The effect of Rashba spin-orbit interaction on persistent current in a chain of two Holstein-Hubbard rings

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Abstract. We study the persistent current in a chain of two coupled quantum rings threaded by an Aharonov-Bohm flux in the presence of electron-phonon and Rashba spin-orbit interactions. In the framework of the Holstein-Hubbard model, the equations for ground state energy, persistent current and Drude weight were derived. Obtaining of the expression for the ground state energy allowed us to obtain the persistent current by a conventional way (differentiating). The influence of the magnetic flux and the Rashba spin-orbit interaction on the ground state energy, persistent current and Drude weight was studied numerically.

1. Introduction

The Rashba effect is a momentum-dependent splitting of spin bands due to spin-orbit interaction and asymmetry of the potential, in particular, in the direction perpendicular to the two-dimensional plane (e.g., for plane heterostructures). It has many applications related to operating electron spins by electric fields in spintronics, quantum computing, anisotropic magnetoresistance. Discovery of giant Rashba effect in bulk crystals and in a number of low-dimensional systems bears a promise of creating devices operating electrons spins at nanoscale and possessing short operational times [1, 2]. We are interested in influence of spin-orbit interaction on persistent current in two coupled normal metal rings threaded by a magnetic flux. Quantum mechanics predicts that the electrons’ many-body ground state may itself contain a “persistent” current which flows through the resistive circuit without dissipating energy [3, 4]. There were a number of experiments and several models [5, 6] in the field. The Hubbard model turns out to be a suitable model to investigate the persistent current in a quantum ring consisting of discrete sites [7]. The effects of spin-orbit interactions [8] are also found to be pronounced in quantum rings. Recently the effect of electron-phonon interaction on the persistent current in a correlated quantum ring in the presence of Rashba spin-orbit interaction has been studied by Monisha et al. [9] using the 1D Holstein-Hubbard model. The advantage with the Rashba interaction is that it can be manipulated by tuning the external magnetic field and concomitantly the persistent current can be controlled. This is needed for spintronics. In the present work we study the effect of Rashba spin-orbit interaction on persistent current in a chain of two Holstein-Hubbard rings threaded by an Aharonov-Bohm flux. The energy spectrum is periodic in flux. Hence, the persistent current which is the change in ground state energy with respect to the magnetic flux is also periodic in flux.
2. Model of two Holstein-Hubbard rings in the presence of Rashba spin-orbit interaction

The chain of two Holstein-Hubbard rings considered in this work is shown in figure 1. Both of the rings consist of discrete lattice sites and the electrons can hop from one site to another.

Figure 1. Chain of two Holstein-Hubbard rings. The intersection point is 0, points 1, 12, 21, N1, N2 are its nearest neighbours.

The Hamiltonian for a chain of two Holstein-Hubbard rings threaded by a magnetic flux is written in the presence of Rashba spin-orbit interaction as:

\[ H = H_{el} + H_p + H_{ep} + H_{so}, \]

with

\[ H_{el} = \varepsilon_0 \sum_i c_i^\dagger c_i + \varepsilon_0 \sum_k c_k^\dagger c_k - t e^{i\theta} \sum_{\langle i,j \rangle} (c_i^\dagger c_j + h.c) - \frac{t}{2} e^{i\theta} \sum_{\langle k,l \rangle} (c_k^\dagger c_l + h.c) + U \sum_i n_i n_i^\dagger + U \sum_k n_k^\dagger n_k^\dagger \]

\[ H_p = \hbar \omega_0 \sum_i \left( b_i^\dagger b_i + \frac{1}{2} \right) + \hbar \omega_0 \sum_k \left( b_k^\dagger b_k + \frac{1}{2} \right) \]

\[ H_{ep} = g_1 \sum_i n_i (b_i + b_i^\dagger) + g_1 \sum_k n_k (b_k + b_k^\dagger) + g_2 \sum_{\langle i,j \rangle} (b_i + b_i^\dagger) + g_2 \sum_{\langle k,l \rangle} (b_k + b_k^\dagger) \]

\[ H_{so} = -t \sum_{\langle i,j \rangle} e^{i\theta} c_i^\dagger c_j + h.c - \frac{1}{2} e^{i\theta} \sum_{\langle k,l \rangle} c_i^\dagger c_j + h.c, \]

