Stability study of flow in a 90 ° bend based on the energy gradient theory

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Abstract. The energy gradient theory is used to analyze numerical simulation of the flow in a 90 degree bend with square cross-section. The Reynolds number based on the cross-sectional width and the averaged velocity is 158, 394 and 790, respectively. It is found that at Re = 790, the value of the energy gradient function K increases with the fluid entering the curved section, causing flow instability and forming a pair of secondary vortices; then the secondary vortices gradually stabilizes and the value of K decreases. At the exit of the bend, the total pressure distribution in the cross-section presents serious distortion, which leads to a transition of two vortices to four vortices. With the flow ahead, the maximum of K rises again, resulting in the transition of four vortices to eight vortices. At the low Reynolds number (Re = 158 and Re = 394), there is only one pair of vortices due to the low value of K, which are stable. This study shows that the occurrence of instability is closely related to the evolution of energy gradient function K.

1. Introduction
The flow in a bend is very complex due to curvature, geometric shape, Mach number and many other factors in industrial applications. Secondary flow may occur in the elbow of bent, which is a common phenomenon and important pattern in fluid mechanics. When the fluid flows through the elbow of the bent, because of influences of pressure gradient and centrifugal force, secondary flow may be generated which is perpendicular to the streamwise direction. With the increase of flow curvature and pressure difference between inside and outside wall of the elbow, the pattern of secondary flow on the cross-section of the bend may generate complicated variation and result in complex flow pattern.

There is a lot of researches on the flow inside the bend in the literature [1-4]. Taylor et al. measured the two orthogonal components of velocity and associated Reynolds stress in a square-sectioned using Laser Doppler Velocimetry [1]. Sudo et al. investigated the turbulent
flow in a circular-sectioned 90 degree bend experimentally. The longitudinal, circumferential and radial components of mean and fluctuating velocities, and the Reynolds stresses in the pipe cross section at several longitudinal stations were obtained [2]. Abhari et al. used experimental and numerical methods to obtain flow pattern in a 90° bend. The results of the experimental data and the numerical simulation showed that the flow pattern in the channel bend is influenced widely by the secondary flow and centrifugal force [3]. Zhan et al. studied the pressure drop changing with the variation of the Dean number experimentally and numerically, the results showed that the pressure drop increases with the mainstream velocity, and the impact is different for different flow states [4]. These researches analyzed the flow in the bend mainly through the study of velocity and pressure, while, the physical mechanism of the flow in the bend is still not fully understood, and further study is needed.

2. Revisiting energy gradient theory

The energy gradient theory was proposed based on the Newtonian mechanics [5-7]. When a fluid particle is disturbed, it will start an oscillation motion. From the classical theory of the Brownian motion, the fluid particles exchange energy and momentum all the time via collisions. The disturbed fluid particle will collide with other particles in transverse directions as it flows along its streamline, and this particle would obtain energy expressed as $\Delta E$ after many cycles; at the same time, the particle would drop energy due to viscosity along the streamline; with the same periods, the energy loss expressed as $\Delta H$ would be considerable. Consequently, there exists a critical value of the ratio of $\Delta E$ and $\Delta H$, above which the particle would leave its equilibrium by moving to a new streamline with higher energy or lower energy and below which the particle would not leave its streamline for its oscillation would be balanced by the viscosity force along the streamline. Referring to [5-7], we can express the criteria of stability as follows:

$$F = \frac{\Delta E}{\Delta H} = \left( \frac{\partial E}{\partial n} \right)_{2A} \left( \frac{\partial H}{\partial s} \right)_{\omega_m} u = \frac{2}{\pi^2} K \frac{A \omega_m}{u} = \frac{2}{\pi^2} K \frac{v_m'}{u} < Const \quad (1)$$

where

$$K = \frac{\partial E/\partial n}{\partial H/\partial s} \quad (2)$$

Here, $F$ is a function of coordinates which expresses the ratio of the energy gained in a half period by the particle and the energy loss due to viscosity in the half period. $K$ is a dimensionless field variable (function) and expresses the ratio of transversal energy gradient and the rate of the energy loss along the streamline. $E = p + 1/2 \rho v^2$ is the kinetic energy per unit volumetric fluid, $s$ is along the stream wise direction, and $n$ is along the transverse direction. $H$ is the loss of the total mechanical energy per unit volumetric fluid along the streamline for finite length, which can be calculated from the Navier-Stokes equations. Further, $\rho$ is the fluid density, $u$ is the stream wise velocity of main flow, $A$ is the amplitude of the disturbance distance, $\omega_m$ is the frequency of the disturbance, and $v_m' = A \omega_m$ is the amplitude of the disturbance of velocity.

