New Charmonium States in QCD Sum Rules: a Concise Review

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Abstract

In the past years there has been a revival of hadron spectroscopy. Many interesting new hadron states were discovered experimentally, some of which do not fit easily into the quark model. This situation motivated a vigorous theoretical activity. This is a rapidly evolving field with enormous amount of new experimental information. In the present report we include and discuss data which were released very recently. The present review is the first one written from the perspective of QCD sum rules (QCDSR), where we present the main steps of concrete calculations and compare the results with other approaches and with experimental data.
1 Introduction

We are approaching the end of a decade which will be remembered as the “decade of the revival of hadron spectroscopy”. During these years several $e^+e^-$ colliders started to operate and produce a large body of experimental information. At the same time new data came from the existing $p-\bar{p}$ colliders and also from the $e-p$ accelerators. In Table 1 we give a list of the new charmonium states observed in these accelerators.

Table 1: Charmonium states observed in the last years.

| state     | production mode | decay mode | ref. |
|-----------|-----------------|------------|------|
| $X(3872)$ | $B \to KX(3872)$ | $J/\psi \pi$ | [1]  |
| $X(3915)$ | $\gamma\gamma \to X(3915)$ | $J/\psi \omega$ | [2]  |
| $Z(3930)$ | $\gamma\gamma \to Z(3930)$ | $DD^*$ | [3]  |
| $Y(3930)$ | $B \to KY(3930)$ | $J/\psi \pi$ | [4]  |
| $X(3940)$ | $e^+e^- \to J/\psi X(3940)$ | $D\bar{D}^*$ | [5]  |
| $Y(4008)$ | $e^+e^- \to \psi\psi Y(4008)$ | $J/\psi \pi$ | [6]  |
| $Z_1^+(4050)$ | $B^0 \to KZ_1^+(4050)$ | $\chi_{c1}\pi^+$ | [7]  |
| $Y(4140)$ | $B \to KY(4140)$ | $J/\psi \phi$ | [8]  |
| $X(4160)$ | $e^+e^- \to J/\psi X(4160)$ | $D^*\bar{D}$ | [9]  |
| $Z_2^+(4250)$ | $B^0 \to KZ_2^+(4250)$ | $\chi_{c1}\pi^+$ | [10] |
| $Y(4260)$ | $e^+e^- \to \psi\psi Y(4260)$ | $J/\psi \pi$ | [11] |
| $X(4350)$ | $\gamma\gamma \to X(4350)$ | $J/\psi \phi$ | [12] |
| $Y(4360)$ | $e^+e^- \to \psi\psi Y(4360)$ | $\psi'\pi$ | [13] |
| $Z^*(4430)$ | $B^0 \to KZ^*(4430)$ | $\psi'\pi^*$ | [14] |
| $X(4630)$ | $e^+e^- \to \psi\psi Y(4630)$ | $\Lambda^c\Lambda^c$ | [14] |
| $Y(4660)$ | $e^+e^- \to \psi\psi Y(4660)$ | $\psi'\pi$ | [15] |

In what follows we will review and comment all this information.

The study of spectroscopy and the decay properties of the heavy flavor mesonic states provides us with useful information about the dynamics of quarks and gluons at the hadronic scale. The remarkable progress on the experimental side, with various high energy machines has opened up new challenges in the theoretical understanding of heavy flavor hadrons.

1.1. New experiments

The $B$-factors, the PEPII at SLAC in the U.S.A., and the KEKB at KEK in Japan, were constructed to test the Standard Model mechanism for CP violation. However, their most interesting achievement was to contribute to the field of hadron spectroscopy, in particular in the area of charmonium spectroscopy. They are $e^+e^-$ colliders operating at a CM energy near 10,580 MeV. The $BB$ pairs produced are measured by the BaBar (SLAC) and Belle (KEK) collaborations. Charmonium states are copiously produced at the $B$-factories in a variety of processes. At the quark level, the $b$ quark decays weakly to a $c$ quark accompanied by the emission of a virtual $W^-$ boson. Approximately half of the time, the $W^-$ boson materializes as a $c\bar{c}$ pair. Therefore, half of the $B$ meson decays result in a final state that contains a $c\bar{c}$ pair. When these $c\bar{c}$ pairs are produced...
close to each other in phase space, they can coalesce to form a \( c\bar{c} \) charmonium meson.

The simplest charmonium producing \( B \) meson decay is: \( B \to K(c\bar{c}) \). Another interesting form to produce charmonium in \( B \)-factories is directly from the \( e^+e^- \) collision, when the initial state \( e^+ \) or \( e^- \) occasionally radiates a high energy \( \gamma \)-ray, and the \( e^+e^- \) subsequently annihilate at a corresponding reduced CM energy, as illustrated in Fig. 1.

When the energy of the radiated \( \gamma \)-ray (\( \gamma_{\text{ISR}} \)) is between 4000 and 5000 MeV, the \( e^+e^- \) annihilation occurs at CM energies that correspond to the range of mass of the charmonium mesons. Thus, the initial state radiation (ISR) process can directly produce charmonium states with \( J^{PC} = 1^- \).

1.2. New states

Many states observed by BaBar and Belle collaborations, like the \( X(3872), Y(3930), Z(3930), X(3940), Y(4008), Y(4050), Y(4140), X(4160), Z(4250), Y(4260), Y(4360), Z^*(4430) \) and \( Y(4660) \), remain controversial. A common feature of these states is that they are seen to decay to final states that contain charmed and anticharmed quarks. Since their masses and decay modes are not in agreement with the predictions from potential models, they are considered as candidates for exotic states. By exotic we mean a more complex structure than the simple quark-antiquark state, like hybrid, molecular or tetraquark states. The idea of unconventional quark structures is quite old and despite decades of progress, no exotic meson has been conclusively identified. In particular, those with \( q\bar{q} \) quantum numbers should mix with ordinary mesons and are thus hard to understand. Therefore, the observation of these new states is a challenge to our understanding of QCD.

In 2003 a great deal of attention was given to pentaquark states, which may be exotic objects \([42, 43]\). After many negative results the study of these states was abandoned. However, in that same year the discovery by BELLE of the unusual charmonium state named \( X \) prompted a lively and deep discussion about the existence of exotic states in the charmonium sector (i.e. non pure \( c - \bar{c} \) states), exactly where, up to that moment, the calculations based on potential models had worked so well.

Establishing the existence of these states means already a remarkable progress in hadron physics. Moreover it poses the new question: what is the structure of these new states? The debate on exotic hadron structure was strongly revived during the short “pentaquark era”. Unfortunately, because of the uncertainties on the experimental side, it was very soon aborted. However, some interesting ideas were passed along to the subsequent discussion about the nature of the \( X, Y \) and \( Z \) states, which is still in progress.

1.3. New quark configurations

Concerning the structure we can say that there are still attempts to interpret the new states as \( c - \bar{c} \). One can pursue this approach introducing corrections in the potential, such as quark pair creation. This “screened potential” changes the previous results, obtained with the unscreened potential and allows to understand some of the new data in the \( c - \bar{c} \) approach \([44]\). Departing from the \( c - \bar{c} \) assignment, the next option is a system composed by four quarks, which can be uncorrelated, forming a kind of bag, or can be grouped in diquarks which then form a bound system \([45, 46, 47]\). These configurations are called tetraquarks. Alternatively, these four quarks can form two mesons which then interact and form a bound state. If the mesons contain only one charm quark or antiquark, this configuration is referred to as a molecule \([48, 49, 50]\). One if the mesons is a charmonium, then the configuration is called hadro-charmonium \([51]\). Another possible configuration is a hybrid charmonium \([52, 53]\). In this case, apart from the \( c - \bar{c} \) pair, the state contains excitations of the gluon field. In some implementations of the hybrid, the excited gluon field is represented by a “string” or flux tube, which can oscillate in normal modes \([54]\).

The configurations mentioned in the previous paragraph are quite different and are governed by different dynamics. In quarkonia states the quarks have a short range interaction dominated by one gluon exchange and a long range non-perturbative confining interaction, which is often parametrized by a linear attractive potential. In tetraquarks besides these two types of interactions, we may have a diquark-antidiquark interaction, which is not very well known. In molecules and hadro-charmonium the interaction occurs through meson exchange. Finally, in some models inspired by lattice QCD results, there is a flux tube formation between color charges and also string junctions. With these building blocks one can construct very complicated “stringy” combinations of quarks and antiquarks and their interactions follow the rules of string fusion and/or recombination \([54, 55]\). In principle, the knowledge of the interaction should be enough to determine the spatial configuration of the system. In practice, this is only feasible in simple cases, such as the charmonium in the non-relativistic approach, where having the potential one can solve the Schrödinger equation and determine the wave function. In other approaches the spatial configuration must be guessed and it may play a crucial role in the production and decay of these states.

1.4. New mechanisms of decay and production

The theoretical study of production and decays of the new quarkonium states is still in the beginning. Some of the states are quite narrow and this is difficult to understand in some of the theoretical treatments, as for example, in the molecular approach.
The narrowness of a state might be, among other possibilities, a consequence of its spatial configuration. Assuming, for example, that the \(X(3872)\) is a set of \(c\bar{c}\) quarks confined in a bag but with sufficient (or nearly sufficient) energy to decay into the two mesons \(D\) and \(\bar{D}\), why is it so difficult for them to do so? What mechanism hinders this seemingly simple coalescence process, which is observed in other hadronic reactions? In a not so distant past the same questions was asked in the case of the \(\Theta^+\) pentaquark. Now we know that the existence of this particle is, to say the least, not very likely. Nevertheless, as mentioned in a previous subsection, some of the ideas advanced in the pentaquark years might be retracted now in a different context to answer the question raised above. In fact, the pentaquark decay \(\Theta^+(uudds) \rightarrow n(udd) + K^+(u\bar{s})\) was energetically “superallowed”, since there was enough phase space available and no process of quark pair creation nor annihilation. Why was this quark rearrangement and hadronization difficult? It has been conjectured \[56\] that the \(\Theta^+\) was in a diquark \((ud)\) - diquark \((ud)\) - antidiquark \((\bar{c}\bar{s})\) configuration, such that, due to the not very well understood repulsive and/or attractive diquark interactions, the two diquarks were far apart from each other with the antiquark standing in the middle. In this configuration it was difficult for one diquark to capture the missing \(d\) quark (to form a neutron), which was very far. These words were implemented in a quantitative model and this configuration was baptized “peanut”, since the three bodies were nearly aligned. Another pentaquark spatial configuration with small decay width was the equilateral tetrahedron with four quarks located in the corners and the antiquark in the center \[57\]. An other interesting conjecture based on lattice arguments was that, due to the dynamics of string rearrangement, the pentaquark constructed with three string junctions had, during the decay process, to pass through a very energetic configuration. Since its initial energy would be much less than that, this passage would have to proceed through tunneling and would therefore be strongly suppressed \[54\].

These ideas are relevant for the new charmonium physics because if they are multiquark states, as it seems to be the case, their spatial configuration will play a more important role both in production and decay.

The production of some of these states (such as, for example, the \(X\)) has been observed both in \(e^+ e^-\) and \(p\bar{p}\) colliders, the latter being much more energetic. The \(X\) produced in electron - positron collisions comes from \(B\) decays, whereas in \(p\bar{p}\) reactions it must come from a hard gluon splitting into a \(c\bar{c}\) pair, which then undergoes some complicated fragmentation process. In theoretical simulations \[58\], it was shown that it is very difficult to produce \(X\) if it is composed by two bound mesons.

Some reviews about these states can be found in the literature \[60, 61, 62, 63, 64, 65, 66, 67, 68\]. There are at least two reasons for writing a report on the subject. The first one is because this is a rapidly evolving field with enormous amount of new experimental information coming from the analysis of BELLE and BABAR accumulated data and also from CLEO and BES which are still running and producing new data. New data are also expected to come from the LHCb, which will start to operate soon and will be generating data with very high statistics for the next several years. In the present report we include and discuss data which were not yet available to the previous reviewers.

The other reason which motivates us to write this report is that, on the theory side each review is biased and naturally emphasizes the approach followed by the authors. Thus, in Refs. \[60\] and \[64\] the authors present the available data and then discuss their theoretical interpretations making a sketch of the existing theories. In \[60\] attention is given to the conventional quark antiquark potential model and to models of the interaction between mesons which form a molecular state. In \[66\] a very nice overview of different theoretical approaches such as potential models, QCD sum rules and lattice QCD was presented. The author works out some pedagogical examples, using general considerations and simplified assumptions. The present review is the first one written from the perspective of QCD sum rules, where we present the main steps of concrete calculations and compare the results with other approaches and with experimental data.

In the next sections we discuss the experimental data and the possible interpretations for the recently observed \(X\), \(Y\) and \(Z\) mesons.

2. QCD sum rules

2.1. Correlation functions

QCD sum rules are discussed in many reviews \[69, 70, 71, 72\] emphasizing various aspects of the method. The basic idea of the QCD sum rule formalism is to approach the bound state problem in QCD from the asymptotic freedom side, \(i.e.,\) to start at short distances, where the quark-gluon dynamics is essentially perturbative, and move to larger distances where the hadronic states are formed, including non-perturbative effects “step by step”, and using some approximate procedure to extract information on hadronic properties.

The QCD sum rule calculations are based on the correlator of two hadronic currents. A generic two-point correlation function is given by

\[
\Pi(q) \equiv i \int d^4 x e^{iqx} \langle 0 | T [j(x) j^\dagger(0)] | 0 \rangle ,
\]

where \(j(x)\) is a current with the quantum numbers of the hadron we want to study.

The fundamental hypothesis of the sum rules approach is the assumption that there is an interval in \(q\) over which the correlation function may be equivalently described at both the quark and the hadron levels. The former is called QCD or OPE side and the latter is called phenomenological side. By matching the two sides of the sum rule we obtain information on hadron properties.

2.2. The OPE side

At the quark level the complex structure of the QCD vacuum leads us to employ the Wilson’s operator product expansion (OPE) \[73\].
In QCD we only know how to work analytically in the perturbative regime. Therefore, the perturbative part of \( \Pi(q) \) in Eq. (1) can be reliably calculated. However, this does not yet imply that all important contributions to the QCD side of the sum rule have been taken into account. The complete calculation has to include the effects due to the fields of soft gluons and quarks populating the QCD vacuum. A practical way to calculate the vacuum-field contributions to the correlation function is through a generalized Wilson OPE. To apply this method to the correlation function \( \Pi \), one has to expand the product of two currents in a series of local operators:

\[
\Pi(q) = i \int d^4 x e^{iq \cdot x} \langle 0 | T[j(x) j^\dagger(0)] | 0 \rangle = \sum_n C_n(Q^2) \hat{O}_n, \tag{2}
\]

where the set \( \{\hat{O}_n\} \) includes all local gauge invariant operators expressible in terms of the gluon fields and the fields of light quarks. Eq. (2) is a concise form of the Wilson OPE. The coefficients \( C_n(Q^2) \) (\( Q^2 = -q^2 \)), by construction, include only the short-distance domain and can, therefore, be evaluated perturbatively. Non-perturbative long-distance effects are contained only in the local operators. In this expansion, the operators are ordered according to their dimension \( n \). The lowest-dimension operator with \( n = 0 \) is the unit operator associated with the perturbative contribution: \( C_0(Q^2) = \Pi^{pert}(Q^2) \), \( \hat{O}_0 = 1 \). The QCD vacuum fields are represented in \( \{\hat{O}_n\} \) in the form of vacuum condensates. The lowest dimension condensates are the quark condensate of dimension three: \( \hat{O}_1 = \langle \bar{q} q \rangle \), and the gluon condensate of dimension four: \( \hat{O}_4 = (g^2 G^2) \). For non exotic mesons, i.e. normal quark-antiquark states, the contributions of condensates with dimension higher than four are suppressed by large powers of \( \Lambda_{QCD}^2/Q^2 \), where \( 1/\Lambda_{QCD} \) is the typical long-distance scale. Therefore, even at intermediate values of \( Q^2 (\sim 1 \text{ GeV}^2) \), the expansion in Eq. (2) can be safely truncated after dimension four condensates. However, for molecular or tetraquark states, the mixed-condensate of dimension five: \( \hat{O}_5 = \langle \bar{q} q \bar{g} g G_q \rangle \), the four-quark condensate of dimension six: \( \hat{O}_6 = \langle \bar{q} q \bar{q} q \rangle \) and even the quark condensate times the mixed-condensate of dimension eight: \( \hat{O}_8 = \langle \bar{q} q \bar{q} q \bar{g} g G_q \rangle \), can play an important role. The gluon-condensate of dimension-six: \( \hat{O}_6 = (g^2 G^2) \) can be safely neglected, since it is suppressed by the loop factor 1/16\( \pi^2 \).

In the case of the (dimension six) four-quark condensate and the (dimension eight) quark condensate times the mixed-condensate, in general factorization assumption is assumed and their vacuum saturation values are given by:

\[
\langle \bar{q} q \bar{q} q \rangle = \langle \bar{q} q \rangle^2, \quad \langle \bar{q} q \bar{q} q \bar{g} g G_q \rangle = \langle \bar{q} q \rangle \langle \bar{q} q \bar{g} g G_q \rangle. \tag{3}
\]

Their precise evaluation requires more involved analysis including a non-trivial choice of factorization scheme \([74]\). In order to account for deviations of the factorization hypothesis, we will use the parametrization:

\[
\langle \bar{q} q \bar{q} q \rangle = \rho \langle \bar{q} q \rangle^2, \quad \langle \bar{q} q \bar{q} q \bar{g} g G_q \rangle = \rho \langle \bar{q} q \rangle \langle \bar{q} q \bar{g} g G_q \rangle. \tag{4}
\]

where \( \rho = 1 \) gives the vacuum saturation values and \( \rho = 2.1 \) indicates the violation of the factorization assumption \([71, 75, 76]\).

