Zero-bias anomalies of point contact resistance due to adiabatic electron renormalization of dynamical defects

V.I.Kozub\textsuperscript{1,2} and A.M.Rudin\textsuperscript{1,3*}

\textsuperscript{1}A.F.Ioffe Physico-Technical Institute, St.-Petersburg, Russia
\textsuperscript{2}Delft Institute of Microelectronics and Submicron Technology, TU Delft, 2628 CJ Delft, The Netherlands
\textsuperscript{3}Institut für Festkörperforschung, Forschungszentrum Jülich, D52425 Jülich, Germany

We study effect of the adiabatic electron renormalization on the parameters of the dynamical defects in the ballistic metallic point contact. The upper energy states of the “dressed” defect are shown to give a smaller contribution to a resistance of the contact than the lower energy ones. This holds both for the “classical” renormalization related to defect coupling with average local electron density and for the “mesoscopic” renormalization caused by the mesoscopic fluctuations of electronic density the dynamical defects are coupled with. In the case of mesoscopic renormalization one may treat the dynamical defect as coupled with Friedel oscillations originated by the other defects, both static and mobile. Such coupling lifts the energy degeneracy of the states of the dynamical defects giving different mesoscopic contribution to resistance, and provides a new model for the fluctuator as for the object originated by the electronic mesoscopic disorder rather than by the structural one. The correlation between the defect energy and the defect contribution to the resistance leads to zero-temperature and zero-bias anomalies of the point contact resistance.

A comparison of these anomalies with those predicted by the Two Channel Kondo Model (TCKM) is made. It is shown, that although the proposed model is based on a completely different from TCKM physical background, it leads to a zero-bias anomalies of the point contact resistance, which are qualitatively similar to TCKM predictions.

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I. INTRODUCTION

Recent advances in nanofabrication technology have made it possible to visualize single defects with internal degrees of freedom - “fluctuators” \[1,2\], which lead to a “telegraph” resistance noise of nanometer scale systems. In metals these defects are believed \[3\] to be some structural defects, which at low temperatures are seen as the well-known two-level tunneling states (TLS) \[8\]. TLSs are the objects typical for strongly disordered amorphous solids \[8\] that switch by tunneling between their two possible configurations. Although the microscopic nature of the fluctuators stays unclear (especially for ballistic devices made of pure metals), the experiments allow to study various phenomenological parameters of these objects, in particular, the interlevel spacing.

Kondo \[9\] was the first, who has pointed out that in metals the parameters of the dynamical defects are strongly renormalized by electrons. One can discriminate between adiabatic electron “dressing”, which is related to a static electron response on the defect potential, and non-adiabatic one, which affects the tunneling process and leads to a renormalization of TLS tunneling matrix element (“dissipative tunneling”). It is this non-adiabatic effect that has attracted most attention (see

\[\textsuperscript{*}\text{Present address: Theoretical Physics Institute, University of Minnesota, Minneapolis, MN 55455}\]
e.g. Ref. [10]) due to its evident importance for the defect dynamics. As for the adiabatic renormalization of the defect parameters, it is customary to include it into the bare values. This procedure is usually justified by the fact that the adiabatic effects are related to a response of the whole electron systems while only a small strip of electron energies, close to the Fermi surface, is responsible for transport properties and sensitive to external factors like temperature or fields applied. However, as it was first demonstrated in Refs. [4, 6], the adiabatic “dressing” of the fluctuator, in particular, the adiabatic renormalization of the fluctuator energy splitting $E$, can be important and depends on the state of the electron system (e.g. on superconducting properties). Furthermore, very recently the surprising “magnetic tuning” of the TLS interlevel spacing observed [11] for TLSs in Bi nanoconstrictions was explained [12] as a direct result of the adiabatic renormalization of TLS parameters by electrons, which states are affected strongly by the magnetic field.

The purpose of the present paper is to show that the adiabatically renormalized energy of the fluctuator state correlates with the fluctuator contribution to the resistance. Namely, the conductance is larger for the fluctuator on its upper level. This fact causes, in particular, Kondo-like zero-temperature and zero-bias anomalies in differential conductance of the metallic point contacts. Indeed, an increase of temperature leads, in average, to an increase of occupation numbers of the fluctuator upper levels. The abovementioned correlation causes a corresponding conductance increase with temperature, which imitates the Kondo-like behavior. The same holds for an applied bias increase.

II. ADIABATIC RENORMALIZATION OF THE PARAMETERS OF THE DYNAMICAL DEFECTS IN METALS

Two mechanisms of the adiabatic renormalization can be considered. First one is due to possible difference of electron-fluctuator coupling potentials $V$ for the two of the fluctuator states ($|V(1)| \neq |V(2)|$) and was studied in Ref. [4, 6, 12]. We will show that for this mechanism (which will be referred to as “classical” one) the abovementioned correlation is due to the fact that expressions for the conductance and for the electron contribution to defect energy include the same strength of the electron-defect coupling.

Second mechanism of adiabatic renormalization of dynamical defects parameters was suggested by Altshuler and Spivak [13]. It implies mesoscopic electron density fluctuations, which lead to a difference, even for $|V(1)| = |V(2)|$, of electron-fluctuator coupling strengths for different fluctuator states due to their spatial resolution. The same correlation does occur for this “mesoscopic” contribution, as for the “classical” one. We will
show it for the experimentally important situation of ballistic point contact, where a description in terms of “local” interference, which involves finite number of scatterers, is possible.

