Composite fermions in the half-filled lowest Landau level: a macroscopic justification

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(March 16, 2022)

An effective Hamiltonian for spinless electrons in the lowest Landau level (LLL) close to half filling is derived. As opposed to the standard treatment in Chern-Simons theories (CS) we first project to the LLL and only then apply a CS-transformation on the Hamiltonian. The transformed field operators act in the lowest Landau level only and have fermionic commutation relations for small wavenumbers ignoring gauge field fluctuations. When acting on the Hamiltonian at half filling the gauge transformation removes the monopole term in the interaction and does not eliminate the magnetic field.

PACS numbers: 73.40.Hm, 71.10.-w, 71.45.-d

Ten years ago J. Jain made the discovery that the fractional quantum Hall effect of interacting electrons may be understood in terms of the integer quantum Hall effect of Composite Fermions (CF) [1]. The key property of this species is that they are fermions, their mutual interaction is small and their interaction with the external magnetic field is reduced as compared to electrons. One way the latter is frequently visualized is by saying that CFs have charge $\pm e\hbar$ and interact with an effective magnetic field that equals the external one reduced by two flux quanta per particle: $B_{\text{eff}} = B_{\text{ex}} - 2\phi_0 n$.

By now the experimental evidence for the existence of CFs is overwhelming [2,3] and generally undisputed. In particular, the existence of a well defined Fermi surface near filling fraction $1/2$, where the effective magnetic field vanishes has been established. In contrast, the theoretical understanding, especially near half filling which is what we focus on in the present letter, is still somewhat incomplete. Important insights have come from numerical work, in particular from the analysis of trial wave functions for finite systems [4,5]. However, despite of the fact that a number of attempts have been made to develop effective theories [4,6] the efforts have been only partially successful.

Let us briefly review two of these approaches. The most prominent one has been developed by Halperin, Lee and Read (HLR) [7]. Originally, the most intriguing feature of the theory was the prediction of a well defined Fermi surface for the CF’s at $\nu = 1/2$. It manifested itself in the RPA-density response that explained an anomaly in surface acoustic wave experiments [8]. However, as has been pointed out e.g. by Simon [9], the theory is not capable of properly accounting for zero order physics in high magnetic fields unless these effects are put in by hand. For example, a well known problem is that up to now it has not been possible to demonstrate how it happens within this formalism that the bare mass $m_b$ describing the energy dispersion of free electrons is replaced by an effective mass $m^*$ describing the dispersion of the CFs. Since the latter is due electron interactions only, $m^*$ must be independent of $m_b$.

The difficulty lies in the very nature of the approach itself: As the first step HLR perform a singular gauge transformation (Chern-Simons transformation). It cancels the external magnetic field at half filling on average and virtually all of the physics related to the actual existence of a strong magnetic field must therefore come from gauge field fluctuations. Apparently, already RPA theory can give proper account of the structure of the density response, but many other features, like the effective mass, have not been properly restored from the fluctuations up to now.

An interesting recent attempt in this direction has been made by Shankar and Murthy [8]. However, success in their theory depends on a parameter $Q$ that is not fixed within the theory itself. For a choice $Q = k_F$ the effective mass is indeed independent of $m_b$, at the expense of a density response that is incompressible in disagreement with what one expects for the Fermi liquid [10]. Stern et. al. advocate another choice $Q \ll k_F$ which reproduces the expected density response and also an effective Hamiltonian with a desirable structure but now the mass is on the wrong scale, again [11].

The second type of approach considers the fermions as bosons at integer filling carrying one flux quantum [12]. The theory deals with the Hamiltonian projected to the LLL anddiscards the idea of a singular gauge transformation, altogether. Generally, the Hamiltonian of a projected theory is formulated in terms of projected field operators that are not easily dealt with since they do not anticommute. The method for handling the projected field operators chosen in this type of theory is to replace the projected operators by the unprojected counterpart and complement the Hamiltonian with a constraint. Then the main problem is to deal with the constraint. Read has calculated the density response and his result is structurally identical with the HLR response.

As it stands, the main disadvantage of this theory is that it has been formulated for bosons at filling fraction unity and the relation to electrons at half filling is not
entirely obvious. Moreover, it has not been extended to filling fractions other than ν = 1, yet.

