Research on Transmission Performance of A Seven-gear Three-degree-of-freedom Planetary Gear Automatic Transmission Based on Graph Theory and Matrix Equation

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Abstract. The transmission performance analysis of planetary gear automatic transmission is helpful for the following structural size optimization, hydraulic design and electric control design. The current graphic method has the disadvantages of low calculation accuracy and lack of intuition in numerical analysis method. This paper proposes a method to analyze the transmission performance based on graph theory and matrix equation. Firstly, a graph theory analysis model is established for the self-designed seven-gear three-degree-of-freedom planetary gear automatic transmission, and the structural characteristics of the transmission are analyzed. Then, on the basis of analyzing the shift actuator worksheet, the motion analysis model and torque analysis model of typical gears are designed by taking the first gear and the reverse gear as examples, and the speed and torque values of all gears of the transmission are calculated by using the corresponding matrix equation. Finally, the power flow diagram of the typical gear is established by taking the first gear and the fourth gear as examples, and the direction and numerical value of power flow are identified on the diagram. The results show that the method can realize the combination of qualitative and quantitative analysis for the transmission performance analysis of the planetary gear automatic transmission with multiple gears and multiple degrees of freedom, which is more intuitive, accurate and efficient.

1. Introduction
With the development of technology in automobile enterprises, the requirements of consumers, energy conservation and emission reduction and other requirements, major automobile enterprises are committed to the development of automatic transmission with excellent performance [1-3]. Planetary-gear-type automatic transmission has a high usage rate in automobiles due to its compact structure and low failure rate. The design and development of automatic transmission covers mechanical, hydraulic, electric control and other aspects, especially the synthesis and analysis of the planetary gear structure in the field of mechanical design [4-5]. After the topological structure of the planetary gear structure is determined from various feasible structures that meet the design requirements, the performance analysis of the mechanism is required, especially the analysis of the motion, stress, power of the coaxial rotating elements of each planetary row of the automatic transmission, because the analysis of these factors will provide the key foundation for the subsequent tooth matching calculation, structural strength analysis, part material selection, etc. Common methods of transmission performance analysis for planetary gear automatic transmissions with multiple gears and multiple degrees of freedom are the graphic method and the numerical analysis method [6-7]. The graphic method is clear and intuitive but lacks accurate numerical results, while the numerical analysis method is suitable for accurate calculation by computer but the analysis process is not intuitive enough. Therefore, by combining the advantages of the graphic
method and the numerical analysis method, this paper proposes a joint mode based on graph theory and matrix equation. For the self-designed seven-gear three-degree-of-freedom planetary gear automatic transmission, the graph theory analysis model is firstly established. On this basis, the corresponding graph theory model and performance matrix are used for motion analysis, torque analysis and power analysis. Then, the corresponding results are analyzed qualitatively and quantitatively.

2. Structural Diagram and Graph Theory Analysis Model of the Seven-gear Three-degree-of-freedom Automatic Transmission

Figure 1 is a structural diagram of a seven-speed planetary gear automatic transmission proposed in reference [8]. It includes three planetary gear rows (the corresponding sun gears, gear rings and planet carriers are S1, R1, P1, S2, R2, P2, S3, R3 and P3 respectively), four clutches (C1, C2, C3 and C4), and two brakes (B1 and B2). The connection relationship of each element is shown in Figure 1. The first planetary row is a single-row single-stage planetary gear row, and the second and third planetary rows are Simpson-type structures composed of single-row single-stage planetary rows. The number of teeth of the sun gear and the gear ring in the first planetary row are $Z_{S1} = 43$, $Z_{R1} = 91$ respectively. The number of teeth of the sun gear and the gear ring in the second planetary row are $Z_{S2} = 33$, $Z_{R2} = 78$ respectively. And the number of teeth of sun gear and gear ring in the third planetary row are $Z_{S3} = 27$, $Z_{R3} = 62$ respectively. The characteristic parameters in the planetary row are:

$$\alpha_1 = \frac{Z_{R1}}{Z_{S1}} = \frac{91}{43} = 2.116$$  \hspace{1cm} (1)

$$\alpha_2 = \frac{Z_{R2}}{Z_{S2}} = \frac{78}{33} = 2.364$$  \hspace{1cm} (2)

$$\alpha_3 = \frac{Z_{R3}}{Z_{S3}} = \frac{62}{27} = 2.296$$  \hspace{1cm} (3)

Figure 1. Structural Diagram of the Seven-gear Three-degree-of-freedom Automatic Transmission.

