Real-time chiral dynamics from a digital quantum simulation

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The chiral magnetic effect in a strong magnetic field can be described using the chiral anomaly in (1 + 1)-dimensional massive Schwinger model with a time-dependent θ-term. We perform a digital quantum simulation of the model using an IBM-Q digital quantum simulator, and observe the corresponding vector current induced in a system of relativistic fermions by a global “chiral quench” – a sudden change in the chiral chemical potential or θ-angle. At finite fermion mass, there appears an additional contribution to this current that stems from the non-anomalous relaxation of chirality. Our results are relevant for the real-time dynamics of chiral magnetic effect in heavy ion collisions and in chiral materials, as well as for modeling high-energy processes at hadron colliders.

I. INTRODUCTION

Quantum theories possess a multi-dimensional Hilbert space that becomes very large for relativistic and/or many-body systems. This is why in addressing the real-time dynamics, the use of quantum simulations potentially provides an exponential advantage over the classical simulations in terms of required computational time and memory. Quantum simulations are also free from the sign problem that obstructs the use of Markov chain Monte-Carlo methods. Because of this, along with the rapid development of digital and analog quantum computers, quantum simulations become a valuable source of information about the real-time behavior of relativistic and many-body systems [1–37].

The Chiral Magnetic Effect (CME) is a generation of electric current in an external magnetic field induced by the chiral asymmetry between the right- and left-handed fermions [38], see [39, 40] for reviews and references. It is a non-equilibrium phenomenon stemming from the relaxation of chiral asymmetry via the chiral anomaly [41, 42]. In high-energy heavy ion collisions, the CME can reveal topological fluctuations in QCD matter [43]. These fluctuations are akin to the electroweak sphalerons in the Early Universe that induce the baryon asymmetry. The experimental study of the effect is ongoing at Relativistic Heavy Ion Collider at BNL and the Large Hadron Collider at CERN, see [44] for review. This effect can also be studied in three-dimensional chiral materials (Dirac and Weyl semimetals) subjected to parallel electric and magnetic fields, and the CME has been observed in a Dirac semimetal ZrTe5 [45] and other materials.

In a constant magnetic field and a given chiral chemical potential (the difference between the chemical potentials of the right- and left-handed fermions), the magnitude of the CME current is completely fixed by the chiral anomaly. However this is not so for a time-dependent magnetic field [46], or for a time-dependent chiral chemical potential. One of the most important effects that determine the real-time dynamics of the CME is the chirality flipping – the transitions between the right- and left-handed fermions that are not related to the anomaly. The simplest mechanism producing such transitions arises from the finite masses of the fermions, since they induce non-conservation of the axial current. The mass effects are important for applications since the quarks in QCD are massive, and quasiparticles in many Dirac materials possess a finite gap.

In the limit of a strong magnetic field, the dynamics of chiral fermions becomes (1+1)-dimensional, since the fermions are frozen at the lowest Landau levels that are not degenerate in spin, and are thus chiral. Indeed, the real-time dynamics of CME, and of the “chiral magnetic wave” can be described [47] within the (1+1)-dimensional QED, the Schwinger model [48]. The chiral anomaly relation in (1+1)-dimensions has the form

\[ \partial_\mu J_5^\mu = \frac{1}{\pi} E + 2i m \bar{\psi} \gamma_5 \psi, \]

where \( J_5^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi \) is the axial current, \( E \) is electric field, and \( m \) is the fermion mass. The first term represents the chiral anomaly, and the second one is due to non-conservation of chirality induced by the masses of the fermions. It is well known [49–51] that a background electric field \( E_{cl} \) in the Schwinger model can be intro-
duced through the $\theta$-angle:
\begin{equation}
E_{c1} = g \frac{\theta}{2\pi}.
\end{equation}

The real-time dynamics of CME can thus be studied in the Schwinger model by considering the time-dependent $\theta$-angle. While the CME stems from the anomaly term, it is very important for applications to understand the role of the second term in (1) in the relaxation of the vector current. This is the goal of our work.

