The density-dependence of the meson-baryon couplings can induce instabilities such as the appearance of (anti)kaon condensates at the core region of neutron stars. In this work, we study the (anti)kaon condensation in the framework of covariant density functional theory. The functionals in the kaonic sector are constrained by experimental and theoretical atomic data, kaon-nucleon scattering data, and data from heavy-ion collisions. We also study the effect of the self-interaction terms for the scalar meson fields has been incorporated in this model. In addition, we incorporate density functional theories and realistic nuclear potentials to explain the interaction in high density regimes. But at those regimes due to the dependence on higher orders of fields, instabilities may arise.

I. INTRODUCTION

Recent measurements of neutron star mass from several candidates (PSR J1614−2230, PSR J0348 + 0432, MSP J0740 + 6620) set the lower bound on the maximum possible mass for this class of compact objects $\sim 2 M_\odot$. Existence of stars with high mass brings the possibility of existence of exotic matter (hyperons, meson condensates) at the core region of the objects. In this work, we investigate the (anti)kaon $(K^-, \bar{K}^0)$ condensation in $\beta$-equilibrated nuclear matter within the framework of covariant density functional theory. The functionals in the kaonic sector are constrained by the experimental studies on $K^-$ atomic, kaon-nucleon scattering data. We find that the equation of state softens with the inclusion of (anti)kaon condensates, which lowers the maximum mass of neutron star. In one of the density-independent coupling cases, the $K^-$ condensation is through a first-order phase transition type, which produces a 2 $M_\odot$ neutron star. The first-order phase transition results in mixed phase region in the inner core of the stars. While $\bar{K}^0$ condensation appears via second-order phase transition for all the models we consider here.

Keywords: neutron stars; equation of state; anti-kaon condensates; mixed phase

(DDRH model) is another possible approach to reduce the instabilities as in the later case the higher orders field terms are not brought into consideration in the effective field equations, a re-arrangement term in thermodynamic consistency, a re-arrangement term is considered which contributes explicitly to the matter pressure consequently influencing the equation of state at higher densities.

Nuclear matter is composed of mainly neutrons with small admixture of protons and electrons in $\beta$-equilibrium condition and fraction of protons are electrons are equal to keep the condition of charge neutrality. With the increase of neutron density the electron Fermi momentum increases to keep the charge neutrality. However, s-wave $\pi N$ scattering potential being repulsive the effective ground state mass of $\pi$ meson increases opposing the possibility of $\pi$ meson appearing. But, effective ground state mass of $K$-meson decreases due to attractive interaction with nucleons opening the possibility of $K$-meson appearing. Kaplan and Nelson for the very first time demonstrated that antikaon $K^-$ may undergo Bose-Einstein condensation in dense matter formed in heavy-ion collisions. Furthermore, other evidence such as the $K^-$ atomic data, kaon-nucleon scattering data studied by several authors in chiral perturbation theory also encouraged the concept of $K^-$ condensed phase presence in the interior of neutron star. The in-medium energy of (anti)kaon $K^-$ mesons decreases in the dense matter due to the lowering of effective mass. Finally, the onset of s-wave $K^-$ condensation occurs when the chemical potential of $K^-$ ($\omega_{K^-}$) equates the electron chemical potential ($\mu_e$). The s-wave $\bar{K}^0$ appears when its chemical potential ($\omega_{\bar{K}^0}$) equates to zero. The threshold density of (anti)kaon appearance is very sensitive to the
optical potential in nuclear symmetric matter. Studies \cite{15, 25, 27} reveal that $K^+$ mesons develop a repulsive optical potential nature in the nuclear matter. Thus, it may be concluded that kaon $K^+$ condensation is not favored inside the neutron star. Phase transitions from hadronic to kaonic phases in dense matter may be either a first (mixed phase) or second order type depending on the (anti)kaon optical potential depths. Various observational features of neutron star evolution such as the spin down rates, cooling, glitches may be affected by the alterations of weak interaction rates, transport properties of matter interior to neutron star due to phase transitions \cite{28, 29}. The first order phase transition cannot be explained by merely the Maxwell’s construction accounting for only one charge conservation because neutron stars have two conserved charges viz. baryon number conservation and global charge neutrality. The Gibbs conditions are employed to adequately explain the mixed phase regime of the neutron star interior \cite{1}.

