quad \chi^2_{\text{prior}}(p) = \frac{(p_0 - p)^2}{\sigma_0^2},$$

(5.11)

where $\chi^2 = \chi^2_+ + \chi^2_-$ as in Eq. (5.9), $p$ is the parameter on which a prior is included and $p_0$ and $\sigma_0$ are its known best fit and 1$\sigma$ errors.
The sensitivity reach of ICAL with projected priors on $|\Delta m_{\text{eff}}^2|$, $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$, as discussed in the text [11]. The $|\Delta m_{\text{eff}}^2|$ is seen to increase almost linearly with exposure. This is not hard to understand as the hierarchy sensitivity comes from the difference in the number of events between normal and inverted hierarchies due to earth matter effects. Since this is a small difference, the relevant statistics in this measurement is small. As a result the mass hierarchy analysis is statistics dominated and the $\Delta \chi^2$ increases linearly with exposure.

5.2.1.2 Impact of systematic uncertainties

The systematic uncertainties, mainly due to the uncertainties in the atmospheric neutrino fluxes, have already been included in the above analysis, through the method of pulls [108, 109] as described in Sec. 5.2.1. An analysis of the extent of the impact of these uncertainties will give us an idea of how much the reduction in these uncertainties will help. In Fig. 5.6 we show the mass hierarchy sensitivity with and without systematic uncertainties in the ICAL analysis. The $\Delta \chi^2$ is shown as a function of the number of years of exposure of the experiment. The data was generated at the benchmark oscillation point.

The effect of taking systematic uncertainties is to reduce the statistical significance of the analysis. We have checked that of the five systematic uncertainties, the uncertainty on overall normalization of the fluxes and the cross-section normalization uncertainty have minimal impact on the final results. The reason for that can be understood from the fact that the atmospheric neutrinos come from all zenith angles and over a wide range of energies. The overall normalization uncertainty is the same for all bins, while the mass hierarchy dependent earth matter effects, are important only in certain zenith angle bin and certain range of energies. Therefore, the effect of the overall normalization errors get cancelled between different bins. On the other hand, the tilt error could be used to modify the energy
5.2.1.3 Impact of the true value of $\sin^2 \theta_{23}$

The mass hierarchy sensitivity of the ICAL will depend strongly on the actual value of $\theta_{23}$. The amount of earth matter effects increases with increase in both $\theta_{13}$ and $\theta_{23}$. In the previous plots, we have seen the mass hierarchy sensitivity for different allowed values of $\sin^2 2\theta_{13}(\text{true})$, while $\sin^2 \theta_{23}(\text{true})$ was fixed at maximal mixing. In Fig. 5.7 we show the sensitivity to the neutrino mass hierarchy as a function of number of years of running of ICAL for different values of $\sin^2 \theta_{13}(\text{true})$ as well as $\sin^2 \theta_{23}(\text{true})$. As seen in the previous subsection, the $\Delta \chi^2$ for the wrong mass hierarchy increases with $\sin^2 2\theta_{13}(\text{true})$ for a given value of $\sin^2 \theta_{23}(\text{true})$ and ICAL exposure. A comparison of the $\Delta \chi^2$ for different values of $\sin^2 \theta_{23}(\text{true})$ reveals that the $\Delta \chi^2$ also increases with $\sin^2 \theta_{23}(\text{true})$.

5.2.1.4 (In)sensitivity to the CP violating phase

In the analyses above, the true value for the CP-violating phase $\delta_{\text{CP}}$ has been taken to be $0^\circ$, and this parameter has not been marginalised over. The reason for this is that atmospheric neutrino data is insensitive to this phase [18, 110]. In order to illustrate this, we show Fig. 5.8 where the data with 500 kt-yr exposure is generated for $\delta_{\text{CP}} = 0$, and the fit is tried for all $\delta_{\text{CP}}$ values. It can be observed that the hierarchy sensitivity is not affected by what value of $\delta_{\text{CP}}$ we choose to fit the data with. We have checked that the same results hold for any actual $\delta_{\text{CP}}$ value, that is the hierarchy sensitivity of ICAL is independent of the actual value of $\delta_{\text{CP}}$.
Figure 5.7: Same as Fig. 5.5 but for $\sin^2 \theta_{23} (\text{true}) = 0.4$ (green band) and $\sin^2 \theta_{23} (\text{true}) = 0.6$ (red band). The width of each of the bands is mapped by increasing the value of $\sin^2 2\theta_{13} (\text{true})$ from 0.08, through 0.1, and up to 0.12.

Figure 5.8: The hierarchy sensitivity at different assumed values of $\delta_{\text{CP}}$ with 500 kt-yr of ICAL data, when the data is generated with the actual value $\delta_{\text{CP}} = 0$.

The reason behind the insensitivity of ICAL to the actual value of $\delta_{\text{CP}}$ lies in the fact that the muon neutrinos at ICAL come dominantly from the original unoscillated muon neutrinos, while the muon neutrinos coming from the oscillated electron neutrinos are a smaller fraction, partly due to the smallness of the electron neutrino flux, but mostly due to the small value of the conversion probability $P_{e\mu}$ owing to the smallness of $\theta_{13}$ and $\Delta m_{21}^2 / \Delta m_{31}^2$ (see Appendix B). The survival probability $P_{\mu\mu}$ therefore controls the oscillations detected at ICAL. In $P_{\mu\mu}$, the CP-violating phase $\delta_{\text{CP}}$ appears as a subdominant term that oscillates as $\cos \Delta$, where the oscillation phase $\Delta \equiv \Delta m_{31}^2 L / (4E)$ (see Appendix B). If the distance $L$ travelled by the neutrino is uncertain by $\delta L$, this phase becomes uncertain by $\delta \Delta \approx \Delta (\delta L / L)$. For a few GeV neutrinos travelling distances of a few thousands of km through the earth matter, $\Delta \sim 10$. Hence only a 10% uncertainty in $L$ can wipe out the information about the phase $\Delta$, and hence about the dependence on $\delta_{\text{CP}}$. It is indeed difficult to determine the direction of incoming neutrino, and hence the value of $L$, for an atmospheric neutrino. Another added factor is the uncertainty in energy, which also contributes to the uncertainty.
in $\Delta$ as $\delta \Delta \approx \Delta(\delta E/E)$ in a similar way.

Note that this insensitivity of ICAL to the $\delta_{CP}$-dependent oscillating term is already a part of our analysis, so the good capability of ICAL to distinguish between the two hierarchies is in spite of this disadvantage. This may be contrasted with the hierarchy sensitivities of fixed baseline experiments like T2K or NO$\nu$A, whose sensitivities to the mass hierarchy depend crucially on the actual $\delta_{CP}$ value. This issue will be discussed later in Chapter 6.

### 5.2.2 Precision measurement of $|\Delta m^2_{32}|$ and $\sin^2 \theta_{23}$

The reach of ICAL for the parameters $\sin^2 \theta_{23}$ and $|\Delta m^2_{32}|$ separately is shown in Fig. 5.9 in terms of the $\Delta \chi^2$ compared to the best-fit value to the simulated data. The precision on these parameters may be quantified by

$$\text{precision} = \frac{p_{\text{max}} - p_{\text{min}}}{p_{\text{max}} + p_{\text{min}}},$$

where $p_{\text{max}}$ and $p_{\text{min}}$ are the largest and smallest value of the concerned oscillation parameter, determined at the given C.L. from the atmospheric neutrino measurements at ICAL, for a given exposure. We find that after 5 years of data taking, ICAL would be able to measure $\sin^2 \theta_{23}$ to a precision of 20% and $|\Delta m^2_{32}|$ to 7.4% at 1$\sigma$. With 10 years exposure, these numbers are expected to improve to 17% and 5.1% for $\sin^2 \theta_{23}$ and $|\Delta m^2_{32}|$, respectively. The precision on $\sin^2 \theta_{23}$ is mainly governed by the muon reconstruction efficiency and is expected to improve with it. It will also improve as the systematic uncertainties are reduced. If the flux normalization error were to come down from 20% to 10%, the precision on $\sin^2 \theta_{23}$ would improve to 14% for 10 years of exposure. Reducing the zenith angle error from 5% to 1% would also improve this precision to $\sim 14\%$. On the other hand, the precision on $|\Delta m^2_{32}|$ is governed by the ability of the detector to determine the value of $L/E$ for individual events accurately. This depends on the energy- and $\cos \theta$- resolution of the detector.

A few more detailed observations may be made from the $\chi^2$ plots in Fig. 5.9. From Fig. 5.9a, one can notice that the precision on $\theta_{23}$ when it is in the first octant ($\sin^2 \theta_{23} < 0.5$) is slightly better than when it is in the second octant ($\sin^2 \theta_{23} > 0.5$), even though the muon neutrino survival probability depends on $\sin^2 2\theta_{23}$ at the leading order. This asymmetry about $\sin^2 \theta_{23} = 0.5$ stems mainly from the full three-flavor analysis that we have performed in this study. In particular, we have checked that the non-zero value of $\theta_{13}$ is responsible for the asymmetry observed in this figure. On the other hand, $\chi^2$ asymmetry about the true value of $|\Delta m^2_{32}|$ observed in Fig. 5.9b is an effect that is present even with a two-flavor analysis.

The precisions obtainable at ICAL for $\sin^2 \theta_{23}$ and $|\Delta m^2_{32}|$ are expected to be correlated. We therefore present the correlated reach of ICAL for these parameters in Figs. 5.10a and 5.10b. As noted above, our three-neutrino analysis should be sensitive to the octant of $\theta_{23}$. Therefore we choose to present our results in terms of $\sin^2 \theta_{23}$ instead of $\sin^2 2\theta_{23}$. Though the constant-$\chi^2$ contours still look rather symmetric about $\sin^2 \theta_{23} = 0.5$, that is mainly due to the true value of $\sin^2 \theta_{23}$ being taken to be 0.5. The values of $\sin^2 \theta_{23}$ away from 0.5 would make the contours asymmetric and would give rise to some sensitivity to the octant of $\theta_{23}$, as we shall see later.
Figure 5.9: The panel (a) shows the $\chi^2$ as a function of $\sin^2 \theta_{23}$ for $|\Delta m^2_{32}| = 2.4 \times 10^{-3}$ eV$^2$ and $\sin^2 \theta_{23}(\text{true}) = 0.5$. The panel (b) shows the $\chi^2$ as a function of $|\Delta m^2_{32}|$ for $\sin^2 \theta_{23} = 0.5$ and $|\Delta m^2_{32}|(\text{true}) = 2.4 \times 10^{-3}$ eV$^2$. Only muon information has been used [12].

5.2.2.1 Impact of finer binning

In order to compare the impact of binning on the reach of ICAL, we compare two binning schemes: one where we take the $(E_\mu, \cos \theta_\mu)$ bins of size (1 GeV, 0.1), and the other where these bins have the size (0.5 GeV, 0.05). The energy and direction resolutions for ICAL are better than what these binning schemes need, and one is restricted mainly by statistics, with the need to ensure that there are significant number of events in each bin. We present our results in Fig. 5.11 for the precision measurements reach of the ICAL with an exposure of 500 kt-yr. We observe that beyond a point, the finer bins bring only a marginal improvement in the precision measurement of both $\sin^2 \theta_{23}$ and $|\Delta m^2_{32}|$.

5.2.3 Sensitivity to the octant of $\theta_{23}$

Earth matter effects in atmospheric neutrinos can be used to distinguish maximal from non-maximal $\theta_{23}$ mixing and can lead to the determination of the correct $\theta_{23}$ octant [111, 112, 113]. We show in Fig. 5.12 the potential of 10 years of ICAL run for distinguishing a non-maximal value of $\theta_{23}$ from maximal mixing in the case where $\sin^2 2\theta_{23} = 0.90$ ($\sin^2 \theta_{23} = 0.342, 0.658$) and $\sin^2 2\theta_{23} = 0.95$ ($\sin^2 \theta_{23} = 0.388, 0.612$). The figure shows that, if the value of $\theta_{23}$ is near the current 3$\sigma$ bound and in the first octant, then it may be possible to exclude maximal mixing to 99% C.L. with this 2-parameter analysis. If $\theta_{23}$ is in the second octant, or if $\sin^2 2\theta_{23}$ is larger than 0.9, the exclusion of the maximal mixing becomes a much harder task.

Figure 5.12 can also be used to quantify the reach of ICAL for determining the correct octant of $\theta_{23}$, if the value of $\sin^2 2\theta_{23}$ is known. This can be seen by comparing the $\chi^2$ value corresponding to the true value of $\sin^2 \theta_{23}$, but in the wrong octant, with that corresponding to the true value of $\sin^2 \theta_{23}$. We find that, for $\sin^2 2\theta_{23} = 0.9$, i.e. just at the allowed 3$\sigma$ bound, the octant can be identified at >95% C.L. with 10 years of ICAL run if $\theta_{23}$ is in the
Figure 5.10: The precision reach expected at ICAL in the $\sin^2 \theta_{23} - |\Delta m_{32}^2|$ plane at various confidence levels, using only muon information. The black (broken), blue (dotted) and red (solid) lines show 68%, 90% and 99% C.L contours. The true values of $\sin^2 \theta_{23}$ and $|\Delta m_{32}^2|$ used for generating data are shown by the black dots. The true values of other parameters used are given in Table 1.1. Panel (a) is for five-year running of the 50 kt detector while (b) is for ten years exposure [12].

first octant. However if $\theta_{23}$ is in the second octant, the identification of the octant would be much harder: $\theta_{23}$ in the wrong octant can be disfavored only to about 85% C.L.. The situation is more pessimistic if $\sin^2 2\theta_{23}$ is closer to unity.

The precision on $|\Delta m_{32}^2|$ will keep improving with ongoing and future long baseline experiments. The inclusion of the information may improve the chance of ICAL being able to identify deviation of $\theta_{23}$ from maximal mixing and its octant to some extent.

We would like to remind the reader that all the results in this section (Sec. 5.2.1) use only the information on muon energy and direction. The addition of hadron information is expected to improve the physics reach of ICAL in all aspects, and this will be explored in the next sections.

5.3 Analysis with $E_{\nu}$ and $\cos \theta_{\mu}$

The addition of the value of hadron energy $E'_{\text{had}}$, calibrated with respect to the number of hadronic hits as described in Chapter 4, to the muon energy $E_{\mu}$ reconstructs the incoming neutrino energy in a charged-current interaction. Since it is the neutrino energy that appears in the neutrino oscillation probabilities, it is conceivable that direct access to neutrino energy will improve the reach of the ICAL to the oscillation parameters.

The analyses in this section and the next that use the hadron energy information assume that the hits created by a muon and hadron can be separated with an 100% efficiency by the ICAL particle reconstruction algorithm. Then, whenever a muon is reconstructed, the corresponding hadron hits can always be considered to be a hadron shower, so that the energy $E'_{\text{had}}$ can be reconstructed. This implies that the neutrino event reconstruction efficiency is
Figure 5.11: Effect of binning on the precision reach expected from the analysis of 50 kt × 10 years data of atmospheric neutrinos in ICAL. The black, blue and red lines show 68%, 90% and 99% C.L contours. The solid lines show the contours for \((E_\mu, \cos \theta_\mu)\) bin widths of (1 GeV, 0.1) while the broken lines show the contours for \((E_\mu, \cos \theta_\mu)\) bin widths of (0.5 GeV, 0.05). Only muon information has been used \[12\].

the same as the muon reconstruction efficiency. (Note that the calibration of \(E'_{\text{had}}\) against the number of hadron shower hits also allows for the possibility of no hits observed in the hadron shower.) Finally, the background hits coming from other sources such as the neutral-current events, charged-current \(\nu_e\) events, cosmic muons, noise, have not been taken into account so far. The systematics due to these effects will have to be taken care of in future, as the understanding of the ICAL detector improves.

The neutrino energy \(E_\nu\) is reconstructed as the sum of the reconstructed muon energy \(E_\mu\) (Sec. 4.1) and the calibrated hadron energy \(E'_{\text{had}}\) (Sec. 4.2). The muon energy resolutions and zenith angle resolutions have been implemented by smearing the true muon energy and direction of each \(\mu^+\) and \(\mu^-\) event, as discussed earlier. Energy of hadron events has also been smeread separately, following the discussion in Sec 4.2. The reconstructed neutrino energy is then taken as the sum of reconstructed muon energy and hadron energy, with the uncertainties calculated separately for each event. The events are then binned in \((E_\nu, \cos \theta_\mu)\) bins: 15 equal \(E_\nu\) bins in the range [0.8, 5.8] GeV, 5 equal \(E_\nu\) bins in the range [5.8, 10.8] GeV, and 20 equal \(\cos \theta_\mu\) bins in the range \([-1.0, 1.0]\). The same analysis as in Sec. 5.2 is then performed, marginalizing over the 3σ allowed ranges of \(|\Delta m^2_{32}|\), \(\sin^2 \theta_23\), and \(\theta_{13}\), with a 10% prior on \(\theta_{13}\). The systematic errors have been taken into account using the method of pulls as before.

