Contact forces in Brazil nut effect phenomenon of boulders on the asteroid surface

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Abstract. Granular particles can be found on the asteroids in the form of regolith. Regolith is a pile of boulders and gravels that covers the surface of an asteroid and have various sizes. Brazil Nut Effect (BNE) is one of the well-known phenomena that happened in this sort of system. This phenomenon leads to occur a tendency for larger boulders to come up to the surface. Some asteroids like Eros and Itokawa show that BNE supposes to happen also in a low gravity environment. BNE in asteroid may be generated by seismic vibration that causes inter-particle collisions. The collisions are represented by contacts among particles, so that contact forces need to be counted in modeling BNE. This study aims to build a modeling of BNE in asteroid involving contact forces caused by inter-particle collisions during the seismic vibration. This study shows that contact forces have a positive role in BNE by inter-particle contacts. The contacts accommodate the system to keep larger boulders staying in the elevated height. In the model that only involves static friction (without rolling), the vector of normal forces dominates over the tangential one for the resultant forces. Uprising of the larger boulders has been observed in the simulation, but most of them are still buried underneath the smaller ones. It is predicted that a seismic vibration with large enough energy is needed to make all of the larger boulders to come up to the surface. With that result, the larger boulders observed on the surface of some asteroids nowadays are reasonably the result of BNE which caused by heavy bombardment by meteoroids in the early stage of Solar System formation that drives some high-energy seismic vibration.

1. Introduction

1.1 BNE in Asteroid

BNE is one of the granular dynamics phenomena that can be found in a system of particles with various sizes. This phenomenon leads the larger particles to come up to the surface [1]. The surface of an asteroid is covered by a pile of boulders and gravels called regolith. This regolith can be considered as a system of granular particles. So, the phenomenon that happened in a system of granular particles can also happen in asteroid’s regolith, including BNE. In this case, seismic vibration may be the main cause of BNE in asteroid. The process of size sorting in BNE has a vibration velocity threshold that is
proportional to $\sqrt{g}$, where $g$ is the gravitational acceleration [2]. Because of this, it is predicted that BNE will be easier to happen in an asteroid since its value of $g$ is smaller than those of planets.

Space missions carried out by several spacecraft support this fact. Spacecraft NEAR found almost 7,000 boulders larger than 15 meters on the surface of asteroid 433 Eros by direct imaging. Another spacecraft, Hayabusa took images of Itokawa and found that the surface of Itokawa is full of large boulders and fine grains [3]. The total volume of the boulders is larger than the volume of craters. It possibly happens as a result of the reaccumulation of the fragments during the formation of Itokawa. Another scenario is those boulders came up to the surface as the result of the size-sorting process generated from seismic vibration. Nonetheless, this process doesn’t show the complete separation since we can find fine grains in the region with the large boulders [4].

Some factors affecting the phenomenon of BNE are the coefficient of restitution ($\epsilon$) and the coefficient of friction ($\mu$). From the previous study, it is stated that the time needed for larger boulders come up to the surface does not depend on the value of $\epsilon$, except for the case with the value of $\epsilon = 0.9$ which shows that the large boulders come up to the surface in a shorter time [5]. The previous study stated that the reason for this phenomenon is still unclear.

The coefficient of friction can be separated into two types, i.e. static friction ($\mu_s$) and rolling friction ($\mu_r$). These two types of friction give different actions on the BNE. Static friction will help the large boulders to come up to the surface and in contrast, rolling friction will resist the boulders to come up since it makes the boulders rolling in their places. The value of friction generated from the process modeled in this work is explained in Section 1.2 below.

1.2 Contact Forces
The contact between two particles will provide contact force, whose components are shown in Figure 1 [6]. This force comprises forces in normal and tangential directions. Normal force has the same direction as the line that connects the centers of those two particles. On the other hand, the tangential force is in the perpendicular direction to the normal force. The total contact force $F_{ij}$ works in those two particles can be presented in Equation 1,

$$F_{ij} = F_{ij}^n + F_{ij}^t, \text{ if } \psi_{ij} > 0, \text{ others,}$$

where $F_{ij}^n$ and $F_{ij}^t$ are normal and tangential forces, respectively. $\psi_{ij}$ is the value of deformation which characterizes the contact between the particles and is written in Equation 2,

$$\psi_{ij} = r_i + r_j - |r_i - r_j|,$$

where $r_i$ and $r_j$ are the radius of particles $i$ and $j$, respectively, while $r_i$ and $r_j$ are the vectors of position.

