Adaptive Tracking Control for a Class of Nonlinear MIMO Systems: A Quasi-Fast Finite-Time Design Technique

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ABSTRACT In this paper, we investigate the problem of adaptive finite-time tracking control for a class of nonlinear multi-input and multi-output (MIMO) systems with a state coupling relationship between subsystems, which is called the structure of nonstrict-feedback. Based on the quasi-fast finite-time design technique, an adaptive controller is obtained for the considered MIMO systems. The advantages of the presented control scheme lie in that the “explosion of complexity” problem in the backstepping process is solved by utilizing the command filter technique, and the design difficulties caused by nonstrict-feedback structure are also overcome. It is shown that all the signals in the closed-loop system are bounded and the convergence performance with the bounded tracking errors is guaranteed in a finite time. Finally, a practical example is given to show the effectiveness of the proposed method.

INDEX TERMS Adaptive control, backstepping, finite-time control, nonlinear multi-input and multi-output systems.

I. INTRODUCTION

As is known to all, the research on adaptive control of nonlinear systems has gained great popularity in the last few decades. Particularly, the research on single-input and single-output (SISO) nonlinear systems has made a host of achievements such as the results in [1]–[7]. Nevertheless, many controlled plants in practice are MIMO nonlinear systems. Due to the state coupling between different subsystems, as well as the uncertainties and external disturbances of the system itself, control of a multi-variable system becomes quite difficult. Therefore, adaptive control of MIMO nonlinear systems has always been a difficult problem in the field of control although some significant results have been published [8], [9]. For example, the work [8] studied the design of adaptive robust controllers for the systems that can be converted into two types of MIMO systems in semi-strict-feedback form. The work [9] presented an event-triggered-based adaptive output feedback tracking control scheme for a class of MIMO nonaffine nonlinear systems.

It needs to be emphasized that the traditional control methods in [10], [11] are difficult to be used to solve the control problem of complex systems with high uncertainties and completely unknown nonlinearities, then intelligent control techniques such as fuzzy logic systems (FLSs) and neural networks (NNs) provide new means for solving the problem [12]–[20]. The reason is that FLSs and NNs have universal approximation capability so that they can be used to model unknown nonlinear functions [1], [21]–[27]. The work in [21] solved the issue of controller design for a class of MIMO nonlinear systems with event-triggered inputs by combining Lyapunov function with backstepping design technique and the system predictability is guaranteed. The adaptive fuzzy output-feedback controller was presented for a class of feedback linearizable nonlinear systems with immeasurable states in [22], in which the unknown functions are approximated by FLSs.

Nevertheless, the computation complexity of the above control schemes is too big to obtain the partial derivatives.
of the virtual controls in the frame of the backstepping procedure. In order to solve the above computational complexity problem, the concept of the command filter technique in [28]–[34] was proposed. By using the command filter method to calculate the derivatives of the virtual controllers, the computational complexity of controller design procedure can be reduced. For instance, the work [28] presented a robust adaptive command filter method to deal with the tracking control issue for a class of MIMO nonlinear systems. The NN-based adaptive control design scheme was employed in [30] for a class of pure-feedback nonlinear systems with uncertainties by using the command filter method. Nonetheless, the above control schemes are stable when the time approaches infinity. However, in order to reduce the energy consumption in practical systems, it is necessary to make the system stable in a limited time.

Since the middle of the 20th century, the control theory has got unprecedented development and a majority of excellent control methods have been presented, including pole assignment, state feedback and observer, optimal control, fuzzy control and adaptive control, etc. In general, most of the existing design methods, such as the method based on the Lyapunov stability theory, can only achieve the asymptotic stability, namely the states of the system converge to the origin when the time goes to infinity. However, in many practical applications, we want to achieve the control effect in a finite time. Based on the definition of finite-time stability was first proposed in the 1960s [35], [36], abundant research and application results have been obtained. Since then, the study of finite-time control for nonlinear systems has attracted a large number of scholars and has been continuously developed and improved [37]–[44]. In fact, finite-time control for MIMO nonlinear systems is rarely reported to the best of our knowledge. The possible reason is that the complexity of MIMO systems leads to the extreme lack of control methods.

