An automatic hybrid numerical scheme for global RANS-LES approaches

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Abstract. A new automatic high-order hybrid central-difference/upwind scheme is developed for the finite-volume approximation of the inviscid fluxes within global hybrid RANS-LES approaches. Performance of the scheme is illustrated by examples of computations of three types of flow: a flow with massive separation with the use of Delayed Detached-Eddy Simulation (DDES), a flow with separation and reattachment with the use of DDES with shear-layer adapted subgrid length scale, and a fully attached flow with the use of Improved DDES (IDDES).

1. Introduction

Nowadays hybrid RANS-LES approaches to turbulence modeling are considered as a compromise between the high quality of Large Eddy Simulation (LES) in separated flow regions and low computational costs of the methods based on the Reynolds Averaged Navier-Stokes equations (RANS) in the near-wall regions of the flow. These methods are powerful tools of computational fluid dynamics and can be used in a wide range of applications. However their performance strongly depends upon the choice of discretization scheme for the governing equations.

In the zonal RANS-LES approaches that choice is dictated by their zonal nature: in LES regions the use of low-dissipative schemes is essential to ensure an acceptable resolution of small turbulent scales, while in RANS sub-domains more dissipative schemes should be used to avoid non-physical oscillations of the solution on the coarse, highly distorted grids typically used in these subdomains.

In the global hybrid approaches the RANS and LES sub-domains are defined automatically, and the position of the RANS-LES interface depends not only on the distance to the nearest wall and grid size but also on the solution. As a result, for global approaches the use of automatic weighted schemes is the most appropriate technique.

For the Detached Eddy Simulation (DES, [1]) such a scheme was proposed in the work [2] where it was demonstrated that for the “natural” DES applications (massively separated flows) this scheme ensures both high resolution of turbulent structures and stability (later on, this conclusion was supported in a large number of numerical studies). However, in the present work this scheme is shown to be sub-optimal in the framework of enhanced modifications of DES, namely, IDDES [3] and DDES with the shear-layer-adapted (SLA) subgrid scale [4], and a new hybrid scheme is proposed which performs equally well within all these DES versions. The scheme was tested in the framework of three types of simulations. First, in order to make sure that for the natural DES application area it performs not worse than the scheme [2], the scheme was applied for DDES of a massively separated flow. After that, a comparison of performance of the proposed scheme and scheme [2] was carried out for a flow...
with separation and reattachment in the framework of DDES combined with SLA definition of the subgrid length-scale and for a fully attached flow in the framework of the IDDES.

2. Formulation of automatic hybrid scheme

The idea of the proposed scheme is similar to that of the scheme [2]. In particular, it consists of the use of a weighted central-difference (CD) and upwind-biased (UB) schemes with a solution-dependent weight function. However the specific UB scheme and the weight function proposed in the present study differ from those of the scheme [2]. In particular, the weight function results in an effectively CD scheme not only in the detached flow areas, as in [2], but also in the attached boundary layers with resolved turbulent structures. In the inviscid (irrotational) areas of the flow and in the boundary layers with no turbulent content (RANS zone), the weight function, just as the weight function of [2], reduces the scheme to an UB one, but the UB scheme chosen in the present work is the so called BCD scheme [5] rather than fifth or third order upwind schemes, as in [2]. The BCD scheme works as an upwind or a central-difference scheme depending on whether the convection boundedness criterion is satisfied or not. Thus, the proposed scheme for the inviscid fluxes reads as:

\[ F = (1 - \sigma)F_{CD} + \sigma F_{BCD} \]  \hspace{1cm} (1)

Here \( F_{CD} \) and \( F_{BCD} \) are the inviscid fluxes approximated with the 4th order centred scheme and with the BCD scheme respectively, the latter presenting a combination of the 3rd order upwind and 4th order centered schemes, and \( \sigma \) is the weight function defined as follows:

\[ \sigma = \max(f_{inv}, f_u, f_{2DBL}) \]  \hspace{1cm} (2)

The functions \( f_{inv} \), \( f_u \) and \( f_{2D BL} \) are aimed at identifying different regions of the flow where the more stable BCD scheme should be used. The first one, \( f_{inv} \), serves for identification of the inviscid regions of the flow and is defined as follows:

\[ f_{inv} = 1 - \tanh\left(B^3\right), \]  \hspace{1cm} (3)

where \( B = C_H \Omega \max(S, \Omega)/\max\left\{S^2 + \Omega^2\right\}/2 \Omega_{max} \) is a function borrowed from the original hybrid scheme [2]. \( S \) and \( \Omega \) are magnitudes of the mean strain and vorticity, \( C_H = 2.0 \), \( \Omega_{min} = \tau^{-1} \) and \( \tau \) is a characteristic convective time. The functionality of \( f_{inv} \) is presented in Figure 1a, where the field of this function for the hump flow (see section 3.3 below) is shown.

