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A study on the spread of COVID 19 outbreak by using mathematical modeling

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ARTICLE INFO

Keywords:
- Mathematical model
- COVID-19
- Corona virus
- Numerical Scheme

ABSTRACT

Mathematical models are mainly used to depict real world problems that humans encounter in their daily explorations, investigations and activities. However, these mathematical models have some limitations as indeed the big challenges are the conversion of observations into mathematical formulations. If this conversion is inefficient, then mathematical models will provide some predictions with deficiencies. A specific real-world problem could then have more than one mathematical model, each model with its advantages and disadvantages. In the last months, the spread of covid-19 among humans have become fatal, destructive and have paralized activities across the globe. The lockdown regulations and many other measures have been put in place with the hope to stop the spread of this deadly disease that have taken several souls around the globe. Nevertheless, to predict the future behavior of the spread, humans rely on mathematical models and their simulations. While many models, have been suggested, it is important to point out that all of them have limitations therefore newer models can still be suggested. In this paper, we examine an alternative model depicting the spread behavior of covid-19 among humans. Different differential and integral operators are used to get different scenarios.

Introduction

Since the outbreak of the Corona virus COVID-19 in January 2020, the virus has affected most countries and taken the lives of several thousands of people worldwide. By March 2020, the World Health Organization (WHO) declared the situation a pandemic, the first of its kind in our generation. To date, many countries and regions have been locked-down and applied strict social distancing measures to stop the virus propagation [1–6]. From a strategic and health care management perspective, the propagation pattern of the disease and the prediction of its spread over time are of great importance, to save lives and to minimize the social and economic consequences of the disease. Within the scientific community, the problem of interest has been studied in various communities, including mathematical epidemiology, biological systems modeling, signal processing and control engineering [7–12]. In this study, epidemic outbreaks are studied from an interdisciplinary perspective, by using an extension of the susceptible-exposed-infected-recovered (SEIR) model, which is a mathematical compartmental model based on the average behavior of a population under study. The objective is to provide researchers a better understanding of the significance of mathematical modeling for epidemic diseases. Table 1. Table 2.

Mathematical models have been insensitively used in the last months starting from January 2020, with the aim to predicting the spread of the fatal infectious disease called covid-19. The outbreak started in December 2019, in a Chinese city called Wuhan. Since then, the virus has spread exponentially in many countries around the globe due to the connections between cities, countries and even continents. While several governmental structures have undertaken fight against the spread of this disease, by imposing several restrictions, researchers in different fields have done serious research with the main aim to understand, analysis and help stop the spread within humans, in addition even provide a vaccine that will help prevent the spread. Mathematicians on their turn have now suggested from collected data many mathematical models that can be used to understand theoretically the behavior of this disease. In many instances, these mathematical models have predicted several situations that are likely to be observed in near future. For example, some mathematical model have predicted the second wave spread in Europe a situation that is being observed today in many European countries [14–18]. There is no doubt that mathematical models although not all accurate are able to help humans to see what could happen in near future. While many mathematical models have been suggested in the last few months, it is important to not that many mathematical model...
will still be suggested especially those mathematical models with new trend of fractional differentiation and integrations. In this paper, we present an analysis of a mathematical model depicting a spread of the fatal disease in a given population. The model is extended using a new trends of fractional differentiation and integration. In this study, epidemic outbreaks are studied from an interdisciplinary perspective, by using an extension of the susceptible-exposed-infected-recovered (SEIR) model, which is a mathematical compartmental model based on the average behavior of a population under study [13]. The objective is to provide researchers a better understanding of the significance of mathematical modeling for epidemic diseases.

Mathematical model for Covid19 disease:

In this section we suggest a mathematical model, i.e. SEQIR model of the Novel Corona virus of Covid-19 in Indian environment [13]. This model of covid-19 tells the dynamics of five populations, namely susceptible (s(t)), Infected but not defected by testing population represented by E(t), Q(t) represent a quarantined person, I(t) represent who under the treatment of isolated, R(t) represents the recovered persons who have been treated of covid-19 [13] (Fig. 1).

The model takes the following form

\[
\frac{dS}{dt} = \Lambda - \alpha SE - \beta_1 S - \sigma_1 S - d_1 S
\]

\[
\frac{dE}{dt} = \alpha SE - r_1 E - \beta_2 E - d_2 E
\]

\[
\frac{dQ}{dt} = \beta_1 S + \beta_2 E - r_2 Q - \sigma_2 Q - d_1 Q
\]

\[
\frac{dI}{dt} = r_1 E + r_2 Q - \sigma_3 I - d_1 I - d_2 I
\]
$\frac{dR}{dt} = \sigma_3 S + \sigma_3 Q + \sigma_3 I - d_1 R$

where the initial conditions are
$S(0) > 0$, $E(0)\geq 0$, $I(0)\geq 0$, $Q(0)\geq 0$, $R(0) > 0$

The above mathematical model can be rewritten as [13]

$\frac{dS}{dt} = \Lambda - aSE - AS$

$\frac{dE}{dt} = aSE - BE$

$\frac{dQ}{dt} = \beta_1 S + \beta_2 E - CQ$

$\frac{dI}{dt} = r_1 E + r_2 Q - DI$

$\frac{dR}{dt} = \sigma_3 S + \sigma_3 Q + \sigma_3 I - d_1 R$

where

$A = (\beta_1 + \sigma_3 + d_1)S$

$B = r_1 + \beta_2 + d_2$

$C = (r_2 + \sigma_2 + d_2)Q$

$D = (\sigma_3 + d_3 + d_2)I$

Model equations in proportions

To simplify the model we normalize the model by transforming the
model equations into proportions.

The model equations are transformed into proportions as follows [13]

\begin{align*}
\frac{dN(t)}{dt} & = A - AS - BE + \beta_3 S + \beta_2 E - CQ + r_1 E + r_2 Q - DI + \sigma_3 S + \sigma_3 Q + \sigma_3 I - d_1 R \\
\frac{dQ}{dt} & = A + (\sigma_3 - A + \beta_3)S + (\beta_2 + r_2 + B)E + (r_2 + \sigma_2 - C)Q + (\sigma_3 - D)I = -d_1 R
\end{align*}

(11)

$s = \frac{S}{N}, e = \frac{E}{N}, i = \frac{I}{N}, r = \frac{R}{N}, q = \frac{Q}{N}$

Then the normalized system is as follows [13]

\begin{align*}
\frac{ds}{dt} & = \frac{1}{N} \left[ dS(t) - SdN(t) \right] \\
\frac{de}{dt} & = \frac{1}{N} \left[ dE(t) - EdN(t) \right] \\
\frac{dI}{dt} & = \frac{1}{N} \left[ dI(t) - IdN(t) \right] \\
\frac{dR}{dt} & = \frac{1}{N} \left[ dR(t) - RdN(t) \right]
\end{align*}

Subtracting (1) and (11) and using (12)

\begin{align*}
\frac{ds}{dt} & = \frac{A}{N} - \frac{s}{N} + (\sigma_3 - A + \beta_3)s^2 + (\beta_2 + r_2 + B + r_1)s + (r_2 + \sigma_2 - C)q + (\sigma_3 - D)si - d_1 rs \\
& - C(qs + (\sigma_3 - D)si - d_1 rs)
\end{align*}

