Space Dynamics in Global Time as an Effective Alternative to General Relativity

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The fundamental physical object of the Global Time Theory is a three-dimensional curved space dynamically developing in global time. The equations of its dynamics are derived from the Lagrangian, and the Hamiltonian of gravitation turns out to be nonzero. The General Relativity solutions are shown to be a subset of the GTT solutions with zero energy density. In Global time Theory, the quantum theory of gravitation can be built on the basis of the Schrödinger equation, as for other fields. The quantum model of the Big Bang is presented in some details.

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I. DYNAMIC GEOMETRY

General relativity (GR) is built on the Riemannian geometry of space-time [1, 2]. Solutions of its basic equations, the Einstein equations, determine full four-dimensional space-time, past, present and future simultaneously. In contrast, The Global Time Theory (GTT) considers a three-dimensional configurational space as a fundamental physical object that develops dynamically in global time which is common for all points of the space.

From times of Ancient Greece, geometry was build on the basis of observations on positions and motion of objects with respect to each other, without any connection to time. The geometry of Euclid and Lobachevsky, and the geometries of Gauss and Riemann do not contain the concept of time. These geometries are intended for undisturbed or slowly moving observers contemplating placed in front of them blueprints or still objects.

The Newton mechanics has introduced to mathematical descriptions of nature the notion of time. However, Newton assumed space to be Euclidean what was natural for his time. Therefore, introduced by him moving systems also were Euclidean spaces moving in absolute space (since the Euclidean space admits, from the point of view of modern mathematics, motion).

The very first description of dynamics with inhomogeneous velocity fields belongs to Euler. In the process of formulating his famous hydrodynamical equations, Euler applied in a local frame connected to the moving fluid the second Newton’s law:

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho} \nabla p,$$

Then he transferred the time derivative from a system in which the fluid elementary volume is at rest ($\mathbf{v} = 0$), to the laboratory system replacing the time derivative by the so called substantial derivative

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + (\mathbf{v}(\mathbf{r}, t) \cdot \nabla).$$ (1)

However, such transformation is valid for a scalar quantity and (by a coincidence) for field of velocities.

The anzats for transformation of tensors of any rank due the time shift was prepared by Sophus Lie when he introduced for an arbitrary infinitesimal transformations of coordinates $\bar{x}^i = x^i - \xi^i(x)$ so called a Lie-variation of a tensor field (for example, for the covariant vector field $A_i$):

$$\delta_\xi A_i = \xi^s A_{is} + \xi^i_{,i} A_s.$$. (2)

In Dynamic geometry the space is represented by a set of all its points. The frame of references in which coordinates of the space points ($\bar{x}^i$) do not change with time, we shall term as an absolute inertial frame. Then, in some other frame of references where point coordinates are connected to their coordinates in an absolute inertial frame
by a time-dependent transformation \( x^i = f^i(\tilde{x}, t) \), there exists a field of absolute velocities

\[
V^i = \frac{\partial x^i}{\partial t}.
\]

(3)

absent in an absolute inertial frame (what is the distinct feature of an absolute inertial frame). An infinitesimal increment of time \( dt \) generates an infinitesimal variation of coordinates

\[
\delta \xi(x, t) = V^i(x, t) \, dt.
\]

The latter transforms the time derivative from a tensor field, taking into account the Lie derivative of the tensor field with respect to Time \( \xi \), from the non-inertial frame to the inertial one, forming the covariant derivative of the tensor field with respect to Time \( \xi \) (3)). The derivative contains \( r + 2 \) elements, where \( r \) is a rank of the tensor. In particular, for a scalar field \( (r = 0) \)

\[
D_t f(x, t) = \frac{\partial f}{\partial t} + V^i \partial_i f,
\]

and for a covariant field of vectors \( A_i(x, t) \)

\[
D_t A_i = \frac{\partial A_i}{\partial t} + V^s A_{is} + V^s_i A_s.
\]

Especially important in the dynamics of space is the time covariant derivative from a metric tensor

\[
D_t \gamma_{ij} = \frac{\partial \gamma_{ij}}{\partial t} + V_{ij} + V_{ji}.
\]

(4)

II. THE GLOBAL TIME THEORY

The Global Time Theory originates from the following physical concepts of space and time:

**Space** is the material carrier of geometrical properties. It is three-dimensional.

**Global time** is a proper time of the space, uniform for all of its points. It flows everywhere and always equally uniformly, it is itself the measure of the uniformity.

