Study on cargo-swing reduction of general gantry crane using hybrid optimal input shaper

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Abstract. The gantry crane is widely used, but easy to accidents, which mostly due to the invalid control of cargo-swing. Based on the appropriate dynamic model and the characteristics of gantry crane, the Hybrid Optimal (HYO) input shaper is designed by interpolation method. HYO input shaper combines the advantages of Zero Vibration Derivation (ZVD) input shaper on the slightly cargo-swing and the Extra Insensitivity (EI) input shaper on the greatly cargo-swing. The comprehensive property of HYO input shaper is better than ZVD and EI input shaper. HYO input shaper effectively suppresses the cargo-swing of gantry crane within 0.018 rad, largely avoids the accidents caused by unstable control of the gantry crane.

1. Introduction
The 3-dimensional transportation of the cargo makes the gantry crane to be a typical variable-parameter flexible mechanical system with high control difficulty. Establishing a reasonable dynamic model and accurate analyzing its dynamic characteristics are the basis for effectively controlling the operation of gantry crane, according to the dynamic principle, the nonlinear dynamic model of the gantry crane is established and reasonable linearization is carried out.

The input shaping technology originated from the Posicast technique proposed by Smith et al. in 1957, which aims to suppress the oscillatory component, such as flexible spacecraft [1]. In the 1990s, the input shaping technology was used to control flexible spacecraft by Singhose, Derezinski and Singer, they proposed several extra insensitive input shapers to further improve the control performance of the input shaper [2,3]. In recent years, LI Minzhi and LIANG Chunyan designed the input shaper to realize the cargo-swing control of the crane based on the optimization idea [4]. DONG Mingxiao and MEI Xuesong study on the time-delay filtering theory and its engineering application [5]. At present, researchs on this technology for cargo-swing reduction of gantry crane has not been found.

2. Control model and dynamic analysis of gantry crane

2.1. Control model
The gantry crane pulls the cargo through the wire rope to complete the lifting, transportation and placing. The general type A5-75/20t double-girder gantry crane is studied, as shown in Fig. 1. The span of the main cart is 26m, the lifting heights of the main and auxiliary hooks are 11m and 13m. The rated lifting speeds of the main and auxiliary hooks are 4.6m/min and 9.23m/min, the rated running speeds of the trolley and the main cart are 38.4m/min and 40.62m/min. The working level is A5. The main cart moves 1-dimensionally, the trolley moves 2-dimensionally, drags the cargo through the rope, during the start and stop, the cargo-swings due to the inertia.
The Cartesian coordinate system \( \{x_0, y_0, z_0\} \) and spherical coordinate system \( \{e_\ell, e_\theta, e_\phi\} \) are established[5], as shown in Fig. 2. The origin \( O_0 \) of the Cartesian coordinate system is taken at one end of the main cart, the coordinates of \( O \) in the Cartesian coordinate system are \((x, y, z)\), the spherical coordinates of the cargo are defined as \((l, \theta, \phi)\). The degrees of freedom of the gantry crane is 5, therefore, the 5 independent generalized coordinates are defined as \(x \) and \(y \) indicating the displacements of the trolley on the \(x\)-axis, the \(y\)-axis direction, the lifting length of the wire rope \(l\), \(\phi\) and \(\theta\), which are used to determine the particle location of the system[5].

\[
\begin{align*}
\text{Figure 1.} & \text{ Type A 5-75/20t double-girder gantry crane.} \\
\text{Figure 2.} & \text{ Cargo-swing diagram of Gantry crane.}
\end{align*}
\]

The generalized forces corresponding to \(x\), \(y\), and \(l\) are defined as \(Q_x\), \(Q_y\) and \(Q_l\), indicating the driving force of the trolley, main cart and the lifting force of the wire rope.

