Direct CP asymmetries of three-body $B$ decays in perturbative QCD

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We propose a theoretical framework for analyzing three-body hadronic $B$ meson decays based on the perturbative QCD approach. The crucial nonperturbative input is a two-hadron distribution amplitude for final states, whose involved time-like form factor and rescattering phase are fit to relevant experimental data. Together with the short-distance strong phase from the $b$-quark decay kernel, we are able to make predictions for direct CP asymmetries in, for example, the $B^\pm \to \pi^+\pi^-\pi^\pm$ and $\pi^+\pi^-K^\pm$ modes, which are consistent with the LHCb data in various localized regions of phase space. Applications of our formalism to other three-body hadronic and radiative $B$ meson decays are mentioned.

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Three-body hadronic $B$ meson decays have been studied for a long time $[1,4]$. They attracted a lot of attentions recently, after the LHCb Collaboration measured sizable direct CP asymmetries in localized regions of phase space $[5,6]$, such as

$$A_{CP}^{\mathrm{reg}}(\pi^+\pi^-\pi^+) = 0.584 \pm 0.082 \pm 0.027 \pm 0.007,$$  \hspace{1cm} (1)

for $m_{\pi^+\pi^-}\text{high} > 15$ GeV$^2$ and $m_{\pi^+\pi^-}\text{low} < 0.4$ GeV$^2$, and

$$A_{CP}^{\mathrm{reg}}(\pi^+\pi^-K^+) = 0.678 \pm 0.078 \pm 0.032 \pm 0.007,$$  \hspace{1cm} (2)

for $m_{K^+\pi^-}\text{high} < 15$ GeV$^2$ and $0.08 < m_{K^+\pi^-}\text{low} < 0.66$ GeV$^2$. Theoretical attempts to understand these data were made: the above CP asymmetries were attributed to the interference between a light scalar and intermediate resonances in $[7]$; the relations among the above CP asymmetries in the $U$-spin symmetry limit were examined in $[9]$; SU(3) and $U$-spin symmetry breaking effects were included into the amplitude parametrization in $[10]$; in $[11]$ the non-resonant contributions were parameterized in the framework of heavy meson chiral perturbation theory $[12]$, and the resonant contributions were estimated by means of the usual Breit-Wigner formalism.

Viewing the experimental progress, it is urgent to construct a corresponding framework based on factorization theorem, in which perturbative evaluation can be performed systematically with controllable nonperturbative inputs. Motivated by its theoretical self-consistency and phenomenological success, we shall generalize the perturbative QCD (PQCD) approach $[13,14]$ to three-body hadronic $B$ meson decays. A direct evaluation of hard $b$-quark decay kernels, which contain two virtual gluons at leading order (LO), is not practical due to an enormous number of diagrams. Besides, the contribution from two hard gluons is power-suppressed and not important. In this region all three final-state mesons carry momenta of $O(m_B)$, and all three pairs of them have invariant masses of $O(m_B^2)$, $m_B$ being the $B$ meson mass. The dominant contribution comes from the region, where at least one pair of light mesons has the invariant mass below $O(\Lambda m_B)$ $[11]$, $\Lambda = m_B - m_b$ being the $B$ meson and $b$ quark mass difference. The configuration involves two energetic mesons almost collimating to each other, in which the dynamics associated with the pair of mesons can be factorized into a two-meson distribution amplitude $\phi_{h_1 h_2}$ $[15]$. It is evident that $\phi_{h_1 h_2}$

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FIG. 1: Single-pion emission diagrams for the $B^+ \rightarrow \pi^+\pi^\mp\pi^+$ decay, where $M_s$ stands for the pion pair.

FIG. 2: Two-pion emission diagrams, where $q$ denotes a $u$ or $d$ quark.

FIG. 3: Annihilation diagrams.

FIG. 4: More annihilation diagrams.

describes the nonperturbative dynamics of a two-meson system appropriately in the localized region of phase space, say, $m_{\pi^+\pi^-}^2 < 0.4$ GeV$^2$.

With the introduction of a two-meson distribution amplitude, the LO diagrams for three-body hadronic $B$ meson decays reduce to those for two-body decays, as displayed in Figs. 1-4. The PQCD factorization formula for a $B \rightarrow h_1 h_2 h_3$ decay amplitude is then written as

$$A = \phi_B \otimes H \otimes \phi_{h_1 h_2} \otimes \phi_{h_3},$$

where the hard kernel $H$ contains only a single hard gluon. The $B$ meson ($h_1 h_2$ pair, $h_3$ meson) distribution amplitude $\phi_B (\phi_{h_1 h_2}, \phi_{h_3})$ absorbs nonperturbative dynamics characterized by the soft scale $\Lambda$ (the invariant mass of the meson pair, the $h_3$ meson mass). Fig. 1 involves transition of the $B$ meson into two light mesons. The amplitude from Fig. 2 is expressed as a product of a heavy-to-light form factor and a time-like light-light form factor in the heavy-quark limit. In Figs. 3 and 4 a $B$ meson annihilates completely, and three light mesons are produced.

Take Fig. 1(a) for the $B^+ \rightarrow \pi^+\pi^-\pi^+$ decay as an example, in which the $B^+$ meson momentum $p_B$, the total momentum $p = p_1 + p_2$ of the pion pair, and the momentum $p_3$ of the second $\pi^+$ meson are chosen, in light-cone coordinates, as

$$p_B = \frac{m_B}{\sqrt{2}} (1, 1, 0_T), \quad p = \frac{m_B}{\sqrt{2}} (1, \eta, 0_T), \quad p_3 = \frac{m_B}{\sqrt{2}} (0, 1 - \eta, 0_T),$$

with the variable $\eta = \omega^2 / m_B^2$, $\omega^2 = p^2$ being the invariant mass squared. The momenta $p_1$ and $p_2$ of the $\pi^+$ and $\pi^-$ mesons in the pair, respectively, have the components

$$p_1^+ = \zeta m_B \frac{m_B}{\sqrt{2}}, \quad p_1^- = (1 - \zeta) m_B \frac{m_B}{\sqrt{2}}, \quad p_2^+ = (1 - \zeta) m_B \frac{m_B}{\sqrt{2}}, \quad p_2^- = \zeta m_B \frac{m_B}{\sqrt{2}},$$
with the $\pi^+$ meson momentum fraction $\zeta$. The momenta of the spectators in the $B$ meson, the pion pair, and the $\pi^+$ meson read, respectively, as

$$k_B = \left(0, \frac{m_B}{\sqrt{2}}x_B, k_B T\right), \quad k = \left(\frac{m_B}{\sqrt{2}}z, 0, k_T\right), \quad k_3 = \left(0, \frac{m_B}{\sqrt{2}}(1 - \eta)x_3, k_3 T\right). \quad (6)$$

