A robust registration algorithm for 3D point cloud data with scale stretches and outliers

Jinlong Li\textsuperscript{1*}, Ni Zeng\textsuperscript{1}, Jingan Meng\textsuperscript{1}, Xiaorong Gao\textsuperscript{1} and Yu Zhang\textsuperscript{1}

\textsuperscript{1} School of Physical Science and Technology, Southwest Jiaotong University, Chengdu, 610031, China
\textsuperscript{*}Corresponding author’s e-mail: jinlong_lee@126.com

Abstract. Generally, the point cloud data obtained by 3D scanner cannot express the overall information of objects at a single time due to the limited field of view, so registration algorithm is needed to obtain the complete information. For the registration of point cloud data sets with large scale stretches and outliers, this paper proposed a new iterative nearest point algorithm named Improved-ICP. In the case of isotropic point extension, the iterative reweighted least squares method is constructed by incorporating a weight function to the minimum error function. In calculation, the weight function is equivalent to increasing the weight the point pairs, and the weights are obtained by an M-estimation criterion. In the paper, the initial registration was optimized to get global convergence for the algorithm, and compared the Improved-ICP algorithm with the ICP algorithm and the Scale-ICP algorithm to verify the effectiveness of the algorithm. To demonstrate the robustness of the Improved-ICP algorithm, we performed several comparative experiments with different scales and different noise between the Scale-ICP and the proposed algorithm in the presence of abnormal points. Experiment results illustrate the Improved-ICP algorithm has high accuracy and strong robustness to scale and abnormal points.

1. Introduction

The task of point clouds registration is to solve the limitation of obtaining data within the limited field of view of the sensor and obtain a complete 3D scene. It is widely used in many fields, such as 3D object recognition [1], 3D reconstruction [2-3], medical 3D image processing [4], to name a few. 3D point cloud registration is a problem of estimating the transformation matrix between two scanning point clouds. According to the transformation matrix, we can combine partial scanned point clouds of the same 3D scene or object into a complete 3D point clouds [5].

In many traditional registration algorithms, iterative closest point (ICP) algorithm is the most classic, because ICP registration effect is significant and widely used [6]. However, ICP algorithms still have some shortcomings, so many ICP variants have been proposed so far. In the development of point cloud registration, for experimental data lacking in features and structures, Feng Li proposed an algorithm to automatically establish the correspondence between the overlapping views without knowing initial information to solve the rigid, disordered and scattered target cloud registration [6]. To solve the problem that most variant ICP algorithms require the users to select a threshold parameter to reject the outlier pairs manually, Dan Lv proposed a method to determine the reasonable points-pairs in planar regions by using the similarity of each plane point of K adjacent features [7], and the distance is also achieved by finding a reasonable point pair in the plane area. Weighting of point-pairs is a robust point cloud registration method [8]. The mass value of the data points or the distance between the points can be used to assigned the weight of point-pairs [9]. Fitzgibbon A W proposed a method, named LM-ICP,
this method can be directly applied to the sum of squares of the shortest distance, so it is often used in
the rigid body transformation stages [10]. M. Torabi proposed a new registration method for the point
clouds acquired by a laser scanning system in [11]. Bingwei He proposed a novel curvature-based
automatic registration algorithm to solve the registration problem with partial overlapping point clouds
[12]. In recent years, there are many researches on ICP algorithm and its variants for point cloud
registration. we also refer to Pottmann [13], Liu [14], Ezra et al [15], Maier-Hein [16] and Zhu [17].

The ICP algorithm does not consider the influence of the scale factor in the process of registration
[18], but the actual situation is more complex, there are large numbers of scale changes in the registration
process. In other words, there is a problem that different distances or angles from the sensor to the
surface of the object will make the scanning resolution different. This problem can be solved by
estimating the motion parameters between the scale parameters and the point cloud in the registration
algorithm [19]. In the actual registration experiments, there is another problem that the outliers can affect
the effect of registration algorithm and outliers may come from the deformation of the models, the
measuring error and loss of false data in the model. Partial outliers are produced because the point sets
represent the overlapping parts rather than the identical parts of the same object. In short, outliers are
unavoidable. If no robust enhancement methods are used in the registration process, the outliers will
have a bad impact on the registration effect, since the least squares algorithms are highly sensitive to
errors in the data, such as outliers [8]. We choose iteratively re-weighted least squares instead of least
squares in this paper. Through the constraint function of the weight function, the influence of outliers
on the registration effect is reduced as much as possible. In summary, the registration algorithm
proposed in this paper is suitable for the point clouds with large-scale stretches or outliers in the initial
point sets.

