Some Subtle Concepts in Fundamental Physics

Ying-Qiu Gu∗

School of Mathematical Science, Fudan University, Shanghai 200433, China

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In this paper, we discuss some subtle concepts, such as coordinate, measurement, simultaneity, Lorentz-FitzGerald contraction, singularity in fundamental physics. The explanations of these concepts in textbooks are usually incomplete and lead to puzzles. Some long-standing paradoxes such as the Ehrenfest one are caused by misinterpretation of these concepts. The analysis shows these concepts all have simple and naive meanings, and can be well understood along suitable logical procedure. The discussion may shed lights on some famous paradoxes, and provide some new insights into the structure and feature of a promising unified field theory.

Key Words: simultaneity, singularity, Lorentz-FitzGerald contraction, Ehrenfest paradox, energy, entropy

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I. COORDINATE VS. MEASUREMENT

The effects of the special relativity such as length contraction and time dilation of a moving object are still puzzling and controversial problems, which result in some famous paradoxes like the Ehrenfest one. To clarify the concepts of measurement and observation, there were once some careful discussions, e.g. by Terrell[1], Penrose[2], Weisskopf[3]. Under some approximation, these researches reached the result that the photograph of a moving object will shows it turns a little angle, which is call the Penrose-Terrell rotation, but the length contraction can not be recorded by the film[4]. The picture of a moving ball is still a ball. However, this problem is not finally resolved, and some discussions still carry on[5, 6, 7]. As pointed out by M. Pardy in [7], the investigation of relativity axioms as the

*Electronic address: yqgu@fudan.edu.cn
noncontradictory system was to our knowledge not published, although we believe that the relativistic system of axioms is not contradictory.

There maybe some basic concepts are confused and misused. In what follows, we try to clarify this problem through detailed calculation. To discuss the effects of relativity, we usually refer to ‘measurement’ and ‘observation’, where the measurement actually corresponds to the ‘coordinates’, and ‘observation’ to the process of taking a snapshot of the moving object. We examine the coordinates and the photographs of a ball, a screen and a curve moving with respect to the camera.

Assuming $S(T, X, Y, Z)$ is the stationary coordinate system, and $O(t, x, y, z)$ is a coordinate system moving along $X$ at speed $V$ with respect to $S$, then we have the following Lorentz transformation for coordinates

\[
T = t \cosh \xi + x \sinh \xi, \quad X = t \sinh \xi + x \cosh \xi, \quad Y = y, \quad Z = z, \tag{1.1}
\]
\[
t = T \cosh \xi - X \sinh \xi, \quad x = X \cosh \xi - T \sinh \xi, \quad y = Y, \quad z = Z, \tag{1.2}
\]

where $\tanh \xi = V$ stands for rapidity, and we set $c = 1$ as the unit of speed. (1.1) and (1.2) keep the 4-dimensional length of a vector invariant

\[
T^2 - X^2 - Y^2 - Z^2 = t^2 - x^2 - y^2 - z^2. \tag{1.3}
\]

The coordinates $(T, X, Y, Z)$ and $(t, x, y, z)$ are mathematical maps which are labelled to each point of space-time beforehand by different researchers according to some rules.

At first, we consider a color sphere of radius $R$ locating at the origin of the static coordinate system $S$, then an idealized stereographic camera can record each point of the sphere. The equation of the sphere is described by

\[
X^2 + Y^2 + Z^2 = R^2, \tag{1.4}
\]

with any constant $T = T_0$. Substituting (1.1) into (1.4) we get the equation of the sphere in the moving system $O$ as

\[
\frac{(x + Vt)^2}{1 - V^2} + y^2 + z^2 = R^2. \tag{1.5}
\]

So for any given moment $t$ in $O$, the sphere becomes an ellipsoid due to the Lorentz-FitzGerald contraction. (1.5) is only related to mathematical concept ‘coordinate’, but independent of physical process.
Figure 1: The photograph series of a unit sphere recorded by the camera

Now we take a photograph of the sphere, which involves the physical process of propagation of photons. Setting a camera at the origin of the moving coordinate system $O$, it take a snapshot of the sphere at the moment $t_1$, then the photons received by the film are emitted from $\vec{r} = (x, y, z)$ at time $t = t(\vec{r})$, where $t(\vec{r})$ depends on the point $\vec{r}$. Note the velocity of the light is constant $c = 1$, so we have relation

$$t = t_1 - \frac{r}{c} = t_1 - \sqrt{x^2 + \rho^2}, \quad \rho \equiv \sqrt{y^2 + z^2}. \quad (1.6)$$