To accomplish this goal, we need to separate the effect of the anomaly from the effect caused by the masses of the fermions in (1). This can be achieved by first applying the chiral rotation $\psi \rightarrow e^{i\gamma_5\theta/2}\psi$ (with $\theta = \theta(t)$) to the fermion fields $\psi$, and then by putting the coupling constant $g$ to zero. In this limit of the theory, the time variation in $\theta$ does not induce a background electric field (see (2)), but it does induce a chiral imbalance between the left- and right-handed fermions through the chiral chemical potential
\begin{equation}
\mu_5 = -\frac{\dot{\theta}}{2}.
\end{equation}

Since the term describing the non-anomalous chirality flipping in (1) vanishes at $m = 0$, in this limit of the theory the chiral chemical potential can induce the vector current only at finite fermion mass. In other words, a chirally imbalanced state with $\mu_5 \neq 0$ at $m = 0$ cannot relax to a state with $\mu_5 = 0$. This can be seen formally by observing that the Hamiltonian of the model in the chiral limit of $m = 0$ commutes with the vector current operator, even at $\mu_5 \neq 0$. This means that a chiral imbalance indeed cannot induce a vector current in the chiral limit of massless fermions.

The situation changes when the fermions become massive. In this case, a chirally imbalanced state can relax to the true ground state with $\mu_5 = 0$, and generate a vector current during this relaxation process. It is clear that this is a real-time phenomenon, and we need to introduce a time-dependent chiral perturbation to study it. We will do this by subjecting the system to two different types of global (spatially independent) “chiral quenches”:

1. Prepare the system in the state with $\theta = 0$ at times $t < 0$. Then, starting at $t = 0$, rotate the $\theta$-angle according to $\theta = -2\mu_5 t$, corresponding to a constant chiral chemical potential (3).

2. As before, prepare the system in the state with $\theta = 0$ at times $t < 0$. Then abruptly change the $\theta$-angle at $t = 0$ to a finite constant value corresponding to $\mu_5 = 0$; see [52] for an earlier study of this quench type.

We will refer to these two global quench protocols as the “$\mu_5$ quench” and the “$\theta$ quench”, respectively.

II. FREE FERMION MODEL WITH THE $\theta$ TERM

We choose the following basis for the gamma matrices,
\begin{equation}
\gamma_0 = Z, \quad \gamma_1 = iY, \quad \gamma_5 = \gamma_0\gamma_1 = X,
\end{equation}
where $X \equiv \sigma_x$, $Y \equiv \sigma_y$, $Z \equiv \sigma_z$ are the Pauli matrices.

The action of the massive Schwinger model with $\theta$ term in (1+1)-dimensional Minkowski space is
\begin{equation}
S = \int d^2x \left[ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g\theta}{4\pi} \epsilon^{\mu\nu} F_{\mu\nu} + \overline{\psi}(i\gamma_\mu D_\mu - m)\psi \right],
\end{equation}
with $D_\mu = \partial_\mu - igA_\mu$. Note that the gauge field $A_\mu$ and the coupling constant $g$ have mass dimensions 0 and 1, respectively. In what follows, we fix the gauge by putting $A_0 = 0$. From the action (5) the canonical momentum conjugate to $A_1$ can be read off as $P = A_1 - \frac{g\theta}{2\pi}$. The corresponding Hamiltonian is then given by
\begin{equation}
H = \int dx \left[ \frac{1}{2} \left( P + \frac{g\theta}{2\pi} \right)^2 + \overline{\psi}(i\partial_x \gamma_1 + m)\psi \right],
\end{equation}
with commutation relations $[A_1(x), P(y)] = i\delta(x-y)$, and $\{\psi(x), \overline{\psi}(y)\} = \gamma_0\delta(x-y)$. Therefore, the term $g\theta/2\pi$ can be identified with a classical contribution to the total electric field $E = A_1$ (in agreement with (2)), while $P$ is the quantum contribution.

