The effect of torque feedback exerted to driver’s hands on vehicle handling – a hardware-in-the-loop approach

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In this paper, road forces on tire are exerted on driver’s hands via an equivalent torque applied to the steering wheel using a hardware-in-the-loop method to analyze the effect of steering torque feedback in vehicle handling. An electrical torque-feedback steering system is used for experimental validation. A 14-degree-of-freedom vehicle dynamic model, including engine, tire, and steering system mechanism are simulated. The required inputs such as throttle angle, brake demand, and steering wheel angular position are transmitted to the computer via an I/O interface card. Tire forces and steering gear torque are solved. This torque is then sent via an I/O interface card to a DC motor connected to the steering shaft. All equations are solved in real time. To investigate the influence of torque feedback on vehicle handling, several experiments are executed on 25 users. For this purpose, an experimental protocol is defined. In the experiments, the users had to drive along a specific path with constant speed using the designed electrical torque-feedback steering system. During the tests, the driving pattern of each user was recorded and the simulator’s instantaneous position was compared with its desirable value. The results show that the torque feedback improved the driver’s perception from the surrounding environment and enables her/him to handle the vehicle satisfactorily.

Keywords: vehicle handling; electrical steering; hardware-in-the-Loop

1. Introduction

Virtual reality systems expose humans to simulated environments and through interaction with the five senses they can simulate physical presence of the real world for the users (Burdea & Coiffet, 2003). Generally, driving simulators are divided into three categories including high-cost, medium-cost and low-cost (Gregersen, Falkmer, Dol, & Pardo, 2001). About 81% of driving simulators are used in research projects and the remaining 19% serve as training simulations for novice drivers (Straus, 2005). The importance of steering system with torque feedback in driving simulators makes it preferable even in low-cost simulations. Such a system increases the environmental interaction between the user and the simulator, and intensifies the user’s sense of immersion and enhances the process of driver training. It is not only essential for training of novice drivers, but also can be utilized for evaluation and validation of the electrical steer control designs in brand-new vehicles in the car industry. Car manufacturers can use this system to evaluate and modify their conceptual designs without bearing the cost of initial modeling.

Replacement of mechanical and hydraulic systems by electronic ones has led to the by-wire technology. Enhancement of efficiency, increased safety and reliability, and reduction of manufacturing costs are the main advantages of electronic steering systems. If the mechanical steering system is fully replaced by a steer-by-wire system, the following benefits are achieved:

1. Removal of steering shaft simplifies the internal design of the vehicle. Removal of various parts allows more space around the engine. Hence, there will be fewer difficulties for installation of the combustion engine and the steering system can be designed and installed as small units.

2. There is no direct physical connection between tires and steering wheel and in the case of accidents, fewer impacts are transferred to the driver via steering wheel.

3. The specifications of the steering system can be easily changed. Hence, desirable steer response and steering feel can be achieved based on the specified needs.

4. Safety can be improved by providing computer-controlled intervention of vehicle controls with systems such as Electronic Stability Control, Adaptive Cruise Control, and Lane Keeping System.

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A considerable number of research works have already been carried out on steer-by-wire systems. Kim and Song (2002) worked on the electronic power steering (EPS) control system. They focused on designing a control logic, which was able to reduce the torque applied to the driver, produce a different steering feel, and improve return-to-center performance. They implemented their proposed approach on a hardware-in-the-loop system to validate it. A robust control technique was used by Chen and Ulsoy (2002) to control the EPS system and provide the driver with desirable driving experiences. They conducted experiments in both time and frequency domains. The obtained results revealed that the robust control technique can reduce a vehicle’s lateral position deviation and provide slight improvement to the marginal stability of the vehicle. This controller was implemented on the steering system of a human-in-the-loop driving simulator. Park, Han, and Lee (2005) studied steer-by-wire system control. They intended to improve the driving experience and enhance a vehicle stability and investigated the advantages of the removal of the steering shaft in the steer-by-wire system and creation of more space around the engine. A state feedback controller was developed by Yih and Gerdes (2004) with lateral slip as the state variable to improve handling of the vehicle. To measure lateral slip, an observer was designed using steering torque, steering angle, and yaw angle rate. The state feedback controller was implemented on a vehicle with the steer-by-wire system. They estimated the steering torque using the steer system’s motor current and calculated the yaw angle rate based on a linear model of the vehicle and the steering system. Segawa, Nakano, Nishihara, and Kumamoto (2001) designed a controller to improve the stability of the vehicles with a steer-by-wire system and implemented it on a driving simulator. They demonstrated that the proposed controller is able to reduce the effect of the lateral winds on the vehicle’s stability. Moreover, based on some experiments, they showed that the controller improves vehicle’s stability in the case of driving on low-friction roads where behavior of the vehicle is different from the driver’s input command. Yao (2006) transformed various driving scenarios, such as experiencing different steering feel, return-to-center performance, and fast and accurate wheel angle tracking based on driver’s input into the control problems. Modern control techniques, system parameter identification, and state variable estimation were used to execute these scenarios. Finally, Yao implemented different control loops on seven vehicles with steer-by-wire systems and eight hardware-in-the-loop systems.

