Towards Fair Deep Clustering With Multi-State Protected Variables

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Abstract

Fair clustering under the disparate impact doctrine requires that population of each protected group should be approximately equal in every cluster. Previous work investigated a difficult-to-scale pre-processing step for $k$-center and $k$-median style algorithms for the special case of this problem when the number of protected groups is two. In this work, we consider a more general and practical setting where there can be many protected groups. To this end, we propose Deep Fair Clustering, which learns a discriminative but fair cluster assignment function. The experimental results on three public datasets with different types of protected attribute show that our approach can steadily improve the degree of fairness while only having minor loss in terms of clustering quality.

1. Introduction

Problem Setting. The purpose of clustering is often given as trying to find groups of instances that are similar to each other but dis-similar to others. Common clustering methods include the spectral clustering (Ng et al., 2001; von Luxburg, 2007) that tries to effectively maximize the edges within a sub-graph (clusters), the Louvain method (Blondel et al., 2008) that tries to maximize the number of edges per cluster, and $k$-means style algorithms (Lloyd, 1982; Ostrovsky et al., 2006) that attempt to minimize the distortion.

An alternative more pragmatic view of clustering is to find equivalence classes where every object in the cluster is effectively equivalent to every other object. This is often how clustering is used in practice: customers in the same cluster are marketed/retained the same way, people in the same community are candidates for the same friendship invitations and images in the same group are given similar tags. In most clustering settings the view of the means justifying the ends is sufficient. That is, any clustering is justifiable so long as it better improves the end result of maximizing click through rate, maximizing number of friendships and so on.

Limits of Existing Work. However, when the clusters are of people or other entities deserving moral status (Warren, 1997) then the issue of fairness of the clustering must be considered. The standard way of ensuring fairness is to declare some variables as protected and these variables are not used by the algorithm when forming the clusters but rather are given to the algorithm to ensure the clustering is fair. To our knowledge there is only one theoretical paper on the topic (Chierichetti et al., 2017) which shows a clever way of pre-processing data set into chunks. This step guarantees that when $k$-center and $k$-median algorithms are applied to the chunks they produce fair clusters. However, this pre-processing step is time consuming and experimental results are limited to one thousand or less sized instances. Furthermore, existing work even in the supervised learning explores using only Boolean protected variables such as gender, ethnicity and attempted to balance them so that each cluster/class has equal (within mathematical limits) of males and females, non-whites and whites etc. The area of multi-state protected variables for clustering has not been studied to our knowledge but many protected status variables are in fact not binary. Consider the classic eight tenants of protected status: sex, race, age, disability, color, creed, national origin, or religion. None are now considered binary with each having more than two responses in most forms used to collect data.

Challenges and Notion of Fairoids. The area of fair clustering raises several challenges. A core challenge is that optimizing some measure of good clustering and a good measure of fairness is a challenging concept as the two are fundamentally different. Then the key challenge is to find a formulation that can optimize both objectives simultaneously. In this work, we explore the potential of incorporating fairness consideration in deep clustering models. The core idea of our method is to use deep learning to learn an embedding that achieves two aims: i) separating the instances into clusters and ii) making the composition of those clusters uniform with respect to the protected attribute. This is achieved by exploiting the idea that the instances with the same protected status form a protected group and inherently a “fairoid” (fairness centroid). That is if the protected status is race, there would be a fairoid for each of white,
The contributions of this paper are summarized as follows:

- We try out method in a deep learning setting and demonstrate its effectiveness against a number of classic baselines.

2. Related Work

2.1. Deep Clustering

Recent deep clustering work could be broadly categorized into two-phase and one-phase approaches. The two-phase approaches (Law et al., 2017; Shaham et al., 2018) learn a parameterized mapping function $F_{W} : \mathbb{R}^{D} \rightarrow \mathbb{R}^{d} \ (D \gg d)$ from original data or high-dimensional feature space to a latent space that is good for classical clustering methods like k-means and then apply k-means on that space. Alternatively, one-phase approaches (Xie et al., 2016; Yang et al., 2016) directly learn an inductive assignment function to allocate data points into $K$ clusters. A representative one-phase work is deep embedded clustering (Xie et al., 2016), which trains the network parameters based on self-training strategy. This paradigm has been adopted by others (Yang et al., 2017; Dizaji et al., 2018). In this work, we focus on the fairness issue in one-stage deep clustering, which to our knowledge has not been investigated before.

