Statistical description with anisotropic momentum distributions for hadron production in nucleus-nucleus collisions

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Abstract

The various experimental data at AGS, SPS and RHIC energies on hadron particle yields for central heavy ion collisions are investigated by employing a generalized statistical density operator, that allows for a well-defined anisotropic local momentum distribution for each particle species, specified by a common streaming velocity parameter. The individual particle ratios are rather insensitive to a change in this new intensive parameter. This leads to the conclusion that the reproduction of particle ratios by a statistical treatment does not imply the existence of a fully isotropic local momentum distribution at hadrochemical freeze-out, i.e. a state of almost complete thermal equilibrium.

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I. INTRODUCTION AND MOTIVATION

A principal question of the physics of (ultra-)relativistic heavy ion collisions is whether the strong interacting hadronic, or initially even deconfined partonic matter achieves an intermediate or final reaction stage of almost thermal and chemical equilibrium. One direct way to check for chemical equilibration is to consider and analyze experimental hadronic abundances data by fixing macroscopic intensive quantities such as temperature and chemical potentials within a statistical model of an equilibrated hadron gas \[1, 2, 3, 4, 5\]. The seminal idea of employing such a statistical description to hadron abundances goes back to Hagedorn for describing elementary reactions and to Hagedorn and Rafelski for the situation of relativistic heavy ion collisions \[6\]. By now, numerous results, initiated with the work of Braun-Munzinger et al \[1\], imply that a high degree of hadrochemical equilibration is achieved at a so called stage of chemical freeze-out, occurring in the central reactions of the heavy ion experiments carried out at the AGS at Brookhaven, at the SPS at CERN and at the RHIC at Brookhaven.

The analyses are based upon rather restrictive assumptions (for two reviews having a critical look at the statistical model see \[7, 8\]): The standard statistical model – often dubbed ‘thermal’ model – assumes a complete chemical equilibrium (see \[9\] and references therein for allowing chemical non-equilibrium) as well as thermal equilibrium at hadrochemical freeze-out. The latter assumption implies the existence of isotropic momentum distributions locally at the individual space-time points within the fireball. The remarkable success of such ‘thermal’ models popularized the belief that the applicability of such approaches also proves for having established a state of nearly complete thermal equilibrium, i.e. a state that has (locally) isotropic distributions in momentum space. In principle, however, a statistical description is more general. The statistical operator describes our knowledge about the system rather than the system itself, making use of incomplete information \[10\]. Still, to best of our knowledge, so far there has not been any investigation, concerning particle abundances, that starts from possible and appropriate statistical operators which do not simply resemble those of assumed (local) thermal equilibrium. This states the motivation for the present study, where we follow some general ideas of constructing statistical distributions based on information theory \[10\]. Our picture is completely different from those that assume local isotropic momentum distributions supplemented by some hydrodynamical expansion.
profile in space-time \cite{11}.

A possibility to probe the degree of local equilibration is to study the various stages of the reaction using microscopic transport models \cite{12, 13, 14} (see also for lower bombarding energies the work \cite{15, 16, 17}). As it turns out, when testing ‘thermalization’, present transport models do not support the idea that a thermal system has been achieved at intermediate or even at later stages of the reaction, but show incomplete local momentum equilibration within the center of mass frame of a small but finite central cell \cite{12, 14}.

To meet this insight we propose in the following to relax the constraint of assuming \textit{locally} fully isotropic, statistical distributions for the comparison to hadron abundances, i.e. of assuming complete kinetic equilibrium. Certainly, at the onset of a heavy ion reaction, the local momentum distributions of the various particles are strongly non-isotropic. Moreover, the various measured rapidity spectra point towards a global picture of the fireball which shows still a considerable longitudinal momentum excess as the spectra are much broader than those of a pure thermal and static source. Part of this broadening could be attributed to the buildup of a collective hydrodynamical expansion, but the other part can in principle originate from locally anisotropic momentum distributions. When constructing a generalized statistical operator, we will follow this latter possibility that the system might not have developed fully isotropic momentum distributions locally in space-time at the point of chemical freeze-out.