This paper is divided into five parts, the second part introduces the principle of the improved-ICP
algorithm. The third part introduces the convergence analysis. The fourth part is the experimental
simulation of the algorithm. The fifth part included conclusion and further discussion.

2. The principle of Improved-ICP algorithm
In order to solve the problems mentioned above, the ICP algorithm with scale factor is added to register
the initial experiment data with different scale coefficients, when there are scale differences in the initial
experimental data. On the basis, Gaussian noise is added to the initial experimental data, and the LMS
is not used in minimizing the error function, instead a weight function is added to the process.

2.1. The Principle
A robust criterion function $\zeta(r)$ can reduces the influence of data with Gaussian errors on the parameter
estimation process. The derivative is defined as $\psi(r)$. As follows:

$$\psi(r) = \zeta'(r) \quad (1)$$

The function $\zeta$ belongs to the robust function set $\Omega$ only when the following conditions are true [21]:
1) $\zeta(r)$ is an even function and when $r = 0$, $\zeta(r) = 0$; 2) $\zeta(r)$ is decreases monotonically on $(-\infty, 0)$
and increases monotonically on $(0, \infty)$; 3) $\psi(r)$ is differentiable, $\psi(r)/r$ is decreases monotonically
and bounded above on $(0, \infty)$.

The weighting function is defined as follows:

$$\omega(r) = \begin{cases} 
\frac{\psi(r)}{r} & r \neq 0 \\
\lim_{r \to 0} \frac{\psi(r)}{r} = \psi'(0) & r = 0 
\end{cases} \quad (2)$$

In this paper, for the case of outliers, we select Tukey’s bi-weight function. 
Tukey’s bi-weight function $\zeta_{Tu}(r)$, the partial derivative $\psi_{Tu}(r)$ and the corresponding weighting
function $\omega_{Tu}(r)$ are as follows:
\[
\zeta_{Tu}(r) = \begin{cases} 
\frac{\kappa^2_{Tu}}{6} \left[ 1 - \left( 1 - \frac{r^2}{\kappa^2_{Tu}} \right)^3 \right] & |r| \leq \kappa_{Tu} \\
\frac{\kappa^2_{Tu}}{6} & |r| > \kappa_{Tu}
\end{cases}
\]  
(3)

\[
\psi_{Tu}(r) = \begin{cases} 
\frac{r}{\kappa^2_{Tu}} & |r| \leq \kappa_{Tu} \\
0 & |r| > \kappa_{Tu}
\end{cases}
\]  
(4)

\[
\omega_{Tu}(r) = \begin{cases} 
\frac{1}{\kappa^2_{Tu}} & |r| \leq \kappa_{Tu} \\
0 & |r| > \kappa_{Tu}
\end{cases}
\]  
(5)

where, \( \kappa_{Tu} = 7.0589 \).

Here, the robust criterion function is added to process the data with occasional Gaussian errors in the parameters estimation. Outliers will have a bad influence on the overall registration results, so the Tukey’s bi-weight function can adjust this impact to some extent. By adding a robust criterion function, the influence of outliers on the registration process could be reduced.

