ANALYSIS OF MEASUREMENT UNCERTAINTY OF (SIMPLE) PITCH DIAMETER CALIBRATION WHEN SELECTED SIZES OF ZEISS WIRES ARE USED

Summary

The calibration of screw thread gauges is a conventional procedure in calibration laboratories and National Metrology Institutes. Proper measurement and reliable uncertainty analysis of the screw thread gauge calibration are complex and demanding tasks.

Different technical documents and international standards in screw metrology define calibration categories, explain measurement plans, measuring elements, nominal diameters and their tolerances, and propose guidelines for the measurement uncertainty estimation. Therefore, the first part of this paper provides a detailed overview of different technical documents and international standards in screw thread metrology.

The second part of this paper aims to draw attention to critical points of the pitch diameter calibration procedure that lead to a significant increase in measurement uncertainty. Most calibration laboratories do not measure the thread angle during the pitch diameter calibration. Therefore, the selection of the measurement wires can contribute greatly to the measurement uncertainty.

Even though the international standards propose the best size wires to avoid this problem, many manufacturers produce measuring wires according to DIN 2269 (Zeiss). In this paper, the authors do simulations of the contribution of measuring wires to the uncertainty in the calibration of the pitch diameter of metric M threads. A simulation is created for each thread gauge according to ISO 1502, and critical threads, in which a significant increase in measurement uncertainty is expected, are highlighted.

The authors propose a simple solution for avoiding this unwanted increase in measurement uncertainty of pitch diameter calibration; the solution involves replacing the DIN 2269 (Zeiss) wires with the best-size wires only for the highlighted critical threads.

Key words: thread gauge calibration, measuring wires, two-ball stylus, pitch diameter calibration, measurement uncertainty

1. Introduction

The calibration of screw thread gauges is an important issue from both an economic and a technical point of view. Measurement uncertainties of a few micrometres that are usually obtained for screw thread gauges do not appear to be very demanding when compared to those
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obtained for gauge blocks or ring gauges, for example. However, proper measurement and its evaluation as well as a reliable uncertainty analysis of the results of calibration of screw thread gauges are difficult and complicated tasks. [1] Results of thread gauge calibration must be traceable to the SI metre definition [2].

Conventional methods and procedures for pitch diameter calibration have been well known since the 19th century; these methods involve the use of universal length measuring machines [3].

Classical metrology of thread gauges is based on the work of Georg Berndt [4][5], who has established a mathematical model for pitch diameter determination. Pitch diameter is calculated from measurement results obtained by the three-wire method for thread plugs and the two-ball method for thread rings. [6] His work has laid a foundation for the international calibration guide, cg-10, published by the European Association of National Metrology Institutes (EURAMET) [1].

This guide promotes a consistent approach to good measurement practice leading to and supporting laboratory accreditation. Although the cg-10 guidelines are not mandatory, they are widely used by calibration laboratories and National Metrology Institutes (NMIs). The document also proposes different categories of calibration and gives procedures for measurement uncertainty estimation considering all relevant uncertainty sources.

The calibration of parallel thread gauges according to the EURAMET cg-10 guide will be discussed in detail in this paper, but other technical documents and international standards will be presented as well.

2. Measurement method

Thread plug is calibrated between two flat surfaces (sing three calibrated wires of the same diameter as the probing element. Wires are inserted in the thread grooves, as shown in Fig. 1 and Fig. 2, and measurement over wires, \( \Delta L \), is used to determine the pitch diameter according to Eq. (1).

\[
\Delta L = m + d_D
\]

In this equation, \( d_2 \) is the pitch diameter of the thread plug, \( \Delta L \) is the measured distance, \( d_D \) is the wire diameter, \( \theta \) is the thread angle, \( P \) is the thread pitch, while \( A_1 \) and \( A_2 \) are the corrections that must be made due to the rake and the measuring force.

