Central inclusive dijets production by double pomeron exchange.
Comparison with the CDF results

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Using the Landshoff-Nachtmann model of the pomeron, rapidity and transverse momentum distributions of gluon jets produced by double pomeron exchange in $pp$ ($p\bar{p}$) collisions are calculated. The comparison with the CDF Run I results on the central inclusive dijets production cross sections is performed. We find the model to give correct order of magnitude for the measured cross sections.

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1 Introduction

The subject of Higgs boson production by double pomeron exchange (DPE) has drawn noticeable interest in recent years [1–11].

The lack of solid QCD framework for diffraction makes the determination difficult. Despite the distinct progress in the recent years the serious uncertainties are still present.

One way to reduce these uncertainties is to study other double pomeron exchange processes and compare them with existing data. A particularly illuminating process is the DPE production of two jets (dijets). Such a process was originally discussed at the Born level in [12]. Later the dijets production was studied in [5, 13] and in [8–11, 14].

Recently, using the Landshoff-Nachtmann model of the pomeron, the cross-section for gluon jets production was calculated [15]. The obtained results together with those for quark-antiquark jets calculated some time ago [16, 17] give the full cross-section for dijet production in double pomeron exchange reactions.\(^1\)

\(^1\)It is generally accepted that DPE dijets mainly consist of gluon jets. For that reason we will use gluon jets and dijets interchangeably.
In the present paper we calculate the differential cross-section for gluon jets production hoping that this will allow a more precise comparison of the model with experiment.

The comparison with the CDF Run I results on the central inclusive dijets production cross sections [18] is also performed.

Figure 1: Production of two gluons by double pomeron exchange.

2 Matrix element and notation

In the Landshoff-Nachtmann model [19] the pomeron is approximated by an exchange of two non-perturbative gluons coupled to one of the quarks of the colliding hadrons.

The matrix element for gluon jets by double pomeron exchange in such model is given by the sum of the three diagrams shown in Fig. 2 (plus analogous emissions from the second gluon)

Figure 2: Three diagrams contributing to the amplitude of the process of gluon pair production by double pomeron exchange. The dashed lines represent the exchange of the non-perturbative gluons.

where the inner quark lines are put on shell. In this model, it was shown that the square of the matrix element (averaged and summed over spins and polarizations)
for the diffractive production of two gluons is of the form [15]:

$$|M_{pp}|^2 = 81|M_{qq}|^2 [F(t_1)F(t_2)]^2,$$  \hspace{1cm} (1)

where $|M_{qq}|^2$ is the dijet production amplitude squared for colliding quarks$^2,^3$:

$$|M_{qq}|^2 = C s^2 (u_1)^2 (u_2)^2 \delta_1^{2-2\alpha(t_1)} \delta_2^{2-2\alpha(t_2)} \exp (2\beta (t_1 + t_2)) R^2.$$  \hspace{1cm} (2)

Transverse momenta of the produced gluons are denoted by $u_1$ and $u_2$. The constants $C$ and $R$ are defined later. $\alpha(t) = 1 + \epsilon + \alpha' t$ is the pomeron Regge trajectory with $\epsilon \approx 0.08$, $\alpha' = 0.25 \text{ GeV}^{-2}$ ($t_1, t_2$ are marked in Fig. 1). $F(t) = \exp(\lambda t)$ is the nucleon form-factor with $\lambda = 2 \text{ GeV}^{-2}$. $\delta_1, \delta_2$ are defined as $\delta_{1,2} \equiv 1 - k_{1,2}/p_{1,2}$ ($k_1, k_2, p_1, p_2$, are marked in Fig. 1). The factor $\exp (2\beta (t_1 + t_2))$ with $\beta = 1 \text{ GeV}^{-2}$ [20] takes into account the effect of the momentum transfer dependence of the non-perturbative gluon propagator given by ($p^2$ is the Lorentz square of the momentum carried by the non-perturbative gluon):

$$D(p^2) = D_0 \exp \left( \frac{p^2}{\mu^2} \right).$$  \hspace{1cm} (3)

