Hybrid prediction method of ECG signals based on VMD, PSR, and RBF neural network

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Abstract

Background: It is significant for doctors and body area networks (BANs) to predict ECG signals accurately. At present, the prediction accuracy of many existing ECG prediction methods is generally low. In order to improve the prediction accuracy of ECG signals in BANs, a hybrid prediction method of ECG signals is proposed in this paper.

Methods: The proposed prediction method combines variational mode decomposition (VMD), phase space reconstruction (PSR), and a radial basis function (RBF) neural network. First, the embedding dimension and delay time of PSR are calculated according to the trained set of ECG data. Second, the ECG data are decomposed into several intrinsic mode functions (IMFs). Third, the phase space of each IMF is reconstructed according to the embedding dimension and the delay time. Fourth, an RBF neural network is established and each IMF is predicted by the network. Finally, the prediction results of all IMFs are added to realize the final prediction result.

Results: To evaluate the prediction performance of the proposed method, simulation experiments are carried out on ECG data from the MIT-BIH Arrhythmia Database. The experimental results show that the prediction index RMSE (root mean square error) of the proposed method is only $10^{-3}$ magnitude and that of some traditional prediction methods is $10^{-2}$ magnitude.

Conclusions: Compared with some traditional prediction methods, the proposed method improves the prediction accuracy of ECG signals obviously.

Keywords: Prediction method, ECG signal, VMD, PSR, RBF neural network

Background

It is of significance to predict ECG accurately. On the one hand, it can let doctors know the patient's condition in advance. On the other hand, it can reduce the energy of sensors in body area networks (BANs). In BANs, some sensors are placed under the skin or inside the body, and their batteries are inconvenient to replace. Therefore, it is very necessary to reduce the
energy consumption and prolong the lifetime of the sensors. Assuming that a prediction model is established in both sensor node and sink node, when the prediction error exceeds the specified threshold value, the sensor node will send the measured data, otherwise, it will not send the measured data, and the sink node will use the predicted data as the measured data. In this way, data transmission can be reduced. There are many time-series signals such as the ECG, body temperature, and blood pressure in BANs. If these time series signals can be accurately predicted from the past and current data, it will greatly reduce the amount of data transmission and the energy consumption of the sensor.

Improving prediction accuracy is the core problem of a time-series prediction algorithm. The main factors that affect the accuracy of time-series prediction are data pre-processing and the prediction model.

The common data pre-processing techniques in time-series prediction algorithms include wavelet transform (WT), empirical mode decomposition (EMD), and variational mode decomposition (VMD). WT decomposes a signal into one low frequency approximation subset and several high frequency detail subsets. Unlike WT, EMD decomposes a signal into several intrinsic mode functions (IMFs). Compared with WT, EMD has obvious advantages in dealing with non-stationary nonlinear data. The disadvantages of EMD are modal aliasing and endpoint effect. In order to overcome the disadvantages of EMD, another signal decomposition method VMD was proposed by Dragomiretskiy et al. [1] in 2014. Similar to EMD, VMD decomposes a signal into several IMFs, but the bandwidth of VMD is finite, which overcomes the problems of EMD. VMD has attracted more and more scholars' attention. A proposed signal de-noising method combining VMD and de-trended fluctuation analysis (DFA) [2] performs better than EMD-based de-noising methods and discrete wavelet threshold filtering methods. Zhang et al. [3] presented a hybrid model for wind speed prediction that combined a neural network, VMD, and Lorenz disturbance. The prediction model not only improves prediction accuracy and forecasting stability, but restores the characteristics of wind speed. A proposed hybrid model for short-term wind power forecasting consists of VMD, k-means clustering, and a LSTM (long short term memory) network [4]. The model has obvious advantages over seven methods such as BP neural network, against which it was compared. Maheshwari et al. [5] proposed a novel method for an automated diagnosis of glaucoma using digital fundus images. In the method, VMD was used in an iterative manner for image decomposition.

