Pair Creation of Dilaton Black Holes in Extended Inflation

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Abstract

Dilatonic Charged Nariai instantons mediate the nucleation of black hole pairs during extended chaotic inflation. Depending on the dilaton and inflaton fields, the black holes are described by one of two approximations in the Lorentzian regime. For each case we find Euclidean solutions that satisfy the no boundary proposal. The complex initial values of the dilaton and inflaton are determined, and the pair creation rate is calculated from the Euclidean action. Similar to standard inflation, black holes are abundantly produced near the Planck boundary, but highly suppressed later on. An unusual feature we find is that the earlier in inflation that the dilatonic black holes are created, the more highly charged they can be.

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1 Introduction

If nature is described by a higher-dimensional theory such as string theory, a dilaton field $\phi$ must be included in the low energy action. This can alter the properties of black hole solutions significantly. In standard Einstein-Maxwell theory with a cosmological constant $\Lambda$, there is a family of charged non-rotating static black hole solutions, the Reissner-Nordström-de Sitter (RNdS) solutions. In dilatonic theories, a natural generalisation of a cosmological constant term is a Liouville potential,

$$V(\phi) = \Lambda e^{2\phi}.$$

(1.1)

Poletti and Wiltshire showed that all RNdS solutions that approach de Sitter space asymptotically have no analogue in dilatonic theories with a Liouville potential [1]. The solutions of highest mass for given charge, however, have a different asymptotic behaviour from all other RNdS solutions. These “Charged Nariai” solutions are the product of 1+1-dimensional de Sitter space with a round two-sphere. They can be generalised to include a dilaton, which then has a constant value [2]. The Charged Nariai black holes admit a smooth Euclidean section (an instanton), which is given by the topological product of two round two-spheres, not necessarily of the same radius. Therefore these black holes can be spontaneously created, if an appropriate background spacetime is available.

In the non-dilatonic case, this background is de Sitter space. The cosmological pair creation of non-dilatonic black holes is well understood [3, 4, 5]. One finds that pair creation is highly suppressed unless the cosmological constant is on the order of the Planck value. Since the universe does not have such a large cosmological constant, an alternative background was used in Refs. [6, 7]: a chaotic inflationary universe. In this case the potential of an inflaton field $\sigma$ acts as an effective cosmological constant [8, 9]:

$$V(\sigma) = m^2 \sigma^2.$$

(1.2)

(For definiteness, we use a massive scalar field, but none of our results depend qualitatively on this choice.) The inflationary universe looks very much like de Sitter space, except that the cosmological constant is not fixed, but decreases slowly. As a consequence, the Euclidean solutions must be chosen slightly complex, so that the minisuperspace variables are perfectly real in the Lorentzian regime [4]. Black holes can be pair created abundantly near the Planck era, when the effective cosmological constant is large, but they become highly suppressed later on. Most neutral black holes evaporate before inflation ends, and magnetically charged black holes, which
are topologically conserved, are strongly diluted by the inflationary expansion [7]. Thus there is no conflict with present day observations.

In the dilatonic case, on the other hand, the Charged Nariai instantons do not readily correspond to a pair creation process, since there is no static de Sitter solution that could act as a background. Even if the dilaton is allowed to become time-dependent, it is pushed towards negative infinity by the fixed cosmological constant. An effective cosmological constant

\[ V(\phi, \sigma) = e^{2b\phi} m^2 \sigma^2, \]

however, is large only for a finite time. Therefore, an inflationary background is still available; the dilaton will decrease only until inflation ends. This extended model of chaotic inflation was introduced by Linde [10] and has received considerable attention [11, 12, 13, 14, 15, 16, 17, 18]. In this paper we investigate the pair creation of dilatonic black holes during extended chaotic inflation through Charged Nariai instantons. It will be interesting to see how the process differs from pair creation during standard inflation. But what provides a more fundamental motivation for this work is that inflation is the simplest background on which cosmological dilaton black holes can be pair created at all.

The paper is organised as follows. The theories we consider are presented in Sec. 2. In Sec. 3 we make a minisuperspace ansatz for the background dilatonic inflationary universe. By choosing appropriate complex initial values, we find solutions that satisfy the no boundary proposal at the origin of Euclidean time, and that are exactly real in the Lorentzian region. We determine some observational constraints on the parameters of the solution space. In Sec. 4 we go through the same procedure for an inflationary universe containing dilatonic Charged Nariai black holes. We find two approximate solutions, which are valid in different regions of the dilaton-inflaton phase space. The first corresponds to a sequence of the kind of fixed-Λ-solutions found in Ref. [2]. In the second, the black hole charge is negligible and the dilaton evolves as if no Maxwell term were present. In Sec. 5 we analyse the pair creation process during inflation. We consider black holes created with a given charge at a given point in inflation, and show that they are all attracted by one of the two approximate solutions. We calculate the Euclidean action and pair creation rate in all cases, and show that pair creation is suppressed except near the Planck boundary. We show that unlike in standard inflation, the black holes created earliest have the highest charge. A summary is provided in Sec. 6.
2 Dilaton Lagrangian

In the string frame, the gravitational Lagrangian with a dilaton and a massive inflaton field is given by:

\[ L_{\text{string}} = (-\bar{g})^{1/2} \left[ e^{-b\phi} R - 2e^{-b\phi} \bar{g}^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - 2\bar{g}^{\mu\nu} (\partial_\mu \sigma)(\partial_\nu \sigma) - 2m^2 \sigma^2 \right]. \quad (2.1) \]

