Calculation of lateral stresses for uniaxial compression of geomaterial samples

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Abstract. A method is proposed for calculating the transverse stresses during uniaxial compression of geomaterial samples from the measured load and mutually perpendicular deformations. Analytical expressions connecting the indicated parameters are obtained. The dependences of the change in the calculated transverse stresses on time are plotted for various values of Poisson's ratio. The difference in transverse stresses demonstrates a much greater sensitivity to mechanical stress than each transverse stress separately. Sharp changes in the values of the difference in transverse stresses are observed, which coincide with bursts of AE activity.

1. Introduction
The similarity of processes occurring in complex geological structures of the earth's crust and deformation processes observed in laboratory experiments on loaded rock samples [1-6] creates a prerequisite for studying processes occurring in the depths of the earth's crust by physical modeling on rock samples. Modeling of inelastic deformation, structure formation and fracture by testing samples of geomaterials on rheological presses has already demonstrated its effectiveness for a number of problems in seismology and tectonophysics [7-10].

In experiments on the destruction of geomaterials, installations with uniaxial compression are traditionally used. In this case, in addition to measuring acoustic emission (AE) and axial load, as a rule, measurements are made of the deformations of the sample under study in three mutually perpendicular directions. In order to have a complete picture of the stress-strain state, it is necessary, in addition to the principal stress, to obtain also the values of the transverse stresses.

The purpose of this work is to calculate the transverse stresses during uniaxial compression of the sample.

1.1. The condition for the occurrence of transverse stresses.
Uniaxial loading of a rectangular specimen is considered (Fig. 1). The sample is positioned in a Cartesian coordinate system centered in the middle. Three mutually perpendicular directions can always be drawn through any point of the deformable body, the shifts between which are taken to be zero.
When a load is applied to a sample of square cross-section, mechanical stresses arise in it, the mathematical description of which is given, respectively, by the tensors of deformations and stresses:

\[
\sigma_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}, \quad \varepsilon_{kl} = \begin{bmatrix} \varepsilon_x & \frac{1}{2} \gamma_{xy} & \frac{1}{2} \gamma_{xz} \\ \frac{1}{2} \gamma_{xy} & \varepsilon_y & \frac{1}{2} \gamma_{yz} \\ \frac{1}{2} \gamma_{xz} & \frac{1}{2} \gamma_{yz} & \varepsilon_z \end{bmatrix}
\]

The relationship between the components of strain and stress tensors (1) is described by the generalized Hooke's law in the form:

\[
\sigma_{ij} = \sum_{k=1}^{M} \sum_{l=1}^{N} c_{ijkl} \cdot \varepsilon_{kl}
\]

where \(c_{ijkl}\) is the fourth-order stiffness tensor, \(N\) and \(M\) are the number of rows and columns of the deformation tensor in the Cartesian coordinate system. The stress tensor \(\sigma\) is usually represented as the sum of the spherical stress tensor \(\sigma_s\) and the stress deviator \(\sigma_d\):

\[
\sigma = \sigma_s + \sigma_d
\]

In what follows, only the components of the spherical tensor will be considered. The stress spherical tensor is called the mean pressure at a point and characterizes all-round uniform compression or tension:

\[
\sigma_s = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}
\]

After transforming and simplifying the components of the stiffness tensor \(c_{ijkl}\) in accordance with the Voigt notation, equation (2) can be represented in matrix form:

\[
\begin{align*}
E \varepsilon_x &= \sigma_x - \mu \sigma_y - \mu \sigma_z \\
E \varepsilon_y &= -\mu \sigma_x + \sigma_y - \mu \sigma_z \\
E \varepsilon_z &= -\mu \sigma_x - \mu \sigma_y + \sigma_z
\end{align*}
\]
i.e., lateral stresses are absent or negligible. The condition for the occurrence of transverse deformations:

\[ \varepsilon_x - \varepsilon_y > 0 \]  

(7)

The fulfillment of condition (7) allows deriving from the system of equations (5) the design formulas for transverse stresses:

\[ \sigma_x = \frac{\sigma_z(1+\mu) - E\varepsilon_x + E\mu(\varepsilon_x - \varepsilon_y - \varepsilon_z)}{2\mu(1+\mu)} \]  

(8)

\[ \sigma_y = \frac{\sigma_z(1+\mu) - E\varepsilon_x - E\mu(\varepsilon_x + \varepsilon_y - \varepsilon_z)}{2\mu(1+\mu)} \]  

(9)

The transverse stresses obtained from formulas (8) and (9) are functions of time:

\[
\begin{cases}
\sigma_x = \sigma_x(t) \\
\sigma_y = \sigma_y(t)
\end{cases}
\]  

(10)

If condition (7) is met, the difference in lateral deformations can be converted to the difference in the occurring transverse stresses, which carry important information about the mechanical processes during compression of the sample. Differences in transverse deformations and stresses can be converted to a more convenient form:

\[ \varepsilon_x - \varepsilon_y = \Delta\varepsilon_{xy} \]  

