The Bekenstein Formula
and
String Theory
(N-brane Theory)

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Abstract

A review of recent progress in string theory concerning the Bekenstein formula for black hole entropy is given. Topics discussed include $p$-branes, D-branes and supersymmetry; the correspondence principle; the D- and M-brane approach to black hole entropy; the D-brane analogue of Hawking radiation, and information loss; D-branes as probes of black holes; and the Matrix theory approach to charged and neutral black holes. Some introductory material is included.

To be published as a Topical Review in Classical and Quantum Gravity.

December 30, 1997.
1 Introduction

Studies of black hole physics during the late 1960’s and early 1970’s yielded laws of black hole mechanics [1] that bear a striking resemblance to the laws of thermodynamics.

| Law     | Thermodynamics                     | Black Holes                      |
|---------|------------------------------------|----------------------------------|
| Zeroth  | Temperature $T$ constant           | Surface gravity $\kappa$ constant|
|         | over body in thermal equilibrium   | over horizon of stationary black hole |
| First   | $dE = TdS + \text{ work terms}$   | $dM = \kappa dA/\kappa + \text{ rotation, charge terms}$ |
| Second  | $\delta S \geq 0$ in any process  | $\delta A \geq 0$ in any process |
| Third   | Impossible to achieve $T = 0$      | Impossible to achieve $\kappa = 0$|
|         | via physical processes            | via physical processes           |

Motivated by these parallels, and the fact that the irreducible mass [2] of a black hole is related to the area of the horizon, Bekenstein [3] argued that the entropy of the black hole should be directly proportional to the area of the event horizon measured in Planck units. A starting point was the no-hair theorem for gravity coupled to a Maxwell field, which states that the only information available to observers outside a black hole is a set of conserved quantities: the ADM mass $M$, the charge $Q$, and the angular momentum $J$. The presence of the black hole event horizon thus motivated Bekenstein’s definition of the black hole entropy as a measure of information about the black hole interior that is inaccessible to outside observers. Bekenstein then argued, on the basis of gedankenexperiments involving infalling particles and then merging black holes, that the entropy-area relation should be linear.

Bekenstein also proposed a generalized second law in which the total entropy, given by the sum of the entropy of the black hole and the common entropy of outside stuff, is nondecreasing. This generalized second law can be used to argue that the entropy of a self-gravitating system of a given spatial extent and given $(M, Q, J)$ can never exceed that of a black hole. Alternatively phrased in terms of information theory, this says that the maximum amount of information in a self-gravitating system is, up to a constant of order unity, a single bit per unit Planck area. Although Bekenstein’s arguments were made in four dimensions, they carry over to other dimensions as well.

Hawking [4] discovered that black holes emit radiation, due to quantum pair production in their gravitational potential gradient and the presence of the event horizon. The emitted radiation has a thermal spectrum, with deviations from a perfect blackbody spectrum, the greybody factors, determined by the frequency dependence of the gravitational potential barriers outside the event horizon. The thermodynamic temperature is given in terms of the geometrical surface gravity at the event horizon,

$$T_H = \frac{\hbar \kappa}{2\pi}.$$  \hfill (1)

An immediate consequence of this identification of the temperature is that the proportionality constant between the entropy and the area is fixed

$$S_{BH} = \frac{A}{4\hbar G_d},$$  \hfill (2)

$^1$We have used units in which $k_B = c = 1$. We now set $\hbar = 1$ also.
where $G_d$ is the $d$-dimensional Newton constant. Notice that this entropy relation is quite different from the relation for a non-gravitating system, where the entropy scales as the volume.

Hawking’s subsequent conjecture that the thermal spectrum of black hole radiation implies information loss in quantum theory created a puzzle which is unsolved even today. The approximations used in finding the entropy and temperature of black holes were semiclassical. Physicists who found the idea of loss of unitarity unacceptable thus looked toward better approximation schemes for a way out. Over the years, various attempts were made to improve the approximations, such as including some backreaction of the black hole geometry when stuff is thrown into a black hole.

The other puzzle remaining was the unknown statistical origin of the thermodynamic black hole entropy. A semiclassical calculation of the partition function of Euclidean quantum gravity was performed in [6], and yielded the Bekenstein-Hawking entropy. However, it was difficult to justify ignoring quantum corrections. In addition, the continuation from Lorentzian to Euclidean signature in quantum gravity is not well understood. The nature of the microscopic degrees of freedom giving rise to the black hole entropy was therefore still obscured from view.

The entropy and information puzzles are connected [7, 8, 9, 10]. From study of both of them, it gradually became clear that their resolutions may be found only in a fully unified quantum theory of gravitation and matter. The leading candidate for a consistent, dynamical theory of quantum gravity is string theory.

String theory has a massless spectrum including the graviton, and at low energy it gives supergravities as effective theories. Black holes therefore appear as classical solutions of low energy string theory. These classical spacetimes receive corrections where curvatures are large, e.g. at the classical singularity, from essentially stringy degrees of freedom.

The perturbative massive spectrum of string theory is an infinite tower of states, with the energy gap between adjacent states set by the tension of the string. The degeneracy at a given mass level increases rapidly with mass, and depends on other quantized conserved quantities such as charge and angular momentum. Black hole entropy, which in a true quantum theory of gravity should arise from the logarithm of a degeneracy of states, also increases with mass. It was therefore proposed that black holes might be identified with string states [11, 7] and thus the entropy could be calculated [7, 12, 13]. This proposal turned out to be incomplete because the logarithm of the string degeneracy did not agree with the entropy of the black hole over the whole parameter space unless some unknown strong-coupling physics was invoked.

The first attempt to precisely match the black hole and string state entropies was made for extremal electric black holes in heterotic string theory in [14]. An obstruction which arose was that black holes with only electric charges have zero classical horizon area and thus zero entropy. This problem was sidestepped via the assumption that stringy corrections smear out the horizon area of these black holes to order unity in string units. The entropy calculated at this “stretched horizon” [14] then scales correctly to agree with the logarithm of the degeneracy of fundamental string states [14].

Nonperturbative aspects of string theory have received much attention during approximately the last three years. Among new discoveries were dualities relating different string theories to one another and to an eleven dimensional theory known as M theory (see e.g.
D-branes (see e.g. [19, 20]), and more recently a proposal for the nonperturbative definition of M theory via a Matrix model now known as Matrix theory (see e.g. [21, 22]).

These discoveries have led to a great deal of activity in the direction of black holes. Many new classical black hole spacetimes have been constructed and classified. Most spectacularly, for the first time there has been progress in identifying the microscopic degrees of freedom responsible for the Bekenstein-Hawking entropy [23]. Recent developments in understanding the nature of the microscopic degrees of freedom behind the Bekenstein-Hawking entropy of black holes will be the subject of this review.

We begin in Section 2 with a discussion of the supergravity actions in ten dimensions and the kinds of $p$-brane classical solutions arising from them. We mention how black holes arise upon dimensional reduction of branes. We then discuss the supersymmetry bound, the different types of supersymmetric (BPS) $p$-branes, their connection to eleven dimensional BPS M-branes, and how their gravitational fields behave when the gravitational coupling becomes weak. We then briefly introduce D-branes and the supersymmetric gauge theory actions describing their dynamics, and mention T- and S-duality.

Section 3 is concerned with the correspondence principle. We first discuss black holes with two electric NS-NS charges, and their correspondence to excited closed strings. We then move on to black holes with one R-R charge, and their correspondence to open or closed strings. We also comment on how the NS-NS and R-R correspondences are related to one another, on the transition between arrays and products, and on the endpoint of Hawking evaporation of a neutral black hole.

In section 4 we discuss the very successful D-brane computation of the microscopic entropy corresponding to BPS and near-BPS black holes. We concentrate mainly on $d=5$ black holes in maximal supergravity, and discuss fractionation. We next remark on $\mathcal{N}=2, d=4$ black holes and the microscopic entropy computation from the point of view of M-branes. We then turn to D-brane analogues of near-BPS black holes in $d=5$.

Section 5 is a discussion of the D-brane analogue of Hawking radiation. We discuss successes and some difficulties of the effective string model for emission and absorption, and how in one system a discrepancy is fixed via the correspondence principle. We then discuss nonrenormalisation arguments for near-BPS systems, and the information puzzle.

In section 6 we discuss D-branes as probes, and Matrix theory. We discuss D-probes from both the supergravity and supersymmetric gauge theory viewpoints. We then give a short introduction to Matrix theory, and move to the computation of the Matrix theory microscopic entropy for $d=5$ BPS black holes. Finally, we discuss recent studies of neutral black holes in Matrix theory.

We end with a brief outlook for the future and acknowledgements in section 6.4. A Note Added at the very end contains a few remarks on the AdS/CFT Correspondence.

## 2 Branes

There are five different superstring theories in $d=10$. We will concentrate on Type II theories, which possess $d=10, \mathcal{N}=2$ supersymmetry, because the associated supergravities have the largest variety of extended objects relevant to black hole applications.
2.1 Supergravity Actions and $p$-branes

The massless modes of Type-II strings separate into two sectors: NS-NS and R-R. The two Type-II theories are distinguished by the relative chiralities of the two supersymmetry generators; the IIB theory is chiral while IIA is not. The NS-NS sector is common to both and contains the metric $G_{\mu \nu}$, antisymmetric tensor potential $B^{(2)}$, and the dilaton $\phi$. The R-R sector for each theory contains antisymmetric tensor potentials $C^{(n)}$ which are of even degree for IIB and odd for IIA.

We give here the ten dimensional low-energy effective actions for the bosonic fields only, in the conventions\cite{24}. For IIA the independent R-R potentials are $C^{(1)}$, $C^{(3)}$ and the action is

$$S_A = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left[ R + 4 \left( \partial \phi \right)^2 - \frac{3}{4} \left( \partial B^{(2)} \right)^2 \right] + \frac{1}{4} \left( 2\partial C^{(1)} \right)^2 + \frac{3}{4} \left( \partial C^{(3)} - 2\partial B^{(2)} C^{(1)} \right)^2 \right\} + \frac{1}{64} \partial C^{(3)} \partial C^{(3)} B^{(2)}.$$

(3)

For IIB the R-R potentials are $C^{(0)}$, $C^{(2)}$, $C^{(4)}$ and the action is\cite{24}

$$S_B = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left[ R + 4 \left( \partial \phi \right)^2 - \frac{3}{4} \left( \partial B^{(2)} \right)^2 \right] - \frac{1}{2} \left( \partial C^{(0)} \right)^2 - \frac{3}{4} \left( \partial C^{(2)} - C^{(0)} \partial B^{(2)} \right)^2 \right\} + (\text{self-dual five-form}) .$$

(4)

In these actions, we have set

$$\alpha' \equiv \ell_s^2 = 1 \ ,$$

(5)

where $\tau_{F1} = 1/(2\pi \alpha')$ is the tension of the fundamental string. The gravitational coupling $\kappa_{10}$ is given in terms of the closed string coupling $g$ by

$$2\kappa_{10}^2 = 16\pi G_{10} = \left( 2\pi \right)^7 g^2 \ell_s^8 .$$

(6)

By analogy with point-particle couplings in electromagnetism, extended objects can couple to R-R and NS-NS antisymmetric tensor fields. Extended objects of spatial dimension $p$ are known as $p$-branes, and they thus have a $(p+1)$ dimensional worldvolume. Electric $p$-branes couple to $(p+1)$-form potentials, while magnetic $p$-branes couple to $(7-p)$-form potentials. Their classical spacetimes, the black $p$-branes, have horizons and are asymptotically flat \cite{25}. Thus in ten dimensions, we may have NS-NS strings (NS1) and fivebranes (NS5) which couple to $B^{(2)}$, and R-R $p$-branes (Rp) for $p = 0, 2, 4, 6$ for IIA and $p = 1, 3, 5$ for IIB which couple to $C^{(n)}$. There is also a gravitational wave $W$, and the Kaluza-Klein monopole KK6 with NUT charge. Rp-branes for $p = 7, 8, 9$ are not asymptotically flat; in these cases there are too few dimensions transverse to the $p$-brane for a Coulomb field to fall off as a power law. For an early review of extended objects in string theory, see \cite{26}.

\footnote{Antisymmetrization is done with weight one, and indices are suppressed. The signature of spacetime is mostly minus.}

\footnote{For the sake of brevity, we have suppressed subtleties concerning the self-dual five-form field strength; for details in resolving these, see \cite{24,28,29}.}
String theories in dimensions below ten are obtained by compactifying unwanted directions on small manifolds. Compactification of Type-II theories on tori produces maximally supersymmetric theories, such as $\mathcal{N} = 8$ supergravity in four dimensions, while compactification on less supersymmetric manifolds produces lower dimensional theories with fewer supersymmetries. For compactifications on tori, the actions in lower dimensions may be derived by applying the Kaluza-Klein procedure to the ten dimensional action, and symmetries of these theories may be seen [30, 31]. For bosonic Lagrangians for all lower dimensional maximal supergravities via dimensional reduction from $d = 11$, see [32]. A complete classification of supergravities in diverse dimensions may be found in [33].

The Newton constant in dimensions lower than ten is obtained from the ten dimensional quantity by dimensional reduction. It is

$$G_d = \frac{G_{10}}{(2\pi)^{10-d}V_{10-d}} ,$$  \hspace{1cm} (7)$$

where $(2\pi)^{10-d}V_{10-d}$ is the volume of the compactification manifold. In $d$ dimensions, the Newton constant has units of $(\text{length})^{d-2}$. We define the Planck length in $d$ dimensions, $\ell_d$, via

$$16\pi G_d \equiv 2\kappa_d^2 \equiv (2\pi)^{d-3}\ell_d^{d-2} .$$  \hspace{1cm} (8)$$

Note that the relation (7) implies a neat consistency of the Bekenstein-Hawking formula (2) in various dimensions. For example, we may consider a $d$-dimensional black hole also as a $(d+p)$ dimensional $p$-brane, by uncompactifying $p$ dimensions. Then the relation (7) ensures that the entropy is the same whether it is calculated in the higher or lower dimension:

$$S_{BH} = \frac{A_d}{4G_d} = \frac{A_{d+p}}{4G_{d+p}} .$$  \hspace{1cm} (9)$$

Upon dimensional reduction, tensors in both the NS-NS and R-R sectors give rise to lower dimensional gauge fields, whose charges may be carried by black holes.

Methods are available in string theory for generating new black hole and $p$-brane solutions algebraically from known ones [34]. Solutions so constructed may be straightforwardly checked against the differential equations of motion. For black holes in Type-II string theory, these methods may be applied directly by starting from the higher dimensional generalizations of Kerr black holes [35]. For a recent comprehensive review of new solutions in various dimensions, methods for constructing them, $p$- and M-brane origin of black hole geometries, classifications, intersection rules, branes at angles, and references, see [36].

## 2.2 Supersymmetry and the String Coupling

Of central importance in a supergravity theory is the supersymmetry algebra, which describes how supersymmetry generators intertwine with one another and with generators of the Poincaré group. In fact, anticommutators of the supergenerators $Q_\alpha$ give back not only the momentum but can also produce central terms. Schematically,

$$\{Q_\alpha, Q_\beta\} \sim (CT^\mu)_{\alpha\beta} P_\mu + (CT^{\mu_1...\mu_p})_{\alpha\beta} Z_{[\mu_1...\mu_p]} ,$$  \hspace{1cm} (10)$$
where $C$ is the charge conjugation matrix and $\Gamma$’s are antisymmetric combinations of gamma matrices. From this we see that objects carrying $Z$-charge are $p$-dimensional extended objects, i.e. $p$-branes. A recent survey of supergravities in various dimensions and the kinds of black objects that can carry various central charges, relevant to D-brane comparisons, may be found in [37]. For a discussion of the importance of the superalgebra in the M theory context see [38].

If we choose the rest frame, sandwiching physical states around the above relation gives rise to a positivity bound on the mass per unit $p$-volume of a $Z$-carrying state in that theory. This bound may be represented schematically by

$$M \geq c|Z|,$$

where the constant $c$ depends on the theory and its couplings. By inspecting the positivity bound, we see that there are special states in this theory, as in any supersymmetric theory, which saturate the bound. They are known as BPS states and preserve some fraction of the supersymmetry of the theory. A supersymmetric nonrenormalisation theorem protects the mass-charge equality from quantum corrections. There is also a nonrenormalisation theorem that protects the degeneracy of BPS states with given $Z$ from quantum corrections if couplings are varied adiabatically [39].

The Reissner-Nordstrom black hole solution can be embedded into e.g. Type-II supergravity. When so embedded, the zero-temperature extremal Reissner-Nordstrom black hole is also supersymmetric; this can be seen by inspecting the supersymmetry variations of the supergravity fields. The link between extremality and supersymmetry extends to many known extremal black $p$-branes of superstring theory. There are exceptions, however, such as rotating black holes in $d = 4$, and extremal but non-BPS black holes.

Superstring theories are all related via dualities to an eleven dimensional theory known as M theory. The low energy limit of M theory, $d = 11$ supergravity, is related [40, 16, 41, 17] to $d = 10$ IIA string theory via compactification of the eleventh coordinate $x^9$ on a circle of radius

$$R_2 = g\ell_s,$$

where $g$ is the string coupling. At weak coupling this is a very small radius and the eleventh dimension is invisible; at strong coupling it opens up and becomes large. The length scale associated to $d=11$ supergravity is the eleven dimensional Planck length, which from (6,8,12) is

$$\ell_{11} = g^{1/3}\ell_s.$$  

BPS $p$-branes play a special rôle in duality relations: because their mass-charge relationship is unchanged by quantum corrections, we can follow them into strong coupling. The BPS $p$-branes occurring in Type-II superstring theories then have an M theory interpretation [11]: $p$-brane solutions of $d = 11$ supergravity were first found in [12]. There are four basic BPS M-theory objects: the gravitational wave (MW), the membrane (M2), the fivebrane (M5) and the KK monopole (MK). Of these, the MW and MK are purely gravitational, while the M2-(M5-)brane carries electric (magnetic) charge associated to the 3-form antisymmetric tensor potential of $d = 11$ supergravity.

\[\text{We use Townsend's convention for labeling the eleventh coordinate } x^9.\]
Consider the BPS M2. In $d = 11$ it may extend in the $x^5$ direction or not; these are termed longitudinal and transverse M2-branes respectively. If the M2 is longitudinal, then in weak-coupling IIA string theory we see a one-dimensional object; if it is transverse, we see a two-dimensional object. In this way, M-theory BPS objects give rise to IIA BPS objects: the M2 gives rise to the NS1 and R2 of IIA, the MW gives rise to R0 and W, the M5 to R4 and NS5, and the MK to R6 and KK6.

Let us now consider the bound (11) in the context of Type-II supergravities. The constant $c$ in the bound is different for different BPS charge-carrying objects [17, 16]:

$$c_{NS1} \sim 1, \quad c_{Rp} \sim \frac{1}{g}, \quad c_{NS5} \sim \frac{1}{g^2}. \quad (14)$$

From this we see that the NS1 is a fundamental object, while there are two qualitatively different kinds of solitonic objects.

 Corrections to the flat metric for a BPS black $p$-brane are of the form \[ \delta G_{\mu\nu} \sim GM/r^{7-p}. \] For the NS1- and Rp-branes, from (6,11,14) we see that at fixed charge $|Z|$ corrections to the flat metric vanish as we turn off the string coupling $g \to 0$ ($\ell_s = 1$):

$$\delta G_{\mu\nu}^{[NS1]} \sim g^2 |Z| \frac{1}{r^{7-p}} \to 0, \quad \delta G_{\mu\nu}^{[Rp]} \sim g^2 |Z| \frac{1}{g r^{7-p}} \to 0. \quad (15)$$

For the solitonic NS5-brane, $c_{NS5} \sim 1/g^2$ and the metric does not become flat for $g \to 0$; likewise for the solitonic Kaluza-Klein monopole.

