On the Optimum Energy Efficiency for Flat-fading Channels with Rate-dependent Circuit Power

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Abstract—This paper investigates the optimum energy efficiency (EE) and the corresponding spectral efficiency (SE) for a communication link operating over a flat-fading channel. The EE is evaluated by the total energy consumption for transmitting per message bit. Three channel cases are considered, namely static channel with channel state information available at transmitter (CSIT), fast-varying (FV) channel with channel distribution information available at transmitter (CDIT), and FV channel with CSIT. The link’s circuit power is modeled as \( \rho + \kappa\phi(R) \) Watt, where \( \rho > 0 \) and \( \kappa \geq 0 \) are two constants and \( \phi(R) \) is a general increasing and convex function of the transmission rate \( R \geq 0 \). For all the three channel cases, the tradeoff between the EE and SE is studied. It is shown that the EE improves strictly as the SE increases from 0 to the optimum SE, and then strictly degrades as the SE increases beyond the optimum SE. The impact of \( \kappa, \rho \) and other system parameters on the optimum EE and corresponding SE is investigated to obtain insight. Some of the important and interesting results for all the channel cases include: (1) when \( \kappa \) increases the SE corresponding to the optimum EE should keep unchanged if \( \phi(R) = R \), but reduced if \( \phi(R) \) is strictly convex of \( R \); (2) when the rate-independent circuit power \( \rho \) increases, the SE corresponding to the optimum EE has to be increased. A polynomial-complexity algorithm is developed with the bisection method to find the optimum SE. The insight is corroborated and the optimum EE for the three cases are compared by simulation results.

Index Terms—Energy efficiency, spectral efficiency, flat-fading channels, quasiconvexity, resource allocation.

I. INTRODUCTION

Energy-efficient communication and signal processing techniques play important roles in applications where devices are powered by batteries [1]–[7]. For a communication system, its energy efficiency (EE) can be evaluated by either the total energy consumption for transmitting per message bit (TEPB), or the number of message bits transmitted with per-Joule total energy consumption (NBPE). A higher EE is represented by either a smaller TE_pb or a greater NBPE. Note that due to the scarcity of spectral resource, there already existed traditional and intensive research on increasing spectral efficiency (SE) as an important goal in the field of wireless communications. Therefore, it becomes a very important research topic to study the relationship between the optimum EE and the corresponding SE, as well as the impact of system parameters on them for wireless communication systems.

Early works studying the EE of communication systems only considered transmission energy but ignored circuit energy consumption. For instance, approximate expressions of per message bit transmission energy were derived in [8] as a function of the spectral efficiency for flat-fading channels in wideband regime, and some strategies to reduce the TEPB were discussed in [9], [10]. In these works, only the transmission energy consumed for radiating radio-frequency signals was taken in account while the circuit energy consumption was neglected, which makes sense for long-distance communication related application scenarios. The major finding is that the SE has to be reduced to improve the EE when only the transmission energy is considered, i.e., the SE and EE are contradictory performance metrics since improving one leads to degradation of another one.

For the high-EE design of short-distance communication systems, which have many promising applications and thus attracted much research interest, the circuit energy consumption however cannot be ignored [11]. For instance, data transmission within a wireless body area network is mainly over short distance, which leads to small transmission energy consumption comparable to the circuit energy consumption [12]. In such a case, the circuit energy must be taken into account. In view of the above fact, the circuit energy was taken into account to optimize the EE of communication systems in recent works. For instance, modulation schemes were optimized in [13] for communication links operating over flat-fading channels, and link adaptation algorithms were developed in [14]–[19] for multi-carrier systems transmitting over frequency-selective channels. General frameworks for energy efficiency optimization were proposed in [20], [21]. In [13], [14], [17], [19]–[21], the circuit power was assumed to remain fixed independently of the bit transmission rate. In [15], [16], [18], the circuit power was assumed to be linear with the transmission rate. In general, the circuit power is an increasing function of the transmission rate, since a greater bit rate indicates that a bigger codebook is used which usually incurs higher power for encoding and decoding on baseband circuit boards [22], [23]. Note that static channels with channel state information at the transmitter (CSIT) were studied in [13]–[17], [19], while both static and fast-varying channels.
with CSIT were studied in [20], [21]. The major finding in these works is that when taking into account the circuit energy consumption, the relationship between the SE and EE is fundamentally different from that when the circuit energy is ignored. In particular, the EE usually first improves then degrades as the SE increases from zero [19].

In this paper, we study the optimum EE and the corresponding SE for flat fading channels with rate-dependent circuit power in a more general form than those studied previously. Even though the flat-fading channel model seems simple, it deserves research effort due to the following reasons. First, it has been widely used in practice especially for low-power applications, e.g., in wireless sensor networks where highly energy-efficient transmission is needed. Second, there exist different cases when using the flat-fading channel, which depend on the condition of channel variation and availability of channel knowledge. For these cases, the optimum EE and the corresponding SE performance deserve much attention and need to be thoroughly investigated. Motivated by the above fact, we consider three different cases for using the flat-fading channel, namely

1) Case 1: static channel with CSIT;
2) Case 2: fast-varying channel with channel distribution information at transmitter (CDIT);
3) Case 3: fast-varying channel with CSIT.

For each case listed above, we model the link’s total power consumption as the sum of the power amplifier’s power consumption and circuit power. The circuit power is modeled as the sum of a constant power and a rate-dependent part which is a general increasing and convex function of the transmission rate. This circuit-power model is more general than those studied in the literature since either fixed circuit power or the circuit power as a linear function of transmission rate was studied previously. We formulate the EE-SE function and make EE-SE tradeoff study. The impact of system parameters on the optimum EE and SE is then studied. In particular, insight is obtained from the theoretical analysis, which may help practical system design to improve its EE. A polynomial-complexity algorithm is developed with the bisection method to find the optimum EE and SE. Finally, we show simulation results to corroborate the insight obtained from the theoretical analysis and compare the optimum EEs for the different channel cases.

The rest of this paper is organized as follows. The system models are described in the next Section. After that, the EE-SE function is formulated and the EE-SE tradeoff analysis is made in Section III. The impact of system parameters on the optimum EE and SE is investigated in Section IV. The algorithm is developed in Section V, and simulation results are shown in Section VI to illustrate the obtained insight. Some conclusions are made in Section VII.

Notations: $E_x\{f(x)\}$ represents the ensemble average of the function $f(x)$ over the probability density function of the random variable $x$. $y'(x)$ and $y''(x)$ indicate the first-order and second-order derivatives of $y(x)$ with respect to $x$, respectively.

II. System models

Consider a point-to-point communication link transmitting over a flat-fading channel using a bandwidth $B$ Hz. The baseband channel model is formulated as

$$y = hx + n$$

where $x$ is the complex symbol emitted by the transmitter, and $y$ represents the corresponding symbol received at the receiver’s baseband processor. $h$ is the channel coefficient. $n$ is the sum of additive white Gaussian noise and the cochannel interference. We assume $n$ is a random variable with circularly symmetric complex Gaussian distribution with zero mean and variance $\sigma^2$, which keeps invariant during data transmission. $h$ and $\sigma^2$ are assumed to be known by the receiver.

