Study on calibration of 3-PR(4R)R three-degree-of-freedom parallel mechanism (February 2022)

HAIBING FENG 1,2, JIAQING YAN, ZHENLIN JIN 1,2, AND ZIMING CHEN

1 Hebei Province Heavy Intelligent Manufacturing Equipment Technology Innovation Center, Yanshan University, Qinhuangdao, 066004, Hebei, China
2 Heavy-duty Intelligent Manufacturing Equipment Innovation Center of Hebei Province, Yanshan University, Qinhuangdao, 066004, Hebei, China

Corresponding author: Zhenlin Jin (e-mail: zljin@ysu.edu.cn).

This research has been supported by the National Key R&D Program of China (2018YFB1308302) and National Natural Science Foundation of China (51775474).

ABSTRACT A kinematic calibration approach is proposed for a triple-planar 3-PR(4R)R parallel mechanism whose attitude error cannot be corrected by motion. First, the kinematic principle of the 3-PR(4R)R parallel mechanism is presented. Then, an error mapping function model is designed based on the mechanism’s closed-loop vector equations. The model reveals the presence of coupling errors in the mechanism, i.e., the coupling of attitude and position errors, and is used to analyze error sensitivity as an indicator of component machining and assembly accuracy. A kinematic inverse solution is used to build a zero-point calibration and a full calibration method. Calibration is performed by a laser tracker-based calibration system, which generates much lower error values and delivers much improved accuracy. Experiments demonstrate that the proposed approach can potentially be used to calibrate parallel mechanisms with non-compensable error geometry sources.

INDEX TERMS Parallel mechanism; Error mapping; Coupling error; Calibration;

I. INTRODUCTION

Parallel mechanisms are often used for fine-tuning in practical engineering due to their high precision and high stability, and such fine-tuning mechanisms have been studied intensively by many researchers. Besides, six-degree-of-freedom (6-DOF) mechanisms are usually adopted for attitude adjustment due to their high stiffness, high precision, and high stability. An example is the automatic attitude adjustment system based on the Stewart parallel mechanism with six degrees of freedom for attitude adjustment applications. Similarly, the 3-PR(4R)R three-degree-of-freedom (3-DOF) parallel mechanism in this paper can also be applied to fine tuning or attitude adjustment.

Many studies have been conducted on the calibration of parallel mechanisms. Using a 6-DOF parallel manipulator as the research structure model, Hu Y et al.[10] analyzed the geometric error sources that affected the reachable space of the mechanism and established a positional error mode to improve the motion accuracy of six-axis hybrid machine tools. Chu W et al.[11] proposed a calibration method for a redundant actuated parallel mechanism for attitude adjustment, and a calibration method based on force closed-loop feedback was used to calibrate each branch chain of the mechanism.

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/
robot in 3-URU space adopting spiral theory. In addition to the common methods mentioned above, G. Chen[14] also presented a parallel robot error modeling method based on the Product-of -Exponentials (POF) Formula.

In this paper, we explore a 3-PR(4R)R mechanism from the perspective of kinematic calibration. First, we develop a calibration algorithm based on the mechanism’s characteristics and build an experimental calibration platform with a laser tracker. Next, we conduct theoretical and experimental analysis for zero-point calibration as well as full calibration. Finally, the accuracy of the fine-tuning mechanism is evaluated.

II. KINEMATIC PRINCIPLE AND ERROR ANALYSIS

A. KINEMATIC PRINCIPLE

A sketch of the fine-tuning parallel mechanism is presented in Fig. 1. The mechanism is composed of two platforms and three branched chains with identical structures. The three branched chains containing closed-loop structures are evenly distributed between the fixed and moving platforms. Specifically, P denotes the moving sub and R denotes the rotating sub. The three branches of the moving sub \(P_i (i=1, 2, 3)\) are evenly distributed circumferentially on the fixed platform. The upper short side \(A_{11}A_{12}\) of the parallelogram is connected to the moving sub slider by rotating sub \(R_{11}\), the lower short side \(B_{11}B_{12}\) of the parallelogram is connected to the moving platform by rotating sub \(R_{12}\), and the four sides of the parallelogram\(A_{11}A_{12}B_{11}B_{12}\) are connected by rotating sub\(R_{i13}, R_{i45}, R_{i45}\) and \(R_{i46}\). The geometric relationships are \(P_i \parallel R_{11} \parallel R_{12}, R_{13} \parallel R_{i4}, R_{i45}, R_{i45} \perp R_{i13}\).

The coordinate system of the mechanism is established, with point \(O\) taken as the origin of the fixed coordinate system, and the axis of \(R_{11}\) on the first branch chain as \(X\)-axis. The \(Z\)-axis is established perpendicularly to the fixed platform, being vertical upward, and \(Y\)-axis is determined by the right-hand rule. Point \(O'\) is set as the origin of the reference coordinate system of the moving platform and the axis of the rotating sub \(R_{12}\) on the first branch chain is taken as the \(X'\)-axis. The \(Z'\)-axis is established perpendicularly to the fixed platform, being vertical upward, and the \(Y'\)-axis is determined by the right-hand rule. \(A_1\) is the midpoint of \(A_{11}A_{12}\), and \(B_1\) is the midpoint of \(B_{11}B_{12}\).

\(l_i (i = 1, 2, 3)\) is defined as the length of \(OA_i, b\) as the length of \(O'B_i, h\) as the length of \(A_iB_i\), and the angles \(\theta_i\) between \(OA_i\) and the positive direction of the \(X\)-axis of the fixed coordinate system \(\{O\}\) as 0°, 120°, and 240°, respectively. The angle between the plane in which the parallelogram is located and the negative direction of the \(Z\)-axis is defined as \(\alpha_i (i = 1, 2, 3)\), and the angle between \(A_i B_i\) and \(A_1 A_2\) is defined as \(\beta_i (i = 1, 2, 3)\).

