Can Conformally Invariant Modified Gravity Solve The Hubble Tension?

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The discrepancy between early-Universe and direct measurements of the Hubble constant, known as the Hubble tension, recently became a pressing subject in high precision cosmology. As a result, a large variety of theoretical models have been proposed to relieve this tension. In this work we analyze a conformally-invariant modified gravity (CIMG) model of an evolving gravitational constant due to the coupling of a scalar field to the Ricci scalar, which is theoretically advantageous as it has only one free parameter and its influence is concentrated around matter-radiation equality, as required for solutions to the Hubble tension based on increasing the sound horizon at recombination. Inspired by similar recent analyses of so-called early-dark-energy models, we constrain the CIMG model using a combination of early and late-Universe cosmological datasets. In addition to the Planck 2018 cosmic microwave background (CMB) anisotropies and weak lensing measurements, baryon acoustic oscillations and the Supernova H0 for the Equation of State datasets, we also use large-scale structure (LSS) datasets such as the Dark Energy Survey year 1 and the full-shape power spectrum likelihood from the Baryon Oscillation Spectroscopic Survey, including its recent analysis using effective field theory, to check the effect of the CIMG model on the (milder) $S_8$ tension between the CMB and LSS. We find that the CIMG model can slightly relax the Hubble tension, with $H_0 = 69.6 \pm 1.6$ km/s/Mpc at 95% CL, while barely affecting the $S_8$ tension. However, current data does not exhibit strong preference for CIMG over the standard cosmological model. Lastly, we show that the planned CMB-S4 experiment will have the sensitivity required to distinguish between the CIMG model and the more general class of models involving an evolving gravitational constant.

I. INTRODUCTION

The standard Λ cold-dark-matter (CDM) cosmological model has been tested by numerous probes and has provided a remarkable explanation for cosmological observations such as the cosmic microwave background (CMB) anisotropies and the baryon acoustic oscillations (BAO). However, despite the immense successes of the ΛCDM model, there has been a growing discrepancy between the measured values of the Hubble constant $H_0$—the current expansion rate of the Universe—as inferred from early-Universe probes, which assume the ΛCDM model, and late-Universe probes, which do not assume such a model.

Most of the late-Universe measurements constrain the value of $H_0$ by applying the distance-ladder method [1]. This method uses parallax measurements to characterize nearby stars (e.g., Cepheid-variable stars, “tip of the red giant branch” stars, Miras, etc.), which are then used to calibrate the luminosity of nearby Type-Ia supernovae (SNe), allowing distant SNeIa to be used to estimate the Hubble flow. The Supernova H0 for the Equation of State (SH0ES) collaboration, which uses Cepheids, recently obtained $H_0 = 74.03 \pm 1.42$ km/s/Mpc [2]. Other distance-ladder measurements lead to other values, most of them in rough agreement with SH0ES.

The measurement of CMB anisotropies, assuming the ΛCDM model, allows an indirect inference of the Hubble constant. Inferring the angular size of the sound horizon and constraining the matter and baryon energy densities directly from the CMB temperature, polarization and lensing power spectra, allows the Planck 2018 collaboration to determine $H_0 = 67.36 \pm 0.54$ km/s/Mpc [3]. A similar early-Universe approach can be taken using a combination of measurements without including CMB anisotropies: Big Bang nucleosynthesis (BBN); BAO; the FIRAS CMB global temperature and late-Universe measurement (e.g., galaxy-lensing based) of the matter density. The result of such analyses agrees quite precisely with that of the CMB [4]. The value of $H_0$ inferred from the CMB measurements is in 4.8σ tension with the value reported by SH0ES, and a similar discrepancy is present in the majority of the $H_0$ values inferred from other variations of early and late Universe measurements [4].

Various theoretical solutions were hitherto suggested to solve the $H_0$ discrepancy, which can crudely be divided into two approaches: pre-recombination and post-recombination solutions. A recent review of the Hubble tension [5] argued that the pre-recombination solutions are more likely to work, mainly due to the fact that post-recombination solutions affect only the inferred value of $H_0$, while the combined data from BAO and local $H_0$ measurements implies that a reduction of the sound horizon at last scattering is required as well (see, however, Ref. [6]). In particular, it was argued that the critical epoch for achieving such reduction of the sound horizon takes place just prior to recombination. An increasing number of models aim to realize such solutions.

Recent analyses [7–11] of a popular subclass of these models, referred to as “early dark energy” (EDE) [12–20], showed that while they reduce the $H_0$ discrepancy, full cosmological concordance is not restored due to their tendency to increase the $S_8$ discrepancy between CMB and large-scale-structure observables, as described below.

In this work we focus on another model suggested to resolve the $H_0$ tension, based on a subclass of scalar-tensor theory. This modified gravity (MG) family of models [21–24], implemented by the coupling of a homogeneous scalar field to the Ricci scalar, acts to increase...
Newton’s gravitational constant $G$ prior to matter-radiation equality $z_{eq}$ (that takes place just before recombination), which increases the Hubble parameter (i.e. the expansion rate) prior to recombination. The increase in $H(z)$ prior to recombination reduces the sound horizon $r_s$ and increases the inferred value of $H_0$. The scalar field, initially frozen at some initial value, subsequently decays to zero, lowering the value of $G_N$ to its current value during post-recombination era. We emphasize that this “natural” occurrence is in contrast to what happens in EDE models, where a fine-tuned parameter $z_c$ determines when the EDE component becomes dominant.

The MG model is parameterized by the initial value of the field $\phi_i$ and the coupling constant $\xi$. Together, these parameters determine the deviation in Newton’s constant $\Delta G_N \approx -\xi \phi_i^2 / M_P^2$. We will focus here on a special case of a conformally-invariant (CI) MG model, for which $\xi \equiv -1/6$ is fixed. The CIMG model thus introduces only one additional parameter, $\phi_1$, compared to ΛCDM (and two fewer than the popular EDE model).

Although it offers a more natural and simple realization of the solution to the $H_0$ discrepancy, the CIMG model exhibits most of the deficiencies manifested in the EDE models, however to a lower extent. In EDE models, some of the ΛCDM parameters shift significantly in order to preserve the fit to the CMB data, while the CIMG model tends to more delicate shifts of these parameters.

The increase in the Hubble parameter generally acts to slightly suppress the the growth of perturbations, for the modes within the sound horizon, during the period of enhanced expansion. In the EDE scenario this suppression forces a shift upward in $\Omega_m h^2$, so as to compensate for the loss of efficiency in the perturbations growth, while $n_s$ shifts upwards due to the localized contribution of the EDE component to the expansion rate, as detailed in Ref. [7]. On the other hand, the increase in the gravitational strength, in the CIMG scenario, already acts to compensate for this loss and therefore allows a smaller shift in the value of $\Omega_m h^2$ [21, 22]. Furthermore, since the deviation in $G_N$ under the CIMG model is not as localized in redshift-space as the dominant period of the EDE component, $n_s$ also does not shift as much. Another impact of CIMG, due to the stronger gravitation, is the downwards shift of the matter density $\Omega_m$ which reduces significantly, compared to the EDE scenario.

