Doppler reflectometry studies of plasma gradient instabilities in L-2M stellarator

N N Skvortsova1,2,9, A Yu Chirkov3, A A Kharchevsky4, D V Malakhov5,6, A K Gorshenin7 and V Yu Korolev8

1 Leading research associate, Prokhorov General Physics Institute of the Russian Academy of Sciences, Vavilova 38, 119991, Moscow, Russia
2 Professor, National Research Nuclear University MEPhI (MEPhI), Kashirskoe shosse 31, Moscow, 115409 Russian Federation
3 Professor, Bauman Moscow State Technical University
4 Ph. D. Student, Moscow State Institute of Radio Engineering, Electronics and Automation
5 Senior research associate, Prokhorov General Physics Institute of the Russian Academy of Sciences
6 Associate professor, Bauman Moscow State Technical University
7 Senior research associate, The Institute of Informatics Problems of the Russian Academy of Sciences
8 Professor, Moscow State University

E-mail: mukudori@mail.ru, 89168766306@mail.ru

Abstract. Paper reports on a broadband spectrum analysis of density fluctuations in the edge plasma of the L-2M stellarator, measured using the Doppler reflectometry diagnostics. The evolution of the electron temperature-gradient (ETG) and ion temperature-gradient (ITG) instabilities in the edge plasma of the stellarator is analyzed. Two additional harmonics in the spectrum are found apart from those determined by the Doppler shift caused by poloidal rotation of plasma. The Doppler shifts of these harmonics are determined from the phase velocities of fluctuations generated by the ETG and ITG instabilities. Thus, local measurements of spectra using the Doppler reflectometry enabled us both to evaluate the poloidal rotational velocity of the plasma and describe the evolution of low-frequency plasma instabilities.

1. Introduction

It is known that plasma is a state of matter with big number of degrees of freedom so the interpretation of plasma turbulence spectra is an ill-posed problem. Studies of low-frequency plasma fluctuations in the L-2M stellarator showed that the structural turbulence is observed throughout the plasma volume. Previously it was shown that it is possible to determine the number of unstable plasma processes which form such a turbulence [1, 2].

Spectra of plasma density fluctuations measured using the Doppler reflectometry (DR) diagnostics in the edge plasma of the L-2M stellarator are broadband (the half-width exceeds maximum frequency) and correspond to structural turbulence spectra [3]. Measurement of the Doppler frequency

To whom any correspondence should be addressed
shift in such spectra is difficult because the shift can depend not only on the poloidal rotation velocity of plasma but also the phase velocity of fluctuations (provided the velocities are of the same order of magnitude). The latter conditions are typical for the DR measurement in many toroidal facilities, e.g. ASDEX and TEXTOR tokamaks [4]. In stellarators, fluctuations in the spectral range measured by the DR may occur due to the gradient drift instabilities: ion temperature-gradient (ITG) instability, electron temperature-gradient (ETG) instability and the trapped electron mode (TEM) [5]. Data obtained using the DR diagnostics can be treated as local ones and, hence, comparing it to the calculated local characteristics of instabilities would be most appropriate.

In this paper the broadband spectrum measured by the DR in the L-2M stellarator was analyzed assuming that the particular random processes are responsible for formation of the measured spectra. In our study, the ITG, ETG and TEM instabilities are analyzed as contributing to the DR measurements. For all the spectra analyzed the spectral components, produced by poloidal rotation of plasma or structural turbulence modes, were extracted. The components related to poloidal plasma rotation, determined by the radial electric field, and structural turbulence modes resulted from the ITG and ETG instabilities.

2. Experimental setup and reflectometry diagnostics description
The experiments were carried out for a standard magnetic configuration of the L-2M stellarator (major radius $R = 100$ cm, averaged plasma radius $a = 11.5$ cm) when the ratio between the amplitude of fundamental harmonic of helical field and the amplitude of longitudinal field on the axis of the facility amounts to 0.228. In this case the rotational transformation angles on the magnetic axis and the boundary magnetic surface for vacuum magnetic configuration are equal to $\varphi(0)/2\pi = 0.185$ and $\varphi(a)/2\pi = 0.78$, respectively. The detailed description of the facility is given in [6]. The electron cyclotron (EC) heating is performed on the second harmonic of electron gyrofrequency (75.3 GHz) using a gyrotron with the power $P$ up to 400 kW at pulse duration of 10–15 ms. Average plasma density measured along central chord was equal to $2 \cdot 10^{13}$ cm$^{-3}$. Average plasma density measured at injection angles of 4, 8, 12, 17$^\circ$ and $\approx 1–2$ cm away from the last closed magnetic surface. Temperature in the plasma core lies within the 300–1000 eV range, depending on the power of the EC heating. It should be noted that at a high heating power, the “dip” on the density profile in the plasma core occurs [7].

