The dynamic analysis of the rotorcraft with robotic landing gear in the landing process on uneven ground

Lianquan Wu, Yongling Liang and Qi Li

Abstract
In order to avoid the hard landing on the uneven ground, several robotic landing gears of the rotorcraft are designed. In previous studies, most literature focuses on the control methods of the robotic landing gears. There has been very little research on the dynamic model of the rotorcraft with the robotic landing gears in the landing process. In this work, based on the Lagrangian formulation, the dynamic model of the rotorcraft considering different parameters in the landing process is derived, where the angle of the slope of the uneven ground and the angle of the attack of the rotorcraft are considered. Then an approach is presented to determine the bounds of the dynamic response of the rotorcraft with uncertainties parameters. The results of numerical examples show that in order to maintain the safety, the robotic landing gear of the rotorcraft would be controlled to adjust the condition of the ground, while the initial velocities are restricted to be less than certain value; the large deviations of the landing parameters would may resulted in an overload of the rotorcraft.

Keywords
Interval parameters, landing process, dynamic response, rotorcraft robotic landing gear, dynamic model

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Introduction
It is well known that the mobility and flexibility are the principal advantages of rotorcrafts, compared to the fixed-wing-aircraft. The rotorcrafts are invaluable tools in many fields such as the emergency rescue missions, the exploration of the disaster areas, or the mountainous environments. The crucial virtue of rotorcrafts is their vertical takeoff and landing ability. A classical case which is frequently encountered is that when a disaster is happening, the environment is harmful or dangerous to persons. The disaster is far away from where the rotorcrafts can take off. The rotorcrafts need to carry out a long distance flight to explore the disaster or the emergency situation. Because of the limitation of the power of the battery, the rotorcrafts cannot continually hover over for a long time; otherwise the rotorcrafts would not be capable of leaving the disaster area. The rotorcrafts would change the spots to get entire information with several landing and takeoff processes. In this situation, the rotorcrafts are unavoidable to land on the uneven terrain. It is very difficult for the rotorcrafts to land on the uneven surface safely, which would result in the behavior of the dynamic rollover or the hard landing. The hard landing, an

1Department of Police skills and Tactics Training, Criminal Investigation Police University of China, Shenyang, P. R. China
2College of Public Security Information Technology and Intelligence, Criminal Investigation Police University of China, Shenyang, P. R. China
3State Key Laboratory of Robotics, Shenyang Institute of Automation, Institutes for Robotics and Intelligent Manufacturing, Chinese Academy of Sciences, Shenyang, P. R. China

Corresponding author:
Qi Li, State Key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences, No.114 Nanta Street, Shenhe District, Shenyang, 110200, Liaoning Province, P.R.China.
Email: lq1231010@163.com

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unacceptable way under normal conditions, means that an aircraft relatively rapidly descents and drops on the ground with a high speed, usually resulting in serious fuselage damage, structural failure, or cargo damage. Therefore, the autonomous landing of the rotorcrafts on the uneven surface is a demanding problem.

In order to avoid the dynamic rollover or the hard landing of the rotorcrafts, several robotic landing gears are proposed by researchers, which would be controlled to adjust the condition of the ground to insure the safe landing. In 2015, a robotic landing gear for the helicopter is developed by the Georgia Institute of Technology under the DARPA’s Mission Adaptive Rotor (MAR) program, which consists of four articulated robotic legs. Each leg has two rotational degrees of the freedom. The legs can fold up against the body of the helicopter after takeoff; while during the landing process, the legs can be extended to land, where the angles of the joints of the legs can be controlled to adapt to the condition of the ground well. A multi-body model considering the system motions, landing loads, and stresses of legs is presented to simulate the dynamics of a rotorcraft with robotic gear landing on the sloped terrain. The dynamic simulation shows that a robotic legged landing gear system can be used for landing on the sloped surface. For the above configuration Kiefer et al. provide a detail multibody dynamic dynamics model based on the Lagrangian equation.

Although there are several dynamic models for the robotic landing gear in the above literature, they do not consider the different situations of the rotorcraft and the different uneven ground conditions in the landing process. In this study, based on the Lagrangian formulation, we provided a dynamic model of the rotorcraft with different number of legs contacting with the ground in the landing process, where the angle of the slope of the uneven ground and the angle of the attack of the rotorcraft were considered. In practical engineering, the parameters of the above model may be uncertain due to the complexity of structures, the uneven ground conditions, the accuracy of control, and the inaccuracy in measurement, etc. To the authors’ knowledge there has been very little research to investigate the effect of these uncertainties on the dynamic response of the rotorcraft in the landing process.

Depending on the prior knowledge about the uncertainties, there are several uncertain models such as the probabilistic model, the fuzzy model, and the interval model. We adopted the interval model to study the effect of uncertainties on the dynamic response of the rotorcraft in the landing process, where uncertainties are uncertain-but-bounded in intervals. The first-order interval analysis method was used to investigate the effect of uncertainties on the dynamic response of the rotorcraft in the landing process due to its efficiency and simplicity. Based on the numerical experiments, we performed comparisons of the dynamic responses of the rotorcraft robotic landing gear with the different legs in the landing process, the different initial velocities, the different angles of the sloped ground, and the angles around the hip joints and the knee joints.