The definitions of the two-pion distribution amplitudes in terms of hadronic matrix elements of nonlocal quark operators up to twist 3 can be found in \[1, 13, 16\]. We parameterize them at the leading partial waves as

$$\phi_{3/2}^{\pi\pi}(z, \zeta, \omega^2) = \frac{3F_{\pi\pi}(\omega^2)}{\sqrt{2N_c}}z(1 - z)(2\zeta - 1),$$

$$\phi_{1/2}^{\pi\pi}(z, \zeta, \omega^2) = \frac{3F_{\pi\pi}(\omega^2)}{\sqrt{2N_c}}z(1 - z),$$

with the number of colors $N_c$, where the factor $2\zeta - 1$ arises from the Legendre polynomial $P_l(2\zeta - 1)$ for $l = 1$. The PQCD power counting indicates the scaling of the vector-current form factor in the asymptotic region, $F_\pi(w^2) \sim 1/w^2$, and the relative importance of the scalar-current and tensor-current form factors, $F_s(w^2)/F_\pi(w^2) \sim m_\pi^2/w$, where $m_\pi^2 = m_{\pi}^2/(m_u + m_d)$ is the chiral scale associated with the pion, $m_\pi$, $m_u$, and $m_d$ being the masses of the pion, the $u$ quark, and the $d$ quark, respectively. To evaluate the nonresonant contribution in the arbitrary range of $w^2$, we propose the parametrization for the complex time-like form factors

$$F_\pi(w^2) = \frac{m^2\exp[i\delta_1^\pi(w)]}{w^2 + m^2}, \quad F_t(w^2) = \frac{m_\pi^2m^2\exp[i\delta_1^\pi(w)]}{w^3 + m_\pi^2m^2}, \quad F_s(w^2) = \frac{m_\pi^2m^2\exp[i\delta_0^\pi(w)]}{w^3 + m_\pi^2m^2}, \quad (9)$$

in which the parameter $m = 1$ GeV is determined by the fit to the experimental data $m_{J/\psi}^2|F_\pi(m_{J/\psi}^2)|^2 \sim 0.9$ GeV$^2$ \[17\], $m_{J/\psi}$ being the $J/\psi$ meson mass. The resultant $w^2$ dependence of $F_\pi(w^2)$ also agrees with the low-energy data of the time-like pion electromagnetic form factor for $w < 1$ GeV \[18\], and with the next-to-leading-order (NLO) PQCD calculation \[19\]. The strong phases $\delta^\pi_1$ are chosen as the phase shifts for the $S$ wave ($I = 0, l = 0$) and $P$ wave ($I = 1, l = 1$) of elastic $\pi\pi$ scattering \[16\] according to Watson’s theorem. We simply parameterize the data of these strong phases \[20, 22\] for $2m_\pi < w < 0.7$ GeV as

$$\delta_0^\pi(w) = \pi(w - 2m_\pi), \quad \delta_1^\pi(w) = 1.4\pi(w - 2m_\pi)^2, \quad (10)$$

in which $2m_\pi$ represents the $\pi\pi$ threshold. The increase of $\delta_1^\pi$ with $w$ in the above expression is consistent with the NLO PQCD result of the time-like pion electromagnetic form factor \[19\].

The $B$ meson, pion, and kaon distribution amplitudes are the same as those widely adopted in the PQCD approach to two-body hadronic $B$ meson decays. We have the $B$ meson distribution amplitude

$$\phi_B(x, b) = N_Bx^2(1 - x)^2\exp\left[-\frac{1}{2}\left(xm_B\omega_B^2 - \frac{\omega_B^2b^2}{2}\right)^2\right], \quad (11)$$

with the shape parameter $\omega_B = 0.45 \pm 0.05$ GeV, and the normalization constant $N_B = 73.67$ GeV being related to the $B$ meson decay constant $f_B = 0.21$ GeV via $\lim_{b \to 0} \int dx \phi_B(x, b) = f_B/(2\sqrt{2N_c})$. The pion and kaon distribution amplitudes up to twist 3, $\phi^P_i(x)$ and $\phi^{P, T}_i(x)$ for $i = \pi, K$, are chosen as \[23\]

$$\phi^P_i(x) = \frac{3f_i}{\sqrt{6}}x(1 - x)\left[1 + a_1C_1^{3/2}(t) + a_2C_2^{3/2}(t) + a_4C_4^{3/2}(t)\right],$$

$$\phi^{P, T}_i(x) = \frac{f_i}{\sqrt{2}}\frac{1}{\sqrt{6}}\left[1 + \left(30\eta_3 - \frac{5}{2}\rho_1^2\right)C_1^{1/2}(t) - 3\left(\eta_3\omega_3 + \frac{9}{20}\rho_1^2(1 + 6a_2)\right)C_1^{1/2}(t)\right],$$

$$\phi^T_i(x) = \frac{f_i}{\sqrt{2}}\frac{1}{\sqrt{6}}\left[1 + \left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_1^2 - \frac{3}{5}\rho_2^2a_2\right)C_2^{3/2}(t)\right],$$

with the pion (kaon) decay constant $f_\pi = 0.13$ ($f_K = 0.16$) GeV, the variable $t = 2x - 1$, the Gegenbauer polynomials

$$C_1^{3/2}(t) = 3t, \quad C_1^{1/2}(t) = \frac{1}{2}(3t^2 - 1), \quad C_2^{3/2}(t) = \frac{1}{2}(5t^2 - 1), \quad C_2^{1/2}(t) = \frac{1}{8}(3 - 30t^2 + 35t^4), \quad C_4^{3/2}(t) = \frac{15}{8}(1 - 14t^2 + 21t^4), \quad (15)$$

