Friedberg-Lee Symmetry for Quark Masses and Flavor Mixing

Ping Ren *
Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract

We point out a generic correlation between the Friedberg-Lee symmetry of quark mass operators and the vanishing of quark masses. We make phenomenological explorations on two textures of quark mass matrices with the broken Friedberg-Lee symmetry. We present a new pattern of quark mass matrices in agreement with current experimental data. Both analytical and numerical results of our calculations are discussed in detail.

PACS number(s): 11.30.Er, 12.15.Ff

*E-mail: renp@mail.ihep.ac.cn
I. INTRODUCTION

Recent experiments [1] have provided us with precise data on the Cabibbo-Kobayashi-Maskawa (CKM) flavor mixing matrix [2] and quark masses. In order to interpret the hierarchical structure of the observed quark mass spectrum and that of the observed flavor mixing parameters, as well as the observed CP violation in hadronic weak interactions, many theoretical and phenomenological models of quark mass matrices have been proposed [3,4]. Among them, the scenarios based on possible flavor symmetries are particularly simple, suggestive and predictive.

In this paper, we focus our interest on a new symmetry of quark mass operators proposed recently by Friedberg and Lee (FL) [5]. While for the lepton sector this symmetry has been explored a lot [6,7], its possible significance for the quark sector has to be seen. Without loss of generality, the quark mass matrices $M_u$ (up-type) and $M_d$ (down-type) can always be taken to be Hermitian in the standard model or its extensions which have no flavor-changing right-handed currents [8]. Thus the quark mass term in the Lagrangian can be written as

$$-L_{\text{mass}} = (u^c t) M_u \begin{pmatrix} u \\ c \\ t \end{pmatrix} + (d^s b) M_d \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{1}$$

where $M_u$ and $M_d$ are both Hermitian matrices. As defined by FL [5], when $L_{\text{mass}}$ has the FL symmetry, it should be invariant under the transformations

$$u \rightarrow u + \lambda^1_uz, \quad c \rightarrow c + \lambda^2_uz, \quad t \rightarrow t + \lambda^3_uz;$$
$$d \rightarrow d + \lambda^1_qw, \quad s \rightarrow s + \lambda^2_qw, \quad b \rightarrow b + \lambda^3_qw. \tag{2}$$

Here $\lambda^i_q$ (for $i = 1, 2, 3$ and $q = u, d$) are complex constants, while $z$ and $w$ are arbitrary space-time independent elements of the Grassmann algebra. Considering this arbitrariness of $z$ and $w$, we find that this invariance requires $\det M_u = 0$ and $\det M_d = 0$, and the vector $\lambda_q = (\lambda^1_q, \lambda^2_q, \lambda^3_q)^T$ (for $q = u$ or $d$) to satisfy $M_q\lambda_q = 0$. In other words, the FL symmetry implies the existence of zero mass eigenvalues of $M_u$ and $M_d$. On the other hand, if there are zero mass eigenvalues for both $M_u$ and $M_d$, then there will be $\lambda_q = (\lambda^1_q, \lambda^2_q, \lambda^3_q)^T$ (for $q = u$ or $d$) as the solution for $M_q\lambda_q = 0$, and $L_{\text{mass}}$ will have the FL symmetry under the transformations in Eq. (2). In one word, the FL symmetry of quark mass operators is equivalent to the vanishing of quark mass eigenvalues. This conclusion can be easily extended to the lepton sector.

A specific parametrization of quark mass matrices proven to be convenient in the analysis of the FL symmetry is

$$M_q = \begin{pmatrix} \gamma_q + \beta_q|\eta_q|^2 & -\gamma_q\eta_q & -\gamma_q\zeta_q^* \\ -\beta_q\eta_q^* & \beta_q + \alpha_q|\xi_q|^2 & -\alpha_q\xi_q \\ -\gamma_q\zeta_q^* & -\alpha_q\xi_q^* & \alpha_q + \gamma_q|\xi_q|^2 \end{pmatrix} \tag{3}$$

for $q = u$ or $d$. Among the parameters, $\alpha_q$, $\beta_q$ and $\gamma_q$ are real, while $\xi_q$, $\eta_q$ and $\zeta_q$ are...
complex. One can find

$$\det M_q = \alpha_q \beta_q \gamma_q |1 - \xi_q \eta_q \zeta_q|^2. \tag{4}$$

For $\det M_q = 0$ we obtain two cases: 1) $\alpha_q = 0$, $\beta_q = 0$ or $\gamma_q = 0$; and 2) $\xi_q \eta_q \zeta_q = 1$. In the second case, if we have $\arg(\xi_q) = \arg(\eta_q) = \arg(\zeta_q)$, then the matrix $U = \text{diag} \{e^{i[\arg(\xi_u) + \arg(\eta_u)]}, e^{i[\arg(\xi_d) + \arg(\eta_d)]}, 1\}$ can make $M_u$ and $M_d$ real through the unitary transformations $U^\dagger M_u U$ and $U^\dagger M_d U$, which implies no CP violation in the quark sector. Furthermore, if we have $\beta_u = \gamma_u |\zeta_u|^2$ and $\beta_d = \gamma_d |\zeta_d|^2$ together with $\xi_u = \xi_d$ and $\eta_u = \eta_d$, the commutator of $M_u$ and $M_d$ will be zero, which denotes no flavor mixing in the quark sector.

According to the experimental data about quark masses, the FL symmetry should be broken at low energy scales. In the first case, the texture zeros are responsible for maintaining the FL symmetry, and breaking the texture zeros will bring nonzero masses to the quark mass spectrum. In the second case, either $\arg(\xi_q) + \arg(\eta_q) + \arg(\zeta_q) \neq 2n\pi$ ($n = 0, \pm 1, \pm 2, ...$) [5] or $|\xi_q||\eta_q||\zeta_q| \neq 1$ can help break the FL symmetry.