The remainder of this paper is organized as follows. First, a statement is given for the problem of the dynamic response of the rotorcraft in the landing
process. Based on the Lagrangian formulation, the dynamic model for the above problem is developed. Then in Section 3, the first-order interval analysis method considering the uneven ground conditions, the accuracy of control, and the inaccuracy in measurement is introduced. Furthermore, comparisons with the different initial conditions are contained in Section 4. Finally, in Section 5, some concluding remarks are presented.

Problem statement

In order to land on the uneven ground and avoid the hard landing, the rotorcraft robotic landing gears were designed such as the robotic landing gears in Sarkisov et al. and Yashin et al. The typical design of the rotorcraft robotic landing gear was shown in Figure 1.

As can be seen in Figure 1, the rotorcraft robotic landing gear which attaches to the bottom of the rotorcraft has four legs. Each leg consists of a thigh, a supporting leg, and a pad; each leg has two degrees of freedom. The shapes of the legs are controlled by the servomotors in the hip joints and the knee joints. The passive footpads are connected to the bottom of the supporting legs, which provide the damping. More detailed information about the rotorcraft can be found in Sarkisov et al.

The dynamic analysis of the robotic landing gear in the landing process

Usually, when considering the dynamic response of the rotorcraft robotic landing gear in the landing process, the dynamic model may be given in Figure 2. In the above model, $m_{pi}$, $m_{di}$, $m_{si}$, $m_{ti}$, $i = 1, 2, 3, 4$ denote the masses of the bottom of the pad, the top of the pad, the supporting leg, and the thigh of each leg of the rotorcraft, respectively. $m_{mi}$ denotes the mass of the body of the rotorcraft. $k_{pi}$, $k_{di}$, $k_{si}$, $k_{ti}$, $i = 1, 2, 3, 4$ denote the stiffnesses of the bottom of the pad, the top of the pad, the supporting leg, and the thigh of each leg of the rotorcraft, respectively. $k_{mi}$, $i = 1, 2, 3, 4$ denote the stiffness between each thigh and center of the mass of the body. $c_{pi}$, $c_{di}$, $c_{si}$, $c_{ti}$, $i = 1, 2, 3, 4$ denote the dampings of the bottom of the pad, the top of the pad, the supporting leg, and the thigh of each leg of the rotorcraft, respectively. $c_{mi}$, $i = 1, 2, 3, 4$ denotes the damping between each thigh and center of the mass of the body. $\gamma_{i1}, \gamma_{i2}$, $i = 1, 2, 3...4$ were defined as the...
angles around the hip joints and the knee joints, respectively.

The kinetic energy of the rotorcraft in the landing process may be written as follows:

\[
D = 0.5 \sum_{i=1}^{4} c_{di} (v_{di}(t) - v_{pi}(t)) (r_{di})^2 + 0.5 \sum_{i=1}^{4} c_{ni} (v_{ni}(t) - v_{pi}(t)) (r_{ni})^2 + 0.5 \sum_{i=1}^{4} c_{mi} (v_{mi}(t) - v_{pi}(t)) (r_{mi})^2 + 0.5 \sum_{i=1}^{4} f_{ps} <v_{pi}(t), u_{pi}>^2
\]

where \(v_{pi}(t) = (v_{p1}(t), v_{p2}(t), v_{p3}(t))\), \(v_{di}(t) = (v_{di1}(t), v_{di2}(t), v_{di3}(t))\), \(v_{ni}(t) = (v_{ni1}(t), v_{ni2}(t), v_{ni3}(t))\), \(v_{mi}(t) = (v_{mi1}(t), v_{mi2}(t), v_{mi3}(t))\), \(i = 1, 2, 3, 4\) denote the velocity vectors of the bottom of the pad, the tip of the pad, the supporting leg, and the thigh of each leg of the rotorcraft in the coordinate system with the origin on the ground, respectively. \(r_{di}(t)\), \(r_{ni}(t)\), and \(r_{mi}(t)\) denote the direction vector of the direction from the top of the center of the pad to the bottom of the center of the pad. The above direction vectors are determined by the structure of the rotorcraft and the angles around the hip joints and the knee joints. The definitions of \(r_{pi}\) = \((r_{p1i}, r_{p2i}, r_{p3i})\), \(r_{ni}\) = \((r_{ni1}, r_{ni2}, r_{ni3})\), \(r_{mi}\) = \((r_{mi1}, r_{mi2}, r_{mi3})\), \(i = 1, 2, 3, 4\) are analogous. \(\mathbf{x}_m(t) = (x_{m1}(t), x_{m2}(t), x_{m3}(t))\) denotes the displacement vector of the body in the coordinate system. \(\mathbf{r}_m = (r_{m1}, r_{m2}, r_{m3})\), \(i = 1, 2, 3, 4\) is the direction from the center of the body to the top of the center of the thigh. \(\mathbf{I} = (l_1, l_2, l_3)\) is the vector of the flight attitude. For example, when the angle of the slope of the ground is \(10^\circ\), the flight attitude can be controlled as \(\mathbf{I} = (0, \sin(10^\circ), \cos(10^\circ))\) to adjust the landing on the uneven ground. When the rotorcraft is landing, the rotor would provide the lift force about \(2/3\) total weight. \(g\) denotes the gravitational acceleration. We defined \(m_{M} = \sum_{i=1}^{4} m_{pi} + \sum_{i=1}^{4} m_{di} + \sum_{i=1}^{4} m_{ni} + \sum_{i=1}^{4} m_{mi}\) as the total mass, then \(\frac{1}{2} m_{M} g <\mathbf{x}_m, \mathbf{I}>\) is term caused by the lift force. The damping energy of the rotorcraft in the landing process may be written as follows:

\[
D = 0.5 \sum_{i=1}^{4} c_{di} (v_{di}(t) - v_{pi}(t)) (r_{di})^2 + 0.5 \sum_{i=1}^{4} c_{ni} (v_{ni}(t) - v_{pi}(t)) (r_{ni})^2 + 0.5 \sum_{i=1}^{4} c_{mi} (v_{mi}(t) - v_{pi}(t)) (r_{mi})^2 + 0.5 \sum_{i=1}^{4} f_{ps} <v_{pi}(t), u_{pi}>^2
\]

where \(v_{pi}(t) = (v_{p1}(t), v_{p2}(t), v_{p3}(t))\), \(v_{di}(t) = (v_{di1}(t), v_{di2}(t), v_{di3}(t))\), \(v_{ni}(t) = (v_{ni1}(t), v_{ni2}(t), v_{ni3}(t))\), \(v_{mi}(t) = (v_{mi1}(t), v_{mi2}(t), v_{mi3}(t))\), \(i = 1, 2, 3, 4\) denote the velocity vectors of the bottom of the pad, the top of the pad, the supporting leg, and the thigh of each leg of the rotorcraft in the coordinate system on the ground, respectively. \(f_{ps}\), \(i = 1, 2, 3, 4\) is the friction coefficient. \(u_{pi} = (u_{p1i}(t), u_{p2i}(t), u_{p3i}(t))\), \(i = 1, 2, 3, 4\) is tangential direction vector of the sliding velocity. We defined \(\mathbf{x}(t) = (x_1(t), x_2(t), \ldots, x_{51}(t))^T\) as the global displacement vectors, where \(x_1 + 12(i-1)(j-1) = x_{pi}(t)\), \(x_j + 12(i-1)(j-1) = x_{di}(t)\), \(x_j + 12(i-1)(j-1) = x_{ni}(t)\), \(x_j + 12(i-1)(j-1) = x_{mi}(t)\), \(i = 1, 2, 3, 4\) denote the displacement vectors of the bottom of the pad, the top of the pad, the supporting leg, and the thigh of each leg of the rotorcraft in the coordinate system on the ground, respectively; \(\mathbf{r}_{di} = (r_{di1}, r_{di2}, r_{di3})\), \(\mathbf{r}_{ni} = (r_{ni1}, r_{ni2}, r_{ni3})\), \(\mathbf{r}_{mi} = (r_{mi1}, r_{mi2}, r_{mi3})\), \(i = 1, 2, 3, 4\) denote the direction vectors of the pad, the supporting leg, and the thigh of each leg of the rotorcraft in the above coordinate system, respectively. \(\mathbf{x}_m(t) = (x_{m1}(t), x_{m2}(t), x_{m3}(t))\) denotes the displacement vector of the body in the coordinate system. \(\mathbf{r}_m = (r_{m1}, r_{m2}, r_{m3})\), \(i = 1, 2, 3, 4\) is the direction from the center of the body to the top of the center of the thigh. \(\mathbf{I} = (l_1, l_2, l_3)\) is the vector of the flight attitude. According to the Lagrangian formulation, the dynamic model of the rotorcraft in the landing process can be described as follows:
\begin{align}
\frac{d}{dt} \left( \frac{\partial L}{\partial y_i(t)} \right) - \left( \frac{\partial L}{\partial x_j(t)} \right) + \left( \frac{\partial D}{\partial y_j(t)} \right) &= 0, \quad j = 1, 2, \ldots, 51 \\
\end{align}

where \( L = T - V \). Then we have the governing equation for the dynamic response of the rotorcraft in the landing process as follows

\[ Kx(t) + Cx(t) + M\dot{x}(t) = F(t) \]

where the matrix \( M = (m(i,j)) \) is the \( 51 \times 51 \) mass matrix, which is a diagonal matrix in which

\[ m(j + 12(i-1), j + 12(i-1)) = m_{pi}, \quad i = 1, 2, 3, 4, \] 
\[ j = 1, 2, 3; \]
\[ m(j + 3 + 12(i-1), j + 3 + 12(i-1)) = m_{ds}, \quad i = 1, 2, 3, 4, j = 1, 2, 3; \]
\[ m(j + 6 + 12(i-1), j + 6 + 12(i-1)) = m_{ri} , \quad i = 1, 2, 3, 4, j = 1, 2, 3; \]
\[ m(j + 9 + 12(i-1), j + 9 + 12(i-1)) = m_{ti} , \quad i = 1, 2, 3, 4, j = 1, 2, 3; \]
\[ m(j + 48, j + 48) = m_m, \quad j = 1, 2, 3. \]

