Analysis of high temperature fatigue lifetime of GH4133B superalloy used in turbine disk of aero-engine

C L Ren¹, R G Zhao¹², Y F Liu³, N Ji¹, L Y Deng¹ and X M Li¹

¹ College of Civil Engineering and Mechanics, Xiangtan University, Xiangtan 411105, China
² Key Laboratory of Dynamics and Reliability of Engineering Structures of College of Hunan Province, Xiangtan University, Xiangtan 411105, China
³ College of Aerospace Engineering, Chongqing University, Chongqing 401331, China

Corresponding author: zhaorongguo@163.com

Abstract. Based on the S-H cavity model theory and the thermodynamic diffusion equation, the high temperature fatigue lifetime equation is deduced, and the influence of stress amplitude and mean stress on fatigue lifetime is quantitatively analyzed. At high temperature of 650°C, according to the test data of fatigue lifetime of GH4133B superalloy under different stress ratios or alternatively at various maximum stress levels, the nonlinear regression analysis method is applied to identify the material parameters in the fatigue lifetime equation, and a 3D \( N-\sigma-m \) curve surface is plotted. The comparison between theoretic fatigue lifetime \( N_t \) and test one \( N \) indicates that the fatigue lifetime equation derived from the microstructure evolution of metallic materials can accurately predict the fatigue lifetime of GH4133B superalloy under different cyclic loading parameters. Finally, a parameter \( \gamma \) is introduced to characterize the effect of mean stress \( \sigma \) and stress amplitude \( \sigma_m \) on fatigue lifetime \( N \) of GH4133B superalloy. It is suggested that the effect of mean stress \( \sigma \) on \( N \) is larger than that of stress amplitude \( \sigma_m \) on \( N \) under the condition of tensile-tensile fatigue loading.

1. Introduction

With regard to the study of high temperature fatigue lifetime of nickel-base superalloy, a lot of researches have been done. Wang [1] carried out the fatigue and creep-fatigue tests for a forged and precipitation hardened nickel-based superalloy GH4169 at 650°C, and investigated the effects of inhomogeneous microstructure and loading waveform on creep-fatigue behaviour. It was found that the \( \Sigma 3 \) CSL boundaries play an important role in the inhomogeneous effect, the tensile dwells show an intergranular damage caused by precipitate-assist micro-voids, and the compressive dwells show an oxidation-assisted damage and slip-band-induced cracks. Chen [2] studied the low cycle fatigue and creep-fatigue interaction behavior of nickel-base superalloy GH4169 at 650°C. Yu [3] studied the creep and low cycle fatigue behaviour of a nickel-base superalloy. It is found that the creep curves show an obvious primary creep stage followed by a short steady-state creep stage and then an accelerating creep stage until leading to failure at 700°C. While at 900°C, the creep curves demonstrate a shorter primary stage, and a longer accelerating creep stage without steady-state creep stage. Simultaneously, the creep and low cycle fatigue properties degenerate with increasing temperature. Holländer [4] investigated the isothermal and thermo-mechanical fatigue behavior of the nickel-base superalloy IN738LC by using the standardized and advanced test methods. Based on the
continuum damage mechanics. Shi [5] carried out a creep and fatigue lifetime analysis of directionally solidified superalloy, and its brazed joints at elevated temperature.

In this work, a high temperature fatigue lifetime prediction model is constructed via the S-H cavity model theory and the thermodynamic diffusion equation, a 3D curve surface of fatigue lifetime is plotted, and the theoretical predicted value and test one are compared. A parameter is introduced to characterize the effect of mean stress and stress amplitude on fatigue lifetime.

2. Cavity nucleation theory

2.1. Fatigue nucleation theory
During the process of fatigue cycle, with the accumulation of plastic deformation, the vacancies are generated and aggregated at the grain boundaries to form the original voids. The void number produced during a cycle is proportional to the stress amplitude $\sigma$, which is a half of the sum of maximum stress $\sigma_{\max}$ and minimum one $\sigma_{\min}$, i.e., $\sigma=(\sigma_{\max}+\sigma_{\min})/2$.

Therefore, the nucleation number $n$ of the voids in the unit volume at grain boundaries can be expressed as

$$n_v = P\sigma^m N$$

where, $P$ is the void nucleation factor, $N$ is the fatigue cycles, and $m$ is a material parameter.

