BEP Enhancement for Semi-Femtocell MIMO
Systems Employing SC-QICs and OSTBCs

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Abstract—In mobile cellular networks, it is estimated that more
than 60% of voice and data services occur indoors. Therefore,
cellular network operators have shown an unprecedented interest
in research on femtocell systems from various aspects to extend
the indoor wireless coverage for providing high-quality and high
data-rate wireless multimedia services contents. In an effort for
reducing the bit-error probabilities (BEPs) and also increasing
bit/symbol capacity of bandwidth limited error-prone wireless
channels in femtocell propagation areas, this paper presents the
performance of a promising candidate technology designed based
on the state-of-the-art techniques. The performance of powerful
space-time turbo codes (STTCs) based on serial concatenation of
quadratic interleaved codes (SC-QICs) with the optimal and also
suboptimal decoding algorithms, in conjunction with orthogonal
space-time block codes (OSTBCs) have been presented in this
contribution for wireless multiple-input single-output (MISO),
and multiple-input multiple-output (MIMO) semi-femtocells.

Index Terms—cellular radio network; diversity; femtocell,
forward error correction; multiple-antenna; space-time coding.

I. INTRODUCTION

Femtocell networks bring significant advantages for both
the mobile cellular network operators and the end users. They
not only provide indoor coverage for places where macrocell
cannot, but they are also capable of offloading traffic from its
layer and improve the capacity while significantly save user
equipment’s power [1]. However, for supporting the demands
for high data-rate video, voice, and data services, it is difficult
to achieve simultaneously high transmission quality and high
information date-rate in a bandwidth-limited wireless channels.
In response to these ever-increasing demands, multiple-antenna
systems and technologies based on space-time codes [2-4] have
been the most important technological breakthrough in the last
two decades [5], which shown to result in substantially higher
capacity than their single-antenna counterparts as it is pointed
out by Foschini [6]. They have shown to result in very low bit-
error probabilities (BEPs) in highly-faded wireless channels,
while increasing the data-rates significantly. To provide more
coding gains for future mobile systems, the information and
coding community also proposed to concatenate the space-time
codes with high performance channel codes. In [7-10], space-
time codes in cooperation with turbo codes [11] which are the
first class of channel codes that perform within one decibels of
Shannon were proposed. Hence, these researches motivated us
to employ the integrated space-time codes and also turbo codes
for improving the physical layer (PHY) performance of generic
femtocell wireless communication systems [12], while making
further potential enhancements through the following methods:
(1) improving the BEP performance in error floor region by
using serial concatenation (SC) instead of conventional parallel
concatenation schemes; (2) improving the BEP performance in
both the waterfall and the floor regions by using permutation
algorithm based on the property of quadratic congruence.

II. CODED M-QAM MIMO SYSTEM MODEL: ANALYSIS

A wireless MIMO system is considered where transmitter
and receiver are equipped with n and m antennas respectively
as show in fig. 1. Data are first encoded by turbo encoder, and
afterwards, modulated by the multilevel quadrature amplitude
(M-QAM) signaling block. Then, the modulated symbols are
mapped using space-time block encoder, and transmitted over
the channel. At the receiver, the combiner combines received
signals that are then sent to the detector. The detected symbols
are demodulated at the M-QAM demodulator signaling unit
and, are then decoded at the iterative near maximum likelihood
(ML) decoder in order to recover the transmitted signal bits.

A. Coded M-QAM System Model: Construction of SC-QICs

The advent of turbo codes [11] that facilitate the operation
of communications systems near Shannon limit was perhaps of
the most crucial historic breakthrough after Shannon’s seminal
work. Since turbo code’s invention, they have reached maturity
within just two decades due to their interesting near-capacity
performance. The initial results showed that the turbo codes
could achieve energy efficiencies within only a half decibel of
the Shannon capacity. This result was taken into account as an
extraordinary performance and first it was met with skepticism,
but further research efforts of turbo coding community not only
clarified their outstanding performance and characteristics, but
also introduced a wealth of practical industrial solutions for
wireless communication systems and services [13, 14]. In this
work, the focus has been on the turbo codes based on serial
concatenation [15] as they have shown to yield performance
comparable to the classic parallel code schemes; they are also
referred to as serial concatenation of interleaved codes [12, 16].
As it is noted in [17], the most critical part in the design of turbo codes is the interleaver design. The design of permutation pattern for interleaver has a key role in turbo coded system’s exceptional BEP output since it remains scarce. In this case, the permutation of a sequence of bits is based on the property of quadratic congruence over an integers ring modulo powers of two, the serial turbo codes utilizing this algorithm is referred to as the serial concatenation of quadratic interleaved codes (SC-QICs) as it has been employed in semi-femtocell.

