Wave-induced stability of gassy seabed with different gas existing forms

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Abstract: For highly saturated seabed soils, gas mainly exists in two forms: one is in the form of discrete small bubbles in pore water (referred to as "small bubble seabed"); the other is independent of the pore water, in the form of large bubbles surrounded by saturated soil matrix (referred to as "large bubble seabed"). For the two different kinds of gassy soil, based on different theoretical models, the wave-induced stability of gassy seabed has been investigated using finite element method. It has been shown that the wave-induced momentary liquefaction depth of the large bubble seabed is less than that of the small bubble seabed. However, the shear failure depth of the large bubble seabed is greater than that of the small bubble seabed, and the difference could be up to about 80%.

1. Introduction
The in-situ degree of saturation of an unsaturated marine sediment is usually between 0.85 to 1[1]. For these cases, the gas exists in discontinuous occlusion bubbles. Nageswaran[2] also observed this situation in the laboratory. They conducted experiments on soils with discrete occlusion bubbles and found that the saturation of the soil was between 0.85 and 1. Wheeler[3] proposed two existing forms of gas in soil under high saturation conditions. One is that the gas exists as small bubbles in the pore water of the soil matrix (referred to as "small bubble seabed"). The other is that the gas is surrounded by the saturated soil matrix and exists in the form of large bubbles (referred to as "large bubble seabed").

According to Biot’s consolidation theory[4], Yamamoto[5] and Hsu and Jeng[6] both proposed an analytical solution of the transient response of the seabed with small bubbles under waves. Thomas[7] investigated the behavior of large bubble seabed and proposed a mathematical model for the large bubble seabed. The model has been verified by Thomas[8]. At present, there are lots of studies on the response of small bubble seabed under waves. Whereas few studies on the response of the large bubble seabed under waves. At the same time, the comparative analysis on the response of seabed with small and large bubbles has not been conducted. Therefore, this paper investigates the stability of small bubble seabed and the large bubble seabed under wave loadings, including the characteristics of liquefaction and shear failure, and the influence of gas existing form on the stability of seabed soil to waves.

2. Numerical model and validation

2.1 Numerical model of gassy seabed under waves
The sketch of wave and seabed interaction is shown in Figure 1, where the wavelength is $L$, the water
depth is \( d \), and the wave height is \( H \). The horizontal direction of the seabed is \( x \)-axis, and the vertical direction to the surface of the seabed is \( z \)-axis. The normal effective stress is positive for tension.

According to Biot’s consolidation theory, the governing equations of small bubble seabed soil are:

\[
G V^2 u + \frac{G}{1 - 2\mu} \frac{\partial \varepsilon}{\partial x} = \frac{\partial p}{\partial x} \quad (1)
\]

\[
G V^2 v + \frac{G}{1 - 2\mu} \frac{\partial \varepsilon}{\partial y} = \frac{\partial p}{\partial y} \quad (2)
\]

\[
k V^2 \sigma = \gamma_n n' \frac{\partial p}{\partial t} + \gamma_n' \frac{\partial \varepsilon}{\partial t} \quad (3)
\]

Where \( u \) and \( v \) are the horizontal and vertical displacements of the soil respectively; \( \varepsilon \) is the volumetric strain of the soil; \( \varepsilon = \varepsilon_0 \frac{\partial u}{\partial x} + \varepsilon_0' \frac{\partial v}{\partial y} \). \( G \) is shear modulus, \( \mu \) is Poisson's ratio, \( n \) is porosity, \( k \) is the permeability coefficient of the seabed, \( \beta \) is the compressibility of the pore fluid, and \( \gamma_n \) is the unit weight of water. \( \beta = 1/K_w + (1 - S_r)/P_{sw} \), \( S_r \) is degree of saturation, \( P_{sw} \) is the hydrostatic pressure, and \( K_w \) is the bulk modulus of water.

