The characteristic analysis of the built-in vector atomic magnetometer in a nuclear magnetic resonance oscillator

Cite as: AIP Advances 9, 045117 (2019); https://doi.org/10.1063/1.5082152
Submitted: 19 November 2018 . Accepted: 03 April 2019 . Published Online: 12 April 2019

Qiyuan Jiang, Jiajia Li, Zhiguo Wang, Yi Zhang, and Hui Luo

ARTICLES YOU MAY BE INTERESTED IN

Magnetocardiography with a 16-channel fiber-coupled single-cell Rb optically pumped magnetometer
Applied Physics Letters 114, 143702 (2019); https://doi.org/10.1063/1.5094339

Angular evolution of thickness-related unidirectional magneto-resistance in Co/Pt multilayers
AIP Advances 9, 045016 (2019); https://doi.org/10.1063/1.5079894

Ultrahigh sensitivity magnetic field and magnetization measurements with an atomic magnetometer
Applied Physics Letters 97, 151110 (2010); https://doi.org/10.1063/1.3491215
The characteristic analysis of the built-in vector atomic magnetometer in a nuclear magnetic resonance oscillator

Cite as: AIP Advances 9, 045117 (2019); doi: 10.1063/1.5082152
Submitted: 19 November 2018 • Accepted: 3 April 2019 •
Published Online: 12 April 2019

Qiyuan Jiang, Jiajia Li, Zhiguo Wang, Yi Zhang, and Hui Luo

AFFILIATIONS
1 College of Advanced Interdisciplinary Studies, National University of Defense Technology, Changsha 410073, China
2 Interdisciplinary center of quantum information, National University of Defense Technology, Changsha 410073, China

ABSTRACT
We analyze the amplitude-frequency, phase-frequency, signal amplification, linear range, and vector characteristics of the built-in vector atomic magnetometer operating at extreme off-resonance condition in a nuclear magnetic resonance oscillator, which makes possible its performance improvement by a balanced strategy in optimizing the parameters based on the proposed model. The experiment validates our prediction of the amplitude-frequency characteristic, and the numerical simulation indicates that the applied carrier field with following demodulation procedure holds the potential to give one order of magnitude, which is experimentally-validated to have at least twice, signal enhancement and enable the vector characteristic, where a large longitudinal static field and an appropriate transverse relaxation time are preferred to have optimized characteristics depending on different applications.

I. INTRODUCTION
High performance nuclear magnetic resonance (NMR) oscillators benefit various applications such as the magnetic resonance imaging (MRI),1–3 the NMR spectroscopy,4–7 and the NMR gyroscope.8–11 The Larmor precession frequency of the detected nuclear species is utilized to enable the signal detection of the NMR oscillator, which is further achieved by measuring nuclear spins with the built-in magnetometer.12

The arguably most mature set-ups are the atomic magnetometers13–15 and the superconducting quantum interference devices (SQUID) magnetometers,16,17 yet the miniaturized atomic magnetometers facilitating ultra-sensitive detection of magnetic field with no needs of cryogenic condition, are more promising than SQUID magnetometers for the application of the built-in magnetometer in the NMR oscillator. Furthermore, the vector atomic magnetometer (VAM) is especially preferred as the built-in magnetometer which enables the compensation of the stray magnetic field along multiple axes in the NMR oscillator.18 Therefore, understanding the characteristics of the built-in VAM is crucial in the NMR oscillator.

The electron paramagnetic resonance (EPR) amplitude-frequency response of the VAM has been reported experimentally, as well as the linear range,19 while corresponding theoretical characteristic analysis of the VAM in the NMR oscillator has not been systematically given.

In this paper, we make characteristic analysis of the built-in VAM in the NMR oscillator, including EPR amplitude-frequency, EPR phase-frequency, signal amplification by the carrier field, linear range, and vector characteristics, where the magnetometer mainly operates at its extreme off-resonance condition, to enable the performance improvement by optimizing the corresponding parameters and facilitate its application in the NMR oscillator.

II. THEORETICAL MODEL
The built-in vector atomic magnetometer (VAM) in a nuclear magnetic resonance (NMR) oscillator is enabled by detecting the Faraday rotation angle of the probe beam polarization, which is proportional to the transverse magnetization vector of the alkali atoms
in the vapor cell. A bias magnetic field is required along the longitudinal direction to form a macroscopic magnetization vector with the help of optical pumping for the ensemble of atoms, while its transverse magnetization vector contains the information of the detected magnetic field along the corresponding axis, which enables the transverse magnetic field detection by the magnetometer.