2.2. Creep nucleation theory
The theoretic analysis and the experimental results indicate that the cavities are continuously nucleated with the creep time, which is sometimes called as a continuous nucleation. The previous theoretical research shows that the void number $n_c$ on grain boundaries per unit area is proportional to the creep strain $\varepsilon$, that is

$$n_m = \alpha_1 \varepsilon$$

where, $\alpha_1$ is a material parameter. Under the condition of one-order approximation, the parameter $\alpha_1$ is independent of the stress. The nucleation rate, which represents the number of voids formed on grain boundaries per unit area in per unit time, can be written as

$$\dot{n}_m = \alpha_1 \dot{\varepsilon}$$

where, $\dot{n}_m$ is the nucleation rate, $\dot{\varepsilon}$ is the creep rate. Supposing that there exists a relation expression between the creep rate and the mean stress, that is

$$\dot{\varepsilon} = k\sigma_m^n$$

where, $\sigma_m$ is the mean stress, $k$ and $n$ are the material parameters, the Equation (3) is rewritten as

$$\dot{n}_m = \alpha \sigma_m^n$$

where, $\alpha' = \alpha k$, which represents the product of the material parameter $\alpha$ and $k$.

Under the condition of stress control mode, the variables in Equation (5) are separated, and then the integral calculation on both sides of equation is operated, the following equation is

$$n_m = \alpha'\sigma_m^n t$$

Due to the relation expression between the fatigue cycle $N$ and the test frequency $f$, i.e., $N=ft$, Equation (6) can be addressed as

$$n_m = (\alpha'/f)\sigma_m^n N = \alpha'\sigma_m^n N$$
where, $\alpha'^*=\alpha'/f=\alpha k/f$, which denotes the ratio of the parameter $\alpha'$ to the test frequency $f$.

2.3. Creep-fatigue nucleation theory

Under the condition of stress control mode, considering the influence of mean stress $\sigma$, and stress amplitude $\sigma_v$ on nucleation rate and as well the influence of creep-fatigue interaction on nucleation rate in the metallic material, the total nucleation number $n$ of the voids in the unit volume at grain boundaries can be expressed as

$$n = \alpha''(\sigma_m + \sigma_v)^n N$$

where, $\alpha''$ is a parameter containing an operated parameter as frequency $f$, and $n$ is the material parameter. In Equation (8), the terms of $\alpha''\sigma; N$, $\alpha''\sigma N$ and $\alpha''(\sigma + \sigma_v)N - \alpha''\sigma N - \alpha''\sigma_v N$ denote the creep nucleation term, the fatigue nucleation term and the creep-fatigue nucleation term.

3. Cavity growth theory

3.1. Volume Diffusion

According to Fick’s first law of diffusion, the mass flux on the unit section area perpendicular to the diffusion direction in the unit time is proportional to the concentration gradient at the cross section. Its mathematical expression is

$$J = -D(\partial C/\partial x)$$

where, $J$ denotes the diffusion flux, $D$ is the diffusion coefficient, $C$ is the volume concentration of diffusion component, and $x$ represents a position coordinate in the linear coordinates.

According to the S-H cavity growth model, the voids are homogeneously generated in the whole grain boundary volume, and the void concentration satisfies Fick’s second law of diffusion, that is

$$\frac{\partial C}{\partial t} = D_v\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r}\right) + \beta$$

where, $D_v$ is the void diffusion coefficient, $\beta$ is the void formation rate, $t$ is the time, and $r$ is a polar diameter in the polar coordinates. In the steady state, i.e., $\partial C/\partial t = 0$, Equation (10) is simplified as

$$\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\beta}{D_v} = 0$$

The solution can be obtained from Equation (11), that is

$$C = -\beta r^2/(4D_v) + A \ln r + B$$

3.2. Cavity growth rate

In the case of uniaxial tension, the elastic strain energy is neglected, and the free enthalpy change $\partial G$ of crystal in this process is expressed as

$$\partial G = \mu_v \partial N - \sigma_m Q \partial N^* - P \Delta V \partial N^*$$

where, $\partial N$ is the void number, $\sigma$ is the tensile stress, $Q$ is the atomic volume, $\Delta V$ is the volume contraction caused by the void formation, and $P$ is the hydrostatic pressure whose value is of $-\sigma/3$.

