Modeling of Economic Data using Bivariate MGARCH Models

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Authors’ contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Abstract

Aims: The aim of this study is to examine Economic data using the multivariate GARCH model.

Study design: The study used monthly data of Nigerian crude oil prices (dollar Per Barrel) and Consumer price Index.

Methodology: This work covers time series data on crude oil price and consumer price Index rural obtained from Central bank of Nigeria (CBN) from 2000 to 2019. To achieve the aim of the study, bivariate VECH and BEKK model were applied.

Results: The results confirmed that returns on economic data were correlated. Also, diagonal multivariate VECH model confirmed one of the properties that it must be ‘positive semi-definite’ and the BEKK also confirmed the volatility spillover effects among the economic data.

Conclusion: From the results obtained, it was confirmed that conditional variances depends only on own lags and own lagged square returns and conditional covariances depends only on own lags and own lagged cross products of returns. As for cross-volatility effects, past innovations in crude oil price have greatest influence on future volatility of returns on economic data. It was also confirmed that time varying covariance displays among these economic data and lower degree of persistence and based on Model selection criteria using the Akaike information criteria (AIC) diagonal VECH model is better fitted than the BEKK model.

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1 Introduction

The term Economic data is used to describe or explain the actual economy in the time past and present. It is one of the key policies that are relevant in the drive of economic growth and business innovation. It is a collection of information on various aspects of a country’s economy.

Nigeria has emerged as African’s largest economy as a result of oil economy which has been the dominant source of government revenues. However, regulatory constraints and security risks have shown that there are new area of investment outside oil and natural gas. These areas and the data generated are dominant source of government revenues which is referred to as economic data. In recent years economic data has become a significant piece of numerous nation’s economy, the expanding esteem has roused business analysts to anticipate costs of stocks and other monetary returns. The assessment of economic data has an important role to play among investors and policy makers. It helps to get data on how different areas are influenced by a country's macroeconomic factors.

Economic data help in the stabilization of the economy [1] and [2].

Today’s economy revolves around data. The world has moved into digital age and more data is produced daily, every hour and minute. Economic data holds an enormous potential in various fields and is essential for economic growth, job creation and societal progress. It has also helped in the process of information for the public and private sector to create added value for third parties or the public for a better and more effective decision-making process.

The accessibility of dependable and modern financial information likewise consoles global financial backers by permitting them to screen monetary turns of events and deal with their venture hazard. It permits firms and people to settle on legitimate business choices with certainty that they comprehend the generally macroeconomic climate [3].

It has been authenticated in other related works that the economic data of a country reflects the well-being of the country. It reveals the economic and financial relations of the country with the other countries in the world. Consequently, it plays a solid determining role on economic decisions and expectancies in developing countries which consistently make efforts to ensure that earnings obtained from exportation are greater than the monies paid for importation of goods and services.

With the expansion in the intricacy of instrument in the danger the board field, high requests for different models can reenact and mirror the qualities of Economic information. One significant component of monetary data is its volatility. This is because volatility is the proportion of danger looked by financial backers, strategy creators, people and monetary establishment. It is realized that unpredictability of Economic information shifts over the long haul and will in general group in periods. While breaking down the co-developments of profits on financial information, it is crucial for gauge, build, assess and conjecture the co-unpredictability.

Therefore, there is need to examine an appropriate model to be used in modeling Economic data. It is against this background that the study used statistical modeling of economic data using Bivariate MGARCH model.

Economic data is very important in the growth of any economy, [4] states that economic data assumes a significant part in the assembly of monetary assets for long haul speculation through monetary intermediation. The danger related with financial information is its unpredictability and related consequences for other miniature and macroeconomic pointers. Economic Data are used by the government to measure policies put in place for the development of different activities in the economy to upgrade the speculation, development and improvement.

Business analysts have contradiction on the impact of cash supply on monetary development [5]. Some others accept that most monetary development relies upon the amount of cash and the efficiency and effectiveness of
the Economic data available by the country that devotes additional time in contemplating conduct of total cash supply in their economic activities [6].

GARCH means Generalized Autoregressive Conditional Heteroscedasticity. It was created by Robert F. Engle in 1982, was the first person to explain a method of estimating volatility in monetary markets. MGARCH simply means Multivariate Generalized Autoregressive Conditional Heteroscedasticity.