In the present paper we combine ideas underlying the earlier theories. By projecting to the LLL we first implement the basic physics of strong magnetic fields: we eliminate the bare kinetic energy in the Hamiltonian and introduce the guiding center coordinate as a dynamical variable in the interaction term. Then we carry out a Chern-Simons transformation which introduces the CFs. The resulting theory reproduces correct results from the earlier CS-approaches in a straightforward manner but is free from their problems. The coupling to disorder is different however and might lead to experimentally observable differences from the standard theory.

In contrast to all previous CS-theories our motivation for applying the CS-transformation focuses on the fermionic character of the resulting quasiparticles: our ultimate goal is to formulate a theory in terms of quasiparticles that live in the LLL only and have fermionic statistics, at least in the small wavenumber limit. In principle, other choices for the statistics are possible, but fermions are preferable since they obey the Pauli principle which automatically favors configurations with low interaction energy independent of details of the (repulsive) interaction. In order to see how to construct the quasiparticles, consider the anticommutator

\[ [\psi_P(x), \psi_P^\dagger(x')]_+ = \frac{1}{2\pi} e^{-i/2 \mathbf{x} \times \mathbf{x}' - |\mathbf{x} - \mathbf{x}'|^2/4} \] (1)

of the projected field operator \( \psi_P(x) \) defined below. (Throughout the paper we use the symmetric gauge, choose units such that \( \hbar = e^2/c = 1 \) and \( x \times x' = x_1 x'_2 - x_2 x'_1 \).) The argument of the complex exponential is the phase an electron picks up when moving around the triangle \( 0, x, x' \). One can introduce new particles with the gauge transformation

\[ \chi(x) = \exp(-i\tilde{\phi}\int d\mathbf{r} \arg(x - \mathbf{r}) \psi_P^\dagger(\mathbf{r})\psi_P(\mathbf{r})) \psi_P(x) \] (2)

where \( \arg(\mathbf{r}) \) denotes the angle between the argument and the \( x \)-axis and \( \tilde{\phi} \) is the number of flux quanta that are attached. The Pauli principle requires it to be an even number, e.g. \( \tilde{\phi} = 2 \) is the proper choice at half filling. The key observation is that these particles \( \chi \) obey the anticommutator

\[ [\chi(x), \chi(x')^\dagger]_+ = \frac{1}{2\pi} e^{-|\mathbf{x} - \mathbf{x}'|^2/4} \] (3)

without a complex phase factor in an approximation where the effective transverse gauge field (operator)

\[ A(x) = \tilde{\phi} \nabla \int d\mathbf{r} \arg(x - \mathbf{r}) \chi^\dagger(\mathbf{r})\chi(\mathbf{r}) - A_{\text{ex}}(x) \] (4)

is neglected \[\text{[13]}\]. This corresponds to mean field approximation at half filling. As long as the mean particle distance is large as compared to the width \( 2\ell \) of the Gaussian they can be considered fermions. Note, that in the literature \[\text{[3,4]}\] the analog object for the standard gauge transformation is frequently called \( a(x) \) \[\text{[4]}\].

We have introduced a different symbol in order to stress that \( a(x) \) is related to fluctuations of the electron density whereas \( A(x) \) relates to the quasiparticle density \( \nabla \wedge A(x) = 2\pi\rho(x)\chi^\dagger(x)\chi(x) - B_{\text{ex}} \). These are different quantities in the present theory and we will come back to this crucial point, below.