This paper focuses on the effect of torque feedback — exerted to driver’s hands — on vehicle handling. An electrical torque-feedback steering system is designed and implemented in a driving simulator for experimental tests. The developed system is a segment of a by-wire control system and hence can be used for the construction and control of the steer-by-wire systems in domestic vehicles. Other applications of the proposed system include tele-operation of vehicles and driver behavior investigations such as works done in Friedrichs and Yang (2010) and Samiee, Azadi, Kazemi, Nahvi, and Eichberger (2014).

In this paper, forces applied to the driver’s hand via the steering wheel will be identified and reproduced for implementation on the hardware. Vehicle’s dynamics is modeled using equations of motion. Dynamic equations of tires are also solved instantaneously to obtain friction forces. Furthermore, geometry of the steering mechanism and tires are simulated to transform the generated forces on the tires to an equivalent torque on the steering wheel. Then, the equivalent torque is sent to the motor driver using an interface card to produce the required torque by providing the equivalent current. The generated torque is measured by a sensor, installed on the output of the gearbox, and is sent back to the computer using another interface card. Hence, the generated torque is compared with the reference (desirable) value and the error signal is fed into the controller.

All of the aforementioned calculations and the data transfers between computer and hardware must be carried out in real time. The used hardware and software are able to perform real-time calculations, generate graphical pictures, and communicate with interface cards simultaneously. The vehicle is modeled by a 14-degree-of-freedom (DOF) model. The dynamic model of tires is of a Fiala type. Other dynamics of the system are ignored and only kinematics is modeled for the generation of torques on the steering wheel based on the generated tire forces. A classical proportional-integrator-derivative (PID) control is used in this paper which demonstrates favorable performance according to the experiments results.

2. Dynamic modeling of vehicle in ADAMS

In this paper, the virtual vehicle’s dynamic equations, tires dynamic equations, and geometry of the front wheels are validated by a full car model with 251-DOF, implemented in ADAMS/Car, which had been previously validated using experimental results of a real car. The characteristics of the subsystems of the vehicle modeled in ADAMS are as follows:

- Front wheel’s suspension system is of McPherson type with linear spring.
- The compound suspension system with nonlinear spring is considered for rear wheels.
- The steering system is of rack-pinion type.
- Disk-type braking system is modeled for each of the four wheels.
- The power generation and transmission units are modeled by a series of mathematical equations, with throttle, brake, and gear as the inputs and the torque on the wheels as the output.
The tires are represented by the Fiala model. The vehicle’s body is modeled as a concentrated mass on the center of gravity of the vehicle.

By proper interconnection of the vehicle subsystems in ADAMS/Car, the full dynamic model of the vehicle is obtained. Figure 1 illustrates the vehicle model.

3. Simulation of vehicle’s dynamics

A 14-DOF model is used to represent the vehicle’s dynamics. The main advantage of the 14-DOF model against other simpler models (e.g. three- and seven-DOF models) is that it is composed of full six-DOF models for vehicle’s body and suspension system dynamics. Figure 2 shows the suspension system of the 14-DOF model. It is assumed that each wheel has its own independent suspension system. For each wheel, there is a spring and damper, located between the body and suspension arm.

The parameters in Figure 2 are defined as follows:

- The distance from wheel $i$ to the connection point of spring and damper to the suspension arm is indicated by $a_i$.
- The distance between the connection point of the spring and damper to the suspension arm of wheel $i$ and the connection point of the suspension arm to the vehicle’s body is denoted by $b_i$.
- The distance from the spring and damper of wheel $i$ to the longitudinal axis is shown by $d_i$.
- Parameters $M_w$ and $Z_u$ represent the $i$th unsprung mass and its height from the ground, respectively, and $Z_r$ indicates the height of the road beneath the $i$th wheel.
- Parameters $K_u$ and $C_u$ are the spring stiffness and equivalent damping coefficient of the $i$th tire, respectively.
- The spring stiffness and equivalent damping coefficient of the $i$th wheel are designated by $K_s$ and $C_s$, respectively.
- And $Z_s$ shows the distance from the center of gravity to the ground.

After deriving the equations of linear and angular motion for the vehicle and some algebraic manipulations, the 14-DOF equations of motion can be expressed as follows (Bastow, Howard, & Whitehead, 2004).

