Microscopic description of Gamow-Teller transitions in middle $pf$–shell nuclei by a realistic shell model calculation

H Nakada† and T Sebe‡

† Department of Physics, Chiba University, Inage-ku, Chiba 263, Japan
‡ Department of Applied Physics, Hosei University, Koganei, Tokyo 184, Japan

Abstract. GT transitions in $N = 28 \sim 30$ nuclei are studied in terms of a large-scale realistic shell-model calculation, by using Towner’s microscopic parameters. $B$(GT) values to low-lying final states are reproduced with a reasonable accuracy. Several gross properties with respect to the GT transitions are investigated with this set of the wavefunctions and the operator. While the calculated total GT$^-$ strengths show no apparent disagreement with the measured ones, the calculated total GT$^+$ strengths are somewhat larger than those obtained from charge-exchange experiments. Concerning the Ikeda sum-rule, the proportionality of $S_{GT}$ to $(N - Z)$ persists to an excellent approximation, with a quenching factor of 0.68. For the relative GT$^-$ strengths among possible isospin components, the lowest isospin component gathers greater fraction than expected by the squared CG coefficients of the isospin coupling. It turns out that these relative strengths are insensitive to the size of model space. Systematics of the summed $B$(GT) values are discussed for each isospin component.
Short title: GT transitions in middle pf-shell nuclei

March 31, 2022
1. Introduction

Gamow-Teller (GT) transitions pose a challenging problem for nuclear structure theories. They play an important role in many astrophysical phenomena. Middle pf-shell nuclei form a starting point of the chain of the s- and r-processes, in which the GT transitions compete with the neutron capture. Almost all heavier nuclei in nature are synthesized via the s- and/or r-processes. Moreover, the GT transition rates of middle pf-shell nuclei themselves are significant inputs in the description of supernova explosions. On the other side, the GT transitions provide us with a stringent test to nuclear many-body wavefunctions, since they are sensitive to some details of the wavefunctions.

In the sd-shell region, a realistic shell-model calculation is successful in describing the GT transition strengths\cite{1}. Realistic shell-model approaches to the GT transitions in the middle pf-shell nuclei have been desired. The extent of excitation out of the 0f7/2 orbit is crucial to the GT transition rates in the middle pf-shell region. However, in most of the shell-model wavefunctions available so far, the amount of the 0f7/2 excitation has not been inspected sufficiently. We have recently reported one of the most successful realistic shell-model calculations for the N = 28 \sim 30 nuclei\cite{2}. Not only the energy levels, but also the electromagnetic properties are reproduced with the parameters derived from microscopic standpoints. We seem to have a reasonable amount of the leakage out of 0f7/2 in these shell-model wavefunctions, since the E2 and M1 properties are reproduced well by the parameters on the microscopic ground. In this article, we extend this spectroscopically tested shell-model approach to the GT transitions, by using Towner’s single-particle parameter-set\cite{3}.

In addition to the \(\beta\)-decay strengths between low-lying states, the summed GT strengths have also been interested in. Even with a reasonable quenching factor for the GT transition operator, there still remain discrepancies between calculations and
measurements in some pf-shell nuclei. While the final states of the GT$^+$ transitions have a unique isospin value, the GT$^-$ strength is distributed over a few isospin values, because of the neutron excess. In many cases, the s– or r–process should be dominated by the lowest isospin components. Furthermore, the GT strengths with a specific isospin transfer are required for some special topics. For instance, in the double-$\beta$-decays only the lowest isospin states can act as intermediate states. It is also important to predict correctly the summed GT strength with a specific isospin transfer. However, the isospin distribution of the GT strength has not been investigated so well. Only recently it has become possible to acquire some information from experiments\cite{4}, on the isospin composition of the GT strength. Whereas shell effects are expected on the relative strength of each isospin component, we have lacked systematic study based on realistic calculations. We shall investigate the total GT strength and its isospin partition, as well as the decay strengths to low-lying states, by using the realistic shell-model wavefunctions in the middle pf-shell.

2. Model space and GT operator

The configuration space of the shell-model calculation\cite{2} is as follows: In the pf–shell on top of the $^{40}$Ca inert core, we take the space consisting all of the $k \leq 2$ configurations, where $k$ represents number of nucleons excited from $0f_{7/2}$. Namely, $k$ is defined as

$$(0f_{7/2})^{n_1-k}(0f_{5/2}1p_{3/2}1p_{1/2})^{n_2+k},$$

with $n_1 = (Z - 20) + 8$ and $n_2 = N - 28$ for $20 < Z \leq 28 \leq N$ nuclei. The $k \leq 2$ configuration space leads to the dimension over 100 000 in the $M$–scheme, for several $N = 30$ nuclei. The Kuo-Brown hamiltonian\cite{5} is diagonalized in this model space, by using the code VECSSE\cite{6}. In Reference \cite{2}, it has been shown that the energy levels of the $N = 28 \sim 30$ nuclei are reproduced within typical deviation of 0.3MeV. The wavefunctions have been tested via the E2 and M1 transition strengths.
and moments, for which we have employed single-particle parameters derived from microscopic standpoints.

