Robust Prescribed Performance Control of Uncertain Vehicle Lateral System under State Constraints

Jing Yu
School of Transportation, Inner Mongolia University, Hohhot 010000, China.
yujing@imu.edu.cn

Abstract. In the study of the zero-error tracking control problem for vehicle lateral control systems under full-state constraints and nonparametric uncertainties, the zero-error tracking control problem is presented in this paper. A neural adaptive tracking control scheme is proposed by combining the error transformation of the vehicle lateral control system with the barrier Lyapunov function, which realizes that the tracking error of the vehicle lateral control converges to a prescribed compact set at a controllable or specified convergence rate in a specified finite time. The scheme has the following significant characteristics: 1) Based on the Nussbaum gain, the preset new energy finite-time control algorithm, the tracking error of the vehicle lateral control system with nonparametric uncertainty and external disturbance decreases to zero with $t \to \infty$. In addition, it also has the control ability to cope with the presence or even unknown moment of inertia of the system. 2) Barrier Lyapunov function (BLF) ensures the bounded input of the neural network during the whole system envelope, and ensures the stable learning and approximation of the neural network. Furthermore, the bounded stability of the closed-loop system is proved by Lyapunov analysis. Finally, the effectiveness and superiority of the proposed control method are verified by simulation.

Keywords: Vehicle lateral system; Full-state constraint; Barrier Lyapunov function; Nussbaum gain; Neural network.

1. Introduction
Vehicles have become an indispensable means of transportation in the world today, but [1], [2], [3], [4] shows that the mobility brought by vehicles is costly. According to the World Health Organization, approximately 1.3 million people worldwide die each year as a result of road traffic accidents, with 20 to 50 million people suffering from non-fatal injuries, many of whom are disabled for this reason. Road traffic injuries have brought huge economic losses to individuals, families and the whole country, including the treatment costs of the dead and the injured, as well as the labor lost by the dead, the disabled and the families who need to occupy work or study time to care for the injured. These losses can account for 3 % of the GDP of most countries. [5] found that 72 % of traffic accidents can be traced back to human errors. With the rapid development of artificial intelligence and automobile technology, Automatic driving vehicles are expected to bear more burden and pressure from human drivers, so as to enhance safety and reduce the workload of drivers. The automatic driving vehicles are the product of
multidisciplinary knowledge and theory, among which environmental recognition system, decision-making system and motion control system are three main components of the software system. Many studies, such as [6], [7], show progress in the overall architecture and feasibility of automatic driving vehicles technology. This paper focuses on the motion control system of automatic driving vehicles.

One of the most important considerations in designing the motion control scheme of automatic driving vehicles is to eliminate the lateral control tracking error while ensuring the vehicle stability. In general, automatic driving vehicles tracking control can be achieved by longitudinal and lateral control based on the current vehicle state and road information. Longitudinal control refers to speed control, which can be attributed to throttle or brake control. Lateral control refers to the control of vehicle steering under different speeds and loads, which makes the vehicle position at the center line of the desired trajectory. This problem can be attributed to the control of the steering wheel. Compared with the longitudinal control, the lateral control is more important. The reason is that the lateral control not only guides the vehicle to move along the desired path, but also directly determines the performance of tracking. It is the basis for ensuring the safety and stability of automatic driving vehicles. The control algorithm is highly required. This paper only studies the lateral control of automatic driving vehicles.

Aiming at the lateral control problem of autonomous vehicles, a new lateral control framework composed of path tracking system and direct yaw moment control system is proposed in [8], and estimated the vehicle sideslip angle by low-cost GPS and INS data fusion. [9] proposed an improved fuzzy dynamic sliding mode lateral control strategy to deal with the uncertainty of autonomous ground vehicle system. [10] proposed a robust lateral motion control system for driving limit of automatic driving vehicles based on adaptive neural network (ANN) to ensure the uniform ultimate boundedness of the closed-loop lateral control system, in which the adaptive neural network (ANN) approximator was used to estimate the uncertainty of tire lateral stiffness. In [11], a nonlinear coordinated steering and braking control system are proposed for the problem of steering and braking coupling control of vision-based automatic driving vehicles in emergency obstacle avoidance. [12] proposed an adaptive cascade nonlinear lateral control method for overactuated autonomous electric vehicles, and designed a hyperbolic tangent function to approximate the saturation function to deal with the input saturation problem. In [13], [14], a lateral model predictive controller (MPC) considering feasible road area and vehicle shape is proposed, in which the model mismatch caused by road condition change and small angle assumption is considered in the form of measurable disturbance. In [15], an alternative control framework integrating local path planning and MPC-based path tracking is proposed. The controller plans trajectories composed of position and velocity states, which follow the best desired path and remain in two secure envelopes.

This paper presents a robust adaptive neural control method for vehicle lateral control system. The main contributions of this paper compared to existing work can be summarized as follows:

1) The vehicle lateral control tracking error not only converges to a prescribed compact set in a finite time that can be predetermined, but also the convergence rate is controllable and can be clearly predetermined in a finite time interval.

2) In the case of non-parametric uncertainty and external disturbance in the vehicle lateral system, the prescribed performance finite-time control algorithm based on Nussbaum gain is proposed in this paper also has the control ability to deal with the uncertainty or even unknown inertia of the system.

3) Furthermore, this paper ensures the bounded input of the neural network through the designed barrier Lyapunov function (BLF), which ensures the stable learning and approximation of the neural network.

2. Dynamic Modeling of Vehicle Lateral System

In the highway scene, the speed is usually maintained at high speed. In this scene, the curvature of most bends is very small, and the front wheel angle is very small. At this time, the sideslip angle of the tire is small, but the influence cannot be ignored, so the kinematics model is not applicable. This paper first establishes the dynamic model.
Based on the monorail model, the body is regarded as a rigid body, and the tire is subjected to the ground reaction force along the $y$-axis of the vehicle coordinate system. As shown in Fig. 1, the lateral forces on the front and rear wheels are $F_{yf}$ and $F_{yr}$, respectively. The distance from the front wheel to the center of mass is $l_f$, the distance from the rear wheel to the center of mass is $l_r$, the velocity deviation angle of the center of mass is $\beta$, and the rotation angle of the front wheel is $\delta_f$. According to Newton's second law, the force balance along $y$-axis and the moment balance along $z$-axis are satisfied in vehicle coordinate system:

$$
\begin{align*}
\mathbf{m}\ddot{v}_y + \dot{\theta}_v \dot{v}_x &= F_{yf} + F_{yr} \\
I_z \ddot{\psi} &= l_f F_{yf} - l_r F_{yr}
\end{align*}
$$

In the formula, $I_z$ is the moment of inertia of the vehicle along the $z$-axis of the vehicle coordinate system, $v_x$ is the longitudinal velocity, $v_y$ is the transverse velocity, and $\psi$ is the heading angle in the global coordinate system. The velocity deflection angles $\theta_{vf}$ and $\theta_{vr}$ of the front and rear wheels are computed as follows:

$$
\begin{align*}
\theta_{vf} &= \arctan\left(\frac{v_y + l_f \dot{\psi}}{v_x}\right) \\
\theta_{vr} &= \arctan\left(\frac{v_y - l_r \dot{\psi}}{v_x}\right)
\end{align*}
$$

Ignoring the tire longitudinal force, in addition, when the cornering angle is small, the tire lateral force is proportional to the cornering angle, and the formula of lateral force can be obtained:

$$
\begin{align*}
F_{yf} &= 2C_f \alpha_f \cos(\delta_f) \\
F_{yr} &= 2C_r \alpha_r
\end{align*}
$$

The relationship between the front and rear wheel side deflection angles $\alpha_f$, $\alpha_r$ and the velocity deflection angle is:

$$
\begin{align*}
\alpha_f &= \delta_f - \theta_{vf} \\
\alpha_r &= -\theta_{vr}
\end{align*}
$$

The system equation in the vehicle coordinate system can be obtained by combining the above equations:

$$
\begin{align*}
\mathbf{m}(\dot{v}_y + \dot{\psi}v_x) &= 2C_f \left[\delta_f - \arctan\left(\frac{v_y + l_f \dot{\psi}}{v_x}\right)\right] \cos(\delta_f) - 2C_r \arctan\left(\frac{v_y - l_r \dot{\psi}}{v_x}\right) \\
I_z \ddot{\psi} &= 2l_f C_f \left[\delta_f - \arctan\left(\frac{v_y + l_f \dot{\psi}}{v_x}\right)\right] \cos(\delta_f) - 2l_r C_r \arctan\left(\frac{v_y - l_r \dot{\psi}}{v_x}\right)
\end{align*}
$$

![Fig. 1 Vehicle lateral force](image)
The state in the vehicle coordinate system is defined as the transverse velocity $v_y$ and the yaw rate $\psi$, then the state differential equation is:

$$\begin{aligned}
\dot{v}_y &= \frac{2C_f}{m} \left[ \delta_f - \arctan \left( \frac{v_y + l_f \psi}{v_x} \right) \right] \cos(\delta_f) - \frac{2C_r}{m} \arctan \left( \frac{v_y - l_r \psi}{v_x} \right) - \psi v_x \\
\dot{\psi} &= \frac{2l_f C_r}{I_z} \left[ \delta_f - \arctan \left( \frac{v_y + l_f \psi}{v_x} \right) \right] \cos(\delta_f) - \frac{2l_r C_r}{I_z} \arctan \left( \frac{v_y - l_r \psi}{v_x} \right)
\end{aligned}$$

Due to the high nonlinearity of the above formula, the linear processing is needed to make the subsequent calculation feasible. In the highway scene, the speed deviation angle and the front wheel angle of the front and rear wheels are small angles, and the system equation becomes:

$$\begin{bmatrix}
\dot{v}_y \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
- \frac{2C_f + 2Cr}{mv_x} & \frac{2C_r l_r + 2Cr l_f}{mv_x} & -v_x \\
- \frac{2l_f C_f - 2l_r Cr}{I_z v_x} & \frac{2l_f^2 C_f + 2l_r^2 Cr}{I_z v_x}
\end{bmatrix}
\begin{bmatrix}
v_y \\
\psi
\end{bmatrix} +
\begin{bmatrix}
\frac{2C_f}{m} \\
\frac{2l_f C_f}{l_z}
\end{bmatrix} \delta_f$$

The relationship between the coordinates $X$, $Y$ in the global coordinate system and the state in the vehicle coordinate system is:

$$\begin{aligned}
\dot{Y} &= v_x \sin(\psi) + v_y \cos(\psi) \\
\dot{X} &= v_x \cos(\psi) - v_y \sin(\psi)
\end{aligned}$$

Partially linearized vehicle lateral system dynamics model can be obtained by simultaneous (10) and (11).