According to the Lagrange motion equation

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k} = Q_k \quad (k = 1,2,3,4,5)
\]

(1)

\[
L = T - V
\]

(2)

\[
T = \frac{1}{2} m[(\dot{x} + l \sin \theta \cos \phi) - l \dot{\phi} \sin \theta \sin \phi + l \dot{\theta} \cos \phi \cos \theta)^2 + (\dot{y} + l \dot{\phi} \cos \phi + l \dot{\phi} \cos \phi)^2 +
\]

\[
(l \cos \phi \cos \theta - l \dot{\phi} \cos \theta \sin \phi - l \dot{\phi} \sin \theta \cos \phi)^2 + \frac{1}{2} M_1 \dot{x}^2 + \frac{1}{2} M_2 \dot{y}^2
\]

(3)

\[
V = -mg \ell \cos \theta \cos \phi
\]

(4)

The where \( k \) is the number of degrees of freedom, \( q_k \) is the generalized displacement of the system, \(Q_k\) is the generalized force of the system, \(L\) is the Lagrange function. \(T\) is the kinetic energy of particle system, including the kinetic energy of the main cart, trolley and cargo. \(V\) is the potential energy of the particle system, including the potential energy of the cargo only.

The nonlinear dynamic model of the gantry crane is established. In order to facilitate the effective control of the gantry crane in engineering, according to the linearization theory, near the cargo balance position \( \theta = 0^\circ \), \( \phi = 0^\circ \) and swing in a small range, the nonlinear model is reasonably linearized, then the linearized model is

\[
M_1 \ddot{x} + b_x \dot{x} - mg \theta + ml \ddot{\theta} = Q_x
\]

(5)
\[ M_2 \ddot{y} + b_y \dot{y} - mg \phi + m \ddot{\phi} = Q_y \]

\[ m(\ddot{l} - g + \dot{x} \theta + \dot{y} \phi) + b_l \dot{l} = Q_l \]

\[ l \ddot{\theta} + 2l \dot{\theta} + g \theta = -\ddot{x} \]

\[ l \ddot{\phi} + 2l \dot{\phi} + g \phi = -\ddot{y} \]

where \( M_1, M_2 \) and \( m \) are the masses of the trolley, main cart and cargo; \( b_x, b_y \) and \( b_l \) are the equivalent damping coefficients of the trolley, main cart and hoisting motion; \( g \) is the gravitational acceleration; \( \dot{l} \) and \( \ddot{l} \) are the cargo lifting speed and hoisting acceleration; \( \dot{\theta}, \dot{\phi} \) and \( \ddot{\theta}, \ddot{\phi} \) are the angular velocity and acceleration; \( \dot{x}, \ddot{x} \) and \( \dot{y}, \ddot{y} \) are the corresponding velocity and acceleration.

2.2. Dynamic analysis of gantry crane

The model is composed of the motion equations of the trolley and main cart, Eqs. (5), (6), the equations of hoisting and cargo motion, Eqs. (7), (8) and (9). The cargo motion are about 2-order oscillation of the cargo-swing angle \( \theta \) and \( \phi \), which describes the kinematic relationship between the trolley, main cart and cargo motion. The accelerations of the trolley and main cart are the inputs, and the cargo-swing angle is the output. The driving forces \( Q_x, Q_y \) and \( Q_l \) drive the trolley, main cart and lifting mechanism, the wire rope is a flexible body, therefore, the lifting motion makes the system a weak damping flexible system containing a rigid mode. The system is time-varying 2-order nonlinear system, state variables are coupled to each other. The frequency of the cargo-swing \( \omega_n = \sqrt{g/l} \) is related to the length of the rope, the damping ratio \( \xi = l/\sqrt{gl} \) is related to the length of the rope and lifting speed, the amplitude of the swing is related to the acceleration of the trolley.

During the simulation, it is required to lift the cargo from the ground to 1.25m, then maintain this height, and finally release the cargo to the ground. The gantry is required to accelerate for 2s, run at a constant speed for 17s, then decelerate for 2s, then stop.

