Pauli–Villars regularization of non-Abelian gauge theories

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(Dated: November 5, 2021)

Abstract

As an extension of earlier work on QED, we construct a BRST-invariant Lagrangian for SU(N) Yang-Mills theory with fundamental matter, regulated by the inclusion of massive Pauli-Villars (PV) gluons and PV quarks. The underlying gauge symmetry for massless PV gluons is generalized to accommodate the PV-index-changing currents that are required by the regularization. Auxiliary adjoint scalars are used, in a mechanism due to Stueckelberg, to attribute mass to the PV gluons and the PV quarks. The addition of Faddeev–Popov ghosts then establishes a residual BRST symmetry. Although there are drawbacks to the approach, in particular the computational load of a large number of PV fields and a nonlocal interaction of the ghost fields, this formulation could provide a foundation for renormalizable nonperturbative solutions of light-front QCD in an arbitrary covariant gauge.

\footnote{Based on an invited talk presented at the Lightcone 2015 workshop, Frascati, Italy, September 21-25, 2015.}
I. INTRODUCTION

Nonperturbative Hamiltonian approaches to quantum field theories \[1, 2\] in more than two dimensions require regularization. The standard approach to regularization of non-Abelian gauge theories is dimensional regularization \[3\], but this is inherently perturbative. For Abelian theories, Pauli–Villars (PV) regularization has proven useful \[4, 5\]. However, the ordinary PV regularization of non-Abelian theories fails. Gauge invariance is violated, blocking any hope of BRST invariance, which confounds proofs of renormalizability \[6\]. To implement a PV regularization \[7\], we repair gauge invariance and then break it properly with a gauge-fixing term. Ghost fields \[8\] are added to build a BRST invariance.

The causes of gauge non-invariance in ordinary PV regularization are the PV gluon mass, the non-degenerate PV quark masses, and the PV-index changing currents for null interactions. A mass is acceptable for PV photons in an Abelian theory \[9\], but the approach used does not generalize to non-Abelian theories. Null interactions (meaning interactions that involve only null combinations of positive and negative metric fields) are necessary to provide the subtractions that regulate theory. So, how can one restore gauge invariance?

The Higgs mechanism is, of course, one possible approach, but investigations of this invariably found only null combinations of gluon fields to be massless, rather than a massless physical gluon. There is, however, another mechanism for generating masses \[10–12\], by coupling the PV gluons and PV quarks to a PV scalar, with all removed from the spectrum in the infinite-mass limit. The mixing of currents can be overcome by extension of the gauge transformation to include mixing PV fields; the original gauge transformation is obtained when the fields are removed from the spectrum. The construction is done in such a way as to facilitate quantization in an arbitrary covariant gauge, as has been done previously for QED \[3\]. The remaining gauge invariance can be tested by studying the dependence of physical quantities on the gauge-fixing parameter.

In the remainder of this paper, we present the PV-regulated Lagrangian term by term, along with the extended gauge transformation and the BRST transformations. We then discuss light-front quantization in an arbitrary covariant gauge and the light-front coupled-cluster method \[13\] for solution of the Hamiltonian eigenvalue problem in Fock space.

II. A PAULI–VILLARS REGULATED LAGRANGIAN

For the purposes of discussion, we divide the PV-regulated Lagrangian into four terms

\[
\mathcal{L} = \mathcal{L}_{\text{massless}} + \mathcal{L}_{\text{gluon}} + \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{ghost}}.
\]  

Here \(\mathcal{L}_{\text{massless}}\) is a gauge-invariant Lagrangian for massless gluons and quarks, with null interactions; \(\mathcal{L}_{\text{gluon}}\) is the mass and gauge-fixing term for gluons and auxiliary scalars; \(\mathcal{L}_{\text{quark}}\) is the mass term for quarks; and \(\mathcal{L}_{\text{ghost}}\) is the Faddeev-Popov ghost term. The first is given by

\[
\mathcal{L}_{\text{massless}} = -\frac{1}{4} \sum_k r_k F_{\mu
u} F_{\mu\nu} + \sum_i \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i + g \sum_{ijk} \beta_i \beta_j \xi_k \bar{\psi}_i \gamma^\mu T_a A_{ak\mu} \psi_j,
\]  

where the field tensor is

\[
F_{\mu\nu} = \partial^\mu A_{ak}^\nu - \partial^\nu A_{ak}^\mu - r_k \xi_k g \sum_{lm} \xi_l \xi_m A_{al\mu} A_{cm}^\nu.
\]  