II. GENERALIZED STATISTICAL OPERATOR FOR LONGITUDINALLY DEFORMED CONFIGURATIONS IN MOMENTUM SPACE

A concept of a statistical description which incorporates at least partially the initial longitudinal momentum excess has been proposed by Neise \cite{18, 19}, some time ago in order to allow for a generalization of the nuclear equation of state at finite density and temperatures for momentum anisotropic phase space distributions. Originally, the idea of two interpenetrating cold fluids goes back to Lovas \cite{20}, who considered both an ellipsoidally deformed Fermi sphere and two separated Fermi spheres by introducing an additional parameter \( v \) which effectively alters the momentum distribution. This latter parametrization has been considered for various studies concerning the effective in-medium interaction and relativistic potentials experienced by nucleons inside a non-equilibrium environment \cite{21}. For relativistic...
heavy ion collisions at GSI energies, i.e. for Au+Au–collisions at 1 AGeV, the relative velocity \( v_{\text{rel}} \) has been analyzed within a microscopic transport model and was found to be \( \approx 0.6 c \) at the stage of maximum compression [17], manifesting a full nonequilibrium situation at intermediate reaction times.

The main idea in [18] is to interpret the velocity parameter as an additional statistical Lagrange multiplier \( \nu \) by introducing a generalized grand canonical potential

\[
\Omega = E - TS - \mu N - \nu O ,
\]

where \( O \) represents the (thermodynamic) expectation value of an observable \( O \), which the system should assume within the grand canonical average as an additional constraint. With this statistical approach various selfconsistency relations are automatically fulfilled. If the observable \( O \) is given by a one-particle operator, \( O = \sum_i n_i O_i \), minimization of (1) for a gas of independent hadronic particles with a given set of one-particle energy states \( \epsilon_i \) results in the generalized distribution [18, 19, 22]

\[
\frac{\partial \Omega}{\partial n_i} = 0 \implies n_i = \frac{1}{\exp[\beta(\epsilon_i - \mu - \nu O_i)] \pm 1} ,
\]

depending on the quantum statistics of the considered hadronic particle. The formalism can be easily generalized to allow for self-consistent mean-field type interactions [18, 19, 21]. To allow for a specified momentum anisotropy, one can choose an adequate one-particle operator \( O \) characterizing the momentum anisotropy by taking the difference between the mean value of the momenta with a positive and negative longitudinal (z) component [18] respectively, i.e.

\[
O[p] = \sum_p [p_z \theta(p_z) - p_z \theta(-p_z)] a^+_p a_p = \sum_p |p_z| a^+_p a_p .
\]

According to [2] one ends up with the generalized momentum distribution

\[
n_p = \frac{1}{\exp[\beta(\epsilon_p - v|p_z| - \mu)] \pm 1} ,
\]

with the conjugated intensive quantity \( \nu \equiv v \).

A more appropriate relativistic notation for the distribution can be obtained by the following more intuitive and explicit construction of the same density operator [19]. Suppose that the homogeneous medium consists of two subsystems in momentum space (dubbed ‘projectile’ and ‘target’ component indicating their origin), which are specified by their
collective flow velocity \( v^{(P)\mu} = \gamma(1,0,0,v) \) and \( v^{(T)\mu} = \gamma(1,0,0,-v) \) in the combined center of mass frame with \( \gamma = (1-v^2)^{-1/2} \). It becomes obvious that the intensive parameter \( v \) corresponds to the velocity of each subsystem. In the special c.m. frame only the positive z-component, respectively negative z-component, of the (assumed) single-particle momenta contribute to the projectile subsystem, respectively target subsystem, i.e. both subsystems are separated by the plane \( p_z = 0 \). For the grand canonical potential one takes the sum of the two components, i.e. 
\[
\Omega = v^{(P)\nu} P^{(P)}_{\nu} - T_0 S^{(P)} + \mu_0 N_B^{(P)} + v^{(T)\nu} P^{(T)}_{\nu} - T_0 S^{(T)} - \mu_0 N_B^{(T)},
\]
from which the statistical density operator immediately results as 
\[
\rho = \frac{1}{Z} \exp \left\{ -\beta_0 \left( v^{(P)\nu} P^{(P)}_{\nu} + v^{(T)\nu} P^{(T)}_{\nu} \right) + \beta_0 \mu_0 N_B \right\} .
\]
For a noninteracting resonance gas of independent hadronic particles one has for the total momentum 
\[
P^{(P/T)}_{\mu} = \sum_h \sum_p p^h_{\mu} n^h_p \theta(+/-p_z) .
\]
A similar expression holds for the entropy and baryon number. For the grand canonical potential \( \Omega \) the contributions for the entropy and the baryon number are entering simply additively, and, as \( \theta(p_z) + \theta(-p_z) = 1 \), the explicit decomposition is only relevant for the sum \( v^{(P)\nu} P^{(P)}_{\nu} + v^{(T)\nu} P^{(T)}_{\nu} \). Putting everything together, from \( \rho \) one ends up with the expression 
\[
n^h_p = \frac{1}{\exp \left\{ \beta_0 \gamma (e^h_p - v|p_z|) - \beta_0 \mu_0 \right\} + 1}
\]
for the individual momentum distributions. \( \rho \) is completely equivalent to \( \Omega \): \( \beta_0 \) and \( \mu_0 \) represent the true inverse ‘temperature’ and (baryo-)chemical potential defined in the local rest frame of each component, whereas \( \beta \) and \( \mu \) in \( \Omega \) are effective parameters modified by \( \gamma \).
A straightforward consideration shows that the momentum distribution becomes elongated in the longitudinal direction by the Lorenz factor \( \gamma \). As an illustration, fig. \( \text{I} \) depicts a situation typical for central Pb+Pb collisions at SPS energies for the case of nucleons, pions and kaons with an intensive parameter chosen to be \( v = 0.4 \) c. The heavier the hadronic particle, the more pronounced the two peaks \( \pm m^h \gamma v \) in the distribution of the ‘two fluids’ manifest.