(11)

\[ \sigma_x - \sigma_y = \Delta\sigma_{xy} \]  

(12)

The relationship between the differences in stresses and strains comes from equation (5) taking into account transformations (11) and (12):

\[ \Delta\varepsilon_{xy} = \frac{E}{1+\mu}\Delta\sigma_{xy} \]  

(13)

2. Results

For the calculation, we used the experimental data obtained in the course of experiments on the destruction of samples of geomaterials with a linearly increasing load under uniaxial compression. In the course of calculations, the values of the parameters were synchronized over the entire considered time interval. The complete cycle of the experiment was chosen as the time interval - from the beginning of loading to the destruction of the sample. This time synchronization makes it possible to observe the behavior of deformations, calculated transverse stresses and compare the obtained values with other parameters, in particular, with the activity of acoustic emission (AE).
The installation for loading rock samples is a gravity-lever press with a linearly increasing load, the maximum value of which is 250 kN (Fig. 2). The uniaxial compression experiment was carried out using water leakage [11]. The location of the strain gauges corresponds to the Cartesian coordinate system.

The samples were made from natural sandstone from the Kegety deposit, Kyrgyzstan, and had the shape of a parallelepiped with dimensions of $24.5 \times 25.5 \times 61$ mm. Poisson's ratio of sandstone is in the range 3.3 ÷ 7.5. The Young's modulus required for calculations was taken equal to 0.37 MPa, and was considered constant throughout the experiment. Linear differential transformers with the following ranges of registered linear displacements were used as sensors recording longitudinal and lateral deformations: ± 0.127 mm; ± 0.254 mm and ± 1.27 mm. During the uniaxial compression experiment, the load on the sample varied from 36.30 kN to 87.24 kN.

In Fig. 3 shows the dependences of transverse and longitudinal deformations on time for the entire measurement period. Their values were calculated as relative deformations. Longitudinal deformations during the entire measurement time are linear, transverse deformations retain it only partially, mainly until the moment of destruction. The deformation along the x axis has a large steepness in relation to the deformation of the y axis, intersects with it at the point corresponding to the onset of destruction of the sample. This can be explained by such factors as the localization of microfractures at the site of the formation of the main crack, and the formation of the main rupture of the sample. By the time $12 \times 10^4$ s after the start of the experiment, the final destruction of the sample occurs.

![Figure 3. Dependences of longitudinal and transverse deformations on time.](image)

Fig. 4 shows a diagram of the deformation of a sandstone specimen under linearly increasing uniaxial loading.
The calculation of the values of transverse deformations was carried out on the basis of expressions (8) and (9). In this case, not one value of Poisson's ratio was used, but the interval from 3.3 to 7.5 with a step of 0.53. In Fig. 5 and 6 show the dependences of the obtained transverse stresses along the x and y axes, respectively, for different values of Poisson's ratio.

The dependences shown in Fig. 5 remain linear throughout the measurement time. The calculated transverse stresses in the x and y directions are similar.
Fig. 6 shows the graphs of the difference in transverse stresses, expression (12), for various values of Poisson's ratio and the AE activity throughout the experiment.

The difference in transverse stresses demonstrates a much greater sensitivity to mechanical stress than each transverse stress separately. Sharp changes in the values of $\Delta \sigma_{\text{xy}}$ are observed, which coincide with bursts of AE activity. The AE activity, in turn, demonstrates the rate of defect formation in the sample under study. It can be argued that an increase in transverse stresses leads to the formation of new defects, which, in turn, is accompanied by an increase in the AE activity. When approaching the moment of destruction, starting from $9.5 \times 10^4$ seconds of observation, the curves intersect at one point, which corresponds to the moment of destruction. Further bursts of AE activity and a sharp slope of the difference in transverse stresses from $11.5 \times 10^4$ to $12.02 \times 10^4$ s indicate the final destruction of the sample.

3. Conclusion
Analytical expressions have been obtained that make it possible to calculate the transverse stresses during uniaxial compression of the sample from the measured mutually perpendicular deformations and the value of the axial load. The obtained transverse stresses make it possible to obtain a complete picture of the stress-strain state of the sample under study. Changes in transverse stresses in time are plotted. The results of experimental data processing demonstrate an increase in transverse stresses with increasing load. The deformation along the $x$ axis has a large steepness in relation to the deformation of the $y$ axis, intersects with it at the point corresponding to the onset of destruction of the sample. This is due to factors such as the location of the main crack and roughness of the sample surface. Comparison of the difference between the transverse stresses and the activity of acoustic emission demonstrates the general picture of the behavior of the parameters: there are sharp changes in the values of $\Delta \sigma_{\text{xy}}$, mass formation of defects, accompanied by bursts of AE activity.

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