Quantum Routing for Emerging Quantum Networks

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Abstract

Quantum routing, the entanglement of an input quantum signal over multiple output paths, will be an important aspect of future quantum networks. Implementation of such routing in near-term quantum networks via the noisy quantum devices currently under development is a distinct possibility. Quantum error correction, suitable for the arbitrary noisy quantum channels experienced in the routing process, will be required. In this work, we design a combined circuit for quantum routing and quantum error correction, and carry out an implementation of such a circuit on a noisy real-world quantum device. Under the assumption of statistical knowledge on the channel, we experimentally verify the quantum nature of the error-corrected quantum routing by determining the path-entanglement through quantum state tomography, measuring also its probability of success. The quantum error correction deployed is identified as successful in terms of improving the routing. Our experiments validate, for the first time, that error-corrected quantum routing in near-term quantum networks is feasible, provided enough statistical information on the noise intrinsic to the routing device is a priori known.

Introduction

A quantum router,1 an important element of emerging quantum networks, can transmit a quantum signal from a singular input path to a coherent superposition of multiple output paths [2], [3], [4], [5], [6]. This key entanglement feature of the router offers remarkable opportunities relative to classical routers, such as increased throughput over noisy channels, and security enhancements enabled by the distributed nature of the quantum information within the network [4]. Beyond its unique routing functionality, a quantum router also provides the only known technique that enables Quantum Random Access Memory (QRAM). This non-classical form of memory access has important implications for quantum networks for two main reasons: one being its ability to form memory-calls in superposition of addresses, the second being its ability to remotely create a superposition of states (e.g., a qubit) drawn from a distributed classical memory [7]. Although not discussed here, quantum routing also has implications for quantum machine learning [8].

In near-term quantum networks, quantum routers can be deployed using Noisy Intermediate-Scale Quantum (NISQ) devices. In principle, such noisy devices can create, receive, transmit, and route qubits over quantum channels. They can be manufactured with various qubit types: e.g., superconducting, trapped ions, photonic, or silicon-based qubits. Of particular interest to the wider community are the superconducting quantum devices developed by IBM. Presently, these NISQ devices can be considered as first-generation quantum computers-made available to the wider community through a cloud platform called the IBM Quantum Experience (IBM Q) [9]. This availability allows us to test directly the performance of quantum routing on real devices of the type that may be deployed in a future network. Although quantum routers are mainly shown to be implemented using linear optics, these optical quantum routers are probabilistic. The success probability of optical quantum routers is significantly less than unity [2], [3], [4], [5]. Deterministic quantum routers have been demonstrated on a superconducting quantum device via the IBM Q [6]. Therefore, the first near-term quantum communication network is likely to be constructed with a superconducting quantum router deployed as a node, by adopting one of the many techniques available for the conversion between photons and superconducting qubits. For the purposes of this article we will simply assume such qubit inter-conversion facilities are made available in a future network setting.2

However, noisy quantum channels and the intrinsic noise of the NISQ device introduce unwanted errors, which can lead to poor quantum-routing performance. A plethora of quantum error correction protocols have been proposed to eliminate the errors caused by noisy quantum channels, e.g., [10], [11]. One of our previous works embedded a general error-correcting code (i.e., correcting an arbitrary one-qubit error) into a quantum router realized on a NISQ device [12]; the experimental results of which confirm that general error correction methods are not feasible on current NISQ devices. In this work, we develop and experimentally test a quantum router with a non-general error correction protocol - a protocol well suited to noise conditions with similarities to the amplitude damping channel.

There are two main aims in this article. (i) First, we wish to point out how error correction in the context of quantum routing may still be fruitful for near-term networks - if enough a priori statistical knowledge on the errors intrinsic to a NISQ device

1 The quantum router in this work indicates a quantum-only phenomenon and is different from the concept of classical-routing decisions for entanglement distribution [1].

