Scheduling of Hard Real-Time Multi-Thread Periodic Tasks

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Abstract

In this paper we study the scheduling of parallel and real-time recurrent tasks. Firstly, we propose a new parallel
task model which allows recurrent tasks to be composed of several threads, each thread requires a single processor
for execution and can be scheduled simultaneously. Secondly, we define several kinds of real-time schedulers that
can be applied to our parallel task model. We distinguish between two scheduling classes: hierarchical schedulers
and global thread schedulers. We present and prove correct an exact schedulability test for each class. Lastly, we also
evaluate the performance of our scheduling paradigm in comparison with Gang scheduling by means of simulations.

1. Introduction

In this research, we consider the preemptive scheduling of hard real-time tasks on identical multiprocessor
platforms (see [3, 2]). We deal with parallel real-time tasks, the case where each task instance (process in the
following) may be executed on different processors simultaneously. More specifically, each process is composed
of several independent threads, each thread requires one processor to be executed, consequently the process
can progress upon several processors simultaneously. In this research we study the schedulability problem of
recurring multi-thread tasks.

Nowadays, the design of parallel/multi-thread programs is common thanks to parallel programming paradigms
like Message Passing Interface (MPI [9, 10]). Even better, sequential programs can be parallelized using tools
like OpenMP (see [4] for details).

Related Work. The literature concerning the scheduling of hard real-time and parallel recurring tasks is rela-
tively poor. We can only report few models of parallel tasks and few results (schedulers and schedulabil-
ity/feasibility tests). Manimaran et al. in [16] consider the non-preemptive EDF scheduling of periodic tasks.
Han et al. in [12] considered the scheduling of a (finite) set of real-time jobs allowing job parallelism while we
consider the scheduling of either an infinite set of jobs (actually processes in our terminology) or equivalently
a set of periodic tasks. In a seminal work we contributed to the feasibility problem of parallel tasks. In [5]
we provided a task model which integrates job parallelism. We proved that the time-complexity of the feasibil-
ity problem of these systems is linear relatively to the number of (sporadic) tasks. In [14] Lakshmanan et al.
consider the fork-join task model where each task is an alternate sequence of sequential code and \( m \) parallel
segments. They provided a partitioning algorithm and a competitive analysis for EDF and the fork-join task
model. Regarding the schedulability of recurrent real-time tasks, and to the best of our knowledge, we can only
report results about the Gang scheduling, where the execution requirement of processes corresponds as a \( C \times v \)
rectangle, with the interpretation that a process requires exactly \( v \) processors simultaneously for a duration of
\( C \) time units. Kato et al. (see [13] for details) considered the EDF Gang scheduling and provided a sufficient
schedulability condition. We studied Fixed Task Priority (FTP) Gang scheduling [8] and we provided an exact
schedulability test for periodic tasks.
This Research. In this paper we introduce a more realistic parallel task model, i.e., we study the scheduling of periodic and (parallel) multi-thread tasks. Our main contribution is exact schedulability tests for Fixed Subprogram Priority (FSP) and (FTP, FSP) schedulers. Additionally, we show that Gang scheduling and multi-thread scheduling are incomparable and we present an empirical study which shows that, in most of the cases, multi-thread scheduling dominates Gang scheduling. Last but not least, in the further work section, we consider a realistic extension of our task model which allows tasks to be a sequence of parallel phases (each one having its own number of threads) and we present a first negative result.

Paper Organization. This paper is organized as follows. Section 2 provides our formal task model and important definitions. In Section 3 we present a taxonomy of schedulers dedicated to our parallel task model. We present exact schedulability tests in Section 4 for two classes of schedulers. In Section 5 we present an empirical study which shows that in most of the cases thread scheduling dominates Gang scheduling. Lastly, in Section 6 we conclude and consider a realistic extension of our parallel task model and present a first negative result.