Tsay [7] conveyed an examination on transmission of spot power costs and value instability was done on five territorial power markets in Australian utilizing a MGARCH model, the outcome shows the presence of positive own mean overflows in just few business sectors. Engle and Kroner [8] said one distinct feature of financial time series is the non-constant volatility of the data. Alexander and Lazar [9,10] proposed univariate mixed normal GARCH process which has been shown to be well suited for analyzing and forecasting financial volatility, they find out that it was not suitable any longer so MGARCH was used and both models were time varying and asymmetric. The MGARCH generates a more reliable model than the univariate models. It is significant to predict the dependence in the co-movement of future returns in a portfolio. Mansor [11] used a bivariate GARCH parameterization for cash and future markets with a flexible functional form for time-varying volatility that is suitable for testing whether the optimal hedge ratio constant and if it is due to deterministic time-to-maturity effects. Hartman and Sedlak [12] examined a ten year exchange rate data using the Multivariate GARCH BEKK and DCC model based on the MAE and RMSE, the result found shows that BEKK model performed better than the DCC model.

Gounopoulos [13] examined the linkages between stock returns and currency exposures of US, UK and Japanese banks and insurance companies by using a BEKK model. Bailey [14] studied on the relationship between china and international stock markets, the results reveal that the return on China’s B shares has little or no correlation with international equity returns. Bae and Karolyi [15] used three asymmetric GARCH models to investigate spillovers in volatility between Japan and the U.S, their results suggest bidirectional relations in volatility between the two markets. Ojo [16] examined dynamic conditional correlation model in the foreign exchange rates of the Indian rupee and four other prominent foreign currencies to measure volatility spillover across these exchange rates. Chevalier [17] investigates dynamic nature of correlation among oil, gas and CO2 of European climate exchange using BEKK, CCC and DCC models. Lebotsa [18] examines price and volatility spillover effect of monthly data of natural gas of US, Europe and Japan in a VEC-Multivariate GARCH model framework.

2 Materials and Methods

2.1 Data

The data used for this study are Monthly data were sourced from the Central Bank of Nigeria (CBN) statistical database website for 20yrs, (2000-2019). (www.cbn.gov.ng). E-views software was used. The variables used are Crude Oil Price and Consumer Price Index.

2.2 Generalized Vech – Garch Model

VECH means Vector Error Conditional Heteroskedasticity. It was the first MGARCH Model that was introduced by Bollerslev [19]. Every conditional variance and covariance is a function of all lagged conditional variances and covariances, as well as lagged squared returns and cross-products of returns. Because it is difficult to impose positive definiteness of the variance-covariance matrix in this model, Bollerslev, Engle & Wooldridge developed a simplified so-called diagonal VECH Model. According to Bollerslev [19], the diagonal VECH GARCH is given as:

\[ VECH(H_t) = C + AVECH(ε_{t-1}ε_{t-1}') + BVECH(H_{t-1}) \] (3.1)

Where \( H_t \) is an \( N \times N \) conditional variance-covariance matrix. 
\( C \) is an \( (N(N+1)/2) \times 1 \) vector. 
\( A_i \) and \( B_i \) are \( (N(N+1)/2) \times (N(N+1)/2) \) parameter matrices. 
\( N \) represents the number of variables.
A diagonal VECH model conditional variance-covariance has been restricted to the form developed by Bollerslev [19] in which A and B are assumed to be diagonal, this implies that there is no direct volatility spillovers from one series to another. This considerably reduces the number of parameters to be estimated to nine in a bivariate case. (Now A and B have 3 elements). i.e. If N= 2, C will be a 3 x 1 parameter vector and A and B will be 3x3 parameter matrix.

For a simple case of diagonal VECH model, when N=2, where p=q=1. The matrix will reduce the number of parameters to nine from twenty one.

\[
\begin{bmatrix}
    h_{11,t} \\
    h_{21,t} \\
    h_{22,t}
\end{bmatrix} =
\begin{bmatrix}
    C_{11} \\
    C_{21} \\
    C_{31}
\end{bmatrix} +
\begin{bmatrix}
    a_{11} & 0 & 0 \\
    0 & a_{22} & 0 \\
    0 & 0 & a_{33}
\end{bmatrix}
\begin{bmatrix}
    \epsilon_{1,t-1}^2 \\
    \epsilon_{2,t-1}^2 \\
    \epsilon_{3,t-1}^2
\end{bmatrix} +
\begin{bmatrix}
    b_{11} & 0 & 0 \\
    0 & b_{22} & 0 \\
    0 & 0 & b_{33}
\end{bmatrix}
\begin{bmatrix}
    h_{11,t-1} \\
    h_{21,t-1} \\
    h_{22,t-1}
\end{bmatrix}
\]  

