Supersymmetric Yang–Mills theories with local coupling:
The supersymmetric gauge

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Abstract

Supersymmetric pure Yang-Mills theory is formulated with a local, i.e. space-time dependent, complex coupling in superspace. Super-Yang-Mills theories with local coupling have an anomaly, which has been first investigated in the Wess-Zumino gauge and there identified as an anomaly of supersymmetry. In a manifest supersymmetric formulation the anomaly appears in two other identities: The first one describes the non-renormalization of the topological term, the second relates the renormalization of the gauge coupling to the renormalization of the complex supercoupling. Only one of the two identities can be maintained in perturbation theory. We discuss the two versions and derive the respective $\beta$ function of the local supercoupling, which is non-holomorphic in the first version, but directly related to the coupling renormalization, and holomorphic in the second version, but has a non-trivial, i.e. anomalous, relation to the $\beta$ function of the gauge coupling.
1 Introduction

Local couplings have always been a useful tool for defining local operators in quantum field theory [1, 2]. They came back to the center of interest from string theory and served for understanding the non-renormalization theorems of chiral vertices in supersymmetric theories [3, 4]. Even more it turned out that local couplings allow to prove rather easily the Adler-Bardeen non-renormalization theorem [5] by giving a precise definition for the renormalization of the topological term $\tilde{G}G$ [6, 7, 8]. In this way local gauge coupling in supersymmetric gauge theories yields also the non-renormalization of the coupling beyond one loop order [9], but it is seen that the renormalization of the topological term also induces a one-loop anomaly in presence of local coupling [7, 10].

By now one has applied the construction to supersymmetric gauge theories in the Wess-Zumino gauge [6, 7, 11]. There the anomaly could be be put into the Ward identity of supersymmetry as obtained out of a generalized Slavnov–Taylor identity. Its coefficient has been calculated as a function of the one-loop corrections to the topological term [10] and it turns out to be independent of the gauge parameter and the scheme. It has been also related to the ratio of the one and 2-loop $\beta$ function of the gauge coupling of pure super-Yang-Mills theories.

In the present paper we repeat the analogous analysis for pure super-Yang-Mills theories formulated in terms of superfields, hence with linear realization of supersymmetry, which permits BPHZ or Wilsonian regularization as an invariant scheme of supersymmetry. We also modify the introduction of the local coupling: We couple the gauge invariant Lagragians of the super-Yang-Mills action to a chiral and an antichiral field as in the Wess-Zumino gauge, but define the gauge coupling by a constant shift in the lowest component of the external fields. Then the renormalization of the coupling is related to the renormalization of the external fields by the shift equation.

For the supersymmetric invariant schemes it turns out that the anomaly is shifted to other symmetry identities. We consider two versions (section 4 and 5): In the first one the anomaly induces non-holomorphic terms in that equation which defines the non-renormalization of the topological term, in the second the shift equation that relates the renormalization of external fields to the renormalization of the gauge coupling is modified by the anomaly. Accordingly we find in the first version non-holomorphic terms in external fields in the $\beta$ function, whereas the second version yields a holomorphic $\beta$ function in the external fields (section 6). From the construction we finally deduce a closed expression for the gauge $\beta$ function [12, 13] and identify the coefficient of the anomaly with the scheme and gauge independent ratio $\beta_{g^2}^{(2)}/\beta_{g^2}^{(1)}$. In the Appendix we construct the complete basis of invariant operators, which contribute in the Callan–Symanzik equation.
A discussion of previous results and a comparison with the Wess-Zumino gauge can be found in the conclusions.

2 Super-Yang-Mills with local gauge coupling

For introducing the local gauge coupling we proceed similar as in the Wess-Zumino gauge \cite{7}. We introduce a chiral and antichiral field $\eta$ and $\bar{\eta}$,

$$
\eta = \eta + \theta \chi + \theta^2 f, \quad \bar{\eta} = \bar{\eta} + \bar{\theta} \chi + \bar{\theta}^2 \bar{f},
$$

and couple them to the gauge invariant Lagrangians $W^\alpha W_\alpha$ and $\bar{W}_\alpha \bar{W}^\alpha$:

$$
\Gamma_{\text{SYM}} = -\frac{1}{128} \text{Tr}(\int dS \eta W^\alpha W_\alpha + \int d\bar{S} \eta \bar{W}_\alpha \bar{W}^\alpha).
$$

with $W^\alpha$ the supersymmetric field strength tensor:

$$
W^\alpha \equiv \bar{D}D(e^{-\phi} D^\alpha e^\phi),
$$

with $\phi = \phi_a \tau_a$ and $\text{Tr} \tau_a \tau_b = \delta_{ab}$.

In this form the action does not have a well-defined free field action for the vector superfield. There are two possibilities to proceed: First we could redefine the vector superfield

$$
\phi \to (\eta + \bar{\eta})^{-1} \phi
$$

identifying the real part of the lowest component with the local coupling. This is analogous to the construction of the Wess-Zumino gauge. Alternatively – and this is the procedure we will follow in the present paper – we can shift the lowest component of the external superfields fields by a constant, which is the gauge coupling:

$$
\eta \to \hat{\eta} = \eta + \frac{1}{2g^2}, \quad \bar{\eta} \to \hat{\bar{\eta}} = \bar{\eta} + \frac{1}{2g^2}.
$$

Then (2) has a well defined free field action and we can treat $\eta$ and $\bar{\eta}$ as ordinary external fields.

The fields $\eta$ and $\bar{\eta}$ are dimensionless. As such they can appear in arbitrary powers in higher orders of perturbation theory. However, as they have been introduced here they satisfy several constraints which restrict their appearance in higher orders:
1. The property that the gauge coupling has been introduced by a shift in the lowest component of the dimensionless superfield gives rise to the identity:

$$\frac{1}{2} \left( \int dS \frac{\delta}{\delta \eta} + \int d\bar{S} \frac{\delta}{\delta \eta} \right) \Gamma = -g^4 \partial_{g^2} \Gamma.$$  

(6)

It relates the renormalization of the gauge coupling to the renormalization of the external fields and makes $\eta$ and $\bar{\eta}$ to local supercouplings.

