Comparison of Spread Foundation Design in Case of Inhomogeneous Subsoil

Giang Nguyen 1,2, Martina Holickova 1
1 University of Žilina, Faculty of Civil Engineering, Univerzitná 8215/1, 010 26 Žilina, Slovakia
2 University of Bielsko-Biela, Willowa 2, 43-309 Bielsko-Biała, Poland

Abstract. Paper deals with designing spread foundation on inhomogeneous subsoil by PN-81/B-03020, by Meyerhof and Hanna and by procedure often applied in Slovakia. The approach using substitute foundation, the approach using punching shear failure and the approach using shear surface are introduced. Three approaches were applied to design foundation on inhomogeneous subsoil for two cases: stronger soil underlain by weaker soil and vice versa. In case stronger soil underlain by weaker soil, PN-81/B-03020 prescribes to determine size of substitute foundation based on soil types (cohesive or cohesionless), distance from foundation base to the weaker soil top and also width of real foundation. Meyerhof and Hanna prescribe to determine adhesive force and passive force per unit length of the vertical planes crossing foundation sidewall. By Meyerhof and Hanna, adhesive force depends on cohesion of stronger layer and also on ratio between bearing capacity of stronger soil and weaker soil. Meyerhof and Hanna also introduce punching shear coefficient, depending on angle of internal friction of stronger soil and ratio between bearing capacity of stronger soil and weaker soil. In case weak layer underlain by strong layer, PN-81/B-03020 prescribes to ignore bearing capacity of strong layer, Meyerhof and Hanna prescribe to determine bearing capacity of both layers. Meyerhof and Hanna prescribe to take into account distance from foundation base to the stronger soil and also depth of failure surface beneath the foundation in the thick bed of the upper weaker soil layer. The procedure often applied in Slovakia does not differ above mentioned two cases and prescribes for both cases to found probably shear surface, to determine average values of subsoil shear strength parameters and unit weight, based on which a bearing capacity of inhomogeneous subsoil will be determined. The results show that in the case stronger soil underlain by weaker soil, foundation sizes obtained by three approaches are different. Results also show that in case weaker layer underlain by stronger layer, neglecting the stronger layer leads to uneconomical design.

1. Introduction
In geotechnical practice we can have a situation, when subsoil under spread foundation is inhomogeneous by the way that subsoil is layered. Generally, one can differ two cases of inhomogeneous subsoil: stronger layer underlain by weaker layer and vice versa. For the case when a stronger layer underlain by weaker layer, the Polish Standard [1] (abbreviated as “old PN”), withdrawn from 01.04.2010, prescribes to design the foundation to suite a bearing capacity of the stronger layer and to check whether a bearing capacity of the weaker layer is sufficient for a substitute foundation. For the case when a weaker layer underlain by a stronger layer, it is a common practice in
Poland, that spread foundation is designed to suite a bearing capacity of the weaker layer, not taking account a bearing capacity of the stronger layer.

In the Polish Standard PN-EN 1997-1:2008 (the Polish version of Eurocode 7, Part 1, abbreviated as “PN-EN”) [2], no design method for spread foundation on inhomogeneous subsoil is specified. The Polish author Pulastates this fact in [3] and suggests applying above mentioned method of substitute foundation also in the future (of course, only a principle of the method is applied; verification of limit states, e. g. GEO shall be carried out in accordance with [2, 13]).

In many other countries, in case of inhomogeneous subsoil, design method for spread foundation by [4,5], posted e. g. in [6] is applied. For the case when a stronger layer underlain by weaker layer, authors in [4, 5] prescribe to determine adhesive force and passive force per unit length of the vertical planes crossing foundation sidewall and punching shear coefficient. In case weak layer underlain by strong layer, authors in [4, 5] prescribe to determine bearing capacity of both layers.

Concerning design of spread foundation, the Slovak Technical Standard STN 731001 “Foundation of structures. Subsoil under shallow foundations” [7] (abbreviated as “old STN”) had been used until 31.03.2010, when it was replaced by the new Slovak Technical Standard STN 731001 “Geotechnical structures. Foundation” [8] (abbreviated as “new STN”). The new STN respects the design approach number 2 (DA2) of Eurocode 7, Part 1 but modified it for Slovak condition. In the both Standards [7] and [8], no design method for spread foundation on inhomogeneous subsoil is specified. The Standards only state that for inhomogeneous subsoil, an individual approach should be applied. In Slovakia, in the past and also in the present, the method using shear surface, posted in [9] is often applied when designing spread foundation on inhomogeneous subsoil. By this method, average values of shear strength parameters (angle of internal friction and cohesion) and unit weight calculated using shear strength parameters and unit weights of the layers will be applied.

