Calculation of the shock response spectrum of the prestressed steel cable

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Abstract. The abrupt fracture of prestressed steel strand will form a great unloading impact force, which will influence the displacement and internal force of the remaining structure. Shock response spectrum method can reduce the computation and ensure the accuracy. In order to obtain the shock response spectrum of the steel strand at the moment of failure, this paper proposes to solve the dynamic response of a single degree of freedom structure after cable break through programming under the premise of knowing the failure time of the strand and the corresponding cable tensioning. We select several unloading curves that are the closest to the test, and solve the response spectrum of the broken cable under different failure time and damping, finally, the response spectrum method is compared with the finite element method by numerical simulation. It can be found that the dynamic coefficient will show different trends due to the change of unloading methods. In the same unloading method, the failure time has little impact on the shock response spectrum, but the impact of damping on the shock response spectrum is very obvious.

1. Introduction
Prestressed cables have been widely used in bridge engineering, large-span buildings, and towering structures. The prestress of the cable makes the structure contain huge energy. The impact time history of cable breaks, like other dynamic load schedules, also includes three factors: time duration, amplitude, and shape characteristics. However, time duration is the most important thing. At present, there is very little research [1][2], which needs to be solved.

The US Code [3][4] specifies the upper limit of the removal time (1/10 residual structure period) when removing the failed member by the removal member method, but this is the result of the research of the ordinary frame. For the cable that is instantaneously broken, The upper limit of the specification is too large and lacks practicality. Mozos [5] used an electro-hydraulic servo actuator to stretch the steel strand at an equal strain rate, and the average rupture time of the undamaged steel strand was about 5.5×10^{-3} s, at the same time, the damaged steel strands with 1.0 mm and 1.9 mm deep circumferential nicks of the two outer ring wires were compared. The author [6] uses MTS with a 15.2mm diameter 1860 steel strand. and the data acquisition frequency is 1600Hz. The tensile fracture time history of two steel strands with intact cable body and circumferential uniform damage was studied. And investigated the influence of loading rate on the time history and duration of cable breakag.
Yang Yongqiang \cite{7} simplified adjacent buildings into single-degree-of-freedom systems when studying the collapse caused by collisions in adjacent buildings with insufficient spacing, calculated the peaks of collision forces at different natural frequencies, and plotted the collision force response spectrum. Wang Junjie \cite{8} proposed a design impact spectrum method for bridges under hull impact, using finite element to simulate different tonnage ships striking rigid walls at different speeds to obtain the impact time history, and calculating the single degree of freedom system under the action of these impact forces. Based on the representative cable force unloading curve obtained by the test, the impact response spectrum of cable fracture is studied by taking the cable break time and damping ratio as key parameters.

2. Program Solving and Verification of Shock Response Spectrum

For a single-degree-of-freedom mass spring damping system, when its common foundation is subjected to impact excitation, its response peak is a function of the natural frequency of the single-degree-of-freedom system. The graph drawn by this function is called Shock Response Spectrum. There are two main calculation methods for the calculation and derivation of the shock response spectrum, namely Laplace transform method and convolution integral method.

2.1 Matlab program for compiling the shock response spectrum

The maximum response of each impact load to a single-degree-of-freedom structure depends only on the ratio of the duration of the pulse to the natural period of the structure \(t_1/T\). Therefore, for various impact load forms, the graph of the displacement amplification factor \(\beta\) and the \(t_1/T\) function, that is, the response spectrum, can be drawn.

The Matlab control toolbox is used to solve the structural dynamic response, and the impact response spectrum of the structure is drawn. Figure 1 is the calculation flow chart of the corresponding shock response spectrum.

![Flow chart of shock response spectrum](image)

2.2 Matlab program verification

Select the rectangular pulse force shown in Figure 2, the above-mentioned source program is used to calculate the impulse response spectrum under the rectangular pulse force \cite{9}.

![Rectangular impact force](image)
The structural dynamics equation under a rectangular pulse is:

\[
\begin{align*}
    m\ddot{u} + ku &= p(t) = \\
    &= \begin{cases} \\
    p_0 & t \leq t_1 \\
    0 & t > t_1 
    \end{cases}
\end{align*}
\]  

(1)

The initial conditions are: \( u(0) = \dot{u}(0) = 0 \). The theoretical analysis is divided into two stages of free and forcing vibration, and the overall maximum response can be obtained as:

\[
\beta = \frac{u_0}{(u_0)_0} = \begin{cases} \\
    2\sin \pi t_1 / T & t_1 / T \leq 1 / 2 \\
    2 & t_1 / T > 1 / 2 
    \end{cases}
\]  

(2)

Figure 3 plots the comparison between the calculated results of the Matlab program and the theoretical shock response spectrum. When \( t_1 / T > 4 \), the program calculated value has a slight fluctuation separation phenomenon near the theoretical calculated value 2; when \( t_1 / T < 4 \), the theoretical calculation value is basically consistent with the calculated value of the program.

