3D hybrid model of foundation-soil-foundation dynamic interaction

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3D dynamic interaction of two adjacent elastic foundations embedded in a finite layered soil region rested in a homogeneous elastic isotropic half-space with a transient dynamic source is studied. The hybrid computational model, the corresponding numerical scheme and the accompanied software are developed, verified and inserted in detail parametric study. The hybrid approach is based on (a) finite element method (FEM) describing the scattered wave field in a finite layered soil profile with two foundations; (b) boundary element method (BEM) considering waves radiating from a dynamic transient source in elastic semi-infinite range. The aim is to propose an efficient hybrid methodology for evaluation of the dynamic response of a foundation-soil-foundation system, taking into account (a) the whole wave path from the dynamic source, through the layered soil region, till the underground structures; (b) the damage state of the geological material. The BEM model is based on the 3D elastodynamic fundamental solution in Fourier domain and it is applied in order to obtain frequency-dependent stiffness matrix and load vector of the dynamically active semi-infinite geological zone. Once the BEM model is formulated, it is inserted as a macro-finite element in the FEM software package ABAQUS. The frequency-dependent FEM model describing the wave field in finite layered soil profile with two elastic foundations is realized by ABAQUS. Solutions in time domain are obtained through application of the inverse fast Fourier transform. As a final result an efficient hybrid model comprising all in one: dynamic (seismic or other type) source-homogeneous elastic half-space-finite heterogeneous by layers and foundations soil profile, is presented. Parametric study illustrating the sensitivity of the dynamic field to key factors such as the type and properties of the load, the soil layering, the material properties of the far- and near-field soil regions, the damaged state of the geological material and the foundation-soil-foundation interaction is shown and discussed.

KEYWORDS
3D elastodynamics, foundation-soil-foundation interaction, hybrid BEM-FEM model, macro-element concept, soil damage state, synthetic signals

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1 | INTRODUCTION

The soil-structure interaction (SSI) problem is a collection of phenomena in the response of structures caused by the soil flexibility, as well as in the response of soils caused by the presence of foundation-structure system. Dynamic structural response is affected by the interaction between structure, foundation and the supporting soil region, which is known as soil-foundation-structure interaction phenomenon. The effect of adjacent foundations/buildings on the response of a structure during an earthquake or other type of dynamic excitation is known as structure-soil-structure interaction and can be considered as a branch of SSI problem. The dynamic response of closely spaced structures is affected by the interaction between the soil and each one of the structures. This type of interaction occurs in adjacent buildings in a large city, in adjacent structures of a nuclear power plant complex, or in a system of close-vicinity machine foundations. The dynamic foundation-soil-foundation interaction phenomenon has long been recognized as an important factor in the seismic and machine-vibrational response of critical facilities and other closely spaced structures or portions of structures. European standards for earthquake resistant buildings ignore more complex effects of the soil-structure-interaction due to the lack of an efficient computational tools. It has been recognized that SSI effects may have a significant impact, especially in cases involving heavier structures, adjacent structures and soft soil conditions. A brief historical overview of the used mechanical models treating the SSI problem starts with the empirical Winkler type models in [77] replacing the supporting soil by a bed of elastic springs and dashpots resting on a rigid base. Later, Lamb’s solution in 1904 formed the basis for the study of oscillation of footings resting on a surface of half-space [11, 56, 57, 64]. With these publications started the modern study of the dynamic SSI, see state of the art in [19, 33, 47, 73, 79]. An almost complete review of the research pertinent to the dynamic analysis of rigid and flexible foundations on an elastic homogeneous soil can be found in [34] and [60]. The results for shallow foundations are discussed in [78], while for the embedded ones in [4], [66] and [67]. SSI problem in layered half-space is discussed in [7, 18, 30, 61, 62, 65]. Special attention in regard of 3D hybrid computational schemes based on the FEM and BEM is presented in [8, 25, 27, 45, 51, 58, 74, 75]. Another field of increasingly interest among SSI engineering society concerns offshore platforms, deep foundations (including pile foundation), see [12, 13, 15, 17, 31, 32, 44, 51, 59, 69].

There exist the following main methodologies for solution of SSI problems in time and in frequency domains: direct approach, substructure or multi-step methodology, hybrid approach, and hybrid approach based on the macro-element concept. Direct method is one in which the soil and structure are modeled together in a single step, which allows the possibility to treat both linear as well as physical and geometrical nonlinear problems. The main disadvantage is that this approach is expensive in the case of large nonlinear models or in 3D layered soils. The application of the direct approach can be seen in [38, 71] who used Finite Difference Method; [2, 20, 41, 43, 50, 80] applying Finite Element Method; [3, 22, 35, 60, 68, 82] working with Boundary Element Method; [81] proposing the Scaled Boundary Finite Element Method. Sub-structure method [85] is one in which the analysis is broken down into several steps that the principal of superposition is used to isolate the two primary causes of soil-structure interaction: (a) inability of foundation to match the free field deformation and (b) the effect of dynamic response of structure foundation system on the movement of supporting soil. Detailed reviews of the subject can be found in [29, 48, 52]. Hybrid approach is based on the fact that the SSI problem involves sub-regions with different characteristics, and it seems to be a natural approach to combine different computational tools within the sub-domains, thus making use of their respective advantages.

Many numerical tools exist for the analysis of each sub-domain, depending on the complexity of the model, and ranging from simple equivalent mass-spring-dashpot systems, for example, in [6, 72], to more complex finite element or boundary element models in [36, 53, 54]. In the current analysis we are interested only in a coupling based on the finite and boundary element methods. Existing coupling approaches can be classified roughly into two main groups: (a) FEM hosted, where the nodal forces are expressed through the boundary element domain nodal tractions and the entire BEM formulation is converted to FEM-like approach, see [28, 76]; (b) BEM hosted, where the FEM formulation is transformed to BEM matrix system of equations, see [9]. A recent review of existing macro-element models for shallow foundations is illustrated in [16, 23]. In [70], a hybrid model is presented for describing 2D seismic response of a soil-underground structure system. The numerical technique is based on the hybrid BEM-FEM via the sub-structure approach with insertion of the BEM model of the seismically active unbounded geological media as a macro-element in the FEM commercial program ANSYS. The analogous problem is considered in [5] taking into account the poroelastic soil properties and the hybrid FEM hosted model is implemented in the ABAQUS software.

The above state of the art shows that there is a lack of results for 3D dynamic behavior of foundation-soil-foundation system accounting for the interaction between foundations and between them and soil layering, taking into account all components along the wave path. These components are dynamic source with seismic or other type of characteristics,
laterally inhomogeneous and heterogeneous wave path, local site with its specific geometrical and mechanical properties. This motivate the authors to formulate here the following main aim: to develop, verify and insert in parametric study an efficient 3D hybrid model of the FEM hosted type based on the macro-element concept. The created computational tool in frequency and time domain is based on the BEM model for the far-field unbounded zone which is converted to a macro-finite element and FEM for the finite layered substructure with two foundations. Additionally, the damage state of the soil and its influence on the foundation-soil-foundation interaction is taken into consideration.

The paper is organized as follows: The statement of the mechanical problem is defined in Section 2. The boundary value problem is formulated in Subsection 2.1. The model describing the damage state of the geological material is discuss in Subsection 2.2. The hybrid FEM-BEM formulation in frequency domain based on the macro-element concept is presented in Section 3. The verification study of the hybrid numerical scheme is discussed in Section 4. A parametric study is shown in section 5, followed by conclusions in Section 6.

2 | PROBLEM STATEMENT

In coordinate system $Ox_1x_2x_3$ consider 3D finite geological region $\Omega_1$ which is embedded in a semi-infinite elastic isotropic media $\Omega_0$ with transient dynamic load comprising either (a) an incident plane wave or (b) waves generated by an embedded source (caused by seismic or other type of dynamic events) at point $X_0(X_{01},X_{02},X_{03})$, see Figure 1. The interface boundary between both regions $\Omega_1$ and $\Omega_0$ is with notation $\Gamma_{int}$, while the surface $\Gamma_f$ and $\Gamma_a$ are the free-surface of ranges $\Omega_1$ and $\Omega_0$, respectively. The boundary of the finite region $\Omega_1$ is $\Gamma_{\Omega_1} = \Gamma_{int} \cup \Gamma_f$, whereas the boundary of the external semi-infinite zone $\Omega_0$ is $\Gamma_{\Omega_0} = \Gamma_{int} \cup \Gamma_a$. The normal vectors to boundary $\Gamma_{int}$ for each aforementioned domain is in opposite direction.

