Estimating Intrinsic Camera Parameters Using the Sum of Cosine Distances

Ye V Goshin¹,²

¹Samara National Research University, Moskovskoe Shosse 34A, Samara, Russia, 443086
²Image Processing Systems Institute of RAS - Branch of the FSRC "Crystallography and Photonics" RAS, Molodogvardejskaya street 151, Samara, Russia, 443001

e-mail: goshine@yandex.ru

Abstract. The paper presents a method for the estimation of camera motion parameters from a set of corresponding points by solving the problem of nonlinear optimisation. The parameters of camera motion are represented in the form of Euler–Rodrigues parameters. This approach makes it possible to avoid redundancy and further answer correction. Furthermore, the use of an angle as a measure of proximity enables us to reduce the problem to unconstrained optimisation, which provides a wider choice of its possible solutions.

1. Introduction

To solve practical problems related to the use of mobile vision devices, it is necessary to determine unknown motion parameters from images obtained from these devices [1–3]. In general, it is assumed that scene objects are static.

The described task is greatly simplified in the case where in the camera itself provides opportunities for the formation of a three-dimensional scene model. For this, RGB-d cameras [4] and stereo cameras [5] can be used. RGB-d cameras are equipped with devices for measuring the distance to objects. In this case, the problem is reduced to the comparison of two cloud points and the calculation of the corresponding three-dimensional transformation. On stereo cameras, two images are formed simultaneously, which enables estimation of the distance to objects and reduces the task to the previous one. In this paper, we consider one of the most complicated and urgent problems — estimation of the motion parameters of a device with a monocular camera.

Most of the papers dedicated to this issue have studied different calibration methods. Some papers [6] propose methods for computation of both extrinsic and intrinsic camera parameters, including distortion parameters. However, camera calibration is carried out through the use of a specific pattern (e.g. a chessboard) and is not feasible in a real-life environment.

Traditionally, the problem of estimating the parameters of rotation and translation is reduced to estimation of the so-called fundamental matrix and the subsequent calculation of the unknown matrices [7]. However, this approach has several drawbacks. In particular, when most of the preliminarily defined points of the images are located on the same plane, fundamental matrix estimation is performed with significant errors, which, in turn, leads to errors in determining the extrinsic camera parameters.
The main idea of this paper is to determine rotation and translation of the cameras directly from the corresponding points, excluding the stage of fundamental matrix estimation. In this paper we use information on the direction of the translation of the corresponding points in an image relative to the other. It allows us to accurately identify the translation and rotation even in the case where in all of the initial points are located very close to one plane.

2. Problem definition

For 3D scene reconstruction from stereo images, the model of the camera obscura is used [8]. It is assumed that stereo images are obtained with one camera from two different points. Camera intrinsic parameters are known and set by matrix $K$:

$$
K = \begin{bmatrix}
    f & 0 & u_0 \\
    0 & f & v_0 \\
    0 & 0 & 1 \\
\end{bmatrix},
$$

where $f$ is the camera focal length and $(u_0, v_0)$ are camera principal point coordinates in the coordinate system connected with the camera.

Assume that $M$ is some point in the global coordinate system, reconstructed from a pair of stereo images. Transformation from the global coordinate system to homogeneous image coordinates is performed as follows:

$$
\begin{bmatrix}
    X \\
    Y \\
    Z \\
    1
\end{bmatrix} = \begin{bmatrix}
    r_{11} & r_{12} & r_{13} & t_1 \\
    r_{21} & r_{22} & r_{23} & t_2 \\
    r_{31} & r_{32} & r_{33} & t_3 \\
\end{bmatrix} \begin{bmatrix}
    X \\
    Y \\
    Z \\
    1
\end{bmatrix},
$$

where $u, v$ are homogeneous image coordinates on a view, $X, Y, Z$ are global coordinates of point $M$, and $(R | t)$ is the camera extrinsic matrix comprising translation and rotation parameters.

Taking into account the following coordinate connection:

$$m = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix},
$$

equation (1) can be rewritten as follows:

$$
\begin{bmatrix}
    x \\
    y \\
    1
\end{bmatrix} = \begin{bmatrix}
    r_{11} & r_{12} & r_{13} & t_1 \\
    r_{21} & r_{22} & r_{23} & t_2 \\
    r_{31} & r_{32} & r_{33} & t_3 \\
\end{bmatrix} \begin{bmatrix}
    X \\
    Y \\
    Z \\
    1
\end{bmatrix},
$$

where $x, y$ are homogeneous image coordinates on a view, $X, Y, Z$ are global coordinates of point $M$, and $(R | t)$ is the camera extrinsic matrix comprising translation and rotation parameters.

