The Numerical Simulation of the Freshwater-Seawater Mixing in the Single Phase Coastal Aquifer

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Abstract. The interaction between freshwater and seawater usually occurs in the coastal aquifer. To simply explain this condition, describing the interface between them is conducted. This approach is a multiphase condition. Unfortunately, this condition is on immiscible condition, whereas the miscible condition should be obtained. The miscible condition of freshwater and seawater along coastal aquifer occurs on the single phase. This condition neglects the concept of the interface between them. The purpose of this research is to simulate the mixing of freshwater and seawater in the coastal aquifer by using the numerical method. Groundwater flow equation is governed involving the effect of contaminant concentration. On the other hand, the conservative groundwater contaminant transport equation is used. The discretization process uses the Finite-Difference Method. Coupling two equations will obtain the spreading of contaminant concentration along the model domain. The result shows that the freshwater-seawater interface does not appear and the change of concentration can be reviewed in every section and every point of the computational domain.

Keywords: Coastal Aquifer; Contaminant Transport; Groundwater; Numerical Model

1. Introduction
Groundwater contaminant in the coastal aquifer for example seawater intrusion occurs on the single phase, due to under the miscible condition between freshwater and seawater [1]. The classical model of seawater intrusion does not accommodate the value of contaminant concentration as shown in Figure 1 [2]. The seawater and freshwater are bounded by the interface between them. This condition is immiscible.

To simulate groundwater contaminant on the single phase, the groundwater flow model and groundwater contaminant transport equation are necessary. Coupling both of them will give the result without the interface. Hence to achieve that condition, the groundwater flow equation must be developed involving the influence of contaminant concentration. There are some example of groundwater flow model coupled to contaminant transport, like as USGS Sutra [3] and USGS Seawat [4]. The one of term of USGS Sutra model consist the change of water saturation. Therefore, this condition is immiscible. In other hand, USGS Seawat is on miscible condition and taking the fluid density as a function of
concentration and pressure. The groundwater flow mathematical model which coupled to groundwater contaminant transport is stated as groundwater flow in the dependent-density condition.

Many researchers have conducted the numerical simulation of freshwater-seawater mixing. Gambolati (1999) through Bear et.al (1999) develop the dependent-density groundwater flow model entitled as Codesa model [5]. Numerical simulation through the Henry Problem conducted by Sherif, et.al (2012) [6]. The Feflow model simulation have conducted by Yang et.al (2013) [7]. The objective of this research is to perform the numerical simulation of freshwater and seawater mixing in the single phase. The dependent-density groundwater flow model developed using the fluid density as only the function of concentration [8]. The spreading of contaminant along the aquifer after simulation is the main achievement on this research.

![Figure 1. The Interface on the Immiscible Condition](image)

2. Method

The first step for simulation freshwater-seawater mixing in the single phase, governing equation must be conducted. There are two equations that must be governed, consist of groundwater flow equation and groundwater contaminant transport equation. Both of the equations are governed based on eulerian mass conservation law which is written as [9]

\[
\int \frac{\partial}{\partial t} \rho d\mathbf{v} + \int \rho \mathbf{v} \cdot \nabla dA = 0 \tag{1}
\]

Using the calculus, the equation (1) can be transformed to:

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} = - \left[ \frac{\partial (\rho q_x)}{\partial x} + \frac{\partial (\rho q_y)}{\partial y} + \frac{\partial (\rho q_z)}{\partial z} \right] dx dy dz \tag{2}
\]

Equation (2) is the basic equation to govern groundwater flow and contaminant transport equation. The difference between them are the term of flux \(q\) and density \(\rho\). In this research, the term of flux \(q\) is determined by Darcy Law for groundwater flow equation. Therefore, the term of flux \(q\) is determined by conservative advection-dispersion flux for the groundwater contaminant transport equation.

2.1 Groundwater Flow Equation

The groundwater flow equation is governed as a part of the contaminant transport equation due the both of equations will be coupled later. Confined, Isotropic, homogenous and isothermal condition are basic assumption taken in this step. Recognize the Darcy law as the term of flux \(q\), will be written as [10] :

\[
q = -K \frac{\partial h}{\partial l} \tag{3}
\]

where, \(h\) is groundwater piezometric head and \(l\) is spatial component. That equation will be substituted into equation (2) and in other hand the left-hand side equation (2) will be expanded using compressibility theories. The result of left-hand side equation (2) will state as
Based on equation (4), \( \rho_f \) is a constant value of freshwater density, \( g \) is gravitational acceleration, \( \alpha \) is porous media compressibility and \( \beta \) is fluid compressibility. The final groundwater flow equation is obtained by substituting equation (3) and equation (4) into equation (2). On two dimensional (2-D) case, the groundwater flow equation will be simply written as:

\[
\rho_f^2 g (n\beta + \alpha) \frac{\partial h}{\partial t} + nE \frac{\partial C}{\partial t} = \rho_f K \left[ \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right] + KE \left[ \frac{\partial h}{\partial x} \frac{\partial C}{\partial x} + \frac{\partial h}{\partial y} \frac{\partial C}{\partial y} \right]
\]

where, \( q_{s,o} \) is the term of sink and source, \( K \) is hydraulic conductivity then \( C \) is contaminant concentration. \( E \) is the gradient of groundwater density to contaminant/solute concentration. Because the contaminant/solute in this research is NaCl, the gradient \( E \) can be determined as 0.7143 \[4\]. The value of \( E \) will be vary depends on the solute which contaminate the groundwater.