This effect called the “density pump-out” can be observed in tokamaks and stellarators at high power densities of external heating. In the L-2M, the “density pump-out” is distinct at the power density of EC heating of the value of $P > 2$ MW/m$^3$.

Long-wavelength fluctuations of plasma density ($k_\perp \rho_s \approx 0.1–0.2$, where $\rho_s$ is the ion cyclotron radius) near the separatrix ($r/a = 0.8–0.9$) were studied by the DR diagnostics. The design of the DR diagnostics device in the L-2M stellarator, implemented in a classic two-channel scheme with Gunn diode serving as a source of probing radiation, is given in [8]. In our experiments the measurements were conducted using an optimized Doppler reflectometer. The probing radiation is injected through a quasioptical system and the signal is registered within the broad band up to 2.5 MHz [9]. The Fourier spectra measurement error (half-width and intensity) evaluated from the signals of the optimized Doppler reflectometer in the steady-state stage of plasma discharge is of 20–30%.

In the experiment the measurements were conducted at two probing frequencies of 31.2 and 34.3 GHz, at injection angles of 4$^\circ$, 8$^\circ$, 12$^\circ$, 17$^\circ$ and $\approx 1–2$ cm away from the last closed magnetic surface. Radiation polarization corresponds to an ordinary wave in plasma. Scattering of the probing wave occurred at long-wavelength fluctuations of plasma with poloidal components of wave vector $k_\perp \approx 0.9–4.2$ cm$^{-1}$.

3. Mathematical model of instabilities in the edge plasma in stellarator
The class of gradient drift instabilities includes the following modes: ITG, ETG and TEM. In classical works, the characteristic parameters of the ITG and ETG modes are defined as follows: for ITG, frequency $\omega \sim \omega_{ci}$, transverse (with respect to the magnetic field) wave number $k_\perp \sim 1/\rho_{Ti}$; for ETG, $\omega \sim \omega_{ce}$, $k_\perp \sim 1/\rho_{Te}$, where $\omega_{ci}$ and $\omega_{ce}$ are the frequencies of the diamagnetic drift of ions.
and electrons, $\rho_i$ and $\rho_e$ are the cyclotron radii of ions and electrons, calculated according to their thermal velocities [13]. The real part of the frequency of the ion mode $\omega_R < 0$, for electron mode one has $\omega_R > 0$. For tokamaks the following classification is used: for ITG, $k_\perp \rho_x \lesssim 0.5$; for TEM, $k_\perp \rho_x \gtrsim 2$; for ETG, $k_\perp \rho_x > 0.5$, where $\rho$ is the ion cyclotron radius, calculated from the electron temperature [14]. These classifications are conditional. Depending on the conditions of the particular magnetic configurations the parameters of modes and directions of their propagation may differ significantly from the above [15]. Also there’s not always a clear border between the ranges of ion and electron modes, because their overlapping is possible [5]. Within gyrokinetics approach the instability in plasma is associated with the resonant denominator in the perturbation of the distribution function:

$$\omega + \omega_{Dj} - k_|| v || \approx 0$$

where $\omega$ is the complex value of frequency, $\omega_{Dj}$ is the orbit-averaged frequency of magnetic drift of particle of the species $j$ $(j = i, e)$, $k_||$ is the longitudinal (relative to the magnetic field) component of the wave vector, $v ||$ is the longitudinal component of the particle velocity.