2 It is also possible for quantum routing to be implemented directly on native hardware without conversion of any photonic qubits via one of the many photonics-based NISQ devices currently under development.
are known. We investigate channels with characteristics similar to the amplitude damping channel to make this point.\( ^3 \) (ii) Second, we wish to introduce the concept of quantum routing to the generic networking community, discussing how its inclusion in near-term quantum networks may be possible even for more general noise conditions (through error mitigation techniques). With these two aims in mind, we summarize our three main contributions as follows. (i) We design a novel quantum circuit for quantum-error-corrected quantum routing, based on noisy superconducting qubits. (ii) We experimentally execute the quantum circuit on a seven-qubit NISQ device, the ibmq_jakarta, accessed through the IBM Q. (iii) We verify the quantum nature of the quantum router by identifying the generation of the path-entanglement via quantum state tomography. We also discuss possible applications of path-entanglement for networks and the use of enhanced error mitigation techniques in improving quantum routing.

The rest of this paper is organized as follows. In Section II, we introduce possible near-term quantum communication networks, as well as the basic principles of quantum routing and QRAM. Section III discusses the experimental setup, the quantum circuits needed, and the results. Section IV concludes the paper.

**FUNDAMENTALS**

**Quantum Routing in Future Networks**

An overview of a large-scale quantum communication network is depicted in Fig. 1. A signal qubit is transmitted from a sender to the two end users simultaneously via quantum routers. Only one end user will receive the signal qubit upon collapse of the superposition of the paths. Although not discussed in any detail here, Fig. 1 also includes an important component of any future quantum network - a quantum repeater. Due to the short coherence time of qubits, and the loss/noise inherent in most quantum channels, quantum repeaters will be needed to realize long-distance transmission of quantum signals. We anticipate quantum routers and quantum repeaters to co-exist in emerging quantum networks, complementing and enhancing each other’s functionality (but one does not require the other). A simple model of the quantum router is depicted in the bottom of Fig. 1. Here, the control qubit \( |\phi\rangle_c \) is a superposition state and contains the control information that directs the path of a signal qubit \( |\phi\rangle_s \) which carries the signal information and is received by the quantum router via path 1. The quantum router requires an ancillary qubit \( |\phi\rangle_n \), which is initially in the \( |0\rangle \) state at path 2 and contains no signal information. The output of the quantum router \( |\Phi\rangle_f \) is an entanglement between the control qubit and the two paths. The signal qubit is routed to the path 1 and 2 when \( |\phi\rangle_c \) is in the \( |0\rangle_c \) and \( |1\rangle_c \) state, respectively. When \( |\phi\rangle_c \) is a superposition state, the two paths both “possess” the signal qubit.

Fig. 2 outlines how quantum routing functionality is used within QRAM. As mentioned already, QRAM is the key technique for reading and writing quantum data through quantum memories and is one of the important network applications.

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3 We do not claim such channels are representative of any current NISQ device - we simply wish to show how some knowledge on a quantum channel can sometimes greatly assist the error correction.
of the quantum router. The QRAM could be situated in any router-node within the network, or even within a quantum-repeater node. Compared to a classical RAM, QRAM has the ability to store quantum data and read multiple data simultaneously. Fig. 2a illustrates the initial situation of a QRAM which has 8 memory cells from $D_{000}$ to $D_{111}$. Each memory cell can store either a classical bit or a qubit. The QRAM shown has 7 ancillary qubits that are initialized in the $\ket{0}$ state and invokes a binary decision tree with 3 levels. The tree receives the input signals, which is a quantum state containing a superposition of memory addresses. In principle, all 8 memory cells shown can be accessed simultaneously, but to provide a detailed example, we assume that only $D_{010}$ and $D_{100}$ would be queried. Fig. 2b provides the working process of querying the two data stored in these two memory cells. The address state in this example is a superposition of $\ket{010}$ and $\ket{100}$ (it is by this means the address is embedded in a state). The root node makes a binary decision based on the received qubits: the left (right) child (next-level node) will be chosen if the first qubit in the received state is read as zero (one). This procedure is repeated at each level (reading the appropriate qubit) until the memory cells referenced $\ket{010}$ and $\ket{100}$ are accessed. At this point, the data stored in $D_{010}$ and $D_{100}$ will be transferred to the output qubit (which was initialized in the $\ket{0}$ state). The output qubit will then become a superposition of the two retrieved data and will be entangled with the address state. Note that, with the increasing number of memory cells, the number of qubits in the address state grows, but it remains the case that only one output qubit is required for the QRAM.

