Electroweak baryogenesis via bottom transport: complementarity between LHC and future lepton collider probes

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We study the complementarity between the Large Hadron Collider (LHC) and future lepton colliders in probing electroweak baryogenesis induced by an additional bottom Yukawa coupling $\rho_{bA}$. The context is general two Higgs doublet model (g2HDM) where such additional bottom Yukawa coupling can account for the observed baryon asymmetry of the Universe if $\text{Im}(\rho_{bA}) \gtrsim 0.058$. We find that LHC would probe the nominal $\text{Im}(\rho_{bA})$ required for baryogenesis to some extent via $bg \rightarrow bA \rightarrow bZh$ process if $300 \text{ GeV} \lesssim m_A \lesssim 450 \text{ GeV}$, where $A$ is the CP-odd scalar in g2HDM. We show that future electron positron collider such as International Linear Collider with $500 \text{ GeV}$ and $1 \text{ TeV}$ collision energies may offer unique probe for the nominal $\text{Im}(\rho_{bA})$ via $e^+e^- \rightarrow Z^* \rightarrow AH$ process followed by $A, H \rightarrow b\bar{b}$ decays in four b-jets signature. For complementarity we also study the resonant diHiggs productions, which may give an insight into strong first-order electroweak phase transition, via $e^+e^- \rightarrow Z^* \rightarrow AH \rightarrow Ahh$ process in six b-jets signature. We find that 1 TeV collision energy with $\mathcal{O}(1) \text{ ab}^{-1}$ integrated luminosity could offer an ideal environment for the discovery.

I. INTRODUCTION

The discovery of the 125 GeV Higgs boson ($h$) \cite{Aad:2012tfa} was a truly watershed moment that established the standard model (SM) as a correct effective theory at electroweak scale. While the SM has withstood all experimental tests so far, cosmological problems such as baryon asymmetry of the Universe (BAU) and dark matter still remain open and a more fundamental theory must exist in nature.

For the BAU generation, one has to satisfy so-called Sakharov’s conditions \cite{Sakharov:1967dj} (for reviews, see Ref. \cite{Buchmuller:2002rq}), whose core mechanism is already built-in even in the SM. However, the observed parameters in the SM turn out to be inconsistent with successful EWBG due to the insufficient magnitude of CP violation and absence of a first-order electroweak phase transition (EWPT). Generally, various new-physics models are conceivable to circumvent those two issues. Among them, a general two-Higgs-doublet model (g2HDM) \cite{Einhorn:1997ct} is one of the most attractive models from the viewpoints of renormalizability, generality, and testability. It is shown that extra Yukawa couplings of the second and third generation quarks and leptons, which can be complex and flavor violating, could provide CP violation sufficient for BAU. The most efficient EWBG scenario would be a case that the top quark has the $\mathcal{O}(1)$ extra Yukawa coupling, followed by a case in which the sizable top-charm-changing Yukawa coupling is present \cite{Modak:2021kth}. Thoroughgoing study of those collider signatures can be found in Refs. \cite{deGouvea:2010rr,Brüning:2010tb,Modak:2015dyx,Modak:2016dka,Modak:2016qpm,Modak:2017jij,Modak:2018slm}.

Under generous assumptions for bubble wall profiles, the bottom quark could also drive the sufficient BAU if the size of the extra bottom Yukawa coupling is larger than the SM bottom Yukawa coupling to some degree. This bottom-Yukawa-driven EWBG can be significant in a case that the aforementioned Yukawa couplings in the up-type quark sector happen to be real or tiny. In Ref. \cite{Modak:2020zqy}, the present authors studied phenomenological consequences of the bottom-Yukawa-driven EWBG in detail assuming that both extra top and bottom Yukawa couplings are present but the former is real and the latter is roughly twice larger than a necessary bare minimum for BAU. It was found that Large Hadron Collider (LHC) with 1000 fb$^{-1}$ integrated luminosity could examine the scenario, primarily via the process $bg \rightarrow bA \rightarrow bZh$ with final states comprising of 3b-jets and a lepton pair, where $A$ is the CP-odd scalar and $h$ is the 125 GeV Higgs boson in the g2HDM. In Ref. \cite{Modak:2020zqy} it was also shown that the $bg \rightarrow bA \rightarrow bZh$ process would provide a sensitive test for the case of $m_A > m_H + m_Z$, where $H$ is the CP-even heavy scalar. While these processes provide a unique probe to bottom-Yukawa-driven EWBG, they become insensitive if $m_A < m_Z + m_h$ and/or $m_A < m_Z + m_H$. Furthermore, if $m_A > 2m_t$, an achievable significance diminishes if the extra top Yukawa coupling is $\mathcal{O}(1)$.

In this work, we further pursue the bottom-driven EWBG scenario with particular emphasis on complementarity between LHC and the International Linear Collider (ILC). After taking the theoretical and experimental constraints into accounts, we investigate a discovery potential of the EWBG scenario assuming a necessary bare minimum of the extra bottom Yukawa coupling and absence of the extra top Yukawa coupling, which is diagonal parameter space investigated in Ref. \cite{Modak:2020zqy}. In this scenario, we examine the $bg \rightarrow bA \rightarrow bZh$ process at the LHC, and compare with the results in Ref. \cite{Modak:2020zqy}.
also proceed to study detectability of EWBG signatures at the ILC assuming 500 GeV and 1 TeV center-of-mass (CM) energies. We consider the process $e^+e^- \rightarrow AH$ with the $A/H \rightarrow b\bar{b}$ decay as well as the $H \rightarrow hh$ decay, leading to $4b$-jets and $6b$-jets final states, respectively.

The paper is organized as follows. In Sec. II we outline the model framework and the available parameter space for our study. Sec III is dedicated for finding discovery prospect of the $bg \rightarrow bA \rightarrow bZb$ process. In Sec IV we discuss sensitivity of $e^+e^- \rightarrow AH \rightarrow 4b$. We also study the $e^+e^- \rightarrow AH \rightarrow Ahh$ process and the corresponding vertex correction for the trilinear $Hhh$ coupling. We summarize our results with some discussions in Sec. V.

