DIS ‘98 STRUCTURE FUNCTIONS SUMMARY, PART II

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Recent results presented in the structure functions working group are briefly summarized for the following topics: The theoretical treatment of heavy quarks in structure functions, higher-order corrections for the leading-twist evolution (including small-\(x\) resummations), the present status of the proton’s parton densities, and the impact of higher twists on determinations of the strong coupling constant. The reader is referred to Part I

1 Heavy quarks in structure functions

The charm structure functions, especially \(F_c^2\), have attracted considerable interest over the past years. Unlike in the fixed-target regime, \(F_c^2\) makes up a sizeable fraction, up to about a quarter, of the total \(F_2\) in the HERA small-\(x\) region. Despite being suppressed, it contributes significantly as well to the scaling violations in the kinematic range covered by the NMC data.

At low scales, \(Q^2 \approx m_c^2\), \(F_c^2\) is uniquely calculated from the light parton densities via the \(\gamma^*g \rightarrow cc\) Bethe-Heitler process and its \(O(\alpha_s^2)\) corrections

\[ C_{2,L} \]

without invoking the concept of a charm parton distribution (we neglect here a possible intrinsic charm component, which seems to be relevant only at high \(x\)). For \(Q^2 \gg m_c^2\) large logarithms appear in the coefficient functions \(C_{2,L}\), which may require a resummation. At \(x < 10^{-2}\) these logarithms dominate \(C_2\) already for \(Q^2 > 20 \text{ GeV}^2\), but \(C_L\) only above \(10^3 \text{ GeV}^2\). Previous leading-order results

\[ \text{ref.} \]

for this resummation have been extended to higher orders in \(\text{ref.} \]

This leads to a high-\(Q^2\) description in terms of four massless flavours, with the charm distribution uniquely specified by the light parton densities.

The problem of the transition between both approaches, i.e., the construction of a variable flavour-number scheme (VFNS) has also been addressed

\[ \text{ref.} \]

There seems to be agreement that a unique construction does not exist, and indeed the prescription of \(\text{ref.} \)

\[ \text{ref.} \]

differ at non-asymptotic values of \(Q^2\). Note, however, that this ambiguity concerns only the coefficient functions, as the parton evolution can be kept strictly massless without any loss of generality.
Results for the VFNS prescription of ref.\textsuperscript{5} are compared to the non-resummed calculation (usually called fixed flavour-number scheme, FFNS) in Fig. 1. The differences are typically 10% at small $x$, i.e., they are of the same size as the factorization-scale dependence of the FFNS calculations\textsuperscript{8}, shown for one value of $Q^2$ in Fig. 1 as well. Hence also the latter approach seems to be applicable at the present level of accuracy in the HERA small-$x$ region. On the other hand, somewhat larger effects are possible for other VFNS prescriptions\textsuperscript{6}.

![Figure 1: Left part: comparison of $F_2^c$ as obtained in NLO from FFNS and VFNS calculations. Right part: the factorization scale dependence of the FFNS predictions at one representative $Q^2$. The light parton densities in both parts are not the same.](image)

Until now the extraction of $F_2^c$ from $D^*$ production at HERA has been performed only in the non-resummed approach (FFNS), as the required exclusive cross sections have been calculated only in this framework so far\textsuperscript{9}. However, first results have been presented from a VFNS event-generator Monte-Carlo program for semi-inclusive heavy-quark production in DIS\textsuperscript{10}.

As is obvious from Fig. 1, the stability of the calculations deteriorates towards larger $x$, i.e., towards the threshold region. This effect is even more pronounced at lower $Q^2$, and renders present calculations in the range of fixed-target experiments rather unreliable. A Sudakov resummation of leading and next-to-leading threshold logarithms, as discussed in ref.\textsuperscript{11}, may lead to a stable framework also in this regime.
2 Higher-order corrections for structure function evolution

We confine ourselves to the unpolarized singlet case, where new results have been obtained during the last year. The corresponding 2-loop (NLO) splitting functions are fully (at all $x$) known for about two decades. On the other hand, at $O(\alpha_s^3)$ only the four lowest even-integer moments were determined so far. A first step towards the full 3-loop anomalous dimensions, which are required to match the accuracy of present and forthcoming high-precision data, has been taken recently by calculating the finite terms of the 2-loop operator matrix elements. Moreover a first partial result has been derived by means of the large-$N_f$ expansion, namely the $(\alpha_s/4\pi)^3N_f^2C_G$ contribution to $P_{gg}$:

$$P^{3\text{-loop}}_{gg}(x,N_f^2C_A) = -\frac{1}{54} \left[ 87\delta(1-x) + (304 + 172x + 208x^2) \ln x - 48(1+x)\ln^2 x + \frac{32}{[1-x]^+} + 192(1+x)[\psi'(1) - \text{Li}_2(x)] 
+ \frac{4(1-x)}{x}(52 + 19x + 52x^2) \ln(1-x) + \frac{4(1-x)}{3x}(236 + 47x + 236x^2) \right].$$

The first moments of this expression agree with the results of ref. 12.

