Entropy generation in merging galaxy clusters

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1 Introduction

This conference has seen much discussion about non-gravitational heating of the intracluster medium, as required to reduce cooling rates and central densities sufficiently to explain the observed properties of galaxy clusters. The amount of energy input required is often computed by comparing to the entropy profile that would be expected from gravitational processes alone, as determined, for example, from cosmological simulations. Indeed, observations of relaxed clusters show that, except for the innermost regions, their entropy profiles follow a power-law profile that scales self-similarly with the cluster mass (e.g. McCarthy et al., 2004, 2005; Voit et al., 2005).

However, the origin of this default entropy profile and apparent self-similarity is not really understood in detail. Models of smooth, spherical accretion are able to reproduce the power-law slope of the entropy gradient (e.g. Cavaliere et al., 1998; Abadi et al., 2000; Tozzi & Norman, 2001; Dos Santos & Doré, 2002), but the normalization is very sensitive to the initial gas density distribution. In particular, if accretion is lumpy rather than smooth (as expected in a universe dominated by cold dark matter), insufficient entropy is generated to explain the observations (Voit et al., 2003). If there is such a strong dependence on initial density, it is unclear why (or if) numerical simulations with different resolutions (and hence smoothing scales) and implementations (e.g. Eularian or Lagrangian) are able to produce self-similar clusters.

The source of this puzzle is illuminated by writing the equation of hydrostatic equilibrium as

\[
\frac{1}{\rho} \frac{d}{dr} \left( \rho T \right) = - \frac{\mu m_H GM}{k r^2}, \tag{1}
\]

which describes how the gas density and temperature ($\rho$ and $T$) depend on distance from the cluster centre, $r$. If the mass $M$ is dominated by dark matter, then the right hand side is constant. The temperature of the gas must be close to the virial temperature $T_{\text{vir}}$, again because the potential is dominated by an external field (the dark matter). We see, therefore, that for any solution to this equation, an equally valid solution can be found by scaling the density (and the corresponding boundary conditions) by an...
arbitrary factor. If we define the “entropy” as

\[ K = \frac{T}{\rho^{2/3}}, \]

we see that the entropy profile of an isothermal cluster with \( T \sim T_{\text{vir}} \) can also be arbitrarily normalized and still yield a valid solution to the hydrostatic equilibrium equation. Merger shocks, as seen in both observations (e.g., Markevitch et al., 2005) and simulations (e.g., Ryu et al., 2003) have a very complex geometry, and entropy is clearly not generated in a single, strong shock. Why, then, do observed clusters show such a striking uniformity in their entropy normalization? If we are to understand how the effects of early energy injection propagate through the mass assembly of clusters, and if we are to implement self-consistent cooling/feedback processes in semi-analytic models of galaxy formation, we must first understand how entropy is generated in purely gravitational processes. This is the purpose of our work, recently accepted for publication (McCarthy et al., 2007), which we summarize in these proceedings.

2 Simulations

We have executed a number of idealized simulations of two-body cluster mergers, using the Tree-SPH code GADGET-2 (Springel, 2005), as described in detail in McCarthy et al. (2007). By default the code implements the entropy-conserving SPH scheme of Springel & Hernquist (2003), which ensures that the entropy of a gas particle will be conserved during any adiabatic process. The simulations span a range of mass ratios, from 10:1 to 1:1, with the mass of the primary fixed at \( M_{200} = 10^{15} M_\odot \). Initial conditions and orbital parameters were chosen to match the typical conditions seen in cosmological simulations. Initially, the gas is assumed to be in hydrostatic equilibrium with the dark matter, with an entropy profile similar to that seen in observations and cosmological simulations.

In these proceedings we discuss three interesting results from these experiments. These results are fairly general to all our simulations, although here we just focus on those derived from the head-on collisions.

2.1 Two shocks, not one

All of our simulations show that there are two main episodes, during which the gas entropy sharply increases. The first occurs when the cores of the clusters

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4 We have also run simulations without dark matter, which help to identify the role played by the collisionless component. We will not discuss those results here, but note that our interpretation of the simulations including dark matter often depends on what we have learned from the gas-only runs.
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It is only then, and not before, that there is sufficient kinetic energy to drive strong shocks through the gas. Therefore, the energy is not deposited primarily at the outskirts, as assumed by spherical accretion ("onion-skin") models (Tozzi & Norman, 2001; Voit et al., 2003) but rather the cluster is heated from the inside-out.

Following this event is a more extended, gradual increase in entropy, due primarily to the reaccretion of gas that was driven away from the cluster by the shock wave (and overpressurized gas at the centre) produced by the collision of the cores. This phase generates approximately the same amount of entropy as the initial shock, over a longer period of time. This phase is what we refer to as the second shock, although in fact the entropy is being generated in numerous weak shocks (and perhaps turbulent mixing) in the outer regions of the cluster.

