Vehicle Mass Identification Based on Two-axle and Two-mass Vibration Model

Zhenyu Wang¹, Ming Li² and Haoyu Wang³

¹-³ School of automotive engineering, Wuhan University of Technology, Wuhan, Hubei, 430070, China

*Corresponding author’s e-mail: 2640783420@qq.com

Abstract. The mass of the vehicle is an important parameter, through which many other performance indicators of the vehicle can be measured. The traditional method of vehicle mass identification is mainly through fixed platform detection station and weighing in motion. In this paper, a two-axle and two-mass vibration model is established to identify the vehicle mass, which takes the free vibration frequency of suspension as input and outputs the vehicle mass. Firstly, when the vehicle passes a specific excitation, the two-mass model composed of suspension and wheels generates free vibration, and then its main frequency is obtained. And then the mathematic relation between the vehicle mass and the main frequency is established through the two-axle vibration model, and the vehicle mass is obtained. Simulation in TruckSim shows that the average relative error between the identified mass and the actual mass is 6.27%. In practical application, the free vibration frequency of the suspension can be obtained by image recognition or displacement sensor, so as to identify the vehicle mass.

1. Introduction

Vehicle mass is a significant parameter of vehicles. It can be used for vehicle overload identification and intelligent transportation system [1]. The current methods to identify mass mainly cover the embedded method and the non-embedded method [2]. Embedded means to add weight measurement equipment inside the vehicle, while non-embedded means to carry out contact or non-contact weight measurement outside the car.

In the research of embedded method, Phong x. Nguyen has proposed a method based on the relationship between vehicle’s vertical acceleration and mass to identify the vehicle mass through the accelerometer on the phone [2]. And Rongjun Zhang has used on-board sensors to detect the deformation in a certain area of the axle, and converted the strain into the electrical signal representing the axle mass of the vehicles [3]. Furthermore, Li Hui-min has designed a resistance displacement sensor to measure the distance between the vehicle body and the axle, converted the displacement into vehicle mass signal by Hooke Law, and modified the results with inclination Angle, speed and acceleration sensors to apply to different conditions [4].

The non-embedded method is mainly to identify the vehicle mass statically by placing loadometer on the road surface at a fixed position, which is also the principle method of vehicle mass recognition at present. However, this method will affect the efficiency of the road. Therefore, weighing in motion (WIM) method has become the research direction of many scholars. But in the process of passing the dynamic weighing station, the speed and acceleration of the vehicle have a great disturbance to the weight measurement. The average error of the measured axle weight ranges from 5% to 30%, and the
corresponding confidence is 90% or 95% [5]. Besides, it is mainly suitable for low-speed occasions. Artur Andrzejczak has designed a loadometer system using piezoelectric modules, the main innovation of whom is to transmit the results of weight measurement through wireless transmission [6]. Similarly, Mohamed Rehan Karim has studied the WIM dynamic weighing system with an error of ±10% of the static weighing [7]. Moreover, Bolesław Mazurek has designed an inclined plane weighing system based on the principle of deformation-to-piezoelectric signal, which can convert the momentum of the vehicle into the electrical signal by PVDF sensor [8]. Furthermore, Shaokang Xiong has established the relationship between the maximum overshoot in vibration and the static vehicle mass based on BP neural network algorithm [9]. In this paper, a Single-mass vibration model is used, and they assume that the stiffness and damping coefficient of suspension of different vehicles are the same, so the relationship between the maximum overshoot and vehicle mass is linear. In fact, suspension of different automotive are different, ignoring the impact of suspension in practical application is a big deviation.

For vehicle vibration theory is applied to identification of vehicle mass, Benjamin Pence presents a base-excitation model of vertical ride dynamics which treats the unsprung vertical accelerations as the ride dynamics model input [10]. And it employs methods based on polynomial chaos theory and on the maximum likelihood approach to estimate the most likely value of the vehicle sprung mass. It is through the vertical acceleration to extract the characteristic parameters of the vehicle, however, there are few researches on the identification of vehicle mass by the free vibration frequency of suspension and wheel.