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The indices are $k$ for (PV) gluons and $i, j$ for (PV) quarks, with values of zero for the physical fields. The gauge transformations of the fields are

$$A_{ak}^\mu \rightarrow A_{ak}^\mu + \partial^\mu \Lambda_{ak} + r_k \xi_k g f_{abc} \Lambda_b A_{ak}^\mu,$$

(2.4)

$$\psi_i \rightarrow \psi_i + ig s_i \beta_i T_a \Lambda_a \psi,$$

(2.5)

with $\Lambda_a \equiv \sum_k \xi_k \Lambda_{ak}$ and $[T_a, T_b] = if_{abc} T_c$. The regularization is provided by the null field combinations:

$$A_{ak}^\mu \equiv \sum_k \xi_{kak} A_{ak}^\mu, \quad \psi \equiv \sum_i \beta_i \psi_i,$$

(2.6)

with $\sum_k r_k \xi_k^2 = 0$ and $\sum_is_i^2 = 0$. Here $r_k$ and $s_i$ equal $\pm 1$ (e.g. $(-1)^k$ and $(-1)^i$), $A_{ak}^\mu$ is Abelian, and $\psi$ is gauge invariant. With these definitions, this term of the Lagrangian can be written as

$$\mathcal{L}_{\text{massless}} = -\frac{1}{4} \sum_k r_k (\partial^\mu A_{ak}^\nu - \partial^\nu A_{ak}^\mu)^2 + g f_{abc} \partial^\mu A_{ak}^\nu A_{b\mu} A_{c\nu} + \sum_i s_i \bar{\psi} i \gamma^\mu \partial_\mu \psi + g \bar{\psi} \gamma^\mu T_a A_{ak} \psi.$$

(2.7)

The free terms are those for fields with metrics $r_k$ and $s_i$, and the interaction terms involve only null fields. The four-gluon interaction is implicit in the infinite-PV-mass limit through a contraction of two three-gluon interactions.

The null field combinations provide the necessary two PV subtractions to regulate any loop. For example, consider a loop with vertices from interactions of the form $g \bar{\psi} \gamma^\mu T_a A_{ak} \psi$. Assume that there is one gluon contraction and one quark contraction to form the loop. The contraction of the $k$th gluon field yields the metric signature $r_k$ and contraction of the $i$th quark field, $s_i$. The coupling coefficients are selected as $\xi_k, \xi_k, \beta_i, \text{and} \beta_i'$. The loop contribution then contains the factors $r_k \xi_k \xi_k'$ and $s_i \beta_i \beta_i'$. These factors provide the two subtractions, because on their own they sum to zero.

The gluon mass and gauge-fixing term is

$$\mathcal{L}_{\text{gluon}} = \frac{1}{2} \sum_k r_k \left( \mu_k A_{ak}^\mu - \partial^\mu \phi_{ak} \right)^2 - \frac{\zeta}{2} \sum_k r_k \left( \partial_\mu A_{ak}^\mu + \mu_k \frac{\mu_k}{\zeta} \phi_{ak} \right)^2.$$

(2.8)

The first term is gauge-invariant, and the second, gauge-fixing. The scalar fields $\phi_{ak}$ obey the gauge transformation

$$\phi_{ak} \rightarrow \phi_{ak} + \mu_k \Lambda_{ak} + \mu_k r_k \xi_k g f_{abc} \int d^4 x' \Lambda_b(x') A_{ak}^\mu(x').$$

(2.9)

The line integral allows the derivative to transform as

$$\partial^\mu \phi_{ak} \rightarrow \partial^\mu \phi_{ak} + \mu_k \partial^\mu \Lambda_{ak} + \mu_k r_k \xi_k g f_{abc} \Lambda_b A_{ak}^\mu.$$

(2.10)

This part of the Lagrangian can be reduced to the form

$$\mathcal{L}_{\text{gluon}} = \frac{1}{2} \sum_k r_k \mu_k^2 \left( A_{ak}^\mu \right)^2 - \frac{\zeta}{2} \sum_k r_k \left( \partial_\mu A_{ak}^\mu \right)^2 + \frac{1}{2} \sum_k r_k \left[ \left( \partial_\mu \phi_{ak} \right)^2 - \frac{\mu_k^2}{\zeta} \phi_{ak}^2 \right],$$

