Large-Scale Bose-Einstein Condensation in a Vapor of Cesium Atoms at Normal Temperature (T=353K)
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Abstract
Large-scale BEC of cesium at T=353 K was first observed. Until now, scientists have applied magnetic fields and lasers, but never applied electric fields, and atoms are oriented at random, so observation of BEC is very difficult. Our innovation lies in the application of electric fields. We theoretically proved that alkali atom (include Cs) may be polar atom doesn't conflict with quantum mechanics. Variation of the capacitance with temperature offers a means of separating the polar and non-polar atom. Cs vapor was filled in cylindrical capacitor. Our experiment shows that Cs is polar atom because its capacitance is related to temperature. In the past, to realize the phase transition, ultralow temperature is necessary. But now we don’t require ultralow temperature, because we use the critical voltage Vc to achieve the phase transition. From the entropy S=Nk ln (2πme^2/kT) = 2πne^2/kT = 63volts. When V < Vc, S > 0; when V > Vc, S < 0, phase transition occurred. When V=350 volts, the capacitance decreased from C=1.97C0 to C = C0 (C0 is the vacuum capacitance), this result implies that almost all Cs atoms (more than 98.9%), like as dipoles, are aligned with the field. We create BEC with 1.928×1017 atoms, these atoms have the same momentum. Cesium material with purity 99.95% was supplied by Strem Chemicals Co., USA. Both BEC and superconductivity are condensed in the momentum space, therefore these two kinds of condensation can’t be observed with the naked eye. When superconductivity occurs, the resistance R≈0, a simple and direct method to observe superconductivity is to measure the resistance by voltmeter. Similarly, when BEC occurs, the electric susceptibility χ =ε/ε0, -1 = 0, a simple and direct method to observe BEC is to measure the capacitance by cylindrical capacitor. BEC is also a quasi-superconducting state.

Keywords: PACS numbers; Bose-Einstein condensation; Entropy; Polar atom; Cylindrical capacitor; Permanent dipole moment; Quasi-superconducting state

Introduction
BEC experiment is quite intriguing and a challenging task for the experimentalists. In 1925 Einstein pointed out that I maintain that, in this case, a number of molecules steadily growing with increasing density goes over in the first quantum state (which has zero kinetic energy) while the remaining molecules distribute themselves according to the parameter value λ=1. “A separation is effected; one part condenses, the rest remains a ‘saturated ideal gas’ (A=0, λ=1)” [1]. The famous physicist Abraham Pais emphasized that this is now called Bose-Einstein condensation. Now we estimate how many atoms in this part of Bose gas. In order to achieve BEC, the suitable density of Bose gas is 10^{13}–10^{15} (take 1/10 as "one part"). But what is about the actual situation? A typical example of BEC is provided by the dilute gases of alkali-metal atoms that can be prepared inside magnetic ion traps [2-6]. All stable alkali species---Li (2), Na (3), K (4), Rb (5), and Cs (6) ---have been condensed. The atoms, usually only 10^{-9} of them, can be trapped and cooled [2-6]. And therefore the previous condensate fraction is usually less than 10^{-}. This result shows that in the one billion alkali atoms which participating in the experiment, equivalent to only one atom was condensed. Strictly speaking, the results of previous experiments did not conform to Einstein’s predictions.

In the past, there are two ways to achieve phase transition: given atomic density, lower the temperature of Bose gas, making T < Tc = \frac{2\hbar^2}{kn^2} \left( \frac{n}{2.612} \right)^{1/2}; or given temperature, increase the density of Bose gas, making n > n_c (the critical density n_c = 2.612^{1/2} \frac{mkT}{2\hbar^2} ). But the latter method is also not successful, because the density of Bose gas can’t be increased indefinitely, and therefore the laser cooling is now widely used. What is the definition of BEC? "Bose-Einstein condensation is a macroscopic occupation of the ground state" [7]. Until now, scientists have applied magnetic field (used to trap atoms) and laser (used to cool atoms), but never applied electric field, despite the use of many advanced technology, condensed atomic number is still very small. This fact shows that according to the current method, the "macroscopic occupation" is very difficult to achieve.

Can we find a new way to get out of the current predicament? This article provides a new idea for the implementation of BEC. In BEC experiments, scientists never applied an electric field because they think that alkali atoms are non-polar atoms. If alkali atom is non-polar atom, has been verified by experiment! Answer: no! No such experimental results have been reported in the history of physics! In fact, this traditional concept is an untested hypothesis, and it has misled physicists all over the world. Variation of the capacitance with temperature offers a means of separating the polar and non-polar atom. If alkali atom is a non-polar atom, its capacitance should be independent of the temperature due to the nucleus located at the center of the electron cloud. We measured the capacitance at different temperatures. Our experiment proved that Cs atom is polar atom, and it becomes a dipole. When an electric field is applied, the dipoles tend to align with the field. In this case, the capacitance decreases, indicating that almost all Cs atoms have been aligned with the field. We create BEC with 1.928×10^{17} atoms, these atoms have the same momentum. Cesium material with purity 99.95% was supplied by Strem Chemicals Co., USA. Both BEC and superconductivity are condensed in the momentum space, therefore these two kinds of condensation can’t be observed with the naked eye. When superconductivity occurs, the resistance R≈0, a simple and direct method to observe superconductivity is to measure the resistance by voltmeter. Similarly, when BEC occurs, the electric susceptibility χ =ε/ε0, -1 = 0, a simple and direct method to observe BEC is to measure the capacitance by cylindrical capacitor. BEC is also a quasi-superconducting state.
to align with the field because of the torque it experiences. So a large-scale BEC at normal temperature can be observed.

