Topology optimization of electrical devices using Gaussian filter

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Abstract. In this paper, a novel topology optimization method, in which gaussian filter is selected as spatial smoothing method, is presented. By changing standard deviation parameter, strength of filtering operation can be adjusted in the present method. Thanks to this, optimum solution, shape of which has multi-layer shield structure and multi-layer flux barrier, can be obtained. To validate the effectiveness, the present method is applied to shape optimization problem of magnetic shield and synchronous reluctance motor. From the results, it can be seen that the present method can get better solutions than that of the conventional method.

Keywords: Topology optimization, Synchronous reluctance motor, Magnetic shield, Filtering

1. Introduction

In recent years, several studies have investigated topology optimization, wherein optimization technologies are linked with magnetic field analyses owing to the improving capabilities of computer-aided methods. Topology optimization is a technique that can elicit drastic improvements in machine performance and yield solution-oriented tools via assessment of the findings of the designer [1–7]. However, the organization of a topology requires a large number of design variables as compared to that required for parameter optimization, which in turns results in several manufacturing issues.

In previous studies, the above mentioned issues have been addressed via three major approaches:

1. Definition of a shape function with a smooth distribution of a normalized Gaussian function and a level set function, and alteration of the shapes along this function distribution thereafter [1–5]

2. Setting of a low design tolerance by limiting the design surface to the periphery of the shape boundary [6]
3. Elimination of porous shapes using spatial filtering [7].

The shape function used in approach 1 has a smooth space distribution; thus, the changes in shapes owing to the function ensure realization of a smooth and easily manufactured shape. In contrast, this shape function limits the design variables in accordance with manufacturing considerations. This technique limits the changes in shapes with a high degree of freedom, which is the primary advantage of topology optimization. Approach 2 is effective for controlling porous shapes; however, similar to the case in approach 1, the degree of freedom in the shape expression is limited.

Considering the aforementioned factors, we herein examine topology optimization techniques based on approach 3. With the proposed technique, spatial filtering can be conducted without limiting the design variables, whereby design solutions can be manufactured easily. However, if the spatial smoothing function in the spatial filtering process is not adjusted properly, achieving the desired magnetic structures becomes difficult. For example, when spatial filtering is used in applications that require thin multi-layer structures, it is easier to achieve a thick magnetic shape with a smoothing function, similar to the case for magnetic shield topology optimization [8]. Therefore, in the proposed technique, a Gaussian filter is used as a spatial smoothing approach [9]. The Gaussian filter defines a window function along a Gaussian distribution with pre-set parameters. It is possible to adjust the strength or weakness of a smoothing function by adjusting the standard deviation parameter in the Gaussian distribution. Topology optimization techniques such as the Gaussian filter, whereby the spatial smoothing functions can be adjusted, are yet to be proposed, and it is also necessary to verify their efficacies before they can be used in electrical equipment.

In this study, we apply the proposed technique to the magnetic shield topology optimization problem [10] and test the solution search performance by comparing the obtained solution with conventional approach 3. In addition, we apply this method to the synchronous reluctance motor topology optimization problem [11] and examine its efficacy.

2. Optimization method

In this study, we investigate the two-dimensional magnetostatic field problem through a governing equation, which is described as follows:

$$\nabla \cdot \nabla A_z = J$$

Here, $A_z$ and $J$ are the vector potential z-direction component and external current density. In
addition, \( \nu \) is the magnetic reluctivity, which in this study is a constant because a linear magnetic material is subject to analysis. When a finite element method is used and formula (1) is assumed to be a discrete form, the following matrix equation is obtained.

\[
Ka = b
\]  

In formula (2), \( K \) is a finite element matrix, \( a \) is a nodal value of \( A \) subscript \( z \), and \( b \) is a right-side vector. During the optimization process, we compute the evaluation value of each individual by solving the matrix equation shown in formula (2).

### 2.1. Base algorithm

With the proposed technique, a GA-based optimization algorithm is constructed. Parent individuals are selected by tournament selection assuming a normal crossover or uniform crossover. An alternation of generations is applied through a roulette selection, and the filtering process is conducted when individuals are newly generated.

A) Initial individuals \( N_{pop} \) are generated;

B) The spatial filtering process is applied to the entire initial population;

C) Two parent individuals are selected through tournament selection, and one individual slave is then generated through a uniform crossover.

