Hierarchical Control for Multi-Agent Autonomous Racing

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Abstract—We develop a hierarchical controller for multi-agent autonomous racing. A high-level planner approximates the race as a discrete game with simplified dynamics that encodes the complex safety and fairness rules seen in real-life racing and calculates a series of target waypoints. The low-level controller takes the resulting waypoints as a reference trajectory and computes high-resolution control inputs by solving a simplified formulation of a multi-agent racing game. We consider two approaches for the low-level planner to construct two hierarchical controllers. One approach uses multi-agent reinforcement learning (MARL), and the other solves a linear-quadratic Nash game (LQNG) to produce control inputs. We test the controllers against three baselines: an end-to-end MARL controller, a MARL controller tracking a fixed racing line, and an LQNG controller tracking a fixed racing line. Quantitative results show that the proposed hierarchical methods outperform their respective baseline methods in terms of head-to-head race wins and abiding by the rules. The hierarchical controller using MARL for low-level control consistently outperformed all other methods by winning over 85% of head-to-head races and more consistently adhered to the complex racing rules. Qualitatively, we observe the proposed controllers mimicking actions performed by expert human drivers such as shielding/blocking, overtaking, and long-term planning for delayed advantages. We show that hierarchical planning for game-theoretic reasoning produces competitive behavior even when challenged with complex rules and constraints.

Index Terms—multi-agent systems, reinforcement learning, hierarchical control, autonomous racing, Monte Carlo methods

I. INTRODUCTION

Autonomous driving has seen an explosion of research in academia and industry [1]. While most of these efforts focus on day-to-day driving, there is growing interest in autonomous racing. Many advancements in commercial automobiles have originated from projects invented for use in motorsports such as disc brakes, rear-view mirrors, and sequential gearboxes [2]. The same principle can apply when designing self-driving controllers because racing provides a platform to develop these controllers to be highly performant, robust, and safe in challenging scenarios.

Successful human drivers are required to both outperform opponents and adhere to the rules of racing. These objectives are effectively at odds with one another, but the best racers can satisfy both. Prior approaches in autonomous racing usually oversimplify the latter by only considering collision avoidance [3]–[6]. In reality, these racing rules often involve discrete variables and complex nuances [7]. For example, a driver may not change lanes more than a fixed number of times when traveling along a straight section of the track. While it is relatively straightforward to describe this rule in text, it is challenging to encode it in a mathematical formulation that can be solved by existing methods for real-time control. These methods have to compromise by either shortening their planning horizons or simply ignoring these constraints. The resulting behavior is an agent that is not optimal, or an agent that may be quick but is unsafe or unfair.

We develop a hierarchical control scheme that reasons about optimal long-term plans and closely adheres to the safety and fairness rules of a multi-agent racing game. The high-level planner forms a discrete approximation of the general formulation of the game. The solution of the discrete problem produces a series of waypoints that both adhere to the rules and are approximately optimal. The low-level planner solves a simplified, continuous state/action dynamic game with an objective to hit as many of the waypoints and a reduced form of the safety rules. Our structure yields a controller that runs in real-time and outperforms other traditional control methods in terms of head-to-head performance and obedience to safety rules. The control architecture is visualized in Figure 1. Although we develop our controller in the context of a racing game, the structure of this method enables reasoning about long-term optimal choices in a game-theoretic setting with complex constraints involving temporal logic and both continuous and discrete dynamics. Hence, it is possible to apply this method to many other adversarial settings that exhibit the aforementioned properties such as financial systems, power systems, or air traffic control.

II. PRIOR WORK

Because multi-agent racing is inherently a more complex problem, most prior work in autonomous racing is focused on single-agent lap time optimization, with fewer and more recent developments in multi-agent racing.

Single-agent racing approaches utilize a mixture of optimization and learning-based methods. One study uses Monte Carlo tree search to estimate where to position the car around various shaped tracks to define an optimal trajectory [8]. The work in [9] proposes a method that computes an optimal trajectory offline and uses a model predictive control (MPC) algorithm to track the optimized trajectory online. Similarly, the authors of [10] also perform calculations offline by...
creation a graph representation of the track to compute a target path and use spline interpolation for online path generation in an environment with static obstacles. In the category of learning-based approaches, online learning to update parameters of an MPC algorithm based on feedback from applying control inputs is developed in [11]. Further, there are works that develop and compare various deep reinforcement learning methods to find and track optimal trajectories [12], [13].

