Scalar condensate decay in a fermionic heat bath in the early universe

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Abstract. We consider one-loop thermal effects on the decay of a scalar field zero mode initially dominating the energy density of the universe. We assume fermionic decay channels and take into account the effects due to both particle and hole excitations, and present approximate expressions for the absorption and decay rates. We apply the results to the inflaton and solve the Boltzmann equations to find the temperature evolution of the fermionic plasma. We show that the reheat temperature can be greater than the inflaton mass if the inflaton decays into more than one fermionic species.

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1. Introduction

Scalar condensates often play an important role in cosmology. For example, the superluminal expansion of the early inflationary universe is driven by primordial dark energy, which in scalar field inflation is the energy of the zero mode inflaton condensate $\phi$. At the end of inflation the condensate $\phi$ oscillates and then decays roughly speaking when the decay rate $\Gamma_\phi > H$, where $H$ is the Hubble rate. The decay products are assumed to thermalize almost instantaneously by scattering against each other, thus reheating the cold universe. While the inflaton condensate is decaying, the expansion rate of the universe depends on the varying relative magnitudes of the plasma and the condensate energies, which thus affect the decay and reheating processes. It turns out [2, 3, 4] that there is a maximum temperature $T_{\text{MAX}}$ which the plasma can attain; this is reached when the plasma energy is still subdominant. The reheat temperature $T_{\text{RH}}$ marks the time after which the universe is dominated by the relativistic gas of the decay products.

The process of thermalization of a scalar condensate in the heat bath of its decay products in an expanding universe is a generic problem that is relevant not only for inflation but, for example, also for the decay of the squark/slepton condensate of the minimally supersymmetric standard model (MSSM) [5]. In thermalized plasma the dispersion relations of the decay products are modified by what is commonly known as plasma masses [6] (the role of plasma masses in early universe was first noted in [7]). These can affect the decay rate, as was recently discussed in [8] in the context of inflation. There the plasma masses were put in by hand into the decay rate $\Gamma_\phi$ to provide a kinematic suppression factor. The authors found that the inclusion of the plasma masses gives rise to an era of constant temperature during reheating; this happens because the masses of the decay products of the inflaton increase with the temperature and at some point the energy of the decay products would exceed the energy of the inflaton zero mode, rendering the decay kinematically forbidden.

However, a thermal background induces modifications in the scalar condensate decay rate which are more subtle than a simple phase space suppression. If one assumes that the scalar condensate decays mainly into fermions, as we will do in this paper, the plasma has two kinds of elementary fermionic excitations, called particles and holes. Particles are modes which in the limit $T \to 0$ yield ordinary elementary excitations, whereas holes are collective modes present in thermal bath only; they are called "holes" since these excitations correspond to the removal of an antiparticle, creating a state with all the same quantum numbers as a particle state, but with different energy [9]. Thus for instance an inflaton condensate can decay into a pair of particles or a pair of holes, or it can transform into a particle by absorbing a hole from the thermal bath. All
these processes contribute to the total thermalization rate of the inflaton condensate and need to be evaluated.

In this paper we calculate the thermalization rate of a scalar condensate decaying into fermionic particles at finite temperature. The results could be applied to any situation where there is an initially dominant scalar field decaying in an expanding universe. Here we focus on the reheating phase of the early universe.

2. Finite temperature thermalization rates

Let us consider the interaction Lagrangian

\[ \mathcal{L} = g_Y \phi \bar{\psi} \psi, \]  

(1)

where \( \phi \) is a some scalar field that forms a zero mode condensate in the early universe, and \( \psi \) is a fermion. Here \( g_Y \) is the Yukawa coupling between the scalar and the fermion. In what follows we will neglect the self-couplings of \( \phi \) as well as its couplings to other scalars and assume that the fermions \( \psi \) are brought to thermal equilibrium instantaneously after they are produced by virtue of \( \psi \bar{\psi} \) scatterings. We also assume that the rest masses of the fermions are much smaller than the ambient temperature \( T \), so we can take the fermionic dispersion relation to be dominated by the first order temperature correction \( m = C g T \), where \( g \) is a coupling constant appropriate for \( \psi \bar{\psi} \) scattering and \( C \) is some number, which we take to be 1 for simplicity.