Inspired by the aforementioned results and discussions, this paper further aims at the problem of adaptive finite-time tracking control for a class of MIMO nonlinear systems by means of using the quasi-fast finite-time practical stability (QFPS) criterion. In the control scheme, we design an adaptive finite-time controller which guarantees that the closed-loop system is stable and the tracking errors converge to a small neighborhood of the origin in a finite time. The main innovation points of this paper are presented in detail from the following aspects: 1) In order to reduce the unnecessary computation in the calculation process appropriately and effectively, we solve the problem of “explosion of complexity” via the command filter technique in the backstepping procedure. 2) Relative to the existing works [45], [46], this paper presents a QFPS criterion for nonlinear MIMO systems, which makes the calculated setting time more accurate than the ones in [45] and [46]. 3) The variable separation technique is used in the previous literatures [47], [48] to deal with the nonstrict-feedback form, but this technique needs to impose strict assumptions on the system functions and can also increase the computational complexity. Different from the variable separation technique used in [47], [48], the structural characteristics of NNs are used to dispose the problem such that the computational complexity is reduced significantly.

II. PROBLEM STATEMENT

A class of nonstrict-feedback nonlinear MIMO systems is described as follows:

\[
\begin{align*}
\dot{x}_{p,j} &= x_{p,j+1} + f_{p,j}(\bar{x}_{p,j}) + \omega_{p,j}, \\
\dot{x}_{p,m_p} &= u_p + f_{p,m_p}(\bar{x}_{p,m_p}) + \alpha_{p,m_p}, \\
y_p &= x_{p,1},
\end{align*}
\]  

where \( j = 1, 2, \ldots, m_p - 1 \), \( p = 1, 2, \ldots, M \). \( x_p = [x_{p,1}, x_{p,2}, \ldots, x_{p,m_p}]^T \in \mathbb{R}^{m_p}, u_p \in \mathbb{R} \) and \( y_p \in \mathbb{R} \) are the state vectors of the \( p \)th subsystem of system (1), control input and control output, respectively. \( \bar{x}_{p,j} \) and \( \alpha_{p,j} \) are the unknown nonlinear functions and \( \omega_{p,j} \) are the external disturbances.

The control objective of this work is to construct an adaptive quasi-fast finite-time tracking project for system (1) via the command filter technique, which can guarantee that tracking errors can converge to the neighborhood of the origin in a finite time and all signals in the closed-loop system are bounded.

**Assumption 1 ([49]):** The target signal \( y_{r,p}(t) \) and it’s time derivatives \( \dot{y}_{r,p}(t) \) are continuous and bounded satisfying \( |y_{r,p}(t)| \leq \bar{y}_{r,p} \) and \( |\dot{y}_{r,p}(t)| \leq \ddot{y}_{r,p} \), where \( \bar{y}_{r,p} > 0 \) and \( \ddot{y}_{r,p} > 0 \) are constants.

**Assumption 2 ([46]):** The external disturbances are bounded as \( |\omega_{i,j}| \leq \tilde{\omega}_{i,j} \), where \( \tilde{\omega}_{i,j} > 0, j = 1, \ldots, m_i \), are unknown constants.

**Lemma 1 ([52]):** For some constants \( m, n, q, \alpha \), if \( m, n, q > 0 \) and \( 0 < \alpha < 1 \), holds such that

\[
\dot{V} \leq -mV(x) - nV^q(x) + q,
\]

then the trajectory of system \( \dot{x} = f(x) \) is QFPS.

**Lemma 2 ([45]):** For any given positive constants \( r_1, r_2 \) and \( r_3 \), it holds that

\[
|m|^r \leq \frac{r_1}{r_1 + r_2}m^{r_1} + \frac{r_2}{r_1 + r_2}m^{r_2} \leq \frac{r_1}{r_1 + r_2}m^{r_1} + \frac{r_2}{r_1 + r_2}m^{r_2}.
\]

**Lemma 3 ([45]):** For \( x_p \in \mathbb{R}, p = 1, \ldots, M, 0 \leq c \leq 1 \), the following inequality is estimated

\[
\sum_{p=1}^{M} |x_p|^c \leq \sum_{p=1}^{M} |x_p|^{c - 1} \leq \sum_{p=1}^{M} |x_p|^c.
\]

