Bouncing Universe with a Nonminimally Coupled Scalar Field on a Moving Domain Wall

Kourosh Nozari\textsuperscript{a,b} and S. Davood Sadatian\textsuperscript{a,c,d}

\textsuperscript{a}Department of Physics, Faculty of Basic Sciences, University of Mazandaran, P. O. Box 47416-95447, Babolsar, IRAN,
\textsuperscript{b}Research Institute for Astronomy and Astrophysics of Maragha, P. O. Box 55134-441, Maragha, IRAN,
\textsuperscript{c}Nishabour Center of Higher Education, Nishabour, IRAN
and
\textsuperscript{d}Islamic Azad University, Nishabour Branch, Nishabour, IRAN
knozari@umz.ac.ir
d.sadatian@umz.ac.ir

Abstract

We study dynamics of a dark energy component nonminimally coupled to gravity on a moving domain wall. We use this setup to explain late-time accelerated expansion and crossing of the phantom divide line by the equation of state parameter of this non-minimally coupled dark energy component. By analyzing parameter space of the model, we show that this model accounts for accelerated expansion and crossing of the phantom divide line with a suitable fine-tuning of the nonminimal coupling. Then we study the issue of bouncing solutions in this framework.

PACS: 04.50.+h, 98.80.-k

Key Words: Dark Energy, Scalar-Tensor Theories, Braneworld Cosmology
1 Introduction

Recent evidences from supernova searches data [1,2], cosmic microwave background (CMB) results [3-5] and also Wilkinson Microwave Anisotropy Probe (WMAP) data [6,7], indicate an positively accelerating phase of the cosmological expansion today and this feature shows that the simple picture of universe consisting of pressureless fluid is not enough. In this regard, the universe may contain some sort of additional negative-pressure component dubbed dark energy. Analysis of the WMAP data [8-10] shows that there is no indication for any significant deviations from Gaussianity and adiabaticity of the CMB power spectrum and therefore suggests that the universe is spatially flat to within the limits of observational accuracy. Further, the combined analysis of the WMAP data with the supernova Legacy survey (SNLS) [8], constrains the equation of state \( w_{de} \), corresponding to almost 74% contribution of dark energy in the currently accelerating universe, to be very close to that of the cosmological constant value. Moreover, observations appear to favor a dark energy equation of state, \( w_{de} < -1 \) [11]. Therefore, a viable cosmological model should admit a dynamical equation of state that might have crossed the value \( w_{de} = -1 \), in the recent epoch of cosmological evolution. In fact, to explain positively accelerated expansion of the universe, there are two alternative approaches: incorporating an additional cosmological component or modifying gravity at cosmological scale. Multi-component dark energy with at least one non-canonical phantom field is a possible candidate of first alternative. This viewpoint has been studied extensively in literature ( see [12] and references therein ). Another alternative to explain current accelerated expansion of the universe is extension of general relativity to more general theories on cosmological scales. In this view point, modified Einstein-Hilbert action resulting \( f(R)\)-gravity ( see [13] and references therein) and braneworld gravity [14-16] are studied extensively. For instance, DGP ( Dvali-Gabadadze-Porrati) braneworld scenario as an infra-red modification of general relativity explains accelerated expansion of the universe in its self-accelerating branch via leakage of gravity to extra dimension. In this model, equation of state parameter of dark energy never crosses \( \omega(z) = -1 \) line, and universe eventually turns out to be de Sitter phase. Nevertheless, in this setup if we use a single scalar field (ordinary or phantom) on the brane, we can show that equation of state parameter of dark energy component can cross phantom divide line [17]. Also quintessential behavior can be achieved in a geometrical way in higher order theories of gravity [18]. One important consequence in quintessence model is the fact that a single minimally coupled scalar field has not the capability to explain crossing of the phantom divide line, \( \omega_{\phi} = -1 \) [19, 20],(see also [21]). However, a single but non-minimally coupled scalar field is adequate to cross the phantom divide line by its equation of state parameter [12]. On the other hand, in the context of scalar-vector-tensor theories, realizing accelerated expansion and crossing of the phantom divide line with one minimally coupled scalar field in the presence of a Lorentz invariance
violating vector field has been reported [22].