There are many prediction models. The RBF neural network has the advantages of fast training speed and not easy to fall into local minimum, which has been widely used in prediction field. Jiang et al. [6] proposed a prediction model for urban ecological carrying capacity that consists of an RBF neural network that is optimized by an improved artificial bee colony algorithm. The model has good prediction accuracy. Zhang et al. [7] proposed a prediction approach based on an improved RBF neural network and multi-label metamorphic relations (MLMRs). The approach improves prediction accuracy by optimizing the RBF neural network structure by affinity propagation and k-means clustering algorithms. Chen [8] proposed an optimized algorithm for traffic flow prediction using an RBF neural network and an improved artificial bee colony (ABC) algorithm in a big-data environment. The algorithm improves the prediction accuracy of measured traffic flow. A proposed short-term prediction method of electricity and gas demand in a multi-energy system based on an RBF neural network model can effectively consider the coupling relationship between power and gas loads to improve prediction accuracy [9].

Phase space reconstruction (PSR) [10] can solve the input problem of the neural network when the neural network is used for time series prediction. PSR involves the reconstruction of the phase space structure of the original system by a one-dimensional time series. At present, PSR has been applied in many fields, such as time-series prediction.
Li et al. [11] proposed a speech signal nonlinear prediction model with a hidden PSR method. Its prediction accuracy is higher than that of linear prediction coding, the RBF neural network model, and the LSTM network. In [12], Zheng et al. proposed a complex conceptor network for time series prediction based on PSR. He et al. [13] proposed a method to characterize driving behaviours using PSR and a pre-trained convolutional neural network (CNN). The method can distinguish driving styles and abnormal driving behaviours with high accuracy. A method based on PSR was proposed to predict the sintering temperature in a rotary kiln [14]. The method improves the prediction accuracy and robustness. In [15], Wu et al. proposed a new classification method to distinguish gait patterns between anterior cruciate ligament deficient and intact knees. The proposed method is based on PSR, Euclidean distance and an RBF neural network, and has a high correct classification rate.

This paper focuses on the prediction of an ECG signal. In BANs, the ECG signal has the largest amount of data among all physiological signals. The sensor will consume a lot of energy to send vast quantities of raw ECG data. In order to save energy of the sensor, it is necessary to reduce the transmission of ECG data without affecting the normal use of ECG. However, it is not easy to predict an ECG signal accurately. An ECG signal is a nonlinear non-stationary time series signal with an inherent random feature. Although an ECG signal is basically a periodic signal, its period does not keep a fixed value. Few papers in the literature have discussed the prediction of ECG signals. Wei et al. [16] developed a model for highly accurate prediction of ECGs and EEGs by combining CNN and bi-directional recurrent neural network (BRNN). In [17], Sun et al. proposed a prediction method of an ECG signal using an error backpropagation neural network (BPNN) and VMD technology. Sun et al. [18] proposed another prediction method of an ECG signal. The method was based on PSR theory and BPNN, with accuracy close to that of the previously mentioned method. To reduce the amount of data transmission, a waveform prediction lightweight algorithm was proposed [19]. In order to improve the prediction accuracy, the algorithm used WT to pre-process the noise. Liu [20] proposed a data fusion algorithm based on WT and least-squares support vector machine (LS-SVM). The algorithm used LS-SVM model to predict an ECG signal. In [21], Heurtefeux et al. investigated the tradeoff between accuracy and complexity to predict ECG values in BANs. They suggested using autoregressive moving average (ARMA) model to predict ECG value, but they couldn't build an ARMA model to predict ECG in experiment. Sun et al. [22] used an autoregressive integrated moving average (ARIMA) model to analyse ECG data streams, but they did not use ARIMA model to predict an ECG signal. An ECG signal prediction method based on ARIMA model and discrete wavelet transform (DWT) was proposed in [23]. This method obtained a good prediction effect, but it needed a lot of calculation because of using many high-order autoregressive (AR) models. In [24], an ECG signal prediction method was proposed. In the method, the PSR theory and the TS fuzzy model were used to predict an ECG signal. The accuracy of the prediction is close to that of [17] and [18].

In this paper, we propose a hybrid prediction method of ECG signals and compare it to other traditional prediction methods to highlight its advantages. The rest of this paper is organized as follows. Methods section describes in detail an ECG signal prediction method based on VMD, PSR, and an RBF neural network. Results section describes a simulation experiment and the analysis of its results. Discussion section compares the proposed prediction method with some traditional prediction methods. Conclusions section provides concluding remarks.

