The buckling of the physically nonlinear frame-rod structural systems

K O Dubrakova*, S V Dubrakov, F V Altuhov, D H Galaeva
Southwest State University, 50 let Oktyabrya str., Kursk, 305040, Russia

E-mail: dubrakova01-33@yandex.ru

Abstract. The article shows that different elements of timber structural systems lose stability at different stages of their deformation. The graphs of geometrical parameters affecting the critical parameters of stability reaching have been given. The results of the stability analysis for frame-rod structural systems taking in account stress values affecting the mechanical characteristic of timber have been presented. Using comparative analysis, we have shown the loading scheme affecting the critical force value and the length factor. Using numerical simulation, we have got that the stress state is significantly affected by the critical parameters of stability for the considered system.

Introduction
Loads acting on structural elements change their values during the operation process. Also, the environmental factors, dimensions of cross sections, etc. can be changed during the operation [1-9, 13-16]. The critical parameters of stability for the structures with timber load-bearing elements depend on a range of factors including stress state [10-12]. Thus, it is necessary to investigate the deformation of rod element of structural system when noted factors change its values in condition of active bifurcation and it is necessary to determine its affecting to the stability critical parameters.

Defining equations
Let us consider the two-span timber frame, the central frame of which is loaded with concentrated force P, and outer frames are loaded with forces αP (figure 1). Let us determine the critical force value taking in account that the first and the third frames are in the linear creep stage and the second one is in the stationary creep stage, with and without this assumption [9-12]. Calculation of the frame is provided by the quasi-statical method on the basis of the displacement method using the step-iteration procedure.
We carried out the critical force calculation using the special functions of the displacement method. If we accept the rotational angles of the rods $Z_1$, $Z_2$, $Z_3$ as unknown variables (figure 1), then the homogenous system of these equations takes the following form:

$$
\begin{align*}
& r_{11}Z_1 + r_{12}Z_1 + r_{13}Z_3 = 0, \\
& r_{21}Z_1 + r_{22}Z_1 + r_{23}Z_3 = 0, \\
& r_{31}Z_1 + r_{32}Z_1 + r_{33}Z_3 = 0
\end{align*}
$$

(1)

where $r_{11} = 4 \cdot i_4 + 4 \cdot i_1 \cdot \varphi_2 \cdot (v_1)$;

$r_{22} = 8 \cdot i_4 + 4 \cdot i_2 \cdot \varphi_2 \cdot (v_2)$;

$r_{33} = 8 \cdot i_5 + 4 \cdot i_3 \cdot \varphi_2 \cdot (v_3)$;

$r_{12} = r_{23} = 2 \cdot i_4$;

$r_{13} = r_{31} = 0$;

$$
\begin{align*}
& v_i = l \cdot \frac{P_i}{B_{rot}}, \quad (i = 1, 2, 3),
\end{align*}
$$
where $B_{\text{red}}$ is the reduced, depending on the stage of deformation, stiffness of the cross section of the rod, determined according to [10]; $v_i$ is the defining equation parameter.

Determinant of the system (1) can be determined by the following expression:

\[
\text{Det} = \left(4 \cdot i_1 \cdot \varphi_2 (v_1) + 4 \cdot i_4 \right) \left(4 \cdot i_2 \cdot \varphi_2 (v_2) + 8 \cdot i_4 \right) \left(8 \cdot i_1 + 4 \cdot i_3 \cdot \varphi_3 (v_3) \right) - \\
-4 \cdot i_4^2 \left(8 \cdot i_1 + 4 \cdot i_3 \cdot \varphi_3 (v_3) \right) - 4 \cdot i_4^2 \left(4 \cdot i_1 \cdot \varphi_2 (v_1) + 4 \cdot i_4 \right) = 0. 
\]

\[\text{(2)}\]

**Quantitative analysis**

Having solved the equation (2) we determine that the singular real root equals (0.0648) for value of load factor $\alpha = 0.6$ regardless to the deformation stages at which frames are operated. That is $\varphi_2 (v_1) = 0.0648$, $\varphi_2 (v_2) = -2.1804$, and $P_{\text{cr},1} = \frac{11.65 \cdot EI}{l}, P_{\text{cr},2} = \frac{30.88 \cdot EI}{l}$. Taking in account values of acting stresses and accepting the first and the third frames operating in linear creep stage, and the second frame in stationary creep condition, we obtain $\varphi_2 (v_1) = 0.772$, $\varphi_2 (v_2) = -3.6678$. That is $P_{\text{cr},1} = \frac{5.90 \cdot EI}{l}, P_{\text{cr},2} = \frac{33.25 \cdot EI}{l}$.