As we can see, Eurocode 7, Part 1 does not specify a design method for spread foundation on inhomogeneous subsoil; therefore there are many possibilities to choose from. So a comparison of above mentioned design methods for spread foundation on inhomogeneous subsoil can be very helpful. Detail comparisons between the old PN, old STN, new STN, applying both substitute foundation and shear surface procedure can be found in [11]. In this paper we will compare procedures by the old PN, PN-EN, new STN and Mayerhof and Hanna (abbreviated as “MH”).

2. Designing spread foundation on inhomogeneous subsoil by old PN, PN-EN 1997-1:2008, new STN and MH

When designing spread foundation, generally, the bearing capacity of subsoils will predetermine the size of foundation. The evaluation of soil bearing capacity is a matter of wide comprehension since it concerns not only the soils but also the actions and the shape of the foundation. The soils can be also inhomogeneous and there is also the water in the foundation soils. The soils bearing capacity can be evaluated also in drained or in undrained condition etc. More details on various spread foundation design procedures can be found in the specific above mentioned documents [1, 2, 6-9, 11-13]. In the following we will introduce briefly calculation of designed bearing capacity of the foundation soils by the old PN, the PN-EN 1997-1:2008, the new STN and MH.

2.1 Designing spread foundation on inhomogeneous subsoil by the old PN

By the old PN, the designed bearing capacity of subsoil can be calculated by the formula:

\[
Q_{\text{BN}} = B L \left[ 1 + 0.3 \frac{B}{L} N_c \phi' i_e + \left( 1 + 1.5 \frac{B}{L} \right) N_D \rho_D \phi g D_{\text{min}} t_D + \left( 1 - 0.25 \frac{B}{L} \right) N_B \rho_B \phi g B \right]
\] (1)
Meanings of symbols in equations (1) including meanings of dimensionless factors are well-known to geotechnical community so they are not introduced here. By the same way, well-known meanings of symbols in further bearing capacity equations in this paper will not be introduced.

In the case, when stronger layer underlain by weaker layer and surface of weak soil layer is in a depth less than 2B, where B is foundation width (see figure 1), the old PN prescribes to check bearing capacity sufficiency (see equation (1)) for a substitute foundation as it can be seen in figure 1 (substitute foundation base is just on the surface of the weak layer).

![Figure 1. Definitions of substitute foundation parameters by the old PN [1].](image)

It is necessary to calculate new parameters (in figure 1 marked by apostrophe) such as a vertical load $N'_r$, eccentricity $e'_B$, foundation depth $D'_\text{min}$. The values $b$, necessary for calculation of substitute foundation width $B'=B+b$ can be obtained using formula:

1) For cohesive soils:
   
   If $h \leq B$ then $b = \frac{h}{4}$;  
   If $h > B$ then $b = \frac{h}{3}$  

2) For cohesionless soils:
   
   If $h \leq B$ then $b = \frac{h}{3}$;  
   If $h > B$ then $b = \frac{2h}{3}$

where:

$h$– distance from real foundation base to surface of weak layer (m), see figure 1.

For the case when a weaker layer underlain by a stronger layer, it is a common practice in Poland, that spread foundation is designed to suite a bearing capacity of the weaker layer, not taking account a bearing capacity of the stronger layer.

2.2 Designing spread foundation on inhomogeneous subsoil by the PN-EN
The principle is the same as in Chapter 2.1, applying equation:

\[
R_d = \left( c'_N \frac{b}{e} s_i + q'_N \frac{b}{q} q_s i_q + \gamma' \frac{B}{2} N_r \gamma_s \gamma_y \right) / \gamma_{R,Y} \tag{2}
\]

2.3 Designing spread foundation on inhomogeneous subsoil by the new STN
By the new STN, the designed bearing capacity of subsoils can be calculated by the formula:
The equation (3) is applied only for homogeneous subsoil in a range of shear surface which arises if foundation fails. The depth of shear surface $z_s$ under foundation base and its horizontal dosage $l_s$ from foundation axis (see figure 2b) are approximately considered to be:

- $z_s = 2B$, $l_s = 6B$ for soil SW, SP and S-F; GW, GP and G-F;
- $z_s = B$, $l_s = 2.5B$ for other soil classes.

