A General Class of Regular Black Holes based on a Smeared Mass Distribution

Alexis Larraña†

National Astronomical Observatory. National University of Colombia.

Alejandro Cardenas-Avendaño†

National Astronomical Observatory. National University of Colombia and Department of Mathematics. Konrad Lorenz University.

Daniel Alexdy Torres‡

Department of Physics. National University of Colombia.

Abstract

We investigate the behaviour of a new general class of rotating regular black holes based on a non-Gaussian smeared mass distribution. It is shown that the existence of a fundamental minimal length and thus the smeared mass consideration cures the well known problems in the terminal phase of black hole evaporation. We find that there is a finite maximum temperature that the black hole reaches before cooling down to absolute zero, so that the evaporation ends up in a zero temperature extremal black hole which mass and size depends on the value of the fundamental length and on the rotation parameter of the black hole.

PACS: 04.70.Dy, 04.50.Kd, 02.40.Gh

Keywords: physics of black holes, noncommutative geometry

*Electronic address: ealarranaga@unal.edu.co
†Electronic address: alcardenasav@unal.edu.co
‡Electronic address: daatorresba@unal.edu.co
I. INTRODUCTION

Black hole physics plays a really important role in the understanding of quantum gravity. Since Hawking found the thermal radiation emitted by a collapsing black hole using the techniques of quantum field theory in a curved spacetime background [1], extensive studies on this area have been done from several theoretical viewpoints. However, after more than forty years of intensive research in this field, various aspects of the problem still remain unclear. In particular, a satisfactory description of the late stage of the evaporation process is still missing. One of the most interesting proposals comes from the string/black hole correspondence principle [2] which suggests that in the extreme regime of the late stage evaporation, stringy effects cannot be neglected. For example, at this stage string theory predicts that target spacetime coordinates become noncommuting operators on a $D$-brane [3, 4]. The idea of noncommutative spacetimes was introduced by Snyder [5] who showed that it helps to cure the divergences in relativistic quantum field theory. Recently, the interest in noncommutative spacetimes grew due to the work of Nicolini et al. in which they found a noncommutative inspired Schwarzschild black hole [6–8]. This work was extended to include the electric charge [9], extra dimensions [10, 11] and noncommutative black holes in (1+1)-dimensions [12], (2+1)-dimensions [13, 14] and in the Randall-Sundrum braneworld model [15] were also found. All these solutions share the remarkable property of the existence of an extreme mass $M_0$ under which no horizons are present. This fact gives as a result that there is a remnant after the Hawking evaporation finishes, which could, in principle, solve the so-called paradox of black hole information loss. In all these works, the smeared mass is mathematically introduced by replacing the point-like source with a Gaussian distribution. However, in the work of Park [16], it is pointed out that the Gaussianity is not always required and that non-Gaussian smeared mass distribution has not been studied much so far, except for the work of Nicolini [17] and the (2+1) noncommutative black hole solutions found in [18] and [19]. As a deformation of the $\delta$-function source, it is enough to require that the distribution has a sharp peak at the origin and that the integration of the distribution function gives a finite value, so that it always can be normalised to unity.

The main purpose of this paper is to obtain a new general class of rotating black hole solutions by solving Einstein equations in the presence of an anisotropic perfect fluid based on a non-Gaussian smeared mass distribution which include the Gaussian, Rayleigh, and
Maxwell-Boltzmann distributions with moments \( n = 0, 1 \) and 2, respectively, and after that, to apply Janis-Newman algorithm to generate angular momentum. The outline of this paper is as follows: in Section 2, we introduce the smeared mass distribution and the static regular black hole class of solutions of Einstein equations. We also study the thermodynamics of the static regular black holes to show that the evaporation process reaches a maximum value of temperature before cooling down towards a zero temperature remanent. In Section 3, we apply a modification of the well known Janis-Newman algorithm to generate angular momentum and to obtain a family of rotating regular black holes. We calculate the temperature of these black holes to find a behaviour that is completely similar to the obtained in the non rotating case. The final section is for the conclusions.

