Remarks on cosmological issues in some string theoretic brane worlds

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Abstract
We examine, in the context of certain string compactifications resulting in five dimensional brane worlds the mechanisms of self/fine tuning of the cosmological constant and the recovery of standard cosmological evolution. We show that self tuning can occur only as long as supersymmetry is unbroken (unless additional assumptions are made) and that the adjustment of the cosmological constant to zero after supersymmetry breaking and the recovery of standard evolution are the same problem verifying previously made statements in the context of general i.e. not necessarily string theoretic brane worlds. We emphasize, however, that contrary to general brane worlds where the above adjustment requires a fine tuning, stringy brane worlds contain an additional integration constant due to the presence of the compact space thus allowing the adjustment to be done only with integration constants.
1 Introduction

Recent developments in superstring/M theory and supergravity have showed that it is possible to have vacuum states such that our world is located on a 3-brane embedded in a warped 5 dimensional space-time with some or all of the extra dimensions compact. One such case is the Horava-Witten theory \cite{1} compactified on a compact Calabi-Yau (CY) threefold \cite{2}. The compact space is characterized by a set of moduli \( b_i \) describing its shape (squashing modes) and a modulus \( \varphi \), specifying its size (the breathing mode). Integrating out the squashing modes can be done by solving the equations \cite{2} \cite{3} (for simplicity we assume that they are constant)

\[
\zeta_{\text{HW}} b_i = \frac{3}{2} n_i, \tag{1}
\]

where \( \zeta_{\text{HW}} = n_k b^k \) with \( n_k (k = 1, ..., h^{1,1}) \) integers. \( h^{1,1} \) is the number of non trivial 1,1 cycles of the CY. The low energy five dimensional effective action corresponds to an \( S^1/Z_2 \) orbifold, the gauge group carrying three branes being \( M5 \) branes located at the fixed points of the orbifold wrapped around the 1,1 cycles of the CY. Consistency of the compactification requires non-trivial four-form field strength flux which induces the presence of a potential in the five dimensional bulk action. There is \( \mathcal{N} = 2 \) supersymmetry in the bulk and half of it is broken on the branes. We will call this model the HW brane world model.

Another scenario is to start from IIB string theory, compactify it on a squashed five sphere \cite{4} \cite{5} (so that only \( \mathcal{N} = 2 \) supersymmetry is left unbroken) and subsequently compactify on an \( S^1/Z_2 \) orientifold \cite{5}. Tadpole cancellation will induce BPS \( D3 \)-branes which can be placed at the fixed points of the orbifold. The gauge group and the chirality originate from open string states on the \( D3 \)-branes. We can stabilize the squashing modes as before and call the breathing mode \( \varphi \). Consistency of the compactification requires non-trivial fluxes which again induce the presence of a potential in the five dimensional bulk action. In the following we ignore the squashing of the sphere to avoid unnecessary complications (however, see Appendix). The potential in that case depends on two parameters. One is \( \zeta'_{I_5} \), the analogue of \( \zeta_{\text{HW}} \) in the HW model and can be computed in terms of the string scale \( M_s \), the radius of the sphere \( L \) and an integer \( n \) as \cite{5}:

\[
\zeta'_{I_5} = \frac{n}{M_s^4 4\pi L^5}. \tag{2}
\]

It is clearly quantized just as \( \zeta_{\text{HW}} \) is. The other parameter is new, not present in a Ricci flat compactification such as that of the HW theory, and it is basically the Ricci scalar \( R_5 \) of the five sphere. It is not quantized: it can be regarded from the five dimensional point of view as a continuous integration constant. We will call this model the \( I_5' \) brane world model. Notice that what we have done here is first compactify on the sphere and subsequently compactify the resulting five dimensional theory on an interval accompanied by an orientifold projection. An alternative route would be to first orientifold the type IIB theory, which would take us to type I string theory, then T-dualize to get the type I' theory and finally compactify (and wrap the branes and orientifold planes) on \( CP^2 \times S^1 \). This would apparently be precisely the compactification proposed in \cite{6}, provided that the above steps commute. For some more details on this see the Appendix.

