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Effect of a timebase mismatch in two-way optical frequency transfer

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Abstract

Two-way frequency transfer on optical fibers is a powerful technique for the comparison of distant clocks over long and ultra-long hauls. In contrast to traditional Doppler noise cancellation, it is capable of sustaining higher link attenuation, mitigating the need of optical amplification and regeneration and thus reducing the setup complexity. We investigate the ultimate limitations of the two-way approach on a 300 km multiplexed fiber haul, considering fully independent setups and acquisition systems at the two link ends. We derive a theoretical model to predict the performance deterioration due to a bad synchronisation of the measurements, which is confirmed by experimental results. This study demonstrates that two-way optical frequency transfer is a reliable and performing technique, capable of sustaining remote clocks comparisons at the $10^{-19}$ resolution, and is relevant for the development of a fiber network of continental scale for frequency metrology in Europe.

Keywords: fiber frequency transfer, optical clocks comparisons, fiber links, phase noise measurements, optical phase post-processing

(Some figures may appear in colour only in the online journal)
Figure 1. Experimental setup for frequency comparison. L, lasers; FPC, Fabry–Perot cavities; C, fiber couplers; FM, Faraday mirrors; PD, photodiode; AOM, acousto-optic modulators; EDFA, erbium-doped fiber amplifiers; MUX, dense wavelength division multiplexing filters.

Our experiment uses a couple of 150 km-long buried fibers, placed in the same cables between INRIM and the Laboratoire Souterraine de Modane (LSM) on the Italy–France border. The two fibers are joined together at LSM to obtain a 300 km loop with both ends at INRIM. This has been done for convenience, nevertheless two completely independent setups for signal detection and data acquisition have been realized. Our testbed is based on a looped configuration with both ends in the same laboratory and the signal travelling on two parallel fibers. The noise of the two fibers is partially correlated as the fibers are placed in the same cable: hence, the residual noise after processing is two orders of magnitude lower than for a real link where the two end points are spatially separated. However, this does not affect data acquisition and allows us to investigate the ultimate resolution of this technique. On both fibers, mixed dense- and coarse-wavelength division multiplexing transmission is implemented (14 km and 286 km respectively) where channel 44 of the ITU grid is dedicated to our experiment. The total loss of the fiber is 105 dB, as the metropolitan spans have significantly high losses and reflections at the connectors. Five EDFAs are used as intermediate amplification stages, reducing the uncompensated loss to 36 dB.

The experimental setup is shown in figure 1. We use two ultrastable lasers (L1, L2), locked to two high-finesse Fabry–Perot cavities. Both lasers are loosely phase-locked to an H-maser using a frequency comb, to ensure long-term frequency variations $\gamma < 10^{-15}$. An ultimate frequency instability on the remote comparison of the order of $\gamma_0$ had been
observed otherwise, being \( \delta \) the link delay. In our case, with a drift of roughly 0.1 Hz s\(^{-1}\) and \( \delta = 1.5 \) ms, we observed a frequency offset at the level of \( \sim 10^{-18} \).

The lasers are launched in the fiber in opposite directions. At both ends, couplers C1 and C2 are used to produce the beatnotes between local and remote lasers (\( bn1 \) and \( bn2 \) respectively). Two acousto optic modulators (AOM) shift the laser frequencies at each pass, to distinguish the signal from stray backreflections. We carefully isolated the couplers into two separate boxes to reduce noise on uncommon fiber branches. The beatnotes are detected with low-noise photodiodes and redundantly tracked with voltage-controlled oscillators, for cycle slips detection. To maximize the SNR, the polarization between local and remote laser is occasionally optimized using a manual polarization controller.

The phase variations of each beatnote can be written as:
\[
\phi_{bn1}(t) - \phi_{bn2}(t) = \Delta \phi_{\text{link1-2}}(t) \tag{1}
\]
where \( \phi_{bn1}(t) \) and \( \phi_{bn2}(t) \) represent the lasers contributions, while \( \phi_{\text{link1-2}}(t) \) are the contributions of the link in the two directions. By subtracting equations (1) and (2) (time dependence neglected for the sake of clarity):
\[
\phi_{bn1} - \phi_{bn2} = 2(\phi_{L1} - \phi_{L2}) - \phi_{\text{link1-2}} + \phi_{\text{link2-1}} \tag{3}
\]
the correlated part of the fiber phase noise between forward and backward propagation cancels, limited by the propagation delay. Thus we can retrieve the uncorrelated phase noise together with the relative phase variations of the lasers.

In order to remove the lasers contribution and evaluate the residual noise of the system, we generate two additional beatnotes (\( bn3 \) and \( bn4 \)) using the standard relations of Fourier calculus, we obtain
\[
S_{\phi,\text{link}}(f) = \frac{1}{2} \left( \exp^{-i2\pi f \tau} - 1 \right) S_{\phi,\text{fiber}}(f), \tag{6}
\]
in the limit of low frequencies, where \( S_{\phi,\text{fiber}}(f) \) is the PSD of the one-way fiber noise and \( \delta = \frac{\Delta}{2L} \) is the link delay, with \( n \) the refractive index of the fiber and \( L \) its length. According to equation (3), when comparing two optical signals, this contribution is further divided by a factor of four.

