Simulation of economic agents interaction in a trade chain

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Abstract. The mathematical model of economic agents interaction is offered in the work. It allows considering the change of price and sales volumes in dynamics according to the process of purchase and sale in the single-product market of the trade and intermediary network. The description of data-flow processes is based on the use of the continuous dynamic market model. The application of ordinary differential equations during the simulation allows one to define areas of coefficients - characteristics of agents - and to investigate their interaction in a chain on stability.

1. Introduction

The research of the regularities that occur in trade systems attracts interest of all participants: producers, intermediaries, consumers, and the new companies appearing in the market as well. The head of any line-up of a network has to consider these processes to predict the consequences of the administrative resolutions.

A set of conditions and factors influences the dynamic processes that occur in the system. That is why, there is the need of deep and comprehensive study of problems of agents interaction for intermediary systems in the market conditions.

The trade system means a non-steady dynamic system, represented in the form of a set of chains of economic agents where characteristics are changed in the course of time.

The review of scientific works showed a variety of approaches on how to research trade systems [1-3]. When developing this model of economic agents interaction, the data-flow processes proceeding in intermediary chains of the system were considered [4, 5].

2. The model of economic agents interaction

It is given: There are restrictions in the chain of economic agents consisting of the producer, intermediaries and the end user. They are connected with:

• the money given for purchase of goods that depends on the predicted demand for goods according to season fluctuations and other influences of environment;
• the inability to sale and/or to purchase a particular quantity of goods that depends on a type of goods, conditions of its storage, the type and volume of a warehouse.

Economic agents own data about the situation in the market and can influence the demand of $D(t)$ and the supply of $S(t)$ of these goods in the market.

It is required: to develop the model of economic agents interaction of a trade chain.
Task solution: The trade and intermediary chain is represented in the form of a multiagent system where the interaction of agents occurs among themselves, the consumer market and the market of resources (fig. 1).

Figure 1. The trade and intermediary chain.

The consumer market is described by:

\[ \Theta = \{Y(t), y_0, \mu\} \]

where \( Y(t) \) – a set of market conditions, \( y_0 \) – a reference state of the market, \( \mu: Y(t) \times D_{\text{ag}} \rightarrow Y(t) \) – a function of market behavior, \( D_{\text{ag}}(t) = \{D_{\text{ag}}(t) \mid i = 1..n\} \) – a set of actions of chain agents. Interactions between agents occur in the local micromarkets on the basis of mechanisms of information exchange and making the decision of the purchase-sale satisfying the buyer and the seller.

The trade and intermediary chain is described as

\[ T = \{\text{Ag}(t), \Theta, \text{Vs}(t), \text{St}(t)\}, \]

where \( \text{Ag} = \{\text{Ag}_i \mid i = 1..n\} \) – a great number of agents, \( \text{Vs}(t) \) – a set of the relations between agents, \( \text{St}(t) = \{\text{St}_k(t) \times \text{St}_n(t) \times \text{St}_m(t) \} \) – a set of conditions of chain, \( \text{St}_i(t) \) – a condition of the \( i \)-agent.

Each \( i \)-agent is described as follows:

\[ \text{Ag}_i(t) = \{\text{St}_i(t), D_{\text{ag}}(t), \text{Ogr}_i(t), T_i\}, \]

where \( D_{\text{ag}}(t) \) – actions of the \( i \)-agent, \( T_i: \text{St}_i \times D_{\text{ag}} \times \text{Ogr}_i \rightarrow \text{St}_{i+1} \) – a function of transitions from a condition of \( k \) to a condition of \( k+1 \), \( \text{Ogr}_i(t) \) – a set of restrictions of the \( i \)-agent.

3. Mathematical methods of definition of chain characteristics

It is given: Producers, intermediaries and consumers of the market of one product form uniform related retail chain stores which can be presented in the form of connected terminating focused graph \( G=(V, E, H) \), where \( V \) – aset of tops of the count, \( E \) – aset of arches, \( H \) of display \( H: X \rightarrow V \times V \).

