Inflation on the resolved warped deformed conifold

Alex Buchel

*Department of Applied Mathematics*
*University of Western Ontario*
*London, Ontario N6A 5B7, Canada*

*Perimeter Institute for Theoretical Physics*
*Waterloo, Ontario N2J 2W9, Canada*

**Abstract**

Braneworld inflation on the resolved warped deformed conifold is represented by the dynamics of a $D3$-brane probe with the world volume of a brane spanning the large dimensions of the observable Universe. This model was recently proposed as a string theory candidate for slow-roll inflationary cosmology in [hep-th/0511254](https://arxiv.org/abs/hep-th/0511254). During inflation, the scalar curvature of the Universe is determined by the Hubble scale. We argue that taking into account the curvature of the inflationary Universe renders dynamics of the $D3$-brane fast-roll deep inside the warped throat.

January 2006
1 Introduction

Inflation [1–3] is an attractive scenario which solves many important problems in cosmology. The basic idea of its simplest realization is that our Universe went through the stage of the accelerated expansion driven by the potential energy of the slowly rolling inflaton field. In agreement with current observational data such a model naturally predicts a flat Universe and a scale invariant spectrum of density perturbations, provided the inflaton potential is sufficiently flat.

The main problem of implementing inflation in string theory is to identify a string field which has such a flat potential. In the original brane-world model scenario [4–7] (and its more recent warped throat realization [8]) the inflaton field is identified with the geometric position of a D3-brane on the compactification manifold. The four-dimensional world volume of a D3-brane is assumed to span the space-time directions of the observable Universe. In many supersymmetric string theory compactifications (a relevant example here is a warped deformed conifold [9]) a scalar field parameterizing position of a D3-brane on the transverse manifold is an exact modulus. A nontrivial potential for such a scalar can be generated by turning on 3-form fluxes. In the case of the warped deformed conifold one can turn on supersymmetry preserving fluxes [10] (describing the baryonic branch deformation of the deformed conifold [9]) to completely lift\(^1\) a 3-brane flat directions [11]. The resulting D3-brane potential on the resolved warped deformed conifold is rather flat. If \(\Phi\) is a canonically normalized inflaton, \(i.e.,\) the radial position of the D3-brane deep inside the warped throat of the resolved-deformed conifold, Dymarsky, Klebanov and Seiberg (DKS) found [11] that \(V(\Phi)\) satisfies

\[
\frac{d^2V}{d\Phi^2} \sim U^4 \Phi^{-6} \ln \Phi, \quad \Phi^2 \gg |U|, \quad (1.1)
\]

where \(U\) is a baryonic branch deformation parameter [10]. Potential (1.1) appear to be very promising for the slow roll inflation. Specifically, if a Hubble scale during inflation is \(H\), using (1.1) the slow roll parameter \(\eta\) is

\[
\eta \equiv \frac{1}{3H^2} \frac{d^2V}{d\Phi^2} \sim H^{-2} U^4 \Phi^{-6} \ln \Phi, \quad (1.2)
\]

which can be made arbitrarily small by considering inflation sufficiently far inside the

---

\(^1\)Lifting a 3-brane flat directions with fluxes is a well-known phenomena. Indeed, a 3-brane on \(AdS_5 \times S^5\) has a six dimensional moduli space which can be partially lifted [12, 13] by turning on Pilch-Warner [14] fluxes.
warped throat.

Unfortunately, above arguments miss a crucial contribution to the probe brane potential which renders DKS inflationary model\textsuperscript{2} unrealistic. Specifically, the main problem is that the potential \[(1.1)\] was evaluated assuming that the world-volume of the $D3$-brane is Minkowski. As we already mentioned the world volume of the inflationary brane extends over the observable Universe. During slow-roll inflation the Universe has a scalar curvature
\[ R_4 = 12\mathcal{H}^2, \tag{1.3} \]
and thus, the background geometry for the brane inflation is \textit{not} a direct warped product of a four-dimensional Minkowski space with the resolved-deformed conifold. Rather, it should be a direct warped product of a four-dimensional de-Sitter space of curvature \[(1.3)\] with the resolved-deformed conifold. Such clarification is very important because the scalar field representing $D3$-brane radial position on the conifold is a conformally coupled scalar \cite{8,15}. Thus, just from the conformal coupling to the curved background the DKS potential would receive an extra contribution (schematically)
\[ V_{DKS} \implies V_{DKS} + \frac{1}{12} R_4 \Phi^2 = \mathcal{H}^2 \Phi^2, \tag{1.4} \]
which would lead to
\[ \eta_{DKS} \implies \eta_{DKS} + \frac{2}{3}. \tag{1.5} \]
Given that deep inside the warped throat $|\eta_{DKS}| \ll 1$, the \textit{correct} slow roll parameter of the DKS inflation is actually $\frac{2}{3}$.

In the rest of this paper we provide precise evaluation of the cosmological parameter $\eta$ in DKS inflationary model. First, building on \cite{16,17}, we derive supergravity equations of motion describing de-Sitter deformation of the resolved warped deformed conifold of \cite{10}. In section 3 we find asymptotic solution to these background equations of motion. In section 4 we discuss $D3$ probe brane dynamics in de-Sitter deformed resolved warped deformed conifold. We conclude in section 5.

\section{De-Sitter deformed resolved warped deformed conifold}

In this section we construct de-Sitter deformation of the supersymmetric resolved warped deformed conifold \cite{10}.