Looking at multi-agent racing works, both optimization and learning-based control approaches are also used. Authors of [5] use mixed-integer quadratic programming formulation for head-to-head racing with realistic collision avoidance but concede that this formulation struggles to run in real-time. Another study proposes a real-time control mechanism for a game with a pair of racing drones [14]. This work provides an iterative-best response method while solving an MPC problem that approximates a local Nash equilibrium. It is eventually extended to automobile racing [3] and multi-agent scenarios with more than two racers [4]. A faster, real-time MPC algorithm to make safe overtakes is developed in [6], but their method does not consider adversarial behavior from the opposing players. Again, these approaches do not consider racing rules other than simple collision avoidance. The work in [15] develops an autonomous racing controller using deep reinforcement learning that considers the rules of racing beyond just simple collision avoidance. Their controller outperforms expert humans while also adhering to proper racing etiquette. It is the first study to consider nuanced safety and fairness rules of racing and does so by developing a reward structure that trains a controller to understand when it is responsible for avoiding collisions, and when it can be more aggressive.

Finally, hierarchical game-theoretic reasoning is a method that has been previously studied in the context of autonomous driving. A hierarchical racing controller was introduced in [16] that constructed a high-level planner with simplified dynamics to sample sequences of constant curvature arcs and a low-level planner to use MPC to track the arc that provided the furthest progress along the track. A two-level planning system is developed in [17] to control an autonomous vehicle in an environment with aggressive human drivers. The upper-level system produces a plan to be safe against the uncertainty of the human drivers in the system by using simplified dynamics. The lower-level planner implements the strategy determined by the upper level-planner using precise dynamics.

III. GENERAL MULTI-AGENT RACING GAME FORMULATION

To motivate the proposed control design, we first outline a dynamic game formulation of a general multi-agent racing game.

Let there be a set \( N \) of players racing over \( T \) steps in \( \mathcal{T} = \{1, \ldots, T\} \). There is a track defined by a sequence of \( \pi \) checkpoints along the center, \( \{c_i\}_{i=1}^\pi \), whose indices are in a set \( C = \{1, \ldots, \pi\} \). The objective for each player \( i \) is to minimize its pairwise differences of the time to reach the final checkpoint with all other players. In effect, the player targets to reach the finish line with the largest time advantage. The continuous state (such as position, speed, or tire wear) for each player, denoted as \( x_i^t \in X \subseteq \mathbb{R}^n \), and control, denoted as \( u_i^t \in U \subseteq \mathbb{R}^k \), are governed by known dynamics \( f^i \). We also introduce a pair of discrete state variables \( r_i^t \in C \) and \( \gamma_i^t \in \mathcal{T} \). The index of the latest checkpoint passed by player \( i \) at time \( t \) is \( r_i^t \), and it is computed by function \( p : X \rightarrow C \). The earliest time when player \( i \) reaches checkpoint \( c_r \) is \( \gamma_i^t \). Using these definitions, we formulate the objective \( [1] \) and core dynamics \( [2]-[4] \) of the game as follows:

\[
\begin{align*}
\min_{u_0^i, \ldots, u_T^i} & \quad \langle |N| - 1 \rangle \gamma_i^t - \sum_{j \neq i}^N \gamma_j^t \\
\text{s.t.} & \quad x_{i,t+1}^j = f(x_i^t, u_i^t), \quad \forall \ t \in \mathcal{T}, j \in N \quad (2) \\
& \quad r_{i,t+1}^j = p(x_{i,t+1}^j, r_i^t), \quad \forall \ t \in \mathcal{T}, j \in N \quad (3) \\
& \quad r_i^1 = 1, \quad \forall \ j \in N \quad (4) \\
& \quad r_i^t = \tau, \quad \forall \ j \in N \quad (5) \\
& \quad \gamma_i^t = \min \{t \mid r_i^t = \tau \land t \in \mathcal{T}\}, \quad \forall \ j \in N \quad (6)
\end{align*}
\]