The constants $C$ and $R$ are defined as:

$$C = \frac{1}{(2\pi)^4} (D_0 G^2 \mu)^6 \mu^2 \left( \frac{g^2/4\pi}{G^2/4\pi} \right)^2,$$  \hspace{1cm} (4)

$$R = 9 \int d\vec{Q}_\perp \vec{Q}_\perp^2 \exp \left( -3\vec{Q}_\perp^2 \right) = 1.$$  \hspace{1cm} (5)

Here $G$ and $g$ are the non-perturbative and perturbative quark gluon couplings respectively. $\mu$ is the range of the non-perturbative gluon propagator (3) and $D_0$ its magnitude at vanishing momentum transfer. From data on the elastic scattering of hadrons one infers $D_0 G^2 \mu \approx 30 \text{ GeV}^{-1}$ and $\mu \approx 1 \text{ GeV}$.

The constant $R$ shows the structure of the loop integral. $\vec{Q}_\perp$ is the transverse momentum carried by each of the three non-perturbative gluons.

Fig. 1 seems to describe exclusive dijets production. In fact, as was clearly stated in the original paper on the Higgs production [2], our calculation really is a central inclusive one i.e. the radiation is present in the central region of the rapidity.

$^2$Note the Regge-like dependence implied by Eq. (2). There are some controversies if such assumption is justified. The resent results [9, 10] on the inclusive DPE dijets production, however, are quite encouraging.

$^3$It is worth to stress that formula (2) is only valid in the limit of $\delta_{1,2} << 1$ and for small momentum transfer between initial and final quarks.
3 Differential cross-section

Having the matrix element (2) we can write the formula for the differential cross-section:

$$d\sigma = (2s)^{-1} (2\pi)^{-8} |M_{pp}|^2 dPH,$$

where $dPH$ is a differential phase-space factor:

$$dPH = d^4k_1 \delta (k_1^2) d^4k_2 \delta (k_2^2) d^4r_1 \delta (r_1^2) d^4r_2 \delta (r_2^2)$$

$$\times \Theta (k_1^0) \Theta (k_2^0) \Theta (r_1^0) \Theta (r_2^0) \delta^{(4)} (p_1 + p_2 - k_1 - k_2 - r_1 - r_2).$$

Expression (6) is to be integrated over all variables except rapidities and transverse momenta of the produced gluons. Following closely the method used in [17] we obtain the following result for the differential cross-section:

$$d\sigma \frac{d}{d^2u_+ d^2u y_1 dy_2} = C_E (u_1)^{-2} (u_2)^{-2} (\delta_1 \delta_2)^{-2e} \pi \frac{\pi}{L_1 + L_2} \exp \left( -\frac{L_1 L_2}{L_1 + L_2} (u_+)^2 \right).$$

In the above expression $y_{1,2}$ are the rapidities of the produced gluons, $u_+ = u_1 + u_2$, $u = (u_1 - u_2)/2$, $C_E = C \frac{81}{16(2\pi)^5}$ and $L_{1,2} = 2 (\beta + \lambda - \alpha' \ln \delta_{1,2})$. $\delta_1, \delta_2$ are expressed by rapidities and transverse momenta as follows:

$$\delta_1 \sqrt{s} = |u_1| \exp (y_1) + |u_2| \exp (y_2),$$

$$\delta_2 \sqrt{s} = |u_1| \exp (-y_1) + |u_2| \exp (-y_2).$$

The differential cross-section (8) gives a Gaussian cut-off on the total transverse momentum of the produced pair. Putting $u_+ = 0$ i.e $u_1 = -u_2 = u$ everywhere except in the exponent and performing the integration over $(u_+)^2$ we obtain:

$$d\sigma \frac{d}{d(u^2) dy_1 dy_2} = C_E (u)^{-4} (\delta_1 \delta_2)^{-2e} \frac{\pi^3}{L_1 L_2}.$$  