The thermalization rate \( \Gamma_{\phi}(T) \) of a scalar condensate has been computed in the past (see e.g. [10]) and is given by the imaginary part of the zero mode propagator \( \Sigma \); at one-loop level this is the cut of the relevant one-loop diagrams at finite \( T \). In general one can write

\[ \Gamma_{\phi}(T, \omega) = -\frac{\text{Im} \Sigma(\omega, T)}{\omega}, \]  

(2)

where \( \omega \) is the energy of the given mode. \( \Sigma \) receives contributions from all the fermionic modes present at finite temperature.

When considering the thermalization of a scalar field, there are two kinds of processes. First, the scalar zero mode can decay directly into a pair of particles or a pair of holes. The energies of the produced particles are then constrained by

\[ M_\phi = 2 \omega_i(k) ; \ i = p, h, \]  

(3)

where \( p \) denotes particle and \( h \) hole. Mostly we will be interested in a cold condensate dominating the universe, such as the inflaton or the MSSM flat direction condensate. Hence we may ignore thermal corrections to the scalar mass and take \( \omega = M_\phi \).

Second, if the mass of the scalar is small enough (or temperature large enough), the possibility opens for a hole in the heat bath to absorb the scalar to produce a particle.
This could be called thermal fragmentation of the condensate. In this case the energies are constrained by

\[ M_\phi + \omega_h(k) = \omega_p(k). \] (4)

As will be discussed below, it turns out that for thermal fragmentation to occur, the temperature of the thermal bath has to become larger than the scalar rest mass. However, since the scalar-fermion (heat bath) coupling, \( g_Y \), is typically much smaller than the coupling \( g \) relevant for \( \psi \bar{\psi} \) scattering, we can ignore the thermal mass of the scalar.

The dispersion relations for fermionic positive energy hole and particle excitations with thermal masses \( m = gT \) can be written as [9, 10]

\[ \hat{\omega}_p - \hat{k} - \frac{g^2}{k} - \frac{g^2}{2k} \left( 1 - \frac{\hat{\omega}_p}{\hat{k}} \right) \ln \left| \frac{\hat{\omega}_p + \hat{k}}{\hat{\omega}_p - \hat{k}} \right| = 0, \] (5)

\[ \hat{\omega}_h + \hat{k} + \frac{g^2}{k} - \frac{g^2}{2k} \left( 1 + \frac{\hat{\omega}_h}{\hat{k}} \right) \ln \left| \frac{\hat{\omega}_h + \hat{k}}{\hat{\omega}_h - \hat{k}} \right| = 0, \] (6)

where \( \hat{\omega} = \omega/T, \hat{k} = k/T \). The dispersion relation for holes can be also expressed as [9]

\[ \hat{\omega}_h = \hat{k} \coth \left[ \frac{\hat{k}^2}{g^2} + \frac{\hat{k}}{\hat{\omega}_h + \hat{k}} \right]; \] (7)

this form turns out to be more convenient in numerical calculations. The relations Eqs. (5)-(7), together with the energy conditions for absorption and decay, can be solved numerically to obtain \( \omega_h, \omega_p \) and \( k \) as functions of temperature, and the results can be used to find \( \Gamma_\phi(T) \). The imaginary parts needed for the decay rates were computed in [10] and read (\( t = T/M_\phi \))

\[ \text{Im} \Sigma_{\text{abs}} = - \frac{g_Y^2}{2\pi g^4} \left[ \hat{k}^2 t^2 (\hat{\omega}_p^2 - \hat{k}^2)(\hat{\omega}_h - \hat{k}^2) (n_h - n_p) \right] M_\phi \] (8)

for the case of absorption, whereas

\[ \text{Im} \Sigma_{\text{dec}} = - \frac{g_Y^2}{4\pi g^4} \left[ \hat{k}^2 t^2 (\hat{\omega}_p^2 - \hat{k}^2)^2 (1 - 2n_p) + \hat{k}^2 t^2 (\hat{\omega}_h - \hat{k}^2)^2 (1 - 2n_h) \right] M_\phi \] (9)

for the case of decay; here \( g_Y \) is the Yukawa coupling between the scalar and the fermion and

\[ n_{h,p} = \frac{1}{\exp(\hat{\omega}_{h,p}) + 1} \] (10)

is the distribution function.

