Mathematical modeling of processes occurring during deposition of sprayed particles of polymeric powder

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Abstract. The deposition of polymeric powder particles on a surface of a treated body and a layer of particles deposited previously is considered using mathematical modeling. Basic provisions of the impact theory are used. The relationships to evaluate the characteristic parameters of the processes under study are given, the results of their analysis and recommendations to improve the deposition efficiency are presented.

When applying the polymeric powder coatings their operational parameters are greatly influenced by technologies and the equipment; used consumables; shape and surface conditions of a treated body; external conditions in which the appropriate technologies are implemented; technological modes and other circumstances [1, 2]. In case of a jet spraying of polymeric powder materials during coating formation process there are three main stages: deposition of powder particles on a treated body surface, formation of a porous dispersed layer; film formation accompanied by agglutination, coalescence, sintering and spreading of particles; structuring of the material of the formed coating. In turn the following phases can be distinguished as a refinement of these stages: deposition of a polymeric powder on a substrate surface; deformation of particles under impact, action of other factors, their agglutination; coalescence, sintering of particles in the sprayed layer; degassing, gas discharge from the forming layer; wetting of the substrate surface; film formation, material curing; molecular and supramolecular structuring, material polymerization of the formed coating; coating adhesion to the solid.

Each of the named phases of the polymeric coating formation in a certain way influences on its operational parameters. In order to evaluate this influence it is necessary to analyze the phases taking into account the peculiarities of the processes preceding to these phases [3]. The present article discusses the first phase – deposition of polymeric powder particles on a treated solid body surface.

Separate powder particles, probably, clusters moving in the jet of air and flue gases firstly fall on the body surface and then on the layer of already deposited particles. Particles behavior at the moment of impact is largely dependent on the particles size, their shape, conditions of the particles material (solid, fritted, melted), its mechanical characteristics; on conditions of substrate surface, its roughness, temperature; on incidence angle of the particles, velocity of fall, impact force and etc. Extensive materials on the processes occurring at the impact of solid and liquid particles (droplets) with a barrier are given in works [4-10].

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For an impact interaction of material bodies it is typical that for a short time of the impact \( \tau_0 \) the velocity of colliding bodies or one of them changes considerably.

Following [7-10] when analyzing the impact of polymeric powder particles with a treated body surface and layer of previously deposited particles we distinguish the following groups of main parameters.

A. Indicators of particles approaching the surface: sizes, shape, volume concentration of particles in the gas jet approaching a treated body.

B. Impact parameters: the particles mass at impact, impact velocity, incidence angle of the particles.

C. Indicators of the treated body (barrier, substrate): thickness, curvature, surface roughness, material density, Young’s modulus, Poisson’s ratio, tensile and compression waves velocity, melt viscosity, surface tension.

D. Indicators of the sprayed layer: thickness, curvature and roughness of the external surface of the layer; density, porosity, Young’s modulus, Poisson’s ratio of the material layer, compression and shear waves velocity, melt viscosity.

When denoting the indicators of powder particles we shall use the subscript \( \rho \), the sprayed layer – \( \nu \), the coating – \( \varsigma \), the body parameters – index \( \tau \). In order to distinguish the phase state of materials: solid, liquid, gaseous we shall use indexes \( \varsigma \), \( l \), \( g \), respectively.

1. Fall of powder particles on the body surface

Let us consider preliminary a unit volume of the gas jet containing a certain amount of powder particles \( N_p \). For the sake of simplicity we assume that all particles have a shape close to a sphere, their radius \( r_p \). When modeling the particles, in particular, there are distinguished the 50% diameter \( d_{p1} \) which can be found from condition that 50\% of particles are particles with smaller diameter the remaining 50\% – particles of larger diameter and the dominant diameter \( d_{p0} \) defined as the particles diameter that is characteristic for the maximum amount of particles,

\[
d_{p0} = \left( \frac{6V_p}{\pi N_p} \right)^{1/3},
\]

where \( V_p = \sum_{i=0}^{N_p} V_{pi} \), \( V_{pi} \) – volume of \( i \)-th powder particle.

Let us also introduce the volumetric particle size distribution

\[
E_d = V_d / V_p,
\]

where \( V_d \) – the volume of those particles whose diameter is equal to \( d \) or close to it. Hence the number of particles with diameter \( d \) that are contained in the considered unit volume will be:

\[
N_d = 6V_p E_d / (\pi d^3).
\]

The total number of particles \( N_p = \sum_d N_d \).

