Charged LFV in a low-scale seesaw mSUGRA model

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We investigate the influence of the boundary conditions of minimal supergravity (mSUGRA) on the supersymmetric mechanism for lepton flavour violation (LFV) proposed recently \cite{1}, within the framework of the MSSM extended by TeV-scale singlet heavy neutrinos. We find that the consideration of the mSUGRA boundary condition may increase the branching ratios of the muon and tau decaying into three charged leptons by up to a factor of 5, whereas the corresponding branching ratio for their photonic decays remains almost unchanged.

1. SLFV in the MSSM3N

Recently, we proposed \cite{1} a novel, fully supersymmetric mechanism for LFV, which is independent of the soft supersymmetry (SUSY) breaking sector of the theory. The mechanism was called supersymmetric lepton flavour violation (SLFV). To demonstrate the importance of SLFV, we considered a $R$-parity conserving seesaw extension of the MSSM with one singlet heavy neutrino per family (MSSM3N). The leptonic sector of the superpotential of the MSSM3N reads \cite{1}:

$$W_1 = \hat{E}^C h_e \tilde{H}_d \hat{L} + \hat{N}^C h_\nu \tilde{H}_u \hat{L} + \hat{N}^C m_M \hat{N}^C \tag{1}$$

where the complex $3 \times 3$ matrices, $h_e$, $h_\nu$ and $m_M$ represent the electron and the neutrino Yukawa couplings, and the symmetric heavy neutrino Majorana mass matrix, respectively. We assume that the heavy neutrino sector of the model is $SO(3)$ symmetric being broken down to an $U(1)$ lepton symmetry by the Yukawa sector \cite{2}. The approximate breaking of these symmetries lead to almost degenerate heavy neutrinos, $m_M \approx m_N I_3$. The approximate flavour symmetries also assure small light neutrino masses, permitting heavy neutrino mass scale as low as 100 GeV, in a way such that the usual see-saw suppression factor of LFV processes, $m_\nu/m_N$ is avoided. The SLFV effects depend on the LFV parameters

$$\Omega_{\ell\ell'} = \frac{v_u^2}{2m_N^2} (h_\nu^\dagger h_\nu)_{\ell\ell'} \tag{2}$$

In contrast to usual SUSY studies \cite{3} where LFV effects only depend on the flavour structure of the soft SUSY-breaking sector induced by renormalization group (RG) running, SLFV effects only depend on the superpotential heavy neutrino mass scale $m_N$ and the neutrino-Yukawa couplings $h_\nu$. In addition, they depend on $\langle \sqrt{2} H_u \rangle \equiv v_u = v \cos \beta$, with $v \approx 246$ GeV and $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$.

The SLFV amplitudes are induced by heavy neutrinos and heavy sneutrinos at the one-loop level. The loop sneutrino contributions include heavy sleptons and/or heavy squarks, and therefore they do depend on the soft SUSY-breaking sparticle masses. In Ref. \cite{1}, the LFV observables induced by SLFV are evaluated by selecting typical values for the sparticle masses at the electroweak scale. Here, we extend the previous study and evaluate the soft SUSY-breaking parameters, as functions of the heavy neutrino mass scale $m_N$ and the LFV parameters $\Omega_{\ell\ell'}$, using one-loop MSSM3N RG equations \cite{4} with universal boundary conditions at the gauge-coupling unification scale $M_X = 2.5 \times 10^{16}$ GeV. RG analysis confirms that the heavy neutrino sector is supersymmetric almost for the whole parameter space allowed by the perturbative condition on neutrino Yukawa matrices: $\text{Tr} (h_\nu^\dagger h_\nu) < 4\pi$. Specifically, the singlet-neutrino sector is supersymmetric to a good approximation, provided the heavy neutrino mass $m_N$ is of comparable or-
order or larger than the soft SUSY breaking parameters $m_0$ (scalar mass), $M$ (gaugino mass) and $A_0$ (trilinear scalar coupling). In the same kinematic regime, the light left-handed neutrinos are also degenerate. The superpotential $\tilde{H}_u\tilde{H}_d$ mixing term $\mu$ turns out to be typically of order 400 GeV, and therefore smaller than the heavy neutrino mass scale $m_N$, for the largest part of the allowed parameter space. The above justifies the approximations used to obtain the dominant terms of the SLFV amplitudes in Ref. [1].