In this letter, we consider a nonminimally coupled scalar field as a dark energy component on a moving domain wall. In this extension, brane is considered as a moving domain wall in a background 5-dimensional anti de Sitter-Schwarzschild (AdSS) black hole bulk. In other words, we consider a static bulk configuration with two 5-dimensional anti de Sitter-Schwarzschild black hole spaces joined by a moving domain wall. Then we study dynamics of equation of state parameter of a non-minimally coupled scalar field on this moving domain wall. We show that adopting a phenomenologically appropriate ansatz with suitable fine-tuning of the parameters of the model, provide enough room to explain accelerated expansion and crossing of the phantom divide line by the dark energy equation of state parameter. We also investigate the existence of bouncing solutions in this setup. Based on recent observational data, parameters of this model are constrained in the favor of late-time accelerated expansion. The importance of this study lies in the fact that currently, models of phantom divide line crossing are so important that they can realize that which model is better than the others to describe the nature of dark energy. In this respect, possible crossing of the phantom divide line (PDL), \( \omega = -1 \), by equation of state parameter of nonminimally coupled scalar field and existence of bouncing solutions in this braneworld setup are discussed.

2 The Setup

We consider a moving domain wall picture of braneworld [23-25], to discuss the issue of quintessence and late-time acceleration along with the phantom divide line crossing of the equation of state parameter of a non-minimally coupled scalar field on the brane. Following [23], we consider a static bulk configuration with two 5-dimensional anti de Sitter-Schwarzschild (AdSS) black hole spaces joined by a moving domain wall. To embed this moving domain wall into 5-dimensional bulk, it is then necessary to specify normal and tangent vectors to this domain wall with careful determination of normal direction to the brane. We assume that domain wall is located at coordinate \( r = a(\tau) \) where \( a(\tau) \) is determined by Israel junction conditions [26]. In this model, observers on the moving domain wall interpret their motion through the static 5-dimensional bulk background as cosmological expansion or contraction. Now, consider the following line element [23]

\[
dS_{5\pm}^2 = -\left( k - \frac{\eta_\pm}{r^2} + \frac{r^2}{\ell^2} \right) dt^2 + \frac{1}{k - \frac{\eta_\pm}{r^2} + \frac{r^2}{\ell^2}} dr^2 + r^2 \gamma_{ij} dx^i dx^j, \tag{1}
\]

where \( \pm \) stands for left(\(-\)) and right(\(+\)) side of the moving domain wall, \( \ell \) is curvature radius of AdS\(_5\) manifold and \( \gamma_{ij} \) is the horizon metric of a constant curvature manifold with \( k = -1, 0, 1 \) for open, flat and closed horizon geometry respectively and \( \eta_\pm \neq 0 \).
generates the electric part of the Weyl tensor on each side. This line element shows a topological anti de Sitter black hole geometry in each side. Using Israel junction conditions [26] and Gauss-Codazzi equations, we find the following generalization of the Friedmann and acceleration equations [23]

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{\rho}{3} + \frac{\eta}{a^4} + \frac{\ell^2}{36}\rho^2,$$

(2)

$$\frac{\ddot{a}}{a} = -\frac{\rho}{6}(1 + 3w) - \frac{\eta}{a^4} - \frac{\ell^2}{36}\rho^2 (2+3w),$$

(3)

where we have adapted a $\mathbb{Z}_2$-symmetry with $\eta_+ = \eta_- \equiv \eta$ and $\omega$ is defined as $\omega = \frac{2}{\rho}$. We assume there is a scalar field non-minimally coupled to gravity on the moving domain wall. The action of this non-minimally coupled scalar field is defined as [27,28,29]

$$S_\phi = \int d^4x \sqrt{-g} \left[ \frac{1}{k^2} \alpha(\varphi) R[g] - \frac{1}{2} g^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi - V(\varphi) \right].$$

(4)

