A broad pseudovector glueball from holographic QCD

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We study the decay of a pseudovector glueball with quantum numbers $J^{PC} = 1^{+}$, the lightest glueball with spin different from zero or two, in the top-down holographic Witten-Sakai-Sugimoto model. This glueball is dual to the string-theoretic Kalb-Ramond tensor field, which completely fixes all its couplings by gauge invariance and $D$-brane anomaly inflow. The dominant decay channels are determined by a Chern-Simons-like action for the flavor branes; couplings from the Dirac-Born-Infeld action turn out to be subdominant. While the resulting width is parametrically of order $N^{-2} \lambda^{-1}$ for $N$ colors and 't Hooft coupling $\lambda$, when extrapolated to $N = 3$ and with $\lambda$ matched to experimental data, the prediction is that the pseudovector glueball is an extremely broad resonance.

I. INTRODUCTION

Glueballs are a cornerstone prediction of quantum chromodynamics (QCD)\(^1\). Their spectrum has been studied extensively in quenched lattice QCD up to and including spin $J = 6$\(^2\)\(^3\)\(^4\), with results involving dynamical quarks becoming slowly available, too, mostly showing only moderate effects from unquenching \(^5\). Experimentally, however, the situation remains unclear even for the lightest (scalar) glueball state, where phenomenological models continue to be divided on which of the two isoscalar mesons $f_{0}(1500)$ and $f_{0}(1710)$ is to be considered a glueball and to which extent there is mixing with $q\bar{q}$ states \(^6\)\(^7\)\(^8\), even though the two mesons have distinctly different decay patterns. The question of experimental candidates for the next lightest (tensor) glueball, for which lattice QCD predicts a mass about roughly 2.4 GeV, is at least as unclear \(^10\)\(^11\). For the pseudoscalar glueball no suitable candidate appears to be available that would be in the mass range around 2.6 GeV predicted by (quenched) lattice QCD. Of course the hope is that ongoing as well as future experiments, such as the PANDA experiment at FAIR \(^12\), will eventually provide fresh data and shed light on this long-standing issue.

Unfortunately, information on experimental signatures such as width and decay patterns are not available from the mentioned lattice QCD studies, while phenomenological models typically come with a large number of parameters and corresponding uncertainties\(^1\).

A new approach to the question of glueball decay properties has been pioneered in Ref. \(^13\) based on the top-down string-theoretic construction of a gauge/gravity dual to (large-$N_c$) QCD with chiral quarks due to Witten \(^14\) and Sakai and Sugimoto \(^15\), which turns out to reproduce remarkably well, often quantitatively correct within 10–30\%, the known features of low-energy QCD \(^16\), while involving only one free dimensionless parameter – the ‘t Hooft coupling at the mass scale of the model.

In Ref. \(^19\) the calculation of glueball decay rates in the Witten-Sakai-Sugimoto (WSS) model has been revisited for scalar and tensor glueballs. With finite quark masses included, the decay pattern of the predominantly dilaton scalar glueball could be matched to those of the experimental candidate $f_{0}(1710)$ \(^20\) if the glueball coupling to the mass term of quark-antiquark mesons is such as to lead to small $\eta \eta'$ decay rates \(^21\). The lightest tensor glueball, on the other hand, is predicted to be a very broad state with $\Gamma/M \sim 0.5$ for $M \sim 2.4$ GeV \(^19\)\(^22\) and may therby have escaped experimental identification so far.

For the pseudoscalar glueball, however, a sharp state with very restricted decay patterns as well as production channels has been predicted in \(^23\) within the WSS model, where this glueball is represented by a Ramond-Ramond one-form potential with a central role in the realization of the U(1)\(_A\) anomaly.

In the present work, we consider the pseudovector $J^{PC} = 1^{+}$ glueball which is dual to the string-theoretic Kalb-Ramond two-form field, whose coupling to quark-antiquark states in the WSS model is completely determined by gauge invariance and anomaly inflow arguments \(^24\) at the leading order.