Figure 5.13 shows the sensitivity of ICAL to the atmospheric mixing parameters \(|\Delta m^2_{\text{eff}}|\) and \(\sin^2 \theta_{23}\), where \(\Delta m^2_{\text{eff}} \equiv \Delta m^2_{32} - (\cos^2 \theta_{12} - \cos \delta_{\text{CP}} \sin \theta_{32} \sin 2\theta_{12} \tan \theta_{23}) \Delta m^2_{21}\) is the effective value of \(\Delta m^2_{\text{atm}}\) relevant for the two-neutrino analysis of atmospheric neutrino oscillations \[3, 7\]. It is seen that with 10 years of data and the \((E_\nu, \cos \theta_\mu)\) analysis technique, the 50 kt ICAL can measure the magnitude of the atmospheric neutrino mass squared difference to 4% and \(\sin^2 \theta_{23}\) to 13%, at 1σ. A comparison of these numbers against those obtained using the \((E_\mu, \cos \theta_\mu)\) analysis shows that that using the reconstructed neutrino
energy improves the precision on $\sin^2 \theta_{23}$ and $|\Delta m^2_{32}|$ by about 20%. However the addition of the coarsely measured hadron energy to the accurately measured muon energy results in some loss of information, which results in some degradation in the performance for mass hierarchy identification.

5.4 Analysis using correlated Information on $E_\mu$, $\cos \theta_\mu$, and $E'\text{had}$

The preceding analyses have been performed by using only the information on muon direction, and the energy of muons or neutrinos. However in each CC event at ICAL, the information on $E_\mu$, $\cos \theta_\mu$ and $E'\text{had} \equiv E_\nu - E_\mu$ is available independently. Thus, the inelasticity $y \equiv E'\text{had}/E_\nu$ is an additional measurable quantity in each event. This advantage of
Figure 5.13: The reach of the ICAL for precision measurements of $|\Delta m^2_{\text{eff}}|$ and $\sin^2 \theta_{23}$, using information on $E_\nu$ and $\cos \theta_{\mu}$, for an exposure of 500 kt-yr. The contours with 68%, 90% and 99% confidence level are shown [13].

ICAL may be exploited to enhance its physics reach [14], as shall be seen in this section. This is implemented by performing an analysis that employs binning in all these three quantities, so that no information is lost and all correlations are taken care of.

5.4.1 The three-dimensional binning

For event generation and inclusion of oscillation, we use the same procedure as described in Sec. 5.2. After incorporating the detector response for muons and hadrons [8, 9], the measured event distribution in terms of $(E\mu, \cos \theta_{\mu}, E'_{\text{had}})$ is obtained. A three-dimensional binning scheme using the measured quantities $E\mu, \cos \theta_{\mu}$ and $E'_{\text{had}}$ is employed for the $\chi^2$ analysis. In order to ensure significant statistics in each bin and also to avoid large number of bins, we use a non-uniform binning scheme for each polarity as shown in Table 5.2. Thus for each polarity, one has a total of $(10 \times 21 \times 4) = 840$ bins.

| Observable | Range | Bin width | Total bins |
|------------|-------|-----------|------------|
| $E\mu$ (GeV) | 1 – 4 | 0.5 | 6 \{ 10 |
| | 4 – 7 | 1 | 3 | 1 |
| | 7 – 11 | 4 | 1 | |
| $\cos \theta_{\mu}$ | (-1) – (-0.4) | 0.05 | 12 \{ 21 |
| | (-0.4) – 0 | 0.1 | 4 | 5 |
| | 0 – 1 | 0.2 | 1 | |
| $E'_{\text{had}}$ (GeV) | 0 – 2 | 1 | 2 \{ 4 |
| | 2 – 4 | 2 | 1 | 1 |
| | 4 – 15 | 11 | 1 | |

Table 5.2: The binning scheme in $E\mu, \cos \theta_{\mu}$, and $E'_{\text{had}}$ for each polarity
For the statistical analysis, we define the Poissonian \( \chi^2 \) for the \( \mu^- \) events as

\[
\chi^2 = \min_{\xi_l} \sum_{i=1}^{N_{\text{had}}'} \sum_{j=1}^{N_{E\mu}} \sum_{k=1}^{N_{\cos\theta\mu}} \left[ 2(N_{ijk}^{\text{theory}} - N_{ijk}^{\text{data}}) - 2N_{ijk}^{\text{data}} \ln \left( \frac{N_{ijk}^{\text{theory}}}{N_{ijk}^{\text{data}}} \right) \right] + \sum_{l=1}^{5} \xi_l^2,
\]

(5.13)

where

\[
N_{ijk}^{\text{theory}} = N_{ijk}^{0 \text{ theory}} \left( 1 + \sum_{l=1}^{5} \pi_{ijl}^{l} \xi_{l} \right).
\]

(5.14)

Here \( N_{ijk}^{\text{theory}} \) and \( N_{ijk}^{\text{data}} \) are the expected and observed number of \( \mu^- \) events in a given \((E_{\mu}, \cos\theta_{\mu}, E'_{\text{had}})\) bin, with \( N_{E'_{\text{had}}} = 4, \) \( N_{E\mu} = 10, \) and \( N_{\cos\theta\mu} = 21. \) The systematic uncertainties have been included by using the pull method [108, 109], which uses the “pull” variables \( \xi_{l} \) [11, 12].

For \( \mu^+ \) events, \( \chi^2 \) is similarly defined. The total \( \chi^2 \) is obtained as

\[
\chi^2_{\text{ICAL}} = \chi^2_- + \chi^2_+ + \chi^2_{\text{prior}},
\]

(5.15)

where

\[
\chi^2_{\text{prior}} = \left( \frac{\sin^2 2\theta_{13} - \sin^2 2\theta_{13}(\text{true})}{0.08 \times \sin^2 2\theta_{13}(\text{true})} \right)^2.
\]

(5.16)

The 8% prior on \( \sin^2 2\theta_{13} \) corresponds to the current accuracy in the measurement of this quantity. No prior on \( \theta_{23} \) or \( \Delta m^2_{32} \) is used, since these parameters will be directly measured at the ICAL.

While implementing the minimization procedure, \( \chi^2_{\text{ICAL}} \) is first minimized with respect to the pull variables \( \xi_{l} \), and then marginalized over the 3\( \sigma \) allowed ranges of the oscillation parameters \( \sin^2 \theta_{23}, \Delta m^2_{\text{eff}}, \) and \( \sin^2 2\theta_{13}, \) wherever appropriate. We do not marginalize over \( \delta_{\text{CP}}, \Delta m^2_{21}, \) and \( \theta_{12} \) since they have negligible effect on the relevant oscillation probabilities at ICAL [108, 110]. Also, \( \delta_{\text{CP}} = 0 \) throughout the analysis.

### 5.4.2 Mass hierarchy sensitivity

We quantify the statistical significance of the analysis to rule out the wrong hierarchy by

\[
\Delta \chi^2_{\text{ICAL-MH}} = \chi^2_{\text{ICAL}}(\text{false MH}) - \chi^2_{\text{ICAL}}(\text{true MH}).
\]

(5.17)

Here, \( \chi^2_{\text{ICAL}}(\text{true MH}) \) and \( \chi^2_{\text{ICAL}}(\text{false MH}) \) are obtained by performing a fit to the “observed” data assuming true and false mass hierarchy, respectively.

Figure 5.14 shows the mass hierarchy sensitivity of ICAL as a function of the run-time of the experiment. It is found that after including the hadron energy information, 10 years of running of the 50 kt ICAL can rule out the wrong hierarchy with \( \Delta \chi^2_{\text{ICAL-MH}} \approx 9.7 \) (for true NH), and \( \Delta \chi^2_{\text{ICAL-MH}} \approx 9.1 \) (for true IH). In other words, the wrong hierarchy can be ruled out to about 3\( \sigma \) for either hierarchy. If the true values of \( \theta_{23} \) and \( \theta_{13} \) are varied over their allowed 3\( \sigma \) range, the corresponding range for \( \Delta \chi^2_{\text{ICAL-MH}} \) after 10 years is 7–12. Compared to the results without using hadron information, with the same binning scheme, the value of \( \Delta \chi^2_{\text{ICAL-MH}} \) increases by about 40% when the correlated hadron energy information is added. This improvement is not merely due to using additional bins compared to the muon-only analysis, as can be checked by comparing the results with those in Sec. 5.2.
5.4.3 Precision Measurement of Atmospheric Parameters

In order to quantify the precision in the measurements of a parameter $\lambda$ (here $\lambda$ may be $\sin^2 \theta_{23}$ or $|\Delta m^2_{32}|$ or both), we use the quantity

$$\Delta \chi^2_{\text{ICAL-PM}}(\lambda) = \chi^2_{\text{ICAL}}(\lambda) - \chi^2_0,$$

(5.18)

where $\chi^2_0$ is the minimum value of $\chi^2_{\text{ICAL}}$ in the allowed parameter range.

The two panels of Fig. 5.15 show the sensitivity of ICAL to the two parameters $\sin^2 \theta_{23}$ and $|\Delta m^2_{32}|$ separately, where the other parameter has been marginalized over, along with $\theta_{13}$ and the two possible mass hierarchies. While the figure shows the results for NH as the true hierarchy, the results with true IH are almost identical. It may be observed from the figure that with the inclusion of hadron energy information, 500 kt-yr of ICAL exposure would be able to measure $\sin^2 \theta_{23}$ to a 1σ precision of 12% and $|\Delta m^2_{32}|$ to a 1σ precision of 2.9%. This may be compared with the muon-only analysis with identical $(E_\mu, \cos \theta_\mu)$ binning, which gives the precisions of 13.7% and 5.4%, respectively.

Figure 5.16 shows the $\Delta \chi^2_{\text{ICAL-PM}}$ contours in the $\sin^2 \theta_{23}$--$|\Delta m^2_{32}|$ plane (left panel) and in the $\sin^2 \theta_{23}$--$|\Delta m^2_{32}|$ plane (right panel), using the hadron energy information. Here the true value of $\theta_{23}$ has been taken to be maximal, so the contours in the left panel are almost symmetric in $\sin^2 \theta_{23}$.

Figure 5.17 shows the comparison of the projected 90% C.L. precision reach of ICAL (500 kt-yr) in $\sin^2 \theta_{23}$--$|\Delta m^2_{32}|$ plane with other experiments [114, 115, 116]. Using hadron energy information, the ICAL will be able to achieve a $\sin^2 \theta_{23}$ precision comparable to the current precision for Super-Kamiokande [114] or T2K [116], and the $|\Delta m^2_{32}|$ precision comparable to the MINOS reach [115]. Of course, some of these experiments would have collected much more statistics by the time ICAL would have an exposure of 500 kt-yr, so the ICAL will therefore not be competing with these experiments for the precision measurements of these...
mixing parameters. However the ICAL measurements will serve as complementary information for the global fit of world neutrino data. Note that, as compared to the atmospheric neutrino analysis at Super-Kamiokande, the ICAL precision on $|\Delta m^2_{32}|$ is far superior. This is due to the better precision in the reconstruction of muon energy and direction at ICAL.

With a non-maximal true value of $\theta_{23}$, the bounds on $\sin^2 \theta_{23}$ range would be asymmetric about 0.5. Figure 5.18 shows the sensitivity of ICAL for $\sin^2 2\theta_{23} = 0.93$ (i.e. $\sin^2 \theta_{23} = 0.37$, 0.63). It may be observed that for $\theta_{23}$ in the lower octant, the maximal mixing can be ruled out with 99% C.L. with 500 kt-yr of ICAL data. However, if $\theta_{23}$ is closer to the maximal mixing value, or in the higher octant, then the ICAL sensitivity to exclude maximal mixing would be much smaller.

Figure 5.15: The left panel shows $\Delta \chi^2_{\text{ICAL-PM}}(\sin^2 \theta_{23})$ and the right panel shows $\Delta \chi^2_{\text{ICAL-PM}}(|\Delta m^2_{32}|)$, assuming NH as true hierarchy. The lines labelled $(E_\mu, \cos \theta_\mu)$ denote results without including hadron information, while the lines labelled $(E_\mu, \cos \theta_\mu, E'_\text{had})$ denote improved results after including hadron energy information [14].

Figure 5.16: $\Delta \chi^2_{\text{ICAL-PM}}$ confidence level contours (2 dof) in the the $\sin^2 \theta_{23}$—$|\Delta m^2_{32}|$ plane (left panel) and in the $\sin^2 2\theta_{23}$—$|\Delta m^2_{32}|$ plane (right panel), using the hadron energy information, with NH as the true hierarchy. The true choices of the parameters have been marked with a dot [14].
5.4.4 Non-maximal $\theta_{23}$ and its octant

In analogy with the mass hierarchy discovery sensitivity, the statistical significance of the analysis to rule out the wrong octant is quantified as

$$\Delta \chi^2_{\text{ICAL-OS}} = \chi^2_{\text{ICAL}(\text{false octant})} - \chi^2_{\text{ICAL}(\text{true octant})}.$$  (5.19)
Here, $\chi^2_{\text{ICAL}}(\text{true octant})$ and $\chi^2_{\text{ICAL}}(\text{false octant})$ are obtained by performing a fit to the “observed” data assuming the true octant and wrong octant, respectively.

Figure 5.19 shows the sensitivity of ICAL to the identification of the $\theta_{23}$ octant, with and without including the hadron energy information. It may be observed that a 2$\sigma$ identification of the octant is possible with the 500 kt-yr INO data alone only when the true hierarchy is NH and the true octant is LO. In this case, without using the hadron energy information one can get a 2$\sigma$ identification only when $\sin^2 \theta_{23}^{(\text{true})} < 0.365$, which is almost close to the present 3$\sigma$ bound. With the addition of hadron energy information, this task is possible as long as $\sin^2 \theta_{23}^{(\text{true})} < 0.395$. If the true octant is HO or the true mass hierarchy is inverted, then the discrimination of $\theta_{23}$ octant with the ICAL data alone becomes rather difficult. The reach is found not to be much sensitive to the exact value of $\theta_{13}$. Clearly the octant discrimination becomes more and more difficult as the true value of $\sin^2 \theta_{23}$ goes close to the maximal mixing. A combination of atmospheric as well as long baseline experiments is needed to make this measurement [117].

In conclusion, the inclusion of correlated hadronic information improves the sensitivity of ICAL to mass hierarchy, precision measurement of $|\Delta m^2_{32}|$ and $\theta_{23}$, exclusion of maximal mixing, as well as $\theta_{23}$ octant determination. This analysis appears to be the optimal one to extract information from ICAL data. However for the potential of this method to be realized, a very good understanding of the hadron response of the ICAL detector is crucial.

### 5.5 ICAL physics potential: highlights

In summary, the ICAL detector is extremely suitable for determining the neutrino mass hierarchy, due to its capabilities of measurements of muon and hadron energies, and identification of the muon charge. The cleanest and the simplest analysis of the ICAL data uses the information on muons only. However the reach of the detector improves tremendously if
the information on hadron energy is also used in addition. Simply adding this energy to the muon energy to reconstruct the neutrino energy is, however, not enough, as it causes some dilution in the accurately known muon information. The information on the muon energy, muon direction, and hadron energy has to be kept separately and used in the analysis. Such an analysis indicates that in 10 years, a 50 kt ICAL can, by itself, distinguish between the normal and the inverted hierarchy with a significance of more than $3\sigma$.

One important point to emphasize here is that this capability of ICAL is independent of the actual value of $\delta_{CP}$. Hence the results in this chapter are independent of $\delta_{CP}$. Moreover, this feature may be exploited by combining the ICAL information with that from the other CP-sensitive experiments, to improve the mass hierarchy discrimination. Such synergies between ICAL and the other experiments will be explored in the next chapter.
Chapter 6

Synergy with Other Experiments

The whole is greater than the sum of its parts.
– Aristotle

Most of the results in this Report have been given with a 10 year running of the 50 kt ICAL. So far, the expected data from ICAL alone has been used. However many of the ongoing accelerator and reactor experiments will keep on gathering more data, and the reach of INO will be enhanced due to the information available from them. A consistent way of taking the impact of these experiments into account is to include their data in a combined $\chi^2$ fit. In this work we present the impact of the prior data from these experiments on the ICAL physics reach, as well as how the ICAL data will help these experiments remove certain ambiguities from their analysis, and zero in on the actual neutrino mixing parameters.

6.1 Combined mass hierarchy reach at $\delta_{CP} = 0$
for ICAL + T2K + NO$\nu$A

As we have seen in Chapter 5, the 50 kt ICAL by itself can identify the mass hierarchy with a significance of $\Delta \chi^2 \approx 9$ with 10 years of running. This reach is independent of the actual value of the CP-violating phase $\delta_{CP}$. Currently running fixed baseline experiments like T2K and NO$\nu$A would already have obtained some sensitivity to the mass hierarchy during their run. This sensitivity will depend on the actual value of $\delta_{CP}$, as we shall see in Sec. 6.2. In this section, we shall see the effect of using the information from these experiments, in the case where the value of $\delta_{CP}$ is taken to be zero. Note that the contribution of the current reactor experiments to the mass hierarchy measurements is very negligible [13].

