Noise source identification in indoor substation using a sparse equivalent source method

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Abstract. Noise source identification is a key step in the noise control design of the transformer. The sound propagation is influenced by the reverberation effect in the indoor substation, which leads that the traditional method cannot give correct results in such environment. To supress the reverberation effect, a sparse equivalent source method is proposed to realize the noise source identification in indoor substation. This method first establishes the indoor sound transfer function model between the equivalent source surface and the acquisition surface by using wave simulation. Based on the wave simulation and combined with the sound pressure sampling data, the distributed equivalent source is recovered on the equivalent source surface by sparse recovery algorithm, and finally the sound pressure reconstruction on the reconstruction surface is realized. The numerical verification demonstrates that this method is capable of extracting the real noise information in the reverberation sound and giving the real noise source distribution result. This method can be used as an effective method in the noise source identification in the indoor substation.

1. Introduction

In recent years, with the acceleration of urbanization process, the urban power load increases year by year, and the construction of substation gradually moves to the inner city. Due to large noise and long-term persistence of substation noise, its noise pollution problem has become a focus of complaints from surrounding residents[1]. In order to reduce the noise radiation level to the surrounding environment, most of the existing substations in urban areas are built indoors. However, due to the significant low-frequency noise, the noise control of indoor substation is still an important problem.

Accurate noise source identification of transformer and other equipment is the precondition of effective noise control, which helps to design noise reduction measures more accurately according to the type and location of noise source; at the same time, the noise source identification of transformer and other equipment in substation is also helpful to monitor its operation, and find out the problems in equipment operation through locating abnormal acoustic events. Therefore, noise source identification for substation environment is always a research hotspot.

Near field holography (NAH) is the most representative noise source identification technology, which was proposed by Williams and Maynard in 1980. In this technology, the sound pressure in space is measured by the holographic plane sensor array, and the sound pressure is transformed from the time domain to the wave number domain, and the transmission relationship between the holographic surface and the reconstruction surface is established, so as to transfer the measured data to the reconstruction surface[2-3]. Finally, the sound pressure of the reconstruction surface is obtained by
inverse Fourier transform, so as to realize the identification of noise sources. In the measurement process, the near-field acoustic holography can not only measure the propagation wave information, but also record the evanescent wave information which decays rapidly with the increase of distance. Therefore, it has broken through the limitation of Rayleigh criterion and has been widely used. Based on the classical near-field acoustic holography, some improved near-field acoustic holography techniques, such as inverse boundary element method near-field acoustic holography, statistical optimal near-field acoustic holography, equivalent source method near-field acoustic holography, have been put forward continuously, making the near-field acoustic holography technology constantly improved[4-5].

From the theoretical basis, the existing near-field acoustic holography is a theory developed under the framework of free field[6], that is, the propagation mode of sound wave is mainly in free field. However, for the indoor substation noise source identification and sound field reconstruction, the classical acoustic holography technology is no longer applicable, because in the indoor environment, affected by reverberation, the propagation of sound wave has multi-path effect, the sound field at a certain position in the indoor space is composed of direct sound and reverberation sound, so the identification of noise source will be greatly affected by reverberation; at the same time, due to the influence of reverberation in indoor substation, the sound propagation has multi-path effect. The transformer and other equipment belong to large volume equipment, and the acoustic radiation range is wide, so it cannot be equivalent to point sound source as the traditional method. Therefore, noise source identification in indoor environment is more difficult than that in free field environment.

For the above problems in the identification of indoor substation noise sources, based on the equivalent sound source method of classical near-field acoustic holography, a sparse equivalent source method for indoor substation noise source identification is developed. In this method, the equivalent source surface, sound pressure acquisition surface and reconstruction surface are arranged in the space. The equivalent source surface is the equivalent radiation source construction area of transformer radiated noise, the sound pressure acquisition surface is the acoustic signal acquisition area, and the reconstruction surface is the reconstruction area of sound pressure. In the processing of noise source identification, this method first establishes the indoor sound transfer function model between the equivalent source surface and the acquisition surface by using wave simulation. On this basis, combined with the sound pressure sampling data, the distributed equivalent source is recovered on the equivalent source surface by sparse recovery algorithm, and finally the sound pressure reconstruction on the reconstruction surface is realized. The simulation results show that this method can effectively suppress the reverberation problem in the indoor environment, and can accurately equivalent the noise source of the transformer equipment with complex shape, so as to recover the actual noise radiation.

