The spin-up of a neutron star crucially depends on the maximum orbital frequency around it, as do a host of other high energy accretion phenomena in low mass X-ray binaries, including quasi-periodic oscillations (QPOs) in the X-ray flux. We compare the maximum orbital frequencies for MIT-bag quark stars and for neutron stars modeled with the FPS equation of state. The results are based on relativistic calculations of constant baryon sequences of uniformly rotating strange star models, and are presented as a function of the stellar rotational frequency. The marginally stable orbit is present outside quark stars for a wide range of parameters, but outside the FPS neutron stars it is present only for the highest values of mass. This allows a discrimination between quark stars and neutron stars in the resonance theory of kHz QPOs.

1 Introduction

If the recently discovered kHz QPOs are a manifestation of strong gravity, they may be used to constrain the external metric of the compact source and the equation of state of matter at supranuclear densities. Kluźniak et al. suggested that the mass of a neutron star may be derived by observing the maximum orbital frequency, if it occurs in the marginally stable orbit, and that the low frequency QPOs occurring in X-ray pulsars will have their counterpart in LMXBs, at frequencies in the kHz range. Such kHz QPOs have indeed been discovered and their frequency used to derive mass values of about $2M_\odot$, under the stated assumption. In fact, the QPO frequency may correspond to a larger orbit, and hence a smaller mass.

The question whether quark stars may have maximum orbital frequencies as low as the observed kHz QPO frequencies has also been investigated. Bulik et al. showed that slowly rotating strange stars, described by the simple MIT bag model with massless and non-interacting quarks, have orbital frequencies at the marginally stable orbit higher than the maximum frequency of 1.07 kHz in 4U 1820-30, reported by Zhang et al. However, the ISCO frequencies can be as low as 1 kHz when more sophisticated models of quark matter (with massive strange quarks and lowest order QCD interactions) and/or rapid stellar rotation are taken into account. The lowest orbital frequency at the ISCO was found to be attained either for non-rotating massive configurations close to their maximum mass limit, or for configurations at the equatorial mass-shedding limit for a broad range of stellar masses. The effect of the crust, if present, has been investigated for normal evolutionary sequences. Typically the crust increases the
maximum orbital frequency at the Keplerian limit.

If the strange star configurations described by Dey et al. are allowed, the rotational and maximal orbital frequencies are much higher than those for neutron star models, or for the MIT-bag models of quark stars. The maximum orbital frequencies for the Dey et al. models are always higher than the kHz QPO frequencies observed to date, and higher than 1.5 kHz for stars with masses greater than 1 $M_\odot$.

2 Results

Here we summarize our studies of maximal orbital frequency of rotating quark stars. We focus on neutron stars modeled with the FPS e.o.s., and on quark stars described by the simplified MIT bag model ($\alpha_c = m_s = 0$ and $B = 60 \text{ MeV}/\text{fm}^3$). Similar results are obtained for other models of quark stars and neutron stars. We have performed the calculations using two different highly accurate relativistic codes.

A comparison between the relevant properties of rotating quark stars and neutron stars described by the FPS equation of state is afforded by Figures 1 and 2. The dotted lines separate the models with and without a co-rotating marginally stable orbit.

The orbits around neutron stars have different properties than those around quark stars (Figures 1, 2). We see that for quark stars with moderate and high baryon masses the marginally orbit is always present at any rotation rate. For intermediate mass quark stars, stable orbits extend down to the stellar surface for moderate rotation rates (the short-dashed models), but the gap is present for either slowly or rapidly rotating stars. For the lowest mass stars, the relativistic gap is present only at high rotation rates.
Figure 2: Maximum orbital frequency vs. the frequency of rotation $\Omega/2\pi$ for the sequences of Fig. 1. One sequence for a very low mass quark star (with $M_b = 0.01M_\odot$) is shown, the critical point on this sequence for Newtonian dynamical instability to non-axisymmetric perturbations is indicated by an asterisk and dynamically unstable configurations are denoted with the dotted line. The various dashed lines have the same meaning as in Fig. 1.

Note that the marginally stable orbit is present also for rapidly rotating quark stars of very low mass - it has recently been discovered that the innermost stable circular orbit exists also in Newtonian gravity, the gap between the marginally stable orbit and the stellar surface being produced by the oblateness of the rapidly rotating low-mass quark star. This effect of oblateness seems to be responsible for the “pushing outwards” of the marginally stable orbit even for massive rotating quark stars (compare the discussion in Stergioulas et al.\cite{14}).

Another consequence is that the period and the mass of rotating quark stars cannot be even approximately inferred from the orbital frequencies alone — the same frequency in the innermost stable orbit (e.g., 1.25 kHz for the model presented) is obtained for quark stars with rotational periods ranging from infinity to about 0.6 ms, and the mass ranging from that of a planetoid to about three solar masses.\cite{27,11} One should note however that quite likely the most rapidly rotating stars are unstable.\cite{11,29}

3 Astrophysical applications

To illustrate how these results may be used to constrain the e.o.s. of dense matter, let us assume that the results of Figs. 1 and 2 are representative of quark stars and neutron stars. The recently proposed theory of resonant origin of kHz QPOs\cite{30,31} received strong observational support with the discovery of two black hole sources in which the observed QPO frequencies are in a 2:3 ratio.\cite{32} According to that theory, the high frequency QPOs which come in pairs arise in an accretion disk, presumed to be geometrically thin, as a result of parametric resonance between the radial and vertical epicyclic frequencies. For a given metric, this occurs at a specific radius. In the Schwarzschild metric, this resonance occurs at the radius $r_{2:3} = 16.2\text{ km} \times M/M_\odot$ and for moderately rotating neutron stars at somewhat smaller radii.