Assuming the applied magnetic field as \( \vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z} \), and the assumed steady-state magnetization vector along the longitudinal direction is denoted by \( \vec{M}_0 = M_0 \hat{z} \). By assuming that the pure pumping effect of pump light only takes place along Z axis, that is, such an effect contributed by the probe light is negligible, the general form of Bloch equation follows:

\[
\frac{d\vec{M}}{dt} = \frac{1}{\tau} (\vec{M}_0 - \vec{M}) + \vec{M} \times \gamma \vec{B},
\]

where \( \gamma \) denotes the gyromagnetic ratio, \( \tau \) denotes the relaxation time, and \( \vec{M} \) denotes the total magnetization vector of the alkali atoms. For the simplicity of analysis, typical magnetic field components are assumed as follows:

\[
\begin{align*}
B_x &= B_0 \cos(\omega_1 t), \\
B_y &= 0, \\
B_z &= B_0 + B_1 \cos(\omega_1 t).
\end{align*}
\]

(2)

where \( B_0, B_1 \), and \( B_1 \) denote the magnetic field intensity of the longitudinal static field, the longitudinal alternating carrier field and the detected transverse field, respectively. Assuming \( |B_1| \ll |B_0| \), thus the steady-state magnetization vector could be approximately maintained along the longitudinal direction. Both the carrier and transverse field vary periodically in time domain with positive-valued frequencies of \( \omega_C \) and \( \omega_1 \), respectively. The carrier field is applied to modulate the detected signal to high frequency and achieve demodulation procedures with the lock-in amplifier, which takes the advantages of avoiding the low-frequency flicker noise and improving the signal strength under certain cases (e.g. NMR detection of certain nuclear species).\(^{20,21}\)

By replacing \( \tau \) with the longitudinal relaxation time \( T_1 \) and the transverse relaxation time \( T_2 \) along different axis, and transform Eq. (1) into scalar formula, one gets:

\[
\begin{align*}
\frac{dM_x}{dt} &= yB_z M_y - yB_y M_x - \frac{1}{T_2} M_x, \\
\frac{dM_y}{dt} &= yB_z M_x - yB_x M_y - \frac{1}{T_2} M_y, \\
\frac{dM_z}{dt} &= yB_x M_y - yB_y M_x - \frac{1}{T_1} (M_z - M_0),
\end{align*}
\]

(3)

where \( M_x, M_y, \) and \( M_z \) denote the projection of \( \vec{M} \) along \( X, Y, \) and \( Z \) directions, respectively. By setting \( M_z = M_x + iM_y \), the transverse terms in Eq. (3) become:

\[
\frac{dM_z}{dt} = iM_x y(B_x + iB_y) - \frac{1}{T_2} M_x - iB_y M_x,
\]

(4)

which leads to the solution, under the case of \( B_z = 0 \), as follows:

\[
M_x = \frac{iyT_2 M_z}{1 + iyB_0 T_2} (B_x + iB_y),
\]

(5)

from which the expression of \( M_x \) can be obtained by having the real part of Eq. (5) as

\[
M_x = \frac{yT_2 M_z}{1 + (yB_0 T_2)} (yB_0 T_2 B_x - B_y),
\]

(6)

while the solution of Eq. (4) under the case of \( B_z \neq 0 \) is given by utilizing the constant variation method\(^{22}\) and the Jacobi-Anger expansion\(^{23}\) as:

\[
M_x = iyT_2 M_z \sum_{n=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \left\{ J_n(C) J_{n-p}(C) \right\} \frac{\exp(\beta t)}{1 + i(yB_0 + nw_1) T_2^2},
\]

(7)

where \( J_n(C) \) denotes the nth order First-type Bessel function, \( C = yB_0/w_1 \). Note that the resonance occurs when \( yB_0 + nw_1 = 0 \). Thus, taking the characteristic of Bessel function into consideration, \( w_1 = yB_0 \) is preferred to maximize the observed signal, which leads to \( n = -1 \) for the occurred resonance. To obtain the desired signal under the case of \( B_z \neq 0 \), a lock-in amplifier is required to achieve the demodulation procedure by multiplying the real part of Eq. (7) by a term of \( \cos(\omega_1 t + \phi) \) and the transmittance of a low-pass filter (LPF), one gets:

\[
FX = -\frac{\alpha}{2} yT_2 M_z J_1(C) \left\{ -B_z J_2(C) - J_0(C) \right\} \sin \phi \\
+ B_1 J_2(C) \cos \phi,
\]