The GT transition operator within the shell-model framework is

\[
T(\text{GT}^\pm) = \sum_i \left\{ g_A^{\text{eff}}(nl)\sigma_i + g_{lA}^{\text{eff}}(nl)l_i + g_{pA}^{\text{eff}}(nl, n'l') Y^{(2)}(\hat{r}_i)\sigma_i \right\} t_{\pm,i},
\]

where the sum runs over valence nucleons. The \( B(\text{GT}) \) values are connected to the \( ft \) values through the following relation,

\[
ft = \frac{6170}{B(\text{GT})}. \tag{3}
\]

For free nucleons, we have \( g_A = 1.26 \) and \( g_{lA} = g_{pA} = 0 \). It has been pointed out that nuclear medium effects should be incorporated into the \( g \)-parameters\(^3\)\(^,\)\(^7\). We use Towner’s microscopic single-particle parameter-set\(^3\) as in the M1 case, in which effects of the core-polarization (CP) and the meson-exchange-currents (MEC) on top of the \(^{40}\text{Ca}\) core are taken into account. No mass-number dependence is considered for the \( g \)-parameters. Apart from the finite values of \( g_{lA}^{\text{eff}} \) and \( g_{pA}^{\text{eff}} \), the quenching for \( g_A \) in the present parameter-set is estimated to be \( g_A^{\text{eff}}/g_A = 0.82 \) (0.81) for the \( 0f \) (1p) orbit.

In the operator of Equation (2), the MEC between a valence nucleon and the core are taken into consideration. Though the MEC between valence nucleons lead to two-body operators, their contribution is expected to be small, as has been confirmed for the M1 quantities\(^8\). Remark that there is no adjustable parameters in this calculation, as in the calculation of the energy levels and electromagnetic properties in Reference \(^2\).

As has been stated already, the \( B(\text{GT}) \) values are sensitive to the \(^{56}\text{Ni}\)-core excitation (i.e., the excitation out of the \( 0f_{7/2} \) orbit), in the middle \( pf \)-shell region. The GT transitions mainly concern the nucleon-spin degrees-of-freedom. The spin degrees-of-freedom will be active to a certain extent for the middle \( pf \)-shell nuclei, because the splitting of single-particle energies makes the \( 0f_{7/2} \) orbit substantially occupied, while its \( LS \)-partner \( 0f_{5/2} \) apt to be empty. On the other hand, the interaction among nucleons
favors saturation of the nucleon-spin, competing with the energy splitting. Hence the calculated $B(\text{GT})$ values are sensitive to the size of model space; the calculated $B(\text{GT})$ values decrease if larger amount of excitation out of $0f_{7/2}$ is involved. The excitation from $0f_{7/2}$ to the upper $pf$-shell orbits is important also for the E2 and M1 transitions. The similarity of the M1-transition operator to the GT operator has long been noticed\cite{3,7}. For the E2 transitions, the quadrupole collectivity, which is a typical property of the residual nucleon-nucleon interaction, competes with the single-particle energy splitting. Thereby the E2 transitions also have a significant correlation with the excitation from $0f_{7/2}$ to the upper orbits. As discussed in Reference \cite{2}, the E2 and M1 strengths are well described by the present shell-model wavefunctions with the parameters derived from microscopic standpoints. Thus we have a reasonable amount of excitation out of $0f_{7/2}$ in the present wavefunctions, and their application to the GT transitions may be promising.

It is commented that convergence for $k$ with the present shell-model hamiltonian is not evident. On the other hand, the spectroscopic test has a particular importance in assessing the reliability of the wavefunctions. Since it generally depends on the model space whether the effective hamiltonian is appropriate or not, such tests should be done for each set of space and hamiltonian. Whereas the Kuo-Brown interaction has originally been developed for the full $pf$-shell calculations, the comprehensive reproduction of the spectroscopic properties confirms a certain reliability only of the present wavefunctions obtained within the $k \leq 2$ space. For this reason, we shall restrict ourselves in this paper to the results extracted from the $k \leq 2$ wavefunctions, putting aside the convergence problem.
3. GT strengths to low-lying levels

We hereafter restrict ourselves to GT transitions from the ground states of the parent nuclei. The GT-decay strength to individual low-lying state is investigated first. Both the initial and final states in this calculation consist of the $k \leq 2$ configurations, for which the wavefunctions have been well tested. The calculated $B$(GT$^\pm$) values are compared with the measured ones\textsuperscript{[9]}, in Tables I and II. This enables us to assess how appropriate the present GT operator is. Moreover, at the same time, this comparison will be a further test of the shell-model wavefunctions. Several $B$(GT$^-$) values are predicted for possible $\beta$-decays, in Table I. In Reference \textsuperscript{[2]}, calculated energy levels have been inverted in a few cases, in making a correspondence to the observed ones, based on the $B$(E2) and/or $B$(M1) values. The inversion of the lowest two ($\frac{7}{2}^-$) states of $^{53}$Cr, which is already taken into consideration in Table I, is consistent with the GT strengths from $^{53}$V. The $B$(GT$^+$) values from $^{55}$Co to ($\frac{9}{2}^-$) states of $^{55}$Fe suggest that the calculated ($\frac{9}{2}^-_1$) level should correspond to the observed ($\frac{9}{2}^-_2$) state, and vice versa. Although this inversion has not been discussed in Reference \textsuperscript{[2]} and not considered in Table I, it does not give rise to contradictions to the electromagnetic properties. Taking this into consideration, we find good agreement between the calculated and measured GT strengths, as far as the $B$(GT) values exceeding 0.01 are concerned. As a general tendency, the present calculation slightly underestimates the low-lying GT strengths. However, except for a few transitions from $^{56}$Ni and $^{57}$Ni, those relatively large GT strengths are reproduced within 70% accuracy. Furthermore, we have agreement in the order-of-magnitude even for most other small GT strengths.
Table 1. $B(GT^-)$ values. The ‘Cal.’ values are obtained by the present shell-model calculation, and the experimental data (Exp.) are taken from Reference [9].

