Determination of sparticle masses and SUSY parameters

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A case study will be presented to determine the particle masses and parameters of a specific mSUGRA model at the Tesla Linear Collider with high precision.

1 Introduction

If supersymmetry will be realized in nature precise mass measurements of the particle spectrum will be very important in order to determine the underlying theory. The potential of the proposed Tesla Linear Collider will be studied within a R-parity conserving mSUGRA scenario with parameters $m_0 = 100$ GeV, $m_{1/2} = 200$ GeV, $A_0 = 0$ GeV, $\tan \beta = 3$ and $\text{sgn}(\mu) > 0$. In general SUSY signatures are fairly easy to detect. However, for the particle spectrum shown in fig. 1 the sleptons and light gauginos will be produced simultaneously at energies $\sqrt{s} > 360$ GeV and thus constitute the dominant background. The key to disentangle the different particles and their decays is a bottom-up approach with proper choices of the beam energies and polarisations.

Events are generated with *Pythia 6.115* – which also provides masses, branching ratios and cross sections – including QED radiation and beamstrahlung. Polarized cross sections are calculated with *Isajet*. The simulation is based on the detector concept of the Tesla CDR and the analyses are similar to those in ref. The one expects integrated luminosities of 500 (320) fb$^{-1}$ per 1–2 years at energies of $\sqrt{s} = 500$ (320) GeV. It is assumed that both beams are polarized, $P_{e-} = 0.8$ and $P_{e+} = 0.6$, and the luminosity is equally shared between $e_L e_R^-$ and $e_R e_L^+$ runs.

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**Figure 1:** Mass spectrum and decay modes of sleptons and light gauginos
2 Mass determinations

SUSY particles are produced in pairs and decay either directly or via cascades into the stable neutralino $\chi^0_1$ (LSP). Typical signatures are multi-lepton and/or multi-jet final states with large missing energy. The kinematics of the decay chain allow to identify and to reconstruct the masses of the primary and secondary sparticles.

Production of sleptons

The simplest case is $e^-_R e^+_R \rightarrow \tilde{\mu}_R \tilde{\mu}_R \rightarrow \mu^- \chi^0_1 \mu^+ \chi^0_1$ at $\sqrt{s} = 320$ GeV with a small background from $\chi^0_2 \chi^0_1$ production. The energy spectrum of the decay muons is flat, see fig. 2, and the end points can be related to the masses $m_{\tilde{\mu}_R}$ and $m_{\chi^0_1}$ with an accuracy of $\sim 0.3\%$. The partner $\tilde{\mu}_L$ can be identified at $\sqrt{s} = 500$ GeV via a unique 6$l^\pm$ signature: $e^-_L e^+_R \rightarrow \tilde{\mu}_L \tilde{\mu}_L \rightarrow \mu^- \chi^0_2 \mu^+ \chi^0_2$ followed by $\chi^0_2 \rightarrow l^\pm l^- \chi^0_1$. Despite the low cross section of 4 fb such a measurement is feasible at TESLA, giving the masses $m_{\tilde{\mu}_L}$ and $m_{\chi^0_2}$ with a precision of $\sim 0.2\%$, see fig. 2. A final example is sneutrino production, where the flavour is tagged via its charged decay, e.g. $e^-_L e^+_R \rightarrow \tilde{\nu}_e \tilde{\nu}_e \rightarrow e^- \chi^+ \chi^- \chi^0_1$. The subsequent decays $\chi^\pm \rightarrow l^\pm \nu \chi^0_1$ and $q \bar{q}' \chi^0_1$ lead to a clean $3l^\pm + 2j$ topology and can be used to determine the chargino-neutralino mass difference very accurately within 0.05 GeV (fig. 2). The visible cross section is huge, $\sigma_{LR}(\tilde{\nu}_e \tilde{\nu}_e) B = 320$ fb compared to $\sigma_{LR}(\tilde{\nu}_\mu \tilde{\nu}_\mu) B = 5$ fb, giving mass resolutions of $\delta m_{\tilde{\nu}_e} = \delta m_{\chi^\pm} = 0.1$ GeV.

