Observer-based event-triggered control and application in active suspension vehicle systems

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ABSTRACT
This paper focuses on event-trigger control of automotive suspension systems. Firstly, fuzzy T-S systems which suitable for automobile suspension systems are studied. Parameter uncertainties and disturbances are included in the fuzzy T-S systems. Secondly, based on linear matrix inequalities (LMIs) and Lyapunov function, the conditions for the stability of fuzzy T-S systems are given. In the meantime, the controller and observer of the fuzzy T-S systems are designed. Finally, the theory is applied to the automotive suspension systems in the simulation.

1. Introduction
More and more attention has been paid to the quality of automobile performance. Comfort is an important factor to measure the automobile performance. How to improve the driving comfort is a hot issue, the automobile suspension systems are closely related to the comfort. Because of the change of vehicle load and passenger number, the vehicle suspension systems become parameter uncertain systems. It is more suitable to use T-S systems to study automobile suspension systems (Jamal et al., 2019; Li et al., 2011; Wang et al., 2019).

Event-trigger is a core concept in modern industrial information technology. In recent years, the research of event-triggered technology has made rapid progress in the control field (Han & Antsaklis, 2013; Hu & Yue, 2012; Wang et al., 2017). For linear networked systems, Hu and Yue (2012) designed event-triggered control design. Han and Antsaklis (2013) gave event-triggered observer and observer-based controller for networked systems. However, there is no vehicle suspension system involved in the above event-trigger control.

In this paper, for the automotive suspension systems, we study event-trigger control. Firstly, fuzzy T-S systems which parameter uncertainties and disturbances are considered. Using LMIs and Lyapunov function, the conditions of the stability are proposed. In numerical simulation, the fuzzy model is applied to the vehicle suspension systems.

The main contributions of this paper can be summarized as follows:

(1) The paper studies observer-based event-triggered control and application in active suspension vehicle systems. Note that (Han & Antsaklis, 2013; Hu & Yue, 2012) also studied event-trigger control, however, control methods are not applied to the vehicle suspension system.

(2) Event-trigger control methods are studied in active suspension vehicle systems. Note that (Jamal et al., 2019; Wang et al., 2021) also studied the vehicle suspension system, however, Jamal et al. (2019) and Wang et al. (2021) investigated control methods are not event-triggered control methods.

2. Problem statement
The dynamic equation of the quarter-car suspension system is established as Wang et al. (2019)

\[
\begin{align*}
\dot{z}_s(t) + c_i[z_s(t) - z_u(t)] + k_i[z_s(t) - z_u(t)] &= u(t) \\
\dot{z}_u(t) + c_i[z_u(t) - z_t(t)] + k_i[z_u(t) - z_t(t)] &= u(t) \\
+k_i[z_u(t) - z_t(t)] + c_i[z_u(t) - z_t(t)] &= -u(t)
\end{align*}
\]

where \(z_s(t) - z_u(t)\) is relative displacement of vehicle body, \(z_u(t) - z_r(t)\) is relative displacement of vehicle.
wheel, \( \dot{z}_i(t) \) is velocity of vehicle wheel vibration. Denote \( x(t) = [z_i(t) - z_o(t) \ z_u(t) - z_i(t) \ z_r(t) - z_o(t)]^T \in \mathbb{R}^4 \), \( z(t) = [\dot{z}_i(t) \ z_o(t) - z_u(t) \ z_r(t) - z_o(t)]^T \in \mathbb{R}^3 \), \( y(t) = [\dot{z}_r(t) - \dot{z}_o(t)]^T \in \mathbb{R}^2 \). The quarter-car suspension system is shown in Figure 1.

In order to study the automobile suspension system, we first study the following fuzzy system. In the simulation, the theoretical part is applied to the automobile suspension system. The considered T-S fuzzy systems with the following state space equation.

