Sphalerons from the minimal 331 model and baryogenesis

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Abstract. The sphaleron energy and rate are investigated within the minimal 331 model. It is shown, that the constraint leading to the first order phase transition is satisfied for both spontaneous symmetry breaking vacuums of the model. A comparison with the economical model study is also made.

1. Introduction
It is well-known that the Standard Model (SM) of particle physics does not contain all of the ingredients necessary for the electroweak baryogenesis. Consequently, new physics beyond the SM is needed, see for example ref [1].

As a matter of fact, the minimal 331 model is enjoying increasing interest due to its rich phenomenology [2]. In fact, as far as we are interested in, the first-order electroweak phase transition (EWPT) in this model is satisfied. Nevertheless an accurate and precise calculation of the energy of the sphaleron is needed to assess the viability of the EWPT because the energy of the latter is directly proportional to the number of baryons typically written \( \nu_c / T_c > 1 \).

The leading goal of the present work is the calculation of the energy and the rate of sphaleron within the minimal 331 model. It is shown, that the latter has a naturally first order EWPT needed for a successful baryonic asymmetry of the universe. The paper is organized as follow, in section 2, we present briefly the minimal 331 model. Section 3 is dedicated to the EWPT formalism. Section 4, sphaleron energy and rate are calculated and discussed. Finally, in section 5, we draw our conclusions.

2. Particles content of the minimal 331 model
The model proposed in ref.[2] is based on the fact, that each family of leptons comes in triplet, in the quark sector, the first family comes in \( SU(3)_c \) triplet and the two others come in anti-triplet representation :

\[
\begin{bmatrix}
  \nu_i \\
  l \\
  \ell^c
\end{bmatrix}
\]

with \( l = e, \mu, \tau \), \( Q_{it} = \begin{pmatrix} u_i \\ d_i \\ J^{(i)} \end{pmatrix} \), \( Q_{it} = -u_i \) and \( i = 2, 3 \). \hspace{1cm} (1)

The scalar potential is given by:
where \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) are dimensionless coupling constants, and \( \mu_1^2, \mu_2^2 \) are the masses of the two scalar triples \( \rho \) and \( \chi \), where:

\[
\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{**} \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^- \\ \chi^0 \\ \chi^{**} \end{pmatrix}
\]

It is worth mentioning that in this model, symmetry breaking is performed in two steps. The first one when \( \chi^0 \) develops a vacuum expectation value (VeV) \( v_\chi \) and the second one when \( \rho^0 \) gets a (VeV) \( v_\rho \):

\[
SU(3)_L \otimes U(1)_Y \rightarrow SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{QED}}
\]

Moreover, the model we are using contains two neutral Higgses \( H^0_1, H^0_2 \) and a doubly charged Higgs \( h^{++} \) with the following masses:

\[
M^2_{H^0_1} = (\lambda_1 - \frac{\lambda_2^2}{4\lambda_2^2})v_\rho^2, \quad M^2_{H^0_2} = \lambda_2v_\rho^2 + \frac{\lambda_3^2}{4\lambda_2^2}v_\rho^2, \quad M^2_{h^{++}} = \lambda_4(v_\rho^2 + v_\chi^2)/2
\]

The Lagrangian density which gives the masses of gauge bosons is given by:

\[
\mathcal{L} = (D_\nu\chi)^+(D_\nu\chi) + (D_\rho\rho)^+(D_\rho\rho)
\]

where \( D_\mu = \partial_\mu - i(g^{\chi}W^\mu_\rho/2 + g^N B^\mu_\rho) \), with \( \lambda^a \) are the Gell-Mann matrices, \( N \) is a quantum number of the group \( U(1)_Y \). It is to be noted that, this model contains the ordinary gauge bosons of the SM (\( W^\pm, Z, \gamma \)) and a minimal extension in this sector by adding new gauge bosons (\( V^+, U^{\pm}, Z' \)) with the following masses respectively:

\[
M^2_{V^+} = g^2v_\rho^2/4, \quad M^2_{U^+} = g^2v_\rho^2/4, \quad M^2_{U^{\pm}} = g^2v_\rho^2/4c_w^2, \quad M^2_Z = g^2v_\rho^2/4c_w^2, \quad M^2_{Z'} = g^2c_w^2v_\rho^2/(3 - 4s_w^2)
\]

