Laser guiding through an axially non-uniform collisional plasma channel.

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Abstract. This paper presents an investigation of laser guiding through an axially non-uniform collisional plasma channel formed by ionizing laser prepulse. Due to self-defocusing of the ionizing prepulse, an axial non-uniform plasma channel is formed. When delayed second laser beam propagates through such preformed plasma channel, on account of non-uniform intensity distribution of laser beam, non-uniform heating of electrons takes place. Non-uniform heating diffuses the electrons away from the axis which further enhances the plasma channel. Unbalanced diffraction and refraction phenomenon through such an axial non-uniform collisional plasma channel results into periodic beam width variation with the distance of propagation. Second order ordinary differential equations for the beam width parameter of prepulse and the guided beam have been set up by using the moment theory approach. Laser guiding upto several Rayleigh lengths has been observed.

1. Introduction
The propagation of laser beam in plasma is relevant to a wide range of applications, such as laser fusion schemes, X-ray lasers, ultrahigh-gradient electron accelerators, fast ignitor concept and laser induced lightning.[1]-[17] Diffraction broadening of laser radiation is one of the principle phenomenon which limits the significant interaction region of laser beam upto confocal parameter(i.e. two times the Rayleigh length, where Rayleigh length, \( R_d = \frac{\pi r_0^2}{\lambda} \), where \( r_0 \) is the spot size and \( \lambda \) is the laser wavelength) and hence limits the use of laser interaction in these applications. Therefore the guiding of the laser radiation becomes essential for long distance propagation in plasma. Several methods like capillary discharge, self guiding, two pulse technique etc. have been proposed to extend the propagation distance of pulses beyond the diffraction limit as reviewed by Esarey et al [18], [19]

For optical guiding of laser pulses in plasmas, the plasma channel has to be prepared. For perfect guiding, a plasma density with minimum on the axis and parabolic increase toward the edge is required. Two pulse technique is used for guiding the laser pulse having high intensity of the order of \( 10^{12} - 10^{20} \text{W/cm}^2 \). The physics of guiding the laser pulse is as follows, a plasma channel is created by focusing a laser prepulse on picosecond time scale via tunnel ionization. Since the prepulse has a gaussian intensity radial profile, therefore the plasma density that results spontaneously, has a peak on the axis and falls of rapidly with radial distance away from the axis, before the diffusion sets in. As a result, refractive index is minimum on the axis and increases towards the edge and hence the medium behaves as defocusing medium. Therefore plasma channel evolved from such a density profile on a nanosecond time scale, after the laser
prepulse is gone, should also have an axial variation of plasma density. The plasma created by the prepulse diffuses radially away from the axis and therefore the plasma density becomes minimum on the axis and maximum at the edge of the plasma channel. Therefore, the refractive index becomes maximum on the axis and decreases toward edges and as a result the medium behaves like a focusing medium.

In the present paper we have considered an axially nonuniform collisional plasma channel. When a delayed intense second laser pulse is passed through such a preformed plasma channel, then it tends to diverge due to diffraction and converge due to refraction. However, when both the convergence and divergence parameters are equal, then the delayed laser pulse propagates without convergence and divergence, but in an axially nonuniform plasma channel this condition cannot be satisfied throughout the channel and therefore the beam radius changes as it propagates. As the gaussian laser beam propagates through plasma, it causes differential ohmic heating of electrons. Under the effect of temperature gradient, electrons diffuse outwards causing decrease in the density of the electrons on the axis and as a consequence the convergence of the beam takes place.

Laser guidance up to several Rayleigh lengths of guided laser pulse in a plasma channel formed by a laser prepulse has been successfully demonstrated[20],[21]. The self-defocusing of the laser prepulse and laser guiding of the second laser pulse in the evolved axially nonuniform plasma channel has been explained by using paraxial ray approximation[22]. The paraxial theory[23],[24] had been the most popular theory for studying the nonlinear propagation of electromagnetic waves in nonlinear media. Despite its mathematical simplicity, the problem regarding this theory is that it takes into account only the regions close to the beam axis in the self focusing mechanism. Moment theory, which is global approach, does not suffer from such limitations and hence give more accurate results. The importance of non-paraxiality in self focusing phenomenon has already been highlighted[25]. Optical guiding of a laser beam in an axially nonuniform plasma channel has been studied by moment theory approach[26]. However, at the considered intensity regime, collisional nonlinearity plays a significant role in determining the nonlinear propagation of laser beam through plasma channel. Therefore, we have included the collisional nonlinearity in the present work to study the more realistic situation. Ref[18], review the two pulse technique used by Durfee & Milchberg, in which there is no axial variation of plasma density, as the code does not include the self-defocusing of the prepulse. However, in the present study, we have taken into account the self-defocusing mechanism, which plays important role as far as the laser guidance is concerned. In section 2, we have developed a moment theory of self-defocusing of ionizing prepulse. In section 3, we have investigated the guidance of the second pulse in the preformed plasma channel by moment method approach. Equations of beam width parameters are solved numerically using Runge Kutta method. The results are discussed in section 4.