(2.11)

where the gluon mass $\mu_k$ is explicit and the scalar is given a mass $\mu_k/\sqrt{\zeta}$ and inherits the metric $r_k$. This is a non-Abelian extension of a Stueckelberg mechanism [10][12].
The quark mass term is

$$L_{\text{quark}} = \sum_i s_i m_i (\bar{\psi}_i + ig \frac{s_i \beta_i}{\mu_{\text{PV}}} \bar{\phi}_a \psi_T a)(\psi_i - ig \frac{s_i \beta_i}{\mu_{\text{PV}}} \bar{\phi}_a \psi), \quad (2.12)$$

with the combination

$$\bar{\phi}_a \equiv \sum_k \xi_k \frac{\mu_{\text{PV}}}{\mu_k} \phi_{ak} \quad (2.13)$$

made null by the additional constraint $\sum_k r_k \xi_k^2 = 0$ and with $\mu_{\text{PV}} \equiv \max_k \mu_k$. The gauge transformation of the combination is Abelian:

$$\bar{\phi}_a \rightarrow \bar{\phi}_a + \mu_{\text{PV}} \Lambda_a. \quad (2.14)$$

To make all couplings null, we define

$$\bar{\psi} = \sum_i \beta_i \frac{m_i}{m_{\text{PV}}} \psi_i, \quad (2.15)$$

with $m_{\text{PV}} \equiv \max_i m_i$, and impose the constraints $\sum_i s_i m_i^2 \beta_i^2 = 0$ and $\sum_i s_i m_i \beta_i^2 = 0$. The second constraint makes $\bar{\psi}$ and $\psi$ mutually null, to allow couplings between these combinations, and cancels the quartic coupling term. The quark mass term then simplifies to

$$L_{\text{quark}} = -\sum_i s_i m_i \bar{\psi}_i \psi_i - ig \frac{m_{\text{PV}}}{\mu_{\text{PV}}} \left[ \bar{\psi} T_a \bar{\phi}_a \psi - \bar{\psi} T_a \bar{\phi}_a \psi \right] \quad (2.16)$$

A standard construction [14] yields the ghost term [8]

$$L_{\text{ghost}} = \sum_k r_k \partial_\mu \bar{c}_a k \partial^\mu c_{ak} + g f_{abc} \left[ \partial_\mu \bar{c}_a c_{b} A^\mu_{a} - \frac{\mu_{\text{PV}}^2}{\zeta} c_a \int dx' c_b(x') A^\mu_{c}(x') \right], \quad (2.17)$$

for ghosts $c_{ak}$ and anti-ghosts $\bar{c}_{ak}$, with null combinations defined as

$$c_{a} \equiv \sum_k \xi_k c_{ak}, \quad \bar{c}_{a} \equiv \sum_k \xi_k \bar{c}_{ak}, \quad \bar{\bar{c}}_{a} \equiv \sum_k \xi_k \frac{\mu_{\text{PV}}^2}{\mu_k^2} \bar{c}_{ak}. \quad (2.18)$$

For these to be (mutually) null, we require $\sum_k r_k \mu_k^2 \xi_k^2 = 0$ and $\sum_k r_k \mu_k^4 \xi_k^2 = 0$.

As a summary of the various constraints, the following need to be satisfied for the adjoint fields:

$$\sum_k r_k \xi_k^2 = 0, \quad \sum_k r_k \frac{\xi_k^2}{\mu_k^2} = 0, \quad \sum_k r_k \mu_k^2 \xi_k^2 = 0, \quad \sum_k r_k \mu_k^4 \xi_k^2 = 0, \quad (2.19)$$

and for the quark fields:

$$\sum_i s_i \beta_i^2 = 0, \quad \sum_i s_i m_i^2 \beta_i^2 = 0, \quad \sum_i s_i m_i \beta_i^2 = 0, \quad \sum_i s_i m_i \beta_i^2 = 0. \quad (2.20)$$

For the PV masses to be chosen independently, these constraints require four PV gluons, four PV ghosts and antighosts, five PV scalars, and three PV quarks. For pure Yang–Mills theory, the number of PV scalars is four, with the $k = 0$ field dropped and $\mu_0 = 0$; the fields
\( \phi_{a0} \) are used only in splitting the masses of the PV quarks. In either case, the number of PV fields translates to a large, but necessary computational load.