**Noisy Quantum Channel**

While the quantum routing process outlined above assumes zero channel noise, we wish to consider in this work the more realistic situation where noise channels are present. That is, we assume the states $\ket{\phi}_a$ and $\ket{\phi}_b$ are prepared at some sender, and then passed through noisy quantum channels. We build a parameterized noisy quantum channel that has similar characteristics to the amplitude damping channel. The details of the noisy channel are not important in this work, we simply require that we have an effective method within the NISQ device to add arbitrary noise to the qubits, and that the level of that noise can be parameterized with a single parameter.

The qubits that transmitted through the noisy quantum channel can be regarded as an open system that interacts with the environment during the transmission. We add an auxiliary qubit to simulate the environment, which starts from a pure state $\ket{0}_E$. The amplitude damping channel $U$ models energy relaxation from an excited state to the ground state, and the strength of the noise in $U$ can be expressed by $\gamma$. There is no noise when $\gamma$ equals to 0, and with the increase of $\gamma$, the strength of the noise in $U$ grows. The detailed representation of $U$ is given in Fig. 3. To realize the amplitude damping channel on a NISQ device, we would apply $U$ to $\ket{\phi}_a$ and $\ket{0}_E$ and then mathematically trace out the auxiliary qubit that simulates the environment. However, in our experiments, we make a $Z$-basis measurement on the auxiliary qubit after the application of $U$ and only keep the resulting state if the measurement outcome is $\ket{0}$. This post-selection process delivers our required parameterized noisy channel and has a success probability of $p_1$ [10].

To run our experiments (see Section III), we first must “imprint” the noisy quantum channel onto the device, since in the current devices no such channel exists. This imprinted channel mimics noise that would be added to the qubits as they traversed a wider network before entering the router. After imprinting the noise, we “forget” we know the exact channel parameter, $\gamma$, when we attempt the error correction. This is a means to model the realistic situation where we do not know exactly the noise in the channel, only some general characteristics, but must still attempt some form of error correction. In this work, we imprint the noisy channel by $U$, which requires us to set a specific value of $\gamma$ as shown in Fig. 3. We assume the noisy quantum channel to possess a $\gamma$ in the range $0 - 1$, with a uniform distribution. At the error correction phase, we assume only knowledge of the distribution of $\gamma$, not its specific value in any realization. We set our estimate of $\gamma$ which we refer to as $\hat{\gamma}_E = 0.5$, by setting it at the mean of the distribution. Although some real-world channels could be approximated by this process, we do not claim we have truly modeled a real-world channel. We use our channel scheme to simply illustrate that when statistical information on a channel is available, quantum error correction on quantum routing within a NISQ device becomes possible. Other, more complicated, channels will likely exist in the wide range of NISQ devices.
now being produced via multiple technology implementations. While we expect similar outcomes to those reported here for some of these other channels, we should be clear that the explicit results we show are specific to the statistical noise model we have assumed.

**Error Correction Protocol**

The correction protocol [10] we adopt is not designed for an arbitrary error on a qubit, but one which develops for a specific noise model. Our adopted scheme applies to scenarios where some access to the entangled environment may be available [13] or where a weak measurement is done to detect leakage from the system [10], and the loss rate can be estimated.

