Vortex-induced vibration of two elastically connected bodies: experimental verification of lock-in to multiple eigenmodes

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Abstract
This study examines experimentally the vortex-induced vibration (VIV) of a mechanical system with two eigenmodes. A previous experiment setup was refined to enable the experiment, and was placed in a circulating water channel to submerge a movable circular cylinder (cylinder A). This cylinder was subjected to fluid flow while being supported elastically, whereupon VIV occurred. A second movable cylinder (cylinder B) above the water was connected to cylinder A and supported elastically. The displacements of those two cylinders were measured. The underlying hypothesis was that the vortex shedding frequency would become locked to one or other of the two eigenfrequencies, and that which eigenmode the vortex shedding frequency became locked to would depend on the reduced velocity. To test this hypothesis, the experimental setup was refined to withstand the high flow speeds required to allow the vortex shedding frequency to become locked to either eigenfrequency. From the results obtained, the amplitude ratio (the ratio of the amplitude of cylinder B to that of cylinder A) and the frequencies of the two cylinders were determined. It was found that those amplitude ratio and frequencies were close in two ranges of reduced velocity to those calculated theoretically by solving the eigenvalue problem of the 2DOF system. This demonstrated that, depending on the reduced velocity, the vortex shedding frequency could become locked to either the first or second eigenmode to produce a definite vortex-induced vibration.

Keywords: Vortex-induced vibration, Two degrees of freedom, Amplitude ratio, Lock-in, Eigenvalue, Eigenvector

1. Introduction

A circular cylinder placed in a fluid flow typically exhibits oscillatory motion. It is well known that such motion, referred to as vortex-induced vibration (VIV), is induced by vortices generated on and shed behind the oscillating cylinder. The magnitude of VIV depends on the flow speed $V$, the natural frequency $f_n$ of the mechanical system including the cylinder, and the diameter $d$ of the cylinder. This dependency is represented by a single dimensionless parameter known as the reduced velocity $V_r = V/(f_n d)$. Reduced velocities of $V_r = 5–8$ generally produce distinct VIV of the cylinder, the amplitude of which typically reaches the equivalent of the cylinder diameter (Bishop and Hassan 1964; Koopman, 1967; Blevins and Burton, 1976; Bearman, 1984; Khalak and Williamson, 1996; Sakamoto et al., 2004; Sarpkaya, 2004; Williamson and Govardhan, 2004).

Previous studies of systems with one degree of freedom (1DOF), such as a single movable circular cylinder (e.g., Khalak and Williamson, 1999), have reported little evidence of VIV for $V_r > 10$. However, when the movable cylinder is placed together with a fixed one to form a tandem arrangement, remarkable VIV is encountered for $V_r > 10$ as it is induced by an instability of the motion and the wake (e.g., Igarashi, 1981; Zdravkovich, 1985; Summer, 2010).

In the present study, we address a system with two degrees of freedom (2DOF) that is driven by flow-induced forces. In many previous studies (e.g., Jauvtis and Williamson, 2004; Assi, 2014; Bai and Qin, 2014; Gsell et al., 2016), the 2DOF system in questions was a body that oscillated in both the streamwise and spanwise directions. However, the 2DOF system considered herein refers to a mechanical system that comprises two vibrating bodies that are connected elastically. These two types of 2DOF system differ physically in the numbers of eigenfrequencies involved in the
dominant component of their vibrations: the former has one eigenfrequency, whereas the latter (i.e., the present case) has two.

Dynamical systems with more than one degree of freedom are very important in the safety-critical design of structures subjected to fluid flows, such as bridges or slender underwater structures. The huge sizes of such structures means that they respond elastically to flow-induced loads, which is equivalent to saying that they have multiple eigenmodes. Such multiplicity can also be created by connecting multiple rigid bodies elastically. This approach has been adopted by industry to suppress the oscillation of mechanical structures by means of a tuned mass damper (e.g., Kamiya et al., 1996). It has also been seen as a promising way to enhance renewable energy from fluid flows through VIV (Barrero-Gil et al., 2012; Nishi 2013).

The present study is focused on two rigid bodies that are connected elastically, that is, a 2DOF system composed of the two bodies and driven by flow-induced forces. Hereinafter, “2DOF system” refers to this system. Many previous studies addressed vibrating flexible structures subject to fluid flows, but, few ones verified the lock-in of vortex-shedding to multiple eigenmodes through examining both the eigenvalue and eigenvector.