Summarizing, the velocity \( v \) should be thought as the intrinsic (Lagrange-)parameter for characterizing any potential non-equilibrium situation in local longitudinal momentum
space within the statistical description. We consider it as the variable for characterizing a possible intermediate situation of the momentum distributions at hadrochemical freeze-out in the local comoving rest frame of a moving spatial cell within the spatially expanding fireball. This cell is then considered as a characteristic representative of the whole system, like for example within a boost-invariant Bjorken geometry: A boost-invariant but locally momentum anisotropic distribution is given by the standard substitution

\[ n_p^h = n^h(p_\perp, p_z) \rightarrow f^h(p_\perp, y; z, t) = n^h(p_\perp, y - \eta) \]  

(9)
of Lorentz-boosted comoving cells, and assuming the intrinsic parameters \( T, \mu_B \) and \( v \) only to depend on the eigentime \( \tau \).

The parameter \( \gamma v \) is certainly different and considerably smaller than in the very initial situation of the heavy ion reaction due to the (unknown) amount of dissipation occurring in the (violent) initial stages. The mismatch compared to a fully momentum isotropic situation should relax continuously in time due to collisions among the various particles, although the system might never actually become locally completely isotropic \([12, 14]\), as outlined in the introduction. In the following we thus will consider \( v \) as a free parameter.

Finally it is worthwhile to point out the strict meaning of a thermodynamical equilibrium description compared to statistical descriptions in general \([10]\). A statistical operator, relying on the concept of incomplete information, assumes the existence of some average constraints \( \langle O_i \rangle \). The density operator then has the standard form \( \rho = \exp(\Omega - \sum_i \nu_i O_i) \) \([10]\). Generally, the operator \( O_i \) does not need to commute with the Hamiltonian \( H \). Being more restrictive, one has a stationary ensemble when taking only constants of motion for the \( O_i \); in other words, the Liouville equation for the density matrix

\[ \frac{d\rho}{dt} = \{H, \rho\} = 0 , \]

(10)

has to be fulfilled for a true thermodynamical equilibrium situation. This is not the case for the chosen momentum anisotropy operator \( O \) reflecting the fact that the momentum anisotropy defined might actually be a time dependent quantity. As outlined above, collisions among the particles in the (strongly) interacting matter are ultimately destroying the stationarity. A requirement for stationarity is fulfilled when considering spatially homogeneous matter and employing approximate effective one-particle (and thus collision-less) Hamiltonians of mean-field type or a simple quasi-free Hamiltonian for describing the hadron
resonance gas. On the other hand, the statistical assumption is made when postulating the existence and minimization of the generalized grand canonical potential with the constraint that locally a finite mismatch in momenta survives at hadro-chemical freeze-out.

III. INFLUENCE OF THE VELOCITY PARAMETER ON PARTICLE RATIOS

Following the bootstrap idea of incorporating the strong interactions among the constituents (see eg [22]), we assume a ‘noninteracting’ hadron (Hagedorn-like) resonance gas for describing the excited hot hadronic matter inside the fireball [1,2,3,4,5]. The chemical freeze-out of particles is assumed to happen for a fixed temperature $T$. For the sake of simplicity we do not intend to distinguish between a $4\pi$-analysis or an analysis of particle ratios being measured only close to midrapidity: An approximate Bjorken geometry as given in (9) would give the same answer for both analyses.

