ABSTRACT

The large branching ratios for $B \to K\pi$ decays as observed by the CLEO Collaboration indicate that penguin interactions contribute a major part to the decay rates and provide an interference between the Cabibbo-suppressed tree and penguin contributions resulting in a CP-asymmetry between the $B \to K\pi$ and its charge conjugate mode. The CP-averaged decay rates depend also on the weak phase $\gamma$ and give us a determination of this phase. In this talk, I would like to report on a recent analysis of $B \to K\pi$ decays using factorisation model with final state interaction phase shift included. We find that factorisation seems to describe qualitatively the latest CLEO data. We also obtain a relation for the branching ratios independent of the strength of the strong penguin interactions. This relation gives a central value of $0.60 \times 10^{-5}$ for $B(\bar{B}^0 \to \bar{K}^0\pi^0)$, somewhat smaller than the latest CLEO measurement. We also find that a ratio obtained from the CP-averaged $B \to K\pi$ decay rates could be used to test the factorisation model and to determine the weak angle $\gamma$ with more precise data, though the latest CLEO data seem to favor $\gamma$ in the range $90^\circ - 120^\circ$. 

\[ B \to K\pi \] 

DECAYS AND THE WEAK PHASE ANGLE $\gamma$\textsuperscript{†}
With the measurement of all the four $B \to K\pi$ branching ratios, we seem to have a qualitative understanding of the $B \to K\pi$ decays. The measured CP-averaged branching ratios($B$) by the CLEO Collaboration [1] show that the penguin interactions dominates the $B \to K\pi$ decays, as predicted by factorisation. The strong penguin amplitude, because of the large CKM factors, becomes much larger than the tree-level terms which are Cabibbo-suppressed and the non-leptonic interaction for $B \to K\pi$ is dominated by an $I = 1/2$ amplitude. This is borne out by the CLEO data which give $B(B^+ \to K^+\pi^-)\simeq 2B(B^- \to K^-\pi^0)$ and $B(B^- \to K^0\pi^-)\simeq B(\bar{B}^0 \to K^-\pi^+)$ :

$$B(B^+ \to K^+\pi^0) = (11.6^{+3.0+1.4}_{-2.7-1.3}) \times 10^{-6},$$
$$B(B^+ \to K^0\pi^+) = (18.2^{+4.6}_{-4.0} \pm 1.6) \times 10^{-6},$$
$$B(B^0 \to K^+\pi^-) = (17.2^{+2.5}_{-2.4} \pm 1.2) \times 10^{-6},$$
$$B(B^0 \to K^0\pi^0) = (14.6^{+5.9+2.4}_{-5.1-3.3}) \times 10^{-6}. \quad (1)$$

If the strength of the interference between the tree-level and penguin contributions is known, a determination of the weak phase $\gamma$ could be done in principle. Previous works [2, 3] shows that factorisation model produces sufficient $B \to K\pi$ decay rates, in qualitative agreement with the CLEO measured values. Also, as argued in [4], for these very energetic decays, because of color transparency, factorisation should be a good approximation for $B \to K\pi$ decays if the Wilson coefficients are evaluated at a scale $\mu = O(m_b)$. Infact, recent hard scattering calculations with perturbative QCD shows that factorisation is valid up to corrections of the order $\Lambda_{\text{QCD}}/m_b$ [5]. It is thus encouraging to use factorisation to analyse the $B \to K\pi$ decays, bearing in mind that there are important theoretical uncertainties in the long-distance hadronic matrix elements, as the heavy to light form factors for the vector current and the value of the current $s$ quark mass are currently not determined with good accuracy.

In this talk, I would like to report on a recent work [6] on the $B \to K\pi$ decays as a possible way to measure the angle $\gamma$ and to see direct CP violation.