2. The loop expansion is a power series expansion in the coupling. This property follows from simple inspection of loop diagrams and can be summarized in the topological formula,

$$N_{g^2} = (N_\eta + N_{\bar{\eta}}) + (l - 1),$$  

(7)

which is valid in the present form for diagrams with external vector legs and $\eta$ insertions. (For the generalization see (18).)

3. Most important for the non-renormalization properties is the identification of the imaginary part of the field $\eta$ with a space–time dependent \( \Theta \) angle:

$$\Gamma_{\text{SYM}} = -\frac{1}{4 \cdot 16} \text{Tr} \int d^4x \left( \frac{1}{g^2(x)} + i \Theta + \frac{1}{g^2} \circ (\theta) \right) W^{a\alpha}W_{\alpha} \bigg|_{g^2} + \text{c.c.} ,$$  

(8)

with

$$\Theta = -i(\eta - \bar{\eta}).$$  

(9)

The $\Theta$ angle couples to a total derivative, which is expressed in the identity:

$$\mathcal{W}^{\eta - \bar{\eta}} \Gamma_{\text{SYM}} \equiv \left( \int dS \frac{\delta}{\delta \eta} - \int d\bar{S} \frac{\delta}{\delta \bar{\eta}} \right) \Gamma_{\text{SYM}} = 0 .$$  

(10)

This identity together with the topological formula defines the renormalization of the $\Theta$ angle in presence of the local coupling and yields the non-renormalization of the topological term [7, 8].

The identity (10) and eq. (7) govern the dependence on the superfields $\eta$ and $\bar{\eta}$ of the naively formulated perturbation theory. They lead together to the holomorphic action of symmetric counterterms, which is claimed in the literature, i.e.

$$\Gamma_{\text{eff,SYM}} = -\frac{1}{128} \text{Tr} \int dS (\dot{\eta} + z^{(1)}) W^{a\alpha}W_{\alpha} + \text{c.c}.$$  

(11)
Thus the non-renormalization properties of the $\Theta$ angle govern the renormalization of the coupling in loop orders $l \geq 2$ and yield the generalized non-renormalization theorem of the coupling. These findings are in complete analogy to the results of the Wess-Zumino gauge \cite{7}. Without anomaly the counterterm action (11) could be considered as an effective action for supersymmetric Yang-Mills theories, but there is an anomaly which makes quantization and renormalization non-trivial.

3 The Slavnov–Taylor identity

For quantization we have to add to the classical action (2) the gauge fixing term. We choose the following form,

$$\Gamma_{g.t.} = \text{Tr} \int dV (\eta + \overline{\eta} + \frac{1}{g^2})(\xi B\overline{B} + \frac{1}{8} DDB\phi + \frac{1}{8} D\overline{D}\overline{B}\phi) ,$$

which satisfies the defining properties of the Yang-Mills action eqs. (6), (7) and (10) without modifications. Then one replaces gauge transformations by BRS-transformations (see \cite{14} for details):

$$s\phi = Q_s(\phi, c_+, \overline{c}_+) = c_+ + \overline{c}_+ + \frac{1}{2}[\phi, c_+ - \overline{c}_+] + O(\phi^2) ,$$

$$sc_+ = -c_+ c_+ , \quad sc_+ = -\overline{c}_+ \overline{c}_+ ,$$

$$sc_- = B , \quad s\overline{c}_- = \overline{B} ,$$

$$sB = 0 , \quad s\overline{B} = 0 .$$

The Faddeev-Popov ghosts $c_+$ and their corresponding antighost $c_-$ as well as the Lagrange multiplier field $B$ are chiral fields, the respective complex conjugate fields are antichiral. Having formulated the gauge fixing with Lagrange multiplier fields $B$ and $\overline{B}$, BRS transformations are off-shell nilpotent on all fields. Then the ghost and gauge-fixing part can be written as a BRS variation and as such it is BRS invariant:

$$\Gamma_{g.t.} + \Gamma_{\text{ghost}} = s\text{Tr} \int dV (\eta + \overline{\eta} + \frac{1}{g^2})(\frac{\xi}{2} c_- \overline{B} + \frac{1}{8} DDc_- \phi + \text{c.c}) .$$

We want to mention that the extension of the gauge fixing to local coupling is not unique, but can be modified in different ways. We choose one form, which is most practicable for deriving the Callan-Symanzik equation, and quote, that the terms of the gauge fixing being BRS variations cannot have any influence on physical quantities as the gauge $\beta$ function or the anomaly coefficients.
One still has to couple the non-linear BRS-transformations to external fields

\[ \Gamma_{\text{ext.f.}} = \text{Tr} \int dV \ Y_\phi s \phi + \text{Tr} \int dS \ Y_c s c_+ + \text{Tr} \int d\bar S \ Y_\phi s \phi . \]  \hspace{1cm} (15)

Then one can express BRS invariance of the classical action in the Slavnov-Taylor (ST) identity, which takes the conventional form:

\[ S(\Gamma_{\text{cl}}) = \text{Tr} \int dV \frac{\delta \Gamma_{\text{cl}}}{\delta \rho} \frac{\delta \Gamma_{\text{cl}}}{\delta \phi} + \text{Tr} \int dS \left( \frac{\delta \Gamma_{\text{cl}}}{\delta Y_c} \frac{\delta \Gamma_{\text{cl}}}{\delta c_+} + B \frac{\delta \Gamma_{\text{cl}}}{\delta c_-} \right) \]
\[ + \text{Tr} \int d\bar S \left( \frac{\delta \Gamma_{\text{cl}}}{\delta Y_\phi} \frac{\delta \Gamma_{\text{cl}}}{\delta \bar c_+} + \bar B \frac{\delta \Gamma_{\text{cl}}}{\delta \bar c_-} \right) = 0 . \]  \hspace{1cm} (16)

