Leptophobic character of the $Z'$ in an $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model

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Abstract

We show that the extra $Z$ boson predicted within the so-called “3-3-1” model has a leptophobic character, and analyse its effects on $Z$ decay widths and on the $tt$ production cross section in $p\bar{p}$ collisions at the Fermilab Tevatron. Recent model-independent analysis are applied in order to estimate the contribution of this $Z'$ to the observables that will be measured at LEP2.

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The presence of an extra neutral vector boson having relatively small couplings to the lepton sector ("leptophobic") has recently been considered with special interest [1,2]. This has mainly been motivated by the experimental observation of possible deviations from the Standard Model (SM) predictions for the decay of the $Z$ boson to heavy quarks at LEP [3], and the excess of jets at large $E_T$ at CDF [4]. In addition, it has been noticed that such a $Z'$ boson could give rise to measurable effects at LEP2 and NLC, provided that the lepton couplings are nonvanishing [5].

The experimental situation for the mentioned LEP and CDF observations is still under accurate final analysis [6]. However, even in the absence of discrepancies with the SM, models including a leptophobic $Z'$ still represent an interesting subject, specially in connection with the on-going LEP2 experiments. In this regard, a detailed study is performed in Ref. [5], where the possible effects of a general extra $Z$ on the final hadronic channels at LEP2 are calculated, and the results are compared with the expected experimental accuracies. It is also stressed that there is a phenomenologically significant difference between the models where the couplings among the $Z'$ and the leptons are just suppressed, and those where these interactions exactly vanish at the tree level. In the first of these two cases the presence of the $Z'$ could be detected at LEP2 even for $M_{Z'} \sim 1$ TeV, while a “totally leptophobic" $Z'$ is expected to yield no visible effects on the conventional observables.

Several authors have followed the idea of a leptophobic $Z'$, and different models can be found in the literature, containing either approximate or exact suppression mechanisms for the $Z'$-lepton couplings [7]. The purpose of this paper is to study in this context the so-called “3-3-1" model, which was presented a few years ago [8,9] following a quite different motivation. The model is based on an $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge group, and attempts to explain (at least partially) the problem of family replication, since it has the particularity of being anomaly-free only if the number of lepton families is a multiple of the number of colors. We show here that the model contains a $Z'$ which falls into the leptophobic class, and analyse the contributions of $Z - Z'$ mixing to the ratios $R_b, R_c$ measured at LEP and the effect of $Z'$ exchange on the top quark production cross section in high-energy $p\bar{p}$ collisions. The work is concluded by studying the possibility of finding observable effects coming from the exchange of this $Z'$ in $e^+ e^-$ collisions at LEP2.

Let us stress that the 3-3-1 model shows a distinctive feature, which is the fact that the suppression of the lepton couplings is not obtained from an ad hoc imposition, but appears as a direct consequence of the required fermion quantum numbers. From this point of view, the leptophobic character of the $Z'$ in this model can be regarded as “natural”.

Outline of the model. The structure of the 3-3-1 model has been detailed in several articles [8–13]. Let us include here just a brief description in order to clarify the notation (we follow that of Ref. [8]).

The model contains the ordinary SM quarks and leptons, as well as three new quarks $J_1$ and $J_{2,3}$ with charges 5/3 and $-4/3$ respectively. These fermions are organized into $SU(3)_L$ triplets and singlets as follows:
where $l = e, \mu, \tau$, and in each case the first and second entries between the parentheses stand for the $SU(3)_L$ representation and $X$ quantum number respectively. Notice that, forced by the requirement of anomaly cancellation, one of the three quark families transforms under $SU(3)_L$ as a $3_3$, while the other two are in a $3^*$. This “family discrimination” gives rise to flavor changing neutral interactions (FCNI) at the tree level [13, 14].

