A Brief History of the Co-evolution of Supernova Theory with Neutrino Physics

Adam Burrows
Department of Astrophysical Sciences, Princeton University, Princeton, NJ USA 08544

The histories of core-collapse supernova theory and of neutrino physics have paralleled one another for more than seventy years. Almost every development in neutrino physics necessitated modifications in supernova models. What has emerged is a complex and rich dynamical scenario for stellar death that is being progressively better tested by increasingly sophisticated computer simulations. Though there is still much to learn about the agency and details of supernova explosions, whatever final theory emerges will have the neutrino at its core. I summarize in this brief contribution some of the salient developments in neutrino physics as they related to supernova theory, while avoiding any attempt to review the hundreds of pivotal papers that have pushed supernova theory forward. My goal has been merely to highlight the debt of supernova astrophysics to neutrino physics.

1 Introduction

The theory of the violent deaths of massive stars in what are called supernova explosions has a long pedigree that spans more than half a century, has engaged hundreds of researchers, and has proven more elusive than anticipated. However, with the advent of numerically and physically sophisticated codes with which to simulate the onset of explosion in three spatial dimensions, the theoretical community now seems to be zeroing in on the mechanism of explosion. Central to this emerging theory are the neutrinos of all species produced copiously at the high densities and temperatures achieved during and after the collapse of the unstable Chandrasekhar core created in the center of the massive star at the end of its life. A fraction of these emerging neutrinos are absorbed behind the bounce shock wave to drive it into explosion, aided by the turbulence in the outer core driven predominantly by neutrino heating itself. In this context, the interaction of neutrinos with matter is key to the fidelity with which theorists can simulate the explosion phenomenon. Hence, progress in supernova theory has paralleled developments not only in computational capabilities, but also in advances in our understanding of neutrino physics. Developments in nuclear physics over the years have also played (and continue to play) important roles, as have improvements in our understanding of pre-supernova stellar evolution. Nevertheless, if neutrinos prove to be the agents of explosion then their pivotal role deserves to be highlighted. This parallel evolution over the years in our understanding of neutrinos and of supernovae is the subject of this brief paper. I will focus upon rough timelines for important conceptual progress in both spheres and will omit almost any detailed discussion of supernova theory itself. Such can be found in numerous papers in the archival literature.

1,2,3,4,5,6,7.
2 The 1930s

Of course, the 1930s were the decade of the discovery of the neutron, the postulation of the existence of the neutrino by Wolfgang Pauli, and its naming by Enrico Fermi. Fermi also formulated the point interaction to describe $\beta$-decay. In addition, this decade witnessed the publication of the prescient papers by Baade & Zwicky, wherein they postulated the existence of neutron stars, coined the term “supernova,” and connected supernovae with the creation of both neutron stars and cosmic rays. However, the possible connection between supernova and neutrinos was not in the air.

3 The 1940s

This connection was first made by Gamow & Schoenberg in their April 1, 1941 paper. They noted that stellar interiors could be hot and could emit neutrinos that might accelerate evolution by facilitating rapid core contraction. This contraction was to be accompanied by rapid outward motion of the envelope and an explosion, and these explosions could be either novae or supernovae, depending upon the original stellar mass. However, the neutrino emission mechanism was by a cycle of electron capture and subsequent beta decay, the so-called “URCA” process (invented in this paper), which did not result in a net change in composition and is not relevant in the modern contexts, but did result in the loss of energy by volumetric neutrino pair emission. The precise mechanism by which the outer layers were to be expelled was not explained, and it was certainly not by neutrino energy or momentum deposition. Neutrinos did not directly drive the explosion, but their emission was to lead, by a mechanism unexplained, to a dynamical stellar phase. Aside from its intriguing comingling of supernovae with neutrinos, this paper was too short on details and too far from what we now conclude concerning stars, supernovae, novae, and neutrinos to be considered useful. In fact, there followed a gap of ~20 years before the neutrino-supernova saga would reemerge. And as far as neutrino physics itself was concerned, the world in the 1940s was too engaged in other pursuits to generate much of substance in the open literature.

