Selecting a school on the basis of parent’s expectation using intuitionistic fuzzy soft set

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Abstract
In this paper a comparative study have been given for the methods used to solve Intuitionistic Fuzzy Soft Matrix, by taking the problem of selecting a school for their kids according to the parents wish.

Keywords
Intuitionistic fuzzy soft set, Max-Min average composition, Max-Min composition, Distance method.

AMS Subject Classification
03E72.

1. Introduction
Due to various uncertainties associated with our real life problems most of them involve data which are not necessarily crisp, precise and deterministic. In 1976 Sanchez developed an algorithm for medical diagnosis, later De et al (2001) have studied Sanchez’s method for medical diagnosis and applied it in intuitionistic fuzzy set, also many researchers used sanchez’s approach of medical diagnosis in various fuzzy numbers.

Atanassov (1986), introduced intuitionistic fuzzy set, Molodtsov (1999) initiated sot set theory and then Maji et al (2003) applied this theory to several fields. Initially, Rajarajeswari et.al, (2013) introduced max-min composition method to solve the problems of intuitionistic fuzzy sets. Later many researchers applied intuitionistic fuzzy sets for their research and invented many methods. Shanmugasundaram et al. (2014) introduced a new technique called intuitionistic fuzzy soft max-min average composition, he also used it for medical diagnosis process and also given the results. By keeping fuzzy soft max-min average composition as base a new technique revised max-min average composition was introduced by Shanmugasundaram et al in 2014. In 2018, Gandhimathi, used distance between two intuitionistic fuzzy soft matrices.

The result obtained in max-min composition and max-min average composition method is based on the maximum value in the score matrix. And the result obtained in distance method is based on the minimum value in the score matrix. An attempt has been made to provide a formal model to study the selection of Schools by using IFSM theory and implement it in the form of field recommendation system.

2. Preliminaries

Definition 2.1 (Intuitionistic fuzzy soft set). Let \( U = \{C_1, C_2, \ldots, C_m\} \) be the universal set and let \( E = \{e_1, e_2, \ldots, e_n\} \) be the set of parameters. Let \( A \subseteq E \) and let \((F,A)\) be a fuzzy soft set in the fuzzy soft class \((U,E)\). Then fuzzy soft set \((F,A)\) is a matrix form as \(A_{m \times n} = [a_{ij}]_{m \times n} \) or \(A = [a_{ij}], i = 1,2,\ldots,m \) and \(j = 1,2,\ldots,n\), where

\[
a_{ij} = \begin{cases} 
    (\mu_j(c_i), v_j(c_i)) & \text{if } e_j \in A \\
    (0,1) & \text{if } e_j \notin A
\end{cases}
\]

\(\mu_j(c_i)\) represents the membership of \(c_i\) in the intuitionistic fuzzy set \(F(e_j)\) and \(v_j(c_i)\) represents the non membership of \(c_i\) in the intuitionistic fuzzy set \(F(e_j)\).
2.1 Operations on Intuitionistic fuzzy soft matrices

Definition 2.2 (Addition and Subtraction). If $A = [a_{ij}] \in IFMS_{m \times n}$ and $B = [b_{ij}] \in IFMS_{m \times n}$ then we define the addition and subtraction of intuitionistic fuzzy soft matrices of $A$ and $B$ as,

1. $A + B = \{ \max [\mu_A(a_{ij}), \mu_B(b_{ij})], \min [\nu_A(a_{ij}), \nu_B(b_{ij})] \}, \\forall i, j$ \hspace{1cm} (2.1)

2. $A - B = \{ \min [\mu_A(a_{ij}), \mu_B(b_{ij})], \max [\nu_A(a_{ij}), \nu_B(b_{ij})] \}, \\forall i, j$ \hspace{1cm} (2.2)

Definition 2.3 (Max-Min composition). Let $A = [a_{ij}] \in IFMS_{m \times n}$ and $B = [b_{jk}] \in IFMS_{n \times p}$, then max – min composition fuzzy soft matrix relation of $A$ and $B$ is defined as $A \ast B = [c_{ik}]_{m \times p}$ Where,

$$c_{ik} = \left\{ \begin{array}{cl}
\\text{Max} \left\{ \min_j \left[ \mu_A(a_{ij}), \mu_B(b_{jk}) \right] \right\}, & \mu \in \mathbb{R} \\
\text{Min} \left\{ \max_j \left[ \nu_A(a_{ij}), \nu_B(b_{jk}) \right] \right\}, & \forall i, j
\end{array} \right.$$ \hspace{1cm} (2.3)

