A SEARCH FOR LARSON-TYPE RELATIONS IN NUMERICAL SIMULATIONS OF THE ISM: EVIDENCE FOR NONCONSTANT COLUMN DENSITIES

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ABSTRACT

We present results from a statistical study of clouds in two-dimensional numerical simulations of the interstellar medium. The clouds in the simulations exhibit a differential mass spectrum \( dN(M)/dM \sim M^{-1.44 \pm 0.1} \) and a velocity dispersion–size relation \( \Delta v \sim R^{0.41 \pm 0.08} \). However, the clouds do not exhibit a clear density-size relation. At a given mean density, clouds span a range of sizes from the smallest resolved scales up to a maximum given by a Larson-type relation \( R_{\text{max}} \sim \rho^a \), with \( a = -0.81 \pm 0.15 \), although numerical effects cannot be ruled out as being responsible for the latter correlation. The clouds additionally span a range of column densities \( N \) of 2 orders of magnitude, supporting the suggestion that the observational density-size relation may be an artifact of survey limitations. In this case, the \( \Delta v-R \) relation can be interpreted as a direct consequence of a \( k^{-2} \) turbulent spectrum rather than of virial equilibrium of clouds that satisfy a \( \rho \propto R^{-1} \) law. The \( k^{-2} \) spectrum is verified in the simulations and is characteristic of a field of shocks. However, we also discuss the possibility that the clouds are in balance between self-gravity and turbulence but with a scatter of at least a factor of 10 in the \( \Delta v-R \) relation, and of 100 in the density-size relation, according to the equilibrium relation \( \Delta v \sim \left( N R \right)^{1/2} \).

In addition, we compare these results with recent observational data. We propose a simple model suggesting that recent results that find nearly constant column densities for dark IRAS clouds may be an artifact of a temperature gradient within the clouds induced by external radiative heating. As a consequence, we emphasize that IRAS surface brightness maps are not appropriate for measuring column densities.

Subject headings: ISM: clouds — ISM: kinematics and dynamics — ISM: structure — methods: numerical — turbulence

1. INTRODUCTION

Interstellar clouds appear to follow a set of scaling relations first noted by Larson (1981) and then apparently confirmed (with slight modifications) by a number of other workers. These "Larson's relations" have the form

\[
\rho \sim R^a, \quad (1a)
\]

\[
\Delta v \sim R^b, \quad (1b)
\]

where \( R \) is the cloud size, \( \rho \) is the gas density, \( \Delta v \) is the velocity dispersion derived from the line widths, and \( a \) and \( b \) are the constant scaling exponents. In addition, the clouds are found to exhibit a mass distribution of the form

\[
\frac{dN(M)}{dM} \sim M^n. \quad (2)
\]

The most commonly quoted values of the exponents are \( a \approx -1.15 \pm 0.15 \), \( b \approx 0.4 \pm 0.1 \), and \( n \approx -1.55 \pm 0.15 \) (Larson 1981; Torrelles et al. 1983; Dame et al. 1986; Falgarone & Pérault 1987; Myers & Goodman 1988; Falgarone, Puget, & Pérault 1992, hereafter FPP; Fuller & Myers 1992; Miesch & Bally 1994; Wood, Myers, & Daughtery 1994, hereafter WMD; Caselli & Myers 1995; see also the reviews by Scalo 1985, 1987; Blitz 1991). However, significantly discrepant values have also been reported (e.g., Carr 1987; Loren 1989), and the validity of these scaling relations is currently the subject of strong controversy within the community.

The above "standard" values of the exponents for equations (1a) and (1b) have been interpreted in terms of the virial theorem (e.g., Larson 1981; Myers & Goodman 1988; Caselli & Myers 1995). For \( a = -1 \) (which coincidentally implies constant column density), a value \( b = 0.5 \) implies virial balance between self-gravity and the internal velocity dispersion. However, note that for an arbitrary value of the density scaling exponent \( a \), a corresponding virial balance value of \( b \) can always be found (Vázquez-Semadeni & Gazol 1995). Thus the density-size relation (eq. [1a]) remains unexplained. In fact, it has been proposed by Scalo (1990) that this relation may be a mere artifact of the dynamic-range limitations of the observations and does not reflect a real property of interstellar clouds. In particular, in the case of molecular line data, the observations are restricted to column densities large enough that the tracer molecule is shielded against photodissociating radiation. On the other hand, while the proportionality between line-integrated CO intensity and mass surface density has been reliably established for extragalactic observations (Dickman, Snell, & Schloerb 1986), this relationship is only valid for scales at which calibrations have been possible, i.e., larger than a few parsecs. Furthermore, for clumps within molecular clouds, the structures identified in CO often do not correspond to those identified with higher density tracers (e.g., Massi & Lizano 1994; J. Scalo 1996, private communication).

In this paper, we present the mass spectrum of clouds and search for Larson-type correlations in three two-dimensional numerical simulations of turbulence in the interstellar medium (ISM), one from Passot, Vázquez-Semadeni, & Pouquet (1995, hereafter Paper I) and the other two being variants of the former with respectively larger density contrasts and larger resolution. We have
developed a cloud-identifying algorithm that allows us to measure the average density, velocity dispersion, and total mass within well-defined (though arbitrary) cloud boundaries in the density fields of the simulations. The cloud sample thus obtained has the advantage over actual observations that no tracer limitations exist in “detecting” the clouds, although, on the other hand, a form of saturation is present as a result of the relatively small density dynamic range and other numerical limitations of the simulations.

In § 2, we briefly describe the simulations and the cloud-identifying algorithm, and in § 3, we present the resulting statistical cloud properties. In § 4, we analyze the limitations of the simulations, discuss the implications of the absence of a density-size relation, and give a preliminary discussion of the physical mechanisms behind the velocity dispersion-size relation. We also compare the results with recent corresponding observations, in particular those of WMD.

Finally, § 5 summarizes the results.

2. NUMERICAL METHOD

Paper I presented magnetohydrodynamic simulations of the ISM, incorporating model terms for cooling, diffuse heating, and local heating from star formation. The simulations solve the equations for the evolution of the density, velocity, internal energy, and magnetic fields in the presence of self-gravity, namely,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = \rho \nabla^2 \rho ,$$

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{\nabla P}{\rho} - v_8 \nabla^2 u - \left( \frac{J}{M_u} \right)^2 \nabla \phi + \frac{1}{\rho} (\nabla \times B) \times B - 2 \Omega \times u ,$$

$$\frac{\partial e}{\partial t} + u \cdot \nabla e = - (\gamma - 1) e \nabla \cdot u + \frac{\nabla^2 e}{\rho} + \Gamma_d + \Gamma_x - \rho \Lambda ,$$

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) - v_8 \nabla^2 B ,$$

$$\nabla^2 \phi = \rho - 1 .$$

We refer the reader to Vázquez-Semadeni, Passot, & Pouquet (1995a) and to Paper I for full details of the simulations. Here we just point out that the simulations from which the data are extracted represent a square section of the ISM along the Galactic plane of size 1 kpc on a side, with a resolution of 512 grid points per dimension. It is also important to note that all the evolution equations contain dissipative or diffusive terms, which are necessary since the numerical technique used to solve the equations (spectral method) does not produce numerical viscosity, so the dissipation must be included explicitly. The momentum and magnetic field equations contain “hyperviscosity” terms of the form $\nabla^2$, which confine the viscous effects to the very smallest scales in the simulations (Babiano et al. 1987; McWilliams 1984). However, although globally dissipative, hyperviscosity is not everywhere positive definite (Passot & Pouquet 1988), so standard Laplacian terms are used in the continuity and internal energy equations.

