COLORED PREONS

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Abstract. Previous studies have suggested complementary models of the elementary particles as (a) quantum knots and (b) preonic nuclei that are field and particle descriptions, respectively, of the same particles. This earlier work, carried out in the context of standard electroweak ($SU(2) \times U(1)$) physics, is here extended to the strong interactions by the introduction of color ($SU(3)$) charges.
1 Introduction

The study of elementary particles as quantum knots in the context of electroweak physics leads in a natural way to a dual description of the elementary fermions as composite structures composed of three preons. We are here interested in investigating the compatibility of this structure with the strong interactions.

It had been previously shown\textsuperscript{2,3,4} that the four classes of elementary fermions \((e, \mu, \tau; \nu_e, \nu_\mu, \nu_\tau; d, s, b; u, c, t)\), carrying the electroweak quantum numbers \((t, t_3, t_0)\), may be regarded as quantum knots described by irreducible representations, \(D_{-3t_3-3t_0}^{3t}(q|abcd)\), of \(SL_q(2)\) in the manner shown in Table 1.

| \(f_1 f_2 f_3\) | \(t\) | \(t_3\) | \(t_0\) | \(Q\) | \(D_{-3t_3-3t_0}^{3t}\) |
|-----------------|-----|-----|-----|-----|-----------------|
| \(e\mu\tau\)   | \(\frac{1}{2}\) | \(-\frac{1}{2}\) | \(-\frac{1}{2}\) | \(-e\) | \(D_{\frac{3}{2}}^{\frac{3}{2}}\) |
| \(\nu_e\nu_\mu\nu_\tau\) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(-\frac{1}{2}\) | \(0\) | \(D_{\frac{3}{2}}^{\frac{3}{2}}\) |
| \(dsb\)        | \(\frac{1}{2}\) | \(-\frac{1}{2}\) | \(\frac{1}{6}\) | \(-\frac{1}{3}\) | \(D_{\frac{3}{2}}^{\frac{3}{2}}\) |
| \(uct\)        | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\frac{1}{6}\) | \(\frac{2}{3}\) | \(D_{\frac{3}{2}}^{\frac{3}{2}}\) |

Here \(Q = e(t_3 + t_0)\) is the expression for the electric charge in the standard theory.

The \(D_{mm'}^{j}(q|abcd)\) are functions lying in the \(SU_q(2)\) algebra and are given explicitly by\textsuperscript{1}

\[
D_{mm'}^{j}(q|abcd) = \sum_{s \leq n_+, t \leq n_-} A_{mm'}^{j}(q, s, t) \delta(s + t, n'_+) a^s b^{n_+ - s} c^t d^{n_- - t} \quad (1.1)
\]

where

\[
n_\pm = j \pm m
\]

\[
n'_\pm = j \pm m'
\]

and the arguments of \(D_{mm'}^{j}\) obey the algebra of \(SL_q(2)\), the “knot algebra”, as follows:

\[
ab = qba \quad bd = qdb \quad bc = cb \quad ad - qbc = 1 \quad q_1 = q^{-1} \quad (A)
\]

\[
ac = qca \quad cd = qdc \quad da - q_1 cb = 1
\]

The \(A_{mm'}^{j}(q, s, t)\) are numerical functions that do not concern us here.
By (1.1) the explicit form of the different $D_{-3t_3-3t_0}^{3t}(q|abcd)$ shown in Table 1 are given in Table 2. Ignoring the numerical coefficients we see that these functions are simple monomials carrying the quantum numbers that are correct for the different fermion families.

**Table 2.**

| $D_{-3t_3-3t_0}^{3t}(q|a, b, c, d)$ | $e\mu\tau$ | $D_{1/2}^{3/2}$ = $a^3$ |
|---|---|---|
| $\nu_\mu\nu_\tau$ | $D_{3/2}^{3/2} = c^3$ |
| $d_{sb}$ | $D_{3/2}^{3/2} = ab^2$ |
| $u_{ct}$ | $D_{3/2}^{3/2} = cd^2$ |

It has been previously noted that the state functions of these four quantum knots may be interpreted as composite field operators where the $(a, b, c, d)$ are preon creation operators, carrying the values of $(t, t_3, t_0)$ as shown in Table 3.

