The IR obstruction to UV completion for Dante’s Inferno model with higher-dimensional gauge theory origin

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Abstract. We continue our investigation of large field inflation models obtained from higher-dimensional gauge theories, initiated in our previous study [1]. We focus on Dante’s Inferno model which was the most preferred model in our previous analysis. We point out the relevance of the IR obstruction to UV completion, which constrains the form of the potential of the massive vector field, under the current observational upper bound on the tensor to scalar ratio. We also show that in simple examples of the potential arising from DBI action of a D5-brane and that of an NS5-brane that the inflation takes place in the field range which is within the convergence radius of the Taylor expansion. This is in contrast to the well known examples of axion monodromy inflation where inflaton takes place outside the convergence radius of the Taylor expansion. This difference arises from the very essence of Dante’s Inferno model that the effective inflaton potential is stretched in the inflaton field direction compared with the potential for the original field.

Keywords: inflation, extra dimensions, particle physics - cosmology connection, string theory and cosmology

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1 Introduction

Effective field theories\(^1\) allow us to make predictions with desired accuracy without knowing the full details of the underlying UV theory. Traditional attitude to effective field theories was that all the terms allowed by the symmetries should appear in the action, and there is no theoretical constraints on them if one does not know the underlying UV theory but one can estimate natural magnitude of their coefficients. However, this view was challenged by the suggestions that some reasonable properties which any UV theory should satisfy impose certain constraints on effective field theories \([3–5]\). In the context of inflation, one of the most studied such criteria is the weak gravity conjecture \([4]\). It states that in order for an effective field theory with a massless Abelian gauge field to be consistently coupled to gravity, there exists at least one charged particle in the spectrum to which the gauge force acts stronger than the gravitational force. The weak gravity conjecture was proposed to explain why extra-natural inflation \([6]\), in which a higher-dimensional component of a gauge field plays the role of inflaton, appeared to be difficult to realize in string theory. In the simplest single-field extra-natural inflation model, the weak gravity conjecture restricts the inflaton field range to be sub-Planckian, making the model observationally unfavored. The restriction from the weak gravity conjecture in general multi-axion inflation models has been a subject of recent extensive studies \([7–23]\).\(^2\)

In this article, we would like to examine another\(^3\) criterion for effective field theories to be embedded in a consistent UV theory: the IR obstruction of UV completion \([5]\), applied to theories with massive vector fields \([24]\).\(^4\) In \([5]\), it was argued that the pathological behavior of

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\(^1\)For a review of effective field theory, see for example \([2]\).

\(^2\)Since the weak gravity conjecture is not the main target of the current article (though it is related in the broader perspective of constraining effective field theories from UV consistencies), we did not attempt to make a complete list of references on it here. We picked up the articles which had attracted our attention while investigating the main theme of this article.

\(^3\)Possible relation between the weak gravity conjecture and the IR obstruction to UV completion has been speculated in \([4]\). See \([8]\) for an investigation in this direction.

\(^4\)See \([25]\) which discusses the analyticity issue in inflation. Note that our interest is on the IR obstruction to UV completion for effective field theories with massive vector fields \([24]\), which has not been discussed before in the context of inflation as far as we have noticed.
an effective field theory, namely the superluminal propagation of fluctuations around certain backgrounds, is closely related to the obstruction for the effective field theory to be embedded in a UV theory whose $S$-matrix satisfies unitarity and canonical analyticity constraints. The obstruction to the UV completion was probed through the analytic property of the forward scattering amplitude of the effective field theory. In [24], the same type of analyticity property was used to argue that a massive vector field theory which has a Lorentz-symmetry-breaking local minimum cannot be embedded in UV theories whose $S$-matrix satisfies unitarity and canonical analyticity property. Incidentally, the constraints on the coefficients of the potential of the massive vector field found in [24] were the same as the constraints derived by requiring causal propagation of the massive vector field [26]. Thus also in the massive vector field theory, the acausal propagation in the IR appears to be the obstruction to UV completion.

In our previous article [1], we surveyed large-field inflation models obtained from higher-dimensional gauge theories. We discussed naturalness of the parameter values allowed by the observational constraints together with the theoretical constraints from the weak gravity conjecture. We concluded that Dante’s Inferno model was most natural among the models studied in [1]. At the time when we were writing [1], BICEP2 had suggested large tensor-to-scalar ratio $r$ [27], therefore we took $r = 0.16$ as our reference value. However, later analysis indicates that the analysis of [27] underestimated the contribution from polarized dusts [28–30]. These analysis gave lower upper bound on $r$ compared with [27], for example $r < 0.12$ at 95% CL in [28], which is also consistent with the earlier analysis [31]. This updated upper bound on the tensor-to-scalar ratio does not qualitatively change our previous conclusion that Dante’s Inferno model is most preferred in our framework. However, it does make the chaotic inflation with quadratic potential which was used in [1] moderately disfavored [30]. To accommodate the updated upper bound of the tensor-to-scalar ratio, in this article we include quartic term to the potential of massive vector field, and this is the place where the IR obstruction to UV completion is relevant: it constrains the sign of the quartic term in the potential to be negative (in the convention described in the main text). We show that this sign is actually favorable when comparing the model with the updated upper bound on the tensor-to-scalar ratio. These will be discussed in section 2.

In section 3, we examine DBI action which was used in the axion monodromy inflation [33]. DBI action is a low energy effective field theory of string theory whose $S$-matrices satisfy unitarity and canonical analyticity constraints. Therefore, it is a good example for testing whether the arguments of the IR obstruction to UV completion [5] were correct. Indeed in [5] it was shown that the embedding fields satisfy the constraints from IR obstruction to UV completion. In the current work, we are interested in the NS-NS (or RR) two-form field in six-dimensional DBI action on 5-branes, which upon dimensional reduction to five-dimensions gives a massive vector field. The five-dimensional model can be treated in a similar way as in the section 2, but the potential for the massive vector field contains higher order terms. One of the main interests here is the effects of these higher order terms. Using the parameter values allowed by the CMB data obtained in section 2, we show in simple examples that the inflation takes place in the field range which is within the convergence radius of the Taylor expansion of the DBI action. This means that the linear approximation of the potential at large field, which was appropriate in the well known examples of axion

\footnote{While we were finalizing the current article, a new tighter bound on the tensor-to-scalar ratio has been announced by Keck Array & BICEP2 collaborations [32]. As our analysis had already finished with the earlier bound, and we would also like to see if the new bound will be confirmed with other independent experiments, we will not consider the bound given in [32] in this article.}
monodromy inflation \[33\], is not valid in Dante’s Inferno model, in the simple models we study. This difference originates from the very essence of Dante’s Inferno model that the inflaton potential is stretched in the inflaton field direction compared with the potential of the original field due to a field redefinition.

We summarize with discussions on future directions in section 4.