The normal force situated at the inter-particles contact, $F^n$, can be approximated as the function of deformation, $\psi$ (Equation 3),

$$F^n = \frac{2Y}{3(1-\nu^2)} \psi^2,$$

with $Y$ is Young modulus and $\nu$ is Poisson ratio. Effective radius $R_{eff}$ is mathematically defined in Equation 4 below,

$$\frac{1}{R_{eff}} = \frac{1}{r_i} + \frac{1}{r_j}.$$
The tangential force caused by the interaction of those particles is expressed in Equation 5

$$F^t = -\text{sign}(v_{\text{rel}}^t) \mu |F^n|,$$  \hspace{1cm} (5)

with $v_{\text{rel}}^t$ is the velocity of the particles in the tangential direction. In this study, we only focus on static friction only. This study aims to build a BNE modeling in asteroid involving contact forces caused by inter-particle collisions during the seismic vibration.

### 1.3 Seismic Vibration in Asteroid

Vibration generated by seismic activity (external object) will come in the form of three waves, P-waves, S-waves, and surface waves (Rayleigh). These waves are characterized by their velocity of propagation. The velocity of P-waves and S-waves can be written in Equation 6 [7],

\[
V_p = \sqrt{\frac{\gamma}{\rho} \frac{1}{1+\nu}} \frac{1}{1-2\nu},
\]

\[
V_s = \sqrt{\frac{\gamma}{\rho} \frac{1}{2(1+\nu)}},
\]

where $\rho$ is the density of the asteroid model, $V_p$ and $V_s$ are the propagation velocity of P-waves and S-waves. On the other hand, Rayleigh waves as the surface waves can also be modeled by using the approximation of its propagation as follows (Equation 7),

$$V_{Ra} \approx V_s \frac{0.862 + 1.14\nu}{1+\nu},$$  \hspace{1cm} (7)

The three seismic waves are then implemented to the mathematical model of the seismic vibration which follows Equation 8,

$$F_s(t) = F_s \left(1 - \cos \left(\frac{2\pi t}{\tau_s}\right)\right),$$  \hspace{1cm} (8)

Equation 8 provides the function of the magnitude of the force generated by the seismic vibration that can be analogous to the positional change. $F_s(t)$ is the magnitude of the force as a function of time, $F_s$ is the amplitude of seismic vibration, and $\tau_s$ is the duration of the seismic vibration that can be calculated using Equation 9,
with $R$ is the radius of the asteroid and $V$ is the propagation velocity of each type of wave.

2. Model and Simulation

2.1 REBOUND

The software used for the simulation is an open-source software called REBOUND. This software usually is used for conducting N-body simulation including molecular dynamics and granular flow [8]. This software is written in C and can be run at Linux, Unix, and Mac OS. To use the software, the users need to choose the parameters of simulation in order to match with their purposes. The parameters of the simulation used in this study are listed in Table 1. The “periodic” boundary condition applied in order to model the shape of the spherical of the asteroid. The total number of time-steps for one simulation of 500 s is 5 million and we need ~48 hours to complete one simulation.

| Parameter                  | Used Module  |
|----------------------------|--------------|
| Gravity                    | Basic        |
| Collision                  | Direct       |
| Boundary                   | Periodic     |
| Integrator                 | Leapfrog     |
| Time-step of integration   | $10^{-4}$ s  |

2.2 Model of the Asteroid

The physical parameters used for the asteroid model and the boulders are adapted from those of Itokawa, with the shape of perfectly spherical for a simplification. The physical properties of the components follow those of silicate and are listed in Table 2.

| Parameter                        | Value                |
|----------------------------------|----------------------|
| Radius of asteroid model         | 320 m                |
| Radius of large boulders         | 0.03 m               |
| Radius of small boulders         | 0.01 m               |
| Density of the asteroid and the boulders | $2,800$ kg/m$^3$       |
| Coefficient of restitution      | $\nu^{0.274}$        |
| Coefficient of static friction   | 0.6                  |