(8)

where \( FX = \alpha \text{LPF} [M_{n+1} \cos(\omega_1 t + \phi)] \) denotes the obtained optical signal after the demodulation procedure, \( \alpha \) denotes the proportional factor between the optical signal and the transverse magnetization vector, and \( \phi \) denotes the initial phase of the demodulation cosine function.

Note that Eq. (6) and Eq. (8) are approximated solutions of Eq. (4) which are only true under the case of \( \omega_1 \ll yB_0 \), where the magnetometer operates at its extreme off-resonance condition. However, the exact solution of Eq. (4) under the case of \( B_z = 0 \) could be obtained by transforming \( B_x \) in Eq. (2) as \( B_x = B_1 (\sin\omega_1 t + e^{-i\omega_1 t})/2 \), one gets:

\[
M_x' = \frac{iyT_2 M_z B_1}{2} \left\{ \frac{\sin(\omega_1 t)}{1 + i(yB_0 + \omega_1) T_2} + \frac{\exp(-i\omega_1 t)}{1 + i(yB_0 - \omega_1) T_2} \right\}
\]

(9)

which leads to the expression of \( M_x' \) as:

\[
M_x' = \frac{yT_2 M_z B_1 \sqrt{(1 - D^2 + E^2)^2 + 4D^2}}{(1 + D^2 - E^2)^2 + 4D^2} \sin(\omega_1 t + \beta_1),
\]

(10)

where \( \beta_1 = \arccos[2D/(1 - D^2 + E^2)] \) denotes the initial phase of the detected signal, \( D = \omega_1 T_2, E = yB_0 T_2 \). The exact solution of Eq. (4) under the case of \( B_z \neq 0 \) is similarly given as:

\[
M_x' = \frac{iyT_2 M_z B_1}{2} \sum_{n=-\infty}^{+\infty} \sum_{p=-\infty}^{+\infty} \left\{ J_n(C) J_{n-p}(C) \left[ \frac{\exp(\omega_1 t)}{1 + i(yB_0 + nw_1 + \omega_1) T_2^2} \right] \
+ \frac{\exp(-\omega_1 t)}{1 + i(yB_0 + nw_1 - \omega_1) T_2^2} \right\},
\]

(11)

which leads to the obtained optical signal after the demodulation procedure as,

\[
FX' = \frac{\alpha yT_2 M_z B_1 J_1(C)}{2 \sqrt{1 + (\omega_1 T_2)^2}} [J_2(C) - J_0(C)] \sin(\omega_1 t + \beta_2) \sin \phi,
\]

(12)
where $\beta_2 = \arccot(a_1T_2)$ denotes the initial phase of the detected signal under this case.

III. EXPERIMENTAL SETUP

The schematic experimental set-up of the built-in VAM as well as the NMR oscillator is shown in Fig. 1, where a vapor cell filled with condensed Rubidium atoms ($^{87}\text{Rb}$, which has $\gamma = 2\pi \times 6998\text{rad/(s}\mu\text{T})$), and buffer gases consisting of 300 Torr $N_2$ as quencher, was placed in the center. Two distributed feedback lasers (UniQuanta DFB801-D) were orthogonally placed that emitted linearly-polarized laser beams. One of the lasers was utilized to emit on-resonance pump beam on the D1 line of the $^{87}\text{Rb}$ atom at 794.98 nm, with a circularly polarization by a quarter-wave plate, meanwhile the other was utilized to emit off-resonance probe beam, with the polarization being rotated by a half-wave plate. Passing through the vapor cell, the probe beam was by a polarization beam splitter (PBS) separated into two mutually perpendicular linearly polarized beam. The Faraday rotation angle of the probe beam was obtained by two photodiodes (PDs) (Hamamatsu, S6775) forming the configuration of balanced photodetector (BPD) (Thorlabs, PDB210A/M).