For the passing particles, one has $\omega_{Dj} \approx 0$. Therefore, for unstable modes with no trapped particles it is needed that $k_|| \neq 0$. In toroidal plasma for the majority of the trapped ions one has $\omega_{Dj} > 0$, for the majority of trapped electrons, $\omega_{Dj} < 0$. Because of the differences in the velocities, the effect of the trapped electrons is more significant. According to (1), the trapped electrons can cause the instability when $k_|| \approx 0$. As calculated in [15], the growth rate $\gamma$ of such modes is sharply reduced when the aspect ratio $R/a > 5$ ($R$ is major radius, $a$ is plasma minor radius). As the average aspect ratio of the L-2M stellarator is $R/a \sim 5$, the effect of the trapped particles can be neglected in the first approximation. Note that in the stellarator, the particles can be locally trapped due to inhomogeneity of magnetic configuration. Thus, we restrict our analysis to the most significant modes from the class of gradient drift instabilities: ITG and ETG. Given that the ratio of plasma pressure to magnetic pressure $\beta$ $< 0.1$ in the L-2M, we will use the electrostatic approximation. In this case, the dispersion equation that combines the ITG and ETG modes has the form [15]:

$$1 + \left[ 1 - \frac{\omega_{zs}}{\omega} \left( 1 - \frac{3}{2} \eta_e \right) \right] \xi^2 Z(\xi) \Gamma_0(b_e) - \frac{\omega_{ze}}{\omega} \eta_e \xi Z(\xi) \Gamma_0(b_e) - \frac{\omega_{ze}}{\omega} \eta_e \xi b_e \left( \Gamma_1(b_e) - \Gamma_0(b_e) \right) = 0$$

$$= \frac{T_e}{T_i} \left[ 1 + \left( 1 + \frac{\omega_{si}}{\omega} \left( 1 - \frac{3}{2} \eta_i \right) \right) \xi^2 Z(\xi) \Gamma_0(b_i) + \frac{\omega_{si}}{\omega} \eta_i \xi Z(\xi) \Gamma_0(b_i) + \frac{\omega_{si}}{\omega} \eta_i \xi b_i \left( \Gamma_1(b_i) - \Gamma_0(b_i) \right) \right]$$

(2)

Here $\omega_{si} = k_\perp \frac{k_i T_i}{q_i B L_n}$, $\omega_{ze} = k_\perp \frac{k_e T_e}{e B L_n}$, $k_B$ is the Boltzmann constant, $T_i$ and $T_e$ are ion and electron temperatures, $q_i$ is the charge of the ions, $e$ is the electron charge, $B$ is the magnetic induction, $L_n$ is the spatial scale of the density gradient, $\eta_i$ and $\eta_e$ are the relative gradients of ion and electron
temperatures, \( \Gamma_n(b) = I_n(b) \exp(-b) \), \( I_n(b) \) is the modified Bessel function, \( b_i = k_{i,\perp}^2 \rho_{i}^2 \), \( b_e = k_{e,\perp}^2 \rho_{e}^2 \),

\[
Z(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-u^2} \, du
\]
is plasma dispersion function of argument \( \xi_i = \frac{\omega}{k_i \sqrt{2k_{i,\perp} T_i} / m_i} \) (for ions) or
\[
\xi_e = \frac{\omega}{k_e \sqrt{2k_{e,\perp} T_e} / m_e}
\]
(for electrons), \( m_i \) and \( m_e \) are ion and electron masses.

The analysis was performed for different regimes of plasma confinement in the L-2M with different levels of the EC heating power (200 and 400 kW). Figure 1 shows the model profiles of temperature, density and pressure for different modes used in the analysis taking into account the density pump-out effect. In figure, \( r \) is the radial coordinate. The measured temperature profiles are well described by the quadratic parabola with a pedestal at the plasma boundary. The thickness of the boundary layer where the density drops to zero is approximately equal to 1 cm (the boundary layer where the density drops to zero is approximately 1 cm thick). The temperature of ions will be considered as a constant over the cross section, except for the region near the boundary. The regimes 1–3 correspond to different time instants of quasi-steady-state phase of the discharge at a heating power of 400 kW without the limiter. In the regime 4 (with the limiter), the heating power is 200 kW, and the density profile is well approximated by a sixth-order parabola.

**Figure 1.** Top: profiles of temperature (\( a \)), density (\( b \)) and pressure (\( c \)) 1, 2, 3 – at different time instants at quasi-stationary (“flat-top”) phase of discharge in the density pump-out operation mode at high heating power; 4 – 100 kW mode with low heating power of 100 kW (sixth order parabola); 5 – second-order parabola (given for comparison). Bottom: relative density gradient (\( a \)) and the ratio of gradients of electron temperature and density (\( b \)) for the regimes 1–5.