The modeling uses a total of 1,200 particles with the large-to-small boulders ratio of 3:37, which means that there are 90 large boulders and 1,110 small boulders. Then, these particles are distributed into six rows and six columns in three-dimensional Cartesian coordinates with some gaps that follows Equation 10 [9]

\[
\begin{align*}
    d_x &= r_s + r_l + f_x, \\
    d_y &= r_l + f_y, \\
    d_z &= 2 \times r_l + f_z,
\end{align*}
\]

(10)

where $d_x$, $d_y$, $d_z$ are the gaps among every boulder in Cartesian coordinates, $r_s$ and $r_l$ are the radius of small and large boulders, respectively, with an addition of random factor in each axis with the
maximum value of $f_x, f_y, f_z$. Equation 10 provides the particles to be intersected and makes the configuration is not perfectly in symmetry. The configuration of the particles can be seen in Figure 2.

![Figure 2](image)

**Figure 2.** Initial configuration of the particles. The red and blue dots represent small and large boulders.

### 2.3 Model of Seismic Vibration

These particles are then dropped into the surface of the asteroid model until they reach the condition of dynamical relaxation when all of them are no longer move. After the dynamical relaxation is reached, the model asteroid will be shaken by seismic vibration. The model of seismic vibration used has been already reviewed in Section 1.3. P-waves, S-waves, and Rayleigh waves are characterized by their propagation velocity which depends on the physical properties of the material such as Young modulus, the density of the asteroid, and Poisson ratio. This study uses silicate as the material model and its physical properties are listed in Table 3.

After knowing the velocity of each type of wave, the duration of the vibration can be calculated by using Equation 9. The value of $\tau_v$ is then implemented to Equation 8 to build the mathematical model of the seismic vibration. In this study, we use the value of the smaller boulders’ radius as the amplitude of the vibration. Our scenario generated the three seismic waves in every 100 s. Therefore, in a simulation with the time limit of 500 s, there will be five cycles that each of them consists of those waves as illustrated in Figure 3. Figure 3 shows the illustration of seismic vibration by changing the coordinate of the model asteroid in z-direction over time. From Figure 3, we know that the P-wave is the first wave in a set of seismic vibration. So, the configuration of the particles will be most disrupted by this very first wave.

| Table 3. Physical properties that used in calculating seismic vibration propagation velocity. |
|-----------------------------------------------|
| Parameter            | Value     |
| Young modulus        | 72 GPa    |
| Density              | 2,800 kg/m$^3$ |
| Poisson ratio        | 0.17      |

### 2.4 Implementation of the Contact Forces

Contact forces in Section 1.2 are implemented in the simulation as additional forces. In the default program that does not include contact forces, every particle will undergo some changes of velocity after collisions due to their coefficient of restitution. On the other hand, contact forces also contribute to their changes in velocity after collisions. The role of static friction is also represented in the tangential direction of the contact forces.
Impulse calculates the magnitude and duration of the forces induced by collisions. We can express it in mathematical formula in Equation 11,

\[ J = F\Delta t, \]  

where \( J \) is the impulse, \( F \) is the magnitude of the forces which is obtained from Equation (1), and \( \Delta t \) is the duration of the contact. The duration of contact can be expressed as the function of collisional velocity (\( v \)) as written in Equation 12,

\[ \Delta t = 5.84 \left( \frac{\rho(1-\nu^2)}{\gamma} \right)^{0.4} r v^{-0.2}. \]

Meanwhile, impulse is also known as the change of linear momentum as formulated in Equation 13,

\[ J = \Delta p, \]

where \( \Delta p \) is the change of linear momentum. Because of this, if we know the magnitude of the impulse, we can combine it with the mass of the particles to get the change of velocity after collisions.

Contact forces as additional forces are implemented as an additional change in velocity after collisions. The value of additional change in velocity is obtained from Equation 13, thanks to the value of impulse in Equation 11 and the duration of the contact in Equation 12. The resulted value will then be used as an augmentation to the change of velocity due to the restitution.