Let us further refer to point $m$ as $m(x, y)$ in compliance with (2). It is obvious that camera extrinsic matrix $(R | t)$ is necessary for 3D scene reconstruction. The main problem is that these parameters are usually unknown and need to be determined from $N$ corresponding points $m(x, y)$ and $m(x', y')$ on initial views. The present paper is dedicated to a solution to this problem.

Assume that there are two sets of corresponding points on images $P = (u_i, v_i), P' = (u'_i, v'_i)$, where $(u_i, v_i)$ and $(u'_i, v'_i)$ are the coordinates of these points on the first and second images, respectively. If the matrix of camera intrinsic parameters $K$ is known, the coordinates of the pixels of images $(u_i, v_i)$ and $(u'_i, v'_i)$ can be transformed into coordinates $(x_i, y_i)$ and $(x'_i, y'_i)$ on the plane of projection of the first and second cameras as follows:

$$
\begin{bmatrix}
    x \\
    y \\
    1
\end{bmatrix} = \begin{bmatrix}
    u \\
    v \\
    1
\end{bmatrix},
$$

Coordinates $(x_i, y_i)$ and $(x'_i, y'_i)$ will be further used in the paper.
3. Traditional approach

The traditional approach to the calculation of the location and orientation includes the stage of fundamental matrix estimation [9, 10]. The fundamental matrix is a matrix satisfying the following condition:

\[(m')^T F m = (x' \ y' 1) \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \]

and it can be found through the use of an eight-point algorithm [8] from a set of given corresponding points. Through the use of the fundamental matrix with a known matrix of intrinsic shooting parameters \(K\) we can calculate the so-called essential matrix:

\[\varepsilon = K^T F K.\]

The traditional approach is based on singular decomposition of the essential matrix, which gives the following relation:

\[\varepsilon = U \Sigma V^T,\]

where \(U\) and \(V\) are orthogonal matrices, and \(\Sigma\) is a diagonal matrix in the following form:

\[
\Sigma = \begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]

The two diagonal values of \(\Sigma\) have to be equal and the third one equals 0.

If we define matrix \(W\) as follows:

\[
W = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad W^{-1} = W^T = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

then

\[
[t]_s = VW\Sigma V^T, \quad R = UW^{-1}V^T,
\]

where

\[
[t]_s = \begin{pmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{pmatrix}, \quad t = \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}.
\]

Since \(\Sigma\) does not always fully meet the requirements of diagonal value equity, an alternate expression can be used:

\[
[t]_s = VZV^T, \quad \text{where} \quad Z = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]

Previously, the author proposed a method and an algorithm for solving this problem without using the intermediate stage of determining the fundamental matrix [11]. However, the traditional approach to image formation used in that paper has several drawbacks. In particular, the traditional approach loses its stability when all of the initial points are located very close to one plane, becoming completely inapplicable to the initial points located strictly on the same plane.

4. Suggested method

Without the loss of generality, suppose that the coordinate system of the first camera coincides with the global coordinate system and, therefore, has the following parameters:

\[
R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad t_x = 0, \quad t_y = 0, \quad t_z = 0.
\]
Let us denote the parameters of the second camera as follows:

\[
R' = \begin{bmatrix}
R'_{11} & R'_{12} & R'_{13} \\
R'_{21} & R'_{22} & R'_{23} \\
R'_{31} & R'_{32} & R'_{33}
\end{bmatrix}, \quad t'_x = 0, \quad t'_y = 0, \quad t'_z = 0.
\]

Given that the image is a projection of the scene points on the camera plane, the relation between the coordinates of the corresponding points and the motion parameters can be written as follows:

\[
\begin{cases}
x = \frac{X}{Z}, \\
y = \frac{Y}{Z},
\end{cases}
\quad \begin{cases}
x' = \frac{R_{11}X + R_{12}Y + R_{13}Z + t'_x}{R_{31}X + R_{32}Y + R_{33}Z + t'_z}, \\
y' = \frac{R_{21}X + R_{22}Y + R_{23}Z + t'_y}{R_{31}X + R_{32}Y + R_{33}Z + t'_z}.
\end{cases}
\]

