From Kondo Effect to Fermi Liquid

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The Kondo effect has been playing an important role in strongly correlated electron systems. The important point is that the magnetic impurity in metals is a typical example of the Fermi liquid. In the system the local spin is conserved in the ground state and continuity with respect to Coulomb repulsion \(U\) is satisfied. This nature is satisfied also in the periodic systems as far as the systems remain as the Fermi liquid. This property of the Fermi liquid is essential to understand the cuprate high-\(T_c\) superconductors (HTSC). On the basis of the Fermi liquid theory we develop the transport theory such as the resistivity and the Hall coefficient in strongly correlated electron systems, such as HTSC, organic metals and heavy Fermion systems. The significant role of the vertex corrections for total charge- and heat-currents on the transport phenomena is explained. By taking the effect of the current vertex corrections into account, various typical non-Fermi-liquid-like transport phenomena in systems with strong magnetic and/or superconducting fluctuations are explained within the Fermi liquid theory.

1. Introduction

Since the discovery of Kondo Effect, the theory of electron correlation has been developed remarkably. In particular, the study of the strongly correlated electron systems such as cuprates and heavy fermions is actively performed. In this article we discuss the contribution of Kondo theory to the theory of strongly correlated electron systems. The key word is the singlet: The ground state of the single impurity in metals is the singlet where a localized spin is coupled antiferromagnetically with conduction electron spin. The resonating-valence-bond (RVB) state is a spin singlet state and is considered to be the basis of the cuprate high-\(T_c\) superconductivity (HTSC). By using Anderson’s orthogonality theorem, we show that the Fermi liquid state is nothing but the RVB state in metal. As a result, we can reasonably arrive at the pairing theory of superconductivity.

2. Anderson Hamiltonian

In 1961 Anderson presented the following Hamiltonian and explained the appearance of a localized moment in metals by using the Hartree-Fock approximation. [1]

\[
H = \sum_{k,\sigma} \varepsilon_k C_{k,\sigma}^\dagger C_{k,\sigma} + \frac{1}{\sqrt{N}} \sum_{k,\sigma} (V_{kd} C_{k,\sigma}^\dagger d_{\sigma} + V_{dd} d_{\sigma}^\dagger C_{k,\sigma}) + \sum_{\sigma} \varepsilon_d n_{d\sigma} + U n_{d\uparrow} n_{d\downarrow}, \tag{1}
\]

The first term is the energy of conduction electrons and the third term is the d-electron part. The last term is the Coulomb repulsion between two d-electrons and is the only many-body interaction. The symmetric Anderson Hamiltonian possesses the electron–hole symmetry and is easy to understand the physical meaning because of the suppressed charge fluctuations. By assuming the average number of d-electrons is unity and taking the nonmagnetic case \(< n_{d\sigma} >=< n_{d\bar{\sigma}} >= 1/2\),

\[
\varepsilon_d = -\frac{U}{2}, \tag{2}
\]

\[
E_d = \varepsilon_d + U/2 = 0, \tag{3}
\]

where \(E_d\) is the averaged d-level in the nonmagnetic state. When one d-electron occupies the d-orbit, the energy of d-electron is \(-U/2\) and the second d-electron occupies the level at the energy of \(\varepsilon_d + U = U/2\). The second term of the Anderson Hamiltonian (1) represents the mixing term between d- and conduction electrons. Owing to this term the d-electron level possesses the lifetime width \(\Delta\):

\[
\Delta = \frac{\pi}{N} \sum_k |W_{kd}|^2 \delta(\varepsilon_k - E_d) = \frac{\pi \rho |V|^2}{N}. \tag{4}
\]

Here \(\rho\) is the density of states of conduction electrons at the Fermi energy. Hereafter, we assume the band-width of conduction band is infinite and \(\rho\) is constant.

2.1. Kondo effect

Anderson explained the existence of magnetic moment as the effect of electron correlation. However, there existed a long standing mystery for these 30 years. [2] As shown in Fig.1, when we decrease the temperature, the residual resistivity begins to increase and the resistivity shows the minimum at a temperature. Kondo explained it in his paper published in 1964. [3]

He used the following s-d Hamiltonian:

\[
H = \sum_{k,\sigma} \varepsilon_k C_{k,\sigma}^\dagger C_{k,\sigma} + \frac{J}{2N} \sum_{k k' \sigma \sigma'} C_{k',\sigma'}^\dagger \sigma_{\sigma'} C_{k,\sigma} \cdot S = H_0 + H_{sd}. \tag{5}
\]

The s-d interaction in the second term can be derived from Eq. (1) by assuming one d-electron and \(U/(\pi \Delta)\) is
sufficiently large and by expanding with respect to mixing term. In the case \( J \) is written as the following and possesses negative sign (\( J < 0 \)).

\[
J = -2|V|^2\left(\frac{-1}{\varepsilon_d} + \frac{1}{\varepsilon_d + U}\right) = -\frac{8|V|^2}{U}. \tag{6}
\]

In the last equation we have assumed the symmetric case, \( \varepsilon_d = -U/2 \). We calculate the T-matrix by using the s-d interaction of Eq.(5) as perturbation. By calculating up to the second Born term, we obtain the following temperature dependent term,

\[
T(\varepsilon) = -\frac{J}{2N}(S \cdot \sigma)\left[1 + \frac{J}{2N} \sum_k \frac{1 - 2f(\varepsilon_k)}{\varepsilon + i\eta - \varepsilon_k}\right]. \tag{7}
\]

Here \( f(\varepsilon) \) is the Fermi distribution function. By using the T-matrix, we obtain the electrical resistivity,

\[
R = R_B[1 + \frac{2J\rho}{N} \log \frac{k_B T}{D}], \tag{8}
\]

\[
X = \frac{J\rho}{N} \log \frac{k_B T}{D}, \tag{9}
\]

where \( R_B \) is the resistivity obtained from the first term of Eq.(7), that is the resistivity by the Born approximation. \( D \) is the band-width of conduction band, and \( k_B \) is the Boltzmann constant. Equation (8) represents the logarithmic increase of resistivity with decreasing temperature. Thus, the problem of resistance minimum has been solved clearly.

The perturbation theory cannot be used when the second term of Eq.(8) approaches to unity. In that case we need higher order terms. In \( X \), the coupling constant \( \frac{|J\rho|}{N} \ll 1 \) is a small parameter and \( X \) becomes unity when \( T = T_K \):

\[
k_B T_K = D \exp\left[\frac{-N}{|J\rho|}\right]. \tag{10}
\]

From this result, we have to calculate the higher order terms in the vicinity of \( T_K \). This calculation was done by Abrikosov and was published in 1965. [4]. The most divergent terms \( \left[J\rho/N \log(k_B T/D)\right]^n \) are summed up to the infinite order as

\[
R = R_B[1 - \frac{J\rho}{N} \log \frac{k_B T}{D}]^{-2} \tag{11}
\]

This expression also diverges at \( T = T_K \).

