Strongly Universal Hamiltonian Simulators

Leo Zhou (Harvard University) 🟢
& Dorit Aharonov (Hebrew University) 🌙

QIP – Feb 5, 2021
Analog Quantum Simulation: a promising application

Many-Body Localization & Time Crystals

[J. Smith et al., 2016]

Quantum Phase Transition

[M. Endres et al., 2016]

Quantum Chemistry

[C. Wang et al., 2019]

Feynman 1981] [Cirac Zoller 2012]
Some families of Hamiltonian are **Universal**: They can simulate all other Hamiltonians

**Universal Hamiltonians**

\[
\tilde{H} = \sum_{\langle i,j \rangle \in E} J_{ij} \tilde{S}_i \cdot \tilde{S}_j
\]

**MBL**

\[
H = \sum_{i,j} \frac{1}{|i-j|^{\alpha}} X_i X_j + \sum_i h_i Z_i
\]

**SYK**

\[
H = \sum_{i,j,k,\ell} J_{ijk\ell} \gamma_i \gamma_j \gamma_k \gamma_{\ell}
\]

**Molecules**

\[
H = \sum_{i,j} h_{ij} a_i^\dagger a_j + \sum_{i,j,k,\ell} V_{ijk\ell} a_i^\dagger a_j^\dagger a_k a_{\ell}
\]

[Cubitt Montanaro Piddock 2017]
What about **Resources**?

\[ \tilde{H} = \sum_{\langle i, j \rangle \in E} J_{ij} \tilde{S}_i \cdot \tilde{S}_j \]

# particles in simulator
\[ \tilde{n} = \text{poly}(n) \]

Interaction energy
\[ J_{ij} = \text{poly}(n) \]

[Cubitt Montanaro Piddock 2017]
What about Resources?

\[
\tilde{H} = \sum_{\{i,j\} \in E} J_{ij} \vec{S}_i \cdot \vec{S}_j
\]

# particles in simulator
\[\tilde{n} = \text{poly}(n)\]

Interaction energy
\[J_{ij} = 2^{\text{poly}(n)}\]

“Weakly Universal” as it may require \(\exp(n)\) resources

[Cubitt Montanaro Piddock 2017]
Our Result:
There are simple **Strongly Universal Hamiltonians**

"**Strongly Universal**" = Universal + All resources are \( \text{poly}(n) \)
for any target Hamiltonian

vs. \( \exp(n) \) for some target Hamiltonian under weak universality

**Implication:** Analog quantum simulation are realistic for general systems
Overview

1) Defining Hamiltonian Simulation & Universality

2) Review of Previous Universal Hamiltonians

3) Main Results: Strongly Universal Hamiltonians
Defining Hamiltonian Simulation

We say $\tilde{H}$ ($\Delta, \eta, \epsilon$)-simulates $H$ if

$$\| \tilde{H}_{\leq \Delta} - \tilde{V} H \tilde{V}^\dagger \| \leq \epsilon$$

$\tilde{V}$ is an (approximately) local isometry encoding

$$\| \tilde{V} - \bigotimes_i V_i \| \leq \eta$$

[Bravyi Hastings 2014] and [Cubitt Montanaro Piddock 2017]
Defining Universality and Translation-Invariance

Consider a family of Hamiltonians

\[ \mathcal{F} = \left\{ \hat{H} = \sum_{\langle i, j \rangle \in E} J_{ij} \hat{h}(\phi_{ij})_{i,j} \right\} \]

Site-dependent parameter \( \phi_{ij} \)

Interaction energy

operator acting on site \((i, j)\)
Defining Universality and Translation-Invariance

Consider a family of Hamiltonians $\mathcal{F} = \{ \tilde{H} = \sum_{(i,j) \in E} J_{ij} \hat{h}(\phi_{ij})_{i,j} \}$

**Weak and Strong Universality**

$\mathcal{F}$ is Weakly Universal if $\forall H \exists \tilde{H} \in \mathcal{F}$ such that $\tilde{H} (\Delta, \eta, \epsilon)$-simulates $H$

$\mathcal{F}$ is Strongly Universal if weakly universal and $\| \tilde{H} \| \leq \text{poly}(n, \Delta, \eta^{-1}, \epsilon^{-1})$

**Semi- and Full-Translation-Invariant (TI)**

$\mathcal{F}$ is Semi-TI if $\hat{h}(\phi_{ij}) = \hat{h}$ e.g.

$$\tilde{H} = X_1X_2 + 3X_2X_3 + \frac{1}{5}X_3X_4 + \cdots$$

$\mathcal{F}$ is Full-TI if semi-TI and $J_{ij} = J$ e.g.

$$\tilde{H} = 12(X_1X_2 + X_2X_3 + X_3X_4 + \cdots)$$
Previous Results: Weak Universality

Any $O(1)$-local Hamiltonian
e.g. SYK model

$$H = \sum_{i,j,k,\ell} J_{i,j,k,\ell} \gamma_i \gamma_j \gamma_k \gamma_\ell$$

(Perturbative) Gadgets

2D, Semi-TI, Weakly Universal Hamiltonians
[Cubitt Piddock Montanaro 2017]

$$\tilde{H} = \sum_{\langle i,j \rangle \in E} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

