A Gaussian Particle Filter Approach for Sensors to Track Multiple Moving Targets

H. Li

Abstract—In a variety of problems, the number and state of multiple moving targets are unknown and are subject to be inferred from their measurements obtained by a sensor with limited sensing ability. This type of problems is raised in a variety of applications, including monitoring of endangered species, cleaning, and surveillance. Particle filters are widely used to estimate target state from its prior information and its measurements that recently become available, especially for the cases when the measurement model and the prior distribution of state of interest are non-Gaussian. However, the problem of estimating number of total targets and their state becomes intractable when the number of total targets and the measurement-target association are unknown. This paper presents a novel Gaussian particle filter technique that combines Kalman filter and particle filter for estimating the number and state of total targets based on the measurement obtained online. The estimation is represented by a set of weighted particles, different from classical particle filter, where each particle is a Gaussian distribution instead of a point mass.

Index Terms—Kalman Filter, Particle Filter, Gaussian Particle Filter, Multiple Target Tracking

I. INTRODUCTION

The problem of tracking and monitoring targets using position-fixed sensors is relevant to a variety of applications, including monitoring of moving targets using cameras [1], tracking anomalies in manufacturing plants [1], and tracking of endangered species [2]–[4]. The position-fixed sensor is deployed to measure targets based on limited information that only becomes available when the target enters the sensor’s field-of-view (FOV) or visibility region [5]. The sensor’s FOV is defined as a compact subset of the region of interest, in which the sensor can obtain measurements from the targets. In many such sensor applications, filter techniques are often required to estimate unknown variables of interest, for examples, target number and target state.

When the noise in the measurement model is an additive Gaussian distribution, the target state can be estimated from frequent observation sequence using a Kalman filter [6]. This approach is well suited to long-range high-accuracy sensors, such as radars, and to moving targets with a known dynamical model and initial conditions. However, most of these underlying assumptions are violated in modern applications, because the targets’ motion models are unknown, and, possibly, random and nonlinear. Also, due to the use of low-cost passive sensors, measurement errors and noise may be non-additive and non-Gaussian. An extended Kalman filter (EKF) can be used when the system dynamics are nonlinear, but can be linearized about nominal operating conditions [7]. An unscented Kalman filter (UKF) method, based on the unscented transformation (UT) method, can be applied to compute the mean and covariance of a function up to the second order of the Taylor expansion [8].

However, the efficiency of these filters decreases significantly when the system dynamics are highly nonlinear or unknown, and when the measurement noise are non-Gaussian. Recently, a non-parametric method based on condensation and Monte Carlo simulation, known as a particle filter, has been proposed for tracking multiple targets exhibiting nonlinear dynamics and non-Gaussian random effects [9]. Particle filters are well suited to modern surveillance systems because they can be applied to Bayesian models in which the hidden variables are connected by a Markov chain in discrete time. In the classical particle filter method, a weighted set of particles or point masses are used to represent the probability density function (PDF) of the target state by means of a superposition of weighted Dirac delta functions. At each iteration of the particle filter, particles representing possible target state values are sampled from a proposal distribution [10]. The weight associated with each particle is then obtained from the target-state likelihood function, and from the prior estimation of the target state PDF. When the effective particle size is smaller than a predefined threshold, a re-sampling technique can be implemented [11]. One disadvantage of classical particle-filtering techniques is that the target-state transition function is used as the importance density function to sample particles, without taking new observations into account [12]. Recently, an particle filter with Mixture Gaussian representation was proposed by author for monitoring maneuvering targets [13], where the particles are sampled based on the supporting intervals of the target-state likelihood function and the prior estimation function of the target state. In this case, the supporting interval of a distribution is defined as the 90% confidence

The author H. Li is with Zhejiang University of Technology, Hangzhou, 310014, P.R.China. Email: zgdlhj@zjut.edu.cn.
interval \[14\]. The weight for each particle is obtained by considering the likelihood function and the transition function simultaneously. Then, the weighted expectation maximization (EM) algorithm is implemented to use the sampled weighted particles to generate a normal mixture model of the distribution.

