Finite element modeling of buildings with structural and foundation rocking on dry sand

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Abstract
Allowing structures to rock during an earthquake can effectively provide base isolation at a relatively small cost. Rocking limits the base shear demand and provides self-centering, but the rocking response depends on energy dissipation caused by interaction with the soil and impacts during re-centering. This paper addresses the computational modeling of buildings that have either been designed to rock on the soil beneath their foundation (foundation rocking) or at the foundation–structure interface (structural rocking). Within OpenSees, foundation and structural rocking were modeled using a beam-on-a-nonlinear-Winkler-foundation model (BNWF) combined with flat-slider elements for footing–soil and superstructure–footing interactions, respectively. The modified with flat-slider elements BNWF model (mBNWF) involves an uplift-dependent stiffness and viscous damping for both vertical and horizontal directions, and a friction–vertical force coupling. The proposed computational model was used to simulate an extensive set of centrifuge tests involving both structural rocking and foundation rocking with sequential excitations. Generally, the proposed modeling approach, without calibration of built-in parameters, adequately captured the response observed in centrifuge experiments. More specifically, the modeling captured the response amplitude and waveform of story accelerations and building rocking angle in most cases, but including potential nonlinear behavior caused by previous ground excitations was in some cases critical to obtain reasonable predictions. This was more profound for foundation rocking due to its inherent dependency on the soil strength and energy dissipation; for structural rocking previous nonlinear response primarily affected the transition time between full contact and rocking, but had a smaller effect on the prediction of maximum response.

KEYWORDS
BNWF, dry sand centrifuge testing, foundation rocking, impact, OpenSees, structural rocking

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**1 | INTRODUCTION**

Soil conditions play an important role in the earthquake response of structures, and inclusion of soil can either have a beneficial or detrimental effect on the predicted response. As a result, accurate computational modeling of dynamic soil–structure interaction (SSI) is important. At the same time, numerous researchers have demonstrated that allowing a structure to uplift during earthquake response can be beneficial, which has generated the need for accurate computational tools that describe the dynamic superstructure–foundation interaction.

Allowing uplift and rocking, either above the foundation (i.e., structural rocking) or below the foundation (i.e., foundation rocking) can reduce the force imposed on the superstructure due to a lack of resistance between the uplifting part and its support. As a result, the need for a high-ductility superstructure is reduced and thus more economical designs can be achieved. A key difference of the two rocking types stems from the different energy dissipation mechanisms that are provided. Foundation rocking takes advantage of energy dissipation provided by soil nonlinearity. Structural rocking causes energy dissipation at impacts during re-centering and can also be designed to include energy dissipation in the form of yielding elements at the rocking interface, if desired.

This paper addresses the computational modeling of structural and foundation rocking using OpenSees by simulating the response of two different types of buildings under different earthquake excitations. The computational models of the buildings presented here simulate equivalent centrifuge models that were tested side by side on dry sand and subjected to numerous earthquake excitations. To induce structural rocking and allow uplift above the foundation, one model was designed with no connection between the footings and its columns, while the other was designed with a fixed connection between the columns and the footings to provoke footing uplift and rocking below the foundation level.

**2 | BACKGROUND**

**2.1 | Modeling approaches for foundation rocking**

Three main approaches are available for computational modeling of foundation rocking. The first approach is to use continuum models with a soil constitutive law, which can describe sufficiently the nonlinear stress-strain response anywhere within the soil domain. This approach is followed when complicated phenomena are of interest. Such applications are blast-induced ground vibrations, combined failure mechanisms with yielding in both the superstructure and the footing soil or determining the total energy dissipated from the full soil domain for buildings with very large foundations. For a list of soil models for this approach see elsewhere. The second approach describes the behavior of a soil–footing system that uses a constitutive law to link footing displacements with forces that develop in the soil–footing interface. Hence, a macroelement is formed between the footing and a rigid boundary (see elsewhere for some examples of macroelements and applications). A common limitation of the two approaches is that numerous material parameters are required, and the properties cannot always be connected directly to physical soil properties. On the contrary, a third approach involving a beam-on-a-nonlinear-Winkler-foundation (BNWF) requires fewer parameters, and can be implemented relatively quickly to predict response trends. This paper focuses on the third approach.

Ideally, a BNWF model will consist of elements that can detect uplift, remove stiffness and viscous damping in all directions when uplift occurs (i.e., forces drop to zero), provide a sufficiently accurate nonlinear material law with appropriate strength and damping characteristics, and allow for easy calibration based on soil properties. Historically, all these attributes have never been combined before. Early efforts showed that vertical springs can be used to capture uplift of the footing and yielding of the soil. However, for a flexible rocking structure, a BNWF model can significantly affect the first mode. Wotherspoon and Pender (2010) focused on different stiffness distributions across the footings of a two-bay portal frame. By using springs with stiffness dependent on uplift for all directions, they examined the force distribution between extreme and central columns. However, their model was limited, as material nonlinearities were not considered. Raychowdhury and Hutchinson (2009) used constitutive laws calibrated against experiments with single footings to describe more accurately the nonlinear response of soil in both the vertical and horizontal directions. An addition of their model was the inclusion of a passive resistance component for embedded footings, alongside a spring for sliding resistance. The model can adequately predict the moment, rotation, and settlement response of single footings as well as the cyclic behavior.
The BNWF model of this paper is a modified version of the Raychowdhury and Hutchinson model and merges all previous attributes. Specifically, uplift-dependent horizontal stiffness and damping are added, along with the coupling of friction with vertical force at the surface of the footing. This upgrade can capture potential total loss of contact between spread footings and the soil when the building uplifts and rocks (foundation rocking). In addition, it provides viscous damping in both horizontal and vertical directions, which adds radiation effects at contact establishment either between soil and footings (foundation rocking) or between superstructure and footings when impacts occur (structural rocking). Within the OpenSees environment this is achieved by replacing the gap elements below the footings with friction-gap elements. It is demonstrated that this model will provide not only a realistic foundation behavior in general but also will sufficiently reproduce the superstructure’s rocking response as a result of the direct ground shaking, as well as the impacts that occur at the interface of rocking, either below or above the foundation level.

2.2 Modeling approaches for structural rocking

Modeling a rocking superstructure on a rigid surface typically involves the creation of partial hinges at the support points. This type of hinge allows pivoting of the superstructure about a corner point, and then switching the hinge to the opposite corner point to allow rocking in the opposite direction. Upon contact, the stiffness and damping provided by the support during the impact can significantly affect the subsequent response. Generally, an inelastic impact allows a rigid block to maintain a continuous rocking response where rotation alternates between two support points, whereas an elastic impact can lead to bouncing or rebound.

