Structure of Be hyper isotopes

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Abstract. The low-lying level structure of ⁹ΛBe and ¹¹ΛBe was studied with the antisymmetrized molecular dynamics for hypernuclei (HyperAMD). It is found that the first excited state ¹/₂⁺ of ⁹Be is shifted up by about 700 keV in ¹⁰ΛBe. In ¹²ΛBe, the parity-inverted ground state of ¹¹Be is reverted by adding the Λ hyperon. In this article, we discuss the reason for these structure changes and the deformation of ¹⁰ΛBe and ¹²ΛBe.

1. Introduction
We focus on the ground and low-lying states of ¹⁰ΛBe and ¹²ΛBe, and discuss the changes of the level structure caused by the Λ hyperon. Since Be isotopes are composed of 2α cluster structure with surrounding neutrons, the ground and low-lying excited states have various deformation depending on the orbit of the extra neutron(s). For example, in ⁹Be, the deformation of the first excited state ¹/₂⁺ is larger than that of the ground state ³/₂⁻. In ¹¹Be, the ground state is ¹/₂⁺ which is inconsistent with ordinary shell-model-picture where the seven neutrons may have a ¹/₂⁻ state as the ground state. One of the reasons for the parity inversion is molecular-orbit structure of ¹¹Be [1]. In the ¹/₂⁺ state, the extra neutrons occupy the σ orbit around the 2α cluster structure and enhance the 2α clustering. The deformation of the ¹/₂⁻ state is smaller than the ¹/₂⁺ state, because the extra neutrons occupy the π orbit and reduce the clustering. By adding a Λ hyperon to such nuclei, it is expected that the binding energy of Λ is different depending on the deformation, and the level structure will be changed. In this article, we discuss the difference of the Λ binding energy depending on nuclear deformation and the changes of the level structure by Λ. To investigate it, we have applied the antisymmetrized molecular dynamics for hypernuclei (HyperAMD [2]) to ¹⁰ΛBe and ¹²ΛBe.

2. Theoretical Framework of HyperAMD
The Hamiltonian used in this study is given as,

\[ \hat{H} = \hat{T}_N + \hat{T}_\Lambda - \hat{T}_g + \hat{V}_{NN} + \hat{V}_{AN} + \hat{V}_C \]  

(1)

where \( \hat{T}_N, \hat{T}_\Lambda \) and \( \hat{T}_g \) are the kinetic energies of the nucleons, the Λ hyperon and the center-of-mass motion. We have used the Gogny D1S as an effective nucleon-nucleon interaction \( \hat{V}_{NN} \), that has been successfully applied to the stable and unstable nuclei. The YNG-NSC97f [3] and improved-YNG-NF [4] interactions have been employed as the Λ-nucleon interaction \( \hat{V}_{AN} \). Since the YNG interactions depend on the nuclear density through the Fermi momentum \( k_F \), we adopted \( k_F = 0.987 \text{ fm}^{-1} \) and \( k_F = 0.973 \text{ fm}^{-1} \) for ¹⁰ΛBe and ¹²ΛBe, respectively. The Coulomb interaction is approximated by the sum of seven Gaussians.
The total wave function of the hypernuclear system is given as

$$|\Psi\rangle = \sum_n C^{(n)}|\psi^{(n)}_n(\vec{r}_\Lambda)\rangle \otimes \frac{1}{\sqrt{A!}} \det[\phi(\vec{r}_j)].$$  \hspace{1cm} (2)

Note that the single particle wave function of $\Lambda$ is represented by a superposition of the Gaussian packets to describe various orbits of $\Lambda$. The configuration of the nucleons and the $\Lambda$ hyperon is optimized through the frictional cooling method.

After the variational calculation, we perform the angular momentum projection and the generator coordinate method (GCM) calculation to obtain the excitation spectra.

3. Results and Discussions

3.1. Level structure of $^{10}\text{Be}$

![Figure 1. Calculated spectra of $^{9}\text{Be}$ (a) and $^{10}\Lambda\text{Be}$ (b). On panel (c), solid lines show the matter density distributions of the $3/2^-$ and $1/2^+$ states of $^{9}\text{Be}$ and the corresponding states in $^{10}\Lambda\text{Be}$.](image)

Figure 1(a) shows the excitation spectra of $^{9}\text{Be}$ obtained with AMD. The left panels of figure 1(c) show the intrinsic density distributions of the ground state $3/2^-$ and first excited state $1/2^+$. It shows that the deformation of each state is quite different. Namely, the deformation of the $1/2^+$ state is larger than that of the ground state. Indeed, the nuclear quadruple deformation parameters $\beta$ listed in table 1 are quite different between the $3/2^-$ and $1/2^+$ states.