### 3. The Electroweak Phase Transition

The one-loop effective potential at finite temperature in the \( \overline{DR} \) renormalization scheme [2] is given by:

\[
V_{\text{eff}}(v_\rho, v_\chi) = \frac{1}{2}(\mu_\rho^2v_\rho^2 + \mu_\chi^2v_\chi^2) + \frac{1}{4}(\lambda_1v_\rho^4 + \lambda_2v_\rho^2v_\chi^2 + \lambda_3v_\rho^2v_\chi^2)
\]

\[
+ \sum_i n_i \frac{m_i^4}{64\pi^2} \log \frac{m_i^2}{\Lambda^2} - \frac{3}{2} \frac{T^4}{2\pi^2} + \sum_i n_i J_{B/P}(m_i/T^2)
\]

Here \( \Lambda = \nu_\rho = 246 \text{ GeV} \) is the renormalization constant, the functions \( J_{B/P}(\alpha) \) represent thermal bosonic and fermionic contributions [3], \( m_i \) are the field-dependent masses and \( n_i \) are the multiplicities of the field, with \( n_{w^\pm}, v^+, u^{\pm} = 6, \quad n_\rho = n_\chi = 3, \quad n_{H^0_1}, n_{H^0_2} = 1, \quad n_{h^{++}} = 2 \) and \( n_{1,2,3,4} = -12 \).

The effective potential of the first step of (EWPT) at a finite temperature has the following compact form:

\[
V_{\text{eff}}(v_\chi) = D(T^2 - T_0^2)v_\chi - ETv_\chi^2 + \frac{\lambda(T)}{4}v_\chi^4
\]

notice that, this phase transition occurs at TeV scale. The parameters of equation (9) are shown to have the following expressions:
\[
D = \frac{1}{24\nu^2} \left(6m_{\psi}^2 + 6m_{U_{1s}}^2 + 3m_{Z}^2 + 6m_{\psi}^2 + 6m_{\psi}^2 + 2m_{h^{+}}^2 \right)
+ 6m_{\psi}^2 + 6m_{\psi}^2 + m_{H_2^0}^2 + 2m_{h^{+}}^2 \right)
E = \frac{1}{12\pi\nu^2} \left(6m_{\psi}^2 + 6m_{U_{1s}}^2 + 3m_{Z}^2 + m_{H_2^0}^2 + 2m_{h^{+}}^2 \right)
\]

(10)

\[
T_0^2 = \frac{m_{H_2^0}^2}{4D} - \frac{1}{32D\pi^2\nu^2} \left(6m_{\psi}^2 + 6m_{U_{1s}}^2 + 3m_{Z}^2 - 12m_{\psi}^2 - 12m_{\psi}^2 - 12m_{\psi}^2 - 12m_{\psi}^2 \right)
\]
\[
\lambda(T) = \frac{m_{H_2^0}^2}{2\nu^2} - \frac{1}{16\pi^2\nu^2} \left(6m_{\psi}^2 \log \frac{m_{\psi}^2}{A_bT^2} + 6m_{U_{1s}}^2 \log \frac{m_{U_{1s}}^2}{A_bT^2} \right.
+ 3m_{Z}^2 \log \frac{m_{Z}^2}{A_bT^2} - 12m_{\psi}^2 \log \frac{m_{\psi}^2}{A_bT^2} - 12m_{\psi}^2 \log \frac{m_{\psi}^2}{A_bT^2} \right)
\]
\[
- 12m_{\psi}^2 \log \frac{m_{\psi}^2}{A_bT^2} + m_{\psi}^2 \log \frac{m_{H_2^0}^2}{A_bT^2} + 2m_{h^{+}}^2 \log \frac{m_{h^{+}}^2}{A_bT^2} \right)
\]

