Second-Order Consensus of Hybrid Multi-Agent Systems With Unknown Disturbances Via Sliding Mode Control

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ABSTRACT In this paper, we consider the second-order consensus problem of hybrid multi-agent systems with unknown disturbances by using sliding mode control under the leader-follower network. First, the hybrid multi-agent system model with disturbances and nonlinear term is proposed, which is composed of continuous-time dynamic agents and discrete-time dynamic agents. Second, the definition of the second-order consensus of hybrid multi-agent system is given. Then, we assume that the interaction among all agents happens in sampling time and each continuous-time dynamic agent can observe its own states in real time. Based on the equivalent approaching law and the states information among agents, the sliding mode control protocols are designed to achieve the second-order consensus of the hybrid multi-agent system. Some sufficient conditions are given for solving the second-order consensus under the sliding mode control protocols. Finally, some simulations are also given to illustrate the validity of the proposed method.

INDEX TERMS Disturbance and nonlinear, hybrid multi-agent systems, second-order consensus, sliding mode control.

I. INTRODUCTION

The topic of cooperative control for multi-agent system has been widely concerned by many researchers in the past decades. Multi-agent system is composed of interconnected agents, in which the dynamics of each agent are affected by the behaviors of neighboring agents. Among many existing coordination methods, the distributed control provides a lot of references for the research of cooperative control in [1]–[5]. And the leader-follower strategies have been widely studied due to their vast applications in engineering problems in [2], [3], [6]–[8]. At the same time, sliding mode control (SMC) has also been widely used in the coordination problem of multi-agent system because of its robustness and fast response to disturbances change in [1], [2], [8]–[9].

As a fundamental problem of distributed coordination, consensus means all agents reach an agreement by designing appropriate protocols. In [10], the authors proposed a simple model with a novel type of dynamics in order to investigate the emergence of self-ordered motion in systems and then studied the consensus problem of this model. Now, researchers call this model the discrete-time multi-agent system. Subsequently, many studies on discrete-time multi-agent systems appeared. In [11], the authors studied the consensus of discrete-time multi-agent systems with time delay under the leader-follower networks. In [12], the authors investigated the consensus for a class of discrete-time systems via delta operators. In practical systems, the discrete-time multi-agent systems always have some certain constraints, therefore, scholars have begun to use sliding mode control methods to solve these constraints. Lots of results about the time delay, uncertainties, and disturbance on discrete-time systems have been obtained by sliding mode control methods in [13]–[16]. For continuous-time multi-agent systems, the authors considered the consensus of multi-agent systems with time delay in three cases and obtained some necessary and/or sufficient conditions for solving the average consensus in [17]. In [18], the authors considered the scaled consensus...
of second-order nonlinear multi-agent systems with time-varying delays. In [19], the authors considered the problem of information consensus among multi-agents in the presence of limited and unreliable information exchange with dynamically changing interaction topologies. In [20], the authors proved that all continuous-time agents can be guaranteed to asymptotically reach bipartite consensus for any logarithmic quantizer accuracy under connected and structurally balanced topology. Disturbances are common problems in practical continuous-time multi-agent systems, and sliding mode control is a good tool to solve the disturbance problem. In [21], distributed active anti-disturbances output algorithms were applied to achieve the consensus of higher-order multi-agent systems by sliding mode control. Many researchers have also done a lot of works on the consensus of continuous-time systems with disturbances by sliding mode control in [22]–[26].

