QCD hard-scattering and the sign of the spin asymmetry $A_{LL}^\pi$

Barbara Jäger and Marco Stratmann
Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

Stefan Kretzer and Werner Vogelsang
Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA, and
RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA

Recent preliminary PHENIX data are consistent with a negative and sizable longitudinal double-spin asymmetry $A_{LL}^\pi$ for $\pi^0$ production at moderate transverse momentum $p_\perp \sim 1 \div 4$ GeV and central rapidity. By means of a systematic investigation of the relevant degrees of freedom we show that the perturbative QCD framework at leading power in $p_\perp$ produces at best a very small negative asymmetry in this kinematic range.

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INTRODUCTION

The determination of the nucleon’s polarized gluon density is a major goal of current experiments with longitudinally polarized protons at RHIC [1]. It can be accessed through measurement of the spin asymmetries

$$A_{LL} = \frac{d\Delta \sigma}{d\sigma} = \frac{d\sigma^{++} - d\sigma^{-+}}{d\sigma^{++} + d\sigma^{-+}}$$

(1)

for high transverse momentum ($p_\perp$) reactions such as $pp \to \pi X$, jet$X$, $\gamma X$. In Eq. (1), $\sigma^{++}$ ($\sigma^{-+}$) denotes the cross section for scattering of two protons with same (opposite) helicities. High $p_\perp$ implies large momentum transfer, and the cross sections for such reactions may be factorized into long-distance pieces that contain the desired information on the (spin) structure of the nucleon, and short-distance parts that describe the hard interactions of the partons and are amenable to QCD perturbation theory (pQCD). All of the reactions listed above have partonic Born cross sections involving gluons in the initial state and may therefore serve to examine the gluon content of the scattering longitudinally polarized protons.

In this paper, we will consider the spin asymmetry $A_{LL}^\pi$ for high-$p_\perp$ $\pi^0$ production, for which very recently the PHENIX collaboration has presented first preliminary data [2] at center-of-mass (c.m.s.) energy $\sqrt{s} = 200$ GeV and central rapidity. The data are consistent with a sizable (up to a few per cent) negative $A_{LL}^\pi$ in the region $p_\perp \sim 1 \div 4$ GeV. Even though the experimental uncertainties are still large and leave room for a different behavior of $A_{LL}^\pi$, the new data motivate us to entertain the unexpected possibility of $A_{LL}^\pi$ being negative. To our knowledge, there have been no predictions of a substantially negative $A_{LL}^\pi$ in the literature. This is in itself interesting, also because at moderate $p_\perp$ the spin asymmetry $A_{LL}^\pi$ is expected to be particularly sensitive to gluon polarization.

HARD-SCATTERING CALCULATION

We may write the polarized high-$p_\perp$ cross section as

$$\frac{d\Delta \sigma^\pi}{dp_\perp d\eta} = \sum_{a,b,c} \int dx_a \int dx_b \int dz_c \Delta a(x_a, \mu) \Delta b(x_b, \mu) \times \frac{d\Delta \sigma_{ab}^g (p_\perp, \eta, x_a, x_b, z_c, \mu)}{dp_\perp d\eta} D_\pi^c (z_c, \mu) ,$$

(2)

where $\eta$ is the pion’s pseudorapidity. The $\Delta a, \Delta b (a, b = q, \bar{q}, g)$ are the polarized parton densities; for instance,

$$\Delta g(x, \mu) \equiv g_+ (x, \mu) - g_- (x, \mu) ,$$

(3)

(the sign referring to the gluon helicity in a proton of positive helicity) is the polarized gluon distribution. The sum in Eq. (2) is over all partonic channels $a + b \to c + X$, with their associated polarized cross sections $d\Delta \sigma_{ab}^g$. These start at $O(\alpha_s^2)$ in the strong coupling with the QCD tree-level scatterings: (i) $gg \to gg$, (ii) $gg \to q\bar{q}$, (iii) $gg(q\bar{q}) \to q\bar{q}(q\bar{q})$, (iv) $qq \to qq$, $qq \to gg$, $qq \to q\bar{q}$, $q\bar{q} \to q\bar{q}$, $q\bar{q} \to q\bar{q}$. The transition of parton $c$ into the observed $\pi^0$ is described by the (spin-independent) fragmentation function $D_\pi^c$. The functions in Eq. (2) are tied together by their dependence on the factorization/renormalization scale $\mu$ which is of the order of the hard scale $p_\perp$, but not further specified. All next-to-leading order [NLO, $O(\alpha_s^3)$] QCD contributions to polarized parton scattering are known [3]. Corrections to Eq. (2) itself are down by inverse powers of $p_\perp$ and are thus expected to become relevant if $p_\perp$ is not much bigger than typical hadronic mass scales. We neglect such contributions for now, but will briefly return to them later.

