New Early Dark Energy

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New measurements of the expansion rate of the universe have plunged the standard model of cosmology into a severe crisis. In this letter we propose a simple resolution to the problem. We propose that a first order phase transition in a dark sector in the early universe, before recombination, can resolve the problem. This will lead to a short phase of a New Early Dark Energy (NEDE) component and can explain the observations. Fitting our model to measurements of the Cosmic Microwave Background, Baryonic Acoustic Oscillations, and supernovae yields a significant improvement of the best-fit compared with the standard cosmological model without NEDE. We find the mean value of the present Hubble parameter in the NEDE model to be $H_0 = 71.4 \pm 1.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (68\% C.L.).

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INTRODUCTION

Recent measurements of the expansion of the universe have led to an apparent crisis for the standard model of cosmology, the ΛCDM model.

Within the ΛCDM model, we can calculate the evolution of the universe from the earliest times until today, and until recently all our measurements were consistent with the model. In particular, we can use the measurements of the Cosmic Microwave Background (CMB) radiation to infer the present value of the Hubble parameter, $H_0$. If the ΛCDM model is correct, this value will have to agree with the value obtained by directly measuring the expansion rate today using supernovae redshift measurements. Now, the problem is that the measurements, direct and indirect, do not agree, and this puts the ΛCDM model in a crisis.

The most precise measurements we have of the temperature fluctuations, polarization and lensing in the CMB radiation are from the European Space Agency satellite, Planck, which, assuming the ΛCDM model, infers the value of the expansion rate today to be $H_0 = 67.36 \pm 0.54 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Comparing that with the expansion rate measured from Cepheids-calibrated supernovae by the SH0ES team\textsuperscript{2}, $H_0 = 74.03 \pm 1.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$, there is a 4.4 σ discrepancy. Other measurements of the current Hubble rate, such as the $H_0$LiCoW\textsuperscript{3}, are also significantly discrepant with the Planck measurement\textsuperscript{4}.

The Planck measurement of the CMB is a very clean experiment with the systematics well under control, and it is therefore unlikely that there is non-understood systematics in the CMB measurements which can explain the discrepancy. The local supernova observations, on the other hand, involves astronomical distance measurements, which are notoriously difficult, and have been plagued by non-understood systematic errors in the past. Various possible sources of systematics have been considered extensively in the literature already. In\textsuperscript{3} it is shown that adding up different possible systematic errors in the determination of the distance ladder cannot resolve the discrepancy. Also the effect of the local bulk flow was considered in\textsuperscript{3, 4} with the same negative conclusion. So far astronomers have no commonly accepted idea of possible systematic effects to explain the discrepancy, and an often echoed conclusion is that new physics beyond the ΛCDM model is required to resolve the tension (see f.ex.\textsuperscript{5}). While it is important to continue to look for possible systematic effects, in the present letter, we will rather consider a simple solution in terms of new physics.

We will study the possibility that a first order phase transition in a dark sector at zero temperature happened shortly before recombination in the early universe\textsuperscript{1}. Such a phase transition will have the effect of lowering an initially high value of the cosmological constant in the early universe down to the value today, inferred from the measurement of $H_0$. Effectively this means that there has been an extra component of dark energy in the early universe, providing a short burst of additional repulsion. Currently, an extra component of Early Dark Energy (EDE) is the best way to resolve the tension between the early and late measurements of $H_0$\textsuperscript{10, 17}. So far people have typically considered a dynamical EDE component that disappears due to a 2nd order phase transition of a slowly rolling scalar field\textsuperscript{4}. Such scenarios have complications if monomial potentials are used both at background and perturbative level\textsuperscript{12, 20}, as one needs the potential to be steep and anharmonic at the bottom to end up with a sufficiently stiff fluid but also flat initially to achieve a sound speed $c_s^2 < 0.9$ for a large enough range of sub-horizon modes. While this problem can be overcome by

\textsuperscript{1} Provided the string landscape\textsuperscript{21} is the correct framework for understanding the cosmological constant problem, NEDE could be a manifestation of this.

\textsuperscript{2} The general idea of having an early dark energy component is older and dates back to\textsuperscript{13, 19}. 

\textsuperscript{3} see f.ex.