(3.2)

Performing the matrix operation, we obtain

\[
\begin{align*}
    h_{11,t} &= C_{11} + a_{11} \epsilon_{1,t-1}^2 + b_{11} h_{11,t-1} \\
    h_{21,t} &= C_{21} + a_{22} \epsilon_{2,t-1}^2 + b_{22} h_{21,t-1} \\
    h_{22,t} &= C_{31} + a_{33} \epsilon_{3,t-1}^2 + b_{33} h_{22,t-1}
\end{align*}
\]  

(3.3)  

(3.4)  

(3.5)

This shows that the variances depend on past own squared residual ($\epsilon_{i,t-1}^2$) and past values of itself ($h_{i,t-1}$). Each element of the covariance matrix ($h_{12t}$) depends on lagged cross-products of residuals ($\epsilon_{1,t-1}\epsilon_{2,t-1}$) and lagged conditional variance.

### 2.3 The BEKK GARCH model

The BEKK means Baba Engle Kraft Kroer who developed the model in 1995 [18]. The BEKK model is generally more accepted and flexible. The conditional covariance matrices are positive definite. The model is defined according to Huang [20].

\[
H_t = C'C + \sum_{j=1}^{q} \sum_{k=1}^{K} A_{kj}' \epsilon_{t-j} \epsilon_{t-j} A_{kj} + \sum_{j=1}^{p} \sum_{k=1}^{K} B_{kj}' H_{t-j} B_{kj}
\]  

(3.6)

Where

- $A_{kj}$, $B_{kj}$, and C are N x N parameter matrices
- C is a lower triangular matrix.

For Bivariate Diagonal BEKK model can be described as thus:

Let $K$ be a 2 x 2 matrix and equal to the $C'C$

\[
K = C'C
\]  

(3.7)

$N$ is the number of variables

\[
A'(\epsilon_{t-1}\epsilon_{t-1})A = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t-1} & \epsilon_{2,t-1} \\ \epsilon_{2,t-1} & \epsilon_{2,t-1} \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}
\]
\[ A\left(\varepsilon_{t,1}^{e}, \varepsilon_{t,2}^{e}\right) A' = \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_{t,1}^{e} \\ \varepsilon_{t,2}^{e} \end{pmatrix} \begin{pmatrix} 11 & 0 \\ 0 & a_{22} \end{pmatrix} \]
\[ = \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_{t,1}^{2} & \varepsilon_{t,1}^{e} \varepsilon_{t,2}^{e} \\ \varepsilon_{t,2}^{e} \varepsilon_{t,1}^{2} & \varepsilon_{t,2}^{2} \end{pmatrix} \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} \]
\[ = \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_{t,1}^{2} + a_{11} \varepsilon_{t,1}^{e} \varepsilon_{t,2}^{e} + 0 \\ 0 + a_{22}^{2} \varepsilon_{t,2}^{2} \varepsilon_{t,1}^{e} + a_{11}^{2} \varepsilon_{t,1}^{2} \varepsilon_{t,2}^{2} \end{pmatrix} \]
\[ = \begin{pmatrix} a_{11}^{2} \varepsilon_{t,1}^{2} & a_{11}^{2} \varepsilon_{t,1}^{e} \varepsilon_{t,2}^{e} \\ a_{22}^{2} \varepsilon_{t,2}^{2} \varepsilon_{t,1}^{e} & a_{22}^{2} \varepsilon_{t,2}^{2} \varepsilon_{t,1}^{2} \end{pmatrix} \]