Under the condition of thermodynamic equilibrium state, i.e., $\partial G = 0$, Equation (13) is reduced as

$$\mu_v = \sigma_m Q + P \Delta V$$
Moreover, for an arbitrary surface, the normal component force \( P \) of surface tension is written as

\[
P = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \gamma_s
\]

(15)

where, \( \gamma \) is the specific surface energy, \( R \) and \( R \) are the two curvature radii of the surface. So the general expression considering a stress action is deduced as

\[
\mu_v = -\Omega_A \left[ \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \gamma_s + \sigma_m \right]
\]

(16)

For the spherical surface whose curvature radius maintains a constant \( R \), Equation (16) becomes

\[
\mu_v = -\Omega_A (2 \gamma_s / R + \sigma_m)
\]

(17)

Therefore, the boundary conditions of void concentration are determined as

\[
\frac{\partial C}{\partial r} = 0 \quad \mu_v = -\sigma_m \Omega_A \quad \text{at} \quad r = \frac{a}{2}; \quad \text{and} \quad \mu_v = -\frac{2\gamma_s}{R} \Omega_A \quad \text{at} \quad r = R
\]

(18)

where, \( a \) denotes the diameter of grain boundary in the S-H cavity growth model.

For solid solution alloys, the chemical potential energy \( \mu \) of the component \( i \) can be expressed as

\[
\mu_i = \mu_i^0 + RT \ln a_i
\]

(19)

where, \( \mu \) is the initiation chemical potential energy, \( T \) is the thermodynamic temperature, \( R \) is a universal gas constant, and \( a=r_x \), \( r \) represents the matter concentration. According to the boundary conditions, the void formation rate \( \beta \) can be determined as

\[
\beta = \frac{8D_v A}{a^2}, \quad A = \frac{\Omega_A (\sigma_m - 2\gamma_s / R)}{RT \gamma [\ln(a/(2R)) - (1-4R^2/a^2)/2]}
\]

(20)

The cavity growth rate is equal to the voids flow into the cavity per unit time, therefore

\[
\frac{dv}{dt} = 2\pi R \delta_b J = 2\pi R D_v \delta_b \left( \frac{\partial C}{\partial r} \right)_{r=R} = \frac{2\pi R D_v \delta_b \Omega_A (1-4R^2/a^2)}{RT \gamma [\ln(a/(2R)) - (1-4R^2/a^2)/2]} \left( \sigma_m - \frac{2\gamma_s}{R} \right)
\]

(21)

Equation (21) can be rewritten as

\[
\frac{dv}{dt} = K \left( \sigma_m - \frac{2\gamma_s}{R} \right), \quad K = \frac{2\pi D_v \delta_b \Omega_A (1-4R^2/a^2)}{RT \gamma [\ln(a/(2R)) - (1-4R^2/a^2)/2]}
\]

(22)

Contacting Equation (8) and Equation (22), and considering \( dN=fdt \), the total void volume is derived as

\[
V = \int n_i dv = \int \frac{\alpha^m}{f} (\sigma_m + \sigma_a)^n K \left( \sigma_m - \frac{2\gamma_s}{R} \right) NdN
\]

(23)

The integral calculation is carried out for Equation (23), and supposing that the cavity volume reaches a critical volume \( V_c \), the specimen is failure. So the critical volume \( V_c \) can be written as

\[
V_c = \frac{\alpha^m}{2f} (\sigma_m + \sigma_a)^n K \left( \sigma_m - \frac{2\gamma_s}{R} \right) N_i^2
\]

(24)
Then, the fatigue life $N_f$ of the specimen of metallic material can be derived from Equation (24), that is

$$N_f = \sqrt{2} f N_c / \left[ \alpha'' K \left( \sigma_m + \sigma_a \right)^n \left( \sigma_m - 2 \gamma / R \right) \right]$$

(25)

Let $c = 2 f V / (\alpha'' K)$, $\alpha = -n / 2$, and $\gamma = 2 \gamma / R$, then Equation (25) can be simplified as

$$N_f = c \left( \sigma_m + \sigma_a \right)^n / \sqrt{\sigma_m - \gamma}$$

(26)

Equation (26) quantitatively describes the effect of stress amplitude $\sigma_a$ and mean stress $\sigma_m$ on fatigue lifetime $N_f$.