| parent    | daughter | Cal.       | Exp.   |
|-----------|----------|------------|--------|
| $^{50}$Sc | $^{50}$Ti | 0.0001     | —      |
| $^{51}$Ti | ($\frac{3}{2}^-_1$) | 0.081     | —      |
| $^{51}$Ti | ($\frac{5}{2}^-_1$) | 0.037     | —      |
| $^{52}$Ti | $^{52}$V  | 0.361      | 0.56 ± 0.04 |
| $^{52}$V  | $^{52}$Cr | 0.050      | 0.0616 ± 0.0003 |
| $^{53}$V  | ($\frac{7}{2}^-_1$) | 0.156     | 0.15   |
| $^{53}$V  | ($\frac{9}{2}^-_1$) | 0.025     | 0.031  |
| $^{53}$V  | ($\frac{7}{2}^-_2$) | 0.0052    | ~ 0.002 |
| $^{53}$V  | ($\frac{9}{2}^-_1$) | 0.0001    | —      |
| $^{53}$V  | ($\frac{9}{2}^-_2$) | 0.0010    | —      |
Table 2. $B(GT^\pm)$ values. The experimental data are taken from Reference [9].

| parent | daughter | Cal.  | Exp.  |
|--------|----------|------|-------|
| $^{54}\text{Mn}$ | 3$^+_1$ | $^{54}\text{Cr}$ | 2$^+_1$ | 0.0036 | 0.0039 |
| $^{55}\text{Fe}$ | (3$^+_2$) | $^{55}\text{Mn}$ | (5$^+_2$) | 0.0031 | 0.0062 ±0.0003 |
| $^{55}\text{Co}$ | (3$^+_1$) | $^{55}\text{Fe}$ | (5$^+_2$) | 0.0068 | 0.0035 ±0.0003 |
| | | | (3$^+_2$) | 0.0046 | 0.00116±0.00006 |
| | | | (3$^+_2$) | 0.0027 | 0.0101 ±0.0006 |
| | | | (3$^+_2$) | 0.0028 | 0.0013 ±0.0002 |
| | | | (3$^+_2$) | 0.0079 | 0.0048 ±0.0004 |
| $^{56}\text{Co}$ | 4$^+_1$ | $^{56}\text{Fe}$ | 4$^+_1$ | 0.00009 | 0.00001 |
| | | | 4$^+_2$ | 0.000009 | 0.00014±0.00001 |
| | | | 3$^+_1$ | 0.0002 | 0.00066±0.00002 |
| $^{56}\text{Ni}$ | 0$^+_1$ | $^{56}\text{Co}$ | 1$^+_1$ | 0.058 | 0.25 |
| $^{57}\text{Ni}$ | (3$^+_2$) | $^{57}\text{Co}$ | (5$^+_2$) | 0.0079 | 0.0141 ±0.0006 |
| | | | (3$^+_2$) | 0.0039 | 0.0055 ±0.0001 |
| | | | (3$^+_2$) | 0.0027 | 0.0037 ±0.0002 |
| | | | (3$^+_2$) | 0.00002 | 0.0112 ±0.0005 |
| | | | (3$^+_2$) | 0.0028 | 0.00005±0.00001 |

