Non-equilibrium phase transition in the model of human virtual stick balancing

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Abstract
Archetypal stick balancing task represents a wide class of unstable processes under human control. The currently dominant theory of human control in stick balancing is based on the concept of discontinuous, or intermittent control. Traditionally, intermittent control models involve threshold-driven control activation, however, recently it has been demonstrated that, in a simple virtual stick balancing task, some basic properties of human control activation mechanisms can only be reflected by more sophisticated, noise-driven models. The aim of the present paper is to demonstrate that the previously introduced double-well model of noise-driven intermittent control activation can reproduce the experimentally observed human behaviours under various conditions. We show that the model successfully reproduces the experimental distributions of actions points (stick angle values triggering activation of human control) obtained in two previously reported experiments. Moreover, we show that a slight change in the model’s noise intensity parameter leads to a sudden shift of model distributions, that is, a non-equilibrium phase transition is observed. Our results extend the current understanding of the concept of noise-driven control activation, suggesting that it is applicable in a variety of experimental setups. The two discovered phases of the double-well model correspond to two different modes of control activation in human operators; physiological basis of these modes has to be investigated in future studies.

1 Introduction
Humans have to perform the task of balancing dynamic systems near an unstable equilibrium on a daily basis, for instance, when standing upright. Stick balancing (Fig. 1) is an increasingly popular paradigm of studying human control behavior in such situations (see e.g. [1, 2, 3]). In particular, much research has been aimed at understanding the mechanisms of discontinuous, or intermittent control in the context of human balance control [4, 5].

Intermittent control assumes switching between periods of passive (control is off) and active (control is on) behavior of the controller. Traditionally in the models of stick balancing, control is switched on when the stick angle exceeds certain threshold value [4]. Models which incorporate threshold-driven control activation can usually explain much dynamics observed in experiments. However, recent investigations of virtual overdamped stick balancing have revealed that control activations at large deviations occurs much more frequently then predicted by threshold-driven mechanism [6]. Besides, some of the most peculiar phenomena observed in human control behavior still remain unexplained by the models based on threshold-driven control activa-
tion. Importantly, such phenomena include high occurrence of large fluctuations in human-controlled systems, which result, e.g., in falls during stick balancing or quiet standing [8]. All these considerations suggest that in controlling even the simplest unstable systems humans employ a somewhat more complex control activation mechanisms than assumed by conventional threshold-based models.

The present paper deals with one of the recent developments in the theory of human intermittent control, the double-well model of noise-driven control activation [9].

The model describes dynamics of the overdamped inverted pendulum under control of a human operator. The state of the mechanical system is described by stick angle $\theta$ and cart velocity $\upsilon$. Also, we model the cognitive state of the operator with respect to the stick by the order parameter $\xi$, which switches intermittently between two states, $\xi = 0$ (passive control phase) and $\xi = 1$ (active control phase). Its dynamics are driven jointly by the deterministic and random forces. The deterministic dynamics are defined by the potential energy landscape, which possesses two attractors corresponding to $\xi = 0$ and $\xi = 1$. The relative strength of the two attractors is defined by the state of the controlled system (in our case, the stick angle $\theta$). The switching between attractors is caused by the multiplicative noise cofactor.

Model dynamics are defined by the dimensionless equations

$$\dot{\theta} = \theta - \upsilon,$$

$$\dot{\upsilon} = \gamma \theta \xi - \sigma \upsilon,$$  \hspace{1cm} (1)

where $\gamma$ and $\sigma$ are feedback parameters. Using the approach of phase space extension to describe human actions [12], we treat $\xi$ as a separate phase variable, governed by the nonlinear Langevin equation

$$\tau \dot{\xi} = -\frac{\partial H}{\partial \xi} + \sqrt{\epsilon} H \zeta, \hspace{1cm} (2)$$

where $\tau$ is the time scale of the control activation process, and $H(\xi, \theta)$ is the “Hamiltonian” shaping the energy landscape of the control activation process (Fig. 3), $\zeta$ is white noise, and $\epsilon > 0$ is the parameter regulating the noise intensity. Note that the form of the Langevin equation (2) does not depend on the interpretation of stochastic process (see [9] for details). We consider the simplest possible form of $H(\xi, \theta)$ providing two metastable attractors, $\xi = 0$ and $\xi = 1$:

$$H(\xi, a(\theta)) = \frac{1}{12} \left( \frac{\xi^4}{4} - (1 + a) \frac{\xi^3}{3} + a \frac{\xi^2}{2} + 1 - a \right), \hspace{1cm} (3)$$

where

$$a(\theta) = 1/(1 + \theta^2). \hspace{1cm} (4)$$

In (4), $a \approx 1$ if $|\theta| \ll 1$ and $a \approx 0$ when $|\theta| \gg 1$ (we measure the stick angle in the units scaled to operator’s perceptual threshold, so that $|\theta| \ll 1$ means that the stick angle significantly exceed the threshold of the operator). If the stick angle is large ($a \approx 0$), the form of $H(\xi, a)$ is such that the “act” state ($\xi \approx 1$) is most likely. On the other hand, the “wait” state ($\xi \approx 0$) can be expected whenever the stick angle is small ($a \approx 1$).