Let us introduce so-called inverse distance \( k = \frac{1}{Z} \), and then

\[
X = \frac{x}{k}, \quad Y = \frac{y}{k}, \quad Z = \frac{1}{k}.
\]

and

\[
\begin{cases}
x' = \frac{R_{11}x + R_{12}y + R_{13} + kt_x}{R_{31}x + R_{32}y + R_{33} + kt_z}, \\
y' = \frac{R_{21}x + R_{22}y + R_{23} + kt_y}{R_{31}x + R_{32}y + R_{33} + kt_z}.
\end{cases}
\]

Let us introduce supplementary variables

\[
x_r = R_{11}x + R_{12}y + R_{13}, \\
y_r = R_{21}x + R_{22}y + R_{23}, \\
z_r = R_{31}x + R_{32}y + R_{33},
\]

and then

\[
\begin{cases}
x' = \frac{x_r + kt'_x}{z_r + kt'_z}, \\
y' = \frac{y_r + kt'_y}{z_r + kt'_z}.
\end{cases}
\]

We wrote the expressions for the coordinates of corresponding point \( m' \) under the assumption that the distance to point \( M \) is equalto 0, 1 and \(+\infty\).

\[
k = 0: \quad \begin{cases}
x'_0 = \frac{x_r}{z_r}, \\
y'_0 = \frac{y_r}{z_r}.
\end{cases}
\]

\[
k = 1: \quad \begin{cases}
x'_1 = \frac{x_r + t'_x}{z_r + t'_z}, \\
y'_1 = \frac{y_r + t'_y}{z_r + t'_z}.
\end{cases}
\]

\[
k = +\infty: \quad \begin{cases}
x'_{\infty} = \frac{t'_x}{t'_z}, \\
y'_{\infty} = \frac{t'_y}{t'_z}.
\end{cases}
\]

The distance "infinity" (\( k = 0 \)) corresponds to the situation where in the point is so far away that its translation between frames does not depend on the translation of the camera, but only on the rotation. Consider two vectors on a plane. The first vector \( d^I = (d'_x, d'_y) \) connects an initial point \( \left( \frac{x_r}{z_r}, \frac{y_r}{z_r} \right) \) with a terminal point \( (x', y') \): \n
\[
(d'_x, d'_y) = \left( x' - \frac{x_r}{z_r}, y' - \frac{y_r}{z_r} \right).
\]
The second vector $\mathbf{d}^u = (d_x^u, d_y^u)$ connects the same point $\left(\frac{x}{z}, \frac{y}{z}, \frac{z}{z}\right)$ with the projection of the translation vector $\left(\frac{t_x}{z}, \frac{t_y}{z}, \frac{t_z}{z}\right)$:

$$d_x^u = \left(\frac{t_x}{z} - \frac{x}{z}\right), \quad d_y^u = \left(\frac{t_y}{z} - \frac{y}{z}\right).$$

The set of all possible positions of the corresponding point $(x', y')$ is a ray emerging from point $(x_0, y_0')$ in the direction of $(x_{\infty}, y_{\infty})$. Point $(x', y')$ should lie on this ray, so directions of vectors $\mathbf{d}^i$ and $\mathbf{d}^u$ should coincide.

Let us introduce cosine distance [12] $f(\mathbf{d}^i, \mathbf{d}^u)$ between the vectors:

$$f(\mathbf{R}, t'_i, t'_j, t'_k) = f(\mathbf{d}^i, \mathbf{d}^u) = 1 - \frac{\langle \mathbf{d}^i, \mathbf{d}^u \rangle}{\|\mathbf{d}^i\| \cdot \|\mathbf{d}^u\|}.$$

Thus, with a known rotation matrix, the problem can be reduced to the following minimisation with respect to parameters $t'_i, t'_j, t'_k$:

$$\min_{t'_i, t'_j, t'_k} \sum_i [f(t'_i, t'_j, t'_k)]^2 = \min_{t'_i, t'_j, t'_k} \sum_i \left[1 - \frac{\langle \mathbf{d}^i, \mathbf{d}^u \rangle}{\|\mathbf{d}^i\| \cdot \|\mathbf{d}^u\|}\right]^2.$$