3.1. Longitudinal equations of vehicle

Using Newton’s second law, the longitudinal dynamic equation of the vehicle can be presented as Equation (1),

$$ M_t (\ddot{u} + qw - rv) = X_1 + X_2 + X_3 + X_4 - F_{ax}, \quad (1) $$

where $M_t$ is the total vehicle mass, $\ddot{u}$ is the longitudinal acceleration of the vehicle’s center of gravity, $q$ is the pitch rate, $w$ is the vertical velocity of the vehicle’s center of gravity, $r$ is the yaw rate, $v$ is the lateral velocity of the vehicle’s center of gravity, and $F_{ax}$ is longitudinal air resistance force. In addition, $X_i$ is found from Equation (2),

$$ X_i = F_{xi} \cos(\delta_i) - F_{yi} \sin(\delta_i) \quad (2) $$

where $\delta_1 = \delta_{fr}$ is the steering angle of the front right wheel, $\delta_2 = \delta_{fl}$ is the steering angle of the front left wheel, and $\delta_3 = \delta_4 = 0$ are the steering angles of rear wheels, which are always equal to zero. Also, $F_{xi}$ and $F_{yi}$ represent the applied longitudinal and lateral forces on the $i$th tire, respectively.

3.2. Lateral equations of vehicle

Using Newton’s second law, the lateral dynamic equation of the vehicle can be presented as Equation (3),

$$ M_t (\dot{v} + ru - pw) = Y_1 + Y_2 + Y_3 + Y_4 - F_{ay}, \quad (3) $$

where $p$ indicates the roll rate and $F_{ay}$ shows the lateral air resistance force. Furthermore, $Y_i$ can be stated as
Equation (4)
\[ Y_i = F_{s1} \sin(\delta_i) + F_{s1} \cos(\delta_i). \] (4)

3.3. Vertical equations of vehicle
The vertical dynamic equation can be shown by
\[ M_s(\dot{w} + p v - q u) = \sum_{i=1}^{4} \frac{F_{si}}{R_{ii}}, \] (5)
where \( M_s \) represents the sprung mass and \( F_{si} \) is the suspension force of wheel \( i \), expressed by Equation (6),
\[ F_{s1} = K_{s1}(Z_{u1} - Z_s + d_1 \sin(\phi) + C_{s1}(u_{u1} - w + Lq \cos(\theta) + d_1 p \cos(\phi)), \]
\[ F_{s2} = K_{s2}(Z_{u2} - Z_s - d_2 \sin(\phi) + C_{s2}(u_{u2} - w + Lq \cos(\theta) - d_2 p \cos(\phi)), \]
\[ F_{s3} = K_{s3}(Z_{u3} - Z_s - d_3 \sin(\phi) + C_{s3}(u_{u3} - w - Lq \cos(\theta) + d_3 p \cos(\phi)), \]
\[ F_{s4} = K_{s4}(Z_{u4} - Z_s - d_4 \sin(\phi) + C_{s4}(u_{u4} - w - Lq \cos(\theta) - d_4 p \cos(\phi)), \] (6)
where \( \theta \) and \( \phi \) are pitch and roll angles of vehicle’s body, respectively, and \( Lf \) and \( Lr \) indicate the distance of the vehicle’s center of gravity from the front and rear axles, respectively. \( R_{ii} \) is a coefficient related to the geometry of the suspension system and is found from Equation (7),
\[ R_{ii} = \frac{a_i + b_i}{b_i}. \] (7)

3.4. Roll angle
Angular equations of vehicle around longitudinal axle represent Equation (8).
\[ I_{ls} \ddot{\phi} + (I_{zs} - I_{ls}) \dot{\phi} = \sum_{i=1}^{4} \frac{\overline{R_{ii}} F_{si} - h_{cg} \sum_{i=1}^{4} Y_i}{R_{ii}}, \] (8)
where \( h_{cg} \) is the distance of vehicle’s center of gravity from ground in standstill and \( I_{ls} \), \( I_{zs} \), and \( I_{ls} \) indicate the moment of inertia of the sprung mass around longitudinal, lateral, and vertical axes, respectively. In addition, \( \overline{R_{ii}} \) is a coefficient related to the geometry of the suspension and,
\[ \overline{R_{i1}} = -d_1 + (d_1 \cdot b_1) \frac{a_1}{a_1 + b_1}, \]
\[ \overline{R_{i2}} = d_2 - (d_2 \cdot b_2) \frac{a_2}{a_2 + b_2}, \]
\[ \overline{R_{i3}} = -d_3 + (d_3 \cdot b_3) \frac{a_3}{a_3 + b_3}, \]
\[ \overline{R_{i4}} = d_4 - (d_4 \cdot b_4) \frac{a_4}{a_4 + b_4}. \] (9)

3.5. Pitch angle
Vehicle’s equations around lateral axle represent Equation (10)
\[ I_{y3} \ddot{q} + (I_{y3} - I_{y1}) \dot{q} r = -L_f \left( \frac{F_{s1}}{R_{11}} + \frac{F_{s2}}{R_{12}} \right) + L_r \left( \frac{F_{s3}}{R_{13}} + \frac{F_{s4}}{R_{14}} \right) \]
\[ + h_{cg} \sum_{i=1}^{4} X_i \sqrt{2}. \] (10)