2. Formal model and definitions

In this work we consider periodic multi-thread tasks, each task $\tau_i$ is characterized by the tuple

$$(O_i, \{q^1_i, q^2_i, \ldots, q^n_i\}, D_i, T_i),$$

where

- $O_i$ is the arrival instant, i.e., the moment of the first activation of the task since the system initialization;
- $\{q^1_i, q^2_i, \ldots, q^n_i\}$ is the set of the $v_i$ subprograms of $\tau_i$; at run-time these subprograms generate threads which can be executed simultaneously, i.e., we allow task parallelism;
- each subprogram $q^j_i$ (1 $\leq$ $j$ $\leq$ $v_i$) is characterized by an individual worst-case execution time $C^j_i$;
- $T_i$ is the period, i.e., the exact inter-arrival time between two successive activations of the task;
- $D_i$ is the relative deadline, i.e., the time by which the current instance of the task has to finish execution relatively to its arrival.

Throughout this paper, all timing characteristics in our model are assumed to be non-negative integers, i.e., they are multiples of some irreducible time interval (for example the CPU clock cycle is the smallest indivisible CPU time unit).

Task/process & subprogram/thread. In this work we will distinguish between off-line and run-time entities. We consider that a task is defined off-line while its instance exists only at run-time under the denomination process. In the same vein we consider that a subprogram is defined off-line while its instance exists only at run-time under the denomination thread. Consequently the scheduler manages processes and/or threads. Meanwhile the process or thread priority can (or cannot) be based on the static task and subprogram characteristics.

Each task system $\tau$ is composed of $n$ periodic and parallel such tasks: $\tau = \{\tau_1, \ldots, \tau_n\}$. The deadline of each task is less than or equal to the period: $\forall i, D_i \leq T_i$ (constrained deadline model). As each task $\tau_i$ has its own offset $O_i$, the task systems considered are asynchronous.

Since tasks are periodic, their subprograms have a periodic behavior as well. The $j$th thread of the $k$th process of $\tau_i$ is characterized by the following parameters: an arrival time $a^k_j = O_i + (k-1) \cdot T_i$, an execution demand $e^k_j = C^j_i$ and an absolute deadline $d^k_j = a^k_j + D_i$.

The $k$th process of $\tau_i$ is characterized by an arrival time $a_k = O_i + (k-1) \cdot T_i$, an execution demand expressed by the parallel execution demands of its threads $e_k = (e^1_k, \ldots, e^n_k)$ and an absolute deadline $d_k = a_k + D_i$.

A task is said to be active if it has a process with unfinished execution demand.

A task is characterized by the measure called utilization: $u_i = \frac{\sum_{j=1}^{v_i} C^j_i}{T_i}$. This measure represents the portion of the platform capacity ($u_i \leq m$) requested by the task $\tau_i$, when executing. We also denote by $U \overset{\text{def}}{=} \sum_{i=1}^{n} u_i$ the total system utilization. In the following, $P$ denotes the least common multiple of all the periods in the task system $\tau$: $P = \text{lcm}\{T_1, \ldots, T_n\}$.

The considered multiprocessor platform contains $m$ unit-capacity processors.
Gang scheduling versus Thread Scheduling. Figure 1 illustrates a Gang and a thread scheduling for the “same” task set: \( \tau_1 = (0, \{1\}, T_1, D_1) > \tau_2 = (0, \{1, 1\}, T_2, D_2) \). Focusing on \( \tau_2 \), Gang scheduling has to manage the rectangle \( C \times v = 1 \times 2 \) while thread scheduling has to manage two 1-unit length threads.

From our point of view thread scheduling paradigm resolves the following Gang scheduling drawbacks:

1. As exhibited in [8] Gang scheduling suffers from priority inversion, i.e., a lower priority task can progress while an higher priority active task cannot.

2. The number of processors required by a task must be not larger than the platform size.

3. Because of the requirement that the task must execute on exactly upon \( v \) processors simultaneously very often many processors may be left idle while there is active tasks.

4. As shown in [8] Gang FTP schedulers are not predictable. Multi-thread FTP schedulers are proven predictable in Section 4.

In this proposal, the exact schedulability tests are based on feasibility intervals with the following definition.

Definition 1 (Feasibility interval). For any task system \( \tau = \{\tau_1, \ldots, \tau_n\} \) and any \( m \)-unit capacity multiprocessor platform, the feasibility interval is a finite interval such that if no deadline is missed while considering only the processes in this interval no deadline will ever be missed.