A similar method is used for the thread ring calibration using a two-ball stylus. Since there is no difference in contact between cylindrical or spherical probes, the same mathematic model can be used for the thread ring calibration, but with opposite measurement direction, i.e. the opposite signs in Eq. (2).
In the equation above, \( D_2 \) is the thread ring pitch diameter, and \( m \) is the distance between measuring spheres. The thread ring gauge calibration is shown in Fig. 3 and Fig. 4.

\[
D_2 = m + d_\theta \left( \frac{1}{\sin(\frac{\alpha}{2})} \right) = \frac{p}{2} \cdot \cot\left(\frac{\alpha}{2}\right) + A_1 - A_2
\]  

(2)

These calibration methods are recommended by EURAMET cg-10 [1]. A detailed explanation of the measurement plan can be found in DKD-R 4-3 Blatt 4.8 [7] and DKD-R 4-3 Blatt 4.9 [8]. Also, ASME B1.16M [9] gives a detailed explanation of metric thread plug calibration with simplified expressions adopted for pitch diameter calculation.

The EURAMET cg-10 guide [1] has three main categories for pitch diameter calibration: simple pitch diameter, pitch diameter, and virtual pitch diameter. Calibration categories are further divided into subcategories with regard to quantities that must be measured and assumptions made for necessary calculations and estimations of measurement uncertainty.

The symbol \( m \) denotes the measured diameter defined as the distance between the centres.

### Table 1 Calibration categories for the pitch diameter determination [1]

| Parameter          | Measured | Assumed | Taken into account in uncertainty analysis |
|--------------------|----------|---------|-------------------------------------------|
| 1                  | Simple pitch diameter |         |                                           |
| 1a                 | \( m \)  | ✓       | ✓                                        |
|                   | \( \alpha \)     | -       | (tolerance zone)                         |
|                   | \( P \)          | defined normal | -                                        |
| 1b                 | \( m \)  | ✓       | ✓                                        |
|                   | \( \alpha \)     | ✓       | ✓                                        |
|                   | \( P \)          | defined normal | -                                        |
| 2                  | Pitch diameter   |         |                                           |
| 2a                 | \( m \)  | ✓       | ✓                                        |
|                   | \( \alpha \)     | -       | (tolerance zone)                         |
|                   | \( P \)          | ✓       | ✓                                        |
| 2b                 | \( m \)  | ✓       | ✓                                        |
|                   | \( \alpha \)     | ✓       | ✓                                        |
|                   | \( P \)          | ✓       | ✓                                        |
| 3                  | Virtual pitch diameter |         |                                           |
| 3                  | \( m \)  | ✓       | ✓                                        |
|                   | \( \beta,\gamma \) | ✓     | ✓                                        |
|                   | \( P \)          | ✓       | ✓                                        |
The same calibration categories are explained in detail in [1], [7] and [8] with an extended parameter list for virtual pitch diameter. The CCL Length Services Classification [10] recognizes the simple pitch diameter and the pitch diameter as different measurands.

Theoretically, in the case of ideal thread geometry, without any geometry error, the calibration result would be identical for all calibration categories.

If 1a or 1b calibration category is used, then the thread angle is not measured; therefore, the nominal value of thread angle is used for the pitch diameter calculation. If the measured gauge has a significant error in the thread angle and if the optimal diameter of the measuring element is not used, the uncertainty of the calibration result can be significantly increased. This case is discussed further in this paper.

3. **Best-size diameter of measuring wires/balls**

The contact of measuring wire with the thread flank should be as close as possible to the pitch cylinder, i.e. pitch diameter. If the thread rake is neglected, then the optimal contact between the measuring wire and the thread flank for the symmetric profile can be shown as in Fig. 5.

From geometric relationships, the best-size diameter can be calculated using Eq. (3).

\[ d_0 = 2r_0 = \frac{P}{2\cos\left(\frac{\pi}{2}\right)} \]  

(3)

The same expression can be found in international standards and technical guides such as EURAMET cg-10 [1], ASME B1.16 M [9], ASME B89.1.17 [11], and ISO 16239 [12]. This expression is also used to calibrate thread rings; the measuring element in the calibration procedure is a sphere (two-ball stylus) instead of wires.