2 The IR obstruction to UV completion for Dante’s Inferno model with higher-dimensionsional gauge theory origin

Dante’s Inferno model \[34\] is a two-axion model described by the following potential in four dimensions:

\[ V_{DI}(A, B) = V_A(A) + \Lambda^4 \left( 1 - \cos \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right). \]  

(2.1)

The potential (2.1) appears as a leading approximation to the effective potential obtained from the following five-dimensional gauge theory compactified on a circle:

\[ S = \int d^5x \left[ -\frac{1}{4} F^{(A)}_{MN} F^{(A)MN} - V_A(A_M) - \frac{1}{4} F^{(B)}_{MN} F^{(B)MN} 
- i \bar{\psi} \gamma^M (\partial_M + i g_{A5} A_M - i g_{B5} B_M) \psi \right], \]  

\[ (M, N = 0, 1, 2, 3, 5), \]  

(2.2)

where

\[ A_M = A_M - g_{A5} \partial_M \theta, \]  

(2.3)

and the field strengths of the Abelian gauge fields are given as

\[ F^{(A)}_{MN} = \partial_M A_N - \partial_N A_M, \quad F^{(B)}_{MN} = \partial_M B_N - \partial_N B_M. \]  

(2.4)

We consider the diagonal kinetic term for the gauge fields for simplicity. Since the metric convention will be important in the following discussions, we explicitly state here that our convention is

\[ \eta_{MN} = \text{diag}(+ - - - -). \]  

(2.5)

The axion decay constants in four-dimension are related to parameters in the five-dimensional gauge theory as

\[ f_A = \frac{1}{g_A(2\pi L_5)}, \quad f_B = \frac{1}{g_B(2\pi L_5)}, \]  

(2.6)

where \( L_5 \) is the compactification radius of the fifth dimension, and \( g_A \) and \( g_B \) are four-dimensional gauge couplings which are related to the five-dimensional gauge couplings \( g_{A5} \) and \( g_{B5} \) as

\[ g_A = \frac{g_{A5}}{\sqrt{2\pi L_5}}, \quad g_B = \frac{g_{B5}}{\sqrt{2\pi L_5}}. \]  

(2.7)

\[^{6}\text{We used the charged fermion as an example of charged matters. One may consider different matter fields, it does not affect the conclusion qualitatively as long as the charge assignment is similar.}\]
We consider the potential of the vector field $A_M$ given in the power series expansion:

$$V_A(A_M) = v_2 A_M A_M^2 + v_4 (A_M A_M)^2 + v_6 (A_M A_M)^3 + \cdots = \sum_{n=1}^{\infty} v_{2n} (A_M A_M)^n. \quad (2.8)$$

From the effective field theory point of view, the functional form of the potential $V_A(A_M)$ is arbitrary as long as it respects Lorentz symmetry, which is already implemented in (2.8). However, it has been claimed that there are certain constraints on the potential in order for the effective field theory to be derived from a UV theory whose $S$-matrix satisfies unitarity and canonical analyticity constraints [5]. In the case of massive vector field theories which is of our current interest, this issue was taken up by [24]. The following sign constraints were derived from the condition that the effective field theory to be embedded to a unitary UV theory with canonical analyticity property:

$$v_2, v_4 < 0. \quad (2.9)$$

Note that our metric convention (2.5) follows that in [24]. Incidentally, (2.9) is the same condition given in [26] for the massive vector field theory to have causal evolution. As we are interested in a model which has a sound IR behavior as well as an origin in a sane UV theory, below we assume that (2.9) is satisfied.

The naturalness in five-dimension dictates $v_{2n} = c_{2n} / \Lambda_{UV}^{3n-5}$ with $c_{2n} \sim \mathcal{O}(1)$, where $\Lambda_{UV}$ is the UV cut-off scale at which the effective field theory breaks down, if there were no symmetry to forbid these terms. However, if there is an approximate symmetry, it is natural that the coefficients of the terms which violate the symmetry is small [35]. In the current case, $|c_{2n}| \ll 1$ with $g_{A5} \sqrt{\Lambda_{UV}} \ll 1$ is natural because turning off these couplings recovers the U(1) gauge symmetry without the charged matter fields and the Stueckelberg field.

When

$$|A_M A_M| \ll \Lambda_{UV}^3, \quad (2.10)$$

dropping the terms with $n \geq 3$ in (2.8) will be a good approximation. Whether (2.10) is realized or not depends both on $|A_M A_M|$ required for inflation which we will obtain below, and on $\Lambda_{UV}$, which depends on the microscopic origin of the effective field theory.\footnote{One can put the upper bound on $\Lambda_{UV}$ by estimating the magnitude of loop corrections, but new physics can enter before the upper bound is reached.} As the field range of the original fields are restricted in Dante’s Inferno model as we review below, it is natural to expect that (2.10) would hold in many cases, but one should examine it for each microscopic model. In this section we assume that (2.10) is satisfied and set $v_{2n} = 0$ for $n \geq 3$ in (2.8):

$$V_A(A_M) = v_2 A_M A_M^2 + v_4 (A_M A_M)^2. \quad (2.11)$$

We will examine how good the truncation of the potential at the quartic order is in explicit microscopic models based on 5-branes in section 3.

The four-dimensional effective potential for the zero-modes $A$ and $B$ of the fifth components of the gauge fields $A_5$ and $B_5$, respectively is given at one-loop order as follows (the details of the calculations are given in appendix A):

$$V_{1\text{-loop}}(A, B) = V_{cl}(A) + V_g(A) + V_f(A, B). \quad (2.12)$$

Here, the classical part of the potential,

$$V_{cl}(A) = \frac{1}{2} m^2 A^2 - \frac{\lambda}{4!} A_4^4, \quad (2.13)$$
directly follows from the classical potential (2.11) upon dimensional reduction. In (2.13) we introduced parametrization suitable in four-dimension:

\[ -v_2 = \frac{m^2}{2} > 0, \quad -\frac{v_4}{2\pi L_5} = \frac{\lambda}{4!} > 0, \]  

(2.14)

where the sign follows from the constraints from the IR obstruction to UV completion, eq. (2.9). As shown in the appendix A, the one-loop contribution from the fermion \( V_f(A, B) \) in (2.12) is given as

\[ V_f(A, B) = \frac{3}{\pi^2(2\pi L_5)^4} \sum_{n=1}^{\infty} \frac{1}{n^5} \cos \left\{ n \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right\}. \]  

(2.15)

\( V_g(A) \) in (2.12) is the one-loop contribution from the gauge field \( A_M \). As shown in the appendix A, the contribution of this term is sub-leading compared with that of the classical potential \( V_{cl}(A) \) when

\[ 2\pi L_5 \gtrsim 1 \times 10^2, \]  

(2.16)

in the parameter region and the field value of our interest which are to be discussed below. Here and below we use the unit \( M_P = 1 \), where \( M_P \) is the reduced Planck scale \( M_P = (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18} \) GeV. Since the five-dimensional gauge theory is non-renormalizable and should be regarded as an effective field theory, we do not expect the compactification radius \( L_5 \) to be too close to the Planck scale. Therefore (2.16) is a reasonable assumption to make. Below we adopt this assumption and drop \( V_g(A) \) from the analysis below. However, though this is a reasonable assumption, it is also for technical simplicity. Dante's Inferno model may still work even if the contribution from \( V_g(A) \) is not negligible, though the loop expansion should be under control for analyzing such case.

By taking the leading \( n = 1 \) term in (2.15), we obtain the potential for Dante's Inferno model (2.1):

\[ V_{DI}(A, B) = V_A(A) + \Lambda^4 \left\{ 1 - \cos \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right\}, \]  

(2.17)

with

\[ V_A(A) = \frac{m^2}{2} A^2 - \frac{\lambda}{4!} A^4, \]  

(2.18)

and

\[ \Lambda^4 = \frac{3}{\pi^2(2\pi L_5)^4}. \]  

(2.19)

The plot of the potential with typical values of parameters is shown in figure 1.

To describe inflation in Dante’s Inferno model, it is convenient to make a rotation in the field space [34]:

\[ \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix} = \begin{pmatrix} \cos \gamma & -\sin \gamma \\ \sin \gamma & \cos \gamma \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}, \]  

(2.20)

where

\[ \sin \gamma = \frac{f_A}{\sqrt{f_A^2 + f_B^2}}, \quad \cos \gamma = \frac{f_B}{\sqrt{f_A^2 + f_B^2}}. \]  

(2.21)

\[ \text{Remember the metric sign convention in (2.5)!} \]
Figure 1. The plot of $V_{DI}(A, B)$ for a typical values of parameters. In the plot the two ends of the $B$-axis which correspond to $B = 0$ and $B = 2\pi f_B$ are identified.

