Abell 1033: A Radio Halo and Gently Re-Energized Tail at 54 MHz

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ABSTRACT

Context. Abell 1033 is a merging galaxy cluster of moderate mass ($M_{500} = 3.24 \times 10^{14} M_{\odot}$) which hosts a broad variety of diffuse radio sources linked to different astrophysical phenomena. The most peculiar one is an ultra-steep spectrum elongated feature which is the prototype of the category of gently re-energized tails (GReETs). Furthermore, the cluster hosts sources previously classified as a radio phoenix and a radio halo.

Aims. In this work, we aim to improve the understanding of the cosmic ray acceleration mechanisms in galaxy clusters in a frequency and mass range poorly explored so far.

Methods. To investigate the ultra-steep synchrotron emission in the cluster, we perform a full direction-dependent calibration of a LOFAR observation centered at 54 MHz. We analyze this observation together with re-calibrated data of the LOFAR Two-meter Sky Survey at 144 MHz and an archival GMRT observation at 323 MHz. We perform a spectral study of the radio galaxy tail connected to the GReET to test if the current interpretation of the source is in agreement with observational evidence below 100 MHz. Additionally, we employ a Markov chain Monte Carlo code to fit the halo surface-brightness profile at different frequencies.

Results. We report an extreme spectral curvature for the GReET, the spectral index flattens from $\alpha_{323\text{MHz}} \approx -4$ to $\alpha_{54\text{MHz}} \approx -2$. This indicates the presence of a cut-off in the electron energy spectrum. At the cluster center, we detect the radio halo at 54, 144 and at lower significance at 323 MHz. We categorize it as an ultra-steep spectrum radio halo with a low-frequency spectral index $\alpha = -1.65 \pm 0.17$. Additionally, with a radio power of $P_{144\text{MHz}} = 1.22 \pm 0.13 \times 10^{22}$ W Hz$^{-1}$, it is found to be significantly above the radio power-to-cluster mass correlations reported in the literature. Furthermore, the synchrotron spectrum of the halo is found to further steepen between 144 and 323 MHz, in agreement with the presence of a break in the electron spectrum, which is a prediction of homogeneous re-acceleration models.

Key words. galaxies: clusters: individual: Abell 1033 – radio continuum: general — X-rays: galaxies: clusters

1. Introduction

Galaxy clusters are the building blocks of the large scale structures of the Universe. When observed at radio wavelengths, we can detect synchrotron emission which traces the non-thermal plasma in the cluster’s volume (for a review, we refer to van Weeren et al.[2019]). Different phenomena are associated with this emission. There is consensus that diffuse, cluster-scale sources such as radio halos develop in the Intraclusters medium (ICM) as a consequence of mergers between clusters (Brunetti & Jones[2014]). Radio halos are Mpc-scale sources with a steep spectral index of $\alpha \leq -1.2$ located in the central region of clusters, approximately following the thermal ICM brightness distribution as observed in the X-rays. The turbulent re-acceleration model (Brunetti et al.[2001], Petrosian[2001], Brunetti & Lazarian[2007], Minniti[2015], Brunetti & Lazarian[2016]) is the favored scenario to explain the mechanism generating radio halos. In this model, turbulence injected in the ICM by a merger event re-accelerates cosmic ray electrons (CRE) from a population of at least mildly relativistic primary (e.g. accelerated by active galactic nuclei) or secondary (generated in proton-proton collisions) seed electrons (Brunetti & Lazarian[2011], Pinzke et al.[2017]). This is supported by the observed connection between the dynamic state of galaxy clusters and the presence of radio halos (Cassano et al.[2010, 2013], Cuciti et al.[2015], Cuciti et al.[2021]). Another important expectation of this model is the presence of a cut-off in the electron spectrum that depends on the energetics of the merger event and translates to a gradual steepening in the observed synchrotron spectrum. This implies that in general at $\sim 1$ GHz frequency, we would be able to detect those radio halos generated during very energetic merger events, while at lower frequency we could be sensitive to a population of radio halos characterized by very steep radio spectra ($\alpha \approx -1.5$) which are in general referred to as ultra-steep spectrum radio halos (USSRH; Cassano et al.[2006], Brunetti et al.[2008]). Such sources have been observed only in a small number of clusters so far (Brunetti et al.[2008], Macario et al.[2011], van Weeren et al.[2012], Wilber et al.[2013], Duchesne et al.[2021], di Gennaro et al.[2021]).

Other extended radio sources in galaxy clusters, such as gently re-energized tails (GReETs, de Gasperin et al.[2017]) and
radio phoenices (Kale & Dwarkanath 2011; Cohen & Clarke 2011; Duchesne et al. 2021), are thought to trace ancient tails and lobes of radio galaxies that have been re-accelerated. They are smaller (~100 kpc) sources and show a direct connection to individual radio galaxies. The observational difference between radio phoenices and GReETs lies in the morphology, the spectral properties and the presence of a shock in the ICM. These differences can be explained by a difference in the underlying mechanism of re-acceleration. Phoenices are mostly irregular sources with a steep spectrum without clear spatial trends, they are thought to be accelerated by the adiabatic compression of old radio lobes through a shock wave (Enßlin & Gopal-Krishna 2001; Enßlin & Brüggen 2002). In contrast, GReETs are elongated tails of radio galaxies which show highly unusual properties. Starting from the host radio galaxy, spectral aging causes an increasing steepening of the observed synchrotron emission, however, at some point, the tail shows an unexpected re-brightening coinciding with a constant or even flattening spectral index. This is thought to be explained by a gentle energetization mechanism probably connected to micro-turbulence in the tail induced by interactions with the ICM (de Gasperin et al. 2017; van Weeren et al. 2021). The exact nature of the mechanism remains unclear, as currently, only a small number of such sources or candidates have been reported in the literature with very limited spectral information (de Gasperin et al. 2017; Cuciti et al. 2018; Wilber et al. 2018; Botteon et al. 2021; Igenstät et al. 2022).

Observations at low radio frequencies (<1 GHz) are essential to study ultra-steep sources such as USSRHs and GReETs since these sources remain undetectable at higher frequencies due to their spectral properties. The Low-Frequency Array (LOFAR; Van Haarlem et al. 2013) is the largest and most sensitive radio-interferometer operating at low frequencies. As such, it is perfectly suited to study diffuse radio emission in clusters of galaxies.