In the case of an asymmetric thread, the contact with the left and the right flank at the pitch cylinder is not possible [13]. The optimal diameter can be selected in three different ways:

- To choose the diameter of the measuring wire that is in contact with the pitch cylinder on one nominated flank.
- To choose the diameter of the measuring wire that is in contact with one flank above the pitch cylinder and another flank below the pitch cylinder (also proposed by EURAMET cg-10 [1]).
- To choose the diameter of the measuring wire that is in contact with the pitch cylinder on the flank that is considered to have more influence on the pitch diameter.
4. Measurement uncertainty of the pitch diameter calibration

Estimation of measurement uncertainty is a demanding task since thread gauges have complex geometries. The calibration of the pitch diameter depends on various geometric errors, such as the error in thread angle, thread pitch, and measuring element. ASME B1.25 [14] proposes a general approach to the measurement uncertainty estimation for the thread gauge calibration and makes the main contribution to the uncertainty budget. This standard covers the pitch diameter calibration, but also makes contributions to the calibration of minor and major diameters, thread angle, thread pitch, and functional lead. This document provides general guidelines for the calibration of a threaded gauge with a thread angle of 60° but does not address the specifics of the methods and calibration equipment.

The EURAMET cg-10 [1] guide describes the estimation of measurement uncertainty in the determination of the pitch diameter of parallel thread gauges explained in section 2 according to the calibration categories stated in section 3. Measurement uncertainty is discussed according to the Guide to the Expression of Uncertainty (GUM) [15] with examples given for the calibration of thread plug and thread ring.

Contributions considered in the measurement uncertainty budget are related to
- measured displacement
- wire/ball diameter
- thread pitch
- thread angle or flank angles
- measurement force
- rake correction
- form deviation.

If calibration categories 1a and 1b are used, pitch calibration is not required, and it is not considered in the uncertainty budget. For calibration categories 1a and 2a, calibration of thread angle is not required and the tolerance zone is used for the uncertainty budget calculation.

5. Contribution of thread angle to the measurement uncertainty of pitch diameter calibration

A great number of calibration laboratories and even some NMIs have a standard practice for the thread gauge calibration according to categories 1a and 2a.

The tolerance zone for a specific thread gauge can be found in international standards; for example, tolerances for the flank angle $\alpha/2$ can be found in ISO 1502 [16] and ASME B1.16M [9].

Standard uncertainty of thread angle with a presumption of rectangular distribution can be calculated as:

$$u\left( \frac{\alpha}{2} \right) = \frac{Tolerance}{\sqrt{3}}$$ (4)

Sensitivity factor $c_{\alpha/2}$ can be defined as a derivative of Eq. 1:

$$c_{\alpha/2} = \frac{\partial d}{\partial \left( \frac{\alpha}{2} \right)} = d_p \frac{\cos\left( \frac{\alpha}{2} \right)}{\sin^2\left( \frac{\alpha}{2} \right)} - \frac{P}{2\sin^2\left( \frac{\alpha}{2} \right)} = \frac{\cos\left( \frac{\alpha}{2} \right)}{\sin^2\left( \frac{\alpha}{2} \right)} (d_P - \frac{P}{2 \cos\left( \frac{\alpha}{2} \right)}) = \frac{\cos\left( \frac{\alpha}{2} \right)}{\sin^2\left( \frac{\alpha}{2} \right)} (d_D - d_0)$$ (5)

The same coefficient can be used for the thread ring as well.

The contribution of flank angle to the measurement uncertainty of the thread gauge calibration, if all other contributions are neglected, can be expressed as:

$$u(d_2)^* = u(D_2)^* = u\left( \frac{\alpha}{2} \right) c_{\alpha/2}$$ (6)
One of the relevant sources of contribution, which can be very significant, is the use of the correct diameter of measuring element $d_D$. Sensitivity factor $C_{a/2}$ linearly depends on the difference between diameters of probe element $d_D$ and best-size $d_0$. If this difference is significant, it will greatly increase the measurement uncertainty of thread gauge calibration.