The coordinates of the output reference point \(O'\) of the mechanism in the fixed coordinate system \(\{O\}\) are set as \([x\ y\ z]\) and the vector closed-loop equation in the fixed coordinate system \(\{O\}\) is established as:

\[
r = l_i + h_i - b_i.
\]

The inverse solution of the position of the fine-tuning mechanism is obtained by rounding off the case where Equation (2) takes the minus sign:

\[
l_i = b + x \cos \theta_i + y \sin \theta_i \\
+ \sqrt{h^2 - z^2 - (x \sin \theta_i - y \cos \theta_i)^2}
\]

B. GEOMETRIC ERROR SOURCE ANALYSIS

Since the fine-tuning mechanism itself requires high accuracy, it is critical to establish an error model for the fine-tuning mechanism and conduct accuracy analysis to ensure the high accuracy of the system.

To establish a unified error model, the coordinate system of the mechanism is first defined as shown in Fig. 2.
The conjoined coordinate system of the moving platform is defined as \( O' - X'Y'Z' \). The origin of the coordinate system \( O' \) is located at the nominal intersection of the three branches of \( B_1B_2B_3 \) and \( B_3B_2B_1 \). The \( Y' \) axis coincides with \( B_1B_2 \), and the \( Z' \) axis is perpendicular to the plane where \( B_1B_2B_3B_2 \) and \( B_3B_1B_2 \) are located. The \( Y' \) axis is determined according to the right-hand rule.

The coordinate system of the driving branch is defined as \( L_i - X_iL_iY_iZ_i \), and the origin \( L_i \) of the driving branch’s coordinate system coincides with the origin \( O \) of the fixed coordinate system.

The upper short side of the parallelogram near the conjoined coordinate system of the fixed platform \( A_1A_2 \) is defined as \( A_i = x_{Ai}y_{Ai}z_{Ai} \). The origin of the coordinate system of the upper short side is point \( A_i \). The \( x_{Ai} \) axis coincides with \( A_1A_2 \), and the \( y_{Ai} \) axis is parallel to the rotation sub-axis of this branch parallelogram. The \( z_{Ai} \) axis is determined according to the right-hand rule.

To describe the lower short side \( B_1B_2 \) of the parallelogram near the moving platform, the transition coordinate system \( L_i' - X_i'Y_i'Z_i' \) is established.

The lower short side of the parallelogram near the conjoined coordinate system of the moving platform \( B_1B_2 \) is defined as \( B_i = x_{Bi}y_{Bi}z_{Bi} \). The origin of the lower short side coordinate system is the point \( B_i \), and the \( x_{Bi} \) axis coincides with \( B_1B_2 \), the \( y_{Bi} \) axis is parallel to the rotation sub-axis of this branch parallelogram. The \( z_{Bi} \) axis is determined according to the right-hand rule.

The geometric error sources are defined as follows. The error of system \( L_i - X_iL_iY_iZ_i \) generated by the rotation of \( O - XYZ \) around \( Z \) axis is defined as \( \Delta \theta_i \); the error of the length of short side \( A_1A_2 \) is defined as \( \Delta c_i \); the error in the driving branch for the rod length of the vector from point \( O_{Li} \) to point \( A_i \) is defined as \( \Delta l_i \); the attitude error of system \( A_i = x_{Ai}y_{Ai}z_{Ai} \) generated by the rotation of the driving branch’s coordinate system \( L_i - X_iL_iY_iZ_i \) around the \( X_i \) axis is defined as \( \Delta \alpha_i \); the error of the length of the long edge \( A_iB_i \) is defined as \( \Delta b_i \); the error of the unit direction vector of the long edge \( A_iB_i \) is defined as \( \Delta \gamma_i \); the vector error of the point \( O \) in system \( O - XYZ \) is defined as \( \Delta r \); the attitude error of vector system \( O' - X'Y'Z' \) with respect to the system \( O - XYZ \) is defined as \( \Delta \theta \); the error of system \( L'_i - X'_iY'_iZ'_i \) generated by the rotation of the dynamic system \( O' - X'Y'Z' \) around the \( Z' \) axis is defined as \( \Delta \theta_i \); length error of the lower short side \( B_1B_2 \) is defined as \( \Delta l_i \); error vector from point \( L_i \) to point \( B_i \) in system \( L_i' - X'_iY'_iZ'_i \) is defined as \( \Delta b_i \); system \( B_i = x_{Bi}y_{Bi}z_{Bi} \) is rotated by the transition coordinate system \( L_i' - X'_iY'_iZ'_i \) around axis \( X'_i \), and attitude state error is defined as \( \Delta \theta_{Bi} \).

\[ r = O_iR(l_i + c_{ij}) + h_{ij} - O_iR(b_i + d_{ij}) \] (4)

where \( c_{ij} \) is the representation of the vector from point \( A_i \) to point \( A_{ij} \) in the coordinate system \( L_i - X_iL_iY_iZ_i \) of the driving branch, \( h_{ij} \) is the representation of the vector from point \( A_{ij} \) to point \( B_{ij} \) in the fixed coordinate system \( O - XYZ \), and \( d_{ij} \) is the representation of the vector from point \( B_{ij} \) to point \( B_{ij} \) in the transition coordinate system.

After the error is introduced, Equation (4) is subtracted under the small regression condition and the higher-order minimum is omitted, which yields:

\[ \Delta r = O_iR \Delta e_{i1} + sgn(j)O_iR(l_iR \Delta e_{i2}/2e_1 + \Delta e_{sLi} \Delta e_{i1} R/c/2e_1) + \Delta h_{ij}w_i + h \Delta w_i + \Delta e_{sAi} O_iR(l_1 + sgn(j)l_1R/c/2e_1) - \Delta \theta_{Bi} O_iR(b_i + sgn(j)l_1R/c/2e_1) - \Delta \theta_{Bi} O_iR(b_i + sgn(j)l_1R/c/2e_1) \] (5)

where \( l_1R = \text{Rot}(z, \theta_1) = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \), \( l_iR = \text{Rot}(x, \alpha_i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_i & -\sin \alpha_i \\ 0 & \sin \alpha_i & \cos \alpha_i \end{bmatrix} \), \( sgn(j) = \begin{cases} -1, & j = 1 \\ 1, & j = 2 \end{cases} \), \( e_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T \).

