Reliability analysis of Aeroengine Blades Based on Fourier Transform

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Abstract. As a key part of aero-engine, blade is responsible for energy conversion. This paper summarizes the significance of traditional aero-engine blade reliability analysis. The basic principle of Fourier Transform is introduced. According to the Fourier transform, the decomposition method of the actual dynamic load curve of the blade in the working process in the frequency domain is deduced, and the dynamic load is decomposed in the frequency domain. The specific information of the sinusoidal harmonics which constitute the time domain signal is obtained, and then the theoretical method of restoring it to the time domain is deduced. The Fast Fourier Transform (FFT) is applied to the time domain signal by MATLAB. The dynamic load of aero-engine blade is analyzed by Fourier Transform, and the derivation process of Fourier Transform and inverse transform is completed. Fourier Transform can simulate the actual dynamic load curve of aero-engine blade, which can be used as the theoretical basis for the next step of aero-engine blade reliability test-bed loading.

1. Introduction
This paper summarizes the important significance of traditional reliability analysis of Aeroengine Blades\textsuperscript{1}. Turbine blades are one of the core components of aeroengine, which have been in the harsh working environment of high temperature, high speed, high stress and high temperature gas impact corrosion for a long time \textsuperscript{2}. The manufacturing process of turbine blade and the stability and reliability of its performance in service have an important influence on the safety, economy and service life of the engine.

Based on the comprehensive consideration of creep and thermal cycle, the failure modes of blades mainly include fatigue, creep, abrasion oxidation, coating deterioration, surface degradation caused by overheating, corrosion, etc\textsuperscript{3}. These failure modes are concentrated in the blade tip, root, inlet side and outlet side, such as cracks around the cooling hole, cracks and defects at the blade tip, overheating and deformation of the surface of the outlet side, corrosion and oxidation of the inlet side, among which the damage of the inlet side is the most serious. The deterioration degree of turbine blades in actual service varies with the operating temperature, rotating speed, operating mode, service time and blade manufacturing, which is the result of multiple factors\textsuperscript{4}.

2. The basic principle of Fourier transform
With the establishment of the theory of function, limit, calculus and series, the French mathematician Fourier published a paper entitled "the analytic theory of heat"[5]. He proposed that the periodic function \( f(x) \) with the period of \( 2\pi \) can be expanded into the sum of infinite sine and cosine functions, that is

\[
f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx
\]  

(1)

In the equation,

\[
a_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \cos nx \, dx \quad (n=0,1,2,..)
\]  

(2)

\[
b_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \sin nx \, dx \quad (n=1,2,..)
\]  

(3)

This formula is the famous Fourier series.

2.1. Uncertain signal (random signal)
The value is uncertain under given conditions[6]:

\[
x(t) = A \cos(\sqrt{\frac{k}{m}} + \phi_0)
\]  

(4)

A: amplitude, \( \sqrt{\frac{k}{m}} \):frequency, \( \phi_0 \):phase angle

![Figure 1. random signal.](image)

Frequency domain description: a method to describe a signal by the frequency structure of the signal: the signal is regarded as the sum of many harmonics (simple harmonic signal), each harmonic is called a frequency component of the signal[7], and the harmonic of those frequencies contained in the signal, as well as the amplitude and phase angle of each harmonic are investigated.

2.2. The complex exponential form of Fourier series of periodic signal
Time domain expression of periodic signal:

\[
x(t) = x(t + \tau) = x(t + 2\tau) = \cdots = x(t + n\tau) (n = \pm 1, \pm 2, \cdots)
\]  

(5)

T: Cycle. Note the value of N: periodic signal "no beginning, no end"[8].