The increases in $\Omega_m h^2$ and $n_s$ increase the late-time amplitude of the density fluctuations $\sigma_8$, aggravating the current (mild) tension between LSS and CMB inferences of this parameter. We follow the convention in Ref. [25] to quantify the parameter shifts and the associated LSS-CMB tension by the combination of the parameters defined as $S_8 \equiv \sigma_8 (\Omega_m/0.3)^{0.5}$, where $\sigma_8$ is the RMS mass fluctuations in a 8Mpc/h at $z = 0$. LSS experiments [26–29] place a combined constraint of $S_8 = 0.770^{+0.018}_{-0.010}$, which is in about 2.7σ tension with the Planck 2018 CMB result. The results of the analysis of the EDE model in Ref. [7], which considered joint CMB-LSS constraints, showed that the EDE model may increase the tension in up to 35%, compared to the ΛCDM model. We will see that the effect on $S_8$ of the CIMG model is much weaker.

In this work we consider the constraints on the CIMG model from different data sets composed of CMB, LSS and $H_0$ measurements. We find that overall the one-parameter CIMG model exhibits similar properties to those of the three-parameter EDE model, only more moderate. It allows a smaller increase in the value of $H_0$ at the cost of much smaller increase in the value of $S_8$. The CIMG model is not favorable to ΛCDM by Planck primary-CMB data alone, but the inclusion of CMB lensing + BAO + redshift-space distortions (RSD) + SN1a + SH0ES datasets results in a detectable CIMG component (i.e. a non-zero $\phi_1$). When using the EFT-based LSS constraints we find an even more significant CIMG component which results in a better fit to SH0ES data without worsening the fits to CMB and LSS datasets, compared to ΛCDM.

We conclude with a simple Fisher analysis to forecast the constraints on the CIMG model from the planned CMB-S4 experiment. In particular we show that it will be able to constrain $\xi \sim -1/6$ to high accuracy, thus distinguishing the CIMG model from other MG models.

II. MODEL

The inference of $H_0$ from CMB measurement requires the determination of three parameters: the sound horizon $r^*_s$, the angular distance distance $D^*_\Lambda$ and the angular acoustic scale $\theta^*$, where “$\Lambda$” denotes last-scattering. These are related by $\theta^* \equiv r^*_s / D^*_\Lambda$, which is measured by Planck 2018 to about 0.03% precision [3]. Thus, any modified evolution of $H(z)$ must accommodate the fixed ratio between $r^*_s$ and $D^*_\Lambda$. The sound horizon at last-scattering

$$r^*_s = \int_{z_\Lambda}^{\infty} \frac{dz}{H(z)} c_s(z),$$

depends on the evolution of $H(z)$ prior to recombination, while the angular diameter distance

$$D^*_\Lambda = \int_0^{\infty} \frac{dz}{H(z)}$$

depends on its evolution post-recombination (and can be used to set $H_0$). Writing the expression for $\theta^*$ explicitly:

$$\theta^* \equiv \frac{\int_{z_\Lambda}^{\infty} \frac{dz}{\sqrt{G_N(z) \sqrt{\rho_{\text{crit}}(1+z)^3 + \rho_m(1+z)^3 + \rho_\phi}}} c_s(z)}{\int_0^{\infty} \frac{dz}{\sqrt{G_N(z) \sqrt{\rho_{\text{crit}}(1+z)^3 + \rho_m(1+z)^3 + \rho_\phi}}}},$$

it is easy to see how an increase in the value of $G_N$ or an introduction of a new dominant contribution to the energy density budget (as EDE models suggest), prior to recombination, acts to reduce the sound horizon while increasing $H(z)$. The CIMG model introduces a scalar field with non-minimal-coupling (NMC), causing an upward
shift in the value of Newton’s gravitational constant $G_N$ prior to recombination. Around matter-radiation equality the field becomes dynamic and decays, reducing Newton’s constant back to its fiducial value. The increase in the gravitational strength enhances the growth of $H(z)$ during this period, which in turn reduces $r^*_s$ and raises the inferred value of $H_0$. Naively speaking, a deviation of about 15% in the value of $G_N$ (while keeping all the other parameters fixed and neglecting the additional energy component) is enough to reduce $r^*_s$ by 7%, as was suggested in Ref. [5].

The MG model [30, 31] we consider is described by the action

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ \frac{F}{2} R + \partial_\mu \phi \partial^\mu \phi + \mathcal{L}_m \right],$$

where $F(\phi) \equiv M_p^2 \left(1 + \xi \frac{\dot{\phi}^2}{2}\right)$ is the effective Planck mass (i.e. the NMC to the Ricci scalar $R$) and $\mathcal{L}_m$ is the Lagrangian density describing the remaining contents of the Universe. The field $\phi$ is coupled to the Ricci scalar through a dimensionless coupling constant $\xi$, while in the special case of CIMG we fix $\xi = -1/6$, for which the action is invariant under conformal transformations [32] and the number of additional parameters to the $\Lambda$CDM model reduces from two ($\xi$ and $\phi_i$) to one. The dynamics of the field $\phi$ are determined by the equation of motion

$$\ddot{\phi} + 3H \dot{\phi} - \xi R \phi = 0.$$  

We demand that $\xi < 0$, therefore as long as $R \ll H$, the field remains frozen at its initial value $\phi_i$.

The evolution of the Ricci scalar can be derived from Einstein’s equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}.$$  

Since the trace of the stress-energy tensor $T_{\mu\nu}$ vanishes for radiation-like components and is equal to $\rho_m$ for matter-like components, by taking the trace of Eq. (6) we find that $R \propto \rho_m$. Therefore, the Ricci scalar is practically zero—compared to $H^2$—during the radiation-dominated (RD) era. Thus the non-minimally coupled field becomes dynamic only around matter-radiation equality when it acquires an effective mass $m^2_{\phi} \sim \xi R \sim \xi H^2$. Then it begins to roll towards its minimum value, as shown in Fig. 1. From the Friedmann equation

$$3FH^2 = \rho + \frac{\dot{\phi}^2}{2} + \Lambda - 3\dot{F}H \equiv \rho + \rho_\phi,$$

we may associate the extra terms as the energy density of the field $\phi$, so that the energy density of the field,

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 - 6\xi H \dot{\phi} \phi - 3\xi H^2 \dot{\phi}^2,$$

scales as $a^{-4.5}$ during the matter-dominated (MD) era, and thus dilutes faster than radiation.