In (1.6), the coordinate of the sphere is constrained by geometric equation (1.5). Substituting (1.6) into (1.5) we get

$$\frac{[x + V(t_1 - \sqrt{x^2 + \rho^2})]^2}{1 - V^2} + \rho^2 = R^2, \quad (1.7)$$

where $(x, \rho)$ is the coordinate of the sphere record by the film at moment $t_1$, and $t_1$ is a constant. At different moment $t_1$, we get different picture of the sphere. The profiles recorded by the picture are displayed in FIG.(1), which are quite different from the original sphere. The coming sphere looks like an olive, but the departing sphere looks like a disk, which contracts heavier than the Lorentz-FitzGerald contraction.
Setting a screen orthogonal to the $X$-axis, then the coordinates of the screen is given by

$$X \equiv X_0 = t \sinh \xi + x \cosh \xi.$$  

(1.8)

Substituting (1.6) into (1.8), we get the picture of the screen, which becomes a standard hyperboloid,

$$\frac{(x + a)^2}{\sinh^2 \xi} - \rho^2 = (X_0 - t_1 \sinh \xi)^2, \quad a = (X_0 - t_1 \sinh \xi) \cosh \xi.$$  

(1.9)

Indeed, the picture of a piece screen with $\rho > 0$ will rotate an angle. However, the angle is not a constant, which depends on the radial distance $\rho$, speed $V$ and $x_0$.

In the case of a screen orthogonal to the $z$-axis, namely, $z = z_0$, the picture is still a plane. However, the curves in the screen are distorted, so we consider an arbitrary spatial curve in the static coordinate system $S$. The curve is described by

$$X = X(\zeta), \quad Y = Y(\zeta), \quad Z = Z(\zeta),$$  

(1.10)

where $\zeta$ is parameter of the curve. Substituting (1.10) into (1.2), and then inserting the results into (1.6), we can solve

$$T(\zeta) = t_1 \cosh \xi - \sqrt{(X - t_1 \sinh \xi)^2 + Y^2 + Z^2},$$  

(1.11)

which is the time in $S$ between the photons emitting from point $\zeta$ and hitting the film. Again inserting (1.11) into (1.2), we get the coordinates of the point $\zeta$ recorded by the film

$$x = X(\zeta) \cosh \xi - T(\zeta) \sinh \xi, \quad y = Y(\zeta), \quad z = Z(\zeta).$$  

(1.12)

For a rigid bar lie at $Y = Y_0, Z = 0$, we have

$$x = (X - t_1 \sinh \xi) \cosh \xi + \sqrt{(X - t_1 \sinh \xi)^2 + Y_0^2} \sinh \xi.$$  

(1.13)

It is still quite complex. We further assume $Y_0 = 0, X \in [-L, L]$ and $L > t_1 \sinh \xi$, then we have

$$x \in \left[-(L + t_1 \sinh \xi) \frac{1 - V}{\sqrt{1 - V^2}}, (L - t_1 \sinh \xi) \frac{1 + V}{\sqrt{1 - V^2}}\right].$$  

(1.14)

When $t_1 \to 0$, the length of the bar ‘we see’ becomes $2L(1 - V^2)^{-\frac{1}{2}} > 2L$, which is not contracted but even expanded. Similar to the moving sphere, the coming part of the bar looks expansion, but the departing part looks contraction.
The above examples show how the concepts of special relativity work consistently in logic. The point of view is simple and naive: The coordinates \((T, X, Y, Z)\) and \((t, x, y, z)\) are just labels of mathematical map rather than real physical observation, but they are language to express observation. The Lorentz transformation (1.1) or (1.2) is nothing but a mapping rule between two label systems for each point in Minkowski space-time, which is purely geometric rather than kinetic and keeps the metric \(\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)\) invariant. The special relativity reveals the connection between time and space, but they are independent in the Newtonian world.

The moment \(t_1\) of taking snapshot is related to the physical process of light transmission. A static camera and a moving one at the same point to take snapshot will receive different photons from the object, so the pictures look quite different if \(V\) is large enough. The Ehrenfest paradox can not be explained by global Lorentz transformation, which involves the concept of simultaneity. We will give more analysis for this problem in the next section.

