Hexadecapolar Kondo effect in URu$_2$Si$_2$?

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We derive the coupling of a localized hexadecapolar mode to conduction electrons in tetragonal symmetry. The derivation can be easily adapted to arbitrary multipole groups in arbitrary environment. We relate our model to the two-channel Kondo (2CK) model and show that for an $f^2$-configuration, a relevant crystal field splitting in addition to the 2CK interaction is intrinsic to tetragonal symmetry. We discuss possible realizations of a hexadecapolar Kondo effect in URu$_2$Si$_2$. Solving our model we find good agreement with susceptibility and specific heat measurements in Th$_{1-x}$U$_x$Ru$_2$Si$_2$ ($x \ll 1$).

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Introduction. In a seminal paper [1], Cox addressed important differences between U- and Ce-based heavy electron materials in terms of the atomic structure of their f-shell. In Ce-based systems the most probable configuration has one $f$-electron, in contrast to many U-based materials having an $f^2$-many body state as the most probable one. In crystal structures where the U-site has cubic symmetry, the $f^2$-states can give rise to quadrupolar degrees of freedom which when coupled to conduction electrons lead to two-channel Kondo non-Fermi liquid behavior [1, 2]. Meanwhile various multipolar orderings have been observed [3] as well as proposed as candidates for “hidden order” (HO) in materials with clear phase transitions but without an obvious order parameter. A prominent example in this area is provided by URu$_2$Si$_2$. For this material quadrupolar [4], octupolar [5], hexadecapolar [6] and triakontadipolar [7] order parameters have all been put forth. Recent experiments have definitely ruled out quadrupolar order [8], whereas another might implicitly hint at it [9]. On the other hand, the hypothesis of active Uranium hexadecapolar degrees of freedom provides a natural explanation for numerous experiments [10]. In this Letter, we generalize the work of ref. [1] and present a simple construction of low-energy Hamiltonians that describe the coupling between multipoles and conduction electrons in the tetragonal crystal field of URu$_2$Si$_2$. We show that the U hexadecapolar degrees of freedom, couple symmetrically to multiple channels of conduction electrons. We solve the resulting model using the numerical renormalization group, and thereby successfully describe the properties of Th$_{1-x}$U$_x$Ru$_2$Si$_2$ ($x \ll 1$) [12, 13]. Thus hexadecapolar fluctuations also serve as an explanation for the anomalies observed Th$_{1-x}$U$_x$Ru$_2$Si$_2$ ($x \ll 1$).

