On physics of the Poynting-Robertson effect

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Abstract. Detailed discussion of university textbook statements (Harwit 1988) concerning the Poynting-Robertson effect (P-R effect) is presented. Discussion is concentrated on better physical understanding of the P-R effect. References to complete correct equation of motion for real dust particle are also presented.

1. Introduction

Orbital evolution due to the interaction between cosmic dust particle and electromagnetic radiation is – as most astronomers think – well-known as the Poynting-Robertson effect (P-R effect).

Srikanth (1999) offers three physical viewpoints on the corresponding statements presented in astronomical literature which is the most referenced on the P-R effect. However, more detailed physics of the P-R effect was presented by Klačka (1992). The statements of Srikanth (1999) were discussed by Klačka (2000a).

Astronomers will not understand the real physics of the P-R effect if incorrect statements are presented in frequently used university textbooks and scientific papers. The aim of this paper is to discuss the statements presented in Harwit (1988, pp. 176-177).

2. P-R effect and Harwit’s discussion

Harwit (1988) discusses the derivation of the P-R drag in two ways. The first access concentrates to the situation as seen from the frame of reference of the Sun. The second access concentrates to the situation as seen from the frame of reference of the dust particle
– grain. It is supposed that particle absorbs sunlight and re-emits this energy isotropically in its own rest frame.

2.1. Reference frame of the Sun

Harwit (1988) formulates heuristic three point instruction for obtaining loss of orbital angular momentum. The most significant incorrect statement is that “a re-emitted photon carries off angular momentum” in the way that produces P-R drag. Another heuristic statement concerns the fact that the loss of angular momentum is proportional to the velocity of the grain – why? Eqs. (5-45) and (5-46) are incorrect.

If we want to put the statements into a correct physics, we have to stress that photons are re-emitted isotropically – total re-emitted momentum is zero in the proper frame of reference of the particle. As a consequence, the particle (grain) losses the momentum proportional to the velocity of the grain – consequence of the Lorentz transformation.

2.2. Reference frame of the grain

The access of the Harwit is instructive. The only improvement may concern the fact that we do not need to consider relativistic aberration of light if we are considering only terms linear in \( v/c \). Moreover, one should explain the possibility of mixing quantities measured in two different reference frames (of the sun and of the grain).

3. P-R effect and Newton’s laws of motion

Harwit (1988) concentrates to explanation of the P-R drag. However, P-R drag is only a part of the total equation of motion for dust particle due to its interaction with electromagnetic radiation (see also Klačka 2000a with respect to the discussion presented by Srikanth 1999). Thus, complete equation of motion should be derived.

Harwit (1988) uses angular momentum as a relevant physical quantity – its one component. However, angular momentum vector cannot fully describe any general motion. Since the time of Newton we know that equation of motion is significant and it is described by time derivative of momentum.

The physical access of Harwit enables to treat the P-R effect only for near circular orbits. Of course, even this type of orbits is not correctly – completely – described, as it is for the true P-R effect (see Klačka and Kaufmannová 1992; the same results were obtained by Breiter and Jackson 1998 – one must be careful, since the Breiter and Jackson’s result concerning the systematic increase of eccentricity is not physically correct – it is caused by linearizing of the P-R effect, see Klačka 1999).
4. Physics of the velocity decrease

Harwit (1988) states that "the grain velocity decreases on just absorbing the light". This statement is incorrect. We will show it using generalization of Robertson’s (1937) result.

The generalization for the P-R effect is defined by the equation

\[ p'_o = (1 - Q'_{PR}) p'_i , \] (1)

which corresponds to the case when the total momentum per unit time of the “outgoing” radiation \( p'_o \) is proportional to the “incident” momentum per unit time \( p'_i \); primes denote quantities measured in the proper frame of reference of the particle (see Eq. (122) in Klačka 1992a). On the basis of Eq. (1) one comes to the following equation

\[ \frac{dp'^\mu}{d\tau} = p'^\mu_i - \left[ (1 - Q'_{PR}) p'^\mu_i + Q'_{PR} \frac{w E_i}{c^2} u'^\mu \right] , \] (2)

where \( u'^\mu \) denotes four-velocity, \( p'^\mu_i = (E_i/c, p_i) \), \( Q'_{PR} \) is pressure coefficient and the quantity \( w E_i \) is scalar (see Eqs. (30), (133) and (134) in Klačka 1992).

Let the perfect absorption of light occurs.

i) The case of perfect absorption and no reemission is described by Eq. (2) when the only first term on the right-hand side is present. Eq. (2) yields then

\[ \frac{dm}{d\tau} = p'_\nu \frac{u_\nu}{c^2} , \]
\[ m \frac{du'^\mu}{d\tau} = p'^\mu_i - (p'_\nu \frac{u_\nu}{c^2}) u'^\mu . \] (3)

Eq. (3) states that the grain velocity decreases on just absorbing the light (mass of the particle increases due to the absorption of light).

ii) The square bracket terms in Eq. (2) correspond to reemission in the form described by Eq. (1). Let the reemission is in the form that \( Q'_{PR} = 0 \); Eq. (2) yields \( du'^\mu/d\tau = 0 \), then \( (dm/d\tau = 0) \). We see that absorption is not the cause for decreasing of the grain velocity.

We can summarize: the grain velocity decreases due to the fact that \( Q'_{PR} \) is positive value and that Eq. (1) holds.

As another example, one may easily calculate interaction of electromagnetic radiation with spherical particle covered by reflecting mirror (specular reflection): equation of motion is the same as that for perfectly absorbing spherical particle with isotropic reemission, i.e., \( Q'_{PR} = 1 \) – the grain velocity decreases, but no absorption exists.
5. Conclusion

We have put into a correct physics the statements presented in Harwit (1988). As for the correct complete derivation of the P-R effect we refer the reader to Klačka (1992a, 1992b, 1993a, 1993b), as for other discussions to Klačka (1993c, 2000a). As for interaction between electromagnetic radiation and nonspherical particle, we refer to Kocifaj and Klačka (1999) and to Klačka (2000b, 2000c, 2000d).

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