Parameter estimation of variable-parameter nonlinear Muskingum model using excel solver

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Abstract. The Muskingum model is an effective flood routing technology in hydrology and water resources engineering. With the development of optimization technology, more and more variable-parameter Muskingum models were presented to improve effectiveness of the Muskingum model in recent decades. A variable-parameter nonlinear Muskingum model (NVPNLMM) was proposed in this paper. According to the results of two real and frequently-used case studies by various models, the NVPNLMM could obtain better values of evaluation criteria, which are used to describe the superiority of the estimated outflows and compare the accuracies of flood routing using various models, and the optimal estimated outflows by the NVPNLMM were closer to the observed outflows than the ones by other models.

1. Introduction

It is well known that the hydraulic method and the hydrologic method are two primary approaches of the flood routing, which is an important component of hydrology and water resources engineering. The hydraulic method needs topographical information on account of numerically solving the complicated equations, while the hydrologic method is more simple and convenient due to its less need of topographical information [1]. The most frequently-used methods for flood routing are the Muskingum model and Muskingum–Cunge method [2]. The Muskingum–Cunge method needs topographical information and ascertains the parameters of the Muskingum model from the point of hydraulics [3][4]. While some natural river channels have obtained observed inflow and outflow data, but have not yet obtained the topographical information. The issue of flood routing for these natural river channels can be faultlessly solved by the Muskingum model.

The first Muskingum model (named LMM) relates the storage volume to the weighted value of inflow and outflow with two parameters (storage parameter and weighting parameter), which have physical significance, based on an assumption that the storage volume has a linear relation with the weighted value of inflow and outflow. However, the assumption is not always correct in the most actual rivers. Consequently, a common three-parameter nonlinear Muskingum model (named NLMM) was presented [5]. The NLMM not only contains the storage parameter and the weighting parameter, but also contains a new parameter, which was called the exponent parameter in this paper. Furthermore, some other nonlinear Muskingum models were recommended to enhance accuracy of the Muskingum model [6][7].
It is evident that parameters of the LMM and NLMM are constant, but in a natural river channel, these parameters are usually variable. Therefore, some researchers present some Muskingum models with variable parameters by considering parameters of the model are varying with the inflow. Easa considered the exponent parameter of the NLMM was varying with the inflow, and recommended a nonlinear variable exponent parameter Muskingum model (named VEPNLMM), which enhanced the accuracy of flood routing [8]. Niazkar and Afzali proposed fourteen variable-parameter nonlinear Muskingum models (named VPNLMM) by dividing the inflow into two or three levels and considering that the parameters were different to the ones in another diverse level, and the VPNLMM, which divided the inflow into three levels and considered the parameters were all variable, decreased the disparity between the observed outflows and the estimated outflows obviously [9].

In this paper, a variable-parameter nonlinear Muskingum model is recommended. First, the proposed model is described. Secondly, the flood routing procedure of the proposed model is provided. Then, the proposed model is verified by using two real case studies. Finally, the conclusions of this paper are summarized.

2. The proposed nonlinear Muskingum model

A variable-parameter nonlinear Muskingum model (NVPNLMM) was proposed and modeled in this paper by considering three parameters of the NLMM were all varying with inflow levels, and the NVPNLMM was composed of the continuity equation and the storage equation, which were described as follows.

\[
\frac{dS_t}{dt} = I_t - Q_t
\]  

(1)

\[
S_t = k_t\left[w_t I_t + (1 - w_t)Q_t\right]^e_t
\]  

(2)

Where \( k_t, w_t \) and \( e_t \) are the storage parameter, the weighting parameter and the exponent parameter, respectively; \( S_t, I_t \) and \( Q_t \) are respectively the storage volume, inflow and outflow. The NVPNLMM can be categorized into two nonlinear Muskingum models (called the NVPNLMM with the inflow dividing level \( L = 2 \) and the NVPNLMM with the inflow dividing level \( L = 3 \)) based on the number of the inflow dividing levels.

