Gaugings of $N = 4$ three dimensional gauged supergravity with exceptional coset manifolds

Parinya Karndumri

String Theory and Supergravity Group, Department of Physics, Faculty of Science, Chulalongkorn University, 254 Phayathai Road, Pathumwan, Bangkok 10330, Thailand
Thailand Center of Excellence in Physics, CHE, Ministry of Education, Bangkok 10400, Thailand
E-mail: parinya.ka@hotmail.com

Abstract: Some admissible gauge groups of $N = 4$ Chern-Simons gauged supergravity in three dimensions with exceptional scalar manifolds $G_2(2)/SO(4)$, $F_4(4)/USp(6) \times SU(2)$, $E_6(2)/SU(6) \times SU(2)$, $E_7(-5)/SO(12) \times SU(2)$ and $E_8(-24)/E_7 \times SU(2)$ are identified. In particular, a complete list of all possible gauge groups is given for the theory with $G_2(2)/SO(4)$ coset space. We also study scalar potentials for all of these gauge groups and find some critical points. In the case of $F_4(4)/USp(6) \times SU(2)$ target space, we give some semisimple gauge groups which are maximal subgroups of $F_4(4)$. Most importantly, we construct the $SO(4) \ltimes T^6$ gauged supergravity which is equivalent to $N = 4 SO(4)$ Yang-Mills gauged supergravity. The latter is proposed to be obtained from an $S^3$ reduction of $(1,0)$ six dimensional supergravity coupled to two vector and two tensor multiplets. The scalar potential of this theory on the scalar fields which are invariant under $SO(4)$ is explicitly computed. Depending on the value of the coupling constants, the theory admits both dS and AdS vacua when all of the 28 scalars vanish. The maximal $N = 4$ supersymmetric $AdS_3$ should correspond to the $AdS_3 \times S^3$ solution of the $(1,0)$ six dimensional theory. Finally, some gauge groups of the theories with $E_6(2)/SU(6) \times SU(2)$, $E_7(-5)/SO(12) \times SU(2)$ and $E_8(-24)/E_7 \times SU(2)$ scalar manifolds are identified.

Keywords: Extended Supersymmetry, Supergravity Models and Supersymmetric Effective Theories.
1. Introduction

Three dimensional Chern-Simons (CS) gauged supergravity has a very rich structure \cite{1}, \cite{2}. Many possible gauge groups of various types are allowed \cite{3}, \cite{4}. This is due to the duality between scalars and vectors in three dimensions. The propagating bosonic degrees of freedom in the ungauged theory are given by scalar fields. The gauge fields enter the gauged Lagrangian via Chern-Simons terms which do not introduce any extra degrees of freedom. So, unlike in higher dimensional analogues, the dimensions of possible gauge groups are not restricted by the number of vector fields present in the ungauged theory.

Of particular interest are non-semisimple gauge groups of the form $G_0 \ltimes T^{\dim G_0}$. This is on-shell equivalent to a Yang-Mills (YM) gauged supergravity with gauge group $G_0$ usually obtained from a dimensional reduction of some higher dimensional supergravity \cite{5}. Working with the CS gauged supergravity is much simpler than the equivalent YM type theory. This has been emphasized in \cite{5} in which the comparison between a simple Lagrangian of the CS type supergravity and a much more complicated Lagrangian of the YM type supergravity has been pointed out. Therefore, the CS construction is more convenient to work with in a three dimensional framework.

The CS gauged supergravity is generally described by gaugings of the ungauged theory in the form of a non-linear sigma model coupled to supergravity \cite{6}. While pure supergravity in three dimensions is topological, for some earlier construction of CS three dimensional supergravity see \cite{7}, this is not the case for the matter coupled supergravity. The target space for scalar fields in the theory with $N > 4$ is a symmetric space of the form $G/H$ in which the global and local symmetry groups are given by $G$ and $H$, respectively. All symmetric spaces involved in $N = 5, 6, 8, 9, 10, 12, 16$ have been given in \cite{6}. In this paper, we will focus on the $N = 4$ theory whose scalar manifold is generally a product of two (not necessarily symmetric) quaternionic manifolds. Furthermore, we are interested in the case of symmetric target spaces and, in particular, the so-called degenerate case in which the target space contains only one quaternionic manifold.

With the embedding tensor formalism of \cite{1}, gaugings can be studied in a $G$ covariant manner. This is very useful, particularly, in the case of symmetric target spaces in which the classification of gauge groups can be achieved by group theoretical method. In this formalism, the symmetric and gauge invariant embedding tensor $\Theta$ is introduced. It acts as a projector on the global symmetry group $G$ to the corresponding gauge group $G_0 \subset G$. In defining a consistent gauge group, the associated embedding tensor needs to satisfy quadratic and linear constraints coming from the closure of the gauge algebra and the consistency with supersymmetry. The general formulation of this
gauged supergravity for any value of \( N \) has been constructed in [8]. Some semisimple
gauge groups for theories with \( N > 4 \) have also been given.

Gaugings of CS three dimensional gauged supergravity with different value of
\( N \) have been studied in various places [1], [3], [4], [9], [10], [11] and [12]. In [13],
[14] and [15], non-semisimple gaugings of the \( N = 4 \) theory with scalar manifold
\( SO(4,k)/SO(4) \times SO(k) \) have been studied. The higher dimensional origin for some
class of gaugings has also been identified. In the following, we will study \( N = 4 \)
theory with exceptional coset manifolds and identify some of their gauge groups for
both semisimple and non-semisimple types. All the exceptional coset manifolds for the
\( N = 4 \) theory have been given in [8]. These are \( G_2(2)/SO(4), F_4(4)/USp(6) \times SU(2), \)
\( E_6(2)/SU(6) \times SU(2), E_{7(-5)}/SO(12) \times SU(2) \) and \( E_{8(-24)}/E_7 \times SU(2). \)

Since \( G_2(2) \) is a small group, we can give all of the possible gauge groups in a theory
with \( G_2(2)/SO(4) \) coset manifold. We will also study scalar potentials of these gauge
groups. The ungauged version of this theory can be obtained by a \( T^2 \) reduction of the
minimal five dimensional supergravity, see for example [16] and [17]. The \( S^2 \) reduction
of this five dimensional theory should give the gauged version in three dimensions with
\( SO(3) \times T^3 \) gauge group as proposed in [3]. But, as we will see, it can be verified that
this gauge group cannot be embedded in \( G_2(2) \). So, if the corresponding \( S^2 \) reduction
exists, it is very interesting to find a description in term of the three dimensional CS
framework.

In the \( F_4(4)/USp(6) \times SU(2) \) case, we will give some admissible semisimple gauge
groups and a non-semisimple gauge group \( SO(4) \times T^6 \). The latter is one of the interesting
results in this paper and should describe an \( S^3 \) reduction of \((1,0)\) six dimensional
supergravity coupled to two vector and two tensor multiplets. Dimensional reductions
of this six dimensional theory have been studied before. Firstly, the \( SU(2) \) reduction of pure
supergravity in six dimensions has been investigated in [18] and [19]. This gives rise to
\( SO(3) \) YM gauged supergravity in three dimensions coupled to three massive
vector fields. The theory is in turn equivalent to \( SO(3) \times (T^3, \hat{T}^3) \) CS gauged
supergravity as proposed by [3] with the three nilpotent generators of \( T^3 \) describing
the three massive vector fields. Then, the \( SU(2) \) reduction of the same theory coupled
to an anti-symmetric tensor multiplet and \( \dim G_c \) vector multiplets of any semisimple
gauge group \( G_c \) has been constructed in [14]. The resulting three dimensional theory is
\( SO(3) \times G_c \) YM gauged supergravity without any massive vector fields since they are
truncated out in the process of reduction.

In this paper, we give one more example of the possible reduction of this six di-
mensional supergravity coupled to two vector and two tensor multiplets on an \( S^3 \).
The reduction on \( T^3 \) giving rise to the ungauged three dimensional supergravity with
\( F_4(4)/USp(6) \times SU(2) \) has been studied in details in [20]. The reduction on \( S^3 \) would
give the gauged version of the three dimensional theory with $SO(4)$ or $SO(4) \times T^6$ for YM and CS supergravities, respectively. We propose the corresponding reduction by constructing the equivalent CS gauged supergravity directly in three dimensions. As mentioned above, the CS version is much simpler to deal with, so this is a convenient starting point. Moreover, both the six dimensional theory and the resulting three dimensional one are in the so-called magical supergravities whose gaugings have been studied recently in [21].

We end the paper by giving some admissible gauge groups in the remaining cases with $E_6(2)/SU(6) \times SU(2)$, $E_7(-5)/SO(12) \times SU(2)$ and $E_8(-24)/E_7 \times SU(2)$ scalar manifolds. Apart from the $G_2(3)/SO(4)$ case, the lists of gauge groups identified in this paper are by no means complete. Further studies are needed in order to cover a larger class of possible gauge groups.

The paper is organized as follows. In section 2, we review the $N = 4$ three dimensional gauged supergravity with symmetric target spaces. The consistency conditions on the embedding tensor are given. We then focus on a specific case of exceptional cosets. Gaugings for each exceptional symmetric space are identified in section 3, 4 and 5. We also study scalar potentials for all gauge groups in the $G_2(3)/SO(4)$ case and $SO(4) \times T^6$ gauging in the $F_4(4)/USp(6) \times SU(2)$ case. The computations are carried out by using the computer program Mathematica. We finally give some conclusions and comments in section 6. Additionally, there is one appendix in which some useful formulae can be found.

2. $N = 4$ gauged supergravity in three dimensions

In this section, we review the construction of three dimensional gauged supergravities following [8]. We will repeat only relevant information to describe $N = 4$ theory and refer the reader to [8] for the full details.

