Addendum: Ultrahigh-energy cosmic-ray bounds on nonbirefringent modified-Maxwell theory

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Abstract

Nonbirefringent modified-Maxwell theory, coupled to standard Dirac particles, involves nine dimensionless parameters, which can be bounded by the inferred absence of vacuum Cherenkov radiation for ultrahigh-energy cosmic rays (UHECRs). With selected UHECR events, two-sided bounds on the eight nonisotropic parameters are obtained at the $10^{-18}$ level, together with an improved one-sided bound on the single isotropic parameter at the $10^{-19}$ level.

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In Ref. [1], ultrahigh-energy-cosmic-ray (UHECR) bounds have been given for the nine Lorentz-violating “deformation parameters” of nonbirefringent modified-Maxwell theory coupled to standard Dirac particles, where the parameters were restricted to a particular domain. In this addendum, we obtain corresponding results for two sets of nonisotropic parameters outside this domain (the two sets are, respectively, parity-odd and parity-even). These new bounds are essentially two-sided, whereas an improved bound on the single isotropic parameter remains one-sided. For convenience, the final bounds will be presented in terms of the widely-used standard-model-extension (SME) parameters [2, 3].

The ‘Note Added in Proof’ of Ref. [1] used 29 UHECR events [4, 5, 6] which, for completeness, are listed in Table I. From the energies and flight directions of these 29 UHECR events, the following two-σ bound was obtained on the quadratic sum of the nine nonbirefringent

| Year | Day | E [EeV] | RA [deg] | DEC [deg] | Year | Day | E [EeV] | RA [deg] | DEC [deg] |
|------|-----|--------|---------|----------|------|-----|--------|---------|----------|
| 1991 | 288 | 320    | 85.2    | 48.0     | 2006| 81  | 79     | 201.1   | −55.3    |
| 1993 | 337 | 210    | 18.9    | 21.1     | 2006| 185 | 83     | 350.0   | 9.6      |
| 2004 | 125 | 70     | 267.1   | −11.4    | 2006| 296 | 69     | 52.8    | −4.5     |
| 2004 | 142 | 84     | 199.7   | −34.9    | 2006| 299 | 69     | 200.9   | −45.3    |
| 2004 | 282 | 66     | 208.0   | −60.3    | 2007| 13  | 148    | 192.7   | −21.0    |
| 2004 | 339 | 83     | 268.5   | −61.0    | 2007| 51  | 58     | 331.7   | 2.9      |
| 2004 | 343 | 63     | 224.5   | −44.2    | 2007| 69  | 70     | 200.2   | −43.4    |
| 2005 | 54  | 84     | 17.4    | −37.9    | 2007| 84  | 64     | 143.2   | −18.3    |
| 2005 | 63  | 71     | 331.2   | −1.2     | 2007| 145 | 78     | 47.7    | −12.8    |
| 2005 | 81  | 58     | 199.1   | −48.6    | 2007| 186 | 64     | 219.3   | −53.8    |
| 2005 | 295 | 57     | 332.9   | −38.2    | 2007| 193 | 90     | 325.5   | −33.5    |
| 2005 | 306 | 59     | 315.3   | −0.3     | 2007| 221 | 71     | 212.7   | −3.3     |
| 2005 | 306 | 84     | 114.6   | −43.1    | 2007| 234 | 80     | 185.4   | −27.9    |
| 2006 | 35  | 85     | 53.6    | −7.8     | 2007| 235 | 69     | 105.9   | −22.9    |
| 2006 | 55  | 59     | 267.7   | −60.7    |
Lorentz-violating parameters $\alpha^l$ [1]:

$$\vec{\alpha} \in D^{(\text{open})}_{\text{causal}} : |\vec{\alpha}|^2 \equiv \sum_{l=0}^{8} (\alpha^l)^2 < A^2,$$

$$A = 3 \times 10^{-18} \left( \frac{M_{\text{prim}}}{56 \text{ GeV}/c^2} \right)^2,$$

showing explicitly the dependence on the mass of the primary charged particle (taken equal for all events). There are indications [4] that these UHECRs originate predominantly from protons but, in order to be on the safe side, we will later take the mass $M_{\text{prim}}$ to be equal to that of iron, $M_{\text{prim}} = 56 \text{ GeV}/c^2$. Bound (1) as well as all other bounds in this addendum are based on the Cherenkov threshold condition (10) in Sec. II B of Ref. [1] and the reader is referred to this section, in particular, for further details.