II. FORMALISM AND PARAMETER SPACE

A. Formalism

The most general CP-conserving g2HDM potential in the Higgs basis can be written as [17, 18]

$$V(\Phi, \Phi') = \mu_{11}^2|\Phi|^2 + \mu_{22}^2|\Phi'|^2 - (\mu_2 \Phi^\dagger \Phi' + h.c.) + \eta_1|\Phi|^4 + \eta_2|\Phi|^2|\Phi'|^2 + \eta_4|\Phi'|^4 + \eta_5|\Phi|^2|\Phi'|^2 + h.c.,$$

where the parameters $m^2_{11}, m^2_{22}, m^2_{22}$ and $\eta_i$s are all real. We consider Higgs basis where the vacuum expectation value $v(=246.22$ GeV) arises from the doublet $\Phi$, i.e., $(\Phi) = (0, v/\sqrt{2})^T$, whereas $(\Phi') = (0, 0)^T$, assuming $\mu_{22} > 0$. The minimization condition with respect to the CP-even neutral component of $\Phi$ yields $\mu_{11} = -\frac{1}{2}\eta_1 v^2$, while that of $\Phi'$ gives $\mu_{22} = \frac{1}{2}\eta_6 v^2$. The mixing angle between $h$ and $H$ can be expressed as [17, 18]

$$\cos^2 \gamma = \frac{\eta_1 v^2 - m^2_{22}}{m^2_H - m^2_h}, \quad \sin 2\gamma = \frac{2\eta_6 v^2}{m^2_H - m^2_h}. \quad (2)$$

In the following we use shorthand $c_\gamma$ and $s_\gamma$ for $\cos \gamma$ and $\sin \gamma$ respectively while in the alignment limit $c_\gamma \rightarrow 0$ and $s_\gamma \rightarrow -1$.

The Yukawa sector of the g2HDM is given by [17]

$$\mathcal{L} = -\frac{1}{\sqrt{2}} \sum F_{U,D,L} \tilde{F}_i \left[ -(\lambda^{F}_{ij} s_\gamma + \rho^{F}_{ij} c_\gamma) h + (\lambda^{F}_{ij} c_\gamma + \rho^{F}_{ij} s_\gamma) H - i \text{sgn}(Q_F) \rho_{ij}^A A \right] P_R F_j \left[ (V P_D)_{ij} P_R - (\rho^{L}_{ij} V)_{ij} P_L \right] D_{ij} H^+ - \bar{v}^{i} \rho^{F}_{ij} P_R \bar{L}_{ij} \bar{h} + h.c.,$$

where $P_{L,R} \equiv (1 \mp \gamma_5)/2$, $V$ is CKM matrix, $i, j = 1, 2, 3$ are generation indices, and $U = (u, c, t)^T$, $D = (d, s, b)^T$, $L = (e, \mu, \tau)^T$ and $V = (\nu_e, \nu_\mu, \nu_\tau)^T$ are column vectors in the flavor space. The matrices $\lambda^{F}_{ij} (= \sqrt{2m^2_i \delta_{ij}/v})$ are real and diagonal, while $\rho^{F}_{ij}$ are in general complex and non-diagonal. It is pointed out in Ref. [19] that electric dipole moment (EDM) of the electron could be suppressed if the diagonal elements of $\rho^F_{ij}$ follow the similar hierarchical structures of the SM Yukawa couplings, i.e., $|\rho_{ee}/\rho_{\mu\mu}| \sim \lambda_e/\lambda_{\mu}$, which tempt us to conjecture $|\rho_{ij}/\rho_{jj}| \sim \lambda_i/\lambda_j$ for all the flavor indices. We however consider somewhat offset parameter space motivated by the successful $\rho_{bb}$-EWBG mechanism in which $\text{Im}(\rho_{bb}) = 0.058 (\gtrsim \lambda_b \simeq 0.024)$. Circumvention of the electron EDM constraint in this scenario will be addressed in Sec. II B.

Here we should note that $h, H,$ and $A$ are not CP eigenstates any more when including loop corrections that break CP through $\text{Im}(\rho_{ij})$. However, the loop corrections are small enough to regard the neutral Higgs bosons as the CP as well as mass eigenstates.

For all practical purposes we turn off all $\rho_{ij}$ except for $\rho_{bb}$, however their impact will be discussed in Sec. V.

Primary motivation of this article is to probe the nominal value $\text{Im}(\rho_{bb}) = 0.058$ [15] required for $\rho_{bb}$-EWBG. In general, LHC would offer exquisite probe via $bg \rightarrow bA \rightarrow bZb$ process if $\text{Im}(\rho_{bb}) \gtrsim 0.15$ [15] but the process requires $m_A > m_Z + m_b$. The process $bg \rightarrow bA \rightarrow bZH$ would also offer sensitive probe if $m_A > m_H + m_Z$ [16]. We note that the dependence of the $AZh$ and $AZH$ couplings on the mixing angle $\gamma$ can be found from [5]

$$\frac{g_2}{2c_W} Z_\mu \left[ c_\gamma (h \delta^{\mu\nu} A - A \delta^{\mu\nu} h) - s_\gamma (H \delta^{\mu\nu} A - A \delta^{\mu\nu} H) \right].$$

where $c_W$ and $g_2$ are the Weinberg angle and the $SU(2)_L$ gauge coupling respectively. As discussed in Ref. [15], the nonzero $\gamma$ could have non-negligible impacts on $\rho_{bb}$-EWBG. From the interactions (3) and (4), one can see that the production $bg \rightarrow bA$ does not depend on $\gamma$ and the decays $A \rightarrow ZH$ and $A \rightarrow Zh$ are scaled by $s_\gamma$ and $c_\gamma$, respectively. In the vicinity of the alignment limit $\gamma = -\pi/2$, the $bg \rightarrow bA \rightarrow bZh$ process would provide more sensitive probe of the mixing angle through $c_\gamma$. While $bg \rightarrow bA \rightarrow bZh$ process can exclude the nominal $|\text{Im}(\rho_{bb})| = 0.058$ at HL-LHC if $m_A \sim 300$ GeV, it fails to probe the nominal $\text{Im}(\rho_{bb})$ above $m_A > 2m_t$ if $\rho_{tt} \sim 0.5$ [15]. Here we shall revisit potential of $bg \rightarrow bA \rightarrow bZh$ process to probe nominal $\text{Im}(\rho_{bb})$ for scenarios where $m_A > 2m_t$, but for vanishingly small $\rho_{tt}$.

