ELECTRON ACCELERATION BY CASCADING RECONNECTION IN THE SOLAR CORONA.
I. MAGNETIC GRADIENT AND CURVATURE DRIFT EFFECTS

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ABSTRACT

We investigate the electron acceleration by magnetic gradient and curvature drift effects in cascading magnetic reconnection of a coronal current sheet via a test particle method in the framework of the guiding center approximation. After several Alfvén transit times, most of the electrons injected at the current sheet are still trapped in the magnetic islands. A small fraction of the injected electrons precipitate into the chromosphere. The acceleration of trapped electrons is dominated by the magnetic curvature drifts, which change the parallel momentum of the electron, and appears to be more efficient than the acceleration of precipitating electrons, which is dominated by the perpendicular momentum change caused by the magnetic gradient drifts. With the resulting trapped energetic electron distribution, the corresponding hard X-ray (HXR) radiation spectra are calculated using an optically thin Bremsstrahlung model. Trapped electrons may explain flare loop top HXR emission as well as the observed bright spots along current sheets trailing coronal mass ejections. The asymmetry of precipitating electrons with respect to the polarity inversion line may contribute to the observed asymmetry of footprint emission.

Key words: acceleration of particles – magnetic reconnection – magnetohydrodynamics (MHD) – methods: numerical – plasmas – Sun: flares

1. INTRODUCTION

Since the first recorded white light observations of solar flares (Carrington 1859; Hodgson 1859), sophisticated ground-based and space-born solar techniques have been introduced to investigate the physics of the Sun. Recently, space telescopes like the Solar and Heliospheric Observatory (SOHO), Yohkoh, RHESSI, Hinode, and the Solar Dynamics Observatory (SDO) have revealed many detailed observations covering broad wavelength ranges at a high temporal, spatial, and spectral resolution.

Generally, it is accepted that the energy of solar flares comes from stressed, non-potential, current-carrying coronal magnetic fields being released by magnetic reconnection. About 10%–50% of the flare energy may be transferred to energetic electrons and ions (e.g., Lin & Hudson 1976). In some cases, energetic electrons alone carried away 50% of the flare energy (e.g., Miller et al. 1997), and were accelerated to energies up to 10–100 MeV (e.g., Aschwanden 2002).

The prime diagnostic of accelerated electrons in solar flares is the hard X-ray (HXR) radiation, coming from these energetic electrons. Two main components were identified in HXR light curves: a sharply increasing component and a slowly varying one. The sharp increase occurs within 0.5–5 s after the initial flaring (e.g., Holman et al. 2011; Zharkova et al. 2011). This indicates that within sub-seconds electrons are locally accelerated in excess of a few MeV. The slowly varying component lasts as long as flares continue, i.e., electron energization continues.

Using high-resolution imaging, the HXR source location in the Sun has been mapped with a few arcsecond resolution. Solar observations have shown HXR emissions from the footpoints of flaring coronal structures. Recently, based on the classical CSHKP (see Priest & Forbes 2002 for a review) solar flare reconnection model and solar flare observations near the limb of the solar disk, HXRs were also found at flare loop tops (e.g., the famous “Masuda flare” by Masuda et al. 1994 with Yohkoh observations and the later review of coronal HXR sources by Krucker et al. 2008).

Although substantial progress has been made in observations, by which mechanisms the flare electrons are accelerated remains an open question. In general, suggested mechanisms can be divided into three classes: (1) acceleration by direct current (DC) electric fields (see, e.g., Zharkova & Gordovskyy 2004, 2005a, 2005b), (2) stochastic acceleration (see, e.g., Vlahos & Cargill 2009), and (3) shock acceleration (see, e.g., Aschwanden 2002; Benz 2008). Observations also show that different flares produce different HXR spectra changing with time and locations with respect to the polarity inversion line (PIL; e.g., Zharkova et al. 2011). All these observed features can hardly be explained by one single acceleration mechanism. Therefore, flare energetic particles are perhaps accelerated by different mechanisms at different times and in different places as flares evolve.

In order to validate different acceleration mechanisms, it is appropriate to carry out test particle calculations based on MHD simulations. Previously, electron acceleration by the parallel component of DC electric fields \( E = -u \times B + \eta J \) has been extensively studied (Priest & Titov 1996; Zharkova & Gordovskyy 2004, 2005a, 2005b; Wood & Neukirch 2005). However, the prescription of \( \eta \) in the resistive MHD simulations is usually ad hoc and arbitrary. Meanwhile, in the collisionless corona, the concept of collisional resistivity \( \eta \) is largely unapplicable. Microphysical effects must be taken into account. Silin et al. (2005) and Büchner & Elkina (2006), for example, have shown that considering possible micro-turbulence strong parallel electric fields must be confined in...
narrow channels of the ion inertia scale size (see also J. Büchner and W. Daughton 2007, Section 3.5 in Birn & Priest 2007). That was also found in particle in cell (PIC) simulations by Dahlin et al. (2014). Macroscopic MHD simulations, on the other hand, are more appropriate for the investigation of electron acceleration by convective electric fields \( \mathbf{E} = -\mathbf{u} \times \mathbf{B} \). For example, Vekstein & Browning (1997), Guo et al. (2010), and Grady et al. (2012) analyzed the particle acceleration by the convective electric fields near a magnetic null point and in a collapsing magnetic trap, respectively.

In the studies of Vekstein & Browning (1997) and Guo et al. (2010), there were only one magnetic X-point or null point without any magnetic island. Electron acceleration at only one reconnection X-point, however, cannot explain the huge accelerated electron number \( 10^{38} \); see Krucker et al. 2008; Cargill et al. (2012) and flux (more than \( 2 \times 10^{36} \) s\(^{-1} \) above 20 keV; see Holman et al. 2003) inferred from HXR observations. The possibility of particle acceleration by cascading reconnection was mentioned first by Shibata & Tanuma (2001). They conjectured that magnetic islands could be formed at many scales by tearing mode instabilities of the stretching current sheet. Later, this concept was confirmed by theoretical approaches (e.g., Loureiro et al. 2007; Uzdensky et al. 2010), observations (e.g., Hoshino et al. 1994; Karlický 2004), AMR MHD simulations (e.g., Bártá et al. 2011), and PIC simulations (e.g., Karlický et al. 2012).

