Modelling militantism and partisanship spread in the chain and square lattice opinion structures by using q-XY opinion model

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Abstract: In this work we have studied the spreading of militantism in 1D and 2D square lattice by operating the q-XY opinion and its utility optimization as control mechanism for the update process. The average utility and opinion value for a pair of opinion nodes are calculated in the framework of standard statistical mechanics using the q-opinion utility \( U = -\frac{1}{2}(\vec{O}^2 - 2) - \vec{F}_\vec{O} + q \left( \frac{1}{2}(\vec{O}^2 - 2) \cdot \vec{F}_\vec{O} \right) \). The agreement with an incoming zealot opinion is modelled as one by one dialogue starting from a random opinion configuration. Each time step one node updates its opinion value according to a set of rules and by specific probabilities based on local utility improvement. New opinion values are accepted with a Metropolis-like probability calculated by employing q-utilities of the nodes under updating process.

For the chain opinion structure in the interaction type preserving regimes where \( 1 - q \frac{1}{2}(\vec{O}^2 - 2) > 0 \) given \( 0 \leq \theta \leq 1 \) it resulted that that both 1D and 2D structures accept the militant offer only partially. For bounded chain the agreement level is at average values for short chain and low values for bounded long opinion chain. Bounded chain structures are found resistant toward militantism accommodation. For square opinion lattices we obtained also that the militant attitude would never cover all the community. Homogenous-nodes group resulted somewhat deterring in accepting the militant opinion with an agreement level of around 0.6. For heterogeneous communities the edge agreement level closed up to around 0.7-0.8. The model reproduces the fact that there would be always a fraction of opposition whatsoever regime is applied. It produces various outcomes imitating the complexity of the social behaviour.

Keywords: opinion models, XY magnets, militantism spread, sociophysics

1. Introduction

The opinion models based on the sociophysical approach have been able to offer a thorough and multidimensional analysis of the social behaviour, see for example references [1], [2], [3]. In particular, the magnetic systems have been used routinely in opinion modelling. Aside very useful Ising or Pot’s spin replicas, the continuous models are also practical and intriguing. We have elaborated in our previous works [5] and [6] the effect of the copulation of the partial interests among individuals and the potential for abrupt or surprising change of their attitudes by using the utility function \( U = -\sum \vec{O}_i \vec{O}_j - \).
\( \vec{F} \sum \vec{O}_i + q (-\sum \vec{O}_i \vec{O}_j)(-\vec{F} \sum \vec{O}_i) \). This last has been envisaged from the XY magnet Hamiltonian by adding an extra utility term to consider nonlinear interaction and competitions between utilities representing partial interests or unhappiness in terms of discussions in the reference [8]. Therein, \( \vec{O}_i \) are opinion vectors of the unity magnitude, \( \vec{F} \) is the exterior field and \( q \) is the intensity of the pairing of the node-interaction utility with the part of the utility due to the interaction of total opinion with exterior field. In [6] we describe the process as ‘the early stage of opinion formation’ and restrict the model to the strongly linked agents which share the same interests. In [5] we used it as the starting opinion configuration in a system to analyse the route to consensus by employing Deffuant update mechanism which consists in a simple and logical update rules, see [9]. In the following the opinion calculated in [5] and [6] is refereed as q-XY opinion. Specifically, this model demonstrated the capability to explain the dependency of the agreement dynamics from the interior preconditions and the shifting of the agreement behaviour. In this study we propose a specific update mechanism resembling the Sznajd procedure used in [10], but with details and differences which are described below. The proposed model aims an alternative study of the behaviour of a community which is organised with a given topology of links and also specifically the agreement toward a militant offer, sole election alternative etc. Before elaborating the evaluation of partisanship or militant idea over the 1D or 2D lattice–based opinion structure we will present below a short description of the q-XY model.

