Electron spin relaxation in \(n\)-type InAs quantum wires

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We investigate the electron spin relaxation of \(n\)-type InAs quantum wires by numerically solving the fully microscopic kinetic spin Bloch equations with the relevant scattering explicitly included. We find that the quantum-wire size and the growth direction influence the spin relaxation time by modulating the spin-orbit coupling. Due to inter-subband scattering in connection with the spin-orbit interaction, spin-relaxation in quantum wires can show different characteristics from those in bulk or quantum wells and can be effectively manipulated by various means.

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I. INTRODUCTION

Spintronics continues to attract interest because of potential applications to information technology, but also because it has greatly improved our understanding of the role of different spin-dependent interaction mechanisms, i.e., of the spin-orbit coupling, for many-electron systems.2,3 Several spintronic devices have been manipulated that carry the carrier via spin-orbit coupling (SOC),4,5 In recent years, progress in nanofabrication and growth techniques has made it possible to produce high-quality quantum wires (QWRs) and investigate physics in these semiconductor nanostructures.6,7,8,9,10,11,12 The energy spectrum of QWR systems with strong SOC has been studied experimentally13,14,15,16,17,18,19 and theoretically,20,21 For \(p\)-type QWRs, most of the works concentrate on the energy spectrum22,23,24,25,26,27,28,29,30,31 but for \(n\)-type quantum structures, many investigations have been performed with the aim of understanding the electron spin relaxation.32,33,34,35

For QWRs, the spin relaxation time \(T_{SR}\) was measured36 and calculated in the framework of a single-particle model24,27,28,29 and Monte-Carlo simulations.33,34,35 Recently, Liu et al.35 investigated the \(T_{SR}\) for QWRs in the (110) crystal direction, but systematic studies of spin relaxation in \(n\)-type QWRs are scarce. A general way to obtain the nonequilibrium spin dynamics and the spin relaxation time in semiconductors heterostructures has been developed by Wu et al.32,33,34 In this approach, the momentum and spin dependent distribution functions are calculated using microscopic kinetic spin Bloch equations (KSBEs), which include the momentum-dependent Dresselhaus and/or Rashba SOC together with the effect of the relevant scattering mechanisms.32,33,34 Cheng et al. applied this approach to study electron spin relaxation in QWR systems.31 However, the influence of higher subbands was not included in this work. Subsequently, the effects of higher subbands and their coupling via Coulomb scattering were shown to be important for the spin-relaxation of holes in QWRs28 and for the case of electrons in quantum wells.37

In this paper, we study the influence of higher subbands on the spin relaxation in \(n\)-type InAs QWRs. This allows us to investigate QWRs with a wide range of sizes. Especially for larger diameters of the wires, the spin precession and the spin relaxation are expected to show different characteristics from narrow ones, as electrons populate more than one subband and therefore experience different SOCs and undergo intra-subband and inter-subband Coulomb scattering. We find that the inter-subband Coulomb scattering can make an important contribution to the spin relaxation. We also study the influence of the growth direction of the QWRs on the spin relaxation. In particular, we consider QWRs with (001), (110), and (111) growth directions, and show that the SRT depends sensitively on the growth direction, quantum-wire size and the direction of the initial spin polarization.

This paper is organized as follows: In Sec. II we describe our model and the KSBEs. Our numerical results are presented in Sec. III. We conclude in Sec. IV.