![Figure 2: Examples of slepton production. Lepton energy spectra of $\tilde{\mu}_R \rightarrow \mu \chi^0_1$ at 320 GeV (upper left), $\tilde{\mu}_L \rightarrow \mu \chi^0_2$ at 500 GeV (upper right) and $\tilde{\nu}_e \rightarrow e^\pm \chi^\pm_1$ at 500 GeV (lower left). Di-jet mass spectrum of $\chi^\pm \rightarrow q \bar{q}' \chi^0_1$ (lower right).](image)

Production of neutralinos and charginos

The lightest observable neutralino
can be detected via its 3-body decay $\chi_2^0 \rightarrow l^+l^-\chi_1^0$. In a direct production the energy spectrum of the di-lepton system can be used to determine the masses of the primary and secondary neutralino, similar to the slepton case. But $\chi_0^0$'s are also abundantly produced in decay chains and from the upper edge of the di-lepton mass spectrum one gets a very precise measurement of the mass difference $\Delta m(\chi_1^0 - \chi_2^0) = 58.6 \pm 0.05$ GeV, essentially only limited by systematics. Similarly, the copiously produced charginos with decays $\chi_1^\pm \rightarrow q\bar{q}\chi_1^0$ give $\Delta m(\chi_1^+ - \chi_1^0) = 55.8 \pm 0.05$ GeV. The lowest mass neutralino production $e_R^+e_L^- \rightarrow \chi_2^0\chi_1^0$ should be studied below $\hat{t}_R$ threshold, the polarisation suppresses $W^+W^-$ background. The reaction $e_L^+e_R^- \rightarrow \chi_2^0\chi_2^0 \rightarrow 4l^\pm$ at $\sqrt{s} = 320$ GeV, spectra shown in fig. is background free and gives a mass resolution of $\delta m_{\chi_2^0} = 0.3$ GeV. Another example is chargino production $e_L^+e_R^- \rightarrow \chi_1^-\chi_1^+ \rightarrow l^\pm\nu\chi_1^0 q\bar{q}\chi_1^0$ with a small $W^+W^-$ contamination of $<10\%$, see fig. The large visible cross section, $\sigma_{LR}B = 330$ fb, compensates for the worse jet energy resolution and results in an accuracy of $\delta m_{\chi_1^\pm} = 0.2$ GeV.

Figure 3: Di-lepton mass and energy spectra of $\chi_2^0 \rightarrow l^+l^-\chi_1^0$ at 320 GeV (left part) and di-jet mass and energy spectra of $\chi_1^- \rightarrow q\bar{q}\chi_1^0$ at 320 GeV (right part).

Figure 4: Visible cross sections near threshold of the reactions $e_R^+e_L^- \rightarrow \tilde{\mu}_R\tilde{\mu}_R$ (left) and $e_L^+e_R^- \rightarrow \chi_1^-\chi_1^+$ (right). Measurements assume $\mathcal{L} = 10$ fb$^{-1}$ per point.

**Threshold scans** Further improvement on sparticle masses may be achieved through threshold scans. Such measurements are relatively simple, they essentially
count additional signatures over a smooth background. Scans have been simulated by assuming a luminosity of 100 fb$^{-1}$ distributed over 10 equidistant points, this procedure is not optimized. The cross section for gaugino pair production rises $\propto \beta$, while the onset of slepton pair production is slower $\propto \beta^3$, see excitation curves in fig. 4. Excellent mass resolutions below 100 MeV can be obtained for the light charginos/neutralinos, degrading for the heavier $\chi$ states to the per mil level. Similar precisions can be obtained for the first generation sleptons and right smuon. Higher mass states of the second and third generation suffer from low detectable cross sections, but still accuracies of a few per mil are achievable.

The results of the various mass measurements using different techniques are compiled in table 1.

| particle  | mass [GeV] | $\delta m_{cont}$ [GeV] | $\delta m_{scan}$ [GeV] | SUSY parameters |
|-----------|------------|-------------------------|-------------------------|----------------|
| $\tilde{\mu}_R$ | 132.0      | 0.3                     | 0.09                    | $m_0, m_{1/2}, \tan \beta$ |
| $\tilde{\mu}_L$ | 176.0      | 0.3                     | 0.4                     |                |
| $\tilde{\nu}_\mu$ | 160.6      | 0.2                     | 0.8                     |                |
| $\tilde{e}_R$ | 132.0      | 0.2                     | 0.05                    |                |
| $\tilde{e}_L$ | 176.0      | 0.2                     | 0.18                    |                |
| $\tilde{\nu}_e$ | 160.6      | 0.1                     | 0.07                    |                |
| $\tilde{\tau}_1$ | 131.0      | 0.6                     |                         | $m_0, m_{1/2}, \mu, \tan \beta$ |
| $\tilde{\tau}_2$ | 177.0      | 0.6                     |                         |                |
| $\tilde{\nu}_\tau$ | 160.6      | 0.6                     |                         |                |
| $\chi^\pm_1$ | 127.7      | 0.2                     | 0.04                    | $M_2, \mu, \tan \beta$ |
| $\chi^\pm_2$ | 345.8      |                         | 0.25                    |                |
| $\chi^0_1$ | 71.9       | 0.1                     | 0.05                    | $M_1, M_2, \mu, \tan \beta$ |
| $\chi^0_2$ | 130.3      | 0.3                     | 0.07                    |                |
| $\chi^0_3$ | 319.8      |                         | 0.30                    |                |
| $\chi^0_4$ | 348.2      |                         | 0.52                    |                |

