HOT ELECTROMAGNETIC OUTFLOWS. III. DISPLACED FIREBALL IN A STRONG MAGNETIC FIELD

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1. INTRODUCTION

The spectrum of a gamma-ray burst (GRB) peaks at energies around the electron rest mass, and below this peak it is usually much flatter than the Rayleigh–Jeans tail of a blackbody. (See Piran 2004; Mészáros 2006, and Kouveliotou et al. 2012 for reviews.) Very bright bursts peaking in the X-ray band, or at lower photon energies, appear to be absent. We show that these fundamental properties of GRBs have, collectively, a simple and cogent explanation: the outflowing material is strongly magnetized, rich in electron–positron pairs, and depleted in ions. It is heated before and during breakout from a confining medium such as a stellar envelope, or a cloud of neutron-rich debris. The Lorentz factor during breakout is modest, $\Gamma \sim (\text{opening angle})^{-1}$, and the outflow is compact enough that the pairs follow a nearly thermal energy distribution described by a single temperature. No additional species of particles, such as neutrons or pions, are invoked or required.

The observed floor to the spectral peak frequency, first identified by Amati et al. (2002) for long GRBs, is recovered if the outflow carries a total energy that is comparable to the binding energy of a massive stellar CO core. Outflows which are strongly magnetized, but have photospheres dominated by the electron–ion component, are shown to have softer spectra. We identify them with X-ray flashes.

The pairs largely annihilate following breakout, if heating temporarily subsides. This allows the outflow to be accelerated outward by a combination of the Lorentz force and radiation pressure. The compact thermalization phase must, however, be followed by additional heating after the outflow has attained $\Gamma \sim 10^2$–$10^3$. This second phase is considered briefly at the end of this paper. What powers the continuing dissipation, and the production of the relativistic particles that emit the hard tail of the spectrum, depends on finer details of the outflow and is therefore more conjectural. The leading candidates are the reconnection of a reversing magnetic field, or the interaction of the fast, magnetized material with a slower baryonic shell ahead of it (Gill & Thompson 2014). We view the calculations presented here as compelling enough that some popular dissipation channels, such as hydrodynamic collisions between baryonic shells, or inelastic collisions between dilute flows of neutrons and charged ions, can now be disfavored.

The strong baryon depletion of the outflow points to the presence of an event horizon in the central engine. The extreme radiative energy of most GRBs is inconsistent with any known type of stellar magnetic flare, even with those most extreme flares of the soft gamma repeaters. Nonetheless, we find a genuine commonality in the physical properties of the outflows that give rise to GRBs and magnetar flares: they are simultaneously photon-rich and strongly magnetized, and, during a critical phase in the emission process, only mildly relativistic. They differ in overall energy scale, and the degree of rotationally forced collimation.

We also show that the largest isotropic-equivalent luminosities measured in GRBs are inconsistent with a hydrodynamical nozzle forming in a collapsing stellar envelope. Emphasis is placed on the collimating effect of a broader, trans-relativistic outflow from the torus surrounding the engine, which can also limit accretion onto a black hole from a collapsing stellar core. This connects the observed jet energy to the binding energy of the core.

The plan of this paper is as follows. In Section 2 we place our work in the context of the voluminous literature on Comptonization in thermal baryonic plasmas, and previous approaches to the prompt emission of GRBs. Section 3 describes our approach to calculating the photon spectrum, and pair density and temperature. The results of the numerical calculations are
shown in Section 4. These are compared with a semi-analytic scaling solution for the plasma temperature and scattering depth in Section 5. The temperature and spectrum of a magnetized outflow with a baryon-dominated photosphere is considered in Section 6. The effects of bulk Comptonization during jet breakout on the emergent spectrum are shown in Section 7. Finally, we draw together our results on the magnetized pair plasma with a global model of a Poynting-dominated jet in Section 8, showing in Section 9 how they together provide a simple motivation of the Amati et al. boundary. Section 10 summarizes our results. Appendix A reviews the various channels for soft-photon emission, and B presents details of our calculation of $e^\pm$ pair creation and annihilation. Throughout this paper, we use the notation $X = X_\odot \times 10^9$ to denote quantity $X$ in units of $10^9$.

2. PREVIOUS APPROACHES TO HOT COMPTONIZING PLASMAS AND THE SPECTRA OF GRBs

A physical explanation for the $\sim m_e c^2$ peak of GRBs must simultaneously account for the relatively soft shape of the spectrum below the peak, which is mostly inconsistent with a blackbody (Goldstein et al. 2012, 2013). For this last reason alone, a simple fireball model (Goodman 1986; Shemi & Piran 1990) is inadequate. Distributed energy release in baryonic fireballs allows for a wider range of low-frequency spectra (e.g., Vurm et al. 2013), but fine-tuning of the location of the dissipation, and the introduction of more complicated particle distributions, are required. Extending the fireball model to include continuous heating at an effective temperature below $\sim 25$ keV produces a natural buffer in the spectral peak at $\sim 0.1 m_e c^2$ in the rest frame, because the feeding of thermal photons toward the peak is limited by the freezeout of the pairs (Thompson 1997; Ghisellini & Celotti 1999; Eichler & Levinson 2000).

Matter and radiation interact in a GRB outflow over many decades in radius. Most theoretical attempts to understand the spectrum have (1) assumed that the low- and high-frequency parts of the spectrum form in the same region, and (2) that this process is localized to a particular radial zone. Because the high-frequency part of the spectrum requires high Lorentz factors, attempts were made to reproduce simultaneously the spectral peak and low-frequency spectrum at high $\Gamma$ (e.g., Pe'er & Waxman 2004; Giannios & Spruit 2005). This approach immediately rules out a thermalization process, such as is investigated here, because the observed spectral peak must sit well above $\sim 1$ MeV. Another difficulty lies in finding a robust mechanism for localizing the dissipation in radius, given the wide range of possibilities.

The specific approach taken here, following Thompson (2006), is to focus on magnetized outflows and divide the emission process into two sequential steps.

1. The low-frequency spectrum of GRBs, up to and including the peak, is assumed to originate in a separate zone from the high-frequency tail.
2. The Lorentz factor in this inner thermalization zone is much smaller than that in the outer parts of the flow, which generate the hard tail. The compactness $\sigma_T L_{iso}/4\pi m_e c^2 r$ associated with the (isotropic-equivalent) outflow luminosity $L_{iso}$ flowing through radius $r$ can greatly exceed unity even if $r$ is as large as $\sim 10^{11}$–$10^{12}$ cm.
3. In particular, this inner zone is associated with jet breakout from a confining medium, and is significantly displaced from the central engine (Eichler & Levinson 2000; Thompson et al. 2007; Lazzati et al. 2013). Confining material is present at intermediate radii: either a Wolf–Rayet envelope (Woosley 1993; Paczynski 1998; MacFadyen & Woosley 1999) or a neutron-rich wind (Duncan et al. 1986; Eichler et al. 1989; Dessart et al. 2009).
4. The hard tail to the spectrum originates further out in the outflow, due to continuing magnetic dissipation. Magnetized outflows could be driven by rapid rotation of a compact star (Duncan & Thompson 1992; Usov 1992, 1994; Thompson 1994; Meszaros & Rees 1997; Lyutikov & Blandford 2003; McKinney 2006), or possibly by magnetic flaring in an accretion disk (Narayan et al. 1992). A jet emitted by the collapsed remnant of a massive neutron star would ultimately be powered by such accretion.

2.1. Pair Creation in Magnetized Outflows

Guilbert et al. (1983) and Svensson (1987) have considered steady-state electron–positron pair pair creation and annihilation involving relativistic particles into a compact, soft photon source, noting that this will lead to the accumulation of a considerable optical depth in cold pairs.

Pair creation in relativistic outflows has also been studied for some time (Cavallo & Rees 1978; Goodman 1986; Paczynski 1990; Shemi & Piran 1990; Kroll & Pier 1991; Grimsrud & Wasserman 1998). However, when baryonic kinetic energy dominates the outflow luminosity, thermally created pairs are present in negligible concentration except very close to the engine, even in the presence of delayed dissipation (e.g., Beloborodov 2013). Pairs can be regenerated by bulk heating near the photosphere of a turbulent MHD outflow (Thompson 1994, 1997), during collisions between dilute baryonic shells (Ghisellini & Celotti 1999), or by non-thermal particle acceleration at shocks (Mészáros & Rees 2000).

On the other hand, if the baryon concentration in the outflow is pushed to very low values—that is, if it is magnetically dominated—then thermally created pairs can dominate the scattering opacity over many decades of radius. Our focus here is on the region inside the photosphere, as influenced by radially distributed heating. We do not consider non-local pair creation effects which would dominate outside the photosphere (Thompson & Madargul 2000; Beloborodov 2002).

Previous work by Usov (1992) focused on pair-creation near the engine by a unipolar inductor mechanism, although in practice this would be dominated by other pair creation channels such as neutrino collisions (e.g., Eichler et al. 1989; Zalamea & Beloborodov 2011) or damping of hydromagnetic turbulence (Thompson & Blaes 1998). Usov (1994) and Meszaros & Rees (1997) considered a pair gas that is advected passively from the engine out to the photosphere of a magnetized wind or jet, assuming the same radial Lorentz factor profile as a thermal fireball inside the photosphere.

The closest treatment to ours is by Thompson (1997), who studied the equilibrium of continuously heated, thermal pair plasmas in strong magnetic fields but did not make a detailed assessment of thermal cyclo-synchrotron emission. In the context of magnetar flares, Thompson & Duncan (2001) considered thermal pair creation in super-QED magnetic fields, where other photon creation processes contribute.

Outflows with comparable energy flux in toroidal magnetic field and thermal radiation have been investigated by Thompson (1994, 2006), Meszaros & Rees (1997), Drenkhahn & Spruit (2002), Giannios (2006), Giannios & Spruit (2007), and Russo & Thompson (2013a, 2013b). The direct involvement of thermal
radiation in the prompt emission from Poynting-dominated outflows has, by contrast, been discounted by Usov (1994), Lyutikov & Usov (2000), Lyutikov & Blandford (2003), and Zhang & Yan (2011). These authors instead proposed that this component would decouple from the outflow (forming e.g., a soft precursor) and that residual pairs trapped in the magnetic field would act as seeds for synchrotron emission at larger distances from the engine.

2.2. Multiple Compton Scattering

Even though the theory of multiple Compton scattering (Comptonization) in dense baryonic plasmas has a long history (with much of the fundamental work done by the Soviet school: Pozdnyakov et al. 1983), the analogous problem in highly compact, thermal pair plasmas has received remarkably little attention. In part, that may be because pairs in the primeval fireball are only present in a state of enormous optical depth.

Here we calculate in some detail the response of a dilute, and strongly magnetized, pair gas to steady heating. The radiation compactness is still very high ($10^3\sim10^5$), but the equivalent blackbody temperature is low enough ($\langle<25$ keV) that pairs are much less numerous than photons. We evolve the Kompaneets equation coupled to a detailed calculation of pair creation and annihilation, and an exact evaluation of cyclo-synchrotron emission.

Separately we allow for a small fraction ($<10^{-2}$) of the plasma energy to be injected in relativistic particles, which have a small direct effect on the photon spectrum below $\sim m_e c^2$, but can spawn a higher density of cold pairs than expected in equilibrium with a thermal photon gas. As a check of our treatment of pair creation and annihilation, we also evolve the full kinetic equations for photons and pairs in a more dilute plasma with a compactness $\ell \sim 10^3$.

During the approach to blackbody equilibrium in a Comptonizing plasma, one generally finds an intermediate, flat component of the spectrum ($F_\nu \sim $ const), which connects to a distinct Wien peak. Ghisellini & Celotti (1999) noted that this intermediate portion of the spectrum might correspond to the low-frequency spectral slopes of GRBs (see also Thompson 1998). However, they focused on plasmas of relatively low compactness ($\ell \sim 10^2$), with a goal of explaining both the low- and high-frequency components of GRB spectra, and did not consider strong magnetization or the effects of expansion (both which influence the spectral slope below the peak). Pe’er & Waxman (2004) and Vurm et al. (2013) showed that a flat low-frequency spectrum can arise from distributed heating in baryonic outflows with secondary pair creation—but only for a much narrower range of compactness than must be experienced by GRBs, and inconsistent with the high compactness expected at jet breakout. The conditions in which a Wien peak fails to emerge from a compact pair plasma (it is usually absent from GRB spectra) are addressed quantitatively for the first time here.

In the dilute pair plasma considered here, thermalization is limited by a relatively low pair density, and by a finite source of soft photons. We show that the end of heating is followed by rapid pair annihilation and only modest spectral cooling. The rest-frame spectral peak is, therefore, buffered to a value $\sim 0.1 m_e c^2$ over a wide range of compactness.

We show that flattening of the spectrum is strongest when the magnetic pressure dominates the photon pressure. Further flattening is shown to occur as the photons flow through a magnetized jet past its breakout point: here the scattering depth drops precipitously and the jet experiences a strong outward Lorentz force combined with pressure from the collimating radiation field (Russo & Thompson 2013a, 2013b).

2.3. Other Emission Models

Considerable attention has already been given to the emergent synchrotron-self-Compton spectrum in relativistic pair plasmas with a modest compactness $\ell \lesssim 100$: initially in the context of accretion disk coronae (Lightman & Zdziarski 1987), and then for GRB outflows (Pe’er & Waxman 2004; Stern & Poutanen 2004). The main goal in these works was to reproduce all the main components of the spectrum within a dissipation zone of limited (but uncertain) size. In the case of GRBs, the Lorentz factor must be high in the high-frequency emission zone, so it was also assumed to be high in the zone that determines the final spectral peak.

Sometimes a separate blackbody component has been introduced (e.g., Pe’er et al. 2006), representing an adiabatically evolved echo of a fireball phase closer to the engine. Incomplete thermalization inside the scattering photosphere of a baryon-dominated outflow naturally leads to a distinct Wien peak in the spectrum (Beloborodov 2013), but this hardly represents the low-frequency part of a typical GRB. It is possible to combine non-thermal particle acceleration and synchrotron emission at moderate scattering depth in a baryon-dominated fireball to produce a GRB-like spectrum (Pe’er et al. 2006; Vurm et al. 2011, 2013), but this solution appears sensitive to the placement of the dissipation zone, and different choices are shown to give quite different results.

Continuous heating in a relativistic outflow, which has some motivation in the magnetized case (Thompson 1994; Spruit et al. 2001), has been shown to produce promising high-frequency spectral slopes (Giannios 2006). But if the photon seeds are restricted to a blackbody and continuing photon creation is turned off, then the low-frequency spectrum does not deviate much from a Planckian (Giannios 2012) unless the photons have undergone strong adiabatic softening before being reheated (Thompson 1998).

Other approaches to a flat low-frequency spectrum have been considered, including hard-spectrum synchrotron cooling particles (Bykov & Meszaros 1996), decaying magnetic fields (Uhm & Zhang 2014), or blackbody emitting jets with sharp angular gradients (Lundman et al. 2013). Finally, we note that dissipation due to $n$-ion collisions, which is a possible source of non-thermal pairs (Beloborodov 2010; Vurm et al. 2011), is negligible during jet breakout at low $\Gamma$, and especially if the electron–ion component is subdominant to thermal pairs.

3. NEARLY THERMAL PAIR PLASMA IN A STRONG MAGNETIC FIELD

A pair plasma differs in an important respect from baryonic plasmas: as the temperature drops below $\sim0.1 m_e c^2$, the pairs annihilate. This has a strong buffering effect on the rate of Compton scattering, and the upward flux of photons in frequency space. Complete thermalization—the formation of a blackbody spectral distribution—is pushed to a much higher compactness ($>10^5$) than would be the case in a baryonic plasma.