Abell 1033 (PSZ2 G189.31+59.24, hereafter A1033) is a moderately massive ($M_{200} = 3.24^{+0.52}_{-0.38} \times 10^{14} M_{\odot}$) galaxy cluster at a redshift of $z = 0.126$ (Planck Collaboration et al. 2016) which shows recent merger activity and hosts a steep spectrum source previously classified as a radio phoenix (de Gasperin et al. 2015). In addition, the cluster contains an elongated, ultra-steep ($\alpha \approx -4.0$) source connected to a wide-angle tail radio galaxy (WAT), a mechanism of gentle re-acceleration was suggested to explain the peculiar spectral properties (de Gasperin et al. 2017). This source is the prototype of the GReET category, and it remains the most extreme among reported cases of radio galaxy tails which show signs of re-acceleration. Observations in the ultra-low frequency regime (<100 MHz) are necessary for a more detailed study of this source. Furthermore, the presence of a radio halo in the cluster was recently reported in Botteon et al. (2022) based on 144 MHz LOFAR observations.

In this work, we analyze a LOFAR low-band antenna (LBA) observation of A1033 and data of the LOFAR Two-Meter Sky Survey (Shimwell et al. 2017, 2019, 2022; LoTSS) to study the properties and spectral behavior of the cluster’s peculiar radio sources. Throughout this work, we assume a fiducial flat ΛCDM cosmology with $\Omega_m = 0.3$ and $H_0 = 70 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$. All spatial distances are with respect to a redshift of $z = 0.126$, at which one arcsecond corresponds to 2.53 kpc.

### References

(1) Originally published in de Gasperin et al. (2017).

### Table 1: Observations.

| Telescope  | Obs. date | Time [h] | Freq. [MHz] | Bandwidth [MHz] |
|------------|-----------|----------|-------------|-----------------|
| LBA        | 30 June 2018 | 8        | 54          | 48              |
| LOFAR HBA  | 24 Nov. 2015$^1$ | 8        | 144         | 48              |
|            | 24 Dec. 2018 | 8        | 144         | 48              |
| GMRT       | 02 Nov. 2014$^1$ | 5.5      | 323         | 32              |

### 2.1. LOFAR LBA data

The LOFAR LBA observation was conducted in observation mode “LBA_OUTER”, where the outer half of the station dipole is used to minimize electromagnetic cross-talk. During the pre-processing, the data was flagged at resolution of 1 s and 3 kHz to mitigate radio frequency interference and subsequently time- and frequency-averaged to 4 s and 49 kHz. The multi-beam capability of LOFAR LBA allowed it to continuously point one beam towards the calibrator source 3C196 during the observation. This source is a well-behaved standard calibrator for the low-band (Heald et al. 2013). The calibrator data were reduced with the LOFAR LBA calibrator pipeline described in de Gasperin et al. (2019). This pipeline is used to find solutions for station-based direction-independent effects as well as an initial estimate for the ionospheric phase errors. The calibrator solutions revealed bad data quality for the station CS031LBA, thus, we excluded this station from further processing. The calibrator solutions were applied to the target field data set, additionally, we correct the data for the primary beam towards the phase-center. Next, we perform direction-independent self-calibration of the target field to find solutions for the average ionospheric effects. We employ the self-calibration pipeline presented in de Gasperin et al. (2020). Starting from an initial model based on a collection of radio surveys (Intema et al. 2017; TGSS; Condon et al. 1998; NVSS; Rengelink et al. 1997; WENNS; Lane et al. 2014; VLSSr), solutions for the direction-independent ionospheric total electron content (TEC), Faraday rotation, and second order primary beam effect are derived in two rounds of self-calibration. The root-mean-square (rms) background noise of the self-calibrated image is 2.5 mJy beam$^{-1}$ at a resolution of $35''$, the image quality is limited by significant direction-dependent ionospheric errors still present in the data. These errors are addressed in direction-dependent calibration, where we employ the calibration pipeline which is being developed for the LOFAR LBA Sky Survey (LoLSS; de Gasperin et al. 2021) and based on the facet calibration strategy outlined in van Weeren et al. (2016). For the next processing steps, we reduce the data volume by averaging the data to a resolution of 8 s in time. We use the direction-independently calibrated image to...
isolate suitable direction-dependent calibrators. We employ the Python Blob Detector and Source Finder (PyBDSF, Mohan & Rafferty (2015)) as source finder, and select bright and compact sources by thresholding based on flux density and source area. Compact sources in a maximum proximity of 6′ are merged by a grouping algorithm. We select all sources and groups of sources with a flux density above $S_{\text{min}} > 1.0 \text{Jy} \times (v/60 \text{MHz})^{-0.8}$ at $v = 30 \text{ MHz}$ or $v = 54 \text{ MHz}$, the reason for this is that spectral variation, mainly of the primary beam, can make it hard to identify all bright sources at a single frequency. This yielded 29 calibration directions for the first of two major iterations of our strategy. To prepare the data for further calibration, we subtract the model obtained after direction independent calibration to create a data set which is empty up to model inaccuracies. Then, we loop through the calibrators ordered by their flux density at 50 MHz and repeat the following steps. Firstly, the model of the calibrator is re-added to the empty data. Second, the data is phase shifted to the calibrator direction and subsequently averaged further to as resolution of 16 s (if $S_{\text{cal}} > 10 \text{Jy}$) or 32 s (otherwise) in time and 0.39 MHz in frequency. This averaging is possible since we only image the single calibrator and not the whole field of view (FoV), so we are not limited by time or frequency smearing. Third, we correct this averaged data set for the primary beam in the new phase center. Then, we self-calibrate on this data set: By imaging a small field around the calibrator, we obtain a model of the calibrator source(s). The data is smoothed in time and frequency with a baseline-dependent kernel size, here shorter baselines are smoothed with a larger kernel to increase the signal-to-noise ratio. We solve on the smoothed data for scalar phases against the model. The frequency interval of this solve step is 0.39 MHz, the time interval varies from 64/128 s in the first iteration down to 16/32 s in the following minor cycles. These solutions are then applied to the data and the calibrator direction is imaged again to obtain an improved model for the next self-calibration iteration. For sources brighter than 10 Jy, we perform two additional diagonal amplitude calibration steps: one, enforcing constant solutions between stations to capture beam model errors, and a subsequent solve to correct remaining errors. If the rms noise of the image does not improve by more than one percent during a self-calibration iteration and the noise level is lower than the initial value, we consider this direction to be converged, otherwise, it is not further used as a calibrator direction. In this case, the solutions are discarded and the direction will be divided between the neighboring calibrators. We re-subtract the converged calibrator model corrupted with the final solutions in that direction from the data to obtain a cleaner empty data set, artifacts in the image around the calibrator related to direction-dependent errors are now strongly reduced. This procedure was repeated for all 29 calibrator directions.

