Regulating Eternal Inflation

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Abstract: We present an interpretation of the physics of space-times undergoing eternal inflation by repeated nucleation of bubbles. In many cases the physics can be interpreted in terms of the quantum mechanics of a system with a finite number of states. If this interpretation is correct, the conventional picture of these space-times is misleading.
1. Introduction

Eternal Inflation[1] or The Self-Reproducing Universe[2] is one of the most confusing ideas to come out of the attempt to marry general relativity with quantum field theory. There are two classes of eternally inflating universe. The simplest model in the first class consists of a scalar field with a potential like that in Fig. 1, coupled to gravity. The false minimum has a positive value and we will consider all three possibilities for the sign of the energy density in the true minimum $V_T = \sigma |V_T|$, $\sigma = \pm 1$, 0. Bubbles of true vacuum nucleate inside the false[3], but as long as the false vacuum is dS space, the interior of any bubble is causally disconnected from the causal diamonds of a large
class of observers\(^1\). Additional bubbles can nucleate inside these causal diamonds. It is generally claimed that any observer will eventually experience a tunneling to the true vacuum, but there is much confusion about how to define the late time asymptotics of the system. One draws Penrose diagrams with a future space-like \(I^+\) and asserts that it has a fractal nature, divided between regions that asymptote to \(T\) and \(F\). Depending on one’s choice of finite time slices, one can assert that at late times the overwhelming majority of space is still inflating, whence the name.

The second class of eternally inflating models relies on quantum fluctuations in a slow-roll regime\(^2\). The fluctuations drive the field back up the potential in some regions of space. Our understanding of these models in terms of holographic and thermal physics is much less complete, and we will not refer to them further in this paper.

We will argue that if \(\sigma = \pm 1\) there is another interpretation of the Coleman-DeLuccia\(^3\) tunneling process, in terms of a system with a finite number of states. In this view, there is no fractal Penrose diagram, and no time slicing ambiguity. The details of the physics depend on the sign of \(\sigma\). In the particularly interesting case where \(\sigma\) is negative and \(|V_T| \gg |V_F|\) (both are of course well below the Planck scale) we argue that it is extremely improbable for an observer to actually view the tunneling event before it is destroyed by other processes which take place in dS space. The Crunch of Joy foretold by Coleman and DeLuccia is replaced by a more prosaic, if no less grim, demise. The case \(\sigma = 0\) appears to describe a system with an infinite number of states, but the limit is delicate and will be discussed in more detail below.

The next section of this paper (very briefly) reviews our understanding of the properties of a theory of stable dS space\(^4\). We emphasize the existence of two Hamiltonians, \(H\) and \(P^0\), relevant for describing the physics seen by a single time-like observer. \(P^0\) is useful for the description of a small subset of low (compared to the maximal black hole mass) energy localized states. We then turn to the case of tunneling between two dS spaces. We recapitulate the argument\(^5\) that there is an interpretation of the CDL instanton transition in a Hilbert space with a large but finite number of states. Most of the states resemble the dS vacuum with smaller c.c., but there is a small subsystem which resembles the other dS vacuum. Transitions back and forth occur, and obey the law of detailed balance. Recall that any given observer sees precisely such a sequence

\(^1\)Observer = large localized quantum system with many semi-classical observables. An observer is well described by finite volume, cut-off quantum field theory, with a large volume in cut-off units. The semi-classical observables are averages of local fields over large volumes. By their nature, observers in this sense are always massive, and follow time-like trajectories through space-time.

\(^2\)
of transitions. In our interpretation, eternal inflation is (in this case) described by a Hilbert space for only a single observer. Different, causally disconnected observers, correspond to complementary descriptions of the same Hilbert space, using different time evolution operators\(^2\). The evolution operators of two causally disconnected observers do not commute at any time.

Finally, we argue that this mathematical description is practically fictitious when one dS vacuum has very large radius. There is no operational way to measure the consequences of tunneling. For large dS radius in the true vacuum, nucleation of macroscopic black holes on top of the observer occurs at a rate which is superexponentially\(^3\) faster than tunneling to the other vacuum. Quantum fluctuations of the largest macroscopic apparatus that is allowed in dS space, also set in on time scales superexponentially shorter than the vacuum tunneling rate.

The next section generalizes these observations to the case where \(V_T\) is negative, while \(V_F\) remains positive. When \(V_F \ll |V_T|\), we argue that the nomenclature \(T, F\) is misleading. The system has a finite number of states, most of which resemble the dS vacuum. Transitions to the CDL crunch are rare events, which are extremely unlikely to be observed by local observers. In the opposite limit, most of the states of the system are associated with the Crunch, and we must find a more complete description of this singular space-time before we can assess whether the instanton corresponds to a process in a sensible quantum system.

Finally, we investigate the case where \(V_F > V_T = 0\). This is a singular limit, with an infinite number of states, and we must carefully distinguish which subclass of states we keep in the limiting theory.

The title of this paper is explained by the fact that we replace the infinite fractal web of conventional eternal inflation by a finite system. In the conclusions, the reader will also find a remark that may clarify the sense in which the present work is a regulator for the conventional picture. This remark is based on the observation of [28], that the entropy of dS space allows for of order \((R M_p)^{1/2}\) commuting copies of the field theoretic degrees of freedom in a single horizon volume. In [28] it was suggested that this could be organized into a regulated version of the global coordinate picture of dS space in quantum field theory. The current paper may be providing a regulated version of conventional eternal inflation, in much the same sense.

In addition to this remark, the concluding section discusses implications of our con-

\(^2\)In the case of CDL instanton geometries, one of the causally disconnected observers explores only a factor space of the full Hilbert space, with the complementary factor in a fixed state.

\(^3\)We use the term super-exponential to refer to two processes whose time scales behave as \(e^{(RM_p)^{a_1}}\) where \(a_1 - a_2\) is of order one. When \(RM_p > 10^2\) the longer time is essentially the same number measured in Planck units as it is in units of the shorter time scale.
siderations for the String Landscape, and for the theory of stable dS space. Throughout the paper, we will work in four dimensions and let $M_P$ stand for the Planck mass and $m_P$ the reduced Planck mass.

2. The decay of the dominant vacuum

2.1 Stable dS space

Let us briefly review the description of stable dS space[4], restricting our attention to four space-time dimensions. The system has a finite number of states, close to the exponential of the Gibbons Hawking (GH) entropy, $S_{GH} = \pi R^2 M_P^2$. To describe the physics of a localized static observer, we need two Hamiltonians, $H$ and $P_0$. The first is the “real” Hamiltonian of the system. It has a highly degenerate spectrum, below $T_{dS} = \frac{1}{2\pi R}$, with level density $e^{-S_{GH}}$. Geometrically, these states live within a Planck distance of the cosmological horizon. They constitute the de Sitter vacuum ensemble.

The Poincare Hamiltonian $P_0$ has commutator

$$[H, P_0] \sim \frac{1}{R} P_0. \quad (2.1)$$

Its eigenspaces with eigenvalue $\ll R M_P^2$ are approximately conserved by the time evolution generated by $H$, and correspond to states localized near the origin of static coordinates. For black holes, the Poincare eigenvalue is the mass parameter in the Schwarzschild metric:

$$ds^2 = -(1 - \frac{2M}{M_P^2 r} + \frac{r^2}{R^2}) dt^2 + \frac{dr^2}{(1 - \frac{2M}{M_P^2 r} + \frac{r^2}{R^2})} + r^2 d\Omega^2. \quad (2.2)$$

All of the states with non-zero Poincare eigenvalue have smaller entropy than the dS vacuum, and they decay back to it. In fact, in the semiclassical approximation for the gravitational field, the Poincare eigenvalue is just the entropy deficit of the corresponding eigen-space, relative to the dS vacuum. This surprising connection is necessary for the consistency of the description of dS space as a thermal distribution of Poincare eigenstates, which is a consequence of quantum field theory in curved space-time. It can also be verified by inspection of the black hole entropy formula, for those states which are semiclassical black holes.

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4This claim is based on the covariant entropy bound[7]. R. Bousso first made the hypothesis[8] that the CEB for a causal diamond implied a finite number of “degrees of freedom” for that system. Fischler and one of the authors[9][10], argued that this implied a finite number of quantum states for a stable dS space.
In the limit $R \to \infty$, the quantum theory of dS space approaches a theory appropriate to asymptotically flat space. We get the flat space theory by focussing on localizable states with Poincare eigenvalues $\ll RM_P^2$, and considering approximate scattering amplitudes of those states. The approximate S matrix $S_R$ is only approximately unitary, because scattering takes place in a thermal background whose source is the dS vacuum ensemble. However, it approaches a unitary operator as $R \to \infty$, because the states on the horizon decouple from the localized states and the Gibbons-Hawking temperature goes to zero. This limiting scattering matrix is the full set of observables for the Poincare invariant limiting theory. If the hypothesis of Cosmological Supersymmetry Breaking (CSB) is correct, the limit is in fact super-Poincare invariant. We will find one attractive consequence of this hypothesis in the present paper, but for the most part we will ignore it.