By multiplying both sides of the equal sign of Equation (5) by \( w_i \) and using the properties of the vector mixing product, we can get

\[ w_1^T \Delta r = w_1^T O_iR \Delta e_{i1} + sgn(j)w_1^T O_iR l_1R \Delta e_{i2}/2e_1 + sgn(j) l_iR/c/2e_1 \times w_i^T O_iR \Delta e_{sLi} + \Delta h_{ij} + \{ O_iR(l_1 + sgn(j)l_1R/c/2e_1) \times w_i \}^T \Delta \theta_{Ai} - \{ O_iR(b_i + sgn(j)l_1R/c/2e_1) \times w_i \}^T \Delta \theta_{Bi} - \{ O_iR(b_i + sgn(j)l_1R/c/2e_1) \times w_i \}^T \Delta \theta_{Bi} \] (6)

As it can be seen from Equation (6), the position error and attitude error of the moving platform have a certain coupling relationship. With the fine-tuning mechanism only having three degrees of freedom for three-dimensional movement, the resulting attitude error cannot be compensated by the motion.

By subtracting the closed-loop equations representing the error through different parallelogram long sides in the same branch, i.e., Equation (6) for \( j = 2 \) from Equation (6) for \( j = 1 \), and then making it \( l_1R = \begin{bmatrix} o_{Li} \\ n_{Li} \\ e_3 \end{bmatrix} \) simplifies to:

\[ c(o_{Li} \times w_i)^T \Delta \theta = c e_{21} w_i^T o_{Li} + c(e_1 \times w_i)^T O_iR \Delta e_{3i} + \Delta h_{ij} - \Delta h_{i1} + c(o_{Li} \times w_i)^T \Delta \theta_{Ai} - c(o_{Li} \times w_i)^T \Delta \theta_{Bi} \] (7)
By writing Equation (7) in a matrix form, we can obtain the attitude error matrix function of the output of the dynamic platform of the fine-tuning mechanism:

$$\Delta \theta = J_{\theta e} \Delta \varepsilon$$

(8)

where $$J_{\theta e} = J_{\theta e}^{-1} J_{\theta B}$$, $$J_{\theta B} = \begin{bmatrix} c(o_{1,2} x w_1 + o_{2,2} x w_2 + o_{3,2} x w_3) \end{bmatrix}^T$$, $$J_{\theta B} = \text{diag} [J_{\theta B1}]$$, $$J_{\theta B1} = \begin{bmatrix} \Delta \varepsilon \end{bmatrix}$$, $$\Delta \varepsilon = \begin{bmatrix} \Delta \varepsilon_1^T \; \Delta \varepsilon_2^T \; \Delta \varepsilon_3^T \end{bmatrix}^T$$.

From Equation (8), 45 error sources can be observed, but not all of them have an effect on the attitude error in the output of the dynamic platform of the fine-tuning mechanism.

By adding the closed-loop equations representing the error through different parallelogram long sides in the same branch, i.e., Equation (6) for $$j = 1$$ and Equation (6) for $$j = 2$$, we can get:

$$w_i^T \Delta r = w_i^T \Delta \varepsilon_1 + \Delta h_{i1}/2 + \Delta h_{i2}/2 + \begin{bmatrix} (O_{Li} Rl_i x w_i) \end{bmatrix}^T \Delta \theta_{Ai} - \begin{bmatrix} (O_{Li} Rb_i x w_i) \end{bmatrix}^T \Delta \theta_{Bi} - c(o_{Li} x w_i)^T \Delta \theta_{Bi}$$

(9)

Equation (9) is written in a matrix form to obtain the attitude error matrix function of the output of the dynamic platform of the fine-tuning mechanism:

$$\Delta r = J_{r C}^{-1} J_{r D} \Delta \varepsilon + J_{r C}^{-1} J_{r D} J_{\theta e} \Delta \varepsilon = J_{r C}^{-1} J_{r D} \Delta \varepsilon$$

(10)

where $$J_{r C} = J_{r C}^{-1} J_{r D} + J_{r C}^{-1} J_{r D} J_{\theta e} J_{r C} = \begin{bmatrix} w_1 \; w_2 \; w_3 \end{bmatrix}^T$$, $$J_{r D1} = \begin{bmatrix} w_i^T O_{Li} Rl_i x w_i \end{bmatrix}^T$$, $$J_{r D2} = \begin{bmatrix} O_{Li} Rb_i x w_i \end{bmatrix}^T$$, $$J_{r D3} = \begin{bmatrix} O_{Li} Rb_i x w_i \end{bmatrix}^T$$.

From Equation (10), it can be observed that not all of the mechanism errors have a direct effect on the position error of the moving platform in the fine-tuning mechanism, except for the attitude error $$\Delta \theta$$ of the moving platform. $$J_{r D1}$$ in the columns corresponding to error sum $$\Delta \varepsilon_2$$ and $$\Delta \varepsilon_3$$, is 0, which means that there is no direct effect on the position error. However, from Equation (8), it can be observed that the errors $$\Delta \varepsilon_2$$ and $$\Delta \varepsilon_3$$ have an effect on the attitude error $$\Delta \theta$$ of the moving platform, which indirectly affects the position error of the moving platform. This indicates that the coupling of attitude error and position error of the moving platform causes all error sources to have an effect on the position error of the moving platform.