The link’s total power consumption is modeled as the sum of two parts: the power consumed by the transmitter’s power amplifier for emitting coded symbols and circuit power. Specifically, the circuit power is modeled as $\rho + \kappa \phi(R)$ Watt, where $\rho > 0$ and $\kappa \geq 0$ are two constants and $R$ represents the transmission rate in the unit of bits/second. $\rho$ represents the rate-independent circuit power which models the sum power of filters, low-noise-amplifiers, mixers, synthesizers, etc. $\kappa \phi(R)$ models the rate-dependent circuit power, e.g., that consumed by channel encoder and decoder. We assume $\phi(R)$ satisfies that

1) $\phi(0) = 0$, i.e., the rate-dependent circuit power is zero when $R = 0$;
2) $\phi(R)$ is differentiable, strictly increasing and (not necessarily strictly) convex of $R \geq 0$.

Note that the rate-dependent circuit power models studied in the literature, e.g., [13]–[16] are special cases of the model assumed above. Specifically, when the rate-dependent circuit power is negligible as in [13], [14], we can simply set $\kappa = 0$. When the rate-dependent circuit power increases linearly with respect to the rate as studied in [15], [16], we can set $\phi(R) = R$ and $\kappa$ as the increasing rate. Moreover, the model is also applicable for the links where the rate-dependent circuit power is strictly convex of the rate as will be studied later in this paper.

Define $G = |h|^2$ as the instantaneous channel power gain. Three different scenarios for using the communication link will be considered as follows:

- Case 1 (static channel with CSIT): the channel keeps invariant with CSI available at the transmitter, and $G$ is known by the transmitter at the beginning of the transmission.
- Case 2 (FV channel with CDIT): the channel varies during the data transmission and the probability density function (pdf) of $G$ is known a priori by the transmitter.
- Case 3 (FV channel with CSIT): the channel varies during the data transmission and $G$ is known during the transmission.

For Case 1 and Case 2, suppose the average power of transmitted symbols (referred to as transmission power hereafter) is $p$ Watt, i.e., $E_x\{|x|^2\} = P$. Assume the optimum codebook
is used, the maximum SE can be evaluated as:

$$\theta(P) = \begin{cases} \log_2\left(1 + \frac{G P}{\sigma^2}\right) & \text{for Case 1;} \\ E_G\left\{\log_2\left(1 + \frac{G P}{\sigma^2}\right)\right\} & \text{for Case 2.} \end{cases}$$  \hspace{1cm} (2)$$

in the unit of bits/second/Hz. For both cases, the average sum power is

$$\kappa \phi(B\theta(P)) + \frac{P}{\xi} + \rho \text{ (Watt)}$$  \hspace{1cm} (3)$$

where \(\xi\) represents the efficiency of the power amplifier.

For Case 3, the transmission power can be adapted according to CSI. Suppose the transmitter uses \(P(G)\) as the transmission power when the channel power gain is \(G\). Note that any nonnegative function of \(G\) can be assigned to \(P(G)\) as a feasible power-allocation strategy, denoted by \(\mathcal{P} = \{P(G) | G \geq 0\}\) hereafter. Obviously, the set of all feasible strategies is simply the set of all nonnegative functions, denoted by \(\mathcal{S}_\mathcal{P}\) hereafter. Assume the optimum codebook is used, the SE corresponding to using \(\mathcal{P}\) is equal to

$$\theta(\mathcal{P}) = E_G\left\{\log_2\left(1 + \frac{G P(G)}{\sigma^2}\right)\right\}$$  \hspace{1cm} (4)$$

in the unit of bits/second/Hz. The average sum power is

$$\kappa \phi(B\theta(\mathcal{P})) + \frac{E_G\{P(G)\}}{\xi} + \rho \text{ (Watt)}.$$  \hspace{1cm} (5)$$

III. EE-SE TRADEOFF FORMULATION AND ANALYSIS

In this paper, the EE is evaluated as the TEPB as in [13]. Obviously, a smaller TEPB indicates a better EE. For the single-user case as studied in this paper, it is also equivalent to express it as the NBPE. For multi-user scenarios as studied in [17], it might be more convenient to use the NBPE to evaluate the EE.

To facilitate the EE-SE tradeoff analysis, we first formulate in Section III-A for each case the EE-SE function \(\varepsilon(\theta)\), defined as the EE corresponding to a given SE \(\theta\). Denote the optimum EE as \(\varepsilon^*\), i.e.,

$$\varepsilon^* = \min_{\theta \geq 0} \varepsilon(\theta),$$  \hspace{1cm} (6)$$

and the corresponding optimum SE as \(\theta^*\), i.e,

$$\theta^* = \arg \min_{\theta \geq 0} \varepsilon(\theta).$$  \hspace{1cm} (7)$$

Note that for each case under consideration, \(\theta^* > 0\) must hold because when \(\theta = 0\), the \(\varepsilon\) is \(+\infty\) due to the existence of \(\rho > 0\). In Section III-B, we will make theoretical analysis and show geometric interpretation to study properties of \(\varepsilon(\theta)\), \(\varepsilon^*\) and \(\theta^*\), which unveils the EE-SE tradeoff.

A. Formulation of the EE-SE function

To formulate \(\varepsilon(\theta)\), we first derive \(\gamma(\theta)\), defined as the ratio of the minimum transmission power required to achieve \(\theta\) to \(\sigma^2\). It can readily be shown that

- For Case 2:
  $$\gamma(\theta) = \min_{p \geq 0: \log_2(1 + G p) \geq \theta} p = \frac{1}{G} \left(2^{\theta} - 1\right)$$  \hspace{1cm} (8)$$

  \begin{align*}
  &\text{where} \quad \theta(\mathcal{P}) = E_G\left\{\log_2\left(1 + \frac{G P(G)}{\sigma^2}\right)\right\} \\
  &\text{for Case 2.} \\
  &\text{in the unit of bits/second/Hz. For both cases, the average sum power is} \\
  &\kappa \phi(B\theta(P)) + \frac{P}{\xi} + \rho \text{ (Watt)} \quad \text{(3)}
  \end{align*}$$

- For Case 3:
  $$\gamma(\theta) = \min_{\{p(G) | G \geq 0\} \in \mathcal{S}_\mathcal{P}} \left\{E_G\{\log_2(1 + G p)\} \geq \theta\right\}$$

  \begin{align*}
  &\text{where} \quad \theta(\mathcal{P}) = E_G\left\{\log_2\left(1 + \frac{G P(G)}{\sigma^2}\right)\right\} \\
  &\text{for Case 2.} \\
  &\text{in the unit of bits/second/Hz. For both cases, the average sum power is} \\
  &\kappa \phi(B\theta(P)) + \frac{E_G\{P(G)\}}{\xi} + \rho \text{ (Watt)} \quad \text{(5)}
  \end{align*}$$

B. EE-SE tradeoff analysis

To facilitate the tradeoff analysis, we first show the following property of \(\gamma(\theta)\):

**Lemma 1:** For each channel case under consideration, \(\gamma(\theta)\) is strictly increasing and strictly convex of \(\theta \geq 0\).