If \( \sigma_1(t) = F_{i_1} \) and \( \cdots \) and \( \sigma_p(t) = F_{i_p} \), then

\[
\begin{aligned}
\dot{x}(t) &= (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + W_1w_1(t) \\
\dot{z}(t) &= C_1x(t) \\
y(t) &= C_2x(t) + W_2w_2(t)
\end{aligned}
\]  

(2)

where \( F_{i_j} \) denotes fuzzy set; \( \sigma_j(t) \) denotes premise variable vector; \( x(t) \) denotes state vector; \( u(t) \) denotes input vector; \( y(t) \) denotes output vector; \( w_1(t) \) denotes input disturbance; \( w_2(t) \) denotes output disturbance; \( A_i, B_i, C_1, C_2, W_1, W_2 \) denote given matrices; \( \Delta A_i, \Delta B_i \) denote the parametric uncertainties.

The T-S fuzzy systems can be described as follows:

\[
\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\sigma(t)) \{ (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + W_1w_1(t) \} \\
\dot{z}(t) &= \sum_{i=1}^{r} \mu_i(\sigma(t))C_1x_i(t) \\
y(t) &= \sum_{i=1}^{r} \mu_i(\sigma(t)) \{ C_2x_i(t) + W_2w_2(t) \}
\end{aligned}
\]

(3)

where \( \alpha_i(\sigma(t)) = \prod_{j=1}^{p} F_{i_j}(\sigma(t)), \alpha_i(\sigma(t)) \geq 0, \mu_i(\sigma(t)) = \alpha_i(\sigma(t))/\sum_{j=1}^{r} \alpha_j(\sigma(t)), \sum_{i=1}^{r} \mu_i(\sigma(t)) = 1 \).

The fuzzy state observer is expressed by

\[
\begin{aligned}
\dot{\hat{x}}(t) &= \sum_{i=1}^{r} \mu_i(\sigma(t))[A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t))] \\
\dot{\hat{y}}(t) &= \sum_{i=1}^{r} \mu_i(\sigma(t))C_2\hat{x}(t)
\end{aligned}
\]

(4)

The event-trigger condition is designed as follows (Wang et al., 2017)

\[
f(t) = e^T(t)\Gamma e(t) - \rho \dot{x}^T(t)\Gamma \dot{x}(t) < 0
\]

(5)

where \( \rho \) is a positive number, \( \Gamma \) is a designed matrix, \( e(t) \) is estimation error, \( e(t) = \hat{x}(t,h) - \hat{x}(t) \), \( t \in [t_r, t_{r+1}] \).

3. Controller design and main results

The controller is designed based on the sampled observer. If \( \sigma_1(t,h) = F_{i_1} \), and \( \sigma_2(t,h) = F_{i_2} \) and \( \cdots \) and \( \sigma_p(t,h) = F_{i_p} \), then

\[
u(t) = K_t\hat{x}(t,h), \quad i = 1, \ldots, N
\]

(6)

where \( K_t \) are control gain matrices with appropriate dimension. Fuzzy controller is designed as follows:

\[
u(t) = \sum_{i=1}^{r} \mu_i(\sigma(t,h))K_t\hat{x}(t,h)
\]

(7)

The difference between \( \mu_i(\sigma(t)) \) and \( \mu_i(\sigma(t,h)) \) is considered, the following inequalities hold (Zhao et al., 2015)

\[
\begin{aligned}
\tilde{\mu}_i &= \epsilon_i \mu_i, \\
|\tilde{\mu}_i - \mu_i| &\leq \Delta_i, \\
\mu_i &= \mu_i(\sigma(t)), \quad \tilde{\mu}_i = \mu_i(\sigma(t,h))
\end{aligned}
\]

where \( \epsilon_i \) and \( \Delta_i \) are two positive scalars.

