The exploration of the three-dimensional structure of the nucleon, both in momentum and in configuration space, is one of the major issues in high energy hadron physics. Transverse momentum dependent parton densities (TMDs) have recently attracted huge interest, not only for their remarkable spin correlation properties, but mostly because they represent a crucial tool for the investigation of the three-dimensional structure of the nucleon.

Huge amount of experimental data on spin asymmetries in several different processes show that TMD distribution and fragmentation functions exist and are non zero. Much progress has been achieved, for instance, in the phenomenological studies of the Sivers distribution function, which represents the number density of unpolarized partons inside a transversely polarized hadron, and of the Collins fragmentation function, which is related to the probability of a polarized quark fragmenting into an unpolarized hadron.

TMD studies are readily performed in the framework of QCD factorization, within a generalized parton model that incorporates the partonic intrinsic transverse motion in the kinematics of the examined scattering processes. In this simple phenomenological approach, cross sections and spin asymmetries are generated as convolutions of distribution and (or) fragmentation TMDs with elementary scattering cross sections; for instance for semi-inclusive deep inelastic scattering (SIDIS) processes, we have:

\[
\frac{d\sigma}{d^4p \rightarrow d^4p'hX} = \sum_q f_{q/p}(x, k_\perp; Q^2) \otimes \frac{d\sigma}{d^4q \rightarrow d^4q} \otimes D_{h/q}(z, p_\perp; Q^2). \tag{1}
\]
In the $\gamma^* - p$ c.m. frame, see Fig. 1, the measured transverse momentum, $P_T$, of the final hadron is generated by the transverse momentum of the quark in the target proton, $k_\perp$, and of the final hadron with respect to the fragmenting quark, $p_\perp$. At order $k_\perp/Q$ it is simply given by

$$P_T = z k_\perp + p_\perp.$$  \hspace{1cm} (2)

There is a general consensus that such a scheme holds in the kinematical region defined by

$$P_T \approx \Lambda_{QCD} \ll Q.$$  \hspace{1cm} (3)

The presence of the two scales, small $P_T$ and large $Q$, allows to identify the contribution from the unintegrated partonic distribution ($P_T \approx k_\perp$), while remaining in the region of validity of the QCD parton model.

Within this simple scheme we can successfully describe a wide range of unpolarized and polarized experimental data, provided we are able to model and phenomenologically determine the appropriate TMDs, including their scale evolution. Historically, in the Torino-Cagliari standard approach, TMDs are parametrized in a form in which their dependence on the lightcone momentum fraction and on the partonic intrinsic transverse momentum are factorized,

$$f_{q/p}(x, k_\perp) = f_{q/p}(x, Q^2) \frac{e^{-k_\perp^2/(\langle k_\perp^2 \rangle)}}{\pi \langle k_\perp^2 \rangle},$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z, Q^2) \frac{e^{-p_\perp^2/(\langle p_\perp^2 \rangle)}}{\pi \langle p_\perp^2 \rangle},$$  \hspace{1cm} (4, 5)

with a $Q^2$-independent, normalized Gaussian factor giving the intrinsic transverse momentum distribution, multiplied by a collinear unpolarized parton distribution function (PDF) evolving with $Q^2$ according to DGLAP equations; $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$ are free parameters which can be extracted from experiments. A similar parameterization is devised for polarized TMDs, like the Sivers function

$$\Delta^N_{q/p^\uparrow}(x, k_\perp) = 2 N_q(x) h(k_\perp) f_{q/p}(x, k_\perp, Q^2)$$  \hspace{1cm} (6)
with

\[ f_{q/p}(x, k_{\perp}) = f(x, Q^2) e^{-k_{\perp}^2/(M_1^2)} \]

\[ N_q(x) = N_q x^\alpha(1 - x)^\beta \frac{(\alpha_q + \beta_q)(\alpha_q + \beta_q)}{\alpha_q \beta_q} \]

and

\[ h(k_{\perp}) = \sqrt{2e} k_{\perp} e^{-k_{\perp}^2/M_1^2} \]

\[ \langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2, \quad \langle p_{\perp}^2 \rangle = 0.20 \text{ GeV}^2. \]

\[ \langle Q^2 \rangle = 2.4 \text{ GeV}^2 \text{ for HERMES and } \langle Q^2 \rangle = 3.2 \text{ GeV}^2 \text{ for COMPASS}. \]

