Kinematic analysis and simulation of a new type of differential micro-feed mechanism with friction

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Abstract

This article presents a new micro-feed mechanism, whose main transmission component is the nut–rotary ball screw pair. The screw and nut are driven by two motors, and they rotate in the same direction, with their movements enabling micro-feeding. The main contribution of the micro-feed mechanism is to avoid the inevitable low-speed nonlinear creeping phenomenon caused by the inherent properties of traditional electromechanical servo system structure, thus realizing high precision micro-feed. In this study, the motion state of the working ball is analyzed using the principle of differential geometry, the friction at the contact points is calculated, the balance equation for force and moment is established, the influences of the screw and nut on the kinematic parameters of the ball at different velocities and the differences in the motion states of the ball in different drive modes are studied, and the mechanical efficiency of the dual-driven ball screw mechanism is calculated. The potential applications of the new micro-feed mechanism and the results of numerical analysis can be applied to advanced technology fields such as robotics, suspensions, powertrain, national defense, integrated electronics, optoelectronics, medicine, and genetic engineering, so that the new system can have a lower stable speed limit and achieve precise micro-feed control.

Keywords

Micro-feed mechanism, dual-driven ball screw, kinematic analysis, frictional force, mechanical efficiency

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Introduction
In this article, we propose a dual-servo micro-feed mechanism based on the motion synthesis principle. The motor-driven ball screw and motor-driven ball nut of two quasi-equal areas can realize a micro-feed mechanism. The rotary nut and ball screw pair is the key component of movement and force transmission, and their dynamic performance directly determines the precision of the micro-feed mechanism; therefore, the kinematic characteristics of the dual-driven ball screw mechanism should be investigated.

Significant progress has been made in the research on the kinematics of single-driven ball screw feed mechanisms. The kinematic behaviors of ball screws and ball bearings are similar; thus, the analysis method applied to ball bearings can be used to study the kinematic characteristics of a dual-driven ball screw mechanism. Harris\textsuperscript{1,2} analyzed the sliding characteristics between the ball and the inner ring roller, derived the contact stress on the rolling surface, and analyzed the failure mode of the bearing. Lin et al.\textsuperscript{3} presented a theoretical analysis method to analyze the kinematic characteristics of a ball screw mechanism to explain the movement and contact patterns of the ball with the contacting elements using the established equations, and they found that a sliding phenomenon always existed in the elliptical contact area of the balls and screw or nut. Lin et al.\textsuperscript{4} employed three methods to calculate the mechanical transmission efficiency of a system and derived an approximate closed-form solution for the mechanical efficiency of ball screw kinematics, and this solution was used to optimize the mechanism. Huang and Ravani\textsuperscript{5} presented an algorithm using a medial axis transformation to analyze the contact stresses in the ball–nut and ball–screw contact areas. Wei and Lin\textsuperscript{6} analyzed the kinematic characteristics of a single-driven ball screw mechanism by considering the change in the elastic deformation and contact angle. Wei et al.\textsuperscript{7} analyzed the kinematics of the ball screw mechanism with pre-tightening force when the oil lubrication is considered, and the calculation results of the theoretical mechanical transmission efficiency were verified by experimental data. Wei and Lai\textsuperscript{8} analyzed the kinematic state of the ball screw mechanism with pre-tightening force of the single screw nut and double roller operating at high rotational velocities. Wei et al.\textsuperscript{9} proposed a novel two-body wear model combined with motion theory to describe the variation in the axial wear depth with respect to the operating level. On the basis of Jones’ channel control theory,\textsuperscript{10} Yoshida et al.\textsuperscript{11} established an analysis method to determine the movement state and load distribution of the ball and studied the effect of the ball screw geometry and different conditions on the ball movement state. Hu et al.\textsuperscript{12} used a homogeneous transformation matrix to establish a kinematic model for a basic single-driven ball screw mechanism and analyzed the kinematic characteristics and slide–roll ratios at the contact points of the raceways. Nakashima et al.\textsuperscript{13} analyzed the influence of the ball screw pair of a double nut on the elastic deformation of the screw in an ultra-precision positioning test table when the preload was changed. Mu and Feng\textsuperscript{14} analyzed the kinematic parameters of a nut-driven ball screw mechanism, studied the problem of isothermal elastohydrodynamic lubrication in elliptical contacts, and used a multigrid solver to derive the numerical solution for the parameters. Xu et al.\textsuperscript{15} presented a crawling analysis model based on rolling contact theory to obtain the frictional force of a ball screw mechanism and studied the effects of crawling parameters on the distribution of friction resistance in a ball screw mechanism.
Considering the influence of frictional force and wear during operation, Zou and Wang\textsuperscript{16} studied the change in the contact stiffness of linear rolling guides and established the initial and final models for contact stiffness. Wei Li and Jorge Angeles\textsuperscript{17} developed a ball screw driving system of 2 DoF; the kinematic analysis, undertaken with a geometrical method based on screw theory, leads to two Jacobian matrices, whose singularity conditions are investigated.

All the aforementioned studies focus on single-driven ball screw mechanisms. In this study, in order to overcome the disadvantage that the current typical single-drive servo system of screw has difficulty in achieving precise micro-feed because of the low-speed creeping phenomenon, a new type of differential micro-feed mechanism based on nut rotating ball screw pair is proposed. The change in drive mode leads to a change in the kinematic state of the ball and consequently affects the transmission precision. In this study, a kinematic analysis of the dual-driven ball screw mechanism is performed, and the movement track of the ball and the relative velocity at the contact point are obtained. Given that friction is the main factor affecting the positioning accuracy of the mechanism, the balance equations for force and torque are established by calculating the frictional force at the contact point. Numerical analysis is employed to solve for the kinematic and dynamic parameters of the dual-driven system and establish the expressions for mechanical efficiency and other performance parameters. In this article, the object to be analyzed is the rotary ball screw pair of the right-hand screw nut.

**Configuration of the dual-driven mechanism**

Figure 1 shows the mechanical components of the dual ball screw mechanism.

The dual-driven system is composed of a nut and rotary ball screw pair, which is connected to the screw motor shaft and is restricted in the axial and radial directions by the angular contact ball bearing on the motor side. The nut and rotary ball screw pair are each supported by a radial ball bearing to provide axial freedom. The rotations of the nut and the rotary ball screw pair are converted into the linear motion of the table, which is braced by two parallel rolling guides.