Similarly,

\[ B^{1}(H_{t,1})B = \begin{pmatrix} b_{11}^{2}h_{t,1}^{2} & b_{11}^{2}h_{t,1}^{e}h_{t,2}^{e} \\ b_{22}^{2}h_{t,2}^{2}h_{t,1}^{e} & b_{22}^{2}h_{t,2}^{2} \end{pmatrix} \]
\[ \Rightarrow H_{t} = c' + A'\left(\varepsilon_{t,1}^{e}, \varepsilon_{t,2}^{e}\right) A + B^{1}(H_{t,1})B \]
\[ = \begin{pmatrix} c_{11} & c_{12}c_{11} \\ c_{12}c_{11} & c_{22} \end{pmatrix} + \begin{pmatrix} a_{11}^{2} \varepsilon_{t,1}^{2} & a_{11}^{2} \varepsilon_{t,1}^{e} \varepsilon_{t,2}^{e} \\ a_{22}^{2} \varepsilon_{t,2}^{2} \varepsilon_{t,1}^{e} & a_{22}^{2} \varepsilon_{t,2}^{2} \varepsilon_{t,1}^{2} \end{pmatrix} \]
\[ + \begin{pmatrix} b_{11}^{2}h_{t,1}^{2} & b_{11}^{2}h_{t,1}^{e}h_{t,2}^{e} \\ b_{22}^{2}h_{t,2}^{2}h_{t,1}^{e} & b_{22}^{2} \varepsilon_{t,2}^{2} \varepsilon_{t,1}^{2} \end{pmatrix} \]
\[ = \begin{pmatrix} c_{11} + a_{11}^{2} \varepsilon_{t,1}^{2} + b_{11}^{2}h_{t,1}^{2}, & c_{12}c_{11} + a_{11}^{2} \varepsilon_{t,1}^{e} \varepsilon_{t,2}^{e} + b_{11}^{2}h_{t,1}^{e}h_{t,2}^{e} \\ c_{12}c_{11} + a_{22}^{2} \varepsilon_{t,2}^{2} \varepsilon_{t,1}^{e} + b_{22}^{2}h_{t,2}^{2}h_{t,1}^{e}, & c_{22} + a_{22}^{2} \varepsilon_{t,2}^{2} \varepsilon_{t,1}^{2} + b_{22}^{2} \varepsilon_{t,2}^{2} \varepsilon_{t,1}^{2} \end{pmatrix} \]

2.4 Multivariate GARCH model estimation

Following the assumption of a conditional normal distribution, the parameters of multi-variate GARCH models of the above model specification can be done using maximization of a Log-Likelihood function. It assumed that the time series treated should be stationary and the distribution of its residual is pre-defined as a conditional Gaussian distribution. The latter assumption can meanwhile give us hints on how to check the adequacy of the established MGARCH model. It is given as:

\[ L(\theta) = \frac{TN}{2} - \frac{1}{2} \sum_{t=1}^{T} (\log H_{t} + \sum_{i=1}^{n} H_{t}^{-1} \sum_{i=1}^{n}) \]

where \( \theta \) is all the parameters to be estimated,
T is the number of observations and
N is the number of the series.

The maximum likelihood estimates for $\theta$ is asymptotically normal, and thus traditional procedures for statistical inference are applicable.

2.4.1 Estimation procedure

The estimation procedure for all models specified above starts with the following steps:
1. Time plot of the raw data
2. Time plot of the transformation of the return series
3. Descriptive test statistics for Normality Test
4. Unit Root Test

2.4.2 Multivariate GARCH model estimation

This is done on the basis of the coefficients of the selected model. The news impact assessment and test for volatility persistence will be done under model parameter estimations.

2.4.3 Model selection

Model selection is done using Akaike information criteria (AIC), Schwartz information criteria (SIC). The Akaike information criteria (AIC) are defined thus:

$$AIC = 2K - 2\ln(L) = 2K + \ln\left(\frac{RSS}{n}\right)$$  \hspace{1cm} (3.9)

Where $K$ represents the number of parameters used in the model and $N$ is the sample size
$L$ represents maximized value of the likelihood.
$RSS$ represents Residual Sum of Squares.

In general the desirable is the one that minimizes the AIC or SIC of HQ on the significant tests for each parameter. However, the study will place emphasis on the Schwartz information criteria because it levies heavy penalty on models for loss of degree of freedom as revealed in Abdulkarem and Abdulhakeem [21].

2.4.4 Model diagnostic check

In order to be sure that the model selected is test fitted and good enough for estimation, the following confirmatory test shall be carried out by testing conditional heteroscedasticity. There are two different tests for testing Conditional Heteroscedasticity, in this study the portmanteau test and ranked-based test were used.

3 Results and Discussion

3.1 Time plots of the raw data

The time plot in Figs. 1 shows the raw data of crude oil prices. From visual examination, the crude oil prices are trending upward and downward (rise and fall which shows the presence of a trend). Therefore there is need for detrending or removal of the trends to enhance stationarity in the series. However, there are different ways of detrending a non stationary series but we will consider two ways, the calculation of log returns of the series and the differencing using the Augmented Dickey-Fuller (ADF) test. It is used to examine the order of integration in time series.

