Pion Form Factor: Transition From Soft To Hard QCD

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Abstract

We have reexamined the elastic pion form factor over a broad range of momentum transfers with the mass evolution from current quark to constituent quark being taken into account. We have also studied the effect of Sudakov form factors, of anomalous quark magnetic moments and of alternative soft wave functions. Our calculation shows a power-law falloff from the present experimental values to near-asymptotic values in the few GeV$^2$ range.
1 Introduction And Review Of Method

Quark and gluon structure functions of hadrons have been extracted via inclusive processes, using experimental data at a few $GeV^2$ and above. In the analysis of inclusive processes at $Q^2$ of a few $GeV^2$ and above QCD enters mainly in the evolution of the parton distribution functions and in higher-twist scaling violations, both of which are generally treated in perturbative QCD (PQCD). In contrast to exclusive processes, both nonperturbative and perturbative QCD are needed explicitly for the treatment of inclusive processes even at very high $Q^2$. It is important to use exclusive processes for the study of the quark and gluon structure of hadrons, and to learn more about the nature of QCD, but there are a number of theoretical problems which must be solved.

It has been suggested\cite{1} that elastic form factors can be calculated neglecting transverse momentum and using a simple transition operator arising from single-gluon exchange between quark pairs—the "hard scattering" model—for $Q^2$ above a few $GeV^2$ (about 1 $GeV$ for the pion), i.e., at momentum transfers where inclusive form factors scale. The $Q^2$ dependence of the elastic form factors predicted by the hard scattering model is consistent with present experimental data for $Q^2$ greater than about 1 $GeV^2$ for the pion and 3 $GeV^2$ for the nucleon, respectively. However, the magnitudes of both elastic form factors are in strong disagreement with the known theoretical large-$Q^2$ limit by a factor of three for the pion and about two orders of magnitude for the nucleon at the largest $Q^2$ for which there is reliable data. This raises serious questions about the validity of the hard-scattering model for the $Q^2$ regions which have now been reached in experiments.

Moreover, one essential assumption of the hard scattering model is that "soft" contributions are negligible. This is a very controversial subject\cite{2}. Since the soft contributions which are calculated in quark models are determined mainly by the radius of the pion, it is known that the soft contributions are consistent with experiment at values of $Q^2$ at which the hard scattering model is often applied. The hard scattering model is clearly inconsistent at that point. There is an extensive body of literature on the general subject of exclusive processes and QCD, and we shall discuss a number of the most important points for the case
of the pion elastic form factor in the present work.

We have been developing a comprehensive picture based on the light-cone Bethe-Salpeter (LCBS) approach\[3, 4] for the treatment of the elastic pion form factor. The elastic pion form factor is related to pion electromagnetic current by the following equation

\[ < P' | J^\mu(0) | P > = F_\pi(Q^2)(P + P')^\mu \]  

(1)

In a quark-antiquark LCBS picture, this can be expressed as a convolution integral in terms of LCBS amplitudes, \( \Psi(x, k_\perp) \):

\[ < P' | J^+(0) | P > = \sum_{i,j} \int [dx][d^2k_\perp] \frac{\Gamma^+}{16\pi^3} \frac{\bar{u}_i}{\sqrt{k_i^+}} \frac{u_j}{\sqrt{k_j^+}} \Psi^*(x, k'_\perp) \Psi(x, k_\perp) \]  

(2)

where \( \Gamma^+ = f_{q1}\gamma^+ + \frac{i}{2m_q}\sigma^{+\nu}t_\nu f_{q2} \) is the quark vertex function, and \( f_{q1}(t^2) \), \( f_{q2}(t^2) \) are quark form factors. These quark form factors are normalized by \( f_{q1}(t^2 = 0) = e_q \) (quark charge), \( f_{q2}(t^2 = 0) = \kappa_q \) (quark anomalous magnetic dipole moment). We take \( \kappa_q \) as a parameter.

The LCBS equation has the form

\[ \Psi(x, k_\perp) = \int dx'd^2k'_\perp K(x, k_\perp; x', k'_\perp) \Psi(x', k'_\perp) \]  

(3)

where \( K \) is the kernel. The hard scattering model can be obtained from Eqs. (2,3) by using the one gluon exchange operator for \( K \) and neglecting transverse momenta\[1\].