To set the stage for our further considerations, Fig. 1 shows NLO predictions for $A_{LL}^\pi$, for various gluon polarizations $\Delta g_q$, all proposed within the framework of the analysis of data from polarized deeply-inelastic scattering (DIS) in [4]. Despite the fact that the $\Delta g_q$’s used in Fig. 1 are all very different from one another, none of the resulting $A_{LL}^\pi$ is negative in the $p_\perp$ region we display.
As a first step in our more detailed analysis of $A_{LL}^\pi$, we discuss the various ingredients to Eq. (2), other than the unknown $\Delta g$ that one hopes to probe: the $\Delta q$ and $\Delta \bar{q}$ distributions, the calculated partonic cross sections, and the fragmentation functions $D_c^\pi$. They all jointly determine the analyzing power for $\Delta g$ provided by $A_{LL}^\pi$.

To a first approximation, the $\Delta q + \Delta \bar{q}$ channel is “known”. $\Delta q + \Delta \bar{q}$ has been probed extensively in polarized DIS. There certainly remains much room for improvement in our knowledge about them, but this does not affect $A_{LL}^\pi$ too strongly. To give one example: for $\pi^0$ production the $q\bar{q}$ channel depends on the sum $\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d}$, so that uncertainties relating to the SU(2) flavor structure of the polarized sea are not relevant.

We now turn to the partonic cross sections and fragmentation functions. We focus on the lowest order (LO) which is expected to capture the qualitatively important points. Among the reactions (i)-(iii) listed above that have gluons in the initial state, process (ii) has a negative partonic spin asymmetry $\hat{a}_{LL} < -1$, while (i) and (iii) both have $\hat{a}_{LL} > 0$.

We therefore exclude that the $gg \rightarrow q\bar{q}$ channel is instrumental in making $A_{LL}^\pi$ negative, and we thus have to investigate possibilities within $\Delta g$ itself, and its involvement in $gg \rightarrow gg$ and $qg \rightarrow qg$ scattering. Given that the polarized scattering cross sections for these reactions are both positive, and that the first process comes roughly with the square of $\Delta g$, it is immediately clear that a sizable negative asymmetry will not be easily obtained. In the next section, we will demonstrate this for the particularly instructive case of rapidity integrated cross sections.

A LOWER BOUND ON $A_{LL}^\pi$

We consider the LO cross section integrated over all rapidities $\eta$. This is not immediately relevant for comparison to experiment, but it does capture the main point we want to make. From Eq. (2), one obtains:

$$\frac{p_T^3}{\Delta \hat{g}} \frac{d\sigma}{d\hat{p}_T} = \sum_{a,b,c} \int_{x_T^2}^{1} dx_a \Delta a(x_a, \mu) \int_{x_T^2}^{1} dx_b \Delta b(x_b, \mu) \times \int_{\sqrt{s}T}^{1} dz_c \frac{d\sigma_{ab}}{dp_\perp}(z_c, \mu),$$

where $z_T^2 \equiv x_T^2/z_a x_a x_b x_T \equiv 2p_T/\sqrt{s}$. It is then convenient to take Mellin moments in $z_T^2$ of the cross section,

$$\Delta \sigma^\pi(N) \equiv \int_0^1 dx_T^2 (x_T^2)^{N-1} \frac{p_T^3}{\Delta \hat{g}} \frac{d\sigma}{d\hat{p}_T}.$$  \hspace{1cm} (5)

