Interrupted and uninterrupted lane changes: a microscopic outlook of lane-changing dynamics

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ABSTRACT

Field observations suggest that the Lane-Changing (LC) processes are often interrupted by the surrounding vehicles. Understanding these intermediate interruptions and evasive behavior during an LC is necessary for developing realistic simulation tools, collision-warning systems, and human-like LC in self-driving vehicles. However, identifying and classifying LC into interrupted (ILC) and uninterrupted (ULC) is not straightforward. This study proposes a methodology for the automatic identification and classification of LC events. Further, we highlighted the characteristic differences between ILC and ULC by comparing their duration, crash likelihood, and driver discomfort. The likelihood, duration of exposure, and severity of LC crashes were captured using the surrogate safety measures such as Anticipated Collision Time (ACT), Time-Exposed ACT (TE-ACT), and Time-Integrated ACT (TI-ACT). The driver’s discomfort was measured using acceleration noise and the number of sign inversions in the acceleration profile. The Kruskal–Wallis–ANOVA test on the above attributes and the sensitivity analysis of the duration models confirm that ILC and ULC are characteristically different.

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1. Introduction

A comprehensive understanding of lane changing dynamics is important for developing realistic simulation tools, autonomous LC systems with improved safety and comfort, or proactive LC safety assessment tools. The LC maneuver is generally considered as a continuous process from the original to the target lane, without any intermediate interruptions. Such an assumption helps in a deterministic and straightforward LC modeling and trajectory planning. However, recent studies have shown that LC dynamics are not necessarily continuous but disrupted intermittently (Monk et al. 2004; Keyvan-Ekbatani, Knoop, and Daamen 2016; Yang et al. 2018). The disruptions could be attributed to the intermittently perceived risks or discomfort and the associated evasive actions by the driver. The risks could arise due to the closing-in of lag vehicle in the target lane faster than the anticipated rate, emergency braking of the leader vehicle, or personal preference of LC drivers.
Figure 1. Field observed lateral position, speed, and acceleration profiles corresponding to LC maneuver (NGSIM, US-101 data); (a and d) Lateral position profiles; (b and e) lateral speed profiles; (c and f) lateral acceleration profiles; corresponding to vehicle IDs Car-242 and Car-1210, respectively.

Excessive lateral acceleration, steering, or frequent braking might result in driver discomfort. These interruptions and the evasive actions cause a longer LC duration, compared to an uninterrupted LC operation.

The following describes the rationale for this study. Traffic simulation models consider the lane-changing as either instantaneous or a process performed during a fixed period, disregarding the intermediate actions. Similarly, in autonomous LC applications, the trajectory planning depends predominantly on an S-shaped parametric curve (Figure 1a) or a sinusoidal lateral acceleration profile (Figure 1c). A preliminary investigation into the real-field LC scenarios has revealed that the above assumptions are highly restrictive. Figure 1 shows two LC trajectories, which are different in terms of their profile and duration. The existing LC studies primarily assume all LCs to be similar to Figure 1(a). But, for an effective modeling of LC operation, it is necessary to differentiate and gain a detailed understanding of the two types of LC operations shown in Figure 1. The benefits of such an understanding are, but not limited to, (i) train the CAVs to understand interrupted LC operations of human drivers in a mixed traffic environment; (ii) development of more dynamic LC trajectories in the case of autonomous LC; (iii) realistic simulation of LC operations.

The present study performs a detailed investigation of LC operations and proposed to classify them into interrupted LC (ILC) and uninterrupted LC (ULC), besides highlighting their characteristic differences. The scope of this study is limited to a single lane change operation, and the dynamics of multiple LCs are not addressed. This study has three major contributions. First, a methodology to identify the time window (referred to as LC window
here on) within which the LC maneuver takes place. The lateral speed profile of the LC vehicle was segmented to identify the LC window. Second, a method to classify the LC trajectories into ILC and ULC, based on the interruption levels during an LC. The interruption level was quantified based on the cumulative distance traversed by an LC vehicle during the deceleration and acceleration phases, respectively. Third, highlighting the characteristic difference of ILC and ULC. The risk and comfort during ILC and ULC were compared to quantify the characteristic differences. This study has shown that the ILC takes more time compared to ULC, to complete the LC operation. Therefore, it is necessary to develop separate LC duration models for ILC and ULC.

The rest of the paper is organized as follows. Section 2 presents the literature review. Section 3 describes the data used in this study. In Section 4, the method proposed to automatically identify the LC window is presented. Section 5 describes the method proposed to classify the LC into ILC and ULC. The characteristics of ILC and ULC are estimated and compared in Section 6. Section 7 summarizes the work and highlights the contributions and future scopes.

2. Background

LC is one of the complicated driving tasks as it comprises multiple vehicle interactions, tactical decision-making, and low-level trajectory planning (Wang et al. 2019; Mirchevska et al. 2018). The current LC models, which are predominantly deterministic and rule-based (Gipps 1986; Hidas 2002; Kesting, Treiber, and Helbing 2007; Hamdar and Mahmassani 2009), can capture presumed cognitive driving behaviors, but they do not necessarily correspond to reality (Keyvan-Ekbatani, Knoop, and Daamen 2016). In reality, the LC dynamics could be highly stochastic and involve multiple vehicle interactions both in longitudinal and lateral directions, simultaneously. The interruptions due to the surrounding vehicles during LC can cause significant differences in the LC behavior compared to the uninterrupted LC (Yang et al. 2018). Moreover, to the knowledge of the authors, only a few studies differentiated the ULC and ILC for a comprehensive understanding of LC dynamics (Mullakkal-Babu et al. 2020).

Due to the absence of relevant literature on ILC, this review primarily focuses on the microscopic aspects of LC operation, which have been used to characterize ILC and ULC in this study. To understand and model different aspects of the LC dynamics, researchers have used various attributes of LC. For example, comfort and safety are the main attributes
Table 1. Parameters used to identify LC window.

| Author and Year | Parameter |
|-----------------|-----------|
| Jin (2010) and Toledo and Zohar (2007) | The curvature of lateral position profile of LC vehicle |
| Balal et al. (2014) | Time at which LC vehicle touches the lane marker |
| van Winsum, Waard, and Brookhuis (1999) | Steering angle |
| Wang, Li, and Li (2014) and Balal, Long Cheu, and Sarkodie-Gyan (2016) | Lateral speed profile |

Table 2. LC duration reported in various studies.

| Author and Year | Duration statistics |
|-----------------|---------------------|
| Toledo, Koutsopoulos, and Ben-Akiva (2003) | LC duration ranges from 1.0 to 13.3 s, with a mean of 4.6 s. |
| Thiemann, Treiber, and Kesting (2008) | 5–6 s as an average LC duration |
| Ahmed et al. (2019) | Average LC duration of 5.5 s |
| Olsen et al. (2002) | Average LC duration of 6.28 s with a standard deviation of 2.0 s |
| Wang et al. (2020) | LC duration ranges from 1.6 to 20 sec, with a median of 6.6 s, a mean of 7.0 s, and a standard deviation of 2.1 s |
| Yang, Wang, and Quddus (2019) | LC duration varied from 0.7 to 16.1 s |

considered while developing a dynamic lane change path planning algorithm (Yang et al. 2018). The LC duration is an important input for LC modeling (Toledo and Zohar 2007). To extract various attributes from LC trajectories, the time window of LC operation needs to be identified accurately. Table 1 shows some of the parameters used to identify LC window from the trajectory data. It is to be noted that, many past works (Toledo and Zohar 2007; Balal et al. 2014) followed a manual detection of LC window, which could be erroneous due to human error. Evidently, the method used to determine the LC window has a significant impact on the LC analyses, especially on the LC duration (Table 2). Thiemann, Treiber, and Kesting (2008) have stated that ‘since the real beginning of a lane change – the decision for making the lane change – is impossible to measure, and the physical beginning – the moment at which the driver starts to turn the wheel – is very difficult if not impossible to measure...’. The difficulty associated with the LC window estimation is evident from this statement.