Before we discuss some of the properties of the new CFs we derive the full model: The Hamiltonian underlying the problem is \( (A_{\text{ex}} = B/2(-y, x), V; \text{Volume}, e/c = 1) \)

\[ H = \int d\mathbf{x} \psi^\dagger(x) \left( \frac{1}{2m_b} \nabla^2 - A(x) \right) \psi(x) \] (5)

\[ + \frac{1}{V} \sum_{\mathbf{q}} v(\mathbf{q}) \rho(\mathbf{q}) \rho(-\mathbf{q}) \]

\[ \rho(\mathbf{q}) = \int d\mathbf{x} \psi^\dagger(x)e^{i\mathbf{x} \cdot \mathbf{q}} \psi(x). \] The projection to the LLL is accomplished by making use of a standard formalism \[\text{[17]}\]: we introduce single particle eigenstates \( |jm\rangle \) with Landau level index \( j \) and an inner quantum number \( m \) that is related to angular momentum. New field operators \( c_{j,m} = \int d\mathbf{x} \psi(x) |jm\rangle \) may be defined and in terms of these the projected electron field operators are \( \psi_P(x) = \sum_m c_{0,m}(x) |m\rangle \). The projected Hamiltonian reads

\[ H = \omega_c/2 \rho_P(q = 0) + \frac{1}{V} \sum_{\mathbf{q}\neq 0} v_0(\mathbf{q}) \rho_P(\mathbf{q}) \rho_P(-\mathbf{q}) \] (6)

with the effective interaction \( v_0(q) = v(q) \exp(-q^2/2) \) and an operator \( \rho_P(\mathbf{q}) = \int d\mathbf{x} \psi_P^\dagger(\mathbf{x}) e^{i\mathbf{q} \cdot \mathbf{R}} \psi_P(\mathbf{x}) \) representing the electron density. The important feature is that the position operator in the plane wave factor exp \( i\mathbf{q} \cdot \mathbf{R} \) of the ordinary expression for the density operator (below eq. (5)) has been replaced by the guiding center coordinate \( \mathbf{R} = \mathbf{x} + (\nabla / i - A_{\text{ex}}) \) with \( \mathbf{x}_0 = (-y, x) \). It is sensitive to gauge transformations in which the external field is canceled in part by the gauge field which leaves us with \( -A \) instead of \( A_{\text{ex}} \):

\[ \rho_P(\mathbf{q}) = \int d\mathbf{x} \chi^\dagger(\mathbf{x}) e^{i\mathbf{q} \cdot \mathbf{x} + i\phi(\nabla / i + \nabla \wedge A(x))} \chi(\mathbf{x}). \] (7)

Equations (4), (5) and (6) together with fermionic commutators for the quasiparticles \( \chi, \chi^\dagger \) constitute the model for composite fermions that we consider here.

Now, let us compare our theory to the standard ones and see what differences arise due to the different gauge transformations used. 1) In both theories the gauge transformation breaks the symmetry between wavenumber and position, that is the typical effect of magnetic fields and a prerequisite for introducing a mass term. The standard transformation achieves this by acting on the kinetic term of the Hamiltonian eliminating the magnetic field on average. By contrast, our version affects
the interaction term, only. We show below that on mean field level at ν = 1/2 the gauge transformation actually removes the monopole term in the interaction and not the magnetic field. It leaves us with a dipole term χ1(∇χ)χ^†. 2) The transformation of HLR maps fermions to fermions and does not modify the (anti-)commutators. The corresponding field operators couple different Landau levels and hence do not describe the experimentally confirmed “real” CF. On the other hand, our projected field operators cannot be interpreted as proper fermions by themselves since they do not anticommute. We use the gauge transformation in order to recombine these “improper particles” so as to form a composite object with fermionic commutator, at least approximately. As opposed to ordinary metals, in our case the screening cloud adds to the fermionic character of the quasiparticles and not only screens the charge. 3) As already mentioned, the gauge field operator A is related to the quasiparticle density ρχ and not to the physical electron density ρP. As can be seen from equation [ ] these are quite different objects: at ν = 1/2 the physical electron density equals the transverse current mode of the quasiparticles and is not directly related to the quasiparticle density. This is to be contrasted with the standard theory where electron- and quasiparticle density are identical. This difference might be relevant for the interpretation of experiments: electric fields produce fluctuations in the electronic density that in turn induce an inhomogenous effective magnetic field seen by the CF. In the standard theory the effective field is directly coupled to the induced charge density whereas in our setup it couples to the induced quasiparticle density. This opens up a possible mechanism that might be related to quantitative discrepancies occurring in the theoretical description of several experiments [ , ]. It is found that these experiments can be reasonably well understood within the CF picture, however with fitting parameters that point to larger scattering times than expected from estimates based on HLR theory by a factor of 2 ÷ 4. A sufficiently weaker coupling to disorder than predicted by standard theory would explain the difference and appears at least possible within the framework suggested, here.

