An Advanced Digital Predictive Valley Current Control Algorithm for a Boost Converter

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Abstract. An advanced digital predictive valley current control algorithm for a boost converter is proposed. The control algorithm predicts the inductor current in a future period by sampling the voltages including input and output, and inductor current of the current period, which overcomes the problem of hardware periodic delay. Under the premise of ensuring the stability of the system, the control algorithm has a faster response speed as compared to the traditional digital proportional-integral (PI) control algorithm. In addition, it is not only algorithm simple, but also easy to implement on DSP of digital control chips. Finally, the proposed control algorithm is verified using simulations software and experimental measurements.

1. Introduction

Analog current controllers have wide applications in various kinds of power electronic converter field [1]. But, with the operation of the system for a long time, the parameter drift caused by the aging of the simulator makes the performance of analog controller decrease or even unstable [2]. In addition, the modification of control strategy and the improvement of performance indicators require the redesign of analog control system, which greatly limits the application of analog controller. So, with the development of digital devices, the analog controllers are gradually replaced by digital controllers [3]. However, one of the biggest disadvantages of digital controller is the problem of hardware periodic delay which results from sampling retainer, computation, and digital-pulse-width modulation (DPWM). In order to solve the delay problem of the digital controller, the predictive control is becoming a promising control technique now [4]-[7].

The method proposed in [4] only considers the disturbance of the inductor current but not that of the reference current. Therefore, when the reference current is disturbed, the inductor current cannot achieve stability in two switching cycles. In [5], the author put forward an improved current control algorithm but the requirements for the hardware devices are high and this is not conducive to real applications. In particular, an approximate predictive technique using linear extrapolation has been proposed for a buck converter in [6] but the duty ratio can only be updated once in two switching cycles, which reduces the transient performance of the controller. Two predictive digital current control schemes were introduced in [7] but they had drawbacks and limitations.

In this paper, a new predictive valley current algorithm for a boost converter is introduced based on the inductor current in continuous mode. Combined with the small-signal model transfer function of the boost converter, the duty ratio equation is deduced in the discrete-time domain by the small-signal analysis of sampled values. The control algorithm enables the inductor current to achieve dead-beat tracking in one switching period, and making the tracking speed of the inductor current greatly
improve. Eventually, the proposed control technique was easily programmed in a digital signal processor (DSP) and enjoyed a very fast dynamic response.

2. Predictive valley current control method

As shown in Figure 1, the proposed algorithm is developed using a basic boost converter, and the typical current controller consists of an ADC, an inner current control and an outer voltage control loop, and a digital DPWM. \( I_r, V_{in}, V_o \) and \( I_L \) are the reference current, the input and output voltages, and the inductor current, respectively. \( V_{r(n)}, V_{o(n)}, I_{L(n)} \) are the sampled value of the corresponding variables. The duty ratio \( d_{(n)} \) is deduced using predictive valley current control algorithm.

\[ \begin{align*}
    \frac{dx(t)}{dt} &= A_1 x(t) + B_1 u(t) \\
    y(t) &= C_1 x(t) + E_1 u(t)
\end{align*} \]  

(1)

\[ \begin{align*}
    \frac{dx(t)}{dt} &= A_2 x(t) + B_2 u(t) \\
    y(t) &= C_2 x(t) + E_2 u(t)
\end{align*} \]  

(2)

where \( x(t) = [i_L, v_c]^T \), \( y(t) = [i_s, v_o]^T \), \( u(t) = v_{in} \)

2.1 Small-signal model of boost converter

When the boost converter is operation in a continuous current mode (CCM), there are two working states in one switching cycle and shown in Figure 2.

![Figure 1. Typical digital controller for a boost converter](image)

![Figure 2. Two operating states of the boost converter in one switching cycle: (a) switch is on and (b) switch is off](image)
\[ A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{L} & 0 \end{bmatrix} \quad B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C_1 = C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad E_1 = E_2 = 0 \]

According to equations (1) and (2), state-space averaging equation and output equation in one switching cycle can be described as:

\[
\begin{align*}
\frac{d\bar{x}(t)}{dt} &= [d(t)A_1 + d'(t)A_2]|\bar{x}(t) + [d(t)B_1 + d'(t)B_2]|\bar{u}(t) \\
\bar{y}(t) &= [d(t)C_1 + d'(t)C_2]|\bar{x}(t) + [d(t)E_1 + d'(t)E_2]|\bar{u}(t) 
\end{align*}
\]

(3)

where \(\bar{x}(t)\), \(\bar{u}(t)\) and \(\bar{y}(t)\) represent the average value of the corresponding variables in one switching cycle. \(d(t)\) is duty ratio and satisfies \(d(t) + d'(t) = 1\).