Subsoil is considered to be homogeneous if difference between minimal and average values does not exceed 4° (for angle of internal friction), 40% of average value (for cohesion) and 5% of average value (for unit weight). All mentioned conditions should be fulfilled. For layered subsoil and for other cases when conditions for applying equation (3) are not fulfilled, it will be solved individually.

It is common practice in Slovakia, that for layered subsoil, in the beginning, one should construct shear surface base on proposed foundation width $B$ and on an arithmetic mean of angles of internal friction of soils of layers, often using Prandtl’s shear surface, see figure 2a). After having the first shear surface, one should calculate average value of angle of internal friction using formula:

$$
\phi = \frac{\phi_1(l_{1a} + l_{1b}) + \phi_2(l_{2a} + l_{2b}) + \phi_3l_3}{\sum_{i=1}^{3} l_i}
$$

(4)

where:

- $\phi_1$, $\phi_2$, $\phi_3$ – an angle of internal friction of soil of layer No. 1, 2 and 3 (°), see figure 2b,
- $l_{1a}$, $l_{1b}$, $l_{2a}$, $l_{2b}$ and $l_3$ – lengths of shear surface crossing layer No. 1, 2 and 3 (m), see figure 2b.

The value of average angle of internal friction, calculated by the formula (4) is then compared with the average angle of internal friction, obtained from previous step. If a difference between them is more than 3%, iteration should be applied till condition of maximal 3% difference will be fulfilled [9]. The last average value of angles of internal friction will be applied to calculate bearing capacity of subsoil by equation (3).

To calculate bearing capacity of subsoil by equation (3), one should calculate also average value of cohesion by the formula:

$$
c = \frac{c_1(l_{1a} + l_{1b}) + c_2(l_{2a} + l_{2b}) + c_3l_3}{\sum_{i=1}^{3} l_i}
$$

(5)

where:

- $c_1$, $c_2$, $c_3$ – cohesions of soil of layer No. 1, 2 and 3 (kPa), see figure 2b,
- $l_{1a}$, $l_{1b}$, $l_{2a}$, $l_{2b}$, $l_2$ and $l_3$ – lengths of shear surface crossing layer No. 1, 2 and 3 (m), see figure 2b,
- and also average value of unit weights by the formula:
\[ \gamma = \frac{\gamma_1A_1 + \gamma_2A_2 + \gamma_3A_3}{\sum_{i=1}^{3} A_i} \]  

(6)

where:
\( \gamma_1, \gamma_2, \gamma_3 \) – unit weights of soil of layer No. 1, 2 and 3 (kN/m\(^3\)), see figure 2b,
\( A_1, A_2 \) and \( A_3 \) – areas of soil of layer No. 1, 2 and 3 (m\(^2\)), see figure 2b.

Figure 2. a) a construction of shear surface; b) lengths of shear surface in specific layers and areas (\( A_1, A_2 \) and \( A_3 \)) of soil of specific layers in the shear surface range.

The values of average cohesion and unit weight obtained from equation (5) and (6), together with average angle of internal friction obtained from equation (4) will be applied to calculate bearing capacity of subsoil by equation (3) and to design spread foundation (to find e. g. foundation width \( B_1 \)). Using width \( B_1 \), one should repeat all calculation procedures until difference between \( B_{n+1} \) and \( B_n \) is satisfied.

2.4 Designing spread foundation on inhomogeneous subsoil by MH
Designing spread foundation on inhomogeneous subsoil by MH is introduced in [6]. In figure 3 we can see a case of shallow continuous foundation supported by a stronger soil layer, underlain by a weaker soil that extends to a great depth ($Ca$ is adhesive force and $Pp$ is passive force per unit length of the faces $aa'$ and $bb'$).