Construction. To construct a tractable model, valid at very low-energies, we take into account only the two lowest-lying $f$-configurations with double occupancy. This is motivated by recent LDA+DMFT calculations which indicate that, while U has mixed valence in this material with $f$-electron occupancy between 2 and 3, two crystal field singlets with double occupancy and different symmetries have the highest probability [6]. The Uranium degrees of freedom are then described by a $J = 4$ multiplet, split by the tetragonal crystal structure. The ground state and the nearest excited level are, respectively, time-reversal and parity even, $A_{2g}$ and $A_{1g}$ basis states of the group $T \times D_{4h}$ with $T = \{I, T\}$ the group of time-reversal, $I$ the identity, $T$ the time-reversal operator; and $D_{4h}$ the tetragonal point group including parity. Viz., the lowest-lying singlets are: $|A_{2g}\rangle = \frac{1}{\sqrt{2}}(|4\rangle - |−4\rangle)$, and $|A_{1g}\rangle = \frac{\cos \phi}{\sqrt{2}} (|4\rangle + |−4\rangle) + \sin \phi |0\rangle$, given in terms of the eigenvectors, $|J_z\rangle$, of the operator $\hat{J}_z$ in the $J = 4$ multiplet with the quantization axis chosen parallel to the $c$-axis of the crystal. To keep the equations short, we follow refs. [1, 2] and assume, the $f$-shell of the U atom hybridizes predominantly with $l = 3, J = \frac{5}{2}$ conduction electrons. The conduction electrons at the local site, $\psi^\dagger_{f,j,J_z}$, can be classified into the four double-valued or spinor irreducible representations (irreps), $\Gamma_{6p}$, $\Gamma_{7p}$ of the tetragonal double point group, $D_{4h}$, with $p = g/u$ for parity even/odd irreps (i.e. for $l$ even/odd). Under time-reversal symmetry $\mathcal{T}\psi^\dagger_{f,j,J_z}\mathcal{T}^{-1} = |−J_z\rangle\langle J_z|\psi^\dagger_{f,j,J_z}$, We set up a basis so that the $\alpha = \pm$ components of the Kramers doublets, $\psi^\dagger_{\Gamma_{6p}}(\alpha)\psi$, where $n$ enumerates doublets of the same type within the same $J$ multiplet, and $j = 6, 7$ complies with our convention: $\mathcal{T}\psi^\dagger_{\Gamma_{6p}}\mathcal{T}^{-1} = \psi^\dagger_{\Gamma_{7p}}$, implying the same for annihilation operators. For the local conduction electron basis, we choose the following two independent $\Gamma_{7u}$ Kramers doublets for creation operators: $\frac{1}{\sqrt{2}}\left[ \begin{array}{c} \psi^\dagger_{\Gamma_{7u}J_z} \\
\psi^\dagger_{\Gamma_{7u}−J_z} \end{array} \right]$, and one $\Gamma_{6u}$: $\psi^\dagger_{\Gamma_{6u}} = \left[ \begin{array}{c} \psi^\dagger_{\Gamma_{6u}−J_z} \\
\psi^\dagger_{\Gamma_{6u}J_z} \end{array} \right]$, on using the condensed notation: $\psi^\dagger_{\Gamma_{J_z}} \equiv \psi^\dagger_{\Gamma_{7u}J_z} \equiv \psi^\dagger_{\Gamma_{6u}J_z}$, Adjacent doublets with the same transformation properties in the same basis are $\Xi_{\Gamma_{7u}} = \left[ \begin{array}{c} \psi^\dagger_{\frac{1}{2}} \\
\psi^\dagger_{\frac{1}{2}} \end{array} \right]$, $\Xi_{\Gamma_{6u}} = \left[ \begin{array}{c} \psi^\dagger_{\frac{1}{2}} \\
\psi^\dagger_{\frac{1}{2}} \end{array} \right]$, $\Xi_{\Gamma_{6u}} = \left[ \begin{array}{c} \psi^\dagger_{\frac{1}{2}} \\
\psi^\dagger_{\frac{1}{2}} \end{array} \right]$. 

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Kondo Hamiltonians are made up of spin-flip and diagonal processes: $\mathcal{H}_K = \mathcal{H}_\perp + \mathcal{H}_z$. When constructing these two parts connecting the two singlets, the only relevant, non-trivial tensor products of irreps are $\Gamma_6 \otimes \Gamma_6 = \Gamma_7u \otimes \Gamma_7u = A_{1g} \oplus A_{2g} \oplus E_g$ and $A_{2g} \otimes A_{2g} = A_{1g}$ [15]. Inserting the appropriate tetragonal Clebsch–Gordan coefficients, symmetry thus binds the most general form for the spin-flip and diagonal parts to be [15]

$$\mathcal{H}_\perp = \frac{i}{2} \sum_{n,m=1} \sum_{i \in \{1,2\}} \mathcal{J}^{12i}_{n,m} \left( \Psi^\dagger_{i\Gamma_7u} \Sigma^{1}_{\Gamma_7u} + \Psi^\dagger_{i\Gamma_7u} \Xi^{1}_{\Gamma_7u} \right) |A_{1g}\rangle \langle A_{2g}| + h.c.,$$

$$\mathcal{H}_z = \sum_{n,m=1} \sum_{i \in \{1,2\}} \mathcal{J}^{nm}_{n,m} \left( \Psi^\dagger_{i\Gamma_7u} \Sigma^{n}_{\Gamma_7u} - \Psi^\dagger_{i\Gamma_7u} \Xi^{n}_{\Gamma_7u} \right) |A_{1g}\rangle \langle A_{1g}|.$$ 