Based on the above analysis, a two-axle and two-mass vehicle vibration model is established in this paper. Through this vibration model, the paper establishes a functional relationship between the vehicle mass and the suspension frequency, taking the suspension free vibration frequency as the input, and outputting the vehicle mass. Eventually, the identified mass and the actual mass are compared by kinetics simulation, so as to verify the feasibility of identifying the vehicle weight based the two-axle and two-mass vibration model.

2. Model derivation
Model derivation consists of three parts: postulated conditions, single axle and two-mass vibration model, two-axle and two-mass vibration model. The differential equation considering the coupling vibration of the suspension and the wheel is deduced in the single axle and two-mass vibration model, and the main frequency is solved. Two-axle and two-mass vibration model considers the mass center position of the vehicle, and the mass is obtained eventually.

2.1. Postulated conditions
First, it is necessary to set the postulate conditions to simplify the complex vehicle structure. The postulate conditions are as follows:

1. The vehicle is simplified into a 6-DOFs vehicle vibration model, including vertical vibration of \( m_{2f}, m_{2c}, m_{2r}, m_{1f}, m_{1r} \) and longitudinal vibration of \( \varphi \). In addition, it is assumed that the irregularity
of the right and left ruts is the same, in other words, the vehicle is symmetric with its longitudinal axis, so the body does not have roll vibration, lateral displacement and yaw vibration. Therefore, only the one side of the vehicle vibration needs to be considered, as shown in figure 1.

2. Ignore the excitation effect of the engine in the vehicle, and only consider the vibration caused by the road surface.

3. The rear wheel runs on a rut, which is the same as the front wheel, so the excitation of the front and rear wheels is the same except for the difference caused by the axle base.

2.2. Single-axle and two-mass vibration model

![Diagram of single-axle and two-mass vibration model](image)

In figure 2, \( m_2 \) is the sprung mass, \( m_1 \) is the unsprung mass, \( K \) is the suspension stiffness, \( C \) is the shock absorber damping coefficient, \( K_t \) is the wheel stiffness. The wheel damping is ignored, because that is too small. \( q \) is the road roughness function.

In this model, the free vibration generated by passing across a certain excitation is studied, and it can be considered that the roughness of the pavement \( q = 0 \). The vertical displacement coordinates of the wheel and the body are \( z_1 \) and \( z_2 \) respectively. The origin of coordinates is selected at their respective equilibrium positions, so the free vibration motion equation is [11]:

\[
\begin{align*}
\{m_2 \ddot{z}_2 + C(\dot{z}_2 - \dot{z}_1) + K(z_2 - z_1) \} &= 0 \\
\{m_1 \ddot{z}_1 + C(\dot{z}_1 - \dot{z}_2) + K(z_1 - z_2) + K_t z_2 \} &= 0
\end{align*}
\]

(1)

2.2.1. Suspension vibration model

As can be seen from the equations of motion, the vibrations of \( m_1 \) and \( m_2 \) are coupled with each other. If \( m_1 \) is not moved \((z_1 = 0)\), we can write:

\[
m_2 \ddot{z}_2 + C \ddot{z}_2 + Kz_2 = 0
\]

(2)

This corresponds to the single freedom vibration of the vehicle body \( m_2 \). Assume \( 2n = C/m_2 \), \( \omega_0^2 = K/m_2 \), then equation (2) can be turned into:

\[
\ddot{z}_2 + 2n \dot{z}_2 + \omega_0^2 z_2 = 0
\]

(3)

The solution of the differential equation is:

\[
z = Ae^{-nt} \sin(\sqrt{\omega_0^2 - n^2} t + \alpha)
\]

(4)

This solution shows that, in the case of damped free vibration, \( m_2 \) vibrates at the damped natural frequency \( \omega_r = \sqrt{\omega_0^2 - n^2} \). This equation can be reduced to \( \omega_r = \omega_0 \sqrt{1 - \zeta^2} \). The damping ratio of automobile suspension system \( \zeta \) is usually around 0.25, being small damping. \( \omega_r \) only decreases by about 3% compared with \( \omega_0 \), it can be approximately considered that \( \omega_r \approx \omega_0 \), so the natural frequency \( \omega_r = \sqrt{K/m_2} \) [11].

