The strange-metal phase of cuprate superconductors exhibits a linear in temperature resistivity, however, the origin of this remarkable anomaly is still not well understood. Here the linear temperature dependence of the electrical resistivity in the strange-metal phase of cuprate superconductors is investigated from the underdoped to overdoped regimes. The momentum dependence of the transport scattering rate arising from the umklapp scattering between electrons by the exchange of the spin excitation is derived and employed to calculate the electrical resistivity by making use of the Boltzmann equation. It is shown that the antinodal umklapp scattering leads to the linear in temperature resistivity in the low-temperature with the temperature linear coefficient that decreases with the increase of the doping concentration, however, the nodal umklapp scattering induces a deviation from the linear in temperature resistivity in the far lower temperature, and then the quadratic in temperature resistivity in the far lower temperature is generated by both the antinodal and nodal umklapp scattering. The theory also shows that the same spin excitation that acts like a bosonic glue to hold the electron pairs together also mediates scattering of electrons in the strange-metal phase of cuprate superconductors responsible for the linear in temperature resistivity and the associated electronic structure.

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I. INTRODUCTION

The parent compound of cuprate superconductors is identified as a Mott insulator, in which the absence of the electronic conduction is due to the strong electron correlation. Superconductivity then emerges when charge carriers are doped into this Mott insulator. At the temperature above the superconducting (SC) transition temperature $T_c$, the electron is in a normal-state. Although this same strong electron correlation that leads to the Mott insulating state persists into the doped regime, the normal-state retains a metallic character. However, this normal-state is not normal at all, since this normal-state exhibits a number of the anomalous properties in the sense that they do not fit in with the standard Landau-Fermi liquid theory. This is why the phase of cuprate superconductors above $T_c$ is so-called as the strange-metal phase.

Among the fascinating phenomena in the strange-metal phase, the most distinct features are (i) the low-energy electronic structure associated with the single-particle scattering rate, including the formation of the disconnected Fermi arcs due to the collapse of the closed electron Fermi surface (EFS), the peak-dip-hump (PDH) structure in the electron excitation spectrum, and the dispersion kink, and (ii) the low-temperature electrical transport associated with the transport scattering rate. In particular, in the early experimental measurements, it was observed that the variation of the electrical resistivity near the optimal doping is linear with temperature, extending to low temperatures of a few kelvin and extrapolating to zero resistivity at zero temperature. This remarkable behaviour of the low-temperature linear temperature dependence of the electrical resistivity is in a striking contrast to the behaviour in conventional metals, where the low-temperature electrical resistivity follows one of several simple power laws, and the linear temperature domain dominates, then the electrical resistivity decreases quadratically as the temperature decreases to zero. In the latter, this linear in temperature (T-linear) resistivity was detected experimentally in a wide doping range from the underdoped to overdoped regimes. Moreover, the suppression of superconductivity with a magnetic field reveals that the T-linear resistivity persists down essentially to the zero temperature limit. Recently, the systematic experimental results in the heavily overdoped regime yielded the low-temperature T-linear resistivity all the way up to the edge of the SC dome. After intensive investigations over more than three decades, it has now become clear that the long-standing T-linear resistivity are the generic features in the strange-metal phase of cuprate superconductors. In this case, a key question posed by these experimental observations is raised: is there a common bosonic excitation that is responsible for pairing the electrons also dominantly scatters electrons in the strange-metal phase responsible for the T-linear resistivity and the associated electronic structure?

Although the low-temperature T-linear resistivity in the strange-metal phase of cuprate superconductors is well established by now, its origin remains the subject of the active research and debate. Theoretically, several scenarios have been proposed for the origin of the T-linear resistivity. In particular, in the marginal Fermi-liquid phenomenology, a single T-linear scattering rate is introduced responsible for the T-linear resistivity. Moreover, it has been postulated that the T-linear behaviour can be attributed to the strongly interacting critical state anchored at a quantum critical point where
a phase transition is tuned to zero temperature. With the close relation to the physics of the quantum critical point, the T-linear resistivity has been interpreted in terms of the Planckian dissipation in which the relaxation-time achieves a putative universal minimum value, irrespective of the underlying mechanisms. On the other hand, it has been argued that the elastic umklapp scattering processes that directly transfer momentum between the electron sea and the underlying square lattice lead to the resistivity linear in temperature in the strange-metal phase. More specifically, it has been shown recently that the electrical resistance arises from the electron umklapp scattering by the exchange of a critical bosonic mode, where the resistivity is characterized by a highly anisotropic scattering rate. This highly anisotropic scattering rate is T-linear near the umklapp points and becomes quadratic in temperature as one moves away from the umklapp point, which therefore leads to a T-linear resistivity in the low-temperature regime and quadratic in temperature resistivity in the far lower temperature regime. These studies and many others therefore indicate that the umklapp scattering dominates the momentum-relaxation mechanism of the electrical transport. However, up to now, the origin of the T-linear resistivity and of its connection to the unconventional electronic structure have not been discussed starting from a microscopic theory, and no explicit calculations of the doping dependence of the T-linear resistivity has been made so far. Superconductivity with the highest $T_c$ emerges directly as an instability of the strange-metal phase, and it thus has long been recognized that the understanding of the essential physics of the strange-metal phase is crucial for the understanding of the mystery of the unconventional superconductivity.

In the recent works within the framework of the kinetic-energy-driven superconductivity, we have studied the low-energy electronic structure of cuprate superconductors both in the SC phase at the temperature below $T_c$ and strange-metal phase at the temperature above $T_c$, where the electron normal self-energy in the particle-hole channel and electron anomalous self-energy in the particle-particle channel are generated by the coupling of the electrons with the spin excitations. In particular, the electrons are renormalized by the electron normal self-energy, and then all the exotic features of the low-energy electronic structure arise from this renormalization of the electrons. In this paper, we start from this low-energy electronic structure in the strange-metal phase of cuprate superconductors to study the nature of the doping dependence of the T-linear resistivity, where the angular dependence of the transport scattering rate arising from the umklapp scattering between electrons by the exchange of the same spin excitations is derived and employed to calculate the electrical resistivity in terms of the Boltzmann equation. Our results show that the antinodal umklapp scattering, in which the electron at around the antinodal region of EFS is scattered by its umklapp partner at around the antinodal region of the neighboring EFS, leads to the T-linear resistivity in the low-temperature with the T-linear resistivity that decreases with the increase of the doping concentration. On the other hand, the nodal umklapp scattering, in which the electron at around the nodal region of EFS is scattered by its umklapp partner at around the nodal region of the neighboring EFS, induces a deviation from the T-linear resistivity in the far lower temperature, and then the quadratic in temperature resistivity in the far lower temperature is generated by both the antinodal and nodal umklapp scattering. Our results therefore indicate that the same spin excitation that is responsible for pairing the electrons also mediates the scattering of electrons in the strange-metal phase responsible for the T-linear resistivity and the associated electronic structure.

The rest of this paper is organized as follows. In Section II, we begin by a short summary of the unconventional features of the low-energy electronic structure due to the interaction between electrons by the exchange of a strongly dispersive spin excitation, and then within the framework of the Boltzmann transport theory, we formulate the essential aspects of the electron-electron umklapp scattering between a circular EFS and its umklapp partner by the exchange of the same spin excitation for deriving the electrical resistivity. In Section III, the Boltzmann equation is employed to study the doping dependence of the low-temperature electrical resistivity, where we show that the angular dependence of the transport scattering rate presents a similar behavior of the single-particle scattering rate, and is largest at around the antinodes and smallest at around the tips of the Fermi arcs. Although this angular dependence of the transport scattering rate is highly anisotropic in momentum-space, it displays a T-linear behaviour in the low-temperature for all directions. Finally, we give a summary in Section IV. In the Appendix A, we presents the details of the derivation of the electron-electron collision term in the Boltzmann equation.