A preliminary estimate of the combined sensitivity to the neutrino mass hierarchy as a function of number of years of run of the ICAL atmospheric neutrino experiment is shown in Fig. 6.1 [17]. For each set of oscillation parameters, the joint $\chi^2$ from all experiments is given by

$$\chi^2 = \chi^2_{-} + \chi^2_{+} + \sum_i \chi^2_i + \chi^2_{prior},$$

(6.1)

where $\chi^2_{-}, \chi^2_{+}$ are as defined in Eqs. (5.13) and (5.14), $\sum_i \chi^2_i$ is the contribution from the accelerator experiments ($i$ runs over T2K and NO$\nu$A), and $\chi^2_{prior}$ is the prior on $\theta_{13}$ from the
Figure 6.1: Preliminary results on the $\Delta \chi^2$ for the wrong hierarchy obtained from a combined analysis of T2K (a luminosity of $8 \times 10^{21}$ pot in neutrino run), NO$\nu$A (3 years neutrino and 3 years antineutrino run), and ICAL [17]. The left (right) panel is for true normal (inverted) hierarchy. We take $\sin^2 2\theta_{13}\text{(true)} = 0.1, \sin^2 2\theta_{23}\text{(true)} = 0.5$ and $\delta_{CP} = 0$, and all parameters are allowed to vary over their 3$\sigma$ ranges as shown in Table 1.1.

reactor experiments. Here a prior of 4.5% on $\sin^2 2\theta_{13}$ has been taken, which matches the outcome of the recent global fit [50]. This joint $\chi^2$ is computed and marginalized over all oscillation parameters, to determine the minimized joint $\Delta \chi^2$ shown in the figure. Note that the x-axis in this figure shows the number of years of running of ICAL only. We assume a complete T2K run (total luminosity $8 \times 10^{21}$ pot) with neutrinos only, 3 years of NO$\nu$A run with neutrinos, and 3 years of NO$\nu$A run with antineutrinos have already been completed. We use the standard setup of these experiments as in the GLoBES package [118, 119].

The figure shows that the prior information from T2K and NO$\nu$A implies that the target of $\Delta \chi^2 = 9$ significance for the hierarchy identification may now be achieved within 6 years of the ICAL running, as opposed to about 10 years with only the ICAL data. Note that this is not just the effect of the $\Delta \chi^2$ provided by the fixed baseline experiments. These experiments also yield an improved precision in $\theta_{23}, |\Delta m^2_{\text{eff}}|$ (through their disappearance channel), and in $\theta_{13}$ (through their appearance channel). As a result, the impact of marginalization over these parameters in the ICAL analysis is greatly reduced. Indeed, for the 500 kt-yr of ICAL data, the contribution of $\Delta \chi^2_{\text{ICAL-MH}}$ to the total $\Delta \chi^2$ in the case of true normal hierarchy increases from 9.5 (without the data from T2K and NO$\nu$A) to 10.4 (with the data from T2K and NO$\nu$A). Note that the contributions of T2K and NO$\nu$A themselves are expected to be $\Delta \chi^2 \approx 0.12$ and 2.6, respectively.

The fact that the total $\Delta \chi^2$ in the ICAL, T2K and NO$\nu$A experiments is greater than the sum of their individual $\Delta \chi^2$ values is the synergy among these experiments. Though ICAL is the dominant contributor to the hierarchy sensitivity in this case, it clearly benefits tremendously from this synergy. Similary for other physics issues like the precision measurements of mixing parameters, the combination of data from long baseline experiments and ICAL will help improve our overall understanding of these parameter values. Here the long baseline experiments are expected to play a dominant role since they are directly sensitive to $\theta_{23}$ and $|\Delta m^2_{\text{eff}}|$ through their disappearance channel, while ICAL is expected to play a
complementary role.

So far we have discussed the case of $\delta_{CP} = 0$. In the next section, we shall explore the case of nonvanishing $\delta_{CP}$ in detail, since it affects the mass hierarchy sensitivity of fixed baseline experiments.

### 6.2 Ensuring mass hierarchy sensitivity for all $\delta_{CP}$

As seen in Sec. 5.2.1.4, the insensitivity of ICAL to the actual value of $\delta_{CP}$ comes from the fact that the muon neutrinos at ICAL come dominantly from the original unoscillated muon neutrinos, while the muon neutrinos coming from the oscillated electron neutrinos play a subdominant role. The survival probability $P_{\mu\mu}$ (See Appendix E) is therefore more relevant than the conversion probability $P_{\mu e}$. In $P_{\mu\mu}$, the CP-violating phase $\delta_{CP}$ appears as a subdominant oscillating term, whose oscillations, moreover, are averaged out due to the uncertainties in the energies and directions of the incoming neutrinos [19].

On the other hand, the fixed baseline experiments like T2K or NO$\nu$A are sensitive to the conversion probability $P_{\mu e}$, whose dominant term depends on $\delta_{CP}$. Moreover, though this term is oscillating, these experiments have a better knowledge of the neutrino energies, and the neutrino directions are very precisely known. As a result, the data at these experiments is highly sensitive to $\delta_{CP}$. In particular, while performing the fit to the data, the value of $\delta_{CP}$ can be adjusted, along with the value of $|\Delta m^2_{31}|$, to compensate for the wrong hierarchy in half the $\delta_{CP}$ parameter space. As a result, even if the wrong hierarchy is being fitted, a good fit may be obtained, albeit at a wrong value of $\delta_{CP}$. Since the actual $\delta_{CP}$ value is unknown, this worsens the hierarchy sensitivity of the experiment. This may be seen from the left panel of Fig. 6.2 where the actual $\delta_{CP}$ is taken to be vanishing, however NO$\nu$A gives a better fit at another value of $\delta_{CP}$, with a very reduced value of $\delta_{CP}$ as compared to that at the true $\delta_{CP}$ value. In the absence of any knowledge about the actual $\delta_{CP}$ value, this results in a reduced performance of the detector.

The right panel of Fig. 6.2 shows the $\Delta \chi^2$ for the mass hierarchy sensitivity as a function of $\delta_{CP}(\text{true})$. The data are generated for normal hierarchy at each value of $\delta_{CP}(\text{true})$ shown on the $x$-axis, and a fit is performed for inverted hierarchy by marginalizing over all oscillation parameters, including $\delta_{CP}$. Clearly, the reach of NO$\nu$A alone for determining the neutrino mass hierarchy is extremely sensitive to the actual value of $\delta_{CP}$. While the sensitivity is $\Delta \chi^2 \approx 9$ for $\delta_{CP}(\text{true})$ near $270^\circ$, it falls to almost zero for $\delta_{CP}(\text{true}) \approx [50^\circ - 150^\circ]$.

When T2K and all reactor data are added, there is some improvement to the combined sensitivity. In particular, in the $\delta_{CP}(\text{true}) \approx [50^\circ - 150^\circ]$ range where NO$\nu$A by itself gives no mass hierarchy sensitivity, the addition of T2K and reactor data takes $\Delta \chi^2$ to $\simeq 3.5$. The reason for this is the mismatch between the best-fits for different experiments. For the same reason, even the reactor data that are not sensitive to $\delta_{CP}$ but only to $|\Delta m^2_{31}|$ help, albeit marginally, in disfavouring the spurious best-fit minima for the wrong hierarchy, since they do not allow the fit value of $|\Delta m^2_{31}|$ to stray far from the actual one. A combined fit with all accelerator and reactor data thus give a best-fit at a point ($\delta_{CP} = 198^\circ$, $\sin^2 \theta_{23} = 0.48$ and $\sin^2 2\theta_{13} = 0.1$) where the tension between these experiments gives a small hierarchy sensitivity even in the disfavoured $\delta_{CP}(\text{true})$ range ($[0^\circ - 180^\circ]$ for normal hierarchy).

Finally, addition of the ICAL data raises the $\Delta \chi^2$ by a constant amount for all values of
Figure 6.2: The left panel shows the $\Delta \chi^2$ for mass hierarchy sensitivity at NO$\nu$A and ICAL, when the actual value of $\delta_{CP}$ is vanishing, and different test value of $\delta_{CP}$ (shown along the x-axis) are taken. The right panel shows the dependence of mass hierarchy sensitivities of experiments (or their combinations) to the actual $\delta_{CP}$, when the test $\delta_{CP}$ is varied over all its range for minimizing $\Delta \chi^2$. The full proposed runs of the long baseline and reactor experiments are taken. The ICAL exposure is taken to be 500 kt-yr, and only the muon energy and direction are used, ignoring the hadron information [11].

$\delta_{CP}(true)$, and ensures the identification of hierarchy to more than $\Delta \chi^2 \approx 10$ for even those $\delta_{CP}(true)$ values for which the other experiments cannot rule out the wrong hierarchy by themselves. In the best-case scenario, the hierarchy may be identified to $\Delta \chi^2 \approx 20$.

Note that the ICAL results in the figure have been obtained by using only the information on muon energy and momentum. If the hadron information is also added, the combined $\Delta \chi^2$ of all the experiments is expected to be 12–22, depending on whether the actual $\delta_{CP}$ value is favourable to the fixed baseline experiments or not.

### 6.3 Octant of $\theta_{23}$ from the ICAL, NO$\nu$A and T2K data

Combined analysis of all the neutrino oscillation data available disfavors the maximal mixing solution for $\theta_{23}$ at 1.4$\sigma$ confidence level [51, 52] which is mostly driven by the MINOS accelerator data in $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance modes [55]. Now, if $\sin^2 2\theta_{23}$ turns out to be different from unity as suggested by the recent oscillation data, this creates the problem of octant degeneracy of $\theta_{23}$ [56]. In a recent global fit work by F. Capozzi et al. [53], the authors have found an overall preference for the first or lower octant (LO) at 95% confidence level assuming normal hierarchy. In case of inverted hierarchy, the higher octant (HO) seems to be preferred [51].

In this section, we present the preliminary results [120] showing the discovery reach of the octant of $\theta_{23}$ with atmospheric neutrinos at ICAL in combination with projected T2K [121, 122] and NO$\nu$A [63, 66, 67] data. We have performed the analysis for ICAL by using only the information on muon energy and muon direction which has been described in detail in Sec. 5.2. The Earth matter effect in the $P_{\mu\mu}$ channel can be very useful to resolve the octant ambiguity of $\theta_{23}$ [111].
Figure 6.3: $\Delta \chi^2_{\text{ICAL-OS}}$ for octant discovery potential as a function of true $\sin^2 \theta_{23}$. The left (right) panel assumes normal (inverted) hierarchy as true choice. In each panel, the black solid line shows the performance of ICAL with an exposure of 500 kt-yr using only the information on muon energy and muon direction. The green dotted line depicts the combined sensitivity of T2K (integrated luminosity of $4 \times 10^{21}$ pot in neutrinos and $4 \times 10^{21}$ pot in antineutrinos) and NO$\nu$A (3 years of $\nu$ run + 3 years of $\bar{\nu}$ run). The red dashed graph presents the combined results of ICAL, T2K, and NO$\nu$A [120].

The potential of the experiments for excluding the wrong octant as a function of true value of $\sin^2 \theta_{23}$ is shown in Fig. 6.3. For each given value of $\theta_{23}$ (true), we marginalize over all the allowed values of $\theta_{23}$ in the opposite octant, including the maximal mixing value. We take 5% prior on $\sin^2 2\theta_{13}$ with a true value of 0.1, and take $|\Delta m_{\text{eff}}^2|$ (true) = $2.4 \times 10^{-3}$ eV$^2$. No priors have been taken on atmospheric parameters. In the case of T2K and NO$\nu$A, we generate the data with $\delta_{\text{CP}} = 0^\circ$, but in fit, we marginalize over the entire range of $\delta_{\text{CP}}$ between 0 to $2\pi$. For ICAL, we take $\delta_{\text{CP}} = 0^\circ$ both in data and in fit as ICAL atmospheric analysis is not sensitive to $\delta_{\text{CP}}$.

From Fig. 6.3, we can see that 50 kt ICAL in ten years can identify the correct octant at $2\sigma$ if $\sin^2 \theta_{23}$ (true) < 0.38 (0.35) only when the true hierarchy is normal (inverted) and the true octant is LO. The green dotted line depicts the combined sensitivity of T2K (2.5 years of $\nu$ run + 2.5 years of $\bar{\nu}$ run) and NO$\nu$A (3 years of $\nu$ run + 3 years of $\bar{\nu}$ run). The red dashed graph presents the combined results of ICAL, T2K, and NO$\nu$A.

The projected data from T2K and NO$\nu$A will clearly play a crucial role in addressing the issue of $\theta_{23}$ octant. Adding the information from these long-baseline experiments, we can improve the octant discovery reach of ICAL significantly, suggesting the possible synergy between the atmospheric and long-baseline data. The combined atmospheric and long-baseline data can establish the correct octant at $3\sigma$ if $\sin^2 \theta_{23}$ (true) < 0.42 (0.43) assuming normal (inverted) hierarchy and the lower octant is chosen by nature.
6.4 ICAL data for improving CPV discovery potential of T2K and NOνA

The identification of mass hierarchy and measurement of CP violation are intrinsically interconnected at the fixed baseline experiments, due to the leading term in the relevant conversion probability $P_{\mu e}$ as given in Appendix B. Therefore in the absence of knowledge about hierarchy, it is possible that the wrong hierarchy conspiring with an incorrect value of $\delta_{CP}$ may mimic the correct combination of hierarchy and $\delta_{CP}$. As a result, in the unfavourable range of $\delta_{CP}$ as described in Sec. 6.2, i.e. $\delta_{CP} \in [0^\circ, 180^\circ]$ if the actual hierarchy is normal NH and $\delta_{CP} \in [180^\circ, 360^\circ]$ if the actual hierarchy is inverted, the ability of NOνA and T2K to measure $\delta_{CP}$, in particular to discover nonzero $\delta_{CP}$ and hence CP violation, is severely curtailed. The hierarchy sensitivity of an atmospheric neutrino experiment like ICAL, which is independent of the actual $\delta_{CP}$ value, can restore the ability of these fixed baseline experiments to discover CP violation [19, 80].

The ICAL analysis is performed using only the information on muon energy and direction, neglecting hadronic information, and taking an exposure of 500 kt-yr. The input parameter ranges and marginalisation ranges as given in Table 1.1 are used, with the prior of 5% taken on $\theta_{13}$. The discovery potential for CPV is quantified by considering a variation of $\delta_{CP}$ over the full range $[0^\circ, 360^\circ]$ in the simulated data, and comparing it with $\delta_{CP} = 0^\circ$ or $180^\circ$ in the theory expectation. The definition of $\chi^2$ used is the same as in Sec. 5.2.

In Fig. 6.4, we plot the CPV discovery potential of T2K and NOνA with and without information from ICAL. For T2K and NOνA, the same specifications as in Sec. 6.1 have been used. The figure shows that, as expected, the CP-violation sensitivity of the experiments is zero for true $\delta_{CP} = 0$ and $\pi$, while it is close to maximum at the maximally CP violating values $\delta_{CP} = 90^\circ$ or $\delta_{CP} = 270^\circ$, depending on the hierarchy. The CPV discovery with NOνA and T2K suffers a drop in one of the half-planes of $\delta_{CP}$, depending on the true hierarchy — in the region $[0^\circ, 180^\circ]$ if it is NH, and $[180^\circ, 360^\circ]$ if it is IH.

The figure also shows that the additional information from ICAL would increase the sensitivities of these experiments in the unfavourable $\delta_{CP}$ half-plane. This corresponds to an almost two-fold increase in the range of $\delta_{CP}$ values for which CP violation can be discovered by the fixed-baseline experiments, and it happens because the hierarchy sensitivity of ICAL excludes the wrong-hierarchy minimum for the CPV discovery at 2$\sigma$. Thus it is quite possible that, though an atmospheric neutrino experiment like ICAL is not sensitive to the CP phase, the first signature of CP violation may well come by the addition of ICAL data to those of these fixed baseline experiments, possibly even a few years after these experiments have completed their runs [19, 80].

The sensitivities of the fixed baseline experiments to CPV discovery, and the relative improvement due to the addition of ICAL, have a clear dependence on $\theta_{23}$, which may be discerned from Fig. 6.4, where we compare the results with $\theta_{23} = 39^\circ$ and $\theta_{23} = 45^\circ$. In the favourable $\delta_{CP}$ region, the CPV discovery potential worsens with increasing $\theta_{23}$. This is because the $\delta_{CP}$-independent leading term in Eq. (B.9) increases with $\theta_{23}$, giving a higher statistical error, while the CP-dependent term has only a weak dependence on this parameter [78]. In the unfavourable region, on the other hand, the CPV discovery potential improves with increasing $\theta_{23}$. This happens because here the minimum of $\chi^2$ from the long baseline
Figure 6.4: CPV discovery vs true $\delta_{CP}$ for NO$\nu$A +T2K and NO$\nu$A +T2K+ICAL, for $\theta_{23}$ (true) = 39° (upper row) and 45° (lower row). The left and right panels correspond to normal and inverted true hierarchies, respectively. We use $\sin^22\theta_{13} = 0.1$, a prior of $\sigma_{\theta_{13}} = 0.005$, and a 500 kt-yr exposure for the ICAL [123].

experiments comes with the wrong hierarchy, and the atmospheric neutrino data is needed to bring it to the correct hierarchy. The hierarchy identification capability of atmospheric neutrino data increases for larger $\theta_{23}$, hence the improvement in CPV discovery potential with higher $\theta_{23}$, as can be seen in Fig. 6.4. For $\theta_{23} \gtrsim 50^\circ$, the ICAL information is nearly superfluous, since the hierarchy sensitivity of the T2K and NO$\nu$A combination itself is good enough to exclude the wrong hierarchy CPV discovery minimum even for unfavourable $\delta_{CP}$ values.