with

$$C_4^{1/2}(t) = \left(\frac{3}{8}(3 - 30t^2 + 35t^4), \quad C_4^{3/2}(t) = \frac{15}{8}(1 - 14t^2 + 21t^4), \quad (15)$$
and the mass ratio \( \rho_{\pi(K)} = m_{\pi(K)} / m_{\pi(0)} \), where \( m_{\pi(K)} = m_{\pi}^2 / (m_s + m_d) \) is the chiral scale associated with the kaon, \( m_{\pi(K)} \) and \( m_s \) being the masses of the kaon and the \( s \) quark, respectively. The Gegenbauer moments \( a_{\pi,K} \) are set to

\[
a_1^\pi = 0, \quad a_1^K = 0.06 \pm 0.03, \quad a_2^{\pi,K} = 0.25 \pm 0.15, \\
a_3^\pi = -0.015, \quad \eta_3^{\pi,K} = 0.015, \quad \omega_3^{\pi,K} = -3. \quad (16)
\]

The above set of meson distribution amplitudes corresponds to the \( B \to \pi \) transition form factors at maximal recoil \( F^{\pi}_{\pi}(0) = F^{\pi}_{K}(0) = 0.23 \) in LO PQCD, which are consistent with the results derived from other approaches \[23, 24]\.

The \( B^+ \to \pi^+ \pi^- \pi^0 \) decay width in the localized region of \( m_{\pi^+ \pi^-}^{\text{min}} < m_{\pi^0}^\text{min} = 0.4 \text{ GeV}^2 \) and \( m_{\pi^+ \pi^-}^{\text{max}} > m_{\pi^0}^{\text{max}} = 15 \text{ GeV}^2 \) is written as

\[
\Gamma = \frac{G_F^2 m_B}{12 \pi^4} \int_{\eta_{\text{min}}}^{\eta_{\text{max}}} d\eta (1 - \eta) \int_0^{\zeta_{\text{max}}} d\zeta |A|^2,
\]

with the Fermi constant \( G_F = 1.16639^{-5} \text{ GeV}^{-2} \) and the bounds

\[
\eta_{\text{max}} = \frac{m_{\pi^0}^2}{m_B^2}, \quad \eta_{\text{min}} = \frac{4m_\pi^2}{m_B^2}, \quad \zeta_{\text{max}} = 1 - \frac{m_{\pi^0}^2}{(1 - \eta)m_B^2},
\]

where the upper bound \( \zeta_{\text{max}} \) is derived from the invariant mass squared \( \langle p_2 + p_3 \rangle^2 \). The contributions from all the diagrams in Figs. 1-4 to the decay amplitude \( A \) are collected in Appendix. The corresponding formulas for the \( B^+ \to \pi^+ \pi^- K^- \) decay can be obtained straightforwardly.

Employing the input parameters \( \Lambda_{(f=4)}^{(M_S)} = 0.25 \text{ GeV}, m_{\pi^\pm} = 0.1396 \text{ GeV}, m_{K^\pm} = 0.4937 \text{ GeV}, m_{B^\pm} = 5.279 \text{ GeV} \[17, 23]\), and the Wolfenstein parameters in \[17\], we derive the direct CP asymmetries in the region of \( m_{\pi^+ \pi^-}^{\text{low}} < 0.4 \text{ GeV}^2 \) and \( m_{\pi^+ \pi^-}^{\text{low or K+pi-high}} > 15 \text{ GeV}^2 \).

\[
A_{CP}(B^+ \to \pi^+ \pi^- \pi^0) = 0.519^{+0.124}_{-0.219}(\omega_B)^{+0.108}_{-0.091}(a_2^\pi)^{+0.027}_{-0.032}(m_\pi^2), \quad (19)
\]

\[
A_{CP}(B^+ \to \pi^+ \pi^- K^0) = -0.018^{+0.024}_{-0.044}(\omega_B)^{+0.006}_{-0.009}(a_2^\pi)K^{+0.002}_{-0.003}(m_\pi^2) (m_K^2).
\]

The first and second errors come from the variation of \( \omega_B = 0.45 \pm 0.05 \text{ GeV} \) and \( a_2^{\pi,K} = 0.25 \pm 0.15 \), respectively, and the third errors are induced by \( m_0 = 1.4 \pm 0.1 \text{ GeV} \) and \( m_0^K = 1.6 \pm 0.1 \text{ GeV} \). The uncertainties caused by the variation of the Wolfenstein parameters \( \lambda, \rho, \eta \), and of the Gegenbauer moment \( a_1^K = 0.06 \pm 0.03 \) are very small, and have been neglected. While the decay widths are proportional to the decay constants \( f_B, f_\pi \) and/or \( f_K \) quadratically, the CP asymmetries are independent of them.

Obviously, our prediction for \( A_{CP}(B^+ \to \pi^+ \pi^- \pi^0) \) agrees well with the LHCb data. Since the emission contribution and the imaginary annihilation contribution depend on the \( B \) meson distribution amplitude in different ways, the variation of \( \omega_B \) explores the relevance of the short-distance strong phase from the \( b \)-quark decay kernel. The sensitivity of the predicted CP asymmetries to \( \omega_B \) then implies the importance of this strong phase. As the \( P \)-wave rescattering phase associated with the pion electromagnetic form factor decreases to half, the predicted CP asymmetries are also reduced to half. The change of the phases associated with the scalar and tensor form factors does not modify the CP asymmetries much. Therefore, we conclude that the short-distance and long-distance \( P \)-wave strong phases are equally crucial for the direct CP asymmetries in the localized region of phase space. The LHCb data in Eq. \[2\] is dominated by the resonant channel \( B^\pm \to B^0 K^\pm \). It is encouraging that the data confirm the NLO PQCD prediction \( A_{CP}(B^\pm \to B^0 K^\pm) = 0.71^{+0.25}_{-0.35} \[20\] \). We have checked that our prediction in Eq. \[20\] for the localized region of phase space is consistent with the LHCb data in Fig. 2 of \[20\]. Moreover, we have predicted larger \( A_{CP}(B^\pm \to \pi^+ \pi^- \pi^0) = 0.631 \) in the region of \( m_{\pi^+ \pi^-}^{\text{low}} < 0.4 \text{ GeV}^2 \) and \( m_{\pi^+ \pi^-}^{\text{high}} > 20.5 \text{ GeV}^2 \) for the central values of the input parameters, which also matches the data \[6\].