The interactions of localizable states are well approximated by quantum field theory as long as no black holes are formed in scattering. For black holes that are much larger than the Planck scale but much smaller than the horizon radius, the inclusive formulae of Hawking evaporation give a reasonably accurate description and it is unrealistic to hope for a practical calculation which could give more accurate information than these.

The entropy of localized states obeying these constraints is of order $(RM_P^3)^{3/2}$. Consequently, there is a limit to the accuracy of measurements that can in principle be made of $S_R$. It is of order $e^{-c(RM_P)^{3/2}}$, with $c \sim 1$. Another way of saying this is that the pointers of any conceivable measuring apparatus built from these degrees of freedom, will undergo spontaneous quantum jumps between pointer positions, on time scales of order $T_q \equiv e^{c(RM_P)^{3/2}}$ (units are essentially irrelevant for these huge numbers when $RM_P$ is a few orders of magnitude or greater). This time scale is much shorter than the recurrence time[11]. More relevant for our discussion will be that in many cases it is much shorter than the tunneling times between false and true vacua.

We should also note that the probability of an observer making observations on time scales of order $T_q$ is itself extremely small. For example the probability of spontaneous nucleation, at the observer’s position, of a black hole of mass $M$ is just $e^{-2\pi RM}$. For large $RM_P$ the probability of an observer surviving for a time $T_q$ is superexponentially small.

It will be important in the sequel that most of the states of the dS quantum theory decouple from the system when we take the $\Lambda \to 0$ limit. The limiting theory is described by an S matrix, which takes into account only localizable states in a single dS horizon, whose Poincare energy remains finite in the $\Lambda \to 0$ limit.

2.2 dS transitions

The CDL instanton for dS to dS transitions is a compact Euclidean manifold with the
form of an ovoid:

$$ds^2 = dz^2 + \rho^2(z)d\Omega^2,$$

accompanied by a scalar field configuration $\phi(z)$. The $z$ coordinate lives in the interval $[0, L]$ and $\rho(z)$ and $\phi(z)$ vanish at both ends of the interval. The instanton exists unless the maximum of the potential is too flat\(^5\). In general, there will be a finite\(^6\) discrete set of such instanton configurations, but only one where the scalar field makes only a single pass over the maximum separating the two minima, as $z$ ranges over the interval. The field never hits either the false or the true vacuum point. The fact that it does not hit the false vacuum is interpreted physically as the statement that we are describing a thermal system. The instanton represents the most advantageous thermally activated process for jumping over the barrier, rather than simple vacuum tunneling. Mathematically, the same fact is explained by noting that the instanton is compact. It is asymptotic boundary conditions on an infinite $z$ interval, which force the instantons for decay of an asymptotically flat or AdS space to asymptote to the false vacuum.

The Lorentzian continuation of the instanton describes two causally disconnected “bubbles” separated by a static (in appropriate coordinates) wall region, which is typically of microscopic size even if parameters have been tuned to make both dS radii large. Each bubble is an excitation of a de Sitter background, one in the true, and one in the false vacuum. Causal diamonds of observers in these bubbles have finite maximal area, and inside a causal diamond the system quickly reverts to the dS vacuum configuration.

The action of the instanton, $S_I$, is negative and it is converted into a probability by subtraction of the dS action of either the true or false vacua. We thereby compute two transition probabilities

$$P_{T \rightarrow F} = e^{-(S_I - S_T)},$$

and

$$P_{F \rightarrow T} = e^{-(S_I - S_F)}.$$  \(2.5\)

They are related by

$$P_{T \rightarrow F} = e^{-(S_F - S_T)} P_{F \rightarrow T}. \quad (2.6)$$

\(^5\)The interpretation of this is that tunneling occurs for a flat maximum, but the semi-classical approximation breaks down. The system tunnels from a region near one minimum of the potential to a position on the flat maximum, where quantum fluctuations are large, and then rolls off the other end.

\(^6\)In [5] one of the authors (T.B.) made the incorrect claim that there were an infinite number of solutions and that at least one always existed. Many colleagues, including A.Linde, E.Weinberg, and most recently M.Kleban, set him straight on this.
We believe that the paper[20] is probably the first to estimate the inverse transition amplitude from a true dS vacuum to a false one. Recognizing that the absolute value of the Euclidean dS action is equal to dS entropy we find that

\[ P_{T \rightarrow F} = e^{-(\Delta \text{Entropy})} P_{F \rightarrow T}. \]  

(2.7)

This is the degenerate form of the law of detailed balance for systems whose states are all at energies much lower than the temperature. Thus, the CDL analysis is not only consistent with our picture of dS space as a system with a finite number of states. It even confirms the conclusion[4] that the bulk of the states lie at energies below the dS temperature.

There is a class of solutions of the CDL equations, for which the intuition that gravity does not make large contributions to the tunneling amplitude is correct. Usually this is attributed to the fact that the true vacuum bubble size, upon nucleation is much smaller than the dS horizon. We will try to argue that this intuition is incorrect. It is convenient to work with a class of potentials of the form \( \mu^4 v(\phi/M) \), where \( \mu \ll m_P \) and \( \mu \ll M \leq m_P \). \( v \) is a bounded function of its argument, \( x \equiv \phi/M \). Very often, we work with polynomial potentials, motivated by the renormalizability criterion of low energy effective field theory. This is perfectly permissible as long as the processes we study do not force us to explore asymptotically large values of \( \phi/M \). The condition \( \mu \ll M \) is the criterion for validity of the semi-classical approximation: all tunneling probabilities will be very small in this limit. For potentials of this form, and in the absence of gravity, the length scale of variation of the instanton is \( M/\mu^2 \). We will work in dimensionless units, with this parameter being the standard of all length and energy scales. The non-gravitational instanton action is of order \( (M/\mu)^4 \). The parameter \( \epsilon^2 \equiv \frac{M^2}{3\omega_P^2} \) measures the size of the gravitational corrections to the tunneling equations, but we will see that the corrections are singular perturbations.

The metric and scalar field for the instanton have the form

\[ ds^2 = dz^2 + \rho^2(z)d\Omega^2, \]  

(2.8)

\[ \phi = \phi(z). \]  

(2.9)

The metric is that of a lozenge or ovoid, because for dS tunneling the \( z \) interval is compact, \( z \in [0,z_f] \). Define

\[ x \equiv \phi/M, \]  

(2.10)

\[ r \equiv \frac{\mu^2 \rho}{M}, \]  

(2.11)

\[ s \equiv \frac{\mu^2 z}{M}. \]  

(2.12)
and
\[ \epsilon^2 = \frac{M^2}{3m_p^2}. \] (2.13)

Then the CDL equations are
\[ \dot{r}^2 = 1 + \epsilon^2 r^2 \left( \frac{\dot{x}^2}{2} - v(x) \right), \] (2.14)
\[ \ddot{r} = -\epsilon^2 r \left( \dot{x}^2 + v(x) \right), \] (2.15)
\[ \ddot{x} + 3 \frac{\dot{r}}{r} \dot{x} = v'(x). \] (2.16)

Here dots refer to derivatives with respect to \( s \), and Eq. 2.15 is found from Eq. 2.14 and 2.16. We define
\[ E \equiv \frac{1}{2} \dot{x}^2 - v, \] (2.17)
which satisfies
\[ \dot{E} = -3 \frac{\dot{r}}{r} (\dot{x})^2. \] (2.18)

Note that this Euclidean energy decreases when \( r \) increases, and vice versa.

The boundary conditions are that we have a smooth, compact manifold, corresponding to
\[ r(0) = r(s_f) = 0 = \dot{x}(0) = \dot{x}(s_f). \] (2.19)

\( s_f \) is freely chosen to satisfy these conditions. However, the it is more convenient to parametrize the unique boundary condition in terms of \( x(0) \), and let \( s_f \) be determined by the equation \( r(s_f) = 0 \). Later we will write a form of the equations in which \( s \) is eliminated in favor of \( r \).

Note that it is the equation for \( \rho \), which requires these boundary conditions, which, when the false vacuum has positive energy density, are infinitely different from those for non-gravitational instantons. The small parameter \( \epsilon \) is a singular perturbation of the non-gravitational equations. For any non-zero value of \( \epsilon \), and any non-singular solution of the equations, \( \rho \) increases and then decreases back to zero. The easiest way to see this is to note that the equations are identical to the equations for a Lorentzian cosmology with negative cosmological constant, whose generic solution ends in a Big Crunch\(^7\). Here we can avoid the Crunch by exploiting the freedom to choose the value of \( \phi \) at (either one but not both of) the end points. As a consequence, even when one of the dS minima has very small c.c., the solution does not begin or end exactly at the minimum of the potential. Loosely speaking, this is a reflection of the thermal

\(^7\text{But please do not confuse this Big Crunch analogy, with the genuine Big Crunch cosmology which we find in the Lorentzian continuation of the instanton when } v_T < 0.\)
nature of dS space. The CDL instanton reflects both vacuum tunneling and thermal activation. Indeed, given the picture of the static dS Hamiltonian advocated in [4], the two are the same. The dS vacuum is in fact a thermal ensemble of a very dense set of levels, clustered below the dS temperature.