Another form of trigonometric function expansion:
\[ x(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(nw_0 t + \phi_n) \]

\[ A_n = \sqrt{a_n^2 + b_n^2} \]

\[ \phi_n = \arctg \frac{-b_n}{a_n} \]

\( a_0 \): Mean value of signal, DC component;

\( A_n \): Amplitude of Nth harmonic;

\( nw_0 \): Frequency of Nth harmonic;

\( \phi_n \): Phase angle of Nth harmonic;

\[ \text{2.3. The Laplace transform} \]

For the case of general signals, the Laplace transform introduced in this paper should be applied. Laplace transform was proposed by French mathematician Laplace transform in 1782\(^{[9]}\). Laplace transform is a kind of Fourier transform, and it is the earliest one proposed, and it is the earliest one proposed. In order to discuss this transformation, the complex exponential signal with variable modulus is defined as:

\[ Kec(s, t) = e^{\sigma t} \]

\(-\infty < t < +\infty\)

Plural number:

\[ s = \sigma + j\Omega \]

\(-\infty < (\sigma, \Omega) < +\infty\)

For a given \( t \), calculate the integral of the function \( Kec(s, t) \):

\[ Adc(t) = \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} Kec(s, t) ds = \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{\sigma t} e^{j\Omega t} jdu = \]

\[ je^{\sigma t} \int_{-\infty}^{+\infty} e^{j\Omega u} du \]

Make \( u = \frac{1}{j}(s - \sigma_0) \)

\[ Adb(t) = \int_{-\infty}^{+\infty} e^{j\Omega u} d\Omega = 2\pi \delta_{\sigma_0}(t) \]

From the above formula:

\[ Adc(t) = \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{\sigma t} ds = \]

\[ je^{\sigma_0 t} \cdot 2\pi \delta_{\sigma_0}(t) = 2\pi j \delta_{\sigma_0}(t) \]

From the above formula:

\[ \delta_{\sigma_0}(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{\sigma t} ds \]
The above formula shows that the unit impulse function \( \delta_a(t) \) can be obtained from the weighted \( \frac{1}{2\pi j} \) integral of the variable modulus complex index signal \( e^{st} \) on the straight line \((\sigma_0 - j\omega, \sigma_0 + j\omega)\) of the complex plane \( S^{[10]} \), that is, \( \delta_a(t) \) can be decomposed into the sum of infinite variable modulus complex index signals \( \frac{1}{2\pi j} e^{st} \). Similar results are obtained for the general continuous time signal \( x_a(t) \).

Now consider the general continuous time signal \( x_a(t) \). From the impulse invariance of linear convolution, there are:

\[
x_a(t) = x_a(t) * \delta_a(t) = \int_{-\infty}^{\infty} x_a(\tau) \delta_a(t-\tau) d\tau
\]

Substituting (7) into the above formula, we can get:

\[
x_a(t) = \int_{-\infty}^{\infty} x_a(\tau) \left( -\frac{1}{2\pi j} \int_{\sigma_0-j\omega}^{\sigma_0+j\omega} e^{s(t-\tau)} ds \right) d\tau
\]

\[
= \frac{1}{2\pi j} \int_{\sigma_0-j\omega}^{\sigma_0+j\omega} x_a(\tau) e^{-st} d\tau
\]

Therefore, for the general continuous time signal \( x_a(t) \), the Laplace transform for \( x_a(t) \) of integral (8) is called, and the Laplace Inverse transform in \( X_a(s) \) of integral (4) is called\(^{[11]}\):

\[
X_a(s) = \int_{-\infty}^{\infty} x_a(t) e^{-st} dt
\]

\[
x_a(t) = \frac{1}{2\pi j} \int_{\sigma_0-j\omega}^{\sigma_0+j\omega} X_a(s) e^{st} ds
\]

3. Complex form of function Fourier series

3.1. Euler formula

In order to facilitate algebra operation and calculus operation and study Fourier series, it is necessary to mention the famous Euler formula, which is the basis of the derivation of the complex form of Fourier series. The expression of Euler formula is as follows\(^{[12]}\):

\[
e^{j\omega t} = \cos \omega t \pm j \sin \omega t
\]

or

\[
\cos \omega t = \frac{1}{2} \left( e^{-j\omega t} + e^{j\omega t} \right)
\]

\[
\sin \omega t = \frac{j}{2} \left( e^{-j\omega t} - e^{j\omega t} \right)
\]

\[
j = \sqrt{-1}
\]

The complex exponential form of Fourier series

\[
x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jnwot} \quad (n = 0, \pm 1, \pm 2, \pm 3, \ldots)
\]