By inserting (7) into (3), observer equation is expressed as follows:

\[
\dot{\hat{x}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \epsilon_j \mu_j \{ (A_i + B_i K_j)\hat{x}(t) + B_i K_j e(t) \}
\]

(8)
Let $\dot{x}(t) = x(t) - \hat{x}(t)$, we can obtain the fuzzy closed-loop system

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i,j} \left[ \left((A_i + \Delta A_i) + (B_i + \Delta B_i)K_j\right)x(t) - (B_i + \Delta B_i)\dot{x}(t) + (B_i + \Delta B_i)Ke(t) + W_{1i}w_1(t) \right]$$

and

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i,j} \left[ (A_i - \Delta B_i)\hat{x}(t) + (A_i - \Delta B_i)Ke(t) + W_{1i}w_1(t) - L_iC_2\hat{x}(t) - L_iW_2w_2(t) \right]$$

(9)

**Theorem 3.1:** Given scalars $\gamma > 0$, $l$ is identity matrix, if there exist positive definite matrices $P_1$, $P_2$, such that the inequality holds

$$\Theta = \begin{bmatrix}
\Theta_{11} & -P_1B_iK_j & P_1W_{1i} & 0 \\
* & \Theta_{22} & P_2W_{1i} & -P_2L_iW_{2j} \\
* & * & * & -\gamma^2I \\
* & * & * & * -\gamma^2I
\end{bmatrix} < 0 \quad (10)$$

then the fuzzy system (9) is asymptotically stable. The control and observer gain matrices are designed as follows

$$K_j = R_jP_1, \quad L_i = P_2^{-1}N_j$$

**Proof:** Consider Lyapunov function as follows

$$V(x(t), \hat{x}(t)) = \begin{bmatrix} x^T(t) & \hat{x}^T(t) \end{bmatrix} \begin{bmatrix} P_1 & 0 \\
0 & P_2 \end{bmatrix} \begin{bmatrix} x(t) \\
\hat{x}(t) \end{bmatrix} \quad (11)$$

$$\dot{V}(t) = \dot{x}^T(t)P_1x(t) + \dot{\hat{x}}^T(t)P_2\hat{x}(t) + \dot{\hat{x}}^T(t)P_2\hat{x}(t)$$

(12)

By inserting the closed-loop system (9) into (12)

$$\dot{V}(t) \leq \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i,j}x^T(t)\left[P_1(A_i + \Delta A_i) + P_1(B_i + \Delta B_i)K_j + (A_i + \Delta A_i)^T P_1 + K_j^T (B_i + \Delta B_i)^T P_1 \right]x(t)$$

$$+ x^T(t)P_1(B_i + \Delta B_i)\hat{x}(t) - x^T(t)K_j^T (B_i + \Delta B_i)^T P_1 \xi(t) + x^T(t)P_1(B_i + \Delta B_i)Ke(t) + e^T(t)K_j^T (B_i + \Delta B_i)^T P_1 \xi(t)$$

$$+ 2x^T(t)P_1W_{1i}w_1(t) + 2\bar{x}^T(t)P_2(A_i - \Delta B_i)\hat{x}(t)$$

$$+ \bar{x}^T(t)(A_i - \Delta B_i)^T P_2 \hat{x}(t) + \bar{x}^T(t)P_2(\Delta A_i + \Delta B_i)\hat{x}(t)$$

$$+ x^T(t)(\Delta A_i + \Delta B_i)^T P_2 \hat{x}(t) + \bar{x}^T(t)(\Delta A_i + \Delta B_i)^T P_2 \hat{x}(t)$$

$$\pm e^T(t)K_j^T \Delta B_i^T P_2 \hat{x}(t) + 2\bar{x}^T(t) P_2W_1w_1(t)$$

$$- \bar{x}^T(t)(P_2L_iC_2 + C_2^T K_j P_2) \hat{x}(t) - 2\bar{x}^T(t) P_2L_iW_2w_2(t)$$

(13)

The uncertainties of system can be represented as

$$\begin{bmatrix} \Delta A_i \\
\Delta B_i \end{bmatrix} = D_iF_i(t) \begin{bmatrix} E_{i1} \\
E_{i2} \end{bmatrix}$$

where $F_i(t)$ are unknown uncertainties satisfying $F_i(t)F_i(t)^T \leq I$.

