Steering Law Controlling the Constant Speeds of Control Moment Gyros

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Abstract. To enable the agile control of satellites, using control moment gyros (CMGs) has become increasingly necessary because of their ability to generate large amounts of torque. However, CMGs have a singularity problem whereby the torque by the CMGs degenerates from three dimensions to two dimensions, affecting spacecraft attitude control performance. This study proposes a new steering control law for CMGs by controlling the constant speed of a CMG. The proposed method enables agile attitude changes, according to the required task, by managing the total angular momentum of the CMGs by considering the distance to external singularities. In the proposed method, the total angular momentum is biased in a specific direction and the angular momentum envelope is extended. The design method can increase the net angular momentum of CMGs which can be exchanged with the satellite. The effectiveness of the proposed method is demonstrated by numerical simulations.

1. Introduction

Agile control of satellites is required to perform observations of the earth. Control moment gyros (CMGs) can enable this by generating large amounts torque, and their use has increased in recent years.

A CMG is an actuator that generates torque by employing the gyroscopic effect using a swinging wheel that rotates at a constant rate in a gimbal axis. Generally, a CMG has a pyramid arrangement of four single-gimbal CMGs which rotate at the same constant speed for three-axis attitude control of a spacecraft with nominally zero momentum (as shown in Fig. 1). CMGs generate a torque through angular momentum transfer to and from the main spacecraft body; the output torque from CMGs is the time derivative of the angular momentum.

CMGs have a singularity problem whereby the torque generated by the CMGs degenerates from three dimensions to two dimensions, and it may affect spacecraft attitude control performance. There are two types of singularities: “external singularities” and “internal singularities”. For external singularities, the angular momentum of the four CMGs is at its maximum magnitude for its direction. Internal singularities are kinematic phenomena that occur at a specific set of angular momentums and gimbal angles. It is generally difficult to avoid external singularities because they occur at the performance limit of CMGs. In contrast, it is possible to avoid internal singularities by using avoidance steering logics.
To overcome the internal singularity problem, several steering logics have been proposed [1]. Steering logics can be divided into two groups [2]: “singularity escape” and “singularity avoidance”. The logic of singularity escape permits the generation of a torque that is different from the attitude control torque, e.g., the generalised singularity robust (GSR) steering logic [3] and the singular direction avoidance (SDA) steering law [4]. In contrast, singularity avoidance logic avoids singularities without the torque error, e.g., null-motion steering logic [5] and path planning methods.

In path planning methods, some studies focused on singularities and the angular momentum of CMGs and proposed a singularity avoidance method by planning a path in the angular momentum workspace. Singularities and the total angular momentum of CMGs can be expressed simultaneously by using the angular momentum workspace. Takada et al. proposed a steering law which keeps the surface cost function low by adding perturbation torques to the angular momentum trajectories [6]. Sato and Takahashi planned an angular momentum path that avoids singularities and minimises its length by application of the A* algorithm [7].

However, these methods address only internal singularities. A CMG is a momentum exchange device, so when CMGs have higher angular momentum, the total output torque from CMGs to the satellite is increased [8]-[9]. Generally, previous studies of CMG treat the angular momentum of a CMG as a fixed value. Therefore, the total angular momentum of CMGs depends only on the gimbal angles. However, since the total angular momentum depends on the constant speed of each individual CMG, it is expected that the net angular momentum which can be exchanged with the satellite can be increased to manage the total angular momentum by controlling the constant speed of a CMG. The constant speed of a CMG in a steady-state can be selected from several rates [10].

In this study, a design method for an agile satellite attitude control system is proposed, to manage the total angular momentum of CMGs by considering the distance to external singularities. By choosing the constant speed of a CMG, the total angular momentum of the CMGs is biased in a specific direction and the angular momentum envelope is extended. Thus, the design method can increase the net angular momentum of the CMGs, which can be exchanged with the satellite. To achieve this demand, when the constant speed of a CMG is changed, the relationship between the total angular momentum of CMGs and the external singular surface is analysed. Using this analysis, a new steering law which can manage the total angular momentum of CMGs is proposed, according to the required task. The effectiveness of the method is demonstrated by numerical simulations.

