SPM BULLETIN

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1. Editor’s note

A hard-disk (more precisely, disk-on-key) crash I have experienced recently led to loss of some of the announcements, and some mess in the chronological order of the remaining ones. Apology for those.

The special issue of Topology and its Applications, dedicated to SPM, did very well in the downloads statistics: Go to http://top25.sciencedirect.com/ and choose the journal.

Greetings,

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2. Research announcements

2.1. Pseudocompact group topologies with no infinite compact subsets. We show that every Abelian group satisfying a mild cardinal inequality admits a pseudocompact group topology from which all countable subgroups inherit the maximal totally bounded topology (we say that such a topology satisfies property $\sharp$). This criterion is used in conjunction with an analysis of the algebraic structure of pseudocompact groups to obtain, under the Generalized Continuum Hypothesis (GCH), a characterization of those pseudocompact groups that admit such a topology. We prove in particular that each of the following groups admits a pseudocompact group topology with property $\sharp$: (a) pseudocompact groups of cardinality not greater than $2^{2^\omega}$; (b) (GCH) connected pseudocompact groups; (c) (GCH) pseudocompact groups whose torsion-free rank has uncountable cofinality. We also observe that pseudocompact groups with property $\sharp$ contain no infinite compact subsets and are examples of Pontryagin reflexive precompact groups that are not compact.

http://arxiv.org/abs/0812.5033
Jorge Galindo and Sergio Macario

2.2. Selective coideals on $(FIN^\infty_k, \leq)$. A notion of selective coideal on $(FIN^\infty_k, \leq)$ is given. The natural versions of the local Ramsey property and the abstract Baire property relative to this context are proven to be equivalent, and it is also shown that the family of subsets of $FIN^\infty_k$ having the local Ramsey property relative to a selective coideal on $(FIN^\infty_k, \leq)$ is closed under the Souslin operation. Finally, it is proven that such selective coideals satisfy a sort of canonical partition property, in the sense of Taylor.

http://arxiv.org/abs/0901.1688
José G. Mijares and Jesús Nieto

2.3. Entire functions mapping uncountable dense sets of reals onto each other monotonically.

http://www.ams.org/journal-getitem?pii=S0002-9947-09-04924-1
2.4. Effective refining of Borel coverings.
http://www.ams.org/journal-getitem?pii=S0002-9947-09-04930-7

Gabriel Debs; Jean Saint Raymond

2.5. Symmetry and colorings: Some results and open problems. We survey some principal results and open problems related to colorings of algebraic and geometric objects endowed with symmetries.
http://arxiv.org/abs/0901.3356
T. Banakh, I. V. Protasov

2.6. Many partition relations below density. We force $2^\lambda$ to be large and for many pairs in the interval $(\lambda, 2^\lambda)$ a stronger version of the polarized partition relations hold. We apply this to problems in general topology. E.g. consistently, every $2^\lambda$ is successor of singular and for every Hausdorff regular space $X$, $\text{hd}(X) \leq s(X)^{+3}$, $\text{hL}(X) \leq s(X)^{+3}$ and better for $s(X)$ regular, via a half-graph partition relation. For the case $s(X) = \mathfrak{c}$ we get $\text{hd}(X), \text{hL}(X) \leq \mathfrak{c}$ (we can get $\leq \mathfrak{c}_1 < 2^{\mathfrak{c}_0}$ but in a subsequence work).
http://arxiv.org/abs/0902.0440
Saharon Shelah

2.7. Lindelof indestructibility, topological games and selection principles. Arhangel’skii proved that if a first countable Hausdorff space is Lindelöf, then its cardinality is at most $2^{\mathfrak{c}_0}$. Such a clean upper bound for Lindelöf spaces in the larger class of spaces whose points are $G_\delta$ has been more elusive. In this paper we continue the agenda started in F.D. Tall, On the cardinality of Lindelöf spaces with points $G_\delta$, Topology and its Applications 63 (1995), 21 - 38, of considering the cardinality problem for spaces satisfying stronger versions of the Lindelöf property. Infinite games and selection principles, especially the Rothberger property, are essential tools in our investigations.
http://arxiv.org/abs/0902.1944
Marion Scheepers and Franklin D. Tall