Applying Young’s inequality and event trigger condition, (13) converted to

$$\dot{V}(t) \leq \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i,j}x^T(t)[P_1(A_i + \Delta A_i) + P_1(B_i + \Delta B_i)K_j$$

$$+ K_j^T B_i^T P_1 + 4P_1D_iD_i^T P_1 + 2E_{i1}E_{i1} + \rho K_j^T K_j + P_1B_i^T B_i P_1 + (2 + 2\rho)K_j^T E_{i2}E_{i2}K_j$$

$$+ P_1B_i^T B_i P_1 + C_2^T C_2]$$

$$+ P_2^T P_2 + 2P_2D_iD_i^T P_2 + 2K_j^T E_{i2}E_{i2}K_j$$

(14)

Combining (14), we can obtain as follows:

$$\dot{V}(t) \leq \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_{i,j} \xi^T(t) \Theta \xi(t)$$

(15)

where $\xi^T(t, s) = [ x^T(t) \quad \dot{x}^T(t) \quad w_1^T(t) \quad w_2^T(t) ]$.

$$\Theta = \begin{bmatrix}
\Theta_{11} & -P_1B_iK_j & P_1W_{1i} & 0 \\
* & \Theta_{22} & P_2W_{1i} & -P_2L_iW_{2j} \\
* & * & * & 0 \\
* & * & * & 0
\end{bmatrix} < 0 \quad (16)$$

$$\Theta_{11} = P_1A_i + A_i^T P_1 + P_1B_iK_j + K_j^T B_i^T P_1 + 4P_1D_iD_i^T P_1$$

$$+ 2E_{i1}E_{i1} + \rho K_j^T K_j + P_1B_i^T B_i P_1 + (2 + 2\rho)K_j^T E_{i2}E_{i2}K_j + P_1B_i^T B_i P_1$$

$$\Theta_{22} = P_2A_i + A_i^T P_2 + 4P_2D_iD_i^T P_2 + 2K_j^T E_{i2}E_{i2}K_j$$

$$- P_2L_iC_2 - C_2^T K_j P_2$$

$$- C_2^T L_i^T P_2$$

$$H_\infty$$ performance index is taken as follows:

$$J = \int_0^T [z^T(t)z(t) - \gamma^2 w_1^T(t)w_1(t) - \gamma^2 w_2^T(t)w_2(t)] dt$$

(17)
Applying the Schur complement formula, Equation (17) can be converted to

\[
J = \int_0^T [z^T(t)z(t) - \gamma^2 w_1^T(t)w_1(t) - \gamma^2 w_2^T(t)w_2(t)] dt
+ \dot{V}(t) - \dot{V}(t) dt
\]

\[
\leq \int_0^T [x^T(t)C_i^T x(t) - \gamma^2 w_1^T(t)w_1(t) - \gamma^2 w_2^T(t)w_2(t) + \dot{V}(t)] dt
\]

\[
< \gamma^2 \int_0^{\infty} [w_1^T(t)w_1(t) + w_2^T(t)w_2(t)] dt
\]

Finally, we get inequalities (10).

Multiplying the sides of (10) by matrix \( diag \{ P_1^{-1} \ 0 \ 0 \ 0 \} \) and applying Schur complement formula, we obtain the inequalities as follows:

\[
\Theta = \begin{bmatrix}
\hat{\Theta}_{11} & -B_jR_j & W_1P_1^{-1} & 0 \\
* & \Theta_{22} & P_2W_2 & -N_jW_2 \\
* & * & -\gamma^2I & 0 \\
* & * & * & -\gamma^2I
\end{bmatrix} < 0 \quad (18)
\]

\[
\hat{\Theta}_{11} = A_iP_1^{-1} + P_1^{-1}A_i^T + B_jR_j + R_j^T P_1^{-1} + 4D_iD_i^T
+ 2P_1^{-1}E_i^T E_1P_1^{-1} + \rho R_j^T R_j + (2 + 2\rho)R_j^T E_2R_j
+ B_j^T P_1^{-1}C_i^T C_i P_1^{-1}
\]

\[
\Theta_{22} = P_2A_i + A_i^T P_2 + 4P_2D_iD_i^T P_2 + 2K_j^T E_3^T E_2K_j
- N_jC_{2i} - C_j^T N_j^T
\]

