TWO FUNDAMENTAL COSMOLOGICAL LAWS
OF THE LOCAL UNIVERSE

Yurij V. Baryshev¹

¹ Astronomical Department of the Saint Petersburg State University
Universitetskij pr.28, Stary Peterhoff, St. Petersburg, 198504
yubaryshev@mail.ru

Abstract: The Local Universe is the most detail studied part of the observable region of space with the radius R about 100 Mpc. There are two empirical fundamental cosmological laws directly established from observations in the Local Universe independently from cosmological theory: first, the Hubble-Humason-Sandage linear redshift-distance law and second, Carpenter-Karachentsev-deVaucouleurs density-radius power-law. Review of modern state of these empirical laws and their cosmological significance is given. Possible theoretical interpretations of the surprising coexistence of both laws at the spatial scales from 1 Mpc to 100 Mpc are discussed. Comparison of the standard space-expansion explanation of the cosmological redshift with possible global gravitational redshift model is given.

Keywords: cosmology, Local Universe, redshift law, density law

PACS: 98.80.-k

1. Introduction

Cosmology as a physical science is based on observations, experiments and theoretical interpretations. Hubble 1937 [22] put forwarded ”The Observational Approach to Cosmology”. It was developed later by Sandage 1995a [41] who used the term ”Practical Cosmology” to denote the observational study of ”our sample of the Universe”, which delivers possibilities for testing alternative initial hypotheses and main predictions of cosmological models.

Cosmology deals with a number of empirical facts among which one hopes to find fundamental laws. This process is complicated by great limitations and even under the paradigmatic grip of any current standard cosmology. One should distinguish between two kinds of cosmological laws:

• directly measured empirical laws,
• logically inferred theoretical laws.
The empirical laws are directly measured relations between observable quantities, which should be corrected for known selection and distortion effects. The logically inferred theoretical laws (theoretical interpretations) are made on the basis of an accepted cosmological model, e.g. the standard or an alternative cosmological model. Theoretical derivations utilize modern theoretical physics and even its possible extensions, which can be tested by observations.

During one hundred years of intensive investigations of the Local Universe (which can be defined as region of space with radius $R$ about 100 Mpc) two especially important cosmological empirical laws were unveiled (see review in [7], [5], [8]):

- the cosmological linear redshift-distance law $cz = HR$,
- the power-law correlation of galaxy clustering $\Gamma(r) \propto r^{-\gamma}$.

Here $R$ is the distance to a galaxy, $H$ is the Hubble constant, $r$ is the radius of test spheres around each galaxy, $\Gamma(r)$ is the complete correlation function (the conditional density) and $\gamma$ is the power-law exponent.

The empirical laws, being based on repeatable observations, are independent of existing or future cosmological models. However, the derived theoretical laws are valid only in the frame of a specific cosmological model. Good examples are the empirical Hubble linear redshift-distance ($z \propto R$) law and the derived theoretical space-expansion velocity-distance ($V_{sp-exp} \propto R$) law within the Friedmann model.

An analysis of both empirical cosmological facts and theoretical initial assumptions together with main logical inference in the frame of the standard and several alternative cosmological models is presented in our book Baryshev & Teerikorpi 2012 [7]. Below I concentrate on the significance for cosmology the redshift-radius and density-radius empirical cosmological laws.

2. Hubble-Humason-Sandage linear redshift-distance law

The linear relation between cosmological redshift and distance to galaxies was first established by Hubble 1929 [21] using distance estimations for 30 galaxies at very small scales $1 \div 10$ Mpc, corresponding to redshifts $z < 0.003$ or spectroscopic radial velocities $v_{rad} < 1000$ km/s.

The extension of the linearity of the redshift-distance relation up to redshifts about $z < 0.05$ or scales about 150 Mpc was done by Hubble & Humason 1931 [23]. They emphasized that "The interpretation of red-shift as actual velocity, however, does not command the same confidence, and the term "velocity" will be used for the present in the sense of "apparent" velocity, without prejudice as to its ultimate significance."