For the NVPNLMM with \( L = 2 \), the inflow was divided into two levels for the storage parameter by dividing inflow \( u_k \), and the dividing inflow \( u_k \) could be defined as the equation (3). The storage parameter was disparate in different inflow dividing level, and the storage parameter \( k_t \) could be calculated using the equation (4). Similarly, the inflow was divided into two levels for the weighting parameter by dividing inflow \( u_w \), and the inflow was also divided into two levels for the exponent parameter by dividing inflow \( u_e \). The dividing inflows \( u_w \) and \( u_e \) could be defined as the equation (3), and the weighting parameter \( w_t \) and the exponent parameter \( e_t \) could be respectively calculated using the equation (5) and the equation (6). It is worth noting that there is a difference between the dividing inflow of the VPNLMM and the dividing inflows \( u_k, u_w \) and \( u_e \), which are usually disparate. The VPNLMM is a special case of the NVPNLMM. In other words, the VPNLMM can be obtained from the NVPNLMM by setting \( u_k = u_w = u_e \).

\[
\begin{align*}
  u_k &= r_k \cdot I_{\text{max}} \\
  u_w &= r_w \cdot I_{\text{max}} \\
  u_e &= r_e \cdot I_{\text{max}}
\end{align*}
\]  

(3)

Where \( I_{\text{max}} \) is the maximum inflow; \( r_k, r_w \) and \( r_e \) are the specific values of the corresponding dividing inflow and the maximum inflow.
Where \( k_1, k_2, w_1, w_2, e_1 \) and \( e_2 \) are parameters of the NVPNLMM with \( L = 2 \). Analogously, for the NVPNLMM with \( L = 3 \), the dividing inflow \( u_{k1} \) and \( u_{k2} \) divided the inflow into three levels for the storage parameter, the dividing inflow \( u_{w1} \) and \( u_{w2} \) divided the inflow into three levels for the weighting parameter, and the dividing inflow \( u_{e1} \) and \( u_{e2} \) divided the inflow into three levels for the exponent parameter. The dividing inflow \( u_{k1}, u_{k2}, u_{w1}, u_{w2}, u_{e1} \) and \( u_{e2} \) are usually disparate and could be defined as the equation (7).

\[
\begin{align*}
  u_{k1} &= r_{k1} \cdot I_{\text{max}}, & u_{k2} &= r_{k2} \cdot I_{\text{max}} \\
  u_{w1} &= r_{w1} \cdot I_{\text{max}}, & u_{w2} &= r_{w2} \cdot I_{\text{max}} \\
  u_{e1} &= r_{e1} \cdot I_{\text{max}}, & u_{e2} &= r_{e2} \cdot I_{\text{max}}
\end{align*}
\]

(7)

Where \( r_{k1}, r_{k2}, r_{w1}, r_{w2}, r_{e1} \) and \( r_{e2} \) are the specific values of the corresponding dividing inflow and the maximum inflow, and they should be restrained.

\[
\begin{align*}
  r_{k1} &< r_{k2} \\
  r_{w1} &< r_{w2} \\
  r_{e1} &< r_{e2}
\end{align*}
\]

(8)

It is similar to the NVPNLMM with \( L = 2 \) that the storage parameter \( k_t \), the weighting parameter \( w_t \) and the exponent parameter \( e_t \) of the NVPNLMM with \( L = 3 \) could be respectively calculated as follows.

\[
\begin{align*}
  k_t &= \begin{cases} 
    k_1, & I_t \leq r_{k1} \cdot I_{\text{max}} \\
    k_2, & r_{k1} \cdot I_{\text{max}} < I_t \leq r_{k2} \cdot I_{\text{max}} \\
    k_3, & I_t > r_{k2} \cdot I_{\text{max}}
  \end{cases} \\
  w_t &= \begin{cases} 
    w_1, & I_t \leq r_{w1} \cdot I_{\text{max}} \\
    w_2, & r_{w1} \cdot I_{\text{max}} < I_t \leq r_{w2} \cdot I_{\text{max}} \\
    w_3, & I_t > r_{w2} \cdot I_{\text{max}}
  \end{cases} \\
  e_t &= \begin{cases} 
    e_1, & I_t \leq r_{e1} \cdot I_{\text{max}} \\
    e_2, & r_{e1} \cdot I_{\text{max}} < I_t \leq r_{e2} \cdot I_{\text{max}} \\
    e_3, & I_t > r_{e2} \cdot I_{\text{max}}
  \end{cases}
\end{align*}
\]

(9) - (11)

Where \( k_1, k_2, k_3, w_1, w_2, w_3, e_1, e_2 \) and \( e_3 \) are parameters of the NVPNLMM with \( L = 3 \).