The $bg \rightarrow bA \rightarrow bZh$ process would become insensitive for $m_A < m_b + m_Z$. In such scenarios future lepton colliders such as ILC or FCCee would offer unique probe for $\rho_{bb}$-EWBG via $e^+e^- \rightarrow Z^* \rightarrow AH$ process [20] followed by $A/H \rightarrow bb$ decays i.e., in four $b$-jets signature. The signature would also receive contribution from $\rho_{bb}$ induced $e^+e^- \rightarrow Z^* \rightarrow bA/H$ process if $A, H$ decays to $bb$. We remark that a similar search $pp \rightarrow Z^* \rightarrow AH \rightarrow bbb$ at the LHC would suffer from overwhelming QCD multijets backgrounds, which prevents us from probing our scenario.

Given the fact that the strong first-order EWPT needs O(1) Higgs quartic couplings, triple Higgs couplings $\phi_i\phi_j\phi_k$ could be potentially large. A sensitive probe for
$Hhh$ coupling is possible via $e^+e^- \to Z^* \to AH \to Ahh$ process (see Ref. [21] for similar discussion). We study this process in six $b$-jets signature. The final state signature would receive contribution from $e^+e^- \to bbH \to bhhh$ if both the $h$ decays to $bb$. The $Hhh$ coupling is defined as the coefficient of the $h^2H$ term in the Higgs potential, from which it follows that [9]

$$\lambda_{Hhh} = \frac{v}{2} \left[ 3c_\gamma s_\gamma^2 \eta_1 + c_\gamma (3c_\gamma^2 - 2) \eta_{45} + 3s_\gamma (1 - 3c_\gamma^2) \eta_6 + 3s_\gamma c_\gamma^2 \eta_T \right],$$

(5)

with $\eta_{45} = \eta_3 + \eta_4 + \eta_5$. For small $c_\gamma$, $\lambda_{Hhh}$ is reduced to

$$\lambda_{Hhh} \simeq -\frac{c_\gamma}{2v} \left[ m_H^2 - 4\mu_{22}^2 + 3c_\gamma \eta_T + \mathcal{O}(c_\gamma^2) \right],$$

(6)

which implies that $\lambda_{Hhh} \to 0$ as $c_\gamma \to 0$. The approximate expression (6) does not differ from the exact one (5) by more than about 1.5% in our benchmark points (BPs) described below. We also notice that $\lambda_{Hhh}$ is always negative in our chosen BPs, which could be important when discussing one-loop corrections. We primarily focus on tree-level $Hhh$ coupling however we will discuss higher-order corrections to $\lambda_{Hhh}$ and its impact on strong first-order EWPT in Sec. IV B. A probe for $Hhh$ coupling in the context of $\rho_{bb}$-EWBG would be indeed possible at the LHC via $b\bar{b} \to H \to hh$ and $bg \to bH \to bhh$. However we have checked that such processes are beyond the scope of the HL-LHC for nominal value $|\text{Im}(\rho_{bb})| = 0.058$ primarily due to overwhelming SM QCD background such as multi-jets and $tt$+jets.

**B. Constraints and parameter space**

Let us find the allowed parameter space for $m_A$, $m_H$ and $m_{H^\pm}$ such that EWBG is possible. As widely known, $\eta_i v^2$, where $\eta_i$ are some linear combinations of $\eta$'s whose magnitude is $\mathcal{O}(1)$, should be greater than $\mu_{22}^2$ in order to induce the strong first-order EWPT, leading to lower bounds of the heavy Higgs bosons. On the other hand, since the quartic couplings are enforced to satisfy perturbativity and tree-level unitarity, their sizes cannot exceed certain values, e.g., $4\pi$, which sets upper bounds of the heavy Higgs bosons. Therefore, typical mass window for the strong first-order EWPT would be $m_{A,H,H^\pm} \in [200, 600]$ GeV.

The parameters in Eq. (1) are required to satisfy perturbativity, tree-level unitarity and vacuum stability conditions, for which we utilized the public tool 2HDMC [24]. We choose three BPs summarized in Table I that satisfy aforementioned three theoretical constraints, electroweak precision measurements, and strong first-order EWPT as needed for EWBG.

Having fixed the BPs, we now turn our attention to constraints on $\text{Im}(\rho_{bb})$. There exist several indirect and direct searches that can constrain the parameter space for $\text{Im}(\rho_{bb})$. For nonvanishing $\rho_{tt}$, $\text{Im}(\rho_{bb})$ receives meaningful constraints from the branching ratio measurement of $B \to X_s \gamma$ ($B(B \to X_s \gamma)$) and the asymmetry of the CP asymmetry between the charged and neutral $B \to X_s \gamma$ decays ($A_{\Delta A_{\text{CP}}}$) [15]. However as we focus on parameter space where $\rho_{tt}$ is small, such constraints practically allow an order of magnitude larger $\text{Im}(\rho_{bb})$ than that of the nominal value required for $\rho_{bb}$-EWBG. Therefore we do not discuss such constraint here and redirect readers to Refs. [15] for further details.

The Higgs signal strength measurements by ATLAS and CMS would however provide some constraints primarily due to our choice of $c_\gamma = 0.1$, which should be clear from the Yukawa couplings of $h$ in Eq. (3). Although no combined analysis has been performed, both CMS and ATLAS collaborations have updated the $h$ boson coupling measurements with full Run 2 data [25, 26]. The central value and $1\sigma$ error bar is provided for the coupling modifier $\kappa_h$, which is defined as the ratio between the observed and SM partial rates (see Refs. [25, 26] for its definition), by both the collaboration. We identify $\kappa_h$
from Eq. (3) as:
\[ |\kappa_b| = \sqrt{\left(-s_\gamma + \frac{c_\gamma \text{Re}(\rho_{bb})}{\lambda_b}\right)^2 + \left(\frac{\text{Im}(\rho_{bb})c_\gamma}{\lambda_b}\right)^2}, \tag{7} \]