On the other hand, Yosida and Okiji calculated the spin susceptibility due to the localized spin and obtained the following result: [5]

\[
\chi_{imp} = \frac{C}{T}\left[1 + \frac{J\rho}{N} \frac{1}{1 - \frac{J\rho}{N} \log \frac{k_B T}{D}}\right], \tag{12}
\]

\[
C = \frac{(g\mu_B)^2S(S + 1)}{3k_B}, \tag{13}
\]

where \( C \) is the Curie constant and \( g \) and \( \mu_B \) are the \( g \)-value and Bohr magneton. This equation shows that the effective Curie constant becomes small with decreasing temperature from high temperature above \( T_K \). This means that the localized spin becomes small toward \( T_K \).

From this result Yosida proposed a singlet ground state: [7]

\[
\Psi_{\text{singlet}} = \frac{1}{\sqrt{2}}[\varphi_\alpha \chi_\alpha - \varphi_\beta \chi_\beta]. \tag{14}
\]

Here \( \chi_\alpha \) and \( \chi_\beta \) are the wave-functions of localized spin and that for up and down spins, respectively. The wave-functions \( \varphi_\alpha, \varphi_\beta \) are the wave-functions of conduction electrons associated to the localized spin states. \( \varphi_\alpha \) has the down spin and \( \varphi_\beta \) has up spin; they compose the spin singlet state with \( \chi_\alpha \) and \( \chi_\beta \).
We show the electronic states in Fig. 2.

Substituting this wave-function into the s-d Hamiltonian (5) and solving the Schrödinger equation, we obtain

\[ H\Psi_{\text{singlet}} = E\Psi_{\text{singlet}}, \tag{15} \]

\[ E = E_D + \tilde{E}, \tag{16} \]

\[ \tilde{E} = -k_B T_K = -D \exp\left(-\frac{N}{|\rho|}\right), \tag{17} \]

where \( E_D \) is the normal term in the ground state energy which can be obtained by the perturbation expansion with respect to the s-d interaction, starting from the doublet state composed of the Fermi sphere and a localized spin. The singular term \( \tilde{E} \) is the binding energy in the spin singlet state. This result shows that the system gains the energy equal to \( T_K \).

The energy gain of the spin singlet state can be naturally understood as follows. The localized up-spin state \( \chi_\alpha \) and the localized down-spin state \( \chi_\beta \) are degenerate with each other. We can gain the binding energy by combining the two states through the transverse component of s-d exchange interaction \( J_\perp \), and lifting the degeneracy. In order to realize this procedure, it is necessary for the following matrix elements to exist between the two components in the singlet ground-state wave-functions:

\[ < \varphi_\alpha \chi_\alpha | - \frac{J}{N} \sum_{k,k'} C_{k\uparrow}^\dagger C_{k\downarrow} S_+ | \varphi_\beta \chi_\beta > \neq 0, \tag{18} \]

\[ < \varphi_\beta \chi_\beta | - \frac{J}{N} \sum_{k,k'} C_{k\downarrow}^\dagger C_{k\uparrow} S_- | \varphi_\alpha \chi_\alpha > \neq 0. \tag{19} \]

Furthermore, for the matrix elements not to vanish, the wave-functions of conduction electrons should satisfy Anderson’s orthogonality theorem.

The Anderson’s orthogonality theorem is the following. \[ \varphi_1 \] and \( \varphi_2 \) be two wave-functions of conduction electrons corresponding to two different ground states. The phase shifts at the Fermi energy for \( \varphi_1 \) and \( \varphi_2 \) are denoted as \( \delta_1 \) and \( \delta_2 \), respectively. Then, the overlap integral between the two ground states is given by

\[ |< \varphi_1 | \varphi_2 >| \sim N^{-(\delta_1 - \delta_2)^2/(2\pi^2)}. \tag{20} \]

This is the orthogonality theorem by Anderson. \( N \) is the number of related conduction electrons, which is a macroscopic number. As a result, when \( \delta_1 \) and \( \delta_2 \) do not coincide with each other, the two wave-functions are orthogonal and the overlap integral vanishes. According to the Friedel sum rule, the phase shifts \( \delta_1 \) and \( \delta_2 \) are related to the local electron numbers \( n_1 \) and \( n_2 \) as \( n_1 = \delta_1 / \pi \) and \( n_2 = \delta_2 / \pi \). Therefore, for the overlap integral to be finite, the conservation law, \( n_1 = n_2 \), should be satisfied. This theorem holds also for the many-body matrix element. For the matrix elements, (18) and (19), not to vanish, the local electron numbers should satisfy the following condition. Denoting the local conduction electron number with \( \sigma \) spin for localized up(down) spin as \( n_\sigma \), we obtain

\[ n_\alpha = n_\beta - 1, \tag{21} \]

\[ n_\alpha = n_\beta + 1, \tag{22} \]

\[ n_\alpha + n_\alpha = 0, \tag{23} \]

\[ n_\beta = n_\beta^\sigma. \tag{24} \]

Here we have used that \( C_{k\sigma}^\dagger = \frac{1}{\sqrt{N}} \sum_k C_{k\sigma} \) and \( C_{0\sigma} = \frac{1}{\sqrt{N}} \sum_k C_{k\sigma} \) creates and destroys one electron at the origin, respectively. From the above condition, we can determine the local electron numbers as \[ \frac{1}{2} \] or \( -\frac{1}{2} \).

This result means, as shown in Fig. 2, that the up-spin component of localized spin is combined with a half conduction electron with down-spin and a half up-spin conduction hole. The down-spin component of the localized spin is combined with a half up-spin conduction electron. Thus the singlet ground state is realized. The local number of electrons \( \pm 1/2 \) corresponds to the phase shift of \( \pm \pi/2 \), and \( |\sin(\delta)| = 1 \) gives the resistivity at the unitarity limit. This value corresponds to the nonmagnetic case of manganese.

Let us consider the case of the Anderson Hamiltonian given by Eq.(1). \[ [9] \]. In this case the ground state wave-function is written by a linear combination of the following four components of d-electron state.

\[ \Psi_g = A_0 \varphi_0 + A_1 d_\uparrow^\dagger \varphi_1 + A_2 d_\downarrow^\dagger \varphi_2 + A_3 d_\uparrow^\dagger d_\downarrow^\dagger \varphi_0. \tag{27} \]
where \( \phi_0, \phi_\uparrow, \phi_\downarrow \) and \( \phi_2 \) are the conduction electron states corresponding to d-electron states possessing 0, \( \uparrow \), \( \downarrow \) and 2 electrons, respectively. In the Anderson Hamiltonian, the energy gain comes from the mixing term, while the Coulomb repulsion increases the energy. Therefore, the expectation value of the mixing term should have finite value in the ground state. This is the necessary condition for the ground state. If we assume \( V_k = V_u k \), we obtain

\[
< \Psi_g | H_{mix} | \Psi_g > \neq 0,
\]

which is equivalent to

\[
V \sum_\sigma |< A_0 \phi_0 > \frac{1}{\sqrt{N}} \sum_k u_k^\sigma C^\dagger_k \sigma | A_\sigma \phi_\sigma > + < A_2 \phi_2 > \frac{1}{\sqrt{N}} \sum_k u_k \sigma C_k \sigma | A_{-\sigma} \phi_{-\sigma} > + < A_\sigma \phi_\sigma > \frac{1}{\sqrt{N}} \sum_k u_k \sigma C_k \sigma | A_0 \phi_0 > + < A_{-\sigma} \phi_{-\sigma} > \frac{1}{\sqrt{N}} \sum_k u_k^\sigma C^\dagger_k \sigma | A_2 \phi_2 > | \neq 0. \tag{29}
\]

