Materials study on a Telescopic Barbell design using Finite Element Model

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Abstract. A telescopic barbell design is proposed, and its design is optimized by evaluating numerous scenarios based on geometry and materials, using COSMOSWorks® (now Solidworks Simulation®). The product is intended for general purpose strength training use and is designed to cater to the portability limitations of existing barbels, which are bulky and lengthy. The objective of this design study is to minimize the cost of manufacturing, by selecting a suitable economical material, provided the maximum deflection (stiffness criteria) and yield stress (strength criteria) is within the design constraints. The decision is not obvious, when strength, stiffness and cost criteria is to be met since high strength materials will weigh less (for the same load) but will cost more and vice versa. Hence, parametric design simulations must be done to choose the most optimum design, which meets the design constraints. Materials from three different steel categories (mild steel, stainless steel, and high strength steel) are selected and a total of 441 scenarios (147 per material) are simulated by parametric geometry alterations. The most economical design, which satisfies both strength and stiffness criteria, is selected and is further analyzed for contact stresses, to ensure these relatively high stresses do not penetrate deep inside the body. It is concluded in this study, with sufficient evidence, that A-36 (mild steel) is still the most economical material for this design, although it has the lowest yield strength, of the materials simulated. This is because the design should also satisfy the stiffness criteria and the Young’s Modulus for all the three grades of steel is very close. It is also concluded that design of barbells can be done based on global maximum stress values, rather than localized contact stresses, because edge/line contact is only present in the software environment, not in the physical service conditions.

1. Introduction
Barbels are an essential part of power lifting, weight training and a variety of other strength training exercises [1]. In fact, it just might be the single most versatile equipment in a gymnasium. It can be used for a verity of muscle groups including thighs, calves, lower back, back, chest, shoulders in several variations. Quite simply, this single device may be enough for a full body workout. International standardization bodies [2], specify the sizing of bars in Olympics and other competitions. The barbels have a gripping area at the center where the user holds the bar (Figure 1), and sleeves at the extremities which hold the weights, added in variations for performing the exercises. The gripping
area has markings which help in holding the barbell in a balanced fashion at various holding distances. As expected, the bars for Men and Women have different sizing and weight as shown in [1]. The bar to be used by Men has a grip diameter and grip length of 28 mm and 1310 mm with a sleeve length of 415 mm. For women’s bar the corresponding dimensions are 25mm, 1310 mm and 320mm. The weight for a standard Men and Women bar is 20kg and 15kg, respectively [3], as shown in Figure 1. A typical power lifting and weightlifting barbel has rotatable sleeves with bearings and bushes, which keeps the loading to be translational rather than torsional.

![Figure 1. Standard dimensions of Men's bar (top) and Women's bar (bottom) as per IWF [2].](image)

2. Literature Survey

To study the science of successful weightlifting, focus is required on both the “lifter” and the “lifted” (athlete and barbell). Numerous studies were found on both the equipment design (beam design) and the optimization of lifting techniques. Two techniques are widely used in Olympic barbell lifting, the “Snatch” technique and the “Clean and Jerk” technique. The “Snatch” involves lifting the barbell in one continuous motion and the “Clean and Jerk” involves breaking down the full lift into two steps. First, lifting the barbell up to shoulders, and then sweeping it up, with the arms straight above the shoulders. Significant literature was found regarding the kinematics of “Snatch” technique and “Clean and Jerk” technique, which provide insights into the science of successful weightlifting. Lester [4] reviewed the literature and the then, current research on the science of performance weightlifting. At first the research focused predominantly on male weightlifters which was addressed by Ykeda [5] and Hoover [6] by focusing on bio-mechanical kinematics of women weightlifters.

