Luminosity of synchrotron radiation from pulsar magnetospheres

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Abstract. We calculate the maximum luminosity of the synchrotron radiation in X-ray and optical bands, which is originated from the electromagnetic cascade in the magnetosphere of rotation-powered pulsars. We find that for pulsars with spin-down luminosity $L_{\text{sd}} \leq 10^{35}$ erg s$^{-1}$, even if the full allowed energy range of the $\gamma$-ray photon emitted by the accelerated particles is taken into account, the observed non-thermal luminosities are higher than the maximum luminosity of synchrotron radiation from pairs created by two-photon collision. The synchrotron radiation from the inner region could explain the observed non-thermal luminosities, so that the $\gamma$-ray pulsars with $L_{\text{sd}} \leq 10^{35}$ erg s$^{-1}$ would have multiple particle accelerators in the magnetosphere.

1. Introduction

In the magnetosphere of rotation-powered pulsars, charged particles are accelerated and created, and emit pulsed emission in broad energy range. The accelerated particles with Lorentz factor $\gamma > 10^6$ emit $\gamma$-ray photons. A fraction of the $\gamma$-ray photons converts to electrons and positrons. The created particles emit synchrotron radiation in X-ray and optical bands (e.g. [1]). Therefore, the high-energy emission provides useful tool to prove the particle acceleration and creation in the magnetosphere.

Observed $\gamma$-ray pulsed emission comes from the outer magnetosphere [2]. At the outer magnetosphere, the particles are created via two-photon pair creation process. Synchrotron radiation from the created particles could produce the observed non-thermal emission. On the other hand, particles could be created near the stellar surface via magnetic pair creation process (e.g., [3]). These particles could also contribute to the observed non-thermal luminosities. In this proceeding, we calculate the maximum luminosity of synchrotron radiation based on the model of Kisaka & Tanaka [4], which considers synchrotron radiation from particles created by both two-photon collision and magnetic pair creation processes. In section 2, we briefly review the model of Kisaka & Tanaka [4]. The difference of Kisaka & Tanaka [4] is that we take into account the full allowed range of the characteristic energy of the $\gamma$-ray photon $E_{\text{cut}}$. In section 3, we show the maximum luminosity of synchrotron radiation in X-ray and optical bands, and compare with the observations. We also discuss the effect of the particle trajectory on the luminosity of synchrotron radiation.
2. Model

We adopt the model described in Kisaka & Tanaka [4]. The basic picture is as follows. A fraction of the spin-down luminosity $L_{sd}$ (a fraction $\eta$) converts to the energy flux $\gamma_p m_e c^2 N_p$ of the accelerated particles with Lorentz factor $\gamma_p$ at a given radius $r$ from the centre of a neutron star, where $N_p$, $m_e$, and $c$ are the number flux of the accelerated particles, the electron rest mass, and the speed of light, respectively. The accelerated particles emit $\gamma$-ray photons with the characteristic energy $E_{\gamma \text{cur}}$ via curvature radiation. The ratio of the $\gamma$-ray luminosity to the energy flux of the accelerated particles is determined by comparing the advection timescale $t_{ad} \sim r/c$ with the radiative cooling timescale $t_{\text{cool,cur}} \sim \gamma_p m_e c^2 / P_{\text{cur}}$, where $P_{\text{cur}}$ is the power of curvature radiation for a particle. A part of $\gamma$-ray photons converts to electron-positron pairs with Lorentz factor $\gamma_{s,pair}$. For pair creation processes, two-photon collision and magnetic pair creation work in the magnetosphere. In the two-photon collision process, the optical depth $\tau_{\gamma\gamma}$ for $\gamma$-ray photons and the luminosity of curvature radiation determine the energy flux of the created particles. The thermal emission from the heated polar cap is considered as seed photons to create the pairs. In the electromagnetic cascade via magnetic pair creation, higher generation pairs could be created due to high pair conversion efficiency. The maximum number of the created particles at a given $r$ is estimated that the $\gamma$-ray photons with the energy $E_{\gamma \text{cur}}$ are split to the photons with the maximum escapable energy $E_{\gamma \text{esc}}$, and all the photons with $E_{\gamma \text{esc}}$ convert to pairs. Note that in this case, almost all $\gamma$-ray luminosity converts to the energy flux of the created particles. The created pairs have non-zero pitch angle in general, so that the particles emit synchrotron radiation in X-ray and optical bands. The ratio of the luminosity of synchrotron radiation for a given frequency $\nu_{\text{obs}}$ to the energy flux of the created particles is determined by the Lorentz factor $\gamma_{s,pair}$, where $P_{\text{syn}}$ is the power of synchrotron radiation for a particle, and $\gamma_{s,pair}$ is the Lorentz factor of particles whose characteristic frequency of synchrotron radiation equals $\nu_{\text{obs}}$.