The classical action \( \Gamma_{\text{cl}} \) comprises the Yang-Mills part (2), the gauge fixing and ghost part (14) and the external field part (15):

\[ \Gamma_{\text{cl}} = \Gamma_{\text{SYM}} + \Gamma_{\text{g.f.}} + \Gamma_{\text{ghost}} + \Gamma_{\text{ext.f.}} . \]  \hspace{1cm} (17)

It is immediately verified that the complete classical action satisfies the shift equation of the local coupling (6) and the defining equation of the \( \Theta \) angle (10).

The topological formula including all fields has its final form:

\[ N_g \Gamma^{(l)} = \left( N_\eta + N_{\bar \eta} + N_{Y_\phi} + N_{Y_c} + N_{\bar Y_\phi} + (l-1) \right) \Gamma^{(l)} . \]  \hspace{1cm} (18)

### 4 The anomaly

For constant coupling it is well known [15] that super-Yang-Mills theory rendered massive by supersymmetric mass terms for vector and Faddeev-Popov fields is renormalizable in the asymptotic sense, i.e. for momenta much larger than the mass parameters of the model. For the extended model with local coupling we can proceed in the same way: We add to the classical action a vector mass term in agreement with equations (19),(10) and (18):

\[ \Gamma_{\phi,\text{mass}} = \int dV \left( \eta + \bar \eta + \frac{1}{g^2} \right) M^2 \phi^2 . \]  \hspace{1cm} (19)

Since the Faddeev-Popov ghosts are chiral fields, it is not possible to add a mass term in agreement with (11) and (3). We choose the following form,

\[ \Gamma_{\text{mass,}\phi_\pi} = \int dS m^2 \left( \eta + \frac{1}{2g^2} \right) c_- c_+ - \int d\bar S m^2 \left( \bar \eta + \frac{1}{2g^2} \right) \bar c_- \bar c_+ , \]  \hspace{1cm} (20)
which yields a soft breaking of the identity (10) in the classical action, i.e., the identity (10) only holds up to soft terms

\[ W^{\eta - \bar{\eta}}_{cl} = \int dS m^2 c_+ c_+ + \int d\bar{S} m^2 \bar{c}_- \bar{c}_+ \sim 0. \]

(21)

Having avoided the off-shell infrared problem of supersymmetric Yang-Mills theories [16] by introducing the soft breaking terms of gauge symmetry one can establish the ST identity up to soft terms in renormalized perturbation theory,

\[ S(\Gamma) \sim 0. \]

(22)

The supersymmetry Ward identities

\[ W_\alpha \Gamma = 0, \quad \bar{W}_\dot{\alpha} \Gamma = 0, \]

(23)

and the global Ward identity of gauge symmetry hold including the soft terms.

The introduction of the multiplets \( \eta \) and \( \bar{\eta} \) does not change these identities as long as one uses a manifest supersymmetric formulation of the theory, i.e. superfields and supergraphs and a respective invariant scheme as for example the BPHZL scheme, and establishes the ST identity by adding the necessary non-invariant counterterms. Thus, the crucial equation for the anomaly is the identity of the \( \Theta \) angle (11). We have shown in the Wess-Zumino gauge [7] that with unbroken supersymmetry an anomalous term appears in the \( W^{\eta - \bar{\eta}} \) identity in one-loop order (\( \hat{\eta} = \eta + \frac{1}{2g^2} \)),

\[ W^{\eta - \bar{\eta}} \sim \frac{r_1}{256} \left( \int dS \hat{\eta}^{-1} W^\alpha W_\alpha - \int d\bar{S} \hat{\bar{\eta}}^{-1} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \right) + O(h^2). \]

(24)

The right-hand-side is BRS invariant and supersymmetric and satisfies the equation (11) and the topological formula (13) in one-loop order. However it cannot be considered as an ordinary scheme dependent breaking since it is the variation of a field monomial which depends on the logarithm of the coupling

\[ \int dS \hat{\eta}^{-1} W^\alpha W_\alpha - \int d\bar{S} \hat{\bar{\eta}}^{-1} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} = W^{\eta - \bar{\eta}} \left( \int dS \ln \hat{\eta} W^\alpha W_\alpha + \int d\bar{S} \ln \hat{\bar{\eta}} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \right). \]

(25)
The respective counterterm does not fulfill the topological formula. As such it cannot be generated in the subtraction procedure of ultraviolet divergences, since all diagrams depend on powers of the coupling and satisfy the topological formula in a trivial way. From eq. (25) it is possible to prove with the same algebraic methods as in [7] that the coefficient of the anomaly is gauge and scheme independent.

In the Wess-Zumino gauge the coefficient \( r^{(1)}_\eta \) has been calculated from scheme independent and convergent one-loop expressions by using gauge invariance in presence of local couplings [10]. There it has been shown that the anomaly is determined by the one-loop correction to the topological term. It is evident from the construction that it is also the topological term which induces the anomaly in the supersymmetric gauge. The anomalous breaking (24) just describes the adjustment of a finite counterterm to the \( \Theta \) angle in lowest order. For the direct computation of the anomaly coefficient in the supersymmetric gauge one had to proceed in the same scheme-independent way as in the Wess-Zumino gauge. As we will illustrate below, it is possible to shift the anomaly in the supersymmetric gauge to gauge symmetry or to the shift equation of the gauge coupling. Since the consistent supersymmetric schemes, the BPHZ and the Wilsonian regularization, fail to be gauge invariant the anomaly can appear in the ST identity and even in the shift equation. The specific form depends not only on the regularization scheme, but also on the explicit form of the gauge fixing and of the soft breaking terms. Hence, in a specific scheme the coefficient cannot be determined immediately from eq. (24), but we have first to establish the ST identity as well as the equation (6). For this reason we will circumvent the direct calculation and determine the coefficient implicitly from the 2-loop \( \beta \) function constructed in the section 6.