2.2. Fairness in Machine Learning

Recently, inherent unfairness\textsuperscript{1} and algorithmic bias\textsuperscript{2} in machine learning algorithms have attracted much research interest. There are two main doctrines regarding fairness in machine learning: Group Fairness and Individual Fairness. The group notion, also called statistical parity, requires parity of some specific statistical measure across all protected demographic groups. Considering a automatic college admission system, the admission probabilities of Caucasian, Black, Asian, etc. students should be the same. On the other hand, the individual notion requires pair of individuals with similar protected information should also be treated similarly with respect to a task-specific metric. For example, people with similar health characteristics should receive similar treatments (Dwork et al., 2012). The two main notions of fairness from social science and the law are disparate impact and disparate treatment. Disparate treatment means a decision on individual changes when the protected attribute is changed. For example, a job applicants is given a job offer but it later retracted when their gender is revealed. Disparate impact denotes that the decision outcome disproportionately benefits or hurts users belonging to different protected groups. Fairness in this work always refers to group fairness and disparate impact notions as others have (Chierichetti et al., 2017), in particular the parity we require

\textsuperscript{1}http://fairml.how/tutorial/index.html/
\textsuperscript{2}http://approximatelycorrect.com/2016/11/07/the-foundations-of-algorithmic-bias/
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is given by Equation 3.

Our work is related to supervised fair representation learning (Madras et al., 2018; Zemel et al., 2013), which learns fairer latent representation but only for classification or transfer learning tasks hence is not directly comparable. Our work instead directly imposes fairness requirements on cluster assignment function and centroids. The work most related to ours is (Chierichetti et al., 2017). They study the fair clustering problem when protected attribute is binary (e.g., male/female). However, it is worth noting that a lot of protected attributes could not be represented as binary values, like age, income, etc. Besides, there might be multiple binary protected attributes required to be balanced simultaneously (e.g., in each cluster #Male = #Female and #Single = #Married). In fact, both single and multiple binary protected variables are just the special cases of single multi-state protected variable.

Our paper tries to attempt settings not addressed in earlier work (Chierichetti et al., 2017) namely: i) multiple protected variables and ii) scalable fair clustering. We attempt the fairness in clustering problem for multi-state protected attribute which has \( T \geq 2 \) unique values. Moreover, the approach in (Chierichetti et al., 2017) is not scalable as it utilizes \( O(N^2) \) combinatorial algorithm so that it cannot be applied if \( N \) is large. To this end, we propose a scalable algorithm via mini-batch training. Our approach jointly optimize clustering objective from DEC (Xie et al., 2016) and our novel fairness objective. Potentially, Our fairness objective could also be combined with better topology-preserving clustering objectives like those in (Shaham et al., 2018; Shah & Kolotou, 2018) but clustering accuracy is not our major concern.

3. Definition and Preliminary

Let \( X \in \mathbb{R}^{N \times D} \) denote \( N \) data points with \( D \)-dimension features. The aim of clustering \( C \) is to learn an assignment function \( \alpha : \mathcal{X} \rightarrow \{1, \ldots, K\} \), which allocates \( N \) data points into \( K \) disjoint clusters \( C^1, C^2, \ldots, C^K \) (\( \bigcap_{k=1}^{K} C^k = \emptyset \)). In addition let the protected attribute annotations \( A \in \mathbb{R}^{N} \) be available for fairness consideration. If there are \( T \) unique values in \( A \), the data points \( X \) can be naturally partitioned into \( T \) disjoint protected groups \( G^1, G^2, \ldots, G^T \) (\( \bigcap_{t=1}^{T} G^t = \emptyset \)) by another function \( \chi : \mathcal{X} \rightarrow A \). Here \( A \) is a finite set of unique values in protected attribute, \( A = \{a_1, a_2, \ldots, a_T\} \). When considering fairness in the clustering problem, the aim is to get a fair partition w.r.t. specific protected attribute \( A \) but a useful clustering w.r.t. topology of original data.