Here we define creation and annihilation operators of electron at the origin as

\[
C_{0 \sigma} = \frac{1}{\sqrt{N}} \sum_k u_k C_k \sigma, \tag{30}
\]

\[
C_{0 \sigma}^\dagger = \frac{1}{\sqrt{N}} \sum_k u_k^\sigma C^\dagger_k \sigma. \tag{31}
\]

In order for the above matrix elements not to vanish, the local electron numbers have to be conserved. From this condition, we can derive the following relations among conduction electron numbers associated to the i-component of d-electron states. Denoting the number of conduction electrons with \( \sigma \) as \( n_\sigma \), we obtain

\[
n_0^\sigma = n_\sigma + 1, \tag{32}
\]

\[
n_0^{-\sigma} = n^{-\sigma} = n_{-\sigma} = n_0, \tag{33}
\]

\[
n_2^\sigma = n_\sigma^{-\sigma} - 1, \tag{34}
\]

\[
n_2^{-\sigma} = n_{-\sigma}^{-\sigma} = n_2^\sigma. \tag{35}
\]

Finally we obtain

\[
n_\sigma = n_{-\sigma} - 1 \tag{36}
\]

Thus, we have the same relation as \( (21) \) for the s-d Hamiltonian. This fact means that the energy gain in the spin singlet state comes from the mixing term in the Anderson Hamiltonian. This mixing energy exists even in the case of \( U = 0 \). The effect of \( U \) for the s-d Hamiltonian is only suppressing the mixing term and reducing the energy gain.

Now we extend the above discussion to periodic systems such as the Hubbard Hamiltonian and the periodic Anderson Hamiltonian. In this case we assign the d-orbit in the impurity model to a lattice point chosen arbitrarily in the periodic system. We can write the ground-state wave function \( \Psi_g \) as \( (27) \). Here the conduction electrons are replaced by local electrons around the site 0 which do not include 0-site. For the electron at a site and electrons around the lattice point, the relation shown in Fig.2 should hold. In the the periodic Anderson Hamiltonian, the matrix element which should be finite is the mixing term. In the case of Hubbard Hamiltonian, transfer matrices to neighboring sites should not vanish. In these models the energy gain originates from the coherent mixing and transfer terms. That is, the coherent band-like motion is the origin of energy gain. For the mechanism to be possible, "conservation of local electron number" should be satisfied. This condition shows that the electron state at a lattice point always couples in the singlet state with electrons existing neighboring sites. This state is nothing but the resonating-valence-bond (RVB) state. The importance of this state is stressed by Anderson \[10\]. The singlet state is the same as the singlet in the single impurity. Thus, we can understand that the RVB state in metals is nothing but the Fermi liquid state, although Anderson denies the Fermi liquid state as the normal state in HTSC and looks for RVB state. The fact "The Fermi liquid is a RVB state in metals" is the important result obtained from the study of the Kondo effect. The Fermi liquid state is the linear combination of local spin singlet states. Let us consider to shift the electron number uniformly in Fig.2. By the shift, the electronic states change from \( (a) \) to \( (b) \). After shifting the electron number, we can see that when a spin of electron is up at a lattice site, the neighboring electron number of this site possesses one up spin defect. That is, when an electron transfers between lattice sites, the local spin conservation holds always. The Hamiltonian conserves the total spins. In the ground state, spin conservation holds also locally. The local conservation of spin is a stronger condition than the total conservation. This result means when we need to bring spin from far distant region, a coherent electron motion is impossible. An electron possessing a spin can transfer by accompanying the same spin hole around it, and the system keeps the uniform state in spin distribution as a whole. When we separate the wave-function into their local components, the same singlet state as in the Kondo effect is realized. This point is very important. For the heavy fermion system, the same argument is applicable by using the periodic Anderson Hamiltonian. \[11\]

2-2. Perturbation expansion for the Anderson Hamiltonian

We use the following perturbation expansion with respect to \( U \): \[12\]

\[
H = H_0 + H', \tag{37}
\]

where
\[ H_0 = \sum_{k\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{1}{\sqrt{N}} \sum_{k\sigma} (V_{kd} c_{k\sigma}^\dagger d_{\sigma} + V_{dk} d_{\sigma}^\dagger c_{k\sigma}) \]
\[ + \sum_{\sigma} E_{d\sigma} n_{d\sigma} - U < n_{d\uparrow} > < n_{d\downarrow} >, \] (38)
\[ E_{d\sigma} = \varepsilon_d + U < n_{d\sigma} >, \] (39)
\[ H' = U(n_{d\uparrow} - < n_{d\uparrow} >)(n_{d\downarrow} - < n_{d\downarrow} >). \] (40)

We expand the fluctuation term which is neglected in the Hartree-Fock approximation, assuming that \(< n_{d\sigma} > = 1/2\) and the expansion parameter is \(u = U/(\pi \Delta)\). Then, we find that the ground state energy is given by
\[ E_g = E(u = 0) + \pi \Delta (-\frac{u}{4} - 0.0369u^2 + 0.0008u^4 + \cdots). \] (41)

This result is shown in Fig.3. The right figure shows the result with that of the exact solution obtained by Wiegmann [14] and Kawakami and Okiji [17]. The perturbation theory is reliable up to \(u = 4\).

Similarly, we expand the spin susceptibility \(\chi_s\), charge susceptibility \(\chi_c\), and the electronic specific heat coefficient \(\gamma\):
\[ \chi_s = \frac{(g\mu_B)^2}{2} \frac{1}{\pi \Delta} \tilde{\chi}_s, \] (42)
\[ \chi_s = \tilde{\chi}_\uparrow \uparrow + \tilde{\chi}_\downarrow \downarrow = \sum_{n=0} a_n u^n. \] (43)
\[ \chi_c = \sum_{\sigma} \frac{1}{\pi \Delta} \tilde{\chi}_c, \] (44)
\[ \chi_c = \tilde{\chi}_\uparrow \uparrow - \tilde{\chi}_\downarrow \downarrow = \sum_{n} a_n (-u)^n = \tilde{\chi}_s (-u), \] (45)
\[ \gamma = \frac{2\pi^2 k_B^2}{3} \frac{1}{\pi \Delta} \tilde{\gamma}, \] (46)
\[ \tilde{\gamma} = \frac{1}{2} (\chi_s + \chi_c) = \tilde{\chi}_\uparrow \downarrow. \] (47)

Here, \(\tilde{\chi}_\uparrow \uparrow, \tilde{\chi}_\downarrow \downarrow\) are given by correlation functions between parallel and anti-parallel spins. The result obtained by perturbation calculation,
\[ \tilde{\chi}_s = 1 + u + (3 - \frac{\pi^2}{4})u^2 + (15 - \frac{3\pi^2}{2})u^3 + 0.055u^4 + \cdots. \] (48)

The results are shown in Fig.4. \(\tilde{\gamma}, \tilde{\chi}_s, \tilde{R}\) are \(\chi_s^2/4\) and \(\chi_s^2/2\) are shown by (1), (2) and (3), respectively, in the function of \(u = U/\pi \Delta\) [References [9]].