2. Noise source identification based on sparse equivalent source method
Assumed that an indoor substation modelling diagram is shown in figure 1. There is a transformer inside this rectangular-shaped room. The transformer radiates noise to the indoor environment. The sound field at a certain location consists of the direct sound and reverberation sound, which leads that the traditional method is unable to extract the noise of the transformer itself. In order to realize accurate noise source identification in indoor environment, a sparse equivalent source method is proposed in this paper.
Figure 1. Schematic of an indoor substation.

As shown in figure 1, an equivalent source surface $S$ is established on the main sound source side of the transformer, and a series of point sources are distributed on the surface. A reconstruction surface $R$ and a sampling plane $Y$ are constructed as the areas of noise source reconstruction and acoustic signal acquisition. After the signals are received by microphones located on the sampling plane, the initial signals of the equivalent sources are recovered by sparse recovery algorithm and then the sound field reconstruction is realized by using the sound field transformation algorithm.

2.1. Framework of the method

The radiated noise of the transformer reaches a series of sampling points on the sampling plane after the propagation in the room. The measured signals are $y = \begin{bmatrix} y_{11}, y_{12}, \cdots, y_{1n} \\ y_{21}, y_{22}, \cdots, y_{2n} \\ \vdots \\ y_{m1}, y_{m2}, \cdots, y_{mn} \end{bmatrix}$, where $n$ represents the number of sampling microphones. Since the transformer has large size, it cannot be treated as a point sound source, so its noise is simulated by $m$ equivalent sources with the equivalent signals are $s = \begin{bmatrix} s_{11}, s_{12}, \cdots, s_{1n} \\ s_{21}, s_{22}, \cdots, s_{2n} \\ \vdots \\ s_{m1}, s_{m2}, \cdots, s_{mn} \end{bmatrix}$. When the equivalent signals are sent by the equivalent sources and arrive at sampling points through certain propagation paths, the acquisition signal vector $y$ can be expressed as:

$$
\begin{align*}
    y_{11} &= s_{11} * h_{11} + s_{12} * h_{21} + \cdots + s_{1n} * h_{mn} \\
    y_{12} &= s_{11} * h_{12} + s_{12} * h_{22} + \cdots + s_{1n} * h_{mn} \\
    \vdots & \vdots \\
    y_{1n} &= s_{11} * h_{1n} + s_{12} * h_{2n} + \cdots + s_{1n} * h_{mn} \\
    y_{21} &= s_{21} * h_{11} + s_{22} * h_{21} + \cdots + s_{2n} * h_{mn} \\
    y_{22} &= s_{21} * h_{12} + s_{22} * h_{22} + \cdots + s_{2n} * h_{mn} \\
    \vdots & \vdots \\
    y_{2n} &= s_{21} * h_{1n} + s_{22} * h_{2n} + \cdots + s_{2n} * h_{mn} \\
    \vdots & \vdots \\
    y_{mn} &= s_{m1} * h_{1n} + s_{m2} * h_{2n} + \cdots + s_{mn} * h_{mn}
\end{align*}
$$

(1)

where the subscript $t$ of each symbol indicates that the signal is a time-domain signal. * is a time-domain convolution operator. $h_{mn}$ is a room impulse response representing the acoustic wave propagation path from the $m$-th equivalent source point to the $n$-th sampling point. In order to facilitate calculation, the variables in equation (1) are transformed into frequency domain, and the time-domain convolution is converted into frequency-domain multiplication:

$$
\begin{bmatrix}
    y_{s1} \\
    y_{s2} \\
    \vdots \\
    y_{sn}
\end{bmatrix}
= 
\begin{bmatrix}
    h_{s11} + h_{s12} + \cdots + h_{s1n} \\
    h_{s21} + h_{s22} + \cdots + h_{s2n} \\
    \vdots \\
    h_{snn} + h_{s2n} + \cdots + h_{snn}
\end{bmatrix}
\begin{bmatrix}
    s_{1} \\
    s_{2} \\
    \vdots \\
    s_{n}
\end{bmatrix}
$$

(2)
where the subscript $\omega$ of each symbol indicates that the signal is a frequency domain signal. Assuming that there is measurement noise in the real situation, the above model can be expressed as follows:

$$y_{\omega} = h_{\omega} s_{\omega} + n$$

(3)

where $n$ is the measurement noise.

By solving this equation, the equivalent signal can be obtained and the noise source can be identified using the free field Green’s Function.

