Defect Spirograph: Dynamical Behavior of Defects in Spatially Patterned Active Nematics

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Topological defects in active liquid crystals can be confined by introducing gradients of activity. Here, we examine the dynamical behavior of two defects confined by a sharp gradient of activity that separates an active circular region and a surrounding passive nematic material. Continuum simulations are used to explain how the interplay among energy injection into the system, hydrodynamic interactions, and frictional forces governs the dynamics of topologically required self-propelling +1/2 defects. Our findings are rationalized in terms of a phase diagram for the dynamical response of defects in terms of activity and frictional damping strength. Different regions of the underlying phase diagram correspond to distinct dynamical modes, namely immobile defects (ID), steady rotation of defects (SR), bouncing defects (TB), bouncing-cruising defects (BC), dancing defects (DA), and multiple defects with irregular dynamics (MD). These dynamic states raise the prospect of generating synchronized defect arrays for microfluidic applications.

Active nematics represent a class of non-equilibrium active systems with broken rotational symmetry, where orientational ordering, elastic stresses, active stresses, and hydrodynamic forces, interact to produce unique structures and dynamical behaviors [1–7]. Active nematics have drawn considerable interest, and a general understanding of their behavior has begun to emerge from observations across a wide range of experimental systems that include in vitro assemblies of filamentous proteins [8–11], dense suspensions of elongated bacteria [12, 13], living nematics [14], and dense colonies of elongated cellular tissues [15–18]. These experiments have been accompanied by theoretical studies that have sought to explain or anticipate their properties [19–33]. In active nematics, active stresses lead to a constant generation and annihilation of topological defects, giving rise to a loss of long-range nematic ordering. The flows associated with these defects are devoid of any spatiotemporal coherence; even for initially ordered systems, regular dynamics and pattern formation are transient [34, 35]. Attempts to control and design active nematics are still in their infancy, and have been hindered by a limited understanding of the underlying coupling between structure and dynamics.

Several reports have shown that geometrical confinement and the topological features can be used to stabilize the chaotic motion of active units and to harness their energy to generate well-defined flows [9, 36–45]. For example, theory predicts the formation of a circulating pair of +1/2 defects under circular confinement, with the bulk dynamics insensitive to the imposed anchoring condition [46]. In unconfined systems, strategies such as assembling extracts of cytoskeletal filaments and motor proteins on anisotropic soft structured interfaces [47, 48], or such as fixing the underlying nematic in contact with a bacterial suspension through a photopatterning [49–51], have both been shown to tame the otherwise chaotic flows. Past work has also sought to harness these flows by relying on frictional forces [36, 52–55]. Very recently, the patterning of activity has been shown to be effective at trapping and segregating topological charges, thereby providing a means to guide defect motion [56–58].

In this work, we rely on the relatively new concept of activity patterning or localization, along with the frictional damping, to control defect dynamics in a quasi 2D active nematic. Through this strategy, it becomes possible to confine and further manipulate the dynamics of two, like-charge +1/2 topological defects (Fig. 1(b)). These new dynamics should be contrasted with those observed for two defects confined by standard hard walls, where momentum cannot be transferred across the boundary, leading to different outcomes and a more limited set of behaviors.

The spatiotemporal evolution of nematic tensorial order parameter $\mathbf{Q}$ and a flow field $\mathbf{u}$ follow the continuum equation of motion for an active nematic and are solved using a hybrid lattice Boltzmann method [21, 37, 59, 60]. The details of simulations and the full governing equations are provided in the SI. Simulations were performed on a 2D lattice confined in a solid disk, with the homeotropic anchoring and no-slip velocity field were enforced at the boundary. The system was initialized with the director field radially oriented. The regions inside the circular domain within the disk were activated by applying uniform extensile active stresses $\Pi^\theta = -\zeta \mathbf{Q}$, with the activity strength $\zeta > 0$. The domain outside the activated region is passive (Fig. 1(a)).

The Poincaré-Hopf theorem [61] enforces the presence of topological defects of total charge +1 in the current system. As such, the system will develop at least one pair of +1/2 defects, which in the absence of activity experience repulsive interactions that maximize their separation distance. At the same time, the interface created by
the spatially inhomogeneous activity profile repels the defects, leading to their entrapment within the active region and the appearance of new patterns and spatiotemporal states (see SI and Fig. S1 for a discussion of the repulsive interaction between defects and soft activity interface).

