To the study of non-Gaussianity in two-field slow-roll inflation.

N.A. Koshelev
Ulyanov State University, Leo Tolstoy str 42, 432970, Russia
(Dated: December 12, 2013)

The general expression for the second order large scale curvature perturbation in the form of a functional over a background solution is derived. The explicit expressions was obtained for two special forms of the inflationary potential. In the considered cases, it is shown that a significant level of non-Gaussianity can be generated during the super-Hubble evolution only if nonadiabatic perturbations are non-negligible at the end of inflation.

PACS numbers: 98.80.Cq

I. INTRODUCTION.

Primordial non-Gaussianity has emerged as one of important probes of the inflationary era. In single field slow-roll inflation, the non-Gaussianity is suppressed by the slow-roll parameters [1], [2], [3], but it can be sizeable in models with a transient violation of standard slow-roll (for example, due to a step feature in the inflationary potential [4]). Non-adiabatic perturbations produced during multi-field inflation may lead to generation of detectable deviations from Gaussian distribution after inflationary stage. For instance, it can take place at an inhomogeneous end of inflation [5], [6], [7], at the reheating [8], [9], [10] or in the curvaton scenario [11]. It has also shown that significant non-Gaussianity can be obtained during slow-roll multi-field inflation [12], [13]. An important feature of most of the known examples of this type is that large scale curvature perturbation evolves even at the end of the inflationary stage (see, however, [14]). Hence, such models are incomplete without an understanding of the evolution of the cosmological perturbations until they become nearly adiabatic [15]. In particular, it is necessary to consider the non-Gaussianity behavior at reheating. This raises the question about principal possibility of slow-roll models with large non-Gaussianity and adiabatic spectrum.

Most computations of the non-Gaussianity in multi-field inflation have been carried out in the framework of the $\delta N$-formalism [16], [17], [18], [19], [20], using labeling the trajectories by slow-roll integrals of motion [21], [22], [23]. In an attempt to go beyond standard assumptions, some extensions of the $\delta N$-formalism are worked out [24], [25], [26]. Also, the covariant formalism [27], [28], [29], [30] and the long-wavelength formalism [31], [32], [33], [34] are developed. Although all approaches mentioned above are geometrically transparent, here we use method of employing the slow-roll Klein-Gordon equations.

The structure this paper is as follows. Chapter II describes the model and basic equations. In Chapter III, slow-roll equations are written and general slow-roll expression for the curvature perturbation in two-field inflation is obtained. For cases of product and sum potentials, the corresponding expressions are written explicitly. In Chapter IV, using decomposition of perturbations on the adiabatic and entropy ones, it is shown that large values of $f_{NL}$ can be generated only at sufficiently large entropy perturbations at the end of inflationary stage. We summary our results in Section V.

II. MODEL AND BASIC EQUATIONS.

Let us consider two canonical scalar fields $\varphi$ and $\chi$ minimally coupled to gravity

$$ S = \int \left\{ -\frac{R}{16\pi G} - \frac{1}{2} \varphi^{\mu}\varphi_{,\mu} - \frac{1}{2} \chi^{\mu}\chi_{,\mu} - U(\varphi, \chi) \right\} \sqrt{-gd^4x}. $$

(2.1)

The Lagrangian density is given by

$$ \mathcal{L} = -\frac{1}{2} \varphi^{\mu}\varphi_{,\mu} - \frac{1}{2} \chi^{\mu}\chi_{,\mu} - U(\varphi, \chi). $$

(2.2)
The Hilbert energy-momentum tensor is defined as

\[ T^{\mu\nu} = -2 \frac{\partial L}{\partial g_{\mu\nu}} + g^{\mu\nu} L, \]  

which gives

\[ T^\nu_\mu = \varphi^\mu \varphi_\nu + \chi^\mu \chi_\nu - \delta^\mu_\nu \left( U(\varphi, \chi) + \frac{1}{2} \varphi^\mu \varphi_\mu + \frac{1}{2} \chi^\mu \chi_\mu \right). \]  

The energy-momentum tensor \( T^{\mu\nu} \), at least up to second order perturbations, can also be treated as energy-momentum tensor of a perfect fluid

\[ T^\mu_\nu = (\rho + P) u^\mu u_\nu + P \delta^\mu_\nu, \]  

where \( \rho \) is density, \( P \) is pressure and \( u^\mu \) is 4-speed.

We split any quantity into a homogeneous background and small inhomogeneous perturbations

\[ f(\eta, x^i) = f^{(0)}(\eta) + \delta f^{(1)}(\eta, x^i) + \frac{1}{2} \delta^2 f^{(2)}(\eta, x^i) + \ldots, \]  

where \( \eta \) is conformal time, the Latin index \( i \) takes values from 1 to 3, and indices in brackets indicate the order of perturbations.

Including second order perturbations, the line element around a spatially flat Friedmann-Robertson-Walker background has the form [35]

\[ ds^2 = -a^2(\eta) \left\{ (1 + 2\psi^{(1)} + \psi^{(2)})d\eta^2 + (2B^1_1 + B^2_1)d\eta dx^i + \left[ (1 - 2\psi^{(1)} - \psi^{(2)}) \delta_{ij} + 2E^1_{ij} + E^2_{ij} \right] x^i x^j \right\}. \]  

The metrics perturbations can be classified into scalar, vector, and tensor types according to their transformation properties under spatial coordinate transformations on constant-time hypersurface [35]. Here we will consider only the scalar type perturbations, i.e., we set

\[ B^{(1)}_i = B^{(1)}_{i}, \quad B^{(2)}_i = B^{(2)}_{i}, \quad E^{(1)}_{ij} = E^{(1)}_{ij}, \quad E^{(2)}_{ij} = E^{(2)}_{ij}. \]  

The freedom of coordinate choice can be used to impose gauge constraints. For example, one can use the uniform density gauge \( \delta \rho^{(1)} = \delta \rho^{(2)} = 0 \) or the uniform curvature gauge \( \psi^{(1)} = \psi^{(2)} = 0 \).

In the analysis of perturbations, a special role is played by the quantities that conserve on large scales if the pressure perturbation is adiabatic. The first conserved nonlinear gauge-invariant quantity was obtained in ref. [35]. The commonly used quantity is curvature perturbation \( \zeta \) on uniform density hypersurfaces, which is ambiguously defined in nonlinear case. We adopt the definition of refs. [1], [37], [38]. On large scales, this quantity is associated with the variable of Malik and Wands [35]

\[ \zeta^{(1)}_{MW} = \left. \zeta^{(1)} \right|_{\rho^{(1)} = \rho^{(2)} = 0}, \quad \zeta^{(2)}_{MW} = \left. \zeta^{(2)} \right|_{\rho^{(1)} = \rho^{(2)} = 0} \]  

by relations [37], [38]

\[ \zeta^{(1)} = \zeta^{(1)}_{MW}, \quad \zeta^{(2)} = \zeta^{(2)}_{MW} - \zeta^{(1)}_{MW}^2. \]  

Gauge transformations allow to write the second order curvature perturbation \( \zeta^{(2)} \) in the gauge-invariant form [37]

\[ \zeta^{(2)} = -\psi^{(2)} - \frac{\mathcal{H}}{\rho'_{(0)}} \delta \rho^{(2)} + 2 \frac{\mathcal{H}}{(\rho_{(0)})^2} \delta \rho^{(1)} \psi^{(1)} + 2 \frac{\delta \rho^{(1)}}{\rho_{(0)}} \psi^{(1)} \left( \frac{\delta \rho^{(1)}}{\rho_{(0)}} \right)^2 \mathcal{H} \left( \frac{\rho_{(0)}}{\rho_{(0)}} - \mathcal{H} \right)^2. \]  

where \( \mathcal{H} = a'/a \) and prime denotes the derivative with respect to conformal time \( \eta \). Most simply, this expression appears in the uniform curvature (UC) gauge

\[ \zeta^{(2)} = -\frac{\mathcal{H}}{\rho_{(0)}} \delta \rho^{(2)}_{UC} + 2 \frac{\mathcal{H}}{(\rho_{(0)})^2} \delta \rho^{(1)}_{UC} \psi^{(1)}_{UC} + \mathcal{H} \left( \frac{\rho_{(0)}}{\rho_{(0)}} \right)^2 \delta \rho^{(1)}_{UC}. \]  

In the following, all calculations done in the uniform curvature gauge with conditions \( E^{(1)} = E^{(2)} = 0 \), what fix a residual freedom of gauge transformations. In order to avoid cluttering the expressions with too many subscripts, we omit the index "UC" below.
A. Density perturbations.