2. SELF-DEFOCUSBING OF PREPULSE

Consider the propagation of a gaussian laser beam of frequency \( \omega \) through a gas along the z-axis. The laser ionizes the gas via tunnel ionization in a time shorter than the pulse duration of the laser. In the experiment performed by Durfee and Milchberg [20], a laser prepulse(10.6\( \mu \)m, \( \sim \)100ps) forms a channel for the second delayed pulse and the laser intensity used was in the range \( 10^{13} - 10^{14} \)W/cm\(^2\). Sharp plasma density fallout was observed in Durfee and Milchberg’s experiment, which was modelled by Liu and Tripathi [22] as

\[
\omega_p^2 = \omega_{p0}^2 \exp \left( \frac{-E_0'}{|E|} \right)
\]

where \( \omega_p = \left( \frac{4\pi n e^2}{m} \right)^{1/2} \) is the electron plasma frequency, \( \omega_{p0}^2 \) is a constant depending on the
neutral particle density and $E'_a = \frac{2}{3} E_a \left( \frac{E_a}{E_h} \right)^{1/2}$. where $E_a$ is the atomic unit of electric field, $|E|$ is the amplitude of laser field, $E_i \& E_h$ are ionization potential of an atom and of a hydrogen atom.

The slowly varying amplitude $A$ of the electric vector $E = \hat{x}A(r, z)e^{(\omega t - k_0 z)}$ of an electromagnetic wave propagating in homogenous medium satisfies the following eqn.[26],[27],[28]

$$\frac{\partial A}{\partial z} + \frac{\tau}{2k_0} \nabla^2 A + \iota \epsilon(\omega) A = 0 \tag{2}$$

Where $P(A.A^*)$ is given by

$$P(A.A^*) = \frac{k_0}{\epsilon_0}(\epsilon - \epsilon_0) = \frac{k_0}{\epsilon_0}\phi(\epsilon.E^*) \tag{3}$$

and $\epsilon$ is given by.

$$\epsilon = \epsilon_0 + \phi(\epsilon.E^*) \tag{4}$$

where $\epsilon_0 \& \phi(\epsilon.E^*)$ are the linear and nonlinear parts of the dielectric constant.

$$\epsilon_0 = \epsilon|_{r=0} = 1 - \frac{\omega_p^2}{\omega^2} \exp(-E'_a f_1/A_{00}) \tag{5}$$

$$\phi(\epsilon.E^*) = \frac{\omega_p^2}{\omega^2} \left[ \exp\left(-\frac{E'_a f_1}{A_{00}}\right) - \exp\left(-\frac{E'_a f_1}{A_{00}} e^{\frac{\omega_p^2}{\omega^2}}\right) \right] \tag{6}$$

Intensity distribution of the prepulse is assumed to be gaussian and is given by.

$$|A|^2 = (A_{00}^2/f_1^2) \exp(-r^2/r_1^2 f_1^2) \tag{7}$$

where $A_{00}$ is the maximum amplitude of electric field at $r = 0$ i.e. on the axis of the laser beam, $f_1$ is dimensionless beam width parameter and $r_1$ is the initial beam width of the prepulse at $z = 0$, $r_1 f_1$ represents the beam width for $z > 0$.

By using the method of moments [26],[27],[28] , we get normalized Eq. for the beam width parameter $f_1$ of the prepulse.

$$\frac{d^2 f_1}{d\xi^2} = \frac{1}{f_1} + \left( \frac{r_1 \omega_p}{c} \right)^2 \frac{E'_a}{A_{00} f_1^4} I_1 - \frac{1}{f_1} \left( \frac{df_1}{d\xi} \right)^2 \tag{8}$$

where

$$I_1 = \int_0^\infty r e^{-\frac{r^2}{2r_1^2}} \exp\left(-\frac{E'_a f_1}{A_{00}} e^{\frac{\omega_p^2}{\omega^2}}\right) r^2 dr \quad \& \quad \xi = \frac{z}{R_d}.$$ 

Initial conditions for numerically solving Eq.(8) are $f_1 = 1$ and $\frac{df_1}{d\xi} = 0$ at $\xi = 0$