Throughout this paper, densities are expressed in units of $1 \text{ cm}^{-3}$ and velocities in units of $11.7 \text{ km s}^{-1}$, the units used in Paper I. In particular, in the present paper, we will use data from the run labeled run 28 in Paper I. However, the star formation scheme used in the simulations of Paper I assumes that a star is formed wherever the local density exceeds a critical value $\rho_*$. (A “star” in the simulations is a point source of heat.) This naturally imposes an upper limit on the densities reached by the model, since the stellar heating increases the local pressure and causes the gas to expand. Thus densities above $\rho_*$ are very rarely reached. In run 28, we have $\rho_* = 30$, limiting the density contrast $\rho_{\text{max}}/\rho_{\text{min}}$ to values $\sim 1000$. In order to obtain a somewhat larger dynamic range, we have performed an additional run, called run 28bis, which is identical to run 28 up to $t = 6.5 \times 10^7$ yr but afterward has the star formation turned off. This run ends up collapsing gravitationally at $t \sim 8.8 \times 10^7$ yr because of the lack of support from stellar energy injection, but intermediate times provide a good framework for study, exhibiting density maxima $\geq 100 \text{ cm}^{-3}$ and density contrasts $\geq 5000$. Finally, a run similar to run 28bis but at a larger resolution $(800 \times 800$, referred to as run 28.800) was also performed, in order to discuss the effects of dissipation. As an illustration, Figure 1 shows a contour plot of the density field of run 28 at $t = 6.6 \times 10^7$ yr, with the velocity field represented by the arrows. Note that, although the size scales represented by the runs are larger than those studied by most papers concerned with cloud statistics, we expect the results to be applicable since all terms in the equations solved in the simulations are scale-free except for the dissipative terms.

In order to investigate the statistical properties of the clouds in the simulations, we have developed an automated algorithm that identifies and labels clouds. We define clouds as connected sets of pixels with densities above a user-defined density threshold, $\rho_*$. The types of clouds that are

![Fig. 1.—Density (contours) and velocity (arrows) fields for run 28 at $t = 6.6 \times 10^7$ yr. The contours are spaced logarithmically, with increments of 0.3125 in log $\rho$. The minimum density in this plot is 0.04 cm$^{-3}$, and the maximum density is 40 cm$^{-3}$.](image-url)
identified in this way are thus clearly dependent on the value chosen for \( \rho_i \), a situation somewhat similar to performing observations by using various density tracers. As an example, Figures 2a and 2b show the clouds obtained by respectively setting \( \rho_i = 4 \) and \( \rho_i = 16 \), at time \( t = 6.6 \times 10^7 \) yr, in the evolution of run 28.

The extremely irregular shapes of most clouds are noteworthy and, in fact, pose a problem in defining the “size” \( R \) of a given cloud. For simplicity, here we take the size of a cloud to be the square root of the number of pixels it contains. This definition may be somewhat unrealistic if clouds are fractal in such a way that their perimeters are not proportional to the square root of their areas (see, e.g., Scalo 1990; Falgarone, Phillips, & Walker 1991), but for simplicity and similarity to observational work we adopt it here. Also, for convenience, sizes are expressed in pixels in most of the figures below (1 pixel \( \sim 2 \) pc).

For each cloud found by the algorithm, it is then a trivial matter to compute the average density, the mass, and the turbulent velocity dispersion, calculated as \( \Delta v = (\langle u - \langle u \rangle \rangle)^{1/2} \), where \( u \) is the local value of the velocity and \( \langle u \rangle \) is the average over the cloud area. Note that this quantity is not density weighted.

3. STATISTICAL CLOUD PROPERTIES

We have obtained relatively large samples of clouds by considering values \( \rho_i = 3, 4, 6, 8, 11, 16, 23, \) and 29 in run 28 and \( \rho_i = 3, 4, 6, 8, 11, 16, 23, 32, 45, 64, \) and 90 in run 28bis. Clouds obtained with each value of \( \rho_i \) are indicated with a particular symbol type in the plots discussed below. In what follows, the data from run 28 will be considered at \( t = 6.6 \times 10^7 \) yr, and from run 28bis at \( t = 7.15 \times 10^7 \) yr, unless otherwise noted. The cloud samples for these cases contain 158 and 145 clouds, respectively.

Figures 3a and 3b show the logarithmic mass distributions of the clouds for runs 28 and 28bis, respectively. The masses are expressed in the non-dimensional units of the simulations, in which the total mass contained is \( 4\pi^2 \), corresponding to \( 0.36 \times 10^7 M_\odot \) in real units. The turnover at low masses may be attributed to incompleteness due to the finite resolution. The high-mass sides of the distributions, however, exhibit least-squares slopes (fitted for log \( M > 0.8 \) and shown as diagonal lines) that imply indices \( n = -1.51 \) and \(-1.43 \), respectively (cf. eq. [2]). These values are within the range of values reported in various observational results (e.g., WMD; see also the reviews by Scalo 1985, Blitz 1991, Mundy 1994, and references therein).

Figures 4a and 4b show the correlation between velocity dispersion and size for all clouds with sizes \( \gtrsim 2 \) pixels (1 pixel clouds are excluded, as they have no velocity dispersion). Least-squares fits to the data yield slopes (solid lines) \( \beta = 0.37 \) and \( \beta = 0.39 \), respectively, for runs 28 and 28bis, with moderately tight correlation coefficients \( \sim 0.6 \). The large scatter of \( \sim 1 \) order of magnitude in the correlations should be emphasized. This is comparable to the scatter found for molecular line data (e.g., FPP). The derived values of \( \beta = 0.4 \) are slightly lower than the most commonly accepted value of 0.5 (dotted lines) but are remarkably close to determinations that include heterogeneous data sets (Larson 1981; FPP; Fuller & Myers 1992).