Electron acceleration by many reconnection sites and magnetic islands was studied with the test particle simulation method (e.g., Li & Lin 2012 and Gordovskyy et al. 2010a, 2010b), as well as PIC simulations (e.g., Dahlin et al. 2014, 2015 and Li et al. 2015) focused on the electron acceleration in collisionless magnetic reconnection with parameters appropriate for solar flares. Sironi & Spitkovsky (2014) and Guo et al. 2014, 2015 considered the relativistic plasma. Studies with the test particle simulation method, however, must prescribe the ad hoc anomalous resistivity models to produce the accelerating fields. Moreover, the number of X-points in these studies were obtained by periodically repeating the simulation domain with only one X-point and they focused most of their attention on the acceleration from the parallel resistive electric fields. Although PIC simulations can treat the particles and electromagnetic fields self-consistently, they still cannot reach large scales to explain the observed features directly.

In this study, we concentrate on investigating the electron acceleration via magnetic gradient and curvature drift effects in the convective electric fields \( \mathbf{E} = -\mathbf{u} \times \mathbf{B} \) of cascading reconnection current sheets trailing a flaring arcade behind an ejected flux rope (see, for example, Lin & Forbes 2000) obtained from AMR MHD simulations (Bártá et al. 2010, 2011; see also Section 2), using a test particle calculation method in the framework of the guiding center approximation (Northrop 1963, see Section 3). In order to compare with solar flare HXR observations, resulting HXR emissions by energetic electrons are derived using an optically thin Bremsstrahlung method (Brown 1971; Tandberg-Hansen & Emslie 1988). In Section 4, electron acceleration dependence on initial conditions (Sections 4.1.1 and 4.2.1) and acceleration properties in different (parallel and perpendicular) directions (Sections 4.1.2 and 4.2.2) and trajectories (Sections 4.1.3 and 4.2.3) are investigated for both trapped (Section 4.1) and precipitating (Section 4.2) electrons, respectively. Additionally, we also compare with other solar flare observations, for example, bright spots along current sheets trailing coronal mass ejections (CMEs) by trapped electrons (Section 4.1.4) and UV and EUV double brightening ribbons at the feet of solar flare loops by precipitating electrons (Section 4.2.4). Finally, our main results are discussed and conclusions are drawn in Section 5.

2. ELECTROMAGNETIC FIELDS OF CASCADING RECONNECTION

Electromagnetic fields of cascading magnetic reconnection are obtained by means of a 2.5-dimensional (2.5D) AMR MHD simulation (Bártá et al. 2010, 2011). In contrast to traditional MHD simulations where grid size is uniform, in this study we take high-resolution AMR MHD simulations to obtain more precise magnetic field structures where current sheet widths become thinner than the initial coarse grid size.

The AMR algorithm works as follows: if at the time step \( t + \Delta t \) some coarse grids are detected containing a thin current sheet, then they will locally be split into sub-boxes with \( 10 \times 10 \) grid points in the sub-system. After such refined meshes are initialized, the necessary more detailed plasma and field values are obtained by interpolating their parent coarse system values at the last time step \( t \). Then the dynamics of both the newly created and the pre-existing refined meshes evolved with time \( t \to t + \Delta t \) with an accordingly refined time step. After that, the plasma and field values at the parent coarse mesh are replaced by averaging the quantities obtained from its corresponding refined meshes at the time step \( t + \Delta t \). The influence of the global dynamics on the refined meshes are considered by interpolating boundary conditions in time and space. This refinement is repeated until the entire simulation is over (see Bártá et al. 2010).

The AMR MHD simulations are restricted to 2.5D, i.e., two-dimensional geometry but three-dimensional plasma velocities and magnetic fields. This assumption is reasonable since observations have shown that the extension of solar flare arcades along the PIL is typically much larger than across the PIL.

The coordinate system is shown in Figure 1: the \( x \)- and \( y \)-axes are directed perpendicular to and along the current sheet, respectively. The current sheet center is located at \( x = 0 \), while the \( z \)-axis is pointing along the PIL located at \( (x = 0, y = 0) \). In this direction, every value is invariant i.e., \( \partial / \partial z = 0 \). The coarse resolution contains \( 6400 \times 800 \) points in the vertical (\( y \)-axis) and the right half of horizontal (positive \( x \)-axis) direction. A mirroring boundary is used at \( x = 0 \) for the left half box: \( \rho, u_x, u_z, B_x, B_z \), and \( U \) are symmetric while \( u_x \) and \( B_y \) are antisymmetric. For the upper and right sides, free boundary conditions are used: all quantities should satisfy the von Neumann prescription \( \partial / \partial n = 0 \) except of the normal magnetic field \( B_y \) and the total energy density \( U, B_x \) and \( U \) are used to fulfill \( \nabla \cdot \mathbf{B} = 0 \). At the bottom, a symmetric boundary condition \( Q(−x) = Q(x) \) is used for \( \rho, B_x, B_y, U \), and the anti-symmetric relation \( Q(−x) = −Q(x) \) is assumed for \( B_z \). The plasma is always static \( u_z = 0 \) at the bottom.

A generalized Harris-type current sheet is chosen as the initial state of the AMR MHD simulation (Bártá
et al. 2010, 2011):

\[
A(x, y, z; t = 0) = -B_{0y} \ln \left[ \exp \left( \frac{x}{\omega_{cs}(y)} \right) + \exp \left( -\frac{x}{\omega_{cs}(y)} \right) \right],
\]

\[
B_z(x, y, z; t = 0) = B_{0z},
\]

\[
\rho(x, y, z; t = 0) = \rho_0 \exp \left( -\frac{y}{L_G} \right)
\]

(1)

where \( \omega_{cs}(y) \) (Equation (2)) shows the characteristic width at different heights of the initial current sheet and \( L_G = 120 \) Mm is the scale height for a fully ionized hydrogen plasma:

\[
\omega_{cs}(y) = \frac{d \cdot y^2 + y + y_0}{y + y_0}
\]

(2)

and \( B_{0y}, B_{0z}, \rho_0, d, \) and \( y_0 \) are normalized quantities: \( B_{0y} = 0.2, \rho_0 = 1.0, B_{0z} = \sqrt{B_{0y}^2 + B_{0z}^2} = 1.0, d = 0.003, \) and \( y_0 = 20.0 \).

The \( x \) - and \( y \)-components of the magnetic field \( (B_x, B_y) \) are obtained from the magnetic vector potential \( A \) as \( B = \nabla \times A \). Note that the magnetic field strength slightly decreases via \( \omega_{cs}(y) \) with height “\( y \)” corresponding to the magnetic field in the solar corona balancing the gravity force (Equation (6)). The initial magnetic field state is displayed in the leftmost panel of Figure 1.