Within a finite element environment, the most common way to model contact is to use gap elements that have a very large finite stiffness in compression and zero stiffness in tension. Normally, the gap elements are placed in pairs along the vertical and horizontal directions. To account for damping during the impact, a viscous damper either in parallel or in series with the stiffness of the gap element can be used. However, if in series (creating a Maxwell element), the damper leads to a stiffness matrix with zero stiffness elements, which results in equilibrium problems under gravity loading. To avoid this shortcoming, a parallel spring-damper element in series with a gap element can be used. This configuration was used along with additional support masses, to mimic Housner’s plastic impact model and ensure continuous rocking response about two corner points. Moreover, nonlinear constitutive laws can be calibrated to account for an appropriate stiffness and damping to replicate instantaneous energy loss, an approach effectively applied in pounding of large-scale structures. Alternatively, numerical dissipation can be used by appropriately tuning the Hilbert–Hughes–Taylor (HHT) and Newmark time integration methods. If numerical convergence is difficult to achieve when an impact occurs, a nonlinear (displacement-dependent) stiffness can be used.

For inverted T structures with a flexible superstructure and rigid base plate, very stiff frames and rigid blocks rocking on a rigid ground, gap elements can capture sufficiently the rocking response. However, the variation of frictional resistance, or a frame with flexible uplifting columns cannot be combined with these elements. For portal frames with flexible columns, friction-gap elements are preferred instead. These elements define a sliding plane on top of which a column can land anywhere and stick, depending on the local friction properties. Kosbab (2010) showed that sliding of a column before uplift occurs can lead to a base shear demand larger than the one required to cause uplift, thus releasing the columns for uplift (with the realistic assumption of sliding) is not beneficial. By using finite element models, it was also shown that varying the coefficient of friction in the range 0.5–1.5 does not cause a significant effect in the base shear demand. This subject is, however, to experimental validation. Overall, friction-gap elements are adequate to capture the combined sliding–rocking response for idealized portal frames on a rigid surface. This paper takes advantage of friction-gap elements by using them in pairs and in an inclined configuration. This configuration can be easily linked to a footing or a rigid surface and essentially represents a slot functioning as a shear key between a flexible superstructure able to uplift and its supporting base.

3 EXPERIMENTAL BASIS FOR COMPUTATIONAL MODELING

3.1 Building model details and instrumentation

The computational modeling and analysis presented in this paper are based on and compared against a series of centrifuge experiments. For these experiments, two building models were designed to represent structural and foundation rocking
They were tested in an artificial gravitational environment using the Cambridge centrifuge beam with a model scale $N_g = 33g$. To allow structural rocking to develop, one model was designed with a partial hinge connection between the columns and the footings (Figure 1A), and it was thus able to rock above its foundation level (hereafter referred to as RA). For foundation rocking, another model was designed with a fixed connection between the columns and the footings (Figure 1B) and it was able to rock below its foundation level (hereafter referred to as RB). The two models represent three- and four-story buildings with shallow foundations resting on a sand bed of 7.5 m and were designed to remain linear elastic.

The static bearing pressure below each footing was approximately 80 kPa (Table 1) under centrifugal conditions. Care was taken to sufficiently separate the discrete footings of each building model to minimize structure–soil–structure interaction (which for adjacent buildings is known to amplify force demands) by ensuring that any footing to footing (or boundary) distance was more than two times the width $b$ of a single footing (Figure 1D). Overall, the design was chosen to ensure that the pre-uplift response would be as similar as possible, enabling direct comparison between structural rocking (model RA) and foundation rocking (model RB). Before the centrifuge experiments, the damping at the first two modes was measured (Table 1). To obtain these values, the two models had their columns clamped on a rigid base and were excited with small impacts. Therefore, these damping values refer to the damping generated by the structural components only (structural damping).

More information on the design properties and the experimental response of buildings and soil can be found elsewhere. In this analysis, prototype scale units were used for the computational modeling, but results are presented in both centrifuge model and prototype scales.
### TABLE 1  Characteristics of models RA and RB

| Properties                                      | RB                       | RA                       |
|-------------------------------------------------|--------------------------|--------------------------|
| First-mode design period (s)                    | 0.019 0.020              | 0.66 0.70                |
| Experimental first-mode frequency (Hz)          | 53 50                    | 1.6 1.5                  |
| Experimental second-mode frequency (Hz)         | 147 136                  | 4.5 4.1                  |
| Numerical first-mode frequency (Hz)             | 53 50                    | 1.6 1.5                  |
| Numerical second-mode frequency (Hz)            | 152 141                  | 4.6 4.3                  |
| Total mass of uplifting parts                   | 2.4 kg 2.1 kg            | 86 metric tonnes 75 metric tonnes |
| Structural damping $\zeta_1, \zeta_2$ for first and second modes | $\zeta_1 = 0.0209, \zeta_2 = 0.009$ | $\zeta_1 = 0.0053, \zeta_2 = 0.0066$ |
| Soil friction angle $\varphi'$                  | $33^\circ$               |                          |
| Footing dimensions                              | $37.5 \times 160 \times 9.6$ mm ($1.24 \times 5.28 \times 0.32$ m) |
| Factor of safety for vertical loading (for design) | FoS = 2.3, Design approach 1/2 of EC739 |                          |
| Story lumped mass ($m_n, n = 1, 2$)             | $m_n = 0.83$ kg ($m_n = 30$ metric tonnes, prototype scale) |

**FIGURE 2** Examples of input motions for the computational modeling, as recorded below the buildings (data shown from accelerometer 8888, below RA)

### 3.2  Sand density and input motions

The experimental program involved two main sets of centrifuge flights/tests (Table 2). One set was conducted using dry dense sand (Hostun HN31) and another set was conducted using a relatively looser sand. Throughout the paper, the two different densities of sand are referred to as “loose” and “dense” to indicate their relative difference. Each experimental set involved more than one centrifuge flight and each flight involved sequential seismic excitations separated by significant time for all vibrations to cease. Excitations based on real records and single- and multi-cycle harmonic excitations were used. For the computational response, Figure 2 presents some of the relevant input motions, which were used, respectively, as recorded below models RA and RB. To avoid artificial excitation from any electrical noise present in the signal, a low pass filter was used with a cutoff frequency of 1000 Hz (model scale).