Our previous research found that rubidium atom has a non-zero permanent dipole moment (PDM), \( d_{Rb} \geq 8.6 \times 10^{-9} \) e.cm, the saturation polarization of rubidium vapor has been observed \( [8,9] \). In particular, recent report showed that sodium is polar atom, BEC of Na atoms at normal temperature has been observed \( [10] \). This article has been rigorously peer reviewed for up to two months. This reviewer has published three theoretical articles about BEC in Physical Review, and these interesting theoretical results may be useful for the design of BEC experiment \( [11-13] \). This fact indicates that this peer review is authoritative and persuasive. Reviewer comments pointed out that “The author presented a good idea (using the critical voltage) to observe the BEC. Moreover, the author shows that the ultra-low temperature is not a necessary condition to verify the existence of BEC. This paper is interesting and certainly deserves publication in the journal”. This objective evaluation is a great inspiration to us, and encourages us to report the results of new BEC experiment of cesium atoms.

Let us take the iron filings as an example for further explanation. In the absence of magnetic field, iron filings are oriented at random, although use laser cooling technology, but the condensation of these iron filings is still very difficult. When a magnetic field is applied, however, the iron filings immediately arranged along tangent to the magnetic field lines like as little compass needles, and the condensation of these iron filings is easily observed. This example shows the previous research is captured in a misunderstanding because they never applied an electric field, and therefore scientists have missed out on this significant discovery.

**Theoretical Breakthrough**

Normally atoms do not have PDM because of their spherical symmetry; however our article proved that alkali atom forms an exception. \( \nu \) and \( n \nu \) states of alkali atoms are not degenerate, and therefore the expectation value of PDM is zero: \( \langle \psi_n \rangle = 0 \) (\( \psi \) is the dipole moment operator). However, many physicists strictly proved that the state function \( \langle \psi \rangle \) doesn't describe an individual particle but an ensemble of particles with the same energy \( [14-16] \).

"The expectation value is the average of repeated measurements on an ensemble of identically prepared systems, not the average of repeated measurements on one and the same system" \( [17] \). So \( \langle \psi_n \rangle \rightarrow 0 \) only means that the average PDM of large number of alkali atoms is zero, but doesn't mean that the PDM of individual alkali atom is zero.

The hydrogen atom is a typical example. "The shift in the energy levels of an atom in an electric field is known as the Stark effect" \( [14] \). Normally the effect is quadratic in the field intensity, which corresponds to an induced electric dipole moment (EDM) \( [14] \). The quadratic Stark effect occurs in general in all states. But LD Landau once stated that "the hydrogen atom forms an exception; here the Stark effect is linear in the field". "The energy levels of the hydrogen atom, unlike those of other atoms, undergo a splitting proportional to the field (the linear Stark effect)" \( [18] \). Evidently, the linear effect corresponds to a PDM. This effect showed that hydrogen atom (\( n=2 \)) has a PDM of magnitude \(-3a_B=1.59 \times 10^{-10} \) e.cm (\( a_B \) is the Bohr radius) \( [14,18] \). LD Landau once stated that "The presence of the linear effect means that, in the unperturbed state \( \psi_{2lm} \), the hydrogen atom has a dipole moment whose mean value is \( d=3a_B \) " \( [18] \). LI Schiff also stated that "It is also possible, as in the case of the hydrogen atom, that unperturbed degenerate states of opposite parities can give rise to a permanent electric dipole moment" \( [19] \). That is, \( d(\psi_{2l=0})=d(\psi_{2l=0})=0 \) and \( d(\psi_{2l=0})=0 \).

However, quantum mechanical calculations indicated that although \( \psi_2 \) is four-fold degenerate, the expectation value of PDM is zero: \( \langle \psi_2 \rangle = 0! \) That is, \( d(\psi_{2l=0})=d(\psi_{2l=0})=d(\psi_{2l=0})=0 \). Evidently, the zero result is inconsistent with above conclusion. Up to now, no quantum mechanical textbook explains this contradictory result. This fact shows that individual hydrogen atom (\( n=2 \)) has a non-zero PDM, but quantum mechanics can't obtain this non-zero result in any way. It convincingly proved that \( \langle \psi_{2l=0} \rangle = 0! \) only means that the average PDM is zero, but an individual hydrogen (\( n=2 \)) may have a non-zero PDM.

Recall that alkali atoms having only one valence electron in the outermost shell can be described as hydrogen-like atoms \( [20] \). So similar to the first excited state of hydrogen, the ground state of alkali atom may be polar atom doesn't conflict with quantum mechanics. A neutral alkali atom (include Cs) is or is not polar atom must be determined by experiments.

