Differential Spatial Modulation with Gray Coded Antenna Activation Order

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Abstract—Differential spatial modulation (DSM) was recently proposed to overcome the challenge of channel estimation in spatial modulation. In this letter, we propose a gray code order of antenna index permutations for DSM. To facilitate the implementation, the well-known Trotter-Johnson ranking and unranking algorithms are adopted, which results in similar computational complexity to the existing DSM that uses the lexicographic order. The coding gain achieved by the proposed gray code order over the lexicographic order is also analyzed and verified via simulations, which reveals a maximum of about 1.2dB for the case of four transmit antennas. Based on the Gray coding framework, we further propose a diversity-enhancing scheme named intersected gray (I-gray) code order for DSM, where the permutations of active antenna indices are selected directly from the odd (or even) positions of the full permutations in the gray code order. From analysis and simulations, it is shown that the I-gray code order can harvest an additional diversity order at the expense of only one information bit loss for each transmission with respect to the gray code order.

Index Terms—Gray coding, DSM, coding gain, diversity order.

I. INTRODUCTION

The optimal maximum-likelihood (ML) decoding in spatial modulation (SM) requires the knowledge of channel state information (CSI), which complicates the implementation [1], [2]. To solve this problem, recently, differential (D-)SM is proposed, which dispenses with the CSI [3]–[5]. In DSM, the antenna activation orders, which can be specified by the permutations of set \{1, 2, \ldots, N_T\} with \(N_T\) denoting the number of transmit antennas, are used as an information carrying mechanism. Consequently, the difference between antenna activation orders plays an important role in the bit error rate (BER) performance of DSM. In the previous work [3], the antenna activation orders are set to follow the lexicographic order such that similar permutations may lead to huge bit difference between the corresponding information bit sequences. This, however, will degrade the BER performance of DSM as a detection error most probably occurs between similar permutations at high signal-to-noise ratio (SNR).

In this letter, we resort to the idea of Gray coding [6], [7] to improve the BER performance of DSM. In the proposed scheme, each information bit sequence has only one bit difference from the previous and next ones, while its corresponding antenna index permutation has only two antenna indices difference from the previous and next ones. For the ease of implementation in the case of large \(N_T\), we apply the well-known Trotter-Johnson ranking and unranking algorithms to build the relationship between the information bit sequence and the corresponding antenna index permutation. From theoretical analysis, it is revealed that the proposed gray code order can achieve a maximum of about 1.2dB coding gain over the lexicographic coder when \(N_T = 4\) with similar computational complexity. Based on the Gray coding framework, we also propose a new scheme called intersected gray (I-gray) code order to improve the diversity performance of DSM, which takes the odd (or even) positions of the full permutations in the gray code order only for information conveying purpose. We show that the I-gray code order achieves an additional diversity order to the gray code order while sacrificing one bit for each transmission.

Notations: \((\cdot)^H\) stands for Hermitian transpose. The complex number field is represented by \(\mathbb{C}\). \(I_M\) is an identity matrix of size \(M \times M\). \(\Re\{\cdot\}\) represents the real component of the argument. \(Tr\{\cdot\}, \text{rank}\{\cdot\}\) and \(\text{mod}(\cdot, \cdot)\) denote the trace, rank and modulus operations, respectively. \(A(i, j)\) denotes the \((i, j)\)th element of matrix \(A\). \([-\cdot]\) and \([\cdot]\) indicate the floor and ceil operations, respectively.

II. SYSTEM MODEL

We consider a base-band \(N_R \times N_T\) multi-input multi-output (MIMO) system, where \(N_R\) represents the number of receive antennas. Specifically, DSM works as follows. At the transmitter, the information bits are partitioned into transmitted blocks of which each is composed of \(m = \lceil \log_2(N_T!) \rceil + N_T \log_2(M)\) bits and are to be transmitted over \(N_T\) time slots, where \(M\) denotes the cardinality of the constellation \(\mathcal{S}\). For each transmitted block, the proceeding \(m_1 = \lceil \log_2(N_T!) \rceil\) bits are first mapped into an integer \(d \in \{1, 2, \ldots, 2^{m_1}\}\) and then into a permutation of antenna indices \(\{A(d, i)\}_{i=1}^{N_T}\), while the remaining \(m_2 = N_T \log_2(M)\) bits are used to select transmitted signals \(\{s_i\}_{i=1}^{N_T}\) from \(\mathcal{S}\) [4].