Let us write only a few components of the two-field energy-momentum tensor in the uniform curvature gauge. Without imposing gauge conditions, all components of the energy-momentum tensor up to second-order terms can be found in Ref. 39.

The perturbation expansion of energy-momentum tensor (2.1) yield

\[
T_{(0)0}^0 = -\frac{1}{2a^2} \delta \phi(0)^2 - \frac{1}{2a^2} \delta \lambda(0)^2 - U(0)
\]

\[
T_{(0)j}^i = \left[ \frac{1}{2a^2} \delta \phi(0)^2 + \frac{1}{2a^2} \delta \lambda(0)^2 - U(0) \right] \delta_{ij}
\]

\[
\delta T_{(1)0}^0 = \frac{1}{a^2} \left[ \left( \phi'(0)^2 + \lambda'(0)^2 \right) \phi(1) - \phi'(0) \delta \phi(1) - \lambda'(0) \delta \lambda(1) - a^2 U \phi \delta \phi(1) - a^2 U \lambda \delta \lambda(1) \right]
\]

\[
\delta T_{(2)0}^0 = \frac{1}{a^2} \left[ \left( \phi'(0)^2 + \lambda'(0)^2 \right) \phi(2) - \phi'(0) \delta \phi(2) - \lambda'(0) \delta \lambda(2) - a^2 U \phi \delta \phi(2) - a^2 U \lambda \delta \lambda(2) - \delta \phi(1)^2 - \delta \lambda(1)^2 \right]
\]

\[
\delta T_{(1)i}^i = -\frac{1}{a^2} \left( \phi'(0) \delta \phi(1,i) + \lambda'(0) \delta \lambda(1,i) \right)
\]

where we use the shorthand \( U, \phi \equiv \frac{\partial U}{\partial \phi(0)}, U, \lambda \equiv \frac{\partial U}{\partial \lambda(0)}, U, \phi \equiv \frac{\partial^2 U}{\partial \phi(0) \partial \phi(0)}, U, \lambda \equiv \frac{\partial^2 U}{\partial \lambda(0) \partial \lambda(0)}, U, \phi \equiv \frac{\partial^2 U}{\partial \phi(0) \partial \lambda(0)} \).

The alternative notation (2.5) leads to expressions

\[
T_{(0)0}^0 = -\rho(0),
\]

\[
T_{(0)j}^i = P_0 \delta_{ij},
\]

\[
\delta T_{(1)i}^i = -\delta \rho(1),
\]

\[
\delta T_{(2)0}^0 = -\delta \rho(2) - 2 \left( \rho(0) + P_0 \right) v(1,i) \delta v^i(1) - 2 \left( \rho(0) + P_0 \right) B_{(1),i} \delta v^i(1),
\]

\[
\delta T_{(1)i}^i = \left( \rho(0) + P_0 \right) \left( B_{(1),i} + v(1,i) \right)
\]

where \( v \) is the velocity potential.

The comparison of equations (2.13), (2.14), (2.17) and (2.18), (2.19), (2.22) gives

\[
\rho(0) = \frac{1}{2a^2} \phi'(0)^2 - \frac{1}{2a^2} \lambda'(0)^2 + U(0),
\]

\[
P_0 = \frac{1}{2a^2} \phi'(0)^2 + \frac{1}{2a^2} \lambda'(0)^2 - U(0),
\]

\[
v(1) = -\frac{1}{B(1) - \frac{\phi'(0) \delta \phi(1)}{\phi'(0)^2 + \lambda'(0)^2}}
\]

From equations (2.15), (2.18) and (2.20), (2.21), it follows now that

\[
\delta \rho(1) = \frac{1}{a^2} \left[ \left( \phi'(0)^2 + \lambda'(0)^2 \right) \phi(1) + \phi'(0) \delta \phi(1) + \lambda'(0) \delta \lambda(1) + a^2 U \phi \delta \phi(1) + a^2 U \lambda \delta \lambda(1) \right],
\]

\[
\delta \rho(2) = \frac{1}{a^2} \left[ \left( \phi'(0)^2 + \lambda'(0)^2 \right) \phi(2) + \phi'(0) \delta \phi(2) + a^2 U \phi \delta \phi(2) + a^2 U \lambda \delta \lambda(2) - \left( \phi'(0)^2 + \lambda'(0)^2 \right) \phi(2) \right]
\]

\[
- \left( \phi'(0)^2 + \lambda'(0)^2 \right) \left( B_{(1),i} - \frac{4 \phi(1)}{\phi'(0)^2 + \lambda'(0)^2} \right) \delta \phi(1,i) + 4 \phi(0) \delta \phi(1) \delta \phi(1) + 4 \phi(0) \delta \phi(1) + 4 \phi(0) \delta \phi(1) \right) \phi(1)
\]

\[
+ a^2 U \phi \delta \phi(1) + a^2 U \phi \delta \phi(1) + 2a^2 U \phi \delta \phi(1) \delta \phi(1) - 2B_{(1),i} \left( \phi(0) \delta \phi(1) + \lambda(0) \delta \lambda(1) \right)
\]

\[
- 4 \frac{\phi(0) \delta \phi(1) \delta \phi(1,i)}{\phi'(0)^2 + \lambda'(0)^2} \left( \delta \phi(1,i) + \delta \phi(1,i) + \delta \lambda(1,i) \right),
\]
B. Perturbed Einstein equations.

At first order, the $0 - 0$ component of the Einstein equations can be written as

$$2a^2U_0(0)\phi_1 + \psi'\delta\phi_1 + \chi'\delta\chi_1 + a^2U\psi\delta\phi_1 + a^2U\chi\delta\chi_1 + \frac{\mathcal{H}}{4\pi G}\Delta B_1 = 0,$$

(2.28)

where $\Delta$ is the Laplace operator.

The $0 - i$ component is

$$\mathcal{H}\phi_{(i)} = 4\pi G\phi_{(0)}\delta\phi_{(1)} + 4\pi G\chi_{(0)}\delta\chi_{(1)}.$$

(2.29)

The consideration of the off-diagonal $i - j$ components gives

$$B'_{(i)} + 2\mathcal{H}B_{(1)} + \phi_{(1)} = 0.$$

(2.30)

From the trace of $i - j$ components we find

$$\mathcal{H}\phi_{(i)}' + (2\mathcal{H}' + \mathcal{H}^2)\phi_{(i)} + \frac{1}{2}\left(\Delta\phi_{(i)} + 2\mathcal{H}\Delta B_{(1)} + \Delta B'_{(i)}\right) = 4\pi G \left(-a^2U\psi\delta\phi_{(1)} - a^2U\chi\delta\chi_{(1)} + \phi_{(0)}'\delta\phi_{(1)} + \chi_{(0)}'\delta\chi_{(1)} - \left(\phi_{(0)}^2 + \chi_{(0)}^2\right)\phi_{(1)}\right).$$

(2.31)

Using equation (2.29) and background Einstein equations, the latter can be reduced to

$$\mathcal{H}\phi_{(i)}' = 4\pi G \left(-a^2U\psi\delta\phi_{(1)} - a^2U\chi\delta\chi_{(1)} + \phi_{(0)}'\delta\phi_{(1)} + \chi_{(0)}'\delta\chi_{(1)} - 2U_0(0)\phi_{(1)}\right).$$

(2.32)

At second order, the $0 - i$ Einstein equation is given by

$$\mathcal{H}\phi_{(2),i} - 4\mathcal{H}\phi_{(1)}\phi_{(1),i} + 2\mathcal{H}B_{(1),ki}B_{(1),i} + B_{(1),ki}\phi^{(k)}_{(i)} - \Delta B_{1}\phi_{1,i}$$

$$= 4\pi G \left[\phi_{(0)}'\delta\phi_{2,i} + \chi_{(0)}'\delta\chi_{2,i} + 2\phi_{(1)}'\delta\phi_{(1),i} + 2\phi_{(1)}'\delta\chi_{(1),i}\right].$$

(2.33)

We then use the first order $0 - i$ equation (2.29) and take the trace of (2.33), which gives [11]

$$\phi_{(2)} + B_{(1),k}B^{(k)}_{(1)} = 2\phi_{(1)}^2 + \frac{4\pi G}{\mathcal{H}} \left[\phi_{(0)}'\delta\phi_{(2)} + \chi_{(0)}'\delta\chi_{(2)}\right] - \frac{1}{\mathcal{H}}\Delta^{-1} \left(\phi_{(1),ki}B_{(1),i}^{(k)} - \Delta B_{(1)}\Delta\phi_{(1)}\right)$$

$$+ \frac{8\pi G}{\mathcal{H}}\Delta^{-1} \left(\phi_{(1)}^2\delta\phi_{(1)} + \chi_{(1)}^2\delta\chi_{(1)} + \delta\phi_{(1),k}\delta\phi_{(1)}^{(k)} + \delta\chi_{(1),k}\delta\chi_{(1)}^{(k)}\right),$$

(2.34)

where $\Delta^{-1}$ is the inverse Laplacian, $\Delta^{-1}(\Delta f) = f$.

