Decay Constants of Heavy Meson of $0^-$ State in Relativistic Salpeter Method

G. Cvetić, C. S. Kim, Guo-Li Wang and Wuk Namgung

1 Dept. of Physics, Univ. Técnica Federico Santa María, Valparaíso, Chile
2 Department of Physics, Yonsei University, Seoul 120-749, Korea
3 Department of Physics, Dongguk University, Seoul 100-715, Korea

ABSTRACT

The decay constants of pseudoscalar heavy mesons of $0^-$ state are computed by means of the relativistic (instantaneous) Salpeter equation. We solved the full Salpeter equation without making any further approximation, such as ignoring the small component wave function. Therefore, our results for the decay constants include the complete relativistic contributions from the light and the heavy quarks. We obtain $F_{D_s} \approx 248 \pm 27$, $F_D \approx 230 \pm 25 (D^0, D^\pm)$, $F_{B_s} \approx 216 \pm 32$, $F_B \approx 196 \pm 29 (B^0, B^\pm)$, $F_{B_c} \approx 322 \pm 42$ and $F_{\eta_c} \approx 292 \pm 25$ MeV.
1 Introduction

The decay constants of mesons are very important quantities. The study of the decay constants has become an interesting topic in recent years, since they provide a direct source of information on the Cabibbo-Kobayashi-Maskawa matrix elements. In the leptonic or nonleptonic weak decays of $B$ or $D$ mesons, the decay constants play an important role. Further, the decay constant plays an essential role in the neutral $D$ – $\bar{D}$ or $B$ – $\bar{B}$ mixing process.

Up until now, the only experimentally obtained values of the decay constants are those of $F_{D^+}$ and $F_{D_s}$. The first value is $F_{D^+} = 300_{-150}^{+180} \pm 80$ MeV by BES [1], with very large uncertainties. The experimental values of $F_{D_s}$ have been obtained from both $D_s \to \mu \nu_{\mu}$ and $D_s \to \tau \nu_{\tau}$ branching fractions by many experimental collaborations (Refs. [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]). They are shown in Table 1. The central values from various experiments range from 194 to 430 MeV. The experimental uncertainties in each experiment are large, even in the most recent measurement, by ALEPH [11] ($F_{D_s} = 285 \pm 19 \pm 40$ MeV), which has the smallest uncertainty. Further, also in ALEPH’s measurement, the contribution from the decay $D_s \to \mu \nu_{\mu} \gamma$ is ignored. Unlike the tree level case which is helicity-suppressed, this radiative decay does not have the helicity suppression. Therefore, this radiative decay may contribute several percent to the branching ratio [12], and may thus cause a sizeable change in the value of the decay constant $F_{D_s}$. Fortunately, new experiments such as Belle, BaBar, Tevatron Run II and CLEO-c will give us a wealth of precision data for $B$ and $D$ mesons soon, and will determine the decay constants to a few percent.

Many theoretical groups are working on the calculation of the decay constants, using different models, for example, lattice QCD, QCD sum rules, and the potential model. In Fig. 1 (taken from Ref. [13]), as an example, the world average of the quenched lattice results for $F_{B_s}$ [14, 15] is shown. From this Figure, we conclude that they give a stable estimate of the decay constant $F_{B_s}$ over several years, but the uncertainties are not small and still remain unchanged over the last several years of work. More precise predictions are still not available. Decay constants of other mesons calculated by lattice methods face the same problem as $F_{B_s}$, the uncertainties are still large. Precise experimental results with uncertainties of only a few percent will be obtained soon. Therefore, advancing the lattice QCD calculations is an urgent task in the future. Calculations with different models are and will continue to be needed, as a means to cross-check the lattice results.
Table 1: Summary of the experimental determinations of the decay constant $F_{D_s}$.

| Ref.         | $F_{D_s}$ (MeV) |
|--------------|-----------------|
| [2] WA75 '93 | 232±45±52       |
| [3] CLEO I '94 | 344±37±52±42   |
| [4] E653 '96 | 194±35±20±14    |
| [5] L3 '97   | 309±58±33±38    |
| [6] DELPHI '97 | 330±95        |
| [7] BES '98  | 430±130±40      |
| [8] CLEO II '98 | 280±19±28±34   |
| [9] BEATRICE '00 | 323±44±12±34  |
| [10] OPAL '01 | 286±44±41      |
| [11] ALEPH '02 | 285±19±40     |

Figure 1: World average of the quenched lattice estimates of $F_{B_s}$.