Within the above framework, we may calculate the leading SLFV amplitudes close to the SUSY limit in the lowest order of $v_s$ and $m_N^{-1}$. To leading order in $g_W$ and $h_u$, the pertinent LFV amplitudes read [1]:

$$T^{\ell'\ell}_\mu = \frac{\alpha_W g_\mu}{8\pi M_W} \bar{l}_i (F^{\ell'\ell}_\gamma g^2 \gamma_\mu P_L + C^{\ell'\ell}_\gamma \sigma_{\mu\nu} q^\nu m_R) l_j,$$

$$T^{2\ell}_\mu = \frac{g_W^2}{8\pi} \Omega_{\ell'\ell} \bar{P}_\ell \gamma_\mu P_L l_j,$$

$$T^{l'\ell\ell_2}_{l_1} = -\frac{\alpha_W^2}{4M_W^2} F^{l'\ell\ell_2}_{box} \bar{P}_\ell \gamma_\mu P_L l_1 \gamma_\mu P_L l_2,$$

where $q = p_\mu - p_{\ell'}$. The amplitudes $T^{l'\ell_1\ell_2}$ and $T^{l'\ell_1d_2}$ have the same structure as the amplitude $T^{l'\ell\ell_2}_{l_1}$, up to replacements $l_1 \rightarrow u_2 \rightarrow d_i$, $i = 1, 2$. The form factors $F^{l'\ell}_\gamma$, $C^{\ell'\ell}_\gamma$, $F^{\ell'\ell}_Z$, $F^{l'\ell\ell_2}_{box}$, $F^{l'\ell_1u_2}_{box}$ and $F^{l'\ell_1d_2}_{box}$ and $F^{l'\ell_1d_2}_{box}$ receive contributions from both the heavy neutrinos $N_{1,2,3}$ and the right-handed neutrinos $\tilde{N}_{1,2,3}$. To illustrate the importance of the SLFV effects, we give the leading form of the form factors $F^{l'\ell}_\gamma$, $C^{\ell'\ell}_\gamma$, $F^{\ell'\ell}_Z$, $F^{l'\ell\ell_2}_{box}$, $F^{l'\ell_1u_2}_{box}$ and $F^{l'\ell_1d_2}_{box}$ in the Feynman gauge,

$$\left(F^{l'\ell}_\gamma\right)^N = \frac{\Omega_{\ell'\ell}}{6s_\beta^2} \ln \lambda_N,$$

$$\left(F^{l'\ell}_\gamma\right)^{\bar{N}} = \frac{\Omega_{\ell'\ell}}{3s_\beta^2} \sum_{k=1}^2 \frac{V_{\ell'\ell_2}^2}{s_\beta^2} \ln \lambda_{N_k},$$

$$\left(G^{l'\ell}_\gamma\right)^N = \Omega_{\ell'\ell} \left(-\frac{1}{6s_\beta^2} - \frac{5}{6}\right),$$

$$\left(G^{l'\ell}_\gamma\right)^{\bar{N}} = \Omega_{\ell'\ell} \left(\frac{1}{6s_\beta^2} + f\right),$$

$$\left(F^{l'\ell}_Z\right)^N = -\frac{3\Omega_{\ell'\ell}}{2} \ln \lambda_N - \frac{\Omega_{\ell'\ell}^2}{2s_\beta^2} \lambda_N,$$

