Consequences of statistical sense determination for WIMP directional detection

Anne M. Green

School of Physics and Astronomy, University of Nottingham, University Park, Nottingham, NG7 2RD, UK

Ben Morgan

Department of Physics, University of Warwick, Coventry, CV4 7AL, UK

(Dated: February 5, 2008)

We study the consequences of limited recoil sense reconstruction on the number of events required to reject isotropy and detect a WIMP signal using a directional detector. For a constant probability of determining the sense correctly, 3-d read-out and zero background, we find that as the probability is decreased from 1.0 to 0.75 the number of events required to reject isotropy using the mean angle statistic is increased by a factor of a few. As the probability is decreased further the number of events required using this statistic increases sharply, and in fact isotropy can be rejected more easily by discarding the sense information and using axial statistics. This however requires an order of magnitude more events than vectorial data with perfect sense determination. We also consider energy dependent probabilities of correctly measuring the sense, 2-d read-out and non-zero background. Our main conclusion regarding the sense determination is that correctly determining the sense of the abundant, but less anisotropic, low energy recoils is most important for minimising the number of events required.

PACS numbers: 95.35.+d

I. INTRODUCTION

Weakly Interacting Massive Particle (WIMP) direct detection experiments aim to directly detect non-baryonic dark matter via the elastic scattering of WIMPs on detector nuclei, and are presently reaching the sensitivity required to detect the lightest neutralino (which in most supersymmetry models is the lightest supersymmetric particle and an excellent WIMP candidate). Since the expected event rates are very small (O(10^{-5} - 1) counts kg^{-1} day^{-1}) distinguishing a putative WIMP signal from backgrounds due to, for instance, neutrons from cosmic-ray induced muons or natural radioactivity, is crucial. The direction dependence of the WIMP scattering rate (due to the Earth’s motion with respect to the Galactic rest frame) provides a potential WIMP ‘smoking gun’ and low pressure gas time projection chambers (TPCs), such as DRIFT (Directional Recoil Identification From Tracks) and NEWAGE, seem to offer the best prospects for a detector capable of measuring the directions of sub-100 keV nuclear recoils.

Early studies found that in principle as few as 30 events would be required to distinguish a WIMP induced signal from isotropic backgrounds. In reality the number of events, and hence the exposure required, depends on the detector properties including the energy threshold, background event rate, whether the read-out measures the recoil momentum in 2 or 3 dimensions (and if 2-d in which plane) and whether the sense (i.e. the absolute sign +⃗p or −⃗p) of the recoil momentum vectors can be measured. The factor with the biggest effect on the number of events required is whether or not the sense of the recoils can be measured. If the data is axial (i.e. the sense can not be measured) the exposure is increased, compared to the vectorial case, by one order of magnitude for 3-d read-out and at least two orders of magnitude for 2-d read-out in the optimal plane. At low energies, the energy deposition dE/dx of nuclear recoils, and hence the density of ionisation created, is predicted to slowly decrease with decreasing recoil energy. Thus the sense of a recoil track is, in principle, measurable by determining the direction in which the ionisation density decreases along the track.

Previous work on simulating recoil tracks using SRIM2003 indicated that the ionization density distributions reconstructed from recoil tracks were close to uniform due to both fluctuations in the production of ionisation and diffusion during the drift of this ionisation to the read-out plane. It was therefore not possible to determine the absolute signs of the reconstructed recoil vectors in these simulations. It should be noted however that while SRIM2003 predicts sulfur recoil ranges and quenching factors (fraction of recoil energy going into ionisation) in agreement with experimental data, it was not designed to model recoils in gaseous targets.

Whether or not the sense of low energy nuclear recoils can be determined is therefore an important question which needs to be resolved experimentally. Ongoing studies indicate that the expected decrease in dE/dx is observable, but fluctuations in the ionization distribution caused by the ionization process and diffusion may wash out the information in the ionization distribution leading to errors in sense reconstruc-
tion. In other words, if the sense of a recoil of energy $E$ is determined via a hypothesis test on some parameter $M$ derived from its reconstructed ionization distribution, then the PDFs $g(M|\text{forward sense}, E)$ and $g(M|\text{backward sense}, E)$ will not be cleanly separated, i.e. the significance level, $\mathcal{S}(E)$, is greater than zero and the power, $\mathcal{P}(E)$, is less than one. Here, the significance level is the probability of rejecting the forward sense hypothesis when it is true, and the power is the probability of rejecting the backward sense hypothesis when it is false\cite{13}. Provided $\mathcal{S}(E) < 0.5$ and $\mathcal{P}(E) > 0.5$, the correct recoil sense will be reconstructed more often than not with probability $P(\text{correct sense}|E) = 1 - \mathcal{S}(E)$. In this Brief Report we therefore study the effect of this probabilistic sense reconstruction on the number of events required to distinguish a WIMP induced recoil signal from isotropic backgrounds.