Further let us assume that the treated body surface is stationary and its curvature is small. We distinguish on it a unit area \( S_m \) with outward normal \( n \). If in the unit volume of the jet adjacent to this area all particles with diameter \( d \) move to the surface at the same velocity \( \dot{\nu}_d \) then number of particles that have fallen on this area in a time \( \tau \)

\[
N_{dn} = v_{dn}N_d \tau / l,
\]

where \( v_{dn} \) – the projection of the velocity vector \( \dot{\nu}_d \) on the normal \( n \), \( l \) – Length equal to one. Accordingly the number of impacts of all particles in a time \( \tau \) will be equal to:
\[ N_m = \sum_d N_{dm}. \]

We define the intensity of particles deposition on the body surface \( I_m \) as the thickness of the material of particles deposited on it per unit time. Under condition that the number of impacts of the particles on the area \( S_m = l^2 \) per unit time is equal to \( N_m / \tau \) we find:

\[ I_m = \frac{1}{l^2} \sum_d (v_{dn} V_d). \]

In the case where the sizes of all particles are identical \( (d_{p0} = d) \) velocity \( v_{dn} = v_n \) we obtain:

\[ N_m = v_n N_p \tau / l, \quad I_m = \pi d^3 N_m / (6 \pi l^2). \]

2. Evaluation of stresses in the body material with multiple impacts of sprayed particles
In addition to the above assumptions we assume that all particles are spherical, of the same size, when approaching to the body are distributed uniformly have the same velocity of fall its magnitude is relatively small (stress in the body material caused by the impact of a particle has time to significantly decrease before the fall of the next particle) all powder particles located close to the considered part body fall on it.

The powder particles falling at first on the body surface and then on the previously deposited particles exert an impact action on the barrier material. In the one-dimensional approximation an impact pressure corresponds to the pressure arising upon the collision of a semi-infinite flat body with a rigid substrate and is described by the dependence [7]:

\[ p = \rho_p C_p v_n, \]

where \( \rho_p \) – material density of the particles, \( C_p \) – sonic velocity in this material (shock-wave velocity).

Taking into account the mechanical characteristics of the barrier material we obtain:

- at impact with the substrate
  \[ p = \rho_p C_p v_n / (1 + \rho_p C_p / (\rho_m C_m)), \]

- at impact with the deposited layer of the particles
  \[ p = \rho_p C_p v_n / (1 + \rho_p C_p / (\rho_H C_H)). \]

Here \( \rho_m, \rho_H; C_m, C_H \) – density, sonic velocity in the substrate material and sprayed layer of particles, respectively.

According to the theory of Tayruvengadam specific intensity of the energy absorbed by the material of the treated body at multiple impacts of particles is defined as follows:

\[ I_e = \alpha_e \zeta(\tau)(p / (R + R_H))^2 / (\rho_p C_p), \]

where \( \alpha_e \) – proportionality coefficient, \( \zeta(\tau) = 1 - \exp(-(\tau / \tau_0)\beta) \), parameter \( \beta \) depends on the properties of the substrate material and amount of load, \( R \) – depth of the substrate material layer, \( R_H = R_H(\tau) \) – layer thickness of the deposited particles. Under condition that \( R_H / R \geq 1 \), energy intensity:

\[ I_e \approx \alpha_e \zeta(\tau)(1 - 2R_H(\tau) / R)(p / R)^2 / (\rho_p C_p). \]

Since at \( \tau \to 0 \) \( \zeta(\tau) \to 0 \), and at \( R_H(\tau) \to 0.5R \) function \( 1 - 2R_H(\tau) / R \to 0 \), it should be expected that in the given range of parameter variations the energy intensity will be maximum. As a result of a number of simplifications and transformations we find time \( \tau_e \) at which energy intensity \( I_e \) reaches its maximum,

\[ \tau_e \approx \tau_0 (\ln(\tilde{R} / (\tilde{R} + \tilde{R}))^{1/\beta}, \]
where \( \tilde{R} = 2R_{H1}/R \), \( R_{H1} = \tilde{R}_{H} (\tau = 0.5\tau_{i}) \), \( (\tau) \equiv \partial(\tau)/\partial \tau \), \( \tau_{i} \) – spraying time, when \( R_{H}(\tau) = 0.5R \); \( \tilde{\beta}_{i} = 2\beta(\tau_{i}/(2\tau_{i}))^{\beta}/\tau_{i} \).

3. Impact of solid deformable particle with the barrier surface

Let us assume, as before, that before the impact the falling particle has a spherical shape, it is a ball. Two stages are distinguished at direct impact of a deformable ball with a surface of a solid stationary body. During the initial stage the velocity of the mass center of a particle \( v_{p} = v \) decreases to zero, the ball deforms, all the initial kinetic energy converts into its internal potential energy. Then the ball starts to move in the opposite direction reaching the velocity \( u \). In this case part of mechanical energy is consumed to impart the residual deformations to the material of the ball and on its heating. By introducing a parameter \( k = u/v \) called coefficient of restitution, we find the average reaction force of the barrier directed along the outer normal to it, \[ P_{cp} = (1+k)mv/\tau_{0} \] (1)

Here \( v = -v_{n} \), \( v_{n} \) – projection on the normal of the velocity of fall of the body \( \tilde{v} \), \( m = m_{p} \) its mass.

