Nucleon-nucleon elastic scattering analysis to 2.5 GeV

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Abstract

A partial-wave analysis of $NN$ elastic scattering data has been completed. This analysis covers an expanded energy range, from threshold to a laboratory kinetic energy of 2.5 GeV, in order to include recent elastic $pp$ scattering data from the EDDA collaboration. The results of both single-energy and energy-dependent analyses are described.

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I. INTRODUCTION

This analysis of elastic nucleon-nucleon scattering data updates our previous analysis [1] to 1.6 GeV in the laboratory kinetic energy. The present analysis extends to 2.5 GeV, which is the limit for elastic $pp$ differential cross sections measured [2] by the EDDA collaboration using the cooler synchrotron at COSY.

Measurements with a laboratory kinetic energy near 2 GeV are particularly interesting as they correspond to a center-of-mass energy (2.7 GeV) which has been suggested [3] for a dibaryon resonance [4]. Near this energy, a sharp structure has been reported in the polarization observable $A_{yy}$ [5], and this was taken as support for such a resonance. A resonancelike structure, at about the same energy, has also been reported in an analysis by Hoshizaki [6]. The authors of Ref. [2] have considered this possibility, but find no evidence for a resonant excursion in their cross sections. Polarization measurements expected from COSY and SATURNE II will certainly help to clarify this issue.

The database above 1.6 GeV is mainly comprised of cross section measurements, much of this coming from Ref. [2]. In Section II we describe the expanded database, noting the additions below 1.6 GeV as well as the new region from 1.6 GeV to 2.5 GeV. While the most significant changes are seen in our $pp$ partial wave amplitudes, both $pp$ and $np$ data have been analyzed.

In Section III, we briefly review the formalism used in our analyses. Here we present the updated amplitudes and make comparisons with our previous solution (SM94) [1]. Fits with and without the new EDDA data are compared to show the influence of this particular measurement. Representative plots showing the agreement between our analysis (SM97) and cross section data have been generated to illustrate the quality of this fit. These results and the prospect for improvements are summarized in Section IV.

II. THE DATABASE

Our previous $NN$ scattering analyses [1] were based on 12838 $pp$ and 10918 $np$ data. In
The new $pp$ data have been produced mainly at COSY [2]. From this source, we have added differential cross sections ranging from 540 MeV to 2520 MeV in the proton kinetic energy and from 35° to 90° in the cm scattering angle. In addition to this, about 60 high quality polarized data ($P$, $A_{xx}$, $A_{yy}$, and $A_{zz}$) at 200 MeV were produced by the Indiana cooler [8]. Another 35 high accuracy differential cross sections between 490 and 790 MeV were recently published [9]. These measurements were made at LAMPF. We have added an excitation function of cross sections at 90° and between 0.3 and 0.4 MeV. These were measured at the Münster University low-energy machine [10]. We have also added a measurement of $A_{zz}$ at 650 MeV produced by LAMPF [11] but missed in the SAID database [1].

In constructing the data base extension from 1600 MeV to 2500 MeV, we reexamined a number of references in order to include higher energy data which had previously been neglected. This search netted additional data mainly from ANL (450 points) and Saclay (893 points). The complete set is listed [12] – [51] in alphabetical order.

The $np$ database has not been increased significantly and, as a result, we did not extend our analysis of the $I = 0$ system. New $np$ polarized data have been produced mainly by TRIUMF (101 points) [52][53], IUCF (33 points) [54], and LAMPF (49 points) [55]. The ANL–LAMPF–New Mexico University–Texas A & M University collaboration has finalized its analysis of 311 high quality $np$ polarized observables ($A_{xx}$, $A_{zz}$, $A_{yy}$, and $A_{zx}$) between 485 and 790 MeV and ranging from 25° to 180° [56]. These measurements were published
previously in Ref. [57]. A few total cross sections in pure spin states between 4 and 16 MeV were produced by TUNL [58] and Charles University at Prague [59,60]. Recently, the final LAMPF $\Delta \sigma_L$ measurements between 480 and 790 MeV were also published [61]. In addition, some new $\Delta \sigma_L$ measurements above 1190 MeV were made at JINR (Dubna) [62]. Added unpolarized measurements include 15 $np$ differential cross sections at 67 MeV from PSI [63] and 6 differential cross sections at 14 MeV from Tübingen University [64]. A few missed differential cross sections at low energies from LAMPF [65] and at 1240 MeV from Berkeley [66] were also added.

A few data sets were added to the data base but not included in the analysis. These include 82 missed $np$ total cross section measurements between 4 and 231 MeV from LAMPF [67]. We excluded these data from the analysis in order to retain the same database (below 350 MeV) as was used in the Nijmegen analysis [68]. This also applies to a new set of $np$ differential cross sections at 162 MeV and at backward angles which were measured at the Svedberg Facility at Uppsala [69].