Expression of complex Fourier coefficient
\[ c_0 = a_0 = \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt \]
\[ c_n = \frac{a_n - jb_n}{2} = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jn\omega t} dt \]

Among them, the calculation formula of is the same as trigonometric function, except that \( n \) includes all integers. It's usually a complex number. Because it's an even function of \( N \), an odd function of \( N \), so \( a_n = a_{-n}, b_n = -b_{-n} \). That is to say, the real part is equal and the virtual part is opposite, \( c_n \) and \( c_{-n} \) are conjugate\(^{[13]} \). Complex index form of \( \mathcal{X} \) is \( c_n = |c_n| e^{jn\phi_n} \). Conjugation can also be expressed as \( |c_n| = |c_{-n}|, \phi_n = -\phi_{-n} \). Namely \( c_n \) and \( c_{-n} \) have Module equal, phase angle opposite. The Fourier series complex index also describes the frequency structure of the signal. For \( n > 0 \), the relation between it and trigonometric function form is,

\[ |c_n| = \frac{\sqrt{a_n^2 + (-b_n)^2}}{2} = \frac{A_n}{2} \]

(Equal to half of the module of trigonometric function\(^{[14]} \))

\[ \phi_n = -\text{arctg} \frac{-b_n}{a_n} \]

(Equal to the phase angle in the form of trigonometric function)

\[ |c_{-n}| = \frac{A_n}{2} \]

\[ \phi_n = -\text{arctg} \frac{-b_n}{a_n} = \text{arctg} \frac{b_n}{a_n} \]

\[ \delta(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{jn\omega} d\omega \]

(n is Integers)

The above formula shows that the unit impulse sequence \( \delta(n) \) can be obtained from when \(-\pi \leq \omega \leq \pi \) the time unit complex index series \( e^{jn\omega} \) by weighted \( 2\pi \) post integration\(^{[15]} \). That is, \( \delta(n) \) can be decomposed into the sum of infinite complex exponential signals \( \frac{1}{2\pi} e^{jn\omega} \). There are similar results for general sequence \( x(n) \). Now, consider the general sequence \( x(n) \). From the impulse invariance of linear convolution, there are:

\[ x(n) = x(n)^* \delta(n) = \sum_{i=0}^{\infty} x(i) \delta(n-i) \]

Substituting (4) into the above formula, we can get:

\[ x(n) = \sum_{i=0}^{\infty} x(i) \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{jn(\omega-i)} d\omega = \]

\[ \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{jn\omega} \left[ \sum_{i=0}^{\infty} x(i) e^{-jn\omega} \right] d\omega \]

If make \( X(e^{jn\omega}) = \sum_{i=0}^{\infty} x(i) e^{-jn\omega} (\omega \leq \omega < +\infty) \), it can be obtained from formula (26):
\[ x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega \] (27)

(n is Integers)

4. Dynamic load measurement data of aeroengine blades

This paper introduces the installation of dynamic load measurement system for Aeroengine Blades. Combined with three-phase force measuring instrument, charge amplifier and data acquisition system, dynoware software, sa-jz series electric vibration exciter, the force curves of blades in X, y and Z directions and the torque curves around Z direction are measured and obtained \(^{[16]}\). The forced vibration method (resonance method) is adopted. Its principle is to use special vibration exciter. The blade is forced to vibrate by the device, and the free vibration property of the blade structure is measured by resonance phenomenon. The main point of the method is to use the vibration exciter for frequency scanning. Because the exciting frequency of the exciter is continuously adjustable, it is often used to test the resonance method. In the actual test, the measuring points should be arranged in the parts with large deformation \(^{[17]}\).