II. STATIC BLACK HOLE SOLUTION BASED ON A NON-GAUSSIAN SMEARED MASS DISTRIBUTION

The existence of a fundamental minimal length forbids matter to contract into a singular point. The emergence of this minimal length as a fundamental constant on the same ground as \( c \) or \( \hbar \), is a general feature of different approaches to quantum gravity \[3, 4\]. The most remarkable outcomes of this idea are the disappearance of curvature singularities in the solutions \[20\], a regular behaviour of the temperature associated to the black hole which allows to determine the existence of a remanent \[13, 21, 22\], and a different form of the relation between entropy and area of the event horizon.

In this paper we will take into account the effects of the existence of a minimal length by keeping the usual form of the Einstein tensor in the field equations and introducing a modified energy-momentum tensor as a source \[20\]. Specifically, it is possible to replace the point-like mass density, described by a Dirac \( \delta \) function, with a smeared object, for example a Gaussian distribution \[23, 24\]. However, as pointed by Park \[16\], the Gaussianity is not always required, so he proposed a smeared source based on the Maxwell-Boltzmann mass distribution to construct a \( (2 + 1) \) dimensional black hole. Inspired by this result, we propose the general mass density of a static, spherically symmetric, smeared particle-like gravitational source in the form

\[
\rho(r) = Ar^n \exp \left(-\frac{r^2}{\ell^2}\right),
\]  

(1)
where $\ell$ is a characteristic length scale of the matter distribution and $A$ is a normalisation constant. This density profile reproduces the Gaussian distribution of Nicolini et. al. for $n = 0$ and includes non-Gaussian (i.e. ring-type) distributions for higher moments, for example Rayleigh for $n = 1$, Maxwell-Boltzmann for $n = 2$, etc. In order to obtain the constant $A$, we consider the mass enclosed in a volume of radius $r$, which must be determined by integrating the density,

$$m(r) = \int_0^r 4\pi \tilde{r}^2 \rho(\tilde{r}) d\tilde{r} = 2\pi A \ell^{n+3} \gamma\left(\frac{n+3}{2}; \frac{r^2}{\ell^2}\right),$$

where $\gamma\left(\frac{n+3}{2}; \frac{r^2}{\ell^2}\right)$ is the lower incomplete gamma function discussed in the Appendix. Phenomenological results imply that noncommutativity is not visible at presently accessible energies, constraining $\ell < 10^{-16} cm$. At large distances one expects minimal deviations from standard vacuum Schwarzschild geometry. In fact, the mass of the black hole $M$ can be determined in the commutative limit $\ell \to 0$ (or equivalently $\frac{r}{\ell} \to \infty$), where the lower incomplete gamma function becomes the usual gamma function, $\gamma\left(\frac{n+3}{2}; \frac{r^2}{\ell^2}\right) \to \Gamma\left(\frac{n+3}{2}\right)$, and we expect that $m(r) \to M$. This gives the normalisation constant $A = \frac{M}{2\pi \Gamma\left(\frac{n+3}{2}\right) \ell^{n+3}}$ and therefore the density distribution becomes

$$\rho(r) = \frac{M}{2\pi \Gamma\left(\frac{n+3}{2}\right) \ell^{n+3}} r^n \ell^{n+3} \exp\left(-\frac{r^2}{\ell^2}\right),$$

which shows that the black hole mass $M$, instead of being perfectly localised at a single point, is diffused through a region of linear size $\ell$.

Before solving the field equations we will define completely the energy-momentum tensor. In order to do it, we consider the covariant conservation condition $T^\mu_\nu = 0$ which, for a spherically symmetric metric is

$$\partial_\nu T^\nu_r = -\frac{1}{2} g^{00} \partial_r g_{00} \left( T^r_r - T^0_0 \right) - g^{0\theta} \partial_r g_{\theta\theta} \left( T^r_r - T^\theta_\theta \right).$$

We want to preserve the Schwarzschild-like property $g_{00} = -g_r^{-1}$, so we require $T^r_r = -T^0_0 = \rho(r)$. Therefore, the divergence free equation allows a solution for $T^\theta_\theta$ which reads

$$T^\theta_\theta = -\rho(r) - \frac{r}{2} \partial_r \rho(r).$$

Note that, rather than a massive structureless point, the proposed source turns out to
be a self-gravitating, droplet of anisotropic fluid of density $\rho$, radial pressure $p_r = -\rho$ and tangential pressure

$$p_\perp = -\rho - \frac{r}{2} \partial_r \rho(r).$$  \hfill (6)

The equations involving the energy-momentum tensor turn out to mean that, on physical grounds, there is a non-vanishing radial pressure balancing the inward gravitational pull and thus, preventing the collapse of the droplet into a matter point. This is precisely the physical effect on matter caused by the existence of a fundamental length in spacetime and is the origin of all new physics at short distance scales.