At the \(t\)-th transmission duration, the transmitted matrix \(S_t \in \mathbb{C}^{N_T \times N_T}\) is obtained as

\[
S_t = S_{t-1}X_t, \tag{1}
\]

where \(X_t \in \mathbb{C}^{N_T \times N_T}\) is the information matrix, which is determined by the information bits. From above introduction, it is clear that \(X_t(A(d, i), i) = s_i\), where \(i \in \{1, 2, \ldots, N_T\}\). Let \(H_t \in \mathbb{C}^{N_R \times N_T}\) denote the channel matrix with covariance

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Thus, the received signal matrix $Y_t \in \mathbb{C}^{N_R \times N_T}$ can be expressed as:

$$Y_t = H_t X_t + N_t,$$

where $N_t \in \mathbb{C}^{N_R \times N_T}$ is an additive white Gaussian noise matrix with zero mean and covariance $\sigma^2 I_{N_R}$. Assuming quasi-static fading, i.e., $H_{t-1} = H_t$, (2) can be thus expressed by the $(t - 1)$-th received signal matrix as

$$Y_t = Y_{t-1} X_t - N_{t-1} X_t + N_t. \quad (3)$$

Accordingly, the optimal ML detection can be derived as

$$\hat{X}_t = \arg \max_{X_t \in \mathcal{G}} \operatorname{Tr}\{\mathcal{R}\{Y_t^H Y_{t-1} X\}\}, \quad (4)$$

where $\mathcal{G}$ stands for the set composed of all valid information matrices. Finally, the information bits are recovered by the demapping of the estimated antenna activation order and the demodulation of the transmitted signals in $\hat{X}_t$. Note that the optimal ML detection of the antenna activation order in (4) imposes great computational burden to the receiver when $N_T$ goes to a very large value, which occurs in large-scale MIMO systems. To solve this problem, one can refer to the low-complexity detection methods proposed in [4, 5].

### III. INDEX-MAPPING IN GRAY CODE ORDER

In this section, we present the idea of encoding permutations of antenna indices in the gray code order for DSM, which is only related to the $m_1$ information bits.

#### A. Look-Up Table

Table I shows the mapping between the information bits and the corresponding permutations in the lexicographic order and gray code order for $N_T = 3$, respectively, where two out of in total six permutations have been discarded. Note that although not specified, it can be readily figured out that both manners give rise to the same result for $N_T = 2$. From Table I, it is obvious that the discarded permutations in the gray code order are different from those in the lexicographic order. However, one can see surprisingly that the lexicographic order for $N_T = 3$ also satisfies the requirement of the gray code order, i.e., any two permutations having only two indices difference differ from only one bit. This implies that both manners for $N_T = 3$ will result in the same BER performance. For $N_T > 3$, the lexicographic order cannot meet the above requirement any longer and both manners will lead to totally different performance. This will be verified in the sequel.

#### B. Ranking and Unranking Methods

The look-up table method necessitates a storage of all permutations at both the transmitter and receiver, which becomes impractical for large $N_T$. Therefore, it is advisable to create an easy-to-implement one-to-one mapping from the information bits to the corresponding permutations, called ranking, and inverse mapping from the permutations to the corresponding information bits, called unranking.

To begin with, we have to first figure out how to generate permutations in the gray code order. Aiming at this, we resort to the idea presented in [8]. Example 1 gives a solution for $N_T = 4$. From this example, we can expect some important properties of gray code order for a general $N_T$ as follows.