When \( \epsilon \ll 1 \), we see a problem in solving these equations. The interval of \( s \) over which the radius of the ovoid expands and retracts, is of order \( \frac{1}{\epsilon} \), while the natural (Euclidean) time scale of variation of \( x \) is of order 1. Thus, if we are looking for an instanton solution which does not have oscillations of \( x \), we must find a place where \( x \) can sit for an “unnaturally” long time. The only such places are the stationary points of the potential. We will assume that the curvature of \( v \) is of order 1 at all these points. A solution which starts very near the maximum of \( v \) at \( x_H \) will oscillate around the maximum with frequency of order one, and does not describe a single pass instanton.

Suppose we start with a solution which starts\(^8\) with \( x(0) \sim x_F \). We will refer to Figure 2, where \( x_F \) is the leftmost point on the potential, which might be visited by the instanton, and choose \( s = 0 \) to be the coordinates of the leftmost point in the instanton. For \( s \) near zero, we can solve the equations exactly, and find that

\[
x(s) \approx x_F + (x - x_F) \frac{I_1(\omega_F t)}{\omega_F t},
\]

where \( I_1 \) is the Bessel function of imaginary argument, and \( \omega_F \) the curvature of \( v \) near \( x_F \). The solution increases exponentially at large \( \omega t \), where the approximation breaks down. However, we can trust it until the deviation is of order 1. At this point we are a distance of order 1 from \( x_T \), and have a velocity of order 1. Recall that the CDL equations describe a particle moving in the inverse potential \(-v\). The system begins with friction in the \( x \) equation. Anti-friction, which could lead to a singularity, sets in only after a time of order \( 1/\epsilon \) when \( r \) starts to re-contract. Thus, in general, the solution oscillates around \( x_H \), the Hawking-Moss maximum of \( v \) (see Figure 2.). If we want a single pass instanton, we must, for small \( \epsilon \) choose \((x - x_F)\) exponentially small, in order that we do not run away from the minimum before \( r \) expands to its maximum, which is \( r_m = o(\frac{1}{\epsilon}) \), and retracts to \( r = o(1) \).

The Euclidean energy

\[
E = \frac{x^2}{2} - v(x),
\]

starts at \(-V_F\) and stays close to this value until \( r \) hits \( r_m \). Then it begins to increase. If we had instead started very close to \( x_T \) (so that the energy was close to \(-V_T\)) this would have implied a disaster. A non-singular instanton must stop at a point on the potential to the right of \( x_F \) in Figure 2, but a solution stopped at that point has energy \(< -V_T \). Initial conditions very close to \( x_T \) give singular solutions.

\( ^8 \)The oscillating instantons describe transitions with smaller probability than the one which makes a single pass over the maximum of \( v \) at \( x_H \).

\( ^9 \)We can “start” the instanton in the basin of attraction of either \( x_T \) or \( x_F \). These are two different descriptions of the same solution.
However, starting as we have, near $x_F$, there is no problem (assuming the barrier is not too flat) in finding a non-singular solution. Note that although the criteria for the thin wall approximation are not generally satisfied, it remains true, for $\epsilon \ll 1$, that the instanton resembles the false vacuum over most of its volume. Since the solution spends most of its time near $x_F$, the action difference $S_I - S_{dS_F}$ is positive and independent of $\epsilon$ and we get a transition probability of order $e^{-c(M/\mu)^4}$. The exponent is of the order of magnitude we would have expected in a calculation which neglected gravitation. The probability for the inverse transition is, on the other hand, superexponentially suppressed when the true c.c. is exponentially small.

As $\epsilon$ is raised to 1, the initial value $x(0)$ moves further from $x_F$, corresponding to the increasing Gibbons-Hawking temperature. The instanton action is of order $c(m_P/\mu)^4$ with a constant $c$ of order $-1$. The probability of true to false transitions is of order $e^{-d m_P^4}$, with a constant $d$ of order 1. Now suppose we change the potential only in the region closer to $x_F$ than $x(0)$, in such a way that the false dS minimum moves down to negative energy. The instanton we have found is still a solution of the equations for this new potential, and predicts a probability for transition from the small c.c. dS space to a negative energy Big Crunch, which is similarly exponentially suppressed. This is our first indication that the conventional picture of dS to Big Crunch transitions, is flawed. Of course, an advocate of the conventional picture could still contend that there is another instanton, which mediates the transition with higher probability. We will postpone till the next section the rebuttal to this argument.

As a consequence of the law of detailed balance obeyed by $dS \rightarrow dS$ transitions, we proposed in [5][4] a regulated description of eternal inflation. The conventional picture views the false vacuum as dominating the global structure of the universe. It expands much more rapidly than it decays and so volume weighted averages over appropriately chosen spatial slices are dominated by the false vacuum. Inflation is viewed as occurring eternally, “somewhere in the multiverse”. Our picture is very different. The false vacuum is a finite, low entropy subsystem of the dominant true vacuum region, which itself has a finite number of physical states. In the false vacuum
ensemble, most of the degrees of freedom of the universe are frozen into a very special state. This configuration decays relatively rapidly, back to the true vacuum.

The true vacuum on the other hand is essentially stable. For example, once $(R_T M_P)^2 > 10^{23}$ it is no more likely for the true vacuum to jump to the false one than it is for the proverbial gas in a box to spontaneously collect in a corner. Indeed, for large values of $R_T M_P$ it is not clear that there is any operational meaning to these jumps. If we view an observer as a naked world line, then it is true that it will encounter an infinite number of back and forth jumps in the course of its history in dS space. On the other hand, if we insist that an observer be a large localizable system with robust semi-classical observables, then the typical observer will probably be blasted apart by spontaneously produced thermal radiation, swallowed by spontaneously nucleated black holes, or suffer quantum jumps of its pointer variables, an exponentially large number of times before it has to endure a CDL tunneling event.

To conclude, it is our contention that the wildly fluctuating, eternally inflating (at the false Hubble parameter), universe of the conventional description, is a figment of the quantum field theorist’s imagination. The true theory of quantum gravity will view the dS to dS transitions as a more or less conventional thermal description of transitions in a finite system, to and from a low entropy meta-stable state. Note that even within the conventional description, no actual experiment performed by local observers will behave differently than our alternate picture predicts. It is only when we take the God-like view of the field theorist and try to interpret global properties of a multiverse that can never be observed, that we run into confusion.

3. The crunch at the end of the tunnel

We now want to generalize these considerations to the case where the true vacuum has negative c.c. As shown by CDL, the Lorentzian bubble in this case, undergoes a Big Crunch on a time scale determined by the shape of the potential in the region of negative energy density. From the eternal inflation point of view, this is not necessarily a problem. In tunneling from dS space, there is always a second bubble which remains in the dS phase. In the conventional view of eternal inflation, any given observer, with probability one, eventually finds himself in the Big Crunch, but “the overwhelming volume of the multiverse is eternally inflating”. From this point of view, the ratio $r = |V_T|/|V_F|$ does not change the qualitative nature of the physics.

The holographic interpretation of these transitions is quite different, and the ratio $r$ plays a crucial role. We will begin with the regime of large $r$, which is easiest to interpret. We will argue that transition probability from false to true vacuum is, when $R M_P$ goes to infinity, of order $e^{-c(R M_P)^2}$, where $R$ is the radius of the false dS vacuum.
A crucial observation is that within the true vacuum bubble of the CDL instanton, there is a maximal area causal diamond, and the maximal area is of order $\frac{m_P^4}{V_T}$. In making this estimate, we have imagined the same form for the potential as in previous sections: $\mu^4 v(\phi/M)$, where $\mu$ and $M$ are two mass scales with $\mu \ll m_P$ and $M \leq m_P$. $v$ is a function with two non-degenerate local minima. We will later adjust dimensionless parameters in the potential to tune the higher minimum to a very small positive value, while the negative minimum remains at a microphysical scale.

The action of the instanton is negative, because the lozenge has positive curvature, and the Einstein term dominates the matter action as a consequence of the gravitational field equations. Indeed, the Euclidean Einstein equations give

$$m_P^2 (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) = \partial_\mu \phi \partial_\nu \phi - \frac{g_{\mu\nu}}{2} (|\nabla \phi|^2 + V(\phi)), \quad (3.1)$$

which implies

$$m_P^2 R = (\nabla \phi)^2 + 2V, \quad (3.2)$$

and

$$S = - \int \sqrt{g}V = -2\pi^2 \left(\frac{M}{\mu}\right)^4 \int_0^{s_f} [r^3 v], \quad (3.3)$$

for a potential of the form we have described. We will see that the potential is positive over much of the volume of the instanton, so that the action is negative.

It is worth understanding how the solution that describes tunneling of an asymptotically flat space into a negative c.c. region, emerges in the limit of vanishing c.c.. This solution is described by the same differential equations. However, if $v(x_F) = 0$, we see the possibility of a solution where $z$ is non-compact and $r$ asymptotes to infinity like $r = z$. All that is necessary is that $E$ vanishes sufficiently rapidly that $r^2 E \to 0$. In fact, examining the equations in the large $r$ regime, we see that $E \sim e^{-br}$ is the actual behavior. $b$ is proportional to the square root of the curvature of the potential at the false vacuum. For such solutions, the gravitational correction $\epsilon^2 E r^2$ is a uniformly small perturbation. The solution is well approximated by its non-gravitational counterpart. Its action is positive and of order $(\frac{M}{\mu})^4$, with small corrections of order $\epsilon^2$. This answer is very different both in sign and magnitude from the action for any compact instanton. However, we have to recall the subtraction of the action of the false vacuum dS space. An instanton which loiters near the false vacuum, so that it is equal to the false dS space over most of its volume, could have a positive action difference of order $(\frac{M}{\mu})^4$.

