Analysis of the possible $D\bar{D}_{s0}^{*}(2317)$ and $D^{*}\bar{D}_{s1}^{*}(2460)$ molecules with QCD sum rules

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Abstract

In this article, we assume that there exist the pseudoscalar $D\bar{D}_{s0}^{*}(2317)$ and $D^{*}\bar{D}_{s1}^{*}(2460)$ molecular states $Z_{1,2}$ and construct the color singlet-singlet molecule-type interpolating currents to study their masses with the QCD sum rules. In calculations, we consider the contributions of the vacuum condensates up to dimension-10 and use the formula $\mu = \sqrt{M_{X/Y/Z} - (2M_c)^2}$ to determine the energy scales of the QCD spectral densities. The numerical results, $M_{Z_1} = 4.61^{+0.11}_{-0.08}$ GeV and $M_{Z_2} = 4.60^{+0.07}_{-0.06}$ GeV, which lie above the $D\bar{D}_{s0}^{*}(2317)$ and $D^{*}\bar{D}_{s1}^{*}(2460)$ thresholds respectively, indicate that the $D\bar{D}_{s0}^{*}(2317)$ and $D^{*}\bar{D}_{s1}^{*}(2460)$ are difficult to form bound state molecular states, the $Z_{1,2}$ are probably resonance states.

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1 Introduction

In the recent years, many new charmonium-like and bottomonium-like exotic mesons $^{[1]}$ (being bosons like the traditional $q\bar{q}$ mesons), have been observed experimentally, and are labeled as the $XYZ$ states $^{[2]}$. These exotic states, with growing evidences, cannot be the pure $c\bar{c}$ or $b\bar{b}$ states and are considered as good candidates for tetraquark states, which do not fit into the conventional quark model picture $^{[3]}$. In order to decipher their underlying structure, a number of interpretations have been proposed, such as the molecules $^{[4,5]}$, the tetraquark states $^{[6]}$, the hybrid mesons $^{[7]}$, the kinematical effects $^{[8]}$, and so on.

In the molecular picture, a four-quark state is explained as a weakly bound state of two mesons $^{[9]}$. Each constituent meson is bound internally by strong QCD color forces, while the mesons bind to each other by means of a much weaker color-neutral residual QCD force, analogous of the van der Waals attraction in chemistry. Among these observed $XYZ$ states, some lie remarkably close to the meson-meson thresholds. Therefore, the molecular interpretation seems plausible for these states. The most impressive example is the original exotic state, the $X(3872)$ $^{[10]}$, which has been investigated as the $D\bar{D}^*$ molecular state by many theoretical groups $^{[4,11]}$, owing to its mass with $m_{X(3872)} - m_{D^{*0}} - m_{D^*} = +0.01 \pm 0.18$ MeV. The $Z_c(3900)$, observed by the BESIII collaboration firstly in 2013 $^{[12]}$, is also close to the threshold of $D\bar{D}^*$, and is taken as the isovector partner of the established isoscalar bound molecular state $X(3872)$ with the same quantum number $J^P = 1^+$ in some references $^{[13,14]}$. Interestingly, the observed bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$ by the Belle collaboration $^{[15]}$, own the same near-threshold nature and are interpreted successfully as the $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states $^{[16]}$. The successes of the molecular interpretation for some observed exotic states stimulate the further theoretical studies on the analogous open-charmed meson pair system as a bound molecular state, which make several predictions of the possible molecules.

In theoretical techniques, the QCD sum rules method is a powerful tool in studying the hidden-charm (bottom) tetraquark or molecular states and hidden-charm pentaquark states. Here, we make the assumption that there exist the pseudoscalar $D\bar{D}_{s0}^{*}(2317)$ and $D^{*}\bar{D}_{s1}^{*}(2460)$ molecular states, and study their masses with the QCD sum rules to check the existence of the corresponding molecular states. The $D$ and $D^*$ mesons have negative parity, while the parity is positive for

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the $\bar{D}_0(2317)$ and $\bar{D}_{s1}(2460)$ mesons. Based on the theoretical analysis in Ref. [17], there is a Coulomb-like force by exchanging a kaon in S-wave, that might be able to bind the $D\bar{D}_0(2317)$ and $D^*\bar{D}_{s1}(2460)$ systems, respectively. In addition, the mass difference between the $D(D^*)$ and $\bar{D}_0(\bar{D}_{s1})$ is close to the kaon mass, which means that the exchanged kaon will be near the mass shell and the interaction will be unusually large. These are the reasons why we are interested in the $D\bar{D}_0(2317)$ and $D^*\bar{D}_{s1}(2460)$ molecules. In calculations, we consider the contributions of the vacuum condensates up to dimension-10, and use the formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2M_{\pi})^2}$ to determine the energy scales of the QCD spectral densities [13], which can enhance the pole contributions remarkably and improve the convergent behaviors of the operator product expansion in the QCD sum rules for the exotic hadrons [18].

The rest of this article is arranged as follows. In section 2, we consider the $D\bar{D}_0(2317)$ and $D^*\bar{D}_{s1}(2460)$ systems as the pseudoscalar molecules, construct the corresponding color singlet-singlet molecule-type interpolating currents, and extract their masses and pole residues with the QCD sum rules. The numerical results and discussions are performed in section 3. The last section is reserved for our conclusion.

2 QCD sum rules for the possible $D\bar{D}_0(2317)$ and $D^*\bar{D}_{s1}(2460)$ molecular states

Based on our assumption that there exist the pseudoscalar $D\bar{D}_0(2317)$ and $D^*\bar{D}_{s1}(2460)$ molecular states, the corresponding color singlet-singlet molecule-type interpolating currents are written as

$$J_1(x) = \bar{q}^a(x)i\gamma_5c^a(x)c^b(x)s^b(x),$$

and

$$J_2(x) = \bar{q}^a(x)\gamma_\mu\gamma^\nu(x)c^b(x)\gamma_\mu\gamma_5s^b(x),$$

respectively, where $a, b$ are color indexes, and $q$ denotes an up or down quark.

In QCD sum rules, we consider the two-point correlation functions

$$\Pi_{1,2}(p) = \int d^4xe^{ip\cdot x}\langle 0|T\{J_{1,2}(x)J_{1,2}^+(0)\}|0\rangle,$$

which can be obtained in two ways: on the phenomenological side and at the quark level.

On the phenomenological side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_{1,2}(x)$ into the correlation functions $\Pi_{1,2}(p)$ to obtain the hadronic representations. After isolating the ground state contributions from the pole terms, we get the following results,

$$\Pi_{1,2}(p) = \frac{\lambda_{2,1}Z_{2,1}}{M_{Z_{1,2}}^2 - p^2} + \cdots,$$

where the $Z_1, Z_2$ denote the ground states, provisionally, considered as the $D\bar{D}_0(2317), D^*\bar{D}_{s1}(2460)$ molecules, respectively, and the pole residues $\lambda_{1,2}$ are defined by $\langle 0|J_{1,2}(0)|Z_{1,2}(p)\rangle = \lambda_{Z_{1,2}}$, which show the couplings of the currents $J_{1,2}$ to the states $Z_{1,2}$.

