A pulsational approach to near infrared and visual magnitudes of RR Lyrae stars

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ABSTRACT

In this paper we present an improved theoretical scenario concerning near infrared and visual magnitudes of RR Lyrae variables, as based on up-to-date pulsating models. New relations connecting $V$ and $K$ absolute magnitudes with periods, mass, luminosity and metal content are discussed separately for fundamental and first overtone pulsators. We also show that the $V-K$ colors are predicted to supply tight constraints on the pulsator intrinsic luminosity. On this basis, we revisit the case of the prototype variable RR Lyr, showing that the parallax inferred by this new pulsational approach appears in close agreement with HST absolute parallax. Moreover, available $K$ and $V$ measurements for field and cluster RR Lyrae variables with known reddening and metal content are used to derive a relation connecting the $K$ absolute magnitude to period and metallicity ($M_K$-[Fe/H]-log$P$) as well as a new calibration of the $M_V$-[Fe/H] relation. The comparison between theoretical prescriptions and observations suggests that RR Lyrae stars in the field and in Galactic Globular Clusters (GGCs) should have quite similar evolutionary histories. The comparison between theory and observations also discloses a general agreement that supports the reliability of current pulsational scenario. On the contrary, current empirical absolute magnitudes based on the Baade-Wesselink (BW) method suggest relations with a zero-point that is fainter than predicted by pulsation models, together with a milder metallicity dependence. However, preliminary results based on a new calibration of the BW method provided by Cacciari et al. (2000) for RR Cet and SW And appear in a much better agreement with the pulsational predictions.

Key words: globular clusters: distances – stars: evolution – stars: horizontal branch – stars: oscillations – stars: variables: RR Lyrae

1 INTRODUCTION

Cosmic distances are fundamental cornerstones in modern astronomy, and reliable distance indicators can supply tight constraints on several cosmological parameters (see e.g. Goo- bar et al. 2000; Ferrarese et al. 2000). Accordingly, the number of theoretical and empirical investigations looking for new standard candles and/or aimed at improving the classical ones is countless in the recent literature (see e.g. Salaris & Cassisi 1997; Bono et al. 1999; Feast 1999; Walker 1999; Caputo et al. 2000; Carretta et al. 2000; Castellani et al. 2000; Gieren, Fouqu´e, & Storm 2000; Layden 2002). However, the ongoing problems affecting distance determinations are soundly revealed by current uncertainties on the true distance modulus $\mu_0$ of the nearest galaxy, the Large Magellanic Cloud (LMC), which is often taken as a reference for extragalactic distance determinations. In fact, current estimates range from $\mu_{0,LMC} \approx 18.37 \pm 0.23$ mag (RR Lyrae, Luri et al. 1998) to $\approx 18.7$ mag (classical Cepheids, Feast & Catchpole 1997; Bono et al. 1999; Caputo et al. 2002).

In this paper we discuss RR Lyrae stars, the radial pulsators connected with low-mass He-burning stars in the Horizontal Branch (HB) evolutionary phase. The use of these variables as distance indicators appears still hampered by significant uncertainties, and indeed the relation between absolute visual magnitude and iron content, $M_V$-[Fe/H], calibrated according to different empirical or theoretical constraints, supplies absolute magnitudes that for a given metallicity cover $\approx 0.3$ mag. Oddly enough, one also finds that the empirical approach based on the BW method, gives different results when different bands (optical vs near in-
frared) or different approaches (surface brightness vs canonical BW) are used.

In this context, the endeavor to link RR Lyrae pulsation properties, i.e., periods and/or amplitudes of the light curve, to the absolute magnitudes of these stars (see Sandage; Caputo 1997; Caputo et al. 2000; Caputo, Degl’Innocenti & Marconi 2001) appear of great relevance, because periods (and amplitudes) are firm and safe observational parameters, independent of distance and reddening. Empirical investigations (Longmore, Fernley, & Jameson 1986; Longmore et al. 1990) have already suggested that in the K band the RR Lyrae stars in GGCs follow a rather well-defined Period-Luminosity (PLK) relation whose slope is, within observational errors, almost constant when moving from metal-poor to metal-rich clusters. This scenario has been recently supported by the theoretical investigation by Bono et al. (2001, Paper I). In that paper, on the basis of nonlinear, convective RR Lyrae models, we found that the K-magnitude of fundamental (F) and fundamentalized first-overtone (FO) pulsators can be fitted by a common Period-Luminosity-Metallicity (PLZK) relation which appears to be in good agreement with observations.

