Parametric instabilities of large-amplitude parallel propagating Alfvén waves: 2D PIC simulation

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Abstract. We discuss the parametric instabilities of large-amplitude parallel propagating Alfvén waves using the 2D PIC simulation code. First, we confirmed the results from a previous study (Sakai et al 2005 New J. Phys. 7 233) that the electrons are heated due to the modified two-stream instability and that the ions are heated by the parallel propagating ion acoustic waves. However, although the past study argued that such parallel propagating longitudinal waves are excited by transverse modulation of the parent Alfvén wave, we consider these waves are more likely to be generated by the usual, parallel decay instability. Further, we performed other simulation runs with different polarization of the parent Alfvén waves or different ion thermal velocity. Numerical results suggest that electron heating by the modified two-stream instability due to the large amplitude Alfvén waves is unimportant with most parameter sets.

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1. Introduction

Large-amplitude, low-frequency Alfvén waves constitute one of the most essential elements of magnetic fluctuations in the solar corona and solar wind [1, 2]. Due to small collisionless dissipation rates, the waves can propagate long distances and efficiently convey such macroscopic quantities as momentum, energy and helicity. Since loading of such quantities is completed when the waves damp away, it is important to examine how the waves can dissipate. Among various possible dissipation processes of the Alfvén waves, parametric instabilities have been believed to be important [3, 4]. Further, the parametric instabilities of these Alfvén waves can cause the developed turbulence, which is believed to produce in situ the localized structures (shocks and discontinuities) in the solar wind [5, 6].

However, most past studies using kinetic simulation code mainly discussed the parametric instabilities among parallel propagating waves in one-dimensional (1D) systems, although the real solar wind is 3D. Actually, arguments including obliquely propagating waves have been developed by several authors using fluid systems [7]–[13]. These studies clarified that the growth rates involving the oblique waves are usually much smaller than those among parallel propagating waves.

On the other hand, Sakai et al [14] recently performed 2D particle-in-cell (2D PIC) simulations. They reported that the left-hand (LH-) polarized Alfvén waves are dissipated through the transverse modulational instability when ion and electron beta values are very low (≈ 0.01). This is in contrast to past studies using fluid systems which indicated that parametric instabilities including obliquely propagating daughter waves (transverse modulation or filamentation instability) can be dominant only in the high \( \beta_f \) (the squared normalized sound velocity) plasma. One of the purposes of the present study is to clarify the reason for this difference in the type of parametric instabilities between the fluid and the PIC simulations.

Sakai et al [14] also reported electron heating through the modified two-stream instability (MTSI) [15], which is caused by the difference in ion and electron bulk velocities (cross-field currents). In the presence of finite amplitude Alfvén waves, the cross-field currents can initially be given by the Walen relation. However, Sakai et al [14] only discussed the LH-mode in their paper. Another purpose of this study is to confirm the importance of the MTSI driven by Alfvén waves with both right-hand (RH-) and LH-polarizations, and also by a mixture of Alfvén waves (in which the Walen condition is not satisfied).

The paper is organized as follows. In section 2, we present our simulation model. In section 3, we present our simulation results. A summary and conclusions of this paper are given in section 4.

2. Simulation setup

We have carried out a 2D PIC simulation [15] with the system size \( L_x = 1024–2048 \) and \( L_y = 512–1024 \) with 100 super-particles (both ions and electrons) per cell. We use periodic boundary conditions in both x- and y-directions, with a uniform ambient magnetic field (\( b_{x0} = 1 \)) in the x-direction, and a homogeneous plasma (the zeroth-order longitudinal bulk velocity \( u_{xj0} = 0 \) (\( j = i, e \)) and the number density \( n_{i0} = n_e = \text{const} \)). The ion-to-electron mass ratio is \( m_i/m_e = 16 \), and the ratio of the electron plasma to the cyclotron frequency is \( \omega_e/|\Omega_e| = 1 \) (\( \omega_j \) and \( \Omega_j \) are plasma and the cyclotron angular frequencies of electrons (\( j = e \)) and ions (\( j = i \))). The Alfvén velocity \( C_A = 0.25c \), thus, when the beta ratio is 0.01 for both ions and
**Table 1.** Parameters used in simulation runs.