### 4  MODELING ASSUMPTIONS

The OpenSees environment was used to model the buildings and the soil in prototype scale. The full mass and stiffness were modeled in a 2D plane. Each building type was modeled separately with nonlinear dynamic analyses. Throughout the paper, results from both single excitation and sequential excitations are discussed. The initial conditions in either case were specified as zero tilting and settlements, although using experimental values of these could significantly improve the computational response.40 Prior to the dynamic analysis, a nonlinear static analysis was carried out considering only the
### Table 2: Experimental program with soil input motion characteristics

| Experimental set (relative density) | Test (flight #) | Sequence and input motion<sup>a</sup> | Max Acc of input<sup>b</sup> (g) model scale | Max Acc of input (g) prototype scale | PGA<sup>c</sup> (g) model scale | PGA (g) prototype scale |
|-------------------------------------|----------------|--------------------------------------|-----------------------------------------------|-----------------------------------|--------------------------|------------------------|
| Dense sand (<i>D_r = 96%</i>)      | Test-0 (Flight-0) | EQ-1 0.1: Kobe                         | 10.21                                         | 0.31                              | 13.82                    | 0.42                   |
|                                    |                 | EQ-2 0.2: 50 Hz pulse                  | 9.41                                          | 0.29                              | 14.06                    | 0.43                   |
|                                    |                 | EQ-3 0.3: 30 Hz pulse<sup>41</sup>     | 6.84                                          | 0.21                              | 10.07                    | 0.31                   |
|                                    |                 | EQ-4 0.4: 50 Hz cyclic                 | 1.26                                          | 0.04                              | 1.81                     | 0.05                   |
|                                    |                 | EQ-5 0.5: 50 Hz cyclic                 | 6.93                                          | 0.21                              | 8.27                     | 0.25                   |
|                                    |                 | EQ-6 0.6: 30 Hz cyclic                 | 7.10                                          | 0.22                              | 11.60                    | 0.35                   |
| Test-1 (Flight-1)                  | EQ-1 1.1: Kobe                           | 10.04                                         | 0.30                              | 13.85                    | 0.42                   |
|                                    | EQ-2 1.2: Imperial Valley                 | 2.50                                          | 0.08                              | 3.51                     | 0.11                   |
|                                    | EQ-3 1.3: 50 Hz pulse                       | 11.57                                         | 0.35                              | 16.86                    | 0.51                   |
|                                    | EQ-4 1.4: 30 Hz cyclic<sup>42</sup>           | 7.20                                          | 0.21                              | 9.94                     | 0.30                   |
|                                    | EQ-5 1.5: 50 Hz cyclic                     | 9.31                                          | 0.28                              | 13.08                    | 0.40                   |
| Test-2 (Flight-2)                  | EQ-1 2.1: Kobe                           | 9.49                                          | 0.29                              | 13.68                    | 0.42                   |
|                                    | EQ-2 2.2: Imperial Valley                  | 2.52                                          | 0.08                              | 3.68                     | 0.11                   |
|                                    | EQ-3 2.3: 50 Hz pulse                       | 11.41                                         | 0.34                              | 16.67                    | 0.51                   |
|                                    | EQ-4 2.4: 30 Hz cyclic                     | 7.25                                          | 0.22                              | 10.83                    | 0.33                   |
|                                    | EQ-5 2.5: 50 Hz cyclic                     | 11.74                                         | 0.36                              | 17.09                    | 0.52                   |
| Loose sand (<i>D_r = 58%</i>)     | Test-1 (Flight-3) | EQ-1 1.1: Kobe                           | 8.68                                          | 0.26                              | 10.68                    | 0.32                   |
|                                    | EQ-2 1.2: Imperial Valley                  | 2.45                                          | 0.07                              | 3.00                     | 0.09                   |
|                                    | EQ-3 1.3: 50 Hz pulse                       | 10.15                                         | 0.31                              | 15.47                    | 0.47                   |
|                                    | EQ-4 1.4: 30 Hz cyclic<sup>42</sup>           | 6.92                                          | 0.21                              | 10.24                    | 0.31                   |
|                                    | EQ-5 1.5: 50 Hz cyclic                     | 11.02                                         | 0.33                              | 16.08                    | 0.49                   |
|                                    | EQ-6 1.6: 30 Hz Pulse                       | 6.74                                          | 0.20                              | 10.24                    | 0.31                   |
| Test-2 (Flight-4)                  | EQ-1 2.1: Kobe                           | 9.18                                          | 0.28                              | 10.86                    | 0.33                   |
|                                    | EQ-2 2.2: Imperial Valley                  | 2.36                                          | 0.07                              | 2.79                     | 0.08                   |
|                                    | EQ-3 2.3: 50 Hz pulse                       | 10.83                                         | 0.33                              | 15.38                    | 0.47                   |
|                                    | EQ-4 2.4: 30 Hz cyclic                     | 7.03                                          | 0.21                              | 9.15                     | 0.28                   |
|                                    | EQ-5 2.5: 50 Hz cyclic                     | 11.02                                         | 0.33                              | 15.78                    | 0.48                   |
|                                    | EQ-6 2.6: 30 Hz pulse                       | 7.04                                          | 0.21                              | 9.58                     | 0.29                   |

<sup>a</sup>Model scale frequencies 50 and 30 Hz correspond to prototype scale periods 0.66 and 1.10 s, respectively.

<sup>b</sup>Max Acc represents the maximum acceleration of the input ground motion as recorded at the base of the centrifuge box (sensor 10190).

<sup>c</sup>P GA is the maximum acceleration recorded at the free surface of the soil in between the two building models (sensor 8836).

self-weight of the buildings. All calculations for soil and building properties were in prototype scale, just as the models created and analyzed in OpenSees.

### 4.1 Modeling of superstructures

Linear elastic beam/column elements were used, located at the centerlines of the structural components and discretized to allow a more realistic wave propagation from impacts (Figure 3A,B). A lumped mass matrix was used which, for the specified element discretization, distributes satisfactorily the element mass. The values of the first two natural frequen-
FIGURE 3 Basic element discretization (A) and refined configuration (B) for model RA. Modeling with the mBNWF model of an RA footing (C) and an RB footing (D) for structural and foundation rocking, respectively, and typical application of the BNWF model for an RB footing (E).\textsuperscript{22}

FIGURE 4 The friction-gap element \textit{flatSliderBearing} (A) with a force–displacement law relating the sliding and axial responses.\textsuperscript{23} Dense sand Test-0 Eq-6: Trajectory of column ends of model RA as obtained by using inclined friction-gap elements (B).

cies of the model were similar to experimental measurements, when considering soil stiffness. To account for the small difference between the experimental and computational frequencies, rotational springs at the connections between the columns and the slabs were calibrated iteratively, while the connections between the columns and the bracing members were assumed rigid. Finally, the weight of each element was applied uniformly across the element.