In this paper we present improved predictions concerning both F and FO pulsators. Section 2 presents new theoretical relations concerning V and K absolute magnitudes. On this basis, in the same section we discuss improved PLZK relations, showing that V – K colors might allow us to constrain the intrinsic bolometric luminosity of the pulsators. Section 3 deals with the prototype variable RR Lyr, used as fundamental calibrator of the RR Lyrae distance scale, while observational data for field and cluster RR Lyrae stars are discussed in Section 4. In Section 5 we present a new relation connecting the K absolute magnitude to period and metallicity (M_K-[Fe/H]-logP) as well as a refined M_V-[Fe/H] relation. A few final remarks concerning both theoretical models and observations are outlined in Section 6.

2 NEW PLZK AND PLCVFK RELATIONS FOR F AND FO PULSATORS

To properly implement the pulsational framework presented in Paper I, we computed additional pulsating models adopting a finer grid of effective temperatures. We ended up with 132 F and 91 FO nonlinear models that cover a wide metallicity range 0.0001 ≤ Z ≤ 0.02. For each metal abundance the helium content was fixed according to the primordial helium abundance of Yp=0.24 and a helium-to-metal enrichment ratio ΔY/ΔZ ≈ 2.5.

At fixed chemical composition, we explored three different luminosity levels that cover current uncertainties on the ZAHB luminosity as well as post-ZAHB evolutionary effects (see column 3 in Table 1). As for as the pulsator mass is concerned, we adopted for each given metallicity a “reference mass” M_r, roughly following the correlation between metal content and mass of ZAHB models populating the RR Lyrae instability strip, i.e. 3.77 ≤ logT_e ≤ 3.86. These M_r values are listed in Table 1 (column 4), together with the evolutionary mass (with an average uncertainty of ~ 2%) of ZAHB models at logT_e = 3.85 and 3.80, as inferred according to evolutionary computations available in the literature. In order to investigate the dependence of the predicted relations on the pulsator mass, additional models were finally constructed at selected metal contents (Z = 0.0001 and Z = 0.001) but with slightly different reference mass values.

The entire set of fundamental and first overtone nonlinear models were integrated in time till they approach the limit cycle stability. These calculations provided periods, pulsation amplitudes, and bolometric light curves. The light curves were transformed into the observational plane by adopting bolometric corrections and color-temperature relations provided by Castelli, Gratton & Kurucz (1997a,b).

Table 2 and 3 list the predicted periods (in days) and mean (magnitude-weighted) V and K magnitudes for the new F and FO pulsation models, respectively. These new data implement similar predictions already presented in Tables 2 and 3 of Paper I.

Data listed in Table 2 and Table 3 disclose that, for any given mass, luminosity and metal content, the visual magnitude M_V of the pulsators is quite independent of the effective temperature, whereas M_K shows a strong dependence on this parameter (see also Paper I). This effect is the consequence of the fact that in the range of RR Lyrae effective temperatures the visual bolometric correction is roughly constant, whereas in the K-band it is strongly dependent on the effective temperature. It follows that the K-band magnitudes become significantly brighter when moving toward cooler effective temperatures. The pulsation equation P = f(L,M,T_e) connects, for each given chemical composition, periods to luminosity, mass and effective temperature. Therefore, the dependence of the absolute magnitude on T_e (through the BC) can be replaced with a dependence on period. Accordingly, when moving from the blue to the red edge of the instability strip the period increases at roughly constant absolute visual magnitude (δM_V/δlogP ~ −0.14), whereas M_K shows a much steeper correlation (δM_K/δlogP ~ −2.1).

On the other hand, when varying the luminosity for any given value of mass and effective temperature, one has again a relation between absolute magnitude and period. The key feature of the K-band is that the two derivatives (δM_K/δlogP)_{M,L} and (δM_K/δlogP)_{M,T_e} are quite similar. In other words, one finds a sort of degeneracy for which the near-infrared period-magnitude (PLK) relation is only marginally dependent on the bolometric luminosity of pulsators (see Fig. 1 in Paper I).

As for the dependence of the PLK relation on the pulsator mass and metal content, current calculations confirm the results presented in Paper I. However, at variance with the approach adopted in that paper, where F and fundamentalised FO periods (i.e., log P_F = log P_{FO} + 0.127) were taken simultaneously into account, we found that the intrin-
sic accuracy can be improved by independently treating the two pulsation modes.

