Non-Local Triggering in Rock Fracture

Qiquan Xiong1 and Jesse C. Hampton1

1Geomechanics and Damage Group (GeoD), Department of Civil and Environmental Engineering, University of Wisconsin-Madison, Madison, WI, USA

Abstract Combining multiphysics observations in the laboratory, we present novel phenomena for the analogue between rock fracture and seismicity. We show, for the first time in a laboratory setting, how a large-scale flaw in rock can facilitate “non-local triggering”—remotely triggered damage in rock mass. Results prove analogues of rock fracture evolution to natural seismicity including several lab-analogues on seismicity model predictions beyond power law. We observe under specimen-scale criticality a relatively small AE perturbation occurring at a seemingly random location can trigger cascading events whose spatial span can traverse the entire rock fracture system. The inclination angle of the non-local triggering pairs shows dependence on the large-scale flaw, and the subsequent generations of triggered AEs by these non-local triggers decline significantly slower than that of the general triggers. The rock fracture system evolves to a state where the resistance to non-local triggering continuously reduces, bridging the gap between an idealized lab setting and a natural fault. The non-locally triggered events can also be reactivated as the rock fracture system evolves towards increased complexity. These lab-observations as well as the understanding on the analogue relationship between rock fracture and seismicity may shed new light on the insight of the periodical, dynamic, and unpredictable nature of the multiscale rock fracture process.

1. Introduction

Evidence of earthquakes triggered by a large-magnitude predecessor in geologically remote regions (Gomberg et al., 2001; Hill et al., 1993; O’Malley et al., 2018; Parsons et al., 2008; Shan et al., 2013) spurs the investigation on the mechanisms between non-locally triggered damage and a large structural discontinuity (i.e., fault) in rock mass. This phenomenon could intuitively be attributed to the stress field alteration along an underground rock fracture system, that is, the coseismic stress transfer and the changes on the Coulomb failure stress along the structural fault system (McCloskey et al., 2005; Parsons et al., 2008; Stein, 1993; Stein et al., 1997). However, the exact details of the triggering dynamics, such as when and how this non-local triggering takes place, are still elusive to date.

Investigations at the laboratory scale are often valuable to understand complex physical processes at geologic space and time scales. To achieve this, an experimental investigation must first confirm the existence of the same (analogue) phenomena observed at field scales, while also providing additional information that can be interrogated. Recent investigations of acoustic emission (AE) in rock fracture validate the influence of a large-scale rock flaw on inducing triggered events in front of the flaw tip (Davidsen et al., 2017). The AEs originate from elastic waves emitted by a localized and irreversible release of strain energy (Mogi, 2006) and are the laboratory analogue of seismicity from tectonic-scale rock fracture processes. Combining the latest AE sensor in situ calibration techniques (Davi et al., 2013; Xiong & Wong, 2017) with multi-physics observations (e.g., optical, mechanical, and AE), real-time and high-accuracy characterization of rock fractures initiating from a large-scale flaw has become possible (Wong & Xiong, 2018). These experimental advances in data quality and analysis techniques provide the unique opportunity to elucidate the non-local triggering dynamics in rock fracture.

In this manuscript, we combine AE data analysis advancements in the field of triggering phenomena (Davidsen et al., 2017) into the investigation of the real-time and high-accuracy data collected from rock fracture experiments on specimens containing a single large-scale flaw (Wong & Xiong, 2018). The inclined single open flaw represents an idealized friction-free section of a natural fault (i.e., the creeping aseismic zone such as the San Juan Bautista and Parkfield creeping section of the San Andreas fault), sandwiched by the loading-initiated rock fractures (Figure 1b). We validate the existence of triggering phenomenon in two tests and elucidate their analogues to natural earthquakes (i.e., the co-existence of swarm-like as well as burst-like
AE event families, Zaliapin & Ben-Zion, 2013b, and the constant existence of Poisson process events occurring in moderate rate, Zaliapin & Ben-Zion, 2013b). Furthermore, we validate, for the first time from the laboratory analogue tests, the existence of non-local triggering in rock fracture and readily uncover the statistical features of non-local triggering and the spatial migration path as well as other features of the events in which they have triggered. The result implies a single large-scale flaw can initiate a series of non-local rock fracture processes. The roots of large AE-event trees (the swarm-like trees) can in most instances occur under specimen-scale mechanical criticality; their magnitudes appear as small perturbations (i.e., lower than the mean and median magnitude of all AEs), and they spatially occur at seemingly random locations. The triggered events, however, can spatially traverse the entire rock fracture system in a cascading manner, where the physical specimen boundary provides the only limit.

Observations of the seemingly random nature in which AEs are generated during mechanical criticality are a universally observed phenomenon (Mogi, 2006), while its physical meaning and correlation with rock fracture process(es) can be understood more distinctly after the confirmation of non-local triggering dynamics here. In addition to the laboratory analogues to natural seismicity and/or to modeling predictions, we statistically prove a large-scale flaw can facilitate a series of non-local rock fracture processes. The roots of large AE-event trees (the swarm-like trees) can in most instances occur under specimen-scale mechanical criticality; their magnitudes appear as small perturbations (i.e., lower than the mean and median magnitude of all AEs), and they spatially occur at seemingly random locations. The triggered events, however, can spatially traverse the entire rock fracture system in a cascading manner, where the physical specimen boundary provides the only limit.

Non-locally triggered events in subsequent generations tend to linger near common locations for several generations, and the offspring population of these subsequent generations of non-locally triggered AE events decreases at a significantly slower rate than that of general triggers. As the rock fracture evolution continues, the changes on the features of non-local triggering AEs imply the rock fractures evolve towards a complex phenomenon that may combine cascading triggering progression, non-local triggering dynamics, and rock

Figure 1. (a) AE source locations of rock fractures, featured with their moment tensor classifications after Wong and Xiong (2018) represented as shape and color alterations. A coordinate system is illustrated in the lower left insert; (b) optical evidence of rock fractures; (c) illustration of alternative rock fracture processes—-independent development (indicated by red dashed lines), or interactive development (indicated by magenta-curved arrows). In subfigure (c), events of smallest and largest magnitudes within this illustration are highlighted in red, with their magnitudes labeled—approximately −0.86 and +0.16, respectively. All events are sized linearly with their magnitudes.
fracture evolution. On the one hand, the greatest distance between non-local triggering pairs continuous to increase as the rock fracture extends, indicating the newly created fractures can also trigger non-local events similarly to the open flaw; on the other hand, the newly created fractures evolve towards a friction-reduction state—the magnitude differences between non-local triggers and their triggered AEs systematically decrease. The observation of these two rock fracture evolution tendencies bridges the gap between the idealized frictionless experimental setting and its natural representation, for example, a large structural fault. One natural consequence of this continuation of evolution (i.e., increase of complexity and lowered resistance to non-local triggering) is that they ensured the periodical occurrences of massive energy releases in rock fractures. The work in this manuscript thus sheds new light on the understanding of triggered earthquakes in remote, or far field, regions. Also, by establishing the correspondence between triggering dynamics and a large-scale structure in rock, the work provides a feasible way to systematically and multi-dimensionally capture information from AE signals beyond several empirical power laws. It is hoped that such analysis of triggering dynamics could boost the relevant research in understanding rock fracture processes across lab-and tectonic-scales.

2. Experiments, Pre-Analysis of AE Observations, and Methodologies

2.1. Previous Experiments

We apply the data analysis method of triggering phenomenon (Davidsen et al., 2017) into the relevant previous experiments (Wong & Xiong, 2018). Those tests were conducted on prismatic Carrara marble specimens containing a single central flaw, and they were referred as $\beta = 70^\circ$ test and $\beta = 60^\circ$ test by Wong and Xiong (2018). The flaw of these inclination angles induces macro-scale shear rock fractures symmetrically (or quasi-symmetrically) from the tips of the central inclined flaw (i.e., Figure 1b), and their initial inclination angles against the maximum loading direction (i.e., 20° to 30°) are within the modeling predicted fault-slip nucleation angle against maximum stress (Reches & Lockner, 1994). The specimens were uniaxially loaded (Wong & Xiong, 2018), which may provide a weaker analogue to the in situ stress condition than the rock fracture AE tests under triaxial loading (i.e., Goebel, 2013; Lei, 2003; Lockner et al., 1991; Stanchits et al., 2009; Zang et al., 1998, etc.). However, we believe the uniaxial loading condition will not fundamentally deteriorate the discovery of the data analysis as, on the one hand, the newly created rock fractures are generally under shear stressing condition; on the other hand, according to the data analysis results, the newly created rock fractures have displayed certain evolution tendencies which are suggesting that these rock fractures initiated under shear stress can also play the role of facilitating the non-local triggering.

