A quark loop model for heavy mesons¹

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Abstract. I consider a model based on a quark–meson interaction Lagrangian. The transition amplitudes are evaluated by computing diagrams in which heavy and light mesons are attached to quark loops. The light chiral symmetry relations and the heavy quark spin-flavour symmetry dictated by the heavy quark effective theory are implemented. The model allows to compute the decay form factors and therefore can give predictions for the decay rates, the invariant mass spectra and the asymmetries.

INTRODUCTION

The increasing number of available data on heavy meson processes demands theoretical predictions for these processes to be compared with experiment. I consider a simple model, based on an effective constituent quark-meson Lagrangian containing both light and heavy degrees of freedom, constrained by the known symmetries of QCD in the limit \( m_Q \to \infty \) and the light chiral symmetry relations. I write a Lagrangian at the meson-quark level [1]. This allows to deduce from a small number of parameters the heavy meson couplings and form factors, with a considerable reduction in the number of free parameters with respect to the Lagrangian written in terms of meson fields only [2].

The part of the quark-meson effective Lagrangian involving heavy and light quarks and heavy mesons is:

\[
\mathcal{L}_{ht} = \bar{Q}_v i v \cdot \partial Q_v - \left( \bar{\chi} (\hat{H} + \hat{S} + i \frac{D_\mu}{\Lambda_\chi} T_\mu) Q_v + h.c. \right) \\
+ \frac{1}{2G_3} \text{Tr}[(\hat{H} + \hat{S})(H - S)] + \frac{1}{2G_4} \text{Tr}[\bar{T}_\mu T_\mu] 
\]

(1)

where \( Q_v \) is the effective heavy quark field, \( \chi \) is the light quark field, \( G_3, G_4 \) are coupling constants and \( \Lambda_\chi (= 1 \text{ GeV}) \) is a dimensional parameter. The Lagrangian (1) is heavy spin and flavour symmetric. Note that the fields \( H \) and \( S \) have the same coupling constant. By putting these two coupling constants equal, one assumes that the effective quark-meson Lagrangian can be obtained from a four quark interaction of the NJL type [3].

The cut-off prescription is part of the dynamical information regarding QCD which is introduced in the model. The idea is to mimic the QCD behaviour in a simple and calculable way. In the infrared the model is not confining and its range of validity can

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not be extended below energies of the order of \( \Lambda_{QCD} \). In practice one introduces an infrared cut-off \( \mu \), to take this into account.

Models related to the one discussed here, with different regularization prescriptions and different approaches are \([4, 5]\). The cut-off prescription used here is implemented via a proper time regularization. After continuation to the Euclidean it reads, for the light quark propagator:

\[
\int d^4k_E \frac{1}{k_E^2 + m^2} \rightarrow \int d^4k_E \int_{1/\mu^2}^{1/\Lambda^2} ds e^{-s(k_E^2 + m^2)}
\]

(2)

where \( \mu \) and \( \Lambda \) are infrared and ultraviolet cut-offs.

The cut-off prescription is similar to the one used in \([3]\), with \( \Lambda = 1.25 \) GeV; the numerical results are not strongly dependent on the value of \( \Lambda \). The constituent mass \( m \) in the NJL models represents the order parameter discriminating between the phases of broken and unbroken chiral symmetry and can be fixed by solving a gap equation, which gives \( m \) as a function of the scale mass \( \mu \) for given values of the other parameters. Here I take \( m = 300 \) MeV and \( \mu = 300 \) MeV.

**HEAVY-TO-HEAVY FORM FACTORS**

As an example of the quantities that can be analytically calculated in the model, one can examine the Isgur-Wise function \( \xi \):

\[
\langle D(v')|\bar{c}\gamma_\mu(1-\gamma_5)b|B(v)\rangle = \sqrt{M_B M_D} C_{cb} \xi(\omega)(v_\mu + v'_\mu)
\]

(3)

where \( \omega = v \cdot v' \) and \( C_{cb} \) contains logarithmic corrections depending on \( \alpha_s \); within the approximations used here, it can be put equal to 1. At leading order \( \xi(1) = 1 \). The same universal function \( \xi \) also parameterizes \( B \to D^* \) semileptonic decay. One finds:

\[
\xi(\omega) = Z_H \left[ \frac{2}{1 + \omega} I_3(\Delta_H) + \left( m + \frac{2\Delta_H}{1 + \omega} \right) I_5(\Delta_H, \Delta_H, \omega) \right].
\]

(4)

where:

\[
I_3(\Delta) = -\frac{i N_c}{16\pi^3} \int_{\text{reg}} \frac{d^4k}{(k^2 - m^2)(v \cdot k + \Delta + i\varepsilon)}
\]

(5)

\[
= \frac{N_c}{16 \pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} \frac{ds}{s^{3/2}} e^{-s(\sigma^2 - \Delta^2)} \left( 1 + \text{erf}(\Delta \sqrt{s}) \right)
\]

\[
I_5(\Delta_1, \Delta_2, \omega) = \frac{i N_c}{16\pi^4} \int_{\text{reg}} \frac{d^4k}{(k^2 - m^2)(v \cdot k + \Delta_1 + i\varepsilon)(v' \cdot k + \Delta_2 + i\varepsilon)}
\]

