A sequentially advancing algorithm based on multi-value dynamic programming for the cut-off grade optimization in open-pit metalliferous deposits

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Abstract

In techno-economic concern, cut-off grade (COG) optimization is the key for efficient mineral liquidation from the huge metalliferous surface mining sector. In this paper, a sequentially advancing algorithm based on exact multi-value dynamic programming (MDP) has been developed to determine the optimum COG of an open-pit metalliferous deposit. The proposed COG optimization algorithm aims to overcome the limitations of straightforward classical techniques in determining the optimum COG. This discrete COG-MDP model is the first of its kind and has the novelty of dealing with the simulation of eight dynamic possibilities to achieve the maximal Net Present Value (NPV). A high-level programming language (Python) has been used to develop the computer model to deal with the complexity of handling a minimum of 500 series of dynamic variables with a precision value of 0.01% in grade bins. This model can generate results in polynomial-time from the complex mine, mill, and smelter and refinery system corresponding to various limiting conditions. The prime objective considered in the model is to optimize the COG of a metalliferous deposit. The model validation has been done using a real-life case study of an open-pit copper mine in India (Malanjkhand Copper Mine, HCL), considering the fixed yearly output of the mining, milling, and smelting and refining. In this study, the optimum COG for the Malanjkhand copper deposit has been found to be (0.33%, 0.23%, 0.52%, 0.26%, 0.27%, 0.22%, 0.24%) with a maximum NPV of ₹ (12204, 14653, 16948, 14609, 21454, 26717, 38821) million corresponding to various scenarios. The findings also show that the present value of net cash-flow grows in the early years, peaks at a specified mid-life time, and then drops as the reserve is depleted. The present value gradually hits zero after the project’s life cycle, confirming the typical pattern of other mining firms.

Keywords ● Multi-value dynamic programming ● Sequentially advancing algorithm ● Cut-off grade (COG) ● Open-pit mining ● NPV maximization.

1 Introduction and critical review of previous work done

Mineral resources are essential for contemporary society’s socioeconomic development. Even though these commodities have been widely cross-referenced as “finite,” their metal output has continuously risen throughout the ages. The twenty-first century has seen exponential resource extraction, corresponding to global economic expansion; consequently, long-term sustainable development is crucial. Global metal utilization is expanding at a rate of roughly 3.2% per year, boosting trade and commerce [1, 2]. Metal mining has consistently shown to be a significant source for meeting metal demands in infrastructure, health, information systems, and other services that can support economic growth [3]. Thus, COG plays a significant role in
achieving the objective of attaining techno-economic sustainability, which strives to maximize recovery utilization. The field of knowledge, exploration, findings, innovations, and technological progress required to meet a predetermined COG has an impact on the ultimate achievable resource capacities. The worldwide average COG for copper is approximately 0.49% [4–6].

Mineral economics is not a new field; this can be followed back 200 years to the research of classical economists like as Smith [7], Ricardo [8], and Jevons [9], among others. Smith [7] noted that the demand for mineral resources appears to be unlimited due to customers’ preferences and desire for manufactured products, which need a large number of fossil fuels and mineral resources to be exploited. Ricardo [8] highlighted that all mineral commodities are exposed to significant fluctuations, either temporary or permanent in nature, and of significant economic impact.

Surface mine planning is a dynamic and complicated system with a host of variables and multi-stages (Fig. 1), making it a complex problem that is hard to solve and demands a continuous financial risk assessment. Open-pit scheduling intends to find the best schedule of calendar programming across the mineral value cycle [10–12]. This is because the value of a mining project is usually affected by various underlying economic and physical variables, such as commodity prices, grades, expenses, timetables, volumes, and environmental concerns, which are unknown with confidence at the project’s inception.

![Surface Mining Operational Flow Diagram](image.png)

**Fig. 1** A sequential multi-stages Surface mining operational flow diagram

Mining is a capital-intensive industry in which economic concerns dominate all activities. This is necessary for its long-term viability and growth. The COG of a mineral deposit is determined based on the techno-economic viability of production and profit obtained from the time a mine is opened until it is closed. COG is the grade below which the ore cannot be processed profitably. Therefore, this criteria creates a boundary inside a particular mineral deposit between ore and waste, where ore must be delivered to a mill for processing, and waste is either left undisturbed or sent to waste disposal [13, 14].

In a number of contexts, mineral resources are classified as either ore or waste. The use of COG is a standard route to explore the ore-waste ratio and the total mineral content. All the COG optimization techniques can be divided broadly into traditional and modern methods. Traditional methods may further be divided into four major sub-categories: mathematical programming, integer programming, linear programming, and nonlinear programming. On the other hand, modern techniques may be divided broadly into two major categories: the evolutionary algorithm and the Monte Carlo simulation.
Out of all the COG techniques, the maximum number of work may be attributed to the analytical approaches, but none can yield the optimal results. Henning [15] was the first who brought the idea of COG optimization into the mining business and a year later the seminal work of Lane [16] was introduced. Lane determined optimal COGs based on limited capacities, supporting Henning’s [15] prediction that ideal COGs tend to drop with time. Henning’s method keeps certain operational parameters fixed throughout the project’s life cycle and does not account for the potential of capacity expansion. Following Lane’s work, the amount of literature on the topic has expanded exponentially.

Through, globally the breakeven COG and Lane's [16, 17] algorithm are being used commonly to maximize profit. However, maintaining a steady breakeven COG across the mine life would have serious consequences, potentially exposing the mining operation to considerably sub-optimal operating results. Lane’s method also solved the breakeven model’s flaws. It considered the production restrictions of various phases and the relevance of the grade-tonnage characteristics.

Unfortunately, Lane’s [16, 17] method has been widely criticized for COG determination that may be more suitable in certain circumstances, making it possible to disregard the optimal estimates for COGs [18]. Lane’s technique has stayed empirical, necessitating the development and implementation of a rational optimization technique [19, 20]. The application of mathematical programming to open-pit mine planning traces back to the 1960s. Acknowledging the limitations of Lane’s approach, a few attempts have been made to overcome them through an alternative analytical framework of optimization [20–23]. The basics of Lane's [16, 17] technique have been expanded in most optimization approaches. Several studies employed time-value-of-money-based optimization approaches. The COG optimization algorithms that involve NPV maximization lead to better outcomes than those that maximizes the profit function.

Lane [17] effectively used the deterministic dynamic programming concepts to the optimization of COG policies. However, he advocated to find an optimal operating strategy and the associated maximum present value in a specific condition, the sum of the maximum present value surface is irrelevant. Every point along the optimal strategy path must be optimal, and all related current values must be maxima. Assuming that future prices would vary, he proposed a dynamic programming method that can assure that the best COG is picked at all times. However, it can only be approximated by forecasting, which is fraught with risk. Dowd [24] demonstrated the technique that might be used for stochastic data with the objective function of maximizing the present value of future net profits. To optimise pit scheduling, several conventional and meta-heuristic techniques are being applied extensively. Later, Wang [25] and Tolwinski [26] studied dynamic programming to study block sequencing. Due to the periodic extraction of finite resources, the efficiency of dynamic programming approaches have shown to be useful in scheduling mining operations for simultaneous optimization of several policies [27]. Linear programming, for example,
might be used to maximize cash flow locally, whereas dynamic programming could be used to improve NPV on a global level.

Cetin and Dowd [28] applied the genetic algorithm to optimize COG for a multi-mineral deposit. The selection of optimum COG for metalliferous deposits is a fundamental problem the mine planner face because of the variation in cost of operation, metallurgical characteristics, and stripping ratios for the different open-pit deposits. Other algorithms, such as genetic algorithms, provide acceptable solutions, but it cannot be said that the solution is optimal.

King [29] presented a variation of COG policy considering the operating and administrative costs. He suggested that the future improvements in project value result from successful strategic planning, and are more significant than the minimal costs. Asad and Dimitrakopoulos [30] extended Lane’s heuristic approach for optimal COG optimization to open-pit mining operations with multiple ore processing facilities and accounting for geological uncertainty based on a stochastic framework. Narrei and Osanloo [31] studied the consequences of reclamation costs on COGs. Baidowi et al. [32] established a mathematical model for COG optimization in the open-pit metalliferous deposits based on overall profit maximisation, incorporating reclamation income and expenditures. The authors proposed for the consideration of environmental variables in the determination of COGs.