The space is the carrier of geometrical properties because geometrical properties are determined by a metric tensor, six components of which are major field variables of the space.

Bodies move in space, fields dynamics (for example, electromagnetic field) develop in space. For each moving point the absolute velocity relative to the space is defined.

There exists an absolute motion with respect to the space, or from different perspective, there exists a velocity field in some frame of references. Thus, the dynamics of space is described by six components of the metric tensor field \( \gamma_{ij}(x, t) \), determining its geometrical properties in each given moment of time, and by three components of absolute velocities \( V^i(x, t) \), determining how each space point moves in each moment of time with respect to the chosen frame of references.

The space is the material carrier of geometrical properties, because equations of the dynamics of the metric tensor and the velocity field are obtained from the Lagrangian equations and define, along with other fields (for example, electromagnetic), energy of the system.

The metric dynamics and velocity field equations are derived from a variation principle, where the gravitational action is presented as a difference between kinetic (quadratic with respect to velocities of the metric deformation) and potential energy of the space (proportional to the scalar curvature of space):

\[
S = \frac{c^4}{16 \pi k} \int (\mu^2_i - (\mu^j_i)^2 + R) \sqrt{\gamma} \, dx \, dt + S_m.
\]

(5)

Here \( S_m \) is the action of all other matter, and \( \mu_{ij} \) is a tensor of space deformation velocities consisting of time covariant derivatives of the metric tensor

\[
\mu_{ij} = \frac{1}{2c} D_t \gamma_{ij} = \frac{1}{2c} (\dot{\gamma}_{ij} + V_{ij} + V_{ji}).
\]

(6)

Note that the action contains the field of absolute velocities \( V^i \) but only through the tensor above.

Variation of the action with respect to the metric tensor \( \gamma_{ij} \) leads to six dynamic equations. Also variation with respect to the absolute velocity field generates three to the equations of connectivity. Thus the set of equations of the space dynamics consists of nine partial differential equations of the second order.

From the action the energy density and the Hamiltonian \( H_v = \int \rho \sqrt{\gamma} \, dx \) are determined as usually in field theory:

\[
\rho = \frac{c^4}{16 \pi k} (\mu^2_i - (\mu^j_i)^2 - R).
\]

(7)

An important feature of this Hamiltonian is its indeterminate sign.

A. The proper time of the moving observer

In GR, as Einstein constantly emphasized, the special relativity theory is valid in local: in small, the space and time are described by the Minkowski metric (tangential space-time).

The global construction of GTT does not superimpose any restrictions on local properties. The special relativity theory, as local structure of space-time, can be also naturally incorporated in GTT. For any moving observer its proper time is defined via a global time and velocity with respect to the space \( v^i = \dot{x}^i - V^i \):

\[
d\tau = dt \sqrt{1 - \frac{1}{c^2} \gamma_{ij} v^i v^j}.
\]

(8)
III. SIMILARITIES BETWEEN GTT AND GR

A. ADM-representation

Similarity and differences between GTT and GR can be the best traced in so called ADM-representation of GR. Arnowitt, Deser and Misner [4] have expressed ten components of the four-dimensional metric tensor through six components of the metric tensor of the three-dimensional space $\gamma_{ij}$, the three-dimensional vector $V^i$ (in notations of GTT), and the function of a course of time $f(x,t)$:

$$g_{00} = f^2 - \gamma_{ij}V^iV^j; \quad g_{0i} = \gamma_{ij}V^j; \quad g_{ij} = -\gamma_{ij}. \quad (9)$$

Components of an inverse metric tensor are respectively

$$g^{00} = \frac{1}{f^2}; \quad g^{0i} = \frac{V^i}{f^2}; \quad g^{ij} = \frac{V^iV^j}{f^2} - \gamma^{ij}. \quad (10)$$

Ten Einstein equations are obtained then as variation equations for ten components of the metric tensor $g_{\alpha\beta}$ of the Hilbert action

$$S_G = \frac{c^4}{16\pi k} \int R \sqrt{g} d^4x, \quad (11)$$

where $R$ is a scalar curvature of the four-dimensional space-time.

