CP-Violation in $b \to s \ell^+ \ell^-$ transition Beyond the Standard Model

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July 4, 2018

Abstract
In this study the CP-asymmetry in the $b \to s \ell^+ \ell^-$ transition was investigated in minimal extension of the Standard Model where $C_9^{\text{eff}}$ receives an extra weak phase due to the new physics effects. We observed that CP-Violation asymmetry can be measurable in the framework of scenario mentioned above.

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1 Introduction

Rare B meson decay induced by the flavor changing neutral current (FCNC) $b \rightarrow s(d)$ transition provides potentially the most sensitive and precise test for the standard model (SM) in the flavor sector at loop level since such transitions are forbidden in SM at tree level. At the same time, these decays are very sensitive to the new physics beyond the SM.

Experimental investigation of these decays will provide a more precise determination of the elements of the Cabbibo-Kobayashi-Maskawa (CKM) matrix such as $V_{tb}, V_{ts}, V_{td}$ and $V_{ub}$. Moreover, they can provide better insight into understanding the origin of CP-violation.

Rare semileptonic decays $b \rightarrow s(d)\ell^+\ell^-$ are more informative for this aim, since these decays are relatively clean compared to pure hadronic decays. Note that the semileptonic $b \rightarrow q\ell^+\ell^-$ transition has been extensively studied in numerous works [1]–[18] in the framework of the SM and its various extensions. The matrix elements of the $b \rightarrow s\ell^+\ell^-$ transition contain terms describing the virtual effects induced by the $tt, cc$ and $uu$ loops, which are proportional to $|V_{tb}V_{ts}^*|, |V_{cb}V_{cs}^*|$ and $|V_{ub}V_{us}^*|$, respectively. Using the unitarity condition of the CKM matrix and recalling that one can neglect $|V_{ub}V_{us}^*|$ in comparison to $|V_{tb}V_{ts}^*|$ and $|V_{cb}V_{cs}^*|$, it is obvious that the matrix element for the $b \rightarrow s\ell^+\ell^-$ transition involves only one independent CKM matrix element, namely $|V_{tb}V_{ts}^*|$, so the CP-violation in this channel is strongly suppressed in the SM.

However, the possibility of CP-violation as the result of new physics effects in $b \rightarrow s$ transition has been studied in supersymmetry [19]–[20], and in SM with fourth generation [21]. In another study, this has been studied with the addition of CP odd phases to Wilson coefficients [22].

Situation for $b \rightarrow d\ell^+\ell^-$ is totally different from $b \rightarrow s\ell^+\ell^-$ transition. In this case, all CKM matrix elements $|V_{td}V_{tb}^*|, |V_{cd}V_{cb}^*|$ and $|V_{ud}V_{ub}^*|$ are in the same order and for this reason the matrix element of $b \rightarrow d\ell^+\ell^-$ transition contains two different amplitudes with two different CKM elements and therefore it is expected to have a large CP violation.

In order to get CP violation not only do we have to have two different amplitudes but also these amplitudes must contain pieces which transform under CP transformation in different ways, i.e. these amplitudes must contain weak and strong phases. It is clear that the weak phase changes its sign under CP transformation but strong phase doesn’t.

Let’s briefly recall the situation in $b \rightarrow s\ell^+\ell^-$ transition in standard model. In the SM, the Wilson coefficients $C_7$ and $C_{10}$ are real, while $C_{9}^{eff}$ contains strong and weak phases. The $C_{9}^{eff}$ is usually parameterized in the following form:

$$C_{9}^{eff} = \xi_1 + \lambda_u \xi_2$$ (1)
where
\[ \lambda_u = \frac{|V_{ub} V_{us}^*|}{|V_{tb} V_{ts}^*|} \]  
(2)

As we have noted, this quantity is very small, therefore, it is usually neglected in calculations and for this reason CP violation in this channel is strongly suppressed.

As we mentioned above, the $b \rightarrow s(d)\ell^+\ell^-$ transition is a promising candidate looking for new physics beyond the SM. New physics effects can appear in rare decays when the Wilson coefficients take values different from their SM counterpart or new operator structures in effective Hamiltonian which are absent in the SM.

In the present work, we focus on CP asymmetry in $b \rightarrow s\ell^+\ell^-$ transition. This asymmetry is very small in SM; therefore, any deviation of this asymmetry from SM prediction clearly indicates existence of new physics. We should note that the first measurement of the $b \rightarrow s\ell^+\ell^-$ decay reported by BELLE [23] is:
\[ B(B \rightarrow X_s \ell^+\ell^-) = (6.1 \pm 1.4^{+1.4}_{-1.1}) \times 10^{-6}. \]  
(3)

An experiment recently done by BELLE Collaboration[24] examined the measurement of forward–backward asymmetry and determination of Wilson coefficients in $B \rightarrow K^{*}\ell^+\ell^-$ decay. In future, a similar measurement will be done for CP violating asymmetry. Although all Wilson coefficients($C_7^{eff}$, $C_9^{eff}$ and $C_{10}^{eff}$) can take CP odd phases, we will discuss the possibility elsewhere. We will here discuss the situation where $C_9^{eff}$ will get a new weak phase. We call this minimal extension of Standard Model since, with this extension, CP asymmetry can appear in $b \rightarrow s$ transition. As we consider the minimal extension of the SM, we assume that the strong phase is the same as the SM case, because it appears in imaginary part of polarization operator.