We then start again with the averaged data after direction-independent calibration. From this data, we peel all calibrator sources outside of a 2.5′ region around the pointing center, meaning that we subtract the model after corrupting it with the corresponding direction-dependent solutions without re-adding it to the data at any point. This strongly reduces the impact of bright sources outside of the primary beam. This data is then imaged using DDFacet (Tasse et al. 2018), this imager simultaneously applies calibration solutions for the remaining 22 calibrator directions based on Voronoi-tesselated facets around the calibrators. The resulting image and model are the products of the first major iteration of our direction-dependent calibration pipeline. We perform a second major iteration using the improved model as input, this time searching for calibrators on the DDE-calibrated image and going 20% lower (to 0.8 Jy) in terms of minimal calibrator flux. We find 25 suited directions for the second iteration, which are now all inside the primary beam main lobe. Again we loop over these sources ordered according to their brightness and repeat the individual self-calibration cycles as described above. Since A1033 is a somewhat special source by being both very bright (>10 Jy at 54 MHz) and extended (around 5′), our default algorithm had problems in correctly picking up all emission associated with A1033 as a calibrator direction. Therefore, for this target we employ an LBA-specific implementation of the extraction-pipeline described in van Weeren et al. (2021). This pipeline subtracts all sources outside of a 0.2′ radius around A1033 from the data using the model and calibration solutions from the direction-dependent pipeline. Then, it performs multiple rounds of self-calibration solving for TEC and a constant phase in each time interval. This $\text{tecadphase}$-constraint reduces the number of free parameters compared to the phase-solve used in direction-dependent calibration. We image the final target-extracted data set using a robust weighting of $-0.3$ and $-1.0$, the images have a background noise of 1.4 and 1.7 mJy beam$^{-1}$ at a resolution of $21'' \times 13''$ and 12'' $\times$ 9'', they are displayed in Figure 1.

### 2.2. LOFAR HBA data

A1033 was covered by the pointings P155+27 and P157+35 observed in the context of LoTSS (Shimwell et al. 2017, 2019, 2022). The analysis of P157+35 has been already presented in de Gasperin et al. (2017). Here, we exploit both data sets, taking advantage of the latest developments in the processing pipelines, to increase the sensitivity of the images towards the cluster. LoTSS is observing the entire northern sky with 8 h observations per pointing covering the frequency range 120-168 MHz. LoTSS pointings are processed by the LOFAR Surveys Key Science Project, using the automated direction-independent and dependent calibration pipelines described in van Weeren et al. (2016), Williams et al. (2016), de Gasperin et al. (2019), Tasse et al. (2018) and Tasse et al. (2021). A1033 is located within the newly released area of LoTSS Data Release 2 (DR2, Shimwell et al. 2022), and, being a cluster in the PSZ2 catalog (Planck Collaboration et al. 2016), it is also included in the sample of PSZ2/LoTSS-DR2 clusters recently analyzed by Botteon et al. (2022). Here, we used the reprocessed dataset presented in Botteon et al. (2022), in which the data quality has been improved using the extraction and self-calibration procedure described in van Weeren et al. (2021). We used WSCLEAN (Offringa et al. 2014), Offringa & Snirnov (2016) to re-image the data and study the properties of the diffuse emission in the cluster. The final HBA image is shown in Figure 1c, it has a resolution of 12×6 and with a noise level of 110 mJy beam$^{-1}$, it is around 40% deeper than the one previously presented in de Gasperin et al. (2017).

| Table 2: Parameters of the images displayed in Figure 1 | The Briggs value is the parameter for the baseline-dependent weighting of the visibility data during imaging. |
| --- | --- |
| **Label** | **Telescope** | **Freq. [MHz]** | **Briggs** | **Resolution ["×"]** | **Noise [mJy beam$^{-1}$]** |
| Fig. 1a | LBA | 54 | -0.3 | $21 \times 14$ | 1.4 |
| Fig. 1b | LBA | 54 | -1.0 | $12 \times 9$ | 1.7 |
| Fig. 1c | HBA | 144 | -0.3 | $12 \times 6$ | 0.11 |
| Fig. 1d | GMRT 323 | -0.3 | $10 \times 6$ | 0.034 |

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3. Results

In Table 2 we summarize the mid- and high-resolution LBA as well as the HBA and GMRT images of A1033 displayed in Figure 1. In Figure 2, the radio sources in the cluster are labeled. A connection between the wide-angle tail radio galaxy (WAT) in the East of the cluster and the GReET is visible at 54 and 144 MHz. A second radio galaxy with a head-tail morphology is marked as “HT” in the figure.

For our analysis, we assume a systematic flux scale uncertainty of 10% for the LOFAR LBA, LOFAR HBA as well as...
WAT between 144 and 325 MHz, which is fairly uniform for a projected

we measure an extremely steep spectral index of

steepen further and remains approximately constant along the
tail. Along the tail, the spectral index grad-
tainty. To investigate the spectral shape, we create curvature

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we create spectral index maps in the ranges 54-144 MHz and

next, all images are convolved to the

matching point sources to account for potential astrometric er-

sure the images are comparable. For each image, a point-source
catalog is created using

uv
dra et al. 2004). For all radio images, a common lower uv-cut of

100 kpc. The green path A and the yellow path B highlight the
location of possible interrupted structures, the dashed section is
the discontinuity.

GMRT (de Gasperin et al. 2021; Shimwell et al. 2022; Chan-
dra et al. 2004). For all radio images, a common lower uv-cut of
100 kpc was used during imaging, and all imaging was carried out
using WSCLEAN. When combining information from multiple fre-

quencies for spectral index mapping or spectral shape-modeling, it is required to carry out a number of pre-processing steps to en-
sure the images are comparable. For each image, a point-source
catalog is created using PyBDSF, the images are aligned by cross-
matching point sources to account for potential astrometric er-

rors at a level below 5". Next, all images are convolved to the
smallest common circular beam. Last, they are re-gridded on the
same pixel layout.

3.1. Spectral index maps

We create spectral index maps in the ranges 54-144 MHz and
144-323 MHz. For this, we discard all pixels with a surface
brightness significance below 3σ rms in at least one of the two
relevant images. We use the linear least squares method in log-
space to determine the spectral index values. The uncertainties
were estimated according to Gaussian propagation of uncertain-
ty. To investigate the spectral shape, we create a curvature
map from the spectral index maps. The curvature is defined as

∆α = α144 − α144. The high-resolution spectral index and curv-

ature maps are displayed in Figure 3, the corresponding uncer-

ainty maps are provided in Appendix B.1. At the position of the
WAT host galaxy, we measure a spectral index of α ≈ −0.65 in
both frequency intervals. Along the tail, the spectral index grad-

ually steepens, the steepening is stronger at higher frequency. At
the location of the bend in the tail, the spectral index does not
steepen further and remains approximately constant along the
GReET for both frequency ranges. For this section of the tail, we
measure an extremely steep spectral index of ∼−3.8 ± 0.18 be-
tween 144 and 325 MHz, which is fairly uniform for a projected
distance of 300 kpc. Towards ultra-low frequencies, the spectrum of
this region shows extreme curvature (Δα = 1.8 ± 0.23).

