Determining equivalent charges on flow and balance in individual account pension systems

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Abstract

In this article we determine a charge on balance that is equivalent to a certain fixed charge on flow for a particular utility-maximizer affiliate participating in a defined-contribution pension fund under the system of individual accounts. We also prove, under market completeness, that the equivalent charge on balance depends only on the current level of the charge on flow, the length of the accumulation period and the risk free rate of return.

Keywords: Pension fund, Defined benefit, Individual account, Charge on balance, Charge on flow

1. Introduction

Two important characteristics of a defined-contribution (DC) pension fund are that affiliates borne the risk derived from fluctuations in the value of assets and that imposed administrative charges have a direct and significant impact on the terminal wealth of the corresponding individual accounts (IA)\(^1\). Those charges have received a great deal of attention from the pension

\(^1\)Murthi et al. (1999) estimate that in the U.K. over 40% of the IA’s value is dissipated through fees and charges. Whitehouse (2001) determines that a levy of one per cent of assets adds up to nearly 20% of the final pension value.
supervisory agencies, policy-makers and researchers, especially in countries that have partially or totally transformed their public defined-benefit pension systems into individual capitalization ones².

As mentioned in James et al. (2001), Whitehouse (2001) and Mitchell (1998) the high charges of IA systems is one of their main criticisms since they discourage participation (as people consider contributions as taxes instead of savings), damage the reputation of the system, reduce future pensions, and increase future costs for the government whether there is guaranteed minimum pension. Devesa-Carpio et al. (2003) consider that the charge scheme adopted by the IA system is very important since fund accumulation process is exponential and targeted for long horizons. Following Kritzer et al. (2011), the most common administrative charges in IA pension systems are proportional on flow (or a percentage of the affiliate’s salary), fixed on flow, proportional on assets (balance) and proportional over excess returns³. However, this article will focus only on charges that are proportional on balance and flow since they are by far the most popular and important. Queisser

²The most familiar and documented example is Chile. The reader can find main aspects of such reform in Arrau et al. (1993), Diamond and Valdés-Prieto (1994), Edwards (1998), Arenas de Mesa and Mesa-Lago (2006). An analysis of the Peruvian pension system’s reform and its current state can be found in Marthans and Stok (2013). Queisser (1998), Sinha (2000), Kay and Kritzer (2001), Mesa-Lago (2006) and Kritzer et al. (2011) provide a good references for the reform, situation and perspective of pension systems in Latin America. Chlon and Rutkowski (1999), Chlon (2000) and Holzman et al. (2003) provide information about the reforms in Poland and Europe.

³Analysis and comparison of administrative charges across different countries can be found in James et al. (2001), Whitehouse (2001), Gómez-Hernández and Stewart (2008), Corvera et al. (2006), Tapia and Yermo (2008) and Devesa-Carpio et al. (2003). Moreover, Sinha (2001), Masías and Sánchez (2007) and Martínez and Murcia (2008) analyze in detail (although there have been some modifications) the administrative charges in Mexico, Peru and Colombia, respectively.
(1998) considers that the charge on flow is more advantageous for the pension fund administrators (PFA) in the initial stages of the system, and although the charge on balance aligns the PFA’s objectives in terms of increasing the fund’s profitability, it tends to be more expensive in the long-run as personal accounts grow in size. On the other hand, Shah (1997) mentions that the charge on flow generates distortions and undesirable tendencies like promoting high start-up costs for the PFAs, discouraging competition in the system and generating losses for older affiliates.

Asset allocation, performance and risk of a DC pension plan during its accumulation and decumulation phases have received a considerable attention in the literature. Nonetheless, methodologies to compare administrative charges in a DC pension fund with IA during its accumulation period have not received a comparable level of attention in the literature, specially in a continuous-time stochastic setting. Therefore, we fill that gap by developing

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a methodology (in the aforementioned environment) to determine equivalent charges on flow and balance. We consider a risk-averse affiliate who wants to maximize her expected utility of terminal wealth in a complete financial market. Then, we determine the equivalent charges by equating the maximum terminal certainty equivalent that can be achieved under both kind of charges. Moreover, under certain assumptions, we prove that the equivalent charges on balance and flow depend only on the length of the accumulation period and the risk-free rate of return; and, to the best of our knowledge this relationship between charges is new in the literature.