Higher optical depths can develop if a modest fraction of the dissipation is in relativistic particles, which we show does tend to harden the low-frequency spectrum. In this way, measurements of GRB spectra offer constraints on the intermittency of the heating process.
The thermal and magnetic energy densities are conveniently parameterized in terms of the compactness,

\[ \ell_{\text{th}} \equiv \frac{\sigma_T U_{\text{th}} c t}{m_e c^2}, \quad \ell_B \equiv \frac{\sigma_T (B^2 / 8\pi) c t}{m_e c^2}. \]  

(1)

We work with comoving quantities in Sections 3–7 unless otherwise stated. Here \( t \) is a characteristic flow time, \( \sigma_T \) is the Thomson cross section, \( m_e \) the electron rest mass, and \( c \) the speed of light. A high compactness suppresses the temperature of the pairs and allows their rapid thermalization, even though the scattering depth does not exceed \( d \tau_T / d \ln(t) = n_e \sigma_T ct \sim 10–100 \). We do not use the magnetization \( \sigma = B^2 / 4\pi \rho_{\text{ion}} c^2 \) to describe the magnetic energy in the jet because the ion rest mass density \( \rho_{\text{ion}} \) is indeterminate in most of our calculations. Above a critical magnetization, which we evaluate in Section 4.5, the ions and neutralizing electrons play a negligible role in the spectral evolution. The magnetization derived from the pair inertia evolves in a relatively complicated way.

By contrast, calculations of more dilute relativistic plasmas, such as Blazar jets, are complicated by uncertainty in the input spectrum of relativistic particles, and the mechanism by which they are accelerated. These uncertainties partly disappear in the problem examined here, because relativistic particles cool much too rapidly to contribute significantly to the Comptonization process, and because hard photons lose energy by recoil.

Seed relativistic particles Compton cool on a timescale

\[ t_C(\gamma) \sim \frac{1}{\ell_{\text{th}} \gamma}. \]  

(2)

Supposing that a fraction \( f_{\text{rel}} \) of the radiation energy is supplied by these particles (with the remainder by gradual heating of thermal particles), the time-averaged energy density in relativistic particles is

\[ U_{\text{rel}} \sim f_{\text{rel}} \frac{t_C}{t} U_{\text{th}}, \]  

(3)

and the time-averaged compactness is

\[ \ell_{\text{rel}} = \frac{\sigma_T U_{\text{rel}} ct}{m_e c^2} \sim f_{\text{rel}} \frac{\gamma}{\gamma-1} \ll 1 \ll \ell_{\text{th}}. \]  

(4)

The equilibrium Compton parameter is

\[ \gamma_{C,\text{rel}} \sim \gamma^2 \sigma_T \frac{U_{\text{rel}}}{\gamma m_e c^2} c t \sim f_{\text{rel}}, \]  

(5)

which is tiny compared with the Compton parameter of the thermal plasma. As a result, the emergent spectrum is determined almost entirely by a competition between soft-photon emission and multiple Compton scattering by thermal particles.

We therefore focus on thermal cyclo-synchrotron emission and absorption. The mean energy of the pairs is in a range, \((0.05–0.2)m_e c^2\), where an exact calculation of the emission spectrum is required. The details are reviewed in Appendix A, and the result shown in Figure 1. This emission channel dominates if the magnetic energy density exceeds the thermal energy density. Other soft photon sources (bremsstrahlung and double Compton) are included for completeness.

### 3.1. Compactness and Co-moving Energy Density at Jet Breakout

In this section we motivate our normalization of the radiation and magnetic compactness in the spectral calculations in Section 4. Our focus is on the state of a relativistic jet before and during breakout from a confining medium such as a Wolf–Rayet envelope or neutron-rich debris cloud. Here we establish lower bounds to the compactness and energy density in the jet, given that its Lorentz factor remains low (comparable to 1/\( \theta \)).

The breakout of a relativistic jet must be accompanied by a broader, trans-relativistic cocoon (e.g., Ramirez-Ruiz et al. 2002; Lazzati et al. 2009). This cocoon provides pressure that helps to confine the jet, at least out to a radius

\[ R_{\text{trans}} \sim c t_{\text{col}} \sim 3 \times 10^{11} \left( \frac{t_{\text{col}}}{10 \text{ s}} \right) \text{ cm}, \]  

(6)

where \( t_{\text{col}} \) is the collapse time of the material powering the jet.

A similar effect is present in a binary neutron star merger, due to the generation of an intense, neutron-rich wind (Dessart et al. 2009). This wind propagates a distance \( \sim (c/3)t_{\text{col}} \sim 10^8(t_{\text{col}}/100 \text{ ms}) \) cm before the merger remnant collapses to form a black hole.

There is a downward gradient in Lorentz factor away from the relativistic jet and into the cocoon. As fresh relativistic material continues to be injected from the engine, this gradient can be maintained on an angular scale \( \delta \theta \lesssim 1/\Gamma \), so that causal contact is maintained across the gradient.

Material of an intermediate Lorentz factor \( \Gamma_2 \) provides confinement for a faster core out to a distance \( \sim 2(\Gamma_2^2/R_{\text{trans}}) \). Therefore complete deconfinement of a Lorentz factor \( \Gamma \) jet is delayed out to a radius

\[ R_{\text{shear}}(\Gamma) \sim 2\Gamma^2 R_{\text{trans}}. \]  

(7)

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1 Even in a part of this heated boundary layer that is strongly depleted in baryons, we find optical depths large enough to suppress the diffusion of photons across the layer (see Figure 10). The photons therefore do not see the gradient, as envisaged by Lundman et al. (2013). The available shear kinetic energy can still couple to the photons through higher-frequency Kelvin–Helmholtz modes.
Evidence for this type of extended structure is seen in the two-dimensional hydrodynamic simulations of Lazzati et al. (2009).

Even after taking into account this re-scaling, the radiation compactness remains very high at breakout. The net isotropic luminosity, including contributions from both thermal energy and toroidal magnetic field, is

\[
L_{\text{iso}} = \left[ \frac{4}{3} \Gamma^2 U_{\text{th}} + \left( \frac{G B_0}{4\pi} \right)^2 \right] 4\pi r^2 c
\]

\[
= \frac{4}{3} \Gamma^2 U_{\text{th}} \left( 1 + \frac{3}{2} \delta_{\text{th}} \right) 4\pi r^2 c. \quad (8)
\]

Here, as elsewhere, \( U_{\text{th}} \) is the comoving thermal energy density, which is dominated by photons. Normalizing the energy density to \( m_e c^2 / \lambda_c^3 \) in the co-moving frame, one finds

\[
U_{\text{th}} = 5.3 \times 10^{-13} \frac{L_{\gamma \text{iso}, \odot, 51}}{r_{\text{col,1}}^2} \left( \frac{\Gamma}{3} \right)^{-6} \frac{m_e c^2}{\lambda_c^3}. \quad (9)
\]

The radiation field is dilute in the sense that a thick pair gas cannot be maintained in full thermodynamic equilibrium:

\[
T_{\text{bb}} = \left( \frac{U_{\text{th}}}{\sigma_\text{SB}} \right)^{1/4} = 0.48 \frac{L_{\gamma \text{iso}, \odot, 51}^{1/4}}{r_{\text{col,1}}^{1/2}} \left( \frac{\Gamma}{3} \right)^{-3/2} \text{keV}. \quad (10)
\]

In spite of this, the bulk-frame compactness is still large:

\[
\ell_B = \frac{\sigma_T U_{\text{th}} r / \Gamma}{m_e c^2} = 1.1 \times 10^{-7} \frac{L_{\gamma \text{iso}, \odot, 51}}{r_{\text{col,1}}} \left( \frac{\Gamma}{3} \right)^{-5}. \quad (11)
\]

Combining this result with the model of a magnetized jet discussed in Section 8, for which the isotropic Poynting luminosity \( L_{P, \text{iso}} \propto \Gamma^{-2} \) and breakout Lorentz factor \( \Gamma \sim 1/\theta \), one finds that more luminous jets may be less compact at breakout:

\[
\ell_B \propto L_{P, \text{iso}}^{-3/2}. \quad (12)
\]

### 3.2. Plasma Dynamics

In addition to these regulating effects, we find that expansion, combined with continued heating and photon creation, has a significant regulating effect on the low-frequency spectrum. A sharp thermal peak formed in an initial thermalization event at very high compactness is noticeably reduced if the plasma expands and there is a continuing upward flux of soft photons.

We choose a simple model representing a conical jet expanding at a constant Lorentz factor inside breakout. The plasma conditions are determined by the comoving compactness \( \ell_B \) and energy density \( U_{\text{th}} \), and what matters here is the range of these quantities during a heating episode. The output spectrum has only a mild dependence on them, and therefore similar results can be expected for jets of a different shape. The assumption of constant Lorentz factor is more central to our approach, since a low Lorentz factor at breakout is needed if the boosted thermal peak is to sit in the frequency range observed in GRBs.

The proper energy density scales with radius \( r \) and bulk-frame time coordinate \( t \sim r/c B^2 / 8\pi \sim r^{-1} \), where \( \delta = 2 \). Heat is injected at a rate of \( dU_{\text{th}} / dt = \delta U_{\text{th},0} (t / t_0)^{-\delta} \) per unit volume, where \( U_{\text{th},0} \) is the initial thermal energy density (in photons, rest energy of pairs, and thermal energy of all material particles). Parameterizing

\[
U_{\text{th}} = f_{\text{th}} \frac{B^2}{8\pi}, \quad (13)
\]

we explore the regime of moderately strong magnetization, \( f_{\text{th}} = 10^{-1} - 1 \).

In the Kompaneets calculations, we must start a simulation with a finite energy density so that the seed electrons do not overheat (become relativistic). Then as time evolves

\[
U_{\text{th}}(t) = (t / t_0)^{-4\delta/3} U_{\text{th},0} + t^{-4\delta/3} \int_{t_0}^{t_f} d\tilde{t} \tilde{t}^{4\delta/3} \frac{dU}{dt} \quad (14)
\]

The instantaneous compactness (1) scales as \( \ell(t) \propto t^{-1} \) for \( \delta = 2 \). In a given simulation, the fraction of the final thermal energy that results from distributed heating is

\[
f_{\text{dis}} \simeq 1 - \frac{1}{3}(t / t_0)^{-\delta/3}. \quad (15)
\]

Generalizing to the case where the jet opening angle varies with radius, but maintaining \( \Gamma \sim 1/\theta \), one finds a comoving energy density \( U \propto (\Gamma^0 \theta)^{-2} \propto r^{-2} \) and a compactness \( \ell(r) \propto U r / \Gamma \propto \theta / r \). A decrease in compactness with radius is therefore a more generic feature of confined jets than the simple model explored here.

The simplest, and perhaps most generic, form of heating involves the damping of hydromagnetic turbulence. Small-scale irregularities in the flow could be triggered by ideal or resistive MHD instabilities; or by an interaction of the magnetized jet material with denser baryonic material. While almost all work on magnetic reconnection focuses on non-ideal effects near current sheets, it should be emphasized that reconnection has simpler effects, by changing the topology of the magnetic field and converting magnetic energy to bulk kinetic energy. In the presence of a dense photon gas and at moderate scattering depths, Compton drag effectively damps the differential motion of all particle species with respect to the mean flow (Thompson 1994).

Nonetheless, Alfvén waves can easily cascade to higher wavenumbers before damping by Compton drag. In a strongly magnetized plasma, they anisotropically heat the electrons and positrons: either because the wave current becomes charge starved (Thompson & Blaes 1998); or, if the particle density is high enough, by Landau damping on the motion of \( e^\pm \) parallel to the magnetic field (Thompson 2006; see Quataert & Gruzinov 1999 for related work on baryonic plasmas). In Sections 9.2 and 9.3 we consider the implications of our thermal plasma solution for reconnection and charge-starvation effects.

### 3.3. Formation of Quasi-thermal Peak, \( T_e, T_B \ll m_e c^2 \)

We evolve the photon spectrum in the diffusion approximation, including stimulated and recoil effects. The Kompaneets equation for the photon occupation number \( N(\omega) \) is

\[
\frac{\partial N}{\partial t} + \omega \frac{\partial N}{\partial \omega} = \frac{\partial N}{\partial t}_{\text{cyc}} + \frac{\partial N}{\partial t}_{\text{drift}} + \frac{\partial N}{\partial t}_{\text{acc}} + \omega \frac{\partial N}{\partial \omega} \left[ T_e \frac{\partial N}{m_e c^2 \partial \ln \omega} + \frac{\pi \omega}{m_e c^2} N(1 + N) \right]. \quad (16)
\]
Here \( n_e \equiv n_e^+ + n_e^- \) is the proper density of scattering particles, and the random particle motion is described by a single temperature. As is usual, stimulated scattering makes a net contribution only to the recoil term, but cancels from the Doppler upscattering term.

The source terms include cyclo-synchrotron emission and absorption, and (non-magnetic) free–free and double-Compton emission and absorption; they are reviewed in Appendix A. The effect of pair creation and annihilation on the spectrum is handled indirectly through additional heating and cooling of the pairs, as specified in Equation (40) and explained further in Section 3.8.

We solve (16) in flux-conservative form, meaning that the variable evolved is \( N(\omega) \equiv \omega^2 N(\omega) \). The equation is solved by the method of lines, with a second-order differencing in frequency and fourth-order Runge–Kutta evolution in time.

Both static and expanding plasmas are considered. In the expanding case, it is essential to consider large expansion factor \( \gg 10^2 \). For example, a jet propagating through the inner core of a Wolf–Rayet star may encounter resistance over such a range of radius, and continue to interact with entrained stellar material even beyond its photosphere (see Section 3.1).

Adiabatic expansion in the outflow rest frame corresponds to a dilution \( n_e \propto t^{-4} \), with \( \delta = 2 \). Then \( \omega = (\delta/3)\omega_0/t \) on the left-hand side of (16). The calculation is stopped if the outflow optical depth \( d\tau_T/d\ln(t) < 1 \) (which generally happens only after heating is turned off).

For completeness, we also include the relative drift between photons and pairs, which is driven by the Lorentz force in a photon phase space, one finds, as usual

\[
\frac{dU_{\gamma}}{dt} = \frac{1}{m_e c^2} \left[ 4 T_e - \frac{\langle \omega^2 \rangle}{\langle \omega \rangle} \right] \sigma_T n_e c U_{\gamma}
\]

Here \( T_C \) is the Compton temperature.

### 3.4. Validity of a Single Temperature

In what follows, we restrict the particle distribution to relativistic Boltzmann,

\[
\frac{dn_e}{d\gamma} = \frac{n_e}{T_e K_2(\gamma/T_e)} \beta T_e^{\gamma} \gamma^{-\gamma/T_e}; \quad T_e \equiv T_e/m_e c^2,
\]

with temperature \( T_e \lesssim 0.2 \), and isotropic pitch angles.

In this situation, the timescales for heating and cooling of the pairs are both very short and in near balance, with cooling being primarily by Compton scattering of the thermal photon field:

\[
\frac{t_{\text{heat}}}{t} \sim \frac{3n_e T_e/2}{f_{\text{th}} B^2/8\pi} = \frac{3T_e d\tau_T/d\ln(t)}{2f_{\text{th}} \beta_B};
\]

\[
\frac{t_{\text{cool}}}{t} \sim \frac{3m_e c}{4\sigma_T U_{\gamma} t} = \frac{3}{4\ell_{\text{th}}},
\]

A thermal distribution presupposes the exchange of energy between the charged particles on a shorter timescale. In the absence of such a process, the pair energy distribution will peak around

\[
E_{\text{q,eq}} = \frac{3m_e c^2}{4d\tau_T/d\ln(t)}. \tag{21}
\]

A monoenergetic distribution is approached in the idealized case of uniform heating and cooling.

An important point is highlighted by Equation (21): the bulk of the pair population remains sub-relativistic during a heating episode, \( m_e c^2 > T_e > T_C \), only if the plasma starts off at a large scattering depth—even if the outflow is still very compact. As we discuss in Section 10, this provides a distinction between an early heating phase (before jet breakout) when the low-frequency part of the GRB spectrum is formed, and a secondary phase (after breakout) that produces the high-energy tail.

