Information transfer using a single particle path-spin hybrid entangled state

T. Pramanik\textsuperscript{*1}, S. Adhikari\textsuperscript{1,1}, A. S. Majumdar\textsuperscript{1,1}, Dipankar Home\textsuperscript{2} and Alok Kumar Pan\textsuperscript{1,2}

1 S. N. Bose National Centre for Basic Sciences, Salt Lake, Kolkata 700 098, India and 2 CAPSS, Department of Physics, Bose Institute, Sector-V, Salt Lake, Kolkata 700 091, India
(Dated: February 4, 2022)

The path-spin entangled state of a single spin-1/2 particle is considered which is generated by using a beam-splitter and a spin-flipper. Using this hybrid entanglement at the level of a single particle as a resource, we formulate a protocol for transferring of the state of an unknown qubit to a distant location. Our scheme is implemented by a sequence of unitary operations along with suitable spin-measurements, as well as by using classical communication between the two spatially separated parties. This protocol, thus, demonstrates the possibility of using intraparticle entanglement as a physical resource for performing information theoretic tasks.

PACS numbers: 03.65. Ta, 03.67.-a,03.67.Ac, 03.67.Bg

Quantum entanglement is a crucial ingredient in the storage and distribution of quantum information in the currently vibrant research area of quantum information\textsuperscript{[1]}. Historically, the first fundamental implication of entanglement was noticed in terms of position-momentum variables\textsuperscript{[2]}, and was later extended for the discrete spin variables\textsuperscript{[3]}. In recent times, the theory of entanglement has been much studied for systems described by Hilbert spaces for the discrete variables\textsuperscript{[4]} on the one hand, and for those corresponding to continuous variables\textsuperscript{[5]} on the other. Several interesting information processing protocols such as quantum teleportation\textsuperscript{[6]}, dense-coding\textsuperscript{[7]} and cryptography\textsuperscript{[8]} have been developed for spin entangled states, as well as for position-momentum entangled states\textsuperscript{[9,10]}

Further, it needs to be noted that in the larger context of investigating various ramifications of quantum entanglement and its applications to a wide range of diverse phenomena such as phase transitions in condensed matter systems\textsuperscript{[11]} and black hole physics\textsuperscript{[12]}, the study of hybrid entanglement between the dynamical variables belonging to different Hilbert spaces such as those corresponding to path (or linear momentum) variables on one hand, and spin variables on the other, is particularly relevant. However, surprisingly, even though quantum mechanics allows for the existence of hybrid entangled states connecting Hilbert spaces with distinctly different properties, the possibility of physical realization of such states is only beginning to be appreciated\textsuperscript{[13,14]}. More interesting is the idea of generation of intraparticle entanglement between different degrees of freedom of the same particle. The entanglement between the polarization and the linear momentum of a single photon\textsuperscript{[14]}, and also the polarization and the angular momentum of a single photon\textsuperscript{[15]} has been demonstrated experimentally. Further, it has been shown recently how flavor oscillations of neutrinos could be related to multimode entanglement of single particle states\textsuperscript{[16]}. The idea of creating entanglement between the path and the spin degrees of freedom for a single spin-1/2 particle was proposed earlier in order to demonstrate contextuality in quantum mechanics\textsuperscript{[17]}. Such path-spin hybrid entangled states for single neutrons have also been realized experimentally\textsuperscript{[18]}. Recently, it has been shown how intraparticle hybrid entanglement could be swapped onto the standard inter-particle entanglement of two qubits\textsuperscript{[19]}

Since intra-particle entanglement between different degrees of freedom is confined locally with a single particle, it should be easier to preserve, at least in principle, against dissipative effects. It is then natural to ask the question as to whether such hybrid entanglement between different degrees of freedom of the same particle could be used as resource for information processing. At the outset such an idea seems difficult to implement, since this entanglement is not delocalised between two regions in a way that is amenable for exploiting as a resource. It may be relevant to recall here the interesting debate in the literature regarding demonstration of nonlocality at the level of an entangled state of a single particle\textsuperscript{[20]}. In this Letter we devise a protocol for using hybrid entanglement at the level of a single particle for quantum information processing. We show how the path-spin entanglement of a single spin-1/2 particle could be used as a resource for transferring the state of an unknown qubit at a distant location. In order to realize such a scheme, we first discuss a method to set-up the intra-particle hybrid path-spin entanglement using a beam-splitter and a spin-flipper. Our protocol for information transfer then proceeds with a series of operations performed by the two distant parties (Alice and Bob) including unitary transformations, appropriate measurements using Stern-Gerlach devices, and classical communications. Our scheme is pictorially illustrated in Fig. 1.