Three dimensional ungauged supergravities are described by nonlinear sigma models coupled to supergravity. Coupling to $N$ extended supergravity requires the existence of $N - 1$ hermitean almost complex structures, $f^P$, $P = 2, \ldots, N$, on the target space of the sigma model. The $f^{IJ}$, $I, J = 1, \ldots, N$, tensors constructed from $f^P$ via

$$f^{1P} = -f^{P1} = f^P, \quad f^{PQ} = f^{[P} f^{Q]}$$

are generators of $SO(N)$ R-symmetry. For $N = 4$ theory, the tensor $J = \frac{1}{6} \epsilon_{PQR} f^P f^Q f^R$ commutes with the almost complex structures and satisfies

$$J^2 = 1, \quad J_{ij} = J_{ji}, \quad J = \frac{1}{24} \epsilon^{IJKL} f^{IJ} f^{KL}.$$
Here and from now on, indices $i, j = 1 \ldots d$ label coordinates on the target space whose dimension is $d$. We will also use the flat target space indices $A, B, \ldots$ with the vielbein defined in the appendix. The product structure of the target space is implied by the fact that $J$ is covariantly constant. So, in general, the target spaces of $N = 4$ three dimensional gauged supergravity are products of two quaternionic spaces. The dimension of the target space is thus given by $d = d_+ + d_-$ with $d_\pm$ being the dimensions of the corresponding two subspaces. Each subspace describes one of the inequivalent multiplets. Although, in three dimensions, there are inequivalent supermultiplets for $N = 4 \bmod 4$, the requirement that the maximal compact subgroup $SO(N) \times H' \subset G$ must act irreducibly on the target space allows only one type of the multiplets [6]. This is not the case for $N = 4$ because the R-symmetry $SO(4)$ is not a simple group and decomposes according to $SO(4) \sim SO(3) \times SO(3)$, and each factor acts on the two subspaces, separately. So, $N = 4$ theory is special in many aspects compared to theories with other values of $N$.

Unlike theories with $N > 4$, the scalar manifolds of the $N = 4$ theory need not be symmetric. However, in this work we are interested in the case of symmetric target spaces of the form $G/H$ in which the maximal compact subgroup $H$ contains the R-symmetry $SO(N)$ or $H = SO(N) \times H'$. As explained above, for the $N = 4$ theory, we have $H = SO(3) \times H' \sim SU(2) \times H'$ for each subspace of the full target space. In this work, we are particularly interested in the exceptional coset spaces which are of the form “non-compact real form of some exceptional group/maximal compact subgroup”. They are quaternionic manifolds $G_2(2)/SO(4)$, $F_4(4)/USp(6) \times SU(2)$, $E_6(2)/SU(6) \times SU(2)$, $E_7(-5)/SO(12) \times SU(2)$ and $E_8(-24)/E_7 \times SU(2)$. Furthermore, we will consider only the degenerate case namely $d_+d_- = 0$. In this case, there is only one quaternionic manifold in the target space. The relevant formulae and useful relations for a symmetric target space are reviewed in the appendix.

Gaugings are implemented by promoting some isometries of the target space to a local symmetry. This procedure is easily dealt with by introducing the so-called embedding tensor, $\Theta_{MN}$, which is gauge invariant. In order to describe a consistent gauging, the embedding tensor needs to satisfy two consistency conditions. The first condition called the quadratic constraint is imposed by the requirement that the gauge generators $J_M = \Theta_{MN}t^N$ with $t^N$ being generators of $G$ form an algebra. This constraint is given by

$$
\Theta_{PL}^{KL}(\Theta_{MN})_{KL} = 0. \tag{2.3}
$$

Furthermore, in order to be consistent with supersymmetry, there is a projection constraint on the T-tensor

$$
T^{IJ,KL} = T^{[IJ,KL]} - \frac{4}{N-2}\delta^{[I[K}T^{L]M,MJ]} - \frac{2}{(N-1)(N-2)}\delta^{[I[K}\delta^{L]J}T^{MN,MN}, \tag{2.4}
$$

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or equivalently,
\[ \mathbb{P}_{\Box} T^{IJ,KL} = 0. \] (2.5)

The T-tensor itself is defined by the image of the embedding tensor under the map \( \mathcal{V} \)
\[ T_{AB} = \mathcal{V}^{\mathcal{M}}_{\mathcal{A}} \Theta_{\mathcal{M}\mathcal{N}} \mathcal{V}^{\mathcal{N}}_{\mathcal{B}} \] (2.6)

with the index \( \mathcal{A} = \{IJ, \alpha, A\} \). The linear constraint means that supersymmetry requires the absence of the \( \Box \) representation of \( SO(N) \) in the T-tensor. The projector \( \mathbb{P}_{\Box} \) projects the \( SO(N) \) representation on to the \( \Box \) representation.

For symmetric target spaces, the condition (2.5) can be lifted to the consistency condition on the embedding tensor and the map \( \mathcal{V} \), which is now an isomorphism, can be obtained from the coset representative \( L \), see the relevant formulae in the appendix. The embedding tensor lives in the symmetric product of the adjoint representation of \( G \). This product can be decomposed into irreducible representations of \( G \) as
\[ (R_{\text{adj}} \times R_{\text{adj}})_{\text{sym}} = 1 \oplus \left[ \bigoplus_i R_i \right], \]
or
\[ \Theta_{\mathcal{M}\mathcal{N}} = \theta \eta_{\mathcal{M}\mathcal{N}} + \sum_i \mathbb{P}_{R_i} \Theta_{\mathcal{M}\mathcal{N}}. \] (2.7)

\( \eta_{\mathcal{M}\mathcal{N}} \) is the Cartan-Killing form of \( G \), and \( \mathbb{P}_{R_i} \) is the G-invariant projector onto the representation \( R_i \). Only one representation of \( R_i \)'s denoted by \( R_0 \) contains the \( \Box \) representation of \( SO(N) \) in the branching under \( SO(N) \). The condition (2.3) can then be written as
\[ \mathbb{P}_{R_0} T_{AB} = 0. \] (2.8)

Using the fact that this condition is G-covariant, we can write it as the condition on \( \Theta \)
\[ \mathbb{P}_{R_0} \Theta_{\mathcal{M}\mathcal{N}} = 0. \] (2.9)

For symmetric spaces in the form of exceptional coset spaces, the embedding tensor takes the simple form
\[ \Theta_{\mathcal{M}\mathcal{N}} = \theta \eta_{\mathcal{M}\mathcal{N}} + \mathbb{P}_{R_1} \Theta_{\mathcal{M}\mathcal{N}}. \] (2.10)

This is because there are only three representations appearing in the decomposition (2.7) with one of them being the forbidden representation \( R_0 \). With this simple structure of the embedding tensor, we can use group theoretical argument given in [8] to find admissible gauge groups \( G_0 \subset G \). The arguments simply says that a semisimple subgroup \( G_0 \subset G \), which is a simple group, is admissible if the decomposition of \( R_0 \) under \( G_0 \) does not contain a singlet, and a semisimple subgroup \( G_0 \subset G \) of the form
$G_1 \times G_2$ is admissible if the decomposition of $R_0$ under $G_1 \times G_2$ contains precisely one singlet with a fixed ratio of the coupling constants. In the next sections, we will use these useful conditions to determine some admissible gauge groups of the $N = 4$ gauged supergravity with exceptional cosets mentioned above. The relevant group theory decompositions can be found in [22] and [23].

For conveniences, we also repeat the exceptional quaternionic spaces together with the decomposition of the corresponding embedding tensor as well as the representation $R_0$ from [8].

| $G/H$   | $d$ | $R_{adj}$ | $(R_{adj} \times R_{adj})_{sym}$ |
|---------|-----|-----------|----------------------------------|
| $\frac{G_{2(2)}}{SO(4)}$ | 8   | 14        | $1 + 27 + 77$                   |
| $\frac{F_4(4)}{SO(4)}$   | 28  | 52        | $1 + 324 + 1053$               |
| $\frac{USp(6)\times SU(2)}{S(U(6)\times SU(2))}$ | 40  | 78        | $1 + 650 + 2430$              |
| $\frac{E_7(-5)}{SO(12)\times SU(2)}$ | 64  | 133       | $1 + 1539 + 7371$            |
| $\frac{E_8(-24)}{E_7\times SU(2)}$ | 112 | 248       | $1 + 3875 + 27000$           |

Table 1: Symmetric spaces for $N = 4$ supergravity and the corresponding $R_0$ representation denoted by the underlined representation.

3. Gaugings in $G_{2(2)}/SO(4)$ coset manifold

In this case, the group $G$ is given by $G_{2(2)}$, the split form of the exceptional group $G_2$. The representation $R_0$ is the 77 representation. The embedding tensor lives in the representation $1 + 27$. We find that the appropriate real forms, which can be embedded in $G_{2(2)}$, of $A_2$ and $A_1 \times A_1$ subgroups can be gauged since the 77 representation of $G_2$ contains none and one singlet when branched under $A_2$ and $A_1 \times A_1$, respectively.

From this, we obtain the admissible gauge groups:

- $SU(2, 1)$
- $SL(3, \mathbb{R})$
- $SO(4) \sim SU(2) \times SU(2)$
- $SO(2, 2) \sim SL(2, \mathbb{R}) \times SL(2, \mathbb{R}) \sim SO(2, 1) \times SO(2, 1)$.

Since the $G_2$ is more tractable, we can also check all other subgroups whether they can be gauged. First of all, one of the two $SU(2)$’s in the $SO(4)$ cannot be gauged as well as their diagonal subgroup $SU(2)_{\text{diag}} \subset (SU(2) \times SU(2))_{\text{diag}}$. The $U(1)$ and
\( U(1) \times U(1) \subset SU(2) \times SU(2) \) cannot be gauged. The non-semisimple group \( SO(2) \ltimes T^2 \) and the nilpotent \( T^3 \), see below, cannot be gauged either. Therefore, there are no other gauge groups apart from those listed above.