Hybrid systems are composed of interconnected continuous-time systems and discrete-time systems. Hybrid systems appear in a variety of applications including manufacturing, communication networks, automotive engine control, computer synchronization, chemical processes, and traffic control in [27]–[28]. Hybrid multi-agent system is a research branch of hybrid system. Hybrid multi-agent systems are dynamical systems, which are composed of continuous-time and discrete-time dynamic agents. In the practical systems, the dynamics of the agents coupled with each other are different, i.e., the dynamics of agents are hybrid. In general, hybrid means heterogeneous in nature or composition. There are some works about heterogeneous systems. In [29], [30], the authors studied the consensus problem of heterogeneous multi-agent system with the linear consensus protocol and the saturated consensus protocol, respectively. The consensus tracking of heterogeneous interdependent group systems was also considered in [31]–[33]. For hybrid systems, the author proposed the sufficient condition for the stability of the hybrid system, which the Lyapunov function of the subsystem is a continuous positive definite function with partial derivatives in [34]. For hybrid multi-agent systems, the authors proposed the hybrid multi-agent systems model and three kinds of consensus protocols are presented for the hybrid multi-agent system with first-order dynamic agents. Then, some necessary and sufficient conditions are established for solving the consensus problem in [35]. At the same time, the second-order consensus of hybrid multi-agent systems was also considered by some researchers. In [36], the second-order consensus of hybrid multi-agent systems was researched by using algebraic graph theory and system transformation method. By analyzing the interactive mode of different dynamic agents, two kinds of effective consensus protocols are proposed for the hybrid multi-agent system. Recently, the authors designed a game to depict the interacting behaviors among the agents in hybrid multi-agent systems and proposed a new consensus protocol based on the designed game in [37]. Some researchers have also done research on hybrid consensus control. In [38], the authors considered the distributed consensus problem of the second-order multi-agent system where the agents are connected via distance-dependent networks and then established the sufficient conditions for the hybrid consensus of the second-order dynamics system. The hybrid consensus-based formation control of agents with second-order dynamics was considered by using neural network in [39].

To the best of our knowledge, however, the existing methods for the consensus analysis are on hybrid multi-agent systems without disturbances and nonlinear term. For example, the consensus of first-order hybrid multi-agent systems without disturbances and nonlinear term was considered in [35], and the second-order consensus of hybrid multi-agent systems without disturbances and nonlinear term was also investigated in [36]. In real system, unknown disturbances are common problem, therefore when hybrid multi-agent systems with second-order dynamic agents and unknown disturbances coexist, it is important to study the second-order consensus problem. In addition, continuous-time dynamic agents in hybrid multi-agent systems are all linear and without generality in [35], [36]. Therefore, it is more practical to study hybrid multi-agent systems with disturbances and nonlinear term. In the above literatures, sliding mode control (SMC) is almost used in heterogeneous or discrete-time/continuous-time multi-agent systems. Almost no sliding mode control method is used to study the consensus of hybrid multi-agent systems with disturbances and nonlinear term. Therefore, based on the above considerations, we propose the consensus protocols for hybrid multi-agent systems with unknown disturbances by using sliding mode control. We assume that the sum of disturbances in the system is bounded, and the sliding mode control protocols are designed by using the bounds of the sum of disturbances. The main contributions of this paper are reflected in three aspects. First, compared with the results of [35] and [36], the consensus problem of hybrid multi-agent systems with unknown disturbances is studied in this paper. We propose the control protocols for unknown disturbances in hybrid multi-agent systems, and the control protocols contain the states information of continuous-time and discrete-time dynamic agents. Second, there are often nonlinear terms in hybrid multi-agent systems, therefore, we consider the continuous-time dynamic agents in hybrid multi-agent systems are nonlinear, and the control protocols can solve the nonlinear problem in the system. Third, the robustness of the hybrid multi-agent systems to external disturbances and nonlinear term is greatly improved by using sliding mode control.

The rest of this paper is organized as follows. In Section 2, we introduce the necessary notations and some mathematical preliminaries. In Section 3, we design the sliding mode control protocols for the hybrid multi-agent systems with disturbances under the leader-follower network, and some sufficient conditions for the second-order consensus are proved. In Section 4, numerical simulations are given to validate the efficiency of the proposed consensus protocols. Finally, some conclusions are drawn in Section 5.
II. PRELIMINARIES

A. NOTATIONS

In this paper, \( \mathbb{N} \) is positive integer, \( \mathbb{R} \) denotes the set of real number and \( \mathbb{R}^{n \times m} \) denotes the \( n \times m \)-dimensional real vector matrix. \( I_m = \{1, 2, \ldots, m\} \) represents the set of continuous-time dynamic agents, \( I_n \cup I_m = \{m + 1, m + 2, \ldots, n\} \) represents the set of discrete-time dynamic agents. \( I_n = \{1, 2, \ldots, m, m + 1, \ldots, n\} \) represents the set of all dynamic agents. Denote by \( I_n \) (or \( 0_n \)) column vector with all entries equal to one (or all zeros). \( I_n \) is an \( n \)-dimensional identity matrix.

