Off-shell and non-static contributions to heavy-quarkonium production

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Abstract. We have shown that if one relaxes the constraint that the quarks in a heavy quarkonium are at rest and on-shell, new contributions to the discontinuity of the production amplitude appear. These can be seen as a s-cut in the amplitude and are on the same footage as the classical cut of the Colour-Singlet Model (CSM), where the heavy quarks forming the quarkonium are put on-shell by hypothesis. We treat this cut in a gauge-invariant manner by introducing necessary new 4-point vertices, suggestive of the colour-octet mechanism. We have further shown that this cut contributes at least as much as the LO CSM at large \( P_T \). However, the 4-point vertices cannot be totally constrained and an ambiguity remains to what concerns their actual contribution. Theoretical insights from meson photoproduction are discussed in that context.

Keywords: heavy-quarkonium production, vector-meson production, gauge invariance, relativistic effects, non-static extension

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More than ten years ago, the CDF collaboration \cite{1,2} brought to light the “\( \psi' \) anomaly”, \textit{i.e.} an excessively large experimental cross section for \( \psi' \) (and \( J/\psi \)) production compared with theoretical expectations. No totally conclusive solution to this problem has been proposed so far (for recent reviews see \cite{3,4}). Even though the Colour-Octet Mechanism (COM), coming from the application of NRQCD to heavy quarkonium, looked as a very promising solution, it appears clearly that as long as fragmentation is the dominant production contribution and the velocity-scaling rules of NRQCD hold, it cannot accommodate the polarisation measurements of CDF \cite{5}, which show a non-polarised, if not slightly longitudinal, production.

In that context, we have felt the necessity to reconsider the appropriateness of the static and on-shell approximation of the Colour-Singlet Model (CSM) \cite{6}.

In order to study properly non-static and off-shell effects, we have used a vertex function as an input for the bound-state characteristics, whereas the Schrődinger
FIGURE 1. The first family (a) has 6 diagrams and the second family (b) 4 diagrams contributing the discontinuity of $gg \to {}^3S_1 g$ at LO in QCD. (c): the gauge-invariance restoring vertex, $\Gamma^{(4)}$.

wave function at the origin is used in the CSM and Long Distance Matrix Elements (LDME) of NRQCD enter the COM. We emphasise again that we probe all the internal phase space of the quarkonium, and thus need a function, where these two approaches simply need a constant factor.

In the case of $^3S_1$ quarkonium (noted $Q$) production in high-energy hadronic collisions, we are to consider gluon fusion $gg \to Qg$. Using the Landau equations [7], we have shown in [8] that there are two families of contributions (see Fig. 1 (a) and (b)): the first is the usual colour-singlet mechanism, where in the context of our model, we use a 3-point function $\Gamma^{(3)}(p, P) = \Gamma(p, P)\gamma_\mu$ at the $Q\bar{Q}Q$ vertex; the second family was never considered before. To simplify the study, we set $m > M/2$ so that the first cut does not contribute. The functional form of $\Gamma(p, P)$ (gaussian or dipole) and its parameters have been discussed in details in [9].

In addition to the second family, one is driven – to preserve gauge invariance (GI) – to introduce new contributions arising from the presence of 4-point vertices. Beside restoring GI, these vertices have to satisfy specific constraints [8, 10, 11]. For the following simple choice for $\Gamma^{(4)}_{\mu\nu}(c_1, c_2, P, q)$

$$-ig_s T^a_{\kappa \lambda} (\Gamma_1 - \Gamma_2) \left[ \frac{c_1^{\lambda}}{(c_1 - q)^2 - m^2} + \frac{c_2^{\lambda}}{(c_2 + q)^2 - m^2} \right] \gamma_\mu, \quad (1)$$

where $\Gamma_1 \equiv \Gamma(2c_1 - P, P)$, $\Gamma_2 \equiv \Gamma(2c_2 - P, P)$, the momenta and indices are as in Fig. 1 (c), the results obtained for $J/\psi$ and $\psi'$ production at the Tevatron are exposed in [8]. For the $J/\psi$, we saw that the $s$-channel cut contributes at least as much as the classical cut of the CSM at large $P_T$. In the $\psi'$ case, we employed the ambiguity upon the vertex-function normalisation [9] due to the node position to show that agreement with the data at low $P_T$ was conceivable. We further noticed that the $P_T$ slope was only slightly different from that of the data. This is at variance with what is widely believed since COM fragmentation (with a typical $1/P_T^2$ behaviour) processes are in agreement with experimental measurements.

Another possible Ansatz for $\Gamma^{(4)}_{\mu\nu}(c_1, c_2, P, q)$ [12], inspired from studies of meson photoproduction [13, 14, 15] and which possesses a better behaviour at low
momenta, is

\[-ig_sT^{\mu}_{ki}(c_1 - q)\nu(c_1 - q)^2 - m^2(\Gamma_2 - F) + \frac{(2c_2 + q)^\nu}{(c_2 + q)^2 - m^2(\Gamma_1 - F)}\gamma^\mu\]  \hspace{1cm} (2)

with \( F = \Gamma_0 - h(\Gamma_0 - \Gamma_1)(\Gamma_0 - \Gamma_2) \) (\( \Gamma_0 \) is the value of the vertex function when \((c_1 - q)^2 = (c_2 + q)^2 = m^2\)) and \( h \) an arbitrary crossing-symmetric function of the momenta. This will be studied in a future work. Applications to \( \eta_c \) and \( \eta_c' \) decays could also be relevant in view of the possible anomaly of \( \eta_c' \to \gamma\gamma \) decay \[16\].

However, there exist further other choices for the GI restoring vertex. Interesting results are indeed obtained by studying the effects of autonomous vertices, which link different suitable choices: they are GI alone and a priori unconstrained in normalisation. The latter can be fitted to described data from \[1, 2, 17, 18\] as shown in \[10, 19\].

In conclusion, we have shown that it is possible to go beyond the on-shell and static approximations of the CSM. It may also be possible to extend the COM in the same manner. This necessitates the introduction of 4-point vertices due to the non-local 3-point vertex relevant for the non-static and off-shell contributions.

By going deeper in the analysis, we have seen that the form of these 4-point vertices is not absolutely constrained, even after imposing necessary conditions to conserve crossing symmetry and the analytic structure of the amplitude. By exploiting this lack of constraint, we have been able to reproduce the direct-production cross section for the \( J/\psi, \psi' \) and \( \Upsilon(1S) \) as measured at the Tevatron by CDF (and also at RHIC by PHENIX for \( J/\psi \)).

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