where \( \langle i,j \rangle \) is any pair of two nearest neighbors on the ring, except at the point of intersection and its nearest neighbors. Pairs \( \langle i,j \rangle \) include the point of intersection and its nearest neighbors. \( H_{el} \) given by Eq. (2) is the electronic Hamiltonian which consists of six terms. The first two terms stand for the site energies of the two rings, \( \varepsilon_0 \) referring to the on-site energy, \( c_i = \begin{pmatrix} c_i^\uparrow \\ c_i^\downarrow \end{pmatrix} \) denoting the creation (annihilation) operator for an electron at site \( i \) with spin \( \sigma \), \( i \) taking values 1, 2, 3, 4, ... \( N \), where \( N \) is the total number of sites in each ring except the point of intersection and its nearest neighbors, \( k \) is the number of point of intersection and its nearest neighbors (points 0, 1, 12, 21, N1, N2 on figure 1). Next two terms describe the kinetic energies, \( t \) being the hopping integral and \( \theta = (2\pi \Phi / n) \) the Aharonov-Bohm phase arising from the quantized magnetic flux \( \Phi = m \Phi_0 \) where \( m \) is an integer and \( \Phi_0 = h c / e \) is the elementary flux quantum. The last two terms represent the onsite Coulomb repulsion with \( U \) measuring the strength of the repulsion and \( n_{i\sigma} = c_i^\dagger c_i \) refers to the operator corresponding to the electron number at site \( i \) with spin \( \sigma \). \( H_p \) given by Eq. (3) is the sum of the Hamiltonians for non-interacting phonons in the two rings, \( b_i^\dagger (b_i) \) being the phonon creation (annihilation) operator at site \( i \) and \( \omega_0 \) the phonon frequency which is assumed to be dispersionless. \( H_{ep} \) given by Eq. (4) describes the on-site and nearest-neighbor electron-phonon interactions with \( g_1, g_2 \) denoting the corresponding coupling strengths. We assume that \( g_2 \) is smaller than \( g_1 \) by an
order or so. Finally, Eq. (5) represents the Rashba spin-orbit interaction with $t_{so}$ as the spin-orbit coupling constant given by: $t_{so} = i\alpha(\sigma_x\cos\varphi + \sigma_y\sin\varphi)$, where $\sigma_m$ is the $m$-th component of the Pauli matrix $\sigma$, $\varphi_{ij} = (\varphi_i + \varphi_j)/2$ with $\varphi_i = 2\pi(i-1)/N$, $i$ being the site index along the azimuthal direction of the ring.

3. Analytical results

To eliminate the phonons we carry out the Lang-Firsov transformation [10] on the Hamiltonian (1) with the generator

$$R = \frac{1}{\hbar\omega_0}(g_1\sum_{i\sigma} n_{i\sigma}(b_i^+ - b_i) + g_1\sum_{k\sigma} n_{k\sigma}(b_k^+ - b_k) + g_2\sum_{(i,j)\sigma} n_{ij\sigma}(b_i^+ - b_i) + \frac{g_2}{2}\sum_{(k,l)\sigma} n_{kl\sigma}(b_k^+ - b_k)).$$

(6)

Then we remove the spin-dependence by performing the unitary transformation and diagonalize the effective electronic Hamiltonian by using the Hartree-Fock approximation. After some algebraic simplifications following Cabib and Callen [12], we obtain the equation for ground state energy:

$$E_{GS} = \sum_i \frac{2N-1}{2N} E_i f(E_i) + \sum_i \frac{1}{2N} E_i f(E_i) + 2K_1 + K_0, \ f(E_i) = \left[e^{\beta(E_i - \mu)} + 1\right]^{-1}. \quad (7)$$

The persistent current and the Drude weight can be evaluated from the following relations:

$$I_{pc} = -\frac{1}{2\pi}\left(\frac{\partial E_{GS}}{\partial \Phi}\right), \quad (8)$$

$$DW = \frac{N}{4\pi^2}\frac{\partial^2 E_{GS}}{\partial \Phi^2} = \Phi_m, \quad (9)$$

where $\Phi_m$ is the location minimum of $E_{GS}$ and can take values 0 or $1/2$ depending on the parity of the number of electrons.

4. Numerical results

For the next computations we shall use the natural units. So we set $t = 1, K_B T = 0.1, U = 0, g_1 = g_2 = 0$ and measure all energies in units of $\hbar\omega_0$.