In order to obtain a black hole solution, we solved the Einstein equations

$$G_\mu^\nu = R_\mu^\nu - \frac{1}{2} g_\mu^\nu R = 8\pi T_\mu^\nu$$  \hfill (7)

with (3) as the matter source and using the line element

$$ds^2 = f(r) dt^2 - \frac{dr^2}{f(r)} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$  \hfill (8)

This gives the function

$$f(r) = 1 - \frac{M}{4\pi r \Gamma \left(\frac{\nu+3}{2}\right) \gamma \left(\frac{n+3}{2}, \frac{r^2}{\ell^2}\right)},$$  \hfill (9)

which is the same result obtained by simply substituting the mass function (2) into the mass term of Schwarzschild metric. Therefore the classical Schwarzschild metric is obtained from this solution in the limit $\ell \rightarrow \infty$.

The line element (8) describes a regular black hole and should give us useful insights about possible spacetime noncommutativity effects on Hawking radiation. Possible horizon(s) of this solution can be obtained by solving the equation $f(r) = 0$, that is

$$r_H = \frac{M}{4\pi \Gamma \left(\frac{\nu+3}{2}\right) \gamma \left(\frac{n+3}{2}, \frac{r_H^2}{\ell^2}\right)}.$$

This equation cannot be solved for $r_H$ in a closed form, but, by plotting $f(r)$ one can determine numerically the existence of horizon(s) and their radius by reading the intersections with the $r$-axis. From Figure (1) is easy to note that the smeared mass distribution introduces new behaviour with respect to Schwarzschild black hole. In fact, instead of a single event horizon, for each value of $n$ in $f(r)$, there are three different possibilities:
1. two distinct horizons for $M > M_0$

2. one degenerate horizon in $r_0$, corresponding to an extremal black hole

3. no horizon for $M < M_0$.

Figure 1: $f(r)$ vs $r$, for various values of $n$. Intercepts on the horizontal axis give radii of the event horizons. Figure a) shows the Gaussian distribution, $n = 0$, figure b) corresponds to the Rayleigh distribution, $n = 1$ and figure c) is the Maxwell-Boltzmann distribution, $n = 2$. In all cases the continuous line represents a black hole with two horizons, $M > M_0$, the dashed line corresponds to the extreme black hole with one degenerate horizon, $M = M_0$, and the dotted line shows a solution with no horizons $M < M_0$. The specific value of $M_0$ depends on the value of the length $\ell$ and on the exponent $n$.

Therefore, we conclude that there is no black hole if the original mass is less than the minimal mass $M_0$ which depends on the value of the length $\ell$ and on the exponent $n$. Furthermore, contrary to the usual Schwarzschild black hole, there can be two horizons for large masses. It also can be seen that for $M \gg M_0$, the inner horizon shrinks to zero while the outer one approaches the Schwarzschild horizon, located at $r_S = 2M$.

A. Thermodynamics

The temperature associated with the black hole is given by
\[ T = \left( \frac{1}{4\pi} \frac{df}{dr} \right)_{r=r_H} = \frac{1}{4\pi r_H} \left[ 1 - 2 \frac{r_H^{n+3}}{\ell^{n+3}} e^{-\frac{r_H^2}{\ell^2}} \frac{1}{\gamma \left( \frac{n+3}{2}, \frac{r_H^2}{\ell^2} \right)} \right], \quad (11) \]

where we have used the derivative of the lower incomplete gamma function described in the Appendix and equation (10) to write \( M \) in terms of \( r_H \). It is clear that the second term inside the brackets is the correction arising from the smeared distribution. For large black holes, i.e. \( \frac{r_H^2}{\ell^2} \gg 1 \), equation (11) recovers the standard Hawking temperature for the Schwarzschild black hole,

\[ T_H = \frac{1}{4\pi r_H}. \quad (12) \]

In Figure (2) we plot the temperature (11) as a function of \( r_H \) and we find that at the initial state of evaporation the black hole temperature increases while the horizon radius is decreasing. Here, the interesting point is to investigate what happens as we reach the final state of the process, i.e. when \( r_H \to \ell \). As is well known, in the standard case the Hawking temperature \( T_H \) diverges as \( M \to 0 \), or equivalently \( r_H \to 0 \). However, in our solution the temperature (11) deviates from the standard hyperbola (12) and instead of exploding, it reaches a maximum value and then it quickly drops to zero for \( r_H = r_0 \), leaving a frozen extremal black hole. In the region \( r_H < r_0 \) there is no black hole, because physically \( T \) cannot be negative. As is easily observed, the Hawking paradox is circumvented by the smeared mass distribution.