By accounting also for the mass dependence, a linear regression through the models gives:

\[ M_K^F = 0.511 - 2.102 \log P + 0.095 \log Z - 0.734 \log L - 1.753 \log M/M_* \]  

\[ M_K^{FO} = -0.029 - 2.265 \log P + 0.087 \log Z - 0.635 \log L - 1.633 \log M/M_* \]  

where mass and luminosity are in solar units and the rms scatter is \( \sigma = 0.016 \) mag. As already discussed in Paper I, the predicted slope at constant luminosity and metallicity appears in good agreement with empirical data for GGC variables, as given, e.g., by the RR Lyrae stars in M3 collected by Longmore et al. (1990).

The present enlarged set of pulsating models confirms that over the explored range of metallicity and luminosity the \( M_K - \log P \) correlation is only mildly dependent on the luminosity, a variation of 0.1 dex implying a change of \( \delta M_K \sim 0.07 \) mag, as well as on the metallicity, a variation of 0.3 dex implying a change of \( \delta M_K \sim 0.03 \) mag. The predicted magnitudes are thus expected to be only mildly dependent on uncertainties introduced by these theoretical or observational parameters. On the other hand, the non-negligible dependence on the pulsator mass should only introduce a marginal uncertainty in the error budget since we can use quite firm theoretical constraints on such evolutionary parameter. As a fact, HB models available in the recent literature supply slightly discordant luminosity values, but very similar predictions concerning the mass.

As well known, fundamental (RRab) and first-overtone (RRc) RR Lyrae populate the red and the blue side of the pulsation region, respectively. Therefore one may assume for F and FO pulsators the predicted stellar mass listed in Table 1 for ZAHB models at log \( T_e = 3.80 \) and 3.85, respectively. The total uncertainty introduced by this assumption is of the order of 3.5% if we simultaneously account for the predicted spread in mass between F and FO pulsators and for the occurrence of post-ZAHB evolutionary effects.

By inserting these mass values into the previous relations, one eventually derives the following near-infrared Period-Luminosity-Metallicity (PLZ) relations:

\[ M_K^F = 0.565 - 2.101 \log P + 0.125 \log Z - 0.734 \log L \]  

\[ M_K^{FO} = -0.016 - 2.265 \log P + 0.096 \log Z - 0.635 \log L \]  

with a rms scatter of \( \sigma_K = 0.031 \) mag and 0.025 mag, respectively, including in quadrature an uncertainty of \( \sim 0.03 \) mag, as given by the adopted spread in mass (3.5%) at fixed metal abundance.

By following a similar procedure, one finds that also
the $T_c$ dependence of the pulsator color can be replaced by a period dependence. In particular, since the $V$ magnitudes are strongly correlated with $L$, whereas the $K$ magnitudes do not, we expect that the $V - K$ color is strongly dependent on luminosity, thus providing useful constraints to this unknown parameter.

As a fact, using the whole set of pulsating models, we derive:

\[(M_V - M_K)^F = 4.014 + 1.986 \log P - 0.134 \log Z - 1.662 \log L + 1.564 \log M_K\]  
(5)

\[(M_V - M_K)^{FO} = 5.195 + 2.518 \log P - 0.159 \log Z - 2.158 \log L + 1.815 \log M_K\]  
(6)

with a rms scatter of 0.027 mag and 0.023 mag, respectively (see Fig. 1).

By assuming the predicted masses at $\log T_c = 3.80$ and 3.85 for F and FO pulsators, respectively, the previous relations yield the Period-Luminosity-Color relations ($PLC_{V,K}$) in the $V,K$ bands for both F and FO pulsators, as given by:

\[(M_V - M_K)^F = 3.963 + 1.986 \log P - 0.162 \log Z - 1.662 \log L\]  
(7)

\[(M_V - M_K)^{FO} = 5.180 + 2.518 \log P - 0.168 \log Z - 2.158 \log L\]  
(8)

3 THE CASE OF RR LYR ITSELF

As already discussed by Bono et al. (2002, Paper II), the predicted $PLZ_K$ relation for F pulsators can be used to provide for the prototype variable RR Lyr a "pulsational" parallax that appears in close agreement with the absolute value recently measured by Benedict et al. (2002, hereafter B02) on the basis of new astrometric data collected with FGS 3, the interferometer on board of HST. In that paper, we assumed an intrinsic luminosity of RR Lyr in the range $\log L/L_\odot = 1.65-1.80$ to cover current uncertainties of HB models. On this basis, we derived $M_K = -0.54 \pm 0.062$ mag and a "pulsation" parallax $\pi_{\text{puls}} = \pm 0.03$ mag.