**II. CALCULATIONS**

We use the same statistical techniques and methods for calculating the directional nuclear recoil spectrum as in Refs.\cite{8,10,12}. We briefly summarise these procedures here, for further details see these references and Ref.\cite{20}. We consider the simplest possible model for the Milky Way halo, an isotropic sphere with local density $\rho = 0.3\ \text{GeV cm}^{-3}$ and a Maxwellian velocity distribution with dispersion $\sigma_v = 270\ \text{km s}^{-1}$, and fix the WIMP mass at $m_{\chi} = 100\ \text{keV}$. Our simulated detector is a TPC filled with 0.05 bar CS$_2$ gas, a 200 $\mu$m pitch micropixel readout plane, a 10 cm drift length over which a uniform electric field of 1 kV cm$^{-1}$ is applied, and is based on the design of the DRIFT-I/II detector\cite{4}. We use the SRIM2003\cite{15} package to generate sulfur recoil tracks and for 3-d read-out recoil directions are reconstructed as the principal axis of the charge distributions recorded by the pixels. 2-d read-out would measure the projection of the recoil momentum vector into a plane fixed on the Earth. The degree of anisotropy of the 2-d recoil angles (and hence the detectability of a WIMP signal) depends on the orientation of the read-out plane\cite{10,11,12}. Here we focus on the optimal case of a read-out plane with normal perpendicular to the spin axis of the Earth. No simulations of the angular resolution function of a 2-d detector are available\cite{21} we therefore assume perfect resolution in this case (and hence the resulting numbers of events are lower limits on the number which would be required in reality). For primary recoil energies below $E_{\text{th}} = 20\ \text{keV}$ the short track length (3-4 pixels) and multiple scattering make it impossible to reconstruct the track direction in our simulated detector, we therefore only consider events with energies above this threshold. Zero background is the goal of the next generation of experiments made from low activity materials with efficient gamma rejection and shielding, located deep underground\cite{21}, however we investigate the effect of non-zero isotropic background by varying the ratio of the background and signal event rates.

Recoil directions in 3-d and 2-d constitute points on the unit sphere and circle respectively. For 3-d data the most powerful test for rejecting isotropy uses the average of the cosine of the angle between the direction of solar motion and the recoil direction, $\langle \cos\theta \rangle$\cite{4}. For 2-d data the most powerful test is the Rayleigh test\cite{22} which uses the mean resultant length of the projected recoil vectors which, modulo fluctuations, should be zero for data drawn from an isotropic distribution\cite{10}.

We calculate the probability distribution of the relevant statistic, for a given number of events $N$, by Monte Carlo generating $10^5$ experiments each observing $N$ recoils drawn from our simulated 3-d or 2-d distributions. We then compare this with the null distribution of the statistic, under the assumption of isotropy and calculate the rejection and acceptance factors, $R$ and $A$, at each value $T$ of the statistic. The rejection factor is the probability of measuring a value of the statistic less than $T$ if the null (isotropic) hypothesis is true or equivalently the confidence with which the null hypothesis can be rejected given that measured value of the statistic. The acceptance is the probability of measuring a value of the statistic larger than $T$ if the alternative hypothesis is true or equivalently the fraction of experiments in which the alternative hypothesis is true that measure a larger absolute value of the test statistic and hence reject the null hypothesis at confidence level $R$. Clearly a high value of $R$ is required to reject the null hypothesis, while a high $A$ is also required, otherwise any one experiment might not be able to reject the null hypothesis at the given $R$ or the null hypothesis might be erroneously rejected. We therefore find the number of events required for $A = R_c = 0.95$, $N_{\text{95}}$.

We consider several functional forms for the energy dependence of the probability of correctly determining the recoil sense, $P(\text{correct sense}|E)$ (hereafter $P_{cs}(E)$). Our simplest model assumes an energy independent probability, $P_{cs}(E) = P_{100\text{keV}}$, with $0.55 \leq P_{100\text{keV}} \leq 1.0$. In reality it is likely that it will be easier to determine the sense of higher energy recoils due to their longer track lengths (c.f. Ref.\cite{18}). We therefore also consider linearly increasing probability $P_{cs}(E) = aE + b$. We parametrise this function in terms of the values of $P_{cs}(E_{\text{th}} = 20\ \text{keV})$, where $E_{\text{th}}$ is the threshold energy as above, and $P_{cs}(E = 100\ \text{keV}) = P_{100\text{keV}}$. We consider two parameter sets for this parametrization, $P_{cs}(E_{\text{th}}) = 0.50$ with $0.55 \leq P_{100\text{keV}} \leq 1.0$, and $P_{cs}(E_{\text{th}}) = 0.75$ with $0.75 \leq P_{100\text{keV}} \leq 1.0$, the latter being more optimistic about reconstruction at low energies. In each case for $E > E_{\text{lim}}$, where $P_{cs}(E_{\text{lim}}) = 1.0$, we set $P_{cs}(E > E_{\text{lim}}) = 1.0$.