**Lemma 4 ([48]):** Choose the basis function vector of an RBF NN \( S(\bar{x}_k) = [S_1(\bar{x}_k), \ldots, S_L(\bar{x}_k)]^T \). For \( k \leq j \), the following inequality holds:

\[
||S(\bar{x}_j)||^2 \leq ||S(\bar{x}_k)||^2.
\]
Lemma 5 ([11]): On a compact set $\Omega_Z \subseteq \mathbb{R}^q$, the RBF NNs can approximate unknown function $f_p(Z)$ such that

$$f_p(Z) = W^*_p S_p(Z) + \delta_p(Z), \forall Z \in \Omega_Z \subseteq \mathbb{R}^q,$$

where $W^*_p$ is the weight vector and is expressed as follows

$$W^*_p = \arg \min_{W_p \in \mathbb{R}^q} \left\{ \sup_{Z \in \Omega_Z} \left| f_p(Z) - W^*_p S_p(Z) \right| \right\},$$

where $\delta_p(Z)$ and $l$ represent the approximation error and the number of nodes in the NNs, respectively. Furthermore, there exists a constant $\varepsilon_p > 0$ that $|\delta_p(Z)| < \varepsilon_p$. $S(Z) = [S_1(Z), S_2(Z), \ldots, S_p(Z)]$ represents the basis function vector and is expressed as follow

$$S_p(Z) = \exp \left( -\frac{(Z - \mu_0)^T (Z - \mu_0)}{p_0^2} \right), \ \forall \mu_0 = 1, 2, \ldots, l,$$

where $\mu_0$ on behalf of the center of Gussian function and $p_0$ stands for the width of Gussian function.

III. MAIN RESULT

A. CONTROLLER DESIGN

Firstly, the coordinate transformation is introduced as follow

$$\begin{align*}
\hat{z}_{p,1} &= x_{p,1} - y_{r,1}, \\
\hat{z}_{p,j} &= x_{p,j} - \hat{v}_{p,j},
\end{align*}$$

where $j = 2, \ldots, m_p$, $z_{p,j}$ are the tracking errors, $y_{r,1}$ is the reference signal, $\hat{v}_{p,j}$ are the output of the first-order command filter for the virtual controllers $v_{p,j}$, which are define as

$$\tilde{v}_{p,j} + \chi_{p,j} \hat{v}_{p,j} = v_{p,j} - \hat{v}_{p,j}(0) = v_{p,j}(0), \quad j = 2, \ldots, m_p,$$

where $X_{p,i}$ are the positive constants. Then, we define a constant as follows

$$\theta_p = \max_{j} \left\{ \frac{\|W_{p,j}\|^2 + a^2_{p,1}}{2} \right\},$$

where $j = 1, 2, \ldots, m_p$, $\hat{\theta}_p$ is the estimate of $\theta_p$ and the estimate error is $\tilde{\theta}_p = \theta_p - \hat{\theta}_p$.

The backstepping design process is described below.

Step p, 1: Combining (1) and (8), we can get

$$\begin{align*}
\dot{z}_{p,1} &= x_{p,2} + f_{p,1}(\hat{x}_{p,1}) + \omega_{p,1} - \dot{y}_{r,1} \\
&= z_{p,2} + \tilde{v}_{p,2} + v_{p,2} - v_{p,2} \\
&\quad + f_{p,1}(\hat{x}_{p,1}) + \omega_{p,1} - \dot{y}_{r,1}.
\end{align*}$$

To deal with the influence of the $(\tilde{v}_{p,2} - v_{p,2})$, we give the following definition of $\varphi_{p,1}$ :

$$\dot{\varphi}_{p,1} = -c_{p,1} \varphi_{p,1} - v_{p,2} + \tilde{v}_{p,2} + \varphi_{p,2} - s_{p,1} \text{sign}(\varphi_{p,1}),$$

where $\varphi_{p,1}(0) = 0$, $c_{p,1} > 0$ and $s_{p,1} > 0$ are known constants.