We aimed to develop an experimental setup that would realize both eigenmodes of the 2DOF system. These involve two eigenfrequencies: the lower first one and the higher second one. This fact suggests that VIV could occur even for \( V_r = 10 \) because the vortex shedding frequency could become locked to the second eigenfrequency, being distinctly different from the VIV with a single eigenfrequency which has been substantially studied.

Although the 2DOF system treated herein is structurally simple, a deliberate examination of the lock-in to the multiple eigenmodes can lay a foundation of the mechanics of flow-induced vibration, and can be useful in relation to safety-critical design or higher efficient renewable energy conversion.

Experimental verification of the above supposition was difficult to achieve previously because it required us to generate very fast flows associated with \( V_r > 10 \). Having identified a structural weakness in our previous experimental design that prevented sufficient results at such high flow speeds, we implement an improved design herein. By analyzing the newly improved experimental results, we are able to determine the eigenvalues and eigenvectors of the two eigenmodes. These eigenvalues and eigenvectors represent the locked-in vibratory frequency and the amplitude ratio of the two bodies, respectively. They are determined from the measurement data, and are compared with their theoretical equivalents to specify which eigenmode the vortex shedding frequency is locked to at various flow speeds.

2. Experimental method

2.1 Support of the two cylinders

The experimental 2DOF setup (Fig. 1) included two movable cylinders and three springs to support the cylinders (Fig. 1a).

One of the cylinders, a circular cylinder (cylinder A) with a diameter of 0.1 m and a longitudinal length of 1.0 m, was supported by a spring (spring A) and placed underwater in a circulating water channel. Cylinder A was supported by spring A.

The circulating water channel (V2-15A, West Japan Fluid Engineering Laboratory Co., Ltd.) had a transparent section that was 3.0 m long, 1.2 m wide, and 0.85 m deep, in which the water depth was 0.6 m. The flow was intentionally accelerated near the water surface to maintain flow uniformity in the transparent section, ensuring that the flow speed deviated from its mean by between −0.9% and +1.5%.

The other cylinder (cylinder B) was placed in air and supported by spring B. Another spring (spring AB) was inserted between cylinders A and B. The mass of cylinder B is the most important parameter specifying the mechanical response of the setup, and therefore was regulated by adding weights to cylinder B.

The parameters related to the design of the setup are listed in Table 1. The stiffness of each spring was determined so that the required range of reduced velocity could be covered.

2.2 Frames for constraining cylinder A to the vertical direction

Being subject to the fluid flow, cylinder A is subject to hydrodynamic loads, and exhibits some directional components of motions. The component we attempt to examine is only the vertical one (Fig. 1b). To restrict the motion of cylinder A to the vertical component, and to prevent cylinder A from drifting downstream because of the drag component of the loads, some aluminum frames were used. Those frames maintained the parallelogram arrangement
(Fig. 1b) while they rotated around pivots like levers. This arrangement constrained cylinder A to move mainly vertically. Horizontal motion was also permitted, but to an extent that was negligible in comparison to the vertical motion. Hereinafter, those frame is referred to as constraint frames.

The experiments of Nishi et al. (2017) suffered from cylinder A drifting downstream because of the drag, thereby preventing complete measurements for $V_r > 10$. The previous design employed two vertical rods (hereinafter referred to as lower rods; Fig. 2) whose bottom ends were welded to cylinder A to transmit its motion to the constraint frames and whose top ends were connected to the constraint frames via set screws. This form of connection posed no problems for $V_r < 9$, but for $V_r > 10$ the lower rods were forced unexpectedly to rotate because the moment exerted by the drag at their top ends surpassed the maximum moment that the set screws could withstand. To eliminate this difficulty, flat plates were employed instead of set screws: two flat plates were welded at the top ends of the lower rods, and others were welded at the bottom ends of newly added rods (hereinafter referred to as upper rods; Figs. 1c and 2). These plates were connected together via screws, thereby increasing the resistance to the drag-exerted moment at higher flow speeds.