Kreucher proposed joint multitarget probability density (JMPD) \[15\] to estimate the number of total targets in workspace and their state, where targets are moving. By using JMPD, the data association problem is avoided, however, the JMPD results in a joint state space, the dimension of which is the dimension of a target state times number of total targets. Since the number of total targets is unknown, the joint space size remain unavailable. To overcome this problem, it is assumed the number of total targets has a maximum value. Therefore, when the maximum number of targets is large, the joint state space becomes intractable.

Inspired by \[16\], this paper presents a novel filter technique which combines Kalman filter and particle filter for estimating the number and state of total targets based on the measurement obtained online. The estimation is represented by a set of weighted particles, different from classical particle filter, where each particle is a Gaussian instead of a point mass. The weight of each particle represents the probability of existing a target, while its probability density function (PDF). There are sampled weighted particles to generate a normal mixture model of the distribution.

II. Problem Formulation

There \(N\) targets are moving in a two dimensional workspace denoted as \(\mathcal{W}\) where \(N\) denotes the unknown number of total targets. For simplicity, there is zero obstacle in the workspace. The goal of the sensor is to obtain the state estimation for all the targets, denoted as \(X_k\), and target number estimation, denoted as \(T_k\), at time step \(k\). The states for total targets at \(k\) is denoted as \(X_k = [x_k^1, x_k^2, \ldots, x_k^N]\) that has \(N\) state vectors. The estimation of total target state at \(k\) is denoted as \(X^k = [\hat{x}_k^1, \hat{x}_k^2, \ldots, \hat{x}_k^N]\). Let \(T^k\) denote the estimation of total target number at \(T^k\). The \(i\)th target is modeled as

\[
x_k^i = F_k x_{k-1}^i + \nu_k,
\]

where

\[
\nu_k \sim N(0, Q_k).
\]

Furthermore, \(F_k\) and \(Q_k\) are assumed known.

In standard estimation theory, a sensor that obtains a vector of measurements \(z_j^k\) in order to estimate an unknown state vector set \(X^k \in \mathbb{R}^n\) at time \(k\) is modeled as,

\[
z_k = h(X^k, \lambda^k),
\]

where \(h: \mathbb{R}^{n+\nu} \to \mathbb{R}^r\) is a deterministic vector function that is possibly nonlinear, the random vector \(\lambda^k \in \mathbb{R}^\nu\) represents the sensor characteristics, such as sensor action, mode \[17\], environmental conditions, and sensor noise or measurement errors. In this paper, the sensor is modeled as

\[
z_k = \frac{1}{N} \sum_{i=1}^{N} x_i + \omega_k.
\]

It is further assumed that the whole workspace is visible to a position fixed sensor (not shown).

A. Particle Filter Methods

The particle filter is a recursive model estimation method based on sequential Monte Carlo Simulations. Because of their recursive nature, particle filters are easily applicable to online data processing and variable inference. More importantly, it is applicable to nonlinear system dynamics with non-Gaussian noises. The PDF functions are represented with properly weighted and relocated point-mass, known as particles. These particles are sampled from an importance density that is crucial to the particle filter algorithm and is also referred to as a proposal distribution. Let \(\{x_{j,p}^k, w_{j,p}^k\}_{p=1}^N\) denote the weighted particles that are used to approximate the posterior PDF \(f(x_j^k | z_j^k)\) for the \(j\)th target at \(t_n\), where \(Z_j^k = \{z_j^1, z_j^2, \ldots, z_j^T\}\) denotes the set of all measurements obtained by sensor \(i\), from target \(j\), up to \(t_n\). Then, the posterior probability density function of the target state, given the measurement at \(t_n\) can be modeled as,

\[
f(x_j^k | z_j^k) = \sum_{p=1}^N w_{j,p}^k \delta(x_j^k, p), \quad \sum_{p=1}^N w_{j,p}^k = 1
\]

where \(w_{j,p}^k\) is non-negative and \(\delta\) is the Dirac delta function. The techniques always consist of the recursive
propagation of the particles and the particle weights. In each iteration, the particles \( x_{j,p}^k \) are sampled from the importance density \( q(x) \). Then, weight \( w_{j,p}^k \) is updated for each particle by

\[
w_{j,p}^k \propto p(x_{j,p}^k) / q(x_{j,p}^k)
\]

where \( p(x_{j,p}^k) \propto f(x_{j,p}^k \mid Z_j^k) \). Additionally, the weights are normalized at the end of each iteration.