Three mutually perpendicular placed magnetic coils were used around the vapor cell in producing magnetic fields, where the longitudinal field ($B_1$ and $B_2$) and transverse field ($B_0$) were produced through a custom-made circuit board connected to the Z-axis and X-axis magnetic coils, respectively. A high performance five-layer nested magnetic shield, originally made of high permeability material (Mumetal), was used outside the magnetic coils in mitigating the impact of the stray magnetic field and its fluctuations, especially the contribution of the earth magnetic field, benefiting from which the detection of the given magnetic field becomes more accurate.

Moreover, a Kapton heating film (Minco HK5565) was applied as the heater to make the atoms vaporized in the vapor cell. A thermistor was used to measure and stabilize the temperature at $100^\circ\text{C}$ around the vapor cell through a feedback program. Finally, a lock-in amplifier (Zurich Instruments HF2LI 1070) was applied as a LPF and the demodulation scheme with the internal synchronized reference signal.

IV. RESULTS AND DISCUSSION

A. EPR amplitude-frequency characteristic

The EPR amplitude-frequency characteristic of the VAM could be analyzed from Eq. (10) under the case of $B_1 = 0$ and Eq. (12) under the case of $B_1 \neq 0$, where the numerical simulations and corresponding experiments are made with different $B_0$ and $T_2$, as depicted in Fig. 2 and Fig. 3, respectively. The EPR amplitudes are all normalized in Fig. 2 and Fig. 3 by dividing the value of EPR amplitude under the case of $f_1 = 0\text{Hz}$ where $f_1 = \omega_1/2\pi$.

Fig. 2 shows that the normalized EPR amplitude decreases with increasing frequency under most cases, where the impact of $B_0$ is illustrated in Fig. 2(a) that the tendency of decrease becomes smaller with increasing $B_0$ and even becomes slight increasing tendency with relatively large $B_0$, while the impact of $T_2$ is much smaller with inverse effect for the curves which keep almost constant through the simulated frequency range as depicted in Fig. 2(b), where the curves overlap when $T_2 \geq 163\mu\text{s}$. This phenomenon could be explained as $D \ll E$ is true under the case of $f_1 \leq 1200\text{Hz}$ and $B_0 = 10\mu\text{T}$, which makes the normalized amplitude of Eq. (10) approximated as $1/[1 + (1/E^2)] \approx 1$ with $T_2 \geq 10\mu\text{s}$ as in Fig. 2(b); on the other hand, $D \ll E$ is not satisfied anymore under the case of $T_2 = 163\mu\text{s}$ and $B_0 \leq 0.1\mu\text{T}$, which makes the effect of $1/\omega_1 I$ in the normalized amplitude of Eq. (10) obvious, thus to have the tendency of decrease as in Fig. 2(a).

On the other hand, similar tendencies under the case of $B_1 \neq 0$ in two different $T_2$ can be seen in Fig. 3, while no comparison of $B_0$ is made as it is not included in Eq. (12). The experimental results in Fig. 3 were obtained with same experimental conditions except for two different vapor cells with distinguishable $T_2$. The simulation results agree well with experimental results in various cases. Fig. 3 shows that the decrease tendency with increasing frequency becomes

![Fig. 1. Experimental set-up. Blue line: pump beam; Magenta line: probe beam; PBS: polarization beam splitter; PD: photodiode; BPD: balanced photo-detector; DAQ: data acquisition board; PC: personal computer.](image-url)
even bigger with increasing $T_2$, which is much faster than that in Fig. 2 with same $T_2$.

Thus, for the application in high frequency range, it is better and more stable to work under the case of $B_c = 0$ with large $B_0$, or under the case of $B_c \neq 0$ with small $T_2$.

**B. EPR phase-frequency characteristic**

The definition of $\beta_1$ and $\beta_2$ in Eq. (10) and Eq. (12) lead to possible analysis of the EPR initial phase-frequency characteristic under the case of $B_c = 0$ and $B_c \neq 0$, where the numerical simulations are made with different $B_0$ and $T_2$, as depicted in Fig. 4 and Fig. 5, respectively.