| Run | Polarization | $\omega_0 / \Omega_i$ | $k_0 C_A / \Omega_i$ | $v_{th,i} / c$ | $u_{j0}$ |
|-----|--------------|------------------------|-----------------------|----------------|---------|
| 1   | L            | -0.503                 | -0.736                | 0.025          | (3)     |
| 2   | R            | 0.56                   | 0.491                 | 0.025          | (3)     |
| 3   | (Mixed)      | (Mixed)                | -0.736                | 0.025          | $u_{j0} = 0$ |
| 4   | L            | -0.503                 | -0.736                | 0.1            | (3)     |

electrons, the electron thermal velocity is $v_{th,e} = \sqrt{2 T_e / m_e} = 0.1 c$, and the ion thermal velocity is $v_{th,i} = \sqrt{2 T_i / m_i} = 0.025 c$. The Debye length $v_{th,e} / \omega_e = 1$ (grid size) and the electron skin depth $c / \omega_e = 10$. As initial conditions, finite amplitude, monochromatic Alfvén waves are given

$$b_p = b_0 \exp(-ik_0 x),$$

$$u_{jp} = u_{j0} \exp(-ik_0 x),$$

where $b = b_x + ib_z$ (complex transverse magnetic field) and $u_j = u_{jy} + iu_{jz}$ (complex transverse bulk velocity of each species), $b_0 = 0.5$, and $u_{j0}$ is given by the Walen relation [16]

$$u_{j0} = -\frac{\omega_0 \Omega_j}{k_0 (\omega_0 + \Omega_j)} b_0.$$  

(3)

The variables $\omega_0$ and $k_0$ satisfy the dispersion relation of the parallel propagating electromagnetic waves in the two fluid (electron–ion) system [17],

$$\frac{k^2 e^2}{\omega^2} = 1 - \frac{\omega_e^2}{\omega (\omega + \Omega_e)} - \frac{\omega_i^2}{\omega (\omega + \Omega_i)}.$$  

(4)

We adopt the notation that the positive (negative) $\omega_0$ corresponds to the RH-(LH-) polarized waves. In our PIC simulation, charge neutrality cannot be assumed and displacement current cannot be neglected due to the values assigned to $C_A$ and $\omega_e / \Omega_e$. While these parameters used in the present study are different from those in past studies that used the hybrid simulation code [4], [18]–[20], our results can basically be explained within the framework of the Hall–MHD theory as will be seen later.

We performed four simulation runs with different parameters as given in table 1. The system size is $L_x = 1024$ and $L_y = 512$ in runs 1, 3 and 4, and $L_x = 2048$ and $L_y = 512$ in run 2. We have confirmed that the results remain essentially the same when we extended the transverse system size to $L_y = 1024$. Run 1 is almost the same as the run in Sakai et al [14] (LH-parent wave satisfying the Walen relation). In run 2 the RH-parent waves are given, and in run 3, magnetic and velocity perturbations are given independently (and thus the Walen relation is not satisfied) in order to model the fluctuations in the chromosphere. In run 4, we introduce higher ion temperature in order to examine the roles of kinetic modulational instability [4, 19, 20]. Initial Alfvén waves are all forward propagating in runs 1, 2, and 4, whereas forward and backward propagating waves are mixed in run 3.

To discuss the propagating direction of Alfvén waves, 2D ($x$–$y$) spacial Fourier transforming analyses were performed in this study. Since we focus on the low-frequency waves in this paper, the vertical and horizontal axes ($k_x$ and $k_y$) of the figures indicating the Fourier analyses (figures 3, 5, 7) are normalized by the ion skin depth ($c / \omega_i = 40$).
Figure 1. Time histories of magnetic field energy (thick line), ion (dashed line) and electron (thin line) total energies in run 1.

Figure 2. Snapshots of longitudinal electric field $e_x$ at (a) $t = 32 \Omega_i^{-1}$ and (b) $t = 64 \Omega_i^{-1}$ in run 1. The vertical and horizontal axes ($y$ and $x$) are normalized by the Debye length $\lambda = v_{th,e}/\omega_e$.

3. Numerical results

First, we show the results of run 1, in which physical parameters correspond to those in Sakai et al [14]. Figure 1 shows the time history of magnetic field energy, ion and electron total energies, respectively. As discussed in Sakai et al [14], the electron energy is enhanced by the excitation of MTSI (the arrow in figure 1). Figure 2(a) clearly shows the obliquely propagating...
waves with parallel electric field, corresponding to figure 3(b) in Sakai et al \cite{14}. We have also confirmed that the propagation angle of the waves is about 60°, which agrees with the simulation results in Sakai et al \cite{14} and the linear analysis in Wu et al \cite{21}. (We note that while Wu et al \cite{21} discuss the case $\omega_e \gg |\Omega_1|$, the growth rate of the MTSI is not sensitive to $\omega_e/|\Omega_1|$ \cite{22}.) The initial perpendicular velocity ($V_0$) is obtained from equation \cite{3}
\begin{equation}
V_0 = (u_{i0} - u_{e0}) = -v_{\phi 0} b_0 \Omega_1 \frac{(1 + r) \omega_0}{(\omega_0 + \Omega_j)(\omega_0 - r \Omega_j)}, \tag{5}
\end{equation}
where $v_{\phi 0} = \omega_0/k_0$ and $r = m_i/m_e$. $V_0$ in run 1 is $V_0 = 0.355 C_A$. We remark that the kinetic effects of electrons are important for the MTSI observed here, since $v_{th,e} > V_0$ \cite{21}.