2.3 Free damping test

Prior to measurements with fluid flow, free damping tests were conducted to determine the eigenfrequencies of the vibratory system. The movable part of the setup was displaced artificially then released. The frequencies of the free damped vibration were measured for each mass ratio. The eigenfrequencies so measured were those of the first mode because, in those tests, cylinders A and B were in phase. The mass damping parameter $m^* \zeta$ (Khalak and Williamson, 1996) was estimated using the measured damping ratio, where $m^*$ is the ratio of the mass of the submerged body to that of the displaced water, and $\zeta$ is the damping ratio; herein, we have $m^* \zeta = 7.39 \times 10^{-2}$, which is categorized as low mass damping.

| Symbol | Definition | Value |
|--------|------------|-------|
| $d$    | Diameter of cylinder A | 0.1 m |
| $k_A$  | Stiffness of spring A  | 160.8 N/m |
| $k_B$  | Stiffness of spring B  | 1200.0 N/m |
| $k_{AB}$ | Stiffness of spring AB | 1525.3 N/m |
| $m_A$  | Mass of movable part  | 27.5 kg |
| $f_A$  | Eigenfrequencies for four cases of mass ratio | 0.708 Hz ($\mu = 0.52$) |
| $\mu$  | Mass ratio | 0.52, 0.70, 0.71, and 0.72 |
| $\zeta$ | Damping ratio (average) | 0.021 |

2.4 Data acquisition and analysis

The displacements of cylinders A and B were measured with laser displacement sensors (IL-600, Keyence Co., Ltd.). After being amplified with an amplifier (IL-1000, Keyence Co., Ltd.), the sensor signals were transmitted to a data recorder (NR-600, Keyence Co., Ltd.) to be stored as a time series. The zero points of this time series were adjusted to eliminate contaminations, followed by processing with a fast Fourier transform to determine the amplitude and frequency of the vibratory components in the time series.
3. Theoretical eigenmode analysis

The eigenfrequencies and eigenvectors of the 2DOF system were computed theoretically for comparison with those measured experimentally. In the theoretical model used herein, the rotational part of the experimental setup is simplified as a single rotating bar (Fig. 1b) into whose inertial moment we incorporate the inertial effect of cylinder A. Furthermore, we incorporate the damping effects involved in the vibratory system into the damping between cylinders A and B, which we represent in bulk fashion by a single damping coefficient $c_{AB}$ that is related to the damping ratio $\zeta$. The rotation angle of the bar is denoted by $\theta$, the spring constants (stiffness) are denoted by $k_A$, $k_B$, and $k_{AB}$, and the displacement of cylinder B is denoted by $x_B$. Applying the above simplifications, the free vibrations of the rotating bar and cylinder B are expressed, respectively, as
To calculate the eigenfrequencies of the 2DOF system, the two cylinders are assumed to vibrate harmonically, as expressed by
\[ A \ddot{x}_A + k_A x_A + c_{AB} (\dot{x}_B - \dot{x}_A) = 0, \]
\[ B \ddot{x}_B + k_B x_B + c_{AB} (\dot{x}_B - \dot{x}_A) = 0. \]

Substituting these forms into Eqs. (2) and (3), the eigenfrequencies \( \omega_n \) (where the subscript \( n \) indicates the sequence number of the mode) were computed (Fig. 3). The ratio \( \frac{x_B^*}{x_A^*} \) represents the ratio of the amplitude of cylinder B to that of cylinder A, as well as the phase difference between the displacements of cylinders A and B. Herein, the absolute value of \( \frac{x_B^*}{x_A^*} \) is referred to as the amplitude ratio.

The ratio \( \frac{x_B^*}{x_A^*} \) is positive for the first mode, and almost constant with variation of the mass ratio (Fig. 3c), whereas the same ratio is negative for the second mode, and its magnitude decreases with increasing mass ratio. Such positive (resp. negative) values of the amplitude ratio indicate that the oscillatory phase of cylinder B is the same as (resp. opposite to) that of cylinder A.