Because of the NMC, Newton’s constant is replaced by an effective Newton constant $G'_N \equiv (8\pi F)^{-1}$, and the deviation from General Relativity (GR) can be parameterized by $\Delta G_N \equiv (G'_N/G_N - 1)$. The deviation from GR can also be parameterized by means of the so-called Post-Newtonian (PN) parameters [33]

$$\gamma_{PN} = 1 - \frac{F^2}{F + 2F^2}, \quad \beta_{PN} = 1 + \frac{FF^2}{8F + 12F^2} \frac{d\gamma_{PN}}{d\phi},$$

where the prediction from GR, i.e. $\gamma_{PN} = \beta_{PN} = 1$, is tightly constrained by Solar System experiments, as $\gamma_{PN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ and $\beta_{PN} - 1 = (4.1 \pm 7.8) \times 10^{-5}$ at 68% CL [34, 35]. Other studies of a varying cosmological constant place constraints on its value and its variation rate [36, 37]. Recent analysis, Ref. [21], showed that a deviation of about 2% in $G_N$ at early times is enough to raise the Hubble constant to a value of $H_0 = 70.56$ km/s/Mpc, without conflicting with Solar System measurements.
III. METHODOLOGY AND DATASETS

We implement the CIMG model by modifying the public code for scalar-tensor theories hi-class [38, 39], which in turn is based on the public Boltzmann code CLASS [40, 41]. In particular, we modified the Brans-Dicke model to match the CIMG model by replacing the Brans-Dicke related $G_2$ and $G_4$ functions with $G_2 = X - \Lambda$ and $G_4 = (1 + \xi \phi^2)/2$ (and $G_3 = G_5 = 0$), and their corresponding derivatives, where $X \equiv -\partial_\phi \phi^0 \phi^i / 2$ and $\Lambda$ plays the role of the cosmological constant.

We also added an extraction of $f_s(z)$ (taken from the public code CLASS-EDF [42]), where $f \equiv d \log D / d \log a$ is the linear growth rate which is needed for implementing the RSD likelihoods in our analyses. In all likelihoods requiring calculations of the non-linear matter power spectrum, we used the “Halofit” prescription implemented in CLASS. We followed the analyses in [7], preforming Markov-Chain-Monte-Carlo (MCMC) analyses, sampling the posterior distributions using the Metropolis-Hasting algorithm [43-45], implemented in the public code Cobaya [46], with Gelman-Rubin [47] convergence criteria $R - 1 < 0.05$. We used a uniform prior for the CIMG parameter $\phi_i = [0, 0.005, 1]$ and Gaussian priors for the ACME cosmological parameters, centered around the ACME fiducial values. For the dataset combination which includes the EFT of LSS, we used MontePython [48, 49] for the MCMC analysis, along with the public code PyBird [9], which implements the EFT likelihood. We also used the public code GetDist [50] to analyze the MCMC chains: extract the best-fit parameters, mean values and errors, and plot the correlations in parameter-space as well as the maximized posteriors.

We used the same datasets used in Ref. [7], which include: Planck 2018 CMB temperature and polarization anisotropies power spectra (TT + TE + EE) and the CMB lensing (P18 and lensing) [3], Baryonic Acoustic Oscillations (BAO) [51-54], redshift-space distortion from SDSS BOSS DR12 (RSD) [51, 55], Type-Ia Supernovae (SNIa) [56], SH0ES 2019 $H_0$ measurements [2] and the Dark Energy Survey Year 1 (DES Y1) [26, 57], with the exception of additional LSS data from the Ki-Degree Survey [27, 28] (KiDS) and the Hyper Suprime-Cam [29] (HSC) survey, which we used only as reference. We also included an additional dataset, comprising of the effective field theory (EFT) of LSS [9, 58-60] applied to the full shape power spectrum of the BOSS/SDSS galaxies clustering DR12 [61, 62] and the BAO post-reconstruction measurements from BOSS, combined with covariance between EFT-BOSS and anisotropic BAO analysis. We did not use the south galactic cap (SGC) field of LOWZ, as in Ref. [59]. This dataset is henceforth referred to as BAO+EFT (not to be confused with the BAO dataset without EFT).

IV. RESULTS

We adopt the Planck convention, holding the sum of neutrino masses fixed to 0.06 eV, with one massive eigenstate against two massless eigenstates, and we fix the effective number of relativistic species to $N_{\text{eff}} = 3.046$. We also fit the $\Lambda$CDM model to each data set as a benchmark for comparison. A summary of the results is tabulated in Table IX.

A. CIMG Meets Primary CMB Alone

The first analysis we performed includes only the temperature and polarization anisotropies power-spectrum data from Planck 2018. While there is a small contribution to this dataset from LSS, due to gravitational lensing of the power spectra, the overall constraints are dominated by information from the recombination epoch. This analysis tests for evidence for the CIMG model using early-Universe data alone.

The results in Table I (see also Table IX) show very weak evidence for the CIMG model. The single CIMG parameter is constrained by an upper bound $\phi_i < 0.213 M_P$ (also indicated by the posterior shown in Fig. 4) and $\Delta G_N$ comprises 0 within 1σ. It seems that primary CMB data alone does not prefer the CIMG model over the standard $\Lambda$CDM model, and indeed the shift in the cosmological parameters is negligible (below 1σ of the $\Lambda$CDM benchmark). We also note that introducing an additional parameter beyond $\Lambda$CDM to the fit in the CIMG parameter model does not improve the fit as one might expect. On the contrary, we find $\Delta \chi^2 = +4.8$, as shown in Table II. We conclude that there is no preferred region, compared to the $\Lambda$CDM model, within the CIMG parameter space when considering the primary CMB data alone.

B. Expanding the analysis to also include CMB lensing, BAO, RSD, SNIa, and SH0ES

Following previous analyses [7, 13] of the EDE model, we include Planck 2018 CMB lensing, BAO, RSD, supernova, and local distance-ladder data in SH0ES 2019. This data set is considered a conclusive combination of early-Universe, LSS and SNeIa distance-ladder data.