II. EQUATIONS VS. REALITY

All physical laws are represented by equations and relations, but a real physical process is just one solution of these equations[8]. In some sense, our symbolic system is much larger than the real world. Nevertheless, some related concepts are usually confused. Now we examine the simultaneity in relativity.

Again we consider two coordinate systems of Minkowski space-time with relative speed \(V\), that is, \(S\) and \(O\) with the coordinate transformation (1.1) and (1.2). If two events occur simultaneously in system \(S\), we have \(\delta T = 0\). It is common knowledge that the two events occur at different time in system \(O\) due to the Lorentz transformation (1.2), and we have \(\delta t = -\delta x \sinh \xi\). If we examine the whole world in system \(S\), the world and the space-time are evolving, and at time \(T = 0\), we have a map of the space simultaneously, which is a hyperplane corresponding to the line \(A'B'\) in FIG.(2). The hyperplane \(A'B'\) evolves into \(C'D'\) at time \(T = T_0\) and into \(E'F'\) at time \(T = T_1\). The corresponding hyperplanes in the moving coordinate system \(O\) are \(AB, CD\) and \(EF\) respectively. The evolution of the hyperplanes forms an evolving block universe rather than a fixed one[9, 10].

From FIG.(2) we find that, only in one special coordinate system, the real world can define the concept of global simultaneity, otherwise it will result in contradiction. We show this by
detailed discussion. Assume that the global simultaneity holds in coordinate system $S$, that is the world evolves from $T = 0$ to $T = T_0$ as shown in FIG. (2). In this case, according to the Lorentz transformation (1.2), the real space evolves from the hyperplane $AB$ to $CD$ in $O$, which is a tilting line without simultaneity. If we toughly define the simultaneity $t = t_0$ in system $O$, we find that we actually erase the history in the region $x < 0$, and fill up the future in the region $x > 0$. Of course such treatment is absurd, because the evolution of the world is even not uniquely determined, and we can not exactly forecast the future. In this sense, the hyperplanes $T = T_0$ and $t = t_0$ describe two different worlds. This evolving characteristic of Lorentz manifold is essentially different from that of Riemann manifold. This distinction is similar to the characteristics of the solutions to the hyperbolic partial differential equation and elliptic one.

The key of the problem is that, the real world is just one solution of the dynamical equations. The covariance of the equations just means they are suitable for all cases of the evolving world, and identical for all researchers. Of course, this is consistent with causality, because any space-like hyperplane in one coordinate system is space-like in other ones, and any suitable initial data given in one hyperplane suitable for the dynamical equations in one
coordinate system are also suitable for the dynamical equations in other ones. This is an astonishing but natural characteristic of the truths. A fundamental postulate of the unified field theory should have such characteristic.

In curved space-time, the real world certainly has one and only one unified cosmic time and a simultaneous hypersurface orthogonal to the time coordinate. The global simultaneity is just valid in this coordinate system. The Cosmic Microwave Background Radiation proves the existence of such coordinate system. Theoretically, for any smooth Lorentz manifold, such coordinate system exists locally, which is the so called Gaussian normal coordinate system\cite{12}. So the covariance of the physical laws does not contradict the existence of a special coordinate system for the real world.

The lack of universal simultaneity is related to the approximation of the classical mechanics\cite{11}, because the classical energy and momentum of a particle is defined as the spatial integrals of some Nöther’s charges, which require the hyperplane with simultaneity. The electric charge is special, which is exactly conserved in all coordinate system for each spinor due to gauge invariance, but the momentum of the spinors can be exactly calculated in all coordinate system only if the spinors take eigenstate, because in this case the Nöther’s charge becomes time independent, otherwise the exchange of energy-momentum exists. The Lorentz transformation, the time dilation and length contraction are just a local effects. The proper time of a moving particle is
\[
\tau = \int_0^t \sqrt{1 - v^2(t)} dt,
\]
even if the motion is accelerating.

