An inventory model with discount for imperfect items and purchasing price dependent demand

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Abstract
A special price discount is used to motivate buyers in nowadays. At the same time, that discount does not affect the profit. So in this model, the one-time only discount is given to the imperfect items. This improves the profit of this model. Because these imperfect items are usually considered as deteriorating items which leads to deterioration loss. Also, demand depends on purchasing price in real life. So, purchasing price dependent demand is considered. The lot size and time length of the cycle are optimized. A sensitivity analysis is presented to illustrate the parameters of this model.

Keywords
Imperfect items, economic order quantity, screening process, one-time-only discount.

1. Introduction
The EOQ (economic order quantity) inventory control model was introduced in the earliest decades of this inventory, it is still widely accepted by many researchers today. Regardless of such an acceptance, the analysis for finding an economic order quantity has several weakness. This has shepherded many researchers to study the EOQ extensively under real life situations.

Ozer Ozturk, Yavug Gazitbey and Orhan Gerdan [5] examined fuzzy optimal production and shortage quantity for fuzzy production inventory with backorder. The two different fuzzy models, one of which includes crisp production and crisp shortage quantity, and the other of which involves those that are fuzzy, have been presented by making use of trapezoidal fuzzy numbers are discussed.

A common unrealistic assumption in using the EOQ is that all units purchased are of good quality. Later, all the defective items, as a result of considering imperfect quality purchase process were considered as deteriorating items. The analysis of deteriorating inventory model is initiated by Ghare and Schrader [3] with a constant rate of decay. After this, more models are developed for deteriorating items. Dr. M. Maragatham, Dr. P.K. Lakshmidevi [2] discussed the fuzzy model with changing deterioration rate. Two different rates of deterioration are allowed at the time of transportation and demand. Tersine and Barman [7] studied the problem of scheduling replenishment orders under the classical EOQ model when both quantity and freight rate discounts are encountered. Yanlai Liang, Fangning Zhou [9] developed a two warehouse inventory model for deteriorating items under conditionally permissible delay in payment.

All the items of imperfect quality, not necessarily deteriorated, could be used. Some people are willing to purchase acceptable imperfect quality items with discounts. So the traditional EOQ model by accounting for imperfect quality items is extended.

Ritha W, Nithya K [6] developed fuzzy inventory model with imperfect items under one-time only discount. Two fuzzy inventory models are discussed with fuzzy parameters for crisp order quantity or fuzzy order quantity in order to extend the order quantity inventory model to the fuzzy environment.

In this paper, an inventory model having imperfect items with the purchasing price dependent demand is discussed. The issue that imperfect quality items are sold as a single batch by the end of the 100% screening process is considered here.
This model is solved analytically to determine the optimal lot size and the optimal cycle time. The numerical examples and sensitivity analysis are provided to illustrate the solution procedure.

## 2. Model formulation

![Diagram](image)

**Figure 1.** The behavior of the inventory model

A lot size is delivered instantaneously with purchasing price $p$ per unit. It is assumed that each lot received contains imperfect items with percentage $d$. These imperfect items are sold as single batch with discounted price. A 100% screening process of the lot is conducted at a rate of $s$ units per unit time. The items of imperfect quality are kept in stock and sold prior to receiving the next shipment as a single batch at a discounted price. The demand is always price dependent in real life. So purchasing price dependent demand is considered in this model.

## 3. Notations

- $T$ - Cycle length
- $I$ - Lot size
- $p$ - Purchasing cost per unit
- $\lambda_1 + \lambda_2 p$ - Demand rate, $\lambda_1, \lambda_2 > 0$
- $d$ - Defective percentage
- $s$ - Screening rate, $s > \lambda_1 + \lambda_2 p$
- $S_1$ - Selling price per unit of good quality
- $S_2$ - Selling price per unit of imperfect quality
- $w$ - Screening cost per unit
- $O$ - Ordering cost per order
- $h$ - Holding cost per unit
- $TR(I)$ - Total revenue per cycle
- $TE(I)$ - Total expenditure per cycle
- $TP(I)$ - Total profit per cycle
- $TPU(I)$ - Total profit per unit time

## 4. The costs involved in this cycle

The ordering cost per cycle $= O$

The purchasing cost per cycle $= pI$

The screening cost per cycle $= wI$

The holding cost per cycle $= h \left[ \frac{I(1 - d)T}{2} + \frac{dI^2}{s} \right]$

$$= h \left[ \frac{I^2(1 - d)^2}{2(\lambda_1 + \lambda_2 p)} + \frac{dI^2}{s} \right]$$

The selling price of good quality items per cycle $= s_1 I(1 - d)$

The selling price of imperfect quality items per cycle $= s_2 I d$

Total revenue per cycle $= s_1 I(1 - d) + s_2 I d$

Total expenditure per cycle $= O + pI + wI + h \left[ \frac{I^2(1 - d)^2}{2(\lambda_1 + \lambda_2 p)} + \frac{dI^2}{s} \right]$

Total profit per cycle $= s_1 I(1 - d) + s_2 I d - \left[ \frac{(\lambda_1 + \lambda_2 p)0}{I(1 - d)} + \frac{p(\lambda_1 + \lambda_2 p)}{1 - d} + \frac{w(\lambda_1 + \lambda_2 p)}{1 - d} + \frac{h(1 - d)}{2} + \frac{hd(\lambda_1 + \lambda_2 p)}{s(1 - d)} \right]$