2.3. The phenomenological side

The generic correlation function in Eq. (1) has a dispersion representation

\[
\Pi(q^2) = -\int ds \frac{\rho(s)}{q^2 - s + i\epsilon} + \cdots, \tag{5}
\]

through its discontinuity, \( \rho(s) \), on the physical cut. The dots in Eq. (5) represent subtraction terms. The discontinuity can be written as the imaginary part of the correlation function:

\[
\rho(s) = \frac{1}{\pi} \text{Im}[\Pi(s)]. \tag{6}
\]

The evaluation of the spectral density (\( \rho(s) \)) is simpler than the evaluation of the correlation function itself, and the knowledge of \( \rho(s) \) allows one to recover the whole function \( \Pi(q^2) \) through the integral in Eq. (5).

The calculation of the phenomenological side proceeds by inserting intermediate states for the hadron, \( H \), of interest. The current \( j (j^\dagger) \) is an operator that annihilates (creates) all hadronic states that have the same quantum numbers as \( j \). Consequently, \( \Pi(q) \) contains information about all these hadronic states, including the low mass hadron of interest. In order for the QCD sum rule technique to be useful, one must parametrize \( \rho(s) \) with a small number of parameters. The lowest resonance is often fairly narrow, whereas higher-mass states are broader. Therefore, one can parameterize the spectral density as a single sharp pole representing the lowest resonance of mass \( m \), plus a smooth continuum representing higher mass states:

\[
\rho(s) = \lambda^2 \delta(s - m^2) + \rho_{\text{conf}}(s), \tag{7}
\]

where \( \lambda \) gives the coupling of the current with the low mass hadron, \( H \):

\[
\langle 0 | j H \rangle = \lambda. \tag{8}
\]

For simplicity, one often assumes that the continuum contribution to the spectral density, \( \rho_{\text{conf}}(s) \) in Eq. (7), vanishes below a certain continuum threshold \( s_0 \). Above this threshold, it is assumed to be given by the result obtained with the OPE. Therefore, one uses the ansatz

\[
\rho_{\text{conf}}(s) = \rho^{\text{OPE}}(s) \Theta(s - s_0). \tag{9}
\]

2.4. Choice of currents

Mesonic currents for open charm mesons are given in Table 2.

| state          | symbol | current | \( J^P \) |
|---------------|--------|---------|----------|
| scalar meson  | \( D_0 \) | \( \bar{q}c \) | 0\( ^+ \) |
| pseudoscalar meson | \( D \) | \( i\bar{q}\gamma_5 c \) | 0\( ^+ \) |
| vector meson  | \( D^0 \) | \( \bar{q}\gamma_\mu c \) | 1\( ^+ \) |
| axial-vector meson | \( D_1 \) | \( \bar{q}\gamma_\mu \gamma_5 c \) | 1\( ^+ \) |

From these currents we can construct molecular currents which can be eigenstates of charge conjugation \( C \) and \( G \)-parity.
Let us consider, as an example, a current with $J^{PC} = 1^{++}$ for the molecular $D^0\bar{D}^*$ system. It can be written as a combination of two currents [77, 78]:

$$j_\mu(x) = [\bar{u}(x)\gamma_5 c(x)][\bar{c}(x)\gamma_\mu u(x)],$$  \hspace{1cm} (10)

and

$$j_\mu(x) = [\bar{u}(x)\gamma_\mu c(x)][\bar{c}(x)\gamma_5 u(x)].$$  \hspace{1cm} (11)

Since the charge conjugation transformation is defined as: $(\bar{q}c) = -q^T C^{-1} = q^T C$ and $(q)c = Cq^T$, we get

$$j^\mu_1) = - (\bar{c}\gamma_5 u)(\bar{\gamma}_\mu c) = j^\mu_0,$$  \hspace{1cm} (12)

$$j^\mu_2) = - (\bar{c}\gamma_\mu u)(\bar{\gamma}_5 c) = -j^\mu_0.$$  \hspace{1cm} (13)

Therefore, the current

$$j_\mu(x) = \frac{1}{\sqrt{2}}(j^\mu_1(x) - j^\mu_2(x)), \hspace{1cm} (14)$$

has positive $C$. However, this current is not a $G$-parity eigenstate. The $G$-parity transformation is an isospin rotation of the charge conjugated current:

$$j^\mu_{1G} = - (\bar{c}\gamma_5 d)(\bar{\gamma}_\mu c), \hspace{1cm} (15)$$

$$j^\mu_{2G} = - (\bar{c}\gamma_\mu d)(\bar{\gamma}_5 c). \hspace{1cm} (16)$$

In the case of a charged molecular $D_1 D^*$ current with $JP = 0^-$, it can also be written as a combination of two currents:

$$j_1 = (\bar{c}\gamma_\mu \gamma_5)(\bar{d}\gamma_\mu c), \hspace{1cm} (17)$$

$$j_2 = (\bar{c}\gamma_\mu u)(\bar{\gamma}_5 c). \hspace{1cm} (18)$$

The charge conjugation transformation in these currents leads to

$$j^\mu_1)C = - (\bar{d}\gamma_\mu \gamma_5)(\bar{\gamma}_\mu c), \hspace{1cm} (19)$$

$$j^\mu_2)C = - (\bar{d}\gamma_\mu c)(\bar{\gamma}_5 \gamma_\mu c), \hspace{1cm} (20)$$

and the isospin rotation gives

$$j^\mu_1)G = (\bar{d}\gamma_\mu \gamma_5 c)(\bar{\gamma}_\mu d) = j_2, \hspace{1cm} (21)$$

$$j^\mu_2)G = (\bar{d}\gamma_\mu c)(\bar{\gamma}_5 \gamma_\mu d) = j_1. \hspace{1cm} (22)$$

Therefore, the current

$$j = \frac{1}{\sqrt{2}}(j_1 + j_2), \hspace{1cm} (23)$$

has positive $G$-parity.

In the case of tetraquark $[cq][\bar{c}\bar{q}]$ currents, they can be constructed in terms of color anti-symmetric diquark states: $\epsilon_{abc}[q^c_\mu C\bar{c}]$, where $a$, $b$, $c$ are color indices of the $SU(3)$ color group, $C$ is the charge conjugation matrix, and $\Gamma$ stands for Dirac matrices. The quantum numbers of the diquark states are given in Table 3.

From these diquarks we can construct tetraquark currents which can be eigenstates of charge conjugation $C$ and $G$-parity.

| Table 3: Currents for charmed diquark states. |
|---------------------------------------------|
| state | current | $J^{PC}$  |
|-------|---------|-----------|
| scalar diquark | $q^c_1 C\bar{y} \gamma_5 c_b$ | 0$^+$ |
| pseudoscalar diquark | $q^c_1 C \bar{c}_b$ | 0$^-$ |
| vector diquark | $q^c_2 C\gamma_5 \gamma_\mu c_b$ | 1$^-$ |
| axial-vector diquark | $q^c_1 C\gamma_\mu c_b$ | 1$^+$ |

In the case of a $J^{PC} = 1^{++}$ current it can be written as a combination of scalar and axial-vector diquarks:

$$j^\mu_1 = \epsilon_{abc} \epsilon_{dec}(q^c_1 C\bar{y} \gamma_5 c_b)(q^d_1 C\gamma_\mu c_c) \bar{C} \Gamma c_b, \hspace{1cm} (24)$$

and

$$j^\mu_2 = \epsilon_{abc} \epsilon_{dec}(q^c_1 C\bar{y} \gamma_5 c_b)(q^d_1 C\gamma_\mu c_c) \bar{C} \Gamma c_b. \hspace{1cm} (25)$$

It is interesting to notice that the structure of the currents in Eqs. (24) and (25) relates the spin of the charm quark with the spin of the light quark. This is very different from the spin structure of the heavy quark effective theory [79]. Heavy quark effective theory, in leading order in 1/M, possesses a heavy-quark spin symmetry. Therefore, hadrons can be classified according to the angular momentum and parity of the light fields only. This gives 0$^-$ and 1$^-$ mesons with identical properties.

Using the charge conjugation transformations one gets:

$$j^\mu_1)C = \epsilon_{abc} \epsilon_{dec}\bar{q}d \gamma_5 \Gamma c_b(c^\mu_1 C\gamma_\mu c_c) = j^\mu_0.$$  \hspace{1cm} (26)

$$j^\mu_2)C = \epsilon_{abc} \epsilon_{dec}\bar{q}d \Gamma c_b(c^\mu_2 C\gamma_\mu c_c) = j^\mu_0.$$  \hspace{1cm} (27)

Therefore, the current

$$j_\mu(x) = \frac{i}{\sqrt{2}}(j^\mu_1(x) + j^\mu_2(x)), \hspace{1cm} (28)$$

has positive $C$. The $i$ was used in Eq. (28) to ensure that $j^\mu_0 = j^\mu_0$. As in the case of the $D^0\bar{D}^*$ molecular current, the current in Eq. (28) is not a $G$-parity eigenstate. However, other combinations of tetraquark currents can be constructed to be $G$-parity eigenstates.

In general, there is no one to one correspondence between the current and the state, since the current in Eq. (28) can be rewritten in terms of a sum over molecular type currents through the Fierz transformation. In the appendix, we provide the general expressions for the Fierz transformation of the tetraquark currents into molecular type of currents as given in Eq. (14). However, as shown in the appendix, in the Fierz transformation of a tetraquark current, each molecular component contributes with suppression factors that originate from picking up the correct Dirac and color indices. This means that if the physical state is a molecular state, it would be best to choose a molecular type of current so that it has a large overlap with the physical state. Similarly for a tetraquark state it would be best to choose a tetraquark current. If the current is found to have a large overlap with the physical state, the range of Borel parameters where the pole dominates over the continuum would be larger, and the OPE for the mass would have a better convergence. These conditions will lead to a better sum rule; this means that the
Borel curve has an extremum or is flat, and the calculated mass is close to the physical value. Therefore, if the sum rule gives a mass and width consistent with the physical values, we can infer that the physical state has a structure well represented by the chosen current. In this way, we can indirectly discriminate between the tetraquark and the molecular structures of the recently observed states. However, it is very important to notice that since the molecular currents, as the one in Eq. (14), are local, they do not represent extended objects, with two mesons separated in space, but rather a very compact object with two singlet quark-antiquark pairs.

When the current is fixed, we proceed by inserting it into Eq. (1). Contracting all the quark anti-quark pairs, we can rewrite the correlation function in terms of the quark propagators. For the light quarks, keeping terms which are linear in the light quark mass $m_q$, this expansion reads:

$$ S_{ab}(x) = \langle 0 | T[\bar{q}_a(x)q_b(0)] | 0 \rangle = \frac{i \delta_{ab}}{2 \pi^2 x^2} - \frac{m_q \delta_{ab}}{4 \pi^2 x^2} - \frac{t^{ab}_G g^{G^a}}{32 \pi^2} \left( \frac{i}{x^2} (4 \sigma^{\mu \nu} + \sigma^{\mu} k - m_q \sigma^{\mu \nu} \ln(-x^2)) \right) - \frac{\delta_{ab}}{12} \langle \bar{q} q \rangle + \frac{i \delta_{ab}}{48} m_q \langle \bar{q} q \rangle k - \frac{x^2 \delta_{ab}}{2^n \pi^3} (\bar{q} g_{\sigma} G q) + \frac{i x^2 \delta_{ab}}{2^n \pi^3} m_q (\bar{q} g_{\sigma} G q) k, $$

where we have used the fixed-point gauge. For heavy quarks, it is more convenient to work in the momentum space. In this case the expansion is given by:

$$ S_{ab}(p) = \frac{i (\not{p} + m)}{p^2 - m^2} \delta_{ab} - \frac{i \langle \bar{q} q \rangle}{4} \frac{t^{ab}_G g^{G^a}}{(p^2 - m^2)^2} + \frac{\delta_{ab}}{12} m_q \langle \bar{q} q \rangle k + \frac{x^2 \delta_{ab}}{2^n \pi^3} m_q (\bar{q} g_{\sigma} G q) k. $$

2.5. The mass sum rule

Now one might attempt to match the two descriptions of the correlator:

$$ \Pi_{\text{phen}}(Q^2) \leftrightarrow \Pi_{\text{OPE}}(Q^2). $$

However, such a matching is not yet practical. The OPE side is only valid at sufficiently large spacelike $Q^2$. On the other hand, the phenomenological description is significantly dominated by the lowest pole only for sufficiently small $Q^2$, or better yet, timelike $q^2$ near the pole. To improve the overlap between the two sides of the sum rule, one applies the Borel transformation:

$$ B_M[\Pi(q^2)] = \lim_{s \to \infty} \frac{(q^2)^{n+1}}{n!} \left( \frac{d}{dq^2} \right)^n \Pi(q^2). $$

Two important examples are:

$$ B_M \left[ \frac{1}{(m^2 - q^2)^n} \right] = 0, \quad \text{for } n > 0. $$

and

$$ B_M \left[ \frac{1}{(m^2 - q^2)^n} \right] = \frac{1}{(n-1)!} \frac{e^{-m^2/M^2}}{(M^2)^{n-1}}, $$

for $n > 0$. From these two results, and , one can see that the Borel transformation removes the subtraction terms in the dispersion relation, and exponentially suppresses the contribution from excited resonances and continuum states in the phenomenological side. In the OPE side the Borel transformation suppresses the contribution from higher dimension condensates by a factorial term.

After making a Borel transform on both sides of the sum rule, and transferring the continuum contribution to the OPE side, the sum rule can be written as

$$ \lambda^2 e^{-m^2/M^2} = \int_{s_{\text{min}}}^{s_{\text{max}}} ds \ e^{-s/M^2} \rho_{\text{OPE}}(s). \quad (35) $$

If both sides of the sum rule were calculated to arbitrary high accuracy, the matching would be independent of $M^2$. In practice, however, both sides are represented imperfectly. The hope is that there exists a range of $M^2$, called Borel window, in which the two sides have a good overlap and information on the lowest resonance can be extracted. In general, to determine the allowed Borel window, one analyses the OPE convergence and the pole contribution: the minimum value of the Borel mass is fixed by considering the convergence of the OPE, and the maximum value of the Borel mass is determined by imposing the condition that the pole contribution should be bigger than the continuum contribution.

In order to extract the mass $m$ without worrying about the value of the coupling $\lambda$, it is possible to take the derivative of Eq. (35) with respect to $1/M^2$, and divide the result by Eq. (35). This gives:

$$ m^2 = \frac{\int_{s_{\text{min}}}^{s_{\text{max}}} ds \ e^{-s/M^2} s \rho_{\text{OPE}}(s)}{\int_{s_{\text{min}}}^{s_{\text{max}}} ds \ e^{-s/M^2} \rho_{\text{OPE}}(s)}. \quad (36) $$

This quantity has the advantage to be less sensitive to the perturbative radiative corrections than the individual sum rules. Therefore, we expect that our results obtained to leading order in $\alpha_s$ will be quite accurate.

2.6. Numerical inputs

In the following sections we will present numerical results. In the quantitative aspect, QCDSR is not like a model which contains free parameters to be adjusted by fitting data. The inputs for numerical evaluations are the following: i) the vacuum matrix elements of composite operators involving quarks and gluons which appear in the operator product expansion. These numbers, known as condensates, contain all the non-perturbative component of the approach. They could in principle, be calculated in lattice QCD. In practice they are estimated phenomenologically. They are universal and, once adjusted to fit, for example, the mass of a particle, they must have always that same value. They are the analogue for spectroscopy of the parton distribution functions in deep inelastic scattering; ii) quark masses, which are extracted from many different phenomenological analyses and used in our calculation; iii) the threshold parameter $s_0$ is the energy (squared) which characterizes the beginning of the continuum. Typically the quantity
\[ m_0 = m (\text{the mass of the ground state particle}) \] is the energy needed to excite the particle to its first excited state with the same quantum numbers. This number is not well known, but should lie between 0.3 and 0.8 GeV. If larger deviations from this interval are needed, the calculation becomes less reliable.

All in all, in QCDSR we do not have much freedom for choosing numbers. In the calculations discussed in the next sections we will use [21, 80, 81, 82]:

\[ 0.13 \pm 0.03 \text{ GeV}, \]

\[ 1.23 \pm 0.05 \text{ GeV}, \]

\[ 4.24 \pm 0.06 \text{ GeV}, \]

\[ -0.23 \pm 0.03 \text{ GeV}^3, \]

\[ 0.8 \pm 0.2 \text{ GeV}^4, \]

\[ m_0^2(\bar{q}q), \]

\[ m_0^2 = 0.8 \text{ GeV}^2. \] (37)

3. The \( X(3872) \) meson

Since its first observation in August 2003 by Belle Collaboration [1], the \( X(3872) \) represents a puzzle and, up to now, there is no consensus about its structure. The \( X(3872) \) has been confirmed by CDF, D0 and BaBar [13, 14, 15]. Besides the discovery production mode \( B^+ \to X(3872)K^- \to J/\psi \pi^+ \pi^- K^- \), the \( X(3872) \) has been observed in prompt \( p\bar{p} \) production [13, 15, 16]. However, searches in prompt \( e^+e^- \) production [18] and in \( e^+e^- \) or \( \gamma \gamma \) formation [19] have given negative results so far. The current world average mass is

\[ M_X = 3871.20 \pm 0.39 \text{ MeV}, \] (38)

and the most precise measurement to date is \( M_X = 3871.61 \pm 0.16 \pm 0.19 \text{ MeV} \), as can be seen by Fig. 2 [20].

The mass of the \( X(3872) \) is at the threshold for the production of the charmed meson pair \( m(D^0D^{\ast 0}) = 3871.81 \pm 0.36 \text{ MeV} \) [21], and this state is extremely narrow: its width is less than 2.3 MeV at 90% confidence level.