In this letter, we present results of a relativistic calculation of decay constants in the framework of full Salpeter equation. The full Salpeter equation is a relativistic equation describing a bound state. Since this method has a very solid basis in quantum field theory, it is very good in describing a bound state which is a relativistic system. In a previous paper [16], we solved the instantaneous Bethe-Salpeter equation [17], which is also called full Salpeter equation [18]. After we solved the full Salpeter equation, we obtained the relativistic wave function of the bound state. We used this wave function to calculate the average kinetic energy of the heavy quark inside a heavy meson in $0^-$ state, and obtained values which agree very well with recent experiments. We also found there that the relativistic corrections are quite large and cannot be ignored [16]. In this letter we use this method to predict the values of decay constants of heavy mesons in $0^-$ state.
2 Decay constants of $0^-$ State

In this Section, we will calculate the decay constants of heavy mesons in $0^-$ state by using the full Salpeter method. In the previous paper[16], we wrote the relativistic wave function of $0^-$ state as:

$$\varphi_{1S_0}(\vec{q}) = \left[ P \varphi_1(\vec{q}) + M_H \varphi_2(\vec{q}) - q_\perp \varphi_2(\vec{q}) \frac{M_H(\omega_Q - \omega_q)}{(m_{\omega_Q} + m_{\omega_q})} + q_\perp P \varphi_1(\vec{q}) \frac{(\omega_Q + \omega_q)}{(m_{\omega_Q} + m_{\omega_q})} \right] \gamma_5. \quad (1)$$

where $m_Q$, $m_q$ and $M_H$ are the masses of heavy quark, light quark, and the corresponding heavy meson, respectively; $p_Q$ and $p_q$ are the momenta of the constituent quarks, and $P$ the total momentum of the heavy meson. $q$ is the relative momentum of the meson defined as

$$q \equiv p_Q - \alpha_1 P \equiv \alpha_2 P - p_q,$$

where

$$\alpha_1 = \frac{m_Q}{m_Q + m_q}, \quad \alpha_2 = \frac{m_q}{m_Q + m_q};$$

the $\omega_Q$ and $\omega_q$ are defined as

$$\omega_Q \equiv \sqrt{m_Q^2 - q_\perp^2}, \quad \omega_q \equiv \sqrt{m_q^2 - q_\perp^2},$$

where the orthogonal part $q_\perp$ of momentum $q$ is defined as

$$q^\mu = q_\parallel^\mu + q_\perp^\mu,$$

$$q_\parallel^\mu \equiv (P \cdot q/M_H^2) P^\mu, \quad q_\perp^\mu \equiv q^\mu - q_\parallel^\mu.$$

In the center-of-mass system of the heavy meson, $q_\parallel$ and $q_\perp$ turn out to be the usual components $(q_0, \vec{0})$ and $(0, \vec{q})$, and $\omega_Q = (m_Q^2 + q_\parallel^2)^{1/2}$ and $\omega_q = (m_q^2 + q_\parallel^2)^{1/2}$. Wave functions $\varphi_1(\vec{q})$ and $\varphi_2(\vec{q})$ represent the eigenfunction of the heavy meson obtained by solving the full Salpeter equation. They will fulfill the normalization condition

$$\int \frac{d\vec{q}}{(2\pi)^3} 4\varphi_1(\vec{q})\varphi_2(\vec{q})M_H^2 \left\{ \frac{\omega_Q - \omega_q}{m_Q - m_q}, \frac{m_Q - m_q}{\omega_Q - \omega_q} + \frac{2q^2(\omega_Q m_Q + \omega_q m_q)}{(\omega_Q m_Q + \omega_q m_q)^2} \right\} = 2M_H. \quad (2)$$