3. Control Objectives and Prior Knowledge

3.1. Control Problem

Consider the vehicle lateral control system described by the following dynamic equation:

$$H(q,p) \dot{q} + N_g(q, \dot{q}, p) \dot{q} + \tau_d(p, t) = u$$

Subject to $\|q\| < k_{c1}$ and $\|\dot{q}\| < k_{c2}$, where $k_{c1}$ and $k_{c2}$ are given positive numbers. In (12), $H(q,p) \in R^{m \times m}$ represents the moment of inertia matrix of the system, $N_g(q, \dot{q}, p) \in R^{m \times m}$ represents the inherent characteristics of the system, $\tau_d(p, t) \in R^m$ represents the interference of the system, $u \in R^m$ is the system control input, $\dot{q}(t)$, $\dot{q}(t)$, $\hat{q}(t) \in R^m$ respectively represents vehicle lateral position, velocity and the derivative vector of velocity, and $p \in R^p$ is the unknown parameter vector. The subsequent theoretical derivation on account of the following assumption: $q(t)$ and $\dot{q}(t)$ are measurable, $H(q,p)$ is uncertain, $N_g(q, \dot{q}, p)$ and $\tau_d(p, t)$ are unknown.

To facilitate theoretical analysis, in the vehicle lateral dynamics equation, let $x_1 = [Y \psi]$, $x_2 = x_1 = [v_y \psi]$, $A = \begin{bmatrix}
- \frac{2C_f + 2Cr}{mv_x} & \frac{2C_r l_r + 2Cr l_f}{mv_x} & -v_x \\
- \frac{2l_f C_f - 2l_r Cr}{I_z v_x} & \frac{2l_f^2 C_f + 2l_r^2 Cr}{I_z v_x}
\end{bmatrix}$, $B = \begin{bmatrix}
\frac{2C_f}{m} \\
\frac{2l_f C_f}{l_z}
\end{bmatrix}$, $u = B \delta_f$, and let $I$ be the unit matrix.

Since there is only zero vector in the zero space of column full rank matrix $B$, $(B^T B)^{-1}$ is an invertible matrix. Then the left inverse of matrix $B$ is $(B^T B)^{-1} B^T = \frac{m^2 I_z}{4 C_f^2 (l_z^2 + m^2 l_z^2)} \begin{bmatrix}
2C_f \\
2l_f C_f
\end{bmatrix}$, and we can get:

$$\delta_f = (B^T B)^{-1} B^T u.$$ 

The vehicle lateral dynamics equation (10) can be simplified to

$$I \dot{x}_2 - A x_2 = u.$$
Define \( q = x_1 \in \mathbb{R}^m, \dot{q} = x_2 \in \mathbb{R}^m \), then (12) can be expressed as
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= F(x_1, x_2, p, t) + G(x_1, p)u
\end{align*}
\]  
where \( G(x_1, p) = H^{-1}(q, p) \) is the control gain matrix, \( F(x_1, x_2, p, t) = -H^{-1}(q, p) \left( N_g(q, q, p) \dot{q} + \tau_d(p, t) \right) \) is the lumped uncertainty. If \( x_1 \) and \( x_2 \) are continuous and bounded, then \( G(\cdot) \) and \( F(\cdot) \) are unknown continuous and bounded.

In this paper, the control objective is to design a robust adaptive neural controller for the vehicle lateral control system, so that the system does not violate the full-state constraint (i.e., for all \( t \geq 0 \), \( \|x_1\| < k_{c1}, \|x_2\| < k_{c2} \)) and all signals are bounded, and the system states \( x_1 \) and \( x_2 \) track the desired reference path \( y_d(t) \) and the virtual control signal \( \alpha_1 \), respectively, so that the specified tracking accuracy can be achieved in the specified convergence rate in a prescribed finite time.

The following assumptions and axioms are introduced to achieve the above control objectives under full-state constraints:

**Assumption 1** [17], [18]: The expected path \( y_d(t) \) and its derivative \( y_d^{(i)}(t)(i = 1, 2, 3) \) are known and bounded, that is, \( y_d(t) \) and \( y_d^{(i)}(t) \) satisfy \( \|y_d(t)\| \leq A_0 < k_{c1} \) and \( \|y_d^{(i)}(t)\| \leq A_1 < k_{c2} \), where \( A_0 \) and \( A_1 \) are positive known constants.

**Lemma 1** [19]: Let \( m \times m \) be a symmetric matrix, \( x \in \mathbb{R}^m \) be a nonzero vector, let \( \rho = (x^T \Gamma x/x^2 \chi), \) then there is at least one eigenvalue \( \Gamma \) in the interior \( (-\infty, \rho) \), and at least one at \( (\rho, \infty) \).

It can be seen from Lemma 1 that for any given nonzero vector \( x \in \mathbb{R}^m \), \( x^T \Gamma x \geq 0 \). By defining \( \overline{a}(t) = (x^T \Gamma x/x^2 \chi), \) there is \( x^T \Gamma x = \overline{a}(t)x^2 \chi \), where \( \overline{a}(t) > 0 \). On the basis of Lemma 1, there are two bounded constants \( \overline{\lambda} \) and \( \overline{\lambda} \) (\( \overline{\lambda} \) and \( \overline{\lambda} \) are positive constants) so that
\[
\overline{\lambda} \leq \lambda_{\text{min}}(t) \leq \overline{a}(t) \leq \lambda_{\text{max}}(t) \leq \overline{\lambda}
\]  
where \( \lambda_{\text{min}}(t) \) and \( \lambda_{\text{max}}(t) \) are the smallest and largest eigenvalue of the symmetric matrix \( G \), respectively. In addition, if \( x = 0 \), then for any nonzero constant \( \theta \in [\overline{\lambda}, \overline{\lambda}] \), \( x^T \Gamma x = \theta x^2 \chi \).

Therefore, it can be concluded that for any given \( x \)
\[
x^T \Gamma x = \alpha(t)x^2 \chi
\]
where \( \alpha(t) = \begin{cases} \overline{a}(t), & \text{if } x \neq 0 \\ \theta, & \text{if } x = 0 \end{cases} \) is a positive bounded function, which is a useful property for the subsequent control design.

In this paper, the Nussbaum gain technique is used to process the unknown function \( \alpha(t) \) to achieve zero-error tracking of vehicle lateral control. If the function \( N(\chi) \) has the following properties [20], it is called Nussbaum function:

\[
\limsup_{\chi \to 0} \int_{\rho_0}^{\chi} N(\chi) \, d\chi = +\infty \\
\liminf_{\chi \to 0} \int_{\rho_0}^{\chi} N(\chi) \, d\chi = -\infty.
\]  

We considered the even Nussbaum-type function \( N(\chi) = e^{\chi^2} \cos(\pi \chi/2) \) in this paper.

**Lemma 2** [20]: Let \( V(t) \) and \( \chi \) be defined as smooth functions on \([0, t_f]\). For any \( t \in [0, t_f]\), \( V(t) \geq 0 \). Let \( N(\chi) = e^{\chi^2} \cos(\pi \chi/2) \) be an even smooth Nussbaum-type function. If the following inequality was established:
\[
V(t) \leq c_0 + \int_{0}^{t} [\alpha(t)N(\chi) + 1] \hat{\chi}e^{-\mu(t)} \, dt
\]
where \( c_0 > 0 \) and \( \mu > 0 \) represent some suitable constants, \( \alpha(t) \) is a time-varying parameter, which takes the value of the unknown closed interval \( A := [\overline{\lambda}, \overline{\lambda}] \) and \( 0 \notin A \). Then \( V(t), \chi, \int_{0}^{t} [\alpha(t)N(\chi) + 1] \hat{\chi} \, dt \) must be bounded on \([0, t_f]\).
Lemma 3 [21]: For any positive constant $k_b$, in the interval $\|x\| < k_b$, for any vector $x \in \mathbb{R}^m$, there are the following inequalities:

$$\log \frac{k_b^2}{k_b^2 - x^T x} \leq \frac{x^T x}{k_b^2 - x^T x}$$

(20)

where $\log(*)$ represents the natural logarithm of $*$. 

