Trinification Phenomenology  
and the structure of Higgs Bosons

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The extension of the Standard Model to $SU(3)_L \times SU(3)_R \times SU(3)_C$ (the trinification group) augmented by the $SO(3)_G$ flavor group is considered. In our phenomenological treatment partly known and partly proposed vacuum expectation values of the scalar Higgs fields play a dominant role. All Higgs fields are taken to be flavor singlets, all flavon fields trinification singlets. We need two flavor (generation) matrices. One determines the mass hierarchy of all fermions, the second one is responsible for all mixings including the CP-violating phase in the CKM matrix. The mixing with higher states contained in the group representation provides for an understanding of the difference between the up quark and the down quark spectrum. There is a close connection between charged and neutral fermions. An inverted neutrino hierarchy is predicted. Examples for the tree-level potential of the Higgs fields are given. To obtain an acceptable spectrum of scalar states, the construction of the potential requires the combination of matrix fields that differ with respect to fermion couplings and flavor-changing properties. As a consequence bosons with fermiophobic components or, alternatively, flavor-changing components are predicted in this model.

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I. THE MODEL

According to present experimental results at the Large Hadron Collider supersymmetry has not been observed [1]. It may not be relevant at the weak scale. The hierarchy problem still persists. This is a serious problem for the vacuum expectation values (vevs) of scalar fields, but only for them. These important momentum independent quantities are presently not understood. They may have their origin at a very high scale. Fortunately however, the particle masses are only indirectly affected by this hierarchy problem, only via their couplings to these vacuum expectation values. Also the well-known quadratic divergence of the Higgs self-energy caused by fermion loops is related to the vev of the Higgs field, a fact which can be demonstrated in the Standard Model.

In view of this situation we treat in this article all vacuum expectation values of scalar fields as fixed basic parameters. In addition, only dimensionless coupling constants are used. The idea is to start with a massless Lagrangian. By introducing vevs for the scalar fields, linear field components show up which have to be canceled. In a $\phi^4$ model this cancellation is performed by adding a term of dimension 2 multiplied by the square of the corresponding vev:

$$\lambda \phi^4 \rightarrow \lambda \phi^4 - 2\lambda \langle \phi \rangle^2 \phi^2.$$  (I.1)

Now the shift $\phi \rightarrow \langle \phi \rangle + \phi'$ can be applied and no linear term in $\phi'$ appears anymore. If log terms can be used more possibilities are open. For instance

$$\lambda \phi^4 \rightarrow \frac{\lambda}{1 + \log(\langle \phi \rangle^4)} \phi^4$$  (I.2)

has no linear term in $\phi'$ either.

The embedding of the Standard Model into a larger group allows us to connect the properties of quarks and charged leptons and their mixings with the properties of neutrinos and their different mixings. It also implies a Higgs sector with more scalar bosons. Here we extend the Standard Model symmetry group $SU(2)_L \times U(1) \times SU(3)_C$ to $SU(3)_L \times SU(3)_R \times SU(3)_C$, the trinification group [2–4], which is a subgroup of $E_6$ [5–8]. In this report we follow to a large extent (with modifications) articles in reference [9–11], but restrict ourselves to the trinification group. As proposed in these articles, we use in addition the generation (flavor) group $SO(3)_G$ and require all fermions to be 3-vectors in generation space. The coupling matrices for fermions are vevs of flavon fields. In our phenomenological treatment these couplings are considered as parameters of the model, without regarding the flavon potential from

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which they originate. We take all Higgs fields to be singlets with respect to the flavor symmetry and all flavon fields to be singlets with respect to the trinification group.

The group $SU(3)_L \times SU(3)_R \times SU(3)_C$ can be unbroken only at and above the scale where the two electroweak gauge couplings $g_1$ and $g_2$ combine. According to the scale dependence of the Standard Model couplings this happens at a scale of about $10^{13}$ to $10^{14}$ GeV. Interestingly, this is just the scale relevant for the small values of the neutrino masses by applying the seesaw mechanism. It is also the place where the self coupling of the Higgs field approaches zero [12]. In this article we do not consider the possible complete unification of $g_1$, $g_2$ and $g_3$ at a still higher scale.

All fermions are described by two-component (left-handed) Weyl fields. They occur in singlet and triplet $SU(3)$ representations of the trinification group. For each generation one has

\[
\begin{align*}
\text{Quarks} : & \quad q(x) = (3,1,\bar{3}), \\
\text{Leptons} : & \quad L(x) = (3,3,1), \\
\text{Antiquarks} : & \quad \bar{q}(x) = (1,3,\bar{3})
\end{align*}
\]

as in applications of $E_6$ [3] [10].

Generation (flavor) indices will be denoted by $\alpha, \beta = 1, 2, 3$. The coupling matrices - originating from trinification singlet flavon fields - are taken to be hermitian $3 \times 3$ matrices. Two coupling matrices are needed: The symmetric matrix $G_{\alpha,\beta}$ and the antisymmetric matrix $A_{\alpha,\beta}$. $G_{\alpha,\beta}$ can be taken to be a diagonal matrix. It is responsible for the flavor hierarchy of all fermion masses. The antisymmetric matrix $A_{\alpha,\beta}$ determines the fermion mixings.