Previous Work With Binary Protected Variables. Previous work (Chierichetti et al., 2017) discussed the definition of fair clustering under the special case \( T = 2 \). The “perfectly balanced” cluster \( C^k \) can be defined as follows, which is colloquially that all protected states are equally likely.

\[
P(x \in G^1 | x \in C^k) = P(x \in G^2 | x \in C^k) \tag{1}
\]

They then define the fairness measure called “balance” inspired by \( p\% \)-rule (Saini, 2006). Here \( P(x \in G^t | x \in C^k) = \frac{N^t_k}{|C^k|}, t = 1, 2 \) and \( N^t_k \) is the cardinality of the set \( \{i | x_i \in C^k \land x_i \in G^t\} \).

\[
\text{balance}(C^k) = \min \left( \frac{N^1_k}{N^2_k}, \frac{N^2_k}{N^1_k} \right) \in [0, 1] \tag{2}
\]

Generalization to Multi-State Protected Variables. If we consider a more general setting, that is the protected attribute is not necessarily binary, we can generalize the definition in Formula 1 into the following:

**Definition 3.1 Condition of Fairest Cluster.** According to the notions of group fairness and disparate impact the \( k^{th} \) cluster \( C^k \) is fair if the chance of encountering any protected status is equal. Formally, \( \forall t_1, t_2 \in 1, 2, \ldots, T: \)

\[
P(x \in G^{t_1} | x \in C^k) = P(x \in G^{t_2} | x \in C^k) \tag{3}
\]

**Proposition 3.1** Definition 3.1 is equivalent to the \( k^{th} \) cluster’s histogram of the protected attribute \( h^k \) having a uniform distribution. That is \( h^k \sim U \left( \inf A, \sup A \right) \).

**Proof:** Value \( h^k_t \) of the histogram \( h^k \) on the bin \( a_t \) for \( k^{th} \) cluster is \( h^k_t = \frac{|I^k_t|}{|C^k|} = \frac{\sum P(x_i \in C^k \land x_i \in G^t)}{\sum P(x_i \in C^k)} \), where \( I^k_t = \{i | \chi(x_i) = a_t \land \alpha(x_i) = k\} \). If \( P(x_i \in G^{t_1} | x_i \in C^k) = P(x_i \in G^{t_2} | x_i \in C^k) \) and \( p(x_i \in G^{t_1}), p(x_i \in C^k) \) are determined from clustering \( C \) and protected attribute \( A \), then \( h^k_{t_1} = h^k_{t_2}, \forall t_1, t_2 \in 1, \ldots, T \). This leads to \( h^k = \frac{1}{T} \), concluding the proof.

In the \( k^{th} \) cluster \( C^k \), there exists \( T \) subgroups \( G^1_k, G^2_k, \ldots, G^T_k \subset G^T \) (\( \forall t, |G^i_k| \geq 0 \)) according to protected attribute annotations. One challenging problem is how to measure disparity among those subgroups \( G^1_k, G^2_k, \ldots, G^T_k \). If \( T = 2 \), we can simply compare the ratios \( |G^1_k|/|C^k| \) and \( |G^2_k|/|C^k| \) through either \( p\% \)-rule or Calders-Verwer score (Calders & Verwer, 2010). The former way is utilized in (Chierichetti et al., 2017) to define balance score in Formula 2 while the latter way could be written as \[ P(x \in G^1 | x \in C^k) - P(x \in G^2 | x \in C^k) \].

When protected attribute \( A \) is not binary, how to measure disparity among those subgroups \( G^1_k, G^2_k, \ldots, G^T_k \) becomes more challenging. Based on Proposition 3.1 above, we define fairness measure of \( C^k \) as the discrepancy between real distribution \( \mathcal{H}_k \) that \( h_k \sim \mathcal{H}_k \) and uniform distribution \( \mathcal{U}(\inf A, \sup A) \).
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Definition 3.2 *Earth-mover Distance as a Fairness Measure*. Let the empirical histogram for the $k^{th}$ cluster be $h_k \sim \mathcal{H}_k$ and the optimal histogram to ensure fairness be $u_k \sim \mathcal{U}(\inf A, \sup A)$. Then a fairness measure is simply the distance between them. We utilize the Wasserstein distance $W_p$ as $D_k(\cdot, \cdot)$:

$$D_k(\mathcal{H}_k, \mathcal{U}) = \left( \inf_{J_k \in \mathcal{J}_k} \int \| h_k - u_k \|^p dJ_k \right)^{1/p}$$ (4)

Here $\mathcal{J}_k(\mathcal{H}_k, \mathcal{U})$ is the joint distribution of $\mathcal{H}_k$ and $\mathcal{U}$.