where \( \lambda_b = \sqrt{2m_b/v} \) with \( m_b \) is the SM mass of \( b \) quark evaluated at \( m_h \). The CMS found \( \kappa_b = 1.18^{+0.19}_{-0.27} \) whereas ATLAS found \( \kappa_b = 0.98^{+0.14}_{-0.13} \) [26]. Allowing 2\( \sigma \) error bars on these measurements we show these limits in Fig. 1 in the Re(\( \rho_{bb} \))-Im(\( \rho_{bb} \)) plane by purple (CMS) and cyan (ATLAS) shaded regions. While finding the limits we simply symmetrized the error bars of CMS and ATLAS measurements. For comparison we also overlay the nominal parameter space for \( \rho_{bb} \)-EWBG (|\( \text{Im}(\rho_{bb}) \)| > 0.058) by the red solid lines in Fig. 1. It is clear that \( \kappa_b \) measurements are not able to cover the entire \( \rho_{bb} \)-EWBG region. This is primary due to the fact that CP-violating term \( \text{Im}(\rho_{bb}) \) does not interfere with the SM part, thereby being more suppressed by the mixing angle \( c_\gamma \), as can be seen from Eq. (7). It would be useful to compare the sensitivity of future \( e^+e^- \) collider in probing \( \kappa_b \). In this regard we focus on the ILC, which is expected to measure \( \kappa_b \) within 1.1% and 0.58% [27] uncertainties at 1\( \sigma \) in its \( \sqrt{s} = 250 \) GeV (denoted as ILC250) and, combined 250 GeV and 500 GeV data (denoted as ILC500). Allowing 2\( \sigma \) error we illustrate these limits in Fig. 1 by blue dotted and solid lines respectively, where in both cases the white crescent shaped regions within the lines are allowed. For comparison the HL-LHC is expected to measure \( \kappa_b \) with \( \approx 6\% \) accuracy [28], which we do not show in Fig. 1. It is clear that sufficient parameter space for \( \rho_{bb} \)-EWBG would survive even after various precise measurements of \( hbb \) coupling.

There also exist some heavy Higgs searches from ATLAS and CMS that also constrain \( \text{Re}(\rho_{bb}) \). E.g., it was found [15, 16] that the most relevant constraints arise from heavy neutral Higgs boson production with at least one \( b \)-jet followed by \( bb \) decay [29] and, heavy charged Higgs searches \( pp \to t(h)H^\pm \) with \( H^+/(H^-) \to t\bar{b}/b\bar{b} \) decays [30, 31] (see also e.g. Refs. [32, 33]). As we primarily focus on parameter space where \( |\text{Im}(\rho_{bb})| \approx 0.058 \) and, the fact that such searches excludes \( \text{Im}(\rho_{bb}) \gtrsim 0.25 \) [15, 16] for the sub-TeV mass range, we refrain a detailed discussion of these here and redirect readers to Refs. [15, 16] for further discussion.

Now we discuss EDM constraint on \( \text{Im}(\rho_{bb}) \) in light of the latest result of ACME Collaboration [34]. This constraint is so overwhelming that one cannot dodge it without relying on some mechanism in any EWBG scenarios in g2HDM. As briefly mentioned below Eq. (3), the electron EDM could be sufficiently suppressed by the build-in cancellation mechanism. For that end, \( \rho_{tt} \) and \( \rho_{ee} \) have to be complex and echo the SM-like Yukawa hierarchy. In the \( \rho_{bb} \)-EWBG scenario, however, \( \rho_{tt} \) is real or small by assumption and the above solution space is the no-go zone. Nonetheless, it is still possible to render the electron EDM small enough to avoid the ACME constraint in concert with \( \rho_{bb} \) and \( \rho_{ee} \) though the cancellation does not manifest any structure. We do not repeat the analysis here and refer the readers to Ref. [15] for more details.

| BP | \( m_1 \) | \( m_2 \) | \( m_3 \) | \( m_4 \) | \( m_5 \) | \( m_6 \) | \( m_7 \) | \( m_{H^\pm} \) | \( m_A \) | \( m_H \) | \( \mu_{22}/\sigma^2 \) | \( c_\gamma \) | \( s_\gamma \) | \( \lambda_{Hhh} \) (GeV) |
|----|-----|-----|-----|-----|-----|-----|-----|-------|-----|-----|-------|-----|-----|-----|
| a  | 0.263 | 3.768 | 3.829 | -2.27 | 0.022 | -0.054 | 0.404 | 341 | 216 | 220 | 0.000373 | 0.1 | -0.995 | -10.95 |
| b  | 0.271 | 3.265 | 5.968 | -3.005 | 0.0135 | -0.132 | 2.291 | 431 | 307 | 310 | 0.078 | 0.1 | -0.995 | -23.62 |
| c  | 0.297 | 3.3 | 4.589 | 1.409 | 0.734 | -0.395 | 2.753 | 435 | 457 | 506 | 0.818 | 0.1 | -0.995 | -21.71 |

TABLE I. Parameter values of the three benchmark points. The masses \( m_{H^\pm}, m_A \) and \( m_H \) are given in GeV.

\[ |\kappa_b| = \sqrt{\left(-s_\gamma + \frac{c_\gamma \text{Re}(\rho_{bb})}{\lambda_b}\right)^2 + \left(\frac{\text{Im}(\rho_{bb})c_\gamma}{\lambda_b}\right)^2}, \tag{7} \]

without relying on any mechanism in any EWBG scenarios in g2HDM. As briefly mentioned below Eq. (3), the electron EDM could be sufficiently suppressed by the build-in cancellation mechanism. For that end, \( \rho_{tt} \) and \( \rho_{ee} \) have to be complex and echo the SM-like Yukawa hierarchy. In the \( \rho_{bb} \)-EWBG scenario, however, \( \rho_{tt} \) is real or small by assumption and the above solution space is the no-go zone. Nonetheless, it is still possible to render the electron EDM small enough to avoid the ACME constraint in concert with \( \rho_{bb} \) and \( \rho_{ee} \) though the cancellation does not manifest any structure. We do not repeat the analysis here and refer the readers to Ref. [15] for more details.

| BP | \( \rho_{bb} \) | \( Z_h \) | \( Z_H \) | \( \Gamma_A \) (GeV) |
|----|-----|-----|-----|-----|
| a  | 1.00 | - | - | 0.043 |
| b  | 0.648 | 0.352 | - | 0.095 |
| c  | 0.301 | 0.699 | - | 0.304 |

TABLE II. The branching ratios and total widths of \( A \) for the benchmark points. Here we assumed \( |\text{Im}(\rho_{bb})| = 0.058 \) and set all \( \rho_{ij} = 0 \). See text for details.

| BP | \( \rho_{bb} \) | \( Hhh \) | \( WW \) | \( ZZ \) | \( tt \) | \( \Gamma_H \) (GeV) |
|----|-----|-----|-----|-----|-----|-----|
| a  | 0.658 | - | 0.244 | 0.098 | - | 0.067 |
| b  | 0.310 | 0.213 | 0.330 | 0.147 | - | 0.199 |
| c  | 0.128 | 0.041 | 0.462 | 0.222 | 0.147 | 0.790 |