**Table 3.**

| Preon | $t$ | $t_3$ | $t_0$ | $Q$ | $D_{-3t_3-3t_0}^{3t}$ |
|---|---|---|---|---|---|
| $a$ | $\frac{1}{6}$ | $-\frac{1}{6}$ | $\frac{1}{6}$ | $-\frac{6}{3}$ | $D_{1/2}^{1/2}$ |
| $b$ | $\frac{1}{6}$ | $-\frac{1}{6}$ | $\frac{1}{6}$ | 0 | $D_{1/2}^{1/2} + \frac{1}{2}$ |
| $c$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $-\frac{1}{6}$ | 0 | $D_{1/2}^{1/2} - \frac{1}{2}$ |
| $d$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{6}{3}$ | $D_{1/2}^{1/2} - \frac{1}{2}$ |

In Table 3 the relation $Q = e(t_3 + t_0)$ of the standard theory is retained.

By Table 3 we see that $a$ and $d$ behave as creation operators for the charged $(-e/3)$ preon and $(e/3)$ antipreon respectively while $b$ and $c$ are neutral with opposite values of both $t_3$ and $t_0$.

The four preon operators are elements of the fundamental representation of $SL_q(2)$ as follows:

$$D_{1/2}^{1/2} : 
\begin{array}{c|cc}
  m \backslash m' & \frac{1}{2} & -\frac{1}{2} \\
  \frac{1}{2} & a & b \\
  -\frac{1}{2} & c & d 
\end{array}$$
The four elementary fermion operators are elements of the \( D^{3/2}_{mm'} \) representation. In both the \( j = 1/2 \) case and the \( j = 3/2 \) case we set

\[
\begin{align*}
  \tilde{j} &= 3t \\
  m &= -3t_3 \\
  m' &= -3t_0
\end{align*}
\]

In this way \( t, t_3, \) and \( t_0 \) are all defined with respect to the \( SL_q(2) \) algebra while in the standard theory \( t, t_3, \) and \( t_0 \) are defined by the \( SU(2) \times U(1) \) algebra. In the case that \( j = 3/2 \) the two definitions agree.

According to these same ideas there should also be bosonic preons corresponding to the adjoint representation \( D_{mm'}^1 \). The operators for these particles are products of two fermionic preon operators just as the operators for leptons, neutrinos, and quarks are products of three fermionic preon operators.

## 2 Gluon Charge

The previous considerations are based on electroweak physics. To describe the strong interactions it is necessary according to standard theory to introduce \( SU(3) \) charge. We shall therefore assume that each of the four preon operators appears in triplicate \((a_i, b_i, c_i, d_i)\) where \( i = R, Y, G \), without changing the algebra \( (A) \). These colored preon operators provide a basis for the fundamental representation of \( SU(3) \) just as the colored quark operators do in standard theory. To adapt the electroweak operators to the requirements of gluon fields we make the following replacements:

- **leptons**: \( a^3 \rightarrow \epsilon^{ijk}a_ia_ja_k \) \hspace{1cm} (2.1)
- **neutrinos**: \( c^3 \rightarrow \epsilon^{ijk}c_ic_jc_k \) \hspace{1cm} (2.2)
- **down quarks**: \( ab^2 \rightarrow a_ig^{jk}b_jb_k \equiv a_i(b^kb_k) \) \hspace{1cm} (2.3)
- **up quarks**: \( cd^2 \rightarrow c_ig^{jk}d_jd_k \equiv c_i(d^kd_k) \) \hspace{1cm} (2.4)

where \( g^{jk} \) is the group metric of \( SU(3) \) and \((i, j, k) = (R, Y, G)\) and \((a_ib_ic_id_i)\) are creation operators for colored preons. Then the leptons and neutrinos are color singlets while the
quark states correspond to the fundamental representation of $SU(3)$, as required by standard theory.

3 The Complementary Models

We have ascribed to the quantum knot the state function $\mathcal{D}^j_{nm'}(q|abcd)$, an irreducible representation of the knot algebra $SL_q(2)$, where the indices $j = \frac{N}{2}$, $m = \frac{w}{2}$, $m' = \frac{r+1}{2}$ are restricted to values of $(N, w, r)$ allowed by the classical (geometrical) knots. The quantum knots have more degrees of freedom than their classical images with the consequence that two quantum knots may be distinguishable when their classical images are not. In particular there are four distinguishable quantum trefoils with $(w, r) = (\pm 3, \pm 2)$ but only two of their classical images $(w, r) = (\pm 3, 2)$ are topologically different. In the physical application $(w, r) = (\pm 3, 2)$ describe the leptons and neutrinos while $(w, r) = (\pm 3, -2)$ describe the two varieties of quarks, i.e., the two additional quantum knots are required to permit the description of colored fermions.

