The Effect of Local Structure and Non-uniformity on Decoherence-Free States of Charge Qubits

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We analyze the robustness of decoherence-free (DF) subspace in charge qubits when there are a local structure and non-uniformity that violate collective decoherence measurement condition. We solve master equations of up to four charge qubits and a detector as two serially coupled quantum point contacts (QPC) with an island structure. We show that robustness of DF states is strongly affected by local structure as well as by non-uniformities of qubits.

I. INTRODUCTION

Decoherence-free (DF) states\[1\] are useful for collective decoherence environment, even if there is a small symmetry breaking perturbation parameterized by $\eta$ in the order of $O(\eta)$ ($\eta \ll 1$)\[2\]. Nowadays, experiments for DF states have been successful up to four qubits in photon system\[3,4\] and in nuclear magnetic resonance (NMR)\[5\]. However, in solid-state qubits, it seems to be more difficult to realize the collective decoherence environment than in the case of optical or NMR qubits. This is because we could not prepare plenty of qubits with mathematically exact size, because the sizes of Cooper-pair box\[6\] or quantum dot (QD)\[7,8,9,10\] are less than hundreds of nm, and moreover, localized trap sites generated in the fabrication process disturb the collective decoherence environment.

We theoretically describe the effect of non-uniform environment on DF states of charge qubits composed of coupled QDs, considering the measurement process, by using time-dependent density matrix (DM) equations. The charge distribution of the qubits changes the QPC current capacitively, resulting in detection and corruption of charged state (backaction)\[11\]. Thus, measurement is a basic and important decoherence process that should be investigated in detail. In Ref\[12\], we discussed the robustness of DF states under non-uniformity of qubits when they are detected by a simple structureless QPC detector. However, generally speaking, the solid-state qubit system is arranged compactly to avoid extra noises, and therefore, qubits have a tendency to be often affected by geometrical local structures such as electrodes or electrical wires. In addition, when there are local structures and large non-uniformities, it is possible that the four-qubit DF states are less robust than non-DF states. In such cases, for example, two qubit non-DF state and the singlet state would be appropriate to constitute two logical states instead of using four-qubit DF states, because defects and fault rates will increase as the number of qubits increases.

In this paper, we discuss the effect of local structures on the DF states combining with the non-uniformity of qubits based on the setup shown in Fig. 1(a) and compare them with two-qubit states shown in Fig. 1(b). We study

\[\begin{align*}
\sum_{i=1}^{N}|\Psi_i(1234)\rangle &= 2^{-1}(|01\rangle - |10\rangle) \otimes (|01\rangle - |10\rangle) \\
\sum_{i=1}^{N}|\Psi_i(234)\rangle &= 1/(2\sqrt{2})(2|0011\rangle - |0101\rangle - |0110\rangle \\
&\quad - |1010\rangle - |1010\rangle + 2|1100\rangle) \\
\sum_{i=1}^{N}|\Psi_i(134)\rangle &= 2^{-1}(|01\rangle - |10\rangle) \otimes (|01\rangle - |10\rangle) \\
&\quad - |1010\rangle - |1010\rangle + 2|1100\rangle)
\end{align*}\]

where $|01\rangle = |1\rangle|0\rangle_2|0\rangle_3|0\rangle_4$ and so on. We also compare these four-qubit DF states with two qubit Bell states: $|a\rangle \equiv (|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)/\sqrt{2}$, $|b\rangle \equiv (|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle)/\sqrt{2}$, $|c\rangle \equiv (|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)/\sqrt{2}$, $|d\rangle \equiv (|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)/\sqrt{2}$ depicted in Fig. 1(b).