In the standard model, the effective Hamiltonian for $B \to K\pi$ decays are given by [7, 8, 9, 10, 11],

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} [V_{ub}V_{us}^*(c_1O_1^u + c_2O_2^u) + V_{cb}V_{cs}^*(c_1O_1^c + c_2O_2^c) - \sum_{i=3}^{10} V_{tb}V_{ts}^*c_iO_i] + \text{h.c.} \quad (2)$$

in standard notation. At next-to-leading logarithms, $c_i$ take the form of an effective Wilson coefficients $c_i^{\text{eff}}$ which contain also the penguin contribution from the $c$ quark loop and are given in [3, 11].

The parameters $V_{ub}$ etc. are the flavor- changing charged current couplings of the weak gauge boson $W^\pm$ with the quarks as given by the Cabibbo-Kobayashi-Maskawa...
(CKM) quark mixing matrix $V$. $V$ is usually defined as the unitary transformation relating the the weak interaction eigenstate of quarks to their mass eigenstate [12]:

$$
\begin{pmatrix}
    d' \\
    s' \\
    b'
\end{pmatrix}
= 
\begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix}
$$

(3)

where $d, s, b$ and $d', s', b'$ are respectively the mass eigenstates and weak interaction eigenstates for the charge $Q = -1/3$ quarks. Since the neutral current is not affected by the unitary transformation on the quark fields, flavor-changing neutral current is absent at the tree-level as implied by the GIM mechanism. The unitarity condition $VV^\dagger = 1$ gives, for the $(db)$ elements relevant to $B$ decays [12]

$$
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0
$$

(4)

This can be represented by a triangle [12] with the three angles $\alpha, \beta$ and $\gamma$ expressed in terms of the CKM matrix elements as [13]:

$$
\alpha = \text{arg}(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)
$$

$$
\beta = \text{arg}(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)
$$

$$
\gamma = \text{arg}(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)
$$

(5)

The angle $\gamma$ enters the $B \to K\pi$ decay amplitudes through the factor $V_{ub}V_{us}^*/V_{tb}V_{ts}^*$ which can be approximated by $-\langle|V_{ub}|/|V_{cb}|\rangle \times \langle|V_{cd}|/|V_{ud}|\rangle \exp(-i \gamma)$ after neglecting terms of the order $O(\lambda^5)$ in the (bs) unitarity triangle, $\lambda$ being the Cabibbo angle in the Wolfenstein parametrisation of the CKM quark mixing matrix. The $B \to K\pi$ decay amplitudes, expressed in terms of the $I = 1/2$ and $I = 3/2$ isospin amplitudes are given by [3],

$$
A_{K^-\pi^0} = \frac{2}{3} B_3 e^{i\delta_3} + \sqrt{\frac{1}{3}} (A_1 + B_1) e^{i\delta_1},
$$

$$
A_{K^0\pi^-} = \frac{\sqrt{2}}{3} B_3 e^{i\delta_3} - \sqrt{\frac{1}{3}} (A_1 + B_1) e^{i\delta_1},
$$

$$
A_{K^-\pi^+} = \frac{\sqrt{2}}{3} B_3 e^{i\delta_3} + \sqrt{\frac{1}{3}} (A_1 - B_1) e^{i\delta_1},
$$

$$
A_{K^0\pi^0} = \frac{2}{3} B_3 e^{i\delta_3} - \sqrt{\frac{1}{3}} (A_1 - B_1) e^{i\delta_1},
$$

(6)

$A_1$ is the sum of the strong penguin $A_1^S$ and the $I = 0$ tree-level $A_1^T$ as well as the $I = 0$ electroweak penguin $A_1^W$ contributions to the $B \to K\pi I = 1/2$ amplitude; similarly $B_1$ is the sum of the $I = 1$ tree-level $B_1^T$ and electroweak penguin $B_1^W$
difference in branching ratios obtained for $\gamma$ to the CLEO measured values, as shown in Fig. 1. where the CP-averaged Eq. (8) is enhanced by the charm quark loop which increases the amplitude by 30% for the contributions from the tree-level and the strong penguin operators at $m$ and $\rho$ respectively, the elastic $B$ and electroweak penguin $B^T_w$ contributions to the penguin operators, for which the Wilson coefficients now have an absorptive part and are given in [9, 11, 14]. The $B \to K\pi$ isospin amplitudes in the factorisation model are given by [2],

