Inverse kinematics based on backbone curve for a hyper-redundant tensegrity bird-neck robotic mechanism

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Abstract. In this paper, we propose a hyper-redundant tensegrity bird-neck robotic mechanism. To study the inverse kinematics of this mechanism, we propose a method with two steps. Firstly, the backbone curve for the bird neck is used to describe the macroscopic shape of the mechanism. Secondly, by coordinating the internal forces of the cables when the bird-neck tensegrity model is in equilibrium, we can get all cable lengths of the bird neck at a certain shape which is the inverse kinematics solution. Simulations are conducted to verify the validity of the proposed inverse kinematics method.

1. Introduction

Tensegrity is a constructing principle that was first described by the architect R. Buckminster Fuller and first visualized by the sculptor Kenneth Snelson [1]. Fuller defines tensegrity systems as structures that stabilize their shape by continuous tension or ‘tensional integrity’ rather than by continuous compression. Tensegrity has some unique characteristics of light weight, impact resistance, and often efficient when employed as certain structure system and the load can be distributed in a nonconcentrated and distributed manner. The musculoskeletal system of the organisms can also be effectively modeled by the tensegrity structure [2], due to its similarity with the compressive rod unit and the tensioned cable unit of the tensegrity. The use of tensegrity structures in the bionic field is one of the promising applications of tensegrity. K.W. Moore et al. have used tensegrity structures to simulate the motion of animal pectoral fins [3]. M. Furet et al. designed and optimized the X-shape Snelson tensegrity mechanism used in a bird neck model, which can move in the two-dimensional plane [4].

In this paper, a bird-neck mechanism is modelled using a tensegrity structure, with the skeletal equivalent of rod and the muscular equivalent of cable. And we derived the inverse kinematic solution of the mechanism, which is the length of cables in the mechanism, in two steps, to control the movement of the end of the bird neck in three dimensions. First, we discrete the trajectory of the mechanism and describe its macroscopic three-dimensional shapes with backbone curves. Then we obtain the inverse kinematic solution by coordinating the internal forces of the cables.

2. Description of the structure

The structure of the hyper-redundant tensegrity bird neck robotic mechanism is shown in Figure 1. The structure consists of nine rigid bodies, which is an X-shaped structure with a torsion angle of 90 degrees. Eight elastic cables (four vertical cables and four saddle cables) are used to connect between each two rigid bodies. There is no rigid contact between the rigid bodies, and the force is distributed to the whole
structure through the tension net, eliminating the moment at the hinge point. The structure conforms to
the concept of tensegrity structure and has the advantages of light weight, low cost, no rigid-rigid joints.

![Figure 1. Structure of the bird neck mechanism](image)

For a three-dimensional tensegrity structure, there are $n$ nodes on it and $s$ cables connecting its
internal rigid bodies. For our structure, $n = 45$ and $s = 64$. The coordinate matrix of nodes is donated
as $\mathbf{N}(\in \mathbb{R}^{n \times 3})$. A connectivity matrix can be used to describe the topology of a tensegrity structure [5]. A
connectivity matrix for the structure is presented by $\mathbf{C}(\in \mathbb{R}^{s \times n})$. The rows of the matrix represent cable
members and the columns represent nodes. Suppose member $k$ connects nodes $i$ and $j$ ($i < j$). Then,
the $i$th and $j$th member of the $k$th row of $\mathbf{C}$ are set to 1 and -1, respectively. The expression is as
follows:

$$\mathbf{C}_{(k,i)} = \begin{cases} 1, & l = i \\ -1, & l = j \\ 0, & l \neq i, l \neq j \end{cases}$$

In our mechanism, the bottom rigid body is fixed and other rigid bodies can move by changing the
length of cables. We define the center of the bottom fixed rigid body as the starting-point and the center
of the top rigid body as the end-point. We study the inverse kinematics of end-point. The bird neck
robotic mechanism in this paper meets the definition of “hyper-redundant” robots [6], which has a large
or infinite motion redundancy [7]. Therefore, it is difficult to calculate its inverse kinematic solution.

3. The backbone curve
To simplified the mechanism, we define the curve formed by the centers of the nine rigid bodies as the
“backbone curve”. By describing the macroscopic shape of the mechanism in terms of the backbone
curve, the inverse kinematic problem of the mechanism is transformed into the problem of determining
the backbone curve.

For a given trajectory of the end-point, there is a series of continuous morphology of the bird neck.
We discretize the trajectory of end-point into a set of points, and obtain the local coordinate system of
each point by the Frenet–Serret formulas. Given a starting-point pose and an end-point pose, a
continuous smooth backbone curve can be determined using the computed results and available
techniques for parametric presentation of space curves [8].