Compressible resistive MHD equations (Equations (3)–(6)) are solved to describe the evolution of the plasma and magnetic fields (e.g., Priest 1984):

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]

(3)

\[
\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g}
\]

(4)

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \mathbf{J})
\]

(5)

\[
\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{S} = \rho \mathbf{u} \cdot \mathbf{g}
\]

(6)

where \( \rho \) is the plasma density, \( \mathbf{u} \) plasma velocity, \( \mathbf{B} \) magnetic field strength, \( \mathbf{E} \) electric field strength, \( \eta \) resistivity, \( \mathbf{g} \) gravitational acceleration at the photospheric level, and \( p \) plasma pressure. The current density \( \mathbf{J} \), total energy density \( U \), and energy flux \( S \) are defined as:

\[
\mathbf{J} = \frac{\nabla \times \mathbf{B}}{\mu_0}
\]

(7)

\[
U = \frac{p}{\gamma - 1} + \frac{1}{2} \rho \mathbf{u}^2 + \frac{B^2}{2\mu_0}
\]

(8)

\[
S = \left( U + p + \frac{B^2}{2\mu_0} \right) \mathbf{u} - \frac{\mathbf{u} \cdot \mathbf{B}}{\mu_0} \mathbf{B} + \eta \mathbf{J} \times \mathbf{B}
\]

(9)

where \( \gamma_0 = \frac{5}{3} \) is the adiabatic coefficient for the adiabatic condition and \( \mu_0 \) is the vacuum magnetic permeability.

Usually, anomalous resistivity \( \eta \) in the Ohm’s law (Equation (5)) and in the energy flux \( S \) (Equation (9)) is chosen ad hoc to describe the sub-grid-scale dissipation effects of microphysical (kinetic) processes.

The AMR MHD simulations (Equations (3)–(6)) are carried out with normalized parameters: the normalizing length scale (half width of the current sheet at \( x = 0 \)) is chosen to be \( L_0 = 6.0 \times 10^5 \) m, the normalizing magnetic field is \( B_0 = 4.0 \times 10^{-2} T \), and the normalizing number density is \( n_0 = 1.25 \times 10^{16} \text{m}^{-3} \) and \( q_0 = |e| \) is taken as the normalizing charge. Other scaling parameters can be derived as: \( V_0 = B_0/\sqrt{|\mu_0| n_0 m_0} = 7.80 \times 10^6 \text{m s}^{-1} \) (where \( m_0 = m_p \) proton mass and \( V_0 \) is the asymptotic value of the Alfven velocity at \( y \to \infty, x = 0 \) and \( t = 0 \)), and the time is normalized by the Alfven transit time \( t_0 = L_0/V_0 = 7.69 \times 10^{-2} \) s. Furthermore, there are \( E_0 = V_0 \) and \( B_0 = 3.12 \times 10^5 \text{V m}^{-1} \). The asymptotic plasma beta parameter is \( \beta = 0.1 \) at \( (x \to \infty, y = 0) \). In the coarse resolution, the mesh sizes are \( \Delta x = \Delta y = 0.045 L_0 \). Hence, the entire simulation domain extends over \((-36, 36) \times (0, 288) L_0^2\) in the \( x \)-\( y \) plane.

Figure 1 depicts the evolution of the in-plane magnetic fields. The total simulation is performed over 520 \( t_0 \) when a CME is ejected through the upper boundary of the box (the last panel of Figure 1). We select a totally fragmented current sheet at \( t = 360 t_0 \) as the background electromagnetic fields since no
more additional magnetic islands are generated after this time step. The anomalous resistivity \( \eta \) does not appear before \( t = 420 \ t_0 \), i.e., the AMR MHD simulations before this time step can be described with the ideal MHD equations \( \eta = 0 \). Magnetic island formations in the condition of \( \eta = 0 \) are mainly due to the nonlinear tearing instabilities in the continually stretched thin current layer (see Báta et al. 2011 for the details of the anomalous resistivity definition and the tearing mode instability functions in the fragmentation of the current layer). Figure 2 shows the magnetic field lines at \( t = 360 \ t_0 \). Most of the magnetic islands are located at \( 0 < y < 108 \ L_0 \), so the test particle calculations are confined below \( y = 108 \ L_0 \) (see the middle panel of Figure 2).

3. METHODS USED

3.1. Test Particle Calculations

If the gyrorADIUS \( r_{gy} = \frac{m_0 v}{qB} \) and gyropEREIOD \( \approx 1/\omega_{gy} = \frac{2m_0 B}{q}\Omega_{gy} \) of the particle are much smaller than the length scale of transverse gradients \( r_{gy} \) and characteristic oscillation periods \( \omega_{gy} \) of the ambient electromagnetic fields (i.e., \( \omega_{gy}/r_{gy} < 1 \) and \( \omega_{gy}/\omega_{gy} \gg 1 \)), a guiding center approximation is valid (Northrop 1963). The gyrorADIUS of only 10 MeV energetic electrons is only 4.4 m with the minimum magnetic field strength 0.19 \( B_0 \) obtained by the AMR MHD simulations. The smallest grid size \( \Delta x = \Delta y = 0.0045 \ L_0 = 2.7 \ m \) is much larger. Hence, here we use the guiding center approximation to trace each electron. In this approximation, the motion of a magnetized charged particle can be decomposed into the motions of its guiding center (including parallel movement and perpendicular drifts) and a gyration around this center (which is replaced by a constant magnetic moment due to average effects).

For a high precision, a relativistic guiding center approximation is used:

\[
\frac{dR}{dt} = v_D + \frac{\gamma v_D}{\gamma} b
\]  

\[
v_D = v_E + m \frac{(\gamma v_E)^2}{\gamma} \left[ b \times (b \cdot \nabla)b \right] \\
+ \frac{m}{q} \left( \gamma v_E \right)^2 \left[ b \times (\nabla(kb)) \right] \\
+ \frac{m}{q} \left( \gamma v_E \right)^2 \left[ b \times (b \cdot \nabla)v_E \right] \\
+ \frac{m}{q} \left( \gamma v_E \right)^2 \left[ b \times (v_E \cdot \nabla)b \right] \\
+ \frac{m}{q} \left( \gamma v_E \right)^2 \left[ b \times (v_E \cdot \nabla)v_E \right] \\
+ \frac{1}{\gamma} \left( \gamma v_E \right)^2 \left[ b \times v_E \right] 
\]  