Then

$$\frac{dT PU(I)}{dI} = \frac{\lambda O}{I^2(1 - d)} - \frac{h(1 - d)}{2} - \frac{hd\lambda}{s(1 - d)}$$

$$\frac{d^2TPU(I)}{dI^2} = \frac{-2\lambda O}{I^3(1 - d)}$$

For maximum profit, $\frac{dT PU(I)}{dI} = 0$

$$(ie), \quad \frac{\lambda O}{I^2(1 - d)} - \frac{h(1 - d)}{2} - \frac{hd\lambda}{s(1 - d)} = 0$$

Then we get

$$I = \sqrt{\frac{2\lambda Os}{h s(1 - d)^2 + 2 d\lambda}}$$

The length of the cycle

$$T = \frac{I(1 - d)}{\lambda}$$

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5. Numerical example

Consider \( p = 12 \) per unit, \( \lambda_1 = 2000 \) units per year, \( \lambda_2 = 200 \) units per year, \( d = 0.1, s_1 = 50 \) per unit, \( s_2 = 30 \) per unit, \( w = 2 \) per unit, \( s = 20000 \) units per year, \( O = 80 \) per cycle, \( h = 0.5 \) per unit.

Then the optimal values

| \( \lambda_1 \) | \( I \) | \( T \) (months) | TPU |
|----------------|-------|-----------------|-----|
| 2000           | 1284  | 3.15            | 150950 |
| 2100           | 1297.8| 3.11            | 154390 |
| 2200           | 1311.3| 3.08            | 157820 |
| 2300           | 1324.7| 3.04            | 161260 |
| 2400           | 1338  | 3.01            | 164700 |

Total profit per unit time \( TPU(I) = 150950 \)

Cycle length \( T = 0.2626 \) year = 3.15 months

Sensitivity analysis on \( \lambda_1 \)

| \( \lambda_2 \) | \( I \) | \( T \) (months) | TPU |
|----------------|-------|-----------------|-----|
| 2000           | 1284  | 3.15            | 150950 |
| 300            | 1438.5| 2.77            | 192200 |
| 400            | 1574.3| 2.5             | 233460 |
| 500            | 1696  | 2.29            | 274720 |
| 600            | 1806.6| 2.12            | 315990 |

Sensitivity analysis on \( \lambda_2 \)

| \( s \) | \( I \) | \( T \) (months) | TPU |
|---------|-------|-----------------|-----|
| 20000   | 1284  | 3.15            | 150950 |
| 15000   | 1273.1| 3.13            | 150940 |
| 10000   | 1252.2| 3.07            | 150930 |
| 8000    | 1237.1| 3.04            | 150930 |
| 5000    | 1195  | 2.93            | 150900 |

Sensitivity analysis on \( s \)

| \( p \) | \( I \) | \( T \) (months) | TPU |
|---------|-------|-----------------|-----|
| 12      | 1284  | 3.15            | 150950 |
| 13      | 1311.3| 3.08            | 152710 |
| 14      | 1338  | 3.01            | 154030 |
| 15      | 1364  | 2.95            | 154910 |
| 16      | 1389  | 2.89            | 155340 |

Sensitivity analysis on \( p \)

| \( d \) | \( I \) | \( T \) (months) | TPU |
|---------|-------|-----------------|-----|
| 0.1     | 1284  | 3.15            | 150950 |
| 0.2     | 1390.7| 3.03            | 142370 |
| 0.3     | 1504.5| 2.87            | 131340 |
| 0.4     | 1620.8| 2.65            | 116630 |
| 0.5     | 1730.8| 2.36            | 96017  |

Sensitivity analysis on \( d \)

References

[1] Chang, H.C, An application of fuzzy sets theory to the EOQ model with imperfect quality items, Computers and Operations Research, 31(2004), 2079–2092.
[2] Dr.M.Maragatham, Dr.P.K.Lakshmidevi, A fuzzy model with changing deterioration rate, International Journal of Engineering Research, 5(11)(2016), 851–855.
[3] Ghare.P.M, Schrader.G.H, A model for exponentially decaying inventory system, Journal of Industrial Engineering, 15(1963), 238–243.
[4] Hsieh. C.H, Optimization of fuzzy production inventory models, Information Science, 146(2002), 29–40.
[5] Ozer Ozturk, Yavuz Gazibey, Orhan Gerdan, Fuzzy optimal production and shortage quantity for fuzzy production inventory with backorder, Advances in Fuzzy Sets and Systems, 19(2)(2015), 73–103.
[6] Ritha. W, Nithya.K, Fuzzy inventory model : Imperfective items under one – time – only discount, Journal of Computing Technologies, ISSN 2278 – 3814.
[7] Tersine.R, Barman.S, Lot size optimization with quantity and freight rate discounts, Logistics and Transportation Review, (27)(4)(1991), 319.
[8] Wen – Kai Kevin Hsu, Hong – Fwu Yu, EOQ model for imperfective items under a one time – only discount, Omega 37, 2009, 1018–1026.
[9] Yanlai Liang, Fangming Zhou, A two warehouse inventory model for deteriorating item under conditionally permissible delay in payment, Applied Mathematical Modelling, 35(2011), 2221–2231.
[10] Zimmermann. H. J, Fuzzy set theory and its applications, 2nd revised ed., Kluwer Academic Publishers, Boston, 1991.