As in the SM case, the scalar sector of the model is responsible for the spontaneous breakdown of the gauge symmetry and the origin of the fermion and gauge boson masses. The scalar fields are organized into one $SU(3)_L$ sextuplet with $X = 0$ (necessary to provide the lepton masses) and three $SU(3)_L$ triplets with $X$ values 1, 0 and -1 respectively. The spontaneous symmetry breakdown follows the hierarchy

$$SU(3)_L \otimes U(1)_X \rightarrow SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$$

where $V$ and $v$ denote the corresponding breaking scales. Finally, the model includes nine gauge bosons, associated with the generators of the gauge group. The charged sector contains two relatively light singly charged particles, which can be identified with the usual SM $W^\pm$ bosons, and four heavier dileptons $Y^\pm$ and $Y^\pm\pm$. The neutral sector, which will be considered below in more detail, includes a new gauge boson $Z'$ besides the ordinary $Z$ and the massless photon.

Neutral gauge bosons and leptonphobia. As usual, the couplings between the fermions and the neutral gauge bosons can be read from the covariant derivative in the fermion kinetic term of the Lagrangian. One has

$$\mathcal{L} = -\sum_{i=1}^{3} \bar{Q}_{iL}(g'X B_{\mu} + g T_3 W_{3\mu} + g T_8 W_{8\mu})\gamma^\mu Q_{iL}$$

$$- g' \sum_{q=u, d, J} \bar{q}_{R} X B_{\mu}\gamma^\mu q_{R} - g \sum_{l=e, \mu, \tau} \bar{\Psi}_{lL}(T_3 W_{3\mu} + T_8 W_{8\mu})\gamma^\mu \Psi_{lL}$$

where $B$ and $W_{3,8}$ are the gauge fields corresponding to the $U(1)_X$ and the diagonal $SU(3)_L$ generators respectively. The change to the $\{A, Z, Z'\}$ basis is obtained by means of the rotation

$$A = \sin \theta \ W^3 + \cos \theta \ (-\sin \phi \ W^8 + \cos \phi \ B)$$

$$Z = \cos \theta \ W^3 - \sin \theta \ (-\sin \phi \ W^8 + \cos \phi \ B)$$

$$Z' = \cos \phi \ W^8 + \sin \phi \ B$$

where the angles $\phi$ and $\theta$ are given by

$$\cos \phi = \frac{\sqrt{1 - 4 \sin^2 \theta}}{\cos \theta}$$

$$\sin \theta = \frac{g'}{g} \sqrt{1 - 4 \sin^2 \theta}$$

$$\tan \theta = \frac{g'}{g}$$
The states $A$ and $Z$ in (3) can now be identified with the ordinary SM neutral gauge bosons. However, the $Z$ is not yet an exact mass eigenstate, but turns out to be slightly mixed with the $Z'$. The corresponding mixing angle $\beta$ is a function of the nonzero vacuum expectation values of the scalar fields, and it is shown to be suppressed by the ratio $v^2/V^2$ between the squares of the above mentioned symmetry breaking scales.

From the resulting couplings between the fermions and the $Z$, it is seen that the angle $\theta$ corresponds to the SM Weinberg angle $\theta_W$. However, in contrast with the SM, the rotation (3) in the 3-3-1 model sets an upper bound for $\sin^2 \theta$, namely 1/4, as it is seen from Eq. (4) [9]. The fact that the value of $\sin^2 \theta_W$ is very close to 1/4 at the $M_Z$ scale is precisely the main reason for the suppression of the $Z'$-lepton couplings. Another important consequence of the former upper bound is that it leads to a constraint for the $SU(3)_L \otimes U(1)_X$ symmetry breaking scale $V$, and thus for the $Z'$ mass [9]. Indeed, taking into account the evolution of $\sin^2 \theta$ with energy, it has been found [10] that the condition $\sin^2 \theta(M_{Z'}) \leq 1/4$ imposes the upper limit $M_{Z'} \leq 3.1$ TeV.