If the series are stationary, it means their mean, variance and covariance are constant overtime and it implies that the results obtained from the analysis are reliable and can be useful in predicting future economic activities [22].
3.2 Time plot of raw data

The time plot in Figs. 2 shows the raw data of consumer prices index. We observe that there is a rise and fall which shows the presence of a trend. The rise and fall in the trend indicates the presence of unit root which is capable of causing biasness in estimation.

Therefore there is need for detrending or removal of the trends to enhance stationarity in the series. However, there are different ways of detrending a non stationary series but we will consider two ways, the calculation of log returns of the series and the differencing using the Augmented Dickey-Fuller (ADF) test. It is used to examine the order of integration in time series If the series are stationary, it means their mean, variance and covariance are constant overtime.
3.3 Time plot of returns series

Figs. 3 – 4 shows the time plot of the return series, they show volatility clustering (rise and fall follows another rise and fall around the origin zero). This simply means the series are stationary. Similarly, after differencing the raw data, the result obtained from the differenced series were used to do a time plot to check for stationarity. This shows that it was stationary which revealed evidence of volatility clustering. The result obtained confirms [23] assertion in the investigation on return and volatility spillovers across equity markets in Mainland China, Hong Kong and the United States. In this study it was shown that the estimated returns on the series were stationary around zero.

Fig. 3. Time plot of the returns on Crude Oil Price

Fig. 4. Time Plot of the Returns on Consumer Price Index (RCPI)
3.4 Descriptive statistics on the returns series

Table 1 show descriptive statistics of returns on the series. All the mean are positive, except that the returns in crude oil prices shows negatively skewed statistics. This is an indication that the returns series are skewed to the left. The probability value of the series (returned) are less than 0.05. This shows that it violate the null hypothesis of normality. The null hypothesis of normality states that the probability value less than 0.05 is not normally distributed while the probability value greater than 0.5 is normally distributed. This was in line with [24] findings in their studied on volatility spillovers in emerging markets during the global financial crises: Diagonal BEKK Approach. In the study, all the series were not normally distributed.

| Text Statistics | RCOP       | RCPI       |
|-----------------|------------|------------|
| Mean            | 0.409576   | 0.982453   |
| Median          | 1.340491   | 0.829724   |
| Maximum         | 18.53161   | 7.162548   |
| Minimum         | -32.10457  | -3.489920  |
| Std. Dev.       | 9.084101   | 1.324827   |
| Skewness        | -0.835426  | 0.527632   |
| Kurtosis        | 3.976282   | 7.186879   |
| Jarque-Bera     | 37.29268   | 185.6586   |
| Probability     | 0.000000   | 0.000000   |

Source: Extract from Eview software Analysis

Table 2 contains the result of unit root test for the raw data series. It shows that all the series were stationary at first difference order 1.

| Variable            | ADF       |
|---------------------|-----------|
| Crude Oil Price     | 1(0)      |
| Consumer Price Index| -2.182    |
|                     | 1(1)      |
|                     | -11.0715***|
|                     | -5.730*** |

Source: Extract from Eviews Software and *** represented 5% Level of Significance

Table 3 shows the lag length selection criteria. This is done to determine the number of lag to be used in the model specification. The acceptable lag length based on the model with the least lag length specification criteria was AIC (20.166) with lag 1.

| Lag  | LogL   | LR      | FPE    | AIC     | SC      | HQ      |
|------|--------|---------|--------|---------|---------|---------|
| 0    | -2343.689 | NA      | 7901.482 | 20.32631 | 20.38592* | 20.35036 |
| 1    | -2309.165 | 67.55460* | 6730.781* | 20.16593* | 20.46397 | 20.28614* |
| 2    | -2297.453 | 22.51101 | 6986.287 | 20.20305 | 20.73954 | 20.41944 |
| 3    | -2285.898 | 21.80837 | 7262.741 | 20.24154 | 21.01646 | 20.55410 |
| 4    | -2277.043 | 16.40789 | 7730.729 | 20.30340 | 21.31675 | 20.71212 |
| 5    | -2263.206 | 25.15737 | 7884.364 | 20.32213 | 21.57392 | 20.82702 |
| 6    | -2255.490 | 13.76218 | 8482.402 | 20.39385 | 21.88408 | 20.99491 |
| 7    | -2246.945 | 14.94496 | 9065.349 | 20.45840 | 22.18706 | 21.15562 |
| 8    | -2237.419 | 16.32909 | 9612.476 | 20.51445 | 22.48155 | 21.30785 |