III. SLOW-ROLL.

As usual, we introduce the two-field slow-roll parameters

$$\epsilon_\varphi \equiv \frac{1}{16\pi G} \left(\frac{U_{\varphi}}{U_{(0)}}\right)^2, \quad \epsilon_\chi \equiv \frac{1}{16\pi G} \left(\frac{U_{\chi}}{U_{(0)}}\right)^2, \quad \eta_{\varphi\varphi} \equiv \frac{1}{8\pi G} \left(\frac{U_{\varphi\varphi}}{U_{(0)}}\right), \quad \eta_{\varphi\chi} \equiv \frac{1}{8\pi G} \left(\frac{U_{\varphi\chi}}{U_{(0)}}\right), \quad \eta_{\chi\chi} \equiv \frac{1}{8\pi G} \left(\frac{U_{\chi\chi}}{U_{(0)}}\right).$$

(3.1)

The slow-roll condition

$$\max\{\epsilon_\varphi, \epsilon_\chi, |\eta_{\varphi\varphi}|, |\eta_{\varphi\chi}|, |\eta_{\chi\chi}|\} \ll 1$$

(3.2)

leads to the relations

$$\varphi_{(0)}'' - \mathcal{H}\varphi_{(0)}' \simeq 0, \quad \chi_{(0)}'' - \mathcal{H}\chi_{(0)}' \simeq 0, \quad \frac{1}{2a^2}\varphi_{(0)}' \simeq \frac{1}{2a^2}\chi_{(0)}' \ll U_0.$$

(3.3)

On large scales, for non-decaying modes of perturbations, there are analogues

$$\delta\varphi_{(1)}'' - \mathcal{H}\delta\varphi_{(1)}' \simeq 0, \quad \delta\chi_{(1)}'' - \mathcal{H}\delta\chi_{(1)}' \simeq 0.$$

(3.4)
\[\delta \varphi''(2) - \mathcal{H} \delta \varphi'(2) \simeq 0, \quad \delta \chi''(2) - \mathcal{H} \delta \chi'(2) \simeq 0.\]  
(3.5)

The basic slow-roll background equations are

\[3\mathcal{H} \varphi'(0) + a^2 U,\varphi = 0, \quad 3\mathcal{H} \chi'(0) + a^2 U,\chi = 0,\]  
(3.6)

\[\mathcal{H}^2 = \frac{8\pi G}{3} a^2 U(0).\]  
(3.7)

Also, the following slow-roll expressions are useful

\[\rho(0) = U(0), \quad \rho'(0) = -\frac{a^2}{3H} \left( U,\varphi^2 + U,\chi^2 \right),\]  
(3.8)

\[\mathcal{H} \rho''(0) - \rho' = \frac{a^2}{3} \left( U,\varphi^2 - \frac{2U,\varphi U,\varphi}{3} + 2 U,\varphi U,\chi + \frac{U,\chi^2}{3} \right).\]  
(3.9)

A. Perturbed slow-roll Klein-Gordon equations.

At first order, for \(N\) scalar fields \(\phi_I\) with the potential \(U(\phi_I)\), \(I = 1, ..., N\), the Klein-Gordon equations can be written as [40]

\[\delta \varphi''_I(1) + 2\mathcal{H} \delta \varphi'_I(1) - \Delta \delta \varphi_I(1) + a^2 \sum_K \left\{ U,\phi_K \phi_I - \frac{8\pi G}{a^2} \left( \frac{a^2 \phi'_K(0) \phi'_I(0)}{\mathcal{H}} \right) \right\} \delta \phi_K(1) = 0.\]  
(3.10)

Considering the non-decaying modes of perturbations on large scales, these equations are reduced in the slow-roll to

\[3\mathcal{H} \delta \varphi'_I(1) + \sum_K \left( a^2 U,\phi_I \phi_K - 24\pi G \phi'_I(0) \phi'_K(0) \right) \delta \phi_K(1) = 0.\]  
(3.11)

For two scalar fields \(\varphi\) and \(\chi\), the equations (3.6), (3.7) and (3.11) give

\[\delta \varphi'_1 = \frac{a^2}{3H} \left( U,\varphi \frac{U,\varphi}{U} - U,\varphi^2 \right) \delta \varphi(1) + \frac{a^2}{3H} \left( \frac{U,\varphi U,\chi}{U} - U,\varphi \chi \right) \delta \chi(1),\]  
(3.12)

\[\delta \chi'_1 = \frac{a^2}{3H} \left( U,\chi \frac{U,\varphi}{U} - U,\chi \varphi \right) \delta \varphi(1) + \frac{a^2}{3H} \left( \frac{U,\chi U,\chi}{U} - U,\chi ^2 \right) \delta \chi(1).\]  
(3.13)

At second order, it is convenient to go to the Fourier representation

\[f(\eta, x') = \int d^3k \delta f_k \exp(ik,x').\]  
(3.14)

Then, for two functions \(f\) and \(h\) the convolution theorem gives

\[[f \ h]_k = \int d^3\lambda f_p h_q\]  
(3.15)

where \(d^3\lambda = \frac{2\pi^3}{(2\pi)^3} \delta^3(k^i - p^i - q^i)\).

The second order Klein-Gordon equations in closed form have been obtained in ref. [41]. Here we use their simplified slow-roll form from ref. [41] (with some additional terms) as a starting point:
\[ \delta \varphi''_{1(2)k} + 2 \dot{H} \delta \varphi'_{1(2)k} + k^2 \delta \varphi_{1(2)k} + \sum_K \left( a^2 U_{\varphi K \varphi I} - 24 \pi G \varphi'_{I(0)} \varphi'_{K(0)} \right) \delta \varphi_{K(2)k} \]

\[ = - \int d^3 \lambda \left\{ a^2 \sum_{K,L} \left( U_{\varphi I K \varphi L} + \frac{8 \pi G}{H} \varphi'_{I(0)} U_{\varphi K \varphi L} \right) \delta \varphi_{K(1)p} \delta \varphi_{L(1)q} \right. \]

\[ + \left( \frac{8 \pi G}{H} \right)^2 \sum_K \varphi'_{K(0)} \delta \varphi_{K(1)p} \left( a^2 U_{\varphi K \varphi I} \sum_K \varphi'_{K(0)} \delta \varphi_{K(1)q} + \varphi'_{I(0)} \sum_K a^2 U_{\varphi K \varphi I} \delta \varphi_{K(1)q} \right) \]

\[ + \frac{16 \pi G}{H} a^2 \sum_K \varphi'_{K(0)} \delta \varphi_{K(1)p} \sum_K U_{\varphi K \varphi I} \delta \varphi_{K(1)q} \}

\[ - \frac{8 \pi G}{H} \int d^1 \lambda \left\{ \frac{p q^4}{k^2} \delta \varphi'_{I(1)p} \sum_K \varphi'_{K(0)} \delta \varphi_{K(1)q} + 2 p^2 \delta \varphi_{I(1)p} \sum_K \varphi'_{K(0)} \delta \varphi_{K(1)q} \right\} \]

\[ + \varphi'_{I(0)} \sum_K \left( \left( \frac{p q^4}{k^2} + \frac{p q^4}{2} \right) \delta \varphi_{K(1)p} \delta \varphi_{K(1)q} + \left( \frac{1}{2} - \frac{q^2 + p q^4}{k^2} \right) \delta \varphi'_{K(1)p} \delta \varphi_{K(1)q} \right) \} \quad (3.16) \]