Several input parameters are needed when we solve the full Salpeter equation. They are similar to those in the usual potential model. In Ref. [16], we obtained the following best fit values of the input parameters by fitting the mass spectra for heavy mesons of $0^-$ states:

$$a = e = 2.7183, \quad \alpha = 0.06 \text{ GeV}, \quad V_0 = -0.60 \text{ GeV}, \quad \lambda = 0.2 \text{ GeV}^2, \quad \Lambda_{QCD} = 0.26 \text{ GeV} \quad \text{and}$$

$$m_b = 5.224 \text{ GeV}, \quad m_c = 1.7553 \text{ GeV}, \quad m_s = 0.487 \text{ GeV}, \quad m_d = 0.311 \text{ GeV}, \quad m_u = 0.305 \text{ GeV}.$$
With this parameter set, we solved the full Salpeter equation and obtained the eigenvalues and the eigenfunction of the ground heavy $0^{-}$ states. We will not show here how the full Salpeter equation is solved and what the calculated mass spectra are, interested reader can find them in Ref. [16]. We can use the obtained eigenfunction of heavy mesons to calculate the decay constant $F_P$. The decay constant is defined as

$$\langle 0|\bar{q}_1\gamma_\mu \gamma_5 q_2|P\rangle = iF_P P_\mu,$$

which can be written in the language of the Salpeter wave functions as:

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 Q|P\rangle = i\sqrt{N_c} \int Tr \left[ \gamma_\mu(1 - \gamma_5)\varphi_1 S_0(\vec{q}) \right] \frac{d\vec{q}}{(2\pi)^3} = i4\sqrt{N_c}P_\mu \int \frac{d\vec{q}}{(2\pi)^3} \varphi_1(\vec{q}).$$

Therefore, we have

$$F_P = 4\sqrt{N_c} \int \frac{d\vec{q}}{(2\pi)^3} \varphi_1(\vec{q}),$$

and the calculated values of decay constants are displayed in Table 2.

In Table 3, we show the theoretical uncertainties of our results for the decay constants. These uncertainties are obtained by varying all the input parameters simultaneously within 10% of the central values, and taking the largest variation of the decay constant.

Table 2: Decay constants of heavy $0^{-}$ meson (in GeV) as predicted by the relativistic Salpeter method

| $F_{B_c}$ | $F_{B_s}$ | $F_{B_d}$ | $F_{B_s}$ | $F_{B_u}$ | $F_{\eta_c}$ | $F_{D_s}$ | $F_{D_d}$ | $F_{D_u}$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 322       | 216       | 197       | 196       | 292       | 248       | 230       | 230       |

Table 3: The theoretical relative uncertainties, obtained as explained in the text, in per cents (%).

|          | $B_c$ | $B_s$ | $B_d$ | $B_u$ | $\eta_c$ | $D_s$ | $D_d$ | $D_u$ |
|----------|-------|-------|-------|-------|----------|-------|-------|-------|
| $\Delta F_P/F_P$ | $\pm 13$ | $\pm 15$ | $\pm 15$ | $\pm 15$ | $\pm 8.6$ | $\pm 11$ | $\pm 11$ | $\pm 11$ |

In Table 4, for comparison, we show recent theoretical predictions for the decay constants as obtained by other methods. For example, we display the recent values from relativistic potential model (PM) [19] based on the quasi-potential approach; most recent value of $F_B$ from another version of using Bethe-Salpeter method (BS) [20], which is also a relativistic result; recent values from the averaged lattice results both in quenched (AQL) and unquenched (AUL) approximation [15]; most recent values from quenched lattice (QL) QCD [21, 22] and unquenched lattice (UL) QCD [23]; and values from QCD Sum
Table 4: Recent calculations by other methods. Here PM means Potential Model, BS means Bethe-Salpeter method, QL means Quenched Lattice, AQL means Average Quenched Lattice, UL means Unquenched Lattice, AUL means Averaged Unquenched Lattice, QSR means QCD Sum Rules. In Ref. [21], the uncertainties are statistical, systematic within the $N_f = 2$ partially quenched approximation, the systematic errors due to partial quenching and the missing virtual strange quark, and an estimate of the effect of chiral logarithms, respectively. In Ref. [23], the uncertainties are from statistics, chiral extrapolation and systematics.