Figure 2. Graph Theory Analysis Model of the Seven-gear Three-degree-of-freedom Automatic Transmission.

The graph theory analysis model drawn according to the structural diagram of the automatic transmission is shown in Figure 2. The planetary sun gear, the gear ring, the planet carrier, the clutch input end, the clutch output end, the brake, the transmission input shaft and the transmission output shaft are respectively indicated by solid circles in the figure. P1 is directly connected with the shell, which is represented by a hollow circle in the graph theory analysis model. The gear meshing relationships of the sun gear, the gear ring and the planet carrier in the planetary row are connected by thick solid lines. The three elements in the planetary row are connected with the clutch, the input shaft and the output shaft by thin solid lines. The input end of the clutch is connected with the output end of the clutch, and the elements of the planetary row are connected with the brake by dotted lines.

The sun gear of the first planetary row is directly connected with the input shaft, the planet carrier is directly connected with the shell, and the output state of the gear ring is constant. However, the first planetary row needs a clutch to transmit power to the subsequent planetary rows, which lacks one degree of freedom. Simpson structure composed of the second planetary row and the third planetary row is a two-degree-of-freedom mechanism. Therefore, this seven-speed automatic transmission is a three-
degree-of-freedom planetary gear automatic transmission. In order to realize the corresponding gear position, two of the six gear shift actuators are required to operate. The operating conditions of the corresponding gear shift actuators are shown in Table 1.

Table 1. Work table of shift actuators for each gear.

| 1st gear | 2nd gear | 3rd gear | 4th gear | 5th gear | 6th gear | 7th gear | Reverse gear |
|----------|----------|----------|----------|----------|----------|----------|--------------|
| C1B2     | C1B1     | C1C3     | C1C4     | C1C2     | C2C4     | C2B1     | C4B2         |

3. Motion Analysis of the Seven-gear Three-degree-of-freedom Automatic Transmission

According to the graph theory analysis model of the transmission in Figure 2 and the work table of shift actuators for each gear in Table 1, the motion analysis model diagram of the corresponding gear can be drawn. In the graph theory analysis model diagram in Figure 2, the thick solid lines between the three elements of the planetary row are deleted. According to the working condition of the gear shift actuators corresponding to the gear positions, the non-working element is deleted, and the thin solid line is substituted between the working elements. Taking the first gear and the reverse gear as examples, the drawn motion analysis model is shown in Figure 3. Since other gears are similar, they will not be described in detail.

From the relative motion, the velocity relation of the three elements in the planetary row can be obtained as follows:

\[
\frac{n_s - n_p}{n_r - n_p} = \pm \frac{Z_r}{Z_s} = \pm \alpha
\]

(4)

On the right side of the equation, a single-row single-stage planetary rows takes "-" and a single-row double-stage planetary rows takes "+", so the corresponding velocity relations can be obtained as follows:

\[
\begin{align*}
    n_s + \alpha n_r - (\alpha + 1)n_p &= 0 \\
    n_s - \alpha n_r + (\alpha - 1)n_p &= 0
\end{align*}
\]

(5) (6)

The matrix describing the transmission performance relation of the three planetary rows of the transmission is represented by \( K_1 \), and the transmission performance matrix is:

\[
K_1 = \begin{bmatrix}
1 & -\alpha_1 & \alpha_1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \alpha_2 & -(\alpha_2 + 1) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \alpha_3 & -(\alpha_3 + 1)
\end{bmatrix}
\]

The matrix describing the connection relationship between the three elements of each planetary row is represented by \( K_2 \). Since the first planet carrier P1 is directly connected to the shell and has a speed of 0, the corresponding value of the row vector is 1 and the others are 0. The sun gear of the second planetary row and the sun gear of the third planetary row are directly connected with each other with equal speed. The planet carrier of the second planetary row and the gear ring of the third planetary row...
are directly connected at the same speed. In the row vector, one element with equal speed is denoted as 1, one as -1, and the other as 0. Therefore, the connection matrix is:

$$K_2 = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1
\end{bmatrix}$$

In different gears, the matrix describing the combination state of the shift actuator is represented by $G_i (i = 1, 2, 3, \ldots)$. When the clutch is engaged, the corresponding two elements have the same speed. One element with the same speed is marked as 1, one as -1, and the other as 0. When the brake is combined, the speed of the element is 0, the corresponding value of row vector is 1, and the others are 0. The input vector of the last behavior of the gear matrix is marked as 1 for the input component and 0 for the others. The gear matrix corresponding to each gear is:

$$G_1 = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad G_2 = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$G_3 = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad G_4 = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$G_5 = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad G_6 = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$G_7 = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad G_R = \begin{bmatrix}
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