A. Renormalization due to difference in defect-electron coupling potentials

Let us start from the “classical” effect. TLS energy splitting $E$ depends on the TLS tunneling parameter $\Delta_0$ and on the TLS asymmetry $\Delta$:

$$E = \sqrt{\Delta_0^2 + \Delta^2}. \quad (1)$$

Interaction of TLS with conduction electrons leads to the renormalization of both TLS asymmetry and of tunneling parameter. For the asymmetrical TLSs with the large enough barrier, $\Delta \gg \Delta_0$, the renormalization of TLS asymmetry gives the major contribution to $E$. As a result, the electron-TLS coupling leads to the renormalization of TLS “bare” energy splitting $E$ which has a form $E \rightarrow E + E_{el}$, where $E_{el} = E_{el,2} - E_{el,1}$,

$$E_{el,i} = \sum_{k,k'} V_{0}^{(i)} \left| \frac{f(\varepsilon_k) - f(\varepsilon_{k'})}{\varepsilon_k - \varepsilon_{k'}} \right| \approx - \frac{|V_{0}^{(i)}|^2}{\varepsilon_F}. \quad (2)$$

Here $V_{0}^{(i)}$ is the electron-TLS coupling constant for $i$-th TLS configuration. The electron bandwidth is assumed to be of order of $\varepsilon_F$, and the electron distribution $f(\varepsilon)$ to have an equilibrium Fermi form $f(\varepsilon) = f_0(\varepsilon) = (\exp(\varepsilon - \varepsilon_F)/T + 1)^{-1}$. Applicability of the second-order perturbation theory approximation is justified for Eq. (2) when $V_{0}^{(i)} \ll \varepsilon_F$. In contrast to Kondo-like corrections, Eq. (2) depends weakly on temperature. The renormalization $E_{el}$ is due to the difference in the values of the total electron energy, renormalized by a presence of TLS, for the TLS in states 1 and 2, respectively. It is important, that $E_{el}$ is not, due to adiabaticity, sensitive to the details of inter-state transition mechanism. Thus the problem is reduced to estimates of energies $E_{el,i}$ corresponding to different configurations of the defect which for this case can be considered as its independent realizations. Therefore, Eq. (2) holds both at low temperatures, when the fluctuator transitions are due to tunneling, and at higher temperatures when thermal activation dominates.

According to Eq. (2), the energy $E$ is lowered from the “bare” value. The lowering is larger the stronger is the coupling of the state with the electrons. On the other hand, a presence of the defect in the state $i$ inside a ballistic point contact with a characteristic size $d$ causes a reduction of the contact conductance

$$\delta G_i \approx - \frac{|V_{0}^{(i)}|^2}{\varepsilon_F^2} \left( \frac{\lambda_F^2}{d^2} \right) G, \quad (3)$$
where $\lambda_F$ is the Fermi wavelength, and $G \approx (e^2/h)(d^2/\lambda_F^2)$ is the Sharvin conductance. Making use of the Eq. (2) one obtains $\delta G_i \approx (e^2/h)(E_{el,i}/\varepsilon_F)$. Therefore, if the defect asymmetry is completely controlled by the conduction electrons, the conductance, which corresponds to the defect in its lower state (larger absolute value of the electron contribution to the defect energy) is smaller than that for the defect in the upper state.

**B. Interference contribution to the renormalization**

Let us consider now the electronic interference. For adiabatic effects different states of “active” defects, fluctuators, can be considered as independent realizations. Thus we can choose some configuration of some active defect as a “reference” scatterer $i$ and consider its properties in the presence of “background” scatterers. We will analyze both the interference contribution to conductance due to the defect $i$ and the “mesoscopic” renormalization of the energy of this defect, which is equal to a change of the electron system energy due to a presence of interference pattern involving defect $i$.

The ballistic point contact contains finite number of scatterers. Therefore the interference contribution to the contact conductance is provided by a local interference (involving trajectories with small number of scatterers) rather than by a global one (which leads to well-known UCF [16]). This “local” interference contribution has been to some extent analyzed in Ref. [17]. In what follows for simplicity we will restrict ourselves mainly to the interference patterns involving only pairs of scatterers. However, as it is shown in Appendix 1a, our results can be generalized for the case of arbitrary number of scatterers.

As it is shown in Appendix 1a [Eq.(27)], the contribution to the conductance due to a pair of scatterers, namely “reference” scatterer $i$ and the “background” $m$, is:

$$
\delta G_{im} = A_{G,im} \xi_{im},
$$

$$
A_{G,im} \approx \frac{e^2 |V_0|^2}{\hbar \varepsilon_F} \left( \frac{\lambda_F}{R_{im}} \right)^2.
$$

Here $\xi_{im} = \cos(2k_F R_{im})$ and $R_{im} \equiv R_i - R_m$ is the vector, which connects two scatterers. For simplicity we assumed that the scattering potentials for all scatterers, both “active” and “passive” ones, depend only on the coordinate $R_i$ of the scatterer: $V_{0kk} = V_0 \exp[i(k - k')R_i]$.