Many years of detail studies of the linearity of the redshift-distance law was performed by Sandage at the Palomar 5m Hale telescope. Sandage developed a special program for 5m telescope to discriminate between selected world models [39].
One of the last papers of Sandage’s team, devoted to analysis of the observed redshift-distance relation, demonstrated linearity of \( z(R) \) law in the interval of redshifts \( 0.001 \div 0.1 \) [43].

Hence for the Local Universe we have observationally established the linear redshift-distance Hubble-Humason-Sandage (HHS) law in the form:

\[
z = \frac{H_{\text{loc}} R}{c} = \frac{V_{\text{app}}}{c}
\]  

(1)

where \( c \) is the velocity of light, \( H_{\text{loc}} \) is the value of the Hubble constant measured in the Local Universe, \( R \) is the measured distance to a galaxy, \( V_{\text{app}} = H_{\text{loc}} \times R \) is the apparent radial velocity which corresponds measured shift of spectral lines \( z \):

\[
z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}}
\]  

(2)

where \( \lambda_{\text{obs}} \) is the observed photon wavelength at the telescope and \( \lambda_{\text{emit}} \) is the wavelength of photon emitted at distance \( R \). The HHS law (1) is also frequently called the Hubble law of redshifts. Note that here \( z \) is the cosmological part of the observed shift of spectral lines after corrections for the solar system motions and averaging over peculiar velocities of galaxies.

The cosmological redshift is a universal physical phenomenon which does not depend on the wavelength of a photon. Very important cosmological question is about the minimal scale where the HHS law is true. Recent studies by Ekholm et al. 2001 [14], Karachentsev et al. 2003 [28] and Karachentsev et al. 2013 [29] demonstrated that according to modern data on 869 galaxy distances in the Local Volume the linear Hubble law well established at small scales \( 1 \div 10 \) Mpc. Remarkably, this is exactly the same interval of scales where Hubble 1929 [21] discovered the redshift-distance law with only 30 galaxies.

In Fig.1 apparent radial velocity-distance relation \( V_{\text{app}} = cz = H_{\text{loc}} R \) for 156 Local Volume galaxies is shown from [28]. The value of the local Hubble constant is \( H_{\text{loc}} = 72 \pm 3 \text{ km/sec/Mpc} \), which is consistent with recent estimations from different Local Universe surveys.

3. Carpenter-Karachentsev-deVaucouleurs density-radius power-law

The rich history of discovery and acute discussions around the density-radius relation for the spatial galaxy distribution in the Local Universe is presented in [5], [6], [7], [45], [47].

Carpenter 1938 [9] was the first who obtained from observations of galaxy systems of different sizes the approximate power-law relation between the number of galaxies \( N \) in a cluster and the size \( r \) of the clusters in the form \( N(r) \propto r^{1.5} \).

Karachentsev 1966, 1968 [26], [27] added an important aspect to Carpenters result. He estimated average properties of 143 systems from binary galaxies to superclusters and found evidence that both luminous and total (virial) mass densities
Figure 1: Apparent radial velocity-distance relation $V_{\text{app}} = cz = H_{\text{loc}}R$ for 156 Local Volume galaxies is shown from [28]. Also the density-radius relation $\Gamma(r) \propto r^{-\gamma}$ from [48] is shown by thin lines for VL2N sample from 2MRS survey [25] and for power-law density-radius relation for exponent $\gamma = 1$.

are decreasing with increasing size of a system. This showed for the first time that the mass radius behavior of the dark mass is also a power law, but the exponent can be different than for the luminous matter.

de Vaucouleurs 1970, 1971 [10], [11] summarized his own and many others works in studies of galaxy systems from pairs to superclusters, including clustering of Abel’s rich galaxy clusters [1], [2]. Based on all available data de Vaucouleurs made the decisive step in recognizing the cosmological significance of the clustering of galaxies as the universal observational power-law density-radius relation [10]. He considered this fundamental cosmological law as the case for a hierarchical cosmology.