$$\left(F^{l'\ell}_Z\right)^{\bar{N}} = \Omega_{\ell'\ell} g \ln \lambda_N,$$

$$F^{l'\ell\ell_2}_{box} = -\left(\delta_{l_1 l_2} \Omega_{\ell'\ell} + \delta_{l_1 l_2} \Omega_{\ell'\ell}\right) + \left(\Omega_{\ell'\ell} \Omega_{\ell_2 l_1} + \Omega_{\ell_2 l_1} \Omega_{\ell'\ell}\right) \frac{\lambda_N}{4s_\beta^2},$$

$$F^{l'\ell_1u_2}_{box} = \left(\delta_{l_1 l_2} \Omega_{\ell'\ell} + \delta_{l_1 l_2} \Omega_{\ell'\ell}\right) h_1,$$

$$F^{l'\ell_1d_2}_{box} = \left(\Omega_{\ell'\ell} \Omega_{\ell_2 l_1} + \Omega_{\ell_2 l_1} \Omega_{\ell'\ell}\right) \frac{\lambda_N}{4s_\beta^2},$$

$$F^{l'\ell_1d_2}_{box} = 4\Omega_{\ell'\ell} h_2,$$

$$F^{l'\ell_1d_2}_{box} = -\Omega_{\ell'\ell} h_2,$$

$$F^{l'\ell_1d_2}_{box} = \Omega_{\ell'\ell} h_1,$$

$$F^{l'\ell_1d_2}_{box} = \Omega_{\ell'\ell} h_2,$$

where $\lambda_N = m_{N_1} - m_{N_2}$, $\lambda_{N_k} = m_{N_1} - m_{N_3}$, $\gamma$ is one of the unitary matrices diagonalizing the chargino mass matrix and $f$, $g$, $h_1$, $h_2$, and $h_3$ are complicated functions of masses and mixing matrices. Detailed results of this study will be given in a forthcoming publication [5].

In the SUSY limit $\tan \beta \rightarrow 1$, $\mu \rightarrow 0$, $m_{\tilde{N}_1} \rightarrow M_W$, $f \rightarrow \frac{5}{2}$, $g \rightarrow \frac{5}{2}$, $h_1 \rightarrow -1$, $h_2 \rightarrow 0$ and $h_3 \rightarrow -1$. We note that the photonic dipole form factor $G^{l'\ell}_\gamma = (G^{l'\ell}_\gamma)^N + (G^{l'\ell}_\gamma)^{\bar{N}}$ vanishes in the SUSY limit. This is a consequence of the SUSY non-renormalization theorem [6]. Beyond the SUSY limit, it strongly depends on the soft SUSY-breaking sector and particulary on the sparticle masses.

In all formfactors, except of $G^{l'\ell}_\gamma$, the $N$- and $\tilde{N}$-loop contributions add constructively. Specifically, in the $M_{SUSY} \gg M_W$ and in the large $M_N$ limit, the following approximate form factor relations are valid: $F^{l'\ell}_{\gamma} \approx 3(F^{l'\ell}_N)^N$, $|G^{l'\ell}_{\gamma}| \approx |(G^{l'\ell}_{\gamma})^N|$, $F^{l'\ell}_Z \approx (F^{l'\ell}_N)^N$, $F^{l'\ell\ell_2}_{box} \approx (F^{l'\ell\ell_2}_{box})^N$, $F^{l'\ell_1u_2}_{box} \approx (F^{l'\ell_1u_2}_{box})^N$. It is important to note that the large $m_N$ limit corresponds to a kinematic region where the neutrino Yukawa couplings $h_u$ are large (see Eq. [6]). In this limit, the $\Omega^{\gamma} l_{i\ell}$ terms dominate in $Z$ and leptonic box amplitudes. More precisely, the terms proportional to $\Omega^{\gamma}$ become larger than those proportional to $\Omega$, if $g^2 < \text{Tr}(h_u^0 h_u)$.
Figure 1. Exclusion contours of $\Omega_{e\mu}$ versus $m_N$ derived from experimental limits on $B(\mu^- \to e^-\gamma)$ (solid), $B(\mu^- \to e^-e^-e^+)$ (dashed) and $\mu \to e$ conversion in Titanium (dash-dotted) and Gold (dash-double-dotted), assuming $\Omega_{ee} = \Omega_{\mu\mu} = \Omega_{e\mu}$ and other $\Omega_{\ell\ell'} = 0$. The upper, middle and lower panel represent the exclusion contours in the SM3N, the MSSM3Nf and the MSSM3NS, respectively. The areas above the contours are excluded; see the text for more details.