Dehghani and Bogdanovic [33] estimated the metal price volatility using a bat algorithm, which provided better results than the classic estimation method for predicting copper price. This gave a trend of price. However, it is found to be unsuitable if there is a significant and sudden change in demand and supply or a technology up-gradation or technology substitution. Under such circumstances, it can change the price dynamics. For example, surface miner’s use is a technology substitution, which lowers the overall investment in equipment by about 50%. It eliminates the need for drilling, blasting, and crushing. Khan and Asad [34] shared a two-stage stochastic linear programming for optimization of COG that accounts for uncertainty in grade and maximizes the NPV subject to the constraint of production capacity over the entire mine life. Franco-sepulveda et al. [35] used a genetic algorithm to optimize the NPV of a mining project. They had analyzed the price of the metal from a historic date to the current date and concluded that for 50% of the time, the sale value was below the estimated value. This method worked only for few variables that too with the average value of either the price or the ore grade. Githiria and Musingwini [36] developed a stochastic and dynamic programming model, extending Lane’s algorithm. They attempted to address the uncertainty by incorporating an assumed value of 2% per period for the case of a gold deposit, which increased annually irrespective of any uncertain futuristic changes related to demand and technology; this, in reality, is not always the case. Saliba and Dimitrakopoulos [37] applied the simultaneous stochastic operation considering the market uncertainty. The price fluctuations of commodity and the problem of optimization of the mining complex under the uncertainty of the market and
supply jointly was the strength of the framework. However, it was limited in optimizing the mining capacity and processing rates with an extension of the destination policy to consider multiple variables in COG decisions.

Su et al. [38] stated that the emergence and collapse of several price bubbles is connected to supposition, macroeconomic instability, a supply-demand imbalance, and economic turmoil. Tahar et al. [39] agreed that commodities prices shocks might have differential effects. The most of commodities’ economies are highly volatile and difficult to anticipate. Considering an erroneous pricing forecast leads to sub-optimal COG selections and implications. Dehghani and Bogdanovic [33] further discussed the rise and fall of China’s market supply and demand during 2011, resulting in price volatility. During that period, the price fluctuations were significantly high, leading to an entirely different level of market dynamics. Such situations are unpredictable, and the authors suggest analyzing them using the average trend. The problem of optimization of COG takes into account a host of variables; thus considering the uncertainty in each of the variables is not a pragmatic decision.

Birch [40] optimized COGs by considering the value of lost ore and the costs of dilution under various degrees of uncertainty using risk and mixed-integer linear programming in Microsoft Excel Solver by maximizing either profit or NPV. Zarsenas and Saeedi [41] used Lane’s algorithm and demonstrated that with an increase of dilution, there was a decrease in average grade and consequently an increase in COG considering dilution. Eugene et al. [42] investigated the strategy of mining options for an ore body using mixed-integer linear programming optimization framework that optimized the NPV of the reserve considering the three cases, namely, (i) open-pit mining (ii) underground mining and (iii) concurrent open-pit and underground mining. The authors concluded that the rate of return on the investments gave better results for underground mining. However, the NPV of the options could not provide a decisive solution. This method fails to give a satisfactory answer when the projects need different levels of investment and with a different economic life of the projects.

However, a few authors have also advocated that the theory of COG optimization supports the ultimate objective of a mining operation through maximization of NPV [43–45]. Ahmadi and Bazzazi [46] proposed that choosing the optimal COG maximizes the NPV and the total profit of the project. The optimization of the COGs considering the maximum achievable NPV over the life of the mine is one of the critical issues in the mining of open pits. The criteria that control the development of COG policy aligns with an operation’s strategic objectives to maximize the discounted value over the life of the process [47]. Russell [48] pioneered the concept of maximising a project’s NPV. The most commonly used economic metric in evaluating financial projects is NPV [49, 50]. It is worth noting that NPV is acutely susceptible to revenue variations [51–54].

Ahmadi [55] optimized the COG based on Lane’s theory to maximize the NPV using MATLAB. The optimum COG is the grade that maximizes the chosen objective function, usually the NPV [56]. Various researchers [57–59] have given several
algorithms to determine optimum COG. They concluded that evaluating the COG based on maximizing the NPV produces more accurate findings than the other methods. The COG that leads to the highest NPV for the project must be selected.

Determination of optimum COG is an essential and crucial aspect to exploit any metalliferous deposits efficiently and economically. Metal prices are dynamic in nature and follow an asymmetrical trend. So, one cannot fix the production schedule having a random COG without optimizing the objective function to reach the maximum NPV. The stage pits can be scheduled strategically and optimally once the ultimate pit has been set, keeping the maximum discounted and undiscounted block value in mind. If one doesn’t optimize or check the COG periodically, there might be a time when the mine can face a cash crisis and needs to be shut off abruptly due to the possibility of negative IRR in the long run. So, the mine planners should plan considering all the price dynamics, COGs, grade-tonnage ratio, reserve, and market scenario. At the end of the exploration stage, the ore body model is first finalized, and then the planning of mine development and project scheduling are carried out accordingly. A planning job requires a multitude of functions. In general, the mine planning comprises of the steps as shown in the sequential flowchart of the whole process given below in Fig. 2.

![Flowchart](image)

**Fig. 2** The flowchart describing the procedural steps in mine planning

## 2 Optimization model framework

The proposed model has been developed considering that the optimum COG for a metalliferous deposit is dynamic in nature. The programming language used for the model has been designed using Python. The complete sequence of mathematical and computational techniques is based on simple algebraic equations, calculus of variation, and iterative algorithm steps fulfilling
the criteria of a sequentially advancing algorithm for the COG optimization and maximization of NPV subject to the condition that the capacities of mining, concentrating, and refining are either limited or unlimited. The model has been validated using real-life data from the study of the Malanjkhand Copper Project of Hindustan Copper Limited (HCL) to demonstrate the effectiveness of the model’s algorithm.

2.1 Operational model

In metalliferous open-pit mine planning and design, the optimum COG is mainly affected by three governing capacity constraints of various operations, i.e., mine, mill, and smelting and refinery. Eight different possibilities can be considered while calculating the optimum COG based on the different limiting conditions the production capacities of the three stages (Table 1).

Table 1 Eight different models of possible combinations based on constraining restrictions

| Models | Mine capacity | Mill capacity | Refinery capacity | Restrictions or constraints (L - Limited and UL – Unlimited) |
|--------|---------------|---------------|------------------|-------------------------------------------------------------|
| 1.     | UL            | UL            | UL               | All capacities are unlimited                                 |
| 2.     | L             | UL            | UL               | Mine capacity is limited                                    |
| 3.     | UL            | L             | UL               | Mill (plant) capacity is limited                            |
| 4.     | UL            | UL            | L                | Refinery (smelter) capacity is limited                      |
| 5.     | L             | L             | UL               | Mine and mill capacities are limited                        |
| 6.     | L             | UL            | L                | Mine and refinery capacities are limited                    |
| 7.     | UL            | L             | L                | Mill and refinery capacities are limited                    |
| 8.     | L             | L             | L                | All (mine, mill and refinery) capacities are limited        |

2.2 Model prerequisites

Prerequisites for the application of the COG optimization model include the development of the ultimate pit limit or pit extent; calculation of mineable ore reserves in terms of mineral grade and tonnage distribution within the pit limit; the mining, processing and refining stage capacities; the operating costs at these stages; and the technical and other economic parameters including the metal price.

2.3 Model assumptions

It has been presumed that the total deposit, as specified by the grade distribution, will be excavated, but only that portion of the excavated material that exceeds or equals the COG should be sent for concentration. The production, concentration, and refining capacities have been assumed to be stable during the project life. Furthermore, it has been assumed that the metal price, operational costs, capital costs, rate of interest, tax rate, and discount rate remain stable over the project life though the metal price index volatility and technical developments can change the entire scenario of mining, milling, and refining costs, as well as, the future revenue. To simplify the problem, it has been assumed that, at least on average, all ore mined within any twelve months will also be processed during that period.
2.4 Model formulation using sequentially advancing algorithm based on multi-value driven dynamic programming

To determine the optimum COG for an open pit metalliferous deposits, one must express the problem as a mathematical programming model. The mathematical models for dealing with multi-varying COG or production scheduling are nonlinear discrete optimization models. The computational complexity of such issues necessitates the development of an efficient solution technique. The creation of the mathematical models and the solution technique have been described below.

The grade tonnage distribution of a metalliferous deposit having an independent domain \( (G, T) \) for input values is shown in Fig. 3. The grade-tonnage distribution of the mineral deposit is distributed in a grade range of \( G (\alpha, \beta) \) and the corresponding tonnage to each range of \( (\alpha, \beta) \) is defined by \( T_{\alpha,\beta} \). The corresponding relation of the function is given by \( f : G \rightarrow T \). For all ‘\( G \)’ belongs to set ‘\( G_{\alpha,\beta} \)’ and for all ‘\( G \)’ values there exists ‘\( T \)’ values belonging to Set \( (T_{\alpha,\beta}) \), such that \( T_{\alpha,\beta} = f (G_{\alpha,\beta}) \) of Range, \( R \in \{ G_{\alpha,\beta}, T_{\alpha,\beta} \} \).

\[ f : G_{\alpha,\beta} \times T_{\alpha,\beta} \]

\[ f_{\alpha,\beta} (G_{\alpha,\beta}) \]

\[ T_{\alpha,\beta} = f (G_{\alpha,\beta}) \]

Fig. 3 Functional structure of grade-tonnage distribution

Here, it is required to select the COG \( (\mathcal{C}) \) for each set of ranges from \( G_{\alpha,\beta} \) and it is required to determine the stages of transition from \( \alpha \) to \( \beta \) for each set of values. The values in each stage are saved and the optimization for the next state proceeds. The grade equal to or above the COG is treated as ore, and the rest is treated as waste and that is sent to the waste dump.