3.1. Experiment versus theory

Both Belle and Babar collaborations reported the radiative decay mode \( X(3872) \to \gamma J/\psi \) [22, 23], which determines \( C = +. \) Belle Collaboration reported the branching ratio:

\[ \frac{\Gamma(X \to J/\psi \gamma)}{\Gamma(X \to J/\psi \pi^+ \pi^-)} = 0.14 \pm 0.05. \] (39)

Recent studies from Belle and CDF that combine angular information and kinematic properties of the \( \pi^0 \pi^- \) pair, strongly favor the quantum numbers \( J^{PC} = 1^{++} \) and \( 2^{++} \) and \( 2^{++} \) are compatible with data. All other possible quantum numbers are ruled out by more than three standard deviations. The possibility \( 2^{++} \) is disfavored by the observation of the decay into \( \psi(2S) \gamma \) [26] and also by the observation of the decays into \( D^0\bar{D}^{\ast 0} \) by Belle and BaBar Collaborations [27, 28]. On the other hand, the possibility \( 1^{++} \) is disfavored by the observation of the decay into \( J/\psi \pi \) by BaBar Collaborations [53]. In the following we will assume the quantum numbers of the \( X(3872) \) to be \( 1^{++} \).

From constituent quark models [84] the masses of the possible charmonium states with \( J^{PC} = 1^{++} \) quantum numbers are: \( 2\Delta P_1(3925) \) and \( 3\Delta P_1(4270) \), and lattice QCD calculations give \( 2\Delta P_1(4010) \) [85] and \( 2\Delta P_1(4067) \) [86]. In all cases the predictions for the mass of the \( 2\Delta P_1 \) charmonium state are much bigger than the observed mass. However, a more recent lattice QCD calculation gives \( 2\Delta P_1(3853) \) [87] and, therefore, this interpretation can not be totally discarded [88]. In any case, the strongest fact against the \( c\bar{c} \) assignment for \( X(3872) \) is the fact that from the study of the dipion mass distribution in the \( X(3872) \to J/\psi \pi^+ \pi^- \) decay, Belle [1] and CDF [24] concluded that it proceeds through the \( X(3872) \to J/\psi \gamma \) decay. Since a charmonium state has isospin zero, it can not decay easily into a \( J/\psi \) final state.

The proximity between the \( X \) mass and the \( D^0\bar{D}^+ \) threshold inspired the proposal that the \( X(3872) \) could be a molecular bound state with small binding energy [48, 49, 50, 90, 91, 92, 24, 25]. As a matter of fact, Tornqvist, using a meson potential model [84], essentially predicted the \( X(3872) \) in 1994, since he found that there should be molecules near the \( D^+D^- \) threshold in the \( J^{PC} = 0^{-+} \) and \( 1^{-+} \) channels. The only other molecular state that is predicted in the potential model updated by Swanson [60] is a \( 0^{--} \) \( D^+D^- \) molecule at 4013 MeV.

In ref. [50] Swanson proposed that the \( X(3872) \) could be a nonet of \( D^0\bar{D}^{\ast 0} \) molecule with a small but important admixture of \( J/\psi \) and \( \omega J/\psi \) components. With this picture the decay mode \( X(3872) \to J/\psi \pi^+ \pi^- \) was predicted at a rate comparable to the \( X(3872) \to J/\psi \pi^+ \pi^- \) mode. Soon after this prediction, Belle Coll. [22] reported the observation of these two decay modes at a rate:

\[ \frac{B(X \to J/\psi \pi^+ \pi^-)}{B(X \to J/\psi \pi^+ \pi^-)} = 1.0 \pm 0.4 \pm 0.3. \] (40)

This observation establishes strong isospin and \( G \) parity violation and strongly favors a molecular assignment for \( X \). However, it still does not completely exclude a \( c\bar{c} \) interpretation for
X since the isospin and G parity non-conservation in Eq. (40) could be of dynamical origin due to $\rho^0 - \omega$ mixing or even due to final state interactions (FSI) containing D loops, such as $X \rightarrow J/\psi \phi \rightarrow D\bar{D} \rightarrow J/\psi \phi$. Although all the ingredients (especially the charm form factors $F_{D^*}$) for the relevant effective field theory are available, there are no quantitative results in the FSI approach yet.

The decay $X \rightarrow J/\psi \phi$ was also observed by BaBar Collaboration (53) at a rate:

$$\frac{B(X \rightarrow J/\psi \pi^+\pi^- \pi^0)}{B(X \rightarrow J/\psi \pi^+\pi^-)} = 0.8 \pm 0.3,$$

which is consistent with the result in Eq. (41).

It is also important to notice that, although a $D^0 \bar{D}^{*0}$ molecule is not an isospin eingenstate, the ratio in Eq. (40) can not be reproduced by a pure $D^0 \bar{D}^{*0}$ molecule. In ref. (98) it was shown that for a pure $D^0 \bar{D}^{*0}$ molecule

$$\Gamma(X(D^0 D^{*0}) \rightarrow J/\psi \pi^+\pi^- \pi^0) \approx 0.15.$$  

In refs. (27, 28) Belle and BaBar Collaborations reported the observation of a near threshold enhancement in the $D^0 \bar{D}^{*0}$ system. The peak mass values for the two observations are in good agreement with each other: (3875.2 ± 1.9) MeV for Belle and (3875.1 ± 1.2) MeV for BaBar, and are higher than in the mass of the $X(3872)$ observed in the $J/\psi \pi^+\pi^-$ channel by (3.8 ± 1.1) MeV. Since this peak lies about 3 MeV above the $D^0 \bar{D}^{*0}$ threshold, it is very awkward to treat it as a $D^0 \bar{D}^{*0}$ bound state. According to Braaten (99), the large mass of the $X$ measured in the $D^0 \bar{D}^{*0}$ decay channel could be explained by the difference between the line shapes of the $X$ into the two decays: $D^0 \bar{D}^{*0}$ and $J/\psi \pi^+\pi^-$. In the decay of a narrow $X$ molecular state into its constituents $D^0 \bar{D}^{*0}$, the width of $D^0$ distorts the decay line shape of the $X(3872)$ (100). Therefore, the peak observed in the $B \rightarrow K D^0 \bar{D}^{*0}$ decay channel could be a combination of a resonance below the $D^0 \bar{D}^{*0}$ threshold from the $B \rightarrow K X$ decay and a threshold enhancement above the $D^0 \bar{D}^{*0}$ threshold. In this case, fitting the $D^0 \bar{D}^{*0}$ invariant mass to a Breit-Wigner does not give reliable values for the mass and width. However, in a new measurement (30) Belle has obtained a mass (3872.6 ± 0.6) MeV in the $D^0 \bar{D}^{*0}$ invariant mass spectrum, which is consistent with the current world average mass for $X(3872)$. Using this new data and taking into account the universal features of the $S$-wave threshold resonance Braaten and Stapleton concluded that the $X(3872)$ is a extremely weakly-bound charm meson molecule (101).

Other interesting possible interpretation of the $X(3872)$, first proposed by Maiani et al. (45), is that it could be a tetraquark state resulting from the binding of a diquark and an antidiquark (95). This construction is based in the idea that diquarks can form bound-states, which can be treated as confined particles, and used as degrees of freedom in parallel with quarks themselves (102 103 104). Therefore, the tetraquark interpretation differs from the molecular interpretation in the way that the quarks are organized in the state, as shown in Fig. 1. The drawback of the tetraquark picture is the proliferation of the predicted states (45) and the lack of selection rules that could explain why many of these states are not seen (105).

The authors of ref. (45) have considered diquark-antidiquark states with $J^{PC} = 1^{+}$ and symmetric spin distribution:

$$X_q = [cq]_{s=1} [\bar{c}q]_{s=0} + [cq]_{s=0} [\bar{c}q]_{s=1}. (43)$$

The most general states that can decay into $2\pi$ and $3\pi$ are:

$$X_l = \cos \theta X_a + \sin \theta X_d, \quad X_h = \cos \theta X_d - \sin \theta X_a. \quad (44)$$

Imposing the rate in Eq. (40), they get $\theta \sim 20^\circ$. It is important to notice that a similar mixture between $D^0 \bar{D}^{*0}$ and $D^*D^{*0}$ molecular states with the same mixing angle $\theta \sim 20^\circ$ (98), would also reproduce the decay rate in Eq. (40).

The authors of ref. (45) also argue that if $X_l$ dominates $B^+$ decays, then $X_h$ dominates the $B^0$ decays and vice-versa. They have also predicted that the mass difference between the $X$ particle in $B^+$ and $B^0$ decays should be $45\pm10$ MeV (45).

There are two reports from Belle (39) and Babar (40) Collaborations for the observation of the $B^0 \rightarrow K^0 X$ and $B^+ \rightarrow K^+ X$ decays that we show in Figs. 3 and 5.

Although the identification of the $X(3872)$ from the decay $B^+ \rightarrow K^+ X$ in these two figures is very clear, this is not the case for the $B^0 \rightarrow K^0 X$ decay, where the evidence for the existence of such a state is not so clear.

In any case, if one accepts the existence of the $X$ in the decay $B^0 \rightarrow K^0 X$, these reports are not consistent with each other. While Belle measures (39):

$$\frac{B(B^0 \rightarrow X K^0)}{B(B^+ \rightarrow X K^+)} = 0.82 \pm 0.22 \pm 0.05,$$

and

$$M(X)_{B^0} - M(X)_{B^+} = (0.18 \pm 0.89 \pm 0.26) \text{ MeV},$$

BaBar measures (40):

$$\frac{B(B^0 \rightarrow X K^0)}{B(B^+ \rightarrow X K^+)} = 0.41 \pm 0.24 \pm 0.05,$$

Figure 3: Cartoon representation for the molecular and tetraquark interpretations of $X(3872)$.
and

$$M(X)_{B^0} - M(X)_{B^+} = (2.7 \pm 1.6 \pm 0.4) \text{ MeV.} \quad (49)$$

In both measurements, the mass difference between the two states is much smaller than the prediction in Eq. (45).

If $X(3872)$ is a loosely bound molecular state, the branching ratio for the decay $B^0 \to K^0 X$ is suppressed by more than one order of magnitude compared to the decay $B^+ \to K^+ X$. The prediction in refs. [60, 107] gives:

$$0.06 \leq \frac{\Gamma(B^0 \to XK^0)}{\Gamma(B^+ \to XK^+)} \leq 0.29, \quad (50)$$

which is, considering the errors, still consistent with the data from BaBar.

Recently BaBar has reported the observation of the decay $X(3872) \to \psi(2S)\gamma$ at a rate:

$$\frac{B(X \to \psi(2S)\gamma)}{B(X \to \psi\gamma)} = 3.4 \pm 1.4, \quad (51)$$

while the prediction from ref. [50] gives

$$\frac{\Gamma(X \to \psi(2S)\gamma)}{\Gamma(X \to \psi\gamma)} \sim 4 \times 10^{-3}. \quad (52)$$

While this difference could be interpreted as a strong point against the molecular model and as a point in favor of a conventional charmonium interpretation [44], it can also be interpreted as an indication that there is a significant mixing of the $c\bar{c}$ component with the $D^0\bar{D}^*$ molecule. As a matter of fact, the necessity of mixing a $c\bar{c}$ component with the $D^0\bar{D}^*$ molecule was already pointed out in some works [89, 108, 109, 110, 111, 112].

In particular, in ref. [110] it was phenomenologically shown that, because of the very loose binding of the molecule, the production rates of a pure molecule $X(3872)$ should be at least one order of magnitude smaller than what is seen experimentally.

In ref. [58], a Monte Carlo simulation of the production of a bound $D^0\bar{D}^*$ state with binding energy as small as 0.25 MeV, obtained a cross section of about two orders of magnitude smaller than the prompt production cross section for the $X(3872)$ observed by the CDF Collaboration. The authors of ref. [58] concluded that $S$-wave resonant scattering is unlikely to allow the formation of a loosely bound $D^0\bar{D}^*$ molecule, thus calling for alternative (tetraquark) explanation of CDF data. However, it was pointed out in ref. [59] that a consistent analysis of $DD^*$ molecule production requires taking into account the effect of final state interactions of the $D$ and $D^*$ mesons.
This observation changes the results of the Monte Carlo calculations bringing the theoretical value of the cross section very close to the observed one. Thus, the question of interpretation of $X(3872)$ production at hadronic machines is not yet settled.

Other interpretations for the $X(3872)$ like cusp [113], hybrids [114, 115], or glueball [116] have already been covered in refs. 66, 61, 62, 63, 64, 65, 66, 67. Here we would like to focus on the QCD sum rules studies of this meson.

3.2. QCDSR studies of $X(3872)$

Considering the $X(3872)$ as a $J^{PC} = 1^{++}$ state we can construct a current based on diquarks in the color triplet configuration, with symmetric spin distribution: $[cq]_{s=1} [\bar{c}\bar{q}]_{s=0} + [cq]_{s=0} [\bar{c}\bar{q}]_{s=1}$, as proposed in ref. [45]. Therefore, the corresponding lowest-dimensional interpolating operator for describing $X_q$ as a tetraquark state is given by:

$$
\mathcal{O}^{(d)}_{\mu} = \frac{i e_a e_{dec}}{\sqrt{2}} \left( q_\mu C \gamma_5 c_b \bar{q}_\nu \bar{c}_e \right) + \left( q_\mu C \gamma_\nu c_b \bar{q}_\nu \bar{c}_e \right),
$$

(53)

where $q$ denotes a $u$ or $d$ quark.

On the other hand, we can construct a current describing $X_q$ as a molecular $DD^*$ state:

$$
\mathcal{O}^{(m)}_{\mu} = \frac{1}{\sqrt{2}} \left[ \left( \bar{q}_\mu (x) \gamma_\nu c_b (x) \bar{c}_e (x) \gamma_\nu \bar{q}_b (x) \right) - \left( \bar{q}_\nu (x) \gamma_\nu c_b (x) \bar{c}_e (x) \gamma_\nu \bar{q}_b (x) \right) \right].
$$

(54)

In general, other four-quark operators with $1^{++}$ are possible. For example, starting from the simple charmed diquark states given in Table II, another tetraquark current with $J^{PC} = 1^{++}$ can be constructed by combining the pseudo scalar $0^-$ and vector $1^-$ diquark. Equivalently, additional current can be constructed for the meson type currents. The number of currents increases further, if one allows for additional color states; color sextet for the diquark and color octet for the molecular states. An extensive study has been carried out for the $0^{++}$ light mesons [117], with their mixing under renormalizations [118] from which one can form renormalization group invariant physical currents. The choice of the current does not matter too much provided that we can work with quantities less affected by radiative corrections and where the OPE converges quite well. This is borne out in the well-known case of baryon sum rules, where a simple choice of operator [119] and a more general choice [120] have been studied. Even though apparently different, mainly in the region of convergence of the OPE, the two choices of interpolating currents have provided the same predictions for the proton mass. In some cases however, particular choices might be preferable over the others.

3.2.1. Two-point function

For the present case, the two currents in Eqs. (53) and (54) were used, in refs. [121] and [122] respectively, to study the $X(3872)$. The correlator for these currents can be written as:

$$
\Pi_{\mu\nu}(q^2) = i \int d^4 x \, e^{iq\cdot x} \langle 0 | T \mathcal{O}^{(d)}_{\mu}(x) \mathcal{O}^{(d)}_{\nu}(0) | 0 \rangle = -\Pi_1(q^2) \frac{q_\mu q_\nu}{q^2} + \Pi_0(q^2) \frac{q_\mu q_\nu}{q^2},
$$

(55)

where, since the axial vector current is not conserved, the two functions, $\Pi_1$ and $\Pi_0$, appearing in Eq. (55) are independent and have respectively the quantum numbers of the spin 1 and 0 mesons.

Using the current in Eq. (53), as an example, the correlation function in Eq. (55) can be written in terms of the quark propagators as:

$$
\Pi_{\mu\nu}(q^2) = -i e_a e_{dec} e_c e_{dec} e_{dec} e_c e' \left\{ \frac{\sqrt{2}}{2(2\pi)^8} \int d^4 x d^4 p_1 d^4 p_2 e^{i q \cdot (p_1 - p_2)} \right\}
$$

$$
\times \text{Tr} \left[ S^{(c)}_{\mu\nu}(p_1) \gamma_5 CS^{(d)}_{\mu\nu}(x) \gamma_5 \right] \times \text{Tr} \left[ S^{(c)}_{\mu\nu}(p_1) \gamma_5 CS^{(e')}_{\mu\nu}(-p_2) \gamma_5 \right]
$$

(56)

In the OPE side, we work at leading order in $\alpha_s$ and consider the contributions of condensates up to dimension eight. To evaluate the correlation function in Eq. (56), we use the momentum-space expression for the charm-quark propagator given in Eq. (50). The light-quark part of the correlation function is calculated in the coordinate-space, using the propagator given in Eq. (29). The resulting light-quark part is combined with the charm-quark part before it is dimensionally regularized at $D = 4$.

The correlation function, $\Pi_1$, in the OPE side can be written as a dispersion relation:

$$
\Pi_1^{(OPE)}(q^2) = \int_{4m_c}^\infty ds \frac{\rho(s)}{s - q^2},
$$

(57)

where the spectral density is given by the imaginary part of the correlation function: $\rho_{\Pi}(s) = \text{Im} \Pi_1^{(OPE)}(s)$. For the current in Eq. (53) we obtain [121]:

$$
\rho^{(d)}(s) = \rho^{(m)}(s) + \rho^{(b\bar{q})}(s) + \rho^{(G\gamma)}(s) + \rho^{(G\gamma)}(s),
$$

(58)
with
\[ \rho_{\text{pert}}(s) = \frac{1}{2 \pi^2} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} \frac{d\beta}{\beta} (1 - \alpha - \beta)(1 + \alpha + \beta) \]
\[ \times \left[ (\alpha + \beta)m_c^2 - a\beta s \right]^4, \]
\[ \rho_{\text{mix}}(s) = -\frac{m_q}{2 \pi^2} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} \frac{d\beta}{\beta} (1 + \alpha + \beta) \]
\[ + \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} \frac{d\beta}{\beta} \left[ (\alpha + \beta)m_c^2 - a\beta s \right] \left[ \frac{m_q}{2 \pi^2} \left( \frac{\langle \bar{q}q \rangle}{2} \right)^2 \right] \]
\[ = -\frac{m_q}{2 \pi^2} \langle \bar{q}q \rangle \left( \frac{1}{2} \right) \left[ (\alpha + \beta)m_c^2 - a\beta s \right]^2, \]
the phenomenological side of Eq. (55) can be written as
\[ \Pi_{\mu\nu}^{\text{pheno}}(q^2) = \frac{\langle \bar{q}q \rangle^2}{M_X^2 - q^2} \left( -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{M_X^2} \right) + \cdots, \tag{63} \]
where the Lorentz structure projects out the 1^{++} state. The dots denote higher axial-vector resonance contributions that will be parametrized, as explained in Sec. 2.3, through the introduction of a continuum threshold parameter \( s_0 \). After making a Borel transform of both sides, and transferring the continuum contribution to the OPE side, the sum rule for the axial vector meson \( X \) up to dimension-eight condensates can be written as:
\[ \langle \bar{q}q \rangle^2 e^{-M_X^2/M^2} = \int_{4\pi}^{s_0} ds e^{-s/M^2} \rho_{\text{OPE}}(s) + \Pi_{\mu\nu}^{\text{mix}(\bar{q}q)}(M^2), \tag{64} \]

Figure 6: The \( f_{\mu\nu}^{(\bar{q}q)} \) OPE convergence in the region \( 1.6 \leq M^2 \leq 2.8 \) GeV^2 for \( \sqrt{m} = 4.17 \) GeV (taken from ref. [133]).

