Numerical inverse estimation of the thermal diffusivity and the adiabatic limit temperature of three types of unfired clay bricks using flash method and global minimization algorithm

Yassine Chihaba\textsuperscript{a}, Saad Raefat\textsuperscript{a}, Mohammed Garouma\textsuperscript{b}, Najma Laaroussi\textsuperscript{a}, Rachid Bouferra\textsuperscript{b}

\textsuperscript{a}Mohammed V University in Rabat, EST de Salé, Materials, Energy and Acoustics TEAM, Morocco.
\textsuperscript{b}Cadi Ayyad University, Marrakech, Morocco.

\textsuperscript{1}yassinechihab@research.emi.ac.ma ; \textsuperscript{2}garoum1@yahoo.fr

Abstract. Clay is an ecological building material used widely in constructions especially in rural areas. In this work an experimental study has been carried out to estimate the apparent thermal diffusivity and adiabatic limit temperature of three types of unfired clay brick using flash method. The three samples were prepared with the same water proportion (picked between the plasticity and the liquidity limits) and exposed to the same drying conditions. The thermal diffusivity and adiabatic limit temperature were estimated using numerical global minimization procedures of the quadratic distance between the theoretical model and the experimental thermogram. The theoretical model is derived from a numerical inversion of Laplace transform using Gaver-Stehfest method. An approach based on the independence between the sensitivity coefficients and the magnitude of these coefficients (Beck criterion) is presented in order to examine the simultaneous estimation of the different parameters related to the theoretical model from the experimental data. For all samples tested, the results show that the estimated parameters give a good agreement between the experimental data and the theoretical model.

1. Introduction

The building sector accounts for 28% of energy consumption and contributes about a third of the CO\textsubscript{2} emissions. In Morocco, the building sector consumes more than 25% of final energy [1]. So, the reducing of the energy cost of building has become a necessity. Unfired Clay is an abundant raw natural resource and it is one of the oldest construction materials needing a very low amount of energy during the manufacturing process.

Many studies have been published concerning the thermal behavior of material based on clay. El Azhary et al. [2] studied the impact of using unfired clay on the energy efficiency of building in the south of Morocco. The thermal properties of fired clay were measured by Laaroussi et al.[3]. Another work of El Azhary et al.[4] studied the effect of mass fraction variation of straw on the thermal properties of composite material clay-straw.

This work is focused on the numerical estimation of the thermal diffusivity and the adiabatic limit temperature for three different unfired clay bricks using the flash method.

2. Experimental approach

2.1 Samples preparation
Three different raw clays extracted from three Moroccan sites (clay of Marrakech (CM), clay of Tamansourt (CT) and clay of Essaouira (CE)) were studied. Figure 1 shows the grain size distribution while the plasticity (PL), the liquidity (LL) limits and the plasticity index (PI) are reported in the Table 1. Three samples were then prepared using the same water proportion. The water to clay ratio was kept equal to 28%. The samples were then dried in an oven at 70°C during three days and packed in plastic films to avoid any moisture contamination.

![Grain size curves of the three clays](image)

**Figure 1.** Grain size curves of the three clays

| sample                  | Liquidity Limit(%) | Plasticity Limit(%) | Plasticity Index(%) |
|-------------------------|--------------------|---------------------|---------------------|
| Clay of Marrakech (CM)  | 37                 | 23                  | 14                  |
| Clay of Tamansourt (CT) | 65                 | 25                  | 40                  |
| Clay of Essaouira (CE)  | 46                 | 26                  | 20                  |

**Table 1.** Atterberg limits of the three raw clays used

2.2 The flash method

This method is mainly used to estimate the thermal diffusivity of materials. Its basic principle is described in Figure 3. The front face of the sample is exposed to a strong heat pulse and the variation of the rise temperature with time is measured at the rear face using a thermocouple type K.

