On the use of the local prior on the absolute magnitude of Type Ia supernovae in cosmological inference

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ABSTRACT

A dark-energy which behaves as the cosmological constant until a sudden phantom transition at very-low redshift \((z < 0.1)\) seems to solve the \(>4\sigma\) disagreement between the local and high-redshift determinations of the Hubble constant, while maintaining the phenomenological success of the \(\Lambda\) cold dark matter (CDM) model with respect to the other observables. Here, we show that such a hockey-stick dark energy cannot solve the \(H_0\) crisis. The basic reason is that the supernova absolute magnitude \(M_B\) that is used to derive the local \(H_0\) constraint is not compatible with the \(M_B\) that is necessary to fit supernova, baryon acoustic oscillations (BAO) and cosmic microwave background (CMB) data, and this disagreement is not solved by a sudden phantom transition at very-low redshift. We make use of this example to show why it is preferable to adopt in the statistical analyses the prior on \(\sigma_B\) as an alternative to the prior on \(H_0\). The three reasons are: i) one avoids potential double counting of low-redshift supernovae, ii) one avoids assuming the validity of cosmography, in particular fixing the deceleration parameter to the standard model value \(q_0 = -0.55\), iii) one includes in the analysis the fact that \(M_B\) is constrained by local calibration, an information which would otherwise be neglected in the analysis, biasing both model selection and parameter constraints. We provide the priors on \(M_B\) relative to the recent Pantheon and DES-SN3YR supernova catalogs. We also provide a Gaussian joint prior on \(H_0\) and \(\sigma_B\) that generalizes the prior on \(H_0\) by SH0ES.

Key words: cosmological parameters–dark energy–cosmology: observations

1 INTRODUCTION

The Hubble constant \(H_0\) – the first cosmographic coefficient in a series expansion of the scale factor – is perhaps the most basic parameter in cosmology. It is then understandable that the \(>4\sigma\) disagreement between the local (Riess et al. 2021) and high-redshift (Aghanim et al. 2020) determinations of the Hubble constant has received much spotlight. Indeed, this tension could very well signal the need of a new standard model of cosmology, although it is not clear which alternative model can successfully explain all available observations (see Knox & Millea 2020; Di Valentino et al. 2021, for details).

A dark-energy which behaves as the cosmological constant until a sudden phantom transition at very-low redshift seems able to solve the \(H_0\) crisis, while maintaining the phenomenological success of the \(\Lambda\) cold dark matter (CDM) model with respect to the other observables. The phenomenology of a late-time transition in the Hubble rate has been first considered by Mortonson et al. (2009), and recently confronted with data by Benevento et al. (2020); Dhawan et al. (2020); Efstathiou (2021), while a low-redshift transition on the dark energy equation of state has been proposed by Alestas et al. (2020) (see also Keeley et al. 2019).

Here, we show that a hockey-stick dark energy \((hsCDM, \text{see Figure 1})\) cannot solve the \(H_0\) crisis. The basic reason is that the supernova absolute magnitude \(M_B\) that is used to derive the local \(H_0\) constraint is not compatible with the \(M_B\) that is necessary to fit supernova, baryon acoustic oscillations (BAO) and cosmic microwave background (CMB) data, and this remains true even with a sudden phantom transition at very-low redshift. Statistically, this becomes evident if one includes the supernova calibration prior on \(M_B\) in the statistical analysis, which would otherwise support \(hsCDM\).

We make use of this example to show in details why it is preferable to adopt the prior on \(M_B\) rather than the prior on \(H_0\) in the cosmological analyses that study the impact of

\[1\] We remind the reader that a hockey stick trend is characterized by a sharp change after a relatively flat and quiet period.
2 Camarena and Marra

Figure 1. Hockey-stick dark energy behaves as the cosmological constant until a sudden phantom transition at very-low redshift.

local $H_0$ on the dark energy properties (see, for instance, the analysis performed in Section 5 of Riess et al. 2016). We also provide the $M_B$ priors relative to the Pantheon and Dark Energy Survey Supernova Program (DES-SN3YR) catalogs, and a joint prior on $H_0$ and $q_0$ that generalizes the one on $H_0$ by the Supernova H0 collaboration.