There has been a lot of discussion on such Randall and Sundrum 1 (RS1) \cite{6} type models. \cite{7} Discussions include the possibility (or not) of having a zero or nearly zero cosmological constant without fine tuning \cite{8}, \cite{9}, \cite{10}, \cite{11} (self tuning of the cosmological constant) and the possibility of recovering a standard cosmological evolution \cite{12}, \cite{13}, \cite{14} even though these phenomena do not occur automatically in general.

\[1\] There is nothing sacred about the squashed five sphere. One could presumably replace it with some other Ricci curved space which yields the appropriate number of supersymmetries and chirality on the branes and thus come up with other examples that have similar properties.

\[2\] In “RS1 type” in this talk we include models with scalar fields in the bulk.
RS1 brane worlds: cancellation between the bulk and the brane cosmological constants and therefore standard cosmological evolution is possible with at least one fine tuning.

In this letter we discuss these issues in the context of the HW and $I'_5$ string models and we hope to clarify first somewhat under what circumstances the various mechanisms can take place and second, we argue that string theory provides us with one additional integration constant compared to non-string theoretic brane worlds due to the presence of the compact space, so that the fine tuning can possibly be avoided.

2 Cosmological issues

The relevant piece of the five dimensional effective action for these compactifications is

$$S_{\text{bulk}} = -\frac{1}{2\kappa_5^2} \int d^5x \sqrt{-G} \left[ R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right],$$

(3)

where $\kappa_5^2 = \frac{1}{M_5}$ with $M_5$ the five dimensional Planck mass, which is related to the ten or eleven dimensional fundamental scale, once the details of the compactification are specified. The potential is

$$V(\varphi) = \frac{1}{6} \zeta_{\text{HW}}^2 e^{2\varphi}$$

(4)

for the compactified Horava-Witten (HW) theory and

$$V(\varphi) = -R_5 e^{\frac{4}{5} \sqrt{3} \varphi} + 8\zeta_{I'_5}^2 e^{2\sqrt{5} \varphi}$$

(5)

for the compactified $I'_5$ theory. The five dimensional space time coordinates are labeled by $\{x^\mu, r\}$ and the corresponding metric is $G_{MN}$, with $M, N = 0, 1, 2, 3, 5$. The action for the branes sitting at $r = 0$ and $r = \pi R$, in the static gauge, is

$$S_{\text{branes}} = \frac{1}{\kappa_5^2} \int d^4x \sqrt{-g} \left[ T_1(\varphi) \delta(r - 0) + T_2(\varphi) \delta(r - \pi R) \right],$$

(6)

with $T_i$ the brane tensions and $g_{\mu\nu}$ the four dimensional induced metric. The above brane action corresponds to an energy momentum tensor (on the brane at $r = 0$ for example, which we will take to be our “Standard Model” (SM) brane) $(T_1, T_1, T_1, T_1, 0)$, to which we will eventually add some additional matter in the form $(\rho, p, p, p, 0)$ for a total energy momentum tensor on the SM brane

$$T^M_N = (-T_1 - \rho, T_1 + p, T_1 + p, T_1 + p, 0).$$

(7)

The matter density $\rho$ and pressure $p$ are assumed to depend only on time and also to be $\rho, p << T_1$. The functional form of the tensions before supersymmetry breaking can be computed directly in the Horava-Witten case to be

$$T_1 = -T_2 = \zeta_{\text{HW}} e^{\varphi} \equiv T(\varphi)$$

(8)

and in the $I'_5$ case it can be fixed by supersymmetry to

$$T_1 = -T_2 = -\sqrt{5} R_5 e^{\frac{4}{5} \sqrt{3} \varphi} + 4\zeta_{I'_5} e^{\sqrt{5} \varphi} \equiv T(\varphi).$$

(9)

*Strictly speaking, this potential corresponds to compactification on the round sphere, so it leaves $N = 8$ supersymmetry in the bulk and $N = 4$ on the branes. By squashing the sphere, we could break half of each of the above supersymmetries, in which case we would have more terms in the potential. This simpler form, however, will be sufficient to illustrate our point.*
We should note here that up today there does not exist a string theoretic calculation that justifies the expression (9) for the five dimensional effective brane tension. In fact, by naively pulling back the ten dimensional $D3$ brane tension to five dimensions in the given background, one finds precisely (but only) the second term in the expression for the five dimensional tension $\delta$. This second term is the one that is analogous to the (quantized) brane tension term in the HW model, proportional to $\zeta_{\text{HW}}$. The problem here is that the additional term in the brane tension proportional to the Ricci scalar can not be reproduced by straightforward reduction arguments. The answer might have to do with effects not completely understood in orientifolding in a non Ricci flat background. Nevertheless, if the compactification is indeed supersymmetric as we claim, the effective brane tension has to be as in (9). We will assume that this is the case and we leave the proof for a later work.