Let us concentrate now on the couplings between the fermions and the $Z'$, which from Eqs. (2) and (3) come out to be

$$\mathcal{L}_{Z'} = -\frac{g}{\cos \theta_W} \left[ \sum_{F=\bar{\psi},Q_i} \bar{F}_L g_L \gamma^\mu F_L + \sum_{q=u_i,d_i,J_i} \bar{q}_R g_R \gamma^\mu q_R \right] Z'_\mu + \text{flavor changing terms}$$

(5)

where

$$g_L = \frac{\sqrt{3} \sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}} X + \sqrt{1 - 4 \sin^2 \theta_W} T_8$$

$$g_R = \frac{\sqrt{3} \sin^2 \theta_W}{\sqrt{1 - 4 \sin^2 \theta_W}} X$$

(6)

Notice that when $\sin^2 \theta_W$ approaches 1/4, the terms containing the $X$ generator enhance, while the $T_8$ part becomes suppressed. The key point is that the leptons are in an $X = 0$ representation, therefore their couplings to the $Z'$ are solely given by the $T_8$ generator term. The quark couplings, on the contrary, will be dominated by the $X$ term, showing a relative enhancement of order $(1 - 4 \sin^2 \theta_W)^{-1}$. Another way of understanding this “leptophobia” is just by looking at the rotation (3): in the $\sin^2 \theta = 1/4$ limit, we get $\cos \phi = 0$, and the $B$ vector boson (corresponding to the $U(1)_X$) decouples, becoming the exact mass eigenstate identified with the $Z'$. As a consequence, its presence is not seen by the leptons, which are invariant under the $U(1)_X$ transformations. Of course, the mass decoupling is possible in this limit because the ratio $g'/g$ in (4) diverges.

Effects on LEP observables and top quark production. In order to analyse the $Z'$ effects within the model under consideration, let us first rewrite the couplings (5) as

$$\mathcal{L}_{Z'} = -\frac{g}{2 \cos \theta_W} \sum_f \bar{f} (g'_{\nu_f} - g'_{\gamma_5} \gamma_5) \gamma^\mu f Z'_\mu + \text{flavor changing terms}$$

(7)

where the sum is now carried out over all the fermions. We have followed here the notation in Refs. [1,5], taking for the $Z'$-fermion couplings the same normalization as the usual one for the $Z$-fermion interactions within the SM.
\[ \mathcal{L}_Z = -\frac{g}{2\cos\theta_W} \sum_f \bar{f}(g_{V_f} - g_{A_f}\gamma_5)\gamma^\mu f Z^\mu \]  

(8)

with \( g_V = T_{3L} - 2Q \sin^2\theta_W \), \( g_A = T_{3L} \). Now, disregarding the exotic quarks \( J_i \) (too massive to give any significant contribution), and using the definitions \( h(x) \equiv \sqrt{1 - 4x} \), \( x \equiv \sin^2\theta_W \), the 3-3-1-model values for \( g'_{V,A_f} \) are found to be

\[ \begin{align*}
g'_{V_l} &= \frac{3h(x)}{2\sqrt{3}}, \\
g'_{V_r} &= \frac{h(x)}{2\sqrt{3}}, \\
g'_{V_{q^+}} &= \frac{1}{\sqrt{3}h(x)}(-\frac{1}{2} + 3x + \delta_{q^+}), \\
g'_{V_{q^-}} &= \frac{1}{\sqrt{3}h(x)}(-\frac{1}{2} + \delta_{q^-})
\end{align*} \]

(9)

\[ \begin{align*}
g'_{A_l} &= -\frac{h(x)}{2\sqrt{3}} \\
g'_{A_r} &= \frac{h(x)}{2\sqrt{3}} \\
g'_{A_{q^+}} &= \frac{1}{\sqrt{3}h(x)}(-\frac{1}{2} - x + \delta_{q^+}) \\
g'_{A_{q^-}} &= \frac{1}{\sqrt{3}h(x)}(-\frac{1}{2} + 2x + \delta_{q^-})
\end{align*} \]

Notice the presence of the set of parameters \( \delta_{q^\pm} \), which contain the non-universal part of the diagonal couplings in (7). This part, as well as the flavor changing terms, arises from the above mentioned “discrimination” between the quark families, thus the \( \delta_{q^\pm} \) are related to the SM quark mixing matrix \( V_{CKM} \). One has

\[ \delta_{q^\pm} = (1 - x) \left[ V^{(\pm)\dagger} \text{diag}(0,0,1) V^{(\pm)} \right]_{i\bar{i}} \]

(10)

where the index \( i \) is equal to 1, 2 or 3 for \( q^\pm \) belonging to the first, second or third quark generation respectively. The matrices \( V^{(\pm)}, 3 \times 3 \) and unitary, introduce new unknown parameters, since they are only constrained by the relation

\[ V^{(\pm)\dagger} V^{(-)} = V_{CKM} \]

(11)

Their unitary character, however, is at least enough to ensure that the parameters \( \delta_{q^\pm} \) lay in the range \( 0 \leq \delta_{q^\pm} \leq 1 - x \).