**Theoretical analysis of the dual-driven ball**

For the investigation on the kinematic characteristics of the dual-driven ball screw mechanism, four coordinate frames (Figure 2) are established to describe the movement of the ball. Given that the angular velocities of the spin and revolution of the ball change with variation in the contact point, the right-hand screw threads and a single nut are analyzed. The rotation direction is clockwise from the $+z'$ axis, and then the nut moves in a straight line along the $-z'$ axis. When the center of the ball moves along the helical groove of the screw, the first (world) coordinate frame, $o-x'y'z'$, is fixed; the $z'$ axis of the world coordinate frame and the screw axis are coincident. In the second (screw rotating) coordinate frame, $o-x_1y_1z_1$, the $z_1$ axis of the ball and the screw axis are also coincident, and they rotate together with the screw. In the third (nut rotating) coordinate frame, $o-x_ny_nz_n$, the $z_n$ axis of the ball and the screw axis are also coincident, and they rotate together with the nut. The fourth coordinate frame, $o'-tnb$, indicates the trajectory of the center
Figure 1. Mechanical components of the dual ball screw mechanism.
1: foundation; 2: rolling guide; 3: nut servo motor; 4: slider; 5: master synchronous belt wheel; 6: nut motor mounting plate; 7: screw servo motor; 8: screw motor mount; 9: ball screw; 10: slave synchronous belt wheel; 11: rotary nut; 12: workbench; 13: bearing block.

Figure 2. The position coordinate of the ball center.
of the ball. With respect to the coordinate $o-x_s,y_s,z_s$, the trajectory along the screw surface is a circular helix with a mean radius of $r_m$. The $b'$-axis (Figure 2) is parallel to the $b$-axis.

**Relationship between coordinate frames**

The relationship between the world coordinate frame ($o-x'y'z'$) and the second or screw-rotating coordinate frame ($o-x_s,y_s,z_s$) can be expressed as follows\textsuperscript{18}

$$X' = T_{1s}X_s$$  \hspace{1cm} (1)

where $X' = [x'y'z']^T$, $X_s = [x_s\ y_s\ z_s]^T$, and $T_{1s} = \begin{bmatrix} C_{\Omega_s} & -S_{\Omega_s} & 0 \\ S_{\Omega_s} & C_{\Omega_s} & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Here $C_{\Omega_s} = \cos(\Omega_s)$, $S_{\Omega_s} = \sin(\Omega_s)$, and $\Omega_s$ denotes the screw’s angular displacement respect to the first coordinate.

Similarly, the relationship between the world coordinate ($o-x'y'z'$) and the nut rotating coordinate ($o-x_n,y_n,z_n$) is given as follows

$$X' = T_{1n}X_n$$  \hspace{1cm} (2)

where $X_n = [x_n\ y_n\ z_n]^T$ and $T_{1n} = \begin{bmatrix} C_{\Omega_n} & -S_{\Omega_n} & 0 \\ S_{\Omega_n} & C_{\Omega_n} & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Here $C_{\Omega_n} = \cos(\Omega_n)$, $S_{\Omega_n} = \sin(\Omega_n)$, and $\Omega_n$ denotes the nut’s angular displacement with respect to the first coordinate.

The relationship between the screw rotating coordinate ($o-x_s,y_s,z_s$) and the nut rotating coordinate ($o-x_n,y_n,z_n$) can be written as follows

$$X_n = T_{ns}X_s$$  \hspace{1cm} (3)

where $S_{\Omega_n-\Omega_s} = \sin(\Omega_n-\Omega_s)$ and $C_{\Omega_n-\Omega_s} = \cos(\Omega_n-\Omega_s)$.

As the ball moves along the helical groove of the screw, the angular displacements of the ball are $\theta_s$ and $\theta_n$ because the screw and nut are each single-driven. Given that the screw and nut are driven by two servo motors and rotate in the same direction, the actual angular displacement, $\theta$, of the dual-driven ball along the helical groove of the screw is $\theta = \theta_s - \theta_n$.

The relationship between the Frenet coordinate frame ($o'-tnb$) and the screw rotating coordinate frame ($o-x_s,y_s,z_s$) is written as

$$X_s = T_2Y$$  \hspace{1cm} (4)
where $Y = [t \, n \, b]^T$ and $T_2 = \begin{bmatrix} -C_\lambda S_\theta & -C_\theta & S_\lambda S_\theta \\ C_\lambda C_\theta & -S_\theta & -S_\lambda C_\theta \\ S_\lambda & 0 & C_\lambda \end{bmatrix}$. Here $S_\lambda$, $C_\lambda$, $S_\theta$, and $C_\theta$ denote $\sin \lambda$, $\cos \lambda$, $\sin \theta$, and $\cos \theta$, respectively, and $\lambda$ is helix angle of the screw.

**Velocity analysis at the contact point**

In this article, when the rotational velocities of the micro-feed mechanism are very low, many parameters can be disregarded. On the basis of the analysis results of Wei et al., the assumptions used in this article are based on four hypotheses: (1) the friction coefficients in the ball–nut and ball–screw contact areas are equal, $\mu_e = \mu_i$. (2) When the rotational velocities of the screw and the nut are both significantly low and the difference between them is minimal, the centrifugal force acting on the ball center can be ignored. (3) When the rotational velocity is low, the gyroscopic angle $\beta$ can be regarded as a constant value, whereas the other gyroscopic angle is $\beta' = 0^\circ$. (4) Pure rolling points exist in the ball–screw and ball–nut contact areas.

The working ball is confined along the helical groove in directions parallel to the normal plane of the trajectory of the ball center; therefore, the ball can only move relative to the screw in the tangential direction of the Frenet frame of the ball center trajectory. Figure 3 shows the position coordinates of the contact points.

The contact point coordinate system $j-X_jY_jZ_j$ ($j = A, B$) between the ball and the raceway is introduced, such that the coordinate origin is located at the center of
the contact ellipse. In this coordinate system, the $X_j Y_j$ plane is located in the elliptical contact plane, the $X_j$-axis is tangential to the ball’s circle formed on the $n-b$ plane (Figure 3), the $Y_j$-axis is parallel to the $t$-axis, and the $Z_j$-axis is normal to the contact surface. The relationship between the Frenet coordinate system and the contact point coordinate system ($j$-$X_j Y_j Z_j$) is given by

$$X_j = T_j Y$$

where $X_j = [x_j, y_j, z_j]^T$, $T_j = \begin{bmatrix} 0 & -S_{\alpha_j} & C_{\alpha_j} \\ 1 & 0 & 0 \\ 0 & C_{\alpha_j} & S_{\alpha_j} \end{bmatrix}$, $S_{\alpha_j} = \sin \alpha_j$, $C_{\alpha_j} = \cos \alpha_j$, and $\alpha_j$ denotes the contact angles at the contact points $A$ and $B$.