Tops of \( v_i \in V, i=1..n, k=0..3 \) are interpreted as economic agents of retail chain stores of the \( k \)-level, arch \((i_k, j_k)\) as the micromarkets where the prices of \( p_{i_k,j_k}(t) \geq 0 \) according to which exchange of production of the volume of \( Q_{i_k,j_k}(t) \geq 0 \) between various tops of the count is established.

It is required: to develop mathematical methods of definition of economic agent characteristics in their interaction.

Task solution: In figure 2, the chain of retail chain stores consisting of two agents of \( k \)- and \( k+1 \)-levels where \( i_k = i \) – the seller, \( j_{k+1} = j \) – the buyer presented to \( Q_{i,j}(t) \) – volumes of the realized goods going on arch \((i_k, j_{k+1})\), \( p(t) \) – the goods price for \( i \)-m the market, \( F_{i,j}(t) \geq 0 \) – the financial means coming from the \( j \)-buyer to the \( i \)-seller.
Figure 2. The line-up of retail chain stores.

Let us use the continuous dynamic market model in the shape of
\[ \frac{dp(t)}{dt} = a_{DS} \left( D(p) - S(p) \right), \]
where \( D(p) \) – a demand function, \( S(p) \) – a supply function, \( a_{DS} \) - a coefficient.

After replacement of \( \Delta Q(p) = D(p) - S(p) \) and \( a = 1/a_{DS} \), we obtain
\[ \Delta Q(p) = a \frac{dp(t)}{dt}. \]

Each \( i \)-agent – an intermediary at any level – can act as a buyer and as a seller, where \( p_i^b(t) \) – the price of goods of the intermediary agent as a buyer, as, \( p_i^s(t) \) – the price of goods of the intermediary agent as a seller.

Main characteristics of subjects of the trade system of the single-product market (end user, buyer, seller):

- a coefficient of goods conductivity of the \( i \)-agent:
  \( R_i(t) = Q_i(t) / p_i(t) \);

- a coefficient of the price change of the agent depends on the change of volumes of consumption of goods in time \([t_i, t_j]\)
  \[ a_i(t) = \frac{\Delta p(t)}{\Delta Q(t)} = \frac{p(t_j) - p(t_i)}{Q(t_j) - Q(t_i)}; \]

- a coefficient of volume change of goods of the agent from the change in price \([t_j, t_k]\).
  \[ a_2(t) = \frac{\Delta p(t)}{\Delta Q(t)} = \frac{p(t_k) - p(t_j)}{Q(t_j) - Q(t_k)}. \]

Goods movement occurs from the producer (seller) to the consumer (buyer) under the influence of forces of supply and demand.

Such economic agents in a trade chain as the end user, the buyer, the seller are considered. Their trial functions of the price and sales volume are presented in table 1.

Table 1. Ratios between the price and sales volume.

|                | Price function | Volume function |
|----------------|----------------|-----------------|
| Seller         | \( p(t) = a_i(t) \frac{dQ(t)}{dt} \) | \( Q(t) = \frac{1}{a_i(t)} \int p(t) dt \) |
| Buyer          | \( p(t) = \frac{1}{a_i(t)} \int Q(t) dt \) | \( Q(t) = a_i(t) \frac{dp(t)}{dt} \) |
| Final customer | \( p(t) = Q(t) \backslash R(t) \) | \( Q(t) = R(t) \cdot p(t) \) |

4. Results and Discussion
The mathematical model of advance of commodity streams is formed for the allocated chains from the system based on preservation regularities of the material stream in a network knot, cost preservation of goods in a network contour and characteristics of each agent according to table 1.

The scheme of a current circuit «the seller – the final customer» is represented in figure 3, where communication of the active seller and the final buyer through the micromarket is simulated.