Results

Data
All ECG data in the simulation experiment are from the MIT-BIH Arrhythmia Database [25]. We randomly selected No.100 ECG data, which consists of 2,768 data points, for the experiment. We used the first 900 as the trained set and the remaining 1,868 as the test set.

Prediction indexes
The common prediction indexes are root mean square error (RMSE), mean absolute error (MAE), and mean square error (MSE), defined as

\[
RMSE = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (X(n) - \bar{X}(n))^2}
\]

(1)

\[
MAE = \frac{1}{N} \sum_{n=1}^{N} |X(n) - \bar{X}(n)|
\]

(2)

\[
MSE = \frac{1}{N} \sum_{n=1}^{N} (X(n) - \bar{X}(n))^2
\]

(3)

where \( \bar{X}(n) \) is the predicted value of \( X(n) \) and \( N \) is the number of \( X(n) \).

VMD of ECG signal
The trained set of No.100 ECG was decomposed into 10 IMFs by VMD as shown in Figure 1. IMF1 is the residual, and IMF2~IMF10 are the components, sorted from low to high frequency.

![VMD of No.100 ECG signal](image)

Figure 1  VMD of No.100 ECG signal
A key problem of VMD is to set the number $k$ of modal components. After many experiments, we found that the minimum predicted RMSE was obtained when the number $k = 10$. Hence, we set $k$ to 10.

**Determination of delay time and embedding dimension**
We determined the embedding dimension $m$ and delay time $\tau$ according to the trained set of No.100 ECG. The embedding dimension $m$ was calculated by FNN (false nearest neighbours) method, as shown in Figure 2.

![Figure 2](image2.png)

**Figure 2** Determining embedding dimension by FNN method

From Figure 2, we got $m = 5$. In the experiment, we also got $m = 5$ by Cao method. The delay time $\tau$ was calculated by MI (mutual information) method, as shown in Figure 3.

![Figure 3](image3.png)

**Figure 3** Determining delay time by MI method

In Figure 3, we got the delay time $\tau = 4$ according to the first local minimum of mutual information.
The parameters \( m \) and \( \tau \) were determined simultaneously by C-C method, as shown in Figure 4.

![Figure 4](image)

**Figure 4** Determining delay time and embedding dimension by C-C method

The C-C method determines the delay time \( \tau \) and delay time window \( \tau_w \), and their relation is \( \tau_w = (m-1)\tau \). If \( \tau \) and \( \tau_w \) are determined, then the embedding dimension \( m \) is calculated. \( \tau \) is the first local minimum of \( \text{deltS} \) and \( \tau_w \) is the global minimum. In Figure 4, we got \( \tau = 4 \) and \( \tau_w = 73 \). We got \( m = 20 \) after calculation.

Different methods lead to different embedding dimension \( m \) and delay time \( \tau \). We conducted experiments on the No. 100 ECG data with various parameter combinations of \( m \) and \( \tau \), which had different prediction results, as shown in Table 1.

| Parameters | RMSE  | MSE      | MAE   |
|------------|-------|----------|-------|
| \( m = 5, \tau = 4 \) | 0.0101 | 1.0152e-04 | 0.0070 |
| \( m = 20, \tau = 4 \) | 0.0077 | 5.9680e-05 | 0.0055 |
| \( m = 5, \tau = 1 \) | 0.0037 | 1.3950e-05 | 0.0029 |

In Table 1, a better prediction index is obtained when \( m = 5 \) and \( \tau = 1 \). Hence, \( m = 5 \) and \( \tau = 1 \) were chosen to reconstruct the phase space. \( m = 5 \) is also the number of input layers of the RBF neural network.

**Prediction results of test set**

We used the proposed method to predict the test set of No. 100 ECG signal. The prediction waveform was shown in Figure 5.
To see more clearly how close original ECG waveform and predicted ECG waveform are, we showed only a part of Figure 5 in Figure 6.

Figure 5 Prediction waveform of test set

Figure 6 A part of Figure 5

Figure 5 and Figure 6 show that the predicted waveform fits well with its original ECG waveform.

