Numerical study of a re-entrant diamond structure under dynamic compression

K P Logakannan\textsuperscript{1,2}, V Ramachandran\textsuperscript{2}, J Rengaswamy\textsuperscript{3} and D Ruan\textsuperscript{1}

\textsuperscript{1} Faculty of Science, Engineering and Technology, Swinburne University of Technology, Hawthorn, VIC 3122, Australia
\textsuperscript{2} Department of Aerospace Engineering, Indian Institute of Technology Madras, Chennai 600036, India
\textsuperscript{3}Department of Engineering Design, Indian Institute of Technology Madras, Chennai 600036, India
Email: druan@swin.edu.au

Abstract. Auxetic materials, due to its negative Poisson’s ratio, shrink laterally when compressed axially and expand laterally when pulled axially. A re-entrant diamond structure was developed by replacing the vertical walls in a conventional re-entrant structure with diamond cells, which featured cross-linking members to make them rigid. The incorporation of the rigid diamond unit cells increased the stiffness, strength, and energy absorption of the structure. A validated finite element (FE) model was adopted from previous work, and the structures were compressed at a speed of 5 m/s in the FE model. The independent geometrical parameters of the re-entrant diamond unit cell were re-entrant wall length ($L_1$), diamond angle ($\theta$), and diamond wall length ($L_2$). The FE model based on these values was thus used to investigate the influence of geometrical parameters ($\theta$ & $L_2/L_1$) on the deformation mode, stiffness, strength, and specific energy absorption (SEA) of the structure. The value of diamond angle ($\theta$) varied from 40° to 90° at intervals of 10°, while the length ratio ($L_2/L_1$) varied from 0.7 to 1.2 in increments of 0.1. The specific strength was used in the discussion to account for differences in the relative density of re-entrant diamond structures.

Keywords: Re-entrant diamond; Auxetic; Axial compression; Deformation mode; Energy absorption.

1. Introduction

“Auxetics” are materials or structures with negative Poisson’s ratios. These materials or structures thus exhibit unique behaviours such as lateral contraction under axial compression and synclastic behaviour under bending. Such materials are useful in fields such as sports [1], medicine [2], and robotics [3]. Lakes [4] reported a foam with a negative Poisson’s ratio, and subsequently, researchers have developed additional auxetic unit cells [5–8]. Grima and Evans [5] achieved auxetic behaviour through the rotation of rigid squares, while Grima and Gatt [6] reported auxetic behaviour in perforated sheets. Bertoldi et al. [7] used elastic instability as the mechanism for achieving auxetic behaviour. They demonstrated auxetic behaviour in an array of circular holes.

Efforts have been made to study and improve the elastic properties of the auxetic structures. Li et al. [8] reported a novel stiff auxetics. Yang et al. [9] studied the elastic properties of a 3D re-entrant unit cell. Fu et al. [10] investigated the shear modulus of re-entrant structures and reported non-linearity in the shear modulus with respect to geometrical parameters. Rayneau-Kirkhope [11] improved the stiffness of the re-entrant structures by replacing conventional beams with hierarchical beams.

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd
For any cellular structure, its performance after elastic region is crucial. Zhang et al. [12] studied the tensile behaviour of a re-entrant structure. Alomarah et al. [13] designed a new auxetic unit cell, a re-entrant chiral unit cell, and studied its properties under dynamic compression [14]. However, literature on the mechanical properties of auxetic structures under dynamic compression remains scarce.

In this work, the performance of a re-entrant diamond structure under dynamic compressive loading was examined. A finite element (FE) model developed and validated in the authors’ previous work [15], was used in this work to investigate the behaviour of the selected structure under dynamic compression without consideration of failure. The FE model was thus employed to study the effects of geometric parameters (length ratio, $L_2/L_1$, and diamond angle, $\theta_2$) on deformation mode, specific plateau stress, specific energy absorption, and Poisson’s ratio.

2. Unit cell feature

A re-entrant unit cell is a widely known auxetic structure as shown in Fig. 1(a). The vertical member (BC) often buckles under compression, which lowers the energy absorption of the structure. To overcome this, the vertical member was replaced with a diamond with a cross-linking member (BDCE) in the re-entrant diamond unit cell shown in Fig. 1(b)). The authors proposed this re-entrant diamond unit cell in their previous work [15]. The geometric parameters of the re-entrant diamond unit cell are length of inclined member ($AB$), $L_i$; length of the diamond member ($BD$), $L_2$; the angle between the inclined member and the diamond, $\theta_1$; and the angle between the members of the diamond, $\theta_2$. All the angles in this work are given in degrees.