These equations can be simplified at small \( k \ (k \ll H) \), insofar as in this case it is possible to consider only the long-wavelength field perturbations at right hand side. Indeed, the classical field perturbations was generated from the quantum field fluctuations at the horizon crossing \([42, 43, 44]\). Hence, at \( p \gg H, q \gg H \), there are negligibly small quantum fluctuations instead of the classical perturbations. As a result, (3.16) is reduced to

\[ \delta \varphi''_{1(2)k} + 2 \dot{H} \delta \varphi'_{1(2)k} + \sum_K \left( a^2 U_{\varphi K \varphi I} - 24 \pi G \varphi'_{I(0)} \varphi'_{K(0)} \right) \delta \varphi_{K(2)k} \]

\[ = - \int d^3 \lambda \left\{ a^2 \sum_{K,L} \left( U_{\varphi I K \varphi L} + \frac{8 \pi G}{H} \varphi'_{I(0)} U_{\varphi K \varphi L} \right) \delta \varphi_{K(1)p} \delta \varphi_{L(1)q} \right. \]

\[ + \left( \frac{8 \pi G}{H} \right)^2 \sum_K \varphi'_{K(0)} \delta \varphi_{K(1)p} \left( a^2 U_{\varphi K \varphi I} \sum_K \varphi'_{K(0)} \delta \varphi_{K(1)q} + \varphi'_{I(0)} \sum_K a^2 U_{\varphi K \varphi I} \delta \varphi_{K(1)q} \right) \]

\[ + \frac{16 \pi G}{H} a^2 \sum_K \varphi'_{K(0)} \delta \varphi_{K(1)p} \sum_K U_{\varphi K \varphi I} \delta \varphi_{K(1)q} \}

\[ - \frac{8 \pi G}{H} \int d^1 \lambda \left\{ \frac{p q^4}{k^2} \delta \varphi'_{I(1)p} \sum_K \varphi'_{K(0)} \delta \varphi_{K(1)q} + 2 p^2 \delta \varphi_{I(1)p} \sum_K \varphi'_{K(0)} \delta \varphi_{K(1)q} \right\} \]

\[ + \varphi'_{I(0)} \sum_K \left( \left( \frac{p q^4}{k^2} + \frac{p q^4}{2} \right) \delta \varphi_{K(1)p} \delta \varphi_{K(1)q} + \left( \frac{1}{2} - \frac{q^2 + p q^4}{k^2} \right) \delta \varphi'_{K(1)p} \delta \varphi_{K(1)q} \right) \} \quad (3.17) \]

One can now go back to the coordinate space. Using equation (3.5) and the background equations (3.6), (3.7) for two canonical scalar fields \( \varphi, \chi \) on large scales we get

\[ \delta \varphi''_{(2)} + \frac{3}{2} \left( \frac{U_{\chi \varphi} - U_{\varphi \chi}}{U_{(0)}} \right) \delta \varphi_{(2)} + \frac{a^2}{2} \left( \frac{U_{\chi \varphi} - U_{\varphi \chi}}{U_{(0)}} \right) \delta \chi_{(2)} = f_\varphi, \quad (3.18) \]

\[ \delta \chi''_{(2)} + \frac{3}{2} \left( \frac{U_{\chi \chi} - U_{\chi 
abla \chi}}{U_{(0)}} \right) \delta \chi_{(2)} + \frac{a^2}{2} \left( \frac{U_{\chi \chi} - U_{\chi \varphi}}{U_{(0)}} \right) \delta \varphi_{(2)} = f_\chi, \quad (3.19) \]

where \( f_1 \) and \( f_2 \) are

\[ f_\varphi = \frac{a^2}{3} \left\{ \left( U_{\varphi \varphi - \frac{U_{\varphi \varphi}}{U_{(0)}}} + \frac{U_{\varphi \varphi}}{U_{(0)}} \right) + \frac{U_{\varphi \varphi}}{U_{(0)}} \right\} \left( \frac{U_{\varphi \varphi}}{U_{(0)}} + \frac{U_{\varphi \varphi}}{U_{(0)}} \right) \delta \varphi_{(1)}^2 \]

\[ + \left( 2 U_{\varphi \varphi} - 4 U_{\varphi \varphi} U_{\varphi \varphi} + 4 U_{\varphi \varphi} U_{\varphi \varphi} - 2 U_{\varphi \varphi} U_{\varphi \varphi} \right) \delta \varphi_{(1)} \delta \chi_{(1)} \]

\[ + \left( U_{\varphi \varphi} - 2 U_{\varphi \varphi} U_{\varphi \varphi} - 2 U_{\varphi \varphi} U_{\varphi \varphi} + 2 U_{\varphi \varphi} U_{\varphi \varphi} \right) \delta \chi_{(1)}^2 \} \quad (3.20) \]
\[ f_x = \frac{a^2}{3H} \left\{ \left( U_{xxx} - 3 \frac{U_x}{U(0)} U_{xx} + 2 \frac{U^3_x}{U^2(0)} \right) \delta \chi^2_{(1)} \right\} + \left( 2U_{,xxx} - 4 \frac{U_x}{U(0)} U_{,xx} + 4 \frac{U^2_x}{U^2(0)} U_{,xx} - 2U_{xxx} \frac{U_x}{U(0)} \right) \delta \chi(1) \delta \varphi(1) \]
\[ + \left( U_{x,xx} - \frac{U_x}{U(0)} U_{,xx} - 2U_{x,xx} \frac{U_x}{U(0)} + 2U_{x,xx} \frac{U^2_x}{U^2(0)} \right) \delta \varphi^2_{(1)} \right\}. \] (3.21)

**B. Slow-roll perturbed Einstein equations.**

Under the slow-roll condition, the equation (2.29) can be rewritten as
\[ \phi_{(1)} = - \frac{1}{2} \frac{U_x}{U(0)} \delta \varphi_{(1)} - \frac{1}{2} \frac{U_{xx}}{U(0)} \delta \chi_{(1)}, \] (3.22)
and the equation (2.31) takes the form
\[ \phi'_{(1)} = \frac{4\pi G}{\mathcal{H}} \varphi'_{(0)} \delta \varphi_{(1)} + \frac{4\pi G}{\mathcal{H}} \left( -a^2 \frac{U_x \varphi'_{(0)} + U_{,xx} \chi_{(0)}}{3\mathcal{H}} + 3\mathcal{H} \varphi_{(0)} \frac{\varphi^2_{(0)} + \chi^2_{(0)}}{a^2 U(0)} \right) \delta \varphi_{(1)} + \frac{4\pi G}{\mathcal{H}} \chi'_{(0)} \delta \chi_{(1)} + \frac{4\pi G}{\mathcal{H}} \varphi'_{(0)} \delta \chi_{(1)} \right\}. \] (3.23)

It is convenient to write the second-order perturbed equations in the Fourier space. At slow-roll leading order, the equation (2.28) gives
\[ k^2 B_{(1)k} = \frac{U_{,\varphi}}{6\mathcal{H} U(0)} \left( a^2 \frac{U^2_{,\varphi}}{2 U(0)} \delta \varphi_{(1)k} - 3\mathcal{H} \delta \varphi'_{(1)k} \right) + \frac{U_{,\chi}}{6\mathcal{H} U(0)} \left( a^2 \frac{U^2_{,\chi}}{2 U(0)} \delta \chi_{(1)k} - 3\mathcal{H} \delta \chi'_{(1)k} \right). \] (3.24)

The equation (2.34) takes the form
\[ \phi_{(2)k} - \int d^3 \lambda \left\{ pq B_{(1)p} B_{(1)q} \right\} = - \frac{1}{2} \frac{U_x}{U(0)} \delta \varphi_{(2)k} - \frac{1}{2} \frac{U_{xx}}{U(0)} \delta \chi_{(2)k} + \int d^3 \lambda \left\{ 2\phi_{(1)p} \phi_{(1)q} \right\} \]
\[ + \frac{(pq)^2 - p^2 q^2 \phi_{(1)p} \phi_{(1)q}}{k^2 q^2 6\mathcal{H}^2} \left[ U_{,\varphi} \left( a^2 \frac{U^2_{,\varphi}}{2 U(0)} \delta \varphi_{(1)q} - 3\mathcal{H} \delta \varphi'_{(1)q} \right) + U_{,\chi} \left( a^2 \frac{U^2_{,\chi}}{2 U(0)} \delta \chi_{(1)q} - 3\mathcal{H} \delta \chi'_{(1)q} \right) \right] \]
\[ + 3\mathcal{H} \frac{1}{a^2 U(0)} \left( \frac{q^2 + pq}{k^2} \delta \varphi_{(1)p} \delta \varphi_{(1)q} + \frac{q^2 + pq}{k^2} \delta \chi_{(1)p} \delta \chi_{(1)q} \right) \right\}. \] (3.25)

**C. Solution of the slow-roll Klein-Gordon equations on large scales.**

The system of the slow-roll Klein-Gordon equations can be solved using the approach of [45]. We turn now to the new variables
\[ x = \frac{U(0)}{U_{,\varphi}} \delta \varphi_{(2)}, \quad y = \frac{U(0)}{U_{,\chi}} \delta \chi_{(2)}. \] (3.26)