2.2.2. Wheel vibration model

If \( m_2 \) does not move \((z_2 = 0)\), we can write:

\[
m_1 \ddot{z}_1 + C \ddot{z}_1 + (K + K_t)z_1 = 0
\]

(5)
Since both the equation (5) and equation (2) are second order homogeneous differential equations, the solution method is the same, and the natural frequency of the wheel part can be obtained by the same method [11]:

\[ \omega_t = \sqrt{(K + K_t)/m_1} \]  

(6)

2.2.3. Two-mass vibration model

The \( \omega_0 \) and \( \omega_t \) obtained above are partial frequencies, and the two vibration are mutually interfering, but if the two vibration are considered comprehensively, the main mode of the two vibration needs to be obtained.

As mentioned above, damping ratio has little effect on frequency, so the damping coefficient \( C \) can be ignored. \( m_1 \) and \( m_2 \) vibrate in simple harmonic mode with frequency of \( \omega_t \) and \( \omega_0 \) respectively, amplitude for \( z_{10} \) and \( z_{20} \). Because the phase Angle will not have an impact on the calculation results, so set the phase Angle is \( \varphi \). We can write:

\[
\begin{cases}
\dot{z}_1 = z_{10} + j\omega_0 z_{10} \\
\dot{z}_2 = z_{20} + j\omega_t z_{20}
\end{cases}
\]

(7)

Because of \( \omega_0 = \sqrt{K/m_2} \), \( \omega_t = \sqrt{(K + K_t)/m_1} \), substitute the above two solutions into the differential equations (1) and get:

\[
\begin{align*}
(\omega_0^2 - \omega^2)z_{20} - \omega_0^2 z_{10} &= 0 \\
-K \frac{m_1}{m_2} z_{20} + (\omega_t^2 - \omega^2)z_{10} &= 0
\end{align*}
\]

\[
\begin{pmatrix}
\omega_0^2 - \omega^2 \\
-K \frac{m_1}{m_2} \omega_0^2
\end{pmatrix}
\begin{pmatrix}
z_{20} \\
z_{10}
\end{pmatrix} = 0
\]

(8)

The condition of the matrix equation having a non-zero solution is the determinant of the coefficients of \( z_{10} \) and \( z_{20} \) is zero. We can write:

\[
(\omega_0^2 - \omega^2)(\omega_t^2 - \omega^2) - \omega_0^2 \frac{K}{m_2} = 0
\]

(9)

Equation 9 is the frequency equation of a two-mass system, whose two roots are the square of the main frequencies \( \omega_1 \) or \( \omega_2 \):

\[
\omega_i^2 = \frac{1}{2} (\omega_t^2 + \omega_0^2) - \frac{1}{4} (\omega_t^2 + \omega_0^2)^2 - \frac{KK_t}{m_2 m_1}
\]

(10)

Low main frequency \( \omega_1 \) is close to \( \omega_0 \), high main frequency \( \omega_2 \) is close to \( \omega_t \), and there is a relationship between them: \( \omega_1 < \omega_0 < \omega_t < \omega_2 \).

Whether the main frequency is \( \omega_1 \) or \( \omega_2 \) is determined by the amplitude of the vehicle body and the wheel. In the case of forced vibration, when the excitation frequency \( \omega \) is close to \( \omega_1 \), low-frequency resonance is produced and the amplitude of the body mass \( m_2 \) is nearly 10 times larger than that of the wheel mass \( m_1 \), known as the body type vibration. When the excitation frequency \( \omega \) is close to \( \omega_2 \), high-frequency resonance is produced and the amplitude of the wheel mass \( m_1 \) is nearly 100 times larger than that of the vehicle mass \( m_2 \), known as wheel type vibration.

