Economic manufacturing quantity model with machine breakdown and deteriorating production process

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Abstract. In this study, the author proposes a jointly setting model for determining the process mean and economic manufacturing quantity (EMQ) under the machine breakdown and deteriorating production process. The system addresses the corrective maintenance, preventive maintenance, and allowable shortage. The quality loss of conforming product is considered and Taguchi’s asymmetric quadratic quality loss function is used for evaluating the product quality. The optimal process mean and economic manufacturing quantity are jointly determined by minimizing the total expected cost of product per unit time including the set-up cost, holding cost, corrective maintenance cost, preventive maintenance cost, shortage cost. A solution procedure is devised to obtain the optimal solution and the sensitivity analysis of key parameters is conducted to investigate the effect on the optimal solution.

1. Introduction
Recently, the supply chain management with quality and reliability design is an important topic for the operational management on the manufacturing/service industry. For the two-level supply chain system, the trade-off problem between the manufacturer and customer should be available for further study. Process mean (process quality level) and production quantity settings are two different problems in the modern quality and inventory management. For the economic manufacturing quantity (EMQ) decision, we need to consider the minimum inventory cost including the set-up cost and holding cost. For the process mean decision, we need to consider the trade-off cost between the conforming and non-conforming product. We can solve the above individual model when the management problem occurs. If we would like to obtain the efficient enterprise resource usage, then we can consider to simultaneously determine the product and process parameters. Hence, we should prefer to adopt the integrated model for solving the above two management problems.

Traditional EMQ model assumes that the perfect product for the manufacturing process. Hence, the defective product for EMQ model has been neglected. Previous researchers, e.g., Porteus [1] and Rosenblatt and Lee [2-3], firstly proposed the imperfect quality of EMQ model. Recently, EMQ model with imperfect quality, inventory control, reliability, and supply chain is the important topic of business, management, and engineering.

There are some works that assured a post-sale warranty cost where imperfection is prevalent in the production system. The works of Djamaludin et al. [4], Yeh and Lo [5], Yeh, et al. [6], Wang and Sheu [7-8], Wang [9-10], and Yeh and Chen [11] can be quoted along this line. Recently, Mujahid and Rahim [12], Rahim and Fareeduddin [13], and Akram et al. [14] have proposed the integrated
determination of production run length and product quality control problem. Sarkar et al. [15], Sana et al. [16], and Sana [17] have addressed the production lot size, sales price, and process maintenance control problem. Lin and Gong [18] considered an integrated manufacturer-buyer supply chain model for vendor’s production system with random breakdown. When a breakdown occurs, it needs to perform the correct maintenance to restore the production system. Economic selection of optimal process mean is a popular topic for modern statistical process control because it has a significant effect on the expected profit/cost per unit product. The study of process mean problem started from Springer [19]. Taguchi [20] redefined the product quality as the loss of society when the product is shipped and sold to the customer. To decrease the society’s loss, he proposed the quadratic quality loss function for evaluating the product quality. Taguchi’s [20] quality loss function can combine with the on-line quality control (production process control) and off-line quality control (product quality design) methods and promote the product quality and performance. His quality loss function has been successfully applied in industry for promoting the product quality and reliability by reducing the product cost. Jeang [21] proposed an integrated model with optimal production quantity, process mean, and process tolerance under the deterioration and breakdown production process. Recently, Chen and Tsai [22] and Chen et al. [23–24] have adopted Taguchi’s [20] quality loss function for determining process mean and production quantity. Their models addressed that the economic model of product will underestimate the expected total relevant cost because the quality loss of conforming product is neglected. Many works about optimum process mean setting problem have been integrated with inspection plan, production and inventory control, maintenance, and supply chain management.