**Proof:** See the Appendix.

We then show an important property of \(\varepsilon(\theta)\) as follows:

**Lemma 2:** For each channel case and any \(\phi(R)\) satisfying the assumptions made in Section II, \(\varepsilon(\theta)\) is a strictly quasi-convex function of \(\theta\).

**Proof:** See the Appendix.

According to the strict quasi-convexity of \(\varepsilon(\theta)\),

$$\max\{\varepsilon(\theta_1), \varepsilon(\theta_2)\} > \varepsilon(\theta)$$  \hspace{1cm} (13)$$

holds \(\forall \theta \in (\theta_1, \theta_2)\) [24, 25]. The strict quasi-convexity is a key feature for \(\varepsilon(\theta)\), based on which we will prove properties of \(\theta^*\) and \(\varepsilon(\theta)\) in the following. To facilitate description, we first derive the derivative of \(\varepsilon(\theta)\) with respect to \(\theta\) as follows:

$$\varepsilon'(\theta) = \frac{1}{B\theta} \left[ P'(\theta) - \frac{P(\theta)}{\theta} \right].$$  \hspace{1cm} (14)$$

**Theorem 1:** For each channel case, the following properties are satisfied:

1) There exists a unique \(\theta^*\) and it satisfies

$$P'(\theta^*) = \frac{P(\theta^*)}{\theta^*}$$  \hspace{1cm} (15)$$

2) \(\varepsilon(\theta)\) is strictly decreasing with \(\theta \in (0, \theta^*]\) and

$$\forall \theta \in (0, \theta^*], P'(\theta) < \frac{P(\theta)}{\theta}. \hspace{1cm} (16)$$

3) \(\varepsilon(\theta)\) is strictly increasing with \(\theta \in [\theta^*, +\infty)\) and

$$\theta \in (\theta^*, +\infty), P'(\theta) > \frac{P(\theta)}{\theta}. \hspace{1cm} (17)$$
Proof: See the Appendix. It is interesting to note that these properties are derived by solely using the strict quasi-convexity of $\varepsilon(\theta)$, without resorting to the derivative of $\varepsilon(\theta)$ with respect to $\theta$ as in [14], [16], [18].

Straightforward geometric interpretations can be presented to intuitively explain the monotonic properties of $\varepsilon(\theta)$ specified in Theorem 1. Specifically, we can plot the line $L_1 = \{X(\theta) = (\theta, P(\theta))| \theta \geq 0\}$ over the two-dimensional plane of coordinates $(\theta, y)$ as shown in Fig. 1. Note that $B_\varepsilon(\theta) = \frac{\varepsilon}{\theta}$ is equal to the slope of the origin-to-$X(\theta)$ line, and $P'(\theta)$ is equal to the slope of the tangent line of $y = P(\theta)$ at $\theta$.

It can be seen from Fig. 1a that as $X(\theta)$ moves away from the origin along the line $L_1$, the slope of the origin-to-$X(\theta)$ line first strictly reduces and then strictly increases, meaning that $\varepsilon(\theta)$ first strictly reduces and then strictly increases. The reason behind this observation is the strict convexity of $P(\theta)$. At $\theta = \theta^*$ where $\varepsilon(\theta)$ is minimized (i.e., the slope of the origin-to-$X(\theta)$ line is the smallest), the origin-to-$X(\theta)$ line coincides with the tangent line of $y = P(\theta)$, indicating that (15) indeed holds. It can be seen that $\theta^*$ is unique due to the strict convexity of $P(\theta)$. For any $\theta \in (0, \theta^*)$, it can be seen from Fig. 1a that the origin-to-$X(\theta)$ line is steeper than the tangent line of $y = P(\theta)$, indicating that (16) indeed holds. For any $\theta \in (\theta^*, +\infty)$, it can be seen from Fig. 1b that the tangent line of $y = P(\theta)$ is steeper than the origin-to-$X(\theta)$ line, indicating that (17) indeed holds. Note that a similar geometric interpretation was exhibited in [20] to derive optimality conditions for solving the problem of maximizing $\frac{y_1(x)}{y_2(x)}$, where $y_1(x)$ is concave and $y_2(x)$ is a nonnegative convex function.

The fundamental insight behind Theorem 1 is that increasing $\theta$ is favorable for improving $\varepsilon$ when $\theta \leq \theta^*$, while $\theta$ has to be sacrificed for better $\varepsilon$ when $\theta \geq \theta^*$. To illustrate the shape of $\varepsilon(\theta)$, the $\varepsilon(\theta)$ is plotted in Fig. 2 for the three channel cases over a typical flat-fading channel. The monotonic properties of $\varepsilon(\theta)$ claimed in Theorem 1 can be seen clearly.

IV. IMPACT OF SYSTEM PARAMETERS ON $\varepsilon^*$ AND $\theta^*$

Collect system parameters for Case 1 into the set $\chi = \{G, \sigma^2, \kappa, \rho\}$, and those for Cases 2 and 3 into the set $\chi = \{\sigma^2, \kappa, \rho\}$. Denote the $\varepsilon^*$ and $\theta^*$ corresponding to a given $\chi$ as $\varepsilon^* (\chi)$ and $\theta^* (\chi)$, respectively. In the following, we will study the impact of system parameters on $\varepsilon^* (\chi)$ and $\theta^* (\chi)$.

We first present some preliminary rules that will play important roles later:

Lemma 3: For each case under consideration, given any $\theta \geq 0$, it must satisfy

$$\theta \begin{cases} < \theta^* (\chi) & \text{iff } \Gamma_\chi (\theta) < 0 \\ = \theta^* (\chi) & \text{iff } \Gamma_\chi (\theta) = 0 \\ > \theta^* (\chi) & \text{iff } \Gamma_\chi (\theta) > 0 \end{cases} \tag{18}$$

where

$$\Gamma_\chi (\theta) = \kappa g (B\theta) + \frac{\sigma^2}{\xi} f(\theta) - \rho, \tag{19}$$

and

$$g(R) = R\phi'(R) - \phi(R), \quad f(\theta) = \theta\gamma'(\theta) - \gamma(\theta). \tag{20}$$
Moreover, the following claims are true:

1) \( g(0) = 0 \) If \( \phi(R) \) is strictly convex of \( R \geq 0 \), \( g(R) \) is strictly increasing of \( R \geq 0 \), while \( g(R) = 0, \forall R \geq 0 \) if \( \phi(R) = R \).

2) \( f(0) = 0 \) and \( f(\theta) \) is strictly increasing of \( \theta \geq 0 \).

**Proof:** See the Appendix.

Straightforward geometric interpretation can be given to corroborate the above properties of \( g(R) \) intuitively (those for \( f(\theta) \) can be interpreted in a similar way and are thus omitted here). Specifically, we can plot the line \( L_2 = \{ \mathbf{X}(R) = (R, \phi(R)) | \forall R \geq 0 \} \) over the two-dimensional plane of coordinates \((R, y)\) when \( \phi(R) \) is strictly convex of \( R \geq 0 \) as shown in Fig. 3. Most interestingly, \( -g(R) = \phi(R) - R\phi'(R) \) is equal to the y-coordinate of the intersection between the line \( R = 0 \) and the tangent line of \( y = \phi(R) \) drawn at the coordinate \( \mathbf{X}(R) \). When \( \mathbf{X}(R) \) moves away from the origin along the line \( L_2 \), \( -g(R) \) strictly decreases, which means that \( g(R) \) indeed strictly increases. When \( \phi(R) = R \), it can be seen that \( g(R) \) is fixed as 0 for any \( R \geq 0 \).

