The analysis of students’ difficulty in learning linear algebra

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Abstract. Linear algebra is one of the subjects that will be taken by students of mathematics education department. Most of students’ score was low. Thus, the purpose of this research was to analyze the students’ difficulty in learning linear algebra. 35 students of mathematics education department were chosen as the participant. In collecting the data, paper and pencil test and interview were used. This research was a qualitative research. The result showed that the students’ difficulty in learning linear algebra was very high which 88.63% students were not able to represent the symbol or notation, 88.11% students had difficulty in using the symbol or notation or ideas of mathematics and logical reasoning, 88,38% students had difficulty in comprehending the symbol or notation used by using logical reasoning, 91.77% students were difficult to check whether the symbol or notation or ideas of mathematics used has been applied correctly or not and use logical reasoning. It indicated that students’ difficulties in learning linear algebra was very high. This is because students' understanding of the material and solving linear algebra problems is quite slow compared to other materials. Lecturers are also not optimal enough in the use of media and learning methods in learning linear algebra in the classroom.

1. Introduction
Basically learning mathematics means learn with an abstract objects. According to [1], mathematics is a science which related or examine the forms or abstract structures and the relationship between its things. Learning Mathematics is a combination in processing the term (name) and notation (symbolic/symbols) by grouping the objects of mathematics into the examples and non-examples. Therefore, the process of teaching mathematics in the schools and universities have difficulties in its to learn.

The outline of mathematics covers algebra, geometry and analysis. One of the concepts of mathematics related to abstract objects is algebra. University students usually learn the concept of algebra starting from semester 3 to 6. One of the lecture algebra that has some relate to other courses such as linear program, structure algebra, and linear algebra. Through algebra linear students are trained to think logically and critically in solving the problem with an abstract objects. Thus, students often find difficulty in learning linear algebra.

Linear algebra is to learn vectors and linear transformation [2]. The concept of vectors and linear transformation that learned in linear algebra is abstracted. Based on the result of an observation, researchers teaching linear algebra courses found that the student had some difficulties in understanding each concept in linear algebra. This fact influenced the student’s learning achievement or the student’s score. At the end of the semester on linear algebra courses of Mathematics Education students of Borneo
University of Tarakan academic year 2014/2015, the lowest score was 34.5 and high score was 88.02 which the mean score was 65.52. Based on the scoring system at the University of Borneo Tarakan, the students’ mean score is C.

According to the [3], students find difficulty in learning linear algebra because of some reasons; concepts are abstract structures, their application areas are unusual for students, most students have yet to learn proof and axiomatic methods, the basic concepts in linear algebra are not shown geometrically, memorizing the concept without actually understanding it, failing in abstract thinking although the topics require abstract thinking, poor conception of definitions, incapacity to interpret verbal expressions, and inability in the readiness level. In addition, [4] stated that the students have no difficulty in learning the concept, definition and formula of linear algebra and operational information, but they have difficulty in implementing the definition and concepts they have learned. Thus the difficulty of learning in the students learn algebra linear vary. As a result, researchers were interested in finding out what were the difficulties and causative factors faced by students in learning linear algebra.

2. Review of literature

The difficulty of learning is a condition in which learners beyond the average (very clever and very stupid) do not get adequate opportunities to develop appropriate capacity[5]. In other definitions, it was said that the difficulty of learning is a condition where a student cannot learn reasonably, due to threats, obstacles or disruptions in learning [6]. The outline the factors cause the onset of trouble learning consist of two kinds:

a) Internal factors

Internal factors are factors that are derived from the Protégé itself. According to [5], internal factors included disruptions or inability of psychophysical kekurang students which includes:

1. The nature of cognitive (the realm of copyright), such as the low intellectual capacity/intelligents is a students
2. Affective nature (the taste of the realm), among others like labilnya emotions and attitudes
3. The nature of psychomotor (realm of karsa), such as disruption of the tools the sense of Viewer and listener (eyes and ears).

b) External factors

The external factors is the factor that comes from outside the individual itself, covering. This factor can be divided three kinds [5]

1. Family environment, such as: the disharmony of the relationship between father and mother, with a low level of economic life of the family.
2. Village/community Environment, such as: the slum areas (slums), and teammates (peer group) who are naughty.
3. The school environment, such as: the condition and location of the school buildings are bad like near the market, adverse conditions as well as learning tools that are of low quality.

Based on research conducted by [7] the difficulty Students learning in linear algebra was divided in three perspectives:

1. Nature of linear algebra – linear algebra is a product of a long process of development and intellectual ways clashes think math from several generations. The result is now an abstract theory "that blends and generalize" with many effects and a widerange of applications.
2. The teaching of Linear Algebra – how it works on linear algebra aims to introduce briefly the general theory and simple looks without giving the students experience the risks mentioned the process of generalizing and alloys and understand its application. The first lesson from the theory of the structure of linear algebra is given at the beginning for students to unite or menggeneralisasai and mengalikasikan theory.
3. How Students learn and face the Linear Algebra — put aside the habit of learning, as a result of two factors above, Students develop an inhibitor formally.

In this research the student was evaluated using three rubrics by [7] as follows:
1. nature of linear algebra
2. The teaching of linear algebra
3. how students learn and deal with linear algebra.

