High-acoustic-index-contrast phononic circuits: numerical modeling

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We numerically model key building blocks of a phononic integrated circuit that enable phonon routing in high-acoustic-index waveguides. Our particular focus is on Gallium Nitride-on-sapphire phononic platform which has recently demonstrated high acoustic confinement in its top layer without the use of suspended structures. We start with systematic simulation of various transverse phonon modes supported in strip waveguides and ring resonators with sub-wavelength cross-section. Mode confinement and quality factors of phonon modes are numerically investigated with respect to geometric parameters. Quality factor up to $10^8$ is predicted in optimized ring resonators. We next study the design of the phononic directional couplers, and present key design parameters for achieving strong evanescent couplings between modes propagating in parallel waveguides. Last, interdigitated transducer electrodes are included in the simulation for direct excitation of a ring resonator and critical coupling between microwave input and phononic dissipation. Our work provides comprehensive numerical characterization of phonon modes and functional phononic components in high-acoustic-index phononic circuits, which supplements previous theories and contributes to the emerging field of phononic integrated circuits.

I. INTRODUCTION

Surface acoustic wave (SAW) devices are widely used in electronic circuits, finding applications such as filters and oscillators in communication devices1 and a range of sensing applications2–4. Recently, SAWs are also exploited for coherent control of various quantum systems, including the electron quantum dots5–7, electron spins in diamond8, superconducting qubits9–11, and integrated photonic devices12–15. Thus, SAW provides a promising platform for hybridized quantum systems16–19, where the traveling SAW phonon can serve as a quantum bus to facilitate quantum state transfer.

The coupling strength between SAW and matter/photons is enhanced with reduced mode volume of the SAW. The enhanced interaction such as the strong coupling regime of cavity quantum acoustodynamics20,21 not only increases coherent coupling rate between quantum bits, but also improve the sensitivity of SAW-based measurement22. Also, the full potential of phononic systems can be only unraveled when maneuverability of phonon becomes comparable to its electrical and optical counterparts. Therefore, strong confinement of the SAW to the diffraction limit and the long lifetime phononic resonators are in high demand. To that end, phonon waveguides23–25 and ring resonators26,27 based on SAW were proposed and experimentally studied in the 1970s. However, in the following several decades, the experiments and detailed theoretical studies of the confinement of itinerant phonons in microstructures were less active, especially for the phonon resonators of radiating wave in nature. Only in recent years, due to the advances of nano-fabrication technologies, ultra-low loss phononic waveguides and resonators been achieved with with bulk acoustic wave28,29 and with SAW30–41, pushing the study of phononic devices, including SAW-related waveguides and resonators back to the frontier of researches as a strong contender for advanced phononic circuits.

Interdigital transducers (IDTs) convert the electrical RF signal into SAW or vice versa by employing the piezoelectric materials. In most applications, IDTs are fabricated on a uniform film, and the lateral size of IDT is much larger than the wavelength of SAW to excite and collect the quasi-collimated SAW. As a primary source or receiver of SAWs, the scaling of IDTs and their integration with other SAW devices is also in critical demand in the development of phononic circuits.

In this paper, we numerically investigate the properties of integrated phononic waveguides and resonators based on Gallium Nitride (GaN)-on-sapphire platform. Different from $\Delta V/V$ confinement effect induced by the metal electrodes30–33,42, we consider the pure geometric confinement of phonon on the surface of the chip, thus the metal-induced loss is eliminated. We prove that the phononic ring resonator can have exceptionally high quality factors ($Q$) with the radius of tens of microns, in which the whispering gallery modes (WGMs) can be excited either by the evanescent acoustic field through the phononic waveguide coupling or by direct IDT excitation. These waveguide and resonator structures will be basic elements in future phononic integrated circuits of both fundamental and practical interests43,44.

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FIG. 1. (a) Schematic illustration of an unsuspended strip phononic waveguide. $w_0$ and $h_0$ represent the width and height of the GaN strip, respectively. (b) The geometry setup in COMSOL simulation. The phononic waveguide modes propagating in the uniform structure along $\mathbf{k}$ with wavevector $\mathbf{k}$ are calculated by applying a periodic boundary condition, and PMLs are employed to absorb radiative acoustic waves in substrate.

