On Spherically Symmetric Breathers in Scalar Theories

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Abstract

We develop an algorithm which can be used to exclude the existence of classical breathers (periodic finite energy solutions) in scalar field theories, and apply it to several cases of interest. In particular, the technique is used to show that a pair of potentially periodic solutions of the 3+1 Sine-Gordon Lagrangian, found numerically in earlier work †, are not breathers. These “pseudo-breather states” do have a signature in our method, which we suggest can be used to find similar quasi-bound state configurations in other theories. We also discuss the results of our algorithm when applied to the 1+1 Sine-Gordon model (which exhibits a well-known set of breathers), and $\phi^4$ theory.
1 Introduction

The leading order chiral lagrangian description of pions, when evaluated on an isospin-polarized field configuration, is identical to the 3+1 Sine-Gordon lagrangian
\[
\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + F_\pi^2 m^2 \cos(\phi/F_\pi). \tag{1}
\]

The motivation for considering isospin-polarization of pion fields arises from the possibility of producing disoriented chiral condensate (DCC) configurations in heavy ion collisions. In previous work \cite{2}, we discussed the numerical properties of a pair of spherically symmetric, long lived classical solutions which arise from this lagrangian (related studies appear in \cite{3} and \cite{4}). The evolution of the two configurations is nearly periodic, with energy localized for hundreds of oscillation cycles. The properties of the lower energy solution are such that both the quantum corrections and higher derivative terms of the Chiral Lagrangian should be negligible. Hence it is likely to correspond to a real physical pion configuration in QCD (pionic breather state, or PBS), which could in principle be produced in heavy ion collisions. Although many of its physical properties have been studied in \cite{2}, some theoretical questions remained unanswered, such as whether truly periodic, finite energy solution might exist in the 3+1 Sine-Gordon model.

In this paper we show that the two PBS solutions do not correspond to true breather solutions, where a breather is defined to be a periodic, finite energy classical solution. Our method can be summarized as follows. First, periodic solutions $\phi(r, t)$ are classified in the large $r$ region based on their asymptotic behavior, in terms of a finite number of parameters. Then a numerical algorithm is used to evolve the configuration using the equations of motion from large $r$ to $r \to 0$. Most choices of asymptotic behavior lead to singular behavior at $r = 0$, and hence do not correspond to finite energy solutions. Using the criteria of finite energy, we can search the parameter space in a systematic manner. In section 2, we review the numerical properties of the two PBS solutions. In section 3, we give an overview of our algorithm. In section 3.1, we analyze the free Klein-Gordon equation and its full set of periodic solutions. In section 3.2, we use the solutions of section 3.1 as asymptotic forms for the full Sine-Gordon theory to search for breather solutions, and present the results. Section 4 contains a proof that interactions do not enlarge the set of asymptotic solutions beyond those of the free Klein-Gordon equation. That is, there is a one-to-one mapping between asymptotic behaviors of the free and interacting equations of motion. In section 5 we analyze the 1+1 Sine-Gordon and $\phi^4$ theories, finding that the former theory contains exact breather solutions while the latter does not. In Section 6 we offer concluding remarks.
2 A review of PBS properties

Both PBSs are spherically symmetric. Imposing spherical symmetry, the equation of motion is

\[ \ddot{\phi} - \phi'' - \frac{2}{r} \phi' = -\sin \phi, \]  

(2)

where \( \dot{f} \equiv \frac{df}{dt} \) and \( f' \equiv \frac{df}{dx} \).

Figures 1 and 2 show the numerically generated evolution of the two breather candidates, exhibiting the \( \phi(r, t) \) vs. \( r \) dependence at different times. The nearly periodic evolution continues virtually unchanged for hundreds of cycles but eventually breaks apart, dissipating energy to radial infinity.

The frequency and approximate lifetime of the lower energy PBS are \( 0.1 \text{ cycles/(fm/c)} \) and \( 850 \text{ fm/c} \). For the higher energy PBS, they are \( 0.0619 \text{ cycles/(fm/c)} \) and \( 4000 \text{ fm/c} \). Additional searches have not indicated a third PBS state.