In this section we have shown that for unfavourable values of $\delta_{CP}$, atmospheric neutrino data from ICAL considerably improves the CPV discovery potential of T2K and NO$\nu$A, and could lead to a significant CPV discovery using existing and upcoming facilities for a large fraction ($\gtrsim 50\%$) of $\delta_{CP}$ values. Adding ICAL muon data to T2K and NO$\nu$A results in an enhanced CPV discovery potential at 2$\sigma$ for almost twice the range of $\delta_{CP}$ values compared to the fixed baseline experiments alone. For maximal CPV the significance of the signal can reach 3$\sigma$ in the unfavourable half-plane also. Indeed, if nature has chosen such unfavourable combinations of parameters then the addition of ICAL to T2K+NO$\nu$A may give us the first signal of leptonic CP violation.
Chapter 7
Exploring New Physics at ICAL

The end of all our exploring will be to arrive where we started
And know the place for the first time.
– T.S. Eliot

In addition to determining the parameters describing neutrino masses and mixing, the neutrino detection at ICAL may be used for probing various sources of new physics that could affect neutrino oscillations. Moreover, while ICAL is primarily designed for detecting muons and hadrons produced from neutrino interactions, it can also be sensitive to more exotic particles like magnetic monopoles or dark matter particles passing through the detector. In this chapter we briefly discuss a few such ideas.

7.1 Probing Lorentz and CPT Violation

Invariance under the product of charge conjugation (C), parity (P) and time reversal (T), i.e. the CPT theorem [124, 125], is an essential feature of quantum field theories that underlie particle physics. This is a consequence of the invariance of the Lagrangian under proper Lorentz transformations. However in a Standard Model Extended (SME) Lagrangian that does not respect the Lorentz trasformation symmetry, the CPT violation (CPTV) may be manifest, which may be measurable at the neutrino oscillation experiments. [126, 127]. On the other hand, if the effects of CPTV are not observed, impressive limits on CPT and Lorentz violation may be obtained.

7.1.1 CPTV effects at the probability level

Lorentz violation may be introduced in the effective Lagrangian for a single fermion field as [128]

\[ \mathcal{L} = i\bar{\psi} \gamma^\mu \partial_\mu \psi - m\bar{\psi} \psi - A_\mu \bar{\psi} \gamma^\mu \psi - B_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi , \]  

where \( A_\mu \) and \( B_\mu \) are real numbers. The terms containing \( A_\mu \) and \( B_\mu \) may be induced by new physics at higher energies, for instance. Since \( A_\mu \) and \( B_\mu \) are invariant under boosts and rotations, they explicitly give rise to Lorentz violation, which in turn leads to CPTV [129]. (CPT violation may also occur if particle and antiparticle masses are different. Such
violation, however, also breaks the locality assumption of quantum field theories \[129\]. This mode of CPTV is not considered here.) The effective CPTV contribution to the neutrino Lagrangian can then be parametrized \[130\] as

$$L_{\nu}^{\text{CPTV}} = \bar{\nu}_L^\alpha \gamma_\mu T_\mu^\alpha \nu_L^\beta,$$

where \( T_\mu^\alpha \) are four Hermitian \( 3 \times 3 \) matrices corresponding to the four Dirac indices \( \mu \), while \( \alpha, \beta \) are flavor indices. The effective Hamiltonian in vacuum for ultra-relativistic neutrinos with definite momentum \( p \) then becomes

$$H \equiv \frac{M M^\dagger}{2p} + b,$$

where \( M \) is the neutrino mass matrix in the CPT conserving limit and \( b \) is the CPT violating term.

For atmospheric neutrinos that may pass through appreciable amounts of matter before reaching the detector, the effective Hamiltonian in the flavor basis, that takes the matter effects and CPTV effects into account, may be written as

$$H_f = \frac{1}{2E} U_0 D(0, \Delta m_{21}^2, \Delta m_{31}^2). U_0^\dagger + U_b D_b(0, \delta b_{21}, \delta b_{31}). U_b^\dagger + D_m(V_e, 0, 0),$$

where \( U_0 \) and \( U_b \) are unitary matrices that diagonalize the \( M M^\dagger \) and \( b \) matrices, respectively, while \( D, D_m \) and \( D_b \) are diagonal matrices with their elements as listed in brackets. Here \( V_e = \sqrt{2} G_F N_e \) with the electron number density \( N_e \), and \( \delta b_{i1} \equiv b_i - b_1 \) for \( i = 2, 3 \), with \( b_1, b_2 \) and \( b_3 \) the eigenvalues of \( b \).

As in the standard convention, \( U_0 \) is parametrized by three mixing angles \( (\theta_{12}, \theta_{23}, \theta_{13}) \) and one phase \( \delta_{\text{CP}} \). The matrix \( U_b \) is parametrized by three mixing angles \( (\theta_{b12}, \theta_{b23}, \theta_{b13}) \) and six phases. Thus, \( H_f \) contains 6 mixing angles \( (\theta_{12}, \theta_{23}, \theta_{13}, \theta_{b12}, \theta_{b23}, \theta_{b13}) \) and seven phases.

The results will clearly depend on the mixing angles in the CPTV sector. In this section, we examine the effects of three different representative sets of mixing angles: (1) small mixing: \( (\theta_{b12} = 6^\circ, \theta_{b23} = 9^\circ, \theta_{b13} = 3^\circ) \), (2) large mixing: \( (\theta_{b12} = 38^\circ, \theta_{b23} = 45^\circ, \theta_{b13} = 30^\circ) \) and (3) identical to the mixing angles in the PMNS matrix: \( (\theta_{b12} = \theta_{12}, \theta_{b23} = \theta_{23}, \theta_{b13} = \theta_{13}) \).

For simplicity, all seven phases have been taken to be zero, and the neutrino oscillation parameters as given in Table 1.1 are used. It is observed that in the probability expressions, \( \delta b_{21} \) always appears with the smaller (by a factor of 30) mass squared difference \( \Delta m_{21}^2 \). Thus its effects on oscillations are subdominant, limiting the capability of atmospheric neutrinos to constrain \( b \), so that no useful constraints on \( \delta b_{21} \) seem to be possible.

For illustration, the oscillograms for the difference of the survival probability of \( \nu_\mu \) with and without CPTV for \( \delta b_{31} = 3 \times 10^{-23} \) GeV are shown in Fig. 7.1. Several general features may be observed. First, the CPTV effects are larger at larger baselines for all energies. This is as expected from the results of the the two-flavour analysis \[88\], which showed that the survival probability difference in vacuum is proportional to \( \sin(\frac{\Delta m_{21}^2 L}{2E}) \sin(\delta b L) \). Second, as is well-known, matter effects are large and resonant for neutrinos and NH, and for antineutrinos with IH. Thus in both these cases, they mask the (smaller) effect of CPTV stemming from \( U_b \). Hence for neutrino events, the CPTV sensitivity is significantly higher if the hierarchy is inverted as opposed to normal, and the converse is true for antineutrino events. Finally,
Figure 7.1: The oscillograms of $\Delta P = (P_{\nu_\mu}^{U_b \neq 0} - P_{\nu_\mu}^{U_b = 0})$ for 3 different mixing cases as described in the text. The left and right panels are for Normal and Inverted hierarchy, respectively. The value of $\delta b_{31} = 3 \times 10^{-23}$ GeV has been taken for illustration [15].
the CPTV effects are largest for the cases 2 and 3, as compared to case 1. This is due to the fact that mixing in case 1 is very small compared to other two. The origin of the difference for the cases 2 and 3 is likely due to the fact that CPT violating effects are smaller when $\theta_{b13}$ is large \cite{15}.

### 7.1.2 Simulation procedure and results

The oscillation probabilities are calculated with the true values of oscillation parameters corresponding to Table 1.1 and assuming no CPTV. The re-weighting algorithm \cite{11} has been used to generate oscillated events. The energy resolutions, efficiencies, and charge misidentification errors are taken into account. Oscillated muon events are binned as a function of muon energy (10 bins) and muon zenith angle (40 bins). These binned data are folded with detector efficiencies and resolution functions. These data (labelled as $N^{ex}$) are then fitted with another set of data (labelled as $N^{th}$), where CPTV is allowed. The statistical significance of the difference between these two sets of data is calculated, using the pull method to include systematic errors as in the analyses in Chapter 5. The $\chi^2$ is calculated separately for the $\mu^+$ and $\mu^-$ events and then added, to exploit the charge identification capability of ICAL. Marginalization has been carried out with respect to the neutrino oscillation parameters $\Delta m^2_{31}$, $\theta_{23}$, $\theta_{13}$, which are varied over their 3$\sigma$ allowed ranges. The CP violating phase $\delta_{CP}$ is varied over its whole range, while $\delta_{b21}$ is marginalized over the range 0 to $5 \times 10^{-23}$ GeV.

![Figure 7.2](image)

**Figure 7.2:** The bounds on $\delta_{b31}$ for different cases for the mixing angles $\theta_b$. The results are marginalized over $\theta_{23}$, $\theta_{13}$, $\delta_{CP}$, $\Delta m^2_{31}$ and $\delta_{b21}$. Left and right panel are for Normal and Inverted hierarchy respectively \cite{15}.

Figure 7.2 illustrates the results of the analysis. It is observed that the best bounds on $\delta_{b31}$ are obtained for both hierarchies in case 3, where mixing in the CPTV sector is the same as the neutrino mixing. Good bounds are also obtainable if the mixing in the CPTV sector is large, as in case 2. While the analysis for all mixing angles of $U_b$ has not been carried out, these results indicate that, as long as the mixing angles are not too small, limits
on $\delta b_{31}$ of the order of $4 \times 10^{-23}$ GeV would be possible at 99% C.L.. It should be noted that the above analysis assumes that the mass hierarchy is known. Indeed, if the marginalization over hierarchy is carried out, the results are considerably weaker.

Note that this study pertains to the type of CPTV that may be parametrized by Eq. 7.1, which stems from explicit Lorentz violation, and the analysis is restricted to the muon detection channel. The CPTV that gives rise to differing masses for particles and antiparticles has not been considered. Given the smaller statistics and flux uncertainties that typify atmospheric data, it would be difficult to obtain good sensitivities to this type of CPTV, which breaks the locality assumption of quantum field theories. Finally, we note that in order to obtain good sensitivity to CPTV, knowing the hierarchy will be an important asset. This will anyway be the focus of many other analyses at ICAL.

### 7.2 Search for Magnetic Monopoles

The possible existence of magnetic monopoles (MM) is predicted by unification theories. These monopoles can have their magnetic charge $g$ quantized via $eg = n\hbar c/2$, where $n$ can take positive or negative integer values [131, 132]. MM solutions of the classical equations of motion for spontaneously broken non-abelian gauge theories [133, 134] lead to a lower bound on the mass of the monopole: $M_{\text{MM}} \geq M_X/G$, where $M_X$ is the mass of the carrier of the unified interaction, and $G$ is the unified coupling constant. The MMs are therefore expected to have large masses. They are expected to be created during the big-bang, and being heavy, their relative speed with respect to the Earth should be of the same order of magnitude as the speed of the galaxy, i.e. $\sim 10^{-3}c$. The MMs that enter the Earth lose their energy gradually by electromagnetic interactions with the surrounding medium [135]. If their mass is small, so will be their kinetic energy and they will be stopped in the Earth matter. The heavier MM’s can however penetrate large distances, and can reach deep inside the Earth, like in the ICAL cavern.

Monopoles would be accelerated in the magnetic field of the galaxy, and hence energy would be drained from the galactic magnetic field. Since the rate of this energy loss should be small over the time scale of regeneration of the galactic magnetic field, an upper bound on the MM flux can be obtained [136]. This bound on the MM flux, called the Parker limit, is about $10^{-15}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ for $M_{\text{MM}} \leq 10^{17}$ GeV. For higher masses, the bound increases linearly. The detectors used for detecting MM are mainly based either on the principle of induction or excitation / ionization. The induction method, where the induced magnetic field is measured using a Superconducting QUantum Interferometer Device (SQUID) [137, 138], has yielded the upper bound on the MM flux to be $\sim 3.8 \times 10^{-13}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$. The ionization method has been used in Cherenkov detectors [139, 140], as well as in scintillators and gaseous detectors [141, 142, 143]. The bounds from these experiments are much tighter – for example, the current best bound for $10^{10}$ GeV $< M_{\text{MM}} < 10^{16}$ GeV is $\sim 2.8 \times 10^{-16}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$, from the MACRO experiment [142]. MMs have also been looked for in accelerator-based experiments [144, 145, 146].

When MMs pass through ICAL, the ionization produced in the RPC detectors can be detected as a pulse which carries the information that there has been a hit, and the time of the hit. Since the momentum of the MMs would be large, they will transfer only a small
fraction of it to the detector, and will travel through the detector in a straight line, almost unaffected. The large surface area of ICAL, combined with the large number of layers the MMs would be able to pass through, makes the MM detection possible. The track of the MM in the ICAL and the characteristic sequence of trigger times of consecutive layers of RPCs will help in identifying the MM against the background. Here we focus on MMs with masses $10^7-10^{17}$ GeV and speeds $10^{-3}c-0.7c$. (The results are presented in terms of $\beta \equiv v/c$ of the particle.) In this parameter space, if the MM flux is near the Parker limit, a few events per year may be expected at ICAL. On the other hand, the non-observation of such events would allow ICAL to put strong bounds on the incoming MM flux.

7.2.1 Monopole simulation for ICAL

The ICAL detector response for the monopole events is simulated using GEANT4, wherein the ICAL detector geometry is defined. A rock mass of density 2.89 g/cc of height 1.3 km from the top surface of the detector is considered in addition to the ICAL itself. Particles are incident on the surface of the rock, and pass through it before being detected in ICAL. An isotropic flux of downward-going MM’s is taken into account by smearing it over the zenith angle ($\cos \theta$) from 0 to $\pi/2$, and uniformly over the $2\pi$ range of the azimuthal angle $\phi$.

The stopping power in the rock of the earth and the iron of the ICAL are taken care of using

$$\frac{-dE}{dX} = \frac{4\pi N_e^2 g^2}{m_e c^2} \left[ \ln \left( \frac{2m_e c^2 \beta^2 \gamma^2}{I_m} \right) - \frac{1}{2} - \frac{\delta_m}{2} - B(\|g\|) + \frac{K(\|g\|)}{2} \right]. \quad (7.5)$$

Particles also lose energy in the active region of the detector which has a gas mixture [R134A (95.15%), Iso-Butane (4.15%) and $SF_6$ (0.34%)], and register hits, whose position and time information is recorded. This allows the reconstruction of the velocity of the particle.

We use the time-of-flight method for identifying the MM using ICAL. For relativistic monopoles, the high energy muons will constitute the main background. We avoid this background by focussing on the events with $\beta < 0.8$. Muons with such a low $\beta$ value are expected to travel less than 5 layers before stopping. In the smaller velocity region, the background will be due to chance coincidences. This can also be minimised by choosing only those events which cross a certain minimum number of layers: if the noise rate per pickup strip is $\mathcal{R}$ and the speed of the particle is $\beta c$, the probability of getting random hits in $n$ consecutive layers in the relevant time window is $n R (nd/\beta c)^{n-1}$, where $d$ is the distance between two layers. Thus for $R \sim 200$ Hz and $\beta = 0.1$, for example, each additional layer would decrease the probability of chance coincidence by $\sim 10^{-5}$. We require the particle to cross at least 10 RPC layers, which would suppress both the high-$\beta$ and low-$\beta$ backgrounds to negligible levels. For each mass and $\beta$, we use a sample of 10,000 events to estimate the detector efficiency. The result is shown in the left panel of Fig. 7.3. It may be observed that the efficiency is almost 90% for $M_{MM} > 10^{12}$ GeV and $10^{-3} < \beta < 0.1$.

7.2.2 Reach for limits on the monopole flux

The event rate expected at ICAL may be estimated by taking the area $A$ of the top surface as the effective area, and the solid angle in which the MM would cross the cut of minimum
Figure 7.3: The left panel shows the efficiency of the detector for MM events, with cuts on the maximum value of $\beta$ and the minimum number of layers to be traversed by the particle. The right panel indicates the 90% upper bounds on the MM flux (in units of $10^{-15} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$) expected from 10 year running of the 50 kt ICAL, if the number of candidate events is zero.

number of layers as the effective solid angle $\Omega$. If the MM flux is $f$ and the detector efficiency is $\epsilon$, the expected number of events $N_{\text{ex}}$ after a running time $T$ of the detector would be

$$N_{\text{ex}} = f (\text{cm}^{-2} \text{sr}^{-1} \text{s}^{-1}) A (\text{cm}^2) \Omega (\text{sr}) T (\text{s}) \epsilon.$$  

(7.6)

If the total observed number of events $N_{\text{obs}}$ is not significantly greater than the expected number of background events $N_{\text{BG}}$, then an upper bound on the MM flux may be obtained. In the right panel of Fig. 7.3 we present the upper bound that will be obtained at the 50 kt ICAL in 10 years, in the scenario $N_{\text{obs}} = N_{\text{BG}} = 0$, using the frequentist method. This is equivalent to by putting $N_{\text{ex}} < 2.3$ for a 90% confidence upper bound. From the figure, it may be observed that an upper bound of $\sim 2 \times 10^{-16} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$ (90% C.L.) should be possible with an exposure of 500 kt-yr of the ICAL detector.
Chapter 8
Concluding Remarks

This report presents the first definitive simulations study of the physics capabilities and potential of the ICAL detector. This includes the response of ICAL to the muons and hadrons produced in the $\nu_\mu$ interactions in the detector, and an understanding of the physics reach of the experiment for the identification of neutrino mass hierarchy and precision measurements of the atmospheric mixing parameters.

The ICAL detector geometry, and its elements such as iron plates, RPCs, the magnetic field, etc. are coded in the GEANT4 simulations framework. At this stage the atmospheric neutrino fluxes used are those at the Kamioka site, however the fluxes at the INO site will soon be incorporated. The atmospheric muon neutrino-induced events are generated using the NUANCE neutrino generator.