After a conformal transformation

\[ g^{\mu\nu} \equiv e^{-b\phi} \bar{g}^{\mu\nu}, \quad (2.2) \]

the Lagrangian takes the Einstein-Hilbert form, but the dilaton field now couples to the inflaton:

\[ L_{\text{Einstein}} = (-g)^{1/2} \left[ R - 2g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - 2e^{b\phi} g^{\mu\nu} (\partial_\mu \sigma)(\partial_\nu \sigma) - 2e^{2b\phi} m^2 \sigma^2 \right]. \quad (2.3) \]

Throughout the paper we shall work in this frame, the Einstein frame. In order to consider charged black holes, a Maxwell term must be added to the Lagrangian:

\[ L = L_{\text{Einstein}} + (-g)^{1/2} \left[ -e^{-2a\phi} F^2 \right], \quad (2.4) \]

where

\[ F^2 = F_{\mu\nu} F^{\mu\nu}. \quad (2.5) \]

This Lagrangian is invariant under the transformation

\[ a \to -a, \quad b \to -b, \quad \phi \to -\phi. \quad (2.6) \]

We fix a gauge by choosing \( a \geq 0 \). In order to obtain slow-roll inflation, we must assume

\[ |b| \ll 1. \quad (2.7) \]

It was shown in Ref. [2] that Nariai-type black hole solutions of this theory exist only for \( 0 < |b| < a \). They are magnetically (electrically) charged for positive (negative) \( b \). We are interested mainly in magnetically charged black holes, since they cannot evaporate. Thus we shall take

\[ 0 < b < a. \quad (2.8) \]

It is easy to adapt our results to the electric case.
3 Inflation Without Black Holes

3.1 Ansatz

We first consider an inflating universe without black holes. We make the metric ansatz corresponding to a spatially closed Friedmann-Robertson-Walker universe. Its spacelike sections are three-spheres of radius $\alpha$:

$$ds^2 = N^2 d\tau^2 + \alpha(\tau)^2 d\Omega_3^2,$$  \hspace{1cm} (3.1)

where $N$ is the lapse function, $\tau$ is the Euclidean time variable, and $d\Omega_3$ is the metric on the unit three-sphere. From the Lagrangian, Eq. (2.3) we obtain the Euclidean action

$$I_{S^3} = -\frac{3\pi}{4} \int N d\tau \left[ \alpha + \frac{\alpha \dot{\alpha}^2}{N^2} - \frac{1}{3} \dot{\alpha}^3 \left( \frac{\dot{\phi}^2}{N^2} + e^{b\phi} \dot{\sigma}^2 + e^{2b\phi} m^2 \sigma^2 \right) \right].$$  \hspace{1cm} (3.2)

Variation with respect to $\alpha$, $\sigma$, $\phi$ and $N$ yields the equations of motion and the Hamiltonian constraint:

$$\frac{\ddot{\alpha}}{\alpha} + \frac{2}{3} \dot{\phi}^2 + \frac{2}{3} e^{b\phi} \dot{\sigma}^2 + \frac{1}{3} e^{2b\phi} m^2 \sigma^2 = 0,$$  \hspace{1cm} (3.3)

$$\ddot{\sigma} + 3\frac{\dot{\alpha}}{\alpha} \dot{\sigma} + b\dot{\phi} \dot{\sigma} - e^{b\phi} m^2 \sigma = 0,$$  \hspace{1cm} (3.4)

$$\ddot{\phi} + 3\frac{\dot{\alpha}}{\alpha} \dot{\phi} - \frac{b}{2} e^{b\phi} \dot{\sigma}^2 - b e^{2b\phi} m^2 \sigma^2 = 0,$$  \hspace{1cm} (3.5)

$$\frac{1}{\alpha^2} - \frac{\dot{\alpha}^2}{\alpha^2} + \frac{1}{3} \dot{\phi}^2 + \frac{1}{3} e^{b\phi} \dot{\sigma}^2 - \frac{1}{3} e^{2b\phi} m^2 \sigma^2 = 0,$$  \hspace{1cm} (3.6)

in the gauge $N = 1$, where an overdot denotes differentiation with respect to $\tau$.

3.2 Solutions

It is convenient to define

$$H^2 = \text{Re} \left( \frac{1}{3} e^{2b\phi} m^2 \sigma^2 \right),$$  \hspace{1cm} (3.7)

$$\theta = e^{-b\phi}. \hspace{1cm} (3.8)$$
We must find solutions that satisfy the no boundary proposal:

\[
\begin{align*}
\alpha &= 0, \quad \dot{\alpha} = 1, \quad \sigma = \sigma_0, \quad \dot{\sigma} = 0, \\
\theta &= \theta_0, \quad \dot{\theta} = 0 \quad \text{for} \quad \tau = 0.
\end{align*}
\] (3.9)

We shall assume that for both the dilaton and inflaton, the initial real value of the field is much greater than the initial imaginary value. An approximate solution near the origin, for \(|H_0\tau| < O(1)|\), is given by

\[
\begin{align*}
\sigma_I(\tau) &= \sigma_0, \\
\theta_I(\tau) &= \theta_0, \\
\alpha_I(\tau) &= H_0^{-1} \sin H_0 \tau.
\end{align*}
\] (3.10-3.12)

This “inner” approximation satisfies the no boundary proposal.