Let us now find a contribution to the energy of “reference” scatterer $i$, which represents one state of some fluctuator, due to its being involved in electron interference along with another scatterer $m$. Following the scheme implied by Eq. (2), it is given by a renormalization of electron system energy due to this pair of scatterers. In second-order perturbation theory approximation we obtain, collecting all terms proportional to $V_{0kk} V_{mkk'}^*$:
E_{el,im} = \text{Re} \sum_{k,k'} |V_0|^2 \exp \left[ i(k - k') R_{im} \right] \frac{f(\varepsilon_k) - f(\varepsilon_{k'})}{\varepsilon_k - \varepsilon_{k'}}.

(5)

A straightforward calculation for a spherical Fermi surface and zero temperatures gives [see Appendix 1a, Eq. (27)]:

E_{el,im} = A_{E,im} \xi_{im}, \quad A_{E,im} \approx \frac{|V_0|^2 \lambda_F^2}{\varepsilon_F R_{im}^3}.

(6)

The obtained renormalization is due to interaction of defect \(m\) with the Friedel oscillation of electron density originated by the defect \(i\).

Both \(\delta G_{im}\), Eq. (3), and \(E_{el,im}\), Eq. (5), are proportional to the same phase factor \(\xi_{im} = \cos(2k_F R_{im})\). Correspondingly,

$$\frac{\delta G_{im}}{(e^2/h)} \cdot \frac{\delta E_{el,im}}{\varepsilon_F} \sim \frac{\lambda_F}{R_{im}} > 0$$

and thus, in analogy with the “classical” effect, the larger is the energy of a configuration, the larger is the contact conductance. As it can be shown (see Appendix 1a) the proportionality to the same phase factor and, therefore, Eq. (3) holds not only for pairs of scatterers but for arbitrary number of scatterers as well.

To estimate a total interference contribution due to the defect \(i\) both to the conductance, \(\delta G_i\), and to the defect energy, \(E_{el,i}\), one must sum over all “background” scatterers \(m\). Due to a random distribution of \(R_{im}\) this results in some mesoscopic fluctuations for both quantities with respect to realizations of the system. However the fact that both quantities are linearly related to the same set of random factors \(\xi_{im}\) leads to the correlation between them, namely

$$\langle \delta G_i E_{el,i} \rangle = C \overline{\delta G E_{el}},$$

where \(\overline{x} \equiv \sqrt{\langle x^2 \rangle}, \quad C \approx 1\), and \(\langle \rangle\) denotes the ensemble average. More detailed argumentation of this is given in Appendix 2. For a given value of \(E_{el,i} = E\) one has

$$\langle \delta G_i \rangle_E = C \overline{\delta G E_{el}}.$$  

(9)

Keeping in mind factors \(1/R_{im}^2\) (for the conductance) and \(1/R_{im}^3\) (for the energy) one may suggest the main contribution to both these quantities to stand from the nearest neighbors. In this case both energy renormalization and contribution to resistance would be related to a few neighbouring defects and thus could be estimated by Eqs. (4) with \(R_{im}\) of the order of most probable interdefect distance \(N_i^{-1/3}\) (where \(N_i\) is the background defect concentration). Note that if we took an average over all possible realizations of the background scatterers we would met the problem with singular behavior of the averaged quantities when \(R_{im} \to 0\). It means that the
average is mainly controlled by (rare) realizations corresponding to very close neighbouring background defect, and one has $R_{im} > k_F^{-1}$, that would give $|\delta G| \sim e^2/h$, $E_{el,i} \sim E_F/(k_Fl)^{1/2}$. However, in the case of small ballistic contact we deal with some given contact realization, so that one deals with the most probable rather than with average quantity (compare with \[15\]).

For the ballistic point contact the number of defects in the contact is small, so that the dominant contribution is expected to be from the trajectories, which involve a boundary of the contact. Assuming the contact to be a short channel with a length $\approx d$, this boundary may be considered as an array of scatterers at a distances $\approx d$ from the defect $i$ with a total number $\approx (d/\lambda_F)^2$. For this case

$$\delta G \approx \left( \frac{\lambda_F}{d} \right) \frac{e^2}{h}, \quad E_{el,i} \approx E_F \left( \frac{\lambda_F}{d} \right)^2.$$  

(10)

Taking values of $d$ typical for nanofabricated ballistic point contacts, $d \approx 5 - 10 \text{nm}$ one gets $|\delta G| \approx (0.05 - 0.1)e^2/h$, $E_{el,i} \approx 30 - 100K$.

The mesoscopic interference renormalization has some special features as compared with the “classical” one. First, mesoscopic disorder lifts energy degeneracy of the defect states, which have different spatial positions. Thus it causes a formation of fluctuators from otherwise symmetric defect configurations (to say, interstitials, which have symmetrical lattice positions).

Then, in this case one expects the temperature and bias behavior of the resistance to depend on an external magnetic field, which affects the electron interference (see e.g. Ref. \[16\]).

In addition, the interference contribution both to the defect energy and to the conductance depends on the electron distribution. The finite applied bias makes this strongly nonequilibrium, which at high enough biases causes a “direct” effect of bias on both quantities.

The physical picture of the mesoscopic renormalization is much richer than that provided by “classical” one, and it is this mechanism that we will concentrate on in the rest of the paper.