This paper is organized as follows. In Section 2 we introduce hockey-stick dark energy, in Section 3 we discuss the prior on $M_B$, while in Section 4 we present the statistical analysis. The results are shown in Section 5 and the conclusions drawn in Section 6.

2 HOCKEY-STICK DARK ENERGY

In order to show the advantages of using a local prior on $M_B$ instead of a local prior on $H_0$ we will consider a model that features a dark energy with the following hockey-stick equation of state (hsCDM):

$$w = \begin{cases} w_x - (1 + w_x) z / z_t & \text{if } z \leq z_t \text{ (the blade)} \\ -1 & \text{if } z > z_t \text{ (the shaft)} \end{cases},$$

which mimics the cosmological constant at higher redshifts and deviates from the latter for $z \leq z_t$, reaching $w_x$ at $z = 0$, see Figure 1. A step equation of state (constant $w_x$ for $z \leq z_t$) shows a very similar phenomenology. Here, we adopt the hockey-stick equation of state as it features the same physical properties of the sources. Models that feature the hockey-stick phenomenology are discussed in Mortonson et al. (2009).

It follows that the expansion rate is, assuming spatial flatness:

$$\frac{H^2(z)}{H_0^2} = \Omega_{M0}(1 + z)^3 + \Omega_{R0}(1 + z)^4 + \Omega_{\Lambda0}(1 + z)^{3\gamma(z)},$$

where $\Omega_{M0} + \Omega_{R0} + \Omega_{\Lambda0} = 1$ and

$$g(z) = \frac{1}{\ln(1 + z)} \int_0^z \frac{1 + w(z')}{1 + z'} dz'$$

$$= \frac{1 + w_x}{z_t \ln(1 + z_t)} \times \begin{cases} (1 + z_t) \ln(1 + z) - z & \text{if } z \leq z_t \\ (1 + z_t) \ln(1 + z_t) - z_t & \text{if } z > z_t \end{cases}.$$

The apparent magnitude is then:

$$m_B(z) = 5 \log_{10} \left[ \frac{d_L(z)}{10 \text{pc}} \right] + M_B,$$

where the luminosity distance is:

$$d_L(z) = (1 + z) \int_0^z \frac{c \, dz'}{H(z')},$$

Finally, the distance modulus is given by:

$$\mu(z) = m_B(z) - M_B.$$

For $z_\text{1} \to \infty$ one recovers the wCDM model with $w = w_x$. We will consider the wCDM model for comparison sake.

3 SUPERNOVA CALIBRATION PRIOR

The determination of $H_0$ by the SH0ES Collaboration is a two-step process (Riess et al. 2016):

(i) First, anchors, Cepheids and calibrators are combined to produce a constraint on the supernova Ia absolute magnitude $M_B$. This step only depends on the astrophysical properties of the sources.

(ii) Second, Hubble-flow Type Ia supernovae in the redshift range $0.23 \leq z \leq 0.15$ are used to probe the luminosity distance-redshift relation in order to determine $H_0$. Cosmography with $q_0 = -0.55$ and $\delta_0 = 1$ is adopted.

The latest constraint by SH0ES reads:

$$H_0^{\text{SH0ES}} = 73.2 \pm 1.3 \text{ km s}^{-1}\text{Mpc}^{-1} \ (\text{Riess et al. 2021}).$$

Usually, one introduces in the cosmological analyses that use an $H_0$ prior the following $\chi^2$ function:

$$\chi^2_{H_0} = \frac{(H_0 - H_0^{\text{SH0ES}})^2}{\sigma_{H_0^{\text{SH0ES}}}^2}.$$

The goal of this paper is to show, using the example of hockey-stick dark energy, that it is preferable to skip step ii) above and adopt directly the local prior on $M_B$ via:

$$\chi^2_{M_B} = \frac{(M_B - M_B^{\text{SH0ES}})^2}{\sigma_{M_B^{\text{SH0ES}}}^2},$$

where $M_B^{\text{SH0ES}}$ is the calibration that corresponds to the $H_0$ prior of equation (7).

