Intersecting M–Fluxbranes

Chiang-Mei Chen, Dmitri V. Gal’tsov and Sergei A. Sharakin

Department of Theoretical Physics, Moscow State University, 119899, Moscow, Russia

New solution to the six–dimensional vacuum Einstein’s equations is constructed as a non-linear superposition of two five-dimensional solutions representing the Melvin–Gibbons–Maeda Universe and its $S$–dual. Then using duality between $D = 8$ vacuum and a certain class of $D = 11$ supergravity configurations we generate $M2$ and $M5$ fluxbranes as well as some of their intersections also including waves and KK-monopoles.

I. INTRODUCTION

As it is well-known when gravity is taken into account, an uniform electromagnetic field gives rise to spacetimes of two different types. One is the Bertotti-Robinson solution which is the product $AdS_2 \times S^2$ and thus is $SO(3)$ symmetric, the field is ‘radially’ directed. The second is the Melvin Universe, which is the static cylindrically symmetric solution to the four-dimensional Einstein-Maxwell theory, the field being directed along the axis of symmetry. In the context of the four-dimensional $N = 2$ supergravity the first solution is fully supersymmetric, while the second is not. It is, however, of interest in view of the possibility to study non-perturbative processes of pair creation of charged black holes in strong electromagnetic field. Similar situation holds in $D = 11$ supergravity where instead of the two-form field one encounters a four-form. Correspondingly, there exists a fully supersymmetric solution $AdS_4 \times S^7$, analogous to the Bertotti-Robinson solution of the four-dimensional Einstein-Maxwell theory, while the Melvin type gives rise to so-called fluxbranes, term suggested in [5]. Simplest explicit fluxbrane solution for the Melvin type gives rise to so-called fluxbranes, term suggested in [5]. Simplest explicit fluxbrane solution for the Melvin type gives rise to so-called fluxbranes, term suggested in [5]. Simplest explicit fluxbrane solution for the Melvin type gives rise to so-called fluxbranes, term suggested in [5]. Simplest explicit fluxbrane solution for the Melvin type gives rise to so-called fluxbranes, term suggested in [5]. Simplest explicit fluxbrane solution for the Melvin type gives rise to so-called fluxbranes, term suggested in [5]. Simplest explicit fluxbrane solution for the Melvin type gives rise to so-called fluxbranes, term suggested in [5]. Simplest explicit fluxbrane solution for the Melvin type gives rise to so-called fluxbranes, term suggested in [5].

II. SIX-DIMENSIONAL SOLUTION

The original Melvin solution representing the gravitational field of a magnetic flux tube in cylindrical coordinates reads

$$ds_4^2 = U^2(-dt^2 + d\chi^2 + d\rho^2) + \frac{\rho^2}{U^2} d\phi^2,$$

$$A_\phi = \frac{a}{U}, \quad U = 1 + \frac{\rho^2}{a^2},$$

where $a = 2/B$, $B$ being the magnetic field strength. Magnetic field is confined within the tube of radius $a$, so that the total flux is equal to $B^{-1}$. The transversal space has an infinite volume, but the circles of constant $\rho$ have the circumferences going to zero at infinity. The same metric describes an electric Melvin solution in view of the continuous electric-magnetic duality of the Einstein-Maxwell system.

If one wishes to consider the Maxwell field as originating from the fifth dimension via Kaluza-Klein reduction, one has to use the following parameterization of the five-dimensional line element

$$ds_5^2 = e^{-\sqrt{3}\phi}(dx^5 + 2A_\mu dx^\mu)^2 + e^{2\sqrt{3}\phi} g_{\mu\nu} dx^\mu dx^\nu,$$

which also contains a dilaton in four dimensions, as it is clear from the reduction of the five–dimensional Einstein–Hilbert action (up to surface terms)

$$S = \int d^4x \sqrt{-g} \left( R - 2(\partial\phi)^2 + e^{-2\sqrt{3}\phi} F^2 \right),$$

with $F = dA$.