In order to evaluate the generality of the proposed method, we used ECG data other than No.100 ECG data to carry out experiments. The experimental results were showed in Table 2.

| ECG numbers | RMSE   | MSE    | MAE   |
|-------------|--------|--------|-------|
| 100         | 0.0037 | 1.3950e-05 | 0.0029 |
| 101         | 0.0041 | 1.7117e-05 | 0.0027 |
| 103         | 0.0048 | 2.2936e-05 | 0.0035 |
| 105         | 0.0030 | 9.0403e-06  | 0.0023 |
| 115         | 0.0041 | 1.6884e-05 | 0.0030 |
Some prediction waveforms were shown in Figure 7.

From Table 2 and Figure 7, the prediction indexes of the proposed method on other ECG data are very close to that of No. 100 ECG data. This illustrates that the generality of the proposed method is good and it is very suitable for ECG signal prediction.

**Discussion**

**Comparison with some traditional prediction methods**

To illustrate the advantages of the proposed prediction method, we compared it with the methods of Sun et al. [17], Sun et al.[18] and Su et al.[24]. We experimented on the same data source, No. 100 ECG, and the experimental results were shown in Table 3 and Figure 8.
Table 3 Comparison with other prediction methods

| Methods         | RMSE   | MAE   |
|-----------------|--------|-------|
| This paper      | 0.0037 | 0.0029|
| Sun et al.[17]  | 0.0233 | 0.0157|
| Sun et al.[18]  | 0.0423 | 0.0240|
| Su et al.[24]   | 0.0146 | 0.0106|

Figure 8 Comparing with other prediction methods

As shown in Table 3, the RMSE and MAE of the proposed method are $10^{-3}$ magnitude, while the RMSE and MAE of [17], [18], and [24] are $10^{-2}$ magnitude. Figure 8 shows that the RMSE and MAE of the proposed method are significantly smaller than those of [17], [18], and [24]. This illustrates that the prediction accuracy of this paper is much higher than that of [17], [18], and [24].

We also compared the proposed method with some traditional prediction models, such as ARMA, LS-SVM, SVM, and Kalman. The experimental results on the data source of No. 100 ECG were shown in Table 4.

Table 4 Comparison with other prediction models

| Prediction models | RMSE   |
|-------------------|--------|
| This paper        | 0.0037 |
| LS-SVM [20]       | 0.2744 |
| SVM [20]          | 0.0669 |
| ARMA [20]         | 0.4630 |
| Kalman [20]       | 0.3967 |

We can see from Table 4 that the RMSE of the proposed method is smaller than that of LS-SVM, SVM, ARMA, and Kalman.

Comparison with some approximate prediction methods
The proposed method (VMD-PSR-RBF for short) was compared with other approximate hybrid methods, such as the method based on WT, PSR, and RBF neural network (WT-PSR-RBF for short), the method based on EMD, PSR, and RBF neural network (EMD-PSR-RBF for short), the method based on VMD, PSR, and BP neural network (VMD-PSR-BP for short), and the method based on VMD, PSR, and GRNN (VMD-PSR-GRNN for short). The experimental data were No. 100 ECG data, and the experimental results were shown in Table 5 and Figure 9.

| Prediction methods          | RMSE   | MAE    | MSE       |
|----------------------------|--------|--------|-----------|
| VMD-PSR-RBF(this paper)     | 0.0037 | 0.0029 | 1.3950e-05 |
| WT-PSR-RBF                  | 0.0060 | 0.0035 | 3.6526e-05 |
| EMD-PSR-RBF                 | 0.0143 | 0.0097 | 2.0423e-04 |
| VMD-PSR-BP                  | 0.0203 | 0.0127 | 4.1180e-04 |
| VMD-PSR-GRNN                | 0.0156 | 0.0101 | 2.4331e-04 |

Table 5 Comparison with some approximate prediction methods

![Figure 9](image-url) Comparing with approximate prediction methods

Table 5 and Figure 9 show that the prediction accuracy is better than several approximate prediction methods. Some prediction waveforms in the local range [35, 50] were shown in Figure 10.
As shown in Figure 10, the prediction waveform of the proposed method is closer to the original ECG waveform than that of WT-PSR-RBF, EMD-PSR-RBF, VMD-PSR-BP, and VMD-PSR-GRNN.

