A comparison of two operational wave assimilation methods

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Abstract

A comparison is carried out between two operational wave forecasting/assimilation models for the North Sea, with the emphasis on the assimilation schemes. One model is the WAM model, in combination with an optimal interpolation method (OIP). The other model, DASWAM, consists of the third generation wave model PHIDIAS in combination with an approximate implementation of the adjoint method.

In an experiment over the period February 19 - March 30, 1993, the models are driven by the same wind field (HIRLAM analysis winds), and the same observation data set is assimilated. This set consists of a) spectra from three pitch-and-roll buoys and b) Synthetic Aperture Radar (SAR) spectra from the ERS-1 satellite. Three analysis/forecast runs are performed: one without assimilation, one with assimilation of buoy measurements only, and one with all data assimilated. For validation, observations from four buoys, altimeter data from ERS-1 and Topex-Poseidon, and scatterometer data from ERS-1 are used.

A detailed analysis of the "Wadden Storm" (February 20-22) shows the very different nature of the two assimilation schemes: the wave and wind field corrections of the WAM/OIP scheme are all in the vicinity of the observations, whereas the DASWAM adjustments are more of a global nature. The impact of some individual buoy and SAR observations is visualized. A comparison of the performance of the two schemes is somewhat obscured by the very different behaviour of the two first-guess runs.

A statistical analysis over the whole 39-day period gives the following results. In a comparison with buoy observations it is shown that a positive impact of wave data assimilation remains until about 12 hours in forecast in both models. At the buoy locations, the impact of OIP assimilation in WAM is larger, both at analysis time and in the short-term forecast. Comparison with altimeter wave heights also shows a slightly larger impact of WAM/OIP than of DASWAM. The impact of assimilation of buoy observations is larger than the impact of satellite SAR observations, at least partly because of the larger amount of buoy data.

The wind speed corrections applied by both assimilation schemes did not significantly improve or deteriorate the quality of the winds, compared to either platform or satellite wind measurements. For DASWAM, this is an indication that a better representation of error covariances in the cost function and a retuning of the wave model can further improve its performance.
1 Introduction

Application of data assimilation to operational wave modelling is a quickly developing subject. It was only a decade ago that first attempts were reported to improve the wave forecast by correcting the wave field with recent observations (Komen, 1985). Since then, the number of near-real time available wave and wind observations has grown drastically because of the launch of earth-observing satellites, notably the ERS-1 and ERS-2. At many forecast centres, this has been the inspiration for the development of methods which can make use of observations in the operational wave forecasting cycle. In this paper, the performance of two of these methods will be compared.

The first line of assimilation schemes consisted of sequential, time-independent methods which were all specifically designed to assimilate integral wave parameters, especially significant wave height (e.g. Thomas, 1988; Janssen et al, 1989; Lionello et al, 1995). These methods are computationally cheap, and some success in improving the wave forecast has been reported (e.g. Günther et al, 1993). This has led to implementing such a system in the operational wave analysis/forecast cycle at the European Centre for Medium-Range Weather Forecasts (ECMWF).

However, in other cases the impact of this type of systems has proved to be small (Burgers et al, 1992; Mastenbroek et al, 1994; Bidlot et al, 1995). As suggested by the last two studies, this may be caused partly by the fact that significant wave height observations alone do not contain sufficient information for a proper update of the wave spectrum. Recently, sequential assimilation systems have been developed which are also capable of assimilating observations of the full wave spectrum (Hassellmann et al, 1994b, 1996b; Voorrips et al, 1996; Breivik et al, 1996). Voorrips et al showed the additional impact of the use of spectral information, by a comparison with a method based on significant wave height assimilation.

Another line of algorithms consists of multi-time-level, variational assimilation schemes, which minimise the misfit between model and observations over a certain time interval (e.g., de Valk and Calkoen, 1989; de Valk, 1994; Barzel and Long, 1994; Bauer et al, 1994; de las Heras et al, 1995; Holthuijsen et al, 1996; Hersbach, 1997). The advantage of multi-time-level methods is that the model dynamics is taken into account explicitly during the assimilation. The dynamics of wind sea and swell are very different, the former being determined largely by the most recent wind forcing, and the latter by wave propagation. This difference affects the statistics of wind sea and swell: the spatial and temporal scales of correlation for swell are variable and generally much larger than for wind sea. Multi-time-level methods can take this
explicitly into account by letting the model completely re-generate the analysis after adjustment of the forcing of the model. This will be an advantage in particular for the analysis of swell, especially if assimilated observations and model output locations of interest are spread over a large area, so the analyses can benefit from remote observations. Another potential advantage of multi-time-level methods is control over the way in which information from observations made at different instants of time is integrated.

The drawback of these variational methods is that they are much more demanding in terms of computer power than the single-time-level methods described above. Therefore, full implementation of the variational technique has so far only been applied to parameter estimation studies (Hersbach, 1997). Schemes which are used for state estimation still need simplifications in order to restrict the computer time. De Valk and Calkoen (1989) and Bauer et al (1994) apply a simplified wave model during the minimization of the cost function. Holthuijsen et al (1996) avoid the iteration in the minimization of the cost by assuming a parabolic dependence of the cost function on the control parameters, and they apply a strongly simplified approximation of the wind field in order to reduce the number of control parameters.

In this paper, we compare the performance of two assimilation schemes, both of which have been implemented in a wave analysis/forecasting cycle for the North Sea. The first scheme, DASWAM, is a multi-time-level variational method based on adjustment of the wind forcing history (de Valk and Calkoen, 1989; de Valk, 1994). The second scheme, OIP (Optimal Interpolation of Partitions: Voorrips et al, 1996), is a sequential method which assimilates wave spectra using the concept of spectral partitioning to reduce the computational burden (see Gerling, 1992, and section 3). The purpose of the comparison is to assess whether the dynamically consistent analyses of the variational scheme lead to more realistic analyzed fields and better forecasts than the OI scheme, which uses fixed covariance fields. Attention is paid not only to the wave fields, but also to the wind field update by the assimilation schemes. Furthermore, the effect of different types of wave measurements is studied.

For a 39-day period in February/March 1993, the wave models are run with the same input winds, and the assimilation schemes are fed with the same wave observations. The period includes the ”Wadden Storm” (20-23 February), which is a highly interesting event, in which significant wave heights up to 10 m were measured in the North Sea. The observations come from directional wave buoys and ERS-1 SAR. The results are validated against buoy measurements, ERS-1 and Topex-Poseidon altimeter measurements, and ERS-1 scatterometer data.

In sections 2 and 3, the two assimilation/forecast models are summarized. Section 4...
describes the comparison experiment, of which the results are discussed in section 4. Some conclusions are drawn in section 6.

2 Adjoint method (DASWAM)

2.1 Introduction

The wave data-assimilation scheme DASWAM developed at Delft Hydraulics is an example of a multi-time-level variational data-assimilation method. It was implemented around 1990 for a wave model covering the North Sea and the neighbouring part of the Atlantic Ocean (de Valk, 1994). Off-line tests were reported in (Delft Hydraulics et al, 1994). In (Delft Hydraulics, 1995), a real-time test producing analyses and forecasts every 6 hours over a period of seven months was reported.

The basis of this approach is that possible errors in model input data are corrected in order to minimize the misfit of the analyses to observations and other available data, with the numerical model as a dynamic constraint (Hasselmann et al, 1994a). The minimization is carried out by an iterative descent method (Fletcher, 1987), using the model itself to evaluate the cost associated to a trial value of the model inputs, and using the adjoint model to compute the gradient of the cost to the model inputs, e.g. (Luenberger, 1969). For numerical models with a finite-dimensional state computed at discrete time levels, the adjoint model is nothing else than the application of the chain rule to keep track of the gradient of the cost to the state, also called adjoint state or costate. This section discusses the wave model under consideration and specific choices made concerning the input data to be corrected (controls), the cost functional measuring the misfit of analyses to observations, and the solution to the minimization problem.

2.2 Model

The wave prediction model applied by Delft Hydraulics is the third generation model PHIDIAS implemented by Delft Hydraulics (van Vledder, 1994). It solves the evolution equation for the wave action density \( A(k, r, t) \):

\[
\left\{ \frac{\partial}{\partial t} + \left( c_g + U \right) \cdot \frac{\partial}{\partial r} - (\nabla \times \Omega) \cdot \frac{\partial}{\partial k} \right\} A = S_{in} + S_{ds} + S_{nl} + S_{bot}
\]

Here \( k \) the wave number, \( r \) position, \( t \) time, \( U \) surface current, \( c_g \) group velocity and \( \Omega \) angular frequency. The right hand side terms \( S_{in}, S_{ds}, S_{nl} \) and \( S_{bot} \) are, respectively, the source terms for wind input, dissipation through white-capping, non-linear wave-wave interactions, and bottom dissipation. They are taken equivalent to the original WAM source terms (WAMDI, 1988).
The model is thus quite similar to the original WAM model, the main difference being that the action density is computed on a wavenumber/direction grid, whereas in WAM, the variance density is computed on a frequency/direction grid. The wavenumber grid consists of 25 grid points, with equal spacing of the logarithm of the wavenumber, between 0.0036 rad/m and 1.01 rad/m, corresponding to 0.03 Hz and 0.5 Hz in deep water. The directional grid consists of 12 points with uniform spacing of 30°.