Most studies treat LC as an instantaneous process or consider a constant duration across all vehicles for simplicity (Wang, Li, and Li 2014; Yang, Wang, and Quddus 2019). In reality, the LC duration varies from one LC to another due to vehicle interactions and driver preferences. Since the LC duration is a critical variable in various LC-related studies, including safety studies, operational analysis (Qiang Liu, Zhang, and Wang 2019; Ferrari 1989; Toledo and Zohar 2007), trajectory planning (Yang et al. 2018), decision-making (Meng et al. 2016), and microscopic simulation, it is vital to determine the necessity of separate duration modeling for ILC and ULC.

3. Data preparation and filtering

This study used the Next Generation Simulation (NGSIM) trajectories to investigate the characteristics of ILC and ULC. The trajectory data collected from both I-80 and US-101 were considered for the study. There was a total of 45 min data from each location, covering 640 m in the case of US-101, and 500 m for I-80. All of the trajectories were reconstructed using the method proposed by Venthuruthiyil and Chunchu (2020) and Venthuruthiyil and Chunchu (2018) to remove the embedded noise in the NGSIM data. The kinematic variables,
such as speed and acceleration, in lateral and longitudinal directions, were estimated from the reconstructed path data.

4. Identification of the time window corresponding to lane change operation

Unlike the existing methods, this study proposes an automated method for defining the LC window as follows:

Let \( \mathbf{X}_j = [\mathbf{p}_1^j, \mathbf{p}_2^j, \ldots, \mathbf{p}_{N_j}^j] \) be the trajectory of \( j \)th vehicle, where \( N_j \) is the length of the trajectory. In this work, we considered each point \( \mathbf{p}_i^j = [t_i^j, x_i^j, y_i^j, v_{x_i}^j, a_{x_i}^j] \) as a vector in five dimensional space, where \( t_i^j \) corresponds to time, \( x_i^j, y_i^j \) and \( v_{x_i}^j, a_{x_i}^j \) correspond to the longitudinal position, lateral position, lateral speed, and lateral acceleration of the vehicle, respectively. \( \mathbf{X}_j \) is the input for both LC widow detection as well as LC classification.

In the first stage, all the trajectories involving LC operation were identified using the lane occupancy count, and the maximum lateral displacement. Here, the lane occupancy count is the number of lanes occupied by the vehicles while traversing the study road stretch. The trajectories with lateral displacements more than 1.5 times the vehicle width were identified, then verified for the lane occupancy count. It is noteworthy that occupying multiple lanes can be corresponding to either LC or overtaking operations. In this study, we consider only the cases corresponding to single lane change. Therefore, the trajectories corresponding to overtaking operations and multiple lane changes are separated. Equation (1) shows the conditions for grouping the lateral maneuvers:

\[
\text{ManeuverType} = \begin{cases} 
1 \text{ (SingleLC), } & \text{if } \Delta y_j > 1.5 \times w_j \& \Phi(l) = 2 \& l(1) \neq l(N_j) \\
2 \text{ (MultipleLC), } & \text{if } \Delta y_j > 1.5 \times w_j \& \Phi(l) > 2 \& l(1) \neq l(N_j) \\
3 \text{ (Overtaking), } & \text{if } \Delta y_j > 1.5 \times w_j \& \Phi(l) > 1 \& l(1) = l(N_j) 
\end{cases}
\]  

(1)

where ‘Maneuver Type’ is a categorical variable; \( \Delta y_j \) is the maximum lateral displacement for vehicle \( j \); \( w_j \) is width of vehicle \( j \); \( l \) is the vector of lane occupancy indicator; and \( \Phi \) is a function that returns the size of the unique values in \( l \). Further, the LC window corresponding to each LC trajectory is defined as follows:

**Definition 4.1 (LC Window):** LC window is the time interval where the following conditions are satisfied. (1) The lateral speed of the vehicle increases from zero state, reaches the maximum, then decreases back to zero. (2) Within this time frame, the vehicle must be occupying exactly two lanes. (3) Maximum lateral displacement during this period should be more than 1.5 times of the vehicle width.

Each LC vehicle trajectory is segmented based on the lateral speed profile and analyzed for the conditions corresponding to LC window. Figure 3 shows a schematic representation of the LC window identification procedure. To be specific, for each trajectory \( \mathbf{X}_j \), the lateral speed profile \( v_y \) is segmented as \( v_y^k = v_y(s_l : s_m) \), where \( s_l \) and \( s_m \) are the start and end times of the segment, and \( k \) is any time instance during the LC operation. The criteria
for segmentation are as follows:

\[ \nu_{y}^{s_{l}m} = \nu_{y}(s_{l} : s_{m}) \subset \nu_{y} \mid s_{l} < s_{m} \quad \forall \ l, m = 1, 2, \ldots, s, \quad \text{and} \quad |\nu_{y}^{s_{l}m}(k)| > 0 \quad \forall \ s_{l} < k < s_{m} \]  

From the identified segments, the LC window is selected if the following criterion (Equation (3)) is satisfied.

\[ \text{LC Segment}_{j} = \{ \Phi \left( \int (s_{l} : s_{m}) \right) = 2 & \Delta y_{j} > 1.5 \times w_{j} \} \]  

Applying Equations (2) and (3), the LC windows corresponding to all the LC operations were identified. Figure 4(a) shows an example of the identified LC window, and Figure 4(b) shows some of the identified (normalized) vehicles paths.

5. Classification of LC trajectories into ILC and ULC

LC trajectories presented in Figure 4(b) clearly shows two patterns: a smooth and S-shaped curve and the other is with a stair-like profile. It is to be noted that a continuous, s-shaped LC profile corresponds to ULC (highlighted using black color in Figure 4(b)). In contrast,
the stair-like profiles correspond to ILC (highlighted using red color in Figure 4 b). The stair-like profile of ILC could be due to intermittent interruptions. Most of the LC path planning studies have used S-shaped parametric curves (Qiang Liu, Zhang, and Wang 2019; Zeng et al. 2019; Li, Luo, and Wu 2019), such as B-spline curve, polynomial curve, circle curve, spiral curve, trapezoidal curves, clothoid, Bezier’s curve, and hyperbolic tangent curve (Berger 2018). However, these models cannot capture the interrupted LC operations (Yang et al. 2018). In order to capture the intermediate interruptions, studies have considered dynamic methods that optimize safety and comfort (Luo et al. 2016; Yang et al. 2018). Thus it is essential to separate ILC and ULC and develop separate models.

