Cosmic evolution in novel-Gauss Bonnet Gravity

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In this short paper we investigate any non-trivial effect the novel Gauss-Bonnet gravity may give rise in the cosmic evolution of the Universe in four spacetime dimensions. We start by considering a generic Friedmann-Lemaître-Robertson-Walker (FLRW) metric respecting homogeneity and isotropicity in arbitrary space-time dimension $D$. The metric depends on two functions: scale factor and lapse. Plugging this metric in novel Einstein-Gauss-Bonnet (EGB) gravity action, doing an integration by parts and then take the limit of $D \to 4$ give us a dynamical action in four spacetime dimensions for scale-factor and lapse. The peculiar rescaling of Gauss-Bonnet coupling by factor of $D - 4$ results in a non-trivial contribution in the action of the theory. In this paper we study this action. We investigate the dynamics of scale-factor and behavior of lapse in an empty Universe (no matter). Due to complexity of the problem we study the theory to first order in Gauss-Bonnet coupling and solve system of equation to the first order. We compute the first order correction to the on-shell action of the empty Universe and find that its sign is opposite of the leading order part. We discuss it consequences.

I. INTRODUCTION

General relativity although is a very good theory of gravity which works over a large range of energy, but it seems to be lacking a complete description of gravity. It is expected to get modified at short distances and/or in deep infrared. At short distances such modification are motivated as the quantum theory of GR has problems: theory is non-renormalizable and lacks the ability to predict.\textsuperscript{1, 7} In deep infrared the observations supporting dark-matter and dark energy\textsuperscript{8–11} are key motivations for modification of gravity at long distances\textsuperscript{12}.

It is generally seen that any kind of modification introduced to Einstein-Hilbert gravity leads to some undesirable problems. For example at high-energies to address issues of renormalizability, a higher-derivative modification although resolves renormalizability\textsuperscript{13–15} but introduces a nasty problem of non-unitarity. Some efforts have been made to tackle it\textsuperscript{[16, 19]} and the direction has seen a recent uprising where the interest has been rekindled following works in asymptotic safety approach\textsuperscript{20–22} and ‘Agravity’\textsuperscript{23}. Issues of unitarity arises in such higher-derivative gravity theories as the equation of motion contains more than two time-derivatives of metric. Lovelock gravity\textsuperscript{24–26} are a special class of higher-derivative gravity theories\textsuperscript{[20–22]} and ‘Agravity’\textsuperscript{23}.

In four spacetime dimension the Lovelock gravity\textsuperscript{27} action is following:

\begin{equation}
S = \frac{1}{16\pi G} \int d^D x \sqrt{-g} \left[ -2\Lambda + R + \frac{\alpha}{D-4} \left( R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) \right],
\end{equation}

where $G$ is the Newton’s gravitational constant, $\Lambda$ is cosmological constant term and $\alpha$ is the Gauss-Bonnet coupling. The Gauss-Bonnet coefficient is defined with a $(D - 4)$ factor in denominator. $G$ has mass dimension $M^{2-D}$, $\Lambda$ has mass dimension $M^2$, while $\alpha$ has mass dimension $M^{-2}$.

By now the action in eq. (1) has been explored in the context of spherical black holes\textsuperscript{30–32}, star-like solutions\textsuperscript{33}, radiating solutions\textsuperscript{34}, collapsing solutions\textsuperscript{40}, even extensions to more higher-curvature Lovelock gravity theories\textsuperscript{40, 41}. There are already a numerous investigation on the thermodynamic behavior\textsuperscript{42–44} of these objects quasi-normal modes\textsuperscript{34}, strong cosmic censorship\textsuperscript{35}, bending of light\textsuperscript{36}, lower-dimensional solutions\textsuperscript{37–38}. Using the recent observations of black-hole shadows constraints on its coupling parameter has also been investigated\textsuperscript{45–48}.