Before proceeding, it is important to point out that supernovae Ia become standard candles only after standardization and that the method used to fit supernova Ia light curves, and its parameters, can influence the inferred value of $H_0$ (e.g., $x_0$, $x_1$, and $c$ in the case of SALT2, Guy et al. 2007). This means that the actual prior on $M_B$ from SH0ES can only be used with the Supercal supernova sample (Scolnic et al. 2015), which is the one adopted by SH0ES in the latest analyses.

Consequently, in order to meaningfully use the local prior on $M_B$, one has to translate it to the light curve calibration adopted by some other dataset $X$. This task can be achieved using the method developed in Camarena & Marra (2020a): the basic idea is to demarginalize the final $H_0$ measurement using for step ii) the supernovae of the dataset $X$ that are in the same redshift range $0.23 \leq z \leq 0.15$. 

MNRA 000, 1–9 (202X)
This procedure, applied to the latest supernova catalogs, produces the priors listed in Table 1. In other words, by adopting the priors given in Table 1 and performing step ii), one recovers the \( H_0 \) determination of equation (7).

It is worth mentioning that supernovae Ia are not perfectly standardizable candles and there are residual correlations with their environment, such as the step correction to \( M_B \) according to the host galaxy mass (Kelly et al. 2010; Lampeitl et al. 2010; Sullivan et al. 2010). The method discussed in this section assumes that these residual corrections have been applied before obtaining the effective prior on \( M_B \).

Correlations between the residuals and the supernova environment have also been used to argue in favor of a possible time evolution of the absolute magnitude (Kang et al. 2020; Kim et al. 2019). Recent analyses suggest that such time evolution is not favored by data (Huang 2020; Koo et al. 2020; Sapone et al. 2020) and could have been produced by systematics (Brout & Scolnic 2021; Rose et al. 2020). Throughout this work we assume that \( M_B \) does not evolve with time.

### 3.1 Local joint \( H_0-q_0 \) constraint

Although, as we argue below, it is preferable to use in the statistical analysis the prior on \( M_B \), it is nevertheless important to determine the local value of \( H_0 \).

The measurement by SH0ES of equation (7) is obtained from the local constraint on \( M_B \) after adopting in the cosmographic analysis the following Dirac delta prior on the deceleration parameter \( q_0 \):

\[
f(q_0) = \delta(q_0 - q_{0,\text{fid}}),
\]

\[
q_{0,\text{fid}} = -0.55, \tag{10}
\]

where the deceleration parameter value takes the relative to the flat concordance ΛCDM model with \( \Omega_{M,0,\text{fid}} = 0.3 \) (Riess et al. 2016). In other words, the constraint of equation (7) uses information beyond the local universe in order to fix the value of \( q_0 \).

One can improve the local determination of \( H_0 \) by adopting an uninformative prior \( f(q_0) = \text{constant} \). Specifically, adopting the \( M_B \) prior relative to the Supercal dataset given in Table 1 and the same 217 Supernovae used by SH0ES, one obtains the joint prior that is given in Table 2 and illustrated in Figure 2. This constraint on \( H_0 \) and the CMB-only constraint from the Planck Collaboration (Aghanim et al. 2020) disagree at the 4.5σ level.

We have used the numerical codes emcee (Foreman-Mackey et al. 2013) and getdist (Lewis 2019).