II. WAVEGUIDE

Efficient confinement and guiding of phonons is essential for scalable operation of phononic devices. In the past decades, SAW has been mostly employed in applications that utilize its vertical confinement at the surface of substrates. However, the lateral confinement to the SAW is seldom studied. Alternatively, three-dimensional confinement of phonon has been mostly achieved in suspended phononic microstructures, which imposes practical constraints in fabrication yield and structural robustness. Here, we numerically study a phononic architecture based on unsuspended phononic waveguides and resonators. Shown in Fig. 1(a) is a typical strip waveguide. The basic requirement for phonon confinement is that the speeds of both transverse and longitudinal waves in strip material are slower than those in the substrate\(^2\). Thus, we choose the material platform of GaN-on-Sapphire satisfying these requirements, with parameters shown in Table I. An equivalent condition is also drawn in Ref. \(^45\) where the waveguide layer material needs to be heavier and less stiff. More detailed analyses about the requirements for the existence of Love wave can be found in Ref. \(^45–47\).

The basic properties of confined guiding modes are studied numerically by three-dimensional finite-element method (COMSOL Multiphysics v5.2). The waveguide geometrical parameters, namely its width $w_0$ and height $h_0$ marked in Fig. 1(a), determine the properties of the waveguide modes such as frequency, confinement, loss, and energy distributions. In this paper, all the simulations are carried out using periodic boundary conditions according to the translational symmetry of the structure, and perfectly matched layers (PMLs) are employed for studying the radiative loss, with the detailed numerical model illustrated in Fig. 1(b). To reveal the basic behaviors of the phononic waveguides, the anisotropy of GaN and Sapphire are neglected.

Shown in Figs. 2(a)-(h) are fundamental to higher-order transverse phonon modes supported in a strip waveguide, with its width set at $w_0 = 5 \mu m$ (for mode $h$, $w_0 = 6 \mu m$ because it doesn’t appear at $w_0 = 5 \mu m$) and $h_0 = 0.5 \mu m$. Wavelength along propagating direction is set at $\lambda = 2 \mu m$. Note that we label the modes by the corresponding characters $a, .., h$ in the following for analyzing the evolution of mode profiles against geometry parameters. Mode $a$ (Fig. 2(a)) is dominated by out-of-plane (z-direction) displacement, which we re-

| Material       | GaN\(^{48,49}\) | Sapphire\(^{50}\) |
|----------------|-----------------|-----------------|
| Density (g · cm\(^{-3}\)) | 6.15            | 3.98            |
| Young’s modulus (GPa)     | 305             | 345             |
| Poisson ratio             | 0.183           | 0.29            |
| Longitudinal wave speed (m/s) | 7350.0          | 10658.0         |
| Transverse wave speed (m/s) | 4578.3          | 5796.4          |

TABLE I. Elastic properties of GaN and Sapphire used in the simulation.
fer as quasi-Rayleigh mode owing to its reminiscence of Rayleigh wave\textsuperscript{51}. Mode $b$ has dominant in-plane ($y$-direction) motion and is a quasi-Love mode because of its similarity to Love wave\textsuperscript{52}. The modes $a$ and $b$ can be regarded as the fundamental phononic modes in the stripe with different polarizations ($z$- and $y$-polarized motion). Due to the lateral confinement and the edges, we found that the phononic modes are distinct from SAW modes. On one hand, the modes do not show pure flexural wave or in-plane shear waves. Especially, at the edge, it is difficult to identify the mode orientation because of the strong hybridization of deformation in all three orthogonal directions. On the other hand, higher-order transverse modes appear due to the lateral confinement (Figs. 2(c)-(h)). For example, compared to fundamental mode $a$ and $b$, in which vibrations on the strip edges are in-phase, the motion of mode $c$ and $d$ on the opposite edges are out-of-phase. In the rest of this paper, we call the symmetric (in-phase) mode $a$ as S-Rayleigh mode and call the anti-symmetric (out-of-phase) mode $c$ as A-Rayleigh mode, as well as A-Love mode $b$ and S-Love mode $d$. We also observe that higher-order modes (Figs. 2(e)-(h)) exhibit more complex field distributions.