Proposition 3.2 *Equivalence of Our Approach For Two Stated Protected Variables*. When protected attribute is binary ($T = 2$) and Wasserstein distance with $p = 1$ is used, Definition 3.2 degenerates into Calders-Verwer score.

Proof: The Calders-Verwer score of cluster $k$ could be written as $|P(x \in G^1 | x \in C^k) - P(x \in G^2 | x \in C^k)| = C_1 |h_k^1 - h_k^2|$ ($C_1$ is a constant and $h_k^1, h_k^2$ denote values on two bins). $D_k(\mathcal{H}_k, \mathcal{U}) = C_2 \sum_{i=1}^{T} |h_k^i - \frac{1}{2}|$. Considering $h_k^1 h_k^2 \geq 0$ and $h_k^1 + h_k^2 = 1$, our fairness measure in Definition 3.2 and Calders-Verwer score are equivalent if $p = 1$ and $T = 2$, concluding the proof. □

As shown in previous work (Zafar et al., 2017), fairness results measured by Calders-Verwer score (Calders & Verwer, 2010) and $\chi^2$ rule show similar trend. Based on Proposition 3.2, we expect that when protected attribute is binary, fairness measured by our protocol should also show similar trend as “balance” defined in (Chierichetti et al., 2017), which is based on $\chi^2$-rule.

4. Deep Learning Formulation

In this section we outline our deep clustering formulation for fairness. We first begin by explaining why finding centroids that are equi-distant from the fairoids is desirable.

4.1. Overview and Use of Fairoids

In our deep clustering formulation, a latent representation $z = F_W(x)$ and cluster assignment function $\alpha : z \rightarrow \{1, ..., K\}$ are simultaneously learned. Thus the $K$ centroids $\mu_1, \mu_2, ..., \mu_K$ of clusters $C^1, C^2, ..., C^K$ are simply $\mu_k = \mathbb{E}_{\alpha(z_i)=k}[z] = \frac{1}{\sum_{i=1}^{N} \alpha_i(\alpha(z_i)=k)} \sum_{i=1}^{N} z_i \alpha_i(\alpha(z_i)=k)$. Each fairoids $\pi_1, \pi_2, ..., \pi_T$ of protected groups $G^1, G^2, ..., G^T$ is represented as $\pi_t = \mathbb{E}_{\chi(i) = a_t}[z] = \frac{1}{|G^i|} \sum_{i=1}^{N} z_i \chi(i = a_t)$, given the protected attribute annotations $A$.

Consider the fairness requirement in Definition 3.1, which requires that in cluster $C^k$, $P(x \in G^1 | x \in C^k) = P(x \in G^2 | x \in C^k)$. When performing clustering on latent representations, we can re-write it into $P(\chi(i) = t_1 | \alpha(z_i) = k) = P(\chi(i) = t_2 | \alpha(z_i) = k)$ where $\chi$ just returns the protected state of an instance. In deep clustering, the requirement that each cluster should have approximately equal number of each protected status individuals can be achieved by making each cluster centroid to be equi-distant to each and every fairoid. Suppose we have a set instances belonging to the $k^{th}$ cluster $X_k = \{x_i | x_i \in C^k\}$ and set of instances belonging to $t^{th}$ protected group $X_t = \{x_i | x_i \in G^t\}$. Both $X_t$ and $X_k$ are encoded into latent representations $Z_t, Z_k$ via the encoder $F_W : X \rightarrow Z$. Larger overlap between instances of $C^k$ and $G^t$ means the $k^{th}$ cluster is more monochromatic towards protected group $G^t$. The overlap could be computed by Maximum Mean Discrepancy (MMD) (Gretton et al., 2006) between $Z_k \sim U_k$ and $Z_t \sim V_t$:

$$\text{MMD}(U_k, V_t) = \left\| \mathbb{E}_{U_k} [F_W(X_k)] - \mathbb{E}_{V_t} [F_W(X_t)] \right\|_Z$$ (5)

It can be seen that $Z_k = F_W(X_k), Z_t = F_W(X_t)$ and $V_t = \{z \in \mathbb{R}^D | \pi_t = \mathbb{E}_{\chi(i) = a_t}[z] \}$ is the joint distribution of $\mathcal{H}_k$ and $\mathcal{U}$.