Let us consider an ensemble of spin-1/2 particles, all corresponding to an initial spin polarized state along the +z axis (denoted by |↑z⟩). Taking into consideration

*tanu.pram99@bose.res.in
satyabrata@bose.res.in
archan@bose.res.in
dhome@bosemain.boseinst.ac.in
apany@bosemain.boseinst.ac.in
the path (or position) variables of the particles, one can write down the joint path-spin state for ensemble as

\[ |S_{ps}^1 \rangle = |\psi_0 \rangle_p \otimes |\uparrow_z \rangle_s \]  

where the subscripts \( p \) and \( s \) refer to the path and spin variables, respectively. The particles are then allowed by Alice to fall on a beam-splitter (BS1). Since the beam-splitter acts only on the path-state without affecting the spin-state of the particles, the input-output relation is described by the unitary transformation

\[
\begin{pmatrix}
|\psi_0 \rangle_p \\
|\psi_0 \rangle_p
\end{pmatrix} = \begin{pmatrix} \alpha & i\beta \\ -i\beta & \alpha \end{pmatrix} \begin{pmatrix} |1 \rangle_p \\
|0 \rangle_p
\end{pmatrix}
\]  

(2)

with \( \alpha^2 + \beta^2 = 1 \) (\( \alpha \) and \( \beta \) are real), where \( \alpha^2 \) and \( \beta^2 \) are the reflection and transmission probabilities respectively, and the reflected or the transmitted channel are designated by \( |0 \rangle_p \) or \( |1 \rangle_p \) respectively. In the following argument a crucial role is provided by the mutually orthogonal path states \( |0 \rangle_p \) and \( |1 \rangle_p \) which are eigenstates of the projection operators \( P(|0 \rangle_p) \) and \( P(|1 \rangle_p) \) respectively. These projection operators can be regarded as corresponding to the observables that pertain to the determination of ‘which channel’ a particle is found to be in. For example, the results of such a measurement for the reflected (transmitted) channel with binary alternatives are given by the eigenvalues of \( P(|0 \rangle_p) \) \( (P(|1 \rangle_p) \); the eigenvalue +1(0) corresponds to a neutron being found (not found) in the channel represented by \( |0 \rangle_p \) \( (|1 \rangle_p \).

The state of a particle emergent from BS1 can be written as

\[ |S_{ps}^1 \rangle = |\psi_0 \rangle_p \otimes |\uparrow_z \rangle_s \]  

We can identify the state vectors \( |0 \rangle_p \), \( |1 \rangle_p \), \( |\uparrow_z \rangle_s \), and \( |\downarrow_z \rangle_s \) as

\[ |0 \rangle_p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1 \rangle_p = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |\uparrow_z \rangle_s = |0 \rangle_s, \quad |\downarrow_z \rangle_s = |1 \rangle_s \]  

(4)

Note that though we are considering here a single particle, the dichotomous path and spin variables enable it to be viewed effectively as two qubits. Next, suppose the particles in the channel corresponding to \( |0 \rangle_p \) pass through a spin-flipper (SF) that contains a uniform magnetic field along, say, the +\( z \)-axis which flips the state |\( \uparrow_z \rangle \) to |\( \downarrow_z \rangle \). The resultant path-spin entangled state (using the above notation) can be written as

\[ |S_{ps}^1 \rangle = U_{CNOT} |S_1 \rangle >_{ps} = \begin{pmatrix} 0 \\ i\beta \\ \alpha \\ 0 \end{pmatrix} \]  

(5)

Or, in other words, as a consequence of introducing a spin-flipper in one of the channels, Alice now possesses the path-spin entangled state given by

\[ |S_{ps}^1 \rangle = \alpha|\uparrow_z \rangle_s \otimes |1 \rangle_p + i\beta|\downarrow_z \rangle_s \otimes |0 \rangle_p \]  

(6)

Note that the above path-spin entanglement between the spin variables and the path observables of a spin-1/2 particle - the ‘intraparticle entanglement’ involved here is distinct from the usually discussed ‘interparticle entanglement’, say, between the spin variables of two spatially separated particles, or even the recently discussed ‘hybrid entanglement’ between the polarization of one photon and the spin of another spatially separated photon. Further, the entangled state generated here is also different from several single particle entangled states involving the entanglement of similar modes (e.g., Fock states \( |m \rangle \), or neutrino flavor states \( |\nu \rangle \) discussed earlier in the literature. The present state corresponds to a single particle hybrid entangled state where the entanglement is between the spin and path variables of the same particle.