Define the error as

$$\eta_{p,1} = z_{p,1} - \varphi_{p,1}.$$  

By combining the equations (11), (12) and (13), one has

$$\begin{align*}
\dot{\eta}_{p,1} &= z_{p,2} - v_{p,2} + \tilde{v}_{p,2} + v_{p,2} + f_{p,1}(\hat{x}_{p,1}) + \omega_{p,1} - \varphi_{p,2} \\
&\quad - \dot{y}_{r,1} + c_{p,1} \varphi_{p,1} - \tilde{v}_{p,2} + v_{p,2} + s_{p,1} \text{sign}(\varphi_{p,1}) \\
&\leq \eta_{p,2} + v_{p,2} + \tilde{v}_{p,1}(\hat{x}_{p,1}) + \omega_{p,1} - \dot{y}_{r,1} \\
&\quad + c_{p,1} \varphi_{p,1} + s_{p,1} \text{sign}(\varphi_{p,1}).
\end{align*}$$

Consider the Lyapunov function as

$$V_{p,1} = \frac{1}{2} \eta_{p,1}^2 + \frac{1}{2 \lambda_p} \hat{\theta}_p^2,$$

where $\lambda_p > 0$ is a known constant. The derivative of $V_{p,1}$ along with the equations (14) and (15) is

$$\dot{V}_{p,1} = \eta_{p,1}(f_{p,1}(\tilde{x}_{p,1}) + \eta_{p,2} + v_{p,2} + \omega_{p,1} - \dot{y}_{r,1} + c_{p,1} \varphi_{p,1} + s_{p,1} \text{sign}(\varphi_{p,1})) + \frac{1}{2 \lambda_p} \hat{\theta}_p^2.$$

Applying Young’s inequality, one has

$$\begin{align*}
\eta_{p,1} s_{p,1} \text{sign}(\varphi_{p,1}) &\leq \frac{1}{2} \eta_{p,1}^2 + \frac{1}{2 \lambda_p} \hat{\theta}_p^2, \\
\eta_{p,1} \omega_{p,1} &\leq \frac{1}{2} \eta_{p,1}^2 + \frac{1}{2 \lambda_p} \hat{\theta}_p^2.
\end{align*}$$

Then, using Lemma 4 - 5, we have

$$\begin{align*}
\eta_{p,1} f_{p,1}(\tilde{x}_{p,1}) &= \eta_{p,1} \left[ W^*_p S_p(\hat{x}_{p,1}) + \delta(\hat{x}_{p,1}) \right] \\
&\leq \left| \eta_{p,1} \right| \left\{ \left\| W^*_p \right\| \left\| S_p(\hat{x}_{p,1}) \right\| + e_{p,1} \right\} \\
&\leq \frac{\theta_p \eta_{p,1}}{2 \lambda_p} \left\| S_p(X_{p,1}) \right\| S_p(X_{p,1}) \\
&\quad + \frac{2 \lambda_p^2}{\lambda_p} \left\| S_{p,1}(X_{p,1}) S_p(X_{p,1}) X_{p,1} \right\|,$$

where $X_{p,1} = \left[ \tilde{x}_{p,1} \right]^T$, $X_{p,1} = [x_{p,1}]^T$.

Now, we can design the virtual controller as

$$\begin{align*}
v_{p,2} &= -c_{p,1} \tilde{v}_{p,1} - l_{p,1} \eta_{p,1}^2 + \dot{y}_{r,1} - \frac{\hat{\theta}_p}{2 \lambda_p} \eta_{p,1} S_{p,1}(X_{p,1}) S_{p,1}(X_{p,1}).
\end{align*}$$

where $l_{p,1} > 0$ is a known constant, $g$ is a positive constant with $0 < g < 1$.