\[
\frac{d(\gamma v_E)}{dt} = \frac{q}{m} E \cdot b - \frac{\mu}{\gamma} \left[ b \cdot \nabla(kb) \right] \\
+ \left( \gamma v_E \right) b \cdot \left[ (b \cdot \nabla)b \right] + \gamma v_E \cdot \left[ (v_E \cdot \nabla)b \right] 
\]  

\[
\gamma = \sqrt{1 - \left( \frac{v_y^2 + v_z^2 + v_D^2}{v^2} \right) / c^2} 
\]  

\[
\frac{d\mu}{dt} = 0
\]  

where \( R, v_D, v_E, \gamma, \) and \( b \) are the guiding center position vector, the perpendicular drift velocity, the velocity along the magnetic field, the relativistic factor \( \left( \frac{\mu}{\gamma^2} \right) \), and the magnetic field direction unity vector \( b = \frac{B}{B} \), respectively. In the expression for the drift velocity \( v_D \) in Equation (11), the term \( v_E \) corresponds to the local \( E \times B \) drift velocity \( v_E = \frac{E \times B}{B^2} \). Other terms are the magnetic curvature drift velocity and the magnetic gradient drift velocity, as well as higher order drifts.
The factor \( k = \sqrt{1 - \frac{v^2}{c^2}} \) relates the electromagnetic field values to the reference frame moving with the velocity \( v_E \).

Finally, \( \mu = \frac{(\gamma v)^2}{2B} \) is the relativistic magnetic moment per mass unit where \( v_\perp \) is the particle gyration velocity perpendicular to \( B \). The electron energy is expressed using the relativistic \( \gamma \)-factor as \( E = (\gamma - 1)mc^2 \). The set of Equations (10)–(14) are solved utilizing a fourth-order Runge–Kutta scheme. The field values between the grid points are interpolated along the electron trajectories with two-dimensional (2D) linear interpolation.

4.752 \times 10^5 test electrons are initially uniformly distributed along the current sheet (0 < y < 108 L_0, x = 0) at 2400 points with 22 different initial velocities from 0.0 to 21.0 \( v_{th} \) and 9 different initial pitch angles from 0 to \( \pi \). (These choices are limited by the computational resources available.) Here \( v_{th} \) is the electron thermal velocity for a typical coronal temperature of 10^6 K. 0.761v_0 \cong 6 \times 10^3 \text{ km s}^{-1}. Every electron is traced for up to 10 \( t_0 \) (~0.769 s) or until it leaves the simulation domain, whichever happens first. Note that this time is shorter than the timescale of essential magnetic field changes in the MHD simulations.

3.2. Spectrum Distribution Function of Accelerated Electrons

To obtain the energetic electron distribution function (or spectrum), we use the fact that the solar corona is practically collisionless. Hence, according to Liouville’s theorem, the particle distribution function keeps constant along the particle trajectory; \( f(E, A, r, t) = f(E_{0}, A_{0}, r_{0}, t_{0}) \). This allows us to calculate the electron distribution function \( f(E, A, r, t) \) at the place where HXRs are expected to be generated by Bremsstrahlung of energetic electrons.

3.3. HXR Emission

Knowing the local plasma number density and electron distribution function, the HXR emissivity \( I(\epsilon) \) integrated over all contributing electrons can be calculated in the framework of the thin target model (Brown 1971) as:

\[
I(\epsilon) = \sum_{E > \epsilon} n(r) v(r) \sigma_B(\epsilon, E)f(E, A, r, t).
\]

Here \( E, A, r, \) and \( v(r) \) are the energy, pitch angle, position, and velocity of the electron at the time \( t \), \( n(r) \) is the local plasma number density, \( \epsilon \) is the radiated photon energy, \( f(E, A, r, t) \) is the electron distribution function at the position of interest place and time, while \( \sigma_B \) is the cross section of the Bremsstrahlung process. For a simple approximation, we take the Bethe–Heitler formula for the Bremsstrahlung cross section (Bethe & Heitler 1934; Brown 1971):

\[
\sigma_B(\epsilon, E) \propto \frac{1}{\epsilon E} \ln \left[ \frac{1 + \sqrt{1 - \frac{\epsilon}{E}}}{1 - \sqrt{1 - \frac{\epsilon}{E}}} \right].
\]

Note that the Bethe–Heitler formula applies only to particle energies less than 100 keV. In this investigation, most of the electrons are accelerated to energies less than 100 keV, i.e., the Bethe–Heitler formula still can give a high accuracy here.

4. RESULTS

Depending on electron locations at the end of test particle calculations 10 \( t_0 \), three groups of electrons can be identified: those trapped in the magnetic islands, those precipitating to the chromosphere, and the ones being ejected into the interplanetary space. There is no electron escaping from the left and right sides of the simulation domain. Here we concentrate our analysis on the trapped (Section 4.1) and precipitating (Section 4.2) electrons.

4.1. Trapped Electrons

More than 80% of simulated electrons are trapped in the magnetic islands along the current sheet by 10 \( t_0 \). This highly complex magnetic field structure provides a very effective trapping mechanism of energetic electrons for the coronal HXR sources.

4.1.1. Acceleration Dependence on Initial Conditions

The acceleration of electrons by the magnetic gradient and curvature drift effects in the convective (or induced) electric field \( \mathbf{E} = -\mathbf{u} \times \mathbf{B} \) is sensitive to the initial position, velocity, and pitch angle of injected electrons. The upper panel of Figure 3 depicts the dependence of the energy gain on the initial conditions together. The lower panels show the corresponding projected results. From left to right, the bottom panels of Figure 3 show the dependence of the acceleration efficiency (\( \Delta E \)) on the electron initial energy, pitch angle, and position, respectively.

The bottom left panel of Figure 3 shows that electron acceleration increases with the increase of the initial energy, which is consistent with the results of Guo et al. (2010) and Figure 2 in Guo et al. (2015). It is interesting to note that both the mean and the standard deviation of the energy gain are roughly proportional to the initial energy. The dependence of the electron acceleration on the initial pitch angle and position, however, is more or less chaotic due to the complex field structures.

The studies of Karlický & Kosugi (2004) found that betatron acceleration process dominates electron acceleration in a collapsing magnetic trap, i.e., electrons with pitch angles closer to 90° can get more energizations. Also there is a good acceleration symmetry around 90°, while here (bottom middle panel of Figure 3) the most energetic electron is no longer associated with an initial pitch angle of 90°. Additionally, acceleration symmetry with respective to pitch angle 90° is broken.