3. The flood routing procedure

Taking the NVPNLMM with \( L = 3 \) for example, the flood routing procedure of the NVPNLMM is described as follows:
Step 1: Assume the parameters \( k_1, k_2, k_3, w_1, w_2, w_3, e_1, e_2, e_3, r_{k1}, r_{k2}, r_{w1}, r_{w2}, r_{e1} \) and \( r_{e2} \), and calculate the storage parameter \( k_t \), the weighting parameter \( w_t \) and the exponent parameter \( e_t \) using the equation (9), the equation (10) and the equation (11), respectively.

Step 2: Set \( t = 1 \), and calculate the initial storage amount \( S_1 \).

\[
S_1 = k_2 \left[ w_2 I_1 + \left(1 - w_2\right) Q_{r,1}\right]^0
\]

Where \( Q_{r,1} \) is the initial routed outflow, and assumed that \( Q_{r,1} = I_1 \).

Step 3: Calculate \( \frac{dS_t}{dt} \).

\[
\frac{dS_t}{dt} = \frac{1}{1 - w_t} I_t - \frac{1}{1 - w_t} \left( \frac{S_{t-1}}{k_t} \right)^{1/e_t} \quad t = 2,3,\ldots,T
\]

Where the mean of \( I_{t-1} \) and \( I_t \) rather than \( I_t \) is recommended to be used [9][10]; \( T \) is the amount of time.

Step 4: Calculate the storage amount \( S_t \).

\[
S_t = S_{t-1} + \frac{dS_t}{dt} \Delta t, \quad t = 2,3,\ldots,T
\]

Step 5: Calculate the routed outflow \( Q_{r,t} \).

\[
Q_{r,t} = \frac{1}{1 - w_t} \left( \frac{S_t}{k_t} \right)^{1/e_t} - \frac{w_t}{1 - w_t} I_t, \quad t = 2,3,\ldots,T
\]

Where \( Q_{r,t} \) is the routed outflows. Analogously, \( I_t \) is replaced by the mean of \( I_{t-1} \) and \( I_t \) in this equation so as to achieve better results.

Step 6: If \( t < T \), execute \( t = t + 1 \) and go to Step 3. Otherwise, end the flood routing procedure of the NVPNLMM.

The sum of the squared deviations (SSD) and the sum of the absolute deviations (SAD) between the observed outflows and the estimated outflows were both considered as evaluation criteria to compare the accuracies of flood routing using various models [8][9]. In addition, minimizing the SSD was adopted as the objective function [8].

\[
\min f(x) = \text{SSD} = \sum_{t=1}^{t=T} \left( Q_{o,t} - Q_{r,t} \right)^2
\]

\[
\text{SAD} = \sum_{t=1}^{t=T} \left| Q_{o,t} - Q_{r,t} \right|
\]

Where \( Q_{o,t} \) is the observed outflows.

Some researchers had estimated the parameters of the nonlinear Muskingum model using the Excel solver and deemed the Excel solver is efficient and convenient [11][12][13]. Therefore, the Excel solver is also used to estimate the parameters of the NVPNLMM in this paper.

4. Model validation

The first case study is from Wilson [14], and some Muskingum models, such as the NLMM, VEPNLMM and VPNLMM, have been applied to this example in correlative papers [8][9][10][15]. The optimal SSD and SAD for this case study by the NVPNLMM with \( L = 3 \) are respectively 2.272 and 5.179, where the corresponding specific values of the dividing inflow and the maximum inflow are \( r_{k1} = 0.342, r_{k2} = 0.389, r_{w1} = 0.410, r_{w2} = 0.685, r_{e1} = 0.166 \) and \( r_{e2} = 0.302 \). Table 1 shows the comparison of the optimal results obtained with the NLMM, VEPNLMM, VPNLMM and NVPNLMM. It is apparent from Table 1 that the optimal SSD and SAD by the NVPNLMM with \( L =\)
3 are the best and lower than the ones by the NVPNLM with $L = 2$, respectively. The optimal SSD by the NVPNLM with $L = 3$ is approximately 93.73%, 90.87%, 76.88% and 43.80% lower than the one by the NLMM, VEPNLMM with $L = 5$, VPNLM with $L = 2$ and VPNLM with $L = 3$, and the corresponding SAD reduction is 76.33%, 74.99%, 50.65% and 13.21%, respectively.