The rest of the article proceeds as follows. Section 2 introduces a methodology to mathematically represent and compare charges on balance and flow. Section 3 discusses an application to the methodology in the Peruvian Private Pension System. Finally, Section 4 draws conclusions.

2. Methodology

Throughout the paper \((\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t \}_{t \geq 0})\) represents a filtered and complete probability space on which a standard \(\{\mathcal{F}_t \}_{t \geq 0}\)-adapted one-dimensional Brownian motion \(B(t)\) is defined. We denote by \(L^2_{(\mathbb{F}, \mathbb{P})}(0,T; \mathbb{R})\) the set of all \(\mathbb{R}\)-valued, measurable stochastic processes \(g(t)\) adapted to \(\{\mathcal{F}_t \}_{t \geq 0}\), such that \(\mathbb{E}[\int_0^T |g(t)|^2 dt] < \infty\).

For any \(t \in [0, T]\), we assume that the Pension Fund Administrator (PFA) can invest the affiliate’s contributions in only two assets which satisfy:

\[
\begin{align*}
\text{for } & P_0(t) = rP_0(t)dt, & P_0(0) = P_0 > 0, \\
\text{for } & P_1(t) = \mu P_1(t)dt + \sigma P_1(t)dB(t), & P_1(0) = P_1 > 0.
\end{align*}
\]

\(4\)
It is clear that $r \geq 0$ is the risk-free rate of return, $\mu$ and $\sigma$ are the risky asset’s growth rate and volatility, respectively. The stochastic differential equation (SDE) in (2) generates a geometric Brownian motion (GBM) which is a common specification to model asset values and it is heavily utilized in stochastic control of DC pension funds as mentioned in Section 1.

2.1. The affiliate’s problem

Consider a particular PFA’s affiliate who has $T > 0$ months before retirement, i.e., $T$ represents the length of her accumulation phase. She already has $W_0 > 0$ ready to be invested in her individual account, and after that initial deposit she will contribute at a constant rate $\theta > 0$ per month for the next $T$ months. Also, for any $t \in [0, T]$ let $x(t) \in L^2(0, T; \mathbb{R})$ be the proportion of her IA that is invested in the risky asset. We assume the adjustments are performed instantly and free of charge. Let $W(t)$ be the affiliate’s wealth in her IA at time $t \in [0, T]$. If the PFA does not charge any administrative fees to the affiliate, then $W(t)$ satisfies

$$dW(t) = [W(t)(x(t)\mu + (1 - x(t))r) + \theta] dt + W(t)x(t)\sigma dB(t),$$

with $W(0) = W_0$.

It is in the affiliate’s interest that the PFA will maximize her expected utility of terminal wealth, $\mathbb{E}[U(W(T))]$, by determining an optimal proportion $x^*(t)$. We assume that $U$ is strictly increasing and concave in its domain.
Therefore, the affiliate wants the PFA to solve problem (P) given by

\[ \begin{align*}
\text{Max} & \quad \mathbb{E}[U(W(T))] \\
\text{st.} & \quad x(t) \in L^2_{\mathcal{F}}(0, T; \mathbb{R}), \\
& \quad dW(t) = [W(t)(\mu - r) + \theta]dt + W(t)x(t)\sigma dB(t), \\
& \quad W(0) = W_0.
\end{align*} \]

Introducing \( V \), the value function of the problem, we have

\[ V(t, W) = \max_{\{x\}} \mathbb{E}[U(W(T)|W(t) = W)], \quad 0 \leq t \leq T. \quad (4) \]

Devolder et al. (2003) proved the optimal control of (P) satisfies

\[ x^*(t) = -\frac{\partial V/\partial W}{\partial^2 V/\partial W^2} \frac{1}{W} \frac{\mu - r}{\sigma^2} \quad (5) \]

while the value function solves the following partial differential equation (PDE):

\[ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial W}(rW + \theta) - \frac{1}{2} \frac{(\mu - r)^2}{\sigma^2} \frac{\partial^2 V/\partial W^2}{\partial^2 V/\partial W^2} = 0, \quad \text{with} \quad V(T, W) = U(W). \quad (6) \]

For example, if we assume that the affiliate has an exponential utility function, then

\[ U(W) = -\frac{1}{c}e^{-cW}, \quad c > 0. \quad (7) \]

