Isospin-symmetry breaking and shape coexistence in A∼70 analogs

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Abstract. Properties of proton-rich nuclei in the A∼70 region could bring insights into fundamental symmetries and interactions and may have relevance for the astrophysical scenarios on the rp-process path. Recent results concerning the interplay between isospin-symmetry-breaking and shape-coexistence effects on isospin-related phenomena in A=70 and A=74 isovector triplets obtained within the beyond-mean-field complex Excited Vampir variational model using an effective interaction obtained from a G-matrix based on the charge-dependent Bonn CD potential in a relatively large model space are presented.

1. Introduction
Proton-rich nuclei in the A∼70 mass region are proper candidates to get insights into fundamental symmetries and interactions and their structure and dynamics play an essential role for the astrophysical scenarios related to rapid proton capture (rp) process in the X-ray burst environment. A fundamental symmetry in nuclei, the isospin symmetry, is broken due to the Coulomb interaction between protons and in the strong interaction due to the differences in the proton-proton, neutron-neutron and neutron-proton interaction strengths because of the mass difference between the up and down quarks and electromagnetic interactions among quarks. The charge-symmetry and charge-independence breaking could be investigated studying different isospin-related phenomena like Coulomb energy differences (CED), mirror energy differences (MED), triplet energy differences (TED) as well as superallowed Fermi β-decay among the triplet T=1 nuclei. Anomalies in the Coulomb energy differences have been identified in the A∼70 mass region for nuclei supposed to manifest shape mixing at low spins [1-5]. The investigation of the structure and dynamics of nuclei near the N=Z line revealed interesting coexistence phenomena generated by the interplay between shape coexistence and mixing, competing like-nucleon and neutron-proton T=1 and T=0 pairing correlations, and isospin-symmetry-breaking interactions.

In the present report first will be presented recent results on the isospin-symmetry-breaking effects on Coulomb energy differences, mirror energy differences, triplet energy differences for the A=70 and A=74 isovector triplets using the beyond-mean-field complex Excited Vampir variational approach based and an effective interaction obtained from a G-matrix based on the charge-dependent Bonn CD potential in a rather large model space. The structure of these nuclei has been intensively investigated both, experimentally and theoretically [5-16]. Effects of the isospin nonconserving forces on the structure of medium mass nuclei have been studied using different theoretical approaches and various effective interactions [2, 3, 4, 5, 17, 18].
Recent studies in the A\textasciitilde70 mass region based on modern shell-model calculations indicated that the experimental trends in MED and TED can be reproduced adding to the Coulomb interaction some phenomenological isospin nonconserving nuclear interactions (INC), but the modern charge-dependent forces cannot account for the phenomenological strengths of the INC force [16, 18].

Superallowed Fermi $\beta$ decays between 0$^+$ T=1 analog states providing tests of the validity of conserved-vector-current (CVC) hypothesis and the unitarity of the Cabibbo-Kobayashi-Maskawa matrix [19] have been intensively investigated in the frame of different theoretical approaches [19-24]. The present study presents recent complex Excited Vampir results on the superallowed Fermi $\beta$ decay of the Z=N+2 members of the A=70 and A=74 isovector triplets, $^{70}$Kr and $^{74}$Sr nuclei, using the same theoretical procedure and ingredients employed in the structure investigations devoted to CED, MED, and TED analysis.

Studies based on the variational approaches of the VAMPIR model family have been successfully performed for the description of a variety of coexistence phenomena in the A\textasciitilde70 mass region, not only in nuclei along the valley of $\beta$-stability, but also in some exotic nuclei close to the proton drip line [24-27]. The complex Excited Vampir approach allows for a unified description of low- and high-spin states including in the projected mean fields neutron-proton correlations in both the T=1 and T=0 channels and general two-nucleon natural- and unnatural-parity correlations. The oblate-prolate coexistence and mixing, the variation of the deformation with mass number, increasing spin, as well as excitation energy have been nicely compared to the available experimental information. Since the Vampir approaches enable the use of rather large model spaces and of general two-body interactions, large-scale nuclear structure studies going far beyond the abilities of the conventional shell-model configuration-mixing approach are possible. I shall briefly describe the complex Excited Vampir variational procedure and define the effective Hamiltonian in the next section. In Section 3 I shall then present results on isospin-symmetry-breaking and shape-mixing effects in the A=70 and A=74 isovector triplets.