The above problem can be handled through the following two iterative steps:

1) From the rotation matrix \( R^k \), the translation vector \( T^k \), and the scale coefficient \( s^k \), search a subset \( Z^k \) of \( Y \), and \( Z^k \) is the same size as \( X \), the goal is to get the minimum value of the objective function \( \varepsilon^k(Z) \):

\[
\varepsilon^k(Z) = \sum_{i=1}^{N} \| s^k \cdot R x_i + T^k - Z_i \|^2 
\]  
(6)

2) Fix \( Z^k \), search the next rotation matrix \( R^{k+1} \), the next translation vector \( T^{k+1} \), and the next scale coefficient \( s^{k+1} \).

\[
\omega_i = \omega \left\| x_i^k - z_i^k \right\|^2 
\]  
(7)

\[
[R^{k+1}, T^{k+1}, s^{k+1}] = \arg\min_{R,T} \sum_{i=1}^{N} \omega_i \left\| s^k \cdot R x_i + T^k - Z_i \right\|_2^2 
\]  
(8)

\[
\bar{\omega} = \sum_{i=1}^{N} \omega_i, \bar{x} = \frac{1}{\bar{\omega}} \sum_{i=1}^{N} \omega_i x_i, \bar{z} = \frac{1}{\bar{\omega}} \sum_{i=1}^{N} \omega_i z_i 
\]  
(9)

where, \( \bar{x}, \bar{z} \) is the weighted average of the data point set \( \{x_i\} \) and the nearest model point set \( \{z_i\} \). Assuming that \( T = -s \cdot R \bar{x} + \bar{z} + u \), where \( u \) is a new independent vector. Eq. 8 can be written as:

\[
\sum_{i=1}^{N} \omega_i \| s R x_i - s R \bar{x} + \bar{z} + u - z_i \|^2
\]  
(10)

Eq. 10 can also be written as:

\[
\sum_{i=1}^{N} \omega_i (s R (x_i - \bar{x}) - (z_i - \bar{z}) + u) \cdot (s R (x_i - \bar{x}) - (z_i - \bar{z}) + u)
\]  
(11)

Since both the sum of all \( \omega_i (x_i - x) \) is equal to zero vector, and so is the sum of all \( \omega_i (y_i - y) \), the formula above can be simplified as:

\[
\sum_{i=1}^{N} \omega_i s^2 \| x_i - \bar{x} \|^2 + \sum_{i=1}^{N} \omega_i \| z_i - \bar{z} \|^2 + \bar{\omega} \| u \|^2 - 2 s \bar{\omega} \text{trace}(RC)
\]  
(12)

Where, \( C = \frac{1}{\bar{\omega}} \sum_{i=1}^{N} [\omega_i x_i z_i^T] - \bar{x} \bar{z}^T \).
The rotation matrix $R$ is obtained by calculating a singular value decomposition of $C$, where $C = U \sum V^T$, and then $R = V \sum U^T$. In some special cases of degeneracy, this may result in a reflection matrix $(\det(VU^T) = -1)$ that can be corrected by recalculating the matrix as $R = V \text{diag}(1,1,-1) U^T$. So

$$ R^{k+1} = \begin{cases} VU^T & \det(VU^T) = -1 \\ V \text{diag}(1,1,-1) U^T & \det(VU^T) = -1 \end{cases} $$

(13)

That is, Eq. 12 is quadric of $s$, and the minimum at $(R^{k+1}, s^{k+1})$ can be found by

$$ \frac{\partial}{\partial s} e^k(R,s) = 0 $$

(14)

solving Eq. 14, we get that

$$ s = \frac{\sum_{i=1}^{N} (Rx_i^k, z_i^k)}{\sum_{i=1}^{N} (x_i^k, z_i^k)} $$

(15)

Where, $x_i^k = x_i - \bar{x}$, $z_i^k = z_i - \bar{z}$.

However $s$, obtained by Eq. 15, may be outside the range of $I$, that is, when $s \leq a$ (or $b$), let

$$ s^{k+1} = a $$

(16)

In summary,

$$ s^{k+1} = \begin{cases} a & s \leq a \\ \frac{\sum_{i=1}^{N} (Rx_i^k, z_i^k)}{\sum_{i=1}^{N} (x_i^k, z_i^k)} & a < s < b \\ b & s \geq b \end{cases} $$

(17)

$$ T^{k+1} = \bar{z}^k - s^{k+1} \cdot R^{k+1} \bar{x} $$

(18)

2.2. Algorithm

Input: Given two 3D data sets $X = \{x_i\}_{i=1}^{N}$ and $Y = \{y_j\}_{j=1}^{N}$, given weight function $\omega$.

Output: Transformation (includes rotation matrix $R$, translation vector $T$, and scale $s$).

1) Initialize rotation $R^0$, translation $T^0$, and scale factor $s^0$, respectively.
2) repeat:
3) $k = k+1$
4) Find the point set $Z^k = \{z_i^k\}_{i=1}^{N}$ in $Y$ closest to $X^k = \{x_i^k\}_{i=1}^{N}$
5) $\omega_i = \omega \|x_i^k - z_i^k\|^2$
6) If any $\omega_i > 0$ then
7) $[R^*, T^*, S^*] = \arg \min_{R,T,S} \sum_{i=1}^{N} \omega_i \|S^k \cdot R x_i^k + T^* - z_i^k\|^2$
8) $x_i^{k+1} = s^k \cdot R^* x_i^k + T^*$, $R^{k+1} = R^* R^k$, $T^{k+1} = R^* T^k + T^*$
9) else $\omega_i = 0$
10) $x_i^{k+1} = x_i^k$, $R^{k+1} = R^k$, $s^{k+1} = s^k$, $T^{k+1} = T^k$
11) terminate iterations
12) end if
13) until convergence
14) return $R^k, T^k$

3. Convergence Analysis

In order to get a better registration result, we use the following method for the initial value: the covariance matrix is used to describe the variability of data sets because many properties of the covariance matrix can be described by eigenvectors and eigenvalues. For example, given two data sets $X = \{x_i\}_{i=1}^{N}$ and $Y = \{y_i\}_{i=1}^{N}$, their covariance matrix can be evaluated respectively by

$$ M_X = \sum_{i=1}^{N} (x_i - \bar{x})(x_i - \bar{x})^T $$

(19)
where, $x_c$ and $y_c$ are the centers of $X$ and $Y$ respectively.

Let $\lambda_1 < \lambda_2 < \lambda_3$ and $\mu_1 < \mu_2 < \mu_3$ be the eigenvalues of $M_X$ and $M_Y$, and let $p_1, p_2, p_3$ and $q_1, q_2, q_3$ be their corresponding normalized eigenvectors. Geometrically speaking, the geometric distributions of $X$ and $Y$ are similar if the following conditions are met:

$$\alpha - \Delta \alpha \leq \frac{\sqrt{\mu_i}}{\lambda_i} \leq \alpha + \Delta \alpha, i = 1, 2, 3$$

Considering some small offset, let $\alpha = \frac{1}{3} \sum \frac{\sqrt{\mu_i}/\lambda_i}{\lambda_i}$. Meanwhile, the distributions of data sets along their corresponding normalized eigenvectors are the eigenvalues. The initial rotation $R^0$ is selected close to the vector $[q_1, q_2, q_3][p_1, p_2, p_3]^{-1}$ and the initial translation $T^0$ is selected close to $y_c - x_c$ respectively. Here, for the related discussion on the selection of the initial translation and initial rotation, see references [12] and [20].

4. Experimental Simulation

In this section, two experiments are used for comparison: 1) The comparison among three algorithms: the ICP, the Scale-ICP and the improved-ICP for the registration of 3D datasets without outliers and scale stretches. 2) In the presence of outliers, the improved-ICP algorithm was compared with the Scale-ICP at different scales.

4.1. Comparison among three algorithms without scale stretch and outliers

Initial experimental data sets selected in this paper are Stanford Bunny models (Bunny045, which has 40097 points and Bunny000, which has 40256 points). The root mean square error curves are obtained by comparing the improved-ICP algorithm with the ICP algorithm and the Scale-ICP algorithm are shown in Fig. 1. The experimental results are got by comparing the ICP algorithm with the Improved-ICP algorithm without outliers or scale stretches. The RMS error is shown in Table 1.

![Figure 1. The RMS error of the three algorithms in 80 iterations.](image)

| Table 1. Registration Results of ICP, Scale-ICP and Improved-ICP |
|------------------|------------------|------------------|
|                  | ICP              | Scale-ICP        | Improved-ICP    |
| Scale            | -                | 0.9805           | 0.9805          |
| RMS error        | 0.00286          | 0.00301          | 0.00266         |
| Loop             | 35               | 81               | 87              |

Results in Fig.1 and Table 1 proves that both algorithms can accomplish the registration without outliers and scale stretches, and the Improved-ICP algorithm has higher registration accuracy.
4.2. Comparison of registration results with scale stretches and outliers

We multiply the point cloud data sets Bunny045 and Bunny000 by different scale coefficients $u$ to generate the registration group between each data set. From the respective initial registration with $u$ of 0.5, 1.0, 2.0, and 10.0, we obtain the corresponding results displayed in Fig. 2 and 3.

Figure 2. Original configuration of two data sets when (a) $u=0.5$, (b) $u=1.0$, (c) $u=2.0$, and (d) $u=10.0$.

Comparison of two data sets after registration with $u$ of 0.5, 1.0, 2.0, and 10. in Fig. 3, $a$ represents the registration of Scale-ICP algorithm and $b$ represents the registration of Improved-ICP algorithm.

Figure 3. Comparison of two data sets after registration when (1) $u=0.5$, (2) $u=1.0$, (3) $u=2.0$, and (4) $u=10.0$.

The comparison of RMS errors between the two algorithms with outliers are shown in Fig. 4, 5 and Table 2. The RMS error curves of the experimental results obtained by multiplying the data with the scale factor $u$ of 0.5, 1.0, 2.0, and 10.0, using the Improved-ICP algorithm, are plotted in Fig. 5.
Figure 4. RMS error registration within 60 iterations by two algorithms (a) $u=0.5$, (b) $u=1.0$, (c) $u=2.0$ and (d) $u=10.0$.

Figure 5. RMS error comparison of the registration results with four scale coefficients.

Table 2. The results of three registration algorithms for data sets with four scale coefficients

| $u$ | Scale-ICP | Improved-ICP | Scale |
|-----|------------|--------------|-------|
| 0.5 | 0.00320    | 0.00312      | 0.4902|
| 1   | 0.00318    | 0.00295      | 0.9805|
| 2   | 0.00306    | 0.00274      | 1.9610|
| 10  | 0.00302    | 0.00266      | 9.8049|

Comparing the experimental results of the Improved-ICP algorithm and the Scale-ICP algorithm horizontally, and it is clear that the Improved-ICP algorithm has a better effect on the initial data containing outliers. The least square method used in ICP algorithm is obviously inferior to the Improved-ICP algorithm proposed, which proves that the Improved-ICP algorithm is robust to the initial data with outliers. The longitudinal comparison of the Improved-ICP algorithm with different scale stretches and outliers proves that the RMS error value is basically unchanged, which shows that the algorithm has little influence on the steps of rotation and translation; four terminal scale values of $s$ are quite close to those of $u$, which further demonstrates the robustness of Improved-ICP algorithm to the scale bias.

5. Conclusions

In this paper, we study the registration of 3D point clouds with outliers and scale stretches. By introducing a robust function into the ICP algorithm with the scale factor, we proposed an Improved-ICP algorithm, the advantages of the algorithm are as follows: 1) For the registration of data sets with the same scale, experimental results showed that the Improved-ICP algorithm has higher accuracy than the ICP without loss of computational efficiency. 2) For the initial data sets with scale stretches, the results of the Improved-ICP algorithm are better than the other two algorithms. 3) In the presence of
large numbers of outliers, the improved ICP algorithm is more robust, but the ICP algorithm is more sensitive to outliers. 4) In most experimental cases, such as the initial experimental data with large-scale stretches and outliers, the Improved-ICP algorithm could be effectively applied. The above is mainly for registration of two point clouds. However, there are also the cases that multiple point clouds are required to register to express complete information about the object, so the registration of multi-perspective point clouds can be studied, and it is possible to combine the Deep-Learning to realize the point clouds registration with scale stretches and outliers in multiple perspectives later.

Acknowledgments
This work was supported by National Nature Science Foundation of China (Grant No. 61471304). And authors also thank School of Southwest Jiaotong University Physical Science and Technology for their kind support in these experiments.