At high values of activity, a transition occurs into a chaotic state, with a fluctuating number of defects, irregular dynamics, continuous formation of bands and their unzipping, and an absence of long-lived defects due to constant defect renovation. As the friction increases, the threshold of activity required for the transition to the MD state becomes larger. The MD state develops when the activity-induced length scale $\sqrt{L/\zeta}$ is smaller than the frictional screening length $\sqrt{\eta/f}$ and the radius of the active circle $R$, where $L$ is the elastic constant, $\eta$ is the medium viscosity, and $f$ the friction coefficient.

In passive systems, the free energy is minimized by the formation of two $+1/2$ defects close to the boundary [62], whose separation distance is determined by the ratio of the surface anchoring strength and the material’s elastic constant. For very low values of activity or for high frictional damping, the active system under consideration behaves as a passive nematic in that it shows two static $+1/2$ defects facing head to head. In contrast to the behavior of a passive system, however, in the ID state the stationary defects’ separation depends on the relative strength of activity and friction. Note that, as the hydrodynamic screening through frictional forces increases and the system further approaches the dry limit, the defects are pushed further apart and the ID state occupies larger regions of the phase space.

Increasing activity leads to a decrease of the defect separation, as depicted in Fig. 2(a). The self-induced flows of the defects for $\pm 1/2$ defects confined into the solid disk are illustrated in Fig. S5; one can appreciate a polar double vortex structure for positive charge defects, which drives a comet-like defect toward its head [24]. In the ID state the director field is static. The active stresses, however, generate four equal-sized vortices of deformed quadrupolar structure with a stagnation point in the centroid (Fig. 2(d)). Increasing activity pushes the defects toward each other, while elastic, friction, and viscous forces keep them apart; all of these forces cancel each other, leading to a static defects state. Note that, despite the defects’ lack of motion, active flows are strongest at the defects’ core and, consequently, viscous forces are imposed that oppose defect movement.

For a given value of friction, above a critical value of activity, the balance between the forces on the defects is broken. Strong elastic repulsions between $+1/2$ defects become dominant, and the active forces are unable to bring the defects closer to each other. The defects tilt away from their line of centers (Fig. 2(b)), and enter a persistent circulation with random direction of orbiting, based on spontaneous symmetry-breaking (see the inset of Fig. 2(a) for a representative trajectory of SR and Movie 1). In contrast to the behavior of defects confined by hard and impermeable walls, the transition to circulatory motion does not require the creation of an additional pair of defects [46]. The ease of reorientation of the director field at the soft activity boundary allows the system to adapt to the defects’ director far-field and their underlying advective flows; circumventing the creation of a pair of new defects, giving rise to a smooth transition from ID to SR. Transitions from quiescence into a moving state [37, 63–65], and the emergence of coherent circular motion [66, 67], are ubiquitous in active matter. Here we have identified an additional lever for control by showing that the right combination of activity level and friction can be used to tune the precise location of the orbiting; it can be positioned, for example, right at the activity interface (Fig. 3(d)). Two of the four vortices in the ID state merge to form a larger vortex in the active region, with two additional vortices farther away (Fig. 2(e)).

In the SR state, increasing activity not only drives the defects to orbits having a larger radius (by a stretching of the middle vortex to reduce the high shear) but, due to the enhanced repulsion between them, they adopt an orientation along the radial vector, trying to escape the active region. In contrast, increasing the friction allows the defects to rotate with a smaller orbiting radius, due to the suppression of momentum propagation, the appearance of localized flows, and a reduction of the effective interaction between defects (Fig. 3(a) and Fig. S6).
FIG. 2: The effect of activity variation on (a) normalized defect separation, (b) average angle of the defect orientation with the radial vector, and (c) average elastic free energy of the system, for ID and SR states. Points are sampled from the red line in Fig. 1(b). (d-f) Velocity field in the domain corresponding to the points marked with I-III. The background color represents the nematic scalar order parameter (S); for the color scale refer to Fig. 1(a). Sample trajectory of two steadily orbiting defects are shown in the inset of Fig. 2(a) (ζ = 0.0008, f = 0.05).