**II. THEORY**

### A. $t$-$J$ model in the fermion-spin representation

The crystal structure of cuprate superconductors is characterized by the square-lattice copper-oxide layers, which are sometimes considered to contain all the essential physics. Immediately after the discovery of superconductivity in cuprate superconductors, it was suggested that the fundamental properties of the doped copper-oxide layer are properly accounted by the square-lattice $t$-$J$ model,

$$H = -\sum_{ill'} t_{ll'} C_{il\sigma}^\dagger C_{ill'} + \mu \sum_{ll\sigma} C_{il\sigma}^\dagger C_{il\sigma} + J \sum_{(ll')} \mathbf{S}_l \cdot \mathbf{S}_{l'},$$

acting on the restricted Hilbert-space with no double electron occupancy $\sum_{\sigma} C_{il\sigma}^\dagger C_{il\sigma} \leq 1$, where the operator...
$C_{l \sigma}^\dagger$ ($C_{l \sigma}$) denotes the creation (annihilation) operator of an electron on site $l$ with spin $\sigma$. $S_l$ is the spin operator of the electron with its components $S_l^x$, $S_l^y$, and $S_l^z$, and $\mu$ is the chemical potential. In this paper, the hopping of the constrained electrons $t_{ll'}$ is restricted to the nearest-neighbor (NN) sites with the hopping amplitude $t_{ll'} = t$ and next NN sites with the hopping amplitude $t_{ll'} = -t'$. The summation $\langle ll' \rangle$ indicates a sum over the NN pairs, while the summation $\langle ll' \rangle'$ is taken over all the NN and next NN pairs. Throughout this paper, we choose the parameters as $t/J = 2.5$ and $t'/J = 0.3$ as in the previous discussions\textsuperscript{60}. The magnitude of $J$ and the lattice constant of the square lattice are the energy and length units, respectively. However, when necessary to compare with the experimental data, we set $J = 1000$ K.

The essence of the strongly correlated physics is reflected in the on-site local constraint of no double occupancy\textsuperscript{62–66}. To avoid the double electron occupancy, we employ the fermion-spin transformation for the parametrization of the constrained electron operators $C_{l \uparrow}$ and $C_{l \downarrow}$ as\textsuperscript{67,68},

$$C_{l \uparrow} = h_{l \uparrow}^{-1}S_{l \uparrow}^{-}, \quad C_{l \downarrow} = h_{l \downarrow}^{-1}S_{l \downarrow}^{+},$$  \hspace{1cm} (2)

respectively, where the spin operator $S_l$ keeps track of the spin degree of freedom of the constrained electron, while the spinful fermion operator $h_{l \sigma} = e^{-i\varphi_{l \sigma}h_{l \uparrow}}$ keeps track of the charge degree of freedom of the constrained electron together with some effects of spin configuration rearrangements due to the presence of the doped hole itself (charge carrier), and then the on-site local constraint of no double occupancy is satisfied in actual analyses. In this fermion-spin representation (2), the original $t$-$J$ model (1) can be rewritten explicitly as,

$$H = \sum_{\langle ll' \rangle \sigma} t_{ll'}(h_{l \uparrow}^{-1}h_{l \uparrow}S_{l \uparrow}^{-}S_{l' \uparrow}^{-} + h_{l \downarrow}^{-1}h_{l \downarrow}S_{l \downarrow}^{+}S_{l' \downarrow}^{+}) - \mu_h \sum_{l \sigma} h_{l \uparrow}^{-1}h_{l \downarrow} h_{l \sigma} + J_{\text{eff}} \sum_{\langle ll' \rangle \sigma} S_l \cdot S_{l'},$$ \hspace{1cm} (3)

where $S_l^{-} = S_l^x - iS_l^y$ and $S_l^{+} = S_l^x + iS_l^y$ are the spin-lowering and spin-raising operators for the spin $S = 1/2$, respectively, $J_{\text{eff}} = (1-\delta)^2 J$, $\delta = \langle h_{l \uparrow}^{-1}h_{l \downarrow} \rangle = \langle h_{l \uparrow}^{-1}h_{l \downarrow} \rangle$ is the doping concentration, and $\mu_h$ is the charge-carrier chemical potential. As a natural consequence, the kinetic-energy term in the $t$-$J$ model (1) has been transferred as the coupling between charge and spin degrees of freedom of the constrained electron, which reflects a basic fact that even the kinetic energy term in the $t$-$J$ model (1) has the strong Coulombic contribution due to the restriction of no double occupancy at any given site, and therefore governs the unconventional features of cuprate superconductors.

### B. Coupling of electrons to the strongly dispersive spin excitation

Starting from the $t$-$J$ model (3) in the fermion-spin representation, we\textsuperscript{68–71} have developed the kinetic-energy-driven superconductivity, where at low temperatures, the charge carriers are bound into the charge-carrier pairs with the d-wave symmetry by the attractive interaction in the particle-particle channel that arises directly from the strong coupling between charge and spin degrees of freedom of the constrained electron in the kinetic energy of the $t$-$J$ model (3) by the exchange of the strongly dispersive spin excitation, then the d-wave electron pairs originated from the d-wave charge-carrier pairs are due to the charge-spin recombination\textsuperscript{71}, and the condensation of these d-wave electron pairs reveals the SC-state with the d-wave symmetry. In particular, the doping dependence of $T_c$ exhibits a dome-like shape with the underdoped and overdoped regimes on each side of the optimal doping, where $T_c$ reaches its maximum. Moreover, this same coupling mediated by the spin excitation that induces the SC-state in the particle-particle channel also leads to the renormalization of the electrons in the particle-hole channel\textsuperscript{60}. Within the framework of this kinetic-energy-driven superconductivity, the exotic features of the low-energy electronic structure in the both SC phase\textsuperscript{57–59} and strange-metal phase\textsuperscript{60} of cuprate superconductors have been investigated, and the obtained results are well consistent with the corresponding experimental observations. Following these previous discussions\textsuperscript{60}, the single-particle propagator in the strange-metal phase of cuprate superconductors can be obtained in the condition of the SC gap parameter $\Delta = 0$,

$$G(k, \omega) = \frac{1}{\omega - \varepsilon_k - \Sigma_{\text{ph}}(k, \omega)},$$ \hspace{1cm} (4)

where $\varepsilon_k = -4t \gamma_k + 4t' \gamma_k' + \mu$ is the electron energy dispersion in the tight-binding approximation, with $\gamma_k = (\cos k_x + \cos k_y)/2$ and $\gamma_k' = \cos k_x \cos k_y$, while the electron normal self-energy $\Sigma_{\text{ph}}(k, \omega)$ sketched in Fig. 1 is expressed as\textsuperscript{60},

![FIG. 1. The skeletal diagram for the electron normal self-energy for scattering electrons from the strongly dispersive spin excitation. The solid-line represents the electron propagator $G$, and the dashed-line depicts the spin propagator $D^{(0)}$, while $\Phi$ describes the bare vertex function $\Lambda$.](image-url)
\[ \Sigma_{ph}(k, i\omega_n) = \frac{t^2}{N^2} \sum_{p,q} A_{p+q+k}^2 \sum_{\alpha} G(p + k, ip_m + i\omega_n) \Pi(p, q, ip_m) = \frac{t^2}{N} \sum_{\alpha} \sum_{p} G(p + k, ip_m + i\omega_n) P^{(0)}(k, p, ip_m) \]

\[ = 2 \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega''}{2\pi} n_B(\omega'') + n_F(\omega') \frac{t^2}{\beta} \sum_{p} A(p + k, \omega') \text{Im} P^{(0)}(k, p, \omega''), \]

where \( N \) is the number of lattice sites, \( A_k = 4\gamma_k - 4(t'/t)' \gamma_k' \) is the bare vertex function, \( \omega_n \) and \( p_m \) are the fermionic and bosonic Matsubara frequencies, respectively, \( n_B(\omega) \) and \( n_F(\omega) \) are the boson and fermion distribution functions, respectively. \( \text{Im} P^{(0)}(k, p, \omega) \) is the imaginary part of \( P^{(0)}(k, p, \omega) \), while \( P^{(0)}(k, p, \omega) \) is so-called as the effective spin propagator, which describes the nature of the spin excitation, and can be expressed as,