Thus, the question we now want to answer is whether, for very small but non-vanishing $v(x_F)$ the instanton converges to the flat space answer. We claimed above that the answer is NO. To see this, begin at $s = 0$ and some point $x(0) = x_I$, in the
basin of attraction of the true vacuum\textsuperscript{10}. Both $\dot{r}$ and $\dot{x}$ are of order one. In general, unless we approach a point where $E$ is very small and $\dot{x}$ can remain small (the only such point is $x_F$), the point at which the expansion of $r$ turns around will occur at $s \sim \frac{1}{\epsilon}$ and $r \sim \frac{1}{\epsilon}$, independent of $v(x_F)$. For such solutions, the maximal radius is of microphysical size, and there is no resemblance to the non-gravitational instanton. A solution with $r_{\text{max}} \sim \frac{1}{\epsilon \sqrt{v(x_F)}}$ can only be obtained by tuning $x_I$ so that one approaches $x_F$ from the right (right and left refer to Figure 2.) with very small velocity, while the radius is still expanding. Friction and the flatness of the potential in the vicinity of $x_F$ will then guarantee that $r$ increases until $\epsilon^2 r^2 v(x_F) \sim 1$. So the turnaround occurs with $x \sim x_F$ and $0 > \dot{x}$ and $|\dot{x}| \ll 1$. But now we may be lost. $r$ begins to decrease, slowly at first, and then at an $o(1)$ rate, so we must wait a long time until $r = 0$ again. $x$ on the other hand, is subject to ever increasing anti-friction. We claim that this will overwhelm the tiny force near the top of the barrier in $-v$. The field will sail past $x_F$ and the solution will be singular.

To be even more explicit, let us note that we can parametrize our solutions by $r_{\text{max}}$, the maximal radius of a cross section of the ovoid, and the value $x_{\text{max}}$ at which this maximum is achieved. The relation

$$\epsilon^2 r_{\text{max}}^2 = \frac{1}{E_{\text{max}}} = \frac{1}{\frac{(\dot{x})^2}{2} - v(x_{\text{max}})},$$

(3.4)
determines the velocity at this point and thus completely determines the solution of the equations. We prefer to start at this point and consider taking the radius to zero separately on the left and right halves of the ovoid. For a single pass instanton, $x$ increases on the right half, and decreases on the left half of the ovoid, as $r$ decreases to zero.

On each branch, we can write the equations for the trajectory $x^\pm(r)$ (+ refers to the left, and − to the right half of the ovoid), as $r$ varies between $r_{\text{max}}$ and 0. They are

$$E_r = -\frac{6}{r}(E + v(x)),$$

(3.5)

$$x^\pm_r = \pm \sqrt{\frac{2(E + v(x))}{1 + \epsilon^2 E r^2}}.$$  

(3.6)

We also have the constraint $E + v(x) \geq 0$. We can solve the first equation to obtain

$$E^\pm = E_{\text{max}} \left(\frac{r_{\text{max}}}{r}\right)^{-6} + 6 \int_r^{r_{\text{max}}} dy \left[y^5 v(x^\pm(y))\right] r^{-6}.$$  

(3.7)

\textsuperscript{10}For this paragraph, and this paragraph only, the Euclidean time runs from the right to the left of the ovoid, starting near $x_T$ and ending near $x_F$ in Figure 2.
This is singular at $r = 0$ unless
\[ E_{\text{max}} = -6r_{\text{max}}^{-6} \int_0^{r_{\text{max}}} dy \left[ y^5 v(x_{\pm}(y)) \right]. \]  
(3.8)

We can then rewrite
\[ E^\pm(r) = -6 \int_0^r dy [y^5 v(x_{\pm}(y))] r^{-6}. \]  
(3.9)

In order to have a geometry which approaches the flat space instanton, we must have $\epsilon^2 r_{\text{max}}^2 = -\frac{1}{E_{\text{max}}} \sim \frac{1}{v(x_F)} \gg 1$. On the other hand, our formula gives
\[ E_{\text{max}} = o(1), \]

unless the instanton loiters near $x_F$ for most of its trajectory. Thus, for large dS radius, this solution must have $x_{\text{max}} \approx x_F$, and of course $x_F < x_{\text{max}} < x_T$. We can certainly find a smooth solution on the left half of the ovoid which has this value of $x_{\text{max}}$, using the same undershoot/overshoot argument that works for the flat space case[12].

However, now we must complete the right half of the ovoid. The equations tell us that near $r_{\text{max}}$, $\frac{dx}{dr}$ is large and positive. Thus, as $r$ is lowered on the right side of the ovoid, $x$ quickly passes to the right of $x_H$. After that it is downhill all the way. From the point of view of the cosmological time $s$, the “universe” is contracting. The resulting anti-frictional force pulls in the same direction as the potential, and we reach infinite negative $E$ in finite time. The curvature of the metric is singular at the same point.

We conclude that no regular geometry can approach the flat space solution. Instead, that solution is gotten by taking the limit of a singular solution of the finite radius equations. The singular half of the large “compact” singular space is thrown away as we take $V(x_F) \to 0$, and we obtain a regular solution with asymptotically Euclidean boundary conditions.

These arguments sound convincing, but are not rigorous. Things are particularly confusing for $\epsilon \ll 1$. In this case, since it takes time of order $1/\epsilon$ for $r$ to expand to its maximum and retract, there are lots of non-singular solutions which oscillate of order $N \sim \frac{1}{\epsilon}$ times around the maximum of $v$ at $x_H$. If $N$ is odd these can be interpreted as CDL tunneling to a true and false vacuum bubble, but there will also be a solution which makes only a single traverse over the maximum. This solution loiters around the false vacuum for a time at least as long as $\frac{1}{\epsilon}$, while $r$ expands to its maximum (also at least of order $\frac{1}{\epsilon}$) and retracts to a value of order 1. It then crosses over to the basin of attraction of $x_F$ in a time of order 1. The question is, if $\epsilon$ is held fixed and parameters tuned so that $V_F$ goes to zero, does the loitering time go like $\frac{1}{\sqrt{V_F}}$, so that the solution looks like the false dS space over most of its volume? Or
does the loitering time asymptote to a constant as $V_F$ goes to zero (and how does the constant depend on $\epsilon$)?

In order to loiter near the false vacuum for a time of order $\frac{1}{\sqrt{V}}$, the value of $x$ at the leftmost zero of $r$ must be exponentially close to $x_F$. Indeed, in this regime we can solve the equations exactly and

$$x(s) = (x(0) - x_F) \frac{I_1(\omega_F s)}{\omega_F s}, \quad (3.10)$$

where $I_1$ is the Bessel function of imaginary argument, $\omega_F$ is the curvature of $v$ at $x_F$.

For application to models of low energy SUSY breaking, we want to tune the false minimum to zero via a formula like

$$v_z(x) \equiv f(x) - (1 + z)f(x_F). \quad (3.11)$$

We claim that as $z \downarrow 0$ $x(0) - x_F$ converges to a non-zero, $\epsilon$ dependent, constant. Unfortunately, we have not been able to come up with a rigorous analytic argument that this is so for this case. Instead, we have resorted to numerical calculations. Results of a preliminary study are reported in the next section. We hope to return to a more detailed numerical investigation of this question in a forthcoming paper[29]

The action for the smooth compact instanton is negative and of order $(\frac{m_P}{\mu})^4$. Thus, as in the case of the $dS \rightarrow dS$ transitions, we must make a subtraction in order to define a probability. For the transition from “false” to “true” vacuum, it is obvious that the correct formula should be

$$P_{F \rightarrow T} \sim e^{-(S_I - S_{dS})}. \quad (3.12)$$

If we are correct that $S_I$ asymptotes to a constant for large de Sitter radius, then this is an incredibly tiny number, essentially the inverse of the recurrence time for the large de Sitter space.

We have gone to great lengths to explain the mathematical behavior of the CDL amplitude for the case $|V_T| \gg V_F > 0$, because we have found so many people who are surprised by the answer. It is worth pointing out however that the result is not new. It was mentioned explicitly in [14] and is at least implicit in many much earlier papers. In particular the paper [13] contains similar estimates and discusses their relation to the entropy of dS space. As far as we know however, there has been no previous attempt to explain the physical basis for the mathematical discontinuity of the result when $V_F = 0$.

There is a simple physical explanation, based on the picture of dS quantum mechanics proposed in [4], of the peculiar mathematical discontinuity of the CDL instanton as the cosmological constant in the “false” vacuum is taken to zero. The flat space limit, in this picture, is obtained by ignoring most of the states of dS space and concentrating
on the low energy localizable states, classified as eigenstates of the operator $P_0$. For finite dS radius, the localized states are unstable, and decay back to the dS vacuum ensemble, but as $R \to \infty$ the lifetime of localizable states becomes infinity and $P_0$ becomes a conserved quantity. The CDL instanton for asymptotically flat space decaying to a Big Crunch describes the properties of the unique zero energy eigenstate of $P_0$.