2.8. Locally precompact groups: (Local) realcompactness and connectedness. A theorem of A. Weil asserts that a topological group embeds as a (dense) subgroup of a locally compact group if and only if it contains a non-empty precompact open set; such groups are called locally precompact. Within the class of locally precompact groups, the authors classify those groups with the following topological properties: Dieudonne completeness; local realcompactness; realcompactness; hereditary realcompactness; connectedness; local connectedness. They also prove that an abelian locally precompact group occurs as the quasi-component of a topological group if and only if it is precompactly generated, that is, it is generated algebraically by a precompact subset.
2.9. The group \textit{Aut(μ)} is Roelcke precompact. Following a similar result of Uspenskij on the unitary group of a separable Hilbert space we show that with respect to the lower (or Roelcke) uniform structure the Polish group \( G = \text{Aut(μ)} \), of automorphisms of an atomless standard Borel probability space \((X, μ)\), is precompact. We identify the corresponding compactification as the space of Markov operators on \( L_2(μ) \) and deduce that the algebra of right and left uniformly continuous functions, the algebra of weakly almost periodic functions, and the algebra of Hilbert functions on \( G \), all coincide. Again following Uspenskij we also conclude that \( G \) is totally minimal.

2.10. An infinite combinatorial statement with a poset parameter. We introduce an extension, indexed by a partially ordered set \( P \) and cardinal numbers \( \kappa, \lambda \), denoted by \((\kappa, <\lambda) \leadsto P\), of the classical relation \((\kappa, n, \lambda) \to ρ\) in infinite combinatorics. By definition, \((\kappa, n, \lambda) \to ρ\) holds, if every map \( F: [\kappa]^n \to [\kappa]^{<\lambda} \) has a \( ρ \)-element free set. For example, Kuratowski’s Free Set Theorem states that \((\kappa, n, \lambda) \to n + 1\) holds iff \( \kappa \geq \lambda + n \), where \( \lambda + n \) denotes the \( n \)-th cardinal successor of an infinite cardinal \( \lambda \). By using the \((\kappa, <\lambda) \leadsto P\) framework, we present a self-contained proof of the first author’s result that \((\lambda + n, n, \lambda) \to n + 2\), for each infinite cardinal \( \lambda \) and each positive integer \( n \), which solves a problem stated in the 1985 monograph of Erdős, Hajnal, Máté, and Rado. Furthermore, by using an order-dimension estimate established in 1971 by Hajnal and Spencer, we prove the relation \((\lambda + (n-1), r, \lambda) \to 2\left[\frac{1}{2}(1-2^{-r})^{-n/r}\right]\), for every infinite cardinal \( \lambda \) and all positive integers \( n \) and \( r \) with \( 2 \leq r < n \). For example, \((\mathbb{N}_{210}, 4, \mathbb{N}_0) \to 32,768\). Other order-dimension estimates yield relations such as \((\mathbb{N}_{109}, 4, \mathbb{N}_0) \to 257\) (using an estimate by Füredi and Kahn) and \((\mathbb{N}_7, 4, \mathbb{N}_0) \to 10\) (using an exact estimate by Dushnik).

2.11. The Schur \( ℓ_1 \) Theorem for filters. We study classes of filters \( \mathcal{F} \) on \( \mathbb{N} \) such that weak and strong \( \mathcal{F} \)-convergence of sequences in \( ℓ_1 \) coincide. We study also analogue of \( ℓ_1 \) weak sequential completeness theorem for filter convergence.