The split form \( G_{2(2)} \) has three commuting generators [24]. It has been proposed in [3] that this theory with non-semisimple gauge group \( SO(3) \ltimes T^3 \) could describe the \( S^2 \) reduction of the minimal five dimensional supergravity. It is well-known that the ungauged \( N = 4 \) theory with coset space \( G_{2(2)}/SO(4) \) can be obtained from \( N = 2 \) supergravity in five dimensions reduced on \( T^2 \). Unfortunately, the group \( SO(3) \ltimes T^3 \) cannot be embedded in \( G_{2(2)} \) as can be explicitly checked from the generators of \( G_{2(2)} \) given in the next subsection. So, \( SO(3) \ltimes T^3 \) is certainly not admissible. Furthermore, the nilpotent gauging, whose example in \( N = 16 \) theory has been given in [4], with the corresponding gauge group \( T^3 \) cannot be gauged. Finally, the group \( G_{2(2)} \) itself can be gauged but the corresponding scalar potential will be a cosmological constant. It is the general fact that the group \( G \) is always admissible with a constant scalar potential. Since the group \( G_{2(2)} \) is quite small, the computation of the corresponding scalar potential for each gauge group is not so difficult to perform. We then study scalar potentials and the associated critical points for all of the admissible gauge groups in the remaining subsections.

3.1 \( SO(4) \) gauging

We first give the structure of the \( G_{2(2)} \) coset. The maximal compact subgroup is \( SO(4) \sim SU(2) \times SU(2) \). Therefore, there are eight scalars. Under \( SO(4) \sim SU(2) \times SU(2) \), they transform as \( (2,4) \). This means that they are four copies of the spinor representation of the first \( SU(2) \). We then identify this group as the R-symmetry. The other \( SU(2) \) would become the group \( H' \) mentioned in section 2.

We will use the explicit generators of \( G_2 \) given in [25]. The corresponding generators of the split from \( G_{2(2)} \) are given in [26]. These generators are denoted by \( Q_i, i = 1, \ldots, 14 \) in [26]. In order to apply our general formalism, we relabel the generators as follow.

- R-Symmetry generators: \( T_{12} = \frac{1}{2} Q_3, T_{13} = -\frac{1}{2} Q_2, T_{23} = \frac{1}{2} Q_1 \)
- Non-compact generators:
  \[
  Y_A = \begin{cases} 
  \frac{1}{2} Q_{A+3}, & A=1,2,3,4, \\
  \frac{1}{2} Q_{A+6}, & A=5,6,7,8.
  \end{cases}
  \]  
  (3.1)

The coset representative can be parametrized by using the Euler angle parametrization given in [25]
\[
L = e^{a_1 Q_1} e^{a_2 Q_2} e^{a_3 Q_3} e^{a_4 Q_4} e^{a_5 Q_5} e^{a_6 Q_6} e^{b_1 Y_1} e^{b_2 Y_2} e^{b_3 Y_3}.
\]  
(3.2)
We find the embedding tensor to be

\[ \Theta = g(\Theta_{SU(2)^{(2)}} - 3\Theta_{SU(2)^{(1)}}). \] (3.3)

In terms of the \( SO(4) \) gauge generators \( Q_i, i = 1, 2, 3, 8, 9, 10 \), the generators for the gauge groups, \( SU(2)^{(1)} \) and \( SU(2)^{(2)} \), are given by

- \( SU(2)^{(1)} \): \( J_i^{(1)} = \frac{1}{2}Q_i, i = 1, 2, 3 \),
- \( SU(2)^{(2)} \): \( J_i^{(2)} = \frac{1}{2}Q_{i+7}, i = 1, 2, 3 \).

Using the formulae in the appendix, the scalar potential is found to be

\[ V = -\frac{27}{2}g^2 \left[ 21 + \cosh(4b_1) + 8(\cosh b_1 + \cosh(3b_1)) \cosh b_2 
\quad + 4 \cosh(2b_2) + 2 \cosh(2b_1)(10 + \cosh(2b_2)) \right] \] (3.4)

which does not depend on \( a_i \). There is only a trivial critical point at \( b_1 = b_2 = 0 \). The critical point preserves \( (4, 0) \) supersymmetry with the value of the potential at the critical point \( V_0 = -1296g^2 \). The full isometry of the corresponding \( AdS_3 \) background is \( SU(1,1|2) \times SU(1,1) \). The bosonic subgroup of \( SU(1,1|2) \) is given by \( SU(1,1) \times SU(1,1) \) \[27\]. This symmetry is the same as the \( N = 4 \) superconformal symmetry in the dual two dimensional CFT in the context of the AdS/CFT correspondence \[28\]. The \( SU(1,1) \) is a part of the \( AdS_3 \) isometry \( SO(2,2) \sim SO(2,1) \times SO(2,1) \sim SU(1,1) \times SU(1,1) \) while the \( SU(2) \) is the R-symmetry. The eight supercharges transform as \((2,2) + (2,2)\) under \( SU(1,1) \times SU(2) \).

### 3.2 \( SO(2,2) \) gauging

For the gauge group \( SO(2,2) \sim SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \), the generators are given by

- \( SL(2,\mathbb{R})^{(1)} \): \( j_1^{(1)} = \frac{1}{2}Q_1, j_2^{(1)} = \frac{1}{2}Q_5, j_3^{(1)} = \frac{1}{4}(Q_3 + \sqrt{3}Q_8) \),
- \( SL(2,\mathbb{R})^{(2)} \): \( j_1^{(2)} = \frac{1}{2}Q_{11}, j_2^{(2)} = \frac{1}{2}Q_{12}, j_3^{(2)} = \frac{1}{4}(Q_3 - \frac{1}{\sqrt{3}}Q_8) \).

Four of the eight scalars are parts of the gauge group while the remaining four correspond to non-compact generators of another \( SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \). The coset representative can be parametrized by

\[ L = e^{a_1 \frac{1}{2}(Q_3 - \sqrt{3}Q_8)} e^{b_1 Y_3} e^{a_2 \frac{1}{2}(\sqrt{3}Q_3 + Q_8)} e^{b_2 Y_7}. \] (3.5)

The embedding tensor is given by

\[ \Theta = g(\Theta_{SL(2)^{(2)}} - 3\Theta_{SL(2)^{(1)}}). \] (3.6)
Notice that the ratio of the coupling constants of the two factors is the same as in the $SO(4)$ case. This emphasizes the fact that the ratio of the two couplings does not depend on the different real forms of the gauge group $[3]$. The resulting scalar potential is given by

\begin{equation}
V = -\frac{3}{2}g^2 \left[ 21 + 4 \cosh(2b_1) - 40 \cosh b_1 \cosh \frac{b_2}{\sqrt{3}} 
+ 2(10 + \cosh(2b_1)) \cosh \frac{2b_2}{\sqrt{3}} + \cosh \frac{4b_2}{\sqrt{3}} - 8 \cosh b_1 \cosh(\sqrt{3}b_2) \right].
\end{equation}

(3.7)

The trivial critical point at $b_1 = b_2 = 0$ is a Minkowski vacuum with $V_0 = 0$ and preserves the full $N = 4$ supersymmetry. The gauge symmetry preserved by this critical point is the maximal compact subgroup of the gauge group $U(1) \times U(1)$. With the relation $\cosh b_1 = 2 \cosh \frac{b_2}{\sqrt{3}}$, there are dS vacua with $V_0 = 18g^2 \cosh^2 \frac{b_2}{\sqrt{3}} \cosh^2 \frac{b_2}{\sqrt{3}}$ depending on the value of $b_2$. For both $b_1$ and $b_2$ non-zero, the critical point breaks all the gauge symmetry. For $b_1 = 0$ or $b_2 = 0$, the $U(1)_{\text{diag}} \subset U(1) \times U(1)$ is preserved. But, $b_1$ cannot be zero since this will give imaginary $b_2$. So, the $U(1)_{\text{diag}}$ point is given by $b_1 = \cosh^{-1} 2, b_2 = 0$ and $V_0 = 18g^2$.

### 3.3 $SU(2,1)$ gauging

By the construction given in \cite{2}, the first eight matrices $c_i$, $i = 1, \ldots, 8$ generate the $SU(3) \subset G_2$. In the split form $G_{2(2)}$, this corresponds to the $SU(2,1)$ subgroup of $G_{2(2)}$. The gauge generators are then given by $Q_i$, $i = 1, \ldots, 8$ in which the maximal compact subgroup $SU(2) \times U(1)$ is generated by $\{Q_1, Q_2, Q_3\}$ and $Q_8$, respectively.

The embedding tensor is given by

\begin{equation}
\Theta = g\eta_{SU(2,1)}
\end{equation}

(3.8)

where $\eta_{SU(2,1)}$ is the Cartan-Killing form of $SU(2,1)$. It does not seem to be possible to find a simple parametrization of the four relevant scalars corresponding to $Y_i$, $i = 5, 6, 7, 8$. Therefore, we simply parametrize the coset representative by

\begin{equation}
L = e^{b_1 Y_5} e^{b_2 Y_6} e^{b_3 Y_7} e^{b_4 Y_8}.
\end{equation}

(3.9)

The potential turns out to be very long and complicated, so we will not attempt to give its explicit form, here. The trivial critical point at $b_1 = b_2 = b_3 = b_4 = 0$ has $(4,0)$ supersymmetry and $V_0 = -16g^2$. The residual gauge symmetry is given by $SU(2) \times U(1)$. Apart from this point, it is most likely that there are no other critical points. However, more detailed investigations are needed in order to have a definite conclusion.
3.4 $SL(3,\mathbb{R})$ gauging

The non-compact form $SL(3,\mathbb{R})$ has $SO(3)$ as its maximal compact subgroup. This subgroup is formed by the diagonal subgroup of the two $SU(2)$'s in $SO(4)$. Recall that the eight scalars transform as $(2, 4)$ under the $SO(4)$, we find that under the $SO(3)_{\text{diag}}$, the scalars transform as

$$2 \times 4 = 3 + 5.$$ 

The $5$ representation forms five non-compact generators and extends the $SO(3)_{\text{diag}}$ to the full $SL(3,\mathbb{R})$ gauge group. The $3$ representation gives the remaining three scalars to be parametrized in the coset representative $L$.