Now, the effective potential of the second step of (EWPT) at a finite temperature reads:
\[
V_{eff}(\nu_\rho) = D'(T^2 - T_0^2)\nu_\rho^2 - E'\nu_\rho^2 + \frac{\lambda'(T)}{4}\nu_\rho^4
\]

(11)

where this second phase transition occurs at the GeV scale. The parameters \(D', E', T_0^2\) and \(\lambda'(T)\) are given by:
\[
D' = \frac{1}{24\nu_\rho^2} \left(6m_{\psi}^2 + 6m_{U_{1s}}^2 + 3m_{Z}^2 + 6m_{\psi}^2 + m_{H_2^0}^2 + 2m_{h^{+}}^2 \right)
E' = \frac{1}{12\pi\nu_\rho^2} \left(6m_{\psi}^2 + 6m_{U_{1s}}^2 + 3m_{Z}^2 + m_{H_2^0}^2 + 2m_{h^{+}}^2 \right)
T_0^2 = \frac{m_{H_2^0}^2}{4D} - \frac{1}{32D\pi^2\nu_\rho^2} \left(6m_{\psi}^2 + 6m_{U_{1s}}^2 + 3m_{Z}^2 - 12m_{\psi}^2 - 12m_{\psi}^2 - 12m_{\psi}^2 \right)
\]
\[
\lambda'(T) = \frac{m_{H_2^0}^2}{2\nu_\rho^2} - \frac{1}{16\pi^2\nu_\rho^2} \left(6m_{\psi}^2 \log \frac{m_{\psi}^2}{A_bT^2} + 6m_{U_{1s}}^2 \log \frac{m_{U_{1s}}^2}{A_bT^2} \right.
+ 3m_{Z}^2 \log \frac{m_{Z}^2}{A_bT^2} - 12m_{\psi}^2 \log \frac{m_{\psi}^2}{A_bT^2} - 12m_{\psi}^2 \log \frac{m_{\psi}^2}{A_bT^2} \right)
\]
\[
- 12m_{\psi}^2 \log \frac{m_{\psi}^2}{A_bT^2} + m_{\psi}^2 \log \frac{m_{H_2^0}^2}{A_bT^2} + 2m_{h^{+}}^2 \log \frac{m_{h^{+}}^2}{A_bT^2} \right)
\]

(12)
For successful electroweak baryogenesis, the condition of a strong first-order (EWPT) should be typically written as \( \frac{\nu_c}{T_c} \geq 1 \) see ref.[4]. In our model, to perform the symmetry breaking, we have two steps. In other words, as it was shown in eq.(4), we have two strong first-order phase transitions, the first occurs at TeV scale with the condition of (EWPT) written as \( \frac{\nu_{c1}}{T_{c1}} \geq 1 \), and the second occurs at GeV scale with the condition of (EWPT) written as \( \frac{\nu_{c2}}{T_{c2}} \geq 1 \).

For the sake of illustration, we have fixed the values of some free parameters of the model, in case:

\[
\begin{align*}
\nu_\rho &= 246 \text{ GeV}, \quad g = 0.65, \quad \lambda_1 = 0.3225, \quad \lambda_2 = 0.1406, \quad \lambda_3 = 0.1882, \\
\lambda_4 &= 0.1063, \quad \lambda^2 = 0.9875, \quad \lambda^{J_1} = 0.2533, \quad \lambda^{J_2} = 0.3979, \quad \lambda^{J_3} = 0.3494,
\end{align*}
\]

notice that, all our dimensionless coupling constants are real and checked the conditions of unitarity and stability. Moreover, the masses of the Higgs \( h_0 \) and those of the gauge bosons \( (Z, W^\pm) \) of the SM as well as the mass of the top quark \( (t) \) are also fixed. In order to perform our calculations, we take \( \nu_c \sim 2 \) TeV. Under these conditions, the other remaining particles acquired the following masses:

\[
\begin{align*}
m_{h_0} &= 125.31 \text{ GeV}, \\
m_{h_1} &\sim 300.8 - 354.96 \text{ GeV}, \quad m_{j_2} \sim 472.51 - 557.53 \text{ GeV}, \quad m_{j_3} \sim 414.98 - 489.65 \text{ GeV} \\
m_{t_1} &\sim 416.55 - 490.04 \text{ GeV}, \\
m_{t_2} &\sim 548.85 - 647.59 \text{ GeV} \\
m_{t_3} &\sim 554.70 - 652.57 \text{ GeV}, \quad m_{j_1} \sim 668.09 - 788.28 \text{ GeV}.
\end{align*}
\]

Notice that, for fermions we have taken only the masses of the top quark \( (t) \) and the new quarks \( (J_1, J_2, J_3) \), since the effect of the other fermions is negligible.

