Twists of Plücker coordinates as dimer partition functions

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Joint work with J. S. Scott.

The homogeneous coordinate ring of the Grassmannian has a cluster structure defined in terms of planar diagrams known as Postnikov diagrams. The cluster corresponding to such a diagram consists entirely of Plücker coordinates. We introduce a twist map on the Grassmannian related to the Berenstein–Zelevinsky twist, and give an explicit Laurent expansion for the twist of an arbitrary Plücker coordinate, in terms of the cluster variables associated with a fixed Postnikov diagram. The expansion arises as a (scaled) dimer partition function (i.e. matching polynomial) of a weighted version of the bipartite graph dual to the Postnikov diagram, modified by a boundary condition determined by the Plücker coordinate.