We choose the description of an isospin symmetric hadronic gas and include baryonic and mesonic resonances of masses up to 2 GeV [5,23]. For the various particle number densities described by the generalized momentum distributions (8) one has

$$
\rho_i = \frac{g_i}{(2\pi)^3} \int d^3p \frac{1}{\exp(\beta_0 \gamma (E_i - v|p_z|) - \beta_0 \mu_i;0) \pm 1},
$$

with spin degeneracy $g_i$, $\beta_0 = \frac{1}{k_B T}$ and $E_i = \sqrt{p_\perp^2 + p_z^2 + m_i^2}$. The chemical potential of each hadronic particle is specified by its baryon number content $N_B$ and its strangeness number content $N_S$ via $\mu_i;0 = N_B \mu_B + N_S \mu_S$, assuming isospin symmetric hadronic matter ($\mu_I = 0$).

Changing the velocity parameter, the various particle number densities do increase with increasing $v$ to almost $2\gamma$ for velocities close to $c$. The total intrinsic energy density, on the other hand, will scale with almost $2\gamma^2$ for such large velocities.

We are now in position to check for the sensitivity in extracting the thermodynamical intensive parameters like the ‘temperature’ parameter $\beta_0^{-1}$ and the chemical potentials when varying the velocity parameter $v$. It is worthwhile to first discuss two limiting situations: For $v = 0$ the standard statistical model description results. For velocity parameters being sufficiently large, the two interpenetrating components fully separate in momentum space, so that the hadronic particle ratios will not be altered when employing the same temperature $\beta_0^{-1}$ and chemical potentials as for $v = 0$. Each of the separate subsystems then describes the situation of a Lorentz boosted momentum isotropic fireball, i.e. the situation for $v = 0$. The
question remains for the true intermediate situations, where one cannot really distinguish among two separate components. As a further limiting case, employing a classical Maxwell-Boltzmann approximation,

\[ n_p \approx \exp(-\beta_0 \gamma (E_i - v|p_z|)) \exp(\beta_0 \mu_i) \]

number ratios of particle and their anti-particles are not altered by a change in the velocity parameter, being solely dependent on the exponential factor containing the chemical potential. We therefore expect only some smaller kinematical sensitivity for particle ratios with unequal masses.

For testing the potential influence of an anisotropic momentum distribution on the hadronic particle ratios the strategy is as follows: We first employ the known values for the temperature \( T_0 \) and potential \( \mu_{B,0} \) of \cite{1,2,3,4} within the present description taking \( v = 0 \) for describing the various situations of heavy ion experiments carried out at the AGS, at the SPS and at RHIC. As one particular example we show in fig. 2 characteristic particle ratios in comparison to the experimental results for central Pb+Pb collisions at SPS energies. We find good agreement to the various hadronic particle ratios (in quality like those given in \cite{1,2,3,4}) and we mention the well-known fact that feeding from all relevant hadrons and resonances to the abundances of ‘stable’ (i.e. detectable) particles is of crucial importance. With the temperature and the chemical potentials held fixed, we now change the velocity parameter in order to see the sensitivity on the particle ratios. In the following we concentrate on the \( \bar{p}/p \) – , the \( p/\pi^+ \) and the \( K^+/K^- \) –ratio. The first two ratios typically fix the temperature and the baryochemical potential. The last fixes the strange chemical potential, which, however, enters as a constraint as the system carries no overall net strangeness. In fig. 3 the actual (non-)dependence of these exemplary particle ratios on varying the velocity parameter is depicted. As expected from the above estimates, the largest deviation, about 6%, one finds for the \( p/\pi^+ \) ratio (for the situation of a Pb-Pb collisions at SPS). We have tested for all the various other particle ratios shown in fig. 2 and find, if at all, only a marginal dependence on the velocity parameter \( v \). \( p/\pi^+ \) shows the ‘strongest’ sensitivity. The same situation is met when looking at ratios measured at AGS or at RHIC. Most of the other ratios, such as the shown \( \bar{p}/p \) and \( K^+/K^- \), only vary by about 1% or even less. In tab. 1 the above three particle ratios as obtained within our investigation are summarized. There is no remarkable change of the particle ratios at any
of the considered collision energies.

Hence, the quality of a possible detailed fit to the particle ratios including the velocity as a free parameter would not be changed: An extraction of $v$ would be completely ambiguous.

IV. SUMMARY AND CONCLUSIONS

We have constructed a statistical description of a hadronic resonance gas allowing for a finite longitudinal momentum excess compared to commonly employed isotropic (‘thermal’) distributions. This is achieved via a velocity parameter acting as a common Lagrange multiplier for fixing a momentum anisotropy for the local momentum distributions. As microscopic transport models do not fully support the idea of a complete local momentum equilibration at any stage of a central heavy ion reaction, the presented approach relaxes this typical constraint of a standard statistical model. From a physics point of view one can argue that the amount of dissipation or collisions is simply not enough to achieve full momentum equilibrium locally in space. Going further with the argument, there exists always the principal question when applying a statistical treatment for hadron production whether it represents a reflection of a true dynamical and stationary equilibrium state or, more generally, whether it is only a reflection of a state minimizing the content of information (i.e. maximizing the ‘entropy’) within a set of appropriate, global constraints, as also employed in this study. This is particularly obvious when applying such concepts to $e^+e^-$, $pp$ or $p\bar{p}$-collisions, as proposed seminally by Hagedorn and described recently by Becattini within a micro-canonical treatment. For such ‘systems’ one can not expect the occurrence of a true dynamical equilibrium state (fulfilling approximately (10)) at any stage of the reaction.