It is noticed in\textsuperscript{49, 52} that under an integration by parts the novel Gauss-Bonnet gravity action gets rid of $(D - 4)$ factors, thereby leading to a well-defined $D \to 4$ limit at the level of action. Their strategy is inspired by Kaluza-Klein compactification where they decompose the metric in two parts $\mathcal{M}_D = \mathcal{M}_4 \otimes \mathcal{M}_{D-4}$:
four-dimensional manifold $\mathcal{M}_4$ and extra-dimension piece $\mathcal{M}_{D-4}$. They write their action under this decomposition and smoothly remove the extra-dimension piece by taking limit. One then gets Horndeski type gravity. A similar study was done in [53] using ADM decomposition. There they noticed that for a well-defined limit and a consistent theory in four dimensions one either break (a part of) the diffeomorphism invariance or have an extra degree of freedom, in agreement with the Lovelock theorem [52].

Inspired by the above work we decided to investigate the cosmological settings in the novel-Gauss-Bonnet gravity. We start by considering the most general metric respecting spatial homogeneity and isotropy in $D$-spacetime dimensions. This is a generalisation of FLRW metric in $D$-dimensions consisting of two unknown time-dependent functions: lapse and scale-factor. This is different from the KK decomposition that has been studied in [49] [52] [53]. Plugging the generalised FLRW metric in action of theory and performing an integration by parts results in action where $D \to 4$ limit can be smoothly taken without encountering divergences. This leaves us with a well-defined action for scale factor and lapse, which gets non-trivial corrections from Gauss-Bonnet gravity part. In this paper we study this and investigate the dynamics of scale-factor and lapse in empty matter free Universe.

The paper is organized as follows: in section II we write the generic FLRW metric and compute its action, in section III we study the action for flat Universe and study the equation of motion perturbatively. We conclude with a discussion in section IV.

**II. FLRW METRIC**

We start by considering generalization of FLRW metric in arbitrary spacetime dimension whose dimensionality is $D$. We write the metric in polar co-ordinates \( \{t_p, r, \theta, \cdots \} \)

\[
ds^2 = -N^2_p(t_p)dt_p^2 + a^2(t_p)\left[\frac{dr^2}{1-kr^2} + r^2d\Omega_{D-2}^2\right],
\]

where $N_p(t_p)$ is lapse function, $a(t_p)$ is scale-factor, $k = (0, \pm 1)$ is the curvature, and $d\Omega_{D-2}$ is the metric corresponding to unit sphere in $D-2$ spatial dimensions. This metric is conformally related to flat metric and hence its Weyl-tensor $C_{\mu\nu\rho\sigma} = 0$. The nonzero entries of Riemann tensor [54] [55] are

\[
R_{00} = -(D-1)\left(\frac{\ddot{a}}{a} - \frac{\dot{a}N_p}{aN_p}\right),
\]

\[
R_{ij} = \left[\frac{(D-2)(kN_p^2 + \dot{a}^2)}{N_p^2a^2} + \frac{\dot{a}N_p - \dot{N}_p}{aN_p}\right]g_{ij},
\]

while the Ricci-scalar is given by

\[
R = 2(D-1)\left[\frac{\dot{a}N_p - \dot{N}_p}{aN_p^3} + \frac{(D-2)(kN_p^2 + \dot{a}^2)}{2N_p^2a^2}\right].
\]

The good thing about FLRW metric is that it is Weyl-flat which implies that Riemann tensor can be expressed in terms of Ricci-tensor and Ricci scalar as following

\[
R_{\mu\nu\rho\sigma} = \frac{R_{\mu\rho\sigma\nu} - R_{\mu\nu\sigma\rho} + R_{\nu\sigma\rho\mu} - R_{\nu\rho\sigma\mu}}{D-2} - \frac{R(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\rho\nu})}{(D-1)(D-2)}.
\]

This identity is valid for all conformally flat metrics and allows one to express

\[
R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{4}{D-2}R_{\mu\nu}R^{\mu\nu} - \frac{2R^2}{(D-1)(D-2)}.
\]

By making use of this identity for conformally flat metrics in the Gauss-Bonnet action one can obtain a simplified action of the theory. In such cases we have

\[
\int d^Dx \sqrt{-g} \left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right) = \frac{D-3}{D-2} \int d^Dx \sqrt{-g} \left(-R_{\mu\nu}R^{\rho\nu} + \frac{DR^2}{D-1}\right).
\]