Figures 5a and 5b show the average density \( \langle \rho \rangle \) versus size for the snapshots of the two runs. Several points should be remarked upon. First, note that in general the average density of the clouds is quite similar to the value of \( \rho_i \) used to define the cloud and, in fact, tends to increase with size at each \( \rho_i \). This is not surprising, because of the presence of dense “cores” inside the largest clouds at each \( \rho_i \), which tend to increase their average density. Second, no clear correlation can be seen in either figure. Instead, at a given \( \rho_i \), clouds down to the smallest possible size are seen. These are small clouds with low densities and, therefore, low

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**Fig. 2a**

Regions ("clouds") with densities larger than an arbitrary threshold \( \rho_i \) in the density field of run 28 at \( t = 6.6 \times 10^7 \) yr. (a) For \( \rho_i = 4 \). (b) For \( \rho_i = 16 \). Note that small clouds with high densities are nested within larger complexes, but small clouds with low densities can be seen in (a) as well.
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FIG. 3

Fig. 3.—Logarithmic mass distributions for (a) run 28 at \( t = 6.6 \times 10^7 \) yr and (b) run 28bis at \( t = 7.15 \times 10^7 \) yr. The diagonal lines show least-squares fits to data bins with \( \log M > 0.8 \), and have slope \(-0.51\) in (a) and \(-0.43\) in (b).

column densities\(^1\) \( N = \rho R \). Third, the size of the largest cloud at each \( \rho_\ell \) is smaller for larger \( \rho_\ell \). In particular, in both figures the set of largest clouds at each \( \rho_\ell \) seems to lie near a

\(^1\) Note that the column density defined here refers to a cut through the clouds on the plane of the simulations, and has units of cm\(^{-2}\). To obtain a column density with units of cm\(^{-1}\), appropriate to our two-dimensional problem, a multiplication by the unit length along the third \( z \)-dimension, perpendicular to the plane of the simulation, should be performed. For simplicity, we omit this constant factor throughout the paper. A similar situation applies to the computation of masses.

\( \rho \sim R^{-1} \) law, similar to the standard exponent in Larson’s relation (eq. [1a]).

In order to test the robustness of the above results, we performed the same analysis at several other times in both runs, namely, at \( t = 7.8, 9.1, 10.4, 11.7, \) and \( 13.0 \times 10^7 \) yr in run 28, and \( t = 7.8 \) and \( 8.6 \times 10^7 \) yr in run 28bis. The average value of the exponent \( \beta \) of the velocity dispersion–size relation is \( \langle \beta \rangle = 0.41 \pm 0.08 \), with typical correlation coefficients \( \sim 0.6 \). For the mass spectrum, an average exponent \( \langle n \rangle = -1.44 \pm 0.1 \) is found. The errors are the stan-

FIG. 4

Fig. 4.—Velocity dispersion vs. size for all clouds in (a) run 28 at \( t = 6.6 \times 10^7 \) yr and (b) run 28bis at \( t = 7.15 \times 10^7 \) yr. Clouds obtained with each value of \( \rho_\ell \) are shown with a specific symbol, as indicated. The solid lines show least-squares fits, with the slopes and correlation coefficients indicated. For reference, a \( \frac{1}{2} \) slope is indicated by the dotted lines.
F.5. Average cloud density vs. size for all clouds in (a) run 28 at $t = 6.6 \times 10^7$ yr and (b) run 28bis at $t = 7.15 \times 10^7$ yr. The lines show a $\rho \propto R^{-1}$ power law. In both cases, the largest cloud at each $\rho_i$ lies near the $\rho \propto R^{-1}$ line, although other times tend to show shallower envelopes (see Fig. 6). The full ensemble of clouds does not show any trend of $\langle \rho \rangle$ with $R$. The cloud labels in (a) and (b) are respectively the same as in Figs. 4a and 4b.

Regarding the density, in all cases small clouds with low densities exist, the plots (not shown) being qualitatively similar to Figures 5a and 5b, and the full ensemble of clouds not exhibiting any correlation with size. The largest clouds at each $\rho_i$, on the other hand, continue to exhibit a nearly power-law relation with size, with average exponent $\langle \alpha \rangle = -0.81 \pm 0.16$. This is smaller than the slopes found in Figures 5a and 5b, which coincidentally seem to have some of the steepest slopes in the distribution. This is illustrated in Figure 6, which shows density versus size for the largest cloud at each value of $\rho_i$ at all times considered above for the two runs. In this figure, clouds obtained with a given value of $\rho_i$ at any one time in either run are shown with the same symbol and joined by a dotted line. The resulting curves have been displaced by increments of 0.2 in log $\langle \rho \rangle$ for clarity. For reference, the solid line shows a $\langle \rho \rangle \propto R^{-1}$ power law. The three uppermost curves in the figure correspond to run 28bis and, because of the larger density dynamic range of this run, exhibit power-law behavior over a larger range of scales while those for run 28 saturate at $\rho \sim 30$. For this reason, only clouds with log $R > 1$ for run 28bis, and with log $R > 1.5$ for run 28, were considered in computing $\langle \alpha \rangle$. The implications of these results are discussed in § 4.2.1. For convenience, in what follows, we drop the brackets when referring to the average density of clouds.

Finally, it should be pointed out that, although clouds with low column densities exist in the simulations, most of the mass still resides in the largest clouds since the distribution of cloud sizes at a given mean density appears to be standard deviations of the set of values found for all times. These results confirm the fact that the simulations show correlations consistent with the observations in both cases.

For 4.2.1, what follows, we drop the brackets when referring to the average density of clouds.
roughly uniform. However, this is possibly an effect of the absence of supernovae in the simulations, since the “expanding H II regions” included do not have enough power to disrupt the largest gravitationally bound complexes.

4. DISCUSSION

4.1. Applicability and Limitations of the Results

The results presented in §3 have important implications provided that the simulations are indeed representative of ISM dynamics. That this is likely to be the case, despite their two-dimensionality, is suggested by the fact that the simulated ISM reproduces both the velocity dispersion—size relation and the mass spectrum of the clouds (see §3), as well as other physical properties of the ISM, such as the mean density of giant cloud complexes (Vázquez-Semadeni et al. 1995a), the cloud and intercloud magnetic field strengths (Paper I), and the rate of formation of massive stars (Vázquez-Semadeni, Passot, & Pouquet 1995b).

However, one important possible criticism due to the two-dimensionality is that a $\rho \sim R^{-1}$ scaling relation in three dimensions might translate into a $\rho \sim R^0$ relation in two dimensions, just because of the elimination of one dimension. This could be at the origin of the near constancy of $\langle \rho \rangle$ observed at every value of $\rho_t$. Closer examination shows that this argument is invalid. Even though the clouds seem to have nearly constant densities at each value of $\rho_t$, this only reflects the fact, due to our cloud-identifying algorithm, that small, dense clouds are not seen at small $\rho_t$ since they are “hidden” within the larger, low-density clouds. But obviously the density is not constant in the simulations, invalidating the possibility of an $R^0$ dependence of the density.

Another important source of concern is that a significant fraction of the clouds in the samples have sizes of only a few pixels, and their properties are thus likely to be affected by viscosity and diffusion. Thus it is important to quantify the extent to which the results of §3 might be influenced by these terms. In particular, the question arises whether the existence of small, low column density clouds might be an artifact of the dissipative terms.