\[ P^{(0)}(k, p, \omega) = \frac{1}{N} \sum_{q} A_{p+q+k}^2 \Pi(p, q, \omega), \]

with the spin bubble \( \Pi(p, q, \omega) \), which is a convolution of two spin propagators, and has been obtained as \( P^{(0)}(k, p, \omega) \)

\[ \Pi(p, q, \omega) = \frac{1}{\omega^2 - \omega^2_{p, q}}, \]

with the MF spin excitation energy dispersion \( \omega_k \) and the weight function of the spin excitation spectrum \( B_k \) that are strongly dispersive, and have been given in Ref. 60. Substituting the above MF spin propagator \( \Pi(p, q, \omega) \) into Eq. (7), the spin bubble \( \Pi(p, q, \omega) \) can be derived as,

\[ \Pi(p, q, \omega) = \frac{W_{pq}^{(1)}}{\omega^2 - \omega^2_{pq} + i\Gamma^2_{pq}} + \frac{W_{pq}^{(2)}}{\omega^2 - \omega^2_{pq} - i\Gamma^2_{pq}}, \]

and then the effective spin propagator \( P^{(0)}(k, p, \omega) \) in Eq. (6) is obtained directly from the above spin bubble (9), where \( \omega_{pq}^{(1)} = \omega_{pq} + |\omega_{pq}^{(1)}| \) and \( \omega_{pq}^{(2)} = \omega_{pq} - |\omega_{pq}^{(2)}| \), and the weight functions of the effective spin excitation spectrum,

\[ W_{pq}^{(1)} = \frac{B_q B_{q+p} \omega_{pq}^{(1)} n_B(\omega_{pq} + |\omega_{pq}^{(1)}|) + n_B(\omega_{pq} + |\omega_{pq}^{(1)}|)}{2\omega_{pq}^2 \omega_{pq}^{(1)}} + 1, \]

\[ W_{pq}^{(2)} = \frac{B_q B_{q+p} \omega_{pq}^{(2)} n_B(\omega_{pq} - |\omega_{pq}^{(2)}|) - n_B(\omega_{pq} - |\omega_{pq}^{(2)}|)}{2\omega_{pq}^2 \omega_{pq}^{(2)} + 1}. \]

With the above effective spin propagator (6), the electron normal self-energy \( \Sigma_{ph}(k, \omega) \) in Eq. (5) has been derived \( \Sigma_{ph}(k, \omega) \) and can be expressed explicitly as,

\[ \Sigma_{ph}(k, \omega) = \frac{t^2}{N^2} \sum_{p, q = 1, 2} (-1)^{\alpha+1} \Omega_{kpq} \]

\[ \times \left( \frac{F_{1kpq}^{(\alpha)}}{\omega + \omega_{pq}^{(\alpha)} - \bar{\sigma}_{p+k}} + \frac{F_{2kpq}^{(\alpha)}}{\omega - \omega_{pq}^{(\alpha)} - \bar{\sigma}_{p+k}} \right), \]

with the renormalized electronic energy dispersion \( \bar{\varepsilon}_k = Z_F \varepsilon_k, \Omega_{kpq} = Z_F A_{p+q+k} B_q B_{q+p}/(4\omega_{pq}q_p + q_p), \) and the functions,

\[ F_{1kpq}^{(\alpha)} = n_F(\bar{\varepsilon}_{p+k})\{1 + n_B(\omega_{q+p}) + n_B(-1)^{\alpha+1}\omega_{q+p}) \]

\[ + n_B(\omega_{q+p})n_B(-1)^{\alpha+1}\omega_{q+p}), \]

\[ F_{2kpq}^{(\alpha)} = [1 - n_F(\bar{\varepsilon}_{p+k})] \{1 + n_B(\omega_{q+p}) + n_B(-1)^{\alpha+1}\omega_{q+p}) \]

\[ + n_B(\omega_{q+p})n_B(-1)^{\alpha+1}\omega_{q+p}), \]

while the single-particle coherent weight \( Z_F \) that has been given explicitly in Ref. 60. In particular, the sharp peaks visible for low-temperature in \( \Sigma_{ph}(k, \omega) \) and \( P^{(0)}(k, p, \omega) \) are actually a \( \delta \)-function, broadened by a small damping used in the numerical calculation for a finite lattice. The calculation in this paper for \( \Sigma_{ph}(k, \omega) \) and \( P^{(0)}(k, p, \omega) \) is performed numerically on a \( 160 \times 160 \) lattice in momentum space, with the infinitesimal \( i\gamma \rightarrow i\Gamma \) replaced by a small damping \( \Gamma = 0.1J \).

C. Electron Fermi surface

The single-particle spectrum function \( A(k, \omega) \) now can be obtained directly from the above single-particle propagator \( \Pi(p, q, \omega) \) as,

\[ A(k, \omega) = -2\text{Im} G(k, \omega) = \frac{2\Gamma_k(\omega)}{\omega - E_k(\omega)^2 + \Gamma_k^2(\omega)}, \]

with the corresponding single-particle scattering rate \( \Gamma_k(\omega) \) and renormalized band structure \( E_k(\omega) \),

\[ \Gamma_k(\omega) = |\text{Im} \Sigma_{ph}(k, \omega)|, \]

\[ E_k(\omega) = \varepsilon_k + \text{Re} \Sigma_{ph}(k, \omega), \]

where \( \text{Re} \Sigma_{ph}(k, \omega) \) and \( \text{Im} \Sigma_{ph}(k, \omega) \) are the real and imaginary parts of the electron normal self-energy \( \Sigma_{ph}(k, \omega) \), respectively.

The shape of EFS has deep consequences for the low-energy electronic properties \( \delta \), and has been also central to addressing electrical transport \( \delta \). In the previous studies \( \delta \), the topology of EFS in the strange-metal phase of cuprate superconductors has been discussed in terms of the intensity map of the single-particle spectral function \( \delta \) at zero energy \( \omega = 0 \), where it has been shown that the highly anisotropic momentum dependence of the single-particle scattering rate \( \Gamma_k(\omega) \) induces a strong redistribution of the spectral weights on EFS. For a convenience in the following discussions of the electrical transport, we plot (a) the EFS map and
FIG. 2. (Color online) (a) The map of the electron Fermi surface and (b) the surface plot of the single-particle spectral function for zero energy $\omega = 0$ at $\delta = 0.15$ with $T = 0.002J$, where the zone center has been shifted by $[\pi, \pi]$, and AN, TFA, and ND indicate the antinode, tip of the Fermi arc, and node, respectively.

(b) the surface plot of the single-particle spectral function $A(k, \omega)$ for zero energy $\omega = 0$ at doping $\delta = 0.15$ with temperature $T = 0.002J$ in Fig. 2, where the Brillouin zone (BZ) center has been shifted by $[\pi, \pi]$, and AN, TFA, and ND indicate the antinode, tip of the Fermi arc, and node, respectively. The most noteworthy in Fig. 2 are the following: (i) the spectral weight at around the antinodal region is suppressed strongly, reflecting a basic fact that EFS at around the antinodal region can not be observed experimentally; (ii) the spectral weight at around the nodal region is suppressed modestly, leading to the formation of the disconnected Fermi arcs; (iii) however, almost all the spectral weight inhabited at around the tips of the Fermi arcs is assembled at around the tips of the Fermi arcs. In other words, the electrons at around the tips of the Fermi arcs have the largest density of states, and then the low-energy electronic properties are largely governed by these electrons at around the tips of the Fermi arcs. In particular, it has been observed experimentally that these characteristic features shown in Fig. 2 in the zero energy case can persist into the case for a finite binding-energy. More importantly, the suppression of the spectral weight at around the antinodal and nodal regions can affect the electrical transport in two ways: through the reduction of the number of current-carrying states, and secondly, through the reduction in the density of electron excitations at around the antinodal and nodal regions.