For most applications, the concerned properties of guided phonon modes are\textsuperscript{24}: dispersion, confinement, radiation loss, energy distributions. Therefore, we numerically evaluate the following properties with varying waveguide geometrical parameters.

1. Mode frequency $f$ for a given wavelength $\lambda$ or wavenumber $k_x = 2\pi/\lambda$ along guided direction.

2. Phase velocity $v_p = 2\pi f/k_x$ and group velocity $v_g = 2\pi df/dk_x$.

3. Mode area

$$A_{\text{eff}} = \frac{1}{L} \frac{\iiint W(x,y,z)\, dx\, dy\, dz}{\max(W(x,y,z))}$$

where $L$ is the waveguide length along the propagation direction, $W(x,y,z)$ is the elastic strain energy density. The integral and maximum are calculated in the full simulated region. The mode area is a concept borrowed from photonics (see Ref.\textsuperscript{53,54}) as a measure of lateral confinement, where a smaller $A_{\text{eff}}$ indicates stronger confinement of phonon.

4. Quality factor

$$Q = 2\pi f \frac{\text{Energy stored}}{\text{Power loss}}.$$  

In this paper, we only calculate the radiation loss into substrate and do not consider other loss mechanisms such as material loss.
5. Energy confinement ratio

\[ \eta = \frac{\iint_{\text{strip}} W(x, y, z) dx dy dz}{\iint_{\text{all}} W(x, y, z) dx dy dz} \quad (3) \]

indicates the ratio between elastic strain energy stored in the phononic waveguide and the total elastic strain energy.

Figures 3(a)-(c) show the modal frequency \( f \), area \( (A_{eff}) \), and energy storage ratio \( (\eta) \) of 8 phonon modes versus the width \( w_0 \) of strip waveguide, with a fixed height \( h_0 = 0.5 \mu m \) and wavelength along propagation direction \( \lambda = 2 \mu m \). To clarify complicated mode hybridization, we now explain the notations and colors of plots that are adopted in this paper. By varying the waveguide geometry by small increments, the mode properties vary smoothly, thus we can track the frequencies or other parameters as continuous curves, which is called “branch” and labeled by different colors. As shown in the frequency plot (Fig. 3(a)), each branch is assigned with one specific color consistent with all other properties plots. As explained above, due to the strong mode hybridization in the strip waveguides, it is very difficult to identify mode orders or polarizations by branches. Therefore, we only label the modes in the regime that can be clearly distinguished according to the modes shown in Fig. 2. Sections labeled with two characters indicate the modes are experiencing hybridization.

The model frequency features versus width \( w_0 \), as shown in Fig. 3(a), indicates a transition between single-mode and multi-mode waveguide. At large \( w_0 \) values, all phonon mode branches asymptotically approach slab phonon modes in GaN-on-sapphire thin films. When \( w_0 \) becomes comparable to \( \lambda \), the curves branch off and the behaviors of each branch can be explained as follows. Since the lateral confinement imposes an approximate quantization condition \( k_y \sim \frac{\pi}{w_0} \), an increasing \( w_0 \) results in a reduction of total phonon wavenumber as \( k = \sqrt{k_x^2 + k_y^2} \) for a fixed \( k_x \), and hence a monotonic reduction of frequency approaching a constant. This intuitive interpretation is valid for most modes, however, fails for the A-Love modes \( b \). According to the mode area and energy confinement ratio given in Figs. 3(b) and (c), the confinement of A-Love mode \( b \) is excellent at small \( w_0 \ll \lambda \) (rapid reducing of mode area), and thereby almost all wave energy is confined in the strip. It could be interpreted as the effective boundary becomes air-GaN for the A-Love mode \( b \) opposed to sapphire-GaN for the other modes, and consequently, it can be treated as an anti-symmetrical flexural Lamb wave of a plate normal to \( y \) direction (Fig. 3(e)). For a plate mode, the vibration frequency \( f \propto \sqrt{\frac{D}{\rho \mu \nu}} \), in which bending stiffness

\[ D = \frac{w_0^2 E}{12(1-\nu^2)} \quad (5) \]

(\( \rho \), \( E \), \( \nu \) are density, Young’s modulus and Poisson ratio, respectively.) Thus the frequency of A-Love mode \( b \) scales with \( w_0 \) and decreases for narrow waveguides.