3.6. Yaw angle
Vehicle’s equations around vertical axle result in Equation (11)
\[ I_{y3} \ddot{q} + (I_{y3} - I_{y1}) \dot{q} p = L_f(Y_1 + Y_2) - L_r(Y_3 + Y_4) \]
\[ + \frac{T_f}{2}(X_1 - X_2) + \frac{T_r}{2}(X_3 - X_4) + \sum_{i=1}^{4} M_{zi}, \] (11)
where \( T_f \) is the distance of front wheels from each other, \( T_r \) is the distance between rear wheels, and \( I_1, I_2, \) and \( I_3 \) represent the moment of inertia around longitudinal, lateral, and vertical axles, respectively.

3.7. Unsprung masses (four equations)
Equations of motion for unsprung masses lead to Equation (12).
\[ M_{wI} \ddot{w}_i = K_{wI}(Z_{ri} - Z_w) + C_{wI}(w_{ri} - w_w) - \frac{F_{si}}{R_{ii}}, \] (12)
where \( w_{ri} \) indicates the rate of change of the road input and \( w_w \) is the rate of change of the motion, which is perpendicular to the unsprung mass.

3.8. Wheels dynamics (four equations)
Considering one DOF for each tire results in Equation (13)
\[ I_{wi} \ddot{\omega}_i = T_{wi} - R_{wi} F_{xi} - T_{Ri} - T_b \] (13)
where \( R_w \) is the effective radius of the tires, and \( T_b \) and \( T_{Ri} \) represent the braking and resistant torques, respectively. The dynamic parameters of the vehicle are presented in Table 1. The 14-DOF model is obtained after performing some algebraic manipulations on the linear and angular equations of motion. To validate the 14-DOF model, a full vehicle model in ADAMS/Car was used.
Two different maneuvers were applied to both the models. In the first maneuver, the vehicle was traveling with a constant velocity of 70 km/h and then a sinusoidal displacement with the frequency of 0.2 Hz and amplitude of 70 degrees is applied to the steering wheel. In the second maneuver, a step input of 70 degrees was applied to
Table 1. Dynamic parameters of the vehicle.

| Parameter                              | Value         |
|----------------------------------------|---------------|
| Distance of vehicle’s center of gravity from front axle | $l_f = 1$ (m) |
| Distance of vehicle’s center of gravity from rear axle | $l_r = 2.6$ (m) |
| Distance between front wheels          | $T_f = 1.401$ (m) |
| Distance between rear wheels           | $T_r = 1.408$ (m) |
| Distance of vehicle’s center of gravity from ground in standstill | $h_{cg} = 0.497$ (m) |
| Mass of vehicle                        | $M_t = 924$ (kg) |
| Coefficient of front spring            | $K_{s1} = 17,650$ (N/m) |
| Coefficient of rear spring             | $K_{s2} = 22,600$ (N/m) |
| Coefficient of front damper            | $C_{s1} = 1600$ (Ns/m) |
| Coefficient of rear damper             | $C_{s2} = 3730$ (Ns/m) |
| Steer ratio                            | $N = 1 : 17.5$ |

the steering wheel of the vehicle, traveling with a constant velocity of 70 km/h.

Figures 3–6 show the results of the comparison between the dynamic parameters of both the models, including the longitudinal and lateral velocities and the rate of yaw angle. These figures demonstrate a close match between the outputs of both the models. The maximum longitudinal and lateral velocity errors for both the models are 3 m/s. Moreover, the 14-DOF model has accurately generated the rate of change of the yaw angle. The maximum error for the 14-DOF model in the case of step input maneuvering is 0.05 rad/s.

4. Dynamic simulation of tire

Correct modeling of tires significantly influences the process of the vehicle dynamic simulation. The applied forces are transferred to the vehicle through the tires. Therefore, the accurate calculation of the tire forces is the very first step toward the simulation of the vehicle. Different tire models have been proposed, which produce dependable estimations for tire forces and torques if the range of applied forces is not broad and tire deflection is slight. This paper uses the Fiala model to simulate the tire. In this model, it is assumed that tire contact to the road is rectangular and camber angle does not affect forces. The Fiala model is described as follows.

4.1. Calculation of longitudinal slip angle of tire

Longitudinal slip happens due to the difference between the velocity of the tire center and the contact patch center. According to Figure 7, theoretical calculation of longitudinal slip is as follows:
The Fiala model uses the following equations for stable estimation of the slip.