For NS1 and Rp branes, at weak coupling, the deviations from flat spacetime are thus negligible. We may then ask what describes the dynamics of these branes at weak coupling. The weak-coupling sister of the BPS NS1-brane is the fundamental string F1 [43, 44]. The weak-coupling sisters of the BPS Rp-branes are the Dp-branes [45, 46, 47]. We now turn to the subject of D-brane dynamics.

### 2.3 D-branes

D-branes are hypersurfaces on which open strings end; for the definitive introduction to D-branes, see [19, 20]. A single Dp-brane carries unit R-R charge [47]. Since D-branes are BPS, two static D-branes exert zero force on one other, because gravitational and dilatonic attraction is cancelled by electrostatic repulsion. So large charges can be built up by stacking many Dp-branes on top of one another.

The dynamics of D-branes is determined by the perturbative dynamics of the open strings ending on them. At low energy, only the massless modes of the open strings are relevant; higher massive modes of the open strings decouple and are absent from the low energy effective theory. The massless bosonic mode of the open string is a gauge potential. Now, the endpoints of open strings can carry gauge labels, known as Chan-Paton factors, at no energy cost. This gives rise to a gauge theory [48] on the D-branes; for $Q$ D-branes the group is $U(Q)$, and the gauge fields are in the adjoint representation.

At this point we mention dualities, which are important in the study of D-branes and other objects in string theory. The basic idea of a duality is that theory $\mathcal{A}$, with its fields and coupling constants, is related to theory $\mathcal{B}$, with its own fields and coupling constants,
by the duality transformation. Sometimes theory $A$ and theory $B$ are the same, and the duality transformation is then a symmetry of that theory. We now mention two duality transformations which will be of interest to us, S-duality and T-duality. S-duality is a symmetry of type IIB and acts in $d = 10$ as

$$S : \quad \tilde{g} = 1/g \quad , \quad \tilde{g}^{1/4}\tilde{\ell}_s = g^{1/4}\ell_s \quad .$$

(16)

S-duality switches R1 with NS1, R5 with NS5, and leaves W invariant while R3 is self-dual.

T-duality is a symmetry of all closed string theories and acts on a compact direction, say $x^9$, with radius $R_9$.

$$T : \quad \frac{\tilde{R}_9}{\ell_s} = \frac{\ell_s}{R_9} \quad , \quad \frac{\tilde{g}}{\sqrt{R_9/\ell_s}} = \frac{g}{\sqrt{R_9/\ell_s}} \quad , \quad \tilde{\ell}_s = \ell_s \quad .$$

(17)

Acting on fundamental strings, T-duality switches winding and momentum modes. On Dp-branes, T-duality changes the dimension of the worldvolume by $\pm 1$, depending on whether or not it acts in a direction perpendicular to the worldvolume. This is in fact one way to see why having an open string sector in a closed string theory requires D-branes [19, 20]. Application of T-duality to systems with nontrivial gravitational fields is more subtle; e.g. for the NS5 we have to smear out the dependence on a transverse coordinate if we want to apply T-duality in that direction. If we then T-dualize, we get the KK6. In a similar way, if we turn on the R_p-brane gravitational fields, we have to smear on a transverse direction before T-dualizing it.

The low-energy effective action for a test D-brane in a supergravity background is in string units [49, 50, 51, 52]

$$I = -\tau_{Dp} \int d^{p+1}\sigma \ e^{-\phi} \sqrt{-\det (G_{\alpha\beta} + [2\pi F_{\alpha\beta} - B_{\alpha\beta}])} - \tau_{Dp} \int e^{2\pi F - B} \wedge \bigoplus_n C(n) \quad ,$$

(18)

where the D-brane tension is [19, 20]

$$\tau_{Dp} = \frac{\sqrt{\pi}}{\kappa_{10} (2\pi\ell_s)^{p-3}} = \frac{1}{(2\pi)^p g \ell_s^{p+1}} \quad .$$

(19)

The action is gauge-invariant, and it is supercovariant in spacetime: all fields are super-space extensions of the ordinary bosonic fields and they are pulled back to the bosonic worldvolume in a spacetime supersymmetric fashion. Suppressing fermions, an example is $G_{\alpha\beta} = G_{\mu\nu} \partial_{[\alpha} X^\mu \partial_{\beta]} X^\nu$. For the lowest components of the superfields, $C(n)$ are the R-R potentials, $B_{(2)}$ is the NS-NS potential, $F_{(2)}$ is the worldvolume gauge field, and $G_{\mu\nu}, \phi$ are the metric and dilaton. Kappa-symmetry of the above action [49, 50, 51, 52] puts the supergravity background on-shell.

At weak coupling, the bulk (supergravity) fields decouple and we are left with a gauge theory on the brane. At weak fields, the Born-Infeld (first) term in the action is then to lowest order $U(1)$ supersymmetric gauge theory. For Q D-branes, the weak-field theory is $U(Q)$ supersymmetric Yang-Mills (SYM). The field content and Lagrangian are determined
by dimensional reduction from the $d = 10$ $A_\mu$, and so on the brane we have a gauge potential $A_\alpha$ and $(9-p)$ scalar fields $X^i \equiv -A_i$. The bosonic Lagrangian is then in string units

$$S_{\text{bos}} \sim \int \frac{1}{g} \text{tr} \left\{ -\frac{1}{4} F^{\alpha\beta} F_{\alpha\beta} + \frac{1}{2} D^\alpha X^i D_\alpha X^i + \frac{1}{4} [X^i, X^j]^2 \right\} .$$

When the branes are pulled apart, the gauge group is broken to $U(1)^Q$ and the $Q$ diagonal entries in the $Q \times Q$ matrices $X^i$ are then the positions of the individual D-branes in the $i$th direction. Higher order Born-Infeld corrections in the gauge theory effective action may in principle be found from string scattering amplitude calculations; see [53] and [54] for the nonabelian generalisation of the Born-Infeld term.

In addition, the Wess-Zumino (second) term allows smaller branes to live inside larger ones [55], and branes to end on branes [56, 57]; for a nice review of the latter from the M theory perspective, see [58]. An example of a configuration which is supersymmetric and has zero binding energy is [19, 20] a D$(p-4)$-brane living inside a D$p$-brane. In this case the smaller brane creates worldvolume gauge fields in the SYM theory on the bigger brane which are analogous to (zero-size) instantons [55].

D-branes are BPS and may be combined by using relatively simple recipes to make intersecting configurations, which break more supersymmetry than plain branes. The overall fraction of supersymmetry preserved may be worked out for boundstates with zero binding energy by imposing each brane’s spinor projection condition in turn, and checking consistency conditions on spinors in a pairwise fashion. For branes at angles and configurations dual to them, the supersymmetry analysis is a little less straightforward [59].

Now, since the dynamics of D-brane configurations at low energy and weak coupling is given in terms of supersymmetric gauge theories, it is possible in some cases to compute the degeneracy of states for a configuration with given quantum numbers. Since D-branes are BPS states, supersymmetry also protects their degeneracy for fixed quantum numbers from quantum corrections as we move adiabatically from weak to strong coupling. This implies [23] that the entropy of a bunch of D-branes should be the same as the entropy of the analogous supersymmetric black $p$-brane configuration. In fact, this idea has a generalisation to non-BPS systems as well.

### 3 The Correspondence Principle

Classical supergravity black hole solutions have length scales associated to them, connected to the parameters of the solution, which determine the length scales over which gravitational fields vary noticeably. These length scales typically depend on $g$, due to (14).

As we will see in detail in later sections, R-R $p$-brane supergravity spacetimes are valid in the regime where these gravitational radii are large in string units, while the D-brane picture is valid in the opposite limit. This complementarity of pictures also holds for the NS1-brane, whose weak-coupling sister is the F1, and thus for many black objects in string theory. We may ask, therefore, if there is some regime where the D-brane/string and supergravity descriptions turn into one another.

An early idea related to the correspondence principle was discussed for four dimensional Schwarzschild black holes in [7]. The Black Hole Correspondence Principle of [64] states
that, upon turning up the string coupling $g$, a D-brane/string state will turn into a black hole when the curvature at the horizon of the corresponding black hole is of order the string scale. This is called the correspondence point. The striking observation of [60] was that if we demand that the masses and other conserved quantum numbers of the two different configurations match at the correspondence point, then the entropies also match. Note that this correspondence generically occurs only at a point in parameter space, and so it resolves the difficulty previously found in early attempts to match string state and black hole entropy. In this section we review the correspondence principle of [60]; see also the review [61].

### 3.1 Two Electric NS-NS Charges

Generic charged black holes in string theory have a position-dependent dilaton field. The string feels a different metric $g_{\mu\nu}$ to the Einstein metric $g_{\mu\nu}^E$; they are related by

$$g_{\mu\nu}^E = e^{-4\phi/(d-2)} g_{\mu\nu} \ .$$

In applying the correspondence principle, we must specify which frame we use in calculating the curvature. The correct frame to use is the natural one, the string frame.

The metric for a $d$-dimensional electrically charged NS-NS black hole is [62] in string frame:

$$ds^2_d = \frac{[1-k(r)]}{f_p(r) f_w(r)} dt^2 - \frac{dr^2}{[1-k(r)]} - r^2 d\Omega^2_{d-2} \ ,$$

where

$$k(r) = \left(\frac{r_H}{r}\right)^{d-3} ,$$

$$f_p(r) = 1 + k(r) \sinh^2 \alpha_p ,$$

$$f_w(r) = 1 + k(r) \sinh^2 \alpha_w .$$

In these equations, $r_H$ is the horizon radius. The $\alpha$ are rapidities corresponding to the algebraic boost transformations used to generate the solutions; without loss of generality we can take them to be positive. The dilaton, and internal modulus field along the string direction, are given by [62]

$$e^{-4\phi} = f_p(r) f_w(r) ,$$

$$G_{99} = f_p(r)/f_w(r) .$$

The ADM mass and gauge charges are [62]

$$M = \frac{(d-3)\omega_{d-2} r_H^{d-3}}{16\pi G_d} \left[ \frac{1}{(d-3)} + \frac{1}{2} (\cosh 2\alpha_p + \cosh 2\alpha_w) \right] ,$$

$$q_{p,w} = \frac{(d-3)\omega_{d-2} r_H^{d-3}}{16\pi G_d} \left[ \frac{1}{2} \sinh 2\alpha_{p,w} \right] .$$

---

5We have changed notation slightly from that work: $\alpha, \beta = \alpha_p \pm \alpha_w$. We have also picked the charge vector to lie along the 9 direction.
where $\omega_n = \text{Vol}(S^n) = 2\sqrt{\pi^{n+1}/\Gamma(n+\frac{1}{2})}$. Classically the rapidities $\alpha$ are continuous parameters, but in the quantum theory the following quantities are integer normalised:

$$Q_p = q_p R_9 \quad , \quad Q_w = \frac{q_w}{R_9} \quad .$$

For neutral black holes $\alpha_{p,w} = 0$. The entropy and Hawking temperature are

$$S_{BH} = \frac{\omega_{d-2} d^{-2}}{4G_d} \cosh \alpha_p \cosh \alpha_w \quad ;$$

$$T_H = \frac{(d-3)}{4\pi R_H} \cosh \alpha_p \cosh \alpha_w \quad .$$

At correspondence, the black hole becomes very small and the curvature at the horizon is large. Then stringy corrections will kick in and modify the metric \cite{22}; for example, the function $[1 - k(r)]$ will get smeared out to order unity. This is the stretched horizon phenomenon \cite{14}. We can also see from the metric that the charges, via the $\alpha$’s, give rise to a redshift of the energy, of order $(\cosh \alpha_p \cosh \alpha_w)$. Then from \cite{27} the $x^9$ direction gets contracted at the horizon by a factor $(\cosh \alpha_w / \cosh \alpha_p)$ from its value at infinity. If the charges are zero, these redshift effects go away.

Let us now determine the correspondence point. From \cite{22} the curvature at the horizon in ten dimensional string frame is $O(r_H^{-2})$, and so the correspondence point occurs at \cite{60}

$$r_H \sim \ell_s \quad .$$

The black hole’s weak-coupling sister is an excited state of a fundamental string wrapped around $x^9$. The string has the same conserved quantum numbers as the black hole, at correspondence. For the left- and right-moving momenta of the string excitations, \cite{83}

$$p_{L,R} = q_p R_9 \mp \frac{q_w R_9}{\ell_s^2} \quad ;$$

while for weak coupling the entropy scales as

$$S_{SS} \sim \sqrt{N_R} + \sqrt{N_L} \quad ,$$

where

$$N_{R,L} \sim \ell_s^2 \left( M^2 - p_{R,L}^2 \right) \quad .$$

In determining $N_{L,R}$ we need to be careful to take account of any redshift effects. Redshifts will be pronounced, i.e. not numbers of order unity, when the rapidities $\alpha$ are large. As we can see from (28,29), large-$\alpha$ corresponds to a near-BPS configuration. To see how this affects determination of $N_{L,R}$ it is simplest \cite{60} to turn off $\alpha_p$; then the energy is redshifted by the same factor as $R_9$ is Lorentz contracted and $N_L = N_R \equiv N$. The energy above extremality is small, and is of order

$$\Delta E \sim M - \frac{Q_w R_9}{\ell_s^2} \sim \frac{N}{Q_w R_9} \quad .$$

From this we see that the effect of the redshift/contraction cancels in determination of $N$. Turning $\alpha_p$ back on does not affect the determination of $N_{L,R}$ either \cite{81}.
Now we are ready to find the entropy of the string state. We use $r_H \sim \ell_s$, together with the charge and mass matching relations and (35, 36), to find the entropy at correspondence [60]:

$$S_{SS} \sim \frac{\ell_s^{d-2}}{G_d} \cosh \alpha_p \cosh \omega ,$$

(38)

up to numbers of order unity, \textit{i.e.} to the accuracy of the correspondence principle. This agrees with $S_{BH}$, as advertised. Note that since the asymptotic string coupling $g$ is taken to be weak, this entropy is large.

Let us figure out the value of the string coupling at correspondence. We have that the internal compactification volume is small; to the accuracy of matching scalings we can ignore its precise numerical value. Then, using $G_d \sim g^2 \ell_s^{d-2}$, and the fact that the local string coupling on the horizon is $g e^{\phi(r_H)}$, we find

$$\left. (e^{\phi_c} g_c) \right|_{r_H} \sim \frac{1}{N^{1/4}} ,$$

(39)

which is a weak coupling. This justifies our assumption that the entropies of left- and right-movers is additive, as in (35). We can calculate the local temperature at the correspondence point; taking account of the redshift it is $\sim 1/\ell_s$.

### 3.2 One R-R Charge

Application of the correspondence principle to systems with one R-R charge shows new features so we now discuss this case. Earlier attempts to understand the entropy of near-extremal R-R $p$-branes may be found in [64, 65]; see also [66].

The metric for a black Rp-brane with worldvolume spatial coordinates $\vec{y}$ is [25] in string frame

$$ds_{10}^2 = f_p(r)^{-1/2} \left[ (1 - k(r)) dt^2 - d\vec{y}^2 \right] - f_p(r)^{1/2} \left[ (1 - k(r))^{-1} dr^2 + r^2 d\Omega_{8-p}^2 \right] ,$$

(40)

where

$$k(r) = \left( \frac{r_H}{r} \right)^{7-p} , \quad f_p(r) = 1 + k(r) \sinh^2 \beta .$$

(41)

To obtain a black hole in $(10 - p)$ dimensions we roll up the $p$-brane on a manifold of volume $(2\pi)^p V_p$. The resulting black hole is known as a parallel $p$-brane black hole; there is a R-R gauge field and the dilaton is

$$e^{\phi} = f_p^{(3-p)/4} .$$

(42)

The mass, R-R charge, entropy and Hawking temperature are given by\footnote{In comparing to other references, such as [64], the reader may wish to note the following conversions: $d = (7 - p)$; $\alpha = (3 - p)/2$; $r_+^{7-p} = r_H^{7-p} \sinh^2 \beta$; $r_-^{7-p} = r_H^{7-p} \cosh^2 \beta$; we have also performed the coordinate transformation $r^{-7-p} := r^{7-p} - r_-^{7-p}$.} \(6\)

$$M = \frac{(7-p)\omega_{8-p} r_H^{-7-p}}{16\pi G_{10-p}} \left[ \frac{1}{2} + \frac{1}{r - p} + \frac{1}{2} \cosh 2\beta \right] ,$$

(43)

$$Q = \frac{(7-p)\omega_{8-p} r_H^{-7-p}}{(2\pi)^{7-p} g} \left[ \frac{1}{2} \sinh 2\beta \right] ,$$

(44)
\[ S_{BH} = \frac{\omega_{8-p}}{4G_{10-p}} \ell_H^{8-p} \cosh(\beta) \quad , \tag{45} \]
\[ T_H = \frac{(7-p)}{4\pi \ell_H \cosh(\beta)} \quad . \tag{46} \]

Note that with these conventions the BPS limit satisfies the relation \( M/V_p = Q/g \), with \( Q \) integer.

We now determine the correspondence point. The maximum curvature at the horizon comes from the angular part of the metric transverse to the \( p \)-brane worldvolume \[60\]; setting it to be of order unity gives
\[ r_H \sim \frac{\ell_s}{\sqrt{\cosh(\beta)}} \quad . \tag{47} \]

Here we see a new wrinkle: the correspondence size of the horizon depends on the boost \( \beta \), not just the string length \( \ell_s \). We will come back to this point later.

As for the NS-NS situation we can find the local coupling on the horizon at correspondence; from \([12,3,13]\) it is \[60\]
\[ (e^{\phi_c} g_c) \bigg|_{r_H} \sim \frac{1}{Q} \quad , \tag{48} \]
which is small for macroscopic charge. For \( p < 3 \), there is a subtlety, that the curvature increases away from the horizon to a maximum before it then decreases to zero at infinity \[60\].

Now, super-stringy curvatures lead to corrections to the classical geometry. It is therefore interesting that taking the horizon curvature to be the relevant one for correspondence is the right prescription. For \( p \geq 3 \), there is no such subtlety; we could imagine resolving the \( p < 3 \) difficulty by T-duality from this case. We have also been informed that the correspondence principle works for rotating black holes \[68\]; there is a similar kind of subtlety encountered there.

For the sister state, the first thing we need is to determine the nature of the degrees of freedom carrying the energy. We know that the BPS \( R_p \)-brane’s sister is the \( D_p \)-brane, but this only carries the BPS energy. We need other degrees of freedom to carry the energy above the BPS energy,
\[ \Delta E \equiv E - \frac{QV}{g\ell_s^{p+1}} \quad . \tag{49} \]

The BPS \( R_p \)-brane has zero classical entropy, so the other degrees of freedom carrying \( \Delta E \) must also carry the entropy for a nonextremal \( p \)-brane. There are two options \[60\]: long (closed) strings in the plane of and near the \( D_p \)-brane, and massless open strings running along the \( D_p \)-brane.

For the former, the energy \( \Delta E \) is proportional to the length, and so is the entropy, and dimensional analysis gives
\[ S_{closed} \sim \ell_s \Delta E \quad . \tag{50} \]

For the latter, the massless open string degrees of freedom are those of a \( U(Q) \) gauge theory on the brane. There are worldvolume gauge fields and transverse scalars, all in the adjoint representation, and so there are \( \sim Q^2 \) degrees of freedom. A first approximation to this system is a free gas, and if we assign the gas a temperature \( T \) we can write the excess energy and entropy as
\[ \Delta E_{open} \sim Q^2 V_p T^{p+1} \quad , \quad S_{open} \sim Q^2 V_p T^p \quad . \tag{51} \]
There are a couple of subtleties to attend to. The first is that there is a redshift. Since all worldvolume metric components involve $f^{-1/2}$, the redshift factor is $\sqrt{\cosh \beta}$, but all worldvolume lengths scale inversely to the excess energy and temperature, so the entropy in (51) is unaffected. The closed string is in the plane of the $p$-brane, and so its entropy (50) is also unaffected by the redshift. This redshift also implies via (46) that the local horizon temperature at correspondence is of order unity in string units.