The $SL(3,\mathbb{R})$ generators are then given by

$$R_1 = \frac{1}{2}(Q_1 + \sqrt{3}Q_9), \quad R_2 = \frac{1}{2}(Q_2 + \sqrt{3}Q_{10}), \quad R_3 = \frac{1}{2}(Q_3 + \sqrt{3}Q_8),$$

$$R_4 = Q_4, \quad R_5 = Q_5, \quad R_6 = Q_{12}, \quad R_7 = \frac{1}{2}(Q_7 + \sqrt{3}Q_{14}),$$

$$R_8 = \frac{1}{2}(\sqrt{3}Q_{13} - Q_6).$$ \quad (3.10)

The first three generators are those of $SO(3)_{\text{diag}}$. The coset representative is given by

$$L = e^{a_1\sqrt{3}Y_5}e^{a_2\sqrt{3}(Y_7 + \sqrt{3}Y_3)}e^{a_3\sqrt{3}(Y_8 - \sqrt{3}Y_4)}.$$ \quad (3.11)

The embedding tensor is similar to that of the $SU(2,1)$, but the Catan-Killing is now

$$\eta_{SL(3)}$$

$$\Theta = g\eta_{SL(3)}.$$ \quad (3.12)
The resulting potential is given by

\[
V = \frac{1}{64} g^2 \left[ -6310 - 1848 \cosh(2a_1) - 66 \cosh(4a_1) - 6 \cosh(4a_1 - 6a_2) \\
- 24 \cosh[2(a_1 - 4a_2)] + 36 \cosh[2(a_1 - 2a_2)] - 6 \cosh[4(a_1 - 2a_2)] \\
+ 18 \cosh(4a_1 - 2a_2) + 912 \cosh[2(a_1 - a_2)] - 21 \cosh[4(a_1 - a_2)] \\
- 1860 \cosh(2a_2) - 30 \cosh(4a_2) + 12 \cosh(6a_2) - 36 \cosh(8a_2) \\
+ 912 \cosh[2(a_1 + a_2)] - 21 \cosh[4(a_1 + a_2)] + 18 \cosh[2(2a_1 + a_2)] \\
+ 36 \cosh[2(a_1 + 2a_2)] - 6 \cosh[4(a_1 + 2a_2)] - 24 \cosh[2(a_1 + 4a_2)] \\
- 6 \cosh(4a_1 + 6a_2) - 48 \cosh^2 a_2 (221 + 80 \cosh(2a_1) + 3 \cosh(4a_1) \\
- (137 + 156 \cosh(2a_1) + 3 \cosh(4a_1)) \cosh(2a_2) + 4 \cosh^2 a_1 \times \\
(-5 + \cosh(2a_1)) \cosh(4a_2) + 8 \cosh^4 a_1 \cosh(6a_2) \cosh(2a_3) \\
+ 96 \cosh^2 a_1 \left(3 + \cosh(2a_1) - 4 \cosh^2 a_1 \cosh(2a_2)\right) \times \\
(\cosh a_2 + \cosh(3a_2))^2 \cosh(6a_3) - 384 \cosh^4 a_1 \cosh^2(2a_2) \cosh(8a_3) \\
+ 6 \cosh(4a_3) (290 + 320 \cosh(2a_1) + 6 \cosh(4a_1) + (325 + 340 \cosh(2a_1) \\
+ 7 \cosh(4a_1)) \cosh(4a_2) - 8 \cosh^4 a_1 \cosh(8a_2) - 8(3 + 7 \cosh(2a_1)) \times \\
\cosh(2a_2) \sinh^2 a_1 - 12 \cosh(6a_2) \sinh^2(2a_1)\right]. 
\]

(3.13)

We find some critical points shown below.

| critical point | \((a_1, a_2, a_3)\) | residual supersymmetry | residual gauge symmetry | \(V_0\) |
|---------------|-----------------|-------------------|----------------------|--------|
| 1             | \((0, 0, 0)\)    | \((4, 0)\)        | \(SO(3)\)           | \(-16g^2\) |
| 2             | \((\cosh^{-1} \sqrt{3}, 0, 0)\) | - | \(SO(2)\) | \(176g^2\) |
| 3             | \((0, \cosh^{-1} \sqrt{1+\sqrt{3}}/2, 0)\) | - | \(SO(2)\) | \(176g^2\) |
|               | or \((0, 0, \cosh^{-1} \sqrt{1+\sqrt{3}}/2)\) | - | - | \(176g^2\) |
| 4             | \((a_0, a_0, 0)\) | - | - | \(176g^2\) |

The \(a_0\) is given by \(\cosh^{-1} \left( \frac{1}{2} \sqrt{\frac{3+3(171-2\sqrt{7053})+3(171+2\sqrt{7053})}{3}} \right)\).

4. Some gaugings in \(F_{4(4)}/U Sp(6) \times SU(2)\) coset manifold

In this symmetric space, we have \(F_{4(4)}\), the split form of \(F_4\), as the global symmetry while the representation \(R_0\) is \(1053\). The embedding tensor lives in the \(1 + 324\)
This case is similar to the $N = 9$ theory in which the coset space is given by $F_{4(-20)}/SO(9)$ studied in [23]. The following subgroups of $F_4$ can be gauged: $SO(9)$, $G_2 \times SU(2)$, $USp(6) \times SU(2)$ and $SU(3) \times SU(3)$. It has been shown in [3] that $SO(p) \times SO(9 - p) \subset SO(9)$, $p = 9, 8, 7, 6, 5$ can also be gauged. Therefore, all real forms of the above subgroups that can be embedded in $F_4(4)$ are admissible. These are given by:

- $SO(5, 4)$, $SO(5, 3)$, $SO(4, 4)$, $SO(5, 2) \times SO(2)$, $SO(4, 3) \times SO(2)$,
  - $SO(5, 1) \times SO(3)$, $SO(4, 2) \times SO(3)$, $SO(4, 1) \times SO(4)$,
  - and $SO(5) \times SO(4)$ or $SO(5, p) \times SO(4 - p)$, $p = 0, 1, 2, 3, 4$,
  - $SO(4, p) \times SO(5 - p)$, $p = 1, 2, 3, 4$ (4.1)

- $G_2(2) \times SL(2, \mathbb{R})$
- $USp(6) \times SU(2)$ and $Sp(6, \mathbb{R}) \times SL(2, \mathbb{R})$
- $SU(3) \times SU(2, 1)$

The maximal compact subgroup $SO(5) \times SO(4) \subset SO(5, 4)$ is embedded in the local symmetry $H$ as $USp(4) \times SU(2) \times SU(2) \sim SO(5) \times SO(4) \subset USp(6) \times SU(2)$. The other $SO$ type gauge groups can be embedded in $SO(5, 4)$. The gauge group $G_{2(2)} \times SL(2, \mathbb{R})$ can be considered as follow: $G_{2(2)}$ is embedded in $SO(4, 3) \subset SO(4, 3) \times SO(2)$ while the $SO(2)$ forms the compact subgroup of the $SL(2, \mathbb{R})$. For $SU(3) \times SU(2, 1)$, we first consider the maximal compact subgroup $SU(3) \times SU(2) \times U(1)$. The $U(3) \sim SU(3) \times U(1)$ is embedded in $U(3) \subset USp(6)$, and the $SU(2)$ is the one appearing in the group $H = USp(6) \times SU(2)$.

Since the computation of the resulting scalar potentials is more complicated in this case, we refrain from giving these potentials in this work. The interested reader can do this computation by using the information about the structure of $F_{4(4)}/USp(6) \times SU(2)$ coset space given in the next subsection and in the appendix.

It has been known that the ungauged $N = 4$ theory with scalar manifold $F_{4(4)}/USp(6) \times SU(2)$ can be obtained from the dimensional reduction on $T^3$ of $N = (1, 0)$ six dimensional supergravity coupled to two tensor and two vector multiplets, see [20] for details. So, we expect to find $SO(4)$ $N = 4$ Yang-Mills gauged supergravity in three dimensions from a dimensional reduction of this six dimensional theory on $S^3$ with $SO(4)$ being the isometry group of the $S^3$. As shown in [3], the resulting three dimensional theory is related to the Chern-Simons gauged supergravity considered in this work with a non-semisimple gauge group of the form $SO(4) \ltimes T^6$. The translational generators $T^6$ transform as an adjoint representation of the $SO(4)$. 

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4.1 $SO(4) \ltimes T^6$ gauging

We now construct $N = 4$ $SO(4) \ltimes T^6$ gauged supergravity with scalar manifold $F_{4(4)}/USp(6) \times SU(2)$. According to [24], the split form $F_{4(4)}$ has nine commuting generators, so it is possible to construct the gauge group $SO(4) \ltimes T^6 \subset F_{4(4)}$. There is a systematic procedure to find non-semisimple gaugings by boosting the admissible semisimple ones, see [4] for details. In this work, we will directly look for this gauging by solving the consistency constraints.

We begin with the identification of the gauge group $SO(4) \ltimes T^6$. An easy way to do this is to consider the embedding of this group inside the $SO(5, 4) \subset F_{4(4)}$. We recall the explicit matrix form of the 52 generators of $F_4$ from [30]. These generators are denoted by $c_i$’s in [30]. We choose the $SO(9)$ subgroup by taking the corresponding generators to be $c_i$ for $i = 1, \ldots, 21, 30, \ldots, 36, 45, \ldots, 52$. It is more convenient to relabel these generators in the form $X_{ij} = X_{[ij]}$, $i, j = 1, \ldots, 9$. The non-compact form $SO(5, 4)$ can be obtained by the Weyl unitarity trick namely multiplying $X_{ij}$ for $i = 1, \ldots, 5$ and $j = 6, \ldots, 9$ by a factor of $i$.