According to the results presented in the previous section, one may revisit the case of RR Lyr, avoiding any assumption on the RR Lyr bolometric luminosity. Adopting $\log P = -0.246$ (Kazarovets, Samus, & Durlevich 2001), $K = 6.54$ mag (Fernley, Skillen, & Burki 1993), $[Fe/H] = -1.39 \pm 0.15$ (Fernley et al. 1998a; Beers et al. 2000) and $V = 7.78$ mag (Hardie 1955), equation (7) gives $log L = 1.642 \pm 0.024 + 0.535A_V$. Thus, for each adopted extinction correction, one derives directly from the observed $V - K$ color the luminosity value to be inserted into equation (3), without using any evolutionary predictions. Once $M_K$ is determined from equation (3), both the intrinsic distance modulus ($\mu_0 = K - 0.11A_V - M_K$) and the absolute visual magnitude ($M_V = V - A_V - \mu_0$) can be easily determined.

Data listed in Table 4 show that the pulsation parallax presents a mild dependence on the adopted extinction correction. In particular, the top panel in Fig. 2 shows that when moving from $A_V = 0.01$ to 0.22 mag, the pulsation parallax (filled circles) decreases from $3.89 \pm 0.07$ to $3.78 \pm 0.07$ mas. However, current pulsational estimates remain well within the errors of the HST absolute parallax (dashed lines), and present a better formal accuracy ($\sigma_\pi/\pi \sim 3\%$ versus $\sim 5\%$). According to this evidence the pulsation parallaxes resemble direct astrometric measurements that are independent of interstellar absorption.

Concerning the RR Lyr predicted absolute magnitude, the middle and the bottom panel in Fig. 2 show that when moving from $A_V = 0.01$ to 0.22 mag $M_K$ changes from $-0.513 \pm 0.041$ mag to $-0.595 \pm 0.041$ mag, while $M_V$ from $0.722 \pm 0.041$ to $0.453 \pm 0.041$ mag, in close agreement with HST results (open circles), but with a better formal accuracy ($\pm 0.041$ versus $\pm 0.114$ mag). In other words, despite the substantial improvement in the accuracy of the RR Lyrae trigonometric parallax provided by HST, when compared
Finally, we note that the color excess given by Burstein & E5, supply a mean reddening of (Kovacs & Walker 2001), together with data listed in Table 2002) or to the Fourier amplitudes of the whole, the weighted mean of extinction corrections available in the literature gives \( <A_V> = 0.087 \pm 0.034 \) mag (see arrow in Fig. 2).

By adopting this mean reddening value, the HST astrometric measurement gives \( M_V = 0.607 \pm 0.114 \) (intrinsic HST uncertainty) \( \pm 0.034 \) mag (\( A_V \) contribution) = 0.607 \( \pm 0.119 \) mag. On the other hand, the pulsational approach gives log \( L = 1.689 \pm 0.030 \), and thus \( M_V = 0.624 \pm 0.041 \) (intrinsic uncertainty on the predicted \( M_K \) magnitude) \( \pm 0.044 \) mag (\( A_V \) contribution) = 0.624 \( \pm 0.060 \) mag.

The final uncertainty on the predicted RR Lyr parameters appears still too large to firmly discriminate among current theoretical or empirical constraints on RR Lyrae intrinsic luminosities and absolute visual magnitude. However, the use of the PLZ\( K \) relation over a large number of well-studied RR Lyrae stars, see next section, that cover a wide metallicity range will provide relevant information on both the slope and, provided that no systematic errors are affecting the pulsating models, the zero-point of the \( M_V \) (RR)-[Fe/H] relation.

### Table 4. The case of RR Lyr itself. For each adopted extinction correction \( A_V \), column 2 gives the luminosity (in solar units) derived using the equation (7). This luminosity is used in equation (3) to evaluate \( M_K(3) \), and in turn to estimate together with \( K = 6.54 \) mag the intrinsic distance modulus, \( \mu_0 \), and the “pulsational” parallax \( \pi_{puls} \). The last two columns give the absolute visual magnitude, based on \( V = 7.784 \) mag as well as on the pulsational and on the HST absolute parallax (\( \pi_{abs}=3.82\pm0.20 \) mas), respectively.