4.1. Fast Fourier transform of measurement data

Based on the previous discussion, 1 second is taken as the sampling time, namely \( L = 1 \) second, \( s = 100 \) times / second, the frequency bandwidth is still \( 2B = 100Hz \), and the total number of sampling is changed to \( n = 2BL = 100 \) times. according to:

\[ t_0 = 0, t_1 = \frac{1}{2B}, t_2 = \frac{2}{2B}, ..., t_{N-1} = \frac{N - 1}{2B} \] (28)

We can assume that the sampling time point is as follows:

\[ t_0 = 0, t_1 = \frac{1}{2B} = 0.01, t_2 = \frac{2}{2B} = 0.02, ..., t_{99} = 0.99 \] (unit: Second)\(^{[18]}\), assuming that the actual dynamic load of blade is the continuous reading function \( f(T) \), the sampling results in a series of discrete values, that is, the dynamic load measured by the measurement system at the time point \( t_k \), \( f(t_k) \) is expressed by, then the computer according to \( f(t_0), f(t_1), ..., f(t_{99}) \) to simulate continuous functions \( f(t) \) \(^{[19]}\).

In this paper, the dynamic load data of X direction in one second are listed in the table 1. In order to show the data more vividly, take the serial number \( k \) as the abscissa and the load value \( f(t_k) \) as the ordinate, draw the corresponding data points and connect them, as shown in Figure 1.

From the perspective of frequency domain, the computer is also equivalent to sampling \(^{[20]}\), that is, the computer samples once every interval \( 1/L = 1Hz \), in the frequency domain of \( (0, 2b) \), so as to ensure that the total number of sampling is still \( n = 2BL = 100 \), which is consistent with the number of sampling in the time domain \(^{[21]}\). The independent variable to be sampled is a series of discrete frequency values:

\[ s_0 = 0, s_1 = \frac{1}{L}, s_2 = \frac{2}{L}, ..., s_{N-1} = \frac{N - 1}{L} = 99 \] (29)

(unit: Hz)
Using MATLAB to fft the time-domain signal, the $f(T)$ is decomposed into a series of circular motions with different frequencies in the frequency domain, corresponding to the sinusoidal harmonics with different frequencies one by one. Frequency and amplitude, frequency and phase, as shown in the fig. 3, fig. 4, fig. 5.
The above completed the Fourier transform of aero-engine blade dynamic load, a series of circular motions with different frequency, radius and phase in the frequency domain \[21\]. Obviously, the harmonic whose frequency is zero is the constant load component, and the harmonic whose frequency is not zero is the dynamic load component. According to the amplitude frequency phase frequency curve, when \( n = 1, 2 \ldots 50 \text{Hz} \), the amplitude and phase of parameter variables in the curve are respectively substituted into the formula to form a series of sinusoidal harmonics \( f_n(t) \) \[22\], such as:

\[
\begin{align*}
  f_0(t) &= a_0 / 2 = 687.2550 \\
  f_1(t) &= A_1 \sin(2\pi \cdot 1 \cdot t + \phi_1) = 3.3551 \sin(2\pi t + 0.6240) \\
  f_2(t) &= A_1 \sin(2\pi \cdot 2 \cdot t + \phi_2) = 2.3245 \sin(4\pi t + 0.7331) \\
  f_3(t) &= A_1 \sin(2\pi \cdot 3 \cdot t + \phi_3) = 1.7275 \sin(6\pi t + 0.6677) \\
  &\vdots
\end{align*}
\]

\( (30) \)
\[ f_{\text{so}}(t) = A_t \sin(2 \pi \cdot 50 \cdot t + \phi_t) \]
\[ = 3.3551 \sin(100 \pi t + 1.5708) \]

If \( f(t) \) is the signal in time domain, \( f(t) \) is calculated by the following formula:
\[ f(t) = f_0(t) + f_1(t) + f_2(t) \quad \text{(31)} \]
\[ + f_3(t) + \ldots + f_{\text{so}}(t) \quad \text{(32)} \]

The above work is done to restore the signal of aeroengine blade in time domain, so that the dynamic load is decomposed in frequency domain \([24]\), and the specific information of the sinusoidal harmonics that constitute the time domain signal is obtained. Fourier transform can simulate the actual dynamic load curve of aeroengine blade.

5. Reliability Estimation Model

The reliability of aero-engine blades is affected by the different factors of engine operating temperature, speed, operation mode, service time and blade manufacture. It is difficult to calculate the reliability directly in the estimation of aero-engine blade reliability. This is due to: (1) the complex structure of the aero blade and the uncertainty of its working environment lead to the failure to obtain the explicit expression of the probability density function of the stress and strength of the parts, which can’t be solved directly according to the formula; (2) the complex function of the parts in the actual working environment. Considering all aspects, the first second order distance method is more suitable for the reliability estimation model of aeroengine blade.