4.2 Modeling of column–footing connection for structural rocking

To model the contact interface between an RA footing and a column, friction-gap elements were used (Figure 3C). This type of element (\textit{flatSliderBearing} element\textsuperscript{23}; Figure 4A) represents a bearing with a flat surface for sliding and allows for uplift. Its main use is for base isolation with sliding.\textsuperscript{43–45} In this case, the friction-gap elements were used in pairs and in an inclined position, which is identical to the shape of the footing slot (groove) in each RA footing. The main input parameters are the shear stiffness and spring stiffness for the axial direction. No rotational stiffness was assigned in the element to prevent linking and constraining the rotation of the column ends with the footing. Simulating uplift is achieved by assigning a no-tension criterion for the axial direction. When contact occurs, a linear elastic response is obtained in the sliding direction, until the point where the shearing force reaches a capacity dependent on the axial
compressive force and the coefficient of friction $\mu$. The order of magnitude of the sliding and axial stiffnesses was estimated by iteratively increasing the stiffness values until the response was no longer affected. The coefficient of friction $\mu = 0.43$ was specified, which is the mean value of static and kinetic coefficients of friction for systems with sliding surfaces made of aluminium.\(^{46}\) Overall, by using the proposed configuration, the local re-centering capability of the column ends can develop. A typical example of the trajectory of the column ends shows that the latter always remain within the RA footing slot (Figure 4B). Thus, the infinitely long sliding planes extending outwards of the footing slot as a result of the elements inherent configuration (Figure 4A) do not interfere with the column end trajectory. Additionally, the column flexibility influences the trajectory, which does not follow an arc as would be the case for the rocking of a rigid block.

### 4.3 Modeling of soil–footing interface

To model the interface between the soil and footings, the BNWF model (Figure 3E) was used initially, whereas it was modified later and the modified version (mBNWF, Figure 3C,D) was used in all the analyses presented here. In its original configuration, the BNWF model distributes a group of vertical springs below the footing to capture vertical and rotational resistance of the soil. In addition, two lateral springs are placed in parallel at one end of the footing to capture sliding and passive resistances. The vertical springs have an asymmetric hysteretic response and allow compression to develop nonlinearly up to a capacity limit, while a reduced or zero strength can develop in tension. The total linear elastic stiffness in the vertical direction can be derived using typical equations from Gazetas,\(^ {47}\) whereas Terzaghi’s bearing capacity equation is used for the total ultimate capacity. For the horizontal direction, the sliding spring is assigned the total lateral soil stiffness calculated by Gazetas,\(^ {47}\) while a capacity based on a Mohr–Coulomb criterion is assigned. The capacity of the passive resistance spring is determined using a passive earth pressure coefficient assuming a linearly varying earth pressure distribution.

Vertical stiffness and capacity are distributed according to the selected number of springs and their area of influence below the footing. Stiffer springs can be placed toward the ends of the footings to represent the initial stress concentration near the footing edges (Figure 3C–E). Note that the end springs have half the capacity and stiffness of their adjacent springs, because their assigned tributary area is half of that of the adjacent springs.\(^ {48}\) Viscous damping for radiation in the vertical direction is distributed uniformly across the number of springs, and is placed in parallel with the linear elastic component of the element (Figure 5A).

The constitutive laws for all the types of BNWF springs were derived initially\(^ {49}\) after performing tests on piles and were later calibrated against a large number of shallow-foundation tests.\(^ {22}\) For brevity, the constitutive law for the vertical springs is presented only (Figure 5B). The initial elastic (tangent) stiffness $k_{in}$ is a fraction of the total vertical stiffness from Gazetas\(^ {47}\) and therefore,

$$ q = k_{in} z, \quad (1) $$

where $q$ is the spring’s load and $z$ is the associated displacement. The extent of the linear elastic region can be adjusted by calibrating the coefficient $C_r$ of the following equation:

$$ q_0 = C_r q_{ult}, \quad (2) $$
where \( q_o \) is the yield load of the spring and \( q_{ult} \) is a fraction of the total bearing capacity of the footing and is the ultimate material resistance. The backbone curve is described by

\[
q = q_{ult} - (q_{ult} - q_o) \left[ \frac{c z_{50}}{c z_{50} - |z^p - z^p_o|} \right]^n,
\]

(3)

where \( z_{50} \) is the displacement at which 50% of \( q_{ult} \) is mobilized, \( z^p \) is the yield displacement, \( z^p \) is the post-yield displacement, and \( c \) and \( n \) are constitutive parameters for tuning the shape of the backbone curve in the post-yield region. A closure and a drag spring are used in parallel to form the gap component. The closure spring has very high and very low stiffnesses in compression and tension, respectively. The drag spring is nonlinear and simulates the small tensional resistance of the soil, with this curve controlled by the equation:

\[
q^d = C_d q_{ult} - (C_d q_{ult} - q^d_0) \left[ \frac{z_{50}}{z_{50} + 2 \left| z^k - z^k_0 \right|} \right],
\]

(4)

where \( q^d \) is the drag force, \( q^d_0 = q \) and \( z^k_0 = z^k \) at the start of the current loading cycle, and \( C_d \) is a ratio of the maximum drag force to the ultimate material resistance \( q_{ult} \). The equations describing the backbone of the springs for sliding and passive resistance are of the same type as Equation (3), and the constants \( C_r, c, n \) for each case can be found elsewhere. The asymmetric behavior of vertical resistance, the gap behavior for the passive resistance and the broad hysteresis typically associated with friction are shown in Figure 6. An example calibration is shown in Figure 5C, where footings of a total bearing capacity \( Q_{ult} \) were loaded with up to a total vertical load \( Q \), and a best fit was obtained with respect to the experimental settlement \( s \) normalized to \( z_{50} \).

A limitation of the original configuration of the BNWF model is that the sliding resistance (friction) is always present, which limits the lateral motion of the footing nodes (Figure 7A). This behavior can be acceptable for footings that always maintain partial contact with the soil, or generally for single-footing structures. However, when the entire footing loses contact with the soil, the sliding resistance (friction) should be zero and the footing should be free to move laterally. Here, this type of response would be expected primarily for the RB model. To account for this behavior (Figure 7B), the flatSliderBearing element is employed, which allows for the development of vertical and horizontal forces only when the soil springs are compressed. In the springs’ axial direction, the constitutive law from the BNWF model is used to account for the vertical and rotational resistance of the soil. For the sliding direction, the stiffness from Gazetas can be distributed uniformly across all elements. Recalling that this element is capable of assigning a failure criterion of the Coulomb type, each horizontal force from each individual element can be linked to the axial (vertical) force with a friction coefficient. In this case, a friction coefficient for dry sand was used. A limitation of the flatSliderBearing is that there is no entry for a viscosity coefficient in the sliding direction that could be used to account for radiation damping laterally. Therefore, the source code of the flatSliderBearing was modified so that the sliding force is represented with both a stiffness and a viscosity coefficient. When the Coulomb failure criterion is applied, the plastic displacements are calculated iteratively based on
the (tangent) stiffness and yielding force (capacity based on friction), while the viscosity coefficient is considered zero. Overall, the modification of the source code allowed to distribute viscous damping for radiation uniformly in the lateral direction, similar to the vertical direction.