B. Curvature Scalars

We approach the regularity problem of the solution by studying the behaviour of two curvature scalars, the Ricci scalar \( R = g_{\mu \nu} R_{\mu \nu} \) with \( R_{\mu \nu} \) the Ricci tensor, and the Kretschmann invariant \( K = R_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma} \) with \( R_{\mu \nu \rho \sigma} \) the Riemann tensor. For our black hole solution, these invariants are

\[ R = \frac{M}{2\pi \Gamma \left( \frac{n+3}{2} \right) \ell^{n+3} e^{-\frac{r_H^2}{\ell^2}}} \left[ 2\frac{r_H^2}{\ell^2} - (n+4) \right], \quad (13) \]

and
Figure 2: Plot of $T$ vs $\frac{r_H}{r}$, for different values of $n$. Intercepts on the horizontal axis give radii of the event horizon of the extremal black hole. $n = 0$, (Gaussian distribution, dot-dashed curve), $n = 1$, (Rayleigh distribution, dashed curve) and $n = 2$ (Maxwell-Boltzmann distribution, dotted curve). For comparison, we also plotted the standard Hawking temperature of Schwarzschild black hole, continuous curve. All temperatures coincide for $\frac{r_H}{r} \gg 1$.

$$K = \left( \frac{M}{2\pi \Gamma \left( \frac{n+1}{2} \right)} \right)^2 \times \left\{ \frac{3}{r^6} \cdot \left( \frac{n+3}{2} \cdot \frac{r_H^2}{r^2} \right) + 2\gamma \left( \frac{n+3}{2} \cdot \frac{r_H^2}{r^2} \right) \left[ \left( n - 2 \right) - 2 \frac{r_H^2}{r^2} \right] \frac{r_H^{n-3}}{\Gamma(n+3)} e^{-\frac{r_H^2}{r^2}} + \left[ 4n^2 - 4n + 2 \right] \frac{r_H^4}{\Gamma(2n+6)} e^{-\frac{2r_H^2}{r^2}} \right\} (14)$$

For $M \neq 0$, these invariants are regular everywhere, including the point $r = 0$. In fact, both invariants vanish at the origin for all values of $n$ except in the case $n = 0$, for which the invariants take a constant value (see Figure 3).
Figure 3: a) Plot of the Ricci invariant $R$ as function of the radial coordinate $r$. b) Plot of the Kretschmann invariant $K$ as function of the radial coordinate $r$. The graphics show the behaviour of the invariants near $r = 0$ for different values of $n$. In both figures the Gaussian distribution $n = 0$, is the continuous curve, the Rayleigh distribution $n = 1$, is the dashed curve and the Maxwell-Boltzmann distribution $n = 2$, corresponds to the dotted curve.

III. ROTATING BLACK HOLES

The Newman-Janis algorithm [25, 26] is a short cut to obtain spinning black hole solutions from the corresponding non rotating ones. According to Drake and Szekeres [27], this algorithm works for vacuum solutions or for solutions with a Maxwell source. It is clear that our black hole [8] is not a vacuum solution, since Einstein equations have an anisotropic fluid as source, and therefore the application of the Newman-Janis procedure is not straightforward. However, Modesto and Nicolini [28] provide a prescription to include non vanishing stress-energy tensors and, using this modification of the algorithm, we will find a general rotating family of black holes based on the smeared mass distribution [8].