4 The 1950s

After the World War there was a great deal of progress in particle physics, nuclear physics, and (not unexpectedly) the theory of nucleosynthesis. The latter is exemplified by the publications of Burbidge, Burbidge, Fower, & Hoyle and Cameron. These papers represented a growing literature in which great strides were made in understanding the origin of the elements in stars, either during quiescent burning or explosively. In physics, the weak interaction experienced a great deal of ferment. Lee & Yang suggested parity violation in the weak interaction and Wu demonstrated it. Goldhaber et al. explored helicity, Pontecorvo posited $e^-/\mu^-$ universality and neutrino oscillations, and Feymann & Gel’Mann introduced $V-A$ theory. Notably, Cowan & Rienes actually detected and measured (anti-)neutrinos from a reactor. However, aside from ongoing speculation, there was not much progress in the theory of supernova explosions.

5 The 1960s

This changed with the study by H.Y. Chiu of thermal neutrino emission processes in stars. Relevant in there own right, these studies also led to discussions between Chiu and Stirling Colgate, which, along with the latter’s participation in the Partial Test Ban Treaty negotiations, inspired Colgate to take the next big step in supernova theory (S. Colgate, private communication). In 1966, Colgate & White published the notion that copious neutrino production in supernova progenitor cores immediately after core collapse could, through the agency of neutrino heating, unbind the mantle in a supernova explosion. In this paradigm, the supernova shock...
wave would be driven by neutrino energy deposition, and this is, in very broad outline and with a few remaining caveats, the currently accepted mechanism. In 1967, Dave Arnett\textsuperscript{23} conducted a numerically and physically more sophisticated “radiation/hydrodynamic” study, also including muon neutrinos, and the stage was set for the permanent association between neutrinos and core-collapse supernova explosions. On the physics front, Lederman et al.\textsuperscript{24} detected the $\nu_\mu$, neutrino and the Weinberg/Salam/Glashow theory of electroweak unification was formulated. The latter postulated the existence of neutral currents.

6 The 1970s

This decade provided most of the remaining progress in the understanding of the weak interaction, neutrinos, and the neutrino-matter interaction necessary to simulate supernova with modern physical fidelity. The weak neutral current, theorized in the last decade, was measured early in this. This led to the calculation by Dicus\textsuperscript{25} of neutral-current scattering of neutrinos off free nucleons, forbidden in the $V-A$ theory of Feynmann and Gel’Mann, as well as the calculation by Freedman\textsuperscript{26} of neutral-current scattering off nuclei. Both these processes can dominate the neutrino-matter scattering rate at various phases of core collapse and explosion and were unknown to earlier supernova theory. In addition, M. Perl and collaborators discovered the $\nu_\tau$ neutrino and Wolfenstein\textsuperscript{27} introduced matter effects into neutrino oscillation theory.

On the supernova front, one of the most important developments was the realization that upon core collapse, at the progressively higher densities and temperatures achieved, not only does the optical depth to neutrinos become large and does neutrino diffusion from the “proto-neutron star”\textsuperscript{28} become relevant, but that electron lepton number becomes trapped. Trapping is not only the achievement of high neutrino optical depths, but the cessation on the dynamical timescales of core collapse and infall of the net loss of electron lepton number ($Y_e = \text{electron/baryon ratio}$). Rather than achieving low neutron-star electron numbers of $\sim 0.03$ before reaching nuclear densities, electron fractions near $\sim 0.3$ are frozen in. This is due to the onset during collapse of the inverse reaction of $\nu_e$ capture onto protons in and out of nuclei that pushes the matter into a “$\beta$” (chemical)-equilibrium and preserves the electrons. The trapped electron neutrinos are then further compressed during the later stages of collapse, but they are now degenerate fermions. Trapping was first recognized by Ted Mazurek\textsuperscript{29} using the old weak-interaction theory and by Kats Sato\textsuperscript{30} using the new theory with neutral currents.