We now find significant evidence for the CIMG model, with this combination of datasets. The initial value of the field, $\phi_i = 0.3^{+0.14}_{-0.24} M_P$ at 95% CL, and the fractional deviation of Newton’s constant, $\Delta G_N = 1.7 \pm 1.7$, both at 95% CL, are detected at $\geq 2\sigma$. As a result, the value of $H_0$ increases to $H_0 = 69.24 \pm 1.4 \mathrm{km/s/Mpc}$,
Table I. The mean (best-fit) ±1σ (68% CL) constraints on the cosmological parameters in ΛCDM and CIMG as inferred from Planck 2018 primary CMB (TT+TE+EE) data alone. The CIMG component is not significant when considering early-Universe information (gravitational lensing have a little influence as well).

| Parameter | ΛCDM | CIMG |
|-----------|------|------|
| \(\log(10^{19} A_s)\) | 3.044 (3.046) ± 0.015 | 3.047 (3.035) ± 0.016 |
| \(n_s\) | 0.9645 (0.9618) ± 0.0042 | 0.9667 (0.9638) ± 0.0045 |
| \(100\theta_s\) | 1.04185 (1.0419) ± 0.00029 | 1.04189 (1.04151) ± 0.00030 |
| \(100 \times \Omega_b h^2\) | 2.235 (2.23284) ± 0.014 | 2.238 (2.23162) ± 0.015 |
| \(\Omega_c h^2\) | 0.1202 (0.1210) ± 0.0013 | 0.1200 (0.1206) ± 0.0014 |
| \(\sigma_8\) | 0.0540 (0.0547) ± 0.0075 | 0.0547 (0.0489) ± 0.0077 |
| \(\phi_i [M_P]\) | – | < 0.213 (0.087) |
| \(H_0 \,[\text{km/s/Mpc}]\) | 67.30 (66.98) ± 0.58 | 67.98 (67.11) ± 0.64 |
| \(\Omega_m\) | 0.3162 (0.3210) ± 0.0081 | 0.310 (0.319) ± 0.012 |
| \(\sigma_8\) | 0.8112 (0.8142) ± 0.0072 | 0.8170 (0.8094) ± 0.022 |
| \(S_8\) | 0.833 (0.842) ± 0.016 | 0.830 (0.834) ± 0.016 |
| \(\Delta G_N (\%)\) | – | 0.68 (0.13) ± 0.14 |

Table II. \(\chi^2\) values for the best-fit ΛCDM and CIMG models, constrained by primary CMB alone. The additional parameter of the CIMG model does not improve the fit to the data as may be expected when increasing the total number of parameters.

\(\chi^2\) statistics from Planck 2018 data only: TT+TE+EE

| Datasets | ΛCDM | CIMG |
|----------|------|------|
| Planck 2018 low-\(\ell\) TT | 24.1 | 23.4 |
| Planck 2018 low-\(\ell\) EE | 396.3 | 395.7 |
| Planck 2018 high-\(\ell\) TT+TE+EE | 2345.1 | 2351.2 |
| Total \(\chi^2\) | 2765.5 | 2770.3 |

Compared to the ΛCDM benchmark, \(H_0 = 68.17 \pm 0.77\) km/s/Mpc at 95% CL. The LSS likelihoods of RSD and BAO have large enough error bars to overlap with the region in parameter space with larger value of \(H_0\), reducing the Hubble tension to 3.1σ. This is due to both the increase in \(H_0\) and its increased errors, emphasising the difficulty of reconciling all the likelihoods in the dataset.

In order to keep the fit with CMB and LSS data, other cosmological parameters shift as well. In particular, \(\Omega_c h^2\) and \(n_s\) shift upwards slightly, a 0.7σ and 0.4σ discrepancy with the benchmark, respectively. The degeneracy between \(H_0\) and \(\Omega_c h^2\) breaks due to the introduction of a new energy density component of the CIMG field, while its degeneracy with \(n_s\) increases (see Fig. 2). We also note a minuscule downward shift in \(\Omega_b h^2\), 0.3σ discrepancy with the benchmark. In addition we find an increase in the value of \(\sigma_8\) and a decrease in \(\Omega_m\), for which the net result is a minor increase in \(S_8\), which translates to a slightly larger tension with the combined LSS constraint: 2.4σ, compared to 2.2σ for the ΛCDM benchmark.

The CIMG component acts to increase the early-Universe expansion rate, as \(G_N^* > G_N\), thus suppressing the matter power spectrum \(P(k)\) for modes within the sound horizon. This suppression requires an upward shift in \(\Omega_c h^2\), which is the driver of the changes in \(P(k)\), translated to a larger \(\sigma_8\). Moreover, as the enhanced expansion of the Universe is localized in time, the shift in the matter power spectra is scale dependent, affecting the value of \(n_s\). Such behavior is expected, to some extent, for every model that acts to increase \(H_0\) in such manner.

Compared to a recent analysis of the EDE model in Ref. [7], the CIMG model exhibits smaller shifts of the cosmological parameters (including \(H_0\)). Since the CIMG component is less localized in time, its free parameter \(\phi_i\) is less correlated with \(n_s\) than the corresponding parameters in the EDE scenario. In addition, the enhanced gravitational strength acts to boost density anisotropies, which counteracts the need for increasing \(\Omega_c h^2\).

As a result, the matter density \(\Omega_m\) is reduced, due to the increased Hubble parameter and the almost unchanged value of the CDM density. That is in contrast to the EDE model, which exhibits a significant increase in \(\Omega_c h^2\) and no shift downwards in \(\Omega_m\), which results in a higher \(S_8\). Therefore the CIMG model offers to relax the \(H_0\) tension, although not as much as the EDE model, but almost without worsening the CMB-LSS tension, when quantified in terms of the well-constrained \(S_8\) parameter.

We find that the additional CIMG parameter improves the total fit to the data, with \(\Delta \chi^2 = -9\), relative to the ΛCDM benchmark, as shown in Table IV. The reduction in \(\chi^2\) is mainly due to the better fit to the SH0ES likelihood which compensates for the degraded fit to the CMB datasets, while the fit to LSS data worsens only slightly, indicating the intrinsic tension between the datasets.

The different shifts in \(H_0\) and \(S_8\) values indicate stronger correlation of the EDE component \(f_{\text{EDE}}\) with \(H_0\) and \(\sigma_8\) than that of the CIMG parameter \(\phi_i\) shown.
Constraints from Planck 2018 TT+TE+EE + CMB Lensing, BAO, RSD, SNIa and SH0ES