As new, more precise and higher statistics data are rapidly becoming available, it is due time to start wondering whether we can find any sign of TMD scale evolution in the experimental measurements we can now access. Interesting studies have been performed by comparing the Sivers asymmetries measured by the HERMES and COMPASS Collaborations, over similar ranges, but different average values of \( Q^2 \):

\[ \langle Q^2 \rangle = 2.4 \text{ GeV}^2 \text{ for HERMES and } \langle Q^2 \rangle = 3.2 \text{ GeV}^2 \text{ for COMPASS}. \]
This scheme of TMD evolution was finalized by Aybat and Rogers\cite{25} for the unpolarized TMDs, who also proposed, together with J. Collins and J. Qiu, a model for the TMD evolution for polarized parton densities, in particular for the Sivers function\cite{26}.

Most of the people mentioned above are participants of this workshop, therefore the interested reader can find more details and better explanations in their contributions to these proceedings.

After the publication of these theory works, the first studies of what can more properly be defined TMD-phenomenology started to be performed: Aybat, Prokudin, Rogers\cite{27} proposed a very elementary phenomenological exercise in which they compared the HERMES and COMPASS Sivers single spin asymmetry $A_{UT}$, 

Fig. 2. Comparison between the Sivers $A_{UT}^{\text{Sins}(\phi_{h}-\phi_{S})}$ asymmetry, upper panel, and Collins $A_{UT}^{\text{Sin}(\phi_{h}+\phi_{S})}$ asymmetry, lower panel, as measured by the HERMES\cite{7,12} and COMPASS\cite{14,18} Collaborations.
calculated at their two fixed values of \( \langle Q^2 \rangle \): 2.4 GeV\(^2\) for HERMES and 3.8 for COMPASS; evolution effects were then compared, as illustrated in Fig. 3. Notice that no \( x \) dependence was taken into account, somehow washing out the most sensitive and interesting information this kind of data can provide on TMD scale evolution.

At the same time, M. Anselmino, S. Melis and myself performed a new, more refined phenomenological analysis of the Sivers TMD evolution, again based on the comparison between HERMES and COMPASS sets of measurements on the Sivers effect, but focussing mostly on the actual \( Q^2 \) and \( x \) dependence. We have shown that TMD-evolution implies a strong variation with \( Q^2 \) of the functional form of the unpolarized and Sivers TMDs, as functions of the intrinsic momentum \( k_\perp \); moreover, our fit of all SIDIS data on the Sivers asymmetry using TMD-evolution, when compared with the same analysis performed with the simplified DGLAP-evolution, exhibits a smaller value of the total \( \chi^2 \), a reduction which mostly originates from the large \( Q^2 \) COMPASS data, which are greatly affected by the TMD evolution, as shown in Fig. 4. We then considered this as a strong indication in favor of the TMD evolution.

Without going into the details of such a complex evolution scheme, there is one particular point that I would like to address. The TMD evolution equation of the unpolarized TMD PDFs, in configuration space, is the following

\[
\tilde{f}_{q/p}(x, b_T; Q) = f_{q/p}(x, Q_0) \tilde{R}(Q, Q_0, b_T) \exp \left\{ -b_T^2 \left( \langle k_\perp^2 \rangle / 4 + \frac{g_2}{2} \ln \frac{Q}{Q_0} \right) \right\},
\]

\[
(12)
\]

where \( g_K(b_T) = \frac{1}{2} g_2 b_T^2 \) with \( g_2 = 0.68 \text{GeV}^2 \) corresponding to \( b_{\text{max}} = 0.5 \text{GeV}^{-1} \).