Substituting equations (17) - (20) into equation (16), we get

$$\begin{align*}
\dot{V}_{p,1} &\leq \eta_{p,1} \eta_{p,2} - (c_{p,1} - 1) \eta_{p,1}^2 + l_{p,1} \eta_{p,1}^2 \\
&\quad + \frac{1}{\lambda_p} \hat{\theta}_p (\frac{2 \lambda_p}{\lambda_p} S_p(X_{p,1}) S_{p,1}(X_{p,1}) \eta_{p,1}^2 - \dot{\theta}_p) \\
&\quad + \frac{a^2_{p,1} + e^2_{p,1} + s^2_{p,1} + \omega^2_{p,1}}{2}.
\end{align*}$$

Step p, q: $(q = 2, \ldots, m_p - 1)$: The time derivative of $\dot{z}_{p,q}$ by using equation (8) is obtained as

$$\begin{align*}
\dot{z}_{p,q} &= \dot{z}_{p,q+1} + \tilde{v}_{p,q+1} - v_{p,q+1} + v_{p,q+1} \\
&\quad + f_{p,q}(\tilde{x}_{p,q}) + \omega_{p,q} - \tilde{v}_{p,q}.
\end{align*}$$
We give the definition of \( \varphi_{p,q} \)

\[
\dot{\varphi}_{p,q} = -c_{p,q}\varphi_{p,q} + \hat{v}_{p,q} - v_{p,q-1} - \varphi_{p,q-1}
+ \varphi_{p,q+1} - s_{p,q}\text{sign}(\varphi_{p,q}), \quad \varphi_{p,q}(0) = 0,
\]

where \( c_{p,q} > 0 \) and \( s_{p,q} > 0 \) are known constants. Define the error as

\[
\eta_{p,q} = z_{p,q} - \varphi_{p,q}.
\]

The time differentiation of \( \eta_{p,q} \) yields

\[
\dot{\eta}_{p,q} \leq \eta_{p,q+1} + v_{p,q+1} + \varphi_{p,q-1} + f_{p,k}(\hat{x}_{p,q}) + \omega_{p,q}
- \hat{v}_{p,q} + c_{p,q}\eta_{p,q} + s_{p,q}\text{sign}(\varphi_{p,q}).
\]

Choose the Lyapunov function as

\[
V_{p,q} = V_{p,q-1} + \frac{1}{2}\eta_{p,q}^2.
\]

The time differentiation of \( V_{p,q} \) gives

\[
\dot{V}_{p,q} = \dot{V}_{p,q-1} + \eta_{p,q}\dot{\eta}_{p,q} + \eta_{p,q+1} + \varphi_{p,q-1}
+ f_{p,q}(\hat{x}_{p,q}) + \omega_{p,q} + \hat{v}_{p,q} - \hat{v}_{p,q} + c_{p,q}\eta_{p,q}
\]

Applying Young’s inequality, we have

\[
\eta_{p,q}\delta_{p,q}\text{sign}(\varphi_{p,q}) \leq \frac{1}{2}\eta_{p,q}^2 + \frac{1}{2}\delta_{p,q}^2, \quad \eta_{p,q}\omega_{p,q} \leq \frac{1}{2}\eta_{p,q}^2 + \frac{1}{2}\omega_{p,q}^2.
\]

Then, using Lemma 4 - 5 and equation (19), we have

\[
\eta_{p,q}\delta_{p,q}\text{sign}(\varphi_{p,q}) \leq \frac{1}{2}\eta_{p,q}^2 + \frac{1}{2}\delta_{p,q}^2,
\]

where \( \hat{x}_{p,q} = [\hat{x}_{p,q}^T, X_{p,q} = [x_{p,1}, \ldots, x_{p,q}]^T \).

Then, the virtual controller as

\[
v_{p,q+1} = -z_{p,q-1} - c_{p,q}\varphi_{p,q} - \lambda_{p,q}\eta_{p,q}^2 + \hat{v}_{p,q}
- \frac{\theta_p}{2\alpha_{p,q}} S_{p,q}(X_{p,q}) S_{p,q}(X_{p,q}) \eta_{p,q}^2.
\]