The scalar bosons (Higgs fields) are described by the matrix fields $H$, $H_A$, $H_{Al}$ and $\tilde{H}$. They transform under the trinification group as $(3,3,1)$ except for $H_{Al}$ which transforms according to $(\bar{3},\bar{6},1)$. These matrices differ with respect to their Yukawa couplings: $H$ is coupled to fermions with the symmetric coupling matrix $G_{\alpha,\beta}$ whereas $H_A$ and $H_{Al}$ couple to fermions with the antisymmetric flavor matrix $A_{\alpha,\beta}$. As deduced from $E_6$, $H_A$ acts on quarks, $H_{Al}$ on leptons only [9, 10]. $\tilde{H}$, on the other hand, does not couple to fermions. The latter property can be achieved by an additional parity-like symmetry $P_G$ with a positive value for $H$, $H_A$, $H_{Al}$ and a negative value for the fermiophobic $\tilde{H}$. In order to obtain a spectrum of the scalar fields with non-vanishing masses (besides the would-be-Goldstone state) the potential has to be formed from at least two different matrix fields. This fact leads us to consider two distinguished cases:

A) The fermiophobic model: the potential is formed by $H$ and $\tilde{H}$ while the fields in $H_A$, $H_{Al}$ are assumed to have a negligible influence on the lower part of the mass spectrum of the scalars. In this case most states will have fermiophobic components.

B) The flavophile model: Here we take $\tilde{H} = 0$ and construct the potential out of the fields $H$ and $H_A$. The contribution of $H_{Al}$ to the potential is assumed not to be relevant for the low mass eigenstates, or $H_{Al}$ has its own additional potential independent of $H$. In this case the scalar bosons can cause flavor changing transitions as we will see.

The vev of $H$ can be taken to be a diagonal matrix while the vev of $\tilde{H}$ will then in general have 5 non-zero elements.

\[
\langle H \rangle = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & M_1 \end{pmatrix}, \quad \langle \tilde{H} \rangle = \begin{pmatrix} v_2 & 0 & 0 \\ 0 & b_2 & b_1 \\ 0 & M_R & M_3 \end{pmatrix}.
\] (1.4)

Also the vev of the matrix field $H_A$ has in general 5 non-zero elements. We write them in the form $^{i}f_{\langle kl \rangle}$. Here $i$ is a left-handed index and $k$ a right-handed one with $i, k = 1, 2, 3$. The matrix field $H_{Al}$ can have 8 non zero elements occurring in the expression $^{i}f^{(kl)}$. (For brevity these indices will often be omitted hereafter). The number of these vevs for $H_A$ and $H_{Al}$ can be reduced since all mixings are supposed to be induced by the mixings of Standard model fields with the high-mass fermions present in the representation (1.3). Since the top has no higher partner we can take $^{1}f_{k} = ^{1}f^{(kl)} = 0$.

Veys carrying the left-handed index 3 like $M_L$ and $M_R$ or $^{3}f_{2}$ break the trinification group down to the Glashow-Weinberg-Salam group. They have to be large compared to the weak scale. $M_1$ is presumably close to the meeting point of $g_1$ and $g_2$ mentioned above. Finally, the small vevs in the first and second rows break the Standard Model group to the electromagnetic $U_e(1)$ symmetry. As a consequence of our scheme it is seen that $v_1$ and $b$ are related to the top mass and the (unmixed) bottom quark mass:

\[
m_t = g_t \, v_1, \quad m_b = g_t \, b \quad i.e. \quad \frac{b}{v_1} = \frac{m_b}{m_t}.
\] (1.5)

The known value $v = 174$ GeV $= \frac{346}{\sqrt{2}}$ GeV for the vev of the Higgs field of lowest mass is related to vevs of $H$, $\tilde{H}$, $H_A$ and $H_{Al}$ :

\[
v^2 = v_1^2 + v_2^2 + b^2 + b_2^2 + b_3^2 + (g^2 f_2)^2 + (g^2 f_3)^2 + (g^2 f^{(13)})^2 + (g^2 f^{(12)})^2.
\] (1.6)
Thus, if the $b_j$ and $f_j$ are similarly small as $b$, ($b \approx m_0 \approx 2.8$ GeV at the scale $m_Z$) the vevs $v_1$ and $v_2$ are restricted according to $v_1^2 + v_2^2 \approx v^2$.

With the flavor matrices $G$ and $A$ and the flavor singlet Higgs fields $H$, $H_A$ and $H_{Al}$ the Yukawa interaction \[ \mathcal{L}_{Y}^{1} = g_{l}G_{\alpha \beta} \left( \psi_{\alpha}^{T}H \psi_{\beta}^{\dagger} \right) + A_{\alpha \beta} \left( \psi_{\alpha}^{T}H_{A} \psi_{\beta}^{\dagger} \right) + \text{h.c.} \] is

The first term gives the up quarks their masses. It also describes parts of the down quark and lepton mass matrices (with Dirac masses for the neutrinos). The second term in (I.7) performs the mixings of the Standard Model quarks as well as their mixings with the heavier quark states occurring in the model. Together, the first and the second term should achieve satisfactory results for the masses of up quarks, down quarks and the Cabibbo-Kobayashi-Maskawa matrix including its CP-violating phase. Similarly, the first term together with the third term should describe masses and mixings of the leptons.

