Tool`s profiling for rotational volumetric deformation - analytical study

G A Costin, N Baroiu, V G Teodor, V Paunoiu and N Oancea
“Dunarea de Jos” University of Galati, Department of Manufacturing Engineering, Domneasca street, no. 111, Galati, Romania
E-mail: nicusor.baroiu@ugal.ro

Abstract. The paper develops in an analytical form an algorithm for the profiling of the active elements of the upper half-die forming which generates by means of plastic rotary volumetric deformation of the frontal teeth. The half die has the revolving axis inclined with respect to the axis of the deformed blank, which rotates around its own axis of symmetry. In the generation process, the generating half die also performs a movement of the blank. The constitutive surfaces of the tooth flanks to be generated are complex surfaces, formed of circular arcs with variable radius and disposed on a conical base surface, on the same axis with the half-die axis. Such teeth are used for front blades in the textile industry or drilling machines which performs, besides the rotational movement of the tool and an alternate rectilinear motion thereof, creating a percussion effect on the work piece. The surfaces of the upper half and the half die teeth may be regarded as reciprocal wrapping surfaces, so that the entire generation process is based on the analytical principles of surface winding. Starting from the constructive shape of the frontal tooth plate, in a classical scheme of the structure of the oscillating motion mechanism of the upper half matrix, the shape of the deformation teeth is determined based on a specific algorithm, based on the general theory of surface winding, and dedicated software products for it.

1. Introduction
The production of cold pressed steel pieces is widely used, even since the last decades of the twentieth century [2]. There are restrictions imposed by the qualities of the deformed material [7], the shape of the piece subjected to deformation processes, related to the type of deformation machine used [4], as well as the level of knowledge and analysis of metal deformation [3], [5], [6], [8], [9], which restricted the applicability domain of specific processes [1].

The types of specialized equipments, on which industrial applications can be developed, as well as the specific constructions of the tools used, constitutes continuous concerns, in the profile industries, in all the developed economies [1].

In the paper, an analysis is made based on which a dedicated algorithm is generated for profiling the shape of the generating tool when processing by volumetric plastic deformation, for a wheel with front teeth, bordered by conical surfaces. The constitutive surfaces of the flanks of the teeth to be generated were geometrically modeled and, based on the generating kinematics, the surfaces families generated in the processing process were determined, whose envelope is the composed surface of the flanks of the tool teeth.
2. Cam geometry with front conical flanks

Figure 2 shows the geometry of the teeth with a composed profile consisting of circle arcs and conical flanks.

It is considered that the flanks of the conical bulbs have as directors, circle arcs of radii $R$, respectively $r$, with centers in $O_e$ and $O_i$. The generators of the conical flanks are rectilinear, forming conical surfaces with the peak at the point $O_e$ a point on the symmetry axis of the wheel with conical bulbs. The radius on which the centers of the directing circles of the conical bulbs are located is defined as $R_o$.

Reference systems are defined, figure 2:
- $XYZ$ – system associated with the front cam axis with the $Z$ axis, given by the $OO_e$ direction, right reference system, associated with the $AB$ profile of the front bulb;
- $X_iY_iZ_i$ – system associated with the bottom, the $BC$ arc of center $O_i$; the $Z_i$ axis is defined by the $OO_i$ direction, right system.

It is noted with $\delta$, respectively $\delta_i$ the angles formed by the generators of the conical surfaces $OM$, respectively $Om$ with the directions $OO_e$, $OO_i$. If it is noted with $\theta$, respectively $\theta_i$ the angles that define on the circles of centers $O_e$, $O_i$ the current point on the circle $AB$, respectively $BC$, then, the conical flanks equations of the front teeth bulbs can be written in the forms:

$AB$ arc of $O_i$ center, see figure 2

\[
\begin{align*}
X &= -u \cdot \sin \delta \cdot \sin \theta; \\
Y &= u \cdot \sin \delta \cdot \cos \theta; \\
Z &= u \cdot \cos \delta,
\end{align*}
\]  
(1)

with $\theta$ and $u$ independent variables ($\tan \delta = R / R_i$).

Also, the conical surface that constitutes the bottom of the front tooth, the $BC$ arc, the circle of radius $r$, with the center in $O_i$, is:

\[
\begin{align*}
X_i &= -u_i \cdot \sin \delta_i \cdot \sin \theta_i; \\
Y_i &= u_i \cdot \sin \delta_i \cdot \cos \theta_i; \\
Z_i &= u_i \cdot \cos \delta_i,
\end{align*}
\]  
(2)

with $\theta_i$ and $u_i$ independent variables ($\tan \delta_i = r / R_i$).

