Accounting for shear deformations with bending a round plate

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Abstract. This paper presents a solution of the stability problem of a round plate taking into account the influence of shear deformations. The critical load of a round plate is determined taking into account shear deformations and without taking into account the shear deformations. The critical load taking into account the transverse shear with a local stability loss of a round plate is less than the critical load without taking into account the transverse shear. The inclusion of shear deformations in the study of the supercritical behavior of a round plate for isotropic materials does not have practical value, but it makes sense with extremely strong anisotropy of the elastic properties of the material.

1. Introduction

In the classical theory of plates, the assumption is used about a material element that is normal to the median surface before deformation [1]. This material element remains rectilinear and perpendicular to it even after deformation.

We used one of the simplest versions of an improved linear theory. The improved linear theory suggests that the material element of the plate is perpendicular to the median plane prior to deformation. This material element remains straight after bending the plate, but the rotation angles of this element are not directly related to the rotation angles of the normal to the median surface [2].

2. The mathematical model

We consider a plate with radius $R$ and thickness $h$. The center of the plate coincides with the origin $oxy$. A thin axisymmetric plate is clamped along the contour. The plate is subject to a uniformly distributed load with intensity $q$. The deformation geometry of a round plate is described by deflection $w = w(r)$ and angle of rotation $\delta_r = \delta_r(r)$.

The deformation components of a layer located at a distance $z$ from the median surface are

$$\varepsilon_r = -z\delta_r'; \varepsilon_0 - z\delta_r/r; \varepsilon_z = 0;$$

$$\gamma_{xy} = 0; \gamma_{xz} = \delta_r - w'; \gamma_{0z} = 0,$$

where $w'$ is the angle of rotation of the normal to the middle surface of the plate. Then the bending strain energy has the form
where \( D \) is flexural stiffness of delamination, \( B \) is transverse shear stiffness:

\[
D = \frac{E_i h^3}{12(1 - \mu_{12}\mu_{21})}, \quad B = G_{12} h.
\]

We assume that the median surface of a round plate is inextensible. The change in the total potential energy of a round plate is determined by the expression

\[
\Delta E = V - 2\pi \int_0^R q \left[ \frac{\partial^2 \psi}{\partial r^2} \right] r dr.
\]

When constructing an approximate solution, we take

\[
\partial_t = \psi(r); \quad w' = c\psi(r),
\]

where \( \psi(r) \) is a function that satisfies the boundary conditions of the task; \( C \) is parameter characterizing the magnitude of the transverse shear.

From the condition \( \Delta E = 0 \) follows

\[
q = \frac{\pi \int_0^R \left[ (\psi' + \psi''/r)^2 - 2(1 - \mu)\psi''/r \right] r dr + \left( c - 1 \right)^2 \int_0^R B \psi'^2 r dr}{c^2 \int_0^R \psi'^2 r dr}.
\]  

(1)

We take \( \psi' = 0 \), where \( \psi^0_t \) is the function corresponding to the solution of the problem without taking into account the transverse shift, then expression (1) has the form

\[
q = q^0_{ct} + B(c - 1)^2,
\]

(2)

where \( q^0_{ct} \) is the critical value of the load found without transverse shear, i.e.

\[
q^0_{ct} = \frac{D_{01}}{2} \left[ \left( \psi^0_t \right)^2 + \psi^0_t/2 \right] - 2(1 - \mu_{12}) \int_0^R (\psi^0_t/2) r dr - \int_0^R (\psi^0_t) / r dr, \quad q^0_{ct} = 16 \frac{D}{R^2}.
\]

Minimizing expression (2) with respect to the parameter \( C \), we find

\[
C_{ct} = 1 + \frac{q^0_{ct}}{B}, \quad q_{ct} = \frac{q^0_{ct}}{1 + q^0_{ct}/B}.
\]

(3)

The critical load taking into account the transverse shear with a local loss of stability is slightly less than the Euler critical load [3]. Taking into account the effects of transverse shear leads to a decrease in the critical load. In particular, for an isotropic material at \( G = \frac{E}{2(1 + \mu)} \) follows
It means that the effects of transverse shear do not significantly affect the accuracy of calculations for a thin layer. In comparison with the study of isotropic plates, taking into account the influence of shear deformations becomes crucial in studies of anisotropic plates.

