TIME AT THE ORIGIN OF THE UNIVERSE: 
FLUCTUATIONS BETWEEN TWO POSSIBILITIES

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Abstract. A variation of Hawking’s idea about Euclidean origin of a non-
singular birth of the Universe is considered. It is assumed that near to zero
moment \( t = 0 \) fluctuations of a metric signature are possible.

1. Introduction

The time in the modern Lorentz - invariant physics is connected with a
metric signature. The spacetime metric for any space can be written as

\[
ds^2 = g_{\mu\nu}dx^\mu dx^\nu = e^a e^b \eta_{ab} = (h^a_\mu dx^\mu) (h^b_\nu dx^\nu) \eta_{ab}
\] (1)

where \( g_{\mu\nu} = h^a_\mu h^b_\nu \eta_{ab} \) is the metric, \( e^a = h^a_\mu dx^\mu \) is 1-form, \( h^a_\mu \) is vier-bein, \( a \) is vier-bein index, \( \mu \) is the coordinate index and \( \eta_{ab} \) is the metric signature
\( \eta_{ab} = \text{diag}\{\sigma, 1, 1, 1\} \). An undefined number \( \sigma \) can be +1 for the Euclidean space and −1 for the Lorentzian spacetime. We see that the difference
between Euclidean and Lorentzian spacetimes is connected with the sign of
\( \sigma = \eta_{00} = \pm 1 \). For \( \eta_{00} = +1 \) we say that there is the Euclidean space and
for \( \eta_{00} = -1 \) the Lorentzian spacetime.

It easy to see that the metric \( g_{\mu\nu} \) has two different degrees of freedom: vier-bein \( h^a_\mu \) and the metric signature \( \eta_{ab} \). \( h^a_\mu \) can be determined
from Einstein’s equations but for the metric signature \( \eta_{ab} \) we have not any
dynamical equations. We put in the metric signature \( \eta_{ab} \) by hand into Ein-
stein’s equations (in the vier-bein formalism). Of coarse, we can determine
the true value of \( \eta_{ab} \) from experiments: in our Universe \( \eta_{ab} = (-1, 1, 1, 1) \).
But on the quantum level (on the Planck level) we can assume that \( \eta_{ab} \) is
fluctuating quantity

\[ \text{diag}(-1,1,1,1) \rightleftharpoons \text{diag}(+1,1,1,1). \]  \hspace{1cm} (2)

Cosmological solutions of Einstein’s equations with ordinary matter have almost without exception a cosmological singularity. The existence of such singularity is one of the most significant problem in the modern physics. There are various approaches to the solution of this problem. Hawking’s point of view [1] is that at the origin of time an Euclidean space emerged from Nothing and the metric signature near to zero moment \( t = 0 \) has the Euclidean value \( \eta_{ab} = \text{diag}(+1,1,1,1) \) but after short time interval (\( \approx t_{Pl} \)) the signature undergoes a quantum jump to the Lorentzian value \( \eta_{ab} \rightarrow \text{diag}(-1,1,1,1) \). Another words: in order to kill singularity we must kill the time. Thus we have the following picture for Hawking’s nonsingular Universe (see, Fig.1). The idea presented here is that near to zero moment there are quantum fluctuations between Euclidean and Lorentzian signatures and after short time (\( \approx t_{Pl} \)) the fluctuations cease and the metric comes to the state with the definite (Lorentzian) metric signature (see, Fig.2).

We can assume that in quantum gravity can be various sort of quantum fluctuations

1. The fluctuations of the metric.
2. The topology fluctuations. This phenomenon is known as a hypothesized spacetime foam.
3. The fluctuations of the metric signature.
4. ... 

Here I will consider only the third kind of quantum gravitational fluctuations. One can say that two different approaches to the problem of the metric signature fluctuations are possible:

1. One can solve the Einstein’s equations with undefined \( \sigma = \pm 1 \) and then we will have fluctuating quantity \( \sigma \) in the solution.
2. Another approach is that every Einstein’s equation (for example, $R_{00} - \frac{1}{2}g_{00}R = 0$) with $\sigma = +1$ and $\sigma = -1$ can have different probability. We can interpret the second approach as follows: in fact Einstein’s equations are some algorithm for calculations of the metric in whole space. In this approach we have an algorithm some parts of which can fluctuate (see, Fig. 3).

We assume that the probability of every version of every Einstein’s equation is connected with a “complexity” of the equation. **What is it the “complexity”?** Of course intuitively it is clear. Here we can recall Einstein’s statement that “Everything should be made as simple as possible, but not simpler.” Our proposal is that the “complexity” is connected with Kolmogorov’s ideas on algorithmic complexity (AC). In this approach any physical system (e.g. the Universe) can be thought of in terms of an algorithm. The longer and more complex the algorithm, the less likely it is for such a system to appear. In particular Universes with different physical laws (field equations) are described by different algorithms. The length of these algorithms then affects the probability that this Universe with a certain set of physical laws will fluctuate into existence. In our case we will search such combination of the Einstein’s equations with the different $\sigma = \pm 1$ so that the solution will be the simplest. At first we will give an exact definition for the complexity.

2. **Kolmogorov’s algorithmic complexity**

The mathematical definition for algorithmic complexity (AC) is

The algorithmic complexity $K(x \mid y)$ of the object $x$ for a given object $y$ is the minimal length of the “program” $P$ that is written as a sequence of the zeros and ones which allows us to construct $x$ starting from $y$:

$$K(x \mid y) = \min_{A(P, y) = x} l(P)$$

(3)
$l(P)$ is length of the program $P$; $A(P,y)$ is the algorithm for calculating an object $x$, using the program $P$, when the object $y$ is given.

3. The 5D Fluctuating Universe

Here I would like to consider the scenario where at the origin of the Universe fluctuations between Euclidean and Lorentzian metrics occur [2] [3]. We start with a vacuum 5D Universe with the metric

$$ds^2(5) = \sigma dt^2 + b(t) (d\xi + \cos \theta d\varphi)^2 + a(r)d\Omega^2 + r_0^2 e^{2\psi(t)} [d\chi - \omega(t) (d\xi + \cos \theta d\varphi)]^2$$

(4)

here $\sigma = \pm 1$ for the Euclidean and Lorentzian signatures respectively. According to the appropriate theorem the multidimensional metric in Eq. (4) has the following electromagnetic potential

$$A = \omega(t) (d\xi + \cos \theta d\varphi) = \frac{\omega}{\sqrt{b}} e^1$$

(5)

which yields an electrical field $E_1$ and a magnetic field $H_1$ like

$$E_1 = F_{01} = \frac{\dot{\omega}}{\sqrt{b}}$$

$$H_1 = \frac{1}{2} \epsilon_{1jk} F_{jk} = -\frac{\omega}{a}$$

(7)

The 5D, vacuum Einstein equations resulting from Eq. (4) are

$$G_{00} \propto 2 \frac{\dot{b} \dot{\psi}}{b} + 4 \frac{\dot{a} \dot{\psi}}{a} + \frac{\ddot{a}}{ab} + \frac{a^2}{a^2} + \sigma \left( \frac{b}{a^2} - \frac{4}{a^2} \right) +$$

$$r_0^2 e^{2\psi} (\sigma H_1^2 - E_1^2) = 0,$$

(8)

$$G_{11} \propto 4 \ddot{\psi} + 4 \dot{\psi}^2 + 4 \frac{\ddot{a}}{a} + \frac{\dot{a} \dot{\psi}}{a} + \sigma \left( \frac{3b}{a^2} - \frac{4}{a^2} \right) -$$

$$\frac{\dot{a}^2}{a^2} + r_0^2 e^{2\psi} (\sigma H_1^2 - E_1^2) = 0,$$

(9)

$$G_{22} = G_{33} \propto 4 \ddot{\psi} + 4 \dot{\psi}^2 + 2 \frac{\ddot{b}}{b} + 2 \frac{\dot{b} \dot{\psi}}{b} - \frac{b^2}{b^2} + 2 \frac{\ddot{a}}{a} +$$

$$2 \frac{\dot{a} \dot{\psi}}{a} + \frac{\ddot{a}}{ab} - \frac{\dot{a}^2}{a^2} - \frac{b}{a} - r_0^2 e^{2\psi} (\sigma H_1^2 - E_1^2) = 0,$$

(10)

$$R_{55} \propto \ddot{\psi} + \dot{\psi}^2 + \frac{\ddot{a}}{a} + \frac{\dot{a} \dot{\psi}}{2b} + \frac{\ddot{b}}{2b} + r_0^2 e^{2\psi} (\sigma H_1^2 + E_1^2) = 0,$$

(11)

$$R_{25} \propto \ddot{\omega} + \dot{\omega} \left( \frac{\dot{a}}{a} - \frac{b}{2b} + 3 \dot{\psi} \right) - \frac{\sigma b}{a^2} \omega = 0$$

(12)
The whole of algorithm is the concatenation of $G_{00} \times G_{11} \times \ldots \times R_{25}$ equations but in our approach every piece of the algorithm (for example $G_{00}$) can fluctuate (here $G_{\bar{A}\bar{B}} = R_{\bar{A}\bar{B}} - \frac{1}{2} \eta_{\bar{A}\bar{B}} R$ is the Einstein tensor). Our basic assumption is that at the Planck scale there can exist regions where quantum fluctuations between Euclidean and Lorentzian metric signatures occur. There are two copies of the classical equations (12): one with $\sigma = +1$ and another with $\sigma = -1$. The basic question under our assumption is how to calculate the relative probability for each pair of equations from (12) (the ones with $\sigma = +1$ versus the ones with $\sigma = -1$).

We will define the probability for each pair of equations in terms of the AC of each pair. We can diagrammatically represent the fluctuations between the Euclidean and Lorentzian versions of Einstein’s equations in the following way

$$\begin{align*}
\sigma = +1 & \quad \longleftrightarrow \quad \sigma = -1 \\
(G^+)_{00} & \quad \longleftrightarrow \quad (G^-)_{00} \\
(G^+)_{11} & \quad \longleftrightarrow \quad (G^-)_{11} \\
(G^+)_{22} & \quad \longleftrightarrow \quad (G^-)_{22} \\
(G^+)_{33} & \quad \longleftrightarrow \quad (G^-)_{33} \\
(R^+)_{55} & \quad \longleftrightarrow \quad (R^-)_{55}
\end{align*}$$

(13)

The signs ± indicates if the equation belongs to the Euclidean or Lorentzian mode. Expression (13) sums up the idea that treating $\sigma$ as a quantum quantity leads to quantum fluctuations between the classical equations: $(R^+)_{\bar{A}\bar{B}} \leftrightarrow (R^-)_{\bar{A}\bar{B}}$ or $(G^+)_{\bar{A}\bar{B}} \leftrightarrow (G^-)_{\bar{A}\bar{B}}$. The probability connected with each pair of equations $(R^\pm_{\bar{A}\bar{B}}$ or $G^\pm_{\bar{A}\bar{B}}$) is determined by the AC of each equation.

**Fluctuation** $(R^+)_{25} \leftrightarrow (R^-)_{25}$. The $R_{25}$ equation in the Euclidean and Lorentzian modes is respectively

$$\ddot{\omega} + \omega \left( \frac{\dot{a}}{a} - \frac{\dot{b}}{2b} + 3\dot{\psi} \right) - \frac{b}{a^2} \omega = 0,
\quad (14)$$

$$\ddot{\bar{\omega}} + \bar{\omega} \left( \frac{\dot{a}}{a} - \frac{\dot{b}}{2b} + 3\dot{\psi} \right) + \frac{b}{a^2} \omega = 0.
\quad (15)$$

Let us consider the $\psi = 0$ case (below we will see that this is consistent with the $R_{55}$ equation). It is easy to see that Eq. (14) can be deduced from the instanton condition

$$E^2_1 = H^2_1 \quad \text{or} \quad \frac{\omega}{a} = \pm \frac{\dot{\omega}}{\sqrt{b}}
\quad (16)$$
The second equation (15) does not have a similar simplification via the instanton condition (16). Based on this simplification from a second order equation (14) to a first order equation (16) we consider the Euclidean equation (14) simpler from an algorithmic point of view than the Lorentzian equation (15). To a first, rough approximation we can take the probability of the Euclidean mode as \( p^+_{25} \approx 1 \) and for the Lorentzian mode as \( p^-_{25} \approx 0 \).

**Fluctuation** \((R^+)_{55} \leftrightarrow (R^-)_{55}\). The \(R_{55}\) equation in the Euclidean and Lorentzian modes is respectively

\[
\ddot{\psi} + \dot{\psi}^2 + \frac{\dot{\psi}}{a} + \frac{\ddot{\psi}}{b} + \frac{r_0^2}{2} e^{2\psi} \left( H^2_1 + E^2_1 \right) = 0, \quad (17)
\]

\[
\ddot{\psi} + \dot{\psi}^2 + \frac{\dot{\psi}}{a} + \frac{\ddot{\psi}}{b} + \frac{r_0^2}{2} e^{2\psi} \left( -H^2_1 + E^2_1 \right) = 0. \quad (18)
\]

The Lorentzian mode (18) has a trivial solution

\[
\psi = 0 \quad (19)
\]

provided the instanton condition \((i.e. \ H^2_1 = E^2_1)\) holds and describes the “frozen” 5 coordinate. Thus for this equation we take the Lorentzian mode as having a smaller AC, and in the contrast with the previous subsection, the Lorentzian mode has the greater probability. Again to a first, rough approximation the probability of the Euclidean mode is \( p^+_{55} \approx 0 \) and consequently for the Lorentzian mode \( p^-_{55} \approx 1 \).

**Fluctuation** \((G^+)_{11} \leftrightarrow (G^-)_{11} \) and \((G^+)_{22} \leftrightarrow (G^-)_{22}\). Taking into account (19) we can write these equations as

\[
4 \dot{a} - \frac{\sigma}{a} b - \frac{\sigma}{a^2} b^2 + \frac{\sigma}{ab} a^2 - \frac{\sigma}{a} b^2 = 0, \quad (20)
\]

\[
2 \ddot{b} - \frac{\sigma}{b} b^2 + \frac{\sigma}{\sigma} b^2 - \frac{\sigma}{ab} a^2 - \frac{\sigma}{a^2} b^2 = 0. \quad (21)
\]

For the Euclidean mode \((\sigma = +1)\) with the instanton condition (16) one can have \(b = a\) (an isotropic Universe) which reduces the two equations of (20) - (21) to only one equation

\[
4 \dot{a} - \frac{\sigma}{a} b - \frac{\sigma}{a^2} b^2 = 0. \quad (22)
\]

\[
\]
For the Lorentzian mode ($\sigma = -1$) $b \neq a$ (an anisotropic Universe) there are still two equations

$$4 \frac{\ddot{a}}{a} - \left( 3 \frac{b}{a^2} - \frac{4}{a} \right) - \frac{\dot{a}^2}{a^2} - r_0^2 \left( H_1^2 + E_1^2 \right) = 0, \quad (23)$$

$$2 \frac{\ddot{b}}{b} - \frac{\ddot{b}^2}{b^2} + 2 \frac{\dot{a}^2}{ab} + \frac{\ddot{a}^2}{a^2} + \frac{b}{a^2} + r_0^2 \left( H_1^2 + E_1^2 \right) = 0, \quad (24)$$

Thus under the instanton condition (16) and $\psi = 0$ we find that the Euclidean mode (22) effectively reduces to one equation which corresponds to an isotropic Universe; the Lorentzian mode (23) - (24) still has two equations which describe an anisotropic Universe. Thus we assign the Euclidean mode the smaller AC and as for the previous equations make the rough approximation $p_{11}^+ \approx 1$ for the Euclidean mode, $p_{11}^- \approx 0$ for the Lorentzian mode.

\textit{Fluctuation} $(G^+)_{00} \leftrightarrow (G^-)_{00}$. The equation $G^\pm_{00} = 0$ has the following form

$$2 \frac{b \dot{\psi}}{b} + 4 \frac{\dot{a} \dot{\psi}}{a} + 2 \frac{\dot{a} b}{ab} + \frac{\dot{a}^2}{a^2} + \sigma \left( -\frac{4}{a} + \frac{b}{a^2} \right) + r_0^2 e^{2\psi} \left( \sigma H_1^2 - E_1^2 \right) = 0 \quad (25)$$

Assuming all the previous conditions (the instanton condition, $\psi = 0$, and $b = a$) the Euclidean mode equations become

$$\frac{\dot{a}^2}{a^2} - \frac{1}{a} = 0 \quad (26)$$

while the Lorentzian mode equations become

$$3 \frac{a^2}{a^2} + \frac{1}{a} - r_0^2 \left( H_1^2 + E_1^2 \right) = 0. \quad (27)$$

The instanton condition again implies that the Euclidean mode has a smaller AC. Thus to a first, rough approximation we take $p_{00}^+ \approx 1$ and $p_{00}^- \approx 0$.

4. Mixed system of the equations.

Under the approximation where the probability associated with each of the equations in (12) is $p \approx 0$ or 1 the \textit{mixed} system of equations which describe
a Universe fluctuating between Euclidean and Lorentzian modes

\[
\frac{\dot{a}}{a^2} - \frac{1}{a} = 0, \quad (28)
\]

\[
\dot{\omega} = \pm \frac{\omega}{\sqrt{a}}, \quad (29)
\]

\[
4\ddot{a} - \frac{\dot{a}^2}{a} - \frac{1}{a} = 0. \quad (30)
\]

Here \(b = a, \psi = 0\) and the instanton condition are all assumed to hold. This system of mixed Euclidean and Lorentzian equations has the following simple solution

\[
a = \frac{t^2}{4}, \quad (31)
\]

\[
\omega = t^2. \quad (32)
\]

We can interpret a small piece (with linear size of the Planck length \(\approx l_{Pl}\)) of our model 5D Universe as a quantum birth of the regular 4D Universe.

5. Conclusions

In this talk we have considered the possibility that Nature can have changing the physical laws. We have postulated that the dynamics of this changing may be connected with the AC of a particular set of laws. This leads to the proposition that an object with a smaller AC has a greater probability to fluctuate into existence. Some physical consequences that can results from this hypothesized fluctuation of physical laws at the Planck scale are:

- the birth of the Universe with a fluctuating metric signature;
- the transition from a fluctuating metric signature to Lorentzian one;
- “frozen” 5th dimension as a consequence of this transition.

References

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