The concrete geometry of the finite region $\Omega_1$ is given in Figure 2a. The finite region $\Omega_1$ consists of two layers: (a) layer 1 in the domain $\Omega_1(L1)$ with boundaries of $\Gamma_f$, $\Gamma_{int}(L1)$, $\Gamma_{S12}$; (b) layer 2 in the domain $\Omega_1(L2)$ with boundaries of $\Gamma_{int}(L2)$, $\Gamma_{S12}$. Note that the normal vectors to the interface boundary between both layers $\Gamma_{S12}$ are equal in magnitude but opposite in sign. There are two identical rectangular foundations in the first layer with length $b$, width $c$ and height $h$. The distance between both foundations is $d$. The top sides of the left and right foundation are free-surfaces and are denoted as $\Gamma_{f1}$ and $\Gamma_{f2}$, while the embedded surfaces are collected in $\Gamma_{g1}$ and $\Gamma_{g2}$, see Figure 2b. Material properties of the geological semi-infinite zone $\Omega_0$ are density $\rho_0$, Lamé constants $\lambda_0, \mu_0$, longitudinal wave velocity $C_{P0} = \sqrt{(\lambda_0 + 2\mu_0)/\rho_0}$, and shear wave velocity $C_{SV0} = \sqrt{\mu_0/\rho_0}$, while the material properties of the both layers in the finite range $\Omega_1$ are as follows: $\lambda_1, \mu_1, C_{P1}, C_{SV1}$ for the layer 1 and $\lambda_2, \mu_2, C_{P2}, C_{SV2}$ for the layer 2. The material properties of the foundations are denoted by $\lambda_f, \mu_f, C_{Pf},$ and $C_{SVf}$.

Comparing Figures 1, 2a and 2b, it can be seen that the free-surface of the finite region $\Omega_1$ denoted by $\Gamma_f$ in Figure 1 corresponds to $\Gamma_f = \Gamma_{f0} \cup \Gamma_{f1} \cup \Gamma_{f2}$ in Figure 2a, whereas $\Gamma_{f0}$ is the free-surface outside the top sides of both foundations. The interface contact boundary $\Gamma_{int}$ between the finite zone $\Omega_1$ and semi-infinite one $\Omega_0$ shown in Figure 1 is the surface $\Gamma_{int}(L1) \cup \Gamma_{int}(L2)$, see Figure 2a.
In resume the boundaries of both zones are as follows:

- In the finite near-field geological zone \( \Omega_1 \):
  - The external boundary is \( \Gamma_{\Omega_1} = \Gamma_{\text{int}} \cup \Gamma_f = \Gamma_{\text{int}} \cup \Gamma_{f0} \cup \Gamma_{f1} \cup \Gamma_{f2} \), where \( \Gamma_{\text{int}} = \Gamma_{\text{int}(L1)} \cup \Gamma_{\text{int}(L2)} \).
  - The internal boundary is \( \Gamma_{S12} \cup \Gamma_{g1} \cup \Gamma_{g2} \).

- In the semi-infinite far-field geological zone \( \Omega_0 \), the boundary is \( \Gamma_{\Omega_0} = \Gamma_u \cup \Gamma_{\text{int}} \).

The aim of the current work is to solve 3D elastodynamic problem for the above described geometry and to evaluate wave field in the finite layered zone \( \Omega_1 \) with two elastic foundations taking into account the influence of the wave field in the dynamically loaded semi-infinite region, type and characteristics of the dynamic source, layering effect in finite zone, foundation-soil-foundation interaction effect and soil damaged state. What follows is to define the initial boundary-value problem describing the posed problem.

### 2.1 Boundary-value problem formulation

The initial boundary-value problem consists of governing equation of 3D motion, initial and boundary conditions discussed below. The equations of motion for \( \Omega_1 \) and \( \Omega_0 \) are as follows:

\[
\sigma_{ij}^{(1)}(x,t) = \rho^{(1)} \frac{\partial^2 u_i^{(1)}(x,t)}{\partial t^2}, \quad x(x_1,x_2,x_3) \in \Omega_1
\]

\[
\sigma_{ij}^{(0)}(x,t) + \Psi F_i(X_0,t) = \rho^{(0)} \frac{\partial^2 u_i^{(0)}(x,t)}{\partial t^2}, \quad x(x_1,x_2,x_3) \in \Omega_0.
\]

Here, \( x(x_1,x_2,x_3) \) is the vector position; \( X_0(X_{01},X_{02},X_{03}) \) is the coordinate vector of the point where the dynamic source is concentrated; \( u_i^{(k)}(x,t) \) and \( \partial^2 u_i^{(k)} / \partial t^2, \ k = 0,1; \ i = 1,2,3, \) are displacement and acceleration components in point \( x(x_1,x_2,x_3) \in \Omega_k \); and \( \sigma_{ij}^{(k)}(x,t) \) are stress tensor components at point \( x(x_1,x_2,x_3) \in \Omega_k \). The notation \( \Psi \) is a conditional multiplier which presents the following terms: \( \Psi = 0 \) if either (a) the dynamic excitation source is within the finite region \( \Omega_1 \) or (b) incident plane wave is considered; \( \Psi = 1 \) when an embedded dynamic source at point \( X_0(X_{01},X_{02},X_{03}) \) (see Figure 1) is considered. In the latter case, the dynamic force having the amplitude \( F_0(X_0) \) and the time history function \( f(t) \) is presented as \( F_i(X_0,t) = F_0 f(t) \delta(x - X_0) \), where \( \delta \) is the Dirac’s delta function and \( X_0 \) is the radius-vector of the point where it is concentrated. Point source approximation is a simple, convenient model to simulate weak seismic events, such as aftershocks or faults with negligible size, compared to the dimension of the seismic zone, see [49, 55]. Other approaches for including the seismic loads in the computational model are the following: (i) simulating the seismic source as a double-couple, see [63]; (ii) simulating the fault rupture as a double-couple sequence (dynamic rupture model), see [26]; (iii) introducing the effects of the seismic source to the model indirectly, that is, this is the case of incident wave loaded the boundary of the object under consideration, see [46].

The initial conditions for displacements and their first derivatives in respect to time are zero. The solution of the problem for transient waves is solved by the use of the following well-known numerical procedure, see [14]: (a) the fast Fourier
transform (FFT) is applied to the governing equations (1) and (2); (b) the corresponding boundary-value problem is solved in frequency domain; and (c) the inverse fast Fourier transform (IFFT) is applied to the solutions in frequency domain and finally solutions in time domain are obtained.

After application of FFT to displacement in respect to the time variable in equations (1) and (2), the frequency-dependent equations of motion have the following form:

\[
\sigma_{ij}^{(1)}(x, \omega) + \rho^{(1)} \omega^2 u_i^{(1)}(x, \omega) = 0, \quad x(x_1, x_2, x_3) \in \Omega_1; \tag{3}
\]

\[
\sigma_{ij}^{(0)}(x, \omega) + \rho^{(0)} \omega^2 u_i^{(0)}(x, \omega) = -\Psi F_0 \tilde{f}(\omega) \delta(x - X_0), \quad x(x_1, x_2, x_3) \in \Omega_0, \tag{4}
\]

where \(\omega\) is the circular frequency in rad/sec. The term \(F_1(X_0, t) = F_0 \tilde{f}(t) \delta(x - X_0)\) becomes \(F_0 \tilde{f}(\omega) \delta(x - X_0)\) in frequency domain, with notation of the term \(\tilde{f}(\omega)\) obtained after FT to the time function \(f(t)\).

Boundary conditions for semi-infinite region \(\Omega_0\) are as follows:

- Along the free-surface \(\Gamma_a\), the tractions \(t_i^{(0)} = \sigma_{ij}^{(0)} n_j^{(0)}\) are zero, \(n_j^{(0)}\) are the components of the outward normal to the surface \(\Gamma_a\).

- Along the interface surface boundary \(\Gamma_{int}\), compatibility and equilibrium conditions of displacements and tractions, respectively, are satisfied, i.e. \(u_i^{(0)} = u_i^{(1)}\) and \(t_i^{(0)} = -t_i^{(1)}\).

- Sommerfeld’s radiation condition is satisfied at infinity.

Boundary conditions for finite region \(\Omega_1\) are:

- Along the free-surface \(\Gamma_f\), the tractions are zero, that is, \(t_i^{(1)} = \sigma_{ij}^{(1)} n_j^{(1)} = 0\), \(n_j^{(1)}\) are the components of the outward normal to the surface \(\Gamma_f\).

- Along the interface surface boundary \(\Gamma_{int}\), compatibility and equilibrium conditions of displacements and tractions are satisfied.

- Along the interfaces between the soil and foundations’ walls and the interface \(\Gamma_{s12}\) between both layers, compatibility and equilibrium conditions of displacements and tractions, respectively, are satisfied.

The solution of the defined mechanical problem in frequency domain satisfies the governing equations (3) and (4) and the boundary conditions discussed above.