In 2010, Sarkar et al. presented a production system with machine failure and production process deterioration. Their model considers shortage and with/without safety stock for determining the optimal machine reliability parameter and production lot size. However, their model did not consider the quality loss of conforming products. In this study, the author proposes a modified Sarkar et al.’s [15] model with product quality loss for minimizing the total relevant cost of product per unit time. Assume that the quality characteristic of the product is normally distributed. Taguchi’s [20] asymmetric quadratic quality loss function will be used for measuring the product quality. The total relevant cost of product per unit time for the modified model includes the set-up cost, corrective cost, preventive cost, holding cost, shortage cost, and expected quality loss of the conforming products and expected cost of the non-conforming products. To our best knowledge, the modified Sarkar et al.’s [15] model with quality loss of the conforming products is a new method for extending the application of Sarkar et al.’s [15] model. The main difference between our modified model and the original Sarkar et al.’s [15] one is that the former addresses the optimal product quality output for the production process and hence should include the trade-off between the quality loss and production cost. The important contribution of the this study is that the determination of optimal production and product parameters is based on the minimization expected total loss of society including the producer’s manufacturing cost and customer’s use cost for product. The management implication of this work is that the manufacturer provides a high quality and reliability product/service will promote the customer’s satisfaction and reduces the total expect cost of the supply chain system. A solution procedure is devised to find the optimal solution for the modified Sarkar et al.’s [15] model and the sensitivity analysis of key parameters is conducted to investigate the effect on the optimal solution.

2. Original Sarkar et al.’s model for determining the reliability parameter and production quantity

Sarkar et al. [15] addressed the unreliable production system with process deterioration, machine breakdown and repair (corrective and preventive maintenance). During long-run process, a certain percentage of defective items are produced which is reworked at a cost immediately. The expected cost function including the corrective cost, preventive cost, set-up cost, holding cost, shortage cost, and rework cost is formulated while shortage for breakdown and with/without safety stock. The product lot size and reliability parameter of the machinery system are considered to be two decision variables. The aim of their model is to generalize economic manufacturing model with unreliable
production system by taking into account jointly the process deterioration, machine breakdown, and maintenance policy.

3. Modified Sarkar et al.’s model for determining the process mean and production quantity

3.1. Notations

| Symbol | Description |
|--------|-------------|
| $D$    | the demand rate in units per unit time |
| $p$    | the production rate in units per unit time, $p > D$ |
| $C_h$  | the holding cost per unit per unit time |
| $C_0$  | the setup cost for each production run |
| $C_1$  | the corrective repair cost per unit time |
| $C_2$  | the preventive repair cost per unit time |
| $C_j$  | the scrap cost for a defective item |
| $C_r$  | the rework cost for a defective item |
| $C_m$  | the material cost per unit product |
| $C_s$  | the shortage cost per unit product |
| $Q$    | the economic manufacturing quantity |
| $C(\mu, Q)$ | the expected total relevant cost of product per unit time |
| $S(\mu, Q)$ | the expected total relevant cost of product |
| $L(Q)$ | the expected cycle length |
| $\rho$ | the elapsed time until production process shifts, it is an exponential distribution with parameter $k$ |
| $f(\rho)$ | the probability density function of $\rho$ |
| $T$    | the elapsed time until the machine fails, it is an exponential distribution with parameter $\phi$ |
| $f(t)$ | the probability density function of $T$ |
| $X_1$  | the elapsed time until the corrective repair completes, it is an exponential distribution with parameter $\lambda_1$ |
| $f(X_1)$ | the probability density function of $X_1$ |
| $X_2$  | the elapsed time until the preventive repair completes, it is an exponential distribution with parameter $\lambda_2$ |
| $f(X_2)$ | the probability density function of $X_2$ |
| $Y$    | the normal quality characteristic of product |
| $f(y)$ | the probability density function of $Y$ |
| $\mu_y$ | the process mean (process quality level) |
| $\sigma_y$ | the process standard deviation |
| $LSL$  | the lower specification limit of product |
| $USL$  | the upper specification limit of product |
| $\Phi(\cdot)$ | the cumulative distribution function for the standard normal random variable |
| $z(\cdot)$ | the probability density function for the standard normal random variable |
| $\alpha$ | the probability of defective product |
| $E(N)$ | the expected number of products in a production cycle |
| $E_r(N)$ | the expected number of rework items in a production cycle |
| Symbol | Description |
|--------|-------------|
| $E_3(N)$ | the expected number of scrap items in a production cycle |
| $E_5(N)$ | the expected number of conforming items in a production cycle |
| $y_0$ | the target value of product |
| $E[Loss(Y)]$ | the expected quality loss per unit product |
| $k_1$ | the quality loss coefficient when $LSL < Y < y_0$ |
| $k_2$ | the quality loss coefficient when $y_0 < Y < USL$ |
| $\Delta_1$ | the deviation from the left-hand side of the target value of the product characteristic |
| $\Delta_2$ | the deviation from the right-hand side of the target value of the product characteristic |