D) The filtering process is conducted on the individual slave generated;

E) Repeat steps C to D until a cross size of \( N_c \) slave individuals is generated;

F) \( N_{pop} \) individuals are selected using roulette selection from \( N_{pop} + N_c \) individuals, and the process migrates to the next generation.

### 2.2. Spatial smoothing

Spatial filtering is conducted through a convolution of the window function. A 5×5 size is selected for the kernel used in a convolution, which is defined through the following formula.

\[
\text{Filter} = \frac{1}{X} \begin{bmatrix}
g_{11} & g_{21} & g_{31} & g_{41} & g_{51} 
g_{12} & g_{22} & g_{32} & g_{42} & g_{52} 
g_{13} & g_{23} & g_{33} & g_{43} & g_{53} 
g_{14} & g_{24} & g_{34} & g_{44} & g_{54} 
g_{15} & g_{25} & g_{35} & g_{45} & g_{55}
\end{bmatrix}
\]  

In (3), \( g_{33} \) is the target element to which the filtering process is applied. The term multiplied by the physical property value of the neighbourhood element is calculated according to its distance from the applied target element. In this study, each term shown in (3) was calculated using the following formula with the kernel locally represented as in Figure 1.
In (4), we can see that the window function differs according to the standard deviation $\sigma$. Through the proposed technique, adjusting the smoothing effect of filtering becomes possible via modification of $\sigma$. However, none of the previous studies based on topology optimization techniques have been able to adjust the smoothing effect in filtering. Consequently, the proposed technique is regarded as superior in terms of its ability to meet design solution standards of various structures through adjustment of the smoothing effect. Equation (4) is based on a locally represented kernel, as shown in Figure 1.

Figure 1. Local coordinates

2.3. Smoothing effect of the spatial smoothing

Figure 2 shows the smoothing effect of the proposed technique. Given the Gaussian distribution per value of $\sigma$, we found that the smoothing effect becomes stronger when $\sigma$ is larger. The distribution at $\sigma = 3.0$, shown in Figure 2(c), is approximately the same as that with the averaging filter described in [7]. A description of the smoothing effect shown in Figure 2 with a window function is shown below.

\[
\text{Filter}^{\sigma_{0.3}} = \frac{1}{5.0} \begin{bmatrix}
0.00 & 0.00 & 0.01 & 0.00 & 0.00 \\
0.00 & 0.12 & 0.50 & 0.12 & 0.00 \\
0.01 & 0.50 & 2.00 & 0.50 & 0.01 \\
0.00 & 0.12 & 0.50 & 0.12 & 0.00 \\
0.00 & 0.00 & 0.01 & 0.00 & 0.00
\end{bmatrix}
\]  

(5-a)

\[
\text{Filter}^{\sigma_{0.8}} = \frac{1}{25.4} \begin{bmatrix}
0.42 & 0.75 & 0.92 & 0.75 & 0.42 \\
0.75 & 1.35 & 1.65 & 1.35 & 0.75 \\
0.92 & 1.65 & 2.00 & 1.65 & 0.92 \\
0.75 & 1.35 & 1.65 & 1.35 & 0.75 \\
0.42 & 0.75 & 0.92 & 0.75 & 0.42
\end{bmatrix}
\]  

(5-b)

\[
\text{Filter}^{\sigma_{3.0}} = \frac{1}{47.3} \begin{bmatrix}
1.79 & 1.87 & 1.89 & 1.87 & 1.79 \\
1.87 & 1.95 & 1.97 & 1.95 & 1.87 \\
1.89 & 1.97 & 2.00 & 1.97 & 1.89 \\
1.87 & 1.95 & 1.97 & 1.95 & 1.87 \\
1.79 & 1.87 & 1.89 & 1.87 & 1.79
\end{bmatrix}
\]  

(5-c)
In (5), when the window function receives a matrix calculation for all terms, it is normalized as 1. During the optimization process, the window function in (5) is convoluted with all design elements, and a magnetic body or air is designated according to the computed values, as per the following formula:

\[
M_e = \begin{cases} 
  \text{iron} & y_e \geq 0.5 \\
  \text{air} & y_e < 0.5 
\end{cases}
\]  

\[
y_e = \sum_i \sum_j g_{ij}(r,s)M_{ij}(r,s)
\]

In (6), \( M_e \) is the number of physical properties of the filtering target element, and \( y_e \) is the gene value of the target element used in the physical property determination defined for all elements within a design surface.