II. MODEL AND DYNAMICAL EQUATIONS

We model the InAs QWR by a rectangular confinement potential, i.e., we assume infinitely high barriers at \(x = \pm a_x\) and \(y = \pm a_y\), and no confinement on the \(z\) direction. The Hamiltonian, which describes the electronic single-particle states in the QWR is then taken to include the confinement potential \(V_C\), the Rashba term \(H_R\) and the Dresselhaus term \(H_D\)

\[
H_e = \frac{\mathbf{P}^2}{2m} + H_R + H_D + V_C(\mathbf{r}) . \tag{1}
\]

The Rashba and Dresselhaus terms are the two contributions to the internal \(k\)-dependent effective field, which
leads to the Dyakonov-Perel’ spin dephasing mechanism. The Rashba term

\[ H_R(k) = \gamma_{41}^{\text{R}} \sigma \cdot k \times \mathcal{E} \]

\[ = \gamma_{41}^{\text{R}} [\sigma_x(k_y \mathcal{E}_z - k_z \mathcal{E}_y) + \sigma_y(k_z \mathcal{E}_x - k_x \mathcal{E}_z) + \sigma_z(k_x \mathcal{E}_y - k_y \mathcal{E}_x)] \]

is due to the inversion asymmetry of the crystal structure. The Dresselhaus term is different for different growth directions. For a (100) InAs QWR, the \( x \), \( y \) and \( z \) axes correspond to the [100], [010] and [001] crystallographic directions, respectively, and the Dresselhaus term can be written as:

\[ H_D^{100} = \frac{\gamma_{41}^{\text{D}}}{2} \{ \sigma_x[k_x(k_y^2 - k_z^2)] + \sigma_y[k_y(k_z^2 - k_x^2)] + \sigma_z[k_z(k_x^2 - k_y^2)] \} . \tag{2} \]

For a (110) QWR, the \( x \), \( y \) and \( z \) directions correspond to the [110], [010] and [110] crystallographic directions, and we have

\[ H_D^{110} = \frac{\gamma_{41}^{\text{D}}}{2} \{ \sigma_x[-\frac{1}{2}k_z(k_y^2 - k_x^2 + 2k_y^2)] + 2\sigma_yk_yk_z + \sigma_z\frac{1}{2}k_z(k_x^2 - k_z^2) \} . \tag{3} \]

For a (111) QWR, the \( x \), \( y \) and \( z \) directions correspond to the [112], [110], and [111] crystallographic directions, and we have

\[ H_D^{111} = \frac{\gamma_{41}^{\text{D}}}{2} \{ \sigma_x[\frac{\sqrt{2}}{3}k_xk_yk_z - \frac{1}{2\sqrt{3}}k_y^3 - \frac{1}{2\sqrt{3}}k_yk_x^2 + \frac{2}{\sqrt{3}}k_yk_z^2 - \frac{\sqrt{2}}{3}k_y^3] + \sigma_y[\frac{1}{2\sqrt{3}}k_z^3 \]

\[ + \frac{1}{2\sqrt{3}}k_xk_y^2 - \frac{1}{\sqrt{6}}k_z^2k_y - \frac{1}{\sqrt{6}}k_y^2k_z + \sigma_z[\frac{\sqrt{3}}{2\sqrt{2}}k_y^3k_z - \frac{1}{\sqrt{6}}k_z^2k_y - \frac{2}{3}k_z^3k_y] \} . \tag{4} \]

As input for the KSBEs we use the basis \{\( \phi_{n_x,n_y,s} \)} of single-particle states, which are obtained from the eigenfunctions of \( p_x^2 + V_{\text{r}}(x) \). For a hard-wall confinement potential which constricts the electrons in the \( x \) and \( y \) directions on mesoscopic length scales, we can employ the envelope function approximation.\(^{39}\) Thus we write the single-particle states in the form

\[ \phi_{n_x,n_y,s} = \psi_{n_x,n_y}(x,y)e^{ik_x} \chi_s \]

with

\[ \psi_{n_x,n_y}(x,y) = \frac{2}{\sqrt{a_xa_y}} \sin\left(\frac{n_x \pi x}{a_x}\right) \sin\left(\frac{n_y \pi y}{a_y}\right) , \tag{5} \]

where \( \chi_s \) denotes the basis vectors in spin space, i.e., eigenstates of \( \sigma_z \). In the envelope function approximation, the effective Hamiltonian acting on the single-particle states is obtained from Eqs. \( 2 \)\(^{34}\) by the replacements \( k_x \rightarrow \langle \psi_{n_x}|k_x|\psi_{n_x'} \rangle = \langle k_x \rangle_{n_x,n_x'} \) and \( k_x^2 \rightarrow \langle k_x^2 \rangle_{n_x,n_x'} \).