### III. PARTIAL-WAVE ANALYSIS

Our first attempts to extend the range of the $NN$ analysis used the parameterization scheme of Ref. [1]. These were unsuccessful. The problem was traced to the basis functions used to expand our K-matrix elements. Many of these become nearly degenerate as the kinetic energy of the incoming nucleon ($T$) increases to 2.5 GeV. As a result, a modified form was used in the present analysis. Apart from this difference, the formalism used here is identical to that used in Ref. [1]. The reader is directed to Refs. [70,71] for more details. In the following we just outline the method used, in order to show how the modified basis functions fit into our parameterization scheme.

For uncoupled partial waves ($^1D_2, ^3F_3, ...$), an S-matrix ($S = S_E S_I$) is used. This product S-matrix is constructed from exchange ($S_E$) and inelastic ($S_I$) pieces. $S_E$ is parameterized in terms of a K-matrix.
\[ S_E = \frac{(1 + iK_E)}{(1 - iK_E)}, \]  

which in turn is expanded as

\[ K_E = \text{Born} + \sum_i \alpha_i A_{li}. \]

Here the Born term gives the single-pion exchange contribution and \( \alpha_i \) are free parameters. The expansion basis elements, \( A_{li} \), are given by

\[ A_{li} = F_{li} \left( \frac{T}{T + T_C} \right)^{i-1}, \]

where the function \( F_{li} \), used as the expansion basis in our previous fits [1], is given by

\[ F_{li}(T) = \frac{4\mu^2}{MT} \int_0^1 Q_l \left( \frac{x_0 - x}{1 - x} \right) x^{i-1/2} \frac{1}{1 - x} dx. \]

Here \( M (\mu) \) is the nucleon (pion) mass and \( x_0 = 1 + (4\mu^2/MT) \). \( Q_l \) is a Legendre function of the second kind. In Eq. (3), \( T_C \) is a parameter which was chosen to be 1 GeV. (The fit was not sensitive to this choice; fits using 0.5 GeV and 1.5 GeV were also attempted.) The basis function given in Eq. (4) was derived in Ref. [71].

To ensure time-reversal invariance, the spin-coupled waves (for example, \( ^3P_2 - ^3F_2 \)) are parameterized as

\[ S(2 \times 2) = S_{E}^{1/2} S_{I} S_{E}^{1/2}, \]

where again the matrix \( S_{E} \) is expanded in terms of a K-matrix with the elements

\[ K_m = \text{Born}_m + \sum_i A_{lm_i}, \]

the subscript \( (m = (+, 0, -)) \) labeling states with \( l_m = (J + 1, J, J - 1) \). As in Ref. [1], the matrix \( S_{I} \) is taken from a Chew-Mandelstam K-matrix coupling the \( NN \) channel to an appropriate \( N\Delta \) state. This has been extensively described in Ref. [70]. The simple modification of the basis elements, displayed in Eq. (3), provided the added flexibility required to extend our analysis to 2.5 GeV.
In Table I, we compare the energy-dependent and single-energy fits over the energy bins used in the single-energy analyses. Also listed are the number of parameters varied in each single-energy solution. A total of 144 parameters were varied in the energy-dependent analysis.

Our single-energy and energy-dependent results for the isovector and isoscalar partial-wave amplitudes are displayed in Figs. 1 and 2. Here we also compare with our previous fit (SM94). In some cases the changes are quite large. This is particularly true near the upper energy limit of SM94, and for the smaller partial waves. The effect of these changes can be clearly seen in Fig. 3, where we show how well the new EDDA data are reproduced by both SM94 and SM97. The influence of this experiment is most pronounced in the forward direction.

In general, we find little structure over the higher energy region. This reflects the smooth, and rather flat, total and reaction cross sections between 1.5 GeV and 2.5 GeV. Our fit to these quantities is displayed in Fig. 4. Note that the reaction cross sections were excluded from our fit. This verifies that the set of total, total elastic (deduced from differential cross sections), and reaction cross sections are self-consistent.

The present analysis actually gives an improved fit to the data below 1.6 GeV. This is due to the altered basis set, found necessary to fit the higher energy data. Numerical comparisons are given in Table II. Here we see that the COSY data comprise a large fraction of the total set above 1.6 GeV. The results of analyses with (SM97) and without (NM97) this data set show how influential these measurements have been in determining the amplitudes. (The fits SM97 and NM97 used identical parameterization schemes. Only the data base was changed.) The COSY data contribute a $\chi^2$/datum of 1.07 when included in the fit. This jumps to 5.6 when we attempt a prediction based on the remaining data.