The generalization of the Melvin solution to Kaluza–Klein theory was constructed by Gibbons and Maeda, in terms of the four–dimensional fields it reads

$$ds_4^2 = U^2(-dt^2 + d\chi^2 + d\rho^2) + U^{-4} \rho^2 d\phi^2,$$

$$e^{-\sqrt{3}\phi} = U, \quad A_\phi = \frac{\rho^2}{2aU},$$

Alternatively, this field configuration represents the solution of the five-dimensional vacuum Einstein’s equations as follows.

*Email: chen@grg1.phys.msu.su
†Email: galtsov@grg.phys.msu.su
‡Email: sharakin@grg1.phys.msu.su
\[ ds^2 = U \left( dz + \frac{\rho^2}{aU} d\phi \right)^2 - dt^2 + d\chi^2 + d\rho^2 + \frac{\rho^2}{U} d\phi^2. \] (5)

In presence of two Killing vectors \( \partial_t, \partial_5 \) the action (4) exhibits a discrete S-duality under interchange of electric and magnetic sectors with the simultaneous change of sign of the dilaton. That way one obtains the following solution

\[ ds^2 = V^{-1}(dy + 2b^{-1} \chi dt)^2 + V(-dt^2 + d\chi^2 + d\rho^2) + \rho^2 d\phi^2, \]
\[ V = 1 + \frac{\rho^2}{b^2}. \]

In terms of four-dimensional fields, it reads

\[ ds^2 = V^{-1}(dy + 2b^{-1} \chi dt)^2 + V^{-1} \rho^2 d\phi^2, \]
\[ e^{-\frac{1}{2} \phi} = V^{-1}, \quad A_t = \frac{\chi}{b}. \] (7)

Remarkably, these two five-dimensional vacuum solutions (4, 7) can be combined through a direct “superposition”, into the following six-dimensional vacuum solution

\[ ds^2 = V^{-1}(dy + 2b^{-1} \chi dt)^2 + U \left( dz + \frac{\rho^2}{aU} d\phi \right)^2 + V(-dt^2 + d\chi^2 + d\rho^2) + \frac{\rho^2}{U} d\phi^2. \] (8)

This solution will be used to generate new classes of 11D supergravity configurations via the following non-locally realized symmetry.

**III. D = 8 VACUUM – D = 11 SUPERGRAVITY CORRESPONDENCE**

One can show [3] that there exist a “duality” between 8D vacuum configurations possessing two commuting space–time Killing vectors and \( D = 11 \) supergravity solutions satisfying a certain ansatz. The eight-dimensional metric possessing two commuting Killing vectors in suitable coordinates reads

\[ ds^2 = h_{ab}(dx^a + A^a_{[1]} dx^\mu)(dx^b + A^b_{[1]} dx^\nu) + (det h)^{-\frac{1}{4}} g_{\mu\nu} dx^\mu dx^\nu, \]
\[ \text{where the } 2 \times 2 \text{ real matrix } h_{ab} \text{ and the one-forms } A^a_{[1]} \text{ depend only on } x^\mu. \]

For the 11D metric one assumes the following ansatz:

\[ ds^2_{11} = g_2^{-1} \delta_{ab} dx^a dx^b + g_3^{-1} \delta_{ij} dy^i dy^j + g_2^{-1} g_3^{-1} g_{\mu\nu} dx^\mu dx^\nu, \]
\[ \text{where the functions } g_2, g_3 \text{ and the six-dimensional metric } g_{\mu\nu} \text{ depend only on transverse space–time coordinates } x^\mu, \]

while for the four-form the corresponding decomposition should hold

\[ \hat{F}_{[4]_{\mu\nu ab}} = \epsilon_{ab} F_{[2]_{\mu\nu}}, \quad \hat{F}_{[4]_{ijkl}} = \epsilon_{ijkl} \partial_\mu \kappa, \]
\[ \hat{F}_{[4]_{\mu_1 \mu_2 \mu_3 \mu_4}} = H_{[4]_{\mu_1 \mu_2 \mu_3 \mu_4}}. \] (11)