By contrast, for finite $R$ the vacuum of the theory is a thermal ensemble with (for large $R$) huge entropy. Our mathematical results suggest that rather than a decay, the CDL instanton for this case describes an unlikely fluctuation to a low entropy meta-stable state. We can find further evidence for this interpretation by applying the covariant entropy bound to the Big Crunch portion of the Lorentzian CDL instanton. For the potential we have chosen, it is easy to see that the area in Planck units of the holographic screen of the maximal causal diamond for observers in this portion of the space-time is of order $(\frac{m_P}{\mu})^4$. To see this use a dimensionless time variable, field, and scale factor, with the same scaling factors we used in the Euclidean calculation above. The dimensionless cosmological time to the crunch is of order $\frac{1}{\epsilon}$, as is the value of the dimensionless scale factor at that time. The actual proper time and scale factor are both of order $\frac{m_P}{\mu^2}$. The coordinate distance to the particle horizon scales like:

$$d_H \sim \int \frac{dt}{a(t)} \sim 1.$$  \hfill (3.13)

The area of the holographic screen of the “last observer standing” is thus $m_P^2 a^2 \sim \frac{m_P}{\mu^2}$. Thus, there is reason to believe that the Big Crunch is a low entropy system, compared to the dS vacuum ensemble, when $\mu$ is a microphysical scale but the “false” vacuum energy is tuned to be near zero.

We would thus advocate a calculation of the probability of the reverse tunneling event: Big Crunch to dS of the form,

$$P_{T \to F} \sim e^{-(S_I + \frac{1}{4} A_{BC})},$$  \hfill (3.14)

where $A_{BC}$ is the area, in Planck units of the largest causal diamond in the Crunch region of the CDL instanton. This formula, together with the one we have given for the inverse transition, would satisfy the law of detailed balance if we assumed that the entropy dominates the free energy for both the dS space and Big Crunch. In the case of dS we have already explained that this is consistent with the spectrum of the dS Hamiltonian advocated in [4]. For the Big Crunch space-time there is no conserved energy, and it is not clear what we mean by free energy. However, it is reasonable to assume that the system there maximizes its entropy, without any constraints[15]. The rapid time dependence of the fields will excite all degrees of freedom available to the system.
If this interpretation of the CDL instanton is correct, we again see a complete disagreement with the logic of eternal inflation. Consider observers in the dS and Crunch regions of the CDL diagram, assuming the covariant entropy bound. The heretics\cite{5} which remain in the false vacuum can access a huge amount of entropy and exist for an extremely long time. The true believers by contrast, exist for a microscopically short period and can access much less information.

The conventional phrases “false and true vacua”, and “instability”, seem misleading in this context. The true vacuum is not a stable state in any sense. Inhomogeneous and anisotropic classical perturbations grow as we approach the Crunch. If we tried to do quantum field theory in the interior of the CDL bubble, using a Gaussian wave functional describing small fluctuations around the bubble as the initial “vacuum state” and polynomials in field operators acting on this state as the excitations, then the state near the Crunch is full of excitations. Everything seems to indicate that the system wants to “thermalize” in the sense of exploring the entire Hilbert space of states available to it. The covariant Entropy bound suggests that this space of states is finite dimensional, with relatively small dimension. On the other hand, the smaller the value of the positive c.c., the more the “meta-stable” false vacuum looks like a stable vacuum state. It has a temperature, which goes to zero with the c.c. This means that excitations occur, but with very small probability.

As small as it is, the probability for even outlandish events like nucleation of a galaxy size black hole, is much more probable for small c.c. (dS radius much larger than a galaxy) than the CDL transition to the “true vacuum”. This is easy to understand: the thermal probability for black hole production, $e^{-m/T}$ just reflects the entropy deficit(!) of black hole states relative to states of the empty dS vacuum\cite{4}. The entropy of the “true vacuum” is much smaller than that of large black holes.

When the absolute value of the negative c.c. is much smaller than the positive false vacuum energy, the maximal causal diamond in the Big Crunch space-time is much larger than the dS horizon size. In this case, it is plausible that the situation is reversed and that the ”true vacuum” really is the state in which the system spends most of its time. However, we are much less certain that there is a real model of quantum gravity corresponding to this semi-classical picture. Indeed, quite generally, when we try to understand the quantum meaning of all possible solutions of general relativity coupled to other matter fields, we may be trying to make sense out of utter nonsense. String theory has taught us that theories of quantum gravity are much more restrictive than classical field theory would lead us to believe. We have no solid evidence that a real theory of quantum gravity will produce any of the situations studied in this paper. It should not surprise us if some of them resist interpretation.

In both the situation where $-V_T \gg V_F$ and $-V_T \ll V_F$ the model contains no
immortal observers. However, Observer’s Doomsday is much more immediate in the latter case. The time before the Big Crunch is power law in $|V_T|$. By contrast, processes which destroy typical observers in the case where the “false” dS vacuum is the ground state (according to our criteria) have probabilities that are superexponentially suppressed as the c.c. goes to zero.

Finally we would like to note that, although we have give a satisfactory explanation of the discontinuity in instanton amplitudes as the dS radius goes to infinity, it might be that this is a fact that does not need an explanation. If the conjecture of [9] is correct and the universe necessarily becomes supersymmetric as the c.c. is sent to zero, then in real quantum theories of gravity, Poincare invariant vacuum tunneling to a Big Crunch never occurs.

4. Some numerical results

In the previous section, we put forward a set of arguments which imply that there is no regular instanton which as we take $V(x_F) \to 0$ approaches the flat space solution. We conjectured that there is a regular instanton which interpolates between the wells of the potential, but in the limit where $V(x_F) \to 0$, this solution will have $r_{\text{max}} \sim \frac{1}{\epsilon}$, independent of the value of $V(x_F)$. Unfortunately, our analytics fall short of a proof, and so in this section we demonstrate some examples of this behavior using numerical methods.

The model we use for the potential is:

$$v_z(x) = f(x) - (1 + z) f(x_F), \tag{4.1}$$

with

$$f(x) = \frac{1}{4} x^4 - \frac{b}{3} x^3 - \frac{1}{2} x^2. \tag{4.2}$$

This auxiliary function, $f(x)$, has a local maximum at the origin, and two negative minima. The potential $v_z(x)$ for three representative values of the parameter $b$, which controls the relative height of the two minima, is shown in Fig. 3. Also shown in this figure (see the inset), is an example of the behavior of a potential (with $b = .3$) as $z$ is taken to zero. In this limit, the false vacuum is a de Sitter space with vanishing cosmological constant.

The equations of motion in this potential are given by Eq. 2.14, 2.15, and 2.16. We will use Eq. 2.15 and 2.16 to evolve, with the initial conditions $x(0) = x_I$, $\dot{x}(0) = 0$, $r(0) = 0$, and $\dot{r}(0) = 1$ (from Eq. 2.14). Our goal is to search for solutions which have two zeros of $r$ with $\dot{x} = 0$ at both and $x$ evolving monotonically between these turning
Figure 3: The potential $v_z(x)$ for $b = \{0.5, 0.3, 1\}$ (red, green, blue) with $z = 0.05$. It can be seen that the parameter $b$ controls the relative heights of the true and false vacuum wells. The inset shows the potential with $b = 0.3$ and $z = \{1, 0.5, 0.2, 0.01\}$ (light to dark). As $z \to 0$, the false vacuum approaches a dS space with vanishing cosmological constant.

points. For a given potential, there is possibly one such solution, uniquely determined by the initial position $x_I$. It is this quantity that we are ultimately solving for.

We have implemented this search numerically, and have preliminary results for the regime of large $\epsilon$ (order one)\(^{11}\). As $\epsilon$ is decreased, the time scale between the zeros of $r$ increases, and since the field can only pause near the extrema of the potential, we are forced to introduce an extreme fine tuning in $x_I$ to induce this unnatural behavior. This is the fundamental limitation on the numerics, but there is a regime where the fine tuning in $x_I$ is reasonable (a few orders of magnitude less than the machine precision). Shown in Fig. 4 and 5 is the evolution of $x(s)$ and $r(s)$ for $\epsilon = 0.6$ in the potential Eq. 4.1 with $b = 0.3$ and $z = \{1, 0.3, 0.1, 0.03, 0.003\}$. These are the unique one-pass solutions, and they satisfy all of the criteria outlined above. As we take $z \to 0$, we find that the

\(^{11}\) Further details of the numerics, including a Mathematica notebook containing some sample calculations, can be found at: \url{http://physics.ucsc.edu/~mjohnson/}. 

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trajectories $x(s)$ and $r(s)$ approach a constant profile (the blue curves). The reason for this behavior can be seen in Fig. 6, where we have plotted $x_I$ as a function of $z$ for a few different values of $\epsilon$. In each case, the initial position approaches a constant as we take $z$ to zero, and since it is the initial position $x_I$ that uniquely determines a solution, we should expect to see the trajectories $x(s)$ and $r(s)$ approach a constant profile.