The composition and equation of state (EOS) of matter inside the compact stars are constrained by recent observations of compact objects in wide range of electromagnetic spectra and in gravitational wave. Most fundamental property of compact objects which constrain the EOS is the observed mass of neutron stars (NSs) which sets the limit of maximum possible mass of NS family. One of the most accurately measured pulsar masses was done by Hulse and Taylor of binary pulsar PSR 1913 + 16 with mass $1.4498 \pm 0.0003 M_\odot$ \cite{30}. In recent years, several new neutron star mass measurements $> 2 M_\odot$ were accomplished viz. the millisecond pulsar (MSP) PSR J1614−2230 of mass $1.97 \pm 0.04 M_\odot$ \cite{31, 32}, PSR J0348 + 0432 $(2.01 \pm 0.04 M_\odot)$ \cite{33} and MSP J0740 + 6620 $(2.14^{+0.20}_{-0.18} M_\odot$ with $95\%$ credibility) \cite{34}. Gravitational wave observations have been also successful in providing bounds on the compact object mass. The recent ‘GW190814’ event observed by the LIGO-Virgo Collaboration (LVC) from a coalescence of a black hole and a lighter companion sets the mass of the former to be $23.2^{+1.1}_{-1.0} M_\odot$ and that of the latter to be $2.59^{+0.08}_{-0.09} M_\odot$ \cite{35}. The nature of the lighter companion to be a heavy neutron star or a light black hole is still not clear. Recently, the space mission NICER (Neutron star Interior Composition ExploreR) also provided mass-radius measurements of PSR J0030 + 0451 to be $1.44^{+0.11}_{-0.14} M_\odot$, $13.02^{+1.24}_{-1.66}$ km \cite{36} and $1.34^{+0.15}_{-0.16} M_\odot$, $12.71^{+1.14}_{-1.19}$ km \cite{37} respectively.

The existence of massive NSs open the possibility of exotic matter appearance in the inner core of the star. The addition of exotic degrees of freedom enhances the softening of equation of states (EOSs). But with the observation of neutron stars possessing mass greater than $2 M_\odot$, the softer EOSs may be mostly discarded as they lead to stars with lower maximum mass \cite{1}. Various non-linear density-independent parametrizations viz. GM1, GM2, GM3 \cite{38}, TM1 \cite{39} evaluates relatively softer EOSs which cannot develop $2 M_\odot$ neutron stars with (anti)kaon condensation \cite{40, 41} undergoing $1^{\text{st}}$ order phase transition. GMT parametrization \cite{41} generates a relatively stiffer EOS thus producing a mixed phase region in the neutron star interior. The GMT set is generated by reproducing the saturation properties of TM1 parametrization with the exclusion of non-linear self-interaction $\omega$-meson (vector) term exclusion. Density-dependent model parametrizations such as DD2 are able to produce stiff EOSs with baryon octet and (anti)kaon condensates as composition leading to neutron stars possessing mass $\geq 2 M_\odot$ \cite{42}.

In this work, we explore the possibility of (anti)kaon condensation in $\beta$-equilibrated nuclear matter in the inner core within the non-linear and density-dependent covariant density functional (CDF) model which shows consistent results with recent astrophysical observations.

The paper is organized in the following manner. In Sec. II we briefly discuss the non-linear CDF and density-dependent relativistic hadron (DDRH) field theory formalism incorporated in this work. Our results are shown in Sec. III and our conclusions are summarized in Sec. IV.

II. FORMALISM

A. Model

In this section, we introduce the non-linear and density dependent CDF model to study the phase transition from hadronic to anti-kaon condensed matter which could be either of the first-order or second-order form. Throughout the work, for the hadronic matter we have considered nucleons ($N \equiv n, p$) alongside electrons and muons. The strong interactions between the baryons as well as the anti-kaons are mediated by the scalar $\sigma$, isoscalar-vector $\omega^\mu$ and isovector-vector $\rho^\mu$ meson fields. We have considered the mean field model with non-linear (NL) scalar meson self interaction as well as density-dependent coupling constants. Throughout the model, the implementation of natural units is incorporated ($h = c = 1$). In general, the total Lagrangian density of the matter is given by \cite{40, 41, 43, 44}

$$\mathcal{L} = \sum_N \bar{\psi}_N (i\gamma_\mu D^\mu - m_N^s) \psi_N + \sum_l \bar{\psi}_l (i\gamma_\mu \partial^\mu - m_l) \psi_l + \bar{D}_\mu^K K D^\mu K - m_K^2 \bar{K} K + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2)
- \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^{\mu\nu} - m_\rho^2 \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{\mu\nu} \cdot \rho^{\mu\nu} - U(\sigma). \quad \text{[Only for NL model]}$$

(1)

Here, $U(\sigma)$ stands for the self interactions of scalar mesons which is present in case of NL model, but not considered in case of density-dependent couplings, the fields $\psi_N, \psi_l$ correspond to the baryon and lepton fields with their bare masses, $m_N$ and $m_l$ respectively. The covariant derivative is given by

$$D_\mu = \partial_\mu + ig_{\omega^l} \omega_\mu + ig_{\rho^j} \tau_j \cdot \rho_\mu$$

