Nzoupe, Fernand Naha; Dikandé, Alain Moïse; Tchawoua, Clement
Phase transition in quantum tunneling and exact statistical mechanics for a model of parameterized double-well potential. (English) Zbl 1482.82003
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Summary: A model for one-dimensional bistable systems characterized by a deformable double-well energy landscape is introduced in order to investigate the effect of shape deformability on the order of phase transition in quantum tunneling and on the quasi-exact integrability of the classical statistical mechanics of these systems. The deformable double-well energy landscape is modeled by a parameterized double-well potential possessing two fixed degenerate minima and a constant barrier height, but a tunable shape of its walls which affects the confinement of the two wells. It is found that unlike bistable models involving the standard $\phi^4$-field model for which the transition in quantum tunneling is predicted to be strictly of second order, a parameterization of the double-well potential also favors a first-order transition occurring above a universal critical value of the shape deformability parameter. The partition function of the model is constructed within the framework of the transfer integral formalism, with emphasis on low-lying eigenstates of the transfer integral operator. Criteria for quasi-exact integrability of the partition function were formulated, in terms of the condition for possible existence of exact eigenstates of the transfer integral operator. The quasi-exact solvability condition is obtained analytically, and from this, some exact eigenstates are derived at several temperatures. The exact probability densities obtained from the analytical expressions of the ground state wavefunctions at different temperatures are found to be in excellent agreement with the probability density obtained from numerical simulations of the Fokker-Planck equation.

MSC:
82B10 Quantum equilibrium statistical mechanics (general)
82B26 Phase transitions (general) in equilibrium statistical mechanics
35Q84 Fokker-Planck equations
81T28 Thermal quantum field theory
81U26 Tunneling in quantum theory
35C08 Soliton solutions

Keywords:
bistable systems; Fokker-Planck equation; quantum equilibrium statistical mechanics; quantum field theory; related classical field theories; solitons; transition in quantum tunneling

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