Substituting the parameters into equation (3), the state-space averaging equation and output equation of the boost converter can be reduced as:

\[
\begin{align*}
\frac{d\bar{I}_i(t)}{dt} &= \begin{bmatrix} 0 & -\frac{d}{L} \\ \frac{d}{C} & -\frac{1}{RC} \end{bmatrix} \bar{I}_i(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \bar{v}_m(t) \\
\bar{v}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \bar{v}_m(t)
\end{align*}
\]

(4)

\[
\bar{y}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{I}_i(t) \\ \bar{v}_m(t) \end{bmatrix}
\]

(5)

The variables are decomposed into DC steady-state value and AC small-signal disturbance, namely.

\[
\bar{I}_i(t) = I_L + \dot{I}_L(t) \quad \bar{V}_i(t) = V_i + \dot{V}_i(t) \quad \bar{V}_m(t) = V_m + \dot{V}_m(t) \quad \bar{y}(t) = Y + \dot{y}(t) \quad d(t) = D + \dot{d}(t)
\]

(6)

where let us assume:

\[
I_L \gg \dot{I}_L(t) \quad V_i \gg \dot{V}_i(t) \quad V_m \gg \dot{V}_m(t) \quad Y \gg \dot{y}(t) \quad D \gg \dot{d}(t)
\]

(7)

Substituting equation (6) in equations (4) and (5):

\[
\begin{align*}
\frac{d(I_L + \dot{I}_L(t))}{dt} &= \begin{bmatrix} 0 & -\frac{(D + \dot{d}(t))}{L} \\ \frac{(D + \dot{d}(t))}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L + \dot{I}_L(t) \\ V_i + \dot{V}_i(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} (V_m + \dot{V}_m(t)) \\
Y + \dot{y}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_L + \dot{I}_L(t) \\ V_i + \dot{V}_i(t) \end{bmatrix}
\end{align*}
\]

(8)

(9)

So, the DC steady-state model can be obtained:

\[
\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{D}{L} \\ D & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L \\ V_i \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} V_m
\]

(10)

Solved equation (10):

\[
\begin{bmatrix} I_L \\ V_i \end{bmatrix} = \frac{V_m}{(1-D)R} \begin{bmatrix} 1 \\ (1-D) \end{bmatrix}
\]

(11)

Then, using approximations we neglect all nonlinear terms such as the second-order terms in equation (8) and obtain once again a linear system, and then we can get small-signal model as follows.


\[
\frac{d}{dt}\begin{bmatrix}
\dot{i}_L(t) \\
\dot{\hat{v}}(t)
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{1-D}{L} \\
1-D & -\frac{1}{RC}
\end{bmatrix}\begin{bmatrix}
\dot{i}_L(t) \\
\dot{\hat{v}}(t)
\end{bmatrix} + \begin{bmatrix}
1 \\
0
\end{bmatrix}\dot{\hat{v}}_m(t) + \begin{bmatrix}
\frac{RD}{L} \\
\frac{1}{C}
\end{bmatrix}\frac{V_m}{(1-D)^2}d(t)
\]

(12)

Duty ratio-to-inductor current transfer function:

\[
\frac{\dot{i}_L(t)}{d(t)} = \frac{V_m(RC_s + 2)}{(1-D)[LCRs^2 + Ls(1-D)^2R]}
\]

(13)

Duty ratio-to-output voltage transfer function:

\[
\frac{\hat{v}(t)}{d(t)} = \frac{V_mR - LV_m^s}{LCRs^2 + Ls(1-D)^2R}
\]

(14)

2.2 The proposed current control algorithm

According to the two operation states of the boost converter under CCM, the waveform of the inductor current in the steady-state is drawn in Figure 3.