$$A_1^T = i\frac{\sqrt{3}}{4} V_{ub} V_{us}^* r a_2,$$
$$B_1^T = i\frac{1}{2\sqrt{3}} V_{ub} V_{us}^* r [ -\frac{1}{2} a_2 + a_1 X ],$$
$$B_3^T = i\frac{1}{2} V_{ub} V_{us}^* r [a_2 + a_1 X ],$$
$$A_1^S = -i\frac{\sqrt{3}}{2} V_{tb} V_{ts}^* r [a_4 + a_6 Y ],\quad B_1^S = B_3^S = 0,$$
$$A_1^W = -i\frac{\sqrt{3}}{8} V_{tb} V_{ts}^* r [a_8 Y + a_{10} ],$$
$$B_1^W = i\frac{3}{4} V_{tb} V_{ts}^* r \left[ \frac{1}{2} a_8 Y + \frac{1}{2} a_{10} + (a_7 - a_9) X \right],$$
$$B_3^W = -i\frac{3}{4} V_{tb} V_{ts}^* r [a_8 Y + a_{10} - (a_7 - a_9) X ]$$

where $r = G_F f_K F_0^{B\pi} (m^2_K)/(m^2_B - m^2_\pi), \quad X = (f_/f_K)[F_0^{B\pi} (m^2_\pi)/(F_0^{B\pi}(m^2_K))(m^2_B - m^2_\pi)/(m^2_B - m^2_\pi)]$ with $q = u, d$ for $\pi^{0, -}$ final states. In this analysis, $f_\pi = 133$ MeV, $f_K = 158$ MeV, $F_0^{B\pi}(0) = 0.33$, $F_0^{B\pi}(0) = 0.38$ [8, 13]; $|V_{cb}| = 0.0395, |V_{cd}| = 0.224$ and $|V_{ub}|/|V_{cb}| = 0.08$ [12]. The value of $m_s$ is not known to a good accuracy, but a value around $(100 - 120)$ MeV inferred from $m_{K^*} - m_\rho, m_{D^+_s} - m_{D^+}$ and $m_{B^0} - m_{B^0}$ mass differences [10] seems not unreasonable and in this work, we use $m_s = 120$ MeV. $a_j$ are the effective Wilson coefficients after Fierz reordering in factorisation model and are given by [3]

$$a_1 = 0.07, \quad a_2 = 1.05,$$
$$a_4 = -0.043 - 0.016i, \quad a_6 = -0.054 - 0.016i,$$  

for the contributions from the tree-level and the strong penguin operators at $N_c = 3$ and $m_b = 5.0$ GeV. The strong penguin contribution $P = a_4 + a_6 Y$, as obtained from Eq. (8) is enhanced by the charm quark loop which increases the amplitude by 30% as pointed out in [4]. This enhancement brings the predicted branching ratios closer to the CLEO measured values, as shown in Fig.1. where the CP-averaged $B \to K\pi$ branching ratios obtained for $\gamma = 70^\circ$ [3], are plotted against the rescattering phase difference $\delta = \delta_3 - \delta_1$.  

4
For a determination of $\gamma$, two quantities obtained from the sum of the two CP-averaged decay rates $\Gamma_{B^-} = \Gamma(B^- \to K^-\pi^0) + \Gamma(B^- \to K^0\pi^-)$ and $\Gamma_{B^0} = \Gamma(B^0 \to K^-\pi^0) + \Gamma(B^0 \to K^0\pi^-)$ which are independent of $\delta$ could be used [6]. As the CP-averaged $B \to K\pi$ decay rates depend on $\gamma$, the computed partial rates $\Gamma_{B^-}$ and $\Gamma_{B^0}$ would now lie between the upper and lower limit corresponding to $\cos(\gamma) = 1$ and $\cos(\gamma) = -1$, respectively. As shown in Fig.2, where the corresponding CP-averaged branching ratios ($B_{B^-}$ and $B_{B^0}$) for $\Gamma_{B^-}$ and $\Gamma_{B^0}$ are plotted against $\gamma$, the factorisation model values with the BWS form factors [15] seem somewhat smaller than the CLEO central values by about $10 - 20\%$. Also, $B_{B^-} > B_{B^0}$ while the data give $B_{B^-} < B_{B^0}$ by a small amount which could be due to a large measured $\bar{B}^0 \to \bar{K}^0\pi^0$ branching ratio.