Since that time the Carpenter-Karachentsev-deVaucouleurs (CKdeV) density-radius empirical cosmological law was discovered and presented in the form

$$\rho(r) = \rho_0 \left( \frac{r}{r_0} \right)^{-\gamma}$$

where $\rho(r)$ is the mass density within a spherical volume of radius $r$ and $\rho_0$ and $r_0$ are the density and radius at the lower cutoff of the structure. The available at that time galaxy data led to the power-law exponent $\gamma = 1.7$.

Intriguingly, at international astronomical conferences, the Great Debate on the existence of very large scale structures in the observed galaxy universe was originated. An acute discussion between homogeneity defenders and inhomogeneity observers (see reviews [6], [7]) is actually ongoing nowadays, though modern data demonstrate the existence of galaxy structures with sizes up to 400 - 1000 Mpc (e.g. [44]). The reason of the hot debates is that in the frame of the standard cosmological model
the homogeneous matter distribution is the basic mathematical assumption for the derivation of linear Hubble law of redshifts [35], [7].

In fact, to understand the observed CKdeV density-radius relation one needs to develop a new mathematical and physical concepts which include discrete fractal stochastic structures. This was done by Mandelbrot 1977 [33] in his theory of fractals, which opens new perspective for description of complex discrete physical systems with properties very different from continues fluid flows. Fractal approach to the analysis of the distribution of galaxies was first used in [33], [38] and developed in [46], [18], [47]. For a detailed review of the history and prospects of the fractal approach to the study of the large-scale distribution of galaxies see [6], [7].

One of the most fundamental statistical properties of the general space distribution of galaxies, which includes complex observed structures (filaments, voids, shells, and walls), is the fractal dimension of the global structure as a whole. According to [18] the fractal dimension \( D \) of a stochastic fractal point process in 3-dimensional space can be inferred from the complete correlation function (conditional density) \( \Gamma(r) \), which has the power-law:

\[
\Gamma(r) = \frac{\langle n(\vec{r}_1)n(\vec{r}_2) \rangle}{\langle n(\vec{r}) \rangle} = kr^{-\gamma} = kr^{-(3-D)}
\]  

where \( n(\vec{r}_i) \), is the particle number density inside volume \( dV_i \) around point \( i \) with the coordinates \( \vec{r}_i \), \( r = | \vec{r}_{12} | = | \vec{r}_1 - \vec{r}_2 | \), the vector of the distance between points 1 and 2, and \( \langle x \rangle \), the ensemble average of \( x \). The second and third equalities are written for isotropic stationary processes, where \( D \) is the fractal dimension and \( \gamma = 3-D \) is called the co-dimension of the fractal. The physical dimension of the \( \Gamma(r) \) is \( 1/cm^3 \) and it is calculated under the condition of all occupied points, this is why it is called the conditional density.

The power-law character of the conditional density (eq 4) is the principal explanation of the CKdeV density-radius law (eq 3). A more detailed analysis will include transition from number density \( n(r) \) to mass density \( \rho(r) \), which should also take into account the luminous and dark matter. Fortunately, conditional density analysis of the real galaxy catalogues shows that it is sufficient for describing the spatial galaxy distribution as a good first approximation.

The statistical estimate of the complete correlation function \( \Gamma(r) \) (conditional density) for the galaxy sample considered is defined as [18]:

\[
\Gamma(r) = \frac{1}{N_c(r)} \sum_{i=1}^{N_c(r)} \frac{N_i(r)}{V(r)}
\]

where \( N_i(r) \) is the number of points inside spherical volume \( V(r) \) around \( i \)-th point and \( N_c(r) \) is the number of centers of test spheres, i.e., the number of points about which this volume is circumscribed. It is important to bear in mind that averaging has to be performed without going beyond the considered sample volume, and this restriction has important effect on the value of the greatest available scale lengths.
This condition strongly restricts the scale-lengths accessible for the analysis of galaxy correlations, because, strictly speaking, in order to reliably compute the conditional density on some selected scale, we must analyze much greater spherical region where all test spheres are completely embedded.

For large galaxy redshift surveys the conditional density $\Gamma(r)$ is a directly determined quantity, which characterizes the spatial, kinematical, and dynamical state of the Local Universe. It can be estimated from the power-law slope $\gamma$ (co-dimension of the fractal) of the complete correlation function $\Gamma(r)$ without invoking any a priori assumptions about the evolution of non-baryonic dark matter and its association with baryonic matter (galaxies) or the form of the distribution of peculiar velocities of galaxies.

Note that the complete correlation function $\Gamma(r)$ has an important advantage over reduced correlation function $\xi(r)$ (Peebles’s two-point correlation function) in that the computation of conditional density requires no assumption about the homogeneity of spatial galaxy distribution within analyzed galaxy sample.

![Figure 2: Conditional density for Volume Limited samples of 2MRS galaxies in the Local Universe](image)

Figure 2: Conditional density for Volume Limited samples of 2MRS galaxies in the Local Universe [48]. The large dots mark the conditional density values where the most reliable slope estimation is possible. The slope $\gamma = 1.0 \pm 0.1$ for all VL samples.

Fig.2 shows the conditional density calculations [48] for the largest complete all-sky galaxy redshift survey 2MRS of the Local Universe [25]. The observed global space distribution of 2MRS galaxies can be described by the power-law complete correlation function of the form $\Gamma(r) = kr^{-\gamma}$ with a slope of $\gamma \approx 1$ over a wide interval of scale-lengths spanning from 0.1 to 100 Mpc. The deeper all-sky volume limited sample is used (from VL1 to VL4) then the larger is the maximum scale-length where the density power-law can be reliably estimated. The shift of the power-law maximum scale-length is consistent with the stochastic fractal model having the
fractal dimension $D = 3 - \gamma \approx 2$ in the whole interval of analyzed scales from 0.1 Mpc up to 100 Mpc.

In the frame of the LCDM theory of large scale structure formation there are two important predictions:

- the galaxy Local Universe is homogeneous after scales about 30 Mpc;
- due to galaxy peculiar velocities there is very large difference between slopes of conditional density calculated for redshift-based distances and real distances independent on $z$.

According to [48], the Fig.3 shows results of the conditional density calculations for the Millemium galaxy catalog (in a sample similar to S1VL2 2MRS) as a function of scale length in real and $z$ space. The predicted slopes are very different for $z$- and $r$-space. Also after scales about 30 Mpc the galaxy distribution becomes homogeneous.

![Figure 3: Conditional density of Millennium mock galaxy catalog in a sample similar to S1VL2 as a function of scale length in real and $z$ space [48]. The slopes are estimated in the $1 < r < 10$ Mpc interval. After scales about 30 Mpc the mock galaxy distribution becomes homogeneous.](image-url)

So for future testing of the nature of the Local Universe galaxy distribution there are two possibilities - first, to get more deep all-sky galaxy redshift surveys (at least up to 500 Mpc) and second, to compare conditional densities measured for redshift and real space: $\Gamma(r_z) \iff \Gamma(r_{\text{real}})$. Hence, very important observational test of the large scale structure origin in the Local Universe is the direct measurements of the peculiar velocities of galaxies. This will require further development of redshift-independent methods for determining galaxy distances and performing
time consuming observational programs aimed to measurement of such distances, like Cosmic Flows surveys [49].

Note that stochastic fractal structures naturally arise in physics as a result of the dynamical evolution of complex systems. Physical fractals are discrete stochastic systems characterized by power-law correlation functions. In particular, fractal structures arise in turbulent flows and in deterministic chaos of nonlinear dynamic systems. Phase transitions and thermodynamics of self-gravitating systems are also characterized by the formation of fractal structures [12], [13], [37]. However, many important aspects of these studies so far remain undiscovered.

4. Physical interpretations of the relation between redshift and density laws

Here I consider two possibilities for explanation of the surprising coincidence of the observed spatial scales where two empirical cosmological laws simultaneously exit (see Fig.4)

![Graph showing the Hubble-deVaucouleurs paradox](image)

Figure 4: Demonstration of the Hubble-deVaucouleurs paradox in the Local Universe. The Hubble-Humason-Sandage linear redshift law $cz = H_{loc}R$ and the fractal Carpenter-Karachentsev-deVaucouleurs density law $\Gamma(r) = k r^{-\gamma}$ with $\gamma \approx 1$ coexist at the same length-scale interval $0.1 \div 100$ Mpc. While in the frame of the SCM the linear redshift-distance relation is the strict consequence of homogeneity [30].

In the frame of the Friedmann model of the Standard Cosmological Model (SCM) there is a deep paradox between Hubble-Humason-Sandage linear redshift-distance law and Carpenter-Karachentsev-deVaucouleurs density-radius power-law. This observational Hubble-deVaucouleurs (HdeV) paradox exists due to the very basis of SCM, which explains the linear HHS law as a strict mathematical consequence of the homogeneity of the matter distribution [36], [35], [4].
For a solution of HdeV paradox within SCM one should assume a large amount of homogeneously distributed non-baryonic dark matter and dark energy. The dominance of homogeneous dark substance density over the usual baryonic matter (galaxies) must start from scales where the linear HHS redshift-distance law already exists. There are also several conceptual problems with interpretation of space-expansion in SCM [3], [17], [20], [19].

Another solution of HdeV paradox can be obtained in the frame of the Fractal Cosmological Model (FCM) [3], presented at the International conference Problems of Practical Cosmology 2008. In the frame of the FCM the space-geometry is static flat Minkowski space-time, the gravitational interaction is described within Feynman’s field gravity approach [15], [16], [7], and the matter is dynamically evolving usual baryonic substance.

The spatial distribution of galaxies in the Local Universe is the stochastic fractal structure with fractal dimension $D \approx 2$ and the cosmological redshift is the new gravitational global effect due to the whole mass within the sphere having radius equals to the distance between the source and the observer. For fractal dimension $D = 2$ the mass of the sphere of radius $r$ grouths as $M(r) \propto r^D \propto r^2$, hence the gravitational potential is $\varphi \propto M/r \propto r^{-1}$ and the cosmological global gravitational redshift is the linear function of distance $z_{gl-gr} \propto r$. It means that the surprising coincidence of length scales for both HHS and CKdeV cosmological laws now is a natural prediction of the fractal cosmological model.

So an important task of Practical Cosmology is to observationally distinct between expanding and static spaces, i.e. to establish the nature of the observed cosmological redshift. Note, that in the classical papers, Hubble 1929 [21] and Hubble & Humason 1931 [23] emphasized that the cosmological part of the measured redshift should be called ”apparent radial velocity” and actually can present the de Sitter effect of ”slowing down of atomic vibrations” - which is actually a kind of the global gravitational effect. During all his life Hubble insisted on the necessity of the observational verification of the nature of the cosmological redshift and suggested several tests together with Tolman [24].

Intriguingly, up to now, after 85 years of observational cosmology there is no crucial experiment which directly measure the real increasing distance with time. In Sandage’s List of 23 Astronomical Problems for the 1995 - 2025 years [42] the first problem of the Practical Cosmology is to test ”Is the expansion real?”.