It is worth noting that the determination of Table 2 only assumes large-scale homogeneity and isotropy and no information from observations beyond the local universe is used. For comparison, we show in Figure 2 also the original constraint of equation (7) that is recovered by fixing \( q_0 = -0.55 \).\(^2\) Note also that \( M_B \) shows basically no correlation with \( q_0 \). In other words, fixing \( q_0 = -0.55 \) (Riess et al. 2021) should not have biased the determination of \( M_B \) via the method of Camarena & Marra (2020a).

\(^2\) To be precise, the constraint of equation (7) adopts third-order cosmography and fixes also \( j_0 = 1 \). As in Figure 2 we use second-order cosmography, fixing \( q_0 = -0.55 \) gives back an \( H_0 \) that is 0.1 km s\(^{-1}\)Mpc\(^{-1}\) higher than the one of equation (7).
4 STATISTICAL INFERENCE

We now discuss the datasets that we adopt in order to constrain the hσCDM model.

4.1 Cosmic Microwave Background

We use the Gaussian prior on \((R, \Omega_B, h^2, n_s)\) derived from the Planck 2018 results (Chen et al. 2019, wCDM model in Table I). We denote with \(\chi^2_{\text{cmb}}\) the corresponding \(\chi^2\) function.

4.2 Baryonic Acoustic Oscillations

We adopt BAO measurements from the following surveys: 6dFGS (Beutler et al. 2011), SDSS-MGS (Ross et al. 2015) and BOSS-DR12 (Alam et al. 2017). 6dFGS and SDSS-MGS provide isotropic measurements at redshifts 0.1 and 0.15, while BOSS-DR12 data constrains \(H(z)\) and \(d_A(z)\) at redshifts 0.38, 0.51 and 0.61. We denote with \(\chi^2_{\text{bao}}\) the corresponding \(\chi^2\) function.

4.3 Supernovae Ia

We consider the Pantheon dataset, consisting of 1048 Type Ia supernovae spanning the redshift range 0.01 < \(z < 2.3\) (Scolnic et al. 2018). We denote with \(\chi^2_{\text{sne}}\) the corresponding \(\chi^2\) function.

4.4 Local constraint

We will consider either the prior on \(H_0\) of equation (8) or the prior on \(M_B\) of equation (9) relative to the Pantheon sample, see Table 1.

4.5 Total likelihood: \(M_B\) vs \(H_0\)

The main goal of this paper is to show how the result of the analysis is biased when using \(\chi^2_{H_0}\) instead of \(\chi^2_{M_B}\). To this end we will build and compare the following two likelihoods:

\[
\chi^2_{\text{tot},H_0}(\theta) = \chi^2_{\text{cmb}} + \chi^2_{\text{bao}} + \chi^2_{\text{sne}} + \chi^2_{H_0},
\]

\[
\chi^2_{\text{tot},M_B}(\theta) = \chi^2_{\text{cmb}} + \chi^2_{\text{bao}} + \chi^2_{\text{sne}} + \chi^2_{\text{M}_B}.
\]

Note that the number of data points is the same for both analyses.

In both cases the parameter vector is:

\[\theta = \{H_0, \Omega_{M_0}, w_s, z_i, M_B, \Omega_B, n_s\} \]

In particular, the posteriors are not marginalized analytically over \(M_B\) so that we can obtain the posterior on \(M_B\). However, it is often computationally useful to marginalize the posterior over \(M_B\) and in the next Section we present the corresponding formulas.

4.6 Posterior marginalized over \(M_B\)

In the case of the \(\chi^2_{\text{tot},H_0}\) of equation (12), it is well known that one can marginalize analytically the posterior over \(M_B\) (Goliath et al. 2001). As we will show below, this is possible also for the \(\chi^2_{\text{tot},M_B}\) of equation (13). For completeness we will present both cases.