**A new formula of atomic PDM is obtained for the first time**

Experiments to search for PDM of atoms began half a century ago. In all experiments, they measured the spin resonance frequency \( \nu \) of individual atom by \( h=2\mu B \mu_0, \mu_0, \mu_0 \) and \( d \) is magnetic and electric dipole moments \( [21,22] \). But "experimental searches for PDM have so far yielded null results" \( [21] \). This fact shows that this formula is not successful, because it measured the microscopic quantity \( d \) by using another microscopic quantity \( \nu \).

However, measuring the average kinetic energy of a gas molecule using the temperature is easy: \( E_k=3kT/2 \). Similarly, we measure the PDM of an atom using the change of the capacitance is easy: \( d(C−C_0)/V \). This formula is easy to verify. The magnitude of the PDM is \( d=C−C_0 \). \( C_0 \) equals the percentage of Cs atoms lined up along an electric field, \( n \) is its density, \( S \) is the plate area. When the electric field is applied, the change of the charge of the capacitor is \( \Delta Q=C−C_0 V \). On the other hand, its volume is \( SH \), the total number of oriented atoms of the capacitor is \( n \). The number of layers of oriented atoms is \( H/\pi \). Because inside the Cs vapor the positive and negative charges cancel out each other, the polarization only gives rise to a net positive charge on one side of the capacitor, and a net negative charge on the opposite side. Therefore \( \Delta Q=SH \), \( n L(a)=n S(n L(a)) \). The number of electric dipole elements is \( n(C−C_0) \).

Our Innovations

**According to W. Ketterle's standard, our experiment is a truly ideal BEC**

Wolfgang Ketterle, in the Nobel Prize winning paper, proposed the objective standard of an ideal BEC: “An ideal Bose condensate shows a macroscopic population of the ground state of the trapping potential. This picture is modified for a weakly interacting Bose gas” \( [3] \). This standard emphasizes that the ground state is the ground state of the trapping potential, which is very important correction. Because this description doesn't involve temperature, and therefore this standard is also suitable for the evaluation of our experiments. Cesium has a non-zero PDM, and it becomes an electric dipole. When an electric field is applied, the dipoles tend to orient in the direction of the field because of the torque it experiences. The magnitude of the torque is \( \tau=\mu B \sin\theta \), and \( \theta \) is the angle between \( d \) and \( E \). When \( \theta=0 \), the dipole is in equilibrium. In our experiment, the trapping potential is the electric
potential energy of Cs atoms: $E = -dE \cos \theta$. When $\theta = 0$, the potential energy is a minimum, which indicates that the dipole is oriented parallel to the field. But atomic collisions tend to disarrange the dipoles. When $V > V_c$, many atoms are in random directions, this state has high entropy $S > 0$; when $V \gg V_c$, the atoms become aligned with the field, this state has low entropy $S < 0$, phase transition occurred. When $V \gg V_c$, $C \approx C_v$, this result implies that the alignment would be perfect, these atoms have the same momentum, this is condensation in momentum space. In effect, BEC is the perfect alignment of bosons. So BEC at normal temperature can be observed. When $V > V_c$, Bose condensate contained up to $1.928 \times 10^{17}$ atoms really achieved “macroscopic population”. The atoms condensed into the ground state of trapping potential, because their electric potential energy is a minimum along the field. Therefore, Ketterle’s standard proved that although we didn’t use laser cooling atoms, but our experiment is an ideal BEC. There are few groups that use electric fields to cool atoms. This article will not discuss this situation.

Variation of the capacitance with temperature offers a means of separating the polar and non-polar atom experimentally. The classical electrodynamics textbook plotted the relationship between $\chi_e$ and $1/T$ [23].

For the polar atom $\chi_e=A + B/T$, for non-polar atom $\chi_e=A$ (1)

Where, A and B is constant [23]. If Cs is polar atom, the form $\chi_e=A + B/T$ should be expected. As a contrast, the capacitance of Hg has been measured, but its capacitance is independent of temperature, Hg is non-polar atom.

Our experimental type is quantitative

The condensate fraction is a very important physical quantity in BEC, but previous experiments didn’t provide a suitable formula, and they are qualitative. We strictly proved that the Langevin function $L(a)$ equals the condensate fraction, and it can be expressed as $L(a) = \frac{1}{\pi} \ln \left( \frac{\pi + a}{\pi - a} \right)$, where $\eta$ is the capacitance constant, and $(C - C_v)$ is the change of the capacitance. For example, when the voltage $V = 350$ volts, $\eta = 192$ pF, $C - C_v = 2$ pF and the coefficient $a = 95$, we obtain $L(a) = 0.9896$. These facts indicate that our experiments not only don’t conflict with their experiments, but also this experimental type is quantitative.