The relation between the $c_i$’s and the $X_{ij}$ has been given in the appendix of [29], and we refer the reader to this work for the explicit form of $X_{ij}$’s. We now give the embedding of $SO(4) \ltimes T^6$ in $SO(5, 4) \subset F_{4(4)}$. The semisimple part $SO(4)$ is given by the diagonal subgroup of the compact subgroup of $SO(4, 5)$, $SO(4) = (SO(4) \times SO(4))_{diag}$ with one of the $SO(4)$ being a subgroup of $SO(5)$ in $SO(5) \times SO(4) \subset SO(4, 5)$. The corresponding generators are given by

$$J^{ab} = X^{ab} + X^{\hat{a}\hat{b}}, \quad a, b = 1, 2, 3, 4 \quad \text{and} \quad \hat{a}, \hat{b} = 6, 7, 8, 9.$$  \hspace{0.5cm} (4.2)

The generators of the translational part $T^6$ are given by

$$t^{ab} = X^{ab} - X^{\hat{a}\hat{b}} + i(X^{\hat{a}b} + X^{\hat{a}\hat{b}}).$$  \hspace{0.5cm} (4.3)

Notice the factor of $i$ indicating the non-compact generators of $F_{4(4)}$. Using the $SO(9)$ algebra satisfied by $X_{ij}$, it can be easily verified that $J^{ab} = J^{[ab]}$ and $t^{ab} = t^{[ab]}$ satisfy the algebra

$$[J^{ab}, J^{cd}] = -4\delta^{[a[c} J^{d]b]}, \quad [J^{ab}, t^{cd}] = -4\delta^{[a[c} t^{d]b]}, \quad [t^{ab}, t^{cd}] = 0.$$  \hspace{0.5cm} (4.4)

This algebra shows that the $t^{ab}$ indeed transform as the adjoint representation of $SO(4)$ generated by $J^{ab}$. Therefore, $J^{ab}$ and $t^{ab}$ generate the non-semisimple group $SO(4) \ltimes T^6$.

Follow [10] in which $SO(4) \ltimes T^6$ $N = 8$ gauged supergravity described the $S^3$ reduction of $N = (2, 0)$ supergravity in six dimensions has been constructed, we now
decompose the generators $J^{ab}$ and $t^{ab}$ in terms of the self-dual and anti-self-dual parts

\[
J^\pm_{ab} = J^{ab} \pm \frac{1}{2} \epsilon_{abcd} J^{cd},
\]
\[
t^\pm_{ab} = t^{ab} \pm \frac{1}{2} \epsilon_{abcd} t^{cd}.
\]

These generate $(SO(3)^+ \times T^3) \times (SO(3)^- \times T^3) \sim SO(4) \times T^6$. As a general result of [5], the corresponding embedding tensor is of the form

\[
\Theta = g_1 \Theta_{AB} + g_2 \Theta_{BB}
\]

where $A$ and $B$ describe the semisimple and translational parts, respectively. In the self-dual and anti-self-dual basis, the ansatz for the embedding is given by

\[
\Theta = g_1^+(\Theta_{A^+B^+} + \Theta_{B^+A^+}) + g_1^- (\Theta_{A^-B^-} + \Theta_{B^-A^-}) \\
+ g_2^+ \Theta_{B^+B^+} + g_2^- \Theta_{B^-B^-}.
\]

The quadratic and linear constraints impose the conditions

\[
g_2^- = -g_2^+ = g_2, \quad \text{and} \quad g_1^- = -g_1^+ = g_1.
\]

This is similar to the $N = 8$ theory considered in [11] in which the consistency conditions also require the relative minus sign between the self-dual and anti-self-dual parts.

We then end up with the embedding tensor of the form

\[
\Theta = g_1 (\Theta_{A^+B^+} - \Theta_{A^-B^-} + \Theta_{B^+A^+} - \Theta_{B^-A^-}) \\
+ g_2 (\Theta_{B^+B^+} - \Theta_{B^-B^-}).
\]

Furthermore, as we will see, it turns out that the existence of the maximal supersymmetric $AdS_3$ vacua at $L = I$ requires the relation $g_2 = g_1$. This is again much similar to the $N = 8$ theory studied in [11].

Since this theory describes an $S^3$ reduction of the $(1, 0)$ six dimensional supergravity coupled to two vector and two tensor multiplets, it is interesting to further study the associated scalar potential. This is certainly useful in the AdS/CFT correspondence. We again refer the reader to the needed formulae in the appendix. There are 28 scalars in the coset manifold $F_4(4)/USp(6) \times SU(2)$. With the 26 dimensional fundamental representation of $F_4$, it is extremely difficult to study the full scalar potential on the 28 dimensional scalar manifold. We then apply the group theory argument of [31] to compute the scalar potential on a submanifold of the full 28-dimensional scalar manifold which is invariant under a certain subgroup of the gauge group.
There are two scalars invariant under the $SO(4)$ part of the gauge group. The coset representative parametrized by these two singlets is given by

$$L = e^{b_1 Y_1} e^{b_2 Y_2}$$  \quad (4.10)

where

$$\bar{Y}_1 = Y_3 - Y_4 - Y_5 - Y_8, \quad \bar{Y}_2 = Y_9 + Y_{11} - Y_{15} - Y_{19} - Y_{21} - Y_{25} + Y_{27}. \quad (4.11)$$

We finally obtain the scalar potential

$$V = 256 \left[ 2 \cosh(\sqrt{2} b_1) \cosh(2 b_2) - 2 \sinh(2 b_2) \right]^2 \left[ 5 g_1^2 - 2 g_2^2 
+ 2 g_2 \left( 2 g_2 \cosh^2(2 \sqrt{2} b_2) + 3 g_2 \cosh(4 b_2) + 7 g_1 \sinh(2 b_2) 
- \cosh(\sqrt{2} b_1) \cosh(2 b_2) (7 g_1 + 8 g_2 \sinh(2 b_2)) \right) \right]. \quad (4.12)$$

From this potential, we find that the potential admits a critical point at $b_1 = b_2 = 0$ or $L = I$ only for

$$g_2 = g_1, \quad g_2 = \frac{5}{16} g_1. \quad (4.13)$$

The first one corresponds to the maximal supersymmetric $AdS_3$ point with $V_0 = -1024 g_1^2$. This can be checked by the condition given in [8]

$$A_1^{IK} A_1^{KJ} e^I = -\frac{V_0}{4} e^I \quad (4.14)$$

which states that the unbroken supersymmetries are given by the eigenvalues of the $A_1$ tensor that satisfy the above condition at the critical point. It can be verified that only $g_2 = g_1$ satisfies this condition.

The second possibility gives $V_0 = 1440 g_1^2$ which is of a dS type. Therefore, this supergravity theory admits both dS and AdS vacua at $L = I$ depending on the value of $g_2$. Our main interest here is for the $g_2 = g_1$ case, so we will further explore this case.

Setting $g_2 = g_1 = g$, we find the potential

$$V = 1024 g^2 \left[ \sinh(2 b_2) - \cosh(\sqrt{2} b_1) \cosh(2 b_2) \right]^2 \left[ 3 + 4 \cosh(2 \sqrt{2} b_1) \cosh^2(2 b_2) 
+ 6 \cosh(4 b_2) - 2 \cosh(\sqrt{2} b_1) (7 \cosh(2 b_2) + 4 \sinh(4 b_2)) \right]. \quad (4.15)$$

Analyzing this potential gives the following non-trivial critical points.

- At $b_1 = 0$ and $b_2 = \frac{1}{2} \ln \frac{16}{5}$, there is a dS critical point with $V_0 = \frac{1125}{8} g^2$. 

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There is a class of Minkowski vacua for \( \cosh(\sqrt{2} b_1) = \tanh(2b_2) \) with \( V_0 = 0 \) and \( N = 4 \) supersymmetry.

We can consider a smaller residual symmetry namely \( SO(3) \) subgroup of \( SO(4) \) generated by \( J^{12}, J^{13} \) and \( J^{23} \). There are five singlets given by

\[
\begin{align*}
\tilde{Y}_1 &= Y_3 - Y_5, \\
\tilde{Y}_2 &= Y_{17} - Y_{27}, \\
\tilde{Y}_3 &= Y_{18} - Y_{28}, \\
\tilde{Y}_4 &= Y_4 + Y_8, \\
\tilde{Y}_5 &= Y_9 + Y_{11} - Y_{15} + Y_{19} - Y_{21} - Y_{25}.
\end{align*}
\]

Unfortunately, the computation of the potential turns out to be extremely difficult. Therefore, we will leave this for future works.

There is another subgroup of \( SO(5, 4) \) which is of the form \( SO(4) \ltimes (T_6, \hat{T}_4) \). The \( \hat{T}_4 \) transform as a vector \( (4) \) of \( SO(4) \) and close onto the translational symmetry \( T_6 \).

According to [5], the theory with this gauge group is on-shell equivalent to the Yang-Mills gauged supergravity with the gauge group \( SO(4) \) coupled to four massive vector fields corresponding to the nilpotent generators of \( \hat{T}_4 \). The generators of this group are those of the \( SO(4) \ltimes T_6 \) together with the four generators of \( \hat{T}_4 \) given by

\[
\hat{T}_a^u = X^{a,5} - iX^{5,a+5}, \quad a = 1, 2, 3, 4.
\]

All of these generator satisfy the above mentioned algebra as can be readily verified.

The embedding tensor for this gauge group is that of the \( SO(4) \ltimes T_6 \) with an additional component of the form \( g_3 \Theta_{\hat{T}_4} \). It turns out that consistency conditions require \( g_3 = 0 \). So, the whole \( SO(4) \ltimes (T_6, \hat{T}_4) \) cannot be gauged.

5. Some gaugings in \( E_6(2)/SU(6) \times SU(2), E_7(-5)/SO(12) \times SU(2) \) and \( E_8(-24)/E_7 \times SU(2) \) coset manifolds

In this section, we consider some admissible gauge groups obtained by applying the general group theory argument presented in section 2. The global symmetry \( G \) in these cases is large, and it is even more difficult than the \( F_4(4)/USp(6) \times SU(2) \) case to give all admissible gauge groups. Therefore, we will not attempt to give an exhaustive list for these target manifolds in this work but simply provide some examples.

5.1 Examples of gaugings in \( E_6(2)/SU(6) \times SU(2) \) coset manifold

In this case, the group \( G \) and representation \( R_0 \) are \( E_6(2) \) and \( 2430 \). The corresponding embedding tensor lives in the representation \( 1 + 650 \). The group theory structure is similar to the \( N = 10 \) theory whose scalar potentials have been studied in [32], but the scalar manifold is described by \( E_6(-14)/SO(10) \times U(1) \) coset space. The suitable real
forms of the $D_5 \times U(1)$, $A_5 \times A_1$, $F_4$ and $G_2 \times A_2$ subgroups can be gauged.

