Structural Evolution of 1D Spectral Function from Low- to High-Energy Limits

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By exactly analyzing the spin-1/2 Luttinger liquid (LL) and numerically solving a model of a mobile impurity electron in the LL, we obtain the one-electron spectral function \( A(p, \omega) \) in a one-dimensional (1D) metal in an entire range of \( p \) at zero temperature. For \( |p| \) near the Fermi point \( p_F \), \( A(p, \omega) \) is featured by two prominent peaks of spinon and (anti)holon representing spin-charge separation, but we also find an additional cusp structure between them. For \( |p| \gg p_F \), this structure evolves as a main peak in \( A(p, \omega) \) by swallowing the antiholon mode and its dispersion relation approaches the one of a free electron, implying the existence of an electron excitation in the whole region, but not quite a quasiparticle in the Fermi liquid due to ever existing power-law decay of the excitation.

PACS numbers: 71.10.Pm, 72.15.Nj, 79.60.–i

The concept of spin-charge separation plays a central role in describing low-energy physics near Fermi points in a 1D interacting electron gas, a typical example of the spin-1/2 Luttinger liquid (LL) \([1]\). This concept may be confirmed in real materials by various experiments \([2]\), including the recent high-resolution angular resolved photoemission spectroscopy in which the one-electron spectral function \( A(p, \omega) \) can be directly measured in the wide range of momentum \( p \) and energy \( \omega \).

If \( |p| \) is not restricted to the region near the Fermi momentum \( p_F \), the linear spectrum approximation, usually adopted in the LL theory, is not sufficient in appropriately obtaining \( A(p, \omega) \). In fact, the effect of the non-linear spectrum on \( A(p, \omega) \) has been intensively studied in recent years \([3-8]\). According to those studies on integrable systems, \( A(p, \omega) \) has singularities for arbitrary \( p \) as \( A(p, \omega) \propto |\omega - \epsilon_s(p)|^{-\mu_s(p)} \) with \( \nu = s \) and \( c \), where \( \epsilon_s(p) \) and \( \epsilon_c(p) \) are energies of spin and charge collective excitations, respectively. In the usual LL theory, the exponent \( \mu_s(p) \) is independent of \( p \), but the nonlinearity in the electron dispersion makes it depend on \( p \) \([5, 8]\). Since the edge of support of \( A(p, \omega) \) is located at \( \omega = \epsilon_s(p) \), \( \mu_s(p) \) determines the power of the threshold singularity in \( A(p, \omega) \) and its actual value has been given from the finite-size spectrum obtained by the Bethe-ansatz method \([6, 8]\). For nonintegrable systems, this threshold singularity remains intact, but the singularity at \( \omega = \epsilon_c(p) \) is smeared into a broad peak \([7]\).

In those preceding works, only the singularities at \( \omega = \epsilon_s(p) \) and \( \epsilon_c(p) \) are discussed on the belief that the electron nature will not sustain in the spin-charge separated system. For \( |p| \) far away from \( p_F \), however, the effect of interactions becomes so weak that we would naively expect that the nature of an injected electron to measure \( A(p, \omega) \) manifests itself as a main peak in \( A(p, \omega) \). Then a natural question arises: Does an electron-like excitation mode actually exist in the 1D interacting electron gas for \( |p| \gg p_F \)? If yes, a related and more intriguing question is: How does the electron-like mode reconcile with the physics of spin-charge separation for \( |p| \approx p_F \)?