One common drawback of particle filters is the degeneracy phenomenon \[12\], i.e., the variance of particle weights accumulates along iterations. A common way to evaluate the degeneracy phenomenon is the effective sample size \( N_e \) \[11\], obtained by,

\[
N_e = \frac{1}{\sum_{p=1}^{N} (w_{j,p}^k)^2}
\]

where \( w_{j,p}^k \), \( p = 1, 2, \ldots, N \) are the normalized weights. In general, a re-sampling procedure is taken when \( N_e < N_s \), where \( N_s \) is a predefined threshold, and is usually set as \( N \). Let \( \{x_{j,p}^k, u_{j,p}^k\}_{p=1}^N \) denote the particle set that needs to be re-sampled, and let \( \{x_{j,p}^k, u_{j,p}^k\}_{p=1}^N \) denote the particle set after re-sampling. The main idea of this re-sampling procedure is to eliminate the particles having low weights by re-sampling \( \{x_{j,p}^k, u_{j,p}^k\}_{p=1}^N \) from \( \{x_{j,p}^k, u_{j,p}^k\}_{p=1}^N \) with the probability of \( p(X_{j,p}^k = x_{j,p}^k) = w_{j,p}^k \). At the end of the resampling procedure, \( u_{j,p}^k, p = 1, 2, \ldots, N \) are set as \( 1/N \).

### B. Kalman Filter Methods

The well known Kalman filter is also a recursive method to estimate system/target state based on a measurement sequence, minimizing the estimation uncertainty. The measurement of the system state with an additive Gaussian noise is given by the sensor. Then, in each iteration, the Kalman filter consists of two precesses: i) it predicts the system state and their uncertainties; ii) it updates the system state and uncertainties with the measurement that newly becomes available. The system dynamics is given as

\[
x_k = F_k x_{k-1} + B_k u_k + \nu_k
\]

where subscript \( k \) and \( k-1 \) denote the current and previous time index, while \( F_k \) is the system discrete transition matrix, and \( B_k \) and \( u_k \) are the control matrix and control input. \( \nu_k \) is the white noise, defined as

\[
\nu_k \approx N(0, Q_k)
\]

where \( \Sigma_k \) is the covariance. At \( k \)th time step, an measurement of the system true state \( x_k \) is made by a sensor, is given by

\[
z_k = H_k x_k + \omega_k
\]

where \( H_k \) is a mapping from system state space to measurement space, and white noise \( Q_k \) is defined as

\[
\omega_k \approx N(0, R_k)
\]

It is assumed that the noise \( \omega_k \) and \( \nu \) at each time step are independent.

Let \( \hat{x}_k \) denote the predicted state estimation given \( \hat{x}_{k-1} \), where \( \hat{x}_{k-1} \) is the updated estimation of system state at \( k-1 \) time step. Furthermore, let \( \Sigma_k \) denote the predicted covariance given \( \hat{x}_{k-1} \), where \( \Sigma_{k-1} \) is the updated estimation covariance. Then, in the predicting step,

\[
\hat{x}_k = F_k \hat{x}_{k-1} + B_k u_k
\]

\[
\hat{\Sigma}_k = F_k \Sigma_{k-1} F_k^T + Q_k
\]

In the updating step, the measurement \( z_k \) is used, together with above predicted state and covariance, to update the state and covariance. The residual, \( y_k \), between measurement and predicted state is given by

\[
y_k = z_k - H_k \hat{x}_k
\]

The innovation covariance \( S_k \) is given by

\[
S_k = H_k \Sigma_k H_k^T + R_k
\]

