Fuzzy Mathematics Comprehensive Forecasting Analysis of Metal Mine Rockburst Based on Multiple Criteria

Nan Hu, Changhong Li*, Yang Liu and Yunfeng Wu
University of Science and Technology Beijing, Beijing, China

*Corresponding author e-mail: lch_ustb@163.com

Abstract. The underground environment in deep mines is complicated and the risk of rockburst is high during excavation. Therefore, it is of great significance to study the tendency of rockburst during excavation. At present, scholars from various countries have analyzed rockburst phenomena from the aspects of strength, stiffness, energy, etc., and have proposed various assumptions and criteria. However, these assumptions and criteria often only consider individual factors, which will lead to one-sidedness and limitations. This article takes the deep mining process of a metal mine as the research background. Based on the results of each independent criterion, fuzzy mathematical theory is introduced to evaluate the rockburst intensity and fuzzy comprehensiveness of various types of rocks in the project. The research results show that the multi-criteria comprehensive evaluation of fuzzy mathematics can well integrate the analysis results of each criterion, and has higher consistency with the actual situation in engineering.

1. Introduction
The study of rockbursts began in the 18th century. So far, many scholars have done a lot of research around related issues. The analysis of rockburst propensity is an important part of it [1,2]. In general, the evaluation methods of rockburst propensity can be summarized into two categories: a single index criterion that considers a major influencing factor and a synthesis Multi-indicator comprehensive criterion for multiple influencing factors. The single index criterion is usually based on theories of strength, stiffness, and energy. For example, Turchaninov summed up the Turchaninov criterion proposed by the mine construction experience in the Hibbin block of Kola Island [3]; the Russense criterion proposed by Norwegian scholars [4]. The energy impact index (A_E) [5], and the elastic energy index (E_T) [6] are all criteria based on energy theory. The single indicator criterion focuses on analyzing the impact of one or several indicators on rockburst. Or it can respond well and predict the occurrence of rockburst under certain circumstances. Therefore, this will have certain one-sidedness and limitations. And comprehensive consideration of all factors will make the study too complicated. The multiple-criteria comprehensive criterion that selects multiple indicators to reflect the tendency of rockbursts makes up the deficiency of the single-index criterion. For example, Tan Yian[7], Wang Yuanhan[8] and others applied fuzzy mathematical theory to construct a fuzzy mathematical method for predicting rockburst tendency. This evaluation method can comprehensively and efficiently reflect the rockburst tendency of rock masses based on engineering experience and actual conditions.

This article takes a deep mining project of a metal mine as an example. The main factors affecting rockburst were selected, and the rockburst propensity was judged by using various criteria. The
comparative analysis results show that there is a big difference in the results of rockburst evaluation for each index. In order to predict rockburst more accurately, the author introduced fuzzy mathematical theory. The fuzzy comprehensive evaluation of rockburst intensity for various types of rocks selected in the project has made up for the shortcoming of only using a single index to judge the rockburst tendency and made the judgment result more accurate.

All manuscripts must be in English, also the table and figure texts, otherwise we cannot publish your paper. Please keep a second copy of your manuscript in your office. When receiving the paper, we assume that the corresponding authors grant us the copyright to use the paper for the book or journal in question. Should authors use tables or figures from other Publications, they must ask the corresponding publishers to grant them the right to publish this material in their paper.

2. Analysis of single indicator criteria

Considering that there are many factors affecting rockburst, the main factors are selected as the object of consideration. The main factors affecting rockburst are in-situ stress, rock strength, and rock energy storage properties. Turchaninov criterion (T), Russense criterion (R), Tao Zhenyu criterion (C), strength and brittleness criterion (B'), linear elasticity criterion (Wₑ), several criteria comprehensively consider the tangential stress of the chamber, the axis of the chamber, There are many factors such as axial stress, maximum principal stress, uniaxial compressive strength of rock, tensile strength of rock, elastic modulus of rock and so on. Therefore, these criteria are selected for comprehensive fuzzy mathematical evaluation.

2.1. Turchaninov criterion analysis

Turchaninov summarized the mine construction experience of the Hibbin block in Kola Island, and proposed that rockburst activity is determined by the ratio of the sum of tangential stress σₐ and axial stress σ₅ in the chamber to the uniaxial compressive strength of the rock σₑ.