Concerning viscosity, because of the hyperviscosity scheme with a $V^8$ operator, its effects on the velocity field are confined to wavenumbers $k$ in the range $\frac{1}{2}k_{\text{max}} \leq k \leq k_{\text{max}}$ with $k_{\text{max}} = 255$ for runs 28 and 28bis, as can be seen from the spectrum in Figure 10 below (see §4.2.2). The same applies to the dissipative term in the magnetic equation (see Fig. 5 of Paper I). Naively, one would then expect the effects of viscosity to be present at scales up to twice the smallest scale of the simulation, i.e., from 1 to 2 pixels. Actually, the correspondence between scale ranges in real and Fourier spaces is not as sharp, and one can expect “leakage” up to possibly 4 pixels. Visual inspection of the velocity field confirms that shocks are typically spread over 4 pixels. Nevertheless, note that viscosity is not effective if the velocity gradients are not large, and thus the 4 pixel estimate is an upper limit to the sizes affected by viscosity. It is worth pointing out that, in Paper I, the range of influence of viscosity was estimated at $\sim 5$ pixels. However, this was an overconservative estimate not based on a detailed analysis of shock widths, and the 4 pixel figure given here should be considered a slightly more precise estimate based on the above considerations.

In order to correct for viscosity effects, clouds of up to 4 pixels in size are excluded from the “corrected” plots shown in the Appendix. Note that, since clouds have in general elongated shapes and sizes are computed as the square root of the number of pixels, the possibility exists that clouds with computed sizes larger than 4 pixels will still be 4 pixels or less across one particular direction. However, we believe this effect may be roughly compensated by the fact that the 4 pixel estimate is an upper limit, and thus we retain all clouds with sizes larger than this.

Of greater concern are the possible effects of mass diffusion, since the standard Laplacian diffusive operator used in the continuity equation causes diffusion to be important over a larger range of scales than the hyperviscosity. Indeed, the characteristic diffusion time $\tau_{\text{diff}}$ can be shorter than the turbulent crossing time $\tau_{\text{NL}}$ for clouds smaller than $\sim 16$ pixels (see Appendix), and diffusion may dominate over turbulent advection for those clouds. In particular, it is possible that the small, low-density clouds reported in the previous section may be a numerical artifact of the diffusion, which tends to reduce density peaks and spread the clouds out or, conversely, to prevent clouds from reaching higher densities and smaller sizes than they do in the simulations.

Note, however, that the effect of mass diffusion is exclusively to damp density gradients originated by the turbulent velocity field, and so diffusion is incapable of forming clouds by itself; instead, it only modifies the properties of clouds formed by turbulence or other processes (gravity or the various instabilities discussed in Paper I). We can thus obtain a crude estimate of the size and maximum density a cloud would have in the absence of diffusion by integrating the diffusion equation backward in time for initial conditions corresponding to the clouds in the simulation, over the length of a nonlinear time $\tau_{\text{NL}}$. This correction overestimates the effects of diffusion since it neglects the advection term entirely, and the nonlinear time is computed by using the turbulent velocity associated with the size of the cloud, as given by the turbulent spectrum in Figure 10 below. However, it is actually turbulent scales larger than a cloud’s size that are most likely to form it (Elmegreen 1993; Vázquez-Semadeni et al. 1995b). In particular, in the simulations at least, clouds often form at the interfaces of expanding shells from previous star formation events, which may have velocities of several kilometers per second, rather than the significantly smaller velocities ($<1$ km s$^{-1}$) indicated by the spectrum, which is a globally averaged quantity. Nevertheless, for robustness, we will use the worst-case correction. The details of this calculation are given in the Appendix. There it is shown that the central density of a cloud varies by factors of $3-5$ in the worst cases under the influence of diffusion. Using this “correction” factor and assuming that the clouds move along lines of constant mass in the density-size plot, one can produce a “corrected” such diagram, shown in Figure 16 in the Appendix for run 28bis. There it can be seen that, although clouds are indeed brought slightly closer to a correlation, at the lowest average densities cloud sizes still vary by factors of $\sim 100$, maintaining the conclusions from §3.

As a further test, we have also produced a preliminary higher resolution run, labeled run 28.800, similar to run 28bis but at a resolution of $800 \times 800$ pixels, the largest that we can perform on the Cray Y-MP of the Dirección General de Servicios de Cómputo Académico (DGSAC), Universidad Nacional Autónoma de México. The density
field for this run is shown in Figure 7. In order to produce this run, the data from run 28 at \( t = 6.5 \times 10^7 \) yr were interpolated to produce initial conditions for the \( 800 \times 800 \) simulation, and then evolved for another \( 0.65 \times 10^7 \) yr, enough to develop the additional small-scale structure corresponding to the larger resolution. Incidentally, it is worth noting that, even though run 28.800 has a resolution only \( \sim 60\% \) larger than runs 28 and 28bis, the computational effort it requires is roughly 5 times larger in CPU time. Figure 8 shows the resulting density-size raw plot for this run, including cloud sizes down to 1 pixel. In the Appendix, Figure 17 shows the corresponding plot incorporating the corrections described above, namely, elimination of clouds with sizes \( \leq 4 \) pixels and the correction for diffusion. In order to maximize the available dynamic range, in the figures in the Appendix we have used values of the density threshold as low as \( \rho_0 = 0.5 \). For run 28.800, it is seen that the clouds with the lowest average densities span a range of roughly a factor of 200 in sizes after the corrections, while the raw data exhibit a range of a factor \( \sim 500 \).

In summary, the discussions from this section suggest that the existence of small clouds with low densities described in § 3 is a true consequence of the dynamics and not an artifact of the dissipative terms used to stabilize the equations.

4.2. Implications of the Results

4.2.1. Density, Size, and Equilibrium

With the above considerations in mind, we have a number of direct implications. First, as already stated above, the absence of a density-size relation implies non-constant column densities for the clouds. Specifically, cloud column densities vary by over 2 orders of magnitude in the simulations. In addition, the result that clouds with sizes down to the smallest scales exist at all values of the mean density implies that the observed density-size relation may indeed be a product of the limited dynamic range of typical surveys. Under these conditions, the derived column densities could appear to show variation over a modest range only, while the observed sizes might span a range larger than 3 orders of magnitude, thus creating an apparent correlation. Observational effects as a possible origin of the density-size relation were first suggested by Larson (1981) himself and later discussed in more detail by Scalo (1990). Another reason for the appearance of a spurious density-size relation may be a selection effect introduced by the tendency of observational work to focus primarily on global intensity maxima of the maps, therefore possibly missing weaker, local maxima, a bias that only recently has started to be avoided (e.g., FPP).

The Larson-type relation defined by the largest clouds, with a scaling exponent \( \alpha \sim -0.8 \pm 0.16 \), is particularly interesting. The immediate question that arises is whether this relation is physical or is induced by numerical constraints. On the physical side, a first consideration is that, if the clouds are hierarchically nested (smaller, denser clouds are part of larger, less dense ones), then mass conservation implies \(-\alpha < 3\) in three dimensions. In our two-dimensional case, this limit becomes \(-\alpha < 2\). This is a physical limit, which in the simulations is pushed closer to the observed relation because of the two-dimensionality.