The interactions between the phonon modes lead to crossings and avoided crossings in Figs. 3(a)-(c). For the avoided crossings, the corresponding mode area and energy confinement ratio show crossing behavior, indicating the coupling between modes and the mode hybridization at these device parameters. For the crossing in frequency, the coupling between modes is negligible, so the changes in mode area and energy confinement ratio is not noticeable. Outside of the hybridization regimes, the confinement of quasi-Rayleigh modes \( (a \text{ and } c) \) weakens (larger mode area) with the increase of width, while that of quasi-Love modes \( (b \text{ and } d) \) remains relatively confined. At very small \( w_0 \), however, the waveguide becomes too narrow to support a fully confined mode \( a \), and its mode frequency rapidly approaches its cutoff frequency which is the frequency of SAW in substrate for \( \lambda = 2 \mu m \). The mode area of A-love mode \( b \), in contrast to mode \( a \), drops because this mode resembles anti-symmetrical flexural Lamb wave at small width, and its confinement is enhanced as width narrows (like a vibrating membrane whose energy is tightly concentrated in the waveguide region thus the influence from substrate is negligible). The quasi-Love modes \( (b,d) \) exhibit stronger confinement than quasi-Rayleigh modes’ \( (a,c) \) due to the much larger \( \eta \) of quasi-Love modes (except the hybridization regime), as shown in Fig. 3(c).
FIG. 5. (a) Schematic illustration of a strip ring resonator. (b) S-Rayleigh (mode a) and (c) A-Love (mode b) in ring resonator. (d) The evolution of mode hybridization profiles with varying $w_0$.

For a strip waveguide of fixed cross-section $h_0 = 0.5 \mu m$ and $w_0 = 5 \mu m$, the dispersion characteristics shown in Fig. 4 are numerically calculated as a function of the mode wavenumber $k_x$. In Fig. 4(b), the quasi-Love modes $b$ and $d$ have faster phase velocities than their corresponding quasi-Rayleigh modes and their behaviors approach the infinite surface according to analytical predictions in Ref. 46,57,58. The cutoffs result in sharp rising of group velocities in Fig. 4(c) at small wavevectors, and meanwhile, in Fig. 4(d), mode areas rise sharply at low frequencies, indicating a remarkable divergence of the confinement when the size of strip becomes much smaller than the wavelength. This is confirmed in Fig. 4(e) that energy confinement ratio decreases, i.e. phonon energy spreads into the substrate, at small $k_x$. In Fig. 4(d), we also found an interesting minimum point of mode area that occur simultaneously for branches of mode $b$ and $d$ around 2.2 GHz. A close comparison with results in Figs. 4(a) and (b) reveals that these minima occur at the avoided crossing point of modes $b$ and $d$ when they are maximally hybridized. Therefore, the mode hybridization presents an unique approach to engineer the phononic modes and might find interesting phononic sensing applications.

III. RING RESONATOR

In this section, we study the properties of ring resonators formed by bending and closing strip waveguide which supports phononic WGMs. The pioneering works of the acoustic WGMs date back to the discovery by Lord Rayleigh59,60, who reported the sound wave travels around a concave boundary in in St Paul’s Cathedral due to the continuous total internal reflection. The concept is generalized to electromagnetic waves61–63, which holds unique characteristics of high Q-factors, small mode volumes, and the ease of fabrication64. These remarkable merits are also possessed by phononic WGMs according to our investigations.

Due to the cylindrical symmetry of the ring resonator, the eigenmode profiles of the elastic differential characteristic equation are in the form as $u(r,z,\phi) = \psi(r,z)e^{im\phi}$, where $(r,z,\phi)$ are the cylindrical coordinators, $\psi(r,z)$ is the field distribution at the cross-section, $m$ is the angular momentum number. Utilizing the symmetry, we can numerically solve the mode profiles of a sector unit in $\phi$ direction in COMSOL, with periodic boundary condition: $u(r,z,\phi) = u(r,z,0)e^{-im\phi}$, where $m = 2\pi R/\lambda$ for an azimuthal wavelength $\lambda = 2 \mu m$ along tangent direction.