| \( A_V \) (mag) | log\( L(7) \) | \( M_K(3) \) (mag) | \( \mu_0 \) (mag) | \( \pi_{puls} \) (mas) | \( M^puls \) (mag) | \( M^{HST} \) (mag) |
|-----------------|-----------------|-----------------|-------------------|-------------------|-------------------|-------------------|
| ±0.024          | ±0.041          | ±0.041          | ±0.072            | ±0.041            | ±0.114            |
| 0.01            | 1.648           | −0.513          | 7.052             | 3.888             | 0.722             | 0.684             |
| 0.04            | 1.664           | −0.524          | 7.060             | 3.872             | 0.684             | 0.654             |
| 0.07            | 1.680           | −0.536          | 7.069             | 3.857             | 0.645             | 0.624             |
| 0.10            | 1.696           | −0.548          | 7.077             | 3.842             | 0.607             | 0.594             |
| 0.13            | 1.712           | −0.560          | 7.086             | 3.827             | 0.568             | 0.564             |
| 0.16            | 1.728           | −0.572          | 7.094             | 3.812             | 0.530             | 0.534             |
| 0.19            | 1.744           | −0.583          | 7.103             | 3.797             | 0.491             | 0.504             |
| 0.22            | 1.760           | −0.595          | 7.111             | 3.783             | 0.453             | 0.474             |

### Table 5. Empirical data for RR Lyr based on the photometry presented by Hardie (1955). The Fourier amplitudes \( A_1, A_3 \) and \( A_5 \) were estimated by fitting the \( V \)-band light curve by means of a cosine series with 7 components.

| \( P \) (d) | \( (V) \) (mag) | \( (B-V) \) (mag) | \( A_3 \) (mag) | \( A_5 \) (mag) | \( A_7 \) (mag) |
|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.5668      | 7.784           | 0.371           | 1.154           | 0.892           | 0.3014          |
|             |                 |                 | 0.1493          |                 | 0.0920          |
from the measured $V - K$ color, without any \textit{a priori} assumption related with the predicted luminosity of horizontal branch models. Once the individual luminosity of RR\textit{ab} variables is derived from equation (7) \textit{or} (8) for RR\textit{c} variables, the predicted PLZ$_K$ \textit{relations} \citep[see equations (3) and (4)]{1998a} provide the absolute near-infrared magnitude. This supplies the intrinsic distance modulus, and in turn the absolute visual magnitude. This means that we can investigate both the $M_K$-\textit{[Fe/H]}-$\log P$ and the $M_V$-\textit{[FeH]} \textit{relations}.

We searched for RR\textit{Lyrae} stars for which accurate $V$ and $K$ light curves, as well as reliable evaluations of both reddening and metallicity, are available in the literature. A total of 39 stars were found, with 30 field and 9 GGC members. Metal abundances have been taken from the homogenous compilation by \cite{1999}. The sample is listed in Table 6 together with key parameters such as the luminosity estimated according to equation (7) \textit{or} (8), the $M_K$ provided by equation (3) \textit{or} (4), and the ensuing $M_V$ magni-
A pulsational approach to near infrared and visual magnitudes of RR Lyrae stars

Figure 3. Estimated individual luminosities of RR Lyrae stars in our sample versus metal abundance (see Table 6). Filled and open circles show field RRab and RRc variables, while filled triangles indicate cluster RRab variables. The asterisk and the open triangle show RR Lyr itself and the peculiar RR Lyrae V49 in 47 Tuc, respectively. The error bars display current uncertainties in metal abundance and in luminosity. Dotted and solid lines refer to old and recent ZAHB models, respectively, while the dashed line depicts the exhaustion of the central He-burning phase. See text for more details.

Figure 4. Pulsational absolute visual magnitudes of RR Lyrae variables in our sample as a function of metal content. The symbols are the same as in Fig. 3.

tudes. The errors associated with luminosity, near-infrared, and visual absolute magnitudes account for the intrinsic dispersion of the predicted relations, as well as for an average uncertainty of ±0.20 dex on the metallicity and of ±0.03 mag on $A_V$.

Figure 3 shows the derived "pulsational" luminosity of RR Lyrae stars in our sample as a function of the metal content. Filled and open circles display field RRab and RRc, respectively. RRab stars in GGCs (M4, M5, and M92) are plotted as filled triangles, while RR Lyr itself with an asterisk. The open triangle marks the position of the RR Lyrae V9 in 47 Tuc which was neglected, since it is peculiar (Storm et al. 1994b). Together with individual RR Lyrae stars, Fig. 3 also shows the predicted log $L_{RR} - [Fe/H]$ relation according to the old ZAHB models by Castellani, Chieffi, & Pulone (1989, 1991, hereinafter CCP, dotted line) as well as the improved computations presented by Cassisi et al. (1998, 1999, hereinafter C99, solid line). The dashed line displays the locus of central He exhaustion according to C99.