At the quark level, we calculate the two-point correlation functions $\Pi_{1,2}(p)$ via the operator product expansion method in perturbative QCD. We contract the $q, s$ and $c$ quark fields with the wick theorem and obtain the following results:

$$\Pi_1(p) = \int d^4xe^{ip\cdot x}\text{Tr} \left[\gamma_5C^{aa'}(x)i\gamma_5Q^{a' a}(-x)\right]\text{Tr} \left[C^{b'b}(x)S^{bb'}(x)\right],$$

$$\Pi_2(p) = \int d^4xe^{ip\cdot x}\text{Tr} \left[\gamma_\mu C^{aa'}(x)\gamma_\mu Q^{a' a}(-x)\right]\text{Tr} \left[\gamma_\nu\gamma_5C^{b'b}(x)\gamma_\nu\gamma_5S^{bb'}(x)\right].$$
where the $Q_{ab}(x)$, $S_{ab}(x)$ and $C_{ab}(x)$ are the full $q$, $s$ and $c$ quark propagators, respectively,

\[
Q_{ab}(x) = \frac{i\bar{d}_a q_f}{2\pi^2 x^4} - \frac{\delta_{ab} \langle \bar{q} q \rangle}{12} - \frac{\delta_{ab} x^2 \langle \bar{q} g_s \sigma G q \rangle}{192} - \frac{\delta_{ab} x^2 \bar{q} q^2}{7776} - \frac{ig_s G_{\alpha\beta} t^n (\dot{f} \sigma^{\alpha\beta} + \sigma^{\alpha\beta} \dot{f})}{32\pi^2 x^2} - \frac{\delta_{ab} x^4 \langle \bar{q} q \rangle (GG)}{27648} \tag{6}
\]

\[
S_{ab}(x) = \frac{i\bar{d}_a q_f}{2\pi^2 x^4} - \frac{\delta_{ab} m_s}{4\pi^2 x^2} \frac{\delta_{ab} \langle \bar{s} s \rangle}{12} + \frac{\delta_{ab} \bar{f} m_s \langle \bar{s} s \rangle}{48} - \frac{\delta_{ab} x^2 \langle \bar{q} g_s \sigma G s \rangle}{192} + \frac{\delta_{ab} x^2 \bar{q} q^2}{7776} - \frac{ig_s G_{\alpha\beta} t^n (\dot{f} \sigma^{\alpha\beta} + \sigma^{\alpha\beta} \dot{f})}{32\pi^2 x^2} - \frac{\delta_{ab} x^4 \langle \bar{s} s \rangle (GG)}{27648} \tag{7}
\]

\[
C_{ab}(x) = \frac{i}{(2\pi)^4} \int d^4 k e^{-ikx} \left\{ \frac{k + m_c + g_s t^n}{k^2 - m_c^2} \delta_{ab} - g_s t^n G_{ab} (k + m_c) \sigma^{\alpha\beta} + \sigma^{\alpha\beta} (k + m_c) \right\} \frac{4(k^2 - m_s^2)^2}{3} \tag{8}
\]

\[
f^{\lambda\alpha\beta} = (k + m_c) \gamma^\lambda (k + m_c) \gamma^\alpha (k + m_c) \gamma^\beta (k + m_c), \tag{9}
\]

\[
f^{\alpha\beta\mu\nu} = (k + m_c) \gamma^\alpha (k + m_c) \gamma^\beta (k + m_c) \gamma^\mu (k + m_c) \gamma^\nu (k + m_c),
\]

$t^n = \frac{\lambda^n}{4}$, the $\lambda^n$ is the Gell-Mann matrix, and $D_\alpha = \partial_\alpha - ig_s G_{\alpha\beta} t^n$ [19]. Then we compute the integrals in the coordinate space for the light quark propagator and in momentum space for the charm quark part. In the operator product expansion, we take into account the contributions of vacuum condensates up to dimension-10, assume vacuum saturation for the higher dimensional vacuum condensates, and keep terms which are linear in the strange quark mass $m_s$. The vacuum condensates are the vacuum expectations of the operators $O_n (\alpha^k)$. We take the truncations $n \leq 10$ and $k \leq 1$ for the operators in a consistent way, and discard the perturbative corrections. In Eqs. 6–7, we retain the terms $\langle \bar{q}_b \sigma_{\mu\nu} q_a \rangle$, $\langle \bar{s}_b \sigma_{\mu\nu} s_a \rangle$, $\langle \bar{q}_b \gamma_\mu q_a \rangle$ and $\langle \bar{s}_b \gamma_\mu s_a \rangle$ originate from the Fierz re-arrangement of the $\langle \bar{q}_b q_a \rangle$ and $\langle s_b s_a \rangle$ to absorb the gluons emitted from the heavy quark lines so as to extract the mixed condensates and four-quark condensates $\langle \bar{q} q \sigma G q \rangle$, $\langle \bar{s} s \sigma G s \rangle$, $g_s^2 \langle \bar{q} q \rangle^2$ and $g_s^2 \langle \bar{s} s \rangle^2$, respectively. One can consult Ref. [20] for some technical details about the operator product expansion. Once the analytical expressions of the correlation functions $\Pi_{1,2}(p)$ are obtained, the QCD spectral densities $\rho_{1,2}(s)$ are given by the imaginary parts of the correlation functions: $\rho_{1,2}(s) = \frac{\text{Im} \Pi_{1,2}(s)}{\pi}$.

According to the quark-hadron duality, we match the correlation functions $\Pi_{1,2}(p)$ obtained on the phenomenological side and at the quark level below the continuum thresholds $s_0$, and perform Borel transform with respect to the variable $p^2 = -t^2$ to obtain the following QCD sum rules:

\[
\frac{\chi^2_{Z_{1,2}} \exp \left( -\frac{M_{Z_{1,2}}^2}{T^2} \right)}{\frac{T^2}{T^2}} = \int_{4m^2}^\infty ds \rho_{1,2} (s) \exp \left( -\frac{s}{T^2} \right), \tag{10}
\]

where

\[
\rho_{1,2} (s) = \rho^0_{1,2} (s) + \rho^1_{1,2} (s) + \rho^2_{1,2} (s) + \rho^3_{1,2} (s) + \rho^4_{1,2} (s) + \rho^5_{1,2} (s) + \rho^6_{1,2} (s) + \rho^7_{1,2} (s) + \rho^8_{1,2} (s) + \rho^9_{1,2} (s), \tag{11}
\]

the superscripts 0, 3, 4, 5, 6, 7, 8, 10 denote the dimensions of the vacuum condensates, and the $T^2$ denotes the Borel parameter. The explicit expressions of the spectral densities $\rho_{1,2}(s)$ are collected in the appendix.
To extract the masses of the states $Z_{1,2}$, we take the derivative of Eq. (10) with respect to $1/T$ and eliminate the pole residues $\lambda Z_{1,2}$:

$$M_{Z_{1,2}}^2 = \frac{\int_{4m_c^2}^{\infty} ds \frac{d}{d(-1/T^2)} \rho_{1,2}(s) \exp\left(-\frac{1}{T^2}\right)}{\int_{4m_c^2}^{\infty} ds \rho_{1,2}(s) \exp\left(-\frac{1}{T^2}\right)}.$$  

(12)

3 Numerical results and discussions

In this section, we perform the numerical analysis. To extract the numerical values of $M_{Z_{1,2}}$, we take the standard values of the vacuum condensates $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1)\langle \bar{q}q \rangle$, $\langle \bar{q}g_sGq \rangle = m_0^2 \langle \bar{q}q \rangle$, $\langle \bar{s}g_sGs \rangle = m_0^2 \langle \bar{s}s \rangle$, $m_0 = (0.8 \pm 0.1)$ GeV, $\langle \bar{u}u, \bar{d}d \rangle = (0.33 \text{GeV})^4$ at the energy scale $\mu = 1$ GeV [19, 21, 22], choose the $\overline{MS}$ masses $m_c(m_c) = (1.28 \pm 0.03)$ GeV, $m_s(\mu = 2$ GeV) = $(0.096 \pm 0.004)$ GeV from the Particle Data Group [2], and neglect the up and down quark masses, i.e., $m_u = m_d = 0$. Moreover, we take into account the energy-scale dependence of the input parameters on the QCD side from the renormalization group equation,

$$\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{1}{\lambda}},$$

$$\langle \bar{s}s \rangle(\mu) = \langle \bar{s}s \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{1}{\lambda}},$$

$$\langle \bar{q}g_sGq \rangle(\mu) = \langle \bar{q}g_sGq \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{1}{\lambda}},$$

$$\langle \bar{s}g_sGs \rangle(\mu) = \langle \bar{s}g_sGs \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{1}{\lambda}},$$

$$m_s(\mu) = m_s(2 \text{ GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2 \text{ GeV})} \right]^{\frac{1}{\lambda}},$$

$$m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{1}{\lambda}},$$

$$\alpha_s(\mu) = \frac{1}{b_0t} \left[ 1 - b_1 \frac{\log t}{b_0^2} + \frac{b_2}{b_0^4 t^2} \left( \log^2 t - \log t - 1 \right) + b_3 \right],$$

(13)

where $t = \log \Lambda^2$, $b_0 = \frac{33 - 2n_f}{12\pi}$, $b_1 = \frac{153 - 19n_f}{24\pi}$, $b_2 = \frac{2857 - 5033n_f + 325n_f^2}{128\pi^3}$, $\Lambda = 213$ MeV, 296 MeV and 339 MeV for the flavors $n_f = 5, 4$ and 3, respectively [2].