The couplings are real and must satisfy $\mathcal{J}^{12i} = \mathcal{J}^{21i}$ to ensure hermiticity, but otherwise arbitrary. We omitted processes including $\Gamma_6$ electrons as they decouple from the impurity. The hexadecapolar, i.e. “spin-flip” fluctuations are thus coupled to four species of conduction electrons, namely to the two independent $\Gamma_7u$ Kramers doublets. $\mathcal{H}_\perp$ has the structure of the two-channel Kondo (2CK) model where the role of spin index is played by the index that distinguishes the two different $\Gamma_7u$’s, and the channels are distinguished by the Kramers indices. To make this correspondence more explicit, we introduce the operators: 

$$\eta_a \equiv \begin{bmatrix} \psi_a \ 
\psi_\beta \end{bmatrix}, \eta_b \equiv \begin{bmatrix} \psi_b \ 
-\psi_\beta \end{bmatrix},$$

and perform the unitary transformation: $|A_{1g}\rangle \rightarrow |A'_{1g}\rangle \equiv i |A_{1g}|$, which allows us to rewrite $\mathcal{H}_\perp$ in the standard notation

$$\mathcal{H}_\perp = \frac{\mathcal{J}^{12} - \mathcal{J}^{21}}{2} \eta_{q\mu}^a \sigma_{\mu\nu}^+ \eta_{q\nu} S^z + h.c. + O S^x,$$

with $q \in \{a,b\}$, $\mu, \nu \in \{\uparrow, \downarrow\}$. Here and in the following, repeated channel ($q$) and spin ($\mu, \nu$) indices are to be summed over; $O = O^\dagger$ contains only conduction electrons [25]; $\sigma^+ \equiv \sigma^x + i \sigma^y$ is composed of Pauli matrices; and $S^+ \equiv S^x + i S^y = |A_{2g}\rangle \langle A_{1g}|; S^- \equiv (|A_{2g}\rangle \langle A_{2g}| - |A'_{1g}\rangle \langle A'_{1g}|)/2; S^z \equiv |A_{2g}\rangle \langle A_{2g}| + |A'_{1g}\rangle \langle A'_{1g}|$.

**Discussion.** Channel symmetry is a consequence of time-reversal symmetry. The operator, $OS^x$ is irrelevant around the 2CK fixed point (and marginal in the free fermion scaling regime), as shown either by NRG calculations or using conformal field theory results [14, 16]. It does not destroy the 2CK state, since it neither breaks channel symmetry, nor lifts the spin degeneracy. Thus we must have $\mathcal{J}^{12}_{\perp} \neq \mathcal{J}^{21}_{\perp}$ in order for overscreening to occur. This asymmetry comes up naturally e.g. if we start off with a spherical symmetric Anderson Hamiltonian, perform the Schrieffer–Wolff transformation to arrive at a Kondo-type of interaction and then project to the crystal field states, $|A_{2g}|, |A_{1g}|$, at strong spin-orbit (i.e. $jj$) coupling, as described in refs. [1, 2, 14].

The diagonal part, $\mathcal{H}_z$ cannot lead to non-Fermi liquid behavior by itself, but it can quite possibly destroy it. Channel symmetry is preserved by time-reversal symmetry. However, the level degeneracies are lifted by the crystal field, both between the $|A_{2(2,1)}|$ states and also in each screening channel between $\Gamma_{7u}^{(n)}$ electrons with different $n$’s. The dangerous terms are

$$\mathcal{H}_{zrel}^{imp} = \Delta_{imp} \eta_{q\mu}^a \eta_{q\mu} S_z,$$

$$\mathcal{H}_{zrel}^{cond} = \Delta_{cond} \eta_{q\mu}^a \sigma_{\mu\nu}^+ \eta_{q\nu} \mathbb{1}.$$ 

Both types of crystal field splittings are relevant around the 2CK fixed point with scaling dimension $\frac{1}{2}$ [14, 16], and present in $\mathcal{H}_z$ with the amplitudes $\Delta_{imp} = (\mathcal{J}^{11} + \mathcal{J}^{12})/2$, and $\Delta_{cond} = (\mathcal{J}^{21} + \mathcal{J}^{22} - \mathcal{J}^{12})/4$. In fact, they are the only possibilities for relevant perturbations, if channel symmetry is intact [16].

Thus for this model to exhibit 2CK scaling in some temperature range, $\Delta_{imp}$ and $\Delta_{cond}$ must fall below the Kondo scale, $T_K$. This necessarily requires fine-tuning, and the basic assumption of the $A_{2g} - A_{1g}$ scenario—and, as we show below, of any other doublet-ground state scenarios—is that this accidental degeneracy is responsible for the unique behavior of URu$_2$Si$_2$ among the large number of U-based heavy fermions. LDA+DMFT calculations for URu$_2$Si$_2$ are indeed consistent with this accidental degeneracy on the scale of $T_K$ [6].