The main purpose of this work is to analyze two specific textures of quark mass matrices with the broken FL symmetry, in the assumption that $\beta_u = \gamma_u |\zeta_u|^2$ and $\beta_d = \gamma_d |\zeta_d|^2$ always hold, and all the parameters keep real before we add CP-violating phases into the off-diagonal elements of $M_u$ and $M_d$ to break the FL symmetry. Note that one of the two textures has been analyzed in Ref. [5]. The present paper is different from the previous one not only because we point out a generic correlation between the FL symmetry of quark mass operators and the vanishing of quark masses but also because our discussions are essentially new in two aspects. (1) We shall follow both analytical and numerical procedures to explore the textures. Our analytical results are more useful for further phenomenological explorations than those obtained in Ref. [5], and our numerical results help find the possible parameter space. (2) We shall propose a new pattern of quark mass matrices. Although Friedberg and Lee have demonstrated a seemingly phenomenologically-allowed texture, we find that it is not completely compatible with current experimental data. In contrast, our new pattern is fully consistent with the experimental data.

The remaining parts of this paper are organized as follows. In Sec. II, we present a phenomenological study of the quark mass matrix texture in the FL ansatz [5], where CP-violating phases are located in (2,3) and (3,2) elements of $M_u$ and $M_d$. In Sec. III, we propose and explore a new ansatz where CP-violating phases are located in (1,2) and (2,1) elements of $M_u$ and $M_d$. Finally, we make a brief summary in Sec. IV.

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1Given real $\xi_q$, $\eta_q$ and $\zeta_q$, Eq. (3) will reproduce the parametrization of the $3 \times 3$ real symmetric matrix in Ref. [5].

2One can verify this point from Eq. (9) in Sec. II, where conditions $\xi_u = \xi_d$ and $\eta_u = \eta_d$ lead to $\theta_u = \theta_d$ and $\phi_u = \phi_d$, and imply an identity CKM matrix with no flavor mixing.
II. THE FL ANSATS

In our assumption, the quark mass matrix with CP invariance is \((q = u \text{ or } d)\)

\[
M_q^0 = \begin{pmatrix}
\beta_q(1 + \xi_q^2)\eta_q^2 & -\beta_q\eta_q & -\beta_q\xi_q\eta_q \\
-\beta_q\eta_q & \beta_q + \alpha_q\xi_q^2 & -\alpha_q\xi_q\eta_q \\
-\beta_q\xi_q\eta_q & -\alpha_q\xi_q & \alpha_q + \beta_q
\end{pmatrix},
\]

(5)

where all the parameters are real. We introduce several new variables by defining

\[
\xi_u = \tan \phi_u, \quad \xi_d = \tan \phi_d, \quad \eta_u = \tan \theta_u \cos \phi_u, \quad \eta_d = \tan \theta_d \cos \phi_d
\]

(6)

with \(\phi_{u,d} \in (-\frac{\pi}{2}, \frac{\pi}{2})\) and \(\theta_{u,d} \in (-\frac{\pi}{2}, \frac{\pi}{2})\). The eigenvalues and eigenvectors are

\[
m_u^0 = 0, \quad m_c^0 = \beta_u \sec^2 \theta_u, \quad m_t^0 = \alpha_u \sec^2 \phi_u + \beta_u, \\
m_d^0 = 0, \quad m_s^0 = \beta_d \sec^2 \theta_d, \quad m_b^0 = \alpha_d \sec^2 \phi_d + \beta_d
\]

(7)

and

\[
|u\rangle_0 = \begin{pmatrix}
\cos \theta_u \\
n \sin \theta_u \cos \phi_u \\
n \sin \theta_u \sin \phi_u
\end{pmatrix},
|c\rangle_0 = \begin{pmatrix}
-n \sin \theta_u \\
n \cos \theta_u \cos \phi_u \\
n \cos \theta_u \sin \phi_u
\end{pmatrix},
|t\rangle_0 = \begin{pmatrix}
0 \\
n -\sin \phi_u \\
n \cos \phi_u
\end{pmatrix},
|d\rangle_0 = \begin{pmatrix}
\cos \theta_d \\
n \sin \theta_d \cos \phi_d \\
n \sin \theta_d \sin \phi_d
\end{pmatrix},
|s\rangle_0 = \begin{pmatrix}
-n \sin \theta_d \\
n \cos \theta_d \cos \phi_d \\
n \cos \theta_d \sin \phi_d
\end{pmatrix},
|b\rangle_0 = \begin{pmatrix}
0 \\
n -\sin \phi_d \\
n \cos \phi_d
\end{pmatrix}.
\]

(8)

Here \(M_u^0\) and \(M_d^0\) can be diagonalized respectively by unitary matrices \(V_u^0 = \{|u\rangle_0, |c\rangle_0, |t\rangle_0\}\) and \(V_d^0 = \{|d\rangle_0, |s\rangle_0, |b\rangle_0\}\) through the unitary transformations \(V_u^0M_u^0V_u^0\) and \(V_d^0M_d^0V_d^0\). Therefore without CP violation, we derive the CKM matrix \(V_{CKM}^0\) from \(V_{CKM}^0 = V_u^0V_d^0\). We have

\[
V_{CKM}^0 = \begin{pmatrix}
\cos \theta_u \cos \theta_d + \sin \theta_u \sin \theta_d \cos \phi & \cos \theta_u \sin \theta_d + \sin \theta_u \cos \theta_d \sin \phi & \sin \theta_u \sin \phi \\
-\sin \theta_u \cos \theta_d + \cos \theta_u \sin \theta_d \cos \phi & \sin \theta_u \sin \theta_d + \cos \theta_u \cos \theta_d \sin \phi & \cos \theta_u \sin \phi \\
-\sin \theta_u \sin \phi & -\cos \theta_u \sin \phi & \cos \phi
\end{pmatrix}
\]

(9)

with \(\phi = \phi_u - \phi_d\).