The equations (3.18), (3.19) can be written as
\[ x' - \left( \ln \left( \frac{U_{,\varphi}}{U(0)} \right) \right)_{,\chi} x' (y - x) = \frac{U(0)}{U_{,\varphi}} f_{,\varphi}, \] (3.27)
\[ y' + \left( \ln \left( \frac{U_{,\chi}}{U(0)} \right) \right)_{,\varphi} x' (y - x) = \frac{U(0)}{U_{,\chi}} f_{,\chi}. \] (3.28)
Their consequence

\[ (x - y)' + \left[ \ln \left( \frac{U_\varphi}{U_{(0)}} \right) \right]_x \chi' + \left[ \ln \left( \frac{U_\chi}{U_{(0)}} \right) \right]_\varphi \varphi' \] \(x - y) = \frac{U_{(0)}}{U_\varphi} f_\varphi - \frac{U_{(0)}}{U_\chi} f_\chi \] (3.29)

has a formal solution

\[ x - y = e^{F(\tau)} \int_{\tau_0}^\tau \left( \frac{U_{(0)}}{U_\varphi} f_\varphi - \frac{U_{(0)}}{U_\chi} f_\chi \right) e^{-F(\tau')} d\tau + \gamma(2)e^{F(\tau)}, \] (3.30)

where \(\gamma(2)\) is a constant and

\[ e^F \equiv \exp \left\{ - \int_{\tau_0}^\tau \left[ \left( \ln \left( \frac{U_\varphi}{U_{(0)}} \right) \right]_x \chi' + \left[ \ln \left( \frac{U_\chi}{U_{(0)}} \right) \right]_\varphi \varphi' \right] d\tau \right\}. \] (3.31)

Here \(\tau_0\) is an arbitrary moment of conformal time. We take \(\tau_0\) to be the moment of horizon crossing \(\tau_*\) when \(k = H\) for the given mode \(k\).

The equations (3.27), (3.28) can be integrated now to give

\[ x = \int_{\tau_*}^\tau \left[ \frac{U_{(0)}}{U_\varphi} f_\varphi - \left( \frac{U_{(0)}}{U_\chi} \right)_x \chi' e^{F(\tau)} (J + \gamma(2)) \right] d\tau + \alpha(2), \] (3.32)

\[ y = \int_{\tau_*}^\tau \left[ \frac{U_{(0)}}{U_\chi} f_\chi + \left( \frac{U_{(0)}}{U_\varphi} \right)_\varphi \varphi' e^{F(\tau)} (J + \gamma(2)) \right] d\tau + \beta(2), \] (3.33)

where \(\alpha(2), \beta(2)\) are constants and

\[ J(\tau) = \int_{\tau_*}^\tau \left( \frac{U_{(0)}}{U_\varphi} f_\varphi - \frac{U_{(0)}}{U_\chi} f_\chi \right) e^{-F(\tau')} d\tau. \] (3.34)

Hence, the second order longwavelength fields perturbations are

\[ \delta \varphi(2) = \frac{U_\varphi}{U_{(0)}} \int_{\tau_*}^\tau \left[ \frac{U_{(0)}}{U_\varphi} f_\varphi - \left( \frac{U_{(0)}}{U_\chi} \right)_x \chi' e^{F(\tau)} (J + \gamma(2)) \right] d\tau'' + \frac{U_\varphi}{U_{(0)}} \alpha(2), \] (3.35)

\[ \delta \chi(2) = \frac{U_\chi}{U_{(0)}} \int_{\tau_*}^\tau \left[ \frac{U_{(0)}}{U_\chi} f_\chi + \left( \frac{U_{(0)}}{U_\varphi} \right)_\varphi \varphi' e^{F(\tau)} (J + \gamma(2)) \right] d\tau' + \frac{U_\chi}{U_{(0)}} \beta(2). \] (3.36)

The consideration of equations (3.32), (3.33) and (3.30) at \(\tau = \tau_*\) gives

\[ x_* - y_* = \gamma(2)e^{F(\tau_*)} = \gamma(2) = \alpha(2) - \beta(2). \] (3.37)

Similarly, we get the first order longwavelength slow-roll solution

\[ \delta \varphi(1) = -\gamma(1) \frac{U_\varphi}{U_{(0)}} \int_{\tau_*}^\tau \left( \frac{U_{(0)}}{U_\varphi} \right)_x \chi' e^{F(\tau')} d\tau'' + \frac{U_\varphi}{U_{(0)}} \alpha(1), \] (3.38)

\[ \delta \chi(1) = \gamma(1) \frac{U_\chi}{U_{(0)}} \int_{\tau_*}^\tau \left( \frac{U_{(0)}}{U_\chi} \right)_\varphi \varphi' e^{F(\tau')} d\tau' + \frac{U_\chi}{U_{(0)}} \beta(1), \] (3.39)

where \(\alpha(1), \beta(1), \gamma(1)\) are constants constrained by

\[ \gamma(1) = \alpha(1) - \beta(1). \] (3.40)

D. Curvature perturbation in slow-roll.

At leading order in slow-roll parameters, we have

\[ \delta \rho(1) = U_\varphi \delta \varphi(1) + U_\chi \delta \chi(1), \] (3.41)
\[
\delta \rho_{(1)} = \left( U, \varphi, \varphi^2 \right) + U, \chi \chi^2 \delta \chi_{(1)},
\]  
\[\delta \rho_{(2)} = U, \varphi \delta \varphi_{(2)} + U, \chi \delta \chi_{(2)} + \int d^3 \lambda \left( U, \varphi, \delta \varphi_{(1)}, \chi^2 \delta \chi_{(1)} + U, \chi, \varphi^2 \delta \chi_{(1)} - 2U, \varphi, \delta \varphi_{(1)} \right) \]  
\[(3.42)\]

Using the background slow-roll equations, the first order equations (3.12), (3.13) and the Einstein equations (3.22) - (3.24), we obtain a simple expression

\[
\delta \rho_{(2)} = \int d^3 \lambda \left( U, \varphi, \delta \varphi_{(1)}, \chi^2 \delta \chi_{(1)} + U, \chi, \varphi^2 \delta \chi_{(1)} - 2U, \varphi, \delta \varphi_{(1)} \right)
\]  
\[(3.43)\]

or

\[
\delta \rho_{(2)} = U, \varphi \delta \varphi_{(2)} + U, \chi \delta \chi_{(2)} + U, \chi \delta \chi_{(1)}^2 + 2U, \varphi, \delta \varphi_{(1)} \delta \chi_{(1)}.
\]  
\[(3.44)\]

The substitution of equations (3.44), (3.12), (3.13) into (2.12) gives

\[
\zeta_{(2)} = \frac{8\pi G U_0(0)}{U, \varphi, U, \chi} \left( U, \varphi, \varphi^2 + U, \chi \delta \chi_{(2)} \right) - \left( U, \varphi, \varphi^2 + U, \chi \delta \chi_{(1)} \right) + \int d^3 \chi \left( U, \varphi, \delta \varphi_{(1)}, \chi^2 \delta \chi_{(1)} + U, \chi, \varphi^2 \delta \chi_{(1)} - 2U, \varphi, \delta \varphi_{(1)} \right)
\]  
\[(3.45)\]

where formal expression for \(\delta \varphi_{(2)}\) and \(\delta \chi_{(2)}\) are given by the equations (3.35) and (3.36).

This equation is the main result of this work. It is follows from (3.35), that the first order fields perturbations give a local contribution to \(\zeta_{(2)}\), both directly and implicitly through the quantities \(\delta \varphi_{(2)}, \delta \chi_{(2)}\).