TABLE III. The branching ratios and total widths of \( H \) for the benchmark points for |\( \text{Im}(\rho_{bb})| = 0.058 \) with all other \( \rho_{ij} = 0 \).
respective branching ratios for the three BPs are summarized in Table II. For the CP-even heavy Higgs boson \( H \), it primarily decays to \( b\bar{b} \), followed by \( WW \) and \( ZZ \) in BP\( a \). In BP\( b \), the \( bb \) and \( WW \) modes comprise about 30% branching ratios, followed by \( hh \) and \( ZZ \). In BP\( c \), \( WW \) is the dominant decay mode, followed by \( ZZ \). The \( t\bar{t} \) channel is also kinematically accessible, which predominates over the \( bb \) and \( hh \) modes. In addition to the above decay modes, the decays such as \( H \rightarrow \tau\tau, H \rightarrow ee, \) etc. would be turned on via nonzero \( |\eta| \) for all the leptons and \( b \)-jets are needed to satisfy \( |\eta| < 2.5 \). Moreover, the separation \( \Delta R \) between the two leptons, any two \( b \)-jets and, a \( b \)-jet and a lepton should be \( \Delta R > 0.4 \). The jets are reconstructed with anti-
\( k_T \) algorithm via default ATLAS-based detector card of Delphes 3.4.2. We veto events with missing transverse energy (\( E_{T\text{miss}} \)) > 35 GeV to reduce the \( t\bar{t}+\text{jets} \) background.

We further require that the invariant mass of the same flavor opposite charge lepton pair (\( m_{\ell\ell} \)) should remain within \( 76 < m_{\ell\ell} < 100 \) GeV, i.e., in the \( Z \) boson mass window. The invariant mass of two \( b \)-jets \( m_{bb} \) in an event to remain within \( |m_{bb} - m_{h} < 25 \) GeV. As each event contains at least three \( b \)-jets more than one \( m_{bb} \) combinations are possible; the one closest to \( m_{h} \) is selected to pass the \( |m_{bb} - m_{h}| < 25 \) GeV cut. We finally require the invariant mass \( m_{\ell\ell bb} \) constructed from the same flavor opposite charge lepton pair that pass the \( 76 < m_{\ell\ell} < 100 \) GeV window and \( b \)-jets combination that passes the \( |m_{bb} - m_{h}| < 25 \) GeV selection to remain within \( |m_{A} - m_{\ell\ell bb}| < 80 \) GeV. Here we adopt the \( b \)-tagging and \( c \)- and light-jets rejection efficiencies ATLAS-based detector card of Delphes 3.4.2. The signal and background cross sections after the selection cuts for the BP\( a \) and BP\( b \) are summarized in Table IV.

We now focus on the achievable significance at HL-LHC using the likelihood for a simple counting experiment [49]

\[
Z(n|n_{\text{pr}}) = \sqrt{-2\ln \left( \frac{L(n|n_{\text{pr}})}{L(n|n)} \right)} = \frac{e^{-\frac{n-n_{\text{pr}}}{n}}}{n!},
\]

where \( n \) and \( n_{\text{pr}} \) are observed and predicted events. For discovery, the signal plus background (\( s+b \)) is compared with the background prediction (\( b \)) with the requirement \( Z(s+b+b) > 5 \), while for the exclusion we demand \( Z(b|s+b) > 2 \) [49]. An evidence would require \( Z(s+b+b) > 3 \). Utilizing the signal and background cross sections in Table IV we find that the achievable significance is \( \sim 2.9\sigma \) for BP\( b \) while \( \sim 2.5\sigma \) for BP\( c \) with 3000 fb\(^{-1} \) integrated luminosity. Therefore we conclude that the discovery is beyond the scope of HL-LHC if the Im(\( \rho_{bb} \)) close to its nominal value 0.058 required for \( \rho_{bb} \) EWBG. We note that Im(\( \rho_{bb} \)) \( \sim 0.15–0.2 \) is still allowed by current data for the sub-TeV mass range as we have already discussed in previous section. We also remark that in BP\( c \), for \( m_{A} > 2m_{\ell} \), discovery is well within the HL-LHC if one considers Im(\( \rho_{bb} \)) \( \sim 0.15 \). This is different from

III. THE \( bg \rightarrow bA \rightarrow bZh \) PROCESS

We first analyze the prospect of discovering nominal Im(\( \rho_{bb} \)) required for EWBG via \( bg \rightarrow bA \rightarrow bZh \) process at HL-LHC. The process can be searched at the LHC via \( pp \rightarrow baX \rightarrow b2ZhX \) [35] followed by \( Z \rightarrow \ell^{+}\ell^{-} \) (\( \ell = e, \mu \)) and \( h \rightarrow b\bar{b} \) i.e., in signature comprising of a pair of same flavor opposite sign leptons (denoted as the \( bZh \) process) and three \( b \)-tagged jets. The process requires that \( m_{A} > m_{Z} + m_{h} \). Therefore BP\( a \) for which \( m_{A} < m_{Z} + m_{h} \) is out of the reach of LHC and we only focus on BP\( b \) and BP\( c \). There exist several SM backgrounds such as \( t\bar{t}+\text{jets}, \) Drell-Yan+jets (DY+jets), \( Wt+jets, tZ+jets, t\bar{h}, tZ+jets, \) whereas subdominant contributions arise from four-top (4\( t \)), \( tW, tWh, tWZ \) and \( WZ+jets \). Backgrounds from \( WW+jets \) is negligibly small and hence not included. We remark that a search can also be performed via \( h \rightarrow \tau\tau \) and \( h \rightarrow \gamma\gamma \) modes, however, they are not as promising as \( h \rightarrow b\bar{b} \).

We generate the signal and SM background event samples at leading order (LO) in \( pp \) collision with \( \sqrt{s} = 14 \) TeV CM energy by MadGraph5_aMC@NLO [36] (denoted as MadGraph5_aMC@NLO) with default NN23LO1 PDF set [37] then interface with Pythia 6.4 [38] for hadronization and showering and, finally fed into Delphes 3.4.2 [39] for the fast detector simulation incorporating the default ATLAS-based detector card. We follow MLM scheme [40, 41] for the matrix element (ME) and parton showering. Note that we do not include backgrounds from the fake and non-prompt sources in our analysis which are typically determined from data and are not properly modeled in the Monte Carlo simulations. The effective model is implemented in FeynRules 2.0 [42] framework.