By an appropriate scaling of \( \phi \) and \( x^\mu \), the Sine-Gordon lagrangian (equation 1) can be written as

\[ \mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \cos \phi. \]  

(3)

In these units, the numerically calculated frequencies of the PBSs are \( \omega = 0.9 \pm 0.05 \) and \( \omega = 0.58 \pm 0.02 \).
3 The Algorithm

For finite energy solutions, $\phi \to 0$ as $r \to \infty$. As $\phi \to 0$, nonlinearities in the equation of motion become negligible, hence the $r \to \infty$ behavior of any breather solution must approach a periodic solution to the noninteracting Klein-Gordon theory.

The strategy of the following algorithm is as follows. First, we solve for the most general periodic, finite energy solutions of the Klein-Gordon equation. These solutions are used as large $r$ asymptotic forms and are evolved inward to the origin using the full Sine-Gordon equation of motion. Only solutions which are finite and differentiable at the origin can correspond to true breather states. Since the space of asymptotic forms is parameterized by a finite number of variables, a comprehensive search can be initiated which in the end can rule out the existence of breathers within a certain frequency range.

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1Sine-Gordon finite energy solutions could converge to any multiple of $2\pi$, but these solutions are equivalent by a translation in $\phi$-space to those with $\phi \to 0$. 
3.1 Asymptotic Forms

It can be shown that the most general periodic, spherically symmetric classical solutions to the free Klein-Gordon equation are linear combinations of

\[ \phi_\omega(r, t) = \frac{1}{r} \sin(kr - \theta_k) \sin(\omega t - \theta_\omega) , \]  

where

\[ w^2 = k^2 + m^2 . \]  

If \( m = 0 \), these solutions have infinite energy for all \( \omega \), and no breathers exist. For nonzero \( m \), the set of solutions for \( 0 < \omega < m \) have imaginary \( k \), corresponding to the finite energy solutions of the form \( \tilde{\phi}_\omega(r, t) = \frac{1}{r} \exp(-\tilde{k}r) \sin(\omega t - \theta) \),

where

\[ \omega^2 + \tilde{k}^2 = m^2 . \]  

(Since these are the only types of solutions we are interested in, we will drop the tilde from now on). The most general finite energy periodic solution is therefore

\[ \phi = \frac{1}{r} \sum_{j=1}^{k_j \text{ real}} a_j \exp(-k_j r) \sin(j \omega_f t - \theta_j) , \]  

where \( \omega_f \) is the fundamental frequency in the Fourier series, and \( k_j = \sqrt{m^2 - (\omega_j)^2} \). Note that in the case that \( 0.5m < \omega_f < m \), this sum has only one term

\[ \phi = \frac{1}{r} a_1 \exp(-k_1 r) \sin(\omega_f t) , \]  

hence the full set of solutions is parameterized by two variables, \( a_1 \) and \( \omega_f \). The frequencies of the two numerical PBS solutions fall well within this frequency range.

In the remainder of this section we will work in units with \( m = 1. \)

3.2 The Search for Breathers

Using the asymptotic form in equation 9 as \( r \to \infty \) boundary conditions, the Sine-Gordon equations of motion can be integrated inward to \( r = 0 \) (see appendix A for computational details). In general, solutions obtained in this way are neither finite nor differentiable at

\[ ^2 \text{These are also not breathers, because } \phi'_\omega(0, t) \neq 0, \text{ and hence there are no true breathers of the free Klein-Gordon equation.} \]
the origin, and therefore do not correspond to physical breather states. For small $r$, the numerically obtained solutions can be fit to the form

$$\frac{A(t)}{r} + B(t) ,$$

and the integral

$$S \equiv \int_0^{2\pi/\omega} dt \ |A(t)|$$

(11)
can be used as a measure of how singular each solution is at the origin. A similar measure of differentiability could be defined (for instance by fitting to a form $A(t) r + b(t)$), but as will soon be shown, in the present case not even the singularity condition is fulfilled, so it is not necessary.