Table 4 contains the result for test of co-integration using trace and max eigen test statistics. This is done to know whether there is a co-integrating relationship within the returns series and from the result obtained, there exist four co-integration equations because the probability is less than 0.05.
Table 4. Test for Co integration Using Trace and Maxeigen Statistic

| Hypothesized No of CE(S) | Eigen Value | Trace Value | 0.05 critical Value | Probability | Max Statistic | 0.5 critical Value | Probability |
|--------------------------|-------------|-------------|---------------------|-------------|---------------|--------------------|-------------|
| None*                    | 0.360       | 302.779     | 47.856              | 0.0001      | 105.376       | 27.584             | 0.0000      |
| Almost 1 *               | 0.303       | 197.403     | 29.797              | 0.0001      | 85.160        | 21.132             | 0.0000      |
| Almost 2 *               | 0.243       | 112.243     | 15.495              | 0.0001      | 65.794        | 14.265             | 0.0000      |
| Almost 3%                | 0.179       | 46.449      | 3.841               | 0.0000      | 46.449        | 3.841              | 0.0000      |

Source: extract from e view Software

Table 5 contains the Test for Heteroskedasticity. It states that residue obtained from a model must obey the assumption of a classical least square regression which says that the residual obtained from a linear regression must obey the assumption of normality (zero mean and constant variance). The probability value in the table shows that it is less than 0.05% which violates the assumption of homoskedasticity.

Table 5. Heteroskedasticity test

| Joint test: | Chi-sq | Df     | Prob. |
|-------------|--------|--------|-------|
| Chi-sq      | 375.9909 | 180    | 0.0000 |

Individual components:

| Dependent | R-squared | F(18,217) | Prob. | Chi-sq(18) | Prob. |
|-----------|-----------|-----------|-------|------------|-------|
| res1*res1 | 0.057762  | 0.739040  | 0.7685| 13.63180   | 0.7528|
| res2*res2 | 0.296031  | 5.069575  | 0.0000| 69.86339   | 0.0000|
| res3*res3 | 0.320037  | 5.674162  | 0.0000| 75.52869   | 0.0000|
| res4*res4 | 0.201182  | 3.06180   | 0.0001| 47.47886   | 0.0002|
| res2*res1 | 0.163208  | 2.351316  | 0.0020| 38.51708   | 0.0033|
| res3*res1 | 0.078638  | 1.028944  | 0.4284| 18.55866   | 0.4195|
| res3*res2 | 0.414085  | 8.520666  | 0.0000| 97.72417   | 0.0000|
| res4*res1 | 0.205426  | 3.116800  | 0.0000| 48.48059   | 0.0001|
| res4*res2 | 0.236654  | 3.737484  | 0.0000| 55.85031   | 0.0000|
| res4*res3 | 0.179888  | 2.644331  | 0.0005| 42.45354   | 0.0010|

Table 6 represents the e-view results of the Bivariate Diagonal VECH-GARCH models. In the VECH Model, the diagonal entries of the ARCH term are all positive and significant at less than 0.1 which is evidence of positive definite condition (variance and co-variance). Also, the variance and co-variance are positive which reveals that each micro economic variable depends on its own lag innovations. Also, return on crude oil price (RCOP) has larger own ARCH effect with the co-efficient value of 0.192^2 While the return on consumer price Index (RCPI) has the smallest own ARCH with the value 0.102^2. This simply means that shock in RCPI do not affect the variance of returns on crude oil price. The GARCH component shows three patterns: Firstly, the first pattern shows that all the variance co-efficient are significant. This means that they depend on their own lag innovations. Secondly, it reveals that own spill over are more than their cross-economic spill over. Similar condition was found in [25] investigating Transmission of equity returns and volatility in Asian developed and emerging markets: A MGARCH analysis. Thirdly, A unidirectional spillover from return on consumer price index (RCPI) to crude oil market. In summary, there is strong evidence of volatility spill over from CPI to crude oil International market.