An LC operation can be divided into two phases. First, a laterally accelerating phase till it reaches the maximum lateral speed ($v_{y}^{\text{max}}$) at time $\tau^*$. In the second phase, the vehicle laterally decelerates till its lateral speed is zero. In ideal conditions, the lateral speed profile corresponding to LC would be perfectly concave, as shown in Figure 1(e). Considering this feature, the present study proposes a method to distinguish the ILC and ULC.

Let $t_{\text{start}}^{\text{LC}}$ be the starting time of the LC operation, and $t_{\text{end}}^{\text{LC}}$ corresponding to the end. The acceleration phase $\Delta t_{\text{acc}}^{\text{LC}}$ can be defined as

$$\Delta t_{\text{acc}}^{\text{LC}} = t_i | t_{\text{start}}^{\text{LC}} \leq t_i \leq \tau^*, \quad i = 1, 2, \ldots, \quad \text{and} \quad |\Delta t_{\text{acc}}^{\text{LC}}| = \tau^* - t_{\text{start}}^{\text{LC}}$$

Similarly, the deceleration phase, $\Delta t_{\text{dec}}^{\text{LC}}$ can be defined as

$$\Delta t_{\text{dec}}^{\text{LC}} = t_i | \tau^* \leq t_i \leq t_{\text{end}}^{\text{LC}}, \quad i = 1, 2, \ldots, \quad \text{and} \quad |\Delta t_{\text{dec}}^{\text{LC}}| = t_{\text{end}}^{\text{LC}} - \tau^*$$

Within $\Delta t_{\text{acc}}^{\text{LC}}$, ideally, it is expected that the vehicle continuously accelerates laterally till the lateral speed is maximum ($\tau^*$). Yet, when there is an interruption to the LC maneuver, the vehicle would decelerate to avoid any potential crash risk with any of the surrounding vehicles or to achieve the desired comfort. Figure 5 shows that from time $t_1$ to $t_2$ and $t_3$ to $t_4$, the LC vehicle is interrupted by the surrounding traffic. Similarly, within $\Delta t_{\text{dec}}^{\text{LC}}$, the LC vehicle is expected to decelerate continuously from $\tau^*$ to a near-zero lateral speed. However, Figure 5 indicates that from time $t_5$ to $t_6$, the LC vehicle accelerated probably due to perceived danger or discomfort.

It is hypothesized that, in the case of an interrupted LC, the vehicle covers some extra distance in order to minimize the crash likelihood or to gain comfort. The proposed method uses the extra cumulative distance traversed by LC vehicles during the lateral acceleration and deceleration phases to classify LC into ILC and ULC. Moreover, this quantity could be used as an indicator for the level of interruption. Shaded rectangles in Figure 5 represent this measure graphically. The level of LC interruption is nothing but the total area of the rectangles. Mathematically the level of interruption ($\Upsilon_{\text{LC}}$) can be defined as follows:

$$\Upsilon_{\text{LC}} = \sum_{i=1}^{n} \Delta v_i \times \Delta t_i + \sum_{j=1}^{m} \Delta v_j \times \Delta t_j$$

where $\Delta v$ is the maximum speed difference due to deceleration/acceleration in the acceleration/deceleration phase; $\Delta t$ is the duration of deceleration/acceleration in the acceleration/deceleration phase; and $n$ is the total number of deceleration instances in the
acceleration phases, and $m$ is the total number of acceleration instances in the deceleration phases, respectively.

For a perfectly uninterrupted LC operation, $\Upsilon_{LC} = 0$. But, in reality, there could be certain deviations. We found that a threshold of $\Upsilon_{LC}^{\text{threshold}} = 0.05$, which was selected after a detailed investigation of the ULC profiles, could be more appropriate for distinguishing ILC and ULC. After grouping the LC trajectories into ILC and ULC, an automatic verification is performed by comparing the LC duration, and the extra distance traveled, to rectify the incorrect classifications. If an LC is classified as uninterrupted, but the duration is higher than 25 s and the area is close to $\Upsilon_{LC}^{\text{threshold}}$, then it is reclassified as ILC. An ILC with the area close to $\Upsilon_{LC}^{\text{threshold}}$ and the duration less than 6 s is reclassified as ULC. Such a fine-tuning would help to minimize the effect of the pre-set threshold value on the classification process. The choice of the durations in the fine-tuning process was from the range of LC duration reported in the literature. Figure 6(a) shows the ULC trajectories extracted using the proposed measure, and Figure 6(b) shows the trajectories corresponding to ILC.

**Figure 5.** Schematic representation LC interruptions reflected in the lateral speed profile.

**Figure 6.** Lane change trajectories corresponding to (a) ULC; (b) ILC.
6. Characteristics of interrupted and uninterrupted lane changes

This section presents the characteristics of ILC and ULC such as the crash likelihood and the discomfort of the driver. Further, the LC duration was compared and separate duration models were developed for ILC and ULC.

6.1. Risk and discomfort associated to ILC and ULC

This study conjunctures that the crash likelihood as well as discomfort level may differ for ILC and ULC. To prove this, we have estimated the crash risk corresponding to ILC and ULC, using a surrogate safety measure, called as the Anticipated Collision Time (ACT) (Venthuruthiyil and Chunchu 2021). ACT considers all the interacting vehicles simultaneously and evaluates the safety of the subject vehicle. Moreover, ACT is a two-dimensional extension of the conventional TTC, suitable for LC risk assessment. The ACT is defined as follows:

\[
ACT = \begin{cases} 
\frac{\delta}{(\partial \delta / \partial t)}, & \text{if } \frac{\partial \delta}{\partial t} > 0 \\
\infty, & \text{otherwise}
\end{cases}
\]  

(7)

where \(\delta\) is the shortest distance between the interacting vehicles; \(\frac{\partial \delta}{\partial t}\) is the closing-in rate along the direction of shortest distance between the vehicles. Considering the ACT profile, two other surrogate safety indicators, namely, the Time Extended ACT (TE-ACT), and Time Integrated ACT (TI-ACT) were estimated as follows:

\[
TEACT_i = \sum_{t=0}^{T} \tau_i(t) \times \Delta t
\]

(8a)

\[
\tau_i(t) = \begin{cases} 
1, & \text{if } \forall 0 \leq ACT_i(t) \leq ACT^* \\
0, & \text{Otherwise}
\end{cases}
\]

(8b)

\[
TIACT_i = \int_{0}^{T} ACT^* - ACT_i(t) \, dt, \quad \forall 0 \leq ACT_i(t) \leq ACT^*
\]

(9)

TE-ACT estimates the duration for which the LC vehicle is exposed to a safety critical scenario during the course of LC. Whereas TI-ACT gives an idea about the severity of the potential crashes during LC. A higher TE-ACT or TI-ACT value indicates a higher crash exposure or crash severity, respectively.

