Sign Reversals of ac Magnetoconductance in Isolated Quantum Dots

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We have measured the electromagnetic response of micron-size isolated mesoscopic GaAs/GaAlAs square dots down to temperature $T = 16 \mu K$, by coupling them to an electromagnetic microresonator. Both dissipative and non dissipative responses exhibit a large magnetic field dependent quantum correction, with a characteristic flux scale which corresponds to a flux quantum in a dot. The real (dissipative) magnetoconductance changes sign as a function of frequency for low enough density of electrons. The signal observed at frequency below the mean level spacing $\Delta$, corresponds to a negative magnetoconductance, which is opposite to the weak localization seen in connected systems, and becomes positive at higher frequency. We propose an interpretation of this phenomenon in relation to fundamental properties of energy level spacing statistics in the dots.

There are several ways of measuring the electrical conductance of a metallic sample. The most common one is to drive a current through it and measure the voltage drop between the connected wires. But, it is also possible to measure the conductance without making any connection to the sample. Capacitance and inductance measurements at finite frequency provide such a way. When a metallic sample of typical size $a$ is inserted into a coil, it acquires a magnetic moment $m \simeq Ia^2$, where $I$ is the eddy current induced in the sample. This changes the impedance of the coil by a quantity: $\delta Z(\omega) \simeq \omega \chi_m(\omega)/V$ where $V$ is the volume of the coil of inductance $L$ and $\chi_m$ is the magnetic susceptibility of the sample related to the conductance $G_m$, the ratio between $I$ and the induced electromotive force by $\delta Z$: $\chi_m(\omega) = \frac{I}{\omega V} G_m(\omega) a^4$ (1)

Similarly the admittance change of a capacitance $C$ due to the insertion of a metallic sample reads: $\delta Z^{-1}(\omega) \simeq \omega \chi_c(\omega)/V$ where $V$ is the volume of the capacitor and $\chi_c(\omega)$, the electrical polarisability of the sample, is related to its effective conductance $G_e$ (defined as ratio between polarization current and unscreened voltage drop through the sample), by $\chi_c(\omega) = \frac{G_e(\omega) a^2}{i \omega \varepsilon_0}$ (2)

In the classical limit at low frequency (such that the size of the sample is much smaller than the skin depth) $G_m$ is real and identical to the Drude conductance $G_D = \sigma a$ within a numerical constant which depends on the geometry of the sample. On the other hand due to screening of the electric field inside the metal $\chi_c$, and for $\omega < \sigma/\varepsilon_0$, $G_e$ is dominated by its imaginary part: $\text{Im} G_e = \omega a \varepsilon_0$. Its real part is equal to:

$$\text{Re} G_e = \frac{(\varepsilon_0 \omega a)^2}{G_D}$$ (3)

In a mesoscopic sample, which length is smaller than the electronic phase coherence length, it was shown theoretically [2] that the conductance strongly depends on the way it is measured. More precisely, it is very sensitive to the coupling between the mesoscopic sample and the macroscopic classical apparatus. When a sample is connected with wires, (that is to say when the coupling is strong), relative quantum corrections on the conductance are of the order of $1/g$ where $g = G_D/\epsilon^2/h$ [9]. This result concerning the dissipative components of the conductance are unchanged for unconnected samples provided that the energy level spectrum is continuous [2] i.e. the typical level width $\gamma$ is much larger than the mean level spacing $\Delta$. Note however that the quantum corrections of Re$G_e$ and Re$G_m$ are opposite in sign. Furthermore Im$G_m$ is expected to be finite and directly related to persistent currents and more generally orbital magnetism in the sample [11 12]. At the same time Im$G_e$ acquires a magnetic field dependence related to quantum corrections to the electrical polarisability [1].

On the other hand, if the discrete spectrum limit $\gamma \ll \Delta$ is achieved, quantum coherence of the sample gives rise to giant magnetoconductance [8 9] whose sign and amplitude are determined by the order relation between the relevant energy scales: $h \omega, \gamma, \Delta, k_BT$. Preliminary experiments [12] on an array of Aharonov-Bohm rings coupled to a resonator, have shown that the investigation of ac conductance on isolated samples in the discrete spectrum limit is indeed possible.

In this letter, we report the measurement of the electromagnetic response of an array of mesoscopic square dots made from GaAs/GaAlAs 2D electron gas at frequencies both below and above energy level spacing. Sign reversals of magnetoconductance were observed which can be interpreted in relation to the sensitivity to magnetic field of level spacings distribution in the dot, according to random matrix theory.

