Pentaquark states in a chiral potential

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Abstract

We discuss a possible interpretation for the pentaquark baryon $\Theta^+$ which has an exotic quantum number of strangeness $S = +1$. The role of the pion which modifies the structure of single-particle levels of quarks is examined. When the pion field is sufficiently strong, the ground state of five quarks for $\Theta^+$ acquires positive parity. If this is the case, an excited state may appear slightly above the ground state with negative parity.

Recently, in the measurement of $K^\pm$ in a photo-induced reaction from the neutron, $\gamma n$(in$^{12}$C) → $K^+K^-n$, Nakano and collaborators (LEPS at SPring-8) has observed a sharp peak in the $K^+n$ invariant mass analysis [1]. It is located at $M \sim 1540 \pm 10$ MeV with a very narrow width $\Gamma < 25$ MeV. Subsequently, DIANA collaboration at ITEP [2] and CLAS collaboration at Jefferson Lab [3] have also confirmed the existence of the resonance structure. The state corresponding to this sharp peak carries unit charge $Q = +1$ and positive strangeness $S = +1$ as a composite system of $K^+n$, and has been identified with an exotic baryon which is now denoted as $\Theta^+$. In a naive valence quark model, such a state can not be described as a conventional three quark state. The flavor quantum numbers of ordinary $qqq$ baryons are dictated by the SU(3) irreducible decomposition, $3 \times 3 \times 3 = 1 + 8 + 8 + 10$. As a consequence, the strangeness of $qqq$ baryons are limited to $S = -3$ ($\Omega$), $-2$ ($\Xi$), $-1$ ($\Sigma, \Lambda$) and 0 ($N$). Hence the $S = +1$ state must require at least five quarks ($qqqq\bar{q}$). Such a state is a member of antidecuplet $\bar{10}$ or even higher dimensional representations such as 27-let. Using flavor conjugate diquarks $(\bar{q}_i)_{qq} \equiv \epsilon_{ijkq_jq_k}$, $\Theta^+$ in the antidecuplet can be constructed as

$$|\Theta^+ \rangle \sim (\bar{s})_{qq}(\bar{s})_{qq\bar{s}}, \quad (\bar{s})_{qq} \sim ud.$$  

The members of the antidecuplet are shown in Fig. [1]. From this construction, the isospin of $\Theta^+$ is $I = 0$. In $\bar{10}$, the seven baryons located at the cites other than the three corners of the triangle may be affected by mixing with octet members. However, the three states on the corners are genuine exotic pentaquark states.

Theoretically, using the chiral quark model, Diakonov et al [4] have already studied the properties of $\Theta^+$ as a member of antidecuplet with predictions of masses and width which are surprisingly very close to what the experimentalists observed. Furthermore, the spin and parity $J^P$ were predicted to be $J^P = 1/2^+$. Once the theoretical method of the chiral quark model are accepted, these consequences can be derived in a straightforward manner. In particular, many of the mass formulae and relations among decay widths are dictated by the symmetry (algebraic) relations [5]. However, if we attempt to understand these properties in the naive quark model, we will be immediately in trouble, since even a very fundamental quantum number such as parity does not agree with the prediction of the chiral quark model. Since the lowest quark state is the $s$ state ($l = 0$), the lowest $qqqq\bar{q}$
state must have negative parity. In another picture of a meson-baryon hybrid (molecule), a $K^+n$ (or $K^0p$) bound state in an $s$ orbit also carries negative parity.

In the latter approach, however, it has been known that the $p$ state rather than the $s$ state becomes lowest for $KN (=K^+n$ and $K^0p)$ and $\bar{K}N$ systems in the bound state approach of the Skyrme model \[6\]. There, the kaon interaction with the hedgehog soliton plays a peculiar but crucially important role to change the ordering of the $s$ and $p$ states of the kaon. As a consequence, the state of $J^P = 1/2^+$ appears as the lowest state both for the $S = \pm 1$ sector. Furthermore, these states are split by the Wess-Zumino-Witten term such that the $S = -1$ sector is lowered, while the $S = +1$ state is pushed up. Now comparing the Skyrme model and the chiral quark soliton model, they must have a common feature which has the origin in the non-perturbative chiral dynamics, since the starting point of their methods is the formation of the hedgehog soliton configuration. The strong correlation generated by the pion must have the crucial role. In fact, in a recent report, Stancu and Riska studied quark energies of the $uudd\bar{s}$ states in a chiral quark model \[7\]. It was shown that flavor dependent interaction through Nambu-Goldstone boson (pion) exchanges affected strongly the structure of the quark levels and interchanged the ordering of the $s$ and $p$ orbits.