Similarly

\begin{align*}
\frac{de}{dt} & = \frac{1}{N} \left[ dE(t) - EdN(t) \right] \\
& - \beta_3 S - \beta_2 E + CQ + r_1 E + r_2 Q - DI
\end{align*}

Subtracting (2) and (11) and using (12)

\begin{align*}
\frac{di}{dt} & = \sigma_3 s + \sigma_3 q + \sigma_3 i - d_1 i - \beta_3 r + (r_3 + \sigma_3 - C)qi + (\sigma_3 - D)i^2 - d_1 i^2
\end{align*}

(15)

Similarly

\begin{align*}
\frac{dq}{dt} & = \frac{1}{N} \left[ dQ(t) - qdN(t) \right] \\
& - (r_2 + \sigma_2 - C)qr + (\sigma_3 - D)i - d_1 i^2
\end{align*}

(16)

However [13]

$s + e + q + i + r = 1$

These equations are the model equation in proportion which defines
prevalence of infection [13].

Equilibrium points and basic reproduction number

In this section we derive the equilibrium points including disease free
of the proposed SEIRQ model by taking $E = 0$, $Q = 0$, $I = 0$ & $R = 0$.

The obtained DFE is $E_{0} = (0, 0, 0, 0, 0)$. To find the Basic reproduction
Number we take the assistance of next generation matrix method
formulation [13].

Assume $y = (E, Q, I, R, S)^T$ then the system can be rewritten as

\begin{align*}
\frac{dy}{dt} & = F(y) - v(y)
\end{align*}

When

\begin{align*}
F(y) & = \begin{bmatrix}
\alpha SE \\
0 \\
0 \\
0
\end{bmatrix} \\
v(y) & = \begin{bmatrix}
BE \\
-\beta_3 S - \beta_2 E + CQ \\
-q_1 E - r_2 Q - DI \\
-\sigma_3 S - \sigma_3 Q - \sigma_3 I - d_1 R \\
-\Lambda + \alpha SE + AS
\end{bmatrix}
\end{align*}

(19)

$v$ is called transition part which describe the alter state $F$ is known as
transmission paper of new infection.

We now define Jacobian matrices of $F(y)$ and $v(y)$ at DFE $E_0$ is
defined by

\begin{align*}
DF(E_0) & = \begin{bmatrix}
F_{1,3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{align*}
\[ \nu(E_0) = \begin{bmatrix} V_{1,1} & 0 & 0 & 0 \\ 0 & -\sigma_2 & -\sigma_3 & d_1 & -\sigma_1 \\ \frac{\alpha\Lambda}{A} & 0 & 0 & 0 & 0 \end{bmatrix} \] 

(20)

\[ V = \begin{bmatrix} \beta_1 & 0 & 0 \\ -\beta_2 & C & 0 \\ -r_1 & -r_2 & D \end{bmatrix} \]

Therefore, \( F V^{-1} \) the next generation matrix of the SEIR model.

Endemic situations by [13]

Existence and uniqueness of disease-free equilibrium state (E0) of the Covid Model

The characteristic equation is given by

\[ \det(J_{E_0} - \lambda I) = 0 \]

where \( \lambda \) is an Eigen value of the matrix \( J_{E_0} \). Therefore, root of (22) are

\[ \lambda_1 = -A < 0 \quad \lambda_2 = \frac{a\Lambda}{A} - B < 0 \quad \lambda_3 = -D < 0 \quad \lambda_4 = -d_1 < 0 \]

Therefore, the given system is locally asymptotically at the E0(0, 0, 0, 0) under the condition \( R_0 < 1 \) [13].

\[ \Lambda - aSE - AS = 0 \]  

(23)

\[ aSE - BE = 0 \]  

(24)

\[ \beta_1 S + \beta_2 E - CQ = 0 \]  

(25)

\[ r_1 E + r_2 Q - DI = 0 \]  

(26)

\[ \sigma_1 S + \sigma_2 Q + \sigma_3 I - d_1 R = 0 \]  

(27)

\[ E' = 0 \quad \text{Or} \quad S' = \frac{B}{a} \]  

(28)

Then in Eq. (23) \( E' = 0 \)

then \( S' = \frac{\Lambda}{A} \)  

(29)

\[ I' = \frac{r_2 Q}{D} \quad \frac{r_2 \beta_1 \Lambda}{CDA} \]  

(30)

\[ R' = \frac{\sigma_1 S' + \sigma_2 Q'}{d_1} + \frac{\sigma_3 I'}{d_1} \]  

(31)

\[ Q' = \frac{\sigma_1 \Lambda}{d_1 A} + \frac{\sigma_2 \beta_1 \Lambda}{d_1 CA} + \frac{\sigma_3 \beta_1 \Lambda}{d_1 DCA} \]  

(32)

Then equilibrium points are given as

\[ E' = (S', 0, Q', I', R') \]  

(33)

\[ S' = \frac{B}{a} \]  

then

\[ E' = \left( \frac{\Lambda}{A} - A \right) \frac{1}{a} \]  

(34)

\[ Q' = \frac{\sigma_1 \Lambda}{d_1 A} + \frac{\sigma_2 \beta_1 \Lambda}{d_1 CA} + \frac{\sigma_3 \beta_1 \Lambda}{d_1 DCA} \]  

(35)

\[ I' = \frac{r_1 E' + r_2 Q'}{D} \]  

(36)

Existence and uniqueness of disease-free equilibrium state (E0) of the Covid Model

Lemma: The Covid model is locally asymptotically stable under the condition \( R_0 < 1 \) and became unstable \( R_0 > 1 \). Therefore, we present the proof by taking the Jacobean system at DFE \( E_0(\frac{\Lambda}{A}, 0, 0, 0) \)
\[ r_1 \left( \frac{x}{x - A} \right)^{\frac{p_1}{r_1}} + \frac{1}{r_2} \left( \frac{\rho \xi}{\xi - A} \right)^{\frac{r_1}{r_2}} \]

Finally

\[ R' = \sigma S' + \sigma Q' \quad \text{and} \quad E' = (S', E', Q', R', I'). \]

**Nonnegative solution and biological feasibility:**

**Lemma:** 
Let initial condition be \( S(t) > 0, E(t) > 0, R(t) > 0 \) as well as \( R(t) > 0 \), each solution of the covid model are positive for all values \( t \) greater than all equal to in the interval \([0, \infty)\).

Proof: We present the proof case by starting with \( C \) by using initial conditions \( S(t), E(t), I(t), R(t) \)
are unique in the interval \([0, \xi)\) where \( 0 < \xi < \infty \).