For illustrative purposes suppose that in a star known to have a rotational period of no less than a few milliseconds, a QPO at 1.37 kHz is detected and identified with the vertical epicyclic
frequency at the resonant radius $r_{2:3}$. It follows, that the mass of the star must be $M < 0.67M_\odot$ and the radius of the star must satisfy $R < r_{2:3} < 10.9 \text{ km}$. The Schwarzschild values would be $M = 0.6M_\odot$ and $R < 9.7 \text{ km}$, respectively. As is evident from the figures, these constraints are not satisfied for any of the neutron star models (for the FPS equation of state), but are easily satisfied by the quark star models. We conclude that, at least in principle, one can use the difference in the orbital properties of quark and neutron stars to distinguish observationally between the two classes of objects.

Acknowledgments

This work has been funded by the following grants: KBN grants 5P03D01721 and 2P03D02117; the Greek-Polish Joint Research and Technology Program EPA N-M.43/2013555 and the EU Program “Improving the Human Research Potential and the Socio-Economic Knowledge Base” (Research Training Network Contract HPRN-CT-2000-00137).

References

1. M. van der Klis, ARA&A 38, 717 (2000).
2. W. Kluźniak, P. Michelson and R. V. Wagoner, ApJ 358, 538 (1990).
3. P. Kaaret, E. C. Ford and K. Chen, ApJ 480, 127 (1997).
4. W. Zhang, A. P. Smale, T.E. Strohmayer and J.H. Swank, ApJ Lett., 500, L171 (1998).
5. W. Kluźniak, ApJ 509, L37 (1998).
6. T. Bulik, W. Kluźniak and W. Zhang, A&A 361, 153 (2000).
7. W. Kluźniak and M.A. Abramowicz, A&A submitted, (2002), [astro-ph/0203314].
8. T. Bulik, D. Gondek-Rosińska and W. Kluźniak, A&A 344, L71 (1999).
9. T. Bulik, D. Gondek-Rosińska and W. Kluźniak, Ap Lett. and Comm. 38, 77 (1999).
10. N. Stergioulas, W. Kluźniak and T. Bulik, A&A 352, L116 (1999).
11. D. Gondek-Rosińska, N. Stergioulas, T. Bulik, et al., A&A 380, 190 (2001).
12. J.L. Zdunik, T. Bulik, W. Kluźniak, P. Haensel, D. Gondek-Rosinska A&A 359, 143 (2000).
13. J.L. Zdunik, P. Haensel, Gondek-Rosińska D. and E. Gourgoullhon, A&A 356, 612 (2000).
14. J.L. Zdunik, P. Haensel, E. Gourgoullhon, A&A 372, 535 (2001).
15. M. Dey, I. Bombaci, J. Dey, S. Ray, B.C. Samanta, Phys. Lett. B 438, 123 (1998).
16. B. Datta, A. V. Thampan, I. Bombaci, A&A 355, L19 (2000).
17. D. Gondek-Rosińska et al., A&A 363, 1005, (2000).
18. D. Gondek-Rosińska, T. Bulik, W. Kluźniak, et al., ESA SP-459, 223 (2001).
19. C.P. Lorenz, D.G. Ravenhall and C.J. Pethick, Phys. Rev. Lett. 70, 379 (1993).
20. J.L. Zdunik, A&A 359, 311 (2000).
21. G.B. Cook, S.L Shapiro and S.A. Teukolsky, ApJ 424, 823 (1994)
22. M.C. Miller, F. K. Lamb, G.B. Cook, ApJ 509, 793 (1998)
23. E. Gourgoullhon et al., A&A 349, 851 (1999).
24. N. Stergioulas and J.L. Friedman, ApJ 444, 30 (1995).
25. W. Kluźniak and R. V. Wagoner, ApJ 297 548 (1985).
26. P. Amsterdamski, T. Bulik, D. Gondek-Rosińska, and W. Kluźniak, A&A 381, L21 (2002).
27. W. Kluźniak, T. Bulik and D. Gondek-Rosińska, ESA SP-459, 301 (2000).
28. J. L. Zdunik and E. Gourgoullhon, Phys. Rev. D 63, 087501 (2001).
29. D. Gondek-Rosińska, E. Gourgoullhon and P. Haensel, in preparation, (2002).
30. W. Kluźniak and M.A. Abramowicz, Acta Phys. Pol. B, 32, 3605 (2001).
31. M. Abramowicz and W. Kluźniak, A&A 374, L19 (2001).
32. R. Remillard, et al., ApJ in press, (2002), [astro-ph/0202305].