### 3 Determination of SUSY parameters

The precise mass measurements of sleptons, neutralinos and charginos constitute an over-constrained set of observables which allow to determine the structure and parameters of the underlying SUSY theory. For example slepton universality can be readily tested to better than a percent. The sensitivity of the particle masses to the MSSM or mSUGRA parameters is indicated in table 1.

In a first test the mSUGRA model is assumed, which is based on the parameters $m_0$, $m_{1/2}$, $A_0$, $\tan \beta$ and sign $\mu$. The renormalisation group equations (RGE) are applied to extrapolate the masses to their common values at the GUT scale, where
the couplings of the gauge groups are related via \( M_i/\alpha_i = m_{1/2}/\alpha_{GUT} \). Using the masses and their errors a \( \chi^2 \) fit is performed to determine in a top-down approach the errors of the mSUGRA parameters assuming the sign of \( \mu \) to be known. The magnitude of \( \mu \) is obtained implicitly by the requirement of electroweak symmetry breaking. The common scalar and gaugino masses \( m_0 \) and \( m_{1/2} \) can be determined very precisely to better than a per mil, \( \tan \beta \) to better than a percent and there is even some sensitivity to the trilinear coupling \( A_0 \). While much of the accuracy on \( m_0 \) and \( m_{1/2} \) can be obtained using particle production which occur below thresholds of about 270 GeV for this model, the accuracy on \( A_0 \) and \( \tan \beta \) requires using thresholds up to about 400 GeV.

In a second fit the GUT relation between \( M_1 \) and \( M_2 \) is relaxed, so that the same model is assumed under less constrained conditions. As result \( M_1 \) and \( M_2 \) as well as \( m_0 \) can still be determined within a per mil accuracy. The other parameters \( \tan \beta \) and \( A_0 \) have slightly enlarged uncertainty.

The results of both fits are summarized in the following table. It is obvious that the inclusion of cross section measurements will improve these results.

| parameter | true value | error  | parameter | true value | error  |
|-----------|------------|--------|-----------|------------|--------|
| \( m_0 \) | 100 GeV    | 0.09 GeV | \( m_0 \) | 100 GeV    | 0.09 GeV |
| \( m_{1/2} \) | 200 GeV | 0.10 GeV | \( M_1 \) | 200 GeV    | 0.20 GeV |
| \( A_0 \) | 0 GeV     | 6.3 GeV | \( A_0 \) | 0 GeV      | 10.3 GeV |
| \( \tan \beta \) | 3       | 0.02   | \( \tan \beta \) | 3         | 0.04   |
| \( \text{sgn}(\mu) \) | +       | not fit | \( \text{sgn}(\mu) \) | +         | not fit |

4 Conclusions

The high luminosity of TESLA allows to disentangle the production and (cascade) decays of the accessible SUSY particle spectrum. Polarisation of both \( e^- \) and \( e^+ \) beams is extremely important for optimizing signal/background ratios. Precision measurements of SUSY particle properties are achievable within reasonable time, e.g. masses with an accuracy of \( \delta m \leq 0.3 \) GeV in the continuum and \( \delta m \leq 0.1 \) GeV at threshold. Such measurements can be used in a RGE analysis of the mass spectrum to determine the underlying SUSY model and parameters very accurately.

References

1. 'Conceptual Design of a 500 GeV \( e^+e^- \) Linear Collider', eds. R. Brinkmann et al., DESY 1997-048 and ECFA 1997-182.
2. E. Accomando et al., Phys. Rep. 299 (1998) 1.
3. T. Sjöstrand, Comp. Phys. Comm. 82 (1994) 74.
4. T. Ohl, Comp. Phys. Comm. 101 (1996) 269.
5. F. Paige et al., [hep-ph/9804321].