The total scalar condensate thermalization rate \( \Gamma_\phi(T, \omega) \equiv \Gamma_{\text{abs}} + \Gamma_{\text{dec}} \) is a complicated function depending on the coupling constant \( g \). However, varying \( g \) and fitting the numerical results with simple functional forms we find that the thermalization
rates are well described by
\[
\Gamma_{\text{dec}}(t, g) = \frac{1}{4\pi} (4 - 6gt) \left( ae^{-b(gt)^2} + c \right) \left( 1 - \frac{2}{e^{1/(2t)} + 1} \right),
\]
(11)
\[
\Gamma_{\text{abs}}(t, g) = \left( (D_2g^2 + D_1g + D_0)t + \frac{E}{g} \right)^{-1},
\]
(12)
where \( a = 0.34; \ b = 5.6546; \ c = -0.09 \) and \( D_2 = 3.746; \ D_1 = -0.7058; \ D_0 = 11.1278; \ E = 1.8015. \)

Since the direct decay channel is the only possible one when \( T \) is small enough, while the absorption channel opens only when \( T \) is sufficiently large, there is a range of temperatures where the both reactions are kinematically forbidden if the scalar couples only to a single fermion. In reality the condensate is more than likely to couple to many species of fermions. For instance, the MSSM flat direction condensate typically couples to many quark and/or lepton generations. Likewise, the inflaton could well couple to several fermionic species. If a condensate couples to more than one species with different thermal masses, it is possible to obtain a piecewise continuous non-zero decay rate. This is because the end point of the direct decay channel and the starting point of the absorption channel depend on the coupling constant: if the scalar condensate decays directly to a fermion with a thermal mass proportional to a coupling constant \( g_1 \) and absorbs a fermion with a thermal mass proportional to a coupling constant \( g_2 \), then in order for the temperature gap in the thermalization rate to vanish the ratio of coupleings must satisfy the relation

\[
\frac{g_2}{g_1} < 0.25,
\]
(13)
which is easily realized in actual models. The Yukawa couplings may of course also vary, but here we keep all the Yukawas equal for simplicity.

To demonstrate the interplay of the decay and absorption channels in case of more than one fermion species, we have calculated the decay rates for a scalar coupled to two fermions with thermal masses proportional to coupling constants \( g_1 = 1, \ g_2 = 0.1 \) and \( g_1 = 1, \ g_2 = 0.2 \); the results are plotted in Fig. 1.

3. Solving the Boltzmann equations

The energy partitioning between the scalar condensate and the relativistic heat bath and its time evolution in the expanding universe are described by the Boltzmann equations,

\[
\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi(T)\rho_\phi = 0,
\]
\[
\dot{\rho}_R + 4H\rho_\phi - \Gamma_\phi(T)\rho_\phi = 0,
\]
(14)
Figure 1. The complete thermalization rates for a scalar zero mode decaying into two fermionic particles with plasma masses proportional to $g_1 = 1$ and $g_2 = 0.1$ (left), and $g_1 = 1$ and $g_2 = 0.2$ (right). The low temperature part of the plot corresponds to the direct decay channels, while the zigzag patterns are due to absorption channels opening first to particles with the largest thermal mass, then to particles with a smaller thermal mass.

where $\rho_\phi$ and $\rho_R$ are respectively the inflaton and the radiation energy densities. Using the dimensionless quantities

$$\Phi \equiv \rho_\phi M_\phi^{-1} a^3; \quad R \equiv \rho_R a^4$$

as was done in e.g. [3], and defining a new variable $x = aM_\phi$, Eq. (14) can be written in a form more suitable for numerical calculations ($' \equiv d/dx$),

$$\Phi' = -\sqrt{\frac{3}{8\pi}} \frac{M_\text{Pl} \Gamma_\phi(T)}{M_\phi^2} \frac{x}{\sqrt{\Phi x + R}} \Phi,$$

$$R' = \sqrt{\frac{3}{8\pi}} \frac{M_\text{Pl} \Gamma_\phi(T)}{M_\phi^2} \frac{x^2}{\sqrt{\Phi x + R}} \Phi.$$  