4.2. A Deep Fair Clustering Model

Based on the motivation above, we propose an inductive model (Figure 2) to directly learn the mapping function $\alpha$ that transforms X into soft assignment $Q \in \mathbb{R}^N \times K$, which is fairer w.r.t. protected attribute $A \in \mathbb{R}^N$ but still discriminative enough. At the beginning, cluster centroids $M = \{\mu_k\}_{k=1}^K$ and fairoids $\Pi = \{\pi_t\}_{t=1}^T$ can be calculated from input data $X$, given protected attributes $A$, as well as the initial encoding function $f_W$. During the training process, clustering objective and fairness objective are jointly optimized until convergence to refine the network parameter $W$, cluster centroids $M$, and fairoids $\Pi$ for improving clustering performance as well as fairness.

As described in motivation part, we need to model probability $P(\alpha(z) = k)$ for clustering objective and conditional probability $P(\chi(z) = t | \alpha(z) = k)$ for fairness objective. By using student’s t-distribution as kernel, the probabilities above are realized by “soft assignments”: Following (Xie et al., 2016), $q_{ik} = P(\alpha(z_i) = k)$ is written as:

$$q_{ik} = \frac{(1 + \|z_i - \mu_k\|^2 / \alpha) - \alpha_i^2}{\sum_{k'}(1 + \|z_i - \mu_{k'}\|^2 / \alpha) - \alpha_i^2}$$ (7)

In this work, we also model the conditional probability $\phi_{kt} = P(\chi(i) = t | \alpha(z_i) = k)$ based on MMD in Formula 5 and kernelized by student’s t-distribution:

$$\phi_{kt} = \frac{(1 + \|\mu(\alpha, z_i) - \pi_t\|^2 / \alpha) - \alpha_i^2}{\sum_t(1 + \|\mu(\alpha, z_i) - \pi_t\|^2 / \alpha) - \alpha_i^2}$$ (8)

Thus, the requirement in Definition 3.1 could be expressed as $\phi_{kt} = \phi_{kt}, \forall t_1, t_2 \in \{1, ..., T\}$.
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4.3. Self-training: Sharpening or Smoothing?

The crucial point is how to choose self-training targets \( P, \Psi \). The overall objective \( \mathcal{L} \) combines \( \mathcal{L}_{fr} \) with the clustering loss \( \mathcal{L}_{cl} \) defined in (Xie et al., 2016):

\[
\mathcal{L}_{fr} = KL(\Psi \| \Phi) = \sum_{k} \sum_{i} \psi_{kt} \log \frac{\psi_{kt}}{\phi_{kt}} \tag{9}
\]

\[
\mathcal{L} = \mathcal{L}_{cl} + \gamma \mathcal{L}_{fr} = KL(\mathcal{P} || \mathcal{Q}) + \gamma \text{KL}(\Psi || \Phi) \tag{10}
\]

The goal of self-training is to impose various constraints on soft assignments \( Q, \Phi \) for various expected properties.

4.4. Mini-batch Training

We also use stacked denoising autoencoder (SDAE) as base model like previous work (Xie et al., 2016; Yang et al., 2017). The initial cluster centroids \( M \in \mathbb{R}^{N \times d} \) are from \( k \)-means++ on initial latent representation \( Z \). After that, we simultaneously optimize the clustering objective and fairness objective via stochastic gradient descent (SGD).