The utility in (7) exhibits constant absolute risk aversion since \(-\frac{U''(W)}{U'(W)} = c\); but, most important, it allows an explicit solution for (P). Following Devolder et al. (2003), the optimal proportion to be invested in the risky
asset is
\[ x^*(t) = \frac{e^{-r(T-t)} \mu - r}{W} \frac{\sigma^2 c}{\sigma^2 c} , \tag{8} \]
and the corresponding value function is
\[ V(t, W) = -\frac{1}{c} \exp \left\{ -c \left( e^{r(T-t)} W + \theta \left( \frac{e^{r(T-t)}}{r} - 1 \right) + \frac{1}{2} \frac{(\mu - r)^2}{\sigma^2 c} (T - t) \right) \right\} . \tag{9} \]

We can observe from (8) that the optimal control does not depend on the contribution rate \( \theta \). If we apply the optimal strategy \( x^*(t) \) stated in (8), then \( W(T) = \overline{W}(T) \) and the maximum expected utility of terminal wealth is
\[ \mathbb{E}[U(\overline{W}(T))] = V(0, W_0) = -\frac{1}{c} \exp \left\{ -c \left( e^{rT} W_0 + \theta \left( \frac{e^{rT}}{r} - 1 \right) + \frac{1}{2} \frac{(\mu - r)^2}{\sigma^2 c} T \right) \right\} . \tag{10} \]
Moreover, the certainty equivalent of \( \overline{W}(T) \), \( CE(\overline{W}(T)) \), is given by
\[ CE(\overline{W}(T)) = e^{rT} W_0 + \theta \left( \frac{e^{rT} - 1}{r} \right) + \frac{1}{2} \frac{(\mu - r)^2}{\sigma^2 c} T . \tag{11} \]
As we can notice from (11), \( CE(\overline{W}(T)) \) is the sum of the future value of \( W_0 \), the future value of a continuous annuity with rate \( \theta \) and a term depending on the market price of risk and the risk aversion parameter but independent of any contribution made to the IA.

Next, we describe in detail the charges that the PFA will apply either on the affiliate’s IA or on her contributions. We will use a structure similar to the one considered in Shah (1997), Diamond (2000), Blake and Board (2000),
2.2. Charge on balance

Let $\delta > 0$ be the monthly charge on balance expressed in continuous time. It is also known as charge on assets or on stock and, in general, it is applied as a percentage of the value of assets under management. The affiliate wants to study the value of her IA under this type of charge. If we denote such wealth as $W_s(t)$, it will satisfy the following SDE:

$$dW_s(t) = [W_s(t)(x_s(t)(\mu - r) + r - \delta) + \theta]dt + W_s(t)x_s(t)\sigma dB(t),$$  \hspace{1cm} (12)\

with $W_s(0) = W_0$. Notice that the charge on balance will diminish the monthly growth rates $\mu$ and $r$ by a quantity equal to $\delta$. We will use control $x_s(t)$ to indicate the fraction of the IA invested in the risky asset under the charge on balance. In this case, the PFA wants to solve the affiliate’s problem $(\text{Pb})$ given by

$$\begin{align*}
\text{Max} & \quad \mathbb{E}[U(W_s(T))] \\
\text{st.} & \quad x_s(t) \in L^2\mathbb{F}(0, T; \mathbb{R}), \\
& \quad dW_s(t) = [W_s(t)(x_s(t)(\mu - r) + r - \delta) + \theta]dt + W_s(t)x_s(t)\sigma dB(t), \\
& \quad W_s(0) = W_0.
\end{align*}$$

If $V_s(t, W_s)$ is the value function of $(\text{Pb})$, then $\mathbb{E}[W_s(T)] = V_s(0, W_0)$ where $W_s(T)$ is the final wealth under the optimal control $x_s^*(t)$.

Based on the results and assumptions regarding the exponential utility
function, the optimal strategy is

$$x^*_s(t) = e^{-(r-\delta)(T-t)} \frac{\mu - r}{W_s} \frac{\mu - r}{\sigma^2 c}, \quad (13)$$

while the maximum certainty equivalent is

$$\text{CE}(W_s(T)) = e^{(r-\delta)T}W_0 + \theta \left( \frac{e^{(r-\delta)T} - 1}{r - \delta} \right) + \frac{1}{2} \left( \frac{\mu - r}{\sigma^2 c} \right)^2 T. \quad (14)$$

Notice that the last term of (14) does not depend on the initial contribution ($W_0$), the charge on balance ($\delta$), and the contribution rate ($\theta$). Next, we describe the charge on flow.