In our previous discussions, it has been shown that the origin of the spectral redistribution to form the Fermi arcs can be attributed to the highly anisotropic momentum dependence of the single-particle scattering rate $\Gamma_k(\omega)$ in Eq. (15a). The EFS contour in momentum space is determined directly by the poles of the single-particle propagator (4) at zero energy: $\tilde{E}_{k_F}(0) = \varepsilon_{k_F} + \text{Re}\Sigma_{ph}(k_F, 0) = 0$, and then the spectral weight of the single-particle spectral function $A(k_F, 0)$ in Eq. (14) at EFS is dominantly governed by the inverse of the single-particle scattering rate $1/\Gamma(\theta)$, where $\Gamma(\theta)$ is defined as $\Gamma(\theta) = \Gamma_{k_F}(0)$, and $\theta$ is the Fermi angle. To see this highly anisotropic $\Gamma(\theta)$ in momentum space more clearly, we plot the angular dependence of $\Gamma(\theta)$ along EFS from the antinode to the node at $\delta = 0.15$ with $T = 0.05J$ in Fig. 3, where the actual minimum of $\Gamma(\theta)$ does not appear at around the nodal region, but resides exactly at around the tip of the Fermi arc. However, the maximal $\Gamma(\theta)$ appears at around the antinodal region, and then $\Gamma(\theta)$ decreases when the Fermi angle is moved away from the antinode. In particular, $\Gamma(\theta)$ at around the nodal region is smaller than that around the antinodal region.

FIG. 3. The single-particle scattering rate $\Gamma(\theta)$ as a function of Fermi angle $\theta$ at $\delta = 0.15$ with $T = 0.05J$ for $\omega = 0$.
flat band is slightly below the Fermi energy. All these obtained results are well consistent with the corresponding ARPES experimental observations. These results also show that the same spin excitation that is responsible for pairing the electrons also dominantly scatters electrons in the strange-metal phase at the temperature above $T_c$ responsible for the unconventional electronic structure.

D. Boltzmann equation

Although the magnitude of the single-particle scattering rate $\Gamma(\theta)$ shown in Fig. 3 at a given Fermi angle is different from that of the corresponding transport scattering rate, both the scattering rates may have a similar behaviour of the angular dependence. In this sense, the result of the angular dependence of $\Gamma(\theta)$ shown in Fig. 3 also indicates that the important electron scattering responsible for the resistivity is mainly concentrated at the antinodes and nodes. For discussions of the anamalous transport properties, it needs to determine how the momentum distribution relaxes in the vicinity of these antinodes and nodes, which can be done by solving the Boltzmann equation with the input of the scattering processes. In the Boltzmann transport theory, the essential behaviour of the electrons is depicted by the distribution function $f(\mathbf{r}, \mathbf{k}, t)$. In this paper, we focus on the dc conductivity in the homogeneous system only, where the position and time dependence in the distribution function are absent, and then the distribution function satisfies the following Boltzmann equation,\(^{12,13}\)

$$
\frac{\partial \mathbf{k}}{\partial t} \nabla_{\mathbf{k}} f(\mathbf{k}) = \left( \frac{df}{dt} \right)_{\text{collisions}} \tag{16}
$$

where the right-hand side is the time rate of change due to the electron-electron collision, while the factor $\partial \mathbf{k}/\partial t$ is equivalent to an acceleration which is equal to the forces on the electrons as,

$$
\frac{\partial \mathbf{k}}{\partial t} = -e \mathbf{E}, \tag{17}
$$

with the charge $e$, where for a convenience in the following discussions, the magnetic field has been dropped, i.e., $\mathbf{H} = 0$, while only an electric field $\mathbf{E}$ is applied to the system. In this case, we substitute Eq. (17) into Eq. (16), and rewrite the Boltzmann equation (16) as,

$$
e\mathbf{v}_k \cdot \nabla_{\mathbf{k}} f(\mathbf{k}) + \left( \frac{df}{dt} \right)_{\text{collisions}} = 0. \tag{18}
$$

Following the discussions in Ref. 79, we now introduce the linear perturbation from the equilibrium in terms of the distribution function as,

$$
f(\mathbf{k}) = n_F(\xi_k) - \frac{dn_F(\xi_k)}{d\xi_k} \Phi(\mathbf{k}), \tag{19}
$$

where $\Phi(\mathbf{k})$ has been interpreted as a local shift of the chemical potential at a given patch of EFS, and satisfies an antisymmetric relation $\Phi(-\mathbf{k}) = -\Phi(\mathbf{k})$. With the help of the above distribution function (19), the Boltzmann equation (18) can be linearized with the result that can be expressed explicitly as,

$$
e\mathbf{v}_k \cdot \nabla_{\mathbf{k}} \xi_k \frac{\partial n_F(\xi_k)}{\partial \xi_k} = - \left( \frac{df}{dt} \right)_{\text{collisions}} = I_{\text{e-e}}, \tag{20}
$$

where $\mathbf{v}_k = \nabla_{\mathbf{k}} \xi_k$ is the electron velocity and $I_{\text{e-e}}$ is the electron-electron collision term.

E. Electron umklapp scattering

For the evaluation of the electron-electron collision in the Boltzmann equation (20), the mechanism of the momentum relaxation needs to be introduced.\(^{12,13}\) After intensive investigations over more than three decades, although the mechanism of the momentum relaxation for the T-linear resistivity still remains controversial, the electron umklapp scattering is believed to be at the heart of the striking behaviour of the electrical transport in the strange-metal phase of cuprate superconductors. In this paper, we adopt the electron umklapp scattering as the mechanism of the momentum relaxation, and then study the electrical transport in the strange-metal phase of cuprate superconductors. For a convenience in the following discussions, the schematic picture for the electron umklapp scattering process is illustrated in Fig. 4, where an electron on a circular EFS (left) is scattered by its partner on the umklapp electron Fermi surface (right), where the intensity map of the electron Fermi surface is the same as shown in Fig. 2a, while the perfect circle (red) is the circle with the radius $k_F^{\text{TFA}}$ with $k_F^{\text{TFA}}$ that is the Fermi wave vector of the tips of the Fermi arcs. An electron on the electron Fermi surface (left) parametrized by the Fermi angle $\theta$ is scattered to a point parametrized by the Fermi angle $\theta'$ on the umklapp electron Fermi surface (right) by the spin excitation carrying momentum $p(\theta, \theta')$. This physical picture is repeated for the other three umklapp electron Fermi surfaces that are closest to the original electron Fermi surface.
radius $k_{F}^{TFA}$, where $k_{F}^{TFA}$ is the Fermi wave vector of the tips of the Fermi arcs. This perfect circle EFS (red) connects all the eight tips of the Fermi arcs, and therefore shows that almost all the spectral weight of the electron excitation spectrum is accommodated on this circle EFS.

In the present case, the umklapp scattering of electrons is mediated by the same spin excitation as in the case of the electron scattering from a critical mode $\phi_{c}$, which defines a contour $\{\phi_{k} \}$. The spectral weight of the electron-electron collision (21) is negligible.

The reason of the electron-electron scattering via the exchange of the effective spin propagator in the present case is the same as in the case discussed in Ref. 53. For the normal scattering ($G=0$), the conservation of the total momentum in Eq. (21) is satisfied straightforwardly, this is because the distribution in the case of the normal scattering will rapidly equilibrate to a fermion distribution function with a shifted overall momentum $\Phi_{k}(\kappa) \propto \kappa \cdot E$, which leads to that its contribution to the integral of the electron-electron collision in Eq. (21) is exactly zero. However, if we consider the scattering between electrons via the emission and absorption of the spin excitations, we would have to keep track of the extra shift boson distribution function as well, which introduces more complications. Moreover, it has been shown that the vanishing of the normal scattering in the electron-electron collision (21) is more general. This is following a basic fact that in order to stay on EFS and conserve the total momentum and energy, the momentum of the normal scattering partner $k'$ must equal to either $k + p$ or $-k$. In the former case, the last two terms in $\{\ldots\}$ in Eq. (21) cancel the first two terms. However, in the latter case, the antisymmetric relation $\Phi(-k) = -\Phi(k)$, the first two terms in $\{\ldots\}$ in Eq. (21) cancel, while the same cancellation is valid for the last two terms corresponding to the outgoing pair $k + p$ and $k' - p$. These results therefore indicate that the contribution from the normal scattering to the integral of the electron-electron collision (21) is negligible.