We define a Lorentzian time variable \(t\) by

\[
\tau = \frac{\pi}{2H_0} + it, \quad (3.13)
\]

For \(H_0 t > O(1)|\) an approximate solution is

\[
\begin{align*}
\sigma(t) &= \bar{\sigma}_0 - \frac{m}{\sqrt{3}} t, \\
\theta(t) &= \bar{\theta}_0 + b^2 \frac{m}{\sqrt{3}} \int_0^t \sigma dt', \\
\alpha(t) &= H^{-1} \cosh \int_0^t H dt'.
\end{align*}
\] (3.14-3.16)

The constants \(\bar{\sigma}_0\) and \(\bar{\theta}_0\) denote the real parts of the values of the two fields in the initial Euclidean region. This “outer” approximation describes an inflating Lorentzian universe, as long as the condition \(|\dot{H}| \ll H^2\) is satisfied, or equivalently, if

\[
\frac{\sigma^2}{\theta} \gg 1. \quad (3.17)
\]

Inflation ends when \(|\dot{H}| \approx H^2\).

As in ordinary chaotic inflation \([3]\), the fields must start out with a non-zero imaginary part, in order to be perfectly real during the Lorentzian inflationary era, i.e. for \(\tau^{\text{Re}} = \frac{\pi}{2H_0}\). This imaginary part can be determined as follows. Because of the initial conditions imposed by the no boundary proposal, Eq. (3.9), both \(\theta\) and \(\sigma\) will
be even functions of $\tau$. Therefore the imaginary parts of both fields will be constant along the imaginary $\tau$-axis, and will be equal to the initial imaginary values:

$$\theta^{\text{Im}} = \theta_0^{\text{Im}}, \quad \sigma^{\text{Im}} = \sigma_0^{\text{Im}}.$$  \hfill (3.18)

In the outer approximation the imaginary parts of the two fields are also constant along the imaginary $\tau$-axis, where they must have the values.

$$\theta^{\text{Im}} = \frac{\pi}{2} b^2 \theta_0, \quad \sigma^{\text{Im}} = -\frac{\pi}{2} \frac{\theta_0}{\sigma_0},$$  \hfill (3.19)

in order to vanish on the line $\tau^{\text{Re}} = \frac{\pi}{2H_0}$. This follows from Eqs. (3.14) and (3.15), where we have neglected the $t^2$-term. Thus it does not matter exactly at which point on the imaginary $\tau$-axis we match the approximations; if we match at some $\tau^{\text{Im}} \approx O(H_0^{-1})$, then Eqs. (3.18) and (3.19) determine $\theta_0^{\text{Im}}$ and $\sigma_0^{\text{Im}}$.

### 3.3 Constraints

Certain constraints are placed on the solution parameters by the requirement that inflation must produce the density fluctuations we observe in today’s universe. For definiteness we shall assume that inflation begins at the Planck boundary,

$$e^{2b\phi_i} m^2 \sigma_i^2 = 1,$$  \hfill (3.20)

and ends, by Eq. (3.17), when

$$\frac{\sigma_e^2}{\theta_e} = 1.$$  \hfill (3.21)

From Eqs. (3.14) and (3.15) it follows that

$$\sigma^2 + \frac{2}{b^2} \theta = \text{const} \equiv \zeta^2.$$  \hfill (3.22)

If we assume that inflation starts in the Planck era, the constant $\zeta^2$ is the only parameter of the solution space. Eq. (3.22) allows us to relate the initial and final values of the inflaton field:

$$\sigma_e^2 \left( 1 + \frac{2}{b^2} \right) = \zeta^2 = \sigma_i^2 + \frac{2}{b^2} m \sigma_i.$$  \hfill (3.23)
The number of e-foldings by which the spatial size of the universe grows during inflation is given by

$$N = \int_{\sigma_i}^{\sigma_e} H dt. \quad (3.24)$$

This can be integrated using Eq. (3.22):

$$N = \frac{1}{b^2} \int_{\sigma_i}^{\sigma_e} \frac{-2\sigma d\sigma}{\zeta^2 - \sigma^2} = \frac{1}{b^2} \left[ \ln(\zeta^2 - \sigma^2) \right]_{\sigma_i}^{\sigma_e}. \quad (3.25)$$

Using Eq. (3.23) to eliminate $\zeta$ and $\sigma_i$, we find

$$N = \frac{1}{b^2} \ln \left( \frac{\sigma_e^2}{m \sigma_i} \right) = -\frac{1}{2} + \frac{1}{b^2} \ln \left( \frac{Y}{-1 + \sqrt{1 + 2Y}} \right), \quad (3.26)$$

where

$$Y \equiv \frac{\sigma_e^2 b^2}{m^2} \left( 1 + \frac{b^2}{2} \right). \quad (3.27)$$

The minimum number of e-foldings of inflation needed to account for the large scale homogeneity of the observable universe is $N_{\text{min}} \approx 60$. This places a lower bound on $Y$:

$$Y > 2X(X - 1), \quad (3.28)$$

where

$$X \equiv \exp \left[ b^2 \left( N_{\text{min}} + \frac{1}{2} \right) \right]. \quad (3.29)$$