After supersymmetry breaking, the tension gets renormalized and we parameterize the effect of supersymmetry breaking on the $i$'th brane by $T = T(\varphi) + \epsilon_i t_i(\varphi)$ where $\epsilon_i$ is the scale of supersymmetry breaking and therefore $\epsilon_i << T$ but also $\rho, p << \epsilon_i$. In the following, we will assume that the breathing mode $\varphi$ depends only on the coordinate $r$. Also, we will denote a time differentiation by a dot, an $r$ differentiation by a prime and a differentiation with respect to $\varphi$ by a hat. The equations of motion corresponding to the actions (3) and (6) are

$$E_{\mu\nu} + \frac{1}{2} G_{\mu\nu} \left[ \frac{1}{2} \varphi^2 + V(\varphi) \right] - \frac{1}{2} \delta_{MN} \delta_{NR} \varphi^2 + g_{\mu\nu} T(\varphi) [\delta_0 - \delta_R] = 0$$  \hspace{1cm} (10)

and

$$\varphi'' + \left( \frac{\sqrt{-G}}{-G} \right)'' \varphi' = \hat{V} + 2 \hat{T} [\delta_0 - \delta_R],$$ \hspace{1cm} (11)

where $E_{\mu\nu}$ are the components of the Einstein tensor and we have defined for simplicity $\delta(r - 0) \equiv \delta_0$ and $\delta(r - \pi R) \equiv \delta_R$.

First, we make the metric ansatz

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2,$$ \hspace{1cm} (12)

which corresponds to a warped five dimensional but flat and static four dimensional world and we set $\rho = p = 0$. The Einstein equations naturally split into the part along the $\mu\nu$ directions and the part along the $r$ direction:

$$E_{\mu\nu} + \frac{1}{2} e^{2A(r)} \eta_{\mu\nu} \left[ \frac{1}{2} \varphi^2 + V + 2T(\delta_0 - \delta_R) \right] = 0,$$ \hspace{1cm} (13)

$$E_{55} - \frac{1}{2} e^{2A(r)} (\varphi')^2 - V = 0.$$ \hspace{1cm} (14)

The non-vanishing Einstein tensor components are

$$E_{\mu\nu} = e^{2A} \eta_{\mu\nu} (6A'^2 + 3A'') \quad \text{ and } \quad E_{55} = 6A'^2.$$ \hspace{1cm} (15)

The equations of motion, in the bulk, after some algebra can be brought in the form

$$A'' = -\frac{1}{6} \varphi'^2, \quad A'^2 = -\frac{1}{12} V(\varphi) + \frac{1}{24} \varphi'^2, \quad \varphi'' + 4A' \varphi' = \hat{V}.$$ \hspace{1cm} (16)

It was suggested in [14] that supersymmetry preserving (BPS) solutions to the above second order set of equations is the (equivalent) first order set

$$\varphi' = \hat{W}, \quad A' = -\frac{1}{6} W, \quad V(\varphi) = \frac{1}{2} \hat{W}^2 - \frac{1}{3} W^2,$$ \hspace{1cm} (17)

\footnote{It might be that a compactification of the HW theory on a non Ricci flat manifold instead of a CY manifold does not have this problem. The challenge is to find manifolds that preserve $\mathcal{N} = 2$ supersymmetry in the 5D bulk. Perhaps some product space of spheres does this.}
where $W$ is a “superpotential” to be computed by solving the last of the above differential equations. We separate these solutions in two classes. One class will be the special solution that does not contain an integration constant and we will denote it by $w$. The other will be the general solution with the integration constant, which we will call $\hat{w}$. For the HW and the $I_5$ models, we have respectively,

$$w = \zeta_{HW} e^\varphi \quad \text{and} \quad w = -\sqrt{5R_5} e^\frac{2}{5} \sqrt{\pi} \varphi + 4\zeta_{I_5} e^{\sqrt{\frac{2}{5}} \varphi}. \quad (18)$$

