The influence of the equivalent hydraulic diameter on the pressure drop prediction of annular test section

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Abstract. The flow behaviour and the pressure drop throughout an annular flow test section was investigated in order to evaluate and justify the reliability of experimental flow loop for wax deposition studies. The specific objective of the present paper is to assess and highlight the influence of the equivalent diameter method on the analysis of the hydrodynamic behaviour of the flow and the pressure drop throughout the annular test section. The test section has annular shape of 3 m length with three flow passages, namely; outer thermal control jacket, oil annular flow and inner pipe flow of a coolant. The oil annular flow has internal and external diameters of 0.0422 m and 0.0801 m, respectively. Oil was re-circulated in the annular passage while a cold water-glycol mixture was re-circulated in the inner pipe counter currently to the oil flow. The experiments were carried out at oil Reynolds number range of 2000 to 17000, covering laminar, transition and turbulent flow regimes. Four different methods of equivalent diameter of the annulus have been considered in this hydraulic analysis. The correction factor model for frictional pressure drop was also considered in the investigations. All methods addressed the high deviation of the prediction from the experimental data, which justified the need of a suitable pressure prediction correlation for the annular test section. The conventional hydraulic diameter method is a convenient substitute for characterizing physical dimension of a non-circular duct, and it leads to fairly good correlation between turbulent fluid flow and heat transfer characteristic of annular ducts.

1. Introduction
A commonly accepted approach to extend the pipe flow equations to annular geometry is by modification of the diameter. The equations that are used in the pipe flow to calculate the friction factor (FF) and Reynolds number (Re) are being used in annular flow by replacing the diameter, as the characteristic geometry in pipe flow equations, by the effective diameter. Different effective diameter definitions have been proposed in the literature both for Newtonian and non-Newtonian fluids. Re and FF relationship is unique to a given effective diameter definition, De. Different values of frictional pressure drop might be obtained according to the De used in characteristic geometry. Several studies have also been proposed to modify the Re definition with shape factor while continuing to use the classical approach of equivalent diameter. Since the results from these studies have significant deviations from each other, the selection of appropriate correlation for the respective fluid and flow regime becomes very important for the accurate estimation of pressure losses in annuli Lawn and Elliott [1].

Dosunmu and Shah [2] developed a turbulent friction factor correlation that includes wall roughness for straight pipe and annular flow. A systematic approach is adopted in which pipe and
annular pressures loss data for two fluids (xanthan gum and guar gum) are used. The first pressure loss data with xanthan gum have been used for correlation development while the second set of data with guar gum have been used for the correlation evaluation. They recommendation that the hydraulic diameter definition of equivalent diameter for an eccentric annulus provided the best annular definition for friction pressure prediction.

In addition to the hydraulic diameter customarily used and accepted as “equivalent” diameter for the annular concentric ducts, there are many other equivalent diameter correlations suggested in the literature. In the past, annuli have been evaluated on the basis of hydraulic diameter as equivalent diameter, but this may not always be the best way to represent the dimension for flow in an annulus. Lawn and Elliott [1], and Jones and Leung [3] recommended the need of further studies to explore the influence of different equivalent diameters in the annular flows. Anifowoshe and Osisanya [4] tested seven different equivalent diameters definitions to estimate their effect on the pressure losses in wellbore hydraulics. The predicted results have been compared with experimental data to select the best model to fit the pressure losses prediction of power low models. The concluded that the pressure losses estimation is significantly affected by the equivalent diameter model. They suggested that the hydraulic diameter definition provided the best estimation of the pressure losses for the power law fluid in laminar flow conditions.

The objective of the present paper is to demonstrate and discuss the pressure drop results, which are predicted theoretically and measured experimentally, for the annulus oil flow using various equivalent diameter models. This effort is part of a hydraulic characterization and correction of unjustified flow loop for wax deposition studies, and to introduce a correction method to adjust the pressure drop prediction. In addition to the commonly used hydraulic diameter model, another three models have been checked, and the pressure drop correction factor model. The investigated region covers all the flow regimes, laminar, transition, and turbulent with Re ranging from 2000 to 17000. The results assisted in understanding the influencing of the equivalent diameter on the prediction of the pressure drop in the deposition section.