Some admissible gauge groups are given by:

- $SO(6, 4) \times U(1)$, $SO(6, 3) \times U(1)$, $SO(6, 2) \times SO(2) \times U(1)$, $SO(4, 4) \times SO(2) \times U(1)$, $SO(6, 1) \times SO(3) \times U(1)$, $SO(4, 3) \times SO(3) \times U(1)$, $SO(6) \times SO(4) \times U(1)$, $SO(4, 2) \times SO(4) \times U(1)$, and $SO(4, 1) \times SO(5)$ or

$$SO(6, p) \times SO(4 - p) \times U(1), \quad p = 0, 1, 2, 3, 4,$$
$$SO(4, p) \times SO(6 - p) \times U(1)^{1 - \delta_{1p}}, \quad p = 1, 2, 3, 4 \quad (5.1)$$

- $SU(6) \times SU(2)$ and $SU(3, 3) \times SL(2, \mathbb{R})$

- $F_4(4)$

- $G_{2(2)} \times SU(2, 1)$.

The embedding of the $SO(6, 4) \times U(1)$ can be given as follow. We first consider the decomposition of $SU(6)$ to $SU(4) \times SU(2) \times U(1)$. Together with the additional $SU(2)$ factor from the group $H$ and using $SU(4) \sim SO(6)$ and $SU(2) \times SU(2) \sim SO(4)$, we find the maximal compact subgroup $SO(6) \times SO(4) \times U(1) \subset SO(6, 4) \times U(1)$. The other real forms can be embedded in $SO(6, 4) \times U(1)$. The $USp(6) \subset SU(6)$ together with the $SU(2)$ form the maximal compact subgroup of $F_4(4)$. Finally, $G_{2(2)} \times SU(2, 1)$ can be embedded in $SO(4, 3) \times SO(3) \times U(1)$ with $G_{2(2)} \subset SO(4, 3)$ and the $SO(3) \times U(1)$ being the maximal compact subgroup of $SU(2, 1)$.

### 5.2 Examples of gaugings in $E_7(-5)/SO(12) \times SU(2)$ coset manifold

With this target manifold, the group $G$ and representation $R_0$ are given by $E_{7(-5)}$ and $7371$. The corresponding embedding tensor lives in the representation $1 + 1539$. This case is the same as $N = 12$ theory in which the scalar manifold is uniquely determined to be $E_{7(-5)}/SO(12) \times SU(2)$ [5]. Some admissible gauge groups have already been given in [5]. We simply repeat them here for completeness.

They are given by:

- $SO(p) \times SO(12 - p) \times U(1)$ for $p = 0, \ldots, 5$, $SO(6) \times SO(6)$, and $SO^*(12) \times SL(2, \mathbb{R})$

- $E_{6(2)} \times U(1)$

- $F_{4(-20)} \times SU(2)$

- $G_{2(2)} \times USp(6)$.
5.3 Examples of gaugings in $E_{8(-24)}/E_7 \times SU(2)$ coset manifold

In this case, the group $G$ and representation $R_0$ are given by $E_{8(-24)}$ and $27000$. The corresponding embedding tensor lives in the representation $1 + 3875$. This case is similar to the maximal $N = 16$ theory but with different real form of $E_8$ namely the group $G$ is $E_{8(8)}$ for the maximal case. The study of scalar potentials for some semisimple gauge groups has been given in [33]. The suitable real forms of the following subgroups can be gauged: $D_4 \times D_4, G_2 \times F_4, E_6 \times A_2$ and $E_7 \times A_1$.

Some admissible gauge groups are given by:

- $SO(4, 4) \times SO(8), SO(7, 1) \times SO(5, 3)$ and $SO(6, 2) \times SO(6, 2)$ or
  
  $SO(8 - p, p) \times SO(4 + p, 4 - p), \quad p = 0, 1, 2$

- $F_4 \times G_{2(2)}, F_{4(4)} \times G_2$ and $F_{4(-20)} \times G_{2(2)}$

- $E_6 \times SU(2, 1), E_{6(2)} \times SU(2, 1)$ and $E_{6(-26)} \times SL(3, \mathbb{R})$

- $E_7 \times SU(2)$ and $E_{7(-25)} \times SL(2, \mathbb{R})$.

The $SO(4, 4) \times SO(8)$ and other real forms can be embedded in the maximal subgroup $SO(12, 4) \subset E_{8(-24)}$. Using the decompositions $E_7 \times SU(2) \rightarrow F_4 \times SU(2) \times SU(2)$ and $E_7 \times SU(2) \rightarrow G_2 \times USp(6) \times SU(2)$, we immediately see the embedding of $F_4 \times G_{2(2)}$ and $F_{4(4)} \times G_2$, respectively. The embedding of the real form $F_{4(-20)} \times G_{2(2)}$ can be seen as follow. We first decompose $SO(12, 4) \rightarrow SO(9) \times SO(3, 4)$. The $SO(9)$ becomes the maximal compact subgroup of $F_{4(-20)}$, and $G_{2(2)}$ is embedded in $SO(4, 3)$. Using the embedding of $E_6 \times U(1) \subset E_7$, we can see the embedding of $E_6 \times SU(2, 1)$ while the decomposition $E_7 \rightarrow SO(12) \times SU(2) \rightarrow U(6) \times SU(2)$ gives the embedding of $E_{6(2)} \times SU(2, 1)$. Finally, $E_{6(-26)} \times SL(3, \mathbb{R})$ is embedded in $E_{8(-24)}$ via the decomposition $E_7 \times SU(2) \rightarrow F_4 \times SU(2) \times SU(2)$ with the maximal compact subgroup $SO(3)$ of $SL(3, \mathbb{R})$ being $SU(2)_{\text{diag}}$. The 112 non-compact generators of $E_{8(-24)}$ transform as $(56, 2)$ under $E_7 \times SU(2)$ which is further decomposed into $(26, 2, 2) + (1, 4, 2)$ under $F_4 \times SU(2) \times SU(2)$. Under $F_4 \times SU(2)_{\text{diag}}$, they transform as

$$(56, 2) \rightarrow (26, 1) + (26, 3) + (1, 3) + (1, 5).$$

The $(26, 1)$ enlarges the $F_4$ to $E_{6(-26)}$ while the $(1, 5)$ becomes five non-compact generators of $SL(3, \mathbb{R})$. 

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In this paper, we have studied gaugings of $N = 4$ gauged supergravity in three dimensions. The scalar target spaces considered here are in the form of the exceptional coset spaces. In the $G_{2(2)}/SO(4)$ case, we have listed all admissible gauge groups as well as study their scalar potentials and some of the corresponding critical points. We have pointed out that the $SO(3) \ltimes \mathbb{T}^3$ cannot be gauged since this gauge group cannot be embedded in the $G_{2(2)}$. This immediately leads to a puzzle. The ungauged version of this theory can be obtained from $T^2$ reduction of the minimal ($N = 2$) supergravity in five dimensions. It has been proposed in [5] that the three dimensional gauged theory with scalar manifold $G_{2(2)}/SO(4)$ and gauge group $SO(3) \ltimes \mathbb{T}^3$ would describe the $N = 2$ five dimensional supergravity reduced on an $S^2$. The spectrum of the Kaluza-Klein reduction has been studied in [35] and [36] together with its dual SCFT. On the other hand, it has been pointed out in [34] that although the symmetry of the $T^2$ reduction of the minimal supergravity in five dimensions get enhanced to $G_{2(2)}$, there does not seem to be a possibility of a consistent $S^2$ reduction. This is precisely in agreement with what has been found here. It could be that the consistent $S^2$ reduction at the full non-linear level may not exist. It is interesting to find the three dimensional description of this reduced theory if the reduction can be achieved.

For the theory with $F_{4(4)}/USp(6) \times SU(2)$, we have identified some gauge groups which are maximal subgroups of $F_{4(4)}$. As one of the main results of this paper, we have constructed an $N = 4$ theory with gauge group $SO(4) \ltimes \mathbb{T}^6$. The resulting theory is on-shell equivalent to Yang-Mills gauged supergravity with $SO(4)$ gauge group according to the general result of [4]. So, we expect that the $SO(4) \ltimes \mathbb{T}^6$ gauged supergravity with $F_{4(4)}/USp(6) \times SU(2)$ scalar manifold can be obtained from the $S^3$ reduction of the $(1,0)$ six dimensional supergravity coupled to two vector and two tensor multiplets. In this case, it is also interesting to study its explicit reduction in the same way as the reduction on the $SU(2)$ group manifold studied in [14], [18] and [19]. From the general results of [34], there do not seem to be any obstacles in this case. Furthermore, we have shown that the theory constructed here admits both AdS and dS vacua at $L = 1$ depending on the value of the coupling constants. The situation is very similar to $N = 4$ gauged supergravity in four dimensions with $SU(2) \times SU(2)$ gauge group in which the relative values of the two couplings determine whether the vacuum is a supersymmetric $AdS_4$ or a non-supersymmetric $dS_4$ solution, see for example the discussion in [37].

We end the paper by considering some examples of admissible gauge groups in the case of $E_{6(2)}/SU(6) \times SU(2)$, $E_{7(-5)}/SO(12) \times SU(2)$ and $E_{8(-24)}/E_7 \times SU(2)$ scalar manifolds. All of the gauge groups presented here are maximal subgroups of the corresponding global symmetry $G$. The detailed study is needed in order to find a larger
class of admissible gauge groups for these theories. It is of interest to study their non- 
semisimple gaugings which might give some insights to the higher dimensional origin 
of these theories.

It could also be interesting to study the scalar potentials as well as the corresponding 
critical points for the gauge groups identified here. This is useful in the study of 
holographic RG flows describing the deformations of the dual SCFT’s. We hope to 
come back to these issues in the future works.