2.2. Modelling of the transfer function in low frequency range

Equation (3) shows that an important step of noise source identification is to recover the equivalent source signal by sampling signal, so the construction of room impulse response becomes a key problem. The room impulse response represents the response of the receiving point to the impulsive sound, which essentially represents the propagation path of the acoustic signal from the sound source to the receiving point, and is a transfer function in the frequency domain. Due to the low frequency noise of transformer and the small space of indoor substation, the wave phenomena such as diffraction and interference are obvious. Therefore, the transfer function between the equivalent source point and the receiving point is constructed by using the wave-based method.

Assuming there is a sound source in the indoor environment. The mass of the medium provided by the source is $q$ in unit time. The acoustic wave equation with a source is given by

$$\nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = -\rho_0 \frac{\partial q}{\partial t}$$

(4)

where $t$ stands for time; $p$ is the sound pressure; $c_0$ is the speed of sound in air; $\rho_0$ is the equilibrium density of air; and $q$ is the volume velocity of the sound source.

Considering the system in frequency domain, the Helmholtz equation can be obtained as follows

$$\nabla^2 p_{\omega} + k^2 p_{\omega} + j\rho_0 \omega q_{\omega} = 0$$

(5)

where $k$ denotes the wave number.

The boundary condition can usually be classified into two kinds as follows

$$\left\{ \begin{array}{ll}
\frac{\partial p_{\omega}}{\partial n} = -j\rho_0 \omega \frac{p_{\omega}}{Z_s} \\
\frac{\partial p_{\omega}}{\partial n} = 0 
\end{array} \right. \quad \Gamma_1$$

$$\quad \Gamma_2$$

(6)

where $n$ denotes outward normal direction, $Z_s$ is the specific acoustic impedance of the boundary. $\Gamma_1$ stands for the damping boundary. $\Gamma_2$ stands for the rigid boundary.

A trial function $\overline{p}_{\omega}$ should be assumed and substituted into the Helmholtz equation and boundary equations according to the weighted residual method. For each equation, there is a residual because the trial function usually is not an exact solution but an estimative one. According to Galerkin weighted residual method and Green’s first identity, the following equation can be obtained

$$\int_\Omega (\nabla \overline{p}_{\omega} \cdot \nabla \overline{p}_{\omega} - k^2 \overline{p}_{\omega} \cdot \overline{p}_{\omega} - j\rho_0 \omega \overline{p}_{\omega} \overline{q}_{\omega}) \, dv + \int_{\Gamma_1} j\rho_0 \omega \overline{p}_{\omega} \cdot \overline{Z_s} \cdot \overline{p}_{\omega} \, ds = 0$$

(7)

The sound pressure of any position, for example $r$, in the problem domain is expressed by
where \( m \) is the number of nodes, \( N_r \) is the vector of the shape functions for position \( r \), and \( p \) is the vector of nodal sound pressure. Substituting the \( \overline{p}_\omega \) in equation (7) with the form described in equation (8), the following equation can be obtained

\[
\int_\Omega [(\nabla N)(\nabla N)^T] \mathrm{d}v \cdot \overline{p}_\omega = \int_\Omega (k^2 N N^T) \mathrm{d}v \cdot p_\omega - \int_\Omega (j \rho_\omega \phi N q_\omega) \mathrm{d}v + \int_\Omega \frac{j \rho_\omega^0}{Z_s} (NN^T) \mathrm{d}v \cdot p_\omega = 0
\]  

(9)

In the above equation, \( K = \int_\Omega (\nabla N)(\nabla N)^T \mathrm{d}v \) is defined as the stiffness matrix, \( M = \frac{1}{c^2} \int_\Omega NN^T \mathrm{d}v \) as the mass matrix, \( C = \frac{\rho_\omega}{Z_s} \int_\Omega NN^T \mathrm{d}s \) as the damping matrix, and \( F = \int_\Omega j \rho_\omega \phi N q_\omega \mathrm{d}v \) as the load matrix. If the sound source is a point source at \( r_0 \), the intensity of the source can be described as \( q_\omega = q_\omega(r_0) \delta(r - r_0) \). By defining \( Q = q_\omega(r_0) \mathrm{d}v \) as the volume velocity, the system equation can be obtained finally

\[
(K + j \omega C - \omega^2 M) p_\omega = F
\]  

(10)

With calculating this equation, the nodal sound pressure as well as that at the receiving point can be obtained and the transfer function from the source to the receiving point is obtained.