III. ZERO-BIAS AND ZERO-TEMPERATURE RESISTANCE ANOMALIES

Let us consider defect $i$ which occupies either of the two neighboring positions, 1 and 2, with close energies. For simplicity we will assume that the energy asymmetry of these defect states is completely determined by the electron renormalization. This object is a sort of two-level fluctuator originated from the electronic disorder rather than from the lattice one. It is important, that due to the correlation discussed above, the upper state of such fluctuator, which corresponds to a defect position with higher energy, gives a smaller contribution to the contact resistance. A conductance increase, which accompanies
a transition from the lower to the upper level of such fluctuator, \( \delta G_i = \delta G_{i1} - \delta G_{i2} \) is, according to Eq. (9), scaled with the energy asymmetry \( E_i = E_{el,i1} - E_{el,i2} \). Here index \( i \) now denotes the fluctuator.

A summation over all fluctuators gives their total contribution to the average contact conductance:

\[
\Delta G = \sum_i \delta G_i n_i = \int \langle \delta G_i \rangle_E P(E) n(E) dE. \tag{11}
\]

Here \( n_i = n(E_{i}) \) is the \( i \)-th fluctuator upper level occupation number and \( P(E) \) a density of states given by statistical properties of \( E_{el,i} \). For the mesoscopic system it is reasonable to take the values of \( E_{el,i} \) for the neighboring defect positions as statistically independent. In this case \( P(E) \) is approximately constant at small \( E \ll E_{el} \). Making use of Eq. (9) and of the expression \( n(E) = [1 + \exp(E/T)]^{-1} \) one has at small temperatures \( T \ll E_{el} \) the conductance enhancement

\[
\Delta G \propto T^\beta,
\tag{12}
\]

where \( \beta = 2 \).

For some defects, like light interstitials or some defect complexes, the probabilities for defect hopping between spatially symmetric positions are relatively high \[18\]. For these “delocalized” defects the effect of electronic disorder provides a many-site “potential relief” instead of two-site fluctuator picture. Assuming that any site can be occupied by only one defect one deals with "Fermi-type" statistics; so at \( T \to 0 \) the sites with lowest energies are occupied by the mobile defects while those with energies higher than the "Fermi level" are free. At finite temperature Eq. (11) can be applied where the site occupation number has again the form \( n(E) = [1 + \exp(E/T)]^{-1} \) if one takes the "Fermi level" as the origin for the energy \( E \). In this case the total number of available sites is much larger than the number of defects \( N \), and for finite temperatures the "Boltzmann-type" statistics holds rather than the "Fermi-type":

\[
n(E) = \frac{N \exp(-E/T)}{\int P(E) \exp(-E/T) dE} \tag{13}
\]

In this case a change of \( T \) does not affect the number of rearranged defects (because any of them can change its energy) and leads only to a change of the average defect energy. As a result, in this case we have in Eq. (12) \( \beta = 1 \) independently of the form of the density of states \( P(E) \).

Let us turn now to the effect of finite bias \( eV \gg T \). For TLS it was first considered in Refs. \[5,19\]. It was shown that for low-energy TLS with small enough energy splitting \( E \), for which the coupling with electrons dominates \[20\], the TLS occupation numbers are sensitive to the electron distribution. For the contact region this is strongly nonequilibrium and for a central point of symmetric contact has a form

\[
f(k) = \theta(k_x) f_0(\varepsilon_k + eV/2) + \theta(-k_x) f_0(\varepsilon_k - eV/2),
\tag{14}
\]
where \( OX \) is the main axis of the contact and \( \theta(x) \) the theta function. The “energy width” of this distribution, \( eV \), plays a role of the effective temperature. In particular, the upper levels of the TLS are empty if \( eV < E \), while for \( eV \gg E \) the occupation numbers of TLS levels are almost equal and \( n(E) \approx (1/2)[1 - (E/eV)] \).

For larger \( E \), the coupling with phonons becomes important due to rapid increase of density of states for the actual phonons with \( E \) increase. For the two-state case the fluctuator relaxation rate due to electron-assisted tunneling is \( W_{el}(E,V) \approx (|V_{0} - V_{0}^{0}|/\varepsilon_{F})^{2}[(eV - E)/\hbar]T \), where \( V_{0} = (V_{0}^{(2)} - V_{0}^{(1)})/2 \).

For the phonon-assisted process \( W_{ph}(E,T) \approx (\Lambda^{2}E^{2}/\mathcal{E} \Theta_{D}^{3})(E/\hbar)\csch(E/2T)T \). Here \( \Lambda \) is the fluctuator deformational potential, \( \mathcal{E} \) and \( \Theta_{D} \) are the atomic and Debye energies, respectively. \( T = \exp(-\lambda) \), where \( \lambda \) is the tunneling constant.

Let us define the characteristic energy \( E^{*} \) for which \( W_{el}(E^{*},eV - E^{*}) \approx W_{ph}(E^{*},T = 0) \); for the reasonable values of parameters (see e.g. Ref. [20]) \( E^{*} \) is expected to be \( \approx 1 - 3K \). For \( eV \gg E^{*} \) a probability of electron-assisted tunneling to the upper level \( W_{el} \) exceeds the probability of phonon-assisted decay of the upper level \( W_{ph} \) up to some threshold energy \( E = E_{th} = E^{*}(eV/E^{*})^{1/3} \) at which an increase of the electron phase volume with bias \( \propto eV \) is compensated by a corresponding increase of phonon phase volume \( \propto E^{3} \). For the crude estimates let us take the occupation numbers \( n(E) \propto \theta(E_{th} - E) \). Now making use of Eq. (14) and assuming density of states \( P(E) \) constant, one obtains the following interpolation formula for the interference contribution to the conductance:

\[
\Delta G \propto \left[ E^{*} \left( \frac{eV}{E^{*}} \right)^{1/3} + T \right]^{\beta} \tag{15}
\]

with \( \beta = 2 \).

