Quantum simulation of open quantum systems in heavy-ion collisions

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We present a framework to simulate the dynamics of hard probes such as heavy quarks or jets in a hot, strongly-coupled quark-gluon plasma (QGP) on a quantum computer. Hard probes in the QGP can be treated as open quantum systems governed in the Markovian limit by the Lindblad equation. However, due to large computational costs, most current phenomenological calculations of hard probes evolving in the QGP use semiclassical approximations of the quantum evolution. Quantum computation can mitigate these costs, and offers the potential for a fully quantum treatment with exponential speedup over classical techniques. We report a simplified demonstration of our framework on IBM Q quantum devices, and apply the Random Identity Insertion Method (RIIM) to account for CNOT depolarization noise, in addition to measurement error mitigation. Our work demonstrates the feasibility of simulating open quantum systems on current and near-term quantum devices, which is of broad relevance to applications in nuclear physics, quantum information, and other fields.

Introduction. Considerable advancements in quantum devices, such as qubit coherence times, have recently been achieved [1–4]. Together with parallel progress in quantum algorithms and executable quantum software, nontrivial quantum computations can be carried out, including hybrid quantum-classical algorithms such as the variational quantum eigensolver [5–10] and fully quantum simulations of the unitary time evolution of closed quantum systems [11, 12]. In high energy and nuclear physics, a variety of quantum computing applications have emerged [13–37]. In particular, quantum simulation can be applied to study dynamics of large size systems that are in principle intractable with classical methods. To perform such simulations, quantum circuits compiled into single- and multi-qubit gates can be implemented on digital quantum computers.

Many physical systems of interest are not closed, but consist of a subsystem interacting with an environment. The dynamics of the subsystem can be formulated as an open quantum system. In the Markovian limit (in which the environment correlation time is much smaller than the subsystem relaxation time), the evolution of the subsystem is governed by a generalization of the Schrödinger equation known as the Lindblad equation [38–40], where instead of keeping track of all of the environmental degrees of freedom, one only needs to record environment correlators that are relevant for the subsystem evolution. A key challenge in extending quantum simulation to open quantum systems is that the Lindblad evolution is non-unitary. During the last decade, algorithms have been developed to overcome this issue, most of which couple the subsystem with auxiliary qubits (whose dimension can be significantly smaller than that of the environment) such that the whole system evolves unitarily [41–47]. More recently, simulations of open quantum systems have been carried out on real quantum devices, but without error mitigation [48].

In this letter, we focus on the application of quantum simulations of open quantum systems to relativistic heavy-ion collisions (HICs). Experiments at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) create a hot (T ≈ 150−500 MeV), short-lived (t ≈ 10 fm/c) quark-gluon plasma (QGP) [49–56]. The QGP is a deconfined phase of QCD matter believed to have existed shortly after the Big Bang [57]. The properties of the QGP can be investigated using jets or heavy quarks [58–62] that involve energy scales much larger than the QGP temperature (“hard probes”).

The evolution of hard probes in the QGP can be treated as an open system evolving in a hot medium. A fully field-theoretical description of hard probes in the medium is challenging and typically various approximations are made. Most studies employ semiclassical Boltzmann or Fokker-Planck (equivalent to Langevin) equations [63–70]; semiclassical transport equations are leading order terms in the gradient expansion of the Wigner transformed Lindblad equation [71, 72]. Recently, several studies have applied Lindblad equations directly to investigate quarkonia [73–80] and jets [81, 82], which are valid if the subsystem and environment are weakly coupled. It is expected that as the size of the subsystem increases (such as the jet radiation phase space, or the number of heavy quarks [83, 84] in the subsystem), solving Lindblad equations would challenge the limits of classical computation. Quantum computing offers a possibility to remove the constraint on the subsystem size, and go beyond the approximations made in semiclassical approaches. Moreover, quantum simulation may provide a solution to the
notoriously difficult sign problem in classical lattice QCD calculations of real time observables \cite{14, 85–87} (the same problem can also appear in open QCD systems).

In this letter, we outline a formulation of the evolution of hard probes in the QGP as a Lindblad equation and explore how simulations on Noisy Intermediate Scale Quantum (NISQ \cite{13}) devices can be used to advance theoretical studies of hard probes in the QGP. Using a quantum algorithm for simulating the Lindblad equation, we study a toy model on IBM Q simulators and quantum devices, and implement error mitigation for measurement and two-qubit gate noise. We demonstrate that quantum algorithms simulating simple Lindblad evolution are tractable on current and near-term devices, in terms of available number of qubits, gate depth, and error rates.

Open quantum system formulation of hard probes in heavy-ion collisions. The Hamiltonian of the full system consists of the hard probe (subsystem) and the QGP (environment) can be written as

\[ H = H_S + H_E + H_I \]

\[ H_S = H_{S0} + H_{S1}. \]

Here \( H_S, H_E \) and \( H_I \) are the Hamiltonians of the subsystem, the environment and their interaction, respectively. A schematic diagram of the setup is shown in Fig. 1. We further split \( H_S \) into the free \( H_{S0} \) and the interacting part of the subsystem \( H_{S1} \). In quantum field theories, Hamiltonians are functionals of fields, which require discretization in position space \cite{16}. Here, instead of simulating the dynamics of fields, we focus on simulating the dynamics of particle states, which is valid for hard probes. If we use multi-particle states \( |p_1, A_1\rangle \otimes \cdots \otimes |p_n, A_n\rangle \) as the basis where \( p_i \) is the four-momentum, \( A_i \) represents all discrete quantum numbers, and \( i = 1, 2, \ldots, n \), then both \( H_{S0} \) and \( H_{S1} \) are matrices and \( H_{S0} \) is diagonal. Note that \( H_{S1} \) is different from \( H_I \): The former is the interaction within the subsystem itself and independent of the environment, while the latter represents the interaction between the subsystem and the environment. For example, for jets in HICs, \( H_{S1} \) can be collinear radiation of collinear particles while \( H_I \) can describe the Glauber exchange between collinear particles (subsystem) and soft fields from the QGP environment \cite{81}.

The total density matrix of the subsystem and the environment evolves under the von Neumann equation. In the interaction picture, this is given by

\[ \frac{d}{dt} \rho^{(\text{int})}(t) = -i[H_I^{\text{int}}(t), \rho^{(\text{int})}(t)]. \]

(3)

The operators are defined by

\[ \rho^{(\text{int})}(t) \equiv e^{i(H_{S0}+H_E)t} \rho(t) e^{-i(H_{S0}+H_E)t} \]

\[ H^{(\text{int})}_{S1}(t) \equiv e^{iH_{S0}} H_{S1} e^{-iH_{S0}} \]

\[ H^{(\text{int})}_I(t) \equiv e^{i(H_{S0}+H_E)t} H_I e^{-i(H_{S0}+H_E)t}. \]

(4)

(5)

(6)

Here \(\beta = 1/T\) is the inverse of the QGP temperature. After the environment is traced out, the reduced evolution of the subsystem density matrix is generally time-irreversible and non-unitary. If the coupling between the subsystem and the environment is weak, the reduced evolution equation can be cast as a Markovian Lindblad equation \cite{38–40}:

\[ \frac{d}{dt} \rho_S(t) = -i[H_{S1}(t) + H_L, \rho_S(t)] \]

\[ + \sum_{j=1}^m (L_j \rho_S(t) L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho_S(t)\}) \]

(9)

where \( H_L \) denotes a thermal correction to \( H_S \) generated by loop effects of \( H_I \), and the \( L_j \) are called Lindblad operators, whose explicit expressions will be given for a toy

\[ 1 \] The backreaction of the QGP medium to jet energy loss \cite{88–97}, which may further modify jet observables is beyond the scope of our considerations here. For a recent review, see Ref. [98].
model below. In general, if the dimension of the subsystem is $d$, (i.e., $\rho_S(t)$ is a $d \times d$ matrix), the number of independent Lindblad operators is $m = d^2 - 1$. When evaluating the Lindblad operators, an environment correlator of the form $\text{Tr}_E [O_E(t_1) O_E(t_2) \rho_E]$ is needed as input, where the $O_E$’s are some environment operators. This correlator can be evaluated perturbatively in thermal field theory if the environment is weakly-coupled. But the construction of the Lindblad equation only requires $H_I$ to be weak. In general $H_E$ itself can be strongly coupled, in which case the correlator has to be computed nonperturbatively using lattice QCD [99–103] or the AdS/CFT correspondence [104–108]. For the nonperturbative computation, one needs to formulate the theory such that the relevant correlator is gauge invariant, where effective field theory can be used. A concrete construction of gauge invariant correlators for quarkonium transport can be found in Refs. [72, 109].

Quantum algorithm. We will apply a quantum algorithm based on the Stinespring dilation theorem, see for example Refs. [44, 110], to simulate the Lindblad equation. The algorithm in terms of the evolution operators $J$, defined below, and $H_S$, is illustrated in Fig. 2. The algorithm couples the subsystem with auxiliary qubits, which are traced out after each time step $\Delta t$. The dimension of the auxiliary register is $m + 1$ and the number of qubits needed in practice for the register is $\lceil \log_2 (m+1) \rceil \equiv [2 \log_2 d]$. Together with the number of qubits required to record the subsystem state, the total number of qubits needed is $[3 \log_2 d]$. We use $\{ |0\rangle_a , |1\rangle_a , \ldots , |m\rangle_a \}$ to label the basis of the auxiliary register, indicated by the subscript $a$.

We assume the initial state $\rho_S(0) = \langle \psi_S(0) | \psi_S(0) \rangle$ is a pure state.$^2$ At the beginning of each cycle at time $t$, the total density matrix of the subsystem and the auxiliary is set to be a $(m+1) \times (m+1)$ block matrix $\rho(t) = |0\rangle_a \langle 0 | \otimes \rho_S(t) = \left( \begin{array}{cccc} \rho_S(t) & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{array} \right)$. (10)

The $J$-operator is also a $(m+1) \times (m+1)$ block matrix

$$J = \left( \begin{array}{cccc} 0 & L_1^\dagger & \cdots & L_m^\dagger \\ L_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_m & 0 & \cdots & 0 \end{array} \right),$$

where each block is a $d \times d$ matrix. One can show that the circuit in Fig. 2 reproduces (9) when $\Delta t \rightarrow 0$. To simulate the evolution from 0 to $t$, the size of the time steps is $\Delta t = t/N_{\text{cycle}}$ where $N_{\text{cycle}}$ is the number of cycles, see Fig. 2.

Toy model and simulation on IBM Q. Simulating real jets and heavy quarks on quantum devices requires a large number of fault-tolerant qubits. As a proof of concept, we consider the following toy model that includes qualitative features of hard probes:

$$H_S = H_{S0} = -\frac{\Delta E}{2} Z$$

$$H_E = \int d^3x \left[ \frac{1}{2} \Pi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda \phi^4 \right]$$

$$H_I = g X \otimes \phi(x = 0),$$

where we use $X, Z$ to denote the single qubit Pauli gates (Pauli matrices). The subsystem Hamiltonian $H_S$ is a two level system with energy difference $\Delta E$. The two levels can correspond to the bound and unbound state of a heavy quark-antiquark pair, exchanging energy with QGP. The environment $H_E$ is a $3 + 1 D$ scalar field theory, that together with (8) mimics the thermal QGP. Here $\Pi$ is the canonical momentum conjugate to $\phi$. The extension to gauge theories requires a gauge invariant formulation of the environment correlator as mentioned earlier. The environment correlator can be calculated nonperturbatively to all orders in $\lambda$. Here for simplicity, we set $m = 0$. Nonvanishing $m$ and $\lambda$ lead to different coefficients of the Lindblad operators but do not alter the quantum algorithm. The interaction strength $g$ between the subsystem and the environment is unitless. In the Markovian limit, two Lindblad operators $j = 0, 1$ are relevant:

$$L_j = \frac{\sqrt{\Gamma_j}}{2} (X - (-1)^j i Y),$$

where $\Gamma_0 = g^2 \Delta E n_B(\Delta E)/(2 \pi)$, $\Gamma_1 = g^2 \Delta E/(2 \pi) + \Gamma_0$ and $n_B(\Delta E) = 1/(\exp(\beta \Delta E) - 1)$ is the Bose-Einstein distribution. We will neglect $H_L$ in this letter. For our numerical studies, we use a unit system where all quantities are counted in units of $T$, the temperature of the

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$^2$ If it is a mixed state, then we decompose it into a linear superposition of pure states. We just need to apply the circuit to each pure state and take the linear superposition in the end.
medium. We initialize the state as $\rho_S(t = 0) = |0\rangle\langle 0|$ and choose $\Delta E = 1(T)$.