On plugging the FRW metric of eq. (2) in the action in eq. (1), one can get an action for $a(t_p)$ and $N_p(t_p)$. On doing integration by parts it is noticed that the resulting terms are independent of factors of $(D-4)$, which cancels off. This resulting action in $D = 4$ is given by

\[
S = \frac{\pi k^{-3/2}}{8G} \int dt_p \left[(3k - \Lambda a)N_p a - \frac{3aa'}{N_p} \right] + 3a\left(\frac{kN_p^2 + \dot{a}^2}{N_p^3} + \frac{4ka^2}{N_p} + \frac{4a^2}{N_p}\right),
\]

where $(\cdot)'$ denotes derivative with respect to $t_p$. Here one notices that the Gauss-Bonnet term gives a non-trivial contribution in $D = 4$ which is possible due to particular style of defining the Gauss-Bonnet coupling parameter. With this action one can do further analysis. To make the theory more appealing one can rescale lapse and scale factor in following manner

\[
N_p(t_p)dt_p = \frac{N(t)}{a(t)}dt, \quad q(t) = a^2(t).
\]
This transformation changes our original metric in eq. (2) into following
\[
ds^2 = -\frac{N^2}{q(t)} dt^2 + q(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2_{D-2} \right],
\]
and our action in eq. (11) changes to following simple form.
\[
S = \frac{\pi k^{-3/2}}{16G} \int dt \left[ (6k - 2\Lambda q) N - \frac{3q^2}{2N} \right.
\]
\[
+ \frac{3\alpha}{8N^3q} (4kN^2 + \dot{q}^2)(4kN^2 + 5q^2),
\]
where () represent derivative with respect to \( t \). It should be noticed that the action doesn’t contain any derivative of \( N \), which happens as we have performed integration by parts previously. This is an interesting higher-derivative action which only depends on \( q \), \( \dot{q} \) and \( N \). This action further acquires simplicity when \( k = 0 \).

III. FLAT SPACE \( k = 0 \)

The action acquires a simplified structure in flat space \( (k = 0) \), this is also motivated physically as our physical Universe is also observed to be spatially flat to a high accuracy. The action in this case is given by,
\[
S_{k=0} = \frac{V_3}{16\pi G} \int dt \left[ -2\Lambda qN - \frac{3q^2}{2N} + \frac{15\alpha q^4}{8N^3 q} \right],
\]
where \( V_3 \) is the volume corresponding to 3-dimensional space. As there is no derivative term corresponding to \( N \) appearing in action so variation of action with respect to \( N \) will result in a constraint. On the other hand, variation of action with respect to \( q(t) \) will give evolution equation for \( q(t) \). In the gauge \( \dot{N} = 0 \) we have \( N(t) = N_c \), and the equation of motion for \( q(t) \) is given by
\[
-2N_c\Lambda + \frac{3\dot{q}}{N_c} + \frac{45\alpha}{8N^3_c} \left( \frac{\dot{q}^4}{q^2} - \frac{4\dot{q}^2\ddot{q}}{q} \right) = 0.
\]
This is a higher-derivative equation. The higher-derivative contribution is novel here which doesn’t arise if the Gauss-Bonnet coupling wasn’t rescaled by factor of \( (D - 4) \). In principle one has to solve for \( q(t) \) from the above equation for the boundary conditions
\[
q(t = 0) = b_0, \quad q(t = 1) = b_1.
\]
Then plug the \( q(t) \)-solution back in to the action in eq. (13) where now we are in constant-\( N \) gauge, integrate with respect to time and obtain an action for the constant lapse \( N_c \). One then look for saddle points solution for \( N_c \) which are obtained by varying this action with respect to \( N_c \). This will be the full saddle point solution of theory.

In practice this is not always possible. In the current case the evolution equation for \( q(t) \) is quite complicated and involve higher-derivatives. We therefore try to solve the system perturbatively. We start by expanding \( q(t) \) in powers of \( \alpha \).
\[
q(t) = q_0(t) + \alpha q_1(t) + \cdots,
\]
where \( q_0 \) is zeroth-order solution while \( q_1 \) is the first order solution.