Coulomb scattering is relatively slow in this context, due to the low particle density:
photon pressure and the Lorentz force, the thermal peak drops by a factor $\sim 10^{-2}$ in the co-moving frame, to $T_e \sim 10^{-3} m_e c^2$. Then $f_{\text{th}}$ rises as the magnetic field dissipates. If $T_c$ is comparable to $T_e$ at the beginning of the heating episode, then it remains much smaller than $T_e$. Perpendicular heating is mainly by non-resonant Compton scattering, with interesting consequences for the angular pattern of the scattered radiation (Thompson 2006).

We describe the volumetric heating of the pairs via

$$\frac{dU_e}{dt} \bigg|_{\text{heat}} = f_{\text{th}} \frac{B^2}{8\pi} = \ell_{\text{th}} \frac{m_e c^2}{\sigma_T c^2 \ell_{\text{tot}}^2}. \quad (26)$$

The compensating change in energy by Compton scattering is the negative of (18). We find that Compton equilibrium is only approximately maintained during heating: $(T_e - T_c)/m_e c^2 \sim \tau_T^{-1}$. There is a rapid approach to equilibrium after the heating turns off, due to the very high compactness.

Even though the photon field cannot be defined by a single temperature if its low-frequency spectrum is flat, the high-frequency spectrum does maintain a thermal form at high compactness: $dU_{\gamma}/d\omega \propto \omega^g e^{-\omega \omega_t}/T_e$, with $g \simeq 3$ and $T_\gamma$ close to $T_e$. For the purposes of constructing simple analytic models of the expanding pair plasma, we will sometimes use

$$\frac{1}{\hbar} \frac{dU_{\gamma}}{d\omega} = K \left( \frac{\omega_t}{c} \right)^3 e^{-\omega_t/T_e} \quad (\omega < \omega_t);$$

$$= K \left( \frac{\omega_t}{c} \right)^3 e^{-\omega_t/T_e} \quad (\omega > \omega_t), \quad (27)$$

which matches smoothly at $\omega_t = 3T_e/\hbar$ with coefficient

$$K = 0.083 \frac{U_{\gamma}}{g g_{\text{th}}} T_e^4. \quad (28)$$

### 3.5. Pair Creation and Annihilation

The density of pairs evolves according to annihilation and creation by photon collisions, $e^+ e^- \leftrightarrow \gamma + \gamma$. In a warm plasma, $T_e \lesssim 0.1$, the annihilation cross section can be approximated by $(\sigma_{\text{ann}} | v_{e^+} - v_{e^-}|) \simeq (3/8) \sigma_T c$, so that

$$\frac{dn_e}{dt} \bigg|_{\text{ann}} \simeq \frac{3}{4} n_e n_e \sigma_T c. \quad (29)$$

The calculation of the rate of pair creation, given by Equation (B4), involves convolutions over the photon distribution function, and is reviewed in Appendix B. In some calculations we include an additional source of cold pairs, derived from non-thermal relativistic particles, through a parameterized term (36) that is described in Section 3.6. In all,

$$\frac{dn_e}{dt} = - \frac{dn_e}{dt} \bigg|_{\text{ann}} + \frac{dn_e}{dt} \bigg|_{\gamma \gamma} + \frac{dn_e}{dt} \bigg|_{\text{nth}}. \quad (30)$$

If both pairs and photons follow thermal distributions with the same temperature $T$, then their densities have a simple relation. The chemical potentials are $\mu_\gamma = \mu_{e^+} = \mu_{e^-} = \mu$, all vanishing in a blackbody gas. Further restricting to $T \ll m_e c^2$, we have

$$n_e = n_{e^+} + n_{e^-} = 2 g_e \left( \frac{m_e T}{2 \pi \hbar^2} \right)^{3/2} e^{(\mu - m_e c^2)/T}. \quad (31)$$

and

$$\frac{1}{\hbar} \frac{dU_{\gamma}}{d\omega} \bigg|_{\text{ann}} = \frac{g_e}{2\pi^2} \left( \frac{m_e c^2}{\hbar} \right)^3 e^{(\mu - m_e c^2)/T}, \quad (32)$$

$$\frac{dE_e}{dt} = \frac{B^2}{8\pi n_e \ell_{\text{heat}}} - \frac{4}{3} g_e \sigma_T c U_{\gamma}. \quad (34)$$

Synchrotron cooling can be neglected here, because (1) $\gamma$ is low enough that the synchrotron emission is self-absorbed; (2) $f_{\text{th}} = 8\pi U_{\gamma}/B^2$ is perhaps as small as $\sim 0.1$, but not much smaller; and (3) the simplest heating mechanism, a cascade of Alfvén waves, creates a strongly anisotropic particle distribution with particle motion primarily along the magnetic field. (This anisotropic distribution is insensitive to cyclotron and firehose...
instabilities, given the extremely small value of the plasma \( \beta = 8\pi n_e k T_e / B^2 \).

At a high radiation compactness, the particles reach an equilibrium Lorentz factor

\[
\gamma^2 - 1 = \frac{3}{4 f_{\text{th}} d\tau_T / d\ln(t)} \left( \frac{\tau_{\text{heat}}}{T} \right)^{-1}.
\]

(35)

Given \( f_{\text{th}} \sim 0.1 \) and \( d\tau_T / d\ln(t) \sim 30 \), we see that \( \tau_{\text{heat}} \) must be shorter than \(-0.03 t \) for \( \gamma^2 \) to exceed \( m_e c^2 / E_{\text{pk}} \sim 10 \). This may be uncomfortably short for a Kelvin–Helmholtz instability driven by velocity shear on a lengthscale \( \sim cT / r \), but not for impulsive bursts of magnetic reconnection.

It is straightforward to incorporate this additional source of pairs into the Kompaneets calculation through an additional source term in Equation (30). The injection of pair rest energy, after averaging over the plasma volume, is described by a single parameter,

\[
m_e c^2 \frac{d n_e}{dt} \Big|_{\text{nth}} = f_{\text{nth}} \frac{d U_e}{dt} \big|_{\text{heat}} \quad (f_{\text{nth}} < f_{\text{rel}}),
\]

(36)

which depends on the energy spectrum and luminosity of the non-thermal particles. After rapidly cooling off the thermal photons, these pairs annihilate at the rate (29), leaving a net optical depth \( d\tau_{\text{nth}} / d\ln(t) \sim (16 f_{\text{nth}} \ell_{\text{th}} / 3)^{1/2} \). This dominates the optical depth of the thermally created pairs if

\[
f_{\text{nth}} > \frac{3}{16 \ell_{\text{th}}} \left[ \frac{d\tau_T}{d\ln(t)} \right]^2.
\]

(37)

A larger Compton parameter can now be maintained, leading to a more strongly peaked photon spectrum and a lower \( E_{\text{pk}} \).

3.7. Validity of the Kompaneets Equation at High Energies

The Kompaneets equation obviously cannot be used to evolve the \( \sim 511 \) keV annihilation feature in a cold pair gas. However, we are considering temperatures low enough that \( n_e m_e c^2 \) is a tiny fraction (typically much less than a percent) of \( U_\gamma \), and the Thomson scattering depth is moderately large. A demonstration that the annihilation line is weak at high \( \ell_{\text{th}} \) is provided by a full kinetic calculation of the photon and electron/positron distributions. (This code will be described in a separate publication.) The development of the spectrum in a non-expanding box up to a final compactness \( \ell_{\text{th}} = 10^3 \) is shown in Figure 3. The annihilation feature indeed becomes negligible; our calculations typically focus on yet higher \( \ell_{\text{th}} \).

We must also consider whether the calculated photon distribution is accurately described by the solution to the Kompaneets equation (16) near the pair-creation threshold. As long as the spectrum has a well-defined thermal peak, the solution to (16) is the same as the thermal equilibrium solution, with the high-frequency expansion \( dU_\gamma / d\omega \sim e^{-\bar{\omega} / T} \). The solution is therefore valid even though the approximation of Thomson scattering breaks down at \( \bar{\omega} \sim m_e c^2 \).

A photon temperature variable is easily extracted from a distribution of the form \( dU_\gamma / d\omega = K_\omega d^\omega e^{-\bar{\omega} / T_\gamma} \). Then \( \langle \bar{\omega} \rangle = \beta T_\gamma \), \( \langle \bar{\omega}^2 \rangle = \beta (\beta + 1) T_\gamma^2 \), and inverting gives

\[
T_\gamma = \frac{\langle \bar{\omega}^2 \rangle}{\langle \bar{\omega} \rangle} - \langle \bar{\omega} \rangle.
\]

(38)

Figure 3. Development of the photon spectrum at final compactness \( 10^3 \), using a full relativistic, kinetic treatment of Compton scattering and pair creation and annihilation. Steady heating, no expansion, with an initial excess of pairs leading to a prominent annihilation line during the early evolution. The line becomes insignificant at \( \ell_{\text{th}} \gtrsim 10^3 \).

(A color version of this figure is available in the online journal.)

A simple check is provided by a Wien distribution, \( \beta = 3 \), for which \( \langle h\omega \rangle = 3 T_\gamma \), \( \langle h\omega^2 \rangle = 12 T_\gamma^2 \). In a GRB-like spectrum (27) with \( \beta \gtrsim 0 \), one has instead

\[
T_\gamma \sim \frac{\langle h\omega^2 \rangle}{\langle h\omega \rangle}.
\]

(39)

3.8. Temperature Evolution of the Pairs

The pairs exchange energy both with thermal photons, and (a much smaller number of) annihilation photons of a somewhat higher frequency. We write

\[
\frac{dT_e}{dt} = \frac{dT_e}{d (K_e)} \left( \frac{d U_e}{dt} \big|_{\text{heat}} - \frac{d U_\gamma}{dt} \big|_{\text{ann}} + \frac{d U_\gamma}{dt} \big|_{\text{ann}} \right) - \frac{d e}{3t d \ln(p_e)},
\]

(40)

where the first two terms on the right-hand side are given by Equations (18) and (26), and \( \langle p_e \rangle \), \( \langle K_e \rangle \) are the mean thermal momenta and kinetic energies. We have also added to (18) a contribution from the net rate of change of photon energy due to pair creation and annihilation:

\[
\frac{d U_\gamma}{dt} \big|_{\text{ann}} = m_e c^2 \left( \frac{d n_e}{dt} \big|_{\text{ann}} - \frac{d n_e}{dt} \big|_{\text{ann}} \right)\left( \frac{d n_e}{dt} \right). 
\]

(41)

The final term in (40) represents adiabatic cooling. We now comment on the signs within the annihilation term.

In this situation, annihilation photons lose energy primarily by the Compton recoil off the colder electrons. Therefore, any energy put into annihilation photons goes quickly into the
kinetic energy of the pairs. Although energetic $e^\pm$ so created will return part of their energy to the photon field by Compton scattering before equilibrating with the thermal pair population by Coulomb scattering, the description of the pair distribution by a single temperature provides a self-consistent way of accounting for the rest energy of pairs created and destroyed. To be consistent, we must account for the creation of $e^\pm$ by high-energy photon collisions by subtracting their rest energy from the thermal energy of the existing particles.

4. RESULTS OF COMPTON EVOLUTION

We now present the results of a numerical solution of Equations (16), (30), and (40), starting with some semi-analytic considerations.

The range of $\ell_{th}$ and $U_{th}$ chosen is motivated by the breakout model of Section 3.1. At breakout, $(U_{th}/a)^{1/4}$ is a factor $\sim 3$–10 below the value ($\sim 25$ keV) where pairs freeze out. For example, a jet of photon luminosity $L_{\gamma,iso} \sim 10^{51}$ erg s$^{-1}$ and breakout Lorentz factor $\Gamma \sim 3$, which finally decouples from the slower sheath at a radius $R_{sheath} \sim 5 \times 10^{12}$ cm, has breakout compactness $\ell_{th} \sim 10^7$. This decreases to $\ell_{th} \sim 10^5$ for a jet with $L_{\gamma,iso} \sim 10^{53}$ erg s$^{-1}$, breakout Lorentz factor $\Gamma \sim 30$, and decoupling radius $R_{sheath} \sim 3 \times 10^{13}$ cm.

4.1. Slope and Normalization of the Low-frequency Spectrum in a Strongly Magnetized Plasma

A flat component of the photon spectrum generally appears at intermediate frequencies in a dilute gas approaching thermodynamic equilibrium by a combination of soft photon emission and Compton scattering (Pozdnyakov et al. 1983). That is the solution to the Kompaneets equation corresponding to a constant flux of photons $F_{\gamma}$ upward in frequency space. Equation (16) can be rewritten as

$$\frac{\partial}{\partial t} \left( \frac{\omega^2}{\pi^2 c^3} N(\omega) \right) = -\frac{\partial F_{\gamma}}{\partial \omega}. \quad (42)$$

Neglecting the stimulated term, which is important in a stationary plasma only on the Rayleigh–Jeans tail, the stationary power-law solution is $N(\omega) \propto \omega^{-3}$, corresponding to $U_\omega = (h\omega^3/\pi^2 c^3) N \sim \text{const}$.

Here we consider how this component connects with a thermal peak (near which Doppler upscattering by thermal $e^\pm$ motions is balanced by recoil energy loss). We show that a distinct Wien peak does not form in a strongly magnetized, dynamic equilibrium by a combination of soft photon emission and pair scattering before equilibrating with the thermal pair population.

At lowest frequencies, the spectrum is blackbody, and breaks to $U_\omega \sim \text{const}$ at a frequency $\omega_\omega$ and harmonic $m_\omega = \omega_\omega/\omega_{ce}$ where the Compton upscattering rate

$$\left. \frac{1}{\omega} \frac{d\omega}{dt} \right|_c \sim 4 \bar{T}_e \sigma_T c = \alpha_{cyc}(\omega)c. \quad (43)$$

Here

$$\alpha_{cyc}(\omega) = \frac{\hbar\omega}{4\pi B_\omega} d^2 N_{cyc} / d\omega dt = \frac{\hbar\omega}{4\pi B_\omega} d^2 N_{cyc} / d\omega dt \quad (44)$$

is the absorption coefficient, $d^2 N_{cyc} / d\omega dt$ is the rate of emission of cyclo-synchrotron photons by a single $e^\pm$ (Figure 1), and the

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.png}
\caption{Break frequency $\omega_\omega = m_\omega \omega_{ce}$ separating the low-frequency Rayleigh–Jeans tail from the intermediate, flat component of the spectrum. Colors correspond to different magnetic energy densities, normalized to the QED magnetic field $B_0 = 4.4 \times 10^{15}$ G. Only temperatures exceeding $(0.1 \times 8^2/\pi \varepsilon_{SB})^{1/4}$ are plotted. The irregularities at low $T_e$ reflect the harmonic structure of the emissivity. (A color version of this figure is available in the online journal.)}
\end{figure}

Planck function $B_\omega = g_\gamma(2\pi)^{-3}\omega^2 T_e/c^2$ when $\hbar \omega \ll T_e$. For completeness, we correct for the enhancement in upscattering by trans-relativistic $e^\pm$, for which

$$\left. \frac{1}{\omega} \frac{d\omega}{dt} \right|_c \sim \frac{4}{3} (\gamma^2 - 1) \sigma_T n_e c = 4 f_{\text{rel}} \bar{T}_e \sigma_T n_e c \quad (45)$$

at low frequencies. If the pairs follow a Boltzmann distribution (19), the correction factor is

$$f_{\text{rel}} \equiv \bar{T}_e + \langle \gamma \rangle; \quad \langle \gamma \rangle = 3 \bar{T}_e \frac{K_1(1/\bar{T}_e)}{K_2(1/\bar{T}_e)}. \quad (46)$$

These equations give the implicit relation for $m_\omega(T_e)$,

$$\frac{1}{m_\omega(T_e) f_{\text{rel}}(T_e) T_e^2} \left. \frac{d^2 N_{cyc}}{d\omega d\omega} \right|_{m_\omega(T_e)} = \frac{32 \alpha_{cyc}^2}{3\pi} \left( \frac{B}{B_0} \right). \quad (47)$$

Here $B_0 = m_e c^3/e\hbar = 4.4 \times 10^{15}$ G is a convenient normalization of the magnetic field. (Near breakout of a GRB jet one may expect $B \sim 10^{-4} B_0$.) The left-hand side of this equation is a function only of $T_e$, showing that $m_\omega(T_e)$ depends weakly on $B$. The solution, for the exact cyclo-synchrotron emissivity, is shown in Figure 4.