Figure 2. Exclusion contours of $\Omega_{e\tau}$ versus $m_N$ derived from present experimental upper limits on $B(\tau^- \to e^-\gamma)$ (solid), $B(\tau^- \to e^-e^-e^+)$ (dashed) and $B(\tau^- \to e^-\mu^-\mu^+)$ (dash-dotted), assuming $\Omega_{ee} = \Omega_{\tau\tau} = \Omega_{e\tau}$ and other $\Omega_{\ell\ell'} = 0$. The upper, middle and lower panel represent the exclusion contours in the SM3N, the MSSM3Nf and the MSSM3NS, respectively. The areas above the contours are excluded; more details are given in the text.
2. Numerical estimates

We now present numerical estimates of LFV observables in three distinct models: (i) the Standard Model with one heavy right-handed neutrino per family (SM3N); (ii) the MSSM3N with fixed superpartner masses (MSSM3NF); (iii) the MSSM with mSUGRA boundary conditions (MSSM3NS).

In the SM3N, the LFV amplitudes depend only on $\Omega_{\ell\ell'}$ and $m_N$. For the SUSY models, the MSSM3NF and the MSSM3NS, the LFV amplitudes are functions of $\Omega_{\ell\ell'}$, $m_N$, $\tan\beta$, $\mu$, the slepton and/or squark masses, and the unitary chargino-mixing mass matrices. We take $\tan\beta = 3$, which is a value close to the SUSY limit value $\tan\beta = 1$. In the MSSM3NF, the lepton and squark matrices and $\mu$ are taken as input parameters. We fix $-\mu = M_Q = M_B = 200$ GeV, $M_{\tilde{W}} = 100$ GeV and $\tan\beta = 3$. In the MSSM3NS, the $\mu$ parameter and the sparticle masses are functions of $m_N$ and $\Omega_{\ell\ell'}$. They are determined by the MSSM3NF RGEs and the universal soft SUSY-breaking parameters $M_t$, $m_0$ and $A_0$ defined at the gauge-coupling unification scale $M_X$. As input values, we take $M = 250$ GeV, $m_0 = 100$ GeV and $A_0 = 150$ GeV.

To simplify our analysis of identifying the regions of parameter space excluded by experimental limits of LFV processes $\mu \to e + X$ and $\tau \to e + X$ (where $X$ indicates generically a photon or 3 charged leptons), we consider three separate conservative scenarios with three non-zero $\Omega_{\ell\ell'}$ parameters: $\Omega_{\mu\mu} = \Omega_{\mu e} = \Omega_{e e'}, \Omega_{\tau\tau} = \Omega_{\tau e} = \Omega_{e e'}, \Omega_{\mu\tau} = \Omega_{\tau \mu} = \Omega_{e \mu}$, respectively.

The exclusion contours of $\Omega_{\ell\ell'}$ versus $m_N$ for $\mu \to e + X$ processes, $\Omega_{\mu\tau}$ versus $m_N$ for $\tau \to e + X$ and $\Omega_{\mu\mu}$ versus $m_N$ for $\tau \to \mu + X$ are given in Figs. 1, 2 and 3 respectively. The areas above the curves are forbidden by the experimental upper bounds on the corresponding processes. The area above the diagonal solid lines represent the nonperturbative regime with $\text{Tr} \left( h_1^T h_1 \right) > 4\pi$, whilst the area above the diagonal dotted lines represent the region where the Yukawa couplings dominate the LFV observables, $\text{Tr} \left( h_1^T h_1 \right) > g_w^2$. The higher values of $\Omega_{\ell\ell'}$ correspond to smaller values of the LFV observables.
the factors multiplying the combinations of $\Omega_{\ell\ell'}$ elements is smaller, if $\Omega_{\ell\ell'}$ needed to satisfy the experimental upper bound is larger.