B. GRAPH THEORY

The communication topologies of multi-agent systems can be modeled by a weighted directed graph \( G = (\mathcal{V}, \mathcal{E}, A) \), where \( \mathcal{V} = \{s_1, s_2, \ldots, s_n\} \) is the vertex set with each node representing one node and \( \mathcal{E} = \{e_{ij} = (s_i, s_j)\} \subseteq \mathcal{V} \times \mathcal{V} \) represents the set of directed edges. Let \( A = [a_{ij}]_{n \times n} \) be the nonnegative matrix, \( A = [a_{ij}]_{n \times n} \) is the weighted adjacency matrix associated with the graph \( G \). The neighbor set of the agent \( i \) is \( N_i = \{j : a_{ij} > 0\} \). The degree matrix \( D = [d_{ij}]_{n \times n} \) is a diagonal matrix with \( d_{ii} = \sum_{j=1}^{n} a_{ij} \). The Laplacian matrix of the graph is defined as \( L = D - A \). A directed tree is a directed graph, where there exists a vertex called the root, such that there exists a unique directed path from this vertex to every other vertex. A directed spanning tree is a directed tree, which consists of all the nodes and some edges in \( G \).

For multi-agent systems under the leader-follower network, the communications between leader and followers are unidirectional. The communication weight between the leader and \( ith \) follower is denoted by \( b_i \), \( b_i > 0 \) if \( ith \) follower can communicate with the leader and \( b_i = 0 \) otherwise.

C. SECOND-ORDER HYBRID MULTI-AGENT SYSTEMS

The hybrid multi-agent system with second-order dynamics is composed of continuous-time and discrete-time dynamic agents. Assume that the interaction among all agents happens in sampling time \( t_k \) and each continuous-time dynamic agent can observe its own state in real time. In the leader-follower network, the dynamics of the leader is \( \dot{x}_0(t) = v_0(t), \dot{v}_0(t) = u_0(t) \), where \( x_0(t), v_0(t), u_0(t) \in \mathbb{R} \) are the position-like and the velocity-like of the leader, \( u_0(t) \) is the control input of the leader. The number of followers is \( n \), labeled 1 through \( n \), where the number of continuous-time dynamic agents is \( m (m < n) \). Without loss of generality, we assume that agent 1 through agent \( m \) are continuous-time dynamic agents. Each agent has dynamics as follows:

\[
\begin{align*}
\dot{x}_i(t) &= f_i(x_i(t), g_i u_i(t) + d_i(t)), \quad i \in I_m \\
\dot{v}_i(t_{k+1}) &= A \xi_i(t_k) + B [u_{id}(t_k) + d_{id}(t_k)], \quad i \in I_n/I_m
\end{align*}
\]  

where \( \dot{x}_i(t) = v_i(t) \), \( v_i(t) \) is the velocity-like of the continuous-time dynamic agents, \( f_i(x_i) \) is smooth nonlinear functions, \( g_i \in \mathbb{R}, t_k = kh, k \in \mathbb{N}, h \) is the sampling period, \( h = t_{k+1} - t_k \), \( \xi_i(t_k) = [x_i(t_k), v_i(t_k)] \), \( x_i(t_k) \) is the position-like of the discrete-time dynamic agents, \( v_i(t_k) \) is the velocity-like of the discrete-time dynamic agents. \( A = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \), \( B = [0, h] \), \( u_{id}(t_k) \in \mathbb{R} \) and \( u_{id}(t_k) \in \mathbb{R} \) are the control input of agents, \( d_i(t) \) and \( d_{id}(t_k) \) are external unknown disturbances. The sum of all the disturbances in the system (1) is \( d_{\text{all}} \). (1) can be converted to

\[
\begin{align*}
\dot{x}_i(t) &= v_i(t) \\
\dot{v}_i(t) &= f_i(x_i) + g_i u_i(t) + d_i(t), \quad i \in I_m \\
x_i(t_{k+1}) &= x_i(t_k) + h v_i(t_k), \\
v_i(t_{k+1}) &= v_i(t_k) + h [u_{id}(t_k) + d_{id}(t_k)], \quad i \in I_n/I_m
\end{align*}
\]  

