Resonant CP-Violating Scalar–Pseudoscalar Transitions at $\mu^+\mu^-$ Colliders

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ABSTRACT

A $\mu^+\mu^-$ collider is an appealing machine to probe resonant CP-violating transitions of a CP-even Higgs particle into the $Z$ boson or into another CP-odd Higgs scalar. These phenomena are studied within a manifestly gauge-invariant approach implemented by the pinch technique. The CP invariance of an extended Higgs sector motivated by supersymmetric $E_6$ models is assumed to be broken radiatively by the presence of heavy Majorana fermions. CP violation originating from Higgs-$Z$ mixing is found to be very modest, whereas CP-number violating transitions involving Higgs scalars only can be resonantly enhanced up to order of unity.

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Recently, much research and theoretical effort have been put into the design and the physics capabilities of a muon collider, which could potentially serve as an important Higgs factory \[1\]. Given the technical facility of a variable centre-of-mass (c.m.) energy for such a machine, it has been argued \[1\] that one could exploit the resonant enhancement of an $s$-channel interaction to copiously produce Standard Model (SM) Higgs-bosons, $H$, in the mass range $100 \leq M_H \leq 200$ GeV, and/or explore the existence of non-SM Higgs bosons with $0.2 \leq M_H \leq 1$ TeV, which couple to $\mu$ significantly. The most compelling extensions of the SM with naturally large $H\mu\mu$ couplings are those having an underlying supersymmetric (SUSY) origin. In order to ensure the absence of the triangle anomalies and give mass to both up and down quarks, the Higgs sector of a SUSY scenario must contain at least two Higgs doublets, giving rise to CP-even and CP-odd Higgs scalars. Therefore, a muon collider may be the most ideal place to search for large Higgs-$Z$-boson mixing effects and, more interestingly, observe transitions between Higgs scalars with opposite CP quantum numbers. In fact, if quantum effects allow for a CP-even Higgs scalar, $H$, to go into the CP-odd $Z$ boson or another CP-odd scalar, $A$ say, such a transition alone would signify CP/T violation in a CPT-invariant theory \[2\]. In the SM, there is no $HZ$ mixing up to two-loop electroweak order. The reason is that a non-trivial CP-odd rephasing invariant combination of Cabbibo-Kobayashi-Maskawa matrix elements is required inside the fermionic loops. However, in natural extensions of the SM, involving Majorana fermions \[3\] or more than one Higgs doublet \[4\], a $HZ$-mixing effect may occur in the decays of the $H$ into top-quark pairs.

In this paper, we study the possibility of CP-violating $HZ$ and/or $HA$ transitions in a model, in which the CP invariance of the Higgs sector is broken radiatively by the presence of heavy Majorana fermions. Models of the kind are the minimal SUSY SM (MSSM), in which Majorana fermions may be identified with the heavy neutralinos, or other scenarios inspired by SUSY-$E_6$ theories, which predict heavy Majorana neutrinos \[3\] at the TeV mass scale. Here, we will work on the latter realization \[7\]. To be specific, we adopt the CP-
violating scenario of [3] for the neutrino sector, which may resemble the model discussed by
the authors in Ref. [6] at the electroweak scale. The model contains three heavy Majorana
neutrinos, denoted here by $N_1$, $N_2$, and $N_3$, from which $N_1$ is predominantly a sequential
isodoublet, whereas $N_2$ and $N_3$ are mainly singlets under SU(2)$_L$. Furthermore, we consider
that the Lagrangians governing the interactions between $N_i$ (with $i = 1, 2, 3$) and $H$, $A$, and the would-be Goldstone boson $G^0$, have the following generic form [8]:

$$L_A = \frac{ig}{4M_W} A \chi_{H}^n \sum_{i,j=1}^{3} \bar{N}_i \left[ \gamma_5 (m_i + m_j) \Re C_{ij} + i(m_j - m_i) \Im C_{ij} \right] N_j,$$

(1)

$$L_H = - \frac{g}{4M_W} H \chi_{H}^n \sum_{i,j=1}^{3} \bar{N}_i \left[ (m_i + m_j) \Re C_{ij} + i\gamma_5 (m_j - m_i) \Im C_{ij} \right] N_j,$$

(2)

where the parameters $\chi_{A,H}^n$ are related with the vacuum expectation values (VEVs) of the
Higgs fields in an extended Higgs sector and $C_{ij}$ are mixing matrices defined in [8]. The
coupling $G^0 N_i N_j$ can be recovered from Eq. (1), if we set $\chi_{H}^n = 1$ and $A \equiv G^0$. This
model has a non-trivial CP-violating phase contained in the rephasing invariant quantity
$\Im m_{N_1 N_2}^2 = \sin \delta_{CP}|C_{N_1 N_2}|^2$ [3], which is taken to be maximum of order one.