The mSUGRA boundary condition has a strong influence on the perturbativity condition: $\text{Tr}(h^i_i h_{\nu}) > 4\pi$. In the SM3N and the MSSM3Nf, the condition $\text{Tr}(h^i_i h_{\nu}) > 4\pi$ is determined at the $M_Z$ scale. In the MSSM3NS, $\text{Tr}(h^i_i h_{\nu}) > 4\pi$ has to be satisfied for any RG scale between $M_Z$ and $M_X$. As $h_{\nu}$ increases with the RG scale, the $\text{Tr}(h^i_i h_{\nu}) > 4\pi$ is determined at the gauge unification scale, when the typical value for $\text{Tr}(h^i_i h_{\nu})$ at the $M_Z$ scale is $\sim 0.3 - 0.45$. Thus, significant part of the SM3N and MSSM3Nf parameter space in the $m_N - \Omega_{\ell\ell'}$ plane gets excluded in the MSSM3NS. Also, the boundary lines $\text{Tr}(h^i_i h_{\nu}) = 4\pi$ and $\text{Tr}(h^i_i h_{\nu}) = g^2_{\nu} B$ come closer to each other. Moreover, the LFV observables cannot be evaluated beyond the perturbativity limit $\text{Tr}(h^i_i h_{\nu}) = 4\pi$, since the RGs rapidly diverge. Instead, in the SM3N and MSSM3Nf, the LFV observables can be computed for any value of $m_N$ and $\Omega_{\ell\ell'}$.

Figures 1 and 2 contain 3 panels. The upper, the middle and the lower panels display exclusion contours obtained in the SM3N, the MSSM3Nf and the MSSM3Nf, respectively.

Figure 1 presents exclusion contours for current experimental limits on and future sensitivities to LFV processes of $\mu \rightarrow e$ transitions: $B(\mu^- \rightarrow e^- \gamma) < 1.2 \times 10^{-11}$ [8] (upper horizontal line), $B(\mu^- \rightarrow e^- \gamma) < 10^{-13}$ [9] (lower horizontal line), $B(\mu^- \rightarrow e^- e^+ e^+) < 10^{-12}$ [9], the constraints from the non-observation of $\mu \rightarrow e$ conversion in $^{48}_{22}$Ti and $^{197}_{75}$Au [10], $R_{\ell\ell e}^{\text{Ti}} > 4.3 \times 10^{-12}$ [11] (dash-dotted) and $R_{\mu e}^{\text{Au}} < 7 \times 10^{-13}$ [12] (dash-double-dotted), as well as potential limits from a future sensitivity to $R_{\mu e}^{\text{Ti}}$ at the $10^{-18}$ level [13] (lower dash-dotted line). Comparing the upper with the middle panel, one can see that $B(\mu \rightarrow e \gamma)$ becomes smaller when the heavy sneutrino contributions are included, while the other observables become larger. The consideration of the mSUGRA boundary condition alter the predictions for the LFV observables in non-trivial manner, i.e. there are no regions of cancelation among terms proportional to $\Omega$ and $\Omega^2$. Furthermore, the theoretical predictions for the LFV observables may increase, especially for $\mu \rightarrow e$ conversion processes.

Figure 2 exhibits exclusion contours for present experimental limits to LFV processes of $\tau \rightarrow e$ transitions: $B(\tau^- \rightarrow e^- \gamma) < 3.3 \times 10^{-8}$ [15] (solid line), $B(\tau^- \rightarrow e^- e^- e^+) < 2.7 \times 10^{-8}$ [14] (dashed line), and $B(\tau^- \rightarrow e^- \mu^- \mu^+) < 2.7 \times 10^{-8}$ [14] (dash-dotted line). The dominance of the heavy neutrino effects in MSSM3NS manifests already at $m_N \sim 200$ GeV and becomes more pronounced than in the MSSM3Nf. The branching ratios for processes, such as $\tau \rightarrow 3$ leptons, can be $\sim 3$ times larger than the one for $\tau \rightarrow e \gamma$ at $m_N \sim 1000$ GeV.

