RE-EXAMINATION OF GENERATION
OF BARYON AND LEPTON NUMBER
ASYMMETRIES BY HEAVY PARTICLE DECAY

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ABSTRACT

It is shown that wave function renormalization can introduce an important
correlation to the generation of baryon and lepton number asymmetries by heavy
particle decay. These terms, omitted in previous analyses, are of the same order
of magnitude as the standard terms. A complete cancellation of leading terms can
result in some interesting cases.
The three key elements for baryogenesis, namely baryon number violation, $C$ and $CP$ violation and departure from thermal equilibrium were clearly identified in Sakharov’s historic paper $^1$ in 1967. Realistic calculations of baryogenesis only became possible however in the late 1970’s, after the introduction of grand unified theories (GUTS), which provided a clear field theoretical model in which baryon number violation occurs.$^2$

The early calculations followed a standard pattern, colloquially referred to as the “drift and decay mechanism”. Pre-existing asymmetries were presumed to be erased before the breaking of the GUT symmetry. A particle $S$, usually a colored Higgs boson, has a long enough lifetime so that it is out of thermal equilibrium when it finally decays. Since the decaying particle has at least two decay modes with different baryon number and its couplings violate $CP$, the ingredients are all in place for baryogenesis.

It was realized almost immediately that one needed to go beyond the tree approximation in calculating decay amplitudes: otherwise $CPT$ invariance leads to a zero baryon asymmetry. Therefore the standard calculation involves an interference between eg. a tree level diagram for $S$ decaying into fermions $S \rightarrow f_1 f_2$ and a one or more loop diagrams for the same process.

Many refinements and elaborations have taken place in the past fifteen years. Departures from the so called “drift and decay mechanism” have been numerous: the most influential one has resulted from the observation by Kuzmin, Rubakov and Shaposhnikov that non-trivial vacuum gauge configurations can lead to a significant baryon number violation at low temperature $^3$ ($\sim 100 \text{ GeV}$). In this note, we will have nothing to say directly about low temperature baryogenesis.$^4$ Our comments are most applicable to the earlier calculations and variations thereof.
It was realized recently\(^5\) that wave function renormalization of a heavy unstable particle can introduce important effects for \(CP\)-violating asymmetries. Baryon number asymmetry is one such particularly interesting example. We have realized that, whereas vertex corrections to the \(S \to f_1 f_2\) decay were treated consistently, external line insertions associated with wave function renormalizations were not. Since in general these are of the same order of magnitude, the calculations change substantially. In one particular example, we will in fact show that the vertex and external line insertions cancel: since, as we said earlier baryon asymmetry is zero at tree level, these corrections are the leading contributions to our process.

To be specific, consider a \(B\)- and \(CP\)-violating interaction (the standard \(SU(5)\) GUT model has two additional interactions of similar form which we have omitted for simplicity)

\[
\mathcal{L}_I = G_\xi \bar{u}_{R,\alpha} e_c R S_{\xi,\alpha} + F_\xi \bar{d}_R,\beta S_{\xi,\gamma} e^{\alpha\beta\gamma} + h.c.,
\]

where \(S_{\xi,\alpha}\) is a heavy scalar belonging to the 5 representation of \(SU(5)\). \(u, d\) and \(e^c\) are the charged fermions of the first generation, \(\alpha, \beta\) and \(\gamma\) are the color indices, and \(\xi = 1, 2, \ldots\) labels different species of \(S\). Complex couplings \(G_\xi\) and \(F_\xi\) are the sources of \(CP\) violation. For simplicity we neglect fermion mixings.

Evidently, baryon number asymmetries generated from \(S_\xi\) decays are determined by the partial rate difference

\[
\Delta S_{\xi} = \Gamma(S_\xi \to e\bar{u}^c) - \Gamma(\bar{S}_\xi \to \bar{e}u^c),
\]

\[
= \Gamma(\bar{S}_\xi \to d\bar{u}^c) - \Gamma(S_\xi \to \bar{d}u^c),
\]