Figure 3 illustrates the role of friction for a constant value of the activity. For low values of friction, where the defects’ orbit is near the activity interface (Fig. 3(d) and Movie 2), the active forces are closely oriented along the radial vector \( \hat{e}_r \), while the elastic forces form obtuse angles with the radial vector, keeping the defects inside the active region. As the friction increases, these forces adopt a perpendicular orientation with respect to the radial vector (Figs. 3(e),(e) and Movie 3) and, for even larger values of friction, the defects become stationary, with forces parallel to the radial vector (Figs. 3 (c),(f)). Interfacial active forces not only confine the defects inside the active region, but also stabilize their sliding along the activity boundary. Defect steering along the interface is also apparent for the case of a flat active/passive boundary (Fig. S2 and detailed discussions in the SI).

In the SR state, increasing the activity reduces the nematic ordering, leading to an increase of the elastic free energy (Fig. 2(c)). For sufficiently large values of friction and activity (see transition from II to III in Fig. 2, and Movie 4), two stable elastic bands (walls) appear between two orbiting defects, which move in phase with the defects. High frictional forces not only allow the two bands that are formed to fit in the active region, but also stabilize them, despite their high elastic cost. Frictional forces reduce the shear stresses across the bands and, consequently, they can approach each other, reducing the effectiveness of long-range hydrodynamic interactions [52, 53].

Along these bands, a Poiseuille-like flow is generated, leading to the formation of additional vortices (Fig. 2(f)). If, in this state, the friction is reduced or the activity is further increased, the elastic penalty incurred by the deformation of these bands increases, causing the bands to unzip by creating a pair of defects on each band (see elastic free energy amplification (reduction) during band formation (unzipping) in Movie 5(c)). While the newly liberated +1/2 defects move along the bands to restore the nematic order, new −1/2 defects annihilate the already existing +1/2 defects, and this cycle continues to form short-lived dancing defects (Movie 5). However, if the advective flows are large enough (further increasing the activity), band splitting occurs at the interface, where the stresses are highest, and two fast-moving +1/2 defects from each band switch off and annihilate the newly created −1/2 ones. The original +1/2 defects remain alive, forming a long-lived dancing state (Figs. 4(a),(b) and Movie 6).

The TB region in the phase diagram occurs for intermediate values of activity and small friction. In this phase, the active forces and the orientation of the defects form an acute angle, with radial vector \( \hat{e}_r \) (Figs. 3(b),(c)), trying to escape the active region. Once they reach the active/passive interface, the active forces drop abruptly and induce a large unfavorable director distortion on the passive side. The system’s need to minimize elastic distortions creates elastic forces that catapult the defects back to the active region, forming a “yin yang” structure (Figs. 4(g),(h) and Movie 8). For higher values of activity, the elastic distortion at the passive side next to the interface increases, and the catapulting effect is stronger (faster) with a sharper reflection angle (Figs. 4(e),(f) and Movie 7). An inner, excluded circular region that defects never cross appears, due to strong repulsive interactions. For higher values of activity, where the elastic-rebound effect is sharper, this excluded area becomes smaller (Movie 7).

Two contributions to the active forces, \( f^a = - (\zeta \nabla \cdot Q + \nabla \zeta \cdot Q) \), arise from: i) the spatial variation of the director field which is most appreciable in the proximity of topological defects and appears in the bulk of the nematic (first term) and, ii) the large activity gradient \( \nabla \zeta \) at the activity interface (second term). The second term, which distinguishes our work from that presented in past studies, is responsible for the rich dynamical states presented here. The direction of this interfacial force de-
We conclude our discussion with a brief mention of a peculiar behavior that we refer to as BC, in which one of the defects undergoes a cruising behavior, and the other shows a bouncing behavior. The viscous forces that drive the long range hydrodynamic interactions, and the frictional forces, compete with each other; the resulting screening length scale $\sqrt{\eta/f}$, defines the width of the elastic bands that form within the active region. Higher values of frictional damping are required to observe the dancing state that is instigated by the pair of oppositely oriented bands; the active region cannot accommodate two bands and, as such, the system develops one band by breaking the symmetry of the flow and the director field. Figures 4(i),(j) show that while one of the defects shows a persistent circular motion, the other one moves toward the interfacial region to induce a radially oriented bending instability. The newly formed band splits and creates a $-1/2$ defect, which rapidly annihilates the original defect, while the new $+1/2$ defect is propelled away from the interface (Movie 9). This behavior shares some similarities with the experimental observation of doubly periodic dynamics of defects, obtained from the slow nucleation of defects at the boundary and fast circulating defects in the bulk of microtubule-based active nematics under circular confinement [43]. The creation of $-1/2$ defects at the solid wall might be indicative of the slippery flow of microtubules at the disk surface, which induces tangential flows that, in turn, drive bending instabilities and creation of a defect pair, similar to the BC state.