Expressions for $\hat{w}$ for these two models can be found in [5] and [15]. The presence of the branes can be taken into account by integrating over $r$ equations (13) and the last of equations (16). Then, one obtains for the SM brane the junction conditions

$$\varphi_0' = \hat{T}_0, \quad A_0' = -\frac{1}{6} T_0, \quad (19)$$

where the subscript 0 reminds us that these conditions have to be obeyed, in principle, only at $r = 0$ (and at $r = \pi R$, since a similar set of junction conditions holds for the other brane as well). Now, notice the similarity between equations (19) and the first two equations in (14). Clearly, if $T_0 = W_0$, the junction conditions are automatically satisfied. Since, however, these conditions have to be satisfied on both branes simultaneously, a particularly convenient way that this can happen is if the relation $T = W$ holds everywhere between and including $r = 0$ and $r = \pi R$. In fact, for the two string theory effective actions we are looking at, this is exactly the case. For instance, in the HW model we have at the supersymmetric point $T = w = \zeta_{HW} e^\varphi$ and therefore the junction conditions (which are just the consistency conditions for a vanishing cosmological constant on our brane), are satisfied identically without the use of any integration constants [2] [15]. For the $I_5$ model as well, it has been shown [16] [17] that supersymmetry requires the relation $T = w$ for all $r$. This mechanism is sometimes called self tuning of the cosmological constant.

In these string theory compactifications, the self tuning mechanism seems to be a direct consequence of supersymmetry and therefore not very surprising. Unfortunately, when supersymmetry is broken, the tension $T$ gets renormalized and the junction conditions, which still need to be obeyed, can only be satisfied for some superpotential different than $w$, namely $\hat{w}$, by the choice of a set of four integration constants, so that $T_0 = \hat{w}_0$. Two independent integration constants come from the solution of (17), one is the size of the orbifold $R$, and one is hidden in $\zeta_{HW}$ ($R_5$) in the HW ($I_5$) case. For now, we will assume that this is always possible for both the HW and the $I_5$ models.

Next, we make the slightly more interesting metric assumption

$$ds^2 = e^{2A(r)}[-dt^2 + e^{\sigma(t)} dx^2] + dr^2, \quad (20)$$

which corresponds to a flat three dimensional space whose volume is changing in time. This is not necessarily the most general assumption (it is too simple; for more realistic metric assumptions see for example [13] and references therein) but it is good enough to demonstrate what we think is the essential point here. We will also allow for supersymmetry breaking, which will be taken into account by $T \rightarrow \hat{T}$. The equations of motion corresponding to the $E_00$, $E_{ij}$, $E_{55}$ components of the Einstein tensor, now become

$$3\dot{a}^2 - e^{2A}(6A'' + 3A'' + \frac{1}{4} \varphi'^2 + \frac{1}{2} V) = e^{2A}(\hat{T} - \rho)(\delta_0 - \delta_R), \quad (21)$$

$$3\dot{a}^2 + 2\dot{a}^2 - e^{2A}(6A'' + 3A'' + \frac{1}{4} \varphi'^2 + \frac{1}{2} V) = e^{2A}(\hat{T} + \rho)(\delta_0 - \delta_R), \quad (22)$$

$$3(2\dot{a}^2 + \ddot{a}) = e^{2A}(6A'' - \frac{1}{4} \varphi'^2 + \frac{1}{2} V), \quad (23)$$

respectively, and the $\varphi$ equation of motion is still

$$\varphi'' + 4A' \varphi' = \dot{\hat{V}} + 2 \dot{\hat{T}}[\delta_0 - \delta_R], \quad (24)$$

where $\hat{V}$ is always possible for both the HW and the $I_5$ models, we have respectively,
Equations (21) and (22) in the bulk immediately imply that

\[ a(t) = c_1 t + c_2 \]  

(25)

and therefore that they are equivalent in the bulk. Explicit bulk solutions for \( A \) and \( \varphi \) can be found in [13] so we will not present any here. Instead, we turn to the physics of the SM brane. Integrating (23), (22) and (24) we obtain the junction conditions

\[ A'_0 = -\frac{1}{6}(\mathcal{T}_0 - \rho), \quad A'_0 = -\frac{1}{6}(\mathcal{T}_0 + \rho), \quad \varphi'_0 = \hat{T}_0. \]  

(26)