We first apply a Hadamard gate $H$ with a parameter $\theta$ to $|0\rangle_a$, which is an ancillary qubit. The representation of $H_\theta$ can be found in Fig. 3. Then, a CX gate (the two-qubit gate in blue in Fig. 3) is performed on the signal qubit and the ancillary qubit, which means that the ancillary qubit would be flipped and unchanged if the signal qubit is in the $|1\rangle$ state and the $|0\rangle$ state, respectively. After that, a controlled-swap gate (the three-qubit gate in purple in Fig. 3) is implemented for realizing the quantum routing process. After the above process, the resulting state of the signal qubit and the ancillary qubit is a superposition of $|0\rangle_s|0\rangle_a$, $|0\rangle_s|1\rangle_a$, $|1\rangle_s|0\rangle_a$, and $|1\rangle_s|1\rangle_a$ states. The last step of the error correction requires a post-selection method applied to the ancillary qubit. This method involves the retention of the post-selected state only when the $Z$-basis measurement result of the ancillary qubit is $|0\rangle$. After the post-selection, the contaminated signal qubit could be recovered to the $|\phi\rangle_s$ state with the success probability $p_2$ if we set $\theta = \arctan(1/\sqrt{1 - \gamma})$ [10]. As we assume $\gamma$ is unknown, we utilize $\gamma_0$ to setup $\theta$ in our experiments. Note that the process is similar when the noisy quantum channel and the error correction are applied on $|\phi\rangle_c$. Here, we only used $|\phi\rangle_s$ as an example for demonstrating the derivations.

**Experimental Results**

**Experimental Setup**

Our experiments are implemented on the IBM quantum device, the *ibmq_jakarta*, which has seven superconducting qubits in a horizontal H-shaped geometry [9]. The quantum circuit of the router (alongside the noisy quantum channel and the error correction) is shown in Fig. 3. This quantum circuit is built via the Quantum Information Science toolKit (Qiskit) [14] - the open-source software development kit for creating and running quantum circuits on the IBM Q.

The three router qubits, (i.e., counting from the top - the first, fourth, and last qubit in the quantum circuit of Fig. 3), are prepared as $|\phi\rangle_c$, $|\phi\rangle_v$, and $|\phi\rangle_n$ via single qubit gates. The two qubits initialized in the $|0\rangle_E$ state are two auxiliary qubits that simulate the environment, and the two qubits initialized in the $|0\rangle_a$ state stand for the ancillary qubits used in the error correction protocol. As discussed earlier, to realize the noisy channel and the error correction on the quantum device, post-selections are performed on these four qubits, as illustrated in Fig. 3. The classical registers $c_0$ and $c_1$ are used for storing the measurement results in the post-selection and the state tomography, respectively.
The basic premise of any error-mitigation method, is to reduce the likelihood of errors occurring, as opposed to error-correction which aims to correct any errors.

measurement outcomes of the four qubits. In our experiments, we set \( \gamma \in \{0, 0.1, 0.2, \ldots, 0.9\} \) for the noisy channel, and for the error correction \( \theta \) is set based on \( \gamma \), i.e., \( \theta = \arctan(\frac{\gamma}{\sqrt{1 - \gamma^2}}) \).

A controlled-swap gate is performed to realize the quantum routing process, which is the core part of the router. The controlled-swap gate acts on the three router qubits: the positions of \(|\Phi\rangle\) and \(|\phi\rangle\) would be swapped and unchanged when \(|\phi\rangle\) is in the \(|1\rangle\) and \(|0\rangle\) state, respectively. The classical register \( c_1 \) contains the measurement outcomes of quantum state tomography, which is utilized to identify the router output. We choose the fidelity
\[
F = \left( \text{Tr} \sqrt{\sigma \sigma'} \right)^2
\]
as one of our performance metrics. Here, \( \sigma = |\Phi\rangle \langle \Phi| \) is the “theoretical” density matrix of the router output, and \( \sigma' \) is a reconstructed density matrix of the three router qubits determined via the quantum state tomography. This fidelity \( F \) estimates the similarity between \( \sigma \) and \( \sigma' \), and ranges from 0 to 1. When \( \sigma \) and \( \sigma' \) are identical to each other, \( F \) equals 1, and when \( \sigma \) and \( \sigma' \) are orthogonal to each other, \( F \) equals 0.