In the usual case, the derivation of the Boltzmann equation starting from the nonequilibrium electron propagator involves integrating over energy $\omega$. However, the electrons at the bottom of the band (then the deep inside EFS) can not be thermally excited, and as a matter of the principle, all the low-temperature conduction processes in the strange-metals should involve only states at around EFS. In particular, in the early discussions, it has been realized to pick a patch of EFS specified by $k(\theta)$ with the range $\theta \in [0, 2\pi]$, which defines a contour along EFS parametrized by the direction $\theta$ of the Fermi momentum vector and integrate the perpendicular momentum and hence over $\varepsilon_{k}$ instead. This is a formula expressed entirely in terms of the EFS property. Furthermore, this method has been employed to study the low-temperature $T$-linear resistivity due to the umklapp scattering from a critical mode. In the present case of the umklapp scattering of electrons mediated by the spin excitation, an electron on EFS parametrized by the Fermi
angle \( \theta \) is scattered to a point parametrized by the Fermi angle \( \theta' \) on the umklapp EFS by the spin excitation carrying momentum \( p(\theta, \theta') \) as shown in Fig. 4. In this case, the usual distribution function \( f(k) \) can be replaced as \( f[k(\theta)] \). However, in the usual formulation, the vector \( k \) is decomposed into \( k(\theta) \) and the momentum in the perpendicular direction, which is then represented by \( \varepsilon_k \). From this EFS parametrization, the standard Boltzmann equation (20) can now be expressed simply, where the component of the momentum \( k \) perpendicular to EFS is replaced by \( \varepsilon_k / v_k \) and \( \varepsilon_k \) in turn is replaced by \( \omega \). After a straightforward calculation [see Appendix A], the electron-electron collision \( I_{\text{ee}} \) in Eq. (21) can be derived explicitly, and then the Boltzmann equation (20) can be obtained as,

\[
e_{v_F}(\theta) \cdot E = -2 \int \frac{d\theta'}{2\pi} \zeta(\theta') F(\theta, \theta') [\Phi(\theta) - \Phi(\theta')],
\]

where \( \Phi(\theta) \) is defined as \( \Phi[k(\theta)] \) and satisfies an antisymmetric relation \( \Phi(\theta) = -\Phi(\theta + \pi) \). \( v_F(\theta) \) is the Fermi velocity at the Fermi angle \( \theta \), and \( \zeta(\theta') = k^2_{\perp}/[4\pi^2 v_F^2] \) is the density of states factor at angle \( \theta' \) with the Fermi wave vector \( k_F \) and Fermi velocity \( v_F \), while the coefficient of \( \Phi(\theta) \) in the first term of the right-hand side of Eq. (23),

\[
\gamma(\theta) = 2 \int \frac{d\theta'}{2\pi} \zeta(\theta') F(\theta, \theta'),
\]

is defined as the scattering out rate, which also is so-called as the angular dependence of the transport scattering rate, while the kernel function \( F(\theta, \theta') \) depends on the Fermi angles \( \theta \) and \( \theta' \) in terms of the magnitude of the momentum transfer \( p(\theta, \theta') \), i.e., \( F(\theta, \theta') \) connects the points \( \theta \) and \( \theta' \) on the umklapp EFS as shown in Fig. 4, and is given by,

\[F(\theta, \theta') = \frac{1}{T} \int \frac{d\omega}{2\pi} \frac{\omega^2}{p(\theta, \theta')} \left| P[k(\theta), p(\theta, \theta'), \omega] \right|^2 \times n_B(\omega)|1 + n_B(\omega)|,\]

where the reduced effective spin propagator \( P[k(\theta), p(\theta, \theta'), \omega] \) has been given in Appendix A. This kernel function \( F(\theta, \theta') \) can be also called as the probability weight or the strength of the umklapp scattering in which the electron at the \( \theta \) point of EFS is scattered by its umklapp partner at the \( \theta' \) point of the neighboring EFS.

### III. LOW-TEMPERATURE LINEAR IN TEMPERATURE RESISTIVITY

The dc conductivity then is evaluated in a standard way by the momentum (then the Fermi angle \( \theta \)) integral of the umklapp scattering process on EFS, where the current density is given by \( J = e_n 1/N \sum_k v_k f(k) \), with the relaxation of the momentum that is generated by the action of the electric field on the mobile electrons at around EFS with the density \( n_0 \). Substituting the distribution function \( f(k) \) in Eq. (19) into the above current density equation (26) and performing the radial integration, the current density now can be obtained as,

\[J = e n_0 \frac{1}{N} \sum_k v_k \frac{d\epsilon_k}{d\epsilon_k} \tilde{\Phi}(k) = -e n_0 \frac{k_F}{v_F} \int \frac{d\theta}{(2\pi)^2} \nu_F(\theta) \Phi(\theta).\]

For the calculation of the dc conductivity, we need to obtain the local shift of the chemical potential \( \Phi(\theta) \). The spectral weight of \( \text{Im} P(k_F, p - k_F, k'_F, \omega) \) in Eq. (21) achieves its maximal value at around the antinodal region [see Fig. 11 in Appendix A], where the scattering probability for two electrons is largest. In other words, the main contribution to the kernel function \( F(\theta, \theta') \) [then the electrical resistivity] comes from such umklapp scattering process in which the electron at around the antinodal region of the circular EFS (left) shown in Fig. 4 is scattered by its partner around the antinodal region of the umklapp circular EFS (right), where the Fermi angle \( \theta' \) is almost identical with the Fermi angle \( \pi - \theta \), and then according to the antisymmetric relation satisfied by \( \Phi(\theta) \), the following relation,

\[\Phi(\theta') = \Phi(\pi - \theta) = -\Phi(\theta),\]

is valid. In this relaxation-time approximation, the local shift of the chemical potential \( \Phi(\theta) \) can be derived straightforwardly from Eqs. (23) and (24) as,

\[\Phi(\theta) = -\frac{e v_F \cos(\theta) E_{\perp}}{2\gamma(\theta)},\]

where the electric field \( E \) has been chosen along the \( \hat{x} \)-axis. Substituting the above result of \( \Phi(\theta) \) into Eq. (27), the dc conductivity therefore is obtained explicitly as,

\[\sigma_{dc}(T) = \frac{1}{2} e^2 n_0 k_F v_F \int \frac{d\theta}{(2\pi)^2} \cos^2(\theta) \frac{1}{\gamma(\theta)},\]

and then the electrical resistivity is obtained directly from the above dc conductivity as,

\[\rho(T) = \frac{1}{\sigma_{dc}(T)}.\]

Now we are ready to discuss the striking features of the electrical transport in the strange-metal phase of cuprate superconductors. We have made a series of calculations for the electrical resistivity \( \rho(T) \) in Eq. (31) at different doping levels, and the results of the electrical resistivity \( \rho(T) \) as a function of temperature at the doping concentrations \( \delta = 0.09 \) (black-line), \( \delta = 0.12 \) (red-line), \( \delta = 0.15 \) (green-line), \( \delta = 0.18 \) (blue-line), and \( \delta = 0.21 \) (yellow-line) are plotted in Fig. 6. Apparently, the experimental results of the doping dependence of the low-temperature electrical resistivity\(^{29-41}\) are qualitatively reproduced, where the highly unconventional
features can be summarized as: (i) the electrical resistivity $\rho(T)$ as a function of temperature is a perfect straight line down to the temperature $T \sim 0.015J = 15K$; (ii) the low-temperature T-linear resistivity extends over a range of doping from the underdoped to overdoped regimes, where the T-linear coefficient (then the strength of the T-linear resistivity) decreases with the increase of the doping concentration; (iii) however, the electrical resistivity deviates from the pure T-linearity at the far lower temperature range $T < 0.015J = 15K$, while our numerical fit indicates that in this far lower temperature range $T < 0.015J = 15K$, the electrical resistivity decreases quadratically as the temperature decreases. The results in Fig. 6 therefore also indicate that the same spin excitation that acts like a bosonic glue to hold the electron pairs together responsible for superconductivity\cite{68-71} also dominates the electron scattering responsible for the low-temperature T-linear resistivity in the strange-metal phase and the associated electronic structure.