The complete dynamical information about spin-dependent single-particle properties is contained in the spin-density matrix \( \rho \). Its matrix elements, in general, defined with respect to the complete set of quantum numbers \( n_x, n_y, k, \sigma \), but because of the translation symmetry in \( z \)-direction, \( \rho \) is diagonal in \( k \), i.e., \( \rho = \rho_{\text{coh}} + \rho_{\text{scat}} \).

We construct the KSBEs by the non-equilibrium Green function method as follows:\(^{32,33,40}\)

\[ \frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial t} \Big|_{\text{coh}} + \frac{\partial \rho}{\partial t} \Big|_{\text{scat}} . \tag{6} \]

The coherent terms can be written as

\[ \frac{\partial \rho_k}{\partial t} \Big|_{\text{coh}} = -i \sum_{\mathcal{Q}} \mathcal{V}\mathcal{Q} \rho_{k-\mathcal{Q}} - \mathcal{I}_{\mathcal{Q}} . \tag{7} \]

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where \( \mathcal{V} = [A, B] = AB - BA \) denotes the commutator, and \( \mathcal{Q} = (q_x, q_y, q) \). \( \mathcal{I}_{\mathcal{Q}} \) is a matrix in \((s,s')\) space and can be considered as a form factor. Its definition reads

\[ \mathcal{I}_{Q,s_1,s_2} = \langle s_1 | e^{iQ\tau} | s_2 \rangle = \delta_{s_1,s_2} F(m_1, m_2, q_y, a_y)F(n_1, n_2, q_x, a_x) \]

where

\[ F(m_1, m_2, q_y, a_y) = \frac{4i \hbar n\gamma_s a_x}{a_x} (1 - \delta_{m,m'}) \]

\[ \langle k_x \rangle = \frac{\hbar^2 \pi^2 n_x^2}{a_x^2} \sin(\pi x/a_x) \]

\[ \langle k_y^2 \rangle = \frac{\hbar^2 \pi^2 n_x^2}{a_x^2} \sin(\pi x/a_x) \]

\[ \langle k_x^2 \rangle_{n_x,n_x'} = \frac{4i \hbar n\gamma_s a_x}{a_x} (1 - \delta_{n,n'}) \]

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\[ \langle k_x^2 \rangle_{n_x,n_x'} = \frac{4i \hbar n\gamma_s a_x}{a_x} (1 - \delta_{n,n'}) \]
The first term in Eq. (10) is the Coulomb Hartree-Fock term, and the second term is the contribution from the single-particle Hamiltonian, i.e., Eq. (1) in (s, s') space, which includes the spin-orbit coupling terms. For small spin polarization, the contribution from the Hartree-Fock term in the coherent term is negligible and the coherent spin dynamics is essentially due to the spin precession around the effective internal fields described by Eqs. (2)-(5). The scattering contributions to the dynamic equation of the spin-density matrix include scatterings between electrons and nonmagnetic impurities, electrons and phonon, and electron and electron scatterings:

\[
F(m_1, m_2, q, a) = 2iaq[e^{i q \cdot \mathbf{r}} + e^{-i q \cdot \mathbf{r}}] 
\]

(12)

in which \(T_{s_1,s_2} = \delta_{s_1,s_2}.\) The statically screened Coulomb potential in the random-phase approximation (RPA) reads

\[
V_q = \sum_{q+q_y} v_q |I_q|^2 / \kappa(q), \quad \kappa(q) = 1 - \sum_{q+q_y} v_q |I_q|^2 \frac{f_{k+q} - f_k}{\epsilon_{k+q} - \epsilon_k}
\]