Because the stiffness of suspension is much smaller than that of wheels, and the excitation frequency is close to 0 in the case of free vibration. Therefore, the vibration amplitude of wheels is smaller than that of the body, generating low-frequency resonance and the main frequency is \( \omega_1 \). We can write:

\[
\omega_1^2 = \frac{1}{2} (\omega_t^2 + \omega_0^2) - \frac{1}{4} (\omega_t^2 + \omega_0^2)^2 - \frac{KK_t}{m_2 m_1}
\]

(11)

It can be seen that through the stiffness and free vibration frequency of the suspension and the wheel, as well as the sprung mass and unsprung mass, the main frequency of the free vibration of the single-axle and two-mass vibration model can be obtained. The solution condition is that the sprung mass is known, but the sprung mass is the variable to be solved, so it is necessary to introduce the two-axle and two-mass Vibration Model.
2.3. Two-axle and two-mass vibration model

Since the amplitude of the wheel is much smaller than that of the suspension, and the main frequency of the suspension and the wheel has been calculated, the influence of the wheel is ignored when solving the sprung mass. Therefore, in figure 1, $K_{1f}$ and $K_{1r}$ are ignored. The sprung mass $m_2$ is decomposed according to equation (12-14) into: concentrated mass $m_{2f}$ allocated to the front axle, connection mass $m_{2c}$ allocated to the center of mass, and concentrated mass $m_{2r}$ allocated to the rear axle, which are connected by a massless rigid rod. In equation (13), $a$ and $b$ are the horizontal distance from the front axle and the rear axle to the center of mass of the vehicle, and $L$ is the axle base of the vehicle. In equation (14), $I_y$ is the moment of inertia of the vehicle body around the Y-axis, and $\rho_y$ is the turning radius of the horizontal Y-axis. The Y-axis passes the center of mass of the vehicle and is perpendicular to the side of the body.

\[ m_{2f} + m_{2r} + m_{2c} = m_2 \]  
(12)

\[ m_{2f}a - m_{2r}b = 0 \]  
(13)

\[ I_y = m_2\rho_y^2 = m_{2f}a^2 + m_{2r}b^2 \]  
(14)

We can write the moment balance equation for the front and rear axles:

\[
\begin{align*}
( m_{2f} \ddot{z}_{2f} + m_{2c} b ( \dot{z}_{2c} a + \ddot{z}_{2f} b ) ) / L^2 + K_{2f} z_{2f} &= 0 \\
( m_{2r} \ddot{z}_{2r} + m_{2c} a ( \dot{z}_{2c} a + \ddot{z}_{2f} b ) ) / L^2 + K_{2r} z_{2r} &= 0
\end{align*}
\]  
(15)

When only the front wheel passes across the excitation, namely, only $z_{2f}$ moves, and $z_{2r} = 0$, the natural circular frequency of the front-end system is:

\[ \omega_{nf}^2 = \frac{K_{2f} L^2}{m_2 (\rho_y^2 + b^2)} \]  
(16)

$K_{2f}$ is the suspension stiffness of the front axle, $L$ is the axle distance of the vehicle, $\rho_y$ is the rotation radius around the transverse axis $y$. $\omega_{nf}$ is equal to the main frequency $\omega_1$ of the two-mass vibration model mentioned above.

Simultaneous equations (11) and (16), and the sprung mass $m_2$ is obtained. Because the unsprung mass of a kind of vehicle is the same, it can be obtained by referring to relevant automobile manuals. The total mass of the vehicle can be obtained by adding the obtained sprung mass and the checked unsprung mass.