Experimental data consisted of continuous measurements of rock fracture evolution at multiple scales via mechanical, AE, and optical techniques (see Wong & Xiong, 2018, for more technical details). AE data quality is increased substantially via the use of a 16-channel acquisition system to provide highly accurate source location information along with the implementation of the latest in-situ-calibration techniques (Davi et al., 2013; Xiong & Wong, 2017) to reduce the uncertainty of the source amplitude from 100% of empirical estimation (McLaskey et al., 2014) to less than 17%. Thus, the migration paths of the triggered AE events based on event-to-event (E2E) AE triggering chains can be readily and accurately obtained.

As rock fractures can be initiated from the flaw during the loading process, traces of AE clusters (Figure 1a) and optical fracture process zones (FPZs, Figure 1b) can be observed. These phenomena provide evidence of continuous and progressive rock fracture evolution (Wong & Xiong, 2018). A natural question on the continuation of the rock fracture process is, will these symmetric fractures extend independently? Alternatively, will they interact? Which raises questions regarding the spatial evolution of natural earthquakes.

2.2. Event-to-Event (E2E) Triggering Model

We validate (or reject) the hypothesis of rock fracture interaction through the re-analysis of the AE sources of these two tests. The tests as well as the rock fracture processes were viewed as planar model tests/rock fracture processes because the third dimension is comparatively small (Wong & Xiong, 2018). As such, we have extended the allowance of qualified AE events in the third dimension (non-planar dimension). This extension of the allowance, however as we have validated, will not change the conclusion of the present paper.
The relative AE moment for each event, which in this manuscript, follows the expression below:

\[ M_{\text{relative}} = \frac{1}{n} \sum_{j=1}^{n} \sqrt{(R_i C_s A_l(x))^2}, \]  

(1)

where \( A_i \), \( C_s \), and \( R_i \) are first-peak amplitude of the waveform signal at sensor \( i \), calibration coefficient of sensor \( i \), and the source-to-sensor distance of the corresponding signal, respectively. The estimation of relative source moment is rooted on the first-peak amplitudes instead of the maximum peaks of the waveform signals. The latter approach (maximum peaks of the waveform) was adopted by many previous studies, for instance, Davidsen et al. (2017), Goebel (2013), Kwiatek et al. (2014), and Zang et al. (1998). Here, first-peak amplitude is used instead of overall signal peak to estimate AE moment because waveform sections long after onset are far more likely to be influenced by a number of factors other than the source strength such as the stacking of multiple phases of elastic waves and the resonance of the sensor. The prismatic sample shape also exacerbates the issue of phase stacking. Moreover, the amplitude of the maximum signal peaks is significantly larger than the amplitude of the first peak, where accurate measurements become problematic due to the reliable voltage ranges (or the so-called dynamic range of the data acquisition system) are a limiting factor. The in situ sensor calibration results of Wong and Xiong (2018) also suggest, the first-peak is a more stable source for AE moment estimation than the overall signal peak, because the uncertainty of the first-peak amplitude can be reduced to as small as less than 17% (Xiong & Wong, 2017).

We convert the relative AE moment into a format analogue to the seismic moment magnitude following the conventional expression:

\[ M' = \frac{2}{3} \log_{10} M_{\text{relative}} - C \]  

(2)

This is for the convention of the following data analysis in seeking the event-to-event (E2E) triggering relationship. For convenience, in the following, this analogue format of “moment magnitude” will be referred as the relative magnitude of AE moment. Because the magnitudes of the AE moment are relative values, parameter \( C \) will not affect the result of the data analysis in this paper. For convenience consideration, we set it to zero. Typical power-law frequency-magnitude distribution can be obtained after the conversion from relative AE moment to relative magnitude of AE moment (see Figure 2b).

To find the trigger, if any, of a given AE event \( j \), we calculate an event-pair metric of spatial and temporal correlation:

\[ n_{kj} = l_{kj}^* \times \tau_{kj}^* \equiv \left( \frac{l_{kj}^*}{t_{kj}^*} \times 10^{-b m_k/2} \right) \times \left( t_{kj}^* \times 10^{-b m_k/2} \right). \]  

(3)

for each of its preceding \( k \) events, where \( l_{kj}^* \) and \( \tau_{kj}^* \) are the normalized spatial and time distances between events \( k \) and \( j \); \( m_k \) is event \( k \)'s relative magnitude of AE moment; \( D_f \), \( b \) are the correlation fractal dimension of AEs, and \( b \)-value of the Gutenberg-Richter (G-R) law, respectively, which are directly estimated from the statistical features of AEs per test. The \( l \)th event that satisfies \( n_{kj}^* = \min_{k} n_{kj} \) is the most probable trigger of \( j \)th event. A very low value of \( n_{kj}^* \) indicates the existence of real E2E triggering. A threshold of \( n_{\text{thresh}} \) can be determined by observing the distribution of shuffled AE data in the normalized space-time plane (Davidsen et al., 2017). Connections between the event-pairs having \( n_{kj}^* > n_{\text{thresh}} \) will be released from the triggering-triggered set of events and instead be categorized as random, non-triggered, Poisson process events (Zaliapin et al., 2008; Zaliapin & Ben-Zion, 2013b). To note, though it may be physically possible for combinations of AE events larger than pairs to remotely trigger other AEs, the work here investigates the statistically more likely scenario of AE pair relationships (e.g., very low probability that multiple AE events occur at exactly the right moment in space and time to induce a non-local triggered damage in a specific location under local criticality).

### 2.3. Model Parameters

As the rock fracture is undergoing symmetric/quasi-symmetric extension (Figure 1), the AE events are divided into two clusters, and the clusters’ centers are obtained using the \( k \)-means algorithm. We obtain
the fractal dimension, \( D_f \), as follows. Using the 60° test as an example (Figure 2a), for both clusters, their distributions of AE density from 12 mm to 42 mm (± 3 mm) to the clusters’ centers display typical power law distribution which ideally fits the definition of the AE fractal dimension, \( \mu(R < r_i) = A_\mu r_i^{-D_f} \), where \( \mu \) is the density and \( r_i \) is the distance. The values of the fractal dimension (\( D_f \)) for both clusters can be obtained by using the AE density data from 12 mm to 42 mm ± 3 mm in a linear regression of \( \log \mu = \text{constant} + D_f \cdot \log r_i \). Results are listed in the upper-right insert table of Figure 2a.

We select a conservative value of 1.6 for \( D_f \). Comparing with the result in the insert table of Figure 2a, it is conservative—the higher the value, the more unfavorable it is for remote triggering. This value of 1.6 has also been used for previous analysis on real seismological data (Zaliapin & Ben-Zion, 2013a). The \( D_f \) value of this example was estimated through the epicenter of the sources, which is primarily in respect to the fact that the tests are viewed as planar model tests. However, as we have validated, using the hypocenter of the sources for obtaining the \( D_f \) value will not change the result of validating (or rejecting) the hypothesis of rock fracture interaction.
The b-value of the AE data set can be estimated through the statistical features of the AE magnitude-frequency distribution. For instance, again using the 60° test as an example, the AE data set produces $b = 1.3155$ according to the maximum likelihood estimation, $b = \log_{10} c / (\bar{M} - M_{\text{min}})$, where $\bar{M}$ and $M_{\text{min}}$ are the average and minimum relative magnitudes of AE moments, respectively. Noticing the number of small magnitude AEs can deviate from the power-law frequency-magnitude distribution (see Figure 2b), a threshold magnitude of AEs exists. A threshold magnitude of $M_{\text{threshold}} = -0.6$ is sufficient for defining the magnitude of completeness—the magnitude above which the AE magnitude-frequency distribution obeys the power law (the G-R law). Of course, AE sources of different magnitude thresholds (i.e., AEs of $M > -0.6$, or AEs of $M > -0.5$) can have different b-values. However, as we have validated through repeating the data analysis under series of different model parameter combinations, any reasonable changes in these model parameters (i.e., $Df$ and b-value) will not change the result of validating (or rejecting) the hypothesis of rock fracture interaction.