### 2.2 Damage model description

The damaged state of the geological material is described by the damage model proposed in [84], where the dispersion phenomenon of elastic waves in a solid permeated by a random distribution of micro-cracks is considered. The material characteristics of solids can be significantly affected by micro-cracks which give rise to stiffness degradation of the solid compared to its originally uncracked state. A radical difference between wave fields in cracked and uncracked media is presented by two phenomena referred as wave dispersion and wave attenuation. In materials with dispersed micro-cracks, the solid is seen by the incident wave as an attenuated and dispersive continuum, even though the cracked solid is still perfectly elastic, see [84]. The essence of the damage model proposed in [84] and its advisability for a solution of the above-defined foundation-soil-foundation interaction problem will be discussed here briefly.

There are the following basic assumptions: (a) the solid is homogeneous, isotropic and linear elastic with distributed slit micro-cracks; (b) the location and orientation of micro-cracks is random; (c) the dilute approximation is adopted, that is, geometrical and dynamical interactions amongst individual cracks are neglected; (d) the crack-faces are not in touch and the traction-free boundary condition is satisfied; and (e) all slit micro-cracks are identical with the same half-length \(a^*\). This approximation has the consequence that the analysis is only appropriate for small crack densities, denoted by \(n\), and less favorable for intermediate and dense concentration of micro-cracks. However, as noted in [84], in most current problems known in material science, the dilute results should be entirely adequate. From a practical point of view, the significance of the dilute approximation is that one can use explicit formulas for computing the attenuation coefficient and the effective wave velocity rather than have to resort to other cumbersome models.
The effective medium approach is applied basing on the homogenization by using a representative volume element (RVE), large enough compared to the dimensions of the micro-cracks and small enough compared to the solid dimensions. The RVE should be able to show the microscopic details and to represent the overall average behavior of the cracked solid. The original heterogeneous cracked solid is presented by a statistically homogeneous elastic solid with macroscopic isotropy, which has the same overall average response as the original one. The effective medium of the cracked solid still remains linear, causal (overall average response of the effective medium can be influenced only by past events) and passive (no energy can be created within the effective solid) as in its originally uncracked state. As in the case of visco-elastic wave propagation in a homogeneous material, the overall average dynamic response can be described by a complex and frequency dependent wave number $K(\omega)$, defined by $K^{\text{eff}}(\omega) = \omega/C^{\text{eff}}(\omega) + i\alpha^{\text{eff}}(\omega)$. Once the complex wave number $K^{\text{eff}}(\omega)$ of the effective medium has been obtained, the effective wave phase velocity $C^{\text{eff}}(\omega) = \omega/\text{Re}[K^{\text{eff}}(\omega)]$ and the attenuation coefficient $\alpha^{\text{eff}}(\omega) = \text{Im}[K^{\text{eff}}(\omega)]$ can be defined by taking the real and imaginary part of $K^{\text{eff}}(\omega)$. In [84], both the theory of Foldy [24] and the causal approach based on [37, 39] relations are applied to define the complex effective wave characteristics $K^{\text{eff}}(\omega), C^{\text{eff}}(\omega)$ and $\alpha^{\text{eff}}(\omega)$, respectively. These effective complex characteristics are obtained in [84] via the help of the BEM numerical procedure for calculation of the scattering cross section of a single micro-crack denoted by $\gamma_{sc}$. The coefficient $\gamma_{sc}$ describes the amount of energy lost by an incident wave due to its scattering by the micro-crack. For the aim of the present work, the authors used the results in [84] for the case of randomly oriented slit cracks but under assumption of small attenuation coefficient. Figures 3a, 3b and 3c show the normalized effective wave phase velocities $C^{\text{eff}}_P/C_P, C^{\text{eff}}_{SV}/C_{SV}$ and normalized attenuation coefficient $\tilde{\alpha} = 4\alpha^{\text{eff}}a^*/\pi\epsilon$ versus $k_s a^*$, where $k_s = \omega/C_{SV}; C_P$ and $C_{SV}$ are the longitudinal and shear wave velocities of the uncracked solid; and $\epsilon = (4n/\pi)(a^*)^2$ is the crack-density parameter introduced in [10].
It can be seen from Figures 3a to 3c that when the dimensionless wave number $k \alpha*\omega$ belongs to the interval $(0.0, 0.5]$, then the normalized attenuation coefficient $\delta$ belongs to the interval $(0.0, 0.05]$, then the normalized effective wave velocity $C^\text{eff}/C_p$ belongs to the interval $[0.865, 0.845]$ at $\epsilon = 0.2$. Thus, at the mentioned frequency interval $(0.0, 0.5]$, the imaginary part $\alpha^\text{eff}(\omega)$ of the complex wave number $K^\text{eff}(\omega)$ is negligibly small compared with its real part $\omega/C^\text{eff}(\omega)$. The numerical results concerning the damaged geological material are obtained in such a frequency interval providing the negligible small attenuation coefficient. The effective material characteristics (elastic moduli and density for damaged material case) are calculated for a fixed crack-density parameter $\epsilon$ following results in Figure 3.

3 HYBRID MODEL IN FREQUENCY DOMAIN BASED ON THE MACRO-ELEMENT CONCEPT

The hybrid approach described in this section is based on 3D-hosted finite element model realized numerically by ABAQUS, the currently one of the most efficient tools in the computational industry having broad mathematical and mechanical library, powerful solvers and user subroutine procedure. The proposed hybrid model exploits the advantages and avoids the disadvantages of both the BEM and the FEM. The BEM model of the dynamically active far-field zone $\Omega_0$ is inserted as a macro-finite element (MFE) in the FEM commercial program ABAQUS, taking advantage of the substructure procedure. The hybrid model is of the FEM-hosted type, where the boundary element model is converted into macro-finite element based on the expression of the nodal forces along the interface between two zones through the nodal tractions obtained by the BEM model and so the entire BEM formulation are converted to FEM-like approach. The model proposed is a continuation of the idea for a hybrid numerical approach presented in [70] but developed here for three dimensional transient elastodynamics.

3.1 Modeling of semi-infinite domain by BEM

Displacement boundary integral equation (BIE) describing the dynamic field in the far-field semi-infinite homogeneous zone $\Omega_0$ with boundary $\Gamma_{\Omega_0} = \Gamma_\alpha \cup \Gamma_{\text{int}}$ is given below, see [46]:

$$
c_{lk}u_k^{(0)}(x, \omega) = \int_{\Gamma_{\Omega_0}} U_{lk}^{\ast}(x, \xi, \omega)u_k^{(0)}(\xi, \omega)\,d\Gamma - \int_{\Gamma_{\Omega_0}} P_{lk}^{\ast}(x, \xi, \omega)\varphi_k^{(0)}(\xi, \omega)\,d\Gamma + \Psi F_0 f(\omega)U_{lk}^{\ast}(x, X_0, \omega), \quad \text{for } x \in \Gamma_{\Omega_0},
$$

where $c_{lk} (l = 1, 2, 3; k = 1, 2, 3)$ is the free-term depending on the geometry at the source point; $x, \xi$ is the source-receiver couple; $U_{lk}^{\ast}(0)$ is the 3D elastodynamic fundamental solution for displacement given in [22] and expressed by the material characteristics of domain $\Omega_0$; and $F_{lk}^{\ast} = C_\text{ipq} U_{rkq} n_q$ is its corresponding function for traction.

BIE (5) transforms into the matrix equation (6) after applied well-known discretization and collocation procedure, see [22]:

$$
\begin{bmatrix}
H^{(0)} & G^{(0)} \\
G^{(0)} & C^{(0)}
\end{bmatrix}
\begin{bmatrix}
u^{(0)} \\
\varphi^{(0)}
\end{bmatrix}
-
\begin{bmatrix}
G^{(0)} & C^{(0)} \\
C^{(0)} & \Phi^{(0)}
\end{bmatrix}
\begin{bmatrix}
t^{(0)} \\
\Phi^{(0)}
\end{bmatrix}
=
\begin{bmatrix}
\Phi^{(0)} \\
\Phi^{(0)}
\end{bmatrix},
$$

where $H^{(0)}$ and $G^{(0)}$ are the influence matrices $3L \times 3L$, $L$ is the number of nodes; $u^{(0)}, t^{(0)}$ are the displacement and traction vectors at nodes on $\Gamma_{\Omega_0}$; and the term $\Phi^{(0)}$ takes into account the contribution of the term $F_{lk} f(\omega)U_{lk}^{\ast}(x, X_0, \omega)$ in the case that an embedded dynamic source at $X_0$ is considered. The detail form of this term in the case when dynamic load is presented by incident plane wave can be seen in [70].