Both 2D and 3D data capture, analysis and visualization were improved by Motion capture, Aerial Performance Analysis System which can digitize the points on barbell and the weightlifter, which was utilized by Hadi [7] to analyze the effects of changing loads on the kinematics of the “Snatch” technique. Hua [8] created a 3D finite element model of lumbar spine and analyzed its mechanics and stress distribution while lifting a barbell. The bio-mechanics of weightlifter and the trajectory of barbell while lifting is also studied by Zehr [9] and Dieen [10]. The effects of different lifting postures and barbell type, on muscle activation and ground reaction forces is also studied by Bentley [11] and Malyszek [12]. The structural response in dynamic loading in beams is studied by Providakis [13], Wang [14], and Huang[15] by different numerical methods such as the Boundary Element Method, Dynamic Stiffness Matrix method, and the Fourier series. Instead of using a constant or stepped cross-section, one can also use topologically optimized material to save weight (for a portability focused barbell) as done by Kim [16] specifically for beams. But such a design will be bound by the
manufacturing capability and cost of the end-product. Contact stress in beams is also studied by Blake [17] including the frictional effects and strain hardening. A contact stress solution is also developed by Sankar [18] by combining the classical beam theory with the elasticity solution, using Fourier transforms.

However, the authors could not locate any study which might help visualize the contact stresses, using FEA simulation, which is a very effective tool to help understand the distribution of overall and contact stresses. Studies also could not be located, which address the parametric optimization of material and cost, by considering multiple materials in a barbell design. Parametric geometry optimization clearly must be done before choosing a material for any product to balance the quality and cost of manufacture. A design must be obtained which addresses the important variables in any design, including cost, strength, stiffness, material availability and quality. Such a design study is missing from literature and an effort is made to address the same. It is evident that both stiffness and strength is of utmost importance in the Barbell design since the springiness or flexing of the barbell will directly affect the lift mechanics. In designing a barbel with telescopic mechanism for compactness, an opportunity is utilized to study the contact stresses, predominantly loaded sections and the effect of strength, stiffness, and cost on material selection in this barbell design. Additionally this new product design (NPD) opportunity is also utilized to develop instructional resources to teach product design in a learn by doing approach as proposed by Santosh [19] and Alok [20] to product design students. The importance of collaborative learning and multi-disciplinary approach in engineering education is also highlighted by Venkateswarlu [21] Srivastava [22].

3. Design thinking and conceptualization

3.1. Need Recognition

Barbell is a necessary part of strength training and it assists in working out almost every muscle group. It can be easily accessed in a gym which usually houses various types on barbells, including general purpose, curl bar, deadlift bar, trap bar [23]. A limitation exists with the conventional barbells which creates an “opportunity gap” because of the lack of portability and an effort is made to rectify this quirk.

3.2. Proposed Solution

A two-step solution is proposed to enhance the portability of a conventional barbell. Firstly, a telescopic mechanism is used to reduce the length of the barbell when needed and secondly, mass is minimized (for individual material) by simulating several scenarios based on dimensions and materials. Also, an economical design is of highest priority in this case since this is aimed at the enthusiast market. In fact, a cheaper low strength material will add the weight which will assist in the workout. In this case, the portability is facilitated by the telescopic setup, which reduces the overall length of the barbell.

4. Methods and Materials

4.1 Study methodology

The objective of this study is to minimize the cost since it is designed for general purpose strength training use. Hence three materials are selected for this study, starting from a general-purpose construction steel (A-36) to a specialized alloy steel (AISI-4130) which. A kitchen utensil grade, stainless steel is also simulated for the static load (AISI-304). The cost of AISI-4130 and AISI-304, when compared to A-36 is approximately twice and thrice, respectively.

The dimensions of the barbel are to be comparable, but not identical to the IWF [2] guidelines (as shown in Figure 1), since the focus here is on portability and cost rather than the weight carrying capacity. The design must satisfy both strength and stiffness criteria, while being cost effective, to be successful. So, three priorities should be met in the present study as listed below:
- The global stress (as against local contact stress) must be less than the yield strength of material (strength criteria).
- The maximum resultant deformation of the barbell should be less than 5mm (stiffen criteria).
- The most economical design/material is selected since this is to be a general-purpose product where cost is the priority for consumers.

Since this is to be a general-purpose strength training bar, the maximum weight carrying capacity is to be 100 kg which is assumed to be sufficient for non-professional users. The intent is to design a portable and economical product; hence priority is given to economical materials which meet the maximum deflection and yield strength criteria. However, to do a cost comparison, candidate materials for this study are mild steel, stainless steel, and alloy steel (varying yield strengths). To keep the weight and hence the cost minimal for a material, numerous scenarios are simulated for each material. The design with minimum weight is selected if it meets the strength and stiffness criteria. It is essential to perform a parametric design evaluation to determine if a “high strength high cost” material will be economical (less material usage) or vice versa. From the three designs finalized (one for each material) the most economical design is selected.