The BRST transformations are

\[
\delta A^\mu_{ak} = \epsilon \partial^\mu c_{ak} + \epsilon r_k \xi_k g f_{abc} A^\mu_c,
\]

\[
\delta \psi_i = i \epsilon g s_i \beta_i T_a c_i \psi, \quad \delta \bar{\psi} = -i \epsilon g s_i \beta_i \bar{\psi} T_a c_i,
\]

\[
\delta \phi_{ak} = \epsilon \mu_k c_{ak} + \epsilon r_k \xi_k \mu_k g f_{abc} \int dx' c_b(x') A^\mu_c(x'),
\]

\[
\delta \partial^\mu \phi_{ak} = \epsilon \mu_k \partial^\mu c_{ak} + \epsilon r_k \xi_k \mu_k g f_{abc} A^\mu_c,
\]

\[
\delta c_{ak} = -\zeta \epsilon \left( \partial_\mu A^\mu_{ak} + \frac{\mu_\zeta}{\zeta} \phi_{ak} \right), \quad \delta c_{ak} = \frac{1}{2} \epsilon r_k \xi_k g f_{abc} c_b c_c,
\]

with \( \epsilon \) a real Grassmann constant, for which \( \epsilon^2 = 0 \). For the various null combinations, we then find

\[
\delta A^\mu_a = \epsilon \partial^\mu c_a, \quad \delta \phi_a = \epsilon \mu_{PV} c_a, \quad \delta c_a = 0, \quad \delta \psi = 0, \quad \delta \bar{\psi} = 0.
\]

The full Lagrangian is invariant with respect to these transformations.

### III. MASSIVE VECTOR QUANTIZATION

The construction of the Lagrangian makes no reference to a choice of quantization coordinates; however, the construction was done in such a way as to keep open the possibility of light-front quantization in an arbitrary covariant gauge. To see how this quantization can take place, consider the case of QED \( E \). The Lagrangian is \( \mathcal{L} = -\frac{1}{4} F^2 + \frac{1}{2} \mu A^2 - \frac{1}{2} \zeta (\partial \cdot A)^2 \), which yields the field equation: \((\Box + \mu^2) A_\mu - (1 - \zeta) \partial_\mu (\partial \cdot A) = 0 \). The light-front Hamiltonian density is

\[
\mathcal{H} = \mathcal{H}|_{\zeta=1} + \frac{1}{2} (1 - \zeta) (\partial \cdot A) (\partial \cdot A - 2 \partial_+ A_+ - 2 \partial_- \cdot \vec{A}_\perp),
\]

with

\[
\mathcal{H}|_{\zeta=1} = \frac{1}{2} \sum_{\mu=0}^3 \epsilon^\mu \left[ (\partial_\perp A^\mu)^2 + \mu^2 (A^\mu)^2 \right]
\]

and \( \epsilon^\mu = (-1, 1, 1, 1) \). From this we obtain the light-front Hamiltonian

\[
\mathcal{P}^- = \int dx \mathcal{H}|_{x=0} = \int dk \sum_{\lambda} \epsilon^\lambda \frac{k_+^2 + \mu^2}{k^2} a^{(\lambda)}(k) a^{(\lambda)}(k)
\]

with \( k = (k^+, \vec{k}_\perp) \), \( \mu^{(\lambda)} = \mu \) for \( \lambda = 1, 2, 3 \), but \( \mu^{(0)} = \tilde{\mu} \equiv \mu / \sqrt{\zeta} \). The nonzero commutator is

\[
[a^{(\lambda)}(k), a^{(\lambda)}(k')] = \epsilon^\lambda \delta_{\lambda \lambda'} (k - k').
\]

The normal-mode expansion for the field is

\[
A_\mu(x) = \int \frac{dk}{\sqrt{16 \pi^2 k^+}} \left\{ \sum_{\lambda=1}^3 \epsilon^{(\lambda)}(k) \left[ a^{(\lambda)}(k) e^{-ik \cdot x} + a^{(\lambda)}(k) e^{ik \cdot x} \right] + \epsilon^{(0)}(k) \left[ a^{(0)}(k) e^{-ik \cdot x} + a^{(0)}(k) e^{ik \cdot x} \right] \right\},
\]

with polarization vectors

\[
e^{(1, 2)}(k) = (0, 2 \hat{e}_{1, 2} \cdot \vec{k}_\perp / k^+, \hat{e}_{1, 2}), \quad e^{(3)}(k) = \frac{1}{\mu} ((k_+^2 - \mu^2) / k^+, k^+, \vec{k}_\perp), \quad e^{(0)}(k) = \vec{k} / \mu.
\]
Here $\tilde{k} = k$, $\tilde{k}^- = (k^2 + \mu^2)/k^+$, and $\hat{e}_{1,2}$ are transverse unit vectors. These vectors satisfy:

$k \cdot e^{(\lambda)} = 0$ and $e^{(\lambda)} \cdot e^{(\lambda')} = -\delta_{\lambda\lambda'}$ for $\lambda, \lambda' = 1, 2, 3$. The first term in $A_\mu$ satisfies $(\Box + \mu^2)A_\mu = 0$ and $\partial \cdot A = 0$. The $\lambda = 0$ term violates each, but the field equation is satisfied [10, 15].

The nondynamical components of the fermion fields satisfy the constraints

$\partial_\mu \psi = 0$ and $\psi^* \gamma_\mu \psi = -\delta_{\mu\lambda}$ for $\lambda = 0, 1, 2, 3$. The first term in $A_\mu$ satisfies

$(\Box + \mu^2)A_\mu = 0$ and $\partial \cdot A = 0$. The $\lambda = 0$ term violates each, but the field equation is satisfied [10, 15].

**IV. LIGHT-FRONT COUPLED-CLUSTER METHOD**

Given a field-theoretic light-front Hamiltonian $\mathcal{P}^-$, we wish to solve the fundamental eigenvalue problem $\mathcal{P}^- |\psi\rangle = M^2 + \mathcal{P}_\perp |\psi\rangle$. The LFCC method [13] is designed to do so in terms of a Fock-state expansion but without the usual truncation of Fock space. Instead, write the eigenstate as $|\psi\rangle = \sqrt{Z}e^T |\phi\rangle$. Here $Z$ controls the normalization, which is fixed as $\langle \phi' | \phi \rangle = \delta (\mathcal{P}' - \mathcal{P})$. The ket $|\phi\rangle$ is the valence state, with norm $\langle \phi' | \phi \rangle = \delta (\mathcal{P}' - \mathcal{P})$. The operator $T$ contains terms that only increase particle number; it does, however, conserve the quantum numbers of the full state, such as $J_\perp$, light-front momentum $\mathcal{P}_\perp$ and charge. Because $p^+$ is always positive, $T$ must include annihilation operators, and powers of $T$ include contractions. With construction of the effective Hamiltonian $\mathcal{P}^- = e^{-T} \mathcal{P}^- e^T$ and the definition of a projection $P_v$ onto the valence Fock sector, we have the coupled system:

$$P_v \mathcal{P}^- |\phi\rangle = M^2 + \mathcal{P}_\perp |\phi\rangle, \quad (1 - P_v) \mathcal{P}^- |\phi\rangle = 0. \quad (4.1)$$

This is still an infinite system of equations. The LFCC approximation is to truncate $T$ at a fixed increase in particle count, but not truncate the exponential of $T$. The projection $(1 - P_v)$ is truncated to a consistent set of Fock sectors above the valence sector, enough to have a finite, but sufficient system of (nonlinear) equations for the functions in $T$ and in the valence state. The leading approximation of $T$ for QCD would include quark $\rightarrow$ quark + gluon, gluon $\rightarrow$ quark + antiquark, and gluon $\rightarrow$ gluon + gluon transitions.

The LFCC approach requires no Fock-space truncation and no sector dependence or spectator dependence of self-energies or bare parameters. It is systematically improvable by adding terms to $T$ and sectors to $(1 - P_v)$ with more constituents. Various applications have been carried out, including a heavy-fermion model [13], $\phi^4$ theory [16], and QED [17].

**V. SUMMARY**

With these tools in place, the time has come for a concentrated assault on QCD. The procedure would be to invoke PV regularization and light-front quantization in an arbitrary
covariant gauge with variable gauge parameter. Any (numerical) approximation\textsuperscript{2, 18, 19} would then be applied to a finite theory where the continuum limit can be taken independent of any chosen renormalization scheme. Fock-space truncation and the resulting uncanceled divergences can be avoided by using the LFCC method. The best places to start would be simpler systems in quenched approximation, such as heavy-quark mesons and glueballs. The computational load and effort are likely comparable to lattice gauge theory; the large number of PV modes and the discretizations being the analogs of lattice size and spacing.

**ACKNOWLEDGMENTS**

This work was done in collaboration with S.S. Chabysheva. Travel to the conference was supported in part by the University of Minnesota Global Programs & Strategy Alliance.

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