Given that the elliptical contact area is significantly smaller than the ball radius, the contact angles are equal in the contact ellipse, that is, $\alpha_e = \alpha_A$ and $\alpha_e = \alpha_B$.

The ball center’s position vector, $oo'$, can be expressed relative to the screw rotating coordinate frame as

$$sR = \begin{bmatrix} r_m C_{\theta} & r_m S_{\theta} & r_m \theta T_{\lambda} \end{bmatrix} X_s = \begin{bmatrix} r_m S_{\lambda} T_{\lambda} \theta - r_m r_m C_{\lambda} T_{\lambda} \theta \end{bmatrix} Y$$

The ball center’s position vector, $oo'$, can be expressed relative to the world coordinate frame as

$$wR = \begin{bmatrix} r_m C_{\theta+\Omega_s} & r_m S_{\theta+\Omega_s} & r_m \theta T_{\lambda} \end{bmatrix} X'$$

where $r_m$ is the mean radius of the screw, $T_{\lambda} = \tan \lambda$, $C_{\theta+\Omega_s} = \cos(\theta + \Omega_s)$, and $S_{\theta+\Omega_s} = \sin(\theta + \Omega_s)$.

The ball center’s velocity relative to the screw rotating coordinate can be calculated by differentiating equation (6) with respect to time

$$s\dot{R} = \begin{bmatrix} -r_m \dot{\theta} S_{\theta} & r_m \dot{\theta} C_{\theta} & \frac{\theta L}{2\pi} \end{bmatrix} X_s = \begin{bmatrix} \frac{\dot{\theta} r_m}{C_{\lambda}} & 0 & 0 \end{bmatrix} Y$$

where $L$ is the lead of the screw.

The ball center’s velocity relative to the world coordinate can be calculated by

$$w\dot{R} = s\dot{R} + \dot{\Omega}_s \times sR = \begin{bmatrix} \frac{\dot{\theta} r_m}{C_{\lambda}} + r_m C_{\lambda} \dot{\Omega}_s & 0 & -r_m S_{\lambda} \dot{\Omega}_s \end{bmatrix} Y$$

Let $\omega_m = \dot{\theta} + \dot{\Omega}_s$ be the angular velocity for the revolution and $\omega_R = (\omega_t, \omega_n, \omega_b)Y$ be the spin angular velocity of the ball. Figure 4 shows the relative sliding speeds, slip angles, and spin angular velocities in the ball–nut and ball–screw contact areas. The angle of $\omega_R$ and $t-b$ plane is $\beta$, and the angle of the projection of $\omega_R$ in the $t-b$ plane and the $b$-axis is $\beta'$; both are gyroscopic angles. The three vector components of $\omega_R$ in the Frenet coordinate system can be written as follows.
\( \omega_t = \omega_R \cos \beta \sin \beta' \) \hspace{1cm} (10)

\( \omega_n = \omega_R \sin \beta \) \hspace{1cm} (11)

\( \omega_b = \omega_R \cos \beta \cos \beta' \) \hspace{1cm} (12)

The instantaneous velocity of the ball’s point \( A \), which is coincident with the contact point between the nut and the ball, can be expressed as

\[
V_{Ab} = \dot{W} \hat{R} + \omega_R \times R_{o'A} = \begin{bmatrix} \frac{\dot{\theta}_m}{C_\lambda} + r_m C_\lambda \dot{\Omega}_s + r_b (\omega_b C_{\alpha A} - \omega_n S_{\alpha A}) \end{bmatrix}^T \]

where \( r_b \) denotes the ball radius.

The instantaneous velocity of the point \( B \) on the ball, which is coincident with the contact point between the screw and the ball, can be expressed as

\[
V_{Bb} = \dot{W} \hat{R} + \omega_R \times R_{o'B} = \begin{bmatrix} \frac{\dot{\theta}_m}{C_\lambda} + r_m C_\lambda \dot{\Omega}_s + r_b (\omega_n S_{\alpha B} - \omega_b C_{\alpha B}) \end{bmatrix}^T \]

**Figure 4.** The velocity coordinate of the contact point.
The instantaneous velocity of the point $A$ on the nut can be expressed as

$$V_{An} = \left[ \begin{array}{ccc} 0 & 0 & \hat{\Omega}_n \end{array} \right] X' \times \hat{R}_{QA} + \left[ \begin{array}{ccc} 0 & 0 & -\frac{(\hat{\Omega}_S - \hat{\Omega}_n)L}{2\pi} \end{array} \right] X'$$

$$= \left[ \begin{array}{c} \frac{\hat{\Omega}_n r_m}{C_\lambda} + \hat{\Omega}_n C_\lambda r_b C_{\alpha A} - r_m S_\lambda T_\lambda \hat{\Omega}_s \\ \hat{\Omega}_n r_b S_\lambda S_{\alpha A} \\ -S_\lambda \left( \hat{\Omega}_n r_b C_{\alpha A} + r_m \hat{\Omega}_s \right) \end{array} \right]^T Y$$

(15)

where $\hat{R}_{QA} = \left[ \begin{array}{ccc} r_m S_\lambda T_\lambda \theta & -r_m - r_b C_{\alpha A} & r_m C_\lambda T_\lambda \theta - r_b S_{\alpha A} \end{array} \right] Y$.

The instantaneous velocity of the point $B$ on the screw can be expressed as

$$V_{BS} = \left[ \begin{array}{ccc} 0 & 0 & \hat{\Omega}_s \end{array} \right] X' \times \hat{R}_{OB} = \hat{\Omega}_s \left[ \begin{array}{c} C_\lambda \left( r_m - r_b C_{\alpha B} \right) \\ -r_b S_{\alpha B} \\ -S_\lambda \left( r_m - r_b C_{\alpha B} \right) \end{array} \right]^T Y$$

(16)

where $\hat{R}_{OB} = \left[ \begin{array}{ccc} r_m S_\lambda T_\lambda \theta & -r_m + r_b C_{\alpha B} & r_m S_\lambda \theta + r_b S_{\alpha B} \end{array} \right] Y$.

