Higher Fock sectors in Wick-Cutkosky model

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Abstract. In the Wick-Cutkosky model we analyze nonperturbatively, in light-front dynamics, the contributions of two-body and higher Fock sectors to the total norm and electromagnetic form factor. It turns out that two- and three-body sectors always dominate. For maximal value of coupling constant $\alpha = 2\pi$, corresponding to zero bound state mass $M = 0$, they contribute 90% to the norm. With decrease of $\alpha$ the two-body contribution increases up to 100%. The form factor asymptotic is always determined by two-body sector.

1 Introduction

In field theory, the state vector $|p\rangle$ is described by an infinite set of the Fock components, corresponding to different numbers of particles. In light-front dynamics \cite{[1][2]} the state vector is defined on the light front plane $\omega \cdot x = 0$, where $\omega$ is the null four-vector ($\omega^2 = 0$). The wave functions are expressed in terms of the variables $k_{\perp}, x$: $\psi = \psi(k_{1\perp}, x_1; k_{2\perp}, x_2; \ldots; k_{n\perp}, x_n)$. The total norm (equalled to 1) is given by sum over all the sectors: $\sum_{n} N_n = 1$, where $n$-body contribution $N_n$ reads:

$$N_n = (2\pi)^3 \int |\psi(k_{1\perp}, x_1; k_{2\perp}, x_2; \ldots; k_{n\perp}, x_n)|^2 \delta^{(2)}(\sum_{i=1}^{n} k_{i\perp}) \delta^{(n)}(\sum_{i=1}^{n} x_i - 1)^2 \prod_{i=1}^{n} \frac{d^2 k_{i\perp} dx_i}{(2\pi)^3 2x_i}. \quad (1)$$

In applications, the infinite set of the Fock components is usually truncated to a few components only. The belief that a given Fock sector dominates (with two or three quarks, for instance) is often based on intuitive expectations and on “experimental evidences” rather than on field-theoretical analysis.

In the Wick-Cutkosky model two massive scalar particles interact by the ladder exchange of massless scalar particles. Two-body sector contains two massive particles. Higher sectors contain two massive and 1, 2, \ldots massless constituents.

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In the present paper, based on the work [3], we present the results of our study in the Wick-Cutkosky model of contributions of the two- and three-body sectors to the total norm. Subtracting them from 1, we get total contribution of all the sectors with \( n \geq 4 \). Besides, we also calculate their contributions to the electromagnetic form factor. Calculations are carried out nonperturbatively in full range of binding energy \( 0 \leq B \leq 2m \).

2 Bethe-Salpeter amplitude in Wick-Cutkosky model

We use Bethe-Salpeter (BS) amplitude known explicitly in the Wick-Cutkosky model [4]. For the ground state with zero angular momentum it reads:

\[
\Phi(k,p) = -i \sqrt{\frac{4}{\pi}} \int_{-1}^{+1} g_M(z) dz \frac{m^2}{(m^2 - M^2/4 - k^2 - zp - i\epsilon)^2},
\]

(2)

where \( k \) and \( p \) are relative and total four-momenta, \( m \) is the massive constituent mass, \( M \) is the mass of the composite system. The representation (2) is valid and exact for the zero-mass exchange. The function \( g_M(z) \) is determined by the integral equation:

\[
g_M(z) = \frac{\alpha}{2\pi} \int_{-1}^{1} K(z,z') g_M(z') dz',
\]

(3)

with the kernel:

\[
K(z,z') = \frac{m^2}{m^2 - \frac{1}{4}(1 - z^2)^2 M^2} \left[ \frac{(1 - z)}{(1 - z') \theta(z - z') + (1 + z)} \theta(z' - z) \right].
\]

Here \( \alpha = g^2/(16\pi m^2) \) and \( g \) is the coupling constant in the interaction Hamiltonian \( H^{int} = -g\phi^2(x)\chi(x) \). In nonrelativistic limit the interaction is reduced to the Coulomb potential \( V(r) = -\frac{\alpha}{r} \).