1. **Product potential, \(U(\varphi, \chi) = V(\varphi)W(\chi)\)**

The formal expressions (3.35), (3.36) and (3.35), (3.36) for product potential can be integrated to give correspondingly

\[
\delta \varphi_{(1)} = \frac{V, \varphi}{V, \chi} V_{(0)}^{(0)} \delta \varphi_{(1)}, \quad \delta \chi_{(1)} = \frac{W, \chi}{W, \varphi} W_{(0)}^{(0)} \delta \chi_{(1)}.
\]  
\[(3.46)\]

and

\[
\delta \varphi_{(2)}(\tau) = \left( \frac{V, \varphi}{V, \chi} \left[ V_{(0)}^{(0)} \right] \right) \delta \varphi_{(2)}, \quad \delta \chi_{(2)}(\tau) = \left( \frac{W, \chi}{W, \varphi} \left[ W_{(0)}^{(0)} \right] \right) \delta \chi_{(2)}.
\]  
\[(3.47)\]

Using the background slow-roll equations and equations (3.46), (3.37), (3.47), the equation (3.45) can be simplified to give

\[
\zeta_{(2)} = \frac{8\pi G V, \varphi^2, V, \chi^2}{V, \varphi, V, \chi} \left( V, \varphi^2 + V, \chi \delta \chi_{(2)} \right)
\]  
\[\left( 3.48 \right)\]

This result is in agreement with ref. [10], where was considered two scalar fields with non-canonical kinetic terms.
At first order we obtain the known result \[55\]
\[
\delta \phi^{(1)} = V_{\phi} C - D W(0) \quad \text{and} \quad \delta \chi^{(1)} = W_\chi C + D V(0)
\] (3.50)
where
\[
C = \frac{V(0)}{V_{\phi}} \delta \phi^{(1)}_* + \frac{W(0)}{W_\chi} \delta \chi^{(1)}_* , \quad D = \frac{\delta \phi^{(1)}_*}{W_{\chi}} - \frac{\delta \phi^{(1)}_*}{V_{\phi}}.
\] (3.51)

The second order equations \[3.35, 3.36\] give
\[
\frac{U(0)}{V_{\phi}} \delta \phi^{(2)} = C^2 \left( \frac{V_{\phi} W(0) - V_{\phi}^2}{U(0)} - \frac{W_{\chi}^2}{U(0)} \right) \frac{\delta \phi^{(1)}_*}{T}, + D^2 \left( \frac{V_{\phi} W(0)}{U(0)} - \frac{V_{\phi}^2 W(0)}{U(0)} - \frac{W_{\chi}^2 V(0)}{U(0)} \right) \frac{\delta \phi^{(1)}_*}{T}, \]
\[
- 2CD \left( \frac{V_{\phi} W(0)}{U(0)} - \frac{V_{\phi}^2 W(0)}{U(0)} + \frac{W_{\chi}^2 V(0)}{U(0)} \right) \frac{\delta \phi^{(1)}_*}{T}, + \gamma_2(2) \frac{W(0) - W(0)_*}{U(0)_*} + \alpha(2),
\] (3.52)
\[
\frac{U(0)}{W_{\chi}} \delta \chi^{(2)} = C^2 \left( \frac{W_{\chi}^2}{U(0)} - \frac{V_{\phi}^2}{U(0)} - \frac{2V_{\phi} W(0)}{U(0)} \right) \frac{\delta \phi^{(1)}_*}{T}, + D^2 \left( \frac{W_{\chi}^2 V(0)}{U(0)} - \frac{W_{\chi}^2 V(0)}{U(0)} - \frac{V_{\phi}^2 W(0)}{U(0)} \right) \frac{\delta \phi^{(1)}_*}{T}, \]
\[
- 2CD \left( \frac{W_{\chi}^2 V(0)}{U(0)} - \frac{V_{\phi}^2 W(0)}{U(0)} + \frac{2V_{\phi} W(0)}{U(0)} \right) \frac{\delta \phi^{(1)}_*}{T}, - \tilde{\gamma}_2(2) \frac{V(0) - V(0)_*}{U(0)_*} + \beta(2),
\] (3.53)

where
\[
\tilde{\gamma}_2(2) = \gamma_2(2) - \frac{1}{2} \left( \frac{V_{\phi}^2 + W_{\chi}^2}{C^2} \right) - D^2 \left( \frac{V_{\phi}^2 W(0)}{U(0)} - \frac{W_{\chi}^2 V(0)}{U(0)} \right) + 2CD \left( \frac{V_{\phi} W(0)}{U(0)} + \frac{W_{\chi} V(0)}{U(0)} \right).
\] (3.54)

The second order curvature perturbation \(\zeta^{(2)}\) is
\[
\zeta^{(2)} = - \frac{8\pi G}{V_{\phi}^2 + W_{\chi}^2} \left( \frac{V_{\phi} W(0) - V_{\phi}^2 (W(0)_* + V(0)_*) - W_{\chi}^2 (V(0) - V(0)_*)}{V_{\phi}^2 + W_{\chi}^2} \right) - V_{\phi}^2 \frac{W(0)}{V_{\phi}^2 + W_{\chi}^2} \left( \frac{1}{2} \gamma_2(2) \frac{W(0) - W(0)_*}{U(0)_*} \right) + \gamma_2(1) \frac{V_{\phi} W(0)}{U(0)} \left( \frac{1}{2} \gamma_2(1) \frac{W(0) - W(0)_*}{U(0)_*} \right),
\]
\[
+ V_{\phi}^2 \frac{W(0)}{W_{\chi}^2} \left( \frac{1}{2} \gamma_2(2) \frac{W(0) - W(0)_*}{U(0)_*} \right) \frac{\delta \phi^{(1)}_*}{V_{\phi}^2} + \gamma_2(1) \frac{V_{\phi} W(0)}{U(0)} \left( \frac{1}{2} \gamma_2(1) \frac{W(0) - W(0)_*}{U(0)_*} \right),
\]
\[
+ \gamma_2(2) \frac{V(0) - V(0)_*}{V_{\phi}^2 + W_{\chi}^2} \frac{\delta \phi^{(2)}_*}{V_{\phi}^2} + 8\pi G \left( \frac{V(0) - V(0)_*}{V_{\phi}^2 + W_{\chi}^2} \right) \frac{\delta \phi^{(2)}_*}{V_{\phi}^2}.
\] (3.55)

This result is identical to that found in ref. \[22\].

**IV. NON-GAUSSIANITY.**

Let us consider the spectrum \(P(k)\) and bispectrum \(B(k_1, k_2, k_3)\), defined by
\[
(\xi_{k_1} \xi_{k_2}) = (2\pi)^3 \delta^{(3)}(k_1 + k_2) P(k_1),
\] (4.1)
\[
(\xi_{k_1} \xi_{k_2} \xi_{k_3}) = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) B(k_1, k_2, k_3).
\] (4.2)
The bispectrum is parametrized in terms of products of the spectrum and the dimensionless nonlinear parameter \(f_{NL}\) as

\[
B_{\zeta}(k_1, k_2, k_3) = 6 f_{NL}(k_1, k_2, k_3) \left( P_{\zeta}(k_1) P_{\zeta}(k_2) + P_{\zeta}(k_2) P_{\zeta}(k_3) + P_{\zeta}(k_3) P_{\zeta}(k_1) \right)
\]  

(4.3)

At leading order, we have

\[
\langle \zeta_{k_1} \zeta_{k_2} \rangle = \langle \zeta(1)_{k_1} \zeta(1)_{k_2} \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2) P_{\zeta(1)}(k_1, k_2)
\]

(4.4)

\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = \frac{1}{2} \langle \zeta(1)_{k_1} \zeta(1)_{k_2} \zeta(2)_{k_3} \rangle + \frac{1}{2} \langle \zeta(1)_{k_1} \zeta(2)_{k_2} \zeta(1)_{k_3} \rangle + \frac{1}{2} \langle \zeta(2)_{k_1} \zeta(1)_{k_2} \zeta(1)_{k_3} \rangle
\]

(4.5)

Since the variable \(\zeta_k\) is Gaussian, the Wick’s theorem yields

\[
\langle \zeta_{k_1} \zeta_{k_2} \zeta^2_{k_3} \rangle = 2(2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) P_{\zeta(1)}(k_1) P_{\zeta(1)}(k_2)
\]

(4.6)

Combining equations 4.2 - 4.6, one can write

\[
\frac{6 \ 5}{f_{NL}(k_1, k_2, k_3)} = \frac{\langle \zeta(1)_{k_1} \zeta(1)_{k_2} \zeta(2)_{k_3} \rangle + c. \ p.}{\langle \zeta(1)_{k_1} \zeta(1)_{k_2} \zeta^2_{k_3} \rangle + c. \ p.}
\]

(4.7)

where c. p. means cyclic permutations.

In the linear perturbation theory, all perturbations are separated on adiabatic and entropy ones. To do this, one can introduce the variables \(\delta s_{(1)}, \delta \sigma_{(1)}\)

\[
\delta s_{(1)} = \cos \Theta \delta \chi_{(1)} - \sin \Theta \delta \varphi_{(1)}, \quad \delta \sigma_{(1)} = \cos \Theta \delta \varphi_{(1)} + \sin \Theta \delta \chi_{(1)}
\]

(4.8)

where

\[
\cos \Theta = \frac{\varphi’(0)}{\sigma’}, \quad \sin \Theta = \frac{\chi’(0)}{\sigma’}
\]

and

\[
\sigma’ = \sqrt{\varphi’^2(0) + \chi’^2(0)}.
\]

Perturbations in \(\delta \sigma_{(1)}\), with \(\delta s_{(1)} = 0\), describe adiabatic field perturbations [48]. Perturbations with \(\delta \sigma_{(1)} = 0\) represent the entropy perturbations.