![Figure 3](image_url)

**Figure 3.** The connection scheme of a chain «the seller – the final customer».

This connection is described in the form of the system of ordinary differential equations of the first order with constant coefficients with starting conditions $Q(0) = Q_0$, $p(0) = p_0$,

$$
\begin{align*}
Q(t) &= a_2(t) + Q'(t) / R(t) = Q_0(t), \\
p(t) &= a_2(t) + p'(t) / R(t) = p_0(t).
\end{align*}
$$

The simulation function is the following:

$$
\begin{align*}
Q(t) &= Q_y + (Q_0 - Q_y) e^{-t / E_S}, \\
p(t) &= p_y + (p_0 - p_y) e^{-t / E_S}, \\
E_S &= \frac{Q_0 - Q_y}{p_0 - p_y}, \\
a_2 &= \frac{p_0}{Q_y}.
\end{align*}
$$

In periods of time $[\Delta t_{1.6}, \Delta t_{1.13}]$, the system simulation with step $\Delta t=1$ is carried out a week with coefficients values $a_2 = \frac{Q_0 - Q_y}{p_0 - p_y} = 79.902 > 0$, $E_S = \frac{Q_0 - Q_y}{p_0 - p_y}$, $\frac{P_y}{Q_y} = 0.949 > 0$.

The transient period made $T = 7$ weeks (Table 3).

**Table 2.** The simulation at step $\Delta t=1$ week.

| $i$ | $\Delta t_{1.6}$ week | Price, $p_i$ | Sales volume, $Q_i$ | Model price, $p_{i,mod}$ | Ratio error, $\delta(p_i)$, % | Model volume, $Q_{i,mod}$ | Ratio error, $\delta(Q_i)$, % |
|-----|-------------------------|--------------|---------------------|--------------------------|----------------------------|-----------------------------|----------------------------|
| 0   | 6                       | 19.00        | 1602.55             | 19.00                    | 1602.55                    |                             |                            |
| 1   | 7                       | 19.36        | 1718.75             | 19.326                   | 0.176                      | 1628.579                    | 5.25                       |
| 2   | 8                       | 19.50        | 1617.80             | 19.439                   | 0.313                      | 1637.649                    | 1.23                       |
| 3   | 9                       | 19.50        | 1560.28             | 19.479                   | 0.108                      | 1640.81                     | 5.16                       |
| 4   | 10                      | 19.50        | 1597.55             | 19.493                   | 0.036                      | 1641.91                     | 2.78                       |
| 5   | 11                      | 19.50        | 1452.55             | 19.497                   | 0.015                      | 1642.295                    | 11.37                      |
| 6   | 12                      | 19.50        | 1395.41             | 19.499                   | 0.005                      | 1642.43                     | 13.07                      |
| 7   | 13                      | 19.50        | 1544.46             | 19.50                    | 0                          | 1642.48                     | 6.35                       |
To estimate the accuracy of the mathematical model, i.e. the hit rate of values of exit pupils of an actual object and values of the same parameters calculated by means of the model, the following relative accuracy has been used:

\[
\delta = \max_{i=1,2} \delta_i = \max_{i=1,2} \left\{ \frac{|Q_{i,\text{mod}} - Q_i|}{Q_i}, \frac{|p_{i,\text{mod}} - p_i|}{p_i} \right\}, \Rightarrow \delta = 13.07\%.
\]

During the change in price of the seller in the course of purchase and sale, we can see the movement to a stable state (fig. 4). Values \(Q_0 < Q_y\) and \(p_0 < p_y\), that is why, the price and volume functions increase up to the established values. The seller and buyer interaction will lead to the equilibrium price and volume, i.e. agents interaction is steady.

5. Conclusion
The developed model of economic agents interaction in a trade and intermediary chain taking into account their dynamics allows one to prove decisions made on the choice of administrative decisions by agents when pricing and sales volumes, and redistribution management of commodity streams as well.

The developed mathematical methods of definition of economic agents characteristics when interacting with an intermediary chain allow tracing the change in price and sales volumes of goods in dynamics, to reveal stability boundaries of interrelation of agents in a network and to estimate potential opportunities in advance of goods.

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References

[1] Oliveira J B, Lima R S, Montveachi J A B 2016 Simulation Modelling Practice and Theory 62 166-191
[2] Groves W, Collins J, Gini M, Ketter W 2014 Decision Support Systems 57 274-284
[3] Denkena B, Henjes J, Henning H 2011 CIRP Journal of Manufacturing Science and Technology 4(1) 9-14
[4] Dulesov A S, Kurynova I A 2011 Modern problems of science and education 5
[5] Dulesov A S, Kurynova I A 2012 Modern problems of science and education 4