Acknowledgments

The author would like to thank the Abdus Salam International Centre for Theoretical 
Physics (ICTP) for hospitality and computing facilities as well as International School 
for Advanced Studies (SISSA) for the support when the early stage of this work was initiated. He also gratefully thanks Henning Samtleben for invaluable cor- 
respondences. This work is partially supported by Thailand Center of Excellence in Physics through the ThEP/CU/2-RE3/11 project and Chulalongkorn University through Ratchadapisek Sompote Endowment Fund.

A. Essential formulae

In this appendix, we collect some useful formulae for three dimensional gauged super-
gravity with symmetric scalar target manifolds. We also give some details about the 
explicit construction of the coset space \( G_2(2)/SO(4) \) and \( F_4(4)/USp(6) \times SU(2) \) which 
are used in the main text and might be useful for further investigations.

We begin with the formulae for a symmetric space of the form \( G/H \) in which \( G \) 
and \( H \) are the global and local symmetry groups, respectively. The \( G \) algebra is given by

\[
[T^{IJ}, T^{KL}] = -4\delta^{I[K}T^{L]J}, \quad [T^{IJ}, Y^A] = -\frac{1}{2}f^{IJ,AB}Y_B, \\
[X^\alpha, X^\beta] = f^{\alpha\beta\gamma}X^\gamma, \quad [X^\alpha, Y^A] = h^\alpha_B Y^B, \\
[Y^A, Y^B] = \frac{1}{4}f^{ABT^{IJ}} + \frac{1}{8}C_{\alpha\beta}h^{\beta AB}X^\alpha
\]  
\text{(A.1)}

\( T^{IJ} \)'s and \( X^\alpha \)'s generate \( SO(N) \times H' \), and \( Y^A \)'s are non-compact generators trans- 
forming in a spinor of \( SO(N) \). We refer the reader to [8] for other notations. The coset 
representative \( L \) transforming under \( G \) and \( H \) by left and right multiplications can be 
used to define the map \( V \) by the relation

\[
L^{-1} t^M L = \frac{1}{2} \gamma^M_{IJ} T^{IJ} + \gamma^M_\alpha X^\alpha + \gamma^M_A Y^A.
\]  
\text{(A.2)}
The metric on the target space \( g_{ij} \) can be computed from the vielbein \( e_i^A \) which is in turn encoded in

\[
L^{-1} \partial_i L = \frac{1}{2} Q_i^{IJ} T^{IJ} + Q_i^a X^a + e_i^A Y^A,
\]

where \( Q_i^{IJ} \) and \( Q_i^a \) are the composite connections for the \( SO(N) \) and \( H' \), respectively.

Given the map \( \mathcal{V} \) from (A.2), the T-tensor can be straightforwardly computed from the embedding tensor by using (2.6). In order to compute the scalar potential, we need to construct the \( A_1 \) and \( A_2 \) tensors. They are given in terms of the T-tensor components by

\[
A_1^{IJ} = -\frac{4}{N-2} T^{IJ,IM} + \frac{2}{(N-1)(N-2)} \delta IJ T^{MN, MN},
\]

\[
A_2^{Ij} = \frac{2}{N} T^{Ij} + \frac{4}{N(N-2)} f^{M(Im, Tj)^M} + \frac{2}{N(N-1)(N-2)} \delta IJ f^{KL m} T^{KL, m}.
\]

These two tensors together with the third one, \( A_3 \), appear in the gauged Lagrangian as fermion mass-like terms [8]. Finally, the scalar potential can be computed by using

\[
V = -\frac{4}{N} g^2 \left( A_1^{Ij} A_1^{Ij} - \frac{1}{2} N g^{ij} A_2^{Ij} A_2^{Ij} \right).
\]

**A.1 Useful formulae for \( G_2(2)/SO(4) \) coset**

We give the explicit form of the various \( \mathcal{V} \) maps and T-tensors for all the gauge groups studied in the case of \( G_2(2)/SO(4) \) coset manifold. These are relevant for computing the corresponding scalar potentials. The repeated indices are summed over the given values.

- **SO(4) gauging:**

  \[
  \mathcal{V}_{(1),(2)}^{i, \dot{J}J} = -\frac{1}{3} \text{Tr}(L^{-1} j_i^{(1),(2)} LT^{\dot{J}J}),
  \]

  \[
  \mathcal{V}_{(1),(2)}^{i, \dot{A}} = \frac{1}{3} \text{Tr}(L^{-1} j_i^{(1),(2)} LY^A), \quad i = 1, 2, 3,
  \]

  \[
  T^{I, \dot{J}J, KL} = g \left( \mathcal{V}_{(2)}^{i, \dot{J}J} \mathcal{V}_{(2)}^{i, \dot{K}L} - 3 \mathcal{V}_{(1)}^{i, \dot{J}J} \mathcal{V}_{(1)}^{i, \dot{K}L} \right),
  \]

  \[
  T^{I, \dot{J}J, A} = g \left( \mathcal{V}_{(2)}^{i, \dot{J}J} \mathcal{V}_{(2)}^{i, \dot{A}} - 3 \mathcal{V}_{(1)}^{i, \dot{J}J} \mathcal{V}_{(1)}^{i, \dot{A}} \right).
  \]

- **\( SL(2, \mathbb{R}) \times SL(2, \mathbb{R}) \) gauging:**

  \[
  \mathcal{V}_{(1),(2)}^{i, \dot{J}J} = -\frac{1}{3} \text{Tr}(L^{-1} j_i^{(1),(2)} LT^{\dot{J}J}),
  \]

  \[
  \mathcal{V}_{(1),(2)}^{i, \dot{A}} = \frac{1}{3} \text{Tr}(L^{-1} j_i^{(1),(2)} LY^A), \quad i = 1, 2, 3,
  \]

  \[
  T^{I, \dot{J}J, KL} = g \left( \mathcal{V}_{(2)}^{i, \dot{J}J} \mathcal{V}_{(2)}^{j, \dot{K}L} - 3 \mathcal{V}_{(1)}^{i, \dot{J}J} \mathcal{V}_{(1)}^{j, \dot{K}L} \right) \eta^{SL(2)}_{ij},
  \]

  \[
  T^{I, \dot{J}J, A} = g \left( \mathcal{V}_{(2)}^{i, \dot{J}J} \mathcal{V}_{(2)}^{j, \dot{A}} - 3 \mathcal{V}_{(1)}^{i, \dot{J}J} \mathcal{V}_{(1)}^{j, \dot{A}} \right) \eta^{SL(2)}_{ij}.
  \]
• $SU(2, 1)$ gauging:

\[ V^{a, IJ} = -\frac{1}{3} \text{Tr}(L^{-1} Q_a L^{IJ}), \quad V^{a, A} = \frac{1}{3} \text{Tr}(L^{-1} Q_a L^A), \quad a = 1, \ldots, 8, \]

\[ T^{IJ, KL} = g V^{a, IJ} V^{b, KL} \eta_{ab}^{SU(2, 1)}, \quad T^{IJ, A} = g V^{a, IJ} V^{b, A} \eta_{ab}^{SU(2, 1)} \]  

(A.8)

• $SL(3, \mathbb{R})$ gauging:

\[ V^{a, IJ} = -\frac{1}{3} \text{Tr}(L^{-1} R_a L^{IJ}), \quad V^{a, A} = \frac{1}{3} \text{Tr}(L^{-1} R_a L^A), \quad a = 1, \ldots, 8, \]

\[ T^{IJ, KL} = g V^{a, IJ} V^{b, KL} \eta_{ab}^{SL(3)}, \quad T^{IJ, A} = g V^{a, IJ} V^{b, A} \eta_{ab}^{SL(3)} \]  

(A.9)

The three almost complex structures are given by

\[ f_2 = -\left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \]

\[ f_3 = -\left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \otimes \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right), \]

\[ f_4 = -\left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right). \]  

(A.10)

The full tensor $f^{IJ}$ can be straightforwardly obtained by the relation (2.1).

\[ f^{IJ} = -\left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \]

\[ f^{IJ} = -\left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \otimes \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right), \]

\[ f^{IJ} = -\left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right). \]  

(A.11)

The full tensor $f^{IJ}$ can be straightforwardly obtained by the relation (2.1).

\[ f^{IJ} = -\left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \]

\[ f^{IJ} = -\left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \otimes \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right), \]

\[ f^{IJ} = -\left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right). \]  

(A.12)
Finally, the 28 non-compact generators are

\[
Y_1 = \frac{i}{2}(c_{22} - c_{38}), \quad Y_2 = \frac{i}{2}(c_{23} + c_{37}), \quad Y_3 = \frac{i}{2}(c_{24} - c_{41}), \\
Y_4 = \frac{i}{2}(c_{25} - c_{44}), \quad Y_5 = \frac{i}{2}(c_{26} + c_{39}), \quad Y_6 = \frac{i}{2}(c_{27} + c_{43}), \\
Y_7 = \frac{i}{2}(c_{28} - c_{42}), \quad Y_8 = \frac{i}{2}(c_{29} + c_{40}), \quad Y_9 = \frac{i}{2}(c_{11} + c_{30}), \\
Y_{10} = \frac{i}{2}(c_{12} + c_{31}), \quad Y_{11} = \frac{i}{2}(c_{13} + c_{32}), \quad Y_{12} = \frac{i}{2}(c_{14} + c_{33}), \\
Y_{13} = \frac{i}{2}(c_{15} + c_{34}), \quad Y_{14} = \frac{i}{2}(c_{16} + c_{45}), \quad Y_{15} = \frac{i}{2}(c_{17} + c_{46}), \\
Y_{16} = \frac{i}{2}(c_{18} + c_{47}), \quad Y_{17} = \frac{i}{2}(c_{19} + c_{48}), \quad Y_{18} = \frac{i}{2}(c_{20} + c_{49}), \\
Y_{19} = \frac{i}{2}(c_{11} - c_{30}), \quad Y_{20} = \frac{i}{2}(c_{12} - c_{31}), \quad Y_{21} = \frac{i}{2}(c_{13} - c_{32}), \\
Y_{22} = \frac{i}{2}(c_{14} - c_{33}), \quad Y_{23} = \frac{i}{2}(c_{15} - c_{34}), \quad Y_{24} = \frac{i}{2}(c_{16} - c_{45}), \\
Y_{25} = \frac{i}{2}(c_{17} - c_{46}), \quad Y_{26} = \frac{i}{2}(c_{18} - c_{47}), \quad Y_{27} = \frac{i}{2}(c_{19} - c_{48}), \\
Y_{28} = \frac{i}{2}(c_{20} - c_{49}). \tag{A.13}
\]