In this paper we consider that the scheduling is priority-driven: the threads are assigned distinct priority levels. According to these priority levels the scheduler decides at each time \( t \) what will be executed on the multiprocessor platform at that time instant: the \( m \) highest (if any) priority threads will be executed simultaneously on the given platform. The thread-processor assignment is univocally determined by the following rule: “higher the priority, lower the processor index”. If less than \( m \) threads are active the processors with the higher indexes are left idle.

We consider preemptive scheduling: a higher priority thread can interrupt the executing lower priority thread.

3. Taxonomy of schedulers

In this work we consider two classes of real-time schedulers for our parallel task model: hierarchical schedulers and global thread schedulers.

- At top-level hierarchical schedulers manage processes with a process-level scheduling rule and use a second (low-level) scheduling rule to manage threads within each process.

- Global thread schedulers assign priorities to threads regardless of the tasks/subprograms that generated them.

In order to define rigorously our Hierarchical and Global schedulers we have to introduce the following schedulers.

Definition 2 (Fixed Task Priority (FTP)). A fixed task priority scheduler assigns a fixed and distinct priority to each task before the execution of the system. At run-time each process priority corresponds to its task priority.

Among the FTP schedulers we can mention Rate-Monotonic (RM) [15] and Deadline-Monotonic (DM) [1].

Definition 3 (Fixed Process Priority (FPP)). A fixed process priority scheduler assigns a fixed and distinct priority to processes upon arrival. Each process preserves the priority level during its entire execution.

The Earliest Deadline First (EDF) [15] scheduler is an example of FPP scheduler.
**Definition 4** (Dynamic Process Priority (DPP)). A dynamic process priority scheduler assigns, at each time $t$, priorities to the active processes according to their run-time characteristics. Consequently, during its execution, a process may have different priority levels.

The *Least Laxity First* (LLF) scheduler is a DPP scheduler since the laxity is a dynamic process characteristic (see [7] for details).

In the same vein, the following schedulers can be defined at thread level:

**Definition 5** (Fixed Subprogram Priority (FSP)). A fixed subprogram priority scheduler assigns a fixed and distinct priority to each subprogram before the execution of the system. At run-time each thread priority corresponds to its subprogram priority.

An example of FSP scheduler is the *Longest Subprogram First* scheduler.

**Definition 6** (Fixed Thread Priority (FThP)). A fixed thread priority scheduler assigns a fixed and distinct priority to threads upon arrival. Each thread preserves the priority level during its entire execution.

As far as we know, no FThP scheduler can be defined based only on the characteristics of the tasks in our model, at least schedulers different than the class FSP.

**Definition 7** (Dynamic Thread Priority (DThP)). A dynamic thread priority scheduler assigns, at time $t$, priorities to the existing threads according to their characteristics. During its execution, a thread may have different priority levels.

An example of DThP is LLF applied at thread level.

### 3.1. Hierarchical schedulers

Hierarchical schedulers are built following the next two steps:

1. at process level, one of the following schedulers is chosen in order to assign priorities to process: FTP, FPP and DPP.

2. for assigning priorities within process, one of the following schedulers will be chosen: FSP, FThP, DThP.

In the following an hierarchical scheduler will be denoted by the couple $(\alpha, \beta)$, where $\alpha \in \{\text{FTP, FPP, DPP}\}$ and $\beta \in \{\text{FSP, FThP, DThP}\}$.

### 3.2 Global thread schedulers

As global thread schedulers, the FSP, FThP and DThP schedulers can be applied to a set of subprograms or threads regardless of the task that they belong to.

Notice that some global thread schedulers are identical to some hierarchical ones. E.g., a total order between threads (i.e., a FThP scheduler) can “mimic” any hierarchical (FTP, FThP) scheduler.

### 4. Exact schedulability tests

In this section, we will present two exact schedulability tests: one for FSP and one for (FTP, FSP) schedulers.

#### 4.1. Exact FSP schedulability test

The first step into defining the schedulability test for FSP schedulers is to prove that their schedules are periodic. The proof is based on the periodicity of FTP schedules when the FTP is applied to systems $\tau'$ with the following task model: a task $\tau'_k \in \tau'$ is characterized by $(O_k, C_k, D_k, T_k)$ [6]. This model will be called in this paper the *sequential task model*. The periodicity of FTP schedules for the sequential task model is stated in Theorem 8. The tasks $\tau'_k$ in $\tau'$ ($1 \leq k \leq n$) are ordered by decreasing priority: $\tau'_1 > \cdots > \tau'_n$.