2.4. Setting the Triggering Threshold

Following Davidsen et al. (2017), we determine the threshold $n_{\text{thresh}}$ by shuffling the time, spatial location, and magnitude information of the AE events independently. The $n_{ij}^*$ values of the shuffled data set are plotted in the normalized space-time coordinate (Figure 2c). The shuffled data set is random in nature and retains certain connection to the original one. As a result, E2E triggering should have no statistical significance in the shuffled data set. The threshold for dividing the existence and non-existence of triggering phenomenon ($n_{\text{thresh}}$) can be determined by observing the distribution of the $n_{ij}^*$ value for the shuffled data set in the normalized space-time coordinate, that is, 60° test as example (Figure 2c). A threshold of $\log(n_{\text{thresh}}) = 8$ is sufficient (indicated as the red line in Figure 2c, which is more than two orders of magnitude lower than the yellow center of the dot cluster) for filtering AEs having no significance in E2E triggering. To avoid bias, multiple data shuffling exercises, that is, 100 times, have been conducted.

3. Results

3.1. Analogue to Natural Seismicity

3.1.1. Existing Triggering

We initially recover the evidence for the existence of triggering under the presence of the large-scale flaw. We analyze the inter-event time (defines as $\Delta t_i$, waiting time between the $i$th and $(i-1)$th AE events (van der Elst & Brodsky, 2010) for the AEs from the start of constant displacement loading stage to the peak stress. This selection of AEs follows the data analysis convention of Davidsen et al. (2017). The probability density function (PDF) of the inter-event time ratio (ITR) $p(R)$ (where $R_i \equiv \Delta t_i / (1 + \Delta t_i)$) can detect (or reject) the existence of triggering and/or anti-clustering phenomena under a temporal sense. Peaks of $p(R)$ close to 0 and 1 suggest the violation of Poisson process, where $p(R)$ should be uniformly distributed between [0, 1] if triggering is non-existent (van der Elst & Brodsky, 2010). Both triggering and anti-clustering phenomena for the 60° test (Figure 3a) and triggering phenomenon for the 70° test (Figure 3b) can be observed. For the 60° test, both triggering and anti-clustering phenomena become pronounced for magnitude-conditioned AEs, which is consistent with result in Davidsen et al. (2017). The PDF of ITR for the 70° test appears more Poisson-like initially and the triggering phenomenon can only be identified after magnitude conditioning (i.e., the purple square for $M > -0.5$ in Figure 3b). However, by seeking the violation of the null hypothesis that event distribution obeys the G-R law (Baiesi & Paczuski, 2004; Davidsen et al., 2017; Zaliapin et al., 2008), that is, E2E of very low $n_{ij}^*$, E2E triggering can be identified in the 70° test (Figure 3c). In Figure 3e, the very low $n_{ij}^*$ event pairs are dominant. This proves the triggering phenomenon in the 70° test exists under the spatiotemporal sense.

The triggering phenomenon of the 60° test under the spatiotemporal sense appears weak for the period from the start of the constant displacement loading stage to the peak stress (Figure 3d). However, as the rock fracture process continues (i.e., taking the AEs from the start of constant displacement loading stage to first AE peak into analysis), the triggering phenomenon of this test will progressively become more pronounced under both temporal (Figure 3c) and spatiotemporal (Figure 3f) senses. This transition phenomenon signifies the significant role of large-scale rock fractures in facilitating triggering phenomenon in the post-stress-peak rock fracture evolution. Indeed, in both tests, the triggering phenomenon will become much
more pronounced as the rock fracture process continues in the post-stress-peak loading, which we will further address in the next subsection.

### 3.1.2. Poisson Process and Swarm-Like Feature

Extending the data analysis of seeking E2E triggering to all AE sources of both tests, we notice the significant growth in the number of the triggered AEs while the number of Poisson process AEs (above $n_{\text{thresh}}$) grows moderately (see Figure 4a). Moreover, we identify, possibly for the first time in laboratory rock fracture tests, two analogue phenomena of natural seismology. The AEs of the entire rock fracture process of these two tests (1) are a combination of continuous moderate Poisson process together with the progressively increasing triggering process (Figure 4c); and (2) topologically, they are of a mixed burst-like and swarm-like feature (Figure 4b).

We build the topology trees of AEs through the connections between triggering-triggered AE pairs (Zaliapin & Ben-Zion, 2013b), where within each tree, an event that has not been triggered is defined as a root, and events having no triggered offspring are defined as leaves. These trees can be characterized by their event number and averaged root-to-leaf depth—averaged number of minimal connections from root to leaves (Zaliapin & Ben-Zion, 2013b). The mixed burst-like and swarm-like AE topology features can immediately be observed for the 60° test (Figure 4b) and the 70° test (Figure 4b insert). The mixture of burst-like and...
swarm-like behavior can be observed in real natural seismicity (Zaliapin & Ben-Zion, 2013b). The burst-like AE topology features are commonly shared by most natural seismicity, where one main shock triggers many primary aftershocks and these only lead to a small number of secondary aftershocks. This leads to a dominant family number of small to medium averaged root-to-leaf depth like the observation (as the analogue) in Figure 4b and its insert. For the swarm-like aspect, dominant and large-size trees with overwhelming populations can be identified in both tests (see Figure 4b, identified with red circle for the 70° test and red box for the 60° test).

By tracing the E2E topological connections (n∗ < nthresh) we are able to define the phenomenon of non-local triggering and as such can better investigate and understand the influence of the pre-flaw and rock fractures on the triggering process. We define the non-local triggering AEs as the triggering-triggered event pairs.

Figure 4. (a) E2E density contours for the 60° and the 70° (the insert) tests for all AEs after 1,000 s testing time. (b) Event population of AE trees versus averaged root-to-leaf depth of AE trees for 60° (main figure) and 70° (insert) tests. Dominant tree or big-size trees having overwhelming populations have been identified with red indications and red legends. (c) Histograms of non-local triggering AEs (orange), all AEs (blue), and rate of Poisson process events (dashed line marked by black squares), for 60° (main figure) and 70° (insert) tests.
crossing the flaw. More specifically, a pair of non-local triggering AEs should be that when the Y axial (parallel to the loading direction, see Figure 1a) coordinate of one event from this pair is larger than the Y coordinate of upper flaw tip, and the Y coordinate of the other event from this pair is lower than the Y coordinate of lower flaw tip. As such, a pair of non-local triggering AEs must consist of AEs from two symmetrically extending rock fractures, and those AEs are topologically connected. The feature of these AE pairs brings us more specific information regarding the cross-scale rock fracture process. Histograms of non-local triggering AEs, all AEs, and the rate of Poisson process events are presented in Figure 4c and its insert for the 60° and the 70° tests, respectively. The non-local triggering histogram validates that non-local triggering phenomenon is not a trivial serendipity phenomenon as each test has more than 100 non-local triggering AE event pairs. A non-negligible amount of non-local triggering can occur under the presence of a large-scale flaw. Poisson events continuously occur with a small-to-moderate rate throughout all rise-and-drops of the AE activities in both tests, proving the co-existence of triggering and Poisson processes throughout the rock fracture process in the current tests. One thing should be noted, the non-local triggering phenomenon exists throughout all combinations of reasonable model parameter selections, and the rejection for the extension of source allowance in the third dimension will also not change this conclusion.

The two features, that is, co-existence of Poisson process as well as triggering process and mixed burst-like and swarm-like feature, are the laboratory analogues to natural seismicity (Gu et al., 2013; Moradpour et al., 2014; Zaliapin et al., 2008; Zaliapin & Ben-Zion, 2013b), but have hardly been reported in laboratory rock fracture AE analyses. Peaks of non-local triggering AEs can precede or mingle with peaks of all AEs (Figure 4c), suggesting a different explanation for the causation of non-local triggering compared to that of the overall AE activity.