Let us assumed that all the solution has the same sign therefore

\[
\frac{dS}{dt} > -AS(t) \quad (35)
\]

\[
S(t) \geq S(0) \exp[-At] > 0
\]

\[
\Rightarrow \frac{dE}{dt} - BE(t)
\]

By given condition we get

\[
E(t) \geq E(0) \exp[-Bt] > 0
\]

Similarly we get

\[
Q(t) \geq Q(0) \exp[-Ct] > 0
\]

and \( I(t) \geq I(0) \exp[-Dt] > 0 \)

Similarly the system turns to

\[
\frac{dR}{dt} > -dR, \quad R(t) \geq R(0)
\]

\[
\exp[-dR(t)] > 0
\]

\[
I(t) \geq I(0) \exp[-Dt] > 0
\]

Thus we have

\[
S(t) > 0, E(t) \geq 0, I(t) \geq 0, R(t) > 0 \quad \forall t > 0.
\]

**Lyapunov stability of SEQIR model**

As we all know that Lyapunov Stability function is used to show the global stability of equilibrium. This is one of the wide applications Lyapunov tells about \( V(x) \)

Is a function unbounded and positively defined and derivative is negative i.e. \( V(x) < 0 \quad \forall x \neq x^* \) where \( x^* \) is an equilibrium and \( V(x) \) is Lyapunov function.

**Lemma:** If \( R_0 < 1 \), then Covid Model DFE \( E_0 = \left( \frac{1}{2}, 0, 0, 0, 0 \right) \) is disease-free equilibrium.

Proof: Let us consider the proposed model for three components \( S,E,I \). If these three equations of the proposed Covid model is disease free equilibrium then \( R,Q \rightarrow 0 \) therefore the Lyapunov function is defined on

\[
V = p \left( S - S^* - S^* \ln \left( \frac{S}{S^*} \right) + E + \frac{I}{r_1} \right)
\]

(40)

where \( S^* = \frac{1}{A} \) and \( p \) is parameter.

Since Eq. (40) is globally stable and disease free.

\[
\dot{V} = \left( S - S^* \right) \frac{\partial V}{\partial S} + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \dot{x}_i + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \dot{x}_i + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \dot{x}_i
\]

(41)

Now we since know from (41) first term is +ve and rest of the term is non negative so \( V \) is globally positive i.e. \( V > 0 \).

Now we do the differentiation we get

\[
\dot{V} = \left( 1 - \frac{S}{S^*} \right) S + \frac{E}{r_1} + \frac{I}{r_1}
\]

(42)

On putting the value of \( S', E', I' \) from given model we get

\[
\dot{V} = 2Ap - \frac{pqAd}{r_1}SI - ApS - \frac{A^2}{r_1} - (A + \frac{ApD}{r_1}) I + \frac{dSID}{r_1} - \frac{D}{r_1} I
\]

(43)

We take \( p = \frac{1}{A} \)

So we have

\[
V = \frac{A}{B} \left( \frac{AS}{A} - \frac{A}{SA} - 2 \right) + \frac{D}{r_1} I(R_0 - 1)
\]

where \( R_0 < 1 \)

\[
\frac{D}{r_1} I(R_0 - 1) < 0
\]

Now suppose that \( \frac{dS}{dt} = x \)

Then we get

\[
x + \frac{1}{x} - 2 = \frac{(x-1)^2}{x} > 0
\]

So here two cases arise. If \( S = S^* = \frac{1}{A} \) so it is equilibrium point and if we take \( x = 1 \) then \( x + \frac{1}{x} - 2 \) is

Zero then we have only \( \frac{2}{5} I(R_0 - 1) < 0 \) which is fully negative.

Therefore, we conclude that \( \dot{V} < 0 \).

Hence with the help of Lyapunov theorem the proposed model id Disease -free equilibrium.

We define the following Lyapunov function

\[
L(S,E,Q,I,R) = \left( S - S^* \right) \ln \left( \frac{S}{S^*} \right) + \left( E - E^* \right) \ln \left( \frac{E}{E^*} \right) + \left( Q - Q^* \right) \ln \left( \frac{Q}{Q^*} \right) + \left( I - I^* \right) \ln \left( \frac{I}{I^*} \right)
\]

(44)

Then the derivative with respect to \( t \) is given as

\[
\frac{dL}{dt}(S,E,Q,I,R) = \left( S - S^* \right) \frac{dS}{dt} + \left( E - E^* \right) \frac{dE}{dt} + \left( Q - Q^* \right) \frac{dQ}{dt} + \left( I - I^* \right) \frac{dI}{dt} + \left( R - R^* \right) \frac{dR}{dt}
\]

Then replacing \( \frac{dS}{dt}, \frac{dE}{dt}, \frac{dQ}{dt}, \frac{dR}{dt} \) and \( \frac{dE}{dt} \), by this we obtain

\[
\frac{dL}{dt}(S,E,Q,I,R) = \left( S - S^* \right) (\Lambda - aSE - AS) + \left( E - E^* \right) (aSE -
\]

(45)
\[ BE + \left( \frac{Q - Q'}{Q} \right) (\beta t S + \beta E - CQ) + \left( \frac{1 - I}{I} \right) (\beta t E + r_2 Q - \Delta I) + \left( \frac{R - R'}{R} \right) (\sigma_1 t S + \sigma_2 Q + \sigma_3 I - d_1 R) \]

\[
\frac{1}{\alpha^2} \left( \frac{S - S'}{S} \right) (\Delta - \alpha(S - S'))
\]

\[ A(S - S') + \left( \frac{E - E'}{E} \right) (\alpha S(E - E') - B (E - E')) + \left( \frac{Q - Q'}{Q} \right) (\beta t S + \beta_2 E - C(Q - Q')) + \left( \frac{1 - I}{I} \right) (r_1 E + r_2 Q - \Delta(I - I')) + \left( \frac{R - R'}{R} \right) (\sigma_1 t S + \sigma_2 Q + \sigma_3 I - d_1 (R - R')) \]

\[
\frac{dL}{dt} = \left( \frac{S - S'}{S} \right) (\Delta - \alpha) - \left( \frac{Q - Q'}{Q} \right) (\Delta - \alpha) - C \frac{Q - Q'}{Q} \frac{(1 - t - I')^2}{I'} - d_1 \]

\[ q + r_1 E + r_2 Q - r_2 Q' + r_1 E' - r_2 Q' + \sigma_1 S + \sigma_2 Q + \sigma_3 I = \frac{\alpha}{\alpha^2} \left( \frac{S}{S} \right) \left( \alpha + \beta + \beta_2 E - \beta_1 S \right) \frac{Q + Q'}{Q} \frac{(1 - t - I')^2}{I'} - d_1 \]

\[ + \sigma_1 S + \sigma_2 Q + \sigma_3 I - \sigma_1 S' + \sigma_3 I' - \sigma_2 Q' + \sigma_3 I' \]

Rearranging, we get

\[ \frac{dL}{dt} = \frac{(S - S')^2}{S} (\alpha - \beta - \beta_2 E) \]

\[ \frac{dL}{dt} = 0 \Rightarrow L_1 = L_2 \]

\[ \frac{dL}{dt} \geq 0 \Rightarrow L_1 \geq L_2 \]

\[ \frac{dL}{dt} \leq 0 \Rightarrow L_1 \leq L_2 \]

### Fractional corona virus model

Before presenting the model in fractional derivative, we give the definition of fractional derivative and their integral below:

**Definition 1.** The Caputo fractional derivative given as

\[ D^{\alpha}_t u(t) = \frac{1}{\Gamma(n - \alpha)} \int^t_0 (t - s)^{n - 1 - \alpha} \frac{d^n}{ds^n} u(s) ds, \]

where \( n - 1 \leq \alpha < n \).