![Figure 1. Sketches of: (a) Re-entrant unit cell; (b) re-entrant diamond with cross-linking member [15]; (c) a finite element model with coarse mesh.](image)

3. Finite element model

The finite element (FE) model used in this work was validated and published by the authors in their previous work [15] and explained here to facilitate understanding. The FE model was built using ABAQUS/Explicit, with the re-entrant diamond structure assumed to be a 3D printed from Polyamide 11 (PA 11). The Young’s modulus, Poisson’s ratio, and density of the printed PA 11 were 1484 MPa, 0.33, and 919 kg/m³, respectively. A bilinear material model without any failure criterion was used to analyse deformation characteristics. The yield strength and tangent modulus of the materials were 34.58 MPa and 135 MPa, respectively, which were obtained from previous experimental tests [15]. The wall thickness of the structure was assumed to be 0.5 mm. The structure was meshed using shell elements (S4R) of size 0.5 mm, while the out-of-plane thickness of the structure was kept to 10 mm for all models used in this work. Rigid plates with the re-entrant diamond structure in between them are shown in Fig. 1(c). A coarse mesh with element size of 2 mm was used in Fig. 1(c) to allow better observation, though a finer mesh with an element size of 0.5 mm was used in the simulation.
4. Deformation mode, stress-strain, and Poisson’s ratio

A typical deformation mode, stress-strain curve, and Poisson’s ratio curve for the structure for a geometric set are shown in Figs. 2 and 3 for better visualisation. The geometrical parameters are $L_1 = 7$ mm, $L_2/L_1 = 1.0$, $\theta_1 = 30^\circ$, $\theta_2 = 60^\circ$, wall thickness = 0.5 mm, and out-of-plane thickness = 10 mm. The number of cells is five in both in-plane directions. Figure 2 shows the deformation mode and its corresponding stress-strain curve. During the initial stages of compression, the bending of the inclined members and rotation of the diamonds results in lateral contraction as shown in image (1) in Fig. 2(a). Image (2) in Fig. 2(a) shows the maximum lateral contraction, and the corresponding stress values are shown in Fig. 2(b). Once the structure shrinks completely in the lateral direction, the stress values increase tremendously. This increase in stress is attributed to the compression of the diamonds. The diamonds need more stress to deform and the densification around a strain of 0.7. Plateau stress is defined as the average value of stress in the strain range of 0.1 to 0.5. The plateau stress obtained from this geometric set is 2.8 MPa. For the same set of geometric parameters, the plateau stress is 2.7 MPa for quasi-static simulation [15], indicating that the structure is not sensitive to speed in the range studied.

Poisson’s ratio is defined as the negative ratio of lateral strain to axial strain, and in this work is discussed in terms of real numbers. A change in the Poisson’s ratio from -1 to -2 is thus considered to be a decrease. The displacement values of the four nodes (highlighted in Fig. 3a) were used to calculate the Poisson’s ratio, as shown in Fig. 3(b). This showed a minimum value of -1.8 initially and fluctuated subsequently up to a strain of 0.2, at which point the structure exhibited the maximum lateral contraction (Point ② in Fig. 2a), and the corresponding Poisson’s ratio was around -1.5. Beyond this point, the Poisson’s ratio increased monotonically. The pattern of the Poisson’s ratio curve obtained from the quasi-static simulation was very similar to the dynamic curve, and thus is not shown here.

Figure 2. (a) Deformation of a re-entrant diamond structure at different stages; (b) stress-strain curve.
5. Size effect

The boundary condition, or number of unit cells of the structure, influences its deformation characteristics. The minimum number of cells in each direction must be identified to eliminate this effect. A size effect study was conducted to examine the number of cells in both in-plane directions, with increases in step of one cell in each direction, starting with 3×3 cells. The results of these models were compared in terms of plateau stress as shown in Fig. 4. For 3×3 cells, the plateau stress was around 3.2 MPa. As this increased, plateau stress converged to 3.5 MPa from 5×5 cells. None of the structures exhibited global buckling, and thus the FE model with 5×5 cells were used in the ensuing parametric study.