II. THE Kalb-Ramond FIELD IN THE WITTEN-SAKAI-SUGIMOTO MODEL AS PSEUDOVECTOR GLUEBALL

An approximation to a holographic dual of low-energy large-$N_c$ Yang-Mills theory that is four-dimensional below the Kaluza-Klein scale $M_{KK}$ is given by Witten’s model \(^14\), which is obtained as the near-horizon geometry of a stack of $N_c$ D4-branes compactified in its coordinate $x^4$ on a circle with radius $R_4 = 1/M_{KK}$ with supersymmetry-breaking boundary conditions. This can be obtained by dimensional reduction of an AdS\(_7\) $\times S^4$ solution to eleven-dimensional supergravity with non-zero four-form flux $F_4$, where $x^{11}$ is similarly compactified, with radius $R_{11} = g_s l_s$, where $g_s$ and $l_s = \sqrt{\alpha'}$ are string coupling and length, respectively. The 11-dimensional

\(^1\) Comparatively specific predictions for the decay patterns of the pseudoscalar and vector glueball have been made, however, in Ref. \(^13\) and \(^12\), respectively, albeit with undetermined total width.
line element is given by
\[ ds^2 = \frac{r^2}{L^2} \left[ f(r) dx^2 + \eta_{\mu\nu} dx^\mu dx^\nu + dx^2_{11} \right] + \frac{L^2}{r^2 f(r)} dr^2 + \frac{L^2}{4} d\Omega_4^2, \]
with \( f(r) = 1 - \frac{r_0^6}{r_{KK}^6} \), where \( L \) is the AdS radius, \( r \in [r_{KK}, \infty) \) is the holographic coordinate bounded from below by \( r_{KK} = M_{KK} L^2/3 \), and \( d\Omega_4^2 \) is the line element of a unit four-sphere. Greek indices indicate 4d Minkowski space with metric \( \eta_{\mu\nu} \).

The spectrum of glueballs of the Witten model can be obtained by solving the equations of motion of the bosonic sector of eleven-dimensional supergravity linearized around the above background \([22]\). This yields towers of glueballs with quantum numbers \( J^{PC} = 0^+, 2^+, 0^-, 1^-, 1^+ \) and masses proportional to \( M_{KK} \). Pseuodvector glueballs with \( J^{PC} = 1^+, 3^+ \) correspond to fluctuations of the three-form field \( A_3 \) which after dimensional reduction yields the Kalb-Ramond two-form field \( B_2 \) of type-IIA string theory. Inserting the ansatz
\[ B_{\mu\nu} = A_{\mu\nu11} = r^3 N_4(r) \tilde{B}_{\mu\nu}(x^0, x), \]
\[ A_{\mu4} = 6 r^2 N_4(r) \epsilon^{a_1b_1c_1d_1} \eta_{\mu(a_1} \partial_{\nu)b_1} \tilde{B}_{c_1d_1}, \]
with \( \epsilon^{a_1b_1c_1d_1} \tilde{B}_{c_1d_1} = 0 \) into the field equations of the 11-dimensional action
\[ 2k_1^2 L_{11}^{(b)} \supset -\frac{\sqrt{-g}}{2 \cdot 4!} F_{a_1...a_4} F_{a_1...a_4} + \frac{\sqrt{g_{4!}}}{2 \cdot 4!} \epsilon^{a_1...a_7} A_{a_1...a_3} F_{a_4...a_7}, \]
where \( 2k_1^2 = (2\pi)^8 l_9^6 g_s^4 \) and the indices \( a_1 \) refer to AdS_7 space, one obtains the mode equation \([23]\)
\[ \frac{d}{dr} r^6 \frac{d}{dr} N_4(r) - (L^4 M^2 r^3 - 27 r^5 + \frac{9 r_{KK}^6}{r}) N_4(r) = 0. \]
With the boundary condition \( N_4(r_{KK}) = 0 \) normalizable modes exist for a discrete mass spectrum with lowest value \( M \approx 2.435 M_{KK} \). Integrating \([3]\) over the radial direction and the sphere \( S^4 \), the four-dimensional effective Lagrangian reads
\[ \mathcal{L}_4 = -\frac{1}{2} \mathcal{C}_B \eta^{a_1b_1} \eta^{c_1d_1} \tilde{B}_{a_1b_1}(M^2 - \square) \tilde{B}_{c_1d_1} + \ldots \]
where
\[ \mathcal{C}_B = R_{11} R_4 L^7 \frac{\pi^4}{3k_1^2} \int dr^3 N_4(r)^2. \]
Setting \( \mathcal{C}_B = 1 \), which yields the condition
\[ N_4(r_{KK})^{-1} = 0.00983838 L^3 \lambda^\frac{2}{3} N_c M_{KK}, \]
corresponds to a canonical normalization of the Kalb-Ramond field, whose transverse polarizations we parametrize by
\[ \hat{B}_{\mu\nu} = \frac{1}{\sqrt{\rho}} \eta^\rho_{\mu} \eta^\kappa_{\nu} \epsilon_{\rho\mu\kappa} \epsilon_\rho \partial_\rho \tilde{V}(\epsilon)(x), \]
where \( \epsilon_\rho \) is a unit spacelike vector. The hermitean field \( \tilde{V}(\epsilon)(x) \) is then canonically normalized, \( \mathcal{L}_4 = -\frac{1}{2} \tilde{V}(\epsilon)(M^2 - \square) \tilde{V}(\epsilon) + \ldots \).