![Figure 3. Bearing capacity of a continuous foundation on layered soil [6].](image)

For rectangular foundations, the following equation is applied for calculation of ultimate bearing capacity [6]:

$$q_a = q_b + \left(1 + \frac{B}{L}\right) \left(2c_i'H - B\right) + \gamma_iH^2 \left(1 + \frac{2D_i}{L}\right) \left(1 + \frac{2D_i}{H}\right) \left(\frac{c_i'}{B} - \gamma_iH\right)$$  

(7)

where:

$$q_b = c_i'N_{c(i)}F_{c(i)} + \gamma_i(D_f + H)N_{q(i)}F_{q(i)} + \frac{1}{2} \gamma_2BN_{\gamma 2}F_{\gamma 2}(i)$$  

(8)

and

$$q_t = c_i'N_{c(t)}F_{c(t)} + \gamma_1(D_f + H)N_{q(t)}F_{q(t)} + \frac{1}{2} \gamma_1BN_{\gamma 1}F_{\gamma 1}(i)$$  

(9)

Adhesion $c_a'$ can be determined graphically based on ratio $q_b/q_t$ and cohesion of the top layer $c_1'$ (figure 5.11 in [6]). Punching shear coefficient $K_s$ is a function of $q_b/q_t$ and angle of internal friction of the top layer $\phi_1'$ and can be determined graphically (figure 5.10 in [6]). Shape factors in equations (8) and (9) can be found in table 4.3 in [6].

When a foundation is supported by a weaker soil layer underlain by a stronger layer (figure 4), ultimate bearing capacity can be given by the empirical equation [6]:

$$q_a = q_t + \left(\frac{H}{D}\right)^2 q_t$$  

(10)

where:

$$q_t = c_i'N_{c(t)}F_{c(t)} + \gamma_1(D_f + H)N_{q(t)}F_{q(t)} + \frac{1}{2} \gamma_1BN_{\gamma 1}F_{\gamma 1}(i)$$  

(11)

and

$$q_b = c_i'N_{c(2)}F_{c(2)} + \gamma_2(D_f + H)N_{q(2)}F_{q(2)} + \frac{1}{2} \gamma_2BN_{\gamma 2}F_{\gamma 2}(2)$$  

(12)
Mayerhof and Hanna [5] (by [6]) suggested that \(D \approx B\) for loose sand and clay, \(D \approx 2B\) for dense sand.

![Figure 4](image-url)  
**Figure 4.** Foundation on weaker soil layer underlain by stronger sand layer [6].

### 3. Examples

To compare above mentioned procedures, in the following we will introduce two examples: in the first example, a strong layer is underlain by a weak layer and in the second example, a weak layer is underlain by a strong layer. The model example is similar to the model introduced by Orr [11].

#### 3.1 Designing spread foundation when a strong layer is underlain by a weak layer

In figure 5 we can see a model example, when strong layer is underlain by a weak layer. Thickness of foundation is 0.8 m, depth of foundation \(D = 0.8\) m. There is a permanent vertical load \(G_{vk} = 900\) kN and a variable vertical load \(Q_{vk} = 600\) kN acting on foundation. The soil of the first layer (strong) is silty sand (in Poland marked as \(P\)) with \(I_D = 0.5\), thickness 1.9 m. The soil of the second layer (weak) is sandy silt (in Poland marked as \(\Pi p\)) with \(I_L = 0.4\), thickness 5.8 m. Soils shear strength parameters are obtained in accordance with the old PN, using \(I_D = 0.5\) and \(I_L = 0.4\). Design of foundation was carried out by procedures mentioned in previous chapter. Results are introduced in the table 1, where “SUBSTFOUND” means that foundation was designed using substitute foundation as in the figure 1 and “SHEARSURFACE” means that foundation was designed using shear surface as in the figure 2.
Figure 5. Model example when a strong layer is underlain by a weak layer.