4. **Evolution of the Fourier spectra of fluctuations during the plasma discharge**

Figure 2 presents the evolution of the Fourier spectra of the plasma density fluctuations in the steady-state phase of the discharge. The dark line on the background of a noisy complex spectrum in each time frame shows the average (stable) spectra. At the quasi-stationary phase of the discharge within 54 to 62 ms interval at a constant average plasma density of \( (1.8–1.9) \times 10^{13} \) cm\(^{-3} \). The shifted broadband
noisy spectra correspond to the steady-state of the discharge. The spectra half-width and intensity, recovered from the averaged values, vary but only slightly (frequency band variation is 20%, the spectral intensity variation is 30%).

**Figure 2.** The evolution of the Fourier spectrum (gray) and the stable Fourier spectrum (black line) of plasma density fluctuations during the discharge in the L-2M stellarator. The time frame for spectrum estimations is 1 ms. The power of the EC heating is 400 kW. Shot number is # 19046 (43491).

Thus, an optimized Doppler reflectometer enabled us to obtain a repetitive complex-valued Fourier spectrum in the steady-state stage of the plasma discharge in the ECR heating mode of L-2M stellarator. The last two spectra in figure 2 (13 ms and 15 ms) correspond to the evolution of fluctuations in the end of discharge with the plasma column rotation velocity of about 150 m/s (the shift is evaluated by the local maximum of the spectrum). Note that in the mode with the high power heating during the formation of the density pump-out in the plasma core, the Doppler shift of the spectrum is reduced compared to the modes with a lower power and a parabolic profile of plasma density [3].

In the course of the research it was found that reducing the error when measuring the spectra with the optimized DR diagnostics has not led to improvements in the poloidal rotation velocity estimation, compared to the non-optimized reflectometer measurement. Figure 2 shows the local maximum value and the corresponding Doppler frequency shift of Fourier spectra at each time frame. Assuming that the poloidal rotation velocity of fluctuations exceeds the phase velocity, this frequency shift may be associated with the velocity of plasma poloidal rotation. Assuming that the radial electric field, which determines the poloidal rotation velocity of the plasma at the steady-state stage, is constant, the Doppler frequency shift is expected to remain the same for a given accuracy of measurement of the spectra. However, the Doppler shift in the 12 ms spectrum is 0.117 MHz, while in the 15 ms spectrum is 0.022 MHz. The Doppler shift, estimated from the spectra’s maximum, varies almost by an order of magnitude at the steady-state stage of the discharge. This may indicate that the Doppler frequency shift is determined not only by the poloidal rotation velocity, but also by its mixture with the phase velocities of the turbulent fluctuations (provided the velocities are of the same order of magnitude). In this case, to determine the poloidal velocity it is necessary to highlight the harmonics associated with
rotation of the plasma on the background of harmonics determined by the phase velocities of the turbulence in the complex-valued spectrum of the Doppler reflectometer signal.

Figure 3 shows the decomposition of the Fourier spectra of fluctuations of the measured DR signal on the components using the method presented in [16, 17]. The decomposition of the two spectra in the stationary stage of the discharge in 52–54 ms and 58–60 ms time windows is given. Initially six processes are set. In the considered spectral range we can distinguish the three processes, exceeding the experimental noise, which has characteristic values of the Doppler frequency shift. Three harmonics are clearly distinct. Two additional modes, besides harmonics related to the plasma rotation, correspond to oscillations propagating in the opposite directions: phase velocity of the first oscillation is directed towards the electron diamagnetic drift, the phase velocity of the second oscillation is directed towards the ion diamagnetic drift.

Figure 3. Decomposition of the Fourier spectra of fluctuations of the measured DR signal. The power of the EC heating is 400 kW.

Figure 4 shows the relative scale of the density gradient $a/L_n$ and the relative gradient of the electron temperature $\eta_e$, that is needed to calculate the development of the ETG and ITG modes in the edge plasma of the L-2M. The ion temperature gradient was assumed to be zero ($\eta_i = 0$) to calculate the ETG mode. The results of the calculations are shown in figure 4. The values of $\omega_0 = k_B T_i/(eB_0 \rho_{Ti}) = 8.5 \times 10^5$ rad/s ($f_0 = 135$ kHz), and $\rho_{Ti} = 0.75$ mm were taken as the reference values. Higher values of the increments correspond to peripheral regions of the plasma column. For the calculation of the ITG-mode the presence of a gradient of ion temperature at the plasma periphery was assumed. We used the relative gradient with $\eta_i = 3$.