For the current in Eq. (53) we show, in Fig. 6, the relative contribution of each term on the OPE expansion of the sum rule. One can see that for \( M^2 > 1.9 \) GeV^2, the addition of a subsequent term of the expansion brings the curve (representing the sum) closer to an asymptotic value (which was normalized to 1). Furthermore the changes in this curve become smaller with increasing dimension. These are the requirements for a good OPE convergence and this fixes the lower limit of the Borel window to be \( M^2 \geq 2 \) GeV^2.

We obtain an upper limit for \( M^2 \) by imposing the constraint that the QCD continuum contribution should be smaller than the pole contribution. The maximum value of \( M^2 \) for which this constraint is satisfied depends on the value of \( s_0 \). The comparison between pole and continuum contributions for \( \sqrt{s_0} = 4.15 \) GeV and \( \rho = 2.1 \) is shown in Fig. 7.

Having the Borel window fixed, the mass is obtained by using Eq. (56). In Fig. 8 we show the obtained \( X \) mass for different values of \( \sqrt{s_0} \) and \( \rho \). We can see by this figure that the effect of the violation of the factorization assumption, given by \( \rho \), is similar to the effect of changing the continuum threshold.

To study the effect of higher dimension condensates in the mass we show, in Fig. 9 the contributions of the individual condensates to \( M_X \) obtained from Eq. (56). From this figure

In Eqs. (60) and (61) the parametrization in Eq. (44) was assumed.

Parametrizing the coupling of the axial vector meson 1^{++}, \( X \), to the current, \( j_{\mu}^{(0)} \), in Eqs. (53) and (54) as:
\[ \langle 0 | j_{\mu}^{(0)}(X) | X^s e_\mu , \tag{62} \]
allowed Borel window and the uncertainties in the parameters, is
values of √s0 those obtained up to dimension-5 are very close
which is compatible with the experimental value of the mass of the X(3872).

The final result for MX, obtained in [121] considering the allowed Borel window and the uncertainties in the parameters, is

\[ MX = (3.92 \pm 0.13) \text{ GeV}, \]  

(65)

\[ M(X_0) - M(X) = (3.3 \pm 0.7) \text{ MeV}, \]  

(66)

in agreement with the BaBar measurement in Eq. (49).

In the case of the current in Eq. (54), the OPE convergence and the pole contribution determine a similar Borel window [122]. The result for the mass obtained in ref. [123] is MX = (3.87 ± 0.07) GeV, in an even better agreement with the experimental mass. However, due to the uncertainties inherent to the QCDSR method, we can not really say that the X(3872) is better described with a molecular type of current than with a tetraquark type of current. To see that we show, in Fig. 10 the double ratio of the sum rules

\[ d_{\text{mol/di}} = \frac{M_{X_{\text{mol}}}}{M_{X_{\text{di}}}}, \]  

(67)

where M_{X_{\text{mol}}} and M_{X_{\text{di}}} are the QCDSR results obtained by using the currents in Eqs. (53) and (54). The ratio plotted in this figure was obtained by using √s0 = 4.15 GeV and by considering the OPE up to dimension-6.

We see from Fig. 10 that the deviations between the two QCDSR results in the allowed Borel window are smaller than 0.01%.

In ref. [123] the same currents given in Eqs. (53) and (54), were used to study the importance of including the width of the state, in a sum rule calculation. This can be done by replacing the delta function in Eq. (67) by the relativistic Breit-Wigner function:

\[ \delta(s - m^2) \rightarrow \frac{1}{\pi} \frac{\Gamma \sqrt{s}}{(s - m^2)^2 + s \Gamma^2}. \]  

(68)

The mass and width are determined by looking at the stability of the obtained mass against varying the Borel parameter M², as usual.

Although the effect of the width is not large, in the case of the molecular current, Eq. (54), it was possible to fit the exper-
imental mass, 3872 MeV, and the width $\Gamma < 2.3$ MeV simultaneously with a continuum threshold, $\sqrt{s_0} = 4.38$ GeV, as can be seen by Fig. 11.

Figure 11: Results for the $X(3872)$ meson mass with a molecular current. The crosses indicate lower and upper limit of the Borel window, respectively (taken from ref. [123]).

3.2.2. Three-point function

One important question, when proposing a multiquark structure for the $X(3872)$, is whether with a tetraquark or molecular structure for the $X(3872)$, it is possible to explain a total width smaller than 2.3 MeV. As a matter of fact, a large partial decay width for the decay $X \rightarrow J/\psi \pi^+ \pi^-$ should be expected in this case. The initial state already contains all the four quarks needed for the decay, and there is no selection rules prohibiting the decay. Therefore, the decay is allowed as in the case of the light scalars $\sigma$ and $\kappa$ studied in [124], where widths of the order of 400 MeV were found. The decay width $\Gamma(X \rightarrow J/\psi \pi\pi)$, with $X(3872)$ considered as a tetraquark state, was also studied in ref. [45]. The authors arrived at $\Gamma(X \rightarrow J/\psi \pi\pi) \sim 5$ MeV. In order to have such small decay width the authors had to make a bold guess about the order of magnitude of the coupling constant in the vertex $XJ/\psi V$ (where $V$ stands for the $\rho$ or $\omega$ vector meson):

$$g_{X\psi V} = 0.475.$$  

(69)

The decay width for the decay $X \rightarrow J/\psi V \rightarrow J/\psi F$ where $F = \pi^+ \pi^- (\pi^+ \pi^- \pi^0)$ for $V = \rho(\omega)$ is given by [45, 125]

$$\frac{d\Gamma}{ds} (X \rightarrow J/\psi F) = \frac{1}{8\pi m_X^2} |M|^2 B_{V \rightarrow F} \times \frac{\Gamma_V m_V}{\pi} \frac{s}{(m_\psi^2 - m_\rho^2)^2 + (m_\psi \Gamma_V)^2} \tag{70}$$

where

$$p(s) = \frac{\sqrt{\lambda(m_X^2, m_\rho^2, s)}}{2m_X}, \tag{71}$$

with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. The invariant amplitude squared is given by:

$$|M|^2 = g_{X\psi V}^2 f(m_X, m_\rho, s), \tag{72}$$

where $g_{X\psi V}$ is the coupling constant in the vertex $XJ/\psi V$ and

$$f(m_X, m_\rho, s) = \frac{1}{3} \left[ 4m_X^2 - m_\rho^2 + \frac{(m_\rho^2 - m_X^2)^2}{2s} \right]$$

$$+ \frac{(m_\rho^2 - s)^2}{2m_\rho^2} \frac{m_X^2 - m_\rho^2}{m_X^2} + s. \tag{73}$$

Therefore, the ratio in Eq. (40) is given by:

$$\frac{\Gamma(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} = \frac{g_{X\psi V}^2 m_\omega \Gamma_\omega B_{\omega \rightarrow \pi \pi \pi} I_\omega}{g_{X\psi V}^2 m_\rho \Gamma_\rho B_{\rho \rightarrow \pi \pi} I_\rho}, \tag{74}$$

where

$$I_V = \int_{(m_\psi - m_\rho)^2}^{(m_\psi - m_\rho)^2} ds \frac{f(m_X, m_\rho, s)}{p(s)} \times \frac{1}{(s - m_X^2)^2 + (m_\psi \Gamma_V)^2}. \tag{75}$$

Using $B_{\omega \rightarrow \pi \pi} = 0.89, \Gamma_\omega = 8.49$ GeV, $m_\omega = 782.6$ MeV, $B_{\rho \rightarrow \pi \pi} = 1, \Gamma_\rho = 149.4$ GeV and $m_\rho = 775.5$ MeV we get

$$\frac{\Gamma(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\Gamma(X \rightarrow J/\psi \pi^+ \pi^-)} = 0.118 \left( \frac{g_{X\psi V}^2}{g_{X\psi \omega}} \right)^2. \tag{76}$$

The coupling constant at the $XJ/\psi V$ vertex can be evaluated directly from a QCD sum rule calculation, which is based on the three-point correlation function:

$$\Pi_{\rho \omega} (p, p', q) = \int d^4xd^4y \, e^{ip' \cdot x} e^{ip \cdot y} \Gamma_{\rho \omega} (x, y), \tag{77}$$

with

$$\Pi_{\rho \omega} (x, y) = \langle 0 | \Pi_{\rho \omega} (x, y) | \Pi_{\rho \omega} (y) \rangle_{a} \langle 0 | 0 \rangle.$$  

(78)
where \( p = p' + q \) and the interpolating fields are given by:

\[
\begin{align*}
\tilde{f}_\mu &= \tilde{c}_\mu \gamma_\mu c_a, \\
\tilde{j}_\mu^\phi &= \frac{1}{2} \bar{u}_a \gamma_\mu u_a - d_a \gamma_\mu d_a, \\
\tilde{j}_\mu^\rho &= \frac{1}{2} \bar{d}_a \gamma_\mu d_a.
\end{align*}
\] (79)

For \( \tilde{f}_\mu^\phi \), it was shown in ref. [98] that if we use the currents in Eqs. (53) or (54), the QCDSR result for the ratio between the couplings is given by:

\[
\frac{\mathcal{G}_{X\phi \omega}}{\mathcal{G}_{X\phi \rho}} = 1.14.
\] (80)

Using this result in Eq. (76) we finally get

\[
\frac{\Gamma(X \to J/\psi^+ \pi^- \pi^0)}{\Gamma(X \to J/\psi^+ \pi^- \pi^-)} = 0.15. \tag{81}
\]

Therefore, to be able to reproduce the experimental result in Eq. (40) it is necessary to use a mixed current:

\[
j_a^X = \cos \theta j_{a1}^{(u)} + \sin \theta j_{a2}^{(d)}, \tag{82}
\]

with \( j_{a1}^{(u)} \) given by Eq. (53) for a tetraquark current or by Eq. (54) for a molecular current. Using the above definitions in Eq. (78), and considering the quarks \( u \) and \( d \) as degenerated, we arrive at

\[
\Pi_{\mu
u}(x, y) = \frac{-iN_c}{2 \sqrt{2}} \left( \cos \theta + (-1)^x \sin \theta \right) \Pi_{\mu
u}^0(x, y), \tag{83}
\]

where \( N_c = 1 \) and \( N_u = 1/3 \), \( I_u = 0 \).

To evaluate the phenomenological side of the sum rule we insert, in Eq. (78), intermediate states for \( X, J/\psi \) and \( V \). Using the definitions:

\[
\begin{align*}
(0, j_\mu^\rho) &\equiv J/\psi(p') = m_\psi f_\psi \epsilon_\rho(p'), \\
(0, j_\mu^\phi) &\equiv V(q) = m_V f_V \epsilon_\phi(q), \\
(X(p)) &\equiv \left( \lambda_\rho \epsilon_\rho(p) \right).
\end{align*}
\]

where \( \lambda_\rho = (\cos \theta + \sin \theta) \lambda^\rho \), with \( \lambda^\rho \) defined in Eq. (62), we obtain the following relation:

\[
\Pi_{\mu\nu}^{(\text{phen})}(p, p', q) = \frac{i(\cos \theta + \sin \theta) \lambda^\rho \epsilon_\rho(p) \epsilon_\phi(q'; p') \lambda_\lambda \epsilon_\lambda(q)}{(2 - m_\psi^2)(2 - m_V^2)(2 - m_X^2)} \times (\epsilon^{\phi\epsilon^\rho} p^\rho + q_\alpha) - \epsilon^{\phi\epsilon^\rho} \frac{p_\rho' q_\alpha}{m_\psi^2} \frac{q_\rho p_\alpha}{m_V^2} \epsilon^{\phi\epsilon^\rho} \lambda_\lambda \lambda_\lambda \lambda_\lambda + \cdots \tag{85}
\]

where the dots stand for the contribution of all possible excited states, and the coupling constant, \( g_{X\phi V} \), is defined by the matrix element, \( \langle J/\psi V | X \rangle \):

\[
\langle J/\psi(p') V(q) | X(p) \rangle = g_{X\phi V} \epsilon^{\phi\epsilon^\rho} p_\rho \epsilon_\rho(q) \epsilon_\phi(p'), \tag{86}
\]

which can be extracted from the effective Lagrangian that describes the coupling between two vector mesons and one axial vector meson [48]:

\[
\mathcal{L} = ig_{X\psi V} \epsilon^{\phi\epsilon^\rho} (\bar{c}_\rho X_\rho) \Psi_\sigma V_\sigma. \tag{87}
\]

With the current in Eq. (82), the ratio between the coupling constants is now given by [98]:

\[
\frac{g_{X\phi \omega}}{g_{X\phi \rho}} = 1.14 \left( \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right). \tag{88}
\]

Using the previous result in Eq. (76) we get

\[
\frac{\Gamma(X \to J/\psi^+ \pi^- \pi^0)}{\Gamma(X \to J/\psi^+ \pi^- \pi^-)} \approx 0.15 \left( \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right)^2. \tag{89}
\]

This is exactly the same relation obtained in refs. [45] [125], that imposes \( \theta \approx 20^0 \) to reproduce the experimental result in Eq. (40). A similar relation was obtained in ref. [127] where the decay of the charm into two and three pions goes through a \( D D^* \) loop.

Using \( \theta \approx 20^0 \), the \( X J/\psi \phi \) coupling constant was estimated from the sum rule with a tetraquark current to be [125]:

\[
g_{X\phi \omega} = 13.8 \pm 2.0, \tag{90}
\]

which is much bigger than the number in Eq. (69), and leads to a much bigger partial decay width:

\[
\Gamma(X \to J/\psi^+ \pi^- \pi^0) = (50 \pm 15) \text{ MeV}. \tag{91}
\]

A similar width was obtained in ref. [98] by using a molecular current like the one in Eq. (54). Therefore, from a QCDSR calculation it is not possible to explain the small width of the \( X(3872) \) if it is a pure four-quark state.

Considering the fact that also the relation in Eq. (61) can not be reproduced with a pure molecular state, in ref. [98] the \( X(3872) \) was treated as a mixture between a \( c\bar{c} \) current and a molecular current, similar to the mixing considered in ref. [126] to study the light scalar mesons:

\[
J_\mu^\rho(x) = (\sin \alpha) J_\mu^{(c\bar{c})}(x) + (\cos \alpha) J_\mu^{(\text{mol})}(x), \tag{92}
\]

with \( J_\mu^{(c\bar{c})}(x) \) given in Eq. (54) and

\[
J_\mu^{(\text{mol})}(x) = \frac{1}{6 \sqrt{2}} \bar{q}q [\tilde{c}_a(x) \gamma_\mu \gamma_5 c_a(x)] \tag{93}
\]

There is no problem in reproducing the experimental mass of the \( X(3872) \) with this current for a large range of the mixture angle \( \alpha \). Considering \( \alpha \) in the region \( 5^\circ \leq \alpha \leq 13^\circ \) they get [98]:

\[
M_X = (3.77 \pm 0.18) \text{ GeV}, \tag{94}
\]

which is in a good agreement with the experimental value. The value obtained for the mass grows with the value of the mixing angle \( \alpha \), but for \( \alpha \geq 30^\circ \) it reaches a stable value being completely determined by the molecular part of the current.

Considering also a mixture of \( D^* D^* \) and \( D^* D^{**} \) components, the most general current is given by

\[
J_\mu^\rho(x) = \cos \theta J_\mu^{(c\bar{c})}(x) + \sin \theta J_\mu^{(\text{mol})}(x), \tag{95}
\]

with \( J_\mu^{(c\bar{c})}(x) \) and \( J_\mu^{(\text{mol})}(x) \) given by Eq. (92).
Using the current in Eq. (95), the $XJ/\psi \omega$ coupling constant obtained in ref. [98] for $\theta = 20^\circ$ and a mixing angle in Eq. (92) $\alpha = 9^\circ \pm 4^\circ$ is:

$$g_{XJ/\psi \omega} = 5.4 \pm 2.4$$

which gives:

$$\Gamma(X \to J/\psi \pi^+ \pi^- \pi^0) = (9.3 \pm 6.9) \text{ MeV}. \quad (96)$$

The result in Eq. (97) is in agreement with the experimental upper limit. It is important to notice that the width and the mass grow with the mixing angle $\alpha$. Therefore, there is only a small range for the values of this angle that can provide simultaneously good agreement with the experimental values of the mass and the decay width, and this range is $5^\circ \leq \alpha \leq 13^\circ$. This means that the $X(3872)$ is basically a $c\bar{c}$ state with a small, but fundamental, admixture of molecular $D\bar{D}^*$ states. By molecular states we mean an admixture between $D\bar{D}^{*0}$, $D\bar{D}^{*0}$ and $D^* D^{*0}$, $D^* D^{*0}$ states, as given by Eq. (53).

