QUANTUM DECOHERENCE AND THE “P(E)-THEORY”

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We point out a close physical and formal similarity between the problems of electron tunneling in the effective environment and the weak localization effects in the presence of interactions. In both cases the results are expressed in terms of the “energy probability distribution function” $P(E)$ which has a finite width even at $T = 0$ due to interactions.

1 Introduction

Recent experiments strongly indicate an intrinsic nature of the low temperature saturation of the electron decoherence time $\tau_\phi$ in disordered conductors, thereby attracting a lot of attention to the fundamental role of interactions in such systems. A theory of the above phenomenon was proposed in where it was demonstrated that electron-electron interactions in disordered systems are indeed responsible for a nonzero electron decoherence rate down to $T = 0$. It was argued that this interaction-induced decoherence has the same physical nature as in the case of a quantum particle with coordinate $q$ interacting with a bath of harmonic oscillators.

The latter problem can be described by the exactly solvable Caldeira-Leggett model. Within this model one can easily observe that the off-diagonal elements of the particle density matrix $\rho(q_1, q_2)$ decay and even at $T = 0$ in equilibrium they are suppressed at a typical length scale $L_d \sim 1/\sqrt{\langle p^2 \rangle}$, where the expectation value for the square of the particle momentum operator $\langle p^2 \rangle$ is determined by the interaction strength and remains nonzero even in the true ground state of the total system “particle+oscillators”. It is important to emphasize that the decoherence length $L_d$ characterizes the behavior of the particle $q$ and not of the (obviously coherent) eigenmodes of the total system obtained by an exact diagonalization of the initial Hamiltonian. However, if the behavior of the particle $q$ (and not of the system eigenmodes) is of interest, the reduced density matrix $\rho(q_1, q_2)$ should be studied and the finite decoherence length $L_d$ will explicitly enter into the expectation values for the corresponding operators.

Similarly, in order to evaluate the conductance of a disordered metal in the presence of electron-electron interactions it is convenient to choose the basis of noninteracting electrons simply because the current operator can be easily expressed in this basis. Then, as in the previous example, the reduced single electron density matrix in the presence of interactions should be evaluated, and after that the expression for the system conductance is readily established. This program was carried out in where the interplay between interaction and weak localization effects was nonperturbatively studied by means of the real time path integral technique.

The aim of this paper is to highlight a similarity between our theory and the so-called “$P(E)$-theory” describing electron tunneling in mesoscopic tunnel junctions in the presence of interaction with the effective electronic environment in metallic electrodes. We will demonstrate that both results naturally follow from the same nonperturbative procedure.

2 Microscopic Path Integral Analysis

Our starting point is the standard Hamiltonian for the interacting electrons in a disordered metal $\hat{H}_{el} = \hat{H}_0 + \hat{H}_{int}$:

$$\hat{H}_0 = \int dr \psi_0^+ (r) \left[ -\frac{\nabla^2}{2m} - \mu + U(r) \right] \psi_\sigma (r),$$

(1)
\[ \hat{H}_{\text{int}} = \frac{1}{2} \int dr \int dr' \psi^+_\sigma(r) \psi^+_\sigma(r') \frac{e^2}{|r-r'|} \psi^\sigma(r') \psi^\sigma(r). \]  

Here \( U(r) \) includes both the random potential of impurities and the potential of a tunnel barrier if the latter is present. Performing the Hubbard-Stratonovich transformation in the interaction term (3) we reformulate the initial many-body problem in terms of a single electron interacting with a two-component quantum field \( V_{1,2} \). The exact electron propagator \( \hat{G} \) (which is \( 2 \times 2 \) matrix in the Keldysh space) is expressed in terms of the propagator \( \hat{G}_V \) for a single electron in the field \( V_{1,2} \) as follows

\[ \hat{G} = \int \frac{dV_1 dV_2}{dV_1 dV_2} \hat{G}_V \frac{e^{iS[V_1,V_2]}}{e^{iS[V_1,V_2]}}, \]

where the dynamics of the field \( V \) is determined by the effective action

\[ iS[V_1,V_2] = 2 \text{Tr} \ln \hat{G}_V^{-1} + i \int_0^t dt' \int dr \frac{(\nabla V_1)^2 - (\nabla V_2)^2}{8\pi}. \]

Note, that eq. (4) is obtained by integrating out the Grassman electron fields and, hence, it explicitly accounts for the Fermi statistics (see for more details).