In GTT, the component $g^{00} = 1$ always and everywhere. This is a function defining the pace of a global time. With this condition, the Hilbert action becomes the action of a configurational space [5]. The component $g^{00}$ can not be varied, and the function which is multiplied by this variation can be arbitrary. It is the function which is an energy density [6].

The variation of all ten components of the space-time metric leads to the basic difference between expressions in GR and GTT: the variation with respect to $f$ produces an extra (comparison with nine equations of GTT) equation

$$H = 0. \quad (12)$$

The density of a complete Hamiltonian (including both space and substance) is equal then to zero, and consequently the Hamiltonian itself is equal to zero in GR.

Solutions of GR therefore define a subset of all GTT solutions with an energy density equal to zero everywhere. It is the tenth equation, in addition to six dynamic equations and three connectivity equations, which cuts out the GR sector from GTT.

B. Reduction of GR to a global time

Cosmological solutions of GR always originate from the metric of the form:

$$ds^2 = c^2 dt^2 - g_{ij}(x,t) dx^i dx^j.$$ 

Here the component of the metric $g^{00} = 1$ what means that the time is global, and $g^{0i} = 0$ what means that the system is globally inertial.

The other solutions of GR (geodesically complete) also can be reduced to the global time. If there is a four-dimensional metric $g_{\alpha\beta}$ in arbitrary coordinates $x^\alpha$, in order to reduce it to a global time one needs to transform coordinates (to pick only new time coordinate $\tau = ct$, more precisely) so that the requirement $g^{00} = 1$ is satisfied. Following tensor transformation rules,

$$g^{00} = g^{\alpha\beta} \frac{\partial \tau}{\partial x^\alpha} \frac{\partial \tau}{\partial x^\beta} = 1. \quad (13)$$

This differential equation for $\tau$ turns out to be the Hamilton-Jacobi equation for trajectories of freely falling mass points (laboratories) for which a common natural time is $t$. Thus, a principle of equivalence, tied an inertial system to the freely falling laboratory, exists in global time. However, in contrast to the Einstein’s elevator, there exists a multitude of such laboratories and the time in them is synchronized. Thus the principle of equivalence becomes global. But the three-dimensional manifold formed by these points-laboratories, is not any more a Euclidean space.

In GTT, the theorem is proved that any static spherically symmetric solution with any kind of matter has an energy equal to zero [8]. Therefore all such solutions in GTT and GR can be reduced to each other. For example, the Schwarschield solution in global time can be presented as

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 + 2\sqrt{\frac{2M}{r}} dt dr \quad (14)$$

$$(dr^2 + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2)).$$

This expression was obtained from the Schwarschield metric by Painleve in 1921 [8]. He carried out in the Schwarschield metric various transformations $\tilde{t} = t + \Phi(r)$ and showed, that the crossections $t = const$ are different at different choices of the function $\Phi(r)$. In particular, he has found a function, at which the spatial crossection is an Euclidean space. This solution can be obtained as a solution of the GTT equations [8].

Solving the equation $H \neq 0$, it is not difficult to reduce to global time ($g^{00} = 1$) and other GR solutions, for example, the Nordström or Kerr metric.
IV. DIFFERENCES BETWEEN GTT AND GR

As was shown above, solutions of GR form a subset of solutions of GTT with an energy density equal to zero. The removal of this restriction leads not only to the appearance of new solutions, but also to the removal of many problems of GR, such as the problem of initial conditions, the problem of a critical density, geodesic completeness, and return to the Shrøedinger theory in quantum area.

A. Cosmological models

For Friedman’s model of space, the three-dimensional sphere of variable radius \( r(t) \), the energy \( E \) in a Planck units system is negative and is given by:

\[
E = -3r \dot{r}^2 - 3r.
\]

Since due to dynamical equations the energy is conserved, it is a differential equation of the first order, the solution for which is a Friedman’s cycloid:

\[
r = \frac{r_m}{2} (1 - \cos \chi); \quad t = \frac{r_m}{2} (\chi - \sin \chi).
\]

The key difference here from the Friedman solution is only physical: in this solution the matter density is absent. Presence of a dust-like matter changes only the constant \( E \). Thus in GTT, a problem of a critical density in a cosmology is absent: the World can be closed or open independently from the matter density.