At this stage, however, the neutrinos are still Dirac neutrinos with masses comparable to the quark masses. Moreover, neutral leptons that are singlets with respect to Standard Model gauge transformations are still massless. An important assumption of our model is therefore the addition of an effective Yukawa interaction $\mathcal{L}_{\text{eff}, 2}$:

\[ \mathcal{L}_{\text{eff}, 2} = \frac{1}{M_N} \left( G_{\alpha \beta} \right)^2 \left( \psi_{\alpha}^{T}H_{1}^{\dagger} \psi_{\beta}^{\dagger} \right) + \text{h.c.} \text{ or } \mathcal{L}_{\text{eff}, 2} = \frac{1}{M_N} \left( G_{\alpha \beta} \right)^2 \left( \psi_{\alpha}^{T}H_{1}^{\dagger} \psi_{\beta}^{\dagger} \right) + \text{h.c.} \] (I.8)

In the fermiophobic model (I.8) couples leptons with $H_{1}^{\dagger}$ and $\tilde{H}_{1}^{\dagger}$ to form trinification singlets. This term could originate from the exchange of a very heavy trinification singlet with mass $M_N \approx M_I$ and an appropriate $U(1)_G$ structure. The second form is taken for the flavophile model where $\tilde{H} = 0$. It is replaced by $H_{A}$. (In a transition to a trinification singlet $H_{1}^{\dagger}$ can act on leptons). Obviously, (I.8) provides masses for neutral leptons only. In particular, they provide a mass matrix with large eigenvalues (because of the large vevs $M_l$, $M_{R_{3}}$, $f_2$) to the neutrinos $2L_3$ and $3L_3$ which are not part of the Standard Model. Through the mixing of these heavy neutrinos with the Standard Model neutrinos the seesaw mechanism takes place. The corresponding flavor matrix (I.8) must be symmetric. Because of the second order form of this third term we choose the matrix $G^2$. As a consequence, the mass hierarchy of the heavy neutrinos is a very strong one \[ \text{10, 11} \]. Thus, depending on the masses $M_N$ and $M_I$ the first generation of the leptons $3L_2$ and $3L_3$ can have masses at or below the TeV region. They are possible dark matter candidates.

The effective Yukawa interaction $\mathcal{L}_{\text{eff}} = \mathcal{L}_{Y}^{1} + \mathcal{L}_{\text{eff}, 2}$ with $G$, $A$ in (I.7, I.8) and the vev configurations of $H$, $H_A$, $H_{Al}$ and $\tilde{H}$ contain all the necessary information about the generation structure and the fermion spectrum. We will show in section 2 that the properties of quarks and charged leptons, their masses and mixings are well described by only a few parameters. The flavor matrices $G$ and $A$ determined in this section are then used in section 3 for the determination of the mass matrix for neutrinos. A generalized seesaw mechanism leads to an inverted hierarchy. Finally, in section 4, tree-level potentials are constructed which give rise to spontaneous symmetry breaking and determine the mass spectrum of the Higgs-like bosons. The corresponding mass eigenstates of the scalar fields will in general have either fermiophobic or flavor-changing components.

II. CHARGED FERMION MASSES AND MIXINGS

A. The up quark mass matrix

The experimentally known up quark masses can be used to construct the flavor matrix $G$. All masses used here and in the following are running masses taken at the scale $m_Z$. The numerical values can be found in ref \[ \text{13} \]. Since $G$ and $\langle H \rangle$ can be chosen to be diagonal matrices, one has from (II.7)

\[ g_{l}v_{1}G = \begin{pmatrix} m_{u} & 0 & 0 \\ 0 & m_{c} & 0 \\ 0 & 0 & m_{t} \end{pmatrix} \] (II.9)

By introducing the small parameter $\sigma = 0.05$, a good phenomenological ansatz of $G$ and thus for the up quark masses a the weak scale is

\[ G = \begin{pmatrix} 1.4 & \sigma^2 & 0 \\ 0 & 1.4 & \sigma^2 \\ 0 & 0 & 1 \end{pmatrix} \] . (II.10)
Interpreting the factor 1.4 as a renormalization factor \[10\] suggests that at the scale \(M_f\) one has \(m_u/m_c = m_c/m_t\). It turns out that the small parameter \(\sigma = 0.05\) can also be used to describe the particle mixing matrix and neutrino properties.

### B. The down quark masses and the Cabibbo-Kobayashi-Maskawa matrix

The first two parts of the effective Yukawa interaction determine the down quark mass matrix. Here one needs the antisymmetric flavor matrix \(A\). It can be described by 3 real parameters. We choose the element \(A^3_2 = i\sigma\) and absorb the remaining multiplication factor into the vev parameter of \(H_A\). This leaves 2 parameters for \(A\). But in the form for \(A\) that we will use here, only one real parameter (\(\tau\)) appears:

\[
A = i \begin{pmatrix}
0 & \sigma & -\sigma \\
-\sigma & 0 & \tau \\
\sigma & -\tau & 0
\end{pmatrix}.
\]

(II.11)

If \(\sigma/\tau\) would be equal to 1, even (odd) permutations of generations would lead to \(A \rightarrow +A\) \((A \rightarrow -A)\). The permutation symmetry with respect to the second and third generation survives for any value of \(\tau\). By fitting the CKM matrix the value \(\tau = 0.45\) turns out to be a good choice.