Figures 5 (b) and (c) display the typical quasi-Rayleigh and quasi-Love WGMs in a ring resonator. Compared to those in waveguide (Figs. 2(a)-(b)), the displacement fields of WGMs in the ring are more concentrated at
values with their calculated modal volumes shown in Figure 7(b). An optimal radius exists for each mode because the lateral confinement (i.e. effective mode area \( A_{eff} \)) becomes better for larger \( R_0 \), while the mode volume is approximately proportional to \( R_0 \) (\( V_{eff} \approx 2\pi R_0 A_{eff} \)). The \( Q \) factors in Fig. 7(c) increase exponentially with increasing \( R_0 \), indicating radiation loss to the substrate caused by bending decreases exponentially with the curvature, similar to the radiation loss of optical WGM in dielectric spheres\(^{61,65} \). We observe an interesting increment of \( Q \) when \( b \) and \( c \) are hybridized at around \( R_0 = 54 \mu m \), suggesting that the mode \( b \) and \( c \) couple to certain common leaky modes in substrate and their destructive interference suppresses the phonon loss.

The dispersion characteristics of ring resonator are summarized in Fig. 8. The \( k_0 = m/R_0 \) in the figure is effective propagating wavenumber along tangential direction. Most branches resemble their counterparts in the strip waveguide (Fig. 4), including \( f - k_0 \) relation, and the decrease of phase velocity and mode volume, and increase of ratio \( \eta \) with increasing \( k_0 \) or \( f \). We also compute \( Q \) factors which grow exponentially at short wavelengths.

### IV. DIRECTIONAL COUPLER

In an integrated phononic circuit, multiport devices transporting and routing phonons between components such as the waveguide-waveguide and waveguide-resonator coupler are indispensable circuit elements. A directional coupler consists of two closely placed parallel waveguides. Here we study the phononic coupling through the coupled-mode theory by analogy to its optical counterpart widely used in the photonic community\(^{66-68} \). The coupling between the modes in dif-
different waveguides arises from the tunneling of the elastic wave between them. The confined waveguide mode has a non-zero evanescent field that overlaps with the other waveguide, leading energy transfer in between.

The spatial mode amplitude evolution along the propagation direction in coupled waveguides for a given input signal frequency are described by the following equations

$$\frac{d}{dz} \alpha_1(z) = -i k_1 \alpha_1(z) - i g_{12} \alpha_2(z), \quad (5)$$

$$\frac{d}{dz} \alpha_2(z) = -i k_2 \alpha_2(z) - i g_{21} \alpha_1(z). \quad (6)$$

Here, the subscript “1”, “2” represent waveguide 1 (width $w_1$) and 2 (width $w_2$), $k_{1(2)}$ is the wavenumber of the mode in waveguide 1(2), and $g_{12}$, $g_{21}$ are the coupling coefficients between two modes. For lossless waveguides, coefficient $g$ must satisfy $g_{12} = g_{21}^*$ due to the power conservation. Setting $g_{12}g_{21} = |g|^2$ and solving Eqs. (5) and (6), we obtain the wavenumber eigenvalues of

$$k_{\pm} = k_1 + k_2 \pm \sqrt{\left(\frac{k_1 - k_2}{2}\right)^2 + |g|^2} \quad (7)$$

for hybrid modes in two waveguides. When $k_1 = k_2 = k$, the difference between $k_{\pm}$ reaches the minimum $2|g|$, and the eigenmodes are the $a_1 \pm a_2$, as an equal superposition of two waveguide modes. If the phonon input at the first waveguide with $a_1(0) = 1$ and $a_2(0) = 0$, we arrive at

$$\alpha_1(z) = \cos (|g|z) e^{-ikz}, \quad (8)$$

$$\alpha_2(z) = \sin (|g|z) e^{-i(kz + \frac{\pi}{2})}. \quad (9)$$

Thus, the phonons in the first waveguide $\alpha_1$ could be fully transported into the second waveguide $\alpha_2$ after a coupling distance of $\pi/2|g|$, or half of the phonons can be transmitted into mode $\alpha_2$ after a coupling distance $\pi/4|g|$. The larger the $|g|$, the quicker the energy exchanges.