In practice, we deal with sampled signals and shall assume that, if acquisitions are not synchronized, we cannot associate the same timestamps to the two data series and directly perform the subtraction. Hence, one of the two sampled series would need to be interpolated and recalculated on a common timebase. The sampled form of \( bn1 \) and \( bn2 \) can be written as \( \phi_{bn1,k} = \phi_{bn1}(t_0 + k\tau) \) and \( \phi_{bn2,k} = \phi_{bn2}(t_0 + \Delta + k\tau) \) with \( k \in \mathbb{N} \), where \( \tau \) is the gate-time of the acquisition, \( t_0 \) is an arbitrary origin and \( \Delta \) is the delay between the two acquisitions. The re-interpolated values of \( bn1 \) on the timebase of \( bn2 \) is:
\[
\phi'_{bn1,k} = \frac{\Delta}{\tau} \phi_{bn1,k+1} + \left( 1 - \frac{\Delta}{\tau} \right) \phi_{bn1,k} \tag{7}
\]
and using the standard relations of Fourier calculus, we obtain the power spectral density of the residual link noise in a two-way scheme where timebase mismatch is considered:
\[
S_{\phi,\text{link}}(f) = \frac{\Delta}{\tau} \left[ \exp^{-i2\pi f \tau} - 1 \right] S_{\phi,\text{fiber}}(f). \tag{9}
\]

3. Experimental results
We show the results obtained on the 300 km test-bed, comparing measurements with and without external trigger. We acquired the beatnotes phase on two independent, dead-time-free phase/frequency counters, one for each side of the link. Figure 2 shows the phase noise PSD of each of the beatnotes between local and fiber-delivered light measured at opposite ends (red line) and of their difference evaluated from PPS-synchronized measurements (blue line). This noise is two orders of magnitude lower than expected from equation (6), since our test-bed is based on two parallel fibers. This is compatible with previous phase-noise measurements we
performed on each of the 150 km fibers, that showed correlation between the two. Indeed, the PSD of the phase-difference between the signals transmitted through the two fibers was 20 dB lower than the PSD of the signal transmitted through each of them in the Fourier frequency range between 0.1 Hz and 100 Hz. This does not affect the results reported here, which aim at determining the ultimate performances of a two-way scheme.

The green spectrum represents the beatnote difference between unsynchronized measurements, where the delay was determined by cross-correlation and data on one counter were interpolated to align the two timebases. Incidentally, we note that for unsynchronized measurements the cross-correlation is never as high as for synchronized measurements; this is a consequence of equation (7). A good agreement is observed with the expected noise as evaluated from equation (9) (black dashed line). The noise is several orders of magnitude higher than that of synchronized measurements, especially at high frequencies.

We stress that high frequency noise is responsible for an increase of the link instability even at long averaging times [15] when the Allan deviation is used as an estimator. This is due to the fact that the Allan deviation cannot improve faster than \( \tau^3 \), where \( \tau \) is the measurement time, therefore the only way to mitigate this effect is to dramatically reduce the measurement bandwidth. As the high-frequency noise contribution increases, more selective filters are needed, which usually require additional computational effort. Therefore, a commonly used compromise, which we also exploit in this work, is to rely on a simple-averaging filter: this is obtained using phase/frequency counters in the so-called ‘averaging’ mode, or \( \Lambda \)-mode.

In figure 3 we show the Allan deviation calculated from phase samples obtained using a commercial counter in \( \Lambda \)-mode with a gate time of 100 ms; we considered multiple acquisitions where \( \Delta \) assumed different values and we interpolated data following the approach detailed in section 2. The measurement noise depends on \( \Delta \) as shown in equation (9) and we calculated it for the various acquisitions. We then calculated the Allan deviation corresponding to a noise of type \( S_\phi(f) = \frac{h f^2}{4 \nu_0^2 \tau a^2} \): \[
\sigma^2(\tau) = \frac{h f_{th}^2}{4 \nu_0^2 \tau a^2} \cdot \tau a^{-2}
\] (10)
where \( f_{th} \) is the measurement bandwidth, \( \nu_0 \) is the absolute frequency of the laser and \( \tau a \) is the averaging time. Merging equations (9) and (10), it is straightforward to derive \( \sigma_\Delta(t_a)/\sigma_0(t_a) \), in agreement with the results shown in figure 3.

Figure 4 shows the instability of the two-way frequency transfer over a 300 km link when the acquisition systems are hardware-synchronised to an external PPS. We removed two cycles-slips from the whole measurement, detected by redundant counting. A deviation from the \( \tau a^{-1} \) slope is observed on the Allan deviation for \( \tau a > 100 \) s, and this two-way implementation achieves an ultimate stability of...
by replacing the long link with a fixed attenuator. The
level of resolution to the instability can be rejected down to an ultimate
us to investigate the ultimate technical limits of the tech-
for light transmission and data acquisition, which enabled
a 300 km testbed using two completely independent setups
We implemented an optical two-way transfer technique on
4. Conclusions
We implemented an optical two-way transfer technique on
10 m fiber link and is considered as the link ultimate uncertainty. No frequency shifts have been measured within
this uncertainty.

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3 × 10−19 in 104 s. The interferometers noise floor accounts for an instability of ∼1 × 10−19, which was measured by replacing the long link with a fixed attenuator. The 3 × 10−19 limit is attributed to polarization mode dispersion along the fiber link and is considered as the link ultimate uncertainty. No frequency shifts have been measured within this uncertainty.