**Conclusions**
In this paper, we reviewed the latest development of ECG prediction methods and proposed a hybrid prediction method of ECG signals. In this method, VMD, PSR and RBF neural network theory were combined to predict an ECG signal. In order to get good prediction results, several methods were compared to determine the embedding dimension and the delay time of PSR. Using the ECG signals from the MIT-BIH Arrhythmia Database as the data source, we evaluated the prediction performance of the proposed method. We compared the proposed method with some traditional prediction methods and some competitive approximate prediction methods. After many experiments, we obtained experimental results showing that the prediction indexes RMSE, MSE, and MAE of this paper were much smaller than those of some traditional prediction methods and models. The proposed prediction method obviously improved the prediction accuracy of an ECG signal.

**Methods**

**Variational mode decomposition**

Variational mode decomposition (VMD) decomposes an input signal into \( k \) discrete band-limited IMFs around the centre frequency, meeting the condition that the sum of the estimated bandwidths of each mode is the smallest. The decomposition steps can be summarized as follows.

**Step 1:** Calculate the analytic signal of each modal function \( u_k(t) \) by Hilbert transform

\[
(\delta(t) + \frac{j}{\pi t}) \ast u_k(t)
\]  

**Step 2:** Multiply the analytical signal by the estimated centre frequency \( e^{-j\omega t} \), and move it to the base frequency spectrum

\[
[(\delta(t) + \frac{j}{\pi t}) \ast u_k(t)]e^{-j\omega t}
\]  

**Step 3:** Estimate the bandwidth of each mode by Gaussian smoothing of the demodulated signal, i.e., the \( L^2 \) norm of the gradient. The constrained variational model is

\[
\min_{\{u_k\}} \{ \sum_k \| (\delta(t) + \frac{j}{\pi t}) \ast u_k(t)]e^{-j\omega t} \|_2^2 \}
\]

subject to \( \sum_k u_k = x \)

where \( x \) is the input signal and \( \| \|_2 \) is the Euclidian distance.

**Step 4:** Turn the constrained variational model into an unconstrained variational model by introducing the quadratic penalty factor \( \alpha \) and Lagrange multiplication operator \( \lambda(t) \). The extended Lagrange expression is

\[
L(\{u_k\}, \{w_k\}, \lambda(t)) = \alpha \sum_k \| (\delta(t) + \frac{j}{\pi t}) \ast u_k(t)]e^{-j\omega t} \|_2^2 + \| x - \sum_k u_k \|^2 + < \lambda(t), x - \sum_k u_k >
\]  

Find the saddle point of the extended Lagrange expression using the alternating direction multiplier method (optimization algorithm) to solve the minimization problem of formula (6). The saddle point is the optimal solution.

**Step 5:** Obtain the saddle point of the extended Lagrange expression by alternately updating \( u_k^{n+1}, w_k^{n+1}, \) and \( \lambda_k^{n+1} \). The formulas for this are
\[
\bar{u}^n_k(w) = \frac{\bar{x}(w) - \sum_{i < j} \bar{u}^{n+1}_i(w) - \sum_{i < k} \bar{u}^n_k(w) + \bar{\lambda}(w)/2}{1 + 2\alpha(w - w^n_i)^2}
\]

\[
W^{n+1}_k = \frac{\int_0^\infty w|\bar{u}^{n+1}_k(w)|^2 dw}{\int_0^\infty |\bar{u}^{n+1}_k(w)|^2 dw}
\]

\[
\bar{\lambda}^{n+1}(w) = \bar{\lambda}(w) + \tau(\bar{x}(w) - \sum_k |\bar{u}^{n+1}_k(w)|)
\]

where \(\bar{u}(w), \bar{\lambda}(w),\) and \(\bar{x}(w)\) are the Fourier transforms of the signals \(u(t), \lambda(t),\) and \(x(t)\), respectively.

Step 6: Repeat step 5 until the convergence condition is reached
\[
\sum_x \|\bar{u}^{n+1}_k - \bar{u}^n_k\|^2 / \|\bar{u}^n_k\|^2 < \varepsilon
\]

**Phase space reconstruction**

Dutch mathematician Floris Takens [26] proved that as long as the embedding dimension is large enough, the reconstructed attractor retains the topological properties of the original attractor. This is the theoretical basis of phase space reconstruction (PSR). PSR is the basis for analysing sequences with chaotic characteristics.