| Parameter                  | ΛCDM               | CIMG               |
|----------------------------|--------------------|--------------------|
| $\log(10^{10} A_s)$        | 3.051 (3.054) $^{+0.013}_{-0.013}$ | 3.051 (3.060) $^{+0.014}_{-0.014}$ |
| $n_s$                      | 0.9689 (0.9691) $^{+0.0035}_{-0.0035}$ | 0.9711 (0.9727) $^{+0.0038}_{-0.0038}$ |
| $100\theta_0$             | 1.04204 (1.04187) $^{+0.00027}_{-0.00027}$ | 1.04199 (1.04212) $^{+0.00028}_{-0.00028}$ |
| $100 \times \Omega_b h^2$ | 2.253 (2.253) $^{+0.013}_{-0.013}$ | 2.247 (2.24705) $^{+0.013}_{-0.013}$ |
| $\Omega_c h^2$            | 0.1183 (0.1185) $^{+0.0009}_{-0.0009}$ | 0.1193 (0.1192) $^{+0.0010}_{-0.0011}$ |
| $\tau_{reio}$             | 0.0503 (0.0618) $^{+0.0065}_{-0.0075}$ | 0.0565 (0.0601) $^{+0.0073}_{-0.0067}$ |
| $\phi_i [M_P]$            | $-$                | 0.297 (0.297) $^{+0.11}_{-0.075}$ |
| $H_0 [\text{km/s/Mpc}]$   | 68.17 (68.07) $^{+0.39}_{-0.39}$ | 69.24 (69.15) $^{+0.60}_{-0.83}$ |
| $\Omega_m$                | 0.3045 (0.3057) $^{+0.0051}_{-0.0051}$ | 0.297 (0.298) $^{+0.006}_{-0.007}$ |
| $\sigma_8$                | 0.8088 (0.8103) $^{+0.0059}_{-0.0059}$ | 0.8242 (0.8267) $^{+0.009}_{-0.012}$ |
| $S_8$                     | 0.815 (0.818) $^{+0.010}_{-0.010}$ | 0.820 (0.823) $^{+0.010}_{-0.010}$ |
| $\Delta G_N (\%)$         | $-$                | 1.70 (1.49) $^{+0.81}_{-1.1}$ |

Table III. The mean (best-fit) ±1σ (68% CL) constraints on the cosmological parameters in the ΛCDM and CIMG scenarios, as inferred from the combination of P18 + lensing + BAO + SNIa + RSD + SH0ES datasets. There is significant evidence for the CIMG model as $\phi_i = 0.3^{+0.24}_{-0.24} M_P$ and $\Delta G_N = 1.7 \pm 1.7$ with 95% CL are detected at $\geq 2\sigma$ significance.

Table IV. $\chi^2$ values for the best-fit ΛCDM and CIMG models, constrained by P18 + lensing + BAO + RSD + SNIa + SH0ES datasets. There is reduction of 9 in the value of $\chi^2$, for the one additional CIMG parameter to ΛCDM, driven almost entirely by the improved fit to SH0ES. However, it is notable that the fit to the LSS data is worse in the CIMG scenario, while the fit to the CMB is not degraded.

\[ \chi^2 = \sum (data - model)^2 / \text{error}^2 \]

in Fig. 4. Placing the CIMG model somewhere between ΛCDM and EDE in context of both $H_0$ and $S_8$ tensions.

C. Considering additional LSS data

We now expand our analysis to include likelihoods from the DES-Y1 dataset [26, 57], in particular the “3x2pt” likelihood, comprised of photometric galaxy clustering, galaxy-galaxy lensing, and cosmic shear two-point correlation functions.

The inclusion of DES data places stronger constraints on $\Omega_m$, which in turn reduces the value of the CIMG parameter $\phi_i$, as shown in Table V (see also Table IX). We find $\phi_i = 0.26^{+0.17}_{-0.23} M_P$ at 95% CL, detected with $\leq 2\sigma$ significance. Meanwhile, the value of $H_0$ shifts further upwards to $H_0 = 69.4^{+1.3}_{-1.2} \text{km/s/Mpc}$ at 95% CL. The reason for that is due to the general shift in $H_0$ when including the DES-Y1 dataset, observed also for the ΛCDM benchmark compared to Table III. Thus the tension with SH0ES measurements is reduced to $3\sigma$ for CIMG, compared to $3.8\sigma$ in the ΛCDM benchmark scenario.

The lower value of $\phi_i$ when DES-Y1 data is included in the combined dataset can be understood in terms of the interplay between $\sigma_8$, $\Omega_m$, $H_0$ and $\phi_i$. The precise DES measurement of $\Omega_m$ breaks the $\Omega_m - H_0$ degeneracy in the ΛCDM fit to the CMB, shifting $H_0$ to larger values. The impact of the DES measurements on the CIMG parameter results in a lower value for $\phi_i$, due to a marked correlation between $\sigma_8$, $H_0$ and $\phi_i$, observed in Fig. 4. The same thing happens in the EDE scenario with $f_{\text{EDE}}$ replacing $\phi_i$, only to greater extent due to its stronger degeneracy with $\sigma_8$ and $H_0$.

It is also notable that the posterior of $\sigma_8$ matches closely that of the fit to primary CMB-only, as shown in Fig. 4, erasing the shift observed without DES. This shift manifests the constraints of LSS on $\phi_i$, due to the correlation between $\sigma_8$ and $\phi_i$, mentioned previously. The
Figure 2. Cosmological parameter constraints from the combination of Planck 2018 primary CMB data (TT+TE+EE); Planck 2018 CMB lensing data; BAO data from 6dF, SDSS DR7, and SDSS DR12; Pantheon SNIa data; the latest SH0ES $H_0$ constraint; and SDSS DR12 RSD data. We do not plot $\tau$, as it is essentially unchanged in the CIMG fit. Some parameters shift by a non-negligible amount in the CIMG fit (compared to ΛCDM), including increases in $\Omega_c h^2$, $n_s$, and $\sigma_8$ as well as broadening of the error bars on these parameters. The increase in $H_0$ is not large enough to reconcile the tension with the SH0ES-only constraint (shown in the grey bands), but it does reduce the tension significantly.

shift in $\sigma_8$ is matched by the shift in $S_8 = 0.809 \pm 0.018$ at 95% CL, which is in 1.9σ tension with combined LSS measurements, negligibly larger than the 1.8σ for ΛCDM.

The $\chi^2$ statistics, tabulated in Table VI, show poor improvement to the fit for the additional CIMG parameter. The CIMG model offers a slightly better fit to SH0ES data alone, compared to the ΛCDM benchmark, while the fit to LSS data worsens. It seems that there is no region in parameter space that is in concordance with all cosmological data sets. This indicates a possible statistical tension between LSS and $H_0$ likelihoods, as each dataset pulls the parameters in the opposite direction.
Constraints from *Planck* 2018 TT+TE+EE + CMB Lensing, BAO, RSD, SNIa, SH0ES and DES-Y1