Substituting equations (28) - (31) into equation (28), we get

\[
\dot{V}_{p,q} \leq \eta_{p,q}\eta_{p,q+1} + \sum_{j=1}^{q} (c_{p,j} - 1)\eta_{p,j}^2 - \sum_{j=1}^{q} l_{p,j}\eta_{p,j}^2
+ \frac{1}{\lambda_p} \theta_p \sum_{j=1}^{m_p} \frac{\lambda_p}{2\alpha_{p,j}} S_{p,q}(X_{p,q}) S_{p,q}(X_{p,q}) \eta_{p,j}^2 - \hat{v}_{p,q}
+ \sum_{j=1}^{q} \frac{\alpha_{p,j}^2 + \epsilon_{p,j}^2 + s_{p,j}^2 + \omega_{p,j}^2}{2}.
\]

**Step p, m:** Choose the Lyapunov function candidate as

\[
V_{p,m} = V_{p,m-1} + \frac{1}{2}\eta_{p,m}^2.
\]

Then, we give the definition of \( \varphi_{p,m} \) as

\[
\varphi_{p,m} = -c_{p,m}\varphi_{p,m} - \varphi_{p,m-1} - s_{p,m}\text{sign}(\varphi_{p,m}).
\]

where \( \varphi_{p,m}(0) = 0, c_{p,m} > 0 \) and \( s_{p,m} > 0 \) are known constants.

According to the equation (8), we define the error as

\[
\eta_{p,m} = z_{p,m} - \varphi_{p,m}.
\]

The time differential of \( \eta_{p,m} \) yields

\[
\dot{\eta}_{p,m} \leq \eta_{p,m+1} + v_{p,m+1} + \varphi_{p,m-1} + f_{p,m}(\hat{x}_{p,m}) + \omega_{p,m} - \hat{v}_{p,m}
+ \varphi_{p,m-1} + c_{p,m}\varphi_{p,m} + s_{p,m}\text{sign}(\varphi_{p,m}).
\]

Combining (33) and (36), we get

\[
\dot{V}_{p,m} = \dot{V}_{p,m-1} + \eta_{p,m}\eta_{p,m} + \lambda_{p,m}\eta_{p,m}^2 + \eta_{p,m}\omega_{p,m}
- \hat{v}_{p,m} + \varphi_{p,m-1} + c_{p,m}\varphi_{p,m}
+ s_{p,m}\text{sign}(\varphi_{p,m}).
\]

Applying Young’s inequality, we have

\[
\eta_{p,m}\lambda_{p,m}\text{sign}(\varphi_{p,m}) \leq \frac{1}{2}\eta_{p,m}^2 + \frac{1}{2}\lambda_{p,m}^2, \quad \eta_{p,m}\omega_{p,m} \leq \frac{1}{2}\eta_{p,m}^2 + \frac{1}{2}\omega_{p,m}^2.
\]

Then, using Lemma 4 - 5 and equation (19), we have

\[
\eta_{p,m}\lambda_{p,m}\text{sign}(\varphi_{p,m}) \leq \frac{1}{2}\eta_{p,m}^2 + \frac{1}{2}\lambda_{p,m}^2,
\]

where \( \hat{x}_{p,m} = [\hat{x}_{p,m}^T, X_{p,m} = [x_{p,1}, \ldots, x_{p,m}]^T \).

Then, the actual controller as

\[
u_{p} = -z_{p,m-1} - \frac{\theta_p}{2\alpha_{p,m}} S_{p,m}(X_{p,m}) S_{p,m}(X_{p,m})
- \lambda_{p,m}\eta_{p,m}^2 + \eta_{p,m}\omega_{p,m}
- l_{p,m}\eta_{p,m}^2 + \hat{v}_{p,m}.
\]

Substituting (38) - (41) into (37), we get

\[
\dot{V}_{p,m} \leq -\sum_{j=1}^{m_p} (c_{p,j} - 1)\eta_{p,j}^2 - \sum_{j=1}^{m_p} l_{p,j}\eta_{p,j}^2
+ \frac{1}{\lambda_p} \theta_p \sum_{j=1}^{m_p} \frac{\lambda_p}{2\alpha_{p,j}} S_{p,m}(X_{p,m}) S_{p,m}(X_{p,m}) \eta_{p,j}^2 - \hat{v}_{p,m}
+ \sum_{j=1}^{m_p} \frac{\alpha_{p,j}^2 + \epsilon_{p,j}^2 + s_{p,j}^2 + \omega_{p,j}^2}{2}.
\]
The adaptive law is defined as

$$\dot{\theta}_p = \lambda_p \sum_{j=1}^{m_p} \frac{1}{2\alpha_p,j} S_s^T p, m_p(X, p, m_p) S_l p, m_p(X, p, m_p) \eta_{p,j}^2 - \rho_p \dot{\theta}_p,$$

(43)

where $\rho_p > 0$ is a known constant.