D. ERROR SENSITIVITY ANALYSIS

The mean and standard deviation of the sensitivity of the attitude error source over the entire operating space are used as a comprehensive criterion for evaluating error sensitivity:

$$\begin{cases} \mu_{\lambda ri} = \frac{\int_{V_r} \lambda ri dV_r}{V_r} \\ \sigma_{\lambda ri} = \sqrt{\frac{\int_{V_r} (\lambda ri - \mu_{\lambda ri})^2 dV_r}{V_r}} \end{cases}$$

(11)

where $$\mu_{\lambda ri}$$ is the mean value of the local sensitivity $$\lambda ri$$ of the position error over the entire operating space, and $$\sigma_{\lambda ri}$$ is the standard deviation of the local sensitivity $$\lambda ri$$ of the position error over the entire operating space.

We use the mean and standard deviation of local sensitivity over the entire operating space as a comprehensive criterion for evaluating error sensitivity:

$$\begin{cases} \mu_{\lambda ri} = \frac{\int_{V_r} \lambda ri dV_r}{V_r} \\ \sigma_{\lambda ri} = \sqrt{\frac{\int_{V_r} (\lambda ri - \mu_{\lambda ri})^2 dV_r}{V_r}} \end{cases}$$

(12)

where $$\mu_{\lambda ri}$$ is the mean value of the local sensitivity $$\lambda ri$$ of the position error over the entire operating space, and $$\sigma_{\lambda ri}$$ is the standard deviation of the local sensitivity $$\lambda ri$$ of the position error over the entire operating space.

First, by traversing all positions in this operating space, local sensitivities $$\lambda_{di}$$ and $$\lambda_{ri}$$ are calculated. Then, using Equations (11) and (12), global error sensitivity analysis is performed on each error source of the fine-tuning mechanism. Finally, the nominal values of each mechanism parameter are calculated based on the values given in Table 1. The results are represented by the error bar graph shown in Fig. 3.

| Table 1. Initial values temporarily given for each parameter of the fine-tuning mechanism |
|---------------------------------|-----|
| Mechanism parameters          | Parameter value |
| b / mm                         | 180 |
| h / mm                         | 260 |
| l_i / mm                       | [280, 430] |

As analyzed with Fig. 3, it can be known that the sensitivities of the attitude error and position error at the end of the fine-tuning mechanism about the distance error source in $$\Delta \varepsilon_1$$ are relatively small, and the sensitivities of both $$\Delta h_1$$ and $$\Delta h_2$$ are relatively large. The position error about the angle error source of the fine-tuning mechanism is more sensitive to $$\Delta \theta_{Ai}$$ than it is to $$\Delta \theta_{Bi}$$. If the standard deviation of the attitude error $$\Delta \theta$$ caused by the error source $$\Delta h_1$$ is expected to be kept no greater than 20 $$\mu$$m, the tolerance of $$h_1$$ must be better than $$T(h_1) = \pm 3 \times 20/9.66 = \pm 6.21$$ $$\mu$$m; if the standard deviation of the position error $$\Delta \varepsilon$$ caused by the error source $$\Delta h_1$$ is expected to be kept no greater than 20 $$\mu$$m, the tolerance of parameter $$h_1$$ must reach $$T(h_1) = \pm 3 \times 20/0.165 = \pm 36$$ $$\mu$$m; if the standard deviation of the position error $$\Delta \varepsilon$$ caused by the desired error source $$\Delta \theta_{Ai}$$ is expected to be kept no greater than 20 $$\mu$$m, the tolerance of this parameter can be relaxed to $$T(h_1) = \pm 3 \times 20/0.2128 = \pm 282$$ $$\mu$$m.

III. CALIBRATION PRINCIPLES AND EXPERIMENTAL STUDIES

A. ANALYSIS OF CALIBRATION PRINCIPLE

The principle of kinematic calibration is to identify the error between the theoretical value of each parameter in the kinematic model and the actual value of each parameter in the real mechanism, and then compensate the error to the
control system, so as to improve the motion accuracy of the mechanism. Generally, the kinematic calibration process involves these steps: establishing an error model, collecting experimental data, identifying the mechanism parameters, compensating the control parameters, and verifying the identification performance. In this part of the study, the error model is established based on the kinematic inverse solution.

### B. ZERO CALIBRATION PRINCIPLE AND ALGORITHM

The derived inverse solution of Equation (3) in a general form as:

\[ 1 = f \left( O', \psi \right) \]  

(13)

where \( l \) is the input parameter, \( O' \) is the end output positional data, \( \psi \) is the mechanism parameters included in the inverse solution, and \( f \left( O', \psi \right) \) is the kinematic inverse solution.

Using the inverse solution of Equation (8), the error between the theoretical value and the true value for the \( j \)th position data can be obtained as:

\[ \Delta l_j = l_j - f \left( O'_{m_j}, \psi_r \right) \]  

(14)

where \( \Delta l_j \) is the error between the theoretical value and the true value of the \( j \)th position, \( O'_{m_j} \) is the measured position data at the \( j \)th position, \( \psi_r \) is the true value of the mechanism parameters included in the inverse solution, and \( l_j \) is the actual input parameters at the \( j \)th position.

The following formula is used to calculate the actual input parameter \( l_j \) of the fine-tuning mechanism at the \( j \)th position:

\[ l_j = f \left( O'_{t_j}, \psi_t \right) \]  

(15)

where \( O'_{t_j} \) is the theoretical positional data output from the end of the mechanism at the \( j \)th position, and \( \psi_t \) is the theoretical value of the mechanism parameters included in the kinematic inverse solution of the fine-tuning mechanism.

The kinematic calibration process of the fine-tuning mechanism is to make the error infinitely close to zero based on the following branch construction equation:

\[ \Delta l_{ij} = L_{ij} \left( l_{i0}, \psi_r \right) = l_{i0} + l_{ij} - f \left( O'_{m_j}, \psi_r \right) = 0 \]  

(16)

where \( \Delta l_{ij} \) is the error of the moving sub position of the \( i \)th branch chain at the \( j \)th position, \( L_{ij} \) is the error function of the \( i \)th branch chain at the \( j \)th position, \( l_{i0} \) is the initial value of the moving sub position of the \( i \)th branch chain, and \( l_{ij} \) is the changed value of the moving sub position of the \( i \)th branch chain at the \( j \)th position.