To enable minibatch training, we need to compute empirical \( \Phi \) based on corresponding centroids \( \mu_{\alpha(z_j)} \in \mathbb{R}^{K \times d} \) in each size-\( n \) minibatch. We calculate \( M = \{ \mu_{\alpha(z_j)} \}_{j=1}^{n} \) from assignment \( Q \in \mathbb{R}^{n \times K} \) and its target \( P \in \mathbb{R}^{n \times K} \), as well as the latent representation \( Z_j \in \mathbb{R}^{n \times d} \) in the \( j^{th} \) minibatch (Bauckhage, 2015):

\[
M_{j} \approx (P_j^T P_j)^{-1} P_j Z_j \tag{12}
\]

4.5. Implementation Details

Regarding network architecture, we follows the design of previous deep autoencoder-based methods with the following dimensions \( D-500-500-2000-d-2000-500-500-D \). The learning rates for pre-training and training stages are 0.1 and 0.01, respectively. During layer-wise pretraining, dropout rate 0.2 is used. All experiments are performed with minibatch size 256. On HAR dataset, we pretrain the stacked denoising autoencoder in greedy layer-wise manner for 150 epochs and global manner for 100 epochs (150 and 75 for Adult dataset, 100 and 50 for D&S dataset). We set the dimension of latent representations \( d \) for three datasets as the number of clusters. There are two hyper-parameters in our work \( \gamma \) in Formula. 9 and \( \beta \) in Formula. 11. In this work we fix \( \beta = 1000 \) (although other values of \( \beta \) might improve performance), \( \gamma \) is the only tunable hyper-parameter in this work. Tuning \( \gamma \) can control the trade-off between two objectives. In Section 5.2.4 we discussed its effects.

Before minibatch training, we store \( M, Q \) and its target \( P \) from \( k \)-means++ initialization and pre-compute \( \Pi, \Phi \) and its target \( \Psi \) according to Formula. 12 and given protected attribute annotations \( A \). During each training epoch, the self-training targets \( P_j, \Psi_j \) in \( j^{th} \) minibatch are directly sampled from \( P \) and \( \Psi \). The overall objective function is optimized with respect to network parameters \( W, \theta \) and cluster centroids \( M \). Our implementation is based on PyTorch library (Paszke et al., 2017). All experimental results are averaged performance of 10 runs with random restarts.

5. Experiments

5.1. Experimental Settings

5.1.1. DATASETS

We evaluate the performance of our proposed model on three datasets with protected attributes. The statistics of
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describe these three datasets are shown in Table 1.

- Human Action Recognition (HAR) (Anguita et al., 2013). This dataset contains 10,299 instances in total with captured action features for 30 participants. There are 6 actions in total which serves as groundtruth for clustering. Identity of 30 persons is used as protected attribute, which is multi-state.

- Adult Income dataset (Adult)\(^3\). It contains 48,842 instances of information describing adults from the 1994 US Census. The groundtruth for clustering is annual income (greater or less than 50). Gender information (Male/Female) is used as protected attribute.

- Daily and Sports Activity (D&S) (Altun et al., 2010). This dataset has 9,120 sensor records of human daily and sports activities. The categorical information such as “Moving on Stairs”, “Playing basketball”, “Rowing”, etc is used as groundtruth for clustering. 8 names of participants are used as protected attribute.

We formulate our train/test data sets by randomly sampling points. Out-of-sample extension is done by allocating each test data point to its nearest centroid.

We evaluate the fairness of 3 deep clustering approaches and compare methods mentioned above, we use the widely used clustering accuracy (ACC) and normalized mutual information (NMI) (Strehl & Ghosh, 2002) metrics. Higher values for both of these metrics indicate better performance.

To evaluate fairness of clustering, we use the protocol defined in Definition. 3.2. We use Wasserstein distance as the distance metric \(D\) named “FWD”, where smaller value denotes better performance. For Adult dataset whose protected attribute is binary, we also evaluate the methods in terms of balance score as described in Equation 2 from previous work (Chierichetti et al., 2017).

| Datasets | Train/Test | Protected Attribute | #cluster |
|----------|------------|---------------------|----------|
| HAR      | 9,099/1,200 | Person Identity (30) | 6        |
| Adult    | 30,165/15,060 | Gender (2) | 2        |
| D&S      | 8,320/800 | Names (8) | 10       |

Table 1. Dataset Statistics. In the third column, (\(-\)) denotes number of unique states in protected attributes.