In case when during the impact a particle is under the influence of external forces, for example, gravity force \( \bar{F}_{g} = mg \) (\( g \) – acceleration of gravity), electrostatic force \( \bar{F}_{e} \) (if a charged particle is in an electrostatic field), force of a particle adhesion to the substrate material \( \bar{F}_{c} \), impact force of the following particle on the previous one \( \bar{F}_{s} \), rebounding velocity \( u \) will depend on both its deformation described by the coefficient of restitution \( k \), and on the action of external force \( \bar{F} = \bar{F}_{g} + \bar{F}_{e} + \bar{F}_{c} + \bar{F}_{s} \). Accordingly,

\[ u = \bar{k}v, \]

where \( \bar{k} = k + k_{f}, k_{f} \) – coefficient characterizing the influence of force \( \bar{F} \) (\( k_{f} = 0 \), if \( \bar{F} = 0 \)) on velocity \( u \). Considering this fact by analogy with (1) we can write:

\[ P_{cp} + F_{em} = (1+\bar{k})mv/\tau_{0}. \]

Hence it follows that the rebounding velocity will be minimum (\( \bar{k} \sim 0 \)) if the projection of the external force average during the impact time is

\[ F_{em} \sim mv/\tau_{0} - P_{cp}. \]

Thus the most favorable situation to decrease the particle rebounding velocity will be the case when gravity force, electrostatic force are directed along the normal to the side of the substrate; surface layer is softened (slightly melted); substrate surface is rough, adhesive (glue) layer is on it; particle following the first one hits it at the end of time period \( \tau_{0} \) (at the beginning of the backward motion of the first particle).

It is known that shock-waves propagate in the material of both the particle and the barrier when particle hits the barrier surface. Basic relationships on the shock-wave are the mass conservation equation and momentum conservation law. As a result of the experimental data analysis it is found that shock-wave velocity \( \tilde{v} \) and velocity of the particle at impact \( v_{p} \) are related by [7]

\[ \tilde{v} \approx v_{p} + \delta \Delta v. \]

Here \( \delta \) – empirical parameter (\( 0 < \delta < 2 \)); \( \Delta v \) – particle velocity jump at the moment of impact. Hence we can obtain the dependence (shock adiabat) for pressure on the shock-wave \( \tilde{p} \):

\[ \tilde{p} \approx \rho_{0}\tilde{v}\Delta v, \]

where \( \rho_{0} \) – initial material density in which the wave propagates. If the barrier is at rest then initial velocity of the barrier particles is equal to zero and impacting particle – \( v_{p} \). Upon particles contact at
the moment of impact their velocities and pressures on both sides in accordance with the consistency condition will be \( v_\ast, p_\ast \). Thus at the initial contact step the velocity jump of the falling particle is \( \Delta v = \Delta v_p = v_p - v_\ast \) and in the barrier – \( \Delta v = v_\ast \).

Pressure \( p_\ast \) (impact stress) that has arisen in the particle and the barrier in the beginning of the impact is spreads along the material of the particles, reaches its opposite surface and is reflected. When shock-wave is approaching the contact surface the velocity of the falling particle and pressure in it decrease rapidly. If the acoustic impedance of the particle characterized by the product of the density of its material on the velocity is small the particle stops the movement inward the barrier. When impedance is high (high density, velocity of the particle) it continues to penetrate deep into the barrier.

A detailed solution to the problem of the impact of spherical body with isotropic elastic barrier is presented in the work of Timoshenko S.P. [6, 8]. Let us give the final calculated ratios not going into details. Provided that the contact duration of the ball and the barrier is high according to the Hertz’s law the impact force will be:

\[
P = \delta_{pm}^{3/2} s^{3/2}.
\]

Here \( \delta_{pm} = 4\sqrt{r_p / (3\pi(k_p + k_m)} \) , \( k_p = (1-v_p^2) / \pi E_p \) , \( k_m = (1-v_m^2) / \pi E_m \) ; \( E_p, E_m; v_p, v_m \) – Young’s modulus, Poisson’s ratio of the particle and barrier material, respectively; \( s \) – convergence of the ball and the barrier, caused by the local compression of the body at the point of contact. Assuming that the barrier is semi-infinite and stationary we find the maximum value of the deformation (the closest convergence of the particle and the barrier)

\[
s_0 = (5m_p v_p^2 / 4\delta_{pm})^{2/5}.
\]

