From $K^+$ in heavy-ion collisions to $K^-$ in kaonic atoms

G. E. Brown

Department of Physics, State University of New York, Stony Brook, NY 11794, USA

C. M. Ko and G. Q. Li

Cyclotron Institute and Physics Department,
Texas A&M University, College Station, Texas 77843, USA

Abstract

Kaon production and kaon flow in heavy-ion collisions at SIS energies are analyzed using the relativistic transport model. The experimental data are found to be consistent with the scenario that kaon scalar (before being cut down by the range term) and vector mean field potentials are about $1/3$ of that of nucleons in the Walecka model. This also explains the strong attraction found for $K^-$ in kaonic atoms, and gives information on the possibility of $K^-$ condensation in dense matter.
I. INTRODUCTION

The study of kaon properties in dense matter is one of the most interesting topics in nuclear physics. In ultrarelativistic heavy-ion collisions, enhanced production of kaon or other strange particles has been proposed as a possible signature for quark-gluon plasma formation in the collisions as the production rate in the quark-gluon plasma has been thought to be order of magnitude higher than in hadronic matter [1]. But the strangeness production rate in hadronic matter is usually estimated using free hadron properties. If the properties of the kaon or other strange particles are modified, then their production rate could be different. Indeed, it has been pointed out in Refs. [2,3] that if the kaon mass is reduced in medium, then its production rate will be significantly enhanced. Also, it has been suggested that the kaon yield from heavy-ion collisions at subthreshold energies is sensitive to the nuclear equation of state at high densities [4] as kaons can not be absorbed due to strangeness conservation. However, to extract information on the nuclear equation of state from the experimental data requires transport model calculations in which the kaon in-medium properties are properly treated. Furthermore, if $K^-$ feels a strong attractive potential as suggested by chiral perturbation theory and supported by the kaonic atom data, then it is possible that kaon condensation can occur in neutron stars [3,4], leading to the possible formation of mini-black holes in galaxies [5].

Recently, Brown and Rho [9] have shown via an effective chiral Lagrangian that, with dropping pion decay constant in medium, the attractive scalar potential (before being cut down by the energy-dependent range term) and the repulsive vector potential acting on a kaon are just $1/3$ of nucleon mean-field potentials in Walecka model. In this paper, we shall reexamine kaon production and flow in heavy-ion collisions at SIS energies using the relativistic transport model, which has been previously generalized to include the kaon mean-field potential, to see if the experimental data can still be consistently explained by the potentials suggested in Ref. [9].

The paper is organized as follows: In Section II, we shall review the kaon in-medium
properties following the ideas in Ref. [9]. Then, kaon production and flow in heavy-ion collisions at SIS energies are studied in Section III. In Section IV, the antikaon potential is discussed in relation to the kaonic atom data [10]. Finally, conclusions are given in Section V.

II. KAON IN NUCLEAR MEDIUM

Since the kaon-nucleon ($KN$) interaction is relatively weak when compared to other hadron-nucleon interactions, the impulse approximation should be reasonable for determining the kaon potential in nuclear matter at low densities. In this approximation, the kaon potential is directly related to the $KN$ scattering length in free space, i.e.,

$$U_{K^+} = -\frac{2\pi}{m_K}(1 + \frac{m_K}{m_N})\bar{a}_{KN}\rho,$$

(1)

where $m_K$ and $m_N$ are the kaon and nucleon masses, respectively, and $\bar{a}_{KN} \approx -0.255$ fm is the isospin-averaged $KN$ scattering length in free space [11]. At normal nuclear matter density $\rho_0 = 0.16$ fm$^{-3}$, the kaon potential is repulsive and has a magnitude of about 30 MeV.

Kaon properties in dense matter have been extensively studied in chiral perturbation theory [12–15]. The most straightforward interaction on a kaon in dense matter is the vector interaction, the so-called Weinberg-Tomozawa term, given in chiral Lagrangian by [9]

$$V_{K^+} = \frac{3}{8f_\pi^2}\rho,$$

(2)

where $f_\pi$ is the pion decay constant and $\rho$ is the nuclear density.

The Weinberg-Tomozawa term is of leading order in the chiral expansion. In the next order come the Kaplan-Nelson term [12] and the range term. The Kaplan-Nelson term can be expressed in terms of a scalar mean field acting on the kaon, either $K^+$ or $K^-$,

$$S_K = -\frac{\Sigma_{KN}}{2m_Kf_\pi^2}\rho_S.$$

(3)
In the above, $\rho_S$ is the nuclear scalar density and has a value of about $\rho_S(\rho_0) \approx 0.93\rho_0$ at normal nuclear matter density. The $KN$ sigma term is denoted by $\Sigma_{KN}$, and its value depends on the nucleon strangeness content and strange quark mass.