For the hadron mass, it is independent of the energy scale because of its observability. However, in calculations, the perturbative corrections are neglected, the operators of the form $O_n(\alpha_s^k)$ with $k > 1$ or the dimensions $n > 10$ are discarded, and the higher dimensional vacuum condensates are factorized into lower dimensional ones therefore the energy-scale dependence of the hadron mass, $m_c$, depending on the energy scale leads to change of integral range $4m_c^2 - s_0$ of the variable $ds$. So we have to consider the energy-scale dependence of the QCD sum rules.

The hidden-charm four-quark system $ccqq'$ could be described by a double-well potential with two light quarks $q'q$ lying in the two wells respectively. In the heavy quark limit, the $c$ quark can be taken as a static well potential, which binds the light quark $q'$ to form a diquark in the color antitriplet channel or binds the light antiquark $\bar{q}$ to form a meson in the color singlet channel (or a meson-like state in the color octet channel). Then the hidden-charm four-quark states are characterized by the effective heavy quark mass $M_c$, and the virtuality $V = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2}$.

The effective mass $M_c$ has uncertainties, the optimal value in the diquark-antidiquark system is
not necessary the ideal value in the meson-meson system. It is natural to take the energy scale \( \mu = V \). In this article, we use the energy-scale formula:

\[
\mu = \sqrt{M_{X/Y/Z}^2 - (2M_0)^2},
\]

with the updated value of the effective \( c \)-quark mass \( M_0 = 1.85 \text{GeV} \) in the meson-meson molecular system to determine the ideal energy scales of the QCD spectral densities [23]. For a better understanding of the energy-scale dependence in Eq. (14), one can refer to Ref. [13, 20, 24], where the authors study the energy-scale dependence of the QCD sum rules for the hidden-charm tetraquark states and molecular states in detail, and suggest the above energy-scale formula for the first time. In our calculations, we observe that the values of the masses \( M \) for the states \( Z \) increase when the energy scales \( \mu \) increase. Thus there exist optimal energy scales, which lead to reasonable masses \( M_{Z_{1,2}} \).

In Eq. (12), there are two free parameters: the Borel Parameter \( T^2 \) and the continuum threshold value \( s_0 \). The extracted hadron mass is a function of the Borel parameter \( T \). To obtain a reliable mass sum rule analysis, we impose two criteria on the hidden-charm molecules to choose suitable working ranges for these two free parameters. The first criterion is the pole dominance on the phenomenological side, which require the pole contributions (PCs) to be about (40 – 60)\%. The PC is defined as:

\[
PC = \frac{\int_{\mu^2}^{s_0} ds \rho_{1,2}(s) \exp\left(-\frac{\mu^2}{s}\right)}{\int_{\mu^2}^{\infty} ds \rho_{1,2}(s) \exp\left(-\frac{\mu^2}{s}\right)},
\]

The second criterion is the convergence of the operator product expansion. To judge the convergence, we calculate the contributions of the vacuum condensates \( D(n) \) in the operator product expansion with the formula:

\[
D(n) = \frac{\int_{\mu^2}^{s_0} ds \rho_{1,2}(s) \exp\left(-\frac{\mu^2}{s}\right)}{\int_{\mu^2}^{\infty} ds \rho_{1,2}(s) \exp\left(-\frac{\mu^2}{s}\right)},
\]

where the index \( n \) denotes the dimension of the vacuum condensates.

To search for the continuum threshold value \( s_0 \) more accurately, we take into account the mass gaps between the ground states and the first radial excited states, which are usually taken as (0.4 – 0.6) GeV in the four-quark sector. For examples, the \( Z(4430) \) is tentatively assigned to be the first radial excitation of the \( Z_c(3900) \) according to the analogous decays, \( Z_c(3900)^\pm \to J/\psi\pi^\pm \), \( Z(4430)^\pm \to \psi'\pi^\pm \) and the mass differences \( M_{Z(4430)} - M_{Z_c(3900)} = 576 \text{MeV}, M_{\psi' - M_{J/\psi}} = 589 \text{MeV} \) [25]; the \( X(3915) \) and \( X(4500) \) are assigned to be the ground state and the first radial excitation of the \( csc\bar{s} \) four-quark states, respectively, and their mass difference is \( M_{X(4500)} - M_{X(3915)} = 588 \text{MeV} \) [26]. The relation

\[
\sqrt{s_0} = M_{X/Y/Z} + (0.4 - 0.6) \text{GeV},
\]

serves as a constraint on the masses of the hidden-charm four-quark states.

In Fig. 1 we show the variations of the pole contributions with respect to the Borel parameters \( T^2 \) for different values of the continuum thresholds \( s_0 \) at the energy scales \( \mu = 2.7 \text{GeV} \) and 2.7 GeV for the states \( Z_1 \) and \( Z_2 \), respectively. From the figure, we can see that the values \( \sqrt{s_0} \leq 4.9 \text{GeV} \) are too small to satisfy the pole dominance condition and result in reasonable Borel windows for these two states \( Z_{1,2} \). To warrant the Borel platforms for the masses, we take the values \( T^2 = (3.7 - 4.1) \text{GeV}^2 \) for the state \( Z_1 \) and \( T^2 = (3.6 - 4.0) \text{GeV}^2 \) for the state \( Z_2 \), respectively. In the above Borel windows, if we choose the values \( \sqrt{s_0} = (5.0 - 5.2) \text{GeV} \), the PCs are about (42 – 60)\% and (42 – 61)\% for the \( Z_{1,2} \), respectively. The pole dominance condition is well satisfied.
Figure 1: The pole contributions with variations of the Borel parameter $T^2$ and the continuum threshold value $s_0$.

Figure 2: The absolute contributions of the vacuum condensates with dimension $n$ in the operator product expansion.

Figure 3: The masses with variations of the Borel parameters $T^2$. 
The central value $M_{Z_1} = 4.61$ GeV is about 260 MeV above the threshold $M_{D^+_s(2317)} = 1870 + 2480 = 4350$ MeV, where the mass of the $D^+_s(2317)$, $M_{D^*_0(2317)} = 2480$ MeV, is taken from the computed results of S. Godfrey and K. Moats about excited charm and charm-strange mesons in Ref. [27], while the central value $M_{Z_2} = 4.60$ GeV is about 130 MeV above the threshold.
\[ M_{D^*+\bar{D}^*_1(2460)} = 2010 + 2460 = 4470 \text{ MeV}. \]

The numerical results indicate that the \( D\bar{D}_{s0}^*(2317) \) and \( D^*\bar{D}^*_1(2460) \) are difficult to form bound state molecular states.

