Critical aging of a ferromagnetic system from a completely ordered state

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We adapt the non-linear \( \sigma \) model to study the nonequilibrium critical dynamics of \( O(n) \) symmetric ferromagnetic system. Using the renormalization group analysis in \( d = 2 + \varepsilon \) dimensions we investigate the pure relaxation of the system starting from a completely ordered state. We find that the average magnetization obeys the long-time scaling behavior almost immediately after the system starts to evolve while the correlation and response functions demonstrate scaling behavior which is typical for aging phenomena. The corresponding fluctuation-dissipation ratio is computed to first order in \( \varepsilon \) and the relation between transverse and longitudinal fluctuations is discussed.

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Aging phenomena have been found in a broad variety of strongly disordered systems such as polymer and spin glasses [1], electronic system of an Anderson insulator [2], array of flux lines pinned by disorder [3], etc. Recently much attention has been attracted by aging of pure (or weakly disordered) systems with slow relaxation dynamics governed, for example, by domain growth [4] as in a ferromagnetic system below the critical temperature \( T_c \) or by critical slowing down as in a ferromagnetic system exactly at criticality [5]. One expects that the critical aging phenomena can be cast into different universality classes of nonequilibrium critical dynamics. Most of studies consider the relaxation of a ferromagnetic system starting from a completely disordered state after quenching it to the fixed temperature \( T \leq T_c \). It was found that the response function \( R(t,s) \) and the correlation function \( C(t,s) \) depend non-trivially on the ratio \( x = t/s \) similar to that found in glassy systems. Here \( s \) and \( t \) are waiting and observation times respectively. The distance from equilibrium can be measured by the fluctuation-dissipation ratio (FDR) \( X(t,s) = TR(t,s)/\partial_s C(t,s) \). It has been argued that for critical aging the limit \( X^\infty = \lim_{s^{-}\rightarrow\infty}X(t,s) \) is a novel universal quantity of critical phenomena [3]. The FDR was computed for the \( d \) dimensional spherical model [3], \( O(n) \) symmetric ferromagnetic model [6] and diluted spin models [7, 8]. In all these systems \( X^\infty \) has values ranging between 0 and 1/2. The mean field calculations show that the aging behavior is modified in the presence of long-range correlations in the initial disordered state [10, 11]. Much less known about the relaxation starting from an ordered state. The numerical simulations show that the correlation function demonstrates behavior which is typical for aging phenomena [8, 12], while the magnetization obeys the long-time scaling behavior almost immediately after the system starts to evolve [13, 14]. This observation was used to develop new effective numerical methods to determine the critical exponents, which are based on the short-time critical dynamics and do not require the time-consuming equilibration of the system [14]. However, up to now there is no any theoretical explanation why the long-time scaling behavior emerges already in the macroscopically early initial stage of relaxation and there is no theoretical framework which allows one to take properly into account the critical fluctuations in this regime of aging.

In this paper we study the nonequilibrium critical dynamics of a ferromagnetic system starting from a completely ordered state. The long-distance properties of the \( O(n) \) symmetric system below the transition point can be related to the non-linear \( \sigma \) model defined by the reduced Hamiltonian [16]

\[
\mathcal{H} = \int d^4x \left[ \frac{1}{2} (\nabla s)^2 - h \cdot s \right], \quad |s(x)|^2 = 1. \tag{1}
\]

In principle, to describe properly the dynamics of a real isotropic magnet one has to consider the Larmor precession of the conserved field \( s \) in the local magnetic field \( h \), i.e. construct the low-temperature version of the model J [17]. However in this work, which to our knowledge is the first analytical study of the critical aging of a finite range system from a completely ordered state, we are interested in the influence of the initial condition rather than in the complications caused by more realistic dynamic model. We restrict consideration to the pure relaxational dynamics (model A) with the non-conserved \( n \)-component order parameter \( s(x) \). It can be described by the Langevin equation \( \partial_t s = -\lambda \delta \mathcal{H}/\delta s + \zeta \), where \( \lambda \) is an Onsager coefficient and \( \zeta \) is a random thermal noise [18, 19, 20]. Because of the constraint \( s^2 = 1 \) the random noise can act only in the tangential direction and thus is \( s \)-dependent. This dependence complicates the explicit expression for thermal noise distribution, however, one can simply formulate the dynamics in terms of a generating functional [20]. The corresponding weight reads

\[
P[s, \hat{s}] = \delta(s^2 - 1)\delta(s \cdot \hat{s}) \exp(-\mathcal{A}), \tag{2}
\]

where \( \hat{s} \) is the response field and the action is given by

\[
\mathcal{A}[s, \hat{s}] = \frac{1}{T} \int d^4 x dt \left( \lambda \hat{s}^2 + i \hat{s} \cdot \partial_t s + \lambda \delta \mathcal{H}/\delta s \right). \tag{3}
\]