We fit the backbone curve with a spline curve, due to spline curve has the characteristics of
continuous, uniform curvature change, and the fitting process is all real-time numerical calculation. This
paper discusses the case of fitting with 5 times spline curves with the following polynomial
representation of the curves:

$$r(u) = \sum_{j=1}^{5} b_j u^{j-1} \quad (0 < u < L)$$

(2)
Where $r$ is the position vector of any point in the spline curve, $b_j$ is the coefficient in the polynomial. $L$ is the length of curve that can be calculates as follows:

$$ L = \begin{cases} \frac{1}{3}L_{\text{init}} & L_x \geq L_{\text{init}} \\ \frac{2}{3}L_x & L_x < L_{\text{init}} \end{cases} $$

(3)

Where $L_{\text{init}}$ is the distance between the starting-point and the end-point, $L_{\text{init}} = 552\text{mm}$ is the initial length of curve.

There are 6 unknown coefficients in the curve polynomial for an end-point. Substitute the position vectors and their derivatives of the start point and end point, $p_0, p_1, p'_0, p'_1, p''_0, p''_1$, into (2), we can obtain the value of the coefficients $b_j$. Let’s define a new variable as follows:

$$ \beta = u/L $$

(4)

(2) can be rewritten as:

$$ r(\beta) = P \cdot f(\beta) \quad (0 \leq \beta \leq 1) $$

(5)

Where: $P = [p_0, p_1, p'_0, p'_1, p''_0, p''_1]$, $f = [f_0, f_1, f_2, f_3, f_4, f_5]$

$$ f_0 = 1 - 10\beta^3 + 15\beta^4 - 6\beta^5 $$

$$ f_1 = 10\beta^3 - 15\beta^4 + 6\beta^5 $$

$$ f_2 = \beta \left( -4\beta^2 + 7\beta^3 - 3\beta^4 \right) L $$

$$ f_3 = \beta \left( 1 - 6\beta^2 + 8\beta^3 - 3\beta^4 \right) L $$

$$ f_4 = \beta \left( \beta / 2 - \beta^2 + \beta^3 / 2 \right) L $$

$$ f_5 = \beta \left( \beta / 2 - \beta^2 + \beta^3 / 2 \right)^2 L $$

We determine nine points on the curve by linearly increasing $\beta$ from the starting-point to the end-point. These nine points correspond to the centers of the nine rigid bodies, which are the origin of the local coordinate systems on rigid bodies.

We define $z$-axis direction of each coordinate system as the tangent direction of the curve at the point. The tangent vector at any point on the curve can be calculated by:

$$ t(\beta) = \frac{r'(\beta)}{\|r'(\beta)\|} $$

(6)

In order to define the direction of the $x$-axis, we define a linear increasing function $\varphi_s(s)$:

$$ \varphi_s(0) = 0, \varphi_s(L) = \varphi_w $$

(7)

Where $\varphi_w$ is the angle between the $x$-axis of the starting-point coordinate system and the end-point coordinate system.

Defining a vector as follows:

$$ n = \frac{n_x \left( 1 - \cos \varphi (s) \right) + \cos \varphi (s)}{n_x \left( 1 - \cos \varphi (s) \right) + \sin \varphi (s)} $$

$$ n = \frac{X_x \times X_y}{\| X_x \times X_y \|} $$

(8)

Where $X_x$ is the unit vector is on the $x$-axis at the starting-point, and $X_y$ is the unit vector is on the $x$-axis at the end-point.

The direction of local coordinate system for each rigid body can be calculated as follows:

$$ z = t_a(\beta_a) $$

$$ y = (t_a(\beta_a) \times v) / \| t_a(\beta_a) \times v \| $$

$$ x = y \times t_a(\beta_a) $$

(9)

To verify the effectiveness of the backbone curve fitting, we give an end-point trajectory as follows:

$$ p(t) = \begin{bmatrix} \sin(2\pi t) & 1 - \cos(2\pi t) & 10 - 2\pi t \end{bmatrix} $$

(10)
Where \( t \) denotes time.

The results of the fitting of the backbone curve with time are shown in Figure 2. The red spiral curve represents the end point trajectory, and the curve between the starting-point and the end-point is the fitted backbone curve, which has nine local coordinate systems corresponding to each rigid body. For display purposes, there are 20 spline curves in the figure. In fact, the discrete widths should be defined according to time.

![Figure 2. Results of curve fitting](image)

We can obtain relative positions of the nine rigid bodies and the coordinate matrix of nodes \( N \) by the backbone curve. Six equilibrium equations can be listed between two adjacent rigid bodies based on forces and moments in three directions. However, since there are eight cables between the two rigid bodies, the force in a cable is multi-soluble.

4. **Inverse kinematics solution**

To solve the problem of multiple solutions of forces on cables, we need to coordinate the internal forces in the structure. The static equilibrium equation of the structure can be represented by:

\[
\mathbf{F}_m = C' \mathbf{QCN}
\]

Where \( \mathbf{F}_m \) denotes the combined external force received by the structure, \( \mathbf{Q} = \text{diag}(q) (\in \mathbb{R}^{3\times3}) \) denotes the force density of the cable members. Usually, the external force is the gravity of the structure itself.

