Semiclassical corrections to a regularized
Schwarzschild metric *

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Abstract
A regular form of the Schwarzschild geometry is proposed. It is more suitable for application in microphysics because the source mass comes out both as a Schwarzschild radius and the Compton wavelength of the mass \( m \). The Komar energy equals \( mc^2 \) in the classical situation \( (\hbar = 0) \).

We propose a regular version of the Schwarzschild metric, to be valid in microphysics. The time-time metric coefficient is modified as [1] (see also [2])

\[-g_{tt} = 1/g_{rr} \equiv f(r) = 1 - \frac{2m}{r} e^{-\frac{1}{kr}}, \tag{0.1}\]

where \( m \) is the object mass, \( k \) is a positive dimensionless constant and has units of length in front of the exponential and \( 1/\text{length} \) at the exponent. We select \( k = 2/e \), so that \( f(r) \) becomes minimal at \( r = k/m = 2/me \). For a horizon to exist, we found that the condition \( m \geq m_P \) should be obeyed [3].

An expansion of \( f(r) \) for \( r >> r_0 = 2/em \) gives us

\[ f(r) \approx 1 - \frac{2m}{r} + \frac{4l_P^2}{er^2}, \tag{0.2}\]

where \( l_P \) is the Planck length. From (0.2) one obtains that \( f(r) \) acquires its Schwarzschild value when \( \hbar = 0 \). The solution (0.1) is not a vacuum solution of Einstein’s equations. The source stress tensor has \( p_r = -\rho \) and fluctuating transversal pressures, where \( \rho \) is the energy density and \( p_r \) is the radial pressure.

The Komar energy associated to the geometry (0.1), with \( k = 2/em \), appears as

\[ W = \left( mc^2 - \frac{2hc}{2m} \right) e^{-\frac{2m}{cmr}}. \tag{0.3}\]

which tends to zero when \( r \to 0 \) and \( W \to mc^2 \) at infinity. The classical situation \( (\hbar = 0) \) leads to the standard result \( W = mc^2 \).

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References

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