In studies of kaon production and flow in heavy-ion collisions at SIS energies, two of the present authors and collaborators have used Eqs. (2) and (3) for the kaon vector and scalar potentials in nuclear matter [16–19], i.e., the kaon mean-field potential is approximately given by

$$U_{K+} \approx \frac{3}{8f_\pi^2}\rho - \frac{\Sigma_{KN}}{2m_Kf_\pi}\rho_S.$$  \hspace{1cm} (4)

Using $\Sigma_{KN} = 350$ MeV and $f_\pi=93$ MeV, the kaon potential used in Refs. [16–19] at normal nuclear matter density $\rho_0$ is about 7 MeV. This is substantially smaller than that implied by the impulse approximation using the KN scattering length.

However, the pion decay constant changes in nuclear matter if we assume that the Gell-Mann–Oakes–Renner relation holds in medium, which is supported by model studies [20]. Then we have

$$\frac{f_{\pi}^*}{f_\pi^2} = \frac{m_\pi^2}{m_\pi^*^2} \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle},$$ \hspace{1cm} (5)

where quantities with "*" denote values in medium.

From the Feynman-Hellmann theorem, one has [21,22]

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \approx 1 - \frac{\Sigma_{\pi N}}{f_\pi^2 m_\pi^2} \rho,$$ \hspace{1cm} (6)

where $\Sigma_{\pi N} = 45$ MeV is the pion-nucleon sigma term. The pion mass is known to change only slightly as the density increases, so putting into Eq. (6) the empirical value determined from $\pi$-mesonic atoms [23], $m_\pi^*(\rho_0)/m_\pi \approx 1.05$, one obtains

$$\frac{f_{\pi}^2(\rho_0)}{f_\pi^2} \approx 0.6.$$ \hspace{1cm} (7)

Since the rho meson mass is connected with the pion decay constant $f_\pi$ through the KFSR relation
\[ m_{\rho}^2 = 2g_V^2f_{\pi}^2, \]  

(8)

where \( g_V \) is the universal vector meson coupling, a decreasing pion decay constant in medium implies that the in-medium rho meson is also reduced as \( g_V \) changes only at loop level. At mean-field level, Brown and Rho \([24]\) has thus introduced the following scaling relation,

\[ \frac{m_{\rho}^*}{m_{\rho}} \approx \frac{m_{\omega}^*}{m_{\omega}} \approx \frac{f_{\pi}^*}{f_{\pi}}, \]  

(9)

The dropping rho meson mass in dense matter is consistent with recent transport model analyses \([25,26]\) of CERES \([27]\) and HELIOS-3 \([28]\) dilepton data from CERN heavy-ion experiments, which show enhanced production of dileptons in the mass region between 300 and 500 MeV.

Including the scaling in \( f_{\pi} \), kaon vector and scalar potentials become

\[ V_{K^+} = \frac{3}{8f_{\pi}^2}m_{\rho}, \]  

(10)

\[ S_K = -\frac{\Sigma_{KN}}{2m_Kf_{\pi}^2}m_{S}. \]  

(11)

For the \( KN \) sigma term \( \Sigma_{KN} \), Brown and Rho have shown in the appendix of Ref. \([20]\) that it has a value

\[ \Sigma_{KN} \approx 450 \pm 30 \text{ MeV}, \]  

(12)

if the nucleon strangeness content is taken from recent lattice QCD calculations \([29,30]\). This value is somewhat larger than what has been used in Refs. \([16-19]\).

Using \( f_{\pi}^2/f_{\pi}^2 \approx 0.6 \) and \( \Sigma_{KN} \approx 450 \text{ MeV} \), then at normal nuclear matter density one has \( V_{K^+} \approx 88 \text{ MeV} \) and \( S_K \approx 100 \text{ MeV} \). In the Walecka model \([31]\), the nucleon mean-field potentials are \( V_N \approx 270 \text{ MeV} \) and \( V_S \approx 330 \text{ MeV} \), one therefore has the approximate relation \([9,20]\)

\[ V_{K^+} \approx \frac{1}{3}V_N, \quad \text{and} \quad S_K \approx \frac{1}{3}S_N. \]  

(13)
Thus, although the Kaplan-Nelson term seems to be connected through $\Sigma_{KN}$ with the explicit breaking of chiral symmetry, an alternative interpretation is that it gives a scalar field on the light quark in the kaon. We thus envisage that the constituent mass of up or down quark in the kaon drops substantially in medium.

Starting from chiral Lagrangian with universal vector coupling, Brown and Rho [9] have thus shown that the factor $(0.6)^{-1}$ in Eq. (7) just brings the scalar and vector mean fields in chiral Lagrangian to that of Walecka theory at nuclear matter density. In a sense, they have derived, working through the density dependence of $f_\pi^*$, the Walecka couplings at $\rho = \rho_0$, beginning from the zero-density couplings in the chiral Lagrangian.