The photon spectrum is $U_\omega = \omega_\omega^2 T_e/\pi^2 c^3$ in the frequency range $\omega_\omega < \omega < E_{pk}/\hbar$. Connecting this with a spectrum of the form (27), up to a peak energy $E_{pk} \simeq 3T_e$, gives

$$f_{th} = \frac{24\alpha_{cyc} m_e^2}{\pi} (\bar{T}_e^2 \bar{T}_e) = 0.056 \left( \frac{T_e}{0.05 m_e c^2} \right)^2 \left( \frac{m_\omega}{20} \right)^2. \quad (48)$$

We may interpret this equation as follows: the low-frequency cyclo-clotron bath does not supply enough photons to form a localized thermal peak (with a low-frequency slope $d \ln F_\omega / d \ln \omega > 0$) as long as the fraction of the magnetic energy density converted to thermal radiation is smaller than (48).
4.2. Constant Heating in a Static Medium

We consider plasmas with a range of final energy density, \( U_{\text{th}} = 10^{-10} - 10^{-8} m_e c^2 / \lambda_c^3 \). This corresponds to \( 10^2 - 10^4 \) times the minimal breakout energy density defined in Section 3.1. The development of the spectrum is shown in Figure 5. The dependence of the output spectrum on compactness is shown in Figure 6. This spectrum is quite flat below the peak in a range of final compactness \( \ell_f \sim 10^2 - 10^6 \), especially for \( f_{\text{th}} = 0.1 \), and is still much flatter than Wien when \( \ell_f = 10^7 \).

4.3. Continuous Heating in an Expanding Medium

We now consider expanding plasma with the thermal profile (14). The net expansion factor is taken to be 30–100, corresponding to a factor \( 10^{-3} - 10^{-4} \) drop in energy density between the onset of heating (occurring at \( \sim 0.01 - 0.03 \) times the breakout radius), and the onset of free expansion following the emergence of the jet. The instantaneous compactness (1) scales as \( \ell(t) \propto t^{-1} \) during heating, and we label the runs by the initial compactness \( \ell_0 \equiv \ell_{\text{th}}(t_0) \).

We stop heating at a fixed time \( t = 0.3t_{\text{tot}} \), after which the plasma suffers adiabatic losses, the temperature drops, and pairs rapidly annihilate. As the scattering depth approaches unity, the differential drift between photons and pairs begins to counterbalance adiabatic cooling. The imprint of bulk Compton scattering on the photons propagating through the photosphere of an accelerating jet is calculated separately in Section 7.

Figure 7 shows the development of the spectrum starting from a seed Wien peak. For an expansion factor \( t_{\text{tot}}/t_0 = 10^2 \), Equation (15) implies that \( \sim 98.5\% \) of the final energy density results from distributed heating. Indeed, the spectrum reaches its final form, modulo changes in amplitude and the position of the Rayleigh–Jeans tail, well before the expansion is complete.

The dependence of the final spectrum on compactness and bulk-frame magnetization (\( \sim f_{\text{th}}^{-1} \)) is shown in Figure 8. In agreement with the analytic argument advanced in Section 4.1, reducing the thermalization efficiency \( f_{\text{th}} \) flattens the connection between the \( F_\omega \) \( \sim \) constant component of the spectrum, and the thermal peak.

Details of the evolution of \( T_e \) and \( \tau_T \) are shown in Figures 9 and 10. The cutoff in heating leads to a rapid drop in electron temperature and annihilation of pairs. The spectral peak frequency experiences a more gradual change. Once \( \tau_T \) drops below \( \sim 5 \), the continuing reduction in scattering depth is driven...
Figure 6. Final photon spectrum, fixed volume, as a function of final compactness. Final energy density $U_{th} = 10^{-10}, 10^{-8} mec^2/\bar{\lambda}^3_c$ (left, right), and magnetization $8\pi U_{th}/B^2 = 0.1, 1$ (top, bottom). The flat part of the spectrum connects more directly to the thermal peak as the magnetization is raised—see Equation (48).

Figure 7. Development of the photon spectrum in an expanding volume with initial (final) thermal energy density $10^{-8}(10^{-12}) mec^2/\bar{\lambda}^3_c$, net expansion factor $t_{tot}/t_0 = 10^2$ and heating turned off at $10^{-0.5} t_{tot}$. After expansion by a decade or so, the spectrum is insensitive to the seed spectrum (solid red line: Wien; dotted red line: GRB-like (27) with low frequency Rayleigh–Jeans cutoff; both $T_0 = 0.03 m e^2$). Initial thermal compactness $10^4$ (left) and $10^6$ (right).

(A color version of this figure is available in the online journal.)
Figure 8. Effect of magnetization and compactness on the final photon spectrum in an expanding, magnetized pair plasma. Initial (final) thermal energy density $10^{-8}(10^{-12})m_e c^2/\bar{\lambda}^3$, net expansion factor $t_{tot}/t_0 = 10^2$, and heating turned off at $10^{-0.5}t_{tot}$. Here we have compensated the effect of adiabatic expansion on the spectral amplitude in models which reach $\tau_T = 1$ at $t < t_{tot}$, so as to afford a direct comparison. (A color version of this figure is available in the online journal.)

Figure 9. Pair temperature corresponding to Figure 8. $T_e$ rapidly adjusts downward to Compton equilibrium after heating turns off at $10^{-0.5}t_{tot}$. Dotted red curves: term (17) included in Kompaneets equation, representing differential acceleration of the magnetic field and entrained $e^\pm$ pairs across the photon field near the scattering photosphere. Solid black curves: drag term turned off. Green curves: Compton temperature corresponding to black curves. (A color version of this figure is available in the online journal.)

Figure 10. Optical depth to Thomson scattering, $d\tau_T/d\ln(t) = n_e(t)\sigma_T c t$. Corresponding temperature evolution in Figure 9. Dotted red curves include term (17) in the Kompaneets equation, solid black curves do not. Solid blue curves: addition non-thermal pair source term (36) in Equation (30). Dashed green curves: semi-analytic model of soft photon creation, Compton upscattering, and pair creation (Section 5). (A color version of this figure is available in the online journal.)

The output values of $T_e$ and $E_{pk} \simeq \langle \hbar \omega \rangle^2/\langle \hbar \omega \rangle$ are shown in Figure 13 as a function of compactness, and the time at which heating is turned off. As long as the gas passes through a brief adiabatic, expansionary phase, we find that the output peak energy clusters around $E_{pk} \sim 0.1m_e c^2$ for final compactness $\gtrsim 10^4 - 10^5$, and extends upward to $\sim 0.2m_e c^2$ for final compactness $\sim 10^2$.

4.4. Effect of Non-thermal Pair Creation

Here we consider the effect of the injection of non-thermal particles on the output spectrum, as parameterized by the
yield (36) of cold pairs that supplement the thermal particle density. To keep the calculation self-consistent, we only consider a high radiation compactness, ranging from $\ell_0 = 10^6$ down to $\ell_f = 10^4$ in the example given. The energy injected directly in cold pairs extends from $f_{nth} = 10^{-5}$ up to $10^{-2}$ of the heat that is deposited gradually in the thermal pairs.

The Compton parameter of the cold pair gas rises significantly at the larger values of the non-thermal energy fraction: see Figure 14. As a result, the spectrum is more strongly peaked (Figure 15), with a harder spectrum right below the peak. While heating is ongoing, the peak of the spectrum is pushed to a lower frequency due to the increased efficiency of soft photon creation and upscattering. We preserve the temporal heating profile that was applied previously to purely thermal plasmas. After heating turns off, at $t = 10^{-0.5}t_{tot}$, the pairs rapidly annihilate and the optical depth through them converges to a common value.

We conclude that moderate rates of non-thermal heating ($f_{nth} \sim 10^{-3} - 10^{-2}$) result in a harder low-frequency spectrum than is usually measured in GRBs.

### 4.5. Minimal Magnetization

After heating turns off and the pair plasma reaches a scattering depth about a few, further annihilation freezes out. We then obtain an estimate of the magnetization in the outflowing material.
The magnetization as a function of time, up until freeze-out, is shown in Figure 16. The cumulative Compton parameter in cold pairs is several times larger than the plotted value, and is \(10^8\) times greater than that supplied directly by the cascading charges (Equation (8)). (A color version of this figure is available in the online journal.)

The magnetization as a function of time, up until freeze-out, is shown in Figure 16. By considering the evolution of a pure pair plasma, we are implicitly setting a lower bound on the magnetization imposed by the ion inertia. Setting the number density of protons in the outflow \(Y_e \rho_{ion}/m_p\) equal to \(n_e^- + n_e^+\) gives the critical magnetization

\[
\sigma_{ion,crit}^{rest} = \frac{B^2 Y_e}{4\pi(n_e^- + n_e^+) m_p c^2}.
\]

Re-writing this in terms of the breakout compactness (11) and transforming to the frame of the engine gives

\[
\sigma_{ion,crit}(\tau_{es}) = 1 = \Gamma \sigma_{ion,crit}^{rest} = \frac{2 \Gamma}{f_{th}} \left( \frac{m_e}{m_p} \right) \ell_{th}
\]

\[
\approx 3.6 \times 10^5 \frac{L_{Y,iso,51}}{f_{th,cool,1}} \left( \frac{\Gamma}{3} \right)^{-3}
\]

at the scattering photosphere \(\sigma_T(n_e^- + n_e^+)(r/\Gamma) = 1\).

5. SCALING SOLUTION FOR OPTICAL DEPTH AND TEMPERATURE IN AN EXPANDING MEDIUM

A useful check of the numerical results described in Section 4 is provided by a simple scaling model. This applies to the initial transient phase during which the plasma reaches a self-similar behavior, and most of the co-moving photon number is accumulated. Thereafter, according to Equation (14), the (differential) rate of photon creation drops off.

The state of an expanding, and continuously heated, thermal pair plasma is conveniently described by \(T_e\) and the optical depth \(d\tau/T / d\ln(t) \equiv \sigma_T n_{e^-} c \ell_{th}\). We consider expansion at constant Lorentz factor, with magnetic energy density \(~\ell^2\), bulk-force volume \(~\ell^3\), and constant ratio of injected thermal to magnetic energy, corresponding to \(\delta = 2\) in Equation (14). Generalization to other expansion profiles is straightforward.

We found that \(T_e\) and \(\tau)\) vary slowly and in opposing ways, so that \(d\tau_c / d\ln(t)\) is nearly constant. The net Compton parameter accumulates logarithmically with time. In the flat portion of the spectrum, the left- and right-hand sides of (16) both approximately vanish. Then one can write

\[
\frac{1}{f^5} \frac{d}{dt} \left( f^2 \frac{\partial n_{\gamma}}{\partial \omega} \right) = - \frac{\partial F_{\gamma}}{\partial \omega} \approx 0, \quad (52)
\]

where \(\partial n_{\gamma}/\partial \omega = N_{\omega} \sigma_T^\gamma c \langle \omega \tau \rangle \) and

\[
F_{\gamma} = n_e \sigma_T c \left( \frac{T_e}{m_e c^2} \right)^3 \frac{3N}{\pi^2} \propto n_e \omega_e^2 \propto t^{-3}
\]

is the rate at which photons flow toward the spectral peak, per unit volume. In the approximation that all the photons are near the peak, these equations integrate to \(n_{e^-}(t) = F_{\gamma}(t)\).

The break frequency \(\omega_b = m_e(T_e) \omega_{ce}\) bounding the low-frequency blackbody tail is given in Figure 4. Further setting \(\langle \omega_{\gamma} \rangle n_{\gamma} = f_{th,B}^2/8\pi\), and writing \(\langle \omega_{\gamma} \rangle = f_{th,T_e}\), one obtains a relation between temperature and optical depth,

\[
\frac{d\tau_c(T_e)}{d\ln(t)} m_{\gamma}^2(T_e) f_{rel}(T_e) \tilde{\tau}_{\gamma} = \frac{\pi}{24\sigma_{em}} \frac{f_{th}}{f_{\omega,C}}.
\]

As long as pair creation and annihilation are nearly in equilibrium, Equation (33) implies,

\[
\frac{d\tau_c(T_e)}{d\ln(t)} = \left( \frac{\pi}{2} \right)^{1/2} \frac{f_{th,T_e}}{f_{\omega,T_e}} \frac{f_{th}}{f_{\omega,C}} e^{-3/2} e^{-1/\tilde{\tau}_{\gamma}},
\]

giving

\[
m_{\gamma}^2(T_e) f_{rel}(T_e) \tilde{\tau}_{\gamma} = 9.6 \left( f_{\omega,C}/f_{\omega,T_e} \right).
\]

The coefficients \(f_{\omega}\) in Equations (54) and (55) are not entirely equivalent. The number density of photons at the pair-creation threshold is determined essentially by \(T_e\) and \(U_p\). The photon energy density receives only a modest supplement from the flat-spectrum band; one finds \(f_{\omega,T_e} \approx 3.9\) for a spectrum of the form (27), as compared with 3 for a Wien spectrum.

On the other hand, Equation (54) is derived in the approximation that all the photons are upscattered to a thermal peak, whereas in fact the flat-spectrum band contributes logarithmically to the total. We find that taking \(f_{\omega,c} = 3/\ln(E_{pk}/m_e \omega_{ce})\) allows an accurate fit of this semi-analytic model to the transient peak, for a compactness varying between \(\sim 10^5\) and \(\sim 10^7\). See the dashed green curves in Figure 10. These values of \(f_{\omega,T_e}, f_{\omega,C}\) are used to construct the plots here and in Section 6.

The electron temperature is shown in Figure 17, and the optical depth and Compton parameter in Figure 18. The differential optical depth \(d\tau_c / d\ln(t)\) varies only slowly with compactness. The cumulative \(\tau_c\) and \(\gamma_c\) are significantly larger: see the dashed lines in Figure 18. Photon creation is dominated by the cyclo-synchrotron channel (Figure 19).
Figure 15. Output spectrum, spectral peak energy, and spectral slope in the simulations of Figure 14. As the optical depth in cold pairs increases, due to the increased efficiency of non-thermal particle injection, the spectrum hardens and forms a more concentrated peak. The peak frequency is pushed lower, due to the increased rate of soft photon creation and upscattering. Processing of this spectrum through the photosphere of an accelerating MHD jet reduces the peak spectral index by $\sim 0.5$ (Section 7).

(A color version of this figure is available in the online journal.)

6. STRONGLY MAGNETIZED OUTFLOW WITH BARYON-DOMINATED PHOTOSPHERE

The presence of baryons in the outflow imposes a lower bound on the scattering depth. This has two effects: first, there is strong adiabatic cooling of any thermal photon gas unless the baryon loading is fine-tuned to a critical value (Shemi & Piran 1990) or, alternatively, unless the outflow is heated continuously out to its photosphere (Thompson 1994; Spruit et al. 2001; Giannios 2006). Second, a large Compton parameter develops at large $\tau_T$, pushing the photons closer to a Wien peak, with a harder low-frequency spectrum.

As an example, consider an expanding plasma with the same heating profile as we have studied previously: a cutoff in heating is followed by adiabatic expansion. But now we introduce baryons into the outflow and allow the expansion to continue well beyond our previous cutoff time $t_{\text{tot}}$, so that the integration is stopped only when $d\tau_T/d\ln(t) < 1$. For heavily baryon-dominated outflows, the plasma must expand by an additional factor $\sim 30$.