![Figure 4](image-url)  

**Figure 4:** The evolution of $x(s)$ in the potential Eq. 4.1 with $b = .3$ for $z = \{1, .3, .1, .03, .003\}$ and $\epsilon = .6$. The red curve is for $z = 1$ and the blue curve is for $z = .003$. Horizontal dotted lines represent $x_F$ (bottom) and $x_T$ (top).

Another observation we make is that as $\epsilon$ decreases, the initial position (both for low- and high-$z$) approaches $x_T$ (the horizontal dotted line in Fig. 6). The time scale over which $r$ evolves between its two zeros will increase as $\epsilon$ decreases (this can easily be seen in Eq. 2.14 and 2.15). We described in section 2.2 that $x$ can only loiter in the neighborhood of the positive extrema of the potential, and therefore as $\epsilon$ is decreased the final field value will approach the position of the false vacuum minimum. Simultaneously, to avoid under-shooting, we must adjust the initial position closer to the position of the true vacuum minimum. As we observed in section 2.2, this corresponds to a decreasing Gibbons-Hawking temperature in the false vacuum as $\epsilon \to 0$. 
We claimed in section 3 that the volume of the instanton in the false vacuum phase will not increase indefinitely as $V_F \to 0$, and therefore there is no regular instanton which approaches the flat space solution in this limit. The result that $r(s)$ and $x(s)$ approach constant trajectories confirms this conjecture, because the time the solution spends near $x_F$ will become independent of $V_F$ after $z$ goes below some critical value. Calculating the Euclidean action of these solutions, given by Eq. 3.3, we see in Fig. 7 that the instanton action asymptotes to a constant value. This shows that the instanton action does not keep pace with the background subtraction term, which is diverging as we take $V_F \to 0$. We therefore conclude that the probability to transition to a big crunch spacetime, given by Eq. 3.12, will not approach the flat space answer, but instead will exponentially approach zero.

We have explored other potentials, running over the range in $b$ shown in Fig. 3, and found the same qualitative behavior in all cases. We have also found the oscillating bounce solutions [5, 6] in each potential, and explicitly verified that they have a higher total Euclidean action (including the background subtraction) than the one-pass solu-
Figure 6: The initial position of the instanton solution, $x_I$, vs $\log_{10}(z)$ for $0.6 < \epsilon < 0.85$ (red curve is for $\epsilon = 0.6$, violet curve is for $\epsilon = 0.85$) in the potential Eq. 4.1 with $b = 0.3$.

This numerical study will be extended to include smaller values of $\epsilon$ in future work [29], but we feel confident that the preliminary results presented above are quite generic.

5. Tunneling to a state with vanishing vacuum energy

The case of dS space tunneling into a space-time with vanishing c.c. is a limit of the cases we have studied, and like all limits, must be approached with care. We will let $R$ stand for the radius, in Planck units, of the dS space whose c.c. is being taken to zero. Let us recall that in this case, as in that of a true vacuum with negative c.c., it is inaccurate to state that we approach flat space, even if the field asymptotes to a finite constant value at a zero energy minimum of the potential. The local geometry indeed approaches that of flat space-time, but the asymptotics are quite different. The space-time does have a null future conformal boundary, where we can imagine organizing the states of the system as “multi-particle scattering states”. However, it was argued in [16] that generic scattering data extrapolate back, not to the smooth CDL space-time
Figure 7: The Instanton action (normalized to $\left(\frac{M}{\mu}\right)^4$) vs $\log_{10}(z)$ for $0.6 < \epsilon < 0.85$ (red curve is for $\epsilon = 0.6$, violet curve is for $\epsilon = 0.85$.) in the potential Eq. 4.1 with $b = 0.3$.

obtained from analytic continuation of the instanton, but to a space-like singularity. In asymptotically flat space-time (which we assume to be exactly supersymmetric[9]) generic scattering data lead to at worst localized singularities, perhaps hidden behind event horizons (the Cosmic Censorship Hypothesis), which eventually give rise to (or evolved from) other scattering data.

This is to be contrasted with the Lorentzian CDL instanton for $dS \to dS$ transitions. Here also the asymptotic geometry is different from that of dS space, but in that case the global geometry is irrelevant to any observation. Causal diamonds remain finite in area and the geometry inside any causal diamond approaches that of the static patch of dS space at late times. There are no analogs of the catastrophic scattering data in these space-times if one makes the hypothesis that dS space has a finite number of states. That is, in exact global dS space, using the formalism of quantum field theory in curved space-time, we can construct “scattering states” at past (future) null infinity which evolve to future (past) Big Crunch (Bang) singularities. According to the conjecture of [9][10] these scattering data are not supposed to be part of the quantum theory of gravity in dS space, which instead one should imagine to be the quantization of the phase space of gravitational solutions which are globally dS in both the past and
the future. Similarly, in the CDL instanton for dS transitions, we reject perturbations of the future asymptotic data, which extrapolate back to Big Bang singularities. This allows all excitations which can be seen inside a dS causal diamond, but not black holes larger than the dS radius. Thus, the structure of CDL instantons for dS transitions fits well with the hypothesis that dS space has a finite number of states.

Now imagine taking a limit in which one of the positive potential energy densities is taken to zero. Referring again to what happens with the limit of pure dS space to flat space-time, we know that we will have to throw away many of the quantum states of the theory in order to have a sensible limit. The entropy of the dS theory at radius $R$ in Planck units is of order $R^2$, but the Poincare invariant limit keeps only of order $e^{R^{3/2}}$ states. The question is, which states do we keep in a theory of CDL tunneling from dS space to negatively curved FRW spaces with vanishing c.c.?

It seems to us that there are two reasonable hypotheses to make, in answer to this question. The first, which corresponds to what passes for conventional wisdom about this problem, was made in [17] [16]: admit the analog of all the (future) scattering states we would keep in taking the limit of dS space to Minkowski space. The result, if it exists, is a cosmology with an infinite number of states. The initial states would all correspond to a singular Big Bang, and most of them would transition to the final FRW state without passing through an intermediate dS phase.

One should also note the proposal of Susskind and Freivogel [17] in which one attempts to interpret the time symmetric Lorentzian CDL instanton as the arena for a scattering theory. Given the observations of [16] it seems to us that this proposal has properties similar to the Big Bang interpretation. It has the added problem of explaining how the Big Crunch which follows from generic past asymptotic data evolves to the Big Bang which preceded generic future asymptotic data.

We think there may be another interpretation of this space-time which makes sense. For very large but finite $R$ we can construct scattering experiments with greater and greater precision in the causal diamonds of the large radius bubble. Following the prescription of [4], the theory contains an operator $P_0$, whose low lying eigenvalues are approximately conserved. This conservation law gets better and better as $R \to \infty$. We can use it to restrict the possible black holes that are produced in scattering experiments inside the causal diamond, to those whose radius is small enough that they fit inside the CDL geometry without causing a Crunch of the entire space-time. Since $P_0$ becomes a conservation law as $R$ goes to infinity, and the horizon states that give rise to thermal fluctuations decouple, we may expect the S-matrix to be unitary in this subspace. If this were the case, we would have a consistent definition of scattering theory in the time symmetric CDL space-time, involving only a finite number of in and out states. Despite the finiteness of the limiting system we could hope that the scattering matrix
was mathematically well defined. In effect, the huge set of localizable states in the large R limit$^{12}$, would act as classical measuring devices for the finite limiting system defining the theory in the CDL instanton background. We will see below that, from the point of view of the Landscape of String Theory, there are advantages to a definition of the CDL instanton space-time, which has only a finite number of states. Note however that in the stringy context there is no way to consider the non-accelerating FRW cosmologies, which arise from CDL instantons, as limits of situations with positive c.c. The Dine-Seiberg regions of moduli space, where these cosmologies find their home, are non-compact and asymptotically supersymmetric. It would contradict the perturbative assumptions on which the string landscape is based, to postulate a positive c.c. in these asymptotic regions.

We want to emphasize that there is no positive evidence that the prescription of keeping only a finite number of states is a consistent one. The problem is that effective field theory in a CDL instanton background is not a self consistent approximation. Innocuous looking perturbations of the future asymptotic data evolved from situations in the past where effective field theory breaks down. We now have at least three proposals for the nature of the theory of these systems: Big Bang, Freivogel-Susskind scattering theory, and the finiteness hypothesis we have just put forward, all of which might be wrong.

6. Applications

6.1 Eternal inflation

We have proposed an entirely new interpretation of the CDL tunneling processes which are usually thought to lead to eternal inflation. This interpretation is holographic and never discusses regions outside the causal horizon of a given observer. According to this interpretation CDL tunneling only makes sense when the “false vacuum” has non-negative (perhaps only positive) cosmological constant$^{13}$. In the case where both minima have positive c.c., the physics is interpreted in the finite dimensional Hilbert space of the dS space with smaller c.c. The instanton splits space-time up into two causally disconnected regions, one of which relaxes back to the true, and one to the

$^{12}$Note that we are proposing to throw most of these localizable states away in the CDL instanton, rather than keeping them as we would in the flat space limit of dS space.