Interactions with quark-antiquark states are introduced by using the construction of Sakai and Sugimoto, which places a stack of \( N_f \) probe DBS-D8-branes in the background described above, located at antipodal points on the \( x^4 \) circle \([17, 18]\). The nonabelian chiral symmetry \( U(N_f)_L \times U(N_f)_R \) is broken spontaneously to \( U(N_f)_L + R \) by the fact that the branes have to connect in the bulk (at \( r = r_{KK} \) for the antipodal embedding); the axil symmetry \( U(1)_A \) is broken by the Witten-Veneziano mechanism \([17, 24, 27]\). The gauge fields \( A_M \) living on the flavor branes are dual to mesonic states. Fluctuations of the background translate into fluctuations of the probe branes, and therefore effective interactions may be derived from the nine-dimensional worldvolume action of the latter. It consists of the Dirac-Born-Infeld (DBI) action
\[ S_{\text{DBI}} = -T_8 \text{Str} \int d^9 x \ e^{-\phi} \sqrt{-\text{det}(g_{MN} + F_{MN})}, \]
where \text{Str} denotes the symmetrized trace, and a Chern-Simons action involving the Ramond-Ramond fields \( C_j \) of type-IIA string theory \([24]\)
\[ S_{\text{CS}} = T_8 \text{Tr} \int e^{\phi} \wedge \sum_j C_{2j+1}, \]
with \( T_8 = (2\pi)^{-8} l_9^{-8} \), \( F_{MN} = 2\pi l_9^2 F_{MN} + B_{MN} \), where \( F_{MN} \) are the components of the \( U(N_f) \) field strength \( F_2 = dA_1 - i A_1 \wedge A_1 \), and \( B_{MN} \) the components of the Kalb-Ramond field \( B_2 \). Upon partial integration, the Chern-Simons action contains a term linear in \( B_2 \), which after integration over \( S^4 \) gives rise to the 5-dimensional integral
\[ S_{\text{CS}}^{(B_2)} = \frac{1}{2} (2\pi s_9)^2 T_8 \int L^3 \pi^2 g_s^{-1} \text{Tr} A_1 \wedge F_2 \wedge B_2. \]
To derive an effective interaction Lagrangian we also need the explicit form of the fluctuations for the gauge field, which for \( N_f = 3 \) are given as usual \([17]\) by
\[ A_Z = \frac{r_{KK}^2}{2L} \phi_0(Z) \Pi^a T^a, \quad A_\mu = \psi_1(Z) V^a_\mu T^a, \]
2 In the following we switch from the 11-dimensional metric to the 10-dimensional string frame metric.
3 Here we are dropping the A-roof genus curvature contributions corresponding to mixed gauge-gravitational anomalies which would give rise to a vertex between two tensor glueballs and a flavor singlet meson \([25]\).
TABLE I. Results for the decay rates of a pseudoscalar glueball of mass \( M = 2311 \text{ MeV} \) into pseudoscalar and vector mesons as determined by Eq. \( \text{(13)} \).

| decay channel | \( \Gamma/M \) |
|---------------|-------------|
| \( \pi \rho \)  | 0.3624 \ldots 0.4803 |
| \( KK^* \)     | 0.1945 \ldots 0.2578 |
| \( \eta \omega \) | 0.0530 \ldots 0.0941 |
| \( \eta \phi \)  | 0.0086 \ldots 0.0076 |
| \( \eta' \omega \) | 0.0168 \ldots 0.0203 |
| \( \eta' \phi \)  | 0.0020 \ldots 0.0079 |
| \( \pi \rho \rho \) | 0.2595 \ldots 0.4556 |
| \( \pi K^*K^* \) | 0.0213 \ldots 0.0375 |
| \( KK^* \rho \)  | 0.0032 \ldots 0.0056 |
| \( KK^* \omega \) | 0.0011 \ldots 0.0019 |
| total          | 0.9225 \ldots 1.3685 |

