Weak Gravity Conjecture from Unitarity and Causality

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mainly based on 1810.03637 w/Y. Hamada, G. Shiu

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Weak Gravity Conjecture

in quantum gravity,

∃ a charged state satisfying

\[ g^2 q^2 \geq \frac{m^2}{2M_{Pl}^2} \]

for each U(1) gauge force

[ArkaniHamed-Motl-Nicolis-Vafa 06']
Weak Gravity Conjecture

in quantum gravity,

\[ \exists \text{ a charged state satisfying} \]

\[ g^2 q^2 \geq \frac{m^2}{2M^2_{\text{Pl}}} \]

for each U(1) gauge force

[AkaniHamed-Motl-Nicolis-Vafa 06’]
Weak Gravity Conjecture

in quantum gravity,

∃ a charged state satisfying

\[ g^2 q^2 \geq \frac{m^2}{2M_{Pl}^2} \quad \text{as} \quad M_{Pl} \to \infty \]

for each U(1) gauge force

[ArkaniHamed-Motl-Nicolis-Vafa 06’]
Weak Gravity Conjecture

in quantum gravity,

\[ \exists \text{ a charged state satisfying} \]

\[ g^2 q^2 \geq \frac{m^2}{2M_{Pl}^2} \]

for each U(1) gauge force

[ArkaniHamed-Motl-Nicolis-Vafa 06’]
in QED, the electron trivially satisfies it:

\[ 10^{-2} \sim g^2 q^2 \geq \frac{m^2}{2M_{Pl}^2} \sim 10^{-44} \]

however, its generalization (ex. axion) constrains models of inflation, dark matter, …
in this talk, I will argue [Hamada-TN-Shiu ’18]

existence of a charged state (BH) satisfying

\[ g^2 q^2 > \frac{m^2}{2M_{\text{Pl}}^2} \]

follows from unitarity & causality

in a wide class of theories

(ex. stingy setups w/dilaton or moduli stabilized below \( M_s \))
plan

1. Introduction: Landscape & Swampland
2. Weak Gravity Conjecture
3. WGC from unitarity and causality
4. Summary and prospects
1. Landscape & Swampland
1. Landscape & Swampland
various QFT models w/ quantum gravity
ex. for particle physics and cosmology

Landscape: string theory has infinitely many vacua!
shape of extra dimensions, brane configurations, …
string theory

= generator of QFT models w/quantum gravity
Q. every QFT model is realized in string theory?
A. NO!!!
no global symmetry in string theory

# continuous symmetries in string theory are gauged!

- world sheet theory analysis [Banks-Dixon ’88, …]

  conserved current $\rightarrow$ gauge boson vertex operator

- if we assume AdS/CFT …

  conserved current $J^\mu$ in CFT $\Leftrightarrow$ gauge field $A_M$ in AdS

# holographic proof including discrete symmetries

[Harlow-Ooguri 18’]
more generally,

black hole (BH) thought experiments motivate

no global symmetry in quantum gravity!
global vs gauge in the BH context

global symmetry
ex. \( B - L \)
gauge symmetry
ex. \( U(1)_{\text{EM}} \) \( Q \)

# no-hair theorem:
event horizon \( \rightarrow \) global symmetry charge is not observable
cf. EM fluxes outside the horizon tell us the EM charge
no global symmetry in quantum gravity

BH

put in particles with $B - L > 0$

BH

Hawking radiation $B - L = 0$

evaporate

global charge is not conserved due to BH evaporation

→ global symmetry is approximate symmetry (if exists)

cf. for gauge symmetry, Hawking radiation is not neutral
in this way, nontrivial constraints on symmetry & matter contents in string theory (quantum gravity in more general) → Landscape & Swampland [Vafa ’06]
swampland: apparently consistent, but not UV completable when coupled to gravity

landscape:
QFT models consistent w/quantum gravity
- where is the boundary?
- phenomenological implications?
web of swampland conjectures

no global symmetry

\[ g \neq 0 \]