In figures (1) and (2), we plot the ratio \( \frac{\nu_c}{T_c} \) as a function of \( T_c \) for each step of (EWPT). In both cases it is clear that the condition \( \frac{\nu_c}{T_c} \geq 1 \) required for a strong first-order (EWPT) is satisfied.

**Figure 1:** the ratio of \( \frac{\nu_{c1}}{T_{c1}} \) In terms of \( T_{c1} \)

**Figure 2:** the ratio of \( \frac{\nu_{c2}}{T_{c2}} \) In terms of \( T_{c2} \)
4. Sphaleron Rate in the minimal $SU(3)$ Model

One has to know that if the transition is done classically, by crossing-over the barrier, from the zero to a non-zero $V_{eV}$, without tunneling, then we talk about sphaleron transitions, or else, the transition occurs from zero vacuum to a non-zero vacuum by quantum tunneling effect, this transition is called instanton.

4.1. Sphaleron Energy

The energy functional in the temporal gauge is given by:

$$\mathcal{E} = 4\pi \int d^3x \left[ \frac{1}{2} (\nabla^2 \nu_x)^2 + \frac{1}{2} (\nabla^2 \nu_\rho)^2 + V_{\text{eff}}(\chi, \rho) \right]$$

we have used the approximation

$$\frac{\partial \nu_x}{\partial t} = \frac{\partial \nu_\rho}{\partial t} = 0$$

and we have two equations of motion for the $V_{eV}$s

$$\nu_x + \nabla^2 \nu_x - \frac{\partial V_{\text{eff}}(\nu_x, T)}{\partial \nu_x} = 0$$
$$\nu_\rho + \nabla^2 \nu_\rho - \frac{\partial V_{\text{eff}}(\nu_\rho, T)}{\partial \nu_\rho} = 0$$

the equations (15), (16) became:

$$\frac{d^2 \nu_x}{dr^2} + \frac{2}{r} \frac{d \nu_x}{dr} - \frac{\partial V_{\text{eff}}(\nu_x, T)}{\partial \nu_x} = 0$$
$$\frac{d^2 \nu_\rho}{dr^2} + \frac{2}{r} \frac{d \nu_\rho}{dr} - \frac{\partial V_{\text{eff}}(\nu_\rho, T)}{\partial \nu_\rho} = 0$$

The sphaleron energies in each phase transition and the relation (13) became:

$$\mathcal{E}_{\text{sph}(SU(3))} = 4\pi \left[ \frac{1}{2} \frac{d^2 \nu_x}{dr^2} + V_{\text{eff}}(\nu_x, T) \right] r^2 dr$$
$$\mathcal{E}_{\text{sph}(SU(2))} = 4\pi \left[ \frac{1}{2} \frac{d^2 \nu_\rho}{dr^2} + V_{\text{eff}}(\nu_\rho, T) \right] r^2 dr$$

with $\mathcal{E}_{\text{sph}(SU(3))}$ and $\mathcal{E}_{\text{sph}(SU(2))}$ are the sphaleron energies in the $SU(3)_L \xrightarrow{\nu_x} SU(2)_L$, $SU(2)_L \xrightarrow{\nu_\rho} U(1)_{\text{QED}}$ phase transitions respectively.