The bottom right panel of Figure 3 shows the magnetic field component \( B_x \) along the current sheet center \( x = 0 \), which can be used to identify magnetic X- and O-points with \( B_x = 0 \). The most efficient acceleration appears to be associated with electrons injected near \( y = 40 \) \( L_0 \), which is located at one end of a magnetic island and contains a large magnetic gradient and curvature acceleration factors (see the left and right panels in the top row of Figure 4). Dahlin et al. (2014) showed that the end of the magnetic island combined with the adjacent X-point exhaust is a good electron acceleration site for parallel magnetic curvature acceleration with PIC simulations.
4.1.2. Energy Gain

The guiding center approach decomposes particle energy into components parallel and perpendicular (gyration) to the magnetic field and the part associated with the guiding center drift in the direction perpendicular to the magnetic field. The maximum drift velocity \( v_D \) (Equation (11)) is less than 1.5 \( v_{th} \) in differently resolved magnetic fields, which is negligible comparing with the other two components.

Figure 3. Dependence of the electron kinetic energy gain (\( \Delta E \)) on the initial pitch angle (\( A_0 \)), velocity (\( v_0 \)), and position (\( y_0 \)). Each point represents one electron. The upper panels show the three-dimensional results with the initial velocity color-coded. The lower panels show the corresponding projected results. The averaged values (* lines) and standard deviations of the energy gain (\( \pm \) lines) in the initial energy and pitch angle spaces are shown in the bottom left and middle panels, respectively. \( B_x \) along the current sheet center is depicted in the bottom right panel. Note that in each lower panel, there are three different scales in the y-axis for \( \Delta E < 0 \), \( 0 < \Delta E < 3 \text{ keV} \), and \( \Delta E > 3 \text{ keV} \).
Considering $E \cdot b = 0$, $\partial B/\partial t = 0$ and $v_D \cong v_E$, $k \cong 1$, Equations (12) and (14) give:

$$\frac{1}{2} \frac{d (\gamma_{\parallel})^2}{dt} = -\gamma_{\parallel} (\gamma_{\parallel})^2 v_E \cdot [ (b \cdot \nabla) b ]$$  (17)

$$\frac{1}{2} \frac{d (\gamma_{\perp})^2}{dt} = \frac{d \mu B}{dt} = \frac{d B}{dt} = \mu \frac{d \gamma_{\perp}}{dt} = \mu v_E \cdot (b \cdot \nabla) b + \mu v_E \cdot \nabla B.$$  (18)

So the energy evolution of an electron in the guiding center limit is given by:

$$\frac{dE_{\parallel}}{dt} \propto \frac{d (\gamma_{\parallel})^2 + (\gamma_{\perp})^2}{dt}$$

$$\propto \frac{dE_E}{dt} \times \mu v_E \cdot \nabla B + (\gamma_{\parallel})^2 v_E \cdot [ (b \cdot \nabla) b ].$$  (19)

Spacial distributions for each acceleration factor in Equations (17) and (18) are shown in the top panels of Figure 4: $(v_E \cdot \nabla B)/(2B)$—top left panel, $(b \cdot \nabla) b$—top middle panel, and $v_E \cdot [ (b \cdot \nabla) b ]$—top right panel. Note that the spacial distribution of $v_E \cdot [ (b \cdot \nabla) b ]$ is quite similar to those shown in Figures 7 and 8 of Dahlin et al. (2014) but here without turbulent structures in magnetic islands. The bottom panel of Figure 4 shows the energy gain rate spectra of the perpendicular gradients $(v_E \cdot \nabla B)/(2B)$ and curvatures $v_E \cdot [ (b \cdot \nabla) b ]$. Note that here only the acceleration timescales less than $10 t_0$ are pitched out.

Equations (17) and (18) show that the parallel magnetic gradient $b \cdot \nabla B$ (or magnetic mirror force) can change both parallel and perpendicular energies, but they cancel out each other in the total energy evolution (Equation (19)). The overall energy change is dominated by the perpendicular magnetic gradient $(\mu v_E \cdot \nabla B)$ and curvature $(\gamma_{\parallel})^2 v_E \cdot [ (b \cdot \nabla) b ]$, which are proportional to the electron energy due to $\mu$ and $(\gamma_{\parallel})^2$, respectively. So the increase of the electron acceleration with
the increasing initial energy is therefore expected (see the lower left panel of Figure 3).

Also due to the combined actions between the magnetic gradient ($\mu$) and curvature $(\nabla v)^2$, acceleration favorable pitch angles (the lower middle panel of Figure 3) are not 0, 180°, or 90°, which correspond to acceleration dominated only by the magnetic curvature $(\gamma v)^2 v_E \cdot [(b \cdot \nabla)b]$ or gradient $(\mu v_E \cdot \nabla)B$, respectively. So in this study, both the magnetic gradient and curvature contribute to electron accelerations, while the acceleration asymmetry around a pitch angle of 90° (the bottom middle panel of Figure 3) is due to the non-symmetric acceleration factors around the current sheet center in $(\mu v_E \cdot \nabla)B$ and $(\gamma v)^2 v_E \cdot [(b \cdot \nabla)b]$ by the third dimension of electromagnetic fields in 2.5D symmetric current sheet geometry.

The top panels of Figure 5 exhibit the symmetry of electron perpendicular and parallel acceleration around a pitch angle of 90°, respectively. One can see in each panel that there is no exactly symmetric pattern around the initial pitch angle of 90°, while the asymmetry in the total energization in the bottom middle panel of Figure 3 is mainly due to the asymmetry in the parallel component by magnetic curvature accelerations.

The bottom panel of Figure 5 shows the distribution of the trapped electrons in the $(E_||, E_\perp)$ plane. These distributions are highly anisotropic. The parallel kinetic energy component dominates among those stronger energetic electrons that indicates that magnetic curvature acceleration is stronger than the gradient acceleration, while the weakly accelerated electrons still roughly keep their initial isotropic distribution. Electrons initially moving along magnetic field lines (with an initial pitch angle 0° or 180°) cannot be accelerated in the perpendicular direction due to the constant magnetic moment $\mu$.

In total, in this study, there is a positive energy gain in the perpendicular direction, since electrons are injected in the middle of the current sheet with the weakest magnetic fields. However, the negative energy gain by the magnetic gradient drifts in the study of Dahlin et al. (2014) was due to stronger magnetic fields in the initial current sheet.
4.1.3. Characteristic Trajectories

To better understand the details of the electron acceleration processes, Figure 6 exhibits the trajectories and energy evolutions of two characteristic electrons: one is the most energetic electron and the other is selected due to its initial position located in the biggest magnetic curvature acceleration region around $y = 90L_0$.