(2)
with \( j \) denoting the nucleons and (anti)kaons. The isospin doublets for kaons are denoted by \( K \equiv (K^+, K^0) \) and that for anti-kaons by, \( \bar{K} \equiv (K^-, \bar{K}^0) \). The effective nucleon (Dirac) and anti-kaon masses in the mean-field approximation are given by

\[
m_N^* = m_N - g_\sigma N \sigma, \quad m_K = m_K - g_\sigma K \sigma
\]  

where \( m_N, m_K \) are the bare nucleon and kaon masses respectively. The field strength tensors for the vector fields in eq. (1) are given by

\[
\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \\
\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu
\]  

The scalar self-interaction terms [45] required in NL model are given by

\[
U(\sigma) = \frac{1}{3} g_\sigma \sigma^3 + \frac{1}{4} g_3 \sigma^4
\]  

where, \( g_3 = b m_N \sigma_N^3 \) and \( g_3 = c g_\sigma^4 \). In the same approximation, the meson fields acquire the ground state expectation values as

\[
\sigma = -\frac{1}{m_\sigma^2} \frac{\partial U}{\partial \sigma} + \sum_b \frac{1}{m_\sigma^2} g_\sigma n_b^0 + \sum_K \frac{1}{m_\sigma^2} g_\sigma K n_K^0, \\
\omega_0 = \sum_b \frac{1}{m_\omega} g_\omega n_b^- - \sum_K \frac{1}{m_\omega} g_\omega K n_K^-, \\
\rho_{03} = \sum_b \frac{1}{m_\rho} g_\rho \tau_3 n_b^0 + \sum_K \frac{1}{m_\rho} g_\rho K \tau_K n_K^0
\]  

The first term in right hand side of eq. (6) (\( \sigma \)-meson field) is required only in the case of NL model. In case of DDRH model, the scalar self-interaction terms are absent [46, 47]. The meson-baryons (anti-kaon) couplings are denoted by \( g_{ij} \), where \( i \) goes over the mesons and \( j \) over the baryons and (anti)kaons, \( \tau_j \) represents the isospin operator. The scalar and baryon(vector) number densities are defined for the baryons as \( n_N = \langle \psi_N \gamma_0 \psi_N \rangle \), \( n_\bar{N} = \langle \bar{\psi}_N \gamma_0 \bar{\psi}_N \rangle \) respectively. In case of the \( s \)-wave (anti)kaons condensates, the number density is given by

\[
n_{K^-, \bar{K}^0} = 2 \left( \omega_K + g_\omega K \omega_0 \pm \frac{1}{2} g_\rho K \rho_{03} \right) = 2 m_K^* K_\bar{K}
\]  

The scalar density and vector number density of the baryon-\( N \) at zero temperature are given by

\[
n_N^* = \frac{m_N^*}{2\pi^2} \left[ p_{FN} E_{FN} - m_N^* \ln \left( \frac{p_{FN} + E_{FN}}{m_N^*} \right) \right], \\
n_N = \frac{m_N}{3\pi^2}
\]  

respectively, where \( p_{FN} \) and \( E_{FN} \) are the Fermi momentum and fermi energy of the \( N \)-th nucleon respectively. The in-medium energies of \( \bar{K} \equiv (K^-, \bar{K}^0) \) for \( s \)-wave condensation are provided by

\[
\omega_{K^-, \bar{K}^0} = m_K^* - g_\omega K \omega_0 + \frac{1}{2} g_\rho K \rho_{03}
\]

with the isospin projections for \( K^-, \bar{K}^0 \) being \(-1/2, +1/2\) respectively.

The chemical potential of the \( N \)-th nucleon is

\[
\mu_N = \sqrt{p_{FN}^2 + m_N^*} + g_\omega N \omega_0 + g_\rho N \tau_3 \rho_{03} + \Sigma',
\]

where, the rearrangement term \( \Sigma' \) is introduced to maintain the thermodynamic consistency in case of DDRH model [47] which is given by

\[
\Sigma' = \sum_n \left[ \frac{\partial g_\omega}{\partial n_{\omega n_N}} \omega_0 n_n^0 + \frac{\partial g_\sigma}{\partial n_{\sigma n_N}} n_N^* + \frac{\partial g_\rho}{\partial n_{\rho n_N}} \rho_{03} \tau_3 n_n^0 \right],
\]

where \( n = \sum_n n_n^0 \) is the total baryon number density. This re-arrangement term contributes explicitly only to the matter pressure. In case of NL CDF model, this term is not required.