A local, $z$-directed magnetic field results in the following leading additions to the Hamiltonian

$$\mathcal{H}_{magz} \propto \mu_B \psi J_z \psi^\dagger \mathbb{1}$$

for the two-singlet part and for the local conduction electrons, respectively. These terms have similar effect as $\mathcal{H}_{zrel}^{c}$. Namely, the impurity part, $\mathcal{H}_{zrel}^{c} + \mathcal{H}_{magz}$, amounts to an effective magnetic field (or crystal field splitting) pointing into other than the $z$-direction. The same holds true for the conduction electrons with the effective magnetic field/crystal field splitting being different in the two channels. Thus while $\mathcal{H}_K$ is not identical to the 2CK Hamiltonian, it flows to the same fixed point when the relevant perturbations, which split apart the two different $\Gamma_7u$ irreps, or the two local singlets, vanish; and the application of magnetic field thus breaks both the channel and the spin symmetry of the 2CK model.

**Comparison with Other Scenarios.** For the order of crystal field levels in URu$_2$Si$_2$, other scenarios have also been put forward in the literature. In turns out that the
structure of the effective low-energy Hamiltonian for any two quasi-degenerate states is rather similar to that of the two-singlet case considered above. This applies also to a product $E_g \otimes \Gamma_5$ of doublet ground state [12, 17–19], formed by $|E_g\rangle$, $|E_g\rangle$. Fluctuations within this doublet can couple to two products of irreps : to $\Gamma_5 \otimes \Gamma_7$ and also to $\Gamma_7 \otimes \Gamma_6$. Importantly, the two different irreps play again the role of spin, while their Kramers indices, connected by time-reversal, play again the role of channel index in the 2CK language. If the two lower-lying irreps were degenerate, the spin symmetry of the 2CK model would be unbroken and 2CK scaling would occur. Since the degeneracy is approximate, the system will eventually flow to a Fermi liquid fixed point.

In the HO phase of URu$_2$Si$_2$, the ref. [6] proposed order parameter, $|\langle A_{2g}\rangle/|A_{1g}\rangle|$, is non-vanishing due to its real part which, in highest order of the multipole expansion, contains the expectation value of the hexadecapolar $A_{2g}$ tensor: $[\langle \hat{J}_x^2 - \hat{J}_y^2 \rangle (\hat{J}_x \hat{J}_y + \hat{J}_y \hat{J}_x) + \langle \hat{J}_x \hat{J}_y + \hat{J}_y \hat{J}_x \rangle (\hat{J}_x^2 - \hat{J}_y^2)]$ [6]. However, the same reasoning can be repeated for $|\langle E_g \rangle/|E_g\rangle|$, whose real part also contains this hexadecapolar ordering. These points are substantiated by the construction of the Hamiltonian for the $E_g$ ground state in the supplementary material section.

Comparison with Experiment. It has long been recognized that $\chi_c$, the (magnetic) dipole susceptibility of Th$_{1-x}$U$_x$Ru$_2$Si$_2$ along the $c$ axis shows log $T$ behavior at low-$T$ (see refs. [12, 19] and Fig. 1) in accord with the 2CK descriptions corresponding to both scenarios. However, susceptibility and resistivity measurements find that the magnetic field ($H$) induced crossover scale to a Fermi liquid depends on $H$ linearly, i.e. $T_H \propto H^\eta$ with $\eta = 1$, which does not agree with the $\eta = 2$ behavior corresponding to the 2CK scaling regime [13]. To make contact with these experiments, we solved the model, Eq. (1) by NRG, and confirmed that it indeed flows to the 2CK fixed point where $O(T)$ is irrelevant. Then we added a magnetic field, mimicked only by Eq. (3), to the 2CK model, and solved this model using an upgraded version of our density matrix-NRG code detailed in ref. [20]. The values of the magnetic field and the Kondo coupling were adjusted to fit the experimental data of refs. [12, 13].