**Definition 2.** The Caputo fractional derivative given as

\[ CF D^{\alpha}_t u(t) = \frac{M(t)}{(1 - \alpha)} \int^t_0 \exp \left( - \frac{t - \tau}{1 - \tau} \right) u^{\alpha}(\tau) d\tau, \]

where \( u \in H^1(a, b), 0 < \alpha < 1 \).

**Definition 3.** The Caputo Fabrizio integral operator given as

\[ CF J^{\alpha}_t (u(t)) = \frac{1}{(1 - \alpha) M(t)} \int^t_0 u(s) ds, \]

where \( 0 < \alpha < 1 \).

**Definition 4.** The Atangana – Baleanu derivative given as

\[ AB D^{\alpha}_t u(t) = \frac{M(t)}{(1 - \alpha)} \int^t_0 \exp \left( - \frac{t - \tau}{1 - \alpha} \right) u^{\alpha}(\tau) d\tau, \]

where \( u \in H^1(a, b), b > a \).

**Definition 5.** The Atangana – Baleanu fractional derivative given as

\[ AB D^{\alpha}_t u(t) = \frac{M(t)}{(1 - \alpha)} \int^t_0 u(s) ds \]

**Definition 6.** The Atangana – Baleanu fractional integral derivative given as

\[ AB D^{\alpha}_t u(t) = \frac{M(t)}{(1 - \alpha)} \int^t_0 u(s) ds \]

where \( M(t) \) is the normalizing function.

**Definition 7.** Suppose that be \( y(t) \) be continuous and fractal differentiable on \( (a, b) \) with order \( \beta \) then be fractal- fractional derivative of \( y(t) \) with order \( \alpha \) in the Riemann – Lowville sense having power law type kernel is defined as follows:

\[ FFP D^{\alpha}_t (y(t)) = \frac{1}{1 (m - \alpha)} \int^t_0 (t - s)^{m - 1} y(s) ds \]

Where \( m - 1 < \alpha, \beta \in \mathbb{N} \) and \( \frac{dy(s)}{ds} = \lim_{s \to \alpha} y(t) - y(s) \)

**Definition 8.** Suppose that be \( y(t) \) be continuous and fractal differentiable on \( (a, b) \) with order \( \beta \) then be fractal- fractional derivative of \( y(t) \) with order \( \alpha \) in the Riemann – Lowville sense having exponentially decaying type kernel is defined as follows:

\[ FFP D^{\alpha}_t (y(t)) = \frac{M(a)}{(1 - \alpha)} \int^t_0 \exp \left( - \frac{a}{1 - \alpha} (t - s) \right) y(s) ds, \]

where \( a > 0, \beta \in \mathbb{N} \) and \( M(0) = M(1) = 1 \).

**Definition 9.** Suppose that be \( y(t) \) be continuous and fractal differentiable on \( (a, b) \) with order \( \beta \) then be fractal- fractional derivative of \( y(t) \) with order \( \alpha \) in the Riemann – Lowville sense having Mittag-Leffler type kernel is defined as follows:

\[ FFP D^{\alpha}_t (y(t)) = \frac{AB(a)}{(1 - \alpha)} \int^t_0 \exp \left( - \frac{a}{1 - \alpha} (t - s) \right) y(s) ds, \]

where \( 0 < a, \beta \leq 1 \) and \( AB(a) = 1 - a + \frac{\beta a}{(1 - \alpha)} \).
kernel is defined as follows:

\[
F_{\text{Fr}} f_{\text{Fr}}(y(t)) = \frac{a_0}{M(\alpha)} \int_0^t s^{\alpha-1} y(s)(t-s)^{\alpha-1} ds + \beta(1-\alpha)s^{\alpha-1}y(t).
\]  

(56)

Numerical schemes

In this section, we have designed three numerical schemes for Caputo Fractal-Fractional, Caputo-Fabrizio Fractal-fractional and the Atangana-Baleanu Fractal-fractional derivative operators.

\[
\begin{align*}
\text{ABC}^\alpha D^\alpha_{\text{Fr}}[S(t)] &= \Lambda - \alpha SE - AS \\
\text{ABC}^\beta D^\beta_{\text{Fr}}[E(t)] &= aSE - BE \\
\text{ABC}^\gamma D^\gamma_{\text{Fr}}[Q(t)] &= \beta_s S + \beta_q E - CQ \\
\text{ABC}^\delta D^\delta_{\text{Fr}}[I(t)] &= r_I E + r_Q D - DI \\
\text{ABC}^\epsilon D^\epsilon_{\text{Fr}}[R(t)] &= \sigma_s S + \sigma_q Q + \sigma_I - \delta R
\end{align*}
\]

(57)

Numerical scheme for Caputo Fractal-fractional derivative

In this section we consider fractal-fractional differential equations in the Caputo sense. Consider the fractional differential equation

\[
S(t) = S(0) + \int_0^t \tau \lambda^{-1}(t-\lambda)^{\alpha-1} f(S, E, Q, I, R, \lambda) d\lambda
\]

Such that our system becomes

\[
\begin{align*}
\text{RL}^\alpha D^\alpha_{\text{Fr}}[S(t)] &= \tau \lambda^{-1}[\Lambda - \alpha SE - AS] \\
\text{RL}^\beta D^\beta_{\text{Fr}}[E(t)] &= \tau \lambda^{-1}[aSE - BE] \\
\text{RL}^\gamma D^\gamma_{\text{Fr}}[Q(t)] &= \tau \lambda^{-1}[\beta_s S + \beta_q E - CQ] \\
\text{RL}^\delta D^\delta_{\text{Fr}}[I(t)] &= \tau \lambda^{-1}[r_I E + r_Q D - DI] \\
\text{RL}^\epsilon D^\epsilon_{\text{Fr}}[R(t)] &= \tau \lambda^{-1}[\sigma_s S + \sigma_q Q + \sigma_I - \delta R]
\end{align*}
\]

(58)