With an unknown rotation matrix, it is possible to express the elements of this matrix through the Euler–Rodrigues rotation parameters [13], which represents as a quadruple of real numbers $(a, b, c, d)$. The rotation matrix can be rewritten through the use of these parameters as follows [14]:

$$\mathbf{R} = \begin{pmatrix}
    r_{11} & r_{12} & r_{13} \\
    r_{21} & r_{22} & r_{23} \\
    r_{31} & r_{32} & r_{33}
\end{pmatrix} = \begin{pmatrix}
    a^2 + b^2 - c^2 - d^2 & 2(bc - ad) & 2(bd + ac) \\
    2(bc + ad) & a^2 + c^2 - b^2 - d^2 & 2(cd - ab) \\
    2(bd - ac) & 2(cd + ab) & a^2 + d^2 - b^2 - c^2
\end{pmatrix}.$$

In this case the parameters of rotation of the two cameras can be written as $(a, b, c, d)$ and $(a', b', c', d')$, respectively. Since it is assumed that the first camera coincides with the global coordinate system, its parameters are as follows:

$$a = 1, \quad b = 0, \quad c = 0, \quad d = 0.$$

Thus, the problem is generalised to minimisation with respect to parameters $a, b, c, d, t'_i, t'_j, t'_k$:

$$\min_{a, b, c, d, t'_i, t'_j, t'_k} \sum_i [f(a', b', c', d', t'_i, t'_j, t'_k)]^2 = \min_{a, b, c, d, t'_i, t'_j, t'_k} \sum_i \left[1 - \frac{\langle \mathbf{d}^i, \mathbf{d}^u \rangle}{\|\mathbf{d}^i\| \cdot \|\mathbf{d}^u\|}\right]^2.$$

During this research the author carried out an experimental study of applicability of the proposed method through the use of the algorithm of bound optimisation by quadratic approximation (BOBYQA) [15].

5. Experimental results

To test the applicability, as well as the accuracy and reliability of the developed computational procedure, an experiment was conducted to establish the relation between the error of the input data (coordinates of points on the camera planes) and the error in the obtained estimates of rotation and translation.

The following intrinsic and extrinsic camera parameters were specified:
In this experiment, the set of initial points with coordinates \([X \ Y \ Z]\) were generated in the following way. The 3D points were located in vertices of a regular square grid with coordinates \([X_{\text{grid}} \ Y_{\text{grid}} \ 1]\). Thereafter, for each point an inverse distance value was randomly generated and coordinates \([X \ Y \ Z]\) were calculated as follows:

\[
\begin{bmatrix}
    X' \\
    Y' \\
    Z'
\end{bmatrix} = \begin{bmatrix}
    t'_{x} \\
    t'_{y} \\
    t'_{z}
\end{bmatrix},
\]

\(t'_{x}, t'_{y}, t'_{z} \in [-6, 6]\).

Based on the generated values of \(X, Y, Z\), the coordinates of the points on both images were calculated. The initial data for the experiments were set on the basis of the computed coordinates on the images with an additive noise. The ratio of the peak signal to noise was fixed, the number of initial points was equal to 36, 100, 400 and 1600, and the inverse distance varied in different ranges, from \([0.2, 0.3]\) to \([0.2, 0.205]\). Each experiment was conducted 1000 times. The result of the rotation estimation was considered correct if the ratio of the norm of the error vector to the norm of the parameter vector did not exceed 0.2. The result of the rotation estimation was considered correct if the angle between the calculated and the actual translation vectors did not exceed 10 degrees. For all of the correct results, the mean value of the error was found, as well as the percentage of correct results among all attempts. The table shows the results of the comparison of the camera parameter determination method proposed in this paper (“PM”) with the method proposed by Hartley using the fundamental (or essential) matrix (“FM”). For the Hartley method there are five different implementations: fundamental matrix estimation through the use of a seven-point algorithm, eight-point algorithm, LMEDS algorithm, and RANSAC algorithm, and direct essential matrix estimation using the RANSAC algorithm. In Tables 1–3 they are presented in the same order.

For the obtained values we calculated the percentage of the results for which both translation and rotation were estimated accurately.