After these obliquely propagating waves excited by the MTSI are damped by electron Landau damping, low-frequency parallel propagating waves become dominant (figure 2(b)). Dissipation of these parallel propagating ion acoustic waves heat the ions as discussed in Sakai et al \cite{14}. Although Sakai et al \cite{14} argued that such parallel propagating longitudinal waves are excited by transverse modulation of the parent Alfvén wave, we consider these waves are more likely to be generated by the usual, parallel decay instability. Figure 3 shows the power spectrum of the complex magnetic field $b (= b_y + ib_z)$ in $k_x - k_y$ wave number space at $t = 96 \Omega_1^{-1}$ in run 1. The vertical and horizontal axes ($k_y$ and $k_x$) are normalized by the ion skin depth $h = c/\omega_i$.

Next, we discuss the RH-mode case (run 2). Figure 4 shows the time history of energies in the same format as figure 1. In contrast to figure 1 (run 1), electron heating due to the excitation of MTSI is not evident. From equation (5), $V_0$ in run 2 is 0.223$C_A$, which is larger than $v_{th,i}$. We find here that even when $V_0 > v_{th,i}$, which is a rough criterion of the MTSI \cite{23}, the cross-field
current is not large enough to excite the MTSI under the particular set of parameters used. On the other hand, the decay instability takes place in a way similar to run 1 (not shown).

In run 3, we discuss the case in which the Walen relation is not satisfied for the initial Alfvén wave (figure 5). These initial conditions are intended to simulate the plasma and the magnetic field fluctuations in the solar wind and solar surface, where both RH- and LH-polarized Alfvén waves are considered to be mixed [2]. In such a case, the amplitudes of the transverse magnetic field and transverse bulk velocities vary in time [20]. Since such linear oscillations modify the magnitude of the cross-field current, they let the system stay in the stable and unstable regimes in turn, so that the MTSI growth is effectively reduced.

Finally, we discuss the results of run 4, in which the parameters are the same as those in run 1 except for $v_{th,i} = 0.1c$. In contrast to run 1, MTSI is suppressed by the ion kinetic effects (figure 6). Further, as figure 7 shows, the decay instability is not the dominant instability in this case, because of the occurrence of ‘kinetic’ modulational instability [20]. In figure 7, both the dominant daughter waves ($k_x \sim -0.25$ and $-1.25$) and the higher harmonic waves are observed.
4. Summary

In this paper, we discussed the parametric instabilities of Alfvén waves using a 2D PIC simulation code. First, we reran the simulation of Sakai et al. [14] using essentially the same set of parameters. While our numerical results are basically the same, we conclude that the generation of longitudinal waves is due to the usual, parallel decay instability, rather than the transverse modulational instability proposed by Sakai et al. [14], since the 1D decay instability is dominant in a low beta plasma as the Hall–MHD theory predicted [10, 13]. Further, we performed several other simulation runs using different parent Alfvén wave polarizations and different ion thermal velocities. The condition to excite the MTSI is easily violated by such modifications of the parameters. Thus, electron heating by the MTSI of the Alfvén waves seems unimportant in the upper chromosphere, since the Alfvén waves excited by the convection in the solar surface are likely to be superpositions of Alfvén waves with different polarizations and propagating directions, but not the monochromatic, LH-polarized wave. Further, a similar process on obliquely propagating waves has also been suggested by Markovskii and Hollweg [24] according to proton heating. A more detailed analysis on this issue is desired.
While the dominant parametric instabilities are quasi-1D in a low beta case, the transverse modulational instability can be more important in different circumstances. For instance, the Alfvén wave excited by the ion beam instabilities can be dominantly dissipated through the transverse modulational instability, which is nonlinearly driven by the obliquely propagating waves excited by the same beam instabilities [25]. Further, even if the parametric instabilities are 1D, 2D or 3D, density structures are excited as a consequence of their nonlinear evolution [11, 26]. When inhomogeneities in the background plasmas exist, heating of electrons by phase-mixed Alfvén waves takes place [27]. We remark that even in uniform background plasmas, electron heating regardless of the MTSI is actually observed in PIC simulations (e.g. Sakai et al [14] and figure 4). The electron heating is possibly caused by numerical noise originating from the super-particles in the PIC simulation. To avoid this and confirm the recent results, electromagnetic Vlasov simulations are planned.

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