Definition 2.4 (Complement function). Let $A = [a_{ij}] \in IFMS_{m \times n}$ where $a_{ij} = (\mu_i(c_i), \nu_i(c_i))$ for all $i, j$. Then $A^C$ is called an intuitionistic fuzzy soft complement matrix if $A^C = [d_{ij}]_{m \times n}$, where,

$$d_{ij} = (v_j(c_j), \mu_j(c_j)), \\forall i, j.$$ \hspace{1cm} (2.4)

Definition 2.5 (Max-Min Composition). If $A = [a_{ij}] \in IFMS_{m \times n}$ and $B = [b_{jk}] \in IFMS_{n \times p}$, then a operation called fuzzy max-min composition for fuzzy soft matrix relation is defined as,

$$A \odot B = \left\{ \text{Max Min} \left\{ \mu_A(a_{ij}), \mu_B(b_{jk}) \right\}, \text{Min Max} \left\{ \nu_A(a_{ij}), \nu_B(b_{jk}) \right\} \right\}, \forall i, j$$ \hspace{1cm} (2.5)

Definition 2.6 (Max-Min average composition). If $A = [a_{ij}] \in IFMS_{m \times n}$ and $B = [b_{jk}] \in IFMS_{n \times p}$, then a new operation called revised intuitionistic fuzzy max-min average composition for fuzzy soft matrix relation is defined as,

$$A \odot B = \left\{ \begin{array}{cl}
\text{Max} \left\{ \frac{\mu_A(a_{ij}) + \mu_B(b_{jk})}{2} \right\}, & \forall i, j \\
\text{Min} \left\{ \frac{\nu_A(a_{ij}) + \nu_B(b_{jk})}{2} \right\}, & \forall i, j
\end{array} \right.$$ \hspace{1cm} (2.6)

Definition 2.7 (Score matrix). If $A = [a_{ij}] \in IFMS_{m \times n}, B = [b_{ij}] \in IFMS_{n \times p}$, and $A^C, B^C$ are the complement of $A$ and $B$ then the score matrix of $A$ and $B$ is defined as,

$$S(A, B) = \frac{V + W}{2},$$ \hspace{1cm} (2.7)

where $V$ is the matrix defined as $V = [(\mu_{A^C} - \nu_{A^C})]$ and $W$ is the matrix defined as $W = [(\mu_{B^C} - \nu_{B^C})]$.

3. Algorithms

Algorithm-1: (Max-Min Average Composition)

Step 1. Input the intuitionistic fuzzy soft set $(F, C), (G, B)$ and obtain the intuitionistic fuzzy soft matrices $K, L$ corresponding to $(F, C)$ and $(G, B)$, respectively.

Step 2. Obtain the intuitionistic fuzzy soft complement matrices $K^C, L^C$ using the formula mentioned in equation (2.4).

Step 3. Compute the intuitionistic fuzzy max-min average compositions $K \phi L$ and $K^C \phi L^C$ using the formula mentioned in equation (2.5).

Step 4. Compute the matrices $X, Y$ and obtain the score matrix $Z$ using the equation (2.7).

Step 5. Identify the maximum score in $Z(A, B)$ for each $A_i$ to select the suitable option.

Algorithm-2: (Max-Min Composition Method)

Step 1. Input the intuitionistic fuzzy soft set $(F, C), (G, B)$ and obtain the intuitionistic fuzzy soft matrices $K, L$ corresponding to $(F, C)$ and $(G, B)$, respectively.

Step 2. Obtain the intuitionistic fuzzy soft complement matrices $K^C, L^C$ using the formula mentioned in equation (2.4).

Step 3. Compute the intuitionistic fuzzy max-min compositions $K \phi L$ and $K^C \phi L^C$ using the formula mentioned in equation (2.5).

Step 4. Compute the matrices $X, Y$ and obtain the score matrix $Z$ using the equation (2.7).

Step 5. Identify the maximum score in $Z(A, B)$ for each $A_i$ to select the suitable option.

Algorithm-3 (Distance method)

Step 1. Input the intuitionistic fuzzy soft set $(F, C), (G, B)$ and obtain the intuitionistic fuzzy soft matrices $K, L$ corresponding to $(F, C)$ and $(G, B)$, respectively.

Step 2. Calculate the distances of intuitionistic fuzzy soft matrices by using distance measure.

Step 3. Identify the smallest distance for each Alternative $B_i$. Then Conclude that the Alternative $B_i$ Suits the criteria $B_i$.