In terms of the rotated fields, the potential (2.17) becomes

$$V_{DI}(\tilde{A}, \tilde{B}) = \frac{m^2}{2} (\tilde{A} \cos \gamma + \tilde{B} \sin \gamma)^2 - \frac{\lambda}{4!} (\tilde{A} \cos \gamma + \tilde{B} \sin \gamma)^4 + \Lambda^4 \left(1 - \cos \frac{\tilde{A}}{f}\right),$$

(2.22)

where

$$f = \frac{f_A f_B}{\sqrt{f_A^2 + f_B^2}}.$$  

(2.23)

Now, the following two conditions are imposed in Dante’s Inferno model:

condition 1 \quad f_A \ll f_B \lesssim 1.  \quad (2.24)

condition 2 \quad |\partial_A V_A(A)|_{A=A_{in}} \ll \frac{\Lambda^4}{f}. \quad (2.25)

Here, $A_{in}$ is the value of the field $A$ when the inflation started. The first hierarchy in the condition 1 is used to achieve effective super-Planckian inflaton travel and implies

$$\cos \gamma \simeq 1, \quad \sin \gamma \simeq \frac{f_A}{f_B}, \quad f \simeq f_A.$$ 

(2.26)

In terms of the variables in the five-dimensional gauge theory, the condition 1 corresponds through (2.6) to the hierarchy between the couplings of the different gauge groups [1]:

$$g_B \ll g_A.$$ 

(2.27)
The second inequality in the condition 1 is motivated by the weak gravity conjecture [4], as mentioned in the introduction. From (2.6) this condition amounts to

$$2\pi L_5 \gtrsim \frac{1}{g_B}.$$  \hspace{1cm} (2.28)

The condition 2 (2.25) is a requirement for the field $\tilde{A}$ to roll down to $\tilde{B}$-dependent local minimum much faster than the field $\tilde{B}$, which is to be identified with the inflaton, rolls down. It imposes the following condition on the parameters of the five-dimensional gauge theory:

$$m^2 A_{\text{in}} - \frac{\lambda}{3!} A_{\text{in}}^3 \ll \frac{A^4}{f_A} = \frac{3g_A}{\pi^2 (2\pi L_5)^3}. \hspace{1cm} (2.29)$$

After $\tilde{A}$ settles down to $\tilde{B}$ dependent local minimum, the motion of $\tilde{B}$ leads to the slow-roll inflation. By redefining $\tilde{B} = \phi$, we obtain the following inflaton potential:

$$V_{\text{eff}}(\phi) = V_A \left( \sin \gamma \tilde{B} \right) = \frac{m_{\text{eff}}^2}{2} \phi^2 - \frac{\lambda_{\text{eff}}}{4!} \phi^4$$

$$= \frac{m_{\text{eff}}^2}{2} \phi^2 \left( 1 - c\phi^2 \right), \hspace{1cm} (2.30)$$

where

$$m_{\text{eff}}^2 := \sin^2 \gamma m^2 \simeq \left( \frac{f_A}{f_B} \right)^2 m^2, \hspace{1cm} (2.31)$$

$$\lambda_{\text{eff}} := \sin^4 \gamma \lambda \simeq \left( \frac{f_A}{f_B} \right)^4 \lambda, \hspace{1cm} (2.32)$$

and

$$c := \frac{\lambda_{\text{eff}}}{12m_{\text{eff}}^2}. \hspace{1cm} (2.33)$$

Compared with the original potential $V_A(A)$ of the field $A$, the potential $V_{\text{eff}}(\phi)$ of the inflaton $\phi$ is stretched in the field space direction, due to the rotation in the field space (2.20), see figure 1. This is the essential feature of the Dante’s Inferno model which allows the super-Planckian excursion of the inflaton while the field ranges of the original fields $A$ and $B$ are sub-Planckian.

The inflaton potential (2.30) is not bounded from below, but we will only consider the region of $\phi$ before the potential starts to go down:

$$|\phi| < |\phi|_{\text{max}} = \frac{1}{\sqrt{2c}}. \hspace{1cm} (2.34)$$

We will not worry about the potential beyond $|\phi| > |\phi|_{\text{max}}$ because this field region is not relevant for the inflation, as we confirm shortly. Actually, in [24] it has been shown that massive vector field theories which can be embedded to a UV theory whose $S$-matrix satisfies unitarity and canonical analyticity constraints do not have a Lorentz-symmetry-breaking vacuum. In such theories, before the potential starts to go down, the contribution from higher order terms in the potential should come in to prevent Lorentz-symmetry-breaking local minimum, assuming that the potential is bounded from below.
Since the inflaton potential (2.30) is symmetric under the reflection $\phi \rightarrow -\phi$, without loss of generality we assume $\phi \geq 0$ below.

We would like to compare our model with the CMB observations. In order for that, we should impose the following condition:

\begin{equation}
\begin{aligned}
\text{condition 3} & \quad \partial^2_A V_{DI}(\tilde{A}, \tilde{B}) \gg H^2,
\end{aligned}
\end{equation}

during the inflation, where $H = \dot{a}(t)/a(t)$, with the dot denoting the derivative with respect to the time $t$. If this condition is satisfied, and there is no other light scalar field with mass below $H$ which we assume to be the case, only inflaton contributes to the scalar power spectrum. Taking into account the condition 2 (2.25), the condition 3 (2.35) reduces to

\begin{equation}
\partial^2_A V_{DI}(\tilde{A}, \tilde{B}) \simeq \frac{\Lambda^4}{f^2} \simeq \frac{\Lambda^4}{f^2} \simeq \frac{3g_A^2}{\pi^2(2\pi L_D)^2} \gg H^2.
\end{equation}

This condition will be examined further later.

From the inflaton potential (2.30), the slow-roll parameters are calculated as

\begin{equation}
\begin{aligned}
\epsilon(\phi) & := \frac{1}{2} \left( \frac{V_{\text{eff}}'}{V_{\text{eff}}} \right)^2 = \frac{2}{\phi^2} \left( \frac{1 - 2c\phi^2}{1 - c\phi^2} \right)^2, \\
\eta(\phi) & := \frac{V_{\text{eff}}''}{V_{\text{eff}}} = \frac{2}{\phi^2} \frac{1 - 6c\phi^2}{1 - c\phi^2}.
\end{aligned}
\end{equation}

The spectral index is given as

\begin{equation}
n_s = 1 - 6\epsilon(\phi_*) + 2\eta(\phi_*),
\end{equation}

where the subscript * refers to the value at the pivot scale 0.002 Mpc$^{-1}$, for which we follow the Planck 2015 analysis [29]. The number of e-fold is given as

\begin{equation}
N(\phi) = \int_{\phi_{\text{end}}}^{\phi} d\phi \frac{V_{\text{eff}}'}{V_{\text{eff}}} = \int_{\phi_{\text{end}}}^{\phi} d\phi \frac{1 - c\phi^2}{2(1 - 2c\phi^2)} = \left[ \frac{\phi^2}{8} - \frac{\ln(1 - 2c\phi^2)}{16c} \right]_{\phi_{\text{end}}}^{\phi},
\end{equation}

where we have defined $\phi_{\text{end}}$ as the field value when $\epsilon(\phi)$ first reaches 1 after the inflation starts. In the parameter region we will consider, this will be determined dominantly by the quadratic part of the potential and given as

\begin{equation}
\phi_{\text{end}} \simeq \sqrt{2}.
\end{equation}

The scalar power spectrum is given by

\begin{equation}
P_s = \frac{V_{\text{eff}}(\phi_*)}{24\pi^2 \epsilon(\phi_*)} = 2.2 \times 10^{-9},
\end{equation}

where the value in the right hand side is from the observation [29]. The tensor-to-scalar ratio is given as

\begin{equation}
r_* = 16\epsilon(\phi_*).
\end{equation}
Figure 2. The effective potential (2.30) and $\phi_*$ for $N_* = 50$ and $N_* = 60$ for different values of $c$.