| Model          | $L$ | Parameters | SSD   | SAD   |
|----------------|-----|------------|-------|-------|
| NLMM<sup>a</sup> | $-$ | 0.0884     | 0.2862| 1.8624| 35.64| 23.00 |
| NLMM<sup>b</sup> | $-$ | 0.6589     | 0.3399| 1.8456| 36.242| 21.88 |
| VEPNLMM        | 5   | 0.483      | 0.266 | 1.906,1.880,1.883,1.889,1.884 | 24.881| 20.711|
| VPNLM          | 2   | 1.293,1.241| 0.405,0.278| 1.705,1.720 | 9.827| 10.495|
| VPNLM          | 3   | 1.804,1.686,1.536| 0.503,0.382,0.300,1.661,1.627,1.676 | 4.043| 5.967 |
| VPNLM          | 3   | 2.021,1.907,1.743| 0.495,0.396,0.302,1.678,1.632,1.646 | 2.272| 5.179 |
| NVPNLM         | 2   | 1.933,1.147| 0.457,0.316| 1.628,1.739 | 6.753| 8.918 |
| NVPNLM         | 3   | 2.021,1.907,1.743| 0.495,0.396,0.302,1.678,1.632,1.646 | 2.272| 5.179 |

<sup>a</sup> The corresponding values are from [15]

<sup>b</sup> The corresponding values are from [10]

The comparison of absolute deviations of the observed outflows and the optimal estimated outflows by various models is depicted in figure 1. As is shown in figure 1 that the absolute deviations by the VPNLM with $L = 3$ and NVPNLM with $L = 3$ are less (closer to the abscissa axis) than the ones by other models. Consequently, taking into account the fact that the optimal SSD and SAD by the NVPNLM with $L = 3$ are lower than the one by the VPNLM with $L = 3$, it is reasonable to perrate that the optimal estimated outflows by the NVPNLM with $L = 3$ provide better data-fitting to the observed outflows than the ones by other models. The optimal estimated outflows by the NVPNLM with $L = 3$ is graphed in figure 2.

![Figure 1. The comparison of absolute deviations of the observed outflows and the optimal estimated outflows by various models for the first case study.](image)

The second case study is from O’Donnel [16]. So far, the existing best SSD and SAD for this case study are respectively 30879.50 and 735.594 in the literatures [6]. The optimal SSD and SAD by the NVPNLM with $L = 2$ are respectively 23231.359 and 713.539, which are 24.77% and 3.00% lower.
than the existing best SSD and SAD, respectively. Furthermore, the optimal SSD and SAD by the NVPNLMM with $L = 3$ are respectively 10367.999 and 421.476, which are 66.42% and 42.70% lower than the existing best SSD and SAD, respectively. Figure 3 shows the comparison of the existing best estimated outflows and the optimal estimated outflows by the NVPNLMM with $L = 3$. It is apparent from figure 3 that the optimal estimated outflows by the NVPNLMM with $L = 3$ provide better data-fitting to the observed outflows than the existing best estimated outflows so far.

Figure 2. The optimal estimated outflows by the NVPNLMM with $L = 3$ for the first case study.

Figure 3. The comparison of the existing best estimated outflows and the optimal estimated outflows by the NVPNLMM with $L = 3$ for the second case study.

5. Conclusions

A variable-parameter nonlinear Muskingum model was proposed and modeled in this paper by considering that three parameters of the NLMM were all varying with inflow levels. The Excel solver was used for the parameter estimation of the NVPNLMM, and then SSD and SAD were considered as evaluation criterions for contrasting various Muskingum models. Two true case studies were applied
to evaluate the performance of the NVPNLMM. Comparisons of the optimal SSD and SAD by various models showed that the SSD and SAD by the NVPNLMM with $L = 3$ have obvious reductions, at least 43.80% and 13.21% for the first case study, in comparison with the ones by the NLMM, VEPNLMM with $L = 5$, VPNLMM with $L = 2$ and VPNLMM with $L = 3$, respectively. The SSD and SAD by the NVPNLMM with $L = 3$ are 66.42% and 42.70% lower than the existing best SSD and SAD for the second case study so far. Moreover, the optimal estimated outflows by the NVPNLMM with $L = 3$ are closer to the observed outflows than the ones by other models for two case studies.

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