Based on the above lemma, we first show the impact of \( \kappa \) on \( \theta^*(\chi) \) and \( \epsilon^*(\chi) \) as follows:

**Theorem 2:** When \( \kappa \) increases, the following claims are true for each case under consideration:

1) \( \epsilon^*(\chi) \) strictly increases.

2) \( \theta^*(\chi) \) strictly decreases if \( \phi(R) \) is strictly convex of \( R \geq 0 \). Moreover,

\[
\lim_{\kappa \to +\infty} \theta^*(\chi) = 0; \quad \lim_{\kappa \to 0} \theta^*(\chi) = \theta_1. \tag{21}
\]

where \( \theta_1 \) satisfies that \( \sigma^2 f(\theta_1) = \rho \).

3) \( \theta^*(\chi) \) is fixed as \( \theta_1 \) if \( \phi(R) = R \).

**Proof:** See the Appendix.

Note that when \( \kappa = 0 \), the power amplifier’s power consumption is the only source for the rate-dependent circuit power, and \( \theta^*(\chi) = \theta_1 \) in such a case. Theorem 2 reveals important insight that the behavior of \( \theta^*(\chi) \) when \( \kappa \) increases depends on the specific model that \( \phi(R) \) follows. If \( \phi(R) = R \), \( \theta^*(\chi) \) keeps unchanged as \( \theta_1 \) when \( \kappa \) increases. However, \( \theta^*(\chi) \) decreases and approaches zero if \( \phi(R) \) is strictly convex of \( R \). It is also interesting to see that \( \theta^*(\chi) \) when \( \phi(R) = R \) is always higher than that when \( \phi(R) \) is strictly convex of \( R \). The impact of \( \sigma^2 \) on \( \theta^*(\chi) \) and \( \epsilon^*(\chi) \) is shown as follows:

**Theorem 3:** When \( \sigma^2 \) increases, the following claims are true for each case under consideration:

1) \( \epsilon^*(\chi) \) strictly increases.

2) \( \theta^*(\chi) \) strictly decreases. Moreover,

\[
\lim_{\sigma^2 \to +\infty} \theta^*(\chi) = 0; \quad \lim_{\sigma^2 \to 0} \theta^*(\chi) = \theta_2. \tag{22}
\]

where \( \theta_2 \) satisfies that \( \kappa g(\theta_2) = \rho \).

**Proof:** This theorem can be proven in a similar way as Theorem 2 thus the proof is omitted here.

Note that when \( \sigma^2 = 0 \), \( \kappa \phi(R) \) is the only source for the rate-dependent circuit power, and \( \theta^*(\chi) = \theta_2 \). Theorem 5 indicates when \( \sigma^2 \) increases, \( \theta^*(\chi) \) decreases and approaches zero.

The impact of \( \rho \) on \( \theta^*(\chi) \) and \( \epsilon^*(\chi) \) is shown as follows:

**Theorem 4:** When \( \rho \) increases, the following claims are true for each case under consideration:

1) \( \epsilon^*(\chi) \) strictly increases.

2) \( \theta^*(\chi) \) strictly increases. Moreover,

\[
\lim_{\rho \to +\infty} \theta^*(\chi) = +\infty; \quad \lim_{\rho \to 0} \theta^*(\chi) = 0. \tag{23}
\]

**Proof:** This theorem can be proven in a similar way as Theorem 2 thus the proof is omitted here.

The above three theorems reveal important insight about the impact of system parameters on \( \theta^*(\chi) \) and \( \epsilon^*(\chi) \) for three cases under consideration. For each case, the optimum EE always increases when any one of \( \sigma^2 \), \( \kappa \) and \( \rho \) increases. For Case 1, the optimum EE degrades when \( G \) increases:

1) \( \epsilon^*(\chi) \) strictly decreases.

2) \( \theta^*(\chi) \) strictly increases. Moreover,

\[
\lim_{G \to +\infty} \theta^*(\chi) = \theta_2; \quad \lim_{G \to 0} \theta^*(\chi) = 0. \tag{24}
\]

**Proof:** This theorem can be proven in a similar way as Theorem 2 thus the proof is omitted here.

The linear rate-dependent circuit power model is considered. At the end of Section III.A in [16], it was said that the linear rate-dependent circuit power model (i.e., \( \kappa R \)) used there can be generalized to a convex model (i.e., \( \kappa \phi(R) \)) as in our work for the static channel with CSIT. However, it was claimed there that after the generalization the optimum SE is still independent of \( \kappa \).

Here we show a different finding that if \( \phi(R) \) is strictly convex of \( R \), the optimum SE decreases as \( \kappa \) increases for all the three cases of the flat-fading channels. The theoretical proof as well as simulation results (see Section VI) are given to corroborate the above new finding.
consumption, the optimum SE has to be decreased, i.e., the link should slower its transmission to operate with the optimum EE.

3) If the rate-independent circuit power \( p \) increases, the optimum SE has to be increased, i.e., the link should transmit at a higher rate to operate with the optimum EE. This observation was also mentioned in [10] when \( \phi(R) = R \) for the static channel case with CSIT.

It is also interesting to compare \( \varepsilon^*(\chi) \) of the three channel cases. It can readily be shown that

1) \( \varepsilon^*(\chi) \) for Case 1 is not higher (i.e., better or same) than that for Case 2 if the average value of the random channel power gain (i.e., \( E_G\{G\} \)) for Case 2 is the same as the fixed channel power gain (i.e., \( G \)) for Case 1. This is because when using the same transmission power \( p \), the SE for Case 2 is not higher than that for Case 1 due to the Jensen’s inequality (i.e., \( E_G\{\log_2(1 + Gp^\ast)\} \leq \log_2(1 + E_G\{G\}p^\ast) \)).

2) \( \varepsilon^*(\chi) \) for Case 3 is not higher than that for Case 2 when the random channel power gain for both cases has the same distribution. The reason is that for any given \( \theta \), the total power for Case 3 is not higher than that for Case 2 since for Case 3 the transmitter has the flexibility to adapt the transmit power according to the available CSI.

We will compare the \( \varepsilon^*(\chi) \) for Case 1 with that for Case 3 by simulation results as will be shown in Section VII.

V. ALGORITHM DESIGN

When the circuit power is a constant, i.e., \( \kappa = 0 \), algorithms have been proposed in [20] to find \( \theta^*(\chi) \) and \( \varepsilon^*(\chi) \) for Cases 1 and 3. When \( \phi(R) = R \), the algorithm proposed in [10] can be used to find \( \theta^*(\chi) \) and \( \varepsilon^*(\chi) \) for Case 1. However, they are not applicable when \( \phi(R) \) is a general convex function of \( R \) as considered in this paper.