Finally we explain the result of the resistivity. The resistivity decreases in terms of \(T^2\) with increasing temperature from unitarity limit which corresponds to the phase shift \(\delta = \pm \pi/2\). [13]. The result is given by
\[ R = R_0 [1 - \frac{\pi^2}{3} (\frac{k_B T}{\Delta})^2 (2\tilde{\chi}_\uparrow \uparrow + \tilde{\chi}_\downarrow \uparrow) + \cdots]. \] (49)

When \(u\) is large, we can use the s-d Hamiltonian. By the relation, \(\tilde{\chi}_\uparrow \uparrow = \tilde{\chi}_\downarrow \downarrow = \chi_s/2\) we obtain
\[ R = R_0 [1 - \frac{\chi_s^2}{4} (\frac{\pi k_B T}{\Delta})^2 + \cdots]. \] (50)

The \(T^2\) dependence can be scaled by Kondo temperature \(T_K\). In that case \(T_K = 2\Delta/\pi \chi_s\).

3. Continuity and Exact Solution
In 1980, the exact solution for the s-d Hamiltonian on the basis of the Bethe hypothesis was obtained by Andrei [14] and Wiegmann [15]. For the Anderson Hamiltonian, Wiegmann [16], and Kawakami and Okiji [17] obtained the exact solution. The results by the exact solution are the following;
\[ \tilde{\chi}_s = \sqrt{\frac{\pi}{2u}} \exp[\frac{\pi^2}{8} - \frac{1}{2u}] + \frac{1}{\sqrt{2\pi u}} \int_{-\infty}^{\infty} \exp[-x^2/2u] \frac{1}{1 + (u + ix)^2} dx, \]
\[ \tilde{\chi}_c = \frac{1}{\sqrt{2\pi u}} \int_{-\infty}^{\infty} \exp[-x^2/2u] \frac{1}{1 + (u + ix)^2} dx. \] (51)
The first term of $\chi_s$ seems to possess a singular term $\exp[-1/u]$ in $u$. However, Zlatić and Horvatić proved that the result is analytic. [18] By using
\[
\chi_s = \exp[\pi^2 u/8] \sqrt{\frac{2}{\pi u}} \int_0^{\infty} \exp[-x^2/2u] \frac{\cos(\pi x/2)}{1-x^2} dx, \tag{52}
\]
\[
\chi_c = \exp[-\pi^2 u/8] \sqrt{\frac{2}{\pi u}} \int_0^{\infty} \exp[-x^2/2u] \frac{\cosh(\pi x/2)}{1+x^2} dx, \tag{53}
\]
we can obtain the expansion coefficients of (43) and (45). The result is given by the following formula.
\[
a_n = (2n-1)a_{n-1} - \left(\frac{\pi}{2}\right)^2 a_{n-2}, \tag{54}
\]
\[
a_0 = a_1 = 1, \tag{55}
\]
from which we obtain the solution,
\[
a_n = \left[\left(\frac{\pi}{2}\right)^{2n+1}/(2n+1)!!\right] P_n, \tag{56}
\]
\[
P_n = \sum_{k=0}^{\infty} \left(\frac{(-1)^k}{k!}\right) \left(\frac{2(n+1)!}{(2n+k+1)!!}\right) \left(\frac{\pi^2}{8}\right)^k. \tag{57}
\]
Here $P_n$ has the following properties,
\[
\frac{2}{\pi} = P_0 < P_n < P_\infty = 1. \tag{58}
\]
When $n$ is large, $a_n$ is given by
\[
a_n \simeq \left(\frac{\pi}{2}\right)^{2n+1}/(2n+1)!!. \tag{59}
\]
As the result we can see that the expansion with respect to $u$ converges with infinite convergence radius, $u = \infty$.

The Wilson ratio is the ratio of the susceptibility to the specific heat coefficient,
\[
R_W = \frac{\chi_s}{2\mu_B^2/\gamma} = \frac{\chi_s}{\gamma} = \frac{2\chi_s}{\chi_s + \chi_c}. \tag{60}
\]
As $u$ increases, the charge fluctuations are suppressed and $\chi_c$ approaches zero. In this case the Wilson ratio $R_W$ approaches from 1 to 2 with increasing $u$. The normal state of the interacting Fermi system is connected continuously from the free Fermi gas with increasing interaction. [19], [20]. As we have stressed here, the magnetic impurity system is a typical Fermi liquid. [21]

4. transport phenomena in high-$T_c$ cuprates and other strongly correlated systems.

The Fermi liquid theory has been developed by many authors to explain various electronic properties in strongly correlated systems like heavy fermion systems, organic superconductors and the high-$T_c$ superconductors (HTSC’s) [9,21]. In the case of HTSC, almost all the physical quantities deviates from the conventional Fermi liquid behaviors observed in usual metals, which are called the non-Fermi liquid behaviors. Recent progress on the Fermi liquid theory has succeeded in explaining various non-Fermi liquid behaviors in HTSC in terms of the Fermi liquid state with strong antiferromagnetic (AF) fluctuations [9,21,22]. Spin fluctuation theories like the SCR theory [22] and the fluctuation-exchange (FLEX) approximation [23] can reproduce $1/T_1T \propto T^{-1}$ and $\rho \propto T$ in HTSC’s, especially the correct superconducting (SC) order parameter, $d_{x^2-y^2}$, as well as the optimum transition temperature $T_c \approx 100K$.

However, the transport phenomena under the magnetic field in HTSC’s, like the Hall coefficient $R_H = (\sigma_{xy}/\sigma_{xx})/B_z$ and the magnetoresistance (MR) $\Delta \rho/\rho_0$, have sometimes been recognized as the hallmark of the breakdown of the Fermi liquid state, because they seem too anomalous to understand in terms of the Fermi liquid theory. For example, $|R_H|$ increases as the temperature decreases below $T_0 \sim 700K$, and $|R_H| \gg 1/ne$ ($n$ being the electron filling number) at lower temperatures [24]. The sign of $R_H$ is positive in hole-doped systems like YBa$_2$Cu$_3$O$_{7-x}$ (YBCO) and La$_{2-x}$Sr$_x$CuO$_4$ (LSCO), whereas it is negative in electron-doped systems like Nd$_{2-x}$Ce$_x$CuO$_4$ (NCCO), although their Fermi surfaces (FS’s) observed in ARPES measurements are hole-like [25]. As for the MR, $\Delta \rho/\rho_0 \propto T^{-4}$ holds for a wide range of temperatures [26,27]. Thus, the Kohler rule derived by a simple relaxation time approximation (RTA) in a single-band model, $R_H \propto n$-const. and $\Delta \rho/\rho_0 \propto \rho_0^{-2} \propto (\propto T^{-2})$ in HTSC, are totally broken in HTSC’s, although it is satisfied in usual metals.