For a comprehensive program starting from the LCBS approach one needs a realistic kernel which includes both the confining and asymptotic freedom aspects of QCD. In our early work\[3\] we used the form

\[ \Psi(x, k_\perp) = \int dx'd^2k'_\perp [K_{conf}(x, k_\perp; x', k'_\perp) + K_{ge}(x, k_\perp; x', k'_\perp)] \Psi(x', k'_\perp) \]  

(4)

and used a relativistic string for the confining kernel, \( K_{conf} \), while the gluon exchange kernel, \( K_{ge} \) is obtained from light-cone PQCD \[4\]. The confining kernel is not known, and even if an accurate phenomenological form were determined through study of the pion, it could not be used for the nucleon, which probably contains important three-quark interactions. In our recent work\[4\] we have avoided the difficult problem of finding a phenomenological
confining potential by using a model for the soft amplitude. I.e., we recognize that the soft BS amplitude, $\Psi_s(x, k_\perp)$ can be considered to be the solution of the equation

$$\Psi^s(x, k_\perp) \equiv \int dx' d^2k'_\perp K_{conf}(x, k_\perp; x', k'_\perp) \Psi^s(x', k'_\perp)$$  \hspace{1cm} (5)$$

Iterating Eq. (5) by inserting $\Psi^s$ for $\Psi$ one obtains the approximate form

$$\Psi(x, k_\perp) \approx \Psi^s(x', k'_\perp) + \int dx' d^2k'_\perp K_{ge}(x, k_\perp; x', k'_\perp) \Psi^s(x', k'_\perp)$$  \hspace{1cm} (6)$$

By using this technique, one can obtain an approximate solution of the Bethe-Salpeter (BS) equation, starting with a relativistic bound state light-cone model wave function. This BS wave function contains both soft and hard ingredients needed to take care of momentum transfer for all $Q^2$ therefore is correctly characterized as including both confinement and asymptotic features of a composite quark system. Inserting the form of Eq. (5) in Eq. (2), one obtains the soft form factor (impulse approximation) from the first term in Eq. (6), a generalization of the hard form factor (the form of the hard form factor with transverse momentum effects retained), and further correction terms.

The application of this approach to the pion form factor has been shown [4] to be in good agreement with the direct BS calculation [3]. We have predicted a power-law falloff behavior for the form factor $Q^2F_\pi(Q^2)$, which reaches asymptotic limit at about 15 $GeV^2$. A check on the next iteration of Eq. (4) using the form of Eq. (5) has shown that the correction is small when used in Eq. (2) to calculate the pion form factor. One nice feature of this approach is that one determines the soft part and the hard part separately, so that one can determine the transition from soft to hard QCD within this LCBS approach.

In this letter, we are going to reexamine our model wave function in the light of recent work by a number of other theorists. In particular, we carry out a study of 1) the evolution of quark mass with momentum, 2) the effect of the quark anomalous magnetic form factor, 3) the effect of the Sudakov form factor [5] and 4) the question of the form of the wave function in the light of the proposed form of Chernyak and Zhitnitsky [6], which has been used in attempts to fit experiment at rather low $Q$ with the hard scattering form, neglecting the soft
2 Quark Mass Evolution

The constituent quark model (CQM) has been successful in explaining many static and low momentum transfer phenomena. But one of the deficiencies of a CQM is that it totally ignores the dynamic aspects of quark masses. We have known for a long time [7] that at high energy (light) quarks are almost massless. On the other hand, the hard scattering model uses massless quark propagators or current quark masses even when applied at rather low momentum transfer.

Since we are interested in getting information for all $Q^2$, it is essential for us to understand the transition from the constituent to the current quark mass. Analyses based on QCD sum rules [8, 9, 10] have indicated that quark mass evolution involves both quark and gluon condensates. The momentum dependence of the light quark masses which results from these studies is of the form

$$m(p^2) = m + g^2 \left( \frac{c_1}{p^2} + \frac{(3m^2 + 4m^3)}{(p^2 + m^2)^3} c_2 \right)$$

(7)

where $c_1 \sim \langle \bar{\psi} \psi \rangle$ (the quark condensate), while $c_2 \sim \langle G_{\mu\nu}^2 \rangle$ (the gluon condensate).