The matrix \( K = (k(i,j)) \) is the \( 51 \times 51 \) stiffness matrix, where

\[ \begin{align*}
\tilde{k}(j + 12(i-1), k + 12(i-1)) &= k_{dr}p_{dj}p_{dk} + A_{ijk}, \\
\tilde{k}(j + 12(i-1), k + 3 + 12(i-1)) &= -k_{dr}p_{dj}p_{dik} + B_{ijk}, \\
\tilde{k}(j + 3 + 12(i-1), k + 12(i-1)) &= -k_{dr}p_{dj}p_{dik}, \\
\tilde{k}(j + 3 + 12(i-1), k + 3 + 12(i-1)) &= k_{dr}p_{dj}p_{dik}, \\
\tilde{k}(j + 6 + 12(i-1), k + 3 + 12(i-1)) &= -k_{sr}p_{sj}p_{sik}, \\
\tilde{k}(j + 6 + 12(i-1), k + 6 + 12(i-1)) &= k_{sr}p_{sj}p_{sik}, \\
\tilde{k}(j + 6 + 12(i-1), k + 6 + 12(i-1)) &= -k_{sr}p_{sj}p_{sik}, \\
\tilde{k}(j + 6 + 12(i-1), k + 6 + 12(i-1)) &= -k_{sr}p_{sj}p_{sik}, \\
\tilde{k}(j + 6 + 12(i-1), k + 6 + 12(i-1)) &= -k_{sr}p_{sj}p_{sik}. \\
\end{align*} \]

The matrix \( C = (\tilde{c}(i,j)) \) is the \( 51 \times 51 \) damping matrix, where

\[ \begin{align*}
\tilde{c}(j + 12(i-1), k + 12(i-1)) &= c_{dR}p_{dj}p_{dik} + D_{ijk}, \\
\tilde{c}(j + 12(i-1), k + 3 + 12(i-1)) &= -c_{dR}p_{dj}p_{dik} + E_{ijk}, \\
\tilde{c}(j + 3 + 12(i-1), k + 12(i-1)) &= -c_{dR}p_{dj}p_{dik}, \\
\tilde{c}(j + 3 + 12(i-1), k + 3 + 12(i-1)) &= c_{dR}p_{dj}p_{dik}, \\
\tilde{c}(j + 6 + 12(i-1), k + 6 + 12(i-1)) &= -c_{sr}p_{sj}p_{sik}, \\
\tilde{c}(j + 6 + 12(i-1), k + 3 + 12(i-1)) &= -c_{sr}p_{sj}p_{sik}, \\
\tilde{c}(j + 6 + 12(i-1), k + 6 + 12(i-1)) &= c_{sr}p_{sj}p_{sik}, \\
\tilde{c}(j + 6 + 12(i-1), k + 3 + 12(i-1)) &= c_{sr}p_{sj}p_{sik}, \\
\tilde{c}(j + 6 + 12(i-1), k + 6 + 12(i-1)) &= -c_{sr}p_{sj}p_{sik}, \\
\tilde{c}(j + 6 + 12(i-1), k + 3 + 12(i-1)) &= -c_{sr}p_{sj}p_{sik}. \\
\end{align*} \]

\[ F(t) = (f_1(t), f_2(t), \ldots, f_{51}(t))^T \]

is the load vector, where

\[ \begin{align*}
\tilde{f}_j &= 12(i-1)(t) = m_{pi}g l_j + C_{ij}, \quad i = 1, 2, 3, 4, j = 1, 2, 3 \\
\tilde{f}_j &= 12(i-1)(t) = m_{ds}g l_j, \quad i = 1, 2, 3, 4, j = 1, 2, 3 \\
\tilde{f}_j &= 12(i-1)(t) = m_{ri}g l_j, \quad i = 1, 2, 3, 4, j = 1, 2, 3 \\
\tilde{f}_j &= 12(i-1)(t) = m_{ti}g l_j, \quad i = 1, 2, 3, 4, j = 1, 2, 3 \\
\tilde{f}_j &= 12(i-1)(t) = m_{mi}g l_j, \quad i = 1, 2, 3, 4, j = 1, 2, 3, 4 \\
\tilde{f}_j &= 12(i-1)(t) = m_{mi}g l_j, \quad i = 1, 2, 3, 4, j = 1, 2, 3 \\
\tilde{f}_j &= 12(i-1)(t) = m_{mi}g l_j, \quad i = 1, 2, 3, 4, j = 1, 2, 3 \\
\tilde{f}_j &= 12(i-1)(t) = m_{mi}g l_j, \quad i = 1, 2, 3, 4, j = 1, 2, 3 \\
\end{align*} \]

In the equation (7)~(9), the terms \( A_{ijk}, B_{ijk}, C_{ij}, D_{ijk}, E_{ijk} \) are related to the friction between the pads and the ground, and the expressions of \( A_{ijk}, B_{ijk}, C_{ij}, D_{ijk}, E_{ijk} \) are given in equation (10). \( L = (l_{i1}(t), l_{i2}(t), l_{i3}(t)), l_{i1}, l_{i2}, l_{i3} \) is the normal direction vector of the sliding velocity, \( \mu \) denotes the friction coefficient between the pads and the ground.