After condensation of the model’s degree of freedom realized by the procedure proposed in [5], the matrix equation (6) can be rewritten in the following form:

$$
\begin{bmatrix}
H^{(0)}_{11} & H^{(0)}_{12} \\
H^{(0)}_{21} & H^{(0)}_{22}
\end{bmatrix}
\begin{bmatrix}
u_1^{(0)} \\
\varphi_1^{(0)}
\end{bmatrix}
-
\begin{bmatrix}
G^{(0)}_{11} & G^{(0)}_{12} \\
G^{(0)}_{21} & G^{(0)}_{22}
\end{bmatrix}
\begin{bmatrix}
t_1^{(0)} \\
\Phi_1^{(0)}
\end{bmatrix}
=
\begin{bmatrix}
\Phi_1^{(0)} \\
\Phi_2^{(0)}
\end{bmatrix},
$$

where $H^{(0)}_{11}$ and $H^{(0)}_{22}$ are the influence matrices $L \times L$, $L$ is the number of nodes; $u_1^{(0)}, t_1^{(0)}$ are the displacement and traction vectors at nodes on $\Gamma_{\Omega_0}$; and the term $\Phi_1^{(0)}$ takes into account the contribution of the term $F_{lk} f(\omega)U_{lk}^{\ast}(x, X_0, \omega)$ in the case that an embedded dynamic source at $X_0$ is considered. The detail form of this term in the case when dynamic load is presented by incident plane wave can be seen in [70].

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$$
\begin{bmatrix}
H^{(0)}_{11} & H^{(0)}_{12} \\
H^{(0)}_{21} & H^{(0)}_{22}
\end{bmatrix}
\begin{bmatrix}
u_1^{(0)} \\
\varphi_1^{(0)}
\end{bmatrix}
-
\begin{bmatrix}
G^{(0)}_{11} & G^{(0)}_{12} \\
G^{(0)}_{21} & G^{(0)}_{22}
\end{bmatrix}
\begin{bmatrix}
t_1^{(0)} \\
\Phi_1^{(0)}
\end{bmatrix}
=
\begin{bmatrix}
\Phi_1^{(0)} \\
\Phi_2^{(0)}
\end{bmatrix},
$$

where $H^{(0)}_{11}$ and $H^{(0)}_{22}$ are the influence matrices $L \times L$, $L$ is the number of nodes; $u_1^{(0)}, t_1^{(0)}$ are the displacement and traction vectors at nodes on $\Gamma_{\Omega_0}$; and the term $\Phi_1^{(0)}$ takes into account the contribution of the term $F_{lk} f(\omega)U_{lk}^{\ast}(x, X_0, \omega)$ in the case that an embedded dynamic source at $X_0$ is considered. The detail form of this term in the case when dynamic load is presented by incident plane wave can be seen in [70].
where the index “1” in the matrices $H^{(0)}$, $G^{(0)}$ and vectors $t^{(0)}$, $u^{(0)}$ refers to the interface boundary $\Gamma_{int}$, while index “2” refers to the boundary $\Gamma_a$, see Figure 1. In resume, notation (0) means that semi-infinite homogeneous elastic isotropic region $\Omega_0$ is under consideration in equation (7); $\mathbf{u}^{(0)}_1$, $\mathbf{t}^{(0)}_1$ are the vectors of nodal displacements and tractions along the interface boundary $\Gamma_{int}$; and $\mathbf{u}^{(0)}_2$, $\mathbf{t}^{(0)}_2$ are the vectors of nodal displacements and tractions along the free-surface boundary $\Gamma_a$.

Applying the condensation procedure in [5], the following relation between traction $\mathbf{t}^{(0)}_1$ and displacement $\mathbf{u}^{(0)}_1$ vectors along the contact boundary $\Gamma_{int}$ is derived:

$$\mathbf{t}^{(0)}_1 = \mathbf{B} \mathbf{u}^{(0)}_1 - \mathbf{p},$$

(8a)

where

$$\mathbf{B} = \left[ G^{(0)}_{11} - H^{(0)}_{12} \mathbf{A}_t \right]^{-1} \left( H^{(0)}_{11} - H^{(0)}_{12} \mathbf{A}_u \right) \quad \text{and} \quad \mathbf{p} = \left[ G^{(0)}_{11} - H^{(0)}_{12} \mathbf{A}_t \right]^{-1} \left( \Phi^{(0)}_{1} - H^{(0)}_{12} \Theta \right).$$

(8b)

$$\mathbf{A}_t = \left[ H^{(0)}_{22} \right]^{-1} G^{(0)}_{21}; \quad \mathbf{A}_u = \left[ H^{(0)}_{22} \right]^{-1} H^{(0)}_{21}; \quad \Theta = \left[ H^{(0)}_{22} \right]^{-1} \Phi^{(0)}_{2}.$$  

(8c)

The conversion of the BEM matrix into FEM-compatible form is performed using the mapping of the nodal traction $\mathbf{t}$ into the nodal force $\mathbf{f}^{(0)}(x \in \Gamma_{int}, \omega)$, which reads

$$\mathbf{f}^{(0)}(x \in \Gamma_{int}, \omega) = [\mathbf{M}^* \mathbf{t}^{(0)}],$$

(9)

where

$$\mathbf{M}^* = \sum_{e=1}^{K} \Lambda_e; \quad \Lambda_e = \int_{\Gamma_e} \mathbf{N}^T \mathbf{N} d\Gamma.$$

Notation $K$ is the number of boundary elements along $\Gamma_{int}$; row vector $\mathbf{N}$ contains the shape functions used for the approximation of field variables, i.e. $\mathbf{N} = [N_1, N_2, \ldots, N_j]$, where $j$ is the number of nodes on element $e$; $\Gamma_e$ is the boundary of the element; $\Lambda_e$ is the elemental mapping matrix; and $\mathbf{T}$ denotes transpose operation.

After substitution of equation (9) into equation (8a), the following generalized expression for the nodal force vector $\mathbf{f}^{(0)}(x \in \Gamma_{int}, \omega)$ is obtained:

$$\mathbf{f}^{(0)} = \mathbf{K}^{(0)} \mathbf{u}^{(0)} - \mathbf{r}^{(0)},$$

(10)

where

$$\mathbf{K}^{(0)} = [\mathbf{M}^*] \mathbf{B}; \quad \mathbf{r}^{(0)} = [\mathbf{M}^*] \mathbf{p}.$$  

Note that equation (10) is complex-valued with complex-valued stiffness matrix, displacement and free-term vectors, that is, we have the followings:

$$\mathbf{K}^{(0)} = \mathbf{K}^{(0)}_{Re} + i \mathbf{K}^{(0)}_{Im}; \quad \mathbf{u}^{(0)} = \mathbf{u}^{(0)}_{Re} + i \mathbf{u}^{(0)}_{Im}; \quad \mathbf{r}^{(0)} = \mathbf{r}^{(0)}_{Re} + i \mathbf{r}^{(0)}_{Im}.$$  

(11)

### 3.2 Modeling of the finite region by FEM

The following classical equation of motion describes the wave field in the finite zone $\Omega_1$:

$$\left[ -\omega^2 \mathbf{M}^{(1)} + i \omega \mathbf{C}^{(1)} + \left( \mathbf{K}^{(1)} + i \mathbf{K}_{s}^{(1)} \right) \right] \mathbf{u}^{(1)} = \mathbf{f}^{(1)},$$

(12)

where $i = \sqrt{-1}$; $\mathbf{M}^{(1)}$, $\mathbf{C}^{(1)}$, and $\mathbf{K}^{(1)}$ are the matrices of mass, viscous damping and stiffness, respectively; $\mathbf{K}_{s}^{(1)}$ is the structural damping matrix; $\mathbf{u}^{(1)}$ and $\mathbf{f}^{(1)}$ are vectors of nodal displacement and nodal force of the FEM mesh, respectively.
3.3 | Hybrid BEM-FEM for the whole model

The method used to couple the BEM into the FEM environment of ABAQUS is based on the substructure generation procedure in which a substructure of the BEM model for semi-infinite domain $\Omega_0$ is created and attached to the FEM model of domain $\Omega_1$ via prescribed tied common nodes or tied surfaces where compatibility and equilibrium conditions are satisfied. ABAQUS model of the whole system includes sub-models concerning near-field $\Omega_1$ and far-field $\Omega_0$ zones in the following form:

$$\begin{bmatrix}
-\omega^2 M + i\omega C + (K + iK_s) \\
\end{bmatrix} u = \tilde{f},$$

(13)

where $M$, $C$, and $K$ are the corresponding mass, viscous damping, and stiffness matrices of the whole global system, respectively; $K_s$ is the structural damping matrix of the whole hybrid model and $\tilde{f}$ is the vector presenting the external load. Comparing and fitting equations (10), (12) and (13) and taking advantage of the substructure procedure available in ABAQUS, we can derive the following relations for the hybrid assembly:

- The global stiffness matrix can be assembled from the stiffness matrix of FEM region $K^{(1)}$ and the real part of the stiffness matrix of BEM region $K_{\text{Re}}^{(0)}$.
- The global structural damping matrix is assembled from the the damping matrix of FEM model $K_{\text{st}}^{(1)}$, which contains the structural damping definition, and the imaginary part of the stiffness matrix of the BEM zone $K_{\text{Im}}^{(0)}$.
- The load vector can be obtained from the assembly of the vector of external loads assigned on the FEM model $\tilde{f}^{(1)}$ and the vector of dynamic force term of the BEM model $r^{(0)}$.
- The global viscous damping matrix is expressed by $C = C^{(1)}$, where viscous damping matrix $C^{(1)}$ contains the damping defined through the Rayleigh and/or proportional damping definition of the FEM model.
- The global mass matrix is $M = M^{(1)}$.
- The unknowns are the nodal displacements along (1) the FEM mesh and (2) the BEM mesh along the FEM-BEM interface.