The term \( m_A l_A^2 \) in Eq. (1) is the moment of inertia of the rotating bar, where \( m_A \) is the mass of the rotating bar and \( l_A \) is its radius of rotation. The mass \( m_A \) was determined by measuring the eigenfrequency of the 1DOF system that includes only cylinder A and spring A. By defining the length \( x_A \equiv l_A \theta \), Eq. (1) can be expressed as

\[ m_A \ddot{x}_A + k_A x_A + c_{AB} (\dot{x}_A - \dot{x}_B) = 0. \]

To calculate the eigenfrequencies of the 2DOF system, the two cylinders are assumed to vibrate harmonically, as expressed by \( x_A = x_A^* e^{i\omega t} \) and \( x_B = x_B^* e^{i\omega t} \). Substituting these forms into Eqs. (2) and (3), the eigenfrequencies \( \omega_n \) (where the subscript \( n \) indicates the sequence number of the mode) were computed (Fig. 3). The ratio \( \frac{x_B^*}{x_A^*} \) represents the ratio of the amplitude of cylinder B to that of cylinder A, as well as the phase difference between the displacements of cylinders A and B. Herein, the absolute value of \( \frac{x_B^*}{x_A^*} \) is referred to as the amplitude ratio.

The ratio \( \frac{x_B^*}{x_A^*} \) is positive for the first mode, and almost constant with variation of the mass ratio (Fig. 3c), whereas the same ratio is negative for the second mode, and its magnitude decreases with increasing mass ratio. Such positive (resp. negative) values of the amplitude ratio indicate that the oscillatory phase of cylinder B is the same as (resp. opposite to) that of cylinder A.

Fig. 3. Frequencies of (a) cylinder A and (b) cylinder B, and (c) the ratio \( \frac{x_B^*}{x_A^*} \) measured for mass ratios of 0.52 (black circles), 0.70 (red circles), 0.71 (black squares), and 0.72 (red squares). Theoretical values are plotted as solid (first eigenmode) and dashed (second eigenmode) lines.

4. Results and discussion

In the figures that follow, displacement and amplitude are given in dimensionless form normalized by \( d \), the diameter of cylinder A, and frequency is normalized by \( \omega_1 \), the eigenfrequency of the first mode.
4.1 Responses against varying reduced velocity

4.1.1 $\mu = 0.52$

For $V_r = 6.0$, cylinders A and B vibrated remarkably in phase (Fig. 4a), although the amplitude of cylinder A was greater than that of cylinder B. Furthermore, the (dimensionless) frequency of cylinders A and B was approximately unity. For $V_r = 12.0$, there was very little response from cylinder A or B (Figs. 4b and 5b).

The VIV of cylinders A and B occurred for $V_r = 5.5–8.0$ (Fig. 6a), wherein the amplitudes of cylinders A and B were roughly 0.7 and 0.3, respectively. As $V_r$ was increased beyond that range, the amplitudes of cylinders A and B gradually decreased, indicating that a mass ratio of 0.52 was too small to elicit a definite response at large reduced velocity. Hence, we proceeded to test larger mass ratios.

4.1.2 $\mu = 0.70$

For $V_r = 6.0$, in a similar manner as for $\mu = 0.52$, cylinders A and B showed clear in-phase VIV (Figs. 4c and 5c). However, the time series of cylinder A (the black curve in Fig. 4c) showed some irregularity. This was probably due to a component with a frequency of 2.8 Hz (Fig. 5c), although its cause was not identified. Distinct differences in VIV were observed for $V_r = 12.0$ (Fig. 4d): cylinder B had greater amplitude than that of cylinder A, in contrast to the case for $\mu = 0.52$, and the vibrations had opposite phases. Spectral analysis (Fig. 5d) revealed that a 1.8-Hz component prevailed in the motion of cylinder B, whereas that of cylinder A comprised components at 0.8 Hz and 1.8 Hz.
For $V_r = 5.0$–8.0, the amplitudes of cylinders A and B were 0.5–0.7 and 0.3–0.4, respectively (Fig. 6b), and their frequencies were approximately 1.0. It should be noted that the amplitudes of cylinders A and B decreased almost to zero at $V_r \approx 9.0$ but increased again at $V_r \approx 11.5$, as seen in the time series for $V_r = 12.0$ (Fig. 4d). In the extreme case of $V_r \approx 12.0$, cylinders A and B differed in their amplitudes: cylinder B had the greater amplitude of 0.4, and cylinder A had an amplitude of 0.2. A difference was seen also in the frequency: approximately 2.5 for cylinder B compared to 1.0 for cylinder A.