2.3. Charge on flow

Let $\alpha > 0$ be the charge on flow and it could be applied as a fraction of the affiliate’s salary or contributions. Whether the affiliate makes a contribution $X$ in a particular month, we assume the charge she will pay to the PFA (at the moment the contribution is made) will be $F = X(1 - e^{-\alpha})$. Considering that $F$ could have been invested in the fund, it is possible to express contribution $X$ as $e^{-\alpha}X$ when adjusted for the opportunity cost of $F$. In the case of a constant rate of contribution, $\theta$, the charge on flow will generate an adjusted contribution rate of $e^{-\alpha}\theta$.

The affiliate wants to study the value of her individual account under this type of charge. We denote such wealth as $W_f(t)$ and it will satisfy the following SDE:

$$dW_f(t) = [W_f(t)x_f(t)(\mu - r) + r + e^{-\alpha}\theta] \, dt + W_f(t)x_f(t)\sigma dB(t), \quad (15)$$
with $W_f(0) = e^{-\alpha}W_0$. Recall that $W_f(T)$ does not represent the “true” wealth of the affiliate at the end of the accumulation phase but the final wealth adjusted by the opportunity cost of the charge on flow. Consequently, random variables $W_f(T)$ and $W_s(T)$ can be compared.

We will use control $x_f(t)$ to indicate the fraction of the IA invested in the risky asset under the charge on flow. Thus, the PFA wants to solve affiliate’s problem $(P_f)$ given by

$$\text{Max } \mathbb{E}[U(W_f(T))]$$

st. $x_f(t) \in L^2_f(0,T;\mathbb{R})$,

$$dW_f(t) = \left[ W_f(t)(x_f(t)(\mu - r) + r) + e^{-\alpha\theta} \right] dt + W_f(t)x_f(t)\sigma dB(t),$$

$$W_f(0) = e^{-\alpha}W_0.$$ 

If $V_f(t,W_f)$ is the value function of $(P_f)$, then $\mathbb{E}[W_f(T)] = V_f(0,e^{-\alpha}W_0)$ where $W_f(T)$ is the final adjusted wealth under the optimal control $x_f^*(t)$.

Based on the previous results and assumptions regarding the exponential utility function, the optimal control and the maximum certainty equivalent are given by

$$x_f^*(t) = \frac{e^{-r(T-t)} \mu - r}{W_f}$$

and

$$\text{CE}(W_f(T)) = e^{-\alpha} \left[ e^{rT}W_0 + \theta \left( \frac{e^{rT} - 1}{r} \right) + \frac{1}{2} \frac{(\mu - r)^2}{\sigma^2}\right] T.$$

Similar to (14), only the last term of (17) depends on the growth rate of the risky asset ($\mu$), its volatility ($\sigma$), and the risk aversion parameter ($c$).
2.4. Comparing charges on balance and flow

The affiliate wants to compare her optimal expected utility of adjusted terminal wealth under the two types of charges considered. Therefore, it is appropriate to contrast both $\text{CE}(\overline{W}_s(T))$ and $\text{CE}(\overline{W}_f(T))$. We define the following ratio to establish such comparison

$$A_{sf} = \frac{\text{CE}(\overline{W}_s(T))}{\text{CE}(\overline{W}_f(T))}.$$  

If $A_{sf} > 1$, the charge on balance will be preferred. If $A_{sf} < 1$ the charge on flow will be preferred. Finally, when $A_{sf} = 1$ the affiliate will be indifferent between both charge schemes.

Under the exponential utility function given by (7), $\text{CE}(\overline{W}_s(T))$ and $\text{CE}(\overline{W}_f(T))$ are given by (14) and (17), respectively. Then, we can express $A_{sf}$ in (18) as

$$A_{sf} = \frac{e^{(r-\delta)T}W_0 + \theta \left( \frac{e^{(r-\delta)T-1}}{r-\delta} \right) + \frac{1}{2} \frac{(\mu-r)^2}{\sigma^2} T}{e^{-\alpha \left[ e^{rT}W_0 + \theta \left( \frac{e^{rT-1}}{r} \right) \right] + \frac{1}{2} \frac{(\mu-r)^2}{\sigma^2} T}.}$$  

Given $\alpha^*$, let $\delta^*$ be the equivalent charge on balance, that is, the value $\delta^*$ such that $A_{sf} = 1$. Thus, both $\alpha^*$ and $\delta^*$ satisfy

$$\alpha^* = \ln \left( \frac{e^{rT}W_0 + \theta \left( \frac{e^{rT-1}}{r} \right)}{e^{(r-\delta^*)T}W_0 + \theta \left( \frac{e^{(r-\delta^*)T-1}}{r-\delta^*} \right)} \right).$$  