Energy-momentum tensor of this non-minimally coupled scalar field is given by

$$T_{\mu\nu} = \nabla_\mu \varphi \nabla_\nu \varphi - \frac{1}{2} g_{\mu\nu} (\nabla \varphi)^2 - g_{\mu\nu} V(\varphi) + g_{\mu\nu} \Box \alpha(\varphi) - \nabla_\mu \nabla_\nu \alpha(\varphi),$$

(5)

where $\Box$ shows 4-dimensional d’Alembertian. We assume a FRW type universe on the brane with line element defined as

$$ds^2 = -dt^2 + a^2(t) d\Sigma_k^2,$$

(6)

where $d\Sigma_k^2$ is the line element for a manifold of constant curvature $k = +1, 0, -1$. Then equation of motion for scalar field $\phi$ is

$$\nabla^\mu \nabla_\mu \varphi = V' - \alpha' R[g],$$

(7)

where a prime denotes the derivative of any quantity with respect to $\varphi$. This equation can be written as

$$\ddot{\phi} + \frac{3}{a} \dot{\phi} + \frac{dV}{d\phi} = \alpha' R[g]$$

(8)

where a dot denotes the derivative with respect to cosmic time, $t$ and Ricci scalar is given by

$$R = 6\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right).$$

(9)

This non-minimally coupled scalar field localized on the brane will play the role of dark energy component in our setup. We assume this field has only time dependence. The energy density and pressure of this non-minimally coupled dark energy component are given as follows

$$\rho = \alpha^{-1}\left(\frac{1}{2} \dot{\varphi}^2 + V(\varphi) - 6\alpha' H \dot{\varphi}\right),$$

(10)
\[ p = \alpha^{-1}\left(\frac{1}{2}\dot{\varphi}^2 - V(\varphi) + 2(\alpha'\dot{\varphi} + 2H\alpha'\dot{\varphi} + \alpha''\varphi^2)\right), \tag{11} \]

where \( H = \frac{\dot{a}}{a} \) is Hubble parameter on the moving domain wall. Now, assuming that brane is tensionless, in which follows we discuss two cases with \( \eta = 0 \) and \( \eta \neq 0 \) separately. Note that \( \eta \) is the coefficient of a term which is called dark radiation term. For \( \eta = 0 \) (the corresponding term can be neglected in late-time due to fast decay), each sub-manifolds of bulk spacetime are exact AdS\( _5 \) spacetimes. With a localized non-minimally coupled scalar field as the only source of energy-momentum on the brane, we discuss late-time acceleration and phantom divide line crossing in this setup. For this purpose, we use energy density and pressure of scalar field defined in equations (10) and (11) to rewrite equation (3) with \( \eta = 0 \) as follows\(^1\)

\[
\frac{\ddot{a}}{a} = \frac{1}{6\alpha}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi) - 6\alpha' H \dot{\varphi} \right) \left( 1 + 3\frac{\dot{\varphi}^2 - 2V(\varphi) + 4(\alpha'\dot{\varphi} + 2H\alpha'\dot{\varphi} + \alpha''\varphi^2)}{\dot{\varphi}^2 + 2V(\varphi) - 12\alpha' H \dot{\varphi}} \right) - \frac{\ell^2}{36\alpha^2}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi) - 6\alpha' H \dot{\varphi} \right)^2 \left( 2 + 3\frac{\dot{\varphi}^2 - 2V(\varphi) + 4(\alpha'\dot{\varphi} + 2H\alpha'\dot{\varphi} + \alpha''\varphi^2)}{\dot{\varphi}^2 + 2V(\varphi) - 12\alpha' H \dot{\varphi}} \right). \tag{12} \]