To analyze CP violation originating from $HZ$ and/or $HA$ mixing, we have to find
an observable sensitive to these effects. Assuming that the facility of having longitudinally
polarized muon beams will be available without much loss of luminosity, we can define the
CP asymmetry

$$A_{CP} = \frac{\sigma(\mu_L^- \mu_L^+ \rightarrow f \bar{f}) - \sigma(\mu_R^- \mu_L^+ \rightarrow f \bar{f})}{\sigma(\mu_L^- \mu_L^+ \rightarrow f \bar{f}) + \sigma(\mu_R^- \mu_L^+ \rightarrow f \bar{f})}. $$

(3)

We must emphasize that $A_{CP}$ is a genuine observable of CP violation if one is able to tag on
the final fermion pair $f \bar{f}$ (e.g., $\tau^+ \tau^-$, $b \bar{b}$, or $t \bar{t}$), since the helicity states $\mu^- \mu^+$ transform into
$\mu^{-} \mu^{+}$ under CP in the c.m. system. Similar CP/T-violating observables based on T-odd
aplanarities at $e^+ e^-$ machines were considered by the authors of Ref. [3], who suggested to
look for CP violation in vector and axial-vector currents. Here, we require, however, that
both muons are left-handed or right-handed polarized. Similar ideas have been applied to
study CP violation in the top-pair production at LHC and TeV-$e^+ e^-$ colliders [10,4,3] and,
more recently, to muon colliders [11] as well.

In our analysis, we consider a manifestly gauge-invariant approach for resonant transitions [12], which is implemented by the pinch technique (PT) [13]. This approach is free from CP-odd gauge artifacts; it reassures the absence of a $HZ$ and/or $HA$ mixing in a CP-invariant and anomaly-free theory, thus preserving the discrete symmetries of the classical action after quantization. Furthermore, we make use of a mechanism for resonant CP violation induced by particle widths in scattering processes, which was discussed in [14] in connection with top-quark production and decay some time ago [15]. This mechanism of CP violation gives rise to a resonant enhancement for certain CP-violating observables, such as $A_{CP}$ in Eq. (3), and so yields measurable effects for a wide range of heavy Higgs masses as we will see below.

We now discuss in short how resummation involving $ZH$ mixing takes place within our approach [12]. There are PT identities that can be employed to convert $ZH$ and $ZZ$ strings into $G^0H$ and $G^0G^0$ ones [16] before resummation occurs. These identities are

$$p^\mu \hat{\Pi}^{ZH}_\mu + i M_Z^0 \hat{\Pi}^{G^0H}_\mu = 0, \quad p^\mu p^\nu \hat{\Pi}^{ZZ}_{\mu\nu} - (M_Z^0)^2 \hat{\Pi}^{G^0G^0} = 0,$$

and

$$p^\mu \Gamma^{Zf\bar{f}}_\mu = -i M_Z^0 \Gamma^{G^0f\bar{f}},$$

where $p^\mu$ of the $Z$ boson always flows into the fermionic vertex and $M_Z^0$ is the bare $Z$-boson mass. If $\hat{\Pi}^{ZH}_\mu(p) = p_\mu \hat{\Pi}^{ZH}(p^2)$ and $\hat{\Pi}^{ZZ}_{\mu\nu}(p) = t_{\mu\nu}(p)\hat{\Pi}^{ZZ}(p^2) + \ell_{\mu\nu}(p)\hat{\Pi}^{ZZ}_L$, with

$$t_{\mu\nu}(p) = -g_{\mu\nu} + p_\mu p_\nu/p^2 \quad \text{and} \quad \ell(p) = p_\mu p_\nu/p^2,$$

then $\hat{\Pi}^{ZH}(p^2) = -i M_Z^0 \hat{\Pi}^{G^0H}(p^2)/p^2$ and $\hat{\Pi}^{ZZ}_L(p^2) = (M_Z^0)^2 \hat{\Pi}^{G^0G^0}(p^2)/p^2$. In this context, it worth stressing the fact that $p^\mu \hat{\Pi}^{G^0}_\mu(p) = p^\mu \hat{\Pi}^{\gamma H}_\mu(p) = 0$, within the PT framework, which implies the absence of $\gamma H$ and $\gamma G^0$ mixing, i.e., $\hat{\Pi}^{\gamma H}_\mu = \hat{\Pi}^{\gamma G^0}_\mu = 0$, independently of whether CP-violating interactions are present in the theory. As a result, one is left with solving the simple coupled Dyson-Schwinger equation system, in which only $H$ and $G^0$ mix, i.e.,

$$[\hat{\Delta}(p^2)]^{-1} = \begin{bmatrix} p^2 + \hat{\Pi}^{G^0G^0}(p^2) & \hat{\Pi}^{G^0H}(p^2) \\ \hat{\Pi}^{HG^0}(p^2) & p^2 - (M_H^0)^2 + \hat{\Pi}^{HH}(p^2) \end{bmatrix}.$$