This paper is organized as follows: In Section 2, we present the theoretical framework for the decay width and CP-violation asymmetry. Section 3 encompasses our numerical results and an estimation of the feasibility of measuring the CP violation and conclusion.

## 2 Theoretical Framework

In this section, we present the theoretical expressions for the decay widths and CP-violation asymmetry. As we mentioned above, we restrict ourselves by considering minimal extension of the SM, more precisely, we extend only $C_9^{eff}$, since in SM only this coefficient has weak and strong phases, i.e. :
\[ C_9^{new} = \xi_1 + (\lambda_u + \lambda_{new})\xi_2 \]  
(4)
where $\lambda_u$ is given by Eq. (2) and $\lambda_{\text{new}}$ is parameterized as:

$$\lambda_{\text{new}} = |\lambda_{\text{new}}| \exp(i\varphi) \quad (5)$$

The explicit expressions of functions $\xi_1$ and $\xi_2$ are respectively:

$$\xi_1 = 4.128 + 0.138\omega(s) + g(m_c, s)(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6)$$
$$- \frac{1}{2}g(m_d, s)(C_3 + C_4) - \frac{1}{2}g(m_b, s)(4C_3 + 4C_4 + 3C_5 + C_6) + \frac{2}{9}(3C_3 + 4C_4 + 3C_5 + C_6) \quad (6)$$

$$\xi_2 = [g(m_c, s) - g(m_u, s)](3C_1 + C_2) \quad (7)$$

Where $\hat{m}_q = m_q/m_b$ and $\hat{s} = \frac{q^2}{m_b}$

As we know, the rare decays are one of the promising classes of decays for new physics beyond the SM.

The QCD corrected effective Hamiltonian describing $b \rightarrow s\ell^+\ell^-$ transitions leads to the matrix element:

$$M = \frac{G_F\alpha V_{tb}^* V_{ts}}{\sqrt{2}\pi} [C_{9}^{\text{new}}(\bar{s}\gamma_\mu P_L b)\bar{\ell}\gamma_\mu \ell + C_{10}(\bar{s}\gamma_\mu P_L b)\bar{\ell}\gamma_\mu \gamma_5 \ell$$
$$- 2C_7^{\text{eff}}i\sigma_{\mu\nu}q^\nu(m_b P_R + m_s P_L)\bar{b}\ell\gamma_\mu \ell], \quad (8)$$

where $q$ denotes the four momentum of the lepton pair. Neglecting the terms of $O(m_q^2/m_W^2)$, $q = u, d, c$, the analytic expressions for all Wilson coefficients, except $C_{9}^{\text{new}}$, can be found in [25-28]. The values of $C_7^{\text{eff}}$ and $C_{10}$ in leading logarithmic approximation are:

$$C_7^{\text{eff}} = -0.315, \quad C_{10} = -4.642 \quad (9)$$

The function $g(\hat{m}_q, \hat{s})$ represents the corrections to the four-quark operators $O_1 - O_6$ and is defined as:

$$g(\hat{m}_q, \hat{s}) = -\frac{8}{9}\ln(\hat{m}_q) + \frac{8}{27} + \frac{4}{9}y_q - \frac{2}{9}(2 + y_q)\sqrt{|1 - y_q|} \left\{ \Theta(1 - y_q) \times \right.$$}
$$\left. \ln \left( \frac{1 + \sqrt{1 - y_q}}{1 - \sqrt{1 - y_q}} \right) - i\pi \right\} + \Theta(y_q - 1) 2\arctan \frac{1}{\sqrt{y_q - 1}}, \quad (10)$$