3.2. 3.2 The Description of Neural Network

NNs is often used for control and modeling of nonlinear systems due to its powerful function approximation, learning and fault tolerance. Among all neural networks (NN), the radial basis function network (RBFNN) because of its simple and linear parametric structure is widely used. In this paper, an unknown continuous nonlinear function $f(Z) : \mathbb{R}^l \rightarrow \mathbb{R}^{m_1}$ can be approximated by RBFNN on compact set $\mathbb{Z} \in \mathbb{R}^l$ as follows:

$$f(Z) = W^T \phi(Z) + \delta(Z)$$

(21)

where $Z \in \Omega_Z \subset \mathbb{R}^l$ is the input vector, $W \in \mathbb{R}^{p_1 \times m_1}$ is the optimal weight matrix of radial basis function neural network (RBFNN), $p_1$ is the number of neurons, $\delta(Z) \in \mathbb{R}^{m_1}$ is the approximation error, $\phi(Z) = [\phi_1(Z), \ldots, \phi_{p_1}(Z)]^T$ is the known smooth basis function vector, $\phi_i(Z) (i = 1, \ldots, p_1)$ is the commonly used Gaussian function with the form

$$\phi_i(Z) = \exp\left[-(Z - \zeta_i)^T(Z - \zeta_i)/\nu_i^2\right]$$

(22)

where $\zeta_i = [\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{il}]^T$ denote the center of the Gaussian basis function and $\nu_i$ denote the width of the Gaussian basis function.

The optimal weight matrix $W$ is a symbol, which is only used for analysis purposes. In general, this ideal weight matrix is an unknown constant matrix. Increasing the number of adjustable weights can reduce the approximation error $\delta(Z)$ of the neural network. The extensive practical application of neural network (NN) shows that if the number of nodes $p_1$ of the neural network (NN) is chosen sufficiently large, then $\|\delta(Z)\|$ can be reduced to an arbitrarily small compact set.

Remark 1: $\phi_M = \max_{Z \in \Omega_Z} \|\phi(Z)\|$. According to the definitions of $\phi(Z)$ and $\delta(Z)$, it is easy to observe that there exist positive numbers $\phi_M$ and $\delta_M$ on compact set $Z$ such that $\|\phi\| \leq \phi_M$ and $\|\delta\| \leq \delta_M$. In addition, let $W_M = \|W\|$, $W_M$ be an unknown positive number.

4. Controller Design

The following speed function is introduced and plays an important role in the development of this method, as described in 4.2.

4.1. Speed Function

Defines a rate function, its form is

$$\kappa(t) = \begin{cases} \left(\frac{T}{T - t}\right)^4 \kappa(t), & 0 \leq t < T \\ \infty, & t \geq T \end{cases}$$

(23)

where for a given or artificially prescribed finite time, $0 < T < \infty$, $\kappa(t)$ is any predominate the diminishing $C^{\infty}$ smooth time function satisfying $\kappa(0) = 1$ and $\dot{\kappa} \geq 0$, that is, $\kappa = 1, 1 + t^2, e^t, 4^t(1 + t^2)$. It should be emphasized that when $t \geq T$, there is $\kappa(t) = C^{\infty}$. To ensure continuity
at \( \overline{\kappa} \) defined in (23), define
\[
\kappa = \lim_{t \to T} (T/(T - t))^4 \kappa(t).
\]
In addition, the following properties can be easily validated from (23).

1) The rate function \( \overline{\kappa}(t) \) is positive in \([0, T)\) and also strictly increasing in \([0, T)\), so \( \overline{\kappa}(t)^{-1} \) is strictly decreasing in \([0, T)\) and \( \overline{\kappa}(0) = 1 \).

2) For all \( t \in [T, \infty) \), \( \overline{\kappa}(t) = \infty \), \( \overline{\kappa}^{-1}(t) = 0 \).

3) \( \kappa(t) \in C^\infty \) is a function of an infinite smooth. For \( t \in [0, T) \), its derivative with respect to the \( n \)th \( (n \to \infty) \) is clearly defined and bounded.

Assumption 2: Set \( T_c > 0 \) for signal processing or computation and transmission for small time interval. For a given finite time \( T \), there is \( T \geq T_c \).

On account of the \( \overline{\kappa}(t) \) defined in (23), the following speed functions can be constructed:

\[
\beta(t) = \begin{cases} 
\frac{1}{(1 - b_f)(\overline{\kappa}(t)^{-1} + b_f)} & 0 
\leq t \leq T \\
\frac{T^4 \kappa(t)}{(1 - b_f)(T - t)^4 + b_f T^4 \kappa(t)} & T 
\geq t \\
\frac{1}{b_f} & \end{cases}
\]  

As Theorem 1 shows, the properties related to speed function \( \beta(t) \) are useful for future control development.

Theorem 1: Set the speed function \( \beta(t) \) by (24) or equivalent (25) to constructe, and define \( \delta = \beta^{-1} \dot{\beta} \). Then the following attributes are established.

\( P_1 \): \( \beta(t) \) is continuously differentiable for all \( t \geq 0 \) and \( \dot{\beta} \geq 0 \) is continuous and bounded everywhere.

\( P_2 \): \( \beta(t) \) is strictly increases and positive for \( t \in [0, T) \) and \( \beta(0) = 1 \) and \( \beta(t) = 1/b_f \) for \( t \geq T \), and \( \beta \in [1, 1/b_f] \) for \( t \in [0, \infty) \).

\( P_3 \): \( \dot{\beta} \) and \( \ddot{\beta} \) are continuously differentiable and bounded for all cases where \( t \geq 0 \). \( \beta^{(3)} \) in any place is continuous and bounded.

\( P_4 \): \( \delta \) and \( \ddot{\delta} = (d/dt)(\beta^{-1} \dot{\beta}) \) are continuously differentiable and bounded, and \( \ddot{\delta} \) in any place is continuous and bounded.

4.2. Controller Design

To realize the goal of the proposed control, we define the \( z \)-coordinate transformation and error transformation as

\[
z_i = x_i - \alpha_{i-1}
\]

\[
\zeta_i = \beta z_i
\]  

for \( i = 1, 2 \), where \( \alpha_0 = y_d \), \( \alpha_1 \) is the control input for the virtual system to be given later, \( \zeta_i(0) = z_i(0) (i = 1, 2) \).

Due to the particularity of \( X \), if the controller can be designed as the vehicle lateral control error \( Y \) after stable transformation, the improved control performance of the actual vehicle lateral control tracking error \( Z \) can be obtained from (27), as shown below.

Due to the particularity of \( \beta(t) \), if the controller can be designed as the vehicle lateral control error \( \zeta_i \) after stable transformation, then an improved control performance of the actual vehicle lateral control tracking error \( z_i \) can be obtained from (27), see below. Consequently, this paper adopts the following two steps based on backstepping technology to construct and analyze the stabilizer of \( \zeta_i \).

Step 1: The \( \zeta_1 \) represents the transformation of the closed loop dynamics is
\[ \dot{\xi}_1 = \beta z_1 + \beta (x_2 - y_d) = \beta (\delta z_1 + x_2 - y_d) \]  
\( \text{where} \quad \delta = \beta^{-1} \hat{\beta}. \) 
Since \( x_2 = z_2 + \alpha_1, \) (28) can be rewritten as
\[ \dot{\xi}_1 = \beta (\delta z_1 + z_2 + \alpha_1 - y_d). \]  

To make sure that the constraint conditions of \( x_1 \) is not violated, on the basis of the properties of the barrier Lyapunov function (BLF) ([22], [23]), the Lyapunov candidate function is constructed as
\[ V_1 = (1/2) \log (k_{b1}^2/(k_{b1}^2 - \xi_1^T \xi_1)). \] 
Define the compact set \( \Omega_{\xi_1} := \{ \xi_1: \| \xi_1 \| < k_{b1} \} \), \( V_1 \) is valid in the set \( \Omega_{\xi_1} \). To satisfy the requirement that \( \| x_1 \| < k_{c1}, k_{b1} \) must be selected in a special way, such that
\[ k_{b1} = k_{c1} - A_0. \]  
\( \text{This is because} \quad \beta \geq 1 \) and \( \xi_1 = \beta z_1, \) \( \| x_1 \| - \| y_d \| \leq \| z_1 \| \leq \| \xi_1 \| < k_{b1} \) in set \( \Omega_{\xi_1} \). Note that \( \| y_d \| \leq A_0, \) from \( z_1 = x_1 - y_d, \) it can be known that \( \| x_1 \| \leq \| x_1 \| + \| y_d \| < k_{b1} + A_0 = k_{c1} - A_0 + A_0 = k_{c1} \), that is, by selecting \( k_{b1} = k_{c1} - A_0, \) the constraint on \( x_1 \) is ensured. Therefore, the key is to ensure \( \| \xi_1 \| < k_{b1}. \) According to the properties of obstacle Lyapunov function (BLF), \( V_1 \) can be bounded. The derivative of \( V_1 \) along (29) is
\[ \dot{V}_1 = -\frac{\xi_1^T \beta}{k_{b1}^2 - \xi_1^T \xi_1} (\delta z_1 + z_2 + \alpha_1 - y_d). \]  
Select virtual system control input \( \alpha_1 \) as
\[ \alpha_1 = -\delta z_1 - c_1 \dot{z}_1 + \dot{y}_d \]  
where \( c_1 > (1/2) \) is the design parameter, (31) is transformed into
\[ \dot{V}_1 = -\frac{c_1 \xi_2^T}{k_{b1}^2 - \xi_1^T \xi_1} + \frac{\xi_2^T \xi_2}{k_{b1}^2 - \xi_1^T \xi_1}. \]  
Let \( \bar{\alpha}_1 \geq \sup_{\| x \| < k_{b1}, \| y_d \| \leq A_1} \max \| \alpha_1 (\delta, z_1, \dot{y}_d) \|. \) \( \delta \) is non-negatively bounded, and then by (32), in the set \( \Omega_{\xi_1} \), there is \( \max \max \| \alpha_1 \| < (\delta + c_1) k_{b1} + A_1. \) Denote that
\[ \bar{\alpha}_1 = (\max \delta) + c_1 k_{b1} + A_1. \]  
It should be noted that since \( \delta \) is artificially designed, the maximum value of \( \delta \) can be computed, so that \( \bar{\alpha}_1 \) can be easily calculated.