A. Coupling between two identical waveguides

For two identical waveguides with $w_1 = w_2$, the wavenumbers of the modes are the same ($k_1 = k_2$), thus all supported modes can couple between the two waveguides without phase mismatching. Here, we set the width $w_0 = 5 \mu m$ and height $h_0 = 0.5 \mu m$ for both waveguides, as shown in Fig. 9(a). From the analysis above, we can estimate the coupling strength $g$ between two waveguides by the splitting of $k_{\pm}$.

Figure 9(b) shows the coupling strength of various waveguide modes with different gap, where the label $a$-g corresponds to the modes introduced in Fig. 3. We set the $\lambda = 2 \mu m$. Eigennodes $a$, $b$ and $c$ in the coupled waveguides are shown in Fig. 9(c), featuring in-phase and out-of-phase hybridized modes in the coupled waveguides as predicted by the coupled-mode theory. Since the evanescent field exponentially decays with the distance to the waveguide, a reduction of the $g$ with increasing gap is expected. As shown by the numerical simulations, there is a clear exponential relation between the coupling strength and the gap, yet the trends of high order modes $e$ and $f$ slightly deviate from the exponential curve at

![FIG. 8. Dispersion characteristics for modes in a ring resonator of radius $R_0 = 50 \mu m$, width $w_0 = 5 \mu m$, and height $h_0 = 0.5 \mu m$. (a) Modal frequencies $f$ versus $k_\phi$—the effective propagating wavenumber defined as $m/R_0$, with $m$ is angular momentum number. (b) and (c) Phase velocity and group velocity versus $k_\phi$. (d-f) Mode volume $V_{eff}$, Q factor, and energy confinement ratio $\eta$ versus $f$, respectively.](image-url)
small/large gaps. This can be attributed to the weaker confinement of the high order modes, for which the perturbative approximation in coupled-mode theory is no longer accurate.

B. Coupling between dissimilar waveguides

The coupled-mode theory is also applicable to dissimilar modes in dissimilar waveguides, therefore opens the possibility for mode conversions with different polarization or mode orders as long as there is a finite mode overlap between them. From Eq. 6, the maximum ratio of energy transmittance between the two modes is \( \text{sinc}^2[1 + (k_1 - k_2)^2 / 4g^2]^{1/2} \). Thus, for efficient energy transfer, we should adjust the coupling width to match the wavenumber of the two dissimilar modes, i.e., fulfill the phase-matching condition. In Fig. 10(a), the width of one waveguide \( (w_1) \) is changed while the width of the other is fixed \( (w_2 = 4.5 \mu m) \) in order to meet with the phase-matching condition

\[
k_1(w_1) = k_2(w_2)
\]

The modal frequencies of coupled waveguides are shown in Fig. 10(a) as the \( w_1 \) is varied from 1 \( \mu m \) to 2.5 \( \mu m \). From the plot, we observe four avoid crossings, corresponding to the four regions of modal coupling: I : \((b_1, a_2)\), II : \((a_1, b_1)\), III : \((b_1, c_2)\), IV : \((a_1, c_2)\). Fig. 10(c) displays the displacement fields of the four coupling regions at the minimum frequency differences. Similar to the identical waveguides, \( g \) values of different modes also exhibit an exponential dependency on \( gap \) (Fig. 10(b)). Among four cases, coupling II is particularly interesting because it represents a special mechanism of “self coupling” between mode \( a \) and \( b \) within waveguide 1, corresponding to a double tunneling process that mode \( a \) and mode \( b \) in waveguide 1 both couple to waveguide 2, which mediates the coupling of mode \( a \) and \( b \). The double tunneling mechanism could also explain the much faster decaying of the coupling strength for case II. With such a mechanism, we would expect the realization of a single-waveguide mode converter in the future with the assistance of an ancillary waveguide.