**Theorem 8** ([6]). For any preemptive FTP scheduling algorithm A, if an asynchronous constrained deadline system $\tau'$ = {$\tau'_1, \ldots, \tau'_n$} is A-feasible, then the A-schedule of $\tau'$ on $m$-unit capacity multiprocessor platform is periodic with a period of $P$ starting from instant $S_n$ where $S_i$ is defined as:

$$
S_1 \equiv O_1,
S_i \equiv \max\{O_i, O_i + \lceil \frac{S_{i-1} - O_i}{T_i} \rceil \cdot T_i\},
\forall i \in \{2, 3, \ldots, n\}.
$$

(Assuming that the execution times of each task is constant.)
In the following, we consider a task system $\tau$ with $r \equiv \sum_{i=1}^{n} v_i$ subprograms; each subprogram $q^i_j$ of the task $\tau_i$ is characterized by $(O_i, C_i, T_i, D_i)$. A FSP scheduler is used to assign priorities to the $r$ subprograms. As these priorities are assigned regardless of the tasks to which the subprograms belong to, we can consider a simpler notation: each subprogram $q^i_j$ is characterized by $(O_i, C_i, T_i, D_i)$ $(1 \leq \ell \leq r)$; if $q^i_\ell$ corresponds to the $j^{th}$ subprogram of $\tau_i$, then $O_\ell = O_i, C_\ell = C_i^j, D_\ell = D_i, T_\ell = T_i$. In the following we assume without loss of generality that subprograms are ordered by FSP decreasing priority: $q^1 > \cdots > q^r$.

**Theorem 9.** For any preemptive FSP scheduling algorithm $A$, if an asynchronous constrained deadline system $\tau$ containing $r$ subprograms (regardless of the tasks they belong to) is $A$-feasible, then the $A$-schedule of $\tau$ on $m$ unit-capacity multiprocessor platform is periodic with a period of $P$ starting from instant $S^*_r$, where $S^*_r$ is defined as follows:

\[
\begin{align*}
S^*_1 & \equiv O_1, \\
S^*_j & \equiv \max\{O_j, O_j + \left\lfloor \frac{S^*_{j-1} - O_j}{r_j} \right\rfloor \cdot T_j\}, \\
& \quad \quad \forall j \in \{2, 3, \ldots, r\}.
\end{align*}
\]  
(Assuming that the execution times of each subprogram is constant.)

**Proof.** When using FSP, a parallel task system $\tau$ with $n$ tasks and $r$ subprograms can be seen a sequential task system $\tau'$ which contains $r$ sequential periodic tasks $\tau' = \{\tau'_1, \ldots, \tau'_r\}$ such that the task $\tau'_\ell$ $(1 \leq \ell \leq r)$ has the same characteristics as the $\ell^{th}$ subprogram of $\tau$ $(\tau'_\ell : (O_\ell, C_\ell, T_\ell, D_\ell))$. From the FSP priority assignment on $\tau$, a FTP priority assignment for $\tau'$ can be defined: if $q^1 > \cdots > q^r$ according to FSP, the corresponding sequential tasks have the following order $\tau'_1 > \cdots > \tau'_r$ according to FTP.

By Theorem 8, we know that the schedule of FTP on $\tau'$ is periodic with a period of $P$ starting with $S_r$. We can observe that $S_r$ has the same value as $S^*_r$. This means that the FSP schedule on $\tau$ is periodic with a period of $P$ starting with $S^*_r$. □

Theorem 9 considers that execution times $C_\ell$ of a subprogram $q^\ell$ $(1 \leq \ell \leq r)$ are constant. In order to define the schedulability test for the FSP schedulers, we have to prove that they are predictable.

**Definition 10 (Predictability).** Let $J$ be the set of threads and $J'$ be the set of threads that differ only with regards to their the execution times: the threads in $J$ have executions times less than or equal to the execution times of the corresponding threads in $J'$. A scheduling algorithm $A$ is predictable if, when applied independently on $J$ and $J'$, a thread in $J$ finishes execution before or at the same time as the corresponding thread in $J'$.