3.2. Spectral aging analysis

In absence of acceleration mechanisms, the electron population in a radio galaxy tail is subject to aging due to synchrotron and
inverse Compton losses. This causes a spatial evolution of the spectral shape along the tail, depending on the magnetic field,
injection history and injection spectral index of the AGN as well as
the projected velocity of the radio galaxy. In the following, we
consider this pure-aging scenario for the WAT and examine if it is in agreement with our multi-frequency observations. This
is the expected scenario if there are no acceleration mechanisms,
usually related to ICM shocks, present in the tail. For our spe-
cial aging model, we probe a path along the brightest part of the

path where we measure the flux densities at 54, 144 and 323 MHz
and the spectral indices between 54-144 and 144-323 MHz in 23
different 12.5" beam-sized regions which are separated by the
beam FWHM. This path is shown in Figure 2 and the flux density
and spectral index evolution as a function of distance is shown in
Figure 4. We consider the Jaffe-Perola (JP) plasma aging model
(Jaffe & Perola 1973), the most commonly used model for the
aging of radio galaxies, to describe the spectral distribution for the first 300 kpc of the tail, before the brightness of the tail
increases again (de Gasperin et al. 2017). We perform a non-linear
least squares fit of the JP model to the spectral index points,
assuming that the projected velocity v⊥ of the galaxy along the

path is constant.

From the spectral index maps, we estimate the injection in-
dex to be α inj = −0.65. The aging history of our model then
depends on the magnetic field B and the velocity v⊥. Unfortu-
nately, the magnetic field and the projected velocity are strongly
correlated, implying that it is not possible to distinguish between
a scenario with a magnetic field close to the minimum aging field
strength and a lower velocity and a scenario with more rapid losses and a higher velocity of the WAT galaxy (as
illustrated in Figure B.1). Therefore, we consider a scenario that
maximizes the life time of the electrons, which provides an upper
limit on the radiative age of the plasma. The life time of the CRe in the relevant energy regime depends on the magnetic
field B as τ age ∝ \sqrt{B} / (B_min + B_{CMB}) (Stroe et al. 2014), where
B_{CMB} = 3.2 \times (1 + z)^2 \mu G (Longair 2011) is the magnetic field
strength equivalent to the cosmic microwave background (CMB)
energy density at redshift z. The expression for τ age takes
its maximum value at the minimal loss magnetic field B = B_min,
where the loss rates due to synchrotron radiation and inverse
Compton scattering with the CMB are equal. This is given by
B_min = B_{CMB} / \sqrt{3}, at the redshift of A1033, B_min = 2.3 μG. We
use the non-linear least squares method to fit a JP model to the
spectral index data points (see Appendix B). By fitting the model
to the spectral index points and not the flux densities, we can
eliminate the normalization factors and hence, reduce dependen-
cies on the injection history of the AGN. The best-fitting model
has a projected velocity of v⊥ = 780 km/s with a reduced χ^2-

 statistic of χ^2/d.o.f. ≈ 29.5/17, in reasonable agreement with the
= 730 km/s found in de Gasperin et al. (2017). This corre-
sponds to a maximum age of the GReET of 800 Myr. The spec-
tral index and flux density data points as well as the best fitting
model are shown in Figure 4 For the flux density models, the
flux normalization is fitted in log-space independently as a free
parameter for each beam-sized region.

Fig. 2: Color composite of the high-resolution LOFAR HBA (or-
ange), the Chandra X-ray (blue) and the SDSS g (white, Alam
et al. 2015) images. The white line corresponds to the path that is
used to analyse the spectral properties of the GReET, the dots mark the distance from the starting point in increments of
100 kpc. The green path A and the yellow path B highlight the
location of possible interrupted structures, the dashed section is
the discontinuity.
Fig. 3: High-resolution spectral index map of A1033. The left image shows the spectral index between 54 and 144 MHz, the right one between 144 and 323 MHz (center). The images share the same color scale and an angular resolution of 12′′ × 9′′. Upper limits below -1.6 are outside the black contours and marked by arrows. The saturation scales inversely with the spectral index uncertainty. The right image shows the corresponding curvature map, defined as the difference between 54-144 and 144-323 MHz.

Fig. 4: Flux density (top) and spectral index (bottom) measured in the 12.5′′ beam-sized regions shown in Figure 2 as a function of distance and estimated age. The continuous lines show the best-fitting model (χ2/d.o.f. = 29.5/17) assuming minimum aging (B = 2.3 μG) with a projected velocity of 780 km/s. The dotted lines show an extrapolation of this model.

To complement the spectral analysis, we create a color-color map. For this, we calculate the 54-144 and 144-323 MHz spectral indices in 5′′ square boxes in a region covering the WAT and GreET and a second region covering the radio phoenix. Here, we consider all boxes within the two regions with a significance above 3σrms at all three frequencies. For comparison, we also show the spectral trajectory of a power law injection and the JP aging model discussed above. For points along the WAT and GreET, we additionally color-code the distance from the injection point along the tail. The resulting diagram is shown in Figure 5. The boxes associated with the radio phoenix occupy a rather confined region in the spectral index space, with no clear trends. Contrary, along the WAT, there is a clear steepening with increased distance from the injection point. The spectral index points departure from the PL-injection spectrum along a trajectory that is in agreement with a JP model with increased distance. After 300 kpc, the steepening between 54-144 and 144-323 MHz stops and the boxes corresponding to the following part of the GreET scatter around α144 = α323.

3.3. Source-subtracted low-resolution images

The mid-resolution images Figure 1a and Figure 1c show the presence of an extended, diffuse radio emission superimposed on the bright sources associated with individual radio galaxies.
the GReET and the radio phoenix. In the LOFAR images, emission with an extension of several 100 kpc is visible at 54 MHz as well as at 144 MHz. This emission is associated with the radio halo detected in A1033 (Botteon et al. 2022). To isolate the emission of the radio halo from the foreground sources, we obtain a model for the compact sources based on high-resolution (Briggs = −1.0) images at 54 MHz, 144 MHz and 323 MHz. This model is restricted to the clean components in a region which only includes the sources present in the high resolution images (so e.g. the radio galaxies, GReET and phoenix) and subtracted from the data, identical regions are used at all three frequencies. We re-image the subtracted data, using baselines in the range 100-20000 λ and tapering the imaging weights to a resolution of 25′ to enhance the extended emission. The resulting images are displayed in Figure 6 together with the regions we used to restrict our subtract model. In the LOFAR maps at 54 and 144 MHz in Figure 6 the extended emission is significantly detected at a surface brightness above 5σ rms for a region with a diameter of ≈ 700 kpc except for the subtracted regions. The shape is approximately circular in both frequencies, however, some morphological differences exist. At 54 MHz, the halo appears to extend further to the Southwest, while a hole in the emission is visible in the Northeast. Given the complex procedure of the calibration of a bright, extended source with complex morphology superimposed on the faint, even more extended halo emission together with the source subtraction, we cannot exclude the possibility that these differences in morphology in the lower-significance regime are not physical but rather artifacts in the data. At 323 MHz, the detected emission appears more patchy, likely due to artifacts caused by calibration errors or radio frequency interference in the GMRT map. In the low-resolution images, we also display the point-source subtracted Chandra X-ray contours from de Gasperin et al. (2015), the shape of the radio emission is approximately circular and mostly confined within the X-ray contours.