The size of the singularity $S$ is calculated for various values of $a_1$ and $\omega$. To fill $a_1 - \omega$ space as uniformly as possible values are randomly chosen in the range $0.5 < \omega_f < 1$ and $a_1 > 0$. In practice an upper limit on $a_1$ has to be set, so a value is chosen which is much larger than that of the two known pseudo-breather states. Also, the lower limit of $a_1$ had to be raised slightly (from 0 to 1.2), because $a_1 \to 0$ corresponds to the trivial solution $\phi(r,t) = 0$, which lowers the value of $S$ to zero across all values of $\omega$.

Figure 3 shows a plot of $S$ vs. $\omega_f$ (the value of $a_1$ is not shown, so the figure can be thought of as a transparent projection plot). As can be seen, this plot has three prominent minima. Two occur at the frequencies of the previously discovered PBSs, $\omega \approx 0.560$ and $\omega \approx 0.865$. Neither minima goes all the way to zero, and hence they do not represent exact breathers. The third minima at $\omega = 1$ corresponds to the trivial solution

$$\phi(r,t) = A \sin(mt - \theta) ,$$

(12)

with infinitesimal $A$.

As further evidence supporting our hypothesis that these minima correspond to the previously discovered PBS states, figure 4 shows the shape of a $\phi(r,t)$ calculated near the minima at $\omega \approx 0.560$. Notice the similarity with figure 2, with the exception of $r \approx 0$ where figure 4 becomes singular.

We now define the solutions corresponding to the minima of the $S$ vs. $\omega_f$ curve as 'pseudo-breather' states (a useful reinterpretation of the acronym 'PBS', which previously

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3Although $S$ is obtained by fitting to a $1/r$ form, a nonzero value of $S$ can be shown to also indicate singularities of the type $\phi \sim 1/r^n$ with $n > 1$. A first derivative fit near the origin would be even more powerful (capable of fitting derivative and logarithmic singularities), but tends to yield ever increasing values of $S$ as the fit is done closer to the origin, which is less manageable. It will therefore be avoided until cases where it is necessary, such as the 1+1 Sine-Gordon equation discussed later in this paper.
Figure 3: The singularity $S$ vs. oscillation frequency $\omega_f$

referred to 'Pionic Breather State'). Although not true breathers, pseudo-breathers can often exhibit very interesting behavior. As was discussed in [2], the two PBS configurations arise readily from multi-domain DCC configurations, have characteristic energies and frequencies, and might extend the lifetime of classical pion configurations by more than an order of magnitude.

4 A More Formal Justification

At this point questions still remain about the comprehensive nature of the solutions obtained in the previous section. It is unclear how the addition of interactions might enlarge the full set of periodic solutions. Perhaps each asymptotic form corresponds to multiple solutions, and integrating inward only uncovers one. In this section we reanalyze the equations of motion by expanding $\phi$ in a power series. Without solving the resulting complicated set of equations, we study its general nature to answer the question just presented, and therefore verify that the previously described algorithm is valid.
In section 4.1, we work out a general form of the solutions. In section 4.2, we use this general form to solve the free Klein-Gordon equations as a test case for the full theory, which we finish with in section 4.3.

### 4.1 A Series Form for the Solutions

First we show that the most general periodic solution with frequency $0.5 < \omega < 1$ is of the form

$$
\phi(r, t) = a \sin(\omega t) \frac{e^{-kr}}{r} + \sum_{j_1=2}^{\infty} \sum_{j_2=2}^{\infty} A_{j_1, j_2}(t) \frac{e^{-j_2kr}}{r^{j_1}},
$$

where the $A_{j_1, j_2}(t)$ are unspecified functions, and $k^2 + \omega^2 = m^2$.

Parameterizing the interaction strength as $\lambda$,

$$
\mathcal{L} = \frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi - \frac{1}{2} m^2 \phi^2 + \lambda V(\phi),
$$

we can now write the solution $\phi$ as a series in $\lambda$,

$$
\phi(r, t) = \sum_{n=0}^{\infty} \lambda^n \phi_n(r, t).
$$

\footnote{In many particular cases such as the Sine-Gordon or $\phi^4$ lagrangians, the sum in this general form starts at $j_1 = 3$ and $j_2 = 3$. This is however a particular case of the form as given, and therefore need not be considered separately.}
We will inductively show that each term is of the form of equation 13.