The ICAL detector is sensitive to muons in the GeV range. The muon momenta are reconstructed using a Kalman filter algorithm that uses the bending of muons in the magnetic field. It enables the reconstruction of muons with an efficiency of more than 80%, and the measurement of its momentum with a precision of $\sim 20\%$ ($10\%$) at 1 GeV (10 GeV). The muon charge is identified correctly with an efficiency of more than 98% for $E_\mu > 4$ GeV, while the zenith angle of the muon at the point of production can be reconstructed to within a degree.

The reconstruction of multi-GeV hadrons is a unique feature of the ICAL. Using calibration against the number of hits in the detector, the energy of a hadron shower can be reconstructed to $\sim 85\%$ ($35\%$) at 1 GeV (15 GeV). The addition of the hadron energy information enhances the reach of ICAL much above that from the muon information alone. The optimal analysis method to obtain the best sensitivity seems to be the analysis that uses these three quantities in each event, without trying to reconstruct the neutrino energy or direction itself.

The simulations studies have included various systematic uncertainties from neutrino fluxes and cross sections. However some detector systematics, like the background contributions from mis-identified neutral current events, have not yet been included in the analyses. The studies show that with 10 years run of the 50 kt ICAL, the mass hierarchy may be identified with a significance of $\chi^2 \geq 9$, i.e. more than $3\sigma$. At the same time, the values of the atmospheric mixing parameters $|\Delta m^2_{32}| (\sin^2 \theta_{23})$ may be determined to a precision of $3\%$ ($12\%$). The identification of the octant of $\theta_{23}$ with the ICAL alone is limited to a $2\sigma$ significance even for favourable ranges of the parameter.
Detection of the atmospheric neutrinos with ICAL is rather insensitive to the value of $\delta_{CP}$. This means that its reach for the mass hierarchy identification does not depend on the actual value of $\delta_{CP}$, unlike the ongoing fixed baseline experiments. Since it provides a data set that is free of the ambiguities due to the unknown $\delta_{CP}$, the addition of the ICAL data to the long baseline data of the ongoing experiments would enhance the $\delta_{CP}$ reach of these experiments. This synergy of the ICAL with other concurrent experiments should be exploited.

Given the multipurpose nature of ICAL, it can also be used for exploring exotic physics scenarios like CPT violation, magnetic monopoles, dark matter, etc., and more such avenues of using the ICAL data are sure to be found in years to come. The simulation studies presented in this report make a strong physics case for the ICAL detector. The codes and algorithms used in the analyses will be improved and fine-tuned to make them more realistic and efficient. This is expected to be a continually ongoing effort.

In parallel with the simulations described in this Report, efforts have gone on to finalise the design and structure of the ICAL detector modules, including the magnet, the RPCs, and their associated electronics. A prototype 1:8 scale model is planned to be built in Madurai, India, where most of the design will be validated. In the meanwhile, many more exciting and novel ideas are in the pipeline, and would be presented in future versions of this report.

The Government of India has recently (Dec 2014) given the final approval for the establishment of INO, giving a big boost to the project. The INO collaboration currently has more than 20 participating institutes from India, and welcomes the participation of high energy physicists from all over the world.
Acknowledgements

The simulation and physics analysis of the ICAL detector would not have been possible without those who have set up the simulation framework for INO on multiple platforms; we would like to especially thank Pavan Kumar and P. Nagaraj, as well as the computing resource personnel in all the institutions where this work was carried out. We are grateful to Dave Casper for making the NUANCE neutrino generator software freely available, and answering a long list of questions on its use in the initial stages. We also thank Sunanda Banerjee for sharing his expertise on GEANT. We would also like to express our gratitude towards M. Honda, who has provided us the detailed theoretical predictions of atmospheric neutrino fluxes, at the SuperKamiokande site as well as at the INO site.

Such a large endaevour is not possible without direct or indirect support from many of our colleagues. While it is not possible to mention the names of everyone here, we would like to thank Debajyoti Choudhury, Anindya Datta, Anjan Joshipura, Namit Mahajan, Subhendu Mohanty, and many of our international colleagues who have worked on phenomenological aspects of atmospheric neutrino oscillations, and whose ideas have been developed into some of the physics analysis issues discussed in this report. Many students and postdocs have contributed to the development and testing of the simulation framework in the initial stages, and to the early results. We would like to make a special mention of Abhijit Bandyopadhyay, Subhendu Chakrabarty, G. K. Padmashree, Arnab Pal, Subhendu Rakshit, Vikram Rentala and Abhijit Samanta, who have worked on simulations related to atmospheric neutrino oscillations.

While this report deals with the simulations of the ICAL detector, these simulations are based on the detector development that has been taking place in parallel, and the feedback of our hardware colleagues has been crucial in the fine-tuning of these simulations. We would therefore like to thank D. N. Badodkar, M. S. Bhatia, N. S. Dalal, S. K. Thakur, S. P. Prabhakar and Suresh Kumar for the development and simulation of the magnet; Anita Behere, Nitin Chandrachoodan, S. Dhuldhaj, Satyajit Jena, S. R. Joshi, N. Krishnapurna, S. M. Lahamge, P. K. Mukhopadhyay, P. Nagaraj, B. K. Nagesh, S. Padmini, Shobha K. Rao, Anil Prabhakar, Mandar Saraf, R. S. Shastrakar, K. M. Sudheer, M. Sukhwani and S. S. Upadhyya for electronics development; Sarika Bhide, Manas Bhuyan, Suresh Kalmani, L. V. Reddy, R. R. Shinde and Piyush Verma for the detector development. We would also like to thank Manoj Kumar Jaiswal, N. Raghuram and Vaibhav Pratap Singh, students who have completed their projects related to ICAL hardware.

We would like to thank K. S. Gothe, Kripa Mahata, Vandana Nanal, Srerup Raychaudhuri and Aradhana Shrivastava for their invaluable help in running the INO Graduate Training Program (INO GTP). We gratefully acknowledge the guidance from C. V. K. Baba (de-
ceased), Pijushpani Bhattacharjee, Ramanath Cowsik, J. N. Goswami, S. K. Ghosh, H. S. Mani, V. S. Narasimham, Sudhakar Panda, Raj Pillay, Amit Roy, D. P. Roy, Probir Roy, Utpal Sarkar, C. P. Singh and Bikash Sinha from time to time.

We thank the Homi Bhabha National Institute (HBNI) for accepting the INO GTP in its ambit. We are grateful to TIFR, which has been the host institute for INO, for making its resources available for INO, including for the GTP. We would also like to acknowledge BARC for all the support it has provided over the years, and for acting as the host Constituent Institution of HBNI for the INO GTP.

Finally, we are grateful to the Department of Atomic Energy and the Department of Science and Technology, Government of India, for funding the INO project. We would also like to acknowledge the funding provided by the University Grants Commission for many of the students working on INO and ICAL, without which this report would not have been possible.
Appendix A

Neutrino Fluxes at the Kamioka and INO sites

In this section, we briefly discuss the preliminary atmospheric neutrino flux calculations corresponding to the INO site, and compare them with those for the Kamioka site. This work is based on Refs. [5] and [92]. The primary cosmic ray flux model based on AMS [150] and BESS [151, 152] data has been used, and the hadronic interactions have been implemented with DPMJET-III [153] above 32 GeV, and JAM below 32 GeV. For the propagation of cosmic rays in the atmosphere, the model NRLMSISE-00 [154], which takes into account the temperatures and densities of the atmosphere’s components and takes care of the position dependence and the time variation in a year, is used. The geomagnetic field model used is IGRF2010 [155]. The atmospheric neutrino flux is obtained using a 3-dimensional scheme below 32 GeV, and a 1-dimensional scheme above that. The flux calculated in both the schemes agree with each other at 32 GeV [5].

Figure A.1 shows the calculated atmospheric neutrino flux averaged over one year by folding over all directions and summing over all types of neutrinos ($\nu_e$, $\bar{\nu}_e$, $\nu_\mu$, $\bar{\nu}_\mu$) for the Super-Kamiokande, INO and South Pole sites. These results, obtained with the NRLMSISE-00 model, agree well with those obtained from the US-standard76 [156] model. However the fluxes with the NRLMSISE-00, as used here, have an advantage in the study of seasonal variations, which could be appreciable at the INO site.

It is observed that the total flux at INO is slightly smaller than that at Kamioka at low energies ($E \lesssim 3$ GeV), but the difference becomes small with the increase in neutrino energy. It may be noted that this is true only for the angle-integrated fluxes. Figure A.2 shows the zenith angle dependence (integrating over all azimuthal angles) of fluxes at Kamioka and at the INO site at two values of energy. It is found that at 1 GeV, there are large up-down asymmetries in the atmospheric neutrino flux. The upward going neutrino flux is larger at the INO site. This asymmetry decreases with the increase in neutrino energy and almost disappears at 10 GeV.

The results for the atmospheric neutrino fluxes as functions of the azimuthal angle $\phi$, in five zenith angle bins, are shown in Fig. A.3 for Kamioka and in Fig. A.4 for the INO site. These results are presented for $\nu_e$, $\bar{\nu}_e$, $\nu_\mu$ and $\bar{\nu}_\mu$, for the (anti)neutrino energy of 3.2 GeV. It is observed that even for such a high energy, the variation of the atmospheric neutrino flux has a complex structure. This is a result of the rigidity cutoff and muon bending in the
Figure A.1: Atmospheric neutrino flux averaged over all directions and summed over \( \nu_e + \bar{\nu}_e + \nu_\mu + \bar{\nu}_\mu \), as a function of neutrino energy for Super-Kamiokande, INO and South Pole sites. This is for the all the energy range of the calculation. The fluxes calculated with US-standard76 are also plotted in dashed line for the Kamioka and the INO sites.

geomagnetic field. At the INO site, the horizontal component of the geomagnetic field is \( \sim 40 \mu T \), larger than that at Kamioka, where it is \( \sim 30 \mu T \), so the azimuthal angle dependence is also more complex. This complex azimuthal angle dependence continues even above 10 GeV for the near horizontal directions.

The atmospheric electron neutrino flux also shows a rapid variation in the azimuth angles, but the statistical errors in the simulations of production of atmospheric neutrinos are still large, and more statistics is needed for a better understanding. Since the INO site is close to the equator, the seasonal variation is also expected, the calculation for which is also in progress. These updates are expected to be reported after the accumulation of sufficient statistics.
Figure A.2: The zenith angle dependence of atmospheric neutrino flux at $E=1.0$ GeV (Left) and $E=3.2$ GeV (Right), averaged over all azimuthal angles calculated for the Super-Kamiokande (top) and the INO (bottom) sites. Here $\theta$ is the arrival direction of the neutrino, with $\cos \theta = 1$ for vertically downward going neutrinos, and $\cos \theta = -1$ for vertically upward going neutrinos.
Figure A.3: The azimuthal angle dependence of atmospheric neutrino flux, averaged over zenith angle bins calculated for the Super-Kamiokande site for $\nu_e$ at $E$=3.2 GeV.
Figure A.4: The azimuthal angle dependence of atmospheric neutrino flux, averaged over zenith angle bins calculated for the INO site for $\nu_e$ at E=3.2 GeV.
Appendix B

Neutrino oscillation probabilities in matter

Neutrino oscillation probabilities in matter are obtained by solving the propagation equation, which may be written in the flavour basis as

\[
\frac{id|\nu_\alpha(x)\rangle}{dx} = H |\nu_\alpha(x)\rangle \quad (B.1)
\]

where \(|\nu_\alpha(x)\rangle = (\nu_e(x), \nu_\mu(x), \nu_\tau(x))^T\). \(H\) is the effective Hamiltonian, given as

\[
H = \frac{1}{2E} U \text{ diag } (0, \Delta m^2_{21}, \Delta m^2_{31}) U^\dagger + \text{ diag } (V(x), 0, 0) . \quad (B.2)
\]

Here \(E\) is the energy of the neutrino and \(\Delta m^2_{ij} = m_i^2 - m_j^2\) is the mass-squared difference between the neutrino mass eigenstates. The PMNS mixing matrix \(U\) relates the neutrino flavour eigenstates and mass eigenstates. \(V(x)\) is the matter potential arising due to the charged-current interaction of \(\nu_e\) with electrons and is given in terms of the electron density \(n_e\) by \(V(x) = \sqrt{2} G_F n_e(x)\). For antineutrinos, \(U \rightarrow U^\ast\) and \(V \rightarrow -V\). In general, for an arbitrary density profile one needs to solve the above equation numerically to obtain the probabilities. However, simplified analytic expressions can be obtained assuming constant matter density. In such cases one can diagonalize the above Hamiltonian to obtain

\[
H = \frac{1}{2E} U^m \text{ diag } (m_1^m, m_2^m, m_3^m) U^{m\dagger} \quad (B.3)
\]

where \(m_i^m\) and \(U^m\) denote the mass eigenvalues and mixing matrix in matter respectively. For a neutrino travelling a distance \(L\), the flavour conversion probability in matter of constant density has an analogous expression as in the case of vacuum [See Eq. (1.3)], and can be expressed as

\[
P_{\alpha\beta}(L) = \delta_{\alpha\beta} - 4 \sum_{j > i} \text{Re} \left( U_{\alpha i}^m U_{\beta i}^m U_{\alpha j}^m U_{\beta j}^m \right) \sin^2 \Delta_{ij}^m + 2 \sum_{j > i} \text{Im} \left( U_{\alpha i}^m U_{\beta i}^m U_{\alpha j}^m U_{\beta j}^m \right) \sin(2\Delta_{ij}^m) , \quad (B.4)
\]
where the quantity $\Delta^m_{ij}$ in presence of matter is defined as

$$\Delta^m_{ij} = \frac{1.27 \ (\Delta m^2_{ij})^m (eV^2) \ L (km)}{E (GeV)},$$

with $(\Delta m^2_{ij})^m = (m^m_i)^2 - (m^m_j)^2$ the difference between the squares of the mass eigenvalues $m^m_i$ and $m^m_j$ in matter.

To obtain tractable expressions, further assumptions need to be made. Many approximate analytic expressions for probabilities exist in the literature. However, different assumptions that lead to different approximate forms have different regimes for validity. To understand the results presented in this Report, the probability expressions obtained under the following two approximations are mostly relevant:

- the one mass scale dominance (OMSD) approximation which assumes $\Delta m^2_{21} = 0$, and
- the double expansion in terms of small parameters $\alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}$ and $\sin \theta_{13}$ [157, 158].

The condition on the neutrino energy and baseline for the validity of both approximations can be expressed as $\Delta m^2_{21} L / E \ll 1$. This translates to $L / E \ll 10^4 \ km / GeV$ for typical values of the solar mass-squared difference $\Delta m^2_{21}$, and hence to $L \leq 10^4 \ km$ for neutrinos of energy $O(\text{GeV})$. Thus these approximations are valid for most of the energy and path-length ranges considered here. The OMSD approximation is exact in $\theta_{13}$ and works better near the resonance region. Below we give the probabilities relevant for the study presented in this report in both OMSD and double expansion approximations, and discuss in which $L$ and $E$ regimes these are appropriate. Note that we give the expressions only for neutrino propagation through a constant matter density. This approximation is not applicable for neutrinos passing through the Earth’s core. However it is enough for an analytic understanding of our arguments. All our numerical calculations take the variation of earth’s density into account through the Preliminary Reference Earth Model (PREM).

### B.1 One Mass Scale Dominance approximation

In this approximation, the Hamiltonian in Eq. (B.2) can be exactly diagonalized analytically. Below we give the expressions for the muon neutrino survival probability $P(\nu_\mu \rightarrow \nu_\mu) \equiv P_{\mu\mu}$ and conversion probability of electron neutrinos to muon neutrinos $P(\nu_e \rightarrow \nu_\mu) \equiv P_{e\mu}$, which are relevant for the atmospheric neutrinos at ICAL since the detector is sensitive to muon flavour. In the OMSD approximation, these can be expressed as

$$P_{\mu\mu} = 1 - \cos^2 \theta^m_{13} \sin^2 2\theta_{23} \sin^2 \left[ 1.27(\Delta m^2_{31} + A + (\Delta m^3_{31})^m)L/2E \right]$$

- $\sin^2 \theta^m_{13} \sin^2 2\theta_{23} \sin^2 \left[ 1.27(\Delta m^2_{31} + A - (\Delta m^2_{31})^m)L/2E \right]$

- $\sin^4 \theta_{23} \sin^2 2\theta^m_{13} \sin^2 \left[ 1.27\Delta m^2_{31}L/E \right], \quad (B.5)$

and

$$P_{e\mu} = \sin^2 \theta_{23} \sin^2 2\theta^m_{13} \sin^2 \left[ 1.27(\Delta m^2_{31})^mL/E \right]. \quad (B.6)$$
Table B.1: Values of $E_{\text{res}}$ at various baselines using the line-averaged PREM [159] density $\rho_{\text{avg}}$. We have used $\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta_{13} = 0.1$.

| L (km) | $\rho_{\text{avg}}$ (g/cc) | $E_{\text{res}}$ (GeV) |
|--------|------------------|-----------------|
| 1000   | 3.00             | 9.9             |
| 3000   | 3.32             | 9.4             |
| 5000   | 3.59             | 8.7             |
| 7000   | 4.15             | 7.5             |
| 10000  | 4.76             | 6.6             |

In the above expressions, $(\Delta m^2_{31})^m$ and $\sin 2\theta_{13}^m$, the mass-squared difference and mixing angle in matter, respectively, are given by

$$
(\Delta m^2_{31})^m = \sqrt{(\Delta m^2_{31} \cos 2\theta_{13} - A)^2 + (\Delta m^2_{31} \sin 2\theta_{13})^2},
$$

$$
\sin 2\theta_{13}^m = \sin 2\theta_{13} \frac{\Delta m^2_{31}}{(\Delta m^2_{31})^m}.
$$

(B.7)

where

$$
A(\text{eV}^2) = 2EV = 2\sqrt{2}G_F n_e E = 0.76 \times 10^{-4} \rho(\text{g/cc}) E(\text{GeV}).
$$

When $A = \Delta m^2_{31} \cos 2\theta_{13}$, we see a resonance. The resonance energy is given by

$$
E_{\text{res}} = \frac{\Delta m^2_{31} \cos 2\theta_{13}}{2\sqrt{2}G_F n_e}
$$

(B.8)

In Tab. B.1 we give the average resonance energies for neutrinos travelling a given distance $L$ through the Earth, for baselines ranging from 1000–10000 km.