\textsuperscript{2}We discuss here DKS inflation only deep inside warped throat. It might be possible to achieve slow-roll inflation at the bottom of the warped resolved deformed conifold of \cite{10}.
Following [16, 17], we consider the following ansatz for the background (Einstein frame) metric

\[ ds_{10}^2 = c_1^2 \left( -dt^2 + e^{2Ht} (d\tilde{x})^2 \right) + c_2^2 (dr)^2 + c_3^2 \sum_{i=1}^2 e_i^2 + c_4^2 \sum_{i=1}^2 \hat{\epsilon}_i^2 + c_5^2 \epsilon_3^2 , \]  

(2.1)

where the \( S^2 \) one-forms \( e_i \) and the \( S^3 \) left-invariant one-forms \( \epsilon_i \) are [18, 10, 11]

\[
\begin{align*}
e_1 &= d\theta_1, & e_2 &= -\sin \theta_1 \, d\phi_1, \\
e_1 &= \sin \psi \, \sin \theta_2 \, d\phi_2 + \cos \psi \, d\theta_2, & e_2 &= \cos \psi \, \sin \theta_2 \, d\phi_2 - \sin \psi \, d\theta_2, \\
e_3 &= d\psi + \cos \theta_2 \, d\phi_2, & \hat{\epsilon}_1 &= e_1 - a \, e_1, \\
\hat{\epsilon}_2 &= e_2 - a \, e_2, & \hat{\epsilon}_3 &= \epsilon_3 + \cos \theta_1 \, d\phi_1, 
\end{align*}
\]

(2.2)

with \( c_i = c_i(r) \) and \( a = a(r) \). The background fluxes take form [18]

\[
\begin{align*}
H_3 &= dB_2, \\
B_2 &= h_1 \left( e_1 \wedge e_2 + e_1 \wedge e_2 \right) + h_2 \left( e_1 \wedge e_2 - e_2 \wedge e_1 \right) + \chi \left( -e_1 \wedge e_2 + e_1 \wedge e_2 \right), \\
F_3 &= P \, \hat{\epsilon}_3 \wedge (e_1 \wedge e_2 + e_1 \wedge e_2) + P \, d \left[ b \left( e_1 \wedge e_1 + e_2 \wedge e_2 \right) \right], \\
F_5 &= \mathcal{F}_5 + \star_{10} \mathcal{F}_5, \quad \mathcal{F}_5 = K \, e_1 \wedge e_2 \wedge e_1 \wedge e_2 \wedge e_3, 
\end{align*}
\]

(2.3)

where \( h_i = h_i(r), \chi = \chi(r), b = b(r), K = K(r), \) and \( P = -\frac{1}{4} \alpha' M \) is determined by the number \( M \) of fractional 3-branes. Finally, there is a dilaton \( \phi \equiv \ln g_s = \phi(r) \), and we assume the asymptotic string coupling to be one, i.e., \( \phi \to 0 \) as \( r \to \infty \).

Type IIB supergravity equations of motion [19] consist of 3-form Maxwell equations, dilaton equation, five-form Bianchi identity and the Einstein equations. The five-form Bianchi identity can be integrated to yield

\[ K = 2P \left( h_1 + bh_2 \right). \]  

(2.4)

The 3-form Maxwell equations reduce to the following coupled ODE’s

\[
0 = b'' + b' \left[ \ln \left( \frac{c_4^2 c_5 g_s}{c_2} \right) \right]' - \frac{b c_2^2 (c_4^2 g_s (c_2^2 + 2c_3^2 a^2) + 2h_2^2)}{c_3^2 c_4^2 c_5^2 g_s} - 2 \frac{c_2^2 h_2 h_1}{c_4^2 c_3^2 c_5^2 g_s} \]  

\[ + \frac{c_3^2 a (c_2^2 + c_4^2 (a^2 + 1))}{c_3^2 c_5^2}, \]  

(2.5)
It is straightforward to verify that the first order constraint \((2.9)\) is consistent with
The dilaton equation is

\[
0 = g_s'' - \frac{(g_s')^2}{g_s} + g_s' \left[ \ln \frac{c_4^2 c_5^4}{c_2} \right]' - \frac{g_s^2 P^2 (b')^2}{c_3^2 c_4^2} + \frac{(\chi')^2}{2c_3^4 c_4^4} \left( (c_4^2 a^2 + c_3^2)^2 - c_4^4 (1 + 2a^2) \right) \\
- \frac{\chi'}{c_3^4 c_4^4} \left( c_3^2 - c_4^2 (1 - a^2) \right) \left( 2c_4^2 ah' + h'_1 (c_3^2 + c_4^2 (1 + a^2)) \right) - \frac{g_s^2 c_2^2 P^2}{2c_3^4 c_4^4} \\
2c_4^2 (c_3^2 + 2c_4^2 a^2)(b - a)^2 + c_4^4 (1 - a^2) (3a^2 + 1) + 4bac_4^4 (a^2 - 1) + c_4^2 h_2^2 \\
+ \frac{(c_3^2 + 2c_4^2 a^2)(h'_2)^2}{c_3^2 c_4^4} + \frac{(h'_1)^2}{2c_3^4 c_4^4} \left( c_4^4 (a^2 + 1)^2 + c_3^2 (c_3^2 + 2c_4^2 a^2) \right) \\
+ 2\frac{ah'_1 h'_2}{c_3^2 c_4^4} \left( c_3^2 + c_4^4 (a^2 + 1) \right),
\]

(2.10)

The Einstein equations are

\[
0 = c_1'' + \frac{3(c_4^2)^2}{c_1} + c_1' \left[ \ln \frac{c_4^2 c_5^4}{c_2} \right]' - \frac{c_1 (\chi')^2}{8c_3^4 c_4^4 g_s} \left( (c_4^2 a^2 + c_3^2)^2 + c_4^4 (1 - 2a^2) \right) \\
+ \frac{c_1 \chi'}{4c_3^4 c_4^4 g_s} \left( c_3^2 + c_4^2 (a^2 - 1) \right) \left( 2c_4^2 ah'_2 + h'_1 (c_3^2 + c_4^2 (a^2 + 1)) \right) - \frac{c_1 P^2 g_s (b')^2}{4c_3^4 c_4^4} \\
- \frac{c_1 (h'_1)^2}{8c_3^4 c_4^4 g_s} \left( c_4^4 (a^2 + 1)^2 + c_3^2 (c_3^2 + 2c_4^2 a^2) \right) - \frac{c_1 (h'_2)^2}{4c_3^4 c_4^4} \left( c_3^2 + 2c_4^2 a^2 \right) \\
- \frac{h'_1 h'_2 c_1 a}{2c_3^4 g_s c_4^4} \left( c_3^2 + c_4^4 (a^2 + 1) \right) - \frac{g_s c_2^2 c_1 P^2}{8c_3^4 c_4^4 c_5^4} \left( 2c_4^2 (c_3^2 + 2c_4^2 a^2)(b - a)^2 \right) \\
+ c_4^4 + c_4^4 (1 - a^2) (3a^2 + 1) + 4bac_4^4 (a^2 - 1) - \frac{c_2^2 c_1 h_2^2}{4c_3^4 c_5^4 g_s} - \frac{c_2^2 c_1 P^2 (h_1 + bh_2)^2}{c_3^4 c_4^4 c_5^4} \\
- 3\frac{c_2^2 \chi^2}{c_1},
\]