### 4.1. Acceleration slip

\[ S_s = \frac{V_t - V_x}{V_t} = \frac{(\omega_{\text{Actual}} - \omega_{\text{Free}})}{\omega_{\text{Actual}}} \]  

(14)

### 4.2. Braking slip

\[ S_s = \frac{V_t - V_x}{V_x} = \frac{(\omega_{\text{Actual}} - \omega_{\text{Free}})}{\omega_{\text{Free}}} \]  

(15)

where \( V_t \) is the linear velocity of the contact point between tire and the road, obtained from Equation (16) and \( V_x \) indicates the image of the velocity of wheel center along the longitudinal axle of the wheel and is found from Equations (17) and (18):

\[ V_t = \omega R_w, \]  

(16)

\[ V_{tr} = V_s - 0.5r \cdot T_r \cos \delta_{tr} + (V_y + r \cdot L_f) \sin \delta_{tr}, \]  

\[ V_{fl} = V_s + 0.5r \cdot T_r \cos \delta_{fl} + (V_y + r \cdot L_f) \sin \delta_{fl}, \]  

(17)

\[ V_{rr} = (V_s - 0.5r \cdot T_r), \]  

\[ V_{rl} = (V_s + 0.5r \cdot T_r). \]  

(18)

In these equations, \( V_{tr} \), \( V_{fl} \), \( V_{rr} \), and \( V_{rl} \) represent the image of the velocity of wheel center along the longitudinal axle of the front right, front left, rear right, and rear left wheels, respectively, and \( R_w \) is the radius of the unloaded wheel.

### 4.3. Calculation of lateral slip angle of tire

The lateral slip angle of the tires is found from the difference between the longitudinal axle of tire and the axis of velocity vector on the tire plane (\( \alpha \) in Figure 8).

The sign of the angle is positive in the upward direction and negative in the downward direction. The slip can be stated based on the state variables such as longitudinal, lateral, and yaw velocities. The lateral slip is expressed by Equation (19)

\[ \alpha_{tr} = \delta_{tr} - a \tan \left( \frac{(V_y + r \cdot L_f)}{(V_t - 0.5r \cdot T_r)} \right), \]  

\[ \alpha_{fl} = \delta_{fl} - a \tan \left( \frac{(V_y + r \cdot L_f)}{(V_t + 0.5r \cdot T_r)} \right), \]  

(19)

\[ \alpha_{rr} = -a \tan \left( \frac{(V_y - r \cdot L_r)}{(V_t - 0.5r \cdot T_r)} \right), \]  

\[ \alpha_{rl} = -a \tan \left( \frac{(V_y - r \cdot L_r)}{(V_t + 0.5r \cdot T_r)} \right). \]

### 4.4. Calculation of vertical force

In the Society of Automotive Engineers coordinate system, vector +Z faces downwards (towards the road). Hence, the vertical force of the road on the tire in the contact point is always negative. The vertical force is expressed by Equation (20):

\[ F_z = \min(0,0, (F_{zk} + F_{zc})) \]  

(20)

where \( F_{zk} \) is the vertical force due to the vertical stiffness of the tire and is found from Equation (21):

\[ F_{zk} = -VS \times TD, \]  

(21)

where VS and TD indicate vertical stiffness and tire deflection, respectively. Moreover, \( F_{zc} \) is the vertical force due
to the tire damping, expressed by Equation (22):

$$F_{xc} = -VD \times \frac{d}{dt}(TD),$$

where $VD$ represents vertical damping of the tire.

### 4.4. Calculation of longitudinal force

The longitudinal force on the tires in any instance depends on the vertical force, $F_z$, instantaneous friction coefficient, $U$, the ratio of the longitudinal slip, $S_s$, and the lateral slip angle $\alpha$ in that instance. Also, total slip ($S_{tot}$), instantaneous friction coefficient ($U$), and critical longitudinal slip ($S_{Critical}$) must be calculated prior to calculation of the longitudinal force. The total slip is expressed by Equation (23),

$$S_{tot} = \sqrt{S_s^2 + \tan^2(\alpha)}.\tag{23}$$

The instantaneous friction coefficient depends on the static friction coefficient ($U_{max}$), dynamic friction coefficient ($U_{min}$), and the total slip. The $U_{max}$ represents the friction coefficient between the tire and the road at zero slip, which does not happen in practice as there always exists non-zero slip. On the other hand, $U_{min}$ indicates friction coefficient between the tire and the road at 100% slip

$$U = U_{min} + (U_{max} - U_{min}) \times S_{tot}.\tag{24}$$

The critical slip ($S_{crit}$) can be expressed by Equation (25)

$$S_{crit} = \frac{U \times F_z}{2 \times C_{slip}},\tag{25}$$

where $C_{slip}$ is the longitudinal slip constant found by the limit of changes in $F_x$ with respect to $S_s$ while slip tends to go to zero. For the critical mode, that is, when the slip is below its critical value ($|S_s| < S_{crit}$), the longitudinal force is calculated by Equation (26):