Substituting $P_n$ into the equations of motion, yields

$$[\partial^2 + m^2] \phi_n(t, r) = P_n[\phi_0(r, t), \phi_1(r, t), ..., \phi_{n-1}(r, t)] ,$$

where $P_n$ are polynomials expressions related to the Klein-Gordon interaction term in a complicated way. The equation for $n = 0$ is just the Klein-Gordon equation of motion

$$[\partial^2 + m^2] \phi_0(t, r) = 0 ,$$

hence its solution is the usual one given by equation 9,

$$\phi = a \sin(\omega t) e^{-kr}r ,$$

This is also the first term in equation 13.

Iteratively substituting in the expressions for $\phi_0, \phi_1, ..., \phi_{n-1}$, we can solve for $\phi_n$.

To show that higher $\phi_n$'s are in the form of the remaining term in equation 13, first note that since $P$ is polynomial whose lowest order term is at least quadratic, inserting $\phi_0, \phi_1, ..., \phi_{n-1}$, yields something of the form

$$\phi(r, t) = \sum_{j_1=2}^{\infty} \sum_{j_2=2}^{\infty} A_{j_1,j_2}(t) \frac{e^{-j_2kr}}{r^{j_1}} ,$$

(this differs from equation 13 only in that it is missing the first term). Equation 16 is a second order linear inhomogeneous differential equation whose most general solution is

$$\phi_n(r, t) = a \sin(\omega t) e^{-kr}r + Q(r, t) ,$$

where $a$ is an undetermined variable and $Q(r, t)$ is any particular solution. It can be shown that $[\partial^2 + m^2] Q$ spans the space given by equation 19 if $Q$ is of the form in equation 13. A $Q$ in the desired form can therefore be chosen to satisfy the differential equation, and by equation 20, $\phi_n$ is also in this form.

4.2 The Klein-Gordon Case

Before we work with the full equation of motion, we will use the form of equation 13 in the free Klein-Gordon Lagrangian to derive the full solution for a given large $r$ behavior of $\phi$ (in the form of equation 9), and show that this is unique. Although this result is trivial, it sets up a formalism which can be naturally extended for the next section when we will repeat this for the full theory including interactions.
Substituting the form of equation 13 into the Klein-Gordon equation of motion yields
the decoupled set of equations
\[ \ddot{A}_{2,j_2}(t) + [-j_2^2 k^2 + m^2]A_{2,j_2}(t) = 0 \] (21)
\[ \ddot{A}_{3,j_2}(t) + [-j_2^2 k^2 + m^2]A_{3,j_2}(t) = 4j_2kA_{2,j_2}(t) \] (22)
and then for \( j_1 > 3 \),
\[ \ddot{A}_{j_1,j_2}(t) + [-j_2^2 k^2 + m^2]A_{j_1,j_2}(t) = 2j_2k(j_1 - 1)A_{j_1-1,j_2}(t) + j_1(j_1 - 1)A_{j_1-2,j_2}(t) \] (23)
These are each second order linear inhomogeneous differential equations with general solution
\[ A_{j_1,j_2}(t) = a_{j_1,j_2} \sin(\omega t - \theta_{j_1,j_2}) + P_{j_1,j_2}(t) \] (24)
where \( a_{j_1,j_2} \) and \( \theta_{j_1,j_2} \) are undetermined constants, \( P_{j_1,j_2}(t) \) is any particular solution, and
\[ \omega^2_j + (kj_2)^2 = m^2 \] (25)
Note however that each \( a_{j_1,j_2} \) is forced to be zero. The \( \sin(\omega t - \theta_{j_1,j_2}) \) is an on-resonance
driving term for the next differential equation (with \( j_1 \) incremented by one), and the associated solution is not periodic. All freedom in the choice of \( a_{j_1,j_2} \) is lost. The driving terms
for each differential equation iteratively become zero and therefore each \( P_{j_1,j_2}(t) = 0 \).