We assume that, as a working hypothesis, the vector and scalar mean fields extrapolate linearly to densities higher than $\rho_0$, i.e., we use $f_\pi^*(\rho_0)$ for higher densities. This is similar to the Walecka model which is applied to higher densities with coupling constants determined at $\rho_0$. Of course, there should be some tendencies for the masses to continue to decrease, and the mean fields go as the inverse squared mass. However, this will be counteracted by higher order corrections, which tend to cut down the mean fields [33].

In the same order of chiral perturbation theory as the Kaplan-Nelson term is the ‘range term’. Much was made [32] of the fact that this range term cancels off all of the attraction corresponding to the Kaplan-Nelson-like term for the pion in pion-nucleon scattering. This range term will also be important for a kaon. Detailed calculations up through and including one loop order in chiral perturbation theory [15] show that the range term can be taken into account by multiplying the Kaplan-Nelson term by a factor $1 - 0.37\omega_K^2/m_K^2$, i.e.

\[
(S_K)_{total} \approx \frac{[1 - 0.37(\omega_K/m_K)^2]\Sigma_{KN}}{2m_Kf_\pi^{s2}}\rho_S \approx -\frac{\Sigma_{KN}}{2m_Kf_\pi^{s2}}\rho_S, \tag{14}
\]

The second expression arrives from the fact that the range term and the scaling in $f_\pi$ approximately cancel each other for the kaon.

The overall kaon potential including the scaling in $f_\pi$ and the range term is then

\[
U_{K^+} \approx \frac{3}{8f_\pi^{s2}}\rho - \frac{\Sigma_{KN}}{2m_Kf_\pi^{s2}}\rho_S. \tag{15}
\]
With again $\Sigma_{KN}=450$ MeV, the kaon potential at $\rho_0$ from Eq. (13) is about 28 MeV, which agrees very well with the empirical value of 30 MeV. Actually the two agree with each other up to $2\rho_0$ as shown in Fig. 1. We also show in Fig. 1 the kaon potential in the case of vanishing scalar attraction.

**III. KAON PRODUCTION AND FLOW AT SIS**

Experimental measurement of kaon production in Au+Au collisions at 1 GeV/nucleon has been carried out by the KaoS collaboration at SIS [34]. Both Fang et al. [16] and Maruyama et al. [35] have analyzed the KaoS data using relativistic transport models. In Ref. [16] a good fit to the data was obtained by using the kaon potential given by Eq. (4) without the range term and medium modification of $f_\pi$. As mentioned above, the kaon potential used in Ref. [16] is less repulsive than both the one from the impulse approximation and the one given by Eq. (15) as suggested by Brown and Rho based on effective chiral Lagrangian with scaling pion decay constant.

Maruyama et al. [35] have independently obtained a good fit to the kaon yield by putting the same Walecka-type scalar mean field on the lambda hyperon as on the nucleon and delta resonance but none on kaon. However, with our rule of nonstrange quark counting, 2/3 of the scalar field should be applied to the lambda hyperon and 1/3 to kaon as was done by Fang et al. [16]. The threshold is thus quite similar in both cases. According to Randrup and Ko [36], kaon production at a given input energy is chiefly determined by the maximum kaon momentum $p_{max}$, which in turn is determined by the threshold. It is therefore quite natural that both Fang et al. and Maruyama et al. could fit equally well the measured kaon yield.

In this work we recalculate the kaon spectra in Au+Au collisions at 1 GeV/nucleon with the kaon potential given by Eq. (15) and including also the contributions from pion-nucleon interactions. The isospin-averaged cross sections for $\pi N \rightarrow \Lambda K$ and $\pi N \rightarrow \Sigma K$ are taken from Ref. [37]. The results are shown in Fig. 2 where the solid and dashed curves
correspond to the results obtained with ($\Sigma_{KN} = 450$ MeV) and without ($\Sigma_{KN} = 0$ MeV) kaon scalar potential. Although the larger repulsive kaon potential used in the present calculation reduces kaon yield from baryon-baryon interactions compared to that of Ref. [16], it is, however, compensated by contributions from the pion-nucleon interactions. The present results with a more repulsive kaon potential than the one used in Ref. [16] thus still agree very well with the experimental data. On the other hand, if we neglect the kaon scalar potential, the theoretical results are reduced by about a factor of 2-5.

Another piece of useful information about $K^+$ properties in dense matter has recently been provided by the FOPI collaboration [38] from measuring the kaon flow in Ni+Ni collisions at 1.93 GeV/nucleon. Li et al. [18,19] have shown that the FOPI data can be well fitted using the vector and scalar mean-field potentials given by Eqs. (4) with $\Sigma_{KN} = 350$ MeV and $f_\pi = 93$ MeV.