**Theorem 11.** FSP schedulers are predictable.

**Proof.** We mentioned in the proof of the Theorem 9 that the task system $\tau$ containing $r$ subprograms can be seen as a system $\tau'$ of $r$ sequential tasks such that a task $\tau'_\ell$ inherits the characteristics of the corresponding subprogram $q^\ell$ of $\tau$ $(1 \leq \ell \leq r)$. A FTP priority assignment for $\tau'$ can be built following the priorities assigned by FSP to the corresponding subprograms in $\tau$: $q^1 > \cdots > q^r$ gives $\tau'_1 > \cdots > \tau'_r$.

In [11] it is proven that for systems like $\tau'$, FTP schedulers are predictable on $m$ unit-capacity multiprocessor platforms. Since $\tau$ is equivalent to $\tau'$ and the FTP scheduler assigns the same priorities to sequential tasks as FSP to the corresponding subprograms, FSP schedulers are also predictable. □

Based on Theorem 9 and 11, we can define an exact feasibility test for FSP schedulers.

**Schedulability Test 1.** For any preemptive FSP scheduler $A$ and for any $A$-feasible asynchronous constrained deadline system $\tau$ containing $r$ subprograms (regardless of the tasks they belong to) on a $m$ unit-capacity multiprocessor platform, $[0,S^*_r + P]$ is a feasibility interval, where $S^*_r$ is defined by equation 2.

**Proof.** This is a direct consequence of Theorems 9 and 11. □

4.2. Exact (FTP, FSP) schedulability test

The first step in the definition of the exact schedulability test for the (FTP, FSP) schedulers is to prove the periodicity of the feasible schedules.

**Theorem 12.** For any preemptive (FTP, FSP) scheduling algorithm $A$, if an asynchronous constrained deadline parallel task system $\tau = \{\tau_1, \ldots, \tau_n\}$ is $A$-feasible, then the $A$-schedule of $\tau$ on $m$ unit-capacity multiprocessor platform is periodic with a period of $P$ starting from instant $S_n$, where $S_i$ is defined by equation 1 and tasks are ordered by decreasing priority: $\tau_1 > \tau_2 > \cdots > \tau_n$.  

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Theorem 12. (FTP, FSP) schedulers are predictable.

Proof. Since based on any (FTP, FSP) scheduler we can define a FSP scheduler as shown in the proof of the Theorem 12 and since, by Theorem 11, FSP schedulers are predictable, (FTP, FSP) schedulers are predictable as well.

We will now define the exact schedulability test for (FTP, FSP) schedulers.

Schedulability Test 2. For any preemptive (FTP, FSP) scheduler \( A \) and for any \( A \)-feasible asynchronous constrained deadline parallel task system \( \tau = \{ \tau_1, \ldots, \tau_n \} \) on a \( m \) unit-capacity multiprocessor platform, \([0, S_n + P)\) is a feasibility interval, where \( S_i \) is defined by equation 3:

\[
S_i = S_{i-1} + \sum_{k=1}^{i-1} \tau_k, \quad \forall j: (1 + \sum_{k=1}^{i-1} \tau_k) \leq j \leq \sum_{k=1}^{i} \tau_k.
\]

and \( 1 \leq i \leq n \).

Furthermore, we can observe that \( S_1 = S_1 = O_1 \). From this fact and equation 4, we can conclude:

\[
S_1 = O_1 \Rightarrow
S_2 = S_2 \Rightarrow
\vdots
S_{n-1} = O_n \Rightarrow
S_n = S_n.
\]

The \( B \)-schedule is then periodic with a period of \( P \) starting from \( S_n \). Since the \( B \)-schedule is the same as the one generated by \( A \), the \( A \)-schedule is also periodic with a period of \( P \) starting from \( S_n \).

We will now prove that the (FTP, FSP) schedulers are also predictable.

Theorem 13. (FTP, FSP) schedulers are predictable.

Proof. Since based on any (FTP, FSP) scheduler we can define a FSP scheduler as shown in the proof of the Theorem 12 and since, by Theorem 11, FSP schedulers are predictable, (FTP, FSP) schedulers are predictable as well.

We will now define the exact schedulability test for (FTP, FSP) schedulers.