As in the previous experiment [12], the mesoscopic samples are coupled to an electromagnetic microresonator, made of two Niobium wires ($d = 4 \mu m$ spaced, $1 \mu m$ width and $20 cm$ long) deposited in a meandering geometry on a sapphire substrate. The superconducting material is used to reduce losses and therefore get a high

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squares by transient illumination of the samples (between 30s and 2mn). The measurements were always done at least one hour after the illumination was stopped, in order to change gradually the number of electrons in the luminescent diode was placed next to the resonator, allowing to detect extremely small variations of frequency in the samples affects the quality factor, whereas the non dissipative response affects the resonance frequency:

$$\frac{\delta f}{f} = -k N_s \chi'(\omega), \delta (Q^{-1}) = k N_s \chi''(\omega)$$

where $$\chi(\omega) = \chi_c(\omega) + \chi_m(\omega)$$ is the average complex susceptibility of a dot [4], $$N_s$$ is the number of samples and $$k$$ is a geometrical coupling coefficient which is of the order of $$1/Ld^2$$, where $$d$$ is the distance between the two Niobium wires. The sample studied in this experiment, was an array of $$10^5$$ disconnected square dots etched, using electron beam lithography techniques, in a semiconductor heterojunction GaAs/GaAlAs. The electron concentration of the two dimensional electron gas (2DEG) before etching was $$n_e = 3.10^{11}$$ cm$$^{-2}$$ and elastic mean free path $$l_e = 10 \mu$$m. This electronic density was strongly depressed in etched samples but could be recovered with illumination. For this purpose an electroluminescent diode was placed next to the resonator, allowing to change gradually the number of electrons in the squares by transient illumination of the samples (between 30s and 2mn). The measurements were always done at least one hour after the illumination was stopped, in order to ensure good stability of the samples. The size of the squares $$a = 1.5 \mu$$m was estimated from weak localization experiments on connected samples made together with the squares. The transport in the samples can thus be considered as ballistic. The mean level spacing in a square $$\Delta = 2 \frac{\hbar}{m^* a}$$, ($$m^*$$ is the effective electron mass) is independent of the electronic density. It is of the order of 38mK which corresponds to a frequency of 760MHz. Resonator and diode are enclosed in a metallic box, protecting from electromagnetic noise, and cooled with a dilution fridge down to 16mK.

We have measured the magnetic field dependence of the resonance frequency and quality factor for several harmonics ($$f_1 = 330$$MHz, $$f_2 = 670$$MHz, $$f_3 = 1065$$MHz) and after successive illuminations. The dc magnetic field was modulated at low frequency ($$\sim 30$$Hz) with a small coil close to the sample. This allows measurement of derivatives of resonance frequency (see fig[1]) and quality factor with respect to magnetic field. The signal of the samples consists in step-like features on the field dependence of $$\delta f/\partial H$$ and $$\delta Q^{-1}/\partial H$$ whose width $$\sim 10$$ Gauss corresponds approximatively to half a flux quantum in a square. In the case of $$\delta f/\partial H$$ the contribution of the empty resonator, linear in magnetic field, has to be sub-
tracted from the signal. After integration, the magnetococonductance $\delta G(H) \propto \delta \chi(H) = \chi(H) - \chi(0)$ of the samples can be deduced from these measurements (see fig. 3). We observe that both real and imaginary components present the striking triangular shape observed in experiments on connected ballistic dots of similar geometries which is believed to be related to the classical integrability of these systems [3].

$\delta \chi'(H)$ and $\delta \chi''(H)$ are found to have very different behaviors as a function of frequency. $\delta \chi'(H)$ is always positive, its amplitude drastically increases with the first successive illuminations which correspond to an increasing number of electrons, and then tends to saturate. For a fixed number of electrons it is nearly independent of frequency [4]. On the other hand $\delta \chi''(H)$ is observed to be negative for the first harmonics and small electronic density and becomes positive with increasing frequency and electronic density. This sign change of the magnetococonductance is the most striking result of our work.

The temperature dependence of the signals were also measured. The temperature decay depicted in fig. 3 is found to be independent of frequency and nearly identical for $\delta \chi'$ and $\delta \chi''$. It cannot be fitted with an exponential as was the case for the ring experiment. The characteristic temperature scale of this decay slightly increases with the number of electrons in the dots.