$^{13}$The point is that AdS minima, with the usual boundary conditions on fluctuations, do not allow instanton solutions. In some cases [18] it has been shown that the CDL instanton corresponds to a boundary field theory with a Hamiltonian which is not bounded from below. It may be possible to stabilize these theories, but it is unlikely that the stable ground state has an interpretation in terms of low curvature bulk space-time.
false vacuum. The false vacuum is interpreted as a low entropy meta-stable state of the high entropy true vacuum system. The word vacuum is a bit of a misnomer because both states are relatively high entropy, low temperature, thermal systems. The instanton describes tunneling back and forth between these states, and the rates are related by the principle of detailed balance. The latter fact is a piece of semi-classical evidence that the current interpretation of instanton physics is the correct one. This interpretation implies that for the case of two positive cosmological constants, there is no sense in which the “false vacuum dominates the global physics of the universe”. That conventional claim is based on attributing independent degrees of freedom to every horizon volume of dS space, into the infinite future in global time. That hypothesis is rejected by the holographic interpretation, and, we claim, at least weakly disfavored by the form of the semi-classical tunneling amplitudes. We do not see how to interpret the detailed balance result in terms of the conventional picture.

The case where the “true vacuum” has negative c.c., much larger in absolute value than the positive c.c. of the “false vacuum”, was studied in this paper. In this case it is true that the false vacuum dominates the physics of the system, but the details are very different than the conventional interpretation would lead us to expect. The situation is again one of a high entropy system (the false vacuum) making rare transitions into a low entropy meta-stable state. However, the false vacuum dominates by virtue of its large entropy, rather than its large expansion rate, so the effect is most marked for very large dS radius, rather than small. This introduces a puzzling discontinuity with the calculation for decay of flat space into the same negative c.c. crunch. We explained that this was due to the very different nature of the asymptotically flat and dS “vacuum states”. The dS vacuum is a highly degenerate mixed state of microscopic degrees of freedom which are mostly beyond the ken of local observers. The flat limit is taken by discarding most of these states and retaining only a set of states localized in a single horizon volume, which become stable in the \( R \to \infty \) limit. They are graded by a new conserved generator, the Poincare Hamiltonian, whose commutator with the static dS Hamiltonian vanishes in this localized subspace in the limit. The flat space vacuum is the non-degenerate zero-eigenstate of the Poincare Hamiltonian. Its decay rate is unaffected by the entropy considerations that make the decay of the dS vacuum improbable, and it is this discontinuity, which is reflected in the CDL calculations.

We should note that if the hypothesis of Cosmological SUSY breaking is correct, then the discontinuity disappears. This hypothesis states that the limit of dS space as the cosmological constant goes to zero is exactly supersymmetric and R symmetric. There is no CDL instanton for the decay of such a supersymmetric state, and supersymmetry leads to a positive energy theorem[19], which implies that the vacuum is stable.
Finally, we recall that there is an entirely different mechanism which is usually covered by the rubric of eternal inflation. This is the *self-reproducing inflationary universe* of Linde[2] (see also earlier work of[21]). Our considerations have nothing explicit to say about these models. However, the self-reproducing universe shares with the instanton based ideas of eternal inflation, a mechanism which depends on the existence of independent degrees of freedom in each horizon volume of dS space at infinite future time. Our reinterpretation of the CDL instanton transitions denies the existence of this infinite set of degrees of freedom. Thus, it is philosophically (though not yet mathematically or physically) contradicting the basis of self-reproduction as well.

### 6.2 A toy landscape

There are two extant proposals for incorporating the observational fact of an accelerating universe into “string theory”. The first is the string theory landscape[22] and the second the theory of stable dS space[4]. Neither proposal is a fully developed mathematical theory.

We will first examine the implications of our interpretation of CDL instantons, for the landscape, under the (false) assumption that there are no Dine-Seiberg regions with potential falling to zero at infinite places in moduli space. This is then an imaginary landscape in which every minimum has either positive or negative energy density. For such a landscape, our interpretation of tunneling phenomena leads to the conclusion that the “stable ground state” is the minimum with maximal area causal diamond in the Lorentzian section of a CDL instanton, which has the field in the basin of attraction of that minimum. For dS minima, this maximal area is the dS horizon area, while for negative c.c. Big Cruncches the area depends on more details of the potential.

The maximal area principle implicitly assumes that the landscape is connected. That is, for each minimum of the potential, we can make a list of all other minima that can communicate with it through a sequence of tunneling events. For a general potential, this might break the space of minima up into disconnected classes, and the maximal area principle would refer to each class independently.

A slightly peculiar feature of this proposal would arise if it turned out that the maximal area causal diamond occurred for negative c.c. In that case the life-time of observers (see the footnoted definition above) in what we call the ground state would be power law in the area. By contrast, if the ground state has positive c.c. an observer’s life-time is an exponential of a power of the area of the maximal causal diamond. Thus, in the case of a negative c.c. maximal diamond we would probably want to invent an ob-
serverphilic principle. The quantum system representing this landscape, might spend most of its time in a Big Crunch ground state, but the most likely situation to be observed (by an exponentially large factor) would be the minimum with smallest positive cosmological constant. Thus, the combination of the maximal area and observerphilic principles predicts that we should expect to see a positive cosmological constant with the smallest value allowed by the landscape. If these principles were applicable to the string landscape, we would have to hope that the current estimates of the number of meta-stable dS ground states is wrong and that the real number is of order $10^{120}$. Furthermore, these principles pick out a unique ground state, so all other features of the low energy physics would be specified, with no recourse to the anthropic principle. Weinberg’s argument would have to be relegated to the category of interesting coincidences. This sort of vacuum selection principle was anticipated in [24].

We should point out that the conclusion that an eternally inflating system could choose the state of lowest positive c.c. has been derived by Linde on a rather different basis. In the stochastic approach to inflation invented by Starobinsky, if one looks for stationary solutions of the equations one concludes that in a given horizon volume, the most probable value of the c.c. is the smallest possible positive one. However, within the context of the global view of eternal inflation, Linde argues that it is by no means clear that one should accept the need for stationary solutions. Followers of the holographic point of view, who reject the existence of degrees of freedom extraneous to the largest causal diamond of any observer in the system, find the argument about stationary distributions more convincing.

6.3 The real string landscape

Of course, the real string landscape is not of the type described above. It has Dine-Seiberg regions in which the potential goes to zero. The CDL instanton geometry in the basin of attraction of those regions describes a space-time which is locally more and more like 10 or 11 dimensional flat space. Supersymmetry is restored asymptotically from the point of view of local physics, and weakly coupled string theory, or low energy 11D SUGRA appears to describe the local low energy physics. Causal diamonds of infinitely large area exist in this space-time.

The conventional wisdom is that there is only one string landscape, all of whose dS minima can tunnel (possibly via multiple steps) into each other and into any of the three asymptotically maximally supersymmetric (AMS) regions of moduli space. We

\[14\text{Unlike the anthropic principle, this principle makes no reference to biology or life. Rather it is a statement about the maximum lifetime of large quantum systems that are well described by quantum field theory, in a system where a typical localized state of high enough energy, is a black hole. Crude estimates of life-times can be made using well understood physics.}\]
should emphasize that while we have only weak evidence that any stringy landscape of dS minima exists we have exactly zero evidence that there is only one connected landscape. We think that anyone who has thought about this at all finds the prospect of three different kinds (IIA, IIB, 11D) of asymptotic regions in the same theory confusing. Is the full Hilbert space a direct sum of Fock spaces of outgoing particle states in these different space-times? Or are they “dual” to each other, whatever that means? In order to avoid having to think about these skull-bursting questions, one might hope that the landscape actually consists of disconnected families of dS minima, with each family tunneling into only one kind of AMS universe.

In fact, if one accepts the conventional wisdom, there is a similar, if slightly more subtle problem for any system where two or more dS minima can tunnel into the same region in field space. The point is that there are then multiple instanton solutions. The Lorentzian continuation of each instanton contains a region of negatively curved FRW universe, which asymptotes to a matter or kinetic energy dominated cosmology. Each FRW universe is slowly varying in the future, so we can introduce an adiabatic Fock space[27] for the quantization of field fluctuations around the classical background. However, because the backgrounds are globally different on the homogeneous slices of constant negative curvature, the Bogoliubov transformation between the two quantum operator algebras is not implemented by a unitary transformation on Fock space\(^{15}\). So even in the simplest model of a landscape we have to deal with the issue of how to include globally different asymptotic space-times in the same quantum theory.

This problem would be avoided if we could make sense of the suggestion that only a finite number of scattering states in each asymptotic region are actually allowed in the context of a landscape. The lack of unitarity of the Bogoliubov transformation is related to the infinite number of degrees of freedom in quantum field theory, and would certainly disappear if we could restrict attention to a finite number of states.

While it is not clear that this is possible, the attraction of the proposal is evident. In particular, if this proposal makes sense, the Stringy Landscape would be similar to the toy landscape of the previous subsection and we could borrow the dynamical selection principle from that discussion. The Stringy Landscape would be a system with a finite number of states\(^{16}\). It could best be viewed from the point of view of the metastable minimum with smallest positive cosmological constant. Tunneling transitions

\(^{15}\)Any two Hilbert spaces of the same (finite or countable) dimension are related by (many) unitary transformations. However, these transformations do not usually map simple observables to each other. The Bogoliubov transformation provides a simple map of observables, but not a unitary map of Hilbert spaces.