As for indicators of difficulty studying linear algebra used in this research was to look at the learners understanding in problem solving linear algebra. Based on the stages by [8], an indicator of the difficulty of studying linear algebra can be outlined as follows:
1. Representation with the appropriate symbol or notation that is revealing what is known and asked correctly and connecting the concepts that exist in the given problem.
2. using symbols/notation or mathematical ideas that are relevant as well as logical reasoning that is drawing up the plan with attention to other problems considering have successfully solved and is similar to a given problem, connecting the concepts that exist in the given issue with the concept that there are similar problems and have been successfully solved before.
3. Interpret the symbol or notation used by using logical reasoning that is implementing a plan that had been made in the previous phase and explain the reason why the completion of measures can be applied.
4. Check whether the symbol or notation or mathematical ideas that are used have been applied correctly and use logical reasoning that is checking the answers obtained by observing whether the measures are applied is in compliance with the planned procedure on the previous phase and check if the answers are retrieved has addressed problems that were given.

3. Research Method

This research were qualitative and quantitative research. This research described students’ difficulties and causative factor in learning linear algebra. The subject of the research was 35 students of Mathematics Education of Faculty teacher training and Education of Borneo University of Tarakan academic year 2015/2016 who was being programmed linear algebra course. In conducting the research, there were some stages as follows:
1. Arranged the research instrument which the essay test was used to see students’ difficulties in answering the item of linear algebra, questionnaires of learning difficulties and interview question.
2. Conducted the essay test, questionnaires and interview to strengthen the analysis of learning difficulty in linear algebra that obtained from instrument essay test of linear algebra
3. Analyzed the results of essay test, questionnaires and interview

The essay was analyzed by using the formula as shown below:

\[ p = \frac{\sum s}{\sum s + \sum b} \times 100\% \]

Where:
- \( p \): percentage of solving error
- \( s \): step of solving error
- \( b \): step of solving correctly

The results of the percentage was used to compared with the table to draw conclusions about the difficulty in solving the linear algebra test [9].
Table 1. The level of difficulty

| The level of difficulty | Category   |
|-------------------------|------------|
| 80 - 100                | Very High  |
| 66 - 79                 | High       |
| 40 - 65                 | Moderate   |
| 0 - 39                  | Low        |

\[ q = \frac{\text{Student's score}}{\text{Maximum Score}} \times 100\% \]

Where, \( q \) is the percentage of causing learning difficulty of linear algebra. The results of the percentage was used to compare with the table to draw conclusions about the cause of solving difficulty in linear algebra test [10].

Table 2. The cause of the learning difficulties

| The level of the Cause | The criteria  |
|------------------------|---------------|
| 81 - 100               | Very weak     |
| 61 - 80                | Weak          |
| 41 - 60                | Enough        |
| 21 - 40                | Strong        |
| 0 - 20                 | Very strong   |

4. The Results and Discussion

The research results was gained as follows:

Table 3. Difficulty learning linear algebra

| Difficulty | Level of Difficulty | Category |
|------------|---------------------|----------|
| Represent with the appropriate notation/symbol | 88.63% | Very High |
| Use the symbol notation or ideas of mathematics which are relevant and logical reasoning | 88.11% | Very High |
| Redefine the symbol or notation that is used by using logical reasoning | 88.38% | Very High |
| Check whether the symbol or notation is used has been applied correctly and using logical reasoning | 91.77% | Very High |

Table 4. Internal factors of learning difficulty

| The Causes                  | Level of Causation | Category |
|-----------------------------|--------------------|----------|
| The learning interest in linear algebra | 75 %              | Weak     |
| The attitude to learning linear algebra | 71 %              | Weak     |
| The attention to learning linear algebra | 74 %              | Weak     |
| The efforts to learn linear algebra | 67.86%            | Weak     |
| The understanding of linear algebra | 60 %              | Enough   |
| The ability to solve linear algebra | 59 %              | Enough   |
| The Causes                                      | Level of Causation | Category |
|------------------------------------------------|--------------------|----------|
| The skill in solving the linear algebra test   | 66.43%             | Weak     |

**Table 5** Externalfactors of learning difficulty

| The Causes                                      | Level of Causation | Category |
|------------------------------------------------|--------------------|----------|
| The tools and books of linear algebra           | 71%                | Weak     |
| Mastering linear algebra material              | 76%                | Weak     |
| The clarity of describing the linear algebra    | 78%                | Weak     |
| The use of learning media                       | 59%                | Enough   |
| Lecturer’s preparation before teaching          | 73.93%             | Weak     |
| The use of teaching methods                     | 59%                | Enough   |
| The atmosphere of linear algebra class          | 64.64%             | Weak     |
| The layout of linear algebra class              | 79%                | Weak     |

Based on the result of learning difficulty algebra linear showed that student’s difficulty was very high. It was also supported by the result of interview which the student reported they do not know the starting point in solving linear algebra test. Consequently, the students were very difficult in determining the symbol/notation that will be used in solving the test. Also, in solving linear algebra students just followed the previous example given by lecturer. Whereas, the tests were different because it was the development of the previous test that has been given.

The cause of learning difficulty linear algebra was presented in table 4 and 5. The internal factors was student’s understanding and solving linear algebra test were slower than the other materials. The other causes did not influence the student in learning linear. While, the external factor had less contribution because the lecturer implemented the media and method in teaching and learning process on enough category. The rest of the aspect was on weak category which means that it did not influence student’s difficulties in learning linear algebra.

5. Conclusion

The result showed that solving a linear algebra was hard for the math students to be done. All the aspects were on very high level of difficulty, representing the symbol or notation, using the symbol or notation or ideas of mathematics which are relevant and logical reasoning, and redefining symbol or notation that is used by using the logic of the logical necessity were above 88%. The significantly hard difficulty was achieved 91.77% that the students were not able to check whether the symbol or notation or ideas of mathematics used has been applied correctly or not and use the logic of the logical necessity. This finding was not only contributed by the internal factor which the students were slower in grabbing and understanding the material than another material but also external that the learning media and method were not adequately implemented by the lecturer to attract and help the students in the learning process.

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