The most efficient acceleration happens to the parallel energy component with slightly increased perpendicular energy when the electrons are trapped in the magnetic island (around $y = 37L_0$) and accelerated again and again with its circulating motions by the positive magnetic curvature $v_E \cdot [(\mathbf{b} \cdot \nabla)\mathbf{b}]$ and gradient $\mu v_E \cdot \nabla B$ in the thin layer around the central current sheet above $y = 40L_0$ (see the left and right panels in the top row of Figure 4). The step-like increased $z$ direction displacement in the central panel of Figure 6 is also due to the positive magnetic curvature $v_E \cdot [(\mathbf{b} \cdot \nabla)\mathbf{b}]$ leading parallel motions along magnetic fields.

In the top right panel of Figure 4, one can see the biggest magnetic curvature acceleration region is located around $y = 90L_0$ (similar to the “dipolar heating” discussed by Dahlin et al. 2014); however, the strongest energetic electron is not from there. That is due to the cancellation between the magnetic curvature acceleration and deceleration when an electron circulates in this magnetic island. Electrons in the second row of Figure 6 are chosen to reveal the acceleration characteristics around $y = 90L_0$. Color-coded trajectory projections of this electron prove the above discussion and its energy gains are mainly due to the parallel acceleration by magnetic curvatures (see Figure 11 and discussion in Dahlin et al. 2014).

Energy oscillations between parallel and perpendicular energies in each characteristic electron energy evolution profile are due to the parallel magnetic gradient $v_E (\mathbf{b} \cdot \nabla B)$ in Equations (17) and (18).

4.1.4. Comparison with Observations

Since the kinetic energy of the most energized trapped electrons at the end of our calculation is already close to half of MeV by the magnetic curvature and gradient drift effects, these energetic trapped electrons have enough energy to produce HXR s by Bremsstrahlung (note that HXR range is 10–400 keV). In the framework of the thin target model (Brown 1971) and using the Bethe–Heitler formula for the
Bremstrahlung cross section (Bethe & Heitler 1934; Brown 1971, see details in Section 3.3), we derive the HXR spectrum of these energetic trapped electrons in order to compare our results with solar flare HXR observations.

Since the initial electron distribution function in the solar atmosphere is not known, we consider four different initial distribution functions as:

\[
f(E_0, A_0, r_0, t_0) = \begin{cases} 
\text{Maxwell–Boltzmann}(100\text{eV}; 0.01m_e c^2) & \text{Power-law E}_0^\alpha (\alpha = 0; 3). \end{cases}
\]

(20)

Here Maxwell–Boltzmann distribution with a temperature of 100 eV may be a natural choice and temperature 0.01 m_e c^2 \approx 5 keV is used to compare with the results in Li et al. (2015), where 0.01 m_e c^2 is the initial temperature of their PIC simulations. 0.01 m_e c^2 implies some pre-heating processes (Liu et al. 2013; Sharykin et al. 2014) are needed before \( t = 360 t_0 \), while the other two power-law initial distributions in the current sheet may be attributed to pre-acceleration or acceleration at smaller scales.

The resulting electron and HXR spectra and their spectral indices at the end of calculations are depicted in the top and bottom panels of Figure 7, respectively. The relationship between the electron (\( \gamma_e \)) and corresponding HXR (\( \gamma_{\text{HXR}} \)) spectral indices agree well with the relationship \( \gamma_{\text{HXR}} = \gamma_e + 1 \) in the thin target model.

Electron and HXR spectral indices, calculated from an initial Maxwell–Boltzmann distribution function for \( T = 10^6 \text{K} \), are too large to explain observations. It is unlikely that magnetic curvature and gradient accelerations are the dominant acceleration processes of low-energy electrons in the solar flares. With an increased temperature 0.01 m_e c^2, the corresponding electron and HXR spectral indices are comparable to those of observed small flares (whose HXR spectral indices can be as soft as \( \gamma_e > 7 \); see Aschwanden 2002; Krucker et al. 2008). With temperatures 0.01 m_e c^2 and the characteristic number density \( n_0 \) and magnetic field strength \( B_0 \) (Section 2), we have plasma \( \beta = n_0 k_B T / (B_0^2 / (2\mu_0)) = 0.016 \), which is between \( \beta = 0.02 \) and 0.007 studied by Li et al. (2015). Here electron spectra with temperatures 0.01 m_e c^2; however, are much softer than those found by Li et al. (2015). That indicates that magnetic curvature and gradient accelerations in the observed MHD scale are weaker than those in the kinetic scale due to the dependence of the magnetic curvature and gradient strength on spatial scales.

For the cases initially having a power-law distribution (e.g., Karlický & Bártá 2006), one may treat the electron acceleration as a diffusion in 2D energy space. Since the diffusion coefficient is approximately proportional to the square of the energy, this explains the difference by one of the spectral indexes of the injected and accelerated electrons below the maximum injection energy of \( \sim 50 \text{keV} \). Interestingly, the electron and HXR spectral indices above 50 keV with power-law initial distributions are very close to those of the Maxwell–Boltzmann initial distribution with a temperature of 0.01 m_e c^2.

Besides the HXR spectra, fine structures (bright spots) along the current sheets trailing CMEs or eruptive filaments were observed (e.g., by Ciaravella et al. 2002; Ko et al. 2003; Savage et al. 2010). These bright spots should come from energetic trapped electrons. Figure 8 shows the locations of trapped electrons with kinetic energies \( > 10 \text{keV} \) at the end of calculation. Structures similar to those found in Figure 8 can also be found in early stage of PIC simulations by Dahlin et al. (2014; see the top left panels in Figures 3 and 4 there) that more strongly accelerated electrons are mainly located at the edge of magnetic islands. Also the locations of these more strongly accelerated electrons correspond with the spatial distribution of the magnetic curvature acceleration rate in the top right panel of Figure 4.
trapped and precipitating electrons are mainly caused by the acceleration in the parallel direction or magnetic curvature acceleration. Precipitating electrons have stronger deceleration than acceleration in the parallel direction, i.e., the acceleration of precipitating electrons are mainly coming from the perpendicular direction. For stronger parallel magnetic curvature accelerations, electrons should stay longer around the current sheet center which has larger magnetic curvatures than other places, while precipitating electrons are ejected out of the current sheet before they can reach higher energies.