**III. RESULTS**

For 3-d read-out, zero background and constant sense determination probability the number of events required
to reject isotropy using the $\langle \cos \theta \rangle$ statistic initially decreases fairly slowly as the probability is decreased (from $N_{95} = 11$ for $P_{100\text{keV}} = 1.0$ to $N_{95} = 39$ for $P_{100\text{keV}} = 0.8$). The increase becomes more rapid as $P_{100\text{keV}}$ is decreased further and $N_{95} \rightarrow \infty$ as $P_{100\text{keV}} \rightarrow 0.5$. The linearly increasing sense determination probability with

$$P_{cs}(E_{th}) = 0.75 \text{ and } P_{100\text{keV}} = 1.0 \left(0.75\right) \text{ requires } \sim 3 \text{ times more (the same number of) events as the constant probability with the same value of } P_{100\text{keV}}.$$ 

For $P_{cs}(E_{th}) = 0.50$ the increase is more dramatic (more than an order of magnitude for $P_{100\text{keV}} = 1.0$). In fact in this case (and also for the constant probability, if $P_{100\text{keV}} < \sim 0.7$) fewer events are needed to reject isotropy using the axial $\langle |\cos \theta| \rangle$ statistic which does not use any information about the sense of the recoil. This is at first glance surprising, however in these cases the low values of $P_{cs}$, especially at low energies where the recoil rate is higher and the anisotropy in directions lower, significantly decrease the anisotropy in the observed recoil momentum vectors and hence the sensitivity of the $\langle \cos \theta \rangle$ statistic. This does not affect the $\langle |\cos \theta| \rangle$ statistic, as it is independent of the value of $P_{cs}$, being sensitive to the concentration of recoil axes, rather than vectors, around the line of solar motion. The number of events required using the axial statistic is, however, an order of magnitude larger than for vectorial data with perfect sense determination. As the signal to noise ratio is decreased the numbers of WIMP events required in each case are increased, by factors of $1.1 - 1.2, 2 - 4$ and $\sim 10$ for $S/N = 10, 1$ and 0.1 respectively.

For 2-d read-out with perfect angular resolution, the numbers of events required are a factor of roughly 2 larger than for the same sense determination probability and signal to noise ratio for 3-d read-out with the recoil direction reconstruction uncertainty taken into account. We caution that projection effects will make angular resolution a more significant factor for 2-d read-out than for 3-d read-out. Estimates based on the projected length of recoil tracks in the planes indicate that the required number of events would increase by at least a further factor of 2 [10].

Finally we examine whether it is more important to correctly determine the sense of the abundant low energy recoils or the rare, but more anisotropic, high energy recoils. To do this we consider step function sense determination probabilities: $P_{cs}(E < E_{\text{step}}) = 0.5 \left(1.0\right), P_{cs}(E > E_{\text{step}}) = 1.0 \left(0.5\right)$ for 3-d read-out and zero background. These correspond to the sense being undetermined (perfectly determined) for low energy recoils and perfectly determined (undetermined) for high energy recoils. It should be noted that neither of these possibilities, in particular the later, are particularly physically plausible. As the energy above which the sense can be (perfectly) determined, $E_{\text{step}}$, is increased the number of events required increases rapidly (by an order of magnitude as $E_{\text{step}}$ is increased from 20 keV to 60 keV). Conversely for the case where the sense of high energy recoils can not be determined as $E_{\text{step}}$ is decreased $N_{95}$ increases slowly initially, and then more rapidly for $E_{\text{step}} < 40 \text{ keV}$. This indicates that determining the sense of the common, but less anisotropic, low energy events is most important for minimising the number of event required to reject isotropy.
IV. SUMMARY

In this Brief Report we have investigated the effect of probabilistic recoil sense determination on the number of events required by a directional detection experiment to reject isotropy, and detect a WIMP signal. We have considered a constant sense determination probability and also increasing probability with increasing recoil energy (higher energy recoils have longer tracks and hence it is likely that it will be easier to determine their sense). For a constant probability of determining the sense correctly, 3-d read-out and zero background, we find that as the probability is decreased from 1.0 to 0.75 the number of events required to reject isotropy using the mean angle statistic is increased by a factor of a few. As the probability is decreased further the number of events required using this statistic increases sharply, and in fact for \(P_{100 \text{ keV}} < 0.7\) isotropy can be rejected more easily by discarding the sense information and using axial statistics (this does however require an order of magnitude more events than vectorial data with perfect sense determination).