We can see from Eq. (7) that as its momentum increases, the quark mass becomes smaller. At the limit where $p^2$ goes to infinity, $m(p^2) = m$ (current quark mass). Although Eq. (7) is only valid above a certain momentum scale, because the sum rule method used in Refs. [8, 9, 10] are not valid in very low momentum region, we can still extract some useful information for the mass evolution out of it. By a direct comparison and through calculations we find almost identical numerical results for pion form factor calculation for $Q^2 > 2 GeV^2$ by using either Eq. (7) or the following form for the quark mass:

$$m(p^2) = m + (M - m) \frac{1 + exp(-\frac{\mu}{\lambda})}{1 + exp(-\frac{\mu}{\lambda})} - 4m \frac{p^2}{(1 - p^2)^2}$$

(8)
where \( m \) and \( M \) are the current and constituent quark mass, respectively. We take the parameters \( \mu = 0.5 \, GeV^2 \) and \( \lambda^2 = 0.2 \, GeV^2 \). Eq. (8) is pure phenomenological. Nevertheless it gives a satisfactory representation of the evolution of the quark mass from the low energy limit of the constituent quark mass to the high energy limit or the current quark mass.

Using Eq. (8) for the mass evolution, we calculate the elastic pion form factor in a broad range of momentum transfer (\( 0 \leq Q^2 \leq 60 \, GeV^2 \)). The results are quite interesting. They are shown in Figs. 1 and 2. Firstly, the quark mass evolution causes only small differences in the calculated results for the pion charge radius, \(<r^2_\pi>\), and the pion decay constant. Secondly, The soft form factors at low momentum transfers (\( Q^2 < 0.5 \, GeV^2 \)) are basically intact though there appears a tiny enhancement around \( Q^2 \sim 0.5 - 2 \, GeV^2 \) region. However, the mass effect becomes increasingly important for larger \( Q^2 \) in that the hard tail from soft contribution is totally suppressed beyond \( Q^2 > 10 \, GeV^2 \). In other words, the hard processes dominate completely above \( \sim 10 \, GeV^2 \). Finally, the running quark mass seems to boost the hard contribution even more at large \( Q^2 \), keeping in mind that the hard wave function is obtained from Eq. (6) where quark mass is one of the essential parameters.

The momentum dependence of the quark mass is one of the most important issues considered in the present work. The general feature of a power falloff from the present experimental values found in Ref.[4] is still found. The suppression of the soft contribution and the enhancement of the hard scattering at large \( Q^2 \) have brought \( Q^2 F_\pi(Q^2) \) very close to its asymptotic value as \( Q^2 \) goes above 10 \( GeV^2 \). This is in good agreement with our previous work although it probably takes a much higher \( Q^2 \) for one to reach the exact asymptotic value. We still conclude that there will be a power-law falloff of \( Q^2 F_\pi(Q^2) \) in the region between about 3 and 10 \( GeV^2 \).

3 Quark Magnetic Form Factor

Before we consider the effect of the momentum dependence of quark form factors, we study the effect of a possible quark anomalous magnetic moments by assuming that \( f_{ql} = e_q \)
and $f_{q2} = \kappa_q$. It was suggested by Chung and Coester [11] that quark anomalous magnetic form factors would play important role in fitting the charge radius and electromagnetic form factors of nucleon. It would be more interesting to see if this is the case for the pion since pion wave function can be more precisely determined for we have known the experimental value of pion decay constant $f_\pi$ very well. The calculation only involves the soft part, so that we use $\Psi^*$ in Eq. (2).