This is a complicated relation and to explain its cosmological implications, we have to consider either some limiting cases or specify \( \alpha(\varphi), V(\varphi) \) and \( \varphi \). Before further discussion, we note that due to existence of several fine-tunable parameters and a combination of plus and minus signs in this relation, essentially it is possible to find a domain of parameter space that satisfies the condition \( \ddot{a} > 0 \) in the favor of positively accelerated expansion. Now, to proceed further, we assume \( \alpha(\varphi) = \frac{1}{2}(1 - \xi \varphi^2) \) which is corresponding to conformal coupling of the scalar field and gravity on the brane. We apply a phenomenologically reliable ansatz (see for instance [30]) so that \( \varphi = \phi_0 e^{-kt} \) and \( a = (t^2 + \frac{\nu t_0}{1 - \nu})^{-\nu} \) where \( k \) and \( t_0 \) are positive constants. Here we assume \( \nu \neq 1 \). Also we set \( V = \lambda \varphi^n \). Defining \( A = \frac{\ddot{a}}{a} \), equation (12) with this ansatz takes the following form

\[
A = -\frac{E_1}{3 - 3\xi \varphi} - \frac{\ell^2 E_1^2}{9 - 9\xi \varphi}, \tag{13} \]

where

\[
E_1 = \frac{1}{2}\phi_0^2 k^2 (e^{-kt})^2 + \lambda \left( \phi_0 e^{-kt} \right)^n - 12\frac{\xi \phi_0^2 (e^{-kt})^2 tk}{(1 - \nu) t^2 + t_0},
\]

and

\[
E_2 = \frac{\phi_0^2 k^2 (e^{-kt})^2 - 2\lambda \left( \phi_0 e^{-kt} \right)^n - 8\xi \phi_0^2 (e^{-kt})^2 k^2 + 16\frac{\xi \phi_0^2 (e^{-kt})^2 kt}{(1 - \nu) t^2 + t_0}}{\phi_0^2 k^2 (e^{-kt})^2 + 2\lambda \left( \phi_0 e^{-kt} \right)^n - 24\frac{\xi \phi_0^2 (e^{-kt})^2 kt}{(1 - \nu) t^2 + t_0}}.
\]

\(^1\)Note that equations (10) and (11) contain additional terms proportional to \( \varphi^2 \) which we have neglected due to exponentially decreasing ansatz for \( \varphi \) used in this paper.
Figure 1 shows the possibility of accelerated expansion \(( A > 0 \) for \( \nu < 1 \) in some appropriate domain of parameter space (for example with \( \lambda = \ell = 1, \xi = 0.15, k = 0.1 \) and \( n = 2 \)). The case with \( \eta \neq 0 \) accounts for accelerated expansion in even more simpler manner due to its wider parameter space. In figure 1, we see that for \( \nu > 1 \) equation (13) has unusual behavior then therefore we restrict ourselves to cases with \( \nu < 1 \). In this braneworld setup, equation of state parameter with above ansatz has the following form

\[
\omega = \frac{p}{\rho} = \frac{\frac{1}{2} \phi_0^2 k^2 (e^{-kt})^2 - \lambda (\phi_0 e^{-kt})^n - 4\xi \phi_0^2 (e^{-kt})^2 k^2 + 8 \frac{\xi \phi_0^2 (e^{-kt})^2 k}{(1-\nu) t^2 + t_0}}{1/2 \phi_0^2 k^2 (e^{-kt})^2 + \lambda (\phi_0 e^{-kt})^n - 12 \frac{\xi \phi_0^2 (e^{-kt})^2 k}{(1-\nu) t^2 + t_0}} \tag{14}
\]

Figure 2 shows the dynamics of equation of state parameter in this model with aforementioned ansatz. As this figure shows, equation of state parameter of this model crosses the phantom divide, \( \omega = -1 \) line. On the other hand, accelerated expansion with \( \eta \neq 0 \) is easily achieved due to wide parameter space of this setup. As we have emphasized in introduction, currently models of phantom divide line crossing are so important that they can realize that which model is better than the others to describe the nature of dark energy. In this sense a non-minimally coupled scalar field on the brane provides a good candidate for explaining accelerated expansion and crossing of the phantom divide line as a suitable candidate of dark energy.
Figure 2: Crossing of the phantom divide line by equation of state parameter of a non-minimally coupled scalar field on the moving brane embedded in AdS$_5$ bulk. Based on analysis of [31,32](see also [35]), we have set $\xi = 0.15$ in numerical calculation. We have set also $\nu = -3$.