The total energy density from the baryonic and leptonic matter is given by

\[
\varepsilon_f = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 + \sum_{n} \frac{1}{\pi^2} \left[ p_{FN} E_{FN}^3 - \frac{m_N^*}{8} \left( p_{FN} + E_{FN} \right) \right]
\]

\[
+ \frac{1}{\pi^2} \sum_i \left[ p_{Fi} E_{Fi}^3 - \frac{m_i^2}{8} \left( p_{Fi} + E_{Fi} \right) \right]
\]

\[
\varepsilon_\bar{K} = m_K^* (n_{K^+} + n_{\bar{K}^0})
\]

Anti-kaons being Bose condensates, there is no direct contribution to the total matter pressure from their ends. The matter pressure is related to the energy density via
the thermodynamic relation (Gibbs-Duhem) as
\[
p_m = \sum_N \mu_N n_N + \sum_i \mu_i n_i - \varepsilon_f, \quad (14)
\]
as there will be no contribution from the anti-kaons. The total energy density is given as \(\varepsilon = \varepsilon_f + \varepsilon_K\).

Initially at the lower density, matter is composed of neutrons \((n)\), protons \((p)\) and electrons \((e)\) in beta-equilibrium and charge neutrality condition. Beta-equilibrium at this stage is satisfied by the chemical potential balance,
\[
\mu_n = \mu_p + \mu_e \quad (15)
\]
With the increasing density, when the chemical potential of electrons becomes equal to the rest mass of muons, the muons appear. Hence the threshold equilibrium condition for the onset of muons is, \(\mu_e = m_\mu\).

Studies \cite{11, 48} shows that strangeness changing processes such as, \(N = N + K\) and \(\varepsilon^- = K^-\) may come into picture inside the neutron star core. Hence kaons may appear by these reactions, when the threshold conditions are satisfied as
\[
\mu_n - \mu_p = \omega_{K^-} = \mu_e, \quad \omega_{K^0} = 0 \quad (16)
\]
The charge neutrality conditions in the hadronic and kaon condensed phases are given by
\[
Q^h = \sum_N q_N n_N^h - n_e - n_{\mu} = 0, \quad (17)
\]
\[
Q^K = \sum_N q_N n_N^K - n_{\bar{K}} - n_e - n_{\mu} = 0
\]
respectively, where \(n_N^h\) and \(n_N^K\) represents the number densities in hadronic and kaon phases respectively, both having the same form.

The appearance of \((anti)\)kaon condensate may occur either through first order or through second order phase transition from the hadronic to kaon phase depending on the optical potential depths of \(K^-\) at nuclear saturation density. In case the transition sets through first order form, the mixed phase comes into picture: the two phases of pure hadronic matter without condensate and with condensate co-exist. Then for this case, the Gibbs conditions alongside global baryon number conservation and charge neutrality can be enforced to determine the mixed phase state \cite{19, 20, 30}. The Gibbs conditions for this state are,
\[
p^h_m = p^K_m, \quad (18)
\]
\[
\mu^h_N = \mu^K_N \quad (19)
\]
where \(h\) and \(K\) superscripts represent the respective quantities in hadronic and anti-kaon condensed phase respectively. In addition, two additional global constraints (viz. global baryon number conservation and charge neutrality) are enforced via the relations,
\[
n_N = (1 - \chi) n_N^h + \chi n_N^K, \quad (20)
\]
\[
(1 - \chi) Q^h + \chi Q^K = 0 \quad (21)
\]
respectively. Here, \(\chi\) is the fraction of anti-kaon \((K^-)\) condensed phase in the mixed phase regime. \(\chi \sim 0,1\) represents the initiation and termination of mixed phase region respectively. Region with \(\chi < 0\) is the pure hadronic phase and \(\chi > 1\) is the pure \((anti)\)kaon condensed phase. In the mixed phase, the total energy density also changes the form to,
\[
\varepsilon = (1 - \chi)\varepsilon^h + \chi\varepsilon^K \quad (22)
\]
where, \(\varepsilon^h, \varepsilon^K\) denotes the total energy density in hadronic and kaon phases respectively.