For wave prediction for the North Sea region, the model was implemented on a rotated spherical grid with pole in Venice; see figure 1. This choice results in relatively high resolution in the southern North Sea, and low resolution near Iceland. As a boundary condition on the open boundaries of the model, for wavenumbers pointing to the interior of the domain a vanishing gradient of the action density in the direction normal to the boundary is imposed. In the absence of information from other sources, this appears to be a reasonable choice, but substantial errors can be expected in the region near the western boundary facing the Atlantic Ocean.

2.3 Controls

In the application of variational inverse modelling within the wave forecasting cycle to estimate the current state of the model, model inputs to be corrected are naturally time-varying inputs (forcing, boundary conditions, etc.). Forcing by wind is generally regarded as a major source of error in numerical wave predictions and analyses. As a consequence, wind fields over a preceding time-interval (the assimilation window) have been chosen as controls. For the regional model for North Sea and north-east Atlantic Ocean described earlier, the assimilation window was fixed at 72 hours. For this regional model, errors in boundary conditions are another major source of error, but the correction of boundary conditions has not been implemented so far.

The most obvious drawback of wind fields as controls is the huge dimension of a wind field sequence. However, this presents only a minor problem in practice (see below). Secondly, there is the problem of constructing physically realistic wind fields. There are limits to the information about wind fields obtainable from a restricted set of wave observations (de Valk, 1994), so we cannot expect that the wind field reconstructions will also be consistent with the physics of the atmosphere. The most complete solution is to couple the wave model to an atmospheric model and develop an assimilation method for the compound model (Hasselmann et al, 1988), provided that the difficult problem of selecting controls for an atmospheric model has been sorted out. A provisional solution is to parameterize the wind field sequence or to incorporate additional constraints on the wind field sequence, such as geostrophy. In the current
implementation of the algorithm, smoothness of wind field corrections is imposed by a tensor product B-spline representation of corrections \( \Delta u \) and \( \Delta v \) to the wind vector components \( u \) and \( v \) (de Boor, 1987):

\[
\Delta u(x, y) = \sum_{j,k} \alpha_{u}^{j,k} B_j^i(x) B_k^k(y)
\]

and

\[
\Delta v(x, y) = \sum_{j,k} \alpha_{v}^{j,k} B_j^j(x) B_k^k(y)
\]

with \( x \) and \( y \) the longitudinal and latitudinal coordinates, respectively. The spline coefficients \( \alpha_{u}^{j,k} \) and \( \alpha_{v}^{j,k} \) represent the perturbation applied to the first-guess wind field and are zero at the start of a data-assimilation run. For each 3-hourly wind field, a different set of spline components is assumed. For each \( j \), \( B_j^j \) is a piecewise cubic function with compact support determined by the knot points, which were placed at every third mesh point of the wave model grid. The B-spline basis functions are, except for shifts, identical in terms of the coordinates of the rotated spherical grid. This results in smoother wind field adjustments in the North. Other constraints are not imposed at present.

2.4 Cost functional

The cost functional should reflect the statistics of errors in observations and in first guesses of model inputs. In general, a suitable choice is the negative of the logarithm of the posterior density of the model inputs, i.e. their conditional probability density relative to the observations. When the cost function contains only terms expressing the misfit to observations, minimizing it is equivalent to maximum likelihood estimation of the model inputs. The misfit of wave analyses to observations and the misfit of wind retrievals to first guess wind fields will appear in separate terms of the cost functional. For most types of spectral wave observations, appropriate cost function terms can be derived straightforwardly by approximating the sea surface as stationary and Gaussian. For directional wave buoys and measured SAR spectra, cost functions have been implemented in this way. However, in the tests reported in this paper, inverted SAR spectra were used for which no error statistics are available (see section 4.1). Also, the available parameters of directional wave buoy spectra did not allow reconstruction of the 1st and 2nd order Fourier coefficients of the directional spreading functions which are used in the cost functional term for these buoys (again, see section 4.1). Therefore, it was decided to formulate cost functionals for both SAR and wave buoy data in terms of inverted directional wave spectra. Directional spectra were derived from the wave buoy data of mean variance density, mean direction and directional spread (Kuik et al, 1988) over 10 distinct frequency intervals.
by assuming a $\cos^2 \phi$ directional distribution for each of the intervals. Then these spectra were transformed to the spectral grid of the model (but now in the frequency/direction domain) using interpolation. The cost functional term for an inverted spectrum from SAR and from a wave buoy used is a simple sum of squares

$$\sum_{i,j} \frac{[F_{\text{pred}}(f_i, \theta_j) - F_{\text{obs}}(f_i, \theta_j)]^2}{\sigma^2}$$

(4)

with $F_{\text{pred}}$ the model prediction of the wave variance spectrum at the grid point and time-level of the observed wave variance spectrum $F_{\text{obs}}$. The variance $\sigma^2$ was taken identical for all spectral bins for both SAR and wave buoys. This simplistic approach has at least the advantage that the effects of spectra from different instruments on the analysis is independent of statistical assumptions, so they can easily be compared. However, from a statistical point of view, it is not an optimal choice.

As mentioned in the previous section, deviations from first guess wind fields were represented by B-splines. Deviations from first guess wind fields were penalized by a cost functional term of the form of a sum of squares of all B-spline coefficients $\alpha_{u,i}^{j,k}$, $\alpha_{v,i}^{j,k}$ of all wind fields, weighed by a single constant. The weight was chosen sufficiently small, so that the magnitudes of wind field corrections did not significantly restrict the influence of the wave observations on wind field reconstructions. This may not be very realistic, but was motivated by the situation that no adequate model of the covariance structure of wind prediction errors had yet been implemented. Aspects which do affect the outcomes are the spatial correlation length, determined by the knot distances of the B-spline basis, and the assumed mutual independence of the spline coefficients of consecutive 3-hourly wind fields. This latter assumption was found unrealistic in previous tests (Delft Hydraulics et al, 1994): time-series of wind field adjustments appeared rather bumpy, and storm systems were often found to be adjusted only at a single instant.

2.5 Numerical solution

To solve large-scale minimization problems, several efficient and storage-effective descent methods are available which require only functional evaluations (based on the forward model) and gradient evaluations (based on the adjoint model). We implemented a limited-memory BFGS method (Liu and Nocedal, 1988).

Still, applying this iteration scheme using a third-generation wave model would require much more computational effort and time than a forecast run, because both the model and its adjoint need to be run over the assimilation window (see $\ref{2.3}$) at least once per iteration.
At this stage, approximation of the model within the minimization procedure seems the only way out. The prerequisite of such an approximate model should be that for wind fields close to the first-guess field, the response of the approximate model should be close to the response of the real third-generation model. In (Bauer et al, 1994), an approximation of the tangent linear model of WAM is used. This is very good for wind fields very close to the first guess, but deteriorates when the disturbance of the wind field becomes larger. We chose instead to use a nonlinear second-generation wave model, tuned to the third-generation model. This should give a better response when the true wind field is rather far away from the first guess.

The wave field in the approximate second-generation model is discretized on the same grid as PHIDIAS. Advection and wave dissipation are modelled in the same way as in PHIDIAS. The main difference from PHIDIAS is the way in which wave growth is modelled, which is the most expensive part of a third-generation model. After advection, the procedure involves (a) extracting wind-sea parameters from the advected spectra in a manner similar to (Janssen et al, 1989); (b) updating these wind-sea parameters from friction velocity, using growth curves tuned to the third-generation model; (c) computing a parameterized wind-sea peak. In parallel, the advected spectrum is dissipated as in WAM to deal with the swell components, and is then combined with the computed wind sea peak by taking the maximum of both.

Some details of the different steps in the computation follow below. In the extraction of wind-sea parameters (a), the parameters extracted are the total variance, mean wave period $T_m$ (defined in section 4.2) and mean direction $\theta_w$ over a wind-sea region of the spectral domain determined from the friction velocity following (Janssen et al, 1989), with friction velocity determined from $U_{10}$ according to the Charnock relationship. In (b), two growth curves are used: one for energy, and the other for period, expressed as a function of wave age (with all variables made dimensionless using the friction velocity). The two growth curves were fitted to calculations with a single-gridpoint re-implementation of WAM without advection (De Valk and Calkoen, 1989, Section 4.2). First an effective wave growth duration is estimated from the wind-sea mean period and energy and the friction velocity, defined as the average of the wave growth durations computed from energy and from mean period. Then the energy and mean period at the next time-level are computed each according to their own growth curves. This procedure ensures that discontinuities in the growth in the case of changing winds are minimized in both mean period and energy. The parameterization of the wind-sea peak (c) is of the JONSWAP form (Hasselmann et al, 1973; Komen et al, 1994, p. 187) but with different coefficients to match the WAM spectra. A $\cos^2$ directional distribution was assumed, independent of frequency. Weaknesses of the current version are the directional relaxation and
the behaviour of the spectral peak shortly after a sudden decrease in wind speed.

When comparing this second-generation model approximation with PHIDIAS or WAM, differences in significant wave height in the order of 0.5-1 m were frequently observed. Therefore, assimilation with this model may be effective for correcting relatively large errors in wind fields but probably not for correcting small errors, if it is assumed that the WAM model results have a smaller error than the observed difference with the 2nd generation approximation.