For ease of exposition, let us define left and right moving directions for number $N_T$ as $\bar{N}_T$ and $\tilde{N}_T$, respectively. Also, define a directional indicator for number $N_T$ as $\mathcal{I}_{\bar{N}_T}$, where $\mathcal{I}_{\bar{N}_T} = 1$ stands for the situation of $\bar{N}_T$ and $\mathcal{I}_{\bar{N}_T} = 0$ for the situation of $\tilde{N}_T$. If the whole swaps between number $N_T$ and the adjacent element in the set $\{1, 2, \ldots, N_T\}$ for the same direction is said to be one round for number $N_T$, one will see that the total number of rounds of number $N_T$, denoted by $l_{N_T}$, is $(N_T - 1)!$. In addition, it can be concluded that $\mathcal{I}_{\bar{N}_T} = 1$ if $l_{N_T}$ is odd and $\mathcal{I}_{\bar{N}_T} = 0$ if $l_{N_T}$ is even, where $l_{N_T}$ is a variable indicating that number $N_T$ is in the $l_{N_T}$-th round. Details on the generation rule can be referred to the Trotter-Johnson ranking and unranking algorithms [8]. In what follows, we summarize them in our notations and mainly focus on their application to DSM.

#### 1) Trotter-Johnson ranking algorithm:

Assume that we have a permutation $a = \{a_1, a_2, \ldots, a_{N_T}\}$ where $u, a_u \in \{1, 2, \ldots, N_T\}$. Define $P_{a_u}$ as the position of the element $a_u$ in $a^{[a_u]}$, where $a^{[a_u]}$ indicates the sub-permutation of $a$, which discards the elements greater than $a_u$. Initially, we will determine whether $a$ is generated from the permutations $\{1, 2\}$ or $\{2, 1\}$ for obtaining $l_3$. It is obvious that $a$ is generated from $\{1, 2\}$ if $P_2 = 2$, which indicates $l_3 = 1$ ($\mathcal{I}_3 = 1$), while $a$ is generated from $\{2, 1\}$ if $P_2 = 1$, which indicates $l_3 = 2$ ($\mathcal{I}_3 = 0$). For the case of $N_T > 3$, $\{1_3\}_{j=4}^{N_T}$ can be calculated recursively by

$$l_j = (l_{j-1} - 1) \cdot (j - 1) + \tilde{I}_{j-1} \cdot P_{j-1} + \bar{I}_{j-1} \cdot (j - P_{j-1}), \quad (5)$$

where $\tilde{I}_{j-1} = 1 - \bar{I}_{j-1}$. Finally, the integer $d \in \{1, \ldots, 2^{m_1}\}$ corresponding to $a$ is calculated by

$$d = (l_{N_T} - 1) \cdot N_T + \bar{I}_{N_T} \cdot P_{N_T} + \mathcal{I}_{N_T} \cdot (N_T - 1 - P_{N_T}). \quad (6)$$

Then, the information bit sequence in the gray code order can be directly obtained from $d$. 

| Bits in LO | Permutations | Bits in GCO | Permutations |
|-----------|--------------|-------------|--------------|
| 00        | (1 2 3)      | 00          | (1 2 3)      |
| 01        | (1 3 2)      | 01          | (1 3 2)      |
| 10        | (2 1 3)      | 11          | (3 1 2)      |
| 11        | (2 3 1)      | 10          | (3 2 1)      |

1 LO=lexicographic order  
2 GCO=gray code order
2) Trotter-Johnson unranking algorithm: The unranking process is the inversion of the ranking process. Firstly, the information bit sequence in the gray code order is converted into an integer \( d \). Then, \( l_{N_T} \) and \( P_{N_T} \) can be derived as
\[
l_{N_T} = \lceil d/N_T \rceil \quad \text{and} \quad P_{N_T} = \text{mod}(d, N_T) + 1.
\]
For the case of \( N_T > 3 \), \( l_k \) and \( P_k \) with \( k = N_T - 1 \) can be calculated by \( l_k = \lfloor (k + 1)/k \rfloor \) and \( P_k = \text{mod}(l_k+1, k) + 1 \), respectively. Then, we set \( k := k - 1 \) and iteratively obtain \( l_k \) and \( P_k \), respectively. The process continues until \( k = 3 \), when \( \{l_k\}_{k=3}^{N_T} \) and \( \{P_k\}_{k=3}^{N_T} \) are all ready. Note that \( \{l_k\}_{k=3}^{N_T} \) can be determined by \( \{l_k\}_{k=3}^{N_T} \). Finally, we can rebuild the permutation from the ranking algorithm.

### C. Coding Gain Analysis

We now analyze the performance of the gray code order by comparing it with that of the lexicographic order. To illustrate the effect of Gray coding, we consider the special case of DSM, i.e., differential space shift keying (DSSK), in which \( \{s_i\}_{i=1}^{N_T} \) are set to 1s for all time.