Even though we neglect long-distance resonance effects in this paper, a more complete analysis of the above decay has to take into account the long-distance contributions, which have their origin in real intermediate $c\bar{c}$ family, in addition to the short-distance contribution. In the case of the $J/\psi$ family, this is usually accomplished by introducing a Breit-Wigner distribution for the resonances through the replacement:

$$g(\hat{m}_c, \hat{s}) \rightarrow g(\hat{m}_c, \hat{s}) - \frac{3\pi}{\alpha^2} \sum_{V = J/\psi, \psi', \ldots} \frac{\hat{m}_V Br(V \rightarrow l^+l^-)\hat{\Gamma}_V}{\hat{s} - \hat{m}_V^2 + i\hat{m}_V \hat{\Gamma}_V^{\text{total}}}, \quad (11)$$

$$3$$
Using the expression of matrix element in equation (8) and neglecting the s-quark mass \( m_s \) \([30]-[31]\), we obtain the expression for the differential decay rate as \([32]\);

\[
\Gamma_0 = \frac{d\Gamma}{ds} = \frac{G_F m_b^5 \alpha^2}{192\pi^3} |V_{tb}V_{ts}^*|^2 (1 - \hat{s})^2 \sqrt{1 - \frac{4\hat{m}_t^2}{\hat{s}}} \Delta \tag{12}
\]

with

\[
\Delta = 4\left(2 + \frac{2\hat{m}_t^2}{\hat{s}}\right) \left[C_7^{eff}\right]^2 + (1 + 2\hat{s}) \left[C_9^{new}\right]^2 + 12(1 + \frac{2\hat{m}_t^2}{\hat{s}}) \text{Re}(C_9^{new}C_7^{eff}). \tag{13}
\]

In the unpolarized case, the CP-Violating asymmetry rate can be defined by

\[
A_{CP}(\hat{s}) = \frac{\Gamma_0 - \Gamma_0^\prime}{\Gamma_0 + \Gamma_0^\prime} \tag{14}
\]

where

\[
\Gamma_0 \equiv \frac{d\Gamma}{ds} = \frac{d\Gamma(b \rightarrow s\ell^+\ell^-)}{ds}, \quad \Gamma_0^\prime \equiv \frac{d\Gamma}{ds} = \frac{d\Gamma(b \rightarrow s\ell^+\ell^-)}{ds} \tag{15}
\]

The explicit expression for the unpolarized particle decay rate \( \Gamma_0 \) has been given in \(12\). Obviously, it can be written as a product of a real-valued function \( r(\hat{s}) \) times the function \( \Delta(\hat{s}) \), given in \(13\); \( \Gamma_0(\hat{s}) = r(\hat{s}) \Delta(\hat{s}) \). Taking the approach in \([33]\), we write the matrix elements for the decay and the anti-particle decay as

\[
M = A + \lambda_{new} B, \quad \overline{M} = A + \lambda_{new}^* \overline{B} \tag{16}
\]

where the CP-violating parameter \( \lambda_{new} \), entering the Wilson coupling \( C_9^{new} \), has been defined in Eq. (4). Consequently, the rate for the anti-particle decay is, then, given by

\[
\bar{\Gamma}_0 = \Gamma_0|_{\lambda_{new} \rightarrow \lambda_{new}^*} = r(\hat{s})\Delta(\hat{s}) ; \quad \bar{\Delta} = \Delta|_{\lambda_{new} \rightarrow \lambda_{new}^*}. \tag{17}
\]

Using \(12\) and \(17\), the CP violating asymmetry is evaluated to be \([33]\)

\[
A_{CP}(\hat{s}) = -\frac{2 |\lambda_{new}| \sin(\varphi) \Sigma}{\Delta + 2 |\lambda_{new}| \cos(\varphi) \Sigma}. \tag{18}
\]

Furthermore, we can do the same approximation done for \( b \rightarrow d\ell^+\ell^- \) \([18] \,[32] \,[33]\), if the \( \lambda_{new} \sim 1 \). So, we can ignore the term proportional to the \( \Sigma \) in the dominator of Eq.\(18\):

\[
A_{CP}(\hat{s}) \approx -2 |\lambda_{new}| \sin(\varphi) \frac{\Sigma}{\Delta}. \tag{19}
\]

Where \( \Delta \) is defined by Eq.\(13\) and \( \Sigma \) is as follows:

\[
\Sigma = \quad Im[\xi_1^* \xi_2]f_+(\hat{s}) + Im(C_7^{eff} \xi_2)f_1(\hat{s})
\]

\[
f_+(\hat{s}) = (1 + 2\hat{s}) \left(1 + \frac{2\hat{m}_t^2}{\hat{s}}\right)
\]

\[
f_1(\hat{s}) = \quad 12(1 + \frac{2\hat{m}_t^2}{\hat{s}}) \tag{20}
\]
3 Numerical analysis

In this section, we examine the dependence of CP-violating asymmetry on $\lambda_{new}$. From the Eq.(14) it follows that CP violating asymmetry depends on both magnitude and phase of $\lambda_{new}$. In order to have an idea about magnitude of $\lambda_{new}$, we assume that the normalized branching ratio can departure from SM result Eq.(3) by about 10 percent in the presence of the new parameters, i.e,

$$-0.1 \leq \frac{B_{new} - B_{SM}}{B_{SM}} \leq 0.1$$  \hspace{1cm} (21)

Note that, the same approach used in [34]. Solving this equation on $|\lambda_{new}|$, we obtain the upper limit for $|\lambda_{new}| \leq 1.16$, so the allowed region for $|\lambda_{new}|$ is:

$$0.0 \leq |\lambda_{new}| \leq 1.16$$  \hspace{1cm} (22)

We, here, see that, under the condition mentioned above, $|\lambda_{new}|$ has to be zero for SM case.