Divide all vectors into \( x, y, z \) directions, the two ends of the equation can be denoted as follows:

\[
\begin{bmatrix}
\mathbf{F}_x \\
\mathbf{F}_y \\
\mathbf{F}_z
\end{bmatrix} =
\begin{bmatrix}
\mathbf{C}' \text{diag}(\mathbf{C}x) \\
\mathbf{C}' \text{diag}(\mathbf{Cy}) \\
\mathbf{C}' \text{diag}(\mathbf{Cz})
\end{bmatrix}
\]

Thus, the equation can be simplified as:

\[
\mathbf{Aq} = \mathbf{F}
\]

The solution of the equation is expressed as:

\[
\mathbf{q} = \mathbf{q}_g + \mathbf{q}_{in}
\]

\[
\mathbf{q}_g = \mathbf{A}^+ \mathbf{F} \quad \mathbf{q}_{in} = (\mathbf{I} - \mathbf{A}^+ \mathbf{A}) \mathbf{w}
\]

Where \( \mathbf{A}^+ \) is the pseudo inverse matrix of \( \mathbf{A} \) and \( \mathbf{w} \) is an arbitrary column vector. \( \mathbf{q}_g \) is the general solution of the equation (13). \( \mathbf{q}_{in} \) is particular solution of (13) which donates the internal force term of the cables. The change of \( \mathbf{q}_{in} \) affects the interaction within the structure and does not deform or move the structure.
In order to minimize energy consumption, we should choose a suitable $w$ to make $q_{\text{in}}$ as small as possible. At the same time, considering the requirement that the cables must be tensioned, the force density of the rope must be bigger than zero. Therefore, the solution needs to satisfy:

$$q_i = A^T F + (I - A^T A) w \geq p_0$$

$$w \Rightarrow (q_i^2)_{\text{min}}$$

(15)

Where $p_0$ denotes the minimum preload force.

The internal force of the cable $f_i$ can be calculated with $f_i = q_i l_i$, when the force density $q_i$ for each cable is found. According to Hooke law and the relationship between the cable lengths represented in the Figure 3. The length of the cables after elongation can be solved by:

$$l_{ui} = l_i - f_i/k_i$$

(16)

Where $l_i$, $f_i$, $k_i$ is the length, internal force and stiffness coefficient of $i$ th cable.

Figure 3. Relationship between the cable lengths

When the end-point is at the specified position, the length of all cables is the inverse kinematic solution we require. We need to verify the correctness of the solution.

5. Simulation of the bird-neck motion

We use NTRT (NASA Tensegrity Robotics Toolkit), which is an open source dynamics simulation software specifically for tensegrity structures, to verify the results of the backbone curve and the inverse solution. The initial model built in NTRT is shown in Figure 4 (a). The length of the 64 cables in the model can be changed by ROS (Robot Operating System).

To verify the effect of the model, we give the position and attitude of the end of the model and input the results of the inverse kinematics solution into NTRT. The comparison between the theoretical poses of the model and the actual poses of the model in NTRT is shown in Figure 4 (b).

Figure 4. Comparison of theoretical and actual poses of the model

To verify the accuracy of the inverse solution during the motion of the mechanism, we gave trajectories of the end-point and programmed for simulation. The flow of the control in the simulation is as follows:

Step1: Initialize the model and discrete end-point trajectory into a set of points.
Step2: Given an end-point pose, calculate the bone curve and obtain the coordinates of nodes.
Step3: Calculate the force density $q$ on each cable according to (14).
Step4: Calculate the length of each cable according to (15).
Step5: Change the cables’ length in the model via ROS.
Step6: Go to Step2 until all discrete points have been used.
Step7: End.
We give a 1-D motion trajectory which is a sine curve and a 3-D motion trajectory which is a spiral curve in the simulation. The Figure 5 shows the comparison between the actual trajectories and the expected trajectories when.

![1-D trajectory](image1)

![3-D trajectory](image2)

Figure 5. Comparison between the actual trajectory and the expected trajectory

It can be seen that the actual trajectories match the expected trajectories and the tracking error is within the reasonable range. Thus, the backbone curve method and internal force coordination method used in this paper can effectively obtain the inverse kinematics solution of the bird-neck mechanism.

6. Conclusion

In this paper, we propose a hyper-redundant tensegrity bird-neck robotic mechanism, according to the configuration of the bird neck and the characteristics of the tensegrity structure. For the inverse kinematics solution of the mechanism, we propose a method which combines the backbone curve method and method of coordinating the internal force of the cables. By this method, we can get the cables length which is the inverse kinematics solution under the condition of ensuring the cables tensioning. Also, the effectiveness of the results of this control method is verified in the simulation in NTRT.

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