An upper bound on the average bit error probability (ABEP) can be derived according to the union bound technique as
\[
P_p \leq \frac{1}{m_1 \cdot 2^{m_1}} \sum_{p=1}^{2^{m_1}} \sum_{q=1}^{2^{m_1}} N(X_p \rightarrow \hat{X}_q) Pr(X_p \rightarrow \hat{X}_q),
\]
where \( Pr(X_p \rightarrow \hat{X}_q) \) is the pairwise error probability accounting for the probability of the information matrix \( \hat{X}_q \) when \( X_p \) is transmitted, and \( N(X_p \rightarrow \hat{X}_q) \) is the number of bits in difference between \( X_p \) and \( \hat{X}_q \). Assuming a rich scattering environment, at high SNR the upper bound in (7) can be further approximated as
\[
P_p \leq c \cdot SNR^{-r_{\text{min}}N_R} \sum_{p,q} N(X_p \rightarrow \hat{X}_q|R_{p,q} = r_{\text{min}}),
\]
where \( c \) is a constant, \( N(\cdot|\cdot) \) is the conditional number of bits in error, \( R_{p,q} = \text{rank}[X_p - \hat{X}_q] \) and \( r_{\text{min}} = \text{min}\ R_{p,q} \). It can be readily figured out that for both the gray code order and lexicographic order we have \( r_{\text{min}} = 1 \), which implies that DSSK achieves unit diversity order regardless of which encoding manner is employed. Therefore, the performance of the gray code order and lexicographic order differ from the coding gain only. To evaluate the value, let us define the total number of bits in difference with \( R_{p,q} = 1 \) for both manners as \( N^G_{\text{error}} = \sum_{p,q} N^G(X_p \rightarrow \hat{X}_q|R_{p,q} = 1) \) and \( N^L_{\text{error}} = \sum_{p,q} N^L(X_p \rightarrow \hat{X}_q|R_{p,q} = 1) \), where the subscripts \( G \) and \( L \) refer to the gray code order and lexicographic order, respectively. With the definitions, from (6) the coding gain achieved by the gray code order over lexicographic order in dB can be thus calculated by
\[
\gamma = 10 \log_{10}(N^L_{\text{error}}/N^G_{\text{error}}).
\]

Recall that \( m_1 = \lceil \log_2(N_T!) \rceil \). The exact coding gain in (2) needs to be evaluated numerically. However, if we relax \( m_1 \) to \( \log_2(N_T!) \), i.e., the full permutations are taken into account, the resulting coding gain can be easily derived from symmetry, which provides insight into the exact result. To see this, let us assume \( m_1 = \log_2(N_T!) \) and define \( \alpha = N_T/L_R \). For \( N_T = 3 \), a word of 2 binary bits are needed to represent a permutation, giving a total of 2 unit word errors in the lexicographic order and 2 unit word errors in the gray code order. The coding gain is thus given by \( \gamma = 10\alpha \cdot \log_{10}(2/2) = 0\text{dB} \), which coincides with the analysis in Section III.A. On the other hand, for \( N_T = 4, 2 \) binary bits are needed with 3 unit word errors in the lexicographic order and 2 unit word errors in the gray code order, which leads to \( \gamma = 10\alpha \cdot \log_{10}(3/2) = 1.17\text{dB} \). Similarly, for \( N_T = 5 \) with 3 binary bits, we have \( \gamma = 10\alpha \cdot \log_{10}(4/3) = 0.26\text{dB} \). From above, it is clear that the coding gain is maximized when \( \alpha \) gets its maximum. However, as \( N_T \) becomes larger beyond 4, \( \alpha \) will become smaller, which implies that the coding gain is maximized for \( N_T = 4 \). In the simulations, we will verify the aforementioned expectation of the exact coding gain for \( m_1 = \lceil \log_2(N_T!) \rceil \).

### IV. A diversity-enhancing scheme based on Gray coding framework

From above analysis, we see that the diversity order achieved by DSM systems depends on \( r_{\text{min}} \). Since in the gray code order two adjacent permutations have two elements difference, the minimum rank equals one and in return the diversity order remains unit. To improve the diversity performance of DSM, in this section we propose a novel scheme, i.e., I-gray code order, based on the gray code order.