3.2. Assumptions
There are some assumptions in the modified model:

1. The elapsed time of production process shifting from an in-control state to an out-of-state, $\rho$, is an exponential distribution with parameter $k$ and probability density function $f(\rho)$.
2. The elapsed time until the machine fails, $T$, is an exponential distribution with parameter $\phi$ and probability density function $f(t)$.
3. The elapsed time until the corrective repair completes, $X_1$, is an exponential distribution with parameter $\lambda_1$ and probability density function $f(x_1)$.
4. The elapsed time until the preventive repair completes, $X_2$, is an exponential distribution with parameter $\lambda_2$ and probability density function $f(x_2)$.
5. The quality characteristic of product, $Y$, is normally distributed with known standard deviation $\sigma_y$ and unknown process mean $\mu_y$.
6. The product is reworked when the quality characteristic is above the $USL$.
7. The product is scrapped when the quality characteristic is below the $LSL$.
8. Taguchi’s asymmetric quadratic quality loss function is adopted for measuring the product quality.
9. Single item is produced in a single machine. During the production run time, the random breakdowns may occur. Only one breakdown in one cycle is considered.
10. If the machine breakdown occurs, then the corrective maintenance starts immediately; otherwise, the preventive maintenance is started as on time.
11. Shortage is considered for the corrective/preventive repair periods.
12. The production rate and demand rate are considered as fixed.
13. The unreliable manufacturing system results in a machine breakdown in which the production is interrupted.
14. The unreliable production system takes into account jointly the process deterioration, machine breakdown, and repair (corrective and preventive maintenance).
15. There is no safety stock.

3.3. Mathematical model
Similar to Sarkar et al. (2010), the corrective/preventive repair time considers the following four periods: (1) the corrective repair time occurs between 0 and $\frac{P-D}{D} \cdot t$; (2) the corrective repair time occurs beyond $\frac{P-D}{D} \cdot t$; (3) the corrective repair time occurs between 0 and $\frac{P-D}{D} \cdot \frac{Q}{p}$; (4) the preventive repair time occurs beyond $\frac{P-D}{D} \cdot \frac{Q}{p}$. In Sarkar et al.’s [15] model, the total relevant cost of product for one production cycle includes the set-up cost, material cost, corrective repair cost,
preventive repair cost, holding cost, shortage cost, and expected non-conforming cost of conforming. However, their model does not consider the expected quality loss of conforming products. Hence, Sarkar et al.’s [15] model will underestimate the expected total relevant cost of products. Our modified model should address the conforming cost of products. Similar to Sarkar et al. [15], the total relevant cost of product per unit time is

\[ C(\mu_y, Q) = \frac{S(\mu_y, Q)}{L(Q)} \]  

(1)