3 Bouncing Solutions

A possible solution of the singularity problem of the standard Big Bang cosmology is the so-called Bouncing Universe. A bouncing universe has an initial contracted state with a non-vanishing minimal radius and then evolves to an expanding phase [30,33]. To have a successful bouncing model in the framework of standard cosmology, the null energy condition is violated for a period of time around the bouncing point. Moreover, for the universe entering into the hot Big Bang era after the bouncing, the equation of state parameter of matter content of the universe must transit from $\omega < -1$ to $\omega > -1$ (see for instance [30]).

As figure 2 shows, in our model equation of state parameter crosses the $\omega = -1$ line from $\omega > -1$ to $\omega < -1$ and this is supported by observations [34]. This is a dynamical model of dark energy which differs from models with just a cosmological constant, pure quintessence field, pure phantom field, K-essence and so on in the determination of the cosmological evolution. However, in some sense this model is similar to quintom dark energy models which consist two fields: one quintessence and the other phantom field [35]. In which follows, we study necessary conditions required for a successful bounce in a model universe dominated by the nonminimally coupled scalar field on the moving brane embedded in AdS$_5$ bulk. During the contracting phase, the scale factor $a(t)$ is decreasing, i.e., $\dot{a}(t) < 0$, and in the expanding phase we have $\dot{a}(t) > 0$. At the bouncing point, $\dot{a}(t) = 0$, and around this point $\ddot{a}(t) > 0$ for a period of time. Equivalently, in the bouncing cosmology the hubble parameter $H$ runs across zero ($H = 0$) from $H < 0$ to $H > 0$. In the bouncing point we have $H = 0$. A successful bounce requires that around
this point the following condition should be satisfied \[30],
\[
\dot{H} = -4\pi G \rho (1 + w) > 0 .
\] (15)

From this relation we see that \(\omega < -1\) in a neighborhood of the bouncing point. After the bounce, the universe needs to enter into the hot Big Bang era, otherwise the universe filled with the matter with an equation of state parameter \(\omega < -1\) will reach the big rip singularity as what happens to the phantom dark energy \[36\]. This requires that the equation of state parameter of dark energy component transits from \(\omega < -1\) to \(\omega > -1\). One can see from figures (2) and (3) that in our setup a non-singular bouncing happens with the Hubble parameter \(H\) running across zero and a minimal non-vanishing scale factor \(a\). At the bouncing point \(\omega\) approaches a finite negative value. So it is possible to realize bouncing solutions in a model universe with non-minimally coupled dark energy component on the moving domain wall in background AdS\(_5\) bulk.

4 Summary

Light-curves analysis of several hundreds type Ia supernovae, WMAP observations of the cosmic microwave background radiation and other CMB-based experiments have shown that our universe is currently in a period of accelerated expansion. In this respect, construction of theoretical frameworks with potential to describe positively accelerated expansion and crossing of the phantom divide line by the equation of state parameter, itself is an interesting challenge. According to existing literature on dark energy models, a \textit{minimally} coupled scalar field in 4-dimension is not a good candidate for dark energy
model that its equation of state parameter crosses the phantom divide line. On the other hand, a scalar field non-minimally coupled to gravity in 4-dimension has the capability to be a suitable candidate for dark energy which provides this facilities. In this paper, we have extended the nonminimal dark energy model to a braneworld setup that brane is considered to be a moving domain wall in a static bulk background of AdS$_5$ type. In this braneworld setup, non-minimally coupled scalar field provides even more reliable candidate for dark energy due to wider parameter space. In fact, non-minimal coupling of the scalar field and gravity arises at the quantum level when quantum corrections to the scalar field theory are considered. Even if for the classical, unperturbed theory this non-minimal coupling vanishes, it is necessary for the renormalizability of the scalar field theory in curved spacetime [27]. Due to complication of dynamical equations, we have restricted our study to some specific form of non-minimal coupling (conformal coupling) and scalar field potentials and also we have considered some special form of time evolution for scale factor and scalar field via a phenomenologically reliable ansatz. Our results show that such a model universe with nonminimally coupled scalar field as dark energy component avoids the problem of the Big Bang singularity. In fact, this model allows for bouncing solution which solves the singularity problem.