5.1.2. BASELINES

We compare our method, in terms of both clustering quality and fairness, with our base model DEC (Xie et al., 2016) and other representative deep or shallow clustering approaches. Shallow clustering baselines including centroid-based \(k\)-means++ (Ostrovsky et al., 2006), density-based DBSCAN (Ester et al., 1996), as well as spectral clustering based SEC (Nie et al., 2009). Apart from DEC, we also compare with other strong baselines that use deep autoencoders including Autoencoder + \(k\)-means (AE+KM) and Deep Cluster Network (DCN) (Yang et al., 2017). We do not compare with previous work (Chierichetti et al., 2017) because: (1) it does not work for datasets with multi-state protected attribute; (2) the complexity of their algorithm is \(O(N^3)\) which is not tractable to our datasets; (3) and their codes are not released.

For DBSCAN and \(k\)-means++, we use their implementations of scikit-learn library (Pedregosa et al., 2011). For DCN, SEC and DEC, we utilize the authors’ open-sourced codes. All hyper-parameters in those algorithms are carefully tuned according to instructions in those papers.

5.1.3. MEASURES

To evaluate the clustering quality of our approach and compared methods mentioned above, we use the widely used clustering accuracy (ACC) and normalized mutual information (NMI) (Strehl & Ghosh, 2002) metrics. Higher values for both of these metrics indicate better performance.

In this section, we present our experimental results that answer the following research questions:

- Q1: How fair are existing representative clustering approaches (both deep and non-deep)?
- Q2: Can our proposed model improve accuracy and fairness over its base model DEC and other representative approaches? When the number of clusters \(K\) is increasing, can our approach steadily learn fairer clusters than baseline methods?
- Q3: When the protected attribute is binary, Does our fairness measure show similar trends to the balance metric defined in (Chierichetti et al., 2017)?
- Q4: When changing hyper-parameter \(\gamma\) of our model, what is the trade-off between clustering accuracy and fairness?

5.2. Results and analysis

We evaluate the fairness of 3 deep clustering approaches and 3 non-deep clustering approaches in terms of FWD on HAR, Adult and D&S datasets (Table 2). It can be seen that there is no necessary relation between clustering quality and fairness. Besides, deep clustering with dimensionality reduction is not necessarily fairer than shallow methods which perform clustering on original features, although deep clustering methods tend to achieve higher clustering accuracy. Among shallow clustering approaches, density-based

\(^3\)https://archive.ics.uci.edu/ml/datasets/adult
Table 2. Experimental results (over 10 trials) on HAR, Adult dataset, and D&S data sets. The “Balance” metric is defined in (Chierichetti et al., 2017). Best performance is in bold black. “↑” and “↓” mean that larger value indicates better performance or vice versa.

To verify that if the fairness measured based on our definition has similar trend to definition and measure in (Chierichetti et al., 2017) when $T = 2$, we also utilize the balance score to evaluate the fairness of our clustering result on Adult data set. As seen in the last column in Table 2, our model dramatically improves the balance of the unfairest cluster by 18% compared to DEC. Moreover, the fairness measured by balance score is verified to has the same trend as FWD for all clustering approaches, which has been theoretically pointed out in Proposition 3.2.

5.2.4. Answer 4: Trade-off between Clustering Accuracy and Fairness

Because the fairness loss could be seen as a “fairness” regularization term for clustering loss, we are interested in how the hyper-parameter $\gamma$ controls the trade-off between clustering accuracy and fairness. Thus we testify our model under different values of $\gamma$ ranging from $10^{-2}$ to $10^{3}$ on Adult dataset, the results can be seen in Figure 7. With larger $\gamma$
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Figure 4. Histograms of the unfairest cluster (with largest FWD) across the training stages on HAR dataset.

Figure 5. t-SNE embeddings of resulting centroids and fairoids from deep embedded clustering (DEC) and our approach.

Figure 6. FWD per cluster with increasing number of clusters.

Figure 7. Trade-off between the clustering accuracy and balance of clusters on Adult dataset when hyper-parameter $\gamma$ varies. The black point shows clustering accuracy and FWD of base model DEC and the blue line shows the performance of ours.

6. Conclusion

In this work, we define the fair clustering problem with multi-state protected variable under group fairness and disparate impact doctrines. Based on the definition, we propose a scalable framework to improve fairness of deep clustering. Our model is fairer but less accurate.

We compare our approach with six representative clustering approaches on three publicly available data sets with different types of protected attributes. The experimental results demonstrate our approach can steadily improve fairness. By controlling one hyper-parameter, our approach can provide flexible trade-off between clustering accuracy and fairness.
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