As a whole, pulsational luminosities appear in reasonable agreement with the result of recent evolutionary computations based on improved input physics. In fact, with the exception of V9 in 47 Tuc, they show a dispersion in luminosity of the order of 0.10 dex (dashed line) above the theoretical ZAHB, in agreement with the predictions of synthetic horizontal branch simulations (see, e.g. Caputo et al. 1993; Demarque et al. 2000). Thus it seems that recent evolution-
Figure 5 gives the following analytical relations (solid lines):

\[ M_V = 0.718(\pm 0.072) + 0.177(\pm 0.069)[\text{Fe/H}] \]  

for RR Lyrae stars with [Fe/H] \( \leq -1.6 \), and

\[ M_V = 1.038(\pm 0.077) + 0.359(\pm 0.027)[\text{Fe/H}] \]  

for RR Lyrae stars with [Fe/H] > -1.6.

Let us now compare our results with current distance determinations based on the BW method. To perform a robust comparison we adopted the absolute visual magnitudes for field RR Lyrae stars derived by Fernley (1994). The key feature of this sample is that the empirical data available in the literature were reduced by the previous author to the homogeneous metallicity scale (Fernley et al. 1998a). We ended up with a sample of 27 RR Lyrae stars. The nine GGC variables were not included, since they appear to be affected by larger intrinsic errors and also because they have not been reduced to the same BW method yet. Figure 6 shows the comparison between these NIR BW absolute magnitudes (filled triangles) and the absolute visual magnitudes for the same objects, as based on the PLZ\(_K\) relation (circles). Data plotted in this figure display quite clearly that for RR Lyrae stars more metal-poor than [Fe/H] \( \sim -1.5 \), the discrepancy is on average of the order of 0.2 mag, in the sense that BW magnitudes are fainter than those provided by current pulsational approach. The discrepancy decreases toward higher metal contents and vanishes in the metal-rich tail. This indicates that the metallicity dependence given by the BW method is milder than predicted by pulsation models.

Note that a new calibration of the BW method has been provided by Fernley et al. (1998b), but the difference over the entire metallicity range is negligible when compared with Fernley (1994). On the contrary, the preliminary analysis provided by Cacciari et al. (2000) of the assumptions currently adopted in the BW method disclosed quite different results. In particular, by using available photometric data together with a large set of new atmosphere models, they found that RR Cet ([Fe/H]=-1.45) is \( \approx 0.12 \) mag brighter than previously estimated, whereas no significant correction was required for SW And ([Fe/H]=-0.24). As a result, these new absolute magnitudes (open squares in Fig. 6), once confirmed, appear fully consistent with the \( M_V - [\text{Fe/H}] \) relation based on pulsation models [equation (10)]. This suggests that the slope of this relation, at least in the metallicity range \(-1.5 \leq [\text{Fe/H}] \leq -0.4\), is steeper than currently adopted (see also McNamara 1999).

Following a referee’s suggestion, one may investigate whether the discrepancy at [Fe/H] < -1.5 between pulsational absolute visual magnitudes and BW magnitudes might be settled by changing the mass value in equations (1) and (5). We found that to remove the difference of
A pulsational approach to near infrared and visual magnitudes of RR Lyrae stars

Figure 6. Comparison between the absolute visual magnitudes estimated using the current pulsation approach and the independent evaluations derived by Fernley (1994) using the NIR BW method (triangles). The two open squares mark the position of RR Cet ([Fe/H]=−1.45) and SW And ([Fe/H]=−0.24) according to the new calibration of the BW method provided by Cacciari et al. (2000). The solid lines are the relations described in the text.

∼ 0.25 mag between BW and pulsational magnitudes at [Fe/H]=−1.8, one should account for a decrease in mass of the order of ∼ 23%, thus predicting a quite unrealistic low-mass, 0.55 $M_\odot$, for these metal-poor (Z ~ 0.0003) horizontal branch stellar structures.

As for K-magnitudes, Fig. 7 shows that the absolute K-magnitudes estimated using the pulsational approach do obey a very tight relation with the metal content, once the dependence on period is removed. Owing to the small number (3) of RRc variables, their periods have been fundamentalized, (log $P_F$=log $P_{FO}$+0.127), thus yielding for the entire sample the following relation:

$$M_K^P = -0.770(\pm 0.044) - 2.101 \log P + 0.231(\pm 0.012)[Fe/H]$$

(11)

The small errors affecting the coefficients of this relation further strengthen the evidence that near-infrared observations could supply accurate distance determinations.