Where

\[
S(\mu_y, Q) = C_0 + C_n Q + \frac{C_1}{\lambda_1} [1 - \exp(-\phi \frac{Q}{p})] + \frac{C_2}{\lambda_2} \exp(-\phi \frac{Q}{p}) + C_n \frac{p(p - D)}{2D} \{(-\frac{Q}{p})^2 \exp(-\phi \frac{Q}{p}) \\
+ \frac{2}{\phi} \frac{Q}{p} \exp(-\phi \frac{Q}{p}) + \frac{1}{\phi} \exp(-\phi \frac{Q}{p})] \\
- \frac{\phi}{\phi + \lambda_1} \frac{p-D}{D} \{(-\frac{Q}{p})^2 \exp((-\phi + \lambda_1) \frac{p-D}{D} Q/p) \\
+ \frac{2}{\phi + \lambda_1} \frac{p-D}{D} \{(-\frac{Q}{p})^2 \exp((-\phi + \lambda_1) \frac{p-D}{D} Q/p) \\
+ \frac{1}{\phi + \lambda_1} \frac{p-D}{D} \{1 - \exp((-\phi + \lambda_1) \frac{p-D}{D} Q/p)\})\} \\
+ \frac{p(p-D)}{2D} \frac{Q}{p} \{1 - \exp(-\lambda_2 \frac{p-D}{D} Q/p)\} \exp(-\phi \frac{Q}{p}) \\
+ \frac{C}{D} \frac{p-D}{p-D} \phi \frac{p-D}{D} \{1 - \exp((-\phi + \lambda_1) \frac{p-D}{D} Q/p)\} \\
+ \frac{1}{\lambda_2} \exp((-\phi + \lambda_2) \frac{p-D}{D} Q/p)\} + C_r p[1 - \Phi(\frac{USL - \mu_y}{\sigma_y})] \frac{1}{\phi} \{1 - \exp(-\phi \frac{Q}{p})\} \\
+ \frac{\phi}{k(k + \phi)} \{1 - \exp(-(k + \phi) \frac{Q}{p})\}] \\
+ C_r p\Phi(\frac{LSL - \mu_y}{\sigma_y}) \frac{1}{\phi} \{1 - \exp(-\phi \frac{Q}{p})\} + \frac{\phi}{k(k + \phi)} \{1 - \exp(-(k + \phi) \frac{Q}{p})\}] \\
+ E[Loss(Y)] p[\Phi(\frac{USL - \mu_y}{\sigma_y}) - \Phi(\frac{LSL - \mu_y}{\sigma_y})] \\
+ \frac{1}{\phi} \{1 - \exp(-\phi \frac{Q}{p})\} \{1 - \exp(-(k + \phi) \frac{Q}{p})\}] \\
\]  

(2)
We adopt the direct search method for obtaining the optimal process mean $\mu^*_y$ and production quantity $Q^*$ which minimizes the total relevant cost of product per unit time. Let

$$L(Q) = \int_0^P \left[ \int_0^{t+P} \frac{f(x_1)}{D} dx_1 \right] f(t) dt + \int_0^P \left[ \int_0^{t+P} f(x_1) dx_1 \right] f(t) dt$$

Substituting the combination of $(\mu^*_y, Q)$ into Eq. (1), we have the following equation:

$$\min_{Q^*} C(\mu^*_y, Q)$$

The mathematical optimization model for determining the decision variables $(\mu_y, Q)$ is to minimize $C(\mu_y, Q) = \frac{S(\mu_y, Q)}{L(Q)}$. One cannot prove that Eq. (1) is convex with respect to $\mu_y$ because of the cumulative distribution function of the standard normal random variable $\Phi(\cdot)$ for a given $Q$. Hence, we haven’t the closed-form solution of $\mu_y$ for a given $Q$. The optimal solution of $\mu_y^*$ for a given $Q$ is found numerically. A one-dimensional direct search method can be used to find the optimal process mean. Let $LSL < \mu_y < USL$. We can adopt the direct search method for solving $\mu_y^*$ with minimum $C(\mu_y, Q)$ under the given $Q$. Substituting the combination of $(\mu_y^*, Q)$ into Eq. (1), we have the resulting total relevant cost of product per unit time.