Besides the down quarks of the Standard Model there exists - according to \[13\] - also a state which is a singlet with respect to Standard Model gauge group transformations. Thus, the down quark mass matrix is a 6 \(\times\) 6 matrix. This new quark \(D\) with \(SU(3)_L\) index \(i = 3\) is very heavy due to the vev \(M_f\) of \(H\). One can integrate out this heavy state if the contributions from \(H_A\) are taken to be small compared to \(M_f\) and \(3f_2\) or \(M_R\). This way one finds the wanted 3 \(\times\) 3 mass matrix for the Standard Model particles. The mixings with the high mass state cannot be neglected. It is seen to be essential for our understanding of the CKM matrix and the deviations of the mass pattern of down quarks from the mass pattern of the up quarks. The light down quark mass matrix is

\[
m_d = m_b^0 G + f^A + f^A_0 \sigma^3 A (G)^{-1} A .
\]

(II.12)

Here \(m_b^0 = g b\) is the value of \(m_b\) before mixing, \(f^A\) is proportional to the vev of \(2(H_A)\) and \(f^A_0\) a parameter resulting from integrating out the heavy \(D\)-quark masses. The factor \(\sigma^3\) serves to cancel the negative powers of \(\sigma\) in \(A(G)^{-1}A\) and thus allows a smooth formal limit \(\sigma \rightarrow 0\). Taking \(m_b^0 = 2.81\) GeV, \(f^A = -0.236\) GeV and \(f^A_0 = 1.18\) GeV (together with the values for \(m_t\), \(\sigma\) and \(\tau\)) an almost perfect representation for all up and down quark masses, and the CP-violating CKM matrix is achieved. The masses at the scale \(m_Z\) agree within error limits with the experimental ones \[13\]. Also the calculated CKM elements describe the data correctly. Only the angle \(\beta\) of the unitary triangle is larger than its experimental value \(\beta \approx 26^\circ\) instead of \(\beta \approx 21^\circ\).

### C. The charged lepton masses and their (not directly observable) mixings

Like the down quarks the charged leptons have heavy partners as well and mix with them via the flavor matrix \(A\). Again, by integrating out these heavy states the 6 \(\times\) 6 mass matrix is reduced to the 3 \(\times\) 3 matrix for the usual leptons. Apart from a sign, its form is the same as for the down quarks.

\[
m_e = -m_e^0 G - f^e A - f^e_0 \sigma^3 A (G)^{-1} A .
\]

(II.13)

Here \(m_e^0\) is the value of \(m_e\) before mixing. \(m_e^0\), \(f^e\) and \(f^e_0\) are now used to fit the masses of \(\tau, \mu\) and the electron. A good fit is obtained by setting \(m_e^0 = 1.64\) GeV, \(f^e = -0.186\) GeV, \(f^e_0 = 3.03\) GeV. These values are not simply related to the corresponding values for the down quarks since for leptons matrix elements of \(H_{AI}\) of \(H_A\) have to be taken \[10\]. The charged lepton mass matrix obtained now allows to calculate the charged lepton mixings, which is a necessary ingredient for the discussion of the neutrino properties.

### III. NEUTRINO MASSES AND MIXINGS.

According to the lepton assignments in \[13\] one has to deal with 5 neutral leptons in each generation. Thus, the matrix for neutral leptons is a 15 \(\times\) 15 matrix. Again, leptons that obtain high masses because of the large vevs in the
effective Yukawa interaction \([17, 18]\) can be integrated out giving rise to a generalized seesaw mechanism. According to our Yukawa interaction the Dirac matrix for the light neutrinos is \(m_l G\), while the heavy neutrinos have masses proportional to \(G^2\). Therefore, a unit matrix is part of the light neutrino mass matrix. Together with a further contribution due to the particle mixing matrix \(A\) one obtains the \(3 \times 3\) matrix for the light neutrinos \([10, 11]\) in terms of the two parameters \(\kappa\) and \(f_g\)

\[
m_\nu \simeq \frac{m_l^2}{M_t} \kappa \mathbf{1} + f_g \sigma^3 \left( A \frac{1}{G} - \frac{1}{G^2} A \right). \tag{III.14}\]

Taking the neutrino mass matrix \(m_\nu\) only up to first order in the small parameter \(\sigma\), one can write \(m_\nu\) in a very simple form: setting \(f_g = 1.40\) \(m_0\) and \(\kappa \frac{m_l^2}{M_t} = m_0 \rho\) one obtains

\[
m_\nu \simeq m_0 \begin{pmatrix} \rho & -i \sigma & i \\ -i \sigma & \rho & -i \tau \sigma \\ i \tau \sigma & i \rho & \rho \end{pmatrix} \tag{III.15}\]

with \(\tau = 0.45\) and \(\sigma = 0.05\) as used for quarks and charged leptons. The eigenvalues of \(m_\nu m_\nu^\dagger\) to first order in \(\sigma\) are

\[
\begin{align*}
(m_2)^2 &\simeq (\rho^2 + 2 + \sqrt{2} \, \tau \, \sigma) m_0^2, \\
(m_1)^2 &\simeq (\rho^2 - 2 - \sqrt{2} \, \tau \, \sigma) m_0^2, \\
(m_3)^2 &\simeq \rho^2 m_0^2. \tag{III.16}
\end{align*}
\]