\textsuperscript{4} for a large enough range of sub-horizon modes. While this problem can be overcome by
using specific terms from the non-perturbative form of the axion potential \[^{10,12}\], it represents a non-generic choice \[^{13}\].

On the other hand, we believe that a first order phase transition holds in it the potential to fully resolve the discrepancy between the early and late measurements of \(H_0\) much more naturally. In addition, a first order phase transition will lead to different experimental signatures, such as gravitational waves; so, there is good reason to believe it might be possible to discriminate between both models by more precise measurements in the future.

Below we explore the simplest NEDE model. For more details and generalizations of the model, as well as a detailed comparison with other models, we refer the reader to our longer companion paper \[^{22}\].

### THE MODEL

In order to have a change in the vacuum energy due to a field that undergoes a first order phase transition, we will consider a scalar field with two non-degenerate minima at zero temperature. However, if the tunneling probability from the false to the true vacuum is initially high, the field will tunnel immediately and NEDE never makes a sizable contribution. On the other hand, once tunneling commences, we need a large rate in order to produce enough bubbles of true vacuum that will quickly collide. If the rate is too small, then part of the universe will be in the true and part of it in the false vacuum, which will lead to large inhomogeneities ruled out by observations. We therefore require an additional sub-dominant trigger field, which suddenly, at the right moment, makes the tunneling rate very high. Analogous to previously considered mechanisms for ending inflation \[^{23,24}\], we will therefore consider models with a general potential of the form,

\[
V(\psi, \phi) = \frac{\lambda}{4} \psi^4 + \frac{1}{2} \beta M^2 \psi^2 - \frac{1}{3} \alpha M \psi^3 + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \lambda \phi^2 \psi^2 ,
\]

where \(\psi\) is the tunneling field and \(\phi\) is the trigger field. The sub-dominant trigger field will be frozen as long as its mass is smaller than the Hubble rate, but as soon as the Hubble rate drops below its mass, it will start decaying and this will trigger the tunneling of the \(\psi\) field. For a second minimum to develop after the point of inflection, we need to impose \(\alpha^2 > 4 \beta \lambda\), \(\beta > 0\). In figure 1, we have included a 3-D visualisation of the evolution of the potential as the trigger field, \(\phi\), starts evolving along the orange path opening up the new vacuum for \(\psi\), to which it tunnels with high probability.

The decay rate per unit volume is \(\Gamma = K \exp (-S_E)\), where \(K\) is a determinant factor which is generically set by the energy scale of the phase transition \[^{25,26}\] and \(S_E\) is the Euclidian action corresponding to a so-called bounce solution \[^{27}\]. While it is possible to find an analytic expression in the thin wall limit for a single field, the general case requires a numerical approach. In \[^{22}\] we argue that a good approximation of the Euclidian action (describing the potential as being effectively one-dimensional) can be written as

\[
S_E \approx \frac{4 \pi^2}{3 \lambda} (2 - \delta_{\text{eff}})^{-3} (\alpha_1 \delta_{\text{eff}} + \alpha_2 \delta_{\text{eff}}^2 + \alpha_3 \delta_{\text{eff}}^3) ,
\]

with numerically determined coefficients \(\alpha_1 = 13.832, \alpha_2 = -10.819, \alpha_3 = 2.0765\) and

\[
\delta_{\text{eff}}(t) = 9 \frac{\lambda}{\alpha^2} \left( \beta + \frac{\phi^2(t)}{M^2} \right) .
\]

The important message from this is that \(S_E\) becomes large as \(\delta_{\text{eff}} \to 2\) and vanishes as \(\delta_{\text{eff}} \to 0\). As a result, the tunneling rate is suppressed when \(\phi\) is frozen at a sufficiently large initial field value (corresponding to \(\delta_{\text{eff}} > 9/4 \sim 2\)) and becomes maximal as \(\phi \to 0\) once the Hubble drag is released (corresponding to \(\delta_{\text{eff}} \to 9 \lambda \beta / \alpha^2 < 9/4\)).

At early times, we require the transition rate to be highly suppressed, which fixes the initial value of the trigger field, \(\phi_{\text{ini}}\), and can be satisfied consistently with the condition that \(\phi_{\text{ini}} / M_{\text{eff}} \ll 1\), which is sufficient to ensure that the contribution of \(\phi\) to the total energy density is sub-dominant.

Now, we also have to ensure that NEDE, given by the potential energy in the \(\psi\) field, gives a sizable contribution to the energy budget at the time \(t_*\) where bubble percolation of the \(\psi\) vacuum becomes efficient. We can quantify it in terms of the ratio \(f_{\text{NEDE}} = \Delta V / \bar{\rho}(t_*)\) where \(\Delta V\) is the liberated vacuum energy and \(\bar{\rho}\) the total energy density. If the transition occurs at a redshift of order \(z \sim 5000, \lambda \sim 0.1, \alpha \sim \beta \sim O(1)\) and \(f_{\text{NEDE}} \sim 0.1\), we have \(M \sim \text{eV}\) and an ultra-light mass scale of order \(m \sim 10^{-27}\text{eV}\). A microphysical model explaining the
mass hierarchy between the $M$ and the $m$ scale would be a model of axion monodromy with two axion fields (see [30] for a field theory version). Here, the masses are protected by softly broken shift symmetries.

We also have to make sure that the nucleation itself happens sufficiently quickly. To that end, we define the percolation parameter $p = T/H^4 \sim M^4/m^4 e^{-5E}$, where we approximated $K \sim M$. Provided $p \gg 1$, a large number of bubbles is nucleated within one Hubble patch and one Hubble time. In fact, for the above choice of parameters, the huge hierarchy between the scalar masses, $M^4/m^4 \sim 10^{108}$, implies that $p \gg 1$ only requires $S_E < 250$, which according to (2) and (3) can be easily satisfied as $\phi \to 0$. This means that percolation is extremely efficient and will cover the entire space with bubbles of true vacuum in a tiny fraction of a Hubble time. Therefore, we can treat it as an instantaneous process on cosmological time scales, which takes place at time $t_*$.

As the space is being filled with bubbles of true vacuum, they expand and start to collide. In our case this happens almost instantly as there is little space left between the bubbles. This coalescence phase is governed by complicated dynamics, which can be studied analytically only in simplified two-bubble scenarios as in [31]. In particular, it leads to the production of density perturbations that are in general not scale invariant and therefore appear to be dangerous. However, this is not a problem, because in a typical scenario bubbles are of a physical size today $\ll$ Mpc when they collide, whereas the CMB is only sensitive to structures with size greater than about $10h^{-1}$Mpc [32]. As part of the collision process the complicated $\psi$ condensate starts to decay. Microscopically, the released free energy gets converted into anisotropic stress on small scales, which we expect, after partially being converted to gravitational radiation, to decay as $1/a^6$ similar to a stiff fluid component.

**MATCHING CONDITIONS**

We use a simple background model describing the instantaneous transition from a background fluid with an equation of state (e.o.s.) parameter that changes from $-1$ to $w_{\text{NEDE}}^*$:

$$w_{\text{NEDE}}(t) = \begin{cases} -1 & \text{for } t < t_* , \\ w_{\text{NEDE}}^* & \text{for } t > t_* , \end{cases}$$

(4)

where the transition happens at time $t_*$. In terms of our field theory model in [11] this corresponds to a situation where all of the liberated vacuum energy is transferred to a fluid with e.o.s. parameter $w_{\text{NEDE}}^*$, where according to the considerations above, we expect $1/3 \leq w_{\text{NEDE}}^* \leq 1$.

**Background Matching**

The above condition fixes the evolution of the background energy density uniquely,

$$\dot{\rho}_{\text{NEDE}}(t) = \frac{\dot{\rho}_{\text{NEDE}}^*}{a(t)} (\frac{a(t)}{a(t_*)})^{3(1+w_{\text{NEDE}}^*)(t)} ,$$

(5)

where $\rho_{\text{NEDE}}^* = f_{\text{NEDE}} \dot{\rho}_* = \text{const.}$ The energy density of NEDE is normalized with respect to the true vacuum and continuous across the transition. In order to denote the discontinuity of a time dependent function $f(t)$ across the transition surface at time $t_*$, we introduce the notation

$$[f]_\pm = \lim_{\epsilon \to 0} [f(t_* - \epsilon) - f(t_* + \epsilon)] \equiv f(-) - f(+) .$$