The slip velocity at the point $A$ between the nut and the ball is given by

$$V_{SA} = V_{Ab} - V_{An} = \left[ \begin{array}{c} \left( \hat{\theta} + \hat{\Omega}_n \right) r_m \left( \omega_b C_{\alpha A} - \omega_n S_{\alpha A} \right) - \hat{\Omega}_n r_b C_\lambda C_{\alpha A} \\ \hat{\Omega}_n r_b S_\lambda S_{\alpha A} \\ -r_b C_{\alpha A} \left( \omega_l - \hat{\Omega}_n S_\lambda \right) \end{array} \right]^T Y$$

(17)

$$= \left[ \begin{array}{c} -r_b \left( \omega_l - \hat{\Omega}_n S_\lambda \right) \\ \left( \hat{\theta} + \hat{\Omega}_n \right) r_m \left( \omega_b C_{\alpha A} - \omega_n S_{\alpha A} \right) - \hat{\Omega}_n r_b C_\lambda C_{\alpha A} \\ \left( \hat{\theta} + \hat{\Omega}_n \right) r_m \left( \omega_b C_{\alpha A} - \omega_n S_{\alpha A} \right) - \hat{\Omega}_n r_b C_\lambda C_{\alpha A} \\ 0 \end{array} \right]^T X_A$$

The slip velocity at the point $B$ between the screw and the ball is given by
\[
V_{SB} = V_{BB} - V_{BS} = \left[ \frac{\dot{r}_m}{C_{\lambda}} + r_b \left( (\bar{\Omega}_B C_{\lambda} - \omega_b) C_{\alpha B} + \omega_n S_{\alpha B} \right) \right]^T Y \\
= \frac{\dot{r}_m}{C_{\lambda}} \left( \omega_i - \dot{\bar{\Omega}} \right) S_{\lambda}^T X_B
\]

(18)

Figure 3 also shows the contact conditions of the ball–nut and ball–screw interfaces. In the figure, \( \omega_e \) and \( \omega_i \) denote the angular velocities of the nut and screw relative to a moving and rotating ball, respectively. They point to the direction parallel to \( b' \) (Figure 2), which is parallel to the \( b \)-axis. According to the fourth assumption, a pure rolling point exists in the contact area between the ball and the nut, and this rolling point is approximate to center point \( A \) at the center of the contact ellipse. At this point of pure rolling, the linear velocities of the ball and the nut must be equal, as shown in Figure 3, and the equivalence is given by

\[
\omega_e r_m + \omega_e r'_e \cos \alpha_e = \left( \omega_b \cos \alpha_e - \omega_n \sin \alpha_e \right) r'_e
\]

(19)

where \( r'_e \) represents the distance from the pure rolling point to the center of the ball in the ball–nut contact area.

Similarly, a point of pure rolling is assumed in the ball screw elliptical contact area, and this point is approximate to point \( B \) at the center of the contact ellipse. Equating the linear velocities of the ball and the screw at this pure rolling point is given by

\[
\omega_i r_m - \omega_i r'_i \cos \alpha_i = \left( \omega_n \sin \alpha_i - \omega_b \cos \alpha_i \right) r'_i
\]

(20)

where \( r'_i \) represents the distance from the pure rolling point to the center of the ball in the ball–screw contact area.

Substituting equations (11) and (12) into equation (19) gives the rearranged form as

\[
\frac{\omega_R}{\omega_e} = \frac{r_m + r'_e \cos \alpha_e}{r'_e \cos (\beta + \alpha_e)}
\]

(21)

Similarly, substituting equations (11) and (12) into equation (20) gives the rearranged form as

\[
\frac{\omega_R}{\omega_i} = \frac{r_m - r'_i \cos \alpha_i}{r'_i \cos (\beta + \alpha_i)}
\]

(22)
In practical applications, the nut moves in the axial direction as it rotates; therefore, the relationship between the angular velocity of the nut or screw and the angular velocity of the ball can be written as

\[ \omega_e = (\omega_u - \omega_m) \cos \lambda \]  

(23)

\[ \omega_i = (\omega_s - \omega_m) \cos \lambda \]  

(24)

where \( \omega_u, \omega_s, \) and \( \omega_m \) denote the nut, screw, and ball’s revolution round the screw axis, respectively.

Similarly, substituting equations (21) and (22) into equations (23) and (24) obtains the screw’s relative angular velocity with respect to the ball, as follows

\[ \omega_i = \frac{(\omega_s - \omega_u) \cos \lambda}{1 + \frac{r_e'(r_m + r_e' \cos \alpha_i) \cos(\beta + \alpha_e)}{r_e'(r_m - r_e' \cos \alpha_i) \cos(\beta + \alpha_e)}} \]  

(25)

The nut’s relative angular velocity with respect to the ball is given by

\[ \omega_e = -\frac{(\omega_s - \omega_u) \cos \lambda}{1 + \frac{r_e'(r_m + r_e' \cos \alpha_i) \cos(\beta + \alpha_e)}{r_e'(r_m - r_e' \cos \alpha_i) \cos(\beta + \alpha_e)}} \]  

(26)

The ball’s revolution around the screw axis is given as

\[ \omega_m = \frac{\omega_s}{1 + \frac{r_e'(r_m + r_e' \cos \alpha_i) \cos(\beta + \alpha_e)}{r_e'(r_m - r_e' \cos \alpha_i) \cos(\beta + \alpha_e)}} + \frac{\omega_u}{1 + \frac{r_e'(r_m + r_e' \cos \alpha_i) \cos(\beta + \alpha_e)}{r_e'(r_m - r_e' \cos \alpha_i) \cos(\beta + \alpha_e)}} \]  

(27)

The ball’s spinning velocity is obtained as

\[ \omega_R = -\frac{(\omega_s - \omega_u)(r_m - r_e' \cos \alpha_i)(r_m + r_e' \cos \alpha_e) \cos \lambda}{r_e'(r_m + r_e' \cos \alpha_e) \cos(\beta + \alpha_i) + r_e'(r_m - r_e' \cos \alpha_i) \cos(\beta + \alpha_e)} \]  

(28)

The elastic deformation of the contact point is significantly smaller than the ball radius, and the pure rolling point is significantly close to the center of the contact ellipse; therefore, \( r_e' \approx r_e' \approx r_b \). The velocities of the nut and screw are both low, and the velocity difference between the two is minimal; thus, the centrifugal force and the gyroscopic moment can be ignored. Under the influence of the curvature, the contact stress between the ball and screw is larger than that between the ball and nut. According to Jones’ theory,\(^{10}\) rolling control can be regarded as the control of the inner ring (i.e. the screw). The gyroscopic angle can be expressed as

\[ \tan \beta = \frac{\sin \alpha_i}{\cos \alpha_i \frac{r_h}{r_m}} \]  

(29)
Analysis of the deformation geometric relationship

Figure 5 shows the deformation geometric relationship. In the figure, $o_e$ is the nut raceway’s groove curvature center, which is assumed to be fixed in the position; $o'_1$ and $o'_2$ are the positions of the ball center before and after load is applied, respectively; $o_{i1}$ and $o_{i2}$ are the groove curvature centers of the screw raceway before and after load is applied, respectively. The distance between the two centers, $o_e$ and $o_{i1}$, before applying the load is given by

$$\overline{BD} = D_b (f_e + f_i - 1)$$  \hspace{1cm} (30)

where $D_b$ is the diameter of the ball. $f_e$ and $f_i$ are the radius coefficients of curvature of the rotary nut and ball screw, respectively, $f_e = r_e / D_b$ , $f_i = r_i / D_b$ . $r_e$ and $r_i$ are the radii of groove curvature of the rotary nut and ball screw, respectively.