By adding complex phases into \((2,3)\) and \((3,2)\) elements, we obtain \(M_q\) \((q = u \text{ or } d)\) in the FL ansatz as

\[
M_q = \begin{pmatrix}
\beta_q(1 + \xi_q^2)\eta_q^2 & -\beta_q\eta_q & -\beta_q\xi_q\eta_q \\
-\beta_q\eta_q & \beta_q + \alpha_q\xi_q^2 & -\alpha_q\xi_q\eta_q \\
-\beta_q\xi_q\eta_q & -\alpha_q\xi_q & \alpha_q + \beta_q
\end{pmatrix}.
\]

(10)

It can be written as \(M_q = M_q^0 + M_q'\), where \(M_q'\) is
\[
M'_q = \alpha_q \tan \phi_q \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 1 - e^{-i\chi_q} \\
0 & 1 - e^{i\chi_q} & 0
\end{pmatrix}.
\] (11)

The eigenvalue equations are
\[
m(m - m^0_e) (m - m^0_t) = \det M_u ,
m(m - m^0_s) (m - m^0_b) = \det M_d
\] (12)

with
\[
\det M_q = 2\alpha_q \beta_q^2 \tan^2 \theta_q \sin^2 \phi_q (1 - \cos \chi_q) .
\] (13)

Neglecting corrections of \(O(m_u/m_e), O(m_u/m_t), O(m_d/m_s)\) and \(O(m_d/m_b)\), we obtain
\[
\alpha_u \approx (m_t - m_s \cos^2 \theta_u) \cos^2 \phi_u , \beta_u \approx m_e \cos^2 \theta_u ,
\alpha_d \approx (m_b - m_s \cos^2 \theta_d) \cos^2 \phi_d , \beta_d \approx m_s \cos^2 \theta_d
\] (14)

from Eqs. (7), (12) and (13) in a good approximation.

In order to obtain the analytical approximations of elements of the CKM matrix with CP violation, we take \(\chi\) from Eqs. (7), (12) and (13) in a good approximation.

According to Eqs. (8) and (11), \(G^q\) reads
\[
G^q = \alpha_q \tan \phi_q (1 - \cos \chi_q) \begin{pmatrix}
\sin^2 \theta_q \sin 2\phi_q & \sin \theta_q \cos \theta_q \sin 2\phi_q & \sin \theta_q \cos 2\phi_q \\
\sin \theta_q \cos \theta_q \sin 2\phi_q & \cos^2 \theta_q \sin 2\phi_u & \cos \theta_q \cos 2\phi_q \\
\sin \theta_q \cos 2\phi_q & \cos \theta_q \cos 2\phi_q & -\sin 2\phi_q
\end{pmatrix}
\]
\[
+ i \alpha_q \tan \phi_q \sin \chi_q \begin{pmatrix}
0 & 0 & \sin \theta_q \\
0 & 0 & \cos \theta_q \\
-\sin \theta_q & -\cos \theta_q & 0
\end{pmatrix}.
\] (15)

Assuming small CP violation and small \(\chi_{u,d}\), we write \(G^q\) to the lowest order in \(\chi_q\) as
\[
G^q = i \alpha_q \tan \phi_q \sin \chi_q \begin{pmatrix}
0 & 0 & \sin \theta_q \\
0 & 0 & \cos \theta_q \\
-\sin \theta_q & -\cos \theta_q & 0
\end{pmatrix} + \mathcal{O} \left(\chi_q^2\right).
\] (16)

The corresponding eigenvectors with the perturbation are given to the first order in \(\chi_{u,d}\) by
\[
|u\rangle = |u\rangle_0 - \frac{i \alpha_u \tan \phi_u \sin \chi_u \sin \theta_u}{m_u - m_t^0} |t\rangle_0 ,
\]
\[
|c\rangle = |c\rangle_0 - \frac{i \alpha_u \tan \phi_u \sin \chi_u \cos \theta_u}{m_c - m_t^0} |t\rangle_0 ,
\]
\[
|t\rangle = |t\rangle_0 + \frac{i \alpha_u \tan \phi_u \sin \chi_u \sin \theta_u}{m_t - m_u^0} |u\rangle_0 + \frac{i \alpha_u \tan \phi_u \sin \chi_u \cos \theta_u}{m_t - m_c^0} |c\rangle_0 ,
\]
\[
|d\rangle = |d\rangle_0 - \frac{i \alpha_d \tan \phi_d \sin \chi_d \sin \theta_d}{m_d - m_u^0} |b\rangle_0 ,
\]
\[
|s\rangle = |s\rangle_0 - \frac{i \alpha_d \tan \phi_d \sin \chi_d \cos \theta_d}{m_s - m_b^0} |b\rangle_0 ,
\]
\[
|b\rangle = |b\rangle_0 + \frac{i \alpha_d \tan \phi_d \sin \chi_d \sin \theta_d}{m_b - m_d^0} |d\rangle_0 + \frac{i \alpha_d \tan \phi_d \sin \chi_d \cos \theta_d}{m_b - m_s^0} |s\rangle_0 .
\] (17)
Furthermore, noticing the hierarchical structure of the observed quark mass spectrum as well as that of the observed flavor mixing parameters, we arrive at the following analytical approximations \(^3\) of the CKM matrix elements:

\[
V_{us} = -\cos \theta_u \sin \theta_d + \sin \theta_u \cos \theta_d \cos \phi \\
+ i \sin \theta_u \cos \theta_d \sin \phi \left[ \sin \chi_u \sin \phi_u \cos \phi_u + \sin \chi_d \sin \phi_d \cos \phi_d \right], \\
V_{cb} = \cos \theta_u \left[ \sin \phi + i \cos \phi \left( \sin \chi_d \sin \phi_d \cos \phi_d - \sin \chi_u \sin \phi_u \cos \phi_u \right) \right], \\
V_{ub} = \sin \theta_u \left[ \sin \phi + i \cos \phi \left( \sin \chi_d \sin \phi_d \cos \phi_d - \sin \chi_u \sin \phi_u \cos \phi_u \right) \right].
\]