![Studied samples](image)

**Figure 2.** Studied samples (a) CT, (b) CE, (c) CM

![Schematic of the flash method](image)

**Figure 3.** Schematic of the flash method

3. Mathematical model

The front face of samples was covered by a thin black painting to have complete absorption of the radiative flux. We assume that the absorbed energy is uniform and the initial temperature $T_0$ of the sample is equal to the ambient one. In order to allow a one-dimension heat transfer across the sample, its lateral faces were insulated. The rise temperature $T$ is governed by the following system (1).
Where \( a \) is the thermal diffusivity, \( h_1 \) and \( h_2 \) are the global heat exchange coefficients on both sides of the sample, \([Q_0 \times f(t)]\) is the finite pulse with duration \( t_d \), \( Q_0 \) (W/m²) is the finite amount of heat absorbed at the front boundary (\( x=0 \)) and \( f(t) \) is the time dependence of the heat generation assimilated as a square function.

For all measurements, the duration of flash \( t_d \) is kept equal to 4s.

The Laplace transform of Eq. (1) allows expressing the Laplace transform \( \Theta \) of the rise temperature as:

\[
\Theta(\varepsilon, T_{ma}, a, b_{11}, b_{12}, p) = T_{ma} F(p) \text{erf}\left(\frac{\varepsilon / a + p t_d}{\sqrt{a}}\right) + \frac{1}{2} \left( 1 - \frac{b_{11} b_{12}}{b_{11} + b_{12}} \right) A(p) + \frac{b_{11} + b_{12}}{b_{11} b_{12}} B(p) + \left( 1 - \frac{b_{11}}{b_{12}} \right) \frac{p t_d}{c}.
\]

\[A(p) = \sinh\left(\frac{\varepsilon / a}{p}\right) \quad \text{and} \quad B(p) = \cosh\left(\frac{\varepsilon / a}{p}\right).
\]

\( e \) is the thickness of the sample, \( b_{11} \) and \( b_{12} \) are the Biot numbers respectively, \( b_{11} = \frac{h_1 e}{\lambda} \) and \( b_{12} = \frac{h_2 e}{\lambda} \).

\( F(p) = \frac{1 - e^{-p t_d}}{p t_d} \) is the Laplace transform of \( f(t) \).

\( T_{ma} = \frac{Q_0}{\rho c e} \) is the adiabatic limit temperature. Where \( c \) is the specific heat and \( \rho \) is the density.

The theoretical thermogram of the rise temperature on the rear face \( T_{th}(e, T_{ma}, a, b_{11}, b_{12}, t) \) is then obtained by inverting numerically the solution (2) using the Gaver-Stehfest method [5].

\[
T_{th}(e, T_{ma}, a, b_{11}, b_{12}, t) = \frac{\ln(2)}{t} \sum_{j=1}^{n} V_j \Theta\left( e, T_{ma}, a, b_{11}, b_{12}, \frac{\ln(2)}{t} \cdot j \right).
\]

The coefficients \( V_j \) are given by the relation:

\[
V_j = (-1)^{\frac{n+j}{2}} \sum_{k=\left[\frac{j+1}{2}\right]}^{\frac{n+1}{2}} \left( \frac{n+1}{2} \right) (k)^{\frac{n}{2}} (2k-1)!
\]

\[= (-1)^{\frac{n+j}{2}} \sum_{k=\left[\frac{j+1}{2}\right]}^{\frac{n+1}{2}} \left( \frac{n+1}{2} \right) (k)^{\frac{n}{2}} (2k-1)!
\]

\( \text{with} \quad n = \frac{\ln(2)}{t} \).

(3)
The even number \( n \) of terms used in the summation must be optimized. Increasing \( n \) increases the precision of the result to a certain limit, and then the precision decreases due to the increase in rounding errors. The Gaver-Stehfest method was chosen for its swiftness and its easier numerical implementation. A special script was written in Mathematica language.[6].