It is now easy to see the following properties of the light neutrinos:

- The neutrino mass spectrum has the form of an inverted hierarchy.
- The ratio between the solar and the atmospheric mass squared differences is independent of the two neutrino parameters \(m_0\) and \(\rho\). This ratio is \(\sqrt{2} \, \tau \, \sigma = 0.031\) in good agreement with experiment.
- The experimentally observed atmospheric mass squared difference can be used to fix the mass parameter \(m_0\) for the light neutrinos. Then \(\rho\) is fixed by the lightest neutrino mass \(m_3\).

\[
m_0 \simeq \frac{1}{\sqrt{2}} \sqrt{\Delta m_{\text{atm}}^2} \simeq 0.035 \, \text{eV}, \\
m_3 \simeq m_0 \rho \simeq 0.035 \, \rho \, \text{eV}. \tag{III.17}
\]

- Without taking account of effects from diagonalizing the charged lepton mass matrix and renormalization, the neutrino mass matrix \(m_\nu\) leads to almost strict bimaximal mixing.

Including the charged lepton mixings obtained from \([11, 13]\) a better, but still not satisfactory agreement with the experimentally determined neutrino mixing angles is achieved. Detailed renormalization group calculations would be necessary, but are not performed here. Instead we introduce a parameter which may in part simulate these effects. It should not change the successful mass pattern obtained so far and is therefore taken to be an orthogonal transformation in generation space. Mixing the first with the third generation by the angle \(\phi\) one gets with \(c = \cos \phi\), \(s = \sin \phi\) the modified neutrino mass matrix:

\[
m_\nu \Rightarrow m_0 \begin{pmatrix} \rho + 2ics & -i(c + \tau \sigma s) & i(c^2 - s^2) \\ -i(c + \tau \sigma s) & \rho & -i(\tau \sigma c - s) \\ i(c^2 - s^2) & i(\tau \sigma c - s) & \rho - 2ics \end{pmatrix}. \tag{III.18}
\]

Since \(m_0, \tau\) and \(\sigma\) are fixed, this matrix depends, for a given mass of the lightest neutrino, only on the angle \(\phi\). Of course, the mixing matrix of charged leptons, obtainable from section 2, has yet to be included.

As an illustrative example we take \(\rho = 1\) and choose \(\phi\) to fit the third neutrino mixing angle \(\theta_{13}' \approx 9^\circ\) in accord with data analysis \([14]\). This leads to \(\phi \simeq \tau/14\). One then finds for the neutrino masses and angles:

\[
m_2 = 0.06012\, \text{eV}, \quad m_1 = 0.05949\, \text{eV}, \quad m_3 = 0.03456\, \text{eV}, \\
\theta_{12}' \approx 36^\circ, \quad \theta_{23}' \approx 49^\circ, \quad \theta_{13}' \approx 9.4^\circ. \tag{III.19}
\]
The CP-violating phase $\delta$, the Majorana angles and the mass parameter for the neutrinoless double $\beta$-decay in this example are

$$
\delta \approx -13^o, \ \frac{\alpha_{21}}{2} \approx -81^o, \ \frac{\alpha_{31}}{2} \approx 89^o, \\
|\langle m_{\beta\beta} \rangle| \approx 0.032 \text{ eV}.
$$

The phase and angles are given according to the standard parametrization [15]. For different values for $\rho$ and even for $\rho \to 0$ the mixing angles shown in (III.19, III.20) are almost unaffected. In view of our simple approach these results for the mixing angles and the mass squared differences are satisfactory. Nevertheless, in case the inverted hierarchy predicted here turns out to be established by experiment, a more detailed study of the model will be necessary. By integrating out heavy states renormalization group effects and the violation of unitarity of the mixing matrices must certainly be incorporated before any final judgement is possible.

**IV. THE SCALAR SECTOR AND THE HIGGS BOSON.**

The embedding of the Standard Model into a larger group implies an extended Higgs structure formed by numerous scalar fields. This is difficult to deal with since only the information about the just discovered Higgs boson [16] can be incorporated. Our aim is to construct in a phenomenological way examples of tree-level potentials for the scalar fields and to calculate the corresponding boson mass spectrum. The tree potential has to be formed from $SU(3)_L \times SU(3)_R$ invariants. Colored fields are not used. Because of the large vevs of color singlet fields the cyclic symmetry between these and colored fields is strongly broken giving the latter a large mass. As mentioned in section 1 our input consists of vacuum expectation values only. They determine the spontaneous symmetry breaking pattern and fix the position of the minimum of the potential.

The presence of Higgs fields with different properties and the necessity of combining gauge group invariants in order to get non-zero masses leads to interesting properties of the bosons obtained. Some will drastically differ from the Standard Model Higgs-like states. We will discuss here two scenarios of interest.