V. COUPLING TO IDT

Aside from the vibrational properties of phononic waveguides and ring resonators, the excitation and detection of phonons are also of practical importance and interests, for example, in phononic implementation of microwave delay lines and filters. Based on the model of ring resonator presented in Sec. III, we add IDT electrodes on the top surface of the ring resonator (Fig. 11(a)), and numerically investigate the coupling between IDT and phononic ring resonators and its efficiency to excite phonons. In this model, only the electrical effect of the IDT electrodes to the resonator is considered,

![Fig. 10](image1.png)

**Fig. 10.** (a) Modal frequencies in directional coupler of dissimilar waveguides, plotted against the waveguide width \( w_1 \), with the other waveguide width and coupling gap fixed \( (w_2 = 4.5 \mu m, gap = 1 \mu m) \). Due to the modal coupling, four avoided-crossing regions \((I, II, III, IV)\) are observed when the frequencies of modes in separate waveguides approach each other. The subscripts of waveguide mode labels \((1, 2)\) denote the waveguide 1 and 2 respectively. (b) The modal coupling strength \( |g| \) as a function of \( gap \). (c) The mode profiles of the directional coupler in the four avoided crossing regions.

![Fig. 11](image2.png)

**Fig. 11.** (a) Schematic illustration of the IDT integrated ring resonator, with the electrodes of IDTs connected to external microwave cable (not shown) for signal input. (b) The influence of the IDT on the mode \( Q \) factors. (c) and (d) The equivalent impedance and the energy transmission (conversion efficiency) from microwave to phonon as functions of the ring radius for \( S \)-Rayleigh and \( A \)-Love modes, respectively.
while the mechanical and other effects due to loading of the electrodes are ignored (such as mass loading).

We first evaluate the quality factor of the mechanical ring resonator under the presence of IDT electrodes. The width of each electrode is 1/4 of the acoustic wavelength and the electrodes cover half of the ring resonator area. For the S-Rayleigh mode with wavelength of 2 \( \mu m \) in a 5 \( \mu m \)-wide ring resonator without IDT, the quality factor increases exponentially with the radius of the resonator as the quality factor is limited by the radiation loss; with the IDT electrodes, the quality factor saturates when the radius is larger than 40 \( \mu m \), as shown in Fig. 11(b). The presence of the electrodes modifies the local phonon velocity, due to the \( \delta v/v \) effect\(^2\). Hence, the IDTs break the cylindrical symmetry of the perfect ring, leading to extra coupling of the phonon mode in ring to the leaky phonon modes in the bulk substrate. The same effect also happens to the Love mode. When the radius is small and radiation loss is dominant, the quality factors with and without IDT electrodes are almost the same. At larger radius in our simulation, the IDT electrodes induced loss becomes comparable with radiation loss and the quality factor with IDT electrodes are slightly lower.

To evaluate the external coupling to the phonon resonator, we also simulate the equivalent impedance of the IDT coupled mechanical ring resonator in order to achieve impedance matching to \( Z_0 = 50 \Omega \) transmission lines for maximal acoustic launching efficiency. In the COMSOL simulation, by assigning voltage \( V \) on the electrodes and measuring the current flow \( I \) on the electrodes at mechanical resonance frequency, the equivalent impedance, \( Z = V/I \), can be extracted. The microwave reflectivity can be expressed by

\[
r = \frac{Z - Z_0}{Z + Z_0} \tag{11}
\]

and the corresponding microwave to mechanical energy conversion efficiency \( T = 1 - |r|^2 \). The simulated results are plotted in Fig. 11(c) and (d) for quasi-Rayleigh and quasi-Love mode, respectively. The results indicate that the effective impedance of the IDT coupled mechanical resonator decreases exponentially with the radius and the impedance matching condition \( Z = Z_0 \) can be satisfied with a certain radius, which gives 100\% microwave-to-phonon conversion efficiency. To explain the dependence of the impedance on the radius, we consider the Butterworth-Van Dyke circuit model of the piezomechanical resonator which consists of a static capacitance \( C_0 \) in parallel with a motional series RLC circuit. The admittance of the circuit is\(^7\)