4. Case Study

In this section we used that revised max-min average composition, Max-min composition and distance method to find the suitable school for the children according to the criteria of parents.

Criteria under Consideration: Here is the list of some important points to consider when it comes to finding the best school: Vision and mission of the school, Curriculum, Faculty, Infrastructure, Extra-curricular activities.
4.1 Illustration

We have presented an application of intuitionistic fuzzy soft set theory using maxmin average composition method for decision making in this section.

In a given set of systems, let \( A = \{A_1, A_2, \ldots, A_m\} \) be the set of \( m \) alternatives-1 and let \( C = \{C_1, C_2, \ldots, C_n\} \) be the set of \( n \) criteria and let \( B = \{B_1, B_2, \ldots, B_k\} \) be the set of \( k \) alternatives-2. Now construct a intuitionistic fuzzy soft set \((F,C)\) over \( A \), where \( F \) is a mapping \( F : C \rightarrow IF^A \) and \( IF^C \) is the collection of all intuitionistic fuzzy subsets of \( A \). This intuitionistic fuzzy soft set gives a matrix \( K \). Then Construct another intuitionistic fuzzy soft set \((G,B)\) over \( C \), where \( G \) is a mapping \( G : D \rightarrow IF^C \) and \( IF^C \) is the collection of all intuitionistic fuzzy subsets of \( C \). This intuitionistic fuzzy soft set gives a matrix \( L \). Obtain the intuitionistic fuzzy soft complement matrices, using the equation (2.4) let it be denoted by \( K^c, L^c \). And then compute \( K\phi L, K^c\phi L^c \) and \( S(K,L) \) using the equation (2.6) and (2.7). Finally find the maximum value for each Alternative-1 \((A_i)\) in the score matrix and then conclude that the alternative \( A_i \) is suitable for the alternative-2’s \( B_j \).

Consider \( A = \{A_1, A_2, A_3\} \) as the universal set, where \( A_1, A_2, A_3 \) represent the set of parents. Then consider \( C = \{C_1, C_2, C_3, C_4, C_5\} \) as the set of criteria, where \( C_1, C_2, C_3, C_4, C_5 \) represent vision and mission of the school, curriculum, faculty, infrastructure, extra-curricular activities respectively for the case study. Let \( B = \{B_1, B_2, B_3\} \) be the set of schools taken for our case study, where \( B_1, B_2, B_3 \) denotes R.C. Fathima, New Bharath, Sai Ram school.

Suppose that intuitionistic fuzzy soft set \((F,C)\) over \( A \), where \( F \) is a mapping \( F : S \rightarrow IF^A \), gives the description of parents expectation on each criteria using intuitionistic fuzzy matrix relation in the form of IFS:

\[
(F,A) = \{(F(A_1)) = \{(A_1,0.8,0.1), (A_2,0.6,0.2), (A_3,0.4,0.1)\}
\]

\[
= \{(F(A_2)) = \{(A_1,0.5,0.3), (A_2,0.8,0.1), (A_3,0.8,0.1)\}
\]

\[
= \{(F(A_3)) = \{(A_1,0.4,0.2), (A_2,0.6,0.2), (A_3,0.9,0.1)\}
\]

\[
= \{(F(A_4)) = \{(A_1,0.4,0.2), (A_2,0.5,0.5), (A_3,0.6,0.1)\}
\]

\[
= \{(F(A_5)) = \{(A_1,0.6,0.3), (A_2,0.4,0.4), (A_3,0.5,0.4)\}
\]

Representing the above intuitionistic fuzzy soft set as intuitionistic fuzzy soft matrix.

\[
K = \begin{bmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 \\
A_1 & (0.8,0.1) & (0.5,0.3) & (0.4,0.2) & (0.6,0.3) \\
A_2 & (0.6,0.2) & (0.8,0.1) & (0.6,0.2) & (0.5,0.5) & (0.4,0.4) \\
A_3 & (0.4,0.1) & (0.8,0.1) & (0.9,0.1) & (0.6,0.1) & (0.5,0.4)
\end{bmatrix}
\]