Further compression elevates the electron neutrino chemical potential ($\sim$Fermi energy), and the average $\nu_e$ neutrino energy at and subsequent to bounce soars to $\sim 150-300$ MeV at the center. Due to the stiffly increasing interaction cross sections with increasing neutrino energy, the optical depth to $\nu_e$ diffusion grows to $\sim 10^5$. Such a large optical depth translates into a time of many seconds for the diffusion of lepton number and energy out of the proto-neutron star (PNS)\textsuperscript{31}. Hence, trapping leads directly to the long emission times of supernova neutrinos. Without the recognition of neutrino trapping, the duration of a supernova neutrino burst would have been predicted to be less than $\sim 100$ milliseconds.

The final overarching physical piece of the collapse puzzle was the recognition that the collapsing core was a Chandrasekhar-mass white dwarf supported by electron degeneracy. This was not obvious until the requisite thermal neutrino heating in the stellar core was implemented in stellar evolution codes and the thermostatic effects of the excited states of nuclei were incorporated into the equation of state. The upshot was the lower entropies and lower temperatures that ensured the core pressures were due to electron degeneracy pressure from the onset of and during core collapse to nuclear densities.
7 The 1980s

The 1980s saw the discovery of the $W$ and $Z$ bosons by the teams led by Rubbia and van de Meer32 and the solidification of the electroweak theory. There were also important developments in the solar neutrino puzzle and the recognition of the reality of neutrino oscillations. On the supernova front, the physics of the neutrino-matter interaction was mature, as reflected in the summary paper by Steve Bruenn33. However, with sophistication in the neutrino sector came puzzles in supernova theory. In particular, spherical models did not explode directly. Jim Wilson broke the logjam with the “delayed” neutrino heating mechanism, wherein the bounce shock stalled for hundreds of milliseconds, only to be revived thereafter. He traced success to the boosting of the driving neutrino luminosities after bounce by “neutron-finger” convection in the inner core. In his calculations, performed using enforced mixing-length convection, the boost was $\sim 25\%$, and this was enough to revitalize the explosion. However, such convection was later shown by Bruenn and Dineva34 to be unphysical. Nevertheless, the potential role of hydrodynamic instabilities and turbulence just interior to the stalled shock wave, driven mostly by neutrino heating itself (from below) was found to be crucial and this basic idea was later developed by among others Herant et al.35 and Burrows et al.36. Such turbulence is now a central, perhaps enabling, facet of supernova theory. However, the manifest hydrodynamic instabilities and turbulence in supernova cores required the development of sophisticated multi-dimensional radiation/hydrodynamic codes to simulate the supernova dynamics in its full multi-dimensional richness. It is reasonably concluded that the development of such complicated codes and the expensive computational platforms they require has set the long timescale of subsequent progress up to the present day.

However, the most exciting event in this decade at the interface between neutrinos and supernovae was the detection in 1987 by Hirata et al.37 in Kamiokande II and by Bionta et al.38 in the IMB of the neutrinos from the supernova SN1987A in the Large Magellanic Cloud $\sim 50$ kiloparsecs away. This first, and to date only, such detection galvanized the astrophysics and physics communities, generated hundreds of papers, and established unambiguously the neutrino/supernova connection. The many-second duration of the event, with average event energies near $\sim 15$ MeV, confirmed that 1) neutrinos are generated and radiated in abundance in supernova cores, 2) neutrinos diffuse out of the dense PNS, 3) the scale of the radiated energy is the binding energy of a neutron star ($\sim 3 \times 10^{53}$ ergs), and 4) electron lepton number is trapped. The latter seems compelling since trapping theory converted a $\sim 50$-100 millisecond event into a multi-second event, a duration that was predicted before 1987. However, the effect of neutrino oscillations on the detected signal still remains to be determined39.