Our goal is to now use this entangled state as a resource for performing the information transfer of the state of another particle (2) held by Alice. The state of particle 2 is given by

\[ |\psi_{in} \rangle^2 = \gamma|0 \rangle_s^2 + \delta|1 \rangle_s^2 \]  

(7)
where $\gamma^2 + \delta^2 = 1$. At this stage we introduce another auxiliary particle possessed by Alice in the state $|0_s^a\rangle$. The role of this particle is to ensure, as will be clear later, that the information about the state to be transferred is not lost even if the particle to be sent to Bob is lost in transit. The state transfer protocol begins with Alice making a CNOT operation, where the first particle’s spin state is the control qubit and the auxiliary particle’s spin state is the target qubit. After the CNOT operation the combined state of the first and the auxiliary particle is given by

$$|S_3\rangle_{ps}^{1a} = (\alpha|1_p^a\rangle|0_s^a\rangle + i\beta|0_p^a\rangle|1_s^a\rangle)$$ (8)

Next, Alice makes another CNOT operation where the first particle’s spin state is the control qubit and the second particle’s spin state is the target qubit. The resultant combined state of the three particles (first, second, and auxiliary) is now given by

$$|\psi\rangle^{12a} = \alpha|1_p^a\rangle|0_s^a\rangle|0_x^a\rangle + i\beta|0_p^a\rangle|1_s^a\rangle|1_x^a\rangle + \alpha\delta|1_p^a\rangle|0_s^a\rangle|0_y^a\rangle + i\beta\delta|0_p^a\rangle|1_s^a\rangle|0_y^a\rangle$$ (9)

Note here that the CNOT operations are performed by Alice on whichever path (transmitted or reflected) that is taken by her particle 1 on emerging from the beam splitter. This has to be done in order to use the path-spin entangled state as the teleportation channel. Performing any such operation before particle 1 falls on the beam splitter is of no use for teleportation, since the path-spin entanglement itself is created after the particle emerges from the beam splitter.

Alice then sends her first particle (in a path-spin entangled state) to Bob. After Bob confirms that he has received the particle, Alice measures the spin of her second particle along the $z$-axis using her Stern-Gerlach device (SG1 in FIG. 1), and also measures the spin of her auxiliary particle also along the $z$-axis using her Stern-Gerlach device (SGA in FIG. 1). Eq (11) can be rewritten as

$$|\psi\rangle^{12a} = \frac{1}{\sqrt{2}}[(\alpha|1_p^a\rangle|0_s^a\rangle + i\beta|0_p^a\rangle|1_s^a\rangle)|0_x^a\rangle + (\alpha|1_p^a\rangle|0_s^a\rangle - i\beta|0_p^a\rangle|1_s^a\rangle)|0_y^a\rangle + (i\beta|0_p^a\rangle|1_s^a\rangle + \alpha\delta|1_p^a\rangle|0_s^a\rangle)|1_y^a\rangle + (-i\beta|0_p^a\rangle|1_s^a\rangle + \alpha\delta|1_p^a\rangle|0_s^a\rangle)|1_x^a\rangle]$$ (10)

using which it follows that the possible spin measurements on Alice’s second particle and the auxiliary particle leads to the corresponding states with the respective probabilities, as listed in the TABLE. I.

Subsequently, Alice classically communicates with Bob to tell him the results about her spin measurements (i.e., spin up or spin down for particle 1 and particle “a”). Now, Bob has to perform the remaining operations in order to recreate the state $\overline{7}$. Bob has two particles—one given by Alice and another particle (assumed to be in a spin up state $|0_s^a\rangle$) which he holds initially. Depending on the measurement results that Alice communicates to Bob, the following operations are performed by him:

**Case I:** When Alice’s spin measurement on her second particle reveals a spin up ($|0_s^a\rangle$) state, and measurement on her auxiliary particle reveals a spin up ($|0_s^a\rangle$) state.