![Figure 3 and Figure 4](image-url)
6. Parametric study

A parametric study was carried out to investigate the influences of length ratio ($L_2/L_1$) and diamond angle ($\theta_2$). In this work, $\theta_2$ is related to $\theta_1$, where $\theta_1 = (60^\circ - \theta_2)/2$. Simulated results were compared in terms of deformation mode, specific stress, SEA, and Poisson’s ratio. Specific stress was used to account for the difference in the density of structures and it is defined as the stress divided by the density of the structure. Density in turn was calculated by dividing the mass of the structure (obtained from the FE model) by the volume enclosed by it. The densities of the structures are listed in Table 1. Specific plateau stress is the average specific stress in the range of 0.1 to 0.5 strain, while energy absorption was calculated as the area under the force-displacement curve. SEA was obtained as the ratio of energy absorption to the mass of the structure. The SEA values at 50% compressed displacement were used for quantification. The length ratio ($L_2/L_1$) varied from 0.7 to 1.2, with an interval of 0.1. Diamond angles were kept in the range of 30° to 80° in steps of 10°.

| $L_2/L_1$ | $\theta_2$ | 30    | 40    | 50    | 60    | 70    | 80    |
|-----------|----------|-------|-------|-------|-------|-------|-------|
| 0.7       | 4.79E+02 | 5.04E+02 | 5.38E+02 | 5.83E+02 | 6.41E+02 | 7.25E+02 |
| 0.8       | 4.24E+02 | 4.47E+02 | 4.73E+02 | 5.09E+02 | 5.60E+02 | 6.26E+02 |
| 0.9       | 3.88E+02 | 4.06E+02 | 4.30E+02 | 4.62E+02 | 5.02E+02 | 5.59E+02 |
| 1         | 3.59E+02 | 3.75E+02 | 3.96E+02 | 4.25E+02 | 4.61E+02 | 5.07E+02 |
| 1.1       | 3.38E+02 | 3.52E+02 | 3.71E+02 | 3.96E+02 | 4.28E+02 | 4.72E+02 |
| 1.2       | 3.19E+02 | 3.33E+02 | 3.49E+02 | 3.73E+02 | 4.02E+02 | 4.42E+02 |

6.1. Effect of length ratio ($L_2/L_1$)

This study kept $L_1$ constant at 7 mm, while $L_2$ was varied to create new ratios; the results are discussed below.

6.1.1. Deformation mode. Figure 5 shows the deformation mode of the re-entrant diamond structure at different strain levels and geometric parameters, with $\theta_2 = 70^\circ$ and $L_2/L_1 = 0.7$ and 1.2. The deformation mode remained the same for all length ratios. Initially, the structure contracted laterally through the bending of inclined cells and rotation of diamonds. Once it attained the maximum lateral contraction (as in Point 2 in Fig. 2a), the diamonds began deforming. The strain at which the maximum contraction achieved is dependent on length ratio ($L_2/L_1$) and diamond angle ($\theta_2$), and this is significant because the stress starts increasing when diamonds start to deform. As shown in Fig. 5, at a strain of 0.3, the diamonds began deforming for $L_2/L_1 = 0.7$. However, for $L_2/L_1 = 1.2$, the diamonds did not deform at that point. Stress starts to increase earlier when $L_2/L_1$ is smaller.

6.1.2. Specific plateau stress and SEA. Figures 6(a) and 6(b) show specific plateau stress with respect to $L_2/L_1$ for different $\theta_2$ values. For $\theta_2 \geq 60^\circ$, specific plateau stress decreased with respect to $L_2/L_1$. As $L_2/L_1$ increased, the length of the diamond wall members increased, which may have weakened the diamonds. As the specific stress is defined as stress divided by the density, while density decreases with respect to $L_2/L_1$, the drop in stress value is more appreciable. For $\theta_2 \leq 50^\circ$, specific plateau stress value increased until $L_2/L_1$ reaches 1.0, then dropped. SEA values are shown in Fig. 6(b). It can be seen that the SEA follows the same pattern as the specific plateau stress.

6.1.3. Poisson’s ratio. The values of Poisson’s ratio that were obtained are shown in Fig. 6(c). In general, Poisson’s ratio decreased as $L_2/L_1$ increased, as lateral strain increases, and axial strain decreases as the $L_2/L_1$ increases. Thus, as $L_2/L_1$ increases, the structure has more freedom to contract laterally, as shown in Fig. 5.
Figure 5. Deformation modes of re-entrant diamond structures ($\theta_2 = 70^\circ$) at different length ratios ($L_2/L_1$) and strains.

