Effect of spin precession on bounding the mass of the graviton using gravitational waves from massive black hole binaries

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Abstract. Observations of gravitational waves from massive binary black hole systems at cosmological distances can be used to search for a dependence of the speed of propagation of the waves on wavelength, and thereby to bound the mass of a hypothetical graviton. We study the effects of precession of the spins of the black holes and of the orbital angular momentum on the process of parameter estimation based on the method of matched filtering of gravitational-wave signals vs. theoretical template waveforms. For the proposed space interferometer \textit{LISA}, we show that precession, and the accompanying modulations of the gravitational waveforms, are effective in breaking degeneracies among the parameters being estimated, and effectively restore the achievable graviton-mass bounds to levels obtainable from binary inspirals \textit{without} spin. For spinning, precessing binary black hole systems of equal masses $10^6 M_\odot$ at 3 Gpc, the lower bounds on the graviton Compton wavelength achievable are of the order of $5 \times 10^{16}$ km.

Summary of the results
The anticipated launch of the Laser Interferometer Space Antenna (\textit{LISA}) in the 2020 time frame will provide a promising new tool for doing astrophysics with massive binary black hole systems. The inspiral and merger of massive black holes (MBH) with masses of the order of $10^5$ - $10^7 M_\odot$ will be detectable to large distances in \textit{LISA}'s sensitive frequency band between $10^{-5}$ and 1 Hz. The detection of gravitational waves (GW) from MBH systems will allow us to infer important astrophysical and astronomical information, such as the masses and spins of the black holes, the location of the system on the sky and its distance from the solar system.

Another important aspect of MBH binaries is the possibility of testing general relativity itself. In previous papers we have studied the bounds that could be placed on alternative theories of gravity such as scalar-tensor theories of the Brans-Dicke type, and theories in which gravitational waves propagate with a wavelength-dependent speed, as if the “graviton” were massive [1, 2, 3, 4, 5, 6]. Specifically, in [5] we showed that the inclusion of aligned, non-precessing spins weakens the bounds obtainable on the graviton mass by almost an order of magnitude. This is because the parameters that characterize the inspiraling binary are highly correlated, so that the addition of parameters (the spins) into the estimation process effectively dilutes the available information, leading to weakened bounds or estimates on most parameters.

However, Vecchio [7] pointed out that when the effects of precession of spins are incorporated into the gravitational waveforms, i.e. when the spins are not aligned with the orbital angular
momentum, the accuracy of parameter estimation can be improved. He studied the so called “simple precession” case where either one of the bodies has zero spin, or the black hole masses are equal, and only spin-orbit interactions are included. The modulations of the amplitude and phase of the gravitational waveform induced by the precession of the spin(s) and by the precession of the orbital plane effectively adds information to the estimation process, partially decouples some of the parameters, and thus leads to restored accuracy. Lang and Hughes [8] extended Vecchio’s work to include arbitrary spins and masses, and also spin-spin interactions, and found significant improvements in the accuracy of mass measurements as well as sky localization. In addition, they showed that the magnitudes of the spins of the binary members, especially for low redshift systems at $z \simeq 1$, could be measured with accuracies of the order of $10^{-2}$.

In this contribution we describe the results of an independent code for analysing binary inspiral with precessing spin and for carrying out parameter estimation based on the method of matched filtering, but extended to include the effects of a massive graviton. In addition to confirming the central conclusions of Lang and Hughes [8], we show that spin precession significantly improve the bounds that can be placed on the mass of the graviton. In parallel work, we have shown that including higher signal harmonics in the PN waveform (but without spins) also leads to improved bounds on the graviton mass [6].

We will not present here all the details of the analysis, but we rather refer the interested reader to the published article [9] for all the computational and method details. Our main conclusion, shown in Figs. 1 and 2 is that the inclusion of spin precession effects increases the lower bound on the graviton Compton wavelength $\lambda_g$ by almost an order of magnitude, on average, with respect to the one calculated for the same non precessing system. Recall that $\lambda_g$ is related to the mass of the graviton by $\lambda_g = h/m_g c$, where $h$ is Planck’s constant and $c$ is the speed of light, so that a lower bound on $\lambda_g$ represents an upper bound on $m_g$.