Definition 1: Hybrid multi-agent system (1) under the leader-follower network is said to reach second-order consensus if for any initial conditions

\[
\begin{align*}
\lim_{t \to \infty} \|x_i(t_k) - x_0(t_k)\| &= 0, \\
\lim_{t \to \infty} \|v_i(t_k) - 0\| &= 0, \quad i \in I_n
\end{align*}
\]  

and

\[
\begin{align*}
\lim_{t \to \infty} \|x_i(t) - x_0(t)\| &= 0, \\
\lim_{t \to \infty} \|v_i(t) - 0\| &= 0, \quad i \in I_m
\end{align*}
\]  

Assumption 1: For hybrid multi-agent system (1) under the leader-follower network, the communication topology of the directed graph \( G \) contains at least one spanning tree from the leader.

Assumption 2: Assume that each agent in hybrid multi-agent system (1) can observe the position-like and velocity-like information of neighboring agents.

Assumption 3: In practical systems, disturbances are not infinite, therefore, the disturbances \( d_i(t) \) and \( d_{id}(t_k) \) are bounded, and the sum of all the disturbances in the system is bounded. \( d_{\text{min}} \) is the minimum value of the \( d_{\text{all}} \) and \( d_{\text{max}} \) is the maximum value of \( d_{\text{all}} \), we make \( d_{\text{min}} \leq d_{\text{all}} \leq d_{\text{max}} \).

III. MAIN RESULTS

The follower agents in hybrid multi-agent system (1) are all affected by the disturbances, it is necessary to design appropriate sliding mode control protocols for solving the second-order consensus problem. In this section, the sliding mode control protocols are designed to realize the second-order consensus of the hybrid multi-agent system with disturbances. Then, some sufficient conditions are given for hybrid multi-agent system to achieve the second-order consensus under sliding mode control protocols.

A. DESIGN SLIDING MODE SURFACES FOR HYBRID MULTI-AGENT SYSTEM

The consensus errors of all agents can be expressed as

\[
e_i(t) = \sum_{j=1}^{n} a_{ij} (x_i(t) - x_j(t_j)) + b_i (x_i(t) - x_0(t)) \\
i \in I_m, \quad j \in I_n
\]
where

\[ x_j(t) = \begin{cases} x_j(t), & j \in I_m \\ x_{j}(t_k), & j \in I_n/I_m \end{cases}, \]

and

\[ \xi_j(t_j) = \begin{bmatrix} [x_j(t) ; v_j(t)] & j \in I_m \\ [x_j(t_k) ; v_j(t_k)] & j \in I_n/I_m \end{bmatrix}. \]

\[ \xi_0(t_k) = [x_0(t_k) ; v_0(t_k)] \]

where \( x_0(t_k) \) and \( v_0(t_k) \) are the position-like and the velocity-like of the leader in sampling time \( t_k \).

**Remark 1:** In hybrid multi-agent system, there may be communication relationships between continuous-time and discrete-time dynamic agents, therefore, we make \( j \in I_n \).

To ensure convergence of the consensus error \( s(t) \) and \( \bar{s}(t) \), the sliding mode surface for the agent \( i \) is considered as

\[ s_i(t) = s_i(t) + \bar{s}_i(t) \]

\[ s_i(t_k) = C_e e_i(t_k), \quad i \in I_n/I_m \]

where \( C_e = \{c_{12}, 1\} \), \( c_{11}, c_{22} \in \mathbb{N} \).

From (5), we have \( \dot{e}_i(t) = \sum_{j=1}^{n} a_{ij} \dot{x}_j(t) - b_i \dot{x}_0(t) \), \( \sigma_{11} = (\sum_{j=1}^{n} a_{ij} + b_i) \). From hybrid multi-agent system (2), we can get \( \sum_{j=1}^{n} a_{ij} \dot{x}_j(t) = \sum_{j=1}^{n} a_{ij} \dot{x}_j(t) + \sum_{j=m+1}^{n} a_{ij} v_{ij}(t_k) \).

**Remark 2:** In this paper, we choose a sufficiently small sampling period \( h > 0 \). Therefore, we can get \( \bar{e}(t) \) and \( \bar{e}_i(t) \).