After receiving the first particle from Alice, Bob sends it through 50 – 50 beam-splitter (BS2 in FIG. 1). The action of the beam-splitter on the states $|0_p\rangle$ and $|1_p\rangle$ is given by

$$|0_p\rangle \rightarrow \frac{1}{\sqrt{2}}[(a)|p\rangle + i(b)|p\rangle]$$
$$|1_p\rangle \rightarrow \frac{1}{\sqrt{2}}[(b)|p\rangle + i(a)|p\rangle]$$ (12)

Now, Bob makes a CNOT operation where spin state of the third particle (held by him from the beginning) is the control qubit and the spin state of the first particle (sent to him by Alice) is the target qubit. The combined state of the two particles after the CNOT operation is given by

$$|\psi\rangle^{13} = \frac{1}{\sqrt{2(\alpha^2 + \beta^2)}}[\alpha|1_p^b\rangle|0_s^b\rangle|0_x^b\rangle - \beta|b_p^b\rangle|1_s^b\rangle|1_x^b\rangle + i\alpha\gamma|b_p^b\rangle|0_s^b\rangle|0_y^b\rangle + i\beta\delta|1_p^b\rangle|0_s^b\rangle|0_y^b\rangle + i\alpha\delta|b_p^b\rangle|1_s^b\rangle|1_y^b\rangle + i\beta\gamma|1_p^b\rangle|1_s^b\rangle|0_y^b\rangle]$$ (13)

Next, Bob measures spin of the first particle using the two sets of the Stern-Gerlach apparatus (placed in both the paths $|a_p\rangle$ and $|b_p\rangle$ along the $x$-axis). There are four possible outcomes of Bob’s measurement, i.e., $|a_p\rangle \otimes |0_s^b\rangle$, $|b_p\rangle \otimes |0_s^b\rangle$, and $|b_p\rangle \otimes |1_s^b\rangle$, and $|b_p\rangle \otimes |1_s^b\rangle$, all of them occurring with equal probability 1/4. Depending on the exact outcome, Bob performs a corresponding unitary operation, described in the TABLE. II, to recreate the state $\overline{7}$ initially held by Alice.

The fidelity of the state transfer process is given by

$$F = |\langle\psi^{in}|\psi^{out}\rangle|^2 = \frac{(\alpha^2 + \beta^2)^2}{\alpha^2\gamma^2 + \beta^2\delta^2}$$ (14)

**Case II:** When Alice’s spin measurement on his second particle reveals a spin down ($|1_s^b\rangle$) state, and measurement on the auxiliary particle reveals a spin down ($|1_s^a\rangle$) state.

Bob follows same procedure similar as in Case I. He first lets the particle sent by Alice fall on a 50 – 50 beam
splitter, makes a CNOT operation with it and the particle held by him, and then measures the spin of the first particle using his Stern-Gerlach apparatus. Finally, depending on his measurement operation, he makes either of the four possible unitary operations as displayed in the TABLE III.

The fidelity of the state transfer process in this case is given by

$$F = |\langle \psi^{\text{in}} | \psi^{\text{out}} \rangle|^2 = \frac{(\beta \gamma^2 + \alpha \delta^2)^2}{\beta^2 \gamma^2 + \alpha^2 \delta^2}$$

(15)

**Case III:** When Alice’s spin measurement on her second particle reveals a spin down ($|1_x\rangle_s^2$) state and measurement on the auxiliary particle reveals a spin up ($|0_x\rangle_s^2$) state.

Bob again follows a similar procedure as in the Cases I and II. The possible unitary operations that he has to perform are listed in TABLE IV. The fidelity of state transfer could also be easily obtained as in the earlier cases.

**Case IV:** When Alice’s spin measurement on her second particle reveals a spin down ($|1_x\rangle_s^2$) state and measurement on the auxiliary particle reveals a spin down ($|1_x\rangle_s^2$) state.

This case is similar to the Case III except in the unitary operations performed by Bob, which are listed in TABLE V.