It is seen from the table that the resonance energy is in the range 6–10 GeV for path lengths in the range 1000–10000 km. These ranges are relevant for atmospheric neutrinos passing through earth and hence provide an excellent avenue to probe resonant earth matter effects. The importance of this can be understood by noting that the resonance condition depends on the sign of $\Delta m^2_{31}$. For $\Delta m^2_{31} > 0$ there is a matter enhancement in $\theta_{13}^m$ for neutrinos, and a matter suppression in $\theta_{13}^m$ for antineutrinos (since $A \rightarrow -A$). The situation is reversed for $\Delta m^2_{31} < 0$. Thus matter effects can differentiate between the two hierarchies and detectors with charge sensitivity (like ICAL) are very suitable for probing this.

For atmospheric neutrinos in ICAL, the most relevant probability is $P_{\mu\mu}$. The significance of the $P_{e\mu}$ channel is less than that of $P_{\mu\mu}$ for two reasons: the number of electron neutrinos produced in the atmosphere is smaller, and more importantly, the probability of their conversion to muon neutrinos is also usually smaller than $P_{\mu\mu}$, so that their contribution to the total number of events in ICAL is small. It is not completely negligible though, since the value of $\theta_{13}$ is moderately large.

Note that $P_{e\mu}$ does not attain its maximum value at $E = E_{\text{res}}$ even though $\sin 2\theta_{13}^m$ achieves its maximum value of unity at this energy, because the mass-squared difference $(\Delta m^2_{31})^m$ hits a minimum [160]. The values of $(\Delta m^2_{31})^m \sin 2\theta_{13}^m$ and $P_{e\mu}$ remain small for path-lengths of $L \lesssim 1000$ km. If $L$ is chosen suitably large so as to satisfy $(1.27\Delta m^2_{31} \sin 2\theta_{13} L/E) \geq \pi/4$, the...
Figure B.1: The oscillograms for the muon neutrino (left panel) and antineutrino (right panel) survival probabilities during their passage through earth in $E$-$\cos \theta_z$ plane. The oscillation parameters used are $\theta_{23} = 45^\circ$, $\delta_{\text{CP}} = 0$, $\Delta m^2_{31} = +2.45 \times 10^{-3} \text{eV}^2$ (NH) and $\sin^2 2\theta_{13} = 0.1$.

then $P_{\mu\mu}$ can reach values $\geq 0.25$ for $\sin^2 2\theta_{23} = 1$. One needs $L \gtrsim 6000 \text{Km}$ to satisfy the above condition. For such baselines and in the energy range 6-8 GeV, the resonant earth matter effects lead to $P_{\mu\mu}$ in matter being significantly greater than its vacuum value \[161\].

The muon neutrino survival probability is a more complicated function and can show both fall and rise above the vacuum value for longer baselines ($\sim 10000 \text{km}$). Thus the energy and angular smearing effects are more for $P_{\mu\mu}$. The maximum hierarchy sensitivity is achieved in this channel when the resonance occurs close to a vacuum peak or dip, thus maximizing the matter effects. This is because when there is resonant matter effect for one hierarchy, the probability for the other hierarchy closely follows the vacuum value. Figure B.1 shows oscillograms for muon neutrino and antineutrino survival probabilities in the case of normal hierarchy, in the plane of neutrino energy and cosine of the zenith angle $\theta_z$. The plot in the left panel shows the resonant effect in the muon neutrino probabilities in the region between $\cos \theta_z$ between -0.6 to -0.8 and energy in the range 6–8 GeV. This feature is not present in the right panel since for the normal hierarchy, the muon antineutrinos do not encounter any resonance effect. The plot also shows the enhanced oscillation features due to the effects of the Earth’s core ($\cos \theta_z$ between -0.8 and -1.0) for neutrinos. For the inverted hierarchy, the muon antineutrino survival probability will show resonance effects, whereas the neutrino probabilities will not. The ICAL detector being charge sensitive can differentiate between neutrino and antineutrino effects and hence between the two hierarchies.

Equations (B.5) and (B.6) can also help us understand the octant sensitivity of atmospheric neutrinos arising due to resonant matter effects. The leading order term in $P_{e\mu}$ in vacuum depends on $\sin^2 \theta_{23} \sin^2 2\theta_{13}$. Although this term is sensitive to the octant of $\theta_{23}$, the uncertainty in the value of $\theta_{13}$ may give rise to octant degeneracies. In matter, the $\sin^2 2\theta^m_{13}$ term gets amplified near resonance, and the combination $\sin^2 \theta_{23} \sin^2 2\theta^m_{13}$ breaks the degeneracy of the octant with $\theta_{13}$. Also, the strong octant-sensitive nature of the term $\sin^4 \theta_{23} \sin^2 2\theta^m_{13}$ near resonance can overcome the degeneracy due to the $\sin^2 2\theta_{23}$-dependent terms. Unfortunately, the muon events in ICAL get contribution from both $P_{\mu\mu}$ channel
and $P_{\mu\mu}$ channel, and the matter effect in these two channels act in opposite directions for most of the baselines. This causes a worsening in the octant sensitivity of muon events at atmospheric neutrino experiments.

The OMSD probabilities are in the limit $\Delta m_{21}^2 = 0$ and have no dependence on the CP phase. These expressions match well with the numerical probabilities obtained by solving the propagation equation in the resonance region, i.e. for the baseline range 6000–10000 km. Accelerator-based experiments like T2K and NO$\nu$A have shorter baselines and lower matter effects, and lie far from resonance. For these experiments, the dominant terms in $P_{\mu\mu}$ are insensitive to the hierarchy and octant. Consequently, the relative change in probability due to the hierarchy/octant-sensitive sub-dominant terms is small. Therefore the $P_{\mu\mu}$ oscillation channel does not contribute much to the hierarchy and octant sensitivity of T2K and NO$\nu$A. However, these experiments get their sensitivity primarily from the $\nu_{\mu} \rightarrow \nu_e$ conversion probability. For this case, the double expansion up to second order in $\alpha$ and $\sin \theta_{13}$ works better.

### B.2 Double expansion in $\alpha$ and $\sin \theta_{13}$

In accelerator experiments, high energy pions or kaons decay to give muons and muon neutrinos/antineutrinos. One can study the muon neutrino conversion probability $P(\nu_{\mu} \rightarrow \nu_e) \equiv P_{\mu e}$ in these with a detector sensitive to electron flavour. In order to study the effect due to $\Delta m_{21}^2$ and the CP phase $\delta_{CP}$ it is convenient to write down the probabilities as an expansion in terms of the two small parameters, $\alpha = \Delta m_{21}^2 / \Delta m_{31}^2$ and $\sin \theta_{13}$, to second order (i.e. terms up to $\alpha^2, \sin^2 \theta_{13}$ and $\alpha \sin \theta_{13}$ are kept) [157, 158].

$$P_{\mu e} = \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 [(1 - \hat{A})\Delta]}{(1 - \hat{A})^2}$$

$$+ \alpha \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos (\Delta + \delta_{CP}) \frac{\sin (\hat{A} \Delta) \sin [(1 - \hat{A})\Delta]}{\hat{A}}$$

$$+ \alpha^2 \sin^2 \theta_{12} \cos^2 \theta_{23} \frac{\sin^2 (\hat{A} \Delta)}{\hat{A}^2} + O(\alpha^3, \alpha^2 s_{13}, \alpha s_{13}^2, s_{13}^3). \quad (B.9)$$

The notations used in writing the probability expressions are: $\Delta \equiv \Delta m_{31}^2 L / 4E$, $s_{ij}(c_{ij}) \equiv \sin \theta_{ij}(\cos \theta_{ij})$, $\hat{A} = 2\sqrt{2}G_F n_e E / \Delta m_{31}^2$. For neutrinos, the signs of $\hat{A}$ and $\Delta$ are positive (negative) for NH (IH). The sign of $\hat{A}$ as well as $\delta_{CP}$ reverse for antineutrinos. This probability is sensitive to all the three current unknowns in neutrino physics, namely — hierarchy, octant of $\theta_{23}$ as well as $\delta_{CP}$ and is often hailed as the golden channel. However the dependences are interrelated and extraction of each of these unknowns depends on the knowledge of the others. Specially the complete lack of knowledge of $\delta_{CP}$ gives rise to the hierarchy-$\delta_{CP}$ degeneracy as well as the octant-$\delta_{CP}$ degeneracy in these experiments, through the second term in Eq. (B.9).

The above expressions reduce to the vacuum expressions for shorter baselines for which $A \to 0$. For such cases there is no hierarchy sensitivity. The hierarchy sensitivity increases with increasing baseline and is maximum in the resonance region. The resonance energy at
shorter baselines is $> 10$ GeV and therefore these experiments cannot probe resonant earth matter effects.

B.3 Probability for Reactor neutrinos

A crucial input in the analysis presented in this Report is the value of $\theta_{13}$, measured by the reactor neutrino experiments. The probability relevant for reactor neutrinos is the survival probability for electron antineutrinos $P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \equiv P_{\bar{\nu}e}$. Since reactor neutrinos have very low energy (order of MeV) and they travel very short distances (order of km), they experience negligible matter effects. The exact formula for the survival probability in vacuum is given by [162, 161]

$$P_{\bar{\nu}e} = 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2 \Delta_{21} - \sin^2 2\theta_{13} \sin^2 \Delta_{31} + \sin^2 2\theta_{13} s_{12}^2 \left[ \sin^2 \Delta_{31} - \sin^2 \Delta_{32} \right].$$

(B.10)

Since this is independent of $\delta_{CP}$ and matter effects, the probability is the same for neutrinos and antineutrinos (assuming CPT conservation, i.e. the same mass and mixing parameters describing neutrinos and antineutrinos).
Appendix C

The Vavilov distribution function

The Vavilov probability distribution function is found to be the suitable one to represent the hit distributions of hadrons of a given energy in the ICAL, as has been observed from Fig. 4.8. The Vavilov probability density function in the standard form is defined by \[ P(x; \kappa, \beta^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \phi(s)e^{xs}ds, \] (C.1)

where

\[ \phi(s) = e^C e^{\psi(s)}, \quad C = \kappa(1 + \beta^2 \gamma), \] (C.2)

and

\[ \psi(s) = s \ln \kappa + (s + \beta^2 \kappa) \left[ \int_0^1 \frac{1 - e^{-st/\kappa}}{t} dt - \gamma \right] - \kappa e^{-s/\kappa}, \] (C.3)

where \( \gamma = 0.577 \ldots \) is the Euler’s constant.

The parameters mean and variance \( (\sigma^2) \) of the distribution in Eq. (C.1) are given by

\[ \text{mean} = \gamma - 1 - \ln \kappa - \beta^2; \quad \sigma^2 = \frac{2 - \beta^2}{2\kappa}. \] (C.4)

For \( \kappa \leq 0.05 \), the Vavilov distribution may be approximated by the Landau distribution, while for \( \kappa \geq 10 \), it may be approximated by the Gaussian approximation, with the corresponding mean and variance.

We have used the Vavilov distribution function \( P(x; \kappa, \beta^2) \) defined above, which is also built into ROOT, as the basic distribution for the fit. However the hadron hit distribution itself is fitted to the modified distribution \( (P_4/P_3) \) \( P((x - P_2)/P_3; P_0, P_1) \), to account for the x-scaling \( P_3 \), normalization \( P_4 \) and the shift of the peak to a non-zero value, \( P_2 \). Clearly \( P_0 = \kappa \) and \( P_1 = \beta^2 \). The modified mean and variance are then

\[ \text{Mean}_{\text{Vavilov}} = (\gamma - 1 - \ln P_0 - P_1) P_3 + P_2, \quad \sigma^2_{\text{Vavilov}} = \frac{(2 - P_1)}{2P_0} P_3^2. \] (C.5)

These are the quantities used while presenting the energy response of hadrons in the ICAL detector.
Appendix D

Hadron energy resolution as a function of plate thickness

A potentially crucial factor in the determination of hadron energy and direction is the thickness of absorber material, namely iron plate thickness in ICAL. In all the simulation studies reported here, we have assumed that the thickness of the iron plate is 5.6 cm, which is the default value. While not much variation in this thickness is possible due to constraints imposed by total mass, physical size, location of the support structure and other parameters like the cost factor etc, we look at possible variation of this thickness in view of optimising the hadron energy resolution [10]. The hadron energy resolution is a crucial limiting factor in reconstructing the neutrino energy in atmospheric neutrino interactions in the ICAL detector. This information is also helpful since ICAL is modular in form and future modules may come in for further improvements using such analyses.

Naively, this can be achieved by simply changing the angle of the propagation of the particle in the simulation, since the effective thickness is \((t/\cos \theta)\). In the case of muons this itself may be sufficient to study the effect of plate thickness. However, in an actual detector, the detector geometry — including support structure, orientation as well as the arrangement of detector elements — imposes additional nontrivial dependence on thickness. Therefore we study hadron energy resolution with the present arrangement of ICAL by varying the plate thickness, while other parameters are fixed. The analysis was done by propagating pions in the simulated ICAL detector at various fixed energies, averaged over all directions in each case.

The hit distribution patterns for 5 GeV pions propagated through sample plate thicknesses in the central region are shown in Fig. D.1. The methodology is already discussed in Chapter 4 and we will not repeat it here. For comparing the resolutions with different thicknesses we use the mean and rms width \((\sigma)\) of the hit distributions as functions of energy.

The hadron energy resolution is parametrised as

\[
\left( \frac{\sigma}{E} \right)^2 = \frac{a^2}{E} + b^2, \tag{D.1}
\]

where \(a\) is the stochastic coefficient and \(b\) is a constant, both of which depend on the thickness. We divide the relevant energy range 2-15 GeV into two sub-ranges, below 5 GeV and above 5 GeV. Below 5 GeV, the quasi-elastic, resonance and deep inelastic processes contribute to
the production of hadrons in neutrino interactions in comparable proportions, while above 5 GeV the hadron production is dominated by the deep inelastic scattering. The results for the energy resolution as a function of plate thickness are shown in Fig. D.2. Note that here we show the square of the resolution instead of the resolution itself.

The stochastic coefficient $a$ as a function of thickness is obtained from a fit to the hadron energy resolution, and is shown in Fig. D.3 as a function of plate thickness for the two energy ranges as in Fig. D.2.

The analysis in the two energy ranges shows that the thickness dependence is stronger than $\sqrt{t}$ which is observed in hadron calorimeters at high energies (tens of GeV) [164]. In fact at the energies of relevance to us, the thickness dependence is not uniform but dependent on the energy. This is borne out by two independent analyses: in the first one we obtain the thickness dependence of the stochastic coefficient $a$ and in the second analysis we directly...
parametrisate the energy resolution as a function of thickness at each energy. Typically instead of $t^{0.5}$, we find the power varying from about 0.65–0.98 depending on the energy.

Finally we compare the ICAL simulations with varying thickness with MONOLITH and MINOS and their test beam runs. This is a useful comparison since no test beam runs with ICAL prototype have been done up to present. We have used the data with these other detectors for validation of the simulations results.

![Figure D.3](image-url)

Figure D.3: The stochastic coefficient $a$ obtained from the fit to Eq. D.1 is shown in the two energy ranges as function of plate thickness [10].

As an example we show the test beam results for 5 cm iron plates (baby MONOLITH) at CERN [165, 166]. The beam energy is in the range 2-10 GeV. These results cannot be compared with the results with smeared vertices since the beam is highly directional and perpendicular to the plate orientation. In order to provide a comparison, we have simulated the ICAL detector with 5cm iron plates with single pions of energy 2-10 GeV incident normally on the detector at a fixed vertex. A comparison of the two is shown in the Fig. D.4.

Figure D.4: The energy response of ICAL detector with 5cm thick iron plates with single pions in the energy range 2-10 GeV, propagated from a fixed vertex in the vertical direction [10] compared with the data from MONOLITH test beam run [165, 166].
For baby MONOLITH, an energy resolution of $68\%/\sqrt{E} \pm 2\%$ was reported and the fit to the ICAL simulation with 5cm plate gives a similar energy resolution of $66.3\%/\sqrt{E} \pm 8.7\%$ was reported testifying to the basic correctness of our approach.

In the test beam run of MINOS with Aluminium Proportional Tubes (APT) active detectors and 1.5 inch (4 cm) steel plates, a hadron energy resolution of $71\%/\sqrt{E} \pm 6\%$ was reported in the range 2.5-30 GeV [167]. ICAL simulation with 4cm iron plates in the same energy range gave $61\%/\sqrt{E} \pm 14\%$. The results are compatible within errors, since the two detector geometries are very different.