The optical depth as a function of time is shown in Figure 20 for a bulk-frame compactness $\ell_\text{th} = 3 \times 10^4$ at the end of heating, thermal energy fraction $f_\text{th} = 0.1$, and magnetization

$$\sigma_{\text{rest}} = \frac{B^2}{4\pi \rho_{\text{ion}} c^2}$$

ranging from 200 down to 2. (Here $\rho_{\text{ion}}$ and $B$ are the proper ion rest mass density and magnetic flux density.) At the higher values of $\sigma_{\text{ion}}$, the scattering depth is dominated by the pairs, but there is a transition to a nearly pair-free gas as the magnetization is reduced (Figure 21). For the highest magnetization runs, the
Figure 16. Magnetization $\sigma_{\text{rest}} = B^2/4\pi(n_{e+} + n_{e-})m_ec^2$ in the rest frame of expanding outflows of initial thermal compactness $\ell_0 = 10^5 - 10^7$, with heating turning off at $10^{-0.5}t_{\text{tot}}$. See also Figures 9–13. When the plasma is pair-dominated, the magnetization due to an electron–ion component is at most a fraction $Y_e/1836$ of this. A somewhat higher compactness, and therefore magnetization, is implied during jet breakout (Equations (11) and (51)) than is provided by our calculations.

Figure 17. Pair temperature in a continuously heated and expanding outflow, as a function of the magnetic compactness $\ell_B$, according to the scaling solution derived in Section 5. Thermal (mainly photon) energy density $U_{\text{th}} = 0.1(B^2/8\pi)$. Temperature is regulated by a competition between two effects: Compton up-scattering of thermal cyclo-synchrotron photons, against the exponential dependence of pair depth on temperature.

(A color version of this figure is available in the online journal.)

The photosphere is reached at a compactness $\ell_{\text{th}} \sim 10^4 (t \sim t_{\text{tot}}$ in the figures). The magnetization below which ions dominate the scattering is obtained from Equation (51) to be $\sigma_{\text{ion, crit}} \sim 1 \times 10^2$, consistent with Figure 21.

Figure 18. Thomson scattering depth and Compton parameter, per logarithm of expansion time $t \propto \ell_B^{-1}$, in the same system described in Figures 4 and 17. The cumulative $\tau_T$ and $y_C$, integrated forward from time corresponding to $\ell_B = 10^6$, is shown in the dashed curves.

(A color version of this figure is available in the online journal.)

The effect on the output spectrum, including the peak energy and the spectral slope below the peak, is shown in Figure 22. A reduction in $E_{\text{pk}}$ without change in slope is mainly caused by the additional expansion. This reduction is concentrated at a magnetization $\sigma_{\text{ion}} \sim 0.1 \sigma_{\text{ion, crit}}$ (strong reduction in $E_{\text{pk}}$).

Eventually, as the magnetization is decreased further, the spectrum hardens below the peak. This is due to the saturation of $y_C$, and the increased soft photon flux toward the peak.

There is an interesting application to X-ray flashes here, which is discussed further in Section 10.
Figure 19. Relative contributions of thermal cyclo-synchrotron, free–free, and double-Compton emission to the creation of soft photons, in the same system described in Figures 4, 17, and 18. (A color version of this figure is available in the online journal.)

Figure 20. Scattering depth in outflows of initial thermal compactness 10^{5.5}, thermal energy density \( f_{th} = 0.1 \) and various bulk-frame magnetizations, \( B_{q}^{2}/4\pi\rho c^{2} = 2 \) (top, red), up to 200 (bottom, blue) in logarithmic increments of 0.1. Flows dominated by the electron–ion component show \( \tau_{T} \propto t^{-1} \), whereas the pair-dominated flows follow a shallower profile after a transient dominated by the annihilation of an initial excess of pairs. (A color version of this figure is available in the online journal.)

6.1. Low Emergent Peak Energy

The transition between pair-dominated and baryon-dominated outflows can also be considered using the semi-analytic model of Section 5. The critical baryonic magnetization is obtained from Equation (50). The result is shown in Figure 23.

Figure 21. Ratio of scattering optical depth to \( e^{+} - e^{-} \) pairs and that due to the electron–ion component, in the same sequence of outflows shown in Figure 20. (A color version of this figure is available in the online journal.)

During a phase of continuous heating, the analog to Equation (54) for the temperature is

\[
m_{\star}^{2}(T_{e}) \frac{T_{e}}{T_{bb}} = \frac{\pi f_{th}}{36\alpha_{em} f_{T}} = \frac{\pi f_{th} \sigma_{rest}^{\text{ion}}(m_{p}/m_{e})}{72\alpha_{em} f_{B}}.
\]

(59)

Here we have taken \( f_{\omega} = \langle \tilde{\omega} \rangle / T_{e} = 3 \) and made use of \( n_{\omega} \simeq (1/2)F_{\omega} \). The result is shown in Figure 24. It is always larger than the equivalent blackbody temperature \( T_{bb} = (f_{th} B_{q}^{2}/8\pi a_{SB})^{1/4} \), except for the flat part of the lowest-magnetization curve.

In contrast with the pair-dominated plasma, the optical depth drops only gradually after heating turns off in a baryon-dominated plasma. Therefore the Wien temperature shown in Figure 24 may significantly exceed the emergent temperature—as is demonstrated by the numerical solutions of the preceding section.

Finally, the soft-photon output through the double-Compton and free–free channels, related to thermal cyclo-synchrotron emission, is shown in Figure 25.

6.2. Neutron-rich Electromagnetic Outflows?

GRB outflows may contain a significant number of neutrons. This is the case if the nuclear composition evolves via weak interactions between nucleons and charged leptons (Derishev et al. 1999; Bahcall & Mészáros 2000; Rossi et al. 2006). The radius at which neutrons and charged ions decouple depends strongly on the Lorentz factor profile of the outflow, as well as on the magnetization (57). The neutron loading of the outflow also will vary with angle: a wind emanating from a torus orbiting a black hole should be neutron-rich. We focus here on the jet core, the source of the prompt gamma-ray emission.

When the magnetization is very large, as considered here, two major changes occur. First, the dissipation associated with n-ion collisions becomes insignificant, because ions carry a negligible fraction of the energy flux. Second, the \( n/p \) ratio in a Blandford–Znajek jet may be very different (significantly lower) than that in unmagnetized matter, because positron capture on
neutrons is greatly accelerated by the large magnetic phase space factor. We ignore this second complication here, and in order to explore the energetics assume comparable numbers of neutrons and protons in the outflow.

A further suppression in the dissipation rate due to $n$-ion collisions arises from a shallow Lorentz factor profile before jet breakout. The dissipation rate per ion is proportional to the Lorentz factor at decoupling between neutrons and ions, so the fraction of the jet energy flux that is dissipated is $\sim \Gamma/\sigma$. A self-similar, magnetized jet relaxes to $\Gamma \sim 1/\theta$ (Lyubarsky 2009) (although strong departures from this are possible in the non-self-similar density profile of a collapsing stellar core). This profile implies a lower Lorentz factor at $n$-ion decoupling, and therefore a lower dissipation rate. The implications of such a Lorentz factor profile for the observed peak energies of GRBs, in combination with our spectral results, are discussed in Section 9.

It is worth separating out these two effects, because it is certainly possible in principle to have a magnetized jet with a higher baryon loading, but a relatively small Lorentz factor at breakout. Neutrons and ions (here idealized as protons) decouple where the optical depth for $n-p$ collisions (cross section $\sigma_{pn} \sim 3 \times 10^{-26}$ cm$^2$) is about unity (Rossi et al. 2006; Beloborodov 2010). In an outflow with large ion magnetization (57), this occurs at a radius

$$R_{pn} \sim \frac{L_{P\text{iso}}\sigma_{pn}}{4\pi \sigma_{ion}\Gamma^2 m_n c^3},$$

where $L_{P\text{iso}}$ is the isotropic Poynting luminosity (70). Beyond this radius, the neutrons and ions develop a relative speed $\sim c$, and inelastic collisions (with a cross section $\sim 0.1\sigma_{pn}$) create pions, and multiple pairs by a cascade process. In an unmagnetized fireball, one sets $L_{P\text{iso}} \rightarrow L_{m\text{iso}}$ (the matter

**Figure 22.** Output spectrum of the same sequence shown in Figures 20 and 21. In more baryon-dominated flows, the spectral slope below the peak rises significantly, and part of the reduction in $E_{pk}$ is due to enhanced upscattering of soft photons ($\gamma_C$ is larger). The remaining reduction in $E_{pk}$ is due to adiabatic cooling, and is therefore model-dependent. Processing of this spectrum through the photosphere of an accelerating MHD jet reduces the peak spectral index by $\sim 0.5$ (Section 7).

(A color version of this figure is available in the online journal.)
Figure 23. Critical magnetization, defined in Equation (50), below which the electron–ion component dominates the scattering optical depth, and pairs largely freeze out. Applies to the scaling solution for an expanding pair plasma derived in Section 5, with scattering depth shown in Figure 18. Critical magnetization at the photosphere is enhanced by a factor $\tau_T$. (A color version of this figure is available in the online journal.)

Figure 24. Limiting temperature resulting from thermal cyclotron emission in a electron–ion dominated plasma. Magnetic compactness varies from $10^4$ down to 1 (blue to black). (A color version of this figure is available in the online journal.)

Figure 25. Relative importance of double-Compton (solid) and free–free (dashed) emission to the production of soft photons, in an electron–ion dominated plasma. Line colors correspond to those in Figure 24. (A color version of this figure is available in the online journal.)

Figure 26. Kinetic luminosity and $\sigma_{\text{ion}} \rightarrow \Gamma$. Then one finds that the compactness at $R_{pn}$ takes a large value,

$$\ell(R_{pn}) = \frac{L_{\text{miso}} \sigma_T}{4 \pi m_e c^3 R_{pn}} \sim \frac{\sigma_T}{\sigma_{\text{ion}}} \frac{m_p}{m_e} \sim 4 \times 10^4,$$

independent of the details of the flow.

The spectral signature of this process has been calculated in detail by Vurm et al. (2011) for a relatively weak magnetization, $\sigma_{\text{ion}} \lesssim \Gamma (\sigma_{\text{rest}} \lesssim 1)$, and in the presence of a seed blackbody radiation field. Synchrotron cooling off the magnetic field prevents the formation of a hard spectral tail when $\sigma_{\text{ion}} \gtrsim 0.1 \Gamma$, but supplements the low-frequency spectrum by self-absorbed synchrotron emission with a frequency-dependent photospheric radius. (Essentially the same mechanism was invoked by Blandford & Königl (1979) to explain flat-spectrum radio emission from relativistic jets with power-law particle distributions.)

The size of this decoupling zone $R_{pn}$ depends sensitively on the acceleration profile of the jet. Beloborodov (2010) considered an unmagnetized, neutron-loaded fireball with Lorentz factor increasing linearly with radius from an engine of size $R_s \sim 100$ km. Then $\Gamma(R_{pn}) \sim 270 L_{\text{iso},51}^{1/4}$ and $R_{pn} \sim 2.7 \times 10^9 L_{\text{iso},51}^{1/4} R_s,7$ cm. In a magnetized jet with $\Gamma \sim 1/\theta$ and $\sigma_{\text{ion}} \gg 1$, this changes to

$$R_{pn} \sim 4 \times 10^{10} L_{\text{iso},51}^{1/3} \sigma_{\text{ion}}^{-1} \left(\frac{\Gamma}{3}\right)^{-2} \text{ cm.} \quad (62)$$

Here we have normalized the magnetization to the level (51) that gives a pair-dominated thermal photosphere. The fraction of the jet energy that is carried by the baryons is tiny,

$$\frac{\Gamma}{\sigma_{\text{ion}}} = 3 \times 10^{-5} \sigma_{\text{ion}}^{-1} \left(\frac{\Gamma}{3}\right), \quad (63)$$

and so dissipation by $n$-ion collisions can be neglected.

How sensitive is this conclusion to the assumed value of $\sigma_{\text{ion}}$? A robust lower bound $\sigma_{\text{ion}} \gtrsim \Gamma \sim 10^2$–$10^3$ is needed to create a GRB. Let us suppose that $\Gamma \sim \theta^{-1}$ out to a distance $R_{\text{exp}} \sim 10^{11}$ cm from the engine, followed by free expansion, $\Gamma \sim \theta^{-1} (r/R_{\text{exp}})$. Then one finds $R_{pn} \sim 4 \times 10^{11} L_{\text{iso},51}^{1/3} (3\theta)^{2/3} R_{\text{exp},11}^{-1/3} \sigma_{\text{ion},3}$ cm, and Equation (63) is replaced by

$$\frac{\Gamma}{\sigma_{\text{ion}}} \sim 4 \times 10^{-3} \left(\frac{L_{\text{iso},51}^{1/3} (3\theta)^{2/3}}{R_{\text{exp},11}^{-1/3} \sigma_{\text{ion},3}}\right). \quad (64)$$
Only $\sim 10\%$ of the neutron kinetic energy is converted to non-thermal pairs through the pion-creating channel. We conclude that a shallow Lorentz factor profile, by itself, tends to suppress the level of dissipation by $n$-ion collisions.

7. SPECTRAL IMPRINT OF BULK COMPTON SCATTERING DURING JET BREAKOUT

As a jet breaks out of a confining medium, it experiences rapid acceleration as magnetic flux surfaces diverge (e.g., Tchekhovskoy et al. 2010). This fast expansion is also plausibly associated with a sudden drop in bulk heating, and a rapid annihilation of electron–positron pairs. In Russo & Thompson (2013a, 2013b), we calculated the imprint of bulk Compton scattering on the outgoing gamma-ray spectrum, assuming that the input spectrum was GRB-like ($F_\omega = \text{const}$ at low frequency, with an exponential cutoff above a frequency $\omega_0$). The net result was that the spectral peak was pushed higher in frequency, along with the low-frequency flat spectrum.

We re-examine this effect here because it further flattens the spectrum below the peak. It is also of interest because some evidence for upscattering during jet breakout may be present in low-energy GRB spectra (Axelsson et al. 2012; Tierney et al. 2013).

Here we replace this simplified input spectrum with the one obtained in Section 4.3 for a continuously heated, and expanding, magnetized pair plasma. We extract the spectrum after heating has stopped, and as the pairs are annihilating. To facilitate comparison, the frequency is normalized to the peak $\omega_0$ of $\omega F_\omega$ in this input spectrum.

The dependence of jet Lorentz factor on radius is calculated using the method of Section 2 in Russo & Thompson (2013b), as plotted in Figures 3–5 of that paper. We consider a range of scattering depth at breakout, varying from 0.1 up to 10, and choose a model with $f_{\text{bh}} = 0.1$. The result, for an initial (final) thermal compactness $\ell_0 = 10^6 (\ell_f = 10^4)$, is shown in Figure 26. The slope of $F_\omega$ is shown in Figure 27, along with the result for the $\ell_0 = 10^5$ plasma. One observes in the slope an extended low-frequency plateau with $d \ln (F_\omega)/d \ln \omega = 0$, and a localized bump that extends to $\sim 0.7–0.9$ at $\omega \sim \omega_0/3$. This peak in $d \ln (F_\omega)/d \ln \omega$ is significantly reduced in the scattered spectrum, by $\sim 0.5$, starting from an input optical depth $\sim 10$.

8. FOCUSED POYNTING-DOMINATED JETS

GRBs emit such extreme fluxes of radiant energy—isotropic energies reaching at least $E_{\gamma,\text{iso}} \sim 10^{55}$ erg in the source rest frame (Abdo et al. 2009)—that purely hydrodynamic modes of energy transport are disfavored. In principle, a jet could be accelerated within a de Laval nozzle (Blandford & Rees 1974) that forms in the envelope material: for example, along the rotation axis in a collapsar, where the ram pressure of the infalling material is reduced.

We first, as an idealized problem, consider such a hydrodynamic jet emerging from a trapped, quasi-spherical bubble of relativistic fluid. Given that the bubble energy cannot exceed the binding energy of the stellar core material, we use realistic pre-collapse stellar models to show that this collimation model is inconsistent with the largest observed burst energies.