**B. Coupling parameters**

In the NL CDF model, we adopt the GMT \cite{41} and NL3 \cite{51} parametrizations for meson-nucleon couplings. For the DDRH model, DD-ME2 \cite{52}, DD2 \cite{20} and PKDD \cite{53} parametrizations are considered for meson-nucleon couplings. Table-I provides the NL coupling parameters implemented in this work.

| Model   | \(g_{\sigma N}\) | \(g_{\omega N}\) | \(g_{\varphi N}\) | \(g_2\) (fm\(^{-1}\)) | \(g_3\) |
|---------|------------------|------------------|------------------|-----------------|--------|
| GMT     | 9.9400           | 12.2981          | 9.2756           | 10.5745         | -24.1907 |
| NL3     | 10.2170          | 12.8680          | 8.9480           | 10.4310         | -28.8850 |

In case of the density-dependent model, the meson-nucleon coupling constants are functions of density as
\[
g_{iN}(n) = g_{iN}(n_0)f_i(x) \quad \text{for } i = \sigma, \omega \quad (23)
\]
where, \(x = n/n_0\), \(n_0\) is the nuclear saturation density and
\[
f_i(x) = a_i \left( 1 + b_i (x + d_i)^2 \right) \left( 1 + c_i (x + d_i)^2 \right) \quad (24)
\]
For the \(\rho\)-meson, the density-dependent coupling constant is given by
\[
g_{\rho N}(n) = g_{\rho N}(n_0)e^{-a_\rho(x-1)} \quad (25)
\]
The density-dependent hadronic model parameters implemented in this work are listed in Table-II.

| Model   | \(g_{\sigma N}\) | \(g_{\omega N}\) | \(g_{\varphi N}\) |
|---------|------------------|------------------|------------------|
| DD-ME2  | 10.5396          | 13.0189          | 7.3672           |
| DD2     | 10.686681        | 13.342362        | 7.25388          |
| PKDD    | 10.7385          | 13.1476          | 8.5996           |

The bare nucleon masses are considered to be identical for both the NL mean field model cases as \(m_N = \ldots\).
In the DDRH models we consider $m_N = 938.9, 938.565, 939.573$ MeV for DD-ME2, DD2, PKDD parametrizations respectively. The bare mass of the (anti)kaons in this calculation is considered to be 493.69 MeV.

The coefficients in eqs. 24–25 are fixed by several parameters. For details we refer the readers to the references [20, 52, 53]. The parameter values employed in the DDRH model to get the density dependence is given in the table-III. The meson-masses for the GMT, NL3, DD-ME2, DD2, PKDD models of the coupling constants incorporated in the calculations are listed in table-IV.

The respective parametrizations satisfy the values of the quantities at $n_0$ as shown in the table-V. In the table, $E/A$, $K_0$, $a_{sym}$ denote the binding energy per nucleon, compression modulus, symmetry energy coefficient.

The meson-(anti)kaon couplings are not considered to be density-dependent in DDRH model. The vector coupling parameters in the kaon sector are evaluated from the iso-spin counting rule and quark model [44] as

$$g_{\omega K} = \frac{1}{3} g_{\omega N}, \quad g_{\rho K} = g_{\rho N}$$

and for the scalar coupling parameters, they are calculated at nuclear saturation density from the real part of $K^-$ optical potential depth as

$$U_K(n_0) = -g_{\sigma K} \sigma(n_0) - g_{\omega K} \omega_0(n_0) + \Sigma_N(n_0)$$ (27)

where, $\Sigma_N(n_0)$ is the contribution from the nucleons alone and is not considered in case of NL CDF model.

Experimental studies [51, 53] show that the kaons experience a repulsive interaction in nuclear matter whereas antikaons experience an attractive potential. Several model calculations [56–60] provide a very broad range of optical potential values as $-120 \leq U_K \leq -40$ MeV. Another calculation from hybrid model [61] suggests the value of $K^-$ optical potential to be in the range $180 \pm 20$ MeV at nuclear saturation density. In this work, we have considered a $K^-$ potential range of $-160 \leq U_K \leq -120$ MeV and the scalar meson-(anti)kaon couplings evaluated for the above potential depth range are listed in the table-VI.
$U_K = -160 \text{ MeV}$. While in case of NL3 parameterization, the phase transition is of second order for the whole range of $K^-$ potential adopted here.

Fig. 1 shows the matter pressure as a function of energy density for both the GMT and NL3 models. The appearance of (anti)kaons to a great extent softens the EOS. The two kinks in the EOSs marks the onset of $K^-$ and $\bar{K}$ respectively. The two kinks are observed to be in higher densities for the NL3 model in comparison to the GMT case referring the delay of (anti)kaons into the matter for the former case. The appearance of (anti)kaon condensation is through first order transition only for $K^-$ with $U_K = -160 \text{ MeV}$ in GMT parametrization. In other cases it is second order phase transition.