A. zeroth-order

At the zeroth order we have
\[
\ddot{q} = \frac{2N^2\Lambda}{3}.
\]
It is a linear second order ODE which can be solved analytically exactly. The solution obeying the boundary condition stated in eq. (15) is given by
\[
q_0(t) = \frac{\Lambda N^2}{3} (t^2 - t) + b_0(1 - t) + b_1 t.
\]
Plugging it back into action in eq. (13) and integrating with respect to \( t \) one gets zeroth-order part of action for \( N_c \). This is given by
\[
S_0 = \frac{V_3}{16\pi G} \left[ -\frac{3(b_0 - b_1)^2}{2N_c} - (b_0 + b_1)\Lambda N_c + \frac{N^3_c\Lambda^2}{18} \right].
\]
Once we have the action for \( N_c \), one can obtain saddle points of this by varying it with respect to \( N_c \) and looking for extrema. Then we see that \( \partial S_0/\partial N_c = 0 \) on solving gives \( N_0 \). Its solutions obey the equation
\[
\frac{3(b_0 - b_1)^2}{2N^2_0} - (b_0 + b_1)\Lambda + \frac{N^3_0\Lambda^2}{6} = 0.
\]
This is a quadratic in \( N^2_0 \) and will consist of four solutions which are given by
\[
(N_0)_{\pm, \pm} = \pm \sqrt{\frac{3}{\Lambda}} \left( b_1 \pm \sqrt{b_0} \right).
\]
At the zeroth order we notice that there are four saddle point solutions. At this level we don’t receive any correction from the Gauss-Bonnet term and they agree with the known saddles in the context of Lorentzian quantum cosmology \[58, 59\]. Corresponding to each \( (N_0)_{\pm, \pm} \) we have corresponding \( (q_0)_{\pm, \pm} \). Each one of them lead to a different FLRW metric. Corresponding to each of them we have an on-Shell action, which is given by
\[
S_{0\text{on-shell}} = \pm \frac{V_3}{4\pi G} \sqrt{\frac{\Lambda}{3}} \left( b^3_1 \pm b^3_0 \right).
\]
B. First order

At first order in $\alpha$ the equations becomes more involved. The evolution of $q(t)$ at first order is dictated by following equation

$$\ddot{q}_1 = -\frac{15}{8N_c^2} \left( \frac{\dot{q}_0^4}{q_0^4} - \frac{4\dot{q}_0^2\ddot{q}_0}{q_0^4} \right),$$

(23)

where $q_0$ is the zeroth order solution to $q(t)$ obtained above. The boundary conditions for $q_1(t)$ can be obtained from eq. (15) and those of $q_0$. This implies that

$$q_1(t = 0) = q_1(t = 1) = 0.$$  

(24)

The ODE for $q_1$ satisfying these boundary conditions can be solved and its solution is given by

$$q_1(t) = \frac{5N_c^2\Lambda^2(t - 1)}{3} - \frac{5U}{4N_c^2} \left[ \left( b_0 - b_1 + \frac{N_c^2\Lambda}{3} \right)(t - 1) \right. \\
\times \tan^{-1}\left( \frac{3(b_0 - b_1) + N_c^2\Lambda}{U} \right) + \left( b_0 - b_1 - \frac{N_c^2\Lambda}{3} \right) \\
\times t \tan^{-1}\left( \frac{3(b_1 - b_0) + N_c^2\Lambda}{U} \right) + (b_1 - b_0) \\
+ \frac{N_c^2\Lambda(2t - 1)}{3} \tan^{-1}\left( \frac{3(b_1 - b_0) + N_c^2\Lambda(1 - 2t)}{U} \right).$$

(25)

where

$$U = \sqrt{6(b_0 + b_1)^2N_c^2\Lambda - 9(b_0 - b_1)^2 - N_c^4\Lambda^2}.$$  

(26)

Once we have obtained the first order correction to $q(t)$, we can plug it back in action in eq. (13) and perform the $t$-integration. This results in a first order corrected action for $N_c$.