In the following we want to compare our results with theoretical predictions on orbital magnetism and polarisability. The orbital susceptibility of ballistic squares has been calculated by several authors, it is found to be paramagnetic in zero field, in agreement with experimental results on the dc magnetization of GaAs/GaAlAs squares [4]. Our results on the other hand indicate unambiguously low field diamagnetism, just like the previous ac experiment on an ensemble of rings [4]. We note however that the amplitude of $\delta \chi'(H)$ measured at saturation correspond to an average orbital susceptibility of the order of the theoretical prediction: $g a^2 \mu_0 e^2 / m^*$. In the case of an electric coupling, the signal can be related to the average magneto-polarisability of the squares. This quantity has been recently calculated [3], its sign (which corresponds to an increase of polarisability with magnetic field) could explain our experimental results on $\delta \chi'(H)$. Nevertheless, according to these predictions, the magneto-polarisability is proportional to $1/g$ and also to the Thomas-Fermi screening length. Thus it should decrease with the number of electrons, which is in complete contradiction with what we measured. So, the understanding of our measurements on $\delta \chi'(H)$ deserves further theoretical investigations.

We now consider the dissipative part of the response. The low frequency classical absorption of a metallic grain in an ac electric field has been shown [3] to be described by the effective conductance $G_e$ (see expression (3)). This result initially derived for 3D systems can easily be extended to 2D samples provided that the electric field lies in their plane. The ratio between “magnetic” losses described by $G_m$ and “electric” losses in the resonator is then simply given by: $r = (Z_0 G_D)^2$, where $Z_0 = \sqrt{\mu_0 / \epsilon_0} = 377 \Omega$ is the vacuum impedance. In our case $r$ varies between 8 and 400 depending on the electronic density in the dots. Therefore, we conclude that the dominant contribution to $\chi''$ comes from $G_m$. Using expression (3) relating $G_m$ to the measured quantity $\delta \chi''(H)$, it is possible to express the magnetoconductance measured for several frequencies and number of electrons in units of $G_D$ as shown in fig. 3. Note that in most cases the magneto-conductance is of the order of $G_D$ as expected for a discrete level system.

The field dependence of the dissipation of Aharonov-Bohm rings in a time dependent flux has been extensively studied [4–7]. It is straightforward to extend these results to singly connected geometries and estimate expected $\delta G_m(H)$ in our case. Let us recall the existence of two different contributions to $\delta G_m(H)$ in the limit where $\gamma \ll \Delta$ and $T > \Delta$: $\delta G_{\text{off}}$ and $\delta G_{\text{di}}$. $\delta G_{\text{off}}$ related to inter-level transitions is expressed in terms of the the level spacing distribution function $R(s)$ which obeys universal rules of random matrix theory [17, 18]. It corresponds for $H = 0$ to Gaussian Orthogonal Ensemble (GOE) and for sufficiently large field, in order that time reversal symmetry is broken, to Gaussian Unitary Ensemble (GUE).

$$\delta G_{\text{off}}(H) = G_D \int \frac{\gamma (R_{\text{GUE}}(s) - R_{\text{GOE}}(s))}{(\hbar \omega - s)^2 + \gamma^2} ds$$  \hspace{1cm} (5)

On the other hand $\delta G_{\text{di}}$ is related to the relaxation of
persistent currents which always yields to positive magnetoconductance:

$$\delta G_{di}(H) = G_{di}(GUE) - G_{di}(GOE) = G_D \frac{\gamma \Delta}{\omega^2 + \gamma^2} (6)$$

In the limit where $\gamma \ll \omega$ the magneto-conductance is dominated by: $\delta G_{off}(H) \sim G_D(R_{GUE}(\omega) - R_{GOE}(\omega))$. The magnetoconductance is negative at $\omega < \Delta$ and changes sign at frequency of the order of $\Delta$ as depicted in fig. We believe that the squares after illumination 1 correspond to this situation. This change of sign is however expected to disappear when $\gamma$ increases such as the condition: $\delta G_2 > |\delta G_1|$ is realized. We suggest that illuminations 3 and 4 correspond to this last situation. The increase of $\gamma$ for successive illuminations is necessary to explain our results. It cannot be due to the contribution of electron-electron interactions (whose contribution decreases with increasing electron density [19]) but possibly to the increase of losses in the etched GaAs substrate. We indeed observed a substantial decrease of $Q$ after each illumination. The electromagnetic noise related to these losses is expected to contribute substantially to the level broadening [20,21]. Theory [6,7] also predicts, for canonical ensemble, changes of sign of the magnetoconductance with temperature at $T < \Delta$ which we have not observed, possibly because of lack of experimental points in the regime where $T \ll \Delta$.

In conclusion these results show that ac conductance measurements on isolated samples reveal indeed new physical features. Giant magnetoconductance has been observed. Sign changes on the real part are in agreement with the pioneer predictions of Gorkov and Eliashberg [18] on the sensitivity of the energy spectrum to time reversal breaking by magnetic field. The imaginary part whose amplitude is of the same order of magnitude than the real one is in principle related to the orbital magnetism in the dots. However its sign is not yet understood.

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FIG. 4. Calculated frequency dependence of $\delta G_m$ for different values of the ratio $\gamma/\Delta$.

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