The average grade of the metal deposit \( \bar{g}_{\alpha,\beta} \) is determined from the grade-tonnage distribution and the concentration of metal contained \( (x) \) in it. It is evaluated using the weighted average formulae given by equation (1).

\[ \bar{g}_{\alpha,\beta} = \frac{\int_{\alpha}^{\beta} G_{\alpha,\beta} \times T_{\alpha,\beta} (x) \, dx}{\int_{\alpha}^{\beta} T_{\alpha,\beta} (x) \, dx} \quad (1) \]

The tonnage of ore \( (T_o) \) and tonnage of waste \( (T_w) \) from the grade-tonnage distribution will be determined using the equations 2(a), 2(b), 2(c) and (3).

\[ T_{\alpha(\alpha,\beta)} = 0, \quad \text{where } G_{\alpha} < \mathcal{C}, \text{ and } G_{\beta} \leq \mathcal{C} \quad 2(a) \]
\[ T_{o(\alpha,\beta)}^2 = \sum_{\alpha, \alpha > G_{\beta} \geq \xi} T - \sum_{\alpha < G_{\beta} \geq \xi} T, \quad \text{where } G_{\alpha} \leq \xi, \text{ and } G_{\beta} > \xi \]  
\[ T_{o(\alpha,\beta)}^3 = \sum_{\alpha, \alpha \geq G_{\beta} \geq \xi} T - \sum_{\alpha < G_{\beta} \geq \xi} T, \quad \text{where } G_{\alpha} \geq \xi, \text{ and } G_{\beta} > \xi \]  
\[ T_{w(\alpha,\beta)}^2 = \sum_{\alpha} T - T_{o(\alpha,\beta)} \]  

Where, \( T \) is the total material \((T_o + T_w)\) to be mined out.

Stripping ratio \((S_{\alpha,\beta})\) is a critical parameter, which plays a significant role in evaluating the \(NPV\) of the deposits. Mine planners play a vital role in ascertaining \(S_{\alpha,\beta}\) values corresponding to their pit progress whether to keep the value uniform or in an increasing trend affecting their \(NPV\)'s. The value of \(S_{\alpha,\beta}\) is determined using equation (4).

\[ S_{\alpha,\beta} = \frac{T_{o(\alpha,\beta)}}{T_{w(\alpha,\beta)}} \]  

Mining companies make money by extracting and processing raw materials into a marketable product. The product of tonnes of ore processed, average mill head grade, % recovery over concentrating and refining, and the product’s selling price are used to determine revenue. The mine complex has both fixed and variable costs during operation. Variable expenses are further divided into those related to mining and processing the ore and those related to digging and disposing of the waste. Drilling and blasting, transportation, disposal, restoration, and other expenditures are examples of variable expenses. General administrative expenditures and any additional costs that are not dependent on output are examples of fixed costs. The \(NPV\) may be calculated by discounting capital expenses and subtracting them from the discounted cash flow. Now using all the parameters mentioned above, sequentially, the equation for the calculation of \(NPV_{\alpha,\beta}\) is determined by equation (5):

\[ NPV_{\alpha,\beta} = PV_{\alpha,\beta} - CC_{\alpha,\beta} \]  

For obtaining the optimized returns \((NPV_{\alpha,\beta}) \rightarrow Q_{m(\alpha,\beta)}, Q_{c(\alpha,\beta)}, Q_{r(\alpha,\beta)}\) are the initial decision variables for all the stages. Decision variables are governed by the limiting capacities (constraints). Based on the cut-off grade \((\xi)\) and the stage transition \((\alpha \leq \psi \leq \beta)\), the amount of production in different stages, i.e., mine, mill and refinery are determined subsequently by any one of the following conditions given in equations (6) through (8):

Subject to:

\[ Q_m \leq M_{(\alpha,\beta)} (\alpha, \psi) \quad \forall \ n \]  
\[ Q_c \leq C_{(\alpha,\beta)} (\alpha, \psi) \quad \forall \ n \]  
\[ Q_r \leq R_{(\alpha,\beta)} (\alpha, \psi) \quad \forall \ n \]  

If \(y\) is the overall metallurgical recovery, then

\[ Y_{\alpha,\beta} = Y_{m(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} \]  

Where:
Generally, COG optimization has been observed as a nonlinear dynamic system and can be solved using Dynamic Programming (DP) to obtain a global optimum. The basic concept behind DP is to break down the optimization issue into a series of sub-problems, each representing the optimization of a single control action at a particular time. The value function, which translates the current state of the system to the NPV of future income, connects these sub-problems. The best control action at each time can be obtained by solving a sub-problem at that moment after the value function has been computed. These sub-problems are, in general, significantly easier than the original optimization issue; each sub-global problem’s optimum may be quantified effectively.

**Module 1: All Unlimited Conditions**

When all the three stages are unlimited, it brings the most exciting and novel part of the optimization combination. This is being the first mathematical approach, which deals with all the unlimited capacities. Here all the three components COG, mine production, and mine-life, have been optimized simultaneously at the same time to determine the optimal COG. Here, for each of the COG — the corresponding ore-waste tonnage, stripping ratio, average grade, and cumulative tonnage has been calculated. Each COG value will be optimized starting with the total tonnage as the mine production from year 1 to 100. This COG — mine life and mine production optimization will give the year-wise production rate for the same COG value for years 1 to 100. Thereafter, the NPV is calculated from year 1 to 100 for the same COG value. This whole process will be repeated for all the COG values of the grade-tonnage distribution curve. After this there will be a final set of data that came from all the COG values having their maximum NPVs with their corresponding optimal mine life and production; among all the optimum COG values, the one having the highest NPV is the most optimal COG.

The following assumptions, as given below, have been made for the sequence of working.

1. Activity time-frame is deterministic.
2. Activity per-period resource requirements and project per-period resource availabilities are fixed and known.

3. The cash flow associated with each activity is not susceptible to change in course, hence no stochastic values have been used for the determination of COG.

4. There is no interruption in the performance of the activity once it has begun (no pre-emption).

5. No cancellations are permitted. Each activity must be finished to check its optimality performance.

6. Each activity is completed following the sequentially advancing algorithm. That paradigm is associated with a predictable activity length, fixed resource requirements, and a set cash flow.

7. When an activity is completed, net non-negative cash flows will occur.

8. The stockpiling option has not been considered.

Under the prior assumptions, the COG optimization issue to maximize the NPV of the project is written as a mathematical programming problem as follows:

**Maximize** → \( NPV_{(\alpha,\beta)} \) subject to \( \sum_{\alpha,\beta} COG_{(\alpha,\beta)} = n \sum_{\alpha,\beta} t_{(\alpha,\beta)} = n \sum_{\alpha,\beta} Q_{m(\alpha,\beta)} \epsilon t_{(\alpha,\beta)} = n \) \( \sum_{\alpha,\beta} t_{(\alpha,\beta)} = n \sum_{\alpha,\beta} Q_{m(\alpha,\beta)} \epsilon t_{(\alpha,\beta)} = n \) \( \sum_{\alpha,\beta} Q_{m(\alpha,\beta)} \epsilon t_{(\alpha,\beta)} = n \)

\[ (10) \]

**Maximize:** \( \sum_{\alpha,\beta} COG_{(\alpha,\beta)} = n \sum_{\alpha,\beta} t_{(\alpha,\beta)} = n \sum_{\alpha,\beta} Q_{m(\alpha,\beta)} \epsilon t_{(\alpha,\beta)} = n \sum_{\alpha,\beta} t_{(\alpha,\beta)} = n \sum_{\alpha,\beta} Q_{m(\alpha,\beta)} \epsilon t_{(\alpha,\beta)} = n \sum_{\alpha,\beta} Q_{m(\alpha,\beta)} \epsilon t_{(\alpha,\beta)} = n \)

\[ (11) \]

Subject to:

\[ COG_{(\alpha,\beta)} = \begin{cases} 0 & \text{for } \sum_{\alpha,\beta} COG_{(\alpha,\beta)} = 0 = NPV_{(\alpha,\beta)} \text{(max)}, \ R \in \{ G_{\alpha,\beta}, T_{\alpha,\beta} \}, \forall 0,0.01,..N \\
\frac{\left( T_{o(\alpha,\beta)} + T_{w(\alpha,\beta)} \right)}{t_{(\alpha,\beta)}} & \text{for } \sum_{\alpha,\beta} Q_{m(\alpha,\beta)} \epsilon t_{(\alpha,\beta)} = n = NPV_{(\alpha,\beta)} \text{(max)}, \forall 1,2,..n \\
\frac{\left( T_{o(\alpha,\beta)} + T_{w(\alpha,\beta)} \right)}{t_{(\alpha,\beta)n}} & \end{cases} \]

Besides the optimization technique mentioned above for the determination of \( Q_{m(\alpha,\beta)} \), the empirical-formulae based mining production rates have also been determined for the comparison and efficacy of the model utilizing the existing thumb rule given by Taylor [14] and the modified Taylor’s rule by Long [60].