2.2. Russense criterion

Norwegian scholar Russense proposed to classify the intensity of rockburst according to the ratio of the maximum tangential stress σₐ of the chamber to the strength Iᵣ of the rock point load. By converting the point load Iᵣ into the uniaxial compressive strength σₑ of the rock, the maximum tangential direction of the chamber can be used. The ratio of stress to uniaxial compressive strength of rock determines the tendency of rockburst.

2.3. Tao Zhenyu criterion

Based on the previous research results and the domestic practical engineering experience, Tao Zhenyu proposed the ratio of the uniaxial compressive strength of the rock σₑ to the maximum principal stress of the surrounding rock σᵣ as a criterion for rockburst occurrence. In order to facilitate the subsequent comprehensive evaluation of fuzzy mathematics, a new index C = 14.5 − σₑ/σᵣ is constructed.

2.4. Strength brittleness coefficient (B)

The ratio of uniaxial compressive strength σₑ to tensile strength of a rock σᵣ is called the brittleness coefficient, and it reflects the brittleness of the rock. Deformation can also be used to define the brittleness coefficient. The ratio of the uniaxial compressive strength to the tensile strength of a rock is the strength brittleness coefficient of the rock.

The calculation formula is:

\[ B = \frac{\sigma_e}{\sigma_r} \] (1)

In the formula: σₑ—uniaxial compressive strength of rock, MPa;
σᵣ—Rock tensile strength, MPa.

In order to facilitate the subsequent comprehensive evaluation of fuzzy mathematics, a new index \( B' = 40 - B \) is constructed.
2.5. Linear elastic energy ($W_e$) criterion
According to the functional principle, the linear elastic energy stored before the rock reaches its peak strength under uniaxial compression can be obtained. According to the elastic energy, the rockburst intensity is divided into four levels.

The formula for calculating the linear elasticity index is as follows:

$$W_e = \frac{\sigma_c^2}{2E_s}$$  \hspace{1cm} (2)

Where: \(\sigma_c\) — uniaxial compressive strength, MPa;
\(E_s\) — Unloading tangent elastic modulus, MPa.

3. Fuzzy mathematics comprehensive forecast analysis of rockburst tendency
Because the causes of rock burst disaster are complex and the influencing factors are numerous, it is inevitable that there will be some limitations and one-sidedness if only some individual factors are considered. Moreover, various factors in geotechnical engineering problems are usually fuzzy. In order to predict rockburst more accurately, the authors introduce the fuzzy mathematics theory to carry out the fuzzy comprehensive evaluation of rockburst intensity of various rocks selected from sanshandao gold mine.

The comprehensive evaluation of fuzzy mathematics gives two finite domains. Specific to the rockburst problem, it is necessary to give a set of evaluation factors for rockburst \(U\) and an evaluation set of rockburst intensity \(V\):

$$U = \{u_1, u_2, u_3, \ldots, u_n\}$$ \hspace{1cm} (3)
$$V = \{v_1, v_2, v_3, \ldots, v_m\}$$ \hspace{1cm} (4)

The fuzzy subset of each factor in the universe determined by the evaluation object on the evaluation result is:

$$R_i = \{r_{i1}, r_{i2}, r_{i3}, \ldots, r_{im}\} i = 1, 2, \ldots, n$$ \hspace{1cm} (5)

The fuzzy subset of each element pair \(V\) in \(U\) constitutes a fuzzy matrix \(R\), which is:

$$R = \begin{bmatrix}
    r_{11} & r_{12} & r_{13} & \cdots & r_{1m} \\
    r_{21} & r_{22} & r_{23} & \cdots & r_{2m} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    r_{n1} & r_{n2} & r_{n3} & \cdots & r_{nm}
\end{bmatrix} \hspace{1cm} (0 \leq r_{ij} \leq 1)$$ \hspace{1cm} (6)

When comprehensively judging things with multiple influencing factors, it is necessary to consider the magnitude of the effect of different factors on the rating level, that is, to assign corresponding rights to each influencing factor. A fuzzy subset of the weighting component set:

$$A = \{a_1, a_2, a_3, \ldots, a_n\}$$ \hspace{1cm} (7)

Where \(a_i\) is the weight corresponding to the \(i\)-th factor \(u_i\) in \(U\) set, and specifies \(\sum_{i=1}^{n} a_i = 1\).

Where \(A\) is the weight corresponding to the \(i\)-th factor \(D\) in the \(B\) set.