The \( \varphi \) junction condition is unchanged since we assumed that \( \rho = \rho(t) \). The first two junction conditions are consistent only if \( p = -\rho \), which is the equation of state for vacuum. Consequently, the equation for the conservation of energy

\[ \dot{\rho} = -3H(\rho + p) \]  

(27)

implies that \( \rho \) is actually constant in time, i.e. matter does not flow out of the brane. Let us now substitute (26) into (23):

\[ \frac{a_0^2}{e^{2A_0}} = \left( \frac{1}{36} \mathcal{T}_0^2 - \frac{1}{24} \mathcal{T}_0^2 + \frac{1}{12} V \right) - \frac{1}{18} \rho \mathcal{T}_0, \]  

(28)

where we have dropped the term \( \mathcal{O}(\rho^2) \). Apparently, standard cosmological evolution is spoiled by the quadratic terms inside of the parenthesis. If the compactification breaks all the supersymmetry, then the brane tension does not take the form of its BPS form plus an order \( \epsilon \) correction and recovery of standard cosmological evolution can be achieved by adjusting integration constants to cancel those terms. This step is basically the RS1 mechanism of canceling the brane against the bulk cosmological constant to 120 orders of magnitude. In our case, however, the compactification preserves some supersymmetry. Supersymmetry breaks at a lower scale, so we can write \( \mathcal{T}_0 = T_0 + \epsilon t_i \). Then, by choosing an integration constant so that \( e^{2A_0} = 1 \) and expanding \( \mathcal{T}_0 \), we obtain, suppressing the brane indices on \( \epsilon \) and \( t \)

\[ H_0^2 = \frac{1}{12} \left( \frac{1}{3} T_0^2 - \frac{1}{2} \hat{T}_0^2 + V \right) + \frac{\epsilon}{12} \left( \frac{2}{3} T_0 t \hat{T}_0 \hat{t} \right) - \rho \left( \frac{1}{18} T_0 \right), \]  

(29)

where we have defined the present Hubble parameter \( a_0 \equiv H_0 \), which by (23) is equal to the constant \( c_1 \) (i.e. no inflation) and dropped a term of \( \mathcal{O}(\epsilon \rho) \). The three groups of terms in the right hand side of (29), (all the terms inside of the parentheses being \( \mathcal{O}(1) \)) are in decreasing order of magnitude, since \( \rho << \epsilon << 1 \). On the other hand, the left hand side \( H_0^2 \) is basically the observed cosmological constant, which is at most of the order of magnitude of the last term in (29). Therefore, agreement with the small observed value of the cosmological constant suggests that the terms of \( \mathcal{O}(1) \) and of \( \mathcal{O}(\epsilon) \) may cancel separately. The cancellation of the \( \mathcal{O}(1) \) terms is indeed not a problem as we saw in the previous section; \( T_0 = w \) in both the HW and the \( I'_5 \) cases and the terms cancel by the original supersymmetry. For the terms of \( \mathcal{O}(\epsilon) \) to vanish, it must be that \( 2\hat{T}_0 t \hat{T}_0 \hat{t} \). This relation, just as before, can be satisfied generically only via the adjustment of the available integration constants, i.e. we can not escape from the cosmological constant problem with supersymmetry, we can only improve it by some fifty orders of magnitude. Again, this is not a surprise. Once these terms cancel, we have standard cosmological evolution with \( H_0^2 \sim \rho \) [11], [12], [13]. Here is in fact where apparently the \( I'_5 \) model seems to have an advantage over the HW model. Recall that in the \( I'_5 \) model we have four continuous integration constants [5], whereas in the HW model we have three continuous and one set of quantized constants (hidden in \( \zeta_{HW} \)) [13]. It seems very hard to satisfy the cancellation of the \( \mathcal{O}(\epsilon) \) terms with a set of discrete parameters.