4. Local sensitivity analysis and global minimization

The local sensitivity analysis permits to know how a small disturbance of the different parameters input of the theoretical model \( T_{th} \) influences the variability of its output. The relative coefficient of sensitivity \( \chi_k \) is defined as:

\[
\chi_k = k \frac{\partial F}{\partial k}
\]

For this model \( F = \frac{\ln(2)}{r} \sum_{j=1}^{n} V_j \left( e^{T_{ma}}, a, b_1, b_2, \frac{\ln(2)}{r}, j \right) \) and \( k = \eta_k \)

\[
\chi_k = \eta_k \frac{\ln(2)}{t} \sum_{j=1}^{n} V_j \left( e^{T_{ma}}, a, b_1, b_2, \frac{\ln(2)}{t}, j \right) \frac{\partial}{\partial \eta_k}, 1 \leq k \leq 4
\]

(5)

Where \( \eta_k \) is unknown parameters which be estimated with \( (\eta_1 = a, \eta_2 = T_{ma}, \eta_3 = b_1, \eta_4 = b_2) \). According to Beck criterion [7], the sensitivity coefficients must be linearly independent. In addition \( \chi_k \) must be different from zero. The analysis of Sensitivity coefficient permits to know if all parameters can be simultaneously estimated from the experimental data with a good precision.

Figure 4 shows an example of relative coefficients of sensitivity curves for the theoretical model \( T_{th} \).

It is noticed that all sensitivity coefficients are different from zero, The analysis of the Figure 4 shows that for low time range \( (t<300s) \) this model is very sensitive to the change in \( a \) and \( T_{ma} \) on the other hand it is very weakly sensitive to the change in \( b_1 \) and \( b_2 \). For time range \( (\geq300s) \), this model becomes sensitive to the parameters \( b_1 \) and \( b_2 \) but the sensitivity coefficients related to these parameters are linearly dependent. We can explain this Interrelationship by the symmetry of existing model for \( b_1 \) and \( b_2 \). In this time range, all parameters are correlated and the theoretical model doesn’t respect the predictability Beck’s criterion.

This model is very weakly sensitive to the change in \( b_1 \) and \( b_2 \), consequently the adopted estimation approach can conduct to several values of these parameters. Therefore, they can't be determined numerically with a good precision from the experimental data. In addition, the sensitivity coefficients related to \( a \) and \( T_{ma} \) meets the beck criterion for low time range \( (t<300s) \), so these tow parameters can be estimated numerically with a good precision in this time range.

However, the four parameters \( T_{ma}, a, b_1, b_2 \) were simultaneously estimated by minimizing the quadratic distance \( M \) between \( T_{th} \) and the experimental thermograms.

\[
M(e, T_{ma}, a, b_1, b_2) = \sum_{j=1}^{N} \left[ T_{exp}(t_j) - T_{th}(e, T_{ma}, a, b_1, b_2, t_j) \right]^2
\]

(6)

\( t_j \) is the time points and \( N \) is the length of the experimental data vector taken into account.

The minimization of the Eq. 6 of the multimodal function \( M \) is solved using Nelder Mead algorithm [8].

5. Results and discussion

5.1 Density:

From the knowledge of the dimensions and masses of the dry samples, we can easily deduce their apparent densities which are presented in Table 2. It is clear that the heaviest types of unfired clay is (CM) with density equal to 1890.256 kg/m³. The clay (CT) and (CE) have almost the same density.
Table 2. Predicted thermal diffusivity, adiabatic limit temperature and samples density