After obtaining the inflaton potential (2.30), our model has two parameters $m_{\text{eff}}$ and $c$ in the potential (2.30), and one choice for the initial condition $\phi_*$. The observational value of the power spectrum (2.42) gives one relation among them, and when the number of e-fold $N$ is specified, (2.40) gives another relation. Then we are left with one independent parameter, for which we choose $c$. The parameter $c$ is further constrained by the observational bounds on the spectral index $n_s$ and the tensor-to-scalar ratio $r$, as shown in the $n_s - r$ plane in figure 3 compared with that given in the Planck 2015 results [30].

From figure 3, we observe that the inclusion of the quartic term in the potential parametrized by positive $c$ of order $\mathcal{O}(10^{-3})$ pushes the model to the observationally favored direction. This is quite as expected: positive $c$ results from $v_2$ and $v_4$ both being negative (2.14). Recall that (2.14) was a condition for avoiding the IR obstruction to UV completion. From this it follows that for larger $c$ the potential (2.30) becomes lower at large inflaton field values, as shown in figure 2. This leads to smaller $r$ through (2.42), which is favored in the latest observations.

The main aim of Dante’s Inferno model is to achieve super-Planckian inflaton excursion in effective field theory while the field ranges of the original fields are sub-Planckian. Thus we further require

$$A_* \lesssim 1.$$  \hfill (2.44)

From figure 2, we observe that $\phi_* \simeq 12 \sim 15$ in the range of the parameter $c$ of our interests. Thus

$$A_* \simeq \frac{f_A}{f_B} \phi_* \gtrsim \frac{g_B}{g_A} \times 15.$$  \hfill (2.45)

Therefore, the condition 4 amounts to

$$g_A \gtrsim 15g_B.$$  \hfill (2.46)

This is compatible with the condition 1, (2.24).
Figure 3. Contour plots of $n_s - r$ for the inflation with potential (2.30), with varying $c$ and with $N_* = 50$ and $N_* = 60$. Compared with the Planck 2015 results [30].

Next we would like to examine the condition 2. Substituting (2.31) and (2.32) into (2.25), we obtain

$$
\frac{3g_B}{\pi^2 (2\pi L_5)^3} \gg \frac{\lambda_{\text{eff}}}{6} \phi_*^3 = \partial_\phi V_{\text{eff}}(\phi_*).
$$

(2.47)

We have used $A_{\text{in}} \sim A_*$ in the above estimate. As an example, we take $N_* = 60$, $c = 0.001$ case which is observationally favorable as shown in figure 3. Then from figure 4 we have $\partial_\phi V_{\text{eff}}(\phi_*) \sim 4 \times 10^{-10}$. Putting this value into (2.47), we obtain

$$
\frac{1}{L_5^3} \gg 3 \times 10^{-7} g_B^{-1}, \quad (N_* = 60, c = 0.001),
$$

(2.48)

or equivalently

$$
\frac{1}{L_5} > 7 \times 10^{-3} g_B^{-1/3}, \quad (N_* = 60, c = 0.001).
$$

(2.49)

On the other hand, from the condition 1 (2.24) we have

$$
2\pi L_5 \gtrsim g_B^{-1}.
$$

(2.50)

Thus we arrive at

$$
7 \times 10^{-3} g_B^{-1/3} < \frac{1}{L_5} \lesssim 2\pi g_B, \quad (N_* = 60, c = 0.001).
$$

(2.51)

Figure 5 shows the allowed values of $L_5$ in (2.51). This figure should be looked together with the condition (2.16), $2\pi L_5 \gtrsim 1 \times 10^2$, which we have imposed to justify neglecting the contribution from the gauge field $V_g(A)$ to the one-loop effective potential. This condition still leaves a large portion of the allowed parameter space. Note that a natural value for $g_A$ is $g_A \lesssim O(1)$, and through (2.46) it means $g_B \lesssim O(10^{-1})$. As shown in figure 5, $L_5$ has allowed region in such values of $g_B$. 

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Finally, let us look back the condition 3 (2.35). From figure 6, we observe $H_* \sim \mathcal{O}(10^{-5})$. Then (2.35) gives only a very mild constraint $g_A \gg \mathcal{O}(10^{-5})$, which is weaker than the bound given from figure 5 and (2.46).

3 Dante’s Inferno model with DBI action of a 5-brane

In this section we study Dante’s Inferno model with a potential for the massive vector field obtained from DBI action of a 5-brane. Our main purpose in this section is to study the effects of higher order terms in the potential comparing with the quartic potential in the previous section, in an explicit model arising from string theory whose $S$-matrix satisfies
Figure 6. The plot of $H_*$ as a function of $c$.

unitarity and canonical analyticity constraints. The low energy effective DBI action of a D5-brane and that of an NS5-brane have been used in the axion monodromy inflation \[33, 36\], and it is also interesting to observe the difference of the Dante’s Inferno model with 5-branes.

In the case of D5-brane, the action is given as

$$S_{D5} = -T_{D5} \int d^6 \sigma \sqrt{- \text{det} \left( G_{ab} + F_{ab} \right)}, \quad (a, b = 0, 1, 2, 3, 5, 6),$$

(3.1)

where

$$F_{ab} = B_{ab} - \partial_a C_b + \partial_b C_a.$$

(3.2)

In (3.1), $G_{ab} = G_{MN} \partial_a X^M \partial_b X^N$ and $B_{ab} = B_{MN} \partial_a X^M \partial_b X^N$ are the pull-back of the target space metric and the NS-NS 2-form field to the D5-brane worldvolume, respectively. $C_a$ is a 1-form gauge field on the D5-brane. The tension of the D5-brane is given by

$$T_{D5} = \frac{1}{(2\pi)^3 g_s \alpha'^3}.$$

(3.3)

The NS-NS 2-form field $B_{MN}$ also has the kinetic term in six-dimension which follows from a compactification of the kinetic term in ten-dimensional bulk:

$$\frac{1}{2(2\pi)^3 g_s^2 (\alpha')^4} \int d^{10} x \, H_{MNL} H^{MNL},$$

(3.4)

$$H_{MNL} = \partial_\mu B_{\mu NL},$$

(3.5)

where [...] denotes the antisymmetrization. The normalization of the kinetic term in six-dimension depends on the volume of the compactified four-dimensional space. Since it is simpler to directly discuss the normalization of the four-dimensional kinetic term after further compactification of two more directions as will be done below, we don’t explicitly write down the six-dimensional kinetic term here.
Let us consider the background
\[ G_{MN} = \text{diag}(++,--) , \quad B_{MN} = 0 , \] (3.6)
in the static gauge \( \sigma^a = x^a \) (\( a = 0, 1, 2, 3, 5, 6 \)).\(^9\) In the perturbative expansions in string coupling, the constant shift of the NS-NS 2-form field
\[ B_{56} \rightarrow B_{56} + 2\pi \frac{2\pi \alpha'}{(2\pi L_5)(2\pi L_6)} , \] (3.7)
is a symmetry. The shift symmetry (3.7) is broken in the existence of the D5-brane. (If we consider all the winding sectors, the shift (3.7) exhibits monodromy.) Upon double dimensional reduction along the sixth direction, the zero-mode of \( B_{M6} \) becomes a massive vector field in five-dimension which we denote as \( a_M \), whereas the zero-mode of the gauge field \( C_M \) becomes the Stueckelberg scalar field which we denote as \( \Theta \):
\[ F_a^6 = 2\pi \alpha' (2\pi L_5)(2\pi L_6) \left( a_M - \partial_M \Theta \right) = 2\pi \alpha' (2\pi L_5)(2\pi L_6) a_M , \] (3.8)
where we will identify \( a_M \) and \( a_M \) with \( A_M \) and \( A_M \) in the previous section up to a proportionality constant, respectively. We will fix the proportionality constants shortly. The five-dimensional potential for \( a_M \) after the double dimensional reduction is given by
\[ \rho T_{DS}(2\pi L_6) \int d^5 x \sqrt{1 + \frac{(2\pi \alpha')^2}{(2\pi L_5)^2(2\pi L_6)^2}} a_M a^M , \quad (M, N = 0, 1, 2, 3, 5) . \] (3.9)
Here, \( \rho \) represents the numerical factor which depends on the detail of the six-dimensional compact space, possibly with a warp factor.