Note that \( \hat{s} \) is a real variable which is connected with the corresponding quantity \( \hat{s} \) introduced in Refs. [18, 19, 20].
by \( \tilde{s} = i\hat{s} \). The first \( \delta \)-function in Eq. (2) is due to the constraint \( s^2 = 1 \) and the additional constraint \( s \cdot \hat{s} = 0 \), imposed by the second \( \delta \)-function, ensures that the thermal noise \( \zeta \) acts only in the tangential direction [2]1.

The effect of a macroscopic initial condition can be taken into account by averaging over the initial configurations \( s_0(x) = s(t = 0, x) \) with a weight \( P[s_0] = \exp(-\mathcal{H}_0[s_0]/T) \) [22]. Taking \( \mathcal{H}_0 = -\int d^2x \; h_0 \cdot s_0(x) \) we specify an initial condition that corresponds to the equilibrium state of the system subjected to an external magnetic field \( h_0 \), which is switched off at \( t = 0 \). There is a deep analogy between the considered model and the renormalization group (RG) description of the surface critical phenomena using expansion about the low critical dimension [22]. We assume that the magnetic fields \( h_0 \) and \( h \) act along the one direction which we will refer as to longitudinal one. We decompose the fields \( s = (\sigma, \pi) \) and \( \hat{s} = (\tilde{\sigma}, \pi) \) into \((n-1)\)-component transverse parts \( \pi \) and \( \tilde{\pi} \) and longitudinal parts \( \sigma = \sqrt{1 - \pi^2} \) and \( \tilde{\sigma} = -\pi \cdot \pi/\sqrt{1 - \pi^2} \), where the constraints on the fields have been used. We now can write the weight functional \( \exp(-A_{\text{tr}}(\pi, \tilde{\pi})) \) for the transverse components only with the action given by

\[
A_{\text{tr}}(\pi, \tilde{\pi}) = \frac{1}{T} \int d^2x \int_0^\infty dt \left\{ \lambda \pi^2 + i\pi(\partial_t - \lambda \nabla^2)\pi \right. \\
+ \frac{i}{2} \tilde{\pi} \left( \frac{\partial_\tilde{\pi} - \lambda \nabla^2}{1 - \pi^2} - \frac{\lambda}{2} \left( \nabla^2 \right) \tilde{\pi} \right) \left. \right\} \left. + \frac{\lambda}{2} \right. \left. \right\} \left. + \frac{2\lambda h \sqrt{1 - \pi^2}}{T} \right\}
\]

We will adopt the dimensional regularization scheme [19] in which the terms generated by the measure in the path integral over \( s \) and \( \tilde{s} \) vanish so that we have omitted these terms in Eq. (4) from the beginning. Let us introduce the connected Green functions \( G_{kk}^{ii}(t, s) \) by \( \langle \langle [\pi]^{k}[\tilde{\pi}]^{k}[\pi_0]^{l}[\tilde{\pi}_0]^{l} \rangle \rangle \), the low temperature expansions of which can be obtained with a loop expansion based on the action [14]. The terms quadratic in \( \pi \) and \( \tilde{\pi} \) give us the free response function and the free correlator:

\[
R_{kk}^0(t, s) = -i\Theta(t-s)G_q(t-s), \quad (5)
\]

\[
C_{kk}^0(t, s) = G_q(|t-s|/h + q^2) \left( h_0^{-1} - \frac{1}{h + q^2} \right) G_q(t+s), \quad (6)
\]

where the notation \( G_q(t) := T e^{-\lambda(h+q^2)t} \) is introduced. The infinite number of higher order terms in the expansion of square roots in Eq. (3) will be treated as interactions. In each order in \( T \) we have to take into account only a fixed number of such terms. The completely ordered initial state corresponds to the limit \( h_0 \to \infty \) in which Eq. (6) becomes a Dirichlet correlator. Although the action has infinite number of vertices proportional to \( h_0 \), which are located at time surface \( t = 0 \), it is easy to show by direct inspection of Feynman diagrams that they do not contribute in the limit \( h_0 \to \infty \). The finite \( h_0^{-1} \) can be treated then as an additional perturbation. The great advantage of the considered nonequilibrium model in comparison with the equilibrium counterpart is that the Green functions are not spoiled by \( \delta \) singularities, so that we can probe the critical domain just taking the limit \( h \to 0 \) without using the less convenient matching procedure [21][22]. However, the theory suffers of the uv divergences which can be converted into poles in \( \varepsilon = d - 2 \) using dimensional regularization. Exploiting the ideas of Refs. [14] and [19] one can prove the renormalizability of the model, which means that all uv divergences can be absorbed into finite number of \( Z \)-factors according to