Figure 3 exhibits exclusion contours for present experimental limits to LFV processes of $\tau \rightarrow \mu$ transitions: $B(\tau^- \rightarrow \mu^- \gamma) < 4.4 \times 10^{-8}$ [15] (solid line), $B(\tau^- \rightarrow \mu^- \mu^- \mu^+) < 2.1 \times 10^{-8}$ [14] (dashed line), and $B(\tau^- \rightarrow \mu^- e^- e^+) < 1.8 \times 10^{-8}$ [14] (dash-dotted line). The exclusion contours in all three panels are very similar to the corresponding contours for $\tau \rightarrow e$ transitions, but the dominance of the heavy neutrinos is slightly more pronounced. In particular, the heavy neutrino dominance in the MSSM3NS manifests before $m_N \sim 200$ GeV and $B(\tau \rightarrow 3$ leptons) can be about 5 times larger than $B(\tau \rightarrow \mu\gamma)$ at $m_N \sim 1000$ GeV.

In summary, we have shown that the incorporation of the mSUGRA boundary condition into the MSSM3N leads to larger theoretical predictions for the LFV observables $R_{\mu e}$, $\mu \rightarrow 3e$ and $\tau \rightarrow 3$ leptons by up to a factor of 5. The branching ratios for the $\ell \rightarrow e\gamma$ processes show a smaller variation; they are slightly larger than those obtained in the MSSM3Nf but smaller than the ones in the SM3N. We plan to present detailed results of this preliminary analysis in the near future [5].

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REFERENCES

1. A. Ilakovac and A. Pilaftsis, Phys. Rev. D \textbf{80} (2009) 091902.
2. A. Pilaftsis, Phys. Rev. Lett. \textbf{95} (2005) 081602; A. Pilaftsis and T.E.J. Underwood, Phys. Rev. D \textbf{72} (2005) 113001.
3. F. Borzumati and A. Masiero, Phys. Rev. Lett. \textbf{57} (1986) 961; J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D \textbf{53} (1996) 2442.
4. P.H. Chankowski and S. Pokorski, Int. J. Mod. Phys. A \textbf{17} (2002) 575; S.T. Petcov et al., Nucl. Phys. B \textbf{676} (2004) 453.
5. A. Ilakovac, A. Pilaftsis and L. Popov in preparation.
6. S. Ferrara and E. Remiddi, Phys. Lett. B \textbf{53} (1974) 347.
7. H. C. Chiang, E. Oset, T. S. Kosmas, A. Faessler and J. D. Vergados, Nucl. Phys. A \textbf{559} (1993) 526.
8. C. Amsler et al., Phys. Lett. B \textbf{667} (2008) 1.
9. S. Ritt [MEG Collaboration], Nucl. Phys. Proc. Suppl. \textbf{162} (2006) 279.
10. To get predictions for $R_{\mu e}$, we use the values for $Z_{\text{eff}} |F(-m_{\mu}^2)|$ and $\Gamma_{\text{capt}}$ given by R. Kitano, M. Koike and Y. Okada, Phys. Rev. D \textbf{66} (2002) 096002.
11. SINDRUM II collaboration, C. Dohmen \textit{et al.}, Phys. Lett. B \textbf{317} (1993) 631.
12. W. Bertl \textit{et al.}, Eur. Phys. J. C \textbf{47} (2006) 337.
13. Y. Kuno, Nucl. Phys. Proc. Suppl. \textbf{149} (2005) 376.
14. K. Hayasaka \textit{et al.}, Belle Collaboration, Phys. Lett. B \textbf{687} (2010) 139.
15. B. Aubert \textit{et al.} Babar Collaboration, Phys. Rev. Lett. \textbf{104} (2010) 021802.