Then, the optimal Kalman gain is calculated as

\[
K_k = \Sigma_k H_k^T S_k^{-1}
\]

Then, the state and covariance can be updated by

\[
\hat{x}_k = \hat{x}_k + K_k y_k
\]

\[
\hat{\Sigma}_k = (I - K_k H_k) \Sigma_k
\]

### IV. Methodology

In this paper, a novel Gaussian particle filter technique follows the main idea of particle filter for estimating the number and state of total targets based on the measurement obtained online. Different from classical particle filter, each particle here is a gaussian instead of a point mass. The estimation for number of total targets and their state is presented by a set of weighted particles. The \( i \)th particle at time \( k \) is denoted as

\[
P_k^i = \{w_k^i, N(x_k^i \mid \mu_k^i, \Sigma_k^i)\}
\]

where \( w_i \) is the probability of existing a target having a state distribution as \( N(\mu, N(x_i \mid \mu, \Sigma_i)) \). By this particle definition, the dimensions of the system state remains the same as the dimensions of each individual target. When these particles are available, the estimated number of total targets can be given as

\[
T = \sum_{i=1}^{N_p} w_i
\]

where \( N_p \) is the particle number.

Notice that the particle representation is different from classical particle filter, where each particle represents a possible value of system state. The updating of each
particle and total weights are also different from classical particle filter. Kalman filter is used to update each particle, the weight and the distribution. Notice that since the measurement at each time step is conditioned on all the targets in the FOV, while in the classical Kalman filter method one measurement is associated with one target, which means data association problem is avoid. Therefore, the Kalman filter is modified to updated the particles which are coupled by one measurement, and some approximations and assumptions are further needed.

Similar to Kalman filters and particle filters, the algorithm proposed in this paper is a recursive method. Assume that at time step \( k \), the measurement \( z_k \) is available, and the estimation of the system at time step \( k \) needed. and some approximations and assumptions are further

Then, by applying Kalman procedure
\[
y_k = z_k - H_k \mu_k
\]  
\[
S_k = H_k \Sigma_k H_k^T + R_k
\]  
\[
K_k = \Sigma_k H_k^T S_k^{-1}
\]  
\[
\mu_k = \mu_k + K_k y_k
\]  
\[
\Sigma_k = (I - K_k H_k) \Sigma_k
\]

Its proof can be found in the appendix.

Once each particle appearing in combination \( E \) has been updated, the weight \( w_c \) is for the particle combination \( E \) can be obtained by
\[
w_c = \Pi(w_i)^{c_i}(1 - w_i)^{1-c_i} \times \frac{1}{(2\pi)^2 || \Sigma_c^{-1} ||} \exp\{- (z_k - \mu_c)^T \Sigma_c^{-1} (z_k - \mu_c)\}
\]

where \( c \in I_E \), where \( I_E \) is the combination index, and \( \mu_c \) and \( \Sigma_c^{-1} \) is given by
\[
\mu_c = H_k \Sigma_k \sum \mu_k
\]  
\[
\Sigma_c = H_k \sum \Sigma_k H_k^T + R_k
\]

Then, insert particle \( \{N^i(\mu_k, \Sigma_k)\} \) into a set \( G_c \) for combination \( c \in I_E \), the set \( G_c \) has a weight \( w_c \). After all \( G_c \) the combination of \( E \) that \( \Pi(w_i)^{c_i}(1 - w_i)^{1-c_i} > \epsilon \) is obtained. Weights are updated by
\[
w_c = \frac{w_c}{\sum w_c}
\]

Then in each group \( G_c \), the weight of \( i \)th particle is updated as
\[
w_c^i = \frac{w_c^i}{\Pi w_k}
\]

The particles in all set \( G_c \) are updated from the same particle in the previous set. If two particles in is close enough, then they are combined as one particle, the weight of which is as the summation of both weights. The distance between \( \mu_i \) and \( \mu_j \) is defined as Mahal-distance
\[
(\mu_i - \mu_j)^T M (\mu_i - \mu_j)
\]

and its covariance is updated as
\[
\Sigma = \sum \Sigma_k^i
\]