Finally, it should be emphasized that the local shift of the chemical potential $\Phi(\theta)$ can be also evaluated directly by the numerical solution of the Boltzmann equation (23) together with an additional electron-impurity collision without making the relaxation-time approximation\cite{33}, where the Fermi angle $\theta'$ variable in Eq. (23) can be discretized, and then the integral-differential equation (23) is converted into the matrix equation. The accurate result of $\Phi(\theta)$ is obtained in terms of the numerical calculation of the inverse of this matrix. In this case, we have also performed a numerical calculation $\Phi(\theta)$ [then $\rho(T)$], and the results show that although the resistivity saturates to a constant $\rho_0(T)$ induced by the impurity, the qualitative behaviour of the electrical resistivity is the same as that obtained in the above relaxation-time approximation except for the subtle difference of the slopes, which is also qualitatively consistent with the results obtained from electrons umklapp scattered by a critical bosonic mode\cite{33}.

An explanation of the above obtained low-temperature T-linear resistivity in the strange-metal phase of cuprate superconductors can be found from the Fermi angle and temperature dependence of the transport scattering rate $\gamma(\theta,T)$ in Eq. (24) obtained directly from the electron umklapp scattering process. To reveal this angular and temperature dependence of $\gamma(\theta,T)$ more clearly, we first plot $\gamma(\theta)$ as a function of Fermi angle $\theta$ at $\delta = 0.15$ with $T = 0.05J$ in Fig. 7. In a comparison with the corresponding angular dependence of the single-particle scattering rate $\Gamma(\theta)$ in Fig. 3, it thus shows that although the magnitude of $\gamma(\theta)$ at an any given Fermi angle is less than that of $\Gamma(\theta)$ at the corresponding Fermi angle, the global behaviour of the angular dependence of $\gamma(\theta)$ is similar to that of $\Gamma(\theta)$, where $\gamma(\theta)$ is largest at around the antinodal region, and smallest at around the tips of the Fermi arcs, which is also consistent with the strong momentum dependence of the effective spin propagator $P(k,p-k,k',\omega)$ shown in Fig. 11 in Appendix A. In other words, both the transport scattering rate $\gamma(\theta)$ and single-particle scattering rate $\Gamma(\theta)$ is similar to the corresponding Fermi angle presents the similar behavior of the effective spin propagator $P(k,p-k,k',\omega)$. The result in Fig. 7 also shows that the electrons at around the tips of the Fermi arcs are mainly responsible for the conductivity, while the transport scattering rates at both the antinodal and nodal regions mainly determine the magnitude of the electrical resistivity and of its behaviour of the temperature dependence.

On the other hand, for any given Fermi angle $\theta$, $\gamma(\theta,T)$ varies strongly with temperature. To see this temperature dependence of $\gamma(\theta,T)$ more clearly, we plot $\gamma(T)$ as a function of temperature for $\delta = 0.15$ at the antinode in Fig. 8. It is surprising that $\gamma(T)$ is entirely T-linear in the low-temperature range $T > 0.015J = 15K$, where it decreases linearly with temperature as the temperature decreases to $T \sim 0.015J = 15K$, while this transport scattering rate $\gamma(T)$ is instead quadratic in temperature (T-quadratic) in the far lower temperature range $T < 0.015J = 15K$. Moreover, although $\gamma(\theta,T)$ is highly anisotropic in momentum-space, this low-temperature T-linear $\gamma(\theta,T)$ occurs at any given
Fermi angle \( \theta \) (then for all directions), in agreement with the experimental observations\(^{41} \). In a comparison with the corresponding results of the temperature dependence of the electrical resistivity shown in Fig. 6, we find that the low-temperature T-linear behaviour of \( \gamma(T) \) together with the low-temperature range and the T-quadratic behaviour of \( \gamma(T) \) together with the far lower temperature range are respectively the same as the corresponding behaviours and ranges in the electrical resistivity \( \rho(T) \), which therefore indicates that the remarkable T-linear resistivity in the low-temperature and the T-quadratic resistivity in the far lower temperature are generated by the T-linear transport scattering rate in the low-temperature and T-quadratic transport scattering rate in the far lower temperature, respectively.

The expression form of the transport scattering rate \( \gamma(\theta, T) \) in Eq. (24) also indicates that the exotic behaviour of the temperature dependence of \( \gamma(\theta, T) \) is mainly determined by the striking features of the temperature dependence of the kernel function (then the probability weight or the strength of the electron umklapp scattering) \( F(\theta, \theta') \) in Eq. (25). With the help of \( \gamma(\theta, T) \) in Eq. (24) and \( F(\theta, \theta') \) in Eq. (25), we now turn to show that (i) the antinodal umklapp scattering, in which the electron at around the antinodal region of EFS is scattered by its umklapp partner at around the antinodal region of the neighboring EFS, leads to the T-linear behaviour of \( \gamma(\theta, T) \) (then the T-linear resistivity); and (ii) the nodal umklapp scattering, in which the electron at around the nodal region of EFS is scattered by its umklapp partner at around the nodal region of the neighboring EFS, tends to induce a deviation from the T-linear behaviour. To see this physical picture more clearly, the surface plot of \( F(\theta, \theta') \) at \( \delta = 0.15 \) with \( T = 0.05J \) is plotted in Fig. 9, where the probability weight of the electron umklapp scattering has been separated into three characteristic regions: (i) the antinodal region, where a particularly large fraction of the probability weight is located, leading to that \( \gamma(\theta) \) is largest at around the antinodal region; (ii) the nodal region, where a small amount of the probability weight is inhabited, leading to that the magnitude of \( \gamma(\theta) \) at around the nodal region is much smaller than that at around the antinodal region; (iii) the region at around the tips of the Fermi arcs, where the strength of the umklapp scattering is anomalously small, leading to the appearance of the weakest scattering at around the tips of the Fermi arcs. The above result in Fig. 9 indicates that the electron umklapp scattering is concentrated at around the antinodes and nodes, and therefore is well consistent with the result of the angular dependence of \( \gamma(\theta, T) \) shown in Fig. 7. However, the strengths of the antinodal and nodal umklapp scattering are strong temperature dependent, which induces a competition between the antinodal umklapp scattering and nodal umklapp scattering. This competition is closely related to the crossover from the T-linear behaviour of \( \gamma(T) \) in the low-temperature into the T-quadratic behaviour in the far lower temperature, and can be well understood in terms of the ratio of the strength of the nodal umklapp scattering to the strength of the antinodal umklapp scattering,

\[
R_F(T) = \frac{F(\theta_{\text{AN}}, \theta_{\text{ND}}, T)}{F(\theta_{\text{AN}}, \theta_{\text{AN}}, T)}.
\]