In the proposed technique, each time an individual is newly generated, (5) and (6) are used to apply spatial filtering.

3. Optimization problem of the magnetic shield

3.1. Optimization problem

Figure 3 shows the optimization problem. Considering the symmetry, 1/4 model of 2D domain is analyzed. The evaluation function is defined by the following formulas such that the average value of the magnetic flux density in the target area in the figure is minimized.

\[
F = \frac{|B|_{\text{avg}}}{B_T}
\]

\[
G = S_{\text{core}} < \frac{S_{\text{design}}}{2}
\]

In (7), \( B \) is the magnetic flux density of the elements within the target area. Moreover, \( B_T = 9.47E-4 \) is a normalized constant, and this value is selected as the average magnetic flux den-
sity in the target area with no magnetic shielding. In addition, $S_{\text{core}}$ is the total magnetic element area of the obtained shield shapes, and a constraint function is defined such that the magnetic shield area is less than 50% of the design region. The design surface was a 4.0. $\times$ 4.0 mm sized quadrilateral element divided into a uniform grid pattern.

Figure 3. Magnetic shield optimization problem [10]

### 3.2. Optimization condition and results

Table 1 lists the optimization parameters and objective function values of the obtained solutions. We selected five standard deviation parameter types to verify the smoothing effect adjustment functions, as shown in Eqn. (5), using the proposed technique. To eliminate the effects of random numbers, we changed the random number type 30 times and computed the results with the highest evaluation value. We then selected a 5 $\times$ 5 sized window function shown in Figure 1.

Figure 4 shows the changes in the evaluation of each elite solution generated during the optimization process. For $\sigma = 0.3$, which has hardly any space smoothing effect, the evaluation function is monotonically reduced. By contrast, a tendency to fall into the local solutions can be seen in the other cases. In the algorithm used for the proposed technique, a smoothing process is applied for the initially created individuals. In addition, the individuals newly created by a crossover again undergo a filtering process. In other words, because a repeated smoothing processing is conducted during the optimization process, there is a tendency for intergenerational individuals to gradually resemble one another.

Figure 5 shows the optimized shapes. The optimal objective function values are obtained when $\sigma = 0.3$, although many porous and floating core elements are seen, which further causes difficulty in attaining the desired shape. When $\sigma = 0.5$, a tri-layer shield structure is formed, and there are few porous core elements. This is more suitable for manufacturing than the case of $\sigma = 0.3$. For $\sigma = 3.0$, a bi-layer shield shape structure is formed, which has a
smaller number of layers and thereby an inferior evaluation as compared to the case where $\sigma = 0.5$. In other words, in terms of the coexistent manufacturability/performance, $\sigma = 0.5$ is an ideal design solution.

The main reason a tri-layer magnetic shield shape is not obtained with $\sigma = 3.0$ is the adjustment of the smoothing effect. Under conditions involving a strong smoothing effect, it is believed that the optimization results that are originally for a multilayer structure will tend toward those for a shield structure. An example of this is the outer-shell shield and intermediate layer, which are joined by filtering and have a particular thickness. As a result, it is believed that the three previous conditions will result in a bi-layer shield structure with a certain thickness. In contrast, it is believed that for $\sigma = 0.5$, the smoothing effect is lower than that for the three previous conditions, resulting in a thin tri-layer shield structure.

From the aforementioned factors, it can be inferred that when the smoothing effect is stronger, the likelihood to fall into the local solutions is higher, and obtaining a multi-layer shield structure is more difficult. In contrast, a certain degree of smoothing effect is required during manufacturing. In the optimization problem of the magnetic shield shape, because a manufacturable tri-layer shield structure can be obtained, $\sigma = 0.5$ is an ideal value.