3. GUIDED PROPAGATION OF SECOND DELAYED PULSE

The plasma moves radially away from the axis after the passage of prepulse. The second gaussian pulse is guided through the channel with a delayed time less than the recombination time. On the arrival of the delayed second laser pulse the density profile of the plasma has been given by Liu and Tripathi[22]

$$\omega_p^2/\omega^2 = \alpha_1(z) + \alpha_2(z) r^2 \tag{9}$$

where $\alpha_1, \alpha_2$ are monotonically decreasing functions of $z$;
\[ \alpha_1 \leq \frac{\omega_{p0}^2}{\omega^2} e^{-\left(\frac{E_p}{A_{00}}\right)f_1}, \quad \alpha_2 \leq \frac{\alpha}{r_0^2 f_1^2} \]

Permittivity of the collisional plasma [24] is given as

\[ \epsilon = \epsilon_0 + \phi(E_0, E_0^*) = 1 - \frac{\omega^2}{\omega_p^2} \left( 1 + \frac{\alpha E_0}{2} \right)^{-5/2} \]

Intensity distribution of the delayed second pulse is given as

\[ E_0, E_0^* = \frac{E_{00}^2}{f_2} e^{\frac{-r^2}{2 f_2^2}} \]

where \( r_2 \) represents beam width at \( z = 0 \) and \( r_2 f_2 \) represents the beam width for \( z > 0 \) of the guided pulse, \( f_2 \) is termed as dimensionless beam width parameter of the guided pulse with \( f_2 = 1 \) at \( z = 0 \).

In Eq.(10) \( \alpha \) is given as \( \alpha = \frac{e^2 M}{6 K T m \omega^2} \), \( T \) being equilibrium temperature of collisional plasma, \( K \) is the boltzmann constant, \( \omega \) is the angular frequency of the incident laser beam, \( M \) & \( m \) refer to masses of ion and electron respectively.

\[ \epsilon_0 = 1 - \alpha_1 \left( 1 + \frac{\alpha E_{00}^2}{2 f_2^2} \right)^{-5/2} \]

\[ \phi(E_0, E_0^*) = \alpha_1 \left( 1 + \frac{\alpha E_{00}^2}{2 f_2^2} \right)^{-5/2} - (\alpha_1 + \alpha_2 r^2) \left( 1 + \frac{\alpha E_{00}^2}{2 f_2^2} \right)^{-5/2} \]

By following the method of moments [26],[27],[28] as in section 2 for the prepulse, we get the following Eq for the beam width parameter \( f_2 \) of the second delayed pulse.

\[ \frac{d^2 f_2}{d\xi^2} = \frac{1}{f_2^3} - \frac{5 \omega^2 \alpha E_{200}^2}{2 e^2 r_2 f_2^4} \left( \alpha_1 I_2 + \alpha_2 I_3 \right) - \frac{2 \omega^2 \alpha_2}{e^2 f_2^4} I_4 - \frac{1}{f_1} \left( \frac{df_1}{d\xi} \right)^2 \]

where

\[ I_2 = \int_0^\infty \left( 1 + \frac{\alpha E_{00}^2}{2 f_2^2} \right)^{-7/2} e^{\frac{-r^2}{2 f_2^2}} r^3 dr \]

\[ I_3 = \int_0^\infty \left( 1 + \frac{\alpha E_{00}^2}{2 f_2^2} \right)^{-7/2} e^{\frac{-r^2}{2 f_2^2}} r^5 dr \]

\[ I_4 = \int_0^\infty \left( 1 + \frac{\alpha E_{00}^2}{2 f_2^2} \right)^{-5/2} e^{\frac{-r^2}{2 f_2^2}} r^3 dr \]

we have numerically solved Eq.(14) by taking initial conditions, \( f_2 = 1, \frac{df_2}{d\xi} = 0 \) at \( \xi = 0 \) and for a model \( \alpha_2(z) \) (Liu and Tripathi, [22]), \( \alpha_2 = \frac{2}{K_0(1+\xi^2)^{1/2}} \).
4. DISCUSSION

The differential Eqs.(8) and (14) for the beam width parameter \( f_1 \) of the laser prepulse and \( f_2 \) of the guided laser pulse respectively have been solved numerically for the following set of parameters.

\[
\frac{\omega_0 \rho_0}{c} = 0.426 \times 10^6 \text{m}^{-1}, \quad \omega = 1.778 \times 10^{15} \text{S}^{-1}, \quad \lambda = 1.06 \mu\text{m}, \quad r_1 = r_2 = 13 \mu\text{m}
\]

and for different values of prepulse and guided beam intensities.