2.2 Groundwater Contaminant Transport Equation

Similar with groundwater flow equation, the groundwater contaminant transport equation will governed based on *Eularian* mass conservation. Special case for groundwater contaminant transport, equation (1) is transformed to \[11\]

\[
\int \frac{\partial}{\partial t} C d\varnothing + \int q dA = 0
\]

This equation concerns to the term of flux (\( q \)) which arranged by advection and dispersion effect. The advection flux in the porous media (\( q_{adv} \)) is defined through equation \[12\]

\[
q_{adv} = n v C
\]

where, \( n \) is the porosity and \( v \) is groundwater velocity. The term of groundwater velocity (\( v \)) in driven based on the result of groundwater flow equation and it can be analysed by equation \[13\]

\[
v_j = - \frac{K}{n} \frac{\partial h}{\partial t}
\]

The dispersion flux in porous media (also called as hydrodynamic dispersion, \( D_h \)) is arranged by molecular diffusion coefficient (\( D_m \)) and mechanics of dispersion (\( D \)). Therefore, the hydrodynamic dispersion (\( D_h \)) equation can be written as

\[
D_h = D + D_m
\]

\( D_m \) is determined depends on the chemical element which contaminates the groundwater. The mechanics of dispersion (\( D \)) is determined by the series \[1\]

\[
D_{ij} = a_1 v i j + a_2 \frac{v_i v_j}{v}
\]

where, \( a_1 \) is called as transversal dispersivity and then \( a_2 \) is called as longitudinal dispersivity. This condition makes the mechanics of dispersion can be divided into two terms follows transversal dispersion (\( D_T \)) and longitudinal dispersion (\( D_L \)). Both of them are written in sequences

\[
D_L = a_L v
\]
Finally, the dispersion flux \( q_{\text{disp}} \) is defined as first fick law and written

\[
q_{\text{disp}} = -nD_h \frac{\partial C}{\partial i}
\]

and the flux of groundwater contaminant transport \( q \) is arranged by advection flux and dispersion flux, thus:

\[
q = nvC + \left( -nD_h \frac{\partial C}{\partial i} \right)
\]

To obtain the groundwater contaminant transport, taking calculus solution equation (6) must be conducted. Then, use equation (15) as the term of flux and the result of this step can be written as

\[
\frac{\partial C}{\partial t} = -\left( v_x \frac{\partial C}{\partial x} + v_y \frac{\partial C}{\partial y} \right) + \left( D_{hx} \frac{\partial^2 C}{\partial x^2} + D_{hy} \frac{\partial^2 C}{\partial y^2} \right) \pm \frac{q_{\text{disp}}}{n} C
\]

All of velocity components (\( v \)) in the equation (15) is equal to equation (8).

3. Simulation, Result and Discussion

The simulation is conducted based on equation (5) and equation (15). Both equations are formed as a discrete equation numerically. The Finite-Difference Method is used in this research. Initial condition and boundary condition used respectively Neumann boundary and Dirichlet boundary as described in Figure 2. Figure 3 describes the model that will be simulated by using numerical and physical parameters as presented in Table 1.
Figure 4 describes the salinity contour along model domain after reach the steady state condition (120 days simulation). The concentration of contaminant can be determined in every locations of computational domain and this is under miscible. The section A-A (Figure 5) explains the induction of seawater at every time step. Figure 6 presented the change of concentration during the simulation in a one point (in this case, point B in Figure 4).

The simulation occurred along groundwater flow equation and groundwater contaminant transport equations are coupled. The first analysis is groundwater flow and then the next analysis is groundwater contaminant transport. Groundwater flow equation gives the influence to advection component of groundwater contaminant transport due the velocity of advection depends on the gradient of the piezometric head. This result can be stated as seawater intrusion without the effect of extraction groundwater resources.

![Figure 4. The Salinity Contour of the Model Domain after 120 Days Simulation](image)

| Parameters                              | Unit     | Values  |
|-----------------------------------------|----------|---------|
| The model width (x-direction)           | m        | 50      |
| The model height (y-direction)          | m        | 25      |
| i x                                     | m        | 2.5     |
| i y                                     | m        | 2.5     |
| i t                                     | day      | 1       |
| Longitudinal dispersivity, $k_L$        | m        | 2.5     |
| Transversal dispersivity, $k_T$         | m        | 0.42    |
| Molecular diffusion coefficient, $D_m$  | $m^2/s$  | $10^{-5}$ |
| Porosity                                |          | 0.35    |
| Hydraulic Conductivity, $K$             | m/s      | $10^{-6}$ |
| Groundwater density, $\rho$             | Kg/m$^3$ | 1000    |
| Gravitational acceleration, $g$         | m/s$^2$  | 9.81    |
| Piezometric head (upstream)             | m        | 121.05  |
| Piezometric head (downstream)           | m        | 120     |
| Relative concentration, $C$             | ppt      | 35      |

Table 1. Numerical and Physical Parameters
The main discussion of this simulation is about the result under miscible condition. If the Figure 4 is compared to Figure 1, the effect of the mixing will appear clearly. Along Figure 4, the salinity of groundwater can be traced. To achieve the stable result like as Figure 5, the stability requirements have to be obtained before the simulation. In this simulation, stability can be controlled by $\Delta x$, $\Delta y$ and $\Delta t$ setup.

4. Conclusion
The coupling between groundwater flow equation and groundwater contaminant equation allow modeling the freshwater and seawater mixing under the miscible condition in single phase. To obtain the value of contaminant along the aquifer, the coupled numerical simulation is necessary. Special case for groundwater flow equation must accommodate the influence of contaminant concentration to groundwater density. Hence, the type of this equation is groundwater flow equation that depends on groundwater density.

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