In this paper, we show that the role of the pion is understood clearly by varying the strength of the interaction and by seeing the structure of the single particle levels of quarks. This is most conveniently demonstrated in the chiral bag model \[8\], where an interaction of the $\vec{\sigma} \cdot \vec{\tau}$ type splits degenerate quark states in the spin and isospin states. Such a point of view provides an interesting way to consider structure of not only three valence quark states but also multi-quark states.

Let us start with a slightly general equation of motion of quarks under the influence of a potential generated by the chiral field (pion):

$$ (i\partial - g(\sigma(\vec{x}) + i\vec{\tau} \cdot \vec{\pi}(\vec{x})\gamma_5)) \psi = 0. $$

where $\sigma(\vec{x})$ and $\vec{\pi}(\vec{x})$ are static scalar sigma and pseudoscalar pion fields which are regarded as background potentials for the quarks. In order to clarify the role of the pion, let us assume the hedgehog configuration in which isospin of the pion field $\vec{\pi}(\vec{x})$ is correlated to the spatial orientation $\hat{r}$, $\vec{\pi}(\vec{x}) = \hat{r} h(r)$. Here $h(r)$ (and $\sigma(\vec{x}) \rightarrow \sigma(r)$) are spherically symmetric functions. The equation of motion with this pion (and sigma) background field affects the motion of the light $u, d$ quarks, but not that of the strange $s$ quark. At this point, we break “spontaneously” the flavor SU(3) symmetry.
Write the four component spinor for the lowest $s$ state,

$$\psi = \begin{pmatrix} u(r) \\ -i\vec{\sigma} \cdot \hat{r}v(r) \end{pmatrix} \chi,$$

(3)

where $\chi$ is a spin-isospin spinor. Then after performing spatial integral the Hamiltonian takes on the form:

$$H = c_1 + c_2 \langle \chi | \vec{\sigma} \cdot \vec{\tau} | \chi \rangle,$$

(4)

where the first term is from the kinetic and $\sigma$ (mass) terms of (2), and the second term from the pion term, with $c_1$ and $c_2$ being numerical constants. A crucial point here is the appearance of the spin and isospin coupling $\sigma\tau$ term. Without that term, there is a four-fold degeneracy in the spin-isospin states, $u \uparrow$, $u \downarrow, d \uparrow$ and $d \downarrow$. Now if the $\sigma\tau$ term is turned on, then a conserved quantity is the grand angular momentum, the sum of the total angular momentum $\vec{j} = \vec{l} + \vec{s}$ and isospin $\vec{\tau}$, $\vec{K} = \vec{j} + \vec{\tau}$. When $\vec{l} = 0$, the above four spin and isospin states form four eigenstates of $\vec{K}$ with $(K, K_z) = (0, 0), (1, -1), (1, 0), (1, 1)$. Since the coefficient $c_2$ is positive for the system of the baryon number $B = 1$, the $K = 0$ state is lowered whereas the $K = 1$ state is pushed up. In the following discussion, we denote eigenstates of $K$ by $h$ instead of $u \uparrow$ and etc.

