Perovskite-structure TlBO₃ (B = Cr, Mn) for thermomechanical and optoelectronic applications: an investigation via a DFT scheme

Wakil Hasan, Adeeb Mahamud Hossain, Md. Rasheduzzaman, Md. Atikur Rahman, Md. Mukter Hossain, K. Rashel Mohammad, Raihan Chowdhury, Khondaker Monower Hossain, M. Moazzam Hossen and Md. Zahid Hasan

First-principles-based DFT calculations have been carried out to analyze the structural, mechanical, elastic anisotropic, Vickers hardness, electronic, optical, and thermodynamic properties of TlBO₃ (B = Cr, Mn) for the first time. We determined the lattice parameters, which are in good agreement with the previous results. The Born criteria was ensured by the elastic constants, which also confirms the ductility of the solid. The elastic constants were also used to evaluate and analyze some related physical properties. The values of Vickers hardness were calculated to determine the hardness and relative application of both TlCrO₃ and TlMnO₃. Though the metallic characteristics were evaluated via the investigation of the electronic band structure and density of states, both TlCrO₃ and TlMnO₃ reveal semiconducting behavior under spin–orbit polarization with up-spin and down-spin configurations. Significant constants such as absorption, conductivity, reflectivity, dielectric, loss function, and refractive index were also considered and determined without spin and with spin. As a result, various possible electronic, optical, and optoelectronic applications were predicted. TlBO₃ (B = Cr, Mn) was also found to be reliable for thermal barrier coating (TBC) as indicated by the evaluated values of thermal conductivity and Debye temperature.

1. Introduction

Perovskite oxides (ABO₃) are a point of interest for scientific researchers for their phase transformation and significant applications in modern technology. ABO₃-structured perovskites have been extensively explored for metal-insulator transitions (MITs) correlated transition metal oxides. La₁₋ₓBₓMnO₃ (B = Mn, Ca) perovskite magnates are well known to show MITs with ferromagnetic–paramagnetic transition.

Perovskite materials are also found to be suitable for solid oxide fuel cells. ABO₃ is considered as an ideal perovskite material, where A is a monovalent otherwise valent cation situated on the edges of the crystal, and B is a tetravalent or pentavalent transition metal. Kim et al. discussed the structural distortion of ABO₃-type perovskite owing to a disparity in the A–O and B–O bond distances. The physical properties of a cation on structural distortion are difficult to investigate, as Kim et al. agreed, because rare earth and yttrium ions bond to oxygen with parallel covalency. As a result, Tl(n), a post-transition metal ion, is a decent choice for A cation in ABO₃ perovskites due to its full occupation of the 4f and 5d orbitals (|XE⟩ = 4f⁴ 5d⁶ 6s² 6p¹) in comparison to rare-earth ions (|XE⟩ = 4fⁿ 5d⁰ 6sⁿ; 0 ≤ n ≤ 14). The transition metals in the B site of ABO₃ perovskite make the material suitable for any application such as high-temperature superconductivity, colossal magnetoresistance (CMR), charge ordering, giant magnetoresistance (GMR), piezoelectricity, and ferroelectricity.

ABO₃ perovskite oxides are also interesting because of their use in the construction of infrared sensors, optoelectronic modulators, infrared detectors, microwave devices, and different electromechanical systems.

ACrO₃ (A = Sc, I, Tl, Bi) perovskite-type chromite is significantly different in its properties from isostructural chromite RCrO₃ (R = rare earth elements), which can result from the distortion of the crystal structure (ScCrO₃, InCrO₃), chemical bonds (TlCrO₃, BiCrO₃) in these compounds, and the electronic structure of A³⁺. RCrO₃ compounds have been found to exhibit spin reorientation transition, good oxygen-ion conduction.
(doped), and sensitivity toward methanol, ethanol, some gases, and humidity.\textsuperscript{26} $\text{R}^{1+}\text{Mn}^{4+}\text{O}_3$ have also been investigated for their multiferroic properties in both perovskites and hexagonal modification\textsuperscript{27–29} and rich magnetic phases.\textsuperscript{30} TiCrO$_3$ (R = Ga, Tb, Er, Tm, Sn, and Y)\textsuperscript{31–34} has shown large polarization at a high temperature equivalent to the Neel temperature ($T_N$) of the Cr subsystem, while LaCrO$_3$ and CeCrO$_3$ display high $T_N$ in RCrO$_3$ structure type materials.\textsuperscript{35} Orthorhombic RMnO$_3$ (R = Tb, Dy)\textsuperscript{36,37} are spiral magnets that display magnetoelectric (ME) coupling. DyFeO$_3$ was investigated by Tokunaga \textit{et al.} and found to be versatile with gigantic ME phenomena.\textsuperscript{38}