\[
\hat{\pi} = Z^{1/2}\pi, \quad \hat{\xi} = Z_{Z-1/2}\xi, \quad \hat{\xi}_0 = (Z_{Z_0})^{1/2}\xi_0, \quad \hat{\pi}_0 = Z_{Z-1/2}\pi_0, \quad \lambda = Z\tilde{\lambda}, \quad \hat{T} = \mu^{-2}K_dZ_T, \quad \hat{h} = \mu^2Z_TZ_{Z-1/2}h_0, \quad \hat{h}_0 = \mu^2Z_T(Z_{Z_0})^{-1/2}h_0. \quad (7)
\]

Here circles denote the bare quantities, \((2\pi)^dK_d\) is the surface area of a \( d \)-dimensional unit sphere, and \( \mu \) is an arbitrary momentum scale. The \( Z \)-factors except for \( Z_0 \) are the same as in equilibrium [18][19] and to order \( T \) given by \( Z = Z_1 + (n-1)(\pi^2/\varepsilon) + Z_{Z_T} = 1 + (n-2)(\pi^2/\varepsilon) \) to correct the canceling divergences arising from the free correlator [4] for \( t \to s \to 0 \). The renormalized Green function reads

\[
G_{kk}^{ii}(t, s) = Z^{k/2}(Z_{Z-1/2})^{k/2}(Z_{Z_0}^{1/2}Z_{Z-1/2})^{k/2}G_{kk}^{ii}(t, s), \quad (8)
\]

and satisfies the RG equation

\[
\left[ \mu \partial_\mu + \beta_T \partial_T + (\hat{\xi} - \zeta)\lambda \partial_\lambda + \rho \partial_\rho + (\rho + \zeta_0/2)\partial_{h_0} \right] G_{kk}^{ii}(t, s) = 0, \quad (9)
\]

with \( \beta_T = \mu \partial_\mu Z_0 \), \( \zeta = \mu \partial_\mu \ln Z_0 \), \( \rho = \beta_T/T + \zeta/2 - d \), where \( \partial_\mu \) denotes the derivative at fixed bare parameters. Note that the equation similar to Eq. (9) holds for longitudinal and mixed Green functions.

The critical behavior of the non-linear \( \sigma \) model is controlled by the unstable \( \varepsilon \) fixed point (FP). For \( n > 2 \) and \( d > 2 \) the beta function has a nontrivial zero \( T_c = \varepsilon/(n-2) + O(\varepsilon^2) \), which is related to the bare critical temperature \( T_c \). The solution of Eq. (9), in conjunction with the simple dimensional analysis, yields the scaling behavior of the Green function at FP as

\[
G_{kk}^{ii}(q, t; T, h, h_0) = \xi(t)^{-d/2}M(T)^{l+k-1-k}M(T)^{l+k} \times M_0(T)^{l+k}F_{kk}^{ii}(q^2, \xi, q^2; hM\xi^d/T, h_0M_0/h). \quad (10)
\]

where \( \xi(T) = \mu^{-1}T^{1/2} \exp(\int_{T_0}^T d\tau(T')/(\beta(T') - 1/\varepsilon(T'))) \) is the correlation length and \( dG = d(k + l + \hat{l} - 1) - 2(\hat{l} + \hat{k}) \) the canonical dimension of \( G_{kk}^{ii} \). The scaling functions \( M, \tilde{M} \) and \( M_0 \) are given by \( \ln M(T) = -
\]

Note that in statics \( M \) has a meaning of the spontaneous magnetization. For \( \tau = T - T_c \to 0^- \) we derive \( \xi \sim |\tau|^{-\nu} \) and \( M \sim |\tau|^{\beta} \), with
critical exponents $\nu = -1/\beta_T(T_c)$ and $\beta = \nu(\zeta(T_c))/2$.

The dynamic exponent reads $z = 2 + \zeta(T_c) - \zeta(T_c)$. To one loop order we have $\nu = 1/\varepsilon$, $\beta = (n-1)/(2(n-2))$ and $z = 2 + O(\varepsilon^2)$.