For the three channel cases under consideration, we propose an algorithm, which is summarized as Algorithm I to find \( \theta^*(\chi) \) and \( \varepsilon^*(\chi) \) based on the bisection method according to Lemma 3. The key to this algorithm is to evaluate \( \Gamma_\chi(\theta) = \kappa g(B\theta) + \frac{p^\ast}{2} f(\theta) - \rho \) corresponding to any given \( \theta \) for each case. To this end, \( g(B\theta) = B\theta \phi'(B\theta) - \phi(B\theta) \) can be computed according to the expression of \( \phi(R) \). Moreover, the following procedures can be taken to compute \( f(\theta) \) for each case:

- Case 1: it has been shown that \( \gamma(\theta) = \frac{1}{G}(2^\theta - 1) \) and \( \gamma'(\theta) = \frac{\ln 2}{G} 2^\theta \), which means that
  \[
  f(\theta) = \frac{1}{G}((\ln 2)\theta - 1)2^\theta + 1. \tag{25}
  \]

- Case 2: note that \( \gamma(\theta) \) can be numerically evaluated as the \( p^\ast \) satisfying
  \[
  E_G\{\log_2(1 + Gp^\ast)\} = \theta \tag{26}
  \]
  with the bisection method. It can readily be shown that
  \[
  \gamma'(\theta) = \frac{\ln 2}{E_G\{\frac{G}{1+Gp^\ast}\}}. \tag{27}
  \]

This means that
  \[
  f(\theta) = \theta - \frac{\ln 2}{E_G\{\frac{G}{1+Gp^\ast}\}} - p^\ast. \tag{28}
  \]

- Case 3: note that problem (10) is a convex optimization problem. By introducing \( \mu \) as the Lagrange multiplier for the constraint, it can readily be seen that the optimum \( p(G) \) is
  \[
  p(G) = \left[ \frac{\mu^*}{\ln 2} - 1 \right]^+, \tag{29}
  \]
  where \( \mu^* \) is the nonnegative value satisfying
  \[
  E_G\{\log_2(1 + Gp(G))\} = \theta. \tag{30}
  \]

Moreover, \( \mu^* \) is equal to the increasing rate of the optimum objective value for problem (10), with respect to \( \theta \) according to the sensitivity analysis in convex optimization theory (see pages 249-253 of [25] for more details.) This means that \( \gamma'(\theta) = \mu^* \) holds. Note that we have already used this sensitivity-analysis based optimization technique in previous works, e.g., [10], [26]–[28], to design resource allocation algorithms in a very efficient and effective way. As a result, \( f(\theta) \) can be evaluated according to
  \[
  f(\theta) = \theta \mu^* - E_G\{p(G)\}. \tag{31}
  \]

Algorithm 1 The algorithm to compute \( \theta^* \) and \( \varepsilon^* \) for each case.

1: \( \theta_1 = 0; \theta_u = 1; \)
2: evaluate \( \Gamma_\chi(\theta_u) \) by using (25), (28) and (31) for Cases 1, 2 and 3, respectively;
3: while \( \Gamma_\chi(\theta_u) < 0 \)
4: \( \theta_\ast = 2 \ast \theta_a \)
5: end while
6: while \( \theta_a - \theta_1 > \delta \)
7: \( \theta = 0.5(\theta_a + \theta_1); \)
8: evaluate \( \Gamma_\chi(\theta) \) by using (25), (28) and (31) for Cases 1, 2 and 3, respectively;
9: if \( \Gamma_\chi(\theta) = 0 \)
10: go to line 15;
11: else if \( \Gamma_\chi(\theta) > 0 \)
12: \( \theta_a = \theta; \)
13: else
14: \( \theta_1 = \theta; \)
15: end if
16: end while
17: output \( \theta^*(\chi) = \theta \) and \( \varepsilon^*(\chi) = \varepsilon(\theta^*(\chi)) \).

In Algorithm I, \( \delta > 0 \) is a prescribed small value to terminate the iteration. It can readily be shown that the worst-case complexity of Algorithm I is \( O(\log_2(\frac{1}{\delta})) \).

VI. SIMULATION RESULTS

In order to corroborate the insight obtained from the theoretical analysis, we have implemented Algorithm I to compute \( \theta^*(\chi) \) and \( \varepsilon^*(\chi) \) for each case, and carried out
simulations for a typical flat-fading communication link whose parameters take practical values very close to those used in [13]. Specifically, $B = 10$ KHz and $\sigma^2 = BN_0N_f$ are used where $N_0 = -170$ dBm/Hz is the noise power spectral density and $N_f$ is the noise figure. The efficiency of the power amplifier efficiency is set as $\xi = 0.4$. When using Algorithm I in the simulations, we set $\delta = 10^{-8}$ and convergence of the algorithm is always observed.

The average channel power gain is evaluated according to

$$\bar{G} = G_0d^{-3.5}$$

(32)

where $d$ is the transmitter-receiver distance, and $G_0 = -70$ dB is chosen as in [13]. $|h|$ is generated as follows:

1) For Case 1 (static channel with CSIT), $|h|$ is fixed as $\sqrt{\bar{G}}$.

2) For Case 2 (FV channel with CDIT) and Case 3 (FV channel with CSIT): $|h|$ is assumed to follow Nakagami distribution with parameter $m$ and $E_h\{|h|^2\} = \bar{G}$. We choose this distribution because the parameter $m$ can easily be adjusted to reflect the severity of fast fading: the larger $m$ indicates less severe fading, i.e., the pdf of $G = |h|^2$ is more compactly concentrated around its average value $\bar{G}$. Specifically, the Nakagami distribution when $m = 1$ is simply the Rayleigh distribution.

A. Illustration of the impact of $\kappa$ on $\theta^*(\chi)$ and $\varepsilon^*(\chi)$

To corroborate the insight about the impact of $\kappa$ on $\varepsilon^*(\chi)$ and $\theta^*(\chi)$, we assume $d = 10$ meter, $N_f = 10$ dB and $\rho = 188$ mW which are the same as that used in [13]. $\varepsilon^*(\chi)$ and $\theta^*(\chi)$ for each channel case have been computed when $\phi(R) = R$ and $\kappa$ increases from $7 \times 10^{-8}$ to $1 \times 10^{-7}$. We have also computed $\varepsilon^*(\chi)$ and $\theta^*(\chi)$ for each channel case with the same parameters except for $\phi(R) = R^{1.3}$. The results are shown in Figure 4.

It can be seen that as $\kappa$ increases, $\varepsilon^*(\chi)$ increases regardless of the form that $\phi(R)$ takes. We have also evaluated by the numerical method that the $\theta_1$ satisfying $x^2 f(\theta_1) = \rho$ is equal to 8.85, 8.73 and 8.16 for Cases 1, 2 and 3, respectively. It is shown in Fig. 4 that the $\theta^*(\chi)$ for the three channel cases when $\phi(R) = R$ keeps fixed at those values when $\kappa$ increases. Moreover, $\theta^*(\chi)$ for each channel case when $\phi(R) = R^{1.3}$ (i.e., $\phi(R)$ is strictly convex of $R$) decreases and is always smaller than the $\theta^*(\chi)$ when $\phi(R) = R$. These observations corroborate the insight obtained in Section IV.