To investigate in more detail the discrepancy between the current approach and the BW method, we compare the pulsational absolute K magnitudes with the absolute K magnitude obtained using the BW true distance modulus and the dereddened apparent K magnitudes listed in column (6) of Table 6. Figure 8 shows the comparison between pulsational $M_K$ values (circles) and results based on the BW method (triangles). Once again, the BW magnitudes at $[Fe/H] \leq -1.5$ are fainter than predicted by the pulsational approach, leading to a milder dependence on [Fe/H] with respect to the PLZ_K predictions (solid line). However, the absolute K-magnitudes derived by Cacciari et al. (2000) for SW And and RR Cet (open squares) appear once again in good agreement with our pulsational approach.

6 SUMMARY AND FINAL REMARKS

Data plotted in Fig. 6 and Fig. 8 clearly show that the pulsational approach provides absolute magnitudes for RR Lyrae stars that are significantly brighter than suggested by the BW method. This difference is more evident for metal-poor ([Fe/H] ≤ −1.5) RR Lyrae variables. The recent revision provided by Cacciari et al. (2000) for RR Cet ([Fe/H]=−1.45) and SW And ([Fe/H]=−0.24) agrees quite well with the pulsational results. This suggests that both the zero-point and the slope of the $M_V$-[Fe/H] and of the $M_K$-[Fe/H]-logP relation based on the old BW calibration need to be revisited.

We have also found that both V and K absolute magnitudes of RR Lyr itself, estimated via the pulsational approach, are in good agreement with the trigonometric parallax recently measured by HST (see Fig. 2). This suggests that at least for the metallicity of RR Lyr ([Fe/H]=−1.39), the pulsational approach is consistent with direct distance determinations.

As far as the metal-dependence is concerned, we finally...
Figure 8. Comparison between the absolute $K$-magnitudes based on the pulsation approach and the BW method. The latter were derived using the true distance moduli given by Fernley (1994) and the $K_0$-magnitudes listed in Table 6. The solid line refers to equation (11).

Note that the effect of metallicity on equation (11) can be revealed by a simple cross-check between predicted $K$-magnitudes and periods. It has been already shown (see Paper I) that $K$-measurements for RR Lyrae stars in M3, namely for variables at quite constant luminosity and metal content, closely follow the pulsational constraints on the period dependence. On the contrary, data plotted in Fig. 9 show that for the current sample of cluster and field RR Lyrae stars the $M_K\log P$ relation is significantly steeper than predicted under the above quoted assumptions (dashed lines). This is the expected aftermath of the fact that metal-poor variables, which have longer periods (see data in Table 6), have also brighter bolometric luminosities (see Fig. 3) than metal-rich ones.

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REFERENCES
Beers T. C. et al., 2000, AJ, 119, 2866
Benedict G. F. et al., 2002, AJ, 123, 473
Blanco V. M., 1992, AJ, 104, 734
Bono G., Caputo F., Castellani V., Marconi M., Storm J. 2001, MNRAS, 326, 1183 (Paper I)
Bono G., Caputo F., Castellani V., Marconi M., Storm J. 2002, MNRAS, 332, 78 (Paper II)
Bono G., Caputo F., Castellani V., Marconi M. 1999, ApJ, 512, 711
Bono G., Caputo F., Stellingwerf R. F., 1995, ApJS, 99, 263
Bono G., Castellani V., Marconi M., 2000, ApJ, 532, 129
Bono G., Caputo F., Cassisi S., Incerpi R., Marconi M., 1997, ApJ, 483, 811
Burstein D., Heiles C., 1978, ApJ, 225, 40
Cacciari C., Clementini G., Fernley J.A., 1992, ApJ, 396, 219
Cacciari C., Clementini G., Castelli F., Melandri F., 2000, in The impact of Large-Scale Surveys on Pulsating Star Research, ASP Conference Series 203, ed. L. Szabados & D. W. Kurtz, 176
Caputo F., 1997, MNRAS, 284, 994
Caputo F., Castellani V., Marconi M., Ripepi V., 2000, MNRAS, 316, 819
Caputo F., Degl’Innocenti S., Marconi M., 2001, in W Centauri, a unique window into astrophysics, ASP Conference Series 265, ed. F. van Leeuwen, J. D. Hughes & G. Pietto, 185
Caputo F., Rinaldis A., Manteiga M., Pulone L., Quarta M. L., 1993, A&A, 276, 41
Caputo F., Marconi M., Musella F., 2002, ApJ, 566, 833
Cardelli J. A., Clayton G. C., Mathis J. S., 1989, ApJ, 345, 245
Carretta E., Gratton R.G., Clementini G., Fusi Pecci F., 2000, ApJ, 533, 215
Cassisi S., Castellani V., Degl’Innocenti S., Weiss A., 1998, A&AS, 129, 267
Cassisi S., Castellani V., Degl’Innocenti S., Salaris M., Weiss A., 1999, A&AS, 134, 103 (C99)
Castellani V., Chieffi A., Pulone L. 1989, ApJ 344, 239