Invoking $\omega/T$ scaling, we fitted the $T$-dependence of $\chi_c$ by the dynamic susceptibility of the 2CK model in magnetic field, as we trust our dynamic correlation functions (produced by the density matrix algorithm at $T = 0$) better than the thermodynamic quantities. Fig. 1 shows convincing agreement between theory and experiment apart from the small discrepancy for $T \geq 30$ K, i.e. for large energies where the resolution of NRG is limited. We obtained $T_K \approx 1.3$ K from the fit (see the caption of Fig. 1 for further details on $T_K$). This finding places the measurements in magnetic fields around the crossover region between the local moment and 2CK scaling regimes. In both regimes, scale invariance entails the hyperscaling relation, $\eta + \nu = 2$ with $\nu$ the critical exponent defined by $\chi \propto H^{-\nu}$. Thus for $H = 0$, the observed $\nu = 2$ gives $\eta = 2$, meaning that for $T$ between 0.1 and 10 K, the system is in the 2CK scaling regime. In contrast, for $H = 1$ to 5 T, the experiments measure $\eta = 1$ resulting $\nu = 1$. Thus, we conclude, these magnetic fields in addition to the ubiquitous, relevant crystal field splitting, are (slightly) larger than $T_K$ and the system flows directly from the local moment regime to a one-channel Kondo fixed point without traversing the 2CK scaling regime. In Fig. 1(a), the ratios of magnetic fields to $T_K$, fitting the susceptibility, further illustrate this point.

By taking a closer look at the specific heat coefficient in Fig. 2(a), we can reinforce these statements, and get another estimate for $T_K$. Two regimes for the given magnetic field values are clearly visible: The curves
that the hypothesis introduced in ref. [6] to describe the symmetrically coupled to two different irreps of conduction electrons is not connected by symmetry, we expect a Fermi liquid to emerge at sufficiently low-energies. Hence the intermediate non-Fermi liquid regime, observed in URu$_2$Si$_2$, is a result of an accidental degeneracy. This accidental degeneracy is responsible for the unique properties of this compound in the dilute and dense limits, among the hundreds of known U-based heavy fermion materials. We found that the scale of the crystal field splitting and the Kondo temperature is smaller in Th$_{1-x}$U$_x$Ru$_2$Si$_2$ ($x \ll 1$) than in URu$_2$Si$_2$. The splitting between the two $\Gamma_7$ irreps should be sensitive to the conduction electron filling, and we expect it to be larger in La$_{1-x}$U$_x$Ru$_2$Si$_2$ [22] where clear Fermi liquid behavior is observed. In this context it is also worth pointing out that in Th$_{1-x}$U$_x$Ru$_2$Si$_2$ ($x \ll 1$), the resistivity follows an approximate log $T$ behavior with a negative coefficient suggesting that a crossover to a Fermi liquid behavior takes place at sufficiently low temperatures. The study of the resistivity, however, will likely require a more realistic model for the diluted URu$_2$Si$_2$, including all bands present in the solid; also a more sophisticated approach to calculating the resistivity in non-Fermi liquid quantum impurity models than the ones present in the literature [23]; and corrections to the very dilute limit as was done recently in the context of the NMR experiments [24].

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+ J_{22}^{22} \eta_{\mu\nu} (\mathbb{I}_{\mu\nu} + \sigma_{\mu\nu}) \eta_{a\nu} - \eta_{\mu\nu} (\mathbb{I}_{\mu\nu} + \sigma_{\mu\nu}) \eta_{b\nu} \]
SUPPLEMENTARY MATERIAL FOR “HEXADECAPOLAR KONDO EFFECT IN URU$_2$SI$_2$?”

Kondo Hamiltonian for a Tetragonal $E_g$ (or $\Gamma_5$) Doublet Ground State

The structure of the Hamiltonian for two singlets, $|A_{2g}\rangle, |A_{1g}\rangle$ and for any other two quasi-degenerate states has many common features. The proposed $E_g$ (or $\Gamma_5$) doublet for the U ion ground state in Th$_{1-x}$U$_x$Ru$_2$Si$_2$ ($x \ll 1$) is claimed to provide full symmetry protection for the 2CK state [12, 17–19]. We show that crystal field fine-tuning is needed in this case as well, to hit the 2CK scaling regime. On the vector space spanned by the $E_g$ ground state doublet

$$|E_g\rangle = \frac{1}{\sqrt{2}} \left[ |\uparrow\rangle - |\downarrow\rangle \right]$$