3.4. Radio halo flux calculation

To get an estimate of the total flux density of the halo, we employ the Halo Flux Density Calculator Halo-FDCA (Boxelaar et al. 2021), a Markov Chain Monte Carlo code to estimate halo flux densities with reduced human bias. The code can handle masks to exclude image regions from the analysis, this allows us to exclude the subtracted regions which do not contain reliable emission. We use the code to fit a spherically-symmetric exponential surface-brightness profile described by

\[ I(r) = I_0 e^{-r/r_e} \]

(1)
to the data. Here, \( I(r) \) is the surface brightness at a distance \( r \) from the profile center, \( I_0 \) is the peak surface brightness and \( r_e \) the e-folding radius (Murgia et al. 2009). This empirical model is oftentimes found in the literature to accurately describe radio halos (e.g. Vacca et al. 2014; Oingsa et al. 2021; Botteon et al. 2021a,b) and it comes with a minimal number of free parameters, beneficial for the halo in A1033 which is partly obscured by other sources. A similar analysis for A1033 in LOFAR HBA is presented in Botteon et al. (2022). To allow a consistent analysis of the LBA and HBA data specifically tailored to A1033, we redo the source subtraction, imaging and fitting procedures also for the HBA data.

The result of the halo flux calculation is a 1.46 ± 0.15 Jy at 54 MHz and 0.29 ± 0.03 Jy at 144 MHz. The detailed parameters are listed in Table 3 and plots of the halo fit result and fit residual are shown in Figure C.1. The location and extension of the best-fitting models at 54 MHz and 144 MHz are in good agreement with e-folding radii of 154 ± 2 and 165 ± 3 kpc, respectively. The LBA model is slightly further south and the radius seven percent smaller. We also manually measure the flux density within the respective 3σ rms contours \( F_{3\sigma} \) of the halo in Figure C.1, obtaining \( F_{3\sigma}^{54\text{MHz}} = 0.87 ± 0.09 \) and \( F_{3\sigma}^{144\text{MHz}} = 0.15 ± 0.02 \). Due to the missing extrapolation of the emission in the masked regions, the manually measured flux is a lower limit for the total flux of the halo. The ratio between the manually measured and the fitted flux density is different between LBA and HBA (0.6 and 0.52), we attribute this to a small difference in the location and size of...
the fitted surface brightness profile. We estimate the total radio power at 150 MHz of the halo based on the Ralo-FDCA results. The resulting value is $P_{150\,\text{MHz}} = 1.22 \pm 0.13 \times 10^{25} \, \text{WHz}^{-1}$, where a spectral index of $\alpha = -1.65$ is used for extrapolation and $k$-correction as will be justified in the following Section 3.5. The independent estimate of the radio power in Botteon et al. (2022) ($P_{150} = 1.09 \pm 0.28 \times 10^{25} \, \text{WHz}^{-1}$) is consistent with ours (at about $1\sigma$), however, their reconstructed morphology differs significantly ($r_c = 165 \pm 3$ to $r_c = 103 \pm 1$), which we interpret as the halo morphology being more susceptible to differences in masking and fitting than the flux density of the halo. In general, the fitted parameters of the halo model are subject to a systematic uncertainty connected to the masking and subtraction procedure, which is hard to assess and thus, not accounted for in our uncertainties.

3.5. Radio halo spectral index

We use the source-subtracted images presented in Figure 6 to create low-resolution spectral index maps and study the spatial variation of the spectral index within the radio halo. The spectral index maps are created as described in Section 3.1, thereby we exclude the regions that were used to subtract the compact sources. These maps are displayed in Figure 7, the uncertainty maps are shown in Figure A.2. In the range of 54-144 MHz, the halo spectral index could be determined significantly across a large area. The northern part of the halo is less steep with a spectral index around $\alpha = -1.5$, the southwestern region is steeper than $\alpha = -2$. The same trend extends to the ultra-steep upper limits in the Southwest and flat-spectrum features expressed by lower limits in the Northeast. As stated in Section 3.3 we cannot exclude the possibility that these features are at least partly a consequence of systematic errors in the calibration. Since only a part of the radio halo is detected above a significance of $3\sigma$ in the 323 MHz GMRT map, we can constrain the spectral index in the range 144-323 MHz only in comparatively small area. In this region to the Northwest of the GReET, the spectral index is $\alpha_{323}^{144} = -2.15 \pm 0.19$, for the majority of the remaining halo area, we can constrain the spectrum to be steeper than $\alpha = -2$. This suggests that the spectrum of the halo is curved.

To estimate the integrated halo spectral index between 54 and 144 MHz, we compare different methods: using the halo fits, we derive a spectral index of $\alpha_{\text{FDCA}} = -1.65 \pm 0.17$ based on the integrated surface brightness. If the flux-density is measured manually in the region where we have $> 5\sigma$ significance at both frequencies, the corresponding integrated spectral index is $\alpha_{\text{int}}^{54} = -1.74 \pm 0.14$. A more conservative estimation using only the area above $10\sigma_{\text{rms}}$ in both images yields $\alpha_{\text{int}}^{144} = -1.69 \pm 0.15$. We will adopt the value based on the Ralo-FDCA fluxes as reference value for the halo since it does not depend on the sensitivity of the observations.

4. Discussion

4.1. Spectral study of the GReET

In Figure 4, we show the flux density and spectral index evolution along the tail together with the best-fitting JP model as described in Section 3.2. After around 200 kpc from the injection point, the spectral index appears to be systematically flatter than the best-fitting model. To some degree, this deviation could be caused by a change in velocity of the WAT. Presumably, the galaxy reached a maximum velocity at the minimum of its trajectory in the cluster’s potential well, and slows down afterwards. This is not accounted for in our constant velocity assumption. However, at around $\approx 350$ kpc form the injection point, the spectrum does not steepen further and a plateau in spectral index associated with the GReET is reached. This constant and even flattening spectral index cannot be explained with the extrapolation of a pure aging model, even when considering a change in the WAT’s velocity. The same stalling in the spectral index trend can also be observed in the color-color diagram in Figure 5 where after initial JP-like aging, the spectral evolution stops and appears frozen for several 100 kpc. Consequently, an additional energization mechanism is required. This mechanism needs to be just efficient enough to compensate the radiation losses, but not so strong that the spectral shape is significantly flattened.