(6)

Applying this operation to the Friedmann equations, we then find

$$[H]_0 = 0 ,$$

(7a)

$$[\dot{H}]_0 = 4\pi G(1 + w_{\text{NEDE}}^*) \dot{\rho}_{\text{NEDE}}^* ,$$

(7b)

where we used the continuity of the background energy density, $[\dot{\rho}]_0 = 0$, which holds due to (5) and the instantaneous character of the transition. The derivation of (7b) also assumes that the e.o.s. of all other fluid components (except for NEDE) is preserved during the transition. Besides the NEDE component, we also track the evolution of the sub-dominant field $\phi$ to turn on the phase transition.

**Perturbation Matching**

It is not enough to implement the modifications on the background level. In fact, neglecting the fluctuations of the NEDE sector would be inconsistent after the decay has occurred, making it mandatory to track their evolution. Before the decay, on the other hand, we can set them to zero as NEDE simply behaves as a (non-fluctuating) cosmological constant. This raises the issue of how to initialize them at time $t_*$. Since the transition is allowed to happen at a relatively late stage in the evolution of the primordial plasma (in the extreme case right before recombination), we cannot assume that all relevant modes are outside the horizon. This makes it quite different to the standard problem of choosing adiabatic initial conditions for super-horizon modes. In the specific case of our two-field model, we use the field value of the trigger field to determine the transition. This is motivated by the $\phi$ dependence of the parameter $\delta_{eq}$ in (3) which controls the exponential in the tunneling rate through (2). The transition surface $\Sigma$ for both sub- and super-horizon modes is therefore defined by a constant
value of the trigger field
\[ \phi(t_*, x)|_\Sigma = \text{const.} \] (8)
As a consequence, fluctuations in \( \phi \) lead to spatial variations of the time \( t' \) at which the decay takes place. These variations, \( \delta \phi \), provide the initial conditions for the fluctuations in the NEDE fluid after the phase transition.

In order to match the conventions used in the Boltzmann code community, we work in synchronous gauge,
\[ ds^2 = -dt^2 + a(t)^2 (\delta_{ij} + h_{ij}) dx^i dx^j, \] (9)
where in momentum space
\[ h_{ij} = \frac{k_i k_j}{k^2} h + \left( \frac{k_i k_j}{k^2} - \frac{1}{3} \delta_{ij} \right) 6 \eta, \] (10)
and \( h = \delta^{ij} h_{ij} \). In the following we will make use of the equations for the metric perturbations that are first order in time derivatives [33],
\[ \frac{1}{2} H \dot{h} - \frac{k^2}{a^2 \eta} = 4 \pi G \delta \rho, \] (11a)
\[ -\frac{k^2}{a^2 \dot{\eta}} = 4 \pi G (\bar{\rho} + \bar{p}) \frac{\theta}{a}, \] (11b)
where \( (\bar{\rho} + \bar{p}) \theta = \int_\ell (\bar{\rho}_i + \bar{p}_i) \theta_i \) and \( \delta \rho = \sum_i \delta \rho_i \) are the total divergence of the fluid velocity and the total energy density perturbation, respectively. The dynamical equations have to be supplemented with Israel’s matching conditions [29, 34]. They relate the time derivatives of \( \eta \) and \( h \) before and after the transition,
\[ \left[ \dot{h} \right]_\pm = -6 [\eta]_\pm = 6 \left[ \dot{H} \right]_\pm \frac{\delta \phi(t_*, k)}{\dot{\phi}(t_*)}, \] (12)
where \( \left[ \dot{H} \right]_\pm \) is specified in (7b), and we used the residual gauge freedom in the synchronous gauge to bring the matching conditions on this simple form. So, we see that the discontinuities of \( \dot{\eta} \) and \( \dot{h} \) are proportional to the spatial variations of the trigger field. We further find that all perturbations without a derivative, including the fluid sector, are continuous, i.e. \( [h]_\pm = [\eta]_\pm = [\dot{\eta}]_\pm = [\theta]_\pm = 0 \), where \( \dot{\delta}_i = \delta \rho_i / \bar{\rho}_i \). This does not apply to NEDE perturbations because the derivation assumed that the e.o.s. of a particular matter component \( i \) is not changing during the transition, in contrast with [4]. In fact, as NEDE behaves as a perfect cosmological constant before the transition, we can consistently set
\[ \delta_{\text{NEDE}} = \theta_{\text{NEDE}} = \sigma_{\text{NEDE}} = 0 \quad \text{for} \quad t < t_*, \] (13)
because these fluctuations are not sourced by the gravitational potential in contrast to ordinary fluids. Here, \( \delta_{\text{NEDE}} \) denotes the anisotropic stress as defined in [35].