The most striking characteristic of BEC is that BEC is condensed in momentum space

Einstein first noticed that Bose gas would condense to the lowest energy state. However, the result of the condensation doesn’t produce crystals, this fact shows that BEC doesn’t occur in the position space but occurs in the momentum space. The term ‘condensation’ often implies a condensation in space, as when liquid water condenses on a cold window in a steamy bathroom. However, for Bose-Einstein condensation it is a condensation in $k$-space, with a macroscopic occupation of the lowest energy state” [7]. Note that the wave vector $k$ equals the momentum $p$ divided by Planck constant $h$, and therefore BEC is a condensation in momentum space. Superconductivity is also the condensation in the momentum space because the Cooper pairs act like bosons, therefore these two kinds of condensation can’t be observed with the naked eye. When superconductivity occurs, the resistance $R=0$ (the resistance of vacuum is zero). Although BCS theory is difficult, however, a relatively simple and direct observation method is to measure the resistance by voltammetry. Similarly, when BEC occurs, the electric susceptibility $\chi_e \propto C/C_v \approx 0$ or $C \approx C_v$, a simple and direct observation method is to measure the capacitance. BEC is quasi superconducting state. Note that the resistance and capacitance are two macroscopic electrical quantities, and their changes are the decisive evidence of these two kinds of condensation. When they($R$ or $C$) are reduced to close to the vacuum value, these two kinds of condensation occur. This fact fully reflects the harmony and unity of nature.

The entropy of a system is a measure of the disorder of molecular or atomic motion. No doubt, it is the most important concept in BEC. Consider a system composed of $N$ cesium atoms which are placed in an electric field $E$, $\theta$ is the angle between $d$ and $E$. Note that the collision between Cs atoms is always through their mass centers, and therefore the nucleus has no contribution to the rotational energy of the atom. When orientation polarization occurs, its rotational energy can be neglected. The potential energy of Cs atom in the field can be expressed as $\varepsilon = -dE \cos \theta$. Unlike the orientation quantization of magnetic moment, the orientation of Cs atoms can be changed continuously in the field. Note that $\beta = 1/kT$ and the chemical potential $\mu = 0$[7], the partition function is given by

$$Z = \int \frac{\frac{1}{\alpha a} \sin \beta \theta}{\beta a} - \int \frac{e^{-a+b} \sin \beta \theta}{\beta a} - \int \frac{e^{-a+b} \sin \beta \theta}{\beta a} = 2 \int \frac{e^{-a+b} \sin \beta \theta}{\beta a} d\theta$$

The entropy is given by $S = N_k \ln Z + \frac{a}{kT} \ln Z$ [7]. Let the coefficient $a = dE/kT = dV/kTH$, we obtain

$$S = N_k \ln \left( \frac{e^{-a+b}}{a} \right) \left( a - a \coth a \right)$$

When $a = dE/kT \gg V$ or $V \approx 37$ volts ($a \approx 10$), $e^{-a+b} = 0$ and $\coth a = 1$, we obtain a simplified formulas

$$S = N_k \ln 2 \pi e / a$$

The critical coefficient is $a = 2 \pi e \approx 17.08$. The formula contains two fundamental constants in nature ($\pi = 3.14159$ and $e = 2.71828$), it reflects the objective laws of BEC.

**Experiment and Interpretation**

The preparatory experiment: we measured the density of Cs vapor. The longitudinal section of the apparatus is shown in Figure 1. This closed glass container resembles a Dewar flask in shape. Its internal and external diameters are $D_1 = 5.6$ cm and $D_2 = 8.12$ cm respectively. The external and internal surfaces was paste with aluminum foil, also can be plated with silver, they form the outer and inner electrode (Figure 1) [10]. Their length is $L = 33.4$ cm. This capacitor is equivalently connected by two capacitors, one is called $C_v$ and can be directly measured. The magnitude of $C_v$ was 0.5%, the measuring frequency was 800Hz, and the definition was 0.1

![Figure 1](image-url)
pF ($C < 200$ pF), $1$ pF ($200$ pF$≤ C ≤ 2$ nF) or $10$ pF ($2$ nF$≤ C < 20$ nF). In order to remove impurities such as oxygen, when the capacitor was empty, it was pumped to vacuum pressure $P=10^{-10}$ torr for $20$ hours. The vacuum capacitance is $C'_{0}=\left(54.0 \pm 0.1\right)$ pF. Next, a small amount of Cs material (about $10$ grams) was put into the container, and it is again pumped to $P ≤ 10^{-10}$ torr, and then it is sealed. We obtain a cylindrical glass capacitor filled with Cs vapor and surplus liquid cesium (Figure 1) [10]. We put the capacitor into a temperature-control stove, raise its temperature slowly, and keep it at $T=473$ K for $6$ hours. It means that these results are obtained under the saturated vapor pressure. We measured the capacitance is $C_{0}=(5140 \pm 10)$ pF. Note that $P=10^{-6.949}$ psi ($473 \leq T ≤ 623$ K, $1$ psi$=6894.8$ Pa) is the saturated vapor pressure of Cs atoms [24]. We obtained $P=481.3$ Pa at $T_{0}=473$ K. From $(\Delta T/T)^{2}$, due to $\Delta T=0.5$ K and $\Delta T/T ≤ 0.001$, $\Delta P=P(T_{0}+\Delta T)-P(T_{0})=9.6$ Pa and $\Delta P/P≈0.02$, so $\Delta n/n ≤ 0.03$. Considering all systematic error, we have that $\Delta n/n ≤ 0.03$, and the density of Cs vapor is $n_{0}=\left(7.37 ± 0.22\right) × 10^{16}$ cm$^{-3}$.

