Positive and negative correlations in computational ghost imaging for a grayscale object

Chao Gao, Xiaqian Wang, Zhifeng Wang, Hongji Cai, Feng Chang and Zhihai Yao

Department of Physics, Changchun University of Science and Technology, Changchun 130022, People’s Republic of China

Email: yaozh@cust.edu.cn

Abstract. Our previous work has studied the positive-negative correlations in computational ghost imaging for a binary object. In this work, we focus on the computational ghost imaging system for a grayscale object. We studied the second-order correlation function of this system, and gave the criterion of the positive and negative correlations. Furthermore, this criterion was verified by using numerical simulations.

1. Introduction
Ghost imaging is an imaging technique which based on the intensity correlation measurements. In a conventional ghost imaging configuration, the beam from a light source is split into 2 equal beams. One beam interacts with the object under test, and the transmitted light is collected by a bucket detector (a detector with no spatial resolution, i.e. a photon-diode), this beam is called the signal beam. The other beam travels the same distance as the signal beam, and its spatial intensity distribution is measured by a CCD, and this beam is called the reference beam. Finally, the image of the object is retrieved by the correlation measurement of the intensity at the two detectors. The first ghost imaging experiment was performed by using entangled photons [1]. But later, it was shown that ghost imaging could also be realized by using a classical thermal light source [2]. In 2008, Shapiro presented the computational ghost imaging [3]. Different from conventional ghost imaging, a programable light source is used so that it is no longer necessary to use a CCD to detect the reference beam. Thus, the reconstruction of the object can be achieved by using a one-pixel detector and a programable light source (a laser and a spatial light modulator, or a digital projector), which simplified the experimental setup of the ghost imaging.

This novel imaging technique displays great potentials because of high lateral resolution imaging [4] and the robustness of atmosphere turbulence [5]. But ghost imaging do has some defects: 1) it takes a long time in the sampling progress. 2) the quality of the reconstructed image needs to be improved. How to efficiently improve the imaging quality is now a very hot topic.

Recently, Wu et al. presented the positive and negative correlation in ghost imaging [6]. Their results have shown that the reconstructed image is the superposition of a positive and a negative image of the object. Obviously, the negative image will decrease the visibility of the reconstructed image. Later in 2012, Wen provided the theoretical explanation of this interesting phenomenon [7]. Wen’s work has proved that, the positive-negative correlations in thermal ghost imaging is due to the relative fluctuations with respect to the average intensity. Furthermore, Wen proposed a criterion to distinguish the positive and negative correlations based on the average intensity of the bucket signal. In 2017, we
studied the positive and negative correlations in a computational ghost imaging system for a binary object. In our work, a criterion was given to distinguish the positive and negative correlations, and it was verified by our simulations and experiments. Also, a comparison between our criterion and the criterion based on the average intensity of the bucket signal was made in our work [8]. In this work, we will focus on the positive-negative correlations of computational ghost imaging for a grayscale object. Our numerical simulations results show that our criterion in [8] works well for a grayscale object.

2. Theory

The schematic diagram for computational ghost imaging is shown in figure 1. We use PC to generate a series of random binary patterns with 50% white pixels and 50% black pixels. And we use a projector to project them on the object. The object is a transmitting grayscale object. Then, the transmitted light is collected by a photon-diode.

![Figure 1. The schematic diagram for computational ghost imaging.](image)

The reconstructed image is given by the second-order correlation function:

\[ G^{(2)}(x, y) = \langle R_n(x, y)B_n \rangle. \]  

(1)

Which \( \langle \ldots \rangle \) stands for the ensemble average. \( R_n(x, y) \) is the binary pattern which illuminates the object. \( B_n \) is the signal collected by the photon-diode, it is called the bucket signal, which can be expressed by:

\[ B_n = \sum_{x', y'} T(x', y')R_n(x', y'). \]  

(2)

Where \( T \) is the transmission function of the object. A grayscale object can be described as the superposition of some binary objects. Let \( T_i \) represent the binary components of the grayscale object \( T \):

\[ T(x, y) = \sum_{i} T_i(x, y). \]  

(3)

The second-order correlation of the grayscale object reads:
\[ G^{(2)}(x, y) = \left\langle \sum_{x', y'} R_n(x', y') \sum_{l} T_l(x', y') \right\rangle \]
\[ = \sum_{l} \left\langle \sum_{x', y'} R_n(x', y') T_l(x', y') \right\rangle \]
\[ = \sum_{l} G^{(2)}_l(x, y) \]  

(4)

Obviously, the ghost image of a grayscale object can be regarded as the superposition of the ghost images of the object’s binary components. Our previous work gives a ‘reverse factor’ to distinguish the positive and negative correlations, it is defined as:

\[ B_{RF} = \frac{\alpha \beta}{M}. \]  

(5)

Where \( \alpha \) is the number of white pixels in each binary pattern. \( \beta / M \) is the total transmission ratio of the object. In the experiments, for every correlation step, we compare the bucket signal with the reverse factor. Correlation steps with the bucket signal which smaller than the reverse factor are the negative correlations, they will reconstruct a negative image of the object.

For every binary component, there is an individual reverse factor, because these components may have different transmission ratios. Thus, it is impossible to find an absolutely correct dividing line of positive and negative correlations. However, we found that, simply replace the total transmission ratio of the binary object \( \beta / M \) into the total transmission ratio of the grayscale object \( t \):

\[ B_{RF}' = \alpha \beta. \]  

(6)

The reverse factor is still accurate enough to divide the positive and negative correlations. In the next section, we will introduce the results of our numerical simulations.