| Parameter                             | AC1DM          | CIMG           |
|---------------------------------------|----------------|----------------|
| log(10^{10} A_s)                     | 3.049 (3.043 ± 0.012) | 3.048 (3.053 ± 0.014) |
| n_s                                  | 0.9705 (0.9701 ± 0.0030) | 0.9722 (0.9728 ± 0.0037) |
| 100θ_s                               | 1.04208 (1.04183 ± 0.00023) | 1.04205 (1.04179) ± 0.00028 |
| 100 × Ω_b h^2                        | 2.259 (2.258) ± 0.011 | 2.255 (2.250) ± 0.013 |
| Ω_b h^2                              | 0.1176 (0.1179) ± 0.0007 | 0.1183 (0.1184) ± 0.0009 |
| τ_reio                               | 0.0591 (0.0544) ± 0.0060 | 0.0569 (0.0563) ± 0.0072 |
| φ_i [M_P]                            | –              | 0.264 (0.308) ± 0.13 ± 0.072 |
| H_0 [km/s/Mpc]                       | 68.52 (68.30) ± 0.30 | 69.40 (69.43) ± 0.69 ± 0.75 |
| Ω_m                                  | 0.2999 (0.3025) ± 0.0039 | 0.2939 (0.2937) ± 0.0058 |
| σ_g                                  | 0.8056 (0.8038) ± 0.0047 | 0.8176 (0.8216) ± 0.0097 ± 0.0110 |
| S_8                                  | 0.805 (0.807) ± 0.007 | 0.809 (0.813) ± 0.009 |
| ΔG_N (%)                             | –              | 1.36 (1.61) ± 0.68 ± 1.1 |

Table V. The mean ±1σ (68% CL) constraints on the cosmological parameters in AC1DM and CIMG, as inferred from the combination of P18 + lensing + BAO + RSD + SNIa + SH0ES + DES-Y1. With the inclusion of DES data the evidence for CIMG decreases, as φ_i = 0.26 ± 0.23 M_P and ΔG_N = 1.4 ± 1.7 with 95% CL, to < 2σ significance.

χ^2 statistics from the fit to *Planck* 2018 TT+TE+EE + CMB Lensing, BAO, RSD, SNIa, SH0ES and DES-Y1

| Datasets                  | LCDM | CIMG |
|---------------------------|------|------|
| CMB TT, EE, TE:           |      |      |
| *Planck* 2018 low-ℓ TT    | 22.4 | 22.3 |
| *Planck* 2018 low-ℓ EE    | 396.0 | 396.3 |
| *Planck* 2018 high-ℓ      | 2350.5 | 2350.5 |
| TT+TE+EE                  |      |      |
| LSS:                      |      |      |
| *Planck* CMB lensing      | 9.4  | 9.2  |
| BAO (6dF)                 | 0.002 | 0.084 |
| BAO (DR7 MGS)             | 1.9  | 2.7  |
| BAO+RSD (DR12 BOSS)       | 5.8  | 7.3  |
| DES-Y1                    | 510.8 | 513.6 |
| Supernovae:               |      |      |
| Pantheon                  | 1034.8 | 1034.8 |
| SH0ES                     | 18.8 | 13.0 |
| Total χ^2                 | 4350.5 | 4349.7 |

Table VI. χ^2 values for the best-fit AC1DM and CIMG models, constrained by CMB + lensing + BAO + RSD + SNIa + SH0ES + DES-Y1 datasets. There is reduction of only 0.8 in χ^2, for the one additional parameter of the CIMG model.

We also test the CIMG model with another LSS dataset, using effective field theory (EFT) applied to BOSS DR12. This dataset is composed of *Planck* 2018 CMB + lensing + EFT with BAO + SH0ES. The results are tabulated in Table VII (and summarized in Table IX). The EFT dataset is less constraining, compared to DES-Y1, as it allows the largest CIMG component of φ_i = 0.33 ± 0.16 M_P at 95% CL, which corresponds to a 2% relative deviation in ΔG_N. The CIMG component now raises the Hubble parameter to the value of H_0 = 69.6 ± 1.6 km/s/Mpc at 95% CL, which is the most significant increase in H_0, compared to the corresponding AC1DM benchmark, of all the datasets tested in this work. The Hubble tension reduces, in this scenario, to 2.7σ, compared to 3.9σ for the AC1DM benchmark. Again, the increase comes at the cost of upward shift of σ_g. But the greater reduction in Ω_m results in a mild increase of S_8, increasing the tension with the combined LSS constraints to just 2.3σ, compared to 2.1σ for AC1DM.

The χ^2 statistics, shown in Table VIII, indicate a reduction of 6.5 in the total χ^2 value, compared to AC1DM. This reduction is once again mainly due to SH0ES, but we also find a reduction in the χ^2 for some of the LSS likelihoods, resulting in a total increase of only 0.5 due to LSS fits, while the fit to CMB is practically not degraded.

This combination of datasets seems to provide a significant relaxation of the Hubble tension without a substantial damage to the fit to CMB and the LSS data, or a notable increase of the tension between these datasets, represented here by S_8.

V. FORECAST FOR CMB-S4 CONSTRAINTS

In this work we analyzed the CIMG model, which is a special case of a MG model with a scalar field coupled to the Ricci scalar (i.e. fixing ξ = −1/6), because it is symmetric and involves only one additional parameter. Previous analysis of the more general model [21] (with ξ free to vary) has found that ξ = −1/6 is allowed by constraints from *Planck* 2018, BAO and H_0 measurements.
Constraints from Planck 2018 TT+TE+EE + CMB Lensing, BAO + EFT and SH0ES

| Parameter          | ACDM            | CIMG            |
|--------------------|-----------------|-----------------|
| log(10^{10} A_s)  | 3.051 (3.040)   | 3.052 (3.059)   |
| n_s               | 0.9690 (0.9683) | 0.9721 (0.9751) |
| 1000\theta_s      | 1.04205 (1.04195) | 1.04204 (1.04189) |
| 100 \times \Omega_b h^2 | 2.252 (2.260) | 2.249 (2.247) |
| \Omega_b h^2      | 0.1182 (0.1184) | 0.1192 (0.1191) |
| \tau_{reio}       | 0.0594 (0.0551) | 0.0573 (0.0630) |
| \phi_i [M_P]      | –               | 0.328 (0.353)   |
| \Omega_m          | 0.3039 (0.3048) | 0.2940 (0.2929) |
| \sigma_s          | 0.8084 (0.8040) | 0.8270 (0.8310) |
| S_8               | 0.814 (0.810)   | 0.818 (0.821)   |
| \Delta G_N (%)    | –               | 1.94 (2.08)     |

H_0 [km/s/Mpc]     | 68.21 (68.18)   | 69.58 (69.67)   |
| \Omega_m          | 0.3039 (0.3048) | 0.2940 (0.2929) |
| \sigma_s          | 0.8084 (0.8040) | 0.8270 (0.8310) |
| S_8               | 0.814 (0.810)   | 0.818 (0.821)   |
| \Delta G_N (%)    | –               | 1.94 (2.08)     |

Table VII. The mean ±1σ (68% CL) constraints on the cosmological parameters in ACDM and CIMG, as inferred from the combination of Planck 2018 primary CMB data (TT+TE+EE); Planck 2018 CMB lensing data; BAO (BOSS DR12) combined with EFT of BOSS and the latest SH0ES H_0 constraint. This combination of datasets yields the strongest evidence for the CIMG model, as \phi_i = 0.33^{+0.16}_{-0.20} M_P and \Delta G_N = 1.9 ± 1.8 with 95% CL, with \gtrsim 2\sigma significance.