(5)
Inverting this matrix, we find
\[
\hat{\Delta}^{G_0}(p^2) = \left\{ p^2 + \hat{\Pi}^{G_0G_0}(p^2) - [\hat{\Pi}^{G_0H}(p^2)]^2/[p^2 - (M_H^0)^2 + \hat{\Pi}^{HH}(p^2)] \right\}^{-1},
\]
\[
\hat{\Delta}^H(p^2) = \left\{ p^2 - (M_H^0)^2 + \hat{\Pi}^{HH}(p^2) - [\hat{\Pi}^{G_0H}(p^2)]^2/[p^2 + \hat{\Pi}^{G_0G_0}(p^2)] \right\}^{-1},
\]
\[
\hat{\Delta}^{G_0H}(p^2) = -\hat{\Pi}^{G_0H}(p^2)\left\{[p^2 - (M_H^0)^2 + \hat{\Pi}^{HH}(p^2)][p^2 + \hat{\Pi}^{G_0G_0}(p^2)] - [\hat{\Pi}^{G_0H}(p^2)]^2 \right\}^{-1}.
\]

The above considerations can be extended to include additional Higgs scalars, such as the physical CP-odd scalar, $A$, and so describe $HA$-mixing effects. In such a case, the inverse propagator in Eq. (5) becomes a $3 \times 3$ matrix.

Within the PT, it is known \cite{17} that the analytic expression of $\Im m\hat{\Pi}^{HH}(s)$ coincides, to one loop, with that of the background field gauge for $\xi_Q = 1$. In this way, we obtain for the different channels
\[
\Im m\hat{\Pi}^{HF}(s) = \frac{\alpha_w N_c f}{8} (\chi_H^f)^2 s \frac{m_f^2}{M_H^2} \left(1 - \frac{4m_f^2}{s}\right)^{3/2} \theta(s - 4m_f^2),
\]
\[
\Im m\hat{\Pi}^{TV}(s) = \frac{n_V \alpha_w}{32} (\chi_V^0)^2 \frac{M_V^4}{M_H^4} \left(1 - \frac{4M_V^2}{s}\right)^{1/2} \times \left[1 + 4 \frac{M_V^2}{M_H^2} - 4 \frac{M_V^2}{M_H^2} (2s - 3M_V^2) \right] \theta(s - 4M_V^2),
\]
where $\alpha_w = g^2/4\pi$, $n_V = 2, 1$ for $V \equiv W, Z$, respectively, and $N_c f = 1$ for leptons and 3 for quarks. In Eqs. (7) and (8), we have parametrized fermionic and bosonic channels by the model-dependent factors $\chi_H^f$ and $\chi_V^{W,Z}$. By analogy, calculation of $\Im m\hat{\Pi}^{AA}(s)$ gives $\chi_V^A = 0$, while $\chi_H^f$ should be replaced by $(1 - 4m_f^2/s)^{-1/2} \chi_A^f$ in Eq. (7). There may also be other channels involving the $HZA$ vertex, which are, however, considered to be phase-space suppressed in the kinematic region $M_H \simeq M_A$, relevant for resonant CP violation. The complete treatment including these effects as well as other SUSY refinements will be given elsewhere \cite{18}.

Considering the Lagrangians (1) and (2), it is straightforward to calculate the CP-
violating $HG^0$ and/or $HA$ mixing in our model, viz.

$$\hat{\Pi}^{AH}(s) = -\frac{\alpha_w}{4\pi} \chi_u^a \chi_H^u \sum_{j>i}^{3} \Im m C_{N_j}^2 \sqrt{\lambda_i \lambda_j} \left[ B_0(s/M_W^2, \lambda_i, \lambda_j) + 2B_1(s/M_W^2, \lambda_i, \lambda_j) \right], \quad (9)$$

where $\hat{\Pi}^{AH}(s) = \hat{\Pi}^{HA}(s)$, $\lambda_i = m_{N_i}/M_W^2$, and $B_0$ and $B_1$ are the usual Veltman-Passarino loop functions, expressed in the convention of Ref. [19]. Again, the $HG^0$ transition is recovered from Eq. (9) by setting $\chi_u^A = 1$.