4. Total GT strengths

The total GT strength is also a significant physical quantity and has long been discussed. There have been several shell-model calculations for the middle $pf$-shell nuclei[10, 11]. We next apply the present shell-model approach to this issue. The total GT strengths obtained in the present shell-model calculation are shown in Table 3. As is well-known, the total GT$^\pm$ strength $\sum B(GT^\pm)$ is equal to the expectation value of $T(GT^\mp) \cdot T(GT^\pm)$ in the initial state. They are therefore a sort of ground-state property. Although the
final-state wavefunctions are not needed to compute the total GT strengths, we here discuss which configurations may come out in the final states. Since the initial state is comprised of the $k \leq 2$ configurations and the GT operator can excite a nucleon from $0f_{7/2}$ to $0f_{5/2}$, the GT$^-$ strength distributes over the $k \leq 3$ configurations in the daughter nucleus, for the transitions from the $N = 28$ isotones. The same holds for the Ni isotopes. For the $N = 29$ and 30 isotones except $^{57}\text{Ni}$ and $^{58}\text{Ni}$, the $k = 4$ configuration also emerges in the final states of the GT$^-$ transitions. This is because the lowest configuration is different between the parent and daughter nucleus. For instance, in $^{52}\text{Ti}$ the $k = 2$ configuration implies $(0f_{7/2})^8(0f_{5/2}1p_{3/2}1p_{1/2})^4$, since $n_1 = 10$ and $n_2 = 2$ (see Equation (1)). The highest configuration generated by the GT$^-$ transition is $(0f_{7/2})^7(0f_{5/2}1p_{3/2}1p_{1/2})^5$. This is the $k = 4$ configuration of $^{52}\text{V}$, the daughter nucleus, because the lowest configuration shifts from $^{52}\text{Ti}$, giving $n_1 = 11$, $n_2 = 1$. Notice that this shift of $n_1$ and $n_2$ does not happen in the $N = 28$ cases. Thus the total GT$^-$ strengths of the $N = 29$ and 30 nuclei are contributed by up to the $k = 4$ configuration. Conversely, the final-state configuration in the GT$^+$ transition is constrained into the $k \leq 2$ space, for any of the nuclei under discussion. The calculated eigenenergies of the ground state may be disturbed, if an admixture of the $k = 3$ and 4 configurations is included explicitly. However, the total GT$^\pm$ strengths are principally ruled by the proton and neutron occupation numbers of the $0f_{7/2}$ orbit[12]. Therefore, besides the convergence problem for $k$, reliable total GT$^-$ strengths can be calculated with the $k \leq 2$ ground-state wavefunctions, as far as the wavefunctions contain the excitation out of $0f_{7/2}$ properly.

The Ikeda sum-rule $S_{\text{GT}} \equiv \sum B(\text{GT}^-) - \sum B(\text{GT}^+) = 3g_A^2(N - Z)$ is derived from the bare GT operator[13]. When we discuss the GT strengths within the $0\hbar\omega$ space, a quenching factor for $g_A$ is sometimes introduced, keeping $g_{lA} = g_{pA} = 0$, from phenomenological viewpoints. Even in that case, $S_{\text{GT}}$ is proportional to $(N - Z)$. The
Table 3. Total GT\(^\pm\) strengths. The experimental data are taken from References [15, 16].

| parent | Cal. | Exp. |
|--------|------|------|
|        | \(\sum B(\text{GT}^-)\) | \(\sum B(\text{GT}^+)\) | \(\sum B(\text{GT}^-)\) | \(\sum B(\text{GT}^+)\) |
| \(^{49}\text{Sc}\) \((\frac{7}{2})_1^-\) | 24.0 | 0.9 | — | — |
| \(^{50}\text{Ti}\) \(0^+_1\) | 21.2 | 1.5 | — | — |
| \(^{51}\text{V}\) \((\frac{7}{2})_1^-\) | 19.5 | 3.0 | 20.0±4.0 | — |
| \(^{52}\text{Cr}\) \(0^+_1\) | 17.4 | 4.3 | — | — |
| \(^{53}\text{Mn}\) \((\frac{7}{2})_1^-\) | 15.9 | 6.1 | — | — |
| \(^{54}\text{Fe}\) \(0^+_1\) | 14.3 | 7.7 | 12.4±3.0 | 5.6±0.8 |
| \(^{55}\text{Co}\) \((\frac{7}{2})_1^-\) | 12.9 | 9.6 | — | — |
| \(^{56}\text{Ni}\) \(0^+_1\) | 11.4 | 11.4 | — | — |
| \(^{50}\text{Sc}\) \(5^+_1\) | 27.0 | 0.9 | — | — |
| \(^{51}\text{Ti}\) \((\frac{3}{2})_1^-\) | 24.2 | 1.3 | — | — |
| \(^{52}\text{V}\) \(3^+_1\) | 22.3 | 2.7 | — | — |
| \(^{53}\text{Cr}\) \((\frac{3}{2})_1^-\) | 20.1 | 3.8 | — | — |
| \(^{54}\text{Mn}\) \(3^+_1\) | 18.6 | 5.6 | — | — |
| \(^{55}\text{Fe}\) \((\frac{3}{2})_1^-\) | 16.9 | 7.2 | — | — |
| \(^{56}\text{Co}\) \(4^+_1\) | 15.5 | 9.1 | — | — |
| \(^{57}\text{Ni}\) \((\frac{3}{2})_1^-\) | 14.0 | 10.9 | — | — |
| \(^{51}\text{Sc}\) \((\frac{7}{2})_1^-\) | 30.1 | 0.8 | — | — |
| \(^{52}\text{Ti}\) \(0^+_1\) | 27.1 | 1.1 | — | — |
| \(^{53}\text{V}\) \((\frac{7}{2})_1^-\) | 24.9 | 2.2 | — | — |
| \(^{54}\text{Cr}\) \(0^+_1\) | 22.4 | 2.9 | — | — |
| \(^{55}\text{Mn}\) \((\frac{5}{2})_1^-\) | 20.8 | 4.6 | — | 2.7±0.3 |
| \(^{56}\text{Fe}\) \(0^+_1\) | 19.1 | 6.2 | 15.7±3.8 | 4.6±0.5 |
| \(^{57}\text{Co}\) \((\frac{7}{2})_1^-\) | 17.5 | 7.9 | — | — |
| \(^{58}\text{Ni}\) \(0^+_1\) | 15.8 | 9.5 | 11.7±2.9 | 6.0±0.6 |
present GT operator, however, has finite values of $g_{lA}^{\text{eff}}$ and $g_{pA}^{\text{eff}}$, because of the CP and MEC effects evaluated from microscopic standpoints. Hence $S_{\text{GT}}$ is not exactly proportional to $(N - Z)$. Figure 1 depicts the $S_{\text{GT}}$ values obtained in the present microscopic calculation. It turns out that the proportionality of $S_{\text{GT}}$ to $(N - Z)$ is still maintained to an excellent approximation. Quenching of $S_{\text{GT}}$ is indicated, resulting in 68% of the bare value as a general trend. The value of $(g_{A}^{\text{eff}}/g_{A})^{2}$ is almost enough to account for this quenching.