An alternative approach for the small-$x$ region has been to resum the most singular small-$x$ terms of $P_{ij}$ to all orders in $\alpha_s$. For the gluonic splitting functions $P_{gq}$ and $P_{gg}$ these terms read $c_k^{Lx} (1/x) \alpha_s^k \ln(1/x)^{k-1}$, and the coefficients $c_k^{Lx}$ were determined long ago as well. More recently also the leading contributions to $P_{qq}$ and $P_{qg}$ were derived, which contain one power of $\ln(1/x)$ less than their gluonic counterparts. These terms dominate the respective splitting functions at some very low values of $x$, depending on the size of the less singular contributions. Until recently estimates of the impact of such terms, which tends to be enhanced substantially by the ubiquitous Mellin convolution, were only possible by educated guesses based on momentum conservation constraints and the structure of the LO and NLO splitting functions.

An inclusion of the leading small-$x$ logarithms into the analysis can lead to very good fits of all small-$x$ $F_2$ and $F_L^2$ HERA data, as demonstrated in a scheme-independent evolution-equation approach as well as within the framework of $k_L$ factorization. However, both approaches seem to require values for observables and parameters – a rather small $F_L$ in the first case and, more notably, a very small $\alpha_s$ in the second one – which may be interpreted as phenomenological indications of large subleading corrections.

During the past year the calculation of $O(\alpha_s^3)$ corrections to the BFKL kernel has been completed. This result fixes the next-to-leading small-$x$
(NLx) piece of $P_{gg}$. In the DIS scheme the presently known terms read

$$P_{gg}^{\text{DIS}}(N, \alpha_s) = \bar{\alpha}_s P_{gg,0}(N) + \bar{\alpha}_s^2 P_{gg,1}(N) + \sum_{l=3}^{\infty} \left( \frac{\bar{\alpha}_s}{N} \right)^l \left( b_{gg,l} \bar{\alpha}_s + N b_{gg,l}^{\text{NLx}} \right),$$

with $\bar{\alpha}_s = 3\alpha_s/\pi$ and $N$ the usual Mellin variable shifted by one unit. The Lx and NLx coefficients $b_{gg,l}$ are compared in Fig. 2, where also the resulting splitting function is shown for $\alpha_s = 0.2$. The NLx corrections turn out to be exceedingly large in the HERA $x$-region, leading to a grossly negative splitting function already above $x \simeq 10^{-3}$. Thus the $\ln(1/x)$ expansion is inapplicable to $P_{gg}$ at any $x$-values of practical interest, a situation which is by no means a special feature of the DIS scheme as demonstrated in ref.\textsuperscript{21}.

Likewise, taking the NLx corrections at face value, the hard pomeron intercept $\omega_P$ would read $\omega_P(\alpha_s) = 2.65 \alpha_s(1 - 6.36 \alpha_s)$ for $N_f = 4$\textsuperscript{21}, leading to negative values for $Q^2$ as high as about 300 GeV$^2$. Hence a reliable extension of structure-function evolution calculations beyond the usual fixed-order perturbation theory, if possible at all, will require new theoretical concepts.
3 Status of parton density parametrizations

Major updates have been presented by the MRS\textsuperscript{23} and GRV\textsuperscript{24} groups, superseding their respective '96 and '94 parton sets\textsuperscript{25}. The CTEQ collaboration has released a dedicated study of the uncertainty of the gluon density\textsuperscript{26}. The present (central) sets refer to values of $\alpha_s(M_Z^2) = 0.116, 0.1175$ and 0.114 for the CTEQ 4\textsuperscript{27}, MRST 1 and GRV (98) distributions, respectively. Note also that heavy quarks are treated differently in these parametrizations.

The present status of the gluon density is shown in Fig. 3. In the small-$x$ part the preliminary H1 and ZEUS error bands from $F_2$ scaling violations\textsuperscript{28}, as well as the recent H1 results from DIS charm production\textsuperscript{29}, are compared to these parametrizations. The difference between CTEQ4M (‘massless charm’) and CTEQ4F\textsuperscript{30} (‘massive charm’) indicates the impact of the heavy quark treatment. The small-$x$ gluon density seems rather well constrained down to $x \simeq 10^{-4}$. Note, however, that relevant theoretical uncertainties (estimated, e.g., by factorization-scale variations) are not taken into account in Fig. 3.