Normally $b$ will be so small that we can approximate

$$X \approx 1 + b^2 N_{\text{min}}. \quad (3.30)$$

We then find the following condition for sufficient inflation:

$$\frac{\sigma_e^2}{m^2} > 2N_{\text{min}}. \quad (3.31)$$
The density perturbation on the horizon scale of the observable universe is given by [13]:

$$\frac{\delta \rho}{\rho} \approx 5 \times 10^{-5}. \quad (3.32)$$

The perturbations generated by the inflationary model we use have been computed elsewhere [11, 12, 15]; here we only quote the result:

$$\frac{\delta \rho}{\rho} \approx \frac{m}{\theta}. \quad (3.33)$$

This leads to the constraint

$$\frac{m}{\theta_e} = \frac{m}{\sigma_S^2} \approx 5 \times 10^{-5}. \quad (3.34)$$

4 Inflation With Black Holes

4.1 Ansatz

We now consider black hole solutions to the inflationary model of Eq. (2.4). The topology of the spatial sections of such solutions is $S^1 \times S^2$. In general, the size of the two-spheres can vary along the one-sphere. In the case where one has a fixed cosmological constant and no dilaton, such solutions approach de Sitter space beyond the cosmological horizon. It was shown by Poletti and Wiltshire [1] that these solutions possess no analogues in dilatonic theories with a Liouville potential, because of the absence of a de Sitter-like solution that could act as a background. The only exception are solutions in which the two-sphere radius is constant along the $S^1$. They correspond to black holes of maximal mass at a given charge, and are called Charged Nariai solutions. They do not approach de Sitter space asymptotically; thus they possess dilatonic analogues.

For a Liouville potential ($\Lambda e^{2\Phi}$) the dilatonic Charged Nariai solutions are given in Ref. [4], where it is shown that in some respects they differ quite radically from their non-dilatonic counterparts. Since they admit a regular Euclidean section, they can mediate black hole pair creation on a suitable de Sitter-like background. For fixed $\Lambda$ there is no such background, because the cosmological constant pushes the dilaton toward negative infinity. The action, Eq. (2.3), however, contains a Liouville potential with an effective cosmological constant $\Lambda_{\text{eff}} = m^2 \sigma^2$. The dilaton will only decrease as long as $\Lambda_{\text{eff}}$ is large; therefore extended chaotic inflation provides
an appropriate background for the pair creation of the dilatonic Charged Nariai solutions of Ref. [2]. They need only be slightly modified to take the time dependence of the cosmological constant into account.

Since the spacelike sections will be the direct product of a one-sphere and a round two-sphere, we make the metric ansatz

\[ ds^2 = N^2 d\tau^2 + \alpha(\tau)^2 d\xi^2 + \beta(\tau)^2 d\Omega_2^2, \] (4.1)

where \( \xi \) has the period 2\( \pi \), and \( d\Omega_2^2 = d\theta^2 + \sin^2 \theta \, d\phi^2 \). The equation of motion due to the Maxwell term in Eq. (2.4),

\[ 0 = \nabla_\mu \left( e^{-2a\phi} F^{\mu\nu} \right), \] (4.2)

can be integrated to yield

\[ F = Q \sin \theta \, d\theta \wedge d\phi \] (4.3)
in the case of magnetically charged black holes. Thus we have \( F^2 = 2Q^2/\beta^4 \), and we can obtain the Euclidean minisuperspace action from Eq. (2.4):

\[ I_{S^1 \times S^2} = -\pi \int N d\tau \left[ \alpha + \frac{\alpha \dot{\beta}^2}{\beta^2} + \frac{2\dot{\alpha} \dot{\beta}}{\beta} + \frac{2\dot{\alpha} \dot{\beta} \dot{\sigma}^2}{\beta^2} \right. \]
\[ - \alpha \beta^2 \left( \frac{\dot{\phi}^2}{\beta^2} + e^{2\phi} \frac{\dot{\sigma}^2}{\beta^2} + e^{2\phi} m^2 \sigma^2 + e^{-2a\phi} \frac{Q^2}{\beta^4} \right) \]
\[ + \pi \left[ -\dot{\alpha} \beta^2 - 2\alpha \beta \dot{\sigma} \right]_{\tau=0}. \] (4.4)

Variation with respect to \( \alpha, \beta, \sigma, \phi \) and \( N \) yields the equations of motion and the Hamiltonian constraint:

\[ \frac{\ddot{\beta}}{\beta} - \frac{\dot{\alpha} \dot{\beta}^2}{\alpha \beta} + \dot{\phi}^2 + e^{2\phi} \dot{\sigma}^2 = 0, \] (4.5)

\[ \frac{\ddot{\beta}}{\beta} + \frac{\dot{\alpha} \dot{\beta}}{\alpha \beta} + \frac{\ddot{\sigma}}{\sigma} + e^{2\phi} \dot{\sigma}^2 + e^{2\phi} m^2 \sigma^2 - e^{-2a\phi} \frac{Q^2}{\beta^4} = 0, \] (4.6)

\[ \ddot{\sigma} + \left( \frac{\dot{\alpha}}{\alpha} + 2\frac{\dot{\beta}}{\beta} \right) \dot{\sigma} + b \dot{\phi} \dot{\sigma} - e^{b\phi} m^2 \sigma = 0, \] (4.7)

\[ \ddot{\phi} + \left( \frac{\dot{\alpha}}{\alpha} + 2\frac{\dot{\beta}}{\beta} \right) \dot{\phi} - \frac{b}{2} e^{b\phi} \dot{\sigma}^2 - be^{2b\phi} m^2 \sigma^2 + a e^{-2a\phi} \frac{Q^2}{\beta^4} = 0, \] (4.8)

\[ \frac{1}{\beta^2} - \frac{\ddot{\beta}^2}{\alpha \beta} + \dot{\phi}^2 + e^{2\phi} \dot{\sigma}^2 - e^{2b\phi} m^2 \sigma^2 - e^{-2a\phi} \frac{Q^2}{\beta^4} = 0, \] (4.9)
in the gauge $N = 1$.