Let us now turn to analyse the possibility of observation of the \( Z' \) effects at LEP. For the allowed range of \( M_{Z'} \), it can be seen that the effects from direct \( Z' \) exchange are negligible at the \( Z \) peak, thus the main contribution to the electroweak observables at LEP comes from the \( Z - Z' \) mixing. In this way, the good agreement between the LEP measurements and the SM predictions for different asymmetries and decay rates serves as a constraint for the \( Z' \) mass and the mixing angle \( \beta \). With the 1992/93 available experimental data, such an analysis has been performed in Ref. [13], leading to the bounds \(-6 \times 10^{-4} < \beta < 4.2 \times 10^{-3}\) and \( M_{Z_2} > 490 \text{ GeV} \), being \( Z_2 \) the exact mass eigenstate nearby the \( Z' \).

As stated above, further measurements found discrepancies with the SM values for the \( Z \to b\bar{b}, c\bar{c} \) decay rates, which could be attributed to this kind of \( Z - Z' \) mixing. A model-independent analysis can be found in Ref. [1], where the authors calculate the minimal allowed bands for the coupling constants \( g'_{V,A_f} \) that are compatible with the LEP and SLC data. This analysis can be applied to the 3-3-1 model, where the lowest order \( Z' \) contributions depend on the \( Z' \) mass, the parameters \( \delta_{b,c} \) and the mixing angle \( \beta \). As expected, the \( Z' \)-quark couplings in this model show a significant enhancement with respect to the \( Z \)-quark ones in the SM, yielding for the ratios \( \xi_{V,A_f} \equiv g'_{V,A_f}/g_{V,A_f} \) the values

\[ \begin{align*}
\xi_{V_{q^+}} &= 2.21 + 8.67 \Delta_{q^+} \\
\xi_{V_{q^-}} &= 3.12 - 4.79 \Delta_{q^-} \\
\xi_{A_{q^+}} &= -3.15 + 3.30 \Delta_{q^+} \\
\xi_{A_{q^-}} &= 0.15 - 3.30 \Delta_{q^-}
\end{align*} \]

(12)
which are obtained from (14) taking for \( \sin^2 \theta_W \) the measured effective value of 0.232. We have also normalized \( \Delta_q \equiv \delta_q/(1 - x) \), so that \( 0 \leq \Delta_q \leq 1 \). In terms of these ratios, the relative deviations of the widths \( \Gamma(Z \rightarrow f \bar{f}) \) with respect to the SM predictions are given by

\[
\frac{\delta \Gamma_{f \bar{f}}}{\Gamma_{f \bar{f}}} = \delta \rho \left( 1 + \frac{4x(1-x)}{1-2x} \frac{Q_f g_{V_f}}{g_{V_f}^2 + g_{A_f}^2} \right) + 2 \beta g_{V_f}^2 \xi_{V_f} + g_{A_f}^2 \xi_{A_f} \left( g_{V_f}^2 + g_{A_f}^2 \right) \tag{13}
\]

where \( \delta \rho \) stands for the mixing contribution to the \( \rho \) parameter, defined as usual by \( \rho \equiv M_{R_V}^2/(M_Z^2(1-x)) \). For a mixing angle \( \beta \ll 1 \), it is easy to show that this contribution can be approximated by

\[
\delta \rho \simeq \left( \frac{M_{Z'}}{M_Z} \right)^2 \beta^2 \tag{14}
\]