![Figure 3. The waveform of the inductor current in CCM](image)

Analyzing the operation principle of the boost converter, we can get the slope of the inductor current for each switching state. Their equations are expressed as:

\[
S_u = \frac{V_m}{L}
\]

(15)

\[
S_d = -\frac{V_m - V_o}{L}
\]

(16)

where \(S_u\) and \(S_d\) represent the increasing and the decreasing slope of the inductor current, respectively.

In the actual hardware equipment, there is a certain delay from sampling to updating duty ratio. Therefore, if the delay is ignored, the system will lose stability. In order to solve the hardware delay problem, the inductor current of the next cycle is predicted in advance so that it can track the reference current at the end of the cycle. According to the sampling values, we can predict the current value at \((n+1)T\) time.

\[
i_{s(n+1)} = i_{s(n-1)} + S_{u(n-1)} \cdot d_{n-1}T_s - S_{d(n-1)} \cdot d_{n-1}T_s + S_{u(n)} \cdot d_nT_s - S_{d(n)} \cdot d_nT_s
\]

(17)

We continue to relax the restrictions for the sampling value and assume that the input and output voltages remain constant in the two switching cycles.
By substituting equation (18) and objective \( i_{c(n)} = i_{c(n+1)} \) into equation (17):

\[
i_{c(n-1)} = i_{s(n-1)} + \frac{V_{m(n)}}{L} \cdot d_{n-1} \cdot T_s + \frac{V_{m(n)} - V_{o(n)}}{L} \cdot T_s + \frac{(1-d_{n-1})}{L} \cdot d_n \cdot T_s + \frac{V_{m(n)} - V_{o(n)}}{L} \cdot (1-d_n) \cdot T_s
\]

By simplifying equation (19), the duty ratio of predictive current control can be reduced as:

\[
d_n = \frac{L}{V_{o(n)} T_s} \left( \hat{i}_{c(n)} - \frac{2(V_{m(n)} - V_{o(n)})}{V_{o(n)}} \right) - d_{n-1}
\]

\[
d_n (1+z^{-1}) = \frac{L}{V_{o(n)} T_s} \left( \frac{2(V_{m(n)} - V_{o(n)})}{V_{o(n)}} \right) (1+z^{-1})^{-1}
\]

The same as the modeling method of the boost converter, the variables are decomposed into DC steady-state value and AC small-signal disturbance, namely.

\[
v_{m(n)} = V_{m(n)} + \hat{v}_{m(n)} \quad V_{o(n)} = V_{o(n)} + \hat{v}_{o(n)} \quad i_{c(n)} = I_{c(n)} + \hat{i}_{c(n)} \quad i_{s(n)} = I_{s(n)} + \hat{i}_{s(n)}
\]

\[
d_n = D + \hat{d}_n
\]

And satisfied,

\[
D \geq \hat{d}_n
\]

Substituting equation (23) in equation (24):

\[
D + \hat{d}_n = \frac{L}{(V_{o(n)} + \hat{v}_{o(n)}) T_s} \left( (z+1)^{-1}(I_{c(n)} + \hat{i}_{c(n)} - I_{s(n)} + \hat{i}_{s(n)}) \right)
\]

\[
- \frac{2((V_{m(n)} + \hat{v}_{m(n)}) - (V_{o(n)} + \hat{v}_{o(n)}))}{(V_{o(n)} + \hat{v}_{o(n)})} \left( (1+z^{-1})^{-1} \right)
\]

Separating equation (25), we can obtain steady-state DC and small-signal AC duty ratio, respectively.

\[
D = \frac{L}{V_{o(n)} T_s} (z+1)^{-1}(I_{c(n)} - I_{s(n)}) - \frac{2(V_{m(n)} - V_{o(n)})}{V_{o(n)}} (1+z^{-1})^{-1}
\]

\[
\hat{d}_n = \frac{L}{V_{o(n)} T_s} (z+1)^{-1}(\hat{i}_{c(n)} - \hat{i}_{s(n)}) + \frac{2((1+z^{-1})^{-1} - D V_{o(n)} - \frac{2}{V_{o(n)}} (1+z^{-1})^{-1} \hat{v}_{m(n)}
\]

By simplifying coefficient of equation (27):

\[
\hat{d}_n = A(z) \cdot (\hat{i}_{c(n)} - \hat{i}_{s(n)}) + B(z) \cdot \hat{v}_{o(n)} + C(z) \cdot \hat{v}_{m(n)}
\]
Where

\[
\begin{cases}
A(z) = \frac{L}{V_{o(n)} T_s} (z + 1)^{-1} \\
B(z) = \frac{2(1 + z^{-1})^{-1} - D}{V_{o(n)}} \\
C(z) = -\frac{2}{V_{o(n)}} (1 + z^{-1})^{-1}
\end{cases}
\] (29)

2.3 Closed-loop transfer function of the inductor current

Through the above theoretical analysis, a closed-loop feedback block diagram of the inductor current is obtained and shown in Figure 4.