Much weaker energization of precipitating electrons can also be found in the bottom panel of Figure 10. In contrast to trapped electrons in the bottom panel of Figure 5, here most precipitating electrons still keep their initial energies shown as stripes parallel to “$E_{i} + E_{e} = 50$ keV,” because initially there are $E_{i0} + E_{e0} = Q$ ($Q$ is discretely distributed between 0 and 50 keV) and more accelerated precipitating electrons have more increased perpendicular energies.

4.2.3. Characteristic Trajectory of the Precipitating Electron

Figure 11 depicts the acceleration properties of the most energized precipitating electron that the acceleration site of this electron is still around the current sheet center by both the magnetic gradients $v_{B} \cdot \nabla B$ and curvatures $v_{B} \cdot [(b \cdot \nabla) b]$. There is no acceleration after it leaves the current sheet center, since the parallel magnetic gradient $v_{B} (b \cdot \nabla B)$ is stronger than the other two terms (see Figure 4) and there is no longer any strong energy change. Also because of the single sign of the parallel magnetic gradients and the direction of the parallel velocity along the precipitating electron trajectory, precipitating electrons do not have energy oscillation as frequently as that of the characteristic trapped electrons in Figure 6.

4.2.4. Comparison with UV and EUV Observations

Weak energizations of precipitating electrons in the convective electric fields cannot cause HXR emissions but they may produce ribbons of UV and EUV brightening (Fletcher et al. 2011).

The top panels of Figure 12 depict the evolution of the spatial distribution of electrons precipitating to the chromosphere. As they show, these two ribbons exhibit an antisymmetric geometry around the PIL, while those more accelerated precipitating electrons (which have final kinetic energies $E_{e0} > 54$ keV) only appear in one arm of the ribbons. The formation of these two ribbons is strongly related to the initial pitch angles of these precipitating electrons: an electron with an initial pitch angle $>90°$ ($<90°$) precipitates into one (the other) branch. That is due to the weak parallel accelerations by magnetic curvatures $(\gamma v_{\parallel}) \nabla v_{\parallel} \cdot [(b \cdot \nabla) b]$ (Equation (17)) which cannot accelerate electrons into the direction anti-parallel to its initial velocity during the calculation, while asymmetry of more accelerated precipitating electron locations is due to the non-symmetric acceleration around initial pitch angle 90° (bottom middle panel of Figure 9). Some of the observed asymmetry between two footpoints therefore may be attributed to some acceleration processes.

The top panels of Figure 12 also reveal that the locations of these precipitating electrons are influenced by their initial
positions: electrons starting closer to the Sun’s surface precipitate closer to the PIL earlier in the chromosphere and have shorter displacements along z-axis (or PIL).

The bottom panels of Figure 12 show that fluxes of electrons with higher energies evolve faster and reach peaks earlier than those of lower energy electrons. Different flux peaks in the light curve of precipitating electrons with $E_x > \text{Max } E_0$ correspond to different acceleration regions and acceleration processes which have a good correspondence to the right panel of Figure 9. The timescales of these flux peaks of precipitating electrons with $E_x > \text{Max } E_0$ indicate that their acceleration timescales are less than $1.0 t_0 < 0.1$ s.
5. CONCLUSIONS AND DISCUSSION

5.1. Conclusions

Complementary to PIC simulations of particle acceleration in magnetic reconnection in the small kinetic scales (Dahlin et al. 2014 and Li et al. 2015) and in contrast to acceleration by parallel electric fields in current sheets which in MHD simulations depend on the choice of the resistivity in the Ohms law, our study focuses on the acceleration due to magnetic gradient and curvature drift effects by large-scale magnetic field structures in the cascading reconnection current sheet.

We found that both electrons trapped near the current sheets and those precipitating onto the chromosphere can be accelerated by the magnetic gradients and curvatures, but trapped electrons can get more efficient accelerations than precipitating ones due to their bouncing motions in the magnetic islands. While magnetic curvatures dominate the energizations of trapped electrons and cause strongly enhanced parallel energies (which was also found by Dahlin et al. 2014 and Li et al. 2015), on the contrary, precipitating electrons have more increased perpendicular energies than their parallel ones due to the magnetic gradient effects. Stronger magnetic curvature accelerations are located at the ends of magnetic islands together with the outflow exhausts from reconnection points which was also found by Dahlin et al. (2014) with PIC simulations. In general, the acceleration rate increases with energy. Large parallel velocities and certain magnetic connectivity are needed for electrons to reach the chromosphere, and the acceleration timescale for precipitating electrons is shorter than 0.1 s.

Due to the asymmetry in the magnetic gradient and curvature acceleration factor around the center of 2.5D current sheet, non-symmetric accelerations around pitch angles of 90° for both trapped and precipitating electrons are found. Trapped energetic electrons can contribute to the formation of bright spots along the current sheets trailing CMEs or eruptive filaments as well as the flare loop top HXR radiation by their Bremsstrahlung. Under the thin target model together with a simple Bethe–Heitler formula for the cross section of Bremsstrahlung and initial distributions function \( \propto E_0^\gamma \) and \( \propto E_0^\gamma \) and Maxwell–Boltzmann distribution with a temperature of 0.01 \( m_e c^2 \), the derived HXR spectral indices are around 5. This is already difficult enough to explain the observed HXR...
spectra in small solar flares. By comparing our electron spectrum with those of Li et al. (2015), we found magnetic gradient and curvature accelerations in large MHD scales are much weaker than those in the kinetic scales.

Precipitating electrons, on the other hand, can produce ribbons of UV and EUV brightening in the solar chromosphere but without HXR radiation since their weak acceleration from magnetic gradients and curvatures only. In the chromospheric ribbon-shape locations of precipitating electrons, electrons starting lower in the solar atmosphere precipitate closer to the PIL. The weak parallel acceleration of precipitating electron leads electrons with initial pitch angles $< 90^\circ$ precipitate to one side of the PIL, while those with initial pitch angles $> 90^\circ$ go to the other side of the PIL. Generally, there is an anti-symmetrical geometry of precipitating electron locations in the chromosphere around the PIL with a weak asymmetry that may contribute to the asymmetry of the observed emission from footpoints.