The same considerations can be applied for the case of “delocalized” defects. Although the probabilities \( W_{el} \) and \( W_{ph} \) for this case can differ from ones for the two-level fluctuators, the scaling \( W_{el}/W_{ph} \sim eV/E^{3} \) (relation between relevant electron and phonon phase volumes) holds for \( eV \gg E \geq T \) and thus Eq. (15) is valid, but with \( \beta = 1 \).

IV. DIRECT EFFECT OF THE APPLIED BIAS ON THE FLUCTUATOR PARAMETERS

When a large enough bias \( V \) is applied to the point contact one should take into account the non-equilibrium electron distribution in course of estimates Eqs. (4), (6), (9). For the distribution given by Eq. (14) one obtains (see Appendix 1b) for \( \Delta E_{el,im} \) the phase factor

\[
\Delta E_{el,im} \propto \cos[2k_{F}R_{im} + \phi(V, R_{im}, k_{F})] \cos(2\Delta kR_{im}), \tag{16}
\]
and for $\delta G_{im}$ a factor

$$
\delta G_{im} \propto \cos[2k_F R_{im} + \phi(V, R_{im}, k_F)] \cos(2\Delta k R_{im})
+ \frac{\partial \phi}{\partial V} \frac{1}{2R(\partial \Delta k/\partial V)} \sin[2k_F R_{im} + \phi(V, R_{im}, k_F)] \sin(2\Delta k R_{im}),
$$

(17)

where $\Delta k \equiv k_F eV/\varepsilon_F$. This is the "direct" effect of bias on the fluctuators parameters in addition to tuning of fluctuator level occupation numbers.

As seen, the first term in Eq. (17) is correlated with the phase factor of Eq. (16) while the second is not and, thus, will sum out. As for the correlated cosine terms, the effect of bias initially (at $\Delta kR \ll 1$) leads to their decrease due to a decrease of the corresponding cosine factors, while for $\Delta kR \gg 1$ (when the factors are random with respect to parameter $R_{im}$) they are suppressed due to additional (with respect to the case $V = 0$) averaging over $R_{im}$:

$$
(\Delta E_{el}, \Delta G) \propto (\Delta kR)^{-1/2}.
$$

Actually we deal here with the well-known energy averaging effect suppressing any mesoscopic phenomena.

It is important to note that these effects can lead to the resistance anomalies even if the defect structure is not rearranged in course of the bias application; the only condition is that the defects occupy the positions with the lowest energies available and thus with largest mesoscopic contribution to resistance $\delta G_{im}$. For these defect configurations one gets in average $\delta G < 0$. Total interference contribution to the conductance due to $N$ defects is:

$$
\Delta G \sim N F(V) \delta G,
$$

(18)

where $F \sim \cos(\Delta kR)$ for $\Delta kR < 1$, and $F \sim (\Delta kR)^{-1/2}$ for $\Delta kR \gg 1$. The result we obtained is that the bias increase leads to a systematic conductance increase. It is interesting that in combination with the effects discussed for relatively small $V$ — occupation of states with higher energies, this "direct" effect can form the configurations with smaller resistances than available for a simple temperature increase. Indeed, it can suppress (negative in average) mesoscopic contribution to conductance due to configurations with large enough energy gap between the available realizations which can not be rearranged at relatively small temperatures when the phonon contribution to resistance (obviously masking the effects in question) is still small. Note that as we have seen above the bias values allowing the same occupation states of the defects and at the same time the same efficiency of electron-phonon processes as in equilibrium state with a temperature $T$ is scaled with $T$ as $eV \approx T(T/E^*)^3$. Thus the energy averaging effects can become to be pronounced for large-gap configurations when the filling of the upper level is still negligible.

Certainly, the temperature increase can also lead to the energy averaging, but the necessary temperatures are too large and correspond to a significant phonon contribution to resistance.
In this section we would like to make several remarks about the limitations and possible complications of our model.

First, until now we considered the defect energy density of states as constant. The limitation of the defect energy band leads to a saturation of $\Delta G(T)$ and $\Delta G(V)$ dependencies at $T > T_{\text{sat}}$ and $V > V_{\text{sat}}$, respectively. These quantities scale as $eV_{\text{sat}} \approx T_{\text{sat}}(T_{\text{sat}}/\varepsilon_F)^3$, and $\Delta G_{\text{sat}} \approx N_f \delta G$, where $N_f$ is a total number of fluctuators. As for the estimate for $T_{\text{sat}}$, taking $\varepsilon_F \approx 10^4 - 10^5$K and $(d/\lambda_F) \approx 50$ and making use of Eq. (10) we obtain $T_{\text{sat}} \approx E_{\text{el},i} \approx 4 - 40$K. Note that the “saturation” value $\Delta G_{\text{sat}}$ corresponds to a random realization of different interference patterns involving the fluctuators, while the values of $\Delta G$ at lower temperatures correspond to preferable occupation of larger resistance states and thus are systematically smaller than typical for mesoscopic disorder.