V. SIMULATION AND RESULTS

As shown in figure [1], \( N \) targets, represented by blue dots, are moving in the 2 dimensional workspace. The whole workspace is visible to a position fixed sensor (not shown) and the workspace is discretized into \( 12 \times 12 \) cells. Each cell represents a \( 1 \times 1 \) rectangular area. The \( i \)th cell, denoted as \( C_i, i \in C \), is defined by \([x_i^l, y_i^l, x_i^r, y_i^r]\), where \( C \) is the cell index set and \((x_i^l, y_i^l)\) and \((x_i^r, y_i^r)\) are up left and down right corner coordinates of the \( i \)th rectangular area respectively.
Only $M$ cells can be measured at each time step $k$, and they don’t have to be adjacent. The goal of the sensor is to estimate the target states and target number at time $k$. In this paper, information value function based $\alpha$ divergence is used to select the best $M$ cells to measure at each step [18]. The estimation of target states and target number at time $k$ is represented by joint multitarget probability density (JMPD) and it is updated after obtaining new measurements.

The target time-discrete state transition function can be written as

$$x_{i}^{k+1} = Fx_{i}^{k} + w_{i}^{k}$$ (36)

where

$$F = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (37)

and $w_{i}^{k}$ is 0 mean Gaussian noise with covariance $Q = \text{diag}(20, 0.2, 20, 0.2)$, and $\tau$ is the time step length, and $i \in \{1, 2, \cdots, T^{k}\}$ [15].

It is further assumed that i) the sensor can measure any cell at time $k$; ii) the sensor can only measure up to $M$ cells at time $k$. The sensor condition $X_{i}^{k}$ represents the signal to noise ratio SNR, currently, it has only one possible value, fixed and known. The measurement $z_{i}^{k}$ is a discrete variable, then joint PMF can be written as

$$f(z_{i}^{k}, X_{i}^{k}, T^{k}, X_{i}^{k}) = f(z_{i}^{k} | X_{i}^{k}, T^{k}, X_{i}^{k})f(X_{i}^{k}, T^{k})f(X_{i}^{k})$$ (38)

When measuring a cell, the imager sensor will give a Raleigh return, either a 0 (no detection) or a 1 (detection) governed by detecting probability, denoted as $p_{d}$, and false alarm probability, denoted as $p_{f}$. According to standard model for threshold detection of Rayleigh returns, $p_{f} = p_{d}^{(1+\text{SNR})}$. When $T$ targets are in the same cell, then the detection probability is $p_{d}(T) = p_{d}^{(1+\text{SNR})/(1+T\times\text{SNR})}$ and the $i$th sensor measurement at time $k$ can be evaluated by

$$p(z_{i}^{k} | X_{i}^{k}, T^{k}, X_{i}^{k}, \lambda_{a,i}, \lambda_{c}^{k}) = \begin{cases} p_{d}(T) & z_{i}^{k} = 1 \\ 1 - p_{d}(T) & z_{i}^{k} = 0 \end{cases}$$

$$T = \sum_{j=1}^{N}(x_{j}^{k} \geq x_{c}^{ui}) \cap (x_{j}^{k} < x_{c}^{dr}) \cap (y_{j}^{k} \geq y_{c}^{ui}) \cap (x_{j}^{k} < x_{c}^{dr})$$

$$c = \lambda_{a,i}^{k}$$ (39)

$$p_{d}(T) = p_{d}^{(1+\lambda_{c}^{k})/(1+T\lambda_{c}^{k})}$$ (40)

where $x_{j}^{k}, y_{j}^{k}$ are two position components of $X_{j}^{k}$ and $T^{k}$ is the target number. Additionally, operators ”$>$” and ”$<$” return either 1 if true or 0 if false, while ”$\cap$” is the Boolean operator ”and”. For example, as shown in figure [1], when $c = k$, $T = 2$, similarly, when $c = j(i)$, $T = 1(0)$.