With eight generators of SU(3) multiplying pseudoscalar and vector octets, \( \Pi^a \equiv \Pi^a(x^a) \) and \( V^a_\mu \equiv V^a_\mu(x^a) \), satisfying \( \text{Tr} T^a T^b = \delta^{ab} \), \( T^a = \frac{i}{2} \lambda^a \) for \( a \in \{1, \ldots, 8\} \); defining \( T^0 = \frac{1}{\sqrt{3}} \mathbb{1} \) the singlet component of the pseudoscalar/-vector meson nonet are included as \( \Pi^0 \equiv \eta_0 \) and \( V^0 \equiv \omega_0 \). The coordinate \( Z \) is defined by \( Z^2 = r^2 / r_{KK}^2 - 1 \), and parametrises the probe brane pairs along the holographic direction from \( Z = -\infty \) to \( +\infty \). In the chiral WSS model, the pseudoscalar octet states are massless whereas the vector mesons form a tower of massive states with alternating parity beginning with \( m_1 = \sqrt{0.699 M_{KK}} \) which sets the scale \( M_{KK} = 949 \text{ MeV} \) when \( m_1 \) is identified with the mass of \( \rho \) and \( \omega \) vector mesons.

Inserting these modes into Eq. \( \text{(13)} \) and integrating out all other dimensions, we obtain the four-dimensional interaction Lagrangian

\[
L_{4, \text{int}}^{CS} \supset b_1 (\Pi^a \partial_\mu V^a_\mu + V^a_\mu \partial_\mu \Pi^a) \tilde{B}^{\mu \nu} - i b_2 \text{Tr}(T^a \{ T^b, T^c \}) \Pi^a V^b_\nu V^c_\nu \tilde{B}^{\mu \nu},
\]

where \( \tilde{B}^{\mu \nu} = \frac{1}{2} \epsilon^{\mu \nu \rho \sigma} B_{\rho \sigma} \),

\[
b_1 = 4C \int r^3 \phi_0 \psi_1 N_4 dZ = 56.027 N_4^{-\frac{1}{2}} \lambda^{-\frac{1}{2}},
\]

and

\[
b_2 = 6C \int r^3 \phi_0 \psi_1 N_4 dZ = 2571.72 N_4^{-\frac{1}{2}} \lambda^{-1},
\]

with \( C = T_8 (2\pi^2 r_{KK} L \pi)^2 / (16 g_s) \).

For an on-shell pseudovector glueball with polarization \( \varepsilon \) one has \( B_{\mu \nu} = \frac{1}{2} (\varepsilon_\mu \partial_\nu - \varepsilon_\nu \partial_\mu) V^{(c)} \). The decay rates for \( V^{(c)} \) into various combinations of pseudoscalar and vector mesons may be evaluated by performing a phase space integral over the corresponding amplitudes and summing/averaging over polarisations of outgoing/ingoing particles. The results are summarised in Table I where we list dimensionless ratios of decay rates to the pseudovector glueball mass for all channels. The glueball mass is taken to be the WSS model result \( M \approx 2311 \text{ MeV} \), \( N_c = 3 \) and we interpolate between two different fits for the ’t Hooft coupling given by \( \lambda = 16.63 \ldots 12.55 \). In the phase space integral we used experimental meson masses \( \text{[29]} \) and thus decays into some combinations of heavier vector mesons than the ones listed in the table are excluded kinematically.

Additionally we have used the relations

\[
\eta = \eta_8 \cos \theta_P - \eta_0 \sin \theta_P
\]

\[
\eta' = \eta_8 \sin \theta_P + \eta_0 \cos \theta_P,
\]

with the mixing angle resulting from combining bare quark masses (introduced e.g. through worldsheet instantons \( \text{[30]} \)) with the Witten-Veneziano mechanism \( \text{[22]} \)

\[
\theta_P = \frac{1}{2} \arctan \left( \frac{2\sqrt{2}}{2} \frac{m_0^2 / (m_K^2 - m_0^2)}{1 - \frac{2}{3} m_0^2 / (m_K^2 - m_0^2)} \right).
\]