The rotational speeds of the sun gear, the gear ring and the planet carrier in the first, second and third planetary rows are $n_{Si}(i = 1, 2, 3)$, $n_{Ri}(i = 1, 2, 3)$, $n_{Pi}(i = 1, 2, 3)$ respectively. According to different gear positions, the corresponding matrix is combined, and the rotation speed of each coaxial element of the planetary row can be calculated by equation 7. If the input rotation speed is set to 2000 $r / \text{min}$, the last value in the right vector of equation 7 is 2000. The calculated rotational speed of each gear is shown in Table 2.
Table 2. Rotational speed of planetary row elements in each gear (r / min).

| Gear    | S1     | R1     | P1    | S2     | R3     | P2    | S3     | R3     | P3     |
|---------|--------|--------|-------|--------|--------|-------|--------|--------|--------|
| 1st     | 2000   | 945.18 | 0     | 2000   | -846.02| 0     | 2000   | 0      | 606.79 |
| 2nd     | 2000   | 945.18 | 0     | 2000   | 0      | 594.53| 2000   | 594.53 | 1020.94|
| 3rd     | 2000   | 945.18 | 0     | 2000   | 498.98 | 945.18| 2000   | 945.18 | 1265.21|
| 4th     | 2000   | 945.18 | 0     | 2000   | 945.18 | 1258.74| 2000   | 1258.74| 1483.63|
| 5th     | 2000   | 945.18 | 0     | 2000   | 498.98 | 945.18| 2000   | 945.18 | 1265.21|
| 6th     | 2000   | 4493.59| 945.18| 2000   | 945.18 | 1258.74| 2000   | 1258.74| 1483.63|
| 7th     | 2000   | 6728   | 0     | 2000   | 6728   | 0     | 2000   | 6728   | 3434.46|
| Reverse | 2000   | 945.18 | 0     | -2234.41| 945.18 | 0      | -2234.41| 0      | -677.91|

In the above table, the corresponding values in columns S1 and P3 represent the input rotational speeds and output rotational speeds in each gear, so the transmission ratio (= n_s1 / n_p3) of each gear calculated is shown in Table 3.

Table 3. Transmission ratio of each gear.

| Gear    | 1st gear | 2nd gear | 3rd gear | 4th gear | 5th gear | 6th gear | 7th gear | Reverse gear |
|---------|----------|----------|----------|----------|----------|----------|----------|------------|
| 3.296   | 1.958965 | 1.580765 | 1.348038 | 1        | 0.725544 | 0.582332 | -2.95023 |

4. Torque Analysis of the Seven-gear Three-degree-of-freedom Automatic Transmission

In Figure 3, by deleting the thin solid lines between the engaged clutches and the vertices corresponding to the disengaged clutches, the torque analysis model diagram of each gear can be obtained. Take the first gear and the reverse gear as examples, as shown in Figure 4. Other gears are similar and will not be repeated here.

The torques of the sun gear, the gear ring and the planet carrier are respectively expressed by T_S, T_R, T_P, then the torque relationships of the three elements of the single-row single-stage planetary row and the single-row double-stage planetary row meet equations 8 and 9, respectively.

\[
\frac{T_S}{1} = \frac{T_R}{\alpha} = \frac{T_P}{-(\alpha + 1)}
\]

(8)

\[
\frac{T_S}{1} = \frac{T_R}{-\alpha} = \frac{T_P}{\alpha - 1}
\]

(9)

Equation 8 can be transformed into

\[
(\alpha + 1)T_S + T_P = 0
\]

(10)

\[
(\alpha + 1)T_R + \alpha T_P = 0
\]

(11)
\[(1-\alpha)T_s + T_p = 0\]  
\[(\alpha - 1)T_s + \alpha T_p = 0\]