The spin component of the wave function can be obtained by conducting a light cone boost on a Melosh rotated quark-antiquark coupled state. See [12], for instance, for a detailed discussion. The term involving quark magnetic moment’s contribution to the form factor can be explicitly written down as

$$F_\pi(Q^2) \sim \int \frac{[dx][d^2k_\perp]}{16\pi^3}(A - B) \frac{t}{m} \kappa$$

where $\kappa = \kappa_u - \kappa_d$, $m$ is quark mass, $t^2 = (1 - x_1)^2 Q^2$, $A$ and $B$ are $\Psi^*_{\uparrow\downarrow}\Psi_{\downarrow\downarrow} + \Psi^*_{\downarrow\uparrow}\Psi_{\uparrow\uparrow}$ and $\Psi^*_{\uparrow\uparrow}\Psi_{\downarrow\downarrow} + \Psi^*_{\downarrow\uparrow}\Psi_{\uparrow\downarrow}$, respectively.

We have tried to adjust $\kappa$ to fit the experimental data at low momentum transfers and found that it does improve the pion charge radius $<r^2_\pi>$ by about 5% (with $\kappa \sim 0.01-0.05$) given the quark mass $m$ and harmonic oscillator parameter $\alpha$. This seems to indicate that at low $Q^2$, there indeed exists an effective quark anomalous moment although the corrections are only within a few per cent as far as the model wave function we choose is concerned. Nevertheless, the range of $\kappa$ we determined here can serve as a guide for further theoretical and experimental investigations of the quark anomalous moment. At large $Q^2$ the quark anomalous moment is negligible.

4 Sudakov Form Factor Effects

There has been a great deal of interest in the introduction of Sudakov form factors in the PQCD treatment of exclusive processes. It has been pointed out by many authors that the inclusion of such form factors is necessary for consistency of the hard scattering assumption.
This has been studied in detail by Sterman and coworkers, who give references to earlier work.[13]

The Sudakov effect[5] results in the presence of double-logs in vertex functions. We introduce these effects by taking the form of the quark form factor as[14]

\[ f_{q1}(q^2) = \exp\left(-\frac{C_F g^2}{8\pi^2} \ln\lambda \ln\tau\right) \] (10)

where \( \lambda = k_2^2/k_1^2 \) and \( \tau = Q^2/k_1^2 \). We find that due to the running coupling constant in QCD the Sudakov suppression is very mild in the domain of \( Q^2 \) where experimental data are available. There is little change in our numerical results from this effect. This is consistent with recent calculations[15] which show that the inclusion of transverse momentum in the soft wave function generates a suppression much stronger than the Sudakov to the hard scattering in the region of present experiment. These results are all very similar to the observations in our previous work[4]. We conclude that it is more essential to take the transverse momentum in the soft wave function into account.[4]

5 Form of Wave Function

The evaluation of the hard scattering form for the form factor requires a quark distribution amplitude. As mentioned above, if one uses the asymptotic value of the quark distribution amplitude or our model, the hard scattering contribution is much smaller than the experimental values of the pion form factor at the largest values of momentum transfer for which there are measurements. There are many published papers which suggest that with a form such as that suggested by Chernyak and Zhitnitsky[6], based on a QCD sum rule analysis, one can use the hard scattering form at rather low \( Q^2 \). Since the method of QCD sum rules does not make use of explicit models of hadronic wave functions, this has been used to justify the hard scattering model for the pion (and nucleon) elastic form factors to fit present

\footnote{The quark distribution amplitude of ours is very close to the asymptotic form. It is known that the Sudakov effect is small for the asymptotic form of the quark distribution amplitude.[13, 15]}
We would like to make two observations about the quark distribution amplitude (QDA). Our spin wave function, apart from an overall factor, is identical to that proposed by Dziembowski\cite{17}. It has been known that this type of wave function, when the parameters are specifically chosen, can generate a corresponding quark QDA that is very similar to the double-humped QDA suggested by Chernyak and Zhitnitsky based on QCD sum rules analysis. One of our findings in this pion form factor calculation is that, under the physical constraints imposed on the wave function, one can never reach this special QDA. Our QDA is more like the asymptotic one (see Fig. 3 for an illustration). This finding is consistent with the recent analyses from both lattice calculation and QCD sum rules\cite{18}.