There have been attempts to extract total GT strengths from charge-exchange (CX) reaction data like $(p, n)$ and $(n, p)$\[14\]. The calculated total GT strengths are compared with the available experimental values of this kind\[15, 16\] in Table 3. Note that $g_{A}$ is included in the GT operator (2), associated with Equation (3). Most of the calculated $\sum B(\text{GT}^{-})$ values are consistent with the data, although they are around the upper bound of the errors of the experimental data. For $\sum B(\text{GT}^{+})$, the present calculation gives somewhat larger values than the experiments. This seems contradictory to the tendency of slight underestimate of the individual low-lying GT strengths. Possible reasons might be:

(i) The GT strength might not fully be detected in the CX experiments. The level density is quite high in the GT resonance region of the middle $pf$-shell, which may make it difficult to measure all the GT strength in this region. In addition, a sizable amount of strength within the $0\hbar\omega$ space could be fragmented beyond the energy under observation. There might be an ambiguity in the background subtraction from the CX data, as is argued for $^{48}\text{Ca}$\[17\].

(ii) The CP and MEC mechanisms could somewhat differ between the weak processes (i.e., $\beta$-decay and electron capture) and the CX reactions, although it is not likely that this gives rise to a big quantitative difference.
There might be a problem with the present shell-model wavefunctions. However, the underestimate in the individual $B(GT)$ suggests that the present wavefunctions and operator involve too much leakage out of the $0f_{7/2}$ orbit, whereas the total $B(GT)$ appears opposite. It is not obvious whether there are any better sets of wavefunctions to dissolve this conflict. One might worry about the influence of the $k = 3$ and 4 configurations on the final-state wavefunctions, which is taken into account for the total GT$^-$ strengths, while neglected for the low-lying strengths. Notice that, however, the major conflict is present in the GT$^+$ strengths, for which we always consider the $k \leq 2$ model space, regardless of the total or the low-lying strengths. It should be mentioned here that the total $B(GT^+)$ values measured in the CX experiments are almost reproduced by a recent full $pf$-shell calculation by the shell-model Monte-Carlo (SMMC) method\cite{11} with the so-called KB3 interaction, despite the difficulty in performing precise spectroscopic tests by the SMMC. Further investigation will be required.

A systematics of the total GT$^+$ strengths has been suggested recently\cite{18}, based on the CX data. It has been pointed out that the summed GT$^+$ strengths are nearly proportional to $(Z - 20) \cdot (40 - N)$. Apart from the difference in absolute value between the present calculation and the CX data, we here test whether or not this systematics is adaptable to the present shell-model results. As is shown in Figure 4, the present results also obey the $(Z - 20) \cdot (40 - N)$ systematics to a good approximation. The main difference between the data and the present calculation is in the coefficient of $(Z - 20) \cdot (40 - N)$. This fact suggests nearly a constant percentage with which the GT$^+$ strengths are overlooked in the experiments or overestimated in the calculation. The calibration may probably be done by a single factor, independent of nuclide, in this region.
5. Isospin partition of $\text{GT}^-$ strengths

While final states of the $\text{GT}^+$ transition are constrained to the lowest isospin, the $\text{GT}^-$ transition strength yields an isospin distribution, owing to the neutron excess. A certain interest is being attracted in the isospin distribution of the $\text{GT}^-$ strength. As mentioned before, the GT strength with a specific isospin transfer may be key to some topical problems. Recently, high-resolution CX experiments have started to give quantitative information on the $\text{GT}^-$ strengths of each isospin component\([4]\). Although there have been a few shell-model calculations\([15]\), no systematic study has yet been performed on this issue. We next turn to a comprehensive study on the isospin distribution of the $\text{GT}^-$ strengths.