It is to be noted that depending on the different uneven ground conditions and the accuracy of control, the rotorcraft may land with one leg, or two legs, or three legs, four legs. There are three cases for each leg. One case is that the leg does not contact with the ground; one case is that the leg contacts with the ground and does not have the sliding velocity, we have \( v_{pi}(t), u_{pi} \neq 0 \); another case is that the leg has contact with the ground and has the sliding velocity \( v_{pi}(t), u_{pi} \neq 0 \).
The relationships of contacts between the pads and the ground can be determined by the location of the bottom of the pad and the slope information of the uneven ground.

\[
A_{ijk} = \begin{cases} 
-\mu k_{ijk} u_{ij} < r_{ dik}, l_{ij} > r_{ dik}, < v_{p}(t), u_{p}> \neq 0 \text{ and contact} \\
-\kappa k_{ijk} u_{ij} < r_{ dik}, l_{ij} > r_{ dik}, < v_{p}(t), u_{p}> = 0 \text{ and contact} , \\
0, \text{ not contact} 
\end{cases}
\]

\[
B_{ijk} = \begin{cases} 
\mu k_{ijk} u_{ij} < r_{ dik}, l_{ij} > r_{ dik}, < v_{p}(t), u_{p}> \neq 0 \text{ and contact} \\
\kappa k_{ijk} u_{ij} < r_{ dik}, l_{ij} > r_{ dik}, < v_{p}(t), u_{p}> = 0 \text{ and contact} , \\
0, \text{ not contact} 
\end{cases}
\]

\[
C_{ij} = \begin{cases} 
-\mu c_{ijk} u_{ij} < l_{ij}, l_{ij} > r_{ dik}, < v_{p}(t), u_{p}> \neq 0 \text{ and contact} \\
-\mu c_{ijk} u_{ij} < l_{ij}, l_{ij} > r_{ dik}, < v_{p}(t), u_{p}> = 0 \text{ and contact} , \\
0, \text{ not contact} 
\end{cases}
\]

\[
E_{ijk} = \begin{cases} 
-\mu c_{ijk} u_{ij} < l_{ij}, l_{ij} > r_{ dik}, < v_{p}(t), u_{p}> \neq 0 \text{ and contact} \\
-\mu c_{ijk} u_{ij} < l_{ij}, l_{ij} > r_{ dik}, < v_{p}(t), u_{p}> = 0 \text{ and contact} , \\
0, \text{ not contact} 
\end{cases}
\]

\[
D_{ijk} = \begin{cases} 
-\mu c_{ijk} u_{ij} < l_{ij}, l_{ij} > r_{ dik}, < v_{p}(t), u_{p}> \neq 0 \text{ and contact} \\
-\mu c_{ijk} u_{ij} < l_{ij}, l_{ij} > r_{ dik}, < v_{p}(t), u_{p}> = 0 \text{ and contact} , \\
0, \text{ not contact} 
\end{cases}
\]

, i = 1, 2, 3, 4, j = 1, 2, 3, k = 1, 2, 3.

The maximum value of the friction coefficient \(\mu\) is about 0.2, for the rubber pad with the ground. Thus \(\gamma\) the angle of the sloped ground is not smaller than arctangent (0.2) about 11.3°. To ensure the safety of landing, the maximum angle of the sloped ground was set to 10°.

The dynamic analysis of the rotorcraft with robotic landing gear with uncertain-but-bounded parameters in the landing process on the uneven ground.

The stiffness matrix \(K\), the mass matrix \(M\), the damping matrix \(C\), and the loading vector \(F(t)\) can be expressed as functions of the structural parameter vector \(b = [b_1, b_2, ..., b_p]\) in which \(p\) is the number of uncertain parameters, that is

\[
K = K(b), \quad M = M(b), \quad C = C(b), \quad F(t) = F(t, b).
\]

Obviously, the structural displacement vector \(x(t)\) is also a function of the structural parameter vector \(b\), and it can be written as

\[
x(t) = x(t, b).
\]

In practical engineering, the parameters of the above model may be uncertain due to complexity of structures, the uneven ground conditions, the accuracy of control, and the inaccuracy in measurement, etc. For this reason, there are uncertainties in parameter vector \(b = [b_1, b_2, ..., b_p]\). We followed the thought of interval analysis theory \(^{17,18}\) and modeled the uncertainties as uncertain-but-bounded parameters. Then assumed that the structural parameter vector \(b = (b_1, b_2, ..., b_p)\) belongs to an interval vector \(b^i = [b^i_1, ..., b^i_p]\). The element of the vector \(b\) holds

\[
b_j \in b_j^i = [\underline{b}_j, \bar{b}_j], j = 1, ..., p, \quad (13)
\]

where \(b_j^i\) is an interval number, and \(\underline{b}_j, \bar{b}_j\) are the lower bound and upper bound of \(b_j^i\), respectively. Then the differential equation of dynamical structures with uncertain parameters may be rewritten as follows

\[
K(b)x(t, b) + C(b)\dot{x}(t, b) + M(b)x(t, b) = F(t, b) \quad (14)
\]

As the uncertainties in \(K(b), M(b), C(b),\) and \(F(t, b)\) are caused by the interval parameter vector \(b\), then the elements of \(K(b), M(b), C(b),\) and \(F(t, b)\) vary within a deterministic interval. Therefore, the dynamic response \(x(t, b)\) of the system (14), which is a function of \(b_j, j = 1, ..., p,\) also varies within a deterministic interval \(x(t, b) = [\underline{x}_j(t), \bar{x}_j(t)]\).