1. The q-XY opinion model and some complementary issues

In the q-XY vector model we have assigned the opinions of the pairs (i, j) by two vectors \{\( \vec{O}_i, \vec{O}_j \)\} of the unity magnitude and employed the standard statistical mechanics calculation for the physical quantities. To account for nonlinearity and copulation of the interest, the utility is proposed

\[
U = -\frac{1}{2} (\vec{O}^2 - 2) - \vec{F} \vec{O} + q (\frac{1}{2} (\vec{O}^2 - 2) * \vec{F} \vec{O}) \tag{1}
\]

In the formulae (1) the third term is fabricated from the weighted product the two classical terms of XY Hamiltonian used in [7]. We have identified only two physical directions for the coordinate system, so the structure is planar. The x-axis is taken along the exterior field \( \vec{F} \) and represents the full agreement with the issue embodied in it. The y-direction represents the state rejecting agreement on \( \vec{F} \) or the “no-interests state” regarding to issue \( \vec{F} \) [5], [6]. In principle the proposed Hamiltonian has the potential to produce complicated dynamics, and therefore is considered as useful for the description of the complexity in the social behaviour. The partition function of the general form

\[
Z = \int -\sum_{i=1}^{N-1} \vec{O}_i \vec{O}_{i+1} - \vec{F} \sum_{i=1}^{N} \vec{O}_i + q (-\sum_{i=1}^{N-1} \vec{O}_i \vec{O}_{i+1}) (\vec{F} \sum_{i=1}^{N} \vec{O}_i) \]

is evaluated by succeeding the steps used in [7] with slight modification mentioned in [6]. The trick consists in using the total vector instead of individual ones which produce the following form for the partition function

\[
Z = \int \prod \delta^{D}(\vec{S}) \delta^{D}(\vec{S} - \sum_{j=1}^{N} \vec{S}_j) \int \prod \delta^{D}(\vec{S} - \sum_{i=1}^{N} \vec{S}_i) \]

This integral can be calculated by contemplating the N-dimension Dirac delta function \( 1 = \prod \delta^{D}(\vec{S} - \sum_{i=1}^{N} \vec{S}_i) \) as proposed initially in [13] for XY magnetic system.

1.1. The q-XY opinion pair

Let’s start from \( Z = \int \prod \delta^{D}(\vec{S}) \delta^{D}(\vec{S} - \sum_{i=1}^{N} \vec{S}_i) \) and \( \int \delta^{D}(\vec{S} - \sum_{i=1}^{N} \vec{S}_i) \) as proposed in [13].

By writing \( \vec{F} \vec{O}_i = k \cos q_1 \) we have

\[
\int \frac{d \vec{O}_1}{(2 \pi)^2} e^{i k \cos (q_1)} d \phi_1 \int \frac{d \vec{O}_2}{(2 \pi)^2} e^{i k \cos (q_2)} d \phi_2 = (2 \pi) I_0(k)^2
\]

where \( I_n(x) \) are the Bessel functions of order \( n \). Also, according to [13] we take

\[
\int \frac{d \vec{O}}{(2 \pi)^2} e^{i k \cos (q)} d \phi = \frac{2}{\pi} \frac{1}{\sqrt{4 - k^2}}
\]

for \( 0 \leq k \leq 2 \). Next,
\[ \int_0^{2\pi} d\theta e^{i \frac{(O^2-2)}{2} + FO \left(1 - q \frac{(O^2-2)}{2}\right)} = e^{-\frac{\beta}{2} \left(1 - \frac{q}{2} (O^2 - 2)\right)} \]

where \( I_n(x) \) are incomplete Bessel function of order \( n \). Bringing those findings together, it results

\[ Z = 8\pi e^{-\beta} \int_0^{\beta FO^2} e^{-\frac{1}{2} \left(1 - \frac{q}{2} (O^2 - 2)\right)} \frac{1}{\sqrt{4 - O^2}} dO \]

From the equation (4) and using statistical thermodynamics formulas for magnets, the average opinion induced in the system and the utility on a given state \( \{J, F, q\} \) are as following

\[ \langle O \rangle_F = \frac{1}{\beta} \frac{\partial \ln(Z)}{\partial F} = \frac{\int_{\beta FO}^{\beta FO^2} \frac{1}{\sqrt{4 - O^2}} I_0 \left(\beta FO \left(1 - \frac{q}{2} (O^2 - 2)\right)\right) dO}{\frac{1}{\sqrt{4 - O^2}} I_0 \left(\beta FO \left(1 - \frac{q}{2} (O^2 - 2)\right)\right)} \]

\[ U = \frac{\partial \ln(Z)}{\partial \beta} = \frac{-\int_{\beta FO}^{\beta FO^2} \frac{1}{\sqrt{4 - O^2}} I_0 \left(\beta FO \left(1 - \frac{q}{2} (O^2 - 2)\right)\right) dO}{\frac{1}{\sqrt{4 - O^2}} I_0 \left(\beta FO \left(1 - \frac{q}{2} (O^2 - 2)\right)\right)} \]

wherein \( A(O) = O \left(1 - \frac{q}{2} (O^2 - 2)\right) \). The above formulae are suitable for numerical calculation. For bounded n-nodes chain we can use similar calculation starting from the \( Z_n \) partition function whereas for unbounded chain we didn’t concluded in an analytic form of the type (4) for the partition function. Note that the extension of the model for large number of individuals reveals the problem of the common interests that we have skipped for the moment. However, for \( n > 3 \) the calculation are tedious but in principle can be performed straightforwardly.