When $m$ is fixed, $\varepsilon^*(\chi)$ for Case 2 (FV channel with CDIT) is always higher than that for Case 1 (static channel with CSIT), which is in agreement with the theoretical analysis in Section IV. When $m$ increases, $\varepsilon^*(\chi)$ for Case 2 approaches that for Case 1. This is because when $m$ takes a large value, the pdf of $G$ becomes more compactly concentrated around the average value $\bar{G}$, hence $E_G\{|G|^2(1 + G\rho)\} \approx \log_2(1 + G\rho)$ holds, meaning that $\gamma(\theta)$ for Case 2 is very close to that for Case 1. These observations indicate that the less severe fading leads to improved EE performance for Case 2.

This means that the rate-dependent circuit energy consumption per bit increases from 70 to 100 nJ, which agrees with those reported in [23]. [24].
It can also be seen that $\varepsilon^*(\chi)$ for Case 3 (FV channel with CSIT) is very close to that for Case 2, which is explained as follows. Note that $\theta^*(\chi)$ for Case 2 and Case 3 are very close and relatively high. For Case 3, the corresponding $\mu^*$ has to take a high value in order to satisfy (30) when $\theta = \theta^*(\chi)$. In such a case, the optimum $p(G)$ is approximately a constant, thus $\gamma(\theta^*(\chi))$ for Case 3 is very close to $\gamma(\theta^*(\chi))$ for Case 2. Therefore, the $\varepsilon^*(\chi)$ for the two cases are approximately equal.

B. Illustration of the impact of $\sigma^2$ on $\theta^*(\chi)$ and $\varepsilon^*(\chi)$

To corroborate the insight about the impact of $\sigma^2$ on $\varepsilon^*(\chi)$ and $\theta^*(\chi)$, we assume $d = 10$ meter, $\rho = 188$ mW, $\kappa = 9 \times 10^8$ and $\phi(R) = R$. $\varepsilon^*(\chi)$ and $\theta^*(\chi)$ for each case have been computed when $N_f$ increases from 10 to 30 dB. The results are shown in Figure 5. It can be seen that as $\sigma^2$ increases due to the increase of $N_f$, $\varepsilon^*(\chi)$ increases while $\theta^*(\chi)$ decreases, which corroborate the insight obtained in Section IV. Moreover, similar points can be observed as said earlier when comparing $\varepsilon^*(\chi)$ of the three channel cases.

C. Illustration of the impact of $\rho$ on $\theta^*(\chi)$ and $\varepsilon^*(\chi)$

To corroborate the insight about the impact of $\rho$ on $\varepsilon^*(\chi)$ and $\theta^*(\chi)$, we assume $d = 10$ meter, $N_f = 10$ dB, $\kappa = 9 \times 10^8$ and $\phi(R) = R$. $\varepsilon^*(\chi)$ and $\theta^*(\chi)$ for each case have been computed when $\rho$ increases from 100 to 300 mW. The results are shown in Figure 6. It can be seen that as $\rho$ increases, both $\varepsilon^*(\chi)$ and $\theta^*(\chi)$ increase, which corroborate the insight obtained in Section IV. Moreover, similar points can be observed as said earlier when comparing $\varepsilon^*(\chi)$ of the three channel cases.

D. Illustration of the impact of $G$ on $\theta^*(\chi)$ and $\varepsilon^*(\chi)$

To corroborate the insight about the impact of $G$ on $\varepsilon^*(\chi)$ and $\theta^*(\chi)$ for Case 1, we assume $N_f = 10$ dB, $\rho = 188$ mW, $\kappa = 9 \times 10^8$ and $\phi(R) = R$. $\varepsilon^*(\chi)$ and $\theta^*(\chi)$ for each channel case have been computed when $d$ increases from 10 to 30 meter. We have also computed $\varepsilon^*(\chi)$ and $\theta^*(\chi)$ for each channel case when $d$ increases from 150 m to 160 meter. The results are shown in Figure 7. It can be seen that as $G$ decreases due to the increase of $d$, $\varepsilon^*(\chi)$ increases while $\theta^*(\chi)$ decreases, which corroborate the insight obtained in Section IV. When $d$ is between 10 and 30 m, points similar as said earlier can be observed when comparing $\varepsilon^*(\chi)$ of the three channel cases.

However, when $d$ is between 150 and 160 meter, different and interesting points can be observed. In such a case, $\theta^*(\chi)$ is relatively small for all three channel cases. $\theta^*(\chi)$ for Case 2 is always higher than that for Case 1 and as $m$ increases, $\theta^*(\chi)$ for Case 2 approaches that for Case 1, meaning that the less severe fading leads to improved EE performance for Case 2 as observed earlier.

However, $\varepsilon^*(\chi)$ for Case 3 is smaller than $\varepsilon^*(\chi)$ for Case 1. The reason is that since $\theta^*(\chi)$ is small, the corresponding average power is also small, and thus the optimum power allocation $p(G)$ for Case 3 is to allocate power and activate transmission only when $G$ is high. In such an opportunistic way, the optimum power allocation for Case 3 can exploit the channel fading in a very efficient way, in the sense that only
the channel states with high channel power gain (which is very likely much higher than $\mathcal{G}$) are used for data transmission. As $m$ increases, $\theta^*(\chi)$ for Case 3 approaches that for Case 1, because the pdf of $G$ becomes more compactly concentrated around $\mathcal{G}$ so that $G$ appears with a higher probability at values very close to $\mathcal{G}$. This means that the less severe fading leads to degraded EE performance for Case 3 when $\theta^*(\chi)$ is small as shown here.

VII. Conclusion

We have investigated the optimum EE and corresponding SE for a communication link over a flat-fading channel. Three cases for the flat-fading channel are considered, namely static channel with CSIT, FV channel with CDIT and FV channel with CSIT. The link’s circuit power is modeled as the sum of a constant and a rate-dependent part as an increasing and convex function of the transmission rate. For all three cases, the tradeoff between the EE and the spectral efficiency (SE) has been studied, and the impact of system parameters on the optimum EE and corresponding SE has been investigated to obtain insight. A polynomial-complexity algorithm has been developed with the bisection method to find the optimum SE. The insight has been corroborated and the optimum EE for the three cases has been compared by simulation results. The insight and algorithm presented in this paper can be applied to guide the practical design of communication links over flat-fading channels for improved EE performance as illustrated by simulation results. In future, we will extend the study to multiuser networks with frequency-selective channels as studied in [29], [30].

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APPENDIX

A. Proof of Lemma 7

For Case 1, it can readily be seen from (8) that $\gamma(\theta)$ is strictly increasing and strictly convex of $\theta \geq 0$.