\[
Y(\omega) = -i\omega C_0 + \frac{1}{1/(i\omega C_m) - i\omega L_m + R_m} = -i\omega C_0 + i\omega^2 C_m \frac{\omega^2 + i\omega \omega_i/Q_i - \omega_s^2}{(\omega^2 - \omega_s^2)^2 + \omega_s^4 \omega^2/Q_i^2} = -i\omega C_0 + \frac{iC_m \omega_s^2 \omega (\omega^2 - \omega_s^2)}{(\omega^2 - \omega_s^2)^2 + \omega_s^4 \omega^2/Q_i^2} + \frac{\omega_s^4 \omega^2 C_m/Q_i}{(\omega^2 - \omega_s^2)^2 + \omega_s^4 \omega^2/Q_i^2} \tag{12}
\]

where \( C_m, L_m \) and \( R_m \) are the motional capacitance, inductance and resistance. The mechanical resonance \( \omega_s = 1/\sqrt{L_mC_m} \) and its intrinsic quality factor \( Q_i = 1/\omega_s R_mC_m \). Note that \( \omega_s \) is the resonant frequency without the presence of the IDT electrodes, and suppose that with IDT on it, the resonant frequency is \( \omega_f \). This resonant frequency shift originates from the \( \delta v/v \) effect and the value \( (\omega_f - \omega_s)/\omega_s \) equals \( \frac{1}{2} \delta v/v \) (as the electrodes cover only half of the ring resonator). In the regime where the quality factor is limited by intrinsic radiation loss instead of the \( \delta v/v \) scattering, we have \( (\omega_f - \omega_s)/\omega_s = \delta v/v \ll 1/Q_i \). In this case, the admittance can be simplified to

\[
Y(\omega_f) = -i\omega_f C_0 + \omega_f C_m Q_i \tag{13}
\]

with the real part being dominant, we get the relation between the equivalent impedance and its quality factor

\[
Z(\omega_f) = \frac{1}{\omega_f C_m Q_i} \tag{14}
\]

We see that the equivalent impedance is inversely proportional to its quality factor. The motional capacitance \( C_m \) is proportional to the electrode area like \( C_0 \), so it is linearly proportional to the radius as the surface area increases while the quality factor increases exponentially with radius. So in Fig. 11(c) and (d), the impedance’s dependence with radius is mostly exponential, consistent with the quality factor simulated in Fig. 11(b). The microwave-to-phonon conversion efficiency can also be understood from coupling condition point of view where the external coupling between the electrodes and the resonator is weak, but by changing the resonator’s intrinsic energy decay rate, critical coupling condition can be satisfied and all the microwave energy can be converted into mechanical energy.

Therefore, the IDT integrated mechanical resonator can enhance microwave-phonon conversion efficiency, boost the performances of applications that require high excitation and collection efficiency and provide coherent interface between phononic and superconducting circuits.

VI. CONCLUSION

In conclusion, we have quantitatively studied phononic mode properties in unsuspended strip waveguides and...
ring resonators. Our numerical results demonstrate efficient confinement of phonon in the practical GaN-on-Sapphire microstructures, which provide a scalable platform of phononic integrated circuits for acoustic signal processing and enhanced phonon-matter or phonon-light interactions. Basic components for a phononic circuit, including the directional coupler made by two waveguides, quasi-Rayleigh mode to quasi-Love mode conversion in coupled non-identical waveguides, ring resonator, as well as microwave photon-to-phonon conversion for efficient input and output coupling. It is worth noting that the studies in this paper can be generalized to other frequencies by simply rescaling the geometry parameters. For example, 10 GHz phononic waveguide modes could be realized with a thickness of 125 nm and a width of 500 nm, whose geometry is compatible with the photonic integrated circuits and promises the applications in optical Brillouin scattering. We believe the phononic waveguides and ring resonators studied in this paper can be applied in future studies on phononic circuits, gyroscopic sensors, integrated acoustic-optics modulators, circulators, integrated delay line and data bus that communicates the quantum bits.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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