Influence of radio frequency harmonics to TAMA300 sensitivity

N Nakagawa¹, K Arai², K Kuroda¹, and TAMA Collaboration

¹ Institute for Cosmic Ray Research, University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba, 277-8582, Japan
² National Astronomical Observatory Japan, Mitaka, Tokyo, Japan
E-mail: nakagawa@icrr.u-tokyo.ac.jp

Abstract. We measured the harmonic noise produced at the radio frequency mixer used to extract gravitational wave signals from a laser interferometer, and set an upper limit of the current sensitivity of TAMA300 applying this measurement to the mixer.

1. Introduction

TAMA300 [1] is a laser interferometer for gravitational wave detection. In order to extract gravitational wave signals, TAMA300 employs a modulation/demodulation technique [2]; phase modulation is applied to the input laser light. The output light from the interferometer is detected by a photodetector, and the photocurrent is demodulated with a demodulator.

In the demodulation process, information of gravitational wave signals that appears in the modulation frequency band is down-converted to an audio frequency band by mixing with a local oscillator signal that is a sinusoidal wave at the modulation frequency. In reality, additional components at the harmonic frequencies of the modulation are present in both the photocurrent and the local oscillator. The harmonic components in both the photocurrent and the local oscillator beat each other and appear in the audio frequency band. Once they are down-converted to the audio frequency band, it is impossible to distinguish them from gravitational wave signals.

In this paper, we described the measurement of the harmonics power of each port at the demodulator, and discussed their influences to the sensitivity of the interferometer.

2. Existence of harmonics and its influence

Modulation and demodulation at a radio frequency band are usually used for the control of a laser interferometer in order to obtain displacement signals [3]. Photodetectors are placed to detect output laser beams from the interferometer. The radio frequency (RF) photocurrent of each photodetector (PD) is demodulated at a mixer. The mixer multiplies the RF input and the local oscillator (LO) signal, resulting in the down-conversion of the interferometer signal to the audio frequency (AF) band. The RF signal is directly connected from PD to the RF port of the mixer, and the LO signal is fed from the local oscillator to the LO port of the mixer through buffers and a phase-shifter (PS) as shown in Fig.1. The PS adjusts the phase of the LO to be coincided with that of the RF signal of the PD.

In a practical system, it is possible that harmonic distortion of each input of the demodulator generates additional contribution to the demodulated signal and affects the sensitivity of the
interferometer. In order to explain by a formula manipulation, we define $\omega_{\text{mod}}$ as an angular frequency of modulation, and $\Delta \omega_j$ as an angular frequency of a variable component with demodulated signal voltage $V_{\text{mod}}$. Gravitational wave (GW) and other information of the interferometer (i.e. seismic noise) appears in this $\Delta \omega_j$. Each output voltage of $V_{\text{LO}}$, $V_{\text{RF}}$, and demodulated signal $V_{\text{dem}}$ are written like these.

\[
V_{\text{LO}} = \sum_{i=1}^{n} a_i \sin(i \times \omega_{\text{mod}})
\]

\[
V_{\text{RF}} = \sum_{j=1}^{n} b_j \sin(j \times \omega_{\text{mod}} - j \times \Delta \omega_j)
\]

\[
V_{\text{LO}} \times V_{\text{RF}} = \frac{1}{2} \sum_{i=1}^{n} a_i b_i \cos(i \times \Delta \omega_i) - \frac{1}{2} \sum_{i \neq j}^{n} a_i b_j \cos((i + j) \times \omega_{\text{mod}} + i \times \Delta \omega_j)
\]

\[
\underbrace{\text{low-pass filter}}_{\frac{1}{2} \sum_{i=1}^{n} a_i b_i \cos(i \times \Delta \omega_i)} \equiv V_{\text{dem}}
\]

That $a_i$ and $b_j$ are amplitude of each voltage. Demodulation process separates the high frequency second term and the low frequency first term. We regard the harmonics ($n \neq 1$) as noise.

Once they are down-converted to the audio frequency band, since the each component of demodulated signal is in the same frequency band, it is impossible to distinguish the harmonic components from gravitational wave signals.

3. Harmonics at the LO port

The LO is usually assumed to produce a monochromatic wave signal. Practically, the LO works as a switch in the mixer, hence it behaves as square wave signal and produces odd-order harmonics. In TAMA300, the LO input power is 11.8dBm at $\times 1.5235(= 1 \times f)$ MHz, and RF input power to demodulator is -18dBm at 1f. This is an enough power ratio for the mixer, since the LO needs at least +10dBm to feed with a sufficiently strong sine wave to fully saturate the diodes in the mixer, and down-converts the interferometer signal of the RF to the AF band.

We checked the harmonics at the LO port of the demodulator of the setup in Fig.1. In this measurement, the RF port was connected to the measurement oscillator, which differs from the local oscillator. The LO input power was also 11.8dBm at $\times 1 \times f$ MHz, and the RF input power was -8.2dBm at $\times n \times f + 0.001$ MHz ($n = 1, 2, 3, \cdots$). We measured the demodulated 1 kHz peak of AF power with $\times n \times f$ MHz RF. This peak was the nth order harmonics of the LO port.