In addition to the core dynamics of the game, there are rules that govern the players’ states. To ensure that the players stay within the bounds of the track we introduce a function, \( q : X \rightarrow \mathbb{R} \), which computes a player’s distance to the closest point on the center line. This distance must be limited to the width of the track \( w \). Therefore, for all \( t \in \mathcal{T} \) and \( j \in N \):

\[
q(x_i^t) \leq w 
\]

Next, we define the collision avoidance rules of the game. We use an indicator function that evaluates if player \( i \) is “behind” player \( j \). Depending on the condition, the distance between every pair of players, computed by function the \( d : X \rightarrow \mathbb{R} \), is required to be at least \( s_1 \) if player \( i \) is behind...
another player \( j \) or \( s_0 \) otherwise. For all \( t \in T, j \in N \), and \( k \in N \setminus \{ j \} \) these rules are expressed by the constraint:

\[
d(x_t^k, x_t^j) \geq \begin{cases} 
1_{\text{player } i \text{ behind player } j} s_1 \\
0 & \text{otherwise}
\end{cases} 
\]

Finally, players are limited in how often they may change lanes depending on the part of the track they are at. We assume that there are \( \lambda \in \mathbb{Z}^+ \) lanes across all parts of the track. If the player’s location on the track is classified as a curve, there is no limit on lane changing. However, if the player is at a location classified as a straight, it may not change lanes more than \( L \) times for the contiguous section of the track classified as a straight. We define a set \( S \) that contains all possible states where a player is located at a straight section. We also introduce a function \( z : X \rightarrow \{1, 2, \ldots, \lambda\} \) that returns the lane ID of a player’s position on the track. Using these definitions, we introduce a variable \( l_t \) calculated by the following constraint for all \( t \in T \) and \( j \in N \):

\[
l_t^j = \begin{cases} 
l_{t-1} + 1 & 1_{x_t^j \in S} = 1_{x_{t-1}^j \in S} \land z(x_t^j) \neq z(x_{t-1}^j) \\
0 & \text{otherwise}
\end{cases} 
\]

This variable effectively represents a player’s count of “recent” lane changes over a sequence of states located across a contiguous straight or curved section of the track. However, the variable is only required to be constrained if the player is on a straight section of the track. Therefore, the following constraint must hold for all \( t \in T \) and \( j \in N \) and if \( x_t^j \in S \):

\[
l_t^j \leq L 
\]

Most prior multi-agent racing formulations [3]–[5] do not include the complexities we introduced through defining constraints \( 8 \)–\( 10 \). They usually have a similar form regarding continuous dynamics and discrete checkpoints \( 2 \)–\( 6 \), and their rules only involve staying on track \( 7 \) and collision avoidance with a fixed distance. However, in real-life racing, there do exist these complexities both in the form of mutually understood unwritten rules and explicit safety rules \( 7 \). As a result, we account for two of the key rules that ensure the game remains fair and safe:

1) There is a greater emphasis on and responsibility of collision avoidance for a vehicle that is following another 

2) The player may only switch lanes \( L \) times while on a straight section of the track \( 9 \)–\( 10 \).

The first rule ensures that a leading player can make a decision without needing to consider an aggressive move that risks a rear-end collision or side collision while turning from the players that are following. This second rule ensures that the leading player may not engage in aggressive swerving or “zig-zagging” across the track that would make it impossible for a player that is following the leader to safely challenge for an overtake. While functions may exist to evaluate these spatially and temporally dependent constraints, their discrete nature suggests that they cannot be easily differentiated. Therefore, most state-of-the-art optimization algorithms would not apply or struggle to find a solution in real time.

IV. Hierarchical Control Design

Traditional optimization-based control methods cannot easily be utilized for the general multi-agent racing game formulated with realistic safety and fairness rules. The rules involve nonlinear constraints over both continuous and discrete variables, and a mixed-integer non-linear programming algorithm would be unlikely to run at rates of 25 Hz-50 Hz for precise control. This inherent challenge encourages utilizing a method such as deep reinforcement learning or trying to solve the game using short horizons.