Paul \textit{et al.} analyzed ABO$_3$ perovskites such as BiNiO$_3$ or PbCrO$_3$ (ref. 48) for MITs, and different investigations on A and B ions led to the proposition of two more candidates in this category, namely, TlMnO$_3$ and InMnO$_3$. Yi \textit{et al.} synthesized TlCrO$_3$ (ref. 31) and TlMnO$_3$ (ref. 49) and investigated them with Mössbauer spectroscopy with high pressure (6 GPa) and high temperature (1500 K), and found separate structural and magnetic properties, where the absence of spin canting suggested that TlCrO$_3$ has a non-trivial magnetic structure. Khlavin \textit{et al.} also studied the magnetic structure of TlMnO$_3$ (ref. 50) under the same conditions of high temperature and pressure by applying the neutron diffraction technique and confirmed anisotropic exchange interactions that revealed that the Jahn–Teller distortion plays a key role in creating the magneto-crystalline anisotropy. Belik investigated manganese oxide-based Bi$_{1−x}$Y$_x$MnO$_3$ with high pressure and high temperature.\textsuperscript{39} R$_2$O$_3$–In$_2$O$_3$ systems were studied at normal pressure, and some ABO$_3$ compounds were found, which contained $\text{I}^{3+}$ or Tl$^{3+}$. With this criteria, Shannon synthesized some perovskites containing In and Tl such as InCrO$_3$, TlCrO$_3$, and TlFeO$_3$.\textsuperscript{40} \textit{Ab initio} study was performed on many perovskites including BaMnO$_3$ (M = Pr, Th, U),\textsuperscript{41} SnTaO$_3$,\textsuperscript{42} ACO$_3$ (A = Ca, Sr),\textsuperscript{43} and SrHfO$_3$.\textsuperscript{44} Z. Hasan \textit{et al.} investigated the ABO$_3$ type perovskite AMoO$_3$,\textsuperscript{45} where A is Ca, Sr, SrBO$_3$ (ref. 58) and B is Cr, Fe, and SrFeO$_3$ with density functional theory (DFT) and found them to be mechanically stable at different pressures along with other physical properties. LnCrO$_3$,\textsuperscript{46,47} which crystallizes in GdFeO$_3$ and BiFeO$_3$ with similar structured perovskites, were studied and found to have applications in thin-film devices. LaNiO$_3$ reveals a metallic nature along with Pauli paramagnetism. LuNiO$_3$ and YNiO$_3$, on the other hand, display Curie–Weiss paramagnetism with antiferromagnetic order, which is below the $T_N$.\textsuperscript{48} The TlNiO$_3$ perovskite was synthesized under high pressure and a relative study was performed with TiNiO$_3$ (T = rare earth).\textsuperscript{49} According to Cao \textit{et al.}, chromium oxide-based CeCrO$_3$,\textsuperscript{50} which is a rare earth orthochromite, shows magnetic properties and anti-ferromagnetism. LaCoO$_3$ with ABO$_3$ structure was studied for a lengthy period for its spin-state transition and electronic or ionic conductivity.\textsuperscript{51} Mn$^{4+}$-based perovskite TiMnO$_3$ has been described to have a triclinic crystal structure with a clear signature of Jahn–Teller distortion.\textsuperscript{52} Ding \textit{et al.} investigated TiCrO$_3$, InCrO$_3$, and ScCrO$_3$ with high pressure and high-temperature settings, which revealed the orthorhombic structure of these materials.\textsuperscript{53}

In this study, further investigation of the ABO$_3$ structure-based TiBO$_3$ (B = Cr, Mn) is done, where Ti is a post-transition metal, Cr and Mn are transition metals, and O is a non-metal. ABO$_3$ structure-based perovskite shows unique physical properties whose diverse types are used in modern technology, motivating us to properly analyze and uncover their physical properties. This paper studies the physical properties such as structural, mechanical, elastic anisotropic, electronic, thermal, Vickers hardness, and optical properties of TiCrO$_3$ and TiMnO$_3$ compounds using DFT-based Cambridge Serial Total Energy Package (CASTEP). The main purpose of this investigation is to analyze the above-mentioned physical properties of the perovskites that can be subjugated to different usages.

2. Computational methods

The investigation of various physical properties such as structural, electronic, mechanical, thermal and optical TiBO$_3$ (B = Cr, Mn) were done with the most commonly used first-principles methodology. Density Functional Theory (DFT) with the periodic ambient condition along with generalized gradient approximation (GGA) and Perdew–Burke–Ernzerhof (PBE) exchange-correlation functional was employed\textsuperscript{44–47} using the Cambridge Serial Total Energy Package (CASTEP) code.\textsuperscript{71} The atomic calculations for Ti – 6s$^2$ 6p$^1$, Cr – 3d$^5$ 4s$^1$, Mn – 3d$^5$ 4s$^2$, and O – 2s$^2$ 2p$^4$ were executed. Brillouin zone k-points sampling was done using the Monkhorst–Pack scheme\textsuperscript{72} with $10 \times 7 \times 10$ grid points for TiCrO$_3$ and $7 \times 5 \times 7$ grid points for TiMnO$_3$. The plane-wave cut-off energy was 450 eV and 400 eV for TiCrO$_3$ and TiMnO$_3$, respectively. The Brydon–Fletcher–Goldfarb–Shanno (BFGS) minimization method, also known as the quasi-Newton method,\textsuperscript{73} was employed as a method of optimization of the crystal. The principles of convergence for the optimization were set to $1 \times 10^{-5}$ eV per atom for total energy, 0.03 eV Å$^{-1}$ for maximum force, and 0.05 GPa for maximum stress. The stress-strain method\textsuperscript{74} was used to attain the elastic stiffness constants of TiBO$_3$ (B = Cr, Mn) using CASTEP. The Young’s modulus, shear modulus, and Poisson’s ratio presented in the 3D anisotropy models were obtained using the ELATE program.\textsuperscript{75} Lastly, the electronics properties were evaluated by setting the above-mentioned parameters.

3. Results and discussion

3.1 Structural properties

Both synthesized TiBO$_3$ (B = Cr, Mn) perovskite materials share the same TiBO$_3$ ionic structure but both share different crystal structure types. Fig. 1 illustrates the crystal structure of TiBO$_3$ (B = Cr, Mn). TiCrO$_3$ crystallizes in an orthorhombic structure in the space group \textit{Pmma} (no. 62). In TiCrO$_3$, the Ti site has 4c Wyckoff position (0.04617, 0.25, 0.98732), the Cr site has 4b Wyckoff position (0, 0.5, 0), and O has two sites, for the first one, the 4c Wyckoff position (0.4504, 0.25, 0.1050) and for the second, the 8d Wyckoff position (03 009, 0.0558, 0.6969).\textsuperscript{76} TiMnO$_3$ crystallizes in the triclinic structure with the space group \textit{Pi} (no. 2). In TiMnO$_3$, Ti site has 2i (0.54745, 0.25794, 0.50994) and 2i (0.04935, 0.24233, 0.97513). The Mn atoms are positioned in 4 sites. Mn1, Mn2, Mn3, and Mn4 has 1d (1, 0.5, 0, 0), 1e (1, 0.5, 0.5, 0), 1b (1, 0, 0, 0.5), and 1g (1, 0, 0.5, 0.5),...
respectively. O atoms are positioned in 6 site O (1–6), which are in the numerical order as follows—2i (for all sites) Wyckoff position (0.9423, 0.2309, 0.3799); (0.4450, 0.2708, 0.1147); (0.2056, 0.5732, 0.2055); (0.8053, 0.0511, 0.8070); (0.2824, 0.4406, 0.6847); (0.6839, 0.9275, 0.2855), respectively. 