Collimation of a magnetized jet begins at the engine. The outflow from the black hole ergosphere is surrounded by a slower and broader outflow from the inner torus, evidence for which is now seen in global disk simulations (e.g., Sadowski et al. 2013). Indeed, collimation of a magnetized jet requires an external medium (Lynden-Bell 2003, and references therein), which generally is not bound to the engine and can carry a comparable energy to the faster jet core.

In a collapsar, this broader outflow would still interact strongly with the collapsing core material. This provides a feedback loop that turns off accretion when the jet energy exceeds a critical value, and allows us to connect the observed (isotropic-equivalent) jet energy to the binding energy of the pre-collapse core (Thompson et al. 2007). This issue is usually side-stepped in setting up numerical models of jet propagation in stellar envelopes, which generally assume strong collimation at the jet base.

The simple model of a magnetized jet described in Section 8.2 fails to account for the uncertain details of the confining medium by relating the apparent jet energy to the magnetic flux threading the black hole, and thence to the binding energy of the stellar core. We note that Poynting-dominated jets also allow for a strong focusing of streamlines toward the jet axis by magnetic pressure gradients (Lynden-Bell 2003, and references therein). In the extreme case of a line current, the magnetic pressure varies as $\theta^{-2}$ with angular distance from the jet axis. The extent to which magnetized jets form such profiles is still ambiguous.

In the last part of this section, we work out the relation between isotropic jet luminosity and opening angle in a steady, axisymmetric, and highly magnetized jet. Combining this with the spectral results of Section 4, we update the results of Thompson et al. (2007) in Section 9 to give a first-principles derivation of the Amati et al. boundary in the plane labeled by observed peak energy $E_{\gamma,\text{pk}}$ and $E_{\gamma,\text{iso}}$.

8.1. Peak Isotropic Luminosity of a Hydrodynamic Jet Flowing from a Confined Bubble

First consider a hydrodynamic jet emerging from a bubble of hot plasma injected into the core of a massive star. The net
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Figure 27. Top panel: spectral slope $d \ln (F_\omega) / d \ln \omega$ corresponding to Figure 26 (compactness $c_j = 10^6$ at jet breakout), and the analogous result for $c_j = 10^2$. Black curves show the effect of raising the radial scattering depth $\tau_*$ at breakout. Effect of delayed dissipation on the hard spectral tail is not included. (A color version of this figure is available in the online journal.)

Figure 28. Binding energy of core material lying outside a given collapsed mass $M_{\text{col}}$, as a function of the zero-age main sequence (ZAMS) mass $M_0$ of the progenitor. $M_{\text{col}}$ enters into Equation (72) for the isotropic Poynting luminosity of a jet emerging from a trapped relativistic bubble in a massive stellar core; and Equation (75) normalizing the spectral peak frequency of a heated jet that freely expands after breakout from a Wolf–Rayet star. Core profiles are obtained from the MESA star integrator (Paxton et al. 2013) in the simplest case of non-rotating single stars of metallicity $Z = 10^{-3}$. The mapping between ZAMS mass and pre-collapse mass profile will vary as these other (uncertain) degrees of freedom are included; the goal here is to probe a plausible range of pre-collapse profiles.

We therefore obtain a strong upper bound on the isotropic jet luminosity,

$$L_{\text{iso}} \leq 4\pi R_c^2 F_{\ast}^{\ast} \frac{8c}{3^{3/2} R_c} E_{\text{bind}}$$

$$\sim 4.6 \times 10^{52} \frac{E_{\text{bind},51}}{R_c,9} \text{ erg s}^{-1}, \quad (65)$$

and on the jet energy

$$E_{P,\text{iso}} \sim L_{P,\text{iso}} t_{\text{col}}$$

$$< 7.3 \times 10^{52} \frac{E_{\text{bind},51}}{R_c,9} 3^{1/2} \left[ \frac{t_{\text{col}}}{2R_c(R_c)} \right] \text{ erg}. \quad (66)$$

Here $M_{\text{col}}$ the collapsed mass inside $R_c$, and $t_{\text{ff}} \sim [R_c^3/2G M_{\text{col}}]^{1/2} = 1.1 R_c^{3/2} (M_{\text{col}}/3 M_\odot)^{-1/2} \text{ s}$ the free-fall time.

The results obtained from MESA stellar models are shown in Figures 29 and 30. For each model, we vary the collapsed mass, taking the net binding energy of all material outside that mass cut. The collapse time $t_{\text{col}}$, estimated to be twice the free-fall time from radius $R_c$, sets a lower bound to the burst duration (Figure 30). The observed isotropic jet energy $E_{P,\text{iso}}$, radiated over a duration $t_{\text{col}} \sim 10$ s, must lie below the value shown.

The ram pressure of infalling core material does not allow a significant increase in jet energy. At a radius $r < R_c$, the confining pressure imparted to a jet of opening angle $\theta_j$ is $P_{\text{ram},\perp} \sim \rho(\theta_j v_r)^2$, where $v_r$ is the net infall velocity. This

\[ binding energy of the CO material is $E_{\text{bind}} \sim 10^{51} \text{ erg outside a radius } R_c \sim (1\text{--}3) \times 10^9 \text{ cm: Figure 28 shows the result for stellar models of various mass constructed using the MESA code (Paxton et al. 2013).} \]

If the bubble material is sufficiently relativistic to drive a GRB, its pressure is dominated by radiation, $P_{\text{rad,0}} \sim E_{\text{bind}}/4\pi R_c^3$. The maximum energy flow out of such a static, confined bubble is (Blandford & Rees 1974) $F_{\ast}^{\ast} \sim (8/3^{3/2}) P_{\text{rad,0}} A^{\ast} c$, where the jet cross section $A$ and energy flux $F_{\text{rad}}$ are measured at the sonic radius $R^{\ast} \sim 2R_c$. The radiation energy flux must decrease from this point outward as the jet expands, because the radiation is still tied to a high density of $e^\pm$ pairs.
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Figure 29. Maximum isotropic jet energy \( (\theta_j^2 R c) E_{\text{bind}} \) carried by relativistic fluid flowing through a nozzle out of a confined bubble of pre-collapse radius \( R_c \) surrounding a collapsed core of mass \( M_{\text{col}} \). These curves set a strong upper bound, because they do not allow for jet expansion outside the nozzle. The bubble pressure \( P_{\text{rad}} \) is determined by setting the bubble energy \( 4\pi R_c^3 P_{\text{rad}} \) equal to the binding energy of material outside the mass cut.

Figure 30. Collapse time of material inside the mass cut \( M_{\text{col}} \), estimated to be twice the free-fall time \( (R_c^3/2GM_{\text{col}})^{1/2} \). This sets a lower bound to the burst duration, which is also influenced by continuing accretion through a collapsed torus.

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Figure 31. Jet transmits energy from the ergosphere of a stellar-mass black hole (not to scale) that has formed by gravitational collapse. A fraction \( f_{B,j} \) of the magnetic flux threading the hole penetrates the surrounding envelope. When this outgoing flux has constant sign, as depicted, it must return through an annulus of shocked jet material (red lines). The remaining flux is trapped in a relativistic bubble of radius \( \gtrsim R_c \sim 10^9 \text{ cm} \), corresponding to the size of the pre-existing CO core in a collapsar, or to the size of the neutron-rich debris cloud that surrounds the remnant of a binary neutron star merger. The connectivity of the trapped magnetic field lines is only schematic: some may still connect to the collapsed torus, and turbulent mixing with some stellar material is likely. The envelope of shocked jet material expands at most trans-relativistically. It provides a confining sheath for the relativistic jet core out to a radius \( R_{\text{trans}} \sim c t_{\text{col}} \) which may exceed the original envelope radius.

8.2. Magnetized, Axially Symmetric, Relativistic Jet

We now turn to a magnetized jet emerging from a black hole in a collapsing core (Figure 31). Even in a very focused jet, the outflowing material extends across a transverse dimension \( \theta_j R_c \) much larger than the light-cylinder radius \( c/\Omega_f \) of the engine.\(^3\) Magnetic field lines tied to the engine rotate, in a steady, axisymmetric jet, with a constant pattern angular velocity \( \Omega_f \). This is the angular velocity of a material star, or 1/2 the angular velocity of a black hole (Blandford & Znajek 1977). The toroidal magnetic field is then

\[
B_\phi = \frac{v_\phi - \Omega_f r \sin \theta}{v_p} B_p \simeq -\frac{\Omega_f r \theta}{c} B_r
\]

in a jet with poloidal flux density \( B_p \) and velocity \( v = \{ v_p, v_\phi \} \). We focus on narrow, relativistic jets, within which \( B_p \simeq B_r \). \( v_p \approx v_r \approx c \) in spherical coordinates.

The radial Poynting flux is

\[
S_r = \frac{E_\phi}{4\pi c} B_\phi \simeq \frac{B_r^2}{4\pi c}
\]

and the isotropic Poynting luminosity

\[
L_{P,\text{iso}} = 4\pi r^2 S_r = \frac{1}{\theta^2} [B_r(r\theta)^2] \frac{\Omega_f^2}{c}.
\]

\(^3\) In this section, all variables refer to the inertial frame in which the jet is propagating.
A simple relation between $L_{\text{Piso}}$ and the core parameters can easily be derived on the premise that some of the poloidal magnetic flux extending from the engine is distributed in a broad fan and captured into a relativistic bubble within the core. The remaining fraction $f_{B,j}$ propagates out of the star behind the jet head. Then,

$$L_{\text{Piso}} = \left( \frac{f_{B,j}}{\theta} \right)^2 B_H^2 R_H^2 \frac{\Omega_f}{c}. \quad (71)$$

Here $B_H$ is the poloidal flux density threading a black hole of radius $R_H$.

As the collapse continues and magnetic flux builds up around the black hole, the total luminosity $L_j$ in the two countercollapsing jets increases with time, until $L_j > E_{\text{bind}}/t_{\text{col}}$. When threaded by a split-monopolar magnetic field, the black hole releases $L_j \sim 2B_H^2 R_H^2 \Omega_f^2/3c$ (Blandford & Znajek 1977; Tchekhovskoy et al. 2010). The isotropic energy of a single jet is then,

$$E_{\text{Piso}} = L_{\text{Piso}} t_{\text{col}} \sim 1.5 \times 10^{51} \left( \frac{f_{B,j}}{\theta} \right)^2 E_{\text{bind},51} \text{ erg.} \quad (72)$$

Even though the isotropic energy flux must vary with angle within the jet, this expression has an important feature: $E_{\text{Piso}}$ depends only on one local variable $\theta$ in addition to the global parameters $f_{B,j}$ and $E_{\text{bind}}$. The effects of jet re-collimation by a stellar envelope (e.g., Lazzati et al. 2013) are factored out as long as $\theta$ can be related to the Lorentz factor in the jet core. This has interesting implications for GRB color-luminosity relations, which we address in Section 9.

9. IMPLICATIONS FOR GRBs

9.1. Observed Relation between Spectral Peak and Isotropic Burst Energy

GRBs with known redshifts can be labeled by the isotropic-equivalent bolometric energy $E_{\gamma,\text{iso}}$ in the hard X-ray/gamma-ray band, and the photon energy $E_{\text{peak}}$ where the spectral energy flux $E^2dN_{\gamma}/dE$ peaks. The measured events generally sit above the Amati et al. line (Amati et al. 2002),

$$E_{\text{obs}}^{\text{pk}} = 100 \text{ keV} \left( \frac{E_{\gamma,\text{iso}}}{10^{52} \text{ erg}} \right)^{1/2}. \quad (73)$$

For a recent re-analysis, which emphasizes this line as a boundary in the $E_{\text{obs}}^{\text{pk}}$-$E_{\gamma,\text{iso}}$ plane, see Heussaff et al. (2013).

Because GRBs typically show extended tails of emission extending above the peak, bursts with high $E_{\gamma,\text{iso}}$ but low $E_{\text{obs}}^{\text{pk}}$ would be detected if they existed (Piran & Narayan 1996). Hence it appears that the peak energy is buffered from below. The simplest candidate mechanism involves a thermal photon gas, which supplies Compton seeds.

We have demonstrated that $E_{\text{pk}}$ in a strongly magnetized pair gas lies well above that encountered in a baryon-dominated outflow of the same compactness, due to the buffering of the Compton parameter at temperatures below $m_c c^2$.

In addition, there is only a modest adiabatic drop in temperature (a factor $\sim 0.5$) after heating ends, because the pairs rapidly annihilate. The bulk-frame peak energy adjusts to $E_{\text{pk}} \sim 0.1 m_c c^2$ for a photospheric compactness $\gtrsim 10^4$ (see Figure 13). The observed peak energy is then

$$E_{\text{obs}}^{\text{pk}} \simeq \frac{4}{3} \Gamma \times 0.1 m_c c^2 = 70 \Gamma \text{ keV.} \quad (74)$$

We have also, in Section 8.1, considered the binding energy of the massive CO cores which are believed to be hosts for long GRBs. This provides a rough upper envelope to the total energy released by a GRB jet, if the accretion time through the black hole is shorter than the collapse time. For the most massive progenitors, the collapse time approaches $\sim 10$ s at an enclosed mass of 4 $M_\odot$ (Figure 30). Longer $t_{\text{eq}}$ burst durations may imply collapse from larger $R_c$ (but with a somewhat smaller $E_{\text{bind}}$); or a long viscous time in a collapsed and centrifugally supported torus, which would require a rapidly-rotating progenitor.

Our goal here is to work out the minimum $E_{\text{obs}}^{\text{pk}}$ corresponding to a given $E_{\gamma,\text{iso}}$. We only consider the GRB emission up to, and including, the peak, with the implication that the bolometric gamma-ray energy would typically be at least $\sim 2$ times larger. This energy also depends on the efficiency of conversion of Poynting flux to photons; we take a maximum value $E_{\gamma,\text{iso}}/E_{\text{Piso}} \sim 1/2$ at breakout. The core binding energy reaches $E_{\text{bind}} \sim 4 \times 10^{51} \text{ erg}$ at a progenitor ZAMS mass 40 $M_\odot$.

Substituting these maximum parameter values into Equation (72) and inverting gives a lower bound

$$E_{\text{obs}}^{\text{pk}} \gtrsim 130 E_{\gamma,\text{iso},52}^{1/2} \frac{\Gamma}{f_{B,j}} \left( \frac{E_{\text{bind}}}{4 \times 10^{51} \text{ erg}} \right)^{-1/2} \text{ keV.} \quad (75)$$

This lies close to Equation (73) if $\Gamma \theta \sim 1$. Note that GRB jets with

1. lower radiative efficiency;
2. lower escaping magnetic flux fraction $f_{B,j}$; and/or
3. originating from CO cores with lower binding energy,

have higher $E_{\text{obs}}^{\text{pk}}$ for a given $E_{\gamma,\text{iso}}$ and sit above the Amati et al. line. Bulk Compton scattering of the thermal emission during breakout also tends to raise the peak energy.

One advantage of this relation is that it does not depend on the distance from the engine, because the comoving $E_{\text{pk}}$ is directly related to the electron rest mass. Related attempts based on a local blackbody approximation do not have this feature (Thompson et al. 2007; Lazzati et al. 2013).

9.2. Implications for Magnetic Reconnection

Magnetic reconnection is a promising source of variability and non-thermal emission in GRB outflows (Thompson 1994; Spruit et al. 2001; Giannios & Spruit 2007; Zhang & Yan 2011; McKinney & Uzdensky 2012), as well as pulsar winds (Coroniti 1990; Lyubarsky & Kirk 2001) and more dilute radio-emitting jets from black holes (Romanova & Lovelace 1992).