The two other functions in the relation (2), \( f_u \) and \( f_{2D BL} \), ensure treatment of the attached boundary layers with no turbulent content with the BCD scheme and the boundary layers with resolved turbulent structures with the CD scheme. The \( f_u \) function is defined as follows:

\[ f_u = 1 - \tanh\left\{2 V_{LES} \right\} \varepsilon \left(\frac{V_{LES}}{V_i} \right)^2, \]  \hspace{1cm} (4)

where \( V_{LES} = \left(\min\left(0.2 \Delta_{max}, 0.4 \ell_d\right)\right) S \) is the eddy viscosity estimated using the Smagorinsky model with a subgrid scale definition \( \Delta_{max} \). This function activates the BCD scheme if the eddy viscosity predicted by the hybrid approach is higher than that predicted by LES.

The \( f_{2D BL} \) function is defined by the following relation:

\[ f_{2D BL} = \tanh\left(8 r_{at}\right)(1 - F_{KH}(<VTM>)), \]  \hspace{1cm} (5)

where \( r_{at} \) is an argument of the DDES shielding function [6] and the \( F_{KH}(<VTM>) \) function identifies 2D flow regions [4]:

\[ F_{KH}(<VTM>) = \max\{F_{KH}^{min}, \min\{F_{KH}^{max}, F_{KH}^{min} + \frac{F_{KH}^{max} - F_{KH}^{min}}{a_2 - a_1}(<VTM>-a_1)\}\}. \]
Here $\mathbf{VTM} = \frac{\sqrt{6} \left| (\mathbf{S} \cdot \mathbf{\omega}) \times \mathbf{\omega} \right|}{\omega^2 \sqrt{3tr(\mathbf{S}^T) - [tr(\mathbf{S})]^2}}$, angular brackets denote averaging over the current and closest neighboring cells, and the empirical constants are: $F_{\text{KH}}^{\max} = 1.0$, $F_{\text{KH}}^{\min} = 0.0$, $a_1 = 0.005$ and $a_2 = 0.01$.

The function equal to $\max\{f_{\nu}, f_{2D, BL}\}$ ensures approximation of the inviscid fluxes in the attached boundary layer with no turbulent content ($x < 0.6c$) with the BCD scheme (Figure 1b).

Finally, a snapshot of the resulting weight function $\sigma$ in the hump flow is presented in Figure 1c. One can see that with this weight function the scheme (1) performs as the BCD scheme in the irrotational flow region and in the boundary layer with no turbulent content and as the centred scheme in the separated region and in the reattached boundary layer with resolved turbulent structures.

Figure 1. Snapshots of the $f_{\nu}$ (a), $\max(f_{\nu}, f_{2D, BL})$ (b), and resulting weigh function $\sigma$ (c) from the simulation of the hump flow (see section 3.3 below)

3. Results and discussion

In this section computational set-ups are presented along with results of simulations of three test-cases chosen for evaluation of performance of the proposed scheme. The scheme was implemented in the incompressible branch of the NTS code, details of which are outlined in [7].

3.1. DES of massively separated flow

The first considered flow is that past the NACA 0021 airfoil at 60 degrees angle of attack studied in the experiment [8]. This is an example of massively separated flow, for which the DES approach was originally proposed and validated. The experimental Reynolds number based on the chord length, $c$, and free stream velocity, $U_0$, was equal to $2.7 \times 10^5$. Computational domain and “O-type” grid in an XY-plane used in the simulations are shown in Figure 2. The radius of the domain is equal to $15c$ and the grid has 211 cells in the azimuthal and 140 cells in the radial direction. It is refined in the vicinity of the leading and trailing edges of the airfoil where enhanced resolution is needed and is clustered towards the airfoil surface so that the first near-wall step in wall units is less than 1.0. The spanwise size of the domain was set equal to the value of $4c$. The 3D grid was generated by extruding of the XY-grid in the spanwise direction with a constant step equal to $0.02c$. As a result, the 3D grid contained nearly $5.5 \times 10^6$ cells. The time step in the simulations was equal to $0.02c/U_0$, which ensures the CFL number is less than 1.0 in the major part of the domain. The simulations were performed using the SA-based DDES.