Schedulability Test 2. For any preemptive (FTP, FSP) scheduler \( A \) and for any \( A \)-feasible asynchronous constrained deadline parallel task system \( \tau = \{ \tau_1, \ldots, \tau_n \} \) on a \( m \) unit-capacity multiprocessor platform, \([0, S_n + P)\) is a feasibility interval, where \( S_i \) is defined by equation 3:

\[
S_i = S_{i-1} + \sum_{k=1}^{i-1} \tau_k, \quad \forall j: (1 + \sum_{k=1}^{i-1} \tau_k) \leq j \leq \sum_{k=1}^{i} \tau_k.
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and \( 1 \leq i \leq n \).

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\[
S_i = S_{i-1} + \sum_{k=1}^{i-1} \tau_k, \quad \forall j: (1 + \sum_{k=1}^{i-1} \tau_k) \leq j \leq \sum_{k=1}^{i} \tau_k.
\]

and \( 1 \leq i \leq n \).

Furthermore, we can observe that \( S_1 = S_1 = O_1 \). From this fact and equation 4, we can conclude:

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S_1 = O_1 \Rightarrow
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The \( B \)-schedule is then periodic with a period of \( P \) starting from \( S_n \). Since the \( B \)-schedule is the same as the one generated by \( A \), the \( A \)-schedule is also periodic with a period of \( P \) starting from \( S_n \).

We will now prove that the (FTP, FSP) schedulers are also predictable.

Theorem 13. (FTP, FSP) schedulers are predictable.

Proof. Since based on any (FTP, FSP) scheduler we can define a FSP scheduler as shown in the proof of the Theorem 12 and since, by Theorem 11, FSP schedulers are predictable, (FTP, FSP) schedulers are predictable as well.

We will now define the exact schedulability test for (FTP, FSP) schedulers.
Taking advantage of the fact that the processors are left idle by τ, the system is schedulable by (DM, IM).

Example 2. This first example presents a task system that is unschedulable by Gang DM, but schedulable by (DM, IM) on a 2-unit capacity multiprocessor platform. The tasks in the system τ = {τ₁, τ₂, τ₃} have the following characteristics: τ₁ : (0, {2}, 3, 3), τ₂ : (0, {3}, 4, 4) and τ₃ : (0, {2}, 12, 12). According to DM τ₁ > τ₂ > τ₃.

We can observe in figure 2 that according to Gang DM, task τ₃ has to wait for 2 available processors simultaneously in order to execute. This is the case at time instant 11; though, at time 12 the task has unfinished execution demand and it misses its deadline.

In the case of (DM, IM), τ₁ and τ₂ execute at the same moments and on the same processors as in Gang DM. The difference is that τ₃ can start executing its first process at time instant 2 since one processor is available. Taking advantage of the fact that the processors are left idle by τ₁ and τ₂ at some moments in time, the first process of τ₃ (which is the only τ₃ process in the interval [0, 12]) finishes execution at time instant 8. No deadline is missed, therefore, the system is schedulable by (DM, IM).

Example 2. The second example presents a task system τ = {τ₁, τ₂, τ₃} which is schedulable with Gang DM, but unschedulable with (DM, IM) on a 3-unit capacity multiprocessor platform. The tasks of the system have the following characteristics: τ₁ : (0, {3, 3}, 4, 4), τ₂ : (0, {1, 1}, 5, 5) and τ₃ : (0, {9}, 10, 10). According to DM τ₁ > τ₂ > τ₃.

In figure 3 we can observe that according to Gang DM, at instant 0, τ₁ is assigned to 2 of the 3 processors in the platform. Since there is only one processor left, τ₂ cannot execute, therefore τ₃ starts its execution on the third processor. At time instant 3, 2 processors are available and, consequently, τ₂ may start executing, etc. No deadline is missed in the time interval [0, 12], therefore the system is Gang DM feasible.

According to (DM, IM), even if τ₁ occupies 2 processors of the 3 in the platform, τ₂ may start executing on the third a first thread from time instant 0 to time instant 1. The second thread of its first process will execute on the third processor from time instant 1 to time instant 2. We can conclude that τ₃ will miss its deadline at time instant 10 since it has 9 units of execution demand and only 6 time units available until its deadline.

5. Empirical Study

The purpose of this empirical study is to evaluate the performance of multi-thread schedulers compared with the one of Gang schedulers. More specifically, the chosen multi-thread scheduler is the (FTP, FSP) scheduler (DM, IM). Among the Gang schedulers, we consider Gang DM.