1.2. Q-XY cellule

Opinion cellule relationship can be reasonable for 2 or 3 nodes. The formal Hamiltonian for N-nodes bounded chain has the form

\[ H = -J \sum_{i=1}^{N-1} O_i O_{i+1} - F \sum_{i=1}^{N} O_i + a \left( - \sum_{i=1}^{N-1} O_i O_{i+1} \right) \left( F \sum_{i=1}^{N} O_i \right) \]

By using N-dimension delta function and \( d^n R = R^{n-1} dR \ast \sin(\varphi_1)^{n-2} d\varphi_1 \sin(\varphi_2)^{n-3} d\varphi_2 \ldots d\varphi_n \) we have

\[ Z = \int_0^{2\pi} d\varphi_1 \ldots \int_0^{2\pi} d\varphi_n \int_0^{\pi} \sin(\theta_1) \int_0^{\pi} \sin(\theta_2) \int_0^{2\pi} e^{i \frac{k}{2} (O^2 - N)} e^{\beta FO \left(1 - a \left(\frac{1}{2} (O^2 - N)\right) \cos \theta\right)} dO \ast \]

\[ \int_0^{\pi} \sin(\theta_1) e^{-ik_1 \cos(\theta_1)} d\theta_1 \sin(\theta_2) e^{-ik_2 \cos(\theta_2)} d\theta_2 \sin(\theta_3) e^{-ik_3 \cos(\theta_3)} d\theta_3 = \]

\[ \int_0^{3} 4\pi O^2 dO \int_0^{\frac{4\pi k^2}{(2\pi)^2}} dk \ast \left( \frac{\sin(k)}{k} \right)^3 e^{\frac{ik}{2} (O^2 - 3)} \ast \int_0^{3} \sin(\theta) e^{\beta FO \left(1 - a \left(\frac{1}{2} (O^2 - N)\right) \cos \theta\right)} \]

\[ \frac{1}{3} \left( \frac{4\pi}{3} \right)^3 \int_0^{3} 4\pi O^2 dO \int_0^{\frac{4\pi k^2}{(2\pi)^2}} dk \ast \left( \frac{\sin(k)}{k} \right)^3 e^{\frac{ik}{2} (O^2 - 3)} \ast \int_0^{3} \sin(\theta) e^{\beta FO \left(1 - a \left(\frac{1}{2} (O^2 - N)\right) \cos \theta\right)} \]

\[ \frac{1}{3} \left( \frac{4\pi}{3} \right)^3 \int_0^{3} 4\pi O^2 dO \int_0^{\frac{4\pi k^2}{(2\pi)^2}} dk \ast \left( \frac{\sin(k)}{k} \right)^3 e^{\frac{ik}{2} (O^2 - 3)} \ast \int_0^{3} \sin(\theta) e^{\beta FO \left(1 - a \left(\frac{1}{2} (O^2 - N)\right) \cos \theta\right)} \]
In our knowledge the form (8) cannot be simplified further analytically so the structures involving 3-node cellules are not included in this work.

1.3. Remarks on the utility : interaction type persevering conditions

Let’s analyse some features of the q-XY model. The utility function can be written

\[ U = -\frac{J(1+2q+2FO)}{2}(O^2 - 2) - FO \equiv \frac{J_{\text{xy}}}{2}(O^2 - 2) - FO \]  \hspace{1cm} (9)

In the equation (9) we have formally the XY–magnet Hamiltonian where the interaction parameter \( J_{\text{new}} \) is transformed in \( \frac{J(1+2q+2FO)}{2} \). So, the effect of the partial utility pairing could be assimilated in the modification of the interaction intensity \( J \). Potentially the FM type interaction would become AFM type when the sign of the effective \( J \) will change. If a steady state is reached, the pair will preserve its initial type if \( 1 + 2q \cdot F \cdot O > 0 \) which reads \( q > -\frac{1}{F\cdot<\theta>_{\bar{x}}} \) or \( F > -\frac{1}{q<\theta>_{\bar{x}}} \) or also if \( \theta_{\bar{x}} > \frac{1}{qF} \).