The claims for Case 2 and Case 3 can be proven in a similar way, hence we only prove the one for Case 3 as follows. Obviously $\gamma(\theta)$ is strictly increasing of $\theta \geq 0$. Proving its strictly convexity is equivalent to show that $\forall \theta_1, \theta_2 \geq 0, \forall \alpha \in (0, 1)$, $\gamma((\alpha\theta_1 + (1 - \alpha)\theta_2) \leq \alpha \gamma(\theta_1) + (1 - \alpha)\gamma(\theta_2)$ To this end, suppose $P_1 = \{p_1(G)\vert V G \geq 0\}$, $P_2 = \{p_2(G)\vert V G \geq 0\}$ and $P_3 = \{p_3(G)\vert V G \geq 0\}$ represent the optimum $\mathcal{P}$ for problem (10) when $\theta = \theta_1$, $\theta = \theta_2$ and $\theta = \alpha\theta_1 + (1 - \alpha)\theta_2$, respectively. This means that $\gamma(\theta_i) = \mathbb{E}_G\{p_i(G)\}$ where $i \in \{1, 2, 3\}$.

Define $f(\mathcal{P}) = \mathbb{E}_G\{\log_2(1 + Gp(G))\}$ where $\mathcal{P} = \{p(G)\vert V G \geq 0\}$. It can readily be shown that $f(\mathcal{P})$ is strictly concave of $\mathcal{P} \in \mathcal{S}_P$. Obviously, the constraint of problem (10) must be saturated at the optimum solution, i.e., $f(P_1) = \theta_1$. 

Fig. 7. The computed $\epsilon^*(\chi)$ and $\theta^*(\chi)$ when $\rho = 188$ mW, $N_f = 10$ dB, $\kappa = 9 \times 10^8$ and $\phi(R) = R$. 

(a) when $d$ increases from 10 to 30 meter

(b) when $d$ increases from 150 to 160 meter
function $f(P_2) = \theta_2$ and $f(P_3) = \alpha \theta_1 + (1 - \alpha) \theta_2$ must hold. According to the strict concavity of $f(P)$,

$$f(\alpha P_1 + (1 - \alpha) P_2) > \alpha f(P_1) + (1 - \alpha) f(P_2) = \alpha \theta_1 + (1 - \alpha) \theta_2$$

follows. This means that $\alpha P_1 + (1 - \alpha) P_2$ is a feasible but not optimum solution for problem (10) with $\theta = \alpha \theta_1 + (1 - \alpha) \theta_2$. On the other hand, $P_3$ is the optimum solution for the same problem. This means that

$$\gamma(\alpha \theta_1 + (1 - \alpha) \theta_2) = \mathbb{E}_G\{p_3(G)\} < \mathbb{E}_G\{(\alpha p_1(G) + (1 - \alpha) p_2(G)\} = \alpha \gamma(\theta_1) + (1 - \alpha) \gamma(\theta_2),$$

which proves the claim.

### B. Proof of Lemma 2

According to Proposition C9 in [24], $\varepsilon(\theta)$ is strictly quasi-convex of $\theta \geq 0$ if

$$\Pi(\gamma) = \{\theta \geq 0 | \varepsilon(\theta) \leq \gamma\}$$

is a strictly convex set for any real value $\gamma$.

Note that $\forall \theta \geq 0$, $\varepsilon(\theta) > 0$, which means that $\Pi(\gamma)$ is empty if $\gamma \leq 0$, hence $\Pi(\gamma)$ is strictly convex since no point lies on the contour of $\Pi(\gamma)$. We now prove the strict convexity of $\Pi(\gamma)$ when $\gamma > 0$. In such a case,

$$\Pi(\gamma) = \{\theta \geq 0 | f(\gamma, \theta) = P(\theta) - \gamma B \theta \leq 0\}.$$

Suppose $\theta$ and $\theta'$ are any two points on the contour of $\Pi(\gamma)$. Obviously $\theta > 0$ and $\theta' > 0$ since $0 \notin \Pi(\gamma)$ due to the fact that $f(\gamma, 0) > 0, \forall \theta \in (\theta, \theta')$,

$$f(\gamma, \theta) < \max\{f(\gamma, \theta'), f(\gamma, \theta)\} \leq 0$$

follows from the strict convexity of $f(\gamma, \theta)$ with respect to $\theta$, meaning that any $\theta$ between any two points on the contour of $\Pi(\gamma)$ must lie in the interior of $\Pi(\gamma)$. Thus, $\Pi(\gamma)$ is a strictly convex set when $\gamma > 0$. Therefore, $\varepsilon(\theta)$ is strictly quasi-convex of $\theta$.

### C. Proof of Theorem 2

To prove the first claim, suppose there exist $\theta'$ and $\theta''$ satisfying $\theta' < \theta''$ and $\varepsilon(\theta') = \varepsilon(\theta'') = \varepsilon^*$. From (13), $\forall \theta \in (\theta', \theta'')$, $\varepsilon^* = \max\{\varepsilon(\theta'), \varepsilon(\theta'')\} > \varepsilon(\theta)$, leading to a contradiction with $\varepsilon^* \leq \varepsilon(\theta)$. Therefore, there must exist a unique $\theta^*$ satisfying $\varepsilon(\theta^*) = \varepsilon^*$. Moreover, $\theta^*$ must satisfy

$$\forall \theta \geq 0, \varepsilon(\theta^*)(\theta - \theta^*) \geq 0 \quad (33)$$

according to Proposition 2.1.2 in [31]. As said earlier, $\theta^* > 0$ must hold, thus $\varepsilon'(\theta^*) = 0$ must hold to satisfy the condition (33). From (13), (14) must hold. This proves the first claim.

We now prove the second claim. For any $\theta_1$ and $\theta_2$ satisfying $0 < \theta_1 < \theta_2 < \theta^*$, $\varepsilon(\theta_1) = \max\{\varepsilon(\theta_1), \varepsilon(\theta_2)\} > \varepsilon(\theta_2)$ follows from (13). This means that $\varepsilon(\theta)$ is strictly decreasing with $\theta \in (0, \theta^*)$. Therefore, $\varepsilon'(\theta) < 0$ must hold $\forall \theta \in (0, \theta^*)$. From (14), (16) must hold. This proves the second claim.

The third claim is proven as follows. For any $\theta_1$ and $\theta_2$ satisfying $\theta_1 > \theta_2 > \theta^*$, $\varepsilon(\theta_1) = \max\{\varepsilon(\theta_1), \varepsilon(\theta_2)\} > \varepsilon(\theta_2)$ follows from (13). This means that $\varepsilon(\theta)$ is strictly increasing with $\theta \in (\theta^*, +\infty)$. Therefore, $\varepsilon'(\theta) > 0$ must hold $\forall \theta \in (\theta^*, +\infty)$. From (13), (17) must hold. This proves the third claim.

### D. Proof of Lemma 2

According to Theorem 1

$$\theta \left\{ \begin{array}{ll} < \theta^*(\chi) & \text{iff } Z_\chi(\theta) < 0 \\ = \theta^*(\chi) & \text{iff } Z_\chi(\theta) = 0 \\ > \theta^*(\chi) & \text{iff } Z_\chi(\theta) > 0 \end{array} \right. \quad (34)$$

must hold where

$$Z_\chi(\theta) = P'(\theta) - P(\theta)$$

$$= \frac{1}{\theta} \left[ \kappa |B\theta \phi'(B\theta) - \phi(B\theta)| + \frac{\sigma^2}{\xi} (\theta \gamma'(\theta) - \gamma(\theta)) \right]$$

$$= \frac{1}{\theta} \left[ \kappa g(B\theta) + \frac{\sigma^2}{\xi} f(\theta) - \rho \right] = \frac{1}{\theta} \Gamma_\chi(\theta)$$

From the above equation, it can readily be seen (34) is equivalent to (18), which proves the first claim.