The DY+jets background cross section is adjusted to the NNLO QCD+NLO EW one by a factor 1.27, which is estimated by FEWZ 3.1 [43, 44], while the \( t\bar{t}+\text{jets} \) background is corrected up to NLO by the \( K \) factor 1.36 [36]. We also normalize the LO \( tW, tZ+jets, t\bar{h}, 4t \) and \( tW^{-}, (ttW^{+}) \) cross sections to NLO ones by the \( K \)-factors 1.56 [45], 1.44 [36], 1.27 [46], 2.04 [36] and 1.35 (1.27) [47] respectively, while both \( tWZ \) and \( tWWh \) are kept at LO.
TABLE IV. The signal and background cross sections (in fb) of the $bZ\ell$ process after selection cuts for the respective BPs at $\sqrt{s} = 14$ TeV LHC. We have assumed $|\text{Im}(\rho_{bb})| = 0.058$ and set all other $\rho_{ij} = 0$ for the signal process. The subdominant backgrounds $4t$, $ttW$, $tW\bar{t}$, $tZW$ and $WZ+$jets are added together and denoted as "Others". The total background yield (Total Bkg.) is given in the last column.

| BP | Signal (fb) | $\ell\bar{\ell}$+ jets | $DY$+ jets | $Wt+$ jets | $t\bar{t}Z$ | Others | Total Bkg. (fb) |
|----|-------------|--------------------------|-----------|------------|----------|--------|----------------|
| $b$ | 0.064       | 0.270                    | 0.702     | 0.404      | 0.017    | 0.01   | 1.403         |
| $c$ | 0.025       | 0.108                    | 0.139     | 0.024      | 0.007    | 0.002  | 0.281         |

IV. THE $e^+e^- \rightarrow Z^* \rightarrow AH$ PRODUCTION

A. The four $b$-jets signature

In this section we investigate the potential for $eeAH$ process i.e., $e^+e^- \rightarrow Z^* \rightarrow AH$ production with $H/A \rightarrow bb$ decays in four $b$-jets signature for two different $e^+e^-$ collision energy $\sqrt{s} = 500$ GeV and 1 TeV. The signature would also receive contribution from $e^+e^- \rightarrow Z^* \rightarrow bbA/H$ process for $A/H \rightarrow bb$ decays, which we have included in our analysis. It is clear from Table I that BPs would be covered by $\sqrt{s} = 500$ GeV while BPb and BPC would require $\sqrt{s} = 1$ TeV.

Although the environment is clean, there indeed exist some SM backgrounds for this process. The dominant backgrounds come from $\ell\bar{\ell}$, four-jets ($4j$) which includes $Zh$ production, with subdominant contribution would arise from $ZZ$ background. The events are generated as in previous section by MadGraph5 aMC followed by showering and hadronization in PYTHIA 6.4, and fed into Delphes 3.4.2 for detector effects. Here we incorporate the default international linear detector card (ILD) of Delphes 3.4.2 for jet reconstruction via anti-$k_T$ algorithm with radius parameter $R = 0.5$ and, for the $b$-tagging and misidentification efficiencies of $c$ and light-jets.

| BP | Signal (fb) | $ZZ$ | $\ell\bar{\ell}$ | $4j$ | Total Bkg. (fb) |
|----|-------------|-----|-----------------|------|----------------|
| $a$ | 0.304       | 0.18| 0.112           | 0.461| 0.753          |

TABLE V. The signal and background cross sections for BPa in fb for $eeAH$ process at $\sqrt{s} = 500$ GeV. Total background is presented in the last column.

The events are selected such that it should contain at least four $b$-jets with all having $p_T > 20$ GeV and $|\eta| < 2.5$. The separations between any two $b$-jets should be $\Delta R > 0.4$. To reduce the backgrounds further, we demand the scalar sum of $p_T$ of all four $b$-jets ($H_T$) should be $> 350$ GeV for BPa, while for BPb and $c$ we require $> 600$ GeV. For illustration we show the normalized $H_T$ distributions in Appendix for BPa and BPb for $\sqrt{s} = 500$ GeV and 1 TeV respectively. The signal and backgrounds after selection cuts for $\sqrt{s} = 500$ GeV and 1 TeV are respectively summarized in Tables V and VI.

We now estimate the significances from the cross sections summarized in Tables V and VI. It is clear that $S/B$ ratios are considerably large for BPa and BPb for the considered CM energies. Utilizing Eq. (8) we find that BPa can be discovered at $\sqrt{s} = 500$ GeV CM energy the with $\sim 250$ fb$^{-1}$ integrated luminosity with evidence emerging with as low as $\sim 80$ fb$^{-1}$ data. The BPb would require $\sqrt{s} = 1$ TeV run and an evidence may come with 120 fb$^{-1}$ but discovery needs 350 fb$^{-1}$ dataset. The BPC is below the sensitivity of even $\sqrt{s} = 1$ TeV lepton collider. Here for all three BPs the signal cross sections are estimated with $\text{Im}(\rho_{bb}) = 0.058$. Therefore we conclude that the nominal value for $\rho_{bb}$-EWBG can be fully covered up to $m_A$, $m_H \lesssim 200$ (400) GeV with moderate integrated luminosity in any future lepton collider if it runs with $\sqrt{s} = 500$ GeV (1 TeV) CM energy.

B. The six $b$-jets signature

We now discuss a resonant diHiggs production $e^+e^- \rightarrow Z^* \rightarrow AH \rightarrow bhh$ in future $e^+e^-$ colliders. We search this process in which both $h$ decays to $bb$ i.e. in six $b$-jets signature. Such final state would also receive contribution from process $e^+e^- \rightarrow AH \rightarrow bhh$ which we have considered as well. For the parameter space described in Table I only BPb and BPC can facilitate $e^+e^- \rightarrow Z^* \rightarrow AH \rightarrow A hh$ and $e^+e^- \rightarrow Z^* \rightarrow A hh$ since $m_H > 2m_h$. Note that discovery may already emerge from four $b$-jets signature discussed in previous subsection while six $b$-jets signature would provide complementarity for $\rho_{bb}$-EWBG.