However, as we have mentioned in subsection II B, the calculation in this paper is performed numerically on a 160 x 160 lattice in momentum space, with the infinitesimal \( \delta_{0+i} \rightarrow i\Gamma \) replaced by a small damping \( \Gamma = 0.1J \), which leads to that the weight of the \( \delta \)-function type peak in \( F(\theta, \theta') \) at the antinode (node) spreads on the extremely small area \( \{\theta_{\text{AN}}\} \{\theta_{\text{ND}}\} \) around the antinode (node) as shown in Fig. 9. In particular, the summation of these spread weights around this extremely small area \( \{\theta_{\text{AN}}\} \{\theta_{\text{ND}}\} \) is less affected by the calculation for a finite lattice. In this case, a more appropriate ratio can be obtained as,

\[
\bar{R}_F(T) = \frac{\bar{F}_{\text{ND}}(T)}{\bar{F}_{\text{AN}}(T)},
\]

for the reduction of the size effect in the finite-lattice environment.
calculation, where \( \bar{F}_{\text{AN}}(T) \) and \( \bar{F}_{\text{ND}}(T) \) are given by,

\[
\bar{F}_{\text{AN}}(T) = \frac{1}{2\pi} \sum_{\theta_{\text{AN}} \in \{\theta_{\text{AN}}\}} F(\theta, \theta', T),
\]

\[
\bar{F}_{\text{ND}}(T) = \frac{1}{2\pi} \sum_{\theta_{\text{ND}} \in \{\theta_{\text{ND}}\}} F(\theta, \theta', T),
\]

with the summation \( \theta_{\text{AN}} \in \{\theta_{\text{AN}}\}, [\theta'_{\text{AN}} \in \{\theta'_{\text{AN}}\}] \) that is restricted to the extremely small area \( \{\theta_{\text{AN}}\} \) \([\theta_{\text{ND}}]\) around the antinode (node). In this case, we have carried out a series of calculation for the ratio \( \bar{R}_F(T) \) at different doping levels, and the result of \( \bar{R}_F(T) \) as a function of temperature at \( \delta = 0.15 \) is plotted in Fig. 10, where \( \bar{R}_F(T) \) decreases monotonically with the increase of temperature. In particular, in the lower ratio range \( \bar{R}_F(T) < 0.395 \), which is corresponding to the low-temperature range \( T > 0.015J = 15K \), the strength of the nodal umklapp scattering is quite weak, while the strength of the antinodal umklapp scattering is particularly strong. This particularly strong antinodal umklapp scattering therefore leads to the low-temperature T-linear behaviour of \( \gamma(T) \). In other words, the low-temperature T-linear behaviour of \( \gamma(T) \) (then the T-linear resistivity) is mainly governed by the antinodal umklapp scattering. However, although both the strengths of the nodal and antinodal umklapp scattering decrease with the decrease of temperature, the decrease of the strength of the nodal umklapp scattering is slower than that of the antinodal umklapp scattering. In particular, in the higher ratio range \( \bar{R}_F(T) > 0.395 \), which is corresponding to the far lower temperature range \( T < 0.015J = 15K \), the antinodal umklapp scattering becomes strong enough to generate a deviation from the T-linear behaviour, and then the T-quadratic behaviour of \( \gamma(T) \) (then the T-quadratic resistivity) in the far lower temperature results from both the antinodal and nodal umklapp scattering.

IV. SUMMARY

In the framework of the kinetic-energy-driven superconductivity, the electrons scattering with the spin excitation forms the strange-metal liquid at the temperature above \( T_c \), where the energy distribution curve exhibits a complicated line-shape at around the nodal and antinodal regions, the electron dispersion has an anomalous dispersion kink along EFS, and more specifically, the highly anisotropic single-particle scattering rate induces an redistribution of the spectral weight on EFS to form the Fermi arcs with almost all the spectral weight assembled at around the tips of the Fermi arcs. We start from this low-energy electronic structure in the strange-metal phase of cuprate superconductors to study the nature of the low-temperature T-linear resistivity. In particular, we derive explicitly the Boltzmann equation, and then employ it to discuss the doping dependence of the electrical resistivity, where the angular dependence of the transport scattering rate originates from the umklapp scattering between electrons by the exchange of the same spin excitations. Our results show that although the magnitude of the angular dependence of the transport scattering rate at any given Fermi angle is smaller than the corresponding value of the single-particle scattering rate, the transport scattering rate presents the similar behaviour of the single-particle scattering rate, where the transport scattering rate is largest at around the antinodal region and smallest at around the tips of the Fermi arcs, indicating that the electrical resistivity is mainly dominated by the antinodal umklapp scattering and nodal umklapp scattering. Moreover, the antinodal umklapp scattering tends to induce the T-linear behaviour of the transport scattering rate, while the nodal umklapp scattering tends to generate a deviation from the T-linear behaviour, therefore there is a competition between the antinodal umklapp scattering and nodal umklapp scattering. However, in the low-temperature range, the nodal umklapp scattering with a quite weak scattering strength is overwhelmed by the antinodal umklapp scattering with a particularly strong scattering strength, and then this particularly strong antinodal umklapp scattering leads to the T-linear behaviour of the transport scattering rate. On the other hand, in the far lower temperature range, the strength of the nodal umklapp scattering becomes strong enough to induce a deviation from the T-linear behaviour, and then both the nodal and antinodal umklapp scattering leads to the T-quadratic behaviour of the transport scattering rate. This T-linear behaviour of the transport scattering rate in the low-temperature and the T-quadratic behaviour in the far lower temperature in turn generate the T-linear resistivity in the low-temperature and the T-quadratic resistivity in the far lower temperature. Our theory also shows that the same spin excitation that acts like a bosonic glue to hold the electron pairs together responsible for the exceptionally high \( T_c \) also mediates the electron scattering in the strange-metal phase responsible for the T-linear resistiv-
ity and the associated electronic structure.

The transport scattering mechanism developed in this paper for the understanding of the T-linear resistivity in the strange-metal phase of cuprate superconductors can be also employed to study the exotic transport in other families of strange metals\textsuperscript{80,81}. In particular, based on this transport scattering theory, we have also discussed the striking T-linear resistivity in the normal-state of the electron-doped cuprate superconductors\textsuperscript{82}, where we show the common mechanism linking the transport in the normal-state of both the hole- and electron-doped cuprate superconductors. These and the related works will be presented elsewhere.

$$I_{\text{e-e}} = \frac{1}{N^2} \sum_{k',p} \frac{2}{T} |P(k, p - k, k', \bar{\varepsilon}_k - \bar{\varepsilon}_{p+G})|^2 [\bar{\Phi}(k) + \bar{\Phi}(k') - \bar{\Phi}(p + G) - \bar{\Phi}(k' + k - p)]$$

$$\times n_F(\bar{\varepsilon}_k) n_F(\bar{\varepsilon}_{k'}) [1 - n_F(\bar{\varepsilon}_{p+G})] [1 - n_F(\bar{\varepsilon}_{k'+k-p})] \delta(\bar{\varepsilon}_k + \bar{\varepsilon}_{k'} - \bar{\varepsilon}_{p+G} - \bar{\varepsilon}_{k'+k-p}),$$

(A1)

with the effective spin propagator $P(k, p - k, k', \omega)$ that can be expressed explicitly as,

$$P(k, p - k, k', \omega) = \frac{1}{N} \sum_{q} A_{p+q} A_{q+k} \Pi(p - k, q, \omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \frac{\text{Im}P(k, p - k, k', \omega')}{\omega' - \omega},$$

(A2)

where the imaginary part of the effective spin propagator $\text{Im}P(k, p - k, k', \omega)$ is directly related to the effective spin spectral function, and is also defined as the scattering probability for two electrons. However, in our pre-

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**Appendix A: Derivation of electron-electron collision**

The aim of this Appendix is to derive the electron-electron collision $I_{\text{e-e}}$ in Eq. (23) of the main text. The electron-electron collision in Eq. (21) can be also rewritten as,

$$\omega = 0, \text{where } \text{Im}P(k, p - k, k', 0) = 0.$$ To see this unusual momentum $p$ dependence of $\text{Im}P(k, p - k, k', \omega)$ more clearly, we plot the intensity map of $\text{Im}P(k, p - k, k', \omega)$ along EFS $k = k' = k_F$ in a $[p_x, p_y]$ plane for energy $\omega = -0.05J$ with $T = 0.002J$ in Fig. 11, where the spectral weight of $\text{Im}P(k_F, p - k_F, k_F', \omega)$ along EFS $k = k' = k_F$ converges on the corresponding EFS $p = k_F$, i.e., $\text{Im}P(k_F, p - k_F, k_F', \omega) \neq 0$ for $p = k_F$, and otherwise $\text{Im}P(k_F, p - k_F, k_F', \omega) = 0$. In particular, the spectral weight of $\text{Im}P(k_F, p - k_F, k_F', \omega)$ exhibits the largest value at around the antinodal region, however, the most striking feature is that the actual minimum of the spectral weight of $\text{Im}P(k_F, p - k_F, k_F', \omega)$ does not appear at around the node, but locates exactly at the tips of the Fermi arcs. This special angular dependence of $\text{Im}P(k_F, p - k_F, k_F', \omega)$ therefore induces an EFS reconstruction to form the Fermi arcs as shown in Fig. 2 with almost all the spectral weight of the electron excitation spectrum that resides at around the tips of the Fermi arcs.

