The digital twin of a warehouse robot for Industry 4.0

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Abstract. One of the most important components in the modern conception Industry 4.0 is the digital twin. The special feature of the mechanical engineering production is the high proportion of time, spent on an interoperalational storage. High efficiency of the enterprise activity is connected with the solution of a flexible automatic warehousing system creation. It is necessary to describe structural and dynamic parameters of a robot-stacker in the mathmalisms. This will allow to find the optimal ratio between the load capacity, cargo speed and statistic, dynamic indicators of positioning accuracy. The obtained results of a mathematical modeling can be used as the algorithms for an integrated environment of a software development for various automation systems of warehouse processes.

1. Introduction
The creation of modern technologically sophisticated equipment, such as a robot-stacker, must take into account several multidirectional requirements [1, 2]. High productivity, positioning accuracy, as well as a flexible response to changing working conditions are required. In this case, the conditions of economic benefit [3], reliability and profit should be observed [4,5]. An effective chain of design, creation and operation of robots depends on multidisciplinary [6] solutions. Mathematical modeling allows not only to abandoning long and laborious tests [7]. The development of digital twin as an innovative method, integrates technical solutions into the end-to-end digital platform of the production cycle immediately. Digital twin algorithms [8,9] are part of the manufacturing execution system as an essential component of cyber-physical systems. In this article, we consider the dynamics of robot-stacker providing the interacting with static and dynamic cells of a rebuilt warehouse of mechanical engineering production. Such innovative solutions allow you to quickly defragment your storage space, optimize cargo handling. Since today in the machining shops the amount in interoperalational warehouses is up to 80-90% of the time, the gain in productivity can be expected to be very significant. In addition, the dynamic accuracy will be provided. It is necessary for the operation of artificial vision systems.
2. Statement of the research problem

The interrelation of parameters of the warehousing zone and those of the robot-stacker [10] can be defined as follows: the structural design of the elements of variable size cells, as well as requirements to warehousing of goods dictate the predetermined accuracy of positioning, while the performance of the warehouse determines the permissible time for correction of the dynamic error. The same groups of parameters [11] exert influence in the reverse direction, which permits their joint development. For theoretical description of the dependences mentioned, a mathematical model [12, 13] of the structure of the prevailing type of the robot-stacker is proposed. It represents a bearing column, traveling on rails (upper and lower) along the warehousing rack. The column bears the lift with vertical positioning mechanisms and control [14] of loading forks, which effect handling of goods and elements of cells.

3. Research method

The study was conducted by mathematical [15,16] modeling. To verify the theoretical model, computer calculations were carried out using real data from modern warehouses of mechanical engineering production.

4. The main formalisms

The static accuracy \( \Delta_{ST} \), is understood here as an index determined by the summary error [17] of positioning depending on both the capabilities of the control system and errors of sensors, as well as the quality of manufacture of mechanical parts (gaps), deformations of structural elements under the action of different loads, etc. The dynamic accuracy \( \Delta_{MOV} \) gives the evaluation of the time [6] of delay of the robot-stacker till the moment, when the oscillation amplitude of the load carrying device attains the value below the permissible [18, 19] one.

The kinematics' model includes the shifts \( x, h \), required for positioning as well as the main sources of oscillations in the given device, caused by the presence of elasticity and influencing the dynamic error: twisting around the axis of the stacker column – \( \alpha \) and shift of the load carrying fork – \( \beta \). After setting-up the expressions for kinematics' and potential energy, and basing on them, Lagrange equations of II degree, the system of differential equations is obtained for calculation of the robot-stacker dynamics.