In this paper we have proposed a promising formalism for three-body hadronic \( B \) meson decays based on the PQCD approach. The calculation is greatly simplified with the introduction of the nonperturbative two-hadron distribution amplitude for final states. The like-time form factors and the rescattering phases involved in the two-pion distribution amplitudes have been fixed by experiments, and the \( B \) meson, pion, and kaon distribution amplitudes are the same as in the previous PQCD analysis of two-body hadronic \( B \) meson decays. Without any free parameters, our results for \( A_{CP}(B^\pm \to \pi^+ \pi^- \pi^\pm) \) and \( A_{CP}(B^\pm \to \pi^+ \pi^- K^\pm) \) accommodate well the recent LHCb data in various localized regions of phase space. It has been observed that the short-distance strong phase from the \( b \)-quark decay kernel and the final-state rescattering phase are equally important for explaining the measured direct CP asymmetries. The success indicates that our formalism has potential applications to other three-body hadronic and radiative \( B \) meson decays \[27\], if phase shifts from meson-meson scattering can be derived in nonperturbative methods \[28, 29\].
Appendix A: Decay amplitudes

In this Appendix we present the PQCD factorization formulas for the diagrams in Figs. 1-4. The sum of the contributions from Figs. (a) and (b) gives

\[ A_{1(a,b)} = V_{ub} V_{ud} F_{B \rightarrow \pi \pi}^{LL} - V_{tb} V_{td} \left( F_{B \rightarrow \pi \pi}^{LL} + F_{B \rightarrow \pi \pi}^{SP} \right), \]

where the amplitudes for the \( B \) meson transition into two pions are written as

\[ F_{B \rightarrow \pi \pi}^{LL} = 8\pi C_F m_B^4 f_\pi \int dx_B dz \int b_B db_B db_B (x_B, b_B)(1 - \eta) \times \left\{ \sqrt{\eta} (2 + z)(\phi_+ + \phi_t) + (1 + z)(\phi_v) \right\} a_1(t_{1a}) E_{1ab}(t_{1a}) h_{1a}(x_B, z, b_B, b) + \sqrt{\eta} (2 - \sqrt{\eta} \phi_v) a_1(t_{1b}) E_{1ab}(t_{1b}) h_{1b}(x_B, z, b_B, b) \right\}, \]

\[ F_{B \rightarrow \pi \pi}^{SP} = F_{B \rightarrow \pi \pi}^{LL} \bigg|_{a_1 \rightarrow a_2} \]

\[ F_{B \rightarrow \pi \pi}^{SP} = -16\pi C_F m_B^4 f_\pi \int dx_B dz \int b_B db_B db_B (x_B, b_B) \times \left\{ \sqrt{\eta} (2 + z)\phi_+ - \sqrt{\eta} \phi_t + (1 + \eta(1 - 2z))\phi_v \right\} a_5(t_{1a}) E_{1ab}(t_{1a}) h_{1a}(x_B, z, b_B, b) + 2\sqrt{\eta}(1 - x_B + \eta)\phi_+ + (x_B - 2\eta)\phi_v a_5(t_{1b}) E_{1ab}(t_{1b}) h_{1b}(x_B, z, b_B, b) \right\}, \]

with arguments \( \phi_+ = \phi_{+}(x, \zeta, \omega^2) \). The Wilson coefficients in the above expressions are defined as \( a_1 = C_1/N_c + C_2, a_2 = C_3/N_c + C_4 + C_5/N_c + C_{10}, \) and \( a_5 = C_5/N_c + C_6 + C_7/N_c + C_8. \) The spectator diagrams in Fig. (c) and (d) lead to

\[ A_{1(c,d)} = V_{ub} V_{ud} M_{B \rightarrow \pi \pi}^{LL} - V_{tb} V_{td} \left( M_{B \rightarrow \pi \pi}^{LL} + M_{B \rightarrow \pi \pi}^{LR} \right), \]

with the amplitudes

\[ M_{B \rightarrow \pi \pi}^{LL} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dx_B dz dx_3 \int b_B db_B db_3 db_3 \phi_B(x_B, b_B) \phi^A_+ (1 - \eta) \times \left\{ \sqrt{\eta} (x_3 + \phi_t) + (1 - \eta)(1 - x_3) - x_B + z\eta)\phi_v \right\} C_1(t_{1c}) E_{1cd}(t_{1c}) h_{1c}(x_B, z, x_3, b_B, b_3) - \left\{ \sqrt{\eta} (\phi_+ - \phi_t + \phi_v) + (x_3(1 - \eta) - x_B)\phi_v \right\} C_1(t_{1d}) E_{1cd}(t_{1d}) h_{1d}(x_B, z, x_3, b_B, b_3) \right\}, \]

\[ M_{B \rightarrow \pi \pi}^{LL} = M_{B \rightarrow \pi \pi}^{LL} |_{C_1 \rightarrow C_{10}} \]

\[ M_{B \rightarrow \pi \pi}^{LR} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dx_B dz dx_3 \int b_B db_B db_3 db_3 \phi_B(x_B, b_B) \times \left\{ \sqrt{\eta} (x_3 + \phi_t) + \sqrt{\eta} (1 - x_3) - x_B)\phi_v \right\} C_1(t_{1c}) E_{1cd}(t_{1c}) h_{1c}(x_B, z, x_3, b_B, b_3) \times (\phi_+ + \phi_v) + (x_3(1 - \eta))\phi_v \right\} C_1(t_{1d}) E_{1cd}(t_{1d}) h_{1d}(x_B, z, x_3, b_B, b_3) \right\}, \]

\[ M_{B \rightarrow \pi \pi}^{LR} = M_{B \rightarrow \pi \pi}^{LR} |_{C_1 \rightarrow C_{10}} \]
and the Wilson coefficients \( a_7 = C_5 + C_7 \) and \( a_9 = C_3 + C_9 \).