\[ \text{Mine} - \text{life}^{(\alpha,\beta)_{Taylor}} \cong 6.5 \times (\text{Ore tonnage in millions})^{1} \]

\[ Q_{m(\alpha,\beta)}^{Taylor} = \frac{\left( T_{o(\alpha,\beta)} + T_{w(\alpha,\beta)} \right)}{\text{Mine} - \text{life}^{(\alpha,\beta)_{Taylor}}} \]

\[ Q_{m(\alpha,\beta)}^{Long} = 0.123 \times \text{tonnage}^{0.649} \times 350 \]
\[
\begin{align*}
    t_{(\alpha, \beta)} &= \begin{cases} 
    1 & \text{for } \sum_{i=1}^{n} t_{(\alpha, \beta)} = \text{NPV}_{(\alpha, \beta)}(\max), \\
    2 & \forall 1,2, \ldots, n 
    \end{cases} \\
    \text{Where, } & j_{at} = \begin{cases} 
    1 & \text{if action } 'a' \text{ is finished in time } 'n', \text{ a multi value — sequential stages} \\
    0 & \text{otherwise, though no interruption in the completion of an action} 
    \end{cases} \\
    \partial &= \text{The discount factor over } n \text{ periods}
\end{align*}
\]

The target function is to maximize the total of the present values of the activities' contributions discounted back to the project's commencement date. The total of the future values of the cash flows connected with the various phases (periods) of that activity, discounted to the end of the action, less the opportunity cost of the recovered investment capital required by the action, is the contribution of that activity. This figure is derived from:

\[
\text{NPV}_{(\alpha, \beta)} = \max_{\alpha \leq \xi \leq \beta} \int_{t_{(\alpha, \beta)}=1}^{t_{(\alpha, \beta)}=n} \int_{t_{(\alpha, \beta)}=1}^{t_{(\alpha, \beta)}=n} \left[ (PV_{(\alpha, \beta)} - CC_{(\alpha, \beta)}) e^{-\partial t} dt \right]
\]  

(15)

According to the mine-life, we calculate the production capacities and the following stage capacities, and then it follows the sequentially advancing optimization approach to determine the optimal COG value. Ultimately, the grade at which it contributes the Global maximal NPV value decides the optimal COG value. The overview of the multi-value driven nested DP algorithmic approach is shown in Fig. 4.

![Fig. 4 An overview of the Multi-value driven nested Dynamic Programming Algorithmic approach](image-url)
Module II: When mining capacity is limited and the rest two stages are unlimited, then the maximum value of \( NPV \) from the array is considered, neglecting that production rate is less than production capacity, taking the corresponding \( COG \), and deciding the final capacities of the rest two stages.

When mining capacity is the governing constraint, then the decision state is, \( Q_m \leq M(\alpha, \beta) \) \( \forall n \); accordingly, the \( Q_m(\alpha, \beta) \), \( Q_c(\alpha, \beta) \), \( Q_r(\alpha, \beta) \) are calculated using equations (16) through (18).

\[
Q_m(\alpha, \beta) = T_o(\alpha, \beta) + T_w(\alpha, \beta) \\
Q_c(\alpha, \beta) = \frac{Q_m(\alpha, \beta)}{1 + S(\alpha, \beta)} \times Y_m(\alpha, \beta) \\
Q_r(\alpha, \beta) = Q_c(\alpha, \beta) \times \hat{g}(\alpha, \beta) \times Y_c(\alpha, \beta) \times Y_r(\alpha, \beta)
\]

Module III: When mining capacity and smelter & refinery capacities are unlimited, and mill capacity is limited, then the maximum value of \( NPV \) from the array is considered where the mill production is less than or equal to the mill capacity and take the corresponding \( COG \), and hence deciding the final output from the rest two unlimited stages.

When milling capacity is the governing constraint, then the decision state is, \( Q_c \leq C(\alpha, \beta) \) \( \forall n \); accordingly, the \( Q_m(\alpha, \beta) \), \( Q_c(\alpha, \beta) \), \( Q_r(\alpha, \beta) \) are calculated using equations (19) through (20).

\[
Q_m(\alpha, \beta) = \left( \frac{T_o(\alpha, \beta) + T_w(\alpha, \beta)}{T_o(\alpha, \beta)} \right) \times Q_c(\alpha, \beta) \times \left( \frac{1}{Y_m(\alpha, \beta)} \right) \\
Q_r(\alpha, \beta) = Q_c(\alpha, \beta) \times \hat{g}(\alpha, \beta) \times Y_c(\alpha, \beta) \times Y_r(\alpha, \beta)
\]

Module IV: When mining capacity and mill capacity are unlimited, and smelter and refinery capacity is limited, then the maximum value of \( NPV \) from the array is considered, which satisfies the condition that corresponding smelter and refinery production is less than or equal to the smelter and refinery capacity and take the corresponding \( COG \), and hence deciding the final output from the rest two unlimited stages.

When smelting & refining capacity is the governing constraint, then the decision state is, \( Q_r \leq R(\alpha, \beta) \) \( \forall n \); accordingly, the \( Q_m(\alpha, \beta) \), \( Q_c(\alpha, \beta) \), \( Q_r(\alpha, \beta) \) are calculated using equations (21) through (22).

\[
Q_c(\alpha, \beta) = \left( \frac{Q_r(\alpha, \beta)}{\hat{g}(\alpha, \beta) \times Y_c(\alpha, \beta) \times Y_r(\alpha, \beta)} \right) \\
Q_m(\alpha, \beta) = \left( \frac{T_o(\alpha, \beta) + T_w(\alpha, \beta)}{T_o(\alpha, \beta)} \right) \times Q_c(\alpha, \beta) \times \left( \frac{1}{Y_m(\alpha, \beta)} \right)
\]

Module V: When mining capacity and mill capacity are limited, and smelter & refinery capacity is unlimited, then the maximum value of \( NPV \) from the array is considered that satisfies the condition that corresponding mine production and mill
production is less than or equal to their capacity and take the corresponding COG, and hence deciding the final smelter and refinery capacity.

When mining and milling capacity are the governing constraints, then the decision state is, \( Q_m \leq M_{(\alpha,\beta)}(\alpha,\psi) \) \( \forall n \) and, \( Q_C \leq C_{(\alpha,\psi)}(\alpha,\psi) \) \( \forall n \);

Accordingly, the all \( Q_{m(\alpha,\beta)} \), \( Q_{C(\alpha,\beta)} \), \( Q_{R(\alpha,\beta)} \) are calculated using equations (23) through (24).

\[
Q_{C(\alpha,\beta)} = \frac{Q_{m(\alpha,\beta)}}{1 + S(\alpha,\beta)} \times Y_{m(\alpha,\beta)}
\]

\[
Q_{R(\alpha,\beta)} = Q_{C(\alpha,\beta)} \times \tilde{g}(\alpha,\beta) \times Y_{C(\alpha,\beta)} \times Y_{R(\alpha,\beta)}
\]

**Module VI:** When mining capacity and smelter capacity are limited, and mill capacity is unlimited, then the maximum value of NPV is considered from the array that satisfies the condition that corresponding mine production and smelter production is less than or equal to their capacity and take the corresponding COG, and hence deciding the mill capacity.

When mining and refining capacity are the governing constraint, then the decision state is, \( Q_m \leq M_{(\alpha,\beta)}(\alpha,\psi) \) \( \forall n \) and, \( Q_r \leq R_{(\alpha,\beta)}(\alpha,\psi) \) \( \forall n \);

Accordingly, the \( Q_{m(\alpha,\beta)} \), \( Q_{C(\alpha,\beta)} \), \( Q_{R(\alpha,\beta)} \) are calculated using equations (25) through (26).

\[
Q_{C(\alpha,\beta)} = \frac{Q_{m(\alpha,\beta)}}{1 + S(\alpha,\beta)} \times Y_{m(\alpha,\beta)}
\]

\[
Q_{R(\alpha,\beta)} = Q_{C(\alpha,\beta)} \times \tilde{g}(\alpha,\beta) \times Y_{C(\alpha,\beta)} \times Y_{R(\alpha,\beta)}
\]

**Module VII:** When milling capacity and smelter and refinery capacity are limited, and mine production capacity is unlimited, then the maximum value of NPV is considered from the array that satisfies the condition that corresponding mill, smelter and refinery production is less than or equal to their capacities and take the corresponding COG, and hence deciding the final mine capacity.