Second, it is important that the picture discussed is sensitive to the external magnetic field. In particular, it is known that in homogeneous diffusive conductors the interference particle-particle channel is suppressed in the strong enough magnetic field $H > \Phi_0/L_c^2$, where $\Phi_0$ is the quantum of the magnetic flux and $L_c$ is a coherence length, instead of which in our case we should use a contact size $d$. This suppression reduces a magnitude of mesoscopic fluctuations nearly twice $[13,16]$. Point contact is a strongly inhomogeneous system and the main contribution to the mesoscopic fluctuations is due to local interference. However, despite the fact, that the effect of magnetic field implies a contribution of configurations which involve more than two scatterers, this contribution is relatively large due to rather high probability of the boundary scattering and lead to decrease of $\Delta G$ with field increase.

Another important feature is related to a coupling between different “active” defects, $i$ and $j$, due to a dependence of the defect $i$ energy on a position of defect $j$. This dependence is given by Eq. (6). For large enough concentration of “active” defects one may expect a formation of self-organized aggregates in the defect system (of spin-glass type). Indeed, the defect positions corresponding to maxima of the Friedel oscillations originated by the other defects become energetically preferable, which introduces some “ordering” into the defect system. Thus a formation of “coherently scattering” aggregates can be possible leading to a significant enhancement of the interference contribution to resistance. The increase of bias is expected first to suppress this contribution to the resistance in a way similar to discussed above. On the other hand, at higher biases the direct bias-induced decrease of “coupling potentials” $E_{\text{el},ij}$ (see Eq. (14)) can destroy such aggregates, which can lead to sharp resistance changes.

Finally, it is instructive to compare results of our model
with the two-channel Kondo model (TCKM) [22,23], which also predicts zero-bias resistance anomalies of a non-magnetic nature. Despite these two models are based on completely different physical background, they predict qualitatively similar resistance behavior at low $T$ and $V$: negative temperature and bias resistance coefficients affected by a magnetic field. However, quantitative predictions of the two models differ. Our model does not predict a singular $T$-behavior at $T \to 0$ - in contrast to TCKM. As for bias dependence, our model predicts singular behavior, $V^{2/3}$ for biases $V \gtrsim 1-3mV$ (see Eq.(15)) and saturation at smaller biases. This saturation can imitate the “restoration of the Fermi liquid behavior” predicted by TCKM. On the other hand, TCKM, being related to non-adiabatic effect, is relevant to the fluctuators of a rather special type (with a small asymmetry and large tunneling probability), while our predictions hold for any sort of mobile defect. Our model also predicts special features at higher temperatures and biases; in particular, the saturation of the zero-bias anomaly at large $V$ and $T$, and a principal possibility to reach larger values of conductance in course of bias increase with respect to ones available for temperature increase.

VI. CONCLUSIONS

To conclude, we predict a new mechanism of zero-bias resistance anomalies in metallic point contacts based on a found correlation between energies of defects with internal degree of freedom and their contributions to resistance. The correlation lifts to some extent the ”random” character of mesoscopic disorder and breaks the symmetry of the defect states with respect to the signs of the mesoscopic contribution to resistance. We suggest a model of a fluctuator related to a purely electronic disorder, which provides a new insight into the nature of fluctuators in the perfect point contacts.

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VII. APPENDIX 1
A. Calculation of the mesoscopic contribution to the conductance and to the defect energy at small applied biases.

To calculate mesoscopic contributions both to conductance and to energy of electron system due to the presence of some finite number of scatterers we will make use of the "wave-optics" approach [24]. The approach is based on perturbation theory in real space. Let us consider an electron, the wave function of which initially is a plane wave with the wave vector $k$. After $n$ successive scattering events involving scatterers $1, \ldots, n$ the electron wave function becomes:

$$|1, \ldots, n> \equiv \psi_{1, \ldots, n}(r) = f^n |r - R_n| \ldots |R_2 - R_1|$$

$$\times e^{ikr_1} \exp (ik(|r - R_n| + \ldots + |R_2 - R_1|)) \quad (19)$$

where $R_1, \ldots R_n$ are the positions of the scatterers. For the short-range scatterers the scattering amplitude $f$ in the Born approximation takes the form:

$$f = -\frac{m^2}{2\pi m^2} \int d^3r V(r) \quad (20)$$

and is assumed the same for all scatterers. Contribution of the scatterers to the conductance is determined by their backscattering efficiency. The interference contribution to the backscattering current due to trajectories involving scatterers $1, \ldots, n$ and $1', \ldots, n'$ is

$$\delta j = \frac{ie\hbar}{2m} (|\langle 1', \ldots, n'|\nabla|1, \ldots, n>| + \text{c.c.}) \quad (21)$$