The first experiment: we measured capacitances of Cs and Hg vapor at different temperatures. A glass cylindrical capacitors fill with cesium at a fixed density $n_{0}$, without liquid cesium, and $n_{0} << n_{s}$. Another cylindrical capacitors fill with Hg vapor at a fixed density $n_{0}$. Their capacitances were still measured by the digital meter, the vacuum capacitance is $C'_{0}=58.6$ pF (for Cs) and $C'_{0}=63.9$ pF (for Hg). Next, a small amount of Cs or Hg material was put into the two capacitors. In order to ensure the number of atoms remained constant, we adopted an ingenious technique (Figure 2) [10]. After fill with Cs or Hg vapor, the capacitances are $C_{1}=64.1$ pF (for Cs) and $C_{0}=64.1$ pF (for Hg) respectively. We measured their capacitances at different temperatures. As was expected, the capacitance of Hg vapor, $C_{0}=64.1$ pF, remains constant at different temperatures (i.e., $B=0$) and $\chi_{e}=A=0.003$. But the capacitance of Cs vapor decreases gradually from $112.7$ pF to less than $96$ pF, and $\chi_{e}=A/B/T!$

The Figure 3 shows the two experimental results. Table 1 gives the electric susceptibility of cesium vapor. By least-square method we obtain $B=282.3$ K and $A=0.007$, because $(\Delta \chi_{e}/\chi_{e})^{2}=(\Delta B)/B)^{2}+(\Delta T/T)^{2}=0.002$, so $(\Delta B)/B<0.004$. They can be expressed as follows

\[ \chi_{e}=0.007 \pm 2.823/T \]  

The two results formed a sharp contrast because Cs atom is polar atom but Hg atom is non-polar.

| T(K)     | 308 | 326 | 345 | 357 | 377 | 392 | 425 | 448 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 1/T(T)   | 3.2468 | 3.0675 | 2.8965 | 2.8011 | 2.6525 | 2.5510 | 2.3529 | 2.2321 |
| C(pF)    | 112.7 | 109.6 | 106.9 | 105.4 | 102.9 | 101.3 | 98.0 | 95.8 |
| $\chi_{e}$ | 0.9232 | 0.8703 | 0.8242 | 0.7986 | 0.7560 | 0.7287 | 0.6724 | 0.6348 |

Table 1: The electric susceptibility $\chi_{e}$ of cesium vapor at different temperature T.

\[ \chi_{e}=0.007 \pm 282.3/T \] for mercury vapor $\chi_{e}=0.003$ (5)

The second experiment: we measured the capacitance of Cs vapor at different voltages under a fixed density $n_{0}$ and $T=353$ K. The vacuum capacitance of the apparatus is $C_{0}=(66.0 \pm 0.1)$ pF, where $H_{e}=6.8$ mm. The Figure 4 shows the experimental method [10]. $C$ is the measured capacitor, which filled with cesium vapor. $C_{0}$ is a reference capacitor. Two signals $V_{c}(t)=V_{co} \cos \omega t$ and $V_{s}(t)=V_{so} \cos \omega t$ were measured by a two channel digital oscilloscope (Tektronix TDS 210 USA). From Figure 4, we have $V_{2} \gg V_{c}$, $V_{s} \gg V_{c}$, $V_{c} \approx C_{0}$, and $V_{2} \approx C_{0}$. The voltages $V_{c}$ could be adjusted from zero to $800$ V. The frequency could be adjusted from one to $10^{6}$ Hz. The measurement was started in $0.01$ volt. When $V_{c} \approx V_{co}$, $C=130.0$ pF is approximately constant. When $V_{c} \approx V_{co}$, the peak difference of the two signals waveform is large, this image means that $C \gg C_{0}$ (Figure 5). When $V_{c}=V_{co}-350$ volts $\gg V_{c}$, $V_{c}=68.0$ pF $\approx C_{0}$. This oscilloscope shows that the peak of the two signals is very close and they almost overlap, this precious image means that large scale BEC at normal temperature has been observed (Figure 6). If almost all the dipoles dipole in a gas were to line up with an external electric field, this effect is called the saturation polarization. The experimental $C$–$V$ curve shows that the saturation polarization of Cs vapor is easily observed when $V_{s} \gg V_{c}$ (Figure 7) [10]. Table 2 gives the measured values of Cs
vapor at different voltages.

RP Feynman, he was awarded the 1965 Nobel Prize, once stated that “The electric field tends to line up the individual dipoles to produce a net moment per unit volume. If all the dipoles in a gas were to line up, there would be a very large polarization, but that does not happen” [25]. Our experiment confirmed his prediction, and it tells us that the saturation polarization of Cs vapor is an unexpected discovery. When the peak of the two signal waveforms is very close, and the two waveforms are almost overlapped, this image means the BEC occurs. The capacitance of the two signal waveforms is very close, and the two waveforms are polarization of Cs vapor is an unexpected discovery. When the peak experiment confirmed his prediction, and it tells us that the saturation would be a very large polarization, but that does not happen” [25].

The electric field tends to line up the individual dipoles to produce a net moment per unit volume. If all the dipoles in a gas were to line up, there would be a very large polarization, but that does not happen. 