The assembly of the global stiffness matrix, structural damping matrix and force vector is illustrated in Figure 4. The hybrid numerical scheme presented here is a 3D generalization of the hybrid scheme in [70] for the case of elastic transient wave propagation in a half-space with embedded seismic source.

3.4 | General scheme of the hybrid approach

The general scheme of the modeling and computation is described in the following steps:

1. The geometries for both the FEM and BEM models are created in the ABAQUS environment. Each model must have surfaces which are in perfect contact with each other and form the inter-regional interface $\Gamma_{\text{int}}$. Note that the common
surfaces do not necessarily share common nodes. Any available type of element as well as elastic mechanical material properties and contact definition can be included in the FEM model.

2. The FEM and BEM models are then meshed in ABAQUS. The BEM model can be meshed using either linear or quadratic shell element. Two input files are generated from the ABAQUS model with 1 input file containing the FEM model description and the other containing the BEM mesh definition.

3. The BEM input file is used as an input for an in-house MATLAB code where user then inserts the material properties for the far-field semi-infinite region $\Omega_0$ and the external dynamic wave definition. The in-house code then consecutively performs the followings:

   - Extraction of the nodes coordinates and elements definition from the BEM input file generated in step 2.
   - In the case that an incident plane wave is considered, computation of the wave field following the procedure in [70].
   - In the case that an embedded dynamic source is considered, application of direct FFT to a prescribed transient load into its frequency-amplitude specter $\hat{f}(\omega)$.

   Computation, condensation and conversion of the influence matrices and the wave field in the far-field semi-infinite region $\Omega_0$, for each frequency $\omega$, into

   - the stiffness and structural damping matrices of the far-field region $\Omega_0$
   - the nodal force vector $\tilde{f}(x \in \Gamma_{int}, \omega)$ along the interface boundary $\Gamma_{int}$ describing the dynamic wave load coming from the semi-infinite region $\Omega_0$ and impinging along $\Gamma_{int}$ on the finite region $\Omega_1$.

4. For each frequency $\omega$, a macro-finite element based on the stiffness and structural damping of the far-field semi-infinite region $\Omega_0$ is generated using substructure procedure in ABAQUS environment.

5. For each frequency $\omega$, the corresponding macro-finite element along with a contact definition for the interface are added to the FEM input file generated in step 2. In result, a group of input files are generated. When either incident plane wave or embedded dynamic load source case is under consideration, the nodal force vectors $\tilde{f}(x \in \Gamma_{int}, \omega)$ are inserted into the input files as well.

6. The updated input files containing the FEM model definition, the macro-finite element representations of the BEM region and the nodal force vectors are then solved using direct-solution steady state dynamic procedure in ABAQUS.

7. Using an in-house post-processing tool, the ABAQUS outputs of the steady state analyzes are then collected and converted into time domain solution through application of inverse-FFT and the solution of the boundary-value problem in time domain formulated in Section 2 is obtained.

The above scheme is not necessarily a manual back-and-forth process since the processing phase in step 3 through 6 can be executed under one set of code written in MATLAB; it is able to send command to the operating system to execute ABAQUS. The hybrid scheme also highlights the following advantages: (1) The BEM and FEM discretized models do not necessarily share common nodes which results in a more flexible modeling; (2) the macro-finite element obtained in step 5 is transferable and reusable which potentially reduces the computational costs and makes it highly attractive for a distributed design work and optimization process.

The numerical solution of the BEM model follows standard discretization and collocation procedure. The surface boundary is discretized into either linear (three-node triangle and four-node quadrilateral) or quadratic (six-node triangles and eight-node quadrilateral) isoparametric boundary elements, using continuous polynomial approximations for the boundary geometry, the displacement, and the traction vectors. Two types of integrals are obtained after discretization of boundary integral equations, depending on whether or not the radial distance $r$ between the source and receiver points is zero: (a) at $r \neq 0$ the integrals are regular, there are no singularities, solution is numerical; (b) at $r = 0$ there are two types of singularities: (*) the displacement-based kernels exhibit a weak singularity of the type $O(1/r)$ and these integrals are solved by appropriate quadrature rule; (**) the traction-based kernels exhibit a strong singularity of the type $O(1/r^2)$ and these integrals are solved using the rigid body motion method, see [22]. Since the rigid body motion method cannot be applied to arbitrary non-smooth nodes of an open polygon (in 2D case) or an open surface (in 3D case) geometry, the discretized half-space is enhanced with dummy elements and/or helper node to create an enclosed geometry. The cost of the additional dummy elements are rather insignificant; they are only required for the computation of the static part of the traction fundamental solution and thus, they are only required to be calculated once.

The well-known problem in hybrid techniques based on BEM and FEM is that FEM leads to sparse symmetric positive definite matrices, while BEM based on collocation technique leads to full, non-symmetric ones. Following the assembly of the stiffness matrix of BEM into the FEM system of equation, as described in the previous section, the stiffness and structural damping matrices of the whole model are both unsymmetric. ABAQUS provides unsymmetric matrix storage and solution scheme which can be activated either by (1) using the syntax “**STEP, UNSYM=YES” in the ABAQUS input
file or by (2) choosing the unsymmetric equation solver in the step editor when using ABAQUS graphical user interface. This option is available for use in a direct-solution steady state dynamic analysis.

4  VERIFICATION OF THE HYBRID COMPUTATIONAL SCHEME

What follows is solution of benchmark examples in order to present the accuracy and convergence study of described above hybrid numerical scheme and to establish its accuracy level. As far as discretization approach is applied in both BEM and FEM models, the relative size of the discretization elements and possible mismatches (i.e., small elements abutting large ones) can cause spurious wave reflections not otherwise present. The accuracy criterion used in discretization procedure states that $\lambda_{SV}/l_{BE} \geq 10$, where $l_{BE}$ is the length of the corresponding element and $\lambda_{SV}$ is the shear wavelength. In the case that quadratic element is in use, it is shown in the following examples that $\lambda_{SV}/l_{BE} \geq 5$ is sufficient.

4.1  Test example 1: wave propagation in a homogeneous half-space

The first benchmark example considers the geometry presented in Figure 1 assuming that the local finite geological region $\Omega_1$ is a square cuboid, see Figure 5, with the same material properties as those of the semi-infinite region $\Omega_0$. This means that the 1st numerical scheme verification concerns the wave propagation in a homogeneous half-space due to incident normal time-harmonic P- or SV-wave. The FEM region, the BEM zone, and the global BEM-FEM model are shown in Figures 5a, 5b, and 5c. The dimension of the surface of the BEM model is 8 by 8 m. The FEM region is a square cuboid which has a width of 2 m and a depth of 1 m. Thus, the BEM-FEM interface is a parallelepiped of the same sizes. In Figure 5, the dummy elements are red colored. The same material properties are applied to both zones. The material properties are as follows: Lamé constants $\lambda = \mu = 4.5$ MPa and Poisson’s ratio of 0.25. The incident P- or SV-wave considered here is a wave propagating in vertical direction, that is, in the direction of the coordinate axis $x_3$ in Figure 1. The shearing direction in the case of SV-wave is in direction of the coordinate axis $x_1$, see Figure 1. The following frequencies are considered: 0.05, 20, and 43.3 Hz for the P-wave; and 0.05, 20, and 40 Hz for the SV-wave. Two mesh sizes, designated as Mesh A and Mesh B, are compared here to show the convergence. The mesh sizes of the boundary and finite elements in both meshes are 1/4 and 1/6 m, respectively, which correspond to 1/5 and 1/7.5 of the shortest shear wavelength ($\lambda_{SV} = 1.25$ m for 40 Hz). The ratio $\kappa$ relates the wavenumber to the length of the discretized free-surface of the half-space ($\Gamma_a$ in Figure 1), written as $\kappa = 2\pi/(\lambda l_{BE})$. The length $l_{BE}$ of the model in Figure 5 is 3 m which corresponds to $\kappa_p = 1.81$ for P-wave case at 43.3 Hz and $\kappa_{SV} = 1.68$ for SV-wave case at 40 Hz. The numbers of elements used for this example, in the case of $\lambda_{SV}/l_{BE} = 5$ and $\kappa_{SV} = 1.68$, are 256 quadratic hexahedral finite elements and 1024 quadratic quadrilateral boundary elements for Mesh A and; 864 quadratic hexahedral finite elements and 2112 quadratic quadrilateral boundary elements for Mesh B.