6. Measurement wires for pitch diameter calibration

Measurement wires are usually part of a set made specifically for the calibration of thread gauges. One set can be suitable for gauges with different thread pitches and even different thread angles. Selection of the correct wire diameter is crucial for the calibration of thread plugs. The value of selected wire diameter should be close to that of best-size diameter $d_0$. The value of $d_0$ is calculated using Eq. 3; it will vary due to different thread angles and thread pitches. Standard sizes of measuring wires are specified in international standards. ISO 16239:2013 [12] defines the material, metric series nominal diameters, diameter tolerances, and designation for screw thread measuring wires. This standard applies to M, UM, G, R, and TR screw threads with standard pitches. A similar standard, ASME B89.1.17 [11], covers measuring wires for the calibration of inch series 60-deg, 29-deg Acme, 7/45-deg Buttress, and metric 60-deg threads. It also provides methods for direct measurement of master and working wires, together with requirements on the geometric quality, important characteristics of comparison equipment, and the means for ensuring that measurements are made with an acceptable uncertainty level. Specifications of measuring wires can also be found in DIN 2269 [17]. This document applies to cylindrical measuring pins with a nominal diameter of 0.1mm to 20 mm; these pins can be used for checking bore holes, cones, distances, tooth forms, screw threads, and for gauging in general. It covers selected sizes for M, BS 84:1956, UN, UNC, UNF, UNEF, and TR threads.

Although many manufacturers make their measuring wire sets in accordance with ASME or ISO standards, which determine that wires should be manufactured with diameters which are close to the best-size diameter, they also produce wires according to the Zeiss recommendation provided by DIN 2269 [17] with a diameter different from the best-size. Also, the Zeiss set can be used for different types of thread gauges (e.g. metric, unified, Whitworth, etc.), but in some cases, the uncertainty of pitch diameter calibration will be significantly increased. To detect this, contribution of thread angle to the measurement uncertainty of pitch diameter calibration should be estimated for each calibrated thread gauge. On the other hand, wires made with diameters close to the best-size diameter are produced exclusively for one type of thread gauge with a specific thread pitch. Therefore, this uncertainty contribution can be neglected.

In standard laboratory practice, uncertainty of the pitch diameter calibration is usually estimated for a thread gauge with specific thread angle and pitch. This estimated value is then used for all other gauges to be calibrated in future. This can lead to underestimated measurement uncertainties. In this research, simulation is made to detect the critical thread gauge plugs (regarding the thread pitch) where an increase in the measurement uncertainty is expected. Simulation is done for metric thread plug gauges only, but the same simulation can easily be done for other thread gauge types as well.

7. Simulation of the thread angle contribution to the pitch diameter calibration

This chapter discusses the contribution of thread angle to the pitch diameter calibration of metric thread plugs. The aim is to detect critical dimensions of measuring wires, which would lead to a significant increase in the measurement uncertainty in calibration category 1a (2a).
The selected size of measuring wires for the M thread calibration according to DIN 2269 [17] are given in Table 2:

### Table 2 Selected sizes of measuring wires for M thread (Zeiss)

| \( P \), mm | \( d_D \), mm | \( P \), mm | \( d_D \), mm | \( P \), mm | \( d_D \), mm |
|---|---|---|---|---|---|
| 0.25 | 0.17 | 0.75 | 0.455 | 3 | 2.05 |
| 0.3 | 0.17 | 0.8 | 0.455 | 3.5 | 2.05 |
| 0.35 | 0.22 | 1 | 0.62 | 4 | 2.55 |
| 0.4 | 0.25 | 1.25 | 0.725 | 4.5 | 2.55 |
| 0.45 | 0.29 | 1.5 | 0.895 | 5 | 3.2 |
| 0.5 | 0.29 | 1.75 | 1.1 | 5.5 | 3.2 |
| 0.6 | 0.335 | 2 | 1.35 | 6 | 4 |
| 0.7 | 0.455 | 2.5 | 1.65 | 8 | 5.05 |

