Synergetic synthesis of tracking control systems

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Abstract. The application of the principles and methods of the synergetic theory of control for the synthesis of automatic controllers that provide a solution to the tracking problem is considered. The study aims to develop analytical procedures for the synthesis of tracking controllers for a general class of nonlinear systems. The proposed approach is based on the idea of the current piecewise linear approximation of the input signal and the synthesis of an asymptotic observer of the slope coefficient of the approximating straight line. To confirm the theoretical conclusions, a computer simulation of the synthesized tracking system with various input signals was carried out.

1. Introduction

Tracking systems are usually allocated to a separate class of automatic control systems. Unlike the most common stabilization systems that perform the task of holding controlled variables in a given constant value, tracking systems must provide a change of the controlled variable \( x^{(c)} \) in accordance with some time signal \( g(t) \) received at the system input. This input signal is considered as a setting action, which is an a priori unknown function of time. The task of the tracking system is to reproduce the input signal with a given accuracy \( |g(t) - x^{(c)}| \leq \varepsilon \).

Currently, the design of tracking systems mainly uses the approaches of the classical theory of automatic control using a linear or linearized mathematical description of controlled processes [1] – [5]. The use of “frequency” methods of analysis and synthesis allows us to solve the tracking problem for a given order of astatism of the system by the input effect. However, this approach encounters serious methodological difficulties in those cases where the dynamics of the controlled object is substantially non-linear. The need to increase the effectiveness of linear tracking controllers determines the use of various areas of modern control theory [6] – [12]: fuzzy logic, adaptive controllers, robust controllers, \( H_{\infty} \) control, etc. But the general “linear” ideology remains dominant.

Synergetic control theory [13] – [15] provides effective methods for the synthesis of controllers for controlling multidimensional and nonlinear objects. At the same time, the synthesized controllers solved the problem of stabilization of the controlled variable, or the problem of generating oscillations of the controlled variable. In [16], [17], a solution to the problem of synergetic synthesis of controllers providing reproduction of a given time signal (periodic or chaotic) and involving the introduction of reference signal generators into the structure of a closed system is presented. It was believed that the characteristics of a temporary signal are a priori determined, and its dynamics can be described by the corresponding differential equations.

This article will show the use of the principles and methods of the synergetic theory of control to solve the problem of the synthesis of tracking automatic control systems in its generally accepted formulation.
2. Synthesis method

Let us formulate the problem of the synthesis of tracking controllers, assuming that the object has one control channel and it is necessary to ensure tracking of one input signal.

Let the dynamics of a controlled object be described by a system of ordinary differential equations:

\[
\dot{x} = F(x, u, g)
\]  

(1)

where \(x\) is the vector of system state variables, \(u\) is the control action, \(g = g(t)\) is the setting action. Any continuous function with a sufficient degree of accuracy can be approximated by a polynomial. If the degree of this polynomial is equal to unity, a linear approximation takes place, which has found wide application in practice.

The reference signal in the tracking system, which is an unknown function of time, can also be considered as a linear signal with a variable slope coefficient of the approximating straight line. These considerations formed the basis for the development of the following methodology for the synergetic synthesis of tracking controllers, which is based on the principle of expanding the state space of the original system by adding differential equations of the reference time signal (a linear function of time) and uses the synthesis technique of an asymptotic observer for the slope of the approximating straight line.

If some value \(g(t)\) is changed linearly in time: \(g(t) = g_0 + g_1t\) then the dynamics of this change is described by a differential equation \(\dot{g} = g_1\). Then the extended system model (synthesis model) takes the following form:

\[
\begin{align*}
\dot{x} &= F(x, u), \\
\dot{y}_1 &= y_2, \\
\dot{y}_2 &= 0,
\end{align*}
\]

(2)

where \(y_1\) is the variable of the reference signal model, and \(y_2\) is the variable characterizing the current value of the slope coefficient of the approximating straight line. From the last equation (2) it follows that \(y_2\) is a piecewise constant quantity, and, therefore, is a piecewise linear function of time.

The synthesis procedure includes two stages. At the first stage, the controller is synthesized under the assumption that all the variables of model (2) are observable. On the second stage, an asymptotic observer is synthesized to evaluate the current value \(y_2\). After that, the estimate of \(\hat{y}_2\) is substituted into the synthesized control law.