### 3.2. Features of Roots for Large Trees, Spatial Migration of Triggered Events

#### 3.2.1. Feature of Roots for Large Trees—Small Perturbation, Random Locations, and Approximating Specimen-Scale Criticality

We have identified the dominant tree for the 60° test and several (three) large-size trees for the 70° test (in Figure 4b and its insert). The roots’ features are summarized in Table 1. We state that the roots of those large trees can be small perturbations (see the fifth column and the “Notice” in Table 1) at seemingly random locations, and their temporal occurrences are mostly after, or near, when specimen-scale criticality has been achieved. To further illustrate/prove this statement, we spatially plot these roots and temporally identify them along the stress curves per test in Figures 5a and 5b, respectively.

In Figure 5a, four roots (1 + 3 roots) from the dominant tree of the 60° test and the three big-size trees of the 70° test have been plotted spatially. Their sizes are linearly normalized within the relative minimum-to-maximum magnitude scales per test. Their information of time and relative magnitude has been labeled in Figure 5a. The preexisting flaws per test have been drawn stacked together by solid lines of different colors in the center of the specimens, and the rock fractures have been roughly drawn by dashed lines of different colors, accordingly. The color of red is for the 60° test and the color of blue for the 70° test. One can see that the spatial locations of those roots can be close to the rock fracture as well as far from it. Two roots appear close to the rock fracture (root 192 and root 731). But they are still a significant distance away from the main rock fracture. The other two roots are quite far away from the flaw or rock fracture (root 458 and root 2572).

| Table 1 |
| --- |
| **Characteristics of the Roots for the Big-Size Trees or the Dominant Tree** |
| Root number | Family size | Occurrence time | Relative magnitude of AE moment (relative per test) | Coordinates (X, Y) |
| --- | --- | --- | --- | --- |
| 60° test | 458 | 1,673 | 3,332.7 s | −0.686 | (76.39, 115.15) |
| 70° test | 192 | 458 | 3,429.8 s | −0.577 | (40.46, 98.07) |
| 70° test | 2572 | 430 | 3,649.5 s | −0.550 | (18.42, 84.86) |
| 70° test | 3,513.2 s | 731 | 458 | 3,513.2 s | −0.551 | (34.00, 24.41) |
| Note. The averaged relative magnitude of AE moment and the median value of them for the 60° test are −0.568 and −0.547, respectively; the averaged relative magnitude of AE moment and the median value of them for the 70° test are −0.533 and −0.546, respectively. |
Overall, it appears no pattern can be found in their spatial occurrence. From the magnitude aspect, one can easily find that the magnitudes of all the roots are lower than the mean and median magnitudes of all AEs per test in Table 1.

The time of root initiation is indicated along the stress curves for each test in Figure 5b. In Figure 5b, the legends and labels for different tests are colored according to the colors of stress curves for the corresponding tests. Specimen-scale criticality should be achieved once the overall peak stress on the loading curves has been reached. Under this consideration, we have the root of the dominant tree for the 60° test and two roots out of the three big-size trees for the 70° test occurring after specimen-scale criticality has been achieved (Figure 5b). In the 60° test, the stress peak is located within 3,306 s to 3,323 s (indicated by red dashed arrows). The axial stresses at these two points are quite close, and from 3,306 to 3,323 s, the stress has only slightly perturbed. Considering the uncertainty of the measurement (which was 1.25 kN accuracy, ~0.526 MPa in 60° test, see Wong & Xiong, 2018), the actual axial stress perturbation around the period of 3,306 to 3,323 s is within this measurement uncertainty range. As such, the specimen-scale criticality should have been achieved in this period even if no clear post-peak stress drop has occurred. Axial stress before and after that period reduces monotonically as the axial stress along the time axis moves away from the perturbation period. In the 70° test, and using the same reasoning, root 731 occurs within the peak stress perturbation period, which suggests its occurrence should occur when specimen-scale criticality has been achieved. For the two remaining roots, one of them is long after the peak stress (see top-left legend for root 2572 in Figure 5b), and the occurrence of the other is also quite close to the stress peak (Figure 5b).

It is worth noting that a stress perturbation has occurred prior to the moment of overall stress peak in the 70° test (indicated by the blue arrow in Figure 5b). Its magnitude of perturbation, especially its downward perturbation, exceeds the uncertainty of measurement. Before this perturbation, the stress rises smoothly; and
after it the stress curve displays a stronger perturbation until the overall stress peak. Although this global perturbation may possibly be induced by local rock heterogeneity, it is also possible that the rock fracture at that perturbation moment has already entered a full-scale criticality state, while due to the laboratory displacement rate and the unpredictability of the local stress release under criticality, the global stress of the specimen still rises slightly.

In addition to the small perturbation feature, the seemingly random spatial feature, as well as the temporal feature of after, or very close to, specimen-scale criticality for the roots of large trees, we will further show that the triggered events of these roots can spatially migrate throughout the entire rock fracture system.

3.2.2. Triggered Event Migration Patterns

Along the root-to-leaf paths of AE trees, patterns of triggered event migration can be discovered. Illustrations of different patterns are, as examples, extracted from the dominant tree of the 60° test and are presented in Figures 6a–6d. Triggered events can migrate within one fracture (Figure 6a), jumping across the large-scale flaw (Figure 6b), jumping back-and-forth between upper and lower fractures (Figure 6c), and progressively migrate across the flaw (Figure 6d). These non-local triggering patterns can be found in both tests. These observed phenomena further delineate the spatial migration features of non-local triggering together with other types of triggering in rock fractures. Bearing in mind the roots of the dominant and big-size trees can only occur after (or very close to) the stress peak, the very existence of these triggering patterns provide evidence of the multifaceted communication between spatially separated fractures occurring when the rock fracture system is evolving under specimen-scale criticality.

Sudden growth on the spatial span of the dominant tree synchronizes well with peaks of overall AEs (Figure 7a). This further signifies the correlation between the spatial triggered event migration and the overall rock fracture process when the rock fracture further evolves under specimen-scale criticality. One interesting phenomenon that should be emphasized, the spatial limit of max tree-span matches well with the optically observed spatial span of the FPZ at all stages (for example, see Figures 6d and 7b). In Figure 6d the triggered event migration path traverses the entire specimen, and the rock FPZ has reached the specimen boundary.
while in Figure 7b the spatial span of one largest tree at the moment just following the peak stress also matches the largest spatial span of the FPZ. This combination of optical and triggered AE event migration pattern observations further confirms the spatially transverse nature of the triggered events within the rock fracture system and, as a consequence, proves the connection between spatially remote events which can be essentially connected by the rock fracture system.

3.3. Large-Scale Flaw Influence in Non-Local Triggering and Gap Between Laboratory Setting and Field-Scale Fault

3.3.1. Large-Scale Flaw Control of Non-Local Triggering

We find the non-local triggering AEs are of larger magnitude than their triggered ones in almost all magnitude-frequency spectrum for both tests (Figure 8a). Inclination angles between these non-local triggering AE event pairs also concentrate along the inclination angle of the large-scale flaw (see Figure 8b, the right-hand side primary peaks in the two histograms). These findings highlight the influence of rock structure in facilitating non-locally triggered events. Regardless of whether the non-local triggering AEs are statistically of larger magnitude in the magnitude-frequency spectrum, there appears to be no observed lower limit for the non-local triggering phenomenon to occur. It is also not necessary that for every non-local triggering its magnitude is larger than that of its non-local triggered event. The magnitude-frequency distributions for both non-local triggering AEs and their triggered ones obey power-law, and the only limit appears to be imposed by the limit of sensitivity for the AE system. Indeed, in the next section we will show that the progressive rock fracture evolution is a significant influential factor for determining the statistical features of non-local triggering. In both tests, there is one secondary non-local triggering inclination angle concentration (the angles between non-local triggering AE event pairs, for example, in Figure 8b, concentrate at a negative inclination angle in the histograms). Indeed, their occurrences are indicative of the rock fracturing process tending towards complexity. We will further explain the physical meaning of this observation in the later section.