For Fig. 2(a) and 2(b), we have
\[
\begin{align*}
\mathcal{A}_{2(a,b)}^{q = u} &= V_{ub}^* V_{ud} F_{B \rightarrow \pi}^{LL} - V_{tb}^* V_{td} \left( F_{B \rightarrow \pi}^{UL} + F_{B \rightarrow \pi}^{LR} \right), \\
\mathcal{A}_{2(a,b)}^{q = d} &= -V_{ib}^* V_{id} \left( F_{B \rightarrow \pi}^{UL} + F_{B \rightarrow \pi}^{LR} + F_{B \rightarrow \pi}^{SP} \right).
\end{align*}
\]

The amplitudes involving the \( B \rightarrow \pi \) transition form factors are expressed as
\[
\begin{align*}
F_{B \rightarrow \pi}^{LL} &= 8\pi C_F m_B^4 F_\pi (\omega^2) \int dx_B dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B)(2\zeta - 1) \\
&\times \left\{ (1 + x_3(1 - \eta))(1 - \eta)\phi_\pi^A + r(1 - 2x_3)(1 - \eta)\phi_\pi^P + r(1 + \eta - 2x_3(1 - \eta))\phi_\pi^T \right\} \\
&\times a_2(t_{2a}) E_{2ab}(t_{2a}) h_{2a}(x_B, x_3, b_B, b_3) \\
&+ \left[ x_B(1 - \eta)\eta \phi_\pi^A + 2r(1 - \eta(1 + x_B))\phi_\pi^P \right] a_2(t_{2b}) E_{2ab}(t_{2b}) h_{2b}(x_B, x_3, b_B, b_3),
\end{align*}
\]
\[
\begin{align*}
F_{B \rightarrow \pi}^{LR} &= F_{B \rightarrow \pi}^{LL} |_{a_2 \rightarrow a_4}, \\
F_{B \rightarrow \pi}^{UL} &= F_{B \rightarrow \pi}^{LL} |_{a_2 \rightarrow a_5}, \\
F_{B \rightarrow \pi}^{LR} &= F_{B \rightarrow \pi}^{UL} |_{a_2 \rightarrow a_5}, \\
F_{B \rightarrow \pi}^{UU} &= F_{B \rightarrow \pi}^{LL} |_{a_2 \rightarrow a_{10}}, \\
F_{B \rightarrow \pi}^{SP} &= 16\pi C_F m_B^4 \sqrt{\pi} F_\pi (\omega^2) \int dx_B dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B, b_B) \\
&\times \left\{ (1 - \eta)\phi_\pi^A + r(2 + x_3(1 - \eta))\phi_\pi^P - r x_3(1 - \eta)\phi_\pi^T \right\} a'_8(t_{2a}) E_{2ab}(t_{2a}) h_{2a}(x_B, x_3, b_B, b_3) \\
&+ \left[ x_B(1 - \eta)\phi_\pi^A + 2r(1 - x_B - \eta)\phi_\pi^P \right] a'_8(t_{2b}) E_{2ab}(t_{2b}) h_{2b}(x_B, x_3, b_B, b_3),
\end{align*}
\]
in which the Wilson coefficients are given by \( a_2 = C_1 + C_2/N_c, \ a_4 = C_3 + C_4/N_c + C_9 + C_{10}/N_c, \ a_6 = C_5 + C_6/N_c + C_7 + C_8/N_c, \ a_8 = C_5 + C_6/N_c - C_7/2 - C_8/(2N_c), \ a'_8 = C_5/N_c + C_6 - C_7/(2N_c) - C_8/2, \) and \( a_{10} = [C_3 + C_4 - C_9/2 - C_{10}/2](N_c + 1)/N_c. \) We derive from Fig. 2(c) and 2(d)
\[
\begin{align*}
\mathcal{A}_{2(c,d)}^{q = u} &= V_{ub}^* V_{ud} M_{B \rightarrow \pi}^{LL} - V_{tb}^* V_{td} \left( M_{B \rightarrow \pi}^{UL} + M_{B \rightarrow \pi}^{SP} \right), \\
\mathcal{A}_{2(c,d)}^{q = d} &= -V_{ib}^* V_{id} \left( M_{B \rightarrow \pi}^{UL} + M_{B \rightarrow \pi}^{LR} + M_{B \rightarrow \pi}^{SP} \right),
\end{align*}
\]
with the amplitudes

\[
M_{B\to\pi}^{LL} = 32\pi C_F m_B^4/\sqrt{2N_c} \int dx_Bdzdx_3 \int b_Bdb_Bdb\phi_B(x_B,b_B)\phi_v \\
\times \left\{ \left[ (1-x_B-z)(1-\eta^2)\phi_c^4 + r x_3 (1-\eta) (\phi_c^P - \phi_c^T) + r (x_B+z)\eta (\phi_c^P + \phi_c^T) \right] \\
-2r\phi_c^P C_2(t_{2c})E_{2cd}(t_{2c})h_{2c}(x_B,z,x_3,b_B,b) \\
- \left[ (z-x_B)x_3 (1-\eta) (\phi_c^P + r (x_B-z)\eta (\phi_c^P - \phi_c^T) - r x_3 (1-\eta) (\phi_c^P + \phi_c^T) \right] \\
x_B(t_{2d})E_{2cd}(t_{2d})h_{2d}(x_B,z,x_3,b_B,b) \right\},
\]

(A19)

\[
M_{B\to\pi}^{RR} = 32\pi C_F m_B^4/\sqrt{2N_c} \int dx_Bdzdx_3 \int b_Bdb_Bdb\phi_B(x_B,b_B) \\
\times \left\{ \left[ (1-x_B-z)(1-\eta) (\phi_s + \phi_t)\phi_c^4 + r (1-x_B-z) (\phi_s + \phi_t) (\phi_c^P - \phi_c^T) \\
+ r x_3 (1-\eta) (\phi_s - \phi_t) (\phi_c^P + \phi_c^T) a_s'(t_{2c})E_{2cd}(t_{2c})h_{2c}(x_B,z,x_3,b_B,b) \\
- [(z-x_B)(1-\eta) (\phi_s - \phi_t)\phi_c^4 + r (z-x_B) (\phi_s - \phi_t) (\phi_c^P - \phi_c^T) \\
+ r x_3 (1-\eta) (\phi_s + \phi_t) (\phi_c^P + \phi_c^T) a_s'(t_{2d})E_{2cd}(t_{2d})h_{2d}(x_B,z,x_3,b_B,b) \right\},
\]