For illustrating that the anomaly can appear in the ST identity we add to the vertex functional \( \Gamma \) of (24) the counterterm,

\[
\Gamma_{\text{ct, noninv}} = -\frac{r^{(1)}_\eta}{256} \text{Tr} \left( \int dS \ln(2\eta) W^\alpha W_\alpha - \int dV \ln(\eta + \eta) e^{-\phi} D^\alpha e^\phi W_\alpha + c.c \right).
\]

The resulting vertex functional satisfies the \( W^{\eta-\eta} \) identity, but breaks the ST identity. Indeed one verifies that the first term cancels the anomaly in (24) whereas the second generates a breaking in the ST identity for local coupling. It is evident that the sum of the two terms does not depend on the logarithm of the coupling and satisfies the topological formula. Since gauge invariance is a fundamental symmetry we will not consider the anomaly in the ST identity in the further discussion, but it might appear there in a specific subtraction scheme and a specific calculation.
It is also interesting that one is able to shift the anomaly to the shift equation (5). Again we start from (24) and add the following counterterm:

\[ \Gamma'_{\text{ct,noninv}} = -\frac{r^{(1)}_\eta}{256} \text{Tr} \left( \int dS \left( \ln(2\eta + \frac{1}{g^2}) + \ln g^2 \right) W^\alpha W_\alpha + c.c. \right) . \]  

(27)

This counterterm also satisfies the topological formula, i.e. it is independent of \( \ln g \). The first term cancels the anomaly in (24), the second term now produces a breaking in (5),

\[ \frac{1}{2} \left( \int dS \frac{\delta}{\delta \eta} + \int d\bar{S} \frac{\delta}{\delta \bar{\eta}} \right) \Gamma' = -g^4 \partial_g^2 \Gamma' - \frac{r^{(1)}_\eta}{256} \text{Tr} \left( \int dS g^2 W^\alpha W_\alpha + c.c. \right) \]  

(28)

where

\[ \Gamma' = \Gamma + \Gamma'_{\text{ct,noninv}} + \mathcal{O}(\bar{h}^2) . \]  

(29)

In the following we will consider the two versions for the appearance of the anomaly: First we take the anomaly in the \( \mathcal{W}^\eta - \bar{\eta} \) identity (24) and leave the shift equation of the coupling (5) in its classical form, and second we take the anomaly in the shift equation (28) and take the \( \mathcal{W}^\eta - \bar{\eta} \) identity in its classical form (24).

## 5 Renormalization

For proceeding with renormalization one has to absorb the anomalous breaking into the symmetry identities. In the Wess-Zumino gauge where the anomaly appeared as a breaking of supersymmetry this was achieved by redefining the supersymmetry transformations. Accordingly, in the supersymmetric gauge where supersymmetry and the ST identity are imposed in their classical form (see (22) and (23)) we have to modify the operators \( \mathcal{W}^\eta - \bar{\eta} \) or the shift equation (5), respectively.

First we consider the anomaly in (24). There the anomalous breaking can be written into the form of an operator acting on the classical super-Yang-Mills action (2):

\[ \mathcal{W}^\eta - \bar{\eta} \Gamma^{(1)} \sim -\frac{r^{(1)}_\eta}{2} \delta \mathcal{W} \Gamma_{\text{SYM}}, \]  

(30)

with

\[ \delta \mathcal{W} = \int dS \left( \eta + \frac{1}{2g^2} \right)^{-1} \frac{\delta}{\delta \eta} - \int d\bar{S} \left( \bar{\eta} + \frac{1}{2g^2} \right)^{-1} \frac{\delta}{\delta \bar{\eta}} . \]  

(31)
Modifying the gauge fixing in an appropriate way (see (60) with (59) and for $H = 0, \Xi = 0$) one is able to establish the identity
\[
W_{\eta}^{\eta - \bar{\eta}} \Gamma \equiv \left( W_{\eta}^{\eta - \bar{\eta}} + \frac{r_{\eta}^{(1)}}{2} \delta W \right) \Gamma \sim 0 ,
\] (32)
to all orders of perturbation theory. It defines together with the ST identity, the topological formula (18) and the classical shift equation (6),
\[
\frac{1}{2} \left( \int dS \frac{\delta}{\delta \eta} + \int d\bar{S} \frac{\delta}{\delta \bar{\eta}} \right) \Gamma = -g^4 \partial g^2 \Gamma ,
\] (33)
the 1PI Green functions of the supersymmetric Yang-Mills theory with local coupling.

If we take the form (28) for the anomaly then we are able to write
\[
\frac{1}{2} \left( \int dS \frac{\delta}{\delta \eta} + \int d\bar{S} \frac{\delta}{\delta \bar{\eta}} \right) \Gamma^{(1)} = -g^4 \partial g^2 \Gamma^{(1)} - r_{\eta}^{(1)} g^6 \partial g^2 \Gamma_{\text{SYM}} .
\] (34)
Again modifying the gauge fixing by counterterms we are able to impose the anomalous shift equation
\[
\frac{1}{2} \left( \int dS \frac{\delta}{\delta \eta} + \int d\bar{S} \frac{\delta}{\delta \bar{\eta}} \right) \Gamma = -g^4 (1 + r_{\eta}^{(1)} g^2) \partial g^2 \Gamma
\] (35)
Now this equation defines together with the classical $W_{\eta}^{\eta - \bar{\eta}}$ identity
\[
W_{\eta}^{\eta - \bar{\eta}} \Gamma \sim 0 ,
\] (36)
a different but equivalent set of 1PI Green functions of supersymmetric Yang-Mills theory with local coupling.