We now focus on the scaling properties of two-times

quantities which describe the critical system evolving from a completely ordered state. To that end we put in what follows $h = h_0 = 0$ and apply to Eq. (10) a

short-time expansion of the fields $\pi$ and $\bar{\pi}$ in terms of the initial fields $\pi_0$ and $\bar{\pi}_0$.

As a result we obtain the scaling behavior of the response and correlation functions for $s^z \to 0$ and $t^z \to 0$:

$$R_q(t, s) = A_R s^{-(z-2+\varepsilon)/2}(t/s)^{\delta} f_R(\lambda q^2(t-s), t/s),$$

$$C_q(t, s) = A_C s^{2-\varepsilon}(t/s)^{\delta} f_C(\lambda q^2(t-s), t/s),$$

where $\eta = 2\beta/\nu - \varepsilon$ is the Fisher exponent. The new dynamic critical exponent is defined as $\theta = (2 - \eta - z - \zeta(0(T_c))/2)/z$, and should be distinguished from the initial slip exponent $\theta$.

To lowest order in $\varepsilon$ it is given by $\theta = -\varepsilon(n-1)/(2(n-2)) + O(\varepsilon^2)$ and coincides with $-\beta/\nu$. Although at this point we do not see any symmetry which ensures this identity, we argue that it holds to all orders in $\varepsilon$ and below give some arguments supporting this conjecture. The functions $f_R$ and $f_C$ are regular functions of both arguments and finite for $s \to 0$.

We have explicitly computed the renormalized response and correlation functions to one-loop order:

$$R_q(t, s) = K_{d-1}^\perp \Omega[t, s] R^0_q(t, s),$$

$$C_q(t, s) = K_{d-1}^\perp \Omega[t, s] C^0_q(t, s) + (n-2)T^2 e^{-\lambda q^2(t+s)} F(2\lambda q^2 s)/2K_d q^2,$$

where we have introduced notations $\Omega[t, s] = 1 - T^2(\gamma + \ln 2\lambda^2 s + n-1/2 \ln z)$ and $F(y) = \gamma + \ln |y| - Ei(y)$. Here $\gamma$ is the Euler constant and $Ei(x)$ the exponential integral, so that $F(0) = 0$ and $F'(0) = -1$. Substituting the FP value $T_c$ in Eqs. (13) and (14) we find that they are consistent with scaling laws (11) and (12). The corresponding non-universal amplitudes to one loop order are given by $A_R = K_{d-1}^\perp (-iT_c)(1 - \gamma T_c/2)/(2\lambda^2)^{-T_c/\nu}$, $A_C = i\lambda A_R$, and the universal (apart from the normalization) scaling functions to the same order read

$$f_R(u, v) = e^{-u},$$

$$f_C(u, v) = (v-1)(e^{-u} - e^{-(u+1)/(v-1)} \times [1 - F(2u/(v-1))/2)]/u.$$

Let us introduce the FDR for a particular mode $q$ as

$$X_q^{-1} = \partial_q C_q(t, s)/i\lambda R_q(t, s).$$

Using Eqs. (13) and (14) we obtain to one loop order that $X_q^{-1} = f_X(2\lambda q^2 s)$ with

$$f_X(u) = 1 + e^{-u} + \varepsilon(n-3)(1-e^{-u})(n-2) - e^{-u}(F(u) - 2F'(u))/2.$$
where \( f(v) = f_R(0,v) \) and \( \psi \) is the solution of Voltera equation \( \int_0^\infty d\epsilon [f'(\epsilon)v \psi(\epsilon) + \psi(\epsilon)] = 0 \) such that \( \psi(\infty) = 0 \). This implies that the solution of the equation of motion can be written as \( M(t) = M_0 f_M(t) \) with \( f_M(t) \) satisfying the condition \( f_M(\infty) = 1 \). Taking into account the known asymptotic behavior for \( t \to \infty \) we conclude that the identity \( \theta = -\beta/\nu z \) holds to all orders. To find the exact form of \( f_M \) we have to solve the corresponding integral equation. Recently, the local scale invariance (LSI) was used to predict the exact scaling form of the response function for a system evolving from a completely disordered state \( f_M = 1 \). We expect that the transverse fluctuations of the considered model share similar scaling properties. In analogy with Ref. \( [26] \) we may expect that \( f(v) = 1 \) and consequently \( \psi(v) = 0 \) and \( f_M(t) = 1 \) to all orders. However, the analysis of two loop graphs suggests that in accordance with Ref. \( [27] \) there should be small corrections to LSI: \( f(v) = 1 + O(\epsilon^2) \). They come from the non-local in time contributions to the vertex function \( \Gamma_q(t,s) \) and give rise to very small corrections in the magnetization \( f_M(t) = 1 + O(\epsilon^4) \) that are likely difficult to observe in simulations.