When milling and refining capacity are the governing constraint, then the decision state is, \( Q_c \leq C_{(\alpha,\psi)}(\alpha,\psi) \) \( \forall n \) and, \( Q_r \leq R_{(\alpha,\beta)}(\alpha,\psi) \) \( \forall n \);

Accordingly, the \( Q_{m(\alpha,\beta)} \), \( Q_{C(\alpha,\beta)} \), \( Q_{R(\alpha,\beta)} \) are calculated using equations (27) through (28).

\[
Q_{m(\alpha,\beta)} = \left( \frac{T_m(\alpha,\beta) + T_{w(\alpha,\beta)}}{T_{o(\alpha,\beta)}} \right) \times Q_{C(\alpha,\beta)} \times \left( \frac{1}{Y_{m(\alpha,\beta)}} \right)
\]

\[
Q_{R(\alpha,\beta)} = Q_{C(\alpha,\beta)} \times \tilde{g}(\alpha,\beta) \times Y_{C(\alpha,\beta)} \times Y_{R(\alpha,\beta)}
\]
Module VIII: When all the three stages are limited, the maximum value of $NPV$ from the array that satisfies the condition corresponding to all three-production stages will be considered. Among the three stages, the minimum capacity stage is the determining factor to decide the final output from the rest two stages, and thus the corresponding calculation will take place. When all the mining, milling and smelting & refining capacity is the governing constraint, then the decision state is, $Q_m \leq M(\alpha, \beta)(\alpha, \psi) \  \forall \ n$ . $Q_r \leq R(\alpha, \beta)(\alpha, \psi) \  \forall \ n$ ; Accordingly, the $Q_{m(\alpha, \beta)}, Q_{c(\alpha, \beta)}, Q_{r(\alpha, \beta)}$ are calculated using equations (29) through (30).

$$Q_{c(\alpha, \beta)} = \frac{Q_{m(\alpha, \beta)}}{(1 + S_{c(\alpha, \beta)})} \times Y_{m(\alpha, \beta)}$$  \hspace{1cm} (29)

$$Q_{r(\alpha, \beta)} = Q_{c(\alpha, \beta)} \times \bar{g}(\alpha, \beta) \times Y_{c(\alpha, \beta)} \times Y_{r(\alpha, \beta)}$$  \hspace{1cm} (30)

Ultimately the grade at which it contributes the maximal $NPV$ value decides the optimal $COG$ value.

The given stage-wise logical conditions are implied to all the eight modules for obtaining the final $Q_{mf(\alpha, \beta)}, Q_{cf(\alpha, \beta)}, Q_{rf(\alpha, \beta)}$ values according to the various limiting conditions from the set of mining, milling, smelting and refining.

Stage – I:

When mining capacity is the governing constraint, then the decision state is, $Q_m \leq M(\alpha, \beta)(\alpha, \psi) \  \forall \ n$ ; Accordingly, the $Q_{m(\alpha, \beta)}, Q_{c(\alpha, \beta)}, Q_{r(\alpha, \beta)}$ are calculated using equations (31) through (33).

$$Q_{m(\alpha, \beta)} = T_{o(\alpha, \beta)} + T_{w(\alpha, \beta)}$$ \hspace{1cm} (31)

$$Q_{c(\alpha, \beta)} = \frac{Q_{m(\alpha, \beta)}}{(1 + S_{c(\alpha, \beta)})} \times Y_{m(\alpha, \beta)}$$ \hspace{1cm} (32)

$$Q_{r(\alpha, \beta)} = Q_{c(\alpha, \beta)} \times \bar{g}(\alpha, \beta) \times Y_{c(\alpha, \beta)} \times Y_{r(\alpha, \beta)}$$  \hspace{1cm} (33)

Where $\bar{g}(\alpha, \beta)$ is the average mill-head grade.

Stage – II:

When milling and refining capacity is the governing constraint, then the decision state is $Q_{c(\alpha, \beta)} \leq C(\alpha, \beta)$ or $Q_{r(\alpha, \beta)} \leq R(\alpha, \beta)$ accordingly, $Q_{m(\alpha, \beta)}$ and $Q_{r(\alpha, \beta)}$, which are calculated using equations (34) and (35):

$$Q_{m(\alpha, \beta)} = \frac{Q_{c(\alpha, \beta)} \times (1 + S_{c(\alpha, \beta)})}{Y_{m(\alpha, \beta)}}$$  \hspace{1cm} (34)

$$Q_{r(\alpha, \beta)} = Q_{c(\alpha, \beta)} \times \bar{g}(\alpha, \beta) \times Y_{c(\alpha, \beta)} \times Y_{r(\alpha, \beta)}$$  \hspace{1cm} (35)

Stage – III:

When refining and mining capacities are the governing constraints, then the decision state is accordingly either.
\[ Q_r(\alpha, \beta) > R_{\alpha, \beta} \quad \text{or} \quad Q_m(\alpha, \beta) > M_{\alpha, \beta} \]
\[ Q_r \leq R_{(\alpha, \beta)}(\alpha, \psi) \quad \forall n \quad : \]

The value of \( Q_c(\alpha, \beta) \) and \( Q_r(\alpha, \beta) \) are calculated using equations (36) and (37).

\[
Q_c(\alpha, \beta) = \frac{Q_r(\alpha, \beta)}{\bar{g}(\alpha, \beta) \times Y_c(\alpha, \beta) \times Y_r(\alpha, \beta)} \quad (36)
\]
\[
Q_r(\alpha, \beta) = \frac{Q_c(\alpha, \beta) \times Y_m(\alpha, \beta) \times (1 + S(\alpha, \beta))}{\bar{g}(\alpha, \beta) \times Y_r(\alpha, \beta)} \quad (37)
\]

After the transition of stages, the decision variables \( (Q_m(\alpha, \beta), Q_c(\alpha, \beta), Q_r(\alpha, \beta)) \) are saved as the final decision variables \( (Q_{mf}(\alpha, \beta), Q_{cf}(\alpha, \beta), Q_{rf}(\alpha, \beta)) \) and now they will become the optimum solutions for each \((\alpha, \beta)\) state of values. These decision variables will be used now to determine the concentrates \((tonnes/yr)\), Tailings \((tonnes/yr)\), Metal content \((tonnes/yr)\) and Slag \((tonnes/yr)\) of the project as mentioned by the equations (38) through (41) as given below. We need to optimize the final value after each stage of the calculation.

\[
\text{Concentrate}(\alpha, \beta) = \frac{Q_c(\alpha, \beta) \times \bar{g}(\alpha, \beta) \times Y_c(\alpha, \beta)}{\epsilon_g \,(\%)} \quad (38)
\]
\[
\text{Tailings}(\alpha, \beta) = Q_m(\alpha, \beta) - \text{Concentrate}(\alpha, \beta) \quad (39)
\]
\[
\text{Metal}(\alpha, \beta) = Q_{rf}(\alpha, \beta) \quad (40)
\]
\[
\text{Slag}(\alpha, \beta) = \text{Concentrate}(\alpha, \beta) - \text{Metal}(\alpha, \beta) \quad (41)
\]

Determination of the Optimum COG is a multi-staged decision problem [61], and output from each state is fed as an input to the next. Each stage has several possible states associated with it, as shown in Fig. 5.

---

**Fig. 5** The dynamic programming multi-stage decision process for ‘n’ stages
The results are optimized across all the stages, and this is governed by Bellman’s principle of Optimality [62]. State variables \((S_n(\alpha,\beta))\) and Decision Variables \((Q_m(\alpha,\beta), Q_c(\alpha,\beta), Q_r(\alpha,\beta))\) have discrete values. A recursive relationship identifies the optimal decision at stage ‘\(n\)’, for the state ‘\(S_n\)’. It gives the decision for each state at stage \((n-1)\). Here, \(f_n(S_n)\) is the optimized objective function value at stage ‘\(n\)’ and ‘\(X_n\)’ is the decision we are making which maximizes over \(\{X_n\}\).

\[
f_n(S_n) = \text{Max} \left\{ R_n(X_n) + f_{n-1} \left( T(S_n, X_n) \right) \right\}
\]

\(R_n\): Return for the decision, ‘\(X_n\)’ in the current state.

\(T(S_n, X_n)\): Transfer function to get the state ‘\(S_{n-1}\)’ corresponding to ‘\(S_n\)’ and ‘\(X_n\)’.