Some constraints are imposed on the model. First, in order to emit synchrotron radiation at a frequency $\nu_{\text{obs}}$, the Lorentz factor $\gamma_{s,syn}$ has to be lower than the Lorentz factor $\gamma_{s,pair}$ determined by the energy of the parent $\gamma$-ray photons, $E_{\gamma \text{cur}}$ or $E_{\gamma \text{esc}}$. Second, the frequency $\nu_{\text{obs}}$ has to be lower than the turnover frequency for the synchrotron radiation, $\nu_{\text{obs}} > \nu_g / \alpha$ [5], where $\nu_g = eB/(2\pi m_e c)$, $e$ is the charge of an electron, $B$ is the magnetic field at the emission region, and $\alpha$ is the pitch angle. The constraint also gives the upper limit on the magnetic field $B \sim 8.6 \times 10^{10} \alpha (h \nu_{\text{obs}}/1\text{keV})$ G at the emission region, where $h$ is the Planck constant. Then, we describe a free electron in a magnetic field in classical regime. Third and forth conditions are the thresholds of the pair creation via two-photon collision and magnetic pair creation, respectively.

In the model, the parameter is only $\eta$ in the dipole magnetic field case. For the pitch angle, we use the approximated value $\alpha \sim \sqrt{R_{lc}}$, where $R_{lc}$ is the radius of the light cylinder. In the non-dipole field case, we take the maximum value of the pitch angle $\alpha = 1$ at any $r$. The strength of the non-dipole magnetic field does not depend on the maximum luminosity of synchrotron radiation [4]. For the radius to the emission region $r$, we take the value which gives the maximum value of the luminosity of synchrotron radiation for given observables such as the surface dipole field $B_s$ and the spin-down luminosity $L_{sd}$.

Different from Kisaka & Tanaka [4], we take into account the full allowed range of the energy $E_{\gamma \text{cur}}$. The observed characteristic energy could depend on the inclination and viewing angles (e.g., [6, 7]). Then, typical energy of parent photons $E_{\gamma \text{cur}}$, which determines the energy of created particles, could be different from the observed $\gamma$-ray photon energy. In the model, there is a lower limit on the energy $E_{\gamma \text{cur}}$ to create the pairs via two-photon collision for a given temperature of the thermal emission from the polar cap surface $T_{pc}$,

$$E_{\gamma \text{cur}} \geq 1.4 \left( \frac{T_{pc}}{10^6.5 \text{K}} \right)^{-1} \text{GeV}. \quad (1)$$
The observed temperature is $T_{pc} \sim 10^6 \cdot 10^{6.5}$ K (e.g., [8]). On the other hand, the full potential drop across the polar cap $\Delta V_{pc}$ gives the upper limit on the energy,

$$E_{cur} \leq 0.29 \frac{3}{4\pi} \frac{hc}{R_{cur}} \gamma_{\text{max}}^3$$

where $R_{cur}$ is the curvature radius of the particle trajectory, and $\gamma_{\text{max}}$ is the maximum Lorentz factor,

$$\gamma_{\text{max}} = \frac{e \Delta V_{pc}}{m_e c^2}.$$  

### 3. Results and Discussion

Figure 1 shows the maximum luminosities of synchrotron radiation $L_{\text{syn}}$ (left) and the ratios $L_{cur}/L_{\text{syn}}$ (right) in the two-photon collision ($\gamma\gamma$) and the magnetic pair creation cases ($B\gamma$), where $L_{cur} = P_{cur} N_p \min\{t_{cool,cur}, t_{ad}\}$ is the luminosity of curvature radiation. For example, the maximum luminosity of synchrotron radiation for conditions $t_{cool,\text{syn}} < t_{ad}$ and $t_{cool,cur} < t_{ad}$ at the emission region are described by

$$L_{\text{syn}} \sim \begin{cases} 
1.5 \times 10^{31} \eta \left( \frac{\nu_{\text{obs}}}{1 \text{ keV}} \right)^{1/2} \left( \frac{E_{cur}}{1 \text{ GeV}} \right)^{-1} \left( \frac{L_{ad}}{10^{35} \text{ erg s}^{-1}} \right)^{15/8} \left( \frac{L_{pc}}{10^{45} \text{ K}} \right)^{-1} \\
4.6 \times 10^{31} \eta \left( \frac{\nu_{\text{obs}}}{1 \text{ keV}} \right)^{5/7} \left( \frac{B_{s}}{10^{12} \text{ G}} \right)^{-1/7} \left( \frac{L_{ad}}{10^{35} \text{ erg s}^{-1}} \right)^{17/14} \text{ erg s}^{-1} (B\gamma, \text{dipole}), \\
4.5 \times 10^{33} \eta \left( \frac{\nu_{\text{obs}}}{1 \text{ keV}} \right) \left( \frac{L_{ad}}{10^{35} \text{ erg s}^{-1}} \right) \text{ erg s}^{-1} (B\gamma, \text{non-dipole}), 
\end{cases}$$

where $L_{pc}$ is the thermal luminosity from the heated polar cap. The plotted synchrotron luminosity in the two-photon collision case is the maximum value for the full allowed range of $E_{cur}$ and the range of the surface magnetic field $B_{s} > 10^{14} \text{G}$. We also plot the observed non-thermal luminosities $L_X$ and $L_{\text{opt}}$ and the flux ratios $F_x/F_X$ and $F_\gamma/F_{\text{opt}}$. For the pulsars with $L_{ad} \leq 10^{35} \text{ erg s}^{-1}$, the observed luminosities and flux ratios are not explained by the two-photon collision case. The maximum luminosities for the magnetic pair creation case with non-dipole field are higher than the all observed values, so that the $\gamma$-ray pulsars would have the accelerator at the inner region of the magnetosphere.

The dependence on $E_{cur}$ is described as follows. Once the energy flux of the accelerated particles is given by the efficiency parameter $\eta$, the energy $E_{cur}$ determines the number flux of the $\gamma$-ray photons in the model. The number flux of the $\gamma$-ray photon and the optical depth give the number flux of created particles. The higher number flux of the created particles gives the higher synchrotron luminosity. The lowest value is $E_{cur} \sim 1.4 \text{ GeV}$ from inequality (1), so that the luminosity $L_{\text{syn}}$ is about twice higher than than that in Kisaka & Tanaka (2017) [4] which fixes $E_{cur} = 3 \text{ GeV}$. On the other hand, for pulsars with $L_{ad} \leq 10^{34} - 10^{36} \text{ erg s}^{-1}$ and the lower value of $E_{cur}$, the upper limit on the radius of the emission region given by the energy constraint $\gamma_{\text{pair}} > \gamma_{\text{syn}}$ becomes smaller than the light cylinder radius. Then, the pitch angle $\alpha$ at the emission region $r < R_{L}$ becomes smaller than unity, so that the higher $E_{cur}$ gives the higher synchrotron luminosity. Note that the energy of the accelerated particle cannot exceed the full potential drop across the polar cap. This gives the upper limit on the energy $E_{cur}$ and the resulting synchrotron luminosity. As a result, the maximum luminosity does not significantly change from the case of Kisaka & Tanaka [4].
Figure 1. Left: Maximum luminosities of synchrotron radiation in 1 keV (upper) and 1 eV (lower) for two-photon collision case ($\gamma\gamma$, blue), magnetic pair creation cases ($B\gamma$) with dipole field (black) and non-dipole fields (red) as a function of the spin-down luminosity $L_{sd}$. Right: Flux ratios $F_\gamma/F_X$ (upper) and $F_\gamma/F_{opt}$ (lower) as a function of $L_{sd}$ for $\gamma$-ray pulsars. Blue curves are the flux ratio of curvature to synchrotron radiation in two-photon collision case. Observed data are taken from [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. Filled green and open magenta circles are radio-loud and radio-quiet $\gamma$-ray pulsars, respectively. Orange squares denote the non-$\gamma$-ray pulsars. Upper and lower limits on the optical luminosity $L_{opt}$ and the flux ratio $F_\gamma/F_{opt}$, respectively, are also plotted as cyan triangles.

For the curvature radius of the trajectory of the accelerated particles, we use the approximation formula, $R_{cur} \sim \sqrt{rR_{lc}}$ [29]. At the outer magnetosphere, the curvature radius of the particle trajectory could be significantly different from that of the magnetic field line (e.g., [6]), which may change curvature radiation from the accelerated particles. However, the curvature radius does not significantly change the resulting synchrotron luminosity from the created particles. As long as the cooling timescale of curvature radiation is shorter than the advection timescale, $t_{cool,cur} < t_{ad}$, the total luminosity of the curvature radiation is $L_{cur} = \eta L_{sd}$, which does not depend on the curvature radius. Most pulsars satisfied the condition $t_{cool,cur} < t_{ad}$.
[4], so that the synchrotron luminosity does not depend on the curvature radius.

In Figure 1, we consider the range of the surface magnetic field $B_s > 10^{11}$ G. Here, we briefly discuss the pulsars with a low surface magnetic field $B_s \sim 10^8$ G such as millisecond pulsars. In the two-photon collision case, since the conditions $\gamma_{s,\text{pair}} > \gamma_{s,\text{syn}}$ and $t_{\text{cool,syn}} > t_{\text{ad}}$ are satisfied at the light cylinder $R_{lc}$, the dependence of the luminosity on the magnetic field is described by $L_{\text{syn}} \propto B_s^{-1/4}$ [4]. Then, the maximum luminosity could be $\sim 10$ times higher than that of the plotted blue curves in Figure 1. On the other hand, in the magnetic pair creation case, the X-ray frequency is higher than $\nu_{s,\text{syn}} / \alpha$ at the surface. Then, the luminosity, $L_{\text{syn}} \propto B_s^{1/8}$, becomes lower for the lower dipole field. If the non-dipole field dominates at the surface, the luminosity could become higher for the higher strength of the non-dipole field. The maximum luminosity is the same as in the case with $B_s > 10^{11}$ G.

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