The misuse of simultaneity will lead to contradictions or paradoxes. Now we consider the famous Ehrenfest paradox in rotational cylindrical coordinate system\cite{13, 14, 15}. The transformation between Born chart \(B(t, r, \phi, z)\) and the static Cartesian chart \(S(T, X, Y, Z)\) is given by
\[
T = t, \quad X = r \cos(\phi + \omega t), \quad Y = r \sin(\phi + \omega t), \quad Z = z.
\]
The line element in \(B(t, r, \phi, z)\) becomes
\[
ds^2 = (1 - r^2 \omega^2) dt^2 - 2r^2 \omega dt d\phi - dr^2 - r^2 d\phi^2 - dz^2.
\]
It includes the ‘cross-terms’ \(dt d\phi\), so the time-like vector \(\partial_t\) is not orthogonal to the spatial one \(\partial_\phi\). This is the key of the paradox. In the opinion of the observer in static coordinate system \(S\), the simultaneity means \(dT = 0\), which leads to \(dt = 0\), and then the spatial length element is given by
\[
dl = |ds|_{dT = 0} = \sqrt{dr^2 + r^2 d\phi^2 + dz^2}.
\]
which is identical to the length in the static cylindrical coordinate system. However, the
temporal length is different. For a clock attached to the rotational coordinate system, we
have $dr = d\phi = dz = 0$, and in this case we certainly have $r|\omega| < 1$. Then the proper time
element is given by

$$d\tau = |ds|_{dt=0} = \sqrt{1 - r^2\omega^2}dt,$$  \hspace{1cm} (2.4)

which shows the relativistic effect, the moving clock slows down. In the region $r|\omega| \geq 1$, the
Born coordinate is still valid, but we can not fix a clock in the rotational system $B(t, r, \phi, z)$
due to $ds$ being imaginary, which means $dt$ becomes space-like element. This is one difference
between mathematical coordinate and physical process.

In the opinion of an observer attached at $(r_0, \phi_0, z_0)$ in the rotational coordinate system,
the spatial vector bases $(\partial_r, \partial_\phi, \partial_z)$ are also orthogonal to each other, and then the line
element between two local points should take the form

$$\delta s^2 = \delta t^2 - \delta r^2 - g_{\phi\phi}\delta \phi^2 - \delta z^2,$$

where $\delta t$ is his local time element. By the universal expression of line element (2.2), we have

$$ds^2 = \delta t^2 - dr^2 - \frac{r^2}{1 - r^2\omega^2}d\phi^2 - dz^2,$$  \hspace{1cm} (2.5)

where $r = r_0$ for this specified observer and $\delta t$ is given by

$$\delta t = \sqrt{1 - r^2\omega^2}dt - \frac{r\omega}{\sqrt{1 - r^2\omega^2}}d\phi.$$  \hspace{1cm} (2.6)

The simultaneity of this observer means $\delta t|_{r=r_0} = 0$. For all observer attached in the
rotational coordinate system, the induced Riemannian line element in the quotient spatial
manifold $(r, \phi, z)$ is generally given by

$$dl^2 = dr^2 + \frac{r^2}{1 - r^2\omega^2}d\phi^2 + dz^2, \hspace{1cm} (r|\omega| < 1),$$  \hspace{1cm} (2.7)

which corresponds to the so called Langevin-Landau-Lifschitz metric. However, the expla-
nations of the metric in textbooks are usually quite complicated.

Since the 1-form (2.6) is not integrable, so we can not define a global time for all observers
attached in the rotational coordinate system. The physical reason is that the underlying
manifold of the Born chart is still the original Minkowski space-time. However, if we toughly
redefine a homogenous time $\tilde{t}$ orthogonal to the hypersurface (2.7), then we get a new curved
space-time equipped with metric

$$ds^2 = d\tilde{t}^2 - dr^2 - \frac{r^2}{1 - r^2\omega^2}d\phi^2 - dz^2, \hspace{1cm} (r|\omega| < 1).$$  \hspace{1cm} (2.8)
Needless to say, (2.8) and (2.5) describe different space-time and are not globally equivalent to each other, but they are equivalent locally. So generally speaking, the Lorentz transformation is only a local manipulation in the tangent space-time of a manifold, and the global Lorentz transformation only holds between Cartesian charts in the Minkowski space-time. The paradoxes usually arise from that we are unaccustomed to calculating 4-dimensional geometry.