The second subtlety is that we have ignored interactions, given by (20). In perturbation theory, interaction vertices on the brane pick up a factor of $Q$ due to Chan-Paton factors, and a factor of the local string coupling. Dimensional analysis then gives the perturbation expansion parameter (20): $(g e^\phi) Q T_p^{-3}$. From (18), and the local temperature, this is of order unity. Thus to the accuracy of the correspondence principle we can neglect interactions. However, if we wanted a more precise calculation we would need to understand these interactions properly. This estimation of the importance of interactions illuminates a puzzle for the threebrane found in (64); there, interactions were neglected, and thus the energy was underestimated and the entropy overestimated.

These two different kinds of degrees of freedom, the closed versus open strings, dominate in different regimes; for large $\beta$ the open string gas dominates and vice versa. Matching the black hole charge and mass to those of the closed or open string configurations, at the correspondence point, yields via (50, 51)

$$S_{\text{open}} \sim \frac{V}{g^2 \ell_p^p} (\tanh \beta)^{2/(p+1)} (\cosh \beta)^{(p-6)/2}, \quad (52)$$

$$S_{\text{closed}} \sim \frac{V}{g^2 \ell_p^p} (\cosh \beta)^{(p-7)/2}. \quad (53)$$

On the other hand, the black hole entropy (13) scales at correspondence as

$$S_{\text{BH}} \sim \frac{V}{g^2 \ell_p^p} (\cosh \beta)^{(p-6)/2}, \quad (54)$$

which agrees qualitatively with whichever of $S_{\text{closed,open}}$ is larger (60), depending on $\beta$.

### 3.3 Connections, and the Endpoint

The correspondence between a near-BPS $R^p$-brane black hole and the open-string gas can be seen to be just the correspondence between a neutral black hole and an excited closed string, viewed in a highly boosted frame (69). In showing this, use is made of the $d=10$ IIA relation with $d=11$ M theory. R-R black holes in $d=(10-p)$ dimensions come from $R^p$-branes wrapped around a $p$-torus. They are therefore related by T-duality to a (smeared) R0-brane in $d=10$. However, a R0-brane is obtained from a $d=10$ Schwarzschild black hole by uncompactifying the latter to a $d=11$ black string stretched along $x^5$, boosting in $x^{0,5}$ by rapidity parameter $\beta$, and compactifying down to $d=10$ again. Then we convert back to the $R^p$-brane black hole by T-duality.

During the boost process, $R_5$ gets Lorentz contracted by the factor $\cosh \beta$, while the $d=11$ Planck scale $\ell_{11}$ and the horizon parameter $r_H$ are unchanged (69). Then from (12, 13) we
find that the new string length is
\[
\ell_s = \left( \frac{\ell_3}{R_z} \right)^{1/2} = \ell_s \sqrt{\cosh \beta}.
\] (55)

Using this new string length in the R-R correspondence point (47), we find that it is precisely the NS-NS correspondence point (33) expressed in terms of the old string length:
\[
r^{(R-R)}_H \sim \frac{\ell_s}{\sqrt{\cosh \beta}} \leftrightarrow r^{(NS-NS)}_H \sim \ell_s.
\] (56)

Other aspects of this mapping are discussed in [69].

Black holes in \(d\) dimensions (BH\(_d\)) can in fact arise in two different ways from \(d + 1\) dimensions. The first is via a \(d + 1\) dimensional black string, which is the direct product of the black hole and the extra dimension \(z\) of radius \(R_z\). The second is to take an array of \(d + 1\) dimensional black holes spaced along \(z\) by \(2\pi R_z\); for a static spacetime the array must be an infinite one [71]. Dimensionally reducing gives approximately BH\(_d\). If the radius of the BH\(_{d+1}\) horizon satisfies \(r_H/R_z \ll 1\) then the array is the correct solution; otherwise the array becomes unstable [72] and turns into the product solution. The reason for this is that, if BH\(_d \times S^1\) and BH\(_{d+1}\) have the same mass, the array has greater entropy than the product if \(r_H/R_z \ll 1\) and vice versa. It was shown in [60] that the properties of D-branes and strings mesh nicely with the correspondence principle in describing this array-product transition.

The conservative way to view the correspondence principle is to apply it in the direction of increasing coupling \(g\): a boundstate at weak coupling of strings and D-branes turns into a black hole as we turn up the string coupling. For neutral black holes, the nature of the approach in the conservative direction has been studied in [73]; the physics depends on \(d\). See also [74].

Viewed in the other direction, however, the correspondence principle is in a sense a radical proposal. It says that when black holes become very small, of order the string scale, they turn into string states. In this way, the correspondence principle provides an answer to the old question regarding the nature of the endpoint of Hawking radiation of a macroscopic Schwarzschild black hole. Namely, that the endpoint is a highly excited string. An interesting fact is that this correspondence point does not occur when the black hole is of order the Planck mass. Taking the compactification volume to be of order the string scale, we find the correspondence mass to be
\[
m_c \sim \frac{\ell_3^{d-3}}{G_d} \sim \frac{1}{\ell_s g^2} \sim \frac{1}{\ell_d g^{2(d-3)/(d-2)}}.
\] (57)

For weak string coupling, this is larger than the Planck mass \(1/\ell_d\); in four dimensions, the correspondence mass is larger by a factor of \(1/g\).

In this section we have not discussed the case where black holes have more than one R-R charge on them, and possibly NS-NS charges as well. The only really new situations occur when two or more R-R charges are large. In these cases, it is found that the matching scale drops out so that exact comparisons can be done for these near-BPS (and BPS) situations. These precise comparisons will be the subject of the next two sections.
4 BPS and Near-BPS Entropy

As mentioned in section 1, the earliest attempt [14] to make precise matchings between black hole entropy and calculable stringy degeneracies faced the obstacle that the area of the relevant classical NS-NS charged black hole vanishes. This problem actually extends further: stringy BPS black holes with nonzero horizon area are not possible for six and higher space-time dimensions in toroidal compactification [65] even with R-R charges included. Roughly speaking, the physical reason for this is that there are not enough independent charges on higher dimensional black holes for the dilaton and moduli to have finite nonzero values at the horizon, and this leads via the classical equations of motion to zero horizon area.

Therefore, the most interesting places to look for nonzero classical BPS black hole entropies are in five and four dimensions. Note in this regard that some authors had previously argued on the basis of semiclassical topological considerations that BPS black holes have zero entropy. However, a good explanation of why this semiclassical result is unreliable in string theory may be found in [75]. Some initial attempts to calculate the microscopic entropy of finite-area extremal black holes from a dual perspective were made in [76].

Agreement of string/D-brane and black hole entropies was first accomplished in a seminal paper by Strominger and Vafa [23] in January of 1996. There, new D-brane technology was used to count the degeneracy of states of a D-brane system with the same quantum numbers as a classical five-dimensional R-R charged BPS black hole. Spectacular, exact agreement was found with the Bekenstein-Hawking entropy. This work was quickly followed by calculations addressing near-extremal black holes [77, 78], rotation [79, 80], and four dimensions [81, 82, 83]. Also discovered were multiparameter agreements between semiclassical Hawking radiation and greybody factors and analogous scattering processes directly calculable in string theory. Agreements between the entropy and particle emission of various four and five dimensional black holes and their corresponding D-brane configurations have also been reviewed in [84, 86], and so we refer the reader there for additional details.

4.1 BPS Black Holes in $d = 5$

In five dimensional maximal supergravity, there are three independent orthogonal charges $Q$ that a black hole can carry. A five dimensional black hole carrying this set of charges can be chosen via a duality transformation to originate from a IIB $d = 10$ configuration which is the intersection of three BPS configurations: a $R_1$-brane of charge $Q_1$, a $R_5$-brane of charge $Q_5$, and a gravitational wave with momentum $Q_p$ along the intersection string. We represent this configuration by the matrix

$$
\begin{bmatrix}
Q_1 & R_1 & 0 & \cdots & 5 & \cdots & \cdots \\
Q_5 & R_5 & 0 & \cdots & 5 & 6 & 7 & 8 & 9 \\
Q_p & W & 0 & \cdots & 5 & \cdots & \cdots
\end{bmatrix}
$$

(58)

Here, each line represents a constituent, and the non-dot numbers correspond to the coordinates which involve that constituent. For example, the D1 is stretched along $x^5$.

This is a supersymmetric state, preserving one of eight supersymmetries, for left-moving momentum. The supersymmetry of this intersecting brane configuration can be seen [84, 86].
from the constituent brane conditions:

\[ \epsilon_L = (\Gamma_0 \Gamma_5 \Gamma_6 \ldots \Gamma_9) \epsilon_R \]  \hfill (R5)  \\
\[ \epsilon_L = (\Gamma_0 \Gamma_5) \epsilon_R \]  \hfill (R1)  \\
\[ \epsilon_L = (\Gamma_0 \Gamma_5) \epsilon_L ; \quad \epsilon_R = (\Gamma_0 \Gamma_5) \epsilon_R \]  \hfill (W)

where the two Majorana-Weyl spinors satisfy \( \epsilon_L = \Gamma_5 \epsilon_L \) and \( \epsilon_R = \eta \Gamma_5 \epsilon_R \) with \( \eta = +1 \) for IIB. We have taken the three charges to be positive.

The five dimensional black hole is obtained by rolling up the string on a circle of radius \( R_5 \), and rolling up the fivebrane on the direct product of the circle and a four-torus \( T^4 \) with volume \((2\pi)^4 V, V = R_6 R_7 R_8 R_9 \). The resulting metric is in five dimensional Einstein frame

\[
\begin{align*}
 ds_5^2 &= (H_1 H_5 H_p)^{-2/3} dt^2 - (H_1 H_5 H_p)^{1/3} \left[ dr^2 + r^2 d\Omega_3^2 \right]
\end{align*}
\]

where \( H_i = 1 + r_i^2/r^2 \) are harmonic functions and

\[
 r_i^2 = \frac{16\pi G_d M_i}{(d-3)\omega_{d-2}} \bigg|_{d=5}
\]

where \( M_i \) is the mass of the \( i \)-th constituent. We have not written out the expressions for the gauge, dilaton, or internal moduli fields. From the gauge fields come the relations

\[
 r_1^2 = \frac{g Q_1 \ell_s^6}{V} , \quad r_5^2 = g Q_5 \ell_s^2 , \quad r_p^2 = \frac{g^2 Q_p \ell_s^8}{R_5^2 V} .
\]

These parameters are the gravitational radii associated to the three charges on the black hole. For the three gravitational radii to be comparable for weak bulk coupling \( g \), we need \( Q_1 \sim Q_5 \ll Q_p \), when \( R_5, V^{1/4} \sim \ell_s \). If all of these scales are smaller than the string scale, the supergravity fields are negligible. In this regime we do not really have a black hole; to get one, we need to grow some gravitational radii above the string scale.

In order for the classical supergravity solution to be valid, we need that closed string loop corrections, which are proportional to \( g^2 \), be small. We can arrange this at fixed \( R_5, V \) while retaining nontrivial gravitational fields by taking the limit

\[
 g \to 0 \quad , \quad Q_1, Q_5, Q_p \to \infty \quad , \quad r_{1,5,p}^2 \text{ fixed} .
\]

We may also ignore stringy \( \alpha' \) corrections to the geometry as long as the gravitational radii are large in string units,

\[
 r_{1,5,p} \gg \ell_s .
\]

Internal dimensions will be inaccessible to outsiders as long as \( R_5, V \) are small. In the supergravity regime, the charges and mass on the black hole are macroscopic in string units.

The Bekenstein-Hawking entropy associated to the black hole geometry is given by

\[
 S_{BH} = \frac{A}{4G_5} = 2\pi \sqrt{Q_1 Q_5 Q_p} .
\]

An important feature of this entropy is that it is independent of the string coupling and of the moduli of the internal compactification manifold. This is in fact a general feature of black
hole entropy; for a recent discussion see e.g. [88]. In addition, the Hawking temperature is zero, as expected for an extremal black hole.

We now turn to the weak-coupling sister D-brane configuration in order to compute the D-brane entropy. The first two charges are carried by $Q_5$ D5-branes with $Q_1$ D1-branes inside. For the third charge, we need massless open strings with endpoints on various D1,5-branes to carry the momentum $P = Q_p/R_5$. The fermionic massless open strings can also each carry $\hbar/2$ of angular momentum. Other configurations have also been studied which are U-dual to this one [85, 86, 89, 90, 91, 92]; U-duality of entropy of four and five dimensional black holes is discussed in e.g. [93] and the comprehensive review [36].

Counting the degeneracy of states for this system of open strings and D-branes is simplest to perform in the limit [23] where we wrap D1,5-branes on a compactification manifold with $R_5 \gg V^{1/4}$, so we get a theory in (1+1) dimensions; it has $\mathcal{N} = (4,4)$ supersymmetry. By analyzing the gauge theory on the brane, one finds [84] that there are $n_b = 4Q_1Q_5$ bosonic and $n_f = n_b$ fermionic degrees of freedom. Roughly, these correspond to the open strings with one endpoint on a D1 and one on a D5, and the 4 arises because the D1 can move only inside the D5. The central charge of this $d = 1+1$ theory is thus $c = n_b + n_f/2 = 6Q_1Q_5$. The more rigorous way to calculate $c$ is via T-duality on $x^5$ where we get the moduli space of instantons on $T^4$. When the charges are macroscopically large, this moduli space is well approximated [94, 95] by the symmetric product space $S^{Q_1Q_5}(T^4)$, and its dimension is $c = 6Q_1Q_5$. Now, the massless bosonic and fermionic degrees of freedom carry left-moving momentum, and energy. Since the supergravity configuration is BPS, the configuration on the brane must also be BPS. Hence the energy and momentum in the (1+1) dimensional theory are related by $E = P_L = Q_p/R_5$. We can then calculate the partition function for this system of bosons and fermions:

$$Z = \left[ \prod_{Q_p=1}^{\infty} \frac{1 + w^{Q_p}}{1 - w^{Q_p}} \right]^{4Q_1Q_5} \equiv \sum \Omega(Q_p) w^{Q_p},$$

(66)

where $\Omega(Q_p)$ is the degeneracy of states at $d = 1+1$ energy $E = Q_p/R_5$. When $Q_p$ is large, the asymptotic behavior of $\Omega(Q_p)$ is at leading order exponential, $\Omega(Q_p) \sim \exp \sqrt{\pi cEL_5/3}$, with $L_5 = 2\pi R_5$. The microscopic entropy is therefore [23]

$$S_{\text{micro}} = 2\pi \sqrt{Q_1Q_5Q_p},$$

(67)

which agrees precisely with the Bekenstein-Hawking entropy of the black hole (65). This agreement between the entropy of the D-brane/string boundstate and the black hole is thus in line with expectations from supersymmetric nonrenormalisation arguments.

Let us now inspect the regime of validity of the D-brane picture [87]. We already have that the closed string coupling is weak, $g \ll 1$. Feynman diagrams involving open strings pick up a factor of $gQ_{1,5}$ because $g_{\text{open}} \sim \sqrt{g}$ and because of the Chan-Paton factors on the open string endpoints. As explained in [96], processes involving $Q_p$ give rise to factors $g^2Q_p$ when propagators hook onto the external state. Therefore, conventional D-brane perturbation theory is good when [87]

$$r_{1,5,p} \ll \ell_s,$$

(68)

which is precisely the opposite regime to [64] where the classical supergravity solution is good. The D-brane/string perturbation theory and black hole regimes are thus complemen-
tary. This feature is related to open-closed string duality. Note in this regard a remark in [96] and the recent work [97] where it is argued that in the near-horizon region there is a duality between the supergravity description and a large-N gauge theory description. We comment more on this in a Note Added at the very end of this article.

A qualitatively different system has been studied which gives rise to BPS $d = 5$ black holes with nonzero entropy [98]. There, the system of branes studied is wrapped D5-branes with self-dual magnetic fluxes on the fivebranes. The degeneracy of states must be counted via the Landau degeneracy of open strings connecting different branes, or alternatively using properties of torons occurring in the supersymmetric gauge theories on the branes. Agreement is found with the Bekenstein-Hawking entropy.

Entropy of a more general BPS six dimensional black string with has also been studied from the string theory perspective in [99, 100, 101, 102, 103]. In these works, left-moving waves on the string [104] are allowed to have large amplitudes. The entropy can be calculated using both string theory and classical gravity. In this case, even though the classical spacetime is singular on the horizon, agreement is again found between the two different approaches. The issue of the importance of quantum corrections to the classical spacetime side of this agreement has not yet been resolved fully.

4.2 Fractionation

An important subtlety arises in the use of the exponential approximation to the formula (66). This approximation is only valid when the energy of the system is such that there is a large degeneracy of states at that given energy, or equivalently when $Q_p \gg Q_{1,5}$. As pointed out in [105], the exponential approximation is no longer valid when $Q_p \sim Q_{1,5}$. The simplest way to see this physically [105] is to recall that the massless open strings constitute a gas of left-movers of order $Q_1 Q_5$ species with average energy $Q_p/R_5$. Introducing a temperature $T_L$ and making the assumption of extensivity we have energy $E \sim Q_1 Q_5 R_5 T_L^2 = Q_p/R_5$ and $S \sim Q_1 Q_5 R_5 T_L$. Eliminating $T_L$ gives $S \sim (Q_1 Q_5 Q_p)^{1/2}$ which is the Bekenstein-Hawking scaling. However, substituting back we find the inverse temperature $T_L^{-1} \sim R_5 (Q_1 Q_5 / Q_p)^{1/2}$, which if all three charges are comparable is longer than the wavelength of a typical quantum in the box of size $R_5$, and so the gas is too cold for thermodynamics to be applicable.

The solution of this problem [105, 84] is to realize that in this regime the intersection string is not multiple copies of singly wound string, but rather a single copy of a multiply wound one. This fractionation was suggested by the work of [106]; fractionation may also be seen in the context of the $S^{Q_1 Q_5} (T^4)$ theory.

Since it is the massless strings running between the D1,5 that dominate the entropy of the configuration [107], the intersection string is wound [105] on a large radius $R_5 Q_1 Q_5$ instead of $R_5$ and thus has an energy gap $\delta E \sim 1/(R_5 Q_1 Q_5)$ rather than the na"ıve $1/R_5$. In addition, the tension of the intersection string becomes fractionated by comparison to the na"ıve D-string tension. Then there are plenty of low energy modes available. Using $E \sim (R_5 Q_1 Q_5) T_L^2$, we find $T_L^{-1} \sim (R_5 Q_1 Q_5)/(Q_1 Q_5 Q_p)^{1/2}$, so that the temperature is plenty hot enough for the equation of state to be valid. The counting then proceeds in a similar manner as before, but replacing $c = 6 Q_1 Q_5$ by $c = 6$ and $L_5$ by $L_5 Q_1 Q_5$. The result again agrees precisely with the Bekenstein-Hawking entropy.
### 4.3 \( \mathcal{N} = 2, d = 4 \) BPS Black Holes

\( d = 4 \) extensions of the IIB \( d = 5 \) results were found in \([81]\) by adding a new ingredient to the \( \{R1, R5, W\} \) system: a KK6. The T-dual system was studied in IIA \([82]\). U-dual \( d = 10 \) configurations were found in \([107, 108]\), in more generality in \([109]\), and in \([107]\) an M-theory configuration was also exhibited. Picking one representative of the duality orbit, we have

\[
\begin{align*}
M_{5_1} & = \begin{pmatrix} 0 & \cdots & 4 & 5 & 6 & 7 & \cdots & 9 \\
M_{5_2} & = \begin{pmatrix} 0 & \cdots & 4 & 5 & 6 & 7 & 8 & 9 \\
M_{5_3} & = \begin{pmatrix} 0 & \cdots & 4 & 5 \\
MW & = \begin{pmatrix} 0 & \cdots & 4 & 5 & 6 & 7 & 8 & 9 \\
\end{align*}
\] (69)

Note that since M5-branes cannot end on any other branes \([58]\), this should really be thought of as fivebranes with parts extending in different orthogonal directions. The four dimensional black hole is recovered by rolling up the above on a manifold \( T^6 \times S^1 \).