### 2.3. Sparse recovery of the equivalent source

After the transfer function of indoor equivalent source point to receiving point is calculated by wave modelling, the equation (3) can be solved to obtain the equivalent signal of equivalent source point. Due to the large volume of transformer in indoor substation, a large number of equivalent sources are needed to simulate its radiated noise. In this case, the underdetermined problem will be formed when the traditional least square method is used to solve the equation (3), which affects the accuracy of the solution. Therefore, in this paper, the sparse recovery method is used to solve the equation. On the one hand, the number of sampling points can be greatly reduced, and on the other hand, the radiation noise of transformer can be simulated with fewer equivalent source points through sparse constraint.

The compressive sensing (CS) theory that has been widely accepted as an efficient recovery tool in acoustic problems is used here. CS theory suggests that if a signal is sparse or compressive and the measurement matrix is highly incoherent with the dictionary, it can be reconstructed from a limited number of measurements by solving a under-determined inverse problem. Thus, it is possible to reduce the number of spatial sampling points or broaden the frequency range.

In equation (3), \( s_\omega \) which expresses the information of sound source is the unknown vector to be determined. If measurement noise is present, equation (3) can be solved by the sparse regularization, i.e., by seeking the solution with minimum \( l_1 \)-norm through the minimization problem

\[
\arg \min_{s_\omega \in \mathbb{C}^n} \|s_\omega\|_1 \text{ subject to } \|y_\omega - h_\omega s_\omega\|_2 \leq \xi
\]  

(11)

where the operator \( \| \cdot \|_p \) indicates the \( l_p \)-norm. The parameter \( \xi \) is an estimation of the upper bound for the noise present in the sensing process.

By using the sparse recovery algorithm, a limit number of the equivalent signals can be recovered. These signals can be considered as the radiated signal in free field without the influence of the
reverberation. Finally, the sound source can be identified through analysing the distribution of the sound field on the reconstruction plane.

3. Numerical verification

In order to verify the proposed method, the indoor substation scenario shown in Figure 1 is used to verify the proposed method. In this simulation, the room size of the indoor substation is 8980*6060*4880mm. The interior of the transformer is set with sound source. The noise radiated by the noise source propagates to the indoor environment through the effect of the transformer shell. In the front of the transformer, the equivalent source surface, reconstruction surface and sampling surface are arranged respectively with an interval of one meter. A total of 25 sampling points are set on the sampling plane with the sampling point spacing of 0.5m; 36 equivalent source points are set on the equivalent source surface with the spacing of 0.4m; and 36 equivalent source points are set on the reconstruction surface, with the spacing of 0.4m. It should be noted that in this simulation, in order to verify the correctness of the method, the scene of indoor substation and transformer are simplified, which does not represent the actual working scene.

In the simulation, firstly, the transfer function from the equivalent source point to the sampling point in indoor environment is simulated by using the fluctuation modelling method, and then the original noise radiation of the transformer is reconstructed on the reconstruction surface using the above method. For comparison and verification, the acoustic radiation at the reconstructed surface of transformer under free field condition is calculated by boundary element method as the standard result. For comparison and verification, the sound radiation in indoor substation is also compared to represent the signal measured under the reverberation.

The sound source identifications at 50Hz, 100Hz and 200Hz are illustrated in figure 2, figure 3 and figure 4.

![Figure 2. Sound field distributions of different methods at 50Hz.](image1)

![Figure 3. Sound field distributions of different methods at 100Hz.](image2)
Figure 4. Sound field distributions of different methods at 200Hz.

Figure 2 to figure 4 illustrate that the proposed method in this paper is capable of giving very close results with the standard data. It means that the method is effective to reduce the influence of the reverberation in indoor substation. With this method, the real sound field radiated by the transformer can be obtained and the noise source identification can be realized. To evaluate the performance of the proposed method, the average absolute errors from 50Hz to 400Hz are calculated and shown in figure 5.

Figure 5. Average absolute errors of the proposed method and the reverberation data without processing

The average absolute errors in figure 5 shows the initial data of the sound radiation in indoor substation has large errors which means that the reverberation has obvious influence on the noise source identification. While the proposed method has errors less than 5dB, which demonstrates that the method has high accuracy.

4. Conclusion
A sparse equivalent source method to identify the noise source in indoor substation is proposed in this method. In the processing of noise source identification, this method first establishes the indoor sound transfer function model between the equivalent source surface and the acquisition surface by using wave simulation. On this basis, combined with the sound pressure sampling data, the distributed equivalent source is recovered on the equivalent source surface by sparse recovery algorithm, and finally the sound pressure reconstruction on the reconstruction surface is realized. The numerical verification demonstrates that this method is capable of extracting the real noise information in the reverberation sound and giving the real noise source distribution result.
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