Based on interval mathematics, the nominal value vector of the interval structural parameter vector \(b^i = [b^i_1, ..., b^i_p]\) is denoted by

\[
b^o = [b^o_1, ..., b^o_p],
\]

where \(b_j^o = 0.5(\underline{b}_j + \bar{b}_j), j = 1, ..., p.\) And the uncertain radius vector of the interval structural parameter vector \(b^i\) is denoted by

\[
\Delta b = (\Delta b_1, \Delta b_2, ..., \Delta b_p),
\]

where \(\Delta b_j = 0.5(\bar{b}_j - \underline{b}_j), j = 1, ..., p.\) Therefore, the structural parameter vector \(b\) can be decomposed into the sum of the nominal value vector \(b^o = [b^o_1, ..., b^o_p]\) and the deviation vector \(\delta = [\delta_1, ..., \delta_p], i.e.

\[
b = b^o + \delta,
\]

or the component form

\[
b_j = b^o_j + \delta_j, j = 1, ..., p,
\]

where \(\delta_j \in [\delta^o_j, \delta^o_j]\) is uncertainty of \(b_j.\) Using the Taylor series expansion, the dynamic response \(x(t, b)\) about \(b^o = [b^o_1, ..., b^o_p]\) was developed as

\[
x(t, b) = x(t, b^o) + \sum_{j=1}^p \frac{\partial x(t, b^o)}{\partial b_j} \delta_j
\]

\[
\dot{x}(t, b) = \dot{x}(t, b^o) + \sum_{j=1}^p \frac{\partial \dot{x}(t, b^o)}{\partial b_j} \delta_j
\]

\[
\ddot{x}(t, b) = \ddot{x}(t, b^o) + \sum_{j=1}^p \frac{\partial \ddot{x}(t, b^o)}{\partial b_j} \delta_j
\]
where \( x(t, b') \) can be obtained by substituting \( b = b' \) into equation (14). The first-order partial derivative \( \partial x(t, b') / \partial b_j \), \( \partial x(t, b') / \partial b_j \), \( \partial x(t, b') / \partial b_j \) can be obtained by
\[
K(b) \frac{\partial x(t, b)}{\partial b_j} + C(b) \frac{\partial x(t, b)}{\partial b_j} + M(b) \frac{\partial x(t, b)}{\partial b_j} = R_i(t, b),
\]
where
\[
R_i(t, b) = \frac{\partial F(t, b)}{\partial b_j} - \frac{\partial K(b)}{\partial b_j} x(t, b) - \frac{\partial C(b)}{\partial b_j} \dot{x}(t, b) - \frac{\partial M(b)}{\partial b_j} \ddot{x}(t, b).
\]
Substituting \( b = b' \) into equation (20) and equation (21), based on the dynamic iteration method, we can obtain \( \partial x(t, b') / \partial b_j \), \( \partial x(t, b') / \partial b_j \), \( \partial x(t, b') / \partial b_j \), and the intervals of the responses as follows
\[
x(t, b) = x(t, b') + \sum_{j=1}^{p} \frac{\partial x(t, b')}{\partial b_j} \delta_j^i,
\]
\[
\dot{x}(t, b) = \dot{x}(t, b') + \sum_{j=1}^{p} \frac{\partial \dot{x}(t, b')}{\partial b_j} \delta_j^i
\]
\[
\ddot{x}(t, b) = \ddot{x}(t, b') + \sum_{j=1}^{p} \frac{\partial \ddot{x}(t, b')}{\partial b_j} \delta_j^i
\]

Although there are the uncertainties in the structural parameters, it is of interest to find the effect of the uncertainties in the angles around the hip joints and the knee joints than the effect of the uncertainties in the structural parameters. The first-order partial derivative about the angles around the hip joints and the knee joints may be obtained by the expression as follows
\[
\frac{\partial K}{\partial \gamma_{st}} = \left( \frac{\partial k(i, j)}{\partial \gamma_{st}} \right), \quad i, j = 1, 2, 3 \ldots 51, s, t = 1, 2,
\]
\[
\frac{\partial C}{\partial \gamma_{st}} = \left( \frac{\partial c(i, j)}{\partial \gamma_{st}} \right), \quad i, j = 1, 2, 3 \ldots 51, s, t = 1, 2.
\]