Contact stresses arise when the barbell is supported on a rack which effectively provides a narrow contact area (slightly bent barbell) when it is loaded. Hence, localized contact stresses are also examined (using an h-adaptive mesh) and it is ascertained that these are limited to a very miniscule volume compared to the total volume of the barbell itself. This is done to ensure that design does not fail because of excessive contact stress which may produce localized yielding on the surface, denting the barbell. If contact stresses, (more than the yield strength) extend considerably beyond the surface of the barbell, dents may develop, rendering the product unusable.

The design is allowed a maximum resultant deflection of 5 mm and the global stress (as against localized contact stress) to be less than the yield strength of material, under a 1000 N load. The final design selected, based on the economy criteria, is analyzed for contact stresses and it is ascertained that localized contact stresses do not occur in a significant volume of the material. Although the contact stresses are visualized and quantified, these are not used for selection of the material. It is obvious that considering the high localized contact stresses for selection of the material will result in over-designing the product, which apparently will be an expensive choice. So, von Mises stress on the topmost edge of bar and the grip is considered in the parametric design study (Figure 2), for selection of a particular design and material.

4.2. Materials Selected
Materials which have suitable stiffness, yield strength, toughness, and hardness (assumed priority order), are to be chosen for a successful mechanical design. However, only strength and stiffness are quantified in this study. For this application, it is preferable to select the materials which have a smaller gap between yield and ultimate tensile strength, since a bar is practically useless even with a slight permanent bend [1]. Bulk yield stress is considered while calculating the factor of safety (FOS) and not the local contact stress between the bar and the rack of a bench press. The local yielding due to these contact stresses is to be countered by choosing the right hardness, which may prevent localized dents at these points.

4.3. Contact and Boundary conditions
Fixed support is provided at a surface with 40 mm width (Figure 2) on the barbell and 500 N load on the other side, which makes this a cantilever setup. The contact condition between the “grip” and the “bar” as shown in Figure 2 is “bonded” which treats these contacts as if these are welded in the simulation environment.
Figure 2. CAD Geometry of the barbell along with the contact, load, and fixture conditions for one of the cases.

4.4. Geometry simplification
Since the geometry is symmetric, only half of the barbell is modelled which converts the simulation setup into a cantilever problem. A fixed support is applied on one end, which imitates holding the entire barbell at the center over a 100 mm length (50 mm for the half cantilever bar). The collars at either end or rotatable sleeves are not modelled. A load of 500 N is applied at the other end over a surface as shown in Figure 3. This is expected to provide unrestricted bending of the barbell under a 1000 N (total) applied load, which shell provided more accurate deflection results.

4.5. Meshing
For the multi-scenario design study a solid mesh is used with the default settings to minimize the computational time as shown in Figure 3. Numerous scenarios are simulated to shortlist the best design which incorporates the design goals (minimize mass and cost) while satisfying the strength and stiffness criteria. While choosing the optimum design for 3 different materials, local contact stress is
neglected and the selection is done based on the deflection and stress on the top edge of bar and grip as shown in Figure 2. A fixed mesh is suitable for this stage of the study.

![Figure 3](image.png)

**Figure 3.** Barbell simplified as a cantilever beam, with load and support applied. Fixed coarse mesh (top) for parametric design study and adaptive localized refinement (bottom) using h-adaptive method for contact stress visualization.

For the second part of the study, an h-adaptive [27] method of mesh refinement is used which refines the mesh in the area, where higher localized stresses (and hence errors) are observed. The mesh is refined in multiple passes of simulation until the “Strain energy norm error” becomes less than 2%. The h-adaptive method is selected since it not only provides the results with less error, but also refines the mesh in the area of stress concentration, which facilitates the visualization of stress hot spots using iso-plots.