Upon the chiral transformation $\psi \rightarrow e^{i\gamma_5\theta/2}\psi$ and $\overline{\psi} \rightarrow \overline{\psi}e^{-i\gamma_5\theta/2}$, the $\theta$-term is absorbed into the phase of the fermion mass term,
\begin{equation}
S = \int d^2x \left[ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \overline{\psi}(i\gamma_\mu D_\mu + \frac{\dot{\theta}}{2}\gamma_5 - me^{i\gamma_5\theta})\psi \right],
\end{equation}
where $\theta = \theta(t)$ is a time-dependent parameter. We then decouple the gauge dynamics by taking the $g \rightarrow 0$ limit to study the free fermionic theory with chiral chemical potential $\mu_5 = -\dot{\theta}/2$ and chirally rotated mass term. The resulting Hamiltonian is given by
\begin{equation}
H = \int dx \overline{\psi} \left[ \gamma_1(i\partial_x - \frac{\dot{\theta}}{2}) + me^{i\gamma_5\theta} \right] \psi.
\end{equation}

Let us consider how the vector current is induced as a result of the chiral quench. In our (1+1) dimensional system, the chiral charge $Q_5 \equiv \overline{\psi}\gamma_5\gamma_0\psi$ and the vector current $J \equiv \overline{\psi}\gamma_1\psi$ are related by $Q_5 = -J$; the vector charge $Q \equiv \overline{\psi}\gamma_0\psi$ and chiral current $J_5 \equiv \overline{\psi}\gamma_5\gamma_1\psi$ are
identical, $Q = J_5$. The Hamiltonian before the quench $H_0$ is obtained from (8) by setting $\theta = 0$ and $\dot{\theta} = -2\mu_5 = 0$. The full Hamiltonian after the quench is

$$H = H_0 - \mu_5 J + m\bar{\psi}(e^{i\gamma_{5}\theta} - 1)\psi.$$  (9)

The vector current is zero in the ground state before the quench ($t < 0$),

$$\langle J \rangle_0 = 0,$$  (10)

where the expectation value $\langle \ldots \rangle_0$ is taken with respect to the ground state of $H_0$. In the massless case, the vector current is identically zero both before and after the quench,

$$\langle e^{iHt} J e^{-iHt} \rangle_0 = 0.$$  (11)

because $H_0$ and $H$ both commute with $J = -Q_5$. With the fermion mass included, $H$ and $J$ no longer commute,

$$[H, J] = [m \cos \theta \bar{\psi} \psi - im \sin \theta \bar{\psi} \gamma_5 \psi, \bar{\psi} \gamma_1 \psi] = 2m \cos \theta \bar{\psi} \gamma_5 \psi - 2im \sin \theta \bar{\psi} \psi.$$

Hence, the current can take a finite value. Indeed, the current does not vanish and at short times behaves as

$$\langle e^{iHt} J e^{-iHt} \rangle_0 = it \langle [H, J] \rangle_0 + O(t^2)$$

$$= 2tm \left( i \cos \theta \langle \bar{\psi} \gamma_5 \psi \rangle_0 + \sin \theta \langle \bar{\psi} \psi \rangle_0 \right) + O(t^2).$$  (13)

A. Lattice spin Hamiltonian

Let us now set up this problem in the lattice form suitable for a digital quantum simulation. We impose a periodic boundary condition, where the 0th and Nth sites are identified and $N$ is an even integer. The staggered Hamiltonian is [53, 54]

$$H = iw \sum_{n=0}^{N-1} \left[ \chi_n^\dagger \chi_{n+1} + \chi_{n+1}^\dagger \chi_n \right] - \frac{m}{2} \sum_{n=1}^{N-1} \left[ \chi_n^\dagger \chi_{n+1} + \chi_{n+1}^\dagger \chi_n \right]
+ m \cos \theta \sum_{n=0}^{N-1} (-1)^n \chi_n^\dagger \chi_n$$

$$+ \frac{m}{2} \sin \theta \sum_{n=0}^{N-1} (-1)^n \left[ \chi_n^\dagger \chi_{n+1} + \chi_{n+1}^\dagger \chi_n \right],$$  (14)

where $a$ is the lattice spacing and $w = (2a)^{-1}$. For the purpose of quantum simulation, we apply the Jordan-Wigner transformation [55],