A typical response of the soil total forces against the rocking angle for each building is shown in Figures 8 and 9. The total force in each direction is the total of all the forces of the elements below the footing while the rocking angle is the rotation of the bottom story slab. For the RA and RB left footings, the vertical soil forces are quantitatively different, although they share the same trend with respect to the rocking angle. For RA, the vertical soil force returns to the footing’s self-weight upon uplift, whereas for RB it diminishes to zero, as expected when the buildings rotate clockwise. However, when the BNWF model is considered for RB, a substantial non-zero lateral force develops even though, for the same clockwise rotations, the vertical force is zero. The same behavior applies for RA with the constant vertical force indicating loss of contact between the superstructure and the footing. Similar observations were made for the right-hand column footing. In contrast, when the mBNWF model is used the lateral forces drop to zero as the loss of contact occurs during the clockwise rotation. The error in terms of residual displacements can be significant, not only between the two computational responses (Figure 10A,B), but also between the numerical and the experimental responses (Figure 10A). If the response is high-pass filtered at 10 Hz (model scale frequency), the residual displacements can be removed and be compared directly to the displacements derived from the experimental tests. Figure 10B shows that the mBNWF model (after high-pass filtering) can better capture the response profile.
5 | INPUT FOR NUMERICAL MODEL

5.1 | Building and soil properties

Most building properties were directly specified to match the centrifuge model. For the soil (Table 3), the BNWF model requires as input an initial value of the small strain shear modulus $G_0$ to derive the stiffness of the spring elements in the linear elastic range of their deformation. The value of $G_0$ may be set equal to the maximum shear modulus $G_{\text{max}}$, which for a given depth and void ratio depends on the effective mean confining stress. When experimental or field measurements are not provided, Equation (5) can be used to empirically estimate $G_{\text{max}}$:

$$G_{\text{max}} = 1000 \cdot K \cdot (\sigma'_m)^{0.5},$$

(5)

where $K$ is a constant based on the void ratio $e$ and $\sigma'_m$ is the effective mean confining stress in lb/ft$^2$. Equation (6) provides a recently modified expression of Equation (5), considering a large number of datasets from literature:

$$G_{\text{max}} = \frac{A(\gamma) \cdot P_a}{(1+e)^3} \cdot \left(\frac{\sigma'_m}{P_a}\right)^{m(\gamma)},$$

(6)

F I G U R E 9  Loose sand Test-1 Eq-5: Total footing forces with respect to building rocking angle when the BNWF model is employed (A) and similarly when the proposed modified model is employed (B) for RB

F I G U R E 10  Loose sand Test-1 Eq-5: Lateral relative displacement of the end node of left column for model RB (see instrument BFH1 in Figure 1)
According to the BNWF model and with regards to the vertical direction, the end length ($R_e$), end stiffness intensity ($R_k$), and spring spacing ($l_e/l$) ratios need to be determined as input parameters. $R_e$ is a percentage of the total width of the footing, $R_k$ is used to describe how many times stiffer are the end springs, and $l_e/l$ is a percentage of the total width of the footing.

### 5.2 Mesh properties for soil

According to the BNWF model with regards to the vertical direction, the end length ($R_e$), end stiffness intensity ($R_k$), and spring spacing ($l_e/l$) ratios need to be determined as input parameters. $R_e$ is a percentage of the total width of the footing, $R_k$ is used to describe how many times stiffer are the end springs, and $l_e/l$ is a percentage of the total width of the footing.

### Table 3: Input properties with default values (in prototype scale)

| Soil properties: | Cohesion $c$ (kPa) | Friction angle ($\phi\max, \phi\crit$) | Soil unit weight $\gamma$ (kN/m$^3$) | Shear modulus $G_0$ (MPa) | Poisson’s ratio $\nu$ | Damping coeff. and ratio, $c_{cx}$ (Ns/m) and $\xi_x$ | Damping coeff. and ratio, $c_{cy}$ (Ns/m) and $\xi_y$ | Tensile strength $T_p$ (kPa) |
|------------------|-------------------|--------------------------------------|---------------------------------|------------------------|-----------------|---------------------------------|---------------------------------|------------------|
| Dense sand       | 0.0               | 53.0                                 | 33.0                            | 16.3                   | 30.0            | 0.375                           | 1.75 × 10$^6$, 0.22              | 2.78 × 10$^6$, 0.63 |
| Loose sand       | 0.0               | 46.0                                 | 33.0                            | 15.0                   | 22.0            | 0.375                           | 1.44 × 10$^6$, 0.21              | 2.29 × 10$^6$, 0.61 |

### Table 3 (continued)

| b) Soil mesh properties: | End length ratio $R_e$ | Stiffness intensity ratio $R_k$ | Spring spacing $l_e/l$ |
|--------------------------|------------------------|-------------------------------|------------------------|
|                          | 0.15                   | 6                             | 0.02                   |

where $A(\gamma)$ and $m(\gamma)$ are dependent on the shear strain $\gamma$ and $p_a$ is the atmospheric pressure. A representative value for $\sigma'_m$ referring to the soil beneath the footing can be estimated based on Equation (7)$^{54,55}$:

$$\sigma'_m = \frac{1}{6} (0.52 - 0.04 \frac{l}{b}) q,$$

where $l$ and $b$ are the footing’s length and width, respectively, and $q$ is the pressure at the footing’s base due to static loading. Alternatively, the mean principal effective stress can be calculated explicitly based on a depth below the footing, but there is no clear consensus on defining that depth, meaning differences can be substantial. For instance, in dense sand when $\sigma'_m$ is calculated at a depth of 1.3b assuming a linear elastic half-space, then $\sigma'_m = 34$ kPa, as opposed to $\sigma'_m = 3.8$ kPa when using Equation (7). Regarding $G_{\max}$, assuming very small strains ($\gamma = 0.0001\%$) then Equations (5) and (6) produce the same result. Overall, using Equations (5) and (7) and assuming very small initial strains, $G_{\max}$ is 30 and 22 MPa for the dense and loose sands, respectively, and these values are adopted for $G_0$.