The optimized lattice parameters \( a \), \( b \), and \( c \) are presented in Table 1 along with the calculated unit cell volume \( V \). The obtained results are associated with the former results and seems to be in good agreement with them, which shows our calculations to be reliable. It is noted that both materials share different crystals structures, but the calculated lattice constants and volumes are quite close.

### 3.2 Elastic constants and mechanical properties

The elastic characteristics of a crystal define various features of the solid under symmetry conditions, as well as solid stability, bonding characteristics, elastic, and machinability. The elastic constants are produced from the first approach via the CASTEP module using a variety of uniform finite value distortions. After the optimization of the internal degrees of freedom, the stresses are therefore determined. The elastic constants for TlBO\(_3\) (B = Cr, Mn) were determined and are presented in Table 2. Nine elastic constants were calculated for orthorhombic TlCrO\(_3\) and twenty-one elastic constants were found for triclinic TlMnO\(_3\). No past results regarding elastic constants were found; thus, no relative investigation has been shown. The current outcomes (elastic constants) must adhere to the Born stability principles by describing the following relationships.

\[
C_{11} - C_{12} > 0, \quad C_{44} > 0, \quad \text{and} \quad C_{11} + 2C_{12} > 0
\]

The elastic constants calculated are positive (Table 2) and uphold the abovementioned stability criteria, which confirms the mechanical stability of TlBO\(_3\) (B = Cr, Mn). The material also satisfies generalized stability criteria for an orthorhombic crystal (TlCrO\(_3\)).

### Table 1  Calculated lattice constants and unit cell volume of TlBO\(_3\) (B = Cr, Mn)

| Compound                  | Lattice constants (Å)    | Volume (Å\(^3\)) | Remarks     |
|---------------------------|--------------------------|-------------------|-------------|
| TlCrO\(_3\) (orthorhombic)| 5.433306 7.693412 5.434486 | 227.16511         | This work   |
| TlMnO\(_3\) (triclinic)  | 5.40318 7.64699 5.30196 | 219.06671         | Exp.\(^{31}\) |
| TlCrO\(_3\) (orthorhombic)| 5.405 7.647 5.302        | 219.14245         | Exp.\(^{29}\) |
| TlMnO\(_3\) (triclinic)  | 5.457218 7.692286 5.463322 | 229.34171         | This work   |
| LaCrO\(_3\) (orthorhombic)| 5.4248 7.9403 5.28650   | 227.713553        | Exp.\(^{49}\) |
| LaMnO\(_3\) (orthorhombic)| 5.41987 7.9250 5.27683  | 226.652881        | Exp.\(^{50}\) |

Fig. 1  Crystal structures of (a) TlCrO\(_3\) and (b) TlMnO\(_3\).
The elastic constants of the compound in Table 2 agree with the above conditions, further confirming that the considered compound TlCrO₃ is mechanically stable in the orthorhombic phase. For both the materials, the C₄₄, C₅₅, and C₆₆ crudely agree with the distortions of the lattice angles (α, β, and γ). It can be concluded that γ is the maximum stable angle since the elastic constant C₆₆ is larger than C₄₄/C₅₅ (Table 2), which is dependent on the changing tendencies of the lattice angles in the crystal.⁴¹ Here, C₁₁ is connected with linear compression resistance in the x-direction and the value C₁₁ is higher than C₁₂, which indicates incompressibility near the a-axis, not near the b-axis. The density of the atom along the [100] direction is higher if compared with the direction of [011]. It is also seen that C₄₄ > C₆₆ for TlMnO₃, which means higher shear deformation resistance along the [010] direction than the [110] direction, while for the TlCrO₃ compound, this value is not so different. The shear component, C₄₄ (TlCrO₃) > C₄₄ (TlMnO₃), indicating higher resistance to shear deformation of TlCrO₃ compared to TlMnO₃. Furthermore, the value of C₄₄ predicts the hardness of the material. Therefore, it assumes that the compound TlCrO₃ should have higher hardness than TlMnO₃. From Cauchy pressure (C₁₁–C₄₄) the metallic or non-metallic (ionic/covalent) features of the compounds are revealed. The materials will be a dominant covalent bond in nature if the Cauchy pressure shown in Table 3 is negative, which is not the case. Thus, both TlCrO₃ and TlMnO₃ have a dominant ionic bond in nature as the Cauchy pressure values are positive.

Voigt–Reuss–Hill (VRH) approximations are employed to evaluate the polycrystalline elastic properties.⁴²–⁴⁴ Bulk modulus (Bᵥ) and shear modulus (Gᵥ) in Voigt approximation is expressed as follows.

\[
Bᵥ = \frac{1}{9} [(2(C₁₁ + C₁₂) + 4C₁₃ + C₃₃)];
\]

\[
Gᵥ = \frac{1}{30} (C₁₁ + C₁₂ - 4C₁₃ + 12C₄₄ + 12C₆₆);
\]

The Bulk modulus Bᵣ and shear modulus Gᵣ in the Reuss approximation is defined as follows.

\[
Bᵣ = \frac{C²}{M}; C² = (C₁₁ + C₁₂)C₃₃ - 2C₁₃²;
\]

\[
M = C₁₁ + C₁₂ + 2C₃₃ - 4C₁₃;
\]

\[
Gᵣ = \frac{\frac{1}{2} [5C²C₄₄C₆₆]}{3BᵣC₄₄C₆₆ + C²(C₄₄ + C₆₆)}; C₆₆ = \frac{C₁₁ - C₁₂}{2}
\]