Now, introducing in Eq. (13) the ratios in (12), we can estimate the relative deviations that correspond to the observables \( R_b, R_c \) and \( \Gamma_h \). These are found to be

\[
\begin{align*}
\frac{\delta R_b}{R_b} &= -0.06 \delta \rho + [-3.46 + 6.21(\Delta_u + \Delta_c)] \beta \tag{15a} \\
\frac{\delta R_c}{R_c} &= 0.12 \delta \rho + [-3.06 + 6.61 \Delta_c - 1.36 \Delta_u] \beta \tag{15b} \\
\frac{\delta \Gamma_h}{\Gamma_h} &= 1.47 \delta \rho + [-1.88 + 1.36(\Delta_u + \Delta_c)] \beta \tag{15c}
\end{align*}
\]

where we have taken into account the relations \( \Delta_u + \Delta_c + \Delta_t = 1 \) and \( \Delta_d + \Delta_s + \Delta_b = 1 \), arising from the unitarity of the matrices \( V^{(\pm)} \). We have also made use of the approximation \( \Delta_b \simeq \Delta_t \), which is obtained from Eq. (11) by neglecting the mixing angles between the third quark family and the other two in the \( V_{CKM} \) matrix.

It can be seen from (15) that a correction of a few per cent in the ratios \( R_b \) and \( R_c \) would require a mixing angle not far from the upper bound quoted above, \( |\beta| \simeq 5 \times 10^{-3} \). On the other hand, a relative deviation of \( \Gamma_h \) below 0.2\% (present experimental accuracy) would be possible either if the mixing angle satisfies \( |\beta| \lesssim 1 \times 10^{-3} \), or if we allow a cancellation between the different terms in (15a). Since the mentioned observed deviations of \( R_b \) and \( R_c \) from the SM predictions are still under revision, we will not insist here on fitting the parameters of the model to the experimental results. However, it is worth to notice that the model-independent analysis in Ref. [3] suggest values of \( |\xi_q| \) of about 3-4 in order to reproduce the possible excess of dijet events at CDF [4] (the \( Z' \) mass in this case is required to be around 800 – 900 GeV). In the case of LEP, the presence of significant effects is also allowed, provided that the mixing angle \( \beta \) has an absolute value of order \( 10^{-3} \) or higher.

We proceed now to consider another potentially important effect of the presence of a \( Z' \), namely the \( Z' \) exchange contribution to the \( t\bar{t} \) production cross section in high-energy \( pp \) collisions. This cross section has been experimentally measured by the CDF and D0 collaborations at Fermilab for a \( pp \) center-of-mass energy of \( \sqrt{s} = 1.8 \) TeV, leading to the values \( \sigma_{t\bar{t}} = 7.5^{+1.9}_{-1.6} \) pb (CDF, \( m_t = 175 \) GeV) [10] and \( \sigma_{t\bar{t}} = 5.5 \pm 1.8 \) pb (D0, \( m_t = 173.3 \) GeV).
GeV \[17\]. The comparison with theoretical results shows that the value quoted by D0 is in good agreement with NLO QCD calculations, while that from CDF turns out to be somewhat higher than expected. Recent theoretical studies have been presented in Refs. \[18\] and \[19\], obtaining respectively \( \sigma_{\bar{u}u} = 4.75^{+0.73}_{-0.62} \) and \( \sigma_{\bar{u}u} = 5.52^{+0.67}_{-0.45} \) (in both cases \( m_t = 175 \) GeV is considered).

It has been argued \[20\] that a \( Z' \) having relatively large couplings to quarks (in particular, to the \( u \) quark) could lead to significant corrections to \( \sigma_{\bar{u}u} \). Indeed, these corrections have been explicitly evaluated in Ref. \[20\] for a particular model containing a leptophobic \( Z' \) \[2\], showing that the effects can be important enough to be experimentally observable. We proceed here along the same lines, performing the relevant calculation for the \( q\bar{q} \to t\bar{t} \) annihilation subprocess, which at the leading order reads

\[
\hat{\sigma}(q\bar{q} \to Z' \to t\bar{t}) = \frac{(G_F M_Z')^2}{6\pi} \frac{\hat{s}}{(\hat{s} - M_{Z'}^2)^2 + (\hat{s} \Gamma_{Z'}/M_{Z'})^2} \times (g_{Vq}^2 + g_{Aq}^2) \left[ \frac{\beta_t^2}{2} (3 - \beta_t^2) g_{Vt}^2 + \beta_t^3 g_{At}^2 \right] \tag{16}
\]

where \( \beta_t^2 = 1 - 4m_t^2/\hat{s} \), and \( \Gamma_{Z'} \) is the total \( Z' \) decay width. This parton-level expression has to be integrated over the corresponding parton distribution functions (PDF) in order to obtain the total \( p\bar{p} \) cross section. We have carried out this calculation taking \( m_t = 175 \) GeV, and using the MRS(A') PDF set \[21\], with \( \alpha_s(M_Z^2) = 0.113 \). A \( K \) factor of 1.3 has also been included \[20\] in order to account for the corresponding next-to-leading order corrections.

Our results for different \( Z' \) mass values are quoted in Table I. We consider two possible \( \Delta_q \) sets, (I) and (II), given by

\[
\begin{align*}
\text{(I)} & \quad \Delta_t = \Delta_b = 1, \quad \Delta_q = 0 \quad \text{for other } q \\
\text{(II)} & \quad \Delta_c = 1, \quad \Delta_s = \cos^2 \theta_c, \quad \Delta_d = \sin^2 \theta_c, \quad \Delta_b = \Delta_u = \Delta_t = 0
\end{align*}
\tag{17a}
\tag{17b}
\]

where \( \theta_c \) is the Cabbibo angle. For the quoted \( M_{Z'} \) values, these sets provide with good approximation the highest (set I) and lowest (set II) contributions to \( \sigma_{\bar{u}u} \) within the allowed parameter space. We find that a \( Z' \) having a mass lower than 700 GeV appears to be excluded, while a \( Z' \) heavier than 900 GeV would need an experimental accuracy of 1 pb or less in \( \sigma_{\bar{u}u} \) in order to produce a visible effect. In the remaining range, i.e. \( M_{Z'} = 700 \) to 900 GeV, the uncertainty introduced by the \( \Delta_q \) parameters, together with the discrepancy between the experimental values obtained by CDF and D0, do not allow a conclusive analysis. However, the above results from both experiments only consider channels with at least one lepton, and the experimental situation can be improved by the inclusion of all the hadronic channels. Then a further reduction in the uncertainty (up to a 10\%) is expected for the second run, by the year 2001 \[1\].

Effects at LEP2. We concentrate now on the possible \( Z' \) effects on the experimental quantities that will be measured at LEP2. The observables of interest in this case are those related to the leptonic cross sections \( e^+e^- \to l^+l^- \), together with three hadronic quantities, namely the \( e^+e^- \to b\bar{b} \) cross section \( \sigma_b \), the total hadronic cross section \( \sigma_h \) and...
the forward-backward asymmetry in the $b\bar{b}$ production $A_{FB,b}$. We see that the presence of a $Z'$ would give rise to deviations from the SM predictions for these observables, which turn out to be negligible for the leptonic channels, but not necessarily for the mentioned hadronic quantities.

By considering the $Z'$ contributions to the *leptonic* observables, it is possible to determine a region in the $(g'_{Vl}, g'_{Al})$ plane where the predictions of the SM cannot be distinguished at LEP2 from those of the models including the extra $Z$. A model-independent analysis shows that (with 95% CL) this region is bounded by

$$
|g'_{Vl}| \lesssim 0.12 \sqrt{\frac{M_{Z'}^2 - s}{s}} \quad \quad |g'_{Al}| \lesssim 0.096 \sqrt{\frac{M_{Z'}^2 - s}{s}}
$$

(18)

where $s$ stands for the squared $e^+e^-$ center-of-mass energy. In the case of the 3-3-1 model, the values for $g'_{Vl,Al}$ contain no free parameters, thus the relations (18) can be unambiguously checked for given values of $s$ and the $Z'$ mass. With $\sin^2 \theta_W = 0.232$, we get from (19)

$$
g'_{Vl} \simeq 0.23 \quad \quad g'_{Al} \simeq -0.08
$$

(19)

Therefore, if the configuration $s_0 = (175 \text{ GeV})^2$ (the most convenient one) is chosen, it is seen that the values in (19) lay within the above non-observability region, even for a “light” $Z'$ of mass $\lesssim 700 \text{ GeV}$.