3. Numerical simulations

We simulated the computational ghost imaging for 5 different grayscale objects, they are shown in figure 2. They have different total transmission ratios. The sizes of the images are \( 64 \times 64 \) pixels, and each of them has 5 levels of grayscale. And we apply 500,000 \( 64 \times 64 \) pixels binary patterns with 50% white and 50% black pixels to modulate the source.

Figure 2. The original objects.

To gain the proper dividing line of positive and negative correlations, in our simulations, we preset the reverse factor from 0 to 2048, respectively (the magnitude of the bucket signal can only be in interval [0, 2048]). For every reverse factor, we gain the positive and negative correlations:

\[ G^{(2)}(x, y) = \left\langle R_n^{(+)}(x, y) B_{RF}^{(+)} \right\rangle + \left\langle R_n^{(-)}(x, y) B_{RF}^{(-)} \right\rangle. \]  

(7)

Where \( R_n^{(+)} \) (\( R_n^{(-)} \)) and \( B_{RF}^{(+)} \) (\( B_{RF}^{(-)} \)) represent the binary patterns and the bucket signals corresponding to the positive (negative) correlations, respectively. When we reconstruct the image of the object, we keep the positive correlations as what they are but reverse the negative correlations in grayscale:

\[ G^{(2)}(x, y) = \left\langle R_n^{(+)}(x, y) B_{RF}^{(+)} \right\rangle + \left\langle [1 - R_n^{(-)}(x, y)] B_{RF}^{(-)} \right\rangle. \]  

(8)
Finally, we examine the visibility of the reconstructed image. The visibility is defined as [9]:

\[
V = \frac{\langle G_{in} \rangle - \langle G_{out} \rangle}{\langle G_{in} \rangle + \langle G_{out} \rangle}.
\]  

(9)

Where \( G_{in} \) and \( G_{out} \) is the average of the second-order correlation function in the transmitting part and the non-transmitting part of the object, in which:

\[
\langle G_{in} \rangle = t \sum_{x,y} T(x, y) G^{(2)}(x, y).
\]  

(10)

\[
\langle G_{out} \rangle = (1-t) \sum_{x,y} [1-T(x, y)] G^{(2)}(x, y).
\]  

(11)

Where \( t \) is the total transmission ratio of the object. If the dividing line is proper, the positive image and the negative image can be separated accurately, thus the visibility of the image reconstructed by equation (8) will be higher in magnitude.

Figure 3. (Colored Online) Visibility of reconstructed images by using different dividing line of positive and negative correlations. The dividing line is changing from 0 to 2048, the dashed black curves are the visibility of images reconstructed by original computational ghost imaging, and the red curves are the visibility of images reconstructed by equation (8). The vertical lines are the reverse factors given by equation (6).
Figure 4. The improved image of computational ghost imaging for a grayscale object by using positive-negative correlations (a) The object (b) The image reconstructed by original computational ghost imaging. (c) The image reconstructed by equation (8) (d) The positive image of the object (e) The negative image of the object.

The results of our numerical simulation are shown in figure 3 and figure 4. It is clear that, when the dividing line is chosen near to the reverse factor, the visibility of the reconstructed image will be higher. Obviously, for computational ghost imaging for a grayscale object, the reverse factor is still accurate enough to be a criterion to distinguish the positive and negative correlations. Figure 4 shows the improvement of the imaging quality by using reverse factor to distinguish the positive and negative correlations. In this simulation, we used a 64×64 pixels image as the object, and we apply 2,500,000 binary patterns with 50% white and 50% black pixels to modulate the source. Compare figure 4(b) with figure 4(c), it is clear that the imaging quality is greatly improved.

4. Summary
Our previous work has studied the positive-negative correlations in computational ghost imaging for a binary object. In this work, according to our numerical simulations, we have verified that, as a criterion to distinguish the positive-negative correlations, reverse factor is still accurate enough in computational ghost imaging for a grayscale object. Once we know the object’s total transmission ratio, by using reverse factor, the quality of the reconstructed image of a grayscale object can be greatly improved.

Acknowledgments
This work is supported by the Projects of Jilin Province Science and Technology Development Plan, with Grants No.20180520165JH.

References
[1] Pittman T B, Shih Y H, Strekalov D V and Sergienko A V 1995 Phys. Rev. A 52 R3429
[2] Bennink R S, Bentley S J and Boyd R W 2002 Physical Review Letters 89 113601
[3] Shapiro J H 2008 Phys. Rev. A 78 061802(R)
[4] Gong W and Han S 2012 Phys. Lett. A 376 1519
[5] Dixon P B, Howland G, Chan K W C, O’Sullivan H, Rodenburg B, Hardy N D, Shapiro J H, Simon D, Sergienko A V and Boyd R W 2011 Phys. Rev. A 83 051803(R)
[6] Wu L A and Luo K H 2011 AIP Conf. Proc. 1384 223
[7] Wen J M 2012 *Journal of the Optical Society of America A Optics Image Science & Vision* **29**(9) 1906-1911
[8] Gao C, Wang X Q, Wang Z F, Li Z, Du G J, Chang F and Yao Z H 2017 *Phys. Rev. A* **96** 023838
[9] Chan K W C, O’Sullivan M N and Boyd R W 2009 *Opt. Lett.* **34** 3343