Combining the fuzzy distribution of the weighted subset \(A\) with the fuzzy matrix \(R\) of each factor of the object being judged, a fuzzy comprehensive evaluation can be performed:

$$B = A \cdot R$$ \hspace{1cm} (8)

Substituting \(A\) and \(R\) into the above formula, find:

$$B = \{b_1, b_2, b_3, \ldots, b_m\}$$ \hspace{1cm} (9)

Thus, the fuzzy subset \(B\) of the evaluation set \(V\) is obtained, and the position of the person with the highest sub-vector value \(b_j\) in \(V\) is the requested evaluation level.

The set of factors \(U\) for judging rockburst intensity are:

$$U = \{T, R, C, B', W_e\} = \{u_1, u_2, u_3, u_4, u_5\}$$ \hspace{1cm} (10)
Table 1. Evaluation criteria for rockburst severity classification

| Index | No rockburst | Slight rockburst | Medium Rockburst | Strong rockburst |
|-------|--------------|-----------------|-----------------|-----------------|
| T     | <0.3         | 0.3~0.5         | 0.5~0.8         | >0.8            |
| R     | <0.2         | 0.2~0.3         | 0.3~0.55        | >0.55           |
| C     | <0           | 0~9             | 9~12            | >12             |
| B'    | <0           | 0~13.3          | 13.3~25.5       | >25.5           |
| W_e   | <40          | 40~100          | 100~200         | >200            |

Table 2. U values of various lithological factor sets

| Lithology           | T    | R    | C    | B'   | W_e  |
|---------------------|------|------|------|------|------|
| Hornblende tonalite | 0.49 | 0.40 | 8.5  | 34.11| 58.51|
| Monzogranite        | 1.48 | 1.17 | 12.4 | 29.78| 92.19|
| Biotite granulite   | 0.58 | 0.49 | 9.7  | 34.07| 118.66|
| Sericite granite    | 0.42 | 0.35 | 7.7  | 32.32| 181.79|
| Sericite            | 1.08 | 0.92 | 11.8 | 33.49| 105.96|
| Footwall granite    | 1.68 | 1.42 | 12.8 | 31.60| 70.25|

The above four indexes respectively graded the rockburst intensity. Based on this, the classification criteria of individual criteria are appropriately adjusted, and the rockburst severity levels are uniformly divided into 4 levels: no rockburst, slight rockburst, medium rockburst, and strong rockburst, that is:

\[ V = \{ \text{no rockburst, slight rockburst, medium rockburst, strong rockburst} \} \]

According to the above individual judgment criteria, the four grade limit values corresponding to the rockburst intensity at each level and the influencing factors are listed in Table 1.

There are many ways to determine the membership function. Statistical analysis of each evaluation index value. According to its distribution characteristics, k times parabolic distribution is selected as the membership function of each evaluation index for the division of rockburst intensity. The standard equation is as follows:

\[
r_1(x_i) = \begin{cases} 1 & x_i \leq a_i \\ \frac{(b_i-x_i)^k}{(b_i-a_i)^k} & a_i < x_i < b_i \\ 0 & x_i \geq b_i \end{cases} \quad (12)
\]

\[
r_2(x_i) = \begin{cases} 1 & a_i \leq x_i \leq b_i \\ \frac{(b_i-x_i)^k}{(x_i-a_i)^k} & x_i > b_i \end{cases} \quad (13)
\]

\[
r_3(x_i) = \begin{cases} 1 & b_i \leq x_i \leq c_i \\ \frac{(c_i-x_i)^k}{(c_i-b_i)^k} & x_i > c_i \end{cases} \quad (14)
\]
Corresponding values of the five impact indicators were measured through field surveys and laboratory tests. According to the critical values of the evaluation criteria listed in Table 1 and the factor set listed in Table 2, substituting them into the above four formulas respectively, a fuzzy relation matrix \( R \) of 5 index factors can be obtained.

According to the importance of each index factor, each factor can be given a corresponding weight. According to the previous engineering data for the specific situation of this study, determine the weight distribution of the main index factors of rockburstT, R, C, B, and \( W_e \) the weight vector A:

\[
A = \{0.2, 0.2, 0.2, 0.15, 0.25\} 
\]

(16)

Comprehensively consider the weight and membership of each factor in the evaluation factor set U, and use the weighted average model to make a fuzzy comprehensive evaluation of rockburst tendency:

\[
B = A \cdot R = \{b_1, b_2, b_3, b_4\} 
\]

(17)

In the formula: B is the fuzzy comprehensive judgment result of the rockburst inclination value of surrounding rocks \( b_1, b_2, b_3, b_4 \) respectively correspond to the tendency values of no rockburst, slight rockburst, medium rockburst, and strong rockburst.