In discussing the isospin composition of the $\text{GT}^-$ strength, a simple argument based on the isospin algebra has often been employed. If we ignore effects of the shell structure, the isospin partition is ruled by the square of the isospin Clebsch-Gordan (CG) coefficients \(\langle T_0 T_0 1 - 1|T_f T_0 - 1\rangle^2\)[19],

\[
\sum B(\text{GT}^-; T_0 - 1) : \sum B(\text{GT}^-; T_0) : \sum B(\text{GT}^-; T_0 + 1) = \frac{2T_0 - 1}{2T_0 + 1} : \frac{1}{T_0 + 1} : \frac{1}{(T_0 + 1)(2T_0 + 1)},
\]

(4)

where \(T_0\) denotes the isospin of the initial state and \(\sum B(\text{GT}^-; T_f)\) represents summed GT strengths with the final isospin \(T_f(= T_0 - 1, T_0, T_0 + 1)\). Although this argument yields qualitative explanation on some points, such as the dominance of the \(T_f = T_0 - 1\) component in heavy nuclei, it seems too simple for quantitative description in the middle \(pf\)-shell and lighter region. We shall look into the relative strengths of each isospin component, which have not been investigated well by realistic calculations.

In order to compute the relative strengths, we first generate the $\text{GT}^-$ state exhausting the transition strength from the initial state; \(|\text{GT}^-\rangle \propto T(\text{GT}^-)|i\rangle\), where \(|i\rangle\) stands for the initial state. In this process, the \(k \leq 3\) space is required for the...
GT$^-$ state generated from the $N = 28$ isotones and the Ni isotopes, while the $k \leq 4$ space should be considered for the other $N = 29$ and 30 isotones. Invoking the isospin-projection technique, we separate each isospin component from the GT$^-$ state. The relative strengths are obtained by the probabilities of the respective isospin component in $|GT^-\rangle$.

In general, the shell structure increases the relative strength of the low isospin component, as will be clarified by the following argument. We shall take $^{54}$Fe as an example, at first. The ground state of $^{54}$Fe ($J^P = 0^+, T_0 = 1$) has the $(\pi 0f_{7/2})^6(\nu 0f_{7/2})^8$ configuration to the first approximation. The GT$^-$ transition produces the $(\pi 0f_{7/2})^7(\nu 0f_{7/2})^7$ and $(\pi 0f_{7/2})^6(\pi 0f_{5/2})^1(\nu 0f_{7/2})^7$ configurations. It is impossible, however, to form a $T_f = T_0 + 1 = 2$ state with the $(\pi 0f_{7/2})^7(\nu 0f_{7/2})^7$ configuration. Moreover, there is no $1^+$ state with $T_f = T_0 = 1$ in the $(\pi 0f_{7/2})^7(\nu 0f_{7/2})^7$ configuration. The $T_f \geq T_0$ strengths are reduced by these mechanisms, indicating an enhancement of the relative strength of the $(T_0 - 1)$ component. A similar discussion has been given for $^{58}$Ni in Reference [4]. The suppression of the $(T_0 + 1)$ fraction is expected also for other nuclei, not restricted to the middle pf–shell region, via the same mechanism; $T_f = T_0 + 1$ is forbidden in the configuration where the proton having been converted by the GT$^-$ transition occupies the same orbit as the initial neutron. For the $N = 28$ isotones, the $T_0$ fraction is suppressed to a certain extent, but less severely than the $(T_0 + 1)$ one; for $T_f = T_0$, one or two angular momenta are not allowed in some configurations with low seniority. For the $Z < 28 < N$ nuclei including most of the $N = 29$ and 30 isotones, an argument about $k$, the nucleon number excited out of $0f_{7/2}$ (see Equation [4]), clearly accounts for the shell-structure effect. The initial state, namely the ground state of the parent nucleus, is dominated by the $k = 0$ configuration. The GT$^-$ transition from this lowest configuration generates the $k = 0$, 1 and 2 configurations of the daughter nucleus. Note that the daughter nucleus has
\[ Z \leq 28 \leq N. \] With the \( k = 0 \) configuration, where the neutron \( 0f_{7/2} \) orbit is fully occupied, only the lowest isospin can be formed. Thus \( T_f \) is restricted only to \((T_0 - 1)\) in the \( k = 0 \) configuration. In the \( k = 1 \) configuration, the maximum possible isospin is \( T_f = T_0 \). The lack of the \( k = 0 \) and \( 1 \) configurations hinders the \((T_0 + 1)\) component greatly, and that of the \( k = 0 \) configuration suppresses the \( T_0 \) fraction moderately. As a result, the \((T_0 - 1)\) fraction becomes larger than estimated from Equation (4).