Based on the LO $HHh$ coupling given in Eq. (5), we first analyze the prospect of $e^+e^- \rightarrow AH \rightarrow bhh$ process with both $h$ decays $bb$ i.e., in six $b$-jets signature with all six $b$-jets having $p_T > 20$ GeV and $|\eta| < 2.4$. Here we...
consider two different CM energy $\sqrt{s} = 500$ GeV and 1 TeV for illustration. The CM energies considered would kinematically allow $e^+e^- \rightarrow AH \rightarrow bhh$ process only for BPb. For event generation we follow the same procedure as in $e^+e^- \rightarrow Z^* \rightarrow AH$ process i.e. generate events via MadGraph5_aMC followed by hadronization and showering in Pythia 6.4 and adopting default ILC card of Delphes for fast detector simulation. The corresponding cross sections $\sqrt{s} = 500$ GeV (1 TeV) before application of any selection cuts reads as $\sim 0.001$ ($\sim 0.2$) fb for BPb with $|\text{Im}(\rho_{bb})| = 0.058$. Following the above mentioned selection cuts, we find 0.0078 fb cross section for $\sqrt{s} = 1$ TeV, but tiny 0.00003 fb for $\sqrt{s} = 500$ GeV. In finding these cross sections we have normalized the $B(h \rightarrow b\bar{b})$ with the modified $hbb$ coupling due to nonvanishing $|\text{Im}(\rho_{bb})| = 0.058$. While no statistically significant cross section is found for 500 GeV run, however one may have $\sim 8$ ($\sim 24$) events with 1000 (3000) fb$^{-1}$ integrated luminosity at $\sqrt{s} = 1$ TeV. In SM, we find such six-b-jets backgrounds to be negligibly small at $e^+e^-$ collider, providing ideal environment for discovery of such signature. This should be compared with the discovery prospect discussed in Sec. IV for BPb via $e^+e^- \rightarrow Z^* \rightarrow AH$ process, which would require $\sqrt{s} = 1$ TeV and $\sim 700$ fb$^{-1}$ data. In finding the six-b-jets cross section here we have not included uncertainties arising from high b-jet multiplicity. Hence, we remark that our six-b-jets cross sections should be treated as exploratory while a more detailed analysis including possible uncertainties arising in $e^+e^-$ collider would be studied elsewhere.

1. The vertex correction for $Hhh$ coupling at $g2HDM$

It is known that one-loop corrections to triple Higgs couplings could be sizable if EWPT is strongly first order [50] (for one-loop calculations to the $hhh$ coupling, see also Ref. [51]). Here we clarify if this argument applies for our $Hhh$ coupling. Dominant one-loop corrections in the $c_s \rightarrow 0$ limit are cast into the form

$$\Delta \lambda_{Hhh} \approx -\frac{\eta_f}{16\pi^2v^2} \left[ 3m_H^2 \left( 1 - \frac{\mu_2^2}{m^2_H} \right)^2 + m_A^2 \left( 1 - \frac{\mu_2^2}{m_A^2} \right)^2 + 2m_H^2 \left( 1 - \frac{\mu_2^2}{m^2_H} \right)^2 \right],$$

(9)

Remarkably, the loop correction would not vanish even in the exact alignment limit $c_s = 0$ due to the presence of the nonzero $\eta_f$, which is in sharp contrast to softlybroken 2HDMs. In our three BPs, moreover, the loop corrections are constructive since $\eta_f$ is positive and tree-level $\lambda_{Hhh}$ is negative. In each case of BPs, we find that

$$\Delta \lambda_{BAA}^{BPb} \approx -4.407 \text{ GeV, } \Delta \lambda_{Bhh}^{BPb} \approx -41.16 \text{ GeV, } \Delta \lambda_{Bhh}^{BPc} \approx -58.55 \text{ GeV.}$$

(10)

One can see that the one-loop corrections are larger than the tree-level values in BPb and BPc. However, this does not necessarily mean that perturbation breaks down since the tree-level $Hhh$ coupling happens to be small by $c_s$, and moreover, some combinations of quartic couplings at one-loop level could be larger than those at tree level though each of quartic couplings is less than 4$\pi$ as seen in Table I. As mentioned in Sec. II B, the tree-level unitarity is not violated either. We note that the $H \rightarrow f\bar{f}$ (with $f$ being fermions) decays are not expected to receive large enhancement from the one-loop corrections since the two of the three vertices in there are not Higgsself couplings (for $h \rightarrow f\bar{f}$ decays, see, e.g., Ref. [52]). Therefore, $B(H \rightarrow hh)$ would be significantly increased at loop level, leading to much larger possibility for discovery at future lepton colliders.

V. DISCUSSION AND SUMMARY

We have analyzed the prospect of probing EWBG induced by additional bottom Yukawa couplings at the LHC and future $e^+e^-$ colliders. We primarily focused on the nominal value $|\text{Im}(\rho_{bb})| = 0.058$ required for $\rho_{bc}$-EWBG. We show that HL-LHC can offer some probe for such parameter space via $bg \rightarrow bA \rightarrow bZh$ process if $300 \leq m_A \leq 450$ GeV. However, the discovery would be beyond even for HL-LHC. In this regard we show that future $e^+e^-$ colliders such as ILC or FCCee would offer exquisite discovery prospect via $e^+e^- \rightarrow Z^* \rightarrow AH$ process at $\sqrt{s} = 500$ GeV and 1 TeV. For parameter space where $m_A < m_h + m_Z$, the $bg \rightarrow bA \rightarrow bZh$ process kinematically insensitive but a 500 GeV run of any $e^+e^-$ collider can discover the $\rho_{bc}$-EWBG via $e^+e^- \rightarrow Z^* \rightarrow AH$ process with $\sim 250$ fb$^{-1}$ data. The discovery for the same process with heavier $m_A$ is also possible when 1 TeV or larger collision energies are available.

For complementarity, we also studied the prospect $e^+e^- \rightarrow AH \rightarrow bhh$ process in six-b-jets signature. Based on our LO order $Hhh$ coupling we found that 1 TeV $e^+e^-$ collider can indeed discover such a process as long as $m_H \sim 300$ GeV. It should be noted that the $Hhh$ coupling could get $O(100\%)$ one-loop correction owing to the sizable Higgs quartic couplings required by the strong first-order EWPT, increasing the significance for the discovery.