**Note:** The centers $O_e$ and $O_i$ are defined as points belonging to the circle of radius $R_o$ with the center in $O$; the plane determined by $O, O_e, O_i$ is the reference plane of the cam with front bulbs.

The coordinates transformation $X_iY_iZ_i \rightarrow XYZ$, see also figure 2, is given by the form:

\[
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix},
\]  
(3)

where the $X_i$ matrix has the meaning given by (2) and

\[
\alpha = \delta + \delta_i.
\]  
(4)

Thus, the two conical surfaces can be defined in the same coordinate system, $XYZ$:

\[
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix} \begin{bmatrix} -u_i \cdot \sin \delta_i \cdot \sin \theta_i \\ u_i \cdot \sin \delta_i \cdot \cos \theta_i \\ u_i \cdot \cos \delta_i \end{bmatrix},
\]  
(5)

or, obviously, in developed form:

\[
\begin{align*}
X &= -u_i \cdot \sin \delta_i \cdot \sin \theta_i; \\
Y &= u_i \cdot \sin \delta_i \cdot \cos \theta_i; \\
Z &= -u_i \cdot \cos \delta_i \cdot \sin \theta_i.
\end{align*}
\]  
(6)

The circular pitch of the teeth on the circle of radius $R_o$ is defined as follows:

\[
P_c = 2 \cdot R_o \cdot (\delta + \delta_i).
\]  
(7)
The maximum values of the linear parameters, \( u \) and \( u_i \), measured along the generators of the conical surfaces are:

\[
\begin{align*}
    u_{\text{max}} &= \sqrt{R_o^2 + R^2}, \quad \text{for the } AB \text{ lobe;} \\
    u_{i\text{max}} &= \sqrt{R_o^2 + r^2}, \quad \text{for the conical surface at the bottom of the teeth, } BC. 
\end{align*}
\]

(8) (9)

The sizes \( R, r \) are constructive sizes.

The minimum value of the parameters \( u, u_i \) is defined for constructive considerations.

The variation limits of the parameters \( \theta \), respectively \( \theta_i \) are:

\[
\begin{align*}
    \theta_{\text{min}} &= -\frac{\pi}{2}; \quad \theta_{\text{max}} = 0; \\
    \theta_{i\text{max}} &= \frac{\pi}{2}; \quad \theta_{i\text{max}} = \pi. 
\end{align*}
\]

Correlations are also defined, see figure 2:

\[
R = R_0 \cdot \sin \delta; \quad r = R_0 \cdot \sin \delta_i. \quad (11)
\]

If it defined with \( Z_{\text{teeth}} \), the number of teeth (\( Z_{\text{teeth}} \) - integer), then there is the relation:

\[
2\pi \cdot R_0 = Z_{\text{teeth}} \cdot P_C \quad \text{sau} \quad P_C = \frac{2\pi \cdot R_0}{Z_{\text{teeth}}}. \quad (12)
\]

3. The generating tool by volumetric deformation

Figure 3 shows the relative position of the reference systems associated with the cam with front conical flanks, \( XYZ \), see also figure 1 and the reference system of the volumetric deformation tool, \( X_2Y_2Z_2 \) (\( Z_2 \) - rotation axis).

The process kinematics is also presented:

- I – rotational movement of the blank around its own axis \( (Y) \), of angle \( \phi_i \);
- II – rotational movement of the volumetric deformation tool around the \( Z_2 \) axis, of angle \( \phi_2 \);
- III – radial feed movement.

The sizes are constructively defined:

- the rolling radius of the volumetric deformation tool, \( R_S \), in correlation with the circular pitch of the teeth, see (12) and the number of tool teeth, \( Z_S \) (integer, constructive value);
- the inclination angle of the rotation axis of the volumetric deformation tool in relation with the reference frontal plane of the generated wheel (the frontal plane of the circle of radius \( R_0 \), the angle \( \tau \), see figure 3).
The reference systems are defined:
- $x_1y_1z_1$ – fixed reference system, $y_1$ axis superimposed on the rotation axis of the blank;
- $x_2y_2z_2$ – fixed system, $z_2$ axis superimposed on the rotation axis of the volumetric deformation tool;
- $X_1Y_1Z_1$ – mobile system associated with the blank;
- $X_2Y_2Z_2$ – mobile system associated with the deforming tool; the $Z_2$ axis is the rotation axis of the tool.