(2.11)
\[ 0 = c_3'' + \left( \frac{c_4'}{c_4} \right)^2 + c_4' \left[ \ln \frac{c_1' c_2' c_3}{c_2} \right]' + \frac{(\chi')^2}{8c_4'g_s c_3^3} \left( 3c_4'(1 - a^2)^2 + c_3^2(2c_4^2 a^2 - c_3^2) \right) \\
+ \frac{\chi'}{4c_4'g_s c_3^3} \left( h_1'(c_3^2(-2c_4^2 a^2 + c_3^2) - 3c_4^4(a^4 - 1)) - 2c_4 a h_2'(3c_4^2(a^2 - 1) + c_3^2) \right) \\
+ \frac{(h_1')^2}{8c_4'g_s c_3^3} \left( 3c_4^4(a^2 + 1)^2 + c_3^2(2c_4^2 a^2 - c_3^2) \right) + \frac{(h_2')^2}{4c_4'g_s c_3^3} \left( c_3^2 + 6c_4^2 a^2 \right) \\
+ \frac{h_1' h_2 a}{2c_4' g_s c_3^3} \left( c_3^2 + 3c_4^2(1 + a^2) \right) + \frac{g_s P^2(b')^2}{2c_3} + \frac{(a')^2 c_4}{2c_3} - \frac{g_s c_2^2 P^2}{8c_4' c_3^3 c_5^3} \left( 2c_4^2(a^2 - 1)^2 + a^2 c_3^2 \right) + \frac{c_3^2 c_2^2 a^2}{2c_3 c_5^3} \\
- \frac{c_3^2(a^2 + 1)}{c_3^3}, \]

\[ 0 = c_4'' + \left( \frac{c_4'}{c_4} \right)^2 + c_4' \left[ \ln \frac{c_1' c_3' c_2}{c_2} \right]' + \frac{(\chi')^2}{8c_4'g_s c_4^3} \left( c_4^2(2c_4^2 a^2 + 3c_3^2) - c_4^4(a^2 - 1)^2 \right) \\
- \frac{\chi'}{4c_4'g_s c_4^3} \left( h_1'(c_3^2(2c_4^2 a^2 + 3c_3^2) + c_4^4(1 - a^4)) + 2c_4 a h_2'(c_3^2 + c_4^2(1 - a^2)) \right) \\
- \frac{(h_1')^2}{8c_4'g_s c_4^3} \left( c_4^4(a^2 + 1)^2 + c_3^2(2c_4^2 a^2 + 3c_3^2) \right) - \frac{(h_2')^2}{4c_4'g_s c_4^3} \left( 2c_4^2 a^2 - c_3^2 \right) \\
+ \frac{h_1' h_2 a}{2c_4' g_s c_4^3} \left( c_3^2 - c_4^2(1 + a^2) \right) + \frac{P^2 g_s(b')^2}{4c_4' c_3^2} - \frac{c_4^4(a')^2}{2c_3} + \frac{g_s c_2^2 P^2}{8c_4' c_3^3 c_5^3} \left( 2c_4^2(-2c_4^2 a^2 + c_3^2)(b - a)^2 - 4bac_4^4(a^2 - 1) + c_4^4(a^2 - 1)(3a^2 + 1) + 3c_4^4 \right) \\
+ \frac{c_3^2 h_2^2}{4c_4' c_3^2 c_5^2 g_s} + \frac{P^2 c_3^2(h_1 + bh_2)^2}{c_3^3 c_4' c_5^3} + \frac{c_3^2 c_2^2}{2c_3 c_4' c_5^3} \left( c_4^2 a^2 + c_3^2 \right) - \frac{c_2}{c_4} - \frac{c_3^2 c_2^2 a^2}{2c_3^3 c_5^2}, \]
\[0 = c_5'' + c_5' \left[ \ln \left( \frac{c_3^2 c_1^2}{c_2} \right) \right] - \frac{c_5(x')^2}{8 g_s c_4^2 c_3^4} \left( c_4^2(a^2 - 1)^2 + c_3^2(c_3^2 + 2c_4^2a^2) \right) + \frac{c_5 x'}{4 g_s c_4^2 c_3^2} \left( c_3^2 + c_4^2(a^2 - 1) \right) \left( 2c_4 a h_2' + h_1'(c_3^2 + c_4^2(a^2 + 1)) \right) - \frac{c_5(h_1')^2}{8 g_s c_4^2 c_3^4} \]
\[c_4^4(a^2 + 1)^2 + c_3^2(c_3^2 + 2c_4^2a^2) \right) - \frac{c_5(h_2')^2}{4 c_3^2 g_s c_4^4} \left( c_3^2 + 2c_4^2a^2 \right) - \frac{c_5 a h_1' h_2'}{2 c_4^2 g_s c_3^4} \left( c_3^2 + c_4^2(1 + a^2) \right) \]
\[- \frac{g_s c_5 P^2(b')^2}{4 c_4^2 c_3^2} + \frac{3 g_s c_2^2 P^2}{8 c_5 c_3^4 c_4^4} \left( 2c_4^2(c_3^2 + 2c_4^2a^2)(b - a)^2 + 4bac_4^2(a^2 - 1) \right) - \frac{c_4^4(a^2 - 1)(3a^2 + 1) + c_3^4}{4 c_4^2 c_3^2 c_5 g_s} + \frac{P^2 c_5^2(h_1 + h_2)^2}{c_5 c_4^4 c_3^4} \]
\[+ \frac{c_2 c_4^2 a^2}{c_3^2 c_5} - \frac{c_2^2 c_3^2}{2 c_1^2 c_3} \left( c_4^4(a^2 - 1)^2 + c_3^2(c_3^2 + 2c_4^2a^2) \right) \]
(2.14)