For the small bubble seabed, the effective stress and shear stress can be expressed as:

\[
\sigma' = 2G \left( \frac{\partial u}{\partial x} + \frac{\mu}{1 - 2\mu} \varepsilon \right) \quad \sigma_{zz}' = 2G \left( \frac{\partial v}{\partial y} + \frac{\mu}{1 - 2\mu} \varepsilon \right) \quad \tau_{xz} = G \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (4)
\]

Thomas[7] proposed the governing equations of large bubble seabed as:

\[
\bar{G} V^2 u + (\lambda + \bar{G}) \frac{\partial \varepsilon}{\partial x} = B \frac{\partial p}{\partial x} \quad (5)
\]

\[
\bar{G} V^2 v + (\lambda + \bar{G}) \frac{\partial \varepsilon}{\partial y} = B \frac{\partial p}{\partial y} \quad (6)
\]

\[
k V^2 p + 2k \frac{\partial^2 p}{\partial x \partial y} = \gamma_n C \frac{\partial \varepsilon}{\partial t} + B \gamma_n' \frac{\partial \varepsilon}{\partial t} \quad (7)
\]

Where \( \lambda = (3K - 2\bar{G})/3 \), \( \bar{G} \) is the shear modulus, \( \bar{G} = (1/G^0 + 1/G')^{-1} \), \( K \) is the bulk modulus, \( K = (1/K^0 + 1/K')^{-1} \), \( K' \) is the bulk modulus of the soil matrix and \( K^0 \) is the gas bulk modulus. \( B \) is the pore pressure coefficient, \( B = 1/(1 + K'/K^0) \), \( K^0 = (\sigma_n + u_e)/n' \), \( \sigma_n \) is the average total stress of
the soil, $u_a$ is the atmospheric pressure, $n_g$ is porosity of gas phase, $n_s = n(1 - S_f)$, $n$ is the porosity; $C$ is the compression coefficient, $C = n_s c_w + 1/(K' + K'^0)$, $n_w$ is porosity of water phase.

For the large bubble seabed, the normal effective stress and shear stress can be expressed as:

$$
\sigma'_x = 2G\left(\frac{\partial u}{\partial x} + \frac{\lambda}{2G}\varepsilon\right) + (1 - \beta)p
\quad \sigma'_y = 2G\left(\frac{\partial v}{\partial y} + \frac{\lambda}{2G}\varepsilon\right) + (1 - \beta)p
\quad \tau_{xz} = G\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)
$$

Boundary conditions on the surface of sea bed are: $\sigma'_z = 0$, $\tau_{z\alpha} = 0$, $P_b = P_0 \cos(awt)$, $P_b$ is the wave pressure on the seabed surface, and $P_0$ is the amplitude of the wave pressure, $P_0 = \gamma_c H / 2 \cos(2\pi d / L)$, $a = 2\pi / L$, $w = 2\pi / T$, $T$ is wave period. The bottom boundary of the seabed is an impermeable boundary.

2.2 Validation

Okusa[9] deduced the analytical solution of the wave-induced pore water pressure in small bubble seabed. The finite element method is used to solve the governing equations of the small bubble seabed in this paper. The parameters are: water depth $d = 20$ m, wavelength $L = 197.42$ m, wave height $H = 5$ m, period $T = 15$ s. Permeability coefficient $k = 10^{-4}$ m/s, porosity $n = 0.5$, shear modulus $G = 7.5 \times 10^5$ Pa, $K = 10^6$ Pa, $C = n\beta$. The comparison of numerical results and analytical solution is shown in Figure 2(a).

Thomas[8] gave the analytical solution of the wave-induced pore water pressure in large bubble seabed. The numerical results were compared with the analytical solution to validate the correctness of the simulation. The parameters are: water depth $d = 20$ m, wavelength $L = 197.42$ m, wave height $H = 5$ m, and period $T = 15$ s. Permeability coefficient $k = 10^{-4}$ m/s, porosity $n = 0.5$, $G' = 6 \times 10^6$ Pa, $K' = 10^6$ Pa, $K'^0 = 3 \times 10^7$ Pa, $1 \times 10^7$ Pa, $6 \times 10^6$ Pa. The results are shown in Figure 2(b).

It can be seen from Figure 2 that the numerical results agree well with the analytical solutions, indicating the reliability of the calculated results in this paper.