Considering all the four cases together, it follows from Eqs. (14) and (15), and the expressions corresponding to the cases III and IV that the average fidelity of state transfer is given by

$$F_{av} = \gamma^4 + \delta^4 + 4 \alpha \beta \gamma^2 \delta^2$$

(16)

If the path-spin entangled state of Alice’s first particle is a maximally entangled state, i.e., $\alpha = \beta = \frac{1}{\sqrt{2}}$, then the average fidelity is equal to 1. In this case our protocol of state transfer is perfect, and the state of Bob’s particle after the completion of the protocol is the same as Alice’s unknown quantum state which she possesses initially.

To summarize, in this work we have shown how the information encoded in the entanglement between two different degrees of freedom of the same particle can be used as a resource for performing the state transfer of an unknown qubit state to a distant location. This protocol is accomplished by a series of operations involving beam-splitters, a spin-flipper, CNOT gates, spin measurements by Stern-Gerlach devices, and unitary transformations. Our protocol may be viewed as a variant of the standard teleportation scheme for a single qubit. The difference here is that since the intra-particle entanglement which is used as a resource here cannot be initially shared between the two distant parties, the particle itself has to be transferred from Alice to Bob at some stage. Note however, that the particle whose state is teleported remains with Alice, and its initial state is destroyed by

| Path and spin measurement | Unitary operation | final state of Bob’s particle $|\psi^{\text{out}}\rangle$ |
|---------------------------|------------------|---------------------------------|
| $|a\rangle_p \otimes |0_x\rangle_s^1$ | $I$ | $|\psi^{\text{in}}\rangle = \frac{(\alpha \gamma + \beta \delta + \alpha \delta - \beta \gamma) |0_x\rangle_s^1 + (\beta \gamma + \alpha \delta + \beta \delta - \alpha \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |
| $|a\rangle_p \otimes |1_x\rangle_s^1$ | $\sigma_z$ | $|\psi^{\text{in}}\rangle = \frac{(\alpha \gamma - \beta \delta + \alpha \delta - \beta \gamma) |0_x\rangle_s^1 + (\beta \gamma + \alpha \delta + \beta \delta + \alpha \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |
| $|b\rangle_p \otimes |0_x\rangle_s^1$ | $\sigma_z$ | $|\psi^{\text{in}}\rangle = \frac{(\alpha \gamma - \beta \delta + \alpha \delta - \beta \gamma) |0_x\rangle_s^1 + (\beta \gamma + \alpha \delta + \beta \delta + \alpha \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |
| $|b\rangle_p \otimes |1_x\rangle_s^1$ | $I$ | $|\psi^{\text{in}}\rangle = \frac{(\alpha \gamma + \beta \delta + \alpha \delta - \beta \gamma) |0_x\rangle_s^1 + (\beta \gamma + \alpha \delta + \beta \delta + \alpha \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |

**TABLE II:**

| Path and spin measurement | Unitary operation | final state of Bob’s particle $|\psi^{\text{out}}\rangle$ |
|---------------------------|------------------|---------------------------------|
| $|a\rangle_p \otimes |0_x\rangle_s^1$ | $\sigma_x$ | $|\psi^{\text{in}}\rangle = \frac{(\beta \gamma + \alpha \delta + \beta \delta - \alpha \gamma) |0_x\rangle_s^1 + (\alpha \gamma + \beta \delta + \alpha \delta - \beta \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |
| $|a\rangle_p \otimes |1_x\rangle_s^1$ | $\sigma_y$ | $|\psi^{\text{in}}\rangle = \frac{(\beta \gamma + \alpha \delta + \beta \delta + \alpha \gamma) |0_x\rangle_s^1 + (\alpha \gamma - \beta \delta + \alpha \delta - \beta \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |
| $|b\rangle_p \otimes |0_x\rangle_s^1$ | $\sigma_y$ | $|\psi^{\text{in}}\rangle = \frac{(\beta \gamma + \alpha \delta + \beta \delta - \alpha \gamma) |0_x\rangle_s^1 + (\alpha \gamma + \beta \delta + \alpha \delta + \beta \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |
| $|b\rangle_p \otimes |1_x\rangle_s^1$ | $\sigma_x$ | $|\psi^{\text{in}}\rangle = \frac{(\beta \gamma + \alpha \delta + \beta \delta + \alpha \gamma) |0_x\rangle_s^1 + (\alpha \gamma - \beta \delta + \alpha \delta - \beta \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |

**TABLE IV:**

| Path and spin measurement | Unitary operation | final state of Bob’s particle $|\psi^{\text{out}}\rangle$ |
|---------------------------|------------------|---------------------------------|
| $|a\rangle_p \otimes |0_x\rangle_s^1$ | $\sigma_x$ | $|\psi^{\text{in}}\rangle = \frac{(\beta \gamma + \alpha \delta + \beta \delta + \alpha \gamma) |0_x\rangle_s^1 + (\alpha \gamma - \beta \delta + \alpha \delta - \beta \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |
| $|a\rangle_p \otimes |1_x\rangle_s^1$ | $\sigma_y$ | $|\psi^{\text{in}}\rangle = \frac{(\beta \gamma + \alpha \delta + \beta \delta - \alpha \gamma) |0_x\rangle_s^1 + (\alpha \gamma + \beta \delta + \alpha \delta - \beta \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |
| $|b\rangle_p \otimes |0_x\rangle_s^1$ | $\sigma_y$ | $|\psi^{\text{in}}\rangle = \frac{(\beta \gamma + \alpha \delta + \beta \delta + \alpha \gamma) |0_x\rangle_s^1 + (\alpha \gamma - \beta \delta + \alpha \delta - \beta \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |
| $|b\rangle_p \otimes |1_x\rangle_s^1$ | $\sigma_x$ | $|\psi^{\text{in}}\rangle = \frac{(\beta \gamma + \alpha \delta + \beta \delta - \alpha \gamma) |0_x\rangle_s^1 + (\alpha \gamma + \beta \delta + \alpha \delta + \beta \gamma) |1_x\rangle_s^1}{\sqrt{2}^2}$ |

**TABLE V:**
Alice’s measurement, thus avoiding any conflict with the no-cloning theorem. It may be noted here that the act of physically sending one or more particles across distances is an unavoidable component of information theoretic protocols involved with setting-up entanglement over distances. Whereas, in the standard teleportation scheme, this process has to be initiated at the beginning in order to set-up a shared entangled state between two parties, in the present scheme involving path-spin entanglement of a single particle, the particle is sent from Alice to Bob in the middle of the protocol.

Now, it is natural to ask the question as to what happens if the particle is lost in transit, i.e., is the information about the state to be teleported lost too? We show here that even if the particle is intercepted, by say, a different receiver Eve (in stead of Bob) it is not possible for Eve to extract the information encoded in the sent qubit. Let us first re-express Eq. [9] as

$$|\psi\rangle^{12a} = \alpha|1\rangle_p^1|0\rangle_s^1 + \beta|0\rangle_p^1|1\rangle_s^1$$

In this scenario, the path-spin entangled qubit (particle-1) is held by Eve and the spin qubit (particle-2) and the auxiliary particle “a” are possessed by Alice. Thereafter, Eve can perform measurement on the state of received path-spin entangled state to extract information about the state given in Eq. (9). But it is clear from the above Eq. (17) that whatever be the outcome of her measurement, she is unable to get any information about the state given in Eq. (9). Note that Eve could have been successful in her task if Alice performs her measurement on the state of particle-2 before sending the particle-1 towards Bob. Further, it is also possible for Alice to retrieve the unknown state, as follows. When Bob confirms to Alice that he didn’t get the particle, Alice makes a spin measurement on her auxiliary particle in the basis \(|0\rangle_s^a, |1\rangle_s^a\). According to the measurement outcome, she performs a suitable unitary transformation on her second particle (i.e., either (i) she does nothing if she gets \(|0\rangle_s^a\), or (ii) she makes the unitary operation \(\sigma_x\) if she gets \(|1\rangle_s^a\) to retrieve the unknown state to be teleported. Note that the role of the auxiliary particle that we have used in this protocol is to ensure that information of the unknown state to be teleported is never lost, even if Alice’s particle 1 is lost in transit.

