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The representation theory of finite sets and correspondences\textsuperscript{1}

Serge Bouc and Jacques Thévenaz

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\textsuperscript{1}This long paper will never appear in this form. It is replaced by the following series of shorter articles:

- Correspondence functors and finiteness conditions, Journal of Algebra 495 (2018), 150-198.
- Correspondence functors and lattices, Journal of Algebra 518 (2019), 453-518.
- The algebra of Boolean matrices, correspondence functors, and simplicity, submitted preprint, 2018.
- Tensor product of correspondence functors, submitted preprint, 2018.
Abstract. We investigate correspondence functors, namely the functors from the category of finite sets and correspondences to the category of $k$-modules, where $k$ is a commutative ring. They have various specific properties which do not hold for other types of functors. In particular, if $k$ is a field and if $F$ is a correspondence functor, then $F$ is finitely generated if and only if the dimension of $F(X)$ grows exponentially in terms of the cardinality of the finite set $X$. In such a case, $F$ has finite length. Also, if $k$ is noetherian, then any subfunctor of a finitely generated functor is finitely generated. When $k$ is a field, we give a description of all the simple functors and we determine the dimension of their evaluations at any finite set.

A main tool is the construction of a functor associated to any finite lattice $T$. We prove for instance that this functor is projective if and only if the lattice $T$ is distributive. Moreover, it has quotients which play a crucial role in the analysis of simple functors. The special case of total orders yields some more specific results. Several other properties are also discussed, such as projectivity, duality, and symmetry. In an appendix, all the lattices associated to a given poset are described.

AMS Subject Classification: 06B05, 06B15, 06D05, 06D50, 16B50, 18B05, 18B10, 18B35, 18E05

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