\[0 = a'' + a' \left[ \ln \left( \frac{c_4^2 c_1^2}{c_2} \right) \right] - \frac{a(x')^2}{g_s c_3^2 c_4^4} \left( c_3^2 + c_4^2(a^2 - 1) \right) + \frac{x'}{g_s c_3^2 c_4^4} \left( h_2'(c_4^2(3a^2 - 1) + c_3^2) \right) + 2h_1'(c_4^2 a^2 + c_3^2) - \frac{a(h_1')^2}{g_s c_3^2 c_4^4} \left( c_3^2 + c_4^2(1 + a^2) \right) - 2 \frac{a(h_2')^2}{g_s c_3^2 c_4^4} \]
\[- \frac{h_1' h_2'}{g_s c_3^2 c_4^4} \left( c_3^2 + c_4^2(3a^2 + 1) \right) - \frac{P^2 c_5^2 g_s(a - b)}{c_3^2 c_4^2 c_5} \left( c_3^2(a^2 - 2ba + 1) + c_3^2 \right) \]
\[- \frac{c_2^2 a c_4^2}{c_3^2 c_4^2} \left( c_3^2 + c_4^2(a^2 - 1) \right) + \frac{c_2^2 a(2c_3^2 - c_4^2)}{c_3^2 c_5} \]
(2.15)
\begin{equation}
0 = 4c_3^2g^2c_5c_4^2\left(c_5c_1^2c_3^2(c_1')^2 + c_5c_3^2c_1^2(c_4')^2 + 4c_5c_3c_1^2c_4c_4' + 8c_5c_3^2c_1c_4c_4 + 8c_5c_3c_1^2c_1c_3
\right.

+ 6c_5c_3^2c_1^2(c_1')^2 + 4c_3^2c_3c_1^2c_1' + 2c_3^2c_3c_1c_3' + 2c_3^2c_3^2c_4c_4' - c_3^2c_5g_s\left(c_4^2(a^2 - 1)^2
\right.

+ c_3^2(2c_3 + 2c_4a^2)\left(\chi^2 + 2c_1^2g_s\left(c_3^2 - c_4^2(1 - a^2)\right)\left(2c_3^2h_2' + h_1'(c_3^2 + c_4^2(a^2 + 1))\right)\chi'
\right.

- c_1c_5g_s\left(c_4^4(a^2 + 1)^2 + c_3^2(c_3^2 + 2c_4a^2)\right)(h_1')^2 - 2c_1c_4^2g_s\left(c_3^2 + 2c_4a^2\right)(h_2')^2

- 4c_1c_4^2c_3^2h_2'g_s\left(c_3^2 + c_4^2(1 + a^2)\right) - c_3c_1c_4^2c_5c_1(g_s^2) - 2g_s^2c_1^2P^2c_4^2c_3^2c_5(b')^2

- 2g_s^2c_3^2c_4^2c_1^2(c_3')^2 - 4g_s^2c_1^4c_2^2c_3^2c_4^2\left(c_3^2 + c_4^2(1 + a^2)\right)c_5^2 + g_s^2c_1^2c_2\left(c_4^4(a^2 - 1)^2
\right.

+ c_3^2(c_3^2 + 2c_4a^2)\right)c_5^2 + 4g_s^2c_4^2c_3^2c_4\left(h_2^2 + g_s^4c_4^2(3a^2 + 1) + c_4^3
\right.

+ P^2g_s^2c_1^2\left(2c_4^2(c_3^2 + 2c_4a^2)(b - a)^2 + 4bac_4^4(a^2 - 1) - c_4^4(a^2 - 1)(3a^2 + 1) + c_4^3
\right.

- 24g_s^2c_3^2c_4^2c_3c_4\right)H^2.

(2.16)
\end{equation}

It is straightforward to verify that the constraint (2.16) is consistent with (2.5)-(2.8), (2.9), (2.10), (2.11)-(2.15). Notice that the Hubble scale $H$ enters only in (2.11) and in the constraint equation (2.16).

As a consistency check, we verified that the supersymmetric baryonic branch deformation of the warped deformed conifold of [10] is indeed a solution of derived supergravity equations of motion with $H = 0$. Comparison with computation of [11] is achieved by parameterizing

\begin{equation}
c_1 = e^{-\phi/4}H^{-1/4}, \quad c_2 = c_5 = e^{-\phi/4}v^{-1/2}e^{x/2}, \quad c_3 = e^{-\phi/4}g^{3/2}e^{2x/2}, \quad c_4 = e^{-\phi/4}g^{-1/2}e^{x/2}.
\end{equation}

where $\{\phi, H, v, g, x\}$ are functions of a radial variable, see Eq. (12.2) of [11]. We further reproduced (again setting $H = 0$) the warped deformed conifold solution of Klebanov and Strassler (KS) [9]. We verified the map between KS background parametrization and the parametrization of the $\mathbb{Z}_2$-symmetric solution of [10], presented in [11].
3 Asymptotic solution

In this section we discuss asymptotic infrared (IR) and ultraviolet (UV) solution to the de-Sitter deformed resolved warped deformed conifold supergravity equations of motion derived in section 2. We find that the IR (the bottom of the warped deformed throat) asymptotic is a smooth deformation\(^3\) of the IR solution of [11] by turning on a nonzero Hubble scale \(\mathcal{H}\). Thus while in [11] the IR geometry is that of \(R^{3,1} \times R^3 \times S^3\), in our case it is \(dS_4 \times R^3 \times S^3\). The UV asymptotic describes a highly warped region of the geometry where following the proposal of [11] we study D3-brane inflation.