(18)

It is well known that nine elements of the CKM matrix \(V\) have six orthogonal relations, corresponding to six triangles in the complex plane \([3]\). Among them, the unitarity triangle defined by 
\[
V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0
\]
is of particular interest for the study of CP violation at \(B\)-meson factories \([1]\). Three inner angles of this triangle are commonly denoted as 
\[
\alpha = \arg \left( -\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right), \\
\beta = \arg \left( -\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}} \right), \\
\gamma = \arg \left( -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right).
\]

(19)

So far the angle \(\beta\) has unambiguously been measured from the CP-violating asymmetry in \(B_d^0\) vs \(\bar{B}_d^0\) \(\rightarrow J/\psi K_S\) decays \([9]\), while current experimental data on \(\alpha\) and \(\gamma\) are not accurate. The approximate result of \(\beta\) for the FL ansatz is

\[
\tan \beta = \tan \phi \sin \theta_u \cos \theta_d \frac{\sin \chi_u \sin \phi_u \cos \phi_u + \sin \chi_d \sin \phi_d \cos \phi_d}{-\sin \theta_u \cos \theta_d + \cos \theta_u \sin \theta_d \cos \phi}.
\]

(20)

Given \(|V_{us}| \sim 0.2\), \(|V_{cb}| \sim 0.04\), \(|V_{ub}| \sim 0.004\) and \(\sin 2\beta \sim 0.7\) \([1]\), we find it difficult for the predictions of this ansatz to agree with the experimental data. In order to satisfy the constraint conditions \(|V_{cb}| \sim 0.04\) and \(|V_{ub}| \sim 0.004\), \(\theta_u \sim 0.1\) and \(\phi \sim 0.04\) must hold. Then we take into consideration the analytical approximation of \(V_{us}\), and find that \(|V_{us}| \sim 0.2\) requires

\[
| -\sin \theta_u \cos \theta_d + \cos \theta_u \sin \theta_d \cos \phi | \approx | -\cos \theta_u \sin \theta_d + \sin \theta_u \cos \theta_d \cos \phi | \sim 0.2.
\]

However, these requirements make it impossible to obtain \(\tan \beta \sim 0.4\) in order to satisfy the constraint \(\sin 2\beta \sim 0.7\). In such a semi-analytical way, we find out that the texture of quark mass matrices proposed by FL \([5]\) is not completely compatible with current experimental data.

In the numerical way, we confirm that this ansatz is disfavored by the experimental data. According to Eqs. (7), (12) and (13), by setting six quark masses as input values, we can

\(^3\)For more details, see Appendix A.
solve $\alpha_{u,d}, \beta_{u,d}$ and $\chi_{u,d}$ in terms of $\theta_{u,d}$ and $\phi_{u,d}$, and put these four variables under restraint of the four experimental observables $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and sin $2\beta$. Our input quark masses at the $M_Z$ scale are [10]

\[
\begin{align*}
    m_u &= 1.28^{+0.50}_{-0.42} \; \text{MeV} \\
    m_c &= 0.624 \pm 0.083 \; \text{GeV} \\
    m_t &= 172.5 \pm 3.0 \; \text{GeV} \\
    m_d &= 2.91^{+1.24}_{-1.20} \; \text{MeV} \\
    m_s &= 55^{+16}_{-15} \; \text{MeV} \\
    m_b &= 2.89 \pm 0.09 \; \text{GeV}.
\end{align*}
\]

And our input experimental constraint conditions on the CKM matrix are [1]

\[
\begin{align*}
    |V_{us}| &= 0.2257 \pm 0.0021 , \\
    |V_{cb}| &= (41.6 \pm 0.6) \times 10^{-3} , \\
    |V_{ub}| &= (4.31 \pm 0.30) \times 10^{-3} , \\
    \sin 2\beta &= 0.687 \pm 0.032 .
\end{align*}
\]

Fig. 1 illustrates the outputs of $|V_{ub}|$ and sin $2\beta$ in this ansatz, which have been constrained by the experimental data on $|V_{us}|$ and $|V_{cb}|$. It is clear that the prediction on $|V_{ub}|$ is too large to agree with the measurement $|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}$. Hence this ansatz is actually ruled out by current experimental data.

III. A NEW ANSATZ

By adding CP-violating phases into (1,2) and (2,1) elements, we obtain our new texture of quark mass matrices as

\[
M_q = \left( \begin{array}{ccc} 
\beta_q(1 + \xi^2)\eta_q^2 & -\beta_q \eta_q e^{-i\chi_q} & -\beta_q \xi q \eta_q \\
-\beta_q \eta_q e^{i\chi_q} & \beta_q + \alpha_q \xi q^2 & -\alpha_q \xi q \\
-\beta q \xi q \eta_q & -\alpha_q \xi q & \alpha_q + \beta_q 
\end{array} \right) \tag{21}
\]

for q = u or d. It can be written as $M_q = M_q^0 + M_q''$, where $M_q''$ is

\[
M_q'' = \beta_q \tan \theta_q \cos \phi_q \left( \begin{array}{ccc}
0 & 1 - e^{-i\chi_q} & 0 \\
1 - e^{i\chi_q} & 0 & 0 \\
0 & 0 & 0
\end{array} \right) . \tag{22}
\]

The equations of eigenvalues and estimations of parameters $\alpha_{u,d}$ and $\beta_{u,d}$ are the same as Eqs. (12)-(14) in Sec. II.