Note that smaller values for the form factors could easily accommodate the latest CLEO measured values, if a smaller value for $m_s$, e.g, in the range $(80 - 100)\text{ MeV}$ is used. What one learns from this analysis is that $B \to K\pi$ decays are penguin-dominated and the strength of the penguin interactions as obtained by perturbative QCD, produce sufficient $B \to K\pi$ decay rates and that factorisation seems to work with an accuracy better than a factor of 2, considering large uncertainties from the form factors and possible non-factorisation terms inherent in the factorisation model and the uncertainties in the penguin amplitude which is sensitive to the current $s$ quark mass. Since the four $B \to K\pi$ decay rates depend on only three amplitudes $A_1$, $B_1$ and $B_3$, it is possible to derive a relation between the decay rates independent of $A_1$. Thus, the quantity $\Delta$ obtained from the decay rates,

$$\Delta = \left\{ \Gamma(B^- \to \bar{K}^0\pi^-) + \Gamma(B^0 \to K^-\pi^+) \\
- 2 \left\{ \Gamma(B^- \to K^-\pi^0) + \Gamma(B^0 \to \bar{K}^0\pi^0) \right\} \tau_{B^0} \right\}
= \left[ - \frac{4}{3} |B_3|^2 - \frac{8}{\sqrt{3}} \text{Re}(B_3^*B_1e^{i\delta}) \right] (C\tau_{B^0})(9)$$

is independent of the strong penguin term. It is given by the tree-level and electroweak penguin contributions. As can be seen from Fig.2, where its values for $\delta = 0$ are plotted against $\gamma$. $\Delta$ is of the order $O(10^{-6})$ compared with $B_{B^-}$ and $B_{B^0}$ which are in the range $(1.6 - 3.0) \times 10^{-5}$. Thus, to this level of accuracy, we can put $\Delta \simeq 0$ and obtain the relation $(r_b = \tau_{B^0}/\tau_{B^-})$.

$$r_bB_{K^0\pi^-} + B_{K^-\pi^+} = 2 \left[ B_{K^0\pi^0} + r_bB_{K^-\pi^0} \right]. \quad (10)$$

which can be used to test factorisation or to predict $B(\bar{B}^0 \to \bar{K}^0\pi^0)$ in terms of the other measured branching ratios. Eq.(14) then predicts a central value $B(\bar{B}^0 \to \bar{K}^0\pi^0) = 0.60 \times 10^{-5}$.  

5
Figure 1: $\mathcal{B}(B \to K\pi)$ vs. $\delta$ for $\gamma = 70^\circ$. The curves (a), (b), (c), (d) are for the CP-averaged branching ratios $B^- \to K^-\pi^0$, $\bar{K}^0\pi^-$ and $\bar{B}^0 \to K^-\pi^+$, $\bar{K}^0\pi^0$, respectively.

Figure 2: $\mathcal{B}_{B^-}$ (a), $\mathcal{B}_{B^0}$ (b), $\Delta$ (c) vs. $\gamma$
Since $B_{K^0\pi^0}$ is not known with good accuracy at the moment, it is useful to use another quantity, defined as

$$r_b B_{K^0\pi^-} + B_{K^-\pi^+} = \left(C_1 \tau_B^0 \right) \left[ \frac{1}{3} |B_3|^2 + (|A_1|^2 + |B_1|^2) - \frac{2}{\sqrt{3}} \text{Re}(B_3^* B_1 e^{i\delta}) \right]$$