We now replace the Riemann-Liouville derivative to Caputo derivative in order to make the use of the integer-order initial conditions, and then we apply the Riemann-Liouville fractional integral on both sides to have the following

\[
S(t) = S(0) + \int_0^t \tau \lambda^{-1}(t-\lambda)^{\alpha-1} f(S, E, Q, I, R, \lambda) d\lambda
\]

\[
E(t) = E(0) + \int_0^t \tau \lambda^{-1}(t-\lambda)^{\alpha-1} g(S, E, Q, I, R, \lambda) d\lambda
\]

\[
Q(t) = Q(0) + \int_0^t \tau \lambda^{-1}(t-\lambda)^{\alpha-1} h(S, E, Q, I, R, \lambda) d\lambda
\]

\[
I(t) = I(0) + \int_0^t \tau \lambda^{-1}(t-\lambda)^{\alpha-1} j(S, E, Q, I, R, \lambda) d\lambda
\]

\[
R(t) = R(0) + \int_0^t \tau \lambda^{-1}(t-\lambda)^{\alpha-1} l(S, E, Q, I, R, \lambda) d\lambda
\]

where

\[
\begin{align*}
f(S, E, Q, I, R, \lambda) &= \Lambda - aSE - AS \\
g(S, E, Q, I, R, \lambda) &= aSE - BE \\
h(S, E, Q, I, R, \lambda) &= \beta_s S + \beta_q E - CQ \\
j(S, E, Q, I, R, \lambda) &= r_I E + r_Q D - DI \\
l(S, E, Q, I, R, \lambda) &= \sigma_s S + \sigma_q Q + \sigma_I - \delta R
\end{align*}
\]

Now applying the numerical scheme of the above system using a new approach at \(t_{n+1}\)

Then we have result as below

\[
S^{n+1} = S^n + \int_0^{t_{n+1}} \lambda^{-1}(t_{n+1} - \lambda)^{\alpha-1} f(S, E, Q, I, R, \lambda) d\lambda
\]

\[
E^{n+1} = E^n + \int_0^{t_{n+1}} \lambda^{-1}(t_{n+1} - \lambda)^{\alpha-1} g(S, E, Q, I, R, \lambda) d\lambda
\]

\[
Q^{n+1} = Q^n + \int_0^{t_{n+1}} \lambda^{-1}(t_{n+1} - \lambda)^{\alpha-1} h(S, E, Q, I, R, \lambda) d\lambda
\]

\[
I^{n+1} = I^n + \int_0^{t_{n+1}} \lambda^{-1}(t_{n+1} - \lambda)^{\alpha-1} j(S, E, Q, I, R, \lambda) d\lambda
\]

\[
R^{n+1} = R^n + \int_0^{t_{n+1}} \lambda^{-1}(t_{n+1} - \lambda)^{\alpha-1} l(S, E, Q, I, R, \lambda) d\lambda
\]

Now by using Lagrange piece-wise interpolation we approximate the function \(x^{-1}f(S, E, Q, I, R, \lambda)\) within the interval \([t_j, t_{j+1}]\) such that

\[
U_j(\lambda) = \frac{\lambda - t_{j+1}}{t_j - t_{j+1}} f(S, E, Q, I, R, \lambda) - \frac{\lambda - t_j}{t_{j+1} - t_j} f(S, E, Q, I, R, \lambda)
\]

\[
W_j(\lambda) = \frac{\lambda - t_j}{t_{j+1} - t_j} g(S, E, Q, I, R, \lambda) - \frac{\lambda - t_{j+1}}{t_j - t_{j+1}} g(S, E, Q, I, R, \lambda)
\]

\[
X_j(\lambda) = \frac{\lambda - t_{j+1}}{t_j - t_{j+1}} h(S, E, Q, I, R, \lambda) - \frac{\lambda - t_j}{t_{j+1} - t_j} h(S, E, Q, I, R, \lambda)
\]

\[
Y_j(\lambda) = \frac{\lambda - t_{j+1}}{t_j - t_{j+1}} j(S, E, Q, I, R, \lambda) - \frac{\lambda - t_j}{t_{j+1} - t_j} j(S, E, Q, I, R, \lambda)
\]

\[
Z_j(\lambda) = \frac{\lambda - t_{j+1}}{t_j - t_{j+1}} l(S, E, Q, I, R, \lambda) - \frac{\lambda - t_j}{t_{j+1} - t_j} l(S, E, Q, I, R, \lambda)
\]

Thus, we obtain

\[
S^{n+1} = S^n + \frac{\lambda}{M(\alpha)} \sum_{j=0}^n \int_0^{t_{j+1}} \lambda^{-1}(t_{j+1} - \lambda)^{\alpha-1} U_j(\lambda) d\lambda
\]

\[
E^{n+1} = E^n + \frac{\lambda}{M(\alpha)} \sum_{j=0}^n \int_0^{t_{j+1}} \lambda^{-1}(t_{j+1} - \lambda)^{\alpha-1} W_j(\lambda) d\lambda
\]

\[
Q^{n+1} = Q^n + \frac{\lambda}{M(\alpha)} \sum_{j=0}^n \int_0^{t_{j+1}} \lambda^{-1}(t_{j+1} - \lambda)^{\alpha-1} X_j(\lambda) d\lambda
\]

\[
I^{n+1} = I^n + \frac{\lambda}{M(\alpha)} \sum_{j=0}^n \int_0^{t_{j+1}} \lambda^{-1}(t_{j+1} - \lambda)^{\alpha-1} Y_j(\lambda) d\lambda
\]

\[
R^{n+1} = R^n + \frac{\lambda}{M(\alpha)} \sum_{j=0}^n \int_0^{t_{j+1}} \lambda^{-1}(t_{j+1} - \lambda)^{\alpha-1} Z_j(\lambda) d\lambda
\]
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operators are fractal–fractional in the Caputo-Fabrizio sense. Thus the
\[ n = \frac{\alpha}{1 + \alpha}(n - j + 2) - n(j - j + 2 + 2\alpha) \]
\[ - \tau_{\lambda} f(S^E, E^I, Q^E, F, R^E, t_j) \times ((n + 1 - j)^{n-1} - (n - j)^{n-1}((n - j + 1 + \alpha))]. \]
\[ Q^{n+1} = Q^n + \frac{\tau(\Delta t)}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} \tau_{\lambda} h(S^E, E^I, Q^E, F, R^E, t_j) \times
\]
\[ ((n + 1 - j)^{n-1} - (n - j)^{n-1}((n - j + 1 + \alpha))]. \]

Numerical scheme for Caputo–Fabrizio–fractal–fractional

Here we consider the caputo models where the fractional differential
operators are fractal–fractional in the Caputo-Fabrizio sense. Thus the
caputo model can be converted to the following
\[ f(S, E, Q, I, R, \lambda) = \Lambda - aSE - AS \]
\[ g(S, E, Q, I, R, \lambda) = aSE - BE \]
\[ h(S, E, Q, I, R, \lambda) = k_{1E} + k_{2E} - CQ \]
\[ j(S, E, Q, I, R, \lambda) = r_1E + r_2Q - DI \]
\[ \sigma_S + \sigma_Q + \sigma_I - d_R \]

Now

\[ R^{n+1} = R^n \frac{\tau}{\Gamma(\alpha)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} \lambda^{n-1}((n + 1 - j)^ {n-1} - \lambda^{n-1}Z(\lambda))d\lambda \]