3.1 IR solution

We assume the same IR boundary conditions as for the warped deformed conifold [9], i.e., we assume that the \(S^2\) of the conifold shrinks to zero size in a smooth way, while all the other warp factors (including the \(S^3\) radius of the conifold) remain finite. It is easy to verify that such boundary conditions guarantee singularity-free geodesically complete space-time. Moreover, as in [9], the curvature invariants of the background can then be made small in string units by choosing fluxes in the IR to be large. So, we search for a small-\(r\) solution to (2.5)-(2.8), (2.9), (2.10), (2.11)-(2.15) subject to the following \(r \to 0\) boundary condition on the geometry

\[
d_{s_{10}}^2 \Rightarrow H_0^{-1/2}\left(-dt^2 + e^{2Ht}(d\vec{x})^2\right) + H_0^{1/2}\left((dr)^2 + r^2 \sum_{i=1}^2 e_i^2 + \omega_0^2(g_5^2 + 2g_3^2 + 2g_4^2)\right), \quad (3.1)
\]

where the one-forms \(\{g_3, g_4, g_5\}\)

\[
g_5 \equiv \hat{e}_3, \quad g_3 = \frac{e_2 + \epsilon_2}{\sqrt{2}}, \quad g_4 = \frac{e_1 + \epsilon_1}{\sqrt{2}}, \quad (3.2)
\]

are defined so as to agree with [9]. The radius-squared of \(S^3\) is \(R^2 = 2H_0^{1/2}\omega_0^2\), while the \(S^2\) parameterized by \(e_i\) smoothly shrinks to zero size. Notice that choosing the IR asymptotic of \(c_2\) to satisfy (3.1) we fixed the rescaling symmetry of the radial coordinate \(r\). Introducing

\[
c_1 \equiv \hat{H}^{-1/4}, \quad c_2 = \hat{H}^{1/4}, \quad c_3 = \hat{H}^{1/4}w_3, \quad c_4 = \hat{H}^{1/4}w_4, \quad c_5 = \hat{H}^{1/4}w_5, \quad (3.3)
\]

\(^3\)In other words, there is no obstruction in the IR for the introduction of the Hubble parameter.
and setting\(^4\) \(g_s(r = 0) = 1\), we find that solution to (2.5), (2.8), (2.9), (2.10), (2.11)-(2.15) subject to (3.1) takes the form

\[
\tilde{H} = H_0 + \sum_{i=1}^{\infty} H_{2i} r^{2i}, \quad w_3 = r \left(1 + \sum_{i=1}^{\infty} w_{3,2i} r^{2i}\right), \quad w_4 = \omega_0 + \sum_{i=1}^{\infty} w_{4,2i} r^{2i},
\]

\[
w_5 = \omega_0 + \sum_{i=1}^{\infty} w_{5,2i} r^{2i}, \quad a = -1 + \sum_{i=1}^{\infty} a_{2i} r^{2i}, \quad g_s = 1 + \sum_{i=1}^{\infty} g_{s,2i} r^{2i},
\]

\[
b = \sum_{i=0}^{\infty} b_{2i} r^{2i}, \quad h_1 = P r \sum_{i=0}^{\infty} h_{1,2i} r^{2i}, \quad h_2 = P r \sum_{i=0}^{\infty} h_{2,2i} r^{2i},
\]

\[(3.4)\]

with \(\chi'\) determined algebraically from (2.9). The most general solution is characterized by seven parameters\(^5\): \(\{H_0, \omega_0, h_{1,0}, h_{1,2}, h_{2,2}, a_2, b_2\}\). We present some of the terms in the perturbative solution (3.4)

\[
b_0 = -1, \quad h_{2,0} = h_{1,0},
\]

\[
H_2 = -\frac{p^2}{12\omega_0^4} - \frac{p^2 h_{1,0}^2}{4\omega_0^4} - \frac{p^2 b_2^2}{\omega_0^2} - 3P^2 (h_{1,2} - h_{2,2})^2 - 2H_0^2 H^2,
\]

\[
g_{s,2} = \frac{p^2}{12H_0\omega_0^6} - \frac{p^2 h_{1,0}^2}{4H_0\omega_0^4} + \frac{p^2 b_2^2}{H_0\omega_0^2} - \frac{p^2}{H_0} (h_{1,2} - h_{2,2})^2,
\]

\[
w_{3,2} = \frac{p^2}{24H_0\omega_0^6} + \frac{p^2 h_{1,0}^2}{24H_0\omega_0^4} - \frac{1}{24H_0\omega_0^2} (4P^2 b_2^2 + H_0) - \frac{3p^2}{2H_0} (h_{1,2} - h_{2,2})^2 - \frac{1}{2} b_2^2 \omega_0^2,
\]

\[
w_{4,2} = -\frac{p^2}{48H_0\omega_0^6} + \frac{7p^2 h_{1,0}^2}{240H_0\omega_0^4} - \frac{1}{120H_0 h_{1,0}\omega_0} (h_{1,0} (14P^2 b_2^2 + 5H_0) + 24P^2 b_2 (h_{1,2} - h_{2,2})) + \frac{\omega_0}{20H_0 h_{1,0}} (27P^2 h_{1,0} (h_{1,2} - h_{2,2})^2 + H_0 (8a_2 h_{1,0} + 10h_{2,2} + 13H_0 h_{1,0} H^2)) + \frac{a_2 \omega_0^3}{10h_{1,0}} (a_2 h_{1,0} + 12h_{1,2} - 12h_{2,2}) - \frac{6a_2 \omega_0^5}{5h_{1,0}} (h_{1,2} - h_{2,2}),
\]

\[
w_{5,2} = -\frac{p^2}{12H_0\omega_0^6} - \frac{11p^2 h_{1,0}^2}{60H_0\omega_0^4} + \frac{1}{15H_0 h_{1,0}\omega_0} (h_{1,0} (5H_0 + 11P^2 b_2^2) + 6P^2 (h_{1,2} - h_{2,2}) b_2)
\]

\[
\quad \quad + \frac{\omega_0}{5H_0 h_{1,0}} (H_0 (-5h_{2,2} - 4a_2 h_{1,0} + H_0 h_{1,0} H^2) + 9P^2 h_{1,0} (h_{1,2} - h_{2,2})^2)
\]

\[
\quad \quad + \frac{a_2 \omega_0^3}{5h_{1,0}} (-3h_{1,2} + 3h_{2,2} + a_2 h_{1,0}) + 12 \frac{a_2 \omega_0^5}{5h_{1,0}} (h_{1,2} - h_{2,2}).
\]

\[(3.5)\]

\(^4\)We can not simultaneously set the string coupling to one both in the UV and in the IR. However, doing so separately in the IR and the UV is possible. Extension to \(g_s(r = 0) \neq 1\) is trivial.