II. FORMULATION

We show the formulation for Fig. 1(a). The Hamiltonian for the combined qubits and the QPC for Fig. 1(a) is written as $H = H_{qb} + H_{qpc} + H_{int}$. $H_{qb}$ describes the interacting four qubits: $H_{qb} = \sum_{i=1}^{N} (\Omega_{i}\sigma_{ix} + \epsilon_{i}\sigma_{iz}) + \sum_{i=1}^{N-1} J_{i,i+1}\sigma_{ix}\sigma_{ix}$. \[\begin{align*}
H_{qpc} &= \sum_{\alpha=L,R} \sum_{\kappa=L,R} \sum_{i=1}^{N}[E_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} + V_{i\alpha}(c_{i\alpha}^\dagger d_{\alpha} + d_{\alpha}^\dagger c_{i\alpha})]
\end{align*}\]

FIG. 1: Qubits that use double dot charged states are capacitively coupled to a QPC detector.
\[ H_{\text{int}} = \sum_{i,x,s} \left[ \sum_{l=1}^{2} \delta V_{i,l,s} \sigma_{iz} \right] (c_{i,l,s} ^{\dagger} d_s + d_s ^{\dagger} c_{i,l,s}) \]
\[ + \sum_{r,s} \left[ \sum_{l=3}^{4} \delta V_{r,i,s} \sigma_{iz} \right] (c_{r,i,s} ^{\dagger} d_s + d_s ^{\dagger} c_{r,i,s}) \]  

(3)

where \( \delta V_{i,a,s} \) (\( \alpha = L, R \)) is an effective change of the tunneling strength between the left and right electrodes. Hereafter we neglect the spin dependence of \( V_{i,a} \), and \( \delta V_{i,a,s} \). We assume that the tunneling strength of electrons weakly depends on the energy \( V_{i,a} = V_{i,a}(E_{i,a}) \) and electrodes are degenerate up to the Fermi surface \( \mu_{a} \). Then qubit states influence the QPC tunneling rate \( \Gamma_L \) and \( \Gamma_R \) by \( \Gamma_L ^{\pm} = \Gamma_L ^{1} \pm \Gamma_L ^{2} \) and \( \Gamma_R ^{1} = \Gamma_R ^{1} \pm \Gamma_R ^{4} \) through \( \Gamma_L ^{\pm} = 2 \pi \nu_{\alpha}(\mu_{\alpha}) \left| V_{i,a}(\mu_{\alpha}) \right| ^2 \) and \( \Gamma_R ^{\pm} = 2 \pi \nu_{\alpha}(\mu_{\alpha} + U) \left| V_{i,a}(\mu_{\alpha} + U) \right| ^2 \), depending on the qubit state \( \sigma_{iz} = \pm 1 \) \( \left( V_{i,a}(\mu_{\alpha}) = V_{i,a}(\mu_{\alpha}) \pm \delta V_{i,a}(\mu_{\alpha}) \right) \) \( \nu_{\alpha}(\mu_{\alpha}) \) is the density of states of the electrodes (\( \alpha = L, R \)). The values of \( \Gamma_L ^{\pm} \) are determined by the geometrical structure of the system. The strength of measurement is parameterized by \( \Delta \Gamma_i \), \( \Gamma^{(\pm)}_i = \Gamma_0 \pm \Delta \Gamma_i \). The measurement strength \( \zeta \) is related to the tunneling rates as \( \Gamma_i = \Gamma_0(1 \pm \zeta) \) (\( \Gamma_0 \) is an unit). We call \( \downarrow \downarrow \), \( \downarrow \uparrow \), \( \uparrow \downarrow \), and \( \uparrow \uparrow \) \( |A \rangle \sim |D \rangle \) respectively, and four-qubit states are written as \( |AA \rangle, |AB \rangle, |BD \rangle, |DD \rangle \). For uniform two qubits, \( \Gamma_A = \Gamma_0(1 - \zeta^2)/2 \), \( \Gamma_B = \Gamma_C = \Gamma_0(1 - \zeta^2)/2 \) and \( \Gamma_D = \Gamma_0(1 + \zeta)/2 \) with \( \zeta = \Delta \Gamma / \Gamma_0 \). 