In the remainder of the letter we show that results from the standard theory and its extension follow in a straightforward manner within the scheme proposed. We begin by deriving an expression valid at small wavenumbers q for the density operator [ ]. Note that attention should be paid to the gauge field: we decompose it into average A = (1 − νφ)Aex and fluctuations A. The derivative of the latter is of order q^0 since ∇ ∧ A = 2πφ : χ^†χ : with : χ^†χ := χ^†χ − n. We perform a double expansion and keep terms up to linear order in q and A:

\[ ρ_P(q) = \int dxe^{iqx} \chi^†(x) (1 + iq ∧ (∇/i + A)) χ(x) + χ^†(x)(1+iq ∧ (∇/i + A))χ(x) iq ∧ A(x + q0/2) + ... \]

After partial integration one obtains

\[ ρ_P ≈ V̂ρδ_q0 + (1 − δ_q0)(1 − νφ) \int dxe^{iqx} : χ^†(x)χ(x) + \int dxe^{iqx}iq ∧ g(x) + 2πφ \int dxe^{iqx}iq ∧ g(x) : χ^†(x)χ(x) : \] (8)

where g = 1/2 (∇/i + A) − (∇/i + A)χ^†χ. The first three terms constitute the expression for the density operator as found and discussed previously by Shankar and Murthy [ ]. It can be interpreted consistently as describing particles with a screened, effective charge e* = e(1 − 2ν) that couple to the external field Aex. In particular, on the plateaus at filling fractions ν = p/(pφ + 1) it is equal to the fractional charge of the quasiparticles e/(pφ + 1). At half integer filling fractions ν = 1/2 the effective charge vanishes and the dipole moment is the leading coupling to electric fields.

The fourth term is new and related to K-invariance [ ]: under the transformation χ(x) → exp(ιKx)χ(x), the density operator acquires a phase factor exp iq ∧ K and the Hamiltonian is unchanged. Its importance has been stressed first by Halperin and Stern [ ]. The additional term ensures the correct transformation properties to leading order and would be absent on mean field level (χ^†χ ≈ 0).

Next we derive an effective Hamiltonian for our model. We restrict ourselves to half integer filling, here. Our strategy is to assume that the quasiparticles χ(x) form a well defined Fermi sphere so that standard concepts of Fermi liquid theory can be carried over. We begin with the usual phase space separation for the Hamiltonian

\[ H = \frac{1}{2V} \sum_q v_0(q) \int dxdx'χ^†(x)e^{iqR + iq ∧ A(x)}χ(x) \]

\[ γ(x)χ^†(x')e^{-iqR' - iq ∧ A(x')}χ(x') \] (9)

and subdivide it into regions with small and large momentum transfer. In the small angle scattering region \( q ≪ k_F \) the expression [ ] for the density operator holds (with A = 0) and we have

\[ H_{dir} = \frac{1}{2} \sum_{q≠0} v_0(q)ρ_P(q)ρ_P(-q). \] (10)

The momentum cutoff Q has been introduced in order to avoid double counting. It satisfies the condition Q ≪ k_F. In a compressible system an appropriate choice for Q is the screening wavenumber [ ]. Although we use a simplified low wavenumber expression this contribution to the energy is still complicated since terms containing up to eight field operators are necessary in order to ensure approximate K-invariance. On the other hand its contribution to the energy is small. For Coulomb interactions it is of the order o(Q)}.
For the exchange (or large angle scattering) contribution to the effective Hamiltonian a cancelation as for the direct channel does not occur. Therefore, we treat the gauge field fluctuations in the simplest approximation and ignore them. This makes the exchange contribution well defined. It can be found after shifting $q \rightarrow q + (x_1 - x_2)$:

$$H_{\text{ex}} = \frac{1}{2} \sum_{q \neq 0} \sum_{k_1, k_2} f(k_1 - k_2)e^{i q \cdot (k_1 - k_2)}$$

$$\chi_{k_1 + q/2, k_2 - q/2}$$

(11)