However, we propose a hierarchical control design involving two parts that work to ensure all of the rules are followed while approximating long-term optimal choices. The high-level planner transforms the general formulation into a game with discrete states and actions where all of the discrete rules are naturally encoded. The solution provided by the high-level planner is a series of discrete states (i.e. waypoints) for each player, which satisfies all of the rules. Then, the low-level planner solves a simplified version of the racing game with an objective putting greater emphasis on tracking a series of waypoints and smaller emphasis on the original game-theoretic objective and a simplified version of the rules. Therefore, this simplified formulation can be solved by an optimization method in real-time or be trained in a neural network when using a learning-based method.

A. High-Level Planner

The high-level planner constructs a turn-based discrete, dynamic game that is an approximation of the general game \( 1 \)–\( 10 \). Continuous components of a players’ states are broken into discrete “buckets” (e.g., speed between 2 m s\(^{-1}\) and 4 m s\(^{-1}\), tire wear between 10% and 15%). In addition, \( \lambda \) (which is the number of lanes) points around each checkpoint are chosen along a line perpendicular to the direction of travel where each point evaluates to a unique lane ID on the track when passed into function \( z(\cdot) \) defined in the general formulation. The left and center of Figure 2 visualize the checkpoints in the original, continuous formulation (in red) expanded into three discrete lanes (green or purple) for the high-level game.

The players’ actions are defined by pairs of lane ID, resolving to a target location near the next checkpoint, and target speed for that location. Therefore, we can apply a simplified inverse approximation of the dynamics to determine the time it would take to transition from one checkpoint to the next and estimate the remaining state variables or dismiss the action if it is dynamically infeasible. This action space also allows us to easily evaluate or prevent actions where rules of the game would be broken. By limiting choices to fixed locations across checkpoints, we ensure that the players always remain on track \( 7 \). Moreover, the players’ actions can be dismissed if they would violate the limit on the number of lane changes by simply checking whether
choosing a lane would exceed their limits or checking if the location is a curve or straight \[10\]. Finally, other actions that could cause collisions can also be dismissed by estimating that if two players reach the same lane at a checkpoint and have a small difference in their time states, there would be a high risk of collision \[9\].

The game is played with each player starting at the initial checkpoint, and it progresses by resolving all players’ choices one checkpoint at a time. The order in which the players take their actions is determined by the player who has the smallest time state at each checkpoint. A lower time state value implies that a player was at the given checkpoint before other players with a larger time state, so it would have made its choice at that location before the others. This ordering also implies that players who arrive at a checkpoint after preceding players observe the actions of those preceding players. Therefore, these observations can contribute to their strategic choices. Most importantly, because the ordering forces the following players to choose last, we also capture the rule that the following players (i.e. those that are “behind” others) are responsible for collision avoidance after observing the leading players’ actions.

The objective of the discrete game is to minimize the difference between one’s own time state at the final checkpoint and that of all other players just like the original formulation \[1\]. Although the discrete game is much simpler than the original formulation, the state space grows as the number of actions and checkpoints increases. Therefore, we solve the game in a receding horizon manner, but our choice of the horizon (i.e. number of checkpoints to consider) extends much further into the future than an MPC-based continuous state/action space controller can handle in real time \[3\]. In order to produce a solution to the discrete game in real-time, we use the Monte Carlo tree search (MCTS) algorithm \[18\]. The solution from applying MCTS is a series of waypoints in the form of target lane IDs (which can be mapped back to positions on track) and the target velocities at each of the checkpoints for the ego player and estimates of the best response lanes and velocities by the adversarial players.

\[\text{B. Low-Level Planner}\]

The low-level planner is responsible for producing the control inputs, so it must operate in real-time. Because we have a long-term plan from the high-level planner, we can formulate a reduced version of the original game for our low-level planner. The low-level game is played over a shorter horizon compared to the original game of just \(\delta\) steps in \(\mathcal{T} = \{1, ..., \delta\}\). We assume that the low-level planner for player \(i\) has received \(k\) waypoints, \(\psi_{i_1}^{j_1}, ..., \psi_{i_{k+1}}^{j_{k+1}}\), from the high-level planner, and player \(i\)'s last passed checkpoint \(r_i^k\).