From Equation (16), it can be seen that the independent variables of the error function include two parts: the initial value of the position of the moving sub of the fine-tuning mechanism \( l_{i0} \), and the mechanism parameters of the fine-tuning mechanism \( \psi_r \).

If only zero calibration is performed on the mechanism, it can be seen from Equation (16) that the independent variables of the three error functions \( l_{ij} \) \((i = 1,2,3)\) corresponding to the \( j \)th position include three unknowns \( l_{i0} \) \((i = 1,2,3)\), and theoretically, only one change is needed to fine-tune the position of the mechanism. To ensure the error of the actual measurement data does not influence the zero calibration process, it is necessary to increase the number of measurements. The zero calibration process after multiple measurements are...
considered will make the initial value of the mechanism input closer to the true value, thus improving the motion accuracy of the mechanism.

Assuming that the positional data of \( n \) fine-tuning mechanisms are measured, \( 3n \) independent equations can be obtained as:

\[
\begin{align*}
& \quad L_{11} (l_{10}) = 0; \quad L_{21} (l_{20}) = 0; \quad L_{31} (l_{30}) = 0 \\
& L_{12} (l_{10}) = 0; \quad L_{22} (l_{20}) = 0; \quad L_{32} (l_{30}) = 0 \\
& \quad \ldots \quad \ldots \quad \ldots \\
& L_{1n} (l_{10}) = 0; \quad L_{2n} (l_{20}) = 0; \quad L_{3n} (l_{30}) = 0
\end{align*}
\]  

(17)

Therefore, the zero calibration problem of the fine-tuning mechanism is transformed into the problem of solving the optimal solution of a nonlinear system of equations including Equation (17). Specifically, the number of equations is much greater than the number of unknowns, and the nonlinear least-squares method is usually used to find the optimal solution of the system of equations.

C. FULL CALIBRATION PRINCIPLE AND ALGORITHM

If the mechanism is to be fully calibrated, it is known from Equation (16) that the three error functions \( l_{ij} (i=1,2,3) \) corresponding to the \( j \)th position contain nine independent variables. Therefore, theoretically only three changes are needed to fine-tune the mechanism’s position. To ensure the error of the actual measurement data does not influence the full calibration process, it is necessary to increase the number of measurements. The full calibration process after multiple measurements is considered will make the initial value of the mechanism input closer to the true value, thus improving the motion accuracy of the mechanism.

Assuming that the positional data of \( n \) fine-tuning mechanisms are measured, \( 3n \) independent equations can be obtained as:

\[
\begin{align*}
& \quad L_{11} (l_{10} h_{1} \theta_{1}) = 0; \quad L_{21} (l_{20} h_{2} \theta_{2}) = 0; \quad L_{31} (l_{30} h_{3} \theta_{3}) = 0 \\
& L_{12} (l_{10} h_{1} \theta_{1}) = 0; \quad L_{22} (l_{20} h_{2} \theta_{2}) = 0; \quad L_{32} (l_{30} h_{3} \theta_{3}) = 0 \\
& \quad \ldots \quad \ldots \quad \ldots \\
& L_{1n} (l_{10} h_{1} \theta_{1}) = 0; \quad L_{2n} (l_{20} h_{2} \theta_{2}) = 0; \quad L_{3n} (l_{30} h_{3} \theta_{3}) = 0
\end{align*}
\]  

(18)

Similarly, the full calibration problem of the fine-tuning mechanism is transformed into the problem of solving the optimal solution of a nonlinear system of equations including Equation (18). Specifically, the number of equations is much greater than the number of unknowns, and the nonlinear least-squares method is usually used to find the optimal solution of the system of equations.

D. CALIBRATION SYSTEM CONSTRUCTION AND CALIBRATION EXPERIMENTS

The built calibration experiment system is shown in Figure 4.

In the experimental calibration system, the coordinate system of the laser tracker is set as the measurement coordinate system \( \{M\} \), with the fixed coordinate system \( \{O\} \) of the fine-tuning mechanism fixed to the fixed platform and the motion coordinate system \( \{O'\} \) fixed to the moving platform. In the measurement process, it is difficult to measure the kinematic system of the fine-tuning mechanism directly. Therefore, a fixed point \( P \) is introduced on the moving platform as an auxiliary measurement point, through which the position of the kinematic system is measured. The position relationship between the three coordinate systems is shown in Fig. 5. Through calibration experiments, the position of the origin of the moving platform in the fixed system \( \{O\} \) is calculated, and according to the position relationship shown in Fig. 5, we can get:

\[
r = OP + PO'
\]  

(19)

where \( r \) is the representation of vector \( OO' \) in system \( \{O\} \), \( OP \) is the representation of the vector of point \( P \) in the system \( \{O\} \), and \( PO' \) is the representation of the vector of point \( P \) to the origin \( \{O'\} \) of the dynamic system of the fine-tuning mechanism in system \( \{O\} \).

Since the data measured in the measurement system are represented in the laser tracker’s coordinate system \( \{M\} \), the data in Equation (19) are transformed into the laser tracker’s coordinate system \( \{M\} \), which is obtained from the coordinate transformation formula:

\[
OP = O_M R (M P - M O), \quad PO' = O_M R M PO'
\]  

(20)

where \( O_M R \) is the rotation matrix from system \( \{M\} \) to system \( \{O\} \), \( M P \) is the vector representation of point \( P \) in system \( \{M\} \), \( M O \) is the vector representation of the fixed system’s origin \( O \) in system \( \{M\} \), and \( M PO' \) is the representation of the vector \( PO' \) in system \( \{M\} \).
corresponding command input value of the mechanism are shown in Table 3.