### B. STABILITY ANALYSIS

**Theorem 1:** Consider the nonlinear MIMO system (1) with Assumptions 1-2, if the control signals (20), (31) and the actual controller (41) are designed, then

1. all signals in the closed-loop system are bounded.
2. tracking errors converge to the neighborhood of the origin in a finite time.

**Proof:** By substituting equations (21), (32) and (42), we get

$$\sum_{p=1}^{M} \dot{\nu}_{p, m_p} = - \sum_{p=1}^{M} \sum_{j=1}^{m_p} (c_{p,j} - 1) \eta_{p,j}^2 - \sum_{p=1}^{M} m_p (\frac{\dot{\eta}_{p,j}^2}{\lambda_p})$$

$$+ \sum_{p=1}^{M} \rho_p \dot{\theta}_p \dot{\theta}_p$$

$$+ \sum_{p=1}^{M} \sum_{j=1}^{m_p} \eta_{p,j}^2 + \eta_{p,j}^2 + \dot{\eta}_{p,j}^2 + \dot{\eta}_{p,j}^2. \quad \text{(44)}$$

Then, we can get the following inequality:

$$\frac{\rho_p}{\lambda_p} \dot{\theta}_p \dot{\theta}_p = \frac{\rho_p}{\lambda_p} \dot{\theta}_p (\theta_p - \ddot{\theta}_p) \leq \frac{\rho_p}{\lambda_p} \dot{\theta}_p^2 - \frac{\rho_p}{\lambda_p} \dot{\theta}_p^2. \quad \text{(45)}$$

Substituting (45) into (44) yields

$$\sum_{p=1}^{M} \dot{\nu}_{p, m_p} = - \sum_{p=1}^{M} \sum_{j=1}^{m_p} (c_{p,j} - 1) \eta_{p,j}^2$$

$$- \sum_{p=1}^{M} \sum_{j=1}^{m_p} \nu_{p,j} \eta_{p,j}^2 + \sum_{p=1}^{M} m_p \dot{\theta}_p^2$$

$$- \sum_{p=1}^{M} \sum_{j=1}^{m_p} \dot{\theta}_p \dot{\theta}_p - \sum_{p=1}^{M} \sum_{j=1}^{m_p} \eta_{p,j}^2$$

$$+ \sum_{p=1}^{M} \sum_{j=1}^{m_p} \eta_{p,j}^2 + \eta_{p,j}^2 + \dot{\eta}_{p,j}^2 + \dot{\eta}_{p,j}^2.$$

Applying Lemma 2 to the term $\sum_{p=1}^{M} \rho_p \dot{\theta}_p \dot{\theta}_p$, one has

$$\sum_{p=1}^{M} \rho_p \dot{\theta}_p \dot{\theta}_p \leq \sum_{p=1}^{M} \frac{\rho_p}{2\lambda_p} \dot{\theta}_p^2 + \sum_{p=1}^{M} \rho_p (1 - g) \eta_{p,j}^{1-g}. \quad \text{(47)}$$

Then, using Lemma 3 and substituting (47) into (46) gives

$$\sum_{p=1}^{M} \dot{\nu}_{p, m_p} \leq - \sum_{p=1}^{M} \sum_{j=1}^{m_p} (c_{p,j} - 1) \eta_{p,j}^2 - \sum_{p=1}^{M} \sum_{j=1}^{m_p} \rho_p \dot{\theta}_p \dot{\theta}_p$$

$$- \sum_{j=1}^{m_p} \rho_p \dot{\theta}_p \dot{\theta}_p - \sum_{p=1}^{M} \sum_{j=1}^{m_p} \eta_{p,j}^2 + \sum_{p=1}^{M} \sum_{j=1}^{m_p} \rho_p \dot{\theta}_p \dot{\theta}_p$$

$$+ \sum_{p=1}^{M} \sum_{j=1}^{m_p} \eta_{p,j}^2 + \eta_{p,j}^2 + \dot{\eta}_{p,j}^2 + \dot{\eta}_{p,j}^2.$$