In this Letter, we have carefully studied the 1D one-electron Green’s function \( G(p, t) \) in momentum space and time and found that for \( p \approx p_F \), its long-time asymptotic form is composed of three independent modes of power-law decay. Two of them correspond to well-known spinon and (anti)holon excitations, but the rest describes the mode of an electron-like particle \( (\text{pseudo} \text{electron}) \) which may be regarded as an electron dressed with a “cloud” of low-lying spin and charge collective excitations. This pseudoelectron does not appear as a main structure in \( A(p, \omega) \) and never leads to a finite jump in the momentum distribution function \( n(p) \), but it is considered as the 1D counterpart of the Landau’s quasiparticle in higher dimensions. As \( p \) goes away from \( p_F \), the pseudoelectron structure gets broader, but with the further increase of \( p \), it becomes less broad and eventually for \( p \gg p_F \), it evolves as a main and divergent peak in \( A(p, \omega) \) by swallowing the antiholon mode. Concomitantly, its dispersion relation approaches the one of a free electron, allowing us to regard the pseudoelectron as a free electron, but actually it is not quite, nor the Landau’s quasiparticle, basically because this excitation is accompanied by power-law decay. Those results clarify the generic feature of \( A(p, \omega) \) in a 1D metal and answer the aforementioned two questions. In the following, we shall substantiate our claim.

Let us consider the spin-1/2 Luttinger model, for which \( G(p, t) = G(p_F + k, t) \) is well known and is given by \([10, 11]\)

\[
iG(p_F + k, t) = \int_{-\infty}^{\infty} \frac{dx}{2\pi} e^{-ikx} \frac{1-\eta iL(x-v_F t)}{\sqrt{1-\eta L(x-u_F t)}} \times [1-i\Lambda(x-u_F t)]^{-\frac{i\pi}{4}} [1+i\Lambda(x+u_F t)]^{-\frac{\pi}{4}},
\]

for \( t > 0 \) with \( \eta \) a positive infinitesimal and \( \Lambda \) a finite momentum-transfer cutoff. Here \( u_F \) and \( u_c \) are the velocities of spinon and (anti)holon, respectively, and \( \partial \) denotes the power of singularity in \( n(p) \). They are related to the velocity of an electron \( v_F \) and the interactions between electrons, \( g_2 \) and \( g_3 \), in the original Hamiltonian through \( u_s = v_F - g_2/2\pi \), \( u_c = (v_F + g_4/2\pi)^2 - (g_2/\pi)^2 \), and \( \partial = (v_F + g_4/2\pi - u_c)/2u_c \) \([12]\). A Galilean-invariant system, \( v_F \) is given by \( v_F = p_F - g_4/2\pi + g_2/\pi \). By eliminating \( g_4 \) from those equations, we obtain \( v_F = [(1+2\theta)u_c + u_s]/2, \)
and thus \( v_F \) as well as \( u_s, u_c, \) and \( \theta \) can be determined by the Bethe-ansatz method for integrable systems.

By regarding the integrand in Eq. (1) as an analytic function of \( x \) to deform the integral path along the real axis into the lower-half complex \( x \) plane and assuming \( u_s < v_F < u_c \), we can evaluate \( iG(p_F + k, t) \) for \( k > 0 \) in the long-time limit of \( t \to \infty \) as

\[
iG(p_F + k, t) = e^{-k/L} \sum_{\nu = s, c, e} \left( \frac{\Lambda}{k} \right)^{1/2 - \gamma_v} e^{-iu_{\nu}kt+i\pi e^{LL}/2L}(\Omega_{\nu}^Ft)^{-1-\mu_{\nu}^{LL}}
+ (\Omega_{\nu}^Ft)^{-1+\mu_{\nu}^{LL}} e^{-iv_Fkt+i\pi e^{LL}/2L}. \tag{2}\]