(13)

with the bare Coulomb potential \(v_q = 4\pi e^2/Q^2\) and

\[
\kappa(q) = 1 - \sum_{q+q_y} v_q |I_q|^2 \left[ \frac{f_{k+q} - f_k}{\epsilon_{k+q} - \epsilon_k} \right]
\]

(14)

In Eq. (14), \(N_i\) is the density of impurities, and \(|U_i|^2\) is the impurity potential. Further, \(|M_{Q,Q'}|^2\) and \(N_{Q,Q'} = \exp(\omega_{Q,Q'}/k_B T) - 1^{-1}\) are the matrix element of the electron-phonon interaction and the Bose distribution function, respectively. The phonon energy spectrum for phonon mode \(Q\) and wavevector \(Q\) is denoted by \(\omega_{Q,Q'}\). For the electron-phonon scattering, we include the electron LO-phonon and electron AC-phonon scattering, for which the explicit expressions can be found in Refs. 33 and 34. Note that, due to the weakness of the SOC, the energy dispersions \(E_{s,k}\) in the scattering terms, Eq. (13), are taken from the diagonalization of \(\frac{p^2}{2m^2} + V_e(\mathbf{r})\) and do not include the SOC, as in Ref. 35. This is different from our previous work in \(p\)-type QWR systems. For holes, the SOC is stronger because holes are derived from \(p\)-orbitals, which experience the spin-orbit interaction directly, and it is therefore necessary to include the SOC in the single-particle energy dispersions.35,36,43

III. NUMERICAL RESULTS

We numerically solve the KSBEs for the spin density matrix \(\rho\), from which we obtain the dynamics of the average spin for electrons with momentum \(k\) via

\[
\langle S \rangle_k(t) = \sum_{\sigma_1,\sigma_2} \rho_{\sigma_1,\sigma_2,\sigma_1,\sigma_2}(t) \langle \sigma | \sigma \rangle,
\]

(15)

where \(\sigma\) is the single-particle spin operator. The SRT \(\tau\) can then be defined by an exponential fit to the envelope of the \(z\) component of the average spin of the ensemble of electrons:\n
\[
\langle S \rangle_z = \sum_k \langle (S)_k \rangle_z(t).
\]

(16)
In all the numerical results, we include the electron-phonon and electron-electron scattering. As initial condition we assume a spin polarization along the z-direction with a small initial spin polarization \( P = (2/h)\langle S \rangle_z/N_e \) where \( N_e \) is the total electron density.

**TABLE I: Material parameters used in the calculation (from Ref. [14] unless otherwise specified).**

| \( \kappa_{ac} \) | 12.25 | \( \kappa_0 \) | 15.15 |
| \( m_e/m_0 \) | 0.023 | \( \Omega_{ac} \) (meV) | 27.0 |
| \( v_{st} \) (km/s) | 4.28 | \( v_{st} \) (km/s) | 1.83 |
| \( \gamma^{Gic}_{xx} \) (e\( \cdot \)Å\(^3\)) | 27.18\(^a\) | \( \gamma^{Gic}_{xx} \) (e\( \cdot \)Å) | 117.1\(^a\) |
| \( \Xi \) (eV) | 5.8 | \( \epsilon_{14} \) (V/m) | 0.35 \times 10^9 |
| \( \Delta_0 \) (eV) | 0.38 | \( E_p \) (eV) | 0.414 |

\(^a\) Ref. [28].