The normalized by incident wave amplitude displacements along the line $x_1 = 0, x_2 = 0$ inside the FEM zone for the case of $\kappa_p = 1.81, \kappa_{SV} = 1.68$ are shown in Figures 6a and 6b. A very good agreements between the analytical solution for free-field wave motion in homogeneous elastic isotropic half-space presented in [1] and solutions obtained by the hybrid BEM-FEM for test example 1 are apparent. In the case of incident P-wave, the relative errors to the analytical solution in the cases of $\kappa_p = 5.44$ and $\kappa_p = 1.81$ for frequency of 43.3 Hz using Mesh A are 2.11% and 2.00%, respectively. The error of the results obtained from Mesh B with $\kappa_p = 1.81$ relative to the analytical solution is 1.36%, meaning that the relative difference between two results of Mesh A and B is 0.65%. In the case of incident SV-wave, the relative errors for the highest considered frequency of 40 Hz and $\kappa_{SV} = 1.68$ are 0.50% and 0.41% for Mesh A and B, respectively. Numerical experiment done using Mesh A in the case of incident plane SV-wave with $\kappa_{SV}$ of 5.03, 1.68, and 1.26 show relative errors of 3.07%, 0.50%, and 0.49%, meaning that $\kappa_{SV} \leq 1.8$ in combination with $\lambda_{SV}/l_{BE} \geq 5$ is sufficient to gain good accuracy.
4.2 Test example 2: response of a rigid massless foundation rested on a homogeneous half-space under vertical harmonic loading

The 2nd test example concerns the dynamic response of a rigid massless foundation resting on a homogeneous elastic isotropic half-space due to vertical harmonic loading applied to the foundation. In this test example, the foundation is included in the FEM region and the half-space is modeled in BEM zone. The dimension of the square foundation is 1 m. The material properties of the half-space are taken from [83] as follows: Lamé constants $\lambda = 180$ GPa, $\mu = 90$ GPa, and Poisson’s ratio of $1/3$. To handle the rigid foundation, rigid body constraint available in ABAQUS, which ties the degree of freedoms of a solid into a reference point, is used and thus, no elastic material definition is required for the foundation. The vertical compliance of the foundation due to vertical harmonic loading is defined as $V_{vv} = u_0 \mu b / (2P)$, where $u_0$ is the displacement at the bottom center of the foundation, $b$ is the width of the foundation, and $P$ is the amplitude of the applied time-harmonic load. To reveal the sensitivity of the result to the size of the discretized free-surface of the half-space, numerical experiment is performed using $\kappa_{SV}$ of $\infty$, 3.5 and 1.8. The case of $\kappa_{SV} = \infty$ means that no free-surface is discretized; only the boundary of the half-space under the foundation is considered. The foundation is modeled using 32 quadratic hexahedral finite elements, 16 of them are in contact with the half-space, while the half-space is modeled using 480, 680, and 1320 quadratic boundary elements for $\kappa_{SV}$ of $\infty$, 3.5 and 1.8, respectively.

Figure 7 shows a comparison between the authors’ results obtained by the hybrid computational approach based on the BEM and FEM with the solutions obtained by 3D pure BEM in [83]. The relative difference between the results of the models with $\kappa_{SV} = \infty$ and $\kappa_{SV} = 3.5$ is 4.73% while the relative difference of the latter with the results of model with $\kappa_{SV} = 1.8$ is 3.29%.
FIGURE 8 Time history (a) and amplitude spectrum, normalized to the maximum absolute value, (b) of the transient excitation.

In sum, the verification study is based on a comparison of the authors’ solutions obtained by the proposed hybrid FEM-BEM with results obtained by other authors and with analytical result for free-field wave motion in homogeneous elastic isotropic half-space under dynamic loads. The comparison shows that the proposed hybrid computational technique works accurately and can be used for simulations presented in the next section.

5 PARAMETRIC STUDY

The aim of this section is to reveal the complex character of 3D wave field that develops in a finite layered soil region containing two elastic foundations and rested in a homogeneous elastic isotropic half-space with an embedded source of dynamic transient excitation. The parametric study consists of two parts. The first part considers a short-range dynamic excitation of a small layered finite region with two embedded foundations (case study 1). It is followed by the second part that is a study regarding a long-range dynamic excitation case of two foundations embedded in a multi-layered sedimentary basin (case study 2).

It is assumed that the time history function of the dynamic force input at the embedded source follows the 90 degree component of the displacement time history recorded by Newhall station during the Northridge earthquake on 17 January 1994. The record has peak displacement of 17.595 cm and it is available at http://www.strongmotioncenter.org. To reduce the computational cost, the time history is trimmed to include only the record in the time range \( t = [1, 26] \) s. It is subsequently filtered using a band-pass of 0.1-25 Hz and then detrended. Thereafter, it is transformed into Fourier domain and then normalized by the maximum absolute value of the amplitude spectrum. The frequency range considered for the analysis is \([0, 8]\) Hz with a resolution of 0.0488 Hz. The time step of the time history function and the subsequent inverse-FFT results is 0.02 s. The treated time history and the normalized absolute amplitude spectrum are shown in Figure 8.

Note, that the time-function of the seismic record is used formally as a time-function of the transient dynamic excitation concentrated in a defined below point \( X_0 \).

Study case 1

The geometry of the model of the first study case is shown in Figure 9. The finite soil region \( \Omega_1 \) (Figure 9a) is a cuboid with widths of 8 and 5 m in \( x_1 \) and \( x_2 \) directions, respectively. It consists of two layers namely \( \Omega_{1(L1)} \) at the top and \( \Omega_{1(L2)} \) at the bottom. The thicknesses of \( \Omega_{1(L1)} \) and \( \Omega_{1(L2)} \) layers are 2.5 and 1.5 m, respectively. Two identical elastic foundations with dimensions of 1.0 x 1.0 x 1.0 m are embedded in \( \Omega_{1(L1)} \) with distance \( d \) between them. The free-surface of the half-space discretized in the BEM model is 2 m at the shortest, which corresponds to ratio \( \kappa_{SV} \) of 0.05.

The material properties considered for the parametric study are laid out in Table 1. Two states of geological material in semi-infinite region \( \Omega_0 \) are compared: undamaged and damaged. The latter is marked \( \Omega_0^* \) in Table 1 and the effective
Figure 9: The geometry of the model for parametric study case 1: (a) FEM region, (b) BEM region and (c) whole BEM-FEM model

Table 1: Material properties considered for parametric study case 1

| Region     | $\lambda$ (MN/m$^2$) | $\mu$ (MN/m$^2$) | Poisson’s ratio | $\rho$ (kg/m$^3$) |
|------------|-----------------------|-------------------|----------------|-------------------|
| $\Omega^*_0$ | 392.82                | 392.82            | 0.25           | 2100              |
| $\Omega_{1(L1)}$ | 62.63             | 43.75             | 0.30           | 1575              |
| $\Omega_{1(L2)}$ | 105.00            | 105.00            | 0.25           | 1680              |
| Foundations | 6666.67              | 10000.00          | 0.20           | 2500              |
| Foundations* | 388.89             | 583.33            | 0.20           | 2500              |
| Foundations** | 95.83               | 143.75            | 0.20           | 2500              |

Phase velocities ratios $C_{effP}/C_P$ and $C_{effSV}/C_{SV}$ are 0.749 and 0.865, respectively, see Figure 3, at the crack-density parameter $\varepsilon = 0.2$ and at $k_s a^*$ close to zero.

The numerical model is discretized using 400 quadratic hexahedral finite elements, 348 active quadratic quadrilateral boundary elements and 104 dummy boundary elements. The maximum element size is 1 m which is approximately 1/20 of the shortest shear wave length $\lambda_{SV}$ of the materials in Table 1 when considering the frequency of 8 Hz as the highest input component.

Two different locations of the embedded dynamic source with amplitude $F_0(0, 0, 10^{10}$ N) are used in the simulations denoted as: dynamic source 1 with location $X_0(0, 0, -10$ m) and dynamic source 2 with location $X_0(1, -0.5, -8$ m). The following arrangements of both foundations are considered: (a) arrangement 1, where the line connecting the centers of the surface walls of both foundations is parallel to the axis $Ox_1$ and the distance between them is $d = 2, 1, 0.5$ m and 0.25 m; (b) arrangement 2, where the line connecting the centers of the surface walls of both foundations is parallel to the diagonal of the free-surface of the finite region $\Omega_1$ and $d = 1$ m.