2. Modelling

2.1. Satellite system

Euler’s rotational equation of motion of a rigid spacecraft with four CMGs is expressed as
\[
\mathbf{I}\dot{\omega} + \omega \times \mathbf{I}\omega = \tau_c + \tau_{\text{ext}}
\]  
(1)

where \(\mathbf{I}\) is the inertia matrix of the satellite, \(\omega\) is the angular rate vector, and \(\tau_{\text{ext}}\) is the external disturbance torque. Then the external disturbance torques, such as the natural environmental torques, can be regarded as zero during the short time maneuver. The attitude control torque \(\tau_c\) is defined as

\[
\tau_c = -\mathbf{h} - \omega \times \mathbf{h}
\]  
(2)

where \(\mathbf{h}\) is the angular momentum of four CMGs.

The linear state feedback controller is defined as

\[
\tau_r = -K_p \mathbf{q}_r - K_d \dot{\mathbf{q}}_r
\]  
(3)

where \(K_p\) is the proportion gain matrix, \(K_d\) is the derivation gain matrix and \(\tau_r\) is the torque command vector. The attitude quaternion error vector \(\mathbf{q}_e\) is calculated from the reference quaternion vector and the current quaternion vector.

2.2. CMG system

The CMG is an actuator that generates a torque employing the gyroscopic effect with a swinging wheel that rotates at a constant rate in a gimbal axis, perpendicular to the axis of wheel rotation. In actual operation, it is necessary to combine several CMGs for redundancy and fault-tolerance. In this study, the CMG system has a pyramid arrangement of four single gimbal CMGs as shown in Fig. 1.

The total angular momentum of four CMGs \(\mathbf{h}\) is expressed in the spacecraft reference frame as

\[
\mathbf{h} = \sum_{i=1}^{4} \mathbf{h}_i
\]

\[
= J \Omega_1 \left[ -c\beta \sin \delta_1 \right] + J \Omega_2 \left[ -c\beta \sin \delta_2 \right] + J \Omega_3 \left[ -c\beta \sin \delta_3 \right] + J \Omega_4 \left[ -c\beta \sin \delta_4 \right]
\]

\[
+ J \Omega_1 \left[ \cos \delta_1 \right] + J \Omega_2 \left[ \cos \delta_2 \right] + J \Omega_3 \left[ \cos \delta_3 \right] + J \Omega_4 \left[ \cos \delta_4 \right]
\]  
(4)

where \(s\beta = \sin \beta, c\beta = \cos \beta; \Omega_i, \delta_i\) is the wheel rate and gimbal angle of the \(i\)-th CMG, \(J\) is the moment of inertia of the wheel, and \(\beta\) is the skew angle of the four CMGs. Generally, the skew angle is usually selected as 54.74 degrees because the maximum angular momentum of the three axes is the same.

The time derivative of the angular momentum of CMGs vector \(\dot{\mathbf{h}}\) is equal to the internal torque vector \(\mathbf{\tau}\) generated by the CMGs and can be obtained as

\[
\dot{\mathbf{h}} = \sum_{i=1}^{4} \frac{\partial \mathbf{h}_i}{\partial \delta_i} \dot{\delta}_i = \mathbf{D}_i \dot{\delta}_i = \mathbf{\tau}
\]  
(5)

where \(\dot{\delta} = [\delta_1, \delta_2, \delta_3, \delta_4]^T\) is the gimbal angle vector, and \(\mathbf{D}_i\) is the Jacobian matrix, written as

\[
\mathbf{D}_i = \begin{bmatrix}
-\Omega_i c\beta \cos \delta_1 & \Omega_i \sin \delta_1 & \Omega_i c\beta \cos \delta_i & -\Omega_i \sin \delta_i \\
-\Omega_2 c\beta \cos \delta_2 & \Omega_2 \sin \delta_2 & \Omega_2 c\beta \cos \delta_2 & -\Omega_2 \sin \delta_2 \\
-\Omega_3 c\beta \cos \delta_3 & \Omega_3 \sin \delta_3 & \Omega_3 c\beta \cos \delta_3 & -\Omega_3 \sin \delta_3 \\
-\Omega_4 c\beta \cos \delta_4 & \Omega_4 \sin \delta_4 & \Omega_4 c\beta \cos \delta_4 & -\Omega_4 \sin \delta_4
\end{bmatrix}
\]  
(6)

The reference gimbal angular velocity that generates the reference torque is obtained as

\[
\dot{\delta}_r = \mathbf{D}_i^T \left( \mathbf{D}_i \mathbf{D}_i^T \right)^{-1} \mathbf{\tau}_r
\]  
(7)

2.3. Singularities

At singularities, the torque \(\mathbf{\tau}\) generated by the CMGs degenerates from three-dimensions to two-dimensions in the case of a specific set of angular momentums and gimbal angles. Singularities are classified into external singularities and internal singularities.
External singularities are phenomena associated with the maximum projection of the total angular momentum of CMGs. For external singularities, the angular momentum of the four CMGs is at its maximum magnitude for its direction.