The values of input parameters which we use in our numerical analysis are: $m_s = 0.15$ GeV, $m_b = 4.8$ GeV, $m_\mu = 105.7$ MeV, $C_7 = -0.314$, $C_{10} = -4.642$, $\alpha = \frac{1}{129}$ [35]. Moreover, we use Wolfenstein parametrization [36] for CKM matrix. The current values of the Wolfenstein parameters are $A = 0.83$ and $\lambda = 0.221$ [37]. Besides, we use the following simple expression of $\xi_1$ and $\xi_2$ in the NLO approximation [32]

$$\xi_1 \simeq 4.128 + 0.138 \omega(\hat{s}) + 0.36 g(\hat{m}_c, \hat{s}), \quad \xi_2 \simeq 0.36 \left[ g(\hat{m}_c, \hat{s}) - g(\hat{m}_u, \hat{s}) \right].$$  \hspace{1cm} (23)

And the explicit expression of the $\omega(\hat{s})$ is:

$$\omega(\hat{s}) = -\frac{2}{9} \pi^2 - \frac{4}{3} \text{Li}_2(\hat{s}) - \frac{2}{3} \ln(\hat{s}) \ln(1-\hat{s}) - \frac{5 + 4\hat{s}}{3(1+2\hat{s})} \ln(1-\hat{s})$$

$$- \frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})} \ln(\hat{s}) + \frac{5 + 9\hat{s} - 6\hat{s}^2}{3(1-\hat{s})(1+2\hat{s})},$$  \hspace{1cm} (24)

In order to eliminate the $\hat{s}$ dependence instead of CP asymmetry in differential decay width, we study CP asymmetry in total decay width by doing numerical integration over $\hat{s}$ in Eq. (19).

$$A_{CP} = -2 |\lambda_{new}| \sin(\varphi) \frac{\int_{(1-m_c^2/m_b^2)}^{(1-m_u^2/m_b^2)} \Sigma d\hat{s}}{\int_{(1-m_c^2/m_b^2)}^{(1-m_u^2/m_b^2)} \Delta d\hat{s}}.$$  \hspace{1cm} (25)

In the figure 1, we present the dependence of CP violating asymmetry on $|\lambda_{new}|$ and $\varphi$, where $|\lambda_{new}|$ varies in the region presented by Eq.22. The figure depicts that $A_{CP}$ is sensitive to the new weak phase and can reach about $\%4.5$ percent. For nominal asymmetry of $\%5$ and branching ratio of $10^{-6}$, a measurement at $3\sigma$ level requires about $10^9$ B mesons [32, 33].
view of clear dilepton signal, such a measurement is quite feasible at future colliders like LHCb, BTeV, ATLAS CMS [38] or ILC [39]. For instance, it is expected to be produced $10^{12}$ $B\bar{B}$ pairs at LHC. So they will be able to measure $b \rightarrow s\ell^+\ell^-$ or exclusive process $B \rightarrow K(K^*)\ell^+\ell^-$. Moreover, the existence of this CP asymmetry in $b \rightarrow s$ transition can be a direct indication of new physics effects since, in SM, this CP asymmetry is near zero [18, 33]. Here a few words about the synergy of LHC and ILC are in order: It is clear that LHC will reach higher energies and can create much more $B\bar{B}$ pairs than ILC. The ILC, on the other hand, can make precision measurements and can be sensitive to the indirect effects of the new particles which can contribute to the penguin diagrams of $b \rightarrow s$ transition even if masses are much higher than the energy of the ILC [40].

In conclusion, this study presented the CP-asymmetry in the $b \rightarrow s\ell^+\ell^-$ transition in minimal extension of the Standard Model where $C_9^{\text{eff}}$ received extra weak phase $\lambda_{\text{new}}$ due to the new physics effects. We imposed %10 of uncertainty to the SM branching ratio of $b \rightarrow s\ell^+\ell^-$ transition and obtained the bound on new parameter $\lambda_{\text{new}}$. Our predictive model showed that the CP-Violation asymmetry could reach to the order of %4.5 which was not only entirely measurable in experiments, but also indicated the new physics effects since, in SM, this CP asymmetry is near zero.

4 Acknowledgement

The author thanks TM Aliev and D. Wyler for their helpful discussions.
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Figure Caption

Fig. (1). The dependence of CP asymmetry $A_{CP}$ on new parameter $\lambda_{new}$ for the $b \rightarrow s \mu^+\mu^-$ transition.
Figure 1: