From the Skyrme model
to hypothetical Skyrmion stars:
Astrophysical implications

R. Ouyed

Department of Physics and Astronomy, University of Calgary, 2500 University Drive NW, Calgary, Alberta, T2N 1N4 Canada

Abstract

We discuss the Skyrme model for strong interactions and the concept of a Skyrmion fluid. The corresponding compact objects, namely Skyrmion stars, constructed from the equation of state describing such a fluid are presented and compared to models of neutron stars based on modern equation of states. We suggest plausible Skyrmion star candidates in the 4U 1636−53 and 4U 1820−30 low mass X-ray binary systems where the suggested masses of the accreting compact companion (∼2.0M⊙) remain a challenge for neutron star models.

1 Introduction

We are closing in on neutron stars both observationally and theoretically. Observationally more and better masses and radii are determined by a number of different methods. The “large” masses implied in few cases (as in the Vela X-1 pulsar where a mass of ∼1.78M⊙ has been suggested; [1]) poses a real challenge to models of neutron stars built using the so-called modern equation of state (EOS) where the uncertainties are reduced by improved two- and three-body forces, relativistic effects and many-body calculations [2]. Even the stiffest EOS so far developed seem to be facing difficulty in accounting for the extreme values (up to 2.4M⊙; Sect. 4). In [3], hereafter OB, as an alternative, we constructed an EOS of dense matter based on the Skyrme model for strong interactions which represents baryons as solitons of classic pionic fields. The resulting compact objects we named Skyrmion stars (SSs) [4] are intrinsically heavier (due to the stiffness of the Skyrmion fluid; hereafter SF) than any other type of compact stars computed using modern EOS. SSs can be as massive as ∼2.8M⊙ leading us to speculate (given the above mentioned observations) that these might exist in nature [2]. The paper is presented as follows: Sect. 2 is devoted to the discussion of the Skyrme model and its link to Quantum-ChromoDynamics (QCD).

1Email: ouyed@phas.ucalgary.ca

2Skyrmion stars, like neutron stars, are likely to be born with masses around 1.5M⊙. We expect only older SSs that have accreted enough mass to reach these extreme masses.
The problem of the *missing attraction* in the Skyrme model is described. In Sect. 3, we discuss the role the dilaton (the glueball potential in QCD) could play in curing such a problem by binding Skyrmions together to form the SF. We end Sect. 3 by reminding the reader of the basic properties of the SF and the resulting EOS used to construct models of SSs. The astrophysical implications follow in Sect. 4 where we compare SSs to stars constructed using modern EOS of dense matter. We conclude in Sect. 5.

2 The Skyrme model

We first give a brief overview of the Skyrme model. The fundamental principles are discussed at a basic level. The interested reader is referred to Ref. [5] for a thorough introduction to the topic.

Skyrme constructed a model of pion interactions consisting of a conventional model of weak meson interactions plus an additional (higher-order) term thought to take into account indirect effects of heavier mesons like the $\rho$-meson. The now well-known Skyrme Lagrangian density is usually written as [6],

$$L_{\text{Skyrme}} = L_2 + L_4$$

where,

$$L_2 = \frac{f_\pi^2}{4} Tr(\partial_\mu U \partial^\mu U^+)$$

is the Skyrme term ($U$ is the chiral field and $f_\pi$ is interpreted as the pion decay constant), and

$$L_4 = \frac{1}{32e^2} Tr([U^+ \partial_\mu U, U^+ \partial_\nu U][U^+ \partial^\mu U, U^+ \partial^\nu U])$$

is the quadratic term introduced by Skyrme to keep the Skyrmion stable against the Derrick instability [7] ($e$ is the Skyrme parameter). Skyrme found that his model contained ‘topologically nontrivial’ configurations (extended objects) of the meson fields, namely topological solitons, which he identified as baryons. For twenty years the Skyrme model was overshadowed by the tremendous success of QCD and only in the early 1980’s after the establishment of its link to low energy QCD that the model was revived.

2.1 Skyrmions and QCD

The success of QCD is limited to the high energy regime, while at low energy it remains virtually intractable. The reason for this is that QCD has a running coupling constant $\alpha_s$; it is a function of momentum transfer, or distance. At short distances of the order of 0.1 fm or less (high energy and momentum transfer of several GeV) QCD is characterized by a small enough $\alpha_s$ that it is treated perturbatively. All of the results obtained in this regime are consistent with experimental data. This is the phase in which the relevant degrees of freedom are quarks and gluons and it is called the asymptotic freedom phase. At large distances of the order of 1 fm or more (low energy and momentum transfer of 1 GeV or less)
\( \alpha_s \) is of the order of unity and QCD is a nonperturbative theory. This is the confinement phase in which quarks are confined inside hadrons and the hadronic degrees of freedom are more relevant. This phase, which is the most practical, is the most mathematically complex. It should provide all properties of hadrons such as masses, sizes, magnetic moments, lifetimes, scattering properties and, in principle, all nuclear phenomena.

The first major step to overcome this problem was taken by 'tHooft [8]. He found that in the limit of a large number of colors (large \( N_c \)), \( 1/N_c \) could be used as an expansion parameter. In this limit, QCD simplifies a great deal and 'tHooft went on to show that at large \( N_c \), QCD is equivalent to a local field theory of mesons and ‘glueballs’ (bound states of gluons, without quarks), with an effective interaction between them of order \( 1/N_c \). The second step was taken by Witten [9]. Assuming confinement, he showed that baryons in large \( N_c \) QCD behave much like solitons in a weakly coupled local field theory of mesons. In this limit, baryon masses scale as \( N_c = 1/g^2 \), where \( g \) is the strength of the meson coupling, while baryon sizes are of order 1. Solitons in weakly coupled theories have masses that scale as \( 1/g^2 \) and sizes that tend to constants as \( g \) tends to zero. Even though the mesons are weakly interacting, the solitons interact strongly as do baryons in QCD.

The next natural step, it seems, is to derive the effective meson Lagrangian from the fundamental QCD Lagrangian. This task, as it turned out, is immensely difficult. Its achievement is equivalent to the solution of the intractable original problem of low energy nonperturbative QCD in the confinement phase. Nevertheless, the form of the resulting effective Lagrangian is being narrowed down under reasonable assumptions. Assuming chiral symmetry is spontaneously broken in QCD (with the physical pions as the resulting Goldstone bosons, and taking the low energy limit in which one expects the Golstone bosons to dominate the dynamics) it has been shown that the first term in the resulting low energy effective Lagrangian is [10]:

\[
L_{\text{eff}} = \frac{N_c f_\pi^2}{16} \text{Tr}(\partial_\mu U \partial^\mu U^+) + ...
\]

which is astonishingly similar to the Skyrme term (Eq. 2)! This picture, which emerged from large-\( N_c \) QCD, is precisely what Skyrme had in mind long before QCD. Further work showed that the similarities between the Skyrme model and to what is described as the nature of mesons and baryons in large \( N_c \) QCD is simply intriguing.

The Skyrme model, however, as it is build was known to predict an isospin independent spin-orbit force with the wrong sign. That is, it predicts a repulsive interaction.

### 2.2 Skyrme model and the missing attraction

The product ansatz for the two-baryon system as suggested in Ref. [6] (Eq. 2), beyond its relative simplicity as compared to other two-baryon field configurations which can be found in the literature [11, 12], becomes exact for large \( N - N \) separation. Unfortunately, it is not the case for the isoscalar component of the spin-orbit force since the standard Skyrme model predicts an isospin independent spin-orbit force with the wrong sign. Namely, it predicts a repulsive interaction while the phenomenological Bonn potential [13] as the
Paris potential \cite{13} gives an attractive one. This came to be known as the problem of the missing attraction. Extensions of the Skyrme model consisted on including higher-order terms in powers of the derivatives of the pion field \cite{13, 16}. Expressed in terms of an $SU(2)$ matrix $U$ which, as we have said characterizes the pion field, a six-order term corresponding to $\omega$-meson exchange \cite{17},

\[ L_6 = -\frac{\beta_\omega}{2\omega_0^2} B_\mu(U) B^\mu(U) \] (5)

where $B^\mu = \epsilon^{\mu\nu\alpha\beta} Tr \left( (\partial_\nu U) (\partial_\alpha U) (\partial_\beta U) (\partial_\gamma U) \right) / 24\pi^2$ is the baryon current, $m_\omega$ the $\omega$-meson mass and $\beta_\omega$ a dimensionless parameter related to the $\omega \rightarrow \pi \gamma$ width, might be a good candidate to solve the problem of the $N - N$ isoscalar spin-orbit force. While it was believed that the inclusion of such a term leads to the correct sign (attractive interaction) for the isoscalar spin-orbit potential \cite{18, 19}, recent calculations (see Ref. \cite{20} for e.g.) proved that by taking into account the second part of of the sixth-order term the anomaly of the Skyrme model (repulsive force instead of attractive) remains. The treatment of the spin-orbit part of the two-pion exchange potential within the Skyrme model needed to be improved in order to correct the anomaly of that sign. Below we explain how and why the dilaton field was suggested as a plausible cure.

3 The Dilaton field in the Skyrme model

3.1 The Dilaton contribution and Skyrmion structure

In Ref. \cite{19} and \cite{22} the authors explored the idea of coupling the Skyrmion to the dilaton field. This idea to account for a scalar field confines the Skyrmion and provides the attractive term missing in the original Skyrme formalism (see OB for more on this).

3.2 The Skyrmion fluid and Skyrmion stars

We start by writing the energy of $N$ Skyrmions per unit volume (parameterized by the density, $\rho_V$) at finite temperature. In the mean field approximation it is given by (where we adopt natural units with $\hbar = c = 1$),

\[ E_V = 2g_N \int \frac{d^3 p}{(2\pi)^3} E_p(n_p + \bar{\rho}_p) + V_\sigma (\sigma_0) - \frac{1}{2} e^{2\sigma_0} m_\omega^2 \omega_0^2 + g_V \omega_0 \rho_V . \] (6)

Here, $g_V$ is the strength of the coupling of the $\omega_0$-meson (of mass $m_\omega$; $\omega_0 = <\omega>$ is the mean-field value) to Skyrmions while $g_N$ represents the isospin degrees of freedom.

\textsuperscript{3}In Ref. \cite{18} and \cite{19} the authors considered only one part of the interaction due to the sixth-order term in their calculations and omitted the second part which arises from the exchange current \cite{21}.
Neutron stars vs Skyrmion stars

4U 1636-53

In Fig. 1 we compare the $M - R$ relation for Skyrmion stars (OB) to the theoretical $M - R$ curve obtained using six recent realistic models for the EOS (UU, BBB1, BBB2, BPAL12, Hyp, and K$^{-1}$). The solid curves labeled SS1 and SS2 are for strange stars (the data was kindly provided to us by the authors of Ref. [26]). The triangle depicts the mass-radius constraint from fits to X-ray bursts in 4U 1636-53. Inside the triangle is the allowed range of $M$ and $R$ which satisfies the compactness constraints as modeled in Ref. [27] (see their Figure 4), and clearly favoring stiffer EOSs. Our modeled stars (OB) cross the triangle suggestive of 4U 1636-53 as a plausible SS candidate.

QPOs and Skyrmion stars: 4U 1820-30

QPOs are neutron stars emitting X-rays at frequencies of the orbiting accreting matter. Such quasi-periodic oscillations (QPO) have been found in 12 binaries of neutron stars with low mass companions. If the QPO originate from the innermost stable orbit [28, 29] of the accreting matter, their observed values imply that the accreting neutron star has a mass of $\simeq 2.4M_\odot$ in the case of 4U 1820-30; this would rule out most modern EOSs allowing only the stiffest ones.

4In the energy equation as given by Eq. (6), the $\rho$ meson coupling has been omitted which reduces the analysis to symmetric nuclear matter only. As such in OB the isospin degrees of freedom ($g_N$) was taken as a free parameter as to allow for the two regimes – pure neutron matter and symmetric matter – to be explored.
Figure 1: The $M - R$ relation for non-rotating Skyrmion stars (OB) as compared to theoretical models of non-rotating neutron stars (UU, BBB1, BBB2, BPAL12, Hyp, and K$^{-1}$) and strange stars (SS1 and SS2); see Ref. [4]. The Schwarzschild radius ($2GM/c^2$) is shown as a dotted line. Inside the triangle is the allowed range of $M$ and $R$ for 4U 1636-53 as modeled in Ref. [27] using fits to X-ray bursts.

SSs is one possibility given that the gravitational mass of the maximum stable non-rotating SS has a value of $\sim 2.8 M_\odot$ (OB). For completeness, one should note that even by making the modern/recent EOS stiffer at high densities in a smooth way, the maximum mass can never exceed 2.3 $M_\odot$ due to the causality condition [30].