We define

$$\tilde{H}^2 = \text{Re} \left( e^{2b\phi} m^2 \sigma^2 \right),$$

(4.10)

$$\tilde{g} = \frac{b}{a}.$$  

(4.11)

The black hole solutions must satisfy the no boundary proposal:

$$\alpha = 0, \quad \dot{\alpha} = 1, \quad \sigma = \sigma_0, \quad \dot{\sigma} = 0,$$

$$\beta = \beta_0, \quad \dot{\beta} = 0, \quad \theta = \theta_0, \quad \dot{\theta} = 0 \quad \text{for} \quad \tau = 0.$$  

(4.12)

Again it is assumed that the initial real values of both fields are much greater than the initial imaginary values, and that we are sufficiently far from the end of inflation, i.e. $\sigma^2/\theta \gg 1$.

In the dilatonic Charged Nariai solutions with a fixed cosmological constant, the dilaton is fixed at the value

$$\phi_{eq} = \frac{1}{2a(1 - \tilde{g})} \ln \left[ \frac{(1 + \tilde{g})^2}{\tilde{g}} Q^2 \Lambda \right],$$

(4.13)

for which the cosmological and Maxwell terms in the dilaton equation of motion cancel out [2]. But now we are dealing with an effective cosmological constant, which decreases slowly. Therefore $\phi_{eq}$ will also change over time:

$$\phi_{eq}(\sigma) = \frac{1}{2a(1 - \tilde{g})} \ln \left[ \frac{(1 + \tilde{g})^2}{\tilde{g}} Q^2 m^2 \sigma^2 \right].$$

(4.14)

There will thus be two cases. One possibility is that the dilaton evolution follows that of $\phi_{eq}$, so that the cosmological and Maxwell terms remain opposite and equal as the inflaton field decreases. In this case the time evolution of the inflationary black hole solution can be approximated as a sequence of static solutions with $\Lambda = \Lambda_{\text{eff}}$. The other possibility is that $\phi_{eq}$ changes too fast for $\phi$ to follow. Correspondingly, we shall give two different approximations for the black hole solutions, valid in different regions of the $(\phi, \sigma)$ phase space. Each will split into an inner and outer approximation.
4.2 Pseudo-static Approximation

First consider the “pseudo-static” case where $\phi \approx \phi_{eq}$. Then the inner approximation, valid for $|\tilde{H}_0(1 - \tilde{g})^{1/2}\tau| < O(1)$, is given by

\begin{align*}
\sigma_I(\tau) &= \sigma_0, \quad (4.15) \\
\phi_I(\tau) &= \phi_{eq}(\sigma_0), \quad (4.16) \\
\alpha_I(\tau) &= \tilde{H}_0^{-1}(1 - \tilde{g})^{-1/2}\sin \tilde{H}_0(1 - \tilde{g})^{1/2}\tau, \quad (4.17) \\
\beta_I(\tau) &= \tilde{H}_0^{-1}(1 + \tilde{g})^{-1/2}. \quad (4.18)
\end{align*}

With the Lorentzian time variable $t$ defined by

\begin{equation}
\tau = \frac{\pi}{2\tilde{H}_0(1 - \tilde{g})^{1/2}} + it, \quad (4.19)
\end{equation}

the outer approximation, valid for $\tilde{H}_0(1 - \tilde{g})^{1/2}t > O(1)$, is given by

\begin{align*}
\sigma(t) &= \tilde{\sigma}_0 - (1 - \tilde{g})^{-1/2}mt, \quad (4.20) \\
\phi(t) &= \phi_{eq}(\sigma), \quad (4.21) \\
\alpha(t) &= \tilde{H}^{-1}(1 - \tilde{g})^{-1/2}\cosh \int_0^t \tilde{H}(1 - \tilde{g})^{1/2}dt', \quad (4.22) \\
\beta(t) &= \tilde{H}^{-1}(1 + \tilde{g})^{-1/2}. \quad (4.23)
\end{align*}

This describes an inflating universe containing a pair of dilatonic black holes of charge $Q$. The imaginary initial value of $\sigma$ can be determined in the way described in the previous section:

\begin{equation}
\sigma_0^{\text{Im}} = -(1 - \tilde{g})^{-1/2}\frac{\pi}{2} \frac{\theta_0}{\sigma_0}. \quad (4.24)
\end{equation}

Then the values of $\phi_0^{\text{Im}}$ and $\beta_0^{\text{Im}}$ follow from Eqs. (4.16) and (4.18).