| Sample | Test | Estimated diffusivity $a \times 10^{-7}$ [m$^2$.s$^{-1}$] | Measurement deviation (%) | Estimated $T_{ma}$ [°C] | Measurement deviation (%) | $\rho$ [kg/m$^3$] |
|--------|------|-------------------------------------------------|--------------------------|--------------------------|--------------------------|---------------------|
| CM     | 1    | 3.660                                          | 1.723                    | 0.964                    | 1.903                    | 1890.256            |
|        | 2    | 3.625                                          | 0.750                    | 0.909                    | 3.911                    |                     |
|        | 3    | 3.598                                          | 2.474                    | 0.965                    | 2.008                    |                     |
|        | Mean value | 3.598                                      |                          | 0.946                    |                          |                     |
| CT     | 1    | 3.285                                          | 1.139                    | 1.034                    | 2.783                    | 1616.83             |
|        | 2    | 3.264                                          | 0.492                    | 1.000                    | 0.596                    |                     |
|        | 3    | 3.197                                          | 1.570                    | 0.985                    | 2.087                    |                     |
|        | Mean value | 3.248                                      |                          | 1.006                    |                          |                     |
| CE     | 1    | 3.004                                          | 0.033                    | 1.047                    | 0.852                    | 1616.16             |
|        | 2    | 3.007                                          | 0.066                    | 1.041                    | 1.420                    |                     |
|        | 3    | 3.006                                          | 0.033                    | 1.081                    | 2.367                    |                     |
|        | Mean value | 3.005                                      |                          | 1.056                    |                          |                     |

5.2 Thermal diffusivity and adiabatic limit temperature:
From the Table 2, it is clear that the thermal diffusivity decreases with the decrease of density, the (CM) has the greatest diffusivity with values equal to 3.598×10$^{-7}$ m$^2$/s, on the other hand, the (CE) has the lowest ones. According to the results of Table 2, it is noticed that the adiabatic limit temperature increases with the decrease of density. In addition, the (CE) has the highest value of the adiabatic limit temperature which means that it has the lowest value of volumetric capacity ($\rho$.c).

Figures (5,6,7) show that the estimated parameters for all samples tested give a good agreement between the experimental data and the theoretical model.

6. Conclusion
This work has presented an estimation of the diffusivity and the adiabatic limit temperature of three types of unfired clay samples using the flash method combined with a global minimization procedure of the quadratic distance between the theoretical model and the experimental thermograms. From the results, it can be concluded that the thermal diffusivity increases with increase of density while the adiabatic limit temperature...
decreases with increase of density. It is showed that the estimated parameters for all tested samples conduct to a good agreement between the experimental data and the theoretical model.

References
[1] Moroccan Agency for the Energy Efficiency (AMEE). Règlement Thermique de Construction au Maroc. 2014. www.amee.ma.
[2] El Azhary, K., Lamrani, A., Raefat, S., Laaroussi, N., Garoum, M., Mansour, M., & Khalfaoui, M. (2017). The improving energy efficiency using unfired clay envelope of housing construction in the south Morocco. Journal of Materials and Environmental Sciences. 8. 3771-3776.
[3] Laaroussi, N., Lauriat, G., Garoum, M., Cherki, A., & Jamot, Y. (2014). Measurement of thermal properties of brick materials based on clay mixtures. Construction and Building Materials. 70. 351-361.
[4] El Azhary, K., Chihab, Y., Mansour, M., Laaroussi, N., & Garoum, M. (2017). Energy Efficiency and Thermal Properties of the Composite Material Clay-straw. Energy Procedia. 141. 160-164.
[5] Stehfest. Numerical inversion of Laplace transforms. Communications of the Association for Computing Machinery. 1970.
[6] Raefat, S., Garoum, M., Laaroussi, N., Thiam, M., & Amarray, K. (2017, July). Thermal diffusivity and adiabatic limit temperature characterization of consolidate granular expanded perlite using the flash method. In IOP Conference Series: Materials Science and Engineering (Vol. 222, No. 1, p. 012004). IOP Publishing.
[7] Beck, J.V. Parameters estimation in engineering and science. Wiley & sons. USA. 1989.
[8] J. A. Nelder and R. Mead, A simplex method for function minimization, Computer Journal 7 (1965), 308–31.