The actual value obtained from this measurement and the corresponding command input value of the mechanism are substituted into Equation (17) to obtain the actual value obtained as follows by fitting the calculation:

$$ r = O^M R (M \mathbf{P} - M O) + O^M R^M \mathbf{P}^O $$  \hspace{1cm} (21)

As analyzed above, it is necessary to measure the fixed coordinate system \{O\}, including the position of its origin \(O\) in system \{M\}, the position of point \(P\) in system \{M\}, and the position of the origin \(O'\) of the fine-tuning mechanism’s motion coordinate system in system \{M\}. The position of each of the measurement points is shown in Fig. 6.

Table 2 shows the three sets of data for the auxiliary measurement point \(P\).

The coordinate system data of the fixed platform of the fine-tuning mechanism and the value of vector \(M \mathbf{P}^O\) can be obtained as follows by fitting the calculation:

$$ M^O R = \begin{bmatrix} 0.530648 & -0.847574 & -0.005559 \\ 0.847556 & 0.530674 & -0.005868 \\ 0.007923 & 0.001598 & 0.999967 \end{bmatrix}, $$

$$ M^O = \begin{bmatrix} 1884.021187 \\ 30.760965 \\ 115.522375 \end{bmatrix}, $$

$$ M \mathbf{P}^O = \begin{bmatrix} 47.583753 \\ 167.196757 \\ 27.232326 \end{bmatrix} $$ \hspace{1cm} (22) \hspace{1cm} (23)

E. ZERO CALIBRATION RESULTS

With the data for auxiliary measurement point \(P\) in Table 2, Equation (22) and Equation (23) are substituted into Equation (21) to obtain the actual value \(r\) output from the fine-tuning mechanism. Then, the actual value obtained from this measurement and the corresponding command input value of the mechanism are substituted into Equation (17) to obtain the zero calibration results of the fine-tuning mechanism, as shown in Table 3.

F. FULL CALIBRATION RESULTS

The actual value \(r\) obtained from the measurement and the corresponding command input value of the mechanism are substituted into Equation (18) to obtain the full calibration results of the fine-tuning mechanism, as shown in Table 4.

IV. ACCURACY EVALUATION

A. POSITION ACCURACY AND POSITION REPEATABILITY EVALUATION

The position accuracy and position repeatability of the fine-tuning mechanism before and after calibration are shown in Table 5 and Table 6. In this paper, the largest position accuracy and position repeatability values obtained after calibration are taken as the final position accuracy and position repeatability of the fine-tuning mechanism, i.e., \(0.59132\) mm for position accuracy and \(0.008164\) mm for position repeatability.

B. POSITION DISTANCE ACCURACY EVALUATION AND POSITION DISTANCE REPEATABILITY EVALUATION

The position distance accuracy and position distance repeatability results of the fine-tuning mechanism before and after calibration are shown in Table 7 and Table 8. In this paper, the largest position distance accuracy and position distance repeatability values obtained after calibration are taken as the final position distance accuracy and position distance repeatability of the fine-tuning mechanism, i.e., \(0.11983\) mm for position distance accuracy and \(\pm 0.14463\) mm for position distance repeatability.

---

TABLE 2. Data of auxiliary measuring point \(P\)

| Group  | \(x\)     | \(y\)     | \(z\)     |
|--------|-----------|-----------|-----------|
| Group 1| 1838.081154 | -136.146847 | -21.375013 |
| Group 1| 1836.348089 | -138.945944 | -24.888753 |
|        | ...       | ...       | ...       |
| Group 2| 1838.082599 | -136.145954 | -21.377125 |
| Group 2| 1841.444856 | -136.206068 | -24.859075 |
|        | ...       | ...       | ...       |
| Group 3| 1838.080897 | -136.148985 | -21.372618 |
| Group 3| 1836.503684 | -133.184228 | -24.846956 |
|        | ...       | ...       | ...       |

Substituting Equation (20) into Equation (19) yields:

$$ r = O^M R (M \mathbf{P} - M O) + O^M R^M \mathbf{P}^O $$

Fig. 7 compares the errors of the moving sub positions before and after calibration in an error bar graph. Specifically, the marked points represent the mean error values and the ranges the standard deviations of the errors. From Fig. 7, it can be seen that the errors of the three branches become closer to the mean error value of 0 after zero calibration or full calibration. Branch 2 exhibits the most obvious change in the mean error value, dropping from \(1.320\,740\) mm before calibration to \(1.263\,188\times10^{-14}\) mm after zero calibration, and to \(1.263\,187\times10^{-14}\) mm after full calibration. Branch 1 exhibits a very small data variation, which indicates that the initial value of Branch 1 is closer to the real value.

IV. ACCURACY EVALUATION

A. POSITION ACCURACY AND POSITION REPEATABILITY EVALUATION

The position accuracy and position repeatability of the fine-tuning mechanism before and after calibration are shown in Table 5 and Table 6. In this paper, the largest position accuracy and position repeatability values obtained after calibration are taken as the final position accuracy and position repeatability of the fine-tuning mechanism, i.e., \(0.59132\) mm for position accuracy and \(0.008164\) mm for position repeatability.

B. POSITION DISTANCE ACCURACY EVALUATION AND POSITION DISTANCE REPEATABILITY EVALUATION

The position distance accuracy and position distance repeatability results of the fine-tuning mechanism before and after calibration are shown in Table 7 and Table 8. In this paper, the largest position distance accuracy and position distance repeatability values obtained after calibration are taken as the final position distance accuracy and position distance repeatability of the fine-tuning mechanism, i.e., \(0.11983\) mm for position distance accuracy and \(\pm 0.14463\) mm for position distance repeatability.