| Table 1 Optimization parameters |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                               | Case A | Case B | Case C | Case D | Case E |
| GA                           | Generation number | 60    | ←     | ←     | ←     | ←     |
|                              | Population number | 80    | ←     | ←     | ←     | ←     |
|                              | Crossed population | 40    | ←     | ←     | ←     | ←     |
|                              | Mutation establishment [%] | 3.0   | ←     | ←     | ←     | ←     |
| Filtering                    | Deviation | 0.3   | 0.5   | 0.8   | 1.5   | 3.0   |
|                              | Kernel size | 5×5   | ←     | ←     | ←     | ←     |
| Results                      | $F$     | 4.30E-6 | 1.72E-5 | 3.28E-5 | 3.85E-5 | 3.42E-5 |
Figure 4. Changes in evaluation function

(a) $\sigma = 0.3$

(b) $\sigma = 0.5$

(e) $\sigma = 3.0$

Figure 5. Optimization results
3.3. Comparison between conventional smoothing methods

In this section, we describe the results of a comparison of proposed method with conventional techniques. The novelty of the proposed technique is that the smoothing effect can be adjusted. From this viewpoint, a comparison between the conventional and proposed methods can be discussed with and without smoothing effect adjustment functions. We applied the filtering process as a conventional technique with the formula below.

$$\text{Filter}^A = \frac{1}{25.0} \begin{bmatrix}
1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\
1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\
1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\
1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\
1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\
\end{bmatrix}$$  \hspace{1cm} (8)

Formula (8) corresponds to an averaging filter, and the kernel size $5 \times 5$, which is the same as that used in the proposed technique. The conventional technique had no smoothing effect adjustment function, and given that the proposed technique is capable of adjusting the smoothing effect according to a desired solution, it is assumed that a higher evaluation will be obtained.

Figure 6 shows the optimization results obtained using the conventional technique. The shield structure in the conventional technique is bi-layer, and we found that it has similar features to those of the proposed technique where $\sigma = 3.0$. This is mainly due to a stronger smoothing effect at work owing to the averaging filter. As a result, the proposed technique yields a higher evaluation value.

Figure 6. Optimization results obtained using a conventional method ($F = 4.30E^{-5}$)

4. Optimization problem of the synchronous reluctance motor

4.1. Optimization problem and results

Figure 7 shows the optimization problem. Considering the symmetry, a quarter region of the
entire model is analyzed. The evaluation function is assumed to be the maximization of the average torque, and we defined the objective function using the following formula:

\[ F = T_{\text{ave}} \rightarrow \text{max.} \]  

In (9), \( T_{\text{ave}} \) indicates the average torque. Table 2 summarizes the optimization parameters and optimization results. For the optimization results, we changed the random number seed 15 times, and noted the highest evaluation value. From Table 2, we found that the optimized solution obtained with \( \sigma = 0.8 \) enables the highest evaluation. Figure 8 shows a comparison of the optimized shapes with conditions \( \sigma = 0.3, 0.8, \) and 1.5. Even in the synchronous reluctance motor optimization problem, a tendency similar to that of the magnetic shield optimization problem is observed, and we found that under condition \( \sigma = 0.3 \), a porous shape will act as a bi-layer flux barrier, which means the air region in the rotor, shape under \( \sigma = 1.5 \). This is primarily dependent on the strength or weakness of the smoothing effect, and a thin flux barrier shape in which the smoothing effect is too strong under the condition \( \sigma = 1.5 \) appears in the outer diameter of the rotor, as seen under condition \( \sigma = 0.8 \), which is filled with the core.

Table 2 Optimization parameters and results

| GA               | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
|------------------|--------|--------|--------|--------|--------|
| Generation       | 100    | ←      | ←      | ←      | ←      |
| Population       | 100    | ←      | ←      | ←      | ←      |
| Crossed population | 50    | ←      | ←      | ←      | ←      |
| Mutation establishment (%) | 3.0   | ←      | ←      | ←      | ←      |

Figure 7. Optimization problem [11]
| Filtering          | Standard deviation | 0.3 | 0.5 | 0.8 | 1.5 | 3.0 |
|-------------------|--------------------|-----|-----|-----|-----|-----|
|                   | Kernel size        | 5×5 | ←   | ←   | ←   | ←   |
| Current condition | Input phase current (Arms) | 3.0 | ←   | ←   | ←   | ←   |
|                   | Load angle (deg)   | 45.0| ←   | ←   | ←   | ←   |
| Results           | Average torque (Nm) | 2.90| 3.17| 3.22| 3.16| 3.09|

Figure 8. Impact on optimized shape of differences in smoothing effect

4.2. Comparison between conventional techniques

Figure 9 shows the comparison results for conventional techniques. The torque performances can be compared by using a tri-layer U-shaped flux barrier as a reference model, shown in Figure 10. In addition, for the proposed technique, we selected results for $\sigma = 0.8$, which had the highest evaluation in Table 2. For spatial filtering with a conventional technique, we applied the averaging filter window function shown in (8). The flux barrier shape in the conventional technique is a bi-layer structure, and is attributable to the smoothing effect being too strong. As a result, it is assumed that the results of the proposed technique have a higher average torque.