Implicitly both equations (32) and (35) define a specific normalization for the coupling in loop orders $l \geq 2$. General normalization conditions can be obtained by carrying out redefinitions of the field $\eta$,
\[
\hat{\eta} \rightarrow \hat{\eta} + \sum_{l \geq 2} z^{(l)} \hat{\eta}^{-l+1} , \quad \hat{\bar{\eta}} \rightarrow \hat{\bar{\eta}} + \sum_{l \geq 2} z^{(l)} \hat{\bar{\eta}}^{-l+1} ,
\] (37)
in version (32) or redefinitions of the coupling,
\[
g^2 \rightarrow g^2 + \sum_{l \geq 2} z^{(l)} g^{2l} ,
\] (38)
in version (35). These redefinitions modify the explicit form of the $W_{\eta}^{\eta - \bar{\eta}}$ identity (32) or of the shift equation (35) but leave the respective non-anomalous identities in their classical form.
We want to conclude the section with the proof that the term (24) is indeed the only anomaly appearing in perturbation theory. For this purpose we again impose (33) in its classical form and list all possible terms which contribute to the breaking of the $W_{\eta - \eta} - \eta$ identity in general loop order $l$. Having established the ST identity asymptotically the hard breakings are $s_{\Gamma_{cl}}$-invariant and supersymmetric. Using parity conservation as well as the topological formula we have the following list of terms ($n, k \geq 1$ in perturbation theory):

$$
\Delta_{SYM}^{(l)} \equiv \text{Tr} \int dS \hat{\eta}^{-l} W^\alpha W_\alpha - \int dS \hat{\eta}^{-l} \tilde{W}_\alpha \tilde{W}^\alpha,
$$

$$
\Delta_{\phi^k}^{(l)} \equiv s_{\Gamma_{cl}} \text{Tr} \int dV (\hat{\eta} - \hat{\rho})^{2n-1} (\hat{\eta} + \hat{\rho})^{-l-2n} Y_\phi^k,
$$

$$
\Delta_{c}^{(l)} \equiv s_{\Gamma_{cl}} \text{Tr}(\int dS \hat{\eta}^{-l-1} Y_c c_+ - \int dS \hat{\eta}^{-l-1} \tilde{Y}_c \tilde{c}_+). \tag{39}
$$

Except for the first class terms with $l = 1$ all terms are variations of counterterms satisfying the topological formula,

$$
W_{\eta - \eta} \text{Tr} \int dS \hat{\eta}^{-l+1} W^\alpha W_\alpha + \int dS \hat{\eta}^{-l+1} \tilde{W}_\alpha \tilde{W}^\alpha = (l - 1) \Delta_{SYM}^{(l)},
$$

$$
W_{\eta - \eta} s_{\Gamma_{cl}} \text{Tr} \int dV (\hat{\eta} - \hat{\rho})^{2n} (\hat{\eta} + \hat{\rho})^{-l+1-2n} \rho \phi^k = 4n \Delta_{\phi^k},
$$

$$
W_{\eta - \eta} s_{\Gamma_{cl}} \text{Tr}(\int dS \hat{\eta}^{-l} \sigma c_+ + \int dS \hat{\eta}^{-l} \tilde{\sigma} \tilde{c}_+) = -l \Delta_{c}^{(l)}. \tag{40}
$$

Thus the only anomaly is the one-loop anomaly of eq. (25).

6 The gauge $\beta$ function

The construction with local coupling provides restrictions on the coefficient functions of the renormalization group and Callan–Symanzik equation. We focus on the construction of the Callan–Symanzik (CS) equation, but want to mention that the coefficient functions of the renormalization group equation are related to the ones of the CS equation in any mass-independent scheme.

In the classical action dilatations are broken by the non-BRS-invariant vector and ghost masses:

$$
\mu \partial_\mu \Gamma_{cl} \sim 0 \quad \mu \partial_\mu \equiv m \partial_m + M \partial_M + \kappa \partial_\kappa, \tag{41}
$$
where $m, M$ are the ghost and vector mass parameters, and $\kappa$ is a normalization point. In higher orders asymptotic scale invariance is broken by the dilatation anomalies,

$$
\mu \partial_\mu \Gamma \sim [\Delta_m]_4 \cdot \Gamma,
$$

(42)

with $\Delta_m$ are integrated field monomials of dimension 4 and with quantum numbers of the classical action. From algebraic consistency one obtains that the breaking is invariant with respect to the defining symmetries of the model: They satisfy the topological formula (38), they are invariant with respect to the linearized ST operator $s_\Gamma$ and are supersymmetric (see (22), (23)):

$$
s_\Gamma([\Delta_m]_4 \cdot \Gamma) \sim 0, \quad W_\alpha \Delta_m = \bar{W}_\alpha \Delta_m = 0.
$$

(43)

The dependence on the external fields $\eta$ and $\bar{\eta}$ is restricted by the $W^{\eta-\bar{\eta}}$ identity in its anomalous (32) or non-anomalous, classical (36) version, i.e.,

$$
W^{\eta-\bar{\eta}}([\Delta_m]_4 \cdot \Gamma) \sim 0, \quad \text{or} \quad W^{\eta-\bar{\eta}}([\Delta_m]_4 \cdot \Gamma) \sim 0.
$$

(44)

The relation of external fields to the local coupling is then defined by the classical identity (33) or the anomalous identity (35), respectively.