For Hill approximation, B and G, which are bulk and shear modulus, respectively, are calculated utilizing the following equations.

\[
B = \frac{1}{2} (Bᵥ + Bᵣ);
\]

\[
G = \frac{1}{2} (Gᵥ + Gᵣ);
\]

As we know, the elastic moduli bulk modulus B, shear modulus G, Young’s modulus Y, and Poisson’s v describe the mechanical performance of solids, which are calculated by VRH approximations.⁴⁵–⁴⁷ B and G is then used to determine the Young’s modulus Y and Poisson’s ratio v via the following equations.

\[
Y = \frac{9BG}{2B + G};
\]

\[
v = \frac{3B - 2G}{2(3B + G)}.
\]

The resistance in contradiction to volume, shear, and longitudinal deformation is represented by bulk modulus B, shear modulus G, and Young’s modulus Y, respectively. Y, G, and B of TlBO₃ were determined for the first time in this investigation and are shown in Table 3. Pugh’s ratio, depicted as B/G, in Table 3 demonstrates if a material is ductile or brittle.⁴⁸

### Table 2: Elastic constants, Cᵢⱼ (GPa) of TlBO₃ (B = Cr, Mn)

| Compound      | C₁₁     | C₁₂     | C₁₃     | C₂₂     | C₂₃     | C₃₃     | C₄₄     | C₅₅     | C₆₆     |
|---------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| TlCrO₃ (orthorhombic) | 298.487 | 252.195 | 298.712 | 104.875 | 42.099  | 104.859 | 150.459 | 85.594  | 150.071 |
| TlMnO₃ (triclinic)   | 298.487 | 252.195 | 298.712 | 104.875 | 42.099  | 104.859 | 150.459 | 85.594  | 150.071 |

### Table 3: The estimated bulk modulus, Bᵣ (Reuss), Bᵥ (Voigt), B (Hill) (GPa), shear modulus, Gᵣ (Reuss), Gᵥ (Voigt), G (Hill) (GPa), Young’s modulus, Y (GPa), Pugh’s ratio, B/G, Poisson’s ratio v, Cauchy pressure, C₁₁–C₄₄, and machinability index μₘ for TlBO₃ (B = Cr, Mn)

| Compound      | Bᵣ     | Bᵥ     | B     | Gᵣ     | Gᵥ     | G     | Y     | B/G    | v      | C₁₁–C₄₄ | μₘ     | Remarks   |
|---------------|--------|--------|------|--------|--------|------|------|--------|--------|---------|--------|-----------|
| TlCrO₃ (orthorhombic) | 179.92 | 180.18 | 180.05 | 68.39  | 81.25  | 74.82 | 197.12 | 2.41   | 0.32   | 45.58   | 1.72   | This work  |
| TlMnO₃ (triclinic)   | 168.62 | 170.64 | 169.63 | 56.79  | 64.67  | 60.73 | 162.76 | 2.79   | 0.34   | 43.97   | 1.93   | This work  |
| CeCoO₃ (cubic)      | 179.12 | 179.12 | 179.12 | 72.66  | 80.39  | 76.53 | 200.97 | 2.34   | 0.32   | 45.12   | 3.05   | Exp.⁴²    |
| CeCuO₃ (cubic)      | 132.09 | 132.09 | 132.09 | 39.94  | 46.55  | 43.24 | 116.96 | 3.05   | 0.35   | 54.51   | 4.25   | Exp.⁴²    |

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which is more than 1.75, indicating the ductile nature of both the materials; otherwise, it behaves in a brittle manner. Likewise, Poisson’s ratio $\nu$ is an additional parameter to determine the ductile or brittle nature of solids.\(^{39}\) The value should be more than 0.25 to be ductile and less to be brittle. As the value of $\nu$ is more than 0.25, as shown in Table 3, it further confirms the result obtained from Pugh’s ratio, suggesting that both TlCrO$\text{3}$ and TlMnO$\text{3}$ are ductile. Furthermore, the value of the Cauchy pressure ($C_{11} - C_{12}$) is positive, indicating the ductile nature of both materials; otherwise, it would be brittle if the value is negative.\(^{41}\)

The machinability index $\mu_M$ describes the cutting capability of a compound, the maximum financial level of machine operation, and plastic strain, which is vital for industrial areas. It is determined as follows.

$$\mu_M = \frac{B}{C_{44}}$$ (16)

For both orthorhombic TlCrO$\text{3}$ and TlMnO$\text{3}$, the value of $\mu_M$ in Table 3 is more than 2, referring to improved lubricating characters and lesser frictions, which has significant applications in different fields.

### 3.3 Elastic anisotropy

The microscopic activity of solids is developed by the amount of elastic anisotropy for single and polycrystalline materials.\(^{33}\) Therefore, the investigation of the outward directional dependence of the elastic tensor is important for the materials involved so that the mechanical resilience, degree of elastic anisotropy, and usage of material under peripheral stress can be heightened. In applied engineering sciences, these appearances will play an equally critical part in the design activities, clear interpretations, along with fundamental crystal physics.\(^{44}\) The Shivakummar Ranganathan model represents elastic anisotropy as follows.\(^{41}\)

$$A^U = 5 \frac{G_V}{G_R} + \frac{B_K}{B_R} - 6 \geq 0.$$ (17)

$A^U = 0$ represents the isotropic performance, although the exemption from this ($A^U \neq 0$) replicates the anisotropic performance of materials. Therefore, the values of $A^U$ are estimated for orthorhombic TlCrO$\text{3}$ ($\sim$0.941) and triclinic TlMnO$\text{3}$ ($\sim$0.702), characterizing both materials as anisotropic. Furthermore, Chung and Buessum presented the conception of percent anisotropy modulus, $A_G$ (shear), and $A_B$ (bulk) as follows\(^{38}\) (in polycrystalline materials).