Those transcendent conditions can be used in the numerical calculation to give a classification of the inner-condition of the system. Another element to be considered is the interaction with exterior field. In this case we may request that the utility term including the exterior field should preserve the sign and we assign this condition as ‘interaction type preservation’ condition. Form the expression

\[ -\frac{J}{2}(O^2 - 2) - FO\left(1 - \frac{q}{q^2}(O^2 - 2)\right) \equiv -\frac{J}{2}(O^2 - 2) - \mu FO \]  \hspace{1cm} (9)

Figure 1. Utility and opinion as function of q-parameter and exterior field

It requires that \( 1-Jq/2 \cdot (O^2-2)>0 \). From the natural condition \( O\leq2 \) we have the boundary for numerical calculation \( O_{\text{max}} = \min\{2, +\sqrt{\pm(2+Jq)/qJ}\} \) given that \( \pm(2+Jq)/qJ>0 \). In the Figure 1 we have displayed the utility and opinion calculated under above conditions. The graphs showed that the optimal values of
each quantity usually do not coincide; therefore the maximization of the utility does not mean a full agreement. Adding to that, the agreement is not result of an equilibrium physical process, so the respective states are non-stationary. Finally, if we include the utility in the selection rules for updating process, it is expected that the militant opinion would advance slowly in the community and always a part of the individuals would not embrace it. Aiming the full agreement, one needs to change the interior parameter \( q \) but if the type interaction must be conserved, this mission is not possible.

2. The dynamics of the spread of militantism from the utility update process

Let consider the process of spreading of extremist opinion in a square lattice. The society of \( N \) individuals could be organized in a topological or in a gaseous network. In the lattice there are strong links and permanent interaction, so when someone would start talking to an individual, this means all the cellule would know it and would react. In the model, if the old happiness on the cellule will be affected by the opinion shift on the contacted node, other members of the cellule could suggest to not follow such a discussion, contracting, etc. So let start the model. At the time \( t \), one individual (zealot) has a militant idea so \( O=1 \) and try to influence the others to adopt his opinion. There are two scenarios. The militant can move in the all topological space where the society lives, trying to convict one-by one the peoples and we classify this as the missionary practice. Otherwise, he can try to convict nearest neighbours and if successful, the other would diffuse the idea. We classify this as the whispering case. Also, the two mechanisms could be combined.

2.1. The updating process

Assume a 1D structure of random opinions and a militant with the opinion \( O_1 \) aligned along the field \( F \) joins the chain in a random position. Temporally, the node in this location \( O_2 \) would interact only with the new-arrived interlocutor. During the interaction, the zealot opinion remains aligned along the field \( F \), so \( O_{1,x} = 1 \), Figure 2. Therefore, only \( O_2 \) would rotate to fulfil the equation

\[
(O_1 + O_2)_x = 2(O_x)
\]  

(10)

Accordingly, we classify the opinion in three groups related to the theoretical \( x \)-component \( < 0 >_x \) calculated with \( q \)-XY model. The opinion with its \( O_{2,x} \) component smaller than \( 2 \times < 0 >_x - 1 \) would be candidate to rotate by such a way that the \( x \)-component would have a random value in the interval \([ \text{Old.Value}, 2 \times < 0 >_x - 1] \). The second group having \( x \)-component in the interval \([ 2 \times < 0 >_x - 1, < 0 >_x ] \) would make a rotation to have \( O_{2,x} = [ < 0 >_x ] \). The last group has nodes with \( O_{2,x} > [ < 0 >_x ] \) so maybe they don’t need to change their old position because they are already fitted in some extent with the militant idea.