The usually considered tests of space expansion

- Tolman’s surface brightness $(1 + z)^4$ test;
- Time dilation with SN Ia $t(z) = t(0)(1 + z)$;
- CMBR temperature $T(z) = T(0)(1 + z)$

can not distinct between space expansion redshift and global gravitational redshift mechanisms.
The crucial test of cosmological space expansion should measure the real increasing distances with time. Nowadays there are at least two proposals for such crucial tests of the expansion of the Universe:

- Sandage’s $z(t)$ test;
- Kopeikin’s $\Delta \nu/\nu$ test in the solar system

It is important to note that on the verge of modern technology there are possibilities for real direct observational tests of the physical nature of the cosmological redshift. First crucial test of the reality of the space expansion was suggested by Sandage[40], who noted that the observed redshift of a distant object (e.g. quasar) in expanding space must be changing with time according to relation $\frac{dz}{dt} = (1 + z)H_0 - H(z)$. In terms of radial velocity, the predicted change $\frac{dv}{dt} \sim 1 \text{ cm s}^{-1}/\text{yr}$. This may be within the reach of the future ELT telescope [34], [32]. In the case of the global gravitational redshift the change of redshift equals zero.

Even within the Solar System there is a possibility to test the global expansion of the universe. According to recent papers by Kopeikin[30, 31] the equations of light propagation used currently by Space Navigation Centers for fitting range and Doppler-tracking observations of celestial bodies contain some terms of the cosmological origin that are proportional to the Hubble constant $H_0$. Such project as PHARAO may be an excellent candidate for measuring the effect of the global cosmological expansion within Solar System, which has a well-predicted blue-shift effect having magnitude $\Delta \nu/\nu = 2H_0\Delta t \approx 4 \times 10^{-15}(H_0/70\text{ km s}^{-1}\text{ Mpc}^{-1})(\Delta t/10^3\text{s})$, where $H_0$ is the Hubble constant, $\Delta t$ is the time of observations. In the case of the non-expanding Universe the frequency drift equals zero.

5. Conclusion

Cosmology at Small Scales is very important part of astronomy. New mathematical and physical ideas in cosmology should be discussed and tested by experiments and observations in the Local Universe from the solar system scales up to the superclusters scales.

Surprises of recent observational cosmology of the Local Universe stimulate its further investigations. A puzzling conclusion is that the Hubble’s law, i.e. the strictly linear redshift-distance relation, is observed just inside strongly inhomogeneous galaxy distribution, i.e. deeply inside fractal structure at scales $1 \div 100$ Mpc. This empirical fact presents a profound challenge to the standard cosmological model where the homogeneity is the basic explanation of the Hubble law, and "the connection between homogeneity and Hubble’s law was the first success of the expanding world model" (Peebles et al. 1991 [36]). However the spectacular observational fact (Fig.4) is that the Hubble’s law is not a consequence of homogeneity of the galaxy distribution, as it was assumed during almost the whole history of cosmology.

New type of global physical laws can appear at cosmological scales which make cosmology especially creative science. Intriguingly, up to now there is no crucial
experiment which directly measure the real increasing distance with time. The global gravitational cosmological redshift can be such new physical phenomenon which should be tested by observations and experiments.

New powerful mathematical methods of fractal structures analysis should be developed for investigation of the large scale structure of the Universe. Even new approaches for description of gravitational interaction in the frame of modern theoretical physics can be tested at all scales from solar system up to the cosmological scales.

This is possible due to very fast development of observational technics and theoretical models which is applied to astronomical objects. Theoretical models utilize modern theoretical physics and even its possible extensions, which can be tested by observations. In conclusion we may say that now we are entering in the golden age of cosmological physics of the Local Universe. So the research in the field of Cosmology at Small Scales is a perspective direction in modern physical science.

Acknowledgements

I am grateful to Pekka Teerikorpi, Georges Paturel, Luciano Pietronero and Francesco Sylos Labini for many years of collaboration in study large scale structure of the Universe. Also I thank Daniil Tekhanovich for joint work in analysis of modern Local Universe galaxy catalogues. This work has been supported by the Saint Petersburg State University (grant No. 6.38.18.2014).