It turns out that the hadronic particle composition is more or less completely insensitive to varying the velocity parameter from zero to nearly speed of light. The extraction of this parameter can not be achieved by looking at particle number ratios. On the other hand, the extraction of the ‘temperature’ parameter as well as the (baryo-)chemical potential, however, are the same as with the standard approach. They are thus the significant and appropriate statistical quantities for dealing with the various number ratios characterizing the intermediate stage of the reaction at hadrochemical freeze-out.

The popular conclusion that the many successes in describing particle ratios within a
'thermal' model give evidence for the existence of a state of thermal equilibrium at hadrochemical freeze-out is, however, not true. Local isotropic momentum distributions reflect merely a simplifying assumption of the applied statistical operator. With this observation in mind, the standard ‘extraction’ of total particle number densities or energy densities, respectively, in absolute terms achieved inside the hadronic fireball at chemical freeze-out becomes also unsteady. Our study underlines the necessity of understanding relativistic heavy ion collision experiments by means of detailed microscopic transport models.

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|       | model for $v = 0$ | model for parameter $v$ with largest deviation in | experiment |
|-------|------------------|-----------------------------------------------|------------|
| **AGS** | $T = 120$ MeV, $\mu_B = 540$ MeV, $\mu_S = 108$ MeV | | |
| $\bar{p}/p$ | 0.000124 | 0.04652 | 0.00045 ± 0.00005 |
| $p/\pi^+$ | 1.0535 | 5.9178 | |
| $K^+/K^-$ | 5.5362 | -0.5315 | 4.4 ± 0.4 |
| **SPS I** | $T = 160$ MeV, $\mu_B = 170$ MeV, $\mu_S = 38$ MeV | | |
| $\bar{p}/p$ | 0.1195 | 0.0048 | 0.12 ± 0.02 |
| $p/\pi^+$ | 0.1863 | 6.1940 | 0.18 ± 0.03 |
| $K^+/K^-$ | 1.51 | -0.2680 | 1.67 ± 0.15 |
| **SPS II** | $T = 168$ MeV, $\mu_B = 266$ MeV, $\mu_S = 71.1$ MeV | | |
| $\bar{p}/p$ | 0.0422 | 0.0104 | 0.055 ± 0.01 |
| $p/\pi^+$ | 0.3294 | 4.7817 | |
| $K^+/K^-$ | 2.0820 | -0.4319 | 1.85 ± 0.09 |
| **RHIC** | $T = 174$ MeV, $\mu_B = 46$ MeV, $\mu_S = 13.1$ MeV | | |
| $\bar{p}/p$ | 0.5845 | 0.8568 | 0.60 ± 0.07 |
| $p/\pi^+$ | 0.1129 | 5.4299 | |
| $K^+/K^-$ | 1.1282 | -0.1297 | 1.136 ± 0.070 |

**TABLE I:** Characteristic particle ratios from various heavy collision experiments (from top to bottom): (1) Si + Au at the AGS, parameters from [1]; (2) S + Au at the SPS, parameters from [2]; (3) Pb + Pb at the SPS, parameters from [3]; (4) Pb + Pb at RHIC, parameters from [4]. The maximal deviation (in percent) by changing the velocity parameter is indicated.
FIG. 1: Generalized statistical momentum space distribution with $v = 0.4$ for the nucleons, the pions and the $K^+$. Temperature parameter $\beta_0$ and chemical potentials are taken as those extracted for $v = 0$ being typical for Pb+Pb collision at SPS energies [3].
FIG. 2: Characteristic particle ratios for Pb+Pb collisions at SPS energies obtained with velocity parameter $v = 0$. Temperature parameter $\beta_0$ and chemical potentials are taken the same as in [3].
FIG. 3: (Weak) Dependence of three typical particle ratios ($\bar{p}/p; p/\pi^+; K^+/K^-$) varying the velocity parameter from $v = 0$ to $v = 0.9c$. Temperature parameter and chemical potentials are taken the same as those of fig. 2 with $v = 0$. 