$$A_G = \frac{G_V - G_R}{2G_H}; \quad A_B = \frac{B_V - B_R}{B_V + B_R}$$ (18)

For $A_G = A_B = 0$, the solid materials represent elastic isotropy. Thus, in the present analysis, both TlCrO$\text{3}$ and TlMnO$\text{3}$ show anisotropic aspects. If we consider the directional requirement on the elastic tensor, TlMnO$\text{3}$ shows a higher degree of anisotropy. In addition, shear anisotropy is more prevalent as it seems to have a greater value (Table 4) for both the materials. In both the materials, TlBO$\text{3}$ ($B = $ Cr, Mn) the bulk modulus is anisotropic, as we observed that $B_{11} \neq B_{22} \neq B_V$. The value of $A_B$ in Table 4 illustrates that the bulk modulus of compounds is anisotropic and TlMnO$\text{3}$ is further anisotropic than TlCrO$\text{3}$, which shows negligible anisotropic character. Both compounds indicate anisotropic character (Table 4) in the measured shear anisotropic factors as well. The anisotropic shear pointers offer a valuation of the degree of anisotropy in the bonding of different planes between atoms. Along the {100}, {010}, and {001} shear planes, the shear anisotropic parameters are stated as follows.

$$A_1 = \frac{4C_{44}}{C_{11} + C_{33} - 2C_{13}}; \quad A_2 = \frac{4C_{44}}{C_{22} + C_{33} - 2C_{23}}; \quad A_3 = \frac{4C_{44}}{C_{11} + C_{22} - 2C_{12}}$$ (19)

In the case of $A_1 = A_2 = A_3$, the materials show isotropic nature or it measures the amount of elastic anisotropy exhibited by the crystal. Since $A_1$, $A_2$, and $A_3$ are not equivalent (Table 4), both orthorhombic and triclinic materials display anisotropic character. The Zener anisotropy index $A$ and the Zener anisotropy equivalent ($A^e$) for both the compounds are calculated to achieve acceptable anisotropy as follows.\(^{95,96}\)

$$A = \frac{2C_{44}}{C_{11} - C_{12}},$$ (20)

$$A^e = \left(1 + \frac{5}{12}A^U\right) + \sqrt{(1 + \frac{5}{12}A^U)^2 - 1}.$$ (21)

In Table 4, the values of $A$ and $A^e$ confirm our previous analysis, reinstating that both the compounds have an anisotropic character. The anisotropy index $A = 1$ suggests the isotropic behavior of solid materials, and anisotropic natures are revealed for more or less than Uni ($> 1$ or $< 1$). The evaluated value of TlBO$\text{3}$ ($B = $ Cr, Mn) shows both materials to be anisotropic, where TlCrO$\text{3}$ manifested a slightly higher degree of elastic anisotropy.

To further study the elastic anisotropy of both the materials, Young’s modulus ($Y$), bulk modulus ($K$), shear modulus ($G$), and

### Table 4 Calculated shear anisotropic factors, $A_i$ ($i = 1–3$), Zener’s anisotropy index ($A$), anisotropy in shear ($A_G$), anisotropy in bulk modulus ($A_B$), universal anisotropy index ($A_U$), and equivalent Zener anisotropy ($A^e$) of TlBO$\text{3}$ ($B = $ Cr, Mn)

| Compound    | Crystal system | $A_1$  | $A_2$  | $A_3$  | $A$    | $A_G$       | $A_B$  | $A_U$     | $A^e$    |
|-------------|----------------|--------|--------|--------|--------|-------------|--------|-----------|----------|
| TlCrO$\text{3}$ | Orthorhombic   | 0.985  | 0.672  | 1.679  | 1.417  | 0.086       | 0.0007 | 0.941     | 2.3605   |
| TlMnO$\text{3}$ | Triclinic      | 1.071  | 0.617  | 1.447  | 1.494  | 0.065       | 0.0059 | 0.702     | 2.1114   |

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Poisson’s ratio ($v$) elastic moduli were constructed for both TlCrO$_3$ and TlMnO$_3$ in Fig. 2 and 3 using ELATE code. 3D and 2D models were used to identify the nature of anisotropy. If the 3D plots are entirely spherical and their projection on different planes is rounded, the materials are isotropic. The nonspherical shape in the 3D plots for both orthorhombic TlCrO$_3$ and...
and triclinic TlMnO₃ signify the extent of anisotropy. As depicted in Fig. 2 and 3, the unconventionality between the 3D surface and the sphere for all the moduli suggests the elastic anisotropy characteristics for TlBO₃ (B = Cr, Mn). This analysis displays good consistency with the results of all the anisotropy indexes discussed above. Furthermore, the 3D plot of different
anisotropy moduli looks somewhat similar in Fig. 2 and 3 for both the compounds.

3.4 Vickers hardness

Vickers hardness is mainly associated with the resistance offered by the material to plastic deformation. The ability of a material to resist plastic deformation can also be estimated by it. Many important applications for practical devices can be predicted by understanding the hardness of a solid. Furthermore, a deep understanding of the mechanical behavior can be achieved through the relationship between elastic poly-crystalline modules and hardness. Vickers hardness can be calculated for metallic compounds by the means of the equation found by Gou et al. If all the individual bond hardness are calculated, their geometric average can be used to evaluate the hardness of a multiband solid. It can be done using the following equations:

\[ H_V = \left[ \prod \pi (H^\mu_n)^{\alpha_n} \right]^{1/\sum \alpha_n} \]  
\[ H^\mu_n = 740(P^\mu - P'^\mu)(v^\mu_b)^{5/3}; \]  
\[ P^\mu = n_{\text{free}}/V; \]  
\[ n_{\text{free}} = \int_{E_F}^{E_{\text{c}} + \varepsilon} N(E) dE \]  
\[ v^\mu_b = \frac{(d^\mu)^3}{\sum \left( \frac{d^\mu}{N^\mu} \right)} \]

here, \( P^\mu \) and \( P'^\mu \) is the Mulliken population and the metallic population of the \( \mu \)-type bond, respectively, \( n_{\text{free}} \) is the number of free electrons between the first pseudo gap and Fermi level, \( V \) is called cell volume (Table 1), \( v^\mu_b \) is the volume of a bond of \( \mu \)-type, \( (d^\mu)^3 \) is the bond length of \( \mu \)-type bond, and \( N^\mu \) is the bond number of \( \mu \) per unit volume.