The most general solution with the given lowest order asymptotic term is therefore
\[ \phi = \frac{1}{r} a \exp(-kr) \sin(\omega t) \] (26)

4.3 Adding Interactions

Now we repeat the analysis in the last section for the case where there is an interaction
term present, and arrive at our final conclusion for this section, that the solution to the full
equation (interaction present) is completely determined by the frequency and size of the first
order asymptotic piece that is a solution to the free Klein-Gordon theory.

Repeating the same analysis with an interaction term changes equation 21-23 to
\[ \ddot{A}_{2,j_2}(t) + [-j_2^2 k^2 + m^2]A_{2,j_2}(t) = \mathcal{P}_{1,j_2}[A_{1,1}, A_{2,2}, ...] \] (27)
\[ \ddot{A}_{3,j_2}(t) + [-j_2^2 k^2 + m^2]A_{3,j_2}(t) = 4j_2kA_{2,j_2}(t) + \mathcal{P}_{2,j_2}[A_{1,1}, A_{2,2}, ...] \] (28)
and then for \( j_1 > 3 \),
\[ \ddot{A}_{j_1,j_2}(t) + [-j_2^2 k^2 + m^2]A_{j_1,j_2}(t) = 2j_2k(j_1 - 1)A_{j_1-1,j_2}(t) + j_1(j_1 - 1)A_{j_1-2,j_2}(t) + \mathcal{P}_{j_1,j_2}[A_{1,1}, A_{2,2}, ...] \] (29)
The actual form of $P_{j_1,j_2}$ is related to the Klein-Gordon interaction term in a complicated way, but two things are clear. First, since the interaction contains no linear terms, $P_{j_1,j_2}$ is at least quadratic in the $A_{j_1,j_2}$'s, and can not explicitly cancel the linear terms already present from equations 21-23. Second, each $P_{j_1,j_2}$ only contains terms $A_{\tilde{j}_1,\tilde{j}_2}(t)$ with $\tilde{j}_1 < j_1$ and $\tilde{j}_2 < j_2$. This allows us to solve iteratively beginning with the lower order solutions, yielding once again simple second order linear inhomogeneous differential equations.

Unlike in the noninteracting case, after inserting in the lower order solutions $A_{j_1,j_2}(t)$, $P_{j_1,j_2}$ may have a term in it proportional to the on-resonance driving piece, $\sin(\omega_{j_2}t - \theta_{j_1,j_2})$. This doesn’t add any freedom in our choices of $a_{j_1,j_2}$ however, in that now these values must be chosen to cancel the resonant pieces. This constraint completely specifies the form of each $A_{j_1,j_2}(t)$, hence there is no additional freedom in the choice of solutions.

The final solution is therefore unique given its lowest order large $r$ asymptotic behavior.

5 Other Examples

We end by applying our method on two other examples, the 1+1 Sine-Gordon equation, and $\phi^4$ theory. The 1+1 Sine-Gordon Lagrangian is a well studied model with known breathers, and is thus used as a test of our algorithm.

Minor modifications had to be made to work in 1+1 dimensions. The asymptotic form
Figure 6: S vs. $\omega$ for the 3+1 $\phi^4$ theory

in equation 9 was changed to

$$\phi = a_1 \exp(-k_1 r) \sin(\omega_f t) .$$  \hspace{1cm} (30)

Also, in 1+1 dimensions, $\phi(r, t)$ does not readily become singular at $r = 0$, hence we tested the differentiability of $\phi$ at zero.

The $S$ vs. $\omega$ plots for each Lagrangian appear in figures 5 and 6. In the 1+1 Sine-Gordon plot the values of $S$ go to zero across all values of $\omega$, indicating a class of breathers as expected. The $\phi^4$ plot stays positive for all values of $\omega$, hence in 3+1 dimensional $\phi^4$ theory, there are no breathers in the frequency range $0.5 < \omega < 1$.