We now briefly discuss the impact of turning on other $\rho_{ij}$ couplings. Current direct and indirect searches still allow $|\rho_{tt}| \sim 0.5$ [15] for sub-TeV $m_A$, $m_H$ and $m_{H^\pm}$. Further $\rho_{cc} \sim 0.3$ is also allowed by direct and indirect searches and flavor physics [13]. A nonvanishing $\rho_{tt}$ motivates one to utilize the conventional $gg \rightarrow A/H \rightarrow tt$ [53] and $gg \rightarrow t\bar{t}A/H \rightarrow t\bar{t}t$ [54] $gb \rightarrow tH^+ \rightarrow t\bar{t}b$ searches [32, 33]. For moderate values of $\rho_{tt}$ and $\rho_{bb}$ one may have $gg \rightarrow bA/H \rightarrow b\bar{t}t$ signature which could be sensitive at the HL-LHC [15]. In this regard it should be reminded that complex $\rho_{tt}$ and $\rho_{bc}$-EWBG mechanism can be found in Refs. [7–14]. In general if such couplings are real they
would not play any role in EWBG, however they would aggravate the signatures that we have discussed so far via suppression in the branching ratios of heavy bosons $A/H$. Nevertheless they would open up several additional direct and indirect probes. Furthermore moderate values of $\rho_{\tau\tau}$ is still allowed by current data though its impact is not as significant as $\rho_u$ and $\rho_C$. We leave out a detailed discussion of EWBG driven by multiple $\rho_{ij}$ couplings and subsequent impacts on collider and flavor physics for future work.

As a first estimate, uncertainties arising from factorization scale ($\mu_F$) and renormalization scale ($\mu_R$) dependences are not included in our LO cross section estimations for $bg \to bA \to bZ\ell\nu$ process. In general, the LO $bg \to bA$ processes have $\sim 25 - 30\%$ scale uncertainties for $m_A \sim (300 - 400)$ GeV as discussed in Ref. [55] (see also Refs. [33, 56–58]). In addition it has been found that [59] the LO cross sections calculated with LO PDF set CTEQ6L1 [60] have relatively higher factorization scale dependence. Therefore, we remark that the LO cross sections in our analysis, which we estimated with LO NN23LO1 PDF set, might have similar uncertainties. A reasonable choice of the factorization scale and renormalization scale has been proposed in Ref. [59], with $\mu_R = m_A$ and varied from $\mu_R = m_A/2$ to $\mu_R = 2m_A$, along with $\mu_F = m_A/4$ and varied from $\mu_F = m_A/8$ to $\mu_F = m_A/2$. There also exist PDF uncertainties for bottom-quark initiated process as discussed in Ref. [61] (see also Ref. [62]). These would typically induce some uncertainties in our results which we leave out for future work.

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**Appendix A: The normalized $H_T$ distributions for the $e^+e^- \to Z^* \to AH$ process**

The normalized $H_T$ distribution is plotted for the $e^+e^- \to Z^* \to AH$ process in Fig. 2.

**Appendix B: Theoretical uncertainties of BAU**

It has been known that the so-called vacuum expectation value-insertion approximation (VIA) that we use in our work tends to give overestimated BAU. We note this issue and add a caveat when interpreting our BAU in Ref. [15]. During the review process of this paper, we came across a paper [63] that points out overlooked errors in VIA-based BAU calculations. We have confirmed that our previous estimated BAU has to be divided by 2 after correcting degrees of freedoms of left-handed fermions in calculating BAU. By this correction, we may take $\text{Im} \rho_{cb} \gtrsim 0.058 \times 2 = 0.116$. We have confirmed that $\text{Im} \rho_{cb}$ can be as large as (0.2-0.25) after taking all the constraints into account (a la Fig. 1). However, this is not only the possible revision if we consider theoretical uncertainties described at the last paragraph in this section.

As noted in Ref. [63], we should be careful about factor 3 when using the strong sphaleron rate in Ref. [64] in order to match the correct normalization. In our calculation, however, the strong sphaleron rate is based on Ref. [65] with a corrected color factor and $\kappa = 1$, which is consistent in itself. Although the numerical difference between the two estimates happens to be around 3, this discrepancy should be regarded as theoretical uncertainties since $\kappa = 1$ is merely a nominal value and systematic error of the lattice calculation could be large [64]. Another comment on criticism made in Ref. [63] is that $\rho_{bb}$-sourced BAU cannot be estimated by a simple scaling $(\lambda_b/\lambda_i)^2$ from $\rho_{tt}$-sourced BAU, where $\lambda_{tt}$ are top and bottom SM Yukawa couplings. With the basis-independent CP-violating form [6, 66], the scaling factor goes like $(\lambda_b/\lambda_i)(\text{Im} \rho_{bb}/\text{Im} \rho_{tt})$, where the latter factor could be greater than 1.

On this occasion, we recapitulate theoretical uncertainties of our BAU detailed in Ref. [15]. In addition to the aforementioned strong sphaleron rate, BAU can be modulated by several factors: (i) bubble wall velocity, (ii) variation of the two Higgs VEVs during EWPT $\Delta \beta$, (iii) critical temperature $T_C$ and corresponding VEV $v_C$, (iv) prescription for UV-divergent piece, (v) CP-conserving source term induced by $\rho_{bb}$, and (vi) an approximation for bubble wall shape. The largest uncertainty may come from (ii) since BAU is proportional to $\Delta \beta$. In the minimal supersymmetric SM, $\Delta \beta = \mathcal{O}(10^{-4} - 10^{-2})$ depending on $m_A$, while in its extensions $\Delta \beta$ could be as large as $\mathcal{O}(0.1)$. Since detailed study on $\Delta \beta$ is absent in g2HDM at this moment, we take $\Delta \beta = 0.015$ for illustration. Our BAU could increase or decrease by an order of magnitude by this factor. The second largest uncertainty could arise from (iv). It is known that temperature-dependent logarithmic divergence exists in the CP-violating source term [67, 68]. A prescription is to remove it by normal ordering or counterterm [67] (for recent criticism on this point, see Ref. [69]). Given the fact that this divergence may be attributed to wrong approximation of thermal damping rate, we adopt another prescription in which a momentum integral is made finite using a cutoff (for details, see Ref. [68]), causing ambiguities in BAU by a couple of factor or more. As for the other uncertainties (i), (iii), (v) and (vi), each of them can increase or decrease BAU by a couple of factor or more. All in all, theoretical uncertainties are not well under control, and the factor 2 error in our previous BAU could be compensated by, for instance, doubling $\Delta \beta = 0.015 \times 2$. In conclusion, parameter space for $\rho_{bb}$-EWBG is still open with generous assumptions and awaits more robust BAU calculation.
FIG. 2. Normalized $H_T$ distributions of the signal and backgrounds for BP$_a$ (left) and BP$_b$ (right) at $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1$ TeV respectively.

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