We can go one step further if in addition we make the stronger assumption that the actual functional form of \( t \) is implied by the vanishing of the \( \mathcal{O}(\epsilon) \) term. This amounts to assuming by analogy to the
supersymmetric $T = w$ relation that $T = Tw$ holds identically after supersymmetry breaking. Substituting in (27) the expressions for $T_0 (= T)$ in the HW and $I'_5$ models and solving for $t$, we obtain a constraint on the functional form of the supersymmetry breaking correction to the brane tension:

\[ t_{HW}(\varphi) \sim e^{4\varphi/3} \quad \text{and} \quad t_{I'_5}(\varphi) \sim e^{5\sqrt{3}\varphi} \left( \frac{4}{\sqrt{3}} e^{\frac{2\sqrt{3}}{3}\sqrt{3}\varphi} + 8\sqrt{\frac{5}{3}} e^{2\sqrt{3}\varphi} \right). \quad (30) \]

It would be interesting to check this last assumption using some popular supersymmetry breaking schemes, since it would be a realization of a self tuning mechanism even after supersymmetry breaking.

Concluding, in this paper we emphasized that in our point of view, the reason that it is possible to have a vanishing cosmological constant and a standard cosmological evolution in RS1 type string theoretic brane worlds such as the HW and $I'_5$ models, is that the four available integration constants are just sufficient to satisfy the four junction conditions. The fourth integration constant ($\zeta_{HW}$ or $R_5$) that is usually not present in non-string theoretic brane worlds, is associated with the existence of the compact space.

3 Appendix

In the Appendix we will give some details of a possible string theory realization of the $I'_5$ model. We emphasize that even though the supergravity realization of this model is rather clear, several stages of its string theory realization are still at this point at a conjectural level and that is why we put it in an Appendix. The picture we have in mind is the following:

In the above, we have of course assumed that the diagram is commuting. Without the intermediate orientifold step $\Omega$ we know that it is [18]. With the orientifold step we do not know it but we assume that it is still a commuting diagram. The $p3$ branes in the picture at the bottom are the $D8/O8$ branes of the original $I'$ theory wrapped around $CP^2 \times S^1$. By squashing the $S^5$ on the left, or the $CP^2$ on the right, we

\[ \text{Even if this is the case though, one has to worry about later phase transitions that might generate additional contributions to the cosmological constant.} \]
expect that the maximal supersymmetry will break to $\mathcal{N} = 2$ in the bulk and to $\mathcal{N} = 1$ on the $p3$ branes in the $I'_5$ theory.

The other important issue is whether the $p3$ branes can support chiral fermions on their world volume. This is necessary for the $I'_5$ model to be potentially fully realistic, i.e. to be able to reproduce the chiral representations of the quarks and the leptons of the Standard Model. Let us start again from type $I'$ string theory and compactify two directions $-a$ and $b$ along the branes on a torus, say $T^2 = S^1_a \times S^1_b$.

Next, T-dualize along both directions, so that the $D8/O8$ branes become $D6/O6$ branes. Then compactify two more directions along the branes on an $S^2$ and one more direction along the branes, say direction $c$, on the circle $S^1_c$. Now, the directions $b$ and $c$ form a torus (with one direction along -direction $c$- and the other direction transverse -direction $b$- to the branes), and the branes can wrap this torus in such a way so that they intersect at non-zero angles. To summarize, what we have at this point is stacks of $D6/O6$ branes wrapped around $S^2$, intersecting at angles on a torus, with a transverse $S^1_b \times S^1_c/Z_2$, where the orbifold $S^1_d/Z_2$ is the orbifold of the original $I'$ theory. The question here is if this system can support chiral fermions. Even if it does not, we expect that some simple modification of the model will. The reason for our expectation is the fact that the $I'$ theory is related to the Horava-Witten theory by a simple circle compactification along the $E_8$ walls, and we know that the HW theory supports chiral fermions on the walls even after its compactification on a smooth manifold (eg a Calabi-Yau). Therefore, we expect that some smooth manifold compactification of type $I'$ must exist that supports chiral fermions on the branes.

To make the connection with the $I'_5$ model, now T-dualize back along directions $a$ and $b$. What we obtain is $D8/O8$ branes wrapped around $S^2 \times T^2 \times S^1_a$, with flux through the torus. Furthermore, the $S^2 \times T^2$ part need not be globally a trivial product. It can be a non-trivial bundle for example. If $CP^2$ can be constructed as a torus bundle over a 2-sphere (this is not clear at all), then we get precisely to the $I'_5$ model and if not, to some cousin of it. Notice finally that in the limit that the flux goes to zero, which in the T-dual language means that the branes become parallel, maximal supersymmetry is restored. In other words, tilting the branes in angles must correspond to the 'squashing' of the $CP^2$ or of its cousin.

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