To proceed further we will evaluate the action (3) outside the tunnel junction within RPA (which amounts to expanding \( \text{Tr} \ln \hat{G}_V^{-1} \) in \( V \) up to terms \( \sim V^2 \)) and find the tunnel barrier contribution to \( S \) by means of the procedure. Then we obtain

\[ iS = i \int \frac{d\omega d^dk}{(2\pi)^{d+1}} V^-(-\omega,-k) \left[ C(\omega,k) + \frac{a^3-d k^2 (\epsilon(\omega,k) - 1)}{4\pi} \right] V^+(\omega,k) - \frac{a^3-d}{2} \int \frac{d\omega d^dk}{(2\pi)^4} V^-(-\omega,-k) \frac{k^2 \text{Im} \epsilon(\omega,k)}{4\pi} \coth \left( \frac{\omega}{2T} \right) V^-(-\omega,k) + S_{\text{AES}}[\varphi^+,\varphi^-]. \tag{5} \]

Here \( a \) is the transverse size of 1d and 2d systems, \( C(\omega,k) \) is the effective capacitance,

\[ \epsilon(\omega,k) = 1 + \frac{8\pi e^2 N_0 D}{-i\omega + Dk^2} \tag{6} \]

is the dielectric susceptibility, \( N_0 = mp_F/2\pi^2 \) is the electron density of states, \( D = v_F l/3 \) and \( S_{\text{AES}} \) describes tunneling through the barrier and has the well known form. The field \( \varphi^\pm \) is related to the jump of the \( V \)-field across the barrier in a standard manner. If the tunnel barrier is absent, the \( V \)-field is continuous, \( \varphi^\pm = 0 \), and \( S_{\text{AES}} = 0 \).

Let us evaluate the conductivity of the system in both cases, i.e. with and without the tunnel barrier. In order to do that we start from the standard quantum mechanical expression for the current operator and after some formal manipulations (see for more details) express the system conductivity in terms of the path integral with the effective action (3), see eq. (49). We will proceed in parallel in order to illustrate the analogy between the two cases at each step of the calculation.

**Tunnel junction.** If the tunnel junction is present in the system we will be interested only in its contribution to the conductance. Expanding the path integral in \( S_{\text{AES}} \) (see e.g. for details) we obtain the expression for the system conductance

\[ G(V) = \frac{1}{R_T^{(0)}} - \frac{2}{\pi R_T^{(0)}} \int_0^{\infty} dt \left( \frac{\pi T}{\sinh \pi T t} \right)^2 \text{Im} \left[ P(t) \right] \cos(\epsilon V_x t). \tag{7} \]

Here \( R_T^{(0)} \) is the “classical” resistance of the tunnel junction, \( V_x \) is the applied voltage and the function \( P(t) \) describes energy smearing for tunneling electrons due to interaction with the
fluctuating field $V$ produced by other electrons. This function has the form

$$P(t) = \left\langle e^{i \int_0^t dt' \left( f^{-} V^+ + f^{+} V^- \right)} \right\rangle_{V^+, V^-}, \quad f^{-} = e, \quad f^{+} = e/2. \quad (8)$$

The expression in the exponent (8) has a simple physical meaning: It describes the phase acquired by a tunneling electron in the field $V$. The correlator for this field at the junction is obtained from the action (8) (with $S_{AES} = 0$) in the limit $k \to 0$. Assuming that the capacitance is dominated by that of a tunnel junction $C$ and defining $V^\pm_\omega = i \omega \varphi_\omega / 2e$ one readily finds

$$\langle |V^\pm_\omega|^2 \rangle = \frac{\omega \coth \left( \frac{\omega C}{2} \right)}{G_s + G_s}, \quad G_s = \sigma^{(0)} a^2 / L, \quad (9)$$

where $\sigma^{(0)} = 2e^2 N_0 D$ and $L$ is the length of a disordered conductor shunting the tunnel junction.

Disordered Conductor. Now let us eliminate a tunnel junction and evaluate the conductivity of a disordered metal in the presence of weak localization and interaction effects. Again making use of eq. (49) of (8) and disregarding the so-called interaction correction we obtain

$$\sigma = \sigma^{(0)} - \frac{2e^2 D}{\pi} \int_{\tau_e}^\infty dt \frac{\langle P(t) \rangle_{\text{diff}}}{(4 \pi D t)^{d/2} a^{3-d}} \quad (10)$$

where (see (8) for details)

$$P(t) = \left\langle e^{i \int_0^t dt' \int dr (f^{-} V^+ + f^{+} V^-)} \right\rangle_{V^+, V^-}, \quad (11)$$

$$f^{-}(t', r) = e\delta(r - r_1(t')), \quad f^{+}(t', r) = \frac{1}{2} \left( e[1 - 2n(\xi_1(t'))] \delta(r - r_1(t')) + e[1 - 2n(\xi_2(t'))] \delta(r - r_2(t')) \right). \quad (12)$$