The low-level objective involves two components. The first is to maximize the difference between its own checkpoint index and the opponents’ checkpoint indices at the end of \(\delta\) steps. The second is to minimize the tracking error, \(\eta_y\), of every passed waypoint \(\psi_{i_{k+1}}^{j_{k+1}}\). The former component influences the player to pass as many checkpoints as possible, which suggests reaching \(c_r\) as quickly as possible. The latter influences the player to be close to the high-level waypoints when passing each of the checkpoints. The objective also includes some multiplier \(\alpha\) that balances the emphasis of the two parts. The objective is written as follows:

\[
\min_{u_1, ..., u_N} \sum_{j \neq i}^{N} \left( \sum_{k=1}^{\delta} r_j^k \right) - \left( |\mathcal{N}| - 1 \right) r_j^k + \alpha \sum_{c=r_i^k}^{r_i^{k+1}} \eta_y
\]

The players’ continuous state dynamics, calculations for each checkpoint, and constraints on staying within track bounds \[12\]-\[15\] are effectively the same as the original formulation.

\[
x_{i+1} = f(x_i, u_i), \quad \forall \ t \in \mathcal{T}, j \in \mathcal{N}
\]

\[
r_{i+1} = p(x_{i+1}, r_i), \quad \forall \ t \in \mathcal{T}, j \in \mathcal{N}
\]

\[
r_j = r_j^*, \quad \forall \ j \in \mathcal{N}
\]

\[
q(x_i^m) \leq w, \quad \forall \ t \in \mathcal{T}, j \in \mathcal{N}
\]

The collision avoidance rules are simplified to just maintaining a minimum distance \(s_0\) as the high-level planner would have already considered the nuances of rear-end collision avoidance responsibilities in \[8\]. As a result, we require the following constraint to hold for all \(t \in \mathcal{T}, j \in \mathcal{N}, \) and \(k \in \mathcal{N} \setminus \{j\}\):

\[
d(x_i^t, x_i^k) \geq s_0
\]

\[\text{Fig. 2. The uncountably infinite trajectories of the general game (left) discretized by the high-level planner (middle). The sequence of target waypoints calculated by the high-level planner (in green) is tracked by the low-level planner (right) and converges to a continuous trajectory (in black).}\]
Finally, we define the dynamics of the waypoint error, $\eta^i_y$, introduced in the objective. It is equivalent to the accumulated tracking error of each target waypoint that player $i$ has passed using a function $h : X \times X \rightarrow \mathbb{R}$ that measures the distance. If a player has not passed a waypoint, then the variable indexed by that waypoint is set to 0. The variable’s dynamics are expressed by the following constraint:

$$\eta^i_y(t) = \begin{cases} \sum_{t=1}^{T} h(x^i_t, \psi^i_t) & \text{if } \exists r^i \geq y \\
 0 & \text{otherwise} \end{cases} \quad \forall y \in \{r^i_1, ..., r^i_{k} + k\} \quad (17)$$

This simplified formulation is similar to the general formulation. However, the constraints introduced by the complex fairness and safety rules are dropped since they are considered by the high-level planner. The center and right of Figure 2 show how the waypoints from the high-level planner (in green) are approximately tracked by the low-level planner producing a continuous trajectory (in black). We consider two methods to solve this low-level formulation. The first method develops a reward structure to represent this simplified formulation for a multi-agent reinforcement learning (MARL) controller. The second method further simplifies the low-level formulation into a linear-quadratic Nash game (LQNG) to compute the control inputs.

1) Multi-Agent Reinforcement Learning Controller: Designing the MARL controller primarily involves shaping a reward structure that models the low-level formulation. The RL agent is rewarded for the following behaviors that would improve the objective function (11):

- Passing a checkpoint with an additional reward for being closer to the target lane and velocity targets.
- Minimizing the time between passing two checkpoints.
- Passing as many checkpoints in the limited time.