2. Theoretical framework

The complex Excited Vampir (EXVAM) model uses Hartree-Fock-Bogoliubov (HFB) vacua as basic building blocks, which are only restricted by time-reversal and axial symmetry. The underlying HFB transformations are essentially complex and do mix proton- and neutron-states as well as states of different parity and angular momentum. The broken symmetries of these vacua (nucleon numbers, parity, total angular momentum) are restored by projection techniques and the resulting symmetry-projected configurations are then used as trial wave functions in chains of successive variational calculations to determine the underlying HFB transformations. Finally the residual interaction between the orthogonal configurations is diagonalized obtaining the configuration mixing. The HFB vacua account for arbitrary two-nucleon correlations including like-nucleon as well as isovector and isoscalar neutron-proton pairing correlations. For nuclei in the A\textasciitilde70 mass region is used a $^{40}$Ca core and the $1p_{1/2}, 1p_{3/2}, 0f_{5/2}, 0f_{7/2}, 1d_{5/2}$ and $0g_{9/2}$ oscillator orbits for both protons and neutrons are introduced in the valence space. We start with an isospin symmetric basis and then introduce the Coulomb shifts for the proton single-particle levels resulting from the $^{40}$Ca core by performing spherically symmetric Hartree-Fock calculations using the Gogny-interaction D1S in a 21 major-shell basis [22].

The effective two-body interaction is constructed from a nuclear matter G-matrix based on the charge-dependent Bonn CD potential. In order to enhance the pairing correlations this G-matrix was modified by adding short-range (0.707 fm) Gaussians with strength of -35 MeV in the T=1 proton-proton and neutron-neutron channel, -20 MeV in the neutron-proton T=1 channel, and -35 MeV in the neutron-proton T=0 channel. In addition, the isoscalar interaction was modified by monopole shifts for the T=0 matrix elements of the form $\langle 1p_1d_{5/2}; IT = 0 | G | 1p_1d_{5/2}; IT = 0 \rangle$, where 1p denotes either the $1p_{1/2}$ or the $1p_{3/2}$ orbit and $\langle 0g_{9/2}0f; IT = 0 | G | 0g_{9/2}0f; IT = 0 \rangle,$
where 0f denotes either the 0f5/2 or the 0f7/2 orbitals. The Hamiltonian includes the two-body matrix elements of the Coulomb interaction between the valence protons.

### 3. Results and discussion

In the isovector triplets the Coulomb energy differences are defined by

\[ CED(J) = E_x(J, T=1, T_z=0) - E_x(J, T=1, T_z=1), \]

the mirror energy differences by

\[ MED(J) = E_x(J, T=1, T_z=-1) - E_x(J, T=1, T_z=1), \]

and the triplet energy differences by

\[ TED(J) = E_x(J, T=1, T_z=-1) + E_x(J, T=1, T_z=1) - 2E_x(J, T=1, T_z=0), \]

where \( E_x \) represents the excitation energy and \( T_z = -1 \) for the proton-proton pair.

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**Figure 1.** The complex Excited Vampir spectra for the analog states in \(^{70}\text{Se},^{70}\text{Br},^{70}\text{Kr}\) compared with the available data.

**Figure 2.** The complex Excited Vampir spectra for the analog states in \(^{74}\text{Kr},^{74}\text{Rb},^{74}\text{Sr}\) compared with the available data.

The results concerning the structure of the 0\(^+\), 2\(^+\), 4\(^+\), and 6\(^+\) states in \(^{70}\text{Se},^{70}\text{Br},^{70}\text{Kr}\) indicate a strong mixing of different prolate and oblate deformed configurations in the intrinsic system in the final wave functions. The oblate-prolate mixing is also significant but weaker in the A=74 nuclei and is changing with spin and excitation energy. In both triplets the amount of mixing is decreasing with increasing spin [5]. It is worthwhile to mention that always the yrast states are dominated by prolate deformed configurations in the intrinsic system and the first excited states by oblate components except for the lowest two 0\(^+\) states in \(^{70}\text{Se}\) which indicate almost equal contribution from prolate and oblate deformed EXVAM configurations.

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**Figure 3.** Occupation of valence spherical orbitals for analog states in \(^{70}\text{Se},^{70}\text{Br},^{70}\text{Kr}\).

**Figure 4.** Occupation of valence spherical orbitals for analog states in \(^{74}\text{Kr},^{74}\text{Rb},^{74}\text{Sr}\).
In figures 1, 2 are presented the complex Excited Vampir spectra for the A=70 and A=74 analogs, respectively, compared with the available data. In figures 3, 4 are depicted the trends in the proton and neutron occupation of the valence spherical orbitals $1p_{3/2}$, $0f_{5/2}$, and $0g_{9/2}$ for the analog states in the A=70 and A=74 triplet, respectively, corroborating the evolution of the shape mixing with increasing spin.