The detail solution procedure for the above Eq. (1) is as follows:

Step 1. Give the maximum $Q, Q_m$.
Step 2. Let $Q = 1$.
Step 3. Let $LSL < \mu_y < USL$. We adopt the direct search method for obtaining the optimal process mean $\mu_y^*$. Substituting the combination of $(\mu_y^*, Q)$ into Eq. (1), we obtain the corresponding $C(\mu_y^*, Q)$.
Step 4. Let $Q = Q + 1$. Repeat Step 3 until $Q = Q_m$. The combination of $(\mu_y^*, Q^*)$ with the minimum total relevant cost of product per unit time $C(\mu_y^*, Q^*)$ is the optimal solution.
4. Numerical example and sensitivity analysis

Some parameters are given as follows: $C_0 = 300, C_p = 10, C_1 = 0.5, C_2 = 5, C_h = 0.5, C_s = 1, C_j = 2, C_r = 3, p = 240, D = 100, \lambda_1 = 2, \lambda_2 = 10, \phi = 0.5, k = 2, \sigma_y = 0.5, LSL = 6, USL = 10, y_0 = 8, k_1 = 2, \text{and } k_2 = 4$. By solving the above Eq. (1), we have the optimal manufacturing quantity $Q^* = 133$ and process mean $\mu^*_y = 7.93$ with the total relevant cost of product per unit time $C(\mu^*_y, Q) = 1379.43$.

Table 1 lists the effect of some parameters on the optimal solution. From Table 1, one has the following conclusions: (1) the process standard deviation, two quality loss coefficients, upper specification limit, and target value of product have a major effect on the process mean; (2) the material cost, production rate per unit time, parameter of the elapsed time until the corrective repair completes, parameter of the elapsed time until the machine fails, process standard deviation, and upper specification limit have a major effect on the manufacturing quantity; (3) the material cost and demand rate per unit time have a major effect on the total relevant cost of product per unit time.

**Table 1.** The sensitivity analysis of parameters for numerical example.

| Parameter | $\mu_y$ | $Q$ | $C(\mu^*_y, Q)$ |
|-----------|---------|-----|-----------------|
| $C_1$     | 0.4     | 133 | 1379.42         |
|           | 0.6     | 133 | 1379.44         |
| $C_2$     | 4       | 133 | 1379.36         |
|           | 6       | 133 | 1379.49         |
| $C_h$     | 0.4     | 134 | 1376.00         |
|           | 0.6     | 132 | 1382.82         |
| $C_s$     | 1.2     | 133 | 1378.39         |
|           | 1.6     | 133 | 1379.43         |
| $C_j$     | 2.4     | 133 | 1379.43         |
|           | 3.6     | 133 | 1379.43         |
| $C_r$     | 2.4     | 133 | 1379.43         |
|           | 8       | 147 | 1160.58         |
|           | 12      | 123 | 1595.18         |
| $p$       | 192     | 120 | 1395.63         |
|           | 288     | 145 | 1362.89         |
| $D$       | 80      | 133 | 1129.16         |
|           | 120     | 133 | 1616.65         |
| $\lambda_1$ | 1.6    | 131 | 1353.23         |
2.4 7.93 135 1396.37
$\lambda_2$ $\mu_y$ $Q$ $C(\mu_y, Q)$
8 7.93 133 1379.31
12 7.93 133 1379.41
$\phi$ $\mu_y$ $Q$ $C(\mu_y, Q)$
0.4 7.93 148 1356.37
0.6 7.93 122 1398.74
$k$ $\mu_y$ $Q$ $C(\mu_y, Q)$
1.6 7.93 134 1375.45
2.4 7.93 133 1382.92
$\sigma_y$ $\mu_y$ $Q$ $C(\mu_y, Q)$
0.4 7.94 136 1369.81
0.6 7.91 131 1390.92
$k_1$ $\mu_y$ $Q$ $C(\mu_y, Q)$
1.6 7.91 134 1377.33
2.4 7.95 133 1381.43
$k_2$ $\mu_y$ $Q$ $C(\mu_y, Q)$
3.2 7.95 134 1376.10
4.8 7.91 133 1382.66
$LSL$ $\mu_y$ $Q$ $C(\mu_y, Q)$
4.8 7.93 133 1379.43
7.2 7.93 133 1379.49
$USL$ $\mu_y$ $Q$ $C(\mu_y, Q)$
8 7.61 130 1395.61
12 7.93 133 1379.43
$y_0$ $\mu_y$ $Q$ $C(\mu_y, Q)$
6.4 6.42 132 1384.79
9.6 9.40 132 1385.33