The breakdown of the Kohler rule suggests that both $R_H$ and $\Delta \rho/\rho_0$ in HTSC’s are strongly enhanced at lower temperatures by the strong correlation effect. In the RTA, the total current $\vec{J}_k$ is assumed as the mean-free-path of the quasiparticle $\vec{v}_k$, therefore $\tau_k$, which is the relaxation time and $\vec{v}_k$ is the quasiparticle velocity. The breakdown of the Kohler rule in HTSC cannot be reproduced satisfactorily within the RTA even if one assume the $k$- and the energy-dependences of $\tau_k$ arbitrarily; therefore many people have considered that the ground state of the HTSC is the non-Fermi liquid in which the concept of the quasiparticle is not valid any more. For example, Anderson considered that the Tomonaga-Luttinger liquid state with two-kinds of relaxation time, $\tau$ and $\tau_1$, is realized [28]. Thus, transport phenomenon has been one of the most strong objection against the Fermi liquid picture in HTSC’s.

However, we stress that the RTA is insufficient for strongly correlated systems because the macroscopic conservation laws are broken. In this section, we study the transport phenomena in HTSC based on Kubo’s linear response theory, by taking the vertex correction (VC) for current into account, following the conserving approximation by Baym and Kadanoff [29]. In correlated electron systems the total current $\vec{J}_k$ is given by the quasi-
particle velocity $\vec{v}_k = \vec{v}(\epsilon_k)$ plus other dragged quasiparticles $\Delta \tilde{J}_k$ induced by interactions. In the Landau-Fermi liquid theory, $\Delta \tilde{J}_k$ is called (a kind of) the back-flow, which is expressed as the the current VC in the microscopic Fermi liquid theory. $\Delta \tilde{J}_k$ is totally dropped in the RTA, However, in various situations, one should take the current VC in the conserving way to avoid unphysical results [29]. In the present article, we show that various anomalous $T$-dependences of the transport phenomena observed in HTSC’s are caused in fact by the current VC’s.

The pseudo-gap phenomena in under-doped systems below $T^* \sim 200K$ is also one of the most important problems in HTSC. Since various anomalous phenomena related to the pseudo-gap cannot be explained by the spin-fluctuation theory, the origin of the pseudo-gap phenomena has been studied intensively for years. Recent theoretical works using the FLEX+T-matrix theory have showed that the strong $d_{x^2-y^2}$-SC fluctuations induced by the AF fluctuations is a significant candidate for the pseudo-gap [21,9,30]. In the present article, we study the transport phenomena below $T^*$ using the FLEX+T-matrix theory with the current VC’s, and show that various anomalous transport coefficients are well reproduced in a unified way. The present work strongly supports that the pseudo-gap phenomena in under-doped HTSC’s are caused by the strong $d_{x^2-y^2}$-SC fluctuations as discussed in Refs. [9,21,30] in detail.

### 4-1. transport phenomena above $T^*$

In the present subsection, we explain the origin of anomalous transport phenomena above the pseudo-gap temperature $T^*$, where the spin-fluctuation theory is meaningful. Based on the Kubo formula, Eliashberg derived the general expression for the DC-conductivity $\sigma_{\mu\nu}$ which is exact within the most divergent terms with respect to $\gamma^{-1}_k$. By generalizing his work, exact expressions for the Hall conductivity $\sigma_{xy}$ [31] and the magneto conductivity $\Delta \sigma_{xx} = \sigma_{xx}(B) - \sigma_{xx}(0)$ [32] have been derived. These expressions contain the total current $\tilde{J}_k$, which is composed of the irreducible four-point vertices ($\Gamma^I$). By imposing the Ward identity $\Gamma^I = \delta \Sigma / \delta G$ in the expressions, we can calculate the Hall coefficient and the magnetoresistance (MR) so as to satisfy the conservation laws.

![Diagram](image)

\[ \tilde{J}_k = \tilde{v}_k + \sum_{k'} \int \frac{d\epsilon}{4\pi i} T^I_{kk'}(0, \epsilon)(G^R_{kk'}(\epsilon))^2 \tilde{J}_{k'}, \]  

where $T_{kk'}^I(0, \epsilon)$ is given by the analytic continuation of $\Gamma_k^I(\epsilon, \epsilon'; \omega)$ from the region $\epsilon_n < 0, \epsilon_n + \omega I$ and $\epsilon_{n'} < 0, \epsilon_{n'} + \omega |[33]$. In the FLEX approximation, $T_{kk'}^I$ is given by three diagrams, which is shown in Fig.5 [34].

Owing to the vertex correction, $\tilde{J}_k$ given by the solution of Eq.(61) is no more parallel to the quasiparticle velocity $\tilde{v}_k$ when the AF fluctuations are prominent, as explained in Ref. [34] for the first time. Apparently, the assumption in the RTA, $\tilde{J}_k \sim \gamma_k \tilde{v}_k$, is invalid. The schematic momentum dependence of $\tilde{J}_k$ is shown in Fig.6. The physical reason for its anomalous $k$-dependence is that the total current behaves as $\tilde{J}_k \propto \tilde{v}_k + \tilde{v}_{k+Q}$ because the quasiparticle at $k$ couples strongly to that at $k+Q$ through the current VC (i.e., the effective interaction between quasiparticles) in the presence of the strong AF fluctuations. The detailed explanation is given in Ref. [34]. Note that the behavior of $\tilde{J}_k$ in Fig.6 will be universal in nearly AF Fermi liquid beyond the FLEX approximation once the Ward identity $\Gamma^I = \delta \Sigma / \delta G$ is satisfied [34].
Here, we discuss the Hall coefficient. The general expression for $\sigma_{xy}$ is given by \[ \sigma_{xy} / B_z = -\frac{e^3}{4} \oint_{FS} dk_\parallel |k_\parallel|^2 \left( \frac{d\theta_k^I}{dk_\parallel} \right) \frac{1}{(\gamma_k^2)} , \] (62)

where $\theta_k^I = \tan^{-1}(J_{kx}/J_{ky})$. $k_\parallel$ is the momentum parallel to the FS, and $\gamma_k = 1/2\tau_k = \text{Im}\Sigma_k(-i\delta)$ represents the quasiparticle damping rate at $k$. As shown in Fig.6, $d\theta_k^I / dk_\parallel$ is positive around point A, whereas it is negative around point B. According to the FLEX approximation, the cold spot, where $\gamma_k$ takes the minimum value on the FS, is located around point A (B) in hole-doped (electron-doped) systems [34]. The cold spot locates at the most distant point from the antiferromagnetic Brillouin zone boundary which connects $(\pi,0)$ and $(0,\pi)$. Because the transport phenomena are governed mainly by the quasiparticles around the cold spot, it is theoretically expected that $R_H > 0$ in hole-doped (electron-doped) systems when the AF fluctuations are strong. We comment that the negative $R_H$ in electron-doped systems had been considered to be an evidence of the non-Fermi liquid state because the curvature of the FS in HTSC is positive everywhere, which directly means that $R_H > 0$ within the RTA. However, it is naturally explained within the Fermi liquid picture as the effect of the current VC [34].

![Figure 6](image)

**FIG. 6.** Schematic behavior of the total current $\vec{J}_k$ and the quasiparticle velocity $\vec{v}_k$ on the Fermi surface. The cold spot is located around the point A (B) in the hole-doped (electron-doped) HTSC’s.