3.3.2. Reduction of Non-Local Triggering Resistance as the Rock Fracture Evolution Continues

Multiple temporal concentrations of non-local triggering occur in the post-stress-peak loading (see the two inserts of Figure 9a, can also see Figure 4c), suggesting a rough categorization for dividing the non-local triggering and triggered AEs into groups. The statistical features of these non-local triggering AE groups can show the evolution tendency of the rock fracture. As the rock fracture continues to evolve, the average
non-local triggering distances (distance between the triggering and triggered events) increase (Figure 9a). The extension of the optical FPZ is simultaneously ongoing with the successive occurrences of non-local triggering concentrations (see Figure 9c, taking the 60° test for illustration). The combination of these observations indicates the newly created rock fractures can also trigger remote events, just as the open, and pre-existing flaw does. Magnitude differences between non-local triggering AEs and their triggered ones display a rise-and-drop behavior from different groups (Figure 9b), indicating the newly created rock fracture evolves towards the friction-reduction tendency. At the initial stage, the open flaw controls the resistance to non-local triggering. One could imagine at this moment the energy required to trigger the remote event is small. As new FPZs are created, resistance of the whole fracture-and-flaw combination for triggering the non-local events will increase because the high resistance of the new FPZs. This combination demands higher energy in triggering remote events. As the rock fracture evolution continues, however, resistance in the fracture-and-flaw combination decreases, and the magnitude differences between non-local triggering AEs and their triggered ones decrease. The observation for the resistance of

Figure 8. (a) Magnitude-frequency distributions of all AEs (blue dashed-line), non-local triggering AEs (red line), and non-local triggered AEs (yellow line) for the 60° test and the 70° test (the insert). (b) Angle-frequency distribution of the non-local triggering-triggered AE pairs for the 60° test and the 70° test (the insert).

Figure 9. (a) Average triggering distance versus triggering AE groups. Inserts are illustrating the grouping of non-local triggering AEs for the 60° test (lower right) and the 70° test (upper left). (b) Magnitude difference between non-local triggering and triggered AEs versus triggering AE group number for the 60° test and the 70° test. (c) Illustration of progressive FPZ extension, as well as widening and brightening (Wong & Xiong, 2018) along concentrations of non-local triggering after peak stress, illustrated by 60° test.
the newly created rock fractures to decrease implies the newly created rock fractures will evolve towards
the friction-reduction tendency. As the initial setting of the experiment, the central flaw, is frictionless,
this observation bridges the gap between an idealized experimental setting where non-local triggering
initially takes place around an open friction-free flaw and the natural rock fractures whose evolution
tendency is towards the friction-reduction direction. For the field-scale analogue of the tests, the large-
tectonic-scale structures in rock mass, validation of the existence non-local triggering phenomenon implies
that a pair of distant earthquakes might substantially correlate by the connection of complex underground
rock fracture systems.

3.4. Further Analysis of Triggering and Non-Local Triggering
3.4.1. Triggered Event Population, General and Non-Local Comparison
The population of non-locally triggered events (including immediately triggered and subsequently triggered
generations) as well as the population of general triggered events decreases just after their immediate trig-
gerated generation for both tests (Figure 10a). This phenomenon is expectable as the branching modeling ana-
lysis has suggested (i.e., Epidemic Type Aftershock Sequence [ETAS] model; Zaliapin & Ben-Zion, 2013b).
While unlike the modeling simulation, no tuning on the modeling parameters and/or tail cuttings of the
ETAS model is necessary to achieve this experimental result. Besides, it is also noted that a pure branching
model (i.e., ETAS) can hardly reproduce the swarm-like feature (the families having larger-leaf depth as well
as large-population) of some natural earthquake observations (Zaliapin & Ben-Zion, 2013b). Comparing the
decreasing tendencies of offspring population between the general triggers (the roots of all families) and the
non-local triggers (whose immediate triggered offspring is non-local), that of the non-local triggers shows
significantly slower rate decreases (see Figure 10a). This phenomenon implies the large-scale flaw cannot
only facilitate the occurrence of non-local triggering, but also facilitate the further growth of triggered off-
spring populations in the subsequent generations. As noted by Zaliapin and Ben-Zion (2013b), alternative
clustering mechanism (other than the pure branching models) may be responsible for a substantial popu-
lation of the clusters (families) having large averaged root-to-leaf depth as well as large cluster size, the present
experimental observation strengthens the hypothesis that “alternative clustering mechanisms” may be cor-
related with the triggering effects of rock fracture systems by large-scale structures.

Figure 10. (a) Population of general triggered events (events triggered from all roots) and non-locally triggered events (events triggered from the non-local triggers) versus their generations for two tests. First generation stands for the general triggers (dashed curves with empty square marks) or the non-local triggers (solid curves with filled square marks). Events from non-local triggers to its fourth generation offspring for (b) the 60° test and (c) the 70° test, respectively. Generations from 1 (the non-local triggers) to 5 (the fourth generation offspring of non-local triggers) are plotted by red, green, blue, cyan, and purple, respectively. Average of 7 mm normalized pre-cracking event density curves for the immediate triggered events by non-local triggers from first non-local triggering concentration for (d) the 60° test and (e) the 70° test, respectively. Homogeneous pre-damage distribution (N ~ r²) are also plotted with black curves of diamond marks.
3.4.2. Spatial Dependence of Large-Scale Structure for the Non-Locally Triggered Events in Immediate and Subsequent Generations

The reliance of the non-local triggering AEs (the non-local triggers) and their subsequent generations of triggered events on the large-scale structure can also be depicted spatially. We plot the non-local triggers and their triggered events from first to fourth generations in Figures 10b and 10c for the 60° test and the 70° test, respectively. At these generations, the difference of the decreasing rate on the offspring population between the non-local triggers and the general triggers is apparent (see Figure 10a). The non-local triggering AEs and their triggered events appear to linger near common locations along the paths of rock fractures for several generations. This helps the AE clusters in Figures 10b and 10c achieve a concentrated delineation on the macro-scale rock fracture paths compared to that of plotting all AEs as in Wong and Xiong (2018). Even if the non-local triggering AEs and their triggered ones in Figures 10b and 10c are only of a fraction of the total AE events (<30%, specifically: 27.36% for the 60° test and 22.52% for the 70° test), clearer delineation of the large-scale rock fracture paths can be achieved. We have suggested in the previous section that the large-scale structure will not only facilitate the non-local triggering but also facilitate the further growth of triggered events in the subsequent generations. It is reasonable to expect that further spatial migration of the subsequent generations of non-locally triggered events tends to linger near and over-damage the common locations along the large-scale rock fractures. This in turn can further facilitate the evolution of rock fractures.

Another spatial observation on the non-local triggered events is, although the non-local triggering-triggered pairs have crossed significant distances, the immediate non-local triggered events tend to occur at the locations surrounded by a higher concentration of pre-cracking events. This observation emphasizes the influence of pre-cracks. We take the first generation triggered events of first non-local triggering concentration (abbreviated as G-1-C-1 in the later description) for illustration. The first non-local triggering concentration (abbreviated as C-1 in the later description) per test refers the first of the three temporal non-local triggering concentrations indicated by the red dashed boxes in the two inserts of Figure 9a. For any AE event, its pre-cracking events are those temporally prior to it. As illustrated in Figures 10d and 10e, for the G-1-C-1 in both tests, the average of normalized pre-cracking event number as a function of distance to the triggered event is higher than the asymptotic equation of homogeneous pre-damage distribution—the closer to the triggered events, the higher the pre-cracking event concentration. This observation implies the spatial preference of the non-local triggered events. They prefer to occur at locations having higher numbers of previous events. The non-local triggering phenomenon, consequently, must involve a series of local rock fracture processes to take place. These local rock fracture processes, described as the pre-cracking events, have not produced the trigger for the non-local triggered event, although pre-cracking events may have closer spatial distance to the non-local triggered event.