(A20)

\[
M_{B\to\pi}^{SP} = 32\pi C_F m_B^4/\sqrt{2N_c} \int dx_Bdzdx_3 \int b_Bdb_Bdb\phi_B(x_B,b_B)\phi_v \\
\times \left\{ \left[ (1-x_B-z + x_3 (1-\eta)) (1-\eta) \phi_c^4 + r \eta (x_B+z) \phi_c^P - \phi_c^T \right] \\
- r x_3 (1-\eta) (\phi_c^P + \phi_c^T) - 2r\phi_c^P a_s'(t_{2c})E_{2cd}(t_{2c})h_{2c}(x_B,z,x_3,b_B,b) \\
- [(z-x_B)(1-\eta^2)\phi_c^4 - r x_3 (1-\eta) (\phi_c^P - \phi_c^T) + r (x_B-z)\eta (\phi_c^P + \phi_c^T) \right] \\
x_B(t_{2d})E_{2cd}(t_{2d})h_{2d}(x_B,z,x_3,b_B,b) \right\},
\]

(A21)

\[
M_{B\to\pi}^{LL} = M_{B\to\pi}^{LL,C_2\to a_4'}/C_2, \\
M_{B\to\pi}^{LLL} = M_{B\to\pi}^{LL,C_2\to a_{10}'}, \\
M_{B\to\pi}^{SP} = M_{B\to\pi}^{SP,a_6\to a_6'},
\]

(A22, A23, A24)

where the Wilson coefficients are defined as \( a_4' = C_4 + C_{10}, \ a_5' = C_5 - C_7/2, \ a_6' = C_6 + C_8, \ a_6'' = C_6 - C_8/2, \) and \( a_{10}' = C_3 + C_4 - C_9/2 - C_{10}/2.\)

The factorizable annihilation diagrams in Fig. 3(a) and 3(b) lead to

\[
A_{3(a,b)} = V_{ub}^* V_{ud} F_{a\pi}^{LL} - V_{tb}^* V_{td} \left( F_{a\pi}^{LL} + F_{a\pi}^{SP} \right),
\]

(A25)
with the three-pion production amplitudes

\[
F^{LL}_{a\pi} = 8\pi C_F m_B^4 f_B \int dz dx_3 \int bdb_2 db_3
\]

\[
\times \left\{ [(x_3(1-\eta) - 1)(1-\eta)\phi_A^A \phi_v + 2r\sqrt{\eta}(x_3(1-\eta)(\phi_P^P - \phi_T^T) - 2\phi_P^P)\phi_s] \right.
\]

\[
\times a_1(t_{3a})h_{3a}(z, x_3, b, b_3)
\]

\[
+ [z(1-\eta)\phi_A^A \phi_v + 2r\sqrt{\eta}\phi_P^P ((1-\eta)(\phi_s - \phi_t) + z(\phi_s + \phi_t))] \left. \right\},
\]

(A26)

\[
F^{LR}_{a\pi} = F^{LL}_{a\pi} |_{a_1 \rightarrow a_3},
\]

(A27)

\[
F^{SP}_{a\pi} = 16\pi C_F m_B^4 f_B \int dz dx_3 \int bdb_2 db_3
\]

\[
\times \left\{ [2r\sqrt{\eta}(1-\eta)\phi_A^A \phi_s + r(1-x_3)(\phi_P^P + \phi_T^T)\phi_v + r\eta((1+x_3)\phi_P^P - (1-x_3)\phi_T^T)\phi_s] \right.
\]

\[
\times a_5(t_{3a})h_{3a}(z, x_3, b, b_3)
\]

\[
+ [2r(1-\eta)\phi_P^P \phi_v + z\sqrt{\eta}((1-\eta)\phi_A^A (\phi_s - \phi_t) + 2r\sqrt{\eta}\phi_P^P \phi_v)] \left. \right\}.
\]

(A28)

The nonfactorizable annihilation diagrams in Fig. 1(c) and 1(d) give

\[
A_{3(c,d)} = V_{ub}^* V_{td} M^{LL}_{a\pi} - V_{tb}^* V_{td} (M^{LL}_{a\pi} + M^{LR}_{a\pi})
\]

(A29)

with the amplitudes

\[
M^{LL}_{a\pi} = 32\pi C_F m_B^4 \sqrt{2N_c} \int dx_B dz dx_3 \int bdb_2 b_3 db_3 \phi_B(x_B, b_B)
\]

\[
\times \left\{ [(1-\eta)(\eta - (1+\eta)(x_B + z))\phi_A^A \phi_v + r\sqrt{\eta}(x_3(1-\eta) + \eta)(\phi_P^P + \phi_T^T)(\phi_s - \phi_t)
\]

\[
r\eta((1-x_3(1-\eta) - \eta(1+x_B - z))\phi_A^A \phi_v - r\sqrt{\eta}(x_B - z)(\phi_P^P + \phi_T^T)(\phi_s - \phi_t)
\]

\[
+ r\sqrt{\eta}(1-\eta)(1-x_3)(\phi_P^P - \phi_T^T)(\phi_s + \phi_t)] C_1(t_{3a})E_{3cd}(t_{3a})h_{3c}(x_B, z, x_3, b, b_3)
\]

\[
\left. \right\},
\]

(A30)

\[
M^{LL}_{a\pi} = M^{LL}_{a\pi} |_{C_1 \rightarrow a_3},
\]