References

[1] Abell, G.O.: The distribution of rich clusters of galaxies. Astrophys. J. Suppl. Ser. 3, 211 (1958)

[2] Abell, G.O.: Evidence regarding second-order clustering of galaxies and interactions between clusters of galaxies. Astron. J. 66, 607 (1961)

[3] Baryshev, Yu. V. Field fractal cosmological model as an example of practical cosmology approach, in ”Practical Cosmology”, Proceedings of the International Conference held at Russian Geographical Society, 23-27 June, 2008, Vol.2, p.60 (2008), (arXiv:0810.0162)

[4] Baryshev, Yu. V. Paradoxes of the Cosmological Physics in the Beginning of the 21-st Century, in Proceedings of the XXX-th International Workshop on High Energy Physics - Particle and Astroparticle Physics, Gravitation and Cosmology - Predictions, Observations and New Projects, WSPC, p.297 (2015), (arXiv:1501.01919)

[5] Baryshev, Yu. & Teerikorpi, P.: Discovery of Cosmic Fractals. World Scientific, Singapore (2002), 408 pp (Polish translation: Poznawanie kosmicznego ladu Wszechswiat,WAM, Krakow, 2005; Italian translation: La scoperta dei frattali cosmici, Bollati Boringhieri, Torino, 2006)
[6] Baryshev, Yu. & Teerikorpi, P., The fractal analysis of the large-scale galaxy distribution. Bull. Spec. Astrophys. Obs., 59, 92 (2006)

[7] Baryshev Yurij & Teerikorpi Pekka, Fundamental Questions of Practical Cosmology: Exploring the Realm of Galaxies, Springer, pp.328 (2012)

[8] Baryshev, Yu., Sylos-Labini, F., Montuori, M., Pietronero, L., Facts and Ideas in Modern Cosmology, Vistas in Astronomy 38, 419 (1994)

[9] Carpenter, E.F.: Some characteristics of associated galaxies I. A density restriction in the metagalaxy. Astrophys. J. 88, 344 (1938)

[10] de Vaucouleurs, G.: The case for a hierarchical cosmology. Science 167, 1203 (1970)

[11] de Vaucouleurs, G.: The large-scale distribution of galaxies and clusters of galaxies. Publ. Astron. Soc. Pac. 83, 113 (1971)

[12] De Vega, H., Sanches, N., Combes, F., Self-gravity as an explanation of the fractal structure of the interstellar medium, Nature 383, 56 (1996)

[13] De Vega, H., Sanches, N., Combes, F., The fractal structure of the universe: A new field theory approach. Astrophys. J. 500, 8 (1998)

[14] Ekholm, T., Baryshev, Yu., Teerikorpi, P., Hanski,M., Paturel, G.: On the quiescence of the Hubble flow in the vicinity of the local group: A study using galaxies with distances from the Cepheid PL-relation. Astron. Astrophys. 368, L17 (2001)

[15] Feynman R., Lectures on Gravitation, California Institute of Technology (1971)

[16] Feynman R., Morinigo F., Wagner W., Feynman Lectures on Gravitation, Addison-Wesley Publ. Comp. (1995)

[17] Francis M.J., Barnes L.A., James J.B., Lewis G.F., Expanding Space: the Root of all Evil? Publ. Astron. Soc. Australia, 24, 95 (2007) (astro-ph/0707.0380)

[18] Gabrielli, A., Sylos Labini, F., Joyce, M., Pietronero, L.: Statistical Physics for Cosmic Structures. Springer, Berlin (2005)

[19] Harrison, E. R., The redshift-distance and velocity-distance laws. ApJ 403, 28 (1993)

[20] Harrison, E. R., Mining energy in an expanding universe. ApJ 446, 63-66, (1995)

[21] Hubble E., A relation between distance and radial velocity among extra-galactic nebulae, Proceed.Nat.Acad.Sci., 15, 168-173 (1929)
[22] Hubble, E.: *The Observational Approach to Cosmology*. Clarendon, Oxford (1937) (68 pp)

[23] Hubble, E., Humason, M.L.: The velocity-distance relation among extra-galactic nebulae. Astrophys. J. 74, 43 (1931)

[24] Hubble E., Tolman R., C., Two methods of investigating the nature of the nebular red-shift. Astrophys. J., 82, 302-337 (1935)

[25] Huchra J. et al., The 2MASS redshift survey - description and data release, Astrophys.J. Suppl. 199, 26 (2012).