Fig. 4 shows that $\beta_1$ decreases with increasing frequency under most cases, where the impact of $B_0$ is illustrated in Fig. 4(a) that the tendency of decrease becomes smaller with increasing $B_0$ and negligible with relatively large $B_0$, while the impact of $T_2$ is much smaller with same effect according to the inserted zooming as depicted in Fig. 4(b).
C. Signal amplification characteristic

To improve the signal to noise ratio (SNR) of the VAM, it is crucial to enhance the signal amplitude. Although a better EPR amplitude-frequency performance is proved under the case with the applied carrier field, the corresponding demodulation procedure should be applied. The analysis on signal amplification could be made by defining the absolute EPR amplitude-frequency response of the VAM under the case of large frequency becomes bigger with increasing frequency. To validate the signal enhancement with the applied carrier field, the corresponding experimental validation under the case of $B_0 = 5.1\mu T$ and $T_2 = 363\mu s$ is achieved as shown in Fig. 6(c). Due to undesired value of $J_1 - J_0$ constant, the experimental results give only twice the signal enhancement with the applied carrier field, which should be around 12 under the optimized case.

On the other hand, similar tendencies of $\beta_2$ in various $T_2$ can be seen in Fig. 5, while no comparison of $B_0$ is made as it is not included in $\beta_2$. Fig. 5 shows that the decrease tendency with increasing frequency becomes bigger with increasing $T_2$, which is similar as in Fig. 2.

In conclusion, both Fig. 4 and Fig. 5 show similar tendencies, and for the application of high frequency range, it is better and more stable to work with large $B_0$ and small $T_2$. The value of $K$ increases with increasing $B_0$ or $T_2$, which reaches even 65 under the case of $B_0 = 10\mu T$ and $T_2 = 1ms$, while decreases with increasing frequency. To validate the signal enhancement with the applied carrier field, the corresponding demodulation scheme under the case of large $B_0$ and $T_2$ in the application of low frequency range, where the boundary condition is determined by $T_2$. The VAM holds the potential to give one order of magnitude signal amplification with the applied carrier field and corresponding demodulation scheme under the case of large $B_0$ and $T_2$.

D. Linear range

The previous discussion is valid based on the assumption of $|B_1| \ll |B_0|$, which makes the longitudinal magnetization vector constant with different $B_1$, leading to a linear relationship between the EPR amplitude and $B_1$. However, with increasing $B_1$, the assumption of $|B_1| \ll |B_0|$ becomes invalid, which leads to the nonlinearity of the VAM. Thus, it is crucial to analyze the linear range of the VAM for its application, which should be based on a modified model. For simplicity, a static magnetic field ($f_1 = 0Hz$) with amplitude $B_1$ is applied along transverse direction instead of an alternating field, which pulls the polarization direction of the macroscopic magnetization vector towards the vector sum of $B_1$ and $B_0$. A new corresponding coordinate system $\alpha$ could be established as,

$$\vec{B}_x^\alpha = \frac{1}{\sqrt{B_1^2 + B_0^2}} (B_0, -B_1); \quad \vec{B}_z^\alpha = \frac{1}{B_0 B_1} (B_1, B_0),$$

where $\vec{B}_x^\alpha$ and $\vec{B}_z^\alpha$ denote the unit vector along new X and Z directions, respectively. The corresponding new expressions of the magnetic field components are as follows:

$$B_x^\alpha = \frac{B_1 B_z \cos \omega_t t}{\sqrt{B_1^2 + B_0^2}}; \quad B_z^\alpha = \sqrt{B_1^2 + B_0^2} \frac{B_0 B_z \cos \omega t}{\sqrt{B_1^2 + B_0^2}},$$

from which the equivalent amplitudes of the magnetic field are given as follows, respectively,

$$B_x^\alpha = \frac{B_1 B_z}{\sqrt{B_1^2 + B_0^2}}; \quad B_0^\alpha = \sqrt{B_1^2 + B_0^2}; \quad B_z^\alpha = \frac{B_0 B_z}{\sqrt{B_1^2 + B_0^2}}.$$
while the expression of the transverse magnetization vector along the detection direction is transformed as Eq. (18), where

$$M_x = M_0^a \frac{B_1}{\sqrt{B_1^2 + B_0^2}} + M_0^a \frac{B_0}{\sqrt{B_1^2 + B_0^2}},$$

which leads to the expression of the optical signal after the demodulation procedure by substituting the definition of

$$FX = \alpha LPF[M_x \cos(\omega_c t + \phi)]$$

into Eq. (18) as,

$$FX = \frac{\alpha y T_2 M_x B_0^a}{4(B_1^2 + B_0^2)} \sum_{n=-\infty}^{\infty} \frac{J_n(C')}{(1 + G^2 - F^2) + 4F^2} \times \left[2F((G^2 - F^2 - 1) \cos \phi + 2G \sin \phi)J_n(C')ight]$$