In this approximation $\phi$ decreases along with $\phi_{eq}$. It is pushed down by the effective potential due to the cosmological and Maxwell terms (the last two terms on the left hand side of Eq. (4.5)): if $\phi > \phi_{eq}$, the Maxwell term will be smaller than the cosmological term, and one obtains $\frac{d\phi}{dt} < 0$. However, the rate of decrease of $\phi$ has a limit corresponding to the dropping of the Maxwell term:

\begin{equation}
\left|\frac{d\phi}{dt}\right| < \min \left\{ (1 - \tilde{g})^{-1/2} b m \sigma e^{b\phi}, \frac{1}{2} (1 - \tilde{g})^{3/2} b m \sigma^3 e^{2b\phi} \right\} \quad (4.25)
\end{equation}
(The two different bounds come from neglecting either one or the other of the two terms in the prefactor of $\dot{\phi}$ in Eq. (4.8).) Thus $\phi$ can only keep pace with $\phi_{eq}$ as long as
\[
\left| \frac{d\phi_{eq}}{dt} \right| = \frac{1}{a} (1 - \tilde{g})^{-3/2} \frac{m}{\sigma}
\]
remains within this limit. This gives us a condition for the validity of the pseudo-static approximation:
\[
\sigma^2 e^{b\phi} \gg \max \left\{ \frac{1}{a^2 \tilde{g}(1 - \tilde{g})}, \left[ \frac{1}{a^2 \tilde{g}(1 - \tilde{g})^3} \right]^{1/2} \right\} .
\]
We call the region of phase space where this equation holds the high field regime.

### 4.3 Neutral Approximation

Since $\sigma^2$ and $e^{b\phi}$ both decrease in the high field regime, the high field condition, Eq. (4.27), will eventually cease to hold. We enter the low field regime, where $\phi$ will start to trail behind $\phi_{eq}$. Then the Maxwell term in Eq. (4.8) will become smaller than the cosmological term and can soon be neglected altogether. Since $b/a < 1$ the Maxwell terms in the other equations of motion can be dropped as well. In this limit we can obtain another approximation, which resembles a neutral black hole solution. This regime is either reached as a second stage after a period of pseudo-static evolution, or it emerges from a Euclidean instanton in a process of pair creation. In the latter case it describes the nucleation of black holes in both the high and low field regimes, as long as the black hole charge is small enough to make the Maxwell term negligible. In order to include the Euclidean section, we begin by giving the inner neutral approximation ($|\tilde{H}_0\tau| < O(1)$):
\[
\begin{align*}
\sigma_I(\tau) &= \sigma_0, \\
\theta_I(\tau) &= \theta_0, \\
\alpha_I(\tau) &= \tilde{H}_0^{-1} \sin \tilde{H}_0\tau, \\
\beta_I(\tau) &= \tilde{H}_0^{-1}.
\end{align*}
\]
With
\[
\tau = \frac{\pi}{2\tilde{H}_0} + it,
\]
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the outer neutral approximation ($\tilde{\mathcal{H}}_0 t > \mathcal{O}(1)$) is given by

\begin{align*}
\sigma(t) &= \bar{\sigma}_0 - mt, \quad (4.33) \\
\theta(t) &= \bar{\theta}_0 + b^2 m \int_0^t \sigma \, dt', \quad (4.34) \\
\alpha(t) &= \tilde{\mathcal{H}}^{-1} \cosh \int_0^t \tilde{\mathcal{H}} \, dt', \quad (4.35) \\
\beta(t) &= \tilde{\mathcal{H}}^{-1}. \quad (4.36)
\end{align*}

This describes an inflationary universe containing a dilatonic black hole pair of negligible charge. As in the FRW solution, the constraint

$$\sigma^2 + \frac{2}{b^2} \theta = \text{const} \quad (4.37)$$

is satisfied. The imaginary parts of the initial values of the inflaton and dilaton are

$$\theta_0^{\text{im}} = \frac{\pi}{2} b^2 \theta_0, \quad \sigma_0^{\text{im}} = -\frac{\pi}{2} \frac{\theta_0}{\sigma_0}. \quad (4.38)$$

The value of $\beta_0^{\text{im}}$ follows from Eq. (4.31).

### 5 Black Hole Pair Creation

We have found instanton solutions for a inflationary universe with a dilaton, and for a black hole pair immersed in this background. Thus black holes can spontaneously appear during extended chaotic inflation. The pair creation rate is given by

$$\Gamma = \exp \left[ - \left( 2I_{\text{bh}}^{\text{Re}} - 2I_{\text{bg}}^{\text{Re}} \right) \right], \quad (5.1)$$

where $I_{\text{bh}}^{\text{Re}}$ and $I_{\text{bg}}^{\text{Re}}$ are the real parts of the Euclidean actions of the black hole and background instantons. We shall discuss pair creation in the high and low field regimes separately.