B. Space dynamics

The removal of the GR restriction about the zero energy density leads to solutions important for space dynamics: to the field of space vertexes \( \mathcal{B} \). These solutions have surprisingly simple mathematical properties (weak superposition principle) and huge energies. Instead of hypothetical "dark matter", "dark energy", and "huge black holes ", significant energetic role in space dynamics plays, from the GTT point of view, a dynamical energy of the space itself.

In \( \mathcal{B} \) the example of a globe having the diameter 20 cm and rotating with 1 turn per second is studied. For draw the space around it to coherent moving it needs to spend the energy of annihilation of 300 000 tons of matter.

C. The Quantum Gravity

In a view of GTT, the quantum gravity theory undergoes the most significant modification. The catastrophic relationship GR \( H = 0 \), putting on hold all quantum dynamics, is removed. In GTT, a quantum gravity, as well as quantum theory of other fields, for example quantum electrodynamics, can be built on the basis of a Schrödinger equation

\[
i \hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi,
\]

defining the dynamics of a state vector of space (and other fields) \( \Psi \) in global time.

The measure of quantum fluctuations is thus defined not in some fixed space, but by the metric of that curved space, in which these fluctuations arise. Thus, in contrast for example to quantum electrodynamics, where the basic problem turns out to be a nonlinearity (during interaction with a field of electrons) but the functional space is flat, in quantum gravity the functional space itself has a curvature \( \mathcal{B} \).

Components of the metric \( \gamma_{ij} \) commute with each other, as well as components of momenta \( \pi^{kl} \). However, an expression for the Hamiltonian can be considerably simplified if one introduces affine momenta \( \pi^i \) which commutes (in sense of Poisson brackets) with each affine momentum:

\[
\pi^i_j = q^i_j + \frac{\delta^i_j}{3}; \quad q^i_i = 0; \quad \pi^i_i = \pi,
\]

In these variables (and at \( V^i = 0 \), the Hamiltonian \( \mathcal{H} \) looks simpler:

\[
H = \frac{1}{\sqrt{\gamma}} \left( 2 q^i_j q^j_i - \frac{\pi^2}{3} \right) - \frac{\sqrt{\gamma}}{2} \frac{(3)}{R},
\]

Note that the metric enters in the kinetic energy only through \( \sqrt{\gamma} \), and this variable commutes with \( q^i_j \), but the latter do not commute with each other:

\[
\{ q^i_j(x), q^k_l(x') \} = \frac{1}{2} (\delta^i_l q^j_k + \delta^i_k q^j_l) \delta(x - x').
\]

This is the commutation relationship for currents of the \( SL(3) \) group, which thus naturally arises in the dynamical theory of gravitation.

These complications arise as a sequence of the background independence of quantum gravity.

D. Quantum cosmology

As a trial stone in this or that approach to a quantum gravity cosmological problems with a finite number of degrees of freedom are often considered.

Let us consider a compact cosmological model of Friedman’s type \( \mathcal{B} \), homogeneous and isotropic with space being a three-dimensional sphere, filled
with matter obeying the equation of state \( \varepsilon = 3p \).
As to the geometry of space, variations of the radius \( r \) are only taken into account.

Hamiltonian

\[
H = -\frac{p^2}{2r} + \frac{q^2}{2r},
\]

where \( q^2 \) characterizes a conserved amount of a ultrarelativistic substance.

The wave function is a function of time and radius of a sphere \( r \). The variables in this problem can be separated. Denoting by a prime the derivative with respect to the radius and symmetrizing \( p^2/r \), we obtain a stationary cosmological wave equation (in Planck units):

\[
u'' - \frac{u'}{r} + (r^2 + q^2)u = 2rEu.
\]