Now we introduce the perturbation matrix $H^q = V_q^0 \tilde{M}_q'' V_q^0$ as

\[
\begin{align*}
H^q &= \beta_q \tan \theta_q \cos \phi_q (1 - e^{-i\chi_q}) \left( \begin{array}{ccc}
\sin \theta_q \cos \theta_q \cos \phi_q & \cos^2 \theta_q \cos \phi_q & -\cos \theta_q \sin \phi_q \\
-\sin^2 \theta_q \cos \phi_q & \sin \theta_q \cos \theta_q \cos \phi_q & \sin \theta_q \sin \phi_q \\
0 & 0 & 0
\end{array} \right) \\
&\quad + \beta_q \tan \theta_q \cos \phi_q (1 - e^{i\chi_q}) \left( \begin{array}{ccc}
\sin \theta_q \cos \theta_q \cos \phi_q & -\sin^2 \theta_q \cos \phi_q & 0 \\
\cos^2 \theta_q \cos \phi_q & -\sin \theta_q \cos \theta_q \cos \phi_q & 0 \\
-\cos \theta_q \sin \phi_q & \sin \theta_q \cos \theta_q \cos \phi_q & 0
\end{array} \right) . \tag{23}
\end{align*}
\]

And the corresponding eigenvectors with the perturbation are
\begin{equation}
|u\rangle = |u\rangle_0 + \frac{H_u^{u_2}}{m_u - m_c^0} |c\rangle_0 + \frac{H_{31}^u}{m_u - m_t^0} |t\rangle_0 ,
\end{equation}
\begin{equation}
|c\rangle = |c\rangle_0 + \frac{H_u^{u_1}}{m_c - m_t^0} |u\rangle_0 + \frac{H_{32}^u}{m_c - m_t^0} |t\rangle_0 ,
\end{equation}
\begin{equation}
|t\rangle = |t\rangle_0 + \frac{H_{u_3}^u}{m_t - m_u^0} |u\rangle_0 + \frac{H_{u_2}^{u_3}}{m_t - m_u^0} |c\rangle_0 ,
\end{equation}
\begin{equation}
|d\rangle = |d\rangle_0 + \frac{H_d^{d_2}}{m_d - m_s^0} |s\rangle_0 + \frac{H_{31}^d}{m_d - m_b^0} |b\rangle_0 ,
\end{equation}
\begin{equation}
|s\rangle = |s\rangle_0 + \frac{H_{d_2}^d}{m_s - m_d^0} |d\rangle_0 + \frac{H_{32}^d}{m_s - m_d^0} |b\rangle_0 ,
\end{equation}
\begin{equation}
|b\rangle = |b\rangle_0 + \frac{H_{d_3}^d}{m_b - m_d^0} |d\rangle_0 + \frac{H_{d_3}^{d_3}}{m_b - m_d^0} |s\rangle_0 .
\end{equation}

In view of the hierarchical structure of the observed quark mass spectrum and that of the observed flavor mixing parameters, we arrive at the following analytical estimations\textsuperscript{4}:

\begin{equation}
V_{us} = - \cos \theta_u \sin \theta_d + \sin \theta_u \cos \theta_d \cos \phi \\
- \sin \theta_u \cos \theta_u \cos^2 \phi_u \left[ \sin \theta_u \sin \theta_d + \cos \theta_u \cos \theta_d \cos \phi \right] \left[ (1 - \cos \chi_u) \cos 2\theta_u + i \sin \chi_u \right] \\
+ \sin \theta_d \cos \theta_d \cos^2 \phi_u \left[ \cos \theta_u \cos \theta_d + \sin \theta_u \sin \theta_d \cos \phi \right] \left[ (1 - \cos \chi_d) \cos 2\theta_d + i \sin \chi_d \right],
\end{equation}
\begin{equation}
V_{cb} = \cos \theta_u \sin \phi \left[ 1 + \sin^2 \theta_u \cos^2 \phi_u \left[ (1 - \cos \chi_u) \cos 2\theta_u + i \sin \chi_u \right] \right],
\end{equation}
\begin{equation}
V_{ub} = \sin \theta_u \sin \phi \left[ 1 - \cos^2 \theta_u \cos^2 \phi_u \left[ (1 - \cos \chi_u) \cos 2\theta_u + i \sin \chi_u \right] \right],
\end{equation}
\begin{equation}
\tan \beta = \sin \theta_u \sin \chi_d \cos \theta_u \cos^2 \phi_d \\
- \sin \chi_u \cos \theta_u \cos^2 \phi_u \left[ \cos \theta_u \cos \theta_d + \sin \theta_u \sin \theta_d \cos \phi \right].
\end{equation}

Given $|V_{us}| \sim 0.2$, $|V_{cb}| \sim 0.04$, $|V_{ub}| \sim 0.004$ and $\sin 2\beta \sim 0.7$\textsuperscript{1}, we find it possible for the predictions of this new ansatz to accord with the experimental data. For instance, if we have $\theta_u \sim 0.1$, $\theta_d \sim 0.26$, $\phi_u \sim 0.02$, $\phi_d \sim -0.02$, $\chi_u = 0$ and $\chi_d \sim 0.72$, the four experimental constraints can be obviously satisfied.

Following the same procedure of numerical calculations in Sec. II, we find out that this new texture of quark mass matrices is consistent with current experimental data. The allowed parameter space is given in Fig. 2, for $\phi_{u,d} \in (0, \frac{\pi}{2})$ and $\theta_{u,d} \in (0, \frac{\pi}{2})$. And the correlations between $m_{u,d}$ and $\chi_{u,d}$ are presented in Fig. 3 for $\chi_{u,d} \in (0, \pi)$, which indicate that $m_u(m_d)$ is physically generated from the CP-violating phase $\chi_u(\chi_d)$, as demonstrated in Eq. (13).