The normalization condition for \( g_M(z) \) is found from the requirement that the full electromagnetic form factor \( F_{full}(Q^2) \) (calculated with full state vector \( |p\rangle \) and, hence, incorporating all the Fock components) equals to 1 at \( Q = 0 \).

Form factor is expressed in terms of the BS amplitude:

\[
(p + p')^\mu F_{full}(Q^2) = -i \int \frac{d^4k}{(2\pi)^4} (p + p' - 2k)^\mu (m^2 - k^2) \Phi \left( \frac{1}{2} p - k, p \right) \Phi \left( \frac{1}{2} p' - k, p' \right).
\]

(4)

We substitute here the BS amplitude [2] and find the normalization of \( g_M(z) \) from the equality \( F_{full}(0) = 1 \). The details of calculations are given in [3].

The function \( g_M(z) \) is found from [3] analytically in the limiting cases of small binding energy \( B = 2m - M \) (\( \alpha \to 0, B \to 0, M \to 2m \)) and of extremely large binding energy (\( \alpha = 2\pi, B = 2m, M = 0 \)). In the case \( M \to 2m \) it reads:

\[
g_M(z) = 8\sqrt{\frac{2}{\pi}} \alpha^{5/2} m^3 \left( 1 + \frac{5\alpha}{\pi} \log \alpha \right) \left[ 1 - |z| + \frac{\alpha}{2\pi} (1 + |z|) \log(z^2 + \alpha^2/4) \right].
\]

(5)

In contrast to the solution found in [3], eq. (5) is calculated to the next \( \alpha \) order, keeping, however, the leading log \( \alpha \) term (i.e., neglecting \textit{const} relative to \( \log \alpha \)).
In the opposite case $M = 0$ $g_{M=0}(z)$ has the form:

$$g_{M=0}(z) = 6\sqrt{30}\pi^{3/2}m^3(1-z^2).$$  \hfill (6)

For arbitrary $M$ the function $g_M(z)$ is found from (3) numerically.

### 3 Two- and three-body contributions

Knowing the BS amplitude, we extract from it the two-body wave function \[1\]:
4 Results

For small $\alpha$, with eq. (5) for $g_M(z)$, the contributions $N_2$ and $N_3$ to the total norm are found analytically (up to $\alpha \log \alpha$ order):

$$N_2 = 1 - \frac{2\alpha}{\pi} \log \frac{1}{\alpha}, \quad N_3 = \frac{2\alpha}{\pi} \log \frac{1}{\alpha}, \quad N_{n \geq 4} = \mathcal{O}(\alpha^2).$$  \hspace{1cm} (10)

For $\alpha = 2\pi$ ($B = 2m$, $M = 0$), with $g_M(z)$ given by (3), we get:

$$N_2 = \frac{9}{14} \approx 64\%, \quad N_3 \approx 26\%, \quad N_{n \geq 4} \approx 10\%.$$  \hspace{1cm} (11)

For $\alpha$ in the interval $2\pi \geq \alpha \geq 0$, corresponding to $0 \leq M \leq 2m$, the values of $N_2$, $N_3$ and $N_{n \geq 4}$ vs. $M$ are found numerically and they are shown in Fig. 2.

![Figure 2. Contributions to the total norm $N_{n=2} + N_{n=3} + N_{n \geq 4} = 1$ of the Fock sectors with the constituent numbers $n = 2$, $n = 3$ and $n \geq 4$ vs. the bound state mass $M$ (in units of $m$).](image_url)

We find that two-body sector always dominates. The sum $N_2 + N_3$ contributes 90% even in the extremely strong coupling case, as we see in (11). This result is non-trivial, since for $\alpha = 2\pi$ one might expect just the opposite relation of the $N_2 + N_3$ and $N_{n \geq 4}$ contributions. For any $\alpha$, asymptotic behavior of the form factor $F_{full}(Q^2)$ is determined by the two-body Fock sector [3].

References

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