$$+ \left[1 + (F - G)^2\right][-\left((F + G) \cos \phi + \sin \phi\right)J_{n-2}(C')]$$

$$+ \left[1 + (F + G)^2\right][G \cos \phi - \sin \phi]J_{n-2}(C'),$$

(19)
where $F = \left(\gamma B_0^a + n\omega_c\right)T_2$, $G = \omega_cT_2$, and $C = \gamma B_0^c/\omega_c$. Thus, the linear range of the VAM could be analyzed from Eq. (19), where the numerical simulations are made with different $T_2$, with the comparison given according to Eq. (12) as the approximated result in this case, as depicted in Fig. 7. The EPR amplitudes are normalized in Fig. 7 by dividing the value of EPR amplitude under the case of $|B_1| = 1\mu T$.

Fig. 7 shows that the simulation results according to Eq. (12) and Eq. (19) agree well within certain range of $|B_1|$ in various cases of $T_2$, where the normalized EPR amplitude varies linearly with $B_1$, while nonlinearity occurs beyond this range, which becomes narrower and the slope of the curve increases with increasing $T_2$. To distinguish the linear range more clearly, the first derivate of the normalized EPR amplitude versus $B_1$ is obtained to show the slope of the curves in Fig. 7, where the corresponding simulation result according to Eq. (19) is illustrated in Fig. 8.

A clear distinguishable linear range could be extracted from Fig. 8 that the approximated linear ranges are $|B_1| \leq 0.5\mu T$ under the case of $T_2 = 100\mu s$, $|B_1| \leq 0.2\mu T$ under the case of $T_2 = 500\mu s$, and $|B_1| \leq 0.1\mu T$ under the case of $T_2 = 1ms$, respectively. Thus, a predicable linear range of the VAM could be obtained from Eq. (19) in various cases. Note that the results in Fig. 7 and Fig. 8 are given under the case of $\varphi = \pi/2$, which would alter with varied $\varphi$ and should be taken into consideration carefully in actual application.

Thus, to have a large linear range, a small $T_2$ is preferred, which is crucial for the application of large field detection.

**E. Vector characteristic**

To achieve the vector characteristic of the VAM, it is crucial to distinguish $B_x$ and $B_y$ from the obtained optical signal, which is not possible under the case of $B_c = 0$ according to Eq. (6). However, the applied carrier field and corresponding demodulation procedure enable its vector characteristic according to Eq. (8) by setting the initial phase $\varphi$ of the demodulation signal to $\pi/2$ or 0. Note that Eq. (8)
is only valid under the case of $\omega_c = yB_0$. Thus, a non-zero $\Delta \omega_z = \omega_z - yB_0$ leads to the modified expression of the optical signal as,

$$FX_m = \frac{\alpha T_z M_y J_{-1}(C)}{2(1 + \Delta \omega_z^2 T_z^2)} \left\{ \left( B_z + B_t \Delta \omega_z T_z \right) \left[ J_{-2}(C) - J_0(C) \right] \sin \varphi \\
+ \left( B_t \Delta \omega_z T_z - B_t \right) \left[ J_{-2}(C) + J_0(C) \right] \cos \varphi \right\}, \tag{20}$$

which requires to modify the value of $\varphi$ to make the coefficient before $B_t$ or $B_z$ as zero in achieving the vector characteristic of the VAM.

**V. CONCLUSION**

We analyze the characteristics of the built-in VAM in the NMR oscillator to study its EPR amplitude-frequency, EPR phase-frequency, signal amplification by the carrier field, linear range, and vector response, where the magnetometer mainly operates at its extreme off-resonance condition. Numerical simulation, being experimentally or theoretically validated, indicates that the applied carrier field with following demodulation procedure holds the potential to give one order of magnitude signal enhancement, where large $B_t$ and $T_2$ are preferred, while a good EPR amplitude-frequency and phase-frequency response with a broad linear range requires large $B_t$ and small $T_2$. The vector characteristic is only achievable by giving a proper $\varphi$ under the case of the applied carrier field, which is impossible without the carrier field. Such a demonstration makes possible the performance improvement for the built-in VAM in the NMR oscillator with optimized parameters, where a validated theoretical analysis is systematically given as the strategy.