The discomfort during LC was quantified using the Acceleration Noise (AN), which is one of the commonly used measures of driver discomfort (Ko, Guensler, and Hunter 2010). The acceleration noise was estimated using the following equation:

\[
AN = \sqrt{\frac{1}{n} (a_i - \bar{a})^2}
\]

(10)

where \(a_i\) is the acceleration of LC vehicle at \(i\)th instance and \(\bar{a}\) is the average acceleration of the vehicle. A higher AN indicates a higher discomfort.

TE-ACT, TI-ACT, and AN distributions are compared to demonstrate the difference in risk and driver comfort associated with ILC and ULC. Figures 7(a–c) show the distribution of ILC,
ULC, and AN, respectively. In the case of ILC, the figure clearly shows that the probability corresponding to longer exposure periods to an unsafe situation is relatively higher (Figure 7a). Similarly, in the case of ILC, the vehicle is more likely to undergo a severe crash (Figure 7b). Using the maximum log-likelihood criterion, the best fitting distributions to all these attributes were identified. It was found that the TE-ACT follows Nakagami distribution for both ILC and ULC. Similarly, TI-ACT follows Generalized Pareto Distribution, for both ILC and ULC, and AN follows Gamma distribution for both ILC and ULC. Table 3 shows the fit statistics along with the parameters. Though the distributions were similar for all the attributes corresponding ILC and ULC, the parameters were different. This indicates that the risk and discomfort probabilities associated to ILC and ULC are different. Therefore, while probabilistic estimation of overall risk during LC activity, as in Park et al. (2018), it essential to consider different probabilities for ILC and ULC.

Further, the Kruskal–Wallis ANOVA test was conducted to highlight the difference between ILC and ULC in terms of crash exposure, crash severity, and driver discomfort. The test hypothesis is as stated below:

### Table 3. Probability Distribution of various LC attributes capturing safety and comfort.

| Attribute       | LC Type | Distribution            | NLogL  | BIC    | AIC    | AICC   | Parameters                                                                 |
|-----------------|---------|-------------------------|--------|--------|--------|--------|-----------------------------------------------------------------------------|
| Acceleration Noise | ILC     | Gamma                   | 618.45 | 1249.59| 1240.89| 1240.92| shape (a) = 2.90, scale (b) = 0.48                                        |
|                 | ULC     | Gamma                   | 389.48 | 790.93 | 782.96 | 782.99 | shape (a) = 4.33, scale (b) = 0.34                                        |
| TE-ACT          | ILC     | Nakagami                | 1164.74| 2341.37| 2333.48| 2333.52| shape (μ) = 0.76, scale (ω) = 117.92, shape (a) = 0.34, scale (b) = 0.48 |
|                 | ULC     | Nakagami                | 1321.83| 2656.19| 2647.66| 2647.68| shape (μ) = 0.83, scale (ω) = 41.76, shape (k) = −0.24, scale (σ) = 22.19 |
| TI-ACT          | ILC     | Generalized Pareto      | 1534.74| 3087.43| 3075.47| 3075.53| shape (k) = −0.24, scale (σ) = 22.19, threshold (θ) = 2.22E − 15           |
|                 | ULC     | Generalized Pareto      | 1862.82| 3744.69| 3731.64| 3731.68| shape (k) = −0.21, scale (σ) = 11.74, threshold (θ) = 2.22E − 15           |
| Duration        | ILC     | Log Normal              | 1412.90| 2838.50| 2829.90| 2829.90| log location (μ) = 2.57, log scale (σ) = 0.31                             |
|                 | ULC     | Log Normal              | 880.01 | 1776.20| 1768.00| 1768.00| log location (μ) = 2.10, log scale (σ) = 0.25                             |

**Figure 7.** Histogram of (a) TE-ACT; (b) TI-ACT; and (c) Acceleration Noise.
| Attribute | LC Type | Average | Standard Deviation | Mean Rank | Sum Rank | Chi-Square | p-Value |
|-----------|---------|---------|--------------------|-----------|----------|------------|---------|
| TE-ACT    | ILC     | 9.3 s   | 5.6 s              | 560.5     | 213579.5 | 110.09     | 9.37E−26|
|           | ULC     | 5.7 s   | 3.1 s              | 375.79    | 197291.5 | 81.91      | 1.43E−19|
| TI-ACT    | ILC     | 17.9 s  | 14.5 s             | 583.08    | 232067.5 | 81.91      | 1.43E−19|
|           | ULC     | 9.8 s   | 8.1 s              | 417.60    | 238867.5 | 81.91      | 1.43E−19|
| AN        | ILC     | 1.6 m/s²| 0.7 m/s²           | 513.05    | 204195   | 6.53       | 0.01    |
|           | ULC     | 1.4 m/s²| 0.8 m/s²           | 466.33    | 266740   | 6.53       | 0.01    |
| Duration  | ILC     | 13.43 s | 5.26 s             | 611.90    | 307175.5 | 392.54     | 2.31E−87|
|           | ULC     | 9.63 s  | 3.06 s             | 264.65    | 108240.5 | 392.54     | 2.31E−87|

Table 4. The Kruskal–Wallis ANOVA test statistics.

$H_0$: The samples come from the same population

Table 4 shows the test statistics. At 95% confidence, the Kruskal–Wallis ANOVA test proves that the safety and comfort during ILC and ULC are significantly different. Moreover, the average TE-ACT value for ILC and ULC indicates that, during ILC the vehicles undergo a prolonged exposure to risk compared to ULC. Similarly, average TI-ACT values indicate that the crash severity is significantly higher for ILC compared to ULC.

Although the distributions for AN corresponding to ILC and ULC are statistically different, their average values do not give any insight into the driver discomfort levels. Therefore, the present study considered another measure to quantify the driver’s discomfort during LC. Notably, a part of the discomfort during LC could have resulted from excessive braking or steering. Braking events can be quantified as the number of sign changes in the longitudinal acceleration profile. In contrast, the steering events can be quantified as the number of sign changes in the lateral acceleration profile. Ideally, in the case of ULC, it was expected to have a single sign change for both the lateral and longitudinal accelerations. While for ILC, there can be multiple sign changes in both lateral and longitudinal acceleration profiles, due to the driver’s evasive actions to reduce the perceived risk. Figure 8(a) presents the average number of sign changes for lateral and longitudinal acceleration profiles corresponding to ILC and ULC. Similarly, Figure 8(b) presents the modes corresponding to the number of sign changes. It is evident from these figures that, during ILC, the vehicle undergoes highest number of sign changes, for both the lateral and longitudinal accelerations. This indicates that during ILC, there would be a frequent braking and steering to ensure safe and comfortable LC operation. Nevertheless, it is most likely that the ULC to have a single sign change.

### 6.2. Duration of ILC and ULC

Further, the LC durations corresponding to both ILC and ULC were compared. Many studies have shown that LC duration follows a Log-Normal distribution (Wang, Li, and Li 2014; Toledo and Zohar 2007; Balal et al. 2014). In line with the literature, this study found that the LC duration for both ILC and ULC follows Log-Normal distribution, though with different parameters. The duration histogram and a probability plot with 95% confidence bound is shown in Figure 9(c and d), respectively for ULC and ILC. The duration distribution parameters of ULC and ILC and the test statistics are shown in Tables 3 and 4, respectively. From the analysis, it is evident that the LC duration for ILC and ULC is statistically different. Therefore, it is inevitable to model the duration corresponding to ILC and ULC separately.
Figure 8. (a) Average number of sign changes in the lateral and longitudinal acceleration profiles; (b) Mode of number of sign changes in the lateral and longitudinal acceleration profiles.