However, the agent is penalized for actions that would violate the constraints:

- Swerving too frequently on straights (10).
- Going off track or hitting a wall (15).
- Colliding with other players (16) with additional penalty if the agent is responsible for avoidance (3).

The rewards capture our low-level formulation objective (11) to pass as many checkpoints as possible while closely hitting the lane and velocity targets (17). The penalties capture the on-track (15) and collision avoidance (16) constraints. However, the penalties also reintroduce the original safety and fairness from the original general game that were simplified away from the low-level formulation (8) and (10). Because these rules are inherently met by satisfying the objective of reaching the high-level planner’s waypoints, their penalties have the weights set much lower than other components of the reward structure. However, we still incorporate the original form of these penalties to reinforce against the possibility that the ego player might be forced to deviate far away from the high-level plan.

The agents’ observations include perfect state information of all players and local observations consisting of 9 LIDAR rays spaced over a 180° field of view centered in the direction that the player is facing.

2) Linear-Quadratic Nash Game Controller: Our second low-level approach solves an LQNG using the Coupled Riccati equations [19]. This method involves further simplifying the low-level formulation into a structure with a quadratic objective and linear dynamics. The continuous state is simplified to just four variables: $x$, $y$, $v$, and $\theta$. The control inputs $u^i$ are also explicitly broken into acceleration, $a^i$, and yaw-rate, $e^i$. The planning horizon is reduced to $\delta$ where $\delta << \delta < T$. To construct our quadratic objective for player $i$, we break it into three components. The first is to minimize the distance to the upcoming target waypoint from the high-level planner $\tilde{\psi}^i$ calculated by the following equation:

$$v^i(\rho_1, \rho_2, \rho_3) = \sum_{t=1}^{\delta} \rho_1((x^i_t - \tilde{\psi}^i_x)^2 + (y^i_t - \tilde{\psi}^i_y)^2) + \rho_2(v^i_t - \tilde{v}^i_x)^2 + \rho_3(\theta^i_t - \tilde{\psi}^i_x)^2 \quad (18)$$

The second component is to maximize each opponent’s distance from the location of estimated target waypoints $\tilde{\psi}^j$ calculated by the following equation:

$$\phi^j(\tilde{\psi}^j, \rho) = \sum_{t=1}^{\delta} \rho((x^j_t - \tilde{\psi}^j_x)^2 + (y^j_t - \tilde{\psi}^j_y)^2) \quad (19)$$

We drop all of the constraints with the exception of collision avoidance, and it is incorporated as the third component and penalty term in the objective where the distance to each opponent should be maximized. This term is calculated by the following equation:

$$\chi^i(x^i_t, y^i_t, \rho) = \sum_{t=1}^{\delta} \rho((x^i_t - x^i)^2 + (y^i_t - y^i)^2) \quad (20)$$

The final quadratic objective aggregates (18)-(20) using weight multipliers ($\rho_i$) to place varying emphasis on the components as follows:

$$\min_{a^i_1, e^i_1, ..., a^i_k, e^i_k} v^i(\rho_1, \rho_2, \rho_3) - \sum_{j \neq i} \phi^j(\tilde{\psi}^j, \rho_4) - \chi^i(x^i_t, y^i_t, \rho_5) \quad (21)$$

Finally, the linear dynamics are time invariant and apply for all players $j \in N$:

$$\begin{bmatrix} x^j_{t+1} \\ y^j_{t+1} \\ v^j_{t+1} \\ \theta^j_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin(\theta^j_{t_0}) \Delta t & \cos(\theta^j_{t_0}) \Delta t \\ 0 & 1 & \cos(\theta^j_{t_0}) \Delta t & \sin(\theta^j_{t_0}) \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x^j_t \\ y^j_t \\ v^j_t \\ \theta^j_t \end{bmatrix} + \begin{bmatrix} a^j_1 \\ a^j_2 \\ e^j_1 \\ e^j_2 \end{bmatrix} \Delta t \quad (22)$$
V. Experiments

The high-level planner is paired with each of the two low-level planners discussed. We refer to our two hierarchical design variants as MCTS-RL and MCTS-LQNG.