The experiment demonstrated applicability of the proposed method for various conditions and the advantage of the proposed approach in comparison with the traditional one when the initial points are located close to the plane.

| \(k_{\text{inv}}\) | \(0.2–0.3\) | 0.0286 |
|-----------------|-------------|--------|
| PM | 0.0458 | 0.0228 | 0.0458 | 0.0547 |
| FM | 0.0400 | 0.0138 | 0.0400 | 0.0439 |
| PM | 0.0254 | 0.0093 | 0.0254 | 0.0370 |
| FM | 0.0299 | 0.0299 | 0.0250 | 0.0266 |
| PM | 0.0282 | 0.0055 | 0.0282 | 0.0266 |
| FM | 0.0282 | 0.0266 |
Table 2. Accuracy of the translation estimation (degrees).

| $k_{inv}$ | The number of points |
|-----------|----------------------|
|          | 36                   | 100      | 400      | 1600     |
|          | PM$^a$    | FM$^b$  | PM$^a$    | FM$^b$  | PM$^a$    | FM$^b$  | PM$^a$    | FM$^b$  |
|          | 0.2–0.3   | 1.2671  | 5.6914    | 4.6926  | 4.0259    | 3.8594  | 3.8951    | 3.9571  |
|          | 3.3910    | 2.0174  | 1.7192    | 1.0885  | 4.0259    | 1.0221  | 3.8594    | 3.8685  |
|          | 0.2–0.25  | 1.6160  | 5.6545    | 6.0578  | 6.0217    | 5.3951  | 2.6835    | 2.6683  |
|          | 5.0069    | 3.7709  | 1.2762    | 1.2698  | 6.0217    | 1.1923  | 5.3951    | 5.7089  |
|          | 5.9512    | 6.1261  | 3.3717    | 3.0055  | 2.1049    | 1.8872  |           |         |
|          | 0.2–0.21  | 1.9474  | 5.5412    | 4.6599  | 4.6558    | 8.7094  |           |         |
|          | 6.8895    | 6.2351  | 1.6757    | 1.3621  | 4.6558    | 1.3928  | 8.7094    | 8.0760  |
|          | 5.5412    | 4.6599  | 8.4435    | 5.6168  | 2.5032    | 2.0056  |           |         |
|          | 3.9016    | 3.6593  |           |         |           |         |           |         |
|          | 0.2–0.205 | 2.0169  | 1.4391    | 6.3954  | 6.2120    | 6.3169  |           |         |
|          | 7.0162    | 6.9368  | 1.4982    | 1.4726  | 6.3954    | 1.4385  | 8.0291    |         |
|          | 1.4391    | 9.0226  | 3.3255    | 3.7775  | 2.4404    | 2.0462  |           |         |
|          | 6.8913    |         |           |         |           |         |           |         |
|          | 3.3255    |         |           |         |           |         |           |         |
|          | 0.2–0.201 | 1.8877  | 7.5282    | 6.8758  | 8.7742    | 6.2381  |           |         |
|          | 7.5282    | 6.3979  | 1.4187    | 1.4787  | 8.7742    | 1.4261  | 6.2381    |         |
|          | 4.5697    |         |           |         | 5.9468    |         |           |         |
|          | 4.0282    | 2.8317  |           |         | 2.6284    | 2.1023  |           |         |

$^a$From top to bottom of cell: seven-point algorithm, eight-point algorithm, LMedS algorithm, RANSAC algorithm, and direct essential matrix estimation using RANSAC algorithm.
| $k_{inv}$ | 36 | 100 | 400 | 1600 |
|----------|----|-----|-----|------|
|          | PM | FM$^a$ | PM | FM | PM | FM | PM | FM |
| 0.2–0.3  | 64.2 | 73.3 | 91.6 | 89.8 |
|          | 89.9 | 96.6 | 99.1 | 96.6 |
|          | 63.7 | 94 | 73 | 95.7 | 91.8 | 95.9 | 90 |
|          | 49.8 | 65.2 | 74.5 | 92.5 |
|          | 68.3 | 88.2 | 94.1 | 98.5 |
| 0.2–0.25 | 24.6 | 44.7 | 53.2 | 65.7 |
|          | 80.8 | 88.5 | 90.7 | 99.1 |
|          | 24.5 | 93.5 | 44.8 | 94.7 | 53.3 | 96.9 | 66.3 |
|          | 16.9 | 32 | 41.3 | 61.6 |
|          | 60.5 | 80.6 | 95.1 | 96.6 |
| 0.2–0.21 | 1.6 | 3.4 | 1.8 | 2.7 |
|          | 10.2 | 21.8 | 28.7 | 33.6 |
|          | 1.5 | 94.6 | 2.6 | 93.7 | 1.5 | 98.5 | 3.2 |
|          | 2.1 | 1.3 | 3.5 | 3.9 |
|          | 35.3 | 47.6 | 51.6 | 65 |
| 0.2–0.205 | 1.1 | 96.1 | 8.8 | 1.9 | 2.7 |
|          | 5 | 94.1 | 6.6 | 15.7 |
|          | 0.6 | 96.6 | - | 2.1 | 98.2 | 1.1 |
|          | 0.5 | 2 | - | - |
|          | 24.2 | 36.8 | 41.4 | 46.6 |
| 0.2–0.201 | 1.9 | 90.5 | 0.8 | 2.5 | 1.8 |
|          | 2.6 | 90.5 | 4.9 | 5.7 | 6 |
|          | 2.1 | 94.4 | 1.1 | 3.4 | 97.8 | 2.6 |
|          | 0.7 | 0 | 4.2 | - |
|          | 28 | 33.9 | 46.9 | 38.7 |