The evaluated values of Vickers hardness of TlBO3 (B = Cr, Mn) are displayed in Table 5. It is clear from Table 5 that TlCrO3 is harder than TlMnO3. However, the overall observation of the values reveals both materials to have much lower hardness compared to diamond, considering the most rigid (Vickers hardness 70 to 150 GPa)\(^{39} \) material among all. As a result, both TlCrO3 and TlMnO3 indicate a flexible and soft nature, making them suitable for thin-film production. The hardness values are 8.729 GPa for TlCrO3 and 5.208 Gpa for TlMnO3, which is close to 2–8 GPa, values of many well-known MAX phase (M is transition metal, A is group 13–15 A element, X is C or N) materials.\(^{1,2} \)

3.5 Electronic properties

The electronic properties of the TlBO3 (B = Cr, Mn) perovskites were studied by measuring the band structure along with the total partial density of states (DOS). The electronic band structures without spin for both orthorhombic TlCrO3 and triclinic TlMnO3 are displayed in Fig. 4(a) and 5(a) correspondingly. The band structure with spin is displayed in Fig. 4(a) and 5(a). The straight (dotted) line at 0 eV in both Fig. 4 and 5 specifies the Fermi level \( (E_F) \) separating the conduction band and the valence band. If the electronic band-structure of both the materials is compared, it is observed that for both materials, energy bands without spin display bulky scattering diagonally to the Fermi level \( (E_F) \); the conduction and valence bands both are superimposed and displayed no bandgap, suggesting both materials to be metals. Moreover, the energy density of TlMnO3 along the Fermi level \( (E_F) \) describes it to be more metallic than TlCrO3. The bandgap \( (E_g = 0.45) \) in Fig. 5(a) reveals possible semiconducting abilities in TlMnO3. Then again, the band structure with spin shows a dramatic transition from metallic to semiconductor for

| Table 5 Theoretical hardness of TlBO3 (B = Cr, Mn) |
| Compounds | Bond | \( n^a \) | \( d^\mu (\text{Å}) \) | \( P^\mu \) | \( P'^\mu \) | \( v^\mu_b (\text{Å}^3) \) | \( H^\mu_n (\text{GPa}) \) | \( H_v (\text{GPa}) \) |
| TlCrO3 | O–Cr(i) | 8 | 1.9201 | 0.55 | 0.043743 | 9.43794 | 8.888 053 | 8.72858 |
| | O–Cr(ii) | 8 | 1.92232 | 0.54 | 0.043743 | 9.47072 | 8.662 297 |
| | O–Cr(iii) | 8 | 1.92342 | 0.54 | 0.043743 | 9.48698 | 8.637 555 |
| TlMnO3 | O–Mn(i) | 2 | 1.95981 | 0.36 | 0.071407 | 5.04956 | 14.36908 5.20793 |
| | O–Mn(ii) | 2 | 1.96337 | 0.36 | 0.071407 | 5.07713 | 14.23928 |
| | O–Mn(iii) | 2 | 1.96358 | 0.36 | 0.071407 | 5.07876 | 14.23167 |
| | O–Mn(iv) | 2 | 1.96733 | 0.35 | 0.071407 | 5.10791 | 13.60809 |
| | O–Mn(v) | 2 | 1.9818 | 0.35 | 0.071407 | 5.22145 | 13.1185 |
| | O–Mn(vi) | 2 | 1.98243 | 0.35 | 0.071407 | 5.22643 | 13.09767 |
| | O–Mn(vii) | 2 | 1.98378 | 0.35 | 0.071407 | 5.23711 | 13.05316 |
| | O–Mn(viii) | 2 | 1.98532 | 0.34 | 0.071407 | 5.24932 | 12.53589 |
| | O–Mn(ix) | 2 | 1.98597 | 0.32 | 0.071407 | 5.25448 | 11.58347 |
| | O–Mn(x) | 2 | 1.98904 | 0.32 | 0.071407 | 5.27888 | 11.49435 |
| | O–Mn(xi) | 2 | 1.99018 | 0.35 | 0.071407 | 5.28797 | 12.54635 |
| | O–Mn(xii) | 2 | 1.99083 | 0.34 | 0.071407 | 5.28935 | 12.53378 |
| | O–Tl(i) | 2 | 2.28171 | 0.1 | 0.071407 | 7.9688 | 0.665 542 |
| | O–Tl(ii) | 2 | 2.30589 | 0.1 | 0.071407 | 8.22484 | 0.631 371 |
| | O–Tl(iii) | 2 | 2.33132 | 0.14 | 0.071407 | 8.49997 | 1.433 789 |
| | O–Tl(iv) | 2 | 2.36947 | 0.12 | 0.071407 | 8.92412 | 0.936 555 |
| | O–Tl(v) | 2 | 2.39337 | 0.12 | 0.071407 | 9.19689 | 0.890 718 |
| | O–Tl(vi) | 2 | 2.41888 | 0.11 | 0.071407 | 9.49411 | 0.670 893 |
both TlCrO$_3$ and TlMnO$_3$, respectively, as no scattering of electrons can be seen along the Fermi level ($E_F$) for both the solids. These results showed the material TlBO$_3$ (B = Cr, Mn) switching from metallic to semiconductor nature. These are very much promising results that can be used to explain the metallic and optoelectronics properties, which may result from both Cr and Mn being magnetic atoms.$^{103,104}$ No previous study on the density of states and band structure has been done.