![Fig. 1: SRT \( \tau \) vs. the QWR width in y direction, \( a_y \), for (100) QWRs at different \( a_x \). The electron density is \( N = 4 \times 10^5 \) cm\(^{-1}\) and \( T = 100 \) K. The arrows mark the densities at which the electron populations in the second and higher subbands are approximately 30 %.](image1)

**A. Influence of the wire size**

In Fig. 1 we plot the SRT as a function of the QWR width in y direction, \( a_y \), for various \( a_x \). We choose (100) QWRs at a lattice temperature of \( T = 100 \) K and a total electron density of \( N_e = 4 \times 10^5 \) cm\(^{-1}\). For a small QWR of width \( a_x = 40 \) nm, the SRT increases monotonously with \( a_y \) because for a small QWR the spacing of the subbands is large, so that for the present conditions only the lowest subband in QWR is appreciably populated. Therefore, as discussed in Sec. II, the effective magnetic field due to the Dresselhaus term contains a longitudinal component, \( B^D_{\parallel}(k) \), that keeps the electronic spins aligned and thus inhibits spin precession, which is mainly due to the Rashba term, thereby effectively reducing the spin relaxation.\(^2\)

An interesting effect arises because \( B^D_{\parallel}(k) \) is proportional to \( (\langle k_x^2 \rangle - \langle k_z^2 \rangle) \), which disappears when the wire widths, and therefore in the present model also the confinement wave functions, in \( x \) and \( y \) directions are identical. For vanishing \( B^D_{\parallel}(k) \), i.e., for \( a_x = a_y \), the SRT reaches a minimum of several 10 ps for the QWRs considered here. Changing the wire size, leads to increasing \( (\langle k_x^2 \rangle - \langle k_z^2 \rangle) \) and therefore to increasing SRT. This is clearly illustrated for the \( a_x = 50 \) nm case. When \( a_x = 60 \) nm, one can also find a minimum of SRT at \( a_x = a_y = 60 \) nm for the reason described above. However, one finds that the SRT decreases with \( a_y \) again when \( a_y > 70 \) nm. This is because the electrons start to populate higher subbands when the wire width increases. Then the effective magnetic field formed by the Dresselhaus term contains not only a longitudinal component but also a transverse component, \( B^D_{\parallel}(k) \), which couples different subbands. This coupling has the same effect as a spin precession, and therefore tends to make the SRT shorter. Moreover, the contribution of the Dresselhaus term becomes as important as the contribution of the Rashba term. When \( a_y > 70 \) nm, \( B^D_{\parallel}(k) \) dominates over \( B^D_{\parallel}(k) \) due to the increasing \( a_y \), and a faster spin relaxation results. The different minima of the SRT at \( a_x = a_y = 60 \) nm and \( a_x = a_y = 50 \) nm are likely due to the different energy gaps between different subbands in the two cases.

**B. (110) and (111) QWRs**

![Fig. 2: SRT \( \tau \) vs. \( E_x \) for (110) QWRs at \( T = 50 \) K and \( N_e = 4 \times 10^5 \) cm\(^{-1}\). ▲: \( a_x = a_y = 30 \) nm with an initial spin polarization along the x-direction; ■: \( a_x = a_y = 30 \) nm with an initial spin polarization along the y-direction; ●: \( a_x = a_y = 50 \) nm with an initial spin polarization along the x-direction.](image2)
the SRT because it influences the different contributions to the SOC. We now investigate QWRs with different growth directions, and start with the case of (110) QWRs. The SOC for (110) QWR is quite different from the SOC for (100) QWR as shown in Eq. (3) and Eq. (4). First we only consider the case of a narrow wire with \( a_x = a_y = 30 \) nm, for which the electronic population is mainly in the lowest subband. In the presence of an electric field of the form \((E_x, E_y, 0)\), the relevant contributions are

\[
H^{110}_R = \gamma_{41}^{06c} \sigma x E_y k_z + \sigma y E_x k_z, \quad \text{(18)}
\]

\[
H^{110}_D = -\frac{1}{2} \gamma_{41}^{06c} \sigma x k_z [(k_x^2) - k_z^2 + 2(k_y^2)]. \quad \text{(19)}
\]