The vertical capacity of each spring is a fraction of the footing’s bearing capacity based on the tributary area below each spring as specified by the BNWF model. The total vertical capacity is therefore dependent on the friction angle of soil $\phi'$, which governs the shear failure interfaces below the footing. Both the maximum friction angle $\phi\max$ and the friction angle at the critical state $\phi\crit$ were considered. For each type of sand, a dilation index $I_R$ was identified. This depends on $\sigma'_m$. Assuming that a footing is loaded with the full building weight during uplift and assuming a reference depth equal to the footing width $b$, $\sigma'_m \approx 45$ kPa. Therefore, low-stress dilation indices were calculated, which resulted in ($I_R, \phi\max \approx (4.0, 53.0\%)$ and (2.6, 46.0\%) for dense and loose (i.e., medium dense) sand, respectively (Table 3). For the resistance in sliding, the BNWF model uses the soil’s friction angle for a total capacity of the footing. In contrast, the mBNWF model assigns a local shearing capacity for each friction-gap element below the footing for the horizontal plane of sliding. The same friction angle was used for both vertical and horizontal capacity. Overall, the selection of an appropriate friction angle is investigated in Section 6.

Next, the dashpot coefficients of the viscous radiation dampers in the horizontal ($c_x$) and vertical ($c_y$) directions were specified following the methodology proposed by Gazetas. When considering the resulting damping ratios $\xi_x, \xi_y$, these translational modes of response, these fall within the expected range of values for the mass and soil stiffness of the buildings considered here. Following this procedure, coefficients $c_x$ and $c_y$ were distributed across the friction-gap elements of mBNWF model, so that each element carries a fraction of the coefficients $c_x$ and $c_y$ in the vertical direction (as initially intended in the BNWF model), and also in the lateral direction (as the mBNWF model allows after modification of the source code of the flatslider element of OpenSees, respectively). This distribution provides a method to account for the reduction of viscous radiation damping due to loss of contact of a footing with soil. This reduction is proportional to the length of the contact loss. However, the fractional values of coefficients $c_x$ and $c_y$ remain constant throughout the analysis, and any variation of these is not considered. It is possible though that the force component of viscous damping is reduced when the soil is in the nonlinear region and the translational or rotational velocity of the footing is low.

### Table 3: Input properties with default values (in prototype scale)

| a) Soil properties: | Cohesion $c$ (kPa) | Friction angle ($\phi\max, \phi\crit$) | Soil unit weight $\gamma$ (kN/m$^3$) | Shear modulus $G_0$ (MPa) | Poisson’s ratio $\nu$ | Damping coeff. and ratio, $c_{cx}$ (Ns/m) and $\xi_x$ | Damping coeff. and ratio, $c_{cy}$ (Ns/m) and $\xi_y$ | Tensile strength $T_p$ (kPa) |
|------------------|-------------------|--------------------------------------|---------------------------------|------------------------|-----------------|---------------------------------|---------------------------------|------------------|
| Dense sand       | 0.0               | 53.0                                 | 33.0                            | 16.3                   | 30.0            | 0.375                           | 1.75 × 10$^6$, 0.22              | 2.78 × 10$^6$, 0.63 |
| Loose sand       | 0.0               | 46.0                                 | 33.0                            | 15.0                   | 22.0            | 0.375                           | 1.44 × 10$^6$, 0.21              | 2.29 × 10$^6$, 0.61 |

### Table 3 (continued)

| b) Soil mesh properties: | End length ratio $R_e$ | Stiffness intensity ratio $R_k$ | Spring spacing $l_e/l$ |
|--------------------------|------------------------|-------------------------------|------------------------|
|                          | 0.15                   | 6                             | 0.02                   |
the footing assigned as an influence area (length) of a spring. In this case, \( R_e = 0.15 \), which is very close to the value of 1/6 recommended.57 For \( R_k \), a value of \(~9.2\) is recommended,57 while for the specific footing aspect ratio a value of \(~2.3\) is obtained.58 Generally, a high value results in smaller settlements, while the moment demand on the footing is not affected.48 A value of \( R_k = 6 \) was selected. Finally, for \( l_e/l < 0.06 \), it is shown48 that the number of elements below the footing leads to convergence for settlement and moment demands. In this case \( l_e/l = 0.02 \) was used, which results in 69 elements below each footing resulting in a very fine mesh.

6  |  EVALUATION OF COMPUTATIONAL MODEL

6.1  |  Simulation with sequential excitations and variable soil friction angle

The objectives of this comparison between experimental and computational results are first, to assess whether the computational model can predict key features of the superstructure’s time-history response as observed in the centrifuge, and second, to assess whether the footing performance in terms of cyclic behavior is realistic.

For the first objective, it is of interest to predict the uplifting and rocking part of the buildings’ response and to provide a reasonable prediction of the amplitude of the demand, and this is done in the form of story accelerations and building rocking angle. Similarly, it is important to predict the part of response with full contact or a transition from full contact to rocking and vice versa. These predictions are then compared against the centrifuge data. The footing force–displacement behavior was not measured directly in the centrifuge experiments, so the effectiveness of foundation modeling will also be evaluated in terms of the effectiveness of superstructure response predictions. A comparison of the footing settlements between the computational and experimental cases is also provided. Each centrifuge flight/test (Table 2) was simulated separately by applying its excitations sequentially, thus mimicking the experiment. After each excitation, zero padding was used to allow the response of the building to dissipate before the start of the next excitation. The default set of values for the soil properties was used for each building (Table 3), with both values of the soil friction angle considered in separate sets of simulations. Where a response mismatch occurred, new simulations were conducted with a friction angle that varied by up to 15% to evaluate the sensitivity of the prediction. This parametric investigation is aimed to evaluate the potential variability of the response based on reasonable values of the soil’s strength and energy dissipation capacity. Both of these properties rely on the soil’s shear strain level, which in this case is difficult to estimate accurately a priori due to the complicated behavior of the footings.

6.2  |  Building response to low-magnitude excitation for dense sand

Test-0, Eq-4 on dense sand is considered here to reveal insights for a small amplitude excitation where full-contact response is mostly expected. This excitation was also at the resonant frequency (Figure 2A). During this test, model RA responded indeed mostly in full contact except for minor uplift and rocking as a result of the resonant excitation (\( t = 5.45–5.5 \) s). The default computational model reproduces the experimental time-history response reasonably well for both friction angles considered here (Figure 11A,B,D,E). Some mismatch in the rocking angle profile here could be either due to double-integrating accelerations to obtain very small displacements in centrifuge or modeling insufficiency.