The first two terms specified by \( \nu = s, c \) represent the contributions from contours along the two branch cuts (which we take parallel to the imaginary axis) associated with the branch points at \( x = u_s, t-i/\Lambda \) and \( u_c, t-i/\Lambda \), showing the well-known time correlation specific to spinon and (anti)holon in the LL with the exponents, \( \mu_{s,c}^{LL} \) and \( \mu_{c}^{LL} \), given respectively as

\[
\mu_{s,c}^{LL} = 1 + \phi_{s,c}^{LL} = 1/2 - \theta
\]

and \( \mu_{c}^{LL} = \phi_{c}^{LL} = (1 - \theta)/2 \). Here \( \gamma_v = 0, \gamma_c = \theta/2, \Omega_{s,c}^F = (|u_s-v_F|^2/2(\omega-u_s)^1/2\sqrt{2}/(1+\theta)\Lambda), \) and \( \Omega_{c}^F = (|2u_c-v_F|^2/2(\omega-u_c)^1/2\Gamma(2+i\theta)\Lambda) \)

with \( \Gamma(z) \) being the Gamma function. The last term stands for the contribution from a simple pole at \( x = v_Ft-i\nu \), seemingly describing an electron moving with the velocity \( v_F \), but not quite a usual quasiparticle in the Fermi liquid because of the existence of the power-law decay with the exponent \( \mu_{\nu} \) (\( \neq 1 \)), where \( \Omega_{\nu} = (|u_{\nu}-v_F|^2/2(\omega-u_{\nu})^{1/2}\sqrt{2}/(1+\theta)\Lambda) \)

and \( \mu_{\nu} = \phi_{\nu} = -\theta \). Thus we shall call it a pseudoelectron. Its decaying behavior persists even at \( k \to 0 \) and eliminates a finite jump in \( n(p) \) at \( p = p_F \).

Casting \( iG(p, t) \) in Eq. (2) into the form

\[
iG(p, t) = \sum_{\ell = s, c, e} [\Omega_{\ell}(p)t]^{-1+\mu_{\ell}(p)}e^{-i\epsilon_{\ell}(p)t+i\pi \phi_{\ell}(p)/2}, \tag{3}\]

and evaluating \( A(p, \omega) \) by \( \pi^{-1} \text{Re} \int_0^\infty dt e^{i\omega t}iG(p, t) \) for \( \omega > 0 \), we can see that \( A(p, \omega) \) has a singularity (peak or shoulder) at \( \omega = \epsilon_{\ell}(p) \) for \( |\mu_{\ell}(p)| < 1 \), in such a way as

\[
A(p, \omega) = \text{const.} + C_{\ell}^+(p)\omega - \epsilon_{\ell}(p) - \mu_{\ell}(p), \tag{4}\]

where \( C_{\ell}^+(p) = \pi^{-1}\Omega_{\ell}(p)^{-1+\mu_{\ell}(p)}\Gamma(\mu_{\ell}(p))\cos(\pi|\mu_{\ell}(p)|/2 + \phi_{\ell}(p)) \)

with the upper (lower) sign for \( \omega \) larger (smaller) than \( \epsilon_{\ell}(p) \). Since the edge of support of \( A(p, \omega) \)

is located at \( \omega = \epsilon_{\ell}(p), \phi_{\ell}(p) \) necessarily equals \( \mu_{\ell}(p) - 1 \).

As for the term \( \ell = e \) in Eq. (3), in which \( e_{c}(p_F+k) = v_Fk, \mu_{e}(p_F) = \mu_F, \) and \( \phi_{e}(p_F) = \phi_F \) with \( \Omega_{e}(p_F) = \Omega_F \)

for the spin-1/2 Luttinger model, we can easily see that \( \partial A(p_F + k, \omega)/\partial \omega \) diverges at \( \omega = v_Fk \) for \( 0 < \theta \leq 1/8 \)