The combined dataset from distant supernovae SNIa, baryon acoustic oscillation peak and the cosmic microwave background radiation show that the non-minimal coupling parameter $\xi$ is closed to its conformal value of $\frac{1}{3}$, [32]. On the other hand recent observations show that crossing of the phantom divide line occurred at $z \approx 0.25$, [12]. This observations can be used to confront our model with observations. In fact, using figure 2 and the relation $1 + z = \frac{a_0}{a(t)}$, we see that in our model crossing of the phantom divide line with $\xi = 0.15$ occurs at $z = 0.263$ which is close to the observationally supported value of $z \approx 0.25$. From another viewpoint, we can obtain a constraint on the value of the non-minimal coupling by assuming that crossing occurs at redshift near to $z \approx 0.25$.

In summary, a model universe with non-minimally coupled scalar field on the moving domain wall in AdS$_5$ bulk allows to explain both early and late-time accelerated expansions. This model realizes crossing of the phantom divide line and solves the standard Big Bang singularity problem by admitting bouncing solutions.

Acknowledgment
This work has been supported partially by Research Institute for Astronomy and Astrophysics of Maragha, Iran.

References

[1] S. Perlmutter et al, Astrophys. J. 517 (1999) 565
[2] A. G. Riess et al, Astron. J. 116 (1998) 1006
[3] A. D. Miller et al, Astrophys. J. Lett. 524 (1999) L1
[4] P. de Bernardis et al, Nature 404 (2000) 955
[5] S. Hanany et al, Astrophys. J. Lett. 545 (2000) L5
[6] D. N. Spergel et al, Astrophys. J. Suppl. 148 (2003) 175
[7] L. Page et al, Astrophys. J. Suppl. 148 (2003) 233; G. Hinshaw et al, [WMAP Collaboration], arXiv:0803.0732
[8] D. N. Spergel et al, Astrophys. J. Suppl. 170 (2007) 377
[9] G. Hinshaw et al, Astrophys. J. Suppl. 170 (2007) 288
[10] L. Page et al, Astrophys. J. Suppl. 170 (2007) 335
[11] A. G. Reiss et al, Astrophys. J 607 (2004) 665; S. W. Allen et al, Mon. Not. R. Astron. Soc. 353 (2004) 457; E. Komatsu et al. [WMAP Collaboration], arXiv:0803.0547
[12] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15 (2006) 1753-1936, [arXiv:hep-th/0603057]; S. Nesseris, L. Perivolaropoulos, JCAP 0701 (2007) 018
[13] S. Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. 4 (2007) 115, [hep-th/0601213]; S. Nojiri and S. D. Odintsov, [arXiv:0807.0685]; T. P. Sotiriou and V. Faraoni, [arXiv:0805.1726]; S. Capozziello and M. Francaviglia, Gen. Relativ. Gravit. 40 (2008) 357
[14] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B 429 (1998) 263
N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Rev. D 59 (1999) 086004
[15] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370
L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690
[16] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B 485 (2000) 208
G. Dvali and G. Gabadadze, Phys. Rev. D 63 (2001) 065007
G. Dvali, G. Gabadadze, M. Kolanović and F. Nitti, Phys. Rev. D 65 (2002) 024031.
[17] H. Zhang and Z.-H. Zhu, Phys. Rev. D 75 (2007) 023510, [arXiv:astro-ph/0611834]
[18] S. Capozziello, Int. J. Mod. Phys. D11 (2002) 483, [arXiv:gr-qc/0201033]; S. Carloni, P. K. S. Dunsby, S. Capozziello and A. Troisi, Class. Quant. Grav. 22 (2005) 4839, [arXiv:gr-qc/0410046].
[19] A. Vikman, Phys. Rev. D 71 (2005) 023515, [astro-ph/0407107].