3.5 RCOP &RCPI (VECH-GARCH)

Table 7 represents the e-view results of the Bivariate Diagonal BEKK-GARCH models. In the Bivariate Diagonal BEKK- GARCH model, in the ARCH term the leading diagonal is positive and significant at 5% level of significance. This simply means that the micro-economic variables are influenced by their past innovations. Also, in the variance term (GARCH) the estimates are significance and positive. Volatility impact is high in the returns on crude oil price than the returns on consumer price index. This shows a clear evidence of the tendency of spillover effect from returns on crude oil price to consumer price Index in a unidirectional order.
### Table 6. Bivariate MGARCH Model

| Coefficient | Std. Error | z-Statistic | Prob. |
|-------------|------------|-------------|-------|
| C(1)        | 0.582490   | 0.589058    | 0.988850 | 0.3227 |
| C(2)        | 0.987480   | 0.021978    | 44.93066 | 0.0000 |

### Variance Equation Coefficients

| Coefficient | Std. Error | z-Statistic | Prob. |
|-------------|------------|-------------|-------|
| C(3)        | 8.149223   | 4.645411    | 1.754253 | 0.0794 |
| C(4)        | 0.001330   | 0.106261    | 0.012518 | 0.9900 |
| C(5)        | -0.003053  | 0.001369    | -2.230484 | 0.0257 |
| C(6)        | 0.192204   | 0.063083    | 3.046852 | 0.0023 |
| C(7)        | 0.129038   | 0.083785    | 1.540106 | 0.1235 |
| C(8)        | 0.102235   | 0.020219    | 5.056284 | 0.0000 |
| C(9)        | 0.715365   | 0.089164    | 8.023028 | 0.0000 |
| C(10)       | 0.725488   | 0.185934    | 3.901858 | 0.0001 |
| C(11)       | 0.904064   | 0.011460    | 78.88990 | 0.0000 |

#### Log likelihood

-1174.040  
Schwarz criterion 10.07666

#### Avg. log likelihood

-2.456150  
Hannan-Quinn criter. 9.981129

#### Equation: RCOP = C(1)

| R-squared | Mean dependent var | 0.409576 |
| Adjusted R-squared | S.D. dependent var | 0.982453 |
| S.E. of regression  | Sum squared resid | 19647.12 |

#### Durbin-Watson stat

1.647867

#### Equation: RCPI = C(2)

| R-squared | Mean dependent var | 0.982453 |
| Adjusted R-squared | S.D. dependent var | 1.324827 |
| S.E. of regression  | Sum squared resid | 417.7356 |

#### Durbin-Watson stat

1.692136

### Covariance specification: Diagonal VECH

\[
GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1)
\]

M is an indefinite matrix

A1 is an indefinite matrix

B1 is an indefinite matrix

### Transformed Variance Coefficients

| Coefficient | Std. Error | z-Statistic | Prob. |
|-------------|------------|-------------|-------|
| M(1,1)      | 8.149223   | 4.645411    | 1.754253 | 0.0794 |
| M(1,2)      | 0.001330   | 0.106261    | 0.012518 | 0.9900 |
| M(2,2)      | -0.003053  | 0.001369    | -2.230484 | 0.0257 |
| A1(1,1)     | 0.192204   | 0.063083    | 3.046852 | 0.0023 |
| A1(1,2)     | 0.129038   | 0.083785    | 1.540106 | 0.1235 |
| A1(2,2)     | 0.102235   | 0.020219    | 5.056284 | 0.0000 |
| B1(1,1)     | 0.715365   | 0.089164    | 8.023028 | 0.0000 |
| B1(1,2)     | 0.725488   | 0.185934    | 3.901858 | 0.0001 |
| B1(2,2)     | 0.904064   | 0.011460    | 78.88990 | 0.0000 |

* Coefficient matrix is not PSD.

### 3.6 RCOP & RCPI (BEKK-GARCH)

Table 8 shows the Diagnostic test which is for autocorrelation using Ljung-Box Qstatistics. The result shows that there is no present of autocorrelation in the standard residuals obtained from the model using the return on the series. Therefore, this shows that the conditional mean return equation are correctly specified with the bivariate VECH-GARCH models.

Table 9 Test for residual Normality using orthogonalization cholesky shows that the model is fitted.
Table 7. Bivariate Model

|                | Coefficient | Std. Error | z-Statistic | Prob.  |
|----------------|-------------|------------|-------------|--------|
| C(1)           | 0.465620    | 0.398806   | 1.167535    | 0.2430 |
| C(2)           | 0.987969    | 0.030797   | 32.08031    | 0.0000 |

**Variance Equation Coefficients**

|                | Coefficient | Std. Error | z-Statistic | Prob.  |
|----------------|-------------|------------|-------------|--------|
| C(3)           | -0.002481   | 0.001703   | -1.456430   | 0.1453 |
| C(4)           | 0.568591    | 0.051033   | 11.14155    | 0.0000 |
| C(5)           | 0.295982    | 0.027821   | 10.63882    | 0.0000 |
| C(6)           | 0.871986    | 0.021463   | 40.62825    | 0.0000 |
| C(7)           | 0.957062    | 0.005361   | 178.5203    | 0.0000 |