(5)

\[
= \int_0^1 dx \frac{1}{1 + 2x^2(1 - \omega) + 2x(\omega - 1)} \times \frac{6}{16\pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} ds \sigma e^{-s(\sigma^2 - \Delta^2)} s^{-1/2} \left( 1 + \text{erf}(\sigma \sqrt{s}) \right) +
\]

\[
\left[ \frac{1}{16\pi^3} \int_{1/\Lambda^2}^{1/\mu^2} ds \sigma e^{-s(\sigma^2 - \Delta^2)} s^{-1/2} \left( 1 + \text{erf}(\sigma \sqrt{s}) \right) + \right]
\]

\[
\frac{6}{16\pi^{3/2}} \int_{1/\Lambda^2}^{1/\mu^2} ds \sigma^2 e^{-s(\sigma^2 - \Delta^2)} s^{-1/2} \left( 1 + \text{erf}(\sigma \sqrt{s}) \right) + \right]
\]
TABLE 1. Form factors and slopes. $\Delta_H$ in GeV.

| $\Delta_H$ | $\xi(1)$ | $\rho_{IW}^2$ | $\tau_{1/2}(1)$ | $\rho_{1/2}^2$ | $\tau_{3/2}(1)$ | $\rho_{3/2}^2$ |
|-----------|----------|-------------|----------------|--------------|----------------|--------------|
| 0.3       | 1        | 0.72        | 0.08           | 0.8          | 0.48           | 1.4          |
| 0.4       | 1        | 0.87        | 0.09           | 1.1          | 0.56           | 2.3          |
| 0.5       | 1        | 1.14        | 0.09           | 2.7          | 0.67           | 3.0          |

\[
\frac{6}{16\pi^2} \int_{1/\Lambda^2}^{1/\mu^2} ds \ e^{-s(m^2-2\sigma^2)} s^{-1}
\]  

(6)

In these equations

\[
\Gamma(\alpha,x_0,x_1) = \int_{x_0}^{x_1} dt \ e^{-t} t^{\alpha-1}
\]  

(7)

is the generalized incomplete gamma function, erf is the error function and

\[
\sigma(x,\Delta_1,\Delta_2,\omega) = \frac{\Delta_1 (1-x) + \Delta_2 x}{\sqrt{1 + 2 (\omega - 1) x + 2 (1 - \omega) x^2}}.
\]  

(8)

One can compute in a similar way the form factors describing the semi-leptonic decays of a meson belonging to the fundamental negative parity multiplet $H$ into the positive parity mesons in the $S$ and $T$ multiplets [1]. Examples of these decays are $B \to D_{VV}^\ast \ell \nu$ where $D_{VV}^\ast$ can be either a $S$ state or a $T$ state. These decays are described by two form factors $\tau_{1/2}, \tau_{3/2}$ [6] which can be computed in the model by a loop calculation similar to the one used to obtain $\xi(\omega)$ [1, 8].

The numerical results for the form factors are in Table 1. The predictions for a few branching ratios calculated in the model are given in Table 2.

TABLE 2. Branching ratios (%) for semileptonic $B$ decays. Theoretical predictions for three values of $\Delta_H$ and experimental results. Units of $\Delta_H$ in GeV.

| Decay mode | $\Delta_H = 0.3$ | $\Delta_H = 0.4$ | $\Delta_H = 0.5$ | Exp. [7] |
|------------|-----------------|-----------------|-----------------|---------|
| $B^0 \to D_{\ell\nu}$ | 3.0            | 2.7             | 2.2             | 2.10 ± 0.19 |
| $B^0 \to D^*_{\ell\nu}$ | 7.6            | 6.9             | 5.9             | 4.60 ± 0.27 |
| $B^0 \to D_{0\ell\nu}$ | 0.03           | 0.005           | 0.003           | –       |
| $B^0 \to D^0_{\ell\nu}$ | 0.03           | 0.008           | 0.0045          | –       |
| $B^0 \to D^\ast_{\ell\nu}$ | 0.27           | 0.18            | 0.13            | < 0.74  |
| $B^0 \to D_2^\ast_{\ell\nu}$ | 0.43           | 0.34            | 0.30            | < 0.65  |