\(^5\)For \(\mathcal{H} = 0\) one can remove \(H_0\) by rescaling the four-dimensional space-time coordinates [10]. With \(\mathcal{H} \neq 0\), such a rescaling is no longer possible.
As in [10], we expect that decoupling of the asymptotic Minkowski region as \( r \to \infty \) would constrain some of these IR parameters. Specifically, in supersymmetric case the presence of the AdS-like boundary determines the size of \( S^3 \) (corresponding to our \( \omega_0 \)) in terms of the baryonic branch parameter (corresponding to our \( a_2 \)). In our case, the presence of the boundary (more detailed analysis are presented below) as \( r \to \infty \) require

\[
\tilde{H} \to 0, \quad \tilde{H}^{1/4} w_i \to \mathcal{O}(\ln^{1/4} r), \quad a \to 0.
\] (3.6)

Thus we expect at least\(^6\) two independent IR parameters. From the perspective of the dual gauge theory we have also at least two independent physical parameters: the ratio of the gauge theory strong coupling scale and the Hubble scale, and the baryonic branch deformation parameter.

### 3.2 UV solution

As in [9] and [10] we choose the radial gauge \( c_2 = c_5 \). Asymptotically as \( r \to \infty \) we expect to recover KS solution [9]. We find it convenient to introduce a new radial coordinate

\[
x = e^{-r/3},
\] (3.7)

which maps the latter boundary to \( x \to 0 \). Introduce\(^7\)

\[
c_1 = \tilde{H}^{-1/4}, \quad c_3 = \tilde{H}^{1/4} w_3, \quad c_4 = \tilde{H}^{1/4} w_4, \quad c_2 = c_5 = \tilde{H}^{1/4} w_5,
\]

\[
\tilde{H} = \frac{4P^2}{\lambda^4} x^4 \hat{h}, \quad w_3 = \lambda x^{-1} \hat{\omega}_3, \quad w_4 = \lambda x^{-1} \hat{\omega}_4, \quad w_5 = \sqrt{\frac{2}{3}} \lambda x^{-1} \hat{\omega}_5,
\]

\[
h_1 = P \hat{h}_1, \quad h_2 = P x^3 \hat{h}_2, \quad b = x^3 \hat{b}, \quad a = x^3 \hat{a},
\] (3.8)

where

\[
\lambda = 2^{-4/3} \epsilon^{2/3},
\] (3.9)

\(^6\)Some of constraints might be redundant. In particular, de-Sitter deformation of the KS cascading gauge theory baryonic branch discussed here might “turn on” extra relevant operators. It would be very interesting to analyze gauge/string theory correspondence in this setup along the lines of [20].

\(^7\)Again, \( \chi' \) is determined algebraically from [20].
is related to the conifold deformation parameter \( \epsilon \) [9]. To have leading KS asymptotics as \( x \to 0 \) we require [11]

\[
\begin{align*}
\hat{h} & \to \frac{3}{32} (-12 \ln x - 1), \quad \hat{\omega}_3 \to 1, \quad \hat{\omega}_4 = \to 1, \quad \hat{\omega}_5 \to 1, \\
\hat{h}_1 & \to -1 - 3 \ln x, \quad \hat{h}_2 \to -2 - 6 \ln x, \quad \hat{b} \to 6 \ln x, \quad \hat{a} \to -2, \quad g_s \to 1.
\end{align*}
\]

(3.10)

Notice that we set the asymptotic string coupling to one. We find the solution to (2.5)-(2.8), (2.9), (2.10), (2.11)-(2.15) subject to (3.10) to take a form of a double series expansion

\[
\begin{align*}
\hat{h} & = \sum_{i=0}^{\infty} x^{2i} \sum_{j=0}^{2i} \hat{h}_{i,j} \ln^j x, \quad \hat{\omega}_3 = \sum_{i=0}^{\infty} x^{2i} \sum_{j=0}^{2i} \hat{\omega}_{3,i,j} \ln^j x, \\
\hat{\omega}_4 & = \sum_{i=0}^{\infty} x^{2i} \sum_{j=0}^{2i} \hat{\omega}_{4,i,j} \ln^j x, \quad \hat{\omega}_5 = \sum_{i=0}^{\infty} x^{2i} \sum_{j=0}^{2i} \hat{\omega}_{5,i,j} \ln^j x, \\
\hat{h}_1 & = \sum_{i=0}^{\infty} x^{2i} \sum_{j=0}^{2i} \hat{h}_{1,i,j} \ln^j x, \quad \hat{h}_2 = \sum_{i=0}^{\infty} x^{2i} \sum_{j=0}^{2i} \hat{h}_{2,i,j} \ln^j x, \\
\hat{b} & = \sum_{i=0}^{\infty} x^{2i} \sum_{j=0}^{2i} \hat{b}_{i,j} \ln^j x, \quad \hat{a} = \sum_{i=0}^{\infty} x^{2i} \sum_{j=0}^{2i} \hat{a}_{i,j} \ln^j x, \\
g_s & = \sum_{i=0}^{\infty} x^{2i} \sum_{j=0}^{2i} g_{s,i,j} \ln^j x.
\end{align*}
\]

(3.11)