$$F_x = -C_{slip} \times S_s.\tag{26}$$

For the full slip mode when slip is higher than its critical value ($|S_s| > S_{crit}$), the longitudinal force is given by

$$F_x = -(F_{x1} - F_{x2}) \times \text{sign}(S_s),\tag{27}$$

where $F_{x1}$ and $F_{x2}$ are stated as,

$$F_{x1} = U \times F_z,\tag{28}$$

$$F_{x2} = \frac{(U \times F_z)^2}{4 \times |S_s| \times C_{slip}}.\tag{29}$$

### 4.5. Calculation of lateral force

The lateral force on a tire is a function of the vertical force and the friction coefficient. Similar to the calculation of the longitudinal force, the Fiala model uses a critical lateral slip ($\alpha_{crit}$), to obtain the lateral force,

$$\alpha_{crit} = a \tan\left(\frac{3 \times U \times |F_z|}{C_a}\right),\tag{30}$$

where $C_a$ is the lateral stiffness, found by the limit of changes in $F_y$ with respect to $\alpha$ when $\alpha$ is close to zero. When the slip angle $\alpha$ reaches its critical value ($\alpha_{crit}$), the lateral force takes its maximum value ($U \times |F_z|$). Furthermore, if $|\alpha| \leq \alpha_{crit}$ then $F_y$ is given by

$$F_y = -U \times |F_z| \times (1 - H^3) \times \text{sign}(\alpha),\tag{31}$$

where $H$ is expressed by

$$H = 1 - \frac{C_a \times |\tan(\alpha)|}{3 \times U \times |F_z|}.\tag{32}$$

For the full slip mode, that is, $|\alpha| > \alpha_{crit}$, $F_y$ is given by

$$F_y = -U \times |F_z| \times \text{sign}(\alpha).\tag{33}$$

Table 2 summarizes some dynamic parameters of the tire.

| Parameter                              | Value          |
|----------------------------------------|----------------|
| Unload tire radius                     | $R_o = 0.283$ (m) |
| Mass of tire                           | $M_{tire} = 14.9$ (kg) |
| Friction coefficient at full slip      | $U_{min} = 0.7$ |
| Friction coefficient at zero slip      | $U_{max} = 0.9$ |
| Vertical damping factor of tire        | $VD = 153,000$ (N/m) |
| Vertical stiffness factor of tire       | $VS = 100$ (N/s/m) |

Figure 9. Longitudinal force on the tire of ADAMS and C# models (sinusoidal input of the steering wheel).
Figures 9–12 illustrate the results of comparison of the tire longitudinal and lateral forces.

It is clear that the simulated model has accurately produced longitudinal and lateral forces. The maximum error for the lateral force is 200 N, obtained in the sinusoidal input maneuvering. The average error over one cycle is less than 100 N, indicating an error level below 5%. This value for longitudinal force is below 50 N.

5. Simulation of front wheels kinematics

The effect of tire forces and torques on the steering wheel must be calculated. In real vehicles, these forces and torques are transferred to the steering wheel through linkage of the front wheels. In the simulated steering system, the linkage connecting the steering box to the steering wheel is incorporated into the control system in a hardware-in-the-loop form and hence its dynamic modeling is not required. However, the linkage from the tire to the steering box does not exist and therefore must be simulated. To perform this simulation, some simplifications are considered to reduce computational burden and allow real-time implementation with insignificant computational error. These simplifications are as follows:

- Wheel angle $\delta$ is calculated by dividing the steering angle by the pinion ratio. Hence, this angle is identical for both wheels. For the real car, the average value of the left and the right wheel’s angle is considered.
- To calculate the torques on the steering wheel, only the influence of the longitudinal and lateral forces, applied from the road to the tire, and the steering angle are taken into account and the effect of other forces and torques is ignored.
- The contact point of the kingpin axle to the ground is considered as the center of rotation.
- The tie-rod shaft is modeled as a bi-force element and its deflection during movement is ignored. Hence, the force applied on the tie-rod is equivalent to the rack force.
- The ground-tire contact point is always constant.

After simplifications, the diagram of the applied forces on the tire is developed as illustrated in Figure 13.

Based on this figure, rack forces are obtained by vector summation of the forces from wheels on the tie-rod.
The forces on the tie-rod are calculated by equations in Equation (34),
\[ A = \left(\frac{1}{\text{TO}_x + \text{CO}_x}\right), \]
\[ B = (F_y + (F_x \cdot \sin \delta))(\text{CO}_x + \text{Caster..Offset}), \]
\[ C = (F_x \cdot \cos \delta \cdot \text{Ground..Offset}), \]
where \( \text{TO}_x \) is the vertical distance of the tie-rod from the longitudinal rotation center of the tire and \( \text{CO}_x \) indicates the vertical distance of the tire–road contact center from the longitudinal rotation center of the tire.