9.2.1. Magnetic Field Geometry

The geometry of the magnetic field depends on the type of source. A striped toroidal field geometry has been demonstrated in force-free calculations of winds from rotating neutron stars with tilted magnetic dipoles (Spitkovsky 2006). This result may not, however, be relevant for GRBs: we argue in this paper that the extreme baryon depletion of GRB outflows points to the rapid formation of an event horizon in the engine. Calculations of jets from black hole magnetospheres which are fed by magnetic flux of variable sign (Beckwith et al. 2008) suggest that the flux threading the horizon rapidly reconnects and maintains a uniform sign that reflects an average over the accretion history. This uniform sign of poloidal field then translates into a uniform sign of the wind-up toroidal field.
The outgoing magnetic flux in the jet core—which connects to the central black hole—is surrounded by an annulus of returning flux (see Figure 31). The outgoing and returning magnetic fields are separated by a cylindrical current sheet. Reconnection could occur at this sheet (McKinney & Uzdensky 2012), although it must be at least partly suppressed by strong radial velocity shear: the return flux contains jet fluid that has shocked at the jet head and then fallen behind it as it escapes the star.

In magnetic tower models (Lynden-Bell 2003; Uzdensky & MacFadyen 2006) the outgoing and returning flux both connect to a differentially rotating object, and are both treated in the force-free approximation. But in the collapsar context, the returning flux will have a significantly different magnetization, temperature, and velocity than the outgoing flux.

Note also that, after the jet fluid escapes the star, and accelerates to $\Gamma \sim 10^2$–$10^3$, a diminishing fraction $\sim (\Gamma \theta_J)^{-1}$ of the toroidal magnetic field sees this cylindrical current sheet. While reconnection at this sheet is a possible source of GRB variability, it is not on energetic grounds likely to be the dominant source.

More complicated radial structure for the magnetic field is possible. Radial reversals in the magnetic field on a characteristic lengthscale $\Delta \sim \pi c/\Omega_j$ are not expected in a Blandford–Znajek jet. More stochastic reversals, occurring on the timescale of the dynamo in the orbiting neutron torus, are still possible (Thompson 2006). For example, a dynamo operating over $\sim 100$ orbital periods ($\Delta t \sim 30$ ms) would generate flips in the magnetic field on a characteristic timescale $\Delta t \sim 10^9$ cm. As long as the jet Lorentz factor remains modest, domains of opposite toroidal field would come into causal contact at a radius $r > 2\Gamma \Delta t \sim 10^{10}$–$10^{11}$ cm. The relevant field geometry is a current sheet stretched out across the width of the jet, and therefore extending beyond the transverse causal scale $\sim r/\Gamma$. The return flux in a neutron-rich annulus surrounding the fast jet core would also vary in size (Figure 32).

Finally, we note that reconnection is possible if the field becomes disorganized due to global current-driven instabilities (e.g., Levinson & Begelman 2013).

9.2.2. Limitations to Reconnection Rate

The rate of reconnection is potentially influenced by two physical effects which we now discuss. The first (which has received less attention in the astrophysical literature because it requires more extreme conditions) involves the trapping of radiation near the sheet. Such an “opacity limit” to the rate of reconnection has been considered in intermediate parts of GRB outflows as they become effectively collisionless (Thompson & Gill 2012). Although it must be at least partly suppressed by strong radial velocity shear, the return flux contains jet fluid that has shocked at the jet head and then fallen behind it as it escapes the star.

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We encounter optical depths $\tau_T \lesssim 10^2$ in continuously heated and strongly magnetized pair plasmas with equivalent black-body temperature $< 0.05 m_e c^2$ (corresponding to distances from the engine larger than $\sim 10^9$ cm). Reconnection is not limited by the build-up of radiation pressure beyond such a distance from the engine, even if it is maximally efficient ($\epsilon_{\rm rec} \sim 0.1$). The back pressure of slowly cooling ions is also irrelevant in this context.

The second constraint on reconnection has been motivated by plasma experiment (Yamada et al. 2010), which shows that X-point reconnection only occurs in electron–ion plasmas of size $L$ if the thickness of a Sweet–Parker current sheet is smaller than the ion plasma length,

$$\delta_{\rm SP} \sim \frac{L}{S^{1/2}} \sim \left( \frac{\eta L c^2}{4\pi V_A} \right)^{1/2} \frac{c}{\omega_{pi}}. \quad (77)$$

Here $\delta_{\rm SP}$ is the Sweet–Parker current sheet thickness, $V_A$ is the Alfvén speed, and $\omega_{pi} = (4\pi n_e e^2/m_p)^{1/2}$ in a hydrogen plasma. The Lundquist number is

$$S = \frac{V_A L}{\eta c^2/4\pi}. \quad (78)$$

If the inequality (77) is not satisfied, then experiments achieving $S \sim 10^{2–3.5}$ find that reconnection is limited by plasma outflow through a narrow current sheet of thickness $\delta_{\rm SP}$.

This leads to the interesting suggestion of delayed reconnection in GRB outflows as they become effectively collisionless.
(McKinney & Uzdensky 2012). When considering such implications, one key issue is whether experiment has achieved large enough $S$ to reflect the behavior of astrophysical systems. If fast X-point reconnection were to take place, then the magnetic Reynolds number of the fluid motions would be

$$R_m = \frac{V_{rec}L}{\eta c^2/4\pi} \sim \epsilon_{rec} S \sim 0.1S.$$  \hspace{1cm} (79)

The point here is that because $\delta S_p$ depends on the size of the plasma, one can always satisfy the inequality (77) by considering a narrower slice closer to the current sheet. However, if $S$ is not too large, then a fluid description of the plasma flow toward these “sub-scale X-points” would break down, and a Sweet–Parker layer should persist.

Numerical experiment shows that X-points do form in weakly magnetized pair plasmas, as long as the box size exceeds $\sim 10^2c/\omega_{pe}$ (Swisdak et al. 2008); and robustly in relativistic pair plasmas (Zenitani & Hoshino 2007). Fast reconnection occurs in spite of the absence of a Hall term in the conductivity (which requires a mass asymmetry between positive and negative charges).

The constraint analogous to (77) in a strongly magnetized pair plasma ($\omega_{pe} = eB/m_ec \gg \omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$, implying $B^2/4\pi \gg n_e m_e c^2$) is, roughly

$$\delta S_p \sim \frac{c}{\omega_{pe}}.$$  \hspace{1cm} (80)

Note also that $V_A \sim c$ on large scales (where radiation and charges are coupled) as long as $B^2/8\pi \gg U_{inh}$; whereas $V_A$ is very close to $c$ on small scales. The resistivity in a magnetized plasma with significant radiation pressure is mainly due to Compton drag on the drift motion of the current-carrying electrons (Uzdensky & Goodman 2008),

$$\eta \sim \left(\frac{4\pi}{3}U_{\gamma}^3\Gamma c \right)\frac{4\pi n_e e^2}{\omega_{pe}}.$$  \hspace{1cm} (81)

Then we find

$$S \sim \frac{27\pi}{2} \frac{\tau_T}{f_{inh} \omega_{em}^2 (B/B_Q)^2} \sim 10^{17} \left(\frac{f_{inh}}{0.1}\right)^{-1} \left(\frac{\tau_T}{10^2}\right) \left(\frac{B}{B_Q}\right)^{-2},$$

corresponding to $\delta S_p \sim 3 \times 10^{-9} (r/\Gamma)$. Given that multiple X-point reconnection does occur sufficiently close to a current sheet at large $S$, the remaining key question involves the non-linear development of this structure. If the magnetic field becomes disorganized, as suggested by recent large-scale particle-in-cell simulations of electron–ion plasmas (Loueiro et al. 2012), then reconnection is sped up by the appearance of multiple Sweet–Parker layers (Lazarai & Vishniac 1999; Kowal et al. 2009). One is led to the picture of an expanding, turbulent current sheet outlined by Thompson (2006). Turbulent reconnection in a strongly magnetized GRB outflow is only limited by causality, that is, by the ability of any given patch of magnetic field to see a current sheet, and by also by radiation pressure close to the engine.

9.3. Charge Starvation of Plasma Currents

A promising mechanism for particle heating involves the damping of MHD turbulence. In the present context, where the plasma is pair dominated and reaches a scattering depth $\sim 10–30$ during the heating phase, the Compton drag time is longer than the wave period. Strongly coupled waves will cascade to a high wave number first.

What happens near the inner scale of this cascade depends essentially on the particle density. At a very low density, Alfvén waves become charge-starved (Thompson 2006), but otherwise they Landau damp on the electrons and positrons. Charge starvation is only possible at a high wavenumber: a large-scale breakdown of MHD is inconsistent with any significant scattering depth through the entrained charges. We reconsider this issue in the context of a dilute pair gas, where charge-starvation effects are enhanced.

Most numerical experiments now find that the energy spectrum of magnetic fluctuations is somewhat flatter than Kolmogorov (Maron & Goldreich 2001; Boldyrev 2006), with scaling $(\delta B)^2 \sim k_0^{-2} = k_0^{-1/2}$. However, we leave the index $\alpha$ open here, and consider also the Kolmogorov scaling $\alpha = 5/3$. The wave shear appears to adjust so that collisions between waves are strongly coupled, and the conserved energy flux in wavenumber space imposes the constraint

$$\omega (\delta B)^2 \equiv k_\parallel V_A (\delta B)^2 \equiv \text{const} \Rightarrow k_\parallel \sim k_\perp^{-1}.$$  \hspace{1cm} (83)

We normalize $k_{\parallel,0} \sim \Gamma/r$ and $k_{\perp,0} \sim (\delta B_0/B)^{-1} \Gamma/r$ at the outer scale.\footnote{If colliding waves are roughly cylindrically symmetric but strongly sheared, $k_{\perp,0} \gg k_{\parallel,0}$, then the condition for this type of “critical balancing” is that the wave amplitude is $\sim k_\perp^{-1}$, corresponding to $(k_{\perp,0}/k_{\parallel,0})(\delta B_0/B) \sim 1$ (Goldreich & Sridhar 1995). There is, however, some numerical evidence that cascading wavepackets become increasingly elongated in the plane transverse to $B$, which changes this condition (Boldyrev 2006).}

To determine whether charge starvation or Landau damping cuts off this spectrum, consider the current fluctuation

$$\delta J \sim \frac{c}{4\pi} k_\perp \delta B \sim \frac{cB}{4\pi \Gamma} \frac{k_{\perp,0}}{k_{\parallel,0}}^{(3-\alpha)/2}.$$  \hspace{1cm} (84)

Defining the charge starvation scale by

$$\delta J \sim e n_e c,$$

one finds

$$k_{\parallel,\text{starve}} \sim \frac{\Gamma}{r} \left(\frac{4\pi e n_e r}{FB}\right)^{2/(3-\alpha)} \frac{B}{\delta B_0}.$$  \hspace{1cm} (85)

When the magnetic energy density dominates the rest energy density of the light charges, as is the case here, Landau damping occurs when the perpendicular wavenumber reaches the electron skin depth, $k_{\perp,}\sim \omega_{pe}/c$. To compare (86) with the Landau-damping scale, we can simply evaluate

$$\left(\frac{k_{\parallel,\text{starve}}}{\omega_{pe}}\right)^2 = \frac{\tilde{\epsilon}_e}{\omega_{pe}} \cdot \left(\frac{B}{B_Q}\right)^{-4/(3-\alpha)} \times \left(\frac{3\tau_T}{2\omega_{em}}\right)^{(1+\alpha)/(3-\alpha)} \left(\frac{B^2}{B_0^2}\right) \frac{\tau_T}{\delta B_0}.$$  \hspace{1cm} (87)

In a jet of isotropic Poynting luminosity $L_{P,iso} = (\Gamma B r)^2 c$ and Lorentz factor $\Gamma$, this gives at a radius $r$$

$$\left(\frac{k_{\parallel,\text{starve}}}{\omega_{pe}}\right)^2 = 0.2 \left(\frac{r}{r_{\text{crit}}}\right)^{5/3} \left(\frac{\Gamma}{3}\right)^{11/3} \frac{L_{P,iso,51}}{L_{P,iso,51}^4 (\delta B_0/B)^2} \left(\frac{\alpha}{3}\right)^2.$$  \hspace{1cm} (88)
Charge starvation effects are therefore potentially quite important inside \( \sim 10^{12} \text{ cm} \) from the engine. Note, however, that the coefficient in (88) is \( \sim 10^3 \) times larger if the wave spectrum is Kolmogorov.

10. DISCUSSION

The repeatable spectral behavior of GRBs deserves a simple and robust explanation. We focus here on the spectral peak and the low-frequency tail below it. Although relativistic beaming introduces complexities by blending together angle, frequency, and time, a simple model of a Planckian emitted by a relativistically boosted photosphere does not come close to reproducing the typical low-frequency spectrum of a GRB: the emergent photon index is only slightly flatter than Rayleigh–Jeans, \( \alpha = 0.4 \) (Beloborodov 2010).

For that reason, it has long been suspected that the spectral peak and low-frequency tail offer essential clues to the emission mechanism (and, thence, to the underlying mechanism of energy transport and particle heating). Here we have demonstrated that by abandoning a longstanding assumption of a significant baryonic component for GRB outflows, but maintaining the strong Poynting flux that is needed to extract energy from the engine at the rates observed, these two essential features of GRBs fall easily into place. Baryons are still need to provide a confining medium, to limit the expansion of the jet to \( \Gamma \sim 1/\theta \) while still confined, and possibly to induce variability via hydrodynamic instabilities at the jet head.

In the framework advanced here, the hard tail of a GRB must originate outside breakout, after the outflow has achieved higher Lorentz factors by a combination of radiation and magnetic stresses. Separating its origin from the remainder of the spectrum is partly motivated by the inability of “one-box” models to avoid fine tuning (of the radius or compactness); and to avoid introducing more complicated, non-thermal particle populations.

Our results are independent of the details of the heating mechanism, as long as it is gradual. Consideration of the lateral structure of the jet, including the presence of slower material, suggests that breakout and full jet acceleration may be delayed to \( \sim 10^5 \) times the gravitational radius of the engine. That is, breakout may even be pushed close to the transition between “jet” and “pancake” geometry. In such a situation, the amount of heating is sensitive to the degree of causal contact both across the jet (e.g., the value of \( \Gamma \theta_j \)), as well as along the jet axis. The \( m = 1 \) modes seen in the 3D jet + torus simulations of McKinney & Blandford (2009), when translated this far out, are a plausible source of the mild heating we require. It is not clear to us that realistic hydromagnetic simulations have yet fully captured the heating effects of radial velocity shear across the jet. Ideal kink modes remain an interesting possibility, although they have only been considered so far in the “magnetic tower” approximation (Lyutikov & Blandford 2003; Giannios & Spruit 2006).

Existing attempts to decompose the spectrum of a GRB into thermal and non-thermal components assume that the low-frequency spectrum of the thermal component is Rayleigh–Jeans, and that the non-thermal components extends above and below the peak (Ryde 2004; Ryde & Pe’er 2009). Based on the present results, such a decomposition should be repeated with a more general low-frequency index in the thermal component (Equation 27).

Our main conclusions can be separated into the prompt emission mechanism, and the physical properties of the engine and the outflow that it generates the following.

10.1. Soft Photon Emission in Magnetized Outflows

*Channels of soft photon emission.* We find that cyclotron synchrotron emission strongly dominates over double-Compton emission when the magnetic energy density is comparable to or larger than the photon energy density. Extrapolating the results of Figure 34 from \( f_{\text{in}} = 8\pi U_{\gamma}/B^2 \approx 0.1, 1 \) shows that cyclotron emission still dominates when \( U_{\gamma} \) is as large as \( \sim 10^2 \) \((B^2/\sigma_8)\). The results derived from our analytic scaling model of an expanding plasma (Section 5) are shown in Figures 19 (pair plasma) and 25 (electron–ion plasma). The numerical solutions of the Kompaneets equation for dilute electron–ion fireballs by Vurm et al. (2013) included cyclotron synchrotron emission, but the broader survey of spectral peak energy by Beloborodov (2013) did not.