After each stage, there are returns, and the return as cash flow \(CF(\alpha,\beta)\) of the ‘\(n\)’ stages has been calculated as follows. The selling price = \(SP(\alpha,\beta)\), operating costs of mine, mill and of refinery = \(\Phi(M(\alpha,\beta), C(\alpha,\beta), R(\alpha,\beta))\), fixed costs of mine, mill and refinery = \(\xi(M(\alpha,\beta), C(\alpha,\beta), R(\alpha,\beta))\), any other costs = \(AC(\alpha,\beta)\), tax rate = \(TR(\alpha,\beta)\) and depreciation = \(D(\alpha,\beta)\) of the commodity – all are time \((t)\) dependent variables, and the \(CF(\alpha,\beta)\) has been calculated for the whole mine life \((n)\) as given below in equation (43).

\[
CF(\alpha,\beta) = \left\{ SP(\alpha,\beta) \times Q_c(\alpha,\beta) \times \bar{g}(\alpha,\beta) \times Y_c(\alpha,\beta) \times Y_r(\alpha,\beta) - \sum \Phi(M(\alpha,\beta), C(\alpha,\beta), R(\alpha,\beta)) - \sum \xi(M(\alpha,\beta), C(\alpha,\beta), R(\alpha,\beta)) - AC(\alpha,\beta) - D(\alpha,\beta)\right\} (1-TR(\alpha,\beta)) + D(\alpha,\beta) \tag{43}
\]

The objective function is to maximize the \(NPV\) for the whole deposit over the whole Mine life \((n)\). It is the cumulative summation of all the cash flows \((CF_0, CF_1, CF_2 \ldots CF_n)\), where \(CF_0\) is the capital invested having a fixed discount rate of \(\delta\) up to \(n^{th}\) year given by equation (44).

\[
NPV(\alpha,\beta) (\text{Max}) = \frac{CF_1}{(1+\delta)} + \frac{CF_2}{(1+\delta)^2} + \ldots + \frac{CF_n}{(1+\delta)^n} - CF_0 = \sum_{t=0}^{n} \frac{CF_t}{(1+\delta)^t} - CF_0 \tag{44}
\]

Therefore, to get the maximum \(NPV(\alpha,\beta)\), sequential formulae for all the intermediate stages up to ‘\(n\)’ are derived using the generalized equation (45).

\[
NPV(\alpha,\beta) = \text{Max} \int_t^n \left\{ \left\{ SP(\alpha,\beta) \times Q_c(\alpha,\beta) \times \bar{g}(\alpha,\beta) \times Y_c(\alpha,\beta) \times Y_r(\alpha,\beta) - \sum \Phi(M(\alpha,\beta), C(\alpha,\beta), R(\alpha,\beta)) \right\} - \sum \xi(M(\alpha,\beta), C(\alpha,\beta), R(\alpha,\beta)) - AC(\alpha,\beta) - D(\alpha,\beta)\right\} (1-TR(\alpha,\beta)) + D(\alpha,\beta) e^{-\delta t} dt \right\} \tag{45}
\]

Using the equation (23), if one starts from stage one, all the intermediate states are evaluated stage by stage, until all the possible values of \(NPV(\alpha,\beta)\) are evaluated. Then, the maximum of \(NPV(\alpha,\beta)\) of all stages is chosen, such that \(NPV(\alpha,\beta)\) corresponding to a particular grade will be the optimum COG of the mill. Here, the sequentially advancing algorithm has used the memorization method to determine the optimum COG.
2.5 Implementation and development of the computer model

A computerized model based on the dynamic programming method has been established for determining the optimal 
\( COG \) using the formulae mentioned. The programming language used to develop the computer package is \textit{Python} 3.7, the 
ideal multi-paradigm programming language for developing scientific and business applications. It imparts a great extent of 
calculation speed and data accuracy. \textit{Python} is garbage-collected and dynamically typed. It supports multiple programming 
paradigms, including structured (mainly procedural), object-oriented and functional programming. A \textit{PyQt} framework, 
version-5, is used for this model. The Graphical User interface (\textit{GUI}) has been developed using \textit{Python} 3.7 and \textit{PyQt} designer 
that comes with \textit{PyQt5 Library}. In particular, there are some of the core packages like \textit{pandas}, \textit{SciPy}, \textit{NumPy} 
and \textit{Matplotlib} that has been used exclusively for this model. The automated model offers more operational capability and a 
graphical user interface (\textit{GUI}) based on the user interface software. The software package consists of three main components: 
input-data, output-result, and result-graphical.

The \textit{input-data} component can be categorized into two parts: Part-1—Mineral inventory data, and Part-2—Costs and other 
parameters Input. The mineral inventory input data consist of grade class intervals (Grade ranges) and the tonnage distribution 
(Quantity of material) for each given class interval. This data can be filled in the data-grid-view (\textit{GUI} form) given on the main 
window. The Costs and other parameters input module can be accessed by clicking on the “Data Input” button on the main 
menu form window. The “Data Input” wizard consists of five sections of different input data: Mine costs input, Mill costs 
input, Refinery costs input, Additional input, and Rates input.

The output-result component gives the final optimum \( COG \) of the deposit in addition to the optimized \( NPV \) of the entire 
operation. The \textit{output} of the result component is divided into four sections: Mining section, Milling section, Smelting & 
Refining section, and Profits section. The Result-graphical component, an essential element of the computer tool and \textit{GUI} 
program, supports the logical conclusion. Following the computation, it displays the result as graphical data in various graphs 
between the provided variables. The user is presented with the following graphical displays by the tool: (1) \textit{Avg. Grade} (%) 
versus \textit{Tonnage} (2) \( COG \) (%) versus \textit{Cumulative Tonnage}, \textit{Average Grade} (%) and (3) \( COG \) (%) versus \textit{NPV}.

3 Validation of outputs using the real case study

To validate the models, the work necessitates a real case study in terms of domain knowledge of an open-pit copper ore mine 
with all or at least three phases of ore production, namely extraction, concentration, refinery, and smelter. The model has been 
validated using data from one of India’s largest open-pit copper deposits.
3.1 Brief Introduction about the Mine

Hindustan Copper Limited (HCL), a public sector company under the administrative supervision of the Ministry of Mines, Government of India, was established on 9th November 1967. It is the only copper producer that is vertically integrated, with operations of mining, beneficiation, smelting, refining, and downstream saleable products. From the run-off mine grade of 0.8 to 1.2% Cu, the ore concentrate of grade 25-26% Cu is prepared. Subsequently, copper concentrate is fed to the smelter of HCL located in Mouhbandar (Ghatsila) to prepare copper bars and plates as per buyers’ specifications (Fig. 6).

Malanjkhand Copper Project (MCP) is a subsidiary of Hindustan Copper Limited, which was commissioned on 12th November 1982. The mining lease was approved in favor of HCL in 1973. The latitude and longitude of this mine are 22°00’59” to 22°02’24” N and 80°41’51” to 80°42’38” E and forms a part of Survey of India Toposheet No. 64 B/12, located in the Balaghat District of Madhya Pradesh in India. The strike length of the deposit is 2600 m. MCP, situated at a distance of 20 km away from the Kanha National Park, contributes about 80 percent of HCL’s overall copper supply. This is the largest copper mine in India (Fig. 7), with an average orebody width of 75 m. It has a total lease area of 479.9 ha.

Fig. 6 The process flow of vertically integrated operations in HCL.

Fig. 7 Site location Map of Malanjkhand Copper Mine, HCL
Malanjkhand Copper Project is a massive metalliferous deposit amounting to 200 Million tonnes \((Mt)\) of copper ore reserves. MCP has rich mineable copper reserves of 143 \(Mt\), which is more than 70\% of the country’s available resources. India’s largest copper mine, Malanjkhand deposit extends vertically from the surface at 580 mean reduced level \((mRL)\) to a depth of \((-300 mRL)\). Open-pit mining was planned initially to a depth of 204 \(m\) \((376 mRL)\), which is now extended to 240 \(m\) \((340 mRL)\). Below the open-pit floor, there exist 160 \(Mt\) reserves of 1.34\% copper grade containing over two million tonnes of copper metal. At present, the central portion of the deposit is being mined out by the open cast mining method, which has reached the Ultimate Pit Limit \((UPL)\).

The general area in the vicinity of the mine is a hilly terrain consisting of narrow basins and hills. Malanjkhand rises in isolated majesty above the undulating terrain and forms a part of the Baihar Plateau. Six peaks with intervening saddles are observed that create an arcuate chain of about 2.6 km long, having an eastward convexity. This highest peak is about 652 \(m\) above mean sea level \((mSL)\), while the ground level of the surrounding area is around 575 above \(mSL\). This describes the general elevation of the Baihar Plateau, which rises about 270 \(m\) above the Balaghat Plains. The digital surface contour terrain of the above area within lease boundary and the surface plan of the project is shown is Fig. 8 \(a\) and \(b\).

![Fig. 8 (a) Initial Surface Contour Terrain (source: HCL) (b) Surface Plan of Malanjkhand Copper Mine (source: HCL)](image)

King [63] initially noted the presence of copper in Malanjkhand in 1886, and Dunn reported it in 1942 [64]. Later, the porphyry character of the copper deposit was discovered by Sikka in 1971, opening up new possibilities in the search for the substantial tonnage of Precambrian polymetallic porphyry copper deposits in Madhya Pradesh, India [65]. The Geological Survey of India (GSI) carried out systematic geological mapping at this site in 1969. Malanjkhand Copper belt consists of a large body of copper ore in granite rocks, ranging in composition from diorite to granite. The rocks of Malanjkhand are from the Proterozoic era and can be divided into two groups, each with unique attributes, distinguished by an unconformity. The oldest group, composed of basement rocks, was formed first, followed by the younger, less metamorphosed rocks of the Chilpi Ghat beds.
Three distinct forms of mineralization are clearly visible. Firstly, the mineralization is primarily correlated with quartz, which is generally limited to the quartz reef as a fracture fill form and can be known as quartz ore. Secondly, the mineralization of the stringer pattern has quartz and calcite joints. This stringer ore is found primarily in granites. Lastly, the disseminated ore inside granites or micro granites is present within the interstices of inherent grains of the related rocks. The most predominant ores are chalcopyrite, chalcocite, and malachite in order of abundance. The secondary and oxidized ore minerals are confined to the upper part of the deposit. Due to hydrothermal vein formation, the Malanjkhand copper ore deposit geometry varies in the strike and dip directions. The host rock is quartz reef and granite.