The motion simulation of the gantry crane model is based on MATLAB. The oscillating angles \( \theta \) and \( \phi \) of the gantry crane nonlinear model and the linearized model have very similar variations in the two directions, as shown in Fig. 3.

\[ \text{Figure 3. Cargo-swing angle of the simulation model of gantry crane.} \]

\[ \text{Figure 4. The errors of cargo-swing angle between the nonlinear and linear model.} \]

Respectively solving the cargo-swing angle errors between the two models, as shown in Fig. 4, the maximum linear error is on the order of \( 10^{-3} \), it is negligible in engineering, so the linear model can accurately describe the dynamic performance of the gantry crane to a certain extent, has the advantage of being physically convenient to implement.

3. Hybrid optimal input shaping
3.1. Input shaping

Input shaping is a control strategy which convolves the reference command with the shaper pulse sequence, uses the resulting shaping command to drive the system. In the case of reasonable design of the shaper pulse amplitude and time lag, the system vibration can be effectively suppressed. After the end of the action of the input shaper, the ratio of the amplitude of the system unit impulse response between with and without the shaper control is called residual vibration [5].

ZVD input shaper is designed, the amplitudes at the model frequency and damping ratio are 0, their differentials to \( \omega_n \) are 0. When the model parameters change in a small range, the ZVD input shaper can effectively suppress the residual vibration of the system [3]. when the control parameters vary widely, the control effect will be greatly reduced, as shown in Fig. 5.

If residual vibration of the shaper is required to be less than a value \( V_{\text{exp}} \) at the undamped natural frequency \( \omega_n \) and the damping ratio \( \xi \), to be 0 at some two frequency points on two sides of \( \omega_n \), and the differential of \( V \) to \( \omega \) at \( \omega_n \) is 0, such shaper is called Extra Insensitivity(EI) input shaper [3], which can ensure that the residual vibration of the controlled system is less than the \( V_{\text{exp}} \) when the system parameters vary widely around \( \omega_n \), as shown in Fig. 5.

\[
V = \frac{1}{(\omega_1 - \omega)(\xi_1 - \xi)} \begin{cases} 0, & \omega \in [\omega_1, \omega_0] \xi \in [\xi_1, \xi_0] \\ \frac{\omega \xi}{(\omega_1 - \omega)(\xi_1 - \xi)} , & \omega \notin [\omega_1, \omega_0] \xi \notin [\xi_1, \xi_0] \end{cases}
\]

Figure 5. 2-order system residual vibration under 3-pulse ZVD and EI control.

The main parameters affecting residual vibration are frequency \( \omega \) and damping ratio \( \xi \), which are determined by the lifting height and speed in the gantry crane system. Such parameters change large and small during the different working processes of the gantry crane.

3.2. Hybrid optimal input shaping

Based on ZVD and EI input shaper, the Hybrid Optimal (HYO) input shaper is designed by interpolation method, which combines the advantages of these two input shapers. An interpolation parameter \( \lambda \) (0<\( \lambda \)<1) is introduced, the time lag and pulse amplitude are

\[
t_{\text{HYO}} = \lambda t_{\text{EI}} + (1 - \lambda) t_{\text{ZVD}}
\]

\[
A_{\text{HYO}} = \lambda A_{\text{EI}} + (1 - \lambda) A_{\text{ZVD}}
\]

Where \( t_{\text{ZVD}}, t_{\text{EI}}, A_{\text{ZVD}}, A_{\text{EI}} \) are action times and amplitudes of the i-th pulse of ZVD and EI shaper.