[20] V. Sahni and Y. Shtanov, JCAP 0311 (2003) 014

[21] Y. H. Wei and Y. Z. Zhang, Grav. Cosmol. 9 (2003) 307; Y.H. Wei and Y. Tian, Class. Quantum Grav. 21 (2004) 5347; F. C. Carvalho and A. Saa, Phys. Rev. D 70 (2004) 087302; F. Piazza and S. Tsujikawa, JCAP 0407 (2004) 004; R-G. Cai, H. S. Zhang and A. Wang, Commun. Theor. Phys. 44(2005) 948; I. Y. Arefeva, A. S. Koshelev and S. Y. Vernov, Phys. Rev. D 72 (2005) 064017; A. Anisimov, E. Babichev and A. Vikman, JCAP 0506 (2005) 006; B. Wang, Y.G. Gong and E. Abdalla, Phys. Lett. B 624 (2005) 141; S. Nojiri and S. D. Odintsov, hep-th/0506212; S. Nojiri, S. D. Odintsov and S. Tsujikawa, hep-th/0501025; E. Elizalde, S. Nojiri, S. D. Odintsov and P. Wang, Phys. Rev. D 71 (2005) 103504; W. Zhao and Y. Zhang, Phys. Rev. D 73 (2006) 123509; I. Ya. Arefeva and A. S. Koshelev, hep-th/0605085.

[22] S. D. Sadatian and K. Nozari, Europhysics Letters, 82 (2008) 49001

[23] Y. S. Myung, Mod. Phys. Lett. A 16 (2001) 1963-1972
Y. S. Myung, Mod. Phys. Lett. A 16 (2001) 2187-2196

[24] S. Mizuno and Kei-ichi Maeda, Phys. Rev. D 64 (2001) 123521

[25] K. Nozari, JCAP, 09 (2007) 003, [arXiv:hep-th/07081611]

[26] S. M. Carroll, An Introduction to General Relativity: Spacetime and Geometry, Addison Wesley, 2004

[27] V. Faraoni, Phys. Rev. D 62 (2000) 023504

[28] M. Bouhamdi-Lopez and D. Wands, Phys. Rev. D 71 (2005) 024010

[29] K. Nozari and S. D. Sadatian, [arXiv:0809.4744], to appear in Euro. Phys. J. C

[30] Y.-Fu. Cai, T. Qiu, Y.-S. Piao, M. Li and X. Zhang, JHEP 0710 (2007) 071

[31] K. Nozari and S. D. Sadatian, Mod. Phys. Lett. A, 23(2008) 2933, [arXiv:0710.0058]

[32] M. Szydlowski, O. Hrycyna and A. Kurek, Phys. Rev. D 77 (2008) 027302, [arXiv:0710.0366]

[33] F. Finelli, JCAP 0310 (2003) 011; M. R. Setare, Phys. Lett. B 602 (2004) 1; M. R. Setare, Eur. Phys. J. C 47 (2006) 851; A. Biswas and S. Mukherji, JCAP 0602 (2006) 002; M. Bojowald, Phys. Rev. D 75 (2007) 081301(R), [arXiv:gr-qc/0608100]; T. Stachowiak and M. Szydlowski, Phys. Lett. B 646 (2007) 209, [arXiv:gr-qc/0610121]; Y.-F. Cai, T. Qiu, Y.-S. Piao, M. Li and X. Zhang, JHEP
0710 (2007)071, [arXiv:0704.1090]; R. Brandenberger, H. Firouzjahi and O. Saremi, [arXiv:0707.4181]; A. Cardoso and D. Wands, Phys. Rev. D 77 (2008) 123538, [arXiv:0801.1667]; M. Novello and S. E. Perez Bergliaffa, [arXiv:0802.1634]

[34] G. -B. Zhao, J. -Q. Xia, H. Li, C. Tao, J. M. Virey, Z.-H. Zhu and X. Zhang, [arXiv:astro-ph/0612728]

[35] K. Nozari, M. R. Setare, T. Azizi and N. Behrouz, [arXiv:0810.1427] and references therein.

[36] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91, (2003) 071301