Step 2: For \( i = 2, \) the derivative of (27) with respect to time is
\[ \dot{\xi}_2 = \beta (\delta z_2 + Gu + F - \dot{\alpha}_1). \]  
The barrier Lyapunov candidate function should then be given as follows in order to make sure that the system state \( x_2 \) is not violated:
\[ V_2 = V_1 + \frac{1}{2} \log \frac{k_{b2}^2}{k_{b2}^2 - \xi_2^T \xi_2}. \]  
Define the compact set \( \Omega_{\xi_2} := \{ \xi_2: \| \xi_2 \| < k_{b2} \} \) and \( (1/2) \log (k_{b2}^2/(k_{b2}^2 - \xi_2^T \xi_2)) \) to be valid in the set \( \Omega_{\xi_2} \). In the same way, to satisfy the condition \( \| x_2 \| < k_{c2}, \) \( k_{b2} \) is selected as
\[ k_{b2} = k_{c2} - \bar{\alpha}_1. \]  
This is because \( \| x_2 \| - \| \alpha_1 \| \leq \| z_2 \| \leq \| \xi_2 \| < k_{b2} \) in the set \( \Omega_{\xi_2} \) can be obtained from (26) and (27), i.e. \( \| x_2 \| < k_{b2} + \| \alpha_1 \| \leq k_{b2} + \bar{\alpha}_1 = k_{c2}. \) On this basis, this paper focuses on the vehicle lateral control tracking scheme which ensures the uniform ultimate boundedness of \( V_1 \) and \( V_2 \), and proves that the scheme not only satisfies the constraints of \( x_1 \) and \( x_2 \), but also achieves the other requirements mentioned in Theorem 2.

Then, along (35), the derivative of \( V_2 \) with respect to time is
\[ \dot{V}_2 = -\frac{c_1 \xi_2^T}{k_{b2}^2 - \xi_1^T \xi_1} + \frac{\beta \xi_2^T}{k_{b2}^2 - \xi_2^T \xi_2} Gu + \frac{\beta \xi_2^T}{k_{b2}^2 - \xi_2^T \xi_2} F (\cdot) \]  
(38)
where \( \left( \frac{\zeta^T \zeta}{k_b^2} - \frac{\zeta^T \zeta_1}{k_b^2} \right) \) is used,
\[
F(\cdot) = \delta z_2 + F(\cdot) - \alpha_1 + \frac{\left( k_b^2 - \zeta^T \zeta_2 \right) z_1}{k_b^2 - \zeta^T \zeta_2} \tag{39}
\]
is concentrated uncertainty. Since \( \alpha_1 \) is a function of variables \( \delta, y_d, y_d \) and \( x_1 \), it is obvious that \( \delta \) is a function of variables \( \delta, x_2, y_d \) and \( y_d \). Moreover, \( F(\cdot) \) is a function of states \( x_1 \) and \( x_2 \). Then by (26)-(27) and (39), it is determined that \( F(\cdot) \) is a function of \( \beta, x_1, x_2, y_d, y_d, \delta \) and \( \delta \), and then denoted \( Z = [\beta, x_1^T, x_2^T, y_d^T, y_d^T, \delta, \delta]^T \in R^{5m+3} \).

Due to the unknown function \( \phi(Z) \) defined in (21) is used to approximate the lumped uncertainty \( F(Z) \) on the compact set \( \Omega_F \subseteq R^{5m+3} \), i.e.
\[
F(Z) = W^T \phi(Z) + \delta_Z(Z). \tag{40}
\]

Then (38) can be written as
\[
\dot{V}_2 = -c_1 \zeta_1^T \zeta_1 + \frac{\beta \zeta_2^T}{k_b^2 - \zeta^T \zeta_2} Gu + \frac{\beta \zeta_2^T}{k_b^2 - \zeta^T \zeta_2} W^T \phi(Z) + \frac{\beta \zeta_2^T}{k_b^2 - \zeta^T \zeta_2} \delta Z(\cdot) \tag{41}
\]
By using Young inequality and on the basis of Remark 1, there is
\[
\frac{\beta \zeta_2^T}{k_b^2 - \zeta^T \zeta_2} W^T \phi \leq \frac{\beta^2 \zeta_2^T \zeta_2}{k_b^2 - \zeta^T \zeta_2} W_M^2 \phi^T \phi + \frac{1}{4} \tag{42}
\]
resulting in
\[
\frac{\beta \zeta_2^T}{k_b^2 - \zeta^T \zeta_2} (W^T \phi + \delta_2) \leq \frac{\beta^2 \zeta_2^T \zeta_2}{k_b^2 - \zeta^T \zeta_2} a \phi + \frac{1}{2} \tag{44}
\]
where
\[
a = \max\{W_M^2, \delta_2^M\} \tag{45}
\]
the virtual constant parameter of \( a \) is unknown,
\[
\phi = \phi^T(Z) \phi(Z) + 1 \tag{46}
\]
the computable function of \( \phi \) is a control design. Therefore, (41) can be further expressed as
\[
\dot{V}_2 \leq -c_1 \zeta_1^T \zeta_1 + \frac{\beta \zeta_2^T}{k_b^2 - \zeta^T \zeta_2} Gu + \frac{\beta^2 \zeta_2^T \zeta_2}{k_b^2 - \zeta^T \zeta_2} a \phi + \frac{1}{2} \tag{47}
\]
It is necessary to illustrate that in the conventional adaptive mechanism for NNs, in the development of the control algorithms, the idealized weight matrix \( W \) need an adaptive law to estimate. As the number of neural network (NN) nodes and the dimension of approximation function increase, a large number of calculations are required. To mitigate this problem, in this paper we construct a simplified adaptation mechanism. Due to the unknown function \( F(\cdot) \) on compact set can use radial basis function neural network (RBFNN) (40) to be approximated, using Young inequality (42)-(44), the unknown constant matrix \( W \) is converted to the unknown scalar dummy parameter \( a \) defined in (45), and in the later controller design, we only use the estimation of the dummy parameter \( a \) and the fundamental function vector \( \phi(Z) \) is used, rather than the estimation of the ideal constant matrix \( W \), which greatly reduces the amount of calculation.

By using the speed function \( \beta(t) \) and the Nussbaum-type function \( N(\chi) \) defined in (24), we construct the following robust adaptive neural control scheme without using any analysis information of unknown control gain matrix \( G(\cdot) \) and uncertain \( F(\cdot) \):
\[
u = N(\chi)(c_2 + \hat{a}\phi) \frac{\beta^2 z_2}{k_{b2}^2 - \zeta_2^2} \tag{48}
\]
\[
\dot{x} = y_x \beta^4(c_2 + \hat{a}\phi) \frac{z_2^2 z_2}{(k_{b2}^2 - \zeta_2^2 z_2^2)} \tag{49}
\]
\[
\dot{a} = y_x \frac{\phi \beta^4 z_2 z_2}{k_{b2}^2 - \zeta_2^2 z_2^2} \alpha \hat{a}, \quad \hat{a}(0) \geq 0 \tag{50}
\]

where \( \hat{a} \) is the estimated value of the virtual unknown parameter \( a \), \( \hat{a}(0) \geq 0 \) is the initial value selected arbitrarily, \( c_2 = c_{21}(\zeta_2^2 - \zeta_2^T z_2) \) is the positive function in the set \( \Omega_{\zeta_2} \), \( c_{21} > 0 \) is the design parameter, \( y_x > 0 \), \( y > 0 \), and \( \sigma > 0 \) are the control parameters selected artificially. It should be noted that when \( \hat{a}(0) \geq 0 \), \( y \left( \phi \beta^4 z_2^2/(k_{b2}^2 - \zeta_2^T z_2)^2 \right) \geq 0 \), then it is considered that when \( t \in [0, \infty) \), \( \hat{a}(t) \geq 0 \). We now intend to give the following results for the robust adaptive neuro-control scheme of the vehicle lateral control system described in (12):

Theorem 2: Considering the vehicle lateral control system model (12) is affected by full-state constraints and external disturbances. Assume that Assumptions 1 and Assumptions 2 hold. On the set \( \Omega_{\zeta_1} \) and \( \Omega_{\zeta_2} \), that is, when the initial conditions satisfy \( \zeta_1(0) = z_1(0) \in \Omega_{\zeta_1} \) and \( \zeta_2(0) = z_2(0) \in \Omega_{\zeta_2} \), the actual controllers with adaptive laws (49) and (50) in (48) are constructed. If \( k_{b1} \) and \( k_{b2} \) are selected as (30) and (37), respectively, the following control objective 1)-3) can be achieved.

1) All signals in the closed-loop system are bounded and do not violate the full-state constraint, that is, \( \forall t \geq 0, \|x_1(t)\| < k_{c1}, \|x_2(t)\| < k_{c2} \), and the control action is \( C^1 \) smooth.

2) Achieving a prescribed tracking performance, such as a prescribed compact set of the vehicle lateral control tracking errors converging to near zero during transients \( [0, T] \) at a specifiable decay rate in a limited time \( T \).

3) The zero-error tracking of vehicle lateral control is realized, that is, when \( t \to \infty, x_1 \to y_d(t), x_2 \to \dot{y}_d(t) \).

Proof: the complete Lyapunov candidate function is selected as \( V = V_2 + (1/2y)\hat{a}^2 \), where \( V_2 \) is defined in (36), and \( \hat{a} = \alpha - \hat{a} \) is the virtual parameter estimation error. The time derivative of \( V \) is

\[
\dot{V} \leq -c_{11}^T \zeta_1 + \beta \zeta_2^2 \frac{\alpha N(\chi)}{k_{b2}^2 - \zeta_2^2} G \dot{u} + \beta \frac{\alpha^2 \tilde{z}_2^2}{(k_{b2}^2 - \zeta_2^2 z_2^2)} \alpha \hat{a} + \frac{1}{2} - \frac{1}{y} \hat{a} \tag{51}
\]

The actual control law (48) is substituted into (51), we can obtain

\[
\dot{V} \leq -c_{11}^T \zeta_1 + \frac{\beta^2 (c_2 + \hat{a}\phi)}{k_{b2}^2 - \zeta_2^2} \frac{\beta \zeta_2^T z_2^2}{(k_{b2}^2 - \zeta_2^2 z_2^2)} \alpha \hat{a} + \frac{1}{2} - \frac{1}{y} \hat{a} \tag{52}
\]

Since \( G \) is symmetric and positive definite, it is easy to obtain \( \zeta_2^2 G \zeta_2 = \alpha(t) \zeta_2^2 \) according to (17), where \( \alpha(t) \) is a time-varying function of bounded scales. Therefore, we can rewrite (52) as

\[
\dot{V} \leq -c_{11}^T \zeta_1 + \frac{\beta^2 (c_2 + \hat{a}\phi)}{k_{b2}^2 - \zeta_2^2} \frac{\beta \zeta_2^T z_2^2 + \alpha \phi \beta^4 \tilde{z}_2^2}{(k_{b2}^2 - \zeta_2^2 z_2^2)} \alpha \hat{a} + \frac{1}{2} - \frac{1}{y} \hat{a} \tag{53}
\]

By adding and subtracting \( \left( \beta^2(c_2 + \hat{a}\phi)\zeta_2^T z_2^2/(k_{b2}^2 - \zeta_2^2 z_2^2) \right) \) on the right side of (53), and note that \( \dot{x} = y_x \left( \left( \beta^2(c_2 + \hat{a}\phi)\zeta_2^T z_2^2/(k_{b2}^2 - \zeta_2^2 z_2^2) \right) \right) \) on the right side of (53), and note that \( \hat{a} = \alpha - \hat{a} \) is the virtual parameter estimation error. The time derivative of \( V \) is

\[
\dot{V} \leq -c_{11}^T \zeta_1 + c_{21} \frac{\beta \zeta_2^T z_2^2}{(k_{b2}^2 - \zeta_2^2 z_2^2)} + \frac{1}{y_x} [\alpha N(\chi) + 1] \dot{x} + \frac{\alpha \phi \beta^4 \tilde{z}_2^2}{(k_{b2}^2 - \zeta_2^2 z_2^2)} \alpha \hat{a} + \frac{1}{2} - \frac{1}{y} \hat{a} \tag{54}
\]

where \( c_2 = c_{21}(k_{b2}^2 - \zeta_2^2 z_2^2), \beta^2 \geq 1 \). Insert the adaptive law of \( \dot{a} \) in (50) into (54) and we have

\[
\dot{V} \leq -c_{11}^T \zeta_1 + c_{21} \frac{\beta \zeta_2^T z_2^2}{(k_{b2}^2 - \zeta_2^2 z_2^2)} + \frac{1}{y_x} [\alpha N(\chi) + 1] \dot{x} - \frac{\sigma}{2y} \alpha \hat{a}^2 + \frac{\sigma}{2y} a^2 + \frac{1}{2} \tag{55}
\]
By using Lemma 3, there is \(-c_1\zeta_1^T\zeta_1/(k_{b1} - \zeta_1^T\zeta_1)\leq -c_1\log\left(k_{b1}^2/(k_{b1} - \zeta_1^T\zeta_1)\right), -c_2\zeta_2^T\zeta_2/(k_{b2}^2 - \zeta_2^T\zeta_2)\leq -c_2\log\left(k_{b2}^2/(k_{b2} - \zeta_2^T\zeta_2)\right).\) Therefore, (55) can be rewritten as

\[
\dot{V} \leq -c_1\log\frac{k_{b1}^2}{k_{b1} - \zeta_1^T\zeta_1} - c_2\log\frac{k_{b2}^2}{k_{b2} - \zeta_2^T\zeta_2} - \frac{\sigma}{2\gamma} \tilde{a}^2 + \frac{1}{\gamma_x} [\alpha N(\chi) + 1] \dot{\chi} + \frac{\sigma}{2\gamma} a^2 + \frac{1}{2} \leq -V + \frac{1}{\gamma_x} [\alpha N(\chi) + 1] \dot{\chi} + C
\]  

(56)

where \(C = \min\{2c_1, 2c_2, \sigma\}\) and \(C = (\sigma/2\gamma)a^2 + (1/2).\) By integrating differential inequality (56) on \([0, t]\), we have

\[
V(t) \leq V(0) + \frac{1}{\gamma_x} \int_0^t [\alpha(x)N(\chi) + 1] \dot{\chi} e^{-(t-\tau)} d\tau
\]  

(57)

From which, the following important results are obtained. First, it has been proved that objection 1 is achieved.

1) Variables \(z_1, z_2, \zeta_1, \zeta_2, x_1, x_2, \dot{\alpha}, \dot{\lambda}, \dot{\chi}, \dot{\theta}\) and \(u\) are bounded. Lemma 2 is applied to (57) to ensure that \(V(t) = \int_0^t [\alpha(x)N(\chi) + 1] \dot{\chi} d\tau, \chi\) is bounded in \([0, t]\), then \(V_1 \in L_{\infty}, V_2 \in L_{\infty}, \dot{\alpha} \in L_{\infty}, \) which means that only when the initial values \(z_1(0) and \zeta_2(0)\) are in the set \(\Omega_{z_1} and \Omega_{\zeta_2}\), \(\zeta_1\) and \(\zeta_2\) are maintained in the set \(\Omega_{\zeta_1}\) and \(\Omega_{\zeta_2}\), respectively. \(\zeta_1 = \beta \zeta_2(i = 1, 2) and \beta\) are bounded functions, so \(z_1\) and \(z_2\) are bounded. Since \(\delta = \beta^{-1}\beta \in L_{\infty}, (32)\) can ensure \(\alpha_1 \in L_{\infty}.\) From (26), when \(y_d\) and \(\dot{y}_d\) are bounded, we have \(x_1 \in L_{\infty}(i = 1, 2),\) which means that \(F(\cdot) and G(\cdot)\) are bounded. Using the argument similar to that in [21], it is concluded that the above conclusion is also valid for \(t = +\infty.\) In addition, as \(\beta\) and \(\beta\) are bounded, \(\zeta_1 \in L_{\infty}.\) It can be seen from Remark 1 that \(\phi(Z) \in L_{\infty}\) in compact set \(\Omega_Z.\) When \(\chi \in L_{\infty}, N(\chi) = e^{x^T \cos(\pi \chi/2)} \in L_{\infty}.\) It is ensured by (48)-(50) that \(u \in L_{\infty}, \dot{\alpha} \in L_{\infty}, \dot{\chi} \in L_{\infty},\) which further implies that \(\dot{x}_2 \in L_{\infty}.\)

2) It turns out that the constraints on the system states \(x_1\) and \(x_2\) are not violated. Notice that \(z_1 = \beta^{-1}z_1\) and \(\zeta_1\) are always maintained in the set \(\Omega_{z_1}\) and \(\Omega_{\zeta_2}\), respectively, then \(\|x_i\| = \beta^{-1}\|\zeta_i\| \leq \|\zeta_i\| < k_{b1}(i = 1, 2) and 0 < \beta^{-1} \leq 1.\) Since \(x_1 = x_1 + y_d, \|y_d\| \leq A_0,\) it can be obtained that \(\|x_1\| \leq \|x_1\| + \|y_d\| < k_{b1} + A_0.\) Since \(k_{b2} = k_{c1} - A_0,\) there is \(\|x_2\| < k_{c1},\) namely \(\|x_1\| < k_{c1}(i = 1, \ldots, m).\) In addition, it is denoted that \(x_2 = x_2 = \alpha_1, \|z_2\| < k_{b2}, \|\alpha_1\| \leq \alpha_1,\) then \(\|x_2\| \leq \|z_2\| + \|\alpha_1\| < k_{b2} + \alpha_1,\) can further represent \(\|x_2\| < k_{c_2}(\|x_1\| < k_{c_2}) as k_{b2} = k_{c_2} - \alpha_1.\) Overall, the constraints on system states \(x_1\) and \(x_2\) are not violated.