It is noteworthy that the deviation from the asymptotic equation of homogeneous pre-damage distribution can be most pronounced for the G-1-C-1 with short normalization distance. This is due to the following two reasons. First, for the G-1-C-1, their triggering phenomenon is mostly influenced by the open flaw. While for the later non-local triggering concentrations, multiple physical processes of rock fracture evolution (e.g., secondary fracture development and triggered event reactivation) may be involved. The deviation from the homogeneous asymptotic equation for the non-local triggered events of the later non-local triggering concentrations may become less pronounced due to the complexity of the mutual influence of the physical processes. Second, longer normalization distances may also contain other scale-dependent physical processes, thus for the same reason may blur the statistical significance. As the open flaw is an idealized large-scale structure, the features of C-1 can be most representative for the cross-scale interaction between the large-scale structure and the triggering phenomenon. Few other physical processes at intermediate scale(s) will be involved. In the next section, the reactivation of non-locally triggered events will also be illustrated by the subsequent generations of the triggered events of C-1.

3.4.3. Reactivation of Non-Locally Triggered Events in Subsequent Generations

In addition to the above-mentioned observation on the slower rate of offspring population decreasing tendency for the non-local triggers, we observe temporally the subsequent generations of non-locally triggered events can be reactivated by other physical processes of the rock fracture evolution. This argument is supported by the observation of the following two phenomena: (1) The subsequent generations of triggered events of C-1 can still occur long after the end of C-1, and (2) temporally, these
subsequent generations can separately occur at different times which are synchronized with several periodically occurring peaks of overall AEs. This can be illustrated by the temporal histograms of the triggered events for those subsequent generations of C-1, which is presented in Figures 11a and 11b for the 60° and the 70° tests, respectively.

In Figures 11a and 11b one can observe that, as the generation number increases, a higher percentage of the offspring population for the subsequent generations can occur at much later times than that of C-1. It is a natural consequence of the increased influence of “other physical processes” of the rock fracture evolution in reactivating the subsequent generation(s) of non-locally triggered events. Another interesting phenomenon has also been observed for the triggering events of C-1 for both tests. The maximum magnitudes of the triggers of C-1 follow a rise-and-drop behavior (Figures 11c and 11d). Existence of this rise-and-drop feature suggests a temporal self-amplification when the non-local triggering phenomenon, occurring as the temporal concentrations in the general rock fracture process, has been facilitated by the large-scale structure.
3.4.4. Off-Fault Direction Rock Fractures—Increasing Complexity

We have mentioned the secondary non-local triggering inclination angle concentration in Figure 8b when we discussed the larger-scale flaw influence on non-local triggering. Those non-local triggering pairs can be referred to as reverse-angled non-local triggering pairs according to their negative inclination angle (see Figure 12a). As the rock fracture evolution continues, the ratio of reverse-angled non-local triggering pairs increases (see Figure 12b). The increase of the ratio of reverse-angled non-local triggering pairs implies that as the resistance of the newly created rock fracture reduces, the shear displacement can induce more off-fault fractures. This is consistent with geophysical interpretation of the unequally developed secondary rock fractures along faults (Reches & Lockner, 1994) and may be explainable by the mechanism of creating secondary fractures.

As Figure 12c illustrates, when the rock fracture extends under shear stress (indicated as the red reversal arrow pair in the figure), a stress shadow will be formed around its tip and can induce rock fracture processes in both the direction parallel to the fault as well as the off-fault direction (Reches & Lockner, 1994). The induced stress shadow at the tips of the large-scale structure is the explanation that Reches and Lockner (1994) have used for explaining the existence of natural inequality of the secondary rock fracture development on the two sides of a shear fault. The development of the secondary rock fractures decreases along the perpendicular direction of the fault, and the development on the two sides are unequal. This explanation is under the perspective of classical fracture mechanics. Meanwhile, it only represents a static stress condition where the induced stress change after the nucleation of secondary off-fault fractures has not been considered.

Once the secondary off-fault fracture has nucleated, further interactions between the shear displacement along the main fault/fracture and the development of the secondary fractures will become much more complex. Continuation of shear displacement along the main fracture may further enhance the secondary fracture extension in the off-fault direction. As such, additional reverse-angled non-local triggering pairs would likely be created. Therefore, the increase of reverse-angled non-local triggering pairs implies shear displacement would induce more off-fault fractures. Consequently, an increase in the ratio of reverse-angled non-local triggering pairs indicates the rock fracture evolution tends towards an enhancement of fracture structure complexity.

**Figure 12.** (a) Schematic drawing of positive angle and negative angle (reverse-angled) non-local triggering pairs. The angle between the green circle events is positive and these events are along the main rock fractures; the angle between the purple box events is negative and these events are in the off-fault direction. Non-local triggering event pairs of negative angles are defined as reverse-angled non-local triggering pairs. (b) Ratio of reverse-angled non-local triggering pairs versus non-local triggering concentration number for both tests (see the two inserts of Figure 9a for the three non-local triggering concentrations). (c) Schematic drawing on the correlation between secondary off-fault fractures and a reverse-angled non-local triggering pair. The schematic drawing of a stress shadow is modeled after Reches and Lockner (1994).
4. Discussion

4.1. Model and Result Stability

The triggering-triggered correlation between event pairs is built by seeking the violation of the null hypothesis that event distribution obeys the G-R law (Davidsen et al., 2017). An intuitive inquiry on the results presented in this paper would be that the model parameters may affect the results. For instance, one can imagine an unrealistically large value for fractal dimension (i.e., $D_f \rightarrow \infty$) can mistakenly trap all AE events into the condition of no-connection with others, as such will be characterized as events of Poisson process. We validate the existence of non-local triggering by systematically repeating the data analysis using all the combinations of the reasonable model parameters. Specifically, for each test $D_f$ can be estimated with epicenters as well as hypocenters, and $b$ value can be estimated from the overall AEs, the AEs above the magnitude of completeness, and the remaining AEs with further higher magnitude condition (i.e., $M > -0.5$). From these combinations, the analysis for each test was repeated six times and in total 12 times for the two tests. Non-local triggering phenomenon can be detected throughout all these combinations for both tests. From these combinations, the population of non-local triggering AEs (the number of non-local triggers) is approximately 3.89% to 5.48% of the total AE population, and the combination of the events for the non-local triggering AE pairs (including the non-local triggers and the non-local triggered events) doubles these percentages. As such it is not an incidental phenomenon. We conclude the results of this manuscript are both real and stable.

Another question regarding the detection of non-local triggering phenomenon in rock fractures is the accuracy of AE source locations. Previously, Wong and Xiong (2018) have stated an estimation for the initial source location uncertainty as <4.5 mm for the primary plane of the specimen and <2 mm for the third dimension. This estimation is based on the comparison between the real center location of sources created by a 5 mm aperture PICO sensor and the calculated source location, and the $P$-wave velocity was measured each specimen before the loading tests. As such, this estimation tends to exaggerate the actual uncertainty, since the source (the PICO sensor) already has a 5 mm aperture. Alternative estimation, such as mentioned in McLaskey and Lockner (2018), may give much smaller source location uncertainty. In McLaskey and Lockner (2018), the uncertainty of source locations was estimated by perturbing the arriving time of the signals within the resolution of sampling rate. The $2\sigma$ of the point cloud (95% confidence) generated by this process was estimated as the source location uncertainty. This approach should give smaller source location uncertainty estimation as the AE system gives 0.33 $\mu$s/point sampling rate. However regardless of the source location uncertainty, one can prove the non-local triggering phenomenon can still be detected above this uncertainty by repeating the analysis for a large number of times and each time perturbing the source location within the estimated uncertainty.