3. Simulation analysis

In order to verify whether the established two-axle and two-mass model can correctly identify the mass of the vehicle, we carry out simulation test by TruckSim. TruckSim is a simulation software for researching the dynamics of commercial vehicles. It has a rich database of vehicles and working conditions, and users can flexibly adjust the structure of vehicles, dynamic performance and working conditions according to their own needs. These characteristics make TruckSim a popular choice for automotive engineers to design their vehicles, as well as helping automobile companies such as GM, Mercedes-Benzes. In this paper, the two-axle commercial vehicle is simulated by TruckSim, and the mass of the vehicle is identified according to the simulation data, and then compared with the actual mass, so as to verify the feasibility of two-axle and two-mass model to identify the mass of the vehicle. Firstly, the vehicle model is established and the commercial vehicle bus is selected as the simulation vehicle. The main parameters of bus are shown in the table 1.

| parameter                  | value |
|----------------------------|-------|
| Sprung mass (kg)           | 6360  |
| Unsprung mass of front axle (kg) | 570   |
| Unsprung mass of rear axle (kg) | 760   |
| Suspension spring rate (N/mm) | 250   |
| Tire spring rate (N/mm)    | 980   |

Table 1. The main parameters of bus.
Then, road and excitation model are established, and smooth concrete pavement is selected for the road. The properties of the road surface are shown in table 2.

Table 2. Main parameters of road.

| Parameters                      | Value |
|---------------------------------|-------|
| Friction                        | 0.85  |
| Tire rolling resistance coefficient | 1     |
| Path Elevation                  | Flat  |

The excitation causes the vehicle to vibrate freely. In this paper, the standard of speed bump is adopted to design the excitation model [12]. The cross section of the speed bumps is shown in figure 3.

Figure 3. Cross section of excitation during simulation.

In the simulation, the driver is set to drive at a constant speed across the road covered with speed bump. The slow speed will lead to excessive lateral vibration of the vehicle, making the vertical vibration of the left and right suspensions inconsistent. If the vehicle is too fast, the vibration of the front suspension will end prematurely, which will be insufficient, and the fitting effect will be poor. Through several simulation, it is found that a speed of about 10km/h can keep the vertical vibration of the left and right suspension consistent, and the vibration is relatively sufficient. So, the driver is controlled to travel at 10km/h.

The figure of the compression of the front suspension across the excitation is shown in figure 4. Before 3.6s, the front suspension compression remains unchanged, and the vehicle is driving on a flat road. At 3.6s, the front wheel hits the excitation, and the suspension begins to compress. The compression of the suspension becomes larger. At 3.9s, the compression of the suspension reaches the minimum value, which means that the front wheel returns to the ground, and the front suspension begins to vibrate freely. The figure presents a sinusoidal curve of attenuation. At 5.4 seconds, the rear wheel hits the excitation. Obviously, the vibration of the rear wheel will interfere with the vibration of the front suspension and ruin its original sinusoidal attenuation curve.

Figure 4. front suspension compression during driving.
Therefore, during 3.9s-5.4s, the front suspension is free to vibrate, and the compression of the suspension presents a relatively regular sinusoidal attenuation curve. The compression data of the front suspension during the free vibration is imported into MATLAB and fitted with equation (4), and the natural frequency of the front suspension $\omega_0$ is 12.63, whose range is [12.15, 13.11], as shown in figure 5.

Since the unsprung mass of a kind of automobile is determined when it is designed and manufactured, it can be searched from the vehicle database. Thus, the natural frequency of the wheel $\omega_t$ can be obtained according to equation 6, which is 46.45.