The basic idea is to assume that the random variable is normal distribution or equivalent to normal distribution, and use Taylor formula to make the state.

The function is expanded at the mean value of each random variable:
\[ Z=G(u_1, u_2, \ldots, u_n) + \sum_{i=1}^{n} (x_i - x_{ui}) \frac{\partial G}{\partial x_i} |_{x_{ui}} \]
\[ u_z = G(u_1, u_2, \ldots, u_n) \]
\[ \sigma_z = \sqrt{\sum_{i=1}^{n} (\sigma_i \frac{\partial G}{\partial x_i}) |_{x_{ui}}} \]
\[ \beta = \frac{u_z}{\sigma_z} \]
\[ R = \Phi(\beta) \quad \text{(33)} \]

In this formula, \( u_i \) is Mean value of random variable \( x_i \); \( \sigma_z \) is Standard deviation \((i=1,2,\ldots,n)\); \( \beta \) is Reliability coefficient. However, there is a great problem in this method, for random variables from the normal distribution, the limit state surface is the same but the state function expression is different, the results will be greatly different. The above problems can be solved by introducing the concept of design point, As shown in figure 6.
Figure 6. Geometric Significance of First Order Second Moment Method on Aviation blade.

\[ G(x_1, x_2) + \sum_{i=1}^{2} (x_i, x_i') \frac{\partial G}{\partial x_i} |_{x_i} = 0 \]

The functions of the tangent plane in the diagram are as follows:

The improved design method takes the design point as the only expansion point in the use of Taylor expansion, which avoids the disadvantage of large difference in calculation results. If the design point is determined is \((x_1^*, x_2^*, ..., x_n^*)\), The expansion of the state function at the design point is as follows:

\[
Z = G(x_1^*, x_2^*, ..., x_n^*) + \sum_{i=1}^{n} (x_i - x_i^*) \frac{\partial G}{\partial x_i} |_{x_i^*},
\]

\[
u_Z = (x_1^*, x_2^*, ..., x_n^*)
\]

\[
\sigma_Z = \sqrt{\sum_{i=1}^{n} (\sigma_{ij} \frac{\partial G}{\partial x_i} |_{x_i^*})^2}
\]

\[
\beta = \frac{\nu_Z}{\sigma_Z}
\]

\[
R = \Phi(\beta)
\]

A comparative analysis of the three types of first order moments is shown in the table below.

Table 2. Comparative analysis of three kinds of first-order quadratic moments.

| Method                | Characteristics                                                                 |
|-----------------------|---------------------------------------------------------------------------------|
| Mean point            | Normal distribution
|                       | Linear limit state function at mean point
|                       | Direct \(\beta\) results without iteration
|                       | \(\beta\) results of different forms of limit state functions are inconsistent |
| Checking Point Method | Normal distribution
|                       | Linear limit state function at checking point
|                       | Checking points are obtained by iterative calculation and \(\beta\) result is obtained |
|                       | \(\beta\) results of different forms of limit state functions are consistent |
| JC Law                | The non-normal distribution of random variables, at the normalization "at the checking point"
|                       | After "equivalent normalization ", the \(\beta\) calculation principle is consistent with the checking calculation |
6. Conclusion and Significance
As a key part of aeroengine, blade is responsible for energy conversion. Therefore, it is very important to analyze the reliability of aeroengine blades. According to the Fourier transform, the decomposition method of the actual dynamic load curve of blades in the working process is deduced in the frequency domain. The signal cannot see its obvious characteristics in the time domain. After the dynamic load is decomposed in the frequency domain, the change trend of amplitude and frequency, phase and frequency can be seen clearly. The specific information of the sinusoidal harmonics which constitute the time domain signal is obtained, and then the theoretical method of restoring it to the time domain is deduced. The fast Fourier transform (FFT) of time domain signal is used in MATLAB to analyze the dynamic load of aeroengine blade, and the derivation process of Fourier transform and inverse transform is clarified. Fourier transform can simulate the actual dynamic load curve of aeroengine blade, which can be used as the theoretical basis for the next step of aeroengine blade reliability test-bed loading.