4.2. Observations Similar to Previous Modeling Predictions

Beyond the existence of several power laws (Baro et al., 2013; Corral, 2003; Davidsen et al., 2007) and two alternative stages (Bak & Tang, 1989; Sornette, 2006) within criticality, Bak and Tang (1989) stated the following predictions (observations) based on their self-organized criticality (SOC) modeling simulation for earthquakes: A fault under criticality can be activated by a small perturbation at a seemingly random location (e.g., butterfly effect); the earthquake can be triggered at all scales in a “chain-reaction” manner, and the only size limit is the system boundary. In our tests, three roots of the four large trees have occurred when specimen-scale criticality has been achieved, and the following triggered event migration can traverse the entire specimen in a cascading manner respecting the specimen boundary as the only limit (Figure 6). Magnitudes of these roots can be lower than the mean and median magnitude of all AEs and spatially occur at seemingly random locations (e.g., Figure 5a). Although the specimen-scale criticality (the stress peak) is not directly equal to the criticality state of the rock fracture system, our observations show reasonable analogue to the above-mentioned modeling predictions. We further note that the small perturbation roots are identified using the contemporary laboratory AE observation limit. Cracking or rock fracture processes at much finer and undetectable scale(s) can also be the trigger of these roots. In principle, it is not physically practical to identify which root “causes” the cascading “chain-reaction” in a chaotic system. However, the present observation has proven that such roots of random and small perturbation features exist in the rock fracture process. In the rock fracture system, the cascading migration of the triggered events will be progressively achieved. The triggering phenomenon under the temporal sense will, as the rock fracture evolution...
continues, progressively become more pronounced (see Figures 3a and 3c). The ratio of triggering population under the spatiotemporal sense (see Figures 3d to 3f and 4a) will also progressively increase with further rock fracture evolution in the post-stress-peak regime. To the best knowledge of the authors, the present observations are the first lab-analogues on these modeling predictions beyond power laws. The large-scale flaw appears to supervise the rock fracture system evolving progressively into a chaos system as such phenomena from the SOC modeling predictions can occur.

Contradictory to the common understanding of “chain-reaction” phenomenon, the population of non-locally triggered events tends to decrease instead of showing an indexed increase (see Figure 10a). This indeed may be due to the arbitrary usage of “chain-reaction” by previous researchers in describing the geophysical phenomenon (Bak & Tang, 1989), and the decreasing tendency is exactly expectable by branching modeling analysis (Zaliapin & Ben-Zion, 2013b). While as Zaliapin and Ben-Zion (2013b) suggested, a pure branching model (i.e., ETAS) can hardly reproduce the swarm-like feature of some natural earthquake observations, we observe a significant slower decreasing tendency of the offspring population for the non-local triggers compared to that of the general triggers (roots). This observation may bring new inspiration to the “alternative clustering mechanism” mentioned by Zaliapin and Ben-Zion (2013b). That “alternative clustering mechanism” was suggested to be responsible for a substantial population of clusters (families) of large averaged root-to-leaf depth as well as large cluster size.

### 4.3. Loading Rate and the Relation to Triggering Phenomenon in Rock Fractures

In laboratory rock mechanics literature, the specimen configuration of the present tests appears as the “common-type” rock mechanics experiments—uniaxial compression tests on pre-flawed specimens which may have been extensively conducted for over half century. However, when comparing the present tests with those of the literature, the present testing is unique in that the possibility to distinguish natural seismicity analogues in observed AE exists. In fact, extensive literature assessment has returned very few of these common-type rock mechanics experiments providing an in-depth discussion regarding AE laboratory analogues to natural seismicity. To understand this, we have conducted an additional 60° test on a specimen of identical flaw and geometrical configuration, using the loading protocol from Wong and Einstein (2009); and the loading rates of similar range have been widely used by many previous literature of this type of rock mechanics tests. The loading rate difference between the previous and the present loading protocols is massive, for example, approximately 10 to 15 min from start to sample failure for Wong and Einstein (2009) versus approximately 1 hr from the start of constant displacement loading stage to failure for the experiments discussed here. This difference can lead to more than 10 times the difference in specimen strain rate for the period from fracture initiation to specimen failure. As such, the tests under the data re-analysis of this paper allow for the rock fracture development and the post-peak rock fracture evolution to be more readily observed.

We collect the AEs of this new 60° test, referred to here as “60° test 2,” using the identical experimental settings (not including the loading protocol) of the tests discussed in this paper, and make efforts to analyze the E2E connections of these AEs following the same procedures outlined above. The result of the $n_{ij}^*$ density according to the AEs from the 60° test 2 has been displayed in Figure 13a. For comparison, one corresponding figure showing the co-existence of Poisson process and triggering process of the 60° test (i.e., Figure 3f) has been reproduced in Figure 13b.

As shown, the AE events of Poisson process have almost disappeared (Figures 13a and 13c) in the test using the loading protocol of Wong and Einstein (2009), while AEs of Poisson process as well as AEs of triggering process co-exist in the plot representing the test of the present paper (Figures 13b and 13d). According to a variety of studies on real natural seismic data (Gu et al., 2013; Moradpour et al., 2014; Zaliapin & Ben-Zion, 2013b; Zaliapin et al., 2008), a natural earthquake catalogue may contain both earthquakes of Poisson process as well as triggering process. As such, one would partially understand, why in previous literature of uniaxial compression tests on pre-flawed rock specimens the post-stress-peak rock fracture evolution has hardly been investigated in detail. The previous “common type” rock mechanics tests under the so-called “commonly practiced” loading protocol may fail to represent a good analogue for large-scale natural seismic processes.

The loading rate for experiments reported in Wong and Xiong (2018) is significantly lower than many previous experiments in pre-flawed specimens. Conversely, the fast loading procedure (Wong & Einstein, 2009) which was adopted by many previous researchers is suspected to eliminate the AEs of Poisson process in
rock fracture. As such, it is lacking one important analogue to real seismicity. This partially explains why many of the discoveries in this manuscript have never been reported through previous experimental results of a widely used specimen-and-loading configuration in laboratory rock mechanics tests. We have illustrated that in the rock fracture process the overall population of events will progressively transfer from Poisson process dominant to triggering process dominant (see Figures 3 and 4a), while the constant Poisson process will exist throughout the entire test with moderate rate (Figure 4c). For the Poisson process, events in rock mass occur randomly and will percolate once their distance is critically close. For the triggering process, the FPZ in rock mass will extend and the large-scale structure acts as the source of damage pervasion. The fast loading procedure may fail to specifically delineate this progressive process thus loses many details within the rock fracture evolution under specimen-scale criticality.

5. Conclusion, Perspective on Non-Local Triggering Mechanism

Summarizing the discoveries in this paper, the mechanism of non-local triggering is as follows:

First, we prove the existence of triggering within the presence of a large-scale structure, showing that analogues such as co-existence of Poisson process and triggering process, as well as the co-existence of burst-like and swarm-like features in the present rock fracture tests. Upon these analogues, we prove the existence of non-local triggering by seeking the E2E triggering connections crossing the rock flaw. The triggered event
migration patterns can also prove the communication between rock fractures exist. Statistically, the inclination angle between non-local triggering and their triggered events are along the orientation of the large-scale structure, and the magnitude-frequency distribution of non-local triggering AEs is larger than that of their directly triggered events. It appears no lower limit exists for the non-local triggering phenomenon to take place. We also observe that a seemingly random small perturbation can cause a cascade of triggered events which can traverse the entire rock fracture system. In the further investigations of the non-local triggering phenomenon, we find the rock fractures evolve towards a state of reduced friction, while also evolving towards enhanced complexity. The immediate non-local triggered events (first generation offspring) occur at locations of higher pre-cracking event concentration and tend to self-amplifying. The decrease of offspring population in the subsequent generations of the non-locally triggered events is significantly slower than that of the general triggered events, and the non-locally triggered events tend to linger near and over-damage common locations along the rock fractures for several generations. These observations further emphasize the influence of the rock fractures, which is not only facilitates non-local triggering taking place, but also facilitates the production of much higher offspring numbers in the subsequent generations of non-locally triggered events. The non-locally triggered events can also be reactivated in later massive energy releases (the AE peaks).

Combining all this information together with the knowledge of classical rock fracture mechanics, we tentatively show a comprehensive perspective of the rock fracture process:

Natural rock fractures are of different magnitudes of scales, which can be submeter-scale rock joints as well as tectonic-scale faults. A large-scale structure in rock mass can induce rock fracture process in both its current orientation as well as in the off-fault orientation. Cracks of smaller scale(s) will primarily occur at the stress concentrations (e.g., large-scale structure tips). The accumulation of these smaller scale cracks will form the FPZ which lays the foundation of future rock fracture paths as well as the foundation for rock fracture evolution at larger scale(s).