Figure 9. LC duration distribution: (a) ILC; (b) ULC; (c) probability plot of Log-Normal distribution of ILC duration; (d) probability plot of Log-Normal distribution of ULC duration.

6.2.1. Lane-change duration modeling
Since the duration is non-negative, the present study has considered the Log-Linear function to model the LC duration.

$$\ln(\Delta t_{LC}) = \beta X + \epsilon$$  \hspace{1cm} (11)

where,

$\Delta t_{LC}$ = Lane change duration

$X$ = Vector of explanatory variables

$\beta$ = Coefficient vector

$\epsilon$ = Error term
Now, the main challenge is the selection of the explanatory variables $X$ to develop the duration model. It is evident from Figure 10 that the LC vehicle has a direct interaction with multiple vehicles (leader and follower vehicles in the current and target lanes). Table 5 shows the characteristics of the surrounding vehicles commonly considered in the LC studies. The descriptive statistics of the variables presented in this table indicate a wide range of conditions under which the LC maneuvers were observed.

This study considers the stepwise multiple linear regression to identify the most significant variables for the LC duration modeling. The $p$-value of an $F$-statistic was used to test the models with and without a potential explanatory variable. The multi-collinearity of the explanatory variables was also checked using the Variable Inflation Factor (VIF). The most commonly used rule of thumb, $VIF \geq 10$, was used to avoid the possible cases of multi-collinearity (O'brien 2007). Considering the most significant explanatory variables, a multiple linear regression was performed to estimate the model parameters. Table 6 presents the test statistics and the model parameters corresponding to ULC, ILC, and combined LC duration models. ANOVA was performed to test the significance of these models. The test results show that all the models are statistically significant at 1% significance.

Though a few explanatory variables are common among the models, each model is different, as indicated by the standardized coefficients. In the case of ULC, the gap between the lead vehicles in the current and target lanes contributes more to the prediction. This variable has a positive weight in the model, indicating that if there is a more gap available in the target lane, the LC vehicle takes relatively more time in the case of ULC. However, the lag vehicle’s speed in the target lane is negatively correlated with the LC duration. This indicates that the driver would be cautious about the lag vehicle’s speed in the target lane, thus completes LC operation quickly. Similarly, the lag vehicle’s acceleration in the current lane also shows a negative correlation with the ULC duration. This could be due to the LC vehicle’s perception of a relatively higher risk of being in the current lane compared to the target lane. However, the lag vehicle’s speed in the target lane is negatively correlated with the LC duration. This indicates the driver would be cautious about the lag vehicle’s speed in the target lane, thus completes LC operation quickly. Similarly, the lag vehicle’s acceleration in the current lane also shows a negative correlation with the ULC duration. This could be due to the LC vehicle’s perception of a relatively higher risk of being in the current lane compared to the target lane. Therefore performs an LC quickly.

In the case of ILC, the LC vehicle’s speed contributing more to the prediction. Moreover, the LC vehicle’s speed correlated negatively to the LC duration. This can be explained with the following hypothetical situation. When the LC vehicle changes lanes with a slower speed, the lag vehicle in the target lane may not yield, assuming that the LC vehicle enters the target lane only after his/her passage. This would lead to a longer LC duration since the

**Figure 10.** Schematic diagram of a lane-changing process.
Table 5. Descriptive statistics of variables influencing LC duration for ULC and ILC.

| Variables | Mean | Std | Median | Min | Max |
|-----------|------|-----|--------|-----|-----|
| LC duration ($\Delta t_{LC}$ (s)) | 9.63 | 13.43 | 5.26 | 12.55 | 25.70 | 40.50 |
| Speed of LC vehicle ($v_{LC}$ (m/s)) | 9.12 | 9.53 | 3.73 | 9.51 | 18.00 | 21.00 |
| Desired speed of LC vehicle ($v_{d,LC}$ (m/s)) | 12.83 | 13.21 | 3.72 | 12.96 | 26.77 | 24.80 |
| Normalized speed of LC vehicle ($v_{normalized,LC}$) | 0.71 | 0.72 | 0.21 | 0.00 | 1.66 | 1.23 |
| Acceleration of LC vehicle ($a_{LC}$ (m/s²)) | 0.15 | 0.12 | 0.56 | 0.16 | 2.00 | 2.65 |
| Speed of lead vehicle in the current lane ($v_{curr,lead}$ (m/s)) | 9.07 | 9.44 | 3.58 | 9.29 | 21.31 | 21.48 |
| Acceleration of lead vehicle in the current lane ($a_{lead,curr}$ (m/s²)) | 0.06 | 0.02 | 0.49 | 0.07 | 2.09 | 1.55 |
| Spacing with lead vehicle in the current lane ($\Delta x_{curr,lead}$ (m)) | 6.22 | 6.59 | 3.50 | 5.92 | 36.65 | 33.25 |
| Relative speed with lead vehicle in the current lane ($\Delta v_{curr,lead}$ (m/s)) | 0.02 | 0.03 | 0.39 | 0.01 | 1.94 | 2.05 |
| TTC with the lead vehicle in the current lane ($TTC_{curr,lead}$ (m/s)) | 76.75 | 77.12 | 161.21 | 29.96 | 1405.00 | 2633.00 |
| Speed of lag vehicle in the current lane ($v_{curr,lag}$) | 8.89 | 9.14 | 3.49 | 8.99 | 17.11 | 17.92 |
| Acceleration of lag vehicle in the current lane ($a_{curr,lag}$) | 0.13 | 0.13 | 0.48 | 0.49 | 2.56 | 1.89 |
| Spacing with lag vehicle in the current lane ($\Delta x_{curr,lag}$ (m)) | 6.84 | 7.14 | 3.31 | 6.49 | 23.61 | 22.67 |
| Relative speed with lag vehicle in the current lane ($\Delta v_{curr,lag}$ (m/s)) | 0.07 | 0.12 | 0.38 | 0.09 | 1.26 | 1.50 |
| TTC with lag vehicle in the current lane ($TTC_{curr,lag}$ (m/s)) | 74.97 | 68.79 | 179.74 | 29.96 | 1333.50 | 1614.75 |
| Speed of lead vehicle in the target lane ($v_{target,lead}$) | 10.35 | 10.20 | 3.63 | 10.08 | 25.30 | 20.00 |
| Acceleration of lead vehicle in the target lane ($a_{target,lead}$) | 0.17 | 0.18 | 0.50 | 0.16 | 1.92 | 1.89 |
| Spacing with lead vehicle in the target lane ($\Delta x_{target,lead}$ (m)) | 5.26 | 6.39 | 4.77 | 6.49 | 24.19 | 44.92 |
| Relative speed with the lead vehicle in the target lane ($\Delta v_{target,lead}$ (m/s)) | -0.38 | -0.20 | 0.74 | 0.01 | 1.85 | 3.51 |
| TTC with the lead vehicle in the target lane ($TTC_{target,lead}$ (m/s)) | 39.89 | 51.50 | 152.49 | 9.27 | 2160.00 | 6878.00 |
| Speed of lag vehicle in the target lane ($v_{target,lag}$) | 9.22 | 9.23 | 3.66 | 9.36 | 2160.00 | 6878.00 |
| Acceleration of lag vehicle in the target lane ($a_{target,lag}$) | 0.21 | 0.19 | 0.53 | 0.16 | 2.76 | 1.75 |
| Spacing with lag vehicle in the target lane ($\Delta x_{target,lag}$ (m)) | 6.55 | 7.15 | 4.53 | 5.92 | 28.46 | 40.83 |
| Relative speed with the lag vehicle in the target lane ($\Delta v_{target,lag}$ (m/s)) | -0.03 | 0.09 | 0.82 | 0.07 | 2.27 | 3.01 |
| TTC with lag vehicle in the target lane ($TTC_{target,lag}$ (m/s)) | 42.76 | 49.78 | 141.31 | 9.27 | 1749.00 | 1739.00 |
| Gap in the target lane ($G_{target}$) | 10.98 | 11.37 | 5.52 | 9.86 | 34.72 | 54.90 |
| Relative speed of lead and lag vehicles in target lane ($\Delta v_{target}$ (m/s)) | 0.34 | 0.29 | 0.55 | 0.31 | 3.32 | 3.06 |
| Gap between lead vehicles in the current and target lanes ($\Delta G_{lead}$) | 1.62 | 1.68 | 5.76 | 7.31 | 31.75 | 30.06 |
### Table 6. Statistics of LC duration models.