B. Coded M-QAM System Model: Space-Time Block Coding

Multiple-antennas based on the space-time codes have been intensively studied since they constitute efficient robust trade-off in terms of their effective throughput, BEP performance, and estimated complexity. After introduction of Alamouti’s twin-antenna space-time codes which appealed in terms of the BER performance and complexity, research was dedicated for developing this scheme for the higher number of transmitters to increase diversity order. Alamouti’s scheme was generalized to arbitrary number of transmitter antennas, leading to the notion of STBCs [3, 13]. In this case, for a system with $n$ and $m$ Tx and Rx antennas, radio signals $c_{ij}, i = 1, 2, ..., n$, are transmitted simultaneously using $n$ transmit antennas. Assuming the path gain from the transmit antenna $i$ to receive antenna $j$ ($a_{ij}$) is constant over a frame of length $l$, at time $t$ the radio signal $r_{ij}^l$ received at the antenna $j$ is given by equation (1) below as:

$$r_{ij}^l = \sum_{i=1}^{n} a_{ij} c_{ij} + y_{ij}^l$$  \hspace{1cm} (1)

where $y_{ij}^l$ are the independent samples of a zero-mean complex Gaussian random variable with variance $1/(2SNR)$ per complex dimension. A STBC is defined transmission matrix $G$ in which its entries are linear combinations of the $x_1, x_2, ..., x_n$ and their conjugates [4]. For Alamouti’s scheme and Tarokh et al.’s rate $3/4$ scheme with three transmitting antennas, the transmission matrices are given as the equations (2) and (3) below [2, 4]:

$$G_{2-Alamouti} = \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix}$$  \hspace{1cm} (2)

$$G_{3-7x}^{rate3/4} = \begin{bmatrix} x_1^2 & x_2^2 \\ -x_2^2 & x_1^2 \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \end{bmatrix}$$  \hspace{1cm} (3)

At the receiver, for detecting symbols of the two transmit antennas (denoted by $s_1$ and $s_2$), the decision metrics given by the equation (4) and the equation (5) have been used as [4]:

$$\begin{align*}
&\left| \sum_{i=1}^{m} (r_{1i} a_{1i} + r_{2i} a_{2i}) - s_1 \right|^2 \\
&+ \left| -1 + \sum_{i=1}^{m} |a_{ij}|^2 \right| |s_1|^2
\end{align*}$$  \hspace{1cm} (4)

$$\begin{align*}
&\left| \sum_{i=1}^{m} (r_{1i} a_{1i}^* + r_{2i} a_{2i}^*) - s_2 \right|^2 \\
&+ \left| -1 + \sum_{i=1}^{m} |a_{ij}|^2 \right| |s_2|^2
\end{align*}$$  \hspace{1cm} (5)

For detecting symbols of the three transmit antennas with rate $3/4$ (denoted by $s_1$, $s_2$, and $s_3$), the maximum likelihood (ML) decoding of STBCs accounts in order to minimize the decision metrics given by the equations (6), (7), and (8) [4]:

$$\begin{align*}
&\left| \sum_{i=1}^{m} (r_{1i} a_{1i} + r_{2i} a_{2i} + \frac{r_{1i}^* - r_{2i}^*}{2} a_{3i}) - \frac{(r_{1i} + r_{2i}) a_{3i}}{2} - s_1 \right|^2 \\
&+ \left| -1 + \sum_{i=1}^{m} |a_{ij}|^2 \right| |s_1|^2
\end{align*}$$  \hspace{1cm} (6)

$$\begin{align*}
&\left| \sum_{i=1}^{m} (r_{1i} a_{1i}^* - r_{2i} a_{2i}^*) - (r_{1i}^* + r_{2i}^*) a_{3i} + \frac{(r_{1i}^* - r_{2i}^*) a_{3i}^*}{2} - s_2 \right|^2 \\
&+ \left| -1 + \sum_{i=1}^{m} |a_{ij}|^2 \right| |s_2|^2
\end{align*}$$  \hspace{1cm} (7)

$$\begin{align*}
&\left| \sum_{i=1}^{m} \frac{(r_{1i} + r_{2i}) a_{3i} + \frac{r_{1i} - r_{2i}}{2} a_{3i}}{\sqrt{2}} + \frac{(r_{1i} + r_{2i}) a_{3i}}{\sqrt{2}} a_{3i} - \frac{(r_{1i}^* + r_{2i}^*) a_{3i}^*}{2} - s_3 \right|^2 \\
&+ \left| -1 + \sum_{i=1}^{m} |a_{ij}|^2 \right| |s_3|^2
\end{align*}$$  \hspace{1cm} (8)
III. MIMO SYSTEM MODEL: SIMULATION AND DISCUSSION