![Block diagram of closed-loop](image)

**Figure 4.** The block diagram of closed-loop

According to Figure 4, we can get the closed-loop transfer function of the reference current-to-inductor current and the input voltage-to-inductor current, respectively.

\[
\left. \frac{i_c^0(s)}{i_c(s)} \right|_{v_i=0} = T_i = \frac{G_{id}(s) \cdot A(s)}{1 - G_{id}(s) \cdot B(s) + G_{id}(s) \cdot A(s)}
\] (30)

\[
\left. \frac{i_i^e(s)}{\hat{v}_m(s)} \right|_{v_i=0} = \frac{G_{id}(s) \cdot C(s)}{1 - G_{id}(s) \cdot B(s) + G_{id}(s) \cdot A(s)}
\] (31)

3. Software simulation

The proposed control algorithm and boost converter were set up in the simulation software with the circuit parameters in Table 1 and shown in Figure 5.

**Table 1.** Boost converter parameters

| parameters           | values     |
|----------------------|------------|
| Input voltage \( (v_i) \) | 20V        |
| Inductor current \( (i_i) \) | 10A (15A)  |
| Inductance \( (L) \)       | 100uH      |
| Capacitance \( (C) \)      | 480uF      |
| Load \( (R) \)             | 3Ω         |
| Switching frequency \( (f_s) \) | 20k        |
Figure 5. The simulation diagram of boost converter

The simulation system consists of four parts, including power-stage circuit, sampling retainer, predictive current controller, and DPWM. The simulation results are shown in Figure 6 and Figure 7.

Figure 6. The transient tracking waveform of the inductor current when the reference current changes:
left waveform is reference current decreasing and right is increasing

From Figure 6, it can be seen that reference current can be tracked completely for two switching cycles, regardless of whether the reference current increases or decreases, and the first switching cycle delay is caused by digital devices and algorithm calculation. In fact, the inductor current needs only one switching cycle adjustment to achieve the final tracking.

Figure 7. The transient tracking waveform of the inductor current when it is disturbed

In Figure 7, a positive inductor current disturbance with an amplitude of 3 A is superimposed on the inductor current at 0.005 seconds. After two switching cycles, namely at 0.0051 seconds, the inductor current achieves the tracking of the reference current again, thus the system stabilizes at this time. Similarly, a negative inductor current disturbance with an amplitude of -3 A at 0.0052 seconds is added to the inductor current. Subsequently, the reference current is tracked in the second switching cycle after a delay of one switching cycle. Eventually, the system returns to the stable state again.
4. Experimental results
The circuit of boost converter and digital controller were built in the laboratory. The algorithm was programmed on a TMS320F28335 DSP chip including a 12-bit AD converter and a 16-bit digital DPWM. The parameters of the boost converter are listed in Table 1.

![Experimental setup diagram]

**Figure 8.** The inductor current waveform when reference current changes: left waveform is reference current increasing and right is decreasing.

It can be seen from the experimental waveform of the inductor current in Figure 8 that experimental results are exactly the same as the simulation results, so they both verify the feasibility of the predictive valley current control algorithm proposed in this paper.

5. Conclusions
According to the operating characteristics of the boost circuit, transfer function of the small-signal model has been derived by using the state-space averaging method. In order to eliminate the effect of the computational time delay and the sampling delay, a predictive valley current control algorithm for the digital operation of the boost converter has been presented in this paper. By predicting the inductor current when the current is disturbed, the reference current can be tracked only in one switching cycle after the delay period ends, this improves the system stability and transient response speed of the inductor current. Finally, the feasibility of the proposed algorithm has been verified by simulation and experiment.

Though taking the boost converter as an example in the paper, the proposed control algorithm is also applicable to other similar DC-DC converter topologies.

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