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References
[1] J. Yang and Y. Hu 2017 Research on Technology of Reproducing Instantaneous Dynamic Load and Reliability Loading Test for Motorized Spindle Based on Fourier Transform Jilin Province Science and technology development plan 11(1) 46–76
[2] Yaohui Lu and C. N. Wang 2018 Analysis of the dynamic response and fatigue reliability of a full-scale car body of a high-speed train Proceedings of the Institution of Mechanical Engineers 232(7) 2006-2013
[3] Tatsuya sakurahara 2020 Global importance measure methodology for integrated probabilistic risk assessment journal of risk and reliability 234(2) 377-86
[4] Miao He and David He 2019 Wind turbine planetary gearbox feature extraction and fault diagnosis using a deep-learning-based approach Journal of Risk and Reliability 233(3) 303-10
[5] Themistoklis Koutsellis. 2019 Parameter estimation of finite failure population model with Weibull distribution Risk and Uncertainty in Engineering Systems 5(2) 2-52
[6] Dooyoul Lee 2020 Analysis of the reliability of a starter-generator using a dynamic Bayesian network Reliability Engineering & System Safety 195(5) 27-31
[7] Kuen-Bae Lee 2019 Effects of Inlet Disturbances on Fan Stability Journal of Engineering for Gas Turbines and Power 141(51014) 1-3
[8] Torsten Heinze 2019 Rotational Speed-Dependent Contact Formulation for Nonlinear Blade Dynamics Prediction Journal of Engineering for Gas Turbines and Power 141(42503) 7-30
[9] RenHong Peng 2019 MATLAB image processing based on Fourier transform Technology Information 17(16) 11-2
[10] Chen Jin 2018 Research on fast Fourier transform Heilongjiang science 9(24) 62-3
[11] Guo Xiaoxi and Zhang Renqun 2019 Research on random vibration data analysis method based on MATLAB Environmental technology 37(03) 12-5
[12] Hong Liang 2015 Effect of dynamic load on bolt strength after sudden loss of aeroengine blade Gas turbine test and research 28(01) 21-4
[13] Sreeeraj K 2020 Comprehensive analysis of effects of dynamic load frequency and hydrogenation to instigate White Etching Areas (WEAs) formation under severe sliding condition of bearing steel Tribology International 144(42503) 7-30
[14] Sun Xuewei. 2019 Dynamic load test and failure analysis of coupler joist of railway freight cars Railway vehicles 57(10) 6-8
[15] Jiexiaobo 2019 Nonlinear vibration analysis of Aeroengine Blades *Journal of dynamics and control* 17(03) 205-12
[16] Zhang Xiandong 2019 Optimization sampling method of aeroengine blade three coordinate measurement *Journal of aerodynamics* 34(01) 168-76
[17] Chen Xi 2018 Fretting fatigue research of tenon and tenon joint structure of aeroengine blade *Science and technology wind* 22 144-5
[18] Sun Fangcheng 2018 Aeroengine Blades and their development trend *Shandong industrial technology* 75(03) 51
[19] Lukas Liehr 2020 On the mathematical validity of the Higuchi method *Physica D: Nonlinear Phenomena* 402(02) 62
[20] Jamal EL Hachem 2020 Modeling, analyzing and predicting security cascading attacks in smart buildings systems-of-systems *The Journal of Systems & Software* 162(02) 34
[21] Yang David Y 2020 Life-cycle management of deteriorating bridge networks with network-level risk bounds and system reliability analysis *Structural Safety* 83(15) 27
[22] Wu Zhanyu 2019 Safety and reliability analysis of power lithium ion batteries *Battery industry* 23(04) 190-7
[23] Zhu Huichao 2019 Reliability analysis of ring stiffened cylindrical shell based on Monte Carlo method *China water transport (second half)* 19(09) 13-5
[24] Wang Xingang 2019 Reliability analysis of main shaft of power tool rest based on impact load, *Mechanical design and manufacturing* 35(12) 181-4