### B. DESIGN SLIDING MODE CONTROL PROTOCOLS FOR HYBRID MULTI-AGENT SYSTEM

We first design sliding mode control protocols for continuous-time and discrete-time dynamic agents in hybrid multi-agent system (1), respectively. Design the control protocol \( u_{ic}(t) \) of the continuous-time dynamic agent as

\[ u_{ic}(t) = [g_i \sigma_{11}]^{-1} [-ksgn(s_i(t)) - \bar{d} s_i(t) - c_{11} \dot{e}_i(t) - \sigma_{11} f_i + \sum_{j=1}^{m} a_{ij} f_j + \sum_{j=1}^{m} a_{ij} g_j u_{jc}(t) + \sum_{j=m+1}^{n} a_{ij} h u_{jd}(t_k) + b_i \dot{x}_0(t) + d_1] \]

(9)

where \( i \in I_m, j \in I_n, k, \bar{d} \in \mathbb{N}, h \) is the sampling period, \( u_{jd}(t_k) \) is the control input of agent \( j \) for \( j \in I_n/I_m \) and \( u_{jd}(t_k) \) will be given below, \( d_1 = (d_{max} - d_{min})/2 \) and \( d_2 = (d_{max} + d_{min})/2 \).

**Remark 3:** In general, the conventional control protocol \( u_{ic}(t) \) only has the states information of continuous-time dynamic agents. In control protocol (9), there has states information of continuous-time and discrete-time dynamic agents, which can better reflect the characteristics of the hybrid multi-agent systems.

**Theorem 1:** For a directed graph \( G \) with spanning tree, if the sliding mode surface \( s_i(t) \) is selected as (7), the consensus control laws are proposed as (9), the second-order consensus of the continuous-time dynamic agents in hybrid multi-agent system (1) is achieved.

**Proof:** Consider the Lyapunov function \( V_0(t) = s_i^2(t)/2 \). Take derivative of \( V_0(t) \), it holds that

\[ \dot{V}_0(t) = s_i(t) \dot{s}_i(t) = s_i(t) (c_{11} \dot{e}_i(t) + \bar{e}_i(t)) \]

\[ = s_i(t) \left[ c_{11} \dot{e}_i(t) + \sigma_{11} \dot{x}_i(t) - \sum_{j=1}^{n} a_{ij} \dot{x}_j(t) - b_i \dot{x}_0(t) \right] \]

(10)

where

\[ \sum_{j=1}^{n} a_{ij} \dot{x}_j(t) = \sum_{j=m+1}^{n} a_{ij} \dot{x}_j(t) + \sum_{j=m+1}^{n} a_{ij} h u_{jd}(t_k) + d_2. \]

Plugging (1) into (10), we can obtain

\[ \dot{V}_0(t) = s_i(t) [c_{11} \dot{e}_i(t) + \sigma_{11} f_i + \sigma_{11} g_j u_{jc}(t) + \sigma_{11} d_{1}(t) - \sum_{j=1}^{m} a_{ij} f_j - \sum_{j=1}^{m} a_{ij} g_j u_{jc}(t) + \sum_{j=m+1}^{n} a_{ij} h u_{jd}(t_k) - \sum_{j=m+1}^{n} a_{ij} d_{1}(t) - b_i \dot{x}_0(t)] \]

(11)

In Assumption 3, we assume the sum of all the disturbances in the system is \( d_{all} \), therefore (11) can be converted to

\[ \dot{V}_0(t) = s_i(t) [c_{11} \dot{e}_i(t) + \sigma_{11} f_i + \sigma_{11} g_j u_{jc}(t) - \sum_{j=1}^{m} a_{ij} f_j - \sum_{j=m+1}^{n} a_{ij} g_j u_{jc}(t) - b_i \dot{x}_0(t) - d_{all}] \]

(12)

Plugging (9) into \( \dot{s}_i(t) \), it holds that

\[ \dot{s}_i(t) = -k sgn(s_i(t)) - \bar{d} s_i(t) + d_2 - d_1 sgn(s_i(t)) - d_{all} \]

(13)

Thus

\[ \dot{V}_0(t) = s_i(t) [-k sgn(s_i(t)) - \bar{d} s_i(t) + d_2 - d_1 sgn(s_i(t)) - d_{all}] \]