2. Experimental apparatus
The experimental set-up used for this study, shown in figure 1, is a high pressure/high temperature (HPHT) flow loop, developed for wax deposition research of waxy crudes. The main test section has an annular flow shape of 3 m long. The test section is fully jacketed with hot glycol-water mixture recirculated in the heating system of the HPHT flow loop from the hot fluid reservoir. While a cold glycol-water mixture is re-circulated in the central pipe and counter-currently to the oil flowing in the annular space. The aim of the cold mixture is to provide a cold conditions for the flowing oil to impose the waxing deposition.

![Figure 1. Schematic of the wax deposition test loop in the HPHT flow loop.](image-url)
The oil is re-circulated through the experimental flow loop by 3” × 3 stage gear pump. The flowrate and density of oil are measured by MICROMOTION flow meter installed downstream the pump. The oil enters and leaves the test section vertically from 2 inch pipe.

A Rosemount differential pressure transducer is installed across the oil section to measure the pressure drop. The inlet of the oil section is equipped by pressure transmitter. The heating jacket temperature, oil temperature and cold glycol-water mixture temperature are monitored at the inlet and outlet of the test section by type-T (copper-constantan) thermocouple probes.

3. Equivalent diameter approaches for non-circular pipe flow

Prediction of Re, FF, and the pressure loses, Δp may differ according to the use of different De. In this contest, several studies have also been proposed to modify the Re definition shape factor while continuing to use the classical approach of equivalent diameter. Since the results from these studies have significant deviations from each other, the selection of appropriate correlation for the respective fluid and flow regime becomes very important for accurate estimation of pressure losses in annuli, Lawn and Elliott [1].

2.1. The Hydraulic Diameter Method, Dh

The common representation of diameter for flow in the non-circular conduits is the hydraulic diameter defined as:

\[ D_h = \frac{4A}{P} \]  

where \( A \) is the flow area and \( P \) is the perimeter. For annular configuration, \( D_h \) becomes,

\[ D_h = d_2 - d_1 \]

In addition to the hydraulic diameter customarily given by equation (1) and accepted as “equivalent” diameter for the annular concentric ducts [5], there are many equivalent diameter correlations also suggested in the literature. These correlations have been checked against the hydraulic diameter to extend the criteria of the pressure drop characterization. By defining the inner and outer diameters of the annulus by \( d_1 \) and \( d_2 \), respectively, the three methods defining the annulus equivalent diameter are as below.

2.1.1. The Petroleum Diameter Method, De, p.

One of the suggested ways of calculating the pressure drop of a non-circular, concentric-annular flow cross-section named “The Petroleum Engineering Method” uses equation (2) for evaluating an equivalent calculation diameter to replace the circular pipe diameter Bertuzzi F [6].

\[ D_{e,p} = \sqrt[3]{\left(\frac{d_2 + d_1}{2}\right) \times \left(\frac{d_2 - d_1}{2}\right)^3} \]  

2.1.2. The Equivalent Area Diameter Method, De, a.

Equation (3), simply, is assuming an equivalent diameter of a circular pipe having the same flow area. The “equivalent area” diameter for annulus is calculated as:

\[ D_{e,a} = \left(\frac{d_2^2 - d_1^2}{d_2 + d_1}\right)^{1/2} \]
2.1.3. The Equivalent Geometry Diameter Method, $D_{eq}$

Another criteria used to obtain an equivalent circular diameter from comparing the geometry in the pressure-loss equations for pipe flow and concentric annulus flow of Newtonian fluid terms is the geometry terms is the “equivalent geometry” diameter given by Bertuzzi F [6]:

$$D_{eq} = \sqrt{d_2^2 + d_1^2 - \frac{d_2^2 - d_1^2}{\ln(d_2/d_1)}}$$  \hspace{1cm} (4)