In ref. [129] a similar mixture between a $c\bar{c}$ state and molecular states (including $J/\psi p$ and $J/\psi \omega$) was considered to study the $X(3872)$ decays into $J/\psi \gamma$ and $\psi(2S)$, using effective Lagrangians. In this approach the authors only needed a small admixture of the $c\bar{c}$ component (equivalent to $\alpha = 78^\circ \pm 2^\circ$ in Eq. (29)) to reproduce the ratio in Eq. (51). It is not clear, however, if with this small $c\bar{c}$ admixture it is possible to obtain the prompt production cross section for the $X(3872)$ observed by the CDF Collaboration [88].

With the mixing angles $\alpha$ and $\theta$ fixed, it is possible to evaluate the width of the radiative decay $X(3872) \to J/\psi \gamma$, to check if it is possible to reproduce the experimental result in Eq. (39). To do that one has just to consider the current in Eq. (95) for the $X(3872)$, and exchange the $j^\gamma_\nu$ current, in Eq. (73), by the photon current $j^\gamma_\nu$:

$$j^\gamma_\nu = \sum_{q=a,d,s} e_q \bar{q} \gamma_\nu q,$$  

with $e_q = \pm \frac{2}{\sqrt{3}} e$ for quarks $u$ and $d$, and $e_q = \pm \frac{1}{\sqrt{2}} e$ for quark $c$ ($e$ is the modulus of the electron charge).

The phenomenological side of the sum rule becomes:

$$\Pi^{\text{phen}}(p, p', q) = \frac{ie(c \cos \theta + \sin \theta) \delta^4 m_{f_{0}} f_{0}}{m_X^2 (p^2 - m_{D}^2)} \times (C \alpha \beta - \beta \alpha) p^\nu q^\nu C$$

$$\times \left[ e^{\nu\nu' q c_{\nu} p} q^\nu (p') q_{\nu} (q) \right] \times X \to \gamma(q) J/\psi(p') \gamma(q). \quad (100)$$

$$\langle \psi(p') | j^\gamma_\nu | X(p) \rangle = i \epsilon_\nu^\gamma (q) \mathcal{M}(X(p) \to \gamma(q) J/\psi(p')) \times (101)$$

with $\langle 111 \rangle$

$$\mathcal{M}(X(p) \to \gamma(q) J/\psi(p')) = e^{\nu\nu' q c_{\nu} p} q^\nu (p') q_{\nu} (q) \times$$

$$\times e_{D} (A g_{\mu} b_{\alpha} p \cdot q + B g_{\mu} g_{\alpha} q_{\alpha} + C g_{\alpha} p_{\alpha} q_{\alpha}). \quad (101)$$

In Eq. (101), $A$, $B$ and $C$ are dimensionless couplings that are determined by the sum rule. Using this relation in Eq. (99), the

$$\Gamma(X \to J/\psi \gamma) = (0.19 \pm 0.13), \quad (104)$$

which is in complete agreement with the experimental result in Eq. (39). Therefore, from a QCDSR point of view, the $X(3872)$ is a mixture of a $c\bar{c}$ state ($\sim 97\%$) and molecular $D\bar{D}^{*0}$, $D\bar{D}^{*0}$ ($\sim 2.6\%$) and $D^* D^{*0}$, $D^* D^{*0}$ ($\sim 0.4\%$) states.

3.3. Summary for $X(3872)$

To summarize, there is an emerging consensus that the $X(3872)$ is not a pure $c\bar{c}$ state neither a pure multi-quark state. From the ratio in Eq. (40), we know that $X(3872)$ is not an isospin eigenstate, therefore, it can not be a pure $c\bar{c}$ state. On the other hand, the binding energy, the production rates and the observed ratio in Eq. (51) are not compatible with a pure molecular state. Considering all the available experimental information, it is very probable that the $X(3872)$ is a admixture of a charmonium state with other multi-quark states: molecular or tetraquark states.

3.4. Predictions for $X_b$, $X'$, $X''$

It is straightforward to extend the analysis done for the $X(3872)$ to the case of the bottom quark. Using the same interpolating field of Eq. (53) with the charm quark replaced by the bottom one, the analysis done for $X(3872)$ was repeated for $X_b$ in ref. [121]. In this case there is also a good Borel window and the prediction for the mass of the state that couples with a tetraquark $(bq)(\bar{b}q)$ with $J^{PC} = 1^{++}$ current is:

$$M_{X_b} = (10.27 \pm 0.23) \text{ GeV}. \quad (105)$$
The central value in Eq. \((147)\) is close to the mass of \(\Upsilon(3S)\), and appreciably below the \(B^+\bar{B}^\pm\) threshold at about 10.6 GeV. For comparison, the molecular model predicts for \(X_b\) a mass which is about 50–60 MeV below this threshold \([60]\), while a relativistic quark model without explicit \((b\bar{b})\) clustering predicts a value of about 133 MeV below this threshold \([130]\). A future discovery of this state, e.g. at LHCb, will certainly test the different theoretical models of this state and clarify, at the same time, the nature of the \(X(3872)\).

In the case of \(X'\) (\(c\bar{c}s[\bar{c}\bar{s}]\)) and \(X'_b\) (\(b\bar{b}[\bar{b}\bar{s}]\)), one has to replace the light quarks in the currents for \(X\) and \(X_b\) by strange quarks. To extract the relatively small mass-splitting, it is appropriate to use the double ratio of moments \([71,131]\):

\[
d^t \equiv \frac{M_X^2}{M_{X'}^2},
\]

which suppresses different systematic errors and the dependence on the sum rule parameters like \(s_0\) and \(M^2\). The result obtained for this ratio in ref. \([121]\) is:

\[
\sqrt{d^t} = 0.984 \pm 0.007.
\]

This leads to the mass splitting:

\[
M_{X'} - M_X = -(61 \pm 30) \text{ MeV}.
\]

Similar methods used in \([71,131]\) have predicted successfully the values of \(M_{D_s}/M_D\) and \(M_{B_b}/M_B\), which is not quite surprising, as in the double ratios, all irrelevant sum rules systematics cancel out.

It is interesting to notice that the \(X'\) mass prediction from ref. \([121]\) is slightly smaller than the \(X(3872)\) mass, which is quite unusual. Such a small and negative mass-splitting is rather striking and needs to be checked using alternative methods. The (almost) degenerate value of the \(X\) and of the \(X'\) masses may suggest that the physically observed \(X(3872)\) state can also have a \(c\bar{c}s\bar{s}\) component.

A similar analysis can be done for the \(X'_b\) (\(b\bar{b}[\bar{b}\bar{s}]\)) giving \([121]\):

\[
\sqrt{d^b} \equiv \frac{M_{X'_b}}{M_{X_b}} = 0.988 \pm 0.018,
\]

and

\[
M_{X'_b} - M_{X_b} = -(123 \pm 182) \text{ MeV}.
\]

We expect that the \(X_b\)-family will show up at LHCb in the near future, which will serve as a test of these predictions.

4. The \(Y(J^{PC}=1^{-{-}})\) family

4.1. Experiment versus theory

The \(e^+e^-\) annihilation through initial state radiation (ISR) is a powerful tool to search for \(1^{-{-}}\) states at the \(B\)-factories. The first state in the \(1^{-{-}}\) family discovered using this process was the \(Y(4260)\). It was first observed in 2005, by the Babar Collaboration \([10]\), in the reaction:

\[
e^+e^- \rightarrow \gamma_{\text{ISR}} J/\psi\pi^+\pi^-.
\]

with mass \(M = (4259 \pm 10)\) MeV and width \(\Gamma = (88 \pm 24)\) MeV. The \(Y(4260)\) was confirmed by CLEO and Belle Collaborations \([2]\).

The BaBar Collaboration also reported a \(\pi\pi\) mass distribution that peaks near 1 GeV, as can be seen in Fig. 12, and this information was interpreted as being consistent with the \(f_0(980)\) decay.

![Dipion mass distribution for Y(4260) -> J/\psi\pi^+\pi^- from ref.][10]

The \(Y(4260)\) was also observed in the \(B^- \rightarrow Y(4260)K^- \rightarrow J/\psi\pi^+\pi^-K^-\) decay \([31]\), and CLEO reported two additional decay channels: \(J/\psi\pi^0\pi^0\) and \(J/\psi K^+K^-\) \([6]\).

![The \(\pi^+\pi^- J/\psi\) invariant mass spectrum from ref.][15]

In an updated report \([32]\), BaBar has confirmed the observation of the \(Y(4260)\), shown in Fig. 13. However, the new \(\pi\pi\) mass distribution shows a more complex structure, as can be seen in Fig. 14.

![The \(\pi^+\pi^- J/\psi\) invariant mass spectrum from ref.][15]

Repeating the same kind of analysis leading to the observation of the \(Y(4260)\) state, in the channel \(e^+e^- \rightarrow \gamma_{\text{ISR}} \Psi(2S)\pi^+\pi^-\), BaBar \([33]\) has identified another broad peak at a mass around 4.32 GeV, which was confirmed by Belle \([12]\). Belle found that the \(\psi'/\psi\) enhancement observed by BaBar was, in fact, produced by two distinct peaks, as can be seen in Fig. 15. The masses and widths obtained by Belle and BaBar from fits to Breit-Wigner resonant shapes are summarized in Table 4.
of freedom has been excited. The nature of this gluonic excitation is not well understood, and has been described by various models. The spectrum of charmonium hybrids has been calculated using lattice gauge theory \[133\]. The result for the mass is approximately 4200 MeV, which is consistent with the flux tube model predictions \[134\]. However, more recent lattice simulations \[135\] and QCD string models calculations \[136\] predict that the lightest charmonium hybrid has a mass of 4400 MeV, which is closer to the mass of the \(Y(4360)\). In any case, a prediction of the hybrid hypothesis is that the dominant open charm decay mode would be a meson pair with one \(S\)-wave \(D\) meson \((D, D^*, D_{10})\) and one \(P\)-wave \(D\) meson \((D_1, D_{11})\) \[137\]. In the case of the \(Y(4260)\) this suggests dominance of the decay mode \(DD_1\). Therefore, a large \(DD_1\) signal could be understood as a strong evidence in favor of the hybrid interpretation for the \(Y(4260)\). In the case of the \(Y(4360)\) and \(Y(4660)\), since their masses are well above the \(DD_1\) threshold, their decay into \(DD_1\) should be very strong if they were charmonium hybrids.

As it is evident from Fig. 15, there is no sign of the \(Y(4260)\) in the \(\psi'\pi^\pm\pi^-\) mass spectrum. The \(\pi\pi\) mass distribution reported in \[12\] for these two resonances can be seen in Fig. 16.

All these three states share the following properties: they are vector states with the photon quantum numbers, they have large total widths, and they have only been observed in charmonium decay modes.

Since the masses of these states are higher than the \(D^{(*)}\bar{D}^{(*)}\) threshold, if they were \(1^-\) charmonium states they should decay mainly to \(D^{(*)}\bar{D}^{(*)}\). However, the observed \(Y\) states do not match the peaks in \(e^+\bar{e} \rightarrow D^{(*)}\bar{D}^{(*)}\) cross sections measured by Belle \[34\] and BaBar \[35, 36\]. Besides, the \(\Psi(3S), \Psi(2D)\) and \(\Psi(4S)\) \(c\bar{c}\) states have been assigned to the well established \(\Psi(4040), \Psi(4160),\) and \(\Psi(4415)\) mesons respectively. The predictions from quark models for the \(\Psi(3D)\) and \(\Psi(5S)\) charmonium states are 4.52 GeV and 4.74 GeV respectively. Therefore, the masses and widths of these three new \(Y\) states are not consistent with any of the \(1^-\) \(c\bar{c}\) states \[63, 64, 132\], and their discovery represents an overpopulation of the charmonium states, as it can be seen in Fig. 17.

An interesting interpretation is that the \(Y(4260)\) is a charmonium hybrid. Hybrids are hadrons in which the gluonic degree of freedom has been excited. The nature of this gluonic excitation is not well understood, and has been described by various models. The spectrum of charmonium hybrids has been calculated using lattice gauge theory \[133\]. The result for the mass is approximately 4200 MeV, which is consistent with the flux tube...
that the mass of a $Y(4940)$ would be 200 MeV higher than the
$Y(4360)$ has a decay with an intermediate state consistent with
$|146\rangle$ and as a 4$^-$ state interpreted as non-resonant manifestations of the Regee zeros
$|144\rangle$, a baryonium state $|141\rangle$, a canonical 5$^-$ S$_1$ c$\bar{c}$ state $|142\rangle$, and a
tetraquark with a $sc$-scalar-diquark and a $\bar{s}\bar{c}$-scalar-antidiquark
in a 2P-wave state $|143\rangle$.

In the case of $Y(4260)$, in ref. $|46\rangle$ it was considered as a
$sc$-scalar-diquark $\bar{s}\bar{c}$-scalar-antidiquark in a P-wave state. Miani
et al. $|46\rangle$ tried different ways to determine the orbital term and they arrived at $M = (4330 \pm 70)$ MeV, which is more consistent with $Y(4360)$. However, from the $\pi\pi$ mass distribution in refs. $|32, 12\rangle$, none of these two states, $Y(4260)$ and $Y(4360)$ has a decay with an intermediate state consistent with $f_0(980)$ and, therefore, it is not clear that they should have an $s\bar{s}$ pair in their structure. Besides, in ref. $|143\rangle$ the authors show that the mass of a $[sc]_S=[\bar{s}\bar{c}]_S=0$ tetraquark in a P-wave state would be 200 MeV higher than the $Y(4260)$ mass. The authors of ref. $|143\rangle$, found that a more natural interpretation for the $Y(4260)$ would be a $[gc]_S=[\bar{q}\bar{q}]_S=0$ tetraquark in a P-wave state. The $Y(4260)$ was also interpreted as a baryonium $\Lambda_\chi - \bar{\Lambda}_\chi$ state $|144\rangle$, a $S$-wave threshold effect $|145\rangle$, as a resonance due to the interaction between the three, $J/\psi\pi\pi$ and $J/\psi K\bar{K}$, mesons $|146\rangle$ and as a $4S$ charmonium state $|147\rangle$. Although there are some arguments against the molecular interpretation $|60, 148\rangle$, the $Y(4260)$ was also considered as a molecular state bound by meson exchange $|149, 150, 151\rangle$. The three $Y$ states were also interpreted as non-resonant manifestations of the Regee zeros $|152\rangle$.

4.2. QCDSR studies for the $Y(J^{PC} = 1^{--})$ states

The $Y(J^{PC} = 1^{--})$ states can be described by molecular or
tetraquark currents, with or without a $s\bar{s}$ pair. In refs. $|153, 154\rangle$
a QCD sum rule calculation was performed using these kind of currents.

The lowest-dimensional interpolating operator to describe a $J^{PC} = 1^{--}$ state with the symmetric spin distribution:
$[cs]_S=0[\bar{c}\bar{s}]_S=1 + [cs]_S=1[\bar{c}\bar{s}]_S=0$ is given by:

$$ j_{\mu} = \frac{ie_0}{\sqrt{2}}(s_d^T C\gamma_{\mu} c_d)(\bar{s}_d\gamma_{\mu} C\bar{c}_d) $$

$$ + (s_d^T C\gamma_{\mu} c_d)(\bar{s}_d\gamma_{\mu} C\bar{c}_d) . $$

$$ (112) $$

From Fig. 15 we see that the OPE convergence for $M^2 \geq 3.2$
GeV$^2$ is better than the one shown in Fig. 6 since the perturba-
tive contribution is the dominant one in the whole Borel region.
Using the dominance of the pole contribution to fix the upper
value of the Borel window, the result for the mass of the state described by the current in Eq. (112) obtained in ref. $|153\rangle$ is:

$$ m_Y = (4.65 \pm 0.10) \text{ GeV}, $$

in excellent agreement with the mass of the $Y(4660)$ me-
son. Therefore, the conclusion in ref. $|153\rangle$ is that the meson
$Y(4660)$ can be described with a diquark-antidiquark tetraquark
current with a spin configuration given by scalar and vector
diquarks. The quark content in the current in Eq. (112) is also consistent with the di-pion invariant mass spectra shown in Fig. 16 which shows that there is some indication that the $Y(4660)$ has a well defined di-pion intermediate state consist-
tent with $f_0(980)$. It is also very interesting to notice that the $f_0(980)$ meson may be itself considered as a tetraquark state $|155\rangle$. This aspect should play an important role in the
decay of the $Y(4660)$ decay since, as shown in ref. $|125\rangle$, it is very hard to explain a small decay width when the initial four-quark state decays into
two final two-quark states.

Replacing the strange quarks in Eq. (112) by a generic light
quark $g$ the mass obtained for a $1^{--}$ state described with the
symmetric spin distribution: $[cq]_S=0[\bar{c}\bar{q}]_S=1 + [cq]_S=1[\bar{c}\bar{q}]_S=0$ is

$$ m_Y = (4.49 \pm 0.11) \text{ GeV}, $$

which is bigger than the $Y(4350)$ mass, but is consistent with it considering the uncertainty.
The $Y$ mesons can also be described by molecular-type currents. In particular a $D_0(b(2317)\bar{D}^*_{1}(2110)$ molecule with $J^{PC} = 1^{--}$, could also decay into $\psi'\pi^+\pi^-$ with a dipion mass spectra consistent with $f_0(980)$. A current with $J^{PC} = 1^{--}$ and a symmetrical combination of scalar and vector mesons is given by:

$$j_\mu = \frac{1}{\sqrt{2}}[\bar{c}_a\gamma_\mu c_a(\bar{s}_b\gamma_5 b_b) + (\bar{c}_a\gamma_\mu c_a)(\bar{s}_b\gamma_5 b_b)].$$ \hspace{1cm} (115)

The mass obtained in ref. [153] for the current in Eq. (115) is $m_{D_0D_s^*} = (4.42 \pm 0.10)$ GeV , \hspace{1cm} (116)
which is in excellent agreement with the mass of the meson $Y(4260)$. Again, in order to conclude if we can associate this molecular state with the meson $Y(4260)$ we need a better understanding of the di-pion invariant mass spectra for the pions in the decay $Y(4260) \to J/\psi\pi^+\pi^-$. From the spectra given in Fig. [14] it seems that the $Y(4260)$ is consistent with a non-strange molecular state $D_0^*\bar{D}$. Using a $D_0$ mass $[21]$ $m_{D_0} = 2352 \pm 50$ MeV, the $D_0^*\bar{D}$ threshold is around 4360 MeV and it is 100 MeV above the mass in Eq. (117), indicating the possibility of a bound state.