In order to match (in the \( \mathcal{H} = 0 \) limit) the probe brane potential computed in [11] we need to find perturbative solution at least up to order \( i = 4 \). To this order we find the solution to be characterized by seven parameters. Upon appropriately adjusting these parameters, we verified that in the \( \mathcal{H} = 0 \) supersymmetric limit our expansion agrees precisely with the asymptotic expansion presented in [10]. As the coefficients of (3.11)-(3.14) are rather involved, we present here only the first nontrivial correction
beyond the KS asymptotic (3.10). We find
\[ \hat{h} = -\frac{3}{32} - \frac{9}{8} \ln x + \frac{27P^2\mathcal{H}^2}{128\lambda^2} x^2 (-96 \ln x + 144 \ln^2 x + 71) + \mathcal{O}(x^4 \ln^3 x), \]
\[ \hat{\omega}_3 = 1 + x^2 \left( \hat{\omega}_{3,(2,1)} \ln x + \frac{117P^2\mathcal{H}^2}{16\lambda^2} - \frac{1}{4} \hat{a}_{(2,0)} \right) + \mathcal{O}(x^4 \ln x), \]
\[ \hat{\omega}_4 = 1 + x^2 \left( \ln x \left[ \frac{27P^2\mathcal{H}^2}{2\lambda^2} - \hat{\omega}_{3,(2,1)} \right] - \frac{207P^2\mathcal{H}^2}{16\lambda^2} + \frac{1}{4} \hat{a}_{(2,0)} \right) + \mathcal{O}(x^4 \ln x), \]
\[ \hat{\omega}_5 = 1 + \frac{9P^2\mathcal{H}^2}{16\lambda^2} x^2 \left( 12 \ln x + 1 \right) + \mathcal{O}(x^4 \ln^2 x), \]
\[ \hat{h}_1 = -1 - 3 \ln x - \frac{27P^2\mathcal{H}^2}{8\lambda^2} x^2 \left( 12 \ln x - 17 \right) + \mathcal{O}(x^4 \ln^3 x), \]
\[ \hat{h}_2 = -2 - 6 \ln x + \frac{27P^2\mathcal{H}^2}{32\lambda^2} x^2 \left( 144 \ln^2 x - 72 \ln x + 245 \right) + \mathcal{O}(x^4 \ln^4 x), \]
\[ \hat{b} = 6 \ln x - \frac{27P^2\mathcal{H}^2}{32\lambda^2} x^2 \left( 144 \ln^2 x - 264 \ln x + 157 \right) + \mathcal{O}(x^4 \ln^4 x), \]
\[ \hat{a} = -2 + x^2 \left( \ln x \left[ \frac{27P^2\mathcal{H}^2}{\lambda^2} - 4\hat{\omega}_{3,(2,1)} \right] + \hat{a}_{(2,0)} \right) + \mathcal{O}(x^4 \ln^3 x), \]
\[ g_s = 1 - \frac{27P^2\mathcal{H}^2}{\lambda^2} x^2 + \mathcal{O}(x^4 \ln^3 x). \]

Notice that to order \( i = 1 \) above only two UV parameters appear. The supersymmetric limit of [10] (to this order) is reproduced setting \( \mathcal{H} = 0 \) and
\[ \hat{\omega}_{3,(2,1)} = -\frac{3}{4} \hat{a}_{(2,0)}. \]

4 \hspace{1cm} D3-brane inflation on resolved warped deformed conifold

Effective action of a \( D3 \) probe brane on the background (2.1) moving along the radial direction takes form [15, 21]
\[ S_{\text{eff}} = T_3 \int d^4\xi \sqrt{-\gamma} \left( \frac{1}{2} c_1^2 c_2^2 \left( \frac{dr}{dt} \right)^2 + C_4 - c_1^4 \right), \]
where \( T_3 \) is the 3-brane tension, \( \gamma_{\mu\nu} \) is the metric of a four-dimensional de-Sitter space
\[ \gamma_{\mu\nu} d\xi^\mu d\xi^\nu \equiv -dt^2 + e^{2Ht}(dx)^2, \]
and the four-form potential \( C_4 \) satisfies
\[ \frac{dC_4}{dr} = K \frac{c_1^4 c_2}{c_3^2 c_4 c_5}. \]
From (4.1) the inflaton potential is

\[ V \equiv T_3 \left( c_1^4 - C_4 \right), \tag{4.4} \]

and the canonically normalized inflaton field \( \Phi \) is given by

\[ d\Phi = \sqrt{T_3 \cdot c_1 c_2} \, dr. \tag{4.5} \]

Using UV solution of the de-Sitter deformed resolved warped deformed conifold of section 3.2 we find for \( x \ll 1 \)

\[
\frac{d^2 V}{d\Phi^2} = 2\mathcal{H}^2 - \frac{27 P^2 \mathcal{H}^4}{2\lambda^2} \, x^2 \left( 3 \ln x + 4 \right) + x^4 \left( \frac{1701 \mathcal{H}^6 P^4}{40 \lambda^4} \left( 120 \ln^2 x + 285 \ln x - 31 \right) - \frac{27 \mathcal{H}^4 P^2}{5 \lambda^2} \left( 40 \hat{\omega}_{3,(2,1)} \ln^2 x + 10 \left( 2 \hat{\omega}_{3,(2,1)} - \hat{a}_{(2,0)} \right) \ln x - 163 \hat{a}_{(2,0)} - 202 \hat{\omega}_{3,(2,1)} \right) + \mathcal{H}^2 \left( 16 \hat{\omega}_{3,(2,1)} \ln^2 x + 8 \left( 4 \hat{\omega}_{3,(2,1)} - \hat{\omega}_{3,(2,1)} \hat{a}_{(2,0)} \right) \ln x - \frac{128}{5} \hat{h}_{1,(4,0)} + \frac{4}{5} \hat{\omega}_{3,(2,1)}^2 \right) - \frac{96}{5} g_{s,(4,0)} - \frac{136}{5} \hat{a}_{(2,0)} \hat{\omega}_{3,(2,1)} - 43 \hat{a}_{(2,0)}^2 \right) + \mathcal{O}(x^6 \ln^5 x). \tag{4.6} \]

Notice that to order \( \mathcal{O}(x^4 \ln^2 x) \) two more UV parameters appear: \( g_{s,(4,0)} \) and \( h_{1,(4,0)} \). Also, \( \frac{d^2 V}{d\Phi^2} \) vanishes to this order whenever \( \mathcal{H} = 0 \). If fact, we find that for the supersymmetric background of [10]

\[
\left. \frac{d^2 V}{d\Phi^2} \right|_{\text{susy}} = \frac{\lambda^2 \hat{a}_{(2,0)}^4}{128 P^2} \, x^6 \left( 8 + 15 \ln x \right) + \mathcal{O}(x^8 \ln x), \tag{4.7} \]

in agreement with the computation of [11].

From (4.6) we conclude that inflation deep inside the warped throat of the resolved deformed conifold is fast-roll. The corresponding cosmological parameter \( \eta \) is

\[ \eta = \frac{2}{3} + \mathcal{O}(\Phi^{-2} \ln \Phi). \tag{4.8} \]

Result (4.8) is expected from the general arguments presented in [21].