Another physical issue is whether the large clouds that follow a density-size scaling law are virialized, so that the standard scenario in which Larson’s relations hold for virialized clouds would apply to the largest clouds. However, examining cloud virialization in our simulations turns out not to be a straightforward task (§ 4.2.2). Here we just check whether the standard \( \Delta v—R \) relation is also satisfied in the large clouds. To this end, some clouds in Figures 4 and 5 have been labeled with numbers so that they can be identified from one figure to another. Interestingly, for run 28 at \( t = 6.6 \times 10^7 \) yr (case a in the figures), the largest clouds, which are very close to the constant column density line \( \rho \sim R^{-1} \), shown as the diagonal line in Figure 5a, are also very close to the line \( \Delta v \sim R^{1/2} \) in Figure 4a, suggesting balance between gravity and turbulence for this set of clouds. However, this is not the case for the clouds in run 28bis at \( t = 7.15 \times 10^7 \) yr (case b in the figures). In this case, although the clouds again show a slope very close to \(-1\) (Fig. 5b), it can be seen from Figure 4b that they all have comparable velocity dispersions. We conclude that even clouds with a \( \rho \propto R^{-1} \) density dependence are not necessarily in equilibrium between turbulence and self-gravity. Possibly, magnetic support is more important for the latter set of clouds, as in the results of Myers & Goodman (1988).

An interesting question is whether the several clouds along the large column density “envelope” of the distribution in the density-size relation are essentially the same cloud seen at several different thresholds \( \rho_0 \) or whether they are truly different clouds. In fact, the answer is that they are neither. This is exemplified in Figure 9, which shows a few selected branches of the cloud hierarchy for run 28bis. One branch off the largest cloud includes all the clouds along the envelope, but both the largest and second largest clouds are seen to also have other branches to daughter clouds off the envelope (dotted lines). The same is true of clouds lying immediately below the envelope (dashed lines), which seem to define a second envelope of similar slope (see Fig. 5).

On the numerical side, the mass diffusion term in the continuity equation may tend to reduce the column density of clouds defined through the threshold density criterion we use here, since the diffusion widens and smooths clouds, the outer parts of which may then be left out of the domain defined by \( \rho_0 \). This effect, plus the plain limitations imposed by the resolution, clearly prevents the formation of very small clouds with very large column densities, causing shallower slopes of the high column density envelope. In fact, while the average slope we obtain implies smaller column densities for smaller clouds, it has been pointed out by Scalo (1985, § III.A) that it is obvious from inspection of Lynds’ dark cloud catalog that smaller clouds are darker. Indeed, in our case, the correction discussed in § 4.1 and in the Appendix tends to bring this envelope toward steeper slopes. In summary, the specific slopes defined by the largest clouds in the \( \log \rho—\log R \) plot cannot be unambiguously attributed to real physical effects. High-resolution three-dimensional simulations are needed to resolve this issue. Unfortunately, the largest simulations of supersonic compressible turbulence known to us (e.g., Porter, Pouquet, & Woodward 1994, using 512\(^2\) grid points), are purely hydrodynamic and do not contain many essential ingredients of ISM dynamics, such as the magnetic field, self-gravity, and stellar (i.e., small-scale, compressible) forcing. Thus the necessary calculations are still a few years in the future.

In any case, regardless of what the specific slope of the envelope turns out to be upon removal of numerical effects, our results suggest that the notion of a density-size scaling
Fig. 7.—Logarithmic gray-scale image of the density field of run 28.800 at time $t = 7.15 \times 10^7$ yr. This run is similar to run 28bis, except that it has larger resolution ($800 \times 800$) and star formation is turned off at $t = 6.9 \times 10^7$ yr. The density extrema are $\rho_{\text{max}} = 129$ and $\rho_{\text{min}} = 3.4 \times 10^{-2}$. 
relation should probably be replaced by that of an “allowed” region in $\rho-R$ space. Whether the high column density boundary is truly a power law, the value of its corresponding index and the physical mechanisms responsible for it are issues that remain to be determined by high-resolution three-dimensional simulations.

4.2.2. Velocity Dispersion–Size Relation and Turbulence

Since the density-size relation is not verified for the clouds in the simulations yet the dispersion-size relation is, the standard argument explaining the $\Delta v-R$ relation, namely, virial equilibrium in clouds satisfying $\rho \propto R^{-1}$, cannot be invoked. This implication is independent of the dimensionality of the simulations, as it only relies on the nonexistence of a density-size relation and not on particular values of the scaling exponents. That is, the two-dimensionality would, at most, change the exponent in the virial equilibrium relation between $\Delta v$ and $\rho$, but not destroy the correlation altogether. Since in the simulations no density-size relation exists, a unique virial equilibrium relation between $\Delta v$ and $\rho$ does not exist either (but see §4.2.3). Note however that a Jeans-type analysis incorporating a “turbulent pressure” (Chandrasekhar 1951; Scalo & Struck-Marcell 1984; Bonazzola et al. 1987; Elmegreen 1991; Vázquez-Semadeni & Gazol 1995) such that

$$\Delta v \sim R^{\rho/2}$$

for clouds of size $R$ equal to their Jeans length. Since the latter is independent of dimensionality, equation (8) also holds independently of dimensionality. Thus the same scaling laws as in three dimensions are expected for two-dimensional clouds in balance between self-gravity and turbulence. This is consistent with a crude estimate of virial balance in which one equates the gravitational energy $W$ to the turbulent kinetic energy $K$. For simple cloud geometries and uniform densities and turbulent velocity dispersions, in three dimensions one obtains $GM^2/R = M(\Delta v)^2$. Taking $M \propto \rho R^3$ yields the usual result $(\Delta v)^2 \propto \rho R^2$. In two dimensions, on the other hand, $M \propto \rho R^2$, but the gravitational energy becomes $W \sim GM^2$, thus preserving the result $(\Delta v)^2 \propto \rho R^2$. However, such a simplified treatment may not be applicable, since in two dimensions logarithmic corrections appear, and also the gravitational term in the virial theorem may differ from the gravitational energy. A detailed analysis of this problem is in progress (Ballesteros-Paredes & Vázquez-Semadeni 1996). Thus the instability analysis is provisionally a more reliable indicator of the equilibrium relation expected in two dimensions.

In the absence of a density-size relation, a plausible origin of the velocity dispersion–size relation is the statistical properties of the turbulence itself. Indeed, an index $\beta = \frac{1}{2}$ is expected for turbulence characterized by an energy spectrum $E(k) \propto k^m$ with spectral index $m = -2$ (see, e.g., Bonazzola et al. 1987; Vázquez-Semadeni & Gazol 1995; Padoan 1995; Fleck 1996), where $k$ is the wavenumber associated with scale $R = 2\pi/k$. Such a spectral index is the signature of a field of random shocks (see Passot, Pouquet, & Woodward 1988 and references therein). Figure 10 shows the spectra for the incompressible (solid line) and irrotational (dashed line) parts of the velocity fields of Figure 1. For comparison, the dotted line shows a $k^{-2}$ power law. It is clear that the spectrum of the incompressible component is remarkably well described by this slope. The irrotational, or compressible, component exhibits somewhat stronger fluctuations (most likely because of the fact that the “stars” in the simulations inject purely compressible energy) but is still very close on average. The 20% discrepancy with the index $\beta \sim 0.4$ found in the simulations may be due to the fact that in the simulations there exists an upper bound to the turbulent velocity dispersion (order a few km s$^{-1}$) that can be
imported to the medium by the stellar heating since, in the model, they only heat the gas to $\sim 10^4$ K. This introduces a "truncation" on the $\Delta v$-$R$ relation, which flattens the resulting slope, as can be seen in Figures 4a and 4b.

