Design of Step-Index Multimode Optical Fiber

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Abstract. In this paper, a step-index fiber with core index 1.445517 and cladding index 1.443157 has been designed and studied. Multimode operation is achieved by using a fiber with core radius 25 µm operating at a wavelength of 1.3 µm. The mode parameters (effective refractive index, phase constant, fractional modal power in the core and cutoff wavelength) were calculated using RP fiber calculator (PRO version 2020). The shapes of the intensity and amplitude distribution of linearly polarized guided modes were shown.

Keywords: Fiber design, Step-index fiber, Multimode fiber, Linearly polarized modes, RP Fiber calculator.

1. Introduction

An optical fiber, which acts as the transmission channel, lies at the heart of an optical communication system [1]. A step-index fiber (SIF) consists of a core with a refractive index ncore that is slightly higher than that of the cladding nclad [2]. Most of the fibers are made from silica glass [3] which can be doped with (e.g., germanium or phosphorus) to increase the refractive index or with (e.g., fluorine or boron) to decrease it [4]. Fibers that support multiple guided modes (LP modes) are called multimode fibers (MMFs) [5]. Typically, the core radius is around 25 µm for MMFs. The three lowest-order guided modes are illustrated in Figure 1.

![Figure 1](image_url)

Figure 1. Intensity profiles for the three lowest linearly polarized (LP) modes [6].

In 1966, Kao and Hockham [7] proposed that glass fibers could be a practical optical transmission medium. They predicted that fiber loss could be reduced below 20 dB/km. The fiber process advanced very quickly after the demonstration of fiber with a loss of about 17 dB/km at a wavelength of 0.633 µm in 1970 [8]. By 1973, the loss of less than 5 dB/km at 0.85 µm was reported [9]. In 1976, the loss of 0.47 dB/km was reported at 1.2 µm [10]. In the late 1970s through the early 1980s, the
telecommunication systems used MMFs along with light emitting diodes (LEDs) or laser transmitters at 0.85 µm and 1.3 µm [4]. In 1988, the first transatlantic fiber cable was made operative at 1.3 µm. By 1996, data transmission at 1 Terabit/second was realized [11, 12]. In 2001, it has been possible to send about 11 Terabit/second through optical fiber [13]. In 2009, Kao received the Nobel prize for his work on fiber optics [14]. In 2018, successful transmission of 1.2 Petabit/second has been achieved [15, 16]. In my previous work [17], a single-mode SIF is designed at 1.31 µm and 1.55 µm. In this work, a MM SIF has been designed at 1.3 µm.

2. Theoretical Background

The V number gathers all of the design parameters characterizing the optical fiber. It is given by [18]:

\[ V = \frac{2n_{\text{eff}}^2 - n_{\text{clad}}^2}{n_{\text{core}}^2 - n_{\text{clad}}^2} \]  

(1)

where \( r \) is the core radius and \( \lambda \) is the vacuum wavelength of operation. Fiber dispersion is the lowest at 1.3 µm wavelength. The V number determines:

- The number of guided modes. If \( V > 2.4048 \) the fiber will be MM.
- The fraction of power guided inside the core.
- The phase constant for some wavelength is the effective refractive index (\( n_{\text{eff}} \)) times the vacuum wavenumber [5]. Guided modes occur if \( n_{\text{clad}} < n_{\text{eff}} < n_{\text{core}} \). Modes with \( n_{\text{eff}} < n_{\text{clad}} \) are called radiation modes. No mode exists when \( n_{\text{eff}} > n_{\text{core}} \) [3].

The normalized phase constant is defined by:

\[ b = \frac{n_{\text{eff}}^2 - n_{\text{clad}}^2}{n_{\text{core}}^2 - n_{\text{clad}}^2} \]  

(2)

For guided modes, \( 0 < b < 1 \).

The allowed discrete values of \( b \) of the guided LP\(_{lm} \) modes are determined by the transcendental equations:

\[ \frac{U J_l(U)}{J_0(U)} = W \frac{K_l(W)}{K_0(W)}, \quad l = 0 \]  

(3)

\[ \frac{U J_{l-1}(U)}{J_l(U)} = -W \frac{K_{l-1}(W)}{K_l(W)}, \quad l \geq 1 \]  

(4)

where \( U = V \sqrt{1-b} \) and \( W = V \sqrt{b} \). Here \( J_l(U) \) and \( K_l(W) \) are the Bessel functions of order \( l \).

There will be a finite number of solutions of Equations (3) and (4) for a given value of \( l \), and the \( m \)th solution (\( m = 1, 2, 3, \ldots \)) is referred to as the LP\(_{lm} \) mode.