Applying the Schur complement formula, \( \hat{\Theta}_{11} < 0 \) is converted to

\[
\Theta_{11} = \begin{bmatrix}
P_1^{-1} E_1 & R_j^T & R_j^T E_2 & P_1^{-1} C_i^T \\
* & -0.5I & 0 & 0 \\
* & * & -\frac{1}{\rho}I & 0 \\
* & * & * & -\frac{1}{2+2\rho}I \\
* & * & * & -I
\end{bmatrix} < 0
\]

\[
\Theta_{11} = A_iP_1^{-1} + P_1^{-1}A_i^T + B_jR_j + R_j^T P_1^{-1} + 4D_iD_i^T + B_j^T B_j
\]

(19)

By solving LMIs (19), the matrices \( P_1 \) and the control gain matrices \( K_j \) are obtained, then put \( P_1 \) and \( K_j \) into (18).

4. Simulation

The related parameters of Equation (3) are listed as follows:

\[
A_1 = \begin{bmatrix}
-\frac{k_s}{m_s + \bar{m}_s} & 0 & -\frac{c_s}{m_s + \bar{m}_s} & \frac{c_s}{m_s + \bar{m}_s} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
-\frac{k_s}{m_s + \bar{m}_s} & 0 & -\frac{c_s}{m_s + \bar{m}_s} & \frac{c_s}{m_s + \bar{m}_s} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
A_3 = \begin{bmatrix}
-\frac{k_s}{m_s + \bar{m}_s} & 0 & -\frac{c_s}{m_s + \bar{m}_s} & \frac{c_s}{m_s + \bar{m}_s} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
A_4 = \begin{bmatrix}
-\frac{k_s}{m_s + \bar{m}_s} & 0 & -\frac{c_s}{m_s + \bar{m}_s} & \frac{c_s}{m_s + \bar{m}_s} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Thus, the simulation results are displayed in Figures 2–7, where Figures 2–5 are the trajectories of states $x_i$ and their estimations $\hat{x}_i$ ($i = 1, 2, 3, 4$); Figure 6 is the curve of control input $u$; Figure 7 are the curves of event-triggered response.

$$W_{11} = W_{13} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s + \hat{m}_s} \\ \frac{1}{m_u + \hat{m}_u} \end{bmatrix}, \quad W_{12} = W_{14} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s + \hat{m}_s} \\ \frac{1}{m_u + \hat{m}_u} \end{bmatrix}, \quad C_{11} = C_{12} = C_{13} = C_{14} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad W_{21} = W_{23} = \begin{bmatrix} 1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}, \quad W_{22} = W_{24} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad \rho = 1, \quad \gamma = 6, \quad \Delta A_i = 0.01 \times D_i \times \text{diag}[\sin t \sin t \sin t \sin t] \times E_{1i}, \quad \Delta B_i = 0.01 \times D_i \times \text{diag}[\sin t \sin t \sin t \sin t] \times E_{2i}, \quad D_i = 0.02l_i, E_{1i} = 0.1l_i, E_{2i} = [0.1; 0.1; 0.1; 0.1], i = 1, \ldots, 4,$$

\begin{align*}
w_2(t) &= \sin(t) / -10, \\
z_i(t) &= \begin{cases} \frac{H}{2}(1 - \cos(\frac{2\pi v}{l})t), & 0 \leq t \leq \frac{l}{v} \\ 0, & t > \frac{l}{v} \end{cases},
\end{align*}

$$w_1(t) = z_i(t), \quad m_s = 350 \text{ kg}, \quad m_u = 30 \text{ kg}, \quad k_s = 10 \text{ kN/m}, \quad k_u = 180 \text{ kN/m}, c_s = 1.5 \text{ kNs/m}, c_t = 17 \text{ Ns/m}.$$
5. Conclusion

This paper has designed event-trigger control for the fuzzy T-S systems with parameter uncertainties and disturbances. The conditions of the asymptotically stability are proposed by LMIs and Lyapunov function. Simulation are given for the vehicle suspension systems.

In the future, we will pay more attention to on multi-agent nonlinear systems such as a separation-based methodology to consensus tracking of switched high-order nonlinear multi-agent systems, consensus in high-power multi-agent systems with mixed unknown control directions and so on. It will be a challenging job.

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