Weak Gravity Conjecture

\[ \geq \rightarrow > \]

non-SUSY AdS

distance conjecture

\[ g \neq 0 \]

tower WGC

sublattice WGC

entropy

dS conjecture

non-SUSY

KK
web of swampland conjectures

- no global symmetry
- $g \neq 0 \Rightarrow$ Weak Gravity Conjecture
- $g^2 q^2 \geq \frac{m^2}{2M_{Pl}^2}$, non-SUSY AdS
- $g \neq 0 \Rightarrow$ distance conjecture
- $g \neq 0 \Rightarrow$ tower WGC, sublattice WGC
- $g \neq 0 \Rightarrow$ entropy
- non-SUSY AdS $\Rightarrow$ dS conjecture
web of swampland conjectures

- no global symmetry
- Weak Gravity Conjecture \( g \neq 0 \)
- non-SUSY AdS \( \geq \rightarrow \rightarrow \)

- distance conjecture
- tower WGC sublattice WGC \( g \neq 0 \)
- dS conjecture non-SUSY

entropy KK
plan

1. Introduction: Landscape & Swampland ✔
2. Weak Gravity Conjecture
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2. Weak Gravity Conjecture

[ArkaniHamed-Motl-Nicolis-Vafa 06’]
global symmetry = gauge symmetry @ $g = 0$

→ any lower bound on gauge coupling $g$ ??
a simplest possibility will be

\[ g^2 q^2 \geq (\text{constant}) \times \frac{m^2}{M_{\text{Pl}}^2} \]
BWs in Einstein-Maxwell theory

1) sub-extremal BH: \( g |Q| < M/\sqrt{2M_{Pl}} \)
   emit Hawking radiation \( (T \neq 0) \) to decay; unstable

2) extremal BH: \( g |Q| = M/\sqrt{2M_{Pl}} \)
   no Hawking radiation \( (T = 0) \)
   \( \rightarrow \) stable unless \( \exists \) some other decay mechanism

\* \( g |Q| > M/\sqrt{2M_{Pl}} \) : naked singularity (cf. cosmic censorship)
a proposal by [ArkaniHamed-Motl-Nicolis-Vafa 06’]:
postulate that extremal BHs have to decay
unless not protected by some symmetry (ex. SUSY)

- “∞ stable states w/o sym protection” seems strange
- revisit from unitarity & causality perspective later
Weak Gravity Conjecture

[ArkaniHamed-Motl-Nicolis-Vafa 06’]

extremal BH

\[ Q = M \]

BH’

\[ Q - q \leq M - m \]

- extremal BH has a decay channel
- the BH’ after decay has no naked singularity

\[ \exists \text{ a charged state satisfying } q \geq m \]

in the unit \( Q_{\text{ext}} = M_{\text{ext}} \) for simplicity
Weak Gravity Conjecture

[ArkaniHamed-Motl-Nicolis-Vafa 06’]

extremal BH

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\[ q \geq m \]