\[
(m_n + m_b + m_r) \ddot{x} + m_n (\ddot{\alpha} r \cos \alpha - \dot{\alpha} \dot{\alpha} r \sin \alpha) + \\
(m_n + m_b + m_r) \ddot{h} + m_n (J_n + J_r + J_\alpha) \dot{\alpha} + m_r (\ddot{\alpha} r \cos \alpha - \ddot{\alpha} r \cos \alpha) - \\
(\dot{x} + \dot{\alpha} r \cos \alpha) \dot{\alpha} \dot{\alpha} r \cos \alpha + \dot{r} r \cos \alpha + \ddot{r} r \sin \alpha + \dot{\alpha} r \cos \alpha) - \\
(m_n + m_b + m_r) \left[ (\dot{\alpha} r + \dot{\beta}) (\cos \alpha - \ddot{\alpha} (\dot{\alpha} r + \dot{\beta} \sin \alpha) \right] r \cos \alpha + \\
+ (m_n + m_b + m_r) \right] (\dot{\alpha} r + \dot{\beta} (\cos \alpha - \ddot{\alpha} (\dot{\alpha} r + \dot{\beta} \sin \alpha) \right) r \cos \alpha - \\
- \dot{r} (\dot{x} + (\dot{\alpha} r + \dot{\beta} \cos \alpha) \dot{\alpha} \sin \alpha) + (\dot{\alpha} r + \dot{\beta}) \dot{\alpha} \cos \alpha \right] r \sin \alpha + \\
+ (\dot{\alpha} r + \dot{\beta}) r \dot{\alpha} \sin 2 \alpha / 2 + m_r (\dot{x} + \dot{\alpha} r \cos \alpha \dot{\alpha} \sin \alpha + (\dot{\alpha} r + \dot{\beta}) \dot{\alpha} \cos \alpha \right] r \sin \alpha - \\
- \dot{\alpha} r \sin 2 \alpha / 2 + \left[ \dot{x} + (\dot{\alpha} r + \dot{\beta} \cos \alpha \right] (\dot{\alpha} r + \dot{\beta}) \dot{\alpha} \cos \alpha - (\dot{\alpha} r + \dot{\beta}) \dot{\alpha} \sin \alpha \right] \cos \alpha + \\
(m_n + m_b + m_r) \left[ \dot{x} + (\dot{\alpha} r + \dot{\beta} \cos \alpha - (\dot{\alpha} r + \dot{\beta}) \dot{\alpha} \sin \alpha \right] \cos \alpha + \\
\right] \dot{x} + (\dot{\alpha} r + \dot{\beta} \cos \alpha \dot{\alpha} \sin \alpha + (\dot{\alpha} r + \dot{\beta}) \dot{\alpha} \sin 2 \alpha / 2 + k_\alpha \alpha = 0 \\
\left[ \dot{x} + (\dot{\alpha} r + \dot{\beta} \cos \alpha \dot{\alpha} \sin \alpha + (\dot{\alpha} r + \dot{\beta}) \dot{\alpha} \sin 2 \alpha / 2 + k_\beta \beta = 0 \\
\right]
\]
where \( m_s, m_r, m_b, m_v \) are accordingly the mass of the stacker columns, lift, load fork, load; \( r_s, r_r \) are distances from the lift axis to the centers of mass of the lift and the load fork with a load; \( J_s, J_r, J_f \) are the central moments of inertia of the lift, load fork and load; \( k_s \) and \( k_r \) are rigidity coefficients corresponding to elastic shifts of \( \alpha \) and \( \beta \); \( Q_s \) and \( Q_r \), are the generalized forces of action of the actuating devices for transfers \( x \) and \( h \).

5. Mathematical model
The above system permits [20] to calculate all the required dynamic parameters of the robot-stackers and determines the interdependence of positioning modes and the required dynamic error set-up time depending on the design characteristics of the robot-stackers. The required maximum permissible error of positioning equal to \( \Delta_{st} + \Delta_{mov} \) can be obtained from consideration of the load placing processes and transfer of the elements of cells ensuring change of their dimensions. The equations and basic calculation method for these processes are considered in paper [21], and for our purposes we shall use the most essential result, that is the influence of the dimensionless coefficients \( k_s \) and \( k_r \) on the summary relative time \( \tau \) of an operating cycle including the duration of approach to the desired cell, time delay for damping of oscillations and load handling. The relative and absolute parameters relation has the following form:

\[
\tau = t_{H_m} / V_m; \quad K_s = k_s r_s / J_s V_m; \quad K_r = k_r r_r / P_r,
\]

where \( H_m \) is the lifting height of the robot-stackers, \( V_m \) is the medium speed, \( J_s \) is the summary reduced moment of the inertia, \( t \) is time, \( P_r \) is the goods weight.