Fig. 7: Source-subtracted spectral index maps at a spatial resolution of 30". The top figure shows the spectral index between 54 MHz and 144 MHz, the bottom figure between 144 MHz and 325 MHz. The $3\sigma$ upper and lower limits are indicated by arrows, red dashed lines highlight the regions where sources are subtracted.
This was previously concluded in de Gasperin et al. (2017), where turbulent re-acceleration of the electrons in the tail by magnetic pumping was proposed as a possible mechanism. The suggested scenario is that turbulence is forced into the tail by interactions with the surrounding ICM which can provide mild acceleration which acts for sufficiently long time scales to explain the homogeneous nature of the GReET. The extreme curvature we detect fully supports the scenario outlined in the previous work: it is a consequence of a cut-off in the electron energy spectrum caused by the balance between the very gentle acceleration and the cooling of CREs, with an acceleration time of the radio-emitting electrons comparable to their cooling time (e.g. Brunetti & Jones 2014).

4.2. Turbulent velocity

In the high-resolution map shown in Figure 2, the tail of the WAT shows disrupted features. We suggest that these can be attributed to large-scale shear-flows and turbulent motions in the ICM. The tail of the WAT appears to be split into three filaments after around 250 kpc from the injection point. The brightest one directly connects to the GReET, the fainter ones, labeled in Figure 2 as A and B, show the presence of a discontinuity. Since the GReET shows a clear bifurcation, where one of the two parts has no clear connection to the WAT, we consider the possibility that a shear motion in the ICM, at a scale close to the injection scale of the turbulence, caused either A or B to disconnect from the southern segment of the GReET. This then allows us to constrain the large-scale turbulent velocity in the ICM: In the diffusion regime, CR are displaced a distance $d$ after a time $t$ according to $d = v_{diff}t$, where $d$ is the spatial diffusion coefficient. For super-Alfvénic turbulence, we can approximate $D \sim v_L L$ where $v_L$ is the turbulent velocity at scale $L$. If the displacement $d \sim L$, the velocity of large scale motions can be estimated as $v_L \sim d/L$, where $t$ can be constrained from the age of the plasma from our limit on the velocity of the WAT. We estimate the age of de Gasperin et al. (2015), which yields:

\[
\frac{L_\text{e}}{\Delta t} \sim \left(\frac{D_\text{off}}{\Delta t}\right)^{1/2} M_1^{1/4} \frac{1}{27L} M_1^{1/4},
\]

where in this case, the Alfvén Mach-number $M_1 \sim 1$ and the injection scale $L \sim L_1 = 10$ kpc is the traverse scale of the GReET. We define the lifetime of the electrons in the GReET as the time where the perpendicular diffusion of electrons can be driven by stochastic diffusion of field lines. The perpendicular diffusion length of the CRE $L_{\perp,\text{ext}}$ after a time $t$ can be estimated as (Lazarian & Yun 2014):

\[
L_{\perp,\text{ext}} \sim \frac{(D_\text{off})^{1/2} M_1^{1/4}}{27L} M_1^{1/4},
\]

\[
B \geq v \sqrt{4\pi\rho}.
\]

For a reference density 100 kpc form the cluster center (at the projected distance to the GReET) of $\rho = 10^{-23}$ kg m$^{-3}$, which is a typical value for clusters similar to A1033, and a velocity in the range derived above from $v_L(10$ kpc $) \geq 89$ km s$^{-1}$ to $v_L(10$ kpc $) \geq 190$ km s$^{-1}$, if this value is below the Alfvén velocity $v_{Alfvén}$, which is typically in the range ~ 100 km s$^{-1}$ (Brunetti & Jones 2014), Reynolds- and Maxwell-stresses of magnetic fields may stabilize the narrow tails. Similar considerations have been put forward to explain the stability of radio filaments in AGN bubbles (Brienza et al. 2021). From the Navier-Stokes equation, the term corresponding to matter motions on a scale $k$: $\rho v^2 k$ competes against magnetic field line tension on the same scale: $B^2 k/(4\pi)$. Thus, for the line tension to dominate, we require:

\[
B \geq v \sqrt{4\pi\rho}.
\]

This was previously concluded in de Gasperin et al. (2017), where turbulent re-acceleration of the electrons in the tail by magnetic pumping was proposed as a possible mechanism. The suggested scenario is that turbulence is forced into the tail by interactions with the surrounding ICM which can provide mild acceleration which acts for sufficiently long time scales to explain the homogeneous nature of the GReET. The extreme curvature we detect fully supports the scenario outlined in the previous work: it is a consequence of a cut-off in the electron energy spectrum caused by the balance between the very gentle acceleration and the cooling of CREs, with an acceleration time of the radio-emitting electrons comparable to their cooling time (e.g. Brunetti & Jones 2014).

Above consideration raises the question of how the tail can resist turbulent diffusion for a timescale of several 100 Myr. Assuming a Kolmogorov cascade, the turbulent velocity at a scale $L$ depends on the size of the scale as $v_L \sim L^{1/3}$. Thus, at a scale equal to the thickness of the filaments, the velocity derived from our estimate should be $v_L(10$ kpc $) \geq 89$ km s$^{-1}$ or $v_L(10$ kpc $) \geq 190$ km s$^{-1}$. If this value is below the Alfvén velocity $v_{Alfvén}$, which is typically in the range ~ 100 km s$^{-1}$ (Brunetti & Jones 2014), Reynolds- and Maxwell-stresses of magnetic fields may stabilize the narrow tails. Similar considerations have been put forward to explain the stability of radio filaments in AGN bubbles (Brienza et al. 2021). From the Navier-Stokes equation, the term corresponding to matter motions on a scale $k$: $\rho v^2 k$ competes against magnetic field line tension on the same scale: $B^2 k/(4\pi)$. Thus, for the line tension to dominate, we require:

\[
B \geq v \sqrt{4\pi\rho}.
\]
4.3. Radio halo

The spectral index of $\alpha = -1.65 \pm 0.17$ between 54 and 144 MHz places the radio halo in A1033 in the USSRH category. This is, together with the further steepening of the spectrum towards higher frequencies, in agreement with the presence of a cut-off in the electron spectrum as predicted by the turbulent acceleration model [Brunetti et al. 2001; Petrovskii 2001; Kuo et al. 2003; Cassano et al. 2006; Brunetti et al. 2008]. It is well known that radio halos in more massive clusters are more powerful [Cassano et al. 2013; van Weeren et al. 2021; Cuciti et al. 2021].