Fig. 2 shows the results of the mass-radius (M-R) relationship for static spherical stars from solution of the Tolman-Oppenheimer-Volkoff (TOV) equations [1] corresponding to the EOSs discussed here and shown in fig. 1. For the crust, we have considered the EOS of Baym, Pethick and Sutherland [62]. Table VII provides the set of maximum mass values, corresponding radius and central density for the nucleons and (anti)kaon EOSs with various values of $U_K$. Inclusion of (anti)kaons leads to reduction of maximum mass of the compact stars.

The variation of particle fractions in the matter with total baryonic number density is shown in the fig. 3 for $U_K(n_0) = -160 \text{ MeV}$ for both the parametrization sets. For GMT parametrization the mixed phase initiates with the onset of $K^-$ at $\sim 2.22 n_0$. In addition to the global conservation rule of baryon number (eq. 20) and charge neutrality (eq. 21), pressure and chemical potential equilibrium conditions between two phases (eqs. 18, 19) determines the mixed phase region. Due to higher rest mass of (anti)kaons compared to the lepton species and being bosons, the former condense in the lowest energy state and so are preferred to maintain the global charge neutrality condition. This results in decrease in electron and muon populations as can be clearly visualized in fig. 3. The mixed phase terminates at $\sim 2.90 n_0$. Further, with the appearance of $\bar{K}$ at $\sim 3.49 n_0$ and ceasing of electron population around $4-4.5 n_0$ the proton and $K^-$ populations becomes equal following the charge neutrality condition. $\bar{K}$ condensates through the second order phase transition. However, for NL3 parametrization the phase transition occurs via second order for both the (anti)kaons ($K^-, \bar{K}$) implying the absence of mixed phase regime. Even though we are fixing the $U_K$ identical to GMT model cases, the (anti)kaons appear at a slightly higher densities compared to the former case.

The threshold densities for the onset of $K^-, \bar{K}$ in the dense nuclear matter for different $K^-$ potentials are pro-

![Diagram](image-url)
TABLE VIII. Threshold densities, $n_{cr}$ (in units of $n_0$) for antikaon condensation in dense nuclear matter for different values of $K^-$ optical potential depths $U_K$ (in units of MeV) at $n_0$.

| $U_K$ (MeV) | GMT $n_{cr}(K^-)$ | GMT $n_{cr}(\bar{K}^0)$ | NL3 $n_{cr}(K^-)$ | NL3 $n_{cr}(\bar{K}^0)$ |
|------------|-----------------|-----------------|-----------------|-----------------|
| $-120$     | 2.87            | 4.45            | 2.77            | 4.35            |
| $-140$     | 2.56            | 3.96            | 2.49            | 3.94            |
| $-160$     | 2.22            | 3.49            | 2.24            | 3.53            |

Fig. 4 shows the extent of mixed phase region inside the neutron star modelled with GMT parametrization. It is evident that the mixed phase regime starts ($\chi \sim 0$) from around matter density of $\sim 2.22 n_0$ which corresponds to star radius 7.34 km, and terminates ($\chi \sim 1$) at around matter density of $2.9 n_0$ or corresponding stellar radius of $\sim 6.13$ km.

The interactions between (anti)kaons and nucleons alters the nucleon effective mass in the mixed phase regime where both the hadronic and kaonic phase co-exist. This effect is shown in Fig. 5. With a difference of $\sim 100$ MeV, the effective nucleon masses are observed to increase in kaonic phase while it decreases in the pure hadronic phase as we move interior towards the pure kaonic phase regime.

The charge densities of each normal and kaon condensed phase in the mixed phase region for the GMT model with $U_K = -160$ MeV as a function of the kaon volume fraction is shown in Fig. 6. The central solid black
line represents the global charge neutrality condition as provided in eq. [21].

The (anti)kaon energies as a function of baryon number density with $U_\bar{K} = -120, -140, -160$ MeV for the two parameter sets (GMT, NL3) are shown in fig. 7. The in-medium energies for both the (anti)kaons decrease with density. The dashed curve representing $\mu_e$ crossing over the $\omega_\bar{K}$ curves marks the end of pure hadronic phase and initiation of pure kaonic phase. $K^-$ condensations initiates once the value of $\omega_K^-$ reaches that of the electron chemical potential and $K^0$ condensation starts when the value of $\omega_K^0$ equates to zero. From fig. 7, it is observed that the threshold condition, $\omega_K^- = \mu_e$ is achieved way earlier than the $\omega_K^0 = 0$ one, leading to earlier appearance of $K^-$. The difference in the energy densities of the hadronic and kaonic phases for GMT parameterization with $U_\bar{K} = -160$ MeV is shown in fig. 8. The total energy density in the mixed phase region is evaluated from eq. (22) and grows monotonically with density.