| Model | Variables | Unstandardized coefficients | Standardized coefficients | Collinearity statistics |
|-------|-----------|-------------------------------|---------------------------|------------------------|
|       |           | B    | Std. Err. | Beta | t    | Sig. | Tolerance | VIF |
| UL C  | (Constant)| 2.215| .038     |      | 59.036| .000 |          |     |
|       | a_LC     | −.079| .024     | −.167| −3.335| .001 | .858      | 1.166|
|       | Δv_{curr}^lead | −.030| .006     | −.443| −4.662| .000 | .238      | 4.193|
|       | Δv_{lead}^targ | .035| .006     | .664 | 5.467 | .000 | .146      | 6.845|
|       | a_targ^lead | .057| .027     | .108 | 2.129 | .034 | .834      | 1.199|
|       | Δv_{targ}^lag | −.016| .003     | −.225| −4.670| .000 | .924      | 1.082|
|       | v_{targ}^lag | .032| .006     | .744 | 5.616 | .000 | .123      | 8.154|
|       | G_{lead} | .032| .006     | .744 | 5.616 | .000 | .123      | 8.154|
| IL C  | (Constant)| 2.750| .044     |      | 62.371| .000 |          |     |
|       | v_LC     | −.037| .004     | −.437| −8.571| .000 | .619      | 1.614|
|       | Δv_{targ}^lag | .012| .003     | .203 | 4.159 | .000 | .676      | 1.479|
|       | Δv_{targ}^lag | .051| .020     | .130 | 2.525 | .012 | .610      | 1.640|
|       | a_{lead}^targ | −.078| .026     | −.126| −3.029| .003 | .927      | 1.079|
|       | Δv_{lead}^targ | .007| .003     | .164 | 2.848 | .005 | .487      | 2.054|
|       | a_{targ}^lead | .012| .003     | .220 | 3.420 | .001 | .389      | 2.573|
|       | G_{lead} | −.007| .003     | −.143| −2.209| .028 | .387      | 2.585|
|       | Δv_{targ}^lag | .010| .005     | .099 | 2.192 | .029 | .788      | 1.270|
| LC    | (Constant)| 2.579| .036     |      | 71.787| .000 |          |     |
|       | v_{targ}^lag | −.030| .003     | −.299| −8.828| .000 | .789      | 1.267|
|       | Δv_{targ}^lag | .014| .003     | .192 | 4.380 | .000 | .474      | 2.109|
|       | a_{targ}^lag | −.073| .023     | −.099| −3.125| .002 | .907      | 1.102|
|       | Δv_{lead}^targ | .014| .004     | .237 | 3.869 | .000 | .242      | 4.133|
|       | Δv_{lead}^targ | .025| .004     | .371 | 6.774 | .000 | .303      | 3.301|
|       | Δv_{targ}^lag | −.011| .003     | −.181| −3.243| .001 | .291      | 3.437|
|       | a_{lead}^targ | −.010| .005     | −.094| −2.161| .031 | .476      | 2.102|
|       | a_{LC}    | −.062| .023     | −.090| −2.685| .007 | .803      | 1.245|
|       | Δv_{curr}^lag | −.064| .027     | −.075| −2.341| .019 | .878      | 1.139|

Maneuver involves evasive actions to avoid conflicts. But, when the LC vehicle is maneuvering at a higher speed, the lag vehicle would yield to avoid a potential crash; thus, the vehicle can complete the maneuver in a lesser time period. The speed difference with the lag vehicle in the target lane is another critical variable that positively affects the ILC duration. It can be inferred that, as the speed difference increases, the lag vehicle would be more confident about his/her passing, thereby delaying the LC maneuver. Another important observation is the impact of the lead vehicle’s acceleration in the target lane. It is to be noted that the lead vehicle’s acceleration in the target lane has a negative impact on the LC duration. If the lead vehicle in the target lane decelerates at a rate that can cause a potential rear-end collision, the LC vehicle might get interrupted. To avoid an unsafe situation, the LC vehicle is forced to perform evasive actions, leading to higher LC duration. A major observation from
this modeling is that in the case of ILC, the lag vehicles’ kinematics in the target lane mainly control the LC duration. Therefore, it is essential to give more importance to the interaction with the lag vehicle in the target lane while LC trajectory planning for an autonomous system, with proper sensing of the lag vehicle’s kinematics.

In the combined model, the gap with the leader vehicle in the target lane contributes more to the model. Most of the variables in the combined model are part of either the ULC model or the ILC model, except the lag vehicle’s acceleration in the target lane and the relative speed with the lead vehicle in the current lane. RMSE corresponding to the combined model is relatively high, indicating a more extensive scattering of the response variable. To better understand the model performance, the relative importance of the model variables was compared using sensitivity analysis.

Sensitivity analysis gives an idea about how the predicted LC duration’s uncertainty can be divided and allocated to different sources of uncertainty in the explanatory variables. Changing one variable at a time (OAT) to see its effect on the output is one of the easiest and most popular ways to perform sensitivity analysis (Murphy et al. 2004). The other variables are held at their baseline (nominal) while one input variable is changed, and the dependent variable’s change is then evaluated. Figure 11 shows the results of the sensitivity analysis. It is evident from the figure that the response variable’s sensitivity in the duration

![Figure 11. Sensitivity analysis of explanatory variables of LC duration models.](image-url)
models corresponding to ILC, ULC, and combined cases is not identical. The combined model, which is commonly used in the existing practices, leads to an unrealistic prediction of LC duration for the ULC maneuver. For example, the explanatory variables common to all three models, such as the gap with the leader vehicle in the target lane ($\Delta x_{\text{targ}}^{\text{lead}}$) (Figure 11c), and the gap between the lead vehicle in the current and target lanes ($\Delta G_{\text{lead}}$)(Figure 11f), affect the LC duration in a differential manner. Therefore, it can be said that the combined model is biased towards the ILC condition, and is significantly away from ULC.