where \(\bar{S}_\xi, \bar{e}\) and \(\bar{u}^c\) are the \(CP\) conjugates of \(S_\xi, e\) and \(u^c\) respectively, and the last step of Eq. (2) follows from \(CPT\). Unless necessary, henceforth we will not display the color indices explicitly.
It follows from Eq. (1) that $S_{\xi}$ has only two decay modes with final states $e\bar{u}^c$ and $d\bar{u}^c$. Adjoining the one-shell $t$-channel final-state scattering $d\bar{u}^c \rightarrow e\bar{u}^c$ to $S_{\xi} \rightarrow d\bar{u}^c$ (Fig. 1b) corresponds to a calculation of an absorptive part of a vertex correction$^6$ (Fig. 1c). The interference of the vertex correction (Fig. 1c) with the tree-level amplitude (Fig. 1a) yields the standard result

$$\Delta S_{\xi}(\text{vertex}) = \frac{M_{\xi}}{32\pi^2} \sum_{\xi'} \text{Im}(G_{\xi}^* G_{\xi'} F_{\xi} F_{\xi'}^*) \left[ 1 - \frac{M_{\xi}^2}{M_{\xi}^2} \ln \left( 1 + \frac{M_{\xi}^2}{M_{\xi'}^2} \right) \right],$$

(3)

where $M_{\xi}$ is the mass of $S_{\xi}$. All fermions are massless at the scale of $M_{\xi}$.

In addition to the $t$-channel scattering, the two final states are also related by an $s$-channel interaction. Adjoining the on-shell $s$-channel amplitude $d\bar{u}^c \rightarrow e\bar{u}^c$ to $S_{\xi} \rightarrow d\bar{u}^c$ corresponds to a calculation of an absorptive part of a wave function renormalization correction (Fig. 1d). If the scalars are not degenerate, the calculation is very simple with the result from the interference of Figs. (1a) and (1d) given by

$$\Delta S_{\xi}(\text{wave}) = -\frac{M_{\xi}}{32\pi^2} \sum_{\xi'} \text{Im}(G_{\xi}^* G_{\xi'} F_{\xi} F_{\xi'}^*) \frac{M_{\xi}^2}{M_{\xi}^2 - M_{\xi'}^2}.$$ 

(4)

In obtaining this result we have assumed for simplicity that $(M_{\xi} - M_{\xi'})^2 \gg (\Gamma_{\xi} - \Gamma_{\xi'})^2$, where $\Gamma_{\xi}$ is the width of $S_{\xi}$. This contribution, which is of the same order as $\Delta S_{\xi}(\text{vertex})$, has been missed by early calculations.

The significance of $\Delta S_{\xi}(\text{wave})$ may be illustrated by its limiting values. When $M_{\xi'} \gg M_{\xi}$ we have from Eqs. (3) and (4)

$$\Delta S_{\xi}(\text{wave}) = 2\Delta S_{\xi}(\text{vertex}).$$

(5)

Thus, neglecting $\Delta S_{\xi}(\text{wave})$ under-estimates $\Delta S_{\xi}(\text{total})$ by a factor of 3 in this
limit. In the opposite limit, i.e., $M_{\xi^\prime} \ll M_\xi$, Eqs. (3) and (4) lead to

$$\Delta S_\xi(wave) = -\Delta S_\xi(vertex), \quad (6)$$

and the total result even cancels to leading order in $(M_{\xi^\prime}^2/M_\xi^2)$.

The relative size and sign of $\Delta S_\xi(vertex)$ and $\Delta S_\xi(wave)$ in the limit $M_{\xi^\prime} \gg M_\xi$ can be understood easily by a Fierz transformation. In general, their final-state scattering amplitude in Figs. (1c) and (1d) is given by

$$A(\bar{d}u^c \rightarrow e\bar{u}^c) = -i G_{\xi^\prime} F_{\xi^\prime}^\alpha e^{a\beta\gamma} \left[ \frac{[\bar{u}_e(p_1)L\gamma(k_1)][\bar{v}_\beta(k_2)Lu_\alpha(p_2)]}{(p_1 - k_1)^2 - M_\xi^2} \right. \left. + \frac{[\bar{u}_e(p_1)Lu_\alpha(p_2)][\bar{v}_\beta(k_2)L\gamma(k_1)]}{(p_1 + p_2)^2 - M_{\xi^\prime}^2} \right], \quad (7)$$

where the $u^\prime$s and $v^\prime$s are the standard Dirac spinors. The first term arises from the $t$-channel (Fig. 1c) and the second is due to the $s$-channel (Fig. 1d). Averaging over the incident momenta in the center of mass frame yields for the $J = 0$ partial wave ($J$ is the total angular momentum)