In Refs. \[28\] \[29\], the authors study the analogous heavy meson systems. In Ref. \[28\], Liu, Luo and Zhu study the S-wave \( D_s\bar{D}^*_0(2317) \) system through the heavy meson chiral perturbation theory, considering the \( \eta \) meson exchange between \( D_s \) and \( \bar{D}_s^0(2317) \), which generates a potential to bind them, and observe that there exists the \( D_s\bar{D}^*_0(2317) \) molecular state. In Ref. \[29\], similarly, using the heavy meson chiral perturbation theory, Sanchez et al study the S-wave \( DD_{s0}^*(2317) \) and \( D^*\bar{D}^*_1(2460) \) systems exchanging a kaon to bind \( D(D^*) \) and \( \bar{D}^*_0(2317) \), and observe that there exists the \( DD_{s0}^*(2317) \) and \( D^*\bar{D}^*_1(2460) \) bound states. Differently, in this article, we construct the color singlet-singlet molecule-type interpolating currents \( J_{\text{singlet}} \) to bind them, and observe that there exists the \( D^*\bar{D}^*_1(2460) \) molecular states, and could couple potentially to the \( D\bar{D}_{s0}^*(2317) \) and \( D^*\bar{D}^*_1(2460) \) scattering states, we get the following results.

Furthermore, the \( Z_{1,2} \) are probably the resonance states, since the constructed color singlet-singlet currents \( J_{1,2}(x) \) may not necessarily correspond the \( DD_{s0}^*(2317) \) and \( D^*\bar{D}^*_1(2460) \) bound state molecular states, and could couple potentially to the \( DD_{s0}^*(2317) \) and \( D^*\bar{D}^*_1(2460) \) scattering states, respectively. At the phenomenological side, we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators \( J_{1,2}(x) \) into the correlation functions \( \Pi_{1,2}(p) \) to obtain the hadronic representations. After isolating the \( DD_{s0}^*(2317) \) and \( D^*\bar{D}^*_1(2460) \) scattering states, we get the following results,

\[
\Pi_1(p) = \frac{i}{(2\pi)^4} \int \frac{d^4k}{k^2} \frac{i}{M^2_{D^*} (k^2)} + \frac{i}{M^2_{\bar{D}^*_1} (k^2)} \left\{ \frac{f^2_D f^2_{\bar{D}^*_1}}{m_c + m_q} \frac{k^4 (k+p)^2}{(m_c + m_q)^2} \right\} + \cdots,
\]

\[
\Pi_2(p) = \frac{i}{(2\pi)^4} \int \frac{d^4k}{k^2} \frac{i}{M^2_{D^*} (k^2)} + \frac{i}{M^2_{\bar{D}^*_1} (k^2)} f^2_D f^2_{\bar{D}^*_1} k^2 (k+p)^2 \left[ g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] \times \left[ g_{\mu\nu} - \frac{(k+p)^\mu (k+p)^\nu}{(k+p)^2} \right] + \cdots,
\]

where the decay constants \( f_D, f_{D^*}, f_{\bar{D}^*_0}, \text{ and } f_{\bar{D}^*_1} \) are defined by

\[
\langle 0|J_1(0)|D_s(k)\bar{D}^*_0(k+p)\rangle = 2 \frac{f_D k^2}{m_c + m_q} f_{\bar{D}^*_1} \sqrt{(k+p)^2} + f_D f_{\bar{D}^*_1} k \cdot (k+p),
\]

\[
\langle 0|J_2(0)|D^*(k)\bar{D}^*_0(k+p)\rangle = f_D f_{\bar{D}^*_1} \sqrt{k^2 (k+p)^2} \varepsilon_\mu \varepsilon^\mu,
\]

the \( \varepsilon_\mu \) are the polarization vectors of the \( D^* \) and \( \bar{D}^*_1 \).
We rewrite the correlation functions $\Pi_{1,2}(p)$ into the following forms through dispersion relation,

$$
\Pi_1(p) = \frac{f_D^2 f_{D^*}^2}{16\pi^2} \int_{(MD + MD^*)^2}^{s_0} \frac{ds}{s-p^2} \left[ \left( \frac{M_{D_s^*}^2 - M_D^2 - s}{s} \right)^2 - \frac{4M_D^2}{s} \right] \times \left[ 4M_D^4 M_{D_s^*}^2 (m_c + m_q)^2 + 2M_D^2 M_{D_s^*} \left( \frac{M_D^2 + M_{D_s^*}^2 - s}{m_c + m_q} \right) + \frac{M_D^2 + M_{D_s^*}^2 - s}{2} \right]^2 + \cdots ,
$$

$$
\Pi_2(p) = \frac{f_D^2 f_{D^*}^2}{16\pi^2} \int_{(MD^* + MD^*)^2}^{s_0} \frac{ds}{s-p^2} \left[ \left( \frac{M_{D_s^*}^2 - M_{D^*}^2 - s}{s} \right)^2 - \frac{4M_{D^*}^2}{s} \right] \times M_{D_s^*}^2 M_{D_{s_1}^*}^2 \left[ 2 + \left( \frac{M_{D_s^*}^2 + M_{D_{s_1}^*}^2 - s}{4M_{D_s^*}^2 M_{D_{s_1}^*}^2} \right) \right] + \cdots .
$$

(20)

In this article, we choose the value $s_0 > (MD + MD_s^*)^2$, $(MD^* + MD_{s_1}^*)^2$, the QCD sum rules can be written as

$$
f_D^2 f_{D^*}^2 \int_{(MD + MD_s^*)^2}^{s_0} \frac{ds}{s-p^2} \left[ \left( \frac{M_{D_s^*}^2 - M_D^2 - s}{s} \right)^2 - \frac{4M_D^2}{s} \right] \times \left[ 4M_D^4 M_{D_s^*}^2 (m_c + m_q)^2 + 2M_D^2 M_{D_s^*} \left( \frac{M_D^2 + M_{D_s^*}^2 - s}{m_c + m_q} \right) + \frac{M_D^2 + M_{D_s^*}^2 - s}{2} \right]^2 \exp \left( -\frac{s}{T^2} \right)
$$

$$
= \kappa_1 \int_{4m_t^2}^{s_0} ds p_1(s) \exp \left( -\frac{s}{T^2} \right),
$$

$$
f_D^2 f_{D^*}^2 \int_{(MD^* + MD_{s_1}^*)^2}^{s_0} \frac{ds}{s-p^2} \left[ \left( \frac{M_{D_{s_1}^*}^2 - M_{D^*}^2 - s}{s} \right)^2 - \frac{4M_{D^*}^2}{s} \right] \times M_{D_s^*}^2 M_{D_{s_1}^*}^2 \left[ 2 + \left( \frac{M_{D_s^*}^2 + M_{D_{s_1}^*}^2 - s}{4M_{D_s^*}^2 M_{D_{s_1}^*}^2} \right) \right] \exp \left( -\frac{s}{T^2} \right)
$$

$$
= \kappa_2 \int_{4m_t^2}^{s_0} ds p_2(s) \exp \left( -\frac{s}{T^2} \right),
$$

(21)

where we introduce a coefficient $\kappa_i$, if $\kappa_i = 1$, the QCD sum rules can be saturated by the scattering states $DD_{s0}^*(2317)$ and $D^*D_{s1}^*(2460)$, respectively. The input parameters are taken as $MD = 1.87 \text{ GeV}$, $MD^* = 2.01 \text{ GeV}$, $MD_{s0}^* = 2.46 \text{ GeV}$ [2], $MD_{s1}^* = 2.48 \text{ GeV}$ [27], $f_D = 0.208 \text{ GeV}$, $f_{D^*} = 0.263 \text{ GeV}$, $f_{D_{s0}^*} = 0.333 \text{ GeV}$, $f_{D_{s1}^*} = 0.245 \text{ GeV}$ [30], $s_0 = 5.1^2 \text{ GeV}^2$. In Fig. 5 we plot the coefficient $\kappa_i$ with variation of the energy scale $\mu$ at $T^2 = 3.9 \text{ GeV}^2$ and $3.8 \text{ GeV}^2$ for the $Z_{1,2}$, respectively. At the vicinities of the energy scale $\mu = 2.3 \text{ GeV}$ and $1.1 \text{ GeV}$, $\kappa_i \approx 1$, however, from the figure, we can see that the coefficient $\kappa_i$ decreases monotonously with increase of the energy scale $\mu$. The reliable QCD sum rules do not depend heavily on the energy scale $\mu$. So, the QCD sum rules can not be saturated by the scattering states $DD_{s0}^*(2317)$ and $D^*D_{s1}^*(2460)$, respectively.