The fuzzy comprehensive evaluation set is also a fuzzy subset on the evaluation set V. According to the established evaluation set and factor evaluation criteria, it is clear that the closer B is to 1, this indicates that the surrounding rocks are more inclined to the intensity. The comprehensive evaluation results are sorted according to size. The position of the largest B in V corresponds to the rockburst intensity tendency of the surrounding rock on the basis of fully considering the weight of each evaluation index and membership. The prediction results are as follows:

| Lithology     | T   | R   | C   | B   | \( W_e \) | Fuzzy |
|--------------|-----|-----|-----|-----|----------|-------|
| Hornblende tonalite | slight | medium | slight | strong | medium | slight |
| Monzogranite   | strong | strong | strong | strong | medium | strong |
| Biotite granulite | medium | medium | medium | strong | medium | medium |
| Sericite granite | slight | medium | slight | strong | strong | medium |
| Sericite       | strong | strong | medium | strong | strong | strong |
| Footwall granite | strong | strong | strong | strong | strong | strong |

Rockburst prediction research is based on in-situ ground stress measurement data and laboratory physical and mechanical test results. Generally speaking, rockbursts are more likely to occur. Because the volume of the rock is smaller than that of the rock mass, and the joints have fewer joints and cracks, the strength measured in the laboratory test sample should be greater than the strength of the rock mass, so the actual rockburst tendency of the surrounding rock should be smaller than predicted.

4. Conclusion
Multi-criteria analysis results of rockburst propensity in deep rock masses show that Turchaninov criterion, Russense criterion, and Tao Zhenyu consider both rock stress state and rock mechanical properties, and the prediction results are similar. The criterion of strength brittleness coefficient only considers the mechanical properties of the rock itself. The linear elastic energy criterion, elastic strain energy storage index criterion and impact energy coefficient criterion predict rockbursts from an energy perspective, and the prediction results are relatively close.
The results of fuzzy comprehensive prediction show that feldspar, sericite, and lower plate granite have a strong tendency to rockburst. The burial depth of these three types of rocks is more than 700m. As the depth increases, the ground pressure increases significantly, increasing the possibility of rockbursts.

Acknowledgments
This work was financially supported by National Key Research and Development Project NO. 15010051.

References
[1] He Manchao, Miao Jingli , Feng Jili. Rockburst process of limestone and its acoustic emission characteristics under true — triaxial unloading conditions,J. International Journal of Rock Mechanics & Mining Sciences, 2010, 47(2):286—298
[2] Kaiser P K, Tannant D D, McCreath D R. Canadian Rockburst Support Handbook,S. Sudbury: Geomechanics Research Centre, Laurentian University, 1996.
[3] Tezuka K,Niitsuma H.Stress Estimated Using Microseismic Clusters and Its Relationship to the Fracture System of the Hijiori Hot Dry Rock Reservoir,J. Engineering Geology, 2000, 56( 1/2) : 47—62.
[4] Russnes B F. Analyses of Rockburst in Tunnels in Valley Sides M Sc . Thesis,J. Trondheim: Norwegian Inst. of Technology, 1974.
[5] TAN Yi’an. Discussion on energy impact index of rockbursting rock,J. Hydrogeology and Engineering Geology, 1992, 19(2): 10-12.(in Chinese)
[6] KIDYBIŃSKI A. Bursting liability indices of coal,J. International Journal of Rock Mechanics and Mining Sciences, 1981, 18(6): 295–304.
[7] TAN Yi’an. Application of comprehensive assessment using fuzzy mathematics to rockburst prediction in underground caverns,C.Proceedings of the 2nd National Congress of Chinese Society for Rock Mechanics and Engineering. Beijing: Knowledge Press , 1989:247 – 253.(in Chinese)
[8] WANG Yuanhan, LI Wodong, LI Qiguang, et al. Method of fuzzy comprehensive evaluations for rockburst prediction,J. Chinese Journal of Rock Mechanics and Engineering, 1998, 17(5): 493 – 501.(in Chinese)