The evaluated total (TDOS) and partial density of states (PDOS) of TlCrO$_3$ without spin and with spin along with atomic contributions of different orbitals around $E_F$ by the CASTEP code are displayed in Fig. 6(a) and (b), respectively. Also, the total (TDOS) and partial density of states (PDOS) of TlMnO$_3$ without spin and with along with the atomic contributions of different orbitals around the Fermi level are displayed in Fig. 7(a) and (b), correspondingly. In Fig. 6(a) and 7(a), the $E_F$ of the TDOS without spin values are about 10 and 24 states/eV for TlCrO$_3$ and TlMnO$_3$, respectively. For both the materials, the Tl-3p orbital seems to be contribute to both the valence band and conduction band. TlCrO$_3$ contributes to both bands in case of Tl-3p orbital at ~0.7 eV (~3.3 states per eV) and ~0.13 eV (3.7 states per eV). TlMnO$_3$ contributes ~0.6 eV (~1.7 states/eV) and ~1.6 eV (~2.4 states per eV). TlCrO$_3$ contributes more to the formation of both the conduction and valence bands, resulting in less overlap of both the bands. As a result, TlCrO$_3$ is less metallic. It can be observed from Fig. 6(a) and 7(a) that major contribution near the Fermi level.

Fig. 4  Electronic band structure of TlCrO$_3$ (a) without spin and (b) with spin.

Fig. 5  Electronic band structure of TlMnO$_3$ (a) without spin and (b) with spin.
level ($E_F$) is derived from Cr-3d and Mn-3d. Here, Cr-3d seems to contribute more to the formation of the conduction band at $\sim$0.6 eV. However, Mn-3d contributes along the Fermi level, making TlMnO$_3$ more metallic. Both TlCrO$_3$ and TlMnO$_3$ compounds also have a fair contribution along with $E_F$ from the O-2p orbital. The orbitals also seem to have significant overlaps in energy without spin, which point toward hybridization and inclination to the creation of covalent bonding. The values of DOS are finite, which indicates TlBO$_3$ (B = Cr, Mn) to be metallic. Moreover, the O-2p contributes to the creation of the valence band at $\sim$2.1 eV (for TlCrO$_3$) and $\sim$2.2 eV (for TlMnO$_3$) below the Fermi level ($E_F$). Hence, both perovskite materials are metallic. Two TDOS peaks for TlMnO$_3$ are found with close values at $\sim$2.1 eV and 0.7 eV. TlMnO$_3$ has the largest TDOS peak at $E_F$, again confirming it to have more metallic characteristics. In case of density of states without spin that is displayed in Fig. 6(a) and with spin displayed in Fig. 6(b), the value of TDOS along the $E_F$ level is $\sim$10 eV and $\sim$2 (both up-spin and down-spin), respectively. For TlMnO$_3$, the value of TDOS along the $E_F$ level is $\sim$24 in case of without spin (Fig. 7(a)) and $\sim$1 (both up-spin and down-spin) with spin (Fig. 7(b)).

3.6 Optical properties

These properties are important for describing the response of solids to an electromagnetic wave and for exploring the potential of photovoltaic and optoelectronic devices. In this context, the performances of the materials to the infrared, visible, and ultraviolet light are particularly noteworthy. Various optical energy-dependent (frequency) parameters, which are the absorption co-efficient $\alpha(\omega)$, conductivity $\sigma(\omega)$, reflectivity $R(\omega)$,
the real part of dielectric function $\varepsilon(\omega)$, loss function $L(\omega)$, and the refractive index $n(\omega)$ are calculated for photon energies up to 50 eV with electric field polarization vectors along the [100] direction to analyze TlBO$_3$ ($B = \text{Cr, Mn}$). Fig. 8 and 9 depicted the different optical properties. The absorption spectrum without spin and with spin is displayed in Fig. 8(a) and (d), which shows expansion at inferior energies from $\sim$0 to 15 eV in case of without spin due to transitions between close-lying energy levels, including the band. Absorption with spin shows a drastic change, which shows expansion of absorption from $\sim$2 eV to $\sim$36 eV. With spin, both materials exhibit possible application as photovoltaic cells.

The peaks associated with the interband transitions exist in the optical conductivity spectrum, which is presented in Fig. 8(b) (without spin) and 8(e) (with spin). A sharp edge can be seen for TlMnO$_3$ near the UV region $\sim$2 eV and expansion can be seen for TlCrO$_3$ near $\sim$4 eV in Fig. 8(b) without spin. However, in the case of with spin in Fig. 8(e), expansion can be seen in conductivity from 0 eV to $\sim$36 eV. This suggests the electrical conductivity in both the materials. Though the peak value is lower than that without spin, it shows a steady peak value rather than a sharp peak. The reflectivity is presented in Fig. 8(c) (without spin) and 8(e) (with spin) for both TlCrO$_3$ and TlMnO$_3$. The zero-frequency reflectivity ($R(0)$) in Fig. 8(c) is $\sim$0.47 and $\sim$0.71 for TlCrO$_3$ and TlMnO$_3$, respectively, but it decreases when evaluated with spin in Fig. 8(f). The zero-frequency reflectivity ($R(0)$) in Fig. 8(f) is only is $\sim$0.0.26 and $\sim$0.28 for TlCrO$_3$ and TlMnO$_3$, respectively. A minor peak in the high-energy region is produced for TlCrO$_3$ due to interband carriers.

We considered the optical properties in the interband region. Dielectric function $\varepsilon(\omega)$ is generally measured by the interband along with interband optical transitions. For this investigation, we overlooked the indirect intraband transition as this comprises phonon with an inadequate scattering cross-section compared to that of direct transition; as a result, phonon scattering is not required to recollect the momentum. The $\varepsilon(0)$ static

Fig. 7 Total and partial DOS of TlMnO$_3$ (a) without spin and (b) with spin.
Fig. 8  Energy-dependent absorption $\alpha(\omega)$, conductivity $\sigma(\omega)$, and reflectivity $R(\omega)$ of TlBO$_3$ (B = Cr, Mn) without spin (a–c) and with spin (d–f).