The above duality goes as follows. One has to build the matrix \( h_{ab} \) and construct two-forms from KK-potentials

\[ h_{ab} = e^{\pm \phi} (e^{-\phi} + \kappa^2 e^\phi)^{-\frac{1}{2}}, \]
\[ F_{[2]} = dA_{[1]}, \quad H_{[2]} = dA_{[1]}. \]
\[ \phi \text{ and } \psi \text{ are defined by } \]
\[ \phi = \frac{1}{2} \ln g_3, \quad \psi = -\frac{3}{4} \ln g_2 - \frac{1}{4} \ln g_3, \]
\[ \text{and } H_{[2]} \text{ is a dual of the four–form field } H_{[4]} \text{ in the transverse space–time defined by } \]
\[ H_{[4]_{\alpha_1 \cdots \alpha_4}} = \frac{1}{2} \sqrt{-g} e^\psi e^\phi \epsilon_{\alpha_1 \cdots \alpha_4 \mu\nu} (H^\mu_{[2]} + \kappa F^\mu_{[2]}), \] (14)

It has to be noted that the mapping from 8D to 11D is a one to many, i.e. an 8D vacuum solution can have several distinct 11D supergravity counterparts depending on the choice and the order of two Killing vectors in eight dimensions.

**IV. SINGLE M–FLUXBRANES**

To use the above procedure for the solution (8) one has to add two extra coordinates \( x_1, x_2 \) so that \( ds^2 = ds^2_8 + dx_1^2 + dx_2^2 \) and then to choose the pair of Killing vectors with respect to which the parameterization of the eight-dimensional metric has been done. Let us start with deriving single \( M_2 \)– and \( M_5 \)–fluxbranes by uplifting either (4) or (5) 5D solutions translated to 8D form by adding three flat space coordinates. Uplifting (4) with respect to the ordered pair of the Killing vectors \( (\partial_2, \partial_x) \) (\( x \) can be any one of the added three flat coordinates) one obtains the \( M_2 \)–fluxbrane

\[ ds^2_{11} = U^{-\frac{1}{2}} \left\{ \rho^2 dx^2 + dy_1^2 + dy^2_2 \right\} + U^{\frac{1}{2}} \left\{ -dt^2 + d\chi^2 + d\rho^2 + dy_3^2 + \cdots + dy_7^2 \right\}, \]
\[ \hat{A}_{\phi_{12}} = \frac{\rho^2}{aU}. \] (15)

Alternatively, one can get the same solution by uplifting (5) with respect to the Killing pair \( (\partial_x, \partial_\phi) \), this is a more complicated way. Note that contrary to the canonical charged \( M_2 \)–brane, the \( M_2 \)–fluxbrane corresponds not to electric, but to the magnetic sector of the form field.

Similarly, the \( M_5 \)–fluxbrane can be derived by uplifting (5) using the following ordered pair of Killing vectors \( (\partial_\phi, \partial_x) \) (simpler way) or uplifting (4) via \( (\partial_x, \partial_\phi) \):
\[ ds_{11}^2 = V^{-\frac{2}{\phi}} \left\{ \rho^2 d\phi^2 + dy_1^2 + dy_2^2 \right\} \]
\[ + V^{-\frac{4}{3\phi}} \left\{ -dt^2 + d\chi^2 + d\rho^2 + dy_6^2 + dy_7^2 \right\}, \]
\[ \hat{A}_{167} = 2b^{-1} \chi. \]

This solution is supported by the electric sector of the four-form, contrary to the standard solitonic five-brane. Both solutions does not possess residual supersymmetry.