We have described a new physical scenario in which the simple spatial patterning of activity leads to defect confinement within a passive/active interface. When coupled to the control of friction at a substrate, we have shown that it becomes possible to program defect motion, thereby turning otherwise chaotic trajectories into precisely sculpted spatiotemporal states and flow fields.

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FIG. 4: (a) Defect trajectory, and (b) snapshots of the director field in the dancing state ($\zeta = 0.0016$, $f = 0.04$). (c-f) Long-time defect trajectories for TB state for different values of activity ($\zeta = 0.0005, 0.0006, 0.0006, 0.0008$) and friction ($f = 0.0, 0.001, 0.002, 0.002$), respectively. Elastic free energy of the system (g) and snapshots of the director with forces exerted on the defect and the defect trajectory (blue) (h), (g)-(h) correspond to (f). Active, elastic, and frictional forces are shown with black, pink, and green arrows, respectively (frictional forces are relatively small). (i) Defect trajectories, and (j) snapshots of the director in the BC state ($\zeta = 0.001$, $f = 0.01$). The orientations of positive (negative) charge defects are identified by purple arrows (a triad of brown arrows). Full blown animations of all of these states are provided in the SI Movie 5–Movie 9.

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Supplementary Information

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**Governing equations and numerical implementation**

The systems considered here are described in terms of a nematic tensorial order parameter \( Q \) and a flow field \( u \). The standard theory of active nematodynamics is used to quantify their spatiotemporal evolution. For uniaxial systems, the nematic order parameter is written in the form

\[
Q = S (n n - I/3)
\]

where unit vector \( n \) is the nematic director field and \( S \) is an order parameter that measures the extent of orientational ordering. The evolution of this non-conserved order parameter obeys strongly non-linear equation \([1]\)

\[
\left( \frac{\partial}{\partial t} + u \cdot \nabla \right) Q = S - \Gamma H,
\]

where the advection term is generalized by:

\[
S = (\xi A + \Omega) \cdot (Q + \frac{I}{3}) + (Q + \frac{I}{3}) \cdot (\xi A - \Omega) - 2\xi (Q + \frac{I}{3}) (Q : \nabla u),
\]

which accounts for the response of the nematic order parameter to the symmetric \( A \), and antisymmetric \( \Omega \), parts of velocity gradient tensor \((\nabla u)\). Here \( \xi \) is the flow aligning parameter, chosen to be \( \xi = 0.7 \) for flow aligning elongated units. The molecular field

\[
H = -\left( \frac{\delta F_{LdG}}{\delta Q} - \frac{I}{3} \text{Tr} \frac{\delta F_{LdG}}{\delta Q} \right),
\]

embodies the relaxational dynamics of the nematic, which drives the system toward the minimum energy configuration with Landau-de Gennes free energy density:

\[
f_{LdG} = A_0 \frac{2}{(1 - U^3)} \text{Tr}(Q^2) - A_0 U \frac{3}{4} \text{Tr}(Q^3) + A_0 U^4 (\text{Tr}(Q^2))^2 + \frac{L}{2} (\nabla Q)^2.
\]

The relaxation rate is controlled by the collective rotational diffusion constant \( \Gamma \). The phenomenological coefficient \( A_0 \) sets the energy scale, \( U \) controls the magnitude of the order parameter, and \( L \) is the elastic constant in the one-elastic constant approximation.

The local fluid density \( \rho \) and velocity \( u \) are governed by the generalized incompressible Navier-Stokes equations, modified by a frictional dissipative term

\[
\nabla \cdot u = 0,
\]

\[
\rho \left( \frac{\partial}{\partial t} + u \cdot \nabla \right) u = \nabla \cdot \Pi - f u.
\]

The total asymmetric stress tensor

\[
\Pi = \Pi^p + \Pi^a,
\]

consists of the sum of a passive and an active stress, and \( f \) is the friction coefficient between the nematic fluid and the underlying substrate. The viscoelastic properties of the nematic are lumped in the passive stress, given by the sum of viscous and elastic terms:

\[
\Pi^p = 2\eta A - P_0 I + 2\xi (Q + \frac{I}{3}) (Q : H) - \xi H \cdot (Q + \frac{I}{3}) - \xi (Q + \frac{I}{3}) \cdot H - \nabla Q : \frac{\delta F_{LdG}}{\delta \nabla Q} + Q \cdot H - H \cdot Q.
\]

The active stress is given by

\[
\Pi^a = -\zeta Q.
\]

Here, \( \eta \) is the isotropic viscosity, \( P_0 \) is the isotropic bulk pressure, and \( \zeta \) measures the activity strength. A flow is generated when \( Q \) experiences a spatial gradient with \( \zeta > 0 \) for extensile systems and \( \zeta < 0 \) for contractile ones. We employ a hybrid lattice Boltzmann method to solve the coupled governing partial differential equations (Eqs. S1, S5) \([2-6]\). The time integration is performed using an Euler forward scheme; the spatial derivatives are carried out using a second order central difference and the coupling is enforced by exchanging local fields between these algorithms at each time step. Simulations were performed on a 150 × 150 two-dimensional lattice confined in a disk of radius...
$R_0 = 75$. The medium viscosity was set to $\eta = 1/6$, and the collective rotational viscosity was set to $\Gamma = 0.3$. We chose the following parameters throughout the simulation $A_0 = 0.01$, $L = 0.01$, $U = 3.0$ (giving $S = 0.5$), homeotropic anchoring conditions with strength $W = 0.01$, and a no-slip velocity field at the surface of the solid disk. The system was initialized with the director field radially oriented. The regions inside the circular domain of radius $R = 25$ were activated by applying uniform extensile active stresses to the nematic. The domain outside the diffusive active/passive interface (tanh profile), of width 2 lattice units, is passive.

The role of active/passive boundary as a soft interface in defects dynamics

In what follows we show how the soft, permeable interface between active and passive regions created by the spatial patterning of activity is fundamentally different from the boundary created by a hard impenetrable wall. In particular, we highlight the effect of active forces, $f^a$, which appear at the interface between active and passive domains. The major difference between a hard boundary and a soft interface resides in the fact that for the latter case the contributions of activity appear as a bulk term, without need for external enforcement of anchoring. At a soft interface, the director field adopts an orientation that is consistent with the minimization of the bulk free energy and the active flows that travel through the interface. In contrast, a hard boundary imposes a no slip boundary condition with zero velocity normal and tangential to the confining walls.

![Diagram](image-url)

FIG. S1: (a)-(d) Variation of interfacial active force as a self-propelled +1/2 defect approaches and crosses the interface in a perpendicular manner. The right half is active (light red) and the left one is passive. The black arrows denote the active forces which emerge at the interface, and the blue lines represent the local nematic director.

Two terms contribute to $f^a$: i) the spatial variation of the director field in the bulk of the nematic $f^a_b$, and ii) the variation in the strength of the activity $f^a_{\text{int}}$, which is given by:

$$f^a = f^a_b + f^a_{\text{int}} = - (\zeta \nabla \cdot Q + \nabla \zeta \cdot Q). \tag{S9}$$

The bulk active force is more significant in the vicinity of defects, and decays as $1/r$ from the core of the defect. In contrast, $f^a_{\text{int}}$ solely emerges from the activity interface. We first consider a case in which the passive liquid crystal occupies the $x < 0$ region, and the active nematic the $x > 0$ region. Note that in simulations, for numerical stability reasons, the interface is of the form $\zeta = 1/2\tilde{\zeta}_0(\tanh(x/\omega)) + 1$, with a small thickness $\omega$. To simplify our analysis we consider an activity pattern of the form $\zeta = \tilde{\zeta}_0 \mathcal{H}(x)$, where $\mathcal{H}(x)$ is a Heaviside function. In 2D, $Q_{ij} = S(n_i n_j - \delta_{ij}/2)$, and the director field $\mathbf{n}(r) = (\cos \theta(r), \sin \theta(r))$, which yields the nematic order:

$$Q = S/2 \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}. \tag{S10}$$

In 2D the director field can be represented by the scalar field $\theta$, which is defined by the angle between the director and the $x$-axis in Cartesian coordinates. It can be shown that the interfacial active force emerging right at the interface
is given by:
\[
f_{\text{int}}^a = -\zeta_0 \delta(x) S/2 (\cos 2\theta \hat{e}_x + \sin 2\theta \hat{e}_y).
\] (S11)

This force has a dramatic influence on the dynamics of the system. In particular, for the case of homeotropic alignment of the director field, \((\theta = 0)\), \(f_{\text{int}}^a\) is anti-parallel to the activity gradient. A tangential anchoring of the director field, \((\theta = \pi/2)\), imposes an active force along the activity gradient (i.e. positive \(x\) axis). It is worth noting that the interfacial anchoring is strongly influenced by the advective flows that are generated by the motion of the defects, as well as the elastic forces. The frictional forces in turn stabilize the existent anchoring at the interface.

To better understand the effects of an active/passive interface we consider the following cases:

A) The +1/2 defect approaches the interface perpendicularly, driven by the active force \(f_{\text{int}}^a = \hat{p}/(2r)\) (Fig. S1). Here \(\hat{p}\) is the unit vector indicating the polarity and the axis of symmetry of the defect, which for this example is towards the \(-x\) axis. The director far field of the +1/2 defect induces a tangential alignment of the director field at the interface (corresponding to \(\theta = \pi/2\) in Eq. S11), resulting in an active interfacial force along the activity gradient. Once the self-generated flow of the defect drives the defect to the passive side, the tail of the comet-shaped defect imposes a homeotropic alignment at the interface (corresponding to \(\theta = 0\) in Eq. S11). These results clearly indicate that the activity interface repels the +1/2 defect that is perpendicular to the interface (see also Movie S1).

B) The positive half-integer charge defect approaches the interface at an angle Fig. S2(a). In the absence of an activity pattern, the defect would move in a straight line. However, here the active/passive interface acts as a barrier, and adapts dynamically to the orientation of the defect to help steer it along the interface Fig. S2. Note the change of director angle from \(\theta < \pi/4\) to \(\theta = \pi/4\), and then to even larger values. The limit \(\theta = \pi/4\) corresponds to the critical value of Eq. S11 that imposes active forces parallel to the interface (pointing anti-parallel to the defect polarity orientation) and hinders the penetration of the defect into the passive region (see Movie S2). This mechanism helps maintain the steady rotation (SR) state observed in Figs. 2(e),(f). The critical inclination angle of the defects above which the sliding of the defects along the interface occurs depends strongly on the activity strength and on the frictional forces. As the activity level increases, the sliding of the defect parallel to the interface requires the defect orientation to move further away from the interface normal. The friction in turn influences the dynamics in an opposite way. Movie S3 displays a case similar to the one in Fig. S2, but with an initial incident defect orientation angle closer to the interface normal. The activity interface fails to prevent the penetration of the defect into the passive side, but imposes a realigning torque on the defect that slowly reorients the defect’s polarity in a direction anti-parallel to the activity gradient.

![Fig. S2: (a)-(e) Trajectory of +1/2 defect approaching an active/passive interface at an angle. The black arrows show the active force that arises at the activity interface, and the blue lines represent the local nematic director. The active domain is shaded in light red.](image)

We next turn to a circular pattern of activity, which is the main focus of the current study. Simulations were performed assuming that the regions enclosed by a circle of radius \(R\) are active, with a diffuse active profile of the form \(\zeta = \zeta_0 (1/2-1/2 \tanh((|r-r_0|-R)/\omega))\). Here, \(r\) and \(r_0\) are the position vector and the position of the system’s center, respectively. For the following analysis, we assume a sharp activity transition, with \(\zeta = \zeta_0 (1-H(|r-r_0|-R))\) profile. It can be shown that, for a radial director field initialization \(n(r) = (\cos \phi, \sin \phi)\), where \(\phi\) is in polar coordinates, the active interfacial force points radially outward (Fig. S3(a)) and has the following form
\[
f_{\text{int}}^r = \zeta_0 S/2 \delta(|r-r_0|-R) \hat{e}_r.
\] (S12)
Once the +1 defect at the center of the system splits into two half-integer defects, the active forces that are strongest at the core of the defect drives them toward the activity interface. This soft and compliant interface adopts the orientation imposed by the head of the +1/2 defect, and the director field switches from radially oriented to circularly oriented. This switching leads to an interfacial active force that is directed radially towards the center, and imposes a repulsive force on the approaching defects (Fig. S3(b)).