### 4.2.3. An Alternative Interpretation

At this point, one important alternative must be pointed out. The large scatter in the velocity dispersion–size relation would be consistent with the clouds' **not** having constant column densities, even if they were in equilibrium between turbulence and self-gravity. Indeed, in equilibrium, the scatter of $\sim 1$ order of magnitude in the velocity dispersion would imply a scatter of $\sim 2$ orders of magnitude in the column density, as can be seen from the equilibrium relation,

$$\Delta v \sim (NR)^{1/2},$$

which is equivalent to equation (8). Thus our results can also be interpreted in the sense that all clouds tend to be virialized, although with a scatter of at least 2 orders of magnitude in the column density. Preliminary results on the energy budget of a smaller cloud sample in run 28 (Ballesteros-Paredes & Vázquez-Semadeni 1995) suggest that the sum of the kinetic and magnetic energies is within a $1$ order of magnitude of the gravitational energy, although surface terms were not considered there. Also, the total gravitational and the turbulent kinetic energies per unit mass in the simulations are almost in equipartition, as shown in Figure 11.

In this alternative interpretation, a density-size correlation may still be present, but missed by the simulations because the scatter is larger than the column density dynamic range in the simulations (and in most observational studies). This scenario would leave the origin of the putative density-size relation unexplained, although it may still be possible that the $\Delta v$-$R$ relation is originated by the turbulent energy spectrum and the $\rho$-$R$ relation is the consequence of virial equilibrium. Although this scenario cannot be ruled out with certainty until significantly higher resolution simulations are performed, it seems unlikely because, in the simulations, clouds with sizes down to the smallest resolved scale are often found even at the lowest values of $\rho$, thus not giving any indication of the presence of a density-size relation. Instead, clouds of similar average densities often span the whole range of scales accessible to them. Also, the global balance between the turbulent and gravitational energies is mostly just a consequence of the presence of a self-regulated cycle of gravitational contraction, star formation, energy injection to the medium and dispersal, and again gravitational contraction, as discussed in Vázquez-Semadeni et al. (1995a), so that near global balance between turbulence and self-gravity is maintained at all times.

Finally, note that here we have not discussed other mechanisms that have been suggested in the literature as being responsible for the $\Delta v$-$R$ relation, such as inverse cascades of angular momentum (Henriksen & Turner 1984), critical thermal pressure equilibria (Chieze 1987), or the contribution from magnetic support (Shu 1987; Myers & Goodman 1988; Mouschovias & Psaltis 1995; Gammie & Ostriker 1996). We will address the role of these processes in the simulations in future work.

### 4.3. Comparison with Observations

The lack of correlation between cloud density and size in the simulations appears to contradict the correlations found in most observational results (Larson 1981; Torrelles et al. 1983; Dame et al. 1986; Falgarone & Pérault 1987; Myers 1990; WMD). However, as discussed in § 4.2.1, we interpret the discrepancy as an effect of the limited column density dynamic range of the observations or of selection effects introduced by focusing exclusively on global intensity maxima in the maps. There are some examples of observations that have intended to avoid these problems, i.e., Carr (1987), Loren (1989), and FPP. The first two authors focused on clumps within a single molecular cloud with extensive star formation, thus better sampling the non-gravitationally bound turbulent transients. The work of
FPP was specifically tailored toward studying low-brightness regions in molecular clouds. In both Loren (1989) and FPP, column densities spanning more than 1.5 orders of magnitude were found.

On the other hand, WMD recently concluded, from an analysis of 60 and 100 μm IRAS maps, that column densities of dark clouds cluster typically at N(H2) ~ 4 × 10^21 cm^-2 (corresponding to a typical 100 μm opacity of τ_100 ≈ 200 μNp), while claiming that the dynamic range of the data would have allowed detection of any significant variations. The discrepancy between these results and those of the present paper seemingly cannot be explained in terms of the limited column density dynamic range of their observations.

In actuality, we believe that the results of WMD may be spurious and attributed to a combination of their selection criteria and the following effect: At 60 and 100 μm, one is observing “warm” dust that could be coming from the “edges” of molecular clouds. If this “warm” dust is being heated by the visible photons from the Galactic stellar radiation field, it is expected that the depth of the “warm” region will be on the order of a few nepers in the visible. This will lead to apparent constant optical depths when determined from observations of the “warm” dust.

Detailed radiative transfer models have been presented by Bernard et al. (1992), who have already warned against using 100 μm surface brightness as a tracer of dust column density. In what follows, we present a simpler model that allows us to give first-order estimates of the scaling of the intensities in the IRAS 60 and 100 μm bands and the apparent dust opacity as a function of the true dust opacity through the cloud.

Similarly to Jarrett, Dickman, & Herbst (1989) and WMD, we assume that the ratio of visible to 100 μm absorption efficiency is (Q_v/Q_100) = 2.0 × 10^4. It is important to emphasize that the 100 μm opacity considered in this ratio is produced by small dust particles that become heated to relatively large temperatures and that are responsible for the observed 100 and 60 μm emission. The absorption efficiency ratio that includes all the dust at 100 μm is about a factor of 10 smaller (Chini, Krugel, & Kreysa 1986). Furthermore, in the range of wavelengths between 60 and 100 μm, the absorption efficiency scales approximately as ν^-1. Using this functional dependence, we can write a crude approximation for the dust temperature as

\[ T_{dust} = (T_{CR} + T_{rad} e^{-\tau_{vis}})^{1/5}, \]

where T_{CR} and T_{rad} are pseudotemperatures that parameterize the heating due to cosmic rays and to radiation, respectively. (Although this approximation is not valid at all wavelengths, we have checked that using slightly different functional dependences does not significantly alter our results.) We assume that cosmic-ray heating is constant for any point in the cloud. Specifically, T_{CR} = 10 K is the temperature that the dust has if heated only by cosmic rays, and T_{rad} is the temperature that the dust would have if heated only by the stellar radiation field (at the edge of the cloud, where no absorption is present). Also, τ_{vis} is the opacity in the visible wavelengths, which increases as we get deeper into the cloud. In this simple model, T_{dust} ≈ T_{rad} at the edge of the cloud, and it tends toward T_{CR} for the inner regions of the cloud (Fig. 12). The intensities at 60 and 100 μm, I_{60} and I_{100}, can be calculated for clouds with different values of τ_{vis} as shown in Figure 13. This calculation is made by assuming that Q_{100}/Q_{60} = 60 μm/100 μm. From this figure it is evident that, for clouds exceeding a few nepers in τ_{vis}, I_{60} and I_{100} do not continue growing with τ_{vis}. An apparent dust temperature, T_{60/100}, can be derived from I_{60} and I_{100}, and the far-infrared opacities are then obtained.