Beginning with the line element (8), we change to the outgoing Eddington-Finkelstein coordinates $\{u, r, \vartheta, \phi\}$, where $u = t - r^*$ and $dr^* = \frac{dr}{f(r)}$, to obtain the metric
\[ ds^2 = f(r) \, du^2 + 2 \, du \, dr - r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right). \]  

This metric is written in terms of null tetrad vectors as

\[ g_{\mu \nu} = l_\mu n_\nu + l_\nu n_\mu - k_\mu \bar{k}_\nu - k_\nu \bar{k}_\mu \]  

where the tetrad vectors are

\[ l^\mu = \delta^\mu_1 \]  
\[ n^\mu = \delta^\mu_0 - \frac{1}{2} f(r)^2 \delta^\mu_1 \]  
\[ k^\mu = \frac{1}{\sqrt{2r^2}} \left[ \delta^\mu_2 + i \sin \theta \delta^\mu_3 \right] \]  

satisfying the relations \( l_\mu l^\mu = k_\mu k^\mu = n_\mu n^\mu = l_\mu k^\mu = n_\mu k^\mu = 0 \) and \( l_\mu n^\mu = -k_\mu \bar{k}^\mu = 1 \), with \( \bar{k} \) the complex conjugate of \( k \). The following step in the Newman-Janis algorithm is to perform the complex increment

\[
\begin{cases}
    r \rightarrow r' = r + ia \cos \theta \\
u \rightarrow u' = u - ia \cos \theta
\end{cases}
\]  

and this is precisely the key point of the algorithm. As stated in [28], if we start from the case \( \ell = 0 \), the metric [5] coincides with the Schwarzschild solution, i.e. the mass function \( m(r) \) becomes \( m(r) = M \), and the usual algorithm can be followed. Note that in this case the mass term in unaffected by the complexification [20] and it is assumed that [27]

\[
\frac{1}{r} \mapsto \frac{1}{2} \left( \frac{1}{r'} + \frac{1}{r'} \right) = \frac{r}{r^2 + a^2 \cos^2 \theta}. \]  

Therefore, the proposal for the complexification of the function \( f(r) \) given in equation [9] is to make \( m(r) \mapsto m \left( \text{Re}(r') \right) = m(r) \) which gives the prescription

\[ f(r) = 1 - \frac{2m(r)}{r} \mapsto 1 - 2m \left[ \text{Re}(r') \right] \left[ \frac{1}{2} \left( \frac{1}{r'} + \frac{1}{r'} \right) \right] = 1 - \frac{2m(r) \, r}{r^2 + a^2 \cos^2 \theta} = F(r, \theta), \]  

or writing the obtained function in terms of the gamma functions,

\[ F(r, \theta) = 1 - \frac{M}{4\pi \Gamma \left( \frac{n+3}{2} \right)} \frac{r}{r^2 + a^2 \cos^2 \theta} \gamma \left( \frac{n+3}{2}, \frac{r^2}{\ell^2} \right). \]
Once we have done this step, the tetrad vectors are

\[ l^\mu = \delta_1^\mu \]

\[ n^\mu = \delta_0^\mu - \frac{1}{2} F (r, \theta) \delta_1^\mu \]

\[ k^\mu = \frac{1}{\sqrt{2 (r^2 + a^2 \cos^2 \theta)}} \left[ i a \sin \theta (\delta_0^\mu - \delta_1^\mu) + \delta_2^\mu + \frac{i}{\sin \theta} \delta_3^\mu \right], \]

from which the metric of the new rotating regular black hole can be read in Boyer-Lindquist coordinates,

\[ ds^2 = F (r, \theta) dt^2 - \frac{\Sigma dr^2}{a^2 \sin^2 \theta + F (r, \theta) \Sigma} + 2 \left( 1 - F (r, \theta) \right) a \sin^2 \theta dt d\phi - \Sigma d\theta^2 \]

\[ - \left[ a^2 \left( 2 - F (r, \theta) \right) \sin^2 \theta + \Sigma \right] \sin^2 \theta d\phi^2 \]

where

\[ \Sigma = r^2 + a^2 \cos^2 \theta. \]

Introducing the quantity

\[ \Delta = \Sigma F (r, \theta) + a^2 \sin^2 \theta = r^2 - 2m (r) r + a^2 \]

we write the new metric in the usual form

\[ ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} dt^2 - \frac{\Sigma dr^2}{\Delta - \Sigma \Delta^2 + 2a \sin^2 \theta \left( 1 - \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \right)} dt d\phi \]

\[ - \left[ \Sigma + a^2 \sin^2 \theta \left( 2 - \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} \right) \right] \sin^2 \theta d\phi^2. \]

Note that this metric recovers the rotating noncommutative inspired black hole reported by Smailagic and Spallucci [29] by taking \( n = 0 \).