![Figure 3](image_url)

**Figure 3.** Illustration of the seismic vibration model. P, S, and, Ra represent P-waves, S-waves, and Rayleigh waves.

### 3. Results and Discussions

#### 3.1 Model without Contact Forces

From Figure 4 we can see that once the system was shaken by the seismic vibration, the upper limit of the height of the large boulder reaches those of the small ones. We can say that in this condition, some of the large boulders already come up to the surface. But after 100 s, these large boulders sink in again because of its large mass. It is predicted that the number of small boulders is not enough to keep the large boulders on the surface.
Figure 4. Height evolution of the boulders in the model without contact forces. Red and blue solid lines indicate the average height of the small and large boulders, while the dashed lines represent the upper and lower limit of the height.

We can see in Figure 5, some of the large boulders already surpassed the height of the small ones. There some bumps on their height due to seismic vibration. To the end of the seismic vibration, the height of each particle is higher than those from the beginning of the vibration.

Figure 5. Height evolution of the particles in the model without contact forces during the seismic vibration
3.2 Model with Contact Forces in Normal Direction

![Figure 6](image1.png)

**Figure 6.** Height evolution of the boulders in the model with contact forces in the normal direction.

The pattern showed in Figure 6 is not much different from that in Figure 4. The most different thing is that large boulders manage to stay on the surface in a longer duration. They stay on the surface until the time of 200 s before sink in again. The height evolution during the vibration showed in Figure 7. The difference to the previous model is that there is a significant bump in height around the time of 29 s. Therefore, contact forces in normal direction can trigger the height bump after the seismic vibration. The contribution of contact forces can be seen after the bump. The previous model (Figure 5) shows a slight decrease in height after the bump. Yet, in this model (Figure 7), this kind of height declining is not observed. The height of boulders are relatively constant after the bump. From this fact, we can conclude that contact forces in normal direction can trigger some bump in height and keep the boulders in a levitated height even after the vibration done.

![Figure 7](image2.png)

**Figure 7.** Height evolution of the particles in the model with contact forces in normal direction during the seismic vibration.
3.3 Model with Contact Forces in Tangential Direction

In Figure 8, we can see that the decrease in the upper limit of the large boulders is not as significant as the previous models. The upper limit of the large boulders is still not too far from the surface. From this, we can infer that contact forces in tangential direction can keep the boulders in elevated height. During the seismic vibration (Figure 9), we can see that the amplitude of the bump is relatively smaller than that in the previous models. This is due to the contribution of the static friction which is implemented in the contact force in the tangential direction (Equation 5). The bump at the end of vibration also happens in this model. We can see it at the time of 26 s. This bump happens at an earlier time than the previous model. The same idea was mentioned in [5] that static friction will cause BNE to happen in a shorter time.

Figure 8. Height evolution of the boulders in the model with contact forces in the tangential direction.

Figure 9. Height evolution of the particles in the model with contact forces in tangential direction during the seismic vibration.
3.4 Model with Contact Forces in Normal and Tangential Direction

![Figure 10](image1)

**Figure 10.** Height evolution of the boulders in the model with contact forces in the normal and tangential direction.

The pattern in Figure 10 is still the same as the previous model. Some of the large boulders are able to come up to the surface and then sink back again after some particular time. From the pattern of height evolution during the seismic vibration (Figure 11) we can see that it is similar to those from the model with normal direction. The value of the amplitude of those models is relatively the same. We can conclude that contact forces in normal direction dominate over the tangential ones. But it differs from the model in the normal direction, the height of the bump in this model occurs at 26 s or similar to the tangential one. This matches the study that the contribution of the static friction will help BNE to happen at an earlier time.

![Figure 11](image2)

**Figure 11.** Height evolution of the particles in the model with contact forces in normal and tangential direction during the seismic vibration.
3.5 Summary of All Model

| Contact Forces          | ‘Sink-in’ time (s) | ‘Sink-in’ value of height (m) |
|-------------------------|--------------------|------------------------------|
| No contact forces       | ~100               | ~0.04                        |
| Normal                  | ~200               | ~0.03                        |
| Tangential              | Relatively stable  | Relatively stable            |
| Normal-Tangential       | ~150               | ~0.025                       |

Table 4. Differences in results between each simulation with different mechanisms of implementing contact forces.