![Figure 7](image)

**FIG. 7.** (i) $R_H$ obtained by the FLEX approximation which takes the current VC’s into account. $R_{H}^{\text{RTA}}$ by the RTA (without VC’s) shows only a weak $T$-dependence. $R_H$ by the FLEX+T-matrix approximation is also shown; it starts to decrease below $T^* \sim 150$K (pseudo-gap behavior). (ii) $\zeta = \Delta\rho / \rho R_H^2 - 1$ takes almost a constant value (the modified Kohler’s rule) by taking account of the current VC’s. On the other hand, $\zeta_{\text{RTA}}$ shows a strong $T$-dependence.

The temperature dependence of $R_H$ in the vicinity of the antiferromagnetic quantum-critical-point (AF-QCP) is...
\[ |R_H| \propto \xi_{\text{AF}}^2, \]  

(63)

which can be derived from the analytical consideration on the current VC [34]. \( \xi_{\text{AF}} \) is the AF correlation length, whose temperature dependence is \( \xi_{\text{AF}} \propto T^{-1/2} \) according to the spin-fluctuation theory (like the FLEX approximation or the SCR theory). As a result, the experimental fact \( R_H \propto T^{-1} \) in HTSC can be successfully reproduced in the present approach.

Similarly, the MR due to the orbital motion of conduction electrons can be studied beyond the RTA, based on the Kubo formula. According to the analytical consideration on the current VC, we obtained the relation [32]

\[
\Delta \rho / \rho_0 \equiv -\Delta \sigma_{xx} / \sigma_{xx}^0 - (\sigma_{xy} / \sigma_{xx}^0)^2 \\
\propto \xi_{\text{AF}}^4 \cdot \rho_0^2,
\]

(64)

which suggests that the Kohler’s rule \( \Delta \rho / \rho_0 \propto \rho_0^2 \) is violated because \( \xi_{\text{AF}}^4 \propto T^{-2} \) holds when the AF fluctuation is strong. Equation (64) explains the experimental relation \( \Delta \rho / \rho_0 \propto T^{-4} \) observed in YBCO and LSCO where \( \rho_0 \propto T \). Moreover, Eqs.(63) and (64) directly mean the “modified Kohler’s rule” [35]

\[
\Delta \rho \cdot \rho_0 \propto R_H^2,
\]

(65)

which is well satisfied in various HTSC compounds although the conventional Kohler’s rule is completely broken [26,27]; therefore its reason has been discussed intensively by many authors. We stress that it is naturally explained in the present study. Figure 7 shows a numerical result by the FLEX approximation including the current VC’s. The coefficient \( \zeta = \Delta \rho / \rho_0 \propto \rho_0 / R_H^2 \) is almost independent for a wide range of temperatures, only when the VC’s are taken into account correctly. The experimental value is \( \zeta = 2 \sim 3 \) in YBCO and Ti-based HTSC, and \( \zeta \sim 13 \) in optimally-doped LSCO, which are well reproduced by the present study which takes into account the difference of the curvature of the FS’s [35].

In summary, the violation of the Kohler’s rule in HTSC had been frequently considered as an evidence of the violation of the quasiparticle picture. However, the present study has solved this long-standing problem in terms of the nearly AF Fermi liquid, by taking the current VC. Moreover, we stress that the anomalous enhancement of the thermoelectric power (TEP), \( S \), is also explained from the standpoint of the Fermi liquid with strong AF fluctuations [36]. Experimentally, \( S \) is positive in hole-doped systems at lower temperatures, whereas it is negative in electron-doped ones. This fact can be reproduced by the FLEX approximation, as a natural consequence of the strong \( \epsilon \)-dependence of \( \tau_k (\epsilon) \) and the difference of the position of the cold spot [36].

4.2. Transport phenomena below \( T^* \)

In under-doped HTSC’s, a deep pseudo-gap in the density of states around the chemical potential emerges below \( T^* \sim 200 \text{K} \), which is called the pseudo-gap region. The mechanism of the pseudo-gap has been a central issue in HTSC for years. Spin-fluctuation theories cannot reproduce it. According to precise ARPES measurements, the \( k \)-dependence of the pseudo gap is equal to that of the SC gap function with \( d_{x^2-y^2} \)-symmetry. Moreover, it changes to the SC gap below \( T_c \) smoothly both in energy and in momentum. This fact strongly suggests an intimate relation between the pseudo-gap and the superconductivity. Motivated by the experimental facts, various strong coupling theories of the SC fluctuations have been studied. Several groups have developed the “FLEX+T-matrix theory”; it is equivalent to the FLEX approximation at higher temperatures, whereas the self-energy correction below \( T^* \) by the strong SC fluctuations (T-matrix) mediated by the AF fluctuations (FLEX) is taken into account self-consistently [9,21,30]. The self-energy in this approximation is shown in Fig.8. The FLEX+T-matrix approximation can reproduce the pseudo-gap in DOS, \( 1/T \tau_T \), and the decrease of \( T_c \) in the under-doped region below \( T^* \).

\[ \Sigma(k,\omega) = \]

\[ \begin{aligned}
\text{AF fluctuations} & \quad \text{+} \\
\text{T-matrix} & \quad \text{+} \\
\end{aligned} \]

\[ \begin{array}{c}
\text{AF flac.} \\
\text{+} \\
\text{...} \\
\end{array} \]

\[ \begin{array}{c}
\text{+} \\
\text{...} \\
\end{array} \]

Fig. 8. Self-energy \( \Sigma_k (\omega) \) in the FLEX+T-matrix approximation. The T-matrix for \( d_{x^2-y^2} \)-wave SC is given by the ladder approximation for the AF fluctuations.

It is possible to study various transport phenomena by the FLEX+T-matrix approximation, taking the current VC’s as discussed in the previous section. As shown in Figs.7 and 9, both \( R_H \) and \( S \) start to decrease below \( T^* \sim 150 \text{K} \) in the FLEX+T-matrix approximation; it is consistent with experiments. It is simply understood from the fact that the AF fluctuations, which gradually increase below \( T_0 = 600 \sim 700 \text{K} \), start to decrease below \( T^* \) reversely by reflecting the pseudo-gap formation. Because \( R_H \) and \( S \) are strongly enhanced due to the AF fluctuations as discussed in the previous section, they decrease below \( T^* \).
(i) $S$ obtained by the FLEX+T-matrix approximation. It starts to decrease below $T^* \sim 150$K, which is consistent with the experimental pseudo-gap behavior. (ii) $\nu$ for LSCO obtained by the FLEX+T-matrix; the abrupt increase of $\nu$ below $T^*$ is reproduced. $\nu_{\text{exp}}$ is cited from [37]. (iii) $\nu$ for NCCO obtained by the FLEX. $\nu_{\text{exp}}$ is cited from [39] [H. Kontani: BUTSURI (Bulletin of the Physical Society of Japan) 58 (2003) 524.].