### V. INTERSECTING M–FLUXBRANES

Now let us derive the 11D counterparts to the six-dimensional solution (1) translated in two flat directions, \( x_1, x_2 \), making different choice of the ordered pairs from three Killing vectors \( \partial_y, \partial_z \) and \( \partial_x \) (\( x \) could be \( x_1 \) or \( x_2 \)). They form a class 11D supergravity solutions of the intersecting fluxbrane type. Different choice of Killing vectors leads to the following intersection:

| Killing Vectors | 11D Solution |
|-----------------|--------------|
| \( \partial_x \) | \( \partial_y \) | \( M2 \perp M2 \)-fluxbranes |
| \( \partial_z \) | \( \partial_z \) | \( M5 \perp M5 \)-fluxbranes |
| \( \partial_x \) | \( \partial_y \) | \( M2 \)-fluxbrane \perp \text{Wave} |
| \( \partial_z \) | \( \partial_x \) | \( M5 \)-fluxbrane \perp \text{Wave} |
| \( \partial_y \) | \( \partial_x \) | \( M2 \)-fluxbrane \perp \text{Monopole} |
| \( \partial_z \) | \( \partial_y \) | \( M5 \)-fluxbrane \perp \text{Monopole} |

First let us consider the case of the pair of Killing vectors \( (\partial_z, \partial_y) \) in some detail. Following (12), the transition to eleven dimension goes through the quantities

\[ \phi = \frac{1}{2} \ln (V^{-1} U^{-1}), \quad \psi = \frac{3}{4} \ln (V^{-1} U), \]

from which the functions \( g_2 \) and \( g_3 \) can be obtained by solving (13):

\[ g_2 = V^{-\frac{2}{3\phi}} U^{-\frac{4}{3}}, \quad g_3 = VU. \]

Then the metric of the corresponding 11D-supergravity solution easily reads from (11)

\[ ds_{11}^2 = V^{-\frac{2}{\phi}} \left\{ \rho^2 d\phi^2 + dy_1^2 + dy_2^2 \right\} \]
\[ + V^{-\frac{4}{3\phi}} \left\{ dy_6^2 + dy_7^2 \right\} \]
\[ + V^{-\frac{4}{3\phi}} \left\{ -dt^2 + d\chi^2 + d\rho^2 + dy_6^2 + dy_7^2 \right\}, \]

while the potential of 11D four–form field can be found according to (14)

\[ \hat{A}_{12} = \frac{\rho^2}{aU}, \quad \hat{A}_{67} = \frac{\rho^2}{bV}. \]

The solution obtained is the intersection of two \( M2 \)-fluxbranes. For the case \( a^{-1} = 0 \) (or \( b^{-1} = 0 \)) it reduces to usual \( M2 \)-fluxbrane located at \( (\phi, y_6, y_7) \) (or \( (\phi, y_1, y_2) \)). Different combination of pair Killing vectors generates through similar calculations other 11D solutions which we present below. The pair \( (\partial_y, \partial_z) \) generates intersecting of two \( M5 \)-fluxbranes:

\[ ds_{11}^2 = V^{-\frac{2}{\phi}} U^{-\frac{4}{3\phi}} \left\{ \rho^2 d\phi^2 + dy_1^2 + dy_2^2 \right\} \]
\[ + V^{-\frac{4}{3\phi}} \left\{ dy_6^2 + dy_7^2 \right\} \]
\[ + V^{-\frac{4}{3\phi}} \left\{ -dt^2 + d\chi^2 + d\rho^2 \right\}, \]
\[ \hat{A}_{12} = 2b^{-1} \chi, \quad \hat{A}_{67} = 2a^{-1} \chi. \]