4.6. Simulation setup

Solidworks® Design study is done to find the most economical and light weight design based on multi-criteria parametric modification of geometry and the material. The software changes the “Parameter” in steps within the maximum and minimum value as specified by the user automatically, each time rebuilding the model and meshing it using the same boundary conditions. The parameters selected to be modified are “Grip ID” and “Grip thickness” which can be identified in Figure 2 (and listed in Table 2, which shows one of the cases, which are simulated. The cases which exhibit von Mises stress, greater than the yield strength of material are rejected and the once accepted are compared. Also, the cases where maximum resultant displacement is greater than 5 mm, are rejected and the once remaining are compared. The goal of the design evaluation study is to minimize the mass and hence, the cost while satisfying the design requirements.
Table 2. Variables, Parameters, and constraints involved in the Parametric design study.

| Parameter       | Constraints | Limits and Steps | Number of cases |
|-----------------|-------------|------------------|-----------------|
| Grip Thickness  | -           | Min: 2 mm, Max: 8 mm, Step: 1 mm | 7               |
| Shaft Diameter  | -           | Min: 20 mm, Max: 40 mm, Step: 1 mm | 21              |
| Material        | 3 Materials | A36, AISI-304, AISI-4130 | 3               |
| Maximum Displacement | < 5mm | Stiffness criteria | -               |
| Maximum Stress  | < Yield Strength | Strength criteria | -               |
| Mass            | Monitor only | Used for cost calculation | -               |

Total number of cases 441

5. Results

5.1. Selected outcomes
The parametric design study is carried out and only the insightful results are shown in this paper. Table 3 shows the material, scenarios, and parameters simulated and the cases selected for each material. In AISI-304, which is a kitchen utensil grade stainless steel [28], a “Bar diameter” of 29 mm and grip thickness of 5 mm comes out to be most economical design, of the cases simulated. In case of A-36 (construction grade steel [24]) and AISI-4130 (high strength steel [26]), case 101 came out to be most economical, while meeting the design criteria (Table 3).

Table 3. Selected scenarios which meet the constraints, for optimal cost.

| Material  | Scenario | Parameters                  | Mass (gram) | Cost reference | Unit cost (USD/kg) | Total cost (USD) |
|-----------|----------|------------------------------|-------------|----------------|-------------------|------------------|
| AISI-304 steel | 73       | Bar Diameter: 29 mm         | 7700.56     | [29]           | 16.446            | 126.60           |
| A-36 steel    | 101      | Bar Diameter: 36 mm, Grip Thickness: 6 mm | 7424.34     | [30]           | 6.318             | 46.90            |
| AISI-4130 steel | 101     | Bar Diameter: 36 mm, Grip Thickness: 6 mm | 7424.34     | [31]           | 12.232            | 90.81            |

The scenario number “101” with the material being A-36 is selected to be the most optimum, of the cases simulated. A-35 is a general-purpose construction steel, also being used for bridges and buildings. The bar can be coated with chrome plating for corrosion resistance, which will still produce the most economical design, which is a priority in this product. The maximum von Mises stress (on the top edge of grip and bar) in this case is 47.91 MPa (nodal stress value) which brings the “Factor of Safety” (FOS) down to 5.21, which is well within the design goal. The deflection is 2.05 mm is well within the design limits as well. The mass of the selected design in case of A-36 material is 5.85 kg for which the cost comes out to be 46.90 USD, which is the decisive factor for its selection at this stage. The cost of A-36 steel is 12.23 USD per kg [30]. The corresponding values for the other two materials are 90.81 USD (for 5.85 kg AISI-4130) and 126.64 USD (for 3.58 kg AISI-304). It can be noted that even though the mass of AISI-304 is the least (of the cases simulated), the cost of A-36 still comes out to be the least, because of the cost difference. The cost of AISI-304 is nearly thrice, when
compared to A-36, and twice when compared to AISI-4130 as shown in Table 3. Since the application is general purpose strength training, the added mass may also serve in the workout.

5.2. Convergence Information and stress results
The h-adaptive method, available in Solidworks® simulation, is used to run multiple loops of mesh refinement, to calculate the results with an acceptable error. The h-adaptive method decreases the mesh element size in the area of higher “strain energy norm” error, which represents the change in strain energy, within that area. Option is also available to focus predominantly in the area of higher strain, versus obtaining accurate results for the whole geometry. The latter refines the mesh for the entire geometry, which may be resource intensive.

![h-Adaptive Convergence Graph](image)

Figure 4. Convergence information of the h-Adaptive method with successive mesh refinement.