For a time series: \(x_1, x_2, \ldots, x_N\), if the delay time \(\tau\) and embedding dimension \(m\) can be properly selected, a new sequence of phase space vectors is expressed as follows:

\[
X_1 = [x_1, x_{1+\tau}, x_{1+2\tau} \ldots x_{1+(m-1)\tau}]
\]

\[
X_2 = [x_2, x_{2+\tau}, x_{2+2\tau} \ldots x_{2+(m-1)\tau}]
\]

\[
\cdots
\]

\[
X_M = [x_M, x_{M+\tau}, x_{M+2\tau} \ldots x_{M+(m-1)\tau}]
\]

where \(M = N - (m-1)\tau\) is the length of the vector sequence. A reconstructed phase space \(X\) as the following matrix:

\[
X = \begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_M
\end{bmatrix}
= \begin{bmatrix}
x_1 & x_{1+\tau} & \cdots & x_{1+(m-1)\tau} \\
x_2 & x_{2+\tau} & \cdots & x_{2+(m-1)\tau} \\
\vdots & \vdots & \ddots & \vdots \\
x_M & x_{M+\tau} & \cdots & x_{M+(m-1)\tau}
\end{bmatrix}
\]

The key to PSR is to correctly select the embedding dimension \(m\) and delay time \(\tau\). There are two views on the determination of the embedding dimension and delay time. The first is that they are unrelated and should be determined independently. From this point of view, the delay time can be determined by the average displacement, autocorrelation function, complex correlation, and mutual information (MI) methods. Methods to determine the embedding dimension include false nearest neighbours (FNN) method, Cao method, and G-P algorithm. The other view is that the two parameters are related, they influence each other, and they should be determined at the same time. Common methods to do this include the window embedding and C-C methods.

**Mutual information method**

Suppose there are two systems, \(S = \{s_1, s_2, \ldots, s_n\}\) and \(Q = \{q_1, q_2, \ldots, q_n\}\). The information entropy of \(S\) and \(Q\) is, respectively.
\[ H(S) = -\sum_{i=1}^{n1} P_s(s_i) \log_2 P_s(s_i) \]  
(14)

\[ H(Q) = -\sum_{j=1}^{n2} P_q(q_j) \log_2 P_q(q_j) \]  
(15)

where \( P_s(s_i) \) and \( P_q(q_j) \) are probabilities of \( s_i \) and \( q_j \), respectively.

The joint entropy of \( S \) and \( Q \) is

\[ I(S,Q) = -\sum_{i=1}^{n1} \sum_{j=1}^{n2} P_{sq}(s_i, q_j) \log_2 P_{sq}(s_i, q_j) \]  
(16)

where \( P_{sq}(s_i, q_j) \) is the joint probability of \( s_i \) and \( q_j \).

The mutual information (MI) between time series \( s(n) \) and its delayed sequence \( q(n) = s(n+\tau) \) is

\[ I(\tau) = I(S,Q) = H(S) + H(Q) - H(S,Q) \]  
(17)

The \( \tau \) corresponding to the first minimum of \( I(\tau) \) is the optimal time delay \( \tau \).

**False nearest neighbours method**

False nearest neighbours (FNN) are phase points that are adjacent in low dimensional space, but not adjacent after mapping to a certain high dimensional space. Suppose \( X_m(i) = (x_i, x_{i+\tau}, ..., x_{i+(m-1)\tau}) \) is a phase vector in m-dimensional reconstruction space, and the nearest neighbour point of \( X_m(i) \) is \( X_m^{NN}(i) \). If \( X_m^{NN}(i) \) satisfies the following inequality, then \( X_m^{NN}(i) \) is the FNN of \( X_m(i) \).

\[ \frac{\| X_{m+1}(i) - X_{m+1}^{NN}(i) \|_2}{\| X_m(i) - X_m^{NN}(i) \|_2} \geq R_T \]  
(18)

where \( \| \cdot \|_2 \) is Euclidian distance, and \( R_T \) is the threshold value.

As the embedding dimension \( m \) increases from small to large, the proportion of FNN points is calculated. When the proportion of FNN points is very small or the number of FNN no longer changes as \( m \) increases, then \( m \) is the best embedding dimension.