With taking into account the transverse shear, the deformations of a round plate with a loss of stability are described by the functions

\[ \vartheta_1(r) = \vartheta_1^0(r), \quad w' = \left(1 + \frac{q_{cr}^0}{B}\right) \cdot \vartheta_1^0(r). \]

We consider the simplest case of anisotropy of the elastic properties of the material of the exfoliated part of the plates [4]. All planes parallel to the median plane of the plate are isotropy planes. We consider \(|\sigma_z| << |\sigma_1|, |\sigma_0|\). We study the nonlinear behavior of circular delamination in a layered anisotropic plate.

The change in the total potential energy has the form

\[ \Delta E = U_2 + V + \Pi_2 + T + U_4. \]

The inclusion of transverse shifts does not affect the formulas for calculating the quadratic deformations of the middle surface \((U_4)\). \(T\) is the potential energy of shear strain.

From the stationary condition of the total potential energy follows

\[ \eta \frac{q_{cr}^0}{\left(1 + \frac{q_{cr}^0}{B}\right)^2} + \eta^3 \frac{3}{14} \frac{Eh}{R^2 \left(1 + \frac{q_{cr}^0}{B}\right)^2} + \eta \left(1 + \frac{q_{cr}^0}{B}\right)^2 \cdot q = 0. \]

We introduce dimensionless quantities

\[ \bar{q} = q/q_{cr}^0; \quad \bar{E} = \frac{E_1}{G_{12}}; \quad \bar{h} = \frac{h}{R}; \quad \bar{\eta} = \frac{\eta}{h}. \]

It can be seen from (3) that taking into account transverse shifts in the exfoliated part gives an order correction \(q_{cr}^0/B\) compared to unit. Determining the value \(q_{cr}^0\), we evaluate this correction (3)

\[ \frac{q_{cr}^0}{B} = 14.68 \mu_1 h^3 \frac{1}{12(1-\mu_{12}) E_1 h} \approx \frac{E_1}{G_{12} R^2}; \quad \bar{c} = 1 + \frac{q_{cr}^0}{B} = 1 + \frac{E h^2}{G_{12}}. \]

Table 1 shows the values of the order correction \(q_{cr}^0/B\) for various materials with different plate thicknesses and the same radius. Given \(\eta \neq 0\), we come to the dependence

\[ \bar{\eta} = \sqrt{\frac{56(1 + \bar{E} h^2)}{9(1 - \mu_{12} \mu_{21})} \left(4 - \frac{4 - 3 \bar{E} h^2 (1 - \mu_{12} \mu_{21})}{4(1 + \bar{E} h^2)}\right)}. \]

Table 2 shows the constant values for the materials.

Figure 1 shows the results of a study of the supercritical behavior of a defect of the type delamination from anisotropic materials.
Table 1. The values of the order correction \( q_{cr}^0/B \) for various materials.

| Material                  | The order correction \( q_{cr}^0/B \) |
|---------------------------|---------------------------------------|
|                           | \( \bar{h} = 0.01 \) | \( \bar{h} = 0.02 \) | \( \bar{h} = 0.03 \) |
| Graphite plastic          | 0.0067 | 0.027 | 0.0604 |
| Kevlar-reinforced plastic | 0.0028 | 0.0112 | 0.0252 |
| Graphite-epoxide          | 0.00205 | 0.0082 | 0.0184 |
| Fiberglass                | 0.0006 | 0.0025 | 0.0055 |

Table 2. The constant values for various materials.

| Material                  | \( E_1 \) (kN mm\(^{-2}\)) | \( E_2 \) (kN mm\(^{-2}\)) | \( G_{12} \) (kN mm\(^{-2}\)) | \( \mu_{12} \) |
|---------------------------|-----------------------------|-----------------------------|-------------------------------|---------------|
| Graphite plastic          | 215                         | 6.5                         | 3.2                           | 0.26          |
| Kevlar-reinforced plastic | 70                          | 4.5                         | 2.5                           | 0.35          |
| Fiberglass                | 53                          | 14.0                        | 8.6                           | 0.26          |

Figure 1. Numerical examples are obtained for: graphitoplastics (curve 3); plastic reinforced by Kevlar (curve 2); isotropic material (curve 1).

3. Analysis of the results of the calculation of a round plate
The inclusion of shear deformations in the study of the supercritical behavior of a round plate for isotropic materials is not practical, but it makes sense with extremely strong anisotropy of the elastic properties of the material. The effect of transverse shear effects leads to a decrease in the critical load. The experimental results are presented in [5-8].

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