In order to see this point slightly in a quantitative manner, we adopt the result from the chiral bag model, where quarks are confined in a spherical bag of a radius $R$, and interact.
with the pion ($\sigma$ and $\pi$) at the bag surface. A mean field solution is then obtained in the hedgehog configuration. For instance, eigenenergies of hedgehog quarks are obtained as functions of the pion strength (chiral angle) at the bag surface $F(r = R) \equiv F$. Here $F$ is defined as a polar angle of the sigma and pion field, $\tan F = h(R)/\sigma(R)$ [8]. In Fig. 2 energy levels for quarks are shown as functions of $F$ for several lower lying states in units of $1/R$. The labels denote $K^P$, where $P$ is parity. When $F = 0$, there is degeneracy in the states of $K^P$ and $(K + 1)^P$. As $F$ is turned on, the degeneracy is resolved. Furthermore, there is a reflection symmetry between the quark and antiquark levels, $E_{\text{quark}}(F, K^P) = E_{\text{antiquark}}(\pi - F, K^P)$ with respect to the axis $F = \pi/2$. At this angle (the magic angle) there appears precisely a zero mode of $0^+$. However, at $F \sim 1$ the level crossing occurs, and for $F \gtrsim 1$

$$E_{0^+} < E_{1^-} < E_{1^+}. \quad (5)$$

This change in the level ordering is crucial when we consider the structure of various baryons including $\Theta^+$. Let us first see implications of the level crossing for $\Theta^+$. A little bit more quantitative discussions will be made shortly. The antistrange quark $\bar{s}$ is not subject to the chiral potential and stays in the $J^P = 1/2^+$ orbit determined by the MIT bag boundary condition ($F = 0$) [9]. $E_\bar{s} \sim 2.3/R$ for $m_\bar{s} \sim 150$ MeV and $R \sim 0.7$ fm. Contrary, the light $h$ ($u,d$) quarks are influenced by $F$. For finite $F$, three $h$ quarks occupy the lowest $K^P = 0^+$ level due to the color degrees of freedom. Now the fourth $h$ quark enters the next level, which is $1^+$ for $F \lesssim 1$ and $1^-$ for $F \gtrsim 1$. Depending on which orbit the fourth quark is in, the total parity of $\Theta^+$ changes; for $F \lesssim 1$ $P = -$, while for $F \gtrsim 1$ $P = +$. The predictions of the chiral quark model and the Skyrme model correspond to the latter case.

To make further discussions in a slightly quantitative manner, we consider a configuration which was found to be optimal for nucleon structure, where $F \sim \pi/2$ and $R \sim -0.7$ fm [8]. In this case the unit of energy is $1/R \sim 300$ MeV. In the following discussions, energy is estimated for the hedgehog configuration. Baryon states of the proper spin and isospin should be obtained by performing spin and isospin projection. The parity of the projected state is, however, not changed from that of the hedgehog.

**Ground state nucleon ($S = 0$)**

Three hedgehog quarks ($h$) occupy the lowest $K^P = 0^+$ states. This quark configuration is denoted as $(0^+_h)^3$. At around $F \sim \pi/2$, the energy of the $0^+$ state is almost zero, and hence the energy contribution from the valence quarks is also almost zero. However, energies are also supplied from the vacuum as the Casimir energy and in the soliton field outside the bag. The total energy of the hedgehog configuration is then about 1.3 GeV. In the chiral bag model, when using the experimental value for the pion decay constant the hedgehog mass is larger than the experimental masses. Also, for physical nucleons, spin and isospin projection has to be performed, which adds a rotational energy as proportional to $I(I + 1)$, where $I$ is the isospin value of the nucleons. Hence the mass of the ground state nucleon is slightly larger than the mass of the hedgehog which is about 1.4 GeV.
\( \Theta^+(S = 1) \):

As we have shown, a tentative quark configuration of \( \Theta^+ \) is \((0_h^+)^2(1_h^+)^1(1/2_s^-)^1\). Here the parity of the antiquark \( \bar{s} \) includes the intrinsic one. The excitation energy of the pentaquark state is about \( 4/R \), which is about 1.2 GeV. Therefore, in a rough estimation, the mass of the 5 quark state is about twice (or slightly less) of the mass of the ground state. If the nucleon mass is normalized to the experimental value of 938 MeV, then the mass of the pentaquark state would be about 1.8 GeV. This value is larger as compared to the observed mass of \( \Theta^+ \). However, we do not yet include the expected hyperfine splitting \([4]\). In fact, 1.8 GeV is close to the mean value of the mass of the antidecuplet in the chiral quark soliton model. From the quark energy level of Fig. 2 we expect that an excited state of five quarks may exist slightly above \( \Theta^+ \) with having negative parity.