This view of four dimensional black holes as composite M5-branes with additional momentum can be extended to compactifications with less supersymmetry. There, instead of wrapping the fivebranes on pairs of 2-cycles in \( T^6 \) we wrap them on 2-cycles in a Calabi-Yau threefold \( CY_3 \), to obtain BPS black holes in \( d = 4, \mathcal{N} = 2 \) supergravity.

Entropy formulæ for the \( \mathcal{N} = 2 \) black holes were obtained in \([110, 111, 112, 113]\); in \([114]\) some quantum corrections were studied. For toroidal compactification a conjecture was made in \([107]\) about how to count states in the microscopic 1+1 dimensional theory on the common string; in \([108]\), essentially the same observation was made independently, in the \( d = 10 \) context. In \([114]\) the microscopic entropy was also computed heuristically for the \( \mathcal{N} = 2 \) case for a restricted class of solutions, and this was extended to the general case in \([116, 117]\). See also the work \([118]\).

Precise methods for counting the microscopic entropy for BPS \( \mathcal{N} = 2, d = 4 \) black holes became available more recently \([119]\), and do not make use of string theory; related work in the IIA context was done in \([120]\). The microscopic computations were done in a regime of parameter space where the M5-branes' gravitational fields were negligible for the purpose of counting states. Certain charge restrictions were also implemented, which meant that the fivebranes were smooth and noncoincident. The dimension of the space of supersymmetric M5-brane deformations, and hence the degeneracy of states, is then calculable in the regime where \( R_5 \gg V_{CY}^{1/6} \), by analyzing a \( \mathcal{N} = (0, 4) \), 1+1 dimensional theory obtained from knowledge of the chiral worldvolume theory for the M5-brane and the Calabi-Yau \( CY_3 \). The microscopic entropy was then calculated, including the first subleading term.

The charge restrictions were also needed in order that semiclassical \( d = 11 \) supergravity could be used for the strongly gravitating side. On the black hole side, the leading correction to the entropy was computed in the Euclidean approach by evaluating the leading correction in the \( d = 4 \) supergravity action, which descended from an \( R^4 \) correction to \( d = 11 \) supergravity, on the classical solution. In the IIA \( d = 10 \) picture, this correction occurs at one string loop, and it was argued that no higher string loops could correct it; \( \alpha' \) corrections were also argued to be subleading due to the charge restrictions. The resulting black hole entropy and its leading quantum correction then agree precisely with the M-brane computation \([119]\).

In this picture, we have seen that the microscopic degrees of freedom behind the Bekenstein-Hawking entropy of the \( \mathcal{N} = 2, d = 4 \) black hole may be thought of as zero-cost
deformations of the M5-brane configuration, or “M5-brane foam” [19]. These are analogous to the D-brane/string moduli space degrees of freedom in the $d = 1 + 1$, $(4,4)$ non-chiral theory from our previous example. An earlier remark about counting black hole entropy by considering “D3-brane foam” was contained in the work [121].

A recent calculation [122] has found qualitative agreement between string state entropy and the sister black hole entropy for $N = 2, d = 4$, in the same spirit as the calculation for maximal supergravities [14]. The undetermined constant for matching in type II is shown to be different to that required for heterotic matching, as expected. Another recent work [123] has considered near-BPS $N = 2$ black holes in the light of the above precise M-brane counting.

We now leave BPS territory and turn to near-BPS configurations.

### 4.4 Near-BPS $d = 5$ Black Holes

In ten dimensional string frame, the metric corresponding to the nonextremal configuration with R-R 1- and 5-brane and momentum charge is [124, 125]

$$
\begin{align*}
    ds_{10}^2 &= f_1(r)^{-1/2} f_5(r)^{-1/2} \left\{ dt^2 - dx_5^2 - k(r) \left( \cosh \alpha_p dt + \sinh \alpha_p dx_5 \right)^2 \\
    &\quad - f_1(r) \left[ dx_6^2 + \ldots + dx_9^2 \right] - f_1(r)f_5(r) \left[ \{1 - k(r)\} dr^2 + r^2 d\Omega_3^2 \right] \right\},
\end{align*}
$$

where $r^2 = \sum_{i=1}^4 (x^i)^2$ and

$$
    k(r) = \frac{r_H^2}{r^2}, \quad f_{1,5}(r) = 1 + k(r) \sinh^2 \alpha_{1,5}.
$$

Here the horizon radius $r_H$ is related to the ADM mass of the pre-boost black hole. Note that the BPS solution may be recovered by taking the limit $r_H \to 0, \alpha_i \to \infty$ in such a way that the product $r_H e^{\alpha_i}$ is finite. To get a $d=5$ black hole we roll up on the five-torus. Then the form of the metric and R-R antisymmetric tensor implies that there are four independent length scales for the five dimensional black hole:

$$
\begin{align*}
    r_1^2 &= \frac{gQ_1 \ell_s^6}{V}, \quad r_5^2 = gQ_5 \ell_s^2, \quad r_H^2 = \frac{2}{\sinh(2\alpha_p)} \frac{g^2 Q_p \ell_s^8}{R_5^2 V}, \quad r_p^2 = \tanh \alpha_p \frac{g^2 Q_p \ell_s^8}{R_5^2 V}.
\end{align*}
$$

$r_{1,5}$ are the scales of variation of the supergravity fields due to R1,5-brane charges. In the BPS limit, $r_p$ is the scale of variation due to the momentum charge.

Arguments similar to those of the last section may be used to calculate the D-brane degeneracy of states. Near-BPS black holes possess a small energy excess over their BPS counterparts, and they break supersymmetry. This added energy needs to be macroscopically measurable but small by comparison to the energy of the BPS system. We can add energy by adding pairs of strings moving with opposite momenta, or by adding pairs of D1,5-branes and anti-D1,5-branes [77].

We work in the regime $R_5 \gg V^{1/4}$ where the theory is effectively $(1+1)$ dimensional, so that $r_H, r_p \ll r_1, r_5$. This means that the contributions to the entropy coming from anti-D1,5-branes are suppressed by comparison to the contributions from left- and right-moving open strings. Then, since the string coupling in five dimensions involves an inverse power
of $R_5$ via (4), interactions between left- and right-movers are weak and we may use a dilute gas approximation \[7\]. In this (1+1) dimensional dilute gas regime, entropies of left and right movers are additive. From the point of view of the supergravity solution, this is where the configuration is really a six dimensional black string.

The lowest energy, and highest entropy, modes for the near-BPS system are then the left- and right-moving open strings. The left-movers for the BPS system have a macroscopically large charge $Q_p = N_L$. To mock up the near-BPS black hole, on the D-brane side we add a few left- and right-movers, $\delta N_{R,L}$, while keeping $\delta N_R = \delta N_L$ in order to keep the charge $Q_p = (N_L + \delta N_L) - (\delta N_R)$ fixed. The microscopic entropy for the near-BPS configuration may then be calculated by adding the left- and right-mover entropies, using fractionation if the charge ratios warrant it. The microscopic entropy of near-BPS D-brane/string system then agrees precisely with the Bekenstein-Hawking entropy,

$$S_{\text{micro}} = S_{\text{BH}} \quad \text{(73)}$$

The case of the near-BPS plain D5-brane was analyzed in \[126\]; previous and subsequent work on BPS fivebranes may be found in \[127\]. It was found that the entropy can be accounted for in terms of a gas of weakly interacting effective instanton strings with fractionated tension. It was, however, difficult to justify the assumption that the gas is weakly interacting. It would be interesting to understand the reason for the precise agreement in this picture.

Surprising agreement for extremal but non-BPS black holes \[128\] has also been found in \[83, 129, 130, 131\].

In recent work \[132\], the connection via previously unexplored duality transformations \[133\] between $d = 4, 5$ (and some higher-$d$) non-extremal black holes and the $d = 3$ BTZ \[134, 135\] black hole has been used to find the microscopic entropy. The degeneracy of states was computed using Carlip’s method \[136\], which makes use of the fact that $d=2+1$ gravity can be described by a Chern-Simons gauge theory. In this way, intriguing agreement with the Bekenstein-Hawking formula was found.

## 5 Emission and Absorption Rates

Classically, black holes are known to gobble up particles crossing their event horizons. Semiclassically, they also Hawking radiate \[4\]. One way to calculate semiclassical emission rates is to begin with the probability of absorption of a particular incoming matter wave. In the case of interest, we solve the wave equation for the field in question, for low-energy modes. In practice, solving this equation can be very difficult technically because of nonlinear mixing between different modes. The wave equation has approximate solutions where the radial coordinate $r$ is related to the gravitational radii $r_i$ by $r \gg r_i$ (asymptotically far away) and $r \ll r_i$ (near the horizon). Matching of the approximate solutions in the overlap region then yields the probability that a wave incident from asymptotically far away will be absorbed by the black hole. This matching procedure can be complicated depending on the hierarchy of scales of the different gravitational radii. The absorption probability for an ingoing wave differs from unity because the curved geometry outside the horizon backscatters part of the incoming wave. The dominant mode for either emission or absorption at low energy...
is the $s$-wave; a general proof that the low-energy $s$-wave cross-section for absorption of minimally coupled scalars by a $d$ dimensional spherically symmetric black hole is the area of the event horizon was given in [137]. The absorption cross-section is then converted via detailed balance to an emission cross-section and then an emission power using the canonical ensemble.

5.1 Emission from D-Branes

In [77] it was argued that Hawking radiation of near-BPS black holes has an analogue in the D-brane system, and an approximate rate for the process was calculated. For clarity we will stay with the five dimensional examples in the $\{Q_1, Q_5, Q_p\}$ system, but results for four dimensional black holes have also been obtained. In this D-brane system, there is a significant population of left-moving excitations and a few right-moving excitations, all treated in the dilute gas approximation. The left-movers and right-movers carry a net momentum and so different temperatures $T_{L,R}$ are assigned to them. In order for thermal equilibrium to be established, some interactions are needed. Now, when a left-moving and a right-moving open string interact, they can scatter elastically or produce a closed string which escapes to the bulk. The latter process is the D-brane analogue of the emission of a Hawking quantum. The dynamics of the D-brane/string system is calculable perturbatively at weak coupling, and so the amplitude for this process may be computed precisely. A review of computation of closed-open string amplitudes relevant to this emission may be found in [138].

Since the number of right-movers is microscopically large, while being macroscopically small, we may make the assumption that the system is in thermal equilibrium, and that it is appropriate to use the canonical ensemble. If the number of right-movers were microscopically small, the loss of one of them to make a bulk degree of freedom would disrupt the ensemble too much and invalidate the assumption. We would also see trouble on the black hole side due to the third law. For a discussion of small corrections to the picture we use, see [139]. Note also that thermalization of left- and right-movers is indeed occurring, through decoherence processes involving the bulk degrees of freedom.

From the partition functions for the left- and right-movers we can then extract in the standard way the distribution functions for the energies and hence the temperatures. The distribution functions are then the usual Bose-Einstein or Fermi-Dirac. From this we can proceed to find the emission rate for closed string quanta into the bulk. We need, as before, to be in the dilute gas regime which is $r_{p,H} \ll r_{1,5}$, and $T_{L,R}^{-1} \gg r_{1,5}$; we also need low energies, $\omega^{-1} \gg r_{1,5}$. For the D-brane action we use the Lagrangian for the effective intersection string, with fractionated tension if needed. In some cases, [102, 140], the effective string picture has been checked explicitly by relating to fundamental strings.

The first precise D-brane calculations [141] were made for emission of bosonic scalars arising from internal components of the ten dimensional graviton $h_{ij}$, which are minimally coupled scalars. It is simplest to use the harmonic gauge for the perturbations. The tree interaction vertex for these modes is then of the form $h_{ij} \partial_\mu X^i \partial^\mu X^j$. The next piece of information needed is the field normalizations, which are obtained from the brane and supergravity actions, remembering fractionation. We can then find the amplitude for the two open strings to collide and produce a bulk $h_{ij}$. To find the emission power from the cross section we average over initial states by using the thermal density of states functions for the
left- and right-movers, and sum over final states. The final answer for the energy emitted in an infinitesimal energy range around $k_0$ per unit time is for the minimally coupled scalars

$$
\int dE(k_0) = \frac{A_H k_0^4 dk_0}{8 \pi^2 e^{k_0/H} - 1},
$$

(74)

where $A_H$ is the area of the event horizon. The temperatures are related to the Hawking temperature by

$$
2 T_H^{-1} = T_R^{-1} + T_L^{-1}.
$$

Absorption processes may be calculated in an entirely similar fashion. Four dimensional $s$-wave results for minimally coupled scalars were obtained in [143]. For a discussion of emission versus absorption in the D-brane context, see [144]. Note that the assumption of thermality for the D-brane ensemble is crucial for reproducing the Hawking rate.

D-brane results in the $s$-wave then agree exactly with the semiclassical black hole results, for emission and absorption of minimally coupled scalars [141], including all coefficients and greybody factors. As with the entropy, we are seeing whole black hole functions reproduced from D-brane physics. This is quite remarkable, because the physical processes are different in the two different approaches.

In string theory the low-energy effective theories are supergravities, and these theories typically have nonminimal couplings between gauge fields and scalars, so that old no-hair theorems do not apply [73]. “Fixed scalars” [110, 145, 146] are scalar fields in the black hole background experiencing a potential near the horizon which fixes their values there. An example of such a field in our $d=5$ example is the dilaton; the emission rate for this was found in [142]. In [147, 148, 149, 150, 151] it was shown that emission of fixed scalars is suppressed by comparison with minimal scalars, both from the supergravity and D-brane perspectives. On the D-brane side this was accomplished by showing that processes involving fixed scalars arise from higher order open-string processes on the brane worldvolume. Precise agreements were found for some situations.

Results have also been obtained for emission of higher angular momentum modes [152, 153], fermionic quanta [154, 155, 156], charged particles [157, 158, 87], intermediate scalars [159], for rotating black holes [160, 161], and for more general charge configurations [162, 163, 164, 165]. These additional cases, including fixed scalars, provide a more stringent test of the effective string picture of near-BPS black hole radiation than the initial calculations. The case of plain D3-branes [166] has also been studied extensively [167, 168, 169, 170, 171, 172] and some precise agreements have been found in that system as well.

### 5.2 Discrepancies, and a Correspondence Principle Fix

Discrepancies between the effective string model for $d=4, 5$ and semiclassical black hole calculations have arisen, in analysis of higher dimension operators, higher partial wave emission, and black holes with generic charge and angular momentum, e.g. in [163, 151, 164, 161, 165]. These discrepancies may be due to insufficient understanding of the effective string model, and/or the lack of an accompanying nonrenormalisation theorem. In addition, in [140] it was shown that the fractionation argument breaks down for D1-branes in isolation and wrapped on $x^5$ when $R_5$ is too small. Assuming that this carries over for the D1-branes in the black hole’s sister boundstate, this means that $R_5$ should not be too small in string units. Otherwise, the excitation energy levels would be smeared into broad resonances and the effective
string model would lose its calculational efficacy. The case of parallel $p$-brane black holes was discussed in [162].

One situation in which there were emission disagreements [173, 164] appeared to clash with the correspondence principle. This was the case of a $d = 5$ black hole with two large NS-NS charges which, as we saw in a previous section, is sister to an excited string state carrying winding and momentum charge. Qualitative features of the emission spectra disagreed between the string and black hole systems at the correspondence point, where they should by rights agree. A resolution of this problem was found in [174].

The essential new input was the realisation that there are other degrees of freedom than stringy excitations which can come into play for the case of a $d = 5$ black hole with charges $Q_w, Q_p$; namely, NS5-branes wrapped around the compactification manifold. Since $M_{NS5} \sim 1/g^2$, these objects are much heavier than strings at weak coupling and are therefore unimportant degrees of freedom. As the coupling is increased, however, the fivebranes become lighter.

Now, from (28,29) we can see that the extra energy above the BPS energy scales in $d = 5$ approximately as

$$\Delta E \sim \frac{r_H^2}{G_d} \sim \frac{R_5 V r_H^2}{g^2 \ell_s^d},$$

which at the correspondence point $r_H \sim \ell_s$ is [174] the mass of a single $NS5 - \overline{NS5}$ pair. However, the fivebranes are fractionated by $1/(Q_w Q_p)$ in this regime, as can be seen by U-duality from the $\{Q_1, Q_5, Q_p\}$ system. To get from there to our situation with $Q_w, Q_p, n_{NS5}$, we perform a sequence of dualities, e.g. $S \cdot T_5 \cdot S \cdot T_{6789} \cdot S$. In the process, additional left- and right-movers $\delta N_R = \delta N_L$ become NS5–$\overline{NS5}$ pairs. Therefore, there are actually many pairs of fractionated fivebranes [174]:

$$n_{5\overline{5}}|_c \sim Q_p Q_w,$$

and a corresponding entropy that is comparable [174] to that carried by strings, at the correspondence point:

$$S_{NS5}|_c \sim \sqrt{Q_w Q_p} \sim S_{F1}|_c.$$

So we see that at the correspondence point the NS5-branes begin to share the excess energy $\Delta E$. At larger couplings the NS5-brane entropy in fact dominates over that of the strings, while at weaker couplings the string entropy dominates.

Now let us recall that gravity does not decouple for NS5-branes (when $\ell_s$ is fixed). Then the fact that the NS5-branes start to contribute to the entropy just at the correspondence point ties in with the fact that the correspondence point is just where the metric deviates enough from flat space to be called a black hole [174]. For four dimensional black holes, the extra branes coming into play are the solitonic Kaluza-Klein monopoles [174].

Then the entropy and the emission properties of the NS-NS black holes can be shown to match at the correspondence point with those of the $\{F1, NS5 – \overline{NS5}\}$ system, with both the F1- and NS5-branes controlling which polarizations may or may not be emitted, just as the D1,5-branes and open strings running around $R_5$ do for the U-dual R-R system [174]. General reasoning of this sort is also suggestive of a way to resolve some difficulties with emission and absorption of higher partial waves [174].
In addition, from (56) we saw that correspondence points of two configurations related by a boost are mapped to each other by the boost. This can be used to relate emission rates in one configuration to emission rates in the other [69, 70].

### 5.3 Near-BPS Nonrenormalisation, and Information

The agreement of the entropy and emission power for near-BPS black holes and the sister D-brane/string states is so striking that we are led to wonder if there is a nonrenormalisation theorem operating. In the BPS case there was a direct nonrenormalisation theorem available to protect the degeneracy of states from quantum corrections as the coupling is increased to take the sister D-brane/string boundstate into a black hole.

For non-BPS configurations, nonrenormalisation theorems may indeed be operating in order to ensure the precise agreement found for entropy and emission for near-BPS black holes in the dilute gas regime. For the $d = 5$ case in the dilute gas regime, an explanation was offered in [173], where it was argued that at energies low by comparison to the inverse gravitational radii $1/r_1,5$, the moduli space description for the effective string is not renormalized perturbatively or nonperturbatively. This means that the entropy and low-energy emission/absorption rates are the same for the black hole and its weak-coupling sister D-brane/string boundstate. For another discussion relating to this system see [176].

Another situation in which a nonrenormalisation argument for low-energy modes in near-BPS backgrounds has been formulated [169, 177] is in the study of absorption and emission of longitudinally polarized gravitons by threebranes. There, the conformal invariance of SYM in $(3 + 1)$ dimensions was used to argue that the central charge, the quantity controlling the leading process for emission and absorption of those low energy modes, is not renormalized.

In addition, in [178] it was argued that some one-loop open string corrections to the tree level parallel D$p$-brane results for particle emission are calculable and small at low energies. From this it is conjectured that the D-brane result may be extrapolated to the black hole regime as long as the energies are kept lower than the inverse of any gravitational radius in the problem.

We have seen in previous sections spectacular agreements between black hole entropy and emission for near-BPS states. This led some optimists in the string theory community to speculate that string theory eliminates the information problem for black holes, because scattering involving D-branes and strings is unitary. However, we believe that this speculation may be premature.