The generalized dependence [22, 23] between the given values allows us to conclude that there is a zone of the most rational design parameters of the robot-stackers, determined by the minimum value of \( \tau \). So, the above results permit to determine the interrelation [24] between the main elements of the rigid automatic warehousing system: the warehousing zone and the robot-stackers serving this zone. Basing on the automated warehouse (AW) volume utilization factor, which is directly connected with the design of the cells, as well as on the performance determined by the robot-stackers characteristics, one can derive a criterion for selection of the flexibility degree, ensuring the maximum obtainable economical effect. It is necessary to take into account [25], that in each particular case the economical calculation is performed with due allowance for the specific features of the automatic warehouse under consideration, which is determined by the selection of the relevant [26] efficiency criterion.

6. Calculation
For illustration, as the simplest criterion, we use the wide-spread linear functional of the type

\[
E = \sum b_i y_i, \quad \text{where } b_i \text{ are the weight coefficients; } y_i \text{ are the characteristic of the AW, which can be taken from the theoretical expressions obtained in the proposed paper: } y_1 \text{ is the volume utilization factor; } y_2 \text{ is the performance, } y_3^{-1} \text{ are the costs of manufacture of the cell elements and the robot-stackers which we adopt in mean root square dependence on the flexibility index and working cycle time accordingly. The use of the present (and any other) criterion requires a joint solution of the equations obtained in the proposed paper by numerical methods with the help of a computer. For search of the AW optimum [27] (as per the criterion max } E \text{ ) data the method of the fastest descent was used. The calculation was based on characteristics of the two most widely used automatic warehouses of the IHI (Japan) and NOKIA (Finland) companies. The data for the calculation are presented in table 1.}

Adopted as characteristics of goods flows were the normal distributions of volumes and specific weights [28] by volume with indices: \( q = 2.5 \), \( \rho = 3 \). As a result of the calculation, zones of optimum values of the discreteness index were obtained. The results obtained for the IHI warehouse allow to
draw a conclusion, that in case of the limited load carrying capacity and less efficient volume utilization of the cells, a rather high index of discreteness (or the order of 8 to 10) is required, then the performance increases by 20-25%, volume utilization factor by 10-12%, which is equivalent to increase in the number of unitary goods by 22%; however, to ensure these capabilities it is necessary to finalize the design of the robot-stacker with the aim of raising its travel speed up to 35-40 meters per minute with simultaneous calculation of the rigidity of its construction within 100 to 200 kg per millimeter. With sufficient load carrying capacity and high utilization index [15] of the cell volume, the characteristics of the IHI automatic warehouse are close to theoretically optimum values, that is the calculated $m$ is close to unity [29, 30].

### Table 1. Data for calculation.

| Warehouse parameters | Number of cells, pc | Cell dimensions, mm | Load carrying capacity, kg | Speed of robot-stacker, m/min | Warehouse height, m |
|----------------------|---------------------|----------------------|-----------------------------|-------------------------------|-------------------|
| IHI                  | 800                 | 2000x1000x1500       | 400                         | 20                            | 20                |
| NOKIA                | 240                 | 1200x600x800         | 250                         | 30                            | 12,4              |

Similar results are obtained for the warehouse of the NOKIA company, whence it follows that with the discreteness indices 3-5 the efficiency will rise by 20%, the volume utilization factor will rise by 8 to 10%, and the number of goods in warehousing at a time can be raised by 15 to 20%.

7. **Conclusions**

The results can be scaled and applied in other industries. Since modern network commerce makes extensive use of distribution center systems, integrated automation will make it possible to increase the density of the stock keeping unit. At the same time, the operation of warehouse equipment occurs under the control of artificial intelligence according to the standards of inter-machine interaction Industry 4.0. On the basis of research of the mathematical model of the automatic warehouse with rearrangeable dimensions of cells and served by a robot stacker, theoretical methods are developed for calculation of basic characteristics of the robot-stacker. The influence of the robot-stacker basic constructional data on the dynamics, as well as on the process of loading and rearrangement of the warehouse cells has been studied. Calculation methods are obtained for the ranges of values of the basic parameters of a flexible automatic warehousing system ensuring its functioning with the maximum efficiency, and the practical calculation has been carried out. Combination of theoretical results obtained with the economical calculation made separately for each particular warehouse will make it possible to develop the flexible automatic warehousing systems with optimum parameters.

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