However, the Nernst coefficient $\nu$ ($\nu \equiv S_{yx}/B_z = -E_y/B_z \nabla_x T$; off-diagonal TEP under the magnetic field) is strongly enhanced in the pseudo-gap region about 100 times larger than the usual metals [37,38]. Such a nontrivial behavior of $\nu$ has attracted much attention as a key phenomenon closely related to the origin of the pseudo-gap. According to the linear response theory, $\nu$ is given by

$$\nu = \left[ \alpha_{xy}/\sigma_{xx}^0 - S_{xyy}/\sigma_{xx}^0 \right]/B_z,$$

where $\alpha_{xy}$ is the off-diagonal Peltier conductivity, which is given by

$$\alpha_{xy} = B_z \cdot \frac{\epsilon^2}{T} \sum_k \int \frac{d\epsilon}{2\pi} \left( \frac{\partial f}{\partial \epsilon} \right) \left| \text{Im} G_k^R (\epsilon)/\epsilon \right|^2 \gamma_k (\epsilon) A_k (\epsilon),$$

$$A_k (\epsilon) = \left( \tilde{Q}_k (\epsilon) \times \frac{\partial}{\partial k_y} \left( J_k (\epsilon)/\gamma_k (\epsilon) \right) \right)_z,$$

$$\tilde{Q}_k (\epsilon) = \epsilon \cdot \tilde{v}_k + \sum_{k'} \int \frac{d\epsilon'}{4\pi i} T_{kk'} (\epsilon, \epsilon') G_{kk'}^R (\epsilon') \left| \gamma_{kk'} (\epsilon') \right|^2 \tilde{Q}_{kk'} (\epsilon'),$$

where $\tilde{Q}_k (\epsilon)$ is the total heat current [40,41]. $\nu$ vanishes in a system with the free-electron like dispersion $\epsilon_k = k^2/2m$ because the first and the second terms in Eq.(66) cancel out with each other within the relaxation time approximation, which is known as the Sommerfeld cancellation. In usual metals, $\nu$ is small because of an approximate Sommerfeld cancellation. However, $\nu$ takes an enhanced value in HTSC because $\alpha_{xy}$ is much enhanced due to the current VC’s given by both the AF- and SC-fluctuations; thus the Sommerfeld cancellation is totally broken below $T^*$, as we will see below.

It is known that $\nu$ takes a huge value in the vortex-liquid state above $H_{c1}$ in a clean two dimensional sample, reflecting the high mobility of vortices. In fact, $\nu$ is frequently used as a sensitive probe for the mixed state. Based on his observation, Ong et al. claimed that spontaneous vortex-antivortex pairs emerge in under-doped systems below $T^*$, and they govern the transport phenomena in the pseudo-gap region. [37]. However, his assumption contradicts with other transport coefficients; for example, flux-flow resistance does not appear below $T^*$.

Based on the FLEX+T-matrix approximation with current VC’s, such an anomalous behavior of $\nu$ can be naturally understood as quasiparticle transport phenomena, without assuming any vortex-like excitations [41].
The enhancement of $\nu$ below $T^*$ comes from the following facts [41]: (i) The total current $\vec{J}_k$ is strongly enhanced by the Maki-Thompson type VC’s by SC fluctuations, except for the nodal direction of the SC gap, i.e., $(\pm \pi, \pm \pi)$-direction for the $d_{x^2-y^2}$-wave SC state. (ii) Due to the current VC’s by the AF-fluctuations, $\vec{J}_k$ is not parallel to the total heat current $\vec{Q}_k$ because the effect of the VC on $\vec{Q}_k$ is not prominent in general, whereas $\vec{J}_k \parallel \vec{Q}_k$ is assumed (without any reason) in the RTA [40,41]. The numerical result by the FLEX+T-matrix approximation is shown in Fig.10 [40]. Although the facts (i) and (ii) is assumed (without any reason) in the RTA [40,41]. The results are shown in Figs. 7 and 9. We found that various anomalous transport phenomena in moderately below $T^*$, whereas (i) and (ii) influence $\rho$, $R_H$ and $S$ only slightly, so they start to decrease moderately below $T^*$.

4.3. $\kappa$-(BEDT-TTF)$_2$X, $\kappa$-(BEDT-TTF)$_2$Cu[N(CN)$_2$]Cl $\kappa$-(BEDT-TTF)$_2$X is a strongly correlated two-dimensional superconductor. Its pressure-temperature (P-T) phase diagram, which is often called the Kanoda phase diagram, the SC phase in the higher pressure region is close to the Mott (antiferromagnetic) insulating phase in the lower pressure region [42]. The maximum SC transition temperature is $T_c \approx 12$K. The simplest effective model for $\kappa$-(BEDT-TTF)$_2$X is the anisotropic triangular Hubbard model at half filling [43]. By analysing this model with reasonable model parameters based on the FLEX approximation, the main feature of the P-T phase diagram is well reproduced [44–46]. The symmetry of the SC state is $d_{x^2-y^2}$-wave according to both the FLEX approximation and the third-order perturbation theory with respect to $U$ [9,21].

The resistivity and the Hall coefficient in $\kappa$-(BEDT-TTF)$_2$Cu[N(CN)$_2$]Cl under $P = 4.5 ~ 10$Kbar was measured in Ref. [47]. In their measurement under high pressures, the effect of the thermal contraction of the sample, which is never negligible at ambient pressure, is expected to be much reduced. They found the approximate relations $\rho \propto T$ and $R_H \propto T^{-1}$ for $T = 30 ~ 100$K. Moreover, the Hall angle $\cot \theta_H \equiv \sigma_{xy}/\sigma_{xx}$ is proportional to $T^2$, as observed in various HTSC’s. A similar behavior has recently been observed in $\kappa$-(BEDT-TTF)$_2$Cu[N(CN)$_2$]Br by Taniguchi et al [48]. These non-Fermi liquid behaviors of the transport properties in $\kappa$-(BEDT-TTF)$_2$X are well reproduced by the FLEX approximation when the current VC’s are taken into account [49].

CeCoIn$_5$ is a quasi two-dimensional heavy fermion superconductor with $T_c = 2.3$K. The symmetry of the SC is $d_{x^2-y^2}$-wave according to the measurement of the thermal conductivity under the magnetic field [50]. It is consistent with the theoretical analysis by the FLEX approximation and the perturbation theory with respect to $U$ [9,21]. Transport phenomena in CeCoIn$_5$ are very remarkable [51]: Under 20K, $\rho \propto T$, $R_H \propto T^{-1}$ and the modified Kohler’s rule $\Delta \rho \propto R_H^2$ is satisfied quite well. These behaviors, which are observed in various HTSC’s, are expected to be reproduced theoretically if one takes the current VC’s caused by the strong AF fluctuations in CeCoIn$_5$. These anomalous transport phenomena, which are expected to appear in nearly AF Fermi liquids, confirm the fact that the spin fluctuations theory can de-
scribe well electronic properties in CeCoIn$_5$.

More interestingly, the relations $\sigma_{xy} \propto B_z$ and $\Delta \sigma_{xx} \propto B_z^2$, both of which are assured by Onsager’s reciprocal theorem when $|B_z|$ is small enough, are violated above the very low characteristic fields $B^*_z (\sim 0.1 \text{Tesla})$, although $\omega^* \tau = (eB_z^*/m^*c)v \ll 1$ is expected to be satisfied. These unexpected anomalous field dependences of $\sigma_{xy}$ and $\Delta \sigma_{xx}$ will come from the fact that CeCoIn$_5$ is very close to the AF-QCP: According to current satisfy. These unexpected anomalous field dependences of $\sigma_{xy}$ and $\Delta \sigma_{xx}$ will come from the fact that CeCoIn$_5$ is very close to the AF-QCP: According to current

$$\xi_{AF} \equiv \xi_{AF}^2 \sim \frac{\Delta}{\mu_B T}$$

$\Delta$ and $\Delta \sigma_{xx} \propto \xi_{AF}^2 \tau^3$, $B_z^2$ are expected. The field dependence of $\xi(Q,0) \propto \xi_{AF}^2$, which will be sensitive to the outer magnetic field in the vicinity of the AF-QCP $[52]$, should cause the deviation from $\sigma_{xy} \propto B_z$ and $\Delta \sigma_{xx} \propto B_z^2$.