\(^{16}\)This assumes that there are a finite number of meta-stable dS vacua, an assertion which is now open to doubt.
would be interpreted in terms of the quantum mechanics of the static Hamiltonian of this dS space. Minima with larger absolute value of the cosmological constant\(^{17}\) are low entropy subsystems, which the system visits rarely, and for periods brief compared to its sojourns in the “ground state”. This would make the string landscape into a predictive model of low energy physics. From many points of view, this conjecture puts the string landscape on the same footing as the equally hypothetical theory of stable dS space. The main difference is the attitude towards SUSY. Part of the hypothesis of [9] is that the scale of SUSY breaking is directly connected with the cosmological constant, through the formula \(m_{3/2} = c\Lambda^{1/4}\). In the Landscape there does not appear to be a similar connection between the value of the c.c. and SUSY breaking.

### 6.4 The theory of stable dS space

The current paper actually arose out of the one of the authors’ (T.B.) consideration of low energy effective theories which could be compatible with the hypothesis of Cosmological SUSY Breaking. It has proven extraordinarily difficult to find any low energy implementation of CSB, which was also compatible with a reasonable phenomenology. The only extant model looks peculiar from the conventional standpoint of low energy SUSY theorists. Once the parameters have been set to accord with the ideas of CSB, the model looks more or less like a conventional model of low energy dynamical SUSY breaking. In particular, much of the dynamics of SUSY breaking is determined by a non-gravitational model of gauge and chiral superfields. This model has, in addition to its SUSY violating vacuum state, a moduli space of SUSic vacua.

In [25] one of the authors (T.B.) made the general observation that non-gravitational theories with SUSY violating meta-stable minima, as well as SUSY vacuum states may be useful for phenomenology. The point is that if we decide to fine tune the c.c. so that it is small and positive at the SUSY violating minimum, then the SUSic vacua have negative c.c. of microphysical scale. This means that they have NOTHING to do with the theory in which the SUSY violating dS space lives. The “meta-stable” SUSY violating vacuum decays into a SUSY violating Big Crunch cosmology, not one of the SUSic vacuum states. The question then becomes one of the lifetime of the “meta-stable” state. This paper gives a surprising answer to that question. It is easy to construct low energy models in which the instanton for decay of a flat, SUSY violating meta-stable minimum, has an action so small that the decay would occur on a time scale shorter than the current age of the universe. This paper shows that the flat space estimate is irrelevant to the calculation of the true vacuum decay lifetime for dS

\(^{17}\)At this point the reader should refer back to the discussion of the observer-philic principle, to recall the extra complication that ensues if the smallest absolute value actually corresponds to a negative c.c.
space, which will be superexponentially longer than the age of the universe if the c.c. is tuned to be exponentially small.

Apart from the numerical/phenomenological significance of this result, these considerations resolve an issue of principle for the hypothesis of CSB. Under that hypothesis, dS space is supposed to be stable. We argued that when the negative c.c. has microphysical scale, the CDL process did not represent an instability. Rather it represents a low probability transition to a low entropy meta-stable state, analogous to all the air in a large room collecting in a small box with a pinhole in it, situated somewhere in the room.

7. Conclusions

The form of the Coleman-DeLuccia probabilities for tunneling to and from a large radius dS space supports a holographic reinterpretation of eternal inflation. When the potential landscape is such that the smallest absolute value of the c.c. occurs for positive c.c. one considers the most stable state of the system to be the dS space with that value of the c.c. Other minima are considered as low entropy meta-stable states of the system, which (for entropic reasons) are visited infrequently, and out of which one tunnels with relative rapidity. For pairs of minima with positive c.c., the success of the detailed balance prediction for the relative rates of transition gives, in our opinion, strong support for this interpretation. The fact that the largest causal diamond in a negative c.c. Big Crunch\textsuperscript{18} has area of microphysical size helps to explain the smallness of the rate for the “decay” of the large radius dS space into this state. It also motivates an estimate of the (much larger) inverse transition rate from the Big Crunch to dS space. These ideas also explain the puzzling discontinuity in the CDL transition rate to a negative c.c. crunch, when the positive c.c. is lowered to zero.

If the lowest absolute value of the c.c. occurs for negative c.c., the fundamental description of the physics is more obscure. Assuming that it can be defined, our generalization of the CDL amplitudes again suggest a finite entropy system, which spends most of its time in the Big Crunch state. However, in this case, and assuming that there is a positive c.c. minimum of very small size, the question of the lifetime of observers comes into play. For our purposes, an observer is a large, localized system, approximately described by cutoff quantum field theory. Such systems can survive in the dS state, for times which are exponential in an inverse power of the positive c.c.

\textsuperscript{18}T.B. cannot resist emphasizing for the Nth time that the AdS solution of the field equations at the negative c.c. minimum has nothing to do with any of the physics discussed in this paper. It is an isolated theory of quantum gravity with AdS boundary conditions, completely described by a conformal field theory.
In the Big Crunch state they survive a time power law in the (absolute value of) the negative c.c. The Big Crunch state is, for most of its history, a state where all degrees of freedom are maximally excited and interacting. The period when observers can exist is short. Thus, even if the landscape has its smallest c.c. negative, one predicts that a typical observer will be found in the region with smallest positive c.c. We emphasize again that real theories of quantum gravity may only realize some (or none) of the toy landscapes we are discussing here.

The most well motivated landscape scenario is based on string theory. Here, the potential has asymptotic Dine-Seiberg regions where flat supersymmetric 10 or 11 dimensional space-time is locally restored. The possibility of regions with vanishing potential is from the point of view presented here, a singular limit in which the number of states of the system might become infinite. The analysis of stable dS space in [4] indicates that such limits have to be taken with great care. Most of the states that exist for finite dS radius, decouple and must be thrown away when considering the Poincare invariant limiting theory. A similar question arises for landscapes with points in field space that have vanishing c.c. We presented several conjectural descriptions for such systems, none of which is without problems. We think that our mathematical results on instantons are correct, and indicate the need for a reinterpretation of eternal inflation. Our suggestion that the string landscape might refer to a quantum system with a finite number of states is much more conjectural.

Our proposed reinterpretation of eternal inflation replaces an infinite and incomprehensible\textsuperscript{19} fractal with a finite system, and suggests different rules for deciding the probability of various observations in an eternally inflating universe. It may be, as advertised in the title of this paper, that these rules can be viewed as a regulated and invariant version of the “volume weighted” counting of probabilities, which has been suggested by the inventors of eternal inflation. In [28] the authors suggested that the global gauge picture of de Sitter space might be approximately justified in the limit of large dS space by the following stratagem: In each horizon volume consider only those localizable states which one would keep in the Poincare invariant limit, and treat them (approximately) by the usual rules of particle physics. The entropy of such states scales as $(RM_P)^{3/2}$. The total dS entropy allows us to have $(RM_P)^{1/2}$ commuting copies of these degrees of freedom, which we can try to associate with disjoint horizon volumes. This would be compatible with the global gauge description of dS space if we enforced an IR cutoff on the global time. Given a form for the potential, we can estimate the probability for a given observer’s Poincare ground state to nucleate a bubble of Big Crunch, by the usual non-gravitational rules. The probability of finding dS

\textsuperscript{19}More precisely, “as yet uncomprehended”
thermal fluctuations of this state is very much smaller than the local bubble nucleation probability, when the dS radius is taken large with microphysical scales fixed\(^{20}\).

But what do we mean by a given observer in the above paragraph? The arrangement of the states of dS space into states of a finite collection of disjoint observers is a gauge choice with no direct physical meaning. All one observer can ever see are its own localizable observables and a collection of degenerate degrees of freedom which form a thermal bath for localizable observations. There are not enough localizable degrees of freedom to measure the precise state of the heat bath. The only predictions of the theory which should be taken seriously are those which refer to averages over the different possible choices of which degrees of freedom we choose to represent the ones that we actually experience. We claim that the typical observer, in this sense, does not experience a vacuum decay with the probability determined by the approximately Poincare invariant calculation in a single horizon volume. Ultimately, the argument for this is the large discrepancy in the maximal entropy allowed by the covariant entropy bound in the two bubbles of the CDL instanton. So the claim is that the mathematically unambiguous CDL calculation of the tunneling probability is taking into account the averaging over different horizon volumes in global dS space, that is invoked in conventional discussions of eternal inflation. Those considerations are modified by taking into account the covariant entropy bound. The global description is cut off in such a way that the entropy attributed to a given dS space never exceeds one quarter of the area of its horizon.

Apart from the radical reinterpretation of some kinds of eternal inflation models, the most significant impact of our results is on dynamical SUSY breaking models. Traditionally, one studies a non-gravitational model and insists that it has no SUSY ground states. Our results suggest that this is much too restrictive. It is sufficient to find a quantum field theory with a metastable SUSY violating ground state. If we then fine tune the c.c. to be very small and positive at the SUSY violating point in field space, then the SUSic ground states are irrelevant. The meta-stable SUSY violating state tunnels to a Big Crunch, but this is a very improbable event, in a system with a finite number of states. The system spends most of its time in the meta-stable SUSY violating state.

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\(^{20}\)Under the hypothesis of Cosmological SUSY Breaking, some microphysical scales are actually tied to the c.c.
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