We further introduce the notation \( \delta^{(\pm)}_{\text{NEDE}} \equiv \delta^*_{\text{NEDE}} \) and \( \theta^{(\pm)}_{\text{NEDE}} \equiv \theta^*_{\text{NEDE}} \) to denote the fluctuations right after the transition. We can now evaluate the discontinuity of Einstein’s equations (11) in order to fix \( \delta_{\text{NEDE}}^* = \theta_{\text{NEDE}}^* \), providing the initial conditions for the NEDE perturbations after the transition. Using (12) and (7b), we have
\[ \delta_{\text{NEDE}} = -3 (1 + w_{\text{NEDE}}^*) H(t_*) \frac{\delta \phi(t_*, k)}{\dot{\phi}(t_*)}, \] (14a)
\[ \theta_{\text{NEDE}} = \frac{k^2}{a(t_*)} \frac{\delta \phi(t_*, k)}{\dot{\phi}(t_*)}. \] (14b)
These two equations together with the junction conditions (12) are the main result of this section. They will allow us to consistently implement our model in a Boltzmann code. In order to close the differential system of the perturbed fluid equations, we assumed the vanishing of the anisotropic stress after the transition, i.e. \( \sigma_{\text{NEDE}} = 0 \) and set the rest-frame sound speed \( c^2_{\text{NEDE}} \) in the NEDE fluid to \( c^2_\text{NEDE} = w_{\text{NEDE}}^* \). Also, we did not need to make any assumption about the spatial momentum \( k \), so the junction conditions apply equally to sub- and super-horizon modes.

**DATA ANALYSIS AND RESULTS**

In order to fit the NEDE model to the CMB data, we have incorporated it into the Boltzmann code [4, 36, 37]. To that end, we made the simplifying assumption that all liberated vacuum energy is ultimately converted to small scale anisotropic stress and gravitational radiation, i.e. \( 1/3 \leq w_{\text{NEDE}}^* \leq 1 \). As a specific choice for our data fit, we take the midpoint \( w_{\text{NEDE}}^* = 2/3 (= c_{\text{NEDE}}^2) \), which we relax in our companion paper [22]. In accordance with our microscopic model the decay is triggered shortly before \( \phi = 0 \), where for definiteness we take \( H/m = 0.2 \) (which avoids a tuning and is still compatible with a quick decay).

The cosmological parameters are then extracted with the Monte Carlo Markov Chain code [39, 40], employing a Metropolis-Hastings algorithm. Compared to \( \Lambda \)CDM we introduce two new parameters: the fraction of NEDE before the decay, \( f_{\text{NEDE}} = \bar{\rho}_{\text{NEDE}} / \bar{\rho}(t_*) \), and the logarithm of the mass of the trigger field \( \log_{10} m \), which defines the redshift at decay time, \( z_* \), via \( H(z_*) = 0.2 \). In total we vary eight parameters \{\( \omega_0, \omega_{\text{cdm}}, \bar{h}, \ln 10^{10} A_s, n_s, \tau_{\text{reio}}, f_{\text{NEDE}}, \log_{10} m \}, \)

3 The adapted CLASS code is publicly available on GitHub: https://github.com/flo1984/TriggerCLASS
4 This scenario also assumes that there are no sizeable oscillations in \( \dot{\phi} \) after the transition, which would give rise to a pressureless fluid component (see a similar study in [38]).
on which we impose flat priors. The neutrino sector is modelled in terms of two massless and one massive species with \( M_\nu = 0.06 \text{eV} \). We impose the initial value \( \phi_{\text{ini}}/M_\nu = 10^{-4} \) to make sure that the trigger field is always sub-dominant and the tunneling rate at early times sufficiently suppressed.