Since the D-brane/string and black hole pictures are complementary, we will obtain little direct insight into the black hole information problem by using the unitary nature of perturbative D-brane/string scattering. We must instead tackle head-on the issue of probing a black hole directly, in the string theory context. The question of whether this can be done is an issue of principle, of whether there is a well-defined prescription for calculating. Whether or not we can subsequently perform the calculation is of less importance.

Among proposals that have been made, two of them [179, 180] suggest that this probing of the internal state of the black hole can indeed be done in principle, and they involve stringy hair on black holes. Some previous suggestions on how stringy hair, such as that associated with the infinite number of gauge symmetries in string theory, might be relevant to the black hole information problem may be found in [181, 182, 183, 184, 185]. In [180],

27
with the assumption of the existence of a local low-energy effective field theory to describe scattering, it was argued that the statistics symmetry group for D-branes may be important for information retrieval. There remain several outstanding issues, however, one of which is that it is unclear whether just the above statistics symmetry can be used to reconstruct the entire quantum state of the black hole. In addition, it is unclear whether information about the black hole may be recovered by using low-energy probes; for a discussion on this, see [179]. In different works [87, 175] it was argued that nonrenormalisation arguments for near-BPS cases in the dilute gas regime may push the information loss problem to energies higher than the inverse gravitational radii. In [186] it was argued that decoherence may be an important means by which mixed states arise. Recent work on near-horizon geometry and string theory such as [177] may also be relevant to studying the information problem.

The upshot is, we believe, that string theory with its current technology has not yet resolved the information problem in black hole physics, although it remains the most promising candidate theory to do so.

6 D-Probes and Matrix Theory

In studies of scattering of strings off D-branes it was found [187, 188, 189, 190, 138] that the strings can probe D-branes down to distances only of order the string scale. This occurs essentially because the D-branes have a “halo” of open strings around and ending on them, which then interact with the closed strings in the bulk with a characteristic scale $\ell_s$.

6.1 D-Branes as Probes

D-branes themselves can also be used as probes [191, 192, 193]. The crucial new feature of using D-branes, rather than strings, as probes, is that the distance scales probed by D-branes depend on the string coupling $g$ [194] and are shorter than $\ell_s$ at weak coupling. The use of D-branes as probes of black holes was first investigated in [96, 195, 196] and later in [197, 198, 199, 200].

Two complementary approaches to the physics of D-brane probes exist. The first is to use the action for a test D-brane in the supergravity background, given by (18). This is a valid scheme when gravitational radii are large by comparison to the string scale. The second method is to use SYM perturbation theory for the D-brane system, and this is valid in the opposite regime. The two approaches may be expected to agree if there is a supersymmetric nonrenormalisation theorem operating. Theorems for systems of interest have not been proven generally; indeed, they are provably unavailable in some situations; see [201] regarding $p = 2, 3$. Recent overviews of the considerable work done in comparing D-brane/(Matrix) and supergravity forces on probes may be found in [198, 202, 203]. We will only touch on these works here.

In order to evaluate the action (18) for a probe Dp-brane in a supergravity background, we need to specify the gauge, i.e. the relation between the worldvolume coordinates $\sigma^\alpha$ and spacetime coordinates $X^\mu$. We choose an obvious generalisation of the static gauge:

\[
X^\alpha = \sigma^\alpha, \quad \alpha = 0, 1, \ldots, p, \\
X^i = X^i(t), \quad i = (p+1), \ldots, 9,
\]

(78)
where the velocities $v^i = dX^i/dt$ are small. We then pull back the spacetime metric in $d=10$ string frame to the $(p+1)$ dimensional worldvolume. This yields a worldvolume metric with time components different to those of the spacetime metric because of the motion of the probe.

For our familiar $\{Q_1, Q_5, Q_p\}$ system, this yields for a test D5-brane probe

$$S_5 = -\tau_{D5} \int d^{5+1}\sigma \frac{1}{H_5(r)} \left[ \sqrt{1 - v^2 H_1(r) H_5(r) H_p(r) - 1} \right], \quad (79)$$

where $H_i = 1 + r_i^2/r^2$ are the familiar harmonic functions describing the black hole background. From the point of view of the $d=5$ black hole, we are imagining that our probe is wrapped around the appropriate compactification manifold, and so it looks like a RR-charged particle. Separating off the kinetic and potential terms, we have for the supergravity potential at lowest order in $v^2$,

$$V^\text{SUGRA}_5 = -\tau_{D5} \frac{v^2}{2} \left[ H_p(r) H_1(r) - 1 \right] \quad . \quad (80)$$

In the moduli space approximation, when we keep only $O(v^2)$ terms, we see that the probe does not see the gravitational field of the constituents of the boundstate which are of the same type as itself. Also, the metric on the moduli space is easily read off from the coefficient of $v^2$ in the action. In \cite{96} the case of the D1-brane probe was analyzed with specific attention to the geodesics on this moduli space. Although the moduli space metric is geodesically complete, there are geodesics that take probes into the black hole.

Analogous probe actions can also be written for non-extremal geometries \cite{197}. Probing of extremal but non-BPS black holes has also been studied in \cite{198,200}.

Now we turn to the computations on the D-brane side. In this case, we are considering a situation where the boundstate of a large number of D-branes/strings is separated by a distance $r$ from the probe D-brane. Open strings stretching between the probe D-brane and the D-branes in the boundstate appear as massive fields in the gauge theory for the probe+boundstate.

For BPS situations these probe+boundstate gauge theories have been analyzed comprehensively in \cite{199,202}, whose notation we follow here. Let us consider the situation where we have $N$ D$p$-branes, possibly with other branes dissolved inside or intersecting them. Then, with the masses of the stretched strings $m = r/\ell_s^2$ providing an infrared cutoff, the general contribution to the $(p+1)$ dimensional gauge theory effective action is, using power counting \cite{199,202,203},

$$\Gamma = \sum_{L=1}^{\infty} \left( g_{YM}^2 N \right)^{L-1} \int d^{p+1}\sigma \sum_n \frac{c_{n,L} F^n}{m^{2n-4-L(p-3)}} \quad , \quad (81)$$

where $c_{n,L}$ are constants. We will compare directly the one-loop term in this expansion with the leading supergravity dependence. To leading order, the term of interest is, at one loop, schematically \cite{199,202,203}

$$\hat{\Gamma}^{(1)} = \frac{c_{7-p}}{(2\pi)^{p+1-r}} \int d^{p+1}\sigma \left\{ -\frac{1}{8} \text{Str} \left[ F^4 - \frac{1}{4} \left( F^2 \right)^2 \right] \right\} \quad , \quad (82)$$

where $c_n = (2\pi)^n/(n\omega_{n+1})$. There is also a two-loop term, part of which is needed for the comparison with supergravity at subleading order in $r$. 

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The way the calculation is done in the SYM theory is as follows \[199, 202\]. The number of D5-branes is \(N\); to represent the D1-branes in the D5-brane worldvolume theory, we turn on self-dual magnetic fields, and we excite waves to represent the last charge. To represent the relative motion of the probe and the boundstate, we turn on a transverse scalar; this actually means we need to include an extra term in \(\hat{\Gamma}^{(1)}\) which is easily obtained by T-duality from \(p \to p + 1\). The precise coefficients for all the gauge theory configurations are fixed by charge normalizations. Then the leading one-loop terms \(\hat{\Gamma}^{(1)}\) may be computed. The result is \[96\]

\[
V_{5}^{\text{SYM}} = -\tau_{D5} \frac{v^2}{2} \left[ \frac{r_1^2}{r^2} + \frac{r_2^2}{r^2} \right],
\]

(83)

which agrees with supergravity at lowest order. The status of the two-loop term in SYM is, as yet, less clear \[96, 199\].

In all of these supergravity/SYM comparisons a subtlety arises, concerning the dependence of the SYM result on the probe-type harmonic function. For precise agreement with SYM, it is found that the probe-type harmonic function of the supergravity boundstate must be altered as \(H \to H - 1\). As pointed out in \[199, 202, 203\], this can be thought of in two different ways. The first is to assume that the number of probe-type branes in the boundstate is large by comparison to any other quantum numbers of the boundstate, so that the \(r\)-dependent term in \(H\) dominates. The second is to dualize the probe branes to D0-branes, and to think of the D0-branes as coming from a null rather than spacelike reduction of an M theory wave MW; this is related to the “DLCQ” prescription of \[204\] which we will discuss shortly.

Other agreements between potentials for D-brane probes interacting with charged backgrounds have been found; see \[199, 202\] for a recent overview with references. In the \(d = 5\) near-BPS black hole case, a new twist arises, namely that in order to obtain agreement for the one-loop results we have to perform a change of radial variable \[197\]. Note that since this is a coordinate transformation it does not contain any physics, and that it could be done term by term in the SYM loop expansion.

### 6.2 Matrix Theory

The low energy limit of M theory is given by eleven dimensional supergravity. Approximately a year ago the proposal was made \[205\] that M theory, the overarching theory of everything, is described in the light front frame by a certain matrix model, known as Matrix theory. This proposal has passed some important tests, such as reproducing some low-energy scattering amplitudes of eleven dimensional supergravity, and the emergence of string perturbation theory in compactification on a circle. Matrix theory also has the virtue that many dualities are derivable from the Hamiltonian. Recent reviews may be found in \[206, 21, 207, 22\]. The ideas of Matrix theory have also been applied to black hole systems. Initially, Matrix theory analogues of D-brane results were discovered for charged black holes \[195, 196, 208\], but recent works have made use of other intrinsic properties of Matrix theory to tackle the case of neutral black holes \[209, 210, 211, 212, 213, 214, 215\].

The basic length scale associated with the dynamics of D0-branes is \[219, 220\]

\[
\ell_{11} = g^{1/3} \ell_s.
\]

(84)
For weak coupling $g$, this is considerably shorter than the string scale. As can be seen from (13), it is also the $d=11$ Planck length.

Now suppose that we wish to do physics with D0-branes at distances $\ell \sim \ell_{11}$. The dynamics is described via the open strings which connect the D0-branes. The mass of an open string stretched between two D0-branes a distance $\ell_{11}$ apart is $m \sim g^{1/3}/\ell_s$. For weak coupling, this is an energy far lower than that of the massive modes of the string. Hence, if we wish to study dynamics at $\ell \sim \ell_{11} \ll \ell_s$, we may neglect massive open string modes and simply use the low-energy effective gauge theory. Closed string loop corrections are also suppressed for weak bulk coupling $g$. Next let us rescale the energies and take weak bulk coupling \[219, 220\] such that

$$g \to 0 \quad , \quad E_f = \frac{g^{1/3}}{\ell_s} = \frac{R_z}{\ell_{11}^2} \quad \text{fixed} \quad . \quad (85)$$

Note that we are sending $\ell_s$ (and $\kappa_{10}$) to zero in this limit. Then the Lagrangian for the D0-branes is obtained by dimensionally reducing $d=9+1$ SYM to $d=1+0$. The resulting Lagrangian is in string units $(A_0 = 0)$

$$\mathcal{L}_0 = \text{Tr} \left\{ \frac{1}{2g} \frac{dX^i}{dt} \frac{dX^i}{dt} + \frac{g}{4} \left[ X^i, X^j \right] ^2 + \theta^T \frac{d\theta}{dt} - \theta^T \gamma_i \left[ \theta, X^i \right] \right\} \quad . \quad (86)$$

The bosonic fields $X^i$ are $N \times N$ matrices and, in physical situations where they commute, the diagonal entries describe the positions in the $i$th direction of the $N$ D0-branes. The fermionic $\theta$ is the 16 component Majorana spinor superpartner of $X^i$. If we then rescale all fields and time by appropriate powers of $\ell_{11}$, \[219, 220\] then we obtain a Hamiltonian in $\ell_{11}$ units with the $g$-dependence as an overall factor:

$$\mathcal{H}_0 = R_z \text{Tr} \left\{ \frac{1}{2} \Pi^i \Pi^i - \frac{1}{4} \left[ X^i, X^j \right] ^2 + \theta^T \gamma_i \left[ \theta, X^i \right] \right\} \quad , \quad (87)$$

where $\Pi^i$ are the canonical momenta for $X^i$.

The BFSS conjecture \[205\] is that the large-$N$ limit of the $d=10$ theory described via $\mathcal{H}_0$ describes $d=11$ M theory in the light front frame. It says that the fundamental degrees of freedom of M theory are the nonrelativistic D0-branes, along with the stretched unexcited strings that connect them.

For an indication of why the BFSS conjecture is reasonable, let us study light front kinematics a little. Consider an M-theory graviton with $N$ units of longitudinal momentum, which in $d=10$ IIA language is a bunch of $N$ D0-branes. The $d=11$ graviton is massless, whereas the $d=10$ D0-branes are massive and BPS. Then

$$P_z = \frac{N}{R_z} \quad . \quad (88)$$

Next, from the usual relation for massless particles in $d=11$, $P_0 = \sqrt{P_+^2 + P_z^2}$, we see that

$$P_0 = P_z + \frac{P_+^2 R_z}{2N} + \mathcal{O} \left( \frac{P_+^4}{N^2} \right) \quad . \quad (89)$$

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Going to light-front coordinates, \( x^\pm = x^0 \pm x^4 \), we find that

\[
P_- \simeq \frac{N}{R_z} , \quad P_+ \simeq \frac{P^2_{11} R_4}{2N} ,
\]  
(90)

and the rough equality becomes exact in the \( N \to \infty \) limit. We can then see by studying \( \mathcal{L}_0, \mathcal{H}_0 \) that the light-front energy \( P_+ \) for the bunch is precisely the energy given by the D0-brane Hamiltonian \( \mathcal{H}_0 \) in (87). Energies for other M-theory objects, such as M2- and M5-branes, may also be obtained from the large-\( N \) Matrix theory Hamiltonian, by arranging the noncommuting matrices \( X^i \) in such a fashion as to reproduce the appropriate central charge in the supersymmetry algebra.

Matrix theory is a quantum mechanics with sixteen supersymmetries. The fermionic superpartners \( \theta \) encode spin information. This extended supersymmetry has remarkable consequences, one of which is the enabling of the identification of asymptotic states such as the graviton and its superpartners [205]. Supersymmetry is also crucial in order for the \( a \) priori short-distance \( (\ell \sim \ell_{11}) \) Matrix theory calculations to reproduce lowest-order long-distance graviton scattering in supergravity \( (\ell \gg \ell_{11}) \) [207].

Now, in order to study lower dimensional black holes, we need to know how to do Matrix theory on compactified manifolds; for simplicity we will concentrate on tori \( T^p \). Then the unexcited open strings connecting the D0-branes may wind around the cycles of the torus, and via T-duality this results in the theory of \( N \) Dp-branes, i.e. \( U(N) \) gauge theory in \( (p+1) \) dimensions. Using (12,13), we can express the usual T-duality relation (17) in terms of the quantities \( R_z, \ell_{11} \):

\[
\tilde{R}_i = \frac{\ell_{11}^3}{R_i R_z} .
\]  
(91)

Following the BFSS conjecture, the conjecture of [204] was made that \( \mathcal{H}_0 \) for finite \( N \) still describes M theory, but in discrete light front\(^1\) quantization, DLCQ, where we compactify on a lightlike circle. Subsequently, [221] and [222] have provided an argument for the relation between finite-\( N \) Matrix theory and lightlike compactified M theory. The way this works is by taking compactification on the usual spacelike circle of radius \( R_5 \), and boosting to a lightlike circle, while holding certain quantities fixed. The resulting theory is called \( M' \) theory; by assuming eleven dimensional Lorentz invariance, we have that \( M \) theory is then the same as \( M' \) theory. To see how this goes, we will follow [222]. We boost up the spatial circle via rapidity \( \delta \), where

\[
sinh \delta = \frac{R_*}{\sqrt{2R_z}} ,
\]  
(92)

and take the infinite boost limit

\[
g \to 0 , \quad R_* \text{ fixed} ,
\]  
(93)

so that the circle becomes null in the limit. For a massive particle carrying \( N \) units of (integer quantized) longitudinal momentum, the light front momenta become precisely

\[
P'_- = \frac{N}{R_*} , \quad P'_+ = \frac{(m^2 + P^2_{11}) R_*}{2N} .
\]  
(94)

\(^7\)The light front is mistakenly, although commonly, referred to as the light cone, so we will stick with the common usage DLCQ to avoid confusion.

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It is important that in this boosting procedure we hold fixed $E_f$ and the M-theory torus radii in $d=11$ Planck units \[222\]

\[E_f = \frac{R_i}{\ell_{11}^2} = E'_f = \frac{R'_i}{(\ell'_{11})^2}, \quad \rho_i \equiv \frac{R_i}{\ell_{11}} = \rho'_i = \frac{R'_i}{\ell'_{11}}. \tag{95}\]

We then get

\[g = \left(\frac{R_3}{\ell_{11}}\right)^{3/2} = R_z^{3/4} \left(E'_f\right)^{3/4} \to 0, \quad \ell_s = \left(\frac{\ell_{11}^3}{R_3}\right)^{1/2} = R_z^{1/4} \left(E'_f\right)^{-3/4} \to 0, \tag{96}\]

as for the limit \[88\] that produced the D0-brane Hamiltonian \[87\].

Going to the dual torus $\tilde{T}^p$ with volume $V_p = \prod_i \tilde{R}_i$, we get from \[91\] the dual radii

\[\tilde{R}_i = 1 = \frac{1}{E_f \rho_i} = \frac{1}{E'_f \rho'_i} = \text{finite}. \tag{97}\]

Then the string coupling associated to the dual torus $\tilde{T}^p$ is, from \[17\],

\[\tilde{g} = g \frac{\tilde{V}_p}{\ell_s^p} = g \frac{\ell_p}{V_p} = R_z^{3-p} \left(E'_f\right)^3 \tilde{V}_p. \tag{98}\]

Let us now inspect the coupling of the SYM theory on the dual torus. It is dimensionful for $p \neq 3$,

\[\tilde{g}^2_{YM} = (2\pi)^{p-2} \tilde{g} \ell_s^{p-3}. \tag{99}\]

From this we find a finite nonzero gauge coupling:

\[\frac{\tilde{g}^2_{YM}}{(2\pi)^{p-2}} = \frac{R_z^{3-p} \ell_{11}^{3(p-2)}}{V_p} = \frac{R_z^{3-p} (\ell'_{11})^{3(p-2)}}{V_p'} = \tilde{V}_p \left(E'_f\right)^3 = \text{finite}. \tag{100}\]

Now, as we can see from \[98\], the T-dual string coupling is well-behaved for $p \leq 3$. However, for $p > 3$ it blows up in the Matrix limit, so the bulk (supergravity) theory does not decouple from the brane (gauge) theory, and we should instead use a dual description \[223, 224\]. In addition, the $d > 3+1$ dimensional gauge theory is strongly coupled in the ultraviolet, so new degrees of freedom are required anyway, in order to make sense of the theory. For $p = 4, 5 \ [222, 221]$ there is a well defined prescription for the theory, at least at large-N \[223\]. However, for $p = 6$ it is not clear whether the bulk theory can be decoupled. In the case of $p = 4$, the appropriate theory is the chiral theory of M5-branes compactified on a circle of finite radius, with $\ell_{11} \to 0$ so that the gravitational field of the M5-branes is negligible. For $p = 5$ an S-duality gives NS5-branes with weak bulk coupling and finite worldvolume coupling; this may be considered as a limit of a “little string theory” (see e.g. \[226\]).

At this point we make the comment that Matrix theory à la BFSS is not yet a complete non-perturbative description of M-theory. The prescription of \[222, 221\] does not produce agreement between Matrix theory and supergravity scattering amplitudes in all situations. A difficulty appears to arise in compactifications on some curved manifolds; a disagreement
which has not yet been ironed out was found for the K3 case by [227]. (See however the positive results of [228].) As we mentioned above, there are also difficulties with compactifications on tori of dimension larger than five, and these cases include toroidal compactification down to the phenomenologically interesting case of four spacetime dimensions. One apparent disagreement [229] seems, however, to have been ironed out recently [230]; see also [231, 232, 233]. Another apparent objection [234] has also been overcome and even used to advantage in e.g. [235]. Lastly, it is not known whether the BFSS conjecture is valid for any finite N (strong conjecture), or whether the limit \( N \to \infty \) is required (weak conjecture).