According to table 1, the position of the center of mass and radius of rotation of the vehicle can be known, and the simultaneous equation (11) and (16) can be used to obtain the main frequency $\omega_0$ of the front suspension to be 11.18 and the sprung mass $m_2$. The fitting front suspension frequency $\omega_0$, the corresponding mass $m_2$ and the relative error are shown in table 3.

| $\omega_0$ | Theoretical value of $m_2$ (kg) | Relative error |
|------------|---------------------------------|----------------|
| Optimal fit | 12.63                           | 5961.3         | 6.27%           |
| Left boundary | 12.15                           | 6433.7         | 1.16%           |
| Right boundary | 13.11                           | 5540           | 12.89%          |
| Minimum error | 12.22                           | 6361.3         | 0.02%           |

It can be obtained from table 3 that the best fit value of $\omega_0$ is 12.63, and the relative error of the corresponding vehicle mass identification value is 6.27%, which can meet the engineering requirements.

The error is mainly caused by the interference effect of tire on suspension vibration. As a result, the actual vibration curve is not the standard sinusoidal attenuation curve. Therefore, there is certain error during the fitting. The following work should be carried out to improve the mathematical model, so that the fitted results better reflect the vibration frequency of suspension.

4. Conclusion

In this paper, a two-axle and two-mass model is proposed to obtain the mass of vehicle. When the front wheel of an automobile passes across a specific excitation, the two-mass model composed of the front suspension and the front wheel generates free vibration. First, the main frequency of the two-mass model is obtained. Then, the mathematic relation between the mass of the vehicle and the main frequency is established through the two-axle vibration model, so as to obtain the mass of the vehicle. The simulation result shows that it is feasible to identify vehicle mass by suspension frequency, and the error is 6.27%.
It should be pointed out that the application of this theory needs to know the sprung mass, the stiffness of the suspension and the tire, and the location of the center of mass. In practical application, the location of the center of mass is difficult to obtain, and other parameters can be obtained by establishing a database. If the influence of the center of mass is ignored, the mass of front and rear axle can be solved by using the single-axle and two-mass model, and then two mass can be added to be gross mass. In this way, the mass can also be solved, but the error will be improved. In practical application, the method of image recognition or displacement sensor is used to identify the free vibration frequency of the suspension, and then the mass of the vehicle is obtained through the two-axle and two-mass model. It has a good application prospect and can provide help for intelligent traffic and overload identification.

References

[1] Wilson, A. Vehicle weight is the key driver for automotive composites. Reinforced Plastics, 2017. 61(2): p. 100-102.

[2] Nguyen, P.X., et al. Vehicle’s Weight Estimation Using Smartphone’s Acceleration Data to Control Overloading. International Journal of Intelligent Transportation Systems Research, 2018. 16(3): p. 151-162.

[3] Zhang R et al. Beidou-based integrated monitoring and management system of vehicle-mounted weight meter. Satellite application, 2015(02): 74-77.

[4] Li H. and li X. Research on vehicle-mounted dynamic weighing algorithm. Computer simulation, 2016.33 (11): 140-143.

[5] Du C. Theoretical analysis of low-frequency dynamic load and algorithm research on improving weighing accuracy. Highway traffic science and technology, 2018.35 (04): 153-158.

[6] Andrzejczak, A, et al. Module hardware structure of wireless vehicle weight measurement system. 2014. Lublin, Poland: IEEE Computer Society.

[7] Karim, M.R., et al. Effectiveness of vehicle weight enforcement in a developing country using weigh-in-motion sorting system considering vehicle by-pass and enforcement capability. IATSS Research, 2014. 37(2): p. 124-129.

[8] Mazurek, B, T. Janiczek and J. Chmielowiec. Assessment of vehicle weight measurement method using PVDF transducers. Journal of Electrostatics, 2001. 51-52: p. 76-81.

[9] Xiong S., et al. Vehicle dynamic weighing technology based on BP neural network. Journal of anhui university of technology (natural science edition), 2014. 31(01): 76-79.

[10] Pence, B., et al. Vehicle sprung mass estimation for rough terrain. International Journal of Vehicle Design, 2013. 61(1-4): p. 3-26.

[11] Yu Z. Automotive theory. machinery industry press, Beijing.

[12] Road rubber speed bumps: JT/T 713-2008[S].