with $f(k) = -\Theta(|k| - |Q|) v_0(k)$. Equation (11) is a two-fermion interaction that is manifestly K-invariant. It gives rise to a self consistency equation for the self energy in Hartree Fock approximation

$$\Sigma_k = -\frac{1}{V} \sum_{|p - k| \geq |Q|} v_0(k - p)n(\Sigma_p - \mu)$$

(12)

where $n(\Sigma_k - \mu)$ is the Fermi function. As already mentioned the contribution of the direct term to $\Sigma_k$ is not easily treated systematically. Since we expect it to be small we neglect it here. From the self energy we deduce an effective mass

$$\frac{1}{m^*} = \frac{1}{k_F} \frac{d \Sigma_k}{dk} = \frac{d \theta_{kp}}{(2\pi)^2} v_0(k - p) \cos \theta_{kp}$$

(13)

where $\theta_{kp}$ denotes the angle between $k$ and $p$ and $|k| = |p| = k_F$. It is of the order of $e^2\ell \ln(Q\ell)$ for Coulomb interaction. A similar result has been reported recently by Read \cite{Read_1993}.

In Fermi liquid theory the standard way to treat a non-local interaction $f(k)$ is a mode decomposition. Thereby Fermi liquid parameters $f_0$ for density-density and $f_1$ for current-current modes are introduced, where $f_1 = -2\pi/m^*$ in the present case. The resulting Hamiltonian reads

$$H = \frac{eB_{\text{ex}}}{m_0 c} \rho \rho (q = 0) + \sum_{k} \frac{k^2}{2m^*} \chi_k \chi_k$$

$$- \frac{1}{2m^*n} \sum_{q \neq 0} g(q)g(-q) + \frac{1}{2} \sum_{q \neq 0} v_0(q) \rho \rho (q) \rho \rho (-q)$$

$$+ \frac{w_0}{2} \sum_{q \neq 0} \rho_0(q) \rho_0(-q) + \frac{w_1}{2} \sum_{q \neq 0} q \cdot \chi(q) \rho_0(-q)$$

(14)

where $\rho_0 = \chi^\dagger \chi$ and $\rho \rho$ as given in equation (8). The coupling constants $w_0$ and $w_1$ equal $f_0$ on the current level of approximation. However, they, as well as $m^*$, might be strongly renormalized by gauge field fluctuations. A Hamiltonian similar in the leading terms has been derived and discussed by Stern et. al., recently \cite{Stern_1993}. Our result is different in two important respects: 1) The terms number five and six stem from an expansion of the exponential in equation (11) and appear to be new. Of special interest for us is the last term since it couples the transverse current (or electron density) to the quasiparticle density. As outlined above this coupling mediates the change in the effective magnetic field (or charge) experienced by the CF when an electric field is applied. 2) Every particle has a bare kinetic energy $eB_{\text{ex}}/cm_0$, where $m_0$ is the bare (or band) mass. The dispersive part of the energy is described through an effective mass $m^*$ independent of $m_0$ given in equation (13).

We turn to our last issue, the density response. In RPA approximation we find for the electron-density response function

$$\chi_{+ +} = \frac{1}{v(q) - \chi_{+ +}^\dagger/n^2 + W - 2i\omega/q^2k_F n}$$

(15)

where $W = -w_0^2/(w_0 + 2\pi/m^*)$. This result is structurally identical with the earlier result by HLRR \cite{Haldane_1997}. The offdiagonal conductivity can be obtained with the notion that $\omega_c(x)$ acts like a chemical potential for a situation with a filling fraction that is kept fixed, locally. This gives rise to a relation between cyclotron current $j_c = h/(2m_0)\nabla V$ and a potential gradient from which we conclude that $\sigma_{xy} = \nu$.

It is a pleasure to acknowledge valuable discussions with W. Apel, Gerland, T. Kopp, Mei-Rong Li, A. D. Mirlin, D. G.Polyakov, A. Rosch, J. Wilke and P. Wölfle. Also, I am grateful to the Graduiertenkolleg \textit{Kollektive Phänomene im Festkörper} der Deutschen Forschungsgemeinschaft for financial support.

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