The effective magnetic field formed by the Dresselhaus term is along the \( x \)-direction, which corresponds to the [110] crystallographic direction, and the effective magnetic field formed by the Rashba term is in the \( x \)-\( y \) plane. If the direction of the total effective magnetic field formed by the SOC is tuned to be exactly the direction of the initial spin polarization, then one can expect an extremely long SRT as pointed out in Refs. 25 and 33. For the QWRs considered in this paper, we can study the physics that gives rise to this effect in the following way: We take the initial spin polarization to be along the \( x \)-direction. In Fig. 2 we plot the SRT as a function of \( E_x = E \cos \theta \) for temperature \( T = 50 \) K, electron density \( N_x = 4 \times 10^8 \) cm\(^{-1}\), and wire geometry \( a_x = a_y = 30 \) nm. The SRT decreases with \( E_x \), which is a measure of the effective magnetic field along \( y \)-direction. As expected, a very long SRT results when \( E_x \) is very small, even though the effective magnetic field along the \( x \)-direction is not zero. In comparison, for an initial spin polarization along the \( y \)-direction a much shorter SRT is obtained, which hardly changes with \( E_x \). This is because the polarized electronic spins precess around the \( x \)-direction and this precession is not influenced strongly by the \( y \)-component of the effective magnetic field. In the inset of Fig. 2, we also show the dependence of the SRT on the angle \( \theta \), which the electric field in the \( x \)-\( y \) plane forms with the \( x \) axis. Assuming \( E = 10 \) kV/cm and an initial spin polarization along the \( x \)-direction, it is found that the SRT decreases with \( \theta \). For small \( \theta \), i.e., for effective magnetic fields close to the \( x \)-direction, the SRT goes up, in agreement with the previous discussion. The result for a larger wire size, \( a_x = a_y = 50 \) nm are also plotted in Fig. 2. In this case the SRT is never longer than 1 ns, even when \( E_x \) is very small because now electrons populate higher subbands, and the effective magnetic field formed by the Dresselhaus term contains not only the longitudinal component but also a transverse component.

We compare the SRT obtained by our calculation with Monte-Carlo results from Ref. 25 in Fig. 3, for a temperature of \( T = 5 \) K. The total electron density and the impurity density are taken to be \( N_e = 4 \times 10^8 \) cm\(^{-1}\) and \( N_i = 3.17 \times 10^6 \) cm\(^{-1}\), and the wire geometry is \( a_x = a_y = 1 \) \( \mu \)m. The results without the Coulomb scattering are actually very close to those obtained by Monte-Carlo simulations. However, when the Coulomb scattering is included, much longer SRTs result. This result underscores the importance of Coulomb scattering for spin relaxation in QWRs.

For the quantitative analysis of the SRT for another growth direction we choose (111) QWRs. Here, we again consider the case of small wire width and low temperature first. The electric field is again taken to be
(E_x, E_y, 0), so that the SOC can be written as
\[ H^{11}_R = \gamma_{41}^{6\text{sc}} [-\sigma_z E_y k_z + \sigma_y E_x k_z] , \]
\[ H^{11}_D = b_{41}^{6\text{sc}} \left\{ -\frac{\sqrt{2}}{3} \sigma_x k_z \langle k_x^2 \rangle - \frac{1}{\sqrt{6}} \sigma_y k_z \left( \langle k_x^2 \rangle + \langle k_y^2 \rangle \right) - \frac{2}{3} \sigma_z k_z \langle k_y^2 \rangle \right\} . \] (21)