Since the functions are not normalized and slightly not orthogonal due to the approximate integration the metric, matrix is calculated

\[
M_{ij} = \int_0^{r_{\text{max}}} u_i(r)u_j(r)\,dr
\]

and the evaluation of matrix operators will be carried out with an inverse matrix \( K_{ij} = M_{ij}^{-1} \). The operator of the radius

\[
r_j = \sum_{s=1}^n K_{is}^{\text{max}} \int_0^{r_{\text{max}}} r u_s(r)u_j(r)\,dr.
\]

is of interest for the analysis of dynamics of the radius. At \( n = 8 \) eigenvalues of this matrix are equal

\[
(0.51, 1.7, 2.6, 3.4, 4.2, 5.0, 5.8, 6.8).
\]

With increase of \( n \) (number of functions) the minimum eigenvalue decreases, but the maximum one grows. The product of the two remains approximately slightly larger than \( \pi \). Therefore quantum effects do not prevent the Big Bang, but accounting for the fact that the Plank’s constant in Hevsyde units has dimensionality of the square of a length, lead to a hypothesis about some a cosmological uncertainty relation:

The product of maximum and minimum radiuses of the World is not less than \( \pi \hbar \).

The wavelet in the space with \( n = 8 \), having a minimum eigenvalue of radius \( r = 0.51 \) is presented in the figure.

However this wavelet is not stationary and begins to (widen) spread. This expansion is not
monotonic: the wavelet (square of the module) breaks into two components, one of which is moving away from zero, but second oscillates near to zero:

From the point of view of quantum mechanics in any problem (for instance solid state) this kind of behavior of the probability density would not cause serious problems. However with respect to the Radius of the World (the unique, single variable) the problems arise. What is the Radius of the World for us? While it is possible to reconcile to average values and small fluctuations around, how then are we supposed to treat the simultaneous probability of two essentially different radiiuses? What should we expect to get if we measure somehow the Radius of the World? Will a reduction of a wavelet in the area of big, or in the area of small radiuses happen?

The possible answer is, that the Radius of the World cannot be measured directly. Hubble has determined it by measuring properties of photons, coming from remote galaxies. These photons also obey quantum theory and their quantum behavior (accessible for our measurements) can be influenced by both, the area of big and small radiuses, but no reduction of a wave function of the Radius of the World will happen at fixing a photon (see [9]).

Overwhelming majority of quantum variables is being observed by nobody. Their quantum mechanical behavior is exhibited only through their influences on a small number of observable variables. From the quantum mechanical point of view, the Radius of the World can have quite spread values, but it can cause only specificity in observed (or not observed yet) behavior of observed variables (photons, space particles). Quite possible that quantum physics has in its basis not a wave function but a density matrix.

V. CONCLUSIONS

Being built on more advanced mathematical framework, GTT introduces into physics a new (in general old, known at a level of philosophy for a long time) physical object: the space. From the point of view of theoretical physics, it is a nine-component field with curved functional space. As well as other fields, for example electromagnetic, it has a density and flux of energy, at the cosmic scale the energy of deformed space is enormous.

The study of properties of this physical object, will probably shed light on modern problems of Cosmic dynamics which are currently being explained by "a dark matter" and similar exotic substances. The study of quantum properties of space will certain advance us in understanding a quantum essence of the World.

Physics of space is not interlinked tightly with the relativism and can be under development to some extend before the special relativity theory. This journey can be made following Niels Bjørn [10], who although is a fiction person, however the work with his articles brought the author to the clear understanding of space dynamics in global time. It is namely him, whom I express my deepest gratitude.

[1] Misner C.W., Thorne K., Wheeler J.A. Gravitation (San Francisco: Freeman, 1974).
[2] L.D. Landau, E.M. Lifshitz, A Field theory.
[3] D.E. Burlankov, Dynamics of space (in Russian) (Nizhni Novgorod: publishing house, 2005)
[4] R. Arnowitt, S. Deser, and C.W. Misner, Phys. Rev. 116, 1322 (1959).
[5] P. Painlevé, C.R. Acad. Sci. (Paris). 173, 677 (1921).
[6] D.E. Burlankov arXiv: gr-qc /0406112 (2004).
[7] D.E. Burlankov, Russian JETP. 51, 842 (1966).
[8] D.E. Burlankov, arXiv: gr-qc /0406110v1 (2004).
[9] K. Blum. Density matrix theory and applications. NY, London, 1981.
[10] D.E. Burlankov, Physics-Uspekhi 47 (8), 833 (2004).