Since A1033 is of considerably lower mass compared to the vast majority of clusters which are known to host a radio halo, it is expected to have a lower radio power. In Figure 3, we compare the 150 MHz radio power of the A1033 halo to the radio halos and halo power-to-cluster mass correlations presented in van Weeren et al. [2021]. Furthermore, we show the two halos detected in the LOFAR deep fields [Osinga et al. 2021] and the low-power halos discovered in Hoang et al. [2021] and Botteon et al. [2021a]. All these radio halos are detected close to 150 MHz and in clusters of the PSZ2 catalog. From this comparison, it is evident that while A1033 is among the lowest mass clusters hosting a radio halo, the halo is quite powerful, it lies considerably above the low-frequency correlations. Compared to the 150 MHz correlations from van Weeren et al. [2021], it is over-luminous by a factor in the range 7 - 55. This is much larger than the intrinsic scatter of the correlations, that is $\sim 3$.

The relatively large radio power is notable given the ultra-steep spectrum nature of the A1033 halo, since in general, USSRH are expected to be less powerful [Cassano et al. 2010; Cuciti et al. 2021].

We investigate whether the power estimate of the halo could be contaminated by other sources. Indeed, we find that, while the region we used to subtract the GREET contains the significant emission in the HBA high-resolution map, images at lower resolution hint a less confined feature with a ten times lower surface brightness in extension of the GREET. This patch of emission is coincident with the brightest region of the unobscured part of the radio halo, labeled $P$ in Figure 6. To ensure that the field $P$ does not contaminate our radio power estimate significantly in case it is not related to the halo, we repeat the flux density calculation, this time with a more conservative mask. The resulting radio power estimate is lower by only 1.3%. Consequently, contamination from embedded sources cannot explain the large observed power.

A radio halo with a power above the correlation should correspond to energetic merger event and/or a merger in an evolutionary phase close to the peak power. In both cases, the X-ray surface brightness distribution should show strong signs of disruption. The X-ray morphological parameters for the cluster were determined in the Chandra analysis [de Gasperin et al. 2015]. While the X-ray concentration parameter $c$ (defined e.g. in Cassano et al. [2013]) does not unambiguously qualify the cluster as a merging one ($c = 0.200 \pm 0.004$), the centroid shift $w = 0.086 \pm 0.006$, a high differential velocity of the brightest cluster galaxy (BCG) and a distinct bimodality in the redshift distribution point towards a clear and possibly strong merger scenario [de Gasperin et al. 2015]. The presence of a strong merger might not fully show in the X-ray morphology parameters since it is likely happening with a significant line-of-sight component, as also concluded in de Gasperin et al. [2015] based on the BCG dynamics and redshift bimodality. Cuciti et al. [2021] found a trend between the distance from the radio power-cluster mass correlation and the X-ray morphological disturbance. Following their definition of the disturbance $d_{X_{\text{corr}}}$, we have $d_{X_{\text{corr}}} = 0.86 \pm 0.03$ which places A1033 in the region of disturbed clusters. While a more energetic merger should lead to a flatter spectral index, the relatively low cluster mass might be the dominant factor determining the spectral properties. An increasing number of spectral studies of halos in low- and intermediate-mass clusters will help to constrain the spectral properties of halos in this mass regime.

A further reason for the comparatively high power and the steep spectrum of the halo in A1033 could lie in a greater abundance of seed electrons in the cluster, possibly accumulated by the various other bright radio sources. The synchrotron luminosity in re-acceleration models is given by [e.g. Brunetti & Vazza 2020; di Gasperin et al. 2021].

$$P \propto F \eta \frac{B^2}{B_{\text{cmb}}^2} + \frac{B^2_{\text{rad}}}{B_{\text{rad}}^2},$$

where $\eta$ is the (re-)acceleration efficiency and $F = \rho v_e^3/L_{\text{inj}}$ is the turbulent energy flux in the emitting volume which depends on the ICM density $\rho$ as well as the injection scale $L_{\text{inj}}$ and turbulent velocity $v_{\text{rad}}$. The efficiency $\eta$ is the fraction of the turbulent energy flux contributing to CRe-acceleration and proportional to the CRe energy density $n_{CRe} \propto \eta \rho v_{\text{rad}}^3$. Bonafede et al. [2022]. Thus, following from Equation 6 for the same CRe energy spectrum, a higher electron density would increase the halo power while not causing a spectral flattening. We speculate that at the low surface brightness patch in extension of the GREET (marked in Figure 6) with spectral properties in-between those of the GREET and the halo, we might possibly witness electrons which were conserved by the GREET entering the CRe population associated with the halo, a similar connection was proposed e.g. in Wilber et al. [2018]. Since only a very small number of halos are detected in clusters with masses similar to or lower than A1033, the shape and scatter of the correlation in this mass range has not been well determined yet. The detailed analysis of the largest sample of radio halos so far, based on the second data release of LoTSS (Botteon et al. 2022), will provide strongly improved constraints in the near future (Cuciti et al. in prep.).

4.4. Radio phoenix

The bright, extended source 150 kpc to the South of the cluster center was previously studied in de Gasperin et al. [2015] and categorized as a radio phoenix, a connection to a bright elliptical galaxy (marked S in Figure 2) that coincides with a radio point source at 1.4 GHz was proposed. The radio phoenix has an irregular and elongated shape with a largest linear size of ~ 380 kpc. Towards the South, it shows a steep gradient, while towards the cluster center, the source is fading slower and at lower frequencies, extends towards the head-tail galaxy and the BCG. In Figure 9, its flux density is compared to the head-tail radio galaxy and the GREET between 54 and 1400 MHz. The source shows spectral curvature, the spectral index flattens from $\alpha_{509}^{144} = -1.58 \pm 0.17$ (de Gasperin et al. [2015]) to $\alpha_{54}^{144} = -0.99 \pm 0.14$. The spatial variation of the spectral properties (Figure 3) does not reveal a clear trend. The spectral curvature we observe is a characteristic feature of radio phoenixes [Enblin & Gopal-Krishna 2001]. Therefore, the ultra-low frequency picture of the source is in line with the phoenix classification. The source differs from the GREET in the spectral shape, its spectrum is significantly less curved between 54 and 323 MHz ($\Delta_{\alpha} \approx 0.3$ to $\Delta_{\alpha} \approx 2.0$), furthermore, its morphology is more irregular and not clearly connected to a radio galaxy.
For the GReET, we detected extreme spectral curvature. The spectrum steepens form $\alpha \approx -4$ between 144 and 323 MHz to $\alpha \approx -2$ between 54 and 144 MHz. This finding is in line with the emission being generated by strongly aged electrons which are re-accelerated by a mechanism with very low acceleration efficiency.