in the unit \( Q_{\text{ext}} = M_{\text{ext}} \) for simplicity
- no rigorous proof, so it is still a conjecture
- but consistent with all known examples in string theory
- if true, various phenomenological implications
  ex. mili-charged dark matter, axion inflation, axion DM, …
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“charge > mass” ⇔ \frac{1}{f} > \frac{S_{\text{inst}}}{M_{\text{Pl}}} ⇔ \frac{f}{M_{\text{Pl}}} \cdot S_{\text{inst}} < 1
```
implications to axion inflation

\[ V(\phi) \propto e^{-S_{\text{inst}}} \left( 1 - \cos \frac{\phi}{f} \right) + \sum_{n \geq 2} e^{-nS_{\text{inst}}} \left( 1 - \cos \frac{n\phi}{f} \right) \]

- negligible higher harmonics ( \( n \geq 2 \) ) \( \rightarrow S_{\text{inst}} > 1 \)
- long enough periodicity \( \rightarrow f > M_{\text{Pl}} \)

※ inconsistent with WGC \( \frac{f}{M_{\text{Pl}}} \cdot S_{\text{inst}} < 1 \)
loophole and prediction
axion monodromy (ex. pure natural inflation) multi-valued potential

\[ V(\phi) = V_{\text{s.r.}}(\phi) + e^{-S_{\text{inst}}} \left( 1 - \cos \frac{\phi}{f} \right) \]

spectator instanton

add an instanton satisfying the WGC bound

\[ V(\phi) = e^{-S_{\text{inst}}} \left( 1 - \cos \frac{\phi}{f} \right) + e^{-S'_{\text{inst}}} \left( 1 - \cos \frac{\phi}{f'} \right) \]

- large field inflation is realized by

\[ V_{\text{s.r.}} \quad \text{or} \quad e^{-S'_{\text{inst}}} \left( 1 - \cos \frac{\phi}{f'} \right) (f' > M_{\text{Pl}}) \]

- WGC is satisfied by the instanton with

\[ S_{\text{inst}} \cdot \frac{f}{M_{\text{Pl}}} \lesssim 1 \]

→ wiggy potential → oscillating feature in power spectrum
plan

1. Introduction: Landscape & Swampland ✓
2. Weak Gravity Conjecture ✓
3. WGC from unitarity and causality
4. Summary and prospects
3. WGC from unitarity and causality

[Hamada-TN-Shiu ’18]
3. WGC from **unitarity and causality**

[Hamada-TN-Shiu ’18]

constraints on signs and/or amplitudes of effective interactions (ex. positivity bound)
There exist heavy BHs satisfying the WVC bound $g |Q| > \frac{M}{\sqrt{2M_{Pl}}}$

if higher derivative interactions have a certain sign

[Kats-Motl-Padi ’06]
\[ S = \int d^4 x \sqrt{-g} \left[ \frac{1}{4} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 
+ \alpha_2 (\tilde{F}_{\mu\nu})^2 + \alpha_3 F_{\mu\nu} F_{\rho\sigma} W^{\mu\nu\rho\sigma} + \cdots \right] \]

※ work in the unit \( 2M_{\text{Pl}}^2 = 1, \ g = 1 \) in the following

※ higher order terms are negligible for heavy BHs

\[ F^2 \sim R \sim 1/M^2 \]
no Hawking radiation

w/Hawking radiation

naked singularity

higher derivative corrections to Einstein-Maxwell theory

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{4} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 + \alpha_2 (\tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \alpha_3 F_{\mu\nu} F_{\rho\sigma} W^{\mu\nu\rho\sigma} + \cdots \right]
\]

\rightarrow \text{modify BH solutions and the horizon structure}

no naked singularity if

\[
\left| \frac{Q}{M} \right| \leq 1 + \frac{2}{5} \frac{(4\pi)^2}{Q^2} (2\alpha_1 - \alpha_3) + O(1/Q^4)
\]
# higher derivative corrections to Einstein-Maxwell theory

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{4} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 + \alpha_2 (\tilde{F}_{\mu\nu})^2 + \alpha_3 F_{\mu\nu} F_{\rho\sigma} W^{\mu\nu\rho\sigma} + \cdots \right] \]

→ modify BH solutions and the horizon structure

no naked singularity if