Now we would like to have a closer look at the IR obstruction to UV completion for the massive vector field theory of \( A_M \). In [5], DBI action was taken as an example which is free from the IR obstruction. [5] focused on the embedding coordinate fields \( X^I(x) \). When there is a small dimensionless expansion parameter, \( g_s \) in this case, unitarity and analyticity of the forward scattering constrains the sign of the coefficients of \( \left( \partial_N X^I \partial_N X^I \right)^n \) to be all positive in the action [5]. A prescription suggested in [24] for massive vector field models was that the constraints from the IR obstruction to UV completion on the sign of the coefficient of the term \( \left( A_N A^N \right)^n \) is identical to that on the coefficient of the term \( \left( \partial_N X^I \partial_N X^I \right)^n \). The five-dimensional action obtained from the six-dimensional DBI action satisfy these conditions, as can be checked from the Taylor expansion of the potential (3.9). As DBI action is a low energy effective action derived from string theory whose \( S \)-matrices satisfy unitarity and canonical analyticity constraints, this supports the prescription proposed in [24].

By further double dimensional reduction along the fifth direction, we obtain the four-dimensional potential for the field \( a \) which is the zero-mode of \( a_4 \):
\[ V_4(a) = \rho T_{DS}(2\pi L_5)(2\pi L_6) \int d^4 x \sqrt{1 + \frac{(2\pi \alpha')^2}{(2\pi L_5)^2(2\pi L_6)^2}} a^2 . \] (3.10)
Recall our metric convention (2.5). Here, \( a \) is normalized so that \( a \rightarrow a + (2\pi) \) corresponds to the shift symmetry (3.7). Thus if we identify this potential with \( V_A(A) \) of the Dante’s Inferno potential (2.1), the proportionality constant between the field \( A \) and \( a \) is fixed as
\[ A = f_A a . \] (3.11)
\(^9\)We will turn on the zero-mode of \( B_{56} \) later.
This means that the kinetic term of $a$ was given as
\[ \int d^4x \frac{f_A^2}{2} \partial_\mu a \partial^\mu a. \tag{3.12} \]

The four-dimensional kinetic term (3.12) follows from the compactification of the ten-dimensional kinetic term (3.4), and $f_A$ depends on the volume of the compactified space.

In terms of the field $A$, the potential (3.10) is written as
\[ V_A(A) = \rho T_{D5}(2\pi L_5)(2\pi L_6) \int d^4x \sqrt{1 + \frac{(2\pi\alpha')^2}{(2\pi L_5)^2(2\pi L_6)^2 f_A^2}} A^2. \tag{3.13} \]

From (3.13) we read off the convergence radius $A_c$ for the Taylor expansion around $A = 0$:
\[ A_c = \frac{f_A(2\pi L_5)(2\pi L_6)}{2\pi\alpha'}. \tag{3.14} \]

As we should assume that $(2\pi L_5), (2\pi L_6) \gg (\alpha')^{1/2}$ in order to justify the suppression of the string corrections, we have
\[ A_c \gg \frac{f_A}{2\pi}. \tag{3.15} \]

Now, we would like to study Dante’s Inferno model as we have done in the previous section for the classical potential (3.13). In order for that, we assume that there is also the field $B$ in (2.1) which may arise from form fields in higher dimensions,\(^\text{10}\) and that the sinusoidal potential in (2.1) is also generated in the same way as in the previous section. The essential ingredient of Dante’s Inferno model is that the effective potential for the inflaton field $\phi = \phi_B$ is stretched by the factor $1/\sin \gamma \simeq f_B/f_A$ in the field space direction compared with the potential for the field $A$:
\[ V_{\text{eff}}(\phi) = V_A(\sin \gamma \phi) \simeq V_A \left( \frac{f_A}{f_B} \phi \right) = \rho T_{D5}(2\pi L_5)(2\pi L_6) \int d^4x \sqrt{1 + \frac{(2\pi\alpha')^2}{(2\pi L_5)^2(2\pi L_6)^2 f_B^2}} \phi^2 \]
\[ = \rho T_{D5}(2\pi L_5)(2\pi L_6) \int d^4x \sqrt{1 + \left( \frac{\phi}{\phi_c} \right)^2}, \tag{3.16} \]
where the convergence radius $\phi_c$ for the Taylor expansion is given by
\[ \phi_c = \frac{f_B A_c}{f_A} = \frac{f_B(2\pi L_5)(2\pi L_6)}{2\pi\alpha'}. \tag{3.17} \]

When $\phi_* \ll \phi_c$, the Taylor expansion of the square root is a good approximation for describing the inflation, while when $\phi_* \gg \phi_c$ the potential (3.16) is approximately a linear potential. The latter was the case studied in [33] for a single axion monodromy model. We examine below which is the case for the current model.

\(^\text{10}\)For example, NS-NS two-form field $B_{M,N}$ with $M = 5$ and $N$ in other extra dimensional direction may do the job.
Figure 7. A comparison between the quartic potential (2.30) and the potential (3.16) obtained from the DBI action.

Let us first truncate the potential (3.16) at the quartic order in the Taylor expansion and apply the results in the previous section. Then, the effective mass $m_{\text{eff}}$ and the effective quartic coupling constant $\lambda_{\text{eff}}$ in the truncated potential are given by

$$m_{\text{eff}}^2 = \frac{\rho T_{D5}(2\pi L_5)(2\pi L_6)}{(2\pi)^2 g_s} \frac{1}{\phi_c^2},$$

$$\lambda_{\text{eff}} = \frac{4!}{8} \rho T_{D5}(2\pi L_5)(2\pi L_6) \frac{1}{\phi_c^4},$$

Thus we obtain

$$c = \frac{\lambda_{\text{eff}}}{12m_{\text{eff}}^2} = \frac{1}{4\phi_c^2}. \tag{3.20}$$

(3.20) gives

$$\phi_c = \frac{1}{2\sqrt{c}} \lesssim |\phi|_{\text{max}} = \frac{1}{\sqrt{2c}}, \tag{3.21}$$

where $|\phi|_{\text{max}}$ was given in (2.34). For example, for $c = 0.001$, $\phi_c \simeq 16 \geq \phi_* \simeq 15$. Therefore, the inflation takes place within the convergence radius of the Taylor expansion in the truncated potential. In figure 7 we compare the quartic potential (2.30) and the potential following from the DBI action (3.16). In figure 7 the parameters of the potential (3.16) are tuned so that it coincides with the quartic potential (2.30) at $\phi = 0$ and $\phi = \phi_*$. What this means is that we regard the quartic potential (2.30) not as a truncation of the Taylor expansion of the potential (3.16) at the quartic order, but as a phenomenological parametrization to fit it. We observe that this phenomenological parametrization is a rather good approximation to the potential (3.16) in the field range where the inflation takes place. Thus although the inflation starts close to the convergence radius and the quartic potential may not be a very
accurate approximation to the potential (3.16), it will be more than enough for qualitative estimates. It is interesting that the inflation does not take place in the field range where the linear approximation at the large field value is valid, which was the case in the well known examples of axion monodromy model [33]. The difference originates from the very essence of Dante’s Inferno model that the potential for the effective inflaton (3.16) is stretched from that for the original field $A$.

Note that the potential obtained from DBI action monotonically increases, differing from the quartic potential which starts to go down from $|\phi| = |\phi|_{\text{max}}$. We do not expect the quartic potential to be a good description in these regions, which however are irrelevant when discussing inflation.