One obtains (we suppress the scale $\mu$ from now on):

$$\Delta \sigma^\pi(N) = \sum_{a,b,c} \Delta a^{N+1} \Delta b^{N+1} \Delta a^{\pi N} \Delta b^{\pi N} D_c^{2N+3} D_c^{2N+3},$$  \hspace{1cm} (6)

where the $\Delta a^{\pi N}$ are the $x_T^2$-moments of the partonic cross sections and, as usual, $f^N \equiv \int_0^1 dx x^{N-1} f(x)$ for the parton distribution and fragmentation functions. We now rewrite Eq. (6) in a form that makes the dependence on the moments $\Delta g$ explicit:

$$\Delta \sigma^\pi(N) = (\Delta g^{N+1})^2 A^N + 2\Delta g^{N+1} B^N + C^N.$$  \hspace{1cm} (7)

Here, $A^N$ represents the contributions from $gg \rightarrow gg$ and $gg \rightarrow q\bar{q}$, $B^N$ the ones from $qg \rightarrow qg$, and $C^N$ those from particular analyses of hadron production in $b\bar{b}$ plus (mostly gluon) jet final states, do constrain $D_c^{2N}$ significantly, even at fairly large $x_c$. For example, the $D_c^{2N}$'s in the sets of $[3]$ and $[5]$ are in reasonable agreement with these data, with the one of $[5]$ arguably setting a lower bound on $D_c^{2N}$. Second, elimination of the $gg \rightarrow gg$ channel would also strongly affect the unpolarized cross section and reduce it by up to an order of magnitude at RHIC energies. However, previous PHENIX measurements of the unpolarized cross section for $pp \rightarrow \pi^0 X$ at $\sqrt{s} = 200$ GeV were found to be in excellent agreement with NLO calculations employing the $D_c^{2N}$ of $[3]$.

We therefore exclude that the $gg \rightarrow q\bar{q}$ channel is instrumental in making $A_{LL}^\pi$ negative, and we thus have to investigate possibilities within $\Delta g$ itself, and its involvement in $gg \rightarrow gg$ and $qg \rightarrow qg$ scattering. Given that the polarized scattering cross sections for these reactions are both positive, and that the first process comes roughly with the square of $\Delta g$, it is immediately clear that a sizable negative asymmetry will not be easily obtained. In the next section, we will demonstrate this for the particularly instructive case of rapidity integrated cross sections.

BASIC OBSERVATIONS

As a first step in our more detailed analysis of $A_{LL}^\pi$, we discuss the various ingredients to Eq. (2), other than the unknown $\Delta g$ that one hopes to probe: the $\Delta q$ and $\Delta \bar{q}$ distributions, the calculated partonic cross sections, and the fragmentation functions $D_c^\pi$. They all jointly determine the analyzing power for $\Delta g$ provided by $A_{LL}^\pi$.

To a first approximation, the $\Delta q$, $\Delta \bar{q}$ may be treated as “known”. $\Delta q + \Delta \bar{q}$ has been probed extensively in polarized DIS. There certainly remains much room for improvement in our knowledge about them, but this does not affect $A_{LL}^\pi$ too strongly. To give one example: for $\pi^0$ production the $q\bar{q}$ channel depends on the sum $\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d}$, so that uncertainties relating to the SU(2) flavor structure of the polarized sea are not relevant.

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Also, for the $p_\perp$ region of interest, one has an average $\langle z_c \rangle \sim 0.3 \div 0.4$, where quarks are more favored than gluons to fragment into pions. A first guess is, then, to attribute a negative $A_{LL}^\pi$ to the negative $gg \rightarrow q\bar{q}$ cross section. However, this expectation is refuted by the numerical hierarchy in the partonic cross sections: at $\hat{\eta} = 0$ in the partonic c.m.s., which is most relevant for the PHENIX data, channel (i) is (in absolute magnitude) larger than (ii) by a factor of about 160. Therefore, if one wanted to suppress the (positive) $gg \rightarrow gg$ contribution, one would effectively have to switch off the gluon-to-pion fragmentation function $D_g^\pi$. Even though our knowledge of $D_g^\pi$ is incomplete, this does not appear to be a sensible solution, for two reasons. First, data from $e^+e^-$ collisions, in particular:

![Graph](image_url)
the (anti)quark scatterings (iv) above; in each case, the appropriate combinations of $\Delta q$, $\Delta \bar{q}$ distributions and fragmentation functions are included.