**B. Density-Dependent CDF model**

For DDRH model, the phase transition from hadronic to kaonic phase for all parametrizations and all (anti)kaon optical potential depths considered in the present work is through the second-order phase transition. The matter pressure as a function of energy density, left panels: GMT, right panels: NL3 parametrization. Upper panels: in-medium energies of $K^0$, lower panels: in-medium energies of $K^-$ with several $U_\bar{K}$. The dashed lines represent the respective electron chemical potentials for each model case. The solid curve exhibits the $U_\bar{K}$ strengths of $-120$ MeV, dash-dotted lines exhibits $-140$ MeV case and dotted lines exhibits the $-160$ MeV case.
Table IX. Threshold densities, \(n_{\epsilon^{-}}\) (in units of \(n_{0}\)) for antikaon condensation in dense nuclear matter for different values of \(K^{-}\) optical potential depths \(U_{K}\) (in units of MeV) at \(n_{0}\) with density-dependent DD-ME2, DD2, PKDD parametrizations.

| \(U_{K}\) (MeV) | DD-ME2 | DD2 | PKDD |
|-----------------|---------|------|-------|
|                 | \(n_{\epsilon^{-}}^{K^{-}}(n_{0})\) | \(n_{\epsilon^{-}}^{K^{-}}(n_{0})\) | \(n_{\epsilon^{-}}^{K^{-}}(n_{0})\) |
| -120            | 3.00    | 3.08 | 3.16  |
| -140            | 2.72    | 2.79 | 2.82  |
| -160            | 2.47    | 2.53 | 2.53  |

Fig. 8. Energy density as a function of baryon number density for the GMT model with \(U_{K} = -160\) MeV for the 3 phases. The solid curve denotes the total energy density variation, while dash-dotted curves represent \(\epsilon^{K^{-}}\) and dash-double dotted curve represent \(\epsilon^{\bar{K}^{0}}\). The three regions demarcation are similar as fig. 5.

Substantially with the inclusion of (anti)kaons. It is observed that configurations with DD-ME2 satisfy the observed maximum mass neutron star constraint of \(\sim 2\, M_{\odot}\) until \(U_{K} = -160\) MeV. For the other two model cases, DD2 satisfy the constraint until \(U_{K} = -140\) MeV, while PKDD providing a further softer EOS satisfy the constraint only up to \(U_{K} = -120\) MeV.

Fig. 11 represents the (anti)kaon energies as a function of baryon number density with \(U_{K} = -120\) MeV for the density-dependent coupling models. The onset of \(K^{-}\) and \(K^{0}\) occurs with \(\omega_{K^{-}}\) crossing over \(\mu_{e^{-}}\) and \(\omega_{K^{0}}\) being equal to zero respectively. Similar behavior is observed in case of DD-ME2 and DD2 parametrization models, while for PKDD model, the \(K^{-}\) in-medium energy is higher compared to the other two cases and \(K^{0}\) in-medium energy is observed to be lower.

The population densities of different species, \(n_{i}\) (in units of \(n_{0}\)) in the neutron star interior as a function of baryon number density for the density-dependent models are provided in fig. 12. (Anti)kaons, being bosons are not constrained by Pauli blocking, resulting in lepton fraction suppression at high density regions. The population behavior in two cases of DD-ME2 and DD2 are observed to be similar with slight difference in the threshold density of \(K^{-}\)-meson appearance. The proton population (subsequently electron and muon) is higher at lower densities for PKDD model case compared to DD-ME2, DD2 models due to higher symmetry energy. For PKDD model, in addition to the early onset of \(K^{0}\) particles, the eradication of lepton species is quite delayed compared to the former parametrizations. This results in further softening of EOS.

The inclusion of hyperons as well as \(\Delta\)-resonances alongwith (anti)-kaons in neutron star matter in case of DD-ME2 coupling parametrization model is shown in fig. 13. In this case, the meson-baryon as well as the meson-(anti-kaon) interactions are considered to be mediated by \(\sigma, \omega, \rho, \sigma^{*}, \phi\)-mesons. The coupling parameters for the meson-hyperon and meson-\(\Delta\) baryon interactions are adapted from reference-63. The EOS with matter composition as nucleons, hyperons, \(\Delta\)-resonances, (anti)-kaons (NY\(\Delta\)K) is observed to be stiffer than the one without \(\Delta\)-baryons (NYK). This happens because of the early appearance of \(\Delta\) resonance particles which pushes the advent of hyperons to higher densities. The phase transition of (anti)-kaons is observed to be second-order form in this matter composition indicating the absence of mixed phase. Detailed discussions on this matter com-
IV. CONCLUSIONS AND OUTLOOK