On solving equation, we obtain the following numerical scheme
\[ S^{n+1} = S^n + \frac{\tau(\Delta t)}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} \tau_{\lambda} f(S^E, E^I, Q^E, F, R^E, t_j) \times
\]
\[ ((n + 1 - j)^{n-1} - (n - j)^{n-1}((n - j + 2 + 2\alpha)) \]
\[ - \tau_{\lambda} f(S^E, E^I, Q^E, F, R^E, t_j) \times ((n + 1 - j)^{n-1} - (n - j)^{n-1}((n - j + 1 + \alpha))]. \]
\[ E^{n+1} = E^n + \frac{\tau(\Delta t)}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} \tau_{\lambda} g(S^E, E^I, Q^E, F, R^E, t_j) \times
\]
\[ ((n + 1 - j)^{n-1} - (n - j)^{n-1}((n - j + 2 + 2\alpha)) \]
\[ - \tau_{\lambda} g(S^E, E^I, Q^E, F, R^E, t_j) \times ((n + 1 - j)^{n-1} - (n - j)^{n-1}((n - j + 1 + \alpha))]. \]
\[ Q^{n+1} = Q^n + \frac{\tau(\Delta t)}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} \int_{t_j}^{t_{j+1}} \tau_{\lambda} h(S^E, E^I, Q^E, F, R^E, t_j) \times
\]
\[ ((n + 1 - j)^{n-1} - (n - j)^{n-1}((n - j + 2 + 2\alpha)) \]
\[ - \tau_{\lambda} h(S^E, E^I, Q^E, F, R^E, t_j) \times ((n + 1 - j)^{n-1} - (n - j)^{n-1}((n - j + 1 + \alpha))]. \]

Applying the Caputo–Fabrizio integral, we obtain
\[ S(t) = S(0) + \frac{(1-a)\tau^{\alpha-1}}{M(a)} f(S, E, Q, I, R, \lambda) + \frac{\alpha \tau}{M(a)} \int_0^t \lambda^{\alpha-1} f(S, E, Q, I, R, \lambda) d\lambda, \]
\[ E(t) = E(0) + \frac{(1-a)\tau^{\alpha-1}}{M(a)} g(S, E, Q, I, R, \lambda) + \frac{\alpha \tau}{M(a)} \int_0^t \lambda^{\alpha-1} g(S, E, Q, I, R, \lambda) d\lambda, \]
\[ Q(t) = Q(0) + \frac{(1-a)\tau^{\alpha-1}}{M(a)} h(S, E, Q, I, R, \lambda) + \frac{\alpha \tau}{M(a)} \int_0^t \lambda^{\alpha-1} h(S, E, Q, I, R, \lambda) d\lambda, \]
\[ I(t) = I(0) + \frac{(1-a)\tau^{\alpha-1}}{M(a)} l(S, E, Q, I, R, \lambda) + \frac{\alpha \tau}{M(a)} \int_0^t \lambda^{\alpha-1} l(S, E, Q, I, R, \lambda) d\lambda, \]
\[ R(t) = R(0) + \frac{(1-a)\tau^{\alpha-1}}{M(a)} r(S, E, Q, I, R, \lambda) + \frac{\alpha \tau}{M(a)} \int_0^t \lambda^{\alpha-1} r(S, E, Q, I, R, \lambda) d\lambda, \]

Taking the difference between the consecutive terms, we obtain
\[ S^{n+1} = S^n + \frac{(1-a)\tau^{\alpha-1}}{M(a)} f(S^n, E^n, Q^n, I^n, R^n, \lambda) - \frac{\tau^{\alpha-1}}{M(a)} \int_0^{t^n} \lambda^{\alpha-1} f(S^n, E^n, Q^n, I^n, R^n, \lambda) d\lambda, \]
\[ E^{n+1} = E^n + \frac{(1-a)\tau^{\alpha-1}}{M(a)} g(S^n, E^n, Q^n, I^n, R^n, \lambda) - \frac{\tau^{\alpha-1}}{M(a)} \int_0^{t^n} \lambda^{\alpha-1} g(S^n, E^n, Q^n, I^n, R^n, \lambda) d\lambda, \]
\[ Q^{n+1} = Q^n + \frac{(1-a)\tau^{\alpha-1}}{M(a)} h(S^n, E^n, Q^n, I^n, R^n, \lambda) - \frac{\tau^{\alpha-1}}{M(a)} \int_0^{t^n} \lambda^{\alpha-1} h(S^n, E^n, Q^n, I^n, R^n, \lambda) d\lambda, \]
\[ R^{n+1} = R^n + \frac{(1-a)\tau^{\alpha-1}}{M(a)} l(S^n, E^n, Q^n, I^n, R^n, \lambda) - \frac{\tau^{\alpha-1}}{M(a)} \int_0^{t^n} \lambda^{\alpha-1} l(S^n, E^n, Q^n, I^n, R^n, \lambda) d\lambda, \]
Now using the Lagrange polynomial piece-wise interpolation and integrating, we obtain $S^{n+1} = S^n + \frac{(1 - \alpha)\tau^{n-1}_M}{M(\alpha)} f(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n) - \frac{\tau^{n-1}_M}{M(\alpha)} f(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n) - \frac{\tau^{n-1}_M}{M(\alpha)} g(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n) - \frac{\tau^{n-1}_M}{M(\alpha)} h(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n) - \frac{\tau^{n-1}_M}{M(\alpha)} (\frac{1}{2}) \left[ \delta(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n) \right]$

$I^{n+1} = I^n + \frac{(1 - \alpha)\tau^{n-1}_M}{M(\alpha)} j(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n)$

$R^{n+1} = R^n + \frac{(1 - \alpha)\tau^{n-1}_M}{M(\alpha)} l(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n)$

$Q^{n+1} = Q^n + \frac{(1 - \alpha)\tau^{n-1}_M}{M(\alpha)} m(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n)$

At $[\tau_{n+1}]$, we get the following

$S^{n+1} = S^n + \frac{(1 - \alpha)\tau^{n-1}_M}{M(\alpha)} f(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n) + \frac{\tau^{n-1}_M}{M(\alpha)} g(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n) + \frac{\tau^{n-1}_M}{M(\alpha)} h(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n) + \frac{\tau^{n-1}_M}{M(\alpha)} (\frac{1}{2}) \left[ \delta(S^n, E^n, Q^n, \Gamma^n, R^n, \tau_n) \right]$

Numerical scheme for Atangana-Baleanu- fractal-fractional

In this section, we consider the models where the fractional differential operator is that of Atangana- Baleanu fractal- fractional derivative. In this investigation the given covid model represent as follows:

$ABC D_T^n [S(t)] = \tau I^{n-1} f(S, E, Q, I, R, t)$

$ABC D_T^n [E(t)] = \tau I^{n-1} g(S, E, Q, I, R, t)$

$ABC D_T^n [Q(t)] = \tau I^{n-1} h(S, E, Q, I, R, t)$

$ABC D_T^n [R(t)] = \tau I^{n-1} I(S, E, Q, I, R, t)$

Now applying the Atangan-Baleanu integral, we have

$S(t) = S(0) + \frac{(1 - \alpha)\tau^{n-1}_M}{AB(\alpha)} f(S, E, Q, I, R, t) + \frac{\tau^{n-1}_M}{AB(\alpha)} g(S, E, Q, I, R, t) + \frac{\tau^{n-1}_M}{AB(\alpha)} h(S, E, Q, I, R, t) + \frac{\tau^{n-1}_M}{2} \left[ \delta(S, E, Q, I, R, t) \right]$
\begin{equation}
S^{n+1} = S^n + \frac{(1 - \alpha)\tau^{n+1}}{AB(a)} f(S^n, E^n, Q^n, F^n, R^n, t_n) + \alpha \int_{(a+1)}^{\alpha+1} \frac{\tau^{n+1}}{AB(\alpha)} \sum_{j=0}^{N-1} \int_{t_0}^{t_j+1} \frac{\tau^{n+1}}{AB(\alpha)} f(S^n, E^n, Q^n, F^n, R^n, t_j) dt_j,
\end{equation}

\begin{equation}
E^{n+1} = E^n + \frac{(1 - \alpha)\tau^{n+1}}{AB(a)} g(S^n, E^n, Q^n, F^n, R^n, t_n) + \alpha \int_{(a+1)}^{\alpha+1} \frac{\tau^{n+1}}{AB(\alpha)} \sum_{j=0}^{N-1} \int_{t_0}^{t_j+1} \frac{\tau^{n+1}}{AB(\alpha)} g(S^n, E^n, Q^n, F^n, R^n, t_j) dt_j,
\end{equation}

\begin{equation}
Q^{n+1} = Q^n + \frac{(1 - \alpha)\tau^{n+1}}{AB(a)} h(S^n, E^n, Q^n, F^n, R^n, t_n) + \alpha \int_{(a+1)}^{\alpha+1} \frac{\tau^{n+1}}{AB(\alpha)} \sum_{j=0}^{N-1} \int_{t_0}^{t_j+1} \frac{\tau^{n+1}}{AB(\alpha)} h(S^n, E^n, Q^n, F^n, R^n, t_j) dt_j,
\end{equation}

\begin{equation}
P^{n+1} = P^n + \frac{(1 - \alpha)\tau^{n+1}}{AB(a)} i(S^n, E^n, Q^n, F^n, R^n, t_n) + \alpha \int_{(a+1)}^{\alpha+1} \frac{\tau^{n+1}}{AB(\alpha)} \sum_{j=0}^{N-1} \int_{t_0}^{t_j+1} \frac{\tau^{n+1}}{AB(\alpha)} i(S^n, E^n, Q^n, F^n, R^n, t_j) dt_j,
\end{equation}

\begin{equation}
R^{n+1} = R^n + \frac{(1 - \alpha)\tau^{n+1}}{AB(a)} j(S^n, E^n, Q^n, F^n, R^n, t_n) + \alpha \int_{(a+1)}^{\alpha+1} \frac{\tau^{n+1}}{AB(\alpha)} \sum_{j=0}^{N-1} \int_{t_0}^{t_j+1} \frac{\tau^{n+1}}{AB(\alpha)} j(S^n, E^n, Q^n, F^n, R^n, t_j) dt_j,
\end{equation}

By using numerical scheme approximating \(\lambda^{n+1}(\tilde{x}, E^{n+1}, Q^{n+1}, F^{n+1}, R^{n+1}, t_{n+1})\), \(\lambda^{n+1}(\tilde{x}, E^{n+1}, Q^{n+1}, F^{n+1}, R^{n+1}, t_{n+1})\), \(\lambda^{n+1}(\tilde{x}, E^{n+1}, Q^{n+1}, F^{n+1}, R^{n+1}, t_{n+1})\), and \(\lambda^{n+1}(\tilde{x}, E^{n+1}, Q^{n+1}, F^{n+1}, R^{n+1}, t_{n+1})\), we get the following:

\begin{equation}
\lambda^{n+1} = \lambda^n + \frac{(1 - \alpha)\tau^{n+1}}{AB(a)} f(S^n, E^n, Q^n, F^n, R^n, t_n) + \alpha \int_{(a+1)}^{\alpha+1} \frac{\tau^{n+1}}{AB(\alpha)} \sum_{j=0}^{N-1} \int_{t_0}^{t_j+1} \frac{\tau^{n+1}}{AB(\alpha)} f(S^n, E^n, Q^n, F^n, R^n, t_j) dt_j,
\end{equation}

\begin{equation}
\lambda^{n+1} = \lambda^n + \frac{(1 - \alpha)\tau^{n+1}}{AB(a)} h(S^n, E^n, Q^n, F^n, R^n, t_n) + \alpha \int_{(a+1)}^{\alpha+1} \frac{\tau^{n+1}}{AB(\alpha)} \sum_{j=0}^{N-1} \int_{t_0}^{t_j+1} \frac{\tau^{n+1}}{AB(\alpha)} h(S^n, E^n, Q^n, F^n, R^n, t_j) dt_j,
\end{equation}

\begin{equation}
\lambda^{n+1} = \lambda^n + \frac{(1 - \alpha)\tau^{n+1}}{AB(a)} i(S^n, E^n, Q^n, F^n, R^n, t_n) + \alpha \int_{(a+1)}^{\alpha+1} \frac{\tau^{n+1}}{AB(\alpha)} \sum_{j=0}^{N-1} \int_{t_0}^{t_j+1} \frac{\tau^{n+1}}{AB(\alpha)} i(S^n, E^n, Q^n, F^n, R^n, t_j) dt_j,
\end{equation}

\begin{equation}
\lambda^{n+1} = \lambda^n + \frac{(1 - \alpha)\tau^{n+1}}{AB(a)} j(S^n, E^n, Q^n, F^n, R^n, t_n) + \alpha \int_{(a+1)}^{\alpha+1} \frac{\tau^{n+1}}{AB(\alpha)} \sum_{j=0}^{N-1} \int_{t_0}^{t_j+1} \frac{\tau^{n+1}}{AB(\alpha)} j(S^n, E^n, Q^n, F^n, R^n, t_j) dt_j.
\end{equation}

Existence and uniqueness under Atangan-Baleanu-fractal-fractional derivative

Here we consider ordinary differential equation with fractal-fractional derivative for general Cauchy problem given as in the Atangan-Balea case.

\[ FF D_{0+}^\alpha(y(t)) = g(t,f(t)) \]

Then by using the definition of \( FF D_{0+}^\alpha(y(t)) \) we can define as follows:

\[ AB(a) \frac{d}{dt} \int_0^t g(\lambda, f(\lambda)) d\lambda - \frac{1}{1-\alpha} \lambda^{\alpha} \int_0^t g(\lambda, f(\lambda)) d\lambda \]

We convert into the following, as integral is differentiable, then we have

\[ \frac{1}{\tau^{n+1}} \frac{d}{dt} \int_0^t g(\lambda, f(\lambda)) d\lambda - \frac{1}{1-\alpha} \lambda^{\alpha} \int_0^t g(\lambda, f(\lambda)) d\lambda \]

Therefore, equation no (4.1) can be written in the following form:

\[ AB(a) \frac{d}{dt} \int_0^t g(\lambda, f(\lambda)) d\lambda - \frac{1}{1-\alpha} \lambda^{\alpha} \int_0^t g(\lambda, f(\lambda)) d\lambda \]

Now we apply the integral in Caputo sense on the right-hand side

\[ f(t) = \frac{(1 - \alpha)}{AB(a)} \tau^{n+1} g(t,f(t)) + \frac{d}{dt} \int_0^t \tau^{n+1} g(\lambda, f(\lambda)) d\lambda + f(0) \]