Here $r_{1, 2}(t)$ are electron trajectories with energies $\xi_{1, 2}$, and $n(\xi)$ is the Fermi function. As in the case of a tunnel junction the (Fourier transformed) function $P(t)$ describes energy smearing for an electron propagating in a disordered conductor and interacting with the fluctuating field $V(\omega, k)$ produced by other electrons. The correlators for this field are again defined by the action (8). E.g. for the correlator $\langle V^+ V^+ \rangle$ in the most interesting limit $C(\omega, k) D \ll \sigma_d$ we find

$$\langle |V^+_k(\omega)|^2 \rangle = a^{3-d} \frac{\omega \coth \left( \frac{\omega}{2} \right)}{(\omega C(\omega, k))^2 + \sigma_d k^2}, \quad \sigma_d = \sigma^{(0)} a^{3-d}. \quad (13)$$

We observe an obvious similarity between the results (8) and (11). In both cases the quantum correction to the conductance is expressed in terms of the function $P(t)$ which has essentially the same form (cf. eqs. (8) and (11)) and the same physical meaning: it accounts for the phase accumulation of the electron propagating in the fluctuating field $V$. The electron trajectories differ in these two cases (they are confined to the junction area in the case of tunneling electrons and they are extended in space for electrons propagating in a disordered conductor), however this difference is purely quantitative and is completely unimportant for our comparison. An additional (and also unimportant) difference is that averaging over diffusive trajectories is carried out in eq. (11) while no such averaging is needed in eq. (8). Finally, we observe that the correlators for the fluctuating field (8) and (13) also have essentially the same form. This equivalence is by no means surprising, since both expressions follow from the same fluctuation-dissipation theorem for disordered conductors. The same equivalence exists between the correlators $\langle V^+ V^+ \rangle$ which we do not present here.
Let us now evaluate the expressions (7) and (10) by averaging over the fluctuating field $V$ in (8) and (11). This averaging is Gaussian in both cases and, hence, can be carried out exactly.

**Tunnel junction.** Integrating out the fluctuating field $V$ in (8) we arrive at the well known result:

$$
\ln P(t) = \frac{2}{g} \int_0^\infty \frac{d\omega}{\omega} \frac{1}{1 + (\omega/\omega_0)^2} \left( \coth \left( \frac{\omega}{2T} \right) \left[ \cos(\omega t) - 1 \right] - i \sin(\omega t) \right) 
$$

$$
\simeq -\frac{2\pi}{g} t T - \frac{2}{g} \ln \frac{1 - e^{-2\pi T t}}{2\pi (T/\omega_0)} - \frac{i \pi}{g}, \quad g = 2\pi G_s/e^2.
$$

In the limit $\max(eV_x, T) \ll \omega_0 = G_s/C$ this equation together with (7) yields

$$
GR_T^{(0)} \propto \max[(T/\omega_0)^{3/g}, (eV_x/\omega_0)^{2/g}].
$$

**Disordered Conductor.** Integrating out the fluctuating field $V$ in (11), substituting the time reversed paths $r_1(t)$ and $r_2(t)$ into (12) and averaging the result over diffusive trajectories we find (see also (13))

$$
(P(t))_{\text{diff}} \simeq \exp(-f_d(t)), \quad f_d(t) =
$$

$$
= \frac{4e^2 D^{1-d/2}}{\sigma_d (2\pi)^d} \left( \int \frac{d^d x}{1 + x^4} \right) \int \frac{d\omega_3 d\omega_4}{(2\pi)^2} \left[ \frac{[\omega_3^{d/2-2} (\omega - \omega') \coth \omega - \omega']^2}{\omega^2} \right. 
$$

$$
\left. + \frac{[\omega_3^{d/2-2} \omega \coth \frac{\omega_3}{\omega} + [\omega_3^{d/2-2} \omega' \coth \frac{\omega_3}{\omega'}]^2}{\omega^2 - \omega'^2} \right] (1 - \cos \omega t).
$$

Let us consider the case of quasi-1d conductors. From (10) we obtain

$$
f_1(t) = \frac{e}{\pi \sigma_1} \sqrt{\frac{2D}{\tau_e}} \left( \frac{2e^2}{\pi \sigma_1} \sqrt{\frac{Dt}{\tau_e}} \ln \left( \frac{2\pi t}{\tau_e} \right) - 6 \right), \quad \pi T t \ll 1,
$$

$$
f_1(t) = \frac{2e^2}{\pi \sigma_1} \sqrt{\frac{D}{\pi}} \left\{ \left( \frac{\pi}{2\tau_e} t + \frac{2}{3} T^{3/2} + \frac{\pi \zeta(1/2)}{\sqrt{2}} t\sqrt{T} - \frac{3\zeta(3/2)}{4\sqrt{2}} \frac{1}{\sqrt{T}} \right) \right.
$$