---

TABLE 3. Zero calibration result

| Parameters | Theoretical values | Calibrated value |
|------------|--------------------|-----------------|
| \(l_{10}/\text{mm}\) | 410 | 410.004 05 |
| \(l_{20}/\text{mm}\) | 410 | 408.679 26 |
| \(l_{30}/\text{mm}\) | 410 | 410.469 35 |

TABLE 4. Full calibration result

| Parameters | Theoretical values | Calibrated value |
|------------|--------------------|-----------------|
| \(l_{10}/\text{mm}\) | 410 | 410.004 05 |
| \(l_{20}/\text{mm}\) | 410 | 408.679 26 |
| \(l_{30}/\text{mm}\) | 410 | 410.469 35 |
| \(h_{1}/\text{mm}\)  | 260 | 260.006 487 |
| \(h_{2}/\text{mm}\)  | 260 | 260.139 361 |
| \(h_{3}/\text{mm}\)  | 260 | 261.235 727 |
| \(\theta_{1}/^\circ\) | 0 | 0.312 582 |
| \(\theta_{2}/^\circ\) | 120 | 120.394 854 |
| \(\theta_{3}/^\circ\) | 240 | 249.698 283 |
V. DISCUSSION OF THE RESULTS

As revealed by the comparative data in Table 5, Table 6, Table 7, and Table 8, the position accuracy, position repeatability, position distance accuracy, and position distance repeatability of the fine-tuning mechanism are improved significantly after calibration. This means that the calibration performance is remarkable and the fine-tuning mechanism meets the requirements for use in terms of each indicator.

VI. CONCLUSION

In this paper, we analyze the kinematic principle of a 3-PR(4R)R 3-DOF fine-tuning mechanism by the position inverse solution of the mechanism, analyze the error sources, establish an error mapping function, and then analyze the sensitivities of the error sources. The kinematic calibration principle is applied to establish a zero calibration model and a full calibration model. The measurement principle of the laser tracker is proposed to build an experimental calibration platform, and the error value is reduced significantly. After
calibration, the position accuracy, position repeatability, position
distance accuracy, and position distance repeatability of the
fine-tuning mechanism are 0.59132 mm, 0.008164 mm,
-0.11983 mm, and ±0.14463 mm. These results suggest that
the proposed approach delivers considerable calibration perfor-
manence and significantly improved accuracy. The proposed
approach offers an inspiring means of calibrating parallel
mechanisms with non-compensable error sources of geometric
effects.

### REFERENCES

[1] Wei W, Li J, Ma S, “Automatic adjustment system for space position and
attitude of complex structural parts,” in 2020 IEEE International
Conference on Information Technology,Big Data and Artificial Intelligence
(ICIBAI), 2020, vol. 1, pp. 1100-1104, Chongqing, China: IEEE, 2020.

[2] J. Yao, B. Han, Y. Dou, Y. Xu, and Y. Zhao, “Fault-tolerant strategy and
workspace of the subreflector parallel adjusting mechanism,” Proc. Inst.
Mech. Eng. C-J. Mech., vol. 233, no. 18, pp. 6656-6667, July 2019.

[3] J. Nella et al., “James Webb Space Telescope (JWST) observatory ar-
chitecture and performance,” in Optical, Infrared, and Millimeter Space
Telescopes, vol. 5487, pp. 576-587, Glasgow, United Kingdom: SPIE, 2004.

[4] L. Zago and S. Droz, “Small parallel manipulator for the active alignment
and focusing of the secondary mirror of the VLTI ATs,” in Optical design,
materials, fabrication, and maintenance, vol. 4003, pp. 450-455, Munich,
Germany: SPIE, 2000.

[5] P. Schipani, S. D’Orsi, D. Fierro, L. Marty, F. Perrotta, and C. Arcidiacono,
“Performance of the VST secondary mirror support system,” Modern
Technologies in Space-and Ground-based Telescopes and Instrumentation,
International Society for Optics and Photonics, vol. 7739, p. 773932,
San Diego, California, United States: International Society for Optics and
Photonics, 2010.

[6] L. Tian, H. Bao, M. Wang, and X. Duan, “Modeling and control of the redundant parallel adjustment mechanism on a deployable antenna panel,” Sensors, vol. 16, no. 10, p. 1632, October 2016.

[7] Z. Ma, S. G. Li, H. Xing, and H. Xiang, “Analytic forward solution for 3-
PPS parallel wing posture adjustment mechanism,” Comp. Integr. Manu.
Syst., vol. 21, no. 2, pp. 449-454, 2015.

[8] P. E. Hsu, Y. L. Hsu, J. M. Lu, and C.-H. Chang, “Seat adjustment design of an intelligent robotic wheelchair based on the Stewart platform,” Int. J.
Adv. Robot. Syst., vol. 10, no. 3, pp. 168, January 2013.

[9] J. Yao, G. Wu, and Y. Zhao, “Configuration optimization and analysis of the parallel adjustment platform for Satellite Integrated Assembly,” in 2013 11th IEEE International Conference on Industrial Informatics (INDIN), pp. 317-320, Bochum, Germany: IEEE, 2013.

[10] Y. Hu, F. Gao, X. Zhao, B. Wei, D. Zhao, and Y. Zhao, “Kinematic cali-
bration of a 6-DOF parallel manipulator based on identifiable parameters
separation (IPS),” Mech. Mach. Theory, vol. 126, pp. 61-78, August 2018.

[11] W. Chu, X. Huang, and S. Li, “A calibration method of redundant actuated parallel mechanism for posture adjustment,” Indus. Robot, vol. 48, no. 4, pp. 494-509, April 2021.

[12] T. Sun, Y. Song, Y. Li, and L. Xu, “Separation of comprehensive geomet-
rical errors of a 3-DOF parallel manipulator based on Jacobian matrix and
its sensitivity analysis with Monte-Carlo method,” Chin. J. Mech. Eng., vol.
24, no. 3, p. 406, March 2011.

[13] A. Frosioli, M. Solazzi, D. Pellegrinetti, and M. Bergamasco, “A new screw theory method for the estimation of position accuracy in spatial parallel manipulators with revolute joint clearances,” Mech. Mach. Theory, vol. 46, no. 12, pp. 1929-1949, December 2011.