Taking into account (9) and the definition of $L_{1,2}$ we finally obtain:

$$d\sigma \frac{d}{d(E_\tau^2) d(\Delta y) dy} = C_E (E_\tau)^{-4} \left( \frac{4E_\tau^2}{s} \cosh^2 \left( \frac{\Delta y}{2} \right) \right)^{-2e}$$

$$\times \frac{\pi^3/4\alpha'^2}{\left( \frac{\lambda + \beta}{\alpha'} - \frac{1}{2} \ln \left( \frac{4E_\tau^2}{s} \cosh^2 \left( \frac{\Delta y}{2} \right) \right) \right)^2 - y^2}.$$  

Here $\Delta y = y_1 - y_2$, $y = (y_1 + y_2)/2$. $E_\tau = |u_1| = |u_2|$ is the transverse energy of one of the produced gluons.

This completes the calculations of the differential cross-section.
Some comments are in order.

(i) Since \( y^2 \ll (\frac{1}{\alpha_c^2})^2 = 144 \) and \( \frac{4E_1^2}{s} \cosh^2(\frac{\Delta y}{2}) = \delta_1 \delta_2 \ll 1 \) the differential cross section (11) depends very weakly on the sum of the dijet rapidities. It is worth to mention that the kinematical limit of the double pomeron exchange region depends on both the sum \( y \) and the difference \( \Delta y \) of the rapidities, as seen from (9).

(ii) The main uncertainty in the expression (11) is the value of \( G^2/4\pi \). Following [2, 16] we take it to be [21] about 1. In fact it should be considered only as an order of magnitude estimate.

4 Comparison with the CDF Run I results

The CDF collaboration has presented [18] results on the central inclusive DPE dijet production cross sections.

At Run I (\( \sqrt{s} = 1.8 \) TeV) the cross section for the central inclusive dijets of \( E_T > 7 \) GeV \([E_T > 10 \) GeV\] is measured to be \( 43.6 \pm 4.4 \text{(stat)} \pm 21.6 \text{(syst)} \) nb \([3.4 \pm 1 \text{(stat)} \pm 2 \text{(syst)} \) nb\]. The kinematics is following: \( 0.01 < \delta_1 \equiv \delta_p < 0.03, 0.035 < \delta_2 \equiv \delta_{\bar{p}} < 0.095 \), jets are confined within \( -4.2 < y < 2.4 \) and the gap requirement \( 2.4 < y_{\text{gap}} < 5.9 \) on the proton side.

It should be noted that in the above experiment the protons were not detected and the DPE events were enhanced by a rapidity gap requirement on the proton side. Thus, in principle, there is no specified cuts on the outgoing proton\(^4\).

Integrating\(^5\) (11) over the appropriate kinematical range we obtain the results shown in Table 1. The running coupling constant \( g^2/4\pi \), appearing in (4), is evaluated at \( 2E_T^{\text{min}} \) \( i.e. \) 0.15 and 0.14 for \( E_T^{\text{min}} = 7, 10 \) GeV respectively. \( G^2/4\pi \) is taken to be 1.

| Transverse energy | CDF        | \( \sigma \) |
|-------------------|------------|-------------|
| \( E_T > 7 \) GeV | 43.6 \pm 26 [nb] | 70 [nb] |
| \( E_T > 10 \) GeV| 3.4 \pm 3 [nb]  | 30 [nb] |

Table 1: Comparison of the results obtained in the present paper with the CDF results on the central inclusive DPE dijets production cross sections. \( G^2/4\pi \) is taken to be 1.