(14)

(i) when \( s_i(t) > 0 \),

\[ \dot{V}_0(t) = s_i(t) [-k - \bar{d} s_i(t) + d_2 - d_1 - d_{all}] \]

(15)

(ii) when \( s_i(t) < 0 \),

\[ \dot{V}_0(t) = s_i(t) [k + \bar{d} |s_i(t)| + d_2 + d_1 - d_{all}] \]

(16)
From the Lyapunov stability theorem we know that the system (1) is stable under the control protocols (9), it means that the states trajectory which outside the sliding mode surface $s_i(t)$ will reach the sliding mode surface $s_i(t) = 0$ in finite time. Therefore, the hybrid multi-agent system states will approach the sliding mode surface $s_i(t) = 0$ in finite time. Then the consensus tracking errors (5) can converge to zero along the sliding mode surface, which implies that continuous-time dynamic agents in hybrid multi-agent system (1) can achieve second-order consensus and the (4) is satisfied. This completes the proof.

Similarly, for the discrete-time dynamic agents in hybrid multi-agent system (1), according to (6) and (8), consensus control laws can be also obtained.

For discrete-time dynamic agent $i (i \in \mathbb{I}_n/\mathbb{I}_m)$, the control protocols are designed as

$$u_{d_i}(t_k) = [Ce\sigma_{\tau 2}B]^{-1}\left[ s_i(t_k) - h [\epsilon \text{sgn}(s_i(t_k)) + q_\nu s_i(t_k)] \right]$$

$$- Ce\sigma_{\tau 2}A\bar{\xi}_i(t_k) + Ce \sum_{j=1}^{n} a_{ij}\bar{\xi}_j(t'_k) + C_e \sum_{j=m+1}^{n} a_{ij}Bu_{jd}(t_k) + b_i\xi_0(t_k) + d_2 - d_1\text{sgn}(s_i(t_k)) \right]$$

where $Ce = [c_{\epsilon 2} 1]$, $\sigma_{\tau 2} = \left( \sum_{j=1}^{n} a_{ij} + b_i \right)$, $\epsilon, q \in \mathbb{N}$, $2 - qh > 0$. $\bar{\xi}_j(t')$ will be given below. $d_1 = \left( d_{\text{max}} - d_{\text{min}} \right)/2$ and $d_2 = \left( d_{\text{max}} + d_{\text{min}} \right)/2$.

Remark 4: In general, the conventional control input $u_{d_i}(t_k)$ only has the states information of discrete-time dynamic agents. In control input (17), there has states information of continuous-time and discrete-time dynamic agents, which can better reflect the characteristics of the hybrid multi-agent systems.

Theorem 2: For a directed graph $G$ with spanning tree, if the sliding mode surface $s_i(t_k)$ is selected as (8), the consensus conditions for the discrete-time dynamic agents in hybrid multi-agent system (1) are achieved.

Proof: Consider the Lyapunov function $V_0(t_k) = s_i^2(t_k)/2$, when the discrete-time dynamic agents meet the condition $\Delta V_0(t_k) = s_i^2(t_{k+1}) - s_i^2(t_k) < 0$ and $s_i(t_k) \neq 0$, the discrete-time dynamic agents in hybrid multi-agent system (1) can achieve second-order consensus. Thus, we can get the stability conditions for the discrete-time dynamic agents

$$\left\{ \begin{array}{c}
\frac{s_i(t_{k+1}) - s_i(t_k)}{s_i(t_k)} \text{sgn}(s_i(t_k)) < 0 \\
\frac{s_i(t_{k+1}) + s_i(t_k)}{s_i(t_k)} \text{sgn}(s_i(t_k)) > 0 
\end{array} \right. \quad (18)$$