The value of \( V \) at which \( b = 0 \) is known as the cutoff value \((V_c)\) of the mode. The cutoff \( V \) values are tabulated in Table 1.

| \( l \) | 1        | 2        | 3        |
|-------|----------|----------|----------|
| 0     | 0        | 3.8317   | 7.0156   |
| 1     | 2.4048   | 5.5201   | 8.6537   |
| 2     | 3.8317   | 7.0156   |          |
| 3     | 5.1356   | 8.4172   |          |
| 4     | 6.3802   | 9.7610   |          |
| 5     | 7.5883   |          |          |
| 6     | 8.7715   |          |          |

The cutoff wavelength \((\lambda_c)\) of the mode is the wavelength where that mode ceases to exist [5]. It can be calculated from [2]:

\[ \lambda_c = \frac{1}{n_{\text{eff}}(V_c)} \]
The range of wavelengths over which mode will propagate is $0 < \lambda < \lambda_c$.

From Equations (1) and (5),

$$\lambda V = \lambda_c V_c$$

A quantity of interest for SIFs is the fractional power carried in the core. It is given by [20]:

$$\frac{P_{core}}{P_{total}} = \frac{W^2}{V^2} + \frac{U^2 K_l^2(W)}{V^2 K_{l+1}(W)K_{l-1}(W)}$$

### 3. Results and Discussion

In this work, a SIF characterized by the parameters in Table 2 has been designed. It operates at a wavelength of 1.3 µm as a multimode fiber.

#### Table 2. Parameters of the fiber.

| $n_{core}$ | $n_{clad}$ | $r$ | $V$ at $\lambda = 1.3$ µm |
|------------|------------|-----|--------------------------|
| 1.445517   | 1.443157   | 25 µm | 9.9766 |

One finds that there are three modes each corresponding to $l = 0$ and $l = 1$, two modes each corresponding to $l = 2$, $l = 3$ and $l = 4$, and one mode each corresponding to $l = 5$ and $l = 6$. The mode properties (Tables 3 to 6) were calculated by using RP Fiber Calculator software (PRO version 2020). From these tables, it can be noted that:

- All values decrease with increasing $l$, $m$ indices of the $LP_{lm}$ modes.
- The fundamental $LP_{01}$ mode is the one with the highest values.
- Effective indices lie between core and cladding indices.
- For any mode, the phase constant is the effective index multiplied by the free space wavenumber.
- Far from cutoff value, the modal power is concentrated in the core.

#### Table 3. Effective refractive indices of $LP_{lm}$ modes.

| $l$ | $m$    |             |             |
|-----|--------|-------------|-------------|
|     | 1      | 2           | 3           |
| 0   | 1.445403 | 1.444921 | 1.444079 |
| 1   | 1.445229 | 1.444561 | 1.443566 |
| 2   | 1.445000 | 1.444151 |             |
| 3   | 1.444772 | 1.443699 |             |
| 4   | 1.444395 | 1.443220 |             |
| 5   | 1.444024 |             |             |
| 6   | 1.443610 |             |             |

#### Table 4. Phase constants (µm$^{-1}$) of $LP_{lm}$ modes.

| $l$ | $m$    |             |             |
|-----|--------|-------------|-------------|
|     | 1      | 2           | 3           |
| 0   | 6.98595 | 6.98362    | 6.97955    |
| 1   | 6.98511 | 6.98188    | 6.97707    |
| 2   | 6.98400 | 6.97990    |             |
| 3   | 6.98266 | 6.97772    |             |
| 4   | 6.98108 | 6.97540    |             |
| 5   | 6.97929 |             |             |
| 6   | 6.97729 |             |             |
Table 5. Fractional powers in the core (%) of LP_{lm} modes.

| l | m=1 | m=2 | m=3 |
|---|-----|-----|-----|
| 0 | 99.6 | 97.5 | 91.9 |
| 1 | 98.9 | 95.6 | 84.6 |
| 2 | 98.0 | 92.9 |       |
| 3 | 96.7 | 88.5 |       |
| 4 | 95.1 | 79.2 |       |
| 5 | 93.0 |       |       |
| 6 | 90.2 |       |       |

Table 6. Cutoff wavelengths (µm) of LP_{lm} modes
(calculated from RP Fiber Calculator).

| l | m=1 | m=2 | m=3 |
|---|-----|-----|-----|
| 0 |      | 3.35659 | 1.84098 |
| 1 | 5.30411 | 2.33976 | 1.49248 |
| 2 | 3.35694 | 1.84102 |       |
| 3 | 2.51500 | 1.53447 |       |
| 4 | 2.02445 | 1.32325 |       |
| 5 | 1.70215 |       |       |
| 6 | 1.47258 |       |       |

Cutoff wavelengths calculated from Equation (5) are listed in Table 7. These values are in good agreement with those in Table 6. Unlike other modes, the LP_{01} mode has no cutoff (V_c = 0). The cutoff wavelengths of LP_{21} and LP_{02} modes are the same (because the cutoff values are the same). Also, the cutoff wavelengths of LP_{22} and LP_{03} modes are the same. Cutoff wavelengths are larger than the operating wavelength (because the V number is larger than the cutoff values). If cutoff value for a particular guided mode approaches V number, then the cutoff wavelength will be close to the operating wavelength.