#### 5.1 High Field Regime

**Pseudo-static Creation**

For the black hole solutions in the high field regime, we found the pseudo-static approximation, Eqs. (4.13) – (4.23). If pseudo-static black holes are created via an
instanton, the black hole charge is entirely fixed by the background: by rewriting Eq. (4.14) the charge can be expressed as a function of the dilaton and inflaton field:

\[ Q_{\text{eq}} = \frac{\tilde{g}^{1/2}}{1 + \tilde{g}} e^{\alpha(1-\tilde{g})\phi} \frac{\sigma}{m}. \]  

(5.2)

In fact black holes of different charge can be also be created, but we leave this discussion until later. In order to become familiar with the scenario, let us first consider the pair creation of a black hole of charge \( Q_{\text{eq}} \) in the high field regime, where Eq. (4.27) holds, and where it can then evolve according to the pseudo-static approximation. This process can be depicted in a dilaton-inflaton phase space diagram (Fig. 1). By Eq. (3.22) the paths traced out by the fields during inflation without a black hole lie on concentric circles around the origin. By Eq. (5.2), the lines of equal charge are hyperbolae. We expect the inflationary universe to start out near the Planck boundary and move counterclockwise along one of the circles. After the pair creation process takes place, the black hole will be described by a pseudo-static approximation. This means that the fields depart from the circle and move instead on the hyperbola corresponding to the black hole charge \( Q_{\text{eq}} \). Eventually the evolution moves out of the high field and into the low field regime, where \( \phi \) can no
longer follow $\phi_{eq}$. The evolution will then be described by the neutral approximation. Therefore the fields will leave the hyperbola and, by Eq. (4.37), move once again on a circle, smaller than the original one, until inflation ends.

By Eq. (5.2),

$$\frac{dQ_{eq}}{dt} = \frac{1}{1 + \tilde{g}} \left( \frac{\tilde{g}}{3} \right)^{1/2} \theta^{-1/\tilde{g}} \left[ \frac{\theta}{\sigma^2} - a^2 \tilde{g} (1 - \tilde{g}) \right].$$

(5.3)

In the high field regime, the right hand side is negative, by Eq. (4.27). Therefore pseudo-static black holes have a higher charge, the earlier in inflation they are created. This feature might seem surprising at first. For black holes pair created in standard chaotic inflation, the maximum charge increases as the cosmological constant runs down [7]. This is because the size of the black holes is inversely proportional to $H$. The black holes created later will thus be larger and can support a higher charge. The sharp contrast with the dilatonic case can be explained by the variability of the Maxwell coupling $e^{-2a\phi}$. This factor increases during inflation, overcompensating for the growth of the black hole radius $\beta$ in the Maxwell term $e^{-2a\phi} Q^2 / \beta^4$. In the low field regime, the right hand side of Eq. (5.3) is positive, and so the black hole charge will increase.

Using the inner solution, Eqs. (3.10) – (3.12), we can calculate the real part of the Euclidean background action, Eq. (3.2):

$$I_{Re}^{S_3} = -\frac{\pi}{2H_0^2}.$$  

(5.4)

In the black hole case the Euclidean action comes entirely from the $\tau = 0$ term in Eq. (4.4) and is given by

$$I_{Re}^{S^1 \times S^2} = \frac{\pi}{H_0^2 (1 + \tilde{g})} = \frac{\pi}{3H_0^2 (1 + \tilde{g})}.$$  

(5.5)

Using Eq. (5.1), the pair creation rate for the pseudo-static black holes can be determined:

$$\Gamma_{pseudo} = \exp \left( -\frac{\pi}{3H_0^2} \frac{1 + 3\tilde{g}}{1 + \tilde{g}} \right).$$  

(5.6)

The result is qualitatively the same as in the non-dilatonic case [4]. Since the exponent is negative, black holes are suppressed relative to the background, as they should be. Near the Planckian regime, where $H \approx 1$, the suppression is weak and many black holes can be created. As the dilaton and inflaton fields decrease, $H$ becomes much smaller than 1, and the pair creation will be exponentially suppressed.
Other Charges

We will now discuss the case where the black holes are created in the high field regime, but not with the charge given by Eq. (5.2). At least initially, they will therefore not be described by the pseudo-static approximation. It is convenient to define the variable $g'$, which takes the role of $\tilde{g}$ in the initial Euclidean section:

$$
\frac{g'}{(1+g')^2} = e^{-2(a-b)\phi} Q^2 m^2 \sigma^2.
$$

(5.7)

By Eqs. (4.6) and (4.9), the radii of the Euclidean $S^2 \times S^2$ will be approximately $\tilde{H}_0^{-1}(1-g')^{-1/2}$ and $\tilde{H}_0^{-1}(1+g')^{-1/2}$. Thus the charge has an upper limit determined by the condition $g' < 1$. If $g' > g$ (or equivalently, $Q > Q_{eq}$) the Maxwell term is larger than the cosmological term in the dilaton equation of motion. Thus the dilaton will increase as the inflaton decreases, and eventually it will reach the value $\phi_{eq}$ given by Eq. (4.14). The evolution of the black hole will then settle down to the pseudo-static approximation (or the neutral approximation, if Eq. (4.27) no longer holds by the time that $\phi = \phi_{eq}$), as shown in Fig. 2.

Figure 2: Creation of a dilaton black hole in the high field regime with $Q > Q_{eq}$. After the black hole is created at C, the dilaton increases to compensate for the high charge. Between P and N the evolution is described by a pseudo-static approximation and after N by a neutral approximation.

If, on the other hand, $g' < g$ ($Q < Q_{eq}$), then the Maxwell term will at first be negligible. The first stage of the black hole evolution will thus be described
by the neutral approximation, with both fields decreasing. Then there are two possibilities. The first is that $\phi$ eventually reaches $\phi_{eq}$. This would lead to a pseudo-static approximation, followed by a neutral approximation (see Fig. 3). The other possibility is that the fields enter the low field regime while $\phi$ is still larger than $\phi_{eq}$. In this case the whole evolution will be described by a neutral approximation (see Fig. 4).