6 Conclusions

In this paper we have developed a numerical search algorithm for spherically symmetric breathers in spin zero theories. This algorithm was used to show that breather-like states found in a previous work were not actual breathers, but rather a new type of object, a 'pseudo-breather', which, while long-lived, is ultimately unstable. We showed that the asymptotic behavior of periodic solutions which might have finite energy can be parameterized in terms of a finite number of parameters. In the case of interest, $0.5m < \omega_f < m$, there are only two parameters: frequency $\omega_f$ and magnitude $a_1$. An extensive search within
the Sine-Gordon model failed to uncover any values of $\omega_f, a_1$ for which the solution is non-singular at the origin. The results for this model are shown in figure 3.

We have, as of yet, left the region $0 < \omega_f < 0.5m$ unexplored ($\omega_f > m$ is automatically excluded by large $r$ energy considerations). It is straightforward in principal to extend the algorithm to arbitrarily small $\omega$. This is done by generalizing the form of the asymptotic solutions. For instance, in the region $0.25m < \omega_f < 0.5m$, the asymptotic form becomes

$$\phi = \frac{1}{r} a_1 \exp(k_1r) \sin(\omega_ft) + \frac{1}{r} a_2 \exp(k_2r) \sin(2\omega_ft - \theta_2).$$

(31)

There is a numerical trade-off, in that to cover smaller regions of $\omega_f$ the number of parameters to describe the asymptotic solution increases, eventually to infinity as $\omega_f \to 0$.

The condition of spherical symmetry can be relaxed slightly by adding higher partial wave terms

$$\phi(\vec{x}, t) = \phi^0_0(r,t) Y^0_0(\theta, \phi) + \phi^{-1}_1(r,t) Y^{-1}_1(\theta, \phi) + \phi^0_1(r,t) Y^0_1(\theta, \phi) + \phi^1_1(r,t) Y^1_1(\theta, \phi) + \cdots.$$  

(32)

Inserting this into the equation of motion yields a coupled set of equations

$$\ddot{\phi}^m_l - \frac{2}{r} \phi^m_l' - \frac{l(l+1)}{r^2} \phi^m_l = -V' \left( \sum_{l,m} Y^m_l \phi^m_l \right).$$

(33)

Again, the number of variables needed to parameterize the space of asymptotic forms increases, thus demanding more computational power for each spherical harmonic added.

Although not discussed here, our algorithm can be modified to work with higher spin theories. The condition of spherical symmetry greatly restricts the asymptotic forms, so in many cases not much more computational power is needed.

Two shortcomings of the analysis in this paper are as follows. First, we can not guarantee that our upper cutoff in $a_1$ has not hidden a true breather, and second, it is possible (as often with a numerical analysis) that some exceptional behavior is concealed in the gaps between calculated values in the $S$ vs. $\omega_f$ graph. Due to the large range of points and small spacing in figure 3, this seems unlikely. However, it prevents the result from being completely rigorous.

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Appendix A: Details of Numerical Evolution

The spherically symmetric 3+1 Sine-Gordon equation of motion is

\[ \ddot{\phi}(r, t) - \phi''(r, t) - \frac{2}{r} \phi'(r, t) = -\sin(\phi) . \]  

(34)

Discretizing, and using the approximations

\[ \frac{df(x_i)}{dx} \approx \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x} \]  

(35)

and

\[ \frac{d^2 f}{dx^2} \approx \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{\Delta x^2} , \]  

(36)

the equations of motion can be written

\[ \phi(r_{i-1}, t_j) = \frac{1}{i-1} \left[ 2\phi(r_i, t_j) i - \phi(r_{i+1}, t_j)(i+1) + \Delta x^2 i f \right] , \]  

(37)

where

\[ f = \frac{\phi(r_i, t_{j+1}) + \phi(r_i, t_{j-1}) - 2\phi(r_i, t_j)}{\Delta t^2} + \sin(\phi(r_i, t_j)) . \]  

(38)

Spatial boundary conditions are fit to an asymptotic form at \( r \to \infty \). In practice this boundary fit is done at a large but finite value \( r = r_0 \). Boundary conditions in the \( t \) direction are of course periodic. The values \( r_0, \Delta x \) and \( \Delta t \) were varied until the resulting evolution (and hence the shape of figure 3) converged.

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