Spontaneous Parametric Down-Conversion Experiment to Measure Both Photon Trajectories and Double-Slit Interference

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Abstract

Recent work using spontaneous parametric down-conversion (SPDC) has made possible investigations of the Einstein-Podolsky-Rosen paradox in its original (position-momentum) form. We propose an experiment that uses SPDC photon pairs to measure through which slit a photon passes while simultaneously observing double-slit interference.
Recently, there has been increasingly sophisticated use of spontaneous parametric
down-conversion (SPDC) to elucidate Einstein-Podolsky-Rosen-type state entanglements.
In general, these experiments take the following form \[1\]. When down-conversion occurs,
conservation of momentum requires that the signal-idler pairs propagate in strongly cor-
related directions that are in some sense “opposite” \[2, 3\]. The pairs are emitted from
two possible activations regions (\(A\) and \(B\)) in a suitable crystal. The signal photons are
subjected to a double slit, in which the slits correspond to \(A\) and \(B\). This process pro-
duces double-slit interference if and only if it is subjected to coincidence counting with an
\(A + B\) idler beam, a phenomenon often called quantum erasure. In an elegant example \[4\],
there are two \(A + B\) idler beams with different phase factors, producing two double-slit
coincidence patterns whose peaks and troughs are perfectly out of phase, and therefore
sum to a perfectly non-double-slit pattern. In order that experiments of this type should
support the standard interpretation of quantum mechanics, it is essential that the sum
total pattern not display double-slit interference.

Srikanth \[5, 6\] has proposed a variant of this type of experiment in which the double-
slit observer may observe interference without coincidence counting, a prospect which
threatens the standard interpretation. It is our intention to boil this type of experiment
down to a few essentials and see whether this threat really exists.

At the core of the experiment is the necessary condition for resolution of a double-slit
pattern,
\[
\frac{w}{d} < \frac{\lambda}{s},
\]  
(1)
where
\[
\begin{align*}
w &= \text{width of the source,} \\
d &= \text{distance from the source to the slits,} \\
s &= \text{slit separation.}
\end{align*}
\]
For strongly correlated photon pairs, we expect that pair detection varies as
\[
\sqrt{2\pi} \frac{\phi}{\phi_0} \exp \left[ \frac{1}{2} \left( \frac{\phi}{\phi_0} \right)^2 \right],
\]  
(2)
where \(\phi\) is the angular deviation from perfect opposition, and \(\phi_0\) is the one-dimensional
standard deviation. It is clear that the experiment harbors a potentially interesting result
only if double-slit interference can be shown to occur when
\[
\frac{s}{d} > \phi_0.
\]  
(3)
So, from \(1\) and \(3\),
\[
\begin{align*}
\frac{\lambda}{w} &> \frac{s}{d} > \phi_0 \\
\frac{\lambda}{\phi_0} &> w.
\end{align*}
\]  
(4)
Using condition (4) and various SPDC results [2, 3, 4], we shall construct an experiment in which

\[
\lambda = 702 \text{ nm}, \quad \phi_0 = 2 \text{ mrad}.
\]

If we set \( \frac{\phi_0}{d} = 6 \phi_0 = 12 \text{ mrad} \) to achieve separation between slits \( A \) and \( B \), Equation 4 is replaced by

\[
w < \frac{\lambda}{6\phi_0} = (83.3)\lambda,
\]

We then set \( w = 25\lambda \) to make the interference pattern easily resolvable. Then if we set \( d = 600 \text{ mm} \), we have

\[
w = 1.75 \times 10^{-2} \text{ mm, and} \quad s = 7.2 \text{ mm},
\]

and the experiment is summarized by Figure 1. Note that because of the small value of \( \frac{\phi_0}{d} \), we have for the sake of clarity used different scales for the horizontal and vertical axes.

The distances \( A'A'' \) and \( B'B'' \) are 1.46 mm each, so we define \( x \equiv A'C = B'D = \frac{1}{2}A'A'' = 0.73 \text{ mm} \). 0.73 mm is close to the usual value for the extent of the activation area (and therefore the coherence length). We therefore choose to work with a crystal (or perhaps just with an activation area) that is 0.73 mm thick. We see from Figure 1 that \( \frac{3}{4} \) of the points that have access to the slits have access to both slits (the shaded area). Therefore, the naive conclusion is that \( \frac{3}{4} \) of the counts beyond the slits are double-slit counts. The total pattern without coincidence counting should be predominantly a double-slit pattern.

We complete the experiment by placing detectors \( \bar{A} \) and \( \bar{B} \) "opposite" slits \( A \) and \( B \), at a distance \( d' \gg d \), so that \( \frac{w}{d'} \) is negligible. The detectors have radius \( 2\phi_0 \).

It is then difficult to escape the conclusion that coincidence counting with either \( \bar{A} \) or \( \bar{B} \) alone will include a significant double-slit component. Note that a time delay for the idlers can be inserted, so that the double-slit counts are registered before \( \bar{A} \) and \( \bar{B} \) are activated.

The one loophole that could avoid this conclusion and still maintain the standard interpretation of quantum mechanics would be if the photon wave functions are composed primarily of waves that appear to radiate from points in zones \( A \) and \( B \) that are outside (the activation area of) the crystal.  

Note that

\[
K_{pe} \equiv \frac{x}{\lambda} = 1.04 \times 10^3
\]

is a measure of the system’s potential entanglement, while

\[
K_{ae} \equiv \frac{\pi/2}{\phi_0} = 0.79 \times 10^3
\]
is a measurement of the *actual entanglement*. We have designed the experiment so that

\[ 0.5 < \frac{K_{ae}}{K_{pe}} < 1. \tag{8} \]

We can determine more generally the conditions for observing the desired effect. We define three parameters \( f \), \( g \), and \( h \) be replacing three inequalities with equations.

\[ K_{pe} > K_{ae}, \]

for degree of entanglement, becomes

\[ K_{pe} = f K_{ae}; \]

\[ \frac{s}{d} > \phi_0, \]

for discriminating A from B, becomes

\[ \frac{s}{d} = 4g\phi_0; \]

and

\[ \frac{\lambda}{s} > \frac{w}{d}, \]

for double-slit pattern resolution, becomes

\[ \frac{\lambda}{s} = 2h\frac{w}{d}. \]

We then find that

\[ \phi_0 = \frac{1}{32\pi(fg^2h)} \tag{9} \]

and

\[ w = \frac{\lambda}{8gh\phi_0}. \tag{10} \]

It appears that the threshold for observing the effect is about \( f \approx 1, g \approx 1, h \approx 1 \) with

\[ \phi_0 = 10 \text{ mrad}, \text{ and} \]

\[ w = (12.5)\lambda, \]

so even in the threshold case, diffraction by the aperture is not a critically limiting factor. Current SPDC results allow us to propose a configuration in which

\[ fg^2h \approx 5 \]

\[ hg^2 = 3.75, \]

with \( \phi_0 = 2 \text{ mrad} \). If we fix \( f \) then \( g^2h = \frac{1}{132\phi_0} \). \( g^2h \) is a visibility parameter for the effect, which improves if the value of \( \phi_0 \) decreases.

We conclude that the experiment is technically viable and consistent, and that an interesting result is expected, provided that the stated loophole does not apply.
References

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Figure 1: A schematic of the experiment.
This figure "figure.gif" is available in "gif" format from:

http://arxiv.org/ps/quant-ph/0106113v1