3. Results and discussion

The momentary liquefaction depth and shear failure depth of the small and the large bubble seabed under waves are compared and analyzed. The wave and seabed parameters are shown in Table 1.
Table 1 Wave and soil parameters

| Parameters          | Value     |
|---------------------|-----------|
| Wave height $H$     | 2.5 m     |
| Water depth $d$     | 10 m      |
| Wave period $T$     | 10 s      |
| Seabed thickness $h$| 20 m      |
| Poisson's ratio $\mu$| 0.3     |
| Porosity $n$        | 0.5       |
| Matrix shear modulus $G$ or $G'$ | $5 \times 10^6$ N/m$^2$ |
| Effective unit weight of soil $\gamma'$ | 7500 N/m$^3$ |
| Unit weight of water $\gamma_w$ | 9800 N/m$^3$ |
| Permeability coefficient $k$ | $10^{-5}$ m/s |
| Degree of Saturation $S_r$ | 0.98, 0.92 |

3.1 Liquefaction characteristics

Liquefaction refers to a state of the seabed when the effective stress of the soil is vanished. Based on the concept of effective stress of soil, Okusa[9] proposed a liquefaction criterion for non-cohesive soil. The criterion can be expressed as: $\gamma' z + \sigma' z \leq 0$. Using the liquefaction criterion, the momentary liquefaction characteristics of the seabed are analyzed. The numerical results for $S_r = 0.98$ and $S_r = 0.92$ are shown in Figure 3.

![Figure 3 Momentary liquefaction zone of gassy seabed under waves](image)

It can be seen from Figure 3 that as the saturation decreases, the liquefaction depth increases. When the saturation decreases from 0.98 to 0.92, the liquefaction depth of the small bubble seabed increases by 0.3 m; while the liquefaction depth of the large bubble seabed increases by 0.1 m. Comparing the relative difference in liquefaction depth between the two kinds of gas existing forms, it has been shown that for $S_r = 0.98$, the relative difference is 63%, and for $S_r = 0.92$, the relative difference is 73%. As the saturation decreases, the relative difference in liquefaction depth would increase.

3.2 Shear failure

When shear stress at a certain point in the soil equals to the shear strength, shear failure will occur at that point, which can be known according to Mohr-Coulomb shear strength criterion. Considering the
wave- and gravity-induced seabed effective normal stress and shear stress, the total effective stress $\bar{\sigma}_x$ and $\bar{\sigma}_z$ and shear stress $\tau_{xz}$ in the x and z directions can be expressed as: 

$$\bar{\sigma}_x = K_0 \gamma z + \sigma_x, \quad \bar{\sigma}_z = \gamma z + \sigma_z, \quad \tau_{xz} = \tau_{xz}; \quad K_0$$

is the coefficient of lateral pressure of soil.

For non-cohesive soil, according to Mohr-Coulomb strength criterion, the stress angle at a certain point in soil can be expressed as:

$$\phi' = \sin^{-1} \sqrt{\left(\frac{\sigma_x' - \sigma_z'}{\sigma_x' + \sigma_z'}\right)^2 + 4\left(\frac{\tau_{xz}'}{\sigma_x' + \sigma_z'}\right)^2}$$  \hspace{1cm} (9)

When the shear stress reaches the shear strength of the soil, the effective internal friction angle of the soil will be the same as the effective soil frictional angle. Therefore, if $\phi' \geq \phi_f$, shear failure occurs in the soil, $\phi_f'$ is the effective internal friction angle of the soil.

For $S_r=0.98$ and $S_r=0.92$, the numerical results are shown in Figure 4.

![Wave-induced shear failure zones of gassy seabed](image)

Figure 4 Wave-induced shear failure zones of gassy seabed

It can be seen from Figure 4 that at wave crest, the shear failure depth of the large bubble seabed is greater than that of the small bubble seabed, whereas it is opposite at the wave trough. The shear depth is almost zero at the nodes. The maximum shear failure depth of the large bubble seabed is always at wave crest, while the position of the maximum shear failure depth of the small bubble seabed could gradually change from wave crest to wave trough as the gas content decreases.

As the saturation decreases, the maximum shear failure depth of the large bubble seabed and the small bubble seabed both decrease. However, the change is not obvious. For the same saturation (indicating the same gas content), the shear failure depth of the large bubble seabed is significantly greater than that of the small bubble seabed, and the difference could be up to more than 80%.

4. Conclusions

Under cyclic wave loadings, the momentary liquefaction depth of the small bubble seabed is greater than that of the large bubble seabed, and the difference increases with the increase of gas content. However, the shear failure depth of the large bubble seabed is greater than that of the small bubble seabed, and the difference could be more than 80%. The maximum shear failure depth of the large bubble seabed locates at wave crest, while the location of the maximum shear failure depth of the small bubble seabed gradually changes from wave crest to wave trough as the saturation decreases.
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