\chi^2 statistics from the fit to Planck 2018 TT+TE+EE + CMB Lensing, BAO + EFT and SH0ES

| Datasets                  | LCDM | CIMG |
|---------------------------|------|------|
| CMB TT, EE, TE:           |      |      |
| Planck 2018 low-\ell TT   | 22.6 | 22.2 |
| Planck 2018 low-\ell EE   | 396.7| 398.3|
| Planck 2018 high-\ell     | 2356.0| 2355.0|
| TT+TE+EE                  |      |      |
| Planck CMB lensing        | 9.2  | 9.1  |
| BAO+EFT (SGC high z)      | 62.7 | 64.2 |
| BAO+EFT (NGC low z)       | 70.8 | 71.0 |
| BAO+EFT (NGC high z)      | 67.1 | 66.0 |
| SH0ES                     | 16.6 | 9.4  |
| Total \chi^2              | 3001.7| 2995.2|

Table VIII. \chi^2 values for the best-fit ACDM and CIMG models, constrained by CMB + CMB Lensing + BAO + EFT + SH0ES. There is reduction of 6.5 in \chi^2, for the one additional parameter of the CIMG model.

Near future experiments may be able to put stronger constraints on the MG model and either exclude or affirm the CIMG model. Here we consider the planned ground-based CMB 'Stage-4' experiment (CMB-S4) [63, 64]. In order to obtain a forecast for CMB-S4 constraints on the MG model we adopt the expected survey performance of CMB-S4\textsuperscript{6} and employ standard Fisher analysis [65–67].

The CMB power spectra can be written as

\[ C^Y_X = (4\pi)^2 \int dk k^2 T^X(k) T^Y(k) P_X(k), \]

where X, Y \{T, E\} stand for temperature and E-mode polarization, and \(T^X\) are their transfer functions.

The forecast on the variance for a set of parameters \(\theta_i\) may be obtained by defining the Fisher matrix as

\[ F_{ij} = \sum_{\ell} \frac{2\ell + 1}{2} f_{sky} \text{Tr} \left[ C_{\ell}^{-1} \partial C_{\ell} \partial C_{\ell} \partial C_{\ell} \partial C_{\ell} \right], \]

where \(f_{sky}\) is the sky-fraction covered and \(C_{\ell}\) are the covariance matrices, which are given by

\[ C_{\ell} = \begin{pmatrix} C^T_T & C^T_E & C^T_d & C^{TE}_T & C^{TE}_E & C^{TE}_d & C^{Td}_T & C^{Td}_E & C^{Td}_d \end{pmatrix}, \]

where we have defined

\[ C^X_Y = C^X_T + N^X_T, \]

where \(N^X_T\) are the noise power spectra, given by

\[ N^T_T = \Delta^2 (\ell+1) \sigma_i^2, \]

\[ N^{EE} = 2 \times N^{TT}_T, \]

\textsuperscript{6} CMB-S4 performance expectations: https://cmb-s4.org/wiki/index.php/Survey_Performance_Expectations
| Parameter | best-fit value | 1σ |
|-----------|----------------|----|
| ξ         | -0.1667        | 0.0115 |
| φi        | 0.297          | 0.005  |

where $\Delta_T$ is the temperature sensitivity, $\sigma_b = \theta_{T,\text{FWHM}}/\sqrt{8 \log 2}$, with the full-width-half-maximum $\theta_{T,\text{FWHM}}$ given in radians. For the lensing noise $N_{\text{ell}}^{dd}$ we follow Ref. [68], constructing it from the E and B modes data, then subtracting from the B-mode data which is in turn used again to construct $N_{\text{ell}}^{dd}$ and so forth, until we reach convergence. We use the 93, 145 and 225 GHz frequencies, with the corresponding sensitivities of $\Delta T = 1.5, 1.5, 4.8 \mu K\text{-arcmin}$, resolution of $\theta_{T,\text{FWHM}} = 2.2, 1.4, 1.0 \text{arcmin}$, over 40% of the sky and a prior on the optical depth of reionization of $\tau = 0.06 \pm 0.01$. The CMB-S4 experiment is expected to observe the $\ell$ range between 30 and 5000 for polarization, although the highest modes will be noise-dominated. We also ignore $\ell > 3000$ for temperature, as higher multipoles would be contaminated by foregrounds.

Finally we define the correlation matrix as $C_{ij} \equiv F_{ij}^{-1}$, thus the variance of each of the parameters $\Theta_i$ is $\sigma_i = \sqrt{C_{ii}}$. For the fiducial values of the parameters we use the best fit values in Table III. The expected constraints on MG parameters from CMB-S4 experiment are shown in Table 3. We find that the CMB-S4 experiment is expected to place strong constraints on $\xi$ and can help determine if CI scenario is preferred. We can also expect CMB-S4 to improve constraints from Planck on the evidence for physics beyond ΛCDM (e.g. EDE, CIMG).

VI. DISCUSSION AND CONCLUSIONS

In recent years the discrepancy between the values of the Hubble constant $H_0$, the current expansion rate of the Universe, inferred from early-Universe measurements, such as Planck 2018 CMB, and late-Universe measurements, such as the SH0ES collaboration distance-ladder measurements, has reached $\gtrsim 4\sigma$ confidence.

A recent review, Ref. [5], of the phenomenology of the Hubble tension suggests that, to restore concordance between recent cosmological data and the cosmological model, increasing the value of $H_0$ alone is not enough, but one should reduce the value of the sound horizon at last scattering $r_s$ as well. It was also suggested that the most promising method to accomplish this goal is by introducing new physics just prior to recombination, at the proximity of matter-radiation equality, which would trigger a rapid increase of the expansion rate throughout this period. A typical way of realizing this methodology is to introduce a new energy component to the cosmological model, so that it will increase $H(z)$ throughout this period and then dilute fast enough to be negligible at later epochs.