(A31)

\[
M^{LR}_{a\pi} = 32\pi C_F m_B^4 \sqrt{2N_c} \int dx_B dz dx_3 \int bdb_2 b_3 db_3 \phi_B(x_B, b_B)
\]

\[
\times \left\{ [\sqrt{\eta}(1-\eta)(2-x_B - z)\phi_A^A (\phi_s + \phi_t) - r(1+x_3)(\phi_P^P - \phi_T^T)\phi_v
\]

\[
- r\eta((1-x_3 - \eta(x_B - z)(\phi_P^P + \phi_T^T) - x_3(\phi_P^P - \phi_T^T) + 2\phi_P^P \phi_v) a_7(t_{3a})E_{3cd}(t_{3a})h_{3c}(x_B, z, x_3, b, b_3)
\]

\[
- [r(1-\eta)(1-x_3)(\phi_P^P - \phi_T^T)\phi_v - \sqrt{\eta}(x_B - z)[r\sqrt{\eta}(\phi_P^P + \phi_T^T)\phi_v
\]

\[
- (1-\eta)\phi_A^A (\phi_s + \phi_t)]] a_7(t_{3a})E_{3cd}(t_{3a})h_{3d}(x_B, z, x_3, b, b_3)
\]

\right\}.
\]

(A32)

Similarly, we derive from Fig. 1(a) and 1(b)

\[
A_{4(a,b)} = V_{ub}^* V_{td} F^{LL}_{a\pi} - V_{tb}^* V_{td} (F^{LL}_{a\pi} + F^{SP}_{a\pi})
\]

(A33)
with the three-pion production amplitudes

\[
F_{a\pi\pi}^{LL} = 8\pi C_F m_B^4 f_B \int dz dx_3 \int bdbb_3 db_3 \\
F_{a\pi\pi}^{SP} = 16\pi C_F m_B^4 f_B \int dz dx_3 \int bdbb_3 db_3 \\
F_{a\pi\pi}^{LL} = F_{a\pi\pi}^{SP} |_{a_1 \to a_3} \\
F_{a\pi\pi}^{LP} = 8\pi C_F m_B^4 f_B \int dz dx_3 \int bdbb_3 db_3 \\
F_{a\pi\pi}^{LL} = F_{a\pi\pi}^{LP} |_{a_1 \to a_3}
\]

and from Fig. 4(c) and 4(d)

\[
A_{4(c,d)} = V_{ab}^* V_{ad} M_{a\pi\pi}^{LL} - V_{cb}^* V_{db} (M_{a\pi\pi}^{LL} + M_{a\pi\pi}^{LR}),
\]

with the amplitudes

\[
M_{a\pi\pi}^{LL} = 32\pi C_F m_B^4 / \sqrt{2N_c} \int dx_B dz dx_3 \int bdbb_3 db_3 \phi_B(x_B, x_B) \\
M_{a\pi\pi}^{SP} = M_{a\pi\pi}^{LL} |_{C_1 \to a_3} \\
M_{a\pi\pi}^{LR} = -32\pi C_F m_B^4 / \sqrt{2N_c} \int dx_B dz dx_3 \int bdbb_3 db_3 \phi_B(x_B, x_B)
\]

The threshold resummation factor \( S_t(x) \) follows the parametrization in 30

\[
S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c,
\]
in which the parameter is set to \( c = 0.3 \). The hard functions are written as

\[
\begin{align*}
    h_{1a}(x_B, z, b_B, b) &= K_0(m_B \sqrt{b_B z_B}) \left[ \theta(b_B - b)K_0(m_B \sqrt{b_B z_B})I_0(m_B \sqrt{b_B z_B} + (b \leftrightarrow b_B)) \right] S_i(z), \\
    h_{1b}(x_B, z, b_B, b) &= K_0(m_B \sqrt{b_B z_B}) \left[ \frac{\pm H_0^{(1)}(m_B \sqrt{b_B + b_B} z_B)}{K_0(m_B \sqrt{b_B + b_B} z_B)} + (b \leftrightarrow b_B) \right] S_i(z), \\
    h_{1c}(x_B, z, x_3, b_B, b_3) &= \left[ \theta(b_B - b_3)K_0(m_B \sqrt{b_B z_B})I_0(m_B \sqrt{b_B z_B} + (b \leftrightarrow b_B)) \right] S_i(z), \\
    h_{1d}(x_B, z, x_3, b_B, b_3) &= \left[ \frac{\pm H_0^{(1)}(m_B \sqrt{b_B + b_B} z_B)}{K_0(m_B \sqrt{b_B + b_B} z_B)} + (b \leftrightarrow b_B) \right] S_i(z), \\
    h_{2a}(x_B, z, b_B, b) &= K_0(m_B \sqrt{b_B z_B}) \left[ \theta(b_B - b)K_0(m_B \sqrt{b_B z_B})I_0(m_B \sqrt{b_B z_B} + (b \leftrightarrow b_B)) \right] S_i(z), \\
    h_{2b}(x_B, z, b_B, b) &= K_0(m_B \sqrt{b_B z_B}) \left[ \frac{\pm H_0^{(1)}(m_B \sqrt{b_B + b_B} z_B)}{K_0(m_B \sqrt{b_B + b_B} z_B)} + (b \leftrightarrow b_B) \right] S_i(z), \\
    h_{2c}(x_B, z, x_3, b_B, b_3) &= \left[ \theta(b_B - b_3)K_0(m_B \sqrt{b_B z_B})I_0(m_B \sqrt{b_B z_B} + (b \leftrightarrow b_B)) \right] S_i(z), \\
    h_{2d}(x_B, z, x_3, b_B, b_3) &= \left[ \frac{\pm H_0^{(1)}(m_B \sqrt{b_B + b_B} z_B)}{K_0(m_B \sqrt{b_B + b_B} z_B)} + (b \leftrightarrow b_B) \right] S_i(z), \\
    h_{3a}(z, x_3, b_3) &= \left( \frac{i\pi}{2} \right)^2 H_0^{(1)}(m_B \sqrt{1 - x_3} z B) S_i(z), \\
    h_{3b}(z, x_3, b_3) &= \left( \frac{i\pi}{2} \right)^2 H_0^{(1)}(m_B \sqrt{1 - x_3} z B) S_i(z), \\
    h_{3c}(x_B, z, x_3, b_3) &= \left( \frac{i\pi}{2} \right)^2 K_0(m_B \sqrt{1 - x_3} z B) S_i(z), \\
    h_{3d}(x_B, z, x_3, b_3) &= \left( \frac{i\pi}{2} \right)^2 H_0^{(1)}(m_B \sqrt{1 - x_3} z B) S_i(z), \\
    h_{4a}(z, x_3, b_3) &= \left( \frac{i\pi}{2} \right)^2 H_0^{(1)}(m_B \sqrt{1 - x_3} z B) S_i(z), \\
    h_{4b}(z, x_3, b_3) &= \left( \frac{i\pi}{2} \right)^2 H_0^{(1)}(m_B \sqrt{1 - x_3} z B) S_i(z), \\
    h_{4c}(x_B, z, x_3, b_3) &= \left( \frac{i\pi}{2} \right)^2 K_0(m_B \sqrt{1 - x_3} z B) S_i(z), \\
    h_{4d}(x_B, z, x_3, b_3) &= \left( \frac{i\pi}{2} \right)^2 H_0^{(1)}(m_B \sqrt{1 - x_3} z B) S_i(z),
\end{align*}
\]
with the Hankel function $H_0^{(1)}(x) = J_0(x) + iY_0(x)$.