The second-generation approximation was not only used in the minimization of the cost function, but also to produce the analyses. The motivation is that due to differences between the second- and third-generation model, analyses produced using these models driven by the same wind fields may be different. Using the second-generation model for the analysis has the advantage that the wind field corrections have been obtained with the same model.

As a result of the approximations made, producing the analysis over a 72 h interval performing 10 iterations to minimize the cost functional requires about twice the computational effort as running the forecast over a 36 h interval using the third-generation model. The overhead of data-assimilation in comparison to a third-generation model forecast without data-assimilation is therefore about 200%.

3 Optimal interpolation of spectral partitions (OIP)

3.1 Introduction

This section describes the wave data assimilation / forecasting system which is currently at a semi-operational stage at KNMI (Voorrips et al, 1996; Voorrips, 1997). Multi-time-level variational assimilation methods have been used at KNMI for parameter estimation (de las Heras et al, 1995; Hersbach, 1997), but research for state estimation and wave forecasting is concentrating on single-time-level, sequential methods. The latter techniques are generally less computationally demanding and have been applied successfully in meteorological forecast models. The drawback of such techniques is, however, that the model dynamics cannot be incorporated explicitly.

Previous experience has been obtained with the optimal interpolation method applied to wave height and wave period measurements (Janssen et al, 1989; Lionello and Janssen, 1990; Burgers et al, 1992; Mastenbroek et al, 1994). Optimal interpolation is a statistical method which determines the minimum error variance solution for the model state, by combining a model first-guess field and observations, with prespecified forecast and observation error covariances.
The method which is used in the present study is also an optimal interpolation method, but an extended version which assimilates observations of full wave spectra. The main spectral characteristics of the spectrum are described by a reduced number of parameters, which limits the computational burden which would be imposed by a full optimal interpolation method. The method was devised and applied to ERS-1 SAR spectra by Hasselmann et al (1994b, 1996b). Voorrips et al (1996) adapted this "Optimal Interpolation of Partitions" (OIP) method for the use of pitch-and-roll buoy data and applied it to the North Sea area. In their experiments, they showed an improvement in wave analysis and forecast performance compared to the scheme of Burgers et al (1992) in the North Sea case, due to the possibility of assimilating spectral and directional details of the wave spectrum.

Since OIP is a sequential method, every assimilation step is performed only for one fixed time level, processing all available observations near that particular time. In a standard operational setting, observations are grouped at 3-hourly intervals, so the assimilation scheme is called every three hours. The computation time needed for the assimilation step is only a few percent of that of a 3-hour run of the WAM model, so the additional cost of assimilation with the OIP scheme is negligible.

The description of the WAM/OIP scheme below is concise. For an extensive description we refer to Voorrips et al (1996).

3.2 Wave model

The wave model which is used is the WAM model, Cycle 4 (WAMDI, 1988; Günther et al, 1992; Komen et al, 1994). It solves the wave transport equation

$$\frac{\partial F}{\partial t} + \nabla \cdot (c_g F) = S_{in} + S_{ds} + S_{nl} + S_{bot}$$

(5)

where $F(f, \theta, r, t)$ is the frequency-directional wave variance density spectrum at location $r$ and time $t$ and $c_g(D(r), f)$ is the group velocity depending on the local depth $D(r)$ and frequency $f$. The right-hand side represents the source terms due to wind input, dissipation through white-capping, nonlinear wave-wave interactions and bottom dissipation, respectively.

The model is implemented on a $\frac{1}{3}$ degree latitude $\times$ $\frac{1}{2}$ degree longitude grid (approximately 32 km grid spacing), which includes the North Sea and part of the Norwegian Sea, see figure [1]. The Norwegian Sea is included mainly in order to capture wave systems which are generated in this area, and which propagate as swell into the North Sea afterwards. At each grid point, the wave variance density spectrum is discretized in 25 frequencies ranging from 0.04 to 0.4 Hz, and in 12 directions.
In the version of the model used in this study, the open boundary of the model is not forced by externally generated wave fields. Instead, the spectra at the boundary are defined to be equal to 90% of the spectrum at the neighbouring grid point within the model region, thus simulating a wave growth with a finite fetch when the wind is blowing into the model region. Currently, tests are being performed with open boundary forcing by the global WAM version which runs at ECMWF.

3.3 Assimilation method

3.3.1 Outline

As mentioned above, computational efficiency of the OIP method is enhanced by projecting the full wave spectrum onto a small number of parameters, corresponding to so-called partitions, before the actual assimilation is performed. This leads to the following step-wise approach:

- **Partitioning** of all observed and model spectra, i.e. division of each spectrum into a few distinct segments. The physical interpretation of each segment ("partition") is that it represents a wave system, corresponding to a certain meteorological event (swell from a distant storm in the past, wind sea which is generated by local wind). Every partition is described by three mean parameters: its total energy, mean direction and mean frequency.

- **Cross-assignment** of partitions of different spectra: connect partitions which are so close to each other in mean spectral parameters that they can be supposed to represent the same wave system.

- **Optimal interpolation** of the mean parameters from observed and model partitions which are cross-assigned. Thus, an analyzed field of partition parameters is obtained.

- **Update** of each spectrum locally, based on the first-guess spectrum and on the analyzed partition parameters.

- **Update** of the driving wind field as well, if there is a wind sea-component in the spectrum.

3.3.2 Spectral partitioning

The concept of spectral partitioning was introduced by Gerling (1992). It is a method to describe the essential features of a two-dimensional wave spectrum $F(f, \theta)$ with only a few parameters, by separating the spectrum into a small number of distinct segments, so-called partitions. The partitioning is a purely formal procedure; however, the partitions can be interpreted physically
as representing independent wave systems. Details of the formalism used to calculate the partitions can be found in Hasselmann et al (1994b, 1996b) and in Voorrips et al (1996).

As said, the physical interpretation of a partition will be that of a wave system, which has a different meteorological origin than other partitions in the spectrum. The data assimilation scheme makes use of this interpretation. The underlying assumption is that components of the discretized spectrum which lie within a partition are fully correlated with each other, whereas components from different partitions are uncorrelated. This assumption is not entirely correct (wave systems may influence each other through dissipation and non-linear interactions), but it is generally a reasonable approximation. Having made this approximation, one can limit oneself to calculating only a few integrated parameters of every partition, and perform the assimilation on these integrated parameters rather than on the full spectrum. The assimilation scheme as designed by Hasselmann et al (1994b, 1996b) uses three parameters per partition: the total energy of each partition, the mean frequency and the mean direction.

Each partition is regarded to be either swell, or wind sea, or mixed wind sea/swell according to a criterion involving the wind speed and the mean wave vector of the partition (cf Voorrips et al, 1996).

### 3.3.3 Partitioning of buoy spectra

The original partitioning scheme (Hasselmann et al, 1994b, 1996b) can only be applied to a full two-dimensional wave spectrum, such as a model spectrum or an inverted SAR spectrum. Pitch-and-roll buoy data, however, contain only the one-dimensional wave spectrum $E(f)$, plus limited information about the directional distribution. To assimilate these data as well, an adapted version of the partitioning scheme was developed (Voorrips et al, 1996) which needs only $E(f)$ and the mean wave propagation direction $\overline{\theta}(f)$ as a function of frequency. Tests with synthetic buoy spectra which were extracted from full spectra showed good agreement between the two partitioning schemes.

### 3.3.4 Cross-assignment of partitions

The next step in the assimilation procedure is to merge the model first-guess and observed partition parameters into an analyzed field of parameters. We have assumed that different partitions within a spectrum are uncorrelated, since they are created by different meteorological events. So, we want to treat these partitions separately from each other in the assimilation. On the other hand, partitions in different spectra (e.g., model and observed spectra, or two model spectra at different locations) are correlated if they are created by the same event. Therefore,
we have to define a cross-assignment criterium between the partitions of two different spectra, in order to decide whether a partition in one spectrum represents the same wave system as a partition in the other spectrum.

The criterium which is used is based on the distance in spectral space between the mean parameters of two partitions. The ones which are closest to each other are cross-assigned. In case the number of partitions in the observed and model spectra do not match, additional assumptions are needed. For details, we refer to Voorrips et al (1996).

3.3.5 Optimal interpolation of partition parameters

When the cross-assignment is done, the mean parameters of the model and observed partitions can be combined to obtain an analyzed field of partition parameters. An important input for the OI procedure are the error covariances of the errors in the observed and the model parameters. The covariances were obtained by calculating long-term statistics of differences between observations and model forecasts. The observation errors were assumed to be spatially independent. For the model forecasts, an error correlation length of 200 km was found. The error variances of observations and model values were taken to be equal and dependent on wave energy (Voorrips et al, 1996).

3.3.6 Update of wave spectra and wind field

The analyzed partition parameters from the optimal interpolation are now combined with the first-guess spectra to obtain analyzed spectra. Every first-guess partition is multiplied by a scale factor and shifted in the \((f, \theta)\) plane such that its mean parameters are equal to the parameters obtained by the optimal interpolation. Small gaps in the spectrum which arise by the different shifts for different partitions are filled by two-dimensional parabolic interpolation.

When a wind sea partition is present in the spectrum, the driving wind field is modified using a simple growth curve relation (Voorrips et al, 1996). The new winds are used until the next wind field is read in, which is half a wind time step later (90 minutes).

4 Comparison of the DASWAM and OIP assimilation methods

4.1 Assimilation experiment

The performance of the DASWAM and WAM/OIP forecasting/assimilation systems (resp. sections 2 and 3) was compared by running the two models for the same period, with the same
input data (wind forcing and assimilated observations), and by evaluating the results against independent observations.

The selected period starts at 0 h GMT, February 19, 1993, and ends at 0 h GMT, March 30, 1993. We chose this relatively long period (39 days) with quickly changing meteorological conditions, as to be able to test the methods in many different situations. The period includes the Wadden Storm (February 20-23), in which wave heights up to 10 m were observed. To create an initial condition at the start of the period, the models were run over the previous two days without assimilation. Wind fields used were 3-hourly analyses from the High Resolution Limited Area Model (HIRLAM; Källberg, 1990), which is the operational weather prediction model at KNMI.

Two types of observations were used for the assimilation:

- wave buoy data: 3-hourly observations of spectral density, mean direction and directional spread over 10 frequency bands between 0.03 and 0.5 Hz, from the locations North Cormorant (NOC), AUK and K13 in the North Sea (see figure [1]).

- 126 ERS-1 wave mode SAR spectra obtained from the Max-Planck Institut für Meteorologie, Hamburg. The SAR spectra were inverted to wave spectra using model output from a global WAM model as a first guess in the inversion following Hasselmann and Hasselmann (1991) and Hasselmann et al (1996a).

Once every 24 hours, starting at 00 h GMT, runs were performed which consisted of two parts: (a) an "analysis" run over the previous time period, in which data were assimilated; and (b) a 24 hours "forecast" run, starting from the analyzed model state obtained after run (a). The time period used for the analysis run (a) differs for the two assimilation methods. For the OIP method, which is a sequential method, it was sufficient to perform a 24-hour analysis, starting from the model state obtained at the end of the previous analysis run. For DASWAM, which is a multi-time-level variational method, an analysis window of 72 hours was chosen, in order to assimilate all relevant observations consistently with the model dynamics (see subsection [2.3]). During the analysis run (a), DASWAM uses the second generation approximate model (cf subsection [2.3]), whereas in the forecast run (b), it uses the third generation model PHIDIAS. WAM/OIP uses the WAM model both in the analysis and in the forecast run (b). Note that the "forecast" runs (b) were performed with analyzed, not forecast wind fields. This was done to ensure that the only difference between an analysis and a forecast lies in the assimilation of wave observations; in this way, the impact of the assimilation is easily monitored.

Three types of analysis runs (a) were performed, each with a subsequent forecast run (b):
• A run without assimilation (NOASS run);

• A run in which only the wave buoy data were assimilated (NOSAR run);

• A run in which both wave buoy and SAR data were used (SAR run).

The NOASS run is used as a reference to study the impact of the real assimilation runs NOSAR and SAR. The SAR run is done in order to study the additional impact of the SAR data, added to the (much larger but less homogeneous) amount of buoy data. Note that whereas the NOASS ”assimilation run” (a) and ”forecast run” (b) are identical for the WAM/OIP system, they are different for DASWAM: in (a), the second-generation wave model is used, in (b) the third-generation PHIDIAS model is used. As will be seen in section 5, the results of these can differ substantially.

Observations which were not assimilated are wave buoy spectra from the location Schiermonnikoog Noord (SON) close to the Dutch coast (figure 1), ERS-1 (OPR) and Topex-Poseidon (MGDR) radar altimeter significant wave height and wind speed data, and ERS-1 (OPR) scatterometer wind velocity data. These data were not not assimilated in order to have a reference for the validation of the results.

4.2 Evaluation method

Results of the various assimilation runs were evaluated based on results for the wave and wind parameters

• significant wave height \( H_s \);

• mean wave period \( T_m = m_0/m_{-1} \) with \( m_i = \int s(f, \theta) f^i df d\theta \) the \( i^{th} \)-order spectral moment, or equivalently, the mean wave frequency \( f_m = 1/T_m \);

• mean wave direction \( \theta_w \);

• low-frequency wave height, \( H_{10} = 4\sqrt{E_{10}} \), where \( E_{10} \) is the total variance of all waves with periods above 10 s;

• wind speed at 10 m above the surface \( U_{10} \).

Model results of these parameters were validated against buoy measurements at NOC, AUK, K13 and SON. The first three buoys were used during the assimilation, so they are mainly of interest for forecast validation. Additionally, significant wave height and wind speed corrections
could be checked against altimeter observations from the ERS-1 and Topex-Poseidon satellites. Finally, the wind velocity corrections were compared to measurements from the ERS-1 scatterometer.

From ERS-1, the OPR products were used. A bias correction to the ERS-1 altimeter wave height data was applied using the formula (Queffeulou et al, 1994)

$$H_s^{corr} = 1.19H_s^{OPR} + 0.19[m].$$  \hfill  \(6\)

From TOPEX-POSEIDON, the GMDR data were used. The TOPEX wind speeds are also known to have a bias compared to buoy observations (Gower, 1996; Komen et al, 1996). Here we apply the correction of Komen et al:

$$U_{10}^{corr} = 0.877U_{10}^{MGDR} + 0.39[m/s].$$  \hfill  \(7\)

The evaluation will be split in two parts. In subsection 5.1, we will study the most interesting part of the investigated period, namely the Wadden storm (20-23 March). Through time series and maps, differences in the characteristics of wave models and assimilation methods, and the impact of SAR observations will be illustrated. In subsection 5.2, the overall performance of the schemes will be presented through a statistical analysis of model results against buoy and satellite data over the full period.

5 Results

5.1 The Wadden Storm, 20-23 February 1993

5.1.1 Wind fields

Figure 2 shows four HIRLAM wind fields at the height of the Wadden storm, from February 20, 15 h GMT until February 21, 9 h GMT. Within 12 hours, the maximum of the north-westerly storm moves quickly from the North of Scotland, through the North Sea to the German Bight. In figure 3, measured and modeled wind speeds are given at four North Sea locations, which are indicated in figure 1. For the southerly locations K13 and SON, the agreement between model and observations is satisfactory. At AUK, and especially at NOC, however, the model wind speed is significantly lower than the platform observations. The underestimation is almost 10 m/s at the maximum of the storm.

Only few satellite wind speed measurements are available during the storm, to complement the platform measurements. From these measurements, no consistent bias of the model wind speed can be inferred, although occasionally, large differences between model and observations
are found. An additional problem here is that the quality of altimeter measurements of wind speeds above 20 m/s (which occur at the height of the storm) is not well-known, due to lack of statistics in most comparisons between conventional and satellite measurements (e.g., Gower, 1996; Komen et al, 1996).

From the above, one can see two good reasons to use the Wadden Storm in a study of wave data assimilation schemes. First, the extreme north-westerly winds will generate high waves which can travel over a large distance into the North Sea. This is a typical situation in which assimilation of data in the northern and central North Sea can improve the wave forecast in the South. Second, the poor quality of the model winds at the height of the storm will leave sufficient room for the assimilation of wave observations to have a large impact on the analysis and prediction of the sea state.

5.1.2 NOASS runs

In this paragraph, we compare the results of the various model runs without assimilation. The difference between the three wave models used will be of importance when comparing the performance of the data assimilation schemes, in which we are mostly interested.

Figures 4 and 5 show time series of, respectively, significant wave height and mean wave period at the same positions. Three types of model runs, all without assimilation, are presented: the WAM NOASS analysis run, the DASWAM NOASS analysis run with the second generation model, and the DASWAM NOASS 24 hour forecast runs with the third generation model PHIDIAS, starting every day at 00 GMT from the NOASS analyzed wave field. Although all model runs have been performed with the same wind field, the results are quite different. The WAM run severely underestimates the wave height at the peak of the storm, at all platforms. The DASWAM NOASS analysis run, on the other hand, gives good results for NOC, K13 and SON, and less underprediction at AUK. The DASWAM NOASS forecast run starting from February 20, 00 GMT, gives again much lower wave heights at the peak of the storm, in fact comparable to the heights obtained with the WAM model. The mean period results (fig. 5) give essentially the same result: good performance by the 2nd generation model, and underestimation by the 3d generation models. The underestimation is largest for the DASWAM NOASS forecast run.

The underestimation of the wave height and period by the third-generation models is at least partly explained by the underprediction of wind speed as was found in paragraph 5.1.1. The absence of bias in the second- generation runs is surprising, given the poor model wind fields. Since the second-generation model is meant to be an approximation to PHIDIAS, it seems
that some retuning is needed. Due to the mismatch between the two models, the assimilation of wave observations in the second-generation model will lead to wind speed corrections which are not optimal for PHIDIAS.

5.1.3 Assimilation runs

Figures 6 and 7 show NOSAR and NOASS results, respectively for WAM/OIP and for DASWAM, versus buoy observations. The model analyses are much closer to the observations than the NOASS results, not only at the three locations of which the data were assimilated, but also at SON (especially for WAM/OIP, figure 3). For both systems, the memory of the assimilation (i.e., the time within which the NOSAR forecast relaxes back the NOASS forecast) varies between 6 hours for the northerly stations and 12 hours for the southerly locations, where the impact of the assimilation "upstream", at AUK and NOC, is felt in the forecast. This period is relatively short: for both systems, results have been reported where the impact lasted over 24 hours (Delft Hydraulics, 1995; Voorrips et al, 1996). Since wind sea dominates in this period, improvements in the wave field are more quickly lost than in a pure swell situation.

Although the analyses seem comparable when looking only at the time series of the analyzed buoys, the nature of the assimilation is quite different. Figure 8 shows the WAM NOASS analyzed wind and wave fields at February 20, 21 GMT, and the increments due to the NOSAR assimilation. The wave height increments are clearly centered around the buoy locations, as is dictated by the fixed error covariance structure in the OI scheme. The wind speed corrections are also quite local. North of NOC, the wind speed is not updated, since the wave field here was computed to be swell in this part of the model. Figure 9 shows the same fields for DASWAM. The corrections here are entirely different. As can be seen from figure 4, the DASWAM NOASS analysis correctly predicts the wave height at NOC, but underpredicts at AUK. The variational DASWAM system responds to this by changing its control, the wind field, between the two stations. In this way, the model manages to increase the wave height at AUK. Thus, although the obtained wave height at AUK is approximately the same for the two schemes, the spatial distribution of the update is entirely different.

At the time of the presented updates, the ERS-1 satellite passed. In figures 8 and 9, the altimeter track is indicated, which passes close to AUK and K13. Figures 10 and 11 compare the measured and modeled wind speed and wave height along the track. Both methods manage to draw the modeled wave height closer to the measurements after assimilation, even though the altimeter measurements are not assimilated themselves. The wind speed is also updated, but both methods clearly do not draw the model wind field closer to the altimeter observations.
Apparently, the assumptions underlying the wind update are not perfect. This is even the case for DASWAM, where the assimilation is consistent with the (second-generation) wave model, but not with the dynamics of the atmosphere (see section 2).

The impact of assimilation of the ERS-1 SAR observations on the results during the Wadden storm was small in general. The chance that the SAR can have an impact is not large: one needs high waves coming from the Norwegian Sea and bad swell prediction by the model, at exactly the time that SAR observations are made, which is only once or twice per day. In one case, however, observations of an important track caused a clear improvement in the Norwegian Sea. Figure 12 shows the track at February 20, 21 GMT, in the left panels. One can see the impact which the SAR observations have on the WAM/OIP analyzed wave height in the Norwegian Sea. The mean wave direction is eastward in this area, so the area of impact travels towards the Norwegian coast. Twelve hours later, a Topex-Poseidon track measures the wave height across the Norwegian Sea (fig. 13). One can clearly see the improvement in the modeled wave height near the Norwegian coast, because of the SAR assimilation 12 hours earlier.

5.2 Statistical results over the period February 19 - March 30, 1993

5.2.1 Buoy measurements

The results of the analysis/forecast runs over the 39-day period February 19 - March 30 have been validated against measurements at the locations NOC, AUK, K13 and SON. Tables 1 - 8 compare the NOASS (no assimilation) with the NOSAR (assimilation of only buoy measurements) runs, in terms of the root mean square error (RMSE)

\[ RMSE(x) = \sqrt{\sum_i (x_{mod}^i - x_{obs}^i)^2} \]  

(8)

Five parameters are compared: significant wave height \( H_s \), low-frequency significant wave height \( H_{10} \), mean wave period \( T_m \), mean wave direction \( \theta_w \), and wind speed \( U_{10} \). Results are given for the last analysis time (+0 h), and for the forecast times +6 h and +12 h. In order to reduce noise due to the relatively low number of runs (39), for each time the results have been combined with those of 3 hours earlier, leading to a maximum of 78 model/observed values for each forecast time. At analysis time, the absolute value of the RMSE is given for the NOASS run. Subsequently, for all three times the ratio of the NOSAR RMSE and the NOASS RMSE is given, expressing the relative impact of the assimilation on the analysis and forecast. For wind speed, only the reduction is given at analysis time, since the assimilation schemes do not make corrections to the forecast wind speed.
For most locations and wave parameters, the performance of the WAM and DASWAM NOASS runs is comparable. The main difference is the larger bias in wave height and period for WAM at the most northerly location, NOC. The difference in RMSE here is mainly caused by the difference in bias (not shown), which has already been noted in the discussion of the Wadden Storm results. The bias of the HIRLAM wind speed compared to platform observations is around -4.5 m/s at NOC, and around -1 m/s for the other locations. Again, the underestimation of wave height and period by the WAM model seems to be consistent with the underestimation of the wind speed. Another reason for the larger underestimation by WAM at NOC may be that the WAM model domain extends not as far westward as the DASWAM domain (fig. 1). Consequently, external swell from the Atlantic Ocean is more easily missed.

The impact of the assimilation on the analysis is locally much larger for the WAM/OIP scheme than for DASWAM: for instance, the RMS error in wave height at K13 is reduced to 37% of the NOASS RMSE (table 5), whereas for DASWAM the reduction is only to 87% (table 6). This is a direct consequence of the different nature of the schemes: optimal interpolation draws the model to the measurements mainly in the vicinity of the observation, whilst the impact of the adjoint method is more global. Comparison of the impacts on wave height at SON (not assimilated), illustrates this. The reduction in RMSE by the WAM/OIP scheme is here 76% of the NOASS RMSE (table 6), which is a much less dramatic improvement then at the assimilated location K13; the reduction by DASWAM is 92% (table 6), which is more comparable to the 87% at K13.

Also in the forecast, the impact of the WAM/OIP scheme on the RMSE is generally higher than the impact of DASWAM, although the difference is much smaller than at analysis time. Averaged over all four wave parameters and over the three locations with roughly the same NOASS quality (AUK, K13, SON), the reduction factor for the RMSE is 75% for WAM/OIP vs. 93% for DASWAM for the +3/+6 h forecasts, and 90% vs 95% for the +9/+12 h forecasts. Clearly, for the short term forecasts, the strong local correction of the OIP scheme is still of importance.

In the period considered, the impact of the assimilation on the forecast is seen up to around 12 hours. This is a shorter period than reported both for DASWAM (Delft Hydraulics, 1995) and for WAM/OIP (Voorrips et al, 1996). Probably, the impact is relatively small because in this period, wind sea was in general the dominating wave system in the North Sea. In periods in which swell is of more importance (this happens for instance in summer periods, see Delft Hydraulics, 1995), the impact period is longer.

Both schemes also correct the wind speed during the assimilation (tables 1 - 8). The cor-
rection does not lead to a significant improvement or deterioration of the model wind compared to the platform observations. At AUK, the reduction (WAM/OIP) or growth (DASWAM) of the RMSE is large, but based on an insufficient number of observations.

The impact of SAR assimilation in addition to the assimilation of buoy observations turned out to be negligible in the comparison with the buoy observations. This is to be expected, since especially in the neighbourhood of the buoys, the influence of the buoy measurements is much larger than the SAR observations, which are sparse and often far away from the buoy location.

5.2.2 Satellite measurements

Tables 9 and 10 show a comparison of WAM/OIP and DASWAM analyzed wave height and wind speed with measurements from the ERS-1 and Topex-Poseidon altimeters. For the comparison two regions are defined (fig. 1): region I covers the North Sea, and region II covers part of the Norwegian Sea. All altimeter measurements within one model grid box are averaged into one “super-observation”. Since the grids of the two model differ, the number of super-observations differs too.

Comparison with both satellites shows a negative bias of the HIRLAM (NOASS) wind of around 0.6 m/s in area II, and a small positive bias (around 0.2 m/s) in region I. These biases are smaller than those found in the platform comparison (-4.5 m/s for NOC in area II, and around -1 m/s for AUK, K13 and SON in area I). Differences between the TOPEX and ERS-1 results are much smaller than those between the comparison with platform measurements (tables 1-8) and with altimeter measurements. This is surprising, since the calibration of the TOPEX winds (7) has been obtained by a comparison with measurements from the same platforms (Komen et al, 1996). One way to explain the apparent discrepancy is the assumption that the model wind speed error is not homogeneous over the analysis areas in this relatively short period. Another reason may be that the altimeters (TOPEX with the correction (7)) underestimate the wind speed at very high winds, like in the Wadden Storm. For the wind speed range above 20 m/s, not enough data are available to make a reliable validation of the altimeter wind speed algorithms.

For both models, the NOASS wave height in the northern region (region II) is negatively biased compared to the altimeter measurements. This is consistent with the buoy comparison at NOC. The negative bias is worse for the WAM model than for DASWAM’s second generation model, which is also in agreement with the buoy comparison results. Apart from the wind speed bias, the wave height bias may be caused by the absence of external boundary information, which is of main importance for the Norwegian Sea (swell entering from the Atlantic).
In the North Sea region, assimilation reduces the mean error in wave height for both models (although the Topex-Poseidon comparison for DASWAM is not convincing). Again, the impact caused by the WAM/OIP scheme is somewhat larger. In the Norwegian Sea, there is still some impact for the WAM/OIP model, but it is much smaller than in region I. The reason that the impact is larger than for DASWAM in region II is probably the fact that the WAM NOASS model results are more negatively biased: the impact in this region is only the reduction of the bias.

The impact of the SAR observations is again very small. Only for WAM/OIP, and only in region II, a small positive impact can be noted. Again, this is the most likely candidate: region II, because the impact from buoys is smaller for the Norwegian Sea, and WAM/OIP, because it has the largest bias to correct.

The correction of the wind speed by the assimilation schemes has no significant impact on the quality, when compared to observations. This result confirms the results of the buoy comparison.

A comparison with the ERS-1 scatterometer wind velocity data shows essentially the same results as the altimeter wind speed measurements.

5.3 Discussion

Summarizing the buoy and satellite comparisons described above, the following main results are obtained. First, both the OIP/WAM and the DASWAM assimilation scheme manage to improve the wave analysis field, and the short term forecast, if not too far from the assimilation sites. Second, the OIP scheme has a somewhat larger effect than DASWAM, both on the analysis and on the forecast. Third, the wind speed corrections applied by both schemes do not significantly affect the quality of the wind field.

The second result is surprising, since the multi-time-level variational method of DASWAM is a more sophisticated assimilation scheme than the rather ad hoc optimal interpolation scheme OIP. The constraint of the model evolution equations to the analysis in DASWAM should lead to a physically consistent correction to the model parameters, while the OIP scheme does not take the assimilation history into account.

The fact that the wind speed corrections of DASWAM do not improve on the (rather poor) first-guess wind field gives an indication why DASWAM does not perform better: apparently, the choice of the control variables (wind speed) is not optimal, or the error statistics attributed to them in the cost function are not correct. Probably, both aspects play a role. The absence of bias in the second-generation NOASS results given the biased wind speed, indicates that
this wave model cannot be assumed to be perfect, as is implicated by the adjoint method (the model equations are a strong constraint in the minimization of the cost function). Hence, the control variables should include wave model errors as well as wind speed. Also, a wind field representation by splines, without correlation between the velocity components and without temporal correlation between consecutive fields, is not optimal.

It is not in itself a problem that the second-generation wave model used for analysis differs from the forecast model in the DASWAM suite, because the wind speed corrections which are determined by the assimilation are never used by the forecast model: only the analyzed wave field after the assimilation is used as an initial condition for PHIDIAS, the third-generation forecast model. However, naturally the model analysis and hence the following forecast will improve when the second-generation model is improved. The results shown here suggest that some retuning of the model may be useful.

The principle of variational assimilation is quite powerful, and it seems that relatively small adaptations to the DASWAM scheme (better representation of the error statistics, retuning of the second-generation wave model) may greatly enhance its performance.

The OIP scheme behaved reasonably well for a rather simple assimilation method, and from the present study it is not evident how its performance could be improved without major modifications. Like with DASWAM, the wind field corrections were not successful. However, in the OIP setup the wind speed correction is only a postprocessing based on simple assumptions (growth curves), and its significance for the wave field analysis is limited. Neither is the wind update in comparable OI schemes necessarily unsuccessful: in the global WAM model, some improvement of the ECMWF first-guess winds was obtained by assimilation of ERS altimeter wave heights using an optimal interpolation scheme (P.A.E.M. Janssen, personal communication). The reason why the wind speed update in the present setup is not successful may be that in these experiments, continually wave observations are assimilated at fixed locations with very little time delay. Thus, the assumption underlying the update scheme that at every assimilation timestep the wave field error is in equilibrium with the wind field error, is in fact violated.

6 Conclusion

In this paper, a close comparison has been carried out between the two wave forecast/assimilation models WAM/OIP and DASWAM, with the emphasis on the assimilation. WAM/OIP is an example of a rather simple, optimal interpolation method. DASWAM is a multi-time-level variational method, with some approximations to reduce the required computation time.
The different properties of the two assimilation schemes are illustrated in an analysis of the most interesting period which was investigated, the "Wadden Storm" of February 20-22, 1993. The corrections of the OIP scheme are localized around the observation sites, whereas the DASWAM scheme corrects the wave and wind field more globally, both in space and time. The performance of the two schemes during the Wadden storm, with respect to buoy observations, is somewhat obscured due to the large negative bias of the first-guess WAM run in the Norwegian Sea, and by the different behaviour of the two wave models which play a role in the DASWAM method. The bias may be caused either by the too low HIRLAM winds (as is supported by the platform measurements at NOC and AUK), or by the fact that the WAM model domain does not extend far enough to the West.

A statistical analysis of the model results against buoy measurements over the period February 19 - March 30, 1993, shows that the OIP scheme draws the model closer to the assimilated observations than DASWAM does. Most of the difference disappears quickly in the forecast, but the impact of OIP remains slightly larger at the observation locations. After 12 hours, most of the impact of the assimilation is lost. This is probably due to the fact that during most of the period, wind sea was the dominating wave system in the North Sea. Comparison of the model analysis results with independent ERS-1 and Topex-Poseidon altimeter wave height measurements confirms the negative bias of WAM in the Norwegian Sea. In the North Sea, a positive impact of the buoy assimilation is seen in the two methods. In the Norwegian Sea, the impact of OIP is larger, probably because the quality of the WAM first-guess is worse.

The wind speed corrections which are the result of the assimilation, do not improve or deteriorate significantly the quality of the wind fields, as compared to measurements from platforms, the ERS-1 and Topex-Poseidon altimeters, or the ERS-1 scatterometer.

The more elaborate variational scheme of DASWAM did not lead to more accurate wave analyses or predictions. In the wind-sea dominated test period, the better tuned statistical basis of OIP proved effective, whereas the potential of DASWAM for making non-local adjustments did not show any clear advantages. It is suspected that improvements in the description of the error statistics in the cost function, together with a retuning of the approximate wave model which is used in the analysis, may show the advantages of the variational method more clearly.

The impact of the SAR assimilation is generally small, especially in the North Sea, where the number of buoy observations is much larger than the number of SAR observations. In the Norwegian Sea, some impact is shown in the WAM/OIP scheme at analysis time. In this region, the number of conventional observations is small, and assimilation of satellite observations is expected to become increasingly important in the future.
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References

[1] Bauer, E., I.R. Young and K. Hasselmann (1994), ”Data assimilation using a Green’s function approach”, in: Komen et al (1994).

[2] Barzel, G., and R.B. Long (1994), ”Wave model fitting using the adjoint technique”, in: Komen et al (1994).

[3] Bidlot, J.-R., F. Ovidio and D. Van den Eynde (1995), ”Validation and improvement of the quality of the operational wave model MU-WAVE by the use of ERS-1 satellite data”, Report MUMM/T3/AR04, MUMM, Brussels, Belgium, 53 p.

[4] Breivik, L.-A., M. Reistad and H. Schyberg (1996), ”Assimilation of ERS SAR wave spectra in a numerical wave prediction model”, DNMI Research Report 31, ISSN 0332-9879, 36 p.

[5] Burgers, G., V.K. Makin, G. Quanduo and M. de las Heras (1992), ”Wave data assimilation for operational wave forecasting at the North Sea”, 3rd International Workshop on Wave Hindcasting and Forecasting, May 19-22, 1992, Montreal, Canada, 202-209.

[6] de Boor, C. (1978), ”A practical guide to splines”, Springer Verlag.

[7] de las Heras, M.M., G. Burgers and P.A.E.M. Janssen (1995), ”Wave data assimilation in the WAM wave model”, J. Marine Systems 6, 77-85.

[8] de Valk, C.F., and C.J. Calkoen (1989), ”Wave data assimilation in a third generation wave prediction model for the North Sea - an optimal control approach”, Report X38, Delft Hydraulics, Delft, Netherlands, 123 p.
de Valk, C.F. (1994), "A wind and wave data assimilation scheme based on the adjoint technique". In: Komen et al (1994).

Delft Hydraulics, HeereMac v.o.f., Shell Internationale Petroleum Maatschappij and Meteo Consult (1994), "ERS-1 pilot project Wavewatch - Impact of wave data from ERS-1 and buoys on wave forecasts and offshore safety. Phase 1: off-line demonstration", Report H1688, Delft Hydraulics, Delft, Netherlands.

Delft Hydraulics (1995), "MAESTRO, A marine information system to support offshore operations - Real-time data-assimilation and wave forecasting", Report H1829, Delft Hydraulics, Delft, Netherlands.

Fletcher, R. (1987), "Practical methods of optimization", John Wiley and Sons, Chichester.

Gerling, T.W. (1992), "Partitioning sequences and arrays of directional ocean wave spectra into component wave systems", J. Atm. Oc. Techn. 9, 444-458.

Gower, J.F.R. (1996), "Intercalibration of wave and wind data from TOPEX/POSEIDON and moored buoys off the west coast of Canada", J. Geophys. Res. 101, 3817-3829.

Günther, H., S. Hasselmann and P.A.E.M. Janssen (1992), "Wamodel Cycle 4", DKRZ Tech. Report 4, DKRZ, Germany, 102 p.

Günther, H., P. Lionello and B. Hanssen (1993), "The impact of the ERS-1 altimeter on the wave analysis and forecast", Report GKSS 93/E/44, GKSS Forschungszentrum Geesthacht, Geesthacht, Germany, 56 p.

Hasselmann, K., T.P. Barnett, E. Bouws, H. Carlson, D.E. Cartwright, K. Enke, J.A. Ewing, H. Gienapp, D.E. Hasselmann, P. Kruseman, A. Meerburg, P. Müller, D.J. Olbers, K. Richter, W. Sell and H. Walden (1973), "Measurements of wind-wave growth and swell decay during the Joint North Sea Wave project (JONSWAP)", Dtsch. Hydrogr. Z. Suppl. A 8(12), 95 p.

Hasselmann, K., S. Hasselmann, E. Bauer, C. Bruning, S. Lehner, H. Graber and P. Lionello (1988), "Development of a satellite SAR image spectra and altimeter wave height data assimilation system for ERS-1", Report 19, Max Planck Institut für Meteorologie, Hamburg, Germany.
[19] Hasselmann, K., and S. Hasselmann (1991), ”On the nonlinear mapping of an ocean wave spectrum into a SAR image spectrum and its inversion”, *J. Geophys. Res.* **C96**, 10713-10729.

[20] Hasselmann, S., G.J. Komen and K. Hasselmann (1994a), ”The general minimization problem”. In: Komen et al (1994).

[21] Hasselmann, S., C. Brüning and P. Lionello (1994b), ”Towards a generalized optimal interpolation method for the assimilation of ERS-1 SAR retrieved wave spectra in a wave model”, *Proc. Second ERS-1 Symp., Oct. 11-14, 1993, Hamburg, Germany, ESA SP-361*, 21-25.

[22] Hasselmann, S., C. Brüning, K. Hasselmann and P. Heimbach (1996a), ”An improved algorithm for the retrieval of ocean wave spectra from synthetic aperture radar image spectra”, *J. Geophys. Res.* **101** (**C7**), 16615-16629.

[23] Hasselmann, S., P. Lionello and K. Hasselmann (1996b), ”A wind- and wave- data assimilation scheme”, to appear in *J. Geophys. Res.*

[24] Hersbach, H. (1997), ”Application of the adjoint of the WAM model to inverse wave modelling”, submitted to *J. Geophys. Res.*

[25] Holthuijsen, L.H., N. Booij, M. van Endt, S. Caires, C. Guedes Soares (1996), ”Assimilation of buoy and satellite data in wave forecasts with integral control variables”, submitted to *J. Marine Systems*.

[26] Janssen, P.A.E.M., P. Lionello, M. Reistad and A. Hollingsworth (1989), ”Hindcasts and data assimilation studies with the WAM model during the Seasat period”, *J. Geophys. Res.* **C94**, 973-993.

[27] Källberg, P. (editor) (1990), ”The HIRLAM forecast model, level 1”, documentation manual, SMHI.

[28] Komen, G.J. (1985), ”Introduction to wave models and assimilation of satellite data in wave models”, in ”The use of satellites in climate models”, *ESA Scientific Publication SP-221*, 21-25.

[29] Komen, G.J., L. Cavaleri, M. Donelan, K. Hasselmann, S. Hasselmann and P.A.E.M. Janssen (1994), ”Dynamics and Modelling of ocean waves”, Cambridge University Press.
[30] Komen, G.J., A.C. Voorrips and J.P.T. Hersbach (1996), "Assimilation of satellite measurements in a sea state forecasting model," *Proc. of the Symposium on Operational Oceanography and Satellite Observation*, Biarritz, France, October 1995, 93-103.

[31] Kuik, A.J., G. Ph. van Vledder and L.J. Holthuijsen (1988), "A method for the routine analysis of pitch-and-roll wave buoy data", *J. Phys. Oceanogr.* **18**, 1020-1034.

[32] Lionello, P., and P.A.E.M. Janssen (1990), "Assimilation of altimeter measurements to update swell spectra in a wave model", *Proc. of the International Symposium on Assimilation of Observations in Meteorology and Oceanography*, Clermont-Ferrand, France, August 1990, 241-246.

[33] Lionello, P., H. Günther and B. Hansen (1995), "A sequential assimilation scheme applied to global wave analysis and prediction", *J. Marine Syst.* **6**, 87-107.

[34] Liu, D.C. and J. Nocedal (1988), "On the limited memory BFGS method for large scale optimization", *Technical report NAM 03*, Northwestern University, Evanston.

[35] Luenberger, D.G. (1969), "Optimization by vector space methods", John Wiley and Sons, Inc., N.Y.

[36] Mastenbroek, C., V.K. Makin, A.C. Voorrips and G.J. Komen (1994), "Validation of ERS-1 altimeter wave height measurements and assimilation in a North Sea wave model", *The Glob. Atm. Oc. System* **2**, 143-161.

[37] Queffeulou, P., A. Bentamy, Y. Quilfen and J. Tournadre (1994), "Validation of ERS-1 and TOPEX-POSEIDON wind and wave measurements", *IFREMER Techn. Rep.* DRO-OS No 94-08.

[38] Thomas, J. (1988), "Retrieval of energy spectra from measured data for assimilation into a wave model", *Q. J. R. Meteorolog. Soc.* **114**, 781-800.

[39] van Vledder, G. Ph. (1994), "PHIDIAS, A program for the computation of wind wave energy", *Report H1861*, Delft Hydraulics, Delft.

[40] Voorrips, A.C., V.K. Makin and S. Hasselmann (1996), "Assimilation of wave spectra from pitch-and-roll buoys in a North Sea wave model", *KNMI Memorandum OO-96-02*. To appear in *J. Geophys. Res.*
[41] Voorrips, A.C. (1997), ”Optimal Interpolation of Partitions: a data assimilation scheme for NEDWAM-4”, KNMI Scientific Report. To appear.

[42] WAMDI group: S. Hasselmann, K. Hasselmann, E. Bauer, P.A.E.M. Janssen, G.J. Komen, L. Bertotti, P. Lionello, A. Guillaume, V.C. Cardone, J.A. Greenwood, M. Reistad, L. Zambresky and J.A. Ewing (1988), ”The WAM model - a third generation ocean wave prediction model”, J. Phys. Oceanogr. 15, 566-592.
| Parameter | N  | NOASS RMSE | NOSAR (%) | NOASS (%) | NOSAR (%) |
|-----------|----|------------|-----------|-----------|-----------|
|           |    | -3h - 0h   | +3h - 6h  | +9h - 12h |
| $H_s$     | 40 | 1.23 m     | 38        | 80        | 93        |
| $H_{10}$  | 40 | 1.31 m     | 39        | 76        | 91        |
| $T_m$     | 40 | 1.63 s     | 36        | 71        | 91        |
| $\theta_w$| 40 | 13.73 deg  | 59        | 78        | 93        |
| $U_{10}$  | 40 | 5.12 m/s   | 107       |           |           |

Table 1: RMS error reduction for WAM/OIP at NOC.

| Parameter | N  | NOASS RMSE | NOSAR (%) | NOASS (%) | NOSAR (%) |
|-----------|----|------------|-----------|-----------|-----------|
|           |    | -3h - 0h   | +3h - 6h  | +9h - 12h |
| $H_s$     | 40 | 0.87 m     | 82        | 92        | 94        |
| $H_{10}$  | 40 | 0.85 m     | 82        | 97        | 95        |
| $T_m$     | 40 | 0.89 s     | 104       | 101       | 102       |
| $\theta_w$| 40 | 18.46 deg  | 80        | 101       | 102       |
| $U_{10}$  | 40 | 5.59 m/s   | 103       |           |           |

Table 2: RMS error reduction for DASWAM at NOC.

| Parameter | N  | NOASS RMSE | NOSAR (%) | NOASS (%) | NOSAR (%) |
|-----------|----|------------|-----------|-----------|-----------|
|           |    | -3h - 0h   | +3h - 6h  | +9h - 12h |
| $H_s$     | 67 | 0.86 m     | 32        | 72        | 89        |
| $H_{10}$  | 67 | 0.91 m     | 33        | 64        | 85        |
| $T_m$     | 59 | 1.00 s     | 36        | 73        | 82        |
| $\theta_w$| 59 | 14.53 deg  | 51        | 96        | 99        |
| $U_{10}$  | 13 | 2.78 m/s   | 77        |           |           |

Table 3: RMS error reduction for WAM/OIP at AUK.

| Parameter | N  | NOASS RMSE | NOSAR (%) | NOASS (%) | NOSAR (%) |
|-----------|----|------------|-----------|-----------|-----------|
|           |    | -3h - 0h   | +3h - 6h  | +9h - 12h |
| $H_s$     | 67 | 0.71 m     | 65        | 84        | 96        |
| $H_{10}$  | 67 | 0.87 m     | 60        | 78        | 96        |
| $T_m$     | 59 | 1.16 s     | 88        | 88        | 96        |
| $\theta_w$| 59 | 25.24 deg  | 86        | 95        | 92        |
| $U_{10}$  | 13 | 2.91 m/s   | 133       |           |           |

Table 4: RMS error reduction for DASWAM at AUK.
| Parameter | $N$ | NOASS RMSE (m) | NOASS (%) | NOASS (%) | NOASS (%) |
|-----------|-----|----------------|-----------|-----------|-----------|
| $H_s$    | 72  | 0.40          | 37        | 69        | 91        |
| $H_{10}$ | 72  | 0.18          | 71        | 54        | 81        |
| $T_m$    | 48  | 0.51          | 47        | 63        | 83        |
| $\theta_w$ | 48  | 17.37        | 36        | 83        | 106       |
| $U_{10}$ | 72  | 1.92          | 98        |           |           |