The pair \( (\partial_z, \partial_z) \) gives the superposition of \( M2 \)-fluxbrane with wave–type solution

\[ ds_{11}^2 = U^{-\frac{4}{3\phi}} \left\{ \rho^2 d\phi^2 + dy_1^2 + dy_2^2 \right\} \]
\[ + U^{-\frac{4}{3\phi}} \left\{ V^{-1}(dy + 2b^{-1} \chi dt)^2 \right\} \]
\[ + V\left\{ -dt^2 + d\chi^2 + d\rho^2 \right\} + dy_6^2 + dy_7^2 \]
\[ \hat{A}_{12} = \frac{\rho^2}{aU}, \]

while the pair \( (\partial_x, \partial_z) \) generates the superposition of \( M5 \)-fluxbrane and wave:

\[ ds_{11}^2 = U^{-\frac{4}{3\phi}} \left\{ \rho^2 d\phi^2 + dy_1^2 + dy_2^2 \right\} \]
\[ + U^{-\frac{4}{3\phi}} \left\{ V^{-1}(dy_2 + 2b^{-1} \chi dt)^2 \right\} \]
\[ + V\left\{ -dt^2 + d\chi^2 + d\rho^2 \right\} + dy_6^2 \]
\[ \hat{A}_{67} = 2a^{-1} \chi. \]

The remaining pair \( (\partial_y, \partial_x) \) generates a superposition of \( M2 \)-fluxbrane with monopole–type solution

\[ ds_{11}^2 = V^{-\frac{2}{\phi}} \left\{ \rho^2 d\phi^2 + U(dz + \frac{\rho^2}{aU} d\phi)^2 \right\} + dy_6^2 + dy_7^2 \]
\[ + V \left\{ -dt^2 + d\chi^2 + d\rho^2 + dy_1^2 + dy_2^2 \right\}, \]
\[ \hat{A}_{\phi 67} = \frac{\rho^2}{bV}. \]

while the reordered pair \( (\partial_y, \partial_x) \) gives a superposition of \( M5 \)-fluxbrane and monopole:

\[ ds_{11}^2 = V^{-\frac{2}{\phi}} \left\{ \rho^2 d\phi^2 + U(dz + \frac{\rho^2}{aU} d\phi)^2 \right\} + dy_6^2 + dy_7^2 \]
\[ + V \left\{ -dt^2 + d\chi^2 + d\rho^2 + dy_1^2 + dy_2^2 \right\}, \]
\[ \hat{A}_{\phi 12} = 2b^{-1} \chi. \]
VI. DISCUSSION

Thus in eleven-dimensional supergravity there exist a variety of fluxbrane solutions originating from $M^2$ and $M^5$ fluxbranes. Contrary to ordinary charged branes, the $M^2$ fluxbrane is magnetic, while the $M^5$ one – electric. Their intersections obey the same intersection rules as those for ordinary $p$-branes. Our class does not include intersections of $M^2$ and $M^5$ fluxbranes between themselves, but these are likely to exist too as intersections of more than two branes. All these solutions are non supersymmetric. Using similar method one can construct more general solutions including combinations of fluxbranes and ordinary charges branes and investigate the creation of the latters in the field of external four-forms.

[1] G. W. Gibbons, Quantized Flux-Tubes in Einstein-Maxwell Theory and Non-Compact Internal Spaces, in Fields and Geometry, Proc. XXII Karpacz Winter School of Theoretical Physics 1986, ed. A. Jadczyk, WS, Singapore.
[2] M.A. Melvin, Phys. Lett. 8 (1964) 65.
[3] H. F. Dowker, J. P. Gauntlett, S. B. Giddings and G. T. Horowitz, Phys. Rev. D50 (1994) 2662.
[4] H. F. Dowker, J. P. Gauntlett, G. W. Gibbons and G. T. Horowitz, Phys. Rev. D53 (1996) 7115.
[5] D.V. Gal’tsov and O.A. Rytchkov, Phys. Rev. D58 (1998) 122001; hep-th/9601160.
[6] G.W. Gibbons and K. Maeda, Nucl. Phys. B298 (1988) 741-775.
[7] C.-M. Chen, D.V. Gal’tsov and S.A. Sharakin, Kaluza-Klein Reduction Revisited: Dualization Alternatives and Non-Local Dualities, in preparation.