The result for this toy model obtained from the IBM Q qiskit simulator [111] is shown in Fig. 3. We measure $P_0(t) \equiv \langle 0|\rho_S(t)|0\rangle$, which can be interpreted as the time-dependent nuclear modification factor. Each time point corresponds to an independent quantum circuit, where the measurement is performed only at the end, as shown in Fig. 2. The results of the quantum algorithm with $N_{\text{cycle}} = 100$ are shown for different values of the coupling $g$. They are consistent with the results obtained with a 4th order Runge-Kutta method that solves Eq. (9) classically. This agreement demonstrates that the circuit successfully solves the Lindblad equation. As expected, the strength of the coupling $g$ controls the rate of approaching thermalization.

In order to run the circuit on a quantum device, we select $N_{\text{cycle}} = 1$ in order to achieve a sufficiently small circuit depth. Modern quantum software packages are available to compile quantum circuits that approximate general unitary operators with minimal error and optimal depth [10, 112–114]. We synthesize a circuit for $\sqrt{\text{cnot}}$ using a leading order zero-noise extrapolation of open quantum systems using quantum devices from IBM [115]. Simulated results were reproduced on the IBM Q Valencia and Santiago devices [117, 118].

Overall, we observe good agreement of the results from the quantum device with the results from the simulator for $N_{\text{cycle}} = 1$ after the error mitigation is applied. The choice of $N_{\text{cycle}} = 1$ is seen to be a reasonable approximation for sufficiently small $t$. Moreover, a modest increase to $N_{\text{cycle}} = 3$, as shown by the simulator in Fig. 4, yields considerably improved convergence, which is promising for near-term applications. These results demonstrate that the simulation of open quantum system dynamics relevant for HICs should be feasible on current and near-term quantum devices.

Conclusions and Outlook. We performed simulations of open quantum systems using quantum devices from IBM Q. In particular, we focused on simulating the non-unitary evolution of a subsystem governed by the Lindblad equation. We demonstrated that digital quantum simulations with a few qubits and a circuit depth of $\sim 70$ gate operations with $\sim 10$ CNOT gates are feasible on
current quantum devices. We used the qsearch compiler to construct the quantum circuit, and implemented two-qubit gate error mitigation using zero noise extrapolation with the Random Identity Insertion Method (RIIM), in addition to readout error mitigation. Simulating open quantum systems is of great importance for theoretical studies of hard probes in heavy-ion collisions. The open quantum system formulation allows one to go beyond semiclassical transport calculations currently used in most phenomenological studies. Future calculations, using a time dependent environment density matrix may allow one to explore a broad range of physical models by varying medium properties such as the initial temperature, microscopic structure, or the probe-medium coupling. Open quantum systems are also relevant for various other systems in nuclear and high-energy physics such as studies of Cold Nuclear Matter effects at the future Electron-Ion Collider [119], the resummation of large logarithms relevant for jet physics [120–123] and studies of the Color Glass Condensate [124, 125].

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FIG. 5. Decomposition of a single cycle of the quantum algorithm in Fig. 2 in terms of single qubit rotations (U1,3) and CNOT gates using the qsearch compiler of Ref. [113]. Here q0 corresponds to the system qubit, q1,2 are the auxiliary qubits and c03 represents three classical bits for the readout. The result of P0(t) from the final trace-out and measurement can be written as 

\[ P_0(t) = \sum_{i,j=0}^{1} \langle 0ij | \rho(t) | 0ij \rangle \]

where \( \langle 0ij | \rho(t) | 0ij \rangle \) is the measurement result for q0 = 0, q1 = i, q2 = j.
FIG. 6. The response matrix of the qubits $q_{0-2}$ of IBM Q Vigo device [115] which is used for the readout error mitigation in Fig. 4. The $2^3$ states are prepared by applying $X$ gates and then corresponding measurements are performed. The error mitigation is implemented using the constrained matrix inversion approach which is implemented in IBM’s qiskit-ignis package [111].