Within the range of system parameter variation, in order to solve the optimal interpolation parameter value \( \lambda \), the change of the system parameters follows the average distribution law, the probability density function is

\[
f(\omega, \xi) = \begin{cases} \frac{1}{(\omega_1 - \omega)(\xi_1 - \xi)}, & \omega \in [\omega_0, \omega_1] \xi \in [\xi_0, \xi_1] \\ 0, & \omega \notin [\omega_0, \omega_1] \xi \notin [\xi_0, \xi_1] \end{cases}
\]

Where \( \omega_0 \) and \( \omega_1 \) are the upper and lower boundaries of frequency, \( \xi_0 \) and \( \xi_1 \) are the upper and lower boundaries of damping ratio.

Combined with parameter changes and residual vibration, the objective function is
\[ J = \int_0^\infty V(\omega, \xi) f(\alpha, \xi) d\omega d\xi \quad (13) \]

Substituting Eqs. (10), (11), (12) into Eq. (13), based on optimization function in Matlab, the special value of \( \lambda \) is obtained which minimize the value of \( J \), at the same time, the residual vibration of the controlled system is also minimized. The parameters of HYO can be obtained by substituting such \( \lambda \) into Eqs. (10), (11). It can be seen that HYO only needs to solve the minimum value of a function with a variable \( \lambda \), which has a more concise optimization process than the general optimization.

3.3. Simulation results of hybrid optimal input shaping

In the normal working condition, the range of rope length is from 1.52m to 12.52m, the hoisting speed is from 0 to 4.6m/min, planning the operation mode that the trolley accelerates for 2s, moves with a uniform speed of 14s and decelerates for 2s. The parameters are substituted into the cargo-swing model for simulation, designing a 3-pulse ZVD input shaper

\[ F(s) = 0.2570 + 0.4960e^{-3.630s} + 0.2400e^{-6.7310s} \quad (14) \]

Taking \( V_{\text{exp}} \) 10%, designing a 3-pulse EI input shaper

\[ F(s) = 0.2630 + 0.4810e^{-3.593s} + 0.2660e^{-6.7260s} \quad (15) \]

Designing a 3-pulse HYO input shaper according to Eqs. (10), (11), (12), (13), (14), (15)

\[ F(s) = 0.2499 + 0.4830e^{-3.656s} + 0.2566e^{-6.7310s} \quad (16) \]

When the gantry crane parameters change slightly, the HYO control performance is close to the ZVD input shaper, the residual swing angle of the cargo is controlled within 0.02 rad. The EI input shaper controls the residual swing angle within 0.08 rad, as shown in Fig. 6.

Figure 6. Cargo-swing angle of gantry crane controlled by the input shapers when the parameters change slightly.

When the parameters change greatly, the HYO input shaper control performance is closer to the EI input shaper, the residual swing angle of the cargo is controlled within 0.018 rad, the ZVD input shaper controls the residual swing angle within 0.051 rad, as shown in Fig. 7.

Figure 7. Cargo-swing angle of gantry crane controlled by the input shapers when the parameters change greatly.

4. Conclusions

According to the analysis of the linearized model and the simulation results, the moving directions of the trolley and main cart are perpendicular to each other, the driving motor only controls the respective motions and does not affect each other. Then, the motions in such directions can be decoupled, the effect on the cargo motion in the corresponding direction is similar. To study the movement of the gantry crane, it is only necessary to study the horizontal movement of the trolley or main cart in the x-
axis or y-axis direction in combination with the lifting motion, which simplifies the space pendulum motion of the cargo into a plane pendulum motion.

The linearized model of the gantry crane can be used as the control object, the motion characteristics of the main cart and trolley are studied separately, then it is easy to realize the effective and stable control of the gantry crane.

In view of the frequent changes of the gantry crane parameters, taking the advantages of ZVD and EI input shaper, the interpolation method is used to design HYO input shaper to suppress the cargo-swing of the type A5-75/20t double-girder gantry crane. The control results show that the control effect of HYO input shaper is better than ZVD and EI input shaper, which achieves the original design purpose that effectively suppressing the cargo swing of gantry crane, largely avoiding the accidents caused by unstable control of the gantry crane.

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