**Numerical examples and discussion**

**Example 1: The dynamic response of the rotorcraft with the robotic landing gear in the landing process**

We assumed that \( m_{ri} = 0.04 \text{kg}, m_{gi} = 0.04 \text{kg}, m_{si} = 0.2 \text{kg}, \) \( m_i = 0.32 \text{kg}, i = 1, 2, 3, 4. \) \( m_m = 6.1 \text{kg} \) \( k_{di} = 9 \times 10^3 \text{Nm}, k_{si} = 6 \times 10^3 \text{Nm}, k_{ai} = 10^4 \text{Nm}, \) \( i = 1, 2, 3, 4. \) \( c_{di} = 0.02 \text{Ns/m}, i = 1, 2, 3, 4, \) \( c_{si} = 0.01 \text{Ns/m}, c_a = 0.01 \) \( k_{ai}, c_m = 0.01 \text{k/m}, i = 1, 2, 3, 4. \) The length of each thigh is 0.2 m. The length of each supporting leg is 0.08 m. The length of each pad is 0.05 m. The length between the mass of each thigh and the mass of the body is 0.2 m. \( r_{m1} = (0, \cos (-10^6), \sin (-10^6)), \) \( r_{m2} = (\cos (-10^6), 0, \sin (-10^6)), \) \( r_{m3} = (0, -\cos (-10^6), \sin (-10^6)), \) \( r_{m4} = (-\cos (-10^6), 0, \sin (-10^6)). \) When the angle of the sloped uneven ground is 8°, the flight attitude can be controlled to adjust the landing on the uneven ground by servomotors in the hip joints and the knee joints.

**Comparison of the dynamic responses with the different legs.** First we considered comparison of the dynamic responses of the rotorcraft with the different number of legs contacting with the ground in the landing process. Set the initial velocity of the rotorcraft as 0.5 \text{m/s}; set the angle of the attack of the rotorcraft as 10°; set the angle of the sloped ground as 10°.

Figures 3 to 5 show the comparisons of the dynamic displacement responses, the velocity responses, and the acceleration responses in the direction of the z-axis of the pad A with the different number of legs, where the pad A is the pad of the leg which is landing on the ground. It can be seen from the above figures that the dynamic displacement responses, the velocity responses, and the acceleration responses decrease as the number of legs are increased from one to four. The magnitudes of the responses for the case of the landing with one leg are very great then other cases.

Figures 6 and 7 show the comparisons of velocity responses, and the acceleration responses of the body with the different number of legs. Thus, in order to maintain the safety, the rotorcraft robotic landing gear would be controlled to adjust the condition of the ground to insure the landing with four legs on the ground at the same time.

**Comparison of the dynamic responses with the different initial velocities.** Then we considered the comparison of the dynamic responses of the rotorcraft robotic landing gear with the different initial velocities in the landing process. Set the angle of the attack of the rotorcraft as 10°; set the angle of the sloped ground as 10°; set the number of legs as four.

Figures 8 and 9 show the comparisons of the velocity responses and the acceleration responses in the direction of the z-axis of the pad A with the different initial velocities. It can be seen from the above figures that the velocity responses and the acceleration responses increase as the initial velocity is increased from 0.2 to 2 \text{m/s}. When the initial velocity is equal to 2 \text{m/s}; the maximum acceleration is greater than 16.1 \text{m/s}^2. For the rotorcraft in this study, the maximum acceleration which the supposing legs suffer is 18 \text{m/s}^2. To maintain or improve the safety, the initial velocities are restricted to be less than 2 \text{m/s}. 

Figure 3. Comparison of the dynamic displacement responses of the pad A with the different legs.

Figure 4. Comparison of the velocity responses of the pad A with the different legs.

Figure 5. Comparison of the acceleration responses of the pad A with the different legs.

Figure 6. Comparison of the velocity responses of the body with the different legs.

Figure 7. Comparison of the acceleration responses of the body with the different legs.

Figure 8. Comparison of the velocity responses of the pad A with the different velocities.
Comparison of the dynamic responses with the different angles of the sloped ground. Then we considered the dynamic responses of the rotorcraft with robotic landing gear with different angles of the sloped ground from $0^\circ$ to $10^\circ$ in the landing process. Set the angle of the attack of the rotorcraft as $10^\circ$, set the number of legs as four. Set the initial velocity of the rotorcraft as $1 \text{ m/s}$. The flight attitude can be controlled to adjust the landing on the uneven ground by servomotors in the hip joints and the knee joints. Figures 10 to 12 show the comparisons of the dynamic displacement responses, the velocity responses, and the acceleration responses in the direction of the $z$-axis of the pad A with the different angles of the sloped ground.

It can be seen from the above figures that if the flight attitude of the rotorcraft can be controlled to adjust the landing on the uneven ground accurately, the dynamic displacement responses, the velocity responses, and the acceleration responses do not have the significant difference as the angles of the sloped ground is increased from $0^\circ$ to $10^\circ$.

Comparison of the dynamic responses with different the angles of attack. Then we consider the dynamic responses of the rotorcraft with the robotic landing gear with different the angles of attack from $0^\circ$ to $10^\circ$ in the landing process. Set the angle of the sloped ground as $10^\circ$; set the number of legs as four; set the initial velocity of the rotorcraft as $1 \text{ m/s}$. The flight attitude can be controlled to adjust the landing on the uneven ground by servomotors in the hip joints and the knee joints. Figures 13 to 15 show the comparisons of the dynamic displacement responses, the velocity responses, and the
acceleration responses in the direction of the z-axis of the pad A with different degrees of the angles.

It can be seen from the above figures that the dynamic displacement response, the velocity responses, and the acceleration responses does not have the significant difference as the angle of the attack of the rotorcraft is increased from $0^\circ$ to $10^\circ$.