The DM equations of four qubits and detector at zero temperature of Fig. 1(a) are derived as in Ref. [13] by

\[ \frac{d \rho_{ij}^{a,b}}{dt} = \left( i [J_{ij} - J_{ji}] - \Gamma_{L}^{(i)} + \Gamma_{L}^{(j)} \right) \rho_{ij}^{a,b} \]
\[ - i \sum_{j=1}^{N} \Omega_{j} (\rho_{g_{j}^{(1)}},s_{j}^{(1)2} - \rho_{a_{j}^{(2)}},s_{j}^{(1)2} + \rho_{a_{j}^{(2)}},s_{j}^{(1)2}) + \sqrt{\Gamma_{L}^{(i)} \Gamma_{R}^{(j)}}, \rho_{ij}^{a,b} \]

(4)

where \( z_1, z_2 = AA, AB, ..., DD \) and, \( \rho_{a_{j}^{(1)}},s_{j}^{(1)2} \) are density matrix elements when no electron, one electron and two electrons exist in the QPC island, respectively. \( J_{AA} = \sum_{i} \xi_{i}+J_{12}+J_{23}, J_{AB} = \sum_{i} \xi_{i}-\xi_{i}+J_{12}+J_{23}, ..., J_{DD} = -\sum_{i} \xi_{i}+J_{12}+J_{23} \) \( g_{z_1}(z_2) \) and \( g_{z_2}(z_1) \) are introduced for the sake of notational convenience and determined by the relative positions between qubit states in Ref. [13]. We have 768 equations for four-qubits. 

To see the decoherence effect explicitly, we study time-dependent fidelity, \( F(t) \equiv Tr[\rho(0)\rho(t)] \) on the rotating coordinate as \( \rho(t) = e^{i \sum \Omega_{j} \sigma_{z,j} t} \rho(0) e^{-i \sum \Omega_{j} \sigma_{z,j} t} \) \( (\Omega_{j} = \sqrt{\Omega_{j}^{2} + \xi_{j}^{2}/4}) \) to eliminate the bonding-antibonding coherent oscillations of free qubits (trace is carried out over qubit states).

III. NUMERICAL RESULTS

Figure 2 shows the effect of the local structure, that is, an island in the QPC detector, on the fidelity of DF states. It is seen that the local structure greatly degrades the fidelity of qubit states. In particular, the degradation is large when the strength of measurement increases. Figure 2 also shows that non-DF two-qubit states are better than four qubit DF states when the strength of measurement is large. This result can be understood if we con-
FIG. 3: Time-dependent fidelity of four-qubit DF states $|\Psi_1^{[4]}\rangle$, $|\Psi_2^{[4]}\rangle$, and $|\Psi_3^{[4]}\rangle$ under various fluctuations: (i) $\Omega = 2\Gamma$, $\epsilon_3 = \eta\Gamma_0$ and $\Gamma_3^{(\pm)} = (1-\eta)\Gamma^{(\pm)}$. (ii) $\Omega = \Omega_0$ and $\epsilon_2 = \epsilon_4 = \eta\Gamma_0$ and $\Gamma_3^{(\pm)} = (1-\eta)\Gamma^{(\pm)}$. (iii) $\Omega = (1-\eta)\Omega$, $\epsilon_2 = \epsilon_4 = \eta\Gamma_0$ and $\Gamma_3^{(\pm)} = (1-\eta)\Gamma^{(\pm)}$. (a) $\eta = 0.01$ and $\zeta = 0.6$ (strong measurement), (b) $\eta = 0.05$ and $\zeta = 0.2$ (weak measurement). $\Omega = 2\Gamma_0$, $J_{ij} = 0$, $\epsilon_i = 0$, $\Gamma_s = \Gamma_0$.

We have solved master equations of four and two qubits with QPC detector, and discuss the robustness of DF states when there are a local structure and non-uniformities. We found that local structure is an obstacle to using DF states other than non-uniformities. We also showed that two-qubit non-DF states are candidates for the logical qubits even when there is some local structure.

FIG. 4: Fidelities of four-qubit DF states at $t = 50\Gamma_0^{-1}$ as a function of non-uniformity $\eta$. (a) Non-uniformity for 2nd and 3rd qubits (case (ii)). (b) Non-uniformity for 4th qubit (case (iii)). $\Omega = 2\Gamma_0$, $J_{ij} = 0$ and $\zeta = 0.2$.

IV. CONCLUSION

We have solved master equations of four and two qubits with QPC detector, and discuss the robustness of DF states when there are a local structure and non-uniformities. We found that local structure is an obstacle to using DF states other than non-uniformities. We also showed that two-qubit non-DF states are candidates for the logical qubits even when there is some local structure.

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