2.12. On uniform asymptotic upper density in locally compact abelian groups. Starting out from results known for the most classical cases of \( \mathbb{N}, \mathbb{Z}^d, \mathbb{R}^d \) or for sigma-finite abelian groups, here we define the notion of asymptotic uniform upper density in general locally compact abelian groups. Even if a bit surprising, the
new notion proves to be the right extension of the classical cases of $\mathbb{Z}^d$, $\mathbb{R}^d$. The new notion is used to extend some analogous results previously obtained only for classical cases or sigma-finite abelian groups. In particular, we show the following extension of a well-known result for $\mathbb{Z}$ of Furstenberg: if in a general locally compact Abelian group $G$ a subset $S$ of $G$ has positive uniform asymptotic upper density, then $S-S$ is syndetic.

http://arxiv.org/abs/0904.1567
Szilard Gy. Revesz

2.13. A $c_0$-saturated Banach space with no long unconditional basic sequences.

http://www.ams.org/journal-getitem?pii=S0002-9947-09-04858-2
J. Lopez-Abad, S. Todorcevic

2.14. Zero subspaces of polynomials on $\ell_1(\Gamma)$. We provide two examples of complex homogeneous quadratic polynomials $P$ on Banach spaces of the form $\ell_1(\Gamma)$. The first polynomial $P$ has both separable and nonseparable maximal zero subspaces. The second polynomial $P$ has the property that while the index-set $\Gamma$ is not countable, all zero subspaces of $P$ are separable.

http://arxiv.org/abs/0903.2374
Antonio Avilés, Stevo Todorcevic

2.15. MAD Families and SANE Player. We throw some light on the question: is there a MAD family (= a family of infinite subsets of $\mathbb{N}$, the intersection of any two is finite) which is completely separable (i.e. any $X \subseteq \mathbb{N}$ is included in a finite union of members of the family or includes a member (and even continuum many members) of the family). We prove that it is hard to prove the consistency of the negation:

1. If $2^{\aleph_0} < \aleph_\omega$, then there is such a family.
2. If there is no such families then some situation related to pcf holds whose consistency is large; and if $a > \aleph_1$ even unknown.

http://arxiv.org/abs/0904.0816
Saharon Shelah

2.16. On the consistency of $d_\lambda > cov_\lambda(\mathcal{M})$. We prove the consistency of: for suitable strongly inaccessible cardinal lambda the dominating number, i.e. the cofinality of $\lambda^\lambda$ is strictly bigger than $cov_\lambda(\mathcal{M})$, i.e. the minimal number of nowhere dense subsets of $2^\lambda$ needed to cover it. This answers a question of Matet.

http://arxiv.org/abs/0904.0817
Saharon Shelah

2.17. $o$-Boundedness of free topological groups. Assuming the absence of $Q$-points (which is consistent with ZFC) we prove that the free topological group $F(X)$ over a Tychonov space $X$ is $o$-bounded if and only if every continuous metrizable image $T$ of $X$ satisfies the selection principle $U_{fin}(O, \Omega)$ (the latter means that for
every sequence \(< u_n >_{n \in w}\) of open covers of \(T\) there exists a sequence \(< v_n >_{n \in w}\) such that \(v_n \in [u_n]^{< w}\) and for every \(F \in [X]^{< w}\) there exists \(n \in w\) with \(F \subset \bigcup v_n\). This characterization gives a consistent answer to a problem posed by C. Hernandes, D. Robbie, and M. Tkachenko in 2000.

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\text{http://arxiv.org/abs/0904.1389}
\]

\textit{Taras Banakh, Dušan Repovš, Lyubomyr Zdomskyy}

2.18. Combinatorial and model-theoretical principles related to regularity of ultrafilters and compactness of topological spaces. V. We generalize to the relations \((\lambda, \mu) \Rightarrow (\lambda', \mu')\) and \(\text{alm}(\lambda, \mu) \Rightarrow \text{alm}(\lambda', \mu')\) some results obtained in Parts II and IV. We also present a multi-cardinal version.

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\text{http://arxiv.org/abs/0903.4691}
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\textit{Paolo Lipparini}

2.19. Menger subsets of the Sorgenfrey line.

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\text{http://www.ams.org/journal-getitem?pii=S0002-9939-09-09887-6}
\]

\textit{Masami Sakai}

2.20. Combinatorial and model-theoretical principles related to regularity of ultrafilters and compactness of topological spaces. VI. We discuss the existence of complete accumulation points of sequences in products of topological spaces. Then we collect and generalize many of the results proved in Parts I, II and IV. The present Part VI is complementary to Part V to the effect that here we deal, say, with uniformity, complete accumulation points and \(\kappa-(\lambda)\)-compactness, rather than with regularity, \([\lambda, \mu]\)-compactness and \(\kappa-(\lambda, \mu)\)-compactness. Of course, if we restrict ourselves to regular cardinals, Parts V (for \(\lambda = \mu\)) and Part VI essentially coincide.