$$\int_{-1}^{1} d\cos \theta \sum_{spin} A = C^\alpha [\bar{u}_e(p_1)Lu_\alpha(p_2)] \left[ 1 - \frac{M_\xi^2}{s} \ln \left( 1 + \frac{s}{M_\xi^2} \right) \right] - \frac{s}{s - M_{\xi^\prime}^2}, \quad (8)$$

where $s = (k_1 + k_2)^2 = M_\xi^2$ and $C^\alpha$ is an overall factor determined by the couplings $G_{\xi^\prime}$ and $F_{\xi^\prime}$ and the normalization constants of the spinors. The ratio between the first and second terms is precisely $\Delta S_\xi(vertex)/\Delta S_\xi(wave)$.

Including fermion mixings the couplings $G_{\xi}$ and $F_{\xi}$ become matrices in flavor space. Hence, generally speaking, $\Delta S_\xi(vertex)$ and $\Delta S_\xi(wave)$ are not simply related as in Eqs. (5) and (6): for vertex corrections one will have a trace over a family
matrix to the fourth power while the wave function correction will have a product of two traces of the square of family matrices. Still, the order-of-magnitudes of $\Delta S_{\xi}(vertex)$ and $\Delta S_{\xi}(wave)$ are the same.

The model of baryogenesis we have considered requires more than one Higgs color triplet ($S_{\xi} \neq S_{\xi'}$) and does not have natural flavor conservation, i.e., both $S_{\xi}$ and $S_{\xi'}$ couple to the two scalar fermion currents. The additional diagrams we have been discussing will not make any contribution without these features.

On the other hand the vertex diagrams also give vanishing contributions to baryogenesis in lowest order for the minimal model; the first non-vanishing contribution arises from a three loop diagram. We have displayed here the simplest $SU(5)$ like model which contributes to baryogenesis in lowest order.

It is interesting to notice that Fig. (1d) is a one-particle-reducible (OPR) diagram. Even though one can introduce a renormalization scheme in which the renormalized self-energy matrix $\Sigma^{(R)}_{\xi\xi'}(p)$ vanishes on-shell, i.e., $\Sigma^{(R)}_{\xi\xi'}(p)|_{p^2=M_{\xi}^2} = \Sigma^{(R)}_{\xi'\xi'}(p)|_{p^2=M_{\xi'}^2} = 0$, that $\Delta S_{\xi}(wave) \neq 0$ is because the kinetic energy part of the renormalized lagrangian will not have the standard normalization, and the renormalized field does not conjugate to its hermitian conjugate (Ref. 5), due to the non-hermiticity of the renormalized effective lagrangian.

It seems that the situation of most interest (Ref. 3) is that in which the heavy particle masses are nearly degenerate, i.e., $(M_{\xi} - M_{\xi'}) - i(\Gamma_{\xi} - \Gamma_{\xi'})/2 \rightarrow 0$. In that case Eq. (4) is invalid. If $CP$ violation still can be treated perturbatively, $\Delta S_{\xi}(wave)$ can be obtained by studying the renormalization effect on unstable particle propagator $\langle 0|TS_\xi(x)S_{\xi'}(y)|0 \rangle$. An analogous example with $\xi = \xi' = 1$ for the $CP$-violating partial rate difference of the decay $t \rightarrow bW^+, bH^+$ is discussed in Ref. 5. Methods useful for degenerate unstable particles with large $CP$-violating
interactions are still unfortunately unavailable.

Wave function renormalization also plays an important role in leptogenesis. Consider the generation of lepton number asymmetries by heavy Majorana neutrino decay.\textsuperscript{8–12} We will assume the neutrinos get their masses by the usual “seesaw” mechanism\textsuperscript{13} so that we have very heavy Majorana neutrinos $N_a$ coupled to, on the $\sim 100 \ GeV$ scale, effectively massless neutrinos $\nu_b$. The coupling between $N_a$ and $\nu_b$ is of the form

$$\mathcal{L}_I = \overline{N}_a (V_{ab} R + V_{ba}^* L) \nu_b \phi + h.c.,$$

where $\phi$ is a neutral scalar meson. The indices $b(a)$ runs from $1(n+1)$ to $n(2n)$, where $n$ is the number of neutrino families, generally taken to be three. The Majorana form of the mass matrix imposes the condition $V_{ab} = V_{ba}$. Although, strictly speaking, Majorana neutrinos do not carry a lepton number, $CP$ violation can nevertheless introduce a partial rate difference

$$\Delta_{ab} = \Gamma(N_a \rightarrow \phi \nu_{R,b}) - \Gamma(N_a \rightarrow \phi \nu_{L,b}).$$

A lepton number asymmetry can therefore be generated if we assign a lepton number $L = \pm 1$ to the left- and right-handed light neutrinos (the latter are of course usually referred to as anti-neutrinos).