In Figure 14, we plot the 100 μm opacity derived in this manner as a function of the cloud’s τ_{vis} for different values of

\[ T_{dust} = (T_{CR} + T_{rad} e^{-\tau_{vis}})^{1/5}, \]

where T_{CR} and T_{rad} are pseudotemperatures that parameterize the heating due to cosmic rays and to radiation, respectively. (Although this approximation is not valid at all wavelengths, we have checked that using slightly different functional dependences does not significantly alter our results.) We assume that cosmic-ray heating is constant for any point in the cloud. Specifically, T_{CR} = 10 K is the temperature that the dust has if heated only by cosmic rays, and T_{rad} is the temperature that the dust would have if heated only by the stellar radiation field (at the edge of the cloud, where no absorption is present). Also, τ_{vis} is the opacity in the visible wavelengths, which increases as we get deeper into the cloud. In this simple model, T_{dust} ≈ T_{rad} at the edge of the cloud, and it tends toward T_{CR} for the inner regions of the cloud (Fig. 12). The intensities at 60 and 100 μm, I_{60} and I_{100}, can be calculated for clouds with different values of τ_{vis} as shown in Figure 13. This calculation is made by assuming that Q_{100}/Q_{60} = 60 μm/100 μm. From this figure it is evident that, for clouds exceeding a few nepers in τ_{vis}, I_{60} and I_{100} do not continue growing with τ_{vis}. An apparent dust temperature, T_{60/100}, can be derived from I_{60} and I_{100}, and the far-infrared opacities are then obtained.

In Figure 14, we plot the 100 μm opacity derived in this manner as a function of the cloud’s τ_{vis} for different values of

\[ T_{dust} = (T_{CR} + T_{rad} e^{-\tau_{vis}})^{1/5}, \]
The apparent warm-dust opacity “saturates” at values on the order of 70–100 µN. Considering that along a line of sight one expects to intersect the front and the back edges of a cloud, the typical 100 µm opacities of $\tau_{100} \approx 200$ µN appear to be explained as a result of this effect. The few clouds with much larger values of $\tau_{100}$ (up to $\tau_{100} \approx 10^4$) reported by WMD are regions of strong star formation activity, which may heat the clouds from the inside, raising $\tau_{100}$. The presence of this effect is also consistent with the limb brightening observed in some clouds in the far-infrared by Snell, Heyer, & Schloerb (1989) and WMD and predicted by the models of Bernard et al. (1992). Note that this effect continues to be applicable even if the clouds are clumpy, since it should hold at the edges of any density peaks, large or small, as long as they have a large enough column density. However, note also that this “saturation” effect in the determination of the apparent $\tau_{100}$ will occur only if reasonably high temperatures ($T_{\text{rad}} \gtrsim 20$ K) are present at the cloud’s surface. In any case, for lower dust temperatures the emission at 60 and 100 µm is very weak and undetectable in practice. One consequence of this effect is that, in order to fully sample the dust from dark clouds, one requires observations at longer wavelengths that will trace the predominant cooler dust component. We intend to verify this effect by comparing column densities obtained with different indicators in future work.

The effect discussed above may clearly impose an upper bound on the column densities derived by WMD. Furthermore, concerning their selection criteria, it should be noted that WMD defined core, cloud, and cloud complex in terms of ranges of extinction. This directly selects against identification of low column density structures. In fact, in their maps small, low-extinction clouds are readily seen, but they were not classified as such precisely because of their low extinctions. Thus the column densities of the cores studied by WMD are bound from above as a result of the “saturation” effect of the cloud edges, and from below by their very definition of a core, rendering their derived column density constancy open to question.

5. SUMMARY AND CONCLUSIONS

In this paper, we have searched for Larson-type (Larson 1981) correlations and cloud mass spectrum slopes in the clouds generated in numerical simulations of the ISM, one from Paper I, a similar one with a larger density dynamic range, and another with higher resolution. We define a cloud as a connected set of pixels in the density field with values larger than an arbitrary threshold $\rho_c$. From the results at several different times in the two simulations, we find that the mass spectrum has the form $dN/dM \propto M^{-1.44 \pm 0.1}$ and the velocity dispersion is related to the cloud size by $\sigma \propto R^{0.41 \pm 0.08}$, where the errors are the standard deviations in the set of values including all the different times. The dispersion-size relation exhibits a scatter of about an order of magnitude, comparable to the scatter observed in real clouds.

The simulated clouds do not exhibit a density-size relation, but instead, at all mean densities, clouds of sizes down to the smallest resolved scales are commonly found. This result implies that the clouds do not have constant column densities but, instead, exhibit a range of ~ 2 orders of magnitude, thus providing strong support for the possibility that the observationally derived density-size relation is an artifact of the limited dynamic range employed by observational surveys, and of the criteria used for selecting the clouds, as first noted by Larson (1981) and then strongly argued for by Scalo (1990). Our results suggest that low column density clouds do exist in the ISM but are systematically missed by most observations. In the simulations, low column density clouds are turbulent transients, as in the suggestion by Magnani, LaRosa, & Shore (1993). Observational work that has used complete cloud samples (e.g., Loren 1989), or specifically looked at structures in the low brightness regions of molecular clouds (FPP), has indeed found a reported column density variability of 1.5 orders of magnitude, and masses well below the virial mass, by factors up to 2 orders of magnitude. However, note that in our simulations most of the mass resides in the largest clouds at each mean density (and hence, with largest column density) since there are comparable numbers of small and large clouds.

The set of largest clouds at every threshold $\rho_c$ exhibits a density-size relation $\rho \sim R^x$, with $x = -0.8 \pm 0.15$. These clouds appear to be close to balance between turbulence and self-gravity on occasions, but not in general. On the other hand, pure mass conservation, which yields an upper limit $-x < 2$ for the two-dimensional case, together with numerical limitations of the simulations, cannot be ruled out as the sole cause responsible for this result. In any case, regardless of what the specific slope of the envelope turns out to be upon removal of numerical effects, our results suggest that the notion of a density-size scaling relation should probably be replaced by that of an “allowed” region in $\rho$-$R$ space. Whether the high column density boundary is truly a power law, the value of its corresponding index and the physical mechanism responsible for it are issues that remain to be determined by high-resolution three-dimensional simulations.

The result that the $\sigma$-$R$ relation is verified in the simulations but the $\rho$-$R$ relation is not supports the interpretation that the former relation is a direct consequence of the sta-
tistical properties of the turbulence, since a turbulent energy spectrum of the form $E(k) \sim k^{-2}$, as observed in the simulations, implies an $R^{1/2}$ scaling for the velocity dispersion. However, we discussed the possibility that a density-size relation does exist, although with a scatter larger than the column density dynamic range spanned by the clouds in the simulations. Such a scatter would be consistent with balance between turbulence and self-gravity, with a scatter of 1 order of magnitude in the velocity dispersion, according to equation (9). In this case, the origin of the $\rho$-$R$ relation would remain unknown, and the turbulent origin of the $\Delta v$-$R$ relation would not have as strong a support. This possibility cannot be ruled out without very high resolution simulations in order to add at least another order of magnitude to the column density dynamic range. However, the simulations do not yield any indication that this may be the case, since cloud sizes down to the smallest resolved scales are found even at the lowest values of $\rho_t$. Besides, the column density dynamic range in the simulations is already larger than that of most observational surveys.