A $J^{PC} = 1^{--}$ molecular current can also be constructed with pseudoscalar and axial-vector mesons. A molecular $D_0^*\bar{D}$ current was used in ref. [154]. The mass obtained with such current is:

$$m_{D_0^*\bar{D}} = (4.27 \pm 0.10) \text{ GeV},$$ \hspace{1cm} (117)

Therefore, considering the errors, the molecular $D_0^*\bar{D}$ assignment for the meson $Y(4260)$ is also possible, in agreement with the findings of ref. [129], where a meson exchange model was used to study the $Y(4260)$ meson. The $D_0^*\bar{D}$ threshold is around 4285 MeV and very close to the $Y(4260)$ mass, indicating the possibility of a loosely bound molecular state.

### 4.3. Summary for $Y(J^{PC} = 1^{--})$ states

To summarize, the discovery of the $Y(4260)$, $Y(4360)$ and $Y(4660)$ appears to represent an overpopulation of the expected charmonium $1^{--}$ states. The absence of open charm production is also inconsistent with a conventional $c\bar{c}$ explanation. Possible explanations for these states include charmonium hybrid, tetraquark state and $D_0^*\bar{D}$ or $D_0\bar{D}^*$ molecular state for $Y(4260)$. The $Y(4660)$ could be a charmonium hybrid, or a tetraquark state with two scalar $[cs]$ diquarks in $P$-wave or with two scalar $[cs]$ diquarks in $P$-wave. The $Y(4660)$ could be a canonical $S^1 S_1$ $c\bar{c}$ state, or a tetraquark state (with symmetrical $[cs]S_{10} [\bar{c}\bar{s}]S_{10}$ or with $[cs]S_{10} [\bar{c}\bar{s}]S_{10}$) in a $P$-wave.

From the QCDSR results described in this Section, the $Y(4260)$ could be a $D_0^*\bar{D}$ or a $D_0^*\bar{D}$ molecular state and the $Y(4660)$ could be a tetraquark state with symmetrical spin distribution: $[cs]S_{10} [\bar{c}\bar{s}]S_{10} + [cs]S_{10} [\bar{c}\bar{s}]S_{10}$. It is not possible to describe the $Y(4360)$ neither as a $D_0^*\bar{D}$ molecular state, nor as a tetraquark state with symmetrical spin distribution: $[cq][s=1][\bar{c}\bar{q}][s=0] + [cq][s=0][\bar{c}\bar{q}][s=1]$. \hspace{1cm} (118)

### 5. The $Z^*(4430)$ meson

All states discussed so far are electrically neutral. The real turning point in the discussion about the structure of the new observed charmonium states was the observation by Belle Collaboration of a charged state decaying into $\psi'\pi^+$, produced in $B^+ \to K\psi'\pi^+$ [13].

#### 5.1. Experiment versus theory

The measured mass and width of this state are $M = (4433 \pm 4 \pm 2)$ MeV and $\Gamma = (45^{+18}_{-13}+30)\text{ MeV}$ [13]. The $B$ meson decay rate to this state is similar to that for the decays to the $X(3872)$ and $Y(3930)$ mesons. There are no reports of a $Z^*$ signal in the $J/\psi\pi^+$ decay channel. Since the minimal quark content of this state is $c\bar{c}ud$, this state is a prime candidate for a multiquark meson. The $Z^*(4430)$ was observed in the $\psi'\pi^+$ channel, therefore, it is an isovector state with positive $G$-parity: $IG = 1^+$. Using the same data sample as in ref. [13], Belle also performed a full Dalitz plot analysis [57] and has confirmed the observation of the $Z^*(4430)$ signal with a 6.4$sigma$ peak significance, as can be seen in Fig. [19]. The updated $Z^*(4430)$ parameters are: $M = (4433^{+15}_{-12}+18)\text{ MeV}$ and $\Gamma = (109^{+86}_{-43}+75)\text{ MeV}$.

Figure 19: Dalitz plot projection for the $\psi'\pi^+$ invariant mass from ref. [57]. The solid (dotted) histograms shows the fit result with (without) a single $\phi\pi^+$ state. The dashed histogram represents the background.

Babar Collaboration [58] also searched the $Z^*(4430)$ signature in four decay modes: $B \to \psi\pi K$, where $B = J/\psi$ or $\psi'$ and $K = K_0^\pm$ or $K^*$. No significant evidence for a signal peak was found in any of the processes investigated, as it can be seen in Fig. [20].

The Babar result gives no conclusive evidence of the $Z^*(4430)$ seen by Belle.

There are many theoretical interpretations of the $Z^*(4430)$ structure [156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172]. Because its mass is close to the $D^*\bar{D}$ threshold, Rosner [156] suggested that it is an $S$-wave threshold effect, while others considered it to be a strong candidate for a $D^*\bar{D}$ molecular state [157, 158, 159, 160, 161, 162].
Other possible interpretations are tetraquark state \([43, 163]\), a cusp in the \(D^*D_1\) channel \([164]\), a baryonium state \([165]\), or a radially excited \(c\bar{s}\) state \([166]\), or a hadro-charmonium state \([66]\). The tetraquark hypothesis implies that the \(Z^*(4430)\) will have neutral partners decaying into \(\psi\pi^0/\eta\). Besides the spectroscopy, there are discussions about its production \([156, 168, 169, 170]\) and decay \([171]\). Considering the \(Z^*(4430)\) as a loosely bound \(S\)-wave \(D^*D_1\) molecular state, the allowed angular momentum and parity are \(J^P = 0^-, 1^-, 2^-\), although the \(2^-\) assignment is probably suppressed in the \(B^+ \to Z^*K\) decay by the small phase space. Among the remaining possible \(0^-\) and \(1^-\) states, the former will be more stable as the later can also decay to \(DD_1\) in \(S\)-wave. Moreover, one expects a bigger mass for the \(J^P = 1^-\) state as compared to a \(J^P = 0^-\) state. There is also a quenched lattice QCD calculation that finds attractive inaraction in the \(D^*D_1\) system in the \(J^P = 0^-\) channel \([173]\). The authors of ref. \([173]\) also find positive scattering length. Based on these findings, they conclude that although the interaction between the two charmed mesons is attractive in this channel, it is unlikely that they can form a genuine bound state right below the threshold.

5.2. QCDSR calculations for \(Z^*(4430)\)

Considering \(Z^*(4430)\) as a \(D^*D_1\) molecule with \(J^P = 0^-\), a possible current describing such state, considered in ref. \([158]\), is given by:

\[
j = \frac{1}{\sqrt{2}} \left[ (\bar{d}_a \gamma_\mu c_u)(\bar{c}_b \gamma_\rho \gamma_5 u_b) + (\bar{d}_a \gamma_\mu \gamma_5 c_u)(\bar{c}_b \gamma_\rho u_b) \right].
\]

(119)

This current corresponds to a symmetrical state \(D^*D_1^0 + \bar{D}^{0\ast}D_1^\ast\), and has positive \(G\)-parity, which is consistent with the observed decay \(Z^*(4430) \to \psi'\pi^0\).

The mass obtained in a QCDSR calculation using such a current was \([158]\):

\[
m_{D^*D_1} = (4.40 \pm 0.10) \text{ GeV},
\]

(120)
in an excellent agreement with the experimental mass.

To check if the \(Z^*(4430)\) could also be described as a diquark-antidiquark state with \(J^P = 0^-\), the following current was considered in ref. \([162]\):

\[
\bar{j}_\mu = \frac{ie_{abc}e_{dec}}{\sqrt{2}} \left[ (u_a^T \gamma_5 c_b)(\bar{d}_d \gamma_\mu \gamma_5 c_e^T) - (u_a^T \gamma_\mu \gamma_5 c_b)(\bar{d}_d \gamma_\mu \gamma_5 c_e^T) \right].
\]

(121)

The mass obtained with this current was \([162]\)

\[
m_{Z_{0^{-}}} = (4.52 \pm 0.09) \text{ GeV},
\]

(122)

which is a little bigger than the experimental value \([13]\), but still consistent with it, considering the uncertainties. Comparing this result with the result in Eq. (120), we see that the result obtained using a molecular-type current is in a better agreement with the experimental value. As mentioned in Sec. II, this strongly suggests that the state is better explained as a molecular state than as a diquark-antidiquark state. This is explicitly borne out in the calculation for the coupling, \(\lambda\), between the state and the current defined in Eq. (5) of Sec. II. We get:

\[
\lambda_{Z_{0^{-}}} = (3.75 \pm 0.48) \times 10^{-2} \text{ GeV}^5,
\]

(123)

\[
\lambda_{D^*D_1} = (5.66 \pm 1.26) \times 10^{-2} \text{ GeV}^5.
\]

(124)

Therefore, one can conclude that the physical particle with \(J^P = 0^-\) and quark content \(c\bar{c}ud\) couples with a larger strength with the molecular \(D^*D_1\) type current than with the current in Eq. (121).

In ref. \([162]\) it was also considered as a diquark-antidiquark interpolating operator with \(J^P = 1^-\) and positive \(G\) parity:

\[
\bar{j}_\mu = \frac{e_{abc}e_{dec}}{\sqrt{2}} \left[ (u_a^T \gamma_5 c_b)(\bar{d}_d \gamma_\mu\gamma_\rho \gamma_5 c_e^T) + (u_a^T \gamma_\rho \gamma_5 c_b)(\bar{d}_d \gamma_\mu \gamma_\rho \gamma_5 c_e^T) \right].
\]

(125)

In this case the Borel stability obtained is worse than for the \(Z^*\) with \(J^P = 0^-\) \([162]\), and the obtained mass was

\[
m_{Z_{1^{-}}} = (4.84 \pm 0.14) \text{ GeV},
\]

(126)

which is much bigger than the experimental value and bigger than the result obtained using the current with \(J^P = 0^-\) in Eq. (122). From these results it is possible to conclude that, while it is also possible to describe the \(Z^*(4430)\) as a diquark-antidiquark state or a molecular state with \(J^P = 0^-\), the \(J^P = 1^-\) configuration is disfavored.

5.3. Summary for \(Z^*(4430)\)

A confirmation of the existence of the \(Z^*(4430)\) is critical before a complete picture can be drawn. If confirmed, the only open options for the \(Z^*(4430)\) structure are tetraquark or molecule. The favored quantum numbers are \(J^P = 0^-\).

5.4. Sum rule predictions for \(B^*B_1\) and \(D^*D_1\) molecules

It is straightforward to extend the analysis done for the \(D^*D_1\) molecule to the case of the bottom quark. Using the same interpolating field of Eq. (119) with the charm quark replaced by the bottom quark, the analysis done for \(Z^*(4430)\) was repeated for \(Z_0\)
The OPE convergence in this case is even better than the one for $Z^*(4430)$ and the predicted mass is

$$m_{Z^*_B} = (10.74 \pm 0.12) \text{ GeV},$$

(127)
in a very good agreement with the prediction in ref. [174].

In the case of the strange analogous meson $Z^*_s$ considered as a pseudoscalar $D_s^* D_s^*$ molecule, the current is obtained by exchanging the $d$ quark in Eq. (119) by the $s$ quark. The predicted mass is [158]:

$$m_{D_s^*D_s} = (4.70 \pm 0.06) \text{ GeV},$$

(128)

which is bigger than the $D_s^* D_s$ threshold $\sim 4.5$ GeV, indicating that this state is probably a very broad one and, therefore, it might be very difficult to be seen experimentally.

### 6. The $Z^*_1(4050)$ and $Z^*_2(4250)$ states

The $Z^*(4430)$ observation motivated studies of other $B^0 \rightarrow K^{-}\pi^+ (\gamma \pi) \chi_{c1}$ decays. In particular, the Belle Collaboration has reported the observation of two resonance-like structures in the $\pi^+ \chi_{c1}$ mass distribution [7].

#### 6.1. Experiment versus theory

The two resonance-like structures, called $Z^*_1(4050)$ and $Z^*_2(4250)$, were observed in the exclusive process $\bar{B}^0 \rightarrow K^- \pi^+ \chi_{c1}$. The significance of each of the $\pi^+ \chi_{c1}$ structures exceeds 5$\sigma$ and, if they are interpreted as meson states, their minimal quark content must be $cc\bar{u}d$. Since they were observed in the $\pi^+ \chi_{c1}$ channel, the only quantum numbers that are known about them are $J^P = 1^-$. The invariant mass distribution, where the contribution of the structure in the $\pi^+ \chi_{c1}$ channel is most clearly seen, is shown in Fig. 21.

Before this observation, it was suggested, in ref. [51], that resonances decaying into $\chi_{c1}$ and one or two pions are expected in the framework of the hadro-charmonium model.

Due to the closeness of the $Z^*_1(4050)$ and $Z^*_2(4250)$ masses to the $D^* D^*(4020)$ and $D_s^* D_s(4285)$ thresholds, these states could also be interpreted as molecular states or threshold effects. Liu et al. [175], using a meson exchange model find strong attraction for the $D^* D^*$ system with $J^P = 0^+$, while using a boson exchange model, the author of ref. [176] concluded that the interpretation of $Z^*_1(4050)$ as a $D^* D^*$ molecule is not favored. In any case, it is very difficult to understand a bound molecular state which is above the $D^* D^*$ threshold. In the case of $Z^*_2(4250)$, using a meson exchange model, it was shown in ref. [149] that its interpretation as a $D_s^* D_s$ or $D_s D_s^*$ molecule is disfavored.

#### 6.2. QCDSR calculations

In ref. [154], the QCD sum rules formalism was used to study the $D^* D^*$ and $D_s^* D_s$ molecular states with $J^P = 1^+$ and $1^-1^-$ respectively. The currents used in both cases are:

$$j_{D^*D^*} = (d_0 \gamma \rho c_0)(\bar{c}_b \gamma^\mu u_b),$$

(130)

$$j_{D_s^*D_s} = \frac{i}{\sqrt{2}} \left[(d_0 \gamma \rho \gamma_5 c_a)(\bar{c}_b \gamma^\mu u_b) + (d_0 \gamma \rho c_a)(\bar{c}_b \gamma^\mu \gamma_5 u_b)\right].$$

(131)

The mass obtained with the current in Eq. (130) is [154]:

$$m_{D^*D^*} = (4.15 \pm 0.12) \text{ GeV},$$

(132)

where the central value is around 130 MeV above the $D^* D^*(4020)$ threshold, indicating the existence of repulsive interactions between the two $D^*$ mesons. Strong interaction effects might lead to a repulsion that could result in a virtual state above the threshold. Therefore, this structure may or may not indicate a resonance.

For the current in Eq. (131), the mass obtained is [154]:

$$m_{D_s^*D_s} = (4.19 \pm 0.22) \text{ GeV},$$

(133)

where the central value is around 100 MeV below the $D_s^* D_s(4285)$ threshold, and, considering the errors, consistent with the mass of the $Z^*_2(4250)$ resonance. Therefore, in this case, there is an attractive interaction between the mesons $D_s$ and $D$ which can lead to a molecular state.

In ref. [123] it was found that the inclusion of the width, in the phenomenological side of the sum rule, increases the obtained mass for molecular states. This means that the introduction of the width in the sum rule calculation, increases the mass of the states that couple to the $D^* D^*$ and $D_s^* D_s$ molecular currents. As a result, using the current in Eq. (131), it is possible to obtain a mass $m_{D_s^*D_s} = 4.25$ GeV with a width $40 \leq \Gamma \leq 60$ MeV, as can be seen in Fig. 22.
On the other hand, the mass of the $D^*D^*$ molecule will be far from the $Z'_1(4050)$ mass. Therefore, the authors of ref. [154] conclude that it is possible to describe the $Z'_1(4250)$ resonance structure as a $D_1 \bar{D}_1$ molecular state with $l\ell' j^P = 1^-\bar{1}^-$ quantum numbers, and that the $D^*D^*$ state is probably a virtual state that is not related with the $Z'_1(4050)$ resonance-like structure. Considering the fact that the $D^*D^*$ threshold (4020) is so close to the $Z'_1(4050)$ mass and that the $\eta'_c(3S_0)$ mass is predicted to be around 4050 MeV [65], it is probable that the $Z'_1(4050)$ is only a threshold effect [65].

7. The $Y(3930)$ and $Y(4140)$ states

7.1. Experiment versus theory

The $Y(3930)$ was first observed by the Belle Collaboration [4] in the decay $B \rightarrow KY(3930) \rightarrow K\omega J/\psi$. It was confirmed by BaBar [63, 83] in two channels $B^+ \rightarrow K^+ \omega J/\psi$ and $B^0 \rightarrow K^0 \omega J/\psi$. The measured mass from these two Collaborations are: $(3943 \pm 11)$ MeV from ref. [4] and $(3919.1^{+2.9}_{-2.8} \pm 2)$ MeV from ref. [83], which gives an average mass of $(3929 \pm 7)$ MeV. This state has positive $C$ and $G$ parities and the total width is $(31^{+10}_{-8} \pm 5)$ MeV [83]. The $m_{J/\psi\omega}$ mass distribution observed by BaBar is shown in Fig. 23 from where we also see the $X(3872)$ observed in its decay into $J/\psi\omega$.

Since the decay $Y \rightarrow J/\psi\omega$ is OZI suppressed for a charmonium state [137], also for $Y(3930)$ it was conjectured that it could be a hybrid $c\bar{c}g$ state [137], a molecular state [50, 109, 175, 178, 179, 177], or a tetraquark state [45].