Being a quadratic form in $\Delta g^{N+1}$, $\Delta \sigma^\pi(N)$ possesses an extremum, given by the condition

$$A^N \Delta g^{N+1} = -B^N.$$  \hspace{1cm} (8)

We note in passing that the same equation may also be derived directly from Eq. (4) by regarding the cross section as a functional of $\Delta g$, using a variational approach, and taking Mellin moments of the resulting stationarity condition. In the following we neglect the contribution from the $gg \to q\bar{q}$ channel which, as we discussed above, is much smaller than that from $gg \to gg$ for the $p_\perp$ we are interested in. The coefficient $A^N$ is then positive, and Eq. (8) describes a minimum of $\Delta \sigma^\pi(N)$, with value

$$\Delta \sigma^\pi(N) \bigg|_{\text{min}} = - \left( B^N \right)^2 / A^N + C^N .$$  \hspace{1cm} (9)

It is then straightforward to perform a numerical Mellin inversion of this minimal cross section:

$$\frac{p_\perp^2 d\Delta \sigma^\pi}{dp_\perp} \bigg|_{\text{min}} = \frac{1}{2\pi i} \int_{\Gamma} dN \left( x_f^2 \right)^{-N} \Delta \sigma^\pi(N) \bigg|_{\text{min}},$$  \hspace{1cm} (10)

where $\Gamma$ denotes a suitable contour in complex-$N$ space.

For the numerical evaluation we use the LO $\Delta q$, $\Delta \bar{q}$ of GRSV [4], the $D_\sigma^\pi$ of [2], and a fixed scale $\mu = 2.5$ GeV. We find that the minimal asymmetry resulting from this exercise is negative indeed, but very small: in the range $p_\perp \sim 1 \div 4$ GeV its absolute value does not exceed $10^{-3}$. The $\Delta g$ in Eq. (9) that minimizes the asymmetry is shown in Fig. 2 compared to $\Delta g$ of the GRSV LO “standard” set [4]. One can see that it has a node and is generally much smaller than the GRSV one, except at large $x$. The node makes it possible to probe the two gluon densities in the $gg$ term at values of $x_a$, $x_b$ where they have different sign, which helps in decreasing $A^\pi_{LL}$.

Even though we have made some approximations in deriving the bound in Eq. (10), we do believe that it exhibits the basic difficulty with a sizable negative $A^\pi_{LL}$ at moderate $p_\perp$: the fact that the cross section is a quadratic form in $\Delta g$ effectively means that it is bounded from below. Note that this bound does not always imply that a negative $A^\pi_{LL}$ is small: at higher $p_\perp$ it does allow a fairly large $A^\pi_{LL}$.

In its details, the qualitative picture drawn by our study is of course subject to a number of corrections. First of all, we have integrated the cross section over all rapidities, whereas for the PHENIX data $|\eta| \leq 0.38$. It is instructive to investigate the qualitative differences associated with this. Figure 3 shows the polarized LO cross section at $p_\perp = 2.5$ GeV versus the “distance” $|x_a - x_b|$ in parton momentum fractions in Eq. (2), for integration over all $\eta$ and for $|\eta| \leq 0.38$. Here we have used the GRSV polarized parton densities. The larger the rapidity range probed, the more likely become collisions of partons with rather different momentum fractions. Indeed, the distribution for $|\eta| \leq 0.38$ is steeper, implying that a node in $\Delta g(x, \mu)$ will now be somewhat less efficient in promoting negative values for the asymmetry. In a more realistic calculation one would also prefer $\mu \sim p_\perp$ to a fixed $\mu$. Furthermore, since the $\Delta q$ and $\Delta \bar{q}$ are coupled to $\Delta g$ via evolution, any change of $\Delta g$ will require a re-tuning of the $\Delta q$, $\Delta \bar{q}$ densities, so that the agreement with the polarized DIS data remains intact. Inclusion of the NLO corrections is important as well.

All these points can be thoroughly addressed only in a “global” NLO analysis of the data, taking into account the results from polarized DIS as well. We will now report on such an analysis. Given that the data are still preliminary, this may seem premature. Our primary goal, however, is to investigate whether the findings of our somewhat idealized case, as summarized by Eqs. (9) and (10), hold true in general.
“GLOBAL” NLO ANALYSIS

The main technical difficulty in a full global NLO analysis of polarized DIS and $pp \to \pi^0X$ data is the numerical complexity of the evaluation of the NLO corrections for the latter cross section. A convenient way to alleviate this problem was presented in \cite{9}. Starting from Eq. (2), one expresses the $\Delta a, \Delta b$ by their Mellin inverses, e.g.,

$$\Delta a(x, \mu) = \frac{1}{2\pi i} \int_{\gamma_N} dN \, x^{-N} \Delta a^N(\mu).$$  
\hspace{1cm} (11)