The measured values of Cs vapor at different voltages $T=353K$.

| $V$ (volt) | 0.01 | 0.3 | 7.1 | 63 | 173 | 232 | 350 | $\infty$ |
|-----------|------|-----|-----|----|-----|-----|-----|-------|
| $C$ (pF)  | 130  | 130 | 118 | 76.6 | 70 | 69 | 68 | 66 |
| $\chi$    | 0.9697 | 0.9697 | 0.7879 | 0.1606 | 0.0606 | 0.0454 | 0.0303 | 0 |
| $P_t$ (cm$^3$) | 7.9 $\times$ 10$^6$ | 2.4 $\times$ 10$^6$ | 4.6 $\times$ 10$^6$ | 6.2 $\times$ 10$^6$ | 8.5 $\times$ 10$^6$ | 8.58 $\times$ 10$^6$ | 6.83 $\times$ 10$^6$ | 8.72 $\times$ 10$^6$ |

Table 2: The measured values of Cs vapor at different voltages $T=353K$.

The electric field acting on a molecule or atom in a gas is almost the same as the external field $E$ [23]. For polar molecules or atoms, $\chi = n\alpha + n d_0 L(a)/\varepsilon_0 E$, where $a=\frac{d_0}{V/kT}$. Langlev function $L(a)=[(e^{-a}+e^{-a})]/(e^{-a}+e^{-a})-1/a=\coth a-1/a$. $L(a)$ equals the average value of $\cos \theta$ [26]:

$$L(a) = \langle \cos \theta \rangle = f \int \cos \theta \exp(d_0 E \cos \theta / kT) \sin \theta d\theta,$$

$$f = \left[ \int \exp(d_0 E \cos \theta / kT) \sin \theta d\theta \right]^{-1}$$

Where, $f$ is normalized constant, $\theta$ is the angle between $d_0$ and $E$ [25]. The electric polarizability of cesium is $\alpha=59.6 \times 10^{-30}$ m$^3$ [27], the density $n<8.0 \times 10^{22}$ m$^{-3}$, and induced susceptibility $\chi = n\alpha < 4.8 \times 10^6$ can be neglected. We obtain $\chi = n d_0 L(a)/\varepsilon_0 E$. Where $a=\frac{d_0}{\varepsilon_0 E}$, and $\chi = n \frac{d_0}{\varepsilon_0 E}$, this is the familiar Langevin formula. The polarization $P$, the average dipole moment per unit volume, is defined as

$$P = \chi E_0 \varepsilon_0 = n d L(a)$$

Formula (6) and (7) are clearly indicated that $L(a)$ equals the condensing fraction of BEC. When $a \gg 1$, $L(a) \approx 1$ and $P_{\text{max}} = n d$, it implies the saturation polarization. Note that $E=V/H$ and the formula of the parallel-plate capacitor $\varepsilon_0 = C_0 H/S$, from $\chi = n d L(a)/\varepsilon_0 E$, we obtain

Condensate fraction: $L(a) = n(C - C_0)/\eta$ or $C = n L(a)/a + C_0$

Where, $\eta = \sqrt{n d^2 / kT}$ is the capacitance constant. When $V=0.3 V$, $a=1$, $L(a)=1/3$ and $\eta = 3(C_0 - C_{C_{\text{max}}}) = 192$ pF. When $V=350 V$, $C_{C_{\text{max}}}=2.0$ pF, $a=\eta L(a)/C_1 - C_{C_{\text{max}}}=95$, $L(95) = 0.9896$. The critical voltage is $V_c = \frac{2n e}{l a} = 62.9 V$.

Table 2: The measured values of Cs vapor at different voltages $T=353K$.
parallel-plate capacitor \( \varepsilon_0 = C_0 H / S \), so the cylindrical capacitor must be regarded as an equivalent parallel-plate capacitor with the plate area \( S = C_0 H / \varepsilon_0 \). In the three experiment, the equivalent area is \( S'_c = C'_c H / \varepsilon_0 = 5.855 \times 10^{-5} m^2 \) and \( S'_c = C'_c H / \varepsilon_0 = 5.069 \times 10^{-5} m^2 \). Since the density \( \eta_s \) is unknown, from equation (8) we obtain \( \eta_s = (C'_c - C_c) V_0 L(\alpha) S'_c n_0 / (C'_c - C_c) V_0 L(\alpha) S'_c n_0 = 5.652 \times 10^4 \text{ cm}^{-3} \), which is a very small number. Therefore, the PDM of an atom would violate time reversal symmetry, it must have two characteristics: the first is very small, \( d \ll 10^{-20} \text{ cm} \), and the second is arises from the nuclear spin.

However, the linear Stark effect of hydrogen atom provides a new example. This effect showed that the hydrogen atom \((n=2)\) has a PDM of magnitude \(-3ea_0=1.59 \times 10^{-9} \text{ e cm}\). Since the nuclear spin was completely irrelevant to the calculation of the PDM, and Bohr radius \((a_0=0.53 \times 10^{-10} \text{ cm})\) is far greater than the nuclear radius \((r_n=10^{-11} \text{ cm})\), so this PDM has nothing to do with the nuclear spin, and only arises from the asymmetrical charge distribution of hydrogen atom. LD Landau once stated that the hydrogen atom has a dipole moment whose mean value is \( d = -3ea_0 \). This is in accordance with the fact, in a state determined by parabolic quantum numbers, the distribution of the charges in the atom is not symmetrical about the plane \( z = 0 \) [18]. So similar to the first excited state of hydrogen atom, the PDM of Cs atom doesn't arise from the nuclear spin but from asymmetrical charge distribution, and it doesn't violate time reversal and parity symmetry [21, 22].

