COMMENTS ON THE LARGE $N_c$ BEHAVIOR OF LIGHT SCALARS

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I review the large $N_c$ behavior of light resonances generated from unitarized one-loop Chiral Perturbation Theory. In contrast with the $\rho$ or $K^*$, the scalar behavior is at odds with a $\bar{q}q$ dominant component. In fact, in the light scalar region, meson-meson amplitudes vanish as $N_c$ increases. Also, the scalar widths, obtained from their associated poles, behave as $O\left(\frac{1}{\sqrt{N_c}}\right) < \Gamma < O(N_c)$. We also clarify on the physical relevance of considering large $N_c$ not too far from real life, $N_c = 3$, and the interpretation of the mathematical $N_c \to \infty$ limit.

1 Introduction

On the one hand, the large $N_c$ expansion is the only analytic approximation to QCD in the whole energy region, providing a clear definition of $\bar{q}q$ states, that become bound, and whose masses and widths behave as $O(1)$ and $O(1/N_c)$, respectively. On the other hand, Chiral Perturbation Theory (ChPT) is the QCD low energy Effective Lagrangian built as the most general derivative expansion respecting SU(3) symmetry and containing only $\pi$, $K$ and $\eta$ mesons. These particles are the QCD low energy degrees of freedom since they are Goldstone bosons of the QCD spontaneous chiral symmetry breaking. For meson-meson scattering ChPT is an expansion in even powers of momenta, $O(p^2), O(p^4), \ldots$, over a scale $\Lambda_\chi \sim 4\pi f_0 \simeq 1$ GeV. Since the $u$, $d$ and $s$ quark masses are so small compared with $\Lambda_\chi$ they are introduced as perturbations, giving rise to the $\pi$, $K$ and $\eta$ masses, counted as $O(p^2)$. At each order, ChPT is the sum of all terms compatible with the symmetries, multiplied by “chiral parameters”, that absorb loop divergences order by order, yielding finite results. The leading order is universal, containing only one parameter, $f_0$, that sets the scale of spontaneous symmetry breaking. Different underlying dynamics manifest with different higher order parameters, called $L_i$, that, once renormalized, depend on a regularization scale as $L_i(\mu^2) = L_i(\mu_1^2) + \Gamma_i \log(\mu_1/\mu_2)/16\pi^2$, where $\Gamma_i$ are constants.

In physical observables the $\mu$ dependence is canceled with that of the loop integrals.

The $\pi, K, \eta$ masses scale as $O(1)$ and $f_0$ as $O(\sqrt{N_c})$. The $L_i$ parameters that determine
meson-meson scattering up to $O(p^4)$ scale\(^2\)\(^3\) as $O(N_c)$ for $i = 1, 2, 3, 5, 8$ whereas $2L_1 - L_2, L_4, L_6$ and $L_7$ scale as $O(1)$. In order to apply the large $N_c$ expansion, the $\mu$ scale, a dependence suppressed by $1/N_c$, has to be chosen between $\mu = 0.5$ and 1 GeV. (In Figure 3 we will see that this estimate yields the correct behavior for light vector mesons, firmly established as $\bar{q}q$ states).

In recent years ChPT has been extended to higher energies by means of unitarization\(^4\)\(^5\). The main idea is that when projected into partial waves of definite angular momentum $J$ and isospin $I$, physical amplitudes $t_{IJ}$ should satisfy an elastic unitarity condition:

$$\text{Im} t_{IJ} = \sigma |t_{IJ}|^2 \Rightarrow \text{Im} \frac{1}{t_{IJ}} = -\sigma \Rightarrow t_{IJ} = \frac{1}{\text{Re} t_{IJ} - i\sigma}. \quad (1)$$

Since the two body phase space $\sigma$ is known, in order to have a unitary amplitude we only need $\text{Re} t^{-1}$, that can be obtained from ChPT: this is the Inverse Amplitude Method (IAM)\(^5\)\(^6\)\(^7\).