![Figure 3: Present small-$x$ and large-$x$ constraints on the proton’s gluon distribution.](image)

Large uncertainties on $xg$ still persist in the large-$x$ region, $x \gtrsim 0.2$. Here the classic constraint has been prompt-photon production in $pp$ collisions\textsuperscript{31}. However, extractions of the gluon density from these data suffer from sizeable scale uncertainties as shown by the gray error band\textsuperscript{32}. In addition, there is the
possibility of a sizeable gluon $k_T$, as recently indicated by results from E706. In fact, the MRST large-$x$ error band on $xg$ stems from varying $k_T$ between 0 (upper curve, set 2) and 640 MeV (lowest curve, set 3) in the fit to the WA70 data. In view of these problems the CTEQ gluon-uncertainty analysis derives its error band from DIS and Drell-Yan data alone. It is interesting to note that all three bands are similarly wide for $x \gtrsim 0.3$. By propagation to high scales, benchmark uncertainties have been derived for gluon-gluon and gluon-quark luminosities at the Tevatron and the LHC.

As the total quark density is quite well constrained by $F_2$ data for $10^{-4} \lesssim x \lesssim 0.7$, the other critical issues are the flavour decomposition and the $x \to 1$ behaviour. In both areas new results have been reported. E866 has published their high-mass data on the $pp/pd$ Drell-Yan asymmetry. As shown in Fig. 4, these results strongly constrain the ratio $\bar{d}/\bar{u}$, especially in the range $0.1 \lesssim x \lesssim 0.3$. Preliminary results on this ratio, inferred from semi-inclusive DIS, as well as data on $F_n^u/F_p^u$ have also been presented by HERMES.

![Figure 4: The E866 Drell-Yan asymmetry data and their impact on the light-quark sea.](image)

$F_n^u/F_p^u$ is a dominant source of information on the $d/u$ ratio. As $F_n^u$ is inferred from deuteron measurements, possible nuclear binding effects enter here. Such effects have been neglected in recent parton parametrization. They can, however, lead to drastic modifications of the valence-quark ratio $d_v/u_v$ at large $x$, as quantified in ref. The region of very large $x$, $x \gtrsim 0.8$, is usually not taken into account either. The DIS and resonance-region data of SLAC have been employed to study the absolute $x \to 1$ behaviour of the valence quarks. A significant flat contribution, as previously discussed in connection with the HERA high $Q^2$ events, is found to be strongly disfavoured.
4 Higher-twist effects and the strong coupling constant

Present-day determinations of $\alpha_s$ from DIS structure functions involve relatively low values of $Q^2$. Hence higher-twist corrections can have appreciable effects. A new result on $\alpha_s(M_2^2)$ from the Gross-LLewellyn-Smith (GLS) sum rule has been reported by CCFR. Their iron data, together with results from other neutrino experiments, in the region $1.3 \leq Q^2/\text{GeV}^2 \leq 5.0$ lead to

$$\alpha_s^{\text{NNLO}}(M_2^2)_{\text{GLS}} = 0.114 \pm 0.009 \text{ (exp.)} \pm 0.05 \text{ (th.)}.$$  

The theoretical error is dominated by the uncertainty of the higher-twist contribution. Nuclear $1/Q^2$ corrections have been studied for this case in ref. 40. Very small corrections to the GLS sum rule are found, but sizeable effects for the incomplete GLS integral, $S_{\text{GLS}}(x \neq 0, Q^2)$, and for the iron-to-nucleon ratio $R_3(x, Q^2) = F_3^\text{Fe}/F_3^N$, see Fig. 5. The latter results may be relevant for $\alpha_s$ determinations from scaling violations in neutrino-nucleus DIS.

New fits to the electromagnetic DIS data of BCDMS, SLAC and NMC at $x > 0.3$ have been presented as well, including target-mass corrections and simple approximations for dynamical higher-twist terms. If the shape of the latter is allowed to vary freely, the fits result in

$$\alpha_s^{\text{NLO}}(M_2^2)_{\partial Q F_2} = 0.114 \pm 0.002 \text{ (exp.)}.$$  

Not surprisingly, this result agrees with previous analyses. On the other hand, a very low value of $\alpha_s(M_2^2) = 0.103 \pm 0.002 \text{ (exp.)}$ is obtained if the shapes of the twist-4 and twist-6 terms are adopted from the renormalon model. Note that this finding seems to be at variance with the analysis of ref. 43, where good agreement between the large-$x$ data and the renormalon approach has been found for a fixed value of $\alpha_s(M_2^2) = 0.120$. 

![Figure 5](image-url)
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