The $f^{IJ}$ tensors can be computed by using $[T^{IJ}, Y^A]$ in the $G$ algebra. With a suitable normalization, they are given by

\[
f_{IJ}^{AB} = -2\text{Tr}(Y^B [T^{IJ}, Y^A]). \tag{A.14}
\]

We now give various components of the $\mathcal{V}$ map. They are computed by

\[
\begin{align*}
\mathcal{V}_A^{ab,IJ} &= -\frac{1}{3} \text{Tr}(L^{-1} J^{ab} L T^{IJ}), \\
\mathcal{V}_B^{ab,IJ} &= -\frac{1}{3} \text{Tr}(L^{-1} t^{ab} L T^{IJ}), \\
\mathcal{V}_A^{ab,A} &= \frac{1}{3} \text{Tr}(L^{-1} J^{ab} L Y^A), \\
\mathcal{V}_B^{ab,A} &= \frac{1}{3} \text{Tr}(L^{-1} t^{ab} L Y^A). \tag{A.15}
\end{align*}
\]

The $T$-tensors are given by

\[
\begin{align*}
T^{IJ,KL} &= g_1 (\mathcal{V}_A^{ab,IJ} \mathcal{V}_B^{cd,KL} + \mathcal{V}_B^{ab,IJ} \mathcal{V}_A^{cd,KL}) \epsilon_{abcd} \\
&\quad + g_2 \mathcal{V}_A^{ab,IJ} \mathcal{V}_B^{cd,KL} \epsilon_{abcd}, \\
T^{IJ,A} &= g_1 (\mathcal{V}_A^{ab,IJ} \mathcal{V}_B^{cd,A} + \mathcal{V}_B^{ab,IJ} \mathcal{V}_A^{cd,A}) \epsilon_{abcd} \\
&\quad + g_2 \mathcal{V}_A^{ab,IJ} \mathcal{V}_B^{cd,A} \epsilon_{abcd}. \tag{A.16}
\end{align*}
\]

In the above equation, rather than using the self-dual and anti-self-dual $SU(2)_{\pm}$ basis, we have used the $SO(4)$ basis, and the appearance of $\epsilon_{abcd}$, instead of the $\delta_{ac} \delta_{bd}$, takes care of the relative minus sign between the $SU(2)_+$ and $SU(2)_-$. With the above given formulae, the scalar potential can be directly obtained.
References

[1] H. Nicolai and H. Samtleben, “Maximal gauged supergravity in three dimensions”, Phys. Rev. Lett. 86 (2001) 1686-1689, arXiv: hep-th/0010076.

[2] H. Nicolai and H. Samtleben, “N = 8 matter coupled AdS3 supergravities”, Phys. Lett. B514 (2001) 165-172, arXiv: hep-th/0106153.

[3] H. Nicolai and H. Samtleben, “Compact and noncompact gauged maximal supergravities in three dimensions”, JHEP 04 (2001) 022, arXiv: hep-th/0103032.

[4] T. Fischbacher, H. Nicolai and H. Samtleben, “Non-semisimple and Complex Gaugings of N = 16 Supergravity”, Commun.Math.Phys. 249 (2004) 475-496, arXiv: hep-th/0306276.

[5] H. Nicolai and H. Samtleben, “Chern-Simons vs Yang-Mills gaugings in three dimensions”, Nucl. Phys. B 638 (2002) 207-219, arXiv: hep-th/0303213.

[6] Bernard de Wit, A. K. Tollsten and H. Nicolai, “Locally supersymmetric D = 3 nonlinear sigma models”, Nucl. Phys. B392 (1993) 3-38, arXiv: hep-th/9208074.

[7] S. Deser and J. H. Kay, “Topologically massive supergravity”, Phys. Lett. 120B (1983) 97-100.

[8] Bernard de Wit, Ivan Herger and Henning Samtleben, “Gauged Locally Supersymmetric D = 3 Nonlinear Sigma Models”, Nucl. Phys. B671 (2003) 175-216, arXiv: hep-th/0307006.

[9] A. Chatrabhuti, P. Karndumri and B. Ngamwatthanakul, “3D N=6 Gauged Supergravity: Admissible Gauge Groups, Vacua and RG Flows”, arXiv: 1202.1043.

[10] H. Nicolai and H. Samtleben, “Kaluza-Klein supergravity on AdS3 x S3”, JHEP 09 (2003) 036, arXiv: hep-th/0306202.

[11] O. Hohm and H. Samtleben, “Effective Actions for Massive Kaluza-Klein States on AdS3 x S3 x S3”, JHEP 05 (2005) 027, arXiv: hep-th/0503088.

[12] M. Berg and H. Samtleben, “An exact holographic RG Flow between 2d Conformal Field Theories”, JHEP 05 (2002) 006, arXiv: hep-th/0112154.

[13] Edi Gava, Parinya Karndumri and K. S. Narain, “AdS3 Vacua and RG Flows in Three Dimensional Gauged Supergravities”, JHEP 04 (2010) 117, arXiv: 1002.3760.

[14] Edi Gava, Parinya Karndumri and K. S. Narain, “3D gauged supergravity from SU(2) reduction of N = 1 6D supergravity”, JHEP 09 (2010) 028, arXiv: 1006.4997.
[15] Edi Gava, Parinya Karndumri and K. S. Narain, “Two dimensional RG flows and Yang-Mills instantons”, JHEP 03 (2011) 106, arXiv: 1012.4953.

[16] M. Possel and S. Silva, “Hidden symmetries in minimal five-dimensional supergravity”, Phys. Lett. B580 (2004) 273-279.

[17] S. Mizoguchi and N. Ohta, “More on the Similarity between $D = 5$ Simple Supergravity and M Theory”, Phys. Lett. B441 (1998) 123-132, arXiv: hep-th/9807111.

[18] H. Lü, C. N. Pope and E. Sezgin, “$SU(2)$ reduction of six-dimensional (1,0) supergravity”, Nucl. Phys. B668 (2003) 237-257, arXiv: hep-th/0212323.

[19] H. Lü, C. N. Pope and E. Sezgin, “Yang-Mills-Chern-Simons Supergravity”, Class. Quant. Grav. 21 (2004) 2733-2748, arXiv: hep-th/0305242.

[20] E. Cremmer, B. Julia, H. Lu and C. N. Pope, “Higher-dimensional Origin of $D = 3$ Coset Symmetries”, arXiv: hep-th/9909099.

[21] M. Gunaydin, H. Samtleben and E. Sezgin, “On the Magical Supergravity in Six Dimensions”, Nucl. Phys. B848 (2011) 62-89, arXiv: 1012.1818.

[22] R. Slansky, “Group Theory for Unified Model Building” Phys. Rep. 79 (1981) 1128.

[23] W. G. McKay, J. Patera, and D.W. Rand, “Tables of Representations of Simple Lie Algebras, Volume I Exceptional Simple Lie Algebras”, Publications CRM, Montréal (1990).

[24] D. Burde and M. Ceballos, “Abelian ideals of maximal dimension for solvable Lie algebras” arXiv: 0911.2995.

[25] S. L. Cacciatori, B. L. Cerchiai, A. Della Vedova, G. Ortenzi and A. Scotti, “Euler angles for $G_2$”, J. Math. Phys. 46 (2005) 083512, arXiv: hep-th/0503106.

[26] Sergio L. Cacciatori and B. L. Cerchiai, “Exceptional groups, symmetric spaces and applications”, arXiv: 0906.0121.

[27] E. S. Fradkin and Y. Ya. Linetsky, “Results of the classification of superconformal algebras in two dimensions”, Phys. Lett. B282 (1992) 352-356, arXiv: hep-th/9203045.

[28] J. M. Maldacena, “The large $N$ limit of superconformal field theories and supergravity”, Adv. Theor. Math. Phys. 2 (1998) 231-252, arXiv: hep-th/9711200.

[29] Auttakit Chatrabhuti and Parinya Karndumri, “Vacua and RG flows in $N = 9$ three dimensional gauged supergravity”, JHEP 10 (2010) 098, arXiv: 1007.5438.
[30] F. Bernardoni, S. L. Cacciatori, B. L. Cerchiai and A. Scotti, “Mapping the geometry of the $F_4$ group”, Adv. Theor. Math. Phys. Volume 12, Number 4 (2008), 889-994, arXiv: 07053978.

[31] N. P. Warner, “Some New Extrema of the Scalar Potential of Gauged $N = 8$ Supergravity”, Phys. Lett. B128 (1983) 169.

[32] Auttakit Chatrabhuti and Parinya Karndumri, “Vacua of $N = 10$ three dimensional gauged supergravity”, Class. Quantum Grav. 28 (2011) 125027, arXiv: 1011.5355.

[33] T. Fischbacher, H. Nicolai and H. Samtleben, “Vacua of Maximal Gauged $D = 3$ Supergravities”, Class. Quant. Grav. 19 (2002) 5297-5334, arXiv: hep-th/0207206.

[34] M. Cvetic, H. Lu and C. N. Pope, “Consistent Kaluza-Klein Sphere Reductions”, Phys. Rev. D62 (2000) 064028, arXiv: hep-th/0003286.

[35] A. Fujii, R. Kemmoku and S. Mizoguchi, “D=5 Simple Supergravity on $AdS_3 \times S^2$ and $N = 4$ Superconformal Field Theory”, Nucl. Phys. B574 (2000) 691-718, arXiv: hep-th/9811147.

[36] Y. Sugawara, “$N = (0,4)$ Quiver SCFT$_2$ and Supergravity on $AdS_3 \times S^2$”, JHEP 06 (1999) 035, arXiv: hep-th/9903120.

[37] C. M. Hull, “Domain Wall and de Sitter Solutions of Gauged Supergravity”, JHEP 11 (2001) 061, arXiv: hep-th/0110048.