Figure 2. Computational domain and grid in XY plane (every other grid line is shown)

Results of the simulations with the current scheme are compared with those obtained with the hybrid scheme [2]. As can be seen from vorticity magnitude snapshots presented on Figure 3, the
The proposed scheme provides somewhat faster shear layer roll up than the scheme [2]. This is explained by the different behavior of schemes in the separation region and in the close vicinity of the leading and trailing edges of the airfoil, where the weight of CD in the new scheme turns out to be higher than that in scheme [2] (Figure 4). Other than that, the weight of CD in the current scheme in the near-wall region within the recirculation bubble is higher than in scheme [2]. As a result, the mean pressure coefficient predicted by the proposed scheme is a bit closer to experimental data than the one predicted by scheme [2] (Figure 5). Note, however, that with the proposed scheme, slightly non-monotonic behavior of the pressure is observed near the edge of the airfoil.

Figure 3. Vorticity snapshots from DDES carried out with the hybrid scheme [2] (left frame) and hybrid scheme (1) (right frame)

Figure 4. Snapshots of the weight function of scheme [2] (left frame) and of scheme (1) (right frame)

Figure 5. Pressure coefficient distribution

3.2. Shear-layer adapted DDES of a flow over the wall-mounted hump

This flow (the so called 2D NASA wall-mounted hump) was studied in the experiments [9]. The Reynolds number based on the hump length $c$ and reference velocity $U_0$ (maximum free stream velocity at the inlet of the domain) in the experiment was equal to $9.36 \times 10^5$, while the height of the hump, $h$, was equal to 0.128$c$. The computational domain and the grid in the XY-plane used in the simulations are shown in Figure 6. The grid has 510 and 126 cells in the $x$- and $y$-directions, respectively. The size of the domain in the homogeneous z direction is 0.4$c$, and the grid-step $\Delta z$ is uniform and equal to $1.0 \times 10^{-2}c$. Thus, the total grid size is about 2.5 Million. The time step in the simulations is equal to $2 \times 10^{-3}c/U_0$ which ensures the CFL number is less than 1.0. Simulations were performed using the SA-based DDES with shear-layer-adapted subgrid scale [4].

Figure 6. Computational domain and grid in XY-plane (every other grid line is shown)

Again, results of the simulations with the hybrid scheme (1) are compared with those obtained with the use of the scheme [2].

As seen in Figure 7, the weight functions of the two schemes in these simulations turn out to be rather different. In particular, in the scheme [2] the weight of the upwind scheme is close to 1.0 in the
near-wall region both in the recirculation zone and in the reattached boundary layer. In contrast to this, in the scheme (1) these regions are treated with the pure CD scheme (Figure 7). As a result, using scheme [2] leads to a bit slower roll-up of the separated shear layer, to a noticeable under-prediction of the skin friction in the recirculation zones and in the reattached boundary layer, and to a strong under-prediction of the resolved Reynolds stresses (see Figure 8). All these lead to a somewhat better agreement of the predictions obtained with the use of scheme (1) with the experimental data.

Figure 7. Snapshots of the weight function of scheme [2] (left frame) and of scheme (1) (right frame). The black line corresponds to non-dimensional vorticity magnitude equal to 20

Figure 8. Distribution of the mean skin friction coefficient (left frame) and profile of resolved normal stress at \(x/c = 0.8\) (right frame)

3.3. IDDES of the wall bounded flow

Zero pressure gradient boundary layer over the flat plate is widely used for validation of turbulence models, both RANS and WMLES. In the present work this test case is used to evaluate performance of the proposed scheme within the framework of IDDES in the WMLES mode.

The Reynolds number based on the momentum thickness of the incoming boundary layer was equal to 1000. The computational domain has a size of \(L_x = 25\delta_0\), \(L_y = 100\delta_0\), and \(L_z = 3\delta_0\) in the streamwise, wall-normal and spanwise directions respectively. The grid \(N_x \times N_y \times N_z = 251 \times 71 \times 61\) was uniform in the \(x\)- and \(z\)-directions (\(\Delta x = 0.1\delta_0\), \(\Delta z = 0.05\delta_0\)) and clustered near the wall in the wall-normal direction with \(y_{\text{min}} = 0.0025\delta_0\). The corresponding steps in wall units are about \(\Delta x^+ = 40\), \(\Delta z^+ = 20\), and \(\Delta y_{\text{min}}^+ = 1.0\). The total size of the grid results in \(1.1 \times 10^6\) cells. The incoming turbulent content was created with the use of a Synthetic Turbulence Generator [11].