Horizons correspond to the solutions of the equation \( \Delta (r_H) = 0 \), i.e.

\[ r_H^2 - \frac{Mr_H}{4\pi \Gamma \left( \frac{n+3}{2} \right)} \gamma \left( \frac{n+3}{2}, \frac{r_H^2}{\ell^2} \right) + a^2 = 0, \]

which can not be solved explicitly. Therefore, we solve equation (10) for the mass parameter \( M (r_H) \) as function of the horizon radius \( r_H \). The plot is given in Figure 4 in \( \ell = 1 \) units,
for the cases \( n = 0, 1, 2 \) and for different values of the spin parameter \( a \). The intersection of each curve with the line \( M = \text{constant} \) determines the position of the horizon(s). Note that the minimum corresponds to the extremal black hole configuration and that increasing the spin parameter \( a \) lifts the minimum upwards.

Figure 4: Plot of the function \( M (r_H) \) for various values of \( n \) and \( a \) in units with \( \ell = 1 \). \( f(r) \) vs \( \frac{r}{\ell} \), for various values of \( n \). The intersections of these curves with the line \( M = \text{constant} \) determines the position of the horizons. Figure a) shows the Gaussian distribution, \( n = 0 \), figure b) corresponds to the Rayleigh distribution, \( n = 1 \) and figure c) is the Maxwell-Boltzmann distribution, \( n = 2 \). In all cases, the minimum of the curve corresponds to the extremal black hole configuration.

A. Thermodynamics

Now we face the problem of the temperature associated with these black holes. It is given in terms of the surface gravity \( \kappa \) as

\[
T = \frac{\kappa}{2\pi}.
\]  

(32)

This time, the surface gravity must be calculated through the relation

\[
\kappa^2 = -\frac{1}{2} \nabla^\mu \psi^\nu \nabla_\mu \psi_\nu
\]

(33)

where \( \psi^\mu \) are null Killing vectors. Since our solution is axisymmetric we will propose the Killing vector
\[ \psi^\mu = \xi^\mu + \Omega \zeta^\mu, \]  
(34)

with \( \xi^\mu = \partial_t \) and \( \zeta^\mu = \partial_\phi \) the Killing vectors associated with time translation invariance and rotational invariance, respectively. The angular velocity \( \Omega \) can be obtained by imposing \( \psi \) to be null,

\[ \psi_\mu \psi^\mu = 0 = g_{tt} + 2\Omega g_{t\phi} + g_{\phi\phi} \Omega^2 \]  
(35)

from which

\[ \Omega = -\omega \pm \sqrt{\omega^2 - \frac{g_{tt}}{g_{\phi\phi}}} \]  
(36)

where we defined

\[ \omega = \frac{g_{t\phi}}{g_{\phi\phi}}. \]  
(37)

For our metric \( \text{(30)} \), \( \Omega \) is

\[ \Omega = -\omega \pm \frac{\Sigma \sqrt{\Delta}}{\{\Sigma^2 + [2\Sigma - (\Delta - a^2 \sin^2 \theta)] a^2 \sin^2 \theta\} \sin \theta} \]  
(38)

with

\[ \omega = -\frac{a [\Sigma - (\Delta - a^2 \sin^2 \theta)]}{\Sigma^2 + [2\Sigma - (\Delta - a^2 \sin^2 \theta)] a^2 \sin^2 \theta}. \]  
(39)

At the event horizon we obtain the angular velocity of the black hole,

\[ \Omega_H = \frac{a}{r_H^2 + a^2}. \]  
(40)

Using this null Killing vector, we calculate the surface gravity at the horizon and the temperature associated to the black hole results to be

\[ T = \frac{r_H}{4\pi (r_H^2 + a^2)} \left[ 1 - \frac{a^2}{r_H^2} - \frac{(r_H^2 + a^2) r_H^{n+1}}{\ell^{n+3}} \frac{2e^{-\frac{r_H^2}{\ell^2}}}{\gamma \left( \frac{n+3}{2}, \frac{r_H^2}{\ell^2} \right)} \right]. \]  
(41)