The Rindler coordinate system is another example. The transformation between the Rindler chart $R(t, x, y, z)$ and the static Cartesian chart $S(T, X, Y, Z)$ of Minkowski space-time is given by[16]

$$T = x \sinh(t), \quad X = x \cosh(t), \quad Y = y, \quad Z = z,$$

which is a 1-1 smooth map in the Rindler wedge \{\(X \in (0, \infty), |T| < X, (Y, Z) \in \mathbb{R}^2\}\}. In the Rindler chart the line element of the space-time becomes

$$ds^2 = x^2 dt^2 - dx^2 - dy^2 - dz^2, \quad x \in (0, \infty), (t, y, z) \in \mathbb{R}^3.$$  \(2.10\)

The above transformation is well defined in mathematics. The problem comes when making simultaneous kinematic calculation. If we acquiesce the hyperplane $T = T_0$ in Cartesian chart describe the real world, most part of the hyperplane $t = t_0$ is still an empty set in physics as shown in FIG(2). The real space only corresponds to the hypersurface $x \sinh(t) \equiv T_0$(constant), but the hyperplane $t = t_0$ is another space-time. So the spatial length element $dL = |ds|_{dT=0}$ can not be confused with $dl = |ds|_{dt=0}$. Accordingly, the Rindler horizon is a meaningless concept in physics. Notice that the Raychaudhuri equation is based on the simultaneity of time-like congruence of the world lines, so it only holds locally and is not suitable for discussion of the large scale structure of the space-time.

The distinction between symmetry of dynamical equations and solutions is also puzzling in field theories. The parity symmetry of the Schrödinger equation or Dirac equation does not means their solutions are certainly even or odd functions of the coordinates, and the rotation symmetry of the equations also does not mean all solutions are spherically symmetric. Even the eigen solution for a single spinor always has a polar axis. We can never establish a coordinate system such that the solution of a water molecule $\text{H}_2\text{O}$ is of parity symmetry or rotation symmetry, but all such symmetries certainly hold for the total dynamical equation system. The permutational symmetry of the many-body Schrödinger equation together with
the Pauli exclusion principle also does not mean the solutions should take the form of Slater determinant. The symmetries of equations and solutions are different, although there are some relationship between them via Lie algebra[17].

III. DIRAC DELTA VS. SINGULARITY

In classical mechanics and electromagnetism we have an idealized model, the Dirac-δ, to describe a point mass or point charge, which is just a mathematical abstraction of the relatively concentrated distribution. Some corresponding singularities, such as infinite of mass density or infinite field intension, can be easily understood as a result caused by the simplification of the models. Even so, some mathematical calculation is still not evident. For example,

\[ \Delta \frac{x}{r^3} = 4\pi \frac{\partial}{\partial x} \delta(\vec{r}) \] (3.1)

is easily confused with \[ \Delta \frac{\vec{\rho}}{r^3} = 0, (r > 0) \]. Only by using the test function \( f(\vec{r}) \in C^\infty \) and calculating the integral \( \int_{R^3} f(\vec{r}) \Delta \frac{\vec{\rho}}{r^3} dV \), we will find out a source at the center and get the right relation (3.1).

In the nonlinear theories such as general relativity, the issue becomes even ambiguous, because we have not a universal definition of generalized function for nonlinear partial differential equations. We take Schwarzschild space-time

\[ ds^2 = b(r)dt^2 - a(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \] (3.2)

as an example to show the problem. In the case of standard solution \( b = a^{-1} = 1 - \frac{R_s}{r} \), the usual opinion believes that the singularity on the horizon \( r = R_s \equiv 2Gm \) is just a coordinate singularity, and it can be removed by suitable coordinate transformation. The Kruskal coordinate is the most popular one,

\[ \tanh \frac{t}{2} = \begin{cases} \frac{\rho}{\rho}, & r > R_s, \\ \frac{\rho}{\tau}, & r \leq R_s, \end{cases} \quad (r - R_s) \exp \left( \frac{r}{R_s} \right) = R_s(\rho^2 - \tau^2). \] (3.3)

\[ ds^2 = \frac{4R_s^3}{r} \exp \left( \frac{r}{R_s} \right) (d\tau^2 - d\rho^2) - r^2(d\theta^2 + \sin^2\theta d\phi^2). \] (3.4)

However, the transformation (3.3) may be invalid globally, because it even has not the first order derivatives at \( r = R_s \). In the viewpoint of differential geometry, if \( r = R_s \) has not
singularity, the space-time in the neighborhood is smooth manifold, and then the consistent local coordinate transformation should be $C^\infty$. In the viewpoint of partial differential equation, the Einstein field equation includes second order derivatives, so a valid coordinate transformation should at least has bounded second order derivatives. In the viewpoint of logic, the exterior Schwarzschild solution has nothing to do with the interior one, except that they are expressed by the same English characters.