Figure 4. The real value of the complex-valued frequency (1, 3, 5) and the growth rate (2, 4, 6) of ETG instability versus transverse wave number at $r/a = 0.95$ (a) and $r/a = 0.6$ (b) for $\eta_i = 0$, $k_B a = 0.05$: 1, 2 – pump-out profile; 3, 4 – sixth-order parabola. The scales are as follows: $\omega_0 = k_B T_i/(eB_0 \rho_{Ti}) = 8.5 \times 10^5$ rad/s ($f_0 = 135$ kHz), $\rho_{Ti} = 0.7$ mm.
The results of calculations are presented in [17]. Under the above conditions the typical increments of the ion and electron modes are close within the order of magnitude. Thus, in the L-2M stellarator, the ITG and ETG modes can be observed simultaneously. Let’s estimate typical values of mode frequency for the detecting wave number $k_\perp \approx 2 \text{ cm}^{-1}$ in the coordinate system rotating in poloidal direction along with the plasma. The frequency shift relative to the poloidal rotation frequency is of the order of 1 MHz for the ETG mode, and 100 kHz for the ITG mode. The estimated values are in a qualitative agreement with the data presented in figure 2.

5. Conclusion
The analysis of the fluctuations spectra and instabilities was performed for various regimes of plasma confinement in the L-2M stellarator at electron-cyclotron heating power of 200 and 400 kW. It is theoretically shown that the development of the ETG and ITG instabilities in the edge of the stellarator plasma for the regimes with a high heating power and the formation of a dip on the density profile in the plasma core (the density pump-out effect) are possible. In all the spectra analysed, one can distinguish the components associated with the poloidal plasma rotation determined by the radial electric field, and the modes of structural turbulence of two types (determined by the ITG and ETG instabilities). It was found out that to reduce the error of the optimized Doppler reflectometry diagnostics of the plasma rotation poloidal velocity it is necessary to identify the harmonic of the complex-valued spectrum, related to the rotation of the plasma, on the background of harmonics determined by the turbulence’s phase velocities. In this case, the local measurements of spectra using the Doppler reflectometry diagnostics enables one not only to measure the plasma rotational velocity, but also to describe the development of low-frequency plasma instabilities.

6. Acknowledgment
Work was partially supported by the Ministry of Education and Science of the Russian Federation, the contract No. 13.2573.2014/K

References
[1] Korolev V Yu and Skvortsova N N 2006 Stochastic Models of Structural Plasma Turbulence (Utrecht: VSP)
[2] Batanov G M et al 2013 JETP Letters 78 502
[3] Akulina D K et al 2008 Plasma Physics Reports 34 1059
[4] Conway G D 2008 Plasma Phys. Control. Fusion 50 124026
[5] Chirkov A Yu and Khvesyuk V I 2011 Plasma Physics Reports 37 437
[6] Kharchev N K et al 2011 Plasma and Fusion Research 6 2402142
[7] Letunov A A et al 2014 Proc. of XLI Int. Conf. of Plasma Phys. and Controlled Fusion (Zvenigorod) 73
[8] Pshenichnikov A A et al 2005 Plasma Physics Reports 31 554
[9] Chernov N A et al 2013 Proc. of XXXIX Int. Conf. of Plasma Phys. and Controlled Fusion (Zvenigorod) 68
[10] Mikhailovskii A B 1977 Theory of Plasma Instabilities 2
[11] Rhodes T L et al 2007 Plasma Phys. Control. Fusion 49 B183
[12] Chirkov A Yu and Khvesyuk V I 2010 Phys. Plasmas 17 01210
[13] Jenko F et al 2000 Phys. Plasmas 7 1904
[14] Chirkov A Yu 2011 J. Fusion Energy 33 139
[15] Lee Y C et al 1987 Phys. Fluids 30 1331
[16] Gorshenin A K et al 2011 Mathematical modelling 23 83
[17] Malakhov D et al 2014 Proc. XXXII Int. Seminar on Stability Problems for Stochastic Models (Trondheim) 68