Pure Strategy or Mixed Strategy?  
An Initial Comparison of Their Asymptotic Convergence Rate and Asymptotic Hitting Time

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Abstract. Mixed strategy evolutionary algorithms (EAs) aim at integrating several mutation operators into a single algorithm. However no analysis has been made to answer the theoretical question: whether and when is the performance of mixed strategy EAs better than that of pure strategy EAs? In this paper, asymptotic convergence rate and asymptotic hitting time are proposed to measure the performance of EAs. It is proven that the asymptotic convergence rate and asymptotic hitting time of any mixed strategy (1+1) EA consisting of several mutation operators is not worse than that of the worst pure strategy (1+1) EA using only one mutation operator. Furthermore it is proven that if these mutation operators are mutually complementary, then it is possible to design a mixed strategy (1+1) EA whose performance is better than that of any pure strategy (1+1) EA using only one mutation operator.

Keywords: Mixed Strategy, Pure Strategy, Asymptotic Convergence Rate, Asymptotic Hitting Time, Hybrid Evolutionary Algorithms.

1 Introduction

Different search operators have been proposed and applied in EAs [1]. Each search operator has its own advantage. Therefore an interesting research issue is to combine the advantages of variant operators together and then design more efficient hybrid EAs. Currently hybridization of evolutionary algorithms becomes popular due to their capabilities in handling some real world problems [2].

Mixed strategy EAs, inspired from strategies and games [3], aims at integrating several mutation operators into a single algorithm [4]. At each generation, an individual will choose one mutation operator according to a strategy probability distribution. Mixed strategy evolutionary programming has been implemented for continuous optimization and experimental results show it performs better than its rival, i.e., pure strategy evolutionary programming which utilizes a single mutation operator [5,6].
However no analysis has been made to answer the theoretical question: whether and when is the performance of mixed strategy EAs better than that of pure strategy EAs? This paper aims at providing an initial answer. In theory, many of EAs can be regarded as a matrix iteration procedure. Following matrix iteration analysis [7], the performance of EAs is measured by the asymptotic convergence rate, i.e., the spectral radius of a probability transition sub-matrix associated with an EA. Alternatively the performance of EAs can be measured by the asymptotic hitting time [8], which approximatively equals the reciprocal of the asymptotic convergence rate. Then a theoretical analysis is made to compare the performance of mixed strategy and pure strategy EAs.

The rest of this paper is organized as follows. Section 2 describes pure strategy and mixed strategy EAs. Section 3 defines asymptotic convergence rate and asymptotic hitting time. Section 4 makes a comparison of pure strategy and mixed strategy EAs. Section 5 concludes the paper.

2 Pure Strategy and Mixed Strategy EAs

Before starting a theoretical analysis of mixed strategy EAs, we first demonstrate the result of a computational experiment.

**Example 1.** Let’s see an instance of the average capacity 0-1 knapsack problem [9,10]:

\[
\text{maximize } \sum_{i=1}^{10} v_i b_i, \quad b_i \in \{0, 1\}, \\
\text{subject to } \sum_{i=1}^{10} w_i b_i \leq C, (1)
\]

where \(v_1 = 10\) and \(v_i = 1\) for \(i = 2, \cdots, 10\); \(w_1 = 9\) and \(w_i = 1\) for \(i = 2, \cdots, 10\); \(C = 9\).

The fitness function is that for \(x = (b_1, \cdots, b_{10})\)

\[
f(x) = \begin{cases} 
\sum_{i=1}^{10} v_i b_i, & \text{if } \sum_{i=1}^{10} w_i b_i \leq C, \\
0, & \text{if } \sum_{i=1}^{10} w_i b_i > C.
\end{cases}
\]

We consider two types of mutation operators:

- s1: flip each bit \(b_i\) with a probability 0.1;
- s2: flip each bit \(b_i\) with a probability 0.9;

The selection operator is to accept a better offspring only.

Three (1+1) EAs are compared in the computation experiment: (1) EA(s1) which adopts s1 only, (2) EA(s2) with s2 only, and (3) EA(s1,s2) which chooses either s1 or s2 with a probability 0.5 at each generation.

Each of these three EAs runs 100 times independently. The computational experiment shows that EA(s1, s2) always finds the optimal solution more quickly than other twos.

This is a simple case study that shows a mixed strategy EA performs better than a pure strategy EA. In general, we need to answer the following theoretical