If in addition to all the previous assumptions we set $W_0 = 0$ (the accumulation phase begins with an amount equal to zero in the affiliate’s individual
account) then equation (20) reduces to

$$\alpha^* = \ln \left( \frac{\bar{s}_T}{\bar{s}_{T(r-\delta^*)}} \right),$$

(21)

where \(\bar{s}_T\) represents the future value at \(T\) of a continuous annuity with unit rate and interest \(r\) and \(\bar{s}_{T(r-\delta^*)}\) is the future value at \(T\) of a continuous annuity with unit rate and interest \(r - \delta^*\). We can observe from (21) that \(\delta^*\) will depend only on \(r\), \(T\) and \(\alpha^*\). Hence, it is independent of the parameters \(\mu\) and \(\sigma\) of the risky asset, the contribution rate \(\theta\) and the risk aversion coefficient \(c\). Notice from (21) that for a fixed \(\alpha^*\) whether \(r\) or \(T\) increases then \(\delta^*\) decreases, i.e., either an increment of the risk-free rate or the length of the accumulation period benefits the charge on flow with respect to the charge on balance.

In the next section we generalize equations (20) and (21) for any risk-averse affiliate as described in Section 2.1.

2.5. Equivalent charges in a complete market

The financial market consisting of the risk-free asset and the risky asset given by (1) and (2) is complete. Also, given \(x_s^*(t)\) and \(x_f^*(t)\), the optimal controls of problems (Pb) and (Pf), we can determine both \(\bar{W}_s(T)\) and \(\bar{W}_f(T)\). The equivalent charges of flow and balance can be obtained by comparing the present values of their corresponding terminal values of the individual accounts under the risk-neutral probability measure \(Q\). For that purpose we define the ratio \(N_{sf}\) as:

$$N_{sf} = \frac{\mathbb{E}_Q[\bar{W}_s(T)]}{\mathbb{E}_Q[\bar{W}_f(T)]}.$$
The ratio in (22) is equivalent to the ratio of the present values of $\overline{W}_s(T)$ and $\overline{W}_f(T)$ using $\mathbb{Q}$ since the factor $e^{-rT}$ appears in both the numerator and denominator of (22). Additionally, if $N_{sf} > 1$, the charge on balance will be preferred. If $N_{sf} < 1$ the charge on flow will be preferred; and, when $N_{sf} = 1$ the affiliate will be indifferent between both schemes.

It is clear that both $p_s = e^{-rT} \mathbb{E}_Q[\overline{W}_s(T)]$ and $p_f = e^{-rT} \mathbb{E}_Q[\overline{W}_f(T)]$ represent the current prices of the affiliate’s IA accounts under the corresponding charges. For example, $p_s$ is the amount of money that the affiliate will receive today in exchange of giving her IA (entirely) at the end of the accumulation phase, in this case it is assumed that the charge is on balance and she will continue to contribute at a rate $\theta$ until $T$.

Under $\mathbb{Q}$ both the risk-free and the risky assets grow at a rate $r$, then for fixed charges $\alpha$ and $\delta$ we have:

$$\mathbb{E}_Q[\overline{W}_s(T)] = e^{(r-\delta)T}W_0 + \theta \left( \frac{e^{(r-\delta)T} - 1}{r - \delta} \right), \quad (23)$$

$$\mathbb{E}_Q[\overline{W}_f(T)] = e^{-\alpha} \left[ e^{rT}W_0 + \theta \left( \frac{e^{rT} - 1}{r} \right) \right]. \quad (24)$$

Given $\alpha^*$, the equivalent charge on balance $\delta^*$ is the value of $\delta$ such that $N_{sf} = 1$, or equivalently, $\delta^*$ must satisfy the following equation:

$$\alpha^* = \ln \left( \frac{e^{rT}W_0 + \theta \left( \frac{e^{rT} - 1}{r} \right)}{e^{(r-\delta^*)T}W_0 + \theta \left( \frac{e^{(r-\delta^*)T} - 1}{r-\delta^*} \right)} \right). \quad (25)$$

Notice that equation (25) coincides with equation (20), and when $W_0 = 0$ it also reduces to equation (21).
3. Numerical application