(11)

which contains a negligible $\delta$-dependent term of the order $O(10^{-7})$. The quantity $R$ defined as

$$R = \frac{\mathcal{B}(B^- \to K^-\pi^0) + \mathcal{B}(B^- \to K^0\pi^-)}{\mathcal{B}(B^- \to K^0\pi^-) + \mathcal{B}(B^0 \to K^-\pi^+)} / r_b.$$  

(12)

is thus essentially independent of $\delta$ and could also be used to obtain $\gamma$, as it does not suffer from large uncertainties in the form factors and in the CKM parameters. As can be seen in Fig. 3, it is not possible to deduce a value for $\gamma$ with the present data which give $R = (0.80 \pm 0.25)$ as the prediction for $R$ lies within the experimental errors. If we could reduce the experimental uncertainties to a level of less than 10%, we might be able to give a value for $\gamma$. Thus it is important to measure $B \to K\pi$ branching ratios to a high precision. Also shown in Fig. 3 are two other quantities more sensitive to $\gamma$, but involved $\mathcal{B}(\bar{B}^0 \to \bar{K}^0\pi^0)$ and are given as

$$R_1 = \frac{\Gamma_{B^-}}{\Gamma_{B^0}}, \quad R_2 = \frac{\Gamma_{B^-}}{(\Gamma_{B^-} + \Gamma_{B^0})}.$$  

(13)

Thus a better way to obtain $\gamma$ would be to use $R_1$ when a precise value for $\mathcal{B}(\bar{B}^0 \to \bar{K}^0\pi^0)$ will be available. The central value of 0.80 for $R$ corresponds to $\gamma = 110^\circ$, close to the value of $(113^{+25}_{-23})^\circ$ found by the CLEO Collaboration in an analysis of all known charmless two-body $B$ decays with the factorisation model [17].

![Figure 3: The curves (a), (b), (c) are for $R$, $R2$, $R1$ respectively.](image-url)
It seems that the CLEO data favor a large $\gamma$ in the range $90^\circ - 120^\circ$. With a large $\gamma$, for example, with the central value of $110^\circ$, as shown in Fig.4, the predicted $B \to K\pi$ branching ratios are larger than that for $\gamma = 70^\circ$ and are closer to the data. The data also show that $B^- \to \bar{K}^0\pi^-$ and $\bar{B}^0 \to K^-\pi^+$ are the two largest modes with near-equal branching ratios in qualitative agreement with factorisation. However, for $\gamma = 70^\circ$, Fig.1 shows that these two largest branching ratios are quite apart, except for $\delta < 50^\circ$ while for $\gamma = 110^\circ$, Fig.4 suggests that these two branching ratios are closer to each other only for $\delta$ in the range $40^\circ - 70^\circ$. With $\gamma < 110^\circ$ and some adjustment of form factors, the current $s$ quark mass and CKM parameters, it might be possible to accommodate these two largest branching ratios with $\delta < 50^\circ$.

The CP-asymmetries, as shown in Fig.3, for $\gamma = 110^\circ$, are in the range $\pm(0.04)$ to $\pm(0.3)$ for the preferred values of $\delta$ in the range $(40 - 70)^\circ$, but could be smaller for $\delta < 50^\circ$. The CLEO measurements however, do not show any large CP-asymmetry in $B \to K\pi$ decays, but the errors are still too large to draw any conclusion at the moment.

In conclusion, factorisation with enhancement of the strong penguin contribution seems to describe qualitatively the $B \to K\pi$ decays. Further measurements will allow a more precise test of factorisation and a determination of the weak angle $\gamma$ from the FSI phase-independent relations shown above.

![Figure 4: $\mathcal{B}(B \to K\pi)$ vs. $\delta$ for $\gamma = 110^\circ$. The curves (a), (b), (c), (d) are for the CP-averaged branching ratios $B^- \to K^-\pi^0$, $\bar{K}^0\pi^-$ and $\bar{B}^0 \to K^-\pi^+$, $\bar{K}^0\pi^0$, respectively.](image)

I would like to thank S. Narison and the organisers of QCD00 for the warm hospitality extended to me at Montpellier.

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