Moreover we have allowed for a mixing of \( \omega \) and \( \phi \) for simplicity with ideal mixing angle \( \theta_V = \arctan(1/\sqrt{2}) \) \text{[31]}. In principle there are also numerous nonvanishing interaction vertices arising from the DBI action of Eq. \( \text{(9)} \). However, as we show in Appendix A they are suppressed parametrically with order \( N_c \lambda^{-2} \) as well as by their numerical value as compared to the contributions from the Chern-Simons term. Besides vertices from terms linear in \( B^{\mu \nu} \) (from higher-order terms in the DBI action), there is also a quadratic term \( \propto \tilde{B}^{\mu \nu} \) from the lowest-order part of the DBI action which has the form of a correction to the mass term of the pseudovector glueball proportional to \( \lambda^2 N_f / N_c \). As previously in Refs. \( \text{[15, 19–21, 23]} \), such corrections to the masses of glueballs are neglected because the decay amplitude derived from the (probe) brane action is itself a quantity of order \( N_f / N_c \) so that the effects of these corrections on glueball decay rates are formally of higher order; the inclusion of such mass corrections would have to be considered together with backreactions of the flavor branes on the supergravity background (e.g. along the lines of Ref. \( \text{[32]} \)), which is beyond the scope of the present paper. (At any rate, as we discuss further in the Appendix, these terms quadratic in \( B^{\mu \nu} \) turn out to be numerically small for our choice of parameters and would only slightly increase our results for the decay rate.)

\footnote{4} If one were to extrapolate the \( 1^{+-} \) glueball mass to values up to \( \sim 3 \text{ GeV} \) as indicated by lattice QCD studies \( \text{[2]} \), the decay width would increase significantly, by a factor of up to \( \sim 2.5 \).
III. DISCUSSION

In previous work [20, 21] we have found that the decay rate and the branching ratios of the predominantly dilatonic scalar glueball calculated in the WSS model with finite quark masses added match remarkably well with existing data for the experimental scalar glueball candid- dates, the isoscalar $f_0(1710)$, which some recent phenomenological models [17, 18] also favor as a meson with dominant glueball content. The coupling of the tensor glueball to quark-antiquark states, which is parametrically of the same order, turns out, however, to lead to a numerically large decay width with $\Gamma/M \sim 0.5$ for $M \sim 2.4$ GeV [19, 22], which is perhaps too large to fit existing experimental candidates among $f_2$ mesons. More recently, two of us have calculated the decay pattern of the next heavier glueball, a pseudoscalar, which is dual to the $C_1$ form field which plays a central role for the $U(1)_A$ anomaly in the WSS model and the appearance of a Witten-Veneziano mass for $\eta$ and $\eta'$ mesons. Here the WSS model predicts a very narrow state [23].

In the present work, we have considered the lightest glueball with spin different from zero or two, a pseudovector which in the WSS model is dual to the Kalb-Ramond two-form field $B_{\mu\nu}$. This has direct couplings to quark-antiquark mesons through the action of D8 branes, which are completely determined by gauge invariance and anomaly inflow arguments [23]. The latter prescribe the structure of the Chern-Simons part of the action that turns out to provide the leading contributions for the decay amplitude of the pseudovector glueball. With the usual set of parameters of the WSS model, the decay width of this glueball turns out to be extremely large, $\Gamma/M \sim 1$ (or even larger when $M$ is extrapolated to the higher values indicated by lattice QCD). This suggests that it would be very difficult to identify the pseudovector glueball in the hadron spectrum (as a separate $h_1$ meson).

By contrast, the vector glueball $J^{PC} = 1^{−+}$, which is carried by the Ramond-Ramond 3-form field [22], is stable to leading order of the glueball decay calculations in the WSS model, and would therefore be predicted to be a narrow resonance (if higher-order corrections to the model are small) [3].

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Appendix A: Pseudovector couplings from the DBI action

Here we calculate the coupling constants of the pseudovector glueball to 3 mesons obtained from the DBI action

\[ S_{\text{DBI}} = -T_8 \int d^9 x e^{-\phi} \text{STr} \sqrt{-\det(g_{MN} + F_{MN})} \]

\[ = -T_8 \int d^9 x e^{-\phi} \sqrt{-g} \text{STr} \left[ 1 + \frac{1}{2} \text{tr} \left( \frac{1}{2} (g^{-1} F)^2 \right) + \frac{1}{3} (g^{-1} F)^3 - \frac{1}{4} (g^{-1} F)^4 \right] + O(\mathcal{F}^5), \]

(A1)

where STr involves symmetrization of the $U(N_f)$ matrices and $\text{tr}$ refers to spacetime indices. Only the term of fourth order in $\mathcal{F}$ contains a vertex of a pseudovector glueball with quark-antiquark states. In the absence of additional metric or dilaton fluctuations, the second-(order mixing) term vanishes because of transversality of $B_{\mu\nu}$. The cubic term is also zero if at least one $\mathcal{F}$ is a singlet because $\text{Tr} (\text{tr} (\mathcal{F} F F)) = \text{tr} (T^P T^P)$ with $T$ being the symmetric matrix $T_{MN} = F_{MN} g^{OP} F_{PN}$.