As a matter of fact, $r = R_s$ may be not only a singular surface, but also a surface with concentrated mass-energy distribution. We make some analysis. Consider the metric generated by a spherical mass shell with thickness $\varepsilon$ and outside radius $R$. The equation governing $a$ is given by

$$\frac{d}{dr} H(r) = 8\pi G \rho r^2, \quad R_s = \int_{R-\varepsilon}^{R} 8\pi G \rho r^2 dr,$$

in which we define $H = r(1 - a^{-1})$ in order to make a linear equation, $\rho$ is the gravitating mass-energy density. Let $\varepsilon \to 0$, we get

$$\frac{d}{dr} H(r) = R_s \delta(r - R),$$

and the solution is given by

$$a = \begin{cases} 1, & r < R, \\ (1 - \frac{R_s}{r})^{-1}, & r \geq R \geq R_s. \end{cases}$$

The interior space-time is Minkowskian, and the exterior space-time is Schwarzschild one. Just like the case of (3.1), the concentrated source is sometimes not evident in the equations, especially for the nonlinear differential equations.

Of course, (3.6) is an extremely simplified model. A realistic model should include the dynamical behavior of the source. The calculation in [18, 19] shows the center of a star is not a balance point of the particles, so the singular surface $r = R_s \neq 0$ may just reflect the intrinsic harmonicity of the Einstein’s field equation.

How to define the singularity in the space-time is a difficult problem. The usual definitions are ‘incomplete geodesic’ or infinite scalar of curvatures such as $R \to \infty$, $R_{\mu\nu}R^{\mu\nu} \to \infty$, $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \to \infty$, etc[12, 20]. As a physical description, the incomplete geodesic is rather difficult for manipulation. Besides, for an evolving space-time, it is not a good choice to use the future property of the manifold, because there are too much information that we
may not command and we can not forecast the future as analyzed above. The singular curvatures also have a disadvantage, that is, it can be easily overlooked or confused with coordinate singularity like the case of Schwarzschild metric. The most convenient definition may be directly using $\rho \to \infty$. Although it is equivalent to $R \to \infty$ in mathematics, they are different in physics. This is because, on the one hand, $\rho \to \infty$ has manifest physical meaning and which can not be overlooked, but $R \to \infty$ is easily overlooked similarly to $\Delta \frac{1}{r} = 0, (r > 0)$. On the other hand, $\rho \to \infty$ occurs before the metric itself becomes singular[18,19], so the space-time can be still treated as a ‘place’[12] when the mass-energy density approaches to infinity.

IV. ENERGY VS. ENTROPY

In physics we have not only primary concepts, such as space-time and field, but also have some profound secondary concepts helping us to understand how the Nature works. The ‘Energy-momentum’ and the ‘Entropy’ are the most important secondary ones. However, the situations for the two concepts are quite different. The energy-momentum of any system can be clearly defined from the dynamical equations according to the Nöther’s theorem. Its general validity and corresponding conservation law are rooted in the underlying principle of covariance. Even in the case that the detailed dynamical equations of a complex system such as a thermodynamic system is unknown, we can also well understand and measure the total energy of the system and the exchange with the environment.

The entropy is also used as a general concept for all system. Indeed, for some idealized statistical models, we have clear definition of entropy[21]. In the case of a reversible thermodynamic process, we have a state function entropy calculated by $dS = \delta Q/T$, where $\delta Q$ is the heat increment received by the system, and we have the energy conservation law or Gibbs-Duhem relation

$$dE = \delta Q + \delta W = TdS - PdV + \sum_k \mu_k dN_k. \quad (4.1)$$

But the definition only holds for idealized reversible process. In the case of equilibrium idea gas with $N$ particles and volume $V$, we can derive the elegant Sackur-Tetrode equation

$$S = Nk \left[ \frac{5}{2} + \ln \left( \frac{V}{Nh^3} \left( \frac{4\pi m E}{3N} \right)^{1 \over 2} \right) \right]. \quad (4.2)$$
But it is also only valid for equilibrium state with some assumptions. Furthermore, we still have more general Boltzmann relation $S = k \ln \Omega$, where $\Omega$ is the number of microstate.