In Figure 5 we see that the profile of the temperature is roughly equivalent to that of the non rotating solution. After a temperature maximum, the black hole cools down to a zero temperature black hole remanent final state. Note that the figure shows how the size and the mass of this remanent is increased with respect to the non rotating case. This can be understood by remembering that this time the rotational kinetic energy is also stored in the final configuration. However, the present analysis does not take into account the loss
Figure 5: Temperature of the rotating black holes as functions of the horizon radius for different values of $n$ and $a$. The behaviour is the same as in the non rotating case but rotation has the effect of increasing the values of the mass and size of the black hole remanent. Figure a) shows the Gaussian distribution, $n = 0$, figure b) corresponds to the Rayleigh distribution, $n = 1$ and figure c) is the Maxwell-Boltzmann distribution, $n = 2$.

of angular momentum due to the Hawking emission and the consequent transition into a Schwarzschild phase. This implies that the evaporation of our rotating black holes is a multi phase process.

**B. Curvature Invariants**

To study the regularity of the rotating black hole we calculate the Ricci $R_{(rot)}$ and Kretschmann $K_{(rot)}$ scalars. This time, the two invariants are regular in the point $r = 0$, $\theta = \frac{\pi}{2}$ for all positive values of $n$, but they are discontinuous (they assume two different values depending of the way one reaches that point, as in [28]).

If we approach the origin moving on the plane $\theta = \frac{\pi}{2}$, we find a regular rotating deSitter-like geometry for $n \geq 0$ because the invariants have the values

$$
\lim_{r \to 0} \left[ \lim_{\theta \to \frac{\pi}{2}} R_{(rot)} \right] = \begin{cases} 
-\frac{2M}{\pi^{3/2}g^3} & \text{for } n = 0 \\
0 & \text{for } n > 0
\end{cases}
$$

and

14
\[
\lim_{r \to 0} \left[ \lim_{\theta \to \frac{\pi}{2}} K_{(rot)} \right] = \begin{cases} 
\frac{4 M^2}{3 \pi \ell^6} & \text{for } n = 0 \\
0 & \text{for } n > 0
\end{cases}.
\] (43)

In order to clarify this behaviour, note that the metric (30) can be expanded for small \(r\) and keeping \(\theta = \frac{\pi}{2}\) as

\[
ds^2 = \left(1 - \frac{\Lambda (n)}{3} r^{n+2}\right) dt^2 - \frac{r^2 dr^2}{r^2 + a^2} - r^2 d\theta^2 + 2a \frac{\Lambda (n)}{3} r^{n+2} dt d\phi
\]
\[
- \left[ r^2 + a^2 - \frac{\Lambda (n)}{3} a^2 r^{n+2} \right] d\phi^2
\] (44)

where

\[\Lambda (n) = \frac{3M}{2\pi (n+3) \Gamma \left( \frac{n+3}{2} \right) \frac{1}{\ell^{n+3}}}\]

and clearly it corresponds to a rotating de Sitter geometry for \(n = 0\).

On the other hand, if we approach the origin along an arbitrary plane, i.e. \(r \to 0\) with \(\theta \neq \frac{\pi}{2}\), we reach the equatorial disk and since there is no matter, we obtain the invariants

\[
\lim_{\theta \to \frac{\pi}{2}} \left[ \lim_{r \to 0} R_{(rot)} \right] = 0
\] (45)

and

\[
\lim_{\theta \to \frac{\pi}{2}} \left[ \lim_{r \to 0} K_{(rot)} \right] = 0.
\] (46)

IV. CONCLUSION

We have shown that the introduction of smeared mass distributions in general relativity as the matter source, inspired by the existence of a fundamental minimal length, gives a general family of regular black holes that reproduces, as special cases, the Gaussian distribution of Nicolini et. al. [6] for \(n = 0\) and includes non-Gaussian (i.e. ring-type) distributions for higher moments, for example Rayleigh for \(n = 1\), Maxwell-Boltzmann for \(n = 2\), etc. The energy-momentum tensor needed for this description is that of an ideal fluid and requires a non-trivial pressure. By applying a modification of the Janis-Newman algorithm we provide
angular momentum to the solutions, obtaining the rotating counter part of these static solutions.

We also investigate the temperature of our regular black holes based on a smeared mass distribution to show that in both cases, rotating and non rotating, there is a maximum value that the black hole can reach before cooling down to absolute zero corresponding to an extreme back hole remanent. The size of this remanent depends on the value of the minimal length \( \ell \), on the value of the exponent \( n \) in the mass distribution and on the value of the spin parameter \( a \). Therefore, we conclude that the smeared source regularises divergent quantities in the final stage of black hole evaporation (in the same way that the noncommutativity approach regularises UV infinities in quantum field theory).