IV. CONCLUSIONS AND FUTURE PROSPECTS

We have extended our $pp$ partial-wave analyses nearly 1 GeV beyond the limit quoted in our previously published results. The present range has been selected to include all
of the recent elastic $pp$ cross section data measured by the EDDA group [2]. We found that it was possible to simultaneously fit the $pp$ total cross section data, in particular the precise data of Ref. [23], along with differential cross sections from the EDDA collaboration [2]. The resulting reaction cross sections, which were not included in the fit, are quite well reproduced. The predicted reaction cross sections are consistent with the results of Ref. [72] at lower energies, but deviate from these and follow the results of Ref. [73] above 1 GeV.

While we find that the partial-wave amplitudes above 1.6 GeV are smooth and structureless, reflecting the behavior seen in the total and elastic cross section data, we have also considered the effect of more localized structures reported in polarization measurements [3,5]. We can add resonancelike structures in individual partial-waves to see their effect on any observable. This will be utilized as more polarization data become available.

As the high energy region was constrained mainly by cross section data, the present solution should be considered as a guide to the expected amplitudes. The EDDA collaboration is planning to measure $P$, $A_{yy}$, $A_{xx}$, and $A_{xz}$ in the near future. This will be crucial to any future analyses.

Further data is also expected from a number of other labs. About 2000 polarized $pp$ measurements are expected above 1000 MeV [74] as the nucleon-nucleon program at SATURNE II is completed. While not included in the present fit, preliminary data [75] from SATURNE II is in reasonable agreement with our predictions. A representative fit to $P$ data, at 2.16 GeV, is given in Fig. 5.

A similar number of polarized quantities from $np$ elastic scattering are expected (between 250 and 560 MeV [76]) from PSI. The Freiburg University group is also planning to publish $np$ measurements which were done at PSI at the beginning of the 1980’s. These data range from 200 to 580 MeV and from $77^\circ$ to $179^\circ$ [77]. Final $np$ differential cross sections between $73^\circ$ and $179^\circ$ measured at Uppsalla [78] are expected to replace data at 96 MeV [79] and 162 MeV [79]. IUCF is also measuring $np$ differential cross sections in the backward direction at about 200 MeV to solve a shape problem in the angular distribution [80]. Other $np$ sources include an extension of $\Delta \sigma_L$ measurements [81] at JINR [62], TRIUMF analyzing
power measurements at 350 MeV \[82\], and TUNL measurements \[83\] of the P parameter and $\Delta\sigma_L$ at 7 and 15 MeV. We will continue to update our energy-dependent and single-energy solutions as the new measurements become available.

Finally we note that by extending our analysis to 2.5 GeV, we may be bridging the gap between the low- and high-energy regions. This is suggested if we plot $d\sigma/dt$ versus $s$, as is shown in Fig. 6. The result expected from dimensional counting at high-energy and fixed cm angle \[84,85\] is

$$
\frac{d\sigma}{dt} \sim \frac{1}{s^{N-2}} = s^{-10},
$$

(7)

where $N$ is the minimum number of fundamental constituents (quarks). While a slightly extended energy range would be more definitive, our results do appear to be consistent with this limit.

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FIGURE CAPTIONS

Figure 1. Isovector partial-wave amplitudes from 0 to 2.4 GeV in the proton kinetic energy. Solid curves give the amplitudes corresponding to the SM97 solution. The real (imaginary) parts of the single-energy solutions are plotted as triangles (squares). For comparison, the previous solution SM94 [1] is plotted with (+) marks. The (x) marks give $\text{Im}T-T^2-T_{sf}^2$ from SM97, where $T_{sf}^2$ is the spin-flip amplitude. All amplitudes are dimensionless.

Figure 2. Isoscalar partial-wave amplitudes from 0 to 1.2 GeV. Notation as in Fig. 1.

Figure 3. Comparison between SM97 (solid curve) and differential cross sections at (a) $\theta^* = 45^\circ \pm 1^\circ$ and (b) $\theta^* = 90^\circ \pm 1^\circ$. Recent COSY measurements [2] are plotted as filled circles. Other data from the SAID data base [7] are plotted as crosses. Our previous solution (SM94) is plotted to 1.6 GeV (dot-dashed line).

Figure 4. Total cross section comparisons. (a) The solid (dashed) curves give the predictions of solution SM97 for the total (total elastic) cross section. Experimental points are from the SAID data base [7]; filled circles are from Ref. [23]. (b) The solid curve gives the total reaction cross section of SM97. Filled circles are estimates from Ref. [72]. Filled triangles are estimates from Ref. [73].

Figure 5. Angular dependence of recent SATURNE II analyzing power ($P$) data [75]. This measurement, at 2.16 GeV, was not included the SM97 analysis. The solid line gives the SM97 prediction. The dashed lines are generated from a single-energy solution and its associated error estimate.