3) It is further proved that \(\dot{z}_1 = \dot{\zeta}_1, \dot{z}_2, \dot{\zeta}_2\) and \(\dot{u}\) are continuous and bounded. Note that \(\dot{z}_1 = x_2 - \dot{y}_d,\) then \(\dot{z}_1\) is continuous and bounded due to \(x_2\) and \(\dot{y}_d,\) further illustrating \(\dot{z}_1 \in L_{\infty}.\) According to (32), there is \(\dot{z}_2 = \beta \dot{z}_2 - \zeta_2, \dot{z}_2\) is continuous and bounded due to \(\dot{z}_2\) and \(\dot{y}_d,\) further illustrating \(\dot{z}_2 \in L_{\infty}.\) Moreover, since the system control input \(u\) is a function of variables \(\chi, \dot{\alpha}, \dot{\beta}, \phi\) and \(z_2,\) then we compute from (48) that

\[
\dot{u} = \frac{\partial u}{\partial N(\chi)} \frac{\partial N(\chi)}{\partial \dot{\chi}} \dot{\chi} + \frac{\partial u}{\partial \dot{\alpha}} \dot{\alpha} + \frac{\partial u}{\partial \beta} \beta + \frac{\partial u}{\partial \phi} \phi + \frac{\partial u}{\partial z_2} \dot{z}_2
\]

with \((\partial u/\partial N(\chi_1))(\partial N(\chi_1)/\partial \dot{X}_1) = (2x_1 \cos((\pi x_1/2)) - (\pi/2) \sin((\pi x_1/2))) e^{x_1^T(c_2)}\) and \((\partial u/\partial \alpha) = (N(\chi) \beta^2 \beta a^2 z_2 - \zeta_2^T \zeta_2), (\partial u/\partial \beta) = (N(\chi) \beta^2 \beta a^2 z_2), (\partial u/\partial \phi) = (N(\chi) \beta^2 \beta a^2 z_2 + N(\chi) \phi), (\partial u/\partial \dot{z}_2) = (N(\chi) \beta^2 \beta a^2), (\partial u/\partial \dot{z}_2) = (N(\chi) \beta^2 \beta a^2) , (\partial u/\partial \dot{z}_2) = (N(\chi) \beta^2 \beta a^2) .\) We can see from the definitions of \(\phi(Z)\) and \(Z\) that \((d\phi/dZ)\) and \(\dot{Z}\) are bounded. \((\dot{z}_2^T z_2/(k_{b2}^2 - \beta^2 z_2^T z_2))\)
\( \beta^2 z_1^2 \) is the function of variables \( \beta \) and \( z_2 \). Since \( \beta \) and \( z_2 \) are bounded, \( (d/dz_2)(\beta^2 z_2 / (k_b^2 - \beta^2 z_2^2 z_2)) \) is bounded. Since all signals, including \( \xi_i(i = 1, 2) \), are bounded, \( \tilde{z}_1, \tilde{z}_2, z_1, x_2, x, \dot{x}, \tilde{a}, \tilde{a}, \beta, \dot{\beta} \) and \( \Phi \) are bounded continuous, obviously \( \dot{u} \) is bounded continuous, i.e., \( u \) is \( C^1 \) smooth.

Next, it is proved that the vehicle lateral control full-state tracking with good transient and steady-state behaviors is obtained.

Since \( z_i = \beta^{-1} \xi_i \) and \( \|\xi_i\| < k_{bi} (i = 1, 2) \) are in the set \( \Omega_{\xi_i} \), then

\[
\|z_i(t)\| = \beta^{-1} \|\xi_i\| < \beta^{-1} k_{bi}
\]

Then from the definition of \( \beta \) in (24), we have

\[
\|z_i(t)\| \leq \left(1 - b_f \right) \left(\frac{T - t}{T}\right)^4 k_{bi} + b_f k_{bi}, \quad 0 \leq t < T
\]

\[
\|z_i(t)\| \leq b_f k_{bi}, \quad t \geq T
\]

which implies that the vehicle lateral control tracking error converges to a prescribed compact set \( \Omega = \{z_i; \|z_i(t)\| \leq b_f k_{bi}\} \) at the rate of decay not less than \((T - t) / T^4 \) in a limited time \( T \).

As seen in (59) and (60), three factors, the adjustment time \( T \), the rate function \( \kappa \), and the design parameter \( b_f \), affect the vehicle lateral control tracking performance, especially for the transient process. Due to \( b_f \) being a free design parameter, it can be chosen arbitrarily as small as possible for \( b_f k_{bi} \) to be as small as possible, so that the vehicle lateral control tracking error converges to a compact set \( \Omega := \{z_i; \|z_i(t)\| \leq b_f k_{bi}\} \). It can be seen from (59) that the transient process of the vehicle lateral control tracking error can be adjusted by choosing to adjust the time \( T \) and the rate function \( \kappa \) in a limited time interval \([0, T]\). In addition, the overshoot of vehicle lateral control tracking error can also be adjusted by selecting the appropriate adjustment time \( T \) and the appropriate rate function \( \kappa \).

At last, it is proved that when \( t \rightarrow \infty \), the full-state tracking error of vehicle lateral control converges to zero, that is, when \( t \rightarrow \infty \), \( x_1 \rightarrow y_d \), \( x_2 \rightarrow \dot{y}_d \).

Note that \( c_2 = c_{21}(k_b^2 - \xi_2^2 \xi_2) \) and \( \tilde{a}(t) \geq 0 \), then it can be known from (49) that

\[
\left(\frac{c_{21} \xi_2^2}{k_b^2 - \xi_2^2 \xi_2}\right) \leq \dot{x}, \quad \text{where} \quad \beta^2 \geq 1.
\]

From the above analysis, in the set \( \Omega_{\xi_2} \), \( 0 < \rho_2n < k_b^2 - \xi_2^2 \xi_2 \leq k_b^2 \), and \( \rho_2n \) is an unknown positive number so that

\[
\frac{(1/k_b^2) \leq (1/(k_b^2 - \xi_2^2 \xi_2)) < (1/\rho_2n),
\]

we further have

\[
\frac{\gamma_{\chi} c_{21} \xi_2^2}{k_b^2} \leq \dot{x}.
\]

Integrating (61) over \([0, t]\), we have

\[
\gamma_{\chi} c_{21} \int_0^t \xi_2^2 \tau d\tau \leq \chi(t) - \chi(0).
\]

Since \( \chi(t) \) is bounded, \( \xi_2 \in L_2 \cap L_{\infty}, \tilde{\xi}_2 \in L_{\infty} \). Then, Barbalat lemma is used to ensure that when \( \xi_2 \rightarrow 0 \) as \( t \rightarrow \infty \). Noted that \( z_2 = \beta^{-1} \xi_2 \) and \( \beta^{-1} > 0 \) are positive bounded functions, then \( z_2 \rightarrow 0 \), that is, \( x_2 \rightarrow \alpha_3 \) as \( t \rightarrow \infty \).

In addition, in the set \( \Omega_{\xi_1} \), \( \xi_1 \xi_2 / (k_b^2 - \xi_1^2 \xi_1) \leq (\xi_1^2 / 2(k_b^1 - \xi_1^2 \xi_1)) + (\xi_1^2 / 2(k_b^1 - \xi_1^2 \xi_1)) \), then (33) can be expressed as

\[
\dot{V}_1 \leq -\left(c_1 - (1/2)\right) \frac{\xi_1^2 \xi_1}{k_b^1} + \frac{\xi_1^2 \xi_1}{2 \rho_1 n},
\]

where \( \rho_1 n \) is an unknown positive number, then

\[
\dot{V}_1 \leq -\left(c_1 - (1/2)\right) \frac{\xi_1^2 \xi_1}{k_b^1} + \frac{\xi_1^2 \xi_1}{2 \rho_1 n}.
\]

Choosing \( c_1 - (1/2) > 0 \), and integrating (62) over \([0, t]\) to have

\[
V_1(t) - V_1(0) \leq \left(-c_1 - (1/2)/k_b^1\right) \int_0^t \xi_1^2 \xi_1 d\tau + (1/2 \rho_1 n) \int_0^t \xi_1^2 \xi_1 d\tau,
\]

from this we have

\[
V_1(t) + (c_1 - (1/2)/k_b^1) \leq V_1(0) + (1/2 \rho_1 n) \int_0^t \xi_1^2 \xi_1 d\tau
\]
\[ k_{b1}^2 \int_0^t \zeta_1^T \zeta_1 \, dt \leq V_1(0) + (1/2 \rho_{in}) \int_0^t \zeta_2^T \zeta_2 \, dt, \] thus we can have \( \zeta_1 \in L_2 \cap L_{\infty}, \zeta_1 \in L_{\infty}, \) and we can get the conclusion of \( \lim_{t \to \infty} \zeta_1 \to 0 \) by using Barbalat lemma, which implies that \( z_1 \to 0, \) that is, \( \lim x_1 \to y_d. \) Then from (32), it is seen that \( \lim \alpha_i \to \dot{y}_d, \) which shows that \( x_2 \to \dot{y}_d \) as \( t \to \infty. \) The proof is completed.

To make it easier for readers to understand the vehicle lateral control system control strategy, this paper proposes the following framework.