Figure 6. Results of the parametric study with respect to the length ratio ($L_2/L_1$): (a) specific plateau stress; (b) SEA; (c) Poisson’s ratio.
6.2. Effect of diamond angle ($\theta_2$)

6.2.1 Deformation mode. The deformation mode of the structure did not change with respect to $\theta_2$. As mentioned in Section 6.1.1, the strain at which the maximum contraction achieved was dependent on $\theta_2$.

6.2.2. Specific plateau stress and SEA. Figure 7 shows the specific plateau stress and SEA with respect to $\theta_2$. Specific plateau stress increased until $\theta_2$ reached 60° and then plateaued, except for $L_2/L_1 = 0.7$. In the case of $L_2/L_1 = 0.7$, the maximum value was noted at $\theta_2 = 70^\circ$. As $\theta_2$ increased, the density of the structure increased, and the plateau stress values also increased with respect to $\theta_2$, the specific plateau stress plateaued out at 60°. The SEA values, shown in Fig. 7(b), indicate that these follows the same trend as specific plateau stress, with maximum SEA value obtained at $\theta_2 = 70^\circ$ and $L_2/L_1 = 0.7$, with a value of 3.5 kJ/kg.

6.2.3. Poisson’s ratio. For smaller length ratios ($L_2/L_1 \leq 0.9$), Poisson’s ratio increases with any increase in $\theta_2$, as shown in Fig. 7(c). For $L_2/L_1 \geq 1.0$, the variation is not linear, however. For smaller $\theta_2$, the structure has more freedom to contract, which results in larger lateral contractions.

7. Conclusion
A validated finite element model based on previous work was used to investigate the deformation mode, strength, and Poisson’s ratio patterns of a re-entrant diamond structure. The differences in plateau stress of the re-entrant diamond structure under quasi-static and dynamic loading were negligible, and the Poisson’s ratio showed similar traits. A parametric study was thus conducted to examine the influences of length ratio ($L_2/L_1$) and diamond angle ($\theta_2$) on both strength and the Poisson’s ratio. Lower $L_2/L_1$ values resulted in higher specific plateau stress, especially when $\theta_2$ was larger, while stress values
decreased when $L_2/L_1$ reached 1.2, irrespective of $\theta_2$ value. $L_2/L_1$ exhibited a greater impact on Poisson’s ratio than $\theta_2$, and structure with $L_2/L_1 = 0.7$ and $\theta_2 = 70^\circ$ exhibited the highest SEA value at 3.5 kJ/kg.

**Acknowledgments**
The authors would like to offer their appreciation for the financial support from the Ministry of Education (MoE), India, through the SPARC scheme (SPARC/2018-2019/P988/SL), that enabled this work.

**References**

[1] Sanami M, Ravirala N, Alderson K, and Alderson A 2014 *Procedia Eng.* 72 453–8

[2] Ali MN, Busfield JJC, and Rehman IU 2014 *J Mater Sci Mater Med.* 25(2) 527–53

[3] Mark AG, Palagi S, Qiu T, and Fischer P 2016 *Proc - IEEE Int Conf Robot Autom.* 2016-June 4951–6

[4] Lakes R 1987 *Science* 235(4792) 1038–40

[5] Grima JN and Evans KE 2000 *J Mater Sci Lett.* 19(17) 1563–5

[6] Grima JN and Gatt R. 2010 *Adv Eng Mater.* 12(6) 460–4

[7] Bertoldi K, Reis PM, Willshaw S, and Mullin T 2010 *Adv Mater.* 22(3) 361–6

[8] Li D, Ma J, Dong L, and Lakes RS 2017 *Mater Lett.* 188 149–51

[9] Yang L, Harrysson O, West H, and Cormier D 2012 *Acta Mater.* 60(8) 3370–9

[10] Fu MH, Xu OT, Hu LL, and Yu TX. 2016 *Int J Solids Struct.* 80 284–96

[11] Rayneau-Kirkhope D 2018 *Sci Rep.* 8(1) 1–10

[12] Zhang J, Lu G, Wang Z, Ruan D, Alomarah A, and Durandet Y 2018 *Compos Struct.* 184 92–101.

[13] Alomarah A, Ruan D, Masood S, Sbarski I, and Faisal B 2018 *Int J Adv Manuf Technol.* 96(5–8) 2013–29

[14] Alomarah A, Xu S, Masood SH, and Ruan D 2020 *Smart Mater Struct.* 29(5) 055031

[15] Logakannan K P. Ramachandran V, Rengaswamy J, Gao Z, and Ruan D *Compos Struct.* 254 112853