The result shown in Fig. 11 therefore indicates that the main contribution in $\text{Im}P(k, p - k, k', \omega)$ comes from the part of the momentum $p = k$. In this case, the term $\Phi(k') - \Phi(k' + k - p) \sim 0$ in the right-hand side of Eq. (A1), and then $\delta(\bar{\varepsilon}_k + \bar{\varepsilon}_{k'} - \bar{\varepsilon}_{p+G} - \bar{\varepsilon}_{k'+k-p})$ in the right-hand side of Eq. (A1) can be replaced by the integral identity as\textsuperscript{53},

![FIG. 11. (Color online) The intensity map of the imaginary part of the effective spin propagator $\text{Im}P(k, p - k, k', \omega)$ along $k = k' = k_F$ in a $[p_x, p_y]$ plane at $\delta = 0.15$ for energy $\omega = -0.05J$ with $T = 0.002J$.](image-url)
\[ \delta(\xi_k + \xi_{k'} - \xi_{p+G} - \xi_{k'+k-p}) = \int_{-\infty}^{\infty} d\omega \delta(\xi_k - \xi_{p+G} - \omega) \delta(\omega + \xi_{k'} - \xi_{k'+k-p}). \] (A3)

On the other hand, the umklapp scattering process occurs mainly at around EFS, i.e., \( k' \approx k'_F \), therefore the momentum \( k' \) in the effective spin propagator \( P(k, p - k, k', \omega) \) can be approximately replaced by the reduced effective spin propagator \( \tilde{P}(k, p - k, \omega) \),

\[ \tilde{P}(k, p - k, \omega) = \frac{1}{W_{sp}} P(k, p - k, k'_F, \omega), \] (A4)

where following the common practice, the scattering probability for two electrons has been normalized with the normalization factor \( W_{sp}^2 = (1/N^2) \sum_{k,p} \int |\text{Im} \tilde{P}(k, p - k, \omega)|^2 d\omega \). Substituting above results in Eqs. (A3) and (A4) into Eq. (A1), the electron-electron collision in Eq. (A1) can be expressed explicitly as\(^{53}\),

\[ I_{e-e} = \frac{1}{N^2} \sum_{k', p} \frac{2}{T} |\tilde{P}(k, p - k, \xi_k - \xi_{p+G})|^2 [\tilde{\Phi}(k) - \tilde{\Phi}(p + G)] n_F(\xi_k) n_F(\xi_{k'}) [1 - n_F(\xi_{p+G})] [1 - n_F(\xi_{k'+k-p})]
\times \int_{-\infty}^{\infty} d\omega \delta(\xi_k - \xi_{p+G} - \omega) \delta(\omega + \xi_{k'} - \xi_{k'+k-p})
= \frac{1}{N^2} \sum_{k', p} \frac{2}{T} |\tilde{P}(k, p, \xi_k - \xi_{p+k+G})|^2 [\tilde{\Phi}(k) - \tilde{\Phi}(p + k + G)] n_F(\xi_k) n_F(\xi_{k'}) [1 - n_F(\xi_{p+k+G})] [1 - n_F(\xi_{k'} - \omega)]
\times \int_{-\infty}^{\infty} d\omega \delta(\xi_k - \xi_{p+k+G} - \omega) \delta(\omega + \xi_{k'} - \xi_{k'-p}). \] (A5)

Now we replace the momentum \( k' \) integration by an integration along EFS and one perpendicular to it, i.e., \((1/N) \sum_{k'} = \int k'dk'/(2\pi)^2\), where the \( \theta_{k'} \) specifies a patch of EFS in the direction \( \theta_{k'} \) as shown in Fig. 4,

\[ I_{e-e} = \frac{1}{2\pi} \frac{2k_F}{TV_F} \frac{1}{N} \sum_p \frac{1}{|p|} \int \frac{d\omega}{2\pi} |\tilde{P}(k, p, \xi_k - \xi_{p+k+G})|^2 [\tilde{\Phi}(k) - \tilde{\Phi}(p + k + G)] n_F(\xi_k) [1 - n_F(\xi_{p+k+G})]
\times \omega [1 + n_B(\omega)] \delta(\xi_k - \xi_{p+k+G} - \omega). \] (A6)

For the obtain of the above equation (A6), the following identity,

\[ \int_{-\infty}^{\infty} d\varepsilon n_F(\varepsilon - \omega) [1 - n_F(\varepsilon)] = \omega [1 + n_B(\omega)], \] (A7)

has been used, where the appearance of the boson distribution function \( n_B(\omega) \) in the right-hand side signals that we are describing a particle-hole effective spin excitation which has the boson statistics\(^{53}\).

Now we turn to evaluate the momentum \( p \) integration, which is quite similar to the evaluation of the momentum \( k' \) integration in Eqs. (A5) and (A6). After a straightforward calculation for the momentum \( p \) integration in Eq. (A6), the electron-electron collision term can be obtained explicitly as

\[ I_{e-e} = \frac{1}{(2\pi)^2} \frac{2k_F^2}{TV_F} \int \frac{d\theta'}{2\pi} \int \frac{d\omega}{2\pi} \sum_{p} \frac{1}{p(\theta', \theta')} |\tilde{P}[k(\theta), p(\theta', \omega)]|^2 [\tilde{\Phi}(\theta) - \tilde{\Phi}(\theta')] n_F(\xi_k(\theta)) [1 - n_F(\xi_k(\theta) - \omega)] \omega [1 + n_B(\omega)], \] (A8)

where \( \tilde{\Phi}(k) \) and \( \tilde{\Phi}(p + k + G) \) in the right-hand side of have been replaced by \( \Phi(\theta) \) and \( \Phi(\theta') \), respectively, and at low-energy regime, the Boltzmann equation in Eq. (20) can be expressed as,

\[ eV_F \cdot \mathbf{E} \frac{\partial n_F(\xi_k(\theta))}{\partial \xi_k(\theta)} = \frac{1}{(2\pi)^2} \frac{2k_F^2}{TV_F} \int \frac{d\theta'}{2\pi} \int \frac{d\omega}{2\pi} \sum_{p} \frac{1}{p(\theta, \theta')} |\tilde{P}[k(\theta), p(\theta', \omega)]|^2 [\tilde{\Phi}(\theta) - \tilde{\Phi}(\theta')] \times n_F(\xi_k(\theta)) [1 - n_F(\xi_k(\theta) - \omega)] \omega [1 + n_B(\omega)]. \] (A9)
Integrating both the left-hand and right-hand sides over the energy $\varepsilon_k(\theta)$, the Boltzmann equation in Eq. (A9) can be obtained explicitly as,

$$
\epsilon v_F \cdot \mathbf{E} = -\frac{1}{(2\pi)^2} \frac{2k_F^2}{T_F^3} \int \frac{d\theta'}{2\pi} \int \frac{d\omega}{2\pi} \frac{1}{p(\theta, \theta')} \left[ \bar{P}[k(\theta), p(\theta, \theta'), \omega] \right]^2 \left[ \Phi(\theta) - \Phi(\theta') \right] \omega^2 n_B(\omega) \left[ 1 + n_B(\omega) \right]
$$

\hspace*{1cm} (A10)

which is the same as quoted in Eq. (23) of the main text.

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