Geometric analyses for Figure 5 give

$$A_1 = \overline{BD} \sin \alpha_e + \delta_e + \theta'R'_i \hspace{1cm} (31)$$

$$A_2 = \overline{BD} \cos \alpha_e + \delta_r \hspace{1cm} (32)$$

$$\cos \alpha_e = \frac{X_2}{(f_e - 0.5)D_b + \delta_e} \hspace{1cm} (33)$$

$$\sin \alpha_e = \frac{X_1}{(f_e - 0.5)D_b + \delta_e} \hspace{1cm} (34)$$
\begin{align*}
\cos \alpha_i &= \frac{A_2 - X_2}{(f_i - 0.5)D_b + \delta_i} 
\sin \alpha_i &= \frac{A_1 - X_1}{(f_i - 0.5)D_b + \delta_i} 
R'_i &= r_m + (f_i - 0.5)D_b \cos \alpha^o
\end{align*}

where \( \alpha^o \) is the initial contact angle in the static state. \( \delta_a \) and \( \delta_r \) are the contact deformations in the axial and radial directions, respectively. \( \theta' \) denotes the angular displacement after applying load. \( \delta_e \) and \( \delta_i \) are defined to be the total elastic deformations of the ball–nut contact point and the ball–screw contact point, respectively, as Figure 3 shows.

Using the Pythagorean theorem in Figure 5, it can be obtained that

\begin{align*}
(A_1 - X_1)^2 + (A_2 - X_2)^2 &= [(f_i - 0.5)D_b + \delta_i]^2 
X_1^2 + X_2^2 &= [(f_e - 0.5)D_b + \delta_e]^2
\end{align*}

According to the above geometric deformation equations, the contact angles of the ball–nut and the ball–screw, \( \alpha_e \) and \( \alpha_i \), can be obtained.

**Analysis of the force balance**

Figure 6 shows the stress analysis on the nut at the ball–nut contact point parallel to the \( Z' \)-axis.

The balance equation for the stress in the nut can be written as

\[ F_{ap} + (F_{Xe} \cos \alpha_e - Q_e \sin \alpha_e) \cos \lambda + F_{Ye} \sin \lambda = 0 \]

where \( F_{ap} \) represents the effect of the axial force on a single ball and \( F_{ap} = F_a / N \), where \( F_a \) is the effect of externally applied axial force on the nut and \( N \) denotes the total number of working balls. Furthermore, \( Q_e \) denotes the effect of the normal force acting on the nut, and \( F_{Xe} \) and \( F_{Ye} \) are the frictional forces that are located in the elliptical ball–nut contact area and are acting on the rotary nut in the \( X_A \)-direction and the \( Y_A \)-direction, respectively.

In the static state, the relation between normal force \( Q_e \) and axial force \( F_{ap} \) is given as

\[ F_{ap} = Q_e \cos \lambda \sin \alpha^o \]

Figure 7 shows that all forces are acting on a ball.

The force balance equation for the ball in the \( t \) direction, which is actually parallel to the \( Y_A \)-axis (or \( Y_B \)-axis), as shown in Figure 4, can be written as

\[ F_{Ye} + F_{Yi} = 0 \]
The force balance in the $n$-direction, as Figure 7 shows, is given as

$$Q_e \cos \alpha_e - Q_i \cos \alpha_i + F_{Xe} \sin \alpha_e + F_{Xi} \sin \alpha_i = 0$$  \hspace{1cm} (43)

The force balance in the $b$-direction is given by

$$Q_e \sin \alpha_e - Q_i \sin \alpha_i - F_{Xe} \cos \alpha_e - F_{Xi} \cos \alpha_i = 0$$  \hspace{1cm} (44)
where $Q_i$ denotes the normal force effecting on a ball by the screw. $F_{X_i}$ and $F_{Y_i}$ are the frictions of the ball–screw area effecting on a ball in the $X_B$-direction and the $Y_B$-direction, respectively.

The hydrodynamic pressure distributions are difficult to obtain by elastohydrodynamic lubrication analysis in the contact areas. Therefore, the Hertz contact stress is used to obtain the approximate solution for the hydrodynamic pressure. Thus, the pressure distribution is

$$P_h(x_h, y_h) = \frac{3Q_h}{2\pi a_h b_h} \sqrt{1 - \left(\frac{x_h}{a_h}\right)^2 - \left(\frac{y_h}{b_h}\right)^2}, (h = i \ or \ e)$$

where $a_h$ and $b_h$ are the long half-axis and the short half-axis of the elliptical contact area, respectively.

They are expressed as

$$a_h = m_{ah} \frac{3}{\sqrt{2\Sigma\rho_h}} \sqrt{\frac{3Q_h}{E_h} \left(\frac{1 - \nu_i^2}{E_i} + \frac{1 - \nu_h^2}{E_h}\right)}$$

$$b_h = m_{bh} \frac{3}{\sqrt{2\Sigma\rho_h}} \sqrt{\frac{3Q_h}{E_h} \left(\frac{1 - \nu_e^2}{E_e} + \frac{1 - \nu_h^2}{E_h}\right)}$$

where $m_{ah}$ and $m_{bh}$ are the coefficients of long half-axis and short half-axis, respectively. $\Sigma\rho_i$ (as $h = i$) represents the curvature sum of the screw and ball. $\Sigma\rho_e$ (as $h = e$) represents the curvature sum of the nut and ball. $\nu_i$, $\nu_e$, and $\nu_h$ denote the Poisson’s ratios of the screw, nut, and ball, respectively. $E_i$, $E_e$, and $E_h$ are the Young’s moduli of the screw, nut, and ball, respectively.

The normal force is applied and accompanied by an elastic deformation arising in the contact area. The relationship is defined as

$$\delta_h = \frac{2K_h}{\pi m_{ah}} \sqrt{\frac{3}{8} \left(\frac{1 - \nu_i^2}{E_i} + \frac{1 - \nu_h^2}{E_h}\right)^2} Q_h^2 \Sigma\rho_h$$

where $2K_h / \pi m_{ah}$ is the coefficient of the elastic modulus.