To obtain a contribution to the conductance one should integrate this equation over $r$ within some reference plane remote from the scatterers system. It is important that only position of last scatterers $n$ and $n'$ are relevant for this integration and one deals with a factor

$$\int d^2\rho \exp [ik(|r - R_n| - |r - R_{n'}|)] .$$

Here $\rho$ is a projection of $r$ on the plane in question. Taking the plane to be normal to $R_n - R_{n'}$ and expanding the exponent as $[\ldots] \sim |R_n - R_{n'}|^2 + |R_n - R_{n'}|^2/r^2$ one gets the result of the integration in the form:

$$\frac{1}{ik|R_n - R_{n'}|} \exp (ik|R_n - R_{n'}|).$$

The next step is the integration over $k$ directions. In its turn, this integration is relevant only to co-ordinates of the "first" scatterers in the chains that is to $R_1$ and $R_{1'}$, which enters the exponential factor $\exp[k(R_1 - R_{1'})]$. Correspondingly, the integration over $\cos \theta$ where $\theta = \angle (k, R_1 - R_{1'})$ gives the factor

$$\frac{1}{ik|R_1 - R_{1'}|} \exp (ik|R_1 - R_{1'}|).$$
Finally one arrives at the following estimate for the mesoscopic contribution to the conductance:

\[
\frac{\delta G}{G} \sim \frac{f^{n+n'}}{k^2a^2R^{n+n'}} \times \cos[\varphi(n, n', n'-1, \ldots, 1'; 1, n, n-1, \ldots, 1)] \tag{22}
\]

where the phase \( \varphi \) is

\[
\varphi(n, n', n'-1, \ldots, 1'; 1, n, n-1, \ldots, 1) = k(|R_{1'} - R_1| + |R_{n'} - R_n| + |R_{n''} - R_{n'-1}| + \ldots + |R_{1''} - R_{1'}| - |R_n - R_{n-1}| - \ldots - |R_2 - R_1|). \tag{23}
\]

Here \( R \) is a typical interscatterer distance within the chains while \( d \) is the contact size appearing as a result of normalization of the backscattering efficiency on the incident electron flow.

The picture discussed can be interpreted as a contribution to scattering due to presence of the scatterer \( n \) affecting the superposition of states formed by successive scattering by chains \( 1, \ldots, n-1 \) and \( 1', \ldots, n' \). The phase \( \varphi \) which after the integration over \( k \) directions is the phase difference for the paths \( n, n', n'-1, \ldots, 1', 1 \) and \( n, n-1, \ldots, 1 \), correspondingly. One should also note that in course of derivation of Eq.(22) we have taken into account that only those electrons with energies close to the Fermi energy contribute to the conductance, and used \( k = k_F \) in Eq.(22).

Now let us estimate the mesoscopic contribution to the electron energy due to the presence of the same system of the scatterers finally affecting the electron state in the position of scatterer \( n \). In the lowest approximation one has

\[
\delta E_{el} = <1', \ldots, n'|V(r - R_n)|1, \ldots, n-1>
\]

where \( V \) is a scattering potential assumed to be short-range: \( V = V_0\delta(r) \). As a result of averaging over the direction of \( k \) we obtain

\[
\delta E_{el} \approx -V_0 \frac{f^{n+n'-1}}{kR^{n+n'+1}} \times \sin[k\varphi(n, n', n'-1, \ldots, 1'; 1, n, n-1, \ldots, 1)] \tag{25}
\]

It is important that due to the same structure of the expression for \( \delta E_{el} \) as of one for the \( \delta G \) the phase difference for the interference pattern \( \varphi \) is exactly the same.

In order to obtain the interference correction to the energy of the whole electron system we should sum Eq.23 over all occupied electronic states. For \( T = 0 \) one has

\[
\delta E_{el} \sim -V_0 \frac{f^{n+n'-1}}{\pi^2 R^{n+n'+1}} \int_0^{k_F} kd\sin[k\varphi(n, n', n'-1, \ldots, 1', 1; n, n-1, \ldots, 1)] =
\]

\[
= -V_0 \frac{f^{n+n'-1}k_F}{\pi^2 R^{n+n'+2}(n+n')} \cos[\varphi(n, n', n'-1, \ldots, 1', 1; n, n-1, \ldots, 1)] \tag{26}
\]

Thus we conclude that the mesoscopic contributions of the same system of scatterers to the conductance and to electron energy renormalization depend on the same phase factor and, therefore, are correlated.

For the simplest case of 2 scatterers (positioned in \( R_1 \) and \( R_{1'} \)) site \( R_1 \) plays at the same time a role of the site \( R_n \). The phase factor in this case is

\[
\cos(k|R_{1'} - R_1| + k|R_{1'} - R_1|) = \cos(2k|R_{1'} - R_1|)
\]

and, correspondingly,

\[
\frac{\delta G}{G} \sim \frac{f^2}{a^2k_F^2R^2} \cos(2kR), \quad \delta E_{el} \sim -\frac{fV_0k_F}{R^3} \cos(2kR)
\]

(27)

which gives Eqs.(11) and (13).
B. Direct effect of bias on the interference contributions

In this subsection we will study the "direct" effect of bias on the renormalization of the fluctuator energy and on the interference contribution to the conductance for the simplest case of 2 scatterers.