7. Summary and conclusion

The present research microscopically examines the characteristics of LC dynamics, after classifying LC into ILC and ULC. To that end, the present study proposed a methodology to automatically identify and classify LC operation into ILC and ULC. In addition, the characteristic differences between ILC and ULC were explored by comparing different LC attributes. We pointed out that, intermediate interruptions during an LC maneuver could increase the crash likelihood, driver discomfort, and duration of the operation.

The main conclusions drawn from this study are:

- The method used for detecting the LC window has a significant impact on the estimated LC duration.
- Field data indicates that the LC process was not always continuous. The surrounding vehicles interrupt the LC vehicle, which can be seen from the lateral position, speed, and acceleration profiles. We recommend a classified analysis of LC to replicate the real field scenario in LC models.
- The interruption level in terms of extra cumulative distance traversed by an LC vehicle in the acceleration and deceleration phases can be used as a criterion for classifying LC scenarios into ILC and ULC.
- Following are the characteristics of ILC and ULC:
  1. Compared to ULC, ILC has a higher crash likelihood. The average risk exposure measured using TE-ACT was found higher during ILC. On an average, during ILC, the vehicle exposed to unsafe situation for 9.3 sec, whereas during ULC it is 3.1 s.
  2. Similarly, when compared the TI-ACT values, the crash severity was also found higher for ILC. On an average, during ILC, the crash severity is 17.9 s$^2$, whereas for ULC it is 9.8 s$^2$. The crashes during ILC shows a severity level double as that of ULC.
  3. During ILC, the drivers undergo higher discomfort owing to the frequent braking and/or steering operations.
  4. The time needed to complete ILC was found higher than that for ULC. Average duration of ILC is 13.43 s, whereas for ULC it is 9.63 s. The duration distributions corresponding to ILC and ULC were statistically different. Since duration is an important variable in most LC models, separate duration models for ILC and ULC needs to be considered for a better representation of the ground truth.
  5. While probabilistic estimation of overall risk during LC activity, it is essential to consider corresponding risk and discomfort probabilities for ILC and ULC.
  6. From the duration models developed in this study, explanatory variables for ILC and ULC were found to be different. Also, the sensitivity of common explanatory variables in ILC and ULC models was not the same.
A major observation from the LC duration modeling is that, the lag vehicles’ kinematics in the target lane mainly control the LC duration for ILC. Therefore, it is essential to give more importance to the interaction with the lag vehicle in the target lane while LC trajectory planning for an autonomous system, with proper sensing of the lag vehicle’s kinematics.

The results from this research illustrate the need for separate analyses of ILC and ULC dynamics in LC studies. Such consideration would enhance LC simulations’ accuracy, collision alert systems, and the performance of human-like autonomous LCs. Nevertheless, a comprehensive ILC investigation with more data is necessary, which is the future scope of this study.

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References

Ahmed, Ishtiak, Alan Karr, Nagui M. Roupail, Gyounghoon Chun, and Shams Tanvir. 2019. “Characterizing Lane Changes Via Digitized Infrastructure and Low-Cost GPS.” Transportation Research Record: Journal of the Transportation Research Board 2673 (8): 298–309. https://doi.org/10.1177%2F0361198119841277

Balal, Esmaeil, Ruey Long Cheu, Thompson Gyan-Sarkodie, and Jessica Miramontes. 2014. “Analysis of Discretionary Lane Changing Parameters on Freeways.” International Journal of Transportation Science and Technology 3 (3): 277–296. https://doi.org/10.1260%2F2046-0430.3.3.277

Balal, Esmaeil, Ruey Long Cheu, and Thompson Sarkodie-Gyan. 2016. “A Binary Decision Model for Discretionary Lane Changing Move Based on Fuzzy Inference System.” Transportation Research Part C: Emerging Technologies 67: 47–61. https://doi.org/10.1016%2Fj.trc.2016.02.009

Berger, Niels. 2018. “Lane Change Path Planning: With State-Dependent Safety Constraints.” http://resolver.tudelft.nl/uuid:03aa26dd-e7e6-4ebb-9cd3-def3e169526f.

Ferrari, Paolo. 1989. “The Effect of Driver Behaviour on Motorway Reliability.” Transportation Research Part B: Methodological 23 (2): 139–150. https://doi.org/10.1016%2F0191-2615%2889%2990037-4

Gipps, P. G. 1986. “A Model for the Structure of Lane-changing Decisions.” Transportation Research Part B: Methodological 20 (5): 403–414. https://doi.org/10.1016%2F0191-2615%2886%2990012-3

Hamdar, Samer H., and Hani S. Mahmassani. 2009. “Life in the Fast Lane.” Transportation Research Record: Journal of the Transportation Research Board 2124 (1): 89–102. https://doi.org/10.3141%2F2124-09

Hidas, Peter. 2002. “Modelling Lane Changing and Merging in Microscopic Traffic Simulation.” Transportation Research Part C: Emerging Technologies 10 (5–6): 351–371. https://doi.org/10.1016%2Fs0968-090x%2802%2900026-8

Jin, Wen-Long. 2010. “A Kinematic Wave Theory of Lane-changing Traffic Flow.” Transportation Research Part B: Methodological 44 (8–9): 1001–1021. https://doi.org/10.1016/j.trb.2009.12.014

Kesting, Arne, Martin Treiber, and Dirk Helbing. 2007. “General Lane-Changing Model MOBIL for Car-Following Models.” Transportation Research Record: Journal of the Transportation Research Board 1999 (1): 86–94. https://doi.org/10.3141%2F1999-10

Keyvan-Ekbatani, Mehdi, Victor L. Knoop, and Winnie Daamen. 2016. “Categorization of the Lane Change Decision Process on Freeways.” Transportation Research Part C: Emerging Technologies 69: 515–526. https://doi.org/10.1016%2Fj.trc.2015.11.012

Ko, Joonho, Randall Guensler, and Michael Hunter. 2010. “Analysis of Effects of Driver/vehicle Characteristics on Acceleration Noise Using GPS-equipped Vehicles.” Transportation Research Part F: Traffic Psychology and Behaviour 13 (1): 21–31. https://doi.org/10.1016%2Fj.trf.2009.09.003
Li, Hongluo, Yutao Luo, and Jie Wu. 2019. “Collision-Free Path Planning for Intelligent Vehicles Based on Bézier Curve.” IEEE Access 7: 123334–123340. https://doi.org/10.1109%2Faccess.2019.2938179

Luo, Yugong, Yong Xiang, Kun Cao, and Keqiang Li. 2016. “A Dynamic Automated Lane Change Maneuver Based on Vehicle-to-vehicle Communication.” Transportation Research Part C: Emerging Technologies 62: 87–102. https://doi.org/10.1016%2Fj.trc.2015.11.011

Meng, Fanlin, Jinya Su, Cunjia Liu, and Wen-Hua Chen. 2016. “Dynamic Decision Making in Lane Change: Game Theory with Receding Horizon.” In 2016 UKACC 11th International Conference on Control (CONTROL). IEEE. https://doi.org/10.1109%2Fcontrol.2016.7737643.