![Figure 2. Opinion vectors](image)

The node in discussion would evaluate the situation and recalculate his new utility. If there is an improvement, the move is accepted. If not, the node (i) supposedly would decide to reject the offer but at the same time, acting genuinely, it would keep it with some chance however. Therefore, a
Metropolis ratio is adapted to control the update process. In the following we employed the acceptance of the move by checking the utility improvement \( U_{\text{new}} - U_{\text{old}} < 0 \) or by using Metropolis-like probability where acceptance = \( \min \left( 1, \exp \left( \frac{U_{\text{new}} - U_{\text{old}}}{T} \right) \right) > \text{random number} \). We discussed two subcases: local disturbances and global disturbances. In the case of local disturbances we take empirically whereas for global disturbances \( T \) is the temperature of the opinion system. Note that in the reference [8] the temperature \( T \) is identified with the effect of all disturbances except the issue in discussion.

### 2.2. Assessment of the opinion agreement for bounded chain

Firstly, the whispering mechanism is not adequate for the chain structure because it could stop without reaching all the peoples in it. We will consider herein the missionary mechanism, so the zealot meets all nodes of the chain. Referring to the utility function (1), one see that for FM couples the values \( q<0 \) will strengthen the unhappiness.

![Figure 3](image)

**Figure 3.** Agreement level in bounded chain, for moderate temperature end fixed field. Blue lines, FM, \( q<0 \); red line FM, \( q>0 \), orange line AFM, \( q>0 \), magenta line AFM, \( q>0 \). Solid lines, \( n=20 \). Dashed lines, \( n=5 \)

Similar situation is the case \( q>0 \) for AFM couples. We call those as promoted states. By running the update process we see that promoted and non-promoted state produce different level of the agreement and different route to it also. Next we observe that long chain and short chain would realise different level of the agreement. We observe that generally the agreement with militant idea remain always low for all combinations of parameters, Figure 3. For short chains it reaches the values 0.6-0.5 whereas for long chain, the agreement level remains typically around 0.3-0.4, Figure 3. Typically for promoted case that is sign \( (q) = \text{sign}(J) \) the agreement is higher, Figure 3. A more realistic picture is the case where each individual represent his inner motivation and so the parameter \( q \) could take different
allowed values but falling in one region (not changing the sign). By analysing various cases of updates and combination of the parameters we conclude that bounded chains are resistant opinion structures. Long chains are very difficult to manage and usually do not agree with an incoming militant opinion. To enable high agreement, a good strategy is to consider fragmented societies that are composed by many short chains. We observe that there is no difference for medium and long chain regarding to the agreement level with a zealot opinion.

2.3. Agreement in a square lattice by whispering process
The idea of opinion square lattice comes with exaggerated assumption for human links topologies but it is simple and methodically instructive for discussion in two dimension social space. Note that real societies have more complicated topologies, see [2], [12]. Herein we constructed the process of opinion formation by the missionary and whispering interaction. In this second case we considered updating steps which are similar with Wolf algorithm [11] in corresponding Monte Carlo procedure, but not identical, as long as the process is not necessarily subject of the detailed balance. We assume that the network structure of the society imposes some interrelationship between nodes, and the preference toward the ideas issued in the system would be discussed between peoples in this subgroup of the community. Next we considered the substance of a social process which is embodied in the interest or the utility. Usually an opinion change could occur if there would be improvement in the utility, but sometimes, ‘adventurous peoples’ explore other alternatives, so we set a probability for a change even if the utility is not improved (decreased). Next, in some specific case related to the very emotional individuals (high initial x-component), the change in the opinion could follow the ‘utility rule’ so those opinion nodes prefer simply to join the zealot community. It resulted the following procedure of opinion updates

1. The zealot contacts the node (i) randomly and start a discussion. He learns the general opinion of the node and classify as candidate for the agreement if opinion of the node has the same sign with q-XY opinion. This process endures among all nearest (first) neighbours until no more candidate are available. Finally we have a cluster of connected nodes–candidates for opinion change.

2. With a given probability the cluster submit an opinion rotation toward the field. Note that the opinion of the zealot is aligned to the field and it produces this last. In this step we apply a rapid convergence rate by appointing the new value to a random number between the old values and the opinion borders which are showed in the figure 2.
   a. If the process is considered temporally local, the opinion is rotated toward F-field if local utility is decreased by this move.
   b. If we consider global approach, the provability is taken of Metropolis algorithm type, hence including the temperature.

3. All opinions with x-component having opposite sign with average opinion were not updated. Opinions having x-component superior than the thermodynamic opinion are shifted toward 1 for ideologised societies were left intact.