Table 1. The sizes of spread foundation on inhomogeneous subsoil in (m) by various design approaches (a strong layer $\pi$ is underlain by a weak layer $\pi_p$).

| Design approach                        | $\phi$ (°) | $c$ (kPa) | $\gamma$ (kN.m$^{-3}$) | $B$ (m) |
|----------------------------------------|------------|-----------|-------------------------|--------|
| Old PN (SUBSTFOUND), (1) Top layer (Bottom layer) | 30.5 (14.53) | 0 (23.69) | 9.44 (10.47) | 2.18   |
| PN-EN (SUBSTFOUND), (2) Top layer (Bottom layer) | 30.5 (14.53) | 0 (23.69) | 9.44 (10.47) | 2.39   |
| New STN (SHEARSURFACE), (3) Average values of $\phi$ and $c$ | 19.54 | 17.01 | 9.95 | 2.66   |
| Meyerhof and Hanna, (4) Top layer (Bottom layer) | 30.5 (14.53) | 0 (23.69) | 9.44 (10.47) | 1.89   |

Difference in size foundation between (1) and (2) in (m) and (%) 
-0.21m (9.6%)

Difference in size foundation between (2) and (3) in (m) and (%) 
-0.27m (11.3%)

Difference in size foundation between (2) and (4) in (m) and (%) 
-0.50m (26.5%)

Difference in size foundation between (3) and (4) in (m) and (%) 
-0.77m (40.7%)

3.2 Designing spread foundation when a weak layer is underlain by a strong layer

In figure 6 we can see a model example, when a weak layer is underlain by a strong layer. Thickness of foundation is 0.8m, depth of foundation $D = 0.8$m. There is a permanent vertical load $G_{vk} = 900$ kN and variable vertical load $Q_{vk} = 600$ kN acting on foundation. The soil of the first layer (weak) is silt with high plasticity (in Slovakia marked as MH) with $I_C = 0.7$, thickness 2.4 m. The soil of the second layer (strong) is silty gravel (in Slovakia marked as GM) with $I_D = 0.5$, thickness 6.0 m. Soils shear strength parameters are obtained in accordance with the old STN, using $I_C = 0.7$ and $I_D = 0.5$. Design of foundation was carried out by procedures mentioned in previous chapter. Results are introduced in the table 2, where “IGNORE-GM” means that foundation was designed not taking into account influence of strong layer GM and “SHEARSURFACE” means that foundation was designed using shear surface as in the figure 2.
Figure 6. Model example when a weak layer is underlain by a strong layer.

Table 2. The sizes of spread foundation on inhomogeneous subsoil in (m) by various design approaches(a weak layer MH is underlain by a strong layer GM).

| Design approach                        | $\phi$ (°) | c (kPa) | $\gamma$ (kN.m$^{-3}$) | B (m) |
|----------------------------------------|------------|--------|------------------------|------|
| Old PN (IGNORE-GM), (1) Top layer      | (30.0)     | (2)    | (19.0)                 | 2.71 |
| Bottom layer                           |            |        |                        |      |
| PN-EN (IGNORE-GM), (2) Top layer       | (30.0)     | (2)    | (19.0)                 | 3.04 |
| Bottom layer                           |            |        |                        |      |
| New STN (SHEARSURFACE), (3) Average    | 25.53      | 5.06   | 20.28                  | 2.40 |
| values of $\phi$ and $c$               |            |        |                        |      |
| Meyerhof and Hanna, (4) Top layer      | (30.0)     | (2)    | (19.0)                 | 2.26 |
| Bottom layer                           |            |        |                        |      |

Difference in size foundation between (1) and (2) in (m) and (%) -0.33m (12.2%)
Difference in size foundation between (2) and (3) in (m) and (%) -0.64m (26.7%)
Difference in size foundation between (2) and (4) in (m) and (%) -0.78m (34.5%)
Difference in size foundation between (3) and (4) in (m) and (%) -0.14m (6.2%)

As we can see from the table 1 and table 2, there are large differences between spread foundations size designed by various procedures. The smallest foundations size is in case of design by Meyerhof and Hanna. In comparison with PN-EN, it is smaller on 0.5m (26.5%) in case a strong layer underlain by a weak layer and up to 0.78m (34.5%) in case a weak layer underlain by a strong layer.

The largest foundations size is in case of design by old PN and PN-EN (a weak layer underlain by a strong layer, table 2). The reason is neglect of influence of strong layer in subsoil bearing capacity. Such neglect causes difference up to already mentioned 0.78m (34.5%), see table 2.

Comparing the procedures applied in Poland in the past and in the present, in both cases (a strong layer underlain by a weak layer and vice versa), spread foundations size designed by the old PN is smaller than by PN-EN, see 0.21m (9.6%) in table 1 and -0.33m (12.2%) in table 2.