The interference of the tree-level amplitude (Fig. 2a) and the vertex correction (Fig. 2b) yields

$$\Delta_{ab(\text{vertex})} = \frac{1}{64\pi^2} \sum_{c,d} \left[ m_c Im(V_{ab} V_{cb}^* V_{dc}^* V_{da}) \left[ 1 - \left( 1 + \frac{m_c^2}{m_a^2} \right) \ln \left( 1 + \frac{m_a^2}{m_c^2} \right) \right] \right]$$

$$+ m_a Im(V_{ab} V_{cb}^* V_{cd}^* V_{ad}) \left[ 1 - \frac{m_c^2}{m_a^2} \ln \left( 1 + \frac{m_a^2}{m_c^2} \right) \right] + \frac{m_a^2}{m_a} \left[ \ln \left( 1 + \frac{m_a^2}{m_c^2} \right) \right],$$

(11)
where $m_a$ and $m_c$ are respectively the masses of $N_a$ and $N_c$, and for simplicity we have neglected scalar masses. The light neutrino masses are also neglected since they are, for all practical purposes, massless. The first term in Eq. (11) corresponds to an internal mass insertion. The second, which has not been included in early studies, is due to an external neutrino mass insertion. The final-state interaction in this vertex correction goes through a $t$-channel with a total angular momentum $J = 1/2$.

Once again wave function renormalization gives a contribution (Fig. 2c) of the same order as $\Delta_{ab}(\text{vertex})$. For non-degenerate heavy neutrinos we find

$$\Delta_{ab}(\text{wave}) = \frac{1}{128\pi^2} \sum_{c,d} \frac{m_a^2}{m_a^2 - m_c^2} \left[ m_c \text{Im}(V_{ab}^* V_{cb}^* V_{da}^*) + m_a \text{Im}(V_{ab}^* V_{cb}^* V_{cd}^*) \right],$$

(12)

where particle widths have been neglected for simplicity. Here the final-state interaction goes through an $s$-channel with $J = 1/2$. Asymptotically, $\Delta_{ab}(\text{vertex})$ and $\Delta_{ab}(\text{wave})$ have the following simple relations

$$\Delta_{ab}(\text{wave}) = \Delta_{ab}(\text{vertex}), \quad \frac{m_c}{m_a} \gg 1,$$

$$\Delta_{ab}(\text{wave}) = \frac{1}{2} \Delta_{ab}(\text{vertex}), \quad \frac{m_c}{m_a} \ll 1.$$

(13)

Thus, neglecting $\Delta_{ab}(\text{wave})$ will under-estimate the lepton number asymmetry by a factor of $2(3/2)$ in the limit $m_c/m_a \gg 1$ ($m_c/m_a \ll 1$).

In conclusion, we have re-examined the generation of baryon and lepton number asymmetries by heavy particle decays. We have shown that an important piece of contribution due to wave function renormalization has been missed by early investigations. This missing piece is generally of the same order of magnitude as the other terms calculated before, and can result, in some interesting cases, in a complete cancellation of the leading terms.
Current thinking often emphasizes the erasure of previously existing baryon and lepton asymmetries at temperatures of $\sim 100 GeV$, which tends to de-emphasize the importance of high temperature phenomena. We do wish to remind the reader, nevertheless, that sphaleron processes (Ref. 4 ) conserve the anomaly $B - L$ so that earlier asymmetries may still be important.$^{14}$
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FIGURE CAPTIONS

Fig. 1. Feynman diagrams for the generation of baryon number asymmetries from the decay of a colored heavy scalar $S_{\xi,\alpha}$ in an $SU(5)$ model.

Fig. 2. Feynman diagrams for the generation of lepton number asymmetries from the decay of a heavy Majorana neutrino $N_{a}$.
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