In the following, we perform Fierz re-arrangement for the currents $J_{1,2}$ both in the color space
and Dirac-spinor space to obtain the results, we cannot distinguish those contributions to study them exclusively. Hence, we infer that the pairs through its components, with the scattering states meson scattering states such as η molecule states and molecule-like states, and embody the net effects. Moreover, for the meson-loops to the correlation function Π

\[ J_1 = -\frac{i}{12} \bar{c}i\gamma_5 cqs - \frac{i}{12} \bar{c}\gamma_\beta \gamma_5 \bar{c}q\gamma_\beta s - \frac{i}{24} \bar{c}\sigma_{\beta\tau} \gamma_5 c\bar{q}\sigma^{\beta\tau} s + \frac{i}{12} \bar{c}\gamma_\beta \bar{c}q\gamma_\beta \gamma_5 s - \frac{1}{12} \bar{c}cqi \gamma_5 s - \frac{i}{8} \bar{c}i\gamma_5 \lambda^a c\bar{q}\lambda^a s - \frac{i}{8} \bar{c}\gamma_\beta \gamma_5 \lambda^a c\bar{q}\sigma^{\beta\tau} \lambda^a s + \frac{i}{16} \bar{c}\sigma_{\beta\tau} \gamma_5 \lambda^a c\bar{q}\sigma^{\beta\tau} \lambda^a s - \frac{1}{8} \bar{c}\lambda^a c\bar{q}i \gamma_5 \lambda^a s , \]

\[ J_2 = -\frac{i}{3} \bar{c}i\gamma_5 cqs + \frac{1}{6} \bar{c}\gamma_\beta \gamma_5 \bar{c}q\gamma_\beta s + \frac{1}{6} \bar{c}\gamma_\beta \bar{c}q\gamma_\beta \gamma_5 s + \frac{i}{3} \bar{c}cqi \gamma_5 s - \frac{i}{2} \bar{c}i\gamma_5 \lambda^a c\bar{q}\lambda^a s + \frac{1}{4} \bar{c}\gamma_\beta \gamma_5 \lambda^a c\bar{q}\gamma_\beta \lambda^a s + \frac{1}{4} \bar{c}\gamma_\beta \lambda^a c\bar{q}\gamma_\beta \gamma_5 \lambda^a s + \frac{i}{2} \bar{c}\lambda^a c\bar{q}i \gamma_5 \lambda^a s . \]  

(22)

The components \( \bar{c}Tc\bar{q}\Gamma' s \) and \( \bar{c}\Gamma \lambda^a c\bar{q}\Gamma' \lambda^a s \) couple potentially to a series of charmonium-light-meson pairs or charmonium-like molecular states or charmonium-like molecule-like states, where \( \Gamma, \Gamma' = 1, 4, 5, 5, 7, 7, \sigma_\beta, \sigma_\beta \gamma_5 \). For example, the current \( J_1 \) couples potentially to the meson pairs through its components,

\[ \bar{c}i\gamma_5 cqs \propto \eta_c K^*_0, \cdots , \]

\[ \bar{c}cqi \gamma_5 s \propto \chi_{c0} K, \cdots , \]

\[ \bar{c}\gamma_\beta \gamma_5 \bar{c}q\gamma_\beta s \propto \chi_{c1} K^*, h_c K^*, \cdots , \]

\[ \bar{c}\gamma_\beta \bar{c}q\gamma_\beta \gamma_5 s \propto J/\psi K_1, \cdots , \]

\[ \bar{c}\sigma_{\beta\tau} \gamma_5 \bar{c}q\sigma^{\beta\tau} s \propto J/\psi K_1, h_c K^*, \cdots . \]

(23)

We cannot distinguish those contributions to study them exclusively. Hence, we infer that the \( Z_{1,2} \) are particular resonance states, which are the special superpositions of the scattering states, molecular states and molecule-like states, and embody the net effects. Moreover, for the meson-meson scattering states such as \( \eta_c K^*_0, \chi_{c0} K, J/\psi K_1, \cdots \) lying below the \( Z_1 \), the \( Z_1 \) can decay to them easily through fall-apart mechanism, and the decays contribute a finite width to the \( Z_1 \). Now, we discuss an effect of the finite width on the predicted mass \( M_{Z_1} \). We consider the contributions of the meson-loops to the correlation function \( \Pi_1 (p) \), as the current \( J_1 (x) \) has non-vanishing couplings with the scattering states \( \eta_c K^*_0, \chi_{c0} K, J/\psi K_1, \) etc.

\[ \Pi_1 (p) = - \frac{\lambda_{Z_1}^2}{p^2 - M_{Z_1}^2 - \Sigma_{\eta_c} K^*_0 (p) - \Sigma_{\chi_{c0}} K (p) - \Sigma_{J/\psi} K_1 (p) + \cdots } , \]

(24)

where the \( \lambda_{Z_1} \) and \( M_{Z_1} \) are bare quantities to absorb the divergences in the self-energies \( \Sigma_{\eta_c} K^*_0 (p) \), \( \Sigma_{\chi_{c0}} K (p) \), \( \Sigma_{J/\psi} K_1 (p) \), etc. The renormalized self-energies contribute a finite imaginary part to
modify the dispersion relation,
\[ \Pi_1 (p) = -\frac{\lambda^2_{Z_1}}{p^2 - M^2_{Z_1} + i\sqrt{p^2\Gamma(p^2)}} + \cdots . \]  
(25)

The finite width effect is considered through the following simple change in the hadronic spectral density,
\[ \delta(s - M^2_{Z_1}) \rightarrow \frac{1}{\pi} \frac{\sqrt{s\Gamma_{Z_1}}(s)}{(s - M^2_{Z_1})^2 + s\Gamma^2_{Z_1}(s)}, \]  
(26)

where
\[ \Gamma_{Z_1}(s) = \Gamma_{Z_1} \frac{M^2_{Z_1}}{s}. \]  
(27)

We take the central values of the input parameters, and \( \Gamma_{Z_1} = 300 \text{MeV}(\text{not small}) \). Then the phenomenological side of the QCD sum rules in Eq. \( \text{(10)} \) changes as follows,
\[ B_{T_1}\Pi_1 = \lambda^2_{Z_1} \exp\left(-\frac{M^2_{Z_1}}{T^2}\right) \]
\[ \rightarrow \frac{\lambda^2_{Z_1}}{\pi} \int_{(M_{j/p} + M_{K^0})^2}^{s_0} ds \frac{\sqrt{s\Gamma_{Z_1}}(s)}{(s - M^2_{Z_1})^2 + s\Gamma^2_{Z_1}(s)} \exp\left(-\frac{s}{T^2}\right) \]
\[ = 0.70\lambda^2_{Z_1} \exp\left(-\frac{M^2_{Z_1}}{T^2}\right) \]  
(28)

and
\[ -\frac{1}{d(1/T^2)} B_{T_2}\Pi_1 = M^2_{Z_1} \lambda^2_{Z_1} \exp\left(-\frac{M^2_{Z_1}}{T^2}\right) \]
\[ \rightarrow \frac{\lambda^2_{Z_1}}{\pi} \int_{(M_{j/p} + M_{K^0})^2}^{s_0} ds \frac{\sqrt{s\Gamma_{Z_1}}(s)}{(s - M^2_{Z_1})^2 + s\Gamma^2_{Z_1}(s)} \exp\left(-\frac{s}{T^2}\right) \]
\[ = 0.70M^2_{Z_1} \lambda^2_{Z_1} \exp\left(-\frac{M^2_{Z_1}}{T^2}\right), \]  
(29)

where the \( B_{T_2} \) denotes the Borel transformation. The numerical factor 0.70 can be absorbed safely into the pole residue \( \lambda_{Z_1} \). Therefore, in this article, when we take the zero width approximation in Eq. \( \text{(10)} \), the predicted masses \( M_{Z_{1,2}} \) are reasonable.