Assuming that at the beginning the condensate energy density dominates, the initial conditions at $x = x_I$ are given by $R(x_I) = 0$ and $\Phi(x_I) = \Phi_I$, where $\Phi_I$ is obtained from the Friedmann equation,

$$\Phi_I = \frac{3}{8\pi} \frac{M_\text{Pl}^2 H_I^2}{M_\phi^2 M_\phi^2} x_I^3.$$  

The temperature of relativistic gas in an expanding universe is given by

$$\frac{T(x)}{M_\phi} = \left( \frac{30}{g_* \pi^2} \right)^{1/4} \frac{R^{1/4}}{x},$$

where $g_* \approx 100$ counts the effective degrees of freedom. The time evolution of temperature as a function of the scale parameter $x$, computed with $\Gamma_\phi(T)$ of Eq. (2) is plotted in Fig. 2 together with the results presented in [3]. We have calculated the temperatures for two sets of initial values following the cases II and III discussed in
Figure 2. Solutions for Boltzmann equations with two different sets of initial values: $g_2^2 = 6 \times 10^{-13}$, $M_\phi = 10^9 \text{GeV}$, $V^{1/4} = 10^{14} \text{GeV}$ (left) and $g_2^2 = 3 \times 10^{-9}$, $M_\phi = 2 \times 10^7 \text{GeV}$, $V^{1/4} = 8 \times 10^{11} \text{GeV}$ (right). The curves are: results obtained by Kolb et al. in [8] (thin solid); the case where thermal corrections are neglected, i.e. $\Gamma_\phi = g_2^2 M_\phi$ (thin dashed); the case where thermal corrections are included with $g_1 = 1$ and $g_2 = 0.1$ (thick dashed); and the case where thermal corrections are included with $g_1 = 1$ and $g_2 = 0.2$ (thick solid).

First with the scalar mass falling between the reheat temperature and the maximum temperature reached by the universe during the reheating, $T_{RH} < M_\phi < T_{MAX}$, with the reasonable numerical values $g_2^2 = 6 \times 10^{-13}$, $M_\phi = 10^9 \text{GeV}$, $V^{1/4} = 10^{14} \text{GeV}$, where $V$ is the value of scalar potential at the end of inflation; when the scalar mass is smaller than the reheat temperature, $M_\phi < T_{RH}$, we adopt the values $g_2^2 = 3 \times 10^{-9}$, $M_\phi = 2 \times 10^7 \text{GeV}$, and $V^{1/4} = 8 \times 10^{11} \text{GeV}$.

As can be seen in Fig. 2, an era of a constant temperature which was found by a partial consideration of the thermal effects relevant for the decay of the inflaton condensate [8], actually vanishes when one takes into account all the interactions between the scalar and the thermal bath at the one-loop level. The inclusion of the absorption decay channel and the possibility to decay into more than one type of fermion allow for the reactions to continue at temperatures greater than the inflaton rest mass. This effect is due to the existence of several fermionic species coupling to the inflaton; in the case of one species only, the result would be almost identical to the case of Kolb et al. [8], with only minor changes due to the hole excitations. As a consequence the thermal history of the heat bath turns out to be qualitatively similar to the case when the thermal masses are ignored. At the end of inflation the rapid decays of inflaton raise the temperature to some maximum temperature, after which most of the time the temperature scales as $T \propto a^{-3/8}$; however, there are short periods during which the temperature drops faster because of absorption channels reaching the level of fastest direct decay rates before closing with decreasing temperature. If the thermal
bath contained also bosons that couple to the inflaton, the thermalization rate would be modified accordingly. The details would again depend on the coupling strengths. Clearly, the thermalization rate and the temperature of the resulting heat bath very much depend not only on the details of the inflaton model itself but also on the reaction rates of the decay products, which determine their plasma masses. Such considerations might also be important for e.g. thermal leptogenesis [11].

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