Defect dynamics for a system with initial circular orientation of director field

Here we probe the sensitivity of defect dynamics to the system’s initial configuration. It can be shown that by initializing the director field with \( n(r) = (−\sin \phi, \cos \phi) \), the active interfacial force points radially inward (Fig. S3(b)), and can be expressed as:

\[
\mathbf{f}_\text{int}^a = -\zeta_0 S/2 \delta(|r - r_0| - R) \hat{e}_r.
\]  

(S13)

Our analysis reveals that the initial circular alignment leads to the expulsion of defects from the active interface. Once the +1/2 defects move out of the active interface, their tails enforce a normal orientation of the director field at the interface. The defects get trapped in the passive region, not only due to the loss of activity, but also to the repulsive interfacial active forces (see Fig. S3(a) for the direction of the force in the case of a director normal to the interface). Interestingly, the circular configuration does not exhibit any self-organized or coordinated defect dynamics. Besides the active turbulent state that emerges for a high level of activity, and the stagnant positive defects that are trapped at the passive region (low activity and high friction systems), other dynamics involves the generation of additional +1/2 defects, which migrate out of active boundary, and the entrapment of −1/2 defects in the passive region (Movie S4). The depletion of +1/2 defects from the active region makes the system prone to bending instabilities. However, the formation of immobile −1/2 defects hinders the appearance of coordinated defect dynamics. This result is in stark contrast with the dynamics reported in [7, 8] for a hard impermeable boundary. In [8], extensile active nematics were confined by a circular boundary, with no-slip boundary conditions and imposed tangential (and other types) anchoring, to demonstrate the insensitivity of spatiotemporal dynamics to the imposed boundary condition. Our study shows that for soft, permeable active/passive interfaces, the dynamics are extraordinarily sensitive to the initialization of the director field; for example, the circulating state observed in [7, 8] does not appear for the case of circular alignment of the director field. Additionally, [7, 8] discussed the accumulation of −1/2 defects at the solid boundary; our study shows entrapment of negative charge defects in the active region.

It is worth noting that, despite the strong dependence of spatiotemporal dynamics observed in our simulations on the initialization of the director field, the dynamics are independent of the anchoring strength imposed at the outer, solid boundary. Movie S5 displays the two-bouncing state (TB) with no anchoring at the solid circular boundary. This results underscores the role of the active/passive soft interface in maintaining well-organized spatiotemporal defect
dynamics, and raises interesting prospects regarding its experimental relevance.

We also simulated a passive system with strong tangential anchoring imposed at the outer solid boundary. Our results show anti-alignment of two $+1/2$ defects, with a center to center distance of $2 \times 5^{-1/4}R_0$, consistent with theoretical and experimental observations in [9], serving to confirm the validity of our simulations.

**Force and torque analysis for $+1/2$ defect near an activity boundary**

As illustrated in Fig. S4, we consider a $+1/2$ defect at $x = d$ near a flat activity interface located at the $y$-axis ($x = 0$). A hard impenetrable wall with homeotropic anchoring conditions located at $x = -b$ is parallel to the soft interface. The defect makes an angle $\theta$ with the $+x$ direction. The elastic energy can be written as [10]

\[
E^e = -\frac{\pi K}{2} \left[ \log \frac{2(d + b)}{a} + \log \cos \frac{\theta}{2} \right],
\]

where $K$ is the elastic constant and $a$ is the defect core size. The above elastic energy captures two key physics, namely (i) the elastic energy diverges when the defect approaches the homoeotropic anchoring wall, and (ii) the $+1/2$ defect facing the $+x (-x)$ direction is the most (least) favorable orientation. This elastic energy expression gives rise to the $x$-components of the elastic force and torque, respectively:

\[
F^e_x = -\frac{\partial E^e}{\partial d} = \frac{\pi K}{2} \frac{1}{d + b},
\]

\[
T^e_x = -\frac{\partial E^e}{\partial \theta} = -\frac{\pi K}{4} \tan \frac{\theta}{2}.
\]

(S14)