Suppose that intuitionistic fuzzy soft set \((G,B)\) over \( C \), where \( G \) is a mapping \( G : D \rightarrow IF^C \), gives the weight of the schools according to each criteria, using the intuitionistic fuzzy matrix relation in the form of IFS:

\[
(G,B) = \{(G(D_1)) = \{(C_1,0.7,0.1), (C_2,0.4,0.2), (C_3,0.1,0.1), (C_4,0.5,0.5), (C_5,0.8,0.1)\}
\]

\[
= \{(G(D_2)) = \{(C_1,0.3,0.3), (C_2,0.8,0.1), (C_3,0.1,0.2), (C_4,0.8,0.1), (C_5,0.5,0.4)\}
\]

\[
= \{(G(D_3)) = \{(C_1,0.4,0.1), (C_2,0.3,0.1), (C_3,0.9,0.1), (C_4,0.6,0.3), (C_5,0.6,0.3)\}
\]

Represent the above mentioned intuitionistic fuzzy soft set as a fuzzy soft matrix,

\[
L = \begin{bmatrix}
B_1 & B_2 & B_3 \\
C_1 & (0.7,0.1) & (0.3,0.3) & (0.4,0.1) \\
C_2 & (0.4,0.2) & (0.8,0.1) & (0.3,0.1) \\
C_3 & (0.1,0.1) & (0.1,0.2) & (0.9,0.1) \\
C_4 & (0.5,0.5) & (0.8,0.1) & (0.6,0.3) \\
C_5 & (0.8,0.1) & (0.5,0.4) & (0.6,0.3)
\end{bmatrix}
\]

Now calculate the intuitionistic fuzzy soft complement matrices as mentioned in step 2.

\[
K^c = \begin{bmatrix}
P_1 & C_1 & C_2 & C_3 & C_4 & C_5 \\
(0.1,0.8) & (0.3,0.5) & (0.2,0.4) & (0.1,0.2) & (0.9,0.1) \\
(0.2,0.6) & (0.1,0.8) & (0.2,0.6) & (0.5,0.5) & (0.4,0.4) \\
(0.1,0.4) & (0.1,0.8) & (0.1,0.9) & (0.1,0.6) & (0.4,0.5)
\end{bmatrix}
\]

\[
L^c = \begin{bmatrix}
P_1 & S_1 & S_2 & S_3 \\
(0.1,0.7) & (0.3,0.3) & (0.1,0.4) \\
(0.2,0.4) & (0.1,0.8) & (0.1,0.3) \\
(0.1,0.1) & (0.2,0.1) & (0.1,0.9) \\
(0.5,0.5) & (0.1,0.8) & (0.3,0.6) \\
(0.1,0.8) & (0.4,0.5) & (0.3,0.6)
\end{bmatrix}
\]

Then to obtain intuitionistic fuzzy max-min average composition relation matrices use equation (2.6). The calculation is as follow, using the equation (2.6), value of \( K\phi L = \) as follows.

\[
K\phi L = \begin{bmatrix}
P_1 & S_1 & S_2 & S_3 \\
(0.35,0.25) & (0.35,0.25) & (0.30,0.40) \\
(0.50,0.35) & (0.40,0.35) & (0.40,0.50) \\
(0.30,0.50) & (0.40,0.35) & (0.35,0.40)
\end{bmatrix}
\]

Then find the matrix \( X \), using the formula,

\[
X = \left[ \left( \mu_{K\phi L} - v_{K\phi L} \right) \right],
\]

as follow,

\[
X = \begin{bmatrix}
P_1 & S_1 & S_2 & S_3 \\
0.50 & 0.40 & 0.25 \\
0.30 & 0.45 & 0.25 \\
0.15 & 0.45 & 0.50
\end{bmatrix}
\]
Then find the matrices $X, Y$ using the formula,

$$ Y = \left[ (\mu_{Kc_{\phi L}} - v_{Kc_{\phi L}}) \right], $$

as follow,

$$ Y = \left[ \begin{array}{ccc} S_1 & S_2 & S_3 \\ P_1 & 0.25 & 0.20 & 0.20 \\ P_2 & 0.35 & 0.20 & 0.30 \\ P_3 & 0.20 & 0.30 & 0.25 \end{array} \right] $$

Using the equation (2.7) the score matrix for intuitionistic fuzzy max-min composition method is,

$$ Z = \frac{1}{2} [X + Y] = \left[ \begin{array}{ccc} S_1 & S_2 & S_3 \\ P_1 & 0.375 & 0.300 & 0.325 \\ P_2 & 0.325 & 0.375 & 0.275 \\ P_3 & 0.175 & 0.375 & 0.375 \end{array} \right] $$

It is clear from the above result that the school $S_1$ satisfies the criteria of parent $P_1$ and school $S_2$ satisfies the expectation of parents $P_2$ and both the schools $S_2, S_3$ satisfies the expectation of $P_3$.