IV. SUMMARY

Keeping with the long-term interest in building phenomenological models of fermion mass matrices based on flavor symmetries and favored by current experimental data, we have

\textsuperscript{4}There are more details in Appendix B.
carried out a further study of the Friedberg-Lee (FL) symmetry of quark mass operators and its explicit breaking. We have illustrated the generic correlation between the vanishing masses and the FL symmetry, and classified the ways to break the FL symmetry and obtain nonzero masses. Using current experimental data, we have analyzed two specific patterns of quark mass matrices: one is proposed by Friedberg and Lee, and the other by the author. The latter is phenomenologically allowed, and may serve as a useful example which might shed light on the underlying dynamics responsible for the generation of fermion masses and the origin of CP violation. While exploring other textures of fermion mass matrices with the broken FL symmetry is still arresting, the FL symmetry at high energy scales and the intensional meaning of the invariance under the fermion field translations are calling for much more attentions of theorists, indeed.

Acknowledgments: The author would like to thank Prof. Z.Z. Xing for stimulating discussions, constant encouragement, and reading the manuscript. He is also grateful to H. Zhang and S. Zhou for helpful discussions. This work was supported in part by the National Natural Science Foundation of China.
APPENDIX A

In Sec. II, noticing Eq. (14) and neglecting corrections of $O(m_u/m_c)$, $O(m_c/m_t)$, $O(m_d/m_s)$, $O(m_s/m_b)$ and $O(m_d/m_b)$, we obtain the eigenvectors as

\[
|u\rangle = |u\rangle_0 + i \sin \chi_u \sin \phi_u \cos \phi_u \sin \theta_u |t\rangle_0 ,
\]

\[
|c\rangle = |c\rangle_0 + i \sin \chi_u \sin \phi_u \cos \phi_u \cos \theta_u |t\rangle_0 ,
\]

\[
|t\rangle = |t\rangle_0 + i \sin \chi_u \sin \phi_u \cos \phi_u \sin \theta_u |u\rangle_0 + i \sin \chi_u \sin \phi_u \cos \phi_u \cos \theta_u |c\rangle_0 ,
\]

\[
|d\rangle = |d\rangle_0 + i \sin \chi_d \sin \phi_d \cos \phi_d \sin \theta_d |b\rangle_0 ,
\]

\[
|s\rangle = |s\rangle_0 + i \sin \chi_d \sin \phi_d \cos \phi_d \cos \theta_d |b\rangle_0 ,
\]

\[
|b\rangle = |b\rangle_0 + i \sin \chi_d \sin \phi_d \cos \phi_d \sin \theta_d |d\rangle_0 + i \sin \chi_d \sin \phi_d \cos \phi_d \cos \theta_d |s\rangle_0 .
\]

(A1)

The CKM matrix is given by $V_{CKM} = V_u^\dagger V_d$, where we define $V_u = \{|u\rangle, |c\rangle, |t\rangle\}$ and $V_d = \{|d\rangle, |s\rangle, |b\rangle\}$. In this approximation, several elements of the CKM matrix are

\[
V_{us} = V_{us}^0 + \sin \chi_u \sin \phi_u \cos \phi_u \sin \theta_u \sin \chi_d \sin \phi_d \cos \phi_d \cos \theta_u V_{tb}^0
\]

\[
+ i \left[ \sin \chi_d \sin \phi_d \cos \phi_d \cos \theta_u V_{ub}^0 - \sin \chi_u \sin \phi_u \cos \phi_u \sin \theta_u V_{ts}^0 \right] ,
\]

\[
V_{cd} = V_{cd}^0 + \sin \chi_u \sin \phi_u \cos \phi_u \cos \theta_u \sin \chi_d \sin \phi_d \cos \phi_d \sin \theta_d V_{tb}^0
\]

\[
+ i \left[ \sin \chi_d \sin \phi_d \cos \phi_d \sin \theta_d V_{ub}^0 - \sin \chi_u \sin \phi_u \cos \phi_u \cos \theta_u V_{ts}^0 \right] ,
\]

\[
V_{cb} = V_{cb}^0 + \sin \chi_u \sin \phi_u \cos \phi_u \cos \theta_u \sin \chi_d \sin \phi_d \cos \phi_d \left( \sin \theta_d V_{td}^0 + \cos \theta_d V_{ts}^0 \right)
\]

\[
+ i \left[ \sin \chi_d \sin \phi_d \cos \phi_d \sin \theta_d V_{td}^0 + \cos \theta_d V_{cs}^0 \right] - \sin \chi_u \sin \phi_u \cos \phi_u \cos \theta_u V_{tb}^0 \right] ,
\]

\[
V_{ub} = V_{ub}^0 + \sin \chi_u \sin \phi_u \cos \phi_u \sin \theta_u \sin \chi_d \sin \phi_d \cos \phi_d \left( \sin \theta_d V_{td}^0 + \cos \theta_d V_{ts}^0 \right)
\]

\[
+ i \left[ \sin \chi_d \sin \phi_d \cos \phi_d \sin \theta_d V_{td}^0 + \cos \theta_d V_{us}^0 \right] - \sin \chi_u \sin \phi_u \cos \phi_u \sin \theta_u V_{tb}^0 \right] ,
\]

\[
V_{ud} = V_{ud}^0 + \sin \chi_u \sin \phi_u \cos \phi_u \sin \chi_d \sin \phi_d \cos \phi_d \left( \sin \theta_u V_{td}^0 + \cos \theta_u V_{cb}^0 \right)
\]

\[
- i \left[ \sin \chi_u \sin \phi_u \cos \phi_u \sin \theta_u V_{td}^0 + \cos \theta_u V_{ud}^0 \right] - \sin \chi_d \sin \phi_d \cos \phi_d \sin \theta_d V_{tb}^0 \right] ,
\]