The linearly increasing sense determination probability with \(P_{cs}(E_{th}) = 0.75\) (0.50) and \(P_{100 \text{ keV}} = 1.0\) requires \(\sim 3\) times (an order of magnitude) more events than the constant probability with the same value of \(P_{100 \text{ keV}}\). This suggests that maximising the probability of correctly determining the sense of the low energy, more common, but less anisotropic events is most important for minimising the number of events required. We have confirmed this conclusion by considering step function sense determination probabilities.

We also considered non-zero background and 2-d read-out. As the signal to noise ratio is decreased the numbers of WIMP events required in each case are increased, by factors of \(\sim 1.1 - 1.2\), \(2 - 4\) and 10 for \(S/N = 10\) and 0.1 respectively. For 2-d read-out with perfect angular resolution, the numbers of events required are roughly a factor of \(\sim 2\) larger than for 3-d read-out with the recoil direction reconstruction uncertainty taken into account (with the same sense determination probability and signal to noise ratio).

Acknowledgments

AMG and BM are supported by STFC. We are grateful to Demitri Muna for computing assistance.

[1] M. W. Goodman and E. Witten, Phys. Rev. D 31, 3059 (1985).
[2] D. N. Spergel, Phys. Rev. D 37, 1353 (1988).
[3] D. P. Snowden-Ifft, C. J. Martoff, and J. M. Burwell, Phys. Rev. D 61, 1 (2000) astro-ph/9904064
[4] G. J. Alner et al., Nucl. Inst. and Meth. A. 535, 644 (2004).
[5] T. Tanimori et al., Phys. Lett. B 578, 241 (2004) astro-ph/0310638
[6] C. J. Martoff, electronic proceedings of SNIC06 Symposium, California, 3-6 July 2006. Available from: http://www-conf.slac.stanford.edu/snic/papers/0188.PDF
[7] C. J. Copi, J. Heo and L. M. Krauss, Phys. Lett. B 461, 43 (1999) astro-ph/990449; C. J. Copi and L. M. Krauss, Phys. Rev. D 63, 043507 (2001) astro-ph/0009467
[8] M. J. Lehner et al., Dark Matter in Astro and Particle Physics, Proceedings of the International Conference DARK2000, Heidelberg, Germany, 2000, p590 ed. H. V. Klapdor-Kleingrothaus, Springer-Verlag (2001).
[9] B. Morgan, A. M. Green and N. J. C. Spooner, Phys. Rev. D 71 103507 (2005) astro-ph/0408087
[10] B. Morgan and A. M. Green, Phys. Rev. D 72, 123501 (2005) astro-ph/0508134
[11] C. J. Copi, L. M. Krauss, D. Simmons-Duffin and S. R. Stroiney, Phys. Rev. D 75, 023614 (2007) astro-ph/0508649
[12] A. M. Green and B. Morgan, JCAP08(2007)022 astro-ph/0609115
[13] J. D. Vergados and A. Faessler, Phys. Rev. D 75 055007 (2007) hep-ph/0611230
[14] A. Hitachi, J. Phys.: Conf Ser. 65, 012013, (2007).
[15] J. F. Ziegler, J. P. Biersack and U. Littmark, The stopping and range of ions in solids, Pergamon Press (1985), http://www.srim.org.
[16] D.P. Snowden-Ifft, T. Ohnuki, E.S. Rykoff and C.J. Martoff, Nucl. Inst. and Meth. A. 498, 155, (2003).
[17] Various talks at “Cygnus 2007: first workshop on direction detection of dark matter”, available at http://pppa.group.shel.ac.uk/cygnus2007/talks/.
[18] D. Dujmic et al. arXiv:0708.2370 (2007).
[19] G. Cowan, Statistical Data Analysis, OUP, (1998).
[20] B. Morgan, Dark Matter Detection With Gas Time Projection Chambers, Ph.D. Thesis, University of Sheffield, (2004).

[21] M. J. Carson et al., Nucl. Inst. and Meth. A. 546, 509 (2005) \texttt{hep-ex/0503017}; M. J. Carson et al., Astropart. Phys. 21, 667 (2004).

[22] Lord Rayleigh, Phil. Mag. 10, 73 (1880); Nature 72, 318 (1905); Phil. Mag. 37, 321 (1918); G. M. Cordeiro and S. Ferrari, Biometrika 78, 573 (1991); K. V. Mardia and P. Jupp, \textit{Directional Statistics}, Wiley, Chichester (2002).

[23] This can be improved to a factor $\sim 30$ if the reduced angles (with the direction of solar motion subtracted) are calculated and analyzed.

[24] A 2-d read-out projects the recoil track into a plane and the effects of this combined with multiple scattering and diffusion will make the angular resolution a function of both the energy and primary recoil direction.