If cesium atom has a large PDM, why its linear Stark effect has not been observed?

This is a challenging question. As two concrete examples, first let us deal with the fine structure and the linear Stark shifts of the hydrogen \((n=2)\). The wavenumber of the fine structure of the hydrogen \((n=2)\) is only \(0.33 \text{ cm}^{-1}\) for the Ha lines of the Balmer series, where \(\lambda=656.3 \text{ nm}\). The splitting is only \(\Delta\lambda=3.33 \times (656.3 \times 10^{-7})=0.014 \text{ nm}\), therefore the fine structure is difficult to observe [28]. The linear Stark shift of the hydrogen \((n=2)\) is proportional to the field intensity: \(\Delta\lambda d = E_c \times 1.59 \times 10^{-8} \text{ e cm}\). When \(E=10^5 \text{ V/cm}\), \(\Delta\lambda = 1.59 \times 10^{-8} \text{ e cm}\), this corresponds to a wavenumber of 12.8 \text{ cm}^{-1}. So the linear Stark shifts of \(\Delta\lambda = E_c \times 1.59 \times 10^{-8} \text{ e cm}\) are easily observed [28]. However, when \(E = 350 \text{ V}\), the most field intensity is \(E_{\text{max}}=V_0/H=515 \text{ V/cm}\), this corresponds to an extremely small \(10^{-6} \text{ e cm}\). When the external electric field increases up to \(E_{\text{max}}\), almost all the Cs atoms (more than 98.9%) were to line up along the field, Cs vapor no longer absorb energy, this Stark effect will not occur. If the PDM of cesium is \(d = 1.54 \times 10^{-8} \text{ e cm}\), the most splitting of the energy levels is \(\Delta\lambda_{\text{max}}=d E_{\text{max}} \times 7.93 \times 10^{-8} \text{ eV}\), this corresponds to a wavenumber of \(\Delta\lambda_{\text{max}} = d E_{\text{max}} \times 7.93 \times 10^{-8} \text{ eV}\), which is not been observed. If the PDM of cesium is \(d = 1.54 \times 10^{-8} \text{ e cm}\), the most splitting of the energy levels is \(\Delta\lambda_{\text{max}}=d E_{\text{max}} \times 7.93 \times 10^{-8} \text{ eV}\), this corresponds to a wavenumber of \(\Delta\lambda_{\text{max}} = d E_{\text{max}} \times 7.93 \times 10^{-8} \text{ eV}\), which is not been observed. Therefore, the PDM does not change. Therefore, the PDM of an atom would violate time reversal symmetry. By the CPT theorem it also implies a violation of CP symmetry [21]. A representative result as follows: \(d(\text{Hg}) = 0.49 \pm 1.29 \text{ cm} \) [22]. In short, if the PDM of atom violates time reversal symmetry, it must have two characteristics: the

**Table 3:** A. L(a), N, and S of Cs vapor at different voltages \((V)\) \(T=353K\).

| V(volt) | 0.01 | 0.30 | 7.1  | 63   | 173  | 232  | 350  | ~  |
|---------|------|------|------|------|------|------|------|---|
| C(p F)  | 130  | 130  | 118  | 76.6 | 70.0 | 69.0 | 68.0 | 66.0|
| a       | 2.7×10^{-2} | 0.0814 | 1.93 | 17.1 | 47.0 | 63.0 | 95.0 | ~ |
| L(a)    | 9×10^{-4} | 0.02714 | 0.5249 | 0.9415 | 0.9787 | 0.9841 | 0.9896 | 1.0 |
| N (10^{17}) | 0.0018 | 0.0529 | 1.023 | 1.834 | 1.906 | 1.917 | 1.928 | 1.948 |
| S (Nk)  | 2.531 | 2.530 | 2.076 | -1.20×10^{-8} | -1.012 | -1.305 | -1.716 | -lnw |

**Table 4:** A sharp contrast between previous and our BEC experiment.

| System | Critical condition of BEC | Number of condensed atoms | Most atoms are in BEC | Macroscopic occupation of ground state | A large-scale BEC |
|--------|---------------------------|---------------------------|---------------------|-------------------------------------|------------------|
|        | T << Tc                   | N ≤ 10^4-10^6            | V >> Vc              | No                                  | No               |
|        | V >> Vc                   | Nc = 1.928×10^{17}       | V >> Vc              | Yes                                 | Yes              |

**Discussion**

If cesium atom has a non-zero PDM, why it does not violate the time reversal and parity symmetry?

According to quantum theory, an atom in its ground state at most has an extremely small PDM, \(d \ll 10^{-30} \text{ cm} \), it points along the nuclear spin axis and arise mainly from the nuclear spin [19, 21, 22]. Under time reversal the direction of the spin changes, while the direction of the PDM does not change. Therefore, the PDM of an atom would violate time reversal symmetry. By the CPT theorem it also implies a violation of CP symmetry [21]. A representative result as follows: \(d(\text{Hg}) = 0.49 \pm 1.29 \text{ cm} \) [22]. In short, if the PDM of atom violates time reversal symmetry, it must have two characteristics: the first is very small, \(d \ll 10^{-20} \text{ cm} \), and the second is arises from the nuclear spin.