\section{Conclusion}

In this article, we assume that there exist the pseudoscalar \( \tilde{D}D^*_0(2317) \) and \( D^*\tilde{D}^*_1(2460) \) molecular states \( Z_{1,2} \), and study their masses with the color singlet-singlet interpolating currents through the QCD sum rule approach. In calculations, we carry out the operator product expansion up to the vacuum condensates of dimension 10 and use the formula \( \mu = \sqrt{M^2_{X/Y/Z} - (2M_c)^2} \) to determine the energy scales of the QCD spectral densities. The numerical results show that the central value of the state \( Z_1 \), \( M_{Z_1} = 4.61 \text{GeV} \), is about 260 MeV above the \( DD^*_0(2317) \) threshold, while, in the case of the \( Z_2 \), the central value \( M_{Z_2} = 4.61 \text{GeV} \) is about 130 MeV above the \( D^*\tilde{D}^*_1(2460) \) threshold, which indicate that the \( DD^*_0(2317) \) and \( D^*\tilde{D}^*_1(2460) \) are difficult to form bound state molecular states. The \( Z_{1,2} \) are probably particular resonance states, which are the special superpositions of the scattering states, molecular states and molecule-like states, and embody the net effects. We expect that these results in our work could be helpful for investigating the \( Z_{1,2} \) experimentally, and would be able to be testified in the future experiments, such as BESIII, LHCb and Belle-II.
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Appendix

The explicit expressions of the QCD spectral densities $\rho_{1,2}(s)$,

$$\rho_1^0(s) = \frac{3}{2048\pi^4} \int_{y_1}^{y_f} dy \int_{z_i}^{1-y} dz \ yz (1-y-z)^3 (s-\tilde{m}_c^2) \ (7s^2 - 6s\tilde{m}_c^2 + \tilde{m}_c^4)$$

$$\rho_1^1(s) = \frac{3m_s m_c}{2048\pi^6} \int_{y_1}^{y_f} dy \int_{z_i}^{1-y} dz \ (y+z) (1-y-z)^2 (s-\tilde{m}_c^2)(5s-2\tilde{m}_c^2), \quad (30)$$

$$\rho_2^0(s) = \frac{3}{1024\pi^4} \int_{y_1}^{y_f} dy \int_{z_i}^{1-y} dz \ yz (1-y-z)^3 (s-\tilde{m}_c^2) \ (7s^2 - 6s\tilde{m}_c^2 + \tilde{m}_c^4)$$

$$\rho_2^1(s) = \frac{3m_s m_c}{1024\pi^6} \int_{y_1}^{y_f} dy \int_{z_i}^{1-y} dz \ (y+z) (1-y-z)^2 (s-\tilde{m}_c^2)^3 (3s-\tilde{m}_c^2)$$

$$\rho_1^2(s) = \frac{3m_s m_c (\langle s\rangle - \langle q\bar{q}\rangle)}{128\pi^4} \int_{y_1}^{y_f} dy \int_{z_i}^{1-y} dz \ (y+z) (1-y-z) (s-\tilde{m}_c^2) \ (2s-\tilde{m}_c^2)$$

$$\rho_2^2(s) = \frac{3m_s m_c (\langle s\rangle - \langle q\bar{q}\rangle)}{64\pi^4} \int_{y_1}^{y_f} dy \int_{z_i}^{1-y} dz \ (y+z) (1-y-z) (s-\tilde{m}_c^2) \ (2s-\tilde{m}_c^2)$$

$$\rho_3^0(s) = \frac{m_s m_c}{2048\pi^4} \left(\frac{\alpha_{GG}}{\pi}\right) \int_{y_1}^{y_f} dy \int_{z_i}^{1-y} dz \ \left(\frac{y}{z^3} + \frac{z}{y^3} + \frac{1}{y^2} + \frac{1}{z^2}\right) (1-y-z)^2 \ [2 + 8\delta(s-\tilde{m}_c^2)]$$

$$\rho_3^1(s) = \frac{m_s m_c}{512\pi^4} \left(\frac{\alpha_{GG}}{\pi}\right) \int_{y_1}^{y_f} dy \int_{z_i}^{1-y} dz \ \left(\frac{z}{y^2} + \frac{y}{z^2}\right) (1-y-z)^3 \ \left[2s-\tilde{m}_c^2 + \frac{2^2}{6}\delta(s-\tilde{m}_c^2)\right]$$

$$\rho_3^2(s) = \frac{m_s m_c}{2048\pi^4} \left(\frac{\alpha_{GG}}{\pi}\right) \int_{y_1}^{y_f} dy \int_{z_i}^{1-y} dz \ \left[\frac{(z-y)}{z^2} + \frac{y}{y^2}\right] (1-y-z) + 4 \ (1-y-z) (3s-\tilde{m}_c^2)$$

$$\rho_3^3(s) = \frac{3}{1024\pi^4} \left(\frac{\alpha_{GG}}{\pi}\right) \int_{y_1}^{y_f} dy \int_{z_i}^{1-y} dz \ (y+z) (1-y-z)^2 (10s^2 - 12s\tilde{m}_c^2 + 3\tilde{m}_c^4), \quad (34)$$
\[
\rho_2^s(s) = \frac{m_c^3}{1024\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{y}{z} + \frac{z}{y} + \frac{1}{y^2} + \frac{1}{z^2} \right) (1 - y - z)^2 \left[ 2 + s\delta(s - \bar{m}_c^2) \right] \\
- \frac{2m_c^2 + m_c}{1024\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{y}{z^2} + \frac{z}{y^2} \right) (1 - y - z)^2 \left( 3s - 2\bar{m}_c^2 \right) \\
- m_c^2 \frac{\alpha_s GG}{256\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( 1 - y - z \right) \left( s - \bar{m}_c^2 \right) \left( 2s - \bar{m}_c^2 \right) \\
+ \frac{1}{1256\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( y + z \right) \left( 1 - y - z \right) \left( s - \bar{m}_c^2 \right) \left( 2s - \bar{m}_c^2 \right) \\
- \frac{1}{1256\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( y + z \right) \left( 1 - y - z \right) \left( s - \bar{m}_c^2 \right) \left( 2s - \bar{m}_c^2 \right) \\
+ \frac{3m_c m_c}{256\pi^4} \frac{\alpha_s GG}{\pi} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( 1 - y - z \right) \left( 3s - 2\bar{m}_c^2 \right), \tag{35}
\]

\[
\rho_3^s(s) = \frac{3m_c \left( \langle \bar{q} g_s G \rangle - \langle \bar{g} q_s G \rangle \right)}{512\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left[ \left( y + z \right) - 2 \left( \frac{y}{z} + \frac{z}{y} \right) (1 - y - z) \right] \\
\left( 3s - 2\bar{m}_c^2 \right) - \frac{3m_c \langle \bar{q} g_s G \rangle}{128\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( 2s - \bar{m}_c^2 + \frac{s^2}{6}\delta(s - \bar{m}_c^2) \right) \\
+ \frac{3m_c \langle \bar{q} g_s G \rangle}{256\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( 3s - 2\bar{m}_c^2 \right) - \frac{3m_c m_c^2 \langle \bar{q} g_s G \rangle}{256\pi^4} \int_{y_i}^{y_f} dy, \tag{36}
\]