The tolerance zone of flank angle for M thread gauge plugs according to ISO 1502 [16] is given in Table 3:

### Table 3 Tolerance for each flank angle of GO and No-Go thread plug

| \( P \), mm | \( \alpha/2 \), min GO (NO-GO) | \( P \), mm | \( \alpha/2 \), min | \( P \), mm | \( \alpha/2 \), min |
|---|---|---|---|---|---|
| 0.25 | ±48 (±48) | 0.75 | ±17 (±17) | 3 | ±9 (±13) |
| 0.3 | ±40 (±40) | 0.8 | ±16 (±16) | 3.5 | ±9 (±12) |
| 0.35 | ±35 (±35) | 1 | ±15 (±16) | 4 | ±8 (±11) |
| 0.4 | ±31 (±31) | 1.25 | ±13 (±16) | 4.5 | ±8 (±11) |
| 0.45 | ±26 (±26) | 1.5 | ±12 (±16) | 5 | ±8 (±11) |
| 0.5 | ±25 (±25) | 1.75 | ±11 (±16) | 5.5 | ±8 (±10) |
| 0.6 | ±21 (±21) | 2 | ±10 (±14) | 6 | ±8 (±10) |
| 0.7 | ±18 (±18) | 2.5 | ±10 (±14) | 8 | ±8 (±10) |

Using Eq. 3, The standard uncertainty of flank angle can be calculated for each pitch.

Considering that M thread has a flank angle of \( \alpha/2 = 30^\circ \), Eq.5. can be written as

\[
e_{\alpha/2} = 3.4641016 \ (d_D - d_0),
\]

where \( d_0 \) is calculated using Eq. 3.

The contribution of the flank angle to the uncertainty of pitch diameter measurement can be calculated for each pitch using Eq. 7. Results are given in Fig. 6.
If wires with diameters close to the best-size diameters are used (ISO 16239 [12] or ASME 89.1.17 [11]), the uncertainty contribution of flank angle will be almost negligible, as shown in Fig. 7.

**Fig. 6** Uncertainty contribution of flank angle to the pitch diameter calibration using the Zeiss dimensions of measuring wires

**Fig. 7** Uncertainty contribution of flank angle to the pitch diameter calibration using the ISO/ASME dimensions of measuring wires

8. **Conclusion**

Although the ISO, ASME, and DIN standards and some technical instructions issued by leading metrology organisations recommend measuring wires (spheres) with diameters close to the best-size diameter, calibration laboratories and even some NMIs use measuring elements specified by the Zeiss instructions.

Typical measurement uncertainties of the pitch diameter of threaded gauges are at the level of only a few micrometres. This uncertainty level can be significantly increased due to the contribution of thread angle error. In this paper, an analysis of the thread angle error contribution is made for the metric thread gauge plugs. If the selected measuring wires have diameters that are close to the best-size diameter, this contribution will not exceed 10 nm and, therefore, can be neglected, as shown in Fig. 7. On the other hand, if the size of measuring wires is selected according to the Zeiss instructions, this contribution can be significant, as shown in Fig. 6. When calibrating threaded gauges with pitches of 2 mm, 2.5 mm, 3 mm, 4 mm, 5 mm, 6 mm, and 8 mm, the uncertainty level of pitch diameter measurements can increase up to 3 μm.
This paper aims to draw attention to critical points in thread gauge metrology that can significantly increase measurement uncertainty. Laboratories that use measuring wires or two-ball stylus with sizes recommended by the Zeiss instructions should be aware of the fact that a significant increase in measurement uncertainty can occur. If the requirements for the calibration of threaded gauges are such that high-level measurement uncertainty is required, it is necessary to calibrate the thread angle or use wire sizes with diameters close to the best-size diameter. Finally, only in the case of critical threads, measuring elements recommended by the ISO 16239 [12] or the ASME 89.1.17 [11] standard can be added to the standard set of Zeiss measuring wires (two-ball stylus).

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