Secondly, the method of QCD sum rules extracts moments of distributions rather than distributions, and these moments contain errors. It has recently been pointed out\cite{19} that if one takes the errors into consideration one cannot actually distinguish between the very asymmetric quark distribution amplitude of Ref\cite{6} and a symmetric one such as that resulting from our model, or even the asymptotic one.

We would like to add another comment here on the soft wave function we used as the starting point to get our approximate solution for BS equation. This concerns the difference of spin wave functions between the light cone form and the instant form. For the pion, we first couple a valence quark and an antiquark in the pion rest frame. A Melosh type Wigner rotation transforms a state from its instant form to the light cone representation. An important feature of this transformation is that it generates two extra helicity components, namely $h_1 \pm h_2 = \pm 1$, apart from the conventional helicity zero components. Contrary to the claims made by several authors\cite{15, 16} regarding the impossibility of properly fitting the pion charge radius while still satisfying the constraints on the pion wave function imposed by the $\pi \to \mu \nu$ decay and $\pi^0 \to \gamma \gamma$ process, we would like to point it out that the unconventional helicity components customarily neglected turn out to be crucial in consistently fitting all the three constraints that the wave function must satisfy. With the parameters given as: $m_q = 0.33 \text{ GeV}, \alpha = 0.32 \text{ GeV}, \kappa = (\kappa_u - \kappa_d) = 0.04$ and $\Lambda_{QCD} = 150 \text{ MeV}$, we get: $<r_\pi^2>$=
$0.45 \text{ fm}^2, f_\pi = 93.4 \text{ MeV}$. The analysis of the contribution to the pure hard process from these unconventional components is under way. We expect some interesting impact on the hard form factors in the region where the PQCD is believed to be dominant. We will report it later in a separate publication.

6 Conclusion

We have calculated pion elastic form factors at momentum transfer from 0 to 60 $\text{GeV}^2$ using the theoretical approach we developed in [4]. Four modifications of our model wave function have been made and tested. An important new result is that quark mass evolution is essential for any theoretical models which expect to include both confinement and asymptotic features in the theory. An empirical formula for the quark mass produces an identical effect for the pion form factor at $Q^2 > 1 \text{ GeV}^2$ as QCD sum rules do and is suitable for extensions to low momentum regions. Our numerical calculation has shown that the effective quark anomalous magnetic moment could show up at low momentum transfer if such a moment exists, but it has a very small effect on the pion form factor in regions of interest for the transition from soft to hard QCD. We have also studied the Sudakov form factor of quarks. We find that a Sudakov form factor has very little effect for the pion form factor at the region of $Q^2$ where experimental data are available currently. We have also studied effects of the form of the soft wave function.

While there is still no agreement in the value of the momentum transfer at which one can reach the asymptotic limit of PQCD for exclusive processes, our work seems to suggest that for the elastic scattering with a pion target the hard contributions enter at about 1 $\text{GeV}^2$, become dominant after about 10 $\text{GeV}^2$ and approach the asymptotic limit at $Q^2$ no lower than 60 $\text{GeV}^2$. Experiments for momentum transfers in the range between 4 to 15 $\text{GeV}^2$ will be a crucial test for our prediction, since we predict a power-law falloff. More accurate data at $Q^2 = 1 \sim 4 \text{ GeV}^2$ are also needed for better model building, for which CEBAF may play a significant role in the near future.
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Figure 1: Quark mass effect on form factors. The solid and dash lines are those with or without quark mass evolution, respectively. The experimental data are taken from [20].

Figure 2: Pion form factors. The parameters are: $m_q = 0.33\, GeV$, $\alpha = 0.32\, GeV$, $\kappa = 0.04$ and $\Lambda_{QCD} = 150\, MeV$. The experimental data are taken from [20].

Figure 3: The quark distribution amplitude. The solid, dash-dot and dash line curves correspond to the harmonic parameters 0.3, 0.55 and 0.58, respectively. We take $m_\pi = 613\, MeV$ [2-17] here merely for an illustration. (For actual calculation of the form factor we use $m_\pi^2 = (k_\perp^2 + m^2)/x(1-x)$, with which the distribution function is about the same as the solid line shown above.)
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