5. Conclusions
In this study, the author has proposed a modified Sarkar et al.’s [15] model for determining the process mean and manufacturing quantity under the machine breakdown and deteriorating production process. The system addresses the corrective maintenance, preventive maintenance, and allowable shortage. The quality loss of conforming product is considered and Taguchi’s asymmetric quadratic quality loss function is used for evaluating the product quality. The optimal process mean and economic manufacturing quantity are jointly determined by minimizing the total expected cost of product per unit time. From the above numerical results, we have the following conclusion: the material cost and demand rate per unit time have a major effect on the total relevant cost of product per unit time. Further study should consider the unknown process standard deviation and address the joint design of manufacturing quantity, specification limits, process mean, and standard deviation.

References
[1] Porteus E L 1986 Operations Research 34 pp 137—144
[2] Rosenblatt M J and Lee H L 1986a IIE Transactions 17 pp 48—54
[3] Rosenblatt M J and Lee H L 1986b IIE Transactions 18 pp 2—9
[4] Dajmaludim V D, Murthy N P and Wilson R J 1994 International Journal of Production
Economics 33 pp 97 – 107
[5] Yeh R H and Lo H C 1998 International Journal of Operations and Quantitative Management 4 pp 265 – 275
[6] Yeh R H, Ho W T and Tseng S T 2000 European Journal of Operational Research 120 pp 575 – 582
[7] Wang C H and Sheu S H 2000 Computer and Mathematics with Applications 40 pp 1297 – 1314
[8] Wang C H and Sheu S H 2003 European Journal of Operational Research 49 pp 131 – 141
[9] Wang C H 2004 Computers & Operations Research 31 pp 2021 – 2035
[10] Wang C H 2006 Naval Research Logistics 53 pp 151 – 156
[11] Yeh R H and Chen T H 2006 European Journal of Operational Research 174 pp 766 – 776
[12] Mujahid S N and Rahim M A 2010 International Journal of Operational Research 9 pp 227 – 240
[13] Rahim M A and Fareeduddin M 2011 International Journal of Industrial and Systems Engineering 8 pp 298 – 325
[14] Akram M A, Al-Saif A W and Rahim M A 2012 International Journal of Industrial and Systems Engineering 11 pp 375 – 405
[15] Sarkar B, Sana S S and Chaudhuri K 2010 International Journal of Mathematics in Operational Research 2 pp 467 – 490
[16] Sana S S, Goyal S K and Chaudhuri K 2007 International Journal of Production Economics 105 pp 64 – 80
[17] Sana S S 2010 An economic production lot size model in an imperfect production system European Journal of Operational Research 201 158–170
[18] Lin G C and Gong D C 2018 A manufacturer-buyer model with random breakdowns Proceeding of the Asia Pacific Industrial Engineering & Management Systems Conference (Hong Kong)
[19] Springer C H 1951 Industrial Quality Control 8 pp 36 – 39
[20] Taguchi G 1986 Introduction to Quality Engineering (Tokyo: Asian Productivity Organization)
[21] Jeang A 2012 Omega 40 pp 774 – 781
[22] Chen C H and Tsai W R 2016 Journal of Industrial and Production Engineering 33 pp 495 – 500
[23] Chen C H, Chou C Y and Kan C C 2015 Journal of Industrial and Production Engineering 32 pp 196 – 203
[24] Chen C H, Khoo M B C, Chou C Y and Kan C C 2015 Journal of Industrial and Production Engineering 32 pp 219 – 224