3. Steel Strand Breakage Shock Response Spectrum

When unloading along the cosine or convex, it represents the unloading process of the rate of change of the cable force from slow to fast; when unloading along the concave mode, it represents the rate of change of the cable force from fast to slow. The specific function corresponding to each unloading method is as follows, the function image is shown in Figure 4, and \( P_0 \) is the cable force value before the cable is broken.

\[
\begin{align*}
    P_1 &= \cos(0.5\pi t) & \text{(Cosine unloading)} \\
    P_2 &= 1 - t & \text{(Slope unloading)} \\
    P_3 &= 1 - t^2 & \text{(Convex unloading)} \\
    P_4 &= (t - 1)^2 & \text{(Groove unloading)} 
\end{align*}
\]  

Fig.4 Breaking function
3.1 Cable break spectrum unloaded along the cosine function path

Figure 5 shows the displacement response spectra for different excitation durations, and Figure 6 shows the corresponding shock spectra. In the $P_1$ unloading mode, the unloading time has no effect on the shock response coefficient of the structure, and the overall trend keeps increasing. When $t_1/T$ is from 0 to 0.5, the $\beta$ rises faster to 1.5, then slowly rises, and the dynamic amplification factor $\beta$ maximum is 1.95 less than 2.0.

![Fig.5 Displacement spectrum of different excitation time under $P_1$](image1)

![Fig.6 Shock response spectrum of different excitation duration under $P_1$](image2)

Fig. 7 plots the shock response spectra of the structural damping ratios $\xi = 0\%$, 2\%, 5\%, 10\%, 20\%, etc. at 2 ms when the impact is held. It can be clearly seen from the figure that as the damping increases, the dynamic amplification factor of the structure gradually decreases, but the basic shape of the curve remains unchanged.

![Fig.7 Shock response spectrum of different damping ratios under $P_1$](image3)

3.2 Cable break spectrum unloaded along the slope function path

Fig. 8 is a displacement response spectrum at different impact durations, and the maximum displacement of the structure also changes significantly as the impact is changed.

Fig. 9 is the impact spectrum of different excitation durations without damping. The five different impact forces have little effect on the shock response coefficient of the structure, and the overall trend keeps increasing, when $t_1/T$ is from 0 to 0.5, $\beta$ gradually decreases and then slowly rises. It can be seen from the figure that when the ratio of the impact holding time to the structural natural period is small, the power amplification factor hardly changes with the change of the impact holding time, however when the impact is close to the natural period of the structure, the impact holding time has a certain influence on the power amplification factor.
Figure 10 is a shock spectrum with different damping ratios of 2ms during impact holding. It can be seen from the figure that when the impact is much shorter than the natural period of the structure, if \( t_i / T < 0.25 \), the damping force is too late to absorb too much energy from the structure, in this way, the power amplification factor is basically the same, and then the effect of damping is gradually reflected.

3.3 Cable break spectrum unloaded along the path of the upper convex function

Figure 11 is a displacement response spectrum of a single degree of freedom structural system at different impact durations. Figure 12 is a shock spectrum for different excitation durations without damping.

Fig. 13 is a shock spectrum with different damping ratios of 2 ms at the time of impact holding. It can be found that the damping effect is gradually manifested when the ratio of the natural period of the impact to the structure is \( t_i / T > 0.25 \).
Fig. 13 Shock response spectrum of different damping ratios under $P_3$

3.4 Cable break spectrum unloaded along the concave function path

Figure 14 is a displacement response spectrum of a single degree of freedom structural system at different impact durations. Figure 15 is the impact spectrum of different excitation durations without damping.

Fig. 14 Displacement spectrum of different excitation duration under $P_4$

Fig. 15 Shock response spectrum of different excitation duration under $P_4$

Fig. 16 is a shock spectrum with different damping ratios of 2 ms at the time of impact holding. It can be seen from the figure that when the impact holding is much smaller than the natural period of the structure, the dynamic amplification factor is basically the same.

Fig. 16 Shock response spectrum of different damping ratios under $P_4$

3.5 Influence of cable force unloading shape on shock spectrum

According to the four cable force unloading methods selected in this paper, the displacement response spectrum (Fig. 17) and shock response spectrum (Fig. 18) of the structural system under the same excitation duration and the same damping ratio are plotted.
From the figure, it can be found that under the four cable force unloading modes, the displacement spectrum and the shock spectrum response corresponding to P3 are the largest. Where P0 is a rectangular excitation load.

4. Conclusion

(1) Under the same excitation duration and the same damping ratio, the displacement response spectrum and the shock response spectrum of different unloading modes are different, and the dynamic coefficient will change due to the unloading mode change. The displacement response spectrum and shock response spectrum corresponding to the convex unloading maximum.

(2) In the same unloading mode, the failure time has a great influence on the displacement response spectrum. The shorter the impact holding time, the more the structure first reaches the maximum displacement, and the failure time has little effect on the shock response spectrum. The influence of damping on the impact spectrum is obvious, especially when the impact period is greater than the natural period ratio of the structure is greater than 0.25.

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