**A. The fermiophobe model.**

The potential responsible for the scalar particle spectrum is constructed from invariants of the fields in $H$ and $\tilde{H}$. From these 36 real fields 21 of them should become massive while leaving 15 would-be-Goldstone particles massless. The remaining fields $H_A$ and $H_{A\tilde{L}}$ are supposed to have little influence on the scalar particle spectrum, at least not in the TeV region or below. Starting from a massless Lagrangian the individual invariants for $\tilde{H}$ and $H$ are

$$
J_1 = (Tr[H^\dagger \cdot H])^2, \ \ J_2 = Tr[H^\dagger \cdot H \cdot H^\dagger \cdot H], \\
J_3 = (Tr[\tilde{H}^\dagger \cdot \tilde{H}])^2, \ \ J_4 = Tr[\tilde{H}^\dagger \cdot \tilde{H} \cdot \tilde{H}^\dagger \cdot \tilde{H}].
$$

However, only $J_1$ and $J_3$ can be modified as in (IV.21) in order to have no linear fields after the appropriate shift $H \to \langle H \rangle + H$ and a similar shift for $\tilde{H}$. Moreover, the masses obtained from these two modified invariants are of order $M_I, M_R$. None of them are of order $v_1$ or $v_2$ like the Higgs boson observed at the LHC. On the other hand, phenomenologically satisfactory results are obtained from a naive ansatz analogue to (IV.22)

$$
V_0 = \kappa \left(c_1 J_1 + c_2 J_2 + c_3 J_3 + c_4 J_4 \right) \\
$$

with

$$
t = 1 + \kappa \log\left[\frac{J_1 J_2 J_3 J_4}{\langle J_1 \rangle \langle J_2 \rangle \langle J_3 \rangle \langle J_4 \rangle}\right].
$$
The condition for the vanishing of all first derivatives of $V_0$ at the point $H = \tilde{H} = 0$ for the shifted fields has a straightforward solution. It determines the parameter $\kappa$ and the coefficients $c_2$, $c_3$, $c_4$ in terms of $c_1$ [19].

$$\kappa = \frac{1}{4}, \quad c_2 = \frac{c_1 \left( b^2 + M_I^2 + v_1^2 \right)}{b^4 + M_I^4 + v_1^4}, \quad c_3 = \frac{c_1 \left( b^2 + M_I^2 + v_1^2 \right)^2}{(b_2^2 + b_3^2 + v_2^2 + M_R^2 + M_I^2)^2}, \quad c_4 = \frac{c_1 \left( b^2 + M_I^2 + v_1^2 \right)^2}{(b_2^2 + b_3^2)^2 + v_2^2 + 2(b_2 M_R + b_3 M_3)^2 + (M_R^2 + M_3^2)^2}.$$ (IV.23)

Obviously, for $M_R^2 + M_3^2 = M_I^2$ one has $c_2 = c_3 = c_4 = c_1$ in the large $M_I$ limit and in case $M_3 < M_R < M_I$ $c_3 \equiv c_4 > c_1$.

From the second derivatives of $\frac{1}{2} V_0$ with respect to all 36 fields at the point $H = \tilde{H} = 0$ of the shifted fields one gets the $36 \times 36$ mass matrix whose eigenvalues - shown here for large $M_I$, $M_R$ and $M_3 \ll M_R$ are

$$m_1^2 = c_1 \left( v_1^2 + b^2 \right), \quad m_2^2 = c_1 \left( v_2^2 + b_3^2 \right) \frac{M_R^4}{M_I^4}, \quad m_3^2 = 4 \, c_1 \, M_I^2, \quad m_4^2 = 4 \, c_1 \, \frac{M_I^4}{M_R^4}, \quad m_i^2 = 0 \quad i = 5, \ldots, 36.$$ (IV.24)

For an easy presentation, we took in these equations $b_2 = 0$.

We note that this result can also be obtained by taking $M_R$ to be proportional to $M_I$ and expanding $V_0$ (after the shift of fields) in terms of $M_I$. After neglecting inverse powers of $M_I$, this (for our purposes equivalent) scalar potential consists of polynomials of fields up to the $4^{th}$ power only.

In [13] we have 2 scalars with low masses [18] [19]. The first one can be identified with the Higgs boson found at the LHC. It is coupled to fermions and gauge bosons. Its mass is $m_{Higgs}^2 = c_1 \left( v_1^2 + b^2 \right)$. The second boson is fermiophobic since it is not directly coupled to fermions, but only to gauge bosons. Its mass depends decisively on the vevs $M_I$, $M_R$ and $M_3$. $v_1$ and $v_2$ are severely restricted by [18] and $v_1$ cannot be too small. Otherwise the top coupling $g_t$ would have to be unreasonably large.

The 32 massless states can be divided into 15 massless Goldstone states and 17 additional states of mass zero. One massless state is due to the so far unbroken general phase transformation of $H$ and $\tilde{H}$. The remaining 16 massless states are due to our provisional neglecting of invariants that connect the fields in $H$ with the fields in $\tilde{H}$. Without them $H$ or $\tilde{H}$ can be independently transformed by $SU(3)_c \times SU(3)_R$ matrices.