**RBF neural network**

Different from a BP neural network, which is a global approximation network, an RBF neural network is a kind of local approximation network. As long as there are enough hidden neurons, an RBF neural network can approximate any continuous nonlinear function with any accuracy. Unlike a BP neural network, an RBF neural network has the advantage of fast training speed, and it does not easily fall into local minima.

The basic idea of an RBF neural network is that the RBF of the hidden layer node transforms the input vector, maps the low-dimensional input data to the high-dimensional space, weights the sums of the output of the node, and maps the results from the high-dimensional space to the low-dimensional space for output [27].

The network structure of an RBF neural network generally consists of an input layer, hidden layer, and output layer, as shown in Figure 11.
In Figure 11, a Gaussian function is usually used as the RBF of a hidden layer as follows.

$$h_j = \exp\left[-\frac{(X - C_j)^2}{2\sigma_j^2}\right], \quad j = 1, 2, \ldots, m,$$

(19)

where $X = [x_1, x_2, \ldots, x_n]$, $C_j$ is the centre of the Gaussian function, $\sigma_j$ is the variance of the Gaussian function and $m$ is the number of hidden layer nodes.

The output of the RBF neural network is

$$y_k = \sum_{j=1}^{m} w_{jk} \exp\left[-\frac{(X - C_j)^2}{2\sigma_j^2}\right], \quad k = 1, 2, \ldots, r,$$

(20)

where $w_{jk}$ is the connection weight from the hidden layer to the output layer.

The learning algorithm of RBF neural network solves for three parameters, which are the centre of the RBF, the variance of the RBF, and the weight of the connection between the hidden layer node and output layer node. Common learning algorithms of RBF neural networks include k-means, the gradient training method, and the orthogonal least square (OLS) product algorithm.

**Proposed ECG signal prediction method**

Based on the study of ECG prediction, this paper proposed a hybrid method of ECG signal prediction based on VMD, PSR, and an RBF neural network. Its flowchart was shown in Figure 12.
The prediction steps of the proposed method are as follows.

Step 1: Calculate the embedding dimension and delay time according to the trained set before VMD. We calculate the embedding dimension by FNN method.

Step 2: Decompose ECG data into \( k \) IMFs by VMD. In the experiment, we take \( k = 10 \).

Step 3: Reconstruct the phase space of each IMF according to the embedding dimension and delay time.

Step 4: According to the trained set of each IMF, establish an RBF neural network and use it to predict the test set of each IMF. The embedding dimension is the number of input layers of the RBF neural network.

Step 5: Add the prediction results of the RBF neural network to obtain the final ECG signal prediction result.

Step 6: Analyse the prediction error and compare it with some traditional prediction methods.

**Abbreviations**

BANs: body area networks; VMD: variational mode decomposition; PSR: phase space reconstruction; RBF: radial basis function; IMFs: intrinsic mode functions; RMSE: root mean square error; MAE: mean absolute error; MSE: mean square error; FNN: false nearest neighbours; MI: mutual information; WT: wavelet transform; EMD: empirical mode decomposition; OLS: orthogonal least square; DFA: de-trended fluctuation analysis; LSTM: long short term memory; MLMRs: multi-label metamorphic relations; ABC: artificial bee colony; PQD: power quality disturbance; BRNN: bi-directional recurrent neural network; BPNN: backpropagation neural network; DWT: discrete wavelet transform; AR: autoregressive; LS-SVM: least-squares support vector machine; CNN: convolutional neural network; ARMA: autoregressive moving average; ARIMA: autoregressive integrated moving average.

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Authors’ contributions

Conception and design of the work: Fuying Huang and Tuanfa Qin. Acquisition of data: Limei Wang. Analysis and interpretation of data: Fuying Huang and Haibin Wan. Drafting the article: Fuying Huang, Tuanfa Qin and Limei Wang. All authors read and approved the final manuscript.

Ethics approval and consent to participate

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Consent for publication

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Availability of data and materials

In this paper, All ECG data are from MIT-BIH Arrhythmia Database. MIT-BIH Arrhythmia Database is available online: https://www.physionet.org/content/mitdb/1.0.0/

Competing interests

The authors declare that they have no competing interests.

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