we have four independent operators defined e.g. as $\mathbb{1}_E \equiv |E_g\rangle \langle E_g| + |E_g\rangle \langle E_g|$, $\sigma^y_E \equiv i (|E_g\rangle \langle E_g| - |E_g\rangle \langle E_g|)$, $\sigma^x_E \equiv |E_g\rangle \langle E_g| - |E_g\rangle \langle E_g|$, $\sigma^z_E \equiv |E_g\rangle \langle E_g| + |E_g\rangle \langle E_g|$. They transform as $A_{1g}, A_{2g}, B_{1g}$ and $B_{2g}$ tensors of $D_{4h}$, respectively [15]. In the Kondo limit, these operators can appear in the Hamiltonian only in certain combinations with the conduction electrons [14]. Namely, the $B_{1g}$ and $B_{2g}$ tensors can only couple to products of $\Gamma_6$ and $\Gamma_7$ electrons following the rule: $\Gamma_6\cup \Gamma_7 = B_{1g} \oplus B_{2g} \oplus E_g$, i.e.

$$\mathcal{H}^{E_g}_{\uparrow} = \sum_{m=1}^{2} \mathcal{J}_{B_{2g}} \left( \Psi^{+}_{\Gamma_6} \Xi_{\uparrow}^{(m)} - \Psi^{+}_{\Gamma_6} \Xi_{\uparrow}^{(m)} \right) \sigma^z_E + \sum_{n,m=1}^{2} \mathcal{J}_{A_{2g}} \left( \Psi^{+}_{\Gamma_6} \Xi_{\uparrow}^{(n)} - \Psi^{+}_{\Gamma_6} \Xi_{\uparrow}^{(n)} \right) \sigma^y_E$$

$$+ \mathcal{J}_{B_{2g}} \left( \Psi^{+}_{\Gamma_6} \Xi_{\uparrow}^{(1)} - \Psi^{+}_{\Gamma_6} \Xi_{\uparrow}^{(1)} \right) + \mathcal{J}_{A_{2g}} \left( \Psi^{+}_{\Gamma_6} \Xi_{\uparrow}^{(2)} - \Psi^{+}_{\Gamma_6} \Xi_{\uparrow}^{(2)} \right) \mathbb{1}_E$$

$$\mathcal{H}^{E_g}_{\downarrow} = \sum_{m=1}^{2} \mathcal{J}_{B_{1g}} \left( \Psi^{+}_{\Gamma_6} \Xi_{\downarrow}^{(m)} - \Psi^{+}_{\Gamma_6} \Xi_{\downarrow}^{(m)} \right) \sigma^z_E + \sum_{n,m=1}^{2} \mathcal{J}_{A_{1g}} \left( \Psi^{+}_{\Gamma_6} \Xi_{\downarrow}^{(n)} - \Psi^{+}_{\Gamma_6} \Xi_{\downarrow}^{(n)} \right) \mathbb{1}_E$$

$$+ \mathcal{J}_{B_{1g}} \left( \Psi^{+}_{\Gamma_6} \Xi_{\downarrow}^{(1)} - \Psi^{+}_{\Gamma_6} \Xi_{\downarrow}^{(1)} \right) + \mathcal{J}_{A_{1g}} \left( \Psi^{+}_{\Gamma_6} \Xi_{\downarrow}^{(2)} - \Psi^{+}_{\Gamma_6} \Xi_{\downarrow}^{(2)} \right) \mathbb{1}_E$$

with arbitrary real couplings. Again, each allowed term describes a 2CK screening process, and channel symmetry is guaranteed by time-reversal symmetry. One apparent difference from the two-singlet case is that $\Gamma_{6g}$ conduction electrons combined with $\Gamma_{7u}$’s can also participate in real 2CK screening processes. Besides there remains also the possibility of the two $\Gamma_{7u}$’s forming the two screening channels. Once we single out the two lowest-lying channels, the two cases become very similar in that the diagonal processes for the $E_g$ ground state also include a relevant splitting between the different crystal field channels of conduction electrons. Thus an $E_g$ doublet ground state does not enjoy level degeneracy protection, and in this respect, it is not distinguished from other, two-singlet ground state scenarios in tetragonal symmetry. Full protection of the 2CK state can be achieved in a setting were cubic symmetry merges the $\Gamma_6$ and $\Gamma_7$ representation into a $\Gamma_8$ quartet as in Cox’s original proposal [1].