Taking as in [18] $b_1$, $f_2$ to be small ($\approx m_b$), an interesting case would be the equality $v_1 = v_2 \approx \sqrt{2} v_1 = 123$ GeV not very different from the Higgs mass found at the LHC [10]. This could happen if vevs of the matrix field $H$ appear also in $\tilde{H}$. It would imply $m_{Higgs} = \sqrt{c_1} \frac{v_1}{\sqrt{2}}$ with $c_1 \approx 1.04$ [19]. But it would also imply the top quark coupling $g_t$ of the field $^1H_1$ to be a factor $\sqrt{2}$ larger than its Standard Model value. Thus, $v_1 \approx 1$, $v_2 \ll v_1$, $c_1 \approx \frac{1}{2}$ is more likely the case.

A close connection between the other vevs of $H$ and $\tilde{H}$ would be the correspondence

$$M_R = M_I, \quad M_3 = 0, \quad b_2, b_3 \text{ of order } b.$$ (IV.25)

($b_2 = 0$ and $b_3 = -b$ corresponds to a $\frac{\pi}{2}$ $U_R$-spin rotation of the vev of $\tilde{H}$ with respect to the vev of $H$). If [17] holds near degeneracies are expected. Even the Higgs at $\approx 125$ GeV could be a state of twins with about equal masses [19]. However, the so far neglected invariants connecting the fields in $H$ with the fields in $\tilde{H}$ will in general change this picture. We need these invariants in order to give masses to the 17 massless bosons obtained above in order to get an acceptable scalar boson spectrum.

There are quite a number of different invariants containing the fields of both multiplets $H$ and $\tilde{H}$ [4]. Restrictions have to guarantee that all new masses are positive and not in conflict with the data. We cannot perform this task in general but will show examples. We take $v_1 = v \cos \beta$, $v_2 = v \sin \beta$ and restrict ourselves to the perhaps unlikely but interesting scheme where [17] holds. Moreover, we will use $b_2 = 0$ and $|b_3| = b$. In order to avoid abrupt discontinuities in the mass matrices constructed below we also require $v_2 > |b_3|$.

The potential to be added to $V_0$ needs 5 new invariants

$$V_S = (r_1 J_1 + r_2 J_2 + r_3 J_3 + r_4 J_4) v_1^2 \frac{M_I^2}{M_I} + r_5 J_5 + r_6 J_6 + r_7 J_7 + r_8 J_8 + r_9 J_9.$$ (IV.26)
The coefficients \( r_1 \ldots r_4 \) are multiplied by the small factor \( \frac{v^2}{M^2} \) since the corresponding invariants appear already in \( V_0 \).

The new invariants are

\[
J_5 = \text{Tr}[H^\dagger \cdot \tilde{H} \cdot \tilde{H}^\dagger \cdot H], \quad J_6 = \text{Tr}[H^\dagger \cdot H \cdot \tilde{H}^\dagger \cdot \tilde{H}],
\]

\[
J_7 = \text{Tr}[H^\dagger \cdot H \cdot \tilde{H}^\dagger \cdot H + \text{Tr}[\tilde{H}^\dagger \cdot H \cdot \tilde{H}^\dagger \cdot \tilde{H}],
\]

\[
J_8 = v (\det H + \det \tilde{H}), \quad J_9 = v (\det \tilde{H} + \det H^\dagger).
\]

(IV.27)

The first seven invariants \( J_1 \) to \( J_7 \) respect the symmetry \( H \to -H \) and \( \tilde{H} \to -\tilde{H} \), while \( J_8 \) and \( J_9 \) break this symmetry. The latter two also break the invariance under a common phase transformation of the fields. Their contribution to the potential will turn out to be very small. The total potential \( V = V_0 + V_S \) should now provide non-zero masses for all fields except the Goldstone ones. After requiring the vanishing of all first derivatives of \( V \) at the proposed minimum, there remain 3 free parameters among the coefficients \( r_i \). We choose \( r_1, r_2, r_7 \). For a range of values for these parameters the Higgs boson in [IV.24] appears again together with an acceptable mass spectrum of the other 20 bosons. Taking \( \tan \beta \approx 0 \), \( M_f = 10^{13} \text{ GeV} \), \( b = 2.8 \text{ GeV} \) and \( c_1 \approx \frac{1}{2} \) the boson at 125 GeV is not degenerate. Its field content is \( 1^1 H_1 \) with a fermiophobic admixture \( 1^1 \tilde{H}_1 \) of about 2\% only. Thus it is very close to the Standard Model Higgs. The masses of the first scalar states above 125 GeV depend in detail on the parameters \( r_i \). Using for example \( r_1 = 1, r_2 = 2, r_7 = -1 \), there are 4 almost degenerate states at \( \simeq 603 \text{ GeV} \): two neutral states formed from \( 2^0 H_3 \) and \( 2^0 \tilde{H}_2 \) fields and a positively and a negatively charged state with the field components \( 1^1 H_3 \) and \( 1^1 \tilde{H}_2 \). Their usual and fermiophobic components are of about equal magnitudes.

For \( \tan \beta \) values different from zero and an appropriate value for \( c_1 \) the single state at 125 GeV is a \( \tilde{H} \) dependent superposition of a usual Higgs and a fermiophobic scalar boson. Because the production and decay properties are now different from the Standard Model Higgs, limits on \( \beta \) may soon be available [20].