\[
V_{tb} = V_{tb}^0 + \sin \chi_u \sin \phi_u \cos \phi_u \sin \chi_d \sin \phi_d \cos \phi_d \left( \sin \theta_u \sin \theta_d V_{ud}^0 + \sin \theta_u \cos \theta_d V_{us}^0
\]

\[
+ \cos \theta_u \sin \theta_d V_{cd}^0 + \cos \theta_u \cos \theta_d V_{cs}^0 \right) + i \left[ \sin \chi_d \sin \phi_d \cos \phi_d \left( \sin \theta_d V_{td}^0 + \cos \theta_d V_{ts}^0 \right) - \sin \chi_u \sin \phi_u \cos \phi_u \left( \sin \theta_u V_{tb}^0 + \cos \theta_u V_{cb}^0 \right) \right] .
\]

(A2)

Since the perturbation in terms of $\chi_u$ and $\chi_d$ is small, we can make further estimations by introducing the experimental data on the CKM matrix. Given current experimental data

[1] $|V_{ud}| = 0.97377 \pm 0.00027$, $|V_{us}| = 0.2257 \pm 0.0021$, $|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}$, $|V_{cd}| = 0.230 \pm 0.011$, $|V_{cs}| = 0.957 \pm 0.017 \pm 0.093$, $|V_{cb}| = (41.6 \pm 0.6) \times 10^{-3}$, $|V_{td}| = (7.4 \pm 0.8) \times 10^{-3}$, $|V_{ts}| = (40.6 \pm 2.7) \times 10^{-3}$, and $|V_{tb}| > 0.78$, we can see the hierarchical structure of the flavor mixing parameters in the CKM matrix and write Eq. (A2) approximately as

\[
V_{us} = - \cos \theta_u \sin \theta_d + \sin \theta_u \cos \theta_d \cos \phi
\]

\[
+ i \sin \theta_u \cos \theta_d \sin \phi \left[ \sin \chi_u \sin \phi_u \cos \phi_u + \sin \chi_d \sin \phi_d \cos \phi_d \right] ,
\]
\[ V_{cd} = -\sin \theta_u \cos \theta_d + \cos \theta_u \sin \theta_d \cos \phi \]
\[ + i \cos \theta_u \sin \theta_d \sin \phi \left[ \sin \chi_u \sin \phi_u \cos \phi_u + \sin \chi_d \sin \phi_d \cos \phi_d \right] , \]
\[ V_{cb} = \cos \theta_u \sin \phi + i \cos \theta_u \cos \phi \left[ \sin \chi_d \sin \phi_d \cos \phi_d - \sin \chi_u \sin \phi_u \cos \phi_u \right] , \]
\[ V_{ub} = i \sin \theta_u \sin \phi + i \sin \theta_u \cos \phi \left[ \sin \chi_d \sin \phi_d \cos \phi_d - \sin \chi_u \sin \phi_u \cos \phi_u \right] , \]
\[ V_{td} = -i \sin \theta_d \sin \phi + i \sin \theta_d \cos \phi \left[ \sin \chi_d \sin \phi_d \cos \phi_d - \sin \chi_u \sin \phi_u \cos \phi_u \right] , \]
\[ V_{tb} = \cos \phi - i \sin \phi \left[ \sin \chi_u \sin \phi_u \cos \phi_u + \sin \chi_d \sin \phi_d \cos \phi_d \right] . \]

(A3)

Then we obtain

\[ -V_{cb}^* V_{cd} V_{td}^* V_{tb} = \cos \theta_u \sin \theta_d | \sin \phi + i \cos \phi \left[ \sin \chi_d \sin \phi_d \cos \phi_d - \sin \chi_u \sin \phi_u \cos \phi_u \right] |^2 \]
\[ \left\{ \cos \phi \left[ -\sin \theta_u \cos \theta_d + \cos \theta_u \sin \theta_d \cos \phi \right] + \right. \]
\[ \left. i \sin \theta_u \cos \theta_d \sin \phi \left[ \sin \chi_u \sin \phi_u \cos \phi_u + \sin \chi_d \sin \phi_d \cos \phi_d \right] \right\} . \] (A4)

Noticing \( \beta = \arg \left( -V_{cb}^* V_{cd} V_{td}^* V_{tb} \right) \), we finally obtain Eqs. (18) and (20) in Sec. II.

**APPENDIX B**

In Sec. III, we notice that \( M''_u \) (\( M''_d \)) is proportional to \( \beta_u (\beta_d) \) and of the order of \( m_u (m_d) \). Instead of assuming small CP violation and calculating to the first order of the CP-violating phases as in Sec. II, here we neglect the contribution of terms \( \mathcal{O}(m_u/m_c) \), \( \mathcal{O}(m_c/m_t) \), \( \mathcal{O}(m_u/m_t) \), \( \mathcal{O}(m_d/m_s) \), \( \mathcal{O}(m_s/m_b) \), and \( \mathcal{O}(m_d/m_b) \), and obtain approximate eigenvectors

\[ |u\rangle = |u\rangle_0 - \sin \theta_u \cos \theta_u \cos^2 \phi_u \left[ (1 - \cos \chi_u) \cos 2\theta_u - i \sin \chi_u \right] |c\rangle_0 , \]
\[ |c\rangle = |c\rangle_0 + \sin \theta_u \cos \theta_u \cos^2 \phi_u \left[ (1 - \cos \chi_u) \cos 2\theta_u + i \sin \chi_u \right] |u\rangle_0 , \]
\[ |t\rangle = |t\rangle_0 , \]
\[ |d\rangle = |d\rangle_0 - \sin \theta_d \cos \theta_d \cos^2 \phi_d \left[ (1 - \cos \chi_d) \cos 2\theta_d - i \sin \chi_d \right] |s\rangle_0 , \]
\[ |s\rangle = |s\rangle_0 + \sin \theta_d \cos \theta_d \cos^2 \phi_d \left[ (1 - \cos \chi_d) \cos 2\theta_d + i \sin \chi_d \right] |d\rangle_0 , \]
\[ |b\rangle = |b\rangle_0 . \] (B1)