Hence, according to (2), the maximum impact force

\[
P_\ast = \delta_{pm}^{2/5} (5m_p v_p^2 / 4)^{3/5},
\]

and the maximum radius of the contact area

\[
a_\ast = \sqrt{r_p (5m_p v_p^2 / 4\delta_{pm}) \frac{1}{5}}.
\]

It is known that in the considered problem the pressure distribution along the area has the form

\[
p = p(r) = p_0 (1-r^2 / a_\ast^2)^{1/2},
\]

where \( p_0 = p_\ast(\tau) \) – surface pressure in the area center, \( r = \sqrt{x^2 + y^2} \); \( x, y \) – Cartesian coordinates on an area with a center in the impact point of the particle. It is easy to show

\[
p_0 = 3P / (2\pi a_\ast^2).
\]

At the moment when deformation stops when \( \dot{s} = 0 \), the radius of the contact area \( a = a_\ast \), force \( P = P_\ast \), pressure

\[
p_0 = p_\ast = \frac{3\delta_{pm}}{2\pi r_p} \left( \frac{5m_p v_p^2}{4\delta_{pm}} \right)^{1/5}.
\]

In order to evaluate the change of function \( s(\tau) \) and also \( a(\tau), p(\tau), P(\tau) \) depending on time it is necessary to integrate the equation that follows from Newton’s second law

\[
\ddot{s}^2 = v_p^2 - 4\delta_{pm} s^{5/2} / (5m_p).
\]

After a number of transformations, integration we get

\[
\tau = \frac{s_0}{v_p} \int_0^\tau \frac{dt}{(1-t^{5/2})^{1/2}},
\]

where \( \overline{s} = s / s_0 \).

In particular, for \( \overline{s} = 1 \) from (5) it is possible to find the impact time:
\[ \tau_0 = 2.94 \left( \frac{5 m_p}{(4 \delta_{pm} V_p^{1/2})} \right)^{2/5}. \]

If \( \overline{s} < 1 \), accordingly, integration variable \( t < 1 \) we obtain:
\[ 1/(1-t^{5/2})^{1/2} \sim 1 + 0.5t^{5/2}. \]

Therefore,
\[ \tau \sim (s_0 / v_p)(1 + \overline{s}^{5/2} / 7), \]
or
\[ s \sim v_p \tau \quad (0 \leq \tau \leq \tau_0) \quad (6) \]

Substituting \( s \) from the dependence (5) into (2), we find the impact force \( P = P(\tau) \), the radius of the contact area
\[ a = a(\tau) = (3\pi(k_p + k_m)r_p P(\tau) / 4)^{1/3}, \quad (7) \]

By the formula (4), (3), respectively, pressure \( p_0 = p_0(\tau) \) and \( p = p(r, \tau) \).

Fig. 1 shows the changes of function \( s(\tau) \) over time \( \tau \) characterizing the convergence of the particle and the barrier, the radius of the contact area \( a(\tau) \) at \( r_p = 8 \times 10^{-3} \) m, \( \rho_p = 900 \) kg/m\(^3\), \( v_p = 15 \) m/s, \( E_p = 1.9 \times 10^8 \) Pa, \( \nu_p = 0.43 \) (high pressure polyethylene); \( E_m = 2 \times 10^{11} \) Pa, \( \nu_m = 0.27 \) (steel). It can be seen that in the beginning of the impact dependence \( s = s(\tau) \) is close to linear whereas \( a = a(\tau) \) is substantially non-linear. However at \( \tau \rightarrow 0.1 \) s both functions asymptotically tend to the limiting values.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Convergence of the particle and the barrier (a), change of the radius of the contact area (b) over time.}
\end{figure}

It is also necessary to take into account that with a high degree of accuracy [8]
\[ s = s_0 \sin(\pi \tau / \tau_0). \]

For small \( \tau / \tau_0 \) approximately
\[ s \sim \pi v_p \tau / 2.94 \equiv 1.1v_p \tau, \]
that, in general, is consistent with (6).

It should be noted that relationships (2)-(7) written for the case of particle impact with the treated body surface can, in principle, be extended on the impact of the particles with previously deposited ones, changing parameters \( E_m, \nu_m \) on Young’s modulus \( E_H \), Poisson’s ratio \( \nu_H \) of the sprayed layer.

Thus analytical dependencies to evaluate the intensity of particles deposition, absorption of the impact energy by the material including taking into account previously deposited powder particles on
the treated body were obtained at impact interaction of solid polymeric powder particles with the barrier. Relationships to calculate the contact time of deformable particles with the barrier, the radius of the contact area, pressure distribution along it, the maximum impact force and other indicators are presented.

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