The whole experimental procedure to determine \( F \) is depicted in Fig. 4. The quantum circuit should first be transpiled to a circuit that only includes basis gates - i.e., gates that can be implemented on the quantum device directly. The quantum state tomography attaches different types of measurements at the end of the transpiled circuit and generates 27 tomography circuits (the 27 measurements \( \{X \otimes X \otimes X \otimes X \otimes X \otimes Y, \ldots, Z \otimes Z \otimes Z \} \) are required for rebuilding the three-qubit state). We then send these tomography circuits with \( \gamma \) and a chosen \( \gamma \) to the quantum device, which executes each tomography circuit 100,000 times. Due to the fact that the quantum device is noisy, we apply a ‘measurement error mitigation’ method provided by Qiskit to reduce errors implicit to the measurement process within the device. These errors are just one type of intrinsic error within the quantum device and are distinct from the errors introduced by the noisy quantum channel (see later discussion). The basic premise of any error-mitigation method, is to reduce the likelihood of errors occurring, as opposed to error-correction which aims to correct any errors.

The measurement error mitigation protocol is applied to three physical qubits of the quantum device, where these physical qubits will be utilized as the three router qubits in the transpiled circuit. The mitigation protocol generates eight calibration circuits, where each circuit prepares one of eight \( Z \)-basis states \( \{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\} \). Then, the \( Z \)-basis measurements are applied to the calibration circuits, and the measurement results are used to build a calibration matrix. The inverse of this matrix is then applied to the experimental outcomes, eliminating the measurement errors in the ideal case. After the application of the measurement error mitigation, we implement the quantum state tomography to reconstruct \( \sigma' \); with the known \( \sigma \) and \( \sigma' \) we then calculate the fidelity \( F \).

RESULTS

Each circle in Fig. 5 represents an outcome averaged over ten repetitions, with an error bar indicating two standard deviations from the mean. Note that we have checked the experiments with more repetitions and found that the results are effectively the same. We see from Fig. 5a that the quantum router with the error correction is feasible when \( \gamma \geq 0.5 \) on the quantum device. Note, a baseline is the case where no noisy channel and no error correction is applied, for which the fidelity of the quantum routing is 0.85 (non-universality as a consequence of intrinsic errors within the device). For current NISQ devices, quantum error correction introduces more qubits and quantum operations, incurring higher intrinsic errors within the device and making error correction unfeasible. However, we note that the error correction improves the performance of the quantum routing within the range of \( \gamma \geq 0.5 \), as the error-corrected results in this range are significantly above the ones without the error correction. We do emphasize, that in this range where the error correction...
partially works, we are not eliminating the noise in the channel, just reducing it. That is, we do not correct fully back to the baseline case. When \( \gamma < 0.5 \), the experimental \( F \) is decreased after the error correction. The reason of this phenomenon is that the noise induced by our noisy quantum channel is smaller than the noise accumulated from the quantum gates. We note that there remains a small probability that this situation observed at low \( \gamma \) is an artifact of experimental noise as evidenced by the error bars shown. The observed overall trend of \( F \) with \( \gamma \) is as expected - the quantum channel introduces more noise for larger \( \gamma \) and the device performs the error correction less efficiently as the noise increases.

Beyond fidelity, we utilize success probability \( P \) of the whole procedure (quantum-error-corrected quantum routing) as another performance metric, as shown (right-hand scale) in Fig. 5a. As the noisy quantum channel and the error correction are implemented on two qubits \(|\phi\rangle_c \) and \(|\phi\rangle_s \), \( P \) should be square of \( p_2 \). Remind that \( p_2 \) is the success probability of the error correction for a noisy qubit derived earlier and the quantum routing process is deterministic. We can observe that higher \( \gamma \) results in lower \( P \) and \( F \), where the experimental \( P \) is consistent with the theory. The success probability, \( P \), decreases with the increase of \( \gamma \) and approaches 0 as \( \gamma \to 1 \), which indicates a tradeoff between \( P \) and the error correction.