A recent acquisition to the list of peculiar states is the narrow structure observed by the CDF Collaboration in the decay $B^+ \rightarrow Y(4140)K^+ \rightarrow J/\psi\phi K^+$. The particle’s signature peak can be seen in Fig. 24. The mass and width of this structure is $M = (4143 \pm 2.9 \pm 1.2) \text{ MeV}$, $\Gamma = (11.7^{+5.8}_{-5.8} \pm 3.7) \text{ MeV}$ [8]. Since the $Y(4140)$ decays into two $I^G(J^{PC}) = 0^-(1^-)$ vector mesons, like the $Y(3930)$ it has positive $C$ and $G$ parities. The possible $J^P$ quantum numbers of a $S$-wave vector-vector system are $0^-$, $1^+$, $2^+$. However, since $C = (-1)^L \delta_S$, the $J^P = 1^+$ is forbidden for a state with $L = 0$ and $C = +1$. Therefore, the possible quantum numbers for $Y(3930)$ and $Y(4140)$ are $J^{PC} = 0^{--}$, $1^{-+}$ and $2^{++}$. At these quantum numbers, $1^{-+}$ is not consistent with the constituent quark model and it is considered exotic.

There are already some theoretical interpretations of this structure. Its interpretation as a conventional $c\bar{c}$ state is disfavored because, as pointed out by the CDF Collaboration [8], it lies well above the threshold for open charm decays and, therefore, a $c\bar{c}$ state with this mass would decay predominantly into an open charm pair with a large total width. It was also shown in ref. [180] that the $Y(4140)$ is probably not a $P$-wave charmonium state: $\chi_{cJ}(J = 0, 1)$. If it were the case, the branching ratio of the hidden charm decay, $Y(4140) \rightarrow J/\psi\phi$, would be
much smaller than the experimental observation.

In ref. [181], the authors interpreted the Y(4140) as the molecular partner of the charmonium-like state Y(3930). They concluded that the Y(4140) is probably a $D_s^*D_s^*$ molecular state with $J^{PC} = 0^{++}$ or $2^{++}$, while the Y(3930) is its $D^*D^*$ molecular partner. This idea is supported by the fact that the mass difference between these two mesons is approximately the same as the mass difference between the $\phi$ and $\omega$ mesons: $m_{Y(4140)} - m_{Y(3930)} \sim m_\phi - m_\omega \sim 210$ MeV. It is also interesting to notice that, if the Y(4140) and the Y(3930) mesons are $D_s^*D_s^*$ and $D^*D^*$ molecular states, the binding energies of these states will be approximately the same: $m_{Y(4140)} - 2m_{D_s^*} \sim m_{Y(3930)} - 2m_{D^*} \sim -90$ MeV. However, with a meson exchange mechanism to bind the two charmed mesons, it seems natural to expect a more deeply bound system in the case that pions can be exchanged between the two charmed mesons, as in the $D^*D^*$, than when only $\eta$ and $\phi$ mesons can be exchanged, as in the $D_s^*D_s^*$ system.

In ref. [182] the authors argue that the Y(4140) can be interpreted either as a $D_s^*D_s^*$ molecular state or as an exotic hybrid charmonium with $J^{PC} = 1^{--}$. A molecular $D_s^*D_s^*$ configuration was also considered in refs. [183, 184, 185, 186, 187]. Using QCD sum rules, the authors of refs. [183, 185, 187] agree that it is possible to describe the Y(4140) with a molecular $D_s^*D_s^*$ current with $J^{PC} = 0^{++}$. Using one boson exchange model the author of ref. [186] showed that the effective potential of the $D_s^*D_s^*$ system supports the explanation of Y(4140) as a molecular state. In ref. [188] the Y(4140) was considered as a $J^{PC} = 1^{++}$ $c\bar{c}c\bar{s}$ tetraquark state. The $J^{PC} = 1^{++}$ assignment reduces the coupling of the Y(4140) with the vector-vector channel and, therefore, a small decay width would be possible in this case but not for a $J^{PC} = 0^{++}$ tetraquark state, as pointed out in ref. [181]. The authors of ref. [189] argue that the $J/\psi\phi$ system has quantum numbers $J^{PC} = 1^{--}$ and that the enhancement observed by CDF does not represent any kind of resonance. There is also a prediction for the radiative open charm decay of the Y(4140) that could test the molecular assignment of this state [190].

7.2. QCDSR calculation for Y(3930) and Y(4140)

Considering the Y(3930) and Y(4140) as $J_c^0J^{PC} = 0^+0^{++}$ states, the possible currents that couple with a $D^*D^*$ and $D_s^*D_s^*$ molecular states are

$$j_Q = (\bar{q}_a\gamma_\mu c_a)(\bar{c}_b\gamma_\mu q_b) \quad \text{and} \quad j_s = (\bar{q}_a\gamma_\mu c_a)(\bar{c}_b\gamma_\mu q_b),$$

respectively. These two currents were considered in a QCDSR study in ref. [185]. Surprisingly the masses obtained were

$$m_{D_s^*D_s^*} = (4.14 \pm 0.09) \text{ GeV},$$

and

$$m_{D^*D^*} = (4.13 \pm 0.11) \text{ GeV}.$$  

There is another QCDSR calculation for the $D^*D^*$ molecular current with $J^{PC} = 0^{++}$ [191], that finds a mass $m_{D^*D^*} = (3.91 \pm 0.11)$ GeV, compatible with the Y(3930) state. However, in ref. [191] the authors have considered only the condensates up to dimension six. We show, in Fig. 25, the contribution of all the terms in the OPE side of the sum rule, up to dimension-six. From this figure we see that only for $M^2 \geq 3.5$ GeV$^2$ the contribution of the dimension-six condensate is less than 20% of the total contribution. Therefore, the lower value of $M^2$ in the sum rule window should be $M^2_{min} = 3.5$ GeV$^2$. The inclusion of the dimension-eight condensate improves the OPE convergence, and its contribution is less than 20% of the total contribution for $M^2 \geq 2.5$ GeV$^2$, as it can be seen in Fig. 26.

Besides, we observe that considering only condensates up to dimension-six, there is no pole dominance, in the Borel range considered in ref. [191], as it can be seen in Fig. 27 and, therefore, no allowed Borel window can be found in this case.

In the case of the $D_s^*D_s^*$ molecular current, the inclusion of the dimension-eight condensate almost does not change the OPE convergence and the value of the mass. This is the rea-
son why the results obtained with the $D_s^*\bar{D}_s^*$ molecular current are the same in refs. [185] and [187].

Therefore, from a QCD sum rule study, the results obtained in refs. [185] and [187] indicate that the $Y(4140)$ narrow structure observed by the CDF Collaboration in the decay $B^+ \rightarrow Y(4140)K^+ \rightarrow J/\psi K^+$ can be very well described by a scalar $D_s^*\bar{D}_s^*$ current. The mass obtained with the $D_s^*\bar{D}_s^*$ scalar current, on the other hand, depends on the dimension of the condensates processes considered in the OPE side, showing that there is still no OPE convergence in the sum rule up to dimension-6 condensate. To test if the convergence is achieved up to dimension-8, it is important to consider higher dimension condensates in the OPE side of the sum rule.

8. The $X(3915)$ and $X(4350)$ states

In recent communications, the Belle Collaboration has reported the observation of two narrow peaks in the two-photon processes $\gamma\gamma \rightarrow \omega J/\psi$ [2] and $\gamma\gamma \rightarrow \phi J/\psi$ [11], as can be seen in Figs. 28 and 29.

These two states were reported in the experimental reviews [68, 192, 193, 194]. When fitted with Breit-Wigner resonance amplitudes, the resonance parameters are:

$$M = (3914 \pm 4 \pm 2) \text{ MeV},$$

$$\Gamma = (28 \pm 12^{+12}_{-9}) \text{ MeV},$$

for $X(3915)$, and

$$M = (4350.6^{+4.9}_{-5.0} \pm 0.7) \text{ MeV},$$

$$\Gamma = (13.3^{+7.9}_{-7.1} \pm 4.1) \text{ MeV},$$

for $X(4350)$.

Since these two states decay into two vector mesons, they have positive $C$ and $G$ parities, like the $Y(3930)$ and $Y(4140)$. As a matter of fact, it is possible that the $X(3915)$ be the same state as the $Y(3930)$, observed by Belle [4] and BaBar [41, 83] in the decay channel $B \rightarrow KY(3930) \rightarrow K\omega J/\psi$, since the Babar mass and width for this state are very close to the result in Eq. (137): $M = (3919.1^{+3.9}_{-3.8} \pm 2 \text{ MeV}$ and $\Gamma = (31^{+10}_{-8} \pm 5) \text{ MeV}$. In any case, a charmonium assignment for this state is difficult [68].

In the case of the $X(4350)$ its mass is much higher than the $Y(4140)$ mass and, as it can be seen in Fig. 29 no $Y(4140)$ signal is observed in the two-photon process $\gamma\gamma \rightarrow \phi J/\psi$. This fact was interpreted, in ref. [192], as a point against the $D_s^*\bar{D}_s^*$ molecular picture for the $Y(4140)$ states. As shown in ref. [184], a $D_s^{*+}\bar{D}_s^{-}$ molecular state should be seen in the two-photon process.

The possible quantum numbers for a state decaying into $J/\psi \phi$ are $J^{PC} = 0^{++}$, $1^{-+}$ and $2^{++}$. At these quantum numbers, $1^{-+}$ is not consistent with the constituent quark model and it is considered exotic. In ref. [11] it was noted that the mass of the $X(4350)$ is consistent with the prediction for a $c\bar{s}c\bar{s}$ tetraquark state with $J^{PC} = 2^{++}$ [188] and a $D_s^{*+}\bar{D}_s^{-}$ molecular state [195]. However, the state considered in ref. [195] has $J^P = 1^{+}$ with no definite charge conjugation. A molecular state with a vector and a scalar $D_s$ mesons with negative charge conjugation was studied by the first time in ref. [153], and the obtained mass was $(4.42 \pm 0.10) \text{ GeV}$, also consistent with the $X(4350)$ mass, but with not consistent quantum numbers. A molecular state with a vector and a scalar $D_s$ mesons with positive charge conjugation can be constructed using the combination $D_s^{*+}\bar{D}_s^{-} - D_s^{-}\bar{D}_s^{*+}$.
Some possible interpretations for this state are: an excited P-wave charmonium state $\Xi_{c2}$; a mixed charmonium-$D^*_sD^*_s$ state $\Upsilon_{c2}$.  

8.1. QCDSR calculation for $X(4350)$

Following Belle Collaboration's suggestion [11], in ref. [198], a QCDSR calculation using a $D^*_sD^*_{s0}$ current with $J^{PC} = 1^{--}$, was considered to test if the new observed resonance structure, $X(4350)$, can be interpreted as such molecular state.

A current that couples with a $J^{PC} = 1^{--} D^*_sD^*_{s0}$ molecular state is given by:

$$ j_{\mu} = \frac{1}{\sqrt{2}} \left[ (\bar{c}_a \gamma_\mu c_a)(\bar{s}_b s_b) - (\bar{c}_a \gamma_\mu s_a)(\bar{s}_b c_b) \right]. \tag{139} $$

For this current the OPE convergence is very good and the dimension-8 condensate (obtained using the factorization hypothesis) is almost negligible, as can be seen by Fig. 30.

**Figure 30:** The OPE convergence for the $J^{PC} = 1^{--} D^*_sD^*_{s0}$ molecule in the region $2.8 \leq M^2 \leq 4.8 \text{ GeV}^2$ for $\sqrt{m} = 5.3 \text{ GeV}$. We plot the relative contributions starting with the perturbative contribution plus the modified correction (long-dashed line), and each other line represents the relative contribution after adding of one extra condensate in the expansion: $\langle \bar{s}s \rangle$, $\langle \bar{c}c \rangle$, and $\langle \bar{s}s \rangle^2$ (dashed line), $\langle \bar{s}s \rangle^2 + \langle \bar{c}c \rangle^2$ (dotted line), $\langle \bar{s}s \rangle^2 + \langle \bar{c}c \rangle^2$ (dash-dotted line), $\langle \bar{s}s \rangle^2 + \langle \bar{c}c \rangle^2$ (dashed line), $\langle \bar{s}s \rangle^2$ (solid line) with $\langle \bar{s}s \rangle^2$ (dotted line), $\langle \bar{s}s \rangle^2$ (dash-dotted line), $\langle \bar{s}s \rangle^2$ (dashed line), $\langle \bar{s}s \rangle^2$ (solid line) with $\langle \bar{s}s \rangle^2$ (dotted line). Taken from ref. [198].

Considering condensates up to dimension-8 and keeping terms which are linear in the strange quark mass $m_s$, the mass obtained in ref. [198] was:

$$ m_{D^*_sD^*_{s0}} = (5.05 \pm 0.19) \text{ GeV}, \tag{140} $$

where the uncertainty was obtained by considering the QCD parameters in the ranges: $m_c = (1.23 \pm 0.05) \text{ GeV}$, $m_s = (0.13 \pm 0.03) \text{ GeV}$, $\langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3 \text{ GeV}^3$, $m_{c}^2 = (0.8 \pm 0.1) \text{ GeV}^2$, and also by allowing a violation of the factorization hypothesis by using $\nu = 2.1$.

The result in Eq. (140) is much bigger than the mass of the narrow structure $X(4350)$ observed by Belle. Therefore, the authors of ref. [198] concluded that it is not possible to interpret the $X(4350)$ as a $D^*_sD^*_{s0}$ molecular state with $J^{PC} = 1^{--}$. It is also interesting to notice that the mass obtained for a state described with a $1^{--} D^*_sD^*_{s0}$ molecular current, given in Eq. (116), is much smaller than the result obtained with the $1^{--} D^*_sD^*_{s0}$ molecular current. This may be interpreted as an indication that it is easier to form molecular states with not exotic quantum numbers.

9. The X(3940), Z(3930), X(4160) and Y(4008) states

The X(3940) and X(4160) were observed by the Belle Collaboration in the analysis of the $M_{\text{rec}(J/\psi)}$ recoil spectrum in $e^+e^\to J/\psi(c\bar{c})$. The X(3940) mass and width are $(3942 \pm 7 \pm 6) \text{ MeV}$ and $\Gamma = (37 \pm 2 \pm 8) \text{ MeV}$, and was observed to decay into $D^*\bar{D}$. The X(4160) was found to decay dominantly into $D^*\bar{D}$ with parameters given as $M = (4156^{+15}_{-20} \pm 15) \text{ MeV}$ and $\Gamma = (139^{+11}_{-6} \pm 21) \text{ MeV}$. The only other known charmonium state observed in $e^+e^\to J/\psi X$ process has $J = 0$. This fact and the absence of these states in the $DD$ decay suggests that these states favor $J^{PC} = 0^{++}$. However, these states are either too low or too high to be the $\eta_c^{(')}$ or the $\eta_c^{('')}$ states [199].

The X(4160) was identified as a $2^{++}$ state in two recent works: generated from a relativistic four quark equations in ref. [200], and from a coupled channel approach using a vector-vector interaction in ref. [177]. In ref. [201] two assignments were found to be possible: $\eta_c(4S)$ and the P-wave excited state $\chi_{c0}(3P)$. So far, no other serious theoretical attempts were performed to investigate these states, and the nature of these states remains a puzzle.

The Z(3930) was observed by Belle collaboration as an enhancement in the $\gamma\gamma \rightarrow D\bar{D}$ event. The observed mass and width are $M = 3929 \pm 5 \pm 2 \text{ MeV}$ and $\Gamma = 29 \pm 10 \pm 2 \text{ MeV}$. The measured properties are consistent with expectations for the previously unseen $\chi'_{c2}$ charmonium state [202, 84].

The Y(4008) was seen by the Belle collaboration as a by-product while confirming the Y(4260) in a two resonance fit to the $e^+e^\to \pi^+\pi^- J/\psi$ reaction [6]. There are theoretical speculations that this state might be the $\psi(3S)$ or is a $D^*\bar{D}^*$ molecular state [203, 176]. However, BaBar could not so far confirm experimentally its existence [32].

10. Other multiquark states

10.1. A $D_sD^*$ molecular state

If the mesons $X(3872)$, $Z^*(4430)$, $Y(4260)$ and $Z^*_2(4250)$ are really molecular states, then many other molecules should exist. A systematic study of these molecular states and their experimental observation would confirm its structure and provide a new testing ground for QCD within multiquark configurations. In this context, a natural extension would be to probe the strangeness sector. In particular, in analogy with the meson $X(3872)$, a $D_sD^*$ molecule with $J^P = 1^+$ could be formed in the $B$ meson decay $B \rightarrow \pi X \rightarrow J/\psi K\pi$. Since it would decay into $J/\psi K^* \rightarrow J/\psi K\pi$, it could be easily reconstructed.
In ref. [122] the QCD sum rules approach was used to predict the mass of the $D_sD^*$ molecular state. Such prediction is of particular importance for new upcoming experiments which can investigate with much higher precision the charmonium energy regime, like the PANDA experiment at the antiproton-proton facility at FAIR, or a possible Super-B factory experiment. Especially PANDA can do a careful scan of the various thresholds being present, in addition to going through the exact form of the resonance curve.

The current used in ref. [122] is very similar to the current in Eq. (54) for the X(3872):

$$j_\mu = \frac{1}{\sqrt{2}} \left[ (\bar{s}_a \gamma_5 c_d)(\bar{c}_d \gamma_{\mu} d_b) - (\bar{s}_a \gamma_{\mu} c_d)(\bar{c}_d \gamma_5 d_b) \right] .$$  

The mass obtained for this state is [122]:

$$M_{D_sD^*} = (3.96 \pm 0.10) \text{ GeV},$$  

which is around 100 MeV bigger than the mass of the X(3872) meson and below the $D_sD^*$ threshold ($M(D_sD^*) = 3980$ MeV).

\[ \text{Figure 31: The } D_sD^* \text{ meson mass as a function of the sum rule parameter } (M') \text{ for } \sqrt{\tilde{M}} = 4.4 \text{ GeV (dashed line), } \sqrt{\tilde{M}} = 4.5 \text{ GeV (solid line) and } \sqrt{\tilde{M}} = 4.6 \text{ GeV (dot-dashed line). The crosses indicate the upper limit in the Borel region allowed by the dominance of the QCD pole contribution (taken from ref. 122).} \]

The Borel curve for the mass of such state is quite stable, as can be seen in Fig. 31 and it has a minimum within the relevant Borel window. Such a stable Borel curve strongly suggests the possibility of the existence of a $D_sD^*$ molecular state with $J^P = 1^+$. Of course if such state exists, there will be no reason why its isospin partners would not exist as well. Therefore, charged states with the quark content $c\bar{c}s\bar{u}$ and $c\bar{c}u\bar{s}$, and with a mass given in Eq. (142), should also be observed in the decay channels: $J/\psi K^+$ and $J/\psi K^-$.  