Taking a value of \( \tan \beta \) close to 1, and \( c_1 \approx 1 \) it is possible to have a twin state at \( \simeq 125 \text{ GeV} \) that is mass degenerate within the experimental resolution. Both members are compositions of fields from \( H \) and from fermiophobic fields from \( \tilde{H} \). In other words, the twin state mentioned earlier may survive the addition of the new invariants [19]. Its existence requires \( r_1 \approx 0.004 \) with the parameters \( r_2 \) and \( r_7 \) remaining unchanged. To exclude or verify a mass degeneracy the method presented in ref [21] could be applied. Our model allows this degeneracy but does not necessarily favor it. Different parameter choices for \( r_1 \) lift this degeneracy.

B. The flavophile model

Here we set \( \tilde{H} = 0 \) and thus have no fermiophobic components in the scalar particle spectrum. Instead, the potential is constructed from the fields occurring in \( H \) and \( H_A \). This is another way to obtain an acceptable boson spectrum with no zero-mass member except the 15 would-be-Goldstone bosons. Unavoidably, it will lead to flavor-changing contributions. The fields from \( H_A \) are supposed either not to be relevant for low-lying states and/or to have their own additional potential not involving the other fields. This is required because the existence of a state coupled to leptons having simultaneously flavor conserving and flavor changing components would be inconsistent with the known strong limits [22] on the decay \( \mu \to e \gamma \).

For the scalar potential we take the same approach as in the fermiophebe model and simply replace \( \tilde{H} \) by \( H_A \). Now we have to set \( c_1 \approx v \), \( v_2 \to \frac{1}{2} f_1 \approx 0 \), \( b_2 \to 2 f_2, b_3 \to 2 f_3, M_R \to 3 f_2 \) and \( M_3 \to 3 f_3 \). As an example we use again \( 3 f_2 = M_f, 3 f_3 = 0, 2 f_2 = 0, 2 f_3 = b = 2.8 \text{ GeV} \) and \( c_1 \approx \frac{1}{2} \). In section 2 we have set \( f_1 \) = 0 since flavor mixing should only occur where mixing with heavy states can take place. We now have to allow a small value for this quantity for stability reasons. As in the fermiophile model \( r_2 \) (now \( 1 f_1 \)) had to be slightly larger than \( |b_3| \). We choose \( 1 f_1 = 3 \text{ GeV} \) but could also take a somewhat larger value without any noticeable effect on the spectrum.

Apart from the very different interpretation one can copy the results from the fermiophebe model. Again there is a range of values for the parameters \( r_1, r_2, r_7 \) for which the state at \( \simeq 125 \text{ GeV} \) coincides with the low mass Higgs boson in [IV.24]. It is barely affected by the heavier bosons. The additional field component \( 1^1 H_A \) that can lead to flavor-changing transitions via the matrix \( A \) is only \( \approx 2\% \) of the field \( 1^1 H_1 \). This boson can hardly be distinguished from the Standard Model Higgs. Nevertheless, a careful analysis like the one performed in [23] would be highly desirable. In almost all higher mass states however, field components from \( H_A \) are present with the same strength as the fields from \( H \). If such states exist one could look for their decays to two quarks of different flavors, for instance to a jet with a leading top quark and a jet with a leading charm quark. An intensive search for such decays as well as for low energy processes induced by virtual boson exchange (in analogy to penguin type processes) is suggested here. Flavor violating Higgs bosons would be of significance for an increased understanding of the CKM matrix.
V. SUMMARY

In this work we considered the generalization of the Standard Model to the trinification group $SU(3)_L \times SU(3)_R \times SU(3)_C$ augmented by the generation symmetry $SO(3)_G$ under which all fermions are 3-vectors in generation space. In our phenomenological approach essential use is made of the vacuum expectation values of scalar Higgs fields. They provide the spontaneous symmetry breaking down to the Standard Model and finally to $U(1)_e$. As in [10, 11] an effective Yukawa interaction is proposed that, beside flavor singlet Higgs fields, contains two flavor (generation) matrices $G$ and $A$. $G$ determines the mass hierarchy of all fermions and $A$ all mixings. The difference between the up quark spectrum and the spectrum of down quarks as well as the structure of the CKM matrix is related to the mixing of fermions with heavy states present in the group representation. Also the form of the neutrino mass matrix is determined by this mixing. Our model leads to an inverted neutrino hierarchy. With the measured atmospheric mass squared difference as input and having only one fit parameter, a satisfactory result for the experimentally observed solar neutrino mass difference and the neutrino mixing pattern is achieved.

Concerning the Higgs sector only simple examples could be dealt with due to the many scalar fields in the representation of the trinification group. Phenomenological tree-level potentials have been constructed giving mass to all fields apart from the 15 would-be Goldstone particles. Due to the combination of different matrix fields required to obtain finite masses, the mass eigenstates have either fermiophobic components or parts that induce flavor changing processes. Within a suitable range of parameters a notable exception is the Higgs-like state at 125 GeV. It has only a very small fraction of those components and is thus barely different from the Standard Model Higgs. However, already the next higher states will strongly differ from usual Higgs-like bosons. These new bosons, if existing at all, would have interesting properties and would allow the study of exciting new processes.

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