\[
\rho_2^s(s) = \frac{3m_c \langle \bar{g} q_s G \rangle - \langle \bar{q} g_s G \rangle}{256\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( y + z \right) \left( 3s - 2\bar{m}_c^2 \right) \\
- \frac{3m_c \langle \bar{q} g_s G \rangle}{64\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( 2s - \bar{m}_c^2 + \frac{s^2}{6}\delta(s - \bar{m}_c^2) \right) \\
- \frac{m_c \langle \bar{g} q_s G \rangle}{128\pi^4} \int_{y_i}^{y_f} dy \left( 1 - y \right) \left( 3s - 2\bar{m}_c^2 \right) - \frac{3m_c m_c^2 \langle \bar{q} g_s G \rangle}{64\pi^4} \int_{y_i}^{y_f} dy, \tag{37}
\]

\[
\rho_1^s(s) = \frac{-m_c^2 \langle \bar{q} q \rangle}{16\pi^2} \int_{y_i}^{y_f} dy \left( 1 + \frac{1}{z} \right) \\
+ \frac{g_s^2 \langle \bar{q} q \rangle^2 + \langle \bar{s} s \rangle^2}{288\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{y}{z} + \frac{z}{y} \right) (1 - y - z) \left( 3s - 2\bar{m}_c^2 \right) \\
- \frac{g_s^2 \langle \bar{q} q \rangle^2 + \langle \bar{s} s \rangle^2}{288\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{y}{z} + \frac{z}{y} \right) (1 - y - z) \left( 3s - 2\bar{m}_c^2 \right) \\
- \frac{-m_c^2 g_s^2 \langle \bar{q} q \rangle^2 + \langle \bar{s} s \rangle^2}{576\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{y}{z^2} + \frac{z}{y^2} \right) (1 - y - z) \left[ 2 + s\delta(s - \bar{m}_c^2) \right] \\
- \frac{-m_c m_c g_s^2 \langle \bar{q} q \rangle \langle \bar{s} s \rangle}{64\pi^2} + \frac{g_s^2 \langle \bar{q} q \rangle^2}{3456\pi^4} \int_{y_i}^{y_f} dy \left[ 2 + s\delta(s - \bar{m}_c^2) \right] \\
+ \frac{m_c m_c^2 g_s^2 \langle \bar{q} q \rangle}{576\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( 1 - y - z \right) \left( 3s - 2\bar{m}_c^2 \right), \tag{38}
\]
\[ \rho_2^6(s) = \frac{g_s^2((\bar{q}q)^2 + (\bar{s}s)^2)}{144\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left[ yz - 2(y+z)(1-y-z) \right] \left[ 2s - \tilde{m}_c^2 + \frac{s^2}{6} \delta(s - \tilde{m}_c^2) \right] \\
- m_c^2(\bar{q}q)(\bar{s}s) \int_{y_i}^{y_f} dy - m_s m_c \left( \frac{(\bar{q}q)(\bar{s}s)}{32\pi^2} + \frac{g_s^2(\bar{q}q)^2}{1728\pi^4} \right) \int_{y_i}^{y_f} dy \left[ 2 + s\delta(s - \tilde{m}_c^2) \right] \\
+ \frac{g_s^2((\bar{q}q)^2 + (\bar{s}s)^2)}{864\pi^4} \int_{y_i}^{y_f} dy (1-y) (3s - 2\tilde{m}_c^2) \\
- \frac{g_s^2((\bar{q}q)^2 + (\bar{s}s)^2)}{288\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{y}{z} + \frac{z}{y} \right) (1-y-z) (3s - 2\tilde{m}_c^2) \\
- \frac{m_c^2 g_s^2((\bar{q}q)^2 + (\bar{s}s)^2)}{864\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{y}{z^2} + \frac{z}{y^2} \right) (1-y-z) \left[ 2 + s\delta(s - \tilde{m}_c^2) \right] \\
+ \frac{m_s m_c g_s^2(\bar{q}q)^2}{288\pi^4} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left[ 2 \left( \frac{1}{y} + \frac{1}{z} \right) + \left( \frac{1}{y^2} + \frac{1}{z^2} \right) m_c^2 \delta(s - \tilde{m}_c^2) \right], \tag{39} \]

\[ \rho_1^7(s) = \frac{4\pi^3}{256\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left[ \frac{(y-z)^2}{z^2} + \frac{z}{y^2} \right] \left( 1 - y - z \right) \left[ 2 + s\delta(s - \tilde{m}_c^2) \right] \\
+ \frac{4\pi^2}{768\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{y}{z^3} + \frac{z}{y^3} + \frac{1}{y^2} + \frac{1}{z^2} \right) \left( 1 - y - z \right) \left( 1 + \frac{s}{T^2} \right) \delta(s - \tilde{m}_c^2) \\
+ \frac{m_c^2(\bar{s}s)(\bar{q}q)}{128\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{1}{y^2} + \frac{1}{z^2} \right) \delta(s - \tilde{m}_c^2) \\
- \frac{m_s m_c^2(\bar{s}s)(\bar{q}q)}{384\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{1}{y^2} + \frac{1}{z^2} \right) \delta(s - \tilde{m}_c^2) \\
- \frac{m_c^4 (\bar{q}q)(\bar{s}s)}{384\pi^2 T^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{1}{y^3} + \frac{1}{z^3} \right) \delta(s - \tilde{m}_c^2) \\
+ \frac{m_c^4 (\bar{s}s)(\bar{q}q)}{256\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{1}{y^3} + \frac{1}{z^3} \right) \delta(s - \tilde{m}_c^2) \\
+ \frac{m_s m_c^2 (\bar{q}q)(\bar{s}s)}{768\pi^2} \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left[ 3 + \left( 2s + \frac{s^2}{2T^2} \right) \delta(s - \tilde{m}_c^2) \right], \tag{40} \]
\[
\rho_2(s) = \frac{m_\epsilon((\bar{q}q) - (\bar{s}s))}{128\pi^2} \left( \frac{\alpha_sGG}{\pi} \right) \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left[ 2 - \left( \frac{y}{z^2 + y^2} \right) (1 - y - z) \right] \left[ 2 + s\delta(s - \bar{m}_c^2) \right] \\
+ \frac{m_c(\bar{q}q) - (\bar{s}s)}{384\pi^2} \left( \frac{\alpha_sGG}{\pi} \right) \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left( \frac{1}{y^2 + \frac{1}{y^2} + \frac{y}{z} + \left( - \frac{1}{y^2 + \frac{1}{y^2} + \frac{z}{y} \right)} (1 - y - z) \right) \left[ 1 + \frac{s}{T^2} \right] \delta(s - \bar{m}_c^2) \\
\left( 1 + \frac{s}{T^2} \right) \delta(s - \bar{m}_c^2) - \frac{m_c(\bar{q}q) - (m_c + m_s)(\bar{s}s)}{768\pi^2} \left( \frac{\alpha_sGG}{\pi} \right) \int_{y_i}^{y_f} dy \left[ 2 + s\delta(s - \bar{m}_c^2) \right] \\
+ \frac{m_c^2(\bar{q}q)}{96\pi^2} \left( \frac{\alpha_sGG}{\pi} \right) \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left[ 3 - \left( \frac{1}{y^2 + \frac{1}{y^2} + \frac{y}{z} + \left( - \frac{1}{y^2 + \frac{1}{y^2} + \frac{z}{y} \right)} (1 - y - z) \right) \left( 1 + \frac{s}{T^2} \right) \delta(s - \bar{m}_c^2) \\
+ \frac{m_c^2(\bar{s}s)}{384\pi^2} \left( \frac{\alpha_sGG}{\pi} \right) \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left[ \frac{y}{z^2 + y^2} + \frac{z}{y^2} \right] (1 - y - z) \left( 1 + \frac{s}{T^2} + \frac{s^2}{2T^4} \right) \delta(s - \bar{m}_c^2) \\
- \frac{m_c^2(\bar{s}s)}{192\pi^2} \left( \frac{\alpha_sGG}{\pi} \right) \int_{y_i}^{y_f} dy \int_{z_i}^{1-y} dz \left[ 3 + \left( 2s + \frac{s^2}{2T^2} \right) \delta(s - \bar{m}_c^2) \\
+ \frac{m_c^2(\bar{q}q)}{192\pi^2} \left( \frac{\alpha_sGG}{\pi} \right) \int_{y_i}^{y_f} dy \left( 1 + \frac{s}{T^2} \right) \delta(s - \bar{m}_c^2) \right], (41)
\]