A. Baseline Controllers

To measure the importance of our design innovations, we also consider three baseline controllers to resemble the other methods developed in prior works.

1) End-to-End Multi-Agent Reinforcement Learning: The end-to-end MARL controller, referred to as “E2E,” represents the pure learning-based methods such as that of [15]. This controller has a similar reward/penalty structure as our low-level controller, but its observation structure is slightly different. Instead of observing the sequence of upcoming states as calculated by a high-level planner, E2E only receives the subsequence of locations from \( \{c_i\}_{i=1}^\tau \) that denote the center of the track near the agent. As a result, it is fully up to its neural networks to learn how to plan strategic and safe moves.

2) Fixed Trajectory Linear-Quadratic Nash Game: The fixed trajectory LQNG controller, referred to as “Fixed-LQNG,” uses the same LQNG low-level planner as our hierarchical variant, but it instead tracks a fixed trajectory around the track. This fixed trajectory is a racing line that is computed offline for a specific track using its geometry and parameters of the vehicle as seen in prior works [9], [10]. However, the online tracking involves game-theoretic reasoning rather than single-agent optimal control in the prior works.

3) Fixed Trajectory Multi-Agent Reinforcement Learning: The fixed trajectory MARL controller, referred to as “Fixed-RL,” is a learning-based counterpart to Fixed-LQNG. Control inputs are computed using a deep RL policy trained to track precomputed checkpoints that are fixed prior to the race.

B. Experimental Setup

Our controllers are implemented in the Unity Game Engine. Screenshots of the simulation environment are shown in Figure 3. We extend the Karting Microgame template [20] provided by Unity. The kart physics from the template is adapted to include cornering limitations and tire wear percentage. Tire wear is modeled as an exponential decay curve that is a function of the accumulated angular velocity endured by the kart. This model captures the concept of losing grip as the tire is subjected to increased lateral loads. Multi-agent support is also added to the provided template in order to race the various autonomous controllers against each other or human players. The high-level planners run at 1 Hz, and low-level planners run at 50 Hz. Specifically, \( \delta \) is set to 0.06 s for the LQNG planner. The implementation of the learning-based agents utilizes a library called Unity ML-Agents [21]. All of the learning-based control agents are trained using proximal policy optimization and self-play implementations from the library. They are also only trained on two sizes of oval-shaped tracks with the same number of training steps.

Our experiments include head-to-head racing on a basic oval track (which the learning-based agents were trained on) and a more complex track shown in Figure 3. Specifically, the complex track involves challenging track geometry with turns whose radii change along the curve, tight U-turns, and turns in both directions. To be successful, the optimal racing strategy requires some understanding of the shape of the track along a sequence of multiple turns. Every pair of controllers competes head-to-head in 50 races on both tracks. The dynamical parameters of each player’s vehicle are identical, and the players start every race at the same initial checkpoint. The only difference in their initial states is the lane in which they start. In order to maintain fairness with respect to starting closer to the optimal racing line, we alternate the starting lanes between each race for the players.

C. Results

Our experiments primarily seek to identify the importance of hierarchical game-theoretic reasoning and the strength of MCTS as a high-level planner for racing games. We count the number of wins against each opponent, average collisions-at-fault per race, average illegal lane changes per race, and a safety score (a sum of the prior two metrics) for the controllers. We also provide a video demonstrating them in action. Based on the results visualized in Figures 4 and 5, we conclude the following key points.

1) The proposed hierarchical variants outperformed their respective baselines.

The results amongst MCTS-RL, Fixed-RL, and E2E show the effectiveness of our proposed hierarchical structure. While all three of the MARL-based agents were only trained on the oval track, the MCTS-RL agent was able to win the most head-to-head races while also maintaining the best safety score by better adapting its learning. Comparing the baselines against each other, Fixed-RL also has more wins and a better safety score than E2E aggregated across both tracks. This result indicates that some type of hierarchical structure is favorable. It suggests that a straightforward task
of trajectory tracking is much easier to learn for a deep neural network than having to learn both strategic planning and respect for the safety and fairness rules.