The same way we obtain the critical parameters for the parameter values $\alpha = 0.8$. Regardless to the values of acting stresses we obtain: $\varphi_2 (v_1) = -1.1646$, $\varphi_2 (v_2) = -0.85265$, and $P_{\text{cr},1} = \frac{22.09 \cdot EI}{l}, P_{\text{cr},2} = \frac{27.56 \cdot EI}{l}$. Accepting the first and the third frames of the frame operating in linear creep conditions, and the second one in stationary creep, we obtain $\varphi_2 (v_1) = 0.0648$, $\varphi_2 (v_2) = -3.6678$. At the same time $P_{\text{cr},1} = \frac{18.66 \cdot EI}{l}, P_{\text{cr},2} = \frac{33.18 \cdot EI}{l}$.

Having solved the equation (2) on the basis of the iteration method at the parameter value $\alpha = 0.9$ regardless to the acting stresses values, we determine $\varphi_2 (v_2) = -1.4181$, $\varphi_2 (v_1) = -0.763$. That gives for the rack’s frame 1 $v_1 = 5.125$, and for the frame 2 $v_2 = 5.40$. The values of critical forces for the frame 2 are equal to $P_{\text{cr},2} = \frac{29.16 \cdot E \cdot l}{I^2}$, and for the frame 1 $P_{\text{cr},1} = \frac{26.27 \cdot E \cdot l}{I^2}$.

In order to analyze the work of frames of the rack at different stages of deformation, we construct the graph for changing the critical forces ratio when the first and the second frames operate in linear and stationary creep stages respectively and when we don’t take in account values of stresses at different $\alpha$ parameter values (figure 3).

The given example shows that taking into account the level of acting stresses has a significant impact on the value of the critical parameters of stability of frame-rod structural systems made of wood. So, in cases when the first and third frame racks are in the linear creep stage, and the second one is in the steady creep stage when the load application parameter is $\alpha = 0.8$, the critical force increases by 2% for all elements of the system under consideration, for $\alpha = 0$, 7 $P_{\text{xp}}$ for rack 2 decreases by 15%, for rack 1 - by 2%.

In addition, for element 1, the free length, which should be used when calculating racks according to SP 64.13330.2016, at a parameter $\alpha = 0.9$ without taking into account the influence of the level of effective stresses will be 0.613 l, and for the rack 2 - 0.581 l. At the same time, assuming that the first and second stands are in the linear and steady creep stages, respectively, after calculating the considered frame-rod structural system, we obtain the free length of the first stand - 0.712 l, for the
stand 2 - 0.566 l. For this example, taking into account the mutual influence of the racks gives for a more loaded rack a greater critical force than for less loaded racks.

Consequently, the free length of racks, which, according to current regulations, is used when checking the strength of their sections, should be determined taking into account the effect of the level of the existing stresses.

In this example, a frame is adopted for analysis, in which the first and the third pillars are in the stage of linear creep, and the second is in the stage of established creep. At the same time, various elements of the structural system may lose stability, being both in the first and in the second stages of deformation. The moment when the frame elements reach critical parameters is significantly influenced by its geometry: the width and height of the cross section and the length of the posts. Figure 4 shows a graph of the deformation stage, in which the rack 1 and 2 frames lose stability, depending on the ratio of the width of the rack cross-section to its length and the coefficient ν.

![Graph showing the deformation stage](image)

**Figure 3.** Changing the ratio \( \frac{P_{cr}}{P} \). For the frames 1 and 2 at different values of load factor α.
Figure 4. Dependencies of the deformation stage, in which the frame supports 1 and 2 lose stability, depending on the ratio of the width of the cross-section of the rack to its length and the coefficient \( \nu \).

To analyze graph 4, let us consider the operation of the frame posts with the ratio of the width of the cross section of the rack \( b \) to its length equal to 0.09 and the value of the coefficient \( \nu = 4.4 \) (Figure 5).