4.6.1 Prior on \(H_0\)

Since, in this case, \(M_B\) enters only the SN likelihood, we will consider only the latter. The \(\chi^2\) function is:

\[
\chi^2_{\text{sne}} = (m_{B,j} - m_B(z_i)) \Sigma_{\text{sne},ij}^{-1} (m_{B,j} - m_B(z_i)),
\]

\[
y_i = m_B - \mu(z_i),
\]

where the apparent magnitudes \(m_{B,i},\) redshifts \(z_i\) and covariance matrix \(\Sigma_{\text{sne}}\) are from the Pantheon catalog (considering both statistical and systematic errors).

In the standard analysis one adopts an improper prior on \(M_B\) and integrate over the latter:

\[
\mathcal{L}^\text{marg}_{\text{sne}} \propto \int_{-\infty}^{+\infty} dM_B \exp\left[-\frac{1}{2} \chi^2_{\text{sne}}\right] = \int_{-\infty}^{+\infty} dM_B \exp\left[-\frac{1}{2} (S_2 - 2M_BS_1 + M_B^2 S_0)\right] \propto \exp\left[-\frac{1}{2} \left(S_2 - \frac{S_1^2}{S_0}\right)\right],
\]

where inconsequential cosmology-independent factors have been neglected and we defined the auxiliary quantities:

\[S_0 = V_1 \cdot \Sigma_{\text{sne}}^{-1} \cdot V_1^T,\]

\[S_1 = y \cdot \Sigma_{\text{sne}}^{-1} \cdot V_1^T,\]

\[S_2 = y \cdot \Sigma_{\text{sne}}^{-1} \cdot y^T,\]

where \(V_1\) is a vector of unitary elements. Equivalently, one can use the following new \(\chi^2\) function instead of \(\chi^2_{\text{sne}}:\)

\[
\chi^2_{\text{sne, marg}} = S_2 - \frac{S_1^2}{S_0},
\]

which does not depend on \(H_0\).

4.6.2 Prior on \(M_B\)

In the case of the \(\chi^2_{\text{tot},M_B}\) of equation (13), \(M_B\) enters the supernova likelihood and the \(M_B\) likelihood, which have to be integrated over at the same time:

\[
\mathcal{L}^\text{marg}_{\text{sne}+\text{loc}} \propto \int_{-\infty}^{+\infty} dM_B \exp\left[-\frac{1}{2} \left(\chi^2_{\text{sne}} + \chi^2_{\text{M}_B}\right)\right] = \exp\left[-\frac{1}{2} \left(S_2 - \frac{S_1^2}{S_0}\right)\right] \times \int_{-\infty}^{+\infty} dM_B \exp\left[-\frac{1}{2} \left(\frac{M_B - \frac{S_1}{S_0}}{S_0^{-1}} + \frac{(M_B - M_B^R)^2}{\sigma_{M_B}^2}\right)\right] = \exp\left[-\frac{1}{2} \left(S_2 - \frac{S_1^2}{S_0} + \frac{(S_1/S_0 - M_B^R)^2}{S_0^{-1} + \sigma_{M_B}^2}\right)\right],
\]

where again inconsequential cosmology-independent factors have been neglected.

Equivalently, one can use the following new \(\chi^2\) function instead of \(\chi^2_{\text{sne}} + \chi^2_{\text{M}_B}:\)

\[
\chi^2_{\text{sne}+\text{loc, marg}} = \chi^2_{\text{sne, marg}} + \chi^2_{\text{loc}},
\]

where

\[
\chi^2_{\text{loc}} = \frac{(S_1/S_0 - M_B^R)^2}{S_0^{-1} + \sigma_{M_B}^2}.
\]
Note that $\chi^2_{\text{loc}}$ does depend on $H_0$. In particular, it is interesting to consider the case of a diagonal covariance matrix $\Sigma_{\text{loc}} = \sigma^2 \mathbb{I}$, where $\mathbb{I}$ is the $n \times n$ identity matrix. In this case one has:

$$\chi^2_{\text{loc}} = \left(5 \log_{10} H_0 - M^B_{\text{loc}} + \frac{1}{2} \sum_i \left(m_{B,i} - 5 \log_{10} d_L(i)_{\text{loc}}\right)\right)^2 / \sigma^2 / n + \sigma^2_{M_B}.$$  \hspace{1cm} (23)

The term within round brackets does not depend on $H_0$ but is a cosmology-dependent intercept that affects the determination of $H_0$ via the local calibration $M^B_{\text{loc}}$. Finally, the error is just the sum in quadrature of the calibration and intercept errors.