These considerations have led us to two complementary models of the elementary particles, namely

(a) quantum knots

(b) preon structures

that are the field and particle descriptions of the same particles. The correspondence may be expressed by the following relations

$$
\begin{align*}
w &= n_a - n_d + n_b - n_c (= 2m = -6t_3) \\
r + 1 &= n_a - n_d - n_b + n_c (= 2m' = -6t_0) \\
N &= n_a + n_b + n_c + n_d (= 2j = 6t)
\end{align*}
$$

Here $(N, w, r)$ describe the number of crossings, the writhe and the rotation of the particle regarded as a quantum knot of field while $(n_a, n_b, n_c, n_d)$ record the number of $(a, b, c, d)$ preons in the dual description of the same structure. We have also described this particle.
by

\[ D^j_{m'n'} = D^{3t}_{-3t_3 - 3t_0} = D^{N/2}_{w_{r+1}/2} \]  (3.4)

as indicated in (3.1)-(3.3).

The knot \((N, w, r)\) and the preon \((n_a, n_b, n_c, n_d)\) descriptions share the same representation of \(SU_q(2)\) as follows.

In terms of \((N, w, r)\) one has

\[ D^j_{mm'} = D^{N/2}_{w_{r+1}/2} \]

where

\[ D^{N/2}_{w_{r+1}/2}(q|abcd) = \left[ \frac{(n'_+)!}{(n_+)!} \frac{(n'_-)!}{(n_-)!} \right]^{1/2} \sum_{0 \leq s \leq n_+}^{n_-} \sum_{0 \leq t \leq n_-}^{n_+} \left\langle \begin{array}{c} n_+ \\ s \end{array} \right\rangle_{q_1} \left\langle \begin{array}{c} n_- \\ t \end{array} \right\rangle_{q_1} \delta(s + t, n'_+) a^s b^{n_+ - s} c^t d^{n_- - t} \]  (3.5)

and again in terms of \((N, w, r)\)

\[ n_+ = \frac{1}{2} [N + w] \]  (3.6)

\[ n'_+ = \frac{1}{2} [N + (r + 1)] \]  (3.7)

The complementary description expressed in terms of the population numbers \((n_a, n_b, n_c, n_d)\) is

\[ D^j_{mm'} = D^{N}_{n_a n_b} \]  (3.8)

where

\[ D^{N}_{n_a n_b} = \left[ \frac{(n_a + n_c)!}{(n_a + n_b)!} \frac{(n_b + n_d)!}{(n_a + n_d)!} \right]^{1/2} \sum_{N \geq n_a, n_b \geq 0}^{n_a + n_b} \sum_{N \geq n_c, n_d \geq 0}^{n_c + n_d} \left\langle \begin{array}{c} n_a + n_b \\ n_a \\ n_b \end{array} \right\rangle_{q_1} \left\langle \begin{array}{c} n_c + n_d \\ n_c \\ n_d \end{array} \right\rangle_{q_1} a^{n_a} b^{n_b} c^{n_c} d^{n_d} \]  (3.9)

The limits on \(\sum\), literally translated from \(D^j_{mm'}\) are shown in the expression for \(D^{N}_{n_a n_b}\) but these limits simply describe the requirement that all population numbers, \(n_i\) satisfy \(N \geq n_i \geq 0\).

Here the exponents \((n_a, n_b, n_c, n_d)\) of \((abcd)\) are

\[ n_a = s \quad n_b = n_+ - s \]

\[ n_c = t \quad n_d = n_- - t \]  (3.10)

They are related to \((n'_+, n'_-, n'_+, n'_-)\) by

\[ n_+ = n_a + n_b \quad n'_+ = n_a + n_c \]

\[ n_- = n_c + n_d \quad n'_- = n_b + n_d \]  (3.11)
Since \( a \) and \( d \) have opposite charge and hypercharge, while \( b \) and \( c \) are neutral with opposite hypercharge, we may define the “preon numbers” \( \nu_a \) and \( \nu_b \) as follows

\[
\nu_a = n_a - n_d \\
\nu_b = n_b - n_c
\]  

(3.12)

Then by (3.1) and (3.2)

\[
\nu_a + \nu_b = 2m = w \\
\nu_a - \nu_b = 2m' = r + 1
\]  