This completes the proof.
In this part, in order to prove the feasibility of the proposed method. We consider a system composed of two inverted pendulums mounted on two carts shown in Figure 1 is studied:

\[
\begin{align*}
\dot{x}_{1,1} &= x_{1,2} + f_1(x) + \omega_{1,2} \\
\dot{x}_{1,2} &= \frac{u_1}{a_1} + f_1(x) + \omega_{1,2} \\
\dot{x}_{2,1} &= x_{2,2} + f_2(x) + \omega_{2,2} \\
\dot{x}_{2,2} &= \frac{u_2}{a_2} + f_2(x) + \omega_{2,2}
\end{align*}
\]  

(54)

where \( f_1(x) = \frac{g_1}{d_0} x_{1,1} - \frac{k(\delta - d_0)}{a_1 m_1} x_{1,1} - \frac{\sin(x_{1,1}) m_1}{m_2} x_{1,2} \)

\( \frac{k(\delta - d_0)}{a_1 m_1} x_{1,1} - \frac{k(d - d_0)}{a_1 m_1} x_{1,1} \) (Fig. 2 - 4)

\( \omega_{1,2} = \frac{\sin(x_{1,1}) m_1}{m_2} x_{1,2} \)

\( X_{1,1}(0) = x_{1,2}(0) = x_{2,2}(0) = 0, x_{2,1}(0) = 5, \theta_1(0) = 1 \) and \( \theta_2(0) = 1 \)

Then, select \( y_{1,1} = \sin(t) \) and \( y_{1,2} = 4 \cos(t) + 0.1 \sin(t) - 6 \) as the reference signal. The distance \( l_2 - l_1 \) is bounded. \( x_{1,1} \) and \( x_{2,1} \) stand for the pendulum angles, \( \dot{x}_{1,1} \) and \( \dot{x}_{2,1} \) show the angular velocities, \( u_1 \) and \( u_2 \) represent the control torques, and \( a = \frac{m_1 + m_2}{m_1} \). Choose \( m_1 = 3, m_2 = 3, l_0 = 2, l_1 = \sin(12t), l_2 = 2 + \sin(15t), g_0 = 9.8, k = 1 \).

From Theorem 1, the parameters in the adaptive QFPS tracking control scheme are selected as \( l_{1,1} = 4, l_{1,2} = 50, l_{2,1} = 2, l_{2,2} = 20, c_{1,1} = 2, c_{1,2} = 20, c_{2,1} = 2, c_{2,2} = 20, a_{1,1} = 1, a_{1,2} = 0.5, a_{2,1} = 1, a_{2,2} = 0.5, s_{1,1} = 1, s_{1,2} = 1, \) and \( \varepsilon_1 = 0.1, \varepsilon_2 = 0.1 \).

The simulation results are shown in Figures 2-5. The tracking performances are shown in Figure 2, and the tracking errors are exhibited in Figure 3. Figure 4 shows the state variables. Figure 5 shows the adaptive parameters. Based on Figures 2-5, it can be inferred that the tracking performance of the proposed method is satisfactory.

V. CONCLUSION

In this paper, the problem of adaptive quasi-fast finite-time tracking control has been studied for a class of nonlinear MIMO systems.
nonstrict-feedback nonlinear MIMO systems. By combining the command filter and backstepping technologies, the issue of “explosion of complexity” has been solved. In addition, the neural-network-based compression technology has been used to address the nonstrict-feedback form during the process of controller design. The presented controller ensures the tracking performance and the closed-loop system is stable well in a finite time. The simulation results have showed the effectiveness of the proposed method in the end. It’s worth noting that in recent years, control problems for nonlinear multi-agent systems have become a hot research topic because of their intensive application in various civilian and military fields. Consequently, in the future we will concentrate on adaptive quasi-fast finite-time tracking control problem of nonlinear multi-agent systems.

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