In this section, we present a numerical application of the methodology to the Peruvian Private Pension System (PPS). We consider a retirement age of 65 years and ignore the mandatory disability insurance fee. We will work with three charges on flow (expressed as percentages of the affiliate’s salary): \( f_{\text{min}} = 1.47\% \), \( f_{\text{max}} = 1.84\% \) and \( f_{\text{avg}} = 1.615\% \) which corresponds to the minimum, maximum and average PPS’s charges on flow as in September 2013. Since dependent workers in Peru have a mandatory contribution of 10% of salary and \( f_i \) is applied to it, we have \( \alpha^*_i = -\ln(1 - 10f_i) \) and therefore \( \alpha^*_\text{min} = 0.1590 \), \( \alpha^*_\text{max} = 0.2033 \) and \( \alpha^*_\text{avg} = 0.1761 \). We will also assume \( r = 0 \) in this application in order to provide the most favorable scenario for the charge on balance. Other scenarios for \( r \) can be either considered or studied in further research.

Figure 1 shows \( \delta^* \) (annualized) for certain ages\(^5\) and three scenarios for the charge on flow: \( f_{\text{min}} \), \( f_{\text{avg}} \) and \( f_{\text{max}} \). As expected from (21), we observe that \( \delta^* \) is strictly increasing in age (decreasing in \( T \)), and strictly decreasing in \( f \) for a fixed \( T \). In the case of a 40-year-old affiliate, or equivalently \( T = (65 - 40) \times 12 = 300 \) months, \( \delta^* \) is 1.462% per year when \( f_{\text{avg}} \) is the corresponding charge on flow. This implies that a charge on balance smaller than 1.462% makes such scheme convenient for the 40-year-old affiliate. The corresponding values for \( f_{\text{min}} \) and \( f_{\text{max}} \) are 1.315% and 1.698% per year, respectively. An important age to consider is 37 years since half of PPS’s affiliates are younger than that age. The corresponding \( \delta^* \) for \( f_{\text{avg}} \) is 1.304%\(^6\).

\(^5\)If \( E \) is the affiliate’s age, then \( T = (65 - E) \times 12 \) months will be the length of the accumulation phase.
per year. Consequently, if the charge on flow is the system’s average (or 1.615% of salary) and the charge on balance is greater than 1.304% per year, then the youngest half of the affiliates in the system will find the charge on balance undesirable. We can observe that $\delta^* \geq 0.7286\%$ for all cases considered in the figure, and that level would make the charge on balance to be preferred for almost all PPS’s affiliates. Recall that we are considering $r = 0$; however, an increment in $r$ will make the values of $\delta^*$ smaller and therefore the charge on balance will become less attractive.

Figure 1: Equivalent annualized charge on balance, $\delta^*$, such that $A_{sf} = 1$ for different ages and charges on flow. We have considered $r = 0$, constant contribution rate, and the following charges on flow $f_{\text{min}} = 1.47\%$, $f_{\text{max}} = 1.84\%$ and $f_{\text{avg}} = 1.615\%$ (the charges are based on salary and assume a mandatory contribution of 10% of the affiliate’s wage).
4. Conclusions and further research

We have developed a methodology to determine equivalent charges on flow and balance for individual account pension systems. We have considered a risk-averse affiliate who wants to maximize her expected utility of adjusted terminal wealth in a complete financial market. Then, we need to solve the corresponding stochastic control problems to find and compare the maximum terminal certainty equivalent (CE) which can be achieved under both charge schemes. Under a fixed contribution rate, an exponential utility function and no initial amount in the IA, we are able to find the equivalent charges (those which make both schemes indifferent in terms of terminal CE) by solving a nonlinear equation involving only the future values of two continuous annuities. Moreover, the results will hold for any utility function and investment strategy since market completeness allows us to work in a risk-neutral environment.

The methodology was applied to the Peruvian Private Pension System (PPS) in order to determine the equivalent charge on balance for different accumulation horizons and three scenarios for the charge on flow. We found that a charge on balance lower than 0.73% per year would make such scheme preferable to the one based on flow for almost all PPS’s affiliates. However, such threshold assumes a risk-free rate of 0%, a constant contribution rate and a fixed charge on flow greater than 1.47% of the affiliate’s salary. Finally, it is possible to extend this methodology to consider a time-varying contribution rate, risk-free rate and charge on balance, as well as other modifications preserving market completeness.

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