After interchange of integrations one obtains

$$\frac{d\Delta \sigma^\pi}{dp_\perp d\eta} = \sum_{a,b} \int_{\gamma_N} dN \int_{\gamma_M} dM \, \Delta a^N(\mu) \, \Delta b^M(\mu) \times \rho_{ab}^M(N, M, p_\perp, \eta, \mu),$$  
\hspace{1cm} (12)

where the $\rho_{ab}^\pi$ contain the partonic cross sections, the fragmentation functions, and all integrations over momentum fractions, with the factors $x_a^{-N}$ and $x_b^{-M}$ as complex “dummy” parton distributions according to Eq. (11). The strength of this approach is that there is no dependence of $\rho_{ab}^\pi$ on the moments $\Delta a^N, \Delta b^M$ of the true parton densities. This means that the $\rho_{ab}^\pi$ can be pre-calculated prior to the analysis on a specific array of the two Mellin variables $N$ and $M$. One chooses a convenient functional form for the parton distributions, depending on a set of free parameters. The latter are then determined from a $\chi^2$ minimization procedure. The double inverse Mellin transformation which finally links the parton distributions with the pre-calculated $\rho_{ab}^\pi$ of course still needs to be performed in each step of the fitting procedure, but becomes extremely fast by choosing the values for $N, M$ on the contours $\Gamma_N, \Gamma_M$ simply as the supports for a Gaussian integration.

Following these lines, we have performed a simultaneous analysis of all data from polarized DIS and of the preliminary PHENIX data for $pp \to \pi^0X$. We have used several different functional forms for the polarized gluon density, in particular allowing it to have a node. The quark densities were allowed to vary as well. We have artificially decreased the error bars of the data points for $A_{LL}^\pi$ in order to see whether the fit can be forced to reproduce a negative $A_{LL}^\pi$ of about $\sim -2\%$ in the region $p_\perp \simeq 1/4$ GeV. We have also slightly shifted individual data points to study the response of the fit. For no case have we been able to find a fit that yielded a negative $A_{LL}^\pi$ with absolute value larger than a few times $10^{-3}$. Even those fits, however, gave a negative $A_{LL}^\pi$ only at the higher end of the $p_\perp$ interval, and invariably they led to a polarized gluon density that had a node and tended to violate positivity $|\Delta g| \leq g$ in certain ranges of $x$. The “global” analysis thus confirms our qualitative finding above that any negative $A_{LL}^\pi$ is also very small.

CONCLUSIONS

Our analysis demonstrates that pQCD at leading power in $p_\perp$ predicts that $A_{LL}^\pi$ is bounded from below by $A_{LL}^\pi \gtrsim O(-10^{-3})$ in the region $p_\perp \simeq 1/4$ GeV. The observation relies on collinear factorization and on exploring the physically acceptable ranges of parton distribution and fragmentation functions.

For now, the data do not allow a compelling conclusion on whether the bound is violated or not. What should one conclude if future, more precise, data will indeed confirm a sizable negative $A_{LL}^\pi$? As indicated earlier, corrections to Eq. (2) are associated with power-suppressed contributions to the cross section. Since $p_\perp$ is not too large, such contributions might well be significant. On the other hand, comparisons of unpolarized $\pi^0$ spectra measured at colliders with NLO QCD calculations do not exhibit any compelling trace of non-leading power effects even down to fairly low $p_\perp \gtrsim 1$ GeV, within the uncertainties of the calculation. It is conceivable that the spin-dependent cross section with its fairly tedious cancelations has larger power-suppressed contributions than the unpolarized one. One may attempt to model the effects by implementing an “innocent” transverse-momentum ($k_\perp$) smearing for the initial partons which generically leads to corrections by powers of $\langle k_\perp/p_\perp \rangle$ with $\langle k_\perp \rangle$ an average $k_\perp$. Such effects were shown to have indeed some potential impact on $A_{LL}^\pi$ at $p_\perp \leq 5$ GeV \cite{10}. A negative $A_{LL}^\pi$ would open up a quite unexpected window on aspects of nucleon structure and limitations of pQCD so far little explored.

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