The results are depicted in the form of graphs in Figure 1 to 3. Eq.(8) is second order nonlinear ordinary differential equation, which governs the variation of beam width parameter \( f_1 \) of the laser prepulse with a dimensionless distance of propagation \( (\xi) \) as shown in Fig. 1. There are three terms on the right hand side of Eq.(8), the first term is due to diffraction divergence and the second term is due to nonlinear refraction, which is responsible for the self-defocusing of the prepulse. Initial contribution due to the third term at \( \xi = 0 \) is zero, which evolves with the distance of propagation and hence counteract the diffraction and self-defocusing of the prepulse.

In Fig. 1, Dotted curve corresponds to the case when only diffraction divergence term is present and the other two terms are ignored. Semidotted curve corresponds to the case when only the first two terms, diffraction divergence and nonlinear refraction are considered. Solid curve corresponds to the case when all the three terms are taken into account. From Figure 1, it is observed that in all the cases, the beam width parameter \( f_1 \) increases monotonically and leads to defocusing of the ionizing pulse. Due to defocusing of prepulse, intensity of the beam decreases and as a result ionization/density of the plasma decreases and we get axially nonuniform plasma channel.

Eq.(14) is also a second order nonlinear ordinary differential equation, which governs the variation of the beam width parameter \( f_2 \) of the guided pulse with a dimensionless distance \( (\xi) \) of propagation. The first term on the right hand side of Eq.(14) is due to diffraction divergence and the second term is due to nonlinear refraction, which counteracts the diffraction and is also responsible for the guidance of the laser pulse. There is a third term \( \frac{1}{f_2} \left( \frac{df_2}{d\xi} \right)^2 \), the initial contribution of this term is zero as \( \frac{df_2}{d\xi} = 0 \) at \( \xi = 0 \). However, as the beam propagates, this term supports the second nonlinear refractive term and hence contributes significantly to the guidance of the laser pulse.

Figure 2, represents the variation of the beam width parameter \( f_2 \) of the second guided pulse with the dimensionless distance of propagation \( \xi \) for two values of the guided pulse intensities, \( 2.3 \times 10^{13} \text{W/cm}^2, 1 \times 10^{14} \text{W/cm}^2 \) and for a fixed value of prepulse intensity, \( 4.0 \times 10^{17} \text{W/cm}^2 \).

It is observed that for a second guided pulse intensity \( 2.3 \times 10^{13} \text{W/cm}^2 \), \( f_2 \) varies in the range \( 0.1816 < f_2 < 1.414 \) as \( \xi \) goes from 0 to 36.60. It is further observed that \( f_2 \) varies in the range \( 0.0844 < f_2 < 1.414 \) as \( \xi \) goes from 0 to 51.20 for a guided pulse intensity \( 1 \times 10^{14} \text{W/cm}^2 \).

So, as we increase the intensity of the guided pulse, collisional nonlinearity becomes more prominent, and hence increases the self focusing of the guided pulse, which further leads to increase in the laser guiding.

Figure 3, presents the variation of the beam width parameter \( f_2 \) of the second guided pulse with the dimensionless distance of propagation \( (\xi) \) for two values of ionizing laser prepulse intensity, \( 1.0 \times 10^{17} \text{W/cm}^2, 4.0 \times 10^{17} \text{W/cm}^2 \) and for a fixed value of second delayed laser pulse intensity, \( 1 \times 10^{14} \text{W/cm}^2 \). It is observed from the figure 3 that for a prepulse intensity, \( 1.0 \times 10^{17} \text{W/cm}^2 \), \( f_2 \) varies in the range \( 0.1079 < f_2 < 1.414 \) as \( \xi \) goes from 0 to 29.40.

It is further observed that for a prepulse intensity \( 4.0 \times 10^{17} \text{W/cm}^2 \), \( f_2 \) varies in the range \( 0.0844 < f_2 < 1.414 \) as \( \xi \) goes from 0 to 51.20. This is due to the fact that the second nonlinear
refractive term in Eq. (15) is very sensitive to the prepulse intensity. So, as we increase the prepulse intensity, nonlinear refractive term dominates over the diffractive term and results in increase in the laser guidance. Therefore from the analysis it is predicted that as we increase the intensity of the prepulse from $1.0 \times 10^{17} \text{W/cm}^2$ to $4.0 \times 10^{17} \text{W/cm}^2$, laser guidance increases from 29.40 Rayleigh length to 51.20 Rayleigh length.

From figures 2 & 3, it is evident that the guided laser pulse propagate through plasma region created by prepulse, without any absorption up to several rayleigh lengths. Results of the present investigation are useful to the physics of laser-induced fusion and other heating experiments, where the guiding of the laser beam in the plasma is very important.
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