Figure 11C,F shows the footing force–displacement from the simulation, with the force being the summation of all the axial (vertical) spring forces of mBNWF model, and the displacement being the vertical displacement at the center of the footing, normalized to the footing width \( b \) (Figure 1, Table 1D). For \( \varphi' = \varphi'_\text{max} \), a linear elastic response is obtained suggesting no yielding of the soil, but some viscous damping is evident as a result of the footing’s high velocity dominating the response of the soil springs. For \( \varphi' = \varphi'_\text{crit} \approx 31^\circ \), the footing force–displacement is nonlinear and a drop in stiffness is clear (Figure 11F). This is a combination of two phenomena, which results in load redistribution below the footings. First, it is the accumulation of large vertical displacements at the extreme ends of the footings as a result of low strength due to low friction angle, resulting in response beyond the yielding point of the springs over many loading cycles. At unloading, the extreme springs lose contact. As a result, during excessive unloading followed by initial reloading, the footing utilizes only the stiffness of the remaining active springs, which are those at its central portion. Subsequently, as the loading matches or exceeds loading from the previous cycle (i.e., at larger settlements), all vertical springs engage and thus the stiffness increases. Second, the accumulation of large settlements at the footing ends could be also due to a developing moment at re-centering. This moment results from the column end sliding in the slot and on the surface of that.
FIGURE 11  Dense sand Test-0 Eq-4: Top story acceleration (A), rocking angle (B) and footing force–displacement behavior (C) for \( \phi' = \phi_{\text{max}} \) and similarly for \( \phi' = \phi_{\text{crit}} \) (D–F) for RA, and for RB (G–L)
first effect is likely larger than the second, but both could contribute. Note that the relatively sudden increase in stiffness exhibited by the mBNWF model would be more gradual in reality due to complex soil behavior that is not captured by the simplified model, for example, redistribution of sand grains below the footing during uplift, but the increase in stiffness is still captured.

For model RB, better results were obtained for $\varphi' = \varphi'_{\text{max}}$ than for $\varphi' = \varphi'_{\text{crit}}$. Specifically, for $\varphi' = \varphi'_{\text{max}}$, a linear elastic response in the soil and a good prediction of the time-history response are obtained (Figure 11G–I). Some damping still exists but this is minor and is the result of the viscosity of soil springs representing radiation. Overall, the response is full-contact with minor rocking but with no local uplift of the footings, as expected. For $\varphi' = 33^\circ$, the response is highly nonlinear with some hysteresis, and with local soil yielding at the edges of the footing allowing local rolling and bigger building rotations (Figure 11J–L). This in turn results in further excitation of the superstructure, as a rotational component is added to the building excitation. While in reality some local yielding will occur in the soil around the footing edges due to higher stresses (Figure 11) as the moment on the footing increases; it is suggested here that for low amplitude excitations this yielding is rather unnecessary in this simulation. Overall, the building response difference between the two types of rocking is practically small (for any pair of friction angles considered in the computational model), and this is expected as it was observed experimentally as well.

### 6.3 Building response to low-frequency excitation for dense and loose sand

A clear rocking response occurred during Test-0, Eq-6 of dense sand for both types of buildings. For model RA and for the friction angles here, the time-history response is similar to the experimental (Figure 12A,B,D,E). Regarding the cyclic behavior of the footing, toward relatively larger settlements the footing adopts a push-down response irrespective of the friction angle; this is indicated by the uniform-step vertical force distribution for $\varphi' = 53^\circ$ (linear elastic response, Figure 12C) and toward larger settlements for $\varphi' = 33^\circ$ (locally nonlinear response, Figure 12F). Overall, this behavior does not affect practically the acceleration and rocking responses but results in larger footing settlements. Considering that generally beyond 0.1% of soil shear strain the soil shear modulus reduces significantly, and using the footing settlement normalized to the footing width $b$ as a proxy to the soil shear strain, then it would be more appropriate to use $\varphi' = 33^\circ$ for large rocking based on Figure 12F, even though only the soil springs at the extreme ends of the footing have yielded as a result of reoccurring push-downs.

For a similar excitation on dense sand (Test-1 and 2, Eq-4), the results for $\varphi' = 28^\circ, 33^\circ$ are presented for RA (Figure 13). The first half of the input excitation contains strong low-frequency content, while the second half is of lower magnitude and the same frequency content (Figure 2E). For $\varphi' = 33^\circ$, rocking is predicted to continue longer than observed in the experiment for both dense and loose sand (Figure 13A–D); for $\varphi' = \varphi'_{\text{max}}$ similar results are obtained (not shown). It is possible that the phase difference between the excitation and the impact sequence is such that enough energy is input to sustain rocking in a steady state mode. For $\varphi' = 33^\circ$ this mismatch is a weakness of the computational model, because the damping provided by the push-down motion of the footings following the impacts is not enough to dissipate this energy and thus change the rocking motion to full-contact response. However, for $\varphi' = 28^\circ$, a better match is achieved (Figure 13F–H). Using $\varphi' = 28^\circ$ provided enough damping for the response switch to full contact. However, in the absence of direct measurement of experimental residual displacements, this value is low and difficult to justify. Finally, a realistic prediction in terms of amplitude and profile is obtained for the vertical displacement time-history of the footings (Figure 13E), enhancing the validity of the force–displacement behavior of the footing presented previously for other tests (Figures 12 and 13). Tuning the superstructure’s damping properties (discussed also in Section 6.4) could also result in capturing the response switch but this is not considered in this paper.

Regarding model RB, the computational response varies significantly with the friction angle. For $\varphi' = 53^\circ$, RB responds in full contact, as shown by the rocking angle, which is contrary to the rocking response observed in the centrifuge (Figure 12G,H). This result is caused by the high strength of soil, which allows for a linear elastic behavior of the footing force (Figure 12I). For $\varphi' = \varphi'_{\text{crit}} \approx 30^\circ$, the cyclic response of the footing is highly nonlinear because of the reduced soil strength. The observed gradual change in stiffness results from progressive contact establishment across the footing width as it lands following the uplift, since an increasing number of soil springs are activated. At building uplift, the footing will ultimately roll on the extreme soft soil springs, but these will accumulate different settlements from each other as the forces redistribute locally across the remaining part of the footing. Therefore, at maximum loading and over many cycles, the force distribution is not consistent (Figure 12L). The relatively better match of response in this case suggests that the soil experiences relatively large values of strain. Therefore, when the RB model experiences large rocking angles, a friction...
FIGURE 12 Dense sand, Test-0, Eq-6: Bottom story acceleration (A), rocking angle (B) and footing force–displacement behavior (C) for \( \varphi' = \varphi'_{\text{max}} \) and similarly for \( \varphi' \approx \varphi'_{\text{crit}} \) (D–F) for RA, and for RB (G–L)
angle similar to that of the critical state is suggested. Following this approach, values of $\varphi' = \varphi'_{\text{crit}} \approx 31^\circ$ in dense sand and $\varphi' = 33^\circ$ in loose sand for Test-2 Eq-4 were found to result in a reasonably good match of the rocking response. Some large high-frequency vibrations are observed for the top story acceleration (Figure 14C), suggesting again that modeling of superstructure damping needs improvement, but this was not considered here. Regarding footing displacements, a realistic prediction of the amplitude is provided (Figure 14E,F), but by adjusting the stiffness ratio $R_k$ (Section 5.2), a better settlement response could be obtained.