2.2. The “Correction Factor” Model, $k_{non-c}$

Another method for predicting the frictional pressure drop for flow in a concentric annular conduits (long pipe stabilized flow assumption) suggests the use of a correction coefficient for non-circular passages, $k_{non-c}$. This factor is to be separately evaluated for laminar and turbulent flow conditions and applied to the frictional pressure drop calculations with the aid of a circular pipe standard correlations but having a diameter equal to the hydraulic diameter, $(D_h = d_2 - d_1)$ of annular conduit Idelchik and Fried [7]. For a rounded annular tube, the correction factor, which is a function of the diameter ratio, can be found for $(Re \leq 2000)$ from:

$$k_{non-c, lam} = \frac{1 - \left(\frac{d_1}{d_2}\right)^2}{1 + \left(\frac{d_1}{d_2}\right)^2 \left[1 - \left(\frac{d_1}{d_2}\right)^2 \ln \left(\frac{d_1}{d_2}\right)^2\right]}$$  \hspace{1cm} (5)

Equation (6), obtained by Jones & Leung (3), is also proposed to predict the correction factor for the laminar flow in the annular passage, as:

$$k_{non-c, lam} = \frac{(d_2 - d_1)^2 \cdot (d_2 - d_1)^2}{(d_2^2 - d_1^2) \cdot (d_2^2 - d_1^2)^2 / \ln(d_1/d_2)}$$  \hspace{1cm} (6)

Both equation (5) and (6) are not a function of Re, they are only function of the diameters of the annular flow conduits.

In the case of turbulent flow, $k_{non-c}$ depends only slightly on the diameter ratio and lies in the range 1.0 – 1.07. The correction factor of such a tube can be calculated from the following formula.

$$k_{non-c, turb} = f \left(\frac{0.02 d_1}{d_2} + 0.98 \left(\frac{1}{f} - 0.27 \frac{d_1}{d_2} + 0.1\right)\right)$$  \hspace{1cm} (7)

where, $d_1$ and $d_2$ are the inner and outer diameters of the annulus passage.

According to the handbook of Hydraulic Resistance [7], the actual frictional pressure drop in a stabilized “long pipe assumption” flow in an annular conduits should be calculated as a product of a correction coefficient and the frictional pressure drop calculated for the hydraulic diameter of the annular conduits.
4. Results and discussion

In order to decide the appropriate model of equivalent diameter, the predicted pressure drop in the test section is compared to the measured pressure drop. A Rosemount differential pressure transducer is installed across the oil section to monitor the pressure drop. The pressure drop is predicted using Darcy equation assuming Newtonian, incompressible and steady flow in the annulus,

\[ \Delta P = f \frac{L}{D_e} \frac{\rho U^2}{2} \]  

(8)

Darcy friction factor, \( f \) is estimated using Churchill’s equation. The Churchill equation combines expressions for friction factor in both, the laminar and the turbulent flow regimes. It is accurate to within the error of the data used to construct the Moody diagram. This equation also provides an estimate for the transition region. The Churchill equation shows very good agreement with the Darcy equation for laminar flow. In the turbulent regime a difference of around 0.5-2% is observed between the Churchill equation and the Colebrook equation, Co [8].

\[ f = 8 \left[ \frac{8}{(Re)^{0.5}} + \frac{1}{(A + B)^{1.5}} \right]^{1/2} \]

\[ A = \left[ 2.45 \ln \left( \frac{7}{(Re)^{0.9}} + 0.27 \frac{e}{D_e} \right) \right]^{16} \]

\[ B = \left( \frac{37530}{Re} \right)^{16} \]

(9)

Re is based on the relevant equivalent diameter, and \( e \) is the surface roughness. Knowing that the annulus inner and outer diameters are 0.0422 and 0.0801 m, respectively, the values of the equivalent diameters, \( D_e \) are obtained from the four models using the set of equation (1) to (4). The calculated values of the equivalent diameters are shown in table 1.