even at \( p \to p_F \). By using the identity \( \int_0^\infty t^{\mu_F-1}e^{it}dt = \Gamma(\mu_F+1)e^{i\pi(\mu_F+1)/2} \), we can also verify that \( A(p, \omega) \)
haves in accord with Eq. (4), exhibiting a structure of the pseudoelectron. In Fig. 4 we have explicitly shown the generic feature of \( A(p, \omega) \) in which a peak with a cusp exists at the pseudoelectron mode (\( \omega = \epsilon_c(p) \)) in addition to the well-known double divergent peaks for \( p \) near \( p_F \). In plotting \( A(p, \omega) \), we need to know concrete values for \( u_s, u_c, \) and \( \theta \); we have determined them by adopting the Yang-Gaudin model \([14\,\text{i.e.} \, 1 \text{D} \text{electron gas with a} \delta \text{-function interaction}], \) described by the Hamiltonian \( H = -\frac{1}{2m} \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2} + V_0 \sum_{i<j}\delta(x_i-x_j) \)

in the intermediate-coupling regime of \( \lambda_0 \equiv mV_0/2\pi p_F = 1 \).

Three comments are in order on the result in Fig. 4 (i) From its definition, \( G(x, t) \), the inverse Fourier transform of \( G(p, t) \), describes the space-time evolution of an additional electron injected into the system at \( x = 0 \) and \( t = 0 \). As long as \( |x|, |\nu t| \ll \Lambda^{-1} \), the injected electron is not much affected by the interactions and behaves like a free electron. This free-electron-like behavior has no effect on the leading singularities in \( n(p) \), density of states, and \( A(p, \omega) \), but it induces the subleading pseudoelectron singularity in \( A(p, \omega) \).

(ii) Irrespective of \( \Lambda \), the limit of \( \eta \to 0^+ \) is required in Eq. (1) to keep the correct electron anticommutation relation \( \{c_{\nu\sigma}, c_{\nu'\sigma'}^\dagger \} = \delta_{\nu\nu'}\delta_{\sigma\sigma'} \) so as to satisfy the sum rule \( \int_{-\infty}^\infty A(p, \omega)d\omega = \{(c_{\nu\sigma}, c_{\nu'\sigma'}^\dagger)\} = 0 \).

Thus the proper limiting procedure in Eq. (1) is to make \( \eta \) zero first with \( \Lambda \) being finite, leading to the result in Fig. 4. If \( \eta \) is replaced by \( \Lambda^{-1} \) as was previously the case, the pole contribution disappears, because the residue vanishes with that choice of \( \eta \), constituting the reason for the omission of the pseudoelectron contribution in preceding studies \([16\,\text{and} \, 17] \).

That choice is physically incorrect, because the anticommutation relation is not globally satisfied, for example, for \( |x| \ll \Lambda^{-1} \), violating the sum rule. (iii) In the course of obtaining \( A(p, \omega) \), we need to deal with a double Fourier transform, which is best implemented by use of a mathematical trick virtually identical to Feynman parameters \([18\,\text{and} \, 19] \). By introducing two Feynman parameters, we can analytically perform the double Fourier transform and then resort to numerical computation of the remaining double Feynman
dependent of Λ and related to $\tilde{\chi}$.

Then, we can calculate the Green’s function for the mobile impurity in the LL with the velocity $v_p = \partial \xi_p / \partial p$.

In Eqs. (6) and (7), phase shifts $\delta$ in momentum-frequency space as

$$(\omega - \xi_p - v_p k) G(p + k, \omega) = 1 + \int \frac{dq d\varepsilon}{(2\pi)^2} \frac{W(q, \varepsilon)}{\varepsilon - v_p q} G(p + k - q, \omega - \varepsilon) e^{-|q|/\Lambda},$$

with Λ the cutoff of momentum transfer $q$ and $W(q, \varepsilon) = (1/4) \sum_{\alpha \beta \nu} \tilde{V}_{\alpha \nu} \chi_{\alpha \beta \nu} (q, \varepsilon) \tilde{V}_{\beta \nu}$ with use of $\chi_{\alpha \beta \nu} (q, \varepsilon)$ the exact correlation functions for the Luttinger model.