Obviously the final choice of the plate thickness depends not only on the behaviour of hadrons but also on the energy range of interest to the physics goals of the experiment. There are also issues of cost, sensitivity to muons and even possibly electrons. The thickness dependence study summarized here provides one such input to the final design.
Bibliography

[1] India-based Neutrino Observatory (INO), http://www.ino.tifr.res.in/ino/.

[2] INO Collaboration, M. S. Athar et al., *India-based Neutrino Observatory: Project Report. Volume I (INO-2006-01).*., 2006.

[3] S. Agostinelli *et al.* [GEANT4 Collaboration], “GEANT4: A Simulation toolkit,” Nucl. Instrum. Meth. A **506**, 250 (2003), http://geant4.cern.ch/.

[4] D. Casper, *The Nuance neutrino physics simulation, and the future*, *Nucl.Phys.Proc.Suppl.* **112** (2002) 161–170, [hep-ph/0208030](http://arxiv.org/abs/hep-ph/0208030).

[5] M. Honda, T. Kajita, K. Kasahara, and S. Midorikawa, *Improvement of low energy atmospheric neutrino flux calculation using the JAM nuclear interaction model*, *Phys.Rev.* **D83** (2011) 123001, [arXiv:1102.2688](http://arxiv.org/abs/1102.2688).

[6] H. Nunokawa, S. J. Parke, and R. Zukanovich Funchal, *Another possible way to determine the neutrino mass hierarchy*, *Phys.Rev.* **D72** (2005) 013009, [hep-ph/0503283](http://arxiv.org/abs/hep-ph/0503283).

[7] A. de Gouvea, J. Jenkins, and B. Kayser, *Neutrino mass hierarchy, vacuum oscillations, and vanishing U(e3)*, *Phys.Rev.* **D71** (2005) 113009, [hep-ph/0503079](http://arxiv.org/abs/hep-ph/0503079).

[8] A. Chatterjee, K. Meghna, R. Kanishka, T. Thakore, V. Bhatnagar, et al., *A Simulations Study of the Muon Response of the Iron Calorimeter Detector at the India-based Neutrino Observatory*, *JINST* **9** (2014) P07001, [arXiv:1405.7243](http://arxiv.org/abs/1405.7243).

[9] M. M. Devi, A. Ghosh, D. Kaur, S. M. Lakshmi, S. Choubey, et al., *Hadron energy response of the Iron Calorimeter detector at the India-based Neutrino Observatory*, *JINST* **8** (2013) P11003, [arXiv:1304.5115](http://arxiv.org/abs/1304.5115).

[10] S. M. Lakshmi, A. Ghosh, M. M. Devi, D. Kaur, S. Choubey, et al., *Hadron energy resolution as a function of iron plate thickness at ICAL*, [arXiv:1401.2779](http://arxiv.org/abs/1401.2779).

[11] A. Ghosh, T. Thakore, and S. Choubey, *Determining the Neutrino Mass Hierarchy with INO, T2K, NOvA and Reactor Experiments*, *JHEP* **1304** (2013) 009, [arXiv:1212.1305](http://arxiv.org/abs/1212.1305).

[12] T. Thakore, A. Ghosh, S. Choubey, and A. Dighe, *The Reach of INO for Atmospheric Neutrino Oscillation Parameters*, *JHEP* **1305** (2013) 058, [arXiv:1303.2534](http://arxiv.org/abs/1303.2534).
[13] D. Kaur, M. Naimuddin, and S. Kumar, *The sensitivity of the ICAL detector at India-based Neutrino Observatory to neutrino oscillation parameters*, Eur.Phys.J. C75 (2015), no. 4 156, [arXiv:1409.2231](https://arxiv.org/abs/1409.2231).

[14] M. M. Devi, T. Thakore, S. K. Agarwalla, and A. Dighe, *Enhancing sensitivity to neutrino parameters at INO combining muon and hadron information*, JHEP 1410 (2014) 189, [arXiv:1406.3689](https://arxiv.org/abs/1406.3689).

[15] A. Chatterjee, R. Gandhi, and J. Singh, *Probing Lorentz and CPT Violation in a Magnetized Iron Detector using Atmospheric Neutrinos*, JHEP 1406 (2014) 045, [arXiv:1402.6265](https://arxiv.org/abs/1402.6265).

[16] N. Dash, V. Datar, and G. Majumder, *Search for Magnetic Monopole using ICAL at INO*, [arXiv:1406.3938](https://arxiv.org/abs/1406.3938). To appear in Astroparticle Physics.

[17] S. K. Agarwalla and T. Thakore, “Synergy between ICAL atmospheric neutrino data and ongoing long-baseline experiments, T2K and NOvA.” Work in progress, 2015.

[18] M. Blennow and T. Schwetz, *Identifying the Neutrino mass Ordering with INO and NOvA*, JHEP 1208 (2012) 058, [arXiv:1203.3388](https://arxiv.org/abs/1203.3388).

[19] M. Ghosh, P. Ghoshal, S. Goswami, and S. K. Raut, *Can atmospheric neutrino experiments provide the first hint of leptonic CP violation?*, Phys.Rev. D89 (2014) 011301, [arXiv:1306.2500](https://arxiv.org/abs/1306.2500).

[20] C. Cowan, F. Reines, F. Harrison, H. Kruse, and A. McGuire, *Detection of the free neutrino: A Confirmation*, Science 124 (1956) 103–104.

[21] C. V. Achar, M. G. K. Menon, V. S. Narasimham, P. V. R. Murthy, B. V. Sreekantan, et al., *Detection of muons produced by cosmic ray neutrinos deep underground*, Phys.Lett. 18 (1965) 196–199.

[22] F. Reines, M. Crouch, T. Jenkins, W. Kropp, H. Gurr, et al., *Evidence for high-energy cosmic ray neutrino interactions*, Phys.Rev.Lett. 15 (1965) 429–433.

[23] J. Davis, Raymond, D. S. Harmer, and K. C. Hoffman, *Search for neutrinos from the sun*, Phys.Rev.Lett. 20 (1968) 1205–1209.

[24] B. Cleveland, T. Daily, J. Davis, Raymond, J. R. Distel, K. Lande, et al., *Measurement of the solar electron neutrino flux with the Homestake chlorine detector*, Astrophys.J. 496 (1998) 505–526.

[25] Kamiokande-II Collaboration, K. Hirata et al., *Real time, directional measurement of B-8 solar neutrinos in the Kamiokande-II detector*, Phys.Rev. D44 (1991) 2241.

[26] Super-Kamiokande Collaboration, Y. Fukuda et al., *Measurements of the solar neutrino flux from Super-Kamiokande’s first 300 days*, Phys.Rev.Lett. 81 (1998) 1158–1162, [hep-ex/9805021](https://arxiv.org/abs/hep-ex/9805021).
[27] **Super-Kamiokande** Collaboration, Y. Fukuda et al., *Evidence for oscillation of atmospheric neutrinos*, Phys.Rev.Lett. 81 (1998) 1562–1567, [hep-ex/9807003].

[28] **SAGE** Collaboration, J. Abdurashitov et al., *Measurement of the solar neutrino capture rate with gallium metal*, Phys.Rev. C60 (1999) 055801, [astro-ph/9907113].

[29] **GALLEX** Collaboration, P. Anselmann et al., *Solar neutrinos observed by gallex at gran sasso.*, Phys. Lett. B285 (1992) 376–389.

[30] **GALLEX, GNO** Collaboration, T. Kirsten, *GALLEX solar neutrino results and status of GNO*, Nucl.Phys.Proc.Suppl. 77 (1999) 26–34.

[31] **SNO** Collaboration, Q. Ahmad et al., *Measurement of the rate of $\nu_e + d \rightarrow p + p + e^-$ interactions produced by $^8B$ solar neutrinos at the Sudbury Neutrino Observatory*, Phys.Rev.Lett. 87 (2001) 071301, [nucl-ex/0106015].

[32] **SNO** Collaboration, Q. Ahmad et al., *Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory*, Phys.Rev.Lett. 89 (2002) 011301, [nucl-ex/0204008].

[33] **KamLAND** Collaboration, K. Eguchi et al., *First results from KamLAND: Evidence for reactor anti-neutrino disappearance*, Phys.Rev.Lett. 90 (2003) 021802, [hep-ex/0212021].

[34] **K2K** Collaboration, M. Ahn et al., *Indications of neutrino oscillation in a 250 km long baseline experiment*, Phys.Rev.Lett. 90 (2003) 041801, [hep-ex/0212007].

[35] **Double Chooz** Collaboration, Y. Abe et al., *Indication for the disappearance of reactor electron antineutrinos in the Double Chooz experiment*, Phys.Rev.Lett. 108 (2012) 131801, [arXiv:1112.6353].

[36] **RENO** Collaboration, J. Ahn et al., *Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment*, Phys.Rev.Lett. 108 (2012) 191802, [arXiv:1204.0626].

[37] **Daya Bay** Collaboration, F. An et al., *Observation of electron-antineutrino disappearance at Daya Bay*, Phys.Rev.Lett. 108 (2012) 171803, [arXiv:1203.1669].

[38] **MINOS** Collaboration, D. Michael et al., *Observation of muon neutrino disappearance with the minos detectors and the numi neutrino beam*, Phys. Rev. Lett. 97 (2006) 191801, [hep-ex/0607088].

[39] **T2K** Collaboration, K. Abe et al., *Indication of Electron Neutrino Appearance from an Accelerator-produced Off-axis Muon Neutrino Beam*, Phys.Rev.Lett. 107 (2011) 041801, [arXiv:1106.2822].

[40] M. Krishnaswamy, M. Menon, V. Narasimham, N. Ito, S. Kawakami, et al., *Evidence for the Production of a New Particle in Neutrino Interactions*, Phys.Lett. B57 (1975) 105.
[41] M. Krishnaswamy, M. Menon, V. Narasimham, N. Ito, S. Kawakami, et al., Evidence for the Production of New Particles in Cosmic Ray Experiments Deep Underground, Pramana 5 (1975) 59–77.

[42] M.R. Krishnaswamy et al, Proc. XXIII Int.Conf. on High Energy Physics, Berkeley(ed.) S Loken (World Scientific, 1986).

[43] M. Murthy, U. Yajnik, K. Balaji, G. Bhattacharyya, A. Dighe, et al., Discussion on a possible neutrino detector located in India, Pramana 55 (2000) 347–355, [hep-ph/0112076].

[44] MONOLITH Collaboration, T. Tabarelli de Fatis, MONOLITH: A High resolution neutrino oscillation experiment, [hep-ph/0106252].

[45] MONOLITH Collaboration, N. Agafonova et al., MONOLITH: A massive magnetized iron detector for neutrino oscillation studies.

[46] S. Behera, M. Bhatia, V. Datar, and A. Mohanty, Simulation Studies for Electromagnetic Design of INO ICAL Magnet and its Response to Muons, [arXiv:1406.3965].

[47] B. Pontecorvo, Mesonium and anti-mesonium, Sov.Phys.JETP 6 (1957) 429.

[48] Z. Maki, M. Nakagawa, and S. Sakata, Remarks on the unified model of elementary particles, Prog.Theor.Phys. 28 (1962) 870–880.

[49] Particle Data Group Collaboration, K. Olive et al., Review of Particle Physics, Chin.Phys. C38 (2014) 090001.

[50] M. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, Updated fit to three neutrino mixing: status of leptonic CP violation, JHEP 1411 (2014) 052, [arXiv:1409.5439].

[51] NuFIT webpage, http://www.nu-fit.org/.

[52] M. Gonzalez-Garcia, M. Maltoni, J. Salvado, and T. Schwetz, Global fit to three neutrino mixing: critical look at present precision, JHEP 1212 (2012) 123, [arXiv:1209.3023].

[53] F. Capozzi, G. Fogli, E. Lisi, A. Marrone, D. Montanino, et al., Status of three-neutrino oscillation parameters, circa 2013, Phys.Rev. D89 (2014) 093018, [arXiv:1312.2878].

[54] D. Forero, M. Tortola, and J. Valle, Neutrino oscillations refitted, [arXiv:1405.7540]

[55] MINOS Collaboration, P. Adamson et al., Measurement of Neutrino and Antineutrino Oscillations Using Beam and Atmospheric Data in MINOS, Phys. Rev. Lett. 110, 251801 (2013) [arXiv:1304.6335].

[56] G. L. Fogli and E. Lisi, Tests of three-flavor mixing in long-baseline neutrino oscillation experiments, Phys. Rev. D54 (1996) 3667–3670, [hep-ph/9604415].
[57] **CHOOZ** Collaboration, M. Apollonio et al., *Initial results from the CHOOZ long baseline reactor neutrino oscillation experiment*, Phys.Lett. **B420** (1998) 397–404, [hep-ex/9711002].

[58] M. Narayan, G. Rajasekaran, and S. U. Sankar, *Three flavor implications of CHOOZ result*, Phys.Rev. **D58** (1998) 031301, [hep-ph/9712409].

[59] **CHOOZ** Collaboration, M. Apollonio et al., *Limits on neutrino oscillations from the CHOOZ experiment*, Phys.Lett. **B466** (1999) 415–430, [hep-ex/9907037].

[60] **Palo Verde** Collaboration, A. Piepke, *Final results from the Palo Verde neutrino oscillation experiment*, Prog.Part.Nucl.Phys. **48** (2002) 113–121.

[61] **T2K** Collaboration, K. Abe et al., *Observation of Electron Neutrino Appearance in a Muon Neutrino Beam*, Phys.Rev.Lett. **112** (2014) 061802, [arXiv:1311.4750].

[62] **Daya Bay** Collaboration, F. An et al., *Spectral measurement of electron antineutrino oscillation amplitude and frequency at Daya Bay*, Phys.Rev.Lett. **112** (2014) 061801, [arXiv:1310.6732].

[63] K. Abe, T. Abe, H. Aihara, Y. Fukuda, Y. Hayato, et al., *Letter of Intent: The Hyper-Kamiokande Experiment — Detector Design and Physics Potential —*, [arXiv:1109.3262].

[64] **IceCube-PINGU** Collaboration, M. Aartsen et al., *Letter of Intent: The Precision IceCube Next Generation Upgrade (PINGU)*, [arXiv:1401.2046].

[65] D. Ayres, G. Drake, M. Goodman, V. Guarino, T. Joffe-Minor, et al., *Letter of Intent to build an Off-axis Detector to study nu(mu) to nu(e) oscillations with the NuMI Neutrino Beam*, [hep-ex/0210005].

[66] **NOvA** Collaboration, D. Ayres et al., *NOvA: Proposal to build a 30 kiloton off-axis detector to study nu(mu) to nu(e) oscillations in the NuMI beamline*, [hep-ex/0503053].

[67] **NOvA** Collaboration, D. Ayres et al., *The NOvA Technical Design Report*, tech. rep., FERMILAB-DESIGN-2007-01.

[68] **LBNE** Collaboration, T. Akiri et al., *The 2010 Interim Report of the Long-Baseline Neutrino Experiment Collaboration Physics Working Groups*, [arXiv:1110.6249].

[69] **LBNE** Collaboration, C. Adams et al., *Scientific Opportunities with the Long-Baseline Neutrino Experiment*, [arXiv:1307.7335].

[70] S. K. Agarwalla, T. Li, and A. Rubbia, *An Incremental approach to unravel the neutrino mass hierarchy and CP violation with a long-baseline Superbeam for large $\theta_{13}$*, JHEP **1205** (2012) 154, [arXiv:1109.6526].
[71] A. Stahl, C. Wiebusch, A. Guler, M. Kamiscioglu, R. Sever, et al., Expression of Interest for a very long baseline neutrino oscillation experiment (LBNO), CERN-SPSC-2012-021, SPSC-EOI-007.

[72] LAGUNA-LBNO Collaboration, S. Agarwalla et al., The mass-hierarchy and CP-violation discovery reach of the LBNO long-baseline neutrino experiment, arXiv:1312.6520.

[73] Y.-F. Li, J. Cao, Y. Wang, and L. Zhan, Unambiguous Determination of the Neutrino Mass Hierarchy Using Reactor Neutrinos, Phys.Rev. D88 (2013) 013008, arXiv:1303.6733.

[74] Y.-F. Li, Overview of the Jiangmen Underground Neutrino Observatory (JUNO), arXiv:1402.6143.

[75] RENO-50 collaboration in International Workshop on RENO-50 toward Neutrino Mass Hierarchy, 2013 http://home.kias.re.kr/MKG/h/reno50/.

[76] T2K Collaboration, K. Abe et al., Neutrino Oscillation Physics Potential of the T2K Experiment, arXiv:1409.7469.

[77] S. K. Agarwalla, S. Prakash, S. K. Raut, and S. U. Sankar, Potential of optimized NOvA for large $\theta_{13}$ and combined performance with a LArTPC and T2K, JHEP 1212 (2012) 075, arXiv:1208.3644.

[78] P. Huber, M. Lindner, T. Schwetz, and W. Winter, First hint for CP violation in neutrino oscillations from upcoming superbeam and reactor experiments, JHEP 0911 (2009) 044, arXiv:0907.1896.

[79] P. Machado, H. Minakata, H. Nunokawa, and R. Z. Funchal, What can we learn about the lepton CP phase in the next 10 years?, arXiv:1307.3248.

[80] M. Ghosh, P. Ghoshal, S. Goswami, and S. K. Raut, Evidence for leptonic CP phase from NOvA, T2K and ICAL: A chronological progression, Nucl.Phys. B884 (2014) 274–304, arXiv:1401.7243.

[81] S. K. Agarwalla, S. Prakash, and S. Uma Sankar, Resolving the octant of $\theta_{23}$ with T2K and NOvA, JHEP 1307 (2013) 131, arXiv:1301.2574.