The process kinematics includes:
- the rotation of the $X_1Y_1Z_1$ system and, joined with it, of the blank:
  \[ x_1 = \omega_1^y (\phi_1) \cdot X_1; \]  
  \[ \omega_1^y \] has the form:
  \[
  \delta = \begin{pmatrix}
  1 & 0 & 0 \\
  0 & \cos \tau & \sin \tau \\
  0 & \sin \tau & \cos \tau
  \end{pmatrix}
  \]  
  \[ \omega_1^y \cdot \delta \cdot \omega_2^y (\phi_1) \cdot X_1 \]  
- the rotation of the $X_2Y_2Z_2$ system joined with the volumetric deformation tool:
  \[ x_2 = \omega_2^y (\phi_2) \cdot X_2; \]  
- the relative position of the fixed systems:
  \[ x_2 = \tau \cdot x_1. \]

Thus, the relative movement of the space $X_1Y_1Z_1$ towards to $X_2Y_2Z_2$, meaning the space movement of the blank to be generated, in relation to the reference system of the volumetric deformation tool, $X_2Y_2Z_2$, is:

\[ X_2 = \omega_3 (\phi_3) \cdot \delta \cdot \omega_2^y (\phi_1) \cdot X_1 \]  
or, developed:
In the transformation (18), the $(X)$ matrix has successively the meanings given by (1) and (6), for the two conical surfaces defined by the directors of the surfaces $AB$, respectively $BC$ and with the common peak of the conical slopes, point $O$, see figure 1.

Following the development of the matrix product (18), it reached at the forms:

$$
X = \left[ \cos \phi \cdot \cos \phi_1 - \sin \phi \cdot \sin \phi_1 \cdot \sin \tau \cdot \sin \phi_1 \right] \cdot X_1 - \sin \phi \cdot \cos \tau \cdot Y_1 + \left[ \sin \phi_1 \cdot \cos \phi_2 + \sin \phi_2 \cdot \sin \tau \cdot \cos \phi_1 \right] \cdot Z_1;
$$

$$
Y = \left[ \cos \phi \cdot \sin \phi_2 + \cos \phi_2 \cdot \sin \phi_1 \cdot \sin \tau \cdot \sin \phi_1 \right] \cdot X_1 + \cos \phi_2 \cdot \cos \tau \cdot Y_1 + \left[ \sin \phi_1 \cdot \sin \phi_2 - \cos \phi_2 \cdot \sin \tau \cdot \cos \phi_1 \right] \cdot Z_1;
$$

$$
Z = -\cos \tau \cdot \sin \phi_1 \cdot X_1 + \sin \tau \cdot Y_1 + \cos \tau \cdot \cos \phi_1 \cdot Z_1.
$$

In further developments, it is accepted that the matrices formed with the coordinates of the two conical surfaces defined by (1) respectively (6), satisfy the condition:

$$
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_{AB, BC} = \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_{AB, BC}
$$

Equations (19), with (1), respectively (2), represent the family of surfaces generated by the blank flanks in relation to the reference system of the volumetric deformation tool.

The envelope of the composed surface family $\Sigma_{AB, BC}$ (19), in which, successively, $\Sigma_{AB}$ depends on the independent variable parameters $\theta$ and $u$ and also $\Sigma_{BC}$ depends on the parameters $\theta_1$ and $u_1$, is obtained by associating the surfaces family (19), (1), for the arc $AB$ and (19), (6) for the arc $BC$, see figure 2, the enwrapping conditions:

- for $AB$:

$$
\begin{bmatrix}
X_{2\theta} \\
Y_{2\theta} \\
Z_{2\theta}
\end{bmatrix} - \begin{bmatrix}
X_{2u} \\
Y_{2u} \\
Z_{2u}
\end{bmatrix} = \begin{bmatrix}
X_{2\theta} \\
Y_{2\theta} \\
Z_{2\theta}
\end{bmatrix} = 0;
$$

- for $BC$:

$$
\begin{bmatrix}
X_{2\theta} \\
Y_{2\theta} \\
Z_{2\theta}
\end{bmatrix} - \begin{bmatrix}
X_{2u} \\
Y_{2u} \\
Z_{2u}
\end{bmatrix} = \begin{bmatrix}
X_{2\theta} \\
Y_{2\theta} \\
Z_{2\theta}
\end{bmatrix} = 0.
$$

The partial derivatives from (1) are:
\[ \dot{X}_{2u} = -\sin \delta \cdot \sin \theta; \]
\[ \dot{Y}_{2u} = \sin \delta \cdot \cos \theta; \]
\[ \dot{Z}_{2u} = \cos \delta; \] (23)

and also,
\[ \dot{X}_{2p} = \dot{X}_{1p} = -u \cdot \sin \delta \cdot \cos \theta; \]
\[ \dot{Y}_{2p} = \dot{Y}_{1p} = -u \cdot \sin \delta \cdot \sin \theta; \]
\[ \dot{Z}_{2p} = \dot{Z}_{1p} = 0. \] (24)