Thus

$$s_i(t_{k+1}) = C_e \sum_{j=1}^{n} a_{ij}(s_i(t_{k+1}) - s_j(t_k)) + b_i(s_i(t_{k+1}) - \xi_0(s_i(t_k)))$$

$$= C_e \sum_{j=1}^{n} a_{ij} s_i(t_{k+1}) - \sum_{j=1}^{n} a_{ij} s_i(t_k) - b_i\xi_0(t_{k+1}) \right]$$

where $\sum_{j=1}^{n} a_{ij} s_i(t_{k+1}) = \sum_{j=1}^{m} a_{ij} s_i(t_{k+1}) + \sum_{j=m+1}^{n} a_{ij} s_i(t_{k+1})$. According to (1) and (2), when $t = t_k$, it can be obtained that

$$\sum_{j=1}^{m} a_{ij} s_i(t_{k+1}) = \sum_{j=1}^{m} a_{ij} \left[ x_j(t_{k+1}) \right]$$

$$= \sum_{j=1}^{m} a_{ij} \left[ \frac{h\xi_j(t) + x_j(t)}{v_j(t)} \right]$$

Thus

$$\sum_{j=1}^{m} a_{ij} s_i(t_{k+1}) = \sum_{j=1}^{m} a_{ij} \left[ \frac{h\xi_j(t) + x_j(t)}{v_j(t)} \right]$$

where $h = \left( \frac{h_j}{d_{\text{j}1}} \right)$, $t = t_k$.

In (21), we make $\xi_j(t') = \left[ \frac{h\xi_j(t) + x_j(t)}{v_j(t)} \right]$. $d_{\text{j}1}(t') = \left[ \frac{h\xi_j(t) + x_j(t)}{v_j(t)} \right]$.

Plugging (17) and (21) into (19), it holds that

$$s_i(t_{k+1}) = s_i(t_k) - h \text{sgn}(s_i(t_k)) - h d_j \text{sgn}(s_i(t_k))$$

$$+ d_2 - d_1 \text{sgn}(s_i(t_k)) - d_{\text{all}} \right] \quad (22)$$

Thus

$$s_i(t_{k+1}) - s_i(t_k) \text{sgn}(s_i(t_k)) = \left[ -h \text{sgn}(s_i(t_k)) - h d_j \text{sgn}(s_i(t_k))$$

$$+ d_2 - d_1 \text{sgn}(s_i(t_k)) - d_{\text{all}} \right] \text{sgn}(s_i(t_k)) \quad (24)$$
Therefore, outside the sliding mode surface function and is reachable, it means that the states trajectory which established, therefore, the discrete-time sliding mode surface exists and is presented in Fig.1. Similarly, for (30), if \( s_i(t_k+1) + s_i(t_k) > 0 \), we can obtain

\[
s_i(t_k) > \frac{\alpha e - (d_{\text{min}} - d_{\text{all}})}{2 - h q}
\]

(28)

Therefore, \( s_i(t_k) > 0 \), and \([s_i(t_k+1) + s_i(t_k)] > 0 \). (ii) when \( s_i(t_k) < 0 \),

\[-[s_i(t_k+1) - s_i(t_k)] = \left(-\alpha + h q |s_i(t_k)| + d_{\text{max}} - d_{\text{all}}\right) < 0
\]

(29)

\[-[s_i(t_k+1) + s_i(t_k)] = \left(-\alpha + (2 - h q) s_i(t_k) + d_{\text{max}} - d_{\text{all}}\right)
\]

(30)

Similarly, for (30), if \(-[s_i(t_k+1) + s_i(t_k)] > 0 \), we can obtain

\[
s_i(t_k) < \frac{-\alpha e - (d_{\text{max}} - d_{\text{all}})}{2 - h q}
\]

(31)

Therefore, \( s_i(t_k) < 0 \), and \(-[s_i(t_k+1) + s_i(t_k)] > 0 \).

From the above analysis we know that the stability conditions (18) for the discrete-time dynamic agents are established, therefore, the discrete-time sliding mode surface exists and is reachable, it means that the states trajectory which outside the sliding mode surface function \( s_i(t_k) \) will reach the sliding mode surface \( s_i(t_k) = 0 \) in finite time. Therefore, the hybrid multi-agent system states will approach the sliding mode surface \( s_i(t_k) = 0 \) in finite time. Then the consensus tracking errors (6) can converge to zero along the sliding mode surface, which implies that discrete-time dynamic agents in hybrid multi-agent system (1) can achieve second-order consensus. This completes the proof.