Finally, the quantum nature of the router is demonstrated by the entanglement generated at the output. We verify the entanglement by reconstructing its density matrix via state tomography, the results of which are in Fig. 6. The theoretical density matrix, \( \Sigma \), is demonstrated in Fig. 6a and d, and the experimental \( \Sigma' \) with the noisy quantum channel performed on \(|\phi\rangle_s \) only. Similarly, c) and f) depict \( \Sigma' \) with the noisy quantum channel and error correction applied on \(|\phi\rangle_s \).

**FIGURE 5.** Fidelity, \( F \), as a function of \( \gamma \) with \( \gamma = 0.5 \), where the noisy quantum channel and the error correction are a) performed on \(|\phi\rangle_c \) and \(|\phi\rangle_s \) or b) only performed on \(|\phi\rangle_s \). Note, EC represents error correction.

**FIGURE 6.** Theoretical and experimental density matrices of \(|\Phi\rangle_f \), the entanglement generated at the output of the quantum router. a) and d) represent the real and imaginary parts of the theoretical density matrix, \( \Sigma \), respectively. b) and c) together represent \( \Sigma' \) without the error correction after the noisy quantum channel performed on \(|\phi\rangle_s \) only. Similarly, c) and f) depict \( \Sigma' \) with the noisy quantum channel and error correction applied on \(|\phi\rangle_s \). For current NISQ devices, quantum error correction introduces more qubits and quantum operations, incurring higher intrinsic errors within the device and making error correction unfeasible.
quantum channel and error correction implemented on $|\phi\rangle$, only, with $\gamma = 0.6$ and $\gamma_f = 0.5$, is illustrated in Fig. 6c and f. For comparison, we also demonstrated $\sigma'$ without the error correction after the noisy quantum channel performed, with $\gamma = 0.6$, on $|\phi\rangle$ (Fig. 6b and e). From the comparison of these figures, the good performance of the error-corrected quantum routing is verified - the corrected state clearly being closer in its matrix elements to the theoretical density matrix elements. Detailed information on the values of these elements can be seen from the range of values shown, and fidelities between the matrices determined. It can be found that $F$ improves from 0.48 to 0.61 after the error correction.

In closing, it is useful to note that we have experimentally implemented elsewhere other error mitigation methods (beyond measurement error mitigation) in the context of quantum routing [15]. In particular, this latter work shows how concatenation of two additional mitigation methods, ‘zero-noise extrapolation’ and ‘probabilistic error cancellation’ (when utilized alongside the measurement error mitigation discussed earlier), leads to almost perfect performance of the quantum routing on the ibmq_jakarta. We refer the reader to this latter work for details; we simply note here that these additional methods expand the viability of quantum routing to noise conditions more general than those investigated in this present work. Although these other methods are somewhat more complex to implement (and not available by default on all NISQ devices), it suffices to say that additional error mitigation methods are available, and as such, the performance levels we have presented here can be considered lower bounds for the noise model adopted. More generally, we can say that advanced error mitigation methods along the lines of those reported in [15] bode well for improved quantum-routing performance over a wide range of noise conditions.

**Conclusion**

In this work, we designed and experimentally demonstrated a quantum router embedded with a quantum error correction scheme. Via quantum state tomography, we verified the quantum nature of the router, and the impact of the error correction on the routing performance. Our results demonstrate, for the first time, that quantum routing with embedded quantum error correction is viable in near-term noisy devices - pointing the way towards the inclusion of quantum routing in quantum networks. Although we have used a specifically-designed noisy channel, our work demonstrates that with use of statistical information only, quantum-error-corrected routing is viable in NISQ devices. Inclusion of quantum routing within near-term quantum networks will enhance the functionality of such networks, allow for deployment of RAM, and provide a pathway to the development of additional network functionality.

We encourage further development of the ideas presented here in the context of other noisy channels on NISQ devices, as well as other use-cases in networks. Quantum routing represents an exciting paradigm shift in the dissemination of information through a connected network. Many new applications of this non-classical routing are bound to be discovered as devices which implement it are integrated into emerging quantum networks. W. Shi is supported by the China Scholarship Council and the Sydney Quantum Academy.

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