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\text{http://arxiv.org/abs/0904.3104}
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\textit{Paolo Lipparini}
3. Unsolved Problems from Earlier Issues

Issue 1. Is \( \binom{\Omega}{1} = \binom{\Gamma}{1} \)?

Issue 2. Is \( \mathcal{U}_{\text{fin}}(\mathcal{O}, \Omega) = \mathcal{S}_{\text{fin}}(\Gamma, \Omega) \)? And if not, does \( \mathcal{U}_{\text{fin}}(\mathcal{O}, \Gamma) \) imply \( \mathcal{S}_{\text{fin}}(\Gamma, \Omega) \)?

Issue 4. Does \( \mathcal{S}_1(\Omega, \Gamma) \) imply \( \mathcal{U}_{\text{fin}}(\Gamma, \Gamma) \)?

Issue 5. Is \( p = p^* \)? (See the definition of \( p^* \) in that issue.)

Issue 6. Does there exist (in ZFC) an uncountable set satisfying \( \mathcal{S}_{\text{fin}}(\mathcal{B}, \mathcal{B}) \)?

Issue 8. Does \( X \notin \text{NON}(\mathcal{M}) \) and \( Y \notin \mathcal{D} \) imply that \( X \cup Y \notin \text{COF}(\mathcal{M}) \)?

Issue 10. Is \( \text{cov}(\mathcal{M}) = \text{od} \)? (See the definition of \( \text{od} \) in that issue.)

Issue 11. Does \( \mathcal{S}_1(\Gamma, \Gamma) \) always contain an element of cardinality \( \text{b} \)?

Issue 12. Could there be a Baire metric space \( M \) of weight \( \aleph_1 \) and a partition \( \mathcal{U} \) of \( M \) into \( \aleph_1 \) meager sets where for each \( \mathcal{U}' \subset \mathcal{U}, \bigcup \mathcal{U}' \) has the Baire property in \( M \)?

Issue 14. Does there exist (in ZFC) a set of reals \( X \) of cardinality \( \text{d} \) such that all finite powers of \( X \) have Menger’s property \( \mathcal{S}_{\text{fin}}(\mathcal{O}, \mathcal{O}) \)?

Issue 15. Can a Borel non-σ-compact group be generated by a Hurewicz subspace?

Issue 16 (MA). Is there an uncountable \( X \subseteq \mathbb{R} \) satisfying \( \mathcal{S}_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma) \)?

Issue 17 (CH). Is there a totally imperfect \( X \) satisfying \( \mathcal{U}_{\text{fin}}(\mathcal{O}, \Gamma) \) that can be mapped continuously onto \( \{0, 1\}^\mathbb{N} \)?

Issue 18 (CH). Is there a Hurewicz \( X \) such that \( X^2 \) is Menger but not Hurewicz?

Issue 19. Does the Pytkeev property of \( C_p(X) \) imply that \( X \) has Menger’s property?

Issue 20. Does every hereditarily Hurewicz space satisfy \( \mathcal{S}_1(\mathcal{B}_\Gamma, \mathcal{B}_\Gamma) \)?

Issue 21 (CH). Is there a Rothberger-bounded \( G \leq \mathbb{Z}^\mathbb{N} \) such that \( G^2 \) is not Menger-bounded?

Issue 22. Let \( \mathcal{W} \) be the van der Waerden ideal. Are \( \mathcal{W} \)-ultrafilters closed under products?

Issue 23. Is the \( \delta \)-property equivalent to the \( \gamma \)-property \( \binom{\Omega}{1} \)?

Previous issues. The previous issues of this bulletin are available online at http://front.math.ucdavis.edu/search?&t=%22SPM+Bulletin%22

Contributions. Announcements, discussions, and open problems should be emailed to tsaban@math.biu.ac.il

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