A snapshot of simulations is shown in Fig. [2] where magenta squares represent positive measurement return and blue dots represent the true targets’ positions. The simulation results are summarized in Fig. [2] where the black curve represents the target state estimation error and the red curve represents the target number estimation error. As shown in Fig. [2] both errors decreases as more measurements become available.

VI. CONCLUSION AND FUTURE WORK

A Gaussian particle filter that combines Kalman filter and particle filter is presented in this paper for estimating the number and state of total targets based on the measurement obtained online. The estimation is represented by a set of weighted particles, different from classical particle filter, where each particle is a Gaussian instead of a point mass. The weight of each particle represents the probability of existing a target, while its Gaussian indicates the state distribution for this target.

Fig. 1. The workspace contains three point targets.

Fig. 2. Simulation Result
This approach is efficient for the problem of estimating number of total targets and their state.

VII. APPENDIX

Without losing generality, $\mathcal{P}_S = \{\bar{P}_k^1, \bar{P}_k^2, \ldots, \bar{P}_k^s\}$, $E = [e_1, e_2, \ldots, e_s]$, for any particle such that $e_j = 1$, its $\mu_k^j$ and $\Sigma_k^j$, given $z_k$ and $\mathcal{P}_S$.

\begin{equation}
    y_k = z_k - \sum_{i=1}^{s} \frac{1}{e_i} x_i^j e_i
\end{equation}

\begin{equation}
    \Sigma_k^j = \text{COV}(x_i^j - \tilde{x}_i^j) = \text{COV}(x_i^j - (\tilde{x}_i^j + K_i^j y_k)) = \text{COV}\left( x_i^j - (\tilde{x}_i^j + K_i^j (\sum_{i=1}^{s} x_i^j e_i) \right)

    \begin{align}
    &+ \nu_k - \sum_{i=1}^{s} \frac{1}{e_i} x_i^j e_i \right)
    
    = \text{COV}\left( (I - \frac{1}{\sum_{i=1}^{s} e_i} IK_k^j) (x_i^j - \tilde{x}_i^j) \right)

    &- K_i^j \nu_k - \sum_{i=1, i\neq j}^{s} K_i^j (x_i^j - \tilde{x}_i^j) 
    
    = (I - \frac{1}{\sum_{i=1}^{s} e_i} IK_k^j) \Sigma_k^j (I - \frac{1}{\sum_{i=1}^{s} e_i} IK_k^j)^T

    &+ \frac{1}{\sum_{i=1}^{s} e_i} IK_k^j \sum_{i=1, i\neq j}^{s} \Sigma_k^j (\frac{1}{\sum_{i=1}^{s} e_i} IK_k^j)^T

    &+ K_i^j R_k K_i^j.
\end{align}

By setting $\partial_{\hat{K}} K_k^j = 0$, therefore

\begin{equation}
    K_k^j = \Sigma_k^j (\frac{1}{\sum_{i=1}^{s} e_i} I)^T

    \times (R_k + \frac{1}{\sum_{i=1}^{s} e_i} I \sum_{i=1}^{s} \Sigma_k^j (\frac{1}{\sum_{i=1}^{s} e_i} I)^T)^{-1}

    = \frac{1}{\sum_{i=1}^{s} e_i} \Sigma_k^j (R_k + \frac{1}{\sum_{i=1}^{s} e_i} I \sum_{i=1}^{s} \Sigma_k^j)_{\frac{1}{\sum_{i=1}^{s} e_i} I})^T)^{-1}
\end{equation}

Then,

\begin{equation}
    \mu_k^j = \mu_k^j + K_k^j y_k
\end{equation}

\begin{equation}
    \Sigma_k^j = \Sigma_k^j - \frac{1}{\sum_{i=1}^{s} e_i} \Sigma_k^j

    \times (R_k + \frac{1}{\sum_{i=1}^{s} e_i} I \sum_{i=1}^{s} \Sigma_k^j (\frac{1}{\sum_{i=1}^{s} e_i} I)^T)^{-1}

    \times \Sigma_k^j
\end{equation}

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