1) The speed function \( \beta \) is constructed by selecting the settling time \( T, \) the rate function \( \kappa \) and the design parameter \( b_f. \)

2) As defined in (26) and (27) are introduced for the z-coordinate transformation and the error transformation.

3) The constant \( k_{bi} \) is computed by using (30) and (37), and the initial condition \( x_i(0) \) is extracted, which makes the initial values of the vehicle lateral control tracking error satisfies the unequal equation \( \| z_i(0) = x_i(0) - \alpha_{i-1}(0) \| < k_{bi}, \) \( i = 1, 2. \)

4) Selecting the design parameters \( c_1, c_{21}, \sigma, \gamma, \gamma_x \) and picking the initial value \( \hat{a}(0), \chi(0). \)

5) Choosing the Gaussian function \( \phi_i(i = 1, 2, \ldots, p) \) as shown in (22) and calculating the core-information function \( \Phi \) defined in (46).

6) The parameter \( \chi \) of the Nussbaum gain and the adaptive parameter estimated value \( \hat{a} \) defined in (49) and (50) respectively are calculated.

7) The virtual controller \( \alpha_i \) and the actual controller \( u \) are obtained by using (32) and (48).

Remark 2: The inputs of all neural network (NN), as a prerequisite for any neural network unit to function, must be maintained in a compact set throughout the control process. In the approach of this paper, the barrier Lyapunov function (BLF) is used to naturally satisfy such conditions, which not only ensures the boundedness of the whole system state but also confines all the closed-loop signals to a compact set throughout the operation. Therefore, the neural network unit can safely enter the loop and act from the time of the system start-up, and perform its learning and approximation role in the process of system operation, thus improving the control performance.

Remark 3: Compared with the adaptive computational torque control method [24], [25], [26] based on parameter decomposition and other methods [21], [27], [28], [29], [30], [31], The proposed control has completely different characteristics.

1) Employing Speed function and error transformation to achieve desired the tracking performance, including the following.

a) In the vehicle lateral control, the tracking error converges to a prescribed compact set in finite time with a convergence rate of not less than \( (T - t)/T \kappa^{-1}. \)

b) The finite time \( T \) is artificially determined, independent of the initial condition of other system initial and independent of other design parameters. A prescribed compact set can be adjusted by an appropriate choice of the design parameter \( b_f, \) since \( b_f \) is a free parameter.

2) Based on Nussbaum technology, in the presence of non-parametric uncertainties and external disturbances, when \( t \to \infty, \) the full-state tracking error of the vehicle lateral control can be driven to zero.

3) The obtained control scheme does not require excessive initial control force, and the control action is \( C^1 \) smooth.

Remark 4: The following control scheme is obtained when \( \beta = 1, \) that is, \( \zeta_i = z_i(i = 1, 2): \)

\[ u = N(\chi)(c_2 + \hat{a} \Phi) \frac{z_2}{k_{b2}^2 - z_2^2} \]

\[ \dot{\chi} = \gamma \chi(c_2 + \hat{a} \Phi) \frac{z_2^T z_2}{(k_{b2}^2 - z_2^2 z_2)^2} \]

\[ \dot{\hat{a}} = \gamma \frac{\Phi \| z_2 \|^2}{(k_{b2}^2 - z_2^2 z_2)^2} - \sigma \hat{a}, \quad \hat{a}(0) \geq 0 \]
\[ \alpha_1 = -c_1 z_1 \]  \hfill (66)

where \( \Phi \) is computed as in (46), \( z_1 \) and \( z_2 \) are defined as in (26), and the above control schemes (63)-(65) are called traditional control methods. In a situation like this, the full-state tracking error of vehicle lateral control with no violation of the full-state constraint converges to zero at \( t \to \infty \). However, from a theoretical analysis point of view, the conventional control method cannot make sure that there is convergence of the vehicle lateral control tracking error to a prescribed compact set \( \Omega_1 \) at an assignable decay rate within a set time, unless the relevant parameters are redesigned by trial-and-error method.

Remark 5: An adaptive control was proposed in [17] [18] for a category of nonlinear systems with full-state constraints. It is not possible to pre-specify the compact set, although the vehicle lateral control tracking error converges to the compact set for a finite time \( T \) (see the proof in [13] for details), but the detailed expression for the finite time \( T \) is not given and is at manual disposal, and the compact set cannot be pre-specified, while the robust adaptive control (48)-(50) proposed in this paper can guarantee that the vehicle lateral control tracking error converges to the specified compact set within a set time \( T \) at a decay rate no less than that specified.

5. Simulation Verification

To verify the effectiveness of the method in this paper, based on the dynamic model of the vehicle lateral system in Chapter 2 and the vehicle lateral control system equation in Section 3.1, the dynamic model of the vehicle lateral system is built by using Car Sim & Simulink simulation environment to verify whether the algorithm can realize the function and meet the control requirements.

5.1. Initial Condition

The whole simulation environment includes vehicle models, road models and control systems. After setting the vehicle and road model in Car Sim, the input and output interface can be set, and the output interface can observe the motion state of the vehicle in real-time. The sub-module of the control system built by Simulink can receive the motion state of the vehicle, and compute the state required for the control algorithm, and feedback to the control algorithm. The output signal of the control algorithm acts on the vehicle model through the input interface of Car Sim to realize the feedback control. The whole simulation process is not real-time calculation but is stored after calculation. The advantage is that the simulation results are not affected by the operation efficiency of the algorithm.

Car Sim vehicle model includes body, powertrain, braking system, steering system, suspend and tire. The body parameters of vehicle models based on Car Sim’s parameter estimation and algorithm simulation in this paper are shown in Fig. 2. The vehicle parameters obtained by parameter estimation are shown in Table 1.

![Fig. 2 Body parameters of Car Sim vehicle](image-url)
Table 1. Vehicle parameters

| Parameters                              | Value | Units  |
|-----------------------------------------|-------|--------|
| Vehicle quality                         | 1412  | kg     |
| Wheelbase                               | 2.910 | m      |
| Distance between the centroid and rear axis | 1.895 | m      |
| Moment of inertia                       | 1536.7| kg·m²  |
| Lateral stiffness of the front wheel    | 148970| N/rd   |
| Lateral stiffness of the rear wheel     | 82204 | N/rd   |

The following results verify that the proposed control method has better control performance than the traditional control method under the same control quantity. It should be noted that the control design only uses the Gaussian function \( \phi(Z) \) defined in (22), and the virtual parameter estimated value \( \hat{m} \) is computed by the algorithm. In the finite time \( T = 68s \) and the rate function \( \kappa = e^t \), the speed function \( \beta \) is as shown in (24). The following comprehensive tests were performed under the same initial conditions.

The same design parameters are used in the simulation: \( c_1 = 1, c_{21} = 6, \gamma = 0.02, \sigma = 0.7, \gamma = 0.03, b_f = 0.5 \). To ensure that the full-state constraint is not violated, \( k_{c1} = 1 \) and \( k_{c2} = 3 \) are given. According to the expected trajectory, we have \( A_0 = 0.9, A_1 = 0.9 \). Then from (30), we have \( k_{b1} = 0.51 \). When \( \kappa = e^t, T = 68s, b_f = 0.7 \). By using MATLAB software we have \( \max\{\delta\} = 3.6 \). Then from (34), we have \( \Omega_1 = 1.78 \), and from (37) we have \( k_{b2} = 0.99 \). According to the initial conditions, from (26) and (27), we have \( \zeta_1(0) = z_1(0) = q(0) - q^*(0) = [0.3, 0.3]^T \) and \( \zeta_2(0) = z_2(0) = \dot{q}(0) - \alpha_1(0) = [0.3(\delta + c_1), 0.3(\delta + c_2)] \), respectively, which ensures that \( \|\zeta_1(0)\| = (0.18)^{1/2} = 0.424 < k_{b1} \) and \( \|\zeta_2(0)\| = \|z_2(0)\| \leq 0.93 < k_{b2} \), which implies that the initial values of the transformation error are in the sets \( \Omega_{\zeta_1} \) and \( \Omega_{\zeta_2} \).

5.2. Curve Analysis

The key characteristics of lateral control are divided into steady-state characteristics and transient characteristics. The steady-state characteristics require that the algorithm is stable and the steady-state error is within a tolerable range. The transient characteristics require the algorithm to minimize the control error in the shortest possible time and ensures certain comfort. To show the advantages of the control method proposed in this paper compared with the traditional method in Remark 4 in terms of transient and steady-state tracking performance. With the same initial conditions and the same design parameters, the simulation results are shown in Fig. 3, Fig. 4 and Fig. 5 show the vehicle path tracking route, vehicle lateral path tracking and tracking error and vehicle heading angle tracking and tracking error under the proposed control method in this paper and the traditional control method respectively.

Comparison of the simulation results of the vehicle path tracking path route under the control method proposed in this paper and the conventional control method, and the results are shown in Fig. 5, \( x_r-y_r \) is the vehicle target path, \( x-y \) is the vehicle tracking path under the traditional control, and \( x_p-y_p \) is the vehicle tracking path under the control method in this paper. The two algorithms have a good effect on path tracking when the path curvature is small, and the difference is not much. However, where the curvature is large, the path tracking effect of the control algorithm in this paper is still better, while the traditional control path tracking error is relatively large. This simulation results show that the control method in this paper has a more stable performance than the traditional control, and can also maintain a smaller control error even when the path curvature is large.
Comparison of the simulation results of the vehicle lateral path tracking and tracking error under the control method proposed in this paper and the conventional control method, and the results are shown in Fig. 4. $y_r$ is the lateral path of vehicle target, $y$ is the lateral path of vehicle tracking under traditional control, and $y_p$ is the lateral path of vehicle tracking under the control method in this paper.
In addition, the changes of the lateral tracking error $e_{cr}$ and $e_{crp}$ under the control method and traditional control in this paper are also given [32]. The results show that under the traditional control, although it does not violate the full-state constraint, the tracking error cannot converge to the prescribed compact set within a given finite time $T$, whereas the proposed adaptive neural network (NN) control has better tracking performance, and the tracking error can quickly converge to the compact set, such as the $e_{crp}$ curve in Fig. 4.