In figure 5 are depicted the complex Excited Vampir results on Coulomb energy differences for the $^{70}$Br - $^{70}$Se and $^{74}$Rb - $^{74}$Kr analogs obtained using the above defined Hamiltonians based on Bonn CD potential compared with available data [6-12]. The trend manifested in the data is reproduced by the EXVAM results for the A=74 pair of nuclei as well as the anomalous behaviour revealed for the $^{70}$Br - $^{70}$Se case.

In figure 6 are presented the complex Excited Vampir predictions on mirror energy differences and triplet energy differences for the A=70 isovector triplet. MED manifest a negative trend, while TED indicate small positive values up to spin 4$^+$ and a small negative value for the spin 6$^+$. In figure 7 are depicted the complex Excited Vampir predictions on mirror energy differences and triplet energy differences for the A=74 isovector triplet. MED manifest a positive trend, while TED indicate a negative trend in agreement with the recent experimental results [16].

Aiming to a unified description of the structure and dynamics of the low-spin states in the nuclei building out the two isovector triplets we extended our investigations to the isospin-mixing and shape coexistence effects on the superallowed Fermi $\beta$-decay of the Z=N+2 members of the A=70 and A=74 isovector triplets, $^{70}$Kr and $^{74}$Sr isotopes. We studied the Fermi $\beta$-decay for
the ground state and yrast $2^+$ state in $^{70}\text{Kr}$ to the analog states in $^{70}\text{Br}$ as well as for the lowest two $0^+$ and $2^+$ states in $^{74}\text{Sr}$ to the analog states in $^{74}\text{Rb}$.

In figure 8 and 9 are depicted the Fermi strength distributions for the decay of the ground state and yrast $2^+$ state in $^{70}\text{Kr}$ to the daughter states in $^{70}\text{Br}$. Our results indicate for the decay of $^{70}\text{Kr}$ a depletion of the ground to ground transition up to 1.95% with the missing strength distributed over many excited $0^+$ states. The strongest nonanalog branches are feeding the fourth and the fifth $0^+$ states in $^{70}\text{Br}$ situated around 3 MeV excitation energy with an upper limit of 0.4% of the total strength. For the decay of the yrast $2^+$ state in $^{70}\text{Kr}$ the depletion of the analog branch amounts to maximum 2.98% and a significant nonanalog branch is feeding the fourth $2^+$ state in $^{70}\text{Br}$ with an upper limit of 1.3% of the total strength.

In figure 10 and 11 are illustrated the Fermi strength distributions for the decay of the ground state and yrast $2^+$ state in $^{74}\text{Sr}$ to the daughter states in $^{74}\text{Rb}$, respectively. For the depletion of the ground to ground decay we found an upper limit of 3%, while for the yrast $2^+$ decay a maximum of 3.6% of the total strength. The missing strength in the decay of the ground to ground state is distributed over many $0^+$ states in $^{74}\text{Rb}$ with an upper limit for the branch to the second and the sixth excited $0^+$ state of 0.8% from the total strength. In the decay of the yrast $2^+$ state in $^{74}\text{Sr}$ two significant nonanalog branches feed the second ($\leq 1.3\%$) and the fourth ($\leq 0.8\%$) $2^+$ states in $^{74}\text{Rb}$ and the rest of the strength is distributed over many $2^+$ states.
For comparison the Fermi strength distributions for the decay of the first excited $0^+$ and second $2^+$ state in $^{74}$Sr are illustrated in figure 12 and 13, respectively. According to complex Excited Vampir results the excitation energy of these states is lower than 1 MeV. Consequently, thermally populated in the X-ray burst environment their decay could have relevance for the astrophysical scenarios concerning the rp-process path.

4. Conclusions
The present report represents the first unitary beyond-mean-field treatment based on an effective two-body interaction constructed from a G-matrix starting from charge dependent Bonn CD potential of isospin-symmetry-breaking and shape coexistence effects on the structure of the A=70 and A=74 isovector triplets of nuclei as well as on the superallowed Fermi $\beta$-decay of their Z=N+2 members, $^{70}$Kr and $^{74}$Sr. Experimental information on the properties of the yrast and non-yrast low-lying states as well as on the predicted significant nonanalog branches are required to support our predictions.

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