Absorbing the hard insertions order by order into differential operators which are symmetric under the defining symmetries in the same way as the insertions $\Delta_m$ one obtains the CS operator with $\beta$ functions and anomalous dimensions and the construction finally yields the CS equation of super-Yang-Mills theories with local coupling:

$$
C \Gamma \sim 0 \quad \text{with} \quad C = \mu \partial_\mu + \mathcal{O}(h).
$$

(45)

The most important result of the present construction with local coupling is the constraint on the $\beta$ function of the gauge coupling. It is evident from the construction that there is only one symmetric differential operator of the superfields $\eta$ and $\bar{\eta}$ satisfying the constraints of the $W^{\eta-\bar{\eta}}$ identity. For the version (32) where the $W^{\eta-\bar{\eta}}$ identity is broken by non-holomorphic contributions the corresponding symmetric CS operator is also non-holomorphic:

$$
D_\eta = \int dS \left( \frac{\delta}{\delta \eta} + \frac{r^{(1)}_\eta}{2} \left( \eta + \frac{1}{2g^2} \right)^{-1} \frac{\delta}{\delta \eta} \right).
$$

(46)

For the version (34), however, the symmetric differential operator of the external field is given by the holomorphic function

$$
D_\eta = \int dS \frac{\delta}{\delta \eta}.
$$

(47)
The operator $\mathcal{D}_\eta$ is the only symmetric differential operator that is not a variation under BRS and it is for this reason singled out from the additional unphysical field redefinitions which we construct in the appendix.

Hence one has

$$C\Gamma = (\mu \partial_\mu - \frac{1}{2} \hat{\beta}_{\eta}^{(1)}(\mathcal{D}_\eta + \mathcal{D}_{\bar{\eta}}) + \text{BRS var.}) \Gamma \sim 0 .$$

(48)

To obtain the $\beta$ function of the gauge coupling $g$ we use the shift equation of the coupling in its classical form (33) or in its anomalous form (28) and eliminate the integrated derivative with respect to external fields by the derivative of the gauge coupling:

$$(\mathcal{D}_\eta + \mathcal{D}_{\bar{\eta}})\Gamma = \begin{cases} -2g^4(1 + r_{\eta}^{(1)} g^2)\partial g^2 \Gamma - r_{\eta}^{(1)} \left( \int dS \frac{g^2 \eta}{\eta + (2g^2)^2} + \text{c.c.} \right) \Gamma \text{ for (30)} \\ -2g^4(1 + r_{\eta}^{(1)} g^2)\partial g^2 \Gamma \text{ for (17)} \end{cases}$$

(49)

From this expression we can read off the $\beta$-function of super-Yang-Mills theories in its closed form [12, 13, 17]:

$$\beta_{g^2} = \hat{\beta}_{\eta}^{(1)} g^4(1 + r_{\eta}^{(1)} g^2) .$$

(50)

Hence the identity (32) and (35) induce both a pure two-loop $\beta$ function, which is determined by the characteristic one-loop coefficients, the one-loop $\beta$ function and the anomaly coefficient $r_{\eta}^{(1)}$. Higher orders are scheme dependent and can be constructed by carrying out finite redefinitions of the superfield $\eta$ (37) or of the coupling (38). They induce modifications of anomalous symmetry identities as well as modifications of the corresponding $\beta$ functions. In general these redefinitions are defined by physical normalization conditions on the coupling.

### 7 Discussion and conclusions

The external fields $\eta$ and $\bar{\eta}$ are used to define insertions of the gauge invariant and supersymmetric Yang-Mills Lagrangian,

$$\frac{\delta}{\delta \eta} \Gamma \equiv [W^a W_a] \cdot \Gamma + \text{BRS var.} ,$$

(51)

and serve as such for a definition of the corresponding local operators. Having consistently constructed the vertex functional $\Gamma$ with these external fields, then one has also uniquely defined the corresponding insertions. In the conclusions we
want to discuss the results of the paper from the point of renormalized insertions which allows a direct comparison with previous results on the topic \cite{17, 9, 23}.

The $\theta^2$ component of $W^\alpha W_\alpha$ in superspace contains the topological term $\tilde{G} G$. In presence of local couplings, i.e. for $\eta, \tilde{\eta} \neq 0$, the higher–order corrections to the topological term are unambiguously determined by gauge invariance and by its property to be a total derivative \cite{8}:

$$\int dS [W^\alpha W_\alpha] \cdot \Gamma - \int d\bar{S} [\bar{W}_\alpha \bar{W}^\alpha] \cdot \Gamma = 0 .$$

(52)

Thus, eq. (52) defines an insertion $[W^\alpha W_\alpha]$ with Adler–Bardeen properties:

$$w \Gamma = r^{(1)}[W^\alpha W_\alpha] ,$$

(53)

with $w$ the chiral part of an anomalous axial symmetry.

On the other hand the integrated insertion $\int dS [W^\alpha W_\alpha]$ is defined at the same time by the derivative with respect to the gauge coupling:

$$\frac{1}{128} \int dS [W^\alpha W_\alpha] \cdot \nabla = g^4 \partial g \Gamma + \text{BRS var.}$$

(54)

In all loop orders except for one loop (52) and (54) can be fulfilled at the same time by adjusting finite counterterms. Hence the topological term yields an unambiguous definition of $[W^\alpha W_\alpha]$ in $l \geq 2$. In one loop order eq. (54) cannot be resolved, since

$$\frac{1}{128} \int dS [W^\alpha W_\alpha] \cdot \Gamma^{(1)} = g^4 \partial g \Gamma^{(1)} = 0 .$$

(55)

for vanishing external fields. Thus eq. (52) and eq. (55) yield two constraints on the renormalized insertion $[W^\alpha W_\alpha]$. For general $N = 1$ supersymmetric theories these constraints are not compatible with each other and lead to the anomaly of super-Yang-Mills theories with local coupling \cite{24} (see Ref. \cite{10} for the direct computation).