4.1. Results of the Pressure Drop using Equivalent Diameter Models

Figure 2 displays the measured and predicted pressure drop across the annulus versus Reynolds number. The actual Re calculated from the measurement in the annulus is ranging from 3000 to 9400. The predicted ranges of Re are extended to 17000 to examine the capacity of each model in simulating the turbulence regime. The results of pressure drop obtained by using the various equivalent diameter models are compared with the measured results. The equivalent diameter has significant effect on the turbulent flow regime length and the pressure drop. Different trends of the pressure drop could be observed for different equivalent diameters used in the prediction. In case of the equivalent area method, which has the lowest values of the pressure drop estimation, the turbulence region is over estimated compared to the realistic estimation. The length of the turbulent flow regime decrease with the increase of the value of the equivalent diameter. For this point, care must be taken in the selection of the appropriate equivalent diameter which can give a good representation for the flow phenomenon in the annular passage. As it is clear in the figure 2, all the models are underestimated the pressure drop compared to the measured values. In fact, deeper investigation has demonstrated that the models are correct if the losses at the inlet and outlet of the annulus test section are taken into consideration. This will be elaborated further in the coming sections.
Figure 2. The effect of the various equivalent diameter models on the prediction of the pressure drop at various turbulent regimes.

Figure 3 shows the regression analysis for the calculated and measured pressure drop using different equivalent diameter models. The “equivalent area” model shows the biggest deviation of 96% between the measured and calculated pressure drop, and the values of the measured pressure drop are 31 times the calculated values with standard error of 3.8. The “equivalent geometry” model showed the smallest deviation of 91% and the measured pressure drop is 11 times the calculated pressure drop with standard error of 3.99.

Figure 3. Calculated versus measured pressure drop and linear regressions; the effect of various equivalent diameter models used for the annulus.
4.2. Results of the Pressure Drop using Correction Factor Model

For the deposition apparatus under consideration in this study, the correction factor for laminar flow, $k_{\text{non-c, lam}}$, according to equation (8), is equal to 1.49. The same value is obtained by using equation (9). For turbulent flow, the correction coefficient, $k_{\text{non-c, turb}}$, is dependent on Re number. This dependency is found to be weak for Re ranging from 2000 to 10000, $k_{\text{non-c, turb}} = 1.058 - 1.060$. This is because of the presence of the transition zone and low turbulence level in this range of Re. Figure 4 illustrates the correction factor effect on the calculation of the pressure drop. The use of Jones & Leung (3) correction factor reduces the root mean square error between the measured and calculated values by 2%, while the standard deviation from the measured values remains the same about 44.33 with or without J-L correction. Therefore the correction factor produces insignificant (positive) effect for improving the calculation routine. However, the correction factor method should be considered for “polishing” calculation results when the hydraulic diameter approximation is used, (3, 5, 7).

![Figure 4. The effect of the correction factor on the calculation of the pressure drop.](image)

Although the equivalent geometry model, equation (4), slightly reduces the discrepancy between the measured and calculated pressure drop results compared to other equivalent diameters, the hydraulic diameter will be used as standard for the calculations and further analysis. This decision is justified by the following reasons:

- The hydraulic diameter is a convenient substitute for the characteristic physical dimension of a non-circular duct, and it leads to fairly good correlation between turbulent fluid flow and heat transfer characteristic of circular and noncircular ducts. The hydraulic diameter is also used for ducts involving laminar flow to provide a consistent basis of comparison with turbulent flow results (1, 3, 4). However, for laminar flow itself, this quantity does not lead to satisfactory correlations between circular and noncircular ducts, Idelchik et al. (7).
- Using a general correlation for the equivalent length in not advised as it demonstrated that the equivalent length is not sufficient for accurate description of the observed behavior. No existing correlation or method has yet found with general acceptance Jones and Leung (3).

5. Conclusion

Annular flow test section, designed originally for wax deposition studies, is investigated experimentally and theoretically to introduce acceptable hydraulic simulation model. Four models, and also the pressure drop correction factor model, have been investigated. The followings are concluded.

- The equivalent diameter models have significant influence on the estimation results of the pressure drop in annuli.
The four models of the equivalent diameters studied in the present work show different trend in estimating the turbulence regions in the annulus.

- The equivalent geometry diameter model under-estimates the turbulence level, while the other models over-estimate the turbulence level.
- The hydraulic diameter is reasonably estimated the turbulence level in the annulus. However, the theoretically obtained results of the frictional pressure losses have shown large deviation compared to the measurement results. Accordingly, a correction method is necessary to reduce the mathematical simulation error of the frictional pressure losses of oil flow in the annular test section in PRSB HPHT flow loop.

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