Since Gang FTP schedulers are not predictable, in this study we consider constant execution times. From [8] and the Schedulability test [2] we know that we have to simulate both Gang FTP and (FTP, FSP) schedulers in
the time interval \([0, S_n + P]\) in order to conclude if the task system is feasible with one of them or with both.

In this empirical study, the execution times of all the subprograms of a task are considered equal.

5.1. Evaluation criteria

Gang DM and (DM, IM) are evaluated according to the following criteria:

- success ratio;
- worst-case response time of the lowest priority task in the system.

**Definition 14** (Response time). The response time of a process represents the duration between the arrival of the process and the moment it finishes execution.

**Definition 15** (Worst-Case Response Time (WCRT)). The worst-case response time of a task is the maximum response time of its processes.

**Success ratio.** The success ratio represents the portion of successfully scheduled task systems and it is defined as follows:

\[
\text{number of successfully scheduled task systems} \quad \frac{\text{total number of considered task systems}}{.}
\]

**Worst-case response time.** If system \(\tau\) is schedulable, the worst-case response times of a task \(\tau_i \in \tau\) for Gang DM and (DM, IM) are calculated according to its processes within the time interval \([0, S_n + P]\).

For each feasible task system with both Gang DM and (DM, IM) we compare the WCRTs (computed for each of the two schedulers) of the lowest priority task. For a given system utilization, we count separately the tasks systems where the (DM, IM) WCRT is strictly inferior to the one computed under Gang DM and inversely. Consequently, the uncounted task systems are those where the computed WCRTs are equal for the two schedulers.

5.2. Task system generation methodology

The procedure for task system generation is the following: individual tasks are generated and added to the system until the total system utilization exceeds the platform capacity \((m)\).

The characteristics of a task \(\tau_i\) are integers and they are generated as follows:

1. the period \(T_i\) is uniformly chosen from \([1, 250]\);
2. the offset is uniformly chosen from \([1, T_i]\);
3. the utilization \(u_i\) of the task is inferior to \(m\) and it is generated using the following distributions:
   - uniform distribution between \([\frac{1}{T_i}, m]\);
   - bimodal distribution: light tasks have an uniform distribution between \([\frac{1}{T_i}, \frac{m}{2}]\), heavy tasks have an uniform distribution between \([\frac{m}{2}, m]\); the probability of a task being heavy is of \(\frac{1}{3}\);
   - exponential distribution of mean \(\frac{m}{4}\);
   - exponential distribution of mean \(\frac{m}{2}\);
   - exponential distribution of mean \(\frac{3m}{4}\).
4. \(v_i\) is uniformly chosen from \([1, m]\);
5. since we consider that all the subprograms of a task \(\tau_i\) have equal execution times, it is sufficient to compute a single execution time value: \(C_i = \frac{u_i \cdot T_i}{v_i}\);
6. the deadline is uniformly chosen between \([C_i, T_i]\).

We use several distributions (with different means) in order to generate a wide variety of task systems and, consequently, to have more accurate simulation results.

The generated systems have a least common multiple of the task periods bounded by 5,000,000. During simulations, we considered multiprocessor platforms containing 2, 4, 8 and 16 processors. A total of 450,000 task systems were generated.
5.3. Results

Firstly, we will analyze the success ratio of the two schedulers.

**Success ratio.** Each of the following figures contains 3 plots: one represents the success ratio of the (DM, IM) multi-thread scheduler, a second one the success ratio of Gang DM and a third one expresses the ratio of task systems successfully scheduled by both of them.

The performance gap between the two schedulers in terms of success ratio is growing as the number of processors grows, as shown in figures 4-7. In the case of the 2 processors platform, they have similar performances. On 4 processors, the (DM, IM) successfully schedules at most 10% more task systems than Gang DM (this is attained at utilization 2.8). On 8 processors it schedules 12% more task systems (at utilization 5.2) and on 16 processors it can schedule 14% more task systems than Gang DM (at instant 10.4).