Similar to the case of (110) QWRs, we expect very long SRTs if the total effective magnetic field points into the direction of the initial spin polarization, at least for narrow wires. For a numerical example of this effect, we choose E_y such that \( \gamma_{41}^{6\text{sc}} E_y + \sqrt{2}/3 b_{41}^{6\text{sc}} \langle k_y^2 \rangle = 0 \) for a small wire with \( a_x = a_y = 10 \) nm, so that the x component of the total effective magnetic field is zero. For an initial spin polarization along the z-direction, which corresponds to the [111] crystallographic direction, we plot the SRT as a function of \( E_x \) in Fig. 4 at \( T = 50 \) K and \( N_e = 4 \times 10^5 \) cm\(^{-1}\). Fig. 4 shows that when \( a_x = a_y = 10 \) nm, there is a pronounced maximum of the SRT at \( E_x = 70 \) kV/cm, which fulfills the relation \( \gamma_{41}^{6\text{sc}} E_x + \frac{1}{\sqrt{6}} b_{41}^{6\text{sc}} \langle k_x^2 \rangle \approx 0 \). Consequently, for this field strength, the direction of the total effective magnetic field is exactly along the direction of the initial spin polarization and this leads to a very long SRT. However, this effect much reduced for larger wire cross sections. When \( a_x = a_y = 20 \) nm, there is still a maximum of the SRT, but the maximum is much less pronounced than for the smaller wire, because now \( \gamma_{41}^{6\text{sc}} E_x + \frac{2}{3} b_{41}^{6\text{sc}} \langle k_y^2 \rangle \) remains finite for all field strengths as \( \langle k_y^2 \rangle \) is changed. This trend continues for wire sizes of \( a_x = a_y = 30 \) nm and \( a_x = a_y = 50 \) nm. Finally, we analyze the the case of \( a_x = a_y = 20 \) nm with the initial spin polarization along x-direction: Here one also does not obtain a maximum of the SRT because the direction of the effective magnetic field is no longer identical to the direction of the initial spin polarization.

C. Doping and temperature dependence

Since the population of higher subbands has been shown to play an extremely important role for the SRT, we next analyze how the electronic population can be changed without using a different wire geometry or growth direction, namely by varying the doping density and/or temperature. In Fig. 5(a) we plot the SRT as a function of \( N_e \) for a (100) QWR of size \( a_x = a_y = 50 \) nm, and \( T = 100 \) K. For the full calculation, the SRT decreases with \( N_e \) because more electrons are present at higher momenta and in higher subbands, so that the effective magnetic fields experienced by these electrons are larger. By the DP mechanism, this results in a faster spin relaxation.\(^2\) To investigate how scattering affects the spin relaxation, we first switch off inter-subband electron-phonon scattering, with the result that the SRT becomes shorter. When we switch off the inter-subband electron-electron scattering, the SRT also becomes shorter. This somewhat counterintuitive effect of scattering on the spin dephasing results because the influence of scattering is different for different regimes, i.e., for strong and weak scattering: If we define \( \Omega = H_R + H_D \) and \( \tau^\ast \) to be the effective momentum relaxation time, then \( \Omega \tau^\ast \) is typically much smaller than 1 for the densities considered here \( (\Omega \tau^\ast = 0.03 \text{ at } N_e = 10 \times 10^5 \text{ cm}^{-2} \text{ if } \tau^\ast \text{ contains only the contribution from the inter-subband electron-electron scattering). This means we are in the strong scattering regime where the motional narrowing picture qualitatively describes the dependence of the SRT on the momentum scattering time as SRT \( \approx 1/\tau^\ast \). (Microscopically this means that the scattering is strong enough to prevent significant deviations from isotropic electron distributions, which are caused by the anisotropic SOC contributions.) Thus switching off scattering contributions effectively leads to a longer momentum relaxation time, and therefore to shorter SRTs, while still remaining in the strong scattering regime. Further, our calculation shows that the inter-subband electron-electron scattering is more important than the inter-subband electron-phonon scattering. When we further exclude the intra-
subband electron-electron scattering, it is found that the SRT becomes shorter when \( N_e < 20 \times 10^5 \) cm\(^{-1}\), but becomes longer when \( N_e > 20 \times 10^5 \) cm\(^{-1}\). Here the most dominant scattering process is suppressed, so that with increasing \( N_e \), the motional narrowing regime is left, because electrons occupy states at higher momenta and higher subbands where the SOC contributions rapidly increase. For higher densities, the weak scattering regime is reached where the anisotropy of the SOC contributions becomes dominant. Additional scattering leads to a more efficient dephasing, so that the SRT increases when we switch off scattering contributions.