As more cracks are accumulated, percolated, and scaled up, and when a point in rock mass is at state of local criticality, a prior cracking event from a remote area can trigger a cracking event via the large-scale rock fracture channel. The rock fracture processes within the FPZs at two tips of the large-scale structure are actually connected and will communicate when the rock fracture has nucleated and extends.

The newly created rock fractures will evolve towards a state of reduced friction. Within its evolution, the remotely triggered event will again be activated (reactivation) and over-damage the common locations with its subsequent generations of triggered events. The self-amplifying of the non-local triggering and the induced stress change dynamically enhance the probability of over-damage to occur due to the stronger perturbations imposed on the locations of local criticalities.

Further rock fracture evolutions will be ensured as subsequent generations of triggered events occur at common locations. Asperities along the newly created rock fractures may be broken and can reduce the roughness of the rock fractures. This ensures the rock fracture evolution towards a state of reduced friction. As shear resistance has been reduced in this evolution, further shear displacement can occur. This displacement facilitates the fracture process in the off-fault direction. As such, the main rock fracture will tend to evolve towards a state of increasing complexity. Along with the rock fracture extension, the rock fracture evolution under full-scale criticality will experience periodical massive energy release states. This massive energy release can even be incidentally initiated via a “small shaking” event at random locations.

Viewing this progress from the micro-to-macro-scales, new large-scale rock fractures have step-by-step evolved from the accumulation of subscale cracks into a new low-friction large-scale rock fracture structure. Small-scale cracking, that is, stress-induced local cracking, dynamically and non-locally triggered remote cracking, and over-damage of the subsequent generations of non-locally triggered AEs, will facilitate the large-scale structure’s extension. Conversely, viewing this progress from macroscale to micro-scale, the shear-sliding displacement, shear resistance reduction, and the tendency of rock fracture evolution towards a state of increasing complexity, in turn assures the further continuation of the rock fracture processes at subscales. The unevenness and anti-synchronism among all these processes at different scales ensure the periodical occurrence of massive energy releases as well as ensure the continuous evolution of the rock fracture. In each of these asynchronous mechanisms, multiscale interactions are involved. For this dynamic and multiscale process, we hope our laboratory analysis protocol as well as
the laboratory observation on the analogue phenomena can cast a light towards comprehensive understanding on the dynamics of rock fractures, and ignite corresponding future research.

Conflict of Interest
No conflicts of interest are present, financial or otherwise.

Data Availability Statement
Readers can find supporting data in CERN’s Zenodo (https://doi.org/10.5281/zenodo.4021391).

References
Baiei, M., & Puzasulti, M. (2004). Scale-free networks of earthquakes and aftershocks. Physical Review E, Statistical, Nonlinear, and Soft Matter Physics, 69(6), 066106. https://doi.org/10.1103/PhysRevE.69.066106
Bak, P., & Tang, C. (1989). Earthquakes as a self-organized critical phenomenon. Journal of Geophysical Research, 94(B11), 15,635–15,637. https://doi.org/10.1029/JB094iB11p15635
Baro, J., Corral, A., Illa, X., Planes, A., Salje, E. K. H., Schranz, W., et al. (2013). Statistical similarity between the compression of a porous material and earthquakes. Physical Review Letters, 110(8), 088702. https://doi.org/10.1103/PhysRevLett.110.088702
Corral, A. (2003). Local distributions and rate fluctuations in a unified scaling law for earthquakes. Physical Review E, Statistical, Nonlinear, and Soft Matter Physics, 68(3), 035102. https://doi.org/10.1103/PhysRevE.68.035102
Davi, R., Vavryčuk, V., Charalampidou, E.-M., & Kwiatek, G. (2013). Network sensor calibration for retrieving accurate moment tensors of acoustic emissions. International Journal of Rock Mechanics and Mining Sciences, 62, 59–67. https://doi.org/10.1016/j.ijrmms.2013.04.004
Davidson, J., Kwiatek, G., Charalampidou, E. M., Goebel, T., Stanchits, S., Dresen, G. (2017). Triggering processes in rock fracture. Physical Review Letters, 119(6), 068501. https://doi.org/10.1103/PhysRevLett.119.068501
Davidson, J., Stanchits, S., & Dresen, G. (2007). Scaling and universality in rock fracture. Physical Review Letters, 98(12), 125502. https://doi.org/10.1103/PhysRevLett.98.125502
Goebel, T. H. W. (2013). Microseismicity, fault structure, and the seismic cycle: Insights from laboratory stick-slip experiments (Doctoral dissertation). Los Angeles, CA: University of Southern California.
Goenberg, J., Reches, Z.e., & Lockner, D. A. (1994). Nucleation and growth of faults in brittle rocks. Journal of Geophysical Research: Solid Earth, 99(16), 31,959–31,974. https://doi.org/10.1029/94JB01049
Parsons, T., Ji, C., & Kirby, E. (2008). Stress changes from the 2008 Wenchuan earthquake and increased hazard in the Sichuan basin. Nature, 454, 483–486. https://doi.org/10.1038/nature07177
Reches, Z.e., & Lockner, D. A. (1994). Nucleation and growth of faults in brittle rocks. Journal of Geophysical Research, 99(B9), 18,159–18,173. https://doi.org/10.1029/94JB00115
Shan, B., Xiong, X., Zheng, Y., Jin, B., Liu, C., Xie, Z., & Hsu, H. (2013). Stress changes on major faults caused by 2013 Lushan earthquake and its relationship with 2008 Wenchuan earthquake. Science China Earth Sciences, 56(7), 1169–1176. https://doi.org/10.1007/s11430-013-4624-1
Stein, R. S. (1993). The role of stress transfer in earthquake occurrence. Nature, 402, 605–609.
Stein, R. S., Barka, A. A., & Dieterich, J. H. (1997). Progressive failure on the North Anatolian fault since 1939 by earthquake stress triggering. Geophysical Journal International, 128(3), 594–604. https://doi.org/10.1111/j.1365-246X.1997.tb05321.x
van der Elst, N. J., & Brodsky, E. E. (2010). Connecting near-field and far-field earthquake triggering to dynamic strain. Journal of Geophysical Research, 115, B07311. https://doi.org/10.1029/2009JB006681
Wong, L. N. Y., & Einstein, H. H. (2009). Systematic evaluation of cracking behavior in specimens containing single flaws under uniaxial compression. *International Journal of Rock Mechanics and Mining Sciences, 46*(2), 239–249. https://doi.org/10.1016/j.ijrmms.2008.03.006

Wong, L. N. Y., & Xiong, Q. (2018). A method for multiscale interpretation of fracture processes in Carrara marble specimen containing a single flaw under uniaxial compression. *Journal of Geophysical Research: Solid Earth, 123*, 6459–6490. https://doi.org/10.1029/2018JB015447

Xiong, Q., & Wong, L. (2017). Comparison of three source types for calibrating AE sensors used in fracture coalescence tests. Paper presented at 51st US Rock Mechanics/Geomechanics Symposium, San Francisco, CA.

Zaliapin, I., & Ben-Zion, Y. (2013a). Earthquake clusters in southern California I: Identification and stability. *Journal of Geophysical Research: Solid Earth, 118*, 2847–2864. https://doi.org/10.1002/jgrb.50179

Zaliapin, I., & Ben-Zion, Y. (2013b). Earthquake clusters in southern California II: Classification and relation to physical properties of the crust. *Journal of Geophysical Research: Solid Earth, 118*, 2865–2877. https://doi.org/10.1002/jgrb.50178

Zaliapin, I., Gabrielyov, A., Keilis-Borok, V., & Wong, H. (2008). Clustering analysis of seismicity and aftershock identification. *Physical Review Letters, 101*(1), 018501. https://doi.org/10.1103/PhysRevLett.101.018501

Zang, A., Wagner, F., Stanchits, S., Dresen, G., Andresen, R., & Haidekker, M. (1998). Source analysis of acoustic emissions in Aue granite cores under symmetric and asymmetric compressive loads. *Geophysical Journal International, 139*(3), 1113–1130. https://doi.org/10.1046/j.1365-246X.1998.00706.x