$$
\left. + \sqrt{T} \ln \left( \frac{1}{4T/\tau_e} \right) + O \left( \frac{1}{T^{1/2}} \right) + 2\pi T^{3/2} e^{-2\pi T t} + O(\sqrt{t} e^{-2\pi T t}) \right\}, \quad \pi T t \gg 1,
$$

where $\tau_e = v_F/l$ and $\zeta(x)$ is the dzeta-function. Making use of (10), (17) and (18) we arrive at the standard form of the weak localization correction in 1d: $\sigma - \sigma^{(0)} \simeq (e^2/\pi a^2) \sqrt{D/\tau_\phi}$, where

$$
1/\tau_\phi = (e^2/\pi \sigma_1) \sqrt{2D/\tau_e} \quad \text{for} \ T \ll 1/\sqrt{\tau_e \tau_\phi}
$$

and

$$
1/\tau_\phi \sim (e^2 D^{1/2} T/\sigma_1)^{2/3} \quad \text{for} \ T \gg 1/\sqrt{\tau_e \tau_\phi}.
$$

### 3 Discussion

As follows from the above analysis, in both physical situations considered here the electron-electron interaction plays essentially the same role: Due to the energy exchange between propagating/tunneling electrons and an intrinsic fluctuating electromagnetic field produced by other electrons the electron energy $E$ is smeared even at $T = 0$. This smearing is described by the function $P(t)$ which Fourier transform $P_E$ plays the role of the energy probability distribution. For the electrons tunneling through the barrier at $T = 0$ and $E \geq 0$ one has

$$
P_E = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt e^{iEt} P(t) \sim E^{-1+2/g},
$$

while for quasi-1d disordered conductors the analogous function evaluated on the time reversed diffusive paths (again at $T = 0$ and $E \geq 0$) reads

$$
P_E = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt e^{iEt} \langle P(t) \rangle_{\text{diff}} \sim \frac{\tau_\phi}{1 + E^2 \tau_\phi^2}.
$$
Note that without interaction the function \( P_E \) reduces to \( P_E = \delta(E) \) in both cases (13) and (20). Electron-electron interaction is responsible for smearing of the energy distribution \( P(E) \) and for a nonzero decoherence rate \( 1/\tau_\varphi \) at \( T=0 \).

It is also important to emphasize that in both cases a nonperturbative analysis of exactly the same type was used in order to account for the interaction effects. These effects can be easily mistreated or even completely missed by insufficient approximations. A clear illustration of this point is provided by the “\( P(E) \)-theory” results (14), (15). E.g. it is obvious that a simple perturbative expansion in \( 1/g \) in (14), (13) (essentially equivalent to the short time expansion) is insufficient and would lead to divergent results at small \( T \) and \( V_x \) because of nonanalyticity in \( 1/g \). On the other hand, the long time expansion (essentially equivalent to the golden-rule-type approximation) would also yield an incorrect result at low \( T \): E.g. smearing (13) at \( T \to 0 \) would not be captured if one would expand the result (14) to any finite order in \( 1/Tt \). This procedure would yield an incorrect conclusion that at low \( T \) the function \( P_E \) has an effective width \( \propto T \) while the remaining term in (14) would be a \( t \)-independent constant \( \sim \ln(\omega_C/T) \). A truly nonperturbative procedure is needed to obtain the correct results (14), (15).

A similar conclusion can be drawn for the weak localization correction in the presence of interactions. In this case the full expression for the \( P \)-function has the form \( \langle P(t) \rangle_{\text{diff}} = A_d(t) \exp(-f_d(t)) \), where \( A_d(t) \) also depends on the interaction. On the relevant time scales \( t \sim \tau_\varphi \) the pre-exponent \( A_d(t) \) depends weakly on time and is completely unimportant for \( \tau_\varphi \) determined solely by the function \( f_d(t) \) in the exponent. However, the pre-exponent \( A_d(t) \) does contribute to a short time expansion of \( \langle P(t) \rangle_{\text{diff}} \) (equivalent to the expansion in the interaction). In the first order of this expansion and at \( T=0 \) the leading term \( \propto t \) from the exponent (17) will be exactly cancelled by the analogous term from \( A_d(t) \). The remaining term \( \propto \sqrt{t} \ln t \) from (17) which also produces (weaker) dephasing is not cancelled (13) but one can mistreat or even miss this term completely if in addition one would expand in \( 1/Tt \) or use the golden rule approximation. In the latter case one would obtain an incorrect conclusion \( 1/\tau_\varphi = 0 \) at \( T = 0 \).

Finally, let us mention that the “\( P(E) \)-theory” is known to well describe the results of various experiments (see e.g. (13)). Here we demonstrated that the low temperature saturation of the junction conductance \( \varphi \) and of the time \( \tau_\varphi \) extracted from the magnetoconductance measurements is of exactly the same physical nature. In both cases it is caused by the electron-electron interaction.

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