5 Conclusion

We gave a brief historical overview of the Skyrme model, its predictions of hadronic interactions and its interesting connection to QCD. Here we showed how the repulsive term in the Skyrme model can be removed by coupling the Skyrmion to the dilaton field. This lead to the concept of the Skyrmion fluid and the related hypothetical stars we called Skyrmion stars. The stiffness of the Skyrmion fluid allows for SSs to be as massive as 2.8 $M_\odot$. The SSs show unique features; for a given mass their radii are in general larger than those of neutron stars constructed using modern EOSs. We discussed examples in astrophysics where SSs might constitute plausible candidates. Future observations constraining the mass-radius plane of compact stars would most likely prove or rule out the existence of SSs in nature.
Acknowledgements. I am grateful to S. Morsink, G. Kälbermann and R. Bhaduri for encouraging help and valuable discussions. The research of R.O. is supported by grants from the Natural Science and Engineering Council of Canada (NSERC).

References

[1] Barziv, O., Kaper, L., van Kerkwijk, M. H., Telting, J. H. & van Paradijs, J. 2001, A&A, 377, 925
[2] Heiselberg, H. & Pandharipande, V. 2000, Ann. Rev. Nucl. & Part. Sci., 50, 481
[3] Ouyed, R. & Butler, M. 1999, ApJ, 522, 453 (OB)
[4] Ouyed, R. 2002, A&A, 2002, 382, 939
[5] Bhaduri, R. K. 1988, Models of the Nucleon: from Quarks to Soliton (Redwood City: Addison Wesley)
[6] Skyrme, T. H. R. 1962a, Proc. R. Soc. London Ser., A260, 127
[7] Skyrme, T. H. R. 1962b, Nucl. Phys., 31, 556
[8] 'tHooft, G., 1974, Nucl. Phys., B72, 461
[9] Witten, E., 1983, Nucl. Phys. B223, 422 ; ibid, 443
[10] Karchev, N. I. & Slavnov, A. A. 1985, Teor. Mat. Fizika., 65, 192 (in russian)
[11] Nyman, E. M. & Riska, D. O. 1986, Phys. Lett., 175B, 392
[12] Amado, R. D., Shao, B. & Walet, N. R. 1993, Phys. Rev. Lett., 314B, 159 (erratum 1994 Phys. Lett. 324B, 467)
[13] Machleidt, R., Holinde, K. & Elster, Ch. 1987, Phys. Rep., 149, 1
[14] Lacombe M., Loiseau B., Richard J. M., et al. 1980, Phys. Rev. C, 21, 861
[15] Moussallam, B. 1993, Ann. Phys., NY, 225, 264
[16] Abada, A. & Merabet, H. 1993, Phys. Rev. D, 48, 2337
[17] Jackson, A. et al. 1985, Phys. Lett., 154B, 101
[18] Riska, D. O. & Schwesinger, B. 1989, Phys. Lett., 229B, 339
[19] Kälbermann, G. & Eisenberg, J. M. 1995, Phys. Lett., 349B, 416
[20] Abada, A. 1996, J. Phys. G: Nucl. Part. Phys., 22, L57-L63
[21] Nyman, E. M. & Riska, D. O. 1987, Nucl. Phys. A, 468, 473
[22] Kälbermann, G. 1997, Nucl. Phys., A612, 359
[23] Walhout, T. S. 1988, Nucl. Phys. A484, 397
[24] Walhout, T. S. 1990, Nucl. Phys. A519, 816
[25] Heusler, M., Droz, S., & Straumann, N. 1992, Phys. Lett. B 285, 21
[26] Li, X-D., Ray, S., Dey, J., Dey, M. & Bombaci, I. 1999, ApJL, 527, L51-L54
[27] Nath, N. R., Strohmayer, T. E., & Swank, J. H. 2002, ApJ, 564, 353
[28] Zhang, W., Strohmayer, T. E. & Swank, J. H. 1997, ApJ, 482, L167
[29] Miller, M. C., Lamb, F. K. & Psaltis, D. 1998, ApJ, 508, 791
[30] Heiselberg, H. & Hjorth-Jensen, M. 2000, Phys. Rep., 328/5-6, 237