Results of the simulations are presented in Figure 9.

Figure 9. Profile of the weight function at \(x = 0.2\)m (left frame) and streamwise distribution of the mean skin friction coefficient in the flat plate boundary layer (right frame)

The figure reveals a drastic difference between the weight functions of the scheme [2] and of the current scheme in this flow. One can see that scheme [2] treats the inner part of the boundary layer up to the distance to the wall about 10 wall units with the upwind scheme, while the current scheme treats this region with virtually CD scheme. As a result, the latter ensures much higher accuracy of the mean
flow prediction. This is illustrated by the comparison of the skin-friction coefficient distributions computed with the use of the two schemes with the similar distribution predicted by the pure 4th order centred scheme (see right frame in Figure 9).

Conclusions
A new hybrid scheme for global RANS-LES approaches is developed and validated in the framework of different DES-like approaches. It is shown that within DDES of the massively separated flow over NACA 0021 airfoil, the scheme performs equally well as the scheme [2]. For a flow with separation and reattachment (NACA wall-mounted hump) computed with the use of DDES combined with shear-layer-adapted definition of the subgrid length-scale, the proposed scheme is superior to the scheme [2].

A major advantage of the proposed scheme over scheme [2] is observed in a fully attached flow (zero pressure gradient boundary layer) treated by IDDES in the wall-modelled LES mode.

Acknowledgments
The results of the work were obtained using computational resources of Peter the Great Sainte-Petersburg Polytechnic University Supercomputing Center (http://www.spbstu.ru).

References
[1] Spalart P R, Jou W-H, Strelets M and Allmaras S R 1997 Comments on the feasibility of LES for wings, and on a hybrid RANS/LES approach Proceedings of first AFOSR international conference on DNS/LES pp. 137–147
[2] Travin A, Shur M, Strelets M and Spalart P R 2002 Physical and numerical upgrades in the detached-eddy simulation of complex turbulent flows Adv. LES Complex Flows Proc. Euromech Colloq. 412 pp. 239–254
[3] Shur M L, P. R. Spalart, M. K. Strelets, and Travin A K 2008 A hybrid RANS-LES approach with delayed-DES and wall-modelled LES capabilities Int. J. Heat Fluid Flow vol. 29 no. 6 pp. 1638–1649
[4] Shur M L, Spalart P R, Strelets M K and Travin A K 2015 An Enhanced Version of DES with Rapid Transition from RANS to les in Separated Flows Flow, Turbul. Combust. vol. 95 no. 4 pp. 709–737
[5] Jasak H, Weller H G and Gosman A D 1999 High resolution NVD differencing scheme for arbitrarily unstructured meshes Int. J. Numer. Methods Fluids vol. 31 no. 2 pp. 431–449
[6] Spalart P R, Deck S, Shur M L, Squires K D, Strelets M K and Travin A 2006 A New Version of Detached-eddy Simulation, Resistant to Ambiguous Grid Densities Theor. Comput. Fluid Dyn. vol. 20 no. 3 pp. 181–195
[7] Shur M L, Strelets M K and Travin A K 2004 High-Order Implicit Multi-Block Navier-Stokes Code: Ten-Year Experience of Application to RANS/DES/LES/DNS of Turbulence http://cfd.spbstu.ru/agarbaruk/c/document_library/DLFE-42505.pdf
[8] Swalwell K, Sheridan J and Melbourne W 2003 Frequency Analysis of Surface Pressures on an Airfoil After Stall 21st AIAA Applied Aerodynamics Conference no. June pp. 1–8.
[9] Greenblatt D, Paschal K B, Yao C-S, Harris J, Schaeffler N W and Washburn A E 2005 A Separation Control CFD Validation Test Case Part 2 - Zero Efflux Oscillatory Blowing AIAA J. pp. 1–24
[10] Shur M L, Spalart P R, Strelets M K and Travin A K 2014 Synthetic turbulence generators for RANS-LES interfaces in zonal simulations of aerodynamic and aeroacoustic problems Flow, Turbul. Combust. vol. 93 no. 1 pp. 63–92