We have redone the calculation using Eq. (15) with $\Sigma_{KN} = 450$ MeV and scaling pion decay constant. The results are shown in Fig. 3 together with the preliminary FOPI data [38]. Three scenarios for the kaon potential in nuclear matter have been considered, i.e., without potential (dotted curve), with vector potential only (dashed curve), and with both scalar and vector potentials (solid curve). It is seen that with both larger vector and scalar potentials than those in Refs. [18,19], the theoretical results are still consistent with the FOPI data. On the other hand, if the kaon scalar potential is neglected, then there appears a clear antiflow of kaons with respect to nucleons, which is seen to contradict the data. Since the magnitude of kaon vector potential seems secure, the lack of kaon antiflow in the experimental data thus indicates that there is an appreciable scalar attraction for kaon in order to cancel partly its strong repulsive vector potential.

IV. KAONIC ATOM

For $K^-$ in nuclear medium, because of G-parity, the vector potential is attractive,

$$V_{K^-} = -\frac{3}{8f_\pi^2}\rho.$$  \hspace{1cm} (16)
The $K^-$ scalar potential is also given by Eq. (14) but with a less important range term than for kaon. Taking, roughly, $\omega_{K^-} = 295$ MeV corresponding to the 200 MeV binding found by Friedman, Gal, and Batty at $\rho \approx \rho_0$ for a $K^-$ meson around a $^{56}\text{Ni}$ atom [10], we find $0.37(\omega_{K^-}/m_K)^2 = 0.13$, so the correction for the range term is only about 13%. Taken literally, this would mean that at $\rho \approx \rho_0$,

$$S_{K^-}(\rho_0) \approx -94 \text{ MeV} \frac{\rho_S}{\rho_0}. \quad (17)$$

Adding vector and scalar interactions gives

$$U_{K^-}(\rho_0) = S_{K^-}(\rho_0) + V_{K^-}(\rho_0) \approx -94 \text{ MeV} \frac{\rho_S}{\rho_0} - 88 \text{ MeV} \approx -175 \text{ MeV}. \quad (18)$$

The magnitude of attraction is thus consistent with the $-200 \pm 20$ MeV found by Friedman, Gal and Batty [10].

Consequently, although the attraction in the scalar mean field needed to produce the observed number of kaons and flow in heavy-ion collisions looks small (about $-65 \text{ MeV} \rho_S/\rho_0$), compared with that found in kaonic atoms (about $-94 \text{ MeV} \rho_S/\rho_0$), once the correction is made including the range term, the scalar attraction needed in the two cases is very similar and is consistent with the one given by Kaplan and Nelson but corrected by a scaling of the pion decay constant.

We note that once the $K^-$ mean-field potential is properly included, then the effect of $\Lambda(1405)$, which can be considered as a bound $\bar{K}N$ state, is suppressed in medium as a result of large energy denominator and Pauli blocking effects [39].

V. CONCLUSIONS

Our conclusion is that basically the nonstrange quark in the kaon feels about 1/3 of the scalar and vector mean field potentials that a nucleon does in the Walecka model. However, corrections for the energy-dependent range term must be made, which is significant for $K^+$, but relatively unimportant for $K^-$. For the vector mean field our argument that $V_K \approx \frac{1}{3}V_N$,
seems quite secure, depending only on the fact that the coupling goes as the nonstrange baryon numbers. For the scalar mean field, both the enhancement of subthreshold kaon yield and the lack of kaon antiflow in heavy-ion collisions at the SIS energies are found to be consistent with the scenario that it is somewhat larger in magnitude than the vector mean field before being cut down by the range term.

In this way one can also understand the very large attraction, \( V_{K^-} = -200 \pm 20 \text{ MeV} \), found by Friedman, Gal and Batty [10] for a \( K^- \) in \(^{56}\text{Ni}\). Furthermore, the sum of absolute values of scalar and vector mean fields, about 600 MeV, that enters into the spin-orbit term in nuclei in the Walecka theory, turns out to be about three times the \( K^- \) potential. We thus believe that our arguments and interpretation of the KaoS and FOPI data give important information for the attractive scalar interaction acting on a kaon in dense matter, which is needed in the calculation of kaon condensation in stars [13,20].

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Figure Captions

Fig. 1: Kaon potential as a function of density from Eq. (1) and Eq. (15) with \( \Sigma_{KN} = 450 \) and 0 MeV, respectively.

Fig. 2: Kaon momentum spectra from Au+Au collisions at 1 GeV/nucleon. The experimental data are from Ref. [34].

Fig. 3: Kaon transverse momentum as a function of center-of-mass rapidity for Ni+Ni collisions at 1.93 GeV/nucleon and \( b \leq 4 \text{ fm} \). The preliminary experimental data are from Ref. [38].
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