#### A. Application to DSSK

In the I-gray code order, the new permutations consist of the ones which locate the odd (or even) positions of the full permutations in the gray code order. The example for \( N_T = 3 \) is given in Table II, where only the odd positions, i.e., 1st, 3rd and 5th, of the full permutations in the gray code order are selected for the purpose. To modulate the information bits, however, only \( 2^{\log_2(N_T!/2)} \) permutations are permitted to be used, which obtains the legitimate permutations (1 2 3) and (3 1 2). It is clear that two adjacent permutations in the I-gray code order will have three elements difference, which improves the diversity order to two in DSSK.

#### B. Application to DSM

The direct application of the I-gray code order to conventional DSM fails to achieve a diversity order of two since the independent symbol-by-symbol transmission scheme will limit \( r_{\text{min}} \) to be unit. To overcome this problem, it is advisable to introduce correlation between two adjacent modulated symbols.
in a DSM block under the framework of the I-gray code order. To this end, we extend the idea of coordinate interleaving design (CID) \[12\] to our design. Specifically, in a DSM block, for a pair of modulated symbols drawn from a phase rotated constellation with angle \(\theta\), the real and imaginary parts of one modulated symbol is combined with the imaginary and real parts of the other modulated symbol. For example, the information matrix for \(N_T = 3, 4\) with antenna activation orders \((3 1 2)\) and \((1 3 4 2)\) can be expressed by

\[
X_t = \begin{bmatrix}
0 & s_2^R + js_3^I & 0 \\
0 & 0 & s_3^R + js_1^I \\
s_1^R + js_2^I & 0 & 0 \\
\end{bmatrix}, \quad (10)
\]

and

\[
X_t = \begin{bmatrix}
s_1^R + js_2^I & 0 & 0 & 0 \\
0 & 0 & 0 & s_4^R + js_5^I \\
0 & s_3^R + js_4^I & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad (11)
\]

respectively, where \(\{s_i = s_i^R + js_i^I\}_{i=1}^4 \in S^\theta\) and \(S^\theta\) denotes the rotated constellation \(S\) with angle \(\theta\). Note that the value of \(\theta\) affects the BER performance of DSM and the optimization of \(\theta\) is considered as our further research.

V. SIMULATION RESULTS AND ANALYSIS

In this section, we conduct simulations to evaluate the BER performance of DSM (DSSK) with the proposed schemes, where slow-varying Rayleigh flat fading channels are assumed.

Fig. 1 shows the comparison results between the BER performances of the gray code order and lexicographic order for \(N_T = 4, 6\) and \(N_R = 1, 2\) in DSSK under the spectral efficiency of 1bps/Hz and 1.5bps/Hz, respectively. At BER \(= 10^{-3}\), it can be seen that the proposed gray code order achieves about 1.2dB and 0.3dB SNR gains over the lexicographic order for \(N_T = 4\) and \(N_T = 6\), respectively. Note that the performance gain in DSM is still impressive though it becomes a little smaller than that in DSSK. Due to page limit, we have not added the results for DSM.

Fig. 2 shows the calculated coding gains achieved by the gray code order over the lexicographic order for \(N_T = 3, 4, 5\) and 6 from \[9\] in DSSK. It is expected that no coding gain is available for \(N_T = 3\) and it achieves the maximum for \(N_T = 4\). On the other hand, one can see that the theoretical results match their simulation counterparts in Fig. 1.

Fig. 3 shows the BER performances of the gray code order and I-gray code order with \(N_T = 4, N_R = 2\) in DSSK and 4QAM DSM, respectively. Similar to \[12\], we set \(\theta = 15^\circ\). We see that the I-gray code order significantly outperforms the gray code order in both configurations at the price of one information bit loss for each transmission. This can be attributed to the SNR gain from a larger minimum Euclidean distance between any two constellation points as well as an additional diversity order.

VI. CONCLUSION

In this letter, we presented the gray code order to improve the performance of DSM. Further, we also proposed the I-gray code order for DSM to achieve an additional diversity order at the cost of one information bit loss for each transmission.
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