[14] G. Chen, L. Kong, Q. Li, H. Wang, and Z. Lin, “Complete, minimal and continuous error models for the kinematic calibration of parallel manipulators based on POE formula,” Mech. Mach. Theory, vol. 121, pp. 844-856, March 2018.

[15] T. Huang, D. G. Chetwynd, D. J. Whitehouse, and J. Wang, “A general and novel approach for parameter identification of 6-DOF parallel kinematic machines,” Mech. Mach. Theory, vol. 40, no. 2, pp. 219-239, February 2005.

[16] L. Yang, X. Tian, Z. Li, F. Chai, and D. Dong, “Numerical simulation of

### TABLE 5. Position accuracy before and after calibration

| Measurement point | Before calibration | After calibration |
|-------------------|--------------------|------------------|
|                   | APP                | APx              | APy              | APz              | APP                | APx              | APy              | APz              |
| P1                | 1.1154             | -0.98904         | -0.3722          | 0.41367          | -0.06441          | 0.40707          | -0.03572         |
| P2                | 1.3581             | -1.1092          | -0.4827          | 0.40186          | -0.01468          | 0.38916          | -0.09915         |
| P3                | 1.2327             | -1.1749          | -0.3452          | 0.31053          | -0.05834          | 0.28625          | 0.10529          |
| P4                | 0.96163            | -0.91365         | -0.28707         | 0.42391          | -0.11974          | 0.40626          | 0.01779          |
| P5                | 1.0498             | 0.57453          | -0.3662          | 0.59132          | -0.08824          | 0.5634           | -0.15638         |

### TABLE 6. Position repeatability before and after calibration

| Measurement point | Before calibration | After calibration |
|-------------------|--------------------|------------------|
|                   | P1                 | P2               | P3               | P4               | P5               |
|                   | 0.00969            | 0.010445         | 0.008529         | 0.008663         | 0.008364         |
|                   | 0.005516           | 0.007261         | 0.005415         | 0.008612         | 0.008164         |

### TABLE 7. Position distance accuracy before and after calibration

| Item              | ADP             | ADx             | ADy             | ADz             | P5               |
|-------------------|-----------------|-----------------|-----------------|-----------------|------------------|
| Before calibration| -0.30457        | -0.17351        | -0.17485        | -0.17917        | 0.008364         |
| After calibration | -0.11983        | -0.066524       | -0.068883       | -0.072139       | 0.008164         |

### TABLE 8. Position distance repeatability before and after calibration

| Item              | RD              | RDx             | RDy             | RDz             | P5               |
|-------------------|-----------------|-----------------|-----------------|-----------------|------------------|
| Before calibration| ±0.49544        | ±0.286          | ±0.28638        | ±0.28575        | 0.008364         |
| After calibration | ±0.14463        | ±0.083573       | ±0.083859       | ±0.083122       | 0.008164         |
calibration algorithm based on inverse kinematics of the parallel mechanism," 
Optik", vol. 182, pp. 555-564, April 2019.

[17] T. Li, F. Li, Y. Jiang, J. Zhang, and H. Wang, “Kinematic calibration of a 3-P (P) S parallel-type spindle head considering the thermal error,” 
Mechatronics", vol. 43, pp. 86-98, May 2017.

[18] J.-F. Wu, R. Zhang, R.-H. Wang, and Y.-X. Yao, “A systematic optimization approach for the calibration of parallel kinematics machine tools by a laser tracker,” 
Int. J. Mach. Tool. Manu.", vol. 86, pp. 1-11, November 2014.

[19] C. Fan, G. Zhao, J. Zhao, L. Zhang, and L. Sun, “Calibration of a parallel mechanism in a serial-parallel polishing machine tool based on genetic algorithm,” 
Int J Adv Manu Technol", vol. 81, no. 1, pp. 27-37, May 2015.

[20] A. Klimchik, A. Pashkevich, D. Chablat, and G. Hovland, “Compliance error compensation technique for parallel robots composed of non-perfect serial chains,” 
Robot. Comp.-Integr. Manu.", vol. 29, no. 2, pp. 385-393, April 2013.

[21] K. Balaji and H. K. B. Shaul, “Kinematic Analysis of Novel 3RRS Parallel Mechanism,” 
Int. J. Sci. Res.", vol. 7, no. 1, pp. 111-114, 2018.

[22] T. A Hess-Coelho, G. Ee Gojtan, and G. P Furtado, “Kinematic Analysis of A 3-DOF Parallel Mechanism for Milling Applications,” 
Open Mech. Eng. J.", vol. 4, no. 1, pp. 69-76, December 2010.

[23] A. A. Selvakumar and M. A. Kumar, “Experimental investigation on position analysis of 3-DOF parallel manipulators,” 
Procédia Eng.", vol. 97, pp. 1126-1134, 2014.

[24] A. Joubair, L. F. Zhao, P. Bigras, and I. Bonev, “Absolute accuracy analysis and improvement of a hybrid 6-DOF medical robot,” 
Ind. Robot.", vol. 42, no. 1, pp. 44-53, January 2015.

[25] Y. C. Yuan and Y. Zhang, “Building of fine-tuning mechanism posture and kinematics model for shield segment erector,” in Advanced materials research. Trans Tech Publications Ltd

ZHENLIN JIN. He was born in Fushun, China, in 1962. He is a Professor at School of Mechanical Engineering, Yanshan University, China. He obtained his PhD degree in Mechanical Engineering from Yanshan University in 2001. In the same year, he joined Robotics Research Institute, Beijing University of Aeronautics and Astronautics, as a Postdoctor. He is currently engaged in research and development on novel force sensor with parallel architecture, parallel manipulator, programmable mechanical press, parallel kinematic machine and their applications.

VOLUME 4, 2016 11

This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/