5 RESULTS

Table 3 shows a comparison between the best fits relative to the analyses of equations (12) and (13). Our results are obtained using the numerical codes CLASS (Blas et al. 2011), MontePython (Audren et al. 2013) and getdist (Lewis 2019).

When using the prior on $H_0$ (Table 3, top), the $h\sigma$CDM model, with an extremely phantom $w_s \simeq -14$, features a significantly lower minimum $\chi^2$ as compared to the $w\sigma$CDM model. In particular, the disagreement with respect to the SH0ES determination of equation (7) is completely resolved. The phantom transition seems to have explained away the $H_0$ crisis. However, the best-fit $M_B$ is $5\sigma$ away from the prior on $M_B$ from Table (1), and this information is not included in the total $\chi^2$. This biases both model selection and the best-fit model. To better illustrate this point, we show in Figure 3 the Hubble rate and the inferred absolute magnitudes $M_{B,i} = m_{B,i} - \mu(z_i)$ for the best-fit $h\sigma$CDM model. Even though the best-fit $H_0$ agrees well with the $H_0$ prior (Figure 3, top), the inferred $M_{B,i}$ do not agree with the local prior on $M_B$ (bottom).

When, instead, the $\chi^2_{M_B}$ of equation (9) is adopted (Table 3, bottom), the $h\sigma$CDM model features the same best-fit $H_0$ of the $w\sigma$CDM model, both $3\sigma$ away from the SH0ES determination of equation (7). Moreover, the $h\sigma$CDM has a worse overall fit to the data as compared to $w\sigma$CDM. In other words, hockey-stick dark energy neither solves the $H_0$ crisis nor manifests any statistical advantage with respect to $w\sigma$CDM.

From these results it is clear what is the source of the Hubble crisis. CMB and BAO constrain tightly the luminosity distance-redshift relation and so the distance modulus $\mu(z)$. The Pantheon dataset constrains the supernova apparent magnitudes $m_{B,i}$. Consequently, CMB, BAO and SNe produce a calibration on $M_B$ which happens to be in strong disagreement with the local astrophysical calibration via Cepheids (see Figure 3 and Table 3). This disagreement was highlighted by Camarena & Marra (2020b, Figure 5) where the inverse-distance ladder technique was used to propagate the CMB constraint on $r_d$ to $M_B$ in a parametric-free way. Figure 4 shows how the constraint on $M_B$ from the
Table 3. Comparison between the best fits relative to the analyses of equations (12) (top) and (13) (bottom). A hat denotes the minimum χ^2. The Δχ^2 values are computed with respect to wCDM. The last two columns give the χ^2-distance (H_B^{cdm} - H_B^{cdm})/σ_H^R21 and (M_B^{cdm} - M_B^{cdm})/σ_M^R21 from the values given in equation (7) and Table (1) (Pantheon), respectively.

| Analysis with prior on H_0 | χ^2 cmb | χ^2 bias | χ^2 me | χ^2 tot | Δχ^2 | best-fit vector | distance from H_B^{cdm} | distance from M_B^{cdm} |
|-----------------------------|---------|----------|--------|---------|-------|-----------------|------------------------|------------------------|
| wCDM                        | 2.9     | 5.1      | 1030.0 | 7.8     | 1045.8| {69.6, 0.29, -1.08, ..., -19.39, 0.046, 0.97} | 2.8                    | 3.8                    |
| hscDM                       | 1.3     | 5.9      | 1027.7 | 0.3     | 1035.1| {72.5, 0.26, -14.4, 0.010, -19.42, 0.043, 0.97} | 0.5                    | 4.9                    |