4.2. Sphaleron Rate

In refs.[5], [6], [7] the sphaleron rate per unit time, per unit volume is given by:
\[ \Gamma \frac{e^{4}}{V} = \alpha^{4}T^{4} \exp(-\varepsilon/T) \]  

where \( V \) is the volume of the EWPT region, \( T \) the temperature, \( \varepsilon \) the sphaleron energy and \( \alpha = 1/30 \) a constant. Comparing the sphaleron rate with the Hubble constant which describe the cosmological expansion rate at the temperature \( T \) and in order to have B violation one has to verify that \( \frac{\Gamma}{V^4} > H \) at \( T > T' \). To estimate the upper bounds of the sphaleron rates, we suppose that the VeVs of the Higgs fields do not change from point to point in the universe, so we have:

\[ \frac{d\dot{v}_{\varphi}}{dr} = \frac{d\dot{v}_{\rho}}{dr} = 0 \]  

the relations (17), (18) became:

\[ \frac{\partial V_{\text{eff}}(\nu_{\varphi}, T)}{\partial \dot{v}_{\varphi}} = 0, \quad \frac{\partial V_{\text{eff}}(\nu_{\rho}, T)}{\partial \dot{v}_{\rho}} = 0 \]  

and the relations (19), (20) can be rewritten as:

\[ \epsilon_{\text{sph}(\text{SU}(3))} = 4\pi \int V_{\text{eff}}(\nu_{\varphi}, T)r^2dr = \frac{4\pi^3}{3} V_{\text{eff}}(\nu_{\varphi}, T) \bigg|_{\nu_{\text{pm}}} \]  

\[ \epsilon_{\text{sph}(\text{SU}(2))} = 4\pi \int V_{\text{eff}}(\nu_{\rho}, T)r^2dr = \frac{4\pi^3}{3} V_{\text{eff}}(\nu_{\rho}, T) \bigg|_{\nu_{\text{pm}}} \]

with \( \nu_{\text{pm}} \) and \( \nu_{\text{pm}} \) are the VeVs at the maximum of the effective potential, and

\[ V = \frac{4\pi^3}{3} \]

the relations (24) become:

\[ \epsilon_{\text{sph}(\text{SU}(3))} \sim \frac{E^4}{T} \frac{T}{4\lambda^2}, \quad \epsilon_{\text{sph}(\text{SU}(2))} \sim \frac{E^4}{4\lambda^3} \]

and

\[ \Gamma_{\text{SU}(3)} = \alpha^4T \exp(-\frac{E^4}{4\lambda^2}T) \]  

\[ \Gamma_{\text{SU}(2)} = \alpha^4T \exp(-\frac{E^4}{4\lambda^3}T) \]  

In figures (3) and (4), we plot the sphaleron rate \( \Gamma(T) \) as a function of the temperature \( T \) for the two steps of phase transitions.

In order to plot the Hubble constant as a function of the temperature \( T \), we use the relation:

\[ H^2 = \frac{\pi^2 g T^4}{90 M^2} \]

where \( g = 106.75 \) is the total number of degrees of freedom and \( M = 2.43 \times 10^{18} \), so we get the figure 5.

From figures (3) and (4), it is clear that the sphaleron rates are linear functions of \( T \). For the first step of the phase transition fig.(4) and with comparison with fig.(5), one can see that \( \Gamma_{\text{SU}(3)} \sim 10^{-3} > H \sim 10^{-13} \),
and for the second step, see fig.(3), $\Gamma_{\text{Sph}(2)} \sim 10^{-4} > H \sim 10^{-13}$. Thus, it is well established as it was mentioned above that we have a B violation.

![Figure 3: the sphaleron rate In terms of $T_{c2}$](image1)

![Figure 4: the sphaleron rate In terms of $T_{c1}$](image2)

![Figure 5: The Hubble constant $H$ as a function of $T$](image3)

5. Conclusion

Through this article, we have investigated the possibility that the minimal 331 model could be a good candidate to explain the violation of baryonic number $B$ in the early universe since this phenomenon is not included in the original formulation of the minimal Standard Model of particles. Indeed, we have calculated the energy and the rate of sphaleron and we have proved that the latter fulfills the third condition of Sakharov of a strong first-order EWPT. In addition, an estimation of the masses of the new particles of the model (exotic quarks, gauge bosons, and Higgses) has been placed.
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