Let us show the application of this technique with a simple example. We pose the problem of synthesizing a tracking controller for a nonlinear object described by the model:

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= (u + f(x_1, x_2))a_1
\end{align*}
\]

(3)

Such structure is seen in the models that describe the dynamics of mechanical systems with one degree of freedom, where the variables \(x_1\) and \(x_2\) correspond to movement and velocity, control \(u\) means the controlling external force, and the function \(f(x_1, x_2)\) characterizes the forces accompanying the movement (friction, gravity, etc.).

In this case, the extended system model takes the form:

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= (u + f(x_1, x_2))a_1, \\
\dot{y}_1 &= y_2, \\
\dot{y}_2 &= 0
\end{align*}
\]

(4)
The synergetic synthesis methods are based on the idea of introducing attractive invariant manifolds \( \psi_1(x) = 0 \) in the state space of a controlled system. The control law in the scalar case (one control channel) is found as a solution to a functional equation written with respect to the corresponding macro variable \( \psi_1 \). This equation is a differential equation that has asymptotic stability with respect to \( \psi_1 = 0 \). Usually, the first-order functional equation \( T_1 \dot{\psi}_1 + \psi_1 = 0, T_1 > 0 \) is used.

The controller should provide an asymptotic convergence of the controlled variable to the variable of the reference signal: \( x_1 \rightarrow y_1 \). Therefore, we introduce an invariant manifold

\[
\psi_1 = x_2 + k(x_1 - y_1) = 0.
\]

In this case, given the \( \psi_1 = 0 \), the dynamics of the controlled variable is described by a differential equation \( \dot{x}_1 = -k(x_1 - y_1) \), which, at \( k > 0 \), has the asymptotic stability with respect to \( y_1 \).

The control law is found as a solution to the functional equation \( T_1 \dot{\psi}_1 + \psi_1 = 0 \) by virtue of the equations of the model (4):

\[
T(\dot{x}_2 + k(\dot{x}_1 - \dot{y}_1)) + x_2 + k(x_1 - y_1) = 0
\]

\[
\iff T((u + f(x_1, x_2))a_1 + k(x_2 - y_2)) + x_2 + k(x_1 - y_1) = 0
\]

\[
\impliedby u = -f(x_1, x_2) - k(x_2 - y_2)/a_1 - (x_2 + k(x_1 - y_1))/(a_1 T).
\]

For the synthesis of the observer, the method of synergetic synthesis of asymptotic observers is used [14]. The model of the extended system is represented in the vector-matrix form:

\[
\dot{v} = g_0(v, u) + G_1(v)w;
\]

\[
w = h_0(v, u) + H_1(v)w,
\]

where \( v \) is the vector of the observed variables, \( w \) is the vector of the observed variables.

In our case:

\[
v = [x_1 \ x_2 \ y_1]^T, \quad w = y_2, \quad g_0 = [x_2 \ (u + f(x_1, x_2))a_1 \ 0]^T, \quad G_1 = [0 \ 0 \ 1]^T, \quad h_0 = 0, \ H_1 = 0.
\]

The observer equations are found from the expression

\[
\dot{z} = Lz - L \int_0^t \Gamma(v)dv - h_0 + \Gamma(v)g_0,
\]

\[
\dot{w} = \int_0^t \Gamma(v)dv - z.
\]

where \( z \) are the observer variables, \( L \) is the matrix selected from the observer stability condition (in our case, it contains one element \( l_1 \), the stability condition is \( l_1 < 0 \)), and the matrix \( \Gamma(v) \) is calculated from the equation \( H_1 - L = \Gamma(v)G_1 \).

By substituting the matrices and vectors into (5), we obtain the desired observer equations:

\[
\dot{z}_1 = l_1z_1 + l_1^2y_1
\]

and the estimates of the slope of the approximating straight line

\[
\dot{y}_2 = -l_1y_1 - z_1
\]

It should be noted that equations (7), (8) are valid for any objects described by model (2).

Taking into account the assessment (8), the control law takes the final form:
\[ u = -f(x_1, x_2) - k(x_2 + l_1 y_1 + z_1)/a_1 - (x_2 + k(x_1 - y_1))/l(a_i T). \]  

(9)

3. Computer modelling

To analyze the effectiveness of the synthesized tracking controller, computer simulation of a closed-loop system (3), (7) – (9) was carried out under various input signals. In real operating conditions, the input of the tracking system receives not the piecewise-linear reference signal \( y_1 \), but the signal \( g(t) \) that varies in time in an arbitrary way. Therefore, for the correct modeling of the system, it is necessary to make a replacement of \( y_1 \leftrightarrow g(t) \) in the equations (7) - (9). The function \( f(x_1, x_2) \) was set as \( f(x_1, x_2) = -x_2 - x_2^3 \).