*GRB-like low-frequency spectrum.* A low-frequency photon index near \( -1 \) naturally arises in a strongly magnetized \((B^2/\sigma_8 \gtrsim U_{\text{th}})\) and nearly thermal pair plasma. The resultant spectral state does not depend significantly on the initial compactness over a range \( \sim 10^{-3} \). Even flatter low-frequency spectra, which are sometimes seen in GRBs, are found at lower values of the compactness \((\lesssim 10^7)\). See Figures 5 and 6.

*Scaling behavior during expansion.* When such a strongly magnetized plasma expands over more than a decade in radius, while being continuously heated, the spectrum remains relatively flat below the peak. This spectral state appears to be an attractor: it does not depend significantly on the initial low-frequency slope of the thermal seed. At a very high initial compactness \( \sim 10^7 \), the photon index reaches a maximum 0 over a narrow (about a factor of a few) frequency band below the peak. See Figures 7 and 8.

*Rapid transition to transparency.* Rapid pair annihilation allows a pair-rich jet that experiences a sudden drop in heating to become transparent, as is shown in Figures 10, 12, and 14. It then feels a strong outward radiation pressure force. This connects the thermal and strongly magnetized plasma state analyzed here with the magnetic jet solutions of Russo & Thompson (2013a, 2013b). As long as heating turns off during breakout and transverse expansion of the jet, then it simultaneously becomes transparent—without any fine tuning of the scattering opacity.

*Spectral flattening during jet breakout.* Modest bulk Compton scattering during this breakout forces some further flattening of the spectrum near its peak: the maximum value of the photon index drops by \( \sim 0.5 \). For example, the photon index maintains an average value \( \sim -0.8 \) from \( 10^{-3}E_{\text{pk}}^{\text{obs}} \) to \( E_{\text{pk}}^{\text{obs}} \) for a final radiation (magnetic) compactness \( \sim 10^6 \). See Figures 26 and 27, and the discussion in Section 7.

*Magentic energy reservoir.* The hard gamma-ray tails seen in GRBs are independent evidence that the magnetic field dominates the energy flux after breakout. For example, in a burst with a high-frequency photon index \( \sim -2.5 \), the flux above the peak is at least comparable to the flux at and below the peak, and it dominates in bursts with harder spectra. The kinetic energy of the entrained pairs and ions is negligible at breakout, and so the magnetic field is identified by default as the energy reservoir for the hard tail. The nearly complete reconnection and thermalization of the magnetic field before breakout (e.g., Levinson & Begelman 2013) is disfavored for the same reason.

*Mapping of observed spectral peak to breakout plasma conditions.* Because the pair plasma experiences only weak adiabatic cooling near breakout, and maintains a relatively low
Lorentz factor, there is a direct connection between the spectral peaks of GRBs and the electron rest mass. In a compact, thermal pair plasma, the bulk-frame spectral peak sits at \( \sim 0.2 m_e c^2 \) at the end of the heating phase. During cooling and pair annihilation, it drops by a factor of only \( \sim 0.5 \) due to adiabatic cooling. The measured spectral peak is therefore largely a measure of the bulk Lorentz factor at breakout: \( \Gamma \sim 3(E_{pk}/200 \text{ keV}) \). See Figure 13.

**Free expansion phase.** This mapping of the spectral peak back to conditions at jet breakout implies a significant (but temporary) drop in the dissipation rate following breakout. The bulk of the jet acceleration occurs during this intermediate phase. An upswing in dissipation, required to explain the presence of the high-frequency spectral tail, is not addressed in this paper. Magnetic reconnection tends to freeze out during the intermediate acceleration phase. The collision between the bulk of the magnetized jet fluid, and a forward shell that is swept up from the confining medium, is also delayed to a larger radius (Thompson 2006).

*Amati et al. boundary.* If \( \Gamma \sim 1/\theta \) during the last stages of jet thermalization, then one obtains the lower bound to \( E_{pk}^{\gamma} \) as a function of \( E_{iso}^{\gamma} \), obtained by Amati et al. from a sample of BeppoSAX bursts (Section 9). Such a relation between \( \Gamma \) and \( \theta \) has been derived for self-similar, cold, magnetized jets (Lyubarsky 2009).

In this derivation we have taken careful account of the expected range of binding energies of CO cores of a range of masses. The Amati et al. boundary corresponds to the most massive cores, to jets with the highest radiative efficiency, and to situations in which a large fraction of the magnetic flux threading the black hole engine escapes the star through the jet. Outflows from less massive cores, with lower radiative efficiency, or lower fractions of the engine output channeled through a relativistic jet, sit above the boundary. Bulk Comptonization during jet breakout also tends to raise the peak energy (Russo & Thompson 2013a, 2013b).

*X-ray flashes from modest baryon loading.* X-ray flashes have a similar duration to GRBs but are spectrally softer. As is shown in Figures 22 and 24, bursts peaking in the X-ray band naturally arise from jets that have a high enough baryon loading that the electron–ion component dominates the scattering opacity at the photosphere.

The transition from GRB to X-ray flash occurs where the breakout magnetization is still very high. We find that the bulk-frame spectral peak energy drops from \( E_{pk}^{\gamma} \sim 0.1 m_e c^2 \) to \( \sim 10^{-2} m_e c^2 \) when \( \sigma_{ion} \) is \( \sim 10 \) times below the minimal value (50) for a pair-dominated photosphere. Consider, for example, a jet of luminosity \( L_{\gamma,iso} \sim 10^{51} \text{ erg s}^{-1} \) and breakout Lorentz factor \( \Gamma \sim 3 \), for which we infer a breakout compactness \( \xi_{bh} \sim 10^5 \) (Equation (11)). Then from Equations (50) and (58), the transition from GRB to X-ray flash is at \( \sigma_{ion} \sim 0.1 \sigma_{ion, crit} \sim 3 \times 10^4 \).

An important consequence of this strong magnetization (\( \sigma_{ion} \gg \Gamma \)) is that there is negligible spreading of the X-ray pulse outside breakout by scattering off the advected electron–ion gas. By contrast, an X-ray pulse produced by off-axis emission, or an increased baryon loading in a baryon-dominated fireball, would typically have a longer duration than the corresponding GRB. Another consequence is that the output spectrum shows a more pronounced thermal peak, with a harder spectrum just below the peak (Figure 22).

We observe similar effects in magnetized plasmas where a modest fraction (\( \sim 10^{-3} -10^{-2} \)) of the dissipation is channeled through relativistic particles (Figure 15). GRB pulses with a low-frequency Rayleigh–Jeans slope (e.g., Crider et al. 1997; Ryde 2004) sometimes appear near the beginning of a burst, and could represent an intermediate level of baryon loading or non-thermal pair creation.

**Absence of high-energy emission.** The “no-high-energy” pulses, which are detected as subcomponents in many GRBs (Pendleton et al. 1997), may correspond to components of GRB jets in which the magnetic field is significantly dissipated before breakout; or in which delayed reheating is somehow suppressed.

**10.3. Engine and Physical Properties of the Jet**

Extreme magnetization. We find that the jet is very strongly magnetized: \( \sigma_{ion} > 10^5 \) for breakout from a Wolf–Rayet star at \( \sim 10^{15} \text{ cm} \) from the engine. See Equation (51) and Figure 23. The outflow from a millisecond magnetar achieves such a high magnetization only after \( \sim 10^2 \) s, as the neutrino-driven wind from the neutron star surface diminishes (Metzger et al. 2011). If the magnetar manages to generate a collimated jet (which is less well motivated than for a black hole engine), then one should see an extended, spectrally soft precursor before the onset of GRB emission.

*Horizon in the engine.* This strong magnetization is most easily generated by the horizon of a stellar-mass black hole. In a collapsar or binary merger, the neutron-rich torus is a strong emitter of neutrinos, whose collisions generate electron–positron pairs (Eichler et al. 1989; Zalamea & Beloborodov 2011). The pressure of this pair plasma in the jet funnel pushes any residual baryons down to the horizon.

*Neutron-poor jet core and neutron-rich jet sheath.* A straightforward consequence of the high magnetization is that any kinetic process involving baryons, in particular inelastic collisions between neutrons and charged ions, plays a negligible role in the prompt emission mechanism (Section 6.2). A shallow Lorentz factor profile inside breakout also tends to suppress the energetic importance of \( n \)-ion collisions. Our conclusions regarding the role of neutrinos in the prompt emission are, therefore, more pessimistic that those of Mészáros & Rees (2011), who considered higher ion densities and somewhat more rapid acceleration inside the \( n \)-ion decoupling radius. Nonetheless, as Figures 31 and 32 illustrate, a jet sheath connecting to a \( n \)-rich torus orbiting a black hole engine may be loaded with a significant density of neutrinos.

*Indirect evidence for a magnetized outflow from luminous GRBs.* We have calculated the maximum isotropic energy that could be carried by a purely hydrodynamic jet escaping through a nozzle from a trapped relativistic bubble inside a collapsing Wolf–Rayet core. For progenitor masses in the range 20–50 \( M_\odot \), this maximum energy is smaller than the largest observed \( E_{iso} \). See Figures 28 and 29. We emphasize the collimating effect of a slower, magnetized outflow from the neutron torus, independent evidence for which is found in global MRI simulations (Sadowski et al. 2013). A broader outflow from the torus also provides feedback that limits accretion from a collapsing stellar core onto a newly formed black hole, establishing a link between the observed jet energy and the binding energy of the core (Thompson et al. 2007).

*Gradual heating points to ideal hydromagnetic instabilities.* We have constrained the heating mechanism in the magnetized jet. The flattest low-frequency spectra are obtained if heating is gradual enough that the pairs remain sub-relativistic. Transient and localized heating could create relativistic particles that generate a higher scattering depth by a pair cascade. In that
case, we show that the low-frequency spectrum is harder than is typical for GRBs (Figures 14 and 15). The spectral peak is also reduced in frequency. We infer that the heating mechanism that forms the low-frequency spectrum is more consistent with the damping of large-scale hydromagnetic modes in a jet, rather than with localized reconnection events.

Questions about collisional effects on magnetic reconnection. The pair plasma is dilute enough that the build-up of radiation pressure does not limit magnetic reconnection at extended current sheets embedded in the outflow. We have also discussed the influence of a high Lundquist number on the development of a tearing instability at a current sheet in Section 9.2. Existing reconnection experiments may overestimate the importance of collisional effects in suppressing the formation of X-points and slowing down the reconnection rate. They should, therefore, be given limited weight in constructing magnetic reconnection models of GRBs.

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APPENDIX A
PHOTON EMISSION AND ABSORPTION PROCESSES

A single electron or positron moving with speed \( \beta c \) (component \( \beta_l = \beta \cos \alpha \)) parallel to the magnetic field \( \mathbf{B} \) emits energy in cyclo-synchrotron photons of frequency \( \omega \) at the rate

\[
\frac{dE_{\text{cyc}}}{dt} \cos(\theta) d\omega = \frac{e^2 \omega^2}{c} \sum_{n=1}^{\infty} \delta \left[ \frac{n \omega_{ce}}{\gamma} - \omega (1 - \mu_{||}) \right] \times \left[ \frac{\cos \theta - \beta_\parallel}{\sin \theta} \right]^2 J_n^2(z) + \beta_\perp^2 J_n^2(z). \tag{A1}
\]

Here \( \theta \) is the emission angle measured with respect to \( \mathbf{B}, \omega_{ce} = eB/m_e c, \) \( z = 2 \beta_\parallel \gamma \sin(\theta/\omega_{ce}), \) and \( \beta_\perp^2 = \beta^2 - \beta_\parallel^2. \) The integral over pitch angle can be eliminated using the delta function, giving

\[
\int d\gamma d\omega = \int_{-1}^{1} \frac{1}{2} d\cos \theta \int_{-1}^{1} d(\cos \theta) \times \int_{1}^{\infty} d\gamma \frac{d\gamma}{\hbar \omega} \int_{1}^{\infty} \frac{dE_{\text{cyc}}}{d\gamma} \cos^2 \theta - X^2 \cos^2 \theta \cos^2 \theta \\
= \frac{\alpha_{\text{em}}}{2} \sum_{n=1}^{\infty} \int_{1}^{\infty} d\cos \theta \int_{1}^{\infty} d\gamma \frac{\sqrt{\cos^2 \theta - X^2}}{\beta \gamma} \times \left[ \frac{(\cos^2 \theta - X^2)}{(\cos \theta \sin \theta)} \right]^2 J_n^2(z) + \left( \beta_\parallel^2 - \frac{X^2}{\cos^2 \theta} \right) J_n^2(z). \tag{A2}
\]

where

\[
X \equiv 1 - \frac{n \omega_{ce}}{\gamma \omega_{ce}}, \quad z = \beta \gamma \sin \theta \frac{\omega}{\omega_{ce}} \sqrt{1 - \frac{X^2}{(\beta \cos \theta)^2}}. \tag{A3}
\]

The output spectrum is shown in Figure 1.
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Figure 34. Relative contribution of cyclo-synchrotron, free–free, and double-Compton soft photon emission to the growth of photon density in an expanding pair plasma, as investigated in Section 4.3. The downward break in the curves occurs where heating is suddenly turned off, at time $10^{-0.5}t_{\text{tot}}$. Bremsstrahlung emission is briefly enhanced near the start of the calculation, as the pair density relaxes to the equilibrium value.

with a correction factor for mildly relativistic temperatures (Svensson 1984),

$$f(\tilde{T}_e) = (1 + 13.91\tilde{T}_e + 11.05\tilde{T}_e^2 + 19.92\tilde{T}_e^3)^{-1}.$$  \hspace{1cm} (A8)

Absorption through all channels is handled by Kirchhoff’s law,

$$\frac{d^2n}{d\omega dt} \rightarrow d^2n \times \left[ 1 - \frac{N(\omega)}{N_{\text{bb}}(\omega)} \right],$$

$$N_{\text{bb}}(\omega) = [\exp(\hbar\omega/T_e) - 1]^{-1}. \hspace{1cm} (A9)$$

The relative net contributions of these emission and absorption processes, integrated over frequency, are shown in Figure 34, for an expanding plasma and two different values of the magnetization.

APPENDIX B

PAIR ANNihilation AND Creation

Here we review the calculation of the creation of pairs by photon collisions, \(\omega + \omega' \rightarrow e^+ + e^-\) (see Svensson 1982 for a detailed treatment in an astrophysical context). A photon of frequency \(\omega\) collides with another photon at a rate

$$\Gamma_{\gamma\gamma}(\omega) = \frac{1}{2} \int_{-1}^{1} d\mu (1-\mu) \int_{\omega_{\text{min}}}^{\infty} d\omega' \sigma_{\gamma\gamma}(\omega, \omega', \mu) \frac{dn_{\gamma}}{d\omega}, \hspace{1cm} (B1)$$

where the cross section

$$\sigma_{\gamma\gamma}(\omega, \omega', \mu) = \frac{3\gamma R}{16} (1 - \beta^2) \left[ 2\beta^2(1 - 2\beta) + (3 - \beta^4) \ln \left( 1 + \frac{\beta}{1 - \beta} \right) \right]. \hspace{1cm} (B2)$$

is defined in terms of the speed \(\beta c\) of the resultant charged particles in the center-of-momentum frame,

$$\frac{m_e c^2}{\sqrt{1 - \beta^2}} = E_{\text{cm}}(\omega, \omega', \mu) = \left[ \frac{1}{2}(\hbar\omega')(\hbar\omega)(1 - \mu) \right]^{1/2}. \hspace{1cm} (B3)$$

The low-energy threshold \(\hbar\omega_{\text{min}}\) of the target photon that results in pair creation is obtained by setting \(E_{\text{cm}} \rightarrow m_e c^2\). The net rate of pair creation per unit volume is

$$\frac{dn_e}{dt} \bigg|_{\gamma\gamma} = \int d\omega \frac{dn_{\gamma}}{d\omega} \Gamma_{\gamma\gamma}(\omega). \hspace{1cm} (B4)$$

Two charged particles are created in each photon collision, but the net collision rate is one-half the integral on the right-hand side of (B4).

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