The instanton action will now be

$$I_{S^1 \times S^2}^{\text{Re}} = -\frac{\pi}{3H_0^2(1 + g')} = -\frac{\pi}{3H_0^2(1 + g')},$$

(5.8)

and the pair creation rate is given by:

$$\Gamma_{\text{other}} = \exp \left( -\frac{\pi}{3H_0^2 \frac{1 + 3g'}{1 + g'}} \right).$$

(5.9)

Figure 3: Creation of a dilaton black hole in the high field regime with $Q < Q_{eq}$. A black hole created at $C$ is initially described by a neutral approximation. It enters a pseudo-static phase at $P$ and another neutral phase at $N$.

Since both $\tilde{g}$ and $g'$ must be positive and less than 1, these results do not differ qualitatively from the pseudo-static case. In particular, the upper bound for the charge, corresponding to $g' = 1$, decreases as the fields roll down, so that the black holes of highest charge are created early in inflation. When this upper bound becomes less than 1, only neutral black holes can be created, since the charge is quantised.
Figure 4: Creation of a dilaton black hole in the high field regime with $Q \ll Q_{\text{eq}}$. A black hole created at $C$ is described by a neutral approximation until inflation ends.

5.2 Low Field Regime

Neutral Creation

For the black hole solutions in the low field regime, we found the neutral approximation, Eqs. (4.28) – (4.36), by neglecting the Maxwell term. If such black holes are created via an instanton, the black hole charge must be small enough to justify this assumption: $g' \approx 0$, or equivalently,

$$Q \ll Q_{\text{eq}},$$

where $Q_{\text{eq}}$ is given by Eq. (5.2). The process is then very simple, as shown in Fig. 5. After the beginning of inflation, by Eq. (3.22), the fields move counterclockwise along a circle around the origin. By Eq. (4.37), when the pair creation takes place, the fields continue to move along the original circle, until inflation ends. The Euclidean action and pair creation rate are given by Eqs. (5.8) and (5.9), with $g'$ set to zero.

Higher Charges

Black holes created in the low field regime with a non-negligible charge, $0 < Q \leq Q_{\text{eq}}$, quickly evolve towards the neutral approximation, since $\phi_{\text{eq}}$ will decrease faster than $\phi$. Even if the charge is nearly maximal, $Q > Q_{\text{eq}}$, the dilaton will only increase
Figure 5: Creation of a dilaton black hole in the low field regime with $Q < Q_{eq}$. A black hole created at $C$ is described by a neutral approximation until inflation ends.

until $\phi = \phi_{eq}$, and then approach the neutral approximation. The corresponding path in phase space is shown in Fig. 5. The Euclidean action and pair creation rate are given by Eqs. (5.8) and (5.9).

6 Summary

We found instantons describing the pair creation of dilatonic black holes during extended chaotic inflation. Working in the Einstein frame, we presented Euclidean solutions for the background and black hole spacetimes that satisfy the no boundary proposal. We determined the complex initial values of the dilaton and inflaton. From the Euclidean action we calculated the pair creation rate.

There is no dilatonic de Sitter solution with a fixed cosmological constant. An inflationary universe, with a slowly decreasing effective cosmological constant, provides the simplest background for the creation of cosmological dilatonic black holes. Such black holes are described by Charged Nariai-type solutions, which are given by the topological product of 1+1-dimensional de Sitter space with a round two-sphere. They were presented recently in the context of a fixed cosmological constant. Adjusting for the time dependence of the effective cosmological constant, we found two types of approximate black hole solutions, which are attractive in different regions of the dilaton-inflaton phase space.
Figure 6: Creation of a dilaton black hole in the low field regime with $Q > Q_{eq}$. Black hole pair creation takes place at $C$. After the dilaton field has increased sufficiently to compensate for the high charge, the black hole enters a neutral phase.

In the pseudo-static approximation, the dilaton evolves such that it balances the Maxwell and the cosmological terms in the dilaton equation of motion. In the neutral approximation, the black hole charge is small, so that the Maxwell term can be neglected; the spatial topology is non-trivial, but the evolution of the dilaton field is the same as in inflation without a black hole. While the inflaton and dilaton fields are still large, i.e. as long as Eq. (4.27) holds, the black hole solutions are attracted to the pseudo-static approximation. Later the Maxwell term rapidly becomes negligible, and the black hole solutions are attracted by the neutral approximation.

We found that dilatonic black holes are plentifully produced near the Planck era. They become highly suppressed as the cosmological constant decreases, similar to the non-dilatonic case. Due to the time-dependence of the Maxwell coupling, however, there is one significant qualitative difference to black hole pair creation in standard inflation: the highest possible black hole charge is maximal at the beginning of inflation, and decreases during the roll-down of the inflaton field. In the late stages of inflation (the low field regime) it increases again.

We have considered Charged Nariai black holes because they are the only RNdS solutions of standard Einstein-Maxwell theory that have an analogue in dilatonic theories with a Liouville term [1, 2]. In standard inflation, however, most black holes
are nucleated via a different instanton, the “lukewarm” solution, which has no exact dilatonic analogue. In order to obtain an estimate of the number of magnetically charged dilaton black holes present in the universe today, it will be necessary to search for an approximate dilatonic equivalent of the lukewarm solution. We hope to make some progress on this question.

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