In fact, experimentally, the ratio $p \equiv (\rho_{xx}(B) - \rho_{xx}(0))/\rho^2_{xy}(B)\rho^2_{xy}(0) [\propto \xi_{AF}^2 \tau^0]$ is nearly constant for $5 \sim 20 \text{K}$ and $0 \sim 4 \text{Tesla}$, although the usual Kohler’s rule is completely broken $[51]$. This fact strongly supports that anomalous transport phenomena in CeCoIn$_5$ can be well described by the current VC’s caused by the strong AF fluctuations.

The Nernst signal $\nu$ in CeCoIn$_5$ is also very anomalous $[53]$. Below 20K, it starts to decreases approximately in proportion to $T^{-1}$, taking an anomalously huge negative value ($\nu \sim -1 \mu V/\text{KT}$) below 4K. This behavior is very similar to the Nernst signal in electron-doped HTSC; it starts to increase below 300K in proportion to $T^{-1}$, and it takes the maximum value $\sim 0.1 \mu V/\text{KT}$, which is still about 100 times larger than that in usual metals. As we have shown above, the temperature dependence and the magnitude of $\nu$ in NCCO are well reproduced by the current VC’s caused by the strong AF fluctuations, even in the absence of the SC fluctuations. (Note that the relation $\nu \propto T^{-1}$ is the hallmark of the importance of the AF fluctuations.) Thus, it is naturally expected that the giant Nernst signal in CeCoIn$_5$ is caused by the current VC’s due to the AF fluctuations. Considering that $\nu$ is proportional to $\tau \propto \rho_0^{-1}$, experimental facts $\nu_{\text{CeCoIn}_5}(4\text{K})/\nu_{\text{NCCO}(100\text{K})} \approx 1(\mu V/\text{KT})/0.1(\mu V/\text{KT}) = 10$ and $\rho_{\text{CeCoIn}_5}(4\text{K})/\rho_{\text{NCCO}(100\text{K})} \approx 5(\mu\Omega)/50(\mu\Omega) = 1/10$ strongly suggest that the origin of the huge $\nu$ in CeCoIn$_5$ is the current VC’s.

Finally, we comment on the effect of the SC fluctuations on the Nernst effect. In hole-doped HTSC’s, $\nu$ increases much faster than $T^{-1}$ below $T^*$, where the SC fluctuations are enhanced. According to the study on the Nernst signal due to the quasiparticle contribution by the FLEX+T-matrix theory, the abrupt increase of $\nu$ in the pseudo-gap region is the cooperative phenomenon between the d-wave SC fluctuations and the AF fluctuations. The SC fluctuations alone will not cause a prominent enhancement of $\nu$ because the total current $J_{\mathbf{k}}$ due to the SC-fluctuations is parallel to $\mathbf{\hat{e}}_{\mathbf{k}}$ (see Eqs. (67), (69), and Ref. [41]).

The effect of SC fluctuations on the quasiparticle transport coefficients in the vicinity of $T_c$, which is called the Maki-Thompson term, has been studied previously for weak-coupling superconductors. The theory of transport phenomena developed by the authors based on the FLEX+T-matrix approximation [41] is an extension of the theory of Maki-Thompson in that (i) the cooperation between the AF- and SC-fluctuations are considered, and (ii) the strong coupling situation where the inelastic quasiparticle damping due to AF- and SC-fluctuations dominates the elastic one. In addition, one have to solve the Bethe-Salpeter equation (61) self-consistently in strongly correlated systems to obtain reasonable results $[34]$. In the case of HTSC’s, the current VC’s are playing important roles in various anomalous transport phenomena which cannot be reproduced within conventional Maki-Thompson terms.

On the other hand, the transport phenomena caused by the short-living Cooper pairs (not by quasiparticles) are called the Azlamazov-Larkin term. More precisely, it is the transport phenomenon due to the amplitude fluctuations of the SC gap function, not by the vortex-like excitations. Recently, the Nernst signal due to the Azlamazov-Larkin term was studied in the particle-hole symmetric case $[54]$. The authors claimed that the abrupt increment of $\nu$ in HTSC below $T^*$ can be explained by the Azlamazov-Larkin term. However, the derived Azlamazov-Larkin term is $\nu \propto O(\tau^{-1})$, whereas the quasiparticle transport term is $\nu \propto O(\tau)$, that is, the latter is $\tau^2$-times larger than the former. Thus, the Azlamazov-Larkin term for $\nu$ can exceed the quasiparticle term only in the close vicinity of $T_c$, when the Sommerfeld cancellation is approximately realized. As we have shown above, however, the Sommerfeld cancellation is totally broken in HTSC’s due to the current VC’s. In the under-doped HTSC’s, the diamagnetism due to the fluctuation of the SC order parameter above $T_c$, which is represented theoretically by the Azlamazov-Larkin term and is known to sensitive to the strength of the field, is observed only in the vicinity of $T_c$: $T_c \lesssim T_c + 5K [37,55]$. In conclusion, the abrupt increment of $\nu$ below $T^*(\gtrsim 100K)$ should be the quasiparticle origin, as discussed in Ref. [41].

4.4. summary of this section

We discussed the recent progress in theoretical studies on the non-Fermi liquid like transport phenomena in HTSC’s, i.e., Hall coefficient, magnetoresistance, electric thermopower and the Nernst coefficient. They are well reproduced by the FLEX or FLEX+T-matrix approximation in a unified way, if one takes the current VC’s into account. In particular, anomalous transport phenomena in the pseudo-gap region are understood successfully by the FLEX+T-matrix approximation, which strongly supports that the origin of the pseudo-gap in HTSC is the strong SC fluctuations induced by the AF fluctuations $[9,21,30]$. Quantitative numerical studies on the transport phenomena, which takes into account the current VC’s, are at the beginning of progress. We expect that fruitful results will be obtained in future on the trans-
port phenomena in various strongly correlated electron systems, not restricted to HTSC’s.

5. Conclusions
In this paper we have stressed that the spin singlet state discovered by Kondo effect is common to all the Fermi liquid irrespectively of impurity or periodic systems. Therefore, the resonating valence bond state in metals is nothing but the Fermi liquid state. When we study physical properties in strongly correlated electron systems, it is very important to consider the vertex correction terms to satisfy the conservation laws. In particular, since electron systems possess the lattice potential, the Umklapp scattering and spin fluctuation originating from electron interactions play the important roles in explaining the anomalous behavior in terms of the Fermi liquid theory. We have shown the typical examples such as Hall coefficient, magnetoresistance and Nernst coefficient.

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