To estimate the importance of these vertices we set $N_f = 1$ and expand the relevant coupling terms

\[ L_{4,\text{int}} = \tilde{b}_1 F^{\rho\sigma} F_{\rho\sigma} F^{\mu\nu} \tilde{B}_{\mu\nu} + \tilde{b}_2 F^{\rho\sigma} F_{\rho\sigma} F^{\mu\nu} \tilde{B}_{\mu\nu} + \tilde{b}_3 \delta^{\rho\sigma} \Pi_{\rho\sigma} T^{\mu\nu} \tilde{B}_{\mu\nu} + \tilde{b}_4 V^{\rho\sigma} \delta^{\mu\nu} T^{\rho\nu} \tilde{B}_{\mu\nu} + \tilde{b}_5 V^{\rho\sigma} F^{\mu\nu} \tilde{B}_{\mu\nu}, \]

(A2)

where $F^{\mu\nu} = \partial^{\mu} V^{\nu} - \partial^{\nu} V^{\mu}$ with coupling constants

\[ \tilde{b}_1 = 36 D \int dZK^{\frac{3}{2}} \psi_1^3 N_4 \]

\[ = 0.000375351 N_4 e^{-\frac{3}{4} \lambda^2}, \]

\[ \tilde{b}_2 = -\frac{b_1}{4}, \]

\[ \tilde{b}_3 = -\frac{D}{18} M_{KK}^6 L^6 \int dZK^{\frac{1}{2}} N_4 \phi_1^2 \psi_1 \]

\[ = -0.000168453 N_4 e^{-\frac{3}{4} \lambda^2}, \]

\[ \tilde{b}_4 = 2 D M_{KK}^4 L^3 \int dZK^{\frac{1}{2}} N_4 \phi_0 \partial Z \psi_1 \]

\[ = -0.00875721 N_4 e^{-\frac{3}{4} \lambda^2}, \]

\[ \tilde{b}_5 = -18 D M_{KK}^2 L \int dZK^{\frac{1}{2}} N_4 \psi_1 (\partial Z \psi_1)^2 \]

\[ = 1.78464 N_4 e^{-\frac{3}{4} \lambda^2}, \]

(A3)

where $D = L^3 N_c/(576 M_{KK}^2 \pi^2)$ and $K = 1 + Z^2$.

These vertices, which couple a pseudovector glueball to three quark-antiquark mesons, are suppressed by an extra factor $\lambda^{-1}$ compared to [15], as well as by a much smaller numerical prefactor; they are therefore negligibly small compared to the vertices obtained from the CS action.

5 Glueballs with spin above two are beyond the supergravity description and require string-theoretic calculations within the WSS model [23].
As mentioned in the text, besides linear terms in $B_{\mu \nu}$, the DBI action also contains a term proportional to $B_{\mu \nu} B^{\mu \nu}$ which would appear as a correction arising from the flavor branes to the mass term of the four-dimensional effective Lagrangian resulting from the 10-dimensional supergravity action. Explicitly, it reads

$$\delta L_4 = -\frac{1}{4} \delta M^2 \eta^\mu \eta^\nu \tilde{B}_{\mu \nu},$$

$$\delta M^2 = 0.0035066 \frac{N_f}{N_c} \lambda^2 M_{KK}^2.$$  \hspace{1cm} (A4)

Taken at face value, this would mean an increase of the pseudovector glueball mass by 8%, from 2411 to 2416...2493 MeV, for our choice of parameters. However, this contribution from the flavor branes, is localized in the 10-dimensional bulk. A complete calculation would require a corrected mode equation for the (bulk) Kalb-Ramond field on a 10-dimensional background with order $N_f/N_c$ backreaction terms along the lines of Ref. [22]. The numerical smallness of the partial result [21] seems to indicate, however, that such an inclusion of backreaction effects may be carried out in a perturbative manner even for parameters which correspond to extrapolations to QCD.

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