$$\chi_n = \frac{X_n - iY_n}{2} \prod_{i=0}^{n-1} iZ_i, \quad \chi_n^\dagger = \frac{X_n + iY_n}{2} \prod_{i=0}^{n-1} (-iZ_i),$$  (15)

which leads to the desired spin Hamiltonian:

$$H = \frac{1}{2} \sum_{n=0}^{N-2} \left( w + \frac{m}{2} (-1)^n \sin \theta \right) [X_n X_{n+1} + Y_n Y_{n+1}]$$

$$- \frac{(-1)^n}{2} \left( w + \frac{m}{2} (-1)^{N-1} \sin \theta \right) [X_{N-1} X_0 + Y_{N-1} Y_0] \prod_{i=1}^{N-2} Z_i$$

$$= \frac{-\theta \dot{w}}{4} \sum_{n=0}^{N-2} [X_n Y_{n+1} - Y_n X_{n+1}]$$

$$+ \frac{(-1)^n \theta w}{4} \left( X_{N-1} Y_0 - Y_{N-1} X_0 \right) \prod_{i=1}^{N-2} Z_i$$
$$+ \frac{m \cos \theta}{2} \sum_{n=0}^{N-1} (-1)^n Z_n.$$  (16)

We are interested in computing the spatial average of vector current, which, in terms of spin operators, is expressed as

$$J = \frac{w}{2N} \sum_{n=0}^{N-2} [X_n Y_{n+1} - Y_n X_{n+1}]$$
$$- \frac{(-1)^n \theta \dot{w}}{2N} \left( X_{N-1} Y_0 - Y_{N-1} X_0 \right) \prod_{i=1}^{N-2} Z_i.$$  (17)

III. SIMULATION

We implement the quantum simulation for the global chiral quench using a IBM Q digital quantum simulator as follows:

1. Prepare the initial state.

We first prepare the ground state of the Hamiltonian (16) at $\theta = 0$ and $\dot{\theta} = 0$ using a python package for exact diagonalization, QuSpin [56].

2. Time evolution with the Hamiltonian at $\theta \neq 0$.

We obtain the evolving state by applying the time-evolution operator,

$$|\psi\rangle = e^{-iHt}|\theta = 0\rangle.$$  (17)

We employ the Suzuki-Trotter decomposition in order to decompose the operator into elementary unitary gates (see Appendix A for details). Upon the

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1 Adiabatic state preparation of the massive Schwinger model at finite $\theta$-angle is studied in detail in [37]
discretization we fix temporal lattice spacing by \( \Delta t = 0.1 \). Here, we consider two protocols for the quench described in the Introduction:

(a) “\( \mu_5 \) quench”:
At \( t = 0 \) we abruptly turn on the chiral chemical potential \( \mu_5 \) and keep it constant during the subsequent evolution.

(b) “\( \theta \) quench”:
At \( t = 0 \) we abruptly change the value of the \( \theta \)-angle, that introduces the pulse in the chiral chemical potential \(-\frac{\theta}{2} \delta(t)\).

3. Evaluation of the vector current.
Using the \(|\psi\rangle\) obtained in the previous step, we evaluate the expectation value of the vector current density \( \langle \psi | J | \psi \rangle \).

IV. RESULTS AND DISCUSSION
Let us begin by discussing the features of our results at short time after the chiral quench:

1. The expectation value of the vector current at short times exhibits an approximately linear dependence on time starting from the second step in time after the quench, as can be seen in Figs 1 and 2.

2. The vector current is also approximately linear in both the chiral chemical potential \( \mu_5 \) and the fermion mass \( m \), as expected from (13) (for the \( \mu_5 \) quench):
\[
\langle e^{iHt}J e^{-iHt}\rangle_0 = 2itm \langle \bar{\psi} \gamma_5 \psi \rangle_0 + \mathcal{O}(t^2).
\]

At larger times, the vector current exhibits a rich dynamics with non-linear dependence on time. Some features of this dynamics can still be understood qualitatively. For the \( \mu_5 \) quench, the current tends to a finite value at late times (see Fig. 1 a), and then, at larger value of the mass, starts to exhibit saturation caused by the relaxation of chirality, see Fig. 2 b. For the \( \theta \) quench, the current after the initial pulse at \( t = 0 \) relaxes back to zero (see Fig. 2), possibly with subsequent oscillations. This relaxation is faster for a large value of the fermion mass, in accord with (1).