\[ \frac{|Q|}{M} \leq 1 + \frac{2}{5} \frac{(4\pi)^2}{Q^2} (2\alpha_1 - \alpha_3) + \mathcal{O}(1/Q^4) \]

\[ BH \text{ with } |Q| > M \quad \text{if } 2\alpha_1 - \alpha_3 > 0 \]
if we can derive $2\alpha_1 - \alpha_3 > 0$,
- an existence proof of WGC (more precisely, mild version)
- decay of heavy extremal BH is kinematically allowed
in the following, I demonstrate
that the inequality follows from unitarity & causality

\[ 2\alpha_1 - \alpha_3 \simeq 2\alpha_1 > 0 \]

in a class of theories including
- theories with light neutral particles (ex. dilaton, moduli)
- open string theory type UV completion
setup and assumptions
- BH dynamics is controlled by photon and graviton at IR
- $\Lambda_{\text{QFT}}$: scale beyond which QFT description breaks down
- assume a weakly coupled UV completion of gravity

ex. $\Lambda_{\text{QFT}} \sim M_s$ in string theory
Sources of higher dimensional operators

(a) neutral light bosons (dilaton, axion)

(b) loop effects (charged particles)

(c) UV effects (stringy states)
Sources of higher dimensional operators

(a) neutral light bosons (dilaton, axion)

\[ \alpha_{1,2} : F^4, \quad \alpha_3 : F^2W \]

\[ |\alpha_i| \gtrsim \frac{1}{m^2} \]

(b) loop effects (charged particles)

(c) UV effects (stringy states)
Sources of higher dimensional operators

(a) neutral light bosons (dilaton, axion)

\[ F_{\mu\nu} \phi \]
\[ \frac{1}{m^2 + p^2} \]
\[ |p^2| \ll m^2 \]
\[ g \]
\[ F_{\rho\sigma} \]
\[ |g^2/m^2| > 0 \]

\[ \alpha_{1,2} : F^4, \quad \alpha_3 : F^2W \]

\[ |\alpha_i| \gtrsim \frac{1}{m^2} \]

(b) loop effects (charged particles)

\[ \mathcal{O}(z^4) \]
\[ \mathcal{O}(z^2) \]
\[ \mathcal{O}(z^0) \]

\[ z = \left| \frac{q}{m} \right| : \text{the charge-to-mass ratio} \]

\[ |\alpha_i| \gg 1 \quad \text{for} \quad z \gg 1 \]
\[ \alpha_i = \mathcal{O}(1) \quad \text{for} \quad z = \mathcal{O}(1) \]

(c) UV effects (stringy states)

\[ \alpha_{1,2} \sim \frac{1}{\Lambda_{QFT}^4}, \quad \alpha_3 \sim \frac{1}{\Lambda_{QFT}^2} \]
loop effects (b) dominates only when \( z \gg 1 \)
- this particle satisfies the WGC bound \( z > 1 \) ( ✔ WGC satisfied)
→ let’s focus on the case either (a) or (c) is dominant
- even in this case \( 2\alpha_1 - \alpha_3 \approx 2\alpha_1 > 0 \) follows from unitarity
\[ 2\alpha_1 - \alpha_3 \approx 2\alpha_1 > 0 \]

\[
\begin{align*}
\alpha_1 : & \ (FF)^2, \\
\alpha_2 : & \ (F\tilde{F})^2, \\
\alpha_3 : & \ F^2W
\end{align*}
\]
Causality constraints

# the FFW coupling $\alpha_3$ is significantly constrained by causality!

[Camacho-Edelstein-Maldacena-Zhiboedov '14]

- generates a new 3pt helicity amplitudes
- leads to causality violation (time-advancement)
  unless $\exists$ an infinite tower of higher spin particles
  with the mass $m \sim \alpha_3^{-1/2}$ just like string theory!

cf. this amplitude is incompatible with SUSY, so $\alpha_3 = 0$ in SUSY theories

$\alpha_1 : (FF)^2, \quad \alpha_2 : (F\tilde{F})^2, \quad \alpha_3 : F^2W$
# Causality constraints

- the FFW coupling $\alpha_3$ is significantly constrained by causality!
  
  [Camanho-Edelstein-Maldacena-Zhiboedov ’14]

- generates a new 3pt helicity amplitudes

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  unless $\exists$ an infinite tower of higher spin particles

with the mass $m \sim \alpha_3^{-1/2}$ just like string theory!

- $\alpha_3$ is tightly bounded by the graviton Regge state scale
  (closed string states)

\[