5.1. Simulation for the left wheel
\[ (F_{\text{Tie..Rod}})_l = A(B + C). \]  

5.2. Simulation for the right wheel
\[ (F_{\text{Tie..Rod}})_r = A(B - C). \]

The geometry of the front wheels was validated using the maneuvers described in Section 3. The developed torques on the pinion of the steering box were compared with those obtained by the ADAMS model during the maneuvers. Figures 14 and 15 illustrate this comparison.

It is seen from these figures that the maximum error for both the maneuvers is about 0.2 Nm. This error can be justified by the fact that only the effect of the longitudinal and lateral forces on development of torques on the steer is considered in the simulated model and the influence of other forces and torques, such as vertical force and rolling resistance torque, is ignored.

6. Hardware-in-the-loop implementation
In this paper, dynamic equations of the vehicle, friction force on tires and the resulting torques on the steering box are simulated in the computer, while elements of the steering system such as torque actuator, sensors, steering shaft, and steering wheel are physically realized in the hardware-in-the-loop system. An illustration of the hardware-in-the-loop system is shown in Figure 16.

This system uses a brushed DC motor, which is directly connected to the end of the steering shaft and produces the torques developed by the friction forces between the tire and the road. The used DC motor has the rated power of 400 W and produced 3.1 Nm torque at the maximum current of 16 A.

The simulated dynamic systems required the instantaneous angular position of the steering wheel to calculate the steering torques. Moreover, the control system needs the motor torque to properly control the applied torque. These requirements are fulfilled by the use of a torque sensor, which is shown in Figure 17.

In this scheme, the angular positions of both ends of the torsion bar are estimated by two 12-bit encoders. The changes in the angles of both ends of the torsion bar are transferred by pulleys and timing belts to the encoders. The pulleys have a 1:2 diameter ratio and are connected to high-resolution encoders (3600 pulse/revolution). The torque sensor resolution is 0.0375 Nm. The sensor works based on a simple principle that the applied torque on a rod is proportional to the difference in rotational angles of the
rod’s two ends (Beer & Johnston, 2008),

\[ T_{gb} = K_{ib} \cdot \Delta \theta = K_{ib} \cdot (\theta_{sw} - \theta_{gb}), \]  

(37)

where \( T_{gb} \) is the output torque of the gearbox, \( K_{ib} \) indicates stiffness coefficient of the torsion bar, which is about 186.6 N m./rad, and \( \theta_{sw} \) and \( \theta_{gb} \) are the rotation angles of the steering wheel and the gearbox, respectively. Hence, the motor torque is calculated and sent as a feedback signal to the computer. The picture of the implemented hardware system is shown in Figure 18 whereas a detailed overview of the torque sensor is illustrated in Figure 19.

7. Simulation of control system

Before hardware implementation of the control system, the controller was simulated in the computer to investigate its functionality and performance. Hence, all elements of the hardware system were modeled by equivalent dynamic equations as shown in Figure 20.

In Figure 20, \( T_{desired} \) is the desired torque which is calculated by the vehicle dynamic model and is compared with the motor torque \( (T_{motor}) \) and the generated error is fed to the controller. The controller’s output signal is amplified by an operational amplifier and then sent to the driver.
Figure 21. Simplified model of the steering system from the steering wheel to the pinion.

of the DC motor to produce the suitable current for generation of the desired torque. The motor torque is passed through the gearbox and then is measured by the torque sensor. The resultant of this torque and the driver torque, $T_{\text{driver}}$, are simultaneously applied to the steering wheel. The rotation angle of the steering wheel is the output of the system, which is re-used to solve the dynamic equations of the vehicle.

To derive the dynamic model of the steering system from the steering wheel to the connection point of motor and gearbox, the overall system is considered by two masses with rotational inertia. Since the mass and inertia of the torsion bar are negligible compared with other system’s components, it is modeled by a torsional spring alone. Hence, the overall system is modeled by two rotational inertias interconnected by a torsion spring as shown in Figure 21.

In this figure, $T_1$ is equivalent to $T_{\text{driver}}$, applied by the driver to the steering wheel and $T_2$ is the motor torque after the gearbox and is expressed by

$$T_2 = T_m \cdot N_1 = T_{\text{gb}}. \quad (38)$$

Moreover, $I_1$ is the sum of the steering wheel inertia ($I_{\text{sw}}$) and the steering shaft inertia ($I_{\text{ss}}$). In addition, $I_2$ indicates the sum of the motor inertia ($I_m$), the gearbox inertia, $I_{\text{gb}}$, and the coupling inertia, $I_{\text{coupling}}$, between the gearbox shaft and the torsion bar (Craig, 2004),