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A pulsational approach to near infrared and visual magnitudes of RR Lyrae stars

Castellani V., Chieffi A., Pulone L. 1991, ApJS 76, 911 (CCP)
Castellani V., Degl’Innocenti S., Girardi L., Marconi M.,
Prada Moroni P.G., Weiss A. 2000, A&A, 354, 150
Castelli F., Gratton R.G., Kurucz R. L., 1997a, A&A, 318, 841
Castelli F., Gratton R.G., Kurucz R. L., 1997b, A&A, 324, 432
Clementini G., Carretta E., Gratton R., Merighi R., Mould J. R.,
McCarthy J. R., 1995, AJ, 110, 2319
Demarque P., Zinn R., Lee Y-W., Yi S., 2000, AJ, 119, 1398
Feast M. W., 1999, PASP, 111, 775
Feast M. W., Catchpole R. M. 1997, MNRAS, 286, 1
Fernley J., 1994, A&A, 284, 16
Fernley J., Barnes T.G., Skillen I., Hawley S.L., Hanley
C.J., Evans D.W., Solano E., & Garrido R., 1998a, A&A, 330, 515
Fernley J., Skillen I., Burki G., 1993, A&AS, 97, 815
Fernley J., Skillen I., Carney B.W., Cacciari C., Janes K., 1998b, MNRAS, 293, 61
Fernley J. A., Skillen I., Jameson R. F., Barnes T. G., Kilkenny
D., Hill G. 1990, MNRAS, 247, 287
Ferrarese L. et al., 2000, ApJ, 529, 745
Gieren W. P., Fouqué P., & Storm J., 2000, in From Extrasolar Planets to Cosmology: The VLT Opening Symposium, ed. J. Bergeron & A. Renzini (Berlin: Springer-Verlag), 217
Goobar A. et al., 2000, Physica Scripta, 85, 47
Hardie R. H., 1955, ApJ, 122, 256
Jones R.V., Carney B.W., Latham D.W. 1988a, ApJ, 326, 312
Jones R.V., Carney B.W., Latham D.W., 1988b, ApJ, 332, 206
Jones R.V., Carney B.W., Storm J., Latham D.W., 1992, ApJ, 386, 646
Kazarovets E. V., Samus N. N., Durlievich O. V., 2001, IBVS, 5135, 1
Kovacs G., Walker A. R., 2001, A&A, 374, 264
Layden A. C., Hanson R. B., Hawley S. L., Klemola, A. R., Hanley C. J. 1996, AJ, 112, 2110
Layden A. C., 2002, in IAU Colloq. 185, Radial and Nonradial Pulsations as Probes of Stellar Physics, ed. C. Aerts, T. Bedding, J. Christensen-Dalsgaard (San Francisco: ASP), 118
Liu T, Janes K.A., 1990a, ApJ, 354, 273
Liu T, Janes K.A., 1990b, ApJ, 360, 561
Longmore A. J., Robson, E. I., Jameson R. F., 1986, MNRAS, 221, 589
Longmore A. J., Dixon R., Skillen I., Jameson R. F., Fernley, J. A., 1990, MNRAS, 247, 684 (L90)
Luri X., Gomez A.E., Torra J., Figueras F., & Mennessier, M. O. 1998, A&A, 335, L81
McNamara D. H., 1999, PASP, 111, 489
Piersimoni A. M., Bono G., Ripepi V., 2002, AJ, 124, 1528
Rey S-C., Lee Y-W., Joo J-M., Walker A., Baird S., 2000, AJ, 119, 1824
Salaris M., Cassisi S., 1997, MNRAS, 289, 406
Sandage A., 1993, AJ, 106, 687
Skillen I., Fernley J.A., Stobie R.S., Jameson R.F., 1993, MNRAS, 265, 301
Solano E., Garrido R., Fernley J., Barnes T. G. 1997, A&AS, 125, 321
Storm J., Carney B.W., Latham D.W., 1994a, A&A, 290, 443
Storm J., Nordström B., Carney B.W., Andersen J., 1994b, A&A, 291, 121
Walker A. R., 1999, in Post-Hipparcos Cosmic Candles, ed. A. Heck & F. Caputo (Dordrecht: Kluwer), 125

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