Figure 5. Stage deformation frame racks when the ratio of the width of the cross section of the rack \( b \) to its length, is equal to 0.09 and the value of the coefficient is \( \nu = 4.4 \)
Analyzing Figures 4, 5 it can be concluded that with the ratio of the width of the cross-section of the frame stand to its length more than 0.122 (zone 1) all the frame stands lose stability, being in the stage of linear creep. If this ratio is in the range from 0.01 to 0.122, when the values of the coefficient $v$ are from 4.2 to 5.22 (zone 2), pillars 1 and 3 are in the first stage, and pillar 2 - in the second. In cases where $b/l$ varies from 0.092 to 0.096, and the parameter of the secular equation greater than 0.522, the operation of the frame pillars changes. The first stand goes into the stage of established creep, and the second - into the linear stage.

The similar analysis of section 4 (see Fig. 4) shows that in cases where the coefficient $v$ is more than 4.24 and the ratio of the width of the cross section to the length of the rack varies from 0.01 to 0.098, all frame elements lose their stability in the steady-state creep stage.

**Summary**

Based on numerical studies, we can draw the following conclusion: depending on the geometrical parameters and the load application scheme, the elements of frame-rod structural systems made of wood may lose stability, being in the stages of linear or steady-state creep. At the same time, taking into account the level of operating, voltages have a significant effect on the value of the critical stability parameters of the systems under consideration. Depending on the load application scheme, the specified parameters can either increase or decrease.

**References**

[1] Aleksandrov A V 2004 *Investigation of the sustainability of structures of the arch covering of the hall using the criteria for identifying the most dangerous elements* 8 14-21.

[2] Aleksandrov A V, Matveev A V 2003 *Criteria for identifying the most dangerous elements and their use in problems of structural stability, Safety of train traffic* (Tr. 4th scientific-practical. conf. Moscow, MIIT) III — 1 — III — 2.

[3] Aleksandrov A V, Travush V I, Matveev A V 2002 *On the calculation of stability of core structures* (Industrial and Civil Construction) 3 16-20.

[4] Aleksandrov A V 2001 *The role of individual elements of the core system in the event of loss of stability* (Bulletin of Moscow Institute of Railway Transport) 5 46.

[5] Dmitrieva K O, Klyueva N V 2016 *Questions of sustainability of core elements of structural systems made of wood under force and environmental loading* (Construction and reconstruction) 4 13-18.

[6] Dubrakova K O 2018 *Questions of sustainability of statically indefinable systems of wood, BST* (Bulletin of construction equipment) 11 54-55.

[7] Klyueva N V, Dmitrieva K O 2016 *Analysis of the stability of core wood structures under power loading and variable humidity* (Scientific Herald of the Voronezh State University of Architecture and Civil Engineering, Construction and architecture) 3 17-24.

[8] Klyueva N V, Dmitriev K O 2016 *Questions of sustainability of the core elements of structural systems of wood of various species under force and environmental loading in conditions of high humidity* (Construction and reconstruction) 5 60-68.

[9] Matveev A V 2002 *Some issues of creating a specialized software for the analysis of bridge structures* (MEIT Herald) 7 76-83.

[10] Pyatikrestovskiy K P, Khuganov H S 2013 *Nonlinear deformations of statically indefinable wooden structures* (Proceedings of higher educational institutions. Construction) 11(12) 21-30.

[11] Pyatikrestovskiy K P 2014 *Nonlinear methods of mechanics in the design of modern wooden structures* (MSCEI, Moscow).

[12] Pyatikrestovskiy K P, Travush V I 2015 *On programming non-linear method of calculating wooden structures* (Academia. Architecture and construction) 2 115-119.

[13] Smorchkov A A, Orlov D A, Kereb S A, Baranovskaya K O 2013 *Safety of constructions from the glued wood at the stage of manufacture* (Industrial and civil construction) 12 74-75.
[14] Travush V I, Kolchunov V I, Dmitriev K O 2015 Long-term strength and stability of compressed rods of wood (Construction and reconstruction) 5 40-46.
[15] Travush V I, Kolchunov V I, Dmitriev K O 2016 The stability of compressed rods of wood with the simultaneous manifestation of power and environmental impact (Construction mechanics and calculation of structures) 2 50-53.
[16] Travush V I, Kolchunov V I, Dmitriev K O 2016 Experimental and theoretical study of the strength and stability of compressed wood rods under power and environmental impact (Proceedings of higher educational institutions. Technology textile industry) 3 280-285.