Accurately computing the electronic properties of a quantum ring

A promising approach to study condensed-matter systems is to simulate them on an engineered quantum platform1–4. However, the accuracy needed to outperform classical methods has not been achieved so far. Here, using 18 superconducting qubits, we provide an experimental blueprint for an accurate condensed-matter simulator and demonstrate how to investigate fundamental electronic properties. We benchmark the underlying method by reconstructing the single-particle band structure of a one-dimensional wire. We demonstrate nearly complete mitigation of decoherence and readout errors, and measure the energy eigenvalues of this wire with an error of approximately 0.01 rad, whereas typical energy scales are of the order of 1 rad. Insight into the fidelity of this algorithm is gained by highlighting the robust properties of a Fourier transform, including the ability to resolve eigenenergies with a statistical uncertainty of $10^{-4}$ rad. We also synthesize magnetic flux and disordered local potentials, which are two key tenets of a condensed-matter system. When sweeping the magnetic flux we observe avoided level crossings in the spectrum, providing a detailed fingerprint of the spatial distribution of local disorder. By combining these methods we reconstruct electronic properties of the eigenstates, observing persistent currents and a strong suppression of conductance with added disorder. Our work describes an accurate method for quantum simulation5,6 and paves the way to study new quantum materials with superconducting qubits.

In condensed-matter systems, the interplay of symmetries, interactions and local fields gives rise to intriguing many-body phases. Insight into these phases of matter comes from both experimental and theoretical developments; however, limitations in both approaches prevent a complete physical picture from emerging8,9. For example, despite great effort, it is still not clear which state is realized at the 5/2 filling of fractional quantum Hall and which interaction would be needed to generate the desired state9,10. Generally, the difficulty stems from the fact that interesting properties of quantum materials arise from subtle interference effects of many particles and small errors can lead to large deviations in observables. Neither numerical methods nor analytics have sufficient accuracy to predict such phenomena in realistic systems. Although conventional experiments provide the most direct approach, the necessary observables—such as correlated measurements—are typically inaccessible and the lack of controllability limits the benefits of such experiments.

To outperform conventional approaches, quantum processors need to overcome two main sources of error: errors from control (unitary) and from decoherence (non-unitary). Here we demonstrate an experimental blueprint for achieving low control error and for the comprehensive mitigation of decoherence. The key insight into this development stems from robust properties of the Fourier transform. Consider a quantum signal that oscillates in time with an envelope that decays due to decoherence. Taking a Fourier transform of the signal results in a fingerprint of the spatial distribution of local disorder. By combining these methods we reconstruct electronic properties of the eigenstates, observing persistent currents and a strong suppression of conductance with added disorder. Our work describes an accurate method for quantum simulation5,6 and paves the way to study new quantum materials with superconducting qubits.

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signal will yield peaks at the oscillation frequencies. Although decoherence (as well as readout errors) will affect the amplitude and width of the peaks, the centre frequencies will remain unaffected; see Supplementary Information section D. On the other hand, small errors in the control parameters will manifest as shifts in the frequency of the peaks, providing a reliable signature from which we can learn these errors. The essence of our work is that studying quantum signals in the Fourier domain enables error mitigation and provides a sensitive probe of control parameters.

We apply this insight both at the level of individual pairs for calibration and at the system level for mitigating decoherence in algorithms. At the level of two qubits, gates can be applied periodically and local observables can be measured as a function of circuit depth. Small errors in the control parameters are inferred from shifts in the Fourier peaks; these errors are then corrected for. In addition, we show that these parameters can be inferred with a statistical precision of less than $10^{-3}$ rad; see Supplementary Information section A. At the system level a similar strategy can be used, in which we apply a multi-qubit unitary periodically and monitor local observables. Here we focus on a simple, exactly-solvable model, and demonstrate an 18-qubit algorithm that consists of more than 1,400 two-qubit gates with a total error in the extracted Fourier frequencies (corresponding to energy eigenvalues) of 0.01 rad and a statistical precision of $10^{-3}$ rad. The 18-qubit ring that is formed in this experiment can be viewed as an Aharonov–Bohm interferometer. The phases associated with single-qubit operations are analogous to disorder in a wire that causes a particle to scatter. The sum of these phases realizes an Aharonov–Bohm flux, which lifts the degeneracy between clockwise- and anticlockwise-propagating particles. In this analogy, precision stems from the sensitivity of wave interference patterns to imperfections. The underlying physics discussed in this work is general and can be adopted by other platforms.