Fortunately, the application of Matrix theory to study the entropy of black holes of interest here does not rely on these subtle details.

In the remainder of this section, we will first discuss the case of charged black holes, and leave the qualitatively different case of neutral black holes until later. Matrix theory compactified on \( T^p \) corresponds to \( d = 11 \) M theory compactified on \( S^1 \times T^p \), so for \( d = 5 \) black holes we need \( p = 5 \). (For \( d = 4 \) black holes we would need \( p = 6 \) which is not yet well enough understood.) For charged black holes, we will assume that the charge represented by \( N \) is large, and use SYM gauge theory as an approximate low-energy effective Matrix theory. This will be suitable for the purposes of identifying the BPS states and their degeneracy.

### 6.3 BPS Configurations

Two different but related approaches have been used in studying \( d = 5 \) BPS black holes in Matrix theory. In both cases, agreement is found with the Bekenstein-Hawking black hole entropy, and parallels with the D-brane approach are seen. We review the Matrix theory pictures of [236, 196] here.

The configuration in M theory which is dual to our familiar \( \{Q_1, Q_5, Q_p\} \) system is

\[
\begin{align*}
M5 & \begin{bmatrix} 0 & \cdots & \cdots & 6 & 7 & 8 & 9 \end{bmatrix} \\
MW & \begin{bmatrix} 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \\
M2 & \begin{bmatrix} 0 & \cdots & 5 & \cdots & \cdots & \cdots \end{bmatrix}
\end{align*}
\]

The \( d = 11 \) metric is [236, 230]

\[
d_{11}^2 = H_{M2}(r)^{1/3}H_{M5}(r)^{2/3} \left\{ H_{M2}(r)^{-1}H_{M5}(r)^{-1} \left[ dx^+ dx^- - (H_{MW}(r) - 1)(dx^-)^2 \right] 
- H_{M2}(r)^{-1} dx_5^2 - H_{M5}(r)^{-1} dx_{6...9}^2 - dr^2 - r^2 d\Omega_3^2 \right\}.
\]

Compactifying \( x^{5...9} \) on \( T^5 \) gives a black string stretched along the \( x^5 \) direction, and upon compactification on \( x^5 \) this yields our \( d = 5 \) BPS black hole metric, with the harmonic functions as before, with \( \{H_{M5}, H_{MW}, H_{M2}\} \to \{H_1, H_5, H_p\} \). The Bekenstein-Hawking entropy is the same calculated in any dimension and we know the answer.

We now wish to compute the degeneracy of states in Matrix theory with the same charges and mass as our supergravity solution. Our theory is \( d = 5 + 1 \) SYM, on the dual torus \( \tilde{T}^5 \), and so various spacetime charges will arise via electric and magnetic fluxes and momentum along internal directions. The dictionary relating M theory quantities to those of the SYM
theory on the dual torus is then \[196, 236\]

\[
\begin{align*}
MW(i) & \quad q_i = \int \text{tr} F_{0i} \\
M2(ij) & \quad m_{ij} = \int \text{tr} F_{ij} \\
M2(i\bar{z}) & \quad m_{i\bar{z}} = \int T_{0i} \\
M5(jklm\bar{z}) & \quad f_i = \int \text{tr} \epsilon_{ijklm} F_{jk} F_{lm} \quad ,
\end{align*}
\]  

(103)

where \( T_{\mu\nu} \) is the energy-momentum tensor in the SYM theory. The case of an M5 wrapped on the \( T^5 \) does not have an analogue in the effective \( d = 5 + 1 \) SYM theory, because it would be a completely delocalized state.

Our familiar configuration corresponds to turning on \( m_5 \), \( f_5 \), and T-dualizing the five-torus. Therefore, our \( Q_1 \) is the number of M5-branes \( f_5 \), our D5-brane charge is \( N \), and our \( Q_p \) is the number of M2-branes \( m_5 \). Now, the M5-branes are represented in the SYM theory by instanton strings. These instanton strings on the torus can be shown \[236\] to have tension fractionated by \( 1/N \). They are therefore the important degrees of freedom in the SYM theory and carry all of the entropy. The light front energy is

\[
P_- = m_5 \frac{R_5 R_5}{\ell_{11}^3} + f_5 \frac{R_5 V}{\ell_{11}^6} = M_{ADM} - \frac{N}{R_5} \quad .
\]  

(104)

From the dictionary (103) and from (6,8,12) we see that the momentum and winding charges on the instanton string are

\[
q_p = m_5 \frac{R_5 R_5}{\ell_{11}^3} \quad ,
\]  

(105)

\[
q_w = f_5 \frac{R_5 V}{\ell_{11}^6} \quad .
\]  

(106)

The BPS instanton string has effective level number and entropy \[236\]

\[
N_L = \frac{1}{2\pi \tau_{\text{eff}}} \left[ P_-^2 - (q_p - q_w)^2 \right] \quad ,
\]  

(107)

\[
S = 2\pi \sqrt{N_L} \quad ,
\]  

(108)

where \( \tau_{\text{eff}} \) is the mass per unit length of the instanton string. We can calculate \( \tau_{\text{eff}} \) \[236\] by using the usual gauge theory formula for the mass of an instanton, \( 4\pi^2/g_{YM}^2 \), and recalling that fractionation occurs. We then use the SYM coupling (100) of the dual torus to get

\[
\tau_{\text{eff}} = \frac{4\pi^2}{g_{YM}^2 N} = \frac{VR_5 R_5^2}{2\pi N \ell_{11}^6} \quad .
\]  

(109)

Putting it all together, we have for the Matrix theory entropy \[236\]

\[
S_{\text{matrix}} = 2\pi \sqrt{f_5 N m_5} \quad ,
\]  

(110)

\[8\]Here, \( R_5, V \) are in fact the T-duals of our familiar torus parameters.
which is indeed the Bekenstein-Hawking entropy. Note that \( R_9 \) has disappeared from the entropy formula, as required\(^9\).

A related approach was studied in [196], where fundamental IIA strings naturally make an appearance; see also [237]. In this approach (the fivebranes are NS5’s), we compactify Matrix theory on a torus which has one small dimension, of radius \( R_9 \). To make contact with IIA strings, any other compact dimensions are taken to be of order \( \ell_s \), which is significantly larger. Thus, on the dual torus there is one large dimension only, and we can therefore dimensionally reduce to \( d = 1+1 \) SYM. The SYM gauge coupling turns out to scale as \( 1/R_9 \), and so it is strong. Then the SYM theory gets stuck on the moduli space, while retaining the option for some SYM fluxes that correspond, via a dictionary similar to (103), to wrapped branes. The entropy counting then proceeds in a way similar to the D-brane counting, and the result agrees precisely with the Bekenstein-Hawking formula. A general entropy formula, invariant under the duality group which remains upon turning off the transverse fivebrane charge, was also derived via the SYM theory in [196].

Before we move on to neutral black holes, let us comment on the near-BPS extension of the above. Both of the approaches [236, 196] yield agreement with \( S_{BH} \) if near-BPS configurations are studied in the naïve \( d = 5+1 \) SYM theory. In the approach of [236], it is argued that all of the available degrees of freedom of the effective Matrix theory can carry the entropy, not only those turned on in the BPS limit. This is somewhat different qualitatively from the D-brane picture of [77]. In addition, in studies of probing near-BPS configurations with D0-branes (MW’s), suggestions are made that a better Matrix theory understanding of the interactions of the probe with the boundstate may give clues useful for information retrieval [195].

### 6.4 Neutral Configurations

We saw above that we needed to have large-\( N \) in order to be able to use SYM gauge theory as our low-energy effective Matrix theory. If there is no large charge on a black hole, as is the case for \( \text{e.g.} \) Schwarzschild-type solutions, then we need a different approach. This case was the subject of [209, 210, 211, 212, 213], and we will discuss it here; see also [22]. For these neutral black holes, the precise coefficient in front of the entropy is not available, but scaling agreement can be found. We will therefore drop all numerical constants; for example, the distinction between the spacelike radius and the lightlike one in DLCQ.

The strategy employed for neutral black holes is to boost the black hole in the longitudinal direction, in order to turn on a significant longitudinal momentum. The idea is then to use Matrix theory degrees of freedom to count the entropy. Suppose we begin with a black hole of mass \( M \). Under the boost, the longitudinal momentum becomes

\[
P_- \sim \frac{N}{R_s} = \frac{N}{M R_s} M.
\]  

The light front energy is from (94)

\[
P_+ \sim \frac{M R_s}{N} M = \frac{M^2 R_s}{N}.
\]  

\(^9\)If we imagined stretching our M2- and M5-branes across the lightlike circle of radius \( R_s \) instead of the spacelike circle of radius \( R_5 \), then the relation (100) ensures that we get the same entropy.
Here $N$ is large but finite.

The object of interest is a $d=(11-p)$ dimensional neutral black hole, where one direction is compactified, namely the longitudinal direction of M theory. Then the appropriate solution is an infinite array of $d$ dimensional neutral black holes; let us space the members of the array in the eleventh dimension by $\sim \hat{R}_*$. When $r_H/\hat{R}_* \ll 1$, this array solution is stable. However, when $r_H/\hat{R}_* \sim 1$, the array becomes unstable and turns into a black string stretched along the longitudinal dimension. When we boost the neutral array or product, the transition point changes [211, 69].

First, let us consider the product solution, i.e. the black string. Let us see what happens to the metric in the boost directions. The longitudinal piece of the metric is, with $\hat{z} \equiv \hat{x}^\natural$,

\[
 ds^2_{||} = \left(1 - \frac{\eta_H^{d-4}}{\eta^{d-4}}\right) dt^2 - dz^2 = dt^2 - dz^2 - \frac{\eta_H^{d-4}}{\eta^{d-4}} dt^2
\]

\[
 = dt^2 - dz^2 - \frac{\eta_H^{d-4}}{\eta^{d-4}} [\cosh \gamma_p dt + \sinh \gamma_p dz]^2
\]

\[
 = dt^2 \left[1 - \frac{\eta_H^{d-4}}{\eta^{d-4}} \cosh^2 \gamma_p\right] - dz^2 \left[1 + \frac{\eta_H^{d-4}}{\eta^{d-4}} \sinh^2 \gamma_p\right] - dt dz \sinh 2\gamma_p \frac{\eta_H^{d-4}}{\eta^{d-4}} , \quad (113)
\]

where $\eta_H$ is the radius of the $(d-1)$ dimensional black hole we used to make the black string. From the metric, we see immediately that if the radius of the $z$ dimension is $R_\star$ at infinity, then at the horizon the radius is

\[
 R_p = \cosh \gamma_p R_\star \sim e^{\gamma_p} R_\star , \quad (114)
\]

and so the longitudinal dimension becomes large at the horizon of the boosted configuration. By direct analogy with [13,14,15], we have at large boosts $\gamma_p$ the black string parameters

\[
 E_p \sim P_p \sim \frac{V_p R_\star \eta_H^{d-4}}{G_{11}} e^{2\gamma_p} , \quad S_p \sim \frac{V_p R_\star \eta_H^{d-3}}{G_{11}} e^{\gamma_p} . \quad (115)
\]

The product solution becomes unstable to forming an array when $\eta_H \sim R_p$. At this transition point, from [14,15,11] we find that the product has entropy [211]

\[
 S_{p|t} \sim P_{p|t} R_\star \sim N . \quad (116)
\]

Next, let us boost the array. Let the asymptotic spacing be, before the boost, $\hat{R}_* = R_\star \cosh \gamma_a$. This ensures that the spacing after the boost is $R_\star$. Then the energy and momentum after the boost are

\[
 E \sim M \cosh \gamma_a \sim M e^{\gamma_a} , \quad P \sim M \sinh \gamma_a \sim M e^{\gamma_a} , \quad (117)
\]

while the entropy is invariant,

\[
 S_a \sim \frac{V_p r_H^{d-2}}{G_{11}} . \quad (118)
\]

The boost also causes [211] an enlargement of the longitudinal radius at the horizon of the black holes in the array. The instability to forming a product solution then sets in at

\[
 r_H \sim e^{\gamma_a} R_\star \equiv R_a . \quad (119)
\]
At this transition point, we find that the entropy of the array is

\[ S_{a|t} \sim P_{a|t} R_s \sim N \sim M r_H \]  

Thus we see that at transition both the supergravity array and product have entropy of order \( N \), the number of units of longitudinal momentum. The array will dominate for larger \( N \), while the product will dominate for smaller \( N \). 

It was argued in [209] that in order to count Matrix degrees of freedom, there is an optimal value of \( N \). The logic goes that if \( N \) is too small, we will not have enough Matrix degrees of freedom to account for the entropy of the neutral black hole, while if \( N \) is too large, we will have too many degrees of freedom and most of them will be frozen into their groundstate in a complicated way. That optimal value is argued to be that given by the common entropy of the array and product at transition [209]. We now turn to examining the nature of the Matrix theory degrees of freedom, near transition.

First let us consider the product side of the transition. The appropriate M theory degrees of freedom are stretched along the longitudinal direction, like the black string. These can only be M2- or M5-branes. The first, and cleanest, Matrix theory calculation for neutral black holes was done [209] for \( d = 8 \), for which we do (DLCQ) Matrix theory on \( \tilde{T}^3 \). Then the modes relevant to the discussion are M2-branes, and they have energy

\[ E_{M2} \sim \frac{R_s R_i}{\ell_{11}^3} \sim \frac{1}{R_i} \]  

These modes therefore correspond via (103) to gluons on the dual torus, with momentum in the \( x_i \) direction. We can find the entropy of these left- and right-moving gluons by using the conformal invariance of 3 + 1 dimensional SYM theory to write an approximate equation of state and the entropy [209]. Since this SYM is the theory of \( N \) coincident D3-branes, we have \( \mathcal{O}(N^2) \) degrees of freedom, and

\[ E_3 \sim N^2 \tilde{V}_3 T^4 \quad , \quad S_3 \sim N^2 \tilde{V}_3 T^3 \]  

Now, from the T-duality relation (111), the light front energy (112) and the fact that \( S \sim N \) around transition, we find upon eliminating the temperature \( T \) [209]

\[ S_{3|t} \sim M (G_8 M)^{1/5} \sim M r_H \]  

which is the entropy of a \( d = 8 \) black hole at transition (120). Therefore, the Matrix theory degrees of freedom have reproduced the entropy of the black hole. We can now go back to the equation of state to check on the temperature at transition. Substituting back, we find

\[ T_t \sim \left( N \tilde{V}_3 \right)^{-1/3} \]  

which is much smaller than the inverse size of the box, and so it seems too cold for the above equation of state to be applicable. However, we have forgotten about fractionation which takes place at low temperatures for wrapped D-branes. At low temperature, the fractionated D3-brane can support fractionated momenta, quantized in units of \( 1/(n_i R_i) \), where \( n_i \) is the winding in the \( i \)th direction. The volume of the fractionated brane is the
same as for $N = n_1 n_2 n_3$ singly wound branes. For the fractionated brane, the above equation of state is then still valid as long as $T > 1/(n_i \tilde{R}_i)$. Therefore, the lowest temperature at which we may apply our equation of state is approximately

$$T \sim \left( n_1 n_2 n_3 \tilde{V}_3 \right)^{-1/3},$$

which is precisely $T_t$. Thus we are right on the edge of validity of (122). Transforming this transition temperature back to the unboosted frame gives the Hawking temperature of the original neutral black hole [209].

In [210] similar reasoning was used to extend the discussion to other $p$. There, somewhat peculiar equations of state were required for the SYM gluon gas in order to reproduce the black hole entropy at transitions for $p \neq 3$. Those equations of state were, however, motivated from studies of parallel $p$-brane black holes, such as [65].

Now we turn to the array side of the transition. When the SYM gas of gluons becomes too cold, which is the case for $N$ larger than its transition value, the gluons freeze out and we are left to do zero mode physics. These zero modes are the motions of the D0-branes themselves. In general, the nature of the boundstate is very complicated, but at $N$ near its transition value there is a simple model [213, 211] which succeeds in reproducing the Bekenstein-Hawking scaling. (For an earlier idea in this direction see [238].)

The effective Lagrangian for the D0-branes was found at one loop SYM in [220] (to two loops in [239]). From the supergravity point of view it is found by letting a probe move in the background of the other D0-branes. To one loop the effective D0-brane Lagrangian is, up to constants of order unity,

$$\mathcal{L}_{0, \text{eff}} \sim \frac{N v^2}{R_*} + \frac{N^2 G_{11} v^4}{R_*^4 V_p r^{d-4}},$$

Note that the potential term is a gravitational attraction, which depends on the velocities of the D0-branes. It is therefore reasonable to assume that the bunch of D0-branes forms a boundstate, with the binding energy coming from the gravitational attraction.

The method of [213, 211] is to use a mean-field approach, and to assume that each D0-brane is in the boundstate of size $r_b$ and saturates the uncertainty bound,

$$\frac{v r_b}{R_*} \sim 1.$$  \hspace{1cm} (127)

Because our D0-branes must be nonrelativistic, we need $r_b/R_* \gg 1$. Then, assuming spherical symmetry and that any spin-dependent forces average to zero, applying the virial theorem then gives a relation between the boundstate size and $N$. Our light front relation (112) then yields

$$M \sim \frac{r_b^{d-3}}{G_d},$$

which reproduces the black hole mass if the boundstate is the same size as the black hole [213, 211]

$$r_H \sim r_b.$$  \hspace{1cm} (129)
From ([111], [127]) and the entropy at transition, we then see that the D0-branes are indeed nonrelativistic at transition. The next step is to assume that the bunch of D0-branes is a Boltzmann gas, so that they are distinguishable. Then the entropy is extensive, of order $N$, and is

$$S \sim N \sim r_H M.$$  \hspace{1cm} (130)

This simple mean-field model is therefore remarkably successful in reproducing the entropy for the $d$ dimensional, longitudinally compactified, black hole at transition. It is also argued in [213] that emitting a D0-brane corresponds to Hawking radiation. In the very recent work [218], a calculation of the rate of this emission was made, and scaling agreement was found with the Hawking emission rate for the sister black hole.

The assumption of distinguishability, however, is difficult to justify. Certainly, for normal D0-branes, we have either Bose-Einstein or Fermi-Dirac statistics, depending on whether any spin is turned on. In a recent work, a model has been proposed [213] in which the distinguishability assumption is satisfied. The analysis of [213] depended on the dimension $d$. The strategy used there was to turn a matrix theory background which does not contribute significantly to the entropy but breaks the D0-brane statistics symmetry and “tethers” each D0-brane to the background in such a way that the D0-branes are distinguishable.

In [214] an all-loop corrected $L_0$ was used to analyze the boundstate of D0-branes, and to study probe interactions with it. The resulting picture is somewhat different but again gives an entropy in a certain regime which agrees with that of the array at transition. D0-probe physics was also studied in [213]. In the recent work [216] an interesting proposal was made that a tachyon instability may signal the existence of the horizon; it would be interesting to find further evidence for this idea. In [217] it was proposed that the entropy counting problem in Matrix theory could be mapped onto the original Gibbons-Hawking calculation, assuming eleven dimensional Lorentz invariance of Matrix theory. Proof of $d = 11$ Lorentz invariance of Matrix theory is however an extremely subtle unsolved problem.

In summary, we see that in regimes where we can calculate in Matrix theory, we reproduce black hole entropy successfully. There remain many interesting issues in the Matrix approach to black holes.