However, the linear Stark effect of hydrogen atom provides a new example. This effect showed that the hydrogen atom \((n=2)\) has a PDM of magnitude \(-3ea_0=1.59 \times 10^{-9} \text{ e cm}\). Since the nuclear spin was completely irrelevant to the calculation of the PDM, and Bohr radius \((a_0=0.53 \times 10^{-10} \text{ cm})\) is far greater than the nuclear radius \((r_n=10^{-11} \text{ cm})\), so this PDM has nothing to do with the nuclear spin, and only arises from the asymmetrical charge distribution of hydrogen atom. LD Landau once stated that the hydrogen atom has a dipole moment whose mean value is \(d = -3ea_0 \). This is in accordance with the fact, in a state determined by parabolic quantum numbers, the distribution of the charges in the atom is not symmetrical about the plane \(z = 0\) [18]. So similar to the first excited state of hydrogen atom, the PDM of Cs atom doesn’t arise from the nuclear spin but from asymmetrical charge distribution, and it doesn’t violate time reversal and parity symmetry [21, 22].
A sharp contrast between previous and our BEC experiment

In order to illustrate the original innovation of our experiments, Table 4 gives a detailed comparison between our and previous BEC experiments [2-6]. From the formula of PDM, if \((C – C_0) = 0\), and is bound to get: \(d = 0\). But in the past few decades, scientists have never measured the capacitance of the alkali atoms, so they missed this significant discovery.

Rigorous mathematical proof that BEC of cesium vapor is bound to occur

From equation (8) \(C = \eta L(a)/a + C_0\), we construct a new function

\[ f(a) = L(a)/a = [(e^a + e^{-a})/a (e^a - e^{-a})] - 1/4a \]

Now we find the inflection point of the function \(f(a)\). From the second order derivative of this function is zero, we obtain \(f''(a) = [(2e^{2a} - 2e^{-2a} + 8ae^a + 8ae^{-a}) / a(e^a - e^{-a})] - 4/(a^3) = 0\). From \(f''(a) = 1/(9.296812010^3) < 0\) and \(f''(a) = 1/(9.296814010^3) > 0\), we obtain its inflection point is \(a = 1.9296813 ≈ 1.93\). The voltages of the inflection point is \(V_p = 7.1V\) and \(lg V_p = 0.85\). By contrast with Fig.4, it is clear that our polarization equation Eq.(8) is correct. This result shows that when \(lg V_{CD} < 0.85\), the C–V curve is upper convex; when \(lg V_{CD} > 0.85\), the curve is down convex. This result shows that when the voltage increases to thousands of volts, C will inevitably approach \(C_0\).

A discussion of many interesting questions, such as the definition of Boltzmann constant, can be found in the reference 10.

Conclusions

In theory, \(\langle \psi | e^{-er} | \psi \rangle = 0\) only means that the average PDM of a large number of alkali atoms is zero, but doesn’t mean that the PDM of individual alkali atom is zero. Despite 6s and 6p states of cesium are not degenerate, but Cs may be polar atom doesn’t conflict with quantum mechanics because it is hydrogen-like atom.

All kinds of alkali atoms are non-polar atom, which is an untested hypothesis, and we must test it by experiments. We measured the capacitance at different temperatures, our experiment proved that Cs atom is polar atom, and it becomes a dipole. But in the past, scientists have never measured the capacitance of alkali atoms, so they missed this significant discovery.

BEC has three main features: BEC is a macroscopic occupation of the ground state; BEC is a condensation in momentum space; Bose gas would undergo a phase transition. Our experiments are fully in line with these three main features, so although we have not used laser cooling techniques, our experiments are an ideal BEC.

Ultra-low temperature is in order to make Bose gas phase transition, and we use the critical voltage \(V_c\) to describe phase transition, and therefore ultra-low temperature is not necessary. Because there are only a handful of laboratories in the world that can achieve ultra-low temperatures, it limits the majority of scientists involved in the study. This new technology will completely change the current situation, and it will be used and praised by the vast majority of scientists.

The entropy of a system is a measure of the disorder of molecular or atomic motion. No doubt, it is the most important concept in BEC. This formula, \(S = Nkln2\pi\hbar/a\), contains two fundamental constants in nature \((\pi = 3.14159\) and \(e = 2.71828\)), it reflects the objective laws of BEC.

The presence of the inflection point proved that the saturation polarization of Cs vapor is inevitable, and therefore BEC of Cs vapor is also a certainty. Including the introduction of entropy, this is the two successful examples of applying mathematics to explain abstruse physical phenomena.

Both BEC and superconductivity are condensed in the momentum space. When the resistance \(R\) or the capacitance \(C\) was reduced to close to the vacuum value, these two kinds of condensation occur. So BEC is also a quasi-superconducting state. This fact fully reflects the harmony and unity of nature.

Our experiments are easily repeated in other laboratories, because the details have been described in this paper. Once scientists completed these measurements, they will obtain the same results as our experiments. They will discover that a large-scale BEC of cesium atoms is easily observed when an electric field is applied!

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