It can be seen in Figure 4, that in successive mesh refinements, the number of nodes keep increasing while the “strain energy norm” error keeps decreasing, which is obvious since results are calculated at higher number of points in subsequent loops. Also, the localized contact stress value has still not converged, even at loop number 12, even though the “strain energy norm” error is just 1.92 %. At this point, we could run further loops and refine the mesh even further until the von Mises stress converge, but our design is not based on the local contact stresses, hence we can stop at this point.

The mesh is refined till the error is within 2%, to visualize the contact stresses and to ensure that these do not occur in a significant volume (Figure 5). With sufficient localized mesh refinement, it can be observed that contact stresses (more than the yield strength) are in a negligible volume, which can be neglected. Also, Figure 6 shows the highlighted areas where the von Mises stress is greater than 250 MPa. This shows the areas which are subjected to higher stresses and based on this insight, necessary steps can be taken to appropriately reinforce the design or select a different material. Also, the h-adaptive method has reduced the element size in the areas of higher “strain energy norm” error, which facilitates the visualization of dominantly stressed regions.
Figure 5. Local contact stress distribution at the support, which exceed the yield strength of A-36.

Figure 6. ISO curve of the von Mises stress, showing stress areas greater than 250 MPa.

5.3. Displacement results

Figure 7 highlights the maximum resultant deflection in the barbell, which is within the design criteria. The deflection result, as expected converged within the first three loops of the h-adaptive method and does not require much refinement of the mesh size.

Stress on the other hand, require localized refinement in the areas of load and support application, since the stresses tend to vary predominantly in these regions. Deflection of a barbell has major impact on the quality of workout since excessive “whippy” behavior may produce oscillations in the bar which are undesirable [1].
Figure 7. Resultant deflection values in the barbell, also pointing out is the maximum value.

6. Discussion

Any design needs to satisfy at least two criteria to be successful, strength criteria and stiffness criteria. As it can be seen in the finally selected design scenario (case 101, A-36 steel), both the global yield stress and the maximum resultant deflection, is within the design requirements. The complicated choice, of selecting a high strength expensive material, or a low strength economical material, is resolved in this design study for this product. It can be seen that a heavier low strength (comparatively) material comes out to be a preferred choice in this design, because of its service requirements and cost being the primary objective. Also, since it is a weight training equipment, added weight (compared to the high strength material) may assist in the workout. It is also observed in this study that if the service conditions of the product permits, the design can be done based on the overall stress values, rather than the maximum localized contact stress values. As is seen in the contact stress results, the material volume in which the von Mises stress exceeds the yield strength, is negligible and is restricted in the vicinity of the boundary surface.

It should also be pointed out that these intensified contact stresses do not exist physically since a perfect line contact condition, does not exist in a physical environment. Also in the convergence plot, shown in Figure 4, it can be seen that resultant displacement value, starts to converge in loop 3 of the h-adaptive method, while the maximum von Mises stress does not, even in loop 12. But further refinement is not required since the design is selected based on the overall stress values. Hence, only the required stress values can be calculated in a design problem, to save time and resources.

Also, the cost difference between the materials is considerable and the heavier A-36 design (which needs more material for the same load) still comes out to be cheaper than the other two costlier materials. Contact stresses are successfully visualized and are beyond the yield strength, in a negligible volume and are confined in the vicinity of the outer surface. It can also be concluded from the results that choosing a sufficiently ‘hard’ material prevents localized yielding at the support or component contacts, and it is needless to go for an expensive material based on these contact stress values, if these are confined in the vicinity of the surface.

7. Conclusion

The multi-criteria parametric design study is successfully done on a telescopic barbell design and it is concluded that in this category of products, design can be done on the overall maximum stress values rather than the local contact stress values, which enable the use of low strength economical materials. To prevent the material from localized denting, a hard-enough material should be chosen, which can resist the surface nicks and dents. It is also noted, that in this category of products, design can be done
based on a simpler multi-criteria parametric design study, rather than an optimization problem, based on complex analytical modeling. It is also signified, that software packages which provide the functionalities of parametric design evaluation, save a lot of time and resources to arrive at a satisfactory result. The time saved in this method, can be spent in doing extensive research on the product materials, components, manufacturing, assembly, procurement, and marketing which are also indispensable part of product design and development.

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