The curvature sum of the screw and ball is defined as

$$\Sigma\rho_i = \frac{2}{r_h} - \frac{1}{r_i} + \frac{\cos \lambda \cos \alpha_i}{r_m - r_h \cos \alpha_i}$$

The curvature sum of the nut and ball is defined as

$$\Sigma\rho_e = \frac{2}{r_h} - \frac{1}{r_e} - \frac{\cos \lambda \cos \alpha_e}{r_m - r_h \cos \alpha_e}$$
From the principal curvature function, the values of coefficients \( m_{ah} \), \( m_{ah} \), and \( 2K_b/\pi m_{ah} \) can be obtained.

**Calculation of the frictional force**

The frictional force in the ball screw mechanism is used to resist torsional moment, which can reduce mechanical efficiency, produce thermal deformation, and consequently degrade positioning accuracy. The kinematic characteristics of the ball screw pair were studied by Wei and Lin and Wei et al., who calculated the friction between the ball and the roller using the Coulomb friction model. To obtain the exact value of friction, Olaru et al. proposed an improved Coulomb friction model. In general, the demands of engineering practice in the low-precision level can be met using the Coulomb friction model. However, this model has some limitations under high-precision and low-velocity conditions; thus, the Coulomb friction model is not a good predictor of the value of friction. By contrast, the LuGre friction model, which was proposed by Canudas de Wit et al. to solve the friction on a table, can accurately describe the static and kinematic characteristics of friction, including pre-slip displacement, friction hysteresis, variable static friction, crawling, and Stribeck effect.

The LuGre friction model visualizes contact surfaces as two rigid bodies that establish contact through bristles, as shown in Figure 8. The stiffness of the lower surface material is greater than that of the upper surface material. When an external force is exerted, the bristles deform like springs, thereby generating the frictional force. State quantity \( z \) represents the average amount of deformation of the contact surface of the mane, and the friction is given by

\[
F_f = \sigma_0 z + \sigma_1 \ddot{z} + \sigma_2 v_r 
\]

\[
\dot{z} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z
\]

\[
g(v_r) = F_c + (F_s - F_c) \exp \left[-\left(\frac{v_r}{v_s}\right)^2\right]
\]

where \( v_r \) is the relative velocity between ball and raceway, \( \sigma_0 \) is the rigidity coefficient, \( \sigma_1 \) is the damping coefficient, \( \sigma_2 \) is the viscous friction coefficient, \( F_c \) is the Coulomb frictional force, \( F_s \) is the maximum static frictional force, \( v_s \) is the Stribeck velocity, and \( g(v_r) \) is the greater than zero and bounded function.

When the system is in steady state, then \( \dot{z} = 0 \), and the relationship of the frictional force and the relative velocity can be obtained as presented from equations (51)–(53)

\[
F_{fss} = \left\{F_c + (F_s - F_c) \exp \left[-\left(\frac{v_r}{v_s}\right)^2\right]\right\} \text{sgn}(v_r) + \sigma_2 v_r
\]
The friction simulation model between the guideway and the table is shown in Figure 9, and the model is completed through Simulink.

The Coulomb frictions of ball–nut contact area in $X_A$-direction and $Y_A$-direction are written respectively as

$$F_{Cxe} = \mu_e Q_e \cos \psi_e$$

$$F_{Cye} = \mu_e Q_e \sin \psi_e$$

where $\mu_e$ represents the friction coefficient produced at contact area of ball–nut. $\psi_e$ is the slip angle of ball–nut at the contact area, whose expression is given by

$$\psi_e = \arctan\left(\frac{V_{YA}}{V_{XA}}\right) + \pi$$

where $V_{XA}$ and $V_{YA}$, obtained from equation (17), are the components of the ball’s sliding velocity relative to the nut in the $X_A$- and $Y_A$-directions, respectively.

The Coulomb frictions of ball–nut contact area in $X_B$- and $Y_B$-directions are written respectively as

$$F_{Cxi} = \mu_i Q_i \cos \psi_i$$

$$F_{Cyi} = \mu_i Q_i \sin \psi_i$$

where $\mu_i$ denotes the friction coefficient produced at contact area of ball–screw. $\psi_i$ is the slip angle at the contact area of ball–screw, whose expression is given by

$$\psi_i = \arctan\left(\frac{V_{YB}}{V_{XB}}\right) + \pi$$
where \( V_{XB} \) and \( V_{YB} \), which can be obtained from equation (18), are the components of the ball’s sliding velocity relative to the screw in the \( X_B \)- and \( Y_B \)-directions, respectively.

**Calculation of the mechanical transmission efficiency**

The mechanical transmission efficiency of the dual-driven ball screw mechanism is equal to the ratio of output power to input power. Taking a single ball as research object, the output power can be written as

\[
W_{\text{output}} = F_ap \cdot V_{\text{axial}}
\]  

(61)

where \( V_{\text{axial}} \) is the linear motion velocity of the nut in the \( Z' \)-axis direction, \( V_{\text{axial}} = ((\omega_s - \omega_u) L) / 2\pi \).

The input power of the dual-driven ball screw mechanism is equal to the sum of the input power of the screw motor and the input power of the nut motor. With a single ball as the research object, the expression of the input power of the screw motor is written as

\[
W_{\text{input}1} = \left( -Q_m r_m \sin \alpha_s \sin \lambda - F_{Yi} r_m \cos \alpha_s \sin \lambda + F_{Yi} r_h \sin \lambda - F_{Yi} r_h \cos \alpha_s \cos \lambda \right) \omega_s
\]  

(62)

Similarly, the expression of the input power of the nut motor is written as

\[
W_{\text{input}2} = \left( Q_e r_m \sin \alpha_e \sin \lambda - F_{Xe} r_m \cos \alpha_e \sin \lambda + F_{Xe} r_h \sin \lambda + F_{Xe} r_h \cos \alpha_e \cos \lambda \right) \omega_u
\]  

(63)

The total input power associated with frictions generated at the contact areas of ball–nut and ball–screw is given by

\[
W_{\text{input}} = W_{\text{input}1} + W_{\text{input}2}
\]  

(64)
The mechanical transmission efficiency of the dual-driven ball screw mechanism is defined as

$$
\eta = \frac{W_{\text{output}}}{W_{\text{input}}} 
$$

(65)

**Simulation and analysis**

In this study, the parameters of the geometric and operating conditions of the dual-driven ball screw mechanism are shown in Table 1. The dual-driven ball screw mechanism realizes micro-feeding; therefore, the velocities of the screw and nut are low, and the difference between these velocities is minimal.