As we have seen, though the truncation of the potential at the quartic order in Taylor expansion may not be a very precise description, it should be good enough for qualitative estimates, so let us proceed with the values obtained in the previous section. From (3.17), the value $\phi_c \approx 16$ can be achieved, for example, $\left(\frac{2\pi L_5}{16}\right)^{1/2} \sim 10^{-1}$, $(2\pi\alpha^{'})^{-1} \sim \mathcal{O}(10^{-1})$ with $f_B \lesssim 1$. For these values, we obtain

$$m^2_{\text{eff}} \gtrsim \frac{\rho}{g_s} \times 10^{-1},$$

$$\lambda_{\text{eff}} \gtrsim \frac{3\rho}{g_s} \times 10^{-3}. \quad (3.23)$$

From figure 8 and figure 9, $m^2_{\text{eff}} \sim \mathcal{O}(10^{-11})$ and $\lambda_{\text{eff}} \sim \mathcal{O}(10^{-13})$ for $c = 0.001$. Therefore, $\rho/g_s \lesssim \mathcal{O}(10^{-10})$ would realize successful Dante’s Inferno model from higher-dimensional gauge theory discussed in the previous section. This value of $\rho/g_s$ may be realizable in appropriate warped geometries, though the study of consistent realization in string theory is beyond the scope of the current article.

The case of DBI action of NS5-brane with RR 2-form field is similar, except for the string coupling dependence. Therefore, we just write down the corresponding formulas. Instead of (3.3), (3.17), (3.18), (3.19) and (3.20), we have

$$T_{\text{NS5}} = \frac{1}{(2\pi)^6 g_s^2 \alpha'^2}, \quad (3.24)$$

$$\phi_c = \frac{f_B (2\pi L_5) (2\pi L_6)}{g_s (2\pi \alpha')}, \quad (3.25)$$

$$m^2_{\text{eff}} = \rho T_{\text{NS5}} (2\pi L_5) (2\pi L_6) \left(\frac{g_s^2 (2\pi \alpha')^2}{(2\pi L_5)^2 (2\pi L_6)^2} \right) \frac{(2\pi L_5)^2 (2\pi L_6)^2 f_B^2}{16}, \quad (3.26)$$

$$\lambda_{\text{eff}} = \frac{4!}{8} \rho T_{\text{NS5}} (2\pi L_5) (2\pi L_6) \left(\frac{g_s^2 (2\pi \alpha')^2}{(2\pi L_5)^2 (2\pi L_6)^2} \right)^2 \frac{(2\pi L_5)^2 (2\pi L_6)^2 f_B^2}{16}, \quad (3.27)$$

and thus

$$c = \frac{\lambda_{\text{eff}}}{12m^2_{\text{eff}}} = \frac{1}{4\phi_c^2}, \quad (3.28)$$

or

$$\phi_c = \frac{1}{2\sqrt{c}} \lesssim |\phi|_{\text{max}} = \frac{1}{\sqrt{2c}}. \quad (3.29)$$

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4 Summary and discussions

In this article, we spotted light on the constraints from the IR obstruction to UV completion [5, 24] on inflation models obtained from higher-dimensional massive vector field theories. While weak gravity conjecture [4] which also discusses constraints from UV completion has been discussed extensively in the past few years, the IR obstruction to UV completion in this context has not been studied previously. In Dante’s Inferno model, which is most promising in this class of models, we have shown that the constraint on the sign of the quartic term in the potential of the massive vector field is in favor of the current observational upper bound on tensor-to-scalar ratio. We also discussed DBI actions on 5-branes in string theory. In particular, we studied the effects of higher order terms in the potential of the massive vector...
field, which arises from NS-NS/RR two-form field on a 5-brane upon dimensional reduction. Interestingly, in these models inflation takes place within the convergence radius of the Taylor expansion. This is in contrast to the well known examples of axion monodromy inflation [33], in which inflaton takes place outside the convergence radius of the Taylor expansion. The difference arises from the very essence of Dante’s Inferno model that the effective inflaton potential is stretched in the inflaton field direction compared with the potential for the original field. The result also tells us that the model with a potential up to the quartic order can approximate the models with higher order terms reasonably well.

While in [5] and [24], canonical analyticity constraints on $S$-matrices in flat space-time were considered, it has been pointed out that analyticity structure of $S$-matrices could be much richer in curved space-time [37–39]. It will be interesting to explore whether further constraints on effective field theories arise in curved space-time, and if they do, what are the implications to inflation models, in particular those which are obtained from higher-dimensional massive vector field theories.

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A The one-loop effective potential

A.1 Calculation of the one-loop effective potential

In this appendix, we evaluate the one-loop effective potential for the zero-modes of $A_5$ and $B_5$. We consider the action in five dimensions consists of the U(1) gauge fields $A_M$ and $B_M$, a massless fermion $\psi$ with charges $\ell$ and $-\ell'$ of U(1)$_A$ and U(1)$_B$, respectively, and the Stueckelberg field $\theta$ associated with the U(1)$_A$ gauge group:

$$S = \int d^5x \left[ -\frac{1}{4} F^{(A)}_{MN} F^{(A) MN} - \frac{1}{4} F^{(B)}_{MN} F^{(B) MN} - V_{\text{cl}}(A_M) + \bar{\psi} \Gamma^M D_M \psi \right],$$

(M, N = 0, 1, 2, 3, 5),

(A.1)

where

$$F^{(A)}_{MN} = \partial_M A_N - \partial_N A_M, \quad F^{(B)}_{MN} = \partial_M B_N - \partial_N B_M,$$

(A.2)

and

$$D_M \psi = \partial_M \psi - i g A_M \ell A_M \psi - i g B_M (-\ell') B_M \psi.$$  

(A.3)

We consider the following classical potential for $A_M$:

$$V_{\text{cl}}(A_M) = v_2 A_M A^M + v_4 (A_M A^M)^2.$$  

(A.4)

The action (A.1) is invariant under the following gauge transformations:

$$A_M \rightarrow A_M + \partial_M \Lambda^{(A)}, \quad B_M \rightarrow B_M + \partial_M \Lambda^{(B)},$$

$$\psi \rightarrow e^{i g A_M \Lambda^{(A)}} e^{i (\ell') g B_M \Lambda^{(B)}} \psi, \quad \theta \rightarrow \theta + g^{-1} A_5 \Lambda^{(A)}.$$  

(A.5)
We assume that the U(1)$_A$ gauge groups are compact, thus the Stueckelberg field $\theta$ is periodically identified as

$$\theta \sim \theta + \frac{2\pi}{g^2_{A5}}. \quad (A.7)$$

To evaluate the one-loop effective potential for the zero-modes of $A_5$ and $B_5$, we expand the action around $x$-independent classical values:

$$A_M(x) = A^0_M + A^q_M(x), \quad B_M(x) = B^0_M + B^q_M(x), \quad \psi = 0 + \psi^q(x), \quad (A.8)$$

where $q$ denotes quantum fluctuation. The expansion of the classical potential around the classical value of $A_M = A^0_M$ up to the quadratic order in the fluctuations is given by

$$V_{cl}(A^0_M + A^q_M) = V_{cl}(A^0_M) + V_K(A^q_M)A^{qK} + \frac{1}{2}V_{KL}(A^0_M)A^{qK}A^{qL} + O(A^{q3}_M), \quad (A.9)$$

where

$$V_{cl}(A^0_M) = v_2A^0_M A^{cM} + v_4(A^0_M A^{cM})^2, \quad (A.10)$$

$$V_K(A^q_M) = \frac{\partial V_{cl}(A^0_M)}{\partial A^{qK}} = 2v_2A^q_K + 4v_4A^0_M A^{cM} A^q_K, \quad (A.11)$$

$$V_{KL}(A^q_M) = \frac{\partial^2 V_{cl}(A^0_M)}{\partial A^{qK} \partial A^{qL}} = 2v_2\eta_{KL} + 4v_4(\eta_{KL} A^q_M A^{cM} + 2A^q_K A^q_L). \quad (A.12)$$