The models with different mechanisms of contact force implementation share the same pattern of large boulders coming up to the surface and then sink back in some value of time. Table 4 shows the differences between each model in terms of ‘sink-in’ time and the value of decreasing height. Model without any contact forces shows a greater value of decreasing height and also earlier time of ‘sink-in’. The factor of static friction is implemented in the tangential direction as shown in Equation 5. In contrast, the model with tangential contact forces shows a relatively stable height since it doesn’t show much decrease in height during the simulation. We predict this happens due to static friction.

| Contact Forces          | Bump time (s) | Bump amplitude (m) |
|-------------------------|---------------|--------------------|
| No contact forces       | 25            | ~0.025             |
| Normal                  | 28            | ~0.01              |
| Tangential              | 26            | ~0.005             |
| Normal-Tangential       | 26            | ~0.01              |

Table 5. Differences in the bump phenomenon of each simulation during the seismic vibration.

Bump phenomenon during the seismic vibration also happens in all models. The differences between the bump in each model are summarized in Table 5. The boulder in the model without contact forces has a different pattern than other models. While the bump in the other models happened at an exact point of time, the particles’ height in the model without contact forces is slowly increasing in a range of time. Besides that, the model without contact forces shows a slightly decreasing of height at the end of seismic vibration while the others not. The model with contact forces in normal direction shows a greater value of bump amplitude that occurred in a longer time than the tangential one. This is due to the static friction that implemented only in the tangential direction. Friction between particles will affect the motion of them and prevent them from going up which results in a smaller value of amplitude. On the other hand, this friction will help the BNE (in this case, the ‘bump’) to happen at an earlier time. The model of contact forces in normal-tangential direction shows the great value of bump amplitude (the same as the normal one) that happened in a shorter time (the same as the tangential one).

4. Conclusions

The contribution of contact forces on the BNE phenomenon has been examined and explained in Section 3. From the explanation above, we can infer that contact forces in every direction, whether in a normal or tangential direction, have a positive role in BNE. In the normal direction, contact between particles gives a greater value of bump amplitude and also the longer and smaller value of the ‘sink-in’ phenomenon. Hence, inter-particles contacts in normal direction will help the larger particles to keep their elevated height from the seismic vibration. On the other hand, static friction that implemented in
the tangential direction will trigger BNE to happen at an earlier time and also prevent the 'sink-in' phenomenon to happen.

The results show that some large boulders that initially placed in the bottom of the pile manage to come up to the surface. However, many others are still buried beneath the surface. It is predicted that we need seismic vibration with larger energy to make all of the large boulders to come up to the surface completely. The domination of the BNE process in observed asteroid nowadays is still unclear. If the large boulders found on the surface of the asteroid are caused by BNE, we need some external seismic cause such as meteoroids impacts, whether a catastrophic one or smaller ones but recurring over time. Both of these scenarios are most likely to happen in the early phase of Solar System formation.

References
[1] Rosato A, Strandbug K J, Prinz F and Swendsen R H 1987 PRL 58 1038-1040
[2] Jiongming Z, Binglu Z and Bin W 1998 Il Nuovo Cimento 20 1443-1448
[3] Lang K R 2011 The Cambridge Guide to the Solar System (Cambridge: Cambridge University Press) p 381
[4] Perera V, Jackson A P, Asphaug E and Ballouz R 2016 Icarus 278 194-203
[5] Matsumura S, Richardson D C, Michel P, Schwartz S R and Ballouz R 2014 MNRAS 443 3368-3380
[6] Tancredi G, Maciel A, Heredia L, Richeri P and Nesmachnow S 2012 MNRAS 420 3368-3380
[7] Quillen A C, Zhao Y, Chen Y, Sanchez P, Nelson R C and Schwartz S R 2019 Icarus 319 312-333
[8] Rein H and Liu S–F 2012 A&A 537 A128
[9] Rozzykin A Z 2019 Modelling Contact Forces in Brazil Nut Effect Phenomenon of Boulders on Asteroid Surface Master’s Program Thesis Institut Teknologi Bandung