In the simulation, the signal \( g(t) \) was defined as a piecewise continuous function of time:

\[
g(t) = \begin{cases} 
-5 + t, & 0 \leq t < 10; \\
1 + 0.5t - 0.1t^2, & 10 \leq t < 20; \\
30\sin 0.3t, & 20 \leq t < 50; \\
10\sin 0.2t - 5\cos 0.3t, & 50 \leq t < 100. 
\end{cases}
\]

Figure 1 shows the transient graphs of a controlled variable and a input signal. Figure 2 shows the transient graphs of the estimation of the slope of the approximating line. It can be seen that the estimate at the first stage tends to a value of 1, which corresponds to the slope coefficient of the linear function \( g(t) = -5 + t \). At the second stage, it changes linearly, which corresponds to a piecewise linear approximation of a parabolic function. At the third and fourth stages, it varies periodically, which corresponds to a periodic input signal.
Figure 2. Transient processes of estimation of the slope of the approximating line in the tracking system

4. Conclusion
By assessing the results of computer modeling, we can conclude that the proposed approach allows us to synthesize controllers that can work out the defining influences of a general class of continuous functions of time and can serve as a theoretical basis for the design of tracking systems for controlling nonlinear dynamic objects.

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References
[1] Akhmetzhanov A A and Kochemasov A V 1986 Tracking systems and regulators (Moscow: Energoatomizdat) p 286
[2] Barsky A G 2009 Optoelectronic Tracking Systems (Moscow: University book) p 200
[3] Nikiforov V O Lukyanova G V 2011 The tracking system of combined control Scientific and Technical Bulletin of the St. Petersburg State University of Information Technologies Mechanics and Optics no 6 (76) pp 39-43
[4] Ivanchura V I Prokopyev A P 2011 Optimization of the tracking system of automatic control Bulletin of the Siberian State Aerospace University no 5 (38) pp 44-49
[5] Shapran A A Ustyugova A A 2013 Synthesis of a control system for a tracking drive of increased accuracy Bulletin of the Ural State University of Railway Engineering no 3 (19) pp 45-49
[6] Bo Xiao, Lam H K, Ge Song and Hongyi Li 2013 Output-Feedback Tracking Control for Interval Type-2 Polynomial Fuzzy-Model-Based Control Systems IEEE Transactions on Industrial Electronics vol 60 no 12 pp 5830-5840
[7] Chang Yeong-Chan 2010 Robust tracking control for nonlinear MIMO systems via fuzzy approaches Automatica vol 36 no 10 pp 1535-1545
[8] Ivanchura V I, Prokopyev A P and Emelyanov R T 2012 A model of a tracking system of automatic control with a fuzzy controller Bulletin of the Siberian State Aerospace University no 3 (43) pp 15-20
[9] Putov V V, Dung Ch A and Kuang F K 2018 Adaptive electromechanical tracking system with two- and three-mass nonlinear elastic objects and neurofuzzy control News of St. Petersburg Electrotechnical University "LETI" no 5 pp 21-24
[10] Reshetnikova G N 2006 Tracking system of adaptive control with a predictive model of lower order Bulletin of Tomsk State University no 290 pp 237-240
[11] Maltsev G N and Afonin G I 2014 Optimization of the adaptive circuit parameters of the tracking system of automatic control based on the analysis of the frequency response News of higher educational institutions. Instrument Engineering vol 57 no 7 pp 26-31
[12] Celentano L 2017 Pseudo-PID robust tracking design method for a significant class of uncertain MIMO systems *IFAC-PapersOnLine* vol 50 no 1 pp 1545–1552

[13] Kolesnikov A A 1994 *Synergetic control theory* (Moscow: Energoatomizdat) p 344

[14] *Modern applied control theory Part 2 Synergetic approach in control theory* 2000 (Taganrog: TRTU publ) p 656

[15] Kolesnikov A A 2019 *Synergetic control methods for complex systems: the theory of system synthesis* (Moscow: URSS Publ) p 244

[16] Popov A N 2012 Synergetic synthesis of regulators for the problems of generation of oscillatory modes in technical systems *News of SFedU. Technical sciences* no 4 (129) pp 156-162

[17] Popov A N 2016 Synergetic synthesis of autopilots for the formation of reference motion trajectories in the horizontal plane *Modern Science and Innovation* no 4, pp 29-36