**Flowchart of numerical analysis**

Figure 10 shows the flowchart illustrating how all the parameters related to the kinematic behavior of the dual-driven ball screw mechanism are obtained sequentially by numerical analyses from the satisfaction of the convergence criterion. Given the parameters of the geometric and operating conditions of the dual-driven ball screw mechanism in Table 1, the study assumes an initial value for the angular displacement ($\theta'$), the elastic deformation ($\delta_a$ and $\delta_r$), and the gyroscopic angle ($\beta$) before initiating the
loop of numerical iterations. The numerical solution for each parameter can be obtained when the convergence criterion is satisfied.

Analysis of the velocity

Figure 11(a) shows the rotational velocity of a single-driven ball screw mechanism when the screw speed is 1 rad/s and the nut does not rotate. Figure 11(b) shows the rotational
velocity of the synthesized dual-driven mechanism, whose screw and nut angular velocities are 11 and 10 rad/s, respectively.

The foremost characteristic of the dual-driven micro-feed mechanism is its capability to demonstrate a stable velocity under a minimal speed difference, without being influenced by the nonlinear friction. Yu and Feng\textsuperscript{22} studied the micro-feed feature of a dual-motor servo system by considering the impact of friction. This study focuses on the low-speed characteristics of the dual-driven ball screw mechanism, and it finds that the speed of the screw demonstrates a noticeable crawling phenomenon when the speed of the screw motor is 1 rad/s under the single-driven condition, according to the repeated simulated system parameters. However, the composite speed of the screw and nut is very stable when this speed is equal to 1 rad/s under the dual-driven condition. As shown in Figure 11, the dual-driven mechanism can output a lower stable velocity and a better low-speed micro-feeding performance than the single-driven mechanism.

**Analysis of the frictional force**

Figure 12(a) and (b) show that the frictional forces at the nut–ball and screw–ball contact points change with a change in the screw rotational velocity under the single-driven condition. Figure 13(a) and (b) display that the frictional forces change at the nut–ball and screw–ball contact points with a change in the composite rate under the dual-driven condition and when the axial force is 500 N.

As shown in Figures 12 and 13, the frictional forces at the two contact points change continuously in the form of negative damping when the screw rotational velocity is lower than 4 rad/s under the single-driven condition; this phenomenon is known as the Stribeck effect. By contrast, the frictional forces at the two contact points increase linearly and slowly when the nut rotational velocity is higher than 2 rad/s and the rotational velocity difference is greater than 0 rad/s ($\omega_s - \omega_n > 0$) under the dual-driven condition. When
the two high-velocity “macro-motions” under the critical speed induce crawling and “micro action” is induced by the rigid drive of the same “rotation-line” rolling screw transmission, the influence of the low-speed nonlinear creeping phenomenon caused by the frictional force of traditional electromechanical servo systems can be avoided in the dual-driven mechanism.
Analysis of the angular velocity for the revolution of the ball

Two angular velocities are produced in a ball when the ball screw mechanism operates under single-driven or dual-driven conditions. These two angular velocities are expressed in absolute values. Figure 14(a) and (b) show the change in the angular velocity for the revolution of the ball when single-driven and dual-driven, respectively.

As shown in Figure 14(a) and (b), the angular velocity for the revolution of the ball increases linearly as the angular velocities for the rotations of the screw and nut increase but does not change with a change in axial force. In addition, the angular velocity for the revolution of the ball is higher under the dual-driven condition than that under the
single-driven condition. Therefore, the dual-driven mechanism has a higher speed and responds more quickly at the same feed rate.

**Analysis of the mechanical transmission efficiency**

The changes in the mechanical transmission efficiency of the dual-driven mechanism as the speed differences between the screw and nut change are shown in Figure 15. When the nut rotational velocity is constant, the mechanical transmission efficiency initially increases and subsequently decreases as the rotational velocities of the screw and nut increase. The linear growth rate of the output power is greater than that of the input power at a low speed; as a result, the mechanical transmission efficiency presents an increasing trend at a low speed. When the speed difference continues to increase, the proportion of consumed power resulting from increased friction increases; thus, when the speed difference is greater than a certain value, the mechanical transmission efficiency is reduced as the speed difference increases.

As shown in Figure 15, when the nut rotational velocity is equal to zero, the mechanical transmission efficiency of the single-driven mechanism is higher than that of the dual-driven mechanism. The dual-driven micro-feed mechanism obtains high accuracy by sacrificing mechanical transmission efficiency; thus, high micro-feed accuracy and transmission efficiency are contradictory.

**Conclusion**

1. In this study, a new dual-driven ball screw micro-feed mechanism is designed. By studying the working ball in the helical raceway and using the principle of
differential geometry to analyze the motion of the ball under the dual-driven condition, we find that the stable output velocity under the dual-driven condition is lower than that under the single-driven condition; therefore, the dual-driven mechanism is more suitable for micro-machining.

2. Comparing the frictional forces at the ball and raceway contact points under the dual-driven condition and under the single-driven condition, we find that the friction changes continuously and demonstrates negative damping, that is, the Stribeck effect, under the single-driven condition. However, the friction at the two contact points increases linearly and slowly as the speed difference increases in the dual-driven mechanism; consequently, the influence of the low-speed non-linear crawling phenomenon caused by friction in traditional electromechanical servo systems can be avoided.

3. The revolution velocity of the ball increases linearly as the angular velocities of the screw and nut increase, and the angular velocity under the dual-driven condition is higher than that under the single-driven condition. Therefore, the dual-driven mechanism has a higher speed and responds more quickly at the same feed rate.

4. When the nut angular velocity is constant, the mechanical transmission efficiency initially increases and subsequently decreases as the angular velocities of the screw and nut increase. The mechanical transmission efficiency of the single-driven mechanism is higher than that of the dual-driven mechanism. However, the dual-driven micro-feed mechanism obtains high accuracy by sacrificing mechanical transmission efficiency, indicating that high micro-feed accuracy and transmission efficiency are contradictory.

Authors’ note

The authors would like to declare that the work described was original research that has not been published previously and is not under consideration for publication elsewhere, in whole or in part.