Open-pit mining will be continued for exploiting the deposit. As per the revised open-pit schedule, mining is contemplated to expand the pit to a depth of 340 mRL. A view of the working pit and the modelled Ultimate Pit design of the Malanjkhand Copper Project is shown in Fig. 9 (a) and (b).

![Working Pit of Malanjkhand Copper Project](image1.png) ![Ultimate Pit design based on 3-D mine planning software](image2.png)

**Fig. 9 (a)** Working Pit of Malanjkhand Copper Project **(b)** Ultimate Pit design based on 3-D mine planning software (*source: HCL*)

The calculation module is based on the grade-tonnage distribution and operational data (mine, mill, and refinery capacities). Financial parameters have been considered as per the Indian scenario (in ₹). The mineral inventory is given in Table 2.

| Grade from (% Cu) | Grade to (% Cu) | Tonnage (in tonnes) |
|------------------|----------------|---------------------|
| 0.20             | 0.45           | 25909429            |
| 0.45             | 0.50           | 3292995             |
| 0.50             | 0.60           | 6222261             |
| 0.60             | 0.70           | 6917390             |
| 0.70             | 0.80           | 6538756             |
| 0.80             | 0.90           | 5674345             |
| 0.90             | 1.00           | 3501976             |
| 1.00             | 1.10           | 3331731             |
| 1.10             | 1.20           | 3283519             |
| 1.20             | 1.30           | 2360291             |
| 1.30             | 1.40           | 1816110             |
| 1.40             | 1.50           | 1630172             |
| 1.50             | 5.60           | 9154230             |
3.2 Model input

Geo-mining details and techno-economic variables are the vital input parameters for optimizing the objective function for maximization of NPV. In addition to the grade-tonnage deposit (mineral inventory) data, the required technical and financial data are given below in Table 3. The following data has been used as input data to determine optimum COG%. This data will be imported into the COG-NPV optimizer tool for the calculation and software input.

| Mining | Values |
|--------|--------|
| Maximum mining capacity (tonnes / yr) | 2540000 |
| Percentage mining recovery (%) | 100 |
| Specific investment cost for mining (₹/tonne / yr) | 160 |
| Unit cost of excavation (₹/tonne ore & waste) | 816.91 |
| Fixed mining cost (₹/tonne) | 28026.48 |
| Average ore transportation cost (₹/tonne / km) | 43.30 |
| Average waste transportation cost (₹/tonne / km) | 43.30 |
| Average ore transportation haul distance (km) | 6 |
| Average waste transportation haul distance (km) | 6 |

| Mill | Values |
|------|--------|
| Maximum mill capacity (tonnes / yr) | 2340000 |
| Mill concentrate grade (%) | 26 |
| Percentage mill Recovery (%) | 92.5 |
| Specific investment cost for mill (₹/tonne / yr) | 150 |
| Unit cost of ore processing (₹/tonne) | 464.42 |
| Fixed mill cost (₹/tonne) | 29314.02 |
| Average concentrate transportation cost (₹/tonne / km) | 1.95 |
| Average mill tailing transportation cost (₹/tonne / km) | 3 |
| Average concentrate transportation haul distance (km) | 180 |
| Average mill tailing transportation haul distance (km) | 4.6 |

| Refinery | Values |
|----------|--------|
| Maximum refinery capacity (tonnes / yr) | 18500 |
| Percentage refinery Recovery (%) | 99.66 |
| Specific investment cost for refinery (₹ / tonne / yr) | 500 |
| Unit cost of refining (₹ / tonne of metal produced) | 57600 |
| Fixed refinery cost (₹ / tonne) | 47800 |
| Average metal transportation cost (₹ / tonne / km) | 3 |
| Average slag transportation cost (₹ / tonne / km) | 3 |
| Average metal transportation haul distance (km) | 0.5 |
| Average slag transportation haul distance (km) | 0.5 |

| Additional Costs | Values |
|------------------|--------|
| Other costs including general administration cost (percentage % of the total operating cost) | 25 |

| Rates | Values |
|-------|--------|
| Selling price of metal (₹ / tonne) | 500000 |
| Tax rate (%) | 34 |
| Discount rate (%) | 11 |

| Grade Interval Precision | Values |
|--------------------------|--------|
| Grade Interval increment (%) | 0.01 |
3.3 Results output

The data mentioned above was provided to the software and processed by the COG optimizer model, after which the final optimized output of every section was generated and listed in Table 4.

Table 4 Summary of the results output of all the eight modules

| Modules | M-1/4/6 | M-2 | M-3 | M-5 | M-7 | M-8A | M-8B | M-8C |
|---------|---------|-----|-----|-----|-----|------|------|------|
| Particulars | values | values | values | values | values | values | values | values |
| **Mining (production in tonnes (t))** | | | | | | | | |
| $Q_m$ (t/yr) | 2132818 | 2540000 | 3788250 | 2340000 | 2340000 | 1772009 | 3809819 | 5352259 |
| OCOG (%) | 0.33 | 0.23 | 0.52 | 0.26 | 0.33 | 0.27 | 0.22 | 0.24 |
| Life (yrs) | 38.00 | 32.00 | 22.00 | 32.00 | 38.00 | 19.00 | 15.00 | 3.00 |
| $T(T_e)$ (t) | 66168302 | 76532073 | 49194329 | 73422942 | 66168302 | 72386565 | 77568451 | 75495696 |
| $T(T_w)$ (t) | 13472903 | 3109131 | 30446876 | 6218263 | 13472903 | 7254640 | 2072754 | 4145509 |
| $S_{a,B}$ | 0.20 | 0.04 | 0.62 | 0.08 | 0.20 | 0.10 | 0.03 | 0.05 |
| **Mill (production in tonnes (t))** | | | | | | | | |
| $Q_e$ (t/yr) | 1772009 | 2440840 | 2340000 | 2340000 | 1772009 | 3809819 | 5352259 | 25165232 |
| $g$ (%) | 1.13 | 1.01 | 1.38 | 1.05 | 1.13 | 1.06 | 1.06 | 1.08 |
| $C_m$ (t/yr) | 71397 | 88320 | 114884 | 87393 | 71397 | 143809 | 198108 | 920194 |
| $T(T_i)$ (t) | 1700613 | 2352521 | 2252516 | 2252607 | 1700613 | 3666010 | 5334452 | 24245038 |
| $T(T_n)$ (t) | 2713071 | 2826231 | 2527440 | 2796564 | 2713071 | 2732376 | 2971619 | 2760583 |
| $T(T_l)$ (t) | 64623286 | 75280661 | 48952560 | 72083436 | 64623286 | 69654189 | 80016774 | 72735114 |
| **Smelter/ Refinery (production in tonnes (t))** | | | | | | | | |
| $Q_o$ (t/yr) | 18500 | 22885 | 29768 | 22645 | 18500 | 37263 | 51333 | 238437 |
| $S_{o}$ (t/yr) | 52897 | 65435 | 85115 | 64748 | 52897 | 106546 | 146775 | 681757 |
| $T(Q_o)$ (t) | 703000 | 732322 | 654900 | 724635 | 703000 | 708002 | 769994 | 715311 |
| $T(Q_p)$ (t) | 2010071 | 2093909 | 1872540 | 2071930 | 2010071 | 2024374 | 2201625 | 2045271 |
| **Profits (€ in million)** | | | | | | | | |
| $T(C_l)$ (€) | 616 | 784 | 972 | 768 | 616 | 1261 | 1764 | 8142 |
| $D_p$ (€/yr) | 16 | 24 | 44 | 24 | 16 | 66 | 118 | 2714 |
| $R_0$ (€/yr) | 9250 | 11443 | 14884 | 11322 | 9250 | 18632 | 25666 | 119219 |
| AOMC (€/yr) | 4322 | 5250 | 6935 | 5185 | 4322 | 8530 | 11783 | 54664 |
| AC (€/yr) | 2848 | 3538 | 4651 | 3493 | 2848 | 5745 | 7943 | 36835 |
| GP (€/yr) | 2154 | 2630 | 3254 | 2621 | 2154 | 4290 | 5823 | 25006 |
| Tax (€/yr) | 732 | 894 | 1106 | 891 | 732 | 1458 | 1980 | 8502 |
| $N_p$ (€/yr) | 1422 | 1736 | 2148 | 1730 | 1422 | 2831 | 3843 | 16504 |
| CF (€/yr) | 1438 | 1760 | 2192 | 1754 | 1438 | 2898 | 3961 | 19218 |
| NPV (€) | 12208 | 14653 | 16948 | 14609 | 12208 | 21454 | 26717 | 38821 |
| $T(R_p)$ (€) | 351500 | 366161 | 327450 | 362317 | 351500 | 354001 | 384997 | 357656 |
| $T(N_p)$ (€) | 54026 | 55552 | 47248 | 55530 | 54026 | 53792 | 57647 | 49511 |

In this model, the input values (operational and economical) have been taken irrespective of the time. Therefore, the values and results will behave accordingly as per the user’s input values. Thus, the mine planners or experts will run their module as per the current situation and get their results to optimize accordingly. If there will be a significant increase in the pricing
dynamics in the coming years, the planning experts will run through the program and optimize accordingly. Therefore, there is no need to introduce the price forecasting technique in this model because there are a host of variables where the cost-escalation factor will be applicable, including the entire mine, mill, and smelter and refinery stages. Therefore, considering all the variables will make things more complex and may lead to an indefinite solution.