- **Degree-reduction** is necessary to map some Hamiltonians to finite-dimensional lattice
- Previous approach uses $O(\log n)$ rounds of perturbative gadgets: $J_{t+1} = [J_t \text{poly}(n)]^c$
  $\rightarrow$ exponential overhead in energy $J_{\text{final}} = 2^{\text{poly}(n)}$
Our Results:
Strongly Universal Hamiltonians in 2D and 1D

2D square lattice, semi-translation-invariant

\[ H = \sum_{\langle i, j \rangle \in E} J_{ij} \hat{h}_{i,j} \]

e.g.

\[ \hat{h}_{ij} = X_i X_j + Y_i Y_j \text{ or } \vec{S}_i \cdot \vec{S}_j \]

1D, nearest-neighbor, 8-dimensional particles

\[ \tilde{H} = \sum_i J_i \hat{h}_{i,i+1} \quad \|\hat{h}_{i,i+1}\| \leq 1 \]

\[ \hat{h}_{i,i+1} \text{ enforces transition rules between configurations} \]

Preprint at bit.ly/universalHam
(and see arXiv on Monday)
Our Results vs. Previous Results

|                        | Spatial dimension | Translation-Invariance | Interaction Energy | Particle Number |
|------------------------|-------------------|------------------------|--------------------|-----------------|
| Cubitt et al. (2017)   | 2D                | semi                   | exp(poly(n))       | poly(n)         |
| Piddock Bausch (2020)  | 2D                | full                   | exp(poly(n))       | exp(poly(n))    |
| Kohler et al. (2020)   | 1D                | full                   | exp(poly(n))       | exp(poly(n))    |
| Kohler et al. (2020)   | 1D                | full*                  | exp(poly(n))       | poly(n)         |
| **Our construction**   | 2D                | semi                   | poly(n)            | poly(n)         |
| **Our construction**   | 1D                | none                   | poly(n)            | poly(n)         |

- Our results are somewhat **tight** since general simulation with $O(1)$ interaction energy and constant degree is **impossible** [Aharonov Zhou 2018]
- strongly universal ⊈ weakly universal (information-theoretical arguments)
Key Ideas in our construction

1) **Non-perturbative** degree-reduction

Degree-reduction with perturbative gadgets requires $\exp(\text{poly}(n))$ energy

Degree-reduction with $O(1)$ interaction energy is impossible in general

Degree-reduction is possible with $\text{poly}(n)$ interaction energy! (via circuits)
Key Ideas in our construction

1) **Non-perturbative** degree-reduction (via circuits)

\[ H = \sum_E E \left| \psi_E \right\rangle \langle \psi_E \right| \]

Perform degree-reduction on circuit

\[ \left| +^m \right\rangle \xrightarrow{\text{QFT}^{-1}} \left| E \right\rangle \]

\[ e^{iH\tau} \left| \psi_E \right\rangle \xrightarrow{\text{ancillas + swaps}} \tilde{H} \]
Key Ideas in our construction

1) **Non-perturbative** degree-reduction (via circuits)

\[
H = \sum_{E} E|\psi_{E}\rangle\langle\psi_{E}|
\]

\[|\psi_{E}\rangle \rightarrow e^{iH\tau} |\psi_{E}\rangle \rightarrow \begin{array}{c}
|+^{m}\rangle \rightarrow QFT^{-1} |E\rangle \rightarrow \tilde{H}
\end{array}\]

perform degree-reduction on circuit

2) Recover eigenvalue structure via **bit-wise energy penalty**

\[
|E\rangle = |E_1 E_2 \cdots E_s \cdots\rangle \quad H_{\text{pen}} = \sum_{b=1}^{s} 2^{-b} |1\rangle\langle 1|_{b} \otimes P_{\text{clock}}(t = T + b)
\]
Structure of Our Proof / Construction

Any $O(1)$-local Hamiltonian

Standard techniques (Trotterization ...)

Phase estimation circuit using NN gates in 1D

Non-perturbative degree-reduction + bit-wise energy penalty

2D “spatially sparse” Hamiltonian
Structure of Our Proof / Construction

Any $O(1)$-local Hamiltonian

Standard techniques (Trotterization ...

Phase estimation circuit using NN gates in 1D

Non-perturbative degree-reduction + bit-wise energy penalty

2D “spatially sparse” Hamiltonian

2D semi-TI Hamiltonian on a square lattice

gadgets

[Oliveira Terhal 2005]
[Piddock Montanaro 2015]
[Cubitt Montanaro Piddock 2017]
Structure of Our Proof / Construction

Any $O(1)$-local Hamiltonian

Standard techniques (Trotterization ...)

Phase estimation circuit using NN gates in 1D

Non-perturbative degree-reduction + bit-wise energy penalty

2D “spatially sparse” Hamiltonian

Modified 1D clock Hamiltonian + bit-wise penalty

1D NN Hamiltonian on 8-dimensional particles

2D semi-TI Hamiltonian on a square lattice

[Aharonov et al 2007]
[Hallgren Nagaj Narayanaswami 2013]
Summary

• We establish that strongly universal analog quantum simulation is possible – can efficiently simulate for any target Hamiltonian
  • 1D and 2D universal systems using poly(n) qubits and interaction energy
  • Tight since impossible to lower interaction energy to $O(1)$ [Aharonov Zhou 2018]

• Analog quantum simulation is relevant for many more systems than previously thought

Open Questions

• 1D semi-TI, strongly universal? Full-TI strongly universal? Fermions?
• Improve the overhead to experimentally relevant regimes?
• Better understand the effects of noise in analog simulation