6 CONCLUSIONS

In this paper we clearly show that a sudden phantom transition at very-low redshift cannot solve the >4σ disagreement between the local and high-redshift determinations of the Hubble constant. This point has been previously made by Benevento et al. (2020) in the contest of a sudden low-redshift discontinuity in the expansion rate, and by Lemos et al. (2019) who showed through an H(z) reconstruction that SN, BAO and r_d constraints do not allow for a higher expansion rate at low redshifts (see also the recent analysis by Efstathiou 2021).

Here, we single out the reason of this failure in solving the H_0 crisis: the supernova absolute magnitude M_B that is used to derive the local H_0 constraint is not compatible with the M_B that is necessary to fit supernova, BAO and CMB data, see Figures 3 and 4. Statistically, this incompatibility is taken into account in the analysis if one adopts the supernova calibration prior on M_B instead of the prior on H_0.

For completeness, we wish to summarize the three reasons why one should use the χ^2_M_B of equation (9) instead of the χ^2_M_B of equation (8):  4

(i) The use of χ^2_M_B avoids potential double counting low-redshift supernovae: for example, there are 175 supernovae in common between the Supercal and Pantheon datasets in the range 0.023 ≤ z ≤ 0.15, and, in the standard analysis, these supernovae are used twice: once for the H_0 determination and once when constraining the cosmological parameters. This induces a covariance between H_0 and the other parameters which could bias cosmological inference.

(ii) The supernova calibration prior on M_B is an astrophysical and local measurement. The determination of H_0 is instead based on a cosmographic analysis and it depends on a) its validity and b) its priors (SH0ES adopts q_0 = -0.55). While one can relax b) and obtain a joint H_0-q_0 prior (see Table 2), the cosmographic analysis may fail for models with sudden transitions such as hscDM. As χ^2_M_B is not based on a cosmographic analysis, it does not suffer from these issues.

(iii) Most importantly, the use of χ^2_M_B guarantees that one includes in the analysis the fact that M_B is constrained by the calibration prior of Table (1).

While, as shown by Table 3 and Figures 5 and 6, the conclusions for wCDM are not changed when χ^2_M_B is adopted, the use of χ^2_M_B becomes compelling when more exotic models are investigated; the use of χ^2_M_B can indeed lead to incorrect conclusions. Given that using χ^2_M_B does not add any statistical complexity to the analysis (see equation (21)), we encourage the community to adopt this prior in all the analyses. However, one should bear in mind that one must adopt the prior on M_B that corresponds to the supernova dataset that one wishes to adopt in their statistical analysis. These can be obtained using the code made available at github.com/valerio-marra/CalPriorSNla, where we

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3 Similar conclusions can be derived from the non-parameter inverse distance ladder analysis of Camarena & Marra (2020b), which shows that the calibration given by CMB and BAO to SN does not agree with the one by Cepheid distances, see Fig. 4.

4 Some of these points were previously raised by Camarena & Marra (2020a); Benevento et al. (2020).
On the use of the prior on the absolute magnitude of supernovae Ia

Figure 5. Marginalized constraints for the $w$CDM model from CMB, BAO, SNe and local observations. The two sets of contours show the analysis that adopts the prior on $M_B$ of equation (9) and the one that adopts the prior on $H_0$ of equation (8).

will keep an updated list of the $M_B$ priors that correspond to the latest supernova catalogs.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.
Figure 6. Marginalized constraints for hockey-stick dark energy ($h\sigma$CDM) from CMB, BAO, SNe and local observations. The two sets of contours show the analysis that adopts the prior on $M_B$ of equation (9) and the one that adopts the prior on $H_0$ of equation (8). As explained in the text, the latter analysis both biases model selection and distorts the posterior.

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