This, and witnessing supernova dynamics in real time, motivates the development of modern supernova neutrino detection capabilities40. It is only by capturing supernova neutrinos and the gravitational waves also generated during collapse that we can see what happens at the time it is happening. Otherwise, the core is shrouded in mystery by the profound opacity to photons of the stellar envelope that surrounds it. Currently, Super-Kamiokande41, IceCube42, and various underground detectors in the Gran Sasso tunnel in Italy stand guard in anticipation of a galactic event, but in the near future JUNO43 and DUNE44 will join them and Hyper-K45 will replace Super-Kamiokande. The per-particle interaction cross sections of importance in these detectors are plotted in Figure 1. For each, it is clear from these plots which reactions dominate. It is worth noting that these modern sentinels could capture hundreds to many thousands of events from a galactic supernova at $\sim 10$ kiloparsecs, whereas we culled but 11 (Kamioka II) and 8 (IMB) events from SN1987A. Clearly, much remains to be learned.

8 The Present

The theory of core-collapse supernova has experienced significant development over the last few decades. This progress has relied upon knowledge of the interaction of neutrinos with matter
via production, absorption, and scattering. A set of important processes now incorporated into modern supernova codes is given in Table 1. With the knowledge represented, theorists have created sophisticated computational capabilities that have enabled the exploration of the supernova mechanism and dynamics in its full multi-dimensional complexity. The neutrino-driven mechanism in its basic form, despite a great deal of change over the decades in our understanding of neutrinos and despite the necessary increase in theoretical sophistication, still holds pride of place — published exploding models in three spatial dimensions are becoming common\textsuperscript{7,3,7,6,7,7,7,5}. Hence, though there is still much to resolve, the centrality of the neutrino in this important astronomical context is assured. Given its weak coupling, modest beginnings in theory, and multi-decade history, one may view its emergence as a pivotal player in one of Nature’s most violent natural phenomena as somewhat of a surprise.

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Table 1: Neutrino-matter reactions of primary relevance in the core-collapse supernova context. BRT refers to Burrows, Reddy, & Thompson\textsuperscript{46}, and BT refers to Burrows & Thompson\textsuperscript{47}, which contains detailed discussions of the handling of inelasticity for both neutrino-electron and neutrino-nucleon scattering and of one approach to nucleon-nucleon bremsstrahlung. TBH refers to Thompson, Burrows, & Horvath\textsuperscript{48}, where a detailed derivation of the nucleon-nucleon bremsstrahlung rates can be found. BS98 refers to Burrows & Sawyer\textsuperscript{49}, which contains the non-relativistic dynamic structure factor formalism that informs an approach to neutrino-nucleon inelastic scattering. R99 refers to Reddy et al.\textsuperscript{50}, where the relativistic formalism for inelasticity in neutrino-electron scattering is provided. H02 refers to Horowitz\textsuperscript{51}, where corrections for weak magnetism are to be found.

\[
\begin{align*}
\nu_i + A & \rightleftarrows \nu_i + A \quad \text{BRT} \\
\nu_i + n, p & \rightleftarrows \nu_i + n, p \quad \text{BRT; BT, BS98} \\
\nu_e + n & \rightleftarrows e^- + p \quad \text{BRT; H02} \\
\bar{\nu}_e + p & \rightleftarrows e^+ + n \quad \text{BRT; H02} \\
\nu_e + A' & \rightleftarrows e^- + A \quad \text{Bruenn}\textsuperscript{43} \\
\nu_i + \bar{\nu}_i & \rightleftarrows e^- + e^+ \quad \text{BRT} \\
\nu_i + e^- & \rightleftarrows \nu_i + e^- \quad \text{BRT; BT, R99} \\
(n, p) + (n, p) & \rightleftarrows (n, p) + (n, p) + \nu_i \bar{\nu}_i \quad \text{BRT; BT, TBH}
\end{align*}
\]

Figure 1 – Above are the energy-dependent cross sections for the neutrino-matter interactions in water (left), scintillator (center), and liquid argon (right), taken from Seadrow et al.\textsuperscript{39}. The water cross sections are relevant to water-Cherenkov detectors such as Super-K and Hyper-K. The scintillator cross sections are relevant to such detectors as JUNO and the Argon cross sections are relevant to DUNE. These cross sections were provided by the SNOwGLoBES software.\textsuperscript{40}.