[26] Karachentsev, I.D.: Some statistical characteristics of superclusters of galaxies. Astrofizika 2, 307 (1966) (English translation in Astrophysics 2, 159)

[27] Karachentsev, I.D.: Average statistical characteristics of systems of galaxies and the problem of the existence of hidden virial mass. Commun. Byurakan Obs. 39, 76 (1968) (in Russian)

[28] Karachentsev, I., Makarov, D.I., Sharina, M.E., et al.: Local galaxy flows within 5 Mpc. Astron. Astrophys. 398, 493 (2003)

[29] Karachentsev I., Makarov D. and Elena I. Kaisina E.: Updated Nearby Galaxy Catalog AJ, 145, 101 (2013)

[30] Kopeikin S., Celestial Ephemerides in an Expanding Universe, Phys. Rev. D86, 064004 (2012)

[31] Kopeikin S., Local gravitational physics of the Hubble expansion, Eur. Phys. J. Plus (2015) 130, 11 (2015) [arXiv:1407.6667]

[32] Liske J. et al., Cosmic dynamics in the era of extremely large telescopes, Mon. Not. R. Astron. Soc. 386, 1192 (2008)

[33] Mandelbrot, B.B.: *Fractals: Form, Chance and Dimension*. W.H. Freeman, New York (1977)

[34] Pasquini L. et al., CODEX: measuring the expansion of the Universe. The Messenger 122, December 2005, p.10 (2005)

[35] Peebles, P.J.E., *Principles of Physical Cosmology*, Princeton Univ.Press, (1993)

[36] Peebles P. et al., The case for the relativistic hot big bang cosmology , Nature 352, 769 (1991).

[37] Perdang, J., Self-gravitational fractal configuration, Vistas Astron. 33, 371 (1990)

13
[38] Pietronero, L.: The fractal structure of the Universe: correlations of galaxies and clusters and the average mass density. Physica A 144, 257 (1987)

[39] Sandage, A.: The ability of the 200-inch telescope to discriminate between selected world models. Astrophys. J. 133, 355 (1961)

[40] Sandage, A., The change of redshift and apparent luminosity of galaxies due to the deceleration of the expanding universes. ApJ 136, 319 (1962)

[41] Sandage A., Practical cosmology: Inventing the past. In: Binggeli, Buser, R. (eds.) The Deep Universe, pp. 1232. Springer, Berlin (1995a)

[42] Sandage A., Astronomical problems for the next three decades. In Key Problems in Astronomy and Astrophysics, Mamaso A. and Munch G. eds., Cambridge University Press (1995b)

[43] Sandage A., Reindl B. and Tammann G., The Linearity of the Cosmic Expansion Field from 300 to 30,000 km s$^{-1}$ and the Bulk Motion of the Local Supercluster with Respect to the Cosmic Microwave Background, Astrophys. J. 714, 1441 (2010).

[44] Shirokov, S. I.; Lovyagin, N. Yu.; Baryshev, Yu. V.; Gorokhov, V. L. Large-scale fluctuations in the number density of galaxies in independent surveys of deep fields, Astron. Rep. 60, 563 (2016)

[45] Sylos Labini F., Inhomogeneities in the universe Classical and Quantum Gravity 28, 164003 (2011).

[46] Sylos Labini, F., Montuori, M., Pietronero, L.: Scale-invariance of galaxy clustering. Phys. Rep. 293, 61 (1998)

[47] Sylos Labini F., Tekhanovich D., and Baryshev Yu., Spatial density fluctuations and selection effects in galaxy redshift surveys, J. of Cosmol. and Astropart. Phys. 7, 035 (2014).

[48] Tekhanovich D. & Baryshev Yu. : Global Structure of the Local Universe according to 2MRS Survey, Astrophys. Bull., 71, 155 (2016).

[49] Tully B. et al., Cosmic Flows-2: the data, Astron. J. 146, 86 (2013).