Mirchevska, Branka, Christian Pek, Moritz Werling, Matthias Althoff, and Joschka Boedecker. 2018. “High-level Decision Making for Safe and Reasonable Autonomous Lane Changing using Reinforcement Learning.” In 2018 21st International Conference on Intelligent Transportation Systems (ITSC). IEEE. https://doi.org/10.1109%2Fitsc.2018.8569448.

Monk, Christopher A., Deborah A. Boehm-Davis, George Mason, and J. Gregory Trafton. 2004. “Recovering From Interruptions: Implications for Driver Distraction Research.” Human Factors: The Journal of the Human Factors and Ergonomics Society 46 (4): 650–663. https://doi.org/10.1518%2Fhfes.46.4.650.56816

Mullakkal-Babu, Freddy Antony, Meng Wang, Bart van Arem, and Riender Happée. 2020. “Empirics and Models of Fragmented Lane Changes.” IEEE Open Journal of Intelligent Transportation Systems 1: 187–200. https://doi.org/10.1109%2Fojits.2020.3029056

Murphy, James M., David M. H. Sexton, David N. Barnett, Gareth S. Jones, Mark J. Webb, Matthew Collins, and David A. Stainforth. 2004. “Quantification of Modelling Uncertainties in a Large Ensemble of Climate Change Simulations.” Nature 430 (7001): 768–772. https://doi.org/10.1038%2Fnature02771

O’Brien, Robert M. 2007. “A Caution Regarding Rules of Thumb for Variance Inflation Factors.” Quality & Quantity 41 (5): 673–690. https://doi.org/10.1007%2Fs11135-006-9018-6

Olsen, Erik C. B., Suzanne E. Lee, Walter W. Wierwille, and Michael J. Goodman. 2002. “Analysis of Distribution, Frequency, and Duration of Naturalistic Lane Changes.” Proceedings of the Human Factors and Ergonomics Society Annual Meeting 46 (22): 1789–1793. https://doi.org/10.1177%2F154193120204602203

Park, Hyunjin, Cheol Oh, Jaepil Moon, and Seongho Kim. 2018. “Development of a Lane Change Risk Index Using Vehicle Trajectory Data.” Accident Analysis & Prevention 110: 1–8. https://doi.org/10.1016/j.aap.2017.10.015

Qiang Liu, Zhi, Teng Zhang, and Yi Fan Wang. 2019. “Research on Local Dynamic Path Planning Method for Intelligent Vehicle Lane-Changing,” Journal of Advanced Transportation 2019: 1–10. https://doi.org/10.1155%2F2019%2F4762658

Thiemann, Christian, Martin Treiber, and Arne Kesting. 2008. “Estimating Acceleration and Lane-Changing Dynamics From Next Generation Simulation Trajectory Data.” Transportation Research Record: Journal of the Transportation Research Board 2088 (1): 90–101. https://doi.org/10.3141/2088-10

Toledo, Tomer, Haris N. Koutsopoulos, and Moshe E. Ben-Akiva. 2003. “Modeling Integrated Lane-Changing Behavior.” Transportation Research Record: Journal of the Transportation Research Board 1857 (1): 30–38. https://doi.org/10.3141%2F1857-04

Toledo, Tomer, and David Zohar. 2007. “Modeling Duration of Lane Changes.” Transportation Research Record: Journal of the Transportation Research Board 1999 (1): 71–78. https://doi.org/10.3141%2F1999-08

van Winsum, W., D. de Waard, and K. A. Brookhuis. 1999. “Lane Change Maneuuvres and Safety Margins.” Transportation Research Part F: Traffic Psychology and Behaviour 2 (3): 139–149. https://doi.org/10.1016%2Fs1369-8478%2899%2900011-x

Venthuruthiyil, Suvin P., and Mallikarjuna Chunchu. 2018. “Trajectory Reconstruction Using Locally Weighted Regression: a New Methodology to Identify the Optimum Window Size and Polynomial Order.” Transportmetrica A: Transport Science 14 (10): 881–900.

Venthuruthiyil, Suvin P., and Mallikarjuna Chunchu. 2020. “Vehicle Path Reconstruction Using Recursively Ensembled Low-pass Filter (RELP) and Adaptive Tri-cubic Kernel Smoother.” Transportation Research Part C: Emerging Technologies 120: 102847. https://doi.org/10.1016/j.trc.2020.102847
Venthuruthiyil, Suvin P., and Mallikarjuna Chunchu. 2021. “Anticipated Collision Time (ACT): A Two-Dimensional Surrogate Safety Indicator for Trajectory-Based Proactive Safety Assessment.” *Transportation Research Part C: Emerging Technologies*. Under review.

Wang, Qi, Zhiheng Li, and Li Li. 2014. “Investigation of Discretionary Lane-Change Characteristics Using Next-Generation Simulation Data Sets.” *Journal of Intelligent Transportation Systems* 18 (3): 246–253. https://doi.org/10.1080%2F15472450.2013.810994

Wang, Chang, Qinyu Sun, Zhen Li, and Hongjia Zhang. 2020. “Human-Like Lane Change Decision Model for Autonomous Vehicles that Considers the Risk Perception of Drivers in Mixed Traffic.” *Sensors* 20 (8): 2259. https://doi.org/10.3390%2Fs20082259

Wang, Junjie, Qichao Zhang, Dongbin Zhao, and Yaran Chen. 2019. “Lane Change Decision-making through Deep Reinforcement Learning with Rule-based Constraints.” In *2019 International Joint Conference on Neural Networks (IJCNN)*, IEEE. https://doi.org/10.1109%2Fijcnn.2019.8852110.

Yang, Minming, Xuesong Wang, and Mohammed Quddus. 2019. “Examining Lane Change Gap Acceptance, Duration and Impact Using Naturalistic Driving Data.” *Transportation Research Part C: Emerging Technologies* 104: 317–331. https://doi.org/10.1016%2Fj.trc.2019.05.024

Yang, Da, Shiyu Zheng, Cheng Wen, Peter J. Jin, and Bin Ran. 2018. “A Dynamic Lane-changing Trajectory Planning Model for Automated Vehicles.” *Transportation Research Part C: Emerging Technologies* 95: 228–247. https://doi.org/10.1016/j.trc.2018.06.007

Zeng, Dequan, Zhuoping Yu Lu Xiong, Junqiao Zhao, Peizhi Zhang, Zhiqiang Li Zhiqiang Fu, Jie Yao, and Yi Zhou. 2019. “A Novel Robust Lane Change Trajectory Planning Method for Autonomous Vehicle.” In *2019 IEEE Intelligent Vehicles Symposium (IV)*, IEEE. https://doi.org/10.1109%2Fivs.2019.8814151.