Several elements of the CKM matrix are given by

\[ V_{us} = V_{us}^0 - \sin \theta_u \cos \theta_u \cos^2 \phi_u \left[ (1 - \cos \chi_u) \cos 2\theta_u + i \sin \chi_u \right] V_{cs}^0 \]
\[ + \sin \theta_d \cos \theta_d \cos^2 \phi_d \left[ (1 - \cos \chi_d) \cos 2\theta_d + i \sin \chi_d \right] V_{cs}^0 , \]
\[ V_{cd} = V_{cd}^0 + \sin \theta_u \cos \theta_u \cos^2 \phi_u \left[ (1 - \cos \chi_u) \cos 2\theta_u - i \sin \chi_u \right] V_{us}^0 \]
\[ - \sin \theta_d \cos \theta_d \cos^2 \phi_d \left[ (1 - \cos \chi_d) \cos 2\theta_d - i \sin \chi_d \right] V_{cs}^0 , \]
\[ V_{cb} = V_{cb}^0 + \sin \theta_u \cos \theta_u \cos^2 \phi_u \left[ (1 - \cos \chi_u) \cos 2\theta_u - i \sin \chi_u \right] V_{ub}^0 , \]
\[ V_{ub} = V_{ub}^0 - \sin \theta_u \cos \theta_u \cos^2 \phi_u \left[ (1 - \cos \chi_u) \cos 2\theta_u + i \sin \chi_u \right] V_{cb}^0 , \]
\[ V_{td} = V_{td}^0 - \sin \theta_d \cos \theta_d \cos^2 \phi_d \left[ (1 - \cos \chi_d) \cos 2\theta_d - i \sin \chi_d \right] V_{ts}^0 , \]
\[ V_{tb} = V_{tb}^0 . \] (B2)
Having paid attention to the hierarchical structure of the flavor mixing parameters in the CKM matrix, we obtain further approximations that lead to Eq. (25) in Sec. III:

\[ V_{us} = - \cos \theta_u \sin \theta_d + \sin \theta_u \cos \theta_d \cos \phi \]
\[ + \sin \theta_u \cos \theta_u \cos^2 \phi_u \left[ (1 - \cos \chi_u) \cos 2\theta_u + i \sin \chi_u \right] \left[ \sin \theta_u \sin \theta_d + \cos \theta_u \cos \theta_d \cos \phi \right] \]
\[ + \sin \theta_d \cos \theta_d \cos^2 \phi_d \left[ (1 - \cos \chi_d) \cos 2\theta_d + i \sin \chi_d \right] \left[ \cos \theta_u \cos \theta_d + \sin \theta_u \sin \theta_d \cos \phi \right], \]
\[ V_{cd} = - \sin \theta_u \cos \theta_d + \cos \theta_u \sin \theta_d \cos \phi \]
\[ + \sin \theta_u \cos \theta_u \cos^2 \phi_u \left[ (1 - \cos \chi_u) \cos 2\theta_u - i \sin \chi_u \right] \left[ \cos \theta_u \cos \theta_d + \sin \theta_u \sin \theta_d \cos \phi \right] \]
\[ - \sin \theta_d \cos \theta_d \cos^2 \phi_d \left[ (1 - \cos \chi_d) \cos 2\theta_d - i \sin \chi_d \right] \left[ \sin \theta_u \sin \theta_d + \cos \theta_u \cos \theta_d \cos \phi \right], \]
\[ V_{cb} = \cos \theta_u \sin \phi + \sin^2 \theta_u \cos \theta_u \cos^2 \phi_u \sin \phi \left[ (1 - \cos \chi_u) \cos 2\theta_u - i \sin \chi_u \right], \]
\[ V_{ub} = \sin \theta_u \sin \phi + \sin^2 \theta_u \cos \theta_u \cos^2 \phi_u \sin \phi \left[ (1 - \cos \chi_u) \cos 2\theta_u + i \sin \chi_u \right], \]
\[ V_{td} = - \sin \theta_d \sin \phi + \sin \theta_d \cos^2 \theta_d \cos^2 \phi_d \sin \phi \left[ (1 - \cos \chi_d) \cos 2\theta_d - i \sin \chi_d \right], \]
\[ V_{tb} = \cos \phi \]

(B3)

and

\[ -V_{cd}^* V_{td} V_{tb} = \cos \theta_u \sin \theta_d \sin^2 \phi \cos \phi \left[ - \sin \theta_u \cos \theta_d + \cos \theta_u \sin \theta_d \cos \phi \right] \]
\[ + i \sin \theta_u \cos \theta_u \sin \theta_d \sin^2 \phi \cos \phi \left[ \sin \chi_u \cos \theta_u \cos^2 \phi_u \right] \]
\[ - \sin \chi_u \cos \theta_u \cos^2 \phi_u \left( \cos \theta_u \cos \theta_d + \sin \theta_u \sin \theta_d \cos \phi \right). \]

(B4)
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FIG. 1. The ranges of the predictions on $|V_{ub}|$ vs $\sin 2\beta$ when the predictions on $|V_{us}|$ and $|V_{cb}|$ are in agreement with current experimental data in the FL antasz in Sec. II.
FIG. 2. The allowed parameter space of variables $\theta_{u,d}$ and $\phi_{u,d}$ in the new ansatz in Sec. III.
FIG. 3. The correlations between $\chi_{u,d}$ and $m_{u,d}$ in the new ansatz in Sec. III.