As can be seen the original Bialas-Landshoff model [2] for DPE diffractive pro-

\(^4\)In principle the result (11) should by multiplied by a factor \( \left(1 - \exp\left(-L_1\frac{s(1-\delta_1)^2}{\exp(2y_{\text{gap}}^\text{max})}\right)\right) \) where \( y_{\text{gap}}^\text{max} \) is the maximum value of the gap. In the present case, \( y_{\text{gap}}^\text{max} = 5.9 \), this factor is close to 1.

\(^5\)Note an identical final state particle phase space factor \( \frac{1}{\pi} \).
duction gives correct order of magnitude for the measured central inclusive cross sections.

5 Discussion

The results shown in Table 1 do not take gap survival effect \( (S_{\text{gap}}^2) \) into account i.e. the probability of the gaps not to be populated by secondaries produced in the soft rescattering. Following [5,22] we take it for the Tevatron energy to be about 0.1 (it is the best we can do). Multiplying the results shown in Table 1 by \( S_{\text{gap}}^2 \) we obtain the results presented in Table 2.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Transverse energy} & \text{CDF} & \sigma \times S_{\text{gap}}^2 & \sigma \times S_{\text{gap}}^2 \\
\text{ } & \text{ } & \frac{G^2}{4\pi} = 1 & \frac{G^2}{4\pi} = 0.4 \\
E_\tau > 7 \text{ GeV} & 43.6 \pm 26 \text{ [nb]} & 7 \text{ [nb]} & 43 \text{ [nb]} \\
E_\tau > 10 \text{ GeV} & 3.4 \pm 3 \text{ [nb]} & 3 \text{ [nb]} & 19 \text{ [nb]} \\
\hline
\end{array}
\]

Table 2: Comparison of our results with the CDF results on the central inclusive DPE dijets production. Gap survival factor is taken into account.

Some comments seem to be in order.

(i) If we assume \( G^2/4\pi = 0.4 \) so that we reproduce the result for \( E_\tau > 7 \) GeV, the result for \( E_\tau > 10 \) GeV overestimates the measured cross section about 5 times. It means that some suppression with \( E_\tau \) is needed.

(ii) The expression (11) \( (+S_{\text{gap}}^2) \) do not include the Sudakov factor i.e. the probability that the rapidity gaps survive QCD radiation. We think that in our particular case, \( E_\tau > 7,10 \) GeV, the Sudakov physics is not so essential, however, the lack of Sudakov \( E_\tau \) suppression can be seen. It is not obvious how the Sudakov factor should be included in the presented model and this problem is currently under our consideration.

(iii) The factor \( S_{\text{gap}}^2 \) is not a universal number. It depends on the initial energy and the particular final state. Theoretical predictions of the survival factor, \( S_{\text{gap}}^2 \), can be found in Ref. [23].

(iv) We work at the partonic level. Thus, in a realistic experimental situation our results correspond to a smaller cone where the jet axes are confined, since the gluon produced just on the boundary of the cone usually produces secondary particles outside the cone and such event is not counted. This effect would decrease the calculated cross section. We checked that taking the range of the rapidity as \(-3.5 < y < 1.7 \) (CDF \(-4.2 < y < 2.4\)) our cross-sections decrease no more than
10%. To cover it fully Monte-Carlo simulations are needed [14].

Finally, let us note that estimates in the present paper, as well as in [2, 9, 10, 15–17], are based on the basis of the pure forward direction. It was first mentioned in [24] that such approach may lead to incorrect results.

6 Conclusions

In conclusion, using the Landshoff-Nachtmann model of the pomeron, we have presented the rapidity and transverse momentum distribution of central inclusive gluon jets produced by double pomeron exchange in $pp$ ($p\bar{p}$) collisions.

We observed a distinct dependence on the difference of gluons rapidities $\Delta y$ and the marginal dependence on their sum $y$. A power-law decrease approximately as $E_T^{-4.3}$, for relatively small $E_T$ production when the Sudakov physics is not essential, with the increasing transverse momentum is observed. We find the model to give reasonable predictions, at least correct order of magnitude for the measured central inclusive cross sections.

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