**Outlook**

We have seen that superstring theory has successfully given an identification of the degrees of freedom giving rise to the Bekenstein-Hawking entropy for black holes. In general situations, the entropy of the strongly gravitating system and that of its weakly gravitating sister system agree qualitatively at the correspondence point, while for BPS and near-BPS systems there are precise agreements for the entropy and low-energy emission/absorption processes over a wider region of parameter space. We have also seen the Matrix theory conjecture reproduces black hole entropy successfully for some cases.

Perhaps the most interesting feature of the stringy approach to black hole entropy in diverse dimensions is that it encompasses many different calculational viewpoints, some consistently related by dualities to one another. We have seen various methods for computing the microscopic entropy, operating in dimensions all the way from eleven to three. In all cases, M-/D-branes and closed/open fundamental strings, have played a crucial rôle.
String/M theory is still a work in progress, and there are several outstanding issues to resolve from studies to date of black hole systems in string theory. One of the most important is the issue of information loss, and how it relates to various microscopic entropy computations. It is our expectation that further progress in string theory will lead to better calculational tools for addressing issues in black hole physics and quantum gravity. We look forward to an exciting future in this area of study.

Acknowledgements
The author wishes to thank Aki Hashimoto, David Gross, Gary Horowitz, Rob Myers and Joe Polchinski for discussions, and especially Simon Ross for discussions and reading over v1 of the manuscript.

This work was supported in part by NSF grant PHY-94-07194.

7 Note Added
The full import of the paper [97] by Maldacena had not been appreciated by the string theory community at the time this article was written. However, this changed early in 1998 when the focus shifted away from Matrix Theory to what has become known as the “AdS/CFT Correspondence”. Here we must be very brief, and so we give only a flavor of this topic without attempting to be comprehensive. A more in-depth appreciation may be gained by consulting e.g. the Strings ’98 Online Conference Proceedings [240].

7.1 The AdS/CFT Correspondence
We noted during the discussion of BPS black holes in subsection 4.1 that the usual perturbative gauge theory and black hole regimes are complementary: they have no common region of validity. We also discussed the decoupling limit in subsection 6.2 where we gave a brief overview of Matrix Theory. In this decoupling limit, although the coupling of the gauge theory on the branes remains finite and nonzero, the coupling between the gauge theory on the branes and the string theory in the bulk is turned off, as is the gravitational field of the branes. The crucial observation made by Maldacena was that the decoupling limit can actually yield a system with a nontrivial gravitational field if in addition one sends the number of branes $N$ to infinity. This reveals a new duality between the large-$N$ limit of a gauge theory and a strongly gravitating system.

In the original paper of [97], the brane systems studied were those with constant (or no) dilaton: the D3 (and M2 and M5), and the D1+D5. In the large-$N$ decoupling limit (an analog of which is readily available in $d=11$) the gravitational influence of the branes is retained, but is frozen out because fluctuations become prohibitively expensive in the limit. In addition, the decoupling limit zooms in on the near-horizon region of the brane geometry. For the D3, this region is $AdS_5 \times S^5$. To see this, let us examine the D3 metric:

$$ds^2 = \frac{1}{\sqrt{H_3}} \left( dt^2 - d\vec{x}_\parallel^2 \right) - \sqrt{H_5} \left( d\vec{x}_\perp^2 \right),$$

(131)
where, with \( r = |\vec{x}_\perp| \),

\[
H_3 = 1 + \frac{4\pi g N (\alpha')^2}{r^4} .
\]  

(132)

The large-\( N \) decoupling limit may be expressed as

\[
\alpha' \to 0 \quad , \quad N \to \infty \quad , \quad g_{YM}^2 \text{ finite} .
\]

(133)

To keep interesting dynamics in the gauge theory we must also keep the masses of strings stretched between branes fixed:

\[
U \equiv \frac{r}{\alpha'} \text{ finite} .
\]

(134)

In taking the decoupling limit we see that we lose the 1 in \( H_3 \), and zoom in on the near-horizon region of the D3 geometry. The length scale set by the geometry, the supergravity radius, is

\[
r_3^4 \equiv 4\pi g N (\alpha')^2 .
\]

(135)

Then with a change of radial coordinate \( z \equiv r_3^2/U \), the D3 geometry becomes in the decoupling limit

\[
ds^2 \to \frac{r_3^2}{z^2} \left( dt^2 - d\vec{x}^2 - dz^2 \right) - r_3^2 d\Omega_5^2 .
\]

(136)

There is also a R-R flux threading the sphere. Thus on the “supergravity [string] side” of this duality we study string theory on \( AdS_5 \times S^5 \), a geometry with isometry group \( SO(4,2) \times SO(6) \). Via standard Kaluza-Klein reduction, this can be viewed as a theory on just the five-dimensional \( AdS_5 \). On the “gauge theory side” of the duality, one has \( \mathcal{N} = 4 \) supersymmetric gauge theory in four dimensions, which is known to be a conformal field theory, at large-\( N \). The \( SO(4,2) \) appears as the conformal group and the \( SO(6) \) as the R-symmetry group of the gauge theory. Maldacena’s conjecture is that these two theories are dual to one another and is known as the “AdS/CFT correspondence”.

As with Matrix Theory, there are weak and strong forms of the conjecture. Defining the \(^{'}\text{t} \)Hooft coupling \( \lambda^2 \equiv g_{YM}^2 N \), we find using (135) and the \( p=3 \) relation \( g_{YM}^2 = 2\pi g \)

\[
\frac{\alpha'}{r_3^2} = \frac{1}{\sqrt{2} \lambda} , \quad g = \frac{\lambda^2}{2\pi} \frac{1}{N} .
\]

(137)

At fixed but large \( \lambda, N \) we see that the strong[\(^{'}\text{t} \)Hooft]-coupling expansion of the gauge theory corresponds to the \( \alpha' \) expansion in string theory, and the \( 1/N \) expansion to the string loop expansion (see e.g. [244]). The strongest conjecture states that the duality holds order by order in \( 1/\lambda \) and in \( 1/N \), while weaker conjectures state that it holds only as \( \lambda \to \infty \) and/or \( N \to \infty \). We see that the AdS/CFT correspondence, if correct, provides us in this D3 case with a nonperturbative definition of string theory on \( AdS_5 \times S^5 \). It also says that the large-\( N \) master field of the gauge theory is provided by supergravity.

Subsequent work has generalized the conjecture in many ways. One generalization [242] handled other BPS Dp-brane systems, an example of which is a large collection of D0-branes. It can then be seen [243] that Matrix Theory is an example of “Maldacena duality”. The original AdS/CFT conjecture was refined significantly by [244] (following earlier studies of absorption/emission of bulk modes by D-brane systems), and independently by [245]. There,
prescriptions were given for computing correlation functions in the large-$N$ gauge theory by using supergravity vertices. Also, it was proposed that the place where the gauge theory lives can be thought of as the boundary at infinity of the $AdS_5$ space. This highlights one of the surprising features of the new duality, namely that of “holography”: the physics of the (KK-reduced) gravitational theory on $AdS_5$ is given by the (dual) large-$N$ gauge theory in four dimensions. For more on holography see [243, 246, 247] and talks in [240]. Subsequent computations of the BPS spectra, anomalies, etc. on both sides of the duality have provided additional evidence for the AdS/CFT conjecture. Another generalization [248] covers the non-BPS case, which is conjectured to give finite-temperature gauge theory. Computations have been performed of everything from Wilson loops to four-point correlations functions to glueball masses, assuming the validity of the AdS/CFT duality conjecture.

So far, more has been learned about gauge theory by using supergravity than the other way around, but we are optimistic that work in the other direction will prove fruitful as well.

### 7.2 Black Hole Entropy and AdS/CFT

New methods for computing the Bekenstein-Hawking entropy of black holes have been investigated since the AdS/CFT correspondence [97] came to light.

We have seen that five-dimensional black holes can be thought of as six-dimensional black strings compactified on a circle. The near-horizon geometry of the black strings turns out to be $AdS_3 \times S^3$. More precisely, the first factor is a BTZ black hole, which is everywhere locally (but not globally) $AdS_3$. A new counting of black hole entropy was done in [249]. (Although the counting was probably motivated by the stringy AdS/CFT correspondence, the method used was actually independent of it.) String theory on $AdS_3$ is dual to a $d=1+1$ CFT, and the isometry group is infinite-dimensional. The entropy was computed using older results on the central charge of the CFT and using Cardy’s formula to get the degeneracy of states. In this approach there were some technical subtleties, and unresolved issues such as the question of where the states being counted reside. The presence of holography makes localization in the radial coordinate of the AdS factor a tricky issue.

A lucent discussion of the subtleties in counting the entropy of the BTZ black hole via different methods, and additional references, may be found in a recent overview article by Carlip. We refer the interested reader to that work [250].

AdS/CFT methods were extended to rotating $d=5$ black holes in [251], and the S-dual NS-NS case was studied in [252]. The BTZ factor was seen [253] in the $d=4$ case by lifting on $x^5$ to M-theory (the other factor being $S^2$), and $d=4$ rotation was discussed in [254]. BPS states were tracked in [255], and a nonperturbative stringy exclusion principle was found to be operating. In that work it was also shown how the discrete identifications giving BTZ from $AdS_3$ give a natural mechanism for seeing thermal effects in the black hole background. The relation between AdS/CFT and perturbations of black holes was studied in e.g. [256]. Some corrections to leading-order results have been investigated in [257].

An explanation for the agreement of the D-brane and black hole entropies for extremal but non-BPS $d=5$ Type-I black holes has been offered in [258]. There it was suggested that restoration of supersymmetry in the large-$N$ limit is responsible for the unexpected agreement. It would be interesting to see whether this proposal carries over to other known extremal non-BPS black holes.
Use of the AdS/CFT correspondence to learn about black holes via gauge theory has also been initiated. In a study of the spectrum of the large-$N$ gauge theory using supergravity, it was argued in [259] that the gauge theory in the D3 case describes spacetime behind the horizon as well as the region outside. In [260], where nonextremal D3-branes were studied, it was further conjectured that large-$N$ gauge theory also offers the possibility of resolving both black hole curvature singularities and the information problem. An AdS$_3$ map between the boundary CFT and the theory in the bulk [261] was argued to be useful for tracking information in black hole spacetimes with the BTZ factor.

We look forward to future progress (e.g. [262]) on black hole issues using the AdS/CFT correspondence and other aspects of M/string theory. Particularly interesting would be insights on the precise nature and the location of states counted by the Bekenstein-Hawking entropy, and hints on solving the information problem.

References

[1] J.M. Bardeen, B. Carter and S.W. Hawking, Comm. Math. Phys. 31 (1973) 161.
[2] D. Christodoulou, Phys. Rev. Lett. 25 (1970) 1596.
[3] J. Bekenstein, Lett. Nuov. Cim. 4 (1972) 737; Phys. Rev. D7 (1973) 2333; Phys. Rev. D9 (1974) 3292.
[4] S.W. Hawking, Nature 248 (1974) 30; Comm. Math. Phys. 43 (1975) 199.
[5] C.G. Callan, S.B. Giddings, J.A. Harvey and A. Strominger, Phys. Rev. D45 (1992) R1005, hep-th/9111056.
[6] G.W. Gibbons, S.W. Hawking, Phys. Rev. D15 (1977) 2752.
[7] L. Susskind, hep-th/9309143.
[8] L. Susskind and J.R. Uglum, PASCOS '94 talk, hep-th/9410074.
[9] G. ’t Hooft, as quoted in [7].
[10] J. Bekenstein, MG7 talk, gr-qc/9409015.
[11] G. ’t Hooft, Nucl. Phys. B335 (1990) 138.
[12] L. Susskind and J.R. Uglum, Phys. Rev. D50 (1994) 2700, hep-th/9401070; hep-th/9511227.
[13] J. Russo and L. Susskind, Nucl. Phys. B437 (1995) 611, hep-th/9405117.
[14] A. Sen, Nucl. Phys. B440 (1995) 421, hep-th/9411187; Mod. Phys. Lett. A10 (1995) 2081, hep-th/9504147.
[15] L. Susskind, L. Thorlacius and J.R. Uglum, Phys. Rev. D48 (1993) 3743, hep-th/9306069.
[16] C.M. Hull and P.K. Townsend, Nucl. Phys. B438 (1995) 109, hep-th/9410167.
[17] E. Witten, Nucl. Phys. B443 (1995) 85, hep-th/9503124.
[18] J.H. Schwarz, Nucl. Phys. Proc. Suppl. 55B (1997) 1, hep-th/9606201.
[19] S. Chaudhuri, C.V. Johnson and J. Polchinski, hep-th/9602052.
[20] J. Polchinski, TASI lectures, hep-th/9611050.
[21] T. Banks, Nucl. Phys. Proc. Suppl. 67 (1998) 180, hep-th/9710231.
[22] D. Bigatti and L. Susskind, hep-th/9712072.
[23] A. Strominger and C. Vafa, Phys. Lett. B379 (1996) 99, hep-th/9601029.
[24] E.A. Bergshoeff, C.M. Hull and T.M. Ortín, Nucl. Phys. B451 (1995) 547, hep-th/9504081.
[25] G.T. Horowitz and A. Strominger, Nucl. Phys. B360 (1991) 197.
[26] M.J. Duff, R.R. Khuri and J.X. Lu, Phys. Rept. 259 (1995) 213, hep-th/9412184.
[27] E. Witten, J. Geom. Phys. 22 (1997) 103, hep-th/9610234.
[28] P. Pasti, D. Sorokin and M. Tonin, Phys. Rev. D55 (1997) 6292, hep-th/9611100.
[29] M. Cederwall and A. Westerberg, J. High Energy Phys. 02 (1998) 004, hep-th/9710007.
[For JHEP, see http://jhep.mse.jhu.edu/ .]
[30] J. Maharana and J.H. Schwarz, Nucl. Phys. B390 (1993) 3, hep-th/9207016.
[31] A. Sen, Int. J. Mod. Phys. A9 (1994) 3707, hep-th/9402002.
[32] H. Lu and C. Pope, Nucl. Phys. B465 (1996) 127, hep-th/9512012.
[33] Supergravities in Diverse Dimensions, Volumes I and II, Ed.s A. Salam and E. Sezgin, North-Holland / World Scientific, 1989.
[34] A. Sen, Phys. Lett. B274 (1992) 34, hep-th/9108011; Phys. Rev. Lett. 69 (1992) 1006, hep-th/9204046.
[35] R.C. Myers and M.J. Perry, Ann. Phys. 172 (1986) 304.
[36] D. Youm, hep-th/9710046, submitted to Physics Reports.
[37] S. Ferrara and J.M. Maldacena, Class. Quant. Grav. 15 (1998) 749, hep-th/9706097.
[38] P.K. Townsend, Cargese lectures, hep-th/9712004.
[39] D. Olive and E. Witten, Phys. Lett B78 (1978) 97.
[40] M.J. Duff, P. Howe, T. Inami and K.S. Stelle, Phys. Lett. B191 (1987) 70.
[41] P.K. Townsend, Phys. Lett. B350 (1995) 184, hep-th/9501068.
[42] R. Güven, Phys. Lett. B276 (1992) 49.
[43] A. Dabholkar and J.A. Harvey, Phys. Rev. Lett. 63 (1989) 478.
[44] A. Dabholkar, G.W. Gibbons, J.A. Harvey and F. Ruiz Ruiz, Nucl. Phys. B340 (1990) 33.
[45] J. Dai, R.G. Leigh and J. Polchinski, Mod. Phys. Lett. A4 (1989) 2073.
[46] R.G. Leigh, Mod. Phys. Lett. A4 (1989) 2767.
[47] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724, hep-th/9510017.
[48] E. Witten, Nucl. Phys. B460 (1996) 335, hep-th/9510135.
[49] M. Cederwall, A. von Gussich, B.E.W. Nilsson, and A. Westerberg, Nucl. Phys. B490 (1997) 163, hep-th/9610148.

[50] M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, Nucl. Phys. B490 (1997) 179, hep-th/9611159.

[51] E. Bergshoeff and P.K. Townsend, Nucl. Phys. B490 (1997) 145, hep-th/9611173.

[52] M. Aganagic, C. Popescu and J.H. Schwarz, Nucl. Phys. B495 (1997) 99, hep-th/9612080.

[53] A.A. Tseytlin, Nucl. Phys. B501 (1997) 41; hep-th/9701125.

[54] D. Brecher and M.J. Perry, hep-th/9801127.

[55] M.R. Douglas, hep-th/9512077.

[56] P.K. Townsend, Nucl. Phys. B283 (1996) 44, hep-th/9512059.

[57] P.K. Townsend, Phys. Lett. B373 (1996) 68, hep-th/9512062.

[58] P.K. Townsend, Nucl. Phys. Proc. Suppl. 58 (1997) 163, hep-th/9609217.

[59] M. Berkooz, M.R. Douglas and R.G. Leigh, Nucl. Phys. B480 (1996) 265, hep-th/9606139.

[60] G.T. Horowitz and J. Polchinski, Phys. Rev. D55 (1997) 6189, hep-th/9612146.

[61] G.T. Horowitz, Presented at Symposium on Black Holes and Relativistic Stars (dedicated to memory of S. Chandrasekhar), Chicago, IL, 14-15 Dec 1996, gr-qc/9704072.

[62] A.W. Peet, Nucl. Phys. B456 (1995) 732, hep-th/9506200.

[63] M.B. Green, J.H. Schwarz and E. Witten, Superstring Theory, Cambridge University Press, 1985.

[64] S.S. Gubser, I.R. Klebanov and A.W. Peet, Phys. Rev. D54 (1996) 3915, hep-th/9602135.

[65] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. B475 (1996) 164, hep-th/9604089.

[66] A. Bytsenko and S. Odintsov, Prog. Theor. Phys. 98 (1997) 987, hep-th/9611151; A. Bytsenko, E. Goncalves and S. Odintsov, JETP Lett. 66 (1997) 11, hep-th/9708130.

[67] R.G. Cai, R.-K. Su and P.K.N. Yu, Phys. Lett. A195 (1994) 307.

[68] R.C. Myers and G.T. Horowitz, to appear.

[69] S.R. Das, S.D. Mathur, S.K. Rama and P. Ramadevi, hep-th/9711003.

[70] S.R. Das, S.D. Mathur and P. Ramadevi, hep-th/9803078.

[71] R.C. Myers, Phys. Rev. D35 (1987) 455.

[72] R. Gregory and R. Laflamme, Phys. Rev. Lett. 70 (1993) 2837, hep-th/9301052.

[73] G.T. Horowitz and J. Polchinski, Phys. Rev. D57 (1998) 2557, hep-th/9707170.

[74] R.R. Khuri, Mod. Phys. Lett. A13 (1998) 1407, gr-qc/9803093.