We start with deriving $E_{el}$. For the step-like electron distribution given by Eq. (14) one obtains for the integral over $k$ (instead of Eq. (26)):

$$\frac{1}{2} \int_{k_F-\Delta k}^{k_F+\Delta k} dk \cdot \sin(2kR + \phi(V, R, k_F)) + \int_0^{k_F-\Delta k} dk \cdot \sin(2kR + \phi(V, R, k_F)) =$$

$$\int_0^{k_F+\Delta k} dk \cdot k \sin(2kR + \phi(V, R, k_F)) - \frac{1}{2} \int_{k_F-\Delta k}^{k_F+\Delta k} dk \cdot k \sin(2kR + \phi(V, R, k_F))$$

(28)

Here $\Delta k = eV/\bar{h}$. An additional $V$-dependent phase $\phi$ is related to a dependence of $k$ on coordinate due to a presence of electric field ($\bar{h}k^2/2m + \varphi(r) = \varepsilon = const$). Calculation of the integral gives the phase factor

$$\cos[2(k_F + \Delta k)R + \phi(V, R, k_F)] + \sin(2k_F R + \phi) \sin(2\Delta k R) = \cos(2k_F R + \phi) \cos(2\Delta k R)$$

(29)

instead of the factor $\cos(2k_F R)$ obtained for $V \to 0$.

In the same way we estimate the contribution to conductance at $T = 0$:

$$\delta I \propto \int_{k_F-\Delta k}^{k_F+\Delta k} \cos(2kR + \phi(V, R, k_F)),$$

$$\delta G = \frac{dI}{dV} \propto \frac{\partial \Delta k}{\partial V} [\cos(2(k_F + \Delta k)R + \phi(V, R, k_F)) + \cos(2(k_F - \Delta k)R + \phi(V, R, k_F))]
- \frac{\partial \phi}{\partial V} \int_{k_F-\Delta k}^{k_F+\Delta k} \sin(2kR + \phi(V, R, k_F))$$

$$\sim \frac{\partial \Delta k}{\partial V} 2 \cos(2k_F R + \phi(V, R, k_F)) \cos(2\Delta k R) + \frac{\partial \phi}{\partial V} \frac{1}{2R^2} \sin(2k_F R + \phi(V, R, k_F)) \sin(2\Delta k R)$$

(30)

Taking into account that $\phi \sim \Delta k R \cdot R/a$, one sees that the second and the first terms in r.h.s. of Eq. (30) are of the same order provided that $R/a \approx 1$. However the first term completely correlates with the corresponding phase factor for the energy renormalization, Eq. (29), while the second term does not.

VIII. APPENDIX 2

For each "active" defect $i$ the interference contribution to the conductance, $G_i$, as well as to the energy, $E_{el,i}$, contains summation over "background" scatterers $m$. Contribution of each scatterer $m$ gives some phase factor $\xi(R_{im}) \equiv \xi_{im} = \xi_{mi}$ which depends on the distance of the scatterer $m$ from the defect. Hence, one can rewrite the expressions for $\delta G_i$ and $E_{el,i}$ in a form:

$$\delta G_i = \sum_m G_{im} \xi_{im} \equiv (\mathbf{G}_i, \mathbf{\xi}_i),$$

(31)

$$E_{el,i} \equiv \sum_m E_{im} \xi_{im} \equiv (\mathbf{E}_i, \mathbf{\xi}_i)$$

(32)
Here we have introduced some "vector space", where vector $\vec{\xi}_i$ contains the set of the corresponding phase factors, and vectors $E_i$ and $G_i$ contain the sets of the prefactors (given by Eqs. (3) and (5), correspondingly). For the ensemble of defects $i$ the vector $\vec{\xi}_i$ should be considered as random, while all components of $E_i$ and $G_i$ are positive. We may rewrite the vectors $E_i$, $G_i$ as

$$E_i = \frac{E_i}{E_i} \bar{E}_i, \quad G_i = \frac{G_i}{G_i} \bar{G}_i \quad (33)$$

where we have introduced the "norms" of the vectors $E_i$ and $G_i$. The correlator $\langle \xi_m, \xi_n \rangle = \gamma \delta_{m,n}$ (where for the cosine phase factors $\gamma = 1/2$), and we obtain

$$\langle \delta G_i, E_{cl,i} \rangle = \langle (G_i, \xi_i)(E_i, \xi_i) \rangle = \gamma \bar{E}_i \bar{G}_i \frac{E_i}{E_i} \frac{G_i}{G_i} \quad (34)$$

The scalar product of the normalized positively-defined vectors in the brackets is of the order of unity, and one comes to the estimate for the average, Eq.8, $\langle ... \rangle = C \delta G \bar{E}_{cl}$ (according to definitions given in front of Eq.7, $\bar{E} = \bar{E}_E$ and $\bar{G} = \delta G$.

Representing vector $G$ as a sum of components ”parallel” and ”normal” to the vector $E$: $G = G_E + G_\perp$ one has $(G_E, E) = (G, E), (G_\perp, E) = 0$, and, finally,

$$G_E = \frac{(G, E)}{E^2} E = C \frac{\delta G}{E} E$$

Decomposing in the same way the random vector $\vec{\xi}_i$ on the components ”parallel” and ”normal” to $E$ ($\xi_E$ and $\xi_\perp$) and taking into account that

$$\langle \delta G \rangle_E = \langle (G_E + G_\perp, \xi_E + \xi_\perp) \rangle_E,$$

$$(G_E, \xi_\perp) = (G_\perp, \xi_E) = 0$$

and $\langle (G_\perp, \xi_\perp) \rangle = 0$ we finally have

$$\langle \delta G \rangle_E = \langle (G_E, \xi_E) \rangle = C \frac{\delta G}{E} (E, \xi_E) = C \frac{\delta G}{E} E$$

which corresponds to Eq.9. It means that $\delta G$ has a linear regression with respect to $E_{cl}$.

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