[82] S. K. Agarwalla, S. Prakash, and S. Uma Sankar, Exploring the three flavor effects with future superbeams using liquid argon detectors, JHEP 1403 (2014) 087, arXiv:1304.3251.

[83] E. Kearns, Physics with Massive Water Cherenkov Detectors, 2013. Talk given at the NNN 2013 Workshop, November 11-13, 2013, Kavli IPMU, Japan, http://indico.ipmu.jp/indico/conferenceDisplay.py?confId=17.

[84] Super-Kamiokande Collaboration Collaboration, M. Ishitsuka, Super-Kamiokande results: Atmospheric and solar neutrinos, hep-ex/0406076.
[85] R. Gandhi and S. Panda, *Probing the cosmic ray 'Knee' and very high energy prompt muon and neutrino fluxes via underground muons*, JCAP 0607 (2006) 011, [hep-ph/0512179].

[86] S. Panda and S. Sinegovsky, *High-energy atmospheric muon flux expected at India-Based neutrino observatory*, Int. J. Mod. Phys. A23 (2008) 2933–2942, [arXiv:0710.3125].

[87] A. S. Joshipura and S. Mohanty, *Constraints on flavor dependent long range forces from atmospheric neutrino observations at super-Kamiokande*, Phys. Lett. B584 (2004) 103–108, [hep-ph/0310210].

[88] A. Datta, R. Gandhi, P. Mehta, and S. U. Sankar, *Atmospheric neutrinos as a probe of CPT and Lorentz violation*, Phys. Lett. B597 (2004) 356–361, [hep-ph/0312027].

[89] A. Chatterjee, P. Mehta, D. Choudhury, and R. Gandhi, *Testing non-standard neutrino matter interactions in atmospheric neutrino propagation*, arXiv:1409.8472.

[90] M. V. N. Murthy and G. Rajasekaran, *Anomalous Kolar events revisited: Dark Matter?*, Pramana 82 (2014) 609–615, [arXiv:1305.2715].

[91] N. Dash, V. Datar, and G. Majumder, *Sensitivity for detection of decay of dark matter particle using ICAL at INO*, arXiv:1410.5182. To be published in Pramana.

[92] M. Honda, T. Kajita, K. Kasahara, S. Midorikawa, and T. Sanuki, *Calculation of atmospheric neutrino flux using the interaction model calibrated with atmospheric muon data*, Phys. Rev. D75 (2007) 043006, [astro-ph/0611418].

[93] G. Barr, T. Gaisser, P. Lipari, S. Robbins, and T. Stanev, *A Three - dimensional calculation of atmospheric neutrinos*, Phys. Rev. D70 (2004) 023006, astro-ph/0403630.

[94] G. Battistoni, A. Ferrari, T. Montaruli, and P. Sala, *The FLUKA atmospheric neutrino flux calculation*, Astropart. Phys. 19 (2003) 269–290, [hep-ph/0207035].

[95] T. Gaisser and M. Honda, *Flux of atmospheric neutrinos*, Ann. Rev. Nucl. Part. Sci. 52 (2002) 153–199, [hep-ph/0203272].

[96] M. Honda, M. Sajjad Athar, T. Kajita, K. Kasahara, and S. Midorikawa, “Atmospheric neutrino flux calculation using MSISIE00 atmosphere model.” To be submitted to Phys. Rev. D, 2015.

[97] M. Sajjad Athar, M. Honda, T. Kajita, K. Kasahara, and S. Midorikawa, *Atmospheric neutrino flux at INO, South Pole and Pyhäsalmi*, Phys. Lett. B718 (2013) 1375–1380, [arXiv:1210.5154].

[98] C. Andreopoulos, A. Bell, D. Bhattacharya, F. Cavanna, J. Dobson, et al., *The GENIE Neutrino Monte Carlo Generator*, Nucl. Instrum. Meth. A614 (2010) 87–104, arXiv:0905.2517.
[99] J. Hewett, H. Weerts, R. Brock, J. Butler, B. Casey, et al., *Fundamental Physics at the Intensity Frontier*, arXiv:1205.2671.

[100] J.A. Formaggio et al., *From eV to EeV: Neutrino cross sections across energy scales*, Rev. Mod. Phys. **84**, 1307.

[101] Infolytica Corp., Electromagnetic field simulation software, http://www.infolytica.com/en/products/magnet/.

[102] B. Satyanarayana, *Design and Characterisation Studies of Resistive Plate Chambers*, PhD thesis, (Department of Physics, IIT Bombay, PHY-PHD-10-701, 2009).

[103] J.S. Marshall, *A study of muon neutrino disappearance with the MINOS detectors and the NuMI neutrino beam*, PhD. Thesis, Univ. of Cambridge (2008).

[104] E. Wolin and L. Ho, *Covariance matrices for track fitting with the Kalman filter*, Nucl.Instrum.Meth. **A329** (1993) 493–500.

[105] H. Bethe and J. Ashkin, “Experimental Nuclear Physics, Ed. E. Segré, J. Wiley, New York, 1953, p. 253.

[106] K. Bhattacharya, A. K. Pal, G. Majumder, and N. K. Mondal, *Error propagation of the track model and track fitting strategy for the Iron CALorimeter detector in India-based neutrino observatory*, Comput.Phys.Commun. **185** (2014) 3259–3268.

[107] J. Beringer et al. (Particle Data Group), Phys. Rev. D **86**, 010001 (2012) ; see also [arXiv:1204.0626v2 [hep-ex]].

[108] G. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, et al., *Solar neutrino oscillation parameters after first KamLAND results*, Phys.Rev. **D67** (2003) 073002, [hep-ph/0212127].

[109] P. Huber, M. Lindner, and W. Winter, *Superbeams versus neutrino factories*, Nucl. Phys. **B645** (2002) 3–48, [hep-ph/0204352].

[110] R. Gandhi, P. Ghoshal, S. Goswami, P. Mehta, S. U. Sankar, et al., *Mass Hierarchy Determination via future Atmospheric Neutrino Detectors*, Phys.Rev. **D76** (2007) 073012, [arXiv:0707.1723].

[111] S. Choubey and P. Roy, *Probing the deviation from maximal mixing of atmospheric neutrinos*, Phys.Rev. **D73** (2006) 013006, [hep-ph/0509197].

[112] V. Barger, R. Gandhi, P. Ghoshal, S. Goswami, D. Marfatia, et al., *Neutrino mass hierarchy and octant determination with atmospheric neutrinos*, Phys.Rev.Lett. **109** (2012) 091801, [arXiv:1203.6012].

[113] D. Indumathi, M. Murthy, G. Rajasekaran, and N. Sinha, *Neutrino oscillation probabilities: Sensitivity to parameters*, Phys.Rev. **D74** (2006) 053004, [hep-ph/0603264].
[114] **Super-Kamiokande** Collaboration, A. Himmel, *Recent Atmospheric Neutrino Results from Super-Kamiokande*, [arXiv:1310.6677](http://arxiv.org/abs/1310.6677).

[115] **MINOS** Collaboration, P. Adamson et al., *Combined analysis of $\nu_\mu$ disappearance and $\nu_\mu \rightarrow \nu_e$ appearance in MINOS using accelerator and atmospheric neutrinos*, *Phys.Rev.Lett.* (2014) [arXiv:1403.0867](http://arxiv.org/abs/1403.0867).

[116] **T2K** Collaboration, K. Abe et al., *Precise Measurement of the Neutrino Mixing Parameter $\theta_{23}$ from Muon Neutrino Disappearance in an Off-axis Beam*, [arXiv:1403.1532](http://arxiv.org/abs/1403.1532).

[117] S. Choubey and A. Ghosh, *Determining the Octant of $\theta_{23}$ with PINGU, T2K, NOvA and Reactor Data*, *JHEP* 1311 (2013) 166, [arXiv:1309.5760](http://arxiv.org/abs/1309.5760).

[118] P. Huber, M. Lindner, and W. Winter, *Simulation of long-baseline neutrino oscillation experiments with GLoBES (General Long Baseline Experiment Simulator)*, *Comput.Phys.Commun.* 167 (2005) 195, [hep-ph/0407333](http://arxiv.org/abs/hep-ph/0407333).

[119] P. Huber, J. Kopp, M. Lindner, M. Rolinec, and W. Winter, *New features in the simulation of neutrino oscillation experiments with GLoBES 3.0: General Long Baseline Experiment Simulator*, *Comput.Phys.Commun.* 177 (2007) 432–438, [hep-ph/0701187](http://arxiv.org/abs/hep-ph/0701187).

[120] S. K. Agarwalla, S. Choubey, and A. Ghosh, “Probing 2-3 Neutrino Mixing at ICAL-INO Combining T2K and NOvA.” In preparation, 2015.

[121] **T2K** Collaboration, Y. Itow et al., *The JHF-Kamioka neutrino project*, [hep-ex/0106019](http://arxiv.org/abs/hep-ex/0106019).

[122] **T2K** Collaboration, K. Abe et al., *The T2K Experiment*, *Nucl.Instrum.Meth.* A659 (2011) 106–135, [arXiv:1106.1238](http://arxiv.org/abs/1106.1238).

[123] A. Chaterjee, M. Ghosh, P. Ghosal, S. Goswami, C. Gupta, S. Raut, and T. Thakore, “ICAL data for improving CPV discovery potential of long-baseline experiments.” In preparation, 2015.

[124] W. Pauli, “Exclusion principle, Lorentz group and reflection of space-time and charge.” London, Pergamon Press, 2015.

[125] G. Grawert, G. Luders and H. Rollnik, “The TCP Theorem and its Applications.” *Fortschr. d. Phys.* 7, 291 (1959).

[126] V. A. Kostelecky and M. Mewes, *Lorentz and CPT violation in the neutrino sector*, *Phys.Rev.* D70 (2004) 031902, [hep-ph/0308300](http://arxiv.org/abs/hep-ph/0308300).

[127] V. A. Kostelecky and M. Mewes, *Lorentz and CPT violation in neutrinos*, *Phys.Rev.* D69 (2004) 016005, [hep-ph/0309025](http://arxiv.org/abs/hep-ph/0309025).

[128] D. Colladay and V. A. Kostelecky, *CPT violation and the standard model*, *Phys.Rev.* D55 (1997) 6760–6774, [hep-ph/9703464](http://arxiv.org/abs/hep-ph/9703464).
[129] O. Greenberg, *CPT violation implies violation of Lorentz invariance*, Phys.Rev.Lett. 89 (2002) 231602, [hep-ph/0201258](http://arxiv.org/abs/hep-ph/0201258).

[130] S. R. Coleman and S. L. Glashow, *High-energy tests of Lorentz invariance*, Phys.Rev. D59 (1999) 116008, [hep-ph/9812418](http://arxiv.org/abs/hep-ph/9812418).

[131] P. A. M. Dirac, *Quantised Singularities in the Electromagnetic Field*, Proc.Roy.Soc. A133 (1931) 60–72.

[132] M. N. Saha, *The Origin of Mass in Neutrons and Protons*, Ind.J.Phys. 10 (1936) 141.

[133] G. t. Hooft, *Magnetic monopoles in unified field theories*, Nucl.Phys. B79 (1974) 276–284.

[134] A. M. Polyakov, *Particle Spectrum in Quantum Field Theory*, JETP Lett. 20 (1974) 194–195.

[135] J. Derkaoui et al., *Energy losses of magnetic monopoles and of dyons in the earth*, Astroparticle Physics 9 (1998) 173–183.

[136] E. Parker, *The Origin of Magnetic Fields*, Astrophys.J. 160 (1970) 383–404.

[137] B. Cabrera, *First Results from a Superconductive Detector for Moving Magnetic Monopoles*, Phys.Rev.Lett. 48 (1982) 1378–1381.

[138] S. Bermon et al., *New limit set on cosmic-ray monopole flux by a large-area superconducting magnetic-induction detector*, Phys.Rev.Lett. 64 (1990) 839–842.

[139] T. Kajita et al., *Search for Nucleon Decays Catalyzed by Magnetic Monopoles*, J.Phys.Soc.Jap. 54 (1985) 4065–4068.

[140] V. Aynutdinov et al., *Search for relativistic magnetic monopoles with the Baikal neutrino telescope*, Astropart.Phys. 29 (2008) 366–372.

[141] Soudan 2 Collaboration, J. L. Thron et al., *Search for magnetic monopoles with the Soudan 2 detector*, Phys.Rev. D46 (1992) 4846–4853.

[142] MACRO Collaboration, M. Ambrosio et al., *Final results of magnetic monopole searches with the MACRO experiment*, Eur.Phys.J. C25 (2002) 511–522.

[143] S. Balestra et al., *Magnetic Monopole Search at high altitude with the SLIM experiment*, Eur.Phys.J. C55 (2008) 57–63.

[144] CDF Collaboration, A. Abulencia et al., *Direct Search for Dirac Magnetic Monopoles in pp Collisions at s=1.96TeV*, Phys.Rev.Lett. 96 (2006) 201801–7.

[145] G. R. Kalbfleisch et al., *Improved Experimental Limits on the Production of Magnetic Monopoles*, Phys.Rev.Lett. 85 (2000) 5292–5295.

[146] ATLAS Collaboration, G. Aad et al., *Search for Magnetic Monopoles in s=7TeV pp Collisions with the ATLAS Detector*, Phys.Rev.Lett. 109 (2006) 261803–18.
[147] S. P. Ahlen, *Stopping-power formula for magnetic monopoles*, Phys.Rev. **D17** (1978) 229–233.

[148] S. P. Ahlen and K. Kinoshita, *Calculation of the stopping power of very-low-velocity magnetic monopoles*, Phys.Rev. **D26** (1982) 2347–2363.

[149] J. Derkaoui, G. Giacomelli, T. Lari, G. Mandrioli, M. Ouchrif, et al., *Energy losses of magnetic monopoles and dyons in scintillators, streamer tubes and nuclear track detectors*, Astropart.Phys. **10** (1999) 339–352.

[150] AMS Collaboration, J. Alcaraz et al., *Cosmic protons*, Phys.Lett. **B490** (2000) 27–35.

[151] T. Sanuki, M. Motoki, H. Matsumoto, E. Seo, J. Wang, et al., *Precise measurement of cosmic ray proton and helium spectra with the BESS spectrometer*, Astrophys.J. **545** (2000) 1135, [astro-ph/0002481].

[152] S. Haino, T. Sanuki, K. Abe, K. Anraku, Y. Asaoka, et al., *Measurements of primary and atmospheric cosmic-ray spectra with the BESS-TeV spectrometer*, Phys.Lett. **B594** (2004) 35–46, [astro-ph/0403704].

[153] S. Roesler, R. Engel, and J. Ranft, *The Monte Carlo event generator DPMJET-III*, [hep-ph/0012252].

[154] http://en.wikipedia.org/wiki/NRLMSISE-00.

[155] http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html.

[156] T. Sanuki, M. Honda, T. Kajita, K. Kasahara, and S. Midorikawa, *Study of cosmic ray interaction model based on atmospheric muons for the neutrino flux calculation*, Phys.Rev. **D75** (2007) 043005, [astro-ph/0611201].

[157] M. Freund, *Analytic approximations for three neutrino oscillation parameters and probabilities in matter*, Phys.Rev. **D64** (2001) 053003, [hep-ph/0103300].

[158] E. K. Akhmedov, R. Johansson, M. Lindner, T. Ohlsson, and T. Schwetz, *Series expansions for three flavor neutrino oscillation probabilities in matter*, JHEP **0404** (2004) 078, [hep-ph/0402175].

[159] A. Dziewonski and D. Anderson, *Preliminary reference earth model*, Phys.Earth Planet.Interiors **25** (1981) 297–356.

[160] M. Banuls, G. Barenboim, and J. Bernabeu, *Medium effects for terrestrial and atmospheric neutrino oscillations*, Phys.Lett. **B513** (2001) 391–400, [hep-ph/0102184].

[161] R. Gandhi, P. Ghoshal, S. Goswami, P. Mehta, and S. U. Sankar, *Large matter effects in nu(mu) to nu(tau) oscillations*, Phys.Rev.Lett. **94** (2005) 051801, [hep-ph/0408361].
[162] S. Choubey and P. Roy, *Testing maximality in muon neutrino flavor mixing*, *Phys.Rev.Lett.* 93 (2004) 021803, [hep-ph/0310316](http://arxiv.org/abs/hep-ph/0310316).

[163] B. Schorr, “Programs for the Landau and the Vavilov distributions and the corresponding random numbers”, Comp. Phys. Commm. 7, 215 (1974); see also, A. Rotondi, P. Montagna, “Fast calculation of Vavilov distribution”, Nucl. Instr. & Meth. B 47, 215 (1990).

[164] Dan Green, *Physics of Particle Detectors* (Cambridge: Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, Cambridge University Press), 2000, p 266–8.

[165] M. Ambrosio, G. Bari, G. Bencivenni, A. Candela, A. Chiarini, et al., *The MONOLITH prototype*, *Nucl.Instrum.Meth.* A456 (2000) 67–72.

[166] G. Bari, A. Candela, M. De Deo, M. D’Incecco, M. Garbini, et al., *Analysis of the performance of the MONOLITH prototype*, *Nucl.Instrum.Meth.* A508 (2003) 170–174.

[167] D. A. Petyt, *A study of parameter measurement in a long-baseling neutrino oscillation experiment*, Ph.D thesis, University of Oxford, Preprint FERMILAB-THESIS-1998-66 (1998).