| Ref. | $F_{B_s}$ | $F_{B_d}$ or $F_{B_u}$ | $F_{D_s}$ | $F_{D_d}$ or $F_{D_u}$ |
|------|-----------|-----------------------|-----------|-----------------------|
| PM[19] | 196±20 | 178±15 | 266±25 | 243±25 |
| BS[20] | | 192 | | |
| QL[21] | 217(6)(+32)(+9)(+17) | 190(7)(+24)(+11)(+8) | 241(5)(+27)(+9)(+5) | 215(6)(+16)(+8)(+4) |
| QL[22] | | | 252±9 | |
| AQL[15] | 200±20 | 173±23 | 230±14 | 203±14 |
| UL[26] | | 190(14)(07)(19) | | |
| AUL[15] | 230±30 | 198±30 | 250±30 | 226±15 |
| QSR[24] | 236±30 | 203±23 | 235±24 | 204±20 |
| QSR[25] | | 206±20 | | 195±20 |

There are other interesting quantities such as the ratios of decay constant $F_{B_s}/F_{B_d}$, $F_{D_s}/F_{D_d}$ and

Table 5: Ratios $F_{B_s}/F_{B_d}$, $F_{D_s}/F_{D_d}$ and the Grinstein ratio by this work and by other methods. In Ref. [26], the first and second uncertainty are the statistical and the systematic errors.

| Ref. | $F_{B_s}/F_{B_d}$ | $F_{D_s}/F_{D_d}$ | R1 |
|------|------------------|------------------|----|
| This work | 1.10±0.01 | 1.08±0.01 | 1.02±0.02 |
| PM[19] | 1.10±0.21 | 1.09±0.22 | 1.01±0.40 |
| QL[21] | 1.16(1)(2)(2)(+4) | 1.14(1)(+2)(3)(1) | 1.02(2)(4)(4)(+4) |
| UL[26] | | | 1.018±0.006±0.010 |
| AQL[15] | 1.15±0.03 | 1.12±0.02 | 1.03±0.05 |
| AUL[15] | 1.16±0.05 | 1.12±0.04 | 1.04±0.08 |
| QSR[27] | 1.16±0.05 | 1.15±0.04 | 1.01±0.08 |

There are other interesting quantities such as the ratios of decay constant $F_{B_s}/F_{B_d}$, $F_{D_s}/F_{D_d}$, and
the Grinstein ratio \[28\] defined as

\[
R_1 = \left( \frac{F_{B_s}}{F_{B_d}} \right) / \left( \frac{F_{D_s}}{F_{D_d}} \right),
\]

which is a quantity sensitive to the light quark mass \( m_s \). In Table 5 we show our values of these ratios and some values obtained by other methods in recent literature. Our uncertainties come from the aforementioned 10 per cent changes of the parameters. The uncertainties of the ratios of decay constants of Ref. \[19\] are large. This is so because the authors of Ref. \[19\] did not give the uncertainties for these ratios. We estimated the uncertainties of these ratios on the basis of their given uncertainties of the decay constants. In the same way we estimated the uncertainties of the Grinstein ratio of other references shown in Table 5, with the exception of those of Ref. \[26\]. From Table 5 one can see that our values of ratios \( F_{B_s}/F_{B_d} \) and \( F_{D_s}/F_{D_d} \) agree with these recent theoretical results. In particular, our central values are very close to those of the relativistic potential model \[19\], and our central value of the Grinstein ratio \( R_1 = 1.02 \) agrees well with the values estimated by other methods.

In conclusion, we calculated the decay constants of heavy \( 0^- \) mesons by means of the relativistic Salpeter method. We obtained \( F_{D_s} \approx 248 \pm 27 \), \( F_D \approx 230 \pm 25 \) \((D^0, D^\pm)\), \( F_{B_s} \approx 216 \pm 32 \), \( F_B \approx 196 \pm 29 \) \((B^0, B^\pm)\), \( F_{B_c} \approx 322 \pm 42 \) and \( F_{\eta_c} \approx 292 \pm 25 \) MeV.

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