Now we consider Picard Lindolf theorem, we let

\[ \Pi^t_{\alpha} = L_{\alpha}(t_0) \times A_\alpha(f_0) \]

where \( L_{\alpha}(t_0) = [t_0-a, t_0+a] \), \( A_\alpha(f_0) = [t_0-b, t_0+b] \)

Now we have,

\[ K = \sup_{ \alpha \in \Pi^t_{\alpha}} \|g\| \]

Then we apply the following operations

\[ \Lambda(C_{\alpha}(t_0), A_{\alpha}(t_0)) \Rightarrow C_{\alpha}(t_0, b), A_{\alpha}(t_0)) \]

\[ \Lambda_{\psi}(t) = f_0 + \frac{(1 - \alpha)}{AB(a)} \tau^{n+1} g(t,f(t)) + \frac{\tau a}{AB(a)\Gamma(\alpha)} \int_0^t \lambda^{\alpha} g(\lambda, \psi(\lambda)) d\lambda \]

\[ \| \Lambda_{\psi}(t) - f_0 \| \leq b \]

\[ \| \Lambda_{\psi}(t) - f_0 \| \leq \frac{(1 - \alpha)}{AB(a)} \tau^{n+1} \| g(t,\psi(t)) \| \]

\[ \Lambda_{\psi}(t) = f_0 + \frac{\tau a}{AB(a)\Gamma(\alpha)} \int_0^t \lambda^{\alpha} g(\lambda, \psi(\lambda)) d\lambda + \frac{\tau a}{AB(a)\Gamma(\alpha)} \int_0^t \lambda^{\alpha} K d\lambda \]

Now we suppose that \( \lambda = ty \) then equation number (71) transform to
\begin{equation}
\leq \frac{(1 - \alpha)}{AB(\alpha)} \tau^{\alpha - 1}K \\
+ \frac{\tau \alpha}{AB(\alpha) \Gamma(\alpha)} \int_{0}^{t} \lambda^{\alpha - 1} (t - \lambda)^{\alpha - 1} d\lambda,
\end{equation}

Let us consider \( \psi_1 \) and \( \psi_2 \) provided that \( \psi_1, \psi_2 \in C I_{\alpha}(t_0, A(t_0)) \) to meet the following inequality we may consider Banach fixed point theorem

\[ \|\Lambda \psi_1 - \Lambda \psi_2\| \leq \beta \|\psi_1 - \psi_2\|_{\infty} \]

where \( \beta \leq 1 \),

Fig. 2. Numerical solution for the class of recovered for different fractional orders.

Fig. 3. Numerical solution for class of infected class for different fractional orders.
\[\|\Lambda\psi_1 - \Lambda\psi_2\| \leq \frac{(1-a)\tau^{r-1}L}{AB(a)} \|\psi_1 - \psi_2\| + \frac{\tau a}{\Gamma(\alpha)AB(a)\Gamma(\alpha)} \int_0^1 (t-\tau)^{r-1} \|g(\lambda, \psi_1 - g(\lambda, \psi_2))\| \, d\tau,\]

\[\|\Lambda\psi_1 - \Lambda\psi_2\| \leq \frac{(1-a)\tau^{r-1}L}{AB(a)} \|\psi_1 - \psi_2\| + \frac{\tau a}{\Gamma(\alpha)AB(a)\Gamma(\alpha)} \int_0^1 (t-\tau)^{r-1} \|g(\lambda, \psi_1 \psi_2)\| \, d\tau,\]

\[\|\Lambda\psi_1 - \Lambda\psi_2\| \leq \frac{(1-a)\tau^{r-1}L}{AB(a)} \|\psi_1 - \psi_2\| + \frac{\tau a}{\Gamma(\alpha)AB(a)\Gamma(\alpha)} \int_0^1 (t-\tau)^{r-1} \|g(\lambda, \psi_1 \psi_2)\| \, d\tau,\]

\[\|\Lambda\psi_1 - \Lambda\psi_2\| \leq \frac{(1-a)\tau^{r-1}L}{AB(a)} \|\psi_1 - \psi_2\| + \frac{\tau a}{\Gamma(\alpha)AB(a)\Gamma(\alpha)} \int_0^1 (t-\tau)^{r-1} \|g(\lambda, \psi_1 \psi_2)\| \, d\tau,\]

Therefore we conclude that

\[\|\Lambda\psi_1 - \Lambda\psi_2\| \leq \frac{(1-a)\tau^{r-1}L}{AB(a)} \|\psi_1 - \psi_2\| + \frac{\tau a}{\Gamma(\alpha)AB(a)\Gamma(\alpha)} \int_0^1 (t-\tau)^{r-1} \|g(\lambda, \psi_1 \psi_2)\| \, d\tau,\]

Now we have seen that \(\Lambda\) has unique solution and this is the complete solution.
Fig. 5. Numerical solution for class of infected class for different fractional orders.

Fig. 6. Numerical solution for the class of recovered for different fractional orders.
Then if this condition satisfied so the equation has a unique solution.

Hence existence and uniqueness under the power law case is complete.

**Numerical simulation**

The numerical simulations presented here give rise to different situation of the spread, it is easy to notice that the use of fractional differentiation and integration provides more room to see different trends of the spread for different classes. For example, the numerical simulations showed that fractional orders play an important role for example, for the infected, recovered, $Q(t)$ and $E(t)$ classes the classical differential operators predict more numbers while the fractional counterparts show less number for different values of fractional orders. Nevertheless, for susceptible class, fractional differentiation predicts more numbers while classical predict less. It is therefore clear that given a set of collected data fractional orders can be used to fit the data.

In this section, using the suggested numerical scheme, we present the numerical solution of the model using the following parameters

The above figures were depicted in the model with Caputo-Fabrizio derivative. The numerical simulations are depicted for different values of fractional order. These solutions are presented in Figs. 2–6. The below Figs. 7–11 are numerical simulations of the model with the Atangana-Baleanu fractional derivative. Here also different classes are depicted.

![Fig. 7. Numerical solution for the class of suspected for different fractional orders.](image1)

![Fig. 8. Numerical solution for the class of infected for different fractional orders.](image2)
for different values of fractional orders. For some classes, including infected classes, the model predicted a lognormal distribution, for the recovered classes, we observed the exponential growth memory process and for the susceptible classes, we observed a fading memory process, an indication that such spread could die off according to the used parameters.

**Conclusion**

Mathematical models depicting a possible spread of covid-19 among humans with five classes have been considered in this paper. These classes included, susceptible, tested population, quarantined, population of those under treatment and recovered classes. We presented section devoted to the study of positive solutions, stability analysis, reproductive number and the conditions under which the possibility of endemic could occur. To further capture more complexities and other possibilities of the spread, different differential and integral operators with non-integer and integer orders were used to modify the classical version. Due to non-linearity of these modified models, different numerical schemes were employed to solve these models numerically. Additionally, some numerical simulations were performed using the obtained numerical solutions.
Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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