We also discussed the recent results of WMD, who have argued in favor of a surprisingly constant column density (to within a factor of a few) in a large sample of cores studied through IRAS 60 and 100 $\mu$m maps while claiming a very large dynamic range in their observational method. We argued that this result may be spurious, by presenting a model in which the warm “skin” of clouds and cores, which is likely to always have visual extinctions of order unity, puts an upper bound to the column densities measured by WMD. From below, WMD’s own selection criteria eliminate low column density clouds, thus mimicking a nearly uniform column density.

Finally we remark that, in this paper, we have limited the discussion to the trends of the density and velocity dispersion with size, omitting any discussion of the role of the magnetic field, which is clearly important for the dynamics of the simulations (Paper I). A detailed account of the energy budget in the simulations including the magnetic energy density, as well as surface terms, is under way (Ballesteros-Paredes & Vázquez-Semadeni 1996).

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APPENDIX

In this appendix, we make an order-of-magnitude estimate of the effect of the numerical diffusion term in the continuity equation. Consider the continuity equation (eq. [3]):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \mu \nabla^2 \rho . \quad (10)$$

In order to estimate the effects of diffusion, we consider the extreme case in which the advection term can be neglected, so that the mass equation becomes the standard diffusion equation:

$$\frac{\partial \rho}{\partial t} = \mu \nabla^2 \rho . \quad (11)$$

For simplicity, we consider a one-dimensional (axisymmetric) problem, the solution of which is

$$\rho(x, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(x') \sqrt{\frac{\pi}{\mu t}} \exp \left[ -\frac{(x-x')^2}{4\mu t} \right] dx' \quad (12)$$

(see, e.g., Habermann 1987), where $f(x')$ is the initial density distribution, which we assume to be Gaussian, $f(x') = \rho_0 \exp \left( -x'^2/2\sigma^2 \right)$. In what follows, we will identify the width $\sigma$ of the Gaussian with the size of the cloud of interest. Equation (12) then becomes

$$\rho(x, t) = \rho_0 \left( \frac{1}{t/t_0 + 1} \right)^{1/2} \exp \left( -\frac{x^2}{2\sigma^2} \right) \frac{t/\sigma^2}{t/t_0 + 1}, \quad (13)$$

where

$$\left( \frac{t_0}{t_{\text{code}}} \right)^2 = \frac{\sigma^2}{2\mu} = 9.41 \times 10^{-3} \left( \frac{\sigma}{1 \text{ pixel}} \right)^2 . \quad (14)$$

is the characteristic diffusion time in units of the code, $t_{\text{code}} = 1.3 \times 10^7$ yr, and the second equality follows from using the value $\mu = 0.008$ (see Paper I) (note that the size of the integration box is $2\pi$ in code units). This value of $t_0$ can be made independent of the resolution of the simulation by choosing $\mu$ such that $\mu k_{\text{max}}^2 = \text{const}$, where $k_{\text{max}}$ is the maximum Fourier wavenumber in the simulation, equal to half the resolution.
Now consider the turbulent crossing (or nonlinear) time for scale \( \sigma \):

\[
\tau_{NL} \sim \frac{\sigma}{u_t},
\]

where \( u_t \) is the turbulent speed associated with the scale \( l \). Note that we are allowing for the possibility that clouds of size \( \sigma \) are generated by turbulent streams of different, typically larger, size \( l \) (see § 4.1). This velocity can be estimated from the turbulent energy spectrum \( E(k) \) as

\[
\frac{1}{2} u_t^2 = \int_{2\pi/l}^\infty E(k) dk,
\]

that is, \( u_t \) is the rms energy per unit mass in scales smaller than \( l \). Using \( E(k) = 0.1 k^{-2} \), as indicated by Figure 10, one obtains

\[
\frac{\tau_{NL}}{t_{code}} = 14.05 \frac{l/(1 \text{ pixel})}{n_{res}/(1 \text{ pixel})},
\]

where \( n_{res} \) is the number of pixels (i.e., the resolution) per spatial dimension of the simulation. From equations (14) and (15), we can now compare the diffusion and nonlinear times, in order to determine the scale at which they are equal. Assuming that \( l = m \sigma \), we obtain

\[
\left( \frac{l_{eq}}{1 \text{ pixel}} \right) = \frac{16.33}{m^{1/2}},
\]

and thus clouds with sizes smaller than \( \sim 16 \) pixels are dominated by the numerical mass diffusion. Therefore it is important to assess the effect of mass diffusion in the results of § 3. To do this, we take the extreme position that for time durations \( \Delta t \leq \tau_{NL} \), a Gaussian cloud is affected exclusively by diffusion and compute the fractional variation of its central density over \( \tau_{NL} \). In Figure 15, we show the evolution of \( \rho(0, t)/\rho_0 \) given by equation (13) over \( \tau_{NL} \) for a cloud of size \( \sigma = 4 \) pixels. Note that this is a worst-case estimate since, on the one hand, in the plots below we have discarded clouds with sizes \( \leq 4 \) pixels in order to avoid viscosity effects (see § 4.1), which are most affected by diffusion, while, on the other hand, we have taken \( m = 1 \).

As we can see from the density decreases by a factor \( \leq 3 \). We can thus expect our density data typically to change by factors smaller than half an order of magnitude.

Using the above results, we can produce a "corrected" density-size plot with the estimated "true" densities \( \rho_{corr} \), calculated as

\[
\rho_{corr} \sim \rho_{data} \left( 1 + \frac{\tau_{NL}/t_0}{1} \right)^{1/2},
\]

where \( \rho_{data} \) are the raw cloud mean densities produced by the simulations, and the "true" sizes are obtained by assuming mass conservation:

\[
R_{corr} \sim \frac{R_{data}}{\left( 1 + \frac{\tau_{NL}/t_0}{1} \right)^{1/4}}.
\]

The "corrected" density-size plots are shown in Figures 16 and 17 for run 28bis and the high-resolution run 28.800, respectively. We see that, although pushed slightly closer to a correlation in the case of run 28bis, the general trend of these
Fig. 16.—“Corrected” density-size plot for run 28bis using the estimates for the “true” density and size of the clouds given by eqs. (17) and (18). Note that a range of ~2 orders of magnitude in size remains for the lowest mean density clouds.

Fig. 17.—Same as Fig. 16, but for run 28.800. Here the sizes of the lowest density clouds vary by nearly 2.5 orders of magnitude. Note that, although only clouds with sizes larger than 4 pixels are retained, the smallest clouds in this figure have sizes smaller than this because of the size “correction,” eq. (18).

plots still supports the main conclusion from § 3, namely, that clouds of sizes down to the resolution are seen at all mean densities.

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