Based on the above mentioned discussions, the simulations in terms of BEP versus the energy per bit/noise power spectral density ($E_b/N_0$) are shown for the systems of interest wherein SC-QICs of length $1024$ bits in conjunction with Alamouti’s twin-antenna space-time codes and also Tarokh et al.’s rate $\frac{3}{4}$ OSTBCs for three transmit antennas are presented. In this case, inner and outer SC-QIC’s constituent codes utilize recursive convolutional codes with generator polynomial metrics $(7,5)_8$ and $(7,5,0; 0,7,5)_8$ with six iterations at the system decoder. The investigation at the system design stage has been carried out for the stochastic channel model. Here, the spectral broadening of the received Rx signal has been modeled based on the Bell spectrum proposed in IEEE 802.11 TGn model given by $S(f) = \frac{1}{1+A(f/f_d)^2}$ where $A$ is a constant used in order to define $0.1S(f)$ at $f_d$ and also being the Doppler spread $|f| \leq f_d$ [22]. Meanwhile, the investigation has been based on the worst case scenario wherein the line-of-sight (LoS) path is blocked; hence, Rayleigh distribution is used for modeling the statistical time varying nature of the Rx received signal’s envelope. In addition, additive noise at the receivers considered to follow the additive Gaussian noise (AWGN). Although assumptions for channel modeling may seem to be not very precise to some extent, it should also be mentioned that using location-specific models at the radio system design stages is not indispensable since stochastic channel modeling is still the most used tool in order to compare the different techniques and algorithms at the system design and comparison stage. The $M = 16$-QAM digital modulation scheme has been employed as it provides a good trade-off in terms of the power and bandwidth efficiencies; and therefore widely adopted in standardized wireless systems such as in digital video broadcasting (DVB), high speed downlink packet access (HSDPA) protocol, and IEEE 80211a PHY layer for modulating OFDM subcarriers. In this case, instead of binary code mapping, gray code mapping has been used which shown to yield additional coding gains. In the fig. 2, the BEP curves for the discussed systems of interest are presented. In order to indicate the resultant coding gains due to deploying additional antenna with the Tarokh’s OSTBCs, we employed one more antenna not only at the transmitter side but also at the receiver side yielding diversity order of six, as compared with that of Alamouti’s MISO scheme with order two. As it can be seen from the figure, while the MISO wireless system results in $\text{BEP} \approx 10^{-4}$ for $\text{SNR} = 20$ dB, and also employing additional antenna at the transmitter and the receiver based on the Tarokh scheme yields $\text{BEP} \approx 10^{-7}$ at the same SNR. The observed difference comes from higher system diversity order which is the main feature of the orthogonal STBCs, and results in the rapid MISO and MIMO wireless STTC systems adaptation.

As it can be seen from the curves, the coding gains due to the utilization of quadratic interleaver is around $4$ dB compared to the optimal matrix-based block interleavers with the same size and under the same simulation parameters. This should be considered as an achievement, since they also yield the more undemanding approach in implementation, and also allow the possibility of analysis due to the structured construction. Since in other works it has been shown the optimal result of matrix-based block interleavers is attainable when the depth and span of are relatively high and comparable, it is predicted that the obtained system coding gains are more astonishing when one compares the designed system with the quadratic interleaver, compared with the same system employing rectangular matrix-based block interleavers. In addition, the performance curves for MISO and MIMO systems are presented for the decoding based on original maximum a posteriori (MAP) algorithm, and its simplified version (Max-Log-MAP algorithm) wherein the computational saving arise by making approximation $\log(e^{x_1} + \ldots + e^{x_n}) = \max(x_1, \ldots, x_n)$ in the forward, backward, and state transition metrics computation for evaluating the a posteriori probability (APP) of information bits. As it can be seen, the performance difference between the MAP decoded and Max-Log-MAP decoded systems is not significant. Therefore, these obtained simulation results confirm and also suggest that since numerical stability problems can arise in computations decision metrics in the MAP system decoding due to the large amount of computational complexities, the Max-Log-MAP decoding is a good overall choice when one considers both the error-rate performance and complexity. Since the delay of turbo codes is the main bottleneck of their utilization in many modern delay-sensitive wireless systems and applications, utilization of the Max-Log-MAP is a promising method while keeping the BEP performance penalties reasonable for the intended systems.

![Figure 2: Bit-Error Probability (BEP) vs. Energy Per Bit/Noise Power Spectral Density ($E_b/N_0$) for Differing Number of Transmitter and Receiver Antennas With $N = 2048$, MAP Max-Log-MAP Algorithms With Six Iterations.](image-url)
space-time codes based on the LDPC codes for the femtocell and also urban microcell [23] areas. The proposed wireless MISO and MIMO systems can further be optimized using the evolutionary computation optimization techniques as in [24].

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