In this way, the IAM generates the $\rho$, $K^*$, $\sigma$ and $\kappa$ resonances not initially present in ChPT, ensures unitarity in the elastic region and respects the ChPT expansion. When inelastic two-meson processes are present the IAM generalizes\(^6\) to $T \simeq (\text{Re} T^{-1} - i\Sigma)^{-1}$, where $T$ is a matrix containing all partial waves between all physically accessible two-body states, whereas $\Sigma$ is a diagonal matrix with their phase spaces, again well known. Using one-loop ChPT calculations, the IAM provides a remarkable description\(^5\) of two-body $\pi$, $K$ or $\eta$ scattering up to 1.2 GeV.

In addition, it generates the $\rho$, $K^*$, $\sigma$, $\kappa$, $a_0(980)$, $f_0(980)$ and the octet $\phi$. Such states are not included in the ChPT Lagrangian, but each one has an associated pole in the second Riemann sheet of its corresponding partial wave. These poles appear already with the $L_i$ set used for standard ChPT, and also with the $L_i$ set obtained from fits to data, which are compatible with each other. For narrow, Breit-Wigner like, resonances, their mass and width is roughly given by $\sqrt{\text{pole}} \sim M_R - i\Gamma_R/2$. Furthermore, the IAM respects the $O(p^4)$ correct low energy expansion, with chiral parameters compatible with standard ChPT. Different IAM fits\(^3\) are mostly due to different ChPT truncation schemes, equivalent up to $O(p^4)$.

Note that the ChPT amplitudes used are fully renormalized, and therefore scale independent. There are no cutoffs or a subtraction constants where a spurious $N_c$ dependence could hide. All the QCD $N_c$ dependence appears correctly through the $L_i$, $f_0$ and the $\pi, K, \eta$ masses.

Recently\(^7\) by rescaling the ChPT parameters, we have studied how those generated resonances behave in the large $N_c$ expansion. Thus, in Fig.1 we see what happens with the $\rho(770)$ and $K^*(892)$ vector mesons. In real life, the modulus of their corresponding partial waves presents a peak, that we have obtained from a fit to data, that becomes narrower as $N_c$ increases whereas the mass remains almost the same. By looking at the mass and width from the pole we see that, for both resonances, they behave as expected for a $\bar{q}q$ state, i.e. $M \sim O(1)$, $\Gamma \sim O(1/N_c)$.

![Figure 1: Left: Modulus of $\pi\pi$ and $\pi K$ elastic amplitudes versus $\sqrt{s}$ for $(I, J) = (1, 1), (1/2, 1)$: $N_c = 3$ (thick line), $N_c = 5$ (thin line) and $N_c = 10$ (dotted line), scaled at $\mu = 770$ MeV. Right: $\rho(770)$ and $K^*(892)$ pole positions: $\sqrt{\text{pole}} \equiv M - i\Gamma/2$ versus $N_c$. The gray areas cover the uncertainty $N_c = 0.5 - 1$ GeV. The dotted lines show the expected $\bar{q}q$ large $N_c$ scaling.](image)

In contrast, in Figure 2 we see the behavior for the $\sigma$ (or $f_0(600)$) and the $\kappa$. The results for the $a_0(980)$ and $f_0(980)$ are roughly similar, but more subtle\(^7\). It is evident that these scalars behave completely different to $\bar{q}q$: The modulus of their partial waves in the resonance region vanish and their widths grow as $N_c$ increases, as $O(N_c^{1/2}) < \Gamma < O(N_c)$.
2 Discussion and conclusions

We have seen that, within the unitarized ChPT approach, $\bar{q}q$ states are clearly identified whereas scalar mesons behave differently as $N_c$ increases. Here I want to emphasize again what can and what cannot be concluded from this behavior and clarify some frequent questions and misunderstandings that I have found in private communications and the literature.

- **The dominant component of the $\sigma$ and $\kappa$ in meson-meson scattering does not behave as a $\bar{q}q$.**

  - Why “dominant”? Because, most likely, scalars are a mixture of different kind of states. If the $\bar{q}q$ was dominant, they would behave as the $\rho$ or the $K^*$ in Figure 1. But it cannot be excluded that there is some smaller fraction of $\bar{q}q$.