In order to define $[W^\alpha W_\alpha]$ in presence of the two constraints (52) and (55) one can pursue different ways:

- One can modify supersymmetry transformations in agreement with the algebra in such a way that (52) and (53) match to each other. This procedure has been carried out in the Wess-Zumino gauge and leads to anomalous supersymmetry transformations for the local coupling \cite{7}. The respective renormalized insertion has Adler–Bardeen properties \cite{53} and fulfils eq. (53) but supersymmetry is not maintained in its classical form.
One can give up the constraint (52) and adjust a finite counterterm in such a way that the renormalization of the topological term matches to eq. (55). The corresponding renormalized insertion has not the Adler–Bardeen property (54), but the coefficient in front of the anomaly contains also higher order corrections. With this definition the equation (52) is modified by non-holomorphic contributions in the fields \( \eta \), which result in non-holomorphic contributions to the \( \beta \) function. Due to relation (55) the field \( \eta \) can be considered as a local supersymmetric coupling. This construction has been performed in the present paper (see (32), (33) and (46)).

One can define \( [W^\alpha W_\alpha] \) in such a way that the topological charge has the Adler–Bardeen property (53) and satisfies (52). If supersymmetry is imposed in its classical form, the relation between the external fields and the coupling (55) is modified (see (35)). With this adjustment one obtains a holomorphic one-loop \( \beta \)-function in the external fields. However, the insertion \( [W^\alpha W_\alpha] \) is not defined in agreement with (55), but eq. (55) gets an one-loop correction (see (35)). This relation induces the 2-loop term in the gauge \( \beta \) function from the one-loop \( \beta \) function of the field \( \eta \) (see (47) and (49)).

The definition with broken supersymmetry is certainly the best motivated one from a physical point of view, since the renormalized insertion \( [W^\alpha W_\alpha] \) has Adler–Bardeen properties as well as an direct relation to the renormalization of the coupling. In this respect we want to point to the effective low energy Higgs Lagrangians for gluon fusion [18], which are defined in such a way that \( \tilde{G}G \) is the non-renormalized Adler-Bardeen insertion and \( G^{\mu \nu} G_{\mu \nu} \) is defined in agreement with the coupling normalization. The effective Lagrangians are indeed not supersymmetric. Vice versa in a related approach to a non-perturbative construction of effective quantum actions for \( N = 1 \) supersymmetric theories it has been shown that for non-trivial configurations supersymmetry is spontaneously broken if the relations (55) and an analog of (52) are imposed at the same time [19, 20].

From an abstract point of view mostly supersymmetric versions with linear supersymmetry have been considered in the past. There the insertion \( [W^\alpha W_\alpha] \) has been defined mainly by the Adler–Bardeen property. (For a different point to view see [21].) In Ref. [17] the closed expression of the gauge \( \beta \) function has been derived in a rigorous and scheme-independent way from the construction of the supercurrent. Here the insertion of \( [W^\alpha W_\alpha] \) is defined from the anomaly of the local R-current in such a way that it satisfies the Adler–Bardeen theorem. Insertions produced by differentiation with respect to the coupling are proven to be expressed in a non-trivial way by the Adler-Bardeen [\( [W^\alpha W_\alpha] \)] [22]. This is
a weaker version of the equation (33) of the present paper taking into account all possible redefinitions of the coupling. In principle the same point of view is taken in those references which use a local coupling and the Wilsonian scheme [9, 23, 24]. Using holomorphicity in the field \( \eta \) is nothing but establishing eq. (52) and defining \( W^\alpha W_\alpha \) with Adler–Bardeen properties resulting in the holomorphic \( \beta \) function.\footnote{See also [25] for the definition of the topological charge in the Wilsonian scheme in a non-supersymmetric context.} Thus, the renormalization of the external field \( \eta \) is not performed in accordance with the interpretation of the local coupling, and the transition from the holomorphic \( \beta \) function to the \( \beta \) function of the gauge coupling has to be carried out in accordance with eq. (35).

Finally it is worth to mention that for \( N = 2 \) supersymmetric Yang-Mills theories eq. (52) and eq. (53) are compatible as such and the theory is free of anomalies and has the holomorphic one-loop \( \beta \) function of the gauge-coupling.

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\section{The Callan–Symanzik equation}

For completeness we construct in this appendix the complete basis of symmetric operators for the CS equation with local coupling. Unfortunately, it is just the unphysical part, which requires quite some technicalities in the construction.

First we want to note that the classical action we have used for the construction is not the most general solution of the ST identity, but the most general solution is obtained by redefining the dimensionless field \( \phi \) by an arbitrary field monomial respecting rigid symmetry (see [14] for details). Hence we have to replace:

\[ \phi_a \rightarrow \phi_a(\phi') = \phi'_a + \sum_{k \geq 2} \sum_{\omega = 1}^{\Omega(k)} G^{k-1} a_{k, \omega}^{a_1 \ldots a_k} \phi_{a_1} \cdots \phi_{a_k}, \]  

with a BRS transformation

\[ s \phi' = \left( \frac{\partial \phi(\phi')}{\partial \phi'} \right)^{-1} Q(c_+, \bar{c}_+, \phi(\phi')). \]
The $s_{a(a_1...a_k)}^a$ are invariant tensors and $\Omega(k)$ is the number of such independent tensors for a given rank $k + 1$. With local couplings also the coefficients $a_{k,\omega}$ are extended to real superfields. They are interpreted as an infinite number of gauge parameters. In higher orders such redefinitions appear in general with arbitrary divergences and we have to use the external fields $a_{k,\omega}$ for absorbing the corresponding terms into the CS operator.