(3.13)

Then the conservation of the writhe and rotation implies the conservation of the preon numbers \( \nu_a \) and \( \nu_b \). According to Table 2 the trefoil solutions of (3.1)-(3.3) are given in Table 4.

| \( \ell \) | \( n_a \) | \( n_b \) | \( n_c \) | \( n_d \) |
|---|---|---|---|---|
| \( \nu \) | 3 | 0 | 0 | 0 |
| \( d \) | 0 | 0 | 3 | 0 |
| \( u \) | 1 | 2 | 0 | 0 |
| \( a \) | 0 | 0 | 1 | 2 |

Since the number of crossings equals the number of preons, one may speculate that there is one preon at each crossing if both preons and crossings are considered pointlike. If the pointlike crossings are labelled \((\vec{x}_1,\vec{x}_2,\vec{x}_3)\), then by (2.1)-(2.4) the wave functions of the trefoils representing leptons \((\ell)\), neutrinos \((\nu)\), down quarks \((d)\), up quarks \((u)\) are as follows:

\[
\Psi_\ell(\vec{x}_1,\vec{x}_2,\vec{x}_3) = \epsilon^{ijk} \psi_i(a|\vec{x}_1)\psi_j(a|\vec{x}_2)\psi_k(a|\vec{x}_3) \\
\Psi_\nu(\vec{x}_1,\vec{x}_2,\vec{x}_3) = \epsilon^{ijk} \psi_i(c|\vec{x}_1)\psi_j(c|\vec{x}_2)\psi_k(c|\vec{x}_3) \\
\Psi_d(\vec{x}_1,\vec{x}_2,\vec{x}_3) = \psi_i(a|\vec{x}_1)\psi^j(b|\vec{x}_2)\psi_j(b|\vec{x}_3) \\
\Psi_u(\vec{x}_1,\vec{x}_2,\vec{x}_3) = \psi_i(c|\vec{x}_1)\psi^j(d|\vec{x}_2)\psi_j(d|\vec{x}_3)
\]

(3.14) \hspace{1cm} (3.15) \hspace{1cm} (3.16) \hspace{1cm} (3.17)

where \( i = (R,Y,G) \) and \( \psi_i(a|\vec{x}) \ldots \psi_i(d|\vec{x}) \) are colored \( \delta \)-like functions localizing the preons at the crossings.

Then the wave function of a lepton describes a singlet trefoil particle containing three preons of charge \((-e/3)\) and hypercharge \((-e/6)\). The corresponding characterization of a neutrino describes a singlet trefoil containing three neutral preons of hypercharge \((-e/6)\).
The wave function of a down quark describes a colored trefoil particle containing one $a$-preon with charge $(-e/3)$ and hypercharge $(-e/6)$ and two neutral $b$-preons with hypercharge $(e/6)$. The corresponding characterization of an up-quark describes a colored trefoil containing two charged $d$-preons with charges $(e/3)$ and hypercharge $(e/6)$, and one neutral $c$-preon with hypercharge $(-e/6)$.

This hypothetical structure would be held together by the fields connecting the charged preons. Since the preons carry both electric charge and hypercharge as well as color charge and color hypercharge, the total Lagrangian that determines these fields, and therefore the preon dynamics, would then be determined by the sum of two non-Abelian Lagrangians, one describing standard electroweak based on local $SU(2) \times U(1)$ and the other describing standard chromodynamics based on local $SU(3)$. Here we may assume that the $SU(2) \times U(1)$ and the $SU(3)$ gauge bosons are also quantum knots described by the adjoint representation $D^1_{mn'}(q|abcd)$ of global $SU_q(2)$. In this way the preons might play a similar role with respect to the tripreon (the basic fermion) that the quarks play with respect to the triquark (the hadron). A search for this kind of substructure depends critically on the mass of the conjectured preon and about which nothing can unfortunately be said with any confidence. Nevertheless we shall try to extend the Higgs idea to a consideration of the mass of the preon.

4 Mass of Preons

Since the preons are necessarily assumed to be pointlike, they must be very heavy. Let us assume that the mass of the preon is computed in the same way as we have computed the mass of the elementary fermions, i.e., by adopting the mass terms of the standard theory, namely

$$
\mathcal{M} = \bar{L} \varphi R + \bar{R} \varphi L \tag{4.1}
$$

where $L$ and $R$ are left and right chiral spinors and $\varphi$ is the Higgs scalar.