4.1.3 $\mu = 0.71$

The time series (Fig. 4e and f) and frequencies (Fig. 5e and f) were similar to those for $\mu = 0.70$. Attention should be paid to the jumps in amplitude for cylinders A and B at $V_r = 11.50$ (Fig. 6c) as for $\mu = 0.70$. For $V_r < 10.0$, the frequencies of cylinders A and B were nearly unity, and jumps in amplitude were accompanied by jumps in frequency; for $V_r > 11.0$, the frequencies of cylinders A and B were 2.3–2.4.

4.1.4 $\mu = 0.72$

The mass ratio was increased further to 0.72 to examine in particular the response at high values of reduced velocity beyond 10.0. Time series and frequency spectra at $V_r = 6.0$ (Figs. 4g and 5g) and 12.0 (Figs. 4h and 5h) are similar to those for $\mu = 0.71$.

For $V_r = 5.0$–7.5, cylinder A had an amplitude around 0.7 whereas cylinder B had an amplitude around 0.4 (Fig. 6d). For $V_r > 10.0$, the amplitude of cylinder B surpassed that of cylinder A, and the phases of cylinders A and B were opposite (Fig. 4h).

![Figure 5: Frequency spectra of cylinders A (black) and B (red). Mass ratio $\mu = 0.52$ (a and b); 0.70 (c and d); 0.71 (e and f); 0.72 (g and h). Reduced velocity $V_r = 6.0$ (a, c, e, and g); 12.0 (b, d, f, and h).](image-url)
It should be noted that the mass ratio of 0.72 yielded a jump in frequency at a smaller reduced velocity of 10.0 compared to the smaller mass ratios. The amplitude of cylinder B tended to grow when the reduced velocity was slightly over 10.0.

![Graphs showing frequency and amplitude vs reduced velocity for different mass ratios](image)

Fig. 6. Frequency and amplitude of cylinders A (black) and B (red) against reduced velocity. Mass ratio: (a) 0.52, (b) 0.70, (c) 0.71, and (d) 0.72. In the panels showing the frequency, the horizontal lines are the first (solid) and second (dashed) eigenfrequencies.

### 4.2 Lock-in to the first or second eigenfrequency

The results presented in subsection 4.1 suggest the following regarding the lock-in of the vortex shedding frequency to the eigenfrequency. Firstly, a definite VIV occurred with a dimensionless frequency of approximately unity for \( V_r = 5.5–8.0 \). This can be interpreted as a lock-in to the first-mode eigenfrequency, as observed in previous studies dealing with a single circular cylinder (e.g., Williamson and Govardhan, 2004). Secondly, the mass ratios of 0.70, 0.71, and 0.72 yielded distinct VIV for \( V_r > 10.0–11.5 \), which is clearly different from the response of a single circular cylinder. The opposite phases of cylinders A and B indicate lock-in to the second-mode eigenfrequency. Thirdly, for \( \mu = 0.70 \) and \( V_r = 11–12 \), and for \( \mu = 0.71–0.72 \) and \( V_r = 10–13 \), the frequencies of cylinders A and B did not agree with each other. This may have been due to transitions between the first and second eigenfrequencies.

To confirm the aspects noted above, we examined the amplitude ratio. For \( \mu = 0.52 \) and \( V_r = 6.5–10.0 \), the amplitude ratio calculated from the experiment was almost constant at around 0.45 (Fig. 7a). The amplitude ratio of the first eigenmode computed by the theoretical model is 0.64.

Greater mass ratios gave amplitude ratios that were distinctly different from those for \( \mu = 0.52 \). For \( \mu = 0.70 \), \( V_r = 4.25–8.00 \) produced the almost constant amplitude ratios of around 0.52 (Fig. 7b). If the reduced velocity exceeded that range, the amplitude ratio decreased to nearly zero then increased again to 2.10 for \( V_r > 10.0 \). The amplitude ratios
For $\mu = 0.71$, the range of reduced velocity within which the amplitude ratio became constant at around 0.50 was $V_r = 5.00–10.00$ (Fig. 7c). For reduced velocities beyond that range, the amplitude ratio was larger, being 3.0 at $V_r = 12.00$ and 2.30 at $V_r = 13.00$. The amplitude ratios computed by the theoretical model are 0.68 (the first eigenmode) and 2.08 (the second eigenmode).

For $\mu = 0.72$, the experiments produced an almost constant amplitude ratio around 0.53 for $V_r = 5.50–9.50$. For $V_r > 9.50$, the amplitude ratio increased to around 2.00. The amplitude ratios computed by the theoretical model are 0.68 for the first eigenmode and 2.05 for the second.