2.2 Distributed heating

If the gas entropy is to scale in a self-similar way following a merger, then $K \propto T \propto M^{2/3}$. For example, a binary merger should result in a $\sim 60$ per cent increase in entropy. One remarkable result is that for all our simulations (with a range of impact parameters and mass ratios, and even the gas-only simulations), most of the gas satisfies this scaling after $\sim 10$ Gyr. The exceptions are at the outer boundary, where there is an excess of entropy generated due to artificial boundary effects, and in the inner $\sim 10$ per cent of gas, which is physically heated to $\sim$twice the self-similar value.

A very interesting result emerges when we simulate the merger of clusters with unequal masses. In this case, the final entropy profile again agrees with the self-similar scaling law, so a 10:1 merger ends up with an entropy that is $\sim 6$ per cent larger than that of the primary, initial halo. One might naively have expected the gas in each component to independently scale in this way; that is, for the gas in the small component to be more strongly shocked than in the primary. However, this is not what happens. Instead, the primary halo is overheated, relative to the self-similar expectation, while the secondary is underheated, as shown in Figure 1. In other words, much of the infall energy associated with the secondary goes into thermalising the gas in the primary, and heating is a distributed, rather than local, process. We find a remarkably robust relation between the energy thermalized in both components (primary and secondary):

$$\frac{E_{T,p}}{E_{T,s}} \approx \left( \frac{M_p}{M_s} \right)^{5/4}. \quad (3)$$

This is not unreasonable, as the Rankine-Hugoniot equations used to determine the post-shock conditions from the pre-shock gas and the Mach number are just a consequence of energy and momentum conservation, and are independent of the path taken between the initial and final state.
As the mass of the primary is increased, the fraction of energy thermalized within its gas also increases. We do not understand why this simple relation arises, but it seems to hold for a wide range of mass ratios and orbital parameters.

2.3 Energy requirements

One would hope to be able to capture most of the relevant physics from these simulations in a simple analytic prescription. The first attempts (e.g. Voit et al., 2003) have assumed all the entropy is generated in a single strong shock. Voit et al. showed that, in a simple, spherical accretion model, this fails to produce sufficient entropy if the infalling matter is clumpy. Our simulations show that this conclusion still holds when we consider more realistic merger models. In particular, for a given mass ratio, we can calculate the maximum energy available to be thermalized, when the cores collide, as shown in Figure 2. If we then assume that all of this energy is thermalized in a single, strong shock, we are unable to produce enough entropy to explain the results of 3:1 or 10:1 mergers.

This is a puzzling result, as our naive expectation had been that a single, strong shock thermalizing all the available energy would yield the maximum entropy. This is true, in that thermalizing the energy in \( N \) weaker shocks does not produce as much entropy, if the system does not evolve between shocks. Where does the real system find the extra energy? The answer appears to be that the gas density actually decreases significantly between the first and second shock events. In McCarthy et al. (2007), we show analytically that if the density drops 20-30\% below its pre-merger value, then the second shock can generate enough entropy for the final cluster to attain its self-similar structure. From the simulations we directly measure that this is indeed what happens: the first shock actually drives gas outward, so that the density in the outer regions ends up lower than it was initially, by 20-30\%.

3 Conclusions

Gravitational shock heating of clusters does not appear to be as simple a process as once envisaged. It is crucial that we understand this mechanism, if we are to improve semi-analytic models of galaxy formation and to understand the heating requirements of real clusters. Our simulations have shown that self-similarity is indeed achieved during cluster mergers, but that this does not happen in a single accretion shock, because there is insufficient energy available. Instead, entropy is generated in two major “shocks”, that heat the gas from the inside-out, in a way that distributes most of the energy within the more massive clump.
Our ultimate goal is to construct an analytic model of cluster growth that self-consistently tracks the entropy of the gas as it is shocked or non-gravitationally heated. There is still much work to be done before we achieve this. Our next steps are to test our analytic, two-shock model against idealized simulations with perturbed initial conditions (e.g. preheating).

![Diagram](image_url)

**Fig. 1.** The resulting $K(M_{\text{gas}})$ distributions (entropy as a function of enclosed gas mass) for the 10:1 mass ratio mergers, normalized to $K_{200}$ (the characteristic entropy of the halo). The dotted line, at $K/K_{200} = 1$, is the entropy distribution expected if the entropy growth is self-similar. We show the final entropy distributions for the primary (short-dashed), secondary (long-dashed), and total (solid) systems for three different orbital cases. Although most of the gas in the final system follows the self-similar expectation, this is achieved by overheating the primary and underheating the secondary.

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Fig. 2. Energy requirements in head-on mergers with three different mass ratios (as shown in the top-left corner of each panel). The horizontal, dashed line shows the amount of energy required to produce the final entropy distribution in the simulations, if all the entropy is generated in a single, strong shock. This is compared with the actual entropy available to be thermalized in the merger. The horizontal, dotted line shows the result of an analytic calculation of the maximum energy available, at the point where the two cores collide. The curved lines show the energy available as measured in the simulations, which reaches a maximum at the time when the cores collide. The different line styles correspond to different assumptions about the interaction between the dark matter and gas, as described in McCarthy et al. (2007). For the 3:1 and 10:1 simulations, there is not enough energy available to produce the observed entropy in a single shock.

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