From Theorem 1 and Theorem 2, we know that hybrid multi-agent system (1) with the control protocols (9) and (17) satisfies the Definition 1, therefore, hybrid multi-agent system (1) can achieve second-order consensus under the leader-follower network.

IV. SIMULATION

In this section, numerical simulations are performed on hybrid multi-agent system (1) with and without disturbances, respectively. Consider a hybrid multi-agent system consisted of 1 leader and 4 followers. The leader is labeled 0, the followers are labeled 1,2,3,4. Follower 1 and 2 represent continuous-time dynamic agents, follower 3 and 4 represent discrete-time dynamic agents, respectively. The communication topology is presented in Fig.1.

The initial state of the leader is \( x_0 = [0, 0] \), the dynamics of the leader is \( \dot{x}_0(t) = \sin t \). The sliding mode surface of the Follower 1 is selected \( s_1(t_k) = 3e_1(t_k) + \hat{e}_1(t_k) \), the sliding mode surface of the Follower 2 is selected \( s_2(t_k) = 4e_2(t_k) + \hat{e}_2(t_k) \). The sliding mode surface of the Follower 3 is selected as \( s_3(t_k) = [3 1] e_3(t_k) \) and the sliding mode surface of the Follower 4 is selected as \( s_4(t_k) = [2 1] e_4(t_k) \). The parameters of the Follower 1 are \( f_i(x_i) = -25x_i(t_k) \) and the disturbance of the Follower 2 are \( f_2(x_2) = -9x_2(t_k) \), \( k = 2 \), \( \vartheta = 10 \). The parameters of the Follower 2 are \( f_2(x_2) = -9x_2(t_k) \), \( k = 2 \), \( \vartheta = 10 \). The disturbance of continuous-time dynamic agents is \( d_{11,21}(t) = 5 \sin (2\pi t) \).
The parametric matrices $A$ and $B$ of the hybrid multi-agent system (1) are $A = \begin{bmatrix} 1 & 0.01; & 0 & 1 \end{bmatrix}$, $B = [0; 0.01]$. The disturbance of discrete-time dynamic agents is $d_{32,42}(t_k) = 1.5 \sin (2 \pi t_k)$. The sum of all the disturbances in the system is $6 \sin (2 \pi t) \leq d_{all} \leq 20 \sin (2 \pi t)$. The parameters of the Follower 3 are $c_{32} = 3$, $\varepsilon = 5$, $q = 30$. The parameters of the Follower 4 are $c_{42} = 3$, $\varepsilon = 5$, $q = 30$, and the sampling time period $h$ is 0.01s. The initial states of continuous-time dynamic agents 1, 2 are selected to be $[-5, 0]$, $[5, 0]$. The initial states of discrete-time dynamic agents 3, 4 are chosen as $[3, 0]$ and $[-3, 0]$.

The response curves of states for the hybrid multi-agent system without and with disturbances are shown from Fig. 2 to Fig. 7. The control signals are shown in Fig. 4 and Fig. 7 for hybrid multi-agent system without and with disturbances. It can be clearly seen through Fig. 2 to Fig. 7 that hybrid multi-agent system (1) achieves second-order consensus via SMC under the leader-follower network.

The model of the hybrid multi-agent system without disturbances is

$$\begin{align*}
\ddot{x}_i(t) &= f_i(x_i) + g_i u_{ic}(t), \\
\dot{\xi}_i(t_{k+1}) &= A\xi_i(t_k) + B [u_{id}(t_k), \quad i \in I_m
\end{align*}$$

The model of the hybrid multi-agent system with disturbances is (1), the control protocols of the system are (9) and (17), the simulation results are Fig. 5, Fig. 6, and Fig. 7.

**V. CONCLUSION**

This paper has studied the second-order consensus of the hybrid multi-agent system with unknown disturbances under the leader-follower network. Firstly, by using SMC, two kinds of sliding mode control protocols have been designed to achieve second-order consensus of the hybrid multi-agent system with disturbances. Secondly, some sufficient conditions are given for hybrid multi-agent system to achieve second-order consensus. Finally, the figures of the hybrid multi-agent system with and without disturbances are presented, which show the robustness of the hybrid multi-agent system is improved by using SMC. The future work will focus on the second-order consensus of hybrid multi-agent systems with time delays, and all agents have nonlinear terms.

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