Going to the non-linear case, it is very convenient to use the equality [48]

\[
\zeta_{(1)} = -H \frac{\delta \sigma_{(1)}}{\sigma’}
\]

(4.9)

and to introduce the rescaled variables

\[
\delta \tilde{s}_{(1)} \equiv \frac{H \delta s_{(1)}}{\sigma’}, \quad \delta \tilde{\sigma}_{(1)} \equiv \frac{H \delta \sigma_{(1)}}{\sigma’}
\]

(4.10)

(4.11)

Using these first order variables (we do not employ the nonlinear generalization of ref. [27]), the second order curvature perturbation can be parameterized as

\[
\zeta_{(2)} = \delta \tilde{s}_{(2)} + C_{\tilde{s} \tilde{s}} \zeta^2_{(1)} - 2C_{\tilde{s} \chi} \zeta_{(1)} \delta \tilde{s}_{(1)} + C_{\chi \chi} \delta \tilde{s}^2_{(1)}
\]

(4.12)

where \(C_{\tilde{s} \tilde{s}}, C_{\tilde{s} \chi}, C_{\chi \chi}\) are some functions of the background fields, and the quantity \(\delta \tilde{s}_2\) is zero if the initial fields perturbations are Gaussian. Neglecting the initial non-Gaussianity of the scalar fields \(\varphi, \chi\), we obtain

\[
\frac{6 f_{NL}(k_1, k_2, k_3)}{5} = C_{\tilde{s} \tilde{s}} - 2 C_{\tilde{s} \chi} \frac{\langle \zeta_{(1)k_1} \zeta_{(1)k_2} \zeta_{(2)k_3} \rangle + c. \ p.}{\langle \zeta_{(1)k_1} \zeta_{(1)k_2} \zeta^2_{k_3} \rangle + c. \ p.} + C_{\chi \chi} \frac{\langle \zeta_{(1)k_1} \zeta_{(1)k_2} \zeta^2_{k_3} \rangle + c. \ p.}{\langle \zeta_{(1)k_1} \zeta_{(1)k_2} \zeta^2_{k_3} \rangle + c. \ p.}
\]

(4.13)
The calculation of the correlation functions yields
\[
\frac{6}{5} f_{NL}(k_1, k_2, k_3) = C_{\delta \delta} + C_{\tilde{\delta} \tilde{\delta}} \frac{K(k_1)K(k_2) + K(k_2)K(k_3) + K(k_1)K(k_3)}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_1)P_\zeta(k_3)} \\
- C_{\tilde{\delta} \tilde{\delta}} \frac{P_\zeta(k_1)K(k_2) + P_\zeta(k_2)K(k_1) + P_\zeta(k_3)K(k_2) + P_\zeta(k_2)K(k_3) + P_\zeta(k_3)K(k_1) + P_\zeta(k_1)K(k_3)}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_1)P_\zeta(k_3)},
\]
(4.14)
where we introduce the notation
\[
K(k) = \langle \xi_{(1)k} \delta_{-k} \rangle.
\]
(4.15)
Spectra and correlation functions are easy to calculate, using the well-known result
\[
\delta \varphi_{(1)k} = \frac{H_*}{\sqrt{2k^3}} e_{\varphi k}, \quad \delta \chi_{(1)k} = \frac{H_*}{\sqrt{2k^3}} e_{\chi k},
\]
(4.16)
where \( e_{\varphi k} \) and \( e_{\chi k} \) are independent Gaussian random variables with zero mean and unit variance.

The analysis of the expression (4.14) is too complicated for general form of potential \( U(\varphi, \chi) \), so let us consider only some particular cases.

**A. Product potential, \( U(\varphi, \chi) = V(\varphi)W(\chi) \).**

In this case we have
\[
\delta \tilde{s}_{(1)} = \frac{8\pi GV_{(0)}W_{(0)}V_{\varphi W_{(0)}}}{V_{\varphi}^2 W_{(0)}^2 + V_{\varphi}^2 W_{(0)}^2} \left( \frac{V_{(0)*}^2}{V_{\varphi}^*} \delta \varphi_{(1)*} - \frac{W_{(0)*}^2}{W_{\chi}^*} \delta \chi_{(1)*} \right),
\]
(4.17)
\[
\delta \tilde{\sigma}_{(1)} = - \frac{8\pi G}{V_{\varphi}^2 W_{(0)}^2 + V_{\varphi}^2 W_{(0)}^2} \left( \frac{V_{(0)*}^2}{V_{\varphi}^*} \delta \varphi_{(1)*} - \frac{W_{(0)*}^2}{W_{\chi}^*} \delta \chi_{(1)*} \right),
\]
(4.18)
The equation (4.19) can be rewritten as
\[
\zeta_{(2)} = \frac{1}{8\pi G} \left[ \left( \frac{V_{\varphi}^2}{V_{(0)*}^2} - \frac{V_{\varphi W_{(0)}}}{V_{(0)*}^2} \right) \cos^2 \Theta + \left( \frac{W_{\chi}^2}{W_{(0)*}^2} - \frac{W_{\chi W_{(0)}}}{W_{(0)*}^2} \right) \sin^2 \Theta \right] \zeta_{(1)} \\
+ \left( \frac{V_{\varphi}^2}{V_{(0)*}^2} - \frac{V_{\varphi W_{(0)}}}{V_{(0)*}^2} \right) \sin^2 \Theta + \left( \frac{W_{\chi}^2}{W_{(0)*}^2} - \frac{W_{\chi W_{(0)}}}{W_{(0)*}^2} \right) \cos^2 \Theta \\
+ 2 \sin \Theta \cos \Theta \left( \frac{V_{\varphi W_{(0)}}}{V_{\varphi}^*} + \frac{W_{\chi W_{(0)}}}{W_{\chi}^*} - \frac{V_{\varphi W_{(0)}}}{V_{(0)*}^2} \right) \delta \tilde{s}_{(1)} \\
+ 2 \sin \Theta \cos \Theta \left( \frac{W_{\chi W_{(0)}}}{W_{\chi}^*} - \frac{W_{\chi W_{(0)}}}{W_{(0)*}^2} \right) \delta \tilde{\sigma}_{(1)} - \delta \tilde{\sigma}_{(2)},
\]
(4.19)
where \( \delta \tilde{\sigma}_{(2)} \) is obtained from \( \delta \tilde{\sigma}_{(1)} \) by replacing \( \delta \varphi_{(1)*} \), \( \delta \chi_{(1)*} \) by \( \delta \varphi_{(2)*} \), \( \delta \chi_{(2)*} \).

The equation above shows that the coefficient \( C_{\delta \delta} \) is of the order of slow-roll parameters, i.e.
\[
|C_{\delta \delta}| \ll 1.
\]
(4.20)
This inequality and equation (4.13) indicate that in models with large \( f_{NL} \) the entropy perturbation \( \delta \tilde{s}_{(1)} \) cannot be neglected. In addition, it follows now that a significant level of non-Gaussianity \( |f_{NL}| \geq 1 \) is possible if and only if
\[
\left| C_{\tilde{\delta} \tilde{\delta}} \frac{P_\zeta(k_1)K(k_2) + P_\zeta(k_2)K(k_1) + P_\zeta(k_3)K(k_2) + P_\zeta(k_2)K(k_3) + P_\zeta(k_3)K(k_1) + P_\zeta(k_1)K(k_3)}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_1)P_\zeta(k_3)} \right| \geq 1.
\]
(4.21)
Consider the case that can be easily implemented
\[
\max\{|C_{\delta\delta}|, |C_{\delta\chi}| \ll 1. \quad (4.22)
\]

Then the mandatory condition of the presence of large non-Gaussianity is
\[
|K(k)| \gg P_{\delta}(k). \quad (4.23)
\]

For this class of models, large \( f_{NL} \) is achieved when two conditions are met: 1) the variables \( \delta \tilde{\sigma}(1) \) and \( \delta \tilde{\chi}(1) \) are highly correlated; 2) entropy perturbations dominate over adiabatic ones, \( P_{\delta}(k) \gg P_{\tilde{\sigma}}(k) \).