V. SUMMARY AND OUTLOOK
We have considered the behavior of a free model of relativistic fermions under the global “chiral quenches” that abruptly change the value of the \( \theta \) angle. It has been observed here that such quenches induce vector currents stemming from the dynamics of chirality relaxation. The resulting real-time dynamics of the vector current appears unexpectedly rich, and is determined by the interplay between the processes of chirality pumping induced by the rotating \( \theta \)-angle and chirality absorption and relaxation.

This pilot study clarifies the effect of explicit breaking of chiral symmetry by fermion mass on the real-time dynamics of the Chiral Magnetic Effect. This is an important problem since in many practical applications the fermions have a finite mass - e.g. the quarks in QCD, or chiral quasiparticles with a finite gap in Dirac semimetals. In addition, the Schwinger model has been used to model the fragmentation of quarks in high-energy collisions [53, 57–62], and the relaxation of chirality is key to the dynamics of this process. Our results show how the fermion mass affects the relaxation to a steady state, and the generation of the vector current.
The operators commute with each other within each set, given that the lattice size $N$ is even. Noting that all operators are time-dependent, we employ the Trotter decomposition for truncating the $U(1)$ gauge field. For example, if $U(1)$ is approximated by $Z_N$, then in general per each site one needs $\log N$ qubits. However in Schwinger model, the absence of propagating gauge field modes allows to use $\log N$ qubits per entire lattice. This should allow to simulate the model with a moderate number of qubits.

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Appendix A: Suzuki-Trotter decomposition

The simplest Suzuki-Trotter decomposition yields the following error:

$$e^{(A+B)\Delta t} = e^{A\Delta t}e^{B\Delta t} + O(\Delta t^2),$$

(A1)

for non-commuting operators $A$ and $B$. This error can be reduced by using

$$e^{(A+B)\Delta t} = e^{A\frac{\Delta t}{2}}e^{B\Delta t}e^{A\frac{\Delta t}{2}} + O(\Delta t^3).$$

(A2)

We employ this improvement to implement the time-evolution operator with the spin Hamiltonian (16). The Hamiltonian consists of five sets of operators,

$$O_Z : \{Z_0,\ldots, Z_{N-1}\},$$
$$O_{XX} : \{X_0X_1, X_1X_2,\ldots, X_{N-2}X_{N-1}, Y_0Z_1Z_2\ldots Z_{N-2}Y_{N-1}\},$$
$$O_{YY} : \{Y_0Y_1, Y_1Y_2,\ldots, Y_{N-2}Y_{N-1}, X_0Z_1Z_2\ldots Z_{N-2}X_{N-1}\},$$
$$O_{XY} : \{X_0Y_1, Y_1X_2,\ldots, Y_{N-3}X_{N-2}, X_{N-2}Y_{N-1}, Y_0Z_1Z_2\ldots Z_{N-2}X_{N-1}\},$$
$$O_{YX} : \{Y_0X_1, X_1Y_2,\ldots, X_{N-3}Y_{N-2}, Y_{N-2}X_{N-1}, X_0Z_1Z_2\ldots Z_{N-2}Y_{N-1}\}.$$
decomposition sketched as follows:

\[
\prod_{s=1}^{n+1} e^{-i(O'_{Xs} + O'_{Ys} + O'_{Xs}^T + O'_{Ys}^T) \Delta t} = e^{i \Omega_{XY}^{n+1} \Delta t} \\
\times \prod_{s=1}^{n} \left( e^{-i(O'_{Xs} + O'_{Ys}) \Delta t} e^{-i O_{XX} \Delta t} e^{-i O_{XY} \Delta t} e^{-i O_{YY} \Delta t} \right) e^{-i \Omega_{XY} \Delta t} + \mathcal{O}(\Delta t^2).
\] (A4)

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