Figures 10-13 compare the results for undamaged and damaged geological material in the semi-infinite region $\Omega_0$, at arrangement 1 of the foundations with $d = 2$ m and in the case of dynamic source 1. Synthetic seismograms at observer points along the line $x_2 = 0, x_3 = 0$ are shown in Figure 10. Note that the normalized amplitudes of displacement component $u_2$ are zero along the lines $x_1$. This figure shows the absolute displacement component normalized by the maximum value of the free-field displacement from the same dynamic source 1 and measured at $x(4$ m, 0, 0), $x(0, 2.5$ m, 0) and $x(0, 0, 0)$ for $u_1$, $u_2$ and $u_3$, respectively. The seismograms in the case of damaged state follow similar form to that of undamaged case but with around 30% increase in amplitudes as shown by the plot of the normalized displacement component $u_3$ versus time at observer point $x(1.5$ m, 0, 0), see Figure 11. Normalized displacement amplitudes along the line $x_2 = 0, x_3 = 0$ are shown in Figure 12 where the results are focused around the displacement peak in the time interval $t = [4.0, 8.62]$ s.

Figure 13 illustrates the effect of the damaged state of the semi-infinite geological region $\Omega_0$ on the normalized displacement amplitudes $|u_3|$ along the free-surface plane $x_3 = 0$ of the finite region $\Omega_1$ at a fixed time moment $t = t^*$, where $t^*$ is the fixed time moment when the response displacement component $u_3$ has its peak.

Figure 14 presents the arrangement effect at different separation distance between foundations on the second displacement component $|u_2|$ along the line $x_1 = 0, x_3 = 0$. The obtained results show that when the foundations are at a distance of 0.25 m to each other, significant change on the displacement wave field is visible. Figure 15 illustrates the differences in the displacement component $u_2$ along the plane $x_3 = 0$ in the finite region $\Omega_1$ at a fixed time $t^*$ between the results of arrangement 1, $d = 1$ m; arrangement 2, $d = 1$ m; arrangement 1, $d = 0.5$ m; and arrangement 1, $d = 0.25$ m. Both Figures 14-15 are obtained for the undamaged state of $\Omega_0$ and show that the types of foundation arrangement and distance between them are responsible factors for the dynamic response.
Further, we consider elastic foundations with strongly reduced stiffness properties due to aging, weathering, cracking, damage or other deteriorating effects, see [54], where the following reduced elastic properties for masonry foundation are given: Poisson’s ratio 0.2 and Young’s modulus $E = 1400$ MPa (Foundations* in Table 1) and 345 MPa (Foundations** in Table 1). These foundation characteristics are used in Figures 16-18. These figures depict normalized amplitudes of displacement components $|u_1|$ (Figure 16), $|u_2|$ (Figure 17) and $|u_3|$ (Figure 18) along the free-surface of the finite region $\Omega_1$ at a fixed time moment of the peak of the displacement component $u_3$ in the cases of the foundations arrangement 1, fixed separation distance $d = 1$ m and the following elastic properties of both foundations: (a) $E = 24000$ MPa, (b) $E = 1400$ MPa and (c) $E = 345$ MPa. Note that for the other considered cases presented in Figures 10-15 and Figure 19, the elastic property of the foundations is $E = 24000$ MPa (Foundations in Table 1).

Normalized displacement components $u_i$ at observer point $x(1,0,0)$ for the model with foundations arrangement 1 and $d = 2$ m due to dynamic signal coming from dynamic source 1 and source 2 are compared in Figure 19.

**Study case 2**

The geometry of the model of the second parametric study case is shown in Figure 20 where a sedimentary basin with a half-sphere geometry with a diameter of 40 m is considered. The basement rock of the basin is represented as semi-infinite region $\Omega_0$ and the finite region $\Omega_1$ consists of 4 layers of elastic isotropic sediment. Two identical elastic foundations with
FIGURE 12 Normalized displacement amplitudes in case study 1 along the line $x_2 = 0, x_3 = 0$ versus $x_1$ in the time interval $[4.0, 8.62]$ s in the cases of undamaged (a, c) and damaged (b, d) soil in semi-infinite region $\Omega_0$ with embedded dynamic source 1 and distance between foundations $d = 2$ m: (a), (b) $|u_1|$; (c), (d) $|u_3|$.

TABLE 2 Material properties considered for parametric study case 2

| Region          | $\lambda$ (MN/m$^2$) | $\mu$ (MN/m$^2$) | Poisson's ratio | $\rho$ (kg/m$^3$) |
|-----------------|----------------------|------------------|----------------|------------------|
| $\Omega_0$     | 1 050.00             | 525.00           | 1/3            | 2 100            |
| $\Omega_0^{*}$ | 392.82               | 392.82           | 0.25           | 2 100            |
| $\Omega_1(L1)$ | 62.63                | 43.75            | 0.30           | 1 575            |
| $\Omega_1(L2)$ | 392.82               | 392.82           | 0.25           | 2 100            |
| $\Omega_1(L3)$ | 62.63                | 43.75            | 0.30           | 1 575            |
| $\Omega_1(L4)$ | 105.00               | 105.00           | 0.25           | 1 680            |
| Foundations    | 6 666.67             | 10 000.00        | 0.20           | 2 500            |

size of 6 m x 6 m and thickness of 1 m are embedded in the top layer with separation distance of 2 m. The largest element size in the BEM and FEM models is 1.5 m which corresponds to 1/13 of the shortest shear wave length. The length of the discretized free-surface is 6 m which leads to $\kappa_{SV}$ of 0.02. Other than consideration regarding the ratio $\kappa_{SV}$, this length is taken to avoid creating a thin beam polygon which leads to numerical inaccuracy, either in static or in dynamic case. The material properties for each layer in the finite region $\Omega_1$, the foundations, the semi-infinite region $\Omega_0$ and its damaged state $\Omega_0^{*}$ are given in Table 2. The same assumption is taken for the damaged state of the basement rock as the one for case study 1 at fixed crack-density parameter $\varepsilon = 0.2$.

The dynamic source, designated as dynamic source 3, is set at location $X_0(0, 0, -5000$ m) with amplitude of $F_0 I(0, 0, 10^{12}$ N) and using the same time function shown in Figure 8. The normalized amplitudes of displacement along the line $x_2 = 0, x_3 = 0$ versus time $t$ (s) are shown in Figure 21 while comparison between the results for undamaged and damaged states of the basement rock at observer point $x(4 m, 0, 0)$ is presented in Figure 22. The latter figure also shows the arrival time delay of the propagating wave in the case of damaged state due to its lower wave velocity. Figure 23 draws normalized amplitudes of displacement $u_1$ at observer points $x(-4 m, 0, 0), x(1 m, 0, 0)$, and $x(7 m, 0, 0)$ in the case both foundations have different elastic properties, that is, $E_{left} = 24000$ MPa for the left-side foundation and $E_{right} = 345$ MPa for the right foundation. The reference values used for normalization are taken from the displacement result of free-field model at points $x(20 m, 0, 0)$ and $x(0, 0, 0)$ for $u_1$ and $u_3$, respectively.
FIGURE 13  Normalized displacement amplitudes $|u_i|$ in case study 1 along the plane $x_3 = 0$ in the cases of undamaged (a, c, e) and damaged (b, d, f) soil in semi-infinite region $\Omega_0$ with embedded dynamic source 1 and distance between foundations $d = 2$ m at a fixed time $t = t^*$, where $t^*$ is the time of the displacement peak: (a), (b) $|u_1|$; (c), (d) $|u_2|$; (e), (f) $|u_3|$.

All results of the parametric study plotted in Figures 10-23 reveal some trends which can be summarized below.

(a) Obviously, the phenomenon of dynamic site effects can be seen in the most of the figures presented in this section, see Figures 10-16, 21-23, due to the well-known fact that the wave field that develops at the free-surface is a result of a complex interplay of geometric and material factors. In most of the figures, there is a great difference between the displacement wave zones near and far-away from the foundations. More specifically, earthquakes and other dynamic events are triggered from a source characteristics that releases energy in the form of waves. These waves filter through geological media on their way to the free-surface and are greatly affected by the material properties and structure of the soil layers with embedded engineering structures and the semi-infinite region under them, including local topography. As a result, the spatial and temporal variation of dynamic signals differs considerably for nearby stations in the same locality and even for the same dynamic event. To date, it has been proven difficult to incorporate site effects in engineering design of underground structures because of the sheer complexity of the problem. This partially reflects in the relative paucity of numerical models capable of handling irregular site geometry, multiple soil deposits and availability of underground engineering facilities.

(b) The local geological soil conditions (layering or soil damage) change the characteristics of the surface dynamic response. One of the important reasons of the dynamic damages is the local soil conditions. The damage state of
Figure 14: Normalized amplitudes of displacement component $|u_2|$ in case study 1 along the line $x_1 = 0, x_3 = 0$ versus $\frac{x_2}{b}$ and versus time $t$ (s) in the case of undamaged soil in semi-infinite region $\Omega_0$ with embedded dynamic source 1 and foundations arrangement as follows: (a) arrangement 1 with $d = 1$ m and (b) arrangement 1 with $d = 0.25$ m.