Quantitatively, the equations \( (4.16), (4.17) \) and \( (4.18) \) lead to the expressions
\[
K(k) = (8\pi G)^3 \sin \Theta \cos \Theta \left( \left( \frac{W_\chi}{W_\psi W(0)} \right)^2 - \left( \frac{V_{\chi}}{V_{\psi} V(0)} \right)^2 \right)^2 \frac{V_{\psi}^2 W_\chi^2 W_\psi^2}{6(V_{\chi}^2 W_\psi^2 + V_{\psi} W_\chi^2) k^3}, \quad (4.24)
\]
\[
P_{\chi}(k) = (8\pi G)^3 \left( \sin^2 \Theta \left( \frac{W_\chi}{W_\psi W(0)} \right)^2 + \cos^2 \Theta \left( \frac{V_{\chi}}{V_{\psi} V(0)} \right)^2 \right) \frac{V_{\psi}^2 W_\chi^2 W_\psi^2}{6(V_{\chi}^2 W_\psi^2 + V_{\psi} W_\chi^2) k^3}. \quad (4.25)
\]

The inequality \( (4.23) \) can be rewritten now as
\[
\left| \left( \frac{W_\chi}{W_\psi W(0)} \right)^2 - \left( \frac{V_{\chi}}{V_{\psi} V(0)} \right)^2 \right| \gg |\tan \Theta| \left( \frac{W_\chi}{W_\psi W(0)} \right)^2 + |\cot \Theta| \left( \frac{V_{\chi}}{V_{\psi} V(0)} \right)^2.
\]

This condition can be satisfied for some inflationary models \( \boxed{13} \).

**B. Sum potential, \( U = V(\varphi) + W(\chi) \).**

In terms of initial fields perturbations, the quantities \( \delta \tilde{\sigma}(1) \) and \( \delta \tilde{\chi}(1) \) are
\[
\delta \tilde{\sigma}(1) = 8\pi G \frac{W_\chi^2 V(0) + W_\psi^2 V(0)}{W_\chi^2 + W_\psi^2} \frac{\delta \varphi^2 (1)_{\psi*} - \delta \chi^2 (1)_{\psi*}}{V_{\psi*}^2} + 8\pi G \left( \frac{W_\chi V(0) \delta \varphi^2 (1)_{\psi*}}{V_{\psi*}^2} + \frac{W_\chi V(0) \delta \chi^2 (1)_{\psi*}}{V_{\psi*}^2} \right).
\]

The equation \( \boxed{8.56} \) yields
\[
\zeta(2) = \left[ \frac{V_{\chi}^2 + W_{\chi}^2}{8\pi G U_{\psi}^2 (0)} \right] - \left[ \frac{V_{\chi}^2 (W_{\psi}^2 + V_{\psi}^2)}{8\pi G U_{\psi}^2 (0)} \right] \left( \frac{W_\chi^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right) \left( \frac{W_{\psi}^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right)
\]
\[
\zeta(2) = \left[ \frac{V_{\chi}^2 (W_{\psi}^2 + V_{\psi}^2)}{8\pi G U_{\psi}^2 (0)} \right] - \left[ \frac{V_{\chi}^2 (W_{\psi}^2 + V_{\psi}^2)}{8\pi G U_{\psi}^2 (0)} \right] \left( \frac{W_\chi^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right) \left( \frac{W_{\psi}^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right)
\]
\[
+ \frac{W_{\chi} (V_{\psi} + V_{\psi}) - V_{\chi} (W_{\psi} - W_{\psi})}{8\pi G U_{\psi}^2 (0)} \left( \frac{W_\chi^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right) \left( \frac{W_{\psi}^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right)
\]
\[
+ \frac{W_{\psi} (V_{\chi} + V_{\chi}) - V_{\psi} (W_{\chi} - W_{\chi})}{8\pi G U_{\psi}^2 (0)} \left( \frac{W_\chi^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right) \left( \frac{W_{\psi}^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right)
\]
\[
+ \frac{V_{\chi}^2 (W_{\psi}^2 + V_{\psi}^2)}{8\pi G U_{\psi}^2 (0)} \left( \frac{W_{\psi}^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right) \left( \frac{W_{\psi}^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right) \left( \frac{W_{\psi}^2 (W_{\psi}^2 + V_{\psi}^2)}{W_\chi^2 + V_{\psi}^2} \right)
\]
\[
\delta \tilde{\sigma}(1) - \delta \tilde{\chi}(2). \quad (4.29)
\]
This equation leads to the same large $f_{NL}$ condition \( \text{(1.21)} \), as in the case of product potentials. For the case of sum potentials, we have

\[
K(k) = \frac{(8\pi G)^3 U(0)_0 V_\phi W_\chi V_{\phi }^2 + W_\chi^2}{6k^2 V_{\phi }^2 W_\chi^2 + W_\chi^2} \left( \frac{W_\chi^2 V(0) - V_\phi^2 W(0)}{W_\chi^2} - \frac{V(0)_* W_\chi^2 - W(0)_* V_\phi^2}{W_\chi^2} \right),
\]
\[\text{(4.30)}\]

\[
P_{\zeta}(k) = \frac{(8\pi G)^3 U(0)_0}{6k^2 V_{\phi }^2 W_\chi^2 (V_{\phi }^2 + W_\chi^2)} \left[ (W_\chi^2 V(0) - V_\phi^2 W(0))^2 W_\chi^2 + V_{\phi }^2 \right. \\
- 2 \left( W_\chi^2 V(0) - V_\phi^2 W(0) \right) (V(0)_* W_{\chi}^2 - W(0)_* V_{\phi }^2) + \left( W(0)_* W_{\chi}^2 + W(0)_* V_{\phi }^2 \right) (W_{\chi}^2 + V_{\phi }^2). \]
\[\text{(4.31)}\]

The inequality \( \text{(1.23)} \) gives

\[
\left| \frac{W_\chi^2 V(0) - V_\phi^2 W(0)}{W_\chi^2 + V_{\phi }^2} - \frac{V_\phi W_{\chi}^2}{V_{\phi }^2 + W_{\chi}^2} \right| \gg \left| \frac{W_\chi^2 + V_{\phi }^2}{U(0)_0 W_\chi} \right| \left( \frac{W_\chi^2 V(0) - V_\phi^2 W(0)}{W_\chi^2 + V_{\phi }^2} \right)^2 \left( \frac{W_\chi^2 + V_{\phi }^2}{W_\chi^2 + V_{\phi }^2} \right)
\]
\[\text{(4.32)}\]

Some examples of slow-roll models satisfying this condition are presented in ref. \[13\].

In general, the coefficients $C_{33}$, $C_{55}$ do not have to be small. For instance, if the field $\chi$ is subdominant during slow roll and

\[
W(0) \leq W(0)_*, \quad \left| \frac{W_\chi}{V_{\phi}} \right| \ll 1, \quad \frac{W_\chi^2 V(0)}{V_{\phi }^2} \ll W(0)_*,
\]
\[\text{(4.33)}\]

then

\[
C_{33} = \frac{V_\phi}{W_\chi} \frac{W(0)_*}{U(0)} \left( \frac{V_{\phi}^2}{8\pi G U(0)_*} - \frac{V_{\phi}^2}{8\pi G U(0)_*} - \frac{W_{\chi}^2}{8\pi G U(0)_*} \right),
\]
\[\text{(4.34)}\]

\[
C_{55} = -\frac{V_\phi}{W_\chi} \frac{W(0)_*}{U(0)} \frac{W_{\chi}^2}{8\pi G U(0)_*}.
\]
\[\text{(4.35)}\]

The coefficients $C_{33}$, $C_{55}$ can be valuable due the enhancing factor $V_\phi/W_\chi$. In this regime, the curvature perturbation $\zeta^{(2)}$ remains constant and entropy perturbation may be small, $P_{\delta\delta}(k) \ll P_{\delta\zeta}(k)$. An example of such solution in the "adiabatic limit" was presented in ref. \[14\]. Although the entropy perturbation $\delta s_{(1)}$ is decaying, it can not be set equal to zero, since then the parameter $f_{NL}$ gets a wrong small value.

V. CONCLUSION.

We presented a derivation of general expression for the second order curvature perturbation $\zeta^{(2)}$ in the form of a functional over a background solution. For two special cases, the explicit expressions was obtained that reproduce known results. It is shown that, in contrast to the linear perturbation theory, the curvature perturbation depend on both the adiabatic and nonadiabatic perturbations. Moreover, a significant level of non-Gaussianity can be generated during the large scale evolution only if nonadiabatic perturbations are non-negligible at the end of inflation. Hence, to compare inflationary theory predictions at large $f_{NL}$ with observational data, one need to investigate the evolution of the large scale curvature perturbation on post-inflationary stages.

Acknowledgments

The author is grateful to Joseph Elliston and David Mulryne for useful discussions and comments.

[1] J. Maldacena, JHEP 0305, 013 (2003).