We also introduce the following gauge fixing term:

$$S_{gf} = \int d^5x \left[ -\frac{1}{2\xi}(\partial_M A^{qM} + \xi m_A^2 \theta^q)^2 - \frac{1}{2\xi}(\partial_M B^{qM})^2 \right], \quad (A.13)$$

where

$$m^2_A := -2v_2 - 4v_4A^0_M A^{cM}. \quad (A.14)$$

We shall choose $\xi = 1$ and $\zeta = 1$ in (A.13). Assuming $A_M$ is at the extremum of the potential, the action up to the quadratic order in the fluctuations is given as

$$S^{(2)} + S_{gf} = \int d^5x \left[ -\frac{1}{4}F^{(A)}_{MN}F^{(A)MN} - \frac{1}{4}F^{(B)}_{MN}F^{(B)MN} + \bar{\psi}^q_i \Gamma^M D_M(A^q_M, B^q_M) \psi^q \right. \right.$$

$$-\frac{1}{2} (\partial_M A^{qM} + g_{A5} m^2_A \theta^q)^2 - \frac{1}{2} (\partial_M B^{qM})^2$$

$$-\frac{1}{2} V_{KL}(A^q_M)(A^{qK} - g_{A5} \partial^K \theta^q)(A^{qL} - g_{A5} \partial^L \theta^q)$$

$$\left. \left] = \int d^5x \left[ \frac{1}{2} X_a M^{ab} X_b + \frac{1}{2} B^N_{MN} \partial^M B^{qN} + \bar{\psi}^q_i \Gamma^M D_M(A^q_M, B^q_M) \psi^q \right. \right], \quad (A.15)$$

where

$$X_a := (A^q_M, g_{A5}^{-1} \theta^q), \quad a = M, \theta,$$

$$M^{ab} := \left( \eta^{MN} (\partial^2_M + m_A^2) - 8v_4 A^{cM} A^{cN} \right. \left. -8v_4 g_{A5}^2 A^{cK} A^{cN} \partial_K \right. \left. -g_{A5}^2 m_A^2 \left( \partial^2_M + \frac{8v_4}{m_A^2} A^c_k \partial^c_k \partial^M + m_A^2 \right) \right). \quad (A.16)$$
We also need to consider the ghost action associated with \( U(1)_A \) gauge fixing since it couples to the Vacuum Expectation Value (VEV) of \( A_M \) via \( m_A^2 \) (A.14) hence contributes to the one-loop effective potential. The ghost action corresponding to the gauge fixing (A.13) is given as

\[
S_{c_A} = \int d^5 x \left[ -\bar{c}_A \left( \partial_M \partial^M + m_A^2 \right) c_A \right].
\]  

(A.17)

The ghosts for the \( U(1)_B \) gauge group, \( c_B \) and \( \bar{c}_B \), are free as usual and decouple from the rest of the calculations.

The fifth dimension is compactified on \( S^1 \) with radius \( L_5 \). The mode expansions of the fields in the fifth direction are given as

\[
A_M(x, x^5) = \frac{1}{\sqrt{2\pi L_5}} \sum_{n=-\infty}^{\infty} A_M^{(n)}(x) e^{i \frac{2\pi n}{L_5} x^5}, \quad \text{same for } B_M, \psi, c_A,
\]

\[
\theta(x, x^5) = \frac{x^5}{g_{A5}^2 L_5} + \sum_{n=-\infty}^{\infty} \theta^{(n)}(x) e^{i \frac{n\pi}{L_5} x^5},
\]  

(A.18)

where \( \theta \) can have integer winding number \( w \), but it can be set to zero by a gauge transformation \( A_5 \to A_5 + k/(g_{A5} L_5), \theta \to \theta + k x^5/(g_{A5}^2 L_5) \) (\( k \) is an integer). In what follows we will fix \( w = 0 \). We consider the following VEVs for \( A_5^{(0)} \) and \( B_5^{(0)} \):

\[
\langle A_5^{(0)} \rangle = A, \quad \langle B_5^{(0)} \rangle = B,
\]  

(A.19)

and the other fields have zero expectation values. Corresponding to the VEV of \( A_M \), the VEV of \( A_M \) is

\[
A_M^c = 0, \quad A_5^c = \frac{1}{\sqrt{2\pi L_5}} \langle A_5^{(0)} \rangle.
\]  

(A.20)

Now we have the quadratic actions for the gauge fields:

\[
S_{A,\theta}^{(2)} = \int d^4 x \sum_{n=-\infty}^{\infty} \frac{1}{2} \bar{X}_{a}^{(n)}(x) M_{ab}^{(n)} X_{b}^{(n)},
\]  

(A.21)

where

\[
\bar{X}_{a}^{(n)} := \langle A_{M}^{(n)}, \bar{\theta}^{(n)} \rangle, \quad \bar{\theta}^{(n)} := (g_{A5}^2/2\pi L_5)^{1/2} \theta^{(n)},
\]

\[
M_{ab}^{(n)} := \begin{pmatrix}
\eta^{MN} \left( \partial_{\mu} + \left( \frac{n}{L_5} \right)^2 + m_A^2 \right)^2 - 8\eta^{4}(A_5^{c})^2 \delta_{5}^{N} \delta_{5}^{M} - \frac{8\eta^{4}}{m_{A} L_5} (A_5^{c})^2 \delta_{5}^{N} \delta_{5}^{M} & - \left( \partial_{\mu} + \left( \frac{n}{L_5} \right)^2 (1 + \frac{8\eta^{4}}{m_{A} L_5} (A_5^{c})^2 + m_A^2) \right)
\end{pmatrix},
\]  

(A.22)

and that for the ghost fields \( c_A \) and \( \bar{c}_A \):

\[
S_{c_A} = \int d^4 x \left[ - \sum_{n=-\infty}^{\infty} \bar{c}_A^{(n)} \left( \partial_{\mu} + \left( \frac{n}{L_5} \right)^2 + m_A^2 \right) c_A^{(n)} \right],
\]  

(A.23)

and that for the fermion:

\[
S_{\psi}^{(2)} = \int d^4 x \sum_{n=-\infty}^{\infty} \bar{\psi}^{(n)} \left( i\Gamma_{\mu} \partial_{\mu} + \ell g_{A} \Gamma^{5} A - \ell' g_{B} \Gamma^{5} B - \Gamma^{5} \frac{n}{L_5} \right) \psi^{(n)}.
\]  

(A.24)
In the above, \( \mu \) denotes the directions in the uncompactified four-dimensional space-time and runs from 0 to 3. Note that \( B_M \) bosons do not contribute to the one-loop effective potential because they do not couple to the background fields at the one-loop level.

We observe that the fermion contribution \( V_f(A, B) \) to the one-loop effective potential \( V(A, B) \) is the same as the previous study [1]:

\[
V_f(A, B) = \text{Tr} \left[ \ln \left( -i \Gamma^\mu \partial_\mu - \ell g_A \Gamma^5 \langle A_5(0) \rangle + \ell g_A \Gamma^5 \langle B_5(0) \rangle + \Gamma^5 \frac{n}{L_5} \right) \right]
\]

where

\[
f_A = \frac{1}{(2\pi g_A \ell L_5)}, \quad f_B = \frac{1}{(2\pi g_B \ell L_5)}.
\]

Employing the \( \zeta \) function regularization, we obtain

\[
V_f(A, B) = \frac{3}{2\pi^2 (2\pi L_5)^3} \sum_{n=1}^{\infty} \frac{1}{n^5} \cos \left[ n \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right].
\]

(A.27)

In (A.27) we have subtracted the constant part by hand. Although the constant term has a physical significance, the huge discrepancy between the theoretically natural value of the constant term and the observationally suggested value of it is the notorious cosmological constant problem, which we do not attempt to address in this article.