From Fig. 1, it is not difficult to see that in the case of $HZ$ mixing, only two diagrams can contribute constructively to $A_{CP}$ through the interference of the $G^0$-exchange graph with the amplitude depending on $\hat{\Delta}_{HG^0}$. The reason is that contraction of a scalar current with a pseudoscalar one vanishes identically. Substituting Eqs. (6)–(9) into Eq. (3) yields

$$A_{CP}(s) = -2 \frac{\hat{\Pi}^{G^0H}(s)}{s} \frac{\Im m \hat{\Pi}^{HH}(s)}{s}. \quad (10)$$

From Eq. (10), we find that CP asymmetries can be large only for heavy Higgs boson masses far above the two-real $W$-boson production threshold, since the Higgs width is then comparable to $M_H$. To give an example, we find $A_{CP} \simeq 2 \times 10^{-2}$ for $M_H = 500$ GeV and $m_{N_1,N_2,N_3} = 0.5, 1.5, 3$ TeV, while the production cross-section is $\sigma \simeq 1$ fb. It seems that one is unlikely to observe $HZ$-mixing effects, even if assuming a high integrated luminosity of $50$ fb$^{-1}$ for the muon collider [1].

The situation changes drastically if the heaviest CP-even $H$ mixes with a CP-odd Higgs scalar, $A$, especially when $M_A > 2M_Z$. The latter is very typical within SUSY unified theories [20]. In our minimally extended SUSY model, $H$ will couple predominantly to fermions, and naturally be degenerate with $A$, i.e. $M_H \simeq M_A$ [21]. Moreover, the coupling parameters will obey the MSSM relation $\chi_H^d = \chi_A^d = 1/\chi_H^u = 1/\chi_A^u = \tan \beta$. Taking these into account, we find that $A_{CP}$ behaves at $s \simeq M_H^2$ as

$$A_{CP} \sim -\frac{2\hat{\Pi}^{AH}(s) \Im m \hat{\Pi}^{HH}(M_H^2) - \Im m \hat{\Pi}^{AA}(M_H^2)}{(M_H^2 - M_A^2)^2 + [\Im m \hat{\Pi}^{AA}(M_H^2)]^2 + [\Im m \hat{\Pi}^{HH}(M_H^2)]^2}. \quad (11)$$

In Fig. 2, we have presented cross sections (solid lines) and CP asymmetries (dotted lines) as a function of the c.m. energy, $\sqrt{s}$, in two different scenarios. We also assume that tuning
the collider energy to the mass of $H$ is feasible, i.e., $\sqrt{s} = M_H$. In our estimates, we take $\tan \beta = 2$, $m_{N_1,N_2,N_3} = 0.5, 1, 1.5$ TeV, and $m_t = 170$ GeV. We analyze two reactions: (a) $\mu^+_L \mu^-_L \rightarrow b \bar{b}$, for $M_A = 170$ GeV and $\chi^V_H = 1$, and (b) $\mu^+_L \mu^-_L \rightarrow t \bar{t}$, for $M_A = 400$ GeV and $(\chi^V_H)^2 = 0.1$. In reaction (a), one can have a significant CP-violating signal if $M_H = 170 \pm 8$ GeV. We also observe the resonant enhancement of CP violation when $M_H = M_A$. More promising is the reaction (b), in which CP violation may be observed for a wider range of Higgs-boson masses, i.e., for $M_H = 350$ GeV $-$ 430 GeV. Again, the mechanism of resonant CP violation is very important to render the observable $A_{CP}$ measurable, as shown in Fig. 2.

Note that resonant CP-violating $HA$ transitions can also take place within the MSSM, in which neutralinos and charginos may play the rôle of heavy Majorana fermions. In the MSSM, the Higgs-mixing mass parameter $\mu$ in the superpotential and the tri-linear soft-SUSY-breaking couplings $A$ can have non-trivial CP-violating phases, which give rise to complex chargino- and neutralino-mass matrices [22] and hence to interactions of the form given in Eqs. (1) and (2). Yet, electric dipole moment bounds on neutron and electron cannot prevent these CP-violating phases from being large [22]. Although multi-Higgs models, such as the Weinberg’s three-Higgs doublet model, may also induce a sizeable $HA$ mixing (cf. Eq. (9)) for a certain corner of their parameter space, the requirement of having resonant CP violation through a relatively small mass difference between $H$ and $A$ will strongly favour only extended Higgs sectors with a SUSY origin [20,21]. Finally, we must stress that, even though building a muon machine with a high degree of polarization may become a difficult task, our analysis will, however, carry over to searches for resonant CP-violating effects in the decay products of the final states, i.e., in the angular-momentum distributions and energy asymmetries of the produced charged leptons and jets [10].

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Figure Captions

Fig. 1: CP-violating $HA$ and $HZ$ (for $A \equiv G^0$) transitions in $\mu^+\mu^-$ collisions.

Fig. 2: Numerical estimates of production cross-sections and CP violation for $\mu^-_{L,R} \mu^+_{L,R} \rightarrow (H, A) \rightarrow f \bar{f}$ in two different SUSY scenarios with heavy Majorana neutrinos (see also text).