Since Eq. (5) is very similar to the one known well in the Luttinger model [10], we are familiar with its solution: by transforming the variables from momentum and frequency into real space and time, we can write down a differential equation for $G(x, t)$, from which we obtain

$$G(p + k, t) = \prod_{\nu = c, s} \prod_{\alpha = \pm} \left[ 1 + i(u_{\nu} - \alpha v_p) \Lambda t \right] \left[ (\delta_{\alpha \nu}/2\pi)^2 \right] ,$$

for $t > 0$, where $\epsilon_\nu (p + k)$ the dressed impurity energy contains an energy shift in proportion to the cutoff Λ as

$$\epsilon_\nu (p + k) = \xi_p + v_p k - \Lambda \sum_{\alpha \nu} (\delta_{\alpha \nu}/2\pi)^2 (u_{\nu} - \alpha v_p)/2.$$  

In Eqs. (5) and (6), phase shifts $\delta_{\alpha \nu} \equiv \delta_{\alpha \nu} (p)$ are independent of Λ and related to $\tilde{V}_{\alpha \nu}$ and $v_p$ through

$$\delta_{\pm c} = \tilde{V}_{\pm c} \sqrt{\frac{1 + \theta}{u_c + v_p}} \sqrt{\frac{1 - \theta}{u_c + v_p}}, \quad \delta_{\pm s} = \tilde{V}_{\pm s} \frac{v_p}{u_s + v_p} .$$

The results in Eqs. (6) and (7) indicate that $\epsilon_\nu (p + k) = v_p k$, $\delta_{+ c} (pF) = 2\pi \sqrt{1 + \theta}$, $\delta_{- c} (pF) = 2\pi \sqrt{\theta}$, $\delta_{+ s} (pF) = 2\pi$, and $\delta_{- s} (pF) = 0$. Thus at $p = pF$, we see that Eq. (6) is reduced to the pole contribution in Eq. (1).

For illustration of the overall behavior of $A(p > p_F, \omega)$ with the change of $p$ and $\omega$, we adopt the Yang-Gaudin model in the weak-coupling region ($\lambda_0 \leq 0.1$) in which analytic expressions such as $\epsilon_p = p_F/m$, $u_s = (1 - \lambda_0) v_F$, $u_c = \sqrt{1 + 2\lambda_0 v_F}$, and $\theta = (\sqrt{1 + 2\lambda_0} - 1)^2/4\sqrt{1 + 2\lambda_0}$ are successfully checked to reproduce the exact Bethe-ansatz results with sufficient accuracy, indicating that we may well specify the mobile impurity model by employing the weak-coupling results of the quadratic dispersion $\xi_p = (p^2 - p_F^2)/2m$ and the interactions, $\tilde{V}_{\pm c} (p) = - \tilde{V}_{+ s} (p) = V_0$, and $\tilde{V}_{- s} (p) = V_0/(1 - 2\lambda_0 \ln[\xi_p / E_i])$, all of which are obtained by the poor man’s scaling with $E_i$ an initial energy scale. Then, from $G(p, t)$ in Eq. (6), we can explicitly compute $A(p, \omega)$ which is insensitive to the choice of $E_i$.

In Fig. 2 $A(p, \omega)$ obtained at $\lambda_0 = 0.1$ is displayed with increasing $p$ from $p_F$ to show its complete structural evolution in the 1D weakly-interacting electron gas with quadratic dispersion. Since we focus on the region of $\omega$ in the very vicinity of $\xi_p$, only the pseudoelectron mode appears as a singular structure in $A(p, \omega)$ in Fig. 2.