The calculated spectra of $^{10}\Lambda\text{Be}$ obtained with YNG-NSC97f [3] is presented in figure 1(b). We have obtained the negative and positive parity states of $^{10}\Lambda\text{Be}$ with $\Lambda$ in s-orbit corresponding to them of $^{9}\text{Be}$. Figure 1(a) and 1(b) show the shift up of the positive parity states by adding $\Lambda$. Indeed, the excitation energy of the $1/2^+$ state is shifted up by about 700 keV. This is because the $\Lambda$ binding energies $B_\Lambda$ are different between the $3/2^-$ and $1/2^+$ states. Here we define $B_\Lambda$ as the sum of the expectation values of the $\Lambda$ kinetic and $\Lambda N$ potential energies. Table 1 shows that $B_\Lambda$ of the $3/2^-$ state is larger than that of the $1/2^+$ state and this difference of $B_\Lambda$ leads to the shift up of the $1/2^+$ state. This is because $\Lambda$ coupled to the $3/2^-$ state with smaller deformation is more deeply bound than that coupled to the largely deformed $1/2^+$ state. This result is consistent with the predictions based on the four-body cluster model calculation [5, 6].

3.2. Parity reversion of the $^{12}\Lambda\text{Be}$ ground state

As shown in figure 2(a), the AMD model successfully describes the parity-inverted ground state of $^{11}\text{Be}$. And the left panels of figure 2(c) present the nuclear density distribution of the $1/2^+$ and $1/2^-$ states. It shows that the deformation of the ground state $1/2^+$ is larger than that of the first excited state $1/2^-$. Since the deformation of the first excited state $1/2^-$ is smaller than that of the ground state $1/2^+$ and the excitation energy of the $1/2^-$ state is rather small, we can expect the parity reversion of the $^{12}\Lambda\text{Be}$ ground state.

Figure 2(b) shows the level structure of $^{12}\Lambda\text{Be}$ with $\Lambda$ in s-orbit obtained with the improved YNG-NF [4]. It is found that the ground state parity of $^{12}\Lambda\text{Be}$ becomes negative, i.e. the parity...
Table 1. Calculated binding and excitation energies $B$ and $E_x$ [MeV], and matter quadruple deformation $\beta$ for the $3/2^-(1/2^-)$ and $1/2^+$ states of $^9$Be ($^{11}$Be) and corresponding states $1_1^-(0^-)$ and $0^+$ of $^{10}_\Lambda$Be ($^{12}_\Lambda$Be). A binding energy $B_\Lambda$ [MeV], and expectation values of the $\Lambda$ kinetic energy $T_\Lambda$ [MeV] and $\Lambda N$ potential energy $V_{\Lambda N}$ [MeV] are also shown in $^{10}_\Lambda$Be ($^{12}_\Lambda$Be).

| $J^\pi$ | $B$ [MeV] | $E_x$ [MeV] | $\beta$ | $B_\Lambda$ [MeV] | $T_\Lambda$ [MeV] | $V_{\Lambda N}$ [MeV] |
|---------|-----------|-------------|--------|-----------------|------------------|------------------|
| $^9$Be 3/2$^-$ | 59.71 | 0.00 | 0.73 |
| $^9$Be 1/2$^+$ | 57.71 | 2.00 | 1.02 |
| $^{10}_\Lambda$Be 1$^-$ | 68.60 | 0.00 | 0.66 | 9.4 | 7.23 | -16.63 |
| $^{10}_\Lambda$Be 0$^+$ | 65.87 | 2.73 | 0.92 | 8.6 | 6.45 | -15.04 |
| $^{11}$Be 1/2$^+$ | 64.45 | 0.32 | 0.52 |
| $^{11}$Be 1/2$^-$ | 64.77 | 0.00 | 0.72 |
| $^{12}_\Lambda$Be 0$^-$ | 74.69 | 0.00 | 0.47 | 10.24 | 6.71 | -16.93 |
| $^{12}_\Lambda$Be 0$^+$ | 74.44 | 0.25 | 0.70 | 9.67 | 6.68 | -16.42 |

The reversion of the $^{12}_\Lambda$Be ground state will occur by adding $\Lambda$. As shown in table 1, $B_\Lambda$ for the $0^-_1$ state is larger than the $0^+_1$ state by about 500 keV. The difference of $B_\Lambda$ mainly comes from in the $\Lambda N$ potential energy $V_{\Lambda N}$, and it originates in the difference of the nuclear deformation.

4. Summary

In summary, we have applied an extended version of AMD for hypernuclei (HyperAMD) to $^{10}_\Lambda$Be and $^{12}_\Lambda$Be. The HyperAMD calculation predicts the positive and negative parity states of $^{10}_\Lambda$Be and $^{12}_\Lambda$Be with $\Lambda$ in $s$-orbit. In $^{10}_\Lambda$Be, it is found that the first excited state $1/2^+$ of $^9$Be is shifted up in $^{10}_\Lambda$Be. In $^{12}_\Lambda$Be, the parity-inverted ground state of $^{11}$Be is reverted by $\Lambda$. These structure changes are mainly due to the difference of the $\Lambda$ binding energy between the ground and first excited states, originated in the difference of their deformation. This is because $\Lambda$ in $s$-orbit is more deeply bound with small deformation.

References

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