### 10.2. A $[cc][\bar{u}\bar{d}]$ state

Considering that the double-charmonium production was already observed in the reaction 3

$$e^+e^- \rightarrow J/\psi + X(3940),$$

it seems that it would be possible to form the tetraquark $[cc][\bar{u}\bar{d}]$. Such state with quantum numbers $I = 0$, $J = 1$ and $P = +1$ which, following ref. [203], we call $T_{cc}$, is especially interesting. Although the process in Eq. (143) will create two $c\bar{c}$ pairs, the production of $T_{cc}$ will further involve combining two light anti-quarks with $cc$ and might be suppressed. On the other hand, heavy ion collisions at LHC might provide another opportunity for its non-trivial production [205].

As already noted previously [204, 206], the $T_{cc}$ state cannot decay strongly or electromagnetically into two $D$ mesons in the $S$ wave due to angular momentum conservation nor in $P$ wave due to parity conservation. If its mass is below the $DD^*$ threshold, this decay is also forbidden, and this state would be very narrow.

Considering $T_{cc}$ as an axial diquark-antidiquark state, a possible current describing it is given by:

$$j_\mu = \frac{i}{2} \epsilon_{abc} \epsilon_{dec}(\bar{c}_d \gamma_{\mu} c_b)(\bar{u}_a \gamma_5 \gamma^T c_d) = \frac{i}{4} \epsilon_{abc} \gamma_\mu c_b[\bar{u}_a \gamma_5 \gamma^T c_d].$$

This current represents well the most attractive configuration expected with two heavy quarks. This is so because the most attractive light antidiquark is expected to be the one in the color triplet, flavor anti-symmetric and spin 0 channel [207, 208, 209]. This is also expected quite naturally from the color magnetic interaction, which can be phenomenologically parameterized as,

$$V_{ij} = \frac{C_H}{m_i m_j} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j.$$  

Here, $m, \lambda, \sigma$ are the mass, color and spin of the constituent quark $i, j$. Depending on the color state, the color factor $\lambda_i \cdot \lambda_j$ is $-\frac{3}{4}, \frac{1}{4}, -\frac{1}{2}$ for quark quark in the color state $\bar{3}, 6$ and for quark-antiquark in the singlet state respectively. The phenomenological value of $C_H$ can be estimated from fits to the baryon and meson mass splitting, from which one finds that two constants $C_M$ and $C_B$ can fit all the mass splitting within the mesons and the baryons respectively. Also, one finds that $C_M$ is about a factor of 3/2 larger than $C_B$ [205, 210]. Therefore, favorable multiquark configuration will inevitably involve diquark configuration with spin zero in color $\bar{3}$; this is a scalar diquark with anti symmetric flavor combination due to anti-symmetry of the total wave function involving color, spin and flavor. The tetraquark picture for the light scalar nonet involves states composed of two scalar diquarks. However, here, one notes that the two pseudo scalar mesons will have a smaller mass and the tetraquark state not stable against strong decay. Possible stable configurations emerge if either the diquark or the anti-diquark in the tetraquark is composed of heavy quarks; this is so because the large attraction between quark-antiquark will be suppressed by the heavy quark mass while the strong attraction in either the diquark or the anti-diquark will remain. Simple estimates within a constituent quark model predict stable configurations with spin zero, where a light diquark combines with a scalar (cb) diquark, and with spin one, where a light diquark combines with spin 1 heavy diquark [210]. The $T_{cc}$ falls into the latter case where heavy anti-diquark is a color triplet $cc$ with...
spins 1, as the pair should combine antisymmetrically. Although the spin 1 configuration is repulsive, its strength is much smaller than that of the light diquark due to the heavy charm quark mass. Therefore a constituent quark picture for $T_{cc}$ would be a light anti-diquark in color triplet, flavor anti-symmetric and spin 0 ($\epsilon_{abc}[\bar{t}_a\gamma_5 C \bar{c}_b]_3$) combined with a heavy diquark of spin 1 ($\epsilon_{abc}[\bar{c}_a C \gamma_\mu \epsilon_b]_3$). The simplest choice for the current which has a non zero overlap with such a $T_{cc}$ configuration is given in Eq. (144). While a similar configuration $T_{ss}$ is also possible [211], we believe that the repulsion in the strange diquark with spin 1 will be larger and hence energetically less favorable.

There are some predictions for the masses of the $T_{QQ}$ states. In ref. [212], the authors use a color-magnetic interaction, with flavor symmetry breaking corrections, to study heavy tetraquarks. They assume that the Belle resonance, $X(3872)$, is a $c\bar{c}q\bar{q}$ tetraquark, and use its mass as input to determine the mass of other tetraquark states. They get $M_{T_{cc}} = 3966$ MeV and $M_{T_{bb}} = 10372$ MeV. In ref. [204], the authors use one-gluon exchange potentials and two different spatial configurations to study the mesons $T_{cc}$ and $T_{bb}$. They get $M_{T_{cc}} = 3876 - 3905$ MeV and $M_{T_{bb}} = 10519 - 10651$ MeV. There are also calculations using expansion in the harmonic oscillator basis [213], and variational method [214]. In ref. [210], the authors use Eq. (155) together with a simple diquark picture to find stable heavy tetraquark configuration with spin zero $T_{bb}$ and spin one $T_{cc}, T_{cb}, T_{bc}$.

In ref. [215], a QCD sum rule calculation was done with the current in Eq. (144). The authors found that the results are not very sensitive to the value of the charm quark mass, neither to the value of the condensates. The QCDSR predictions for the $T_{cc}$ mesons mass is:

$$M_{T_{cc}} = (4.0 \pm 0.2) \text{ GeV},$$  

in a very good agreement with the predictions based on the one gluon exchange potential model [204], and color-magnetic model [212].

It is straightforward to extend the analysis done for the $T_{cc}$ to the bottom sector. The prediction for the $T_{bb}$ mass from ref. [215] is:

$$M_{T_{bb}} = (10.2 \pm 0.3) \text{ GeV},$$  

also in a very good agreement with the results in refs. [204], [212] and [214].

The results from ref. [215] show that while the $T_{cc}$ mass is bigger than the $D^*D$ threshold at about 3.875 GeV, the $T_{bb}$ mass is appreciably below the $B^*B$ threshold at about 10.6 GeV. Therefore, these results indicate that the $T_{bb}$ meson should be stable with respect to strong interactions and must decay weakly. These results also confirm the naive expectation that the exotic states with heavy quarks tend to be more stable than the corresponding light states [216].

11. Summary

We have presented a review, from the perspective of QCD sum rules, of the new charmonium states recently observed by BaBar and Belle Collaborations. As it was seen case by case, this method has contributed a great deal to the understanding of the structure of these new states. When a state is first observed and its existence still needs confirmation, a QCDSR calculation can be very useful. It can provide evidence in favor or against the existence of the state. We have computed the masses of several $X, Y$ and $Z$ states and they were supported by QCDSR. In some cases a tetraquark configuration was favored and in some other cases a molecular configuration was favored. QCDSR does not always corroborate previous indications. In Ref. [217], a comprehensive QCD analysis of light tetraquarks led to the conclusion that their existence (as tetraquarks states) is very unlikely. However, a more positive conclusion was found in the case of tetraquarks with at least one heavy quark.

The limitations in statements made with QCDSR estimates come from uncertainties in the method. However these statements can be made progressively more precise as we know more experimental information about the state in question. One good example is the $X(3872)$, from which, besides the mass, several decay modes were measured. Combining all the available information and using QCDSR to calculate the observed decay widths, we were able to say that the $X(3872)$ is a mixed state, where the most important component is a $c\bar{c}$ pair, which is mixed with a small molecular component, out of which a large fraction is composed by neutral $D$ and $D^*$ mesons with only a tiny fraction of charged $D$ and $D^*$ mesons. This conclusion is very specific and precise and it is more elaborated than the other results presented addressing only the masses of the new charmonia. This improvement was a consequence of studying simultaneously the mass and the decay width. This type of combined calculation will eventually be extended to all states.

We close this review with some conclusions from the results presented in the previous sections. They are contained in Tables 5 and 6.

In Table 5 we present a summary of the most plausible interpretations for some of the states presented in Table 1 described in the previous sections.

| State       | Structure                      | $J^P$  |
|-------------|--------------------------------|--------|
| $X(3872)$   | mixed $c\bar{c} - D\bar{D}^*$  | $1^+$  |
| $Y(4140)$   | $D_s^*\bar{D}_s$ molecule     | $0^+$  |
| $Z(4250)$   | $D_1\bar{D}_1$ molecule       | $1^-$  |
| $\bar{Y}(4620)$ | $D_0 \bar{D}$ or $D\bar{D}_1$ molecule | $1^-$  |
| $Z(4430)$   | $D^*\bar{D}_1$ molecule       | $0^-$  |
| $Y(4660)$   | $[c\bar{s}][\bar{c}\bar{s}]$ tetraquark | $1^-$  |

In Table 6 we present some predictions for bottomonium states, that may be observed at LHCb.

From the last Table we see that the mass predictions for the states $X_0$ and $T_{bb}$ are basically the same. These states, if they exist, may be produced at LHCb. Since the $T_{bb}$ is a charged state, it could be more easily identified. Moreover, since its mass prediction lies below the $B^*B$ threshold, it should be very narrow because it could not decay strongly.
| State     | Structure                  | $J^P$ | Mass (GeV) |
|-----------|----------------------------|-------|------------|
| $X_b$     | $[bq][-b\bar{q}]$ tetraquark or $B^0$ molecule | $1^{++}$ | $10.27 \pm 0.23$ |
| $Z_b$     | $[bq][-b\bar{q}]$ tetraquark or $B^*_{1}$ molecule | $0^-$ | $10.74 \pm 0.12$ |
| $T_{bb}$  | $[bb][-u\bar{d}]$ tetraquark $B^0$ molecule | $1^{++}$ | $10.2 \pm 0.3$ |

Looking at Table 6, we see that the predictions made with tetraquark and molecular currents are the same. Probably, in order to distinguish one from the other, we will need to know the decay width of these states. On the other hand, from the point of view of measurement, this convergence of predictions makes them robust. In other words, even not knowing very precisely their structure, we know that these states must be there.

Table 6 represents the final result of a comprehensive effort and a careful analysis of several theoretical possibilities in the light of existing data. It is an encouraging example of what QCDSR can do. This Table contains a short summary of what we have learned about the new charmonium states in the recent past. Table 6 addresses the near future and shows some predictions.

From what was said above, hadron spectroscopy is and will be a very lively field and QCDSR is a powerful tool to study it.

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### 12. Appendix: Fierz Transformation

In general, the currents constructed from diquark antidiquark type of fields are related to those composed of meson type of fields. However, the relation is suppressed by a typical color and Dirac factors so that we obtain a reliable sum rule only if we have chosen the current well to have a maximum overlap with the physical meson. This is expected to be particularly true for multiquark configuration with special diquark or molecular structures. Therefore, if we obtain a sum rule that reproduces the physical mass well, we can infer the inner structure of the multiquark configuration. For example, if the state is of molecular type, it will have a maximum overlap with a current that is constructed with the two corresponding meson current; the sum rule will then be able to reproduce the physical mass well. On the other hand, if a current with a diquark-antidiquark type of current is used, the overlap to the physical meson will be small and the sum rule will not be able to reproduce the mass well. The opposite will also be true.

To show the suppression, we discuss Fierz transformation of tetraquark current made of diquark-antidiquark fields. The starting relation is given as follows.

$$q_{a}^{'} q_{b}^{'} = \frac{1}{8} \delta_{ab}^{T} \left[ \frac{1}{2} \delta_{ij} (\bar{q}_j \gamma_\mu q_i \Lambda^\mu) + \gamma^\alpha (\bar{q}_j \gamma_\mu \gamma^\alpha \Lambda^\mu) \right]$$

Here, $a, b$ are the color and $i, j$ the Dirac spinor indices respectively. The two color representations come from $3 \otimes \bar{3} = 8 \oplus 1$.

In the formulas to follow, the two color terms can be worked out using the following formulas,

$$e_{abc} e_{dec} (q_{a}^{T} \gamma_{\mu} q_{b}^{T} \gamma_{\nu} q_{c}^{T} \gamma_{\tau} q_{d}^{T}) (q_{d}^{T} \gamma_{\nu} q_{c}^{T} \gamma_{\tau} q_{b}^{T} \gamma_{\mu} q_{a}^{T}) = -2 (\bar{q} \gamma_{\mu} \gamma_{\tau} q) (q \gamma_{\tau} \gamma_{\nu} q)$$

We start by applying the formula to the current composed of two scalar diquarks. After Fierz transformation, we find,

$$j^{\mu} = e_{abc} e_{dec} (q_{a}^{T} C \gamma_{5} \gamma_{\mu} C q_{b}^{T} C \gamma_{5} \gamma_{\eta} C q_{c}^{T} C \gamma_{5} \gamma_{\tau} C q_{d}^{T} C \gamma_{5} \gamma_{\nu} C q_{c}^{T} C \gamma_{5} \gamma_{\tau} C q_{b}^{T} C \gamma_{5} \gamma_{\nu} C q_{a}^{T} C \gamma_{5} \gamma_{\mu} C q_{d}^{T} C \gamma_{5} \gamma_{\tau} C q_{c}^{T} C \gamma_{5} \gamma_{\nu} C q_{b}^{T} C \gamma_{5} \gamma_{\mu} C q_{a}^{T} C \gamma_{5} \gamma_{\mu} C q_{d}^{T})$$

This is a typical example of the expansion. The factor of 1/6 multiplying the color singlet quark-antiquark types of currents, comes from the color and Dirac factors and is responsible for the small overlap to the various quark-antiquark type of currents. In Eq. (150), all meson type of currents contributes. In general however, depending on the current type and charge conjugation, only certain types of meson-meson currents contributes. Several examples for the expansion used in the text are given below.
1. For $X(3872)$ with $J^{PC} = 1^{++}$, we find,

$$f^{(q,di)}_{\mu} = \frac{i e_{abc} e_{dce}}{\sqrt{2}} [q_{a}^{T} C_{\gamma} q_{b}] \langle \bar{q}_{d} \gamma_{\mu} C \bar{e}_{c}^{T} \rangle$$

$$+ \langle \bar{q}_{d} C_{\gamma} q_{b} \rangle \langle \bar{q}_{d} \gamma_{\mu} C \bar{e}_{c}^{T} \rangle]$$

$$= \frac{i}{\sqrt{2}} (-1)^{j} \sum_{j} \left[ \langle \bar{c} \gamma_{\mu} C \bar{\gamma}_{j} \rangle \langle \bar{q}_{d} \gamma_{\mu} C \bar{e}_{c}^{T} \rangle \right]$$

As can be seen from the above equation, the molecular component as given in $f^{(q,med)}_{\mu} = \sum_{\lambda} \langle \bar{c} \gamma_{\mu} q \rangle \langle \bar{q}_{\lambda} \gamma_{\mu} C \bar{e}_{c}^{T} \rangle$ comprises only a small part of the wave function given in Eq. (151). The overlap to the molecular type is suppressed by $\frac{1}{\sqrt{2}}$ so that the overall contribution to the correlation function is suppressed by $\frac{1}{2}$.  

2. For $Z(4430)$

- with $J^{P} = 0^{-}$, we find the following current works well.

$$f^{(di)}_{\lambda_{0^{-}}} = \frac{i e_{abc} e_{dce}}{\sqrt{2}} [u_{a}^{T} C_{\gamma} q_{b}] \langle \bar{d}_{a} \gamma_{\mu} C \bar{e}_{c}^{T} \rangle$$

$$- \langle u_{a}^{T} C_{\gamma} q_{b} \rangle \langle \bar{d}_{a} \gamma_{\mu} C \bar{e}_{c}^{T} \rangle]$$

$$= \frac{i}{\sqrt{2}} (-1)^{j} \sum_{j} \left[ \langle \bar{c} \gamma_{\mu} C \bar{\gamma}_{j} \rangle \langle \bar{d}_{a} \gamma_{\mu} C \bar{e}_{c}^{T} \rangle \right]$$

As can be seen from the above equation, the molecular component as given in Eq. (119), contributes with a suppression factor of $1/3$. It is useful to compare this factor to the ratio of the overlaps found in QCD sum rule analysis and given in Eqs. (123) and (124):

numerically, the ratio $\lambda_{Z_{0}}/\lambda_{D_{s}} = 0.66$. On the other hand, assuming that the $Z(4430)$ has only a molecular component that couples dominantly to the molecular current, the ratio should be $1/3$, as can be seen from Eq. (152). The fact that it is larger than $1/3$ suggest that while the $Z(4430)$ is dominantly of molecular type, there still remains some coupling to other type of non-molecular type of current component, such as the diquark-antidiquark type.

- with $J^{P} = 1^{-}$, we find the following current does not fit any of the observed $Z$ states.

$$j_{\mu} = \frac{i e_{abc} e_{dce}}{\sqrt{2}} [u_{a}^{T} C_{\gamma} q_{b}] \langle \bar{d}_{a} \gamma_{\mu} C \bar{e}_{c}^{T} \rangle$$

$$- \langle u_{a}^{T} C_{\gamma} q_{b} \rangle \langle \bar{d}_{a} \gamma_{\mu} C \bar{e}_{c}^{T} \rangle]$$

$$= \frac{i}{\sqrt{2}} (-1)^{j} \sum_{j} \left[ \langle \bar{c} \gamma_{\mu} C \bar{\gamma}_{j} \rangle \langle \bar{d}_{a} \gamma_{\mu} C \bar{e}_{c}^{T} \rangle \right]$$

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