The domain used in (1a) is defined by

$$D^{(\text{open})}_{\text{causal}} \equiv \{ \vec{\alpha} \in \mathbb{R}^9 : \forall \vec{x} \in \mathbb{R}^3 (\alpha^0 + \alpha^j \hat{x}^j + \tilde{\alpha}^{jk} \hat{x}^j \hat{x}^k) > 0 \},$$

where $\hat{x} \equiv \vec{x}/|\vec{x}|$ denotes a unit vector in Euclidean three-space and the traceless symmetric $3 \times 3$ matrix $\tilde{\alpha}^{jk}$ is defined in terms of the parameters $\alpha^l$ for $l = 4, \ldots, 8$ (see below). The parameter domain (2) allows for vacuum Cherenkov radiation in all directions and, with boundaries added, is believed to constitute a significant part of the physical domain of the theory, where, e.g., unitarity and microcausality hold; cf. Appendix C of Ref. [7]. It may, nevertheless, be of interest to get bounds outside this domain, because modified-Maxwell theory could be only part of the full Lorentz-noninvariant theory.

The crucial observation is that domain (2) shrinks to zero size in the hyperplane $\alpha^0 = \tilde{\alpha}^{jk} = 0$, so that bound (1a) becomes ineffective there. Still, the data from Table II can be used to get the following two–$\sigma$ bound on the three parity-odd nonisotropic parameters in this hyperplane

$$\alpha^0 = \alpha^4 = \alpha^5 = \alpha^6 = \alpha^7 = \alpha^8 = 0 : \sum_{j=1}^{3} (\alpha^j)^2 < \left( 4 \times 10^{-18} \right)^2 \left( \frac{M_{\text{prim}}}{56 \text{ GeV}/c^2} \right)^4 .$$

Similarly, there is a two–$\sigma$ bound on the five parity-even nonisotropic parameters in an orthogonal hyperplane

$$\alpha^0 = \alpha^1 = \alpha^2 = \alpha^3 = 0 : \sum_{l=4}^{8} (\alpha^l)^2 < \left( 4 \times 10^{-18} \right)^2 \left( \frac{M_{\text{prim}}}{56 \text{ GeV}/c^2} \right)^4 .$$

It is, in principle, possible to get other bounds for the eight nonisotropic parameters, but, for the moment, bounds (3) and (4) suffice.

If only a single parameter $\alpha^l$ for $l \in \{1, \ldots, 8\}$ is considered (all seven other nonisotropic parameters and the isotropic parameter $\alpha^0$ being zero), bounds (3) and (4) give a two-sided
bound on that single isolated parameter. Setting $M_{\text{prim}} = 56 \text{ GeV}/c^2$ and showing explicitly the approximate one-$\sigma$ error, these bounds are

$$l \in \{1, \ldots, 8\} : \quad |\alpha| < (2 \pm 1) \times 10^{-18},$$

(5)

for $\alpha^0 = \alpha^m = 0$ with $m \in \{1, \ldots, 8\}$ and $m \neq l$. Incidentally, the possibility of getting certain two-sided Cherenkov bounds from an isotropic set of UHECR events has already been noted in Appendix C of Ref. [7].

For completeness, we also give the following one-sided bound on the single $\alpha^0$ parameter

$$\alpha^0 < (1.4 \pm 0.7) \times 10^{-19},$$

(6)

for $\alpha^m = 0$ with $m \in \{1, \ldots, 8\}$. Bound (6) has been derived by setting $M_{\text{prim}} = 56 \text{ GeV}/c^2$ and using the 148 EeV Auger event from Table II which has a reliable energy calibration [4]. For the Fly’s Eye event with an estimated energy of 320 EeV [5], bound (6) would be reduced by a factor of approximately 5 according to Eq. (10) in Ref. [1].

In order to facilitate the comparison with existing laboratory bounds and future ones, we provide a dictionary between our $\alpha^l$ (or $\tilde{\alpha}^{\mu\nu}$) parameters and the nonbirefringent SME parameters defined by Eq. (11) in Ref. [3]:

$$\tilde{\alpha} \equiv \begin{pmatrix} \alpha^0 \\ \alpha^1 \\ \alpha^2 \\ \alpha^3 \\ \alpha^4 \\ \alpha^5 \\ \alpha^6 \\ \alpha^7 \\ \alpha^8 \end{pmatrix} \equiv \begin{pmatrix} \tilde{\alpha}^{00} \\ \tilde{\alpha}^{01} \\ \tilde{\alpha}^{02} \\ \tilde{\alpha}^{03} \\ \tilde{\alpha}^{11} \\ \tilde{\alpha}^{12} \\ \tilde{\alpha}^{13} \\ \tilde{\alpha}^{22} \\ \tilde{\alpha}^{23} \end{pmatrix} = \begin{pmatrix} 2 \tilde{\kappa}_{\text{tr}} \\ -2 (\tilde{\kappa}_{\alpha+})^{(23)} \\ -2 (\tilde{\kappa}_{\alpha+})^{(31)} \\ -2 (\tilde{\kappa}_{\alpha+})^{(12)} \\ - (\tilde{\kappa}_{e-})^{(11)} \\ - (\tilde{\kappa}_{e-})^{(12)} \\ - (\tilde{\kappa}_{e-})^{(13)} \\ - (\tilde{\kappa}_{e-})^{(22)} \\ - (\tilde{\kappa}_{e-})^{(23)} \end{pmatrix}.$$  

(7)

The Cartesian coordinates employed (cf. Sec. III A of Ref. [3]) are such that the flight-direction vector $\hat{q}$ of an UHECR primary at the top of the Earth atmosphere is given by

$$\begin{pmatrix} \hat{q}_1 \\ \hat{q}_2 \\ \hat{q}_3 \end{pmatrix} = - \begin{pmatrix} \sin(\pi/2 - \delta) \cos \alpha \\ \sin(\pi/2 - \delta) \sin \alpha \\ \cos(\pi/2 - \delta) \end{pmatrix},$$

(8)

in terms of the celestial coordinates RA $\equiv \alpha$ and DEC $\equiv \delta$ from Table I.

Using the dictionary (7), bounds (5) and (6) give the following two-$\sigma$ (95% CL) bounds
on the nine isolated SME parameters of nonbirefringent modified-Maxwell theory:

\[
\left| \left( \tilde{\kappa}_{o+} \right)_{(ij) = (23),(31),(12)} \right| < 2 \times 10^{-18}, \tag{9a}
\]

\[
\left| \left( \tilde{\kappa}_{e-} \right)_{(kl) = (11),(12),(13),(22),(23)} \right| < 4 \times 10^{-18}, \tag{9b}
\]

\[
\tilde{\kappa}_{tr} < 1.4 \times 10^{-19}, \tag{9c}
\]

for the Sun-centered Cartesian coordinate system employed in (8). The Cherenkov bounds (9a), (9b), and (9c) are significantly stronger than the current laboratory bounds at the $10^{-12}$, $10^{-16}$, and $10^{-7}$ levels, respectively; see the third paragraph of Sec. V in Ref. [1] for further discussion and references. It is to be emphasized that these Cherenkov bounds only depend on the measured energies and flight directions of the charged cosmic-ray primaries at the top of the Earth atmosphere.

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[1] F.R. Klinkhamer and M. Risse, “Ultrahigh-energy cosmic-ray bounds on nonbirefringent modified-Maxwell theory,” Phys. Rev. D 77, 016002 (2008), arXiv:0709.2502 [hep-ph].
[2] D. Colladay and V.A. Kostelecky, “Lorentz-violating extension of the standard model,” Phys. Rev. D 58, 116002 (1998), arXiv:hep-ph/9809521.
[3] V.A. Kostelecký and M. Mewes, “Signals for Lorentz violation in electrodynamics,” Phys. Rev. D 66, 056005 (2002), arXiv:hep-ph/0205211.
[4] J. Abraham et al. [Pierre Auger Collaboration], “Correlation of the highest-energy cosmic rays with the positions of nearby active galactic nuclei,” Astropart. Phys. 29, 188 (2008), arXiv:0712.2843v1 [astro-ph].
[5] D.J. Bird et al., “Detection of a cosmic ray with measured energy well beyond the expected spectral cutoff due to cosmic microwave radiation,” Astrophys. J. 441, 144 (1995), arXiv:astro-ph/9410067.
[6] N. Hayashida et al., “Observation of a very energetic cosmic ray well beyond the predicted 2.7–K cutoff in the primary energy spectrum,” Phys. Rev. Lett. 73, 3491 (1994).
[7] C. Kaufhold and F.R. Klinkhamer, “Vacuum Cherenkov radiation in spacelike Maxwell–Chern–Simons theory,” Phys. Rev. D 76, 025024 (2007), arXiv:0704.3255 [hep-th].