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Appendix I

Notation

\( a_h \)  
half-length of the major axis of the elliptical contact area

\( A \)  
balk–nut contact point

\( A_1 \)  
horizontal distance between \( o_e \) and \( o_{i2} \)

\( A_2 \)  
vertical distance between \( o_e \) and \( o_{i2} \)

\( b' \)  
axis passes through origin \( O \) and forms an angle \( \lambda \) from \( z \) axis

\( b_h \)  
half-length of the minor axis of the elliptical contact area

\( B \)  
balk–screw contact point

\( BD \)  
distance between \( o_e \) and \( o_{i1} \), before applying the load

\( D_b \)  
diameter of the ball

\( E_b \)  
Young’s modulus of the ball

\( E_h \)  
Young’s modulus of the screw and nut

\( f_e \)  
dimensionless radius of curvature of nut

\( f_i \)  
dimensionless radius of curvature of screw

\( F_a \)  
externally applied load acting on the nut

\( F_{ap} \)  
axial force applied to one ball

\( F_{cxe} \)  
Coulomb friction of ball–nut contact area in \( X_A \)-direction

\( F_{cxl} \)  
Coulomb friction of ball–nut contact area in \( X_B \)-direction

\( F_{cye} \)  
Coulomb friction of ball–nut contact area in \( Y_A \)-direction

\( F_{cyi} \)  
Coulomb friction of ball–nut contact area in \( Y_B \)-direction

\( F_s \)  
maximum static friction
\( F_{xe} \)摩擦力在球-螺母接触区域的\( X_A \)-方向

\( F_{xi} \)摩擦力在球-螺杆接触区域的\( X_B \)-方向

\( F_{ye} \)摩擦力在球-螺母接触区域的\( Y_A \)-方向

\( F_{yi} \)摩擦力在球-螺杆接触区域的\( Y_B \)-方向

\( g(v_r) \)大于零且有界函数

\( K_h \)弹性模量的系数

\( L \)螺杆的螺距

\( m_{sh} \)主要轴长度系数

\( m_{bh} \)次要轴长度系数

\( N \)球体数量

\( o_1' \)没有施加载荷时球心位置

\( o_2' \)施加载荷时球心位置

\( o_e \)螺母螺纹槽曲率中心

\( o_{i1} \)没有施加载荷时螺纹槽曲率中心

\( o_{i2} \)施加载荷时螺纹槽曲率中心

\( P_h(x_h,y_h) \)压力分布

\( Q_e \)球和螺母之间的正压力

\( Q_i \)球和螺杆之间的正压力

\( r_b \)球半径

\( r_e \)螺母螺纹槽曲率半径

\( r'_e \)纯滚动点到球心的距离

\( r_i \)螺纹槽曲率半径

\( r'_i \)纯滚动点到球心的距离

\( r_m \)螺杆平均半径

\( R'_i \)\( z' \)轴和\( o_{i1} \)轴之间的距离

\( \hat{s}R \)球心的位置向量

\( \hat{s}
\hat{R} \)球心向量与螺杆旋转坐标系
\( W_R \) position vector of the ball center with respect to the world coordinate system

\( W_{\dot{R}} \) velocity of the ball center with respect to the world coordinate system

\( t, n, b \) coordinate direction distance of the Frenet coordinates

\( v_s \) Stibbeck velocity

\( V_{Ab} \) instantaneous velocity of point \( A \) on the ball

\( V_{axial} \) linear velocity in the \( Z \)-axis direction of the nut

\( V_{An} \) instantaneous velocity of point \( A \) on the nut

\( V_{Bb} \) instantaneous velocity of point \( B \) on the ball

\( V_{BS} \) instantaneous velocity of point \( B \) on the screw

\( V_{Sb} \) slip velocity at point \( A \) between the nut and ball

\( V_{SB} \) slip velocity at point \( B \) between the screw and ball

\( V_{XA} \) component of the ball’s sliding velocity relative to the nut in the \( X_A \)-direction

\( V_{Xb} \) component of the ball’s sliding velocity relative to the screw in \( X_B \)-direction

\( V_{YA} \) component of the ball’s sliding velocity relative to the nut in the \( Y_A \)-direction

\( V_{Yb} \) component of the ball’s sliding velocity relative to the screw in \( Y_B \)-direction

\( W_{input1} \) input power of screw motor

\( W_{input2} \) input power of nut motor

\( W_{input} \) total input power related to the frictions

\( W_{output} \) output power

\( x', y', z' \) three coordinate components of the \( X' \) coordinate system

\( x_n, y_n, z_n \) three coordinate components of the \( X_n \) coordinate system

\( x_s, y_s, z_s \) three coordinate components of the \( X_s \) coordinate system

\( X' \) first (world) coordinate system

\( X_1 \) horizontal distance between \( o_e \) and \( o'_2 \)

\( X_2 \) vertical distance between \( o_e \) and \( o'_2 \)

\( X_j, Y_j, Z_j \) three coordinates direction of contact point coordinate system

\( X_n \) nut rotating coordinate system
$X_s$  screw rotating coordinate system  
$Y$  Frenet coordinate system  

$\alpha^\circ$  initial contact angle in the static state  
$\alpha_e$  contact angle formed at the ball–nut contact  
$\alpha_i$  contact angle formed at the ball–screw contact  
$\alpha_j$  contact angle at the contact points $A$ and $B$  

$\beta, \beta'$  two gyroscopic angles; the angle between $\omega_R$ and $t-b$ plane and the angle between the projection of $\omega_R$ in the $t-b$ plane and the $b$-axis  

$\delta_a$  contact deformation in the axial direction  
$\delta_e$  total elastic deformations created at the contact point of the nut  
$\delta_i$  total elastic deformations created at the contact point of the screw  
$\delta_r$  contact deformation in the radial direction  
$\eta$  mechanical efficiency  

$\theta$  actual angular displacement of the dual-driven ball along the helix groove of screw  
$\dot{\theta}$  actual angular velocity of the dual-driven ball revolution along screw  
$\theta'$  angular displacement after applying load  
$\lambda$  helix angle of the screw  

$\mu_e$  friction coefficient produced at ball–nut contact area  
$\mu_i$  friction coefficient produced at ball–screw contact area  
$\nu_b$  Poisson’s ratio of the ball  
$\nu_h$  Poisson’s ratio of the screw and nut  

$\sigma_0$  rigidity coefficient  
$\sigma_1$  damping coefficient  

$\sigma_2$  viscous friction coefficient  
$\Sigma \rho_h$  curvature sum of the ball–nut and ball–screw  

$\psi_e$  slip angle at the ball–nut contact area  
$\psi_i$  slip angle at the ball–screw contact area  

$\omega_e$  angular velocity of the nut relative to a moving and rotating ball  
$\omega_i$  angular velocity of the screw relative to a moving and rotating ball
\( \omega_m \) revolution angular velocity of the ball
\( \omega_R \) spinning angular velocity of the ball
\( \omega_s \) screw’s revolution with respect to the screw axis
\( \omega_t, \omega_b, \omega_b \) components of spinning angular velocity in \( t-, n-, \) and \( b- \) directions, respectively
\( \omega_u \) nut’s revolution with respect to the screw axis
\( \Omega_n \) nut’s angular displacement respect to the world coordinate
\( \dot{\Omega}_n \) angular velocity of the rotating nut
\( \Omega_s \) screw’s angular displacement respect to the world coordinate
\( \dot{\Omega}_s \) angular velocity of the rotating screw