Second, the gauge fixing sector has to be modified in such a way that it satisfies the anomalous identities (32) or (35). For this purpose we define the superfield $\tilde{G}^2$,

$$\tilde{G}^{-2} = \eta + \bar{\eta} + \frac{1}{g^2} + O(h) ,$$

as an invariant under the anomalous symmetry identities. It can be verified that the $\tilde{G}$ that is defined order by order by the following implicit equation fulfills these requirements:

$$\tilde{G}^{-2} - r^{(1)}_\eta \ln (\tilde{G}^{-2} + r^{(1)}_\eta) = \begin{cases} \dot{\eta} - \frac{r^{(1)}_\eta}{2} \ln (2\dot{\eta} + r^{(1)}_\eta) + c.c \text{ for (32)}, \\ \dot{\eta} - \frac{r^{(1)}_\eta}{2} \left( \ln \left( \frac{1}{g^2} + r^{(1)}_\eta \right) + c.c \right) \text{ for (35)}. \end{cases}$$

Then the gauge fixing function,

$$\Gamma_{g.t.}(H, \Xi) = \int dV \, \tilde{G}^{-2} \left( (\Xi + \xi) \bar{B}B + \frac{1}{8} e^H (\phi DDB + \phi D\bar{D}\bar{B}) \right) ,$$

satisfies the defining symmetry identities. For vanishing external fields it just reduces to the classical expression. For later usage we have introduced an external field $H$ and have extended the gauge parameter $\xi$ to an external field $\Xi$ with shift. These fields as well as the gauge parameters $a_k$ couple to BRS-variations and can be extended to BRS doublets $(u, v) = (H, C), (\Xi, X), (a_k, \chi_k)$:

$$su = v , \quad sv = 0 .$$

In addition, the dependence of $\Gamma$ on $H$ is constrained by an integrated identity:

$$\left( \int dS \left( B \frac{\delta}{\delta B} + c_- \frac{\delta}{\delta c_-} \right) + \int d\bar{S} \left( \bar{B} \frac{\delta}{\delta \bar{B}} + \bar{c}_- \frac{\delta}{\delta \bar{c}_-} \right) - 2\xi \partial_\xi - \int dV \, \frac{\delta}{\delta H} \right) \Gamma = 0 .$$

With these ingredients we are able to express all field redefinitions appearing in the breaking of dilatations as field differential operators. Taking into account that the gauge fixing action does not receive loop corrections due to linearity in the auxiliary field $B$ and $\bar{B}$ the operator $\mathcal{D}_H + \mathcal{D}_\eta$ has to be supplemented by the operator:

$$\mathcal{D}_H = -2 \int dV \, \tilde{G}^2 (1 + r^{(1)}_\eta \tilde{G}^2) \left( \frac{\delta}{\delta H} + (\Xi + \xi) \frac{\delta}{\delta \Xi} \right) ,$$

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and
\[(\mathcal{D}_\eta + \mathcal{D}_\pi + \mathcal{D}_H)\Gamma_{\text{g.f.}} = 0.\] (64)

The linear redefinitions of the fields \(\phi\) give rise to the following \(l\)-loop counting operator,
\[\mathcal{N}^{(l)}_\phi = \int dV f^{(l)}(\xi, H, a_{k,\omega}) \tilde{G}^{2l}(\phi \frac{\delta}{\delta \phi} - \rho \frac{\delta}{\delta \rho} - \frac{\delta}{\delta H}),\] (65)

and the renormalization of the parameters \(a_{k,\omega}\) to
\[\mathcal{D}_{a_{k,\omega}} = \int dV h^{(l)}(\xi, H, a_{l,\omega}) \tilde{G}^{2l} \frac{\delta}{\delta a_{k,\omega}}.\] (66)

Having constructed \(\tilde{G}^2\) as a symmetric superfield it is obvious that these operators commute with the corresponding anomalous symmetry operators. Using the BRS-doublet structure of external fields \(H, \Xi, a_k\) it is evident that the operators of eqs. (63), (65) and (66) are BRS-variations.

The complete CS equation is a linear combination of symmetric operators:
\[\left(\mu \partial_\mu - \frac{1}{2} \tilde{\beta}^{(1)}_\eta (\mathcal{D}_\eta + \mathcal{D}_\pi + \mathcal{D}_H) - \sum_l \left( \hat{\gamma}^{(l)} N^{(l)} + \sum_{\omega,k} \hat{\gamma}^{(l)}_{\omega,k} D^{(l)}_{a_{k,\omega}} \right) \right) \Gamma \sim 0.\] (67)

For vanishing external fields \(\eta, \bar{\eta} = 0\) and for \(H = 0\) one can use the identity (62) and replace the field differentiation with respect to \(H\) by a differentiation with respect to the auxiliary fields \(B, \bar{B}\) and the gauge parameter \(\xi\). Using furthermore that \(\tilde{G} \to g\) for vanishing external fields, we find the usual CS equation of ordinary super-Yang-Mills theories with a closed, being in the explicit construction here a pure 2-loop \(\beta\) function of the gauge coupling:
\[\left(\mu \partial_\mu + \beta^{(1)}_{g^2} g^4 (1 + r^{(1)}_\eta g^2) (\partial^2_{\eta} + \xi \partial_\xi - N_B) - \gamma (N_\phi - N_B - N_{c_{\omega}} + 2 \xi \partial_\xi) - \sum_{\omega,k} \gamma_{\omega,k} \partial_{a_{k,\omega}} \right) \Gamma \sim 0 ,\] (68)

where
\[\gamma = \sum_l \hat{\gamma}^{(l)} f^{(l)} g^{2l} \quad \text{and} \quad \gamma_{\omega,k} = \sum_l h^{(l)} g^{2l} \hat{\gamma}^{(l)}_{\omega,k}.\] (69)

\(N_\eta\) are the usual field counting operators, which include in the case of complex fields also the complex conjugated ones. Absence of an anomalous dimension of the Faddeev-Popov ghost \(c_{\xi}\) has its origin in the non-renormalization theorems of chiral vertices.
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