Assuming a maximum lifetime scenario, we found a lower limit for the projected velocity of the WAT of 719 km/s, in agreement with earlier findings. This corresponds to an maximum age of the tail of $\approx 700$ Myr.

We investigated the properties of the radio halo in A1033. We found a radio power of $P_{150\text{MHz}} \approx 1.22 \times 10^{25}$ W Hz$^{-1}$. This power is above the radio power-to-cluster mass correlations by a factor $> 7$. At the same time, we found the spectrum of the halo to be ultra-steep between 54 and 144 MHz ($\alpha \approx -1.69$), with a further steepening at higher frequencies.

We presented possible reasons for both the high luminosity and the ultra-steep spectrum of the halo: The halo power could be explained by an energetic merger that does not fully reveal itself in the X-ray morphology parameters due to a significant line-of-sight component of the merger axis, a scenario which is supported by the redshift bimodality and the high radial velocity of the BCG. This requires that the spectral properties are dominantly controlled by the cluster mass. Alternatively, a particularly high density of cosmic ray electrons, possibly accumulated by the various bright radio sources in the cluster, can explain an increased radio power without spectral flattening. Simulations could be used to investigate whether a significant per-cluster variation of seed electrons can be caused by sources such as the GReET and radio phoenix.

We detected two candidate disrupted filaments in the GReET, so we speculated that large-scale turbulent motions are responsible for the disruption. With the lower limit on the WAT velocity we derived, we constrained the turbulent velocity to be greater than 187 km s$^{-1}$ and 587 km s$^{-1}$, respectively, within a typical range found in simulations. The thin tail itself could be stabilized by magnetic fields if the turbulence at these small scales becomes sub-Alfvénic.

We found that the spectrum of the radio phoenix flattens from $\alpha \approx -1.6$ to $\alpha \approx -1.0$ between 1.4 GHz and 54 MHz, which is a characteristic property of radio phoenices.

With the further advance of low-frequency radio surveys such as LoTSS and LoLSS, it will be possible to perform spectral analyses for a rapidly growing number of cluster radio sources. These will show whether the GReET in A1033 is representative for a greater category of sources or truly special in terms of its low acceleration efficiency. Additionally, systematical analyses of radio halos in clusters of low and moderate mass will lead to a better understanding of the halo power and the spectral index distributions in this mass range.

5. Conclusion

In this work, we presented an analysis of the galaxy cluster Abell 1033 based on new ultra-low frequency data taken with LOFAR LBA at 54 MHz and existing data at higher frequencies (LOFAR HBA data at 144 MHz and GMRT data at 323 MHz). The cluster is an especially interesting target, since despite its comparatively low mass, it hosts a variety of strong and extended radio sources such as a GReET, a radio phoenix, interacting AGN and a radio halo, which show very heterogeneous properties. Our findings are:
Appendix A: Uncertainty maps

Fig. A.1: Uncertainty maps to Figure 3. The left and center figure show the spectral index uncertainties between 54-144 MHz (left) and 144-323 MHz (center). The right image shows the uncertainty of the curvature map.

Fig. A.2: Source-subtracted spectral index uncertainty maps at a spatial resolution of 30". The two figures are for the frequency ranges 54-144 MHz (left) and 144-323 MHz (right). Red dashed lines highlight the regions where sources are subtracted.
Appendix B: Spectral aging model

We employ the standard JP spectral aging model to describe the aging at the first section of the WAT radio galaxy. The flux density of this model at a frequency $\nu$ and after a time $t$ is given by (Harwood et al. 2013):

$$S_{JP}(N_0, \nu, B, t, z, \alpha_{inj}) = N_0 \sqrt{\frac{3\ln(10)e^3}{16\pi\epsilon_0cm(\epsilon^2 + 1)^2}} \int_0^\infty d\delta \sin(\delta)^2 \int d\log(E)EF(x)\nu_c(E, B, t, z, \alpha_{inj}),$$  \hfill (B.1)

where $e$ and $m_e$ are the electron charge and mass, $\epsilon_0$ is the vacuum permittivity, $\delta$ the pitch angle and $F(x)$ the following function:

$$F(x) = x \int_\infty^x K_{5/3}(z)\,dz.$$  \hfill (B.2)

In Equation B.1, the energy-integration is performed in log-space for numerical efficiency. We first fit a spectral index model to the observed spectral index values $\alpha$ to remove dependence on the normalizations $N_0$, the spectral index model is calculated as:

$$\alpha_{JP}(\nu_1, \nu_2, B, t, z, \alpha_{inj}) = \log \frac{S_{JP}(\nu_1, B, t, z, \alpha_{inj})}{S_{JP}(\nu_2, B, t, z, \alpha_{inj})} \frac{\nu_1}{\nu_2}.$$  \hfill (B.4)

The uncertainty for the observed spectral index values $\sigma_\alpha$ are obtained from the uncertainties of the flux densities $\sigma_S$ according to Gaussian propagation of uncertainty:

$$\sigma_\alpha = \frac{1}{\log(\nu_1/\nu_2)} \sqrt{\sigma_{S_1,\text{stat}}^2 + (0.1 \times S_1)^2 \frac{\sigma_{S_2,\text{stat}}^2 + (0.1 \times S_2)^2}{S_2^2}}.$$  \hfill (B.5)

Here we take into account the 10% systematic uncertainty on the flux-scales. We fit the WAT projected velocity $v_\perp$ by minimizing the standard $\chi^2$-statistics assuming a minimum aging magnetic field $B_{min}$ and a linear motion $t = \frac{d_\perp}{v_\perp}$:

$$\chi^2 = \sum_{i=0}^{9} \sum_{j=0}^{1} \left( \frac{\alpha_i(\nu_j, \nu_{j+1}) - \alpha_{JP}(\nu_j, \nu_{j+1}, B_{min}, d_\perp, v_\perp, z, \alpha_{inj})}{\sigma_\alpha(\nu_j, \nu_{j+1})} \right)^2.$$  \hfill (B.6)

Subsequently, a normalization factor $N_{0,i}$ is fitted in log-space for each beam-sized region along the WAT/GReET.

Fig. B.1: Aging model merit function against magnetic field (x-axis) and projected velocity (y-axis) in arbitrary units.
Appendix C: Halo fit results

Fig. C.1: Results of the halo fitting in LBA (a) and HBA (b). The circles in the left panels indicate the position of the best-fit halo on top of the flux density maps, the center panels show the model, and the right panels the residual. The green/red regions are excluded from the fitting.