3. Bend model and mathematical method
The computational geometry is a square-sectioned 90 degree bend. For purposes of analysis, the bend is divided into three sections: upstream, elbow and downstream. The width of the square cross-section is 0.04m (d = 0.04m), the length of upstream and downstream are 0.3m. The radius ratio is 2.3(Re / D = 2.3), where \( Re \) is the mean radius, \( D \) is the hydraulic diameter; the hydraulic diameter of this model is 0.04m. The curvature radius of elbow of the inner wall is 0.072m, and the radius of curvature of the outer wall is 0.112m. Figure 1(left) shows the coordinate system and the origin O is located at the centre of inlet surface. In the figure, \( S_1 \) and \( S_2 \) represent the distance along the longitudinal axis of the bend, \( \theta \) is the polar angle. The cross-section at the starting of the curved section is defined as \( \theta = 0^\circ \). In this paper, hexahedral mesh grid is used. The grid contains about 640000 nodes.

4. Numerical simulation results and analysis

In Figure. 1, E represents the total pressure; K represents the energy gradient function. With the increase of the Reynolds number, the influence of the centrifugal force on the flow in the elbow becomes large. This causes the increase of the gradient of the total pressure of the flow along the streamwise direction in the elbow. The value of K in the elbow increases with the Re. It can also be found in the following that the secondary flow also becomes stronger gradually.

![Image](image_url)

**Figure 1.** Bend model and distribution of total pressure (above) and K (below) on the z=0 plane.

Figure 2 shows the streamline on cross-sections along the streamwise direction at Re = 790. The outer wall of the bend is on left, and the inner wall of the bend is on right. At the cross section \( \theta = 0^\circ \), the velocity of the secondary flow is small and no obvious vortex is formed. With the increase of \( \theta \), effect of transversal pressure gradient due to the curvature appears, forming a pair of secondary vortices. Then, the vortex core moves to the inner wall of the bend and closes to the axis of the bend gradually. At the \( \theta = 90^\circ \) cross section, the secondary vortices become four. With the flow ahead, there is a transition of four vortices to eight vortices. And all of the vortices are symmetrical and vary along the streamwise direction.
Figure 2. Streamline of secondary flows on cross section at Re=790.

Figure 3 shows the distribution of K on the cross sections at Re = 790. The outer wall of the bend is on left, and the inner wall of the bend is on right. Comparing Figure 2 and Figure 3, the following phenomena can be found: (1). From $\theta = 0^\circ$ to $\theta = 60^\circ$, the secondary flow increases because of the centrifugal force. The value of the K has a peak area, and the area is bigger and bigger. (2). On $\theta = 90^\circ$ cross section, the influence of the centrifugal disappear gradually, the peak area decreases. The peak area along the z-direction, results in a transition of two vortices to four vortices. (3). With the flow ahead, on $y=0.02$ cross section; there is a transition of four vortices to eight vortices. On this cross section, the vortex slitting is just located at the peak area. Hence, these phenomena suggest that the occurrence of instability is closely related to the evolution of energy gradient function K.

Further, the distribution of velocity, total pressure, energy gradient function K along the z direction on the centre of the cross section at Re=790 is shown in Figure 4. In Figure 4, the influence of the centrifugal force is small when $\theta$ is less than 30 degree. In these cases, distributions of the velocity and the total pressure are relatively uniform. The influence of the centrifugal force on the flow increases with the increase of $\theta$ angle. It causes the increase of
the velocity and total pressure nearing the outer wall and the decrease of the velocity and total pressure nearing the inner wall. There are two peaks of the value of $K$ at $\theta = 60^\circ$ and the first one is small. The secondary vortices begin to split at this position. At $\theta = 90^\circ$, the total pressure distribution in the cross-section presents serious distortion, which leads to generation of the maximum of $K$. As such, it promotes instability of the flow, and therefore causes a transition of two vortices to four vortices. With the flow ahead, the bigger peak turns to be small and the smaller peak turns to become big, the maximum of $K$ rises again, resulting in the transition of four vortices to eight vortices.

(a) $\theta = 0^\circ$

(b) $\theta = 30^\circ$

(c) $\theta = 60^\circ$

(d) $\theta = 90^\circ$ (y=0)

(f) y=0.02
Figure 4. Distribution of velocity (left), total pressure (middle), K (right) along the z direction on the cross section at Re=790 (l is coordinate along the direction of the curvature radius)

5. Conclusions
The main conclusions of this study are as follows:

(1) At the low Reynolds number (Re = 158 and Re = 394), there is only one pair of vortices due to the low value of K, which are stable.

(2) At Re = 790, the value of the energy gradient function K increases with the fluid entering the curved section, causing flow instability and forming a pair of secondary vortices; then the secondary vortices gradually stabilizes and the value of K decreases. At the exit of the bend, the total pressure distribution in the cross-section presents serious distortion, which leads to generation of maximum of K. As such, it promotes instability of the flow, and therefore causes a transition of two vortices to four vortices. With the flow ahead, the maximum of K rises again, resulting in the transition of four vortices to eight vortices.

From the above, this study shows that the occurrence of instability is closely related to the evolution of energy gradient function K.

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