To write out the active force and torque, we introduce an effective energy for a given activity level $\alpha$:

\[
E^a = \alpha h^2 (R - d) \cos \theta,
\]

where $R$ is the characteristic size of the active region, corresponding to the radius of the pattern in the circular pattern simulation. Parameter $h$ is the thickness or the hydrodynamic screening length of the quasi-2D system, determined by $\sqrt{\eta/\gamma}$, with $\eta$ and $\gamma$ being the isotropic viscosity and friction parameter, respectively. The physical picture is that only the active stress within a circular region of radius $h$ centered at the defect core can mobilize the $+1/2$ defect, due to the hydrodynamic screening provided by the friction in the system. Therefore, we can arrive at the active force

FIG. S4: Schematic of the system. A $+1/2$ defect is near a flat active-passive interface (red dashed line) located at $x = 0$. A parallel impenetrable hard wall with homeotropic anchoring is located at $x = -b$, mimicking the scenario considered in the main text by ignoring curvature effects in the circular pattern simulations. Here $\theta$ and $\phi$ denote the angles that the defect makes with the $+x$ and $-x$ direction, respectively.
and torque:

\[ F^a_x = \alpha h^2 \cos \theta; \]
\[ T^a_x = \alpha h^2 (R - d) \sin \theta. \]  
(S15)

where \( F^a_x \) is simply the active force projected onto the \(+x\) direction. The active torque vanishes when \( \theta = 0 \) or \( \pi \) due to symmetry reasons, and it reaches a maximum when the defect is moving parallel to the soft interface, which corresponds to the most asymmetric scenario. Such asymmetry is also distance dependent. When the defect is sufficiently far from the soft interface, the torque should vanish. Therefore, such a torque has a \((R-d)\) term.

The force and torque balance \( F^e_x + F^a_x = 0 \) and \( T^e + T^a = 0 \) yield

\[ \alpha h^2 \cos \phi = -\alpha h^2 \cos \theta = \frac{\pi K}{2} \frac{1}{d + b}; \]
\[ \alpha h^2 (R - d) \sin \phi = \alpha h^2 (R - d) \sin \theta = \frac{\pi K}{4} \tan \frac{\theta}{2}. \]  
(S16)

The expression above leads to the following inequality:

\[ \frac{2 \sin^2 \left( \frac{\phi}{2} \right)}{1 - 2 \sin^2 \left( \frac{\phi}{2} \right)} = \frac{d + b}{2(R - d)} \geq \frac{b}{2R}, \]

therefore,

\[ 2 \sin^2 \left( \frac{\phi}{2} \right) \geq \frac{1}{1 + 2R/b}. \]

According to Eq. S16, one has

\[ \alpha = \frac{\pi K \tan \frac{\theta}{2}}{h^2 (R - d) \sin \phi} = \frac{\pi K}{4h^2 (R - d) \cdot 2 \sin^2 \left( \frac{\phi}{2} \right)} \leq \frac{\pi K (1 + 2R/b)}{4h^2 (R - d)} \equiv \alpha_c. \]

The above inequality implies that when activity is low, i.e. \( \alpha < \alpha_c \), a solution can be found that satisfies the two balance equations, meaning that the \(+1/2\) defect can glide along the soft interface, in which case we expect a defect rotating state for the circular pattern systems. However, if the activity reaches a critical value \( \alpha_c \), the solution no longer exists, indicating that the defect cannot remain close to the soft boundary, in which case we expect a defect-bouncing state. This critical activity threshold can explain the existence of a phase boundary between the defect-rotation and defect-bouncing states. With simulation parameters \( K = 0.01, R = 25, b = 50, h = \sqrt{\eta/\gamma} \simeq 2 \), we find \( \alpha_c \sim 0.0003 \), which compares favorably with the transition activity \( \alpha_c \simeq 0.001 \) found in full hydrodynamic simulations.
FIG. S5: Self-generated flows of positive (a) and negative (b) half integer charge defects for a system of extensile active nematics. The blue lines in the background denote the director field and the black arrows are the velocity field. The $+1/2$ defect shows asymmetric double vortices which drive the defect towards its head. The $-1/2$ defect forms six-fold symmetric vortices with zero net force at the core of the defect. The activity is uniform, no-slip velocity and strong homeotropic anchoring are imposed at the outer surface of the disk. The director field is initialized with the ansatz to ensure the formation of the corresponding defects at the center.

FIG. S6: Effect of frictional damping forces on the average kinetic energy of the system. The points are associated with the blue line in Fig. 1 of the main text. The average kinetic energy is reduced by increasing the friction coefficient.
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