We discuss the mechanics of lock-in by summarizing the aforementioned results. The amplitude ratios obtained experimentally were almost constant, irrespective of the mass ratios, for $V_r = 5.5–8.0$. They are moderately close to the theoretical ones, demonstrating that VIV occurred in that case as lock-in to the first-mode eigenfrequency.

The larger mass ratios ($\mu = 0.70, 0.71,$ and $0.72$) provided amplitude ratios of approximately 2.00 for $V_r = 12.0–13.0$. Those amplitude ratios agree moderately with the theoretical ones for the second eigenmode.

An amplitude ratio of nearly zero suggests that the vortex shedding frequency was locked-in to neither the first nor the second eigenmode. The experimental amplitude ratios of 0.50–2.00 clearly deviate from those computed theoretically. Combined with the frequencies (Fig. 6b, c, and d), they show that the responses were transient between the first and second eigenmodes.

![Fig. 7. Measured amplitude ratios (circles) against reduced velocity at mass ratios of (a) 0.52, (b) 0.70, (c) 0.71, and (d) 0.72. Horizontal lines are absolute values of the ratio $x_n/\hat{x}_n$ for the first (solid lines) and second (dashed lines) modes calculated theoretically.](image-url)

For $\mu = 0.52$, there was very little indication of VIV locked-in to the second eigenfrequency. This is because the range of flow speed tested in the experiment did not allow the vortex shedding frequency to increase to the second-mode eigenfrequency. This is supported by the theoretical result that the higher the second-mode eigenfrequency, the lower the mass ratio (Fig. 3).

The following point requires further examination. The theoretical frequency and amplitude ratio are in moderate agreement with the experimental ones, but full agreement could not be achieved (Fig. 7). For instance, the amplitude ratios determined experimentally tended to be smaller than those computed theoretically. We have not yet identified the cause of these deviations, but we offer some plausible explanations herein. It may be due to the somewhat simplified treatment of the damping effects in Eqs. (1) and (2), or the accuracy to which the spring stiffness was measured. The latter measurements were made carefully and we believe the associated errors to be small enough. Nevertheless, the fact remains that the spring stiffness has a considerable effect on the estimation of eigenfrequency. In future, we will...
pay more attention to the extension and contraction of the springs as the cylinders oscillate.

The 2DOF VIV addressed in this study could be applied to technologies such as vibration absorbers and renewable energy extraction. Unlike the 1DOF system, the 2DOF one provides remarkable VIV for \( V_r > 10.0 \), forcing us to consider a wider range of flow speed when increasing the freedom of a vibratory system. In the present study, we determined all the eigenmode properties (i.e., the eigenfrequencies and eigenvectors) to verify the lock-in for the wider range of the flow speed in the 2DOF system.

5. Conclusion

In this study, we examined the vibration of a 2DOF system driven by vortex-induced forces acting on a submerged circular cylinder (cylinder A) to verify experimentally the lock-in of the vortex shedding. The 2DOF system comprised the submerged cylinder A and another cylinder (cylinder B) in air, which were connected elastically with springs and supported with other springs. We began by constructing an experimental setup that was operable even at the high flow speeds required for VIV to occur for \( V_r > 10.0 \) when the vortex shedding frequency is locked-in to the second-mode eigenfrequency. The structure supporting the submerged cylinder was required to withstand fairly large drag loading on the cylinder exerted by the high flow speed. While varying the mass ratio (i.e., the ratio of the mass of the submerged cylinder to that of the other one), the displacements of the two cylinders were measured to determine their amplitudes and frequencies of vibration. From the measured results, the ratio of the amplitude of cylinder B to that of cylinder A was calculated, as well as their frequencies, and comparison was made with the equivalent values computed with a theoretical model of the 2DOF vibratory system. It was found consequently that for \( V_r = 5.0–9.0 \), within which the VIV of a 1DOF system typically occurs, the 2DOF system also exhibited VIV with frequencies that were locked-in to the first-mode eigenfrequency. For \( V_r = 12.0–13.0 \), the 2DOF system exhibited VIV with frequencies that were locked-in to the second-mode eigenfrequency. A larger mass ratio permitted the lock-in to the second eigenmode to occur at a lower reduced velocity.

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