Log likelihood: -1183.005
Schwarz criterion: 10.06002
Avg. log likelihood: 9.958198

Equation: RCOP = C(1)
R-squared: 0.000038
Mean dependent var: 0.409576
Adjusted R-squared: 0.000038
S.D. dependent var: 9.084101
S.E. of regression: 9.084274
Sum squared resid: 19640.72
Durbin-Watson stat: 1.648404

Covariance specification: Diagonal BEKK
GARCH = M + A1*RESID(-1)*RESID(-1)*M + B1*GARCH(-1)*B1
M is a scalar
A1 is a diagonal matrix
B1 is a diagonal matrix

Table 8. Diagnostic Check

| Lags | Q-Stat  | Prob. | Adj Q-Stat | Prob. | Df |
|------|---------|-------|------------|-------|----|
| 1    | 33.74407| 0.0000| 33.88585   | 1.0000| 4  |
| 2    | 37.06766| 0.0000| 37.23749   | 0.0000| 8  |
| 3    | 42.65781| 0.0000| 42.89870   | 0.0000| 12 |
| 4    | 44.19141| 0.0002| 44.45840   | 0.0002| 16 |
| 5    | 54.58444| 0.0000| 55.07351   | 0.0000| 20 |
| 6    | 62.07002| 0.0000| 62.75185   | 0.0000| 24 |
| 7    | 70.00408| 0.0000| 70.92530   | 0.0000| 28 |
| 8    | 76.00991| 0.0000| 77.13912   | 0.0000| 32 |
| 9    | 79.65480| 0.0000| 80.92664   | 0.0000| 36 |
| 10   | 80.76680| 0.0001| 82.08720   | 0.0001| 40 |
| 11   | 83.89948| 0.0003| 85.37101   | 0.0002| 44 |
| 12   | 86.70155| 0.0005| 88.32121   | 0.0004| 48 |

*The test is valid only for lags larger than the System lag order.
Table 9. Normality

| Component | Skewness | Chi-sq  | Df | Prob.   |
|-----------|----------|---------|----|---------|
| 1         | -0.622799 | 15.45048 | 1  | 0.0001  |
| 2         | -0.873715 | 30.40790 | 1  | 0.0000  |
| Joint     |          | 45.85838 | 2  | 0.0000  |

| Component | Kurtosis | Chi-sq  | Df | Prob.   |
|-----------|----------|---------|----|---------|
| 1         | 9.017608 | 360.6073 | 1  | 0.0000  |
| 2         | 7.082290 | 165.9565 | 1  | 0.0000  |
| Joint     |          | 526.5638 | 2  | 0.0000  |

| Component | Jarque-Bera | Df | Prob. |
|-----------|-------------|----|-------|
| 1         | 376.0578    | 2  | 0.0000|
| 2         | 196.3644    | 2  | 0.0000|
| Joint     | 572.4222    | 4  | 0.0000|

Table 10 contains Estimation Results for Model Selection for Bivariate MGARCH and it was found that based on Akaike information criteria diagonal bivariate VECH is better fitted than BEKK because it has the least Akaike information.

4 Summary of Findings

Table 10. Estimating results for model selection

|           | BIVARIATE | AIC   | SIC   | LEAST AIC                  |
|-----------|-----------|-------|-------|----------------------------|
| VECH      | RCOP & RCPI | 9.917 | 10.077 | VECH MGARCH (9.917)        |
| BEKK      | RCOP & RCPI | 9.958 | 10.060 |                            |

5 Conclusion

This study mainly focused on the application of Multivariate GARCH model to modeling Nigeria economic data. In order to achieve the aim of the study, two multivariate models were used in the study and the results obtained shows that diagonal multivariate VECH model is better fitted. This confirmed that it is positive definite condition (variance and co-variance). This reveals that there exist a strong evidence of a time-varying conditional covariance and interdependence Nigeria economic data. As for cross-volatility effects, past innovations in crude oil price has greatest influence on future volatility of other returns on economic data. Also, It was confirmed that time varying correlation displays betwen crude oil price and consumer price index, has high degree of persistence during these period under investigation. We confirmed that time varying variance-covariance displays among these economic data and their corresponding level of persistence. Based on Model selection criteria using the Akaike information criteria (AIC) diagonal VECH GARCH model are better fitted than the BEKK MGARCH model.

Competing Interests

Authors have declared that no competing interests exist.

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