The analysis of the linear range could be, in future works, extended to different cases of $\varphi$ for the actual application.

**ACKNOWLEDGMENTS**

This work was supported by the Natural Science Foundation of Hunan (Grant No. 2018J3608), the Research Project of National University of Defense Technology (Grant No. ZK170204), the National Natural Science Foundation of China (Grant Nos. 61671458, 61308059, 61701515), and the China Postdoctoral Science Foundation (Grant No. 2017M613367).

**REFERENCES**

1. T. Härber, D. Schmid-Lorch, F. Reinhard, and J. Wrachtrup, Nat. Nanotechnol. 10, 299 (2015).
2. T. Keijer, A. C. van Rossum, M. J. van Eenige, J. J. Bax, F. C. Visser, J. J. Teule, and C. A. Visser, J. Magn. Reson. Imag. 11, 607 (2000).
3. M. S. Grinolds, S. Hong, P. Maleinskiy, L. Luan, M. D. Lukin, R. L. Walsworth, and A. Yacoby, Nat. Phys. 9, 2543 (2013).
4. S. I. Devience, L. M. Pham, I. Lovchinsky, A. O. Sushkov, N. Bar-Gill, C. Belthangady, F. Casola, M. Corbett, H. Zhang, M. Lukin, H. Park, A. Yacoby, and R. L. Walsworth, Nat. Nanotechnol. 10, 313 (2015).
5. D. Kumar Deelchand, P.-F. Van de Moortele, A. Gregor, I. Ilits, P. Andersen, J. P. Strupp, J. T. Vaughan, K. Ugurbil, and P.-G. Henry, J. Magn. Reson. 206, 1 (2010).
6. H. Chudo, M. Ono, K. Harri, M. Matsuo, J. Ieda, R. Haruki, S. Okayasu, S. Maekawa, H. Yasuoka, and E. Saitoh, Appl. Phys. Express. 7, 063004 (2014).
7. M. Bak, J. T. Rasmussen, and N. Chr. Nielsen, J. Magn. Reson. 147, 296 (2000).
8. D. Meyer and M. Larsen, Gyroscopy. Navigat. 5, 2 (2014).
9. T. W. Kornack, R. K. Ghosh, and M. V. Romalis, Phys. Rev. Lett. 95, 230801 (2005).
10. C. Fang and J. Qin, Sensors 12, 6331 (2012).
11. J. Kitching, S. Knappe, and E. A. Donley, IEEE Sens. J. 11, 9 (2011).
12. I. M. Savukov and M. V. Romalis, Phys. Rev. Lett. 94, 123001 (2005).
13. D. A. Keder, D. W. Prescott, A. W. Conovaloff, and K. L. Sauer, AIP Advances 4, 127159 (2014).
14. W. Chalupczak, R. M. Godun, S. Pustelny, and W. Gawlik, Appl. Phys. Lett. 100, 242401 (2012).
15. H. B. Dang, A. C. Maloof, and M. V. Romalis, Appl. Phys. Lett. 97, 151110 (2010).
16. H. Storm, P. Hömmen, D. Drung, and R. Körber, Appl. Phys. Lett. 110, 072603 (2017).
17 M. Schmelz, V. Zakosarenko, A. Chwala, T. Schönau, R. Stolz, S. Anders, S. Linzen, and H. G. Meyer, IEEE Trans. Appl. Supercond. 26, 5 (2016).
18 T. G. Walker and M. S. Larsen, Adv. At. Mol. Opt. Phys. 65, 373 (2016).
19 Z. Ding, J. Yuan, G. Lu, Y. Li, and X. Long, IEEE Photonics J. 9, 1 (2017).
20 H. C. Torrey, Phys. Rev. 104, 563 (1956).
21 B. C. Grover, Phys. Rev. Lett. 40, 391 (1978).
22 C. C. Tannoudji, J. D. Roc, S. Haroche, and F. Laloé, Revue De Physique Appliquée 5, 95 (1970).
23 A. Cuyt, V. B. Petersen, B. Verdonk, H. Waadeland, and W. B. Jones, Handbook of Continued Fractions for Special Functions (Springer, Berlin, 2008) pp. 343–369.
24 E. J. Eklund, Microgyroscope based on spin-polarized nuclei, Ph.D. thesis, University of California, Irvine (2008) pp. 52-60.