We can even prove all such definitions of entropy are consistent, but unfortunately, there is no method to generalize these statistical definitions to describe a system of internal structure and organization. Different from the energy-momentum, the entropy have not an underlying dynamical principle, although we usually take the second thermodynamic law as a generally valid principle. We find it is actually difficult to understand and to measure entropy for any complex system. The order or disorder of a complex system, the physical essence of entropy, is also an ambiguous statement. Besides the statistical system, in the Nature there are a lot of systems of internal structure, interaction and organization. For example, we put some eggs in a black box and then set the box in a heat bath with suitable temperature. After some time, we will get alive chickens. If the temperature is higher, we will get cooked eggs. How to measure and compare the entropy of the two processes? There are also the contrary processes. Press some vapor into the black box and set it in a cold environment, then we will get beautiful hexagonal snow, whose molecules are automatically organized. So the concept entropy for a complex system is quite ambiguous, and even can not be understood in principle. The misuse of such concept is usually to make puzzles rather than to clarify any physical rule.

We usually use the second thermodynamic law to explain the arrow of time. In this case it is somewhat similar to putting the cart before the horse as pointed out by G. Ellis[10]. The one direction of time is a basic fact and characteristic of the evolving world, so what we should do is to explain why our linearized and simplified time-reversible theories are so successful, and to find out the underlying complete formalisms.

V. UNIVERSALITY VS. EFFECTIVENESS

The universality and effectiveness are always contradictory in logic. In a fundamental theory such as general relativity or unified field theory, such contradictory becomes even more serious. The more universal the postulates, the more invalid messages they contain, but the more concrete the constraints, the greater the risk of failure the framework takes.

To explain the accelerating expansion of the present universe, one prescription is to modify
the Einstein-Hilbert Lagrangian by

\[ f(R) = -2\Lambda + R + \varepsilon_1 R^2 + \varepsilon_2 R^3 + \cdots. \] (5.1)

In \[22, 23\], shifting the emphasis from Einstein’s field equations to a broader picture of spacetime thermodynamics of horizons leads to a series of field equations constructed by different order combinations of metric and Riemann curvature, which includes Lanczos-Lovelock gravity. Of course, these theories are more general than the Einstein’s equation \( G_{\mu \nu} + \Lambda g_{\mu \nu} = \kappa T_{\mu \nu} \). Such theories can never be cancelled by experiments, because we can never perform an experiment with 0 error. When we get an empirical result with relative error \(10^{-8}\), one may argue \(|\varepsilon_n| < 10^{-9}\). On the contrary, the gauge invariance of the particle models such as \(SU(2)\), \(SU(3)\) is strong and narrow constraints in logic, and some possibly correct field equations without such symmetries are arbitrarily excluded. As a working hypothesis for concrete calculation, it may be right or not. However, as a general principle, the risk of being disproved by more accurate experiments and replaced by more general principles is large.

Then what is the ideal feature of the postulates in the unified field theory or the final theory? Recently, there are more and more philosophical considerations on this issue\[8\]-\[11\],\[24\]-\[33\]. Many scientists believe the Principle of Equivalence has a compelling force, and which leads to the metric theory of gravity. However this point of view does not shared by all physicists\[34, 35\]. The principle of equivalence once indeed was a good guidance to introduce curved space-time, but it may be only an approximate principle in strict sense, because the concept ‘mass of a particle’ is just a classical approximation, and the trajectory of a fermion with nonlinear potential is not an exact geodesic\[11\]. The real power of general relativity comes from that the curved space-time is much more general and natural than the rigid Minkowski space-time. The lack of universal simultaneity strongly suggests that the real space of the Universe should be a curved hypersurface. On the contrary, the principle of covariance of the physical laws indeed has an inviolable feature. So the ideal postulates of the final theory should have such philosophical depth, self-evident meanings and an excellent balance between universality and effectiveness. The ‘Singularity-free’ postulate also has such feature, so it is hopeful to become a universal principle of physical laws. According to this principle, we may delete most terms from (5.1) and reduce it to the Einstein-Hilbert formalism.
The balance of universality and effectiveness is also required in choosing and using mathematical tools. The proof of some profound and general theoretical results such as the positive mass theorem indeed needs advanced tools and skills. However, the solution to a specific physical problem is usually only one step away from the fundamental laws, but an advanced mathematical tool is often established on a long queue of abstract definitions and conditions, which will insert dozens of logical steps in the process, and cover the physical essence of the problem under brilliant cloth. Relatively speaking, a correct command of the basic principles and a strict training in logic are more important to solve the usual physical problems.

The above discussions and some previous works of the present author are all based on conventional mature principles and solid logical procedure, so they can not be treated as heterodoxies. In contrast, there are a lot of radicalism and speculation in modern physics. When science is still a logical system of knowledge, the above ideas and treatments may not be overlooked.

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