Figure 10 shows the torque waveform. With the proposed technique, we found that the difference in the largest and smallest torque is lesser, compared to that for the conventional technique. As a result, a higher average torque can be obtained with the proposed technique relative to that for the conventional method. Because only the reluctance torque is used in SyRM, it can be assumed that the stator magnetic flux can be obtained effectively in the proposed optimized shape. The number of stator slots is six slots/pole, and the stator magnetic
flux distribution hardly depends on the number of slots, as shown in Figure 9. To effectively pick up the magnetic flux generated by the stator coils, rotor core shapes are aligned with the stator flux distribution. In addition, to effectively enlarge the reluctance torque, it is necessary to maintain the q-axis inductance and decrease the d-axis inductance. From this viewpoint, in a SyRM electromagnetic design, the thin flux barriers in the radial direction are designed with a tri-layer structure according to the number of slots. As a result, it is thought that the results of the proposed technique have a higher average torque than those of conventional techniques. In addition, we found that the results of the proposed technique are overall higher in torque than the reference model. This is mainly attributable to the magnetomotive force created by the stator. Figure 11 shows the flux line distribution of the model obtained using the proposed technique and the reference model. Under a load angle of 45°, where the reluctance torque is the largest, the flux line distribution is as shown in Figure 11. In this case, the rotor core is generated on the main stator flux path under a load angle of 45°. Thus, the optimized shape obtained by the proposed technique differs in terms of the magnetic path width thickness. From this viewpoint, it can be seen that the optimized shape obtained by the proposed method, for \( \sigma = 0.8 \), is possible to ultimately utilize a stator magnetic flux. Therefore, the average torque of the solution reached by the proposed technique is higher.

(a) Conventional \( (T_{\text{ave}} = 2.90 \text{ Nm}) \)

(b) Present \( (\sigma = 0.8) \)
4.3. Modification of the optimized shape

In this section, we verify the feasibility of the solution obtained using the proposed technique.
Because optimized shapes obtained by the proposed technique barely depend on the mesh segmentation, the resulting solutions may be difficult to manufacture through punching or laser cutting. In addition, because a strength analysis has not been conducted using the optimization process, a study of the centrifugal force resistance strength is applied. In this study, the smoothing process for a flux barrier shape is conducted using spline interpolation. In addition, the strength design is applied using a strength analysis, and a solution that holds for the centrifugal force resistance strength and manufacturability is reached.

Figure 12 summarizes the results of the above study. In the proposed technique, approximately twice the Mises stress is generated with respect to the reference model, whereas in the modified model shown in Figure 12 (b), the bridge thickness of the outer rotor diameter is changed, and thus the maximum stress is approximately the same as that of the reference model. In addition, as shown in Figure 12(c), we found that a smooth shape can be obtained by applying a shape boundary correction using spline interpolation. We also found that by increasing the thickness of the bridge on the outer diameter, the leakage flux increases and the torque decreases, although the final solution exceeds that of the reference model shown in Fig. 12(d).

From the above results, it was found that by applying a motor design using the proposed technique, it is possible to reach a design solution with a highly manufacturable shape, which holds for the strength of the centrifugal force resistance.
5. Conclusions

In this report, we describe a new topology optimization technique using a Gaussian filter. With the proposed technique, it is possible to adjust the smoothing effect during the filtering processes. Further, it is possible to obtain a magnetic shield shape with a tri-layer shield structure, which is difficult via conventional techniques. In addition to the magnetic shield optimization problem, it was found that similar optimization results can be achieved through practical synchronous reluctance motor designs. In addition, these results were confirmed to have favorable properties, even when the shape boundary correction and strength design accounting for the manufacturability are considered. In the future, we plan to apply this technique to address optimization problems in the design of two-dimensional as well as three-dimensional equipment shapes.

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