Comparing the procedures applied in Slovakia and Poland in the present, while in the case a strong layer underlain by a weak layer, spread foundations size designed by the new STN is larger than by PN-EN, see -0.27m (11.3%) in table 1, in the case a weak layer underlain by a strong layer, spread foundations size designed by the new STN is smaller than by PN-EN, see -0.64m (26.7%) in table 2 (the reason is already mentioned neglect of influence of strong layer in subsoil bearing capacity by the procedure applied in Poland).
4. Conclusions

When designing spread foundation on inhomogeneous subsoil with a weak layer underlain by a strong layer, neglecting the strong layer leads to uneconomical design. We would like to recommend in such case to design foundation with average shear strength parameters, obtained e.g. from proposed shear surface.

From introduced analyses, foundation sizes designed by Meyerhof and Hanna are the smallest. However, it is necessary to carry out more analyses to draw general conclusion.

Acknowledgment

The authors gratefully acknowledge the funding by ERANET-CORNET consortium under the international research project PROGEO 2 “Geotextiles from Sustainable Raw Materials and Textile Waste, New Mobile Production Technology and New Application Fields in Drainage and Hydraulic Engineering” DZP/CORENET/1/20/2017.

References

[1] PN-81/B-03020: 1981: “Building soils. Foundation bases. Static calculation and design,” (in Polish language), Warsaw : Polish Committee for Standardization, 1981.
[2] PN-EN 1997-1:2008: “Eurocode 7. Geotechnical design. Part 1: General rules,” (in Polish language), Warsaw : Polish Committee for Standardization, 2008.
[3] O. Puła, “Spread foundation design by Eurocode 7,” (in Polish language). Wrocław : Publishing house „Dolnośląskie Wydawnictwo Edukacyjne”, 2nd ed., pp. 141, 2012.
[4] G. G. Meyerhof, “Ultimate Bearing Capacity of Footings on Sand Layer Overlying Clay,” Canadian Geotechnical Journal, vol. 11, pp. 224–229, 1974.
[5] Meyerhof, G. G. and Hanna, A. M. (1978). “Ultimate Bearing Capacity of Foundations on Layered Soil under Inclined Load,” Canadian Geotechnical Journal, vol. 15, pp. 565–572, 1978
[6] Braja M. Das, „Principles of Foundation engineering,“ Boston : Cengage Learning, 8th ed., 2014.
[7] STN 73 1001: 1987: “Foundation of structures. Subsoil under shallow foundations,” (in Czech language), Prague : Institute for Standardizations and Measurements, 1987.
[8] STN 73 1001: 2010: “Geotechnical structures. Foundation,” (in Slovak language), Bratislava : Slovak Standards Institute, 2010.
[9] P. Turček, I. Slávik, „Foundation engineering,” (in Slovak language), Bratislava: STU Publisher, 2002.
[10] M. Decký, E. Remišová, M. Mečár, L. Bartuška, J. Lizbetine, I. Drevený, I.: In situ Determination of Load Bearing Capacity of Soils on the Airfields. In: Journal Procedia Earth and Planetary Science, p. 11-18, doi:10.1016/j.proeps.2015.08.004, ISSN 1878-5220
[11] G. Nguyen, “Designing spread foundation on inhomogeneous subsoil by various approaches,” Inżynieria morska i geotechnika, vol. 38, p. 26-32, 2017.
[12] M. Drusa, J. Vlček.: Importance of Results Obtained from Geotechnical Monitoring for Evaluation of Reinforced Soil Structure – Case Study, Journal of Applied Engineering Sciences, De Gruyter Open, ISSN: 2247-3769. Vol 6. Issue 1/2016. DOI: 10.1515/jaes-2016-0002
[13] Drusa M, Vlček J., Orininová L.: The role of geotechnical monitoring at design of foundation structures and their verification – Part 1, J. of Civil and Environmental Engineering, De Gruyter Open, eISSN: 2199-6512, Vol. 12, Issue 1/2016, 21-26 DOI: 10.1515/cee-2016-0003
[14] Trevor L.L. Orr, “Model solution for Eurocode 7 workshop examples,” International Workshop on the Evaluation of Eurocode 7, pp. 75-108, 2005.