Figure 15: Normalized amplitude of displacement component $|u_2|$ in case study 1 along plane $x_3 = 0$ at a fixed time $t = t^*$, where $t^*$ is the time of displacement peak, in the case of undamaged soil in semi-infinite region $\Omega_0$ with dynamic source 1. The foundations arrangements are as follows: (a) arrangement 1 with $d = 1$ m, (b) arrangement 2 with $d = 1$ m, (c) arrangement 1 with $d = 0.5$ m and (d) arrangement 1 with $d = 0.25$ m.
FIGURE 16  Normalized amplitudes of displacement component $u_1$ in case study 1 at a fixed time of displacement peak $t = t^*$ in the case of elastic foundations arrangement 1, separation distance between foundations is $d = 1\, \text{m}$ and their elastic properties are different. Observer points are along the plane $x_3 = 0$ in (a), (b), (c) and along the line $x_1 = 1.5\, \text{m}, x_3 = 0$ versus $x_2$ in (d).

FIGURE 17  Normalized amplitudes of displacement component $u_2$ in case study 1 at a fixed time of displacement peak $t = t^*$ in the case of elastic foundations arrangement 1, separation distance between foundations is $d = 1\, \text{m}$ and their elastic properties are different. Observer points are along the plane $x_3 = 0$ in (a), (b), (c) and along the line $x_2 = 0.5\, \text{m}, x_3 = 0$ versus $x_1$ in (d).

The soil material provokes two base phenomena, the wave attenuation and dispersion accompanied with decreasing of soil stiffness. The damaged model of Zhang and Gross [84] proposing the procedure for calculating the phase velocity of waves propagating in a damaged solid containing a distribution of completely randomly oriented micro-cracks is incorporated in a 3D hybrid model described in Section 3. The effect of the damaged state of the soil is visible in almost all of Figures 10-13, 19, and 22. All numerical results obtained reveal the influence of the soil micro-structure state.
FIGURE 18  Normalized amplitudes of displacement component $u_3$ in case study 1 at a fixed time of displacement peak $t = t^*$ in the case of elastic foundations arrangement 1, separation distance between foundations is $d = 1$ m and their elastic properties are different. Observer points are along the plane $x_3 = 0$ in (a), (b), (c) and along the line $x_2 = 0.5$ m, $x_3 = 0$ versus $x_2$ in (d).

FIGURE 19  Normalized displacement components $u_i$ in case study 1 at observer point $x(1 \text{ m, 0, 0})$ versus time $t$ (s) in the cases of undamaged and damaged soil in semi-infinite region $\Omega_0$ with dynamic source 1 and source 2 and separation distance between foundations $d = 2$ m: (a) $u_1$, (b) $u_2$ and (c) $u_3$. 
FIGURE 20  The geometry of the model for parametric study case 2

FIGURE 21  Normalized amplitudes of displacement $|u_i|$ in case study 2 along the line $x_2 = 0$, $x_3 = 0$ versus $\frac{x_1}{b}$ and versus time $t$ (s) in the case of undamaged basement rock in semi-infinite region $\Omega_0$ with embedded dynamic source 3: (a) $|u_1|$ and (b) $|u_3|$.

FIGURE 22  Comparison of the normalized displacement component $u_3$ at observer point $x(4,0,0)$ in case study 2 versus time $t$ (s) between results for undamaged and damaged basement rock in semi-infinite region $\Omega_0$ considering the embedded dynamic source 3.
change on the seismic wave picture. The maximal percentage differences observed between displacements obtained for undamaged and damaged soils in Figure 12 are 30.67 and 34.18% for \( u_1 \) and \( u_3 \), respectively, while the differences between the displacements in Figure 19 are 30.18, 39.88, and 34.31% for \( u_1 \), \( u_2 \) and \( u_3 \), respectively. The maximum difference observed in Figure 22 is 74.80%.

(c) The sensitivity of the dynamic response to the dynamic source location can be seen in the obtained results in Figure 19 for the observer point \( x(1, 0, 0) \) in the cases of two different dynamic source locations when all the rest model parameters are fixed. The source location changes strongly the 3D dynamic response in one and the same observer point, especially when it combines with the damage state of the soil.

(d) In the whole system of key model parameters important role play also the elastic properties of both foundations. This is illustrated in Figures 16-18. The wave pictures for all three displacement components along the free-surface of the finite local geological region reveal a strong sensitivity to the foundation stiffness reduction. There exist observer points for which the foundation stiffness reduction leads to 50% increase of the seismic response. Figure 23 also illustrates that the difference in elastic properties of both foundations plays an important role in the dynamic response of the region under consideration.

(e) The effect of the foundation-soil-foundation dynamic interaction is illustrated in Figures 14-15, where it is visible that different geometrical arrangements of both foundations have significant impact on the wave field along the free-surface. The results obtained in the cases of foundations arrangement 1 with separation distances between foundations of 1, 0.5, and 0.25 m clearly illustrate the influence of foundation-soil-foundation interaction on the dynamic response in Figures 15a, 15c, and 15d. The geometry of diagonal disposition of the foundations at a fixed distance of 1 m is projected in the wave picture shown in Figure 15b. The wave zone near the foundations accurately and visibly reflects the geometry of their mutual location in all Figures 15a, 15b, 15c, and 15d.

FIGURE 23 Comparison of the normalized displacements \( u_1 \) in case study 2 versus time \( t \) (s) between results for the cases where both foundations have the same elastic properties \((E = 24000 \text{ MPa})\) and different elastic properties \((E_{\text{left}} = 24000 \text{ MPa}, E_{\text{right}} = 345 \text{ MPa})\). Embedded dynamic source 3 is considered. The displacements are measured at the following observer points: (a) \( x(-4 \text{ m}, 0, 0) \); (b) \( x(1 \text{ m}, 0, 0) \); (c) \( x(7 \text{ m}, 0, 0) \).
CONCLUSIONS

The study formulates and solves 3D elastodynamic problem for foundation-soil-foundation dynamic interaction in a finite layered soil region rested in a homogeneous elastic isotropic half-space where a transient dynamic source is buried. To this purpose, an efficient hybrid BEM-FEM based on the macro-element concept is developed and corresponding numerical scheme and software are verified. Each of the two computational techniques is applied in the part of the global model where it works more efficiently. The BEM is used as a tool for simulation of wave propagation from the source position to the local laterally inhomogeneous geological profile, while the FEM is applied inside the finite layered soil region with two embedded foundations. The 3D hybrid methodology is of the FEM hosted type, where the boundary element model is converted into macro-finite element, the nodal forces along the interface between two zones are expressed through the boundary element domain nodal tractions and the entire BEM formulation is converted to FEM-like approach. The 3D BEM model of the dynamically active far-field zone is inserted as a macro-finite element in the FEM commercial program ABAQUS. Developed hybrid BEM-FEM methodology based on the macro-element concept removes a need for many simplifying assumptions that are used in the most of the available results, for example rigid foundation, single foundation, homogeneous ground assumption, or neglect of soil layering and damaged state of the soil. Note that the proposed here computational tool allows a consideration of an arbitrary geometry of foundations and also of the all interface boundaries between layers.

The proposed mathematical model and the used hybrid tool are applied for investigation of the following key factors: (a) site effects phenomena due to the impedance contrasts of soil layers, (b) lateral inhomogeneity effects, (c) influence of the earthquake source properties, (d) existence of damage processes in soils, (e) conversion of body waves into surface waves, (f) foundation-soil-foundation dynamic interaction. This is realized by taking into account the mechanical, geometrical and geological features throughout the wave path from the dynamic source to the free-surface of the local geological profile with heterogeneities of natural character or human activity. The results obtained show that in more realistic structural models, the wave pattern at a site is a complex result of different physical and mechanical factors such as non-parallel layering, earthquake source properties, damage state of the soils and SSI phenomena. Most of the results in the literature take into account either one or two of the physical mechanisms controlling the dynamic response, while the hybrid approach we propose here has the potential to study simultaneously the combined effects of different key factors discussed above.

As a future study, the authors plan to develop further the proposed here hybrid technique for solution of more complex boundary value problems modeling nonlinear, inelastic soil, interface and structural material behavior. This will be realized via modeling of dynamic behavior of semi-infinite soil region (macro-element model) via 3D transient Boundary Element-Convolution Quadrature Method based on the following steps: (i) use of fundamental solutions in the Laplace transformed domain; (ii) numerical approximation of the Riemann time convolution integrals in the time-domain BEM equations, by the Lubich [42] quadrature formula; (iii) a linear multi-step method, which provides direct solution in the time domain.

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