Figure 16 shows that when the angle of the attack of the rotorcraft is less than $8^\circ$, the sliding distance slowly increases as the angle of the attack of the rotorcraft is increased. When the angle of the attack of the rotorcraft is greater than $8^\circ$, the sliding distance quickly increases as the degree of the angle of the attack of the rotorcraft is increased.

Example 2: The dynamic response of the rotorcraft robotic landing gear with uncertain-but-bounded parameters in the landing process

Example 2 is similar to example 1, except that uncertain-but-bounded parameters is considered in this example. When the angle of the sloped ground is $10^\circ$, the flight attitude can be controlled to adjust the landing on the uneven ground by servomotors in the hip joints and the knee joints. For example, the angles around the hip joints and the knee joints can be controlled as $\gamma_{11} = 0, \gamma_{21} = 0, \gamma_{31} = 25^\circ, \gamma_{41} = 25^\circ, \gamma_{12} = 0^\circ, \gamma_{22} = 0^\circ, \gamma_{32} = 10^\circ, \gamma_{42} = 10^\circ$ to adjust the landing on the uneven ground. In practical engineering, the above angles have uncertainties, the angles would be modeled...
as, \( \gamma_{11} = 0^0 + [-\theta, + \theta] \), \( \gamma_{21} = 0^0 + [-\theta, + \theta] \), \( \gamma_{31} = 25^0 + [-\theta, + \theta] \), \( \gamma_{41} = 25^0 + [-\theta, + \theta] \), \( \gamma_{12} = 0^0 + [-\theta, + \theta] \), \( \gamma_{22} = 0^0 + [-\theta, + \theta] \), \( \gamma_{32} = 10^0 + [-\theta, + \theta] \), \( \gamma_{42} = 10^0 + [-\theta, + \theta] \), where \( \theta = j^0, j = 0, 1, \ldots, 12 \) is the uncertain factor.

Figures 17 to 19 show the comparisons of the bounds of the dynamic displacement responses, the velocity responses, and the acceleration responses in the direction of the z-axis of the pad A with the different uncertain factors. It can be seen from the above figures that the bounds of the dynamic displacement response, the velocity responses with \( \theta = 8^0 \) are obviously more large then the bounds with \( \theta = 4^0 \). The widths of the bounds of the dynamic displacement response, the velocity responses with \( \theta = 8^0 \) are obviously more large then the widths of the bounds with \( \theta = 4^0 \). Figure 19 shows that the upper bound of the acceleration response with \( \theta = 8^0 \), is more large then the upper bound with \( \theta = 4^0 \). But lower bound of the response for the case with \( \theta = 8^0 \) and the case with \( \theta = 4^0 \) are the same, which is equal to the lower bound of the response without considering uncertainties.

Figures 20 and 21 show the comparisons of the bounds of the dynamic displacement responses and the acceleration responses of the pad A corresponding to uncertain factor \( \theta \). As can be seen from the figures above, the upper bounds of the dynamic displacement response and the acceleration response increase as the uncertain factor is increased. But lower bounds of the responses are unchanged, which is equal to the lower bounds of the responses without considering uncertainties.
These numerical results reveal that the large deviations of the angles around the hip joints and the knee joints would lead to drastic increases of the dynamic displacement response and the acceleration response of the pad, which may resulted in an overload of the rotorcraft.

Conclusions

In order to avoid the hard landing on the uneven ground which is resulted in serious fuselage damage, structural failure, or cargo damage, the robotic landing gears of the rotorcraft are designed. Thus the landing attitude can be changed by controlling the servomotors to change the angles of the thigh and the supporting leg of the robotic landing gears to adjust the landing condition of the uneven ground. Depending on the different uneven ground conditions and the accuracy of control, the rotorcraft may land with different number of legs and different landing conditions. In this work, based on the Lagrangian formulation, the dynamic model of the rotorcraft with different number of legs contacting with the ground in the landing process was derived, where the angle of the sloped ground and the angle of the attack of the rotorcraft were considered. Then, for estimation of the effect of the uncertainties on the response, an approach was presented to determine the lower bound and the upper bound of the dynamic response of the rotorcraft with uncertainties parameters. The results of numerical examples indicate that: (1) Under the same initial condition, the dynamic displacement responses, the velocity responses, and the acceleration responses decrease as the number of legs are increased from one to four. The magnitudes of the responses for the case of the landing with one leg are very great then other cases. (2) In order to maintain the safety, the rotorcraft robotic landing gear would be controlled to adjust the condition of the ground to insure the landing with four legs on the ground at the same time. (3) To maintain or improve the safety, the initial velocities are restricted to be less than certain value; (4) If the rotorcraft robotic landing gear is controlled to adjust the condition of the ground, the dynamic displacement response, the velocity responses, and the acceleration responses does not have the significant difference with different angles of the sloped ground and different angles of attack. (5) The large deviations of the angles around the hip joints and the knee joints would lead to drastic increases of the dynamic displacement response and the acceleration response of the pad, which may resulted in an overload of the rotorcraft.

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ORCID iD

Lianquan Wu https://orcid.org/0000-0002-2025-7466

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