The total number of parameters for the studied system are the following 16: The individual masses of the binary system $\ln(m_i)$, luminosity distance $\ln(L_D)$, the two dimensionless spin parameters $\chi_i = S_i/m_i^2$ ($0 < \chi_i < 1$), the time and phase of coalescence $t_c$, $\phi_c$, two angles for the binary position at the sky, $\phi_S$, $\cos \theta_S$, two angles for the initial angular momentum vectors $\phi_L$, $\cos \theta_L$, four angles for the initial orientations of the spins of the two bodies, $\phi_{S1}$, $\cos \theta_{S1}$, $\phi_{S2}$, $\cos \theta_{S2}$ and finally the massive graviton parameter $\beta_g = \lambda_g^2 D^2/(1+z)^2$, where $D$ is a distance factor, $\mathcal{M}$ is the chirp mass and $z$ the redshift. For the Monte Carlo simulations we fix the pair of black hole masses, the redshift (consequently the luminosity distance) and we take random values for the 8 angle parameters, the 2 dimensionless spin parameters and the phase and time of coalescence.

Indeed, the new bounds, labeled MG+SO+PREC and MG+SO+SS+PREC in Fig. 1, which incorporate spin-orbit (SO) effects only and spin-orbit and spin-spin (SS) effects, respectively, along with the effect of a massive graviton (MG), are comparable to those inferred from an identical system without spin effects at all, labeled MG. This improvement is independent of mass, as seen in Fig. 2, which plots median lower bounds on $\lambda_g$ for systems without spin (MG), with non precessing spins (MG+SO+SS), and with precessing spins (MG+SO+SS+PREC) for various pairs of masses spanning two orders of magnitude in total mass.

In Figures 3,4 we plot the distribution of the correlation coefficients between the massive graviton parameter $\beta_g$, and the two dimensionless spin parameters $\chi_i$ (left panel) and the two masses $m_i$ (right panel) for the $10^6 + 10^8 M_\odot$ black hole case. The correlations are quite mild, with most of the values ranging between 0 and 0.8, in contrast to the nonprecessing case [5], where correlation coefficients larger than 0.9 were routine. This illustrates clearly the strong decorrelating effect of the spin precession.

The effect of choosing arbitrary coalescence times $t_c$ is illustrated in Fig. 5 where the distributions of lower bounds on the graviton Compton wavelength $\lambda_g$ are shown for fixed and random values of $t_c$. It is clear from the graph that randomizing $t_c$, leads to somewhat smaller
lower bounds, with a tail at low values of the bounds, depicting the effect of signal loss in some of the cases. Fig. 5 also shows that using two LISA arm combinations generally leads to improved bounds.

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Figure 1. Distribution of lower bounds on the graviton Compton wavelength $\lambda_g$ (in units of $10^{15}$ km) for $10^4$ equal-mass $(10^6 M_\odot)$ black-hole binaries at redshift $z = 0.55$, or a luminosity distance 3 Gpc, randomly located on the sky. Number of bins is set to 50. First three histograms (narrow lines; red, blue, green in the color version) assume either no spins or aligned spins with spin-orbit (SO) and/or spin-spin (SS) coupling. Final two histograms (thick lines; violet and black in the color version) include precession induced by non-aligned spins.

Figure 2. Median lower bounds on the graviton Compton wavelength $\lambda_g$ (in units of $10^{15}$ km) for $10^4$ black-hole binaries at redshift $z = 0.55$, or a luminosity distance 3 Gpc, randomly located on the sky. Systems contain black holes of mass $(1, 1) \times 10^5$, $(1, 10) \times 10^5$, $(1, 1) \times 10^6$, $(1, 10) \times 10^6$ and $(1, 1) \times 10^7 M_\odot$, from left to right, respectively.
Figure 3. Distribution of the correlation coefficients between spin parameters $\chi_i$ and the massive graviton parameter $\beta_g$.

Figure 4. Distribution of the correlation coefficients between individual masses $m_i$ and the massive graviton parameter $\beta_g$.

Figure 5. Distribution of lower bounds on $\lambda_g$ (in units of $10^{15}$ km) for $10^4$ binaries including spin precession. The system is a $10^6 + 10^6$ $M_\odot$ BBH at $z = 0.55$ (3 Gpc). Red curve is for $t_c$ fixed to one year; black curve is for random values of $t_c$ in the one year interval of the *LISA* mission. Solid (dashed) lines refer to one (two) *LISA* detectors respectively.