We investigate the appearance of the (anti)kaon condensation in $\beta$-equilibrated charge neutral nucleonic matter within the framework of mean field theory with density-independent (NL CDF model) as well as dependent (DDRH model) coupling constants. Observations constrain the maximum mass of neutron star family to be above $2 M_\odot$. Stars with mass near and above $2 M_\odot$ must contain central density above $4 n_0$. At such high density the phase transition to boson condensate within the nuclear matter is highly probable. However, the appearance of (anti)kaon condensation softens the equation of state lowering the maximum mass of the neutron star family. We discuss here certain parametrizations of EOS within RMF model which are stiff enough to provide maximum mass above $2 M_\odot$ with appearance of (anti)kaon condensation will be provided in future works.
TABLE X. Maximum mass, $M_{\text{max}}$ (in units of $M_\odot$), radius (in km), corresponding central density (in units of $n_0$) of nucleon compact stars for different values of $K^-$ optical potential depths $U_K$ (in units of MeV) at $n_0$ in DD-ME2, DD2, PKDD parametrization models.

| Configuration | $U_K$ (MeV) | DD-ME2 | DD2 | PKDD |
|---------------|-------------|--------|-----|------|
| $npe\mu K$    |             |        |     |      |
| 0             | 2.48        | 2.42   | 2.33 |      |
| $npe\bar{K}$  |             | 2.29   | 2.21 | 2.10 |
| 0             | 11.96       | 11.77  | 11.63| 12.31|
| $npe\mu K$    |             | 12.28  | 12.14| 12.44|
| 0             | 5.36        | 5.69   | 5.94 | 5.54 |
| $npe\bar{K}$  |             | 12.37  | 12.23| 12.31|
| 0             | 5.33        | 5.62   | 5.94 | 5.54 |
| $npe\mu K$    |             | 12.43  | 12.29| 12.31|
| 0             | 5.22        | 5.47   | 5.94 | 5.54 |

FIG. 11. The effective energy of (anti)kaons as a function of baryon number density with $U_K = -120$ MeV. Upper panel: for $\bar{K}^0$ and lower panel: for $K^-$ with solid lines denoting DD-ME2, dash-dotted curves representing DD2 and dotted lines representing PKDD parametrization.

FIG. 12. Same as fig.3. Upper panel: DD-ME2, center panel: DD2, lower panel: PKDD parametrization model and $K^-$ potential depth of $-120$ MeV. Solid curves denote neutron ($n$), long-dashed curves proton ($p$), dash-dotted lines electron ($e^-$), dotted lines muon ($\mu^-$), short-dashed curves $K^-$ and short-double dotted lines denote $\bar{K}^0$ population.
phase region ranges for a length of $\sim 1.21$ km from stellar radius of 6.13 to 7.34 km. The outer core region upto 7.34 km is the pure hadronic phase while the region in the inner core from 6.13 km to center of the star is the pure (anti)kaon condensed phase. Moreover, the $K^0$ condensation is a second-order phase transition. The compact star with the mixed phase regime satisfies the bounds set on mass-radius by the various recent astrophysical observations. The EOS (mixed phase) evaluated with GMT model incorporating the $K^0$ condensation which satisfies 2 $M_{\odot}$ criteria can be employed to study the glitch phenomena in pulsars.

In case of density-dependent parametrization models (DD-ME2, DD2, PKDD), the (anti)kaon condensation is through second-order phase transition. Among these parametrizations, DD-ME2 produces the stiffest equation of state for both the cases with only nucleons as well as nucleons and (anti)kaons. All parameter sets explain the 2 $M_{\odot}$ neutron star without the inclusion of (anti)kaons. Higher optical potential leads to early appearance of $K^-$ in the star interior. In case of DD2 model, the configuration with $U_K = -160$ MeV doesn’t satisfy the $\sim 2 M_{\odot}$ maximum mass star. For the PKDD model producing the softest EOS among the considered coupling models, the astrophysical maximum mass constraint is not satisfied with $U_K \geq -140$ MeV. Within the framework model considered in this work, the upper limit for $U_K$ in case of DD-ME2 model is $-160$ MeV. The likelihood of hyperon and $\Delta$-baryons in the neutron star matter beside (anti)-kaons are also studied with DD-ME2 model and $U_K = -140$ MeV. The (anti)-kaons tend to appear at the higher density for the matter with $\Delta$-baryons in comparison to the one without $\Delta$-resonances in the neutron star matter. The mass-radius constraints are observed to be satisfied by this matter composition as well. Further analysis on the effects due to the presence of hyperons as well as $\Delta$-baryons with the appearance of (anti)-kaonic condensation in neutron star matter is beyond the scope of this work.

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