Finally we studied the behaviour of two curvature invariants (Ricci and Kretschmann) to show that they do not present singularities, at least for indices \( n \geq 0 \). This fact, together with the existence of the residual mass \( M_0 \) in the evaporation process, are both manifestations of the de-localisation of the source.

Acknowledgement

This work was partially supported by the Universidad Nacional de Colombia. Hermes Project Code 18140.

Appendix

The lower incomplete gamma function \( \gamma (a, x) \) is given by

\[
\gamma (a, x) \equiv \int_0^x t^{a-1}e^{-t}dt
\]  
(47)

while the upper incomplete gamma function \( \Gamma (a, x) \) is

\[
\Gamma (a, x) \equiv \int_x^\infty t^{a-1}e^{-t}dt
\]  
(48)

and their sum corresponds to the total gamma function

\[
\Gamma (a) = \gamma (a, x) + \Gamma (a, x) = \int_0^\infty t^{a-1}e^{-t}dt.
\]  
(49)

The derivative of the lower incomplete gamma function is
\[ \frac{\partial}{\partial x} \gamma(a, x) = e^{-x}x^{a-1}. \]  

(50)

Finally, the incomplete gamma function has the asymptotic behaviour

\[ \frac{\gamma(a, x)}{x^a} \to \frac{1}{a} \text{ as } x \to 0. \]  

(51)

[1] S. W. Hawking, Comm. Math. Phys. 43, 199 (1975).
[2] L. Susskind, Phys. Rev. Lett. 71, 2367 (1993).
[3] E. Witten, Nucl. Phys. B 460, 335 (1996).
[4] N. Seiberg, E. Witten, JHEP 9909, 032 (1999).
[5] H. S. Snyder, Phys. Rev. 71 38 (1947).
[6] Nicolini P, Smailagic A, Spallucci E. Phys. Lett. B, 632, 547 (2006).
[7] Nicolini P, Smailagic A, Spallucci E. ESA Spec. Publ. 637, 111 (2006).
[8] Nicolini P. J. Phys. A 38, L631 (2005).
[9] Ansoldi S et al. Phys. Lett. B 645, 261 (2007).
[10] Rizzo T G. J. High Energy Phys. 0609, 021 (2006).
[11] Spallucci E, Nicolini P, Smailagic A. Phys. Lett. B 670, 449 (2009).
[12] Mureika J R, Nicolini P. Phys. Rev. D 84, 044020 (2011).
[13] Myung Y S, Kim Y W, Park Y. J. High Energy Phys. 0702, 012 (2007).
[14] Tejeiro J M, Larranage A. Pramana J. Phys. 78, 1 (2012).
[15] A. Larrañaga, C. Benvides and C. Rodriguez. Rom. J. Phys. 59, 57 (2014).
[16] Park M I. Phys. Rev. D 80, 084026 (2009).
[17] P. Nicolini, A. Orlandi and E. Spallucci, Adv. High Energy Phys. 2013, 812084 (2013).
[18] Myung Y S, Yoon M. Eur. Phys. J. C 62, 405 (2009).
[19] Liang, J., Liu Y. C., Zhu Q. Chinese Physics C 38, 2, 025101 (2014).
[20] Nicolini P. Int. J. Mod. Phys. A 24, 1229 (2009).
[21] R. Banerjee, B. R. Majhi and S. Samanta, Phys. Rev. D 77, 124035 (2008).
[22] Y. S. Myung, Y. W. Kim and Y. J. Park, Gen. Relativ. Gravit. 41, 1051 (2009).
[23] A. Smailagic, E. Spallucci, J. Phys. A 36, L467 (2003).
[24] A. Smailagic, E. Spallucci, J. Phys. A 36 L517 (2003).
[25] E. T. Newman and A. Janis, J. Math. Phys. 6, 915 (1965).
[26] L. Herrera and J. Jimenez, J. Math. Phys. 23, 2339 (1982).
[27] S. P. Drake and P. Szekeres, Gen. Rel. Grav. 32, 445 (2000).
[28] L. Modesto and P. Nicolini, Phys. Rev. D 82, 104035 (2010).
[29] A. Smailagic and E. Spallucci. Phys. Lett. B 688, 82 (2010).