We will use the following datasets: the most recent \( SH_0ES \) measurement, which is \( H_0 = 74.03 \pm 1.42 \text{km s}^{-1}\text{Mpc}^{-1} \) \[2\], the Pantheon dataset \[42\] comprised of 1048 SNe Ia in a range \( 0.01 < z < 2.3 \); the BOSS DR 12 anisotropic BAO and growth function measurements at redshift \( z = 0.38, 0.51 \) and 0.61 based on the CMASS and LOWZ galaxy samples \[43\], as well as small-z, isotropic BAO measurements of the 6dF Galaxy Survey \[44\] and the SDSS DR7 main Galaxy sample \[45\] at \( z = 0.106 \) and \( z = 0.15 \), respectively; the Planck 2018 TT,TE,EE and lensing likelihood \[46\] with all nuisance parameters; constraints on the primordial Helium abundance from \[47\].

Performing a likelihood analysis shows that the best-fit improves by \( \Delta \chi^2 = -16.9 \) compared to \( \Lambda\text{CDM} \)[4]. The extracted mean values can be found in Tab.\([1]\) in particular, \( H_0 = 71.4 \pm 1.0 \text{km s}^{-1}\text{Mpc}^{-1} \) (68\% C.L.) with best-fit \( H_0 = 71.5 \text{km s}^{-1}\text{Mpc}^{-1} \). The posteriors for a subset of parameters are depicted in Fig.\([2]\) with the first column showing an overlap of the 68\% C.L. contours between NEDE and \( SH_0ES \). In addition, the decay takes place at \( z_\ast = 4920_{-730}^{+620} \) and there is a non-vanishing NEDE fraction \( f_{\text{NEDE}} = 12.6_{-2.9}^{+3.2} \)\%, excluding \( f_{\text{NEDE}} = 0 \) with a 4.3\( \sigma \) significance.

**CONCLUSIONS**

We have studied a NEDE model where the decay of NEDE happens through a first order phase transition. This makes our model unique compared to older EDE models (which all rely on a second order phase transition) both from a theoretical and phenomenological perspective. The NEDE model holds in it the potential to fully resolve the discrepancy in \( H_0 \) as inferred from early CMB and BAO measurements and late time distance ladder measurements. Our first most simplified implementation of the model (fixing as many free parameters as possible by making simple assumptions) already yields a significant improvement in the fit over the \( \Lambda\text{CDM} \) model of \( \Delta \chi^2 = -15.6 \). It also yields a significant improvement compared to models with extra dark radiation \[48\] \[50\] of \( \Delta \chi^2 = -12.4 \). We expect that the model will fit the data even better when the simplifying assumptions made in the present short letter are dropped in future work.

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\[5\] We assume that chains are converged if the Gelman-Rubin criterion \[41\] yields \( R - 1 < 0.01 \).
TABLE I. The mean and 1σ errors obtained from the cosmological parameter extraction.

| Model | $\omega_b$ | $\omega_{cdm}$ | $H_0 [\text{km/s/Mpc}]$ | ln $10^{10} A_s$ | $n_s$ | $\tau_{sio}$ | $f_{\text{NEDE}}$ | $z_s$ |
|-------|------------|----------------|---------------------------|------------------|-------|-------------|-----------------|-------|
| AC1D | 2.251 $^{+0.014}_{-0.013}$ | 0.1184 $^{+0.0009}_{-0.0009}$ | 68.13 $^{+0.41}_{-0.41}$ | 3.053 $^{+0.014}_{-0.016}$ | 0.9686 $^{+0.0033}_{-0.0037}$ | 0.0599 $^{+0.0079}_{-0.0077}$ | – | – |
| NEDE | 2.292 $^{+0.022}_{-0.024}$ | 0.1304 $^{+0.0034}_{-0.0035}$ | 71.4 $^{+0.10}_{-0.10}$ | 3.067 $^{+0.014}_{-0.016}$ | 0.9889 $^{+0.0007}_{-0.0007}$ | 0.0571 $^{+0.0069}_{-0.0069}$ | 0.126 $^{+0.032}_{-0.029}$ | 4920 $^{+620}_{-740}$ |
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