Also, for the conical surface that has as directories the \( BC \) arc see also (6), it results:
\[ \dot{X}_{2\theta} = -u \cdot \sin \delta \cdot \cos \theta \cdot \cos \alpha; \]
\[ \dot{X}_{2\theta u} = -\sin \delta \cdot \sin \theta \cdot \cos \alpha + \cos \delta \cdot \sin \alpha; \]
\[ \dot{Y}_{2\theta} = -u \cdot \sin \delta \cdot \sin \theta; \]
\[ \dot{Y}_{2\theta u} = \sin \delta \cdot \cos \theta; \]
\[ \dot{Z}_{2\theta} = u \cdot \sin \delta \cdot \cos \theta \cdot \sin \alpha \]
\[ \dot{Z}_{2\theta u} = \sin \delta \cdot \sin \theta \cdot \sin \alpha + \cos \delta \cdot \cos \alpha. \] (25)

The partial derivatives of the surface family (19) are also defined in relation with the parameter \( \phi_i \):
\[ \dot{X}_{2\phi} = \left[ -\sin \phi_1 \cdot \cos \phi_2 - \cos \phi_1 \cdot \sin \phi_2 \cdot \frac{d\phi_1}{d\phi_i} - \frac{d\phi_2}{d\phi_i} \cdot \cos \phi_2 \cdot \sin \theta \cdot \sin \phi_i - \sin \phi_2 \cdot \sin \tau \cdot \cos \phi_i \right] \cdot X_i - \]
\[ -\cos \phi_2 \cdot \frac{d\phi_2}{d\phi_i} \cdot \cos \tau \cdot Y_i + \]
\[ + \left[ \cos \phi_1 \cdot \cos \phi_2 - \frac{d\phi_1}{d\phi_i} \cdot \sin \phi_1 \cdot \sin \phi_2 + \frac{d\phi_2}{d\phi_i} \cdot \cos \phi_2 \cdot \sin \tau \cdot \cos \phi_i - \sin \phi_2 \cdot \sin \tau \cdot \cos \phi_i \right] \cdot Z_i; \] (26)

\[ \dot{Y}_{2\phi} = \left[ -\sin \phi_1 \cdot \sin \phi_2 + \frac{d\phi_1}{d\phi_i} \cdot \cos \phi_1 \cdot \cos \phi_2 - \frac{d\phi_2}{d\phi_i} \cdot \sin \phi_2 \cdot \sin \theta \cdot \sin \phi_i + \cos \phi_2 \cdot \sin \tau \cdot \cos \phi_i \right] \cdot X_i - \]
\[ -\frac{d\phi_2}{d\phi_i} \cdot \sin \phi_2 \cdot \cos \tau \cdot Y_i + \]
\[ + \left[ \cos \phi_1 \cdot \sin \phi_2 + \frac{d\phi_1}{d\phi_i} \cdot \cos \phi_1 \cdot \cos \phi_2 + \frac{d\phi_2}{d\phi_i} \cdot \sin \phi_2 \cdot \sin \tau \cdot \cos \phi_i + \cos \phi_2 \cdot \sin \tau \cdot \cos \phi_i \right] \cdot Z_i; \]

\[ \dot{Z}_{2\phi} = -\cos \delta \cdot \cos \phi_1 \cdot X_i - \cos \tau \cdot \sin \phi_1 \cdot Z_i. \]

**Note:** In the expressions of the partial derivatives (26), \( X_i, Y_i, Z_i \) have the meanings given, successively, by (1), respectively (6), see also the specification (20).

The size of the radius \( R_s \) of the volumetric deformation tool is defined:
\[ 2\bar{u} \cdot R_s = Z_s \cdot P_C, \] (27)

where:
- \( R_s \) – the rolling radius of the deformation tool, see figure 3;
- \( P_C \) – the circular pitch of the teeth on the radius circle \( R_o \), see (7);
- \( Z_s \) – the number of teeth of the deforming tool (constructive).

The inclination angle of the deforming tool axis, \( \delta \), is defined:
\[ \sin \delta = \frac{R_o}{R_s} \tag{28} \]

\( R_o \) – see figure 2; \( R_s \) – constructive, as the size of wheel with front teeth

For the numerical representation of the surfaces of the deforming tool lobes, the condition \( Z_S = H \) is associated, \( H \) - variable between the limits:

\[
\begin{align*}
H_{\max} &= R_o \cdot \cos \delta; \\
H_{\min} &= u_{\min} \cdot \cos \delta,
\end{align*}
\tag{29}
\]

depending on the searched surface element: \( AB \) or \( BC \).

4. Conclusions

The analytical study was developed for a conical tooth with a circular profile, based on the general analytical theorems (Gohman).

The algorithm allowed the determination of a specific enwrapping condition based on three independent variable parameters.

Obviously, a numerical solution to such a problem involves imagining a specific software product.

5. References

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