We now prove the claim about $g(R)$. Obviously $g(0) = 0$ holds. It can readily be verified that $g(R) = 0, \forall R \geq 0$ if $\phi(R) = R$. If $\phi(R)$ is strictly convex of $R \geq 0$,

$$g'(R) = \phi'(R) + R \phi''(R) - \phi'(R) = R \phi''(R), \quad (35)$$

meaning that $g(R)$ is strictly increasing of $R \geq 0$ due to the strict convexity of $\phi(R)$. Hence the claim about $g(R)$ is proven. In a similar way it can be proven that $f(\theta)$ is strictly increasing of $\theta \geq 0$.

### E. Proof of Theorem 2

Suppose $\kappa$ increases from $\kappa_1$ to $\kappa_2$ (i.e., $\kappa_2 > \kappa_1$) while all other parameters are fixed. Denote the $\chi$ when $\kappa = \kappa_1$ and $\kappa = \kappa_2$ as $\chi_1$ and $\chi_2$, respectively.

To prove the first claim, note that

$$\varepsilon^*(\chi_2) = \frac{\kappa_2 \phi(B\theta^*(\chi_2)) + \frac{\sigma^2}{\xi} \gamma(\theta^*(\chi_2)) + \rho}{B\theta^*(\chi_2)}$$

$$> \frac{\kappa_1 \phi(B\theta^*(\chi_2)) + \frac{\sigma^2}{\xi} \gamma(\theta^*(\chi_2)) + \rho}{B\theta^*(\chi_2)}$$

$$\geq \frac{\kappa_1 \phi(B\theta^*(\chi_1)) + \frac{\sigma^2}{\xi} \gamma(\theta^*(\chi_1)) + \rho}{B\theta^*(\chi_1)}$$

$$= \varepsilon^*(\chi_1), \quad (36)$$

where the inequality in the second line is due to the fact that $\kappa_2 > \kappa_1$ and $\theta^*(\chi_2) > 0$ (meaning that $\phi(B\theta^*(\chi_2)) > 0$). The inequality in the third line is due to the fact that $\theta^*(\chi_2)$ is a feasible SE while $\theta^*(\chi_1)$ is the optimum SE minimizing the EE when $\kappa = \kappa_1$. This proves the first claim.

We now prove the second claim. Note that when $\phi(R)$ is strictly convex of $R \geq 0, g(R)$ is strictly increasing of $R \geq 0,$
meaning that \( g(B\theta^*(\chi_1)) > 0 \). Therefore,

\[
\Gamma_{\chi_2}(\theta^*(\chi_1)) = \kappa_2 g(B\theta^*(\chi_1)) + \frac{\sigma^2}{\xi} f(\theta^*(\chi_1)) - \rho \\
\quad > \kappa_1 g(B\theta^*(\chi_1)) + \frac{\sigma^2}{\xi} f(\theta^*(\chi_1)) - \rho \\
\quad = \Gamma_{\chi_1}(\theta^*(\chi_1)) = 0,
\]

(37)

follows. According to Lemma 3 \( \theta^*(\chi_1) > \theta^*(\chi_2) \) holds. This proves the claim that \( \theta^*(\chi) \) strictly decreases when \( \kappa \) increases.

To prove (21), note that \( \theta^*(\chi) \) satisfies that \( \kappa g(B\theta^*(\chi)) + \frac{\sigma^2}{\xi} f(\theta^*(\chi)) = \rho \). On the one hand,

\[
\lim_{\kappa \to +\infty} g(B\theta^*(\chi)) = \lim_{\kappa \to +\infty} \frac{\rho - \frac{\sigma^2}{\xi} f(\theta^*(\chi))}{\kappa} = 0
\]

holds, meaning that \( \lim_{\kappa \to +\infty} \theta^*(\chi) = 0 \). On the other hand,

\[
\lim_{\kappa \to 0} \frac{\sigma^2}{\xi} f(\theta^*(\chi)) = \lim_{\kappa \to 0} [\rho - \kappa g(B\theta^*(\chi))] = \rho
\]

holds, meaning that \( \lim_{\kappa \to 0} \theta^*(\chi) = \theta_1 \). This proves the (21).

To prove the last claim, note that when \( \phi(R) = R \), \( \frac{\sigma^2}{\xi} f(\theta^*(\chi)) = \rho \) holds since \( g(B\theta^*(\chi)) = 0 \). This means that \( \theta^*(\chi) = \theta_1 \) always holds.

\[F. \text{ Proof of Theorem 2}\]

Suppose \( \rho \) increases from \( \rho_1 \) to \( \rho_2 \) (i.e., \( \rho_2 > \rho_1 \)) while all other parameters are fixed. Denote the \( \chi \) when \( \rho = \rho_1 \) and \( \rho = \rho_2 \) as \( \chi_1 \) and \( \chi_2 \), respectively. Note that

\[
\varepsilon^*(\chi_2) = \frac{\kappa \phi(B\theta^*(\chi_2)) + \frac{\sigma^2}{\xi} \gamma(\theta^*(\chi_2)) + \rho_2}{B\theta^*(\chi_2)} \\
> \frac{\kappa \phi(B\theta^*(\chi_2)) + \frac{\sigma^2}{\xi} \gamma(\theta^*(\chi_2)) + \rho_1}{B\theta^*(\chi_2)} \\
\geq \frac{\kappa \phi(B\theta^*(\chi_1)) + \frac{\sigma^2}{\xi} \gamma(\theta^*(\chi_1)) + \rho_1}{B\theta^*(\chi_1)} \\
= \varepsilon^*(\chi_1),
\]

(38)

follows, where the inequality in the third line is due to the fact that \( \theta^*(\chi_2) \) is a feasible SE while \( \theta^*(\chi_1) \) is the optimum SE minimizing the EE when \( \rho = \rho_1 \). This proves the first claim.

We now prove the second claim. Note that

\[
\Gamma_{\chi_2}(\theta^*(\chi_1)) = \kappa g(B\theta^*(\chi_1)) + \frac{\sigma^2}{\xi} f(\theta^*(\chi_1)) - \rho_2 \\
< \kappa g(B\theta^*(\chi_1)) + \frac{\sigma^2}{\xi} f(\theta^*(\chi_1)) - \rho_1 \\
= \Gamma_{\chi_1}(\theta^*(\chi_1)) = 0,
\]

(39)

follows. According to Lemma 3 \( \theta^*(\chi_1) < \theta^*(\chi_2) \) holds. This proves that \( \theta^*(\chi) \) strictly increases when \( \rho \) increases.

To prove (22), note that \( \theta^*(\chi) \) is equal to the \( \theta \) satisfying \( \kappa g(B\theta) + \frac{\sigma^2}{\xi} f(\theta) = \rho \). According to Lemma 13 \( \kappa g(B\theta) + \frac{\sigma^2}{\xi} f(\theta) \) is a strictly increasing function of \( \theta \). Therefore, (24) holds.
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