The situation is more promising for the above mentioned *hadronic* observables, although in this case the $Z'$ couplings depend on the unknown parameters $\Delta q$. In particular, we find that the total hadronic cross section could be increased by about 2% even for a $Z'$ mass of $\sim 900 \text{ GeV}$. In order to estimate the shifts for the hadronic observables with respect to the SM predictions, we use here the so-called “$Z$-peak subtracted” approach, which improves the Born approximation by taking measured $Z$-peak quantities as input parameters (for the case of a $Z'$ having universal couplings to quarks, a similar analysis has already been performed in Refs. [1,5]). In this way, the relative corrections for $s = s_0$ in the 3-3-1 model are found to be

$$
\begin{align*}
\frac{\delta \sigma_h}{\sigma_h} &= \left[ 5 \times 10^{-2} \right] \left\{ \frac{(1 \text{ TeV})^2}{M_{Z'}^2 - s_0} \left[ 0.18 - 0.13 \Delta_b \right] \right\} \\
\frac{\delta \sigma_b}{\sigma_b} &= \left[ 1.4 \times 10^{-2} \right] \left\{ \frac{(1 \text{ TeV})^2}{M_{Z'}^2 - s_0} \left[ 0.65 + 0.35 (\Delta_u + \Delta_c) \right] \right\} \\
\frac{\delta A_{FB,b}}{A_{FB,b}} &= \left[ 10 \times 10^{-2} \right] \left\{ \frac{(1 \text{ TeV})^2}{M_{Z'}^2 - s_0} \left[ 0.19 - 0.59 \Delta_b \right] \right\}
\end{align*}
$$

(20)

where once again the relation $\Delta_d + \Delta_s + \Delta_b = 1$ has been used. For each observable, the corresponding accuracy (two standard deviations) expected at LEP2 has been explicitly factorized out, so that the expressions in curly brackets have to be at least of order one to yield visible effects. It is seen from (21) that the uncertainty introduced by the parameters $\Delta q$ is minimal for $\sigma_h$, where a positive enhancement can be obtained. In fact, in this case the shift from the SM value can be produced even at three standard deviations for a $Z'$ mass below 900 GeV. For the other two observables, the situation is less favourable, although the
use of the combined observables would allow to set improved negative constraints (recall that $\Delta_u + \Delta_c \simeq 1 - \Delta_b$).

Taking now into account the above results from the analysis of the effects on $t\bar{t}$ production, we conclude that a $Z'$ having a mass in the range $700 - 900$ GeV is likely to produce significant contributions to both the top production cross section $\sigma_{t\bar{t}}$ measured at Fermilab and the total hadronic cross section $\sigma_h$ to be measured at LEP2. The shift from the SM predictions appears to be particularly important for the latter, where the effect of the unknown parameters turns out to be minimized and a visible signal can be obtained even for $M_{Z'}$ up to $1$ TeV. Since the hadronic cross section is expected to be measured with relatively good accuracy at LEP2, we believe that the 3-3-1 model should be considered as a potentially interesting one in the forthcoming runs, when the expected luminosity will be achieved.

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TABLE I. Contributions to $\sigma_{t\bar{t}}$ from the extra $Z$ boson, for $\Delta_q$ sets (I) and (II) and different values of the $Z'$ mass.

| $M_{Z'}$ (GeV) | $\sigma_{t\bar{t}}^{(I)}$ (pb) | $\sigma_{t\bar{t}}^{(II)}$ (pb) |
|----------------|-------------------------------|-------------------------------|
| 700            | 4.3                           | 1.7                           |
| 800            | 1.9                           | 0.77                          |
| 900            | 0.82                          | 0.34                          |
| 1000           | 0.39                          | 0.15                          |
| 1200           | 0.11                          | 0.04                          |