### 5 Conclusion

In this paper we analyzed background curvature corrections in DKS inflationary model [11]. We found that due to the conformal coupling to the background, the inflaton
in this model receives Hubble scale mass correction which renders slow-roll inflation impossible. The correct slow-roll parameter $\eta$ in DKS model is given by (4.8). While DKS inflationary scenario in strongly warped region of the geometry does not allow for tuning of $\eta$, the other slow-roll parameter $\epsilon$, 

$$
\epsilon \equiv \frac{1}{18M_{pl}^2} \left( \frac{1}{2} \frac{dV}{d\Phi} \right)^2 = \frac{2}{9} \frac{\Phi^2}{M_{pl}^2} + \mathcal{O} \left( \frac{\mathcal{H}^4}{M_{pl}^2 \Phi^2 \ln^2 \Phi} \right),
$$

(5.1)
can be made small. Indeed, one can always choose a baryonic branch deformation parameter $U$ of [10] such that

$$
|U| \ll M_{pl}^2,
$$

(5.2)
which would allow inflaton to be simultaneously in the highly warped region of the geometry $\Phi^2 \gg |U|$ while having small second slow-roll parameter (5.1).

The fact that conformal coupling of the inflationary 3-brane to the background leads to unacceptably large $\eta$-parameter in generic supergravity backgrounds was emphasized in [21]. The obvious solution to this large-$\eta$ problem proposed in [15] was to turn on appropriate 3-form fluxes to compensate for the 3-brane conformal coupling. Explicit realization of the latter proposal was discussed in [22], where the authors study inflation in de-Sitter deformed $\mathcal{N} = 2^*$ warped throat.

**Acknowledgments**

It is a pleasure to thank Ofer Aharony, Micha Berkooz and Lev Kofman for stimulating discussions. I would like to thank the Weizmann Institute of Science for hospitality during part of this work. Research at Perimeter Institute is supported in part by funds from NSERC of Canada. I gratefully acknowledge support by NSERC Discovery grant.

**References**

[1] A. Guth, “The inflationary universe: a possible solution to the horizon and flatness problems,” Phys. Rev. D 23, 347 (1981)

[2] A. Linde, “A new inflationary universe scenario: a possible solutions of the horizon, flatness, homogeneity, isotropy and primodial monopole problems,” Phys. Lett. B 108, 389 (1982);

$^8$Given (4.8) it can be readily evaluated.
[3] A. Albrecht and P. Steinhard, “Cosmology fro grand unified theories with radiatively induced symmetry breaking,” Phys. Rev. Lett. 48, 1220 (1982).

[4] G. R. Dvali and S. H. H. Tye, “Brane inflation,” Phys. Lett. B 450, 72 (1999) arXiv:hep-ph/9812483.

[5] S. H. S. Alexander, “Inflation from D - anti-D brane annihilation,” Phys. Rev. D 65, 023507 (2002) arXiv:hep-th/0105032.

[6] G. R. Dvali, Q. Shafi and S. Solganik, “D-brane inflation,” arXiv:hep-th/0105203.

[7] C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh and R. J. Zhang, “The inflationary brane-antibrane universe,” JHEP 0107, 047 (2001) arXiv:hep-th/0105204.

[8] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and S. P. Trivedi, “Towards inflation in string theory,” JCAP 0310, 013 (2003) arXiv:hep-th/0308055.

[9] I. R. Klebanov and M. J. Strassler, “Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities,” JHEP 0008, 052 (2000) arXiv:hep-th/0007191.

[10] A. Butti, M. Grana, R. Minasian, M. Petrini and A. Zaffaroni, “The baryonic branch of Klebanov-Strassler solution: A supersymmetric family of SU(3) structure backgrounds,” JHEP 0503, 069 (2005) arXiv:hep-th/0412187.

[11] A. Dymarsky, I. R. Klebanov and N. Seiberg, “On the moduli space of the cascading SU(M+p) x SU(p) gauge theory,” arXiv:hep-th/0511254.

[12] A. Buchel, A. W. Peet and J. Polchinski, “Gauge dual and noncommutative extension of an N = 2 supergravity solution,” Phys. Rev. D 63, 044009 (2001) arXiv:hep-th/0008076.

[13] N. J. Evans, C. V. Johnson and M. Petrini, “The enhancon and N = 2 gauge theory/gravity RG flows,” JHEP 0010, 022 (2000) arXiv:hep-th/0008081.

[14] K. Pilch and N. P. Warner, “N = 2 supersymmetric RG flows and the IIB dilaton,” Nucl. Phys. B 594, 209 (2001) arXiv:hep-th/0004063.
[15] A. Buchel and R. Roiban, “Inflation in warped geometries,” Phys. Lett. B 590, 284 (2004) [arXiv:hep-th/0311154].

[16] A. Buchel and A. A. Tseytlin, “Curved space resolution of singularity of fractional D3-branes on conifold,” Phys. Rev. D 65, 085019 (2002) [arXiv:hep-th/0111017].

[17] A. Buchel, “Gauge / gravity correspondence in accelerating universe,” Phys. Rev. D 65, 125015 (2002) [arXiv:hep-th/0203041].

[18] G. Papadopoulos and A. A. Tseytlin, “Complex geometry of conifolds and 5-brane wrapped on 2-sphere,” Class. Quant. Grav. 18, 1333 (2001) [arXiv:hep-th/0012034].

[19] J. H. Schwarz, “Covariant Field Equations Of Chiral N=2 D = 10 Supergravity,” Nucl. Phys. B 226, 269 (1983).

[20] O. Aharony, A. Buchel and A. Yarom, “Holographic renormalization of cascading gauge theories,” Phys. Rev. D 72, 066003 (2005) [arXiv:hep-th/0506002].

[21] A. Buchel, “Gauge theories on hyperbolic spaces and dual wormhole instabilities,” Phys. Rev. D 70, 066004 (2004) [arXiv:hep-th/0402174].

[22] A. Buchel and A. Ghodsi, “Braneworld inflation,” Phys. Rev. D 70, 126008 (2004) [arXiv:hep-th/0404151].