### 6.4 Building response to pulse-like excitations for dense sand

The response to a pulse-like excitation is considered additionally to examine the amplitude decay at the free vibration during full contact. During Test-2 Eq-3, the pulse excitation led to a single cycle of rocking followed directly by full-contact response, whereas the computational models indicate free rocking for several cycles for $\varphi' = 33^\circ$ (Figure 15A,B,E,F). Again, if the strength of the soil is artificially lowered to $\varphi' = 28^\circ$, then the additional soil damping limits the amplitude of the high-frequency oscillations during the free decay. This comes at the expense of additional oscillations in the overall
building rotation (Figure 15C,D,G,H). Alternatively, by assuming $\varphi' = 33^\circ$ but with increased superstructure damping, a better acceleration response can be obtained without rocking during the free decay (not presented here). Clearly, even though the superstructure damping used was measured from the experimental free-decay response of the buildings for the two first full-contact modes, the computational model requires further refinement for the superstructure damping. Similar results are obtained for the pulse-like Kobe excitations.

### 6.5 Overall model performance

The previous sections showed that the waveform of the experimental response is reasonably well captured by the computational model in most cases, but with the condition that the soil’s friction angle might need calibration. Following this case by case comparison, Figures 16 and 17 compare the maxima noted from the experimental and computational responses across the three main types of response (story accelerations and rocking angle) for dense and loose sand, respectively, considering $\varphi' = 33^\circ$. This assumption implies large deformations in the soil, for which an analysis with expected rocking response over the most part of an excitation sequence is a reasonable selection. Predictions close to the experimental acceleration and rocking angle responses were obtained for model RA for both dense and loose sand. On the contrary, all the predicted response maxima of model RB were conservative, except in the case of the prolonged low-frequency excitation where poor rocking angle predictions were obtained. A possible reason for this limitation is that the mBNWF model was used directly, without calibration of the backbone shape parameters $C_r, c, n$ of the soil springs, as the static behavior of the footings was not known from the centrifuge experiments. However, considering that lack of similar data in the design process is very possible, and that the simulations discussed throughout the paper produced, in general, reasonable amplitude and waveform predictions, the mBNWF model may provide useful predictions.
FIGURE 15  Dense sand, Test-2, Eq-3: Top story (A) and rocking angle (B) responses for $\varphi' = \varphi'_\text{crit} = 33^\circ$ for model RA, RB (C and D) and similarly for $\varphi' = \varphi'_\text{crit} \approx 28^\circ$.

7 CONCLUSIONS

An approach to computationally model the response of flexible buildings with structural and foundation rocking was presented. The computational response was compared against the response of two flexible building models with spread footings on dry sand that were subjected to sequences of earthquake excitations in centrifuge conditions. For the centrifuge experiment, one model was designed to uplift above the footings to allow structural rocking, and the other model was designed with a fixed connection between columns and footings to provoke foundation rocking. A dense sand case was considered, followed by a case of medium dense (loose) sand. The modeling approach presented here involved both the SSI below the footings of the building models and also the superstructure–footing interaction for structural rocking. For SSI, the BNWF was selected and modified within the OpenSees framework using the flatsliderBearing element (mBNWF). The same element was used also to model partial hinges for structural rocking, and specifically to replicate footing slots that act as shear keys. The objective was to predict key response features, such as rocking response, full-contact response, and the transition between these. The critical state and peak strength friction angles were both considered in the analysis in an effort to investigate parametrically the different responses in terms of damping, settlements, and waveform. This effort led to the following conclusions:

1. Modification of the existing BNWF model improved the modeling of the footing’s motion in contact and no contact conditions. The mBNWF imposes no lateral restraint to the footing as its predecessor, allowing for partial or full uplift with freedom in the horizontal direction. Further, this modification ensured that in partial contact the footing has friction only below the in-contact portion, while enabling viscous damping representing radiation to be added in the
FIGURE 16  Maximum response predicted in OpenSees plotted against maximum response obtained in centrifuge for dense sand for RA (top) and RB (bottom) for $\phi' = \phi'_{\text{crit}} = 33^\circ$.

FIGURE 17  Maximum response predicted in OpenSees plotted against maximum response obtained in centrifuge for loose sand for RA and RB for $\phi' = \phi'_{\text{crit}} = 33^\circ$.

Horizontal direction. Consequently, a more realistic development of the vertical and horizontal forces below a footing is obtained when the mBNWF model is used. For sequential excitations, this modification is very important, because the response history (for instance, residual footing displacements) can significantly affect the response to subsequent excitations.
2. Predictions regarding the building model with structural rocking are less dependent on soil strength and energy dissipation than those of the building with foundation rocking, for the soil types considered here. Specifically, the footing settlements during continuous structural rocking were predicted to remain small, but local yielding below the footing edges can still occur increasing overall settlements. This suggested that soil responded mostly with the initial stiffness and viscous damping properties and thus the choice of friction angle is not influential. However, selecting the critical state friction angle is more likely to lead to better waveform predictions when rocking is mixed with full-contact response.

3. The modeling of the building with the foundation rocking captured the rocking and no rocking response for the dense and loose sand cases, respectively, when a critical state soil friction angle was assumed. However, in low-magnitude excitations where mostly full-contact response was expected, better predictions were obtained when the friction angle corresponded to the peak soil strength. Therefore, for realistic predictions, understanding the potential of rocking during an excitation, and consequently, the soil’s strain level, is important when the mBNWF is used for modeling of foundation rocking. An automated process to simulate this could be developed but was not considered here.

4. Overall, the simple nature of the mBNWF model is an advantage in that a relatively small number of model parameters is required to produce a realistic response (accelerations and deformations) for buildings with foundation and structural rocking. On the other hand, effects such as soil dilation or moving linear elastic force distributions with end concentrations cannot be modeled. In situations where a large variation of shear strain can occur, for example in transitioning from large rocking to full-contact response, a more sophisticated modeling approach might be more appropriate.

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