The matrix describing the torque characteristic relation of the three planetary rows of the transmission is represented by \(M\), and the torque characteristic matrix is:

\[
M = \begin{bmatrix}
1 - \alpha & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha - 1 & \alpha & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \alpha_2 + 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \alpha_2 + 1 & \alpha_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \alpha_3 + 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & \alpha_3 + 1 & \alpha_3
\end{bmatrix}
\]

In the torque analysis model in Figure 4, an element with an external torque of 0 is called an independent component without external force, that is, the element is neither connected to the input shaft or the output shaft, nor braked. The matrix representing the stress state of each independent component without external force is represented by a matrix of independent component without external force \(E_i (i=1,2,3,...)\). In the row vector, the independent component without external force is marked as 1, and the other components are 0. The matrices of independent components without external force corresponding to the gear position are as follows:

\[
E_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_5 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_6 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_7 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\]

The torque input matrix is expressed by \(I_i (i=1,2,3,...)\), in which the input member is denoted as 1 and the others as 0, and the torque input matrices corresponding to gear positions are respectively:

\[
I_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad I_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}, \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad I_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}, \quad I_6 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}, \quad I_7 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad I_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

The torques acting on the sun gear, the gear ring, and the planet carrier in the first, second, and third planetary rows are \(T_{S_i} (i=1,2,3) \), \(T_{R_i} (i=1,2,3) \), \(T_{P_i} (i=1,2,3) \) respectively. According to different gear positions, the corresponding matrix is combined, and the torque on each coaxial element of the planetary row can be calculated by equation 14. If the input torque is set to be 250 N•m, the last value in the right vector of equation 14 is 250. The calculated torque of each gear is shown in Table 4.
Table 4. Torques Acting on Planetary Row Elements under Each Gear (Nm).

| Gear | S1 | R1 | P1 | S2 | R3 | P2 | S3 | R3 | P3 |
|------|----|----|----|----|----|----|----|----|----|
| 1st  | 0  | 0  | 0  | 0  | 0  | 0  | 250| 574| -824|
| 2nd  | 0  | 0  | 0  | 101.413| 239.741| -341.155| 148.587| 341.155| -489.741|
| 3rd  | 130.100| -275.291| 145.191| 0  | 0  | 0  | 119.900| 275.291| -395.191|
| 4th  | 77.966| -164.975| 87.01| 69.786| 164.975| -234.762| 102.248| 234.762| -337.01|
| 5th  | 0  | 0  | 0  | 0  | 0  | 0  | 75.85| 174.15| -250|
| 6th  | -61.482| 130.096| -68.614| -55.032| -130.096| 185.128| 55.032| 126.354| -181.386|
| 7th  | 0  | 0  | 0  | 0  | 0  | 0  | 75.85| 174.15| -250|
| Reverse gear | 250| -529| 279| 223.773| 529| -752.773| -223.773| -513.783| 737.557|

5. Power Analysis of the Seven-gear Three-degree-of-freedom Automatic Transmission

The speed and torque of each element of the planetary row calculated from Table 2 and Table 4 can further calculate the size and direction of power P (= torque T × speed n). The power greater than 0 is the input power, indicated by the inward arrows; The power less than 0 is the output power, which is indicated by the outward arrows, and the power value is marked in the figures. Taking the first gear and the fourth gear as examples, the power flow diagram of the typical gear is shown in Figure 5, in which the unit of power is Kw. It can be seen in the figure that in the first gear, only the third planetary row participates in power transmission, while in the fourth gear, all three planetary rows participate in power transmission. Other gears are similar and will not be repeated here.

![Power Flow Diagrams](image-url)
6. Conclusion
In this paper, a method based on graph theory and matrix equation to jointly analyze the transmission performance of the multi-gear multi-degree-of-freedom planetary automatic transmission is proposed for the self-designed seven-gear three-degree-of-freedom planetary automatic transmission. Firstly, the graph theory analysis model is established for the target transmission. Secondly, the motion analysis model and torque analysis model for typical gears are established according to the work table of shift actuators. Then, the speed and torque values of each gear of the transmission are calculated by using the corresponding matrix equation. Finally, the power flow diagram of the typical gear is established, on which the direction and numerical value of power flow are identified.

With the combination of graph theory and matrix equation, the transmission performance of the planetary gear automatic transmission can be displayed intuitively, and the objective of qualitative and quantitative analysis can be achieved.

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