By comparing Eq. (5) in its long-time limit with Eq. (6), we can determine $\mu_\nu (p)$ and $\phi_\nu (p)$ as

$$\mu_\nu (p) = 1 - \sum_{\alpha \nu} (\delta_{\alpha \nu}/2\pi)^2 / 2 ,$$

$$\phi_\nu (p) = - \sum_{\alpha \nu} (\delta_{\alpha \nu}/2\pi)^2 \text{sign}(u_\nu - \alpha v_p)/2 .$$

They are independent of Λ and universal, but they depend on $p$ in the system with nonlinear dispersion [6], as plotted in Fig. 3 in which $\mu_\nu (p)$ and $|\phi_\nu (p)| - 1$ are given by the solid and dotted curves, respectively. The structural feature of $A(p, \omega)$ is specified by both $\mu_\nu (p)$ itself.
and its relative value with respect to $|\phi_e(p)| = 1$. Thus we define two dimensionless constants, $c_1 = 1 - 1/\sqrt{2 + O(\lambda_0)}$ and $c_2 = \sqrt{3} + O(\lambda_0)$, corresponding to the intersections of the solid and dotted curves. By virtue of Eq. (1), $A(p, \omega)$ at $\omega = \epsilon_e(p)$ has a peak with a cusp for $p/p_F - 1 < c_1 \lambda_0$ (the spin-charge separated regime) and a divergent one for $p/p_F - 1 > c_2 \lambda_0$ except for the narrow vicinity of $p/p_F - 1 \approx c_2 \lambda_0$ (the nearly free-electron regime). In the intermediate region of $c_1 \lambda_0 < p/p_F - 1 < c_2 \lambda_0$, on the other hand, $A(p, \omega)$ shows only a broad peak around $\omega = \xi_p$, due to the strong damping effect brought about by the allowed energy-conserving electron-charge excitation scatterings in which $\xi_p - \xi_p - q = \omega$ is satisfied in Eq. (5) (the pseudoelectron damping regime).

In order to give an entire picture of $A(p, \omega)$ including spinon and antiholon modes, we have also plotted the exponents, $\mu_s(p)$ and $\mu_e(p)$, in Fig. 3, which are exact for the Yang-Gaudin model with $\lambda_0 = 0.1$. (Accurate values for those exponents can be derived from the exact Bethe-ansatz results for $\epsilon_s(p)$ and $\epsilon_e(p)$ for Galilean invariant systems [7].) By comparing them with $\mu_e(p)$, we can identify the mode(s) to dominate $A(p, \omega)$: (i) For $|v_p|/v_F - 1 \approx 0$, the situation is characterized by $\mu_s(p) \approx \mu_e(p) \gg \mu_e(p)$, implying the dominance of spinon and antiholon modes. Thus the physics in this regime is well described by the concept of spin-charge separation, but the pseudoelectron is now found to appear as an additional singular cusp structure as depicted in Fig. 1. (ii) For $|v_p|/v_F - 1 \approx \lambda_0$, the pseudoelectron excitation is overdamped as shown in Fig. 3 and sandwiched between the spinon and antiholon divergent peaks, making it difficult to be detected, although we do not expect that its total contribution to the spectral weight is negligible. (iii) For $|v_p|/v_F - 1 \gg \lambda_0$, $\mu_e(p)$ becomes much larger than either $\mu_s(p)$ or $\mu_e(p)$, indicating that the long-time evolution of $G(p, t)$ is controlled by the single mode of pseudoelectron, well defining the nearly free-electron regime. In fact, according to our explicit calculation based on Ref. [7], $\mu_s(p)$ is a decreasing function of $|p|$ and becomes negative for $|p| > 2p_F$, so that the edge of support of $A(p, \omega)$ never diverges. As for the antiholon, $\mu_e(p)$ slowly increases with increasing $|p|$ and reaches $1/2$ at $|p| \to \infty$. In the large-$|p|$ limit with $\lambda_0$ fixed, both $\epsilon_e(p)$ and $\epsilon_s(p)$ approach the free-electron dispersion, but because $\mu_e(p) \approx 2\mu_e(p) \approx 1$, the antiholon peak gets absorbed into the pseudoelectron one [25]. Thus $A(p, \omega)$ is composed of a single divergent peak, reduced to a delta function at $\omega = \xi_p$ at $|p| \to \infty$ where $G(p, t) = -ie^{-i\xi_p t}$ due to $\mu_e(p) \to 1$ and $\phi_e(p) \to 0$ in Eq. (3).