[75] G.T. Horowitz, Talk given at Pacific Conference on Gravitation and Cosmology, Seoul, South Korea, 1996, gr-qc/9604051.
[76] F. Larsen and F. Wilczek, Phys. Lett. B375 (1996) 37, hep-th/9511063; Nucl. Phys. B488 (1997) 261, hep-th/9609084.
[77] C.G. Callan and J.M. Maldacena, Nucl. Phys. B472 (1996) 591, hep-th/9602043.
[78] G.T. Horowitz and A. Strominger, Phys. Rev. Lett. 77 (1996) 2368, hep-th/9602051.
[79] J.C. Breckenridge, R.C. Myers, A.W. Peet and C. Vafa, Phys. Lett. B391 (1996) 93, hep-th/9602063.
[80] J.C. Breckenridge, D.A. Lowe, R.C. Myers, A.W. Peet, A. Strominger and C. Vafa, Phys. Lett. B381 (1996) 423 hep-th/9603078.
[81] C.V. Johnson, R.R. Khuri and R.C. Myers, Phys. Lett. B378 (1996) 78, hep-th/9603061.
[82] J.M. Maldacena and A. Strominger, Phys. Rev. Lett 77 (1996) 428, hep-th/9603060.
[83] G.T. Horowitz, D.A. Lowe and J.M. Maldacena, Phys. Rev. Lett. 77 (1996) 430, hep-th/9603195.
[84] J.M. Maldacena, Ph.D. Thesis, Princeton University, hep-th/9607235; Karpacz lectures, hep-th/9705078.
[85] G. Papadopoulos and P.K. Townsend, Phys.Lett. B380 (1996) 273, hep-th/9603087.
[86] A.A. Tseytlin, Nucl.Phys. B475 (1996) 149, hep-th/9604033.
[87] J.M. Maldacena and A. Strominger, Phys. Rev. D55 (1997) 861, hep-th/9609026.
[88] L. Andrianopoli, R. D'Auria and S. Ferrara, Phys. Lett. B403 (1997) 12, hep-th/9703150; Phys. Lett. B411 (1997) 39, hep-th/9705024.
[89] J.P. Gauntlett, D.A. Kastor, J. Traschen, Nucl. Phys. B478 (1996) 544-560, hep-th/9604179.
[90] N. Khviengia, Z. Khviengia, H. Lu and C.N. Pope, Phys. Lett. B388 (1996) 21, hep-th/9605077.
[91] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. B479 (1996) 319, hep-th/9607107.
[92] M.S. Costa and M. Cvetic, Phys. Rev. D56 (1997) 4834 hep-th/9703204.
[93] V. Balasubramanian, Cargese lectures, hep-th/9712213.
[94] M. Bershadsky, V. Sadov and C. Vafa, Nucl. Phys. B463 (1996) 398, hep-th/9511222.
[95] C. Vafa, Nucl. Phys. B463 (1996) 435, hep-th/9512078.
[96] M.R. Douglas, J. Polchinski and A. Strominger, J. High Energy Phys. 12 (1997) 003, hep-th/9703031.
[97] J. Maldacena, hep-th/9711200.
[98] M.S. Costa and M.J. Perry, Nucl. Phys. B520 (1998) 205, hep-th/9712026; Nucl. Phys. B524 (1998) 333, hep-th/9712160.
[99] G.T. Horowitz and D. Marolf, Phys. Rev. D55 (1997) 835, hep-th/9605224; Phys. Rev. D55 (1997) 846, hep-th/9606113.
[100] N. Kaloper, R.C. Myers and H. Roussel, Phys. Rev. D55 (1997) 7625, hep-th/9612248.
[101] G.T. Horowitz and H.-S. Yang, Phys. Rev. D55 (1997) 7618, hep-th/9701074.
[102] D. Marolf, Phys. Rev. D57 (1998) 2427, hep-th/9705063.
[103] S.F. Ross, hep-th/9710158.
[104] D. Garfinkle, Phys. Rev. D46 (1992) 4286, gr-qc/9209002.
[105] J.M. Maldacena and L. Susskind, Nucl. Phys. B475 (1996) 679, hep-th/9604042.
[106] S.R. Das and S.D. Mathur, Phys. Lett. B375 (1996) 103, hep-th/9601152.
[107] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. B475 (1996) 179, hep-th/9601066.
[108] V. Balasubramanian and F. Larsen, Nucl. Phys. B478 (1996) 199, hep-th/9604189.
[109] V. Balasubramanian, R.G. Leigh and F. Larsen, Phys. Rev. D57 (1998) 3509, hep-th/9704143.
[110] S. Ferrara, R.E. Kallosh and A. Strominger, Phys. Rev. D52 (1995) 5412, hep-th/9508072.
[111] A. Strominger, Phys. Lett B383 (1996) 39, hep-th/9602111.
[112] S. Ferrara and R.E. Kallosh, Phys. Rev. D54 (1996) 1514, hep-th/9602130; Phys. Rev. D54 (1996) 1525, hep-th/9603096.
[113] M. Shmakova, Phys.Rev. D56 (1997) 540, hep-th/9612076.
[114] K. Behrndt, G. Lopez Cardoso, B. de Wit, R.E. Kallosh, D. Lüst and T. Mohaupt, Nucl. Phys. B488 (1997) 236, hep-th/9610105.
[115] D. Kaplan, D.A. Lowe, J.M. Maldacena and A. Strominger, Phys. Rev. D55 (1997), hep-th/9609204.
[116] K. Behrndt and T. Mohaupt, Phys. Rev. D56 (1997) 2206, hep-th/9611140.
[117] J.M. Maldacena, Phys. Lett. B403 (1997) 20, hep-th/9611163.
[118] S.-J. Rey, Nucl. Phys. B508 (1997) 569, hep-th/9610157.
[119] J.M. Maldacena, A. Strominger and E. Witten, J. High Energy Phys. 12 (1997) 002, hep-th/9711053.
[120] C. Vafa, Adv. Theor. Math. Phys. 2 (1998) 207, hep-th/9711067. [For ATMP, see http://www.intlpress.com/journals/ATMP/ .]
[121] C.G. Callan and J.M. Maldacena, Nucl. Phys. B513 (1998) 198, hep-th/9708147.
[122] A. Sen, J. High Energy Phys. 02 (1998) 011, hep-th/9712150.
[123] K. Behrndt, M. Cvetič and W.A. Sabra, hep-th/9712221.
[124] M. Cvetič and D. Youm, Nucl. Phys. B476 (1996) 118, hep-th/9603100.
[125] G.T. Horowitz, J.M. Maldacena and A. Strominger, Phys. Lett. B383 (1996) 151, hep-th/9603109.
[126] J.M. Maldacena, Nucl. Phys. B477 (1996) 168, hep-th/9605016.
[127] R. Dijkgraaf, E. Verlinde and H. Verlinde, Nucl. Phys. B486 (1997) 77, hep-th/9603126; Nucl. Phys. B486 (1997) 89, hep-th/9604053; Nucl. Phys. B484 (1997) 543, hep-th/9607028; Comm. Math. Phys. 185 (1997) 197, hep-th/9608096.
[128] T. Ortín, Karpacz lectures, hep-th/9705093.
[129] A. Dabholkar, Phys. Lett. B402 (1997) 53, hep-th/9702050.
[130] A. Dabholkar, G. Mandal and P. Ramadevi, Nucl. Phys. B520 (1998) 117, hep-th/9705239.
[131] H.J. Sheinblatt, Phys. Rev. D57 (1998) 2421, hep-th/9705054.
[132] K. Sfetsos and K. Skenderis, Nucl. Phys. B517 (1998) 179, hep-th/9711138.
[133] S. Hyun, hep-th/9704005.
[134] M. Bañados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett 69 (1992) 1849, hep-th/9204009.
[135] M. Bañados, M. Henneaux, C. Teitelboim and J. Zanelli, Phys. Rev. D48 (1993) 1506, gr-qc/9302012.
[136] S. Carlip, Phys. Rev. D51 (1995) 632, gr-qc/9409052, Phys. Rev. D55 (1997) 878, gr-qc/9606043.
[137] S.R. Das, G.W. Gibbons and S.D. Mathur, Phys. Rev. Lett. 78 (1997) 417, hep-th/9609052.
[138] A. Hashimoto and I.R. Klebanov, Nucl. Phys. Proc. Suppl. 55B (1996) 231, hep-th/9611214.
[139] E. Keski-Vakkuri and P. Kraus, Nucl. Phys. B491 (1997) 249, hep-th/9610045.
[140] S.D. Mathur, Talk given at Strings '96, July 1996, Santa Barbara, U.S.A., hep-th/9609053.
[141] S.R. Das and S.D. Mathur, Nucl. Phys. B478 (1996) 561, hep-th/9606185.
[142] S.R. Das and S.D. Mathur, Nucl. Phys. B482 (1996) 153, hep-th/9607149.
[143] S.S. Gubser and I.R. Klebanov, Phys. Rev. Lett. 77 (1996) 4491, hep-th/9609076.
[144] A. Dhar, G. Mandal and S.R. Wadia, Phys. Lett. B388 (1996) 51, hep-th/9605234.
[145] R.E. Kallosh, G.W. Gibbons and B. Kol, Phys. Rev. Lett. 77 (1996) 4992, hep-th/9607108.
[146] S. Ferrara, G.W. Gibbons and R.E. Kallosh, Nucl.Phys. B500 (1997) 75, hep-th/9702103.
[147] B. Kol and A. Rajaraman, Phys. Rev. D56 (1997) 983, hep-th/9608126.
[148] C.G. Callan, S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. B489 (1997) 65, hep-th/9610172.
[149] I.R. Klebanov and M. Krasnitz, Phys. Rev. D55 (1997) 3250, hep-th/9612051.
[150] H.W. Lee, Y.S. Myung and J.Y. Kim, hep-th/9708099.
[151] I.R. Klebanov and M. Krasnitz, Phys. Rev. D56 (1997) 2173, hep-th/9703216.
[152] S.D. Mathur, Nucl. Phys. B514 (1998) 204, hep-th/9704156.
[153] S.S. Gubser, Phys. Rev. D56 (1997) 4984, hep-th/9704195.
[154] S.S. Gubser, Phys. Rev. D56 (1997) 7854, hep-th/9706100.
[155] S. Das, A. Dasgupta, P. Majumdar and T. Sarkar, hep-th/9707124.
[156] K. Hosomichi, Nucl. Phys. B524 (1998) 312, hep-th/9711072.
[157] S.S. Gubser and I.R. Klebanov, Nucl. Phys. B482 (1996) 173, hep-th/9608108.
[158] S.W. Hawking and M.M. Taylor-Robinson, Phys. Rev. D55 (1997) 7680, hep-th/9702043.
[159] I.R. Klebanov, A. Rajaraman and A.A. Tseytlin, Nucl. Phys. B503 (1997) 157, hep-th/9704112.
[160] J.M. Maldacena and A. Strominger, Phys. Rev. D56 (1997) 4975, hep-th/9702013.
[161] M. Cvetič and F. Larsen, Phys. Rev. D56 (1997) 4994, hep-th/9705192; Nucl. Phys. B506 (1997) 107, hep-th/9706071; Phys. Rev. D57 (1998) 6297, hep-th/9712118.
[162] S.R. Das, Phys. Lett. B412 (1997) 259, hep-th/9705165.
[163] I.R. Klebanov and S.D. Mathur, Nucl. Phys. B500 (1997) 115, hep-th/9701187.
[164] R. Emparan, Phys. Rev. D56 (1997) 3591, hep-th/9704204.
[165] D. Kastor and J. Traschen, Phys. Rev. D57 (1998) 4862, hep-th/9707157.
[166] A.A. Tseytlin, Nucl. Phys. B469 (1996) 51, hep-th/9602064.
[167] I.R. Klebanov, Nucl. Phys. B496 (1997) 231, hep-th/9702076.
[168] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. B499 (1997) 217, hep-th/9703040.
[169] S.S. Gubser and I.R. Klebanov, Phys. Lett. B413 (1997) 41, hep-th/9708003.
[170] S.S. Gubser and A. Hashimoto, hep-th/9805140.
[171] S.D. Mathur and A. Matusis, hep-th/9805064.
[172] K. Hosomichi, hep-th/9806010.
[173] E. Halyo, B. Kol, A. Rajaraman and L. Susskind, Phys. Lett. B401 (1997) 15, hep-th/9609073.
[174] S.D. Mathur, hep-th/9706154.
[175] J.M. Maldacena, Phys. Rev. D55 (1997) 7645, hep-th/9611125.
[176] S. F. Hassan and S.R. Wadia, Phys. Lett. B402 (1997) 43, hep-th/9703163 hep-th/9712213.
[177] I.R. Klebanov, Strings '97 talk, hep-th/9709160.
[178] S.R. Das, Strings '97 talk, hep-th/9709200.
[179] T. Banks, hep-th/9606026.
[180] A. Strominger, Phys. Rev. Lett. 77 (1996) 3498, hep-th/9606016.
[181] M. Bowick, S. Giddings, J. Harvey, G. Horowitz and A. Strominger, Phys. Rev. Lett. 61 (1988) 2823.
[182] L. Krauss and F. Wilczek, Phys. Rev. Lett. 62 (1989) 221.
[183] S. Coleman, J. Preskill and F. Wilczek, Phys. Rev. Lett. 67 (1991) 1975; Nucl. Phys. B378 (1992) 175.
[184] J. Schwarz, Phys. Lett. B272 (1991) 239.
[185] S. Giddings, J. Harvey, J. Polchinski, S. Shenker and A. Strominger, Phys. Rev. D50 (1994) 6422, hep-th/9309152.
[186] R.C. Myers, Gen. Rel. Grav. 29 (1997) 1217, gr-qc/9705065.
[187] I.R. Klebanov and L. Thorlacius, Phys. Lett. B371 (1996) 51, hep-th/9510200.
[188] C. Bachas, Phys. Lett. B374 (1996) 37, hep-th/9511043.
[189] S.S. Gubser, A. Hashimoto, I.R. Klebanov and J.M. Maldacena, Nucl. Phys. B472 (1996) 231, hep-th/9601057.
[190] M.R. Garousi and R.C. Myers, Nucl. Phys. B475 (1996) 193, hep-th/9603194.
[191] M.R. Douglas and G. Moore, hep-th/9603167.
[192] M.R. Douglas, hep-th/9604198.
[193] T. Banks, M.R. Douglas and N. Seiberg, Phys. Lett. B387 (1996) 278, hep-th/9605199.
[194] S.H. Shenker, hep-th/9509132.
[195] M. Li and E.J. Martinec, Class. Quant. Grav. 14 (1997) 3187, hep-th/9703211.
[196] R. Dijkgraaf, E. Verlinde and H. Verlinde, Nucl. Phys. B506 (1997) 121, hep-th/9704018.
[197] J.M. Maldacena, Phys. Rev. D57 (1998) 3736, hep-th/9705053.
[198] J.M. Pierre, Phys. Rev. D56 (1997) 6710, hep-th/9707102.
[199] A. Chepelev and A.A. Tseytlin, Nucl. Phys. B515 (1998) 73, hep-th/9709087.
[200] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, Phys. Lett. B423 (1998) 238, hep-th/9711010.
[201] M. Dine and N. Seiberg, Phys. Lett. B409 (1997) 239, hep-th/9705057.
[202] A.A. Tseytlin, Strings '97 talk, hep-th/9709123.
[203] J.M. Maldacena, Strings '97 talk, hep-th/9709099.
[204] L. Susskind, hep-th/9704080.
[205] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, Phys. Rev. D55 (1997) 5112, hep-th/9610043.
[206] R. Dijkgraaf, E. Verlinde and H. Verlinde, Strings '97 talk, hep-th/9709107.
[207] M. Berkooz, Strings '97 talk, hep-th/9712012.
[208] M. Li and E.J. Martinec, Class. Quant. Grav. 14 (1997) 3205, hep-th/9704134.
[209] T. Banks, W. Fischler, I.R. Klebanov and L. Susskind, hep-th/9709091.
[210] I.R. Klebanov and L. Susskind, Phys. Lett. B416 (1998) 62, hep-th/9709108.
[211] G.T. Horowitz and E.J. Martinec, Phys. Rev. D57 (1998) 4935, hep-th/9710217.
[212] M. Li, J. High Energy Phys. 01 (1998) 009, hep-th/9710226.
[213] T. Banks, W. Fischler, I.R. Klebanov and L. Susskind, J. High Energy Phys. 01 (1998) 008, hep-th/9711005.
[214] H. Liu and A.A. Tseytlin, J. High Energy Phys. 01 (1998) 010, hep-th/9712063.
[215] E.J. Martinec and M. Li, hep-th/9801070.
[216] D. Kabat and G. Lifschytz, hep-th/9806214.
[217] D.A. Lowe, hep-th/9802173.
[218] T. Banks, W. Fischler and I.R. Klebanov, Phys. Lett. B423 (1998) 54, hep-th/9712236.
[219] D. Kabat and P. Pouliot, Phys. Rev. Lett. 77 (1996) 1004, hep-th/9603127.
[220] M.R. Douglas, D. Kabat, P. Pouliot and S.H. Shenker, Nucl.Phys. B485 (1997) 85, hep-th/9608024.
[221] A. Sen, Adv. Theor. Math. Phys. 2 (1998) 51, hep-th/9709220.
[222] N. Seiberg, Phys. Rev. Lett. 79 (1997) 3577, hep-th/9710009.
[223] M. Berkooz, M. Rozali and N. Seiberg, Phys. Lett. B408 (1997) 105, hep-th/9704089.
[224] N. Seiberg, Phys. Lett. B408 (1997) 98, hep-th/9705221.
[225] J.M. Maldacena and A. Strominger, hep-th/9710014.
[226] R. Argurio and L. Houart, Nucl. Phys. B517 (1998) 205, hep-th/9710027.
[227] M.R. Douglas and H. Ooguri, Phys. Lett. B425 (1998) 71, hep-th/9710178.
[228] S. Kachru, A. Lawrence and E. Silverstein, Phys. Rev. Lett. 80 (1998) 2996, hep-th/9712223.
[229] M. Dine and A. Rajaraman, Phys. Lett. B425 (1998) 77, hep-th/9710174.
[230] Y. Okawa and T. Yoneya, hep-th/9806108.
[231] W. Taylor IV and M. Van Raamsdonk, hep-th/9806066.
[232] M. Fabbrichesi, G. Ferretti and R. Iengo, hep-th/9806018 and hep-th/9806166.
[233] R. Echols and J. Gray, hep-th/9806109.
[234] S. Hellerman and J. Polchinski, hep-th/9711037.
[235] M. Krogh, hep-th/9801034.
[236] M. Li and E.J. Martinec, Strings ’97 talk, hep-th/9709114.
[237] T. Banks and N. Seiberg, Nucl. Phys. B497 (1997) 41, hep-th/9702187.
[238] I.V. Volovich, Talk given at 2nd International Sakharov Conference on Physics, Moscow, Russia, 20-23 May 1996. Published in Sakharov Conference 1996:618-621, hep-th/9608137.
[239] K. Becker, M. Becker, J. Polchinski and A.A. Tseytlin, Phys. Rev. D56 (1997) 3174, hep-th/9706072.
[240] Strings '98, June 1998, Santa Barbara, USA. Online Proceedings (audio recordings and scanned transparencies): http://www.itp.ucsb.edu/online/strings98/.

[241] D.J. Gross and H. Ooguri, hep-th/9805129.

[242] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, Phys. Rev. D58 (1998) 046004, hep-th/9802042. [This journal reference is not a typo. PRD has changed from page numbering to article numbering.]

[243] S.H. Shenker, talk given May 21, during ITP Program on Dualities in String Theory. DiST program online: http://www.itp.ucsb.edu/online/dual/.

[244] S.S. Gubser, A.M. Polyakov and I.R. Klebanov, Phys. Lett. B428 (1998) 105, hep-th/9802109.

[245] E. Witten, hep-th/9802150.

[246] E. Witten and L. Susskind, hep-th/9805114.

[247] J.L.F. Barbón and E. Rabinovici, hep-th/9805143.

[248] E. Witten, hep-th/9803131.

[249] A. Strominger, J. High Energy Phys. 02 (1998) 009, hep-th/9712251.

[250] S. Carlip, hep-th/9806026.

[251] M. Cvetič and F. Larsen, hep-th/9805097.

[252] M. Cvetič and A.A. Tseytlin, hep-th/9806141.

[253] F. Larsen and V. Balasubramanian, hep-th/9802198.

[254] M. Cvetič and F. Larsen, hep-th/9805146.

[255] J.M. Maldacena and A. Strominger, hep-th/9804085.

[256] V. Balasubramanian, P. Kraus and A. Lawrence, hep-th/9805171.

[257] K. Behrndt, I. Brunner and I. Gaida, hep-th/9806195.

[258] J.L.F Barbón, J.L. Mañes and M.A. Vásquez-Mozo, hep-th/9805154.

[259] G.T. Horowitz and H. Ooguri, Phys. Rev. Lett. 80 (1998) 4116, hep-th/9802116.

[260] G.T. Horowitz and S. Ross, J. High Energy Phys 04 (1998) 015, hep-th/9803085.

[261] E.J. Martinec, hep-th/9804111; http://www.itp.ucsb.edu/online/dual/martinec1/.

[262] T. Banks, M.R. Douglas, G.T. Horowitz and E.J. Martinec, to appear.