The evolution factors in the above factorization formulas are given by

$$E_{1ab}(t) = \alpha_s(t) \exp[-S_B(t) - S_{Ms}(t)],$$
$$E_{1cd}(t) = \alpha_s(t) \exp[-S_B(t) - S_{Ms}(t) - S_\pi|_{b_3=b_B}],$$
$$E_{2ab}(t) = \alpha_s(t) \exp[-S_B(t) - S_\pi(t)],$$
$$E_{2cd}(t) = \alpha_s(t) \exp[-S_B(t) - S_{Ms}(t) - S_\pi|_{b_3=b_B}],$$
$$E_{3ab}(t) = \alpha_s(t) \exp[-S_{Ms} - S_\pi(t)],$$
$$E_{3cd}(t) = \alpha_s(t) \exp[-S_B(t) - S_{Ms}(t) - S_\pi|_{b_3=b}],$$
$$E_{4ab}(t) = E_{3ab}(t),$$
$$E_{4cd}(t) = E_{3cd}(t),$$

(A43)

in which the Sudakov exponents are defined as

$$S_B = s \left( x_B \frac{m_B}{\sqrt{2}}, b_B \right) + \frac{5}{3} \int_{1/b_B}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})),$$
$$S_{Ms} = s \left( z \frac{m_B}{\sqrt{2}}, b \right) + s \left( 1 - z \right) \frac{m_B}{\sqrt{2}}, b \right) + 2 \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})),$$
$$S_\pi = s \left( x_3 \frac{m_B}{\sqrt{2}}, b_3 \right) + s \left( 1 - x_3 \right) \frac{m_B}{\sqrt{2}}, b_3 \right) + 2 \int_{1/b_3}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})),$$

(A44)

with the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$. The explicit expressions of the functions $s(Q, b)$ can be found, for
The involved hard scales are chosen in the PQCD approach as

\[
\begin{align*}
t_{1a} &= \max \left\{ m_B \sqrt{z}, 1/b_B, 1/b \right\}, \\
t_{1b} &= \max \left\{ m_B \sqrt{|x_B - \eta|}, 1/b_B, 1/b \right\}, \\
t_{1c} &= \max \left\{ m_B \sqrt{x_B z}, m_B \sqrt{z((1 - \eta)(1 - x_3) - x_B)}, 1/b_B, 1/b_3 \right\}, \\
t_{1d} &= \max \left\{ m_B \sqrt{x_B z}, m_B \sqrt{|x_B - x_3(1 - \eta)|}, 1/b_B, 1/b_3 \right\}, \\
t_{2a} &= \max \left\{ m_B \sqrt{x_3(1 - \eta)}, 1/b_B, 1/b_3 \right\}, \\
t_{2b} &= \max \left\{ m_B \sqrt{x_B(1 - \eta)}, 1/b_B, 1/b_3 \right\}, \\
t_{2c} &= \max \left\{ m_B \sqrt{x_B x_3(1 - \eta), m_B \sqrt{|1 - x_B - z|x_3(1 - \eta) + \eta|}}, 1/b_B, 1/b, \right\}, \\
t_{2d} &= \max \left\{ m_B \sqrt{x_B x_3(1 - \eta), m_B \sqrt{|x_B - z|x_3(1 - \eta)|}}, 1/b_B, 1/b \right\}, \\
t_{3a} &= \max \left\{ m_B \sqrt{1 - x_3(1 - \eta)}, 1/b, 1/b_3 \right\}, \\
t_{3b} &= \max \left\{ m_B \sqrt{z(1 - \eta)}, 1/b, 1/b_3 \right\}, \\
t_{3c} &= \max \left\{ m_B \sqrt{(1 - x_3) z(1 - \eta)}, m_B \sqrt{1 - x_3(1 - x_B - z)(1 - \eta) + (x_B + z - 1)\eta}, 1/b_B, 1/b_3 \right\}, \\
t_{3d} &= \max \left\{ m_B \sqrt{(1 - x_3) z(1 - \eta)}, m_B \sqrt{|x_B - z|(1 - x_3)(1 - \eta)}, 1/b_B, 1/b_3 \right\}, \\
t_{4a} &= \max \left\{ m_B \sqrt{1 - z}, 1/b, 1/b_3 \right\}, \\
t_{4b} &= \max \left\{ m_B \sqrt{\eta + x_3(1 - \eta)}, 1/b, 1/b_3 \right\}, \\
t_{4c} &= \max \left\{ m_B \sqrt{(1 - z)(\eta + x_3(1 - \eta))}, m_B \sqrt{1 - z(1 - x_3)(1 - \eta) - x_B}, 1/b_B, 1/b_3 \right\}, \\
t_{4d} &= \max \left\{ m_B \sqrt{(1 - z)(\eta + x_3(1 - \eta))}, m_B \sqrt{(1 - z)|x_B - \eta - x_3(1 - \eta)|}, 1/b_B, 1/b_3 \right\},
\end{align*}
\]

(A45)

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