A similar equation regulates the evolution of the first derivative of the Sivers function. Eq. (12) shows that the \( Q^2 \) evolution is controlled by the logarithmic \( Q \) dependence of the \( b_T \) Gaussian width, together with the factor \( \tilde{R}(Q, Q_0, b_T) \): for increasing
values of $Q^2$, they are responsible for the typical broadening effect already observed in Refs. [25] and [26]. Notice that the parameter $g_2$, that controls the $b_T$ Gaussian width and its spreading, is not extracted from our fit, but taken as a fixed values from elsewhere [29]. We could have determined its value in our fit, and probably got a smaller value, but it is important to remember that SIDIS asymmetries are very little sensitive to the precise value of $g_2$; therefore, our choice to keep it fixed was motivated by the balance one always has to keep between number of data and number of free parameters, to obtain a reliable fit. Drell Yan processes, instead, where $Q^2$s are much larger and perturbative corrections become important, are extremely sensitive to it. Therefore, one should not simply take the parameters used in this application of TMD evolution to SIDIS processes and apply them blindly to Drell Yan data or other processes. This would require a new, careful, global analysis on all SIDIS and Drell Yan data, re-starting from unpolarized cross sections.

Sun and Yuan [32] have recently applied some CSS-like evolution scheme at one loop, with strong approximations which hold for moderate $Q$ and $Q_0$, to account for the TMD evolution of the unpolarized TMD PDFs, and extended this formalism to the Sivers function as well. In this approximated scheme the evolution does not produce such a strong suppression of the Drell-Yan asymmetries. They then perform a phenomenological study on a rather limited selection of Drell-Yan and SIDIS data, showing that the evolution scheme they propose can satisfactorily describe most of them.

Remaining on Drell Yan processes, a recent preliminary study was performed by S. Melis, fitting the E288 [30] and E605 [31] Drell-Yan data and assuming a Gaussian
$k_\perp$ dependence. The free parameter $\langle k_T^2 \rangle$ was extracted separately for each data sets corresponding to different $\sqrt{s}$ values. The results showed that those Drell Yan data indicate a $\sqrt{s}$ (roughly linear) dependence in addition to the logarithmic $Q^2$ dependence typical of scale evolution. This is shown in Figs. 5 and 6.

Most recently D. Boer has performed a study of the energy scale dependence of the Sivers asymmetry in SIDIS, although on a larger range of $Q^2$ values (3 - 100 GeV): he finds that the peak of the Sivers asymmetry falls off with $Q$ roughly like $(1/Q)^{0.7}$, quite faster than found within the CSS evolution schemes. Moreover, the peak of the asymmetry is located around the initial scale $Q_0$ and moves rather slowly towards higher transverse momentum values as $Q$ increases, which may be due to the absence of perturbative tails of the TMDs.

Important work on TMD evolution has also recently been done by Echevarria, Idilbi, Scimemi in the framework of effective field theory which, however, has not yet reached the stage of feasible phenomenological applications.

As this is one of the opening talks of this workshop, there are no proper conclusions. Rather, I will close with a few remarks on future perspectives.

As far as TMD evolution is concerned, we have recently come a long way. We now have evolution schemes and some first attempts towards a full phenomenological study of the unpolarized distribution and fragmentation TMDs, and of the Sivers
and Collins functions. These are very preliminary studies, which need to be refined and re-thought in a more consistent and appropriate way, especially as far as the parametrization of unknown phenomenological quantities are concerned.

From the experimental side, we certainly need more SIDIS (polarized and unpolarized) data at larger values of $x$ (JLab 12) and spanning a larger $Q^2$ range (EIC) as well as more (and more precise) Drell-Yan data, for which many beautiful experiments are being planned (COMPASS, RHIC, Fermilab, NICA, JPARK). Inclusive hadron production in hadron-hadron scattering processes, as well, represent a very interesting and infinitely challenging field where to sharpen our tools.

With the new experimental data on SIDIS multiplicities coming in, we have to go back one step, re-think and re-perform a solid, global analysis of Drell-Yan as well as SIDIS unpolarized cross sections, to determine the basic parameters for the phenomenological quantities needed for the implementation of the TMD evolution schemes. Afterwards, we can proceed on a firm footing to perform the same analysis for the Sivers, transversity and Collins TMD functions, keeping in mind the importance of finding phenomenological frameworks suitable for all processes.

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