2 Model

For detailed description of the model, we refer the reader to Ref. [9]; here we just briefly review its basic tenets.
Intermediate values of $\theta$ provide bistable dynamics, so both the states are possible.

According to Eq. (1), the order parameter $\xi$ switches intermittently between the states $\xi = 0$ and $\xi = 1$, which are mapped onto the operator’s two cognitive states, “wait” and “act”. In this way, the model captures on-off intermittency observed experimentally in human control.

### 3 Results

Here we confront the model to the results of two experiments on virtual stick balancing conducted previously [6, 11]. In both experiments, the subjects (operators) observed a mechanical system on a computer screen, and had to balance the stick upwards by moving the cart via computer mouse. In experiment 1 (“no mouse” condition) the mouse cursor was not present on the screen, so the operators could only observe the angular deviation of the stick from the vertical position [6]. In experiment 2 (“mouse” condition), the reference point (mouse cursor) was displayed near the upper tip of the stick. Thereby, the operators had additional information about the state of the stick, that is, the linear displacement of the upper tip of the stick from the reference point [11]. This information made it significantly easier for the operators to perform the task, and, importantly, resulted in a markedly different distributions of action points (values of stick angle triggering the operator’s response). However, it is still unclear whether these different patterns of operators’ behavior can be attributed to different perceptual and/or cognitive mechanisms of control activation, or to different modes of the same basic mechanism. The present work aims to address this question.

We numerically simulated the dynamics of the model for a range of values of the noise intensity parameter $\epsilon$. The other parameters were fixed at values $\sigma = 3.5, \gamma = \sigma^2/2, \tau = 0.2$ which, on the one hand, were previously shown to be physically plausible and, on the other hand, provided good fit to the experimental data [9]. The main focus of the present paper is on the properties of control activation mechanism, which manifests itself in the moments when human operators start actively controlling the system, so we only consider here the results of the simulations directly illustrating the patterns of control activation, that is, the action point (AP) distributions. Detailed investigation of the other aspects of system dynamics is left for future work.

Fig. 4 illustrates the distribution of action points (with stick angles z-scored) for the values of the noise intensity parameter from $\epsilon = 0.01$ to $\epsilon = 0.09$ with the step of 0.01, as compared to the distributions obtained experimentally from human operators. For small values of noise intensity, the action point distribution of the model closely follows the experimental distribution for the “mouse” condition. The shape of the distribution in this case is approximately characterized by power law (straight line in the log-log plot 4b), indicating possible involvement of self-organized critical state of the control activation mechanism.

With the noise intensity increasing from $\epsilon = 0.02$ to $\epsilon = 0.03$, the system exhibits a phase transition: the AP distribution abruptly changes from power-law-like to Laplace-like (approximately straight line in the log-log plot 4a), which now matches the experimental distribution for the “no mouse” condition. Further changes in noise intensity (from 0.03 to 0.09) do not change the shape of the distribution, revealing a distinct, robust system phase.

Fig. 3: Two-well energy landscape $H(\xi)$ depending on deviation of the stick from the desired position. Figure file is available under CC-BY [13]
Fig. 4: Distributions of action points (AP) exhibited by human operators in virtual stick balancing (black lines) and produced by the double-well model (blue circles). Dashed black line is the AP distribution in the “mouse” condition (mouse cursor is hidden from the computer screen), and solid black line is the distribution for the “mouse” condition (mouse cursor displayed near the upper tip of the stick provides additional visual feedback to the operator). For model simulations, only parameter $\epsilon$ was varied, with other parameters taking the values $\sigma = 3.5$, $\gamma = \sigma^2/2$, $\tau = 0.2$; (a) x-axis linearly scaled, y-axis log-scaled; (b) both axes log-scaled. Figure file is available under CC-BY [14]

4 Conclusion

The obtained simulation results lead us to the hypothesis that the two phases of the model (1) correspond to two different modes of the single control activation mechanism. The first mode is characterized by Laplace-like action points distribution and high intensity of noise in the control activation mechanism causing repeated switching between on- and off-control periods ($\epsilon \gtrsim 0.03$). This mode is observed experimentally in human operators reacting solely to angular deviations of the stick. The second mode generates power-law-like distributions of action points, and is reproduced by the model for relatively low noise intensity ($\epsilon \lesssim 0.02$). The latter mode is found in the operators supplied with additional sensory feedback. To the best of our knowledge, this is the first control activation model to incorporate these two activation modes. The physiological basis of the two distinct control activation modes needs to be investigated in follow-up studies, which may further shed light on the nature of human intermittent control.

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