|\alpha_3| \lesssim \frac{1}{\Lambda_{QFT}^2} \rightarrow \text{classified into the UV effect (c)}
\]

cf. the same bound is available from conformal bootstrap

[Meltzer-Poland ’17, Afkhami-Jeddi et al ’18]
the Wilson coefficients enjoy the hierarchy \(|\alpha_1|, |\alpha_2| \gg |\alpha_3|\)

- if the effect (a) of light neutral particles is dominant
- if photon and graviton carry different sets of Regge states and the photon Regge state effect is dominant

ex. open string theory

open string Regge stats: \(\alpha_{1,2} \sim \frac{1}{g_s M_s^2}\)  
\(\bigcirc\) disk

closed string Regge stats: \(\alpha_{1,2,3} \sim \frac{1}{M_s^2}\)  
\(\bigcirc\) annulus
\[ 2\alpha_1 - \alpha_3 \simeq 2\alpha_1 > 0 \]

\[ \alpha_1 : (FF)^2, \quad \alpha_2 : (F\tilde{F})^2, \quad \alpha_3 : F^2W \]
Unitarity constraints

# unitarity implies positivity of $\alpha_1 (F_{\mu\nu}F^{\mu\nu})^2$ and $\alpha_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2$!

\[ \alpha_i = \frac{1}{2m^2 f^2} > 0 \]

※ sign of $\alpha$ depends on sign of propagator (norm positivity)

ex. exchange of dilaton and axion

\[ \mathcal{L}_\phi = -\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2 \phi^2 + \frac{\phi}{f_\phi} F_{\mu\nu}F^{\mu\nu} \]
\[ |p^2| \ll m^2 \]
\[ \frac{1}{2m^2 f^2_\phi} (F_{\mu\nu}F^{\mu\nu})^2 \]

\[ \mathcal{L}_a = -\frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m^2 a^2 + \frac{a}{f_a} F_{\mu\nu}\tilde{F}^{\mu\nu} \]
\[ |p^2| \ll m^2 \]
\[ \frac{1}{2m^2 f^2_a} (F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \]
Unitarity constraints

# unitarity implies positivity of $\alpha_1(F_{\mu\nu}F^{\mu\nu})^2$ and $\alpha_2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$!

More generally, we can explicitly show [Hamada-TN-Shiu ’18]

\[ \alpha_1 > 0 \ (\alpha_2 > 0) \text{ follows from unitarity} \]

when photon is coupled to parity even (odd) neutral scalars

or spin $s \geq 2$ neutral particles

by using factorization and UV mildness of scattering amplitudes, and assuming that graviton Regge states are subdominant effects

cf. spinning polynomials basis of [ArkaniHamed-Huang-Huang ’17]
to summarize,

unitarity and causality implies $2\alpha_1 - \alpha_3 \approx 2\alpha_1 > 0$

※ WGC bound is satisfied by heavy extremal BHs!

※ decay of heavy extremal BH is kinematically allowed

in the following classes of theories:

- theories with light neutral particles
  (parity even scalar or spin $s \geq 2$ particles; dilaton, moduli, KK-graviton)

- open string theory type UV completion
  (photon Regge state effect dominates over graviton Regge state one)
4. Summary and prospects
Summary

as a consequence of unitarity & causality,
heavy charged BH satisfies the WGC bound

\[ g^2 q^2 > \frac{m^2}{2M_{Pl}^2} \]

- in theories w/light neutral particles (ex. dilaton, moduli)
- in open string theory type UV completion

✔ extension to higher dimension, multiple U(1)’s
✔ entropy correction is positive in these theories

(cf. [Cheung-Liu-Remmen 18’])
Prospects

# enlarge applicability of our argument
- heterotic string type setup was not covered
  (photon and graviton are from closed string)
- detailed study of Regge amplitudes will be necessary

# connection to non-SUSY AdS conjecture [Ooguri-Vafa ’16]
- AdS = near horizon limit of extremal BHs
- decay of extremal BHs is kinematically allowed
  in the aforementioned theories

# extension to WGC for axion
- corrections to extremality condition of black instantons
Thank you!