Mathematical modeling of the MHD stability dependence on the interpole distance in the multianode aluminium electrolyser

R N Kuzmin, N P Savenkova, A V Shobukhov, A V Kalmykov
Lomonosov Moscow State University, Leninskie Gory, Moscow, 119991, Russia

E-mail: mknandrew@mail.ru, shobukhov@mail.ru, alex_2391@mail.ru

Abstract. The paper deals with investigation of the MHD-stability dependence on the depth of the anode immersion in the process of aluminium electrolysis. The proposed 3D three-phase mathematical model is based on the Navier-Stokes and Maxwell equation systems. This model makes it possible to simulate the distributions of the main physical fields both in horizontal and vertical planes. The suggested approach also allows to study the dynamics of the border between aluminium and electrolyte and the shape of the back oxidation zone.

1. Introduction
An aluminium electrolyser is a complex nonlinear object. Its main volume is a rectangular bath with rounded corners (see Fig.1). At the advanced stage of the process the bath contains a layer of liquid aluminium at the bottom and a layer of melted cryolite with special additives above it. Graphite electrodes are immersed into the cryolite layer from above to a relatively small depth. In the course of the process a gas layer appears under the anodes as a result of the electrochemical reaction.

The graphite electrodes are the anodes, while the liquid aluminium and partly the bottom of the bath are the cathodes. An electric potential difference is applied to the electrodes. Meanwhile, the upper liquid layer (the electrolyte) contains positive and negative ions that are the mobile charge carriers. These ions move up and down to the corresponding electrodes and are discharged on them. As a result of these reactions, the primary aluminium and the gaseous mixture appear. The most important reactions here are the formation of various aluminium salts near the anode and the reduction of these salts to metal near the cathode, actually — at the aluminium-electrolyte border [1].

One of the most important parameters of the industrial electrolyser cell is the anode immersion depth. The greater is the depth, the smaller is the electrical resistance of the bath; thus the electrical energy consumption of the cell decreases. But at the same time the probability of the MHD instability increases, and it may cause losing the whole amount of the primary metal.

2. Description of the Problem
The proposed model is based on the so-called multiliquid approach [2,3]. It is supposed that in the heterogeneous mixture each phase occupies only a fraction of the elementary volume. In this regard we introduce the variables $\alpha_m$, $m=1,2,3$, that characterize the volume fraction of the m-th phase in the mixture.
Figure 1. The cross-section scheme of the multi-anode electrolyser.

Let use denote the macroscopic velocities of the mixture components as \( \mathbf{v}_m = (u_m, v_m, w_m) \) the pressure as \( p \), the magnetic field as \( \mathbf{H}_m = (H_x, H_y, H_z) \), the Lorentz force as \( \mathbf{F}_m = (F_x, F_y, F_z) \), the electric current density as \( \mathbf{J} = (j_x, j_y, j_z) \) and the mixture temperature as \( T \). Then the mass variation of the m-th phase is described by the following equation:

\[
\frac{\partial \alpha_m \rho_m}{\partial t} + \text{div}(\alpha_m \rho_m \mathbf{v}_m) = J_m,
\]

where \( \rho_m \) - the density of the m-th phase - is constant, and \( J_m \) is the source (or sink) of the m-th phase due to the chemical reactions.

The transfer of momentum for each phase is described as:

\[
\frac{\partial \alpha_m \rho_m \mathbf{v}_m}{\partial t} + (\mathbf{v}_m \cdot \nabla)(\alpha_m \rho_m \mathbf{v}_m) = -\alpha_m \nabla p + \mu_m \Delta \mathbf{v}_m + \alpha_m \rho_m \mathbf{g} + \mathbf{P}_{mn} + \alpha_m \mathbf{F}_m,
\]

where \( \mu_m \) is the dynamic viscosity of the media, \( \mathbf{F}_m \) is the volume density of the Lorentz force:

\[
\mathbf{F}_m = \mu_0 [\text{rot} \mathbf{H} \times \mathbf{H}]
\]

and \( \mathbf{P}_{mn} \) is the volume interphase viscous friction force between the n-th and the m-th mixture components.

The equation for pressure may be obtained by summation of the momentum equations for the three phases and by taking into account the balance equations:

\[
\nabla \cdot \left( \sum_{m=1,2,3} \frac{\alpha_m}{\rho_m} \nabla p \right) = \nabla \cdot \mathbf{L},
\]

together with the boundary conditions:
\[ \frac{\partial p}{\partial n} = (L, n) \left/ \sum_{m=1,2,3} \left( \frac{\alpha_m}{\rho_m} \right) \right. \], \tag{5} \]

\[ L = \sum_{m=1,2,3} \left( (v_m \cdot \nabla) (\alpha_m v_m) + \frac{\alpha_m \mu_m}{\rho_m} \Delta v_m + \alpha_m g + \right. \]

\[ + \frac{1}{\rho_m} \sum_{m=n} \alpha_m \alpha_g \left( \alpha_m \mu_m + \alpha_g \mu_n \right) (v_m - v_n) + \frac{\alpha_m}{\rho_m} F_m \right), \tag{6} \]

where \( n \) is the vector normal to the boundary.

The equation for the electric field potential is derived from the electric field divergency equation. The mixture is supposed to be electrically neutral, and thus the equation for the potential looks as follows:

\[ \nabla \cdot \left( \sum_{m=1,2,3} (\alpha_m \sigma_m) \nabla \varphi \right) = \nabla \cdot \left( \mu_0 [v, H] \sum_{m=1,2,3} (\alpha_m \sigma_m) \right) \tag{7} \]

where \( v \) is the average mixture velocity, and \( \sigma_m \) is the conductivity of the \( m \)-th phase. The anode electric potential is set equal to 4 volts. In the nonconductive part of the bath surface the normal derivative of the electric potential equals zero. In whole the boundary conditions look as follows:

\[ \varphi \big|_{\text{anode}} = 4, \varphi \big|_{\text{cathode}} = 0, \left. \frac{\partial \varphi}{\partial n} \right|_f = 0. \tag{8} \]

We get the electric current density from the relation:

\[ J = \sum_{m=1,2,3} (\alpha_m \sigma_m) \left( \mu_0 [v, H] - \nabla \varphi \right) \tag{9} \]

which is due to the media flow, which causes induction of the additional electric field. We neglate the bias current, because the variation of the electric potential with time is small enough.

The magnetic field is the solenoidal field with the vector potential \( A \). We find this potential using the Coulomb calibration:

\[ \nabla A = 0. \tag{10} \]

Using the quazistationary approximation, we get the following Neummann problem for the vector potential:

\[ \begin{cases} \Delta A = -J \\ \frac{\partial A}{\partial n} = 0 \end{cases} \tag{11} \]

Having obtained the vector potential, we can easily find the magnetic field itself.

The problem described above was solved numerically. The finite difference equations were split according to physical processes on the staggered meshes [2,4].
3. Results of Computational Experiments
We've carried out a series of computations that simulated the industrial process of aluminium electrolysis with anode immersion depths varying from 10 to 15 to 20 centimeter. Fig.2-4 present the dynamics of the interpole distance with respect to time.

**Figure 2.** Dynamics of the interpole distance, anode immersion depth equals 10 cm.

**Figure 3.** Dynamics of the interpole distance, anode immersion depth equals 15 cm.
Conclusion
The paper contains the results of mathematical modeling of an industrial aluminium electrolyser. We demonstrate that the amplitude of the liquid aluminium surface oscillations increases together with the depth of the anode immersion. It may cause the development of the MHD instability, but in all three cases shown here the electrolysis process is stable.

References
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