**Excited states of the nucleon \((S = 0)\)**

By lifting one \( h \) quark into a higher level, we can form excited states. At \( F \sim \pi/2 \), the first excited level is \( 1^- \) and the next one is \( 1^+ \). Hence the relevant three quark configurations are \((0_h^+)^2(1_h^-)^1\) and \((0_h^+)^2(1_h^-)^1\), respectively. If we take this result seriously, the first excited state of the nucleon would be \( 1/2^- \) with a mass about \( 2/R \sim 600 \text{ MeV} \) above the ground state, while the second excited state is \( 1/2^+ \) at about a few hundreds MeV above the \( 1/2^- \) state. If we make a rough argument in which a slightly larger bag and small \( F \) would be preferred for excited states due to less number of (two) \( h \) quarks in the lowest \( 0^+ \), then the above conclusion would be slightly modified; the ordering of \( 1/2^\pm \) states would change. Qualitatively, we expect that the two \( 1/2^\pm \) states appear with similar excitation energies. We may identify them with \( N(1440) \) and \( N(1535) \). The nature of \( N(1440) \) is, however, quite different from an ordinary picture of radial excitation. Here, the energy splitting is produced by the interaction with the chiral potential. It would be interesting to consider such a configuration for the positive parity \( N(1440) \) resonance. On the other hand, the \( 1^- \) state is an \( l = 1 \) orbital excitation just as in an ordinary interpretation.

\( \Lambda(g.s.) \) and \( \Lambda(\text{excited states}) \), \((S = -1)\)

For the ground state, the three quark configuration is \((0_h^+)^2(1_h^-)^1\). At \( F \sim \pi/2 \), the energy appears to be higher than the ground state by \( 2/R \sim 600 \text{ MeV} \). In a chiral bag calculation, however, larger hyperon \((hhs)\) bag is preferred as compared to the nucleon \((hhh)\) bag, which brings the mass of the ground state \( \Lambda \) nearly at the right place \([10]\). For excited states, one \( h \) jumps into the first excited orbit, either \( 0^+ \) or \( 1^- \) state depending on the strength of the pion field. The excitation energy is once again of order \( 2/R \sim 600 \text{ MeV} \) (or smaller for larger \( 1/R \)) above the ground state \( \Lambda \). It is not possible to discuss more quantitative discussions such as for the splittings among the flavor singlet (predominantly \( \Lambda(1405) \)), negative parity flavor octets (\( \Lambda(1670) \), \( \Lambda(1690) \)) and the one of positive parity (\( \Lambda(1600) \)). However, the following remarks may deserve being pointed out. When the pion field is strong \( (F > \pi/2) \), the quark configuration \((0_h^+)^2(1_h^-)^1\) for \( \Lambda(g.s.) \) may be interpreted as a meson-baryon bound state (Fig. 3). In this region, the two hedgehog quarks in the \( 0^+ \) orbit dive into the negative energy sea. This situation may be interpreted as a vacuum of baryon number one (hedgehog soliton) and one anti-hedgehog \( (\bar{h}) \) in the \( 0^+ \) orbit. The parity of the anti-hedgehog state here includes both the intrinsic and orbital ones, and therefore, the orbital state of this \( h \) is \( p \) state. The \( p \) state \( h \) and the \( s \) state \( s \) quarks form an anti-kaon \((\bar{K})\) in a \( p \) orbit. A similar interpretation is also possible for the excited \( \Lambda \)'s by lifting \( h \) into a higher orbit. A three quark state or meson-baryon state for \( \Lambda \) is somewhat a matter of interpretation depending on the strength of the pion field.