References
[1] Zhou, S., Xiao, S. (2018). 3D face recognition: a survey. Human-centric Computing and Information Sciences, 8(1), 1-27.
[2] Cao, M., Jia, W., Lv, Z., Li, Y., Xie, W., Zheng, L., & Liu, X. (2019). Fast and robust feature tracking for 3D reconstruction. Optics & Laser Technology, 110, 120-128.
[3] Lu, H., Li, Y., Uemura, T., Kim, H., & Serikawa, S. (2018). Low illumination underwater light field images reconstruction using deep convolutional neural networks. Future Generation Computer Systems, 82, 142-148.
[4] Yang, H., Shi, J., & Carlone, L. (2020). Teaser: Fast and certifiable point cloud registration. IEEE Transactions on Robotics, 37(2), 314-333.
[5] Besl, P. J., & McKay, N. D. (1992). Method for registration of 3-D shapes. In Sensor fusion IV: control paradigms and data structures, International Society for Optics and Photonics., Boston. pp. 586-606.
[6] Li, F., Stoddart, D., & Hitchens, C. (2017). Method to automatically register scattered point clouds based on principal pose estimation. Optical Engineering, 56(4), 044107.
[7] Lv, D., Sun, J., Li, Q., & Wang, Q. (2015). Registration of partially overlapping laser-radar range images. Optical Engineering, 54(10), 103113.
[8] Bergström, P., & Edlund, O. (2014). Robust registration of point sets using iteratively reweighted least squares. Computational optimization and applications, 58(3), 543-561.
[9] Rusinkiewicz, S., Levoy, M. (2001). Efficient variants of the ICP algorithm. In Proceedings third international conference on 3-D digital imaging and modelling., Quebec. pp. 145-152.
[10] Fitzgibbon, A. W. (2003). Robust registration of 2D and 3D point sets. Image and vision computing, 21(13-14), 1145-1153.
[11] Torabi, M., & Younesian, D. (2015). A new methodology in fast and accurate matching of the 2D and 3D point clouds extracted by laser scanner systems. Optics & laser technology, 66, 28-34.
[12] He, B., Lin, Z., & Li, Y. F. (2013). An automatic registration algorithm for the scattered point clouds based on the curvature feature. Optics & Laser Technology, 46, 53-60.
[13] Potthmann, H., Huang, Q. X., Yang, Y. L., & Hu, S. M. (2006). Geometry and convergence analysis of algorithms for registration of 3D shapes. International Journal of Computer Vision, 67(3), 277-296.
[14] Liu, Y. (2004). Improving ICP with easy implementation for free-form surface matching. Pattern Recognition, 37(2), 211-226.
[15] Planitz, B. M., Maeder, A. J., & Williams, J. A. (2005) The correspondence framework for 3d surface matching algorithms. Computer Vision and Image Understanding, 97(3), 347-383.
[16] Maier-Hein, L., Franz, A. M., Dos Santos, T. R., Schmidt, M., Fangerau, M., Meinzer, H. P., & Fitzpatrick, J. M. (2011). Convergent iterative closest-point algorithm to accommodate anisotropic and inhomogenous localization error. IEEE transactions on pattern analysis and machine intelligence, 34(8), 1520-1532.

[17] Zhu, J., Jin, C., Jiang, Z., Xu, S., Xu, M., & Pang, S. (2019). Robust point cloud registration based on both hard and soft assignments. Optics & Laser Technology, 110, 202-208.

[18] Yang, J., & Chen, H. (2017). The 3D reconstruction of face model with active structured light and stereo vision fusion. In 2017 3rd IEEE International Conference on Computer and Communications (ICCC), Ghaziabad. pp. 1902-1906.

[19] Phillips, J. M., Liu, R., & Tomasi, C. (2007). Outlier robust ICP for minimizing fractional RMSD. In Sixth International Conference on 3-D Digital Imaging and Modeling (3DIM 2007), Montreal. pp. 427-434.

[20] Kanatani, K. I. (1994). Analysis of 3-D rotation fitting. IEEE Transactions on pattern analysis and machine intelligence, 16(3), 543-549.