5.2. Discussions

HXR emissions of most flares mainly come from the footpoints of flare loops which implies that a large number of energetic flare electrons should precipitate into the solar chromosphere to produce the observed footpoint radiations. Occasionally, strong HXR emissions are also observed above the flare loops such as the famous Masuda flare. Due to the low density of the solar corona, HXR coronal sources imply high density of energetic electrons and efficient trapping, which may lead to a high-energy electron density comparable to that of the background plasma as shown by Krucker et al. (2008). The efficient trapping of energetic electrons near the current sheet in our study can naturally explain these observations. Magnetic mirroring and turbulence have been suggested as efficient agents for trapping of energetic electrons in flare loops. Our study here shows that the topological structure of the magnetic fields can play an important role in trapping energetic electrons near the current sheet and producing coronal HXR sources.

The trapping efficiency of our study is very high. In reality, the magnetic field evolution, turbulence scattering of particles at smaller scales, and cascade of current sheet toward smaller scales may all enhance the probability of particle precipitation and lead to the escape of energetic electrons trapped near the current sheet on a slightly longer timescale than the acceleration timescale (less than 0.1 s) of directly precipitating electrons. Moreover, particle motion will become chaotic due to nonlinear resonances between particle bounce motion and gyration when the spatial scale collapses to the kinetic one. With the transition to chaos, Buechner & Zelenyi (1989) found that trapped non-adiabatic charged particles can escape due to chaotic pitch angle scattering effects.

The time profiles of differently energized directly precipitating electrons in our calculations (bottom left panel of Figure 12) correspond well with the positive time delay among HXR pulse components in different energy channels in the Masuda flare inferred by Aschwanden et al. (1996). With our calculations, we suggest that the different HXR pulses in an energy channel could be due to different acceleration sites (X-points) of these directly precipitating electron in the solar corona (bottom panels of Figure 12).

Our results, however, still cannot explain the hard HXR spectra from solar flare footpoints which can be as hard as 1.5 in large solar flares. Besides the thick target effect in a high-density region, harder spectra may be produced by electron transport and further acceleration in flare loops. Acceleration of electrons associated with the return current in flare loops may also address the number problem of accelerated electrons that cannot be explained with the high trapping efficiency in our calculations either.

In the studies of Dahlin et al. (2014) and Li et al. (2015), accelerations from parallel electric fields were included and

Figure 11. Trajectory projection (first and second columns) and energy evolution (third column) of the most energized precipitating electron. Its trajectory is color-coded according to the local kinetic energy as those in Figure 6.
discussed. Dahlin et al. (2014) found that the efficiency of parallel electric field acceleration increases with the increase of guiding fields. Also, with a strong guiding field, parallel electric field acceleration will dominate electron energizations. Parallel electric fields in their studies exist on the scale of the electron skin depth only, which cannot be resolved with MHD simulations. In MHD simulations, parallel electric fields come from anomalous resistivity used to reflect microphysical (kinetic) effects (e.g., instabilities) on large MHD scale dynamics. In the later time steps of the AMR MHD simulations (e.g., the last panel of Figure 1), one can find the formation of a CME or plasmoid ejection (which was also observed in the Masuda flare by Shibata et al. 1995). In contrast to the evolution of magnetic islands, the formation of the CME or plasmoid occurs after the appearance of anomalous resistivity (see Báta et al. 2011 for the details of the anomalous resistivity) first turned on at $y = 40$ Mm above the loop footpoints at $t = 420 t_0$ in our MHD simulations. However the nature of this anomalous resistivity and its relation to the kinetic scale parallel electric fields needs to be clarified in the future (Buechner & Zelenyi 1989).

Our cascading reconnection current sheet model connects the coronal HXR sources (or Masuda flares) with CMEs. However, the detection of Masuda-type flares requires imaging with a high dynamical range and the appropriate projection of X-ray sources to avoid dominance by footpoint sources. The 2D geometry of our current sheet may also enhance the trapping efficiency significantly. In reality, CMEs are more easily detected than coronal HXR sources and are not always associated with large-scale 2D current sheet structures. These may explain the more frequent detection of CMEs than Masuda-type flares.

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Figure 12. Top panels: evolution of the chromosphere locations of precipitating electrons ($E_e$, black “∗” for $E_e < 54$ keV and red “∗” for $E_e > 54$ keV). Bottom left panel: light curve of the precipitating electrons for four energy ranges: $E_e < 10$ keV—dashed line, 10 < $E_e < 25$ keV—dotted line, 25 < $E_e < Max E_0$—dash-dot line, $E_e > Max E_0$—solid line (here Max $E_0$ is the maximum value of electron initial energies). Bottom right panel: average initial positions of the corresponding ones in the bottom left panel for electrons with $E_e > Max E_0$.  

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APPENDIX
VALIDATION OF THE ACCURACY BY USING THE CONSERVATION OF THE SECOND ADIABATIC INVARIANT

The conservation of the second adiabatic invariant (Northrop 1963) of trapped electrons can be used to validate the accuracy of the numerical scheme solving Equations (10)–(13):

\[ J_\parallel = m \int_a^b v_\parallel dl \simeq \text{constant.} \quad (21) \]

In Equation (21), the integral is taken along the particle guiding center trajectory between the mirror points “a” and “b.” Figure 13 shows an example electron with conserved \( J_\parallel \).

The upper left panel of Figure 13 depicts the XY-projection of the electron trajectory. Every blue asterisk in the upper right panel of Figure 13 corresponds to the mirror points. The upper right panel shows \( J_\parallel \) values along the trajectory and its red, green, and pink + points show the absolute changes of \( J_\parallel \) during a half-period (deep pink + points) or one-period (red and green + points). Three kinds of velocities (perpendicular gyration velocity—blue line, parallel velocity—red line, and drift velocity—dark line) and energy (perpendicular gyration energy—blue line, parallel energy—red line, and total kinetic energy—dark line) are separately shown in the bottom left and right panels.

Figure 13. Characteristic electron orbit indicating the conservation of the second adiabatic invariant. The upper left panel shows the XY-projection of the trajectory and the blue + points in this panel correspond to the mirror points. The upper right panel shows \( J_\parallel \) values along the trajectory and its red, green, and deep pink + points show the absolute changes of \( J_\parallel \) during a half-period (deep pink + points) or one-period (red and green + points). Three kinds of velocities (perpendicular gyration velocity—blue line, parallel velocity—red line, and drift velocity—dark line) and energy (perpendicular gyration energy—blue line, parallel energy—red line, and total kinetic energy—dark line) are separately shown in the bottom left and right panels.

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The adiabatic invariant. The bottom right panel of Figure 13 indicates that the total kinetic energy is exchanged between the parallel and the perpendicular directed motion.

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