Next, we compare MCTS-LQNG and Fixed-LQNG. Although MCTS-LQNG has a worse safety score, it has twice as many head-to-head wins when aggregated over both tracks. Furthermore, the main difference in their safety score is attributed to the fact that MCTS-LQNG considers multiple trajectories whereas Fixed-LQNG only follows a fixed trajectory that does not involve changing lanes often. Their collision-at-fault numbers, aggregated from both tracks, are similar. Furthermore, MCTS-LQNG was more competitive in its races against E2E and Fixed-RL on the complex track while the Fixed-LQNG had about 9% fewer wins in the races against them combined. MCTS-LQNG considered trajectories that could result in overtakes when opponents made mistakes from any part of the track. However, to overtake, Fixed-LQNG had to rely on opponents making mistakes that were not along its fixed trajectory.

2) **MARL is more successful and robust than LQNG as a low-level planner.**

In both tracks, the MARL-based agents outperformed their LQNG-based counterparts in terms of the two key metrics: head-to-head wins and safety scores. However, this result is likely due to the heuristic simplifications in our LQNG design. Vehicle dynamics are only linearized around the initial state of each agent and are time-invariant, meaning the linear approximation is only valid for a very short time horizon. Therefore, LQNG-based controllers could only rely on braking/acceleration instead of yaw-rate to avoid collisions, thus often conceding in close battles and losing races. If the dynamics were linearized over the state at each timestep \( t \) instead of just the initial state at \( t_0 \), we could use a larger time horizon and expect better performance for the LQNG-based agents. The difference in safety scores primarily arises from increased illegal lane changes by these agents. The high-level planner runs in parallel with the low-level and at a lower frequency. As a result, the calculated high-level plan is sometimes delayed and does not account that the low-level controllers have already made choices that might contradict the initial steps in the plan. It causes the LQNG-based controllers to more often break the lane changing rules by swerving across the track to immediately follow the high-level plan when it is updated. The MARL-based agents are more robust to this situation because they have those safety rules encoded in their reward structures, albeit with smaller weights.

3) **MCTS-RL outperforms all other implemented controllers.**

MCTS-RL has the best safety score and recorded a win rate of over 88% of the 400 head-to-head races it participated in across both tracks. It combined the advantage of having a high-level planner that evaluates long-term plans and a low-level planner that is robust to the possibility that the high-level plans may be out of date. For example, Figure 6 demonstrates how the high-level planner provided a long-term strategy, guiding the agent to give up an advantage at present for a greater advantage in the future when overtaking. The RL-based low-level planner approximately follows the high-level strategy in case stochasticity of the MCTS algorithm yields a waypoint that seems out of place (e.g., the checkpoint between \( t = 3 \) and \( t = 4 \) in Figure 6a).

Furthermore, MCTS-RL is also successful at executing defensive maneuvers as seen in Figure 6b due to those same properties of long-term planning and low-level robustness. Both of these tactics resemble strategies of expert human drivers in real head-to-head racing.

**VI. CONCLUSION**

We developed a hierarchical controller for multi-agent autonomous racing that adheres to safety and fairness rules found in real-life racing and outperforms other common control techniques such as purely optimization-based or purely learning-based control methods. Our high-level planner constructed long-term trajectories that abided by the introduced complex rules about collision avoidance and lane changes. As a result, we design an objective for the low-level controllers to focus on tracking the high-level plan, which is an easier problem to solve compared to the original racing-game formulation. Our method outperformed the baselines both in terms of winning head-to-head races and a safety score measuring obedience to the rules of the game. Finally, they also exhibited maneuvers resembling those performed by expert human drivers.

We are exploring extensions of this hierarchical control design by expanding the game to team-based autonomous racing where the racers have a mix of both cooperative and competitive objectives with one teammate and two opponents. Other future work should also introduce additional high-level and low-level planners and investigate policy-switching hierarchical controllers where we switch between
various high and low-level controllers depending on the state of the game. Lastly, our hierarchical control design can be extended to other multi-agent systems applications where there exist complex rules such as energy grid systems or air traffic control. Constructing a discrete high-level game allows for natural encoding of the complex constraints, often involving discrete components, to find an approximate solution that can warm start a more precise low-level planner.

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