Then to obtain intuitionistic fuzzy max-min composition relation matrices use equation (2.5). Using the values calculated using the equation (2.5), value of $K_{\phi L}$ is as follows,

$$ K_{\phi L} = \left[ \begin{array}{ccc} S_1 & S_2 & S_3 \\ P_1 & (0.70, 0.10) & (0.50, 0.20) & (0.60, 0.10) \\ P_2 & (0.60, 0.20) & (0.80, 0.10) & (0.60, 0.10) \\ P_3 & (0.50, 0.10) & (0.80, 0.10) & (0.90, 0.10) \end{array} \right] $$

Now applying intuitionistic fuzzy max-min average formula for calculating $K^c_{\phi L}$ using equation (2.5) in $K^c$ and $L^c$, Using the values calculated using the equation (2.5) value of $K^c_{\phi L}$ is as follows,

$$ K^c_{\phi L} = \left[ \begin{array}{ccc} S_1 & S_2 & S_3 \\ P_1 & (0.20, 0.40) & (0.30, 0.40) & (0.30, 0.50) \\ P_2 & (0.50, 0.50) & (0.40, 0.50) & (0.30, 0.60) \\ P_3 & (0.10, 0.60) & (0.40, 0.40) & (0.30, 0.40) \end{array} \right] $$

Then find the matrices $X, Y$ using the formula,

$$ X = \left[ (\mu_{K_{\phi L}} - v_{K_{\phi L}}) \right], $$

as follow,

$$ X = \left[ \begin{array}{ccc} S_1 & S_2 & S_3 \\ P_1 & 0.30 & 0.10 & 0.10 \\ P_2 & 0.10 & 0.30 & 0.00 \\ P_3 & -0.40 & -0.10 & 0.00 \end{array} \right] $$

Using the formula,

$$ \text{score} = \frac{1}{2} [X + Y] = \left[ \begin{array}{ccc} S_1 & S_2 & S_3 \\ P_1 & 0.200 & 0.100 & 0.150 \\ P_2 & 0.200 & 0.300 & 0.100 \end{array} \right] $$

For the problem illustrated above, we are using the algorithm-3 mentioned in and the results are as follow, the lowest distance points obtained out a proper diagnosis and the only proper way of calculating the widely used distances for IFSM is take into account all the parameteres; the membership function and non-membership function. To be more precise, the normalized Hamming distance for all the symptoms of the ith patient from the kth diagnosis is equal to

\[
I[(F, E), (G, E)] = \frac{1}{10} \sum_{j=1}^{5} |\mu_j(P_i) - \mu_j(d_k)| + \frac{1}{10} \sum_{j=1}^{5} |v_j(P_i) - v_j(d_k)|
\]

(4.1)

From the calculations above, we have the matrix,

$$ Z = \left[ \begin{array}{ccc} S_1 & S_2 & S_3 \\ P_1 & 0.150 & 0.220 & 0.170 \\ P_2 & 0.200 & 0.170 & 0.180 \\ P_3 & 0.270 & 0.140 & 0.009 \end{array} \right] $$

We have given a table by comparing all the values obtained by three methods and also we have given a diagram representing our result below.

### 5. Conclusion

From the above table, we can conclude that School-1 satisfies the criteria of the parent-1 and it suits for their child, and School-2 Satisfies the criteria of the parent-2 and it suits for their child, and School-3 satisfies the criteria of the parent-3 and it suits for their child. Though their arise a tie for parent-3 by using Min-Max average composition, by comparing all the three methods the conclusion have been taken.
Table 1. Comparison table for three methods

| Methods                        | School-1 | School-2 | School-3 |
|-------------------------------|----------|----------|----------|
| Parent-1                      |          |          |          |
| Min-Max Average Composition   | 0.375    | 0.300    | 0.325    |
| Min-Max Composition           | 0.200    | 0.100    | 0.150    |
| Distance method               | 0.150    | 0.220    | 0.170    |
| Parent-2                      |          |          |          |
| Min-Max Average Composition   | 0.325    | 0.375    | 0.275    |
| Min-Max Composition           | 0.200    | 0.300    | 0.100    |
| Distance method               | 0.200    | 0.170    | 0.180    |
| Parent-3                      |          |          |          |
| Min-Max Average Composition   | 0.175    | 0.375    | 0.375    |
| Min-Max Composition           | -0.05    | 0.350    | 0.350    |
| Distance method               | 0.27     | 0.140    | 0.090    |

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