4. High temperature fatigue lifetime prediction

4.1. High temperature fatigue tests

The fatigue tests for specimens of GH4133B superalloy are carried out at high temperature of 650°C at atmospheric environment on an MTS809 materials testing machine. The fatigue test waveform is sine wave with a frequency of 2Hz.

The tests are divided into two groups. For the first group of high temperature fatigue tests, the maximum stress is controlled as 700MPa, and the stress ratios are set as 0.01, 0.1, 0.2 and 0.4, respectively. For the second group of high temperature fatigue tests, the stress ratio is set as 0.1, and the maximum stress ranges from 900MPa to 550MPa. The fatigue tests are conducted at 650°C, the experimental data of displacement of the specimen are collected, and the fatigue cycle numbers are recorded. The fatigue lifetimes under different operating parameters are shown in Table 1.

4.2. High temperature fatigue lifetime prediction

According to the experimental data of fatigue lifetimes of GH4133B superalloy at different operating parameters listed in Table 4, the theoretical formula derived in this paper, i.e., Equation (26), is applied to predict the fatigue lifetime. Utilizing the nonlinear regression analysis method, the material parameters in Equation (26) are identified, and the theoretical formula of fatigue lifetime is written as

$$N_f = \frac{3.08195 \times 10^{10} \left( \sigma_m + \sigma_a \right)^{-1.78840}}{\sqrt{\sigma_m - 297.77153}}$$

(27)

The curve surface of fatigue lifetime versus mean stress and stress amplitude are shown in Figure 1. The theoretical fatigue lifetimes are calculated by Equation (27), and the comparison between theoretical lifetime $\hat{N}_f$ and experimental one $N_f$ is shown in Figure 2. It can be found from Figure 2 that all of the experimental data points are within the range of ±3 error factor, which suggests that the theoretic formula of Equation (26) derived in this paper, can be used to predict the fatigue lifetime of GH4133B superalloy at different stress operating parameter at 650°C.

Table 1. The fatigue lifetimes of GH4133B superalloy at different operating parameters.

| $R$ | $\sigma_{max}$ (MPa) | $\sigma_{min}$ (MPa) | $\sigma_m$ (MPa) | $\sigma_a$ (MPa) | $N_f$ (cycle) | $R$ | $\sigma_{max}$ (MPa) | $\sigma_{min}$ (MPa) | $\sigma_m$ (MPa) | $\sigma_a$ (MPa) | $N_f$ (cycle) |
|-----|---------------------|---------------------|-----------------|-----------------|---------------|-----|---------------------|-----------------|-----------------|-----------------|---------------|
| 0.01| 700.0               | 7.0                 | 353.5           | 346.5           | 27903         | 0.1 | 900.0               | 90.0            | 495.0           | 405.0           | 3449          |
| 0.1  | 700.0               | 70.0                | 385.0           | 315.0           | 26823         | 0.1 | 800.0               | 80.0            | 440.0           | 360.0           | 10976         |
| 0.2  | 700.0               | 140.0               | 420.0           | 280.0           | 27928         | 0.1 | 600.0               | 60.0            | 330.0           | 270.0           | 54769         |
| 0.4  | 700.0               | 280.0               | 490.0           | 210.0           | 42659         | 0.1 | 550.0               | 55.0            | 302.5           | 247.5           | 178117        |
In order to quantitatively describe the effect of mean stress and stress amplitude on fatigue lifetime, the partial derivatives of fatigue lifetime $N_f$ in Equation (26) to the mean stress $\sigma_m$ and the stress amplitude $\sigma_a$ are individually calculated, and the following equations can be obtained, i.e.,

$$\frac{\partial N_f}{\partial \sigma_m} = \frac{ca(\sigma_m + \sigma_a)^{a-1}}{\sqrt{\sigma_m - \zeta}} - \frac{c(\sigma_m + \sigma_a)^{a}}{2(\sigma_m - \zeta)^{1/2}(\sigma_m - \zeta)^{1/2}} \frac{\partial N_f}{\partial \sigma_a} = \frac{c a (\sigma_m + \sigma_a)^{a-1} \sqrt{\sigma_m - \zeta}}{\sigma_m - \zeta}$$

(28)

A parameter $\gamma$, whose value is the ratio of the partial derivative of $N_f$ to $\sigma$, and that of $N_f$ to $\sigma_a$, is defined, i.e.,

$$\gamma = \frac{\partial N_f}{\partial \sigma_m} \left/ \frac{\partial N_f}{\partial \sigma_a} \right. = 1 - \frac{\sigma_m + \sigma_a}{2a(\sigma_m - \zeta)}$$