  - Also, since scalars could be an admixture of states with different nature and wave functions, it could happen that the small $\bar{q}q$ component could be concentrated in the core and better seen in other reactions whereas in scattering we are seeing mostly the outer region.

- **Two meson and some tetraquark states** have a consistent “qualitative” behavior, i.e., both disappear in the continuum of the meson-meson scattering amplitude as $N_c$ increases (also the glueballs for the $\sigma$ case but not for the $\kappa$). Waiting for more quantitative results, we have not been able to establish yet the nature of that dominant component, but two-meson states or some kind of tetraquarks are, qualitatively, candidates to form that dominant component.

  The IAM results have been later confirmed, since “very similar” numerical results have been reported with other unitarization techniques, and the the $N_c \to \infty$ limit has been studied. This limit is interesting mathematically, and maybe could have some physical relevance if the data and the large $N_c$ uncertainty on the choice of scale were more accurate. Nevertheless

- **Contrary to the large $N_c$ behavior in the vicinity of $N_c = 3$, the mathematical $N_c \to \infty$ limit may not give information on the “dominant component” of light scalars.** The reason was commented above: In contrast to $\bar{q}q$ states, that become bound, two-meson and some tetraquark states dissolve in the continuum as $N_c \to \infty$. Thus, even if we started with an infinitesimal $\bar{q}q$ component in a resonance, there could be a sufficiently large $N_c$ for which it may become dominant, and beyond that $N_c$ the associated pole would behave as a $\bar{q}q$ state although the
original state only had an infinitesimal admixture of $\bar{q}q$. Also, since the mixings of different components could change with $N_c$, a too large $N_c$ could alter significantly the original mixings.

Indeed this can happen\textsuperscript{10} for the $\sigma$ for certain choices of chiral parameters: at a sufficiently high $N_c$ the pole may turn back toward the real axis (see Figure 3). The IAM also yields such numerical result, and for the $\kappa$ too. However, as commented above, it does not mean that the “correct interpretation... is that the $\sigma$ pole is a conventional $\bar{q}q$ meson environed by heavy pion clouds\textsuperscript{10}. That the scalars are not conventional, is simply seen comparing the scalars in Figure 2 with the “conventional” $\rho$ and $K^*$ in Figure 1. A large two-meson component is allowed, but the $N_c \rightarrow \infty$ limit is not unique\textsuperscript{10} given the uncertainty in the chiral parameters: scalar poles can move to negative mass square (quite weird), to infinity or to a positive mass square. But even if the $L_i$ where determined with a much greater precision, it is not clear that we could draw any conclusion at $N_c \rightarrow \infty$: As emphasized above and in\textsuperscript{10}, one loop ChPT amplitudes are independent of the renormalization scale $\mu$, but the $L_i$ are not. Thus, we have to choose a scale between roughly 0.5 and 1 GeV to start our $N_c$ scaling. As seen in Figure 3, that uncertainty is enough to change the $N_c \rightarrow \infty$ behavior, even when starting from exactly the same set of $L_i$.

Therefore, robust conclusions on the dominant light scalar component, can be obtained not too far from real life, $N_c = 3$, for a $\mu$ choice between roughly 0.5 and 1 GeV, and checking that simultaneously the $\rho$ and $K^*$ behave as almost pure $\bar{q}q$ states. That is one of the reasons why in Figures 1 and 2 we have only plotted up to $N_c = 30$, but not 100, or a million.

![Figure 3: Large $N_c$ behavior versus renormalization scale choice. Left: The $\rho$ pole tends to the real axis if $0.5\,\text{GeV} < \mu < 1\,\text{GeV}$, but not for $\mu = 1.2\,\text{GeV}$. Right: The sigma pole behavior changes wildly for $\mu = 1.2\,\text{GeV}$, but always at odds with a $\bar{q}q$ dominant component. Note that the scale here is larger than on the left.](image)

In summary, the dominant component of light scalars as generated from unitarized one loop ChPT scattering amplitudes does not behave as a $\bar{q}q$ state as $N_c$ increases away from $N_c = 3$.

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