The above procedures try to mimic the situation of grouped reaction, stratification by shared beliefs and various rate of agreement between different sub-groups. Formally the general update idea is based on the Deffuant mechanism by keeping the zealot opinion unchanged, so final opinion approaches this last by increasing the total x-component value for the opinions. Remember that a key parameter of the model is the pairing weight q. we can set it fixed for homogenous nodes or lets it be whichever in the allowed domain for heterogenous nodes. All nodes are contacted in a missionary phase but the intermediate communication between them to create a cluster has the character of the whispering mechanism. However, the process is missionary because we do not let opinions to change their values in the intermediate phase of clustering. In another approach we have activated this mechanism producing a mixed process. The utilities used in the update controlling are calculated for nearest neighbour in the sense that for the node contacted we calculate the totalling utility of four pairs. The procedure 1-3 is performed for an epoch for all nodes of the lattice. Taking the real time into the
account, for communities of hundred or more individuals, too many epochs means a very long time spent. We observed again that full consensus for the militant issue could not be realised for the hypothetical society we have constructed by this model. By such assessment the agreement for regular 2D lattice with cyclic boundary condition would be around 0.6-0.8, Figure 6. In the case where the $q$-parameter takes such a value that the interaction preserves the type, the agreement level is obtained around 0.6-0.7. If we set the $q$-values to take all allowed values, the total opinion of the society oscillates with small amplitude around limit levels which is 0.7-0.8 for moderate temperature and field intensity (herein $T=1$, $F=1$).

![Figure 4](image.png)

Figure 4. Agreement level in bounded chain, 30 nodes, by $[J, q]$ combination. Blue lines, FM, $q<0$; red line FM, $q>0$, orange line AFM, $q>0$, magenta line AFM, $q>0$. Dashed lines, $q$ fixed, solid lines, $q$ variable in the allowed domain.

If we refer concrete systems, as seen in this approach, the common support for nondemocratic elections expected to give around 70% con. On other situation the interpretation would take inconsideration the connotation of each element. The model supports varies combinations of the update mechanisms, and system parameters which reflects the multiple features of social behaviour.

**Conclusions**

In this work we have analysed the spread of the militant opinion in one and two dimensional opinion lattices by employing $q$-XY opinion and a utility based update mechanism. Due to the non-monotonic behaviour of quantities {Utility, Opinion} the level of the agreement of the inter-linked opinion nodes is usually moderate. In the short chains this agreement would reach 0.6-0.7 whereas longest one usually the agreement is lower and typically around 0.3-0.4. Therefore, the best agreement reached in the societies being composed of many small chain structures whereas long chain structure are resistant toward new militant idea. For two dimensional square lattices we obtained that usually the agreement will be in the range 0.6-0.8 and in the more realistic configuration it has the values of
around 0.75-0.8. In this sense we obtain an intriguing correspondence between prediction of this model and the self-organization structures where such numerical ratios are common. But the most realistic outcome is related to the predictions of the rejection of the society for full consensus with a militant idea.

References

[1] A Sîrbu, V Loreto, V D P Servedio, F Tria (2016) arXiv:1605.06326 [physics.soc-ph]
[2] C Castellano, S Fortunato, V. Loreto, Rev. Mod. Phys. 81, 591 (2009), and refs therein
[3] S Galam. Sociophysics: arXiv:0803.1800v1 [physics.soc-ph] (2009)
[4] Mantegna, R & Stanley, H. (2007). An introduction to econophysics: correlations and complexity in finance. Cambridge University Press New York, NY, USA.
[5] D. Prenga 2019 J. Phys.: Conf. Ser. 1391 012056
[6] D. Prenga. A model for early stage of opinion formation. SigmaPhi, 2017
[7] O Ciftja, D Prenga, J Magn. Magn. Mat. 416, 220-225 (2016).
[8] D. Stauffer, J. Stat. Phys, 151. 10.1007/s10955-012-0604-9 arXiv:1207.6178v1
[9] G. Deffuant, D. Neau, F. Amblard, G. Weisbuch, Adv. Compl. Sys. 3(1-4), 87 (2000).
[10] K. Sznajd-Weron, Acta Phys. Pol. B 36(8), (2005).
[11] E. Luijten: Introduction to Cluster Monte Carlo Algorithms, Lect. Notes Phys. 703, 13–38 (2006)
[12] S.N. Dorogovtsev, J.F.F. Mendes, A.N. Samukhin, Physics B 666, 396 (2003).
[13] O.Ciftja, M. Luban, Phys. Rev. B 60, 141 (1999).