3.4 Graphs of the results output

The Graphical User Interface of the output data module for the above results output as mentioned in the graphical result module are shown through graphs illustrated in Figures 10, 11, 12 and 13, respectively.

![Graphs of the results output](image)

**Fig. 10 (a) Tonnage versus Grade distribution of deposit (b) Cumulative tonnage, Average grade versus COG**

![Graphs of the results output](image)

**Fig. 11 Showing all the NPV values versus COG(%) related with the eight modules**
Fig. 12  Showing the 3-D line graph of NPV's versus COG(%) versus life of mine of all the eight modules

Fig. 13  Showing the 3-D line graph of NPV's versus COG(%) versus mine production ($Q_m$) of all the eight modules
4. Discussions of results

The results after the simulation from all the eight modules have been presented through tables and figures. The observations according to the given grade-tonnage and techno-economic variations are discussed below in Table 5.

**Table 5** Conclusive summary sheet of the output variables of all the eight modules for the open-pit metalliferous deposit (MCP)

| Sl. No. | Optimum mill COG (%) | NPV (in Millions ₹) | Conditions (L-Limited; UL-unlimited) | Observations (Mine: M; Concentrator: C; Refinery: R) |
|---------|-----------------------|---------------------|--------------------------------------|----------------------------------------------------------|
| 1.      | 0.33                  | 12208               | Module - 1: M, C, R (L, L, L)        | The optimum COG value for the Module 1, 2, 4 and 7 is same. In all the above four modules, there are different limiting conditions, and out of these three limiting conditions, R becomes the final limiting function, and hence it decided the final C and R values. Therefore, in these scenarios, all four modules are getting the same results as the final \( Q_m, Q_c, \) and \( Q_r \) values. In these four modules, same optimal COG(%) of 0.33 with the maximum NPV of ₹ 12208 million have been found. |
| 2.      | 0.23                  | 14653               | Module - 2: M, C, R (L, UL, UL)      | In Module 2, M is the limiting function. It has decided the ultimate C and R values, and correspondingly, the limiting logical conditions on M, C, and R values contributed to the final \( Q_m, Q_c, \) and \( Q_r \) results. This module has given the optimal COG(%) at 0.23 with the maximum NPV of ₹ 14653 million. |
| 3.      | 0.52                  | 16948               | Module - 3: M, C, R (UL, L, UL)      | In Module 3, C is the limiting function. It has decided the ultimate M and R values, and correspondingly the limiting logical conditions on M, C, and R values contributed to the final \( Q_m, Q_c, \) and \( Q_r \) results. This module has given the optimal COG(%) at 0.52 with the maximum NPV of ₹ 16947 million. |
| 4.      | 0.26                  | 14609               | Module - 5: M, C, R (L, L, UL)       | In Module 5, M and C are the limiting function, whereas R has a scope of unlimited capacity. So, limiting conditions of M and C have decided the ultimate values, and correspondingly the limiting logical conditions on M, C, and R values contributed to the final \( Q_m, Q_c, \) and \( Q_r \) values. This module has given the optimal COG(%) at 0.26 with the maximum NPV of ₹ 14609 million. |
When all the capacities (mining, concentrator, and smelter and refinery) are unlimited, the optimal COG (%) and the corresponding maximum NPV, mine production and mine life have been calculated according to Taylor’s [14] mine life rule, by Long’s [60] rule and by the DP based algorithm (the presented COG-MDP model). It has been found that in all the three cases, mining capacity becomes the limiting function. Therefore, the logic sequentially advances and decides the final C and R values, and correspondingly the limiting logical conditions on \( Q_c \) and \( Q_r \) values will contribute to the final results. It may be observed that among the three methods for the unlimited conditions, the DP based algorithm gives the maximum NPV of ₹388201 million at COG(%) of 0.24.

A major advantage of the DP based algorithm is that it optimizes all the four variables – COG%, NPV, mine production and mine life simultaneously.

5. Conclusion and recommendations

The significance of developing a new evaluation technique, such as the one proposed in this study, lies in providing a pragmatic analytical approach to the mining industry that allows the mine planner or analyst to obtain a more realistic estimate of an open-pit project value accounts for the given techno-economic conditions. The three primary stages utilized in COG optimization depend upon the limiting conditions of mining, milling, and smelting and refining. In each of these sectors, several smaller divisions may be established. By individually optimizing mine and COG in planning stages, a guarantee of obtaining the highest NPV is generally lost.

This necessitates developing a suitable approach for finding the optimum COG of ore, and hence a multi-value dynamic programming has been developed and used to compute COG for a metalliferous deposit. In this model, discrete values have been utilized for Dynamic Programming (DP), a more deterministic approach than stochastic one. The developed model is based on the concept of dynamic programming and thus finds the solution much more straightforward than other optimization techniques. The fundamental framework and dynamics guide this model rather than tailoring it to time series. This increases
the model’s generic validation, but it takes a long time to design and parameterize. Its key strength is that it optimizes several policies simultaneously, that the results are connected and mutually reliable, and effectively validates against facts. As in this model, COG optimization has been carried out after finalizing the Ultimate Pit, and the pit optimization follows the objective function of maximizing profits irrespective of time. Therefore, whenever one does mining, it will create wealth, and the economics will behave as per the existing scenario. As a result, mine planners must re-run the COG-MDP optimizer model depending on market conditions and modify their intermediate stages of excavation following demand and economics. In general, the mine planners can choose their optimization in three stages.

- First stage by initially excavating higher-grade ore to recover capital, operating, and running costs and reduce the payback period.
- Second stage, by recovering the depreciation and tax amounts, can mine with the average-grade or lower grade.
- Third stage, when the machines or plants are fully depreciated, they can go for mining the lower grade ore and sustain the working of mines.

This work has the novelty of dealing with eight different optimization possibilities. So far, only six potential outcomes have been dealt with as on date by Lane [16]. The computer can produce the result in seconds from the complex system of mine, mill, and smelter and refinery comprising of eight different possibilities, and thus it will be a useful industry-oriented tool to plan a mining project for a given deposit considering all the possible conditions. An alternative method to Lane’s algorithm and other existing approaches, a technique based on computer programming algorithms, has been presented here as a feasible and general method for the solution of optimal COG and production rate determination.

Open-pit mines such as the Malanjkhand copper mine has diversified production constraints. Besides, these operations will generally have a well-defined operating range, which may vary in actual situations. The model does not consider the uncertainty in economic parameters, especially the uncertainty and fluctuations on metal prices, stockpiling or blending, mining dilution, multi-mineral deposits, and mine rehabilitation. The stockpiling option has not been included as that may increase the complexities enormously and may lead to an incomplete solution. The objective of this model is to introduce a universal approach for resolving the optimality scenario related to COG of open-pit metalliferous deposits.

This method is capable of handling technically more acceptable relationships and definitions. However, the software provides enough flexibility to include or exclude the parameter(s) by changing the design’s code; therefore, there is ample room for the expansion and modification of the model to cover additional scenarios. The intention here is to introduce a general method for the solution with a deterministic mathematical approach.
However, there is a scope to modify the program by including the stochastic behavior of the variables (commodity price, operating and maintenance expenses, fixed cost, hauling distance, discount rate etc.) to make it more applicable but at the cost of increasing the complexities. However, based on mining conditions and economics, the material categorized as waste could become economically viable to process in the future, necessitating a shift in policy. So, the utility and complexity of stockpiling is a more extensive area to explore.

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Compliance with ethical standards

Conflict of interest: The authors declare that they have no conflicts of interest.

Author Contributions

Pritam Biswas: Conceptualization, Methodology, Formal analysis, Investigation, Data curation, Writing-Original draft, Visualization. Rabindra Kumar Sinha: Writing review and editing, Supervision, Project administration, Validation, Resources. Phalguni Sen: Conceptualization, Writing review and editing and Co-supervision.

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**Article Highlights**

- A sequentially advancing algorithm based on multi-value dynamic programming (MDP) is being developed for the cut-off grade \( COG \) optimization model for surface mining applications.

- This discrete \( COG-MDP \) model is the first of its kind in the field of \( COG \) optimization and has the novelty of dealing with the simulation of eight different optimization possibilities to achieve the maximum Net Present Value (NPV) for mineral grades.

- The computer model is developed in Python to contend with the complexities of processing a minimum of 500 series of dynamic variables associated with the complex system of mine, mill, and smelter and refinery with an accuracy value of 0.01% in grade bins.