$$I_1 = I_{\text{sw}} + I_{\text{ss}}, \quad (39)$$

$$I_2 = I_m \cdot N_1^2 + I_{\text{gb}} + I_{\text{coupling}}. \quad (40)$$

The equation governing DC motor is expressed by Equation (41) as stated in Dorf & Bishop (2004),

$$V_a = V_b + R_a I_a + L_a \frac{dI_a}{dt}, \quad (41)$$

where $V_a$, $I_a$, and $R_a$ are armature voltage, current, and resistance, respectively, and $L_a$ is the armature self-inductance. $V_b$ indicates the induced voltage, which is proportional to motor speed. The relationship between the armature current and the output torque is expressed by

$$T_m = K_t I_a,$$

where $K_t$ is the motor constant. The classical PID controller is used in this paper. To analyze the system response, a step input is applied. To find the controller coefficients, the Zeigler–Nichols approach is used. Then the coefficients are tuned for a more accurate response. Since the input of the actual system changes frequently, the transient response of the system is very important. To investigate the transient response, a sinusoidal input with 1 Hz frequency and unit amplitude is applied to the system. The system response reveals that the designed controller is able to desirably follow variable inputs.

8. Experimental results

After investigating the controller performance by simulation inputs, the control was implemented on hardware.

Before doing the main tests, and to evaluate systems performance in a virtual driving condition, a user was asked to use the system and follow the desired route, specified by graphical illustrations.

The desired steering shaft torque, obtained by dynamic equations, along with the real torque generated by the steering system of the driving simulator and measured by the torque sensor are shown in Figure 22.

The generated torque by the dynamic equations and controller is transferred to driver’s hands through the motor connected to the steering shaft. The maximum delay for this case is below 0.05 s.

For more investigation of the steering system performance and the influence of torque feedback on driver’s proper steering of the vehicle, various experiments were executed on 25 users in the driving simulator. To do so, first an experimental protocol and a driving scenario were defined. Then, the users had to drive on the route of Figure 23, using the steering system, as described in this paper.

Figure 22. Desired torque and torque generated with the steering system’s motor in a driving scenario.
Before doing the main tests and to familiarize each participant with the path, each subject was asked to drive in the specified scenario once when there is torque feedback on the steering system (active mode) and another time when there is no torque feedback on the steering system (passive mode). For the main test, the users had to drive along the path with constant speed three times in the active steering mode and three times in the passive steering mode. During the experiment, the driving pattern of each user was recorded and the simulator’s instantaneous position was compared with its desirable value (keeping within the road limits). Figure 24 shows the path passed by one of the users during the driving scenario for both active and inactive steering modes.

The deviation of the user from the desired position is shown in Figure 25. Obviously, it can be seen that there is a small deviation when the steering system with torque feedback is used.

To have a broader analysis, the results obtained for all users are considered. The average error of maximum deviations from the road for all 25 users and the standard deviation of this error for three experiments, in the passive mode, are shown in Figure 26.

In this figure, the white circles indicated the root mean square error of all the users in every experiment and the blue vertical lines show the standard deviation of the error.

It is seen that, on average, the maximum deviation of the users for the first, second, and third stage is 11.5, 12 and 17.2 m, respectively. Clearly, increasing the number of experiments has led to higher deviations due to the driver fatigue. The standard deviation of error has also increased with the number of experiments, which is not desirable.

The average of maximum errors during the three experiments in the active mode (with torque feedback) is depicted in Figure 27.

Comparing Figures 26 and 27 confirms the favorable influence of the torque feedback on error reduction.
Moreover, the torque feedback has improved users’ performance during the experiments. For instance, the average of maximum errors in the first round of experiment is 13 m, whereas this value in the third round of experiments had been reduced to 7 m.

9. Conclusion
This paper presented the design, construction, and control of the electrical steering system for a driving simulator. The simulator can be used in driving simulation to have a deeper sense of immersion by the users as well as for construction of steer-by-wire systems. The system was then used to analyze the effect of torque feedback exerted to driver’s hands on vehicle handling.

To produce the steering torques, the 14-DOF equations of the vehicle, the engine model, the dynamic equations of the wheels, and the steer geometry were simulated. The validity of all the models was investigated by comparison with a 251-DOF car model, developed and validated in ADAMS.

After construction of simulator, it was used by a user to travel along a specified path. The torque calculated by the dynamic equations was compared with those generated by the system actuator and it was seen that the system followed the generated torques. On the other hand, though the PID controller fulfilled the control requirements of the system, it was sensitive to variations of system parameters. Hence, using adaptive control schemes may improve system response.

Various experiments were conducted on several users to investigate the influence of the torque feedback on the steering wheel on proper steering of the vehicle. By comparing the results obtained with and without torque feedback during these experiments, it was shown that the torque feedback improved the driver’s perception from the surrounding environment and enabled her/him to handle the vehicle satisfactorily.

Disclosure statement
No potential conflict of interest was reported by the authors.

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