We shall assign a $SU_q(2)$ singlet structure to $\varphi$ and the preon representation $D^{1/2}_{mn'}$ to both $L$ and $R$. Then we substitute for $L$ and $R$ as follows:

$$
L \rightarrow \chi_L D^{1/2}_{mn'} |0\rangle \tag{4.2}
$$
\[ R \rightarrow \chi_R D_{nn'}^{1/2}|0\rangle \]  \hfill (4.3)

where \( \chi_L \) and \( \chi_R \) are the standard fermionic fields and \( D_{nn'}^{1/2}|0\rangle \) describes the internal structure of the preons. Here \( |0\rangle \) is the ground state of the \( SU_q(2) \) algebra. Then

\[
\mathcal{M} \rightarrow \langle 0|\bar{D}_{nn'}^{1/2}D_{nn'}^{1/2}|0\rangle (\bar{\chi}_L\varphi\chi_R + \bar{\chi}_R\varphi\chi_L) = M(m, m')\bar{\chi}\chi
\]  \hfill (4.4)

where the mass is

\[ M(m, m') = \rho(m, m') \langle 0|\bar{D}_{nn'}^{1/2}D_{nn'}^{1/2}|0\rangle \]  \hfill (4.5)

where \( \rho(m, m') \) is a local minimum of the Higgs potential. We shall assume that there are 4 local minima of the Higgs potential, namely

\[ (m, m') = \left( \pm \frac{1}{2}, \pm \frac{1}{2} \right) \]  \hfill (4.7)

For example, the mass of the \( D_{1/2}^{1/2} \) preon is

\[ M \left( \frac{1}{2}, \frac{1}{2} \right) = \rho \left( \frac{1}{2}, \frac{1}{2} \right) \langle 0|\bar{a}a|0\rangle \]  \hfill (4.8)

The mass of the electron computed in the same way is

\[ M \left( \frac{3}{2}, \frac{3}{2} \right) = \rho \left( \frac{3}{2}, \frac{3}{2} \right) \langle 0|\bar{a}^3a^3|0\rangle \]  \hfill (4.9)

Then the ratio of the preon mass \( (m_p) \) to the electron mass \( (m_e) \) is

\[
\frac{m_p}{m_e} = \frac{\rho \left( \frac{1}{2}, \frac{1}{2} \right)}{\rho \left( \frac{3}{2}, \frac{3}{2} \right)} \frac{\langle 0|\bar{a}a|0\rangle}{\langle 0|\bar{a}^3a^3|0\rangle}
\]  \hfill (4.10)

The factor \( \frac{\langle 0|\bar{a}a|0\rangle}{\langle 0|\bar{a}^3a^3|0\rangle} \) is a simple function of \( q \) and \( |\beta| \).

If the ratio of the Compton wavelengths is greater than unity, i.e., if

\[
\frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e} = \frac{\rho \left( \frac{1}{2}, \frac{1}{2} \right)}{\rho \left( \frac{3}{2}, \frac{3}{2} \right)} \frac{\langle 0|\bar{a}a|0\rangle}{\langle 0|\bar{a}^3a^3|0\rangle} > 1
\]  \hfill (4.11)

then the mass of the preon is compatible with the pointlike extension of the electron. Under the foregoing assumptions this preon model would require several minima in the Higgs potential satisfying relations such as the following:

\[
\rho \left( \frac{1}{2}, \frac{1}{2} \right) > \frac{\langle 0|\bar{a}^3a^3|0\rangle}{\langle 0|\bar{a}a|0\rangle} \rho \left( \frac{3}{2}, \frac{3}{2} \right)
\]  \hfill (4.12)
where $\rho \left( \frac{3}{2}, \frac{3}{2} \right)$ and $\rho \left( \frac{1}{2}, \frac{1}{2} \right)$ are local minima of the Higgs potential. Here $\rho \left( \frac{3}{2}, \frac{3}{2} \right)$ is in the estimated neighborhood of the Higgs mass that appears in the standard theory.

There is a stronger limit on $m_P$ if $\lambda_p$ is of the order of $10^{-16}$ cm as suggested by scattering experiments. In any case $m_P$ is much greater than the masses of the leptons, neutrinos and quarks. According to any nuclear physics model the binding energy must therefore almost totally compensate the sum of the masses of the three preons. On the other hand, according to the model underlying (4.12), the neutrino, lepton, quark, and preon masses are generated by the symmetry breaking Higgs potential.

References

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