Table 7. Cutoff wavelengths (µm) of LP_{lm} modes
(calculated from Equation 5).

| l | m=1 | m=2 | m=3 |
|---|-----|-----|-----|
| 0 |      | 3.38480 | 1.84867 |
| 1 | 5.39320 | 2.34951 | 1.49873 |
| 2 | 3.38480 | 1.84867 |       |
| 3 | 2.52542 | 1.54084 |       |
| 4 | 2.03278 | 1.32871 |       |
| 5 | 1.70915 |       |       |
| 6 | 1.47860 |       |       |

Figures 2 to 4 show 2D mode profiles and plots of the radial dependence. These figures have been produced with the software RP Fiber Calculator. The modes profiles have increasing complexity as their indices are increased. The three lowest-order modes are similar to those illustrated in Figure 1. The intensity distribution of the lowest-order mode (LP_{01}) is circularly symmetric. The LP_{11} mode has two lobes oriented left and right. The higher-order modes have multilobed patterns, with 2l maxima in the angular direction and m maxima in the radial direction. In each profile, the thin circle locates the
core boundary. The amplitude distribution can be obtained from the intensity distribution by taking the square root. It can be seen that only $LP_{0m}$ modes have a profile having a peak in the center of the fiber core. While, the other modes have a peak away from the center.

| $l$ | $m$ |
|-----|-----|
| 0   | 1   | 2   | 3   |
| $LP_{0,1}$ | $LP_{0,2}$ | $LP_{0,3}$ |
| $LP_{1,1}$ | $LP_{1,2}$ | $LP_{1,3}$ |
| $LP_{2,1}$ | $LP_{2,2}$ |   |
| $LP_{3,1}$ | $LP_{3,2}$ |   |
| $LP_{4,1}$ |   |   |
| $LP_{5,1}$ |   |   |
Figure 2. Intensity distributions of LP\textsubscript{ln} modes.

| $l$ | 1 | 2 | 3 |
|-----|---|---|---|
| 0   | ![Intensity distribution for LP\textsubscript{0m} modes](image1.png) | ![Intensity distribution for LP\textsubscript{1m} modes](image2.png) | ![Intensity distribution for LP\textsubscript{2m} modes](image3.png) |
| 1   | ![Intensity distribution for LP\textsubscript{3m} modes](image4.png) | ![Intensity distribution for LP\textsubscript{4m} modes](image5.png) | ![Intensity distribution for LP\textsubscript{5m} modes](image6.png) |
| 2   | ![Intensity distribution for LP\textsubscript{6m} modes](image7.png) | ![Intensity distribution for LP\textsubscript{7m} modes](image8.png) | ![Intensity distribution for LP\textsubscript{8m} modes](image9.png) |
| 3   | ![Intensity distribution for LP\textsubscript{9m} modes](image10.png) | ![Intensity distribution for LP\textsubscript{10m} modes](image11.png) | ![Intensity distribution for LP\textsubscript{11m} modes](image12.png) |
| 4   | ![Intensity distribution for LP\textsubscript{12m} modes](image13.png) | ![Intensity distribution for LP\textsubscript{13m} modes](image14.png) | ![Intensity distribution for LP\textsubscript{14m} modes](image15.png) |
| 5   | ![Intensity distribution for LP\textsubscript{15m} modes](image16.png) | ![Intensity distribution for LP\textsubscript{16m} modes](image17.png) | ![Intensity distribution for LP\textsubscript{17m} modes](image18.png) |
Figure 3. Radial intensity distributions of LP\(_{lm}\) modes.
Figure 4. Radial amplitude distributions of LP$_{tm}$ modes.

4. Conclusions

Using RP Fiber Calculator program, it is possible to calculate cutoff wavelengths and other properties of modes. It is shown that if the operating wavelength is less than the cutoff wavelength, the fiber will be multimoded. The further away a mode is from its cutoff value the more concentrated its power is in the core. This fiber is candidate for practical use because most guided modes have fractional core powers of over 90%.

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