HEAVY-TO-LIGHT FORM FACTORS

The model allows to compute the $B$ semileptonic decay form factor to $\pi, \rho, \text{etc}$. The form factors of $B$ to a vector meson $V$ consist of two kind of contributions. In the first one the current is directly attached to the loop of quarks. In the second, there is an intermediate state between the current and the $B V$ system [9]. For $B \to \pi$ form factors
an extra contribution is also taken into account [10]. Results are in good agreement with available data. For $B \to \pi \ell \nu$ (using $V_{ub} = 0.0032, \tau_B = 1.5610^{-12}$ s):

$$B(\bar{B}^0 \to \pi^+ \ell \nu) = (1.1 \pm 0.5) \times 10^{-4},$$

for $B \to \rho \ell \nu$:

$$B(\bar{B}^0 \to \rho^+ \ell \nu) = (2.5 \pm 0.8) \times 10^{-4},$$

for $B \to a_1 \ell \nu$:

$$B(\bar{B}^0 \to a_1^+ \ell \nu) = (8.4 \pm 1.6) \times 10^{-4}.$$  

In the limit of heavy mass for the initial meson and of large energy for the final one (LEET), the expressions of the form factors simplify and for $B \to V \ell \nu$, they reduce only to two independent functions [11]. The four-momentum of the heavy meson is written as $p = M_H v$ in terms of the mass and the velocity of the heavy meson. The four-momentum of the light vector meson is written as $p' = En$ where $E = v \cdot p'$ is the energy of the light meson and $n$ is a four-vector defined by $v \cdot n = 1, n^2 = 0$. The relation between $q^2$ and $E$ is:

$$q^2 = M_H^2 - 2M_HE + m_V^2$$

The large energy limit is defined as:

$$\Lambda_{QCD}.m_V << M_H, E$$

keeping $v$ and $n$ fixed and $m_V$ is the mass of the light vector meson. The relations between the form factors appearing in the LEET limit constitute a powerful theoretical cross-check of the formulas derived in the model. The result is as follows:

$$A_0(q^2) = \left(1 - \frac{m_V^2}{M_H E}\right) \zeta_{||}(M_H, E) + \frac{m_V}{M_H} \zeta_{\perp}(M_H, E)$$

$$A_1(q^2) = \frac{2E}{M_H + m_V} \zeta_{\perp}(M_H, E)$$

$$A_2(q^2) = \left(1 + \frac{m_V}{M_H}\right) \left[\zeta_{\perp}(M_H, E) - \frac{m_V}{E} \zeta_{||}(M_H, E)\right]$$

$$V(q^2) = \left(1 + \frac{m_V}{M_H}\right) \zeta_{\perp}(M_H, E).$$

The explicit expressions for $\zeta_{||}$ and $\zeta_{\perp}$ are [12]:

$$\zeta_{||}(M_H, E) = \frac{\sqrt{M_H Z_H} \ m_V^2}{2Ef_{V}} \left[I_3\left(\frac{m_V}{2}\right) - I_3\left(-\frac{m_V}{2}\right)\right] + 4\Delta_H m_V Z \sim \frac{\sqrt{M_H}}{E}$$

$$\zeta_{\perp}(M_H, E) = \frac{\sqrt{M_H Z_H} \ m_V^2}{2Ef_{V}} \left[I_3(\Delta_H) + m_V^2 Z\right] \sim \frac{\sqrt{M_H}}{E},$$
where terms proportional to the constituent light quark mass $m$ have been neglected. It is interesting to note that in LEET one can also relate the tensor form factor $T_1$, $T_2$ and $T_3$ to the semileptonic ones and to the $\zeta_\perp$ and $\zeta_\parallel$ form factors of the LEET limit [11]:

\begin{align}
T_1(q^2) &= \zeta_\perp(M_H,E), \\
T_2(q^2) &= \left(1 - \frac{q^2}{M_H^2 - m_V^2}\right)\zeta_\perp(M_H,E), \\
T_3(q^2) &= \zeta_\perp(M_H,E) - \frac{m_V}{E} \left(1 - \frac{m_V^2}{M_H^2}\right)\zeta_\parallel(M_H,E).
\end{align}

$\zeta_\perp$ and $\zeta_\parallel$ obtained in this way agree with those of (18,19) [13]. Concerning the scaling properties of $\zeta_\parallel$ and $\zeta_\perp$, the asymptotic $E$-dependence is not predicted by the large energy limit. As $E \sim M$ at $q^2 = 0$ the Feynman mechanism contribution to the form factors would indicate a $1/E^2$ behaviour rather than the $1/E$ found in the model. Note however that the $E$-dependence is not rigorously established in QCD.

**CONCLUSIONS**

Calculating directly from the QCD Lagrangian remains an extremely difficult task, in spite of the impressive success of lattice QCD calculations. A most promising approach is the one based on heavy meson effective Lagrangians, which incorporate the heavy quark symmetries and in addition the approximate chiral symmetry for light quarks. Although with increasing data such an approach is the best one beyond direct QCD calculations, a large number of parameters have to be fixed before obtaining predictions. An intermediate approach consists in using the effective Lagrangian at the level of mesons and constituent quarks plus few simple assumptions on the QCD dynamics. It allows to compute meson transition amplitudes by evaluating loops of heavy and light quarks. The model describes a number of essential features of heavy meson physics in a simple and compact way, in particular Isgur-Wise scaling in the heavy-to-heavy semileptonic decays and the large energy limit for the heavy-to-light ones.

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