In this work we considered the CIMG model as a candidate for alleviating the Hubble tension. We analyzed it using various combinations of datasets composed of early-Universe data, direct measurements of $H_0$ and LSS data: Planck 2018 CMB and its lensing, BAO (6dF, SDSS DR7 and SDSS DR12), SDSS DR12 RSD, SN distance data from Pantheon, SH0ES distance-ladder measurements of $H_0$, DES-Y1 3x2pt and BAO+EFT (BOSS/SDSS galaxies clustering DR12). We also compared the results to a similar analysis done for the EDE model [7]. The constraints we found on the CIMG model and its influence on other cosmological parameters share many of the characteristics of the EDE model, while the former introduces only one additional parameter to the cosmological model and is not fine tuned as the latter, which requires at least two additional parameters. We find that the CIMG model allows an increase of the Hubble parameter up to $H_0 = 69.58 \pm 0.80$ km/s/Mpc, when considering all types of datasets (summarized results in Table IX and Fig. 4).

Initially we considered primary CMB anisotropies alone: Planck 2018 TT+TE+EE. Although the value of $H_0$ is increased slightly, there is no significant evidence for the CIMG model (Table I). Furthermore, the total fit to the CMB is worsened in the CIMG scenario, compared to ΛCDM. We conclude that the CIMG model is not preferred by primary CMB data alone. In contrast to the EDE scenario, in which the posterior of the EDE component might be biased due to degeneracy of the other parameters of the model (as describe in Ref. [10]), the posterior of $\phi_i$, shown in Fig. 4, indicates accurately the preference of the dataset. For the P18 dataset it is located around zero, indicating it disfavours CIMG.

When we supplement the primary CMB dataset with Planck 2018 lensing + BAO + RSD + SNIa + SH0ES, we find, as shown in Table III, a substantial CIMG...
Table IX. The mean $±1\sigma$ constraints on cosmological parameters in the CIMG scenario from Planck 2018; CMB lensing; BAO; RSD; SNIa; SH0ES; DES-Y1; and a combined BAO+EFT dataset. Only $\phi_i$ is a sampled parameter. The significance of the CIMG component is highly dependent on the datasets: the inclusion of SH0ES tends to increase the value of $\phi_i$, whereas the inclusion of DES-Y1 reduces its value. The right column refers to another dataset composed of the BAO+EFT likelihood, which allows for a larger value for $\phi_i$. Even the highest value found for $H_0$ does not relieve the Hubble tension completely.

Constraints on CIMG parameter from varying sets of data

*Planck 2018; CMB lensing; BAO; SN; SH0ES; DES-Y1*

![Image of constraints summary table]

Figure 4. Constraints on the CIMG parameter from various datasets: primary *Planck 2018*; CMB lensing; BAO; RSD; SNIa; SH0ES; DES-Y1; and a combined BAO+EFT. Here we present a subset of the parameters: the initial condition $\phi_i$ for the CIMG, along with $H_0$ [km/s/Mpc] and $\sigma_8$. The contours show $1\sigma$ and $2\sigma$ posteriors for various dataset combinations, computed with GetDist [50]. The P18 dataset alone (green) yields a posterior for $\phi_i$ which tends to zero, thus disfavoring a significant CIMG component, in contrast to the combined datasets (purple and blue), which feature a significant CIMG component. The EFT dataset seems to put lower constraints on $\sigma_8$ than DES-Y1, as shown in its posteriors for each dataset, which explains the more significant CIMG component when using the EFT instead of DES-Y1 (blue on each side). Here we also include datasets without SH0ES (salmon), in which the CIMG significance is almost completely erased, indicating the low preference of the model by all other datasets.

The tension component, corresponding to $\phi_i = 0.3_{-0.24}^{+0.14} M_P$ and $H_0 = 69.24 \pm 1.4 \text{ km/s/Mpc}$ at $95\%$ CL. The tension with the SH0ES measurements is reduced to $3.1\sigma$, while the tension with LSS data, $2.4\sigma$, is only slightly greater...
than that of the $\Lambda$CDM benchmark. The CIMG model offers a better fit to this combined dataset as $\Delta \chi^2 = -9$. This reduction of $\Delta \chi^2$ is mainly due to the better fit to SH0ES data, as shown in Table IV. We note that the better fit to both CMB and SH0ES data comes at the expense of a worse fit to BAO and lensing data (the LSS part of this dataset), indicating a correlation between the different datasets. As described in Ref. [7] for the EDE model, the introduction of the CIMG component forces some cosmological parameters to shift in order to keep the fit to the CMB data. But, due to the increase in the gravitational strength and the non-localized dynamics of the CIMG model, the shifts in $\Omega_m h^2$ and $n_s$ are very small compared to the EDE model. However, for the same reason, the downward shift in $S_8$ due to the increase in the matter clustering amplitude $\sigma_8$. The correlation between the cosmological parameters is shown in Fig. 2.

Including the DES-Y1 likelihood in the combined dataset, we saw that the posterior of $\phi_i$ is driven slightly backwards (Fig. 4), corresponding to a smaller CIMG component. The inclusion of DES-Y1 likelihood acts to reduce the value of $S_8$, due to the stronger constraints on $\Omega_m$, resulting in smaller tension with the LSS data. Nevertheless, the fit to LSS datasets is worse than that of the $\Lambda$CDM benchmark, as the fit to BAO+RSD (BOSS DR12) dataset worsens compared to the improvement exhibited by the benchmark, as shown in Table VI. Excluding SH0ES from the combined dataset erases the CIMG component, as shown in Fig. 4. These results confirm the correlation between the datasets, which leads to the conclusion that it is not possible to reconcile DES, BAO and SH0ES datasets simultaneously, using the methodology of reducing $r^*_s$, as recently elaborated in Ref. [6].

We also considered a combined dataset which includes the newly published BAO+EFT likelihood. We found a larger CIMG component than in any other combination of datasets, $\phi_i = 0.33^{+0.16}_{-0.20} \, M_p$ at 95% CL, which corresponds to $H_0 = 69 \pm 1.6 \, \text{km/s/Mpc}$. The relatively large CIMG component is followed by a large value for $\sigma_8$, but due to the significant reduction in $\Omega_m$ (Table VII), which is allowed by this dataset, the resulting value of $S_8$ is not increased as one would have expected (compared to the values in Table III, for example). Excluding SH0ES from this dataset also results in significant decrease of the CIMG component, however not to the same extent as in the datasets with DES, as shown in Fig. 4.

Finally, forecasts for the near-future ground experiment CMB-S4 show that we can expect it to place strong constraints on the parameter $\xi$ to distinguish the CIMG model from the more general MG, and also to place stronger constraints on $\phi_i$. In light of the results of our analysis, the CIMG model might present a rather elegant and natural solution to relieve the Hubble tension, compared to EDE, but since no model that acts to increase $H_0$ by reducing $r^*_s$ seems to be able to reconcile BAO, DES and SH0ES at the same time, the search for new physics to explain the Hubble tension has not concluded.

Note added: While this paper was undergoing the last round of text edits, Ref. [69] appeared on the arXiv with overall consistent conclusions in its short discussion of the model analyzed in depth here.

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