$a$ from top to bottom of cell: seven-point algorithm, eight-point algorithm, LMedS algorithm, RANSAC algorithm, and direct essential matrix estimation using RANSAC algorithm.

6. Conclusion

Due to the use of information on the direction of the translation of the corresponding points in an image relative to the other, the proposed method makes it possible to accurately identify the translation and rotation even in the case where in all of the initial points are located very close to one plane. Due to its specificity, the traditional approach using a fundamental matrix under such conditions loses its stability, becoming completely inapplicable to the initial points located strictly on the same plane.

Thus, the advantage of the proposed approach in comparison with the traditional one arises and increases with the approach of the initial points to the plane.

7. References

[1] Aqel M O, Marhaban M H, Saripan M I and Ismail N B 2016 Review of visual odometry: types, approaches, challenges, and applications Springer Plus 5(1) 1897
[2] Konovalenko I A, Miller A B, Miller B M and Nikolaev D P 2015 UAV navigation on the basis of the feature points detection on underlying surface ECMS 499-505
[3] Karpenko S, Konovalenko I, Miller A, Miller B and Nikolaev D 2015 UAV control on the basis of 3D landmark bearing-only observations Sensors 15(12) 29802-29820 DOI: 10.3390/s151229768
[4] Kerl C, Sturm J and Cremers D 2013 Robust odometry estimation for RGB-D cameras IEEE International Conference on Robotics and Automation (ICRA) 3748-3754
[5] Howard A 2008 Real-time stereo visual odometry for autonomous ground vehicles IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS) 3946-3952
[6] Kholopov IS 2017 Projective distortion correction algorithm at low altitude photographing Computer Optics 41(2) 284-290 DOI: 10.18287/0134-2452-2017-41-2-284-290
[7] Ovchinkin A and Ershov E 2016 Statistical analysis of the characteristics of high degree polynomial solving methods used in the five-point algorithm Proc. SPIE International Conference on Robotics and Machine Vision 10253(102530L) 1-5

[8] Hartley R and Zisserman A 2003 Multiple view geometry in computer vision (Cambridge University Press)

[9] Csurka G, Zeller C, Zhang Z and Faugeras O 1997 Characterizing the uncertainty of the fundamental matrix Computer Vision and Image Understanding 68(1) 18-36

[10] Hartley R I 1995 In defence of the 8-point algorithm Proc. of the 5th International Conference on Computer Vision 1064-1070

[11] Goshin Ye V and Fursov V A 2015 3D scene reconstruction from stereo images with unknown extrinsic parameters Computer Optics 39(5) 770-776 DOI: 10.18287/0134-2452-2015-39-5-770-776

[12] Zhang D and Lu G 2003 Evaluation of similarity measurement for image retrieval International Conference on IEEE Neural Networks and Signal Processing 2 928-931

[13] Schwab A L 2002 Quaternions, finite rotation and Euler parameters Cornell University Notes (Ithaca, NY)

[14] Dai J S 2015 Euler–Rodrigues formula variations, quaternion conjugation and intrinsic connections Mechanism and Machine Theory 92 144-152

[15] Powell M J D 2009 BOBYQA algorithm for bound constrained optimization without derivatives (Cambridge NA Report NA2009/06, University of Cambridge) 26-46

Acknowledgements
The reported study was funded by RFBR according to research projects No. 17-29-03112, 16-29-09528.