We conclude by observing that creating the intra-particle path-spin entanglement could be considerably easier using beam-splitters and spin-flippers, as we have shown, than generating inter-particle entanglement through the controlled interaction of two particles. Since one does not have to preserve entanglement between two distant parties, our scheme should be less susceptible to decoherence effects, and thus provides an advantage over the standard scheme using two entangled qubits. The present work, however, is limited to showing the possibility of using intra-particle entanglement as a resource for information transfer, and issues regarding practical feasibility need to be worked out in more details. The path (or linear momentum) degrees of freedom for physical particles are always present in any experimental set-up. Here we have exploited these path variables to first generate path-spin entanglement at the level of a single particle, and then use it as physical resource for performing teleportation. This opens up the possibility of exploiting path-spin intra-particle entanglement for performing further information theoretic tasks. It may be also noted that though our protocol is demonstrated here for spin-1/2 particles such as neutrons, it could be easily implemented to other types of quanta such as photons using suitable optical devices. Finally, our analysis generally reemphasizes the notion that entanglement is a fundamental concept independent of either any particular physical realization of Hilbert space [13], or delocalisability of the involved modes, and specifically highlights that hybrid entanglement at the level of a single particle [17, 18] could be regarded as a real physical resource.

Acknowledgements ASM and DH acknowledge support from the DST project no. SR/S2/PU-16/2007. DH thanks the Centre for Science and Consciousness, Kolkata, India.

[1] A. Peres, Quantum Theory: Concepts and Methods, (Kluwer Academic Publishers, The Netherlands, 1995); D. Home, Conceptual Foundations of Quantum Physics: An overview from modern perspectives, Plenum, New York, 1997; M. Nielsen and I. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, Cambridge 2000.
[2] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. 47 (1935) 777.
[3] D. Bohm, Quantum Theory, Prentice-Hall, Englewood Cliffs, NJ, 1951.
[4] R. Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, arXiv: quant-ph/0702225.
[5] S. L. Braunstein and P. van Loock, Rev. Mod. Phys. 77 (2005) 513; J. Laurat, G. Keller, J-A Oliveira-Huguenin, C. Fabre, T. Coudreau, A. Serafini, G. Adesso and F. Illuminati, Journal of Optics B 7 (2005) S577.
[6] C. H. Bennett, G. Brassard, C. Crpeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70 (1993) 1895.
[7] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. 69 (1992) 2881.
[8] C. H. Bennett and G. Brassard, Proceedings of IEEE International Conference on Computer, Systems and Signal processing, pages 175–179, Bangalore, India, 1984; A. K. Ekert, Phys. Rev. Lett. 67 (1991) 661.
[9] L. Vaidman, Phys. Rev. A 49 (1994) 1473; S. L. Braunstein and H. J. Kimble, Phys. Rev. A 49 (1994) 1567; S.
[10] S. L. Braunstein and P. V. Loock, Rev. Mod. Phys. 77 (2005) 513.

[11] D. Aharonov, Phys. Rev. A 62 (2000) 062311; L.-A. Wu, M. S. Sarandy, D. A. Lidar and L. J. Sham, Phys. Rev. A 74 (2006) 052335.

[12] S. L. Braunstein and A. K. Pati, Phys. Rev. Lett. 98 (2007) 080502; M. Arzano, A. Hamma and S. Severini, arXiv:0806.2145.

[13] M. Zukowski and A. Zeilinger, Phys. Lett. A 155 (1991) 69; X.-S. Ma, A. Qarry, T. Jennewein and A. Zeilinger, Phys. Rev. A 79 (2009) 042101.

[14] D. Boschi, S. Branca, F. De Martini, L. Hardy and S. Popescu, Phys. Rev. Lett. 80 (1998) 1121; M. Michler, H. Weinfurter, and M. Zukowski, Phys. Rev. Lett. 84 (2000) 5457.

[15] J. T. Barreiro, T.-C. Wei and P. G. Kwiat, Nat. Phys. 4 (2008) 282.

[16] M. Blasone, F. Dell’Anno, S. De Siena and F. Illuminati, Europhys. Lett. 85 (2009) 50002.

[17] S. Basu, S. Bandyopadhyay, G. Kar and D. Home, Phys. Lett A 279 (2001) 281; A. K. Pan and D. Home, Phys. Lett. A, 373, 3430(2009).

[18] Y. Hasegawa, R. Loidl, G. Badurek, M. Baron and H. Rauch, Nature 425 (2003) 45.

[19] S. Adhikari, A. S. Majumdar, D. Home and A. K. Pan, arXiv:0909.0425.

[20] S. J. van Enk, Phys. Rev. A 72 (2005) 064306; A. Drezet, Phys. Rev. A 74 (2006) 026301; S. J. van Enk, Phys. Rev. A 74 (2006) 026302; J. Dunningham and V. Vedral, Phys. Rev. Lett. 99 (2007) 180404.