$$S_1 = S_0 + \frac{V_{3\alpha}}{16\pi G} \left[ \frac{5(b_0 - b_1)^2\Lambda}{N_c} - \frac{5(b_0 + b_1)N_c\Lambda^2}{3} \\
+ \frac{10N_c^2\Lambda^3}{27} + \frac{15U^3}{108N_c^3} \tan^{-1}\left( \frac{3(b_0 - b_1) + N_c^2\Lambda}{U} \right) \\
+ \tan^{-1}\left( \frac{3(b_1 - b_0) + N_c^2\Lambda}{U} \right) \right] + \cdots.$$  

(27)

This first order corrected action can be varied with respect to $N_c$ again to obtain the first order correction to saddle points. To obtain this we substitute

$$N_c = N_0 + \alpha N_1 + \cdots.$$  

(28)

Then $N_1$ can be obtained from

$$\frac{\partial S_1}{\partial N_c} \bigg|_{N_c \to (N_0 + \alpha N_1)} = 0.$$  

(29)

From this equation we have

$$N_1 = -\frac{5N_0\Lambda}{2},$$  

(30)

where $N_0$ is given by eq. (20). Once the expression of $N_0$ given in eq. (20) is plugged back in above, we get the full first order corrected lapse which is given by

$$N_c = s_1 \sqrt{\frac{3}{\Lambda} \left( 1 - \frac{5\alpha\Lambda}{2} + \cdots \right) \left( \sqrt{b_1 + s_2\sqrt{b_0}} \right)},$$

(31)

where $s_{1,2} = \{\pm, \pm\}$. The first order corrected on-shell action is given by

$$S_1^{\text{on-shell}} = \left( 1 - \frac{5\alpha\Lambda}{6} + \cdots \right) S_0^{\text{on-shell}},$$

(32)

where $S_0^{\text{on-shell}}$ is the zeroth order on-shell action given in eq. (22).

IV. DISCUSSION AND CONCLUSION

In this short paper we investigate the non-trivial effects that arise in novel-Gauss-Bonnet gravity, where the coefficient in front of the Gauss-Bonnet term in action has been appropriately rescaled by factor of $(D - 4)$. We investigate cosmological spacetimes respecting homogeneity and isotropicty. We consider a most general spacetime respecting such symmetry in $D$-spacetime dimensions. It consists of two functions: scale factor $a(t_p)$ and lapse $N_p(t_p)$. Plugging this in the gravitational action, doing integration by parts and taking the limit $D \to 4$, we are able to obtain a dynamical action for the scale factor $a(t_p)$. We notice that an integration by parts (and discarding a surface term) allows us to obtain an action for the resulting theory where the $D - 4$ terms cancels off completely. This allows us to perform a smooth $D \to 4$ limit without witnessing divergences. Similar observations were also made in [49, 53] where the authors considered KK decomposition of metric. This interesting observation indicates that probably something deep is happening which requires more careful analysis.

Once the action of theory is obtained one can study various aspects of it. We notice that a redefinition of scale factor and lapse lead us to a simplified theory for the new variables: $q(t)$ and $N(t)$. The resulting action is a function of $N$, $q$ and $\dot{q}$ only. The action doesn’t contain any derivative term for the lapse, indicating that its dynamical nature comes from the dynamics of $q(t)$ indirectly. On varying the action with respect to $q(t)$ we obtain equation of motion. This is a non-linear higher-derivative ODE, and it might be hard to find an analytic closed form solution of same. We solve the system perturbatively, doing perturbation in the Gauss-Bonnet coupling. We compute the first order correction to Einstein-Hilbert solution for $q$ and $N$, for the case of empty Universe. Using these we also compute the first order correction to the on-shell action. We notice that the sign of the first order correction to on-shell action is negative. This can have serious consequences in euclidean path-integral as such correction where $\alpha > 0$ will lead to exponential growth. This perhaps maybe considered a shortcoming of this theory.
Moreover, this is will also indicate that only one sign of coupling is allowed.

Given that we have an action of theory in four space-time dimensions it is then natural to investigate the quantum gravity path-integral and the corrections it achieves in the novel Gauss-Bonnet gravity. Full gravitational path-integral although quite complicated to deal with, still many nice features of it can be understood when degree of freedoms are reduced substantially. We will present this in our next publication [60].

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