(29)

It can be found from Equation (27) that the material parameters $\zeta$ and $\alpha$ in Equation (26) are identified as 297.77153 MPa and -1.78840, respectively. For the case of tensile-tensile fatigue test, there must be $(\sigma + \sigma) > 0$ and $(\sigma - \zeta) > 0$. Therefore, under the condition of tensile-tensile loading mode adopting in this paper, the parameter $\gamma$ is always larger than one, which suggests that the effect of average stress on fatigue lifetime is greater than that of stress amplitude on fatigue lifetime. Substituting the values of $\zeta$ and $\alpha$ into Equation (29), the values of $\gamma$ at different operating parameters are calculated. It is found that when the maximum stress is fixed at 700 MPa, and the stress ratio ranges from 0.01, 0.1, 0.2 to 0.4, the values of $\gamma$ are calculated as 3.98, 3.01, 2.48 and 1.96, respectively. The value of $\gamma$ is reverse with increasing stress ratio, which suggests that the effect of creep damage on fatigue lifetime gradually descends. While the stress ratio is fixed at 0.1, and the maximum stress levels ranges from 550 MPa, 600 MPa, 700 MPa, 800 MPa to 900 MPa, the values of $\gamma$ are individually calculated as 2.21, 2.46, 3.01, 4.97 and 11.44. The value of $\gamma$ increases with increasing maximum stress, which suggests that the effect of creep damage on fatigue lifetime of GH4133B superalloy gradually ascends.

Referring to Equation (8), supposing that the creep-fatigue nucleation equation is

$$n_s = k\sigma_m^{m_s} \sigma_a^{n_s} N$$

(30)

Then, the fatigue life $N_f$ of the specimen of metallic material can be derived as

$$N_f = c \sigma_m^{m_s} \sigma_a^{n_s} / \sqrt{\sigma_m - \zeta}$$

(31)

According to Equation (31), the test data are fitted using the nonlinear regression method, the material parameters in Equation (31) are indentified, and the theoretical formula of fatigue lifetime is uncer...
The curve surface of fatigue lifetime versus mean stress and stress amplitude are shown in Figure 3. The theoretical fatigue lifetimes are calculated by Equation (32), and the comparison between theoretical lifetime $N_f$ and experimental one $N_{ft}$ is shown in Figure 4. It can be found from Figure 4 that all the test data points are in the range of ± 2 error factor, and the theoretical results are in good agreement with the experimental data, which indicates that comparing with Equation (26), Equation (31) can more accurately predict the fatigue lifetime.

5. Conclusions
The conclusions are given as following:
(a) The fatigue lifetime equation is deduced, the nonlinear regression analysis method is used to identify the material parameters in the equation, and a 3D $N_f$-$s_m$-$s_a$ curve surface is plotted. The comparison between theoretical fatigue lifetime $N_f$ and test one $N_{ft}$ indicates that the fatigue lifetime equation proposed in this paper can more accurately predict the fatigue lifetime of GH4133B superalloy under different operating parameters.
(b) A parameter $\gamma$ is introduced to characterize the influence of mean stress and stress amplitude on fatigue lifetime $N_f$ of GH4133B superalloy. It is suggested that the effect of mean stress on $N_f$ is larger than that of stress amplitude on $N_f$ under the condition of tensile-tensile fatigue loading.

Acknowledgements
The authors gratefully acknowledge the financial support of the Research Foundation of Education Bureau of Hunan Province, (No.14A144).

References
[1] Wang R Z, Chen B, Zhang X C, Tu S T, Wang J and Zhang C C 2017 Int. J. Fatigue 97 190
[2] Chen G, Zhang Y, Xu D K, Lin Y C and Chen X 2016 Mater. Sci. Eng. A 655 175
[3] Yu J, Sun X, Tao J, Zhao N, Guan H and Hu Z 2010 Mater. Sci. Eng. A 527 2379
[4] Holländer D, Kulawinski D, Thiele M, Damm C, Henkel S, Biermann H and Gampe U 2016 Mater. Sci. Eng. A 670 314
[5] Shi D Q, Dong C L, Yang X G, Sun Y T, Wang J K and Liu J L 2013 Mater. Des. 45 643