In figure 3 we plot the dependence of the ground state energy $E_{GS}$ on the flux $\Phi$ (in figure 3 and next in paper it is pointed as $E_{GS}/F_0$) for different values of Rashba spin-orbit constant $\alpha$ for a chain of two rings (figure 3). One can see that the ground state energy is periodic with $\alpha$.

The variation of persistent current $I_{pc}$ as a function of $\alpha$ is represented in figure 5. The phase of the persistent current changes when $\alpha$ exceeds a critical value $\alpha_c$. When $\alpha > \alpha_c$, $I_{pc}$ with $\alpha$ becomes monotonically increasing.

It’s in interest to compare these results with one-dimensional Holstein-Hubbard model for a single ring, thus we use the model described in [9]. So we plot the dependence of the $E_{GS}$ on the flux $\Phi$ (see figure 2) and the variation of $I_{pc}$ as a function of $\alpha$ (see figure 4) for a single ring. For the case when $\alpha > 0$, in the presence of Rashba spin-orbit interaction, connection of two rings increases the values of $E_{GS}$ and doesn’t changes the period. The variation of persistent current $I_{pc}$ as a function of $\alpha$ is similar for both models except the difference in critical value.
Then we study the effects of temperature, chemical potential, spin-orbit interaction and electron-phonon interaction on the persistent current for a chain of two rings. To understand the effect of electron-electron interaction, we set \( t = 1, K_B T = 0.1, g_1 = g_2 = 0 \).

The variation of persistent current for different values of \( U \) with \( \alpha = 0, 2 \) is shown of figure 6, figure 7. In absence of Rashba spin-orbit interaction when \( \alpha = 2 \) there seem to exist a critical value \( U_c \). When \( U > U_c \) the persistent current decreases when \( U \) increases. The decrease is quite rapid. Such affect appears as when \( U \) increases the electrons experience a larger onsite repulsion and find it more difficult to hop from one site to another and this reduces persistent current. As for the single ring see figure 8, figure 9. The results for a single ring and for a chain are quite similar.
Next we look into the effects of on-site electron-phonon interaction. We set $t = 1, K_B T = 0.1, U = g_2 = 0$. As it’s shown in figure 11, the persistent current decreases when $g_1$ increases. The explanation is when $g_1$ increases, the deepening of the self-trapping polarization potential happens and this causes the localization which will inhibit conduction. This effect is also shown in figure 10 for the case of single ring. But in the case of two chained rings, the gradient of the curve is not monotonic.
The next important step is to study the effects of temperature on persistent current. We set $t=1, U=0, g_1=g_2=0$. Figure 12 and figure 13 show that persistent current decreases as temperature increases as for a single ring so as for a chain. Also there seem to exist a critical value $(k_bT)_c$ and when $k_bT > (k_bT)_c$ the persistent current decreases monotonically.

Finally, we will study the effect of chemical potential on persistent current. We set $t=1, K_b T = 0.1, U = 0, g_1 = g_2 = 0$. The variation of persistent current $I_{pc}$ for different values of $\mu$ in the presence ($\alpha = 2$) and absence ($\alpha = 0$) of Rashba spin-orbit interaction is shown in figure 15. Such a behavior is analogous for the single ring (see figure 14).
5. Conclusions
We studied the effect of Rashba spin-orbit interaction on the persistent current for the model of two discrete coupled Holstein-Hubbard rings. The main steps of the model consideration are as follows. 1) We eliminate the phonons degrees of freedom by the Lang-Firsov transformation; 2) the spin-dependence was removed by performing the unitary transformation; 3) the effective electronic Hamiltonian was diagonalized by using the Hartree-Fock approximation; 4) the equations for ground state energy, persistent current and Drude weight were obtained. We investigated the effects of Aharonov- Bohm flux, temperature, chemical potential spin-orbit interaction and electron-phonon interaction on the persistent current. The model can be modified for other purposes, e.g., for description of tunnelling (using mathematical procedure suggested in [11]). The result can be useful for quantum computing applications.

Acknowledgements
The authors thank Prof. R.Figari and Prof. W.Florek for interesting discussion. The reported study was funded by RFBR, project number 19-31-90164 and partially financially supported by the Government of the Russian Federation (grant 08-08), by grant 16-11-10330 of Russian Science Foundation.

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