Fig. 9  Energy-dependent dielectric function $\varepsilon(\omega)$ ($\varepsilon_1$ real and $\varepsilon_2$ imaginary), loss function $L(\omega)$, and refractive index $n(\omega)$ of TlBO$_3$ (B = Cr, Mn) without spin (a–c) and with spin (d–f).
The optical property, viz., the refractive index, holds a significant role from the standpoint of applications such as solar cells, photonic crystals, and waveguides. The calculated values of the zero-frequency refractive index are ~9 and ~10 for the solids TlCrO₃ and TlMnO₃, respectively, as shown in Fig. 9(c) in case of without spin. As the frequency is increased more than 10 eV, the values of the refractive index decrease for both the materials, and slight non-linear nature can also be seen. In Fig. 9(f) with spin, the value of the refractive index decreases to ~3 and ~3.3 for TlCrO₃ and TlMnO₃, respectively.

### 3.7 Thermodynamic properties

The thermodynamic properties of TlBO₃ (B = Cr, Mn) are significant to find different natures of the solids under different pressures and temperatures. Different variety of thermodynamic properties can be measured such as Debye temperature ($\theta_D$), melting temperature ($T_m$), and minimum thermal conductivity ($K_{min}$) using other properties such as density ($\rho$), longitudinal, transverse, and average sound velocities ($v_l$, $v_t$, and $v_m$). The Debye temperature ($\theta_D$) is evaluated as follows:\(^{105}\)

$$\theta_D = \frac{h}{k_B} \left[ \frac{3m}{4\pi} \frac{N_A \rho}{M} \right]^{1/3} v_m$$  \hspace{1cm} (27)

where $h$, $k_B$, $V$, $v_m$, and $n$ is Planck's constant, Boltzmann constant, unit cell volume, average sound velocity, and the number of atoms in the unite cell, respectively. Here, $v_m$ can be evaluated by the equation

$$v_m = \left[ \frac{1}{3} \left( \frac{2}{v_t^3} + \frac{1}{v_l^3} \right) \right]^{-1/3}$$  \hspace{1cm} (28)

where $v_l$ is the transverse velocity and $v_l$ is the longitudinal sound velocity of the solids. $v_l$ and $v_t$ are evaluated by the formula below.

The melting temperature ($T_m$) is also determined to explore the possible application of the materials at high temperatures. The following equation is used to determine ($T_m$):\(^{106}\)

$$T_m = 354 + \frac{4.5(2C_{11} + C_{33})}{3}$$  \hspace{1cm} (31)

Thermal conductivity, an important parameter to investigate the heat conduction of both TlCrO₃ and TlMnO₃, was determined. The minimum thermal conductivity ($K_{min}$) of a solid material is associated with temperature and will decrease to a certain value with a steady rise in temperature.\(^{107}\) Minimum thermal conductivity ($K_{min}$) is calculated by Clarke formula:\(^{108}\)

$$K_{min} = k_B v_m \left( \frac{M}{\rho N_A n} \right)^{-2/3}$$  \hspace{1cm} (32)

where $M$, $n$, and $N_A$ signify the molecular mass, atoms per molecule, and Avogadro's number, respectively. The calculated thermal properties $\theta_D$, $T_m$, $K_{min}$, $v_m$, $v_l$, and $v_t$ are presented in Table 6. The values of $\theta_D$ and $K_{min}$ for the title compounds are much lower than the commercialized thermal barrier coating (TBC) compound Y₂Al₁₂O₃₉ ($K_{min} = 1.13$ W m⁻¹ K⁻¹; $\theta_D = 564$ K).\(^{109,110}\)

### 4. Conclusions

The physical properties such as structural, mechanical, elastic anisotropic, Vickers hardness, electronic, optical, and thermodynamic properties of TlBO₃ (B = Cr, Mn) structured perovskites were investigated for the first time using ab initio-based DFT calculations. Good agreement was obtained between the calculated values and the obtained elastic constants. Born criteria for both the materials was fulfilled, signifying that both solids are mechanically stable. Both materials are ductile, as defined by all the elastic moduli such as Cauchy pressure, Poisson’s ratio, and Pugh’s ratio. All the different methods employed for studying the anisotropy of TlBO₃ (B = Cr, Mn) obtained the same results. The hardness calculations reveal that both solids are flexible, soft, and suitable for making a thin film. The Vickers hardness value is also very small compared to MAX phase materials. Both solids displayed a transition from
metallic to semiconducting nature without spin and with spin configuration. The investigation of the optical properties of both TlCrO₃ and TlMnO₃ reveals various possible uses such as waveguides, ultralarge integration of integrated circuits, microelectronics, and the reduction of solar heating, where the investigation with spin study exposes both materials’ better absorption and application in solar cells. The solids have lower values of θᵳ and Kᵢ, which implies that they can be used as a thermal barrier coating (TBC) material. We hope that this investigation will help a researcher use these materials for different applications according to their physical properties and inspire scientists both experimentally and theoretically to discover this material in more depth in the future.

Data availability

All data needed to evaluate the conclusion of this study are presented in the paper. Additional data are available from the corresponding author upon reasonable request.

Author contributions

Wakil Hasan: methodology, data curation, formal analysis, writing original draft; Adeb Mahamud Hossain: formal analysis, review-editing; Md. Rasheduzzaman: conceptualization, supervision, formal analysis, review-editing. Md. Atikur Rahman: validation, formal analysis, review-editing; Md. Mukter Hossain: formal analysis, review-editing; supervision, K. Rashel Mohammad: formal analysis, review-editing; Raihan Chowdhury: formal analysis, review-editing; Khandaker Monower Hossain: formal analysis, review-editing; M. Moazzam Hossen: formal analysis, review-editing; Md. Zahid Hasan: investigation, conceptualization, supervision, formal analysis, review-editing.

Conflicts of interest

The authors declare no conflict of interests.

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