This qualitative expectation is confirmed in Figure 3, by the present realistic calculation. In comparison with Equation (4), the low isospin component has an enhancement in its relative strength in any nucleus. This trend is also consistent with the CX experiment in \(^{58}\text{Ni}\). In Figure 4, the isospin partitions in several different model spaces are compared. Typical examples are taken from an even-even \(^{56}\text{Fe}\), a proton-odd \(^{53}\text{V}\) and a neutron-odd \(^{55}\text{Fe}\) nuclei. The space \( A \) is defined so that the initial-state wavefunction should be comprised only of the \((0f_{7/2})^{n_1}(1p_{3/2})^{n_2}\) configuration, where \( n_1 \) and \( n_2 \) are defined below Equation (4). The space \( B \) implies the \( k = 0 \) space for the initial state, whose wavefunction is obtained by diagonalizing the Kuo-Brown hamiltonian. The space \( C \) indicates the present \( k \leq 2 \) model space, for the initial-state wavefunction. All possible configurations are considered for the final states. The total \( GT^- \) strengths appreciably depend on the size of model space. The difference between the spaces \( A \) and \( B \) is negligibly small, as far as the same GT operator is employed. The total \( B(GT^-) \) value decreases considerably if we use Towner’s parameters, into which the effects outside the \( 0\hbar \omega \) space are incorporated. The \( k > 0 \) configurations reduce \( \sum B(GT^-) \) further, to about 60% of the value obtained in the space \( A \) with the bare operator. It turns out, on the other hand, that the relative strengths are quite insensitive to the truncation of the model space. Although the isolation of the \( 0f_{7/2} \) orbit is relaxed as enlarging the model space from \( A \) to \( C \), the shell-structure effect on the relative strength does not seem to become weaker. The simplest wavefunctions in the space \( A \)
provides us with a seed of the shell effect, and their isospin composition is preserved in the higher configurations to be mixed, to a great extent. This suggests that, if the total \( GT^- \) strength is known, we can get a sound and stable prediction on the \( GT^- \) strengths of definite isospin component from wavefunctions with simple configurations. It is also confirmed in the space \( B \) that the modification of the \( GT \) operator from the bare one hardly influences the relative strength.

Since we are dealing with as many as 24 nuclei in the region \( 20 < Z \leq 28 \leq N \leq 30 \), it is possible to study systematics of the summed \( GT^- \) strengths for each isospin component, based on the present shell-model calculation. The principally active orbit in the ground state is \( 0f_{7/2} \) for protons, while \( (0f_{5/2}1p_{3/2}1p_{1/2}) \) for neutrons. Therefore the systematics concerns multiple orbits, not dominated only by \( 0f_{7/2} \). The systematics of \( S_{GT} \) and \( \sum B(GT^+) \) have been discussed already. The \( \sum B(GT^-; T_0 + 1) \) values are related to the \( GT^+ \) strengths through the isospin algebra. We here discuss the systematic behavior of \( \sum B(GT^-; T_0 - 1) \) and \( \sum B(GT^-; T_0) \). It is remembered that the present calculation yields the same \((Z, N)\)-dependence for \( \sum B(GT^+) \) as found in the CX data, besides the overall coefficient. Whether there is a missing of the summed \( GT \) strength in the measurement or an overcounting in the present calculation, the study of systematics will be useful in calibrating either of them, as well as in speculating the summed \( GT \) strengths of surrounding nuclei. One might notice that not all of the four quantities, \( S_{GT}, \sum B(GT^+), \sum B(GT^-; T_0 - 1) \) and \( \sum B(GT^-; T_0) \), are independent. Only three of them can be independent, because \( S_{GT} \) is equal to \( \sum B(GT^-) - \sum B(GT^+) \) by definition, and \( \sum B(GT^-; T_0 - 1) = \sum B(GT^+)/[(T_0 + 1)(2T_0 + 1)] \). It is, however, worth studying their systematics individually, since the systematics is a matter of approximation. Apart from \( S_{GT} \), the systematics is investigated in an empirical manner at present, and its origin is not yet obvious. Moreover, there is a notable difference in magnitude among these quantities, as will be shown below. Thus the precision of the systematics may depend
on the quantity under discussion.

The summed GT$^-$ strengths for the $T_f = T_0 - 1$ and $T_0$ components are presented in Figure 5. The $(T_0 - 1)$ strengths are almost proportional to $2T_0 = N - Z$, with a trivial exception of the $T_0 = 1/2$ case where $T_f = T_0 - 1$ is forbidden. It is also found that the $\sum B(\text{GT}^-; T_0 - 1)$ values are close to $S_{\text{GT}}$. The deviation of $\sum B(\text{GT}^-; T_0 - 1)$ from $S_{\text{GT}}$ is less than 10% in most cases. This seems to be driven, to an appreciable extent, by a cancellation between the GT$^-$ strengths with $T_f \geq T_0$ and the GT$^+$ strength. On the other hand, the summed $T_f = T_0$ strengths behave in quite a different manner. The $\sum B(\text{GT}^-; T_0)$ value decreases as $T_0$ increases, and is roughly proportional to $1/(T_0 + 1)$, which is just the squared isospin CG coefficient. There should be some shell effects in the $T_f = T_0$ component, as is stated above.

The systematics of $\sum B(\text{GT}^-; T_0)$ suggests that the shell effects preserves the isospin dependence of the squared CG coefficients approximately. In this sense, the shell effects on $\sum B(\text{GT}^-; T_0)$ seem to contribute nearly uniformly in this region. It is noted that the $\sum B(\text{GT}^-; T_0 + 1) = \sum B(\text{GT}^+)/[(T_0 + 1)(2T_0 + 1)]$ has another $(Z, N)$ dependence, different from the strengths of the $(T_0 - 1)$ and $T_0$ components.