Table 1 shows the power of radio frequency harmonics of the LO port. This reveals that radio frequency harmonics exist in odd multiple frequencies. The result shows they are due to an
intrinsic character of the mixer based on the power of an ideal square wave. We also investigated about even-order harmonics, and revealed they were due to the phase shifter. However, even-order power of the LO port did not have much power to affect the sensitivity.

| Frequencies                  | 1f   | 2f   | 3f   | 4f   | 5f   |
|-----------------------------|------|------|------|------|------|
| Local oscillator power      | 0    | -50.5| -43.5| -69.7| -42.0| [dB] |
| Demodulated 1kHz power      | 0    | -26.2| -10.8| -28.1| -17.1| [dB] |
| (Ideal square wave power)   | 0    | —    | -9.5 | —    | -14.0| [dB] |

It is important that the power of third harmonics is not much smaller than that of fundamental frequency. This may have influence to the sensitivity of the interferometer.

4. Harmonics at the RF port of TAMA300
It is impossible to prevent the generation of harmonics at the RF port. Gravitational waves (GW) reach an interferometer, and then the output light from the interferometer is detected by a photo detector (RF port). In the RF port, GW signals appear at 1\(f\), while 2\(f\) signals are also generated from the beating of modulation sidebands.

We confirmed that the second-order harmonic strongly existed at the output signal of the PD port of TAMA300. The setup of the TAMA300 interferometer was Recycled Fabry-Perot Michelson (RFPMI), modulated with 15.235MHz, and after the RFPMI was locked, output signal from PD was divided to two ports; one was used to a servo of the interferometer and the other was used for measurement in Fig.2. The photodiode that fends the signal input uses a tuned circuit, whose Q is 23 and resonance frequency is 15.235MHz of LO fundamental frequency. This may be stripping harmonic content from the signal port, which is supposing to make harmonic noise less troublesome against to the local oscillator port. Under this setup, it had no influence to the servo or a sensitivity of the interferometer even if we added any operation to the measurement port.

The power of the RF port was directly measured after the power splitter, and its power is shown as Table 2. This table reveals that the second-order harmonics actually exists and has larger power than 1\(f\) in the RF port. In this measurement, higher order harmonics are also identified. This may also have influence to the sensitivity of the interferometer.
5. The contribution of the harmonics to the sensitivity

Since harmonics were identified in both LO and RF ports, it was necessary to investigate the contribution of the harmonics to the sensitivity. In order to measure the influence, we used the same setup as in section 4. In addition, we used notch filters and buffers to separate the contribution of each harmonic component shown in Fig.3. These buffers reduce the reflection of the signal that may produce electrical noise at the same time.

![Figure 3. The added filters (orange square) and the setup to measure the contribution by harmonics. When we measured the contribution of 2f, 1f notch filter was used since 2f power was dominant at the RF port. When we measured the contribution of higher order harmonics together, 1f and 2f notch were used since higher order harmonics are smaller than those of 1f and 2f.](image)

Table 2. Harmonic distortion of the RF port

| Frequencies | 1f | 2f | 3f | 4f | 5f |
|-------------|----|----|----|----|----|
| RF port power | 0  | +6 | -35| -28| -32| [dB] |

Since the phase of the LO is adjusted to the best sensitivity for the only fundamental frequency, we can not know which phase demodulate the harmonics. Actually, the contribution to the sensitivity by harmonics was measured at two phases so as to determine an upper limit. One was the max influence; it was the highest power in all phases. The other was the influence measured at the lowest power in all phases. And we took root sum squares of them as an upper limit.

The measurement result shows that the influence of harmonics was not limiting current noise. The influence of the 2f harmonics is shown in Fig.4, while the influence of the overall higher order harmonics is reported in Fig.5. This is the upper limit of the influence by harmonics. In spite of the filter that reduces the power of fundamental frequency, it remained a little and appeared as the "Influence of attenuated 1f". On the other hand, electrical noise was caused by the added buffers, which prevent to affect the sensitivity of the servo.

In this measurement, an attenuated 1f noise dominated in frequencies less than 20Hz, whereas, the noise of an electrical circuit dominated in frequencies larger than 70Hz and harmonics noise dominated in the frequency range for 20-70 Hz. However, all these noises were well below the sensitivity curve of TAMA300.

6. Conclusion

We discussed the influence of radio frequency harmonics in LO and RF. In the LO port, the harmonics were detected due to the intrinsic character of the mixer. In the RF port, the harmonics were also detected due to the signal extraction technique of the interferometer.
Experimentally, the contribution to the sensitivity of TAMA300 was investigated. The result showed that the influence of harmonics was not the limiting current noise, and we obtained the upper limit of the influence of radio frequency harmonics.

This result also shows that since the upper limit was close to the current sensitivity, further investigation would be needed when the sensitivity of the interferometer becomes better.
Acknowledgement
This research is supported in part by a Grant-in-Aid for Scientific Research on Priority Areas (415) of the Ministry of Education, Culture, Sports, Science and Technology.

References
[1] Ando M., et al., 2001, *Phys. Rev. Lett.* 86, 3950
[2] Arai K., and the TAMA collaboration, 2002, *Class. Quantum Grav.* 19, 1843-1848
[3] M. W. Regehr, F. J. Raab, and S. E. Whitcomb, 1995, *Opt. Lett.* 20, 1503