Fig. 5 Vehicle heading angle tracking and tracking error

Comparison of the simulation results of the vehicle heading angle tracking and tracking error under the control method proposed in this paper and the conventional control method, and the results are shown in Fig. 5. $\phi_{ir}$ is the vehicle target heading angle, $\phi_{i}$ is the vehicle tracking heading angle under traditional control, and $\phi_{ip}$ is the vehicle tracking heading angle under the control method in this paper. In addition, the changes of heading angle tracking errors $e_{\phi_{i}}$ and $e_{\phi_{ip}}$ under the control method and traditional control in this paper are also given. The results show that the tracking error of heading angle cannot converge to the specified compact set in the given finite time of the traditional control, resulting in a large path tracking error when the curvature is large in Fig. 3, whereas the control method proposed in this paper has better tracking performance and smoother control performance, and also has strict theoretical analysis of the stability of the system, as shown in Theorem 2.

6. Summary
In this paper, robust adaptive control of the vehicle lateral control system with full-state constraints and uncertain dynamics is investigated. Introducing the obstacle Lyapunov function to make sure that the full-state constraints are not violated, and the neural network unit can operate from the start of the system, and safely play its learning and approximation role in the entire system operating envelope. The speed function and error transformation are introduced to achieve transient and steady-state tracking performance so as to make the full-state tracking error converges to a prescribed compact set of approximately zero at a specifiable rate of convergence in a given finite time. It is further demonstrated that the tracking error further converges to zero with time by introducing Nussbaum gain into the loop and using the Barbalet lemma. Moreover, all the closed loop system signals are bounded. Furthermore,
more general nonlinear systems can apply this approach, such as the strict feedback systems with nonparametric uncertainties, which will be the focus of future research.

References

[1] Eskandarian, A. Handbook of Intelligent Vehicles. Springer London, 2012.

[2] Funke J, Brown M, Erlien S. M, et al. Collision Avoidance and Stabilization for Autonomous Vehicles in Emergency Scenarios. IEEE Transactions on Control Systems Technology. Vol. 25 (2017) No. 4, p. 1204-1216.

[3] He X, Yang K, Ji X, et al. Research on Vehicle Stability Control Strategy Based on Integrated-Electro-Hydraulic Brake System. WCX™ 17: SAE World Congress Experience. Detroit, Michigan, U.S., 2017.

[4] Zhang H, Wang J. Active Steering Actuator Fault Detection for an Automatically-Steered Electric Ground Vehicle. IEEE Transactions on Vehicular Technology. Vol. 66 (2017) No. 5, p. 3685-3702.

[5] Thomas P, Morris A, Talbot R, et al. Identifying the causes of road crashes in Europe. Annals of advances in automotive medicine. Vol. 57 (2013), p. 13-22.

[6] Gao Y, Gray A, Tseng H E, et al. A tube-based robust nonlinear predictive control approach to semiautonomous ground vehicles. Vehicle System Dynamics. Vol. 52 (2014) No. 6, p. 802-823.

[7] Kritayakirana K, Gerdes J C. Using the centre of percussion to design a steering controller for an autonomous race car. Vehicle System Dynamics. Vol. 50 (2012) No. sup1, p. 33-51.

[8] Guo J, Luo Y, Li K, et al. Coordinated path-following and direct yaw-moment control of autonomous electric vehicles with sideslip angle estimation. Mechanical Systems and Signal Processing. Vol. 105 (2018), p. 183-199.

[9] Hwang C I, Yang C C, Hung J Y. Path Tracking of an Autonomous Ground Vehicle With Different Payloads by Hierarchical Improved Fuzzy Dynamic Sliding-Mode Control. IEEE Transactions on Fuzzy Systems. Vol. 26 (2017) No. 2, p. 899-914.

[10] Ji X, He X, Lv C, et al. Adaptive-neural-network-based robust lateral motion control for autonomous vehicle at driving limits. Control Engineering Practice. Vol. 76 (2018), p. 41-53.

[11] Guo J, Ping H, Wang R. Nonlinear Coordinated Steering and Braking Control of Vision-Based Autonomous Vehicles in Emergency Obstacle Avoidance. IEEE Transactions on Intelligent Transportation Systems. Vol. 17 (2016) No. 11, p. 3230-3240.

[12] Guo J, Luo Y, Wang J, et al. An adaptive cascade trajectory tracking control for over-actuated autonomous electric vehicles with input saturation. Science China Technological Sciences. Vol. 62 (2019) No. 12, p. 2153-2160.

[13] Guo H, Cao D, Chen H, et al. Model predictive path following control for autonomous cars considering a measurable disturbance: Implementation, testing, and verification. Mechanical Systems and Signal Processing. Vol. 118 (2019), p. 41-60.

[14] Guo H, Liu F, Xu F, et al. Nonlinear Model Predictive Lateral Stability Control of Active Chassis for Intelligent Vehicles and Its FPGA Implementation. IEEE Transactions on Systems, Man, and Cybernetics: Systems. Vol. 49 (2017) No. 1, p. 2-13.

[15] Brown M, Funke J, Erlien S, et al. Safe driving envelopes for path tracking in autonomous vehicles - ScienceDirect. Control Engineering Practice. Vol. 61 (2017) No. 316, p. 307-316.

[16] Zhao K, Song Y, Ma T, et al. Prescribed Performance Control of Uncertain Euler-Lagrange Systems Subject to Full-State Constraints. IEEE Transactions on Neural Networks and Learning Systems. Vol. 29 (2018) No. 8, p. 3478-3489.

[17] Liu Y J, Tong S. Barrier Lyapunov Functions-based adaptive control for a class of nonlinear pure-feedback systems with full state constraints. Automatica. Vol. 64 (2016), p. 70-75.

[18] Liu Y J, Jing L, Tong S, et al. Neural Network Control-Based Adaptive Learning Design for Nonlinear Systems With Full-State Constraints. IEEE Transactions on Neural Networks and Learning Systems. Vol. 27 (2016) No. 7, p. 1-10.
[19] Horn R A, Johnson C R. Matrix Analysis. Cambridge University Press, 1985.
[20] Ge S S, Wang J. Robust adaptive tracking for time-varying uncertain nonlinear systems with unknown control coefficients. Automatic Control IEEE Transactions on. Vol. 48 (2003) No. 8, p. 1463-1469.
[21] He W, Chen Y, Yin Z. Adaptive Neural Network Control of an Uncertain Robot With Full-State Constraints. IEEE Transactions on Cybernetics. Vol. 46 (2017) No. 3, p. 620-629.
[22] Tee K P, Ge S S, Tay E H. Barrier Lyapunov Functions for the control of output-constrained nonlinear systems. IFAC Proceedings Volumes. Vol. 46 (2013) No. 4, p. 449-455.
[23] Tee K P, Ge S S, Li H, et al. Control of Nonlinear Systems with Time-Varying Output Constraints. 2009 IEEE International Conference on Control and Automation IEEE. 2009, p. 524-529,
[24] Craig J J, Ping H, Sastry S S. Adaptive Control of Mechanical Manipulators. The International Journal of Robotics Research. Vol. 6 (1986), p. 190-195.
[25] Sadegh N, Horowitz R. An exponentially stable adaptive control law for robot manipulators. IEEE Transactions on Robotics & Automation. Vol. 6 (1990) No. 4, p. 491-496.
[26] Slotine J, Li W. Composite adaptive control of robot manipulators. Automatica. Vol. 25 (1989) No. 4, p. 509-519.
[27] Yue Y, Song Y, Chen G. Finite-time cooperative-tracking control for networked Euler-Lagrange systems. Iet Control Theory & Applications. Vol. 7 (2013) No. 11, p. 1487-1497.
[28] Song Y D. Adaptive motion tracking control of robot manipulators-non-regressor based approach. Proceedings of the 1994 IEEE International Conference on Robotics and Automation. Vol. 4 (1994), p. 3008-3013.
[29] Yang C, Jiang Y, Li Z, et al. Neural Control of Bimanual Robots with Guaranteed Global Stability and Motion Precision. IEEE Transactions on Industrial Informatics. Vol. 13 (2017) No. 3, p. 1162-1171.
[30] Yang C, Wang X, Long C, et al. Neural-Learning-Based Telerobot Control With Guaranteed Performance. IEEE Transactions on Cybernetics. Vol. 47 (2017) No. 10, p. 3148-3159.
[31] Zheng Z, Xia Y, Fu M. Attitude stabilization of rigid spacecraft with finite - time convergence. International Journal of Robust and Nonlinear Control. Vol. 21 (2011) No. 6, p. 686-702.
[32] Prescribed Performance Control of Uncertain Euler–Lagrange Systems Subject to Full-State Constraints", IEEE Transactions on Neural Networks and Learning Systems, 2018.