We can also observe in each of the figures 4-7 that (DM, IM) and Gang DM are incomparable since the corresponding plots are above the “both” plot in each of these figures. Moreover the amount of additional systems that thread scheduling can manage is quite better. In figure 4 we can see that on a 2 processors platform, the amount of additional systems that (DM, IM) can schedule is at most 2 times higher (at utilization 1.6) than the one of additional systems scheduled by Gang DM. In the case of a 4 processors platform, (DM, IM) can perform 4.3 times better than Gang DM (at utilization 2.8), on 8 processors 5.4 times (at utilization 5.2) and, finally, on 16 processors it can perform 7.5 times better than Gang DM (at utilization 10.4).

![Figure 4. Success Ratio: 2 processors](image1)

**Figure 4. Success Ratio: 2 processors**

![Figure 5. Success Ratio: 4 processors](image2)

**Figure 5. Success Ratio: 4 processors**

**Response time.** In the following we will reference the figures 8-10. The utilization of the considered systems is superior to 25% and inferior to 90% of the platform capacity.
In each figure, there are two plots: one that marks the portion of task systems (among those which are schedulable by both Gang DM and (DM, IM)) where the (DM, IM) WCRT of the lowest priority task is strictly inferior to the one computed under Gang DM; a second plot marks the contrary behavior.

The results of our simulations showed that, for systems executed on 2 processors, the WCRTs of the lowest priority task under the two schedulers are equal.

On 4, 8 and 16-unit capacity multiprocessor platforms, (DM, IM) outperforms Gang DM as we can see in figures 8–10. For all of the three cases, the (DM, IM) WCRT is inferior to the Gang DM WCRT in at most 50% of the considered task systems. The lowest performance gap is observed on the 4 processors platform (at utilization 1.4) and, even in that case, (DM, IM) performs 8% better than Gang DM.

6. Conclusions and future work

In this work we considered the multi-thread scheduling for parallel real-time systems. The main advantage of this model is that it does not require all threads of a same task to execute simultaneously as Gang scheduling does.

We defined in this paper the several types of priority-driven schedulers dedicated to our parallel task model and scheduling method. We distinguished between hierarchical schedulers (that firstly assign distinct priorities at system level and secondly, within each task) and global thread schedulers (that do not take into account the original tasks when priorities are assigned at thread level).

We showed that, contrary to Gang FTP, the hierarchical and global thread schedulers based on FTP and FSP are predictable. Based on this property and the periodicity of their schedules, we defined two exact schedulability tests.

Even though the Gang and multi-thread schedulers are, as we have shown, incomparable, the empirical study confirmed the intuition that our approach outperforms Gang scheduling. In terms of success ratio, the
performance gap increases as the number of processors grows.

**Future work.** While we have shown the benefits of our thread scheduling model in comparison with Gang scheduling, in further work we would like to extend our model to have more realistic parallel task models. In particular we aim to consider multi-phase tasks, in the sense that a task can have an arbitrary number of phases that have to be executed sequentially. Each phase would be characterized by its own degree of parallelism. E.g., the task could be composed of initialization, computing and finalization phases; solely the computing phase can
be parallelized, others are sequential. Unfortunately, we have to report a first negative result, i.e., multi-phase multi-thread hierarchical schedulers are not predictable.

We extend the model as follows, each task \( \tau_i \) is characterized by:

\[
(O_i, \{\phi^1_i, \phi^2_i, \ldots, \phi^\ell_i\}, D_i, T_i)
\]

with the interpretation that the task is composed by a sequence of \( \ell_i \) phases, each phase \( \phi^j_i \) is characterized by a set of subprograms \( q^1_{i,j}, q^2_{i,j}, \ldots, q^{v_{i,j}}_{i,j} \) (\( v_{i,j} \) is the number of subprograms in the phase \( j \) of task \( i \)). Each thread is defined as our original model.

Consider for instance \( \tau_1 = (0, \phi^1_1 = \{2\}, \phi^2_1 = \{2, 2, 2\}) > \tau_2 = (1, \phi^1_2 = \{1\}) \) (we omit here deadline and period characteristics), Figure 11 shows that if we consider the worst-case duration of each thread of \( \tau_1 \) the single thread of \( \tau_2 \) complete at time 2 while if we reduce the duration of the first phase of task \( \tau_1 \) by one time unit then \( \tau_2 \) completes at time 4. The example shows the non predictability of multi-phase multi-thread hierarchical schedulers.

![Figure 11. Multiphase: unpredictability](image)

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