Table 5: RMS error reduction for WAM/OIP at K13.

| Parameter | $N$ | NOASS RMSE (m) | NOASS (%) | NOASS (%) | NOASS (%) |
|-----------|-----|----------------|-----------|-----------|-----------|
| $H_s$    | 72  | 0.41          | 87        | 94        | 94        |
| $H_{10}$ | 72  | 0.33          | 86        | 83        | 87        |
| $T_m$    | 48  | 0.67          | 97        | 103       | 99        |
| $\theta_w$ | 48  | 15.83        | 96        | 97        | 98        |
| $U_{10}$ | 72  | 2.13          | 100       |           |           |

Table 6: RMS error reduction for DASWAM at K13.

| Parameter | $N$ | NOASS RMSE (m) | NOASS (%) | NOASS (%) | NOASS (%) |
|-----------|-----|----------------|-----------|-----------|-----------|
| $H_s$    | 75  | 0.54          | 76        | 76        | 89        |
| $H_{10}$ | 75  | 0.35          | 67        | 58        | 76        |
| $T_m$    | 42  | 0.59          | 94        | 68        | 89        |
| $\theta_w$ | 42  | 15.10        | 93        | 122       | 105       |
| $U_{10}$ | 75  | 2.37          | 98        |           |           |

Table 7: RMS error reduction for WAM/OIP at SON.

| Parameter | $N$ | NOASS RMSE (m) | NOASS (%) | NOASS (%) | NOASS (%) |
|-----------|-----|----------------|-----------|-----------|-----------|
| $H_s$    | 75  | 0.54          | 92        | 104       | 94        |
| $H_{10}$ | 75  | 0.47          | 84        | 98        | 91        |
| $T_m$    | 42  | 0.90          | 78        | 92        | 87        |
| $\theta_w$ | 42  | 16.53        | 104       | 97        | 104       |
| $U_{10}$ | 75  | 2.35          | 100       |           |           |

Table 8: RMS error reduction for DASWAM at SON.
| area | satellite | run | N  | wave height | wind speed |
|------|-----------|-----|----|-------------|------------|
|      |           |     |    | bias | σ | bias | σ |
| I    | ERS-1     | NOASS | 340 | -0.36 | 0.72 | 0.15 | 1.84 |
| I    | ERS-1     | NOSAR | 340 | -0.17 | 0.53 | 0.31 | 1.90 |
| I    | ERS-1     | SAR  | 340 | -0.16 | 0.53 | 0.30 | 1.89 |
| I    | TOPEX     | NOASS | 510 | -0.26 | 0.53 | 0.47 | 1.68 |
| I    | TOPEX     | NOSAR | 510 | -0.12 | 0.43 | 0.49 | 1.69 |
| I    | TOPEX     | SAR  | 510 | -0.14 | 0.42 | 0.50 | 1.68 |
| II   | ERS-1     | NOASS | 662 | -1.37 | 0.78 | -0.86 | 2.29 |
| II   | ERS-1     | NOSAR | 662 | -1.24 | 0.77 | -0.86 | 2.28 |
| II   | ERS-1     | SAR  | 662 | -1.18 | 0.77 | -0.82 | 2.44 |
| II   | TOPEX     | NOASS | 1764 | -1.25 | 0.76 | -0.45 | 2.37 |
| II   | TOPEX     | NOSAR | 1764 | -1.14 | 0.75 | -0.41 | 2.38 |
| II   | TOPEX     | SAR  | 1764 | -1.08 | 0.73 | -0.42 | 2.37 |

Table 9: Altimeter statistics for WAM/OIP.

| area | satellite | run | N  | wave height | wind speed |
|------|-----------|-----|----|-------------|------------|
|      |           |     |    | bias | σ | bias | σ |
| I    | ERS-1     | NOASS | 298 | 0.02 | 0.72 | 0.22 | 1.86 |
| I    | ERS-1     | NOSAR | 298 | 0.08 | 0.55 | 0.21 | 2.03 |
| I    | ERS-1     | SAR  | 298 | 0.11 | 0.56 | 0.27 | 2.06 |
| I    | TOPEX     | NOASS | 496 | 0.21 | 0.64 | 0.40 | 1.67 |
| I    | TOPEX     | NOSAR | 496 | 0.18 | 0.64 | 0.34 | 1.69 |
| I    | TOPEX     | SAR  | 496 | 0.23 | 0.73 | 0.32 | 1.69 |
| II   | ERS-1     | NOASS | 407 | -0.53 | 0.88 | -0.81 | 2.34 |
| II   | ERS-1     | NOSAR | 407 | -0.56 | 0.81 | -1.06 | 2.32 |
| II   | ERS-1     | SAR  | 407 | -0.56 | 0.91 | -1.33 | 2.28 |
| II   | TOPEX     | NOASS | 779 | -0.30 | 0.88 | -0.37 | 2.33 |
| II   | TOPEX     | NOSAR | 779 | -0.38 | 0.79 | -0.44 | 2.27 |
| II   | TOPEX     | SAR  | 779 | -0.34 | 0.85 | -0.40 | 2.31 |

Table 10: Altimeter statistics for DASWAM.
Figure 1: Map of the area. Plusses indicate the DASWAM (Delft Hydraulics model) area. Double solid lines are the boundary of the WAM (KNMI model) domain. Single solid lines indicate areas I and II, which are used for the statistical analysis of model results against altimeter observations. Filled squares indicate the position of four wave buoys. NOC: North Cormorant; AUK: Auk Alpha; SON: Schiermonnikoog Noord.
Figure 2: Wind fields at the maximum of the Wadden storm. Upper left: February 20, 15 h GMT. Upper right: February 20, 21 h GMT. Lower left: February 21, 3 h GMT. Lower right: February 21, 9 h GMT.
Figure 3: Modeled and observed wind speed at four locations for the period February 19-24, 1993 (the "Wadden Storm"). Markers: observations. Solid line: HIRLAM model results.
Figure 4: Modeled and observed significant wave height at four locations for the period February 19-24, 1993. All model runs without assimilation. Markers: observations. Dotted line: NOASS analysis run with WAM. Dashed line: NOASS analysis run with DASWAM (2nd generation model). Solid lines: 24 hour NOASS forecast runs with DASWAM (3d generation model), starting every day at 0 GMT from the DASWAM NOASS analysis.
Figure 5: Modeled and observed mean wave period at four locations for the period February 19-24, 1993. Line types and markers as in figure 4.
Figure 6: WAM/OIP significant wave height results for the period February 19-24, 1993. Markers: observations. Thick solid line: WAM NOSAR analysis run. Thin solid lines: WAM NOSAR forecast runs. Dotted line: WAM NOASS run.
Figure 7: DASWAM significant wave height results for the period February 19-24, 1993. Markers: observations. Thick solid line: DASWAM NOSAR analysis run. Thin solid lines: DASWAM NOSAR forecast runs. Dotted lines: DASWAM NOASS forecast runs.
Figure 8: WAM/OIP model wind and wave height fields at February 21, 21 GMT. Upper left: HIRLAM wind field. Upper right: $H_s$ for the NOASS run. Lower left: increments of $U_{10}$ for the NOSAR assimilation run. Lower right: increments of $H_s$ for the NOSAR run. Also indicated is the ERS-1 altimeter track and the location of the four buoys.
Figure 9: DASWAM model wind and wave height fields at February 21, 21 GMT. Upper left: HIRLAM wind field. Upper right: $H_s$ for the NOASS run. Lower left: increments of $U_{10}$ for the NOSAR assimilation run. Lower right: increments of $H_s$ for the NOSAR run.
Figure 10: Comparison of WAM/OIP and ERS-1 altimeter for the track of February 20, 21 GMT (the track is shown in figure 8). Upper panel: significant wave height. Lower panel: Wind speed. Small square markers: altimeter observations. Stars: corresponding NOASS (analysis) model results. Plusses: NOSAR (analysis) results. Large squares: buoy and platform observations at K13 and AUK.
Figure 11: Comparison of DASWAM and ERS-1 altimeter for the track of February 20, 21 GMT (the track is shown in figure 9). Symbols: as in figure 10.
Figure 12: Impact of SAR assimilation on the significant wave height with the WAM/OIP system. Left panels: wave height fields at February 20, 21 h GMT, when the ERS-1 passed over. Plusses indicate the SAR observations. Upper left: wave field of the SAR analysis run. Lower left: difference between the SAR and the NOSAR analyses at the same time. Right panels: wave fields at February 21, 9 h GMT, when the Topex-Poseidon passed. The thick sold line indicates its altimeter track. Upper right: SAR analysis. Lower right: difference with the NOSAR analysis.
Figure 13: Comparison of WAM/OIP SAR and NOSAR runs with TOPEX-POSEIDON altimeter wave height at February 21, 9 GMT. The altimeter track is shown in figure [12], right panels. Markers: altimeter wave height. Plusses: SAR assimilation results. Stars: NOSAR assimilation results.