\[
\rho_1^S(s) = \frac{(\bar{q}q)(\bar{g}_sG_s) + (\bar{s}s)(\bar{q}g_sG_q)}{64\pi^2} \int_{y_i}^{y_f} dy \left[ m_c^2 \left( 1 + \frac{s}{T^2} \right) - s \right] \delta(s - \bar{m}_c^2) \\
+ \frac{m_c^2(2\bar{q}q)(\bar{g}_sG_s) + 3(\bar{s}s)(\bar{q}g_sG_q)}{384\pi^2} \int_{y_i}^{y_f} dy \left( 1 + \frac{s}{T^2} + \frac{s^2}{2T^4} \right) \delta(s - \bar{m}_c^2) \\
+ \frac{m_c(\bar{q}q)(\bar{g}_sG_s) + (\bar{s}s)(\bar{g}_sG_q)}{64\pi^2} \int_{y_i}^{y_f} dy \left[ 1 - \frac{1}{2y(1-y)} \right] \left( 1 + \frac{s}{T^2} \right) \delta(s - \bar{m}_c^2) \right], (42)
\]

\[
\rho_2^S(s) = \frac{m_c^2(2\bar{q}q)(\bar{g}_sG_s) + 3(\bar{s}s)(\bar{q}g_sG_q)}{192\pi^2} \int_{y_i}^{y_f} dy \left( 1 + \frac{s}{T^2} + \frac{s^2}{2T^4} \right) \delta(s - \bar{m}_c^2) \\
+ \frac{m_c^2((\bar{q}q)(\bar{g}_sG_s) + (\bar{s}s)(\bar{q}g_sG_q))}{16\pi^2} \int_{y_i}^{y_f} dy \left( 1 + \frac{s}{T^2} \right) \delta(s - \bar{m}_c^2) \right], (43)
\]

\[
\rho_1^{10}(s) = -\frac{(\bar{q}q)(\bar{g}_sG_q)}{64\pi^2} \int_{y_i}^{y_f} dy \left( \frac{49s}{48T^2} - \frac{s^2}{2T^4} + \frac{m_c^2s^2}{4T^6} \right) \delta(s - \bar{m}_c^2) \\
+ \frac{\bar{q}q)(\bar{s}s)}{288} \left( \frac{\alpha_sGG}{\pi} \right) \int_{y_i}^{y_f} dy \left\{ \left( \frac{1}{y^3} + \frac{1}{(1-y)^3} \right) \frac{m_c^4}{T^4} - \frac{m_c^2s^2}{T^6} \right\} \delta(s - \bar{m}_c^2) \\
- 3 \left[ \frac{1}{y^2 + \frac{1}{y^2}} \right] \left[ \frac{m_c^2}{T^2} \right] \delta(s - \bar{m}_c^2) \\
- \frac{m_c(\bar{q}q)(\bar{s}s)}{384} \left( \frac{\alpha_sGG}{\pi} \right) \int_{y_i}^{y_f} dy \left\{ \left( \frac{1}{y^2 + \frac{1}{y^2}} \right) \frac{y}{T^4} + \frac{s^3}{6T^8} \right\} \delta(s - \bar{m}_c^2) \\
+ \frac{m_c(\bar{q}g_sG_q)(\bar{g}_sG_q)}{768\pi^2} \int_{y_i}^{y_f} dy \left\{ \left( \frac{1}{y(1-y)} - \frac{s^2}{T^6} - \frac{s^3}{2T^8} \right) \delta(s - \bar{m}_c^2) \\
- \frac{m_c(\bar{q}q)(\bar{s}s)}{1152T^4} \left( \frac{\alpha_sGG}{\pi} \right) \int_{y_i}^{y_f} dy \left[ \frac{1}{y^3} + \frac{1}{(1-y)^3} \right] \left( 1 - \frac{s}{T^2} \right) \delta(s - \bar{m}_c^2) \right], (44)
\]
\[ \rho_{2}^{p}(s) = -\frac{m_{0}^{2}\langle \bar{q}q, \sigma Gq \rangle \langle \bar{q}g, \sigma Gs \rangle}{64\pi^{2}T^{6}} \int_{y_f}^{y_i} dy \left[ s^{2} \delta(s - \hat{m}_{c}^{2}) - \frac{\langle qg, \sigma Gq \rangle \langle \bar{q}g, \sigma Gs \rangle}{256\pi^{2}T^{2}} \int_{y_i}^{y_f} dy \right] \delta(s - \hat{m}_{c}^{2}) \\
- \frac{m_{0}m_{c}\langle \bar{q}q, \sigma Gq \rangle \langle \bar{q}g, \sigma Gs \rangle}{768\pi^{2}T^{8}} \int_{y_{i}}^{y_{f}} dy \left[ s^{2} \delta(s - \hat{m}_{c}^{2}) \right] \\
+ \frac{m_{0}^{2}\langle \bar{q}q \rangle \langle s \bar{s} \rangle}{72T^{2}} \left( \frac{\alpha_{s}GG}{\pi} \right) \int_{y_{i}}^{y_{f}} dy \left\{ \frac{1}{y^{4}} + \frac{1}{(1 - y)^{3}} \right\} m_{c}^{2} - 3 \left( \frac{1}{y^{2}} + \frac{1}{(1 - y)^{2}} \right) \right \delta(s - \hat{m}_{c}^{2}) \\
- \frac{m_{0}m_{c}\langle \bar{q}q \rangle \langle s \bar{s} \rangle}{1152T^{4}} \left( \frac{\alpha_{s}GG}{\pi} \right) \int_{y_{i}}^{y_{f}} dy \left\{ \frac{s^{3}}{T^{4}} + 6s \left( \frac{1}{y^{2}} + \frac{1}{(1 - y)^{2}} \right) - \frac{1}{(1 - y)^{2}} \right\} \delta(s - \hat{m}_{c}^{2}) \\
- \frac{m_{0}m_{c}^{2}\langle \bar{q}q \rangle \langle s \bar{s} \rangle}{576T^{4}} \left( \frac{\alpha_{s}GG}{\pi} \right) \int_{y_{i}}^{y_{f}} dy \left\{ \frac{1}{y^{3}} \right\} \left( 1 - \frac{s}{T^{2}} \right) \delta(s - \hat{m}_{c}^{2}), \quad (45) \right.

where \( y_f = \frac{1 + \sqrt{1 - 4m_{c}^{2}y}}{2}, \quad y_i = \frac{1 - \sqrt{1 - 4m_{c}^{2}y}}{2}, \quad z_{i} = \frac{m_{0}^{2}}{y_{i}(1 - y_{i})}, \quad \hat{m}_{c}^{2} = \frac{m_{c}^{2}}{y_{i}(1 - y_{i})}, \quad \int_{y_{i}}^{y_{f}} dy \rightarrow \int_{0}^{1} \int_{1-y}^{1} dz, \) when the δ functions \( \delta(s - \hat{m}_{c}^{2}) \) and \( \delta(s - \hat{m}_{c}^{2}) \) appear.

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