We now discuss the relation between transverse and longitudinal fluctuations. Let us introduce the observable-independent effective temperature \( \theta = \theta_T x \epsilon_T \) and the longitudinal correlation function \( R_q(t) = \langle \sigma(x,t) \sigma_0(0) \rangle \) and analogously the longitudinal correlation function \( C_q(t,s) \). Expressing \( \sigma \) and \( \partial_\sigma \) in terms of \( \pi \) and \( \bar{\pi} \) we obtain their low temperature expansions. To lowest order the renormalized longitudinal response function is given by

\[
R_q(t,s) = \frac{n-1}{2K_d T} \left[ \ln \frac{t}{t-s} + F(\lambda q^2(t-s)^2/2t) - F(\lambda q^2(t-s)/2) \right] R_q(t,s) + O(T^2 \epsilon, T^3). \tag{20}
\]

The corresponding expression for the longitudinal correlator is too cumbersome so that for the sake of conciseness we write down only its time derivative at \( q = 0 \)

\[
\partial_q C_q=0(t,s) = \frac{n-1}{2K_d T^2} T^2 \ln \frac{t+s}{t-s} + O(T^2 \epsilon, T^3), \tag{21}
\]

which is needed to compute the longitudinal FDR \( X_q = i\lambda R_q(t,s)/\partial_q C_q(t,s) \). For the latter we obtain \( X_q=0 = \frac{1}{3} + O(\epsilon) \). The computation to order \( \epsilon \) requires two-loop calculations and is left for future investigations. However, we expect that the scaling ansatzes \( [11] \) and \( [12] \) are valid also for the longitudinal functions.

After we submitted our preprint to arxiv.org there appeared Refs. \( [28] \) and \( [29] \) which also study the critical aging from a magnetized state but for infinite range models such as the spherical model, long range ferromagnetic model and Ising model in the limit of large dimension. These works confirm that the critical aging from an ordered state is characterized by \( X^\infty > 1/2 \).

The \( d-2 \) expansions of the response and correlation functions obtained in Ref. \( [25] \) for the spherical model are in agreement (up to prefactors) with our results for the longitudinal functions \( [20] \) and \( [21] \). The difference between the spherical limit (\( n \to \infty \)) of the transverse FDR \( [15] \) and the FDR of the spherical model \( [28] \) suggests that \( X^\infty \neq X^\infty \) for finite \( n \) and thus one cannot introduce the observable-independent effective temperature \( T_{\text{eff}} = T/X^\infty \). Rewriting the response function of the spherical model in the form \( [11] \) we obtain \( f_R^{\text{ph}}(0,v) = 1 - (1 - 1/v)^{(d-2)/2} \). Thus in contrast to the transverse response of the \( O(n) \) system with the scaling function \( f_R(0,v) \) being finite in the limit \( v \to \infty \), the scaling function of the spherical model decays as \( f_R^{\text{ph}}(0,v) \propto v^{-1} \). Supposing the same asymptotics for the longitudinal function \( f_R(0,v) \) of the \( O(n) \) system we arrive at \( R_q=0(t,s) \propto s^{-\beta/\nu z} (t/s)^{d-1} \) for \( t \gg s \). This, of course, has to be checked independently.

In summary, we have studied the nonequilibrium critical behavior of a ferromagnetic systems which evolves from a completely ordered state. We have shown that the universal long-time scaling behavior of magnetization emerges in the macroscopically early initial stage of relaxation and that can be considered as a consequence of the LSI. The two-times functions exhibit aging behavior which in contrast to the aging from a disordered state is characterized by the FDR larger than \( 1/2 \). Numerical simulations of Ref. \( [15] \) show that the generalized scaling behavior exists in the short-time regime of the critical relaxation even for an initial state with arbitrary magnetization. However, instead of the critical exponent \( \theta \) there exists a whole universal characteristic function which up to now can be estimated only numerically. We expect that our computation being extended to finite \( h_0 \) can describe this scaling behavior and provide a way to calculate the corresponding characteristic function. We hope that the considered model can also be useful to study the influence of Goldstone modes on phase ordering kinetics at \( T < T_c \).

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