Next we turn to the gauge boson contributions to the one-loop effective potential. We introduce the Euclidean time \( \tau \) and the Euclidean gauge field \( A_E^M \) as follows:

\[
\tau = it, \quad A_E^0 = iA^0, \quad A_E^i = A^i, \quad A_E^5 = A^5.
\]

(A.28)

The gauge boson loops give rise to the effective action of \( A \):

\[
\Gamma_g(A) = -2 \ln \det D^2|_{A_E^\mu} - \frac{1}{2} \ln \det D^2|_{A_E^\mu} - \frac{1}{2} \ln \det D^2|_\partial + \ln \det D^2|_{cA},
\]

where the determinant is that with respect to \( x^\mu \) and \( n \), and

\[
D^2|_{A_E^\mu, A_E^5, cA} = -\partial^2_E + \left( \frac{n}{L_5} \right)^2 + m_A^2,
\]

\[
D^2|_\partial = -\partial^2_E + \left( \left( \frac{n}{L_5} \right)^2 + m_A^2 \right) \left( 1 + \frac{8v_4}{m_A^2} (A_5^c)^2 \right).
\]

(A.30)

The effective potential is given as

\[
V_g(A) = \sum_{n=-\infty}^{\infty} \int \frac{d^4p_E}{(2\pi)^4} \left[ \frac{3}{2} \ln \left\{ p_E^2 + \left( \frac{n}{L_5} \right)^2 + m_A^2 \right\} \right. \\
+ \frac{1}{2} \ln \left\{ p_E^2 + \left( \left( \frac{n}{L_5} \right)^2 + m_A^2 \right) \left( 1 + \frac{8v_4}{m_A^2} (A_5^c)^2 \right) \right\} \right].
\]

(A.31)
The $\zeta$ function regularization yields the following result:

$$V_g(A) = -\frac{1}{4\pi^2(2\pi L_5)^2} \left[ 3 + \left( 1 - \frac{\lambda}{3m_A^2} \right)^2 \right] \sum_{k=1}^{\infty} \frac{m_A^2}{k^3} (1 + 3(kz)^{-1} + 3(kz)^{-2}) e^{-kz}, \quad (A.32)$$

where

$$z := \sqrt{m_A^2(2\pi L_5)^2}, \quad (A.33)$$

and (A.14) is rewritten as

$$m_A^2 = -2v_2 + \frac{4v_4}{(2\pi L_5)^2} A^2 = m^2 - \frac{\lambda}{6} A^2, \quad m^2, \lambda > 0. \quad (A.34)$$

Adding classical potential (A.9) and the one-loop contributions (A.27) and (A.32), we obtain the following effective potential for $A$ and $B$:

$$V_{1-loop}(A, B) = V_{cl}(A) + V_f(A, B) + V_g(A) \quad (A.35)$$

$$= \frac{m^2}{2} A^2 - \frac{\lambda}{4!} A^4 + \frac{3}{\pi^2(2\pi L_5)^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \cos \left[ n \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right]$$

$$- \frac{1}{4\pi^2(2\pi L_5)^2} \left[ 3 + \left( 1 - \frac{\lambda A^2}{3m_A^2} \right)^2 \right] \sum_{k=1}^{\infty} \frac{m_A^2}{k^3} (1 + 3(kz)^{-1} + 3(kz)^{-2}) e^{-kz}.$$

### A.2 The comparison between $V_g(A)$ with $V_{cl}(A)$

In what follows, we examine whether and when the contributions of $V_g(A)$ to the energy density and spectral index are sub-leading compared with $V_{cl}(A)$. For this purpose, it is convenient to change the variable from $A$ to $\phi \sim \frac{f_B}{f_A} A$. To estimate $V_g(\phi)$, taking only $k = 1$ term in (A.35) is a good approximation:

$$V_g(\phi) \sim -\frac{m^2(1 - 2c\phi^2)}{4\pi^2(2\pi L_5)^2} \left[ 3 + \left( 1 - \frac{4c\phi^2}{1 - 2c\phi^2} \right)^2 \right] (1 + 3\phi^{-1} + 3\phi^{-2}) e^{-z}$$

$$+ \frac{3}{\pi^2(2\pi L_5)^4}, \quad (A.36)$$

with

$$z = (2\pi L_5)m(1 - 2c\phi^2)^{1/2}. \quad (A.37)$$

In the above we have added the constant term so that $V_g(0) = 0$ is satisfied, in order to tune the cosmological constant. Since we are mostly interested in the case $|V_g(\phi)| \ll V_{cl}(\phi)$, we subtracted $V_g(0)$ instead of the energy density at the minimum of the total potential for simplicity. To estimate (A.36), we first observe from figure 8 that $m_{eff} \lesssim 7 \times 10^{-6}$. Thus if we take $f_B/f_A \sim 10 - 30$, $m = \frac{f_B}{f_A} m_{eff} \lesssim 7 \times 10^{-5} - 2 \times 10^{-4}$. On the other hand, from figure 5 we observe that $2\pi L_5 \lesssim 10^3$, thus $m(2\pi L_5) \lesssim 2 \times 10^{-1}$. Taking the leading term in the power series expansion in $m(2\pi L_5)$, we obtain

$$V_g(\phi) \sim -\frac{3}{4\pi^2(2\pi L_5)^4} \left\{ 3 + \left( 1 - \frac{4c\phi^2}{1 - 2c\phi^2} \right)^2 \right\} + \frac{3}{\pi^2(2\pi L_5)^4}$$

$$= \frac{3}{\pi^2(2\pi L_5)^4} v_g(x)|_{x = \sqrt{\phi}}, \quad (A.38)$$
where
\[ v_g(x) = \frac{1}{4} \left\{ 3 + \left( 1 - \frac{4x^2}{1 - 2x^2} \right)^2 \right\} - 1. \] (A.39)

To compare (A.38) with \( V_{cl}(\phi) \), we rewrite \( V_{cl}(\phi) \) as
\[ V_{cl}(\phi) = \frac{m^2_{\text{eff}}}{2c} \phi^2 \left( 1 - c\phi^2 \right) = \frac{m^2_{\text{eff}}}{2c} v_{cl}(x)|_{x=\sqrt{c}\phi}, \]
where
\[ v_{cl}(x) = x^2(1 - x^2). \] (A.40)

Near the observationally preferable point \( c = 0.001 \) and \( N^* = 60 \), \( \phi^*_s \lesssim 15 \) and thus \( \sqrt{c}\phi^*_s \lesssim 0.5 \). In the domain \( 0 \leq x < 0.5 \) the functions \( v_g(x) \), \( v_{cl}(x) \) and their derivatives are roughly of order one, thus for a crude comparison between \( V_g(\phi) \) and \( V_{cl}(\phi) \) we can compare the coefficients in front of these functions, \( 3/\pi^2(2\pi L_5)^4 \) and \( m^2_{\text{eff}}/2c \). When \( c = 0.001 \) and \( N^* = 60 \), \( m^2_{\text{eff}}/2c \sim 6 \times 10^{-9} \), thus the contributions of \( V_g(\phi) \) to the energy density and the spectral index compared with those of \( V_{cl}(\phi) \) are sub-leading if
\[ \frac{3}{\pi^2(2\pi L_5)^4} \lesssim 6 \times 10^{-9}, \] (A.41)
or equivalently
\[ 2\pi L_5 \gtrsim 1 \times 10^2. \] (A.42)

Since the five-dimensional gauge theory is not renormalizable and is regarded as an effective field theory, it is natural that the compactification radius \( L_5 \) is not too close to the Planck scale. Therefore, (A.42) is a natural condition to impose and we have assumed this in the main body.

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