We also plot the case of larger wire width with \( a_x = a_y = 70 \) nm in Fig. 5(b). Compared to the case of Fig. 5(a), the electrons are populating higher subbands, so that the Dresselhaus term becomes more important. Therefore, the band structure anisotropy will become important at smaller \( N_e \). Fig. 5(b) shows that the result of the full calculation intersects the one without the electron-electron scattering at \( N_e \approx 8 \times 10^5 \) cm\(^{-1}\). For this wire geometry, the result without the inter-subband electron-electron scattering also crosses the full calculation. To see in more detail how the strength of anisotropic SOC contributions affects the spin relaxation, we also plot the result without the Dresselhaus term. For this somewhat artificial case, the SRT becomes much longer. This indicates that the contribution of the Dresselhaus term is very important for this wire width in contrast to the case of smaller wire width in Fig. 5(a), because the transverse component of the effective magnetic field contributed by the Dresselhaus term is important only for the large wire width when higher subbands are populated.

![Figure 6](image)

**FIG. 6:** SRT \( \tau \) vs. the temperature for (100) QWRs at different wire sizes \( a_x = a_y \) at carrier density \( N = 4 \times 10^5 \) cm\(^{-1}\).

Finally we look at the temperature dependence of the SRT at different wire widths in Fig. 6. We choose (100) QWRs at a representative electron density of \( N_e = 4 \times 10^5 \) cm\(^{-1}\). The temperature affects the SRT in two ways: For the smaller wires \( a_x = a_y \leq 50 \) nm the carriers are confined in the lowest subband without populating higher \( k \)-states where the anisotropic SOC contributions to the bandstructure become much stronger. In this strong scattering regime, the behavior can qualitatively be explained by the motional narrowing: Increasing temperature leads to enhanced scattering with phonons and electrons with higher kinetic energies, so that the effective carrier lifetime becomes shorter, and the SRT increases. Around \( a_x = a_y = 60 \) nm and \( T > 150 \) K this behavior crosses over to the weak scattering limit, in which higher subbands are populated, the precession frequencies around the effective internal fields become higher, and additional scattering leads to a more efficient dephasing, so that the SRT decreases with \( T \).

**IV. CONCLUSION**

In conclusion, we have investigated the spin relaxation of electrons in \( n \)-type InAs QWRs. The SRT is calculated by numerically solving the microscopic KSBEs including multiple subbands. The inclusion of higher subbands allows us to investigate QWRs larger QWRs than in Ref. [31] and we find that the quantum-wire size influences the spin relaxation time via the SOC: The Dresselhaus term contains a longitudinal contribution to the internal effective magnetic field, which can effectively reduce the spin precession, and thereby the spin relaxation. It also contains a transverse component, which provides a contribution to the spin precession involving different subbands and enhances spin relaxation. When the wire width is small and only the lowest subband in the QWR is important, the longitudinal term is dominant. When the wire width is large and higher subbands are populated, the transverse contribution dominates over the longitudinal one, and a faster spin relaxation results. We also studied different growth directions for QWRs. We show that one can obtain long spin relaxation time by optimizing the growth direction, quantum-wire width and the direction of the initial spin polarization. Further, we investigated how the details of the microscopic scattering mechanisms and the spin-orbit effects in the band structure affect the spin relaxation. The population of higher subbands was found to have decisive influence on the behavior of the SRT. For instance, if the geometry and external conditions are such that higher subbands become populated, the dependence of the SRT on temperature is reversed because the motional narrowing regime is left.

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