The conclusion made in the above qualitative arguments, especially for the parity of the pentaquark state \( \Theta^+ \) is very important. This would be a clear evidence in which the
role of the pion is appreciated in explaining very fundamental properties such as parity. In the experiments so far, parity of $\Theta^+$, as well as its spin, has not been determined yet. It is crucially important to know in the next step these quantum numbers in order to further understand the structure of the pentaquark state.

Another important and interesting property of $\Theta^+$ is the narrow decay width $\Gamma \lesssim 25$ MeV. This is once again in agreement with the prediction of the chiral soliton model. The small width can be explained by the small phase space in the $p$-wave coupling of $\Theta^+ \to KN$. The coupling constant extracted from the experimental width of 25 MeV is about $g \sim 4$, in the same order of magnitude as other strong meson-baryon coupling constant, e.g. $g_{\pi NN} \sim 13$.

The finding of $\Theta^+$ reminds us another recent finding of $D_s(2317)$ meson, whose quark content is $c \bar{s}$ [11]. The properties of $\Theta^+$ and $D_s$ have some similarities in that; both of them contain anti-strangeness $\bar{s}$, their masses are significantly smaller than a naive quark model prediction, and their widths are unexpectedly small. For $\Theta^+$, we have argued that these properties may be explained by a strong interaction dynamics driven by the hedgehog pion, associated with the Nambu-Goldstone bosons of spontaneously broken chiral symmetry. Also for mesons, the importance of chiral symmetry in the presence of heavy quarks has been pointed out, giving a reasonable explanation for the properties of $D_s$ mesons [12, 13]. It will be interesting to pursue more the role of chiral symmetry for both meson and baryon dynamics.

The pentaquark state is interesting in its own right, but also it opens a step toward multi-quark states. Further investigations on properties of the pentaquark baryons should shed light on rich structure of hadronic matter.

Note after the submission: After the submission of the manuscript, there appear several related papers. In Ref. [14] and [15], a diquark-triquark and diquark-diquark-$\bar{s}$ configurations, respectively, were considered with color-magnetic interactions. Apart from the spin and parity assignment of $\Theta^+$, they predicted quite different pattern for other resonance
states. In Ref. [16], a QCD sum rule calculation predicted $J^P$ of $\Theta^+$ to be $1/2^-$. Further studies of exotic configurations will certainly be needed.

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References
[1] T. Nakano et. al. (LEPS collaboration), Phys. Rev. Lett. 91, 012002 (2003).
[2] V.V. Barmin et. al. (DIANA collaboration), hep-ex/0304040
[3] S. Stepanyan et. al. (CLAS collaboration), hep-ex/0307018
    S. Stepanyan, Talk given at the Conference on the Intersection of Particle and Nuclear Physics, http://www.cipanp2003.bnl.gov, New York, May 2003.
[4] D. Diakonov, V. Petrov and M. Polyakov, Z. Phys. A359, 305 (1997).
[5] H. Walliser and V.B. Kopeliovich, hep-ph/0304058
[6] C.G. Callan and I. R. Klebanov, Nucl. Phys. B262, 365 (1985);
    C.G. Callan, K. Hornbostel and I. R. Klebanov, Phys. Lett. B202, 269 (1988).
[7] Fl. Stancu and D. O. Riska, hep-ph/0307010
[8] A. Hosaka and H. Toki, Phys. Reports, 277, 65 (1996), and references therein.
[9] A. Chodos and R. L. Jaffe and K. Johnson and C. B. Thorn, Phys. Rev. D10, 2599 (1974).
[10] B.-Y. Park and M. Rho, Z. Phys. A331, 151 (1988).
[11] B. Aubert et. al. (BABAR collaboration), hep-ex/0304021
[12] M. A. Nowak, M. Rho and I. Zahed, hep-ph/0307102, Phys.Rev. D48 (1993) 4370.
[13] W.A. Bardeen, E.J. Eichten and C.T. Hill, hep-ph/0305049
    W.A. Bardeen and C.T. Hill, Phys. Rev. D49 (1994) 409.
[14] M. Karliner and H.J. Lipkin, hep-ph/0307243
[15] R.L. Jaffe and F. Wilczek, hep-ph/0307341
[16] S.L. Zhu, hep-ph/0307345