Internal singularities are kinematic phenomena that occur at a specific set of angular momentums and gimbal angles when the Jacobian matrix is singular. For internal singularities, the torque vectors for each CMG are coplanar, eliminating the possibility of generating torque normal to the plane.

Singularities occur when $\mathbf{D}_d$ meets the conditions

$$\text{rank}(\mathbf{D}_d) < 3 \quad \text{or} \quad \text{rank}(\mathbf{D}_d \mathbf{D}_d^T) < 3 \iff \det(\mathbf{D}_d \mathbf{D}_d^T) = 0.$$ (8)

At singularities, the pseudo-inverse does not exist and the pseudo-inverse steering logic encounters singular states. The singularity parameter $d$ is an indicator of singularities; it approaches zero when the CMGs approaches a singularity and is defined as

$$d = \det(\mathbf{D}_d \mathbf{D}_d^T).$$ (9)

2.4. Singular surface [11]

Since the total angular momentum of CMGs can be obtained uniquely, to plot it in the angular momentum workspace, the relationship between singularities and the state of CMGs is expressed by using the coordinates. The set of singularities is located on a curved surface, which is called the singular surface.

In the angular momentum workspace, analytic expressions for the singular surfaces $(H_x, H_y, H_z)$ can be obtained as

$$H_x = J\Omega_x \frac{c\beta(-a\mathbf{u}_z + c\mathbf{u}_x)}{e_1} + J\Omega_z \frac{u}{e_2} + J\Omega_y \frac{c\beta(a\mathbf{u}_z + c\mathbf{u}_x)}{e_3} + J\Omega_x \frac{u}{e_4},$$

$$H_y = J\Omega_y \frac{a\mathbf{u}_z - c\mathbf{u}_x}{e_1} + J\Omega_x \frac{u}{e_2} + J\Omega_y \frac{c\beta(-a\mathbf{u}_z - c\mathbf{u}_x)}{e_3} + J\Omega_z \frac{u}{e_4},$$

$$H_z = J\Omega_z \frac{a\mathbf{u}_z + c\mathbf{u}_x}{e_1} + J\Omega_y \frac{c\beta(a\mathbf{u}_z - c\mathbf{u}_x)}{e_2} + J\Omega_x \frac{u}{e_3} + J\Omega_z \frac{c\beta(a\mathbf{u}_z + c\mathbf{u}_x)}{e_4}.$$ (10)

$$e_1 = \pm \sqrt{1 - (a \mathbf{u}_x + c \mathbf{u}_z)^2},$$

$$e_2 = \pm \sqrt{1 - (a \mathbf{u}_z + c \mathbf{u}_x)^2},$$

$$e_3 = \pm \sqrt{1 - (-a \mathbf{u}_x + c \mathbf{u}_z)^2},$$

$$e_4 = \pm \sqrt{1 - (-a \mathbf{u}_z + c \mathbf{u}_x)^2}.$$ (11)

where $u_x = \sin \theta_1, u_y = -\sin \theta_1 \cos \theta_2, u_z = \cos \theta_1 \cos \theta_2$ in Fig. 1. The external singular surface is equivalent to the angular momentum envelope in the angular momentum workspace.

From Eq. (4), the time histories of the total angular momentum of CMGs during the task can be plotted as the angular momentum path in the workspace, since the total angular momentum is changed by a set of gimbal angles. Then, from Eq. (5), since the time derivative of the total angular momentum is equal to the internal torque vector generated by the CMGs, the direction in which the plot moves expresses the direction of the torque vector, and the time derivative of the length expresses the magnitude.

3. Design method

In Sec. 2, since a time derivative of the angular momentum of the CMGs vector $\mathbf{h}$ is the same as the torque vector $\tau$ generated from the CMGs, it can appear that the angular momentum is related to
the settling time. From Eq. (5), previous studies of CMG treat the angular momentum of a CMG as a fixed value: \( J\Omega = h \). However, the total angular momentum of CMGs depends on not only the gimbal angles but also the constant speed of each CMG. Therefore, it is expected that the satellite attitude can be changed in a more agile manner, according to the required task, by increasing the net angular momentum of the CMGs.