According to the systematics, the summed strengths $\sum B(\text{GT}^-; T_0 - 1)$ and $\sum B(\text{GT}^-; T_0)$ in this region are functions only of $T_0$, to a relatively good approximation. Since $\sum B(\text{GT}^-; T_0 + 1)$ amounts to less than 10% of the total GT$^-$ strength in the $T_0 \geq 1$ cases, the total strength is also a function of $T_0$ to a rough approximation, although the function-form may be somewhat complicated. As has been mentioned in connection with Equation (4), the $T_f = T_0 - 1$ component is dominant for large $T_0$. It can be shown from the systematics that the dominance of the $(T_0 - 1)$ component grows even more rapidly than estimated from Equation (4), as $T_0$ increases. The isospin algebra (Equation (4)) implies that the ratio $\sum B(\text{GT}^-; T_0 - 1)/\sum B(\text{GT}^-; T_0)$ rises in $O(T_0)$ for increasing $T_0$. The systematics implies, on the other hand, that this ratio goes up
in $O(T_0^2)$. Note that the shell-structure effect is incorporated into the simple functions representing the systematics, in an effective manner. In comparison with the estimate based on the isospin algebra, the systematics implicates that the shell-structure effect becomes the stronger for the larger $T_0$, and gives rise to the faster domination of the low-isospin component.

Whereas the total GT strengths and their isospin distribution have been studied systematically, the energy distribution of the GT strengths is left beyond the scope of this article, except for the decay problems shown in Table [1] and [2]. This is because, despite the dispersion of the GT strength over the $k = 3$ and 4 configurations, spectroscopic test in the enlarged (i.e., $k \leq 3$ or $k \leq 4$) space has not been satisfactory yet. A correction to the energy will be required, when we take into account the admixture of the $k = 3$ and 4 configurations. We do not know a good way to do it at present, and this point is left as a future problem. It should be emphasized that, as stated already, the total GT strength and their isospin partition depend only on the ground-state wavefunction, not concerning the energy correction.

6. Summary

GT transitions in $N = 28 \sim 30$ nuclei have been investigated from a fully microscopic standpoint. For individual low-lying GT strengths, we have reproduced the relatively large $B$(GT) values within 70% accuracy except for $^{56}$Ni and $^{57}$Ni. While the calculated total GT$^-$ strengths are in agreement with the measured ones within the range of experimental errors, the calculated total GT$^+$ strengths are somewhat larger than those extracted from the CX experiments. Although the GT strengths are calculated with Towner’s parameters incorporating medium effects, the Ikeda sum-rule survives with a quenching factor of 0.68. From the calculated GT$^-$ strengths for respective $T_f$ component, it is confirmed that low $T_f$ component gathers larger relative strength than
expected by the squared isospin CG coefficients, because of a shell effect. Intriguingly, it turns out that this shell-structure effect is hardly influenced by the size of model space; the relative strength is insensitive to the extent of the excitation out of $0f_{7/2}$.

The systematics of the summed $B$(GT) values has also been discussed. According to the study on the $\sum B$(GT$^+$) systematics, the discrepancy in the total GT strengths between the present calculation and the CX experiments seems to emerge with nearly a constant percentage. The $T_0$ dependence of $B$(GT$^-$; $T_f$) is quite different among the $T_f = T_0 - 1$, $T_0$ and $T_0 + 1$ components. Rapid growth of the $(T_0 - 1)$ component relative to the $T_0$ one, for increasing $T_0$, is suggested. Although the origin of the systematics is not yet clear enough, this systematics might be useful to predict summed GT strengths in surrounding nuclei without carrying out elaborate computations.

Acknowledgments

The authors are grateful to Dr. Y. Fujita for valuable discussions and comments. They also thank Dr. W. Bentz for careful reading the manuscript. A part of this work has been performed within Project for Parallel Processing and Super-Computing at Computer Centre, University of Tokyo.

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Figure Captions

Figure 1. Calculated $S_{GT}$ values as a function of $(N - Z)$. Plus, circle and diamond symbols stand for $N = 28$, 29 and 30 nuclei, respectively.

Figure 2. Calculated total GT$^+$ strengths as a function of $(Z - 20) \cdot (40 - N)$. See Figure 1 for symbols.

Figure 3. Isospin composition of the GT$^-$ strengths: $T_f = T_0 - 1$ (lightly shaded), $T_f = T_0$ (open) and $T_f = T_0 + 1$ (darkly shaded) probabilities in percentage. Short sticks at the right of each column indicate partitions due to the isospin algebra (Equation (4)).

Figure 4. Model-space dependence of the isospin composition of the GT$^-$ strength for $^{56}$Fe, $^{53}$V and $^{55}$Fe. Each isospin component is expressed by the shaded area as in Figure 3. The labels of the model spaces (A, B and C) are described in the text. The GT operator is the bare operator or the effective operator with Towner’s parameter-set. The cross symbols show the total $B$(GT$^-$) values relative to the value obtained in the space $A$ with the bare operator.
**Figure 5.** Systematic behavior of summed $B(GT^-)$ values for $T_f = T_0 - 1$ and $T_0$ components. Upper: $\sum B(GT^-; T_0 - 1)$ as a function of $2T_0 = N - Z$. Lower: $\sum B(GT^-; T_0)$ as a function of $1/2(T_0 + 1) = 1/(N - Z + 2)$. See Figure [for symbols.}
\[ \sum B(GT) \]

\[ 0.11 \times (Z-20) \times (40-N) \]