Four comments are in order: (i) As is evident from Eqs. (8)–(10), both $\mu_e(p)$ and $\phi_e(p)$ depend on $p$ mainly through $v_p = \partial \xi_p / \partial p$. For non-Galilean invariant systems, we can obtain very similar results for them by suitably choosing the dispersion $\xi_p$ and an interaction parameter $\lambda_0$, at least for $|v_p| \geq \lambda_0$ in the weak-coupling region [26]. (ii) The strong pseudoelectron damping occurs on the condition $|v_p| \approx \lambda_0$, which is allowed in general at $|v_p|/v_F - 1 \sim \min(1, \lambda_0)$. Thus, if $\lambda_0$ lies outside of the weak-coupling region, the spin-charge separated regime appears for $|v_p|/v_F - 1 \ll \min(1, \lambda_0)$. (iii) Our discussion has been limited only for $|p| \approx p_F$, but a similar discussion can be made for $|p| < p_F$ in which electron-hole asymmetry due to nonlinearity in $\xi_p$ may be a matter of interest. (iv) According to the numerical results for $A(p, \omega < 0)$ obtained by the dynamical density-matrix renormalization group (DDMRG) method for the 1D Hubbard model in the intermediate coupling regime of $U = 4.9t$ and the filling $n = 0.6$ (for which $\lambda_0 \approx 1$ [27]), the peaks of spinon, holon, and its shadow band for $|p| < p_F$, which are predicted by the Bethe ansatz, are found, but there appears no signature of an additional noticeable structure, at least for $|p|$ not close to $p_F$. This absence of the pseudoelectron structure in the DDMRG does not contradict our theory, because the data without a strong artificial broadening effect are given only in the pseudoelectron damping regime of $1 - |v_p|/v_F \sim \lambda_0$.

In summary, we have theoretically studied the overall behavior of $A(p, \omega)$ in a 1D metal at zero temperature in order to establish the concept of a “pseudoelectron”, describing the behavior of an electron injected into the LL with either linear or nonlinear dispersion. This pseudoelectron is found to manifest itself in an entire range of $p$, though its importance in the whole structure of $A(p, \omega)$ depends on $p$: for $|p| \approx p_F$, it appears only as an additional cusp structure to the main peaks of spinon and (anti)holon, while for $|p| > p_F$, it provides a main and divergent peak. This pseudoelectron very much resembles a quasiparticle in higher-dimensional Fermi-liquid systems (e.g., see Fig. 3 in Ref. [24]), although it is not quite the same, reflecting the specialty of 1D physics. We hope that this concept of a pseudoelectron will be confirmed in the future through experiment and/or large-scale numerical calculation with deliberately-chosen parameters so as to avoid its overdamping regime.

H.M. thanks H. Fukuyama, M. Nakamura, and K.
In standard notations \[10\], we have taken \(g_{∥} = 0\) and \(g_{∥} = g_{⊥}\) by the Pauli principle and spin-SU(2) symmetry, respectively, and defined \(g_{∥} = g_{∥} = g_{∥}\).

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[The data for \(\mu_{c}(p)\) in \[23\] are in good accord with those obtained in \[8\] and consistent with the results in other works \[4, 9\]. It is, however, noted that the data of \(\mu_{c}(0) = \mu_{c}(0) = 0.70\) and \(\mu_{c}(\pi/10) = 0.44 < \mu_{c}(\pi/10) = 0.56\) in \[23\] with \(\mu_{c}(p)\) being the the shadow-band exponent contradict the results in those works \[4, 9\], where \(\mu_{c}(0) = \mu_{c}(0) > 3/8\) for \(\lambda_{0} > 1\) and \(\mu_{c}(p) > \mu_{c}(p)\) for any \(p\).]