In this section a design method is proposed; firstly the relationship between the total angular momentum of CMGs and the external singular surface by changing the constant speed of a CMG is analysed. Then using this analysis, a new steering law is proposed.

3.1. The relationship between the total angular momentum of CMGs and the external singular surface

When the angular momentum of a CMG is changed by changing the constant speed of a CMG, the relationship between the total angular momentum of CMGs and the external singular surface is analysed.

A typical pyramid arrangement of four CMGs with a skew angle is 54.74 degrees and an initial gimbal angles vector \( \delta_0 = [0 \ 0 \ 0 \ 0]^T \) is considered. From Eq. (4), when the four CMGs each have the same angular momentum (and the same constant speed), the total angular momentum of the CMGs is zero. In contrast, the total angular momentum of the CMGs in a steady-state can be biased in the positive direction of the y-axis by increasing the angular momentum of CMG1, and can be biased in a negative direction of the y-axis by decreasing it. Conversely, the total angular momentum is biased in the negative direction of the y-axis by increasing the angular momentum of CMG3, and is biased in the positive direction of the y-axis by decreasing it. Similarly, the total angular momentum is biased in a negative direction of the x-axis instead of the y-axis by increasing the angular momentum of CMG2, and is biased in the positive direction of the x-axis by decreasing it. The total angular momentum is biased in the positive direction of the x-axis instead of the y-axis by increasing the angular momentum of CMG2, and is biased in the positive direction of the x-axis by decreasing it. In these analyses, the total angular momentum can be biased in an arbitrary direction by managing the angular momentum of a specific CMG.

Conversely, from Eq. (10), the angular momentum envelope is expanded in the y-axis in particular, by increasing the angular momentum of CMG1. The same applies to an increase in the angular momentum of CMG3. The angular momentum envelope is expanded in the x-axis with an increase of the angular momentum of CMG2 and CMG4.

The numerical simulation was conducted for three scenarios, as described below. The parameters are given in Table 1.

- Case 1: the constant speeds of the four CMGs are all 3000 rpm
- Case 2: the constant speed of CMG1 is changed to 4500 rpm from Case 1
- Case 3: the constant speed of CMG2 is changed to 4500 rpm from Case 1

In Case 1, the total angular momentum of the CMGs in a steady-state was zero in the directions of the x, y and z-axes. The maximum angular momentum of the CMGs within the angular momentum envelope was 128.8 Nms in the x-axis and the y-axis, and 133.4 Nms in the z-axis. The skew angle is usually selected as 54.74 degrees because the angular momentum of the three axes is the same.

In Case 2, the total angular momentum was 19.5 Nms in the direction of the y-axis and 0 Nms in the directions of the x and z-axes. The maximum angular momentum was 149.3 Nms in the y-axis and 150.1 Nms in the z-axis.

In Case 3, the total angular momentum was \(-19.5\) Nms in the direction of the x-axis and 0 Nms in directions of the y and z-axes. The maximum angular momentum was 149.3 Nms in the x-axis, 140.6 Nms in the y-axis and 150.1 Nms in the z-axis.

3.2. A design method to manage the total angular momentum of the CMGs

In Sec. 3.1, to change the angular momentum of a specific CMG, the total angular momentum of the CMGs in a steady-state was biased in an arbitrary direction. Also, to increase the angular
momentum of a CMG, the angular momentum envelope was expanded. Using this analysis, a new steering law which can change the satellite attitude is proposed. As an example, a rotation around the roll axis is assumed. When the initial gimbal angles vector is $\delta_0 = [0 \ 0 \ 0 \ 0]^T$ and the CMGs have the same angular momentums, the total angular momentum of the CMGs is zero. If the angular momentum of CMG2 is increased by increasing its speed, the total angular momentum is biased in the negative direction of the x-axis and the angular momentum envelope is expanded (particularly in the x-axis). So, the distance between the total angular momentum in a steady-state and the external singularities is extended. The net angular momentum of the CMGs which can be exchanged with the satellite is increased and the total output torque is increased. Therefore, the satellite can change attitude in a more agile manner.