Figure 6. $d\sigma/dt$ plotted as a function of $s$ at $\theta^* = 90^\circ$. The SM97 solution is plotted as a solid curve. The dash-dotted line gives $d\sigma/dt \sim s^{-10}$. The plotted data are from Ref. [86].
Table I. Comparison of the single-energy (SES) and energy-dependent (SM97) fits to $pp$ and $np$ data. Values of $\chi^2$ are given for the SES and SM97 fits (evaluated over the same energy bins). Also listed is the number of parameters varied in each single-energy solution.

| Energy Range (MeV) | $\chi^2$ SES(SM97)/$pp$ data | $\chi^2$ SES(SM97)/$np$ data | Parameters |
|-------------------|-------------------------------|-------------------------------|------------|
| 4-6               | 22(39)/28                     | 50(66)/53                     | 6          |
| 7-12              | 84(132)/88                    | 221(309)/87                   | 6          |
| 11-19             | 17(49)/27                     | 191(445)/236                  | 8          |
| 19-30             | 123(275)/114                  | 263(286)/295                  | 8          |
| 32-67             | 294(375)/224                  | 667(754)/485                  | 10         |
| 60-90             | 55(64)/72                     | 457(595)/329                  | 10         |
| 80-120            | 161(185)/154                  | 419(487)/353                  | 10         |
| 125-174           | 301(310)/287                  | 328(367)/272                  | 11         |
| 175-225           | 249(354)/212                  | 715(766)/499                  | 13         |
| 225-270           | 66(91)/64                     | 243(270)/236                  | 13         |
| 276-325           | 274(309)/256                  | 571(655)/518                  | 17         |
| 325-375           | 297(320)/246                  | 421(474)/353                  | 17         |
| 375-425           | 555(601)/436                  | 753(843)/549                  | 17         |
| 425-475           | 902(1004)/665                 | 775(799)/629                  | 18         |
| 475-525           | 1322(1484)/1081               | 1252(1419)/787                | 30         |
| 525-575           | 861(972)/754                  | 549(584)/432                  | 31         |
| 575-625           | 1032(1154)/760                | 422(491)/367                  | 36         |
| 625-675           | 891(863)/754                  | 1263(1563)/875                | 36         |
| 675-725           | 838(882)/777                  | 403(473)/386                  | 37         |
Table I. (continued)

| Energy Range (MeV) | $\chi^2$ SES(SM97)/$pp$ data | $\chi^2$ SES(SM97)/$np$ data | Parameters |
|--------------------|-----------------------------|-----------------------------|------------|
| 725-775            | 990(1195)/827               | 512(558)/374               | 37         |
| 775-824            | 1583(1754)/1170             | 1518(1845)/944             | 38         |
| 827-874            | 1195(1358)/939              | 386(497)/366               | 39         |
| 876-924            | 341(412)/389                | 753(920)/628               | 41         |
| 926-974            | 790(945)/679                | 354(498)/352               | 43         |
| 976-1020           | 931(1131)/708               | 300(441)/331               | 43         |
| 1078-1125          | 528(689)/413                | 427(671)/326               | 45         |
| 1261-1299          | 680(972)/507                | 242(377)/248               | 29         |
| 1481-1521          | 139(266)/149                | 262(441)/283               | 29         |
| 1590-1656          | 472(655)/409                | 336(514)/350               | 31         |
| 1685-1724          | 185(293)/118                | 182(291)/117               | 31         |
| 1778-1818          | 404(628)/347                | 348(532)/375               | 31         |
| 1929-1968          | 218(271)/168                | 216(270)/167               | 31         |
| 2065-2104          | 673(1241)/431               | 229(371)/248               | 31         |
| 2176-2224          | 1005(1325)/377              | 375(514)/377               | 31         |
| 2330-2470          | 803(1257)/458               | 803(1257)/458              | 31         |
Table II. Comparison of present and previous solutions. Dataset A was used in the SM94 analysis [1]. Dataset B contains all data (apart from the EDDA data [2]) used in generating solution SM97. See the text for details regarding the SM97 and NM97 fits.

| PWA     | Data                  | $\chi^2/_{pp}$ data | $\chi^2/_{np}$ data |
|---------|-----------------------|---------------------|----------------------|
|         | (0-1600 MeV)          | (0-1300 MeV)        |                      |
| SM94    | (dataset A)           | 22375/12838         | 17516/10918          |
| SM94    | (dataset B)           | 22390/12889         | 18480/10843          |
| SM97    | (dataset B)           | 20910/12889         | 17400/10843          |
|         | (0-2520 MeV)          | (0-2000 MeV)        |                      |
| SM97    | (dataset B)           | 26460/14873         | 17440/10854          |
| SM97    | (EDDA dataset [2])    | 2278/2121           | –                    |
| NM97    | (dataset B)           | 25240/14873         | 17280/10854          |
| NM97    | (EDDA dataset [2])    | 11964/2121          | –                    |