We demonstrate our method using superconducting qubits with adjustable couplers. This is because they enable control over individual frequencies, which set local fields and magnetic flux; and control over couplings, which set kinetic energy or hopping. A schematic of our 54-qubit processor is shown in Fig. 1a. The qubits are depicted as grey crosses and the tunable couplers as blue squares. Eighteen of the qubits are chosen to form a one-dimensional ring. Connecting the qubits in a ring enables us to introduce a controllable synthetic flux using single-qubit gates. A schematic showing the control sequence used in these experiments. Each large vertical grey box indicates a cycle of evolution that we repeat many times. Each cycle contains two sequential layers of $\sqrt{\text{iSWAP}}$ gates (blue), separated by single-qubit $z$-rotations (R$_z$, grey). Periodicity in space leads to eigenstates of the cycle unitary with definite momentum. Periodicity in time introduces conservation of energy. Together, this realizes a digital circuit with well defined physical properties, such as energy, momentum and flux. The gate sequence used in this work is shown in Fig. 1b. Each large vertical grey box indicates a single cycle of evolution that we repeat periodically in time. Each cycle contains two sequential layers of $\sqrt{\text{iSWAP}}$ gates (blue), separated by single-qubit $z$-rotations (grey). Within each cycle, a two-qubit gate is applied between all possible pairs in the loop. The $\sqrt{\text{iSWAP}}$ gates cause a particle (microwave excitation in this case) to hop between adjacent lattice sites (qubits). The $z$-rotations are used to generate local fields and their summation gives rise to an effective magnetic flux that threads the qubit loop. Here we will focus on the dynamics of a single particle; however, our approach enables a straightforward generalization to full many-body systems.

The connectivity and gate sequence are chosen such that the algorithm is translationally invariant in space, resulting in a cycle unitary for which the eigenstates have well defined momentum. Because the control sequence is periodic in time, the cycle unitary will have well defined energies (known as quasi-energies). This enables us to realize a tight-binding Hamiltonian with terms of the form $\sum_{nn'}(\sigma^+_n\sigma^+_m + \sigma_-^n\sigma_-^m)$, where $\sigma^+_n$ ($\sigma^-_n$) are raising (lowering) operators that cause an excitation to propagate along the ring. See Supplementary Information section B4 for details of the model and Supplementary Information section D for the embedding into qubits. The eigenstates $\psi_i$ of this model are plane-waves, and eigenvalues $\omega_i$ can be expressed in terms of the momentum $k$

$$\psi_i(x) = e^{ikx}, \quad \cos \omega_i = \sin k/2 = \cos \Phi \Rightarrow$$

(1)

where $x$ is the position along the ring and $\Phi$ is the flux threading the loop. Combined with the ability to introduce a synthetic magnetic field using $z$-rotations, we realize a digital quantum circuit with robust physical properties of momentum, energy and flux. We investigate the eigenspectrum of this 18-qubit ring using a many-body spectroscopy technique. Peaks in a spectroscopy experiment provide a robust signature of the underlying quantum system. The raw data are shown in Fig. 2a, in which we plot the expectation values of the Pauli x and y operators on a single qubit (denoted $\langle X \rangle$ and $\langle Y \rangle$) as a function of the number of cycles in the control sequence. Although the raw data do not contain particularly intuitive features, the complex Fourier transform of $\langle X \rangle + i\langle Y \rangle$ has the special property that peaks appear only at frequencies that correspond to the energy eigenvalues. The Fourier transform of the time-domain data is shown in Fig. 2b, in which we observe clear, well resolved peaks. 
In the absence of local fields (z-rotations), the dynamics are governed entirely by the kinetic energy (or hopping), and a simple plane-wave model describes the spectrum. This enables us to associate with each peak a corresponding value of momentum by simply noting the index of the peak, starting from 0. The momentum has units of $2\pi/N_x$, where $2\pi/N_x$ corresponds to the lattice spacing in a typical condensed-matter setting and the extra factor of 2 arises from the discrete evolution using gates. In Fig. 2c we show the measured energy as a function of the inferred momentum, realizing an experimental technique for extracting the single-particle band structure. The energies are inferred by fitting the data to the expression

$$\langle X \rangle + i \langle Y \rangle = e^{-i \omega_d t} \sum_k A_k e^{-i \omega_d k},$$

where $d$ are the measured circuit depths, $\omega$ is a damping rate and $A_k$ are Fourier amplitudes. This expression is derived in Supplementary Information section D2. The difference between ideal eigenvalues (given by equation (1)) and the measured eigenvalues is shown in the inset of Fig. 2c and has a typical value of around 0.01 rad; to our knowledge, this level of accuracy is unprecedented for an 18-qubit experiment with more than 1,400 two-qubit gates.

Extracting information from the Fourier domain revealed other salient features that were crucial in arriving at our results. At large circuit depths, decoherence causes the signal to decrease below the noise level of the experiment. Maintaining a high signal-to-noise ratio is therefore key to scalable error mitigation. Fourier transforms have the important property that the statistical uncertainty scales inversely with the length of the time-domain signal. Consider a shallow circuit of depth $D$ in which coherence can be neglected. In this limit, the statistical uncertainty $\sigma$ scales as $1/\sqrt{N} \times 1/D$, where $N$ is the number of measurements. In Supplementary Information section F3 we provide the explicit relation and show that this physics is general and is not affected by the damping rate $\omega$. The factor $1/D$ is expected when taking a Fourier transform, and the factor $1/\sqrt{N}$ is the standard expression for finite-sampling noise. Here we benefit from both factors because we fit the data to equation (2) rather than simply taking a Fourier transform. Experimentally, the statistical uncertainty in the measured eigenvalues are computed using bootstrap resampling and are shown as error bars in Fig. 2c, multiplied by 1,000 so as to be visible. The typical uncertainty in the measured eigenvalues is of the order of $10^{-4}$ rad. This method provides a high-precision tool for probing eigenvalues in large quantum systems.

The energy levels of atoms and materials shift in the presence of an external magnetic field, providing a simple probe of the underlying system. In Fig. 3a, we provide a control sequence for producing a synthetic magnetic field that we will use to probe disorder in the local potentials, $\xi$. By applying a specific pattern of z-rotations around the SWAP gate, we can produce a complex hopping such that a particle hopping between adjacent lattice sites accumulates the phase $\xi$; see Supplementary Information section B2. The sum of these phases across all links produces a magnetic flux, $\Phi = \sum \chi$. This is analogous to the Aharonov–Bohm phase that an electron accumulates when circulating in a conducting ring threaded by a magnetic flux. In addition to flux, the z-rotations can be used to control the phase of the particle in the cases in which it stays on the same site, corresponding to a dynamical phase that a particle would accumulate in a local potential $\xi$.

The measured energy eigenvalues are plotted as a function of flux in Fig. 3b. The data (blue circles) are placed atop the exact spectrum (black lines), and we observe excellent agreement between data and theory. At zero flux, the spectrum is highly degenerate: away from zero flux, the eigenvalues split. This happens because, in the absence of an external flux, a particle has the same energy when travelling clockwise or anticlockwise owing to symmetry. The application of flux breaks this symmetry and results in chirality. Disorder in the local potentials will also break this degeneracy and lead to gaps in the measured spectrum. These gaps enable us to infer the spatial distribution of the disorder through the relation

$$\text{gap}_k \propto \frac{1}{N_q} \sum_{x=1}^{N_q} \xi_x e^{i k x},$$

where $\text{gap}_k$ is the gap at momentum $k$ and $\xi_x$ is the local field at position $x$; the right-hand side of this expression is simply the Fourier transform of the local fields at spatial frequency $k$; Supplementary Information section B4 contains a derivation of this expression that includes over-rotations in the swap angles. This is a notable result: gaps in the spectrum correspond one-to-one to the spatial Fourier components of disorder. This provides a scalable metrology tool for diagnosing control errors in quantum algorithms.

To better understand this effect, we injected disorder into the local fields in a controlled manner. The pattern of local disorder is shown in Fig. 3c. Rather than random disorder, which will open gaps at all values of momentum, we have chosen to add disorder with a single spatial frequency to highlight equation (3). The resulting spectrum with added
The expectation value of the energy eigenstate at momentum $k$ is the energy eigenstate at momentum $k$.

Disorder is shown in Fig. 3d, in which we observe the formation of gaps at the expected transitions. The ability to systematically control the disorder enables us to explore new condensed-matter systems, such as many-body localized phases\textsuperscript{20–22}.

In typical condensed-matter systems, disorder leads to scattering and is the origin of electrical resistance. To study this effect, we focus on the degeneracy near zero flux, as this region of the spectrum is first-order sensitive to disorder. Figure 4a shows a magnified view of the spectrum at $k = \pm 5$. Disorder in the local fields causes a small gap to form between the two levels. In this region, the spectrum is well fit by a simple avoided level crossing model, shown as solid lines. This generic model for the behaviour near a level crossing will enable us to infer electric properties of the eigenstates.

When an external magnetic field $\Phi$ gives rise to a current $I$ in a wire, the Hamiltonian can be written as $H = \Phi l$. This enables us to define the current operator as simply $I = \partial H / \partial \Phi$. The expectation value of the current in an eigenstate is then given by the relation

$$\langle \psi_k | I | \psi_k \rangle = \frac{d \psi_k}{d \Phi}.$$  \hspace{1cm} (4)

Fig. 3 | Synthetic flux as a probe of local disorder. a. Gate decomposition for producing local fields and complex hoppings. The sum of the hopping phases over all pairs realizes a synthetic flux. b. The spectrum of the cycle unitary is plotted as a function of flux in the case of nominally zero disorder. The black lines show the ideal spectrum. Expectation values are estimated using 30,000 samples. c. The pattern of added disorder is plotted as a function of position along the ring of qubits. d. The measured spectrum as a function of flux in the case of added disorder. Only the expected transitions become gapped. This demonstrates the correspondence between gaps in the spectrum and the spatial Fourier components of disorder in the system. In addition, the absence of substantial splitting in the case of native disorder indicates that intrinsic disorder is small.

Fig. 4 | Inferring current and conductance from avoided level crossings. a. Plot of energy against flux for a single value of momentum. Near zero flux, we observe an avoided level crossing caused by intrinsic disorder in the local fields. The solid lines show a fit of the data to a simple avoided level crossing model. b. The derivative of the energy with respect to flux is shown plotted against flux. This quantity corresponds to the expectation value of current in each eigenstate. For small fluxes, a linear response to flux is observed; for large flux, current is independent of flux, corresponding to persistent current states. c. The second derivative of energy is plotted against the amplitude of added disorder; this quantity corresponds to the conductance of the loop. The black line shows the ideal spectrum. We observe a strong suppression of conductance with added disorder. These results demonstrate that expectation values of observables in eigenstates can be extracted with high precision and accuracy.
the slope of the plot of current against flux near zero flux and the density of states; this equation relates the non-equilibrium property conductance to the single-particle spectrum. This quantity is known as Thouless conductance. In Fig. 4c we plot Thouless conductance as a function of added disorder and observe a strong suppression of the conductance with increasing disorder. A numerical simulation is shown as a black line. Because this quantity is computed from eigenvalues, it retains the unprecedented accuracy and precision inherent in using a Fourier transform to process the experimental data.

Quantum processors hold promise to solve computationally difficult tasks beyond the capability of classical approaches. However, in order for these engineered platforms to be considered as serious contenders, they must offer computational accuracy beyond the current state-of-the-art classical methods. Although analytical approaches occasionally provide exact solutions, they quickly lose their relevance upon small perturbations to the underlying Hamiltonian. Numerical methods, in addition to tackling ground-state problems, can handle the dynamics of highly excited states and non-equilibrium phenomena. Currently, the most powerful numerical methods—such as the density matrix renormalization group algorithm—have roots in renormalization group ideas and are successful in one dimension and in quasi-one-dimensional geometries. In dealing with higher spatial dimensions in which entanglement spreads widely in space or grows rapidly in time, all numerical methods resort to approximations, in which parts of the Hilbert space are truncated to make the computation feasible. As a result of these limitations, one can safely claim that, for example, studying dynamics in an 8-by-8 spin lattice with local arbitrary interactions and predicting observables with 1% accuracy is beyond any classical computational method. With the experimental advancements presented here, going beyond this classical horizon could be within reach in the near future.

Online content
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Methods

Here we use the Sycamore quantum processor, which consists of 54 superconducting qubits and 86 tunable couplers in a two-dimensional array\textsuperscript{14}. This processor consists of gmon qubits (transmons with tunable coupling) with frequencies ranging from 5 GHz to 7 GHz. These frequencies are chosen to mitigate various error mechanisms such as two-level defects. Our coupler design enables us to quickly tune the qubit–qubit coupling from 0 MHz to more than 40 MHz. The chip is connected to a superconducting circuit board and cooled down to below 20 mK in a dilution refrigerator.

Each qubit has a microwave control line that is used to drive an excitation and a flux control line to tune the frequency. The processor is connected through filters to room-temperature electronics that synthesize the control signals. We execute single-qubit $X$, $Y$, $X/2$ and $Y/2$ gates by driving 25-ns microwave pulses resonant with the qubit transition frequency. Single-qubit $Z$-rotations are implemented using 10-ns flux pulses. The qubits are connected to a resonator that is used to read out the state of the qubit. The state of all qubits can be read simultaneously using frequency-multiplexing.

Initial device calibration is performed using ‘Optimus’, in which calibration experiments are represented as nodes in a graph\textsuperscript{29}. As well as these initial experiments, we perform a new two-qubit calibration technique that we term ‘Floquet Calibrations.’ This name is based on the idea that we want to calibrate a periodic sequence of gates; the unitary describing one period of evolution is known as the Floquet unitary. This technique is very similar to a Ramsey experiment in which greater precision is achieved at long times. Details of this procedure are presented in the Supplementary Information.

Data availability

The data presented in this study can be found in the Dryad repository located at https://doi.org/10.5061/dryad.4f4qrfj9x.

Code availability

The Python code for processing the data presented in this study can be found in the Dryad repository located at https://doi.org/10.5061/dryad.4f4qrfj9x.

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Author contributions

C.N. designed and executed the experiment. C.N. and P.R. wrote the manuscript. C.N., T.M. and V. Smelyanskiy wrote the Supplementary Information. V. Smelyanskiy, S.B., T.M., Z.J., X.M., L.B.I. and C.N. provided the theoretical support and analysis techniques, the theory of Floquet calibration and the open system model. Y.C., V. Smelyanskiy and H.N. led and coordinated the project. Infrastructure support was provided by the hardware team. All authors contributed to revising the manuscript and the Supplementary Information.

Competing interests

The authors declare no competing interests.

Additional information

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