4. Numerical simulation

The effectiveness of the proposed method was demonstrated by numerical simulation. The purpose of the analysis was to perform a 30 degree roll slew maneuver. The numerical simulation was conducted for Case 1 and Case 3; Case 1 is the comparative method and Case 3 is the proposed method. In both methods, the internal singularity avoidance method was GSR steering logic. The GSR steering logic can be represented as

$$\dot{\delta}_g = D_r^T \left( D_r D_r^T + H_r^2 \lambda \mathbf{E} \right)^{-1} \tau,$$

(12)

$$\mathbf{E} = \begin{bmatrix} 1 & \varepsilon_2 & \varepsilon_1 \\ \varepsilon_2 & 1 & \varepsilon_1 \\ \varepsilon_1 & \varepsilon_2 & 1 \end{bmatrix} > 0$$

(13)

$$\varepsilon_i = \varepsilon_0 \sin(\omega t + \phi_i)$$

(14)

$$\lambda = \lambda_0 e^{-\mu t |\mathbf{H}|}$$

(15)

The parameters are given in Table 1. The parameters of each method were decided by trial-and-error.

Fig. 2–Fig. 5 show the time histories of the Euler angle of the satellite, the singularity parameter, and the angular momentum in each method respectively. In the angular momentum workspace, a yellow point shows the total angular momentum of the CMGs in a steady-state, a red point shows it after the maneuver, and a green line shows the angular momentum path during the maneuver.

As shown in Fig. 2 and Fig. 4, in the comparative method, the total angular momentum of the CMGs in the steady-state was zero. This method approached singularities twice; it approached an internal singularity and escaped by GSR steering logic at 2 s, and reached external singularities between 5 and 16 s. The external singularities were difficult to avoid because of the maximum restriction of the CMGs. The net angular momentum of the CMGs in the x-axis was changed by nearly 130 Nms, between 0 Nms and 128.8 Nms.

In contrast, as shown in Fig. 3 and Fig. 5, in the proposed method the total angular momentum of the CMGs was biased in the negative direction of the x-axis. This method also approached singularities twice. This method approached internal singularities between 2–3 s, and external singularities between 7 and 12 s. The net angular momentum of the CMGs in the x-axis was changed by nearly 170 Nms between −19.5 Nms and 149.3 Nms.

Table 2 shows that the settling time in the proposed method was 3.2 s shorter than in the comparative method. The reason is that the proposed method can change the net angular momentum by nearly 40 Nms more than the comparative method in the x-axis.
Table 1 Parameters and values of experimental setup

| Parameters                                      | Symbols | Values                      |
|-------------------------------------------------|---------|-----------------------------|
| Satellite moment of inertia                     | $I$     | $\text{diag}(5000,5000,3000)\text{ kgm}^2$ |
| Gimbal axis moment of inertia                   | $I_g$   | 0.19 kgm$^2$                |
| Wheel axis moment of inertia                    | $I_w$   | 0.13 kgm$^2$                |
| Skew angle of 4CMG                              | $\beta$ | 54.7 deg                   |
| Initial gimbal angles vector                    | $\delta_0$ | $[0\ 0\ 0\ 0]^T$        |
| Maximum angular velocity of gimbal axis         | $\dot{\delta}_{\text{max}}$ | $\pm 1\ \text{rad/s}$    |
| Maximum angular acceleration of gimbal axis     | $\ddot{\delta}_{\text{max}}$ | $\pm 3\ \text{rad/s}^2$ |
| Control cycle                                   | $\Delta t$ | 0.02 s                   |
| Nominal angular momentum of CMG                 | $H_0$   | 40 Nms                     |
| Parameter                                       | $\epsilon_0$ | 0.01                   |
|                                                 | $\lambda_0$ | 0.01                     |
|                                                 | $\mu$   | $10^{-9}$                  |
|                                                 | $\phi_i$ | $0,\pi/2,\pi$             |

Table 2 Result of simulation

| Cost function | Case1 | Case3 |
|---------------|-------|-------|
| Settling time [s] | 29 | 25.8 |
| Singular time [s] | 12.3 | 7.7 |

Fig. 2 Simulation result of the comparative method

Fig. 3 Simulation result of the proposed method
Fig. 4 Simulation result of the comparative method

Fig. 5 Simulation result of the proposed method
5. Conclusion